Higher-Spin Gauge Models
in the BRST-antifield Formalism

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Abstract

We construct higher-spin gauge models including fermions by applying the BRST-antifield formalism. We introduce massless totally-symmetric rank-$n$ tensor-spinors, which is Dirac spinors in $D$-dimensional spacetime, as well as massless bosonic totally-symmetric tensors. By applying the BRST deformation scheme, the free higher-spin gauge theory is deformed such that the deformed action $S$ would satisfy the master equation $(S, S) = 0$. We obtain vertices of two fermions and one boson, and those of a fermion and a boson in turn. They contain terms of all orders in the deformation parameter and satisfy the master equation exactly. Employing the BRST deformation scheme built on AdS spaces, we derive higher-spin gauge models including fermions on AdS spaces as well. We also consider the case that infinite series contained in our models is convergent.

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1 Introduction

Higher-spin gauge theories are expected to reveal characteristic features of string theory in the high-energy limit [1] (see also [2]). They also provided nontrivial examples of the AdS/CFT correspondence [3]. A bosonic higher-spin gauge theory on a three-dimensional AdS space (AdS$_3$) [4] has been conjectured [5] to be dual to the 't Hooft limit of the $W_N$ minimal model, while Vasiliev’s higher-spin gauge theory on AdS$_4$ [6] is conjectured [7] to be dual to the three-dimensional $O(N)$ sigma-model.

Free higher-spin gauge theories are now well understood. Interacting theories of massless higher-spin gauge fields in flat spacetime are severely constrained by the no-go theorem [8]. To avoid this, spin values and the number of derivatives contained in interaction vertices should be restricted so that the higher-spin gauge fields are not included in the asymptotic states. By using the light-cone formulation\(^2\), a classification of spin values and the number of derivatives of allowed cubic vertices for totally-symmetric bosonic and fermionic fields was given in [16,17]. In constructing interaction vertices explicitly, the Noether’s procedure is used in [18–21], and the BRST approach to impose

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\(^1\)See [9] for an analysis in the field theory framework.

\(^2\)Remarkably, chiral higher-spin gravity [10,11] in four-dimensions is shown to be one-loop finite [12,13]. See also [14,15].
gauge invariances is used in [22–29] on flat spacetime and [30, 31] on AdS spaces (see [32, 33] for review).

In this paper, we employ the BRST-antifield formalism [34] which is known to be very powerful in constructing interactions systematically. Using this cohomological method\(^3\), bosonic interaction vertices are constructed in [35, 36] and those including gauge fermions in [38–40]. The results in [38, 39] are shown to be consistent with the classification [16, 17] and with those obtained in the tensionless limit of open string theory [2] (see also [42–46]).

In [37], applying the BRST deformation scheme, the authors constructed bosonic higher-spin gauge models of massless totally-symmetric tensors which have higher-order vertex more than the cubic vertex. These models contain vertices of all orders in the deformation parameter\(^4\). It is noted that the action \(S\) satisfies the master equation \((S, S) = 0\) and so is BRST-invariant exactly. In constructing vertices, the Fronsdal tensor \([47]\) is employed as a building block. Since the Fronsdal tensor may be defined on anti-de Sitter (AdS) space, these models constructed on flat space are generalized to those on AdS spaces naturally. In the present paper, we see that the BRST deformation scheme works on AdS space also and drives vertices on AdS space.

The purpose of this paper is to generalize [37] to include higher-spin gauge fermions. We introduce massless totally-symmetric rank-\(n\) tensor-spinors, which are Dirac spinors in \(D\)-dimensional spacetime, as well as massless bosonic totally-symmetric tensors. By applying the BRST deformation scheme, a free higher-spin gauge action \(S\) is deformed such that the deformed action would satisfy the master equation. In section 3, we examine the models with two fermions and one boson, while, in section 4, the models with a fermion and a boson are examined. As a result, we obtain the actions including all order vertices for each cases. These actions satisfy the master equation exactly. We also consider the case that infinite series contained in our models is convergent. Furthermore, while each vertex considered in [37] forms an open chain of fields, in this paper, more general case, in which each vertex forms a closed chain of fields, is considered. The results for bosonic higher-spin gauge models are summarized in appendices.

The paper is organized as follows. In the next section, we introduce free higher-spin gauge theories of a bosonic tensor and a fermionic tensor-spinor in the BRST-antifield formalism. The BRST deformation scheme is explained as well. Higher-spin gauge model of two fermions and one boson is derived in section 3, and that of a fermion and a boson is derived in section 4. After introducing objects on AdS spaces, we derive higher-spin gauge models on AdS spaces in the BRST deformation scheme in section 5. The last section is devoted to a summary and discussions. In appendices, we present bosonic higher-spin gauge models on flat and AdS spaces which are generalization of the models found in [37].

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\(^3\)See [41] for a different approach.

\(^4\)These models are free on-shell and the cubic vertex \(S_1\) is BRST-exact as seen in section 6.
2 Free higher-spin gauge theory and BRST deformation scheme

We explain the notations used throughout this paper. We will introduce gauge fermions as well as gauge bosons in $D$-dimensional flat spacetime with $D \geq 4$. Free actions of higher-spin gauge fields are given in the BRST-antifield formalism. We explain the BRST deformation scheme used in constructing vertices.

2.1 Bosonic higher-spin gauge theory

First we introduce a totally-symmetric rank-$n$ tensor bosonic gauge field $\phi_{\mu_1 \cdots \mu_n}$ in $D$-dimensional flat spacetime. The Fronsdal tensor $[47]$ for this field is defined by

$$F_{\mu_1 \cdots \mu_n}(\phi) = \Box \phi_{\mu_1 \cdots \mu_n} - \partial_{(\mu_1} \partial_{\mu_2} \phi_{\mu_3 \cdots \mu_n)} + \partial_{(\mu_1} \partial_{\mu_2} \phi'_{\mu_3 \cdots \mu_n)},$$  \hspace{1cm} (2.1)

where $\Box = \eta^{\mu\nu} \partial_\mu \partial_\nu$ and $\eta^{\mu\nu} = \text{diag}(-1, +1, \ldots, +1)$. A prime on a field represents the trace, namely $\phi'_{\mu_3 \cdots \mu_n} = \phi_{\mu_3 \cdots \mu_n} \rho$. The divergence of $\phi$ is expressed as $\partial_\cdot \phi_{\mu_2 \cdots \mu_n} = \partial_{\cdot \mu_1} \phi_{\mu_2 \cdots \mu_n}$. Under the gauge transformation with a totally-symmetric rank-$(n-1)$ tensor parameter $\xi_{\mu_2 \cdots \mu_n}$,

$$\delta \phi_{\mu_1 \cdots \mu_n} = \partial_{(\mu_1} \xi_{\mu_2 \cdots \mu_n)},$$  \hspace{1cm} (2.2)

the Fronsdal tensor $F_{\mu_1 \cdots \mu_n}$ is transformed to $\delta F_{\mu_1 \cdots \mu_n} = 3 \partial_{(\mu_1} \partial_{\mu_2} \partial_{\mu_3} \xi'_{\mu_4 \cdots \mu_n)}$. Here and hereafter, we require that the gauge parameter $\xi$ is traceless for $n \geq 3$

$$\xi'_{\mu_4 \cdots \mu_n} = 0,$$  \hspace{1cm} (2.3)

which ensures the gauge invariance of the Fronsdal tensor. Furthermore we impose a double traceless constraint on $\phi$ for $n \geq 4$

$$\phi''_{\mu_5 \cdots \mu_n} = 0,$$  \hspace{1cm} (2.4)

so that the Fronsdal equation $F(\phi) = 0$ should describe the propagation of a massless rank-$n$ tensor gauge field. The action which leads to the Fronsdal equation is

$$S_{\phi} = \int d^D x \frac{1}{2} \phi_{\mu_1 \cdots \mu_n} G_{\mu_1 \cdots \mu_n},$$  \hspace{1cm} (2.5)

where $G_{\mu_1 \cdots \mu_n}$ is defined by

$$G_{\mu_1 \cdots \mu_n}(\phi) \equiv F_{\mu_1 \cdots \mu_n} - \frac{1}{2} \eta_{(\mu_1 \mu_2} F'_{\mu_3 \cdots \mu_n)}.$$  \hspace{1cm} (2.6)

\footnote{Our convention for symmetrization of indices is

$$\partial_{(\mu_1} \cdots \partial_{\mu_r} \phi_{\nu_{r+1} \cdots \nu_m)} = \frac{1}{r!(n-r)!} \sum_{\{\mu_1, \ldots, \mu_r\}} \partial_{\mu_1} \cdots \partial_{\mu_r} \phi_{\nu_{r+1} \cdots \nu_m},$$

where $\{\mu_1, \ldots, \mu_r\}$ indicates that the sum is taken over permutations of $\mu_1, \ldots, \mu_r$.}

\footnote{We frequently omit totally-symmetric indices but this may not cause confusion.}
Varying the action $S_{\phi}$ with respect to $\phi$, we obtain the equation of motion $G_{\mu_1...\mu_n} = 0$. A useful relation to derive this equation is\footnote{We drop surface terms throughout this paper.}

$$
\int d^Dx \varphi_1^{\mu_1...\mu_n} \tilde{G}_{\mu_1...\mu_n}(\varphi_2) = \int d^Dx \tilde{G}_{\mu_1...\mu_n}(\varphi_1) \varphi_2^{\mu_1...\mu_n}, \tag{2.7}
$$

where $\varphi_i$ are arbitrary totally-symmetric rank-$n$ tensors. We have defined $\tilde{G}(A)$ by

$$
\tilde{G}_{\mu_1...\mu_n}(A) \equiv G_{\mu_1...\mu_n}(A) + \frac{1}{2} \eta(\mu_1 \mu_2 \partial_{\nu_3} A_{\mu_4...\mu_n}^{\nu}), \tag{2.8}
$$

where $A$ is an arbitrary totally-symmetric rank-$n$ tensor. When the double traceless condition for $\varphi_i$ would be implemented, one obtains $\tilde{G}(\varphi_i) = G(\varphi_i)$. Using the relation (2.7) with $\tilde{G} = G$, we obtain $G(\phi) = 0$ from the variation of (2.5)\footnote{Taking the trace of $G = 0$ leads to $-\frac{1}{4}(D + 2n - 6)F' - \frac{1}{2} \eta F''' = 0$. Assume that $D + 2n - 6 \neq 0$, and then we obtain $F' = 0$ as $F''' = 0$ follows from $\varphi'' = 0$. This implies that $F = 0$. $D + 2n - 6 = 0$ is satisfied for $(D,n) = (6,0)$ and $(4,1)$ as $D \geq 4$. In both cases, $G = 0$ means $F = 0$ as $F' = 0$. As a result, the equation of motion $G = 0$ implies the Fronsdal equation $F = 0$.}.

