COMPARATIVE ANALYSIS OF PHENOMENOLOGICAL APPROXIMATIONS
FOR THE LIGHT CURVES OF ECLIPSING BINARY STARS WITH
ADDITIONAL PARAMETERS

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A comparative analysis is made of special profiles of eclipses used for phenomenological modelling of
the light curves of eclipsing binary stars. Families of functions which generalize local approximations
and functions which are theoretically unlimited in width and based on a Gaussian are examined. The
light curve of the classical Algol subtype ($\beta$ Persei) star V0882 Car=2MASS J11080308-6145589 is used
for the analysis. Dozens of modified functions with additional parameters are analyzed and 14 of these
are chosen based on the criterion of a minimum sum of the squares of the deviations. The best are
functions with an additional parameter which describe a profile that which is limited in phase.

Keywords: eclipsing binary stars: light curves

1. Introduction

Phenomenological modelling of the light curves of variable stars can be used to obtain the parameters needed
to register an object in the General Catalog of Variable Stars (GCVS) [1], the Variable Star Index VSX
(http://aavso.org/vsx), and similar catalogs. It is the start for a small fraction of stars which are later studied by
spectroscopy, polarimetry, and multicolor photometry. For most stars, however, there are no further studies and phenomenological modelling remains as the main source of information on them. The classical methods of studying variable stars are described by Tsesevich [2], for example. Methods of approximating the symmetric and asymmetric extrema are examined in Ref. 3 and polynomial approximations in computer programs in Refs. 4 and 5.

In most cases, the parameters are determined on the basis of separate parts of the light curve. These include the brightnesses at the primary maximum and minimum, and for eclipsing systems, in the secondary minimum and maximum, as indicated in the GCVS [1]. In addition, an obligatory parameter for eclipsing systems is the width D of the minimum, and a desirable parameter is the duration of the full eclipse phase.

The complete light curves are analyzed using graphical smoothing methods and approximation by trigonometric polynomials (truncated Fourier series). For EA (Algol) type stars the number of parameters is very large, and this leads to the appearance of visible waves in the light curve (the Gibbs effect [6]) and to increased statistical error in smoothing the light curve and in the corresponding values at the maximum and minimum.

Smooth approximations (including trigonometric polynomials [7,8] and “symmetric” trigonometric polynomials [9]) cannot be used to determine one of the essential parameters: the eclipse width. Thus, it has become necessary to introduce functions (“special profiles”) which describe the eclipse in a statistically optimal fashion and use a smaller number of parameters. Andronov [10,11] proposed the “New Algol Variable” (NAV) approximation, which has been used for many stars of the Algol (EA) type, as well as two other types EB (β Lyrae) and EW (W Ursae Majoris) [12-15]. Approximations based on Gaussians and their modifications have been discussed by Mikulášek [16,17].

An alternative approach involves physical modelling of the light curves based on the Wilson-Devinney method [18] for which various programs have been developed [19-23]. Physical modelling, however, requires values for parameters (the temperature of at least one of the components and the mass ratio) that can be reliably obtained from spectral observations, and these have been made for ~1% of the known eclipsing binary systems.

Another possibility is the use of a simplified physical model in which stars are assumed to be spherically symmetric, while limb darkening effects are neglected [24]. This model has been used to study and classify eclipsing binary stars [25], both for the OGLE survey [26] and by ourselves [27].

In this paper, we study a modification of phenomenological modelling in order to improve the quality of the approximation by introducing one or more additional parameters. This work has been done as part of the international Inter-longitude Astronomy Project [28,29], Astroinformatika, and the Ukrainian Virtual Observatory [30,31].

2. The data

As an illustration of the application of the proposed functions, we have used observations of an Algol type star (2MASS J11080308-6145589 [32], which has recently acquired the designation V0882 Car [1]). Of the full phase curve [32], we have used the phase interval [-0.08,0.08], within which 120 brightness observations have been made. This makes it possible to study an individual minimum for comparison of the approximations, while we make use
of the full phase curves, including reflection, ellipsoidal, and O’Connell effects, as well as differences in the profiles of the primary and secondary minima when their widths are the same [12-15] to investigate the stars.