Similarly the gauge variation of $S_{\phi}$ vanishes due to the gauge invariance $G(\delta \phi) = 0$.

In the BRST-antifield formalism, corresponding to the gauge parameter $\xi_{\mu_2...\mu_n}$, we introduce a Grassmann-odd ghost field $c_{\mu_2...\mu_n}$ with the same algebraic symmetry. The $c_{\mu_2...\mu_n}$ must be traceless $c'_{\mu_4...\mu_n} = 0$ because of the traceless condition $\xi' = 0$. The gauge invariance of $S_{\phi}$ is encoded to the BRST invariance under the BRST transformation

$$
\delta_B \phi_{\mu_1...\mu_n} = \partial(\mu_1 c_{\mu_2...\mu_n}), \quad \delta_B c_{\mu_2...\mu_n} = 0. \tag{2.9}
$$

The gauge field $\phi$ and the ghost field $c$ are collectively called “fields” and denoted as $\Phi^A$. We further introduce antifields $\Phi_A^* = \{\phi_{\mu_1...\mu_n}^*, c_{\mu_2...\mu_n}^*\}$ which have the same algebraic symmetries but opposite Grassmann parity. Two gradings are introduced. One is the pure ghost number $pgh$, and the other is the antighost number $agh$. The ghost number $gh$ is defined as $gh \equiv pgh - agh$. The grading property is summarized in Table 1.

| $Z$    | $pgh(Z)$ | $agh(Z)$ | $gh(Z)$ |
|--------|----------|----------|---------|
| $\phi_{\mu_1...\mu_n}$   | 0        | 0        | 0       |
| $c_{\mu_2...\mu_n}$       | 1        | 0        | 1       |
| $\phi_{\mu_1...\mu_n}^*$  | 0        | 1        | $-1$    |
| $c_{\mu_2...\mu_n}^*$      | 0        | 2        | $-2$    |

Table 1: Grading properties of fields and antifields
The action $S_\phi$ in (2.5) can be extended to $S^0[\Phi, \Phi^*]$ such that the BRST transformation of a functional $X(\Phi^A, \Phi^*_A)$ is expressed as

$$\delta_B X = (X, S^0).$$

(2.11)

Note that $\delta_B$ acts from the right. The nilpotency $\delta^2_B = 0$ requires the master equation $(S^0, S^0) = 0$. In the present case, $S^0$ may be given as

$$S^0[\Phi, \Phi^*] = S_\phi + \int d^D x \phi^{*\mu_1\ldots\mu_n} \partial_{(\mu_1} c_{\mu_2\ldots\mu_n)},$$

(2.12)

which leads to (2.9) and

$$\delta_B \phi^{*\mu_1\ldots\mu_n} = -G_{\mu_1\ldots\mu_n},$$

(2.13)

$$\delta_B c^{\mu_2\ldots\mu_n} = -n \partial \cdot \phi^{*\mu_2\ldots\mu_n} + \frac{n}{D + 2n - 6} \eta(\mu_2 \mu_3 \cdots \mu_n),$$

(2.14)

when $D + 2n - 6 \neq 0$. The last term on the right-hand side of (2.14) is required for the nilpotency $\delta^2_B c^* = 0$. We note that the key relation for the nilpotency, $-\partial \cdot G_{\mu_2\ldots\mu_n} + \frac{n}{D + 2n - 6} \eta(\mu_2 \mu_3 \cdots \mu_n) = 0$, follows from the double traceless condition (2.4). Since $\delta_B c^* = (c^*, S^0)$, the last term on the right-hand side of (2.14) requires an additional term, $\int d^D x \frac{n}{D + 2n - 6} \eta(\mu_2 \mu_3 \cdots \mu_n) c^{\mu_2\ldots\mu_n}$, in the action $S^0$. However, this term disappears due to $c' = 0$, and leaves the action (2.12) unchanged.

### 2.2 Fermionic higher-spin gauge theory

Next, we introduce a totally-symmetric rank-$n$ tensor-spinor gauge field $\psi_{\mu_1\ldots\mu_n}$ which is a Dirac spinor in $D$-dimensional flat spacetime. The Fronsdal tensor [48] for this field is defined by

$$S_{\mu_1\ldots\mu_n} (\psi) = \bar{\psi} \left( \gamma^\mu \psi_{\mu_1\ldots\mu_n} - \partial_{(\mu_1} \psi^{\mu_2\ldots\mu_n)} \right).$$

(2.15)

A slash of an object denotes the $\gamma$-trace of the object, namely $\bar{\theta} = \gamma^\mu \partial_\mu$ and $\bar{\psi}_{\mu_2\ldots\mu_n} = \gamma^{\mu_1} \psi_{\mu_1\ldots\mu_n}$. The Dirac matrices $\gamma^\mu$ satisfy $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$. We choose $\gamma^0$ as anti-hermite $\gamma^0 = -\gamma^0$ while $\gamma^i$ as hermite $\gamma^i = \gamma^i$, so that $\gamma^{\mu+1} \gamma^0 = -\gamma^0 \gamma^\mu$. The Dirac-conjugate of a spinor $\psi$ is defined by $\bar{\psi} \equiv \psi^\dagger \gamma^0$. Our convention of the hermite conjugate is that $(\bar{\phi}_1 \bar{\phi}_2)^\dagger = \bar{-\phi_2 \phi_1}$, where $\phi_i$ are either Grassmann even spinor or Grassmann odd spinor. Under the gauge transformation with a totally-symmetric rank-$(n-1)$ tensor-spinor parameter $\epsilon_{\mu_2\ldots\mu_n}$,

$$\delta \psi_{\mu_1\ldots\mu_n} = \partial_{(\mu_1} \epsilon_{\mu_2\ldots\mu_n)},$$

(2.16)

the Fronsdal tensor $S$ is transformed to $\delta S_{\mu_1\ldots\mu_n} = -2i \partial_{(\mu_1} \partial_{\mu_2} \epsilon_{\mu_3\ldots\mu_n)}$. Here and hereafter, we require that $\epsilon$ is $\gamma$-traceless for $n \geq 2$.

$$\epsilon_{\mu_3\ldots\mu_n} = 0,$$

(2.17)

9We comment on the case $D + 2n - 6 = 0$, namely $(D, n) = (6, 0)$ and (4, 1). When $(D, n) = (6, 0)$ the gauge transformation (2.2) becomes trivial, and so ghost and anti-fields are not introduced. When $(D, n) = (4, 1)$, the second term on the right-hand side of (2.14) is not needed because $\partial^\mu G_\mu = \partial^\mu (\Box \phi - \partial_\mu \partial^{-} \phi) = 0$.}

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which ensures the gauge invariance of the Fronsdal tensor $S$. Furthermore we impose a triple $\gamma$-traceless constraint on $\psi$ for $n \geq 3$

$$\psi_{\mu_1 \cdots \mu_n} = 0,$$  \hspace{1cm} (2.18)

so that the Fronsdal equation $S(\psi) = 0$ should describe the propagation of a massless rank-$n$ tensor-spinor gauge field. The action which leads to the Fronsdal equation $S = 0$ is given by

$$S_\psi = \int d^Dx \frac{1}{2} \left[ \mathcal{R}_{\mu_1 \cdots \mu_n} \psi_{\mu_1 \cdots \mu_n} - \bar{\psi}_{\mu_1 \cdots \mu_n} \mathcal{R}_{\mu_1 \cdots \mu_n} \right],$$ \hspace{1cm} (2.19)

where we introduced $\mathcal{R}_{\mu_1 \cdots \mu_n}$ as

$$\mathcal{R}_{\mu_1 \cdots \mu_n}(\psi) \equiv S_{\mu_1 \cdots \mu_n} - \frac{1}{2} \gamma_{(\mu_1 S_{\mu_2 \cdots \mu_n})} - \frac{1}{2} \eta_{(\mu_1 \mu_2 S'_{\mu_3 \cdots \mu_n})}.$$ \hspace{1cm} (2.20)

A useful and important relation we frequently use throughout this paper is

$$\int d^Dx \bar{\mathcal{R}}_{\mu_1 \cdots \mu_n}(\psi_1) \psi_{\mu_1 \cdots \mu_n} = -\int d^Dx \bar{\psi}_{\mu_1 \cdots \mu_n} \mathcal{R}_{\mu_1 \cdots \mu_n}(\psi_2),$$ \hspace{1cm} (2.21)

where $\psi_1$ are arbitrary totally-symmetric rank-$n$ tensor-spinors. We have defined $\bar{\mathcal{R}}(A)$ by

$$\bar{\mathcal{R}}_{\mu_1 \cdots \mu_n}(A) \equiv \mathcal{R}_{\mu_1 \cdots \mu_n}(A) - \frac{i}{2} \eta_{(\mu_1 \mu_2} \partial_{\mu_3} \mathfrak{A}_{\mu_4 \cdots \mu_n)},$$ \hspace{1cm} (2.22)

where $A$ is an arbitrary totally-symmetric rank-$n$ tensor-spinor. When the triple $\gamma$-traceless condition for $\psi_1$ would be implemented, one has $\bar{\mathcal{R}}(\psi_1) = \mathcal{R}(\psi_1)$. Varying the action $S_\psi$ with respect to $\bar{\psi}$ and using (2.21), we obtain the equation of motion $\mathcal{R}_{\mu_1 \cdots \mu_n} = 0$. Taking the $\gamma$-trace of $\mathcal{R} = 0$, we obtain $-\frac{1}{2} (D + 2n - 4) S - \frac{1}{2} \eta S' = 0$. When $D + 2n - 4 \neq 0$, this implies that $S = 0$, since $S' = 0$ follows from $\psi' = 0$. Further taking the $\gamma$-trace of $S = 0$, we obtain $S' = 0$. These imply that $S = 0$. When $D + 2n - 4 = 0$, $\mathcal{R} = S$ follows because $(D, n) = (4, 0)$ for $D \geq 4$ and $n \geq 0$. This is because $\mathcal{R} = 0$ means $S = 0$. As a result, the equation of motion $\mathcal{R} = 0$ implies the Fronsdal equation $S = 0$. Similarly the gauge invariance of $S_\psi$ is ensured by the gauge invariance $\mathcal{R}(\delta \psi) = 0$.