3. Basic formulas

The phase is used as the independent variable. The values of the initial epoch, determined by other methods, can, however, be shifted slightly, so for modelling of eclipses it is necessary to use the phase difference \( u = \phi - \phi_0 \), where \( \phi_0 \) is the phase corresponding to the middle of the deduction describing the eclipse. The approximation for a segment of the light curve can be written in general form as

\[
x(\phi) = C_1 - C_2 G(\phi - C_3; C_4; \ldots; C_m),
\]

where \( C_1 \) is the smoothed value of the brightness at phase \( \phi = C_3 \) with the eclipse taken into account, \( C_2 \) is the amplitude of the occultation, and the function \( G \) depends both on the phase and on the additional parameters describing the profile of the eclipse. Of these, the most important is \( C_4 \), which describes the characteristic width of the minimum. If a function \( G \) is used which varies only within the eclipse, then it is convenient to introduce \( C_4 \) as the half width of the eclipse, and the dimensionless parameter \( \varepsilon = z = (\phi - C_3)/C_4 \). In the following we shall use the variables \( u \) and \( \varepsilon \) for profiles that are not bounded in phase and \( z \) for minima with a finite half width \( C_4 \), i.e., \(-1 \leq z \leq 1\).

As opposed to the earlier papers [10,11,13,14] in which we defined \( C_1 \) as the integral mean value of the brightness over the truncated second degree trigonometric polynomial approximation without including the contributions of the eclipses, in this paper we study the quality of the approximation of the minimum itself, and it is more convenient to redefine \( C_1 \) as the value of the brightness at the minimum. This function \( G(\varepsilon) = 1 - H(\varepsilon) \), where \( H(\varepsilon) \) is a function that was used previously [13,14] (Eq. (3)).

The major property of the function \( G(\varepsilon) \) is that \( G(0) = 0 \). For functions that are limited in phase, we use the notation \( z \) instead of \( \varepsilon \) in order to emphasize that \( G(z) = 1 \) for \( |z| > 1 \). For reasons of symmetry, it is appropriate to define the function \( G(\varepsilon) = G(-\varepsilon) = G(\varepsilon) \) as symmetric.

The classical function used, for example, as a first approximation of the profiles of Doppler broadened spectrum lines is the Gaussian

\[
G(u) = 1 - \exp\left(-|C_4|u^2\right).
\]

In this case, \(|C_4| = 1/2\sigma^2\), where \( \sigma \) is the characteristic width, which has the significance of a mean square deviation in probability theory. This function is characterized by only four parameters, but cannot account for the variety of observed profiles of minima.

Table 1 and Fig. 1 list the best approximations that we have chosen in order of deteriorating quality in terms
### TABLE 1. Characteristics of the Best Approximations

| i | Formula                                                                 | SSE    | $\sigma(C_i)$ | $X_c (C_i)$ | $\sigma[X_c]$ | m  |
|---|-------------------------------------------------------------------------|--------|---------------|-------------|---------------|----|
| 1 | $1 - (1 - (l z + C_6 l z (1 - l z)))^{C_1}$, $l z \leq 1$              | 0.00459| -0.00139      | 15.0412     | 0.00142       | 6  |
| 2 | $(1 - \exp(1 - \cosh(z)))^{C_1} / (1 - \exp(1 - \cosh(1)))^{C_1}$, $l z \leq 1$ | 0.00462| -0.00139      | 15.0394     | 0.00129       | 5  |
| 3 | $(1 - C_6 - C_7 - C_8 l z^2 + C_6 z^4 + C_7 z^6 + C_8 z^8)^{C_1}$, $l z \leq 1$ | 0.00462| -0.00139      | 15.0437     | 0.00166       | 8  |
| 4 | $1 - (1 - l z)^{C_1}$, $l z \leq 1$                                    | 0.00462| -0.00139      | 15.0435     | 0.00142       | 6  |
| 5 | $\begin{cases} 0, & \text{if } l u \leq C_6 \\ (1 - (l u - 1) (l u - C_6))^{C_1}, & \text{if } C_6 < l u \leq C_4 \end{cases}$, $l z \leq 1$ | 0.00465| -0.00139      | 15.0338     | 0.00143       | 6  |
| 6 | $1 - (1 - l z)^{C_1}$, $l z \leq 1$                                    | 0.00477| -0.00139      | 15.0485     | 0.00131       | 5  |
| 7 | $1 - \exp(-C_4 (\cosh(e) - 1)^{C_1})$                                  | 0.00480| -0.00139      | 15.0458     | 0.00145       | 6  |
| 8 | $1 - (1 - (1 - \exp(1 - \cosh(e)))^{C_1}) (1 + C_6 e^2 + C_7 e^4)$    | 0.00487| -0.00139      | 15.0455     | 0.00159       | 7  |
| 9 | $1 - \exp(C_5 (1 - \cosh(e)))$                                        | 0.00500| -0.00142      | 15.0377     | 0.00135       | 5  |
| 10| $1 - \exp(1 - \cosh(e))$                                              | 0.00509| -0.00140      | 15.0393     | 0.00121       | 4  |
| 11| $(1 - \exp(-1 C_4 l z^2 + C_6 l z^4 + C_7 l z^6 + C_8 l z^8))^{C_1}$ / $(1 - \exp(-1 C_4 l - 1 C_6 l - 1 C_7 l))^{C_1}$, $l z \leq 1$ | 0.00544| -0.00146      | 15.0550     | 0.00180       | 8  |
| 12| $1 - (1 - l z)^{C_1}$, $l z \leq 1$                                    | 0.00581| -0.00143      | 15.0609     | 0.00145       | 5  |
| 13| $(1 - \exp(-1 C_4 l u^2))^{C_1}$                                       | 0.00599| -0.00143      | 15.0334     | 0.00147       | 5  |
| 14| $(1 - \exp(-1 C_4 l u^2))$                                             | 0.00736| -0.00134      | 15.0456     | 0.00145       | 4  |
Fig. 1. Approximations by various functions of the part near the brightness minimum of the star V0882 Car = 2MASS J11080308-6145589. The numbers on the curves correspond to those in Table 1.
of the criterion of minimum SSE (sum of squared errors). The Gaussian occupies the last (14-th) place. For the series and function studied here, SSE=0.00736. And, although poor convergence of the observations to a Gaussian is observed for almost all eclipsing stars, it continues to be used [33], evidently because of its popularity in statistics and its inclusion in a number of program packages.

In order to account for the fact that real minima have finite duration, Andronov [10,11] has proposed using the function

$$G(z) = 1 - \left(1 - |z|^\beta\right)^{1.5}, \quad (3)$$

for which the exponent 1.5 corresponds to the theoretical asymptotic behavior of the light curve near the limit of the eclipse. This method was called the NAV (New Algol Variable) method. The properties of the test function for this approximation are discussed in Ref. 34.

For the example examined here, this method yields SSE = 0.00477 (rated in 6-th place), or only 4% worse than the best approximation with a large number of parameters.

The Maclaurin series expansion of this function

$$G(z) = 1 - \left(1 - |z|^\beta\right)^{1.5} = \frac{3}{2} |z|^{\beta} - \frac{3}{8} |z|^{2\beta} - \frac{1}{16} |z|^{3\beta} - \frac{3}{128} |z|^{4\beta} + \ldots \quad (4)$$

shows that the asymptote for small $|z|$ is $G(z) \sim |z|^\beta$, and this parameter determines the profile of the function. Since we are concerned with the neighborhood of the minimum ($|z|=0$), in a classical study of the function it is to be expected that the function will have a positive second derivative. In our case, the second derivative is equal to

$$G''(z) = \frac{3}{2} \beta(\beta - 1)|z|^{\beta - 2}, \quad (5)$$

i.e., it is positive and finite only for $\beta = 2$. When $\beta > 2$, $G''(z) = 0$, i.e., the minimum is flatter than expected for most analytic functions with an asymptotically parabolic minimum. $\beta = 1$ corresponds to a “triangular” profile of the minimum, since as $z \to 0$, $G(z) = (3/2)|z|$ asymptotically. In this case, the first derivative has a discontinuity at $z = 0$. The minimum limit is actually $\beta = 1.5$, which corresponds to an instantaneous transition from entering the eclipse to leaving it (an “instantaneous” full eclipse is smaller with respect to the star’s size).

When $\beta \gg 2$ these functions describe “flat” minima or “full eclipses.” However, a value $\beta < 2$ was optimal for many of the stars we have studied; this leads to a discontinuity in the second derivative and a visually “sharp” profile. However, even for these stars $\beta > 1.5$, so that the approximations are physically real, although they differ from the usually studied analytic functions with $\beta = 2$.

Mikulášek, et al. [16], have modified the classical Gaussian in order to obtain a power law asymptote near the minimum. In our notation,
\[ G(u) = \left(1 - \exp \left( -\left| C_5 \right| u^2 \right) \right)' = (1 - \exp(\theta))'. \] (6)

SSE = 0.00599 (13-th place), i.e., introducing the additional parameter \( C_5 = r \) improves the quality of the approximation but it is usually inferior to our NAV method.