Corresponding to the tensor-spinor gauge parameter $\epsilon_{\mu_2 \cdots \mu_n}$, we introduce a Grassmann-even ghost tensor-spinor $\zeta_{\mu_2 \cdots \mu_n}$ with the same algebraic symmetry. The $\zeta_{\mu_2 \cdots \mu_n}$ must be $\gamma$-traceless $\zeta_{\mu_2 \cdots \mu_n} = 0$, just like $\psi_{\mu_2 \cdots \mu_n} = 0$. The gauge invariance of $S_\psi$ is encoded to the BRST invariance under the BRST transformation

$$\delta_B \psi_{\mu_1 \cdots \mu_n} = \partial(\mu_1 \zeta_{\mu_2 \cdots \mu_n}), \hspace{1cm} \delta_B \zeta_{\mu_2 \cdots \mu_n} = 0.$$ \hspace{1cm} (2.23)

In addition to spinor fields $\Psi^A \equiv \{ \psi_{\mu_1 \cdots \mu_n}, \zeta_{\mu_2 \cdots \mu_n} \}$, we introduce spinor antifields $\Psi^*_A \equiv \{ \psi^*_{\mu_1 \cdots \mu_n}, \zeta^*_{\mu_2 \cdots \mu_n} \}$ which have the same algebraic symmetries but opposite Grassmann parity. The grading property is summarized in Table 2.

The action $S_\psi$ in (2.19) can be extended to $S^0[\Psi, \Psi^*]$ such that the BRST transformation of a functional $X(\Psi^A, \Psi^*_A)$ is expressed as $\delta_B X = (X(S^0))$. The antibracket for two functionals, $X(\Psi^A, \Psi^*_A)$ and $Y(\Psi^A, \Psi^*_A)$, is defined by

$$(X, Y) \equiv X \frac{\delta}{\delta \Psi^A} \frac{\delta}{\delta \Psi^*_A} Y - X \frac{\delta}{\delta \Psi^*_A} \frac{\delta}{\delta \Psi^A} Y.$$ \hspace{1cm} (2.24)
Table 2: Grading properties of fields and antifields

|   | pgh(Z) | agh(Z) | gh(Z) |
|---|---|---|---|
| ψ_{μ1...μ_n} | 0 | 0 | 0 |
| ζ_{μ2...μ_n} | 1 | 0 | 1 |
| ψ^*_{μ1...μ_n} | 0 | 1 | -1 |
| ζ^*_{μ2...μ_n} | 0 | 2 | -2 |

In the present case, $S^0$ may be given as

$$S^0[Ψ, Ψ^*] = S_ψ + \int d^Dx (\bar{Ψ}^*_{μ1...μ_n} \partial_{(μ_1 ζ_{μ2...μ_n})} - \partial_{(μ_1 ζ_{μ2...μ_n})} Ψ^*_{μ1...μ_n}),$$

(2.25)

which leads to (2.23) and

$$δ_B ψ^*_{μ1...μ_n} = R_{μ1...μ_n},$$

(2.26)

$$δ_B ζ^*_{μ2...μ_n} = -n\partial \cdot ψ^*_{μ2...μ_n} + \frac{n}{D+2n-4}(\gamma \partial \cdot ψ^* + η \partial \cdot ψ^*'),$$

(2.27)

when $D + 2n - 4 \neq 0$\(^{10}\). Note that the second term on the right-hand side of (2.27) is required for the nilpotency $δ^2_B ζ^* = 0$. We find that the key relation for the nilpotency

$$-\partial \cdot R + \frac{1}{D+2n-4}(\gamma \partial \cdot R + η \partial \cdot R') = 0$$

(2.28)

follows from the triple $γ$-traceless condition (2.18). Since $δ_B ζ^* = (ζ^*, S^0)$, the second term on the right-hand side of (2.27) requires an additional term

$$\frac{n}{D+2n-4} \int d^Dx \left(\bar{ζ}(\gamma \partial \cdot ψ^* + η \partial \cdot ψ^*') - (\gamma \partial \cdot ψ^* + η \partial \cdot ψ^*')ζ\right)$$

(2.29)

in the action $S^0$. However this term disappears due to $ζ' = 0$\(^{11}\), and leaves the action (2.25) unchanged.

### 2.3 BRST deformation scheme

BRST deformation scheme is very useful in constructing vertices systematically [34]. We will explain relevant aspects.

Suppose that $S$ is a deformation of $S^0$ expanded in a deformation parameter $g$,

$$S = S^0 + g S^1 + g^2 S^2 + \cdots.$$  

(2.30)

When $S$ solves the master equation $(S, S) = 0$, $S$ is invariant under the BRST transformation generated by $S$: $δ_B S = (S, S) = 0$. Here we add the letter $g$ to $δ_B$ in order to distinguish it from

\(^{10}\)When $D + 2n - 4 = 0$, namely $(D, n) = (4, 0)$, the gauge transformation (2.16) becomes trivial, and so a ghost and antifields are not introduced.

\(^{11}\) $ξ' = 0$ follows from $ξ = 0$.  

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\( \delta B \) generated by \( S^0 \) considered in the previous subsections. The master equation \((S, S) = 0\) means, at the order of \( g^n \),

\[
\sum_{k=0}^{n} (S^k, S^{n-k}) = 0. \tag{2.31}
\]

The equation for \( n = 0 \), \((S^0, S^0) = 0\), is satisfied by definition. Note that \( S^n \) is determined by the set \( \{S^0, S^1, \ldots, S^{n-1}\} \).

We expand \( \delta B \) as

\[
\delta B = \Delta + \Gamma, \tag{2.32}
\]

where \( \Delta \) reduces \( agh \) by one while \( \Gamma \) leaves it unchanged. These \( \Delta \) and \( \Gamma \) act on fields and antifields as summarized in Table 3. Correspondingly, we expand \( S^n \) with respect to \( agh \). In this paper, we choose a certain cubic vertex as \( S^1 \), and find solutions of the master equation \((S, S) = 0\) in which \( S^n \) is expanded as

\[
S^n = \alpha_n^2 + \alpha_n^1 + \alpha_n^0 \tag{2.33}
\]

where \( agh(\alpha_n^i) = i \). The master equation (2.31) at the order of \( g^n \) reduces to the following three equations with respect to \( agh \)

\[
\begin{align*}
\Delta \alpha_n^2 + \Gamma \alpha_n^2 + \frac{1}{2} \sum_{k=2}^{n-1} (\alpha_n^k, \alpha_n^{n-k}) + \frac{1}{2} \sum_{k=1}^{n-2} (\alpha_n^k, \alpha_n^{n-k}) &= 0, \\
\Delta \alpha_n^1 + \Gamma \alpha_n^1 + \frac{1}{2} \sum_{k=2}^{n-1} (\alpha_n^k, \alpha_n^{n-k}) + \frac{1}{2} \sum_{k=1}^{n-1} (\alpha_n^k, \alpha_n^{n-k}) + \frac{1}{2} \sum_{k=1}^{n-2} (\alpha_n^k, \alpha_n^{n-k}) &= 0, \\
\Delta \alpha_n^0 + \Gamma \alpha_n^0 + \frac{1}{2} \sum_{k=1}^{n-1} (\alpha_n^k, \alpha_n^{n-k}) + \frac{1}{2} \sum_{k=1}^{n-1} (\alpha_n^0, \alpha_n^{n-k}) &= 0.
\end{align*}
\]

In the next section, we consider vertices among rank-\( n_1 \) and rank-\( n_2 \) tensor-spinors and a rank-\( n \) tensor, while in the section 4 vertices between a rank-\( n_1 \) tensor-spinor and a rank-\( n \) tensor are examined.
3 Higher-spin gauge model of two fermions and one boson

We introduce three gauge fields, a rank-\( n_1 \) tensor-spinor \( \psi_1 \), a rank-\( n_2 \) tensor-spinor \( \psi_2 \), and a rank-\( n \) tensor \( \phi \). In addition, we introduce corresponding ghosts, \( \zeta_I \) (\( I = 1, 2 \)) and \( c \), and antifields \( \{ \psi^*_I, \phi^*, \zeta^*_I, c^* \} \) as well. As explained in section 2, the free action for them is given as

\[
S^0[\Phi, \Phi^*, \Psi, \Psi^*] = \int d^D x \left[ \frac{1}{2} \phi^{I_1 \cdots I_n} G_{I_1 \cdots I_n} + \phi^{I_1 \cdots I_n} \partial_{(I_1} c_{I_2 \cdots I_n)} 
+ \sum_{I=1,2} \left( \frac{1}{2} \mathcal{R}_{I_1 \cdots I_{n_I}}(\psi_I) \psi_I^{I_1 \cdots I_{n_I}} - \bar{\psi}_I^{I_1 \cdots I_{n_I}} \mathcal{R}_{I_1 \cdots I_{n_I}}(\psi_I) 
+ \bar{\psi}_I^{I_1 \cdots I_{n_I}} \partial_{(I_1} \zeta_{I_2 \cdots I_{n_I})} - \partial_{(I_1} \bar{\zeta}_{I_2 \cdots I_{n_I})} \psi_I^{I_1 \cdots I_{n_I}} \right) \right]. \tag{3.1}
\]

First we consider \( S^1 = \alpha_2 + \alpha_2 + \alpha_0 \). We choose a cubic vertex as \( S^1 \). As shown in [37], we may set \( \alpha_2 = 0 \). This is because the \( \alpha_2 \) with \( agh = pgh = 2 \) must be \( \Gamma \)-exact, and so it leads to a BRST-exact \( S^1 \). The master equation at the order of \( g^1 \), \( (S^0, S^1) = 0 \), is

\[
\Gamma \alpha_1 = 0, \tag{3.2}
\]
\[
\Delta \alpha_1 + \Gamma \alpha_0 = 0. \tag{3.3}
\]

We shall choose, as \( \alpha_1 \),

\[
\alpha_1 = i \int d^D x s^{IJ} \text{tr} \left[ \bar{\psi}_I^* \tilde{\partial} c \mathcal{R}(\psi_J) + \mathcal{R}(\psi_I) \tilde{\partial} c^T \psi_J^* \right] \tag{3.4}
\]

where \( s^{12} = s^{21} = 1 \) and \( s^{11} = s^{22} = 0 \). For notational simplicity, we used the matrix notation. The first term of the integrand means

\[
i \bar{\psi}_1^{\rho_1 \cdots \rho_r \mu_1 \cdots \mu_p} \partial_{(\mu_1} c_{\mu_2 \cdots \mu_p) \nu_1 \cdots \nu_q} \mathcal{R}^{\nu_1 \cdots \nu_q} \rho_1 \cdots \rho_r (\psi_2) + i \bar{\psi}_2^{\rho_1 \cdots \rho_r \nu_1 \cdots \nu_q} \partial_{(\nu_1} c_{\nu_2 \cdots \nu_q) \mu_1 \cdots \mu_p} \mathcal{R}^{\mu_1 \cdots \mu_p} \rho_1 \cdots \rho_r (\psi_1), \tag{3.5}
\]

while the second term means

\[
i \bar{\mathcal{R}}^{\rho_1 \cdots \rho_r \mu_1 \cdots \mu_p} (\psi_1) \partial_{(\mu_1} c_{\mu_2 \cdots \mu_p) \nu_1 \cdots \nu_q} \psi_2^{\nu_1 \cdots \nu_q} \rho_1 \cdots \rho_r + i \bar{\mathcal{R}}^{\rho_1 \cdots \rho_r \nu_1 \cdots \nu_q} (\psi_2) \partial_{(\nu_1} c_{\nu_2 \cdots \nu_q) \mu_1 \cdots \mu_p} \psi_1^{\mu_1 \cdots \mu_p} \rho_1 \cdots \rho_r. \tag{3.6}
\]

Here \( p + r = n_1, q + r = n_2 \) and \( p + q = n \). In this notation, \( \psi_1 \) and \( \psi_1^* \) are \( d(p) \times d(r) \) matrices, \( \psi_2 \) and \( \psi_2^* \) are \( d(q) \times d(r) \) matrices, where \( d(s) = \frac{(D-1+s)!}{(D-1)!s!} \). The index of the derivative in \( \tilde{\partial} c \) is always contracted with one of the indices of the field sitting to its immediate left.

It is obvious that \( \Gamma \alpha_1 = 0 \) because of the gauge invariance of \( \mathcal{R} \). Acting \( -\Delta \) on \( \alpha_1 \), we derive

\[
-\Delta \alpha_1 = -i \int d^D x s^{IJ} \text{tr} \left[ \mathcal{R}(\psi_I) \tilde{\partial} c \mathcal{R}(\psi_J) + \mathcal{R}(\psi_I) \tilde{\partial} c^T \mathcal{R}(\psi_J) \right] = \Gamma i \int d^D x s^{IJ} \text{tr} \mathcal{R}(\psi_I) \phi \mathcal{R}(\psi_J). \tag{3.7}
\]

As a result, we find

\[
\alpha^0_1 = i \int d^D x s^{IJ} \text{tr} \mathcal{R}(\psi_I) \phi \mathcal{R}(\psi_J). \tag{3.8}
\]
We shall show that the solution of the master equation at the order of $g^n$ is
\begin{equation}
S^n = \alpha_2^n + \alpha_1^n + \alpha_0^n, 
\end{equation}
\begin{equation}
\alpha_2^n = \int d^D x i^n (s^n)^{IJ} \text{tr} \tilde{\psi}_I^J \partial c \Psi^{n-2}[\tilde{\mathcal{R}}(\partial c^T \psi_I^J)], 
\end{equation}
\begin{equation}
\alpha_1^n = \int d^D x i^n (s^n)^{IJ} \text{tr} \left[ i^n \tilde{\psi}_I^J \partial c \Psi^{n-1}[\mathcal{R}(\psi_I^J)] - (-i)^n \Psi^{n-1}[\mathcal{R}(\psi_I^J)] \partial c^T \psi_I^J \right], 
\end{equation}
\begin{equation}
\alpha_0^n = - \int d^D x i^n (s^n)^{IJ} \text{tr} \tilde{\psi}_I^J \Psi^n[\mathcal{R}(\psi_I^J)]. 
\end{equation}
Here we have defined $\Psi$ by $\Psi^0[A] = A$, $\Psi[A] = \mathcal{R}(\phi A)$, $\Psi^2[A] = \mathcal{R}(\phi \mathcal{R}(\phi A))$, and so on. A useful relation we frequently use is
\begin{equation}
\int d^D x \tilde{A} \Psi^k[\mathcal{R}(B)] = (-1)^{k+1} \int d^D x \bar{\Psi}^k[\mathcal{R}(A)] B. 
\end{equation}
Now, suppose that $\{S^1, S^2, \ldots, S^{n-1}\}$ solves the master equation at the order of $g^k$ ($k = 1, \ldots, n-1$). We will derive $S^n$ expanded as (3.9) and show that the $\alpha_2^n$, $\alpha_1^n$ and $\alpha_0^n$ coincide with those given in (3.10), (3.11) and (3.12), respectively.

The master equation (2.31) at the order of $g^n$ reduces to (2.34), (2.35) and (2.36). First we will derive $\alpha_2^n$ from (2.34). Noting that
\begin{equation}
(\alpha_k^n, \alpha_{n-k}^n) = i^n (s^n)^{IJ} \int d^D x \text{tr} \left[ (-1)^k \Psi^{k-1}[\mathcal{R}(\partial c^T \psi_I^J)] \partial c \Psi^{n-k-2}[\tilde{\mathcal{R}}(\partial^c c^T \psi_I^J)] 
+ (-1)^{k+1} \Psi^{n-k-2}[\mathcal{R}(\partial c^T \psi_I^J)] \partial c \Psi^{k-1}[\tilde{\mathcal{R}}(\partial^c c^T \psi_I^J)] \right],
\end{equation}
we obtain
\begin{equation}
\begin{aligned}
- \frac{1}{2} \sum_{k=1}^{n-2} (\alpha_k^n, \alpha_{n-k}^n) 
- \frac{1}{2} \sum_{k=2}^{n-1} (\alpha_k^n, \alpha_1^n) 
= \int d^D x i^n (s^n)^{IJ} \text{tr} \sum_{l=0}^{n-3} (-1) l \Psi^l[\mathcal{R}(\partial c^T \psi_I^J)] \partial c \Psi^{n-3-l}[\mathcal{R}(\partial c^T \psi_I^J)] 
= \Gamma \int d^D x i^n (s^n)^{IJ} \text{tr} \tilde{\psi}_I^J \partial c \Psi^{n-2}[\mathcal{R}(\partial c^T \psi_I^J)].
\end{aligned}
\end{equation}
This implies that $\alpha_2^n$ is given as (3.10).

Next we will derive $\alpha_1^n$ from (2.35). Using (3.10), we derive
\begin{equation}
\begin{aligned}
\Delta \alpha_2^n & = \frac{1}{2} \sum_{k=2}^{n-1} (\alpha_k^n, \alpha_{n-k}^n) 
- \frac{1}{2} \sum_{k=1}^{n-2} (\alpha_0^n, \alpha_2^n) 
= i^n (s^n)^{IJ} \int d^D x \text{tr} \left[ \sum_{k=0}^{n-2} (-1)^{k+1} \Psi^{k+1}[\mathcal{R}(\psi_I^J)] \partial c \Psi^{n-k-2}[\mathcal{R}(\partial c^T \psi_I^J)] 
+ \sum_{l=0}^{n-2} (-1)^l \Psi^l[\mathcal{R}(\partial c^T \psi_I^J)] \partial c^T \Psi^{n-2-l}[\mathcal{R}(\psi_I^J)] \right].
\end{aligned}
\end{equation}
On the other hand, one finds
\[-\frac{1}{2} \sum_{k=1}^{n-1} (\alpha_1^k, \alpha_1^{n-k}) = i^n \sum_{k=1}^{n-1} (s^n)^{IJ} \int d^D x \text{tr} \left( (-1)^k \Psi^{k-1} [\mathcal{R}(\psi_I)] \partial c^T \Psi^{n-k-1} [\mathcal{R}(\partial c^T \psi)_J^*] \right) \]
\[+ (-1)^{k+1} \Psi^{k-1} [\mathcal{R}(\partial c^T \psi)_J^*] \partial c \Psi^{n-k-1} [\mathcal{R}(\psi_J)] \right). \quad (3.16)\]

Combining these results together we obtain
\[-\Delta \alpha_1^n - \frac{1}{2} \sum_{k=2}^{n-1} (\alpha_2^k, \alpha_2^{n-k}) - \frac{1}{2} \sum_{k=1}^{n-1} (\alpha_1^k, \alpha_1^{n-k}) - \frac{1}{2} \sum_{k=1}^{n-2} (\alpha_0^k, \alpha_2^{n-k}) \]
\[= i^n (s^n)^{IJ} \int d^D x \text{tr} \left( \sum_{k=0}^{n-2} (-1)^{k+1} \Psi^{k} [\mathcal{R}(\psi_I)] \partial c \Psi^{n-k-2} [\mathcal{R}(\partial c^T \psi)_J^*] \right) \]
\[\quad + \sum_{l=0}^{n-2} (-1)^{l} \Psi^{l} [\mathcal{R}(\partial c^T \psi)_J^*] \partial c \Psi^{n-2-l} [\mathcal{R}(\psi_J)] \right) \]
\[= \Gamma \int d^D x i^n (s^n)^{IJ} \text{tr} \left( \bar{\psi}_J^* \partial c \Psi^{n-1} [\mathcal{R}(\psi_J)] - (-1)^n \Psi^{n-1} [\mathcal{R}(\psi_I)] \partial c^T \psi_J^* \right). \quad (3.17)\]

This implies that \(\alpha_1^n\) is given as in (3.11).