Expanding this function in a Maclaurin series gives

\[ G(u) = \theta' \left(1 - \frac{r \theta}{2} + \frac{(3r^2 + r) \theta^2}{24} - \frac{(r^3 + r^2) \theta^3}{48} + \ldots \right) \] (7)

e.i., asymptotically \( G(u) - \theta' = \left| C_5 \right| u^2 \). This function is similar to the function obtained for the NAV method with \( \beta = 2r \). The basic difference occurs near the boundaries of the occultation.

In order to improve the approximation without invoking additional parameters, Mikulášek, et al. [17], replaced the parabola in the exponent with a hyperbolic cosine, i.e.

\[ G(\varepsilon) = \left(1 - \exp(1 - \cosh(\varepsilon)) \right)' . \] (8)

For \( r = 1 \) this approximation is No. 10 in Table 1 according to the SSE ratings.

After another 3 parameters were added to the existing 5, the modification [17]

\[ G(\varepsilon) = 1 - \left(1 - \exp(1 - \cosh(\varepsilon)) \right)^{C_5} \left(1 + C_6 \varepsilon^2 + C_7 \varepsilon^4 \right) , \] (9)

was proposed. It is No. 8 in Table 1. Thus, the NAV approximation for this series is better than those examined in Ref. 17.

In Refs. 13 and 14 we examined a modification of the functions proposed in Refs. 10, 11, and 17. Here the list of modifications has been extended and the best are listed in Table 1.

These include the Mikulášek function

\[ \left(1 - \exp \left( C_6 (1 - \cosh(\varepsilon)) \right) \right)^{C_5} , \] (10)

to which we have added an additional parameter \( |C_6| \) (this is left out by Mikulášek, i.e., it can be assumed equal to 1). We note that on the internet page var2.astro.cz, this function (for fixed \( C_6 = 1 \)) is “standard” for determining the times of the minima for eclipsing stars.

A modification with the power in the exponent has been more successful,

\[ 1 - \exp \left( -|C_6|(\cosh(\varepsilon) - 1)^{C_5} \right) . \] (11)
This approximated is rated 7 (SSE = 0.00480).

It is clear that the choice of the hyperbolic cosine instead of an ordinary parabola is related to the sharper reduction in the exponent with distance from zero in the constant sign series:

\[ 1 - \cosh(e) = -\frac{e^2}{2} - \frac{e^4}{24} - \frac{e^6}{720} - \frac{e^8}{40320} - \ldots \]  

(12)

We have also tried a phase limited approximation with an expansion in terms of even powers of the argument

\[ G(z) = \frac{\left(1 - \exp\left(-|C_3|z^2 - |C_6|z^4 - |C_7|z^6\right)\right)^{C_8}}{\left(1 - \exp\left(-|C_3| - |C_6| - |C_7|\right)\right)^{C_8}} \]  

(13)

but this was less successful (rating 11, SSE = 0.00544). The signs for the absolute values indicate that the parameters are positive.

Therefore, the unbounded functions employing an exponential and hyperbolic cosine rate 7 or below, i.e., all of these approximations are inferior in terms of the minimum SSE criterion than the original NAV algorithm (with a rating of 6).

We have, however, tried to improve the NAV method and modify the phase limited eclipse profiles (|z| ≤ 1). Five of the functions (1-5 in Table 1) yielded the same SSE to within a statistically insignificant difference of 1.5%.

In order of improving rating, this piecewise continuous function with a part of the full eclipse

\[ G(u) = \begin{cases} 
0, & \text{if } |u| \leq C_6 \\
1 - \left(1 - |u|/(C_4 - |C_6|)\right)^{C_6}, & \text{if } C_6 < |u| < C_4 
\end{cases} \]  

(14)

with an additional exponent \( C_6 \) (previously taken to equal 1.5)

\[ G(u) = 1 - \left(1 - |u|\right)^{C_6}. \]  

(15)

The higher rated series in even powers of \( z \) (which can be obtained by expanding the exponentials in a series with our modified exponent) is raised to the power

\[ G(z) = \left([C_6 - C_7 - C_8]z^2 + C_6z^4 + C_7z^6 + C_8z^8\right)^{C_8}. \]  

(16)