Finally, we solve (2.36) for \(\alpha_0^n\). It is straightforward to derive
\[-\Delta \alpha_0^n - \frac{1}{2} \sum_{k=1}^{n-1} (\alpha_1^k, \alpha_0^{n-k}) - \frac{1}{2} \sum_{k=1}^{n-1} (\alpha_0^k, \alpha_1^{n-k}) \]
\[= i^n (s^n)^{IJ} \int d^D x \text{tr} \left( \sum_{k=0}^{n-1} (-1)^{k+1} \Psi^{k} [\mathcal{R}(\psi_I)] \partial c \Psi^{n-k-1} [\mathcal{R}(\psi_J)] \right) \]
\[\quad + \sum_{k=1}^{n} (-1)^{k} \Psi^{k-1} [\mathcal{R}(\psi_I)] \partial c^T \Psi^{n-k} [\mathcal{R}(\psi_J)] \right) \]
\[= i^n (s^n)^{IJ} \int d^D x \text{tr} \sum_{k=0}^{n-1} (-1)^{k+1} \Psi^{k} [\mathcal{R}(\psi_I)] \partial c \Psi^{n-k-1} [\mathcal{R}(\psi_J)] \]
\[= \Gamma \int d^D x (-i^n) (s^n)^{IJ} \text{tr} \bar{\psi}_I \Psi^{n} [\mathcal{R}(\psi_J)]. \quad (3.18)\]

This implies that \(\alpha_0^n\) is given as in (3.12).

Summarizing the above results, we have shown that \(S^n\) is definitely given in (3.9)-(3.12).
3.1 BRST-invariant deformed action of three gauge fields

In the above we have derived $S^k$ ($k = 1, 2, \cdots$). The free action $S^0$ is given in (3.1). Combining these together we obtain the total action

$$S = S^0 + \sum_{k=1}^{\infty} g^k S^k$$

$$= \int d^D x \ tr \left[ \frac{1}{2} \phi^T G(\phi) + \phi^* T \partial c + \frac{1}{2} \left( \mathcal{R}(\bar{\psi}_I) \psi_I - \bar{\psi}_I \mathcal{R}(\psi_I) \right) + \bar{\psi}_I^* \partial \zeta_I - \partial \bar{\psi}_I^* 
- \bar{\psi}_I \sum_{k=1}^{\infty} [(igs)^k]^{IJ} \bar{\Psi}^k [\mathcal{R}(\psi_J)] + \bar{\psi}_I^* \partial c \sum_{k=1}^{\infty} [(igs)^k]^{IJ} \bar{\Psi}^k-1 [\mathcal{R}(\psi_J)] 
- \sum_{k=2}^{\infty} [(-igs)^k]^{IJ} \bar{\Psi}^{k-1} [\mathcal{R}(\psi_J)] \partial c^T \psi_I^* + \bar{\psi}_I^* \partial c \sum_{k=2}^{\infty} [(igs)^k]^{IJ} \bar{\Psi}^{k-2} [\mathcal{R}(\partial c^T \psi_J^*)] \right]. \quad (3.19)$$

This action contains terms of all orders in the deformation parameter $g$ and is BRST-invariant exactly. We note that the BRST-invariance of the action is ensured even if $g$ is not small. We consider the general case in which $g$ is not restricted to be small in this paper. More precisely the infinite series contained in the action, say $\sum_{k=1}^{\infty} [(igs)^k]^{IJ} \bar{\Psi}^k [\mathcal{R}(\psi_J)]$, may be expressed formally in a closed form $(\frac{1}{1-igs})^{IJ} \mathcal{R}(\psi_J)$ when $|igs\Psi| < 1$. However we will not restrict ourselves on the case with $|igs\Psi| < 1$. In order to make this point obvious we use the infinite series form throughout this paper\footnote{In [37], the closed form is used for a concise expression, but obviously no restriction is imposed.}.

We find that the action $S$ turns to the form expanded in $agh$ as

$$S = S_0 + S_1 + S_2,$$ \hspace{1cm} (3.20)

$$S_0 = \int d^D x \ tr \left[ \frac{1}{2} \phi^T G(\phi) - \bar{\psi}_I \sum_{k=0}^{\infty} [(igs)^k]^{IJ} \bar{\Psi}^k [\mathcal{R}(\psi_J)] \right], \quad (3.21)$$

$$S_1 = \int d^D x \ tr \left[ \phi^* T \partial c + \bar{\psi}_I^* \partial \zeta_I - \partial \bar{\psi}_I^* 
+ \bar{\psi}_I^* \partial c \sum_{k=0}^{\infty} [(igs)^{k+1}]^{IJ} \bar{\Psi}^k [\mathcal{R}(\psi_J)] - \sum_{k=0}^{\infty} [(igs)^{k+1}]^{IJ} \bar{\Psi}^k [\mathcal{R}(\psi_J)] \partial c^T \psi_I^* \right], \quad (3.22)$$

$$S_2 = \int d^D x \ tr \left[ \bar{\psi}_I^* \partial c \sum_{k=0}^{\infty} [(igs)^{k+2}]^{IJ} \bar{\Psi}^k [\mathcal{R}(\partial c^T \psi_J^*)] \right], \quad (3.23)$$

where $agh(S_i) = i$. This action $S$ contains all orders in $g$, and invariant under the BRST-transformation $\delta_B S = (S, S) = 0$.

We have obtained a BRST-invariant deformed action of three gauge fields by using the BRST-antifield formalism. Here we examine the gauge-invariant action $S_0$ in (3.21). First of all we derive
equations of motion. Varying $S_0$ with respect to $\phi$ and $\bar{\psi}_I$, we obtain

$$G(\phi) + i g s^{IJ} \sum_{k=0}^{\infty} [(i g s)^k]_{IJ} \Psi^k [\mathcal{R} (\psi_I)] \sum_{l=0}^{\infty} [(i g s)^l]_{JL} \Psi^l [\mathcal{R} (\psi_L)] = 0,$$

$$\sum_{k=0}^{\infty} [(i g s)^k]_{IJ} \Psi^k [\mathcal{R} (\psi_J)] = 0. \quad (3.24)$$

Substituting (3.25) into the first equation leads to $G(\phi) = 0$. The gauge transformation can be read off from the BRST transformation. We find that the gauge transformation with a rank-$(n_I - 1)$ tensor-spinor parameter $\epsilon_I$ remains unchanged

$$\delta \phi = 0, \quad \delta \psi_I = \partial \epsilon_I, \quad (3.26)$$

while the gauge transformation with a rank-$(n - 1)$ tensor parameter $\xi$ turns into

$$\delta \phi = \partial \xi, \quad \delta \psi_I = \tilde{\partial} \xi \sum_{k=0}^{\infty} [(i g s)^{k+1}]_{IJ} \Psi^k [\mathcal{R} (\psi_J)]. \quad (3.27)$$

The latter gauge transformation is deformed by vertices. However we note that the gauge algebra remains abelian on-shell. In fact, applying the gauge transformation twice on $\psi_I$ leads to terms which vanish due to (3.25).

4 Higher-spin gauge model of a fermion and a boson

In this section, we consider a model which formed by two gauge fields, a rank-$n_1$ tensor-spinor $\psi$ and a rank-$n$ tensor $\phi$. By introducing corresponding ghosts, $\zeta$ and $c$, and antifields $\{\psi^*, \phi^*, \zeta^*, c^*\}$, the free action is given as

$$S^0[\Phi, \Phi^*] = \int d^D x \left[ \frac{1}{2} g^{\mu_1 \cdots \mu_n} G_{\mu_1 \cdots \mu_n} + \phi^* g^{\mu_1 \cdots \mu_n} \partial_{\mu_1} e_{\mu_2 \cdots \mu_n} \right]$$

$$+ \frac{1}{2} \left( \mathcal{R}_{\mu_1 \cdots \mu_{n_1}} (\psi) g^{\mu_1 \cdots \mu_{n_1}} - \tilde{\psi} \mathcal{R}_{\mu_1 \cdots \mu_{n_1}} (\psi) \right)$$

$$+ \tilde{\psi} g^{\mu_1 \cdots \mu_{n_1}} \partial_{\mu_1} e_{\mu_2 \cdots \mu_{n_1}} - \phi^* g^{\mu_1 \cdots \mu_{n_1}} \partial_{\mu_1} e_{\mu_2 \cdots \mu_{n_1}} \right]. \quad (4.1)$$

Applying the BRST deformation scheme, we can derive a deformed action in the similar manner explained in the previous section. We present the result below. We find that the solution of the master equation (2.31) at the order of $g^n \ (n \geq 1)$ is

$$S^n = \alpha_2^n + \alpha_1^n + \alpha_0^n, \quad (4.2)$$

$$\alpha_2^n = \int d^D x i^n \text{tr} \ \tilde{\psi} \tilde{c} \Psi^{n-2} [\mathcal{R} (\tilde{\partial} c \psi^*)], \quad (4.3)$$

$$\alpha_1^n = \int d^D x \text{tr} \left[ i^n \tilde{\psi} \tilde{c} \Psi^{n-1} [\mathcal{R} (\psi)] - (-i)^n \Psi^{n-1} [\mathcal{R} (\psi)] \tilde{\partial} c \psi^* \right], \quad (4.4)$$

$$\alpha_0^n = - \int d^D x i^n \text{tr} \tilde{\psi} \Psi^n [\mathcal{R} (\psi)]. \quad (4.5)$$
We used the matrix notation in which \( \psi \) and \( \psi^* \) are \( d(p) \times d(r) \) matrices, \( \bar{\psi} \) and \( \bar{\psi}^* \) are \( d(r) \times d(p) \) matrices and \( \phi \) and \( \partial c \) are \( d(p) \times d(p) \) matrices, where \( p + r = n_1 \) and \( 2p = n \).

We find that the action turns to the form expanded in \( agh \) as

\[
S = S_0 + S_1 + S_2, \quad (4.6)
\]

\[
S_0 = \int d^D x \operatorname{tr} \left[ \frac{1}{2} \phi^T G(\phi) - \bar{\psi} \sum_{k=0}^{\infty} i^k \Psi^k [\mathcal{R}(\psi)] \right], \quad (4.7)
\]

\[
S_1 = \int d^D x \operatorname{tr} \left[ \phi^T \partial c + \bar{\psi}^* \partial \xi - \partial \bar{\psi}^* \right. \\
\left. + \bar{\psi}^* \partial c \sum_{k=0}^{\infty} (ig)^{k+1} \Psi^k [\mathcal{R}(\psi)] - \sum_{k=0}^{\infty} (ig)^{k+1} \Psi^k [\mathcal{R}(\psi)] \partial c^T \psi^* \right], \quad (4.8)
\]

\[
S_2 = \int d^D x \operatorname{tr} \bar{\psi}^* \partial c \sum_{k=0}^{\infty} (ig)^{k+2} \Psi^k [\bar{\mathcal{R}}(\partial c^T \psi^*)]. \quad (4.9)
\]

This action \( S \) contains all orders in \( g \), and invariant under the BRST-transformation \( \delta_B S = (S, S) = 0 \).

We have obtained a BRST-invariant deformed action of two gauge fields by using the BRST-antifield formalism. Here we comment on the gauge invariance of the action \( S_0 \) in (4.7). We find that the gauge transformation with a rank-\( (n_1 - 1) \) tensor-spinor parameter \( \epsilon \) remains unchanged

\[
\delta \phi = 0, \quad \delta \psi = \partial \epsilon, \quad (4.10)
\]

while the gauge transformation with a rank-\( (n - 1) \) tensor parameter \( \xi \) changes into

\[
\delta \phi = \partial \xi, \quad \delta \psi = \partial \xi \sum_{k=0}^{\infty} (ig)^{k+1} \Psi^k [\mathcal{R}(\psi)]. \quad (4.11)
\]

The gauge transformation is deformed by the vertex, but the gauge algebra remains abelian on-shell.

## 5 Higher-spin gauge models on AdS spaces

The Higher-spin gauge models obtained in sections 3 and 4 are generalized to those on AdS spaces. For this purpose, we formulate the BRST deformation scheme on AdS spaces.

First of all we introduce the covariant derivative on a tensor-spinor and the Fronsdal tensor on AdS spaces. The covariant derivative\(^{13}\) of a field \( \varphi \) is introduced as

\[
D_{\mu} \varphi = \partial_{\mu} \varphi + \frac{1}{2} \omega^{ab}_{\mu} S_{ab} \varphi,
\]

where \( \omega^{ab} \) is the spin-connection. The total covariant derivative \( \mathcal{D}_{\mu} \) acts on an arbitrary rank-\( n \) tensor-spinor \( \psi_{\mu_1 \ldots \mu_n} \) as

\[
\mathcal{D}_{\mu} \psi_{\mu_1 \ldots \mu_n} = D_{\mu} \psi_{\mu_1 \ldots \mu_n} - \Gamma^{\lambda}_{\mu \nu \mu_1 \ldots \mu_n} \lambda = \nabla_{\mu} \psi_{\mu_1 \ldots \mu_n} + \frac{1}{4} \omega_{\mu}^{ab} \gamma_{ab} \psi_{\mu_1 \ldots \mu_n}, \quad (5.1)
\]

\(^{13}\)Recall that Lorentz transformation law of a field \( \varphi \) is \( \delta \varphi = \frac{1}{2} e^{ab} S_{ab} \varphi \), where \( e^{ab} \) is an antisymmetric parameter and \( a, b \) denote local Lorentz indices. The \( S_{ab} \) is the Lorentz generator satisfying \( S_{ab} \varphi = 0 \) for a spin-0 \( \varphi \), \( S_{ab} \psi = \frac{1}{2} \gamma_{ab} \psi \) for a spin-\( \frac{1}{2} \) \( \psi \) and \( (S_{ab})_{cd} A^d = (\eta_{ab} \eta_{cd} - \eta_{ad} \eta_{bc}) A^d \) for a spin-1 \( A^a \).
The commutation relation of $\mathcal{D}_\mu$ on a tensor-spinor $\psi_{\mu_1...\mu_n}$ turns to

$$[\mathcal{D}_\mu, \mathcal{D}_\nu] \psi_{\mu_1...\mu_n} = R_{\mu\nu(\mu_1\rho\psi_{\mu_2...\mu_n})} + \frac{1}{4} R^{ab}_{\mu\nu} \gamma_{ab} \psi_{\mu_1...\mu_n}$$

$$= - \frac{1}{2l^2} \gamma_{\mu\nu} \psi_{\mu_1...\mu_n} - \frac{1}{l^2} \left( g_{\mu(\mu_1} \psi_{\mu_2...\mu_n)\nu} - g_{\nu(\mu_1} \psi_{\mu_2...\mu_n)\mu} \right)$$

(5.2)

where $l$ denotes the radius of the AdS space. We note that $\mathcal{D}_\mu \gamma^\nu \psi = \gamma^\nu \mathcal{D}_\mu \psi$ \[^{14}\] , which is useful in deriving this equation. In the second equality, we have used the fact that $R_{\mu\nu\rho\sigma} = -\frac{1}{l^2} (g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho})$ for AdS spaces.

Fronsdal tensor for a rank-$n$ tensor-spinor $\psi_{\mu_1...\mu_n}$ on AdS spaces is given as [42]

$$\mathcal{I}_{\mu_1...\mu_n} = i \left( \mathcal{D}_\mu \psi_{\mu_1...\mu_n} - \mathcal{D}_{(\mu_1} \psi_{\mu_2...\mu_n)} + \frac{m}{2l} \gamma_{\mu_1...\mu_n} + \frac{1}{2l} \gamma(\mu_1 \psi_{\mu_2...\mu_n}) \right) ,$$

(5.3)

where $m = D + 2n - 4$. It reduces to $S$ in (2.15) in the limit $l \to \infty$. This is invariant under the gauge transformation with a rank-$(n - 1)$ tensor-spinor gauge parameter $\epsilon$ [42]

$$\delta \psi_{\mu_1...\mu_n} = \tilde{\mathcal{D}}(\mu_1 \epsilon_{\mu_2...\mu_n})$$

(5.4)

if $\epsilon$ satisfies the $\gamma$-traceless condition $\dot{\epsilon} = 0$, which is assumed below. We have introduced $\tilde{\mathcal{D}}$ by

$$\tilde{\mathcal{D}}(\mu_1 \epsilon_{\mu_2...\mu_n}) \equiv \mathcal{D}(\mu_1 \epsilon_{\mu_2...\mu_n}) + \frac{1}{2l} \gamma(\mu_1 \epsilon_{\mu_2...\mu_n}) .$$

(5.5)

The action which leads to the Fronsdal equation $S = 0$ is given by

$$S_{\Psi} = \int d^D x \sqrt{-g} \frac{1}{2} \left[ \tilde{\mathcal{R}}_{\mu_1...\mu_n} \psi^{\mu_1...\mu_n} - \bar{\psi}^\mu_{\mu_1...\mu_n} \mathcal{R}_{\mu_1...\mu_n} \right] ,$$

(5.6)

where we introduced $\mathcal{R}_{\mu_1...\mu_n}$ as

$$\mathcal{R}_{\mu_1...\mu_n}(\psi) \equiv \mathcal{I}_{\mu_1...\mu_n} - \frac{1}{2} \gamma(\mu_1 \mathcal{I}_{\mu_2...\mu_n}) - \frac{1}{2} g(\mu_1 \mu_2 \mathcal{I}_{\mu_3...\mu_n}) .$$

(5.7)

An important relation we frequently use below is

$$\int d^D x \sqrt{-g} \tilde{\mathcal{R}}(\psi_1) \psi_2 = - \int d^D x \sqrt{-g} \psi_1 \tilde{\mathcal{R}}(\psi_2)$$

(5.8)

where $\psi_i$ are arbitrary rank-$n$ tensor-spinors, and $\tilde{\mathcal{R}}(A)$ is defined by

$$\tilde{\mathcal{R}}(A) \equiv \mathcal{R}(A) - \frac{i}{2} g A'$$

(5.9)

\[^{14}\]This can be shown as

$$\mathcal{D}_\mu \gamma^\nu \psi_{\mu_1...\mu_n} = \epsilon^\nu_a \mathcal{D}_\mu (\gamma^a \psi_{\mu_1...\mu_n})$$

$$= \epsilon^\nu_a (\nabla_\mu \gamma^a \psi_{\mu_1...\mu_n} - \omega^a_{\mu\nu} \gamma^b \psi_{\mu_1...\mu_n} + \frac{1}{2} \omega^a_{\mu\nu} \gamma^b \psi_{\mu_1...\mu_n})$$

$$= \epsilon^\nu_a \gamma^a \nabla_\mu \psi_{\mu_1...\mu_n} + \frac{1}{4} \omega^a_{\mu\nu} \gamma^b \psi_{\mu_1...\mu_n}$$

$$= \gamma^\nu \mathcal{D}_\mu \psi_{\mu_1...\mu_n}$$

where $\gamma^\nu = \epsilon^\nu_a \gamma^a$ and $\mathcal{D}_\mu \epsilon^\nu_a = 0$ are used in the first equality.
where $A$ is arbitrary rank-$n$ tensor-spinor. Using this relation, we derive $\mathcal{R} = 0$ from the variation of $S_\psi$. In the manner similar to one explained in section 2.2, $\mathcal{R} = 0$ leads to $\mathcal{I} = 0$. The action $S_\psi$ is gauge invariant because $\mathcal{R}$ is gauge invariant $\mathcal{R}(\delta \psi) = 0$.