Another outstanding result, with a rating of 2, is from the Mikulášek function, which we propose limiting in phase as
\[ G(z) = \frac{(1 - \exp(1 - \cosh(z)))^{C_1}}{(1 - \exp(1 - \cosh(t)))^{C_1}}. \] (17)

The best rated series, however, is the NAV method with a modified argument with an additional parameter \( C \) within the interval (-1,1):

\[ G(z) = 1 - \left( 1 - \left| z \right| + C_6 \left| z \right| (1 - \left| z \right|)^{C_6} \right)^{1.5}. \] (18)

4. Theoretical families of functions

Figure 2 shows families of approximations for the main methods, modifications of which have been studied in this paper. The top shows the phase limited functions (3) proposed by Andronov [10,11] and the bottom, the unlimited Mikulášek functions (10). For comparison with respect to the profile near the eclipse enter, it was assumed

Fig. 2. Top: the effect on the function \( G \) of the parameter \( (\beta = C_5 \text{ in Eq. (3)}) \) for \( \beta = 2r \) and \( r = 0.001, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1, 1.2, 1.4, 1.6, 1.8, 2 \) (increasing width); bottom: the effect of the parameter \( r \) \( (\beta = C \text{ in Eq. (10)}) \) on the shape of the light curves for the same set of values as in the top frame.
that $\beta = 2r$ for the same set of data. The main difference between these families is in the boundedness or unboundedness of the theoretical profiles, while near the eclipse center the profiles become equally stepped. In this case, however, it was preferable to use only the segments with rising and falling branches [35].

Figure 3 shows families of approximations for the best modification (18). Introducing an additional parameter reduced the SSE by 2.5% compared to the original formula (3). Thus, the function is more promising for improving the approximation to the eclipse profile. The natural physical limit is the interval (-1,1); otherwise the function ceases to be monotonically varying from the center of the eclipse to the edges.

5. Discussion

In an earlier paper [10,11], we examined different functions as an approximation. Later [17], several new functions were introduced, but, as before, they were formally infinitely wide. That is, the phenomenological
approximation with a large number of parameters describes a curve with a smaller mean square deviation. When the number of parameters is increased, however, the general problem is the nonorthogonality of the basis functions and the corresponding marked deterioration in the accuracy of the determination of even the previous parameters. We have compared several modifications of the NAV method with methods based on a Gaussian [13,14]. Here the list of approximation functions was considerably extended, but that is roughly a third of the modifications that we checked. To reduce the effect on the approximation owing to the part of the light curve outside the eclipse, we have selected only a total of 120 observations within an interval from -0.08 to 0.08 near the minimum of the complete light curve of the star V0882 Car = 2MASS J11080308-6145589 [32]. The WinCurveFit v1.1.2 (Kevin Raner Software) program was used for a nonlinear least squares approximation; this makes it possible to determine up to 8 parameters. We have examined approximations with from 5 to 8 parameters.

The results are shown in Table 1 and correspond to the smoothed curves of Fig. 1 in order of declining quality of the approximations. The sum of the squared errors was used as the main test function for ranking the approximations. The best turned out to be the approximation by the NAV method with an added correction parameter. However, the very small difference between the test functions meant that it was not possible to establish an indisputable advantage for any of the approximations among the several with essentially the same significance.

The Mikulášek approximation method yields almost the same quality, but, as noted above, the width of the eclipse, where the function describing the profile intersects zero, is formally infinite. We note that rapid searches for periods and automatic classifications of stars usually employ approximations with substantially poorer agreement between theory and the observations, but which can be calculated fairly rapidly. These include the classical method for preliminary study of the Algols, in which the brightness is divided into “eclipse” (brightnesses below some limit) and “outside the eclipse” sections and approximated by a triangle or parabola. We have also analyzed these approximations, but the SSE test function values for them are much poorer, so they are suitable only for preliminary (inaccurate) estimation of the parameters.

6. Conclusion

The functions examined here and their modifications yield a natural improvement in the quality of the approximation in terms of the SSE criterion as the number of parameters is increased, but the effect of adding a parameter differs substantially for different initial functions. Of the 14 best functions, the best approximation without a phase limit only ranks seventh. The six best approximations correspond to a limited width of the minimum (as expected physically) for modifications of the function (3) [10,11] and the function (10) [17]. Both families of modifications have an asymptotic power law dependence near the center of the eclipse, but differ at the edges. The approximation (18) with “phase distortion” is the best for the test series of observations, but the statistical significance of the parameter has to be determined separately for each series of observations.
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