Corresponding to the gauge parameter $\epsilon_{\mu_2 \cdots \mu_n}$, we introduce a Grassmann-even tensor-spinor ghost $\zeta_{\mu_2 \cdots \mu_n}$ which is $\gamma$-traceless $\zeta_{\mu_4 \cdots \mu_n} = 0$. The gauge invariance of $S_\psi$ is encoded to the BRST invariance under the BRST transformation

$$\delta_B \psi_{\mu_1 \cdots \mu_n} = \mathcal{D}_{(\mu_1 \zeta_{\mu_2 \cdots \mu_n})}, \quad \delta_B \zeta_{\mu_2 \cdots \mu_n} = 0.$$  \hspace{1cm} (5.10)

The action $S_\psi$ in (5.6) can be extended to $S^0[\Psi, \Psi^\ast]$ such that the BRST transformation of a functional $X(\Psi^A, \Psi^A_\ast)$ is expressed as $\delta_B X = (X, S^0)$. In the present case, $S^0$ may be given as

$$S^0[\Psi, \Psi^\ast] = S_\psi + \int d^D x \sqrt{-g} \left( \bar{\psi}^{\ast \mu_1 \cdots \mu_n} \mathcal{D}_{(\mu_1 \zeta_{\mu_2 \cdots \mu_n})} - \bar{\mathcal{D}}_{(\mu_1 \zeta_{\mu_2 \cdots \mu_n})} \psi^{\ast \mu_1 \cdots \mu_n} \right),$$  \hspace{1cm} (5.11)

which leads to (5.10) and

$$\delta_B \psi^{\ast \mu_1 \cdots \mu_n} = \mathcal{R}_{\mu_1 \cdots \mu_n},$$  \hspace{1cm} (5.12)
$$\delta_B \zeta_{\mu_2 \cdots \mu_n} = -n \mathcal{D} \cdot \psi^{\ast \mu_2 \cdots \mu_n} + \frac{n}{D + 2n - 4} (\gamma \mathcal{D} \cdot \psi^\ast + g \hat{\mathcal{D}} \cdot \psi^\ast),$$  \hspace{1cm} (5.13)

when $D + 2n - 4 \neq 0$\textsuperscript{15}. We have added the second term on the right-hand side of (5.13) for the nilpotency $\delta^2_B \zeta^\ast = 0$. We find that the key relation for the nilpotency

$$-\mathcal{D} \cdot \mathcal{R} + \frac{1}{D + 2n - 4} (\gamma \hat{\mathcal{D}} \cdot \mathcal{R} + g \hat{\mathcal{D}} \cdot \mathcal{R}') = 0$$  \hspace{1cm} (5.14)

follows from the triple $\gamma$-traceless condition $\psi' = 0$. Since $\delta_B \zeta^\ast = (\zeta^\ast, S^0)$, the second term on the right-hand side of (5.13) requires an additional term

$$\frac{n}{D + 2n - 4} \int d^D x \sqrt{-g} \left( \bar{\zeta} (\gamma \hat{\mathcal{D}} \cdot \psi^\ast + g \hat{\mathcal{D}} \cdot \psi^\ast) - (\gamma \hat{\mathcal{D}} \cdot \psi^\ast + g \hat{\mathcal{D}} \cdot \psi^\ast) \zeta \right),$$  \hspace{1cm} (5.15)

in the action $S^0$. However this term disappears due to $\zeta = 0$\textsuperscript{16}, and leaves the action (5.11) unchanged.

We have introduced free fermionic higher-spin gauge theory on AdS spaces in the BRST-antifield formalism. In appendix A.1, the free bosonic higher-spin gauge theory on AdS spaces is introduced. The action of $\Delta$ and $\Gamma$ on fields and antifields is summarized in Table 4. $\mathcal{I}$ and $\mathcal{R}$ are defined in (A.27) and (5.7), respectively.

Vertices can be constructed by using the BRST deformation scheme, as was done in sections 2.3, 3 and 4. We will not repeat the procedure here but present the results below. The deformed

\textsuperscript{15}When $D + 2n - 4 = 0$, namely $(D, n) = (4, 0)$, the gauge transformation (5.4) becomes trivial.

\textsuperscript{16}$\zeta' = 0$ follows from $\xi = 0$. 

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\[
\begin{array}{ccc}
Z & \Delta(Z) & \Gamma(Z) \\
\phi_{\mu_1 \ldots \mu_n} & 0 & \nabla_{(\mu_1 \phi_{\mu_2 \ldots \mu_n})} \\
c_{\mu_2 \ldots \mu_n} & 0 & 0 \\
\phi_{\mu_1 \ldots \mu_n}^{*} & -\nabla \cdot \phi_{\mu_2 \ldots \mu_n}^* & 0 \\
c_{\mu_2 \ldots \mu_n}^* & -n \nabla \cdot \phi_{\mu_2 \ldots \mu_n}^* + \frac{\mathcal{D} - 2n - 6}{D + 2n - 6} c_{\mu_2 \mu_3} \nabla \cdot \phi_{\mu_4 \ldots \mu_n}^* & 0 \\
\psi_{\mu_1 \ldots \mu_n} & 0 & \mathcal{D}_{(\mu_1 \psi_{\mu_2 \ldots \mu_n})} \\
\zeta_{\mu_2 \ldots \mu_n} & 0 & 0 \\
\psi_{\mu_1 \ldots \mu_n}^* & \mathcal{R}_{\mu_2 \ldots \mu_n} & 0 \\
\zeta_{\mu_2 \ldots \mu_n}^* & -n \mathcal{D} \cdot \psi_{\mu_2 \ldots \mu_n}^* + \frac{\mathcal{D} - 2n - 4}{D + 2n - 4} \gamma_{(\mu_2 \mathcal{D} \cdot \psi_{\mu_3 \ldots \mu_n}) + g_{\mu_2 \mu_3} \mathcal{D} \cdot \psi_{\mu_4 \ldots \mu_n}}^* & 0 \\
\end{array}
\]

Table 4: Action of $\Delta$ and $\Gamma$

The action of tensor-spinors $\psi_I$ ($I = 1, 2$) and a tensor $\phi$ is found to be\(^\text{17}\)

\[
S = S_0 + S_1 + S_2, \quad (5.16)
\]

\[
S_0 = \int d^D x \sqrt{-g} \text{tr} \left[ \frac{1}{2} \phi^T \mathcal{G} \phi - \bar{\psi}_I \sum_{k=0}^{\infty} [(i g s)^k]^J \psi^k \mathcal{R}(\psi_J) \right], \quad (5.17)
\]

\[
S_1 = \int d^D x \sqrt{-g} \text{tr} \left[ \phi^* T \nabla c + \bar{\psi}_I^* \mathcal{D} \zeta_I - \mathcal{D} \bar{\zeta}_I \psi_I^* + \bar{\psi}_I^* \nabla c \sum_{k=0}^{\infty} [(i g s)^{k+1}]^J \psi^k \mathcal{R}(\psi_J) - \sum_{k=0}^{\infty} [(i g s)^{k+1}]^J \psi^k \mathcal{R}(\psi_J) \nabla c^T \psi_I^* \right], \quad (5.18)
\]

\[
S_2 = \int d^D x \sqrt{-g} \text{tr} \bar{\psi}_I^* \nabla c \sum_{k=0}^{\infty} [(i g s)^{k+2}]^J \psi^k \mathcal{R}(\nabla c^T \psi_J^*), \quad (5.19)
\]

where $\Psi$ is defined as $\Psi^0[A] = A$, $\Psi^1[A] = \mathcal{R}(\phi A)$, $\Psi^2[A] = \mathcal{R}(\phi \mathcal{R}(\phi A))$, and so on. To derive this result, we frequently used the relation

\[
\int d^D x \sqrt{-g} \mathcal{A} \psi^k \mathcal{R}(B) = (-1)^{k+1} \int d^D x \sqrt{-g} \psi^k \mathcal{R}(A) B. \quad (5.20)
\]

The obtained action reduces to (3.20)-(3.23) in the flat limit as expected.

\(^\text{17}\)The determinant of the metric, $g$, may not be confused with the coupling $g$. 

Furthermore we obtain the deformed action of a tensor-spinor \( \psi \) and a tensor \( \phi \)
\[
S = S_0 + S_1 + S_2, \quad (5.21)
\]
\[
S_0 = \int d^D x \sqrt{-g} \text{tr} \left[ \frac{1}{2} \phi^T \mathcal{G}(\phi) - \bar{\psi} \sum_{k=0}^{\infty} (ig)^k \bar{\psi}^k [\mathcal{R}(\psi)] \right], \quad (5.22)
\]
\[
S_1 = \int d^D x \sqrt{-g} \text{tr} \left[ \phi^T \nabla_c + \bar{\psi}^* \mathcal{D} \zeta - \mathcal{D} \zeta^* \bar{\psi}^* \right.
\]
\[
+ \bar{\psi}^* \nabla_c \sum_{k=0}^{\infty} (ig)^{k+1} \bar{\psi}^k [\mathcal{R}(\psi)] - \sum_{k=0}^{\infty} (ig)^{k+1} \bar{\psi}^k [\mathcal{R}(\psi)] \nabla_c^T \psi^* \left. \right], \quad (5.23)
\]
\[
S_2 = \int d^D x \sqrt{-g} \text{tr} \bar{\psi}^* \nabla_c \sum_{k=0}^{\infty} (ig)^{k+2} \bar{\psi}^k [\mathcal{R}(\nabla_c^T \psi^*)]. \quad (5.24)
\]

This action reduces to (4.6)-(4.9) in the flat limit as expected. We note that these actions contain terms of all orders in \( g \), and are invariant exactly under the BRST-transformation \( \delta_B S = (S, S) = 0 \). We have obtained BRST-invariant deformed actions of gauge fields on AdS spaces by applying the BRST deformation scheme.

We find that the action \( S_0 \) in (5.17) is invariant under the gauge transformations with rank-\((n_1 - 1)\) tensor-spinor parameters \( \epsilon_I \)
\[
\delta \phi = 0, \quad \delta \psi_I = \mathcal{D} \epsilon_I, \quad (5.25)
\]
and under the gauge transformation with a rank-\((n - 1)\) tensor parameter \( \xi \)
\[
\delta \phi = \nabla \xi, \quad \delta \psi_I = \mathcal{D} \xi \sum_{k=0}^{\infty} (ig)^{k+1} \psi^k [\mathcal{R}(\psi)]. \quad (5.26)
\]

It is also shown that the action (5.22) is invariant under the gauge transformation with a rank-\((n_1 - 1)\) tensor-spinor parameter \( \epsilon \)
\[
\delta \phi = 0, \quad \delta \psi = \mathcal{D} \epsilon, \quad (5.27)
\]
and under the gauge transformation with a rank-\((n - 1)\) tensor parameter \( \xi \)
\[
\delta \phi = \nabla \xi, \quad \delta \psi = \mathcal{D} \xi \sum_{k=0}^{\infty} (ig)^{k+1} \psi^k [\mathcal{R}(\psi)]. \quad (5.28)
\]

We note that the gauge transformations (5.26) and (5.28) are deformed by vertices, but the gauge algebra remains abelian on-shell.

6 Summary and Discussion

Introducing totally-symmetric rank-\( n \) tensor-spinors, which are Dirac spinors in \( D \)-dimensional spacetime, as well as totally-symmetric rank-\( n \) tensors, we constructed higher-spin gauge models
including fermions by applying the BRST deformation scheme. The deformed action $S$ contains terms of all orders in the deformation parameter $g$ and satisfy the master equation exactly. Introducing objects on AdS spaces and writing down the BRST transformation law on AdS spaces in the BRST-antifield formalism, we applied the BRST deformation scheme to derive higher-spin gauge models on AdS-spaces.

Extending the case [37] that each vertex forms an open chain of fields, we constructed the models in which each vertex forms closed chain of fields. The models presented here contain the previous models as special cases. These results are summarized in appendices.

The actions obtained in this paper contain infinite series, say $\sum_{k=1}^{\infty}[(igs)^k]^I_J\Psi^k[\mathcal{R}(\psi_J)]$ in (3.19). This term is expressed in a closed form $(1/(1-igs\Psi))^{I}J\mathcal{R}(\psi_J)$ as long as $|igs\Psi| < 1$. It is not clear what this condition means, as $\Psi$ contains a tensor and differentials. However, let us assume this condition satisfied anyway. In this case, the equation of motion (3.25) is expressed as $(1/(1-igs\Psi))^{I}J\mathcal{R}(\psi_J) = 0$. By applying $(1-igs\Psi)^{I}J$, this reduces formally to $\mathcal{R}(\psi_J) = 0$, which is the free equation of motion.

For bosonic gauge models, infinite series, say $\sum_{k=0}^{\infty}[(gs)^k]^{I}JG(k(\phi_J))$ in (A.13), is expressed as a closed form, $(1/(1-gs\Phi))^{I}JG(\phi_J)$, as long as $|gs\Phi| < 1$. In this case, the equation of motion (A.19) is expressed as $(1/(1-gs\Phi))^{I}JG(\phi_J) = 0$, and reduces to $G(\phi_J) = 0$ formally. As a result, the higher-spin gauge models may reduce to free theory when infinite series contained in these models is convergent.

Furthermore, the higher-spin gauge models up to the cubic vertex reduce to free theory. When we construct the deformed action, we choose $S^1$ to satisfy the master equations at the first order. To avoid a BRST-exact $S^1$, we set $\alpha_1^2 = 0$. This may not be enough. In fact, $S^1$ is shown to be BRST-exact:

$$S^1 = (R, S^0),$$

(6.1)

where, for example,

$$R = -\frac{1}{2} \int d^D x \text{ tr } \left[ \phi^*T \delta c \phi^* + \phi^*T \phi G(\phi_1) \right],$$

(6.2)

in the case with two bosons on flat space. Thus, as $S^1$ can be absorbed into $S^0$ by the field redefinition, the total action may be regarded as a free action. We hope to report this point in another place.

Our models contain the deformation parameter $g$. As the dimension of a tensor $\phi$ is $\frac{D-2}{2}$ and that of a tensor-spinor $\psi$ is $\frac{D-1}{2}$, the mass dimension of $g$ is found to be $-\frac{D+2}{2}$ for bosonic models given in appendices while $-\frac{D}{2}$ for models including fermions. Since the dimensions of $g\Phi$ and $g\Psi$ are zero, vertices have the same dimension as expected.

Our models are BRST-invariant up to surface terms. We have dropped surface terms assuming that there is no boundary. Boundary terms in AdS spaces are examined in [49]. It is interesting to consider a boundary action such that the total action including the boundary action is BRST-invariant.
There are many interesting issues, such as including massive tensors and massive tensor-spinors, including tensors and tensor-spinors with mixed-symmetry, and constructing the unconstrained local version [50–53] of our models. These are left for future investigation.

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Appendix

A Higher-spin gauge model of three bosons

We present a higher-spin gauge model of three bosonic tensors. This is a slight generalization of the model given in [37]. We give a brief derivation here to make this paper self-contained.

We introduce three gauge fields, a pair of rank-\(n_I\) tensors \(\phi_I\) \((I = 1, 2)\) and a rank-\(n\) tensor \(\phi\). As explained in section 2.1, we introduce ghosts, \(c_I\) and \(c\), corresponding to the gauge parameters, and antifields \(\{\phi_I^*, c_I^*, c^*\}\) as well. The free action is

\[
S^0 = \int d^Dx \left[ \sum_{I=1,2} \left( \frac{1}{2} \phi_I^{\mu_1 \cdots \mu_{n_I}} G_{\mu_1 \cdots \mu_{n_I}} (\phi_I) + \phi_I^{*\mu_1 \cdots \mu_{n_I}} \partial_{(\mu_1} c_{I \mu_2 \cdots \mu_{n_I})} \right) + \frac{1}{2} \phi^{\mu_1 \cdots \mu_n} G_{\mu_1 \cdots \mu_n} (\phi) + \phi^{*\mu_1 \cdots \mu_n} \partial_{(\mu_1} c_{\mu_2 \cdots \mu_n)} \right]
\]  

(A.1)

where \(G\) is defined in (2.6). We derive vertices by applying the BRST deformation scheme. We expand \(S^1\) with respect to \(agh\) as \(S^1 = \alpha_2^1 + \alpha_1^1 + \alpha_0^1\). As explained in section 3, we may set \(\alpha_2^1 = 0\). The master equation at the order of \(g^1\), \((S^0, S^1) = 0\), is decomposed with respect to \(agh\) into (3.2) and (3.3). We shall choose \(a_1\) as

\[
\alpha_1^1 = \int d^Dx s^{IJ} \text{tr} G(\phi_I)^T \tilde{\partial} c \phi_J^*.
\]  

(A.2)

For notational simplicity, we used a matrix notation. The integrand means

\[
G^{\rho_1 \cdots \rho_p \mu_1 \cdots \mu_p} (\phi_I) \partial_{(\mu_1} c_{\mu_2 \cdots \mu_p)} \phi_2^{* \nu_1 \cdots \nu_q} \phi_2^{\rho_1 \cdots \rho_r} + G^{\rho_1 \cdots \rho_p \nu_1 \cdots \nu_q} (\phi_2) \partial_{(\nu_1} c_{\nu_2 \cdots \nu_p)} \phi_1^{* \mu_1 \cdots \mu_q} \phi_1^{\rho_1 \cdots \rho_r}.
\]

(A.3)

Here \(p + r = n_1, q + r = n_2\) and \(p + q = n\). The \(\phi_I\) and \(\phi_I^*\) denote \(d(p) \times d(r)\) matrices, while the \(\phi_2\) and \(\phi_2^*\) denote \(d(q) \times d(r)\) matrices. When \(r = 0\), the solution presented here reduces to the one
given in [37]. It is obvious that $\Gamma_1^1 = 0$ because of the gauge invariance of $G$. Acting $-\Delta$ on $\alpha_1^1$, we derive

$$-\Delta \alpha_1^1 = \int d^Dx s^{IJ} \text{tr} G(\phi_I)^T \partial c G(\phi_J) = \Gamma \int d^Dx s^{IJ} \frac{1}{2} \text{tr} G(\phi_I)^T \phi G(\phi_J),$$

(A.4)

which implies that

$$\alpha_0^1 = \int d^Dx s^{IJ} \frac{1}{2} \text{tr} G(\phi_I)^T \phi G(\phi_J).$$

(A.5)

We shall show that the solution at the order of $g^n$ is

$$S^n = \alpha_2^n + \alpha_1^n + \alpha_0^n$$

(A.6)

with

$$\alpha_2^n = \int d^Dx (s^n)^{IJ} \frac{1}{2} \text{tr} (\tilde{\partial} c \phi_J)^T \Phi_n^{-2}[\tilde{G}(\tilde{\partial} c \phi_J)],$$

(A.7)

$$\alpha_1^n = \int d^Dx (s^n)^{IJ} \text{tr} (\tilde{\partial} c \phi_J)^T \Phi_n^{-1}[\tilde{G}(\phi_J)],$$

(A.8)

$$\alpha_0^n = \int d^Dx (s^n)^{IJ} \frac{1}{2} \text{tr} \phi_J^T \Phi_n[\tilde{G}(\phi_J)],$$

(A.9)

where $\Phi$ is introduced as $\Phi^0[A] = A$, $\Phi[A] = \tilde{G}(\phi A)$, $\Phi^2[A] = \tilde{G}(\phi \tilde{G}(\phi A))$ and so on. Now, suppose that $\{S^1, S^2, \cdots, S^{n-1}\}$ solves the master equation at the order of $g^k$ ($k = 1, \cdots, n - 1$). We will show that the $\alpha_2^n$, $\alpha_1^n$ and $\alpha_0^n$ coincide with (A.7), (A.8) and (A.9).

First we will derive $\alpha_2^n$ from (2.34). Because the equation

$$-\frac{1}{2} \sum_{k=2}^{n-1} (\alpha_2^k, \alpha_1^{n-k}) - \frac{1}{2} \sum_{k=1}^{n-2} (\alpha_1^k, \alpha_2^{n-k}) = \Gamma \int d^Dx (s^n)^{IJ} \frac{1}{2} \text{tr} (\tilde{\partial} c \phi_J)^T \Phi_n^{-2}[\tilde{G}(\tilde{\partial} c \phi_J)]$$

(A.10)

follows, $\alpha_2^n$ may be given as (A.7). Next we will derive $\alpha_1^n$ from (2.35). We find that

$$- \Delta \alpha_2^n - \frac{1}{2} \sum_{k=2}^{n-1} (\alpha_2^k, \alpha_0^{n-k}) - \frac{1}{2} \sum_{k=1}^{n-2} (\alpha_1^k, \alpha_1^{n-k}) - \frac{1}{2} \sum_{k=1}^{n-2} (\alpha_0^k, \alpha_2^{n-k})$$

$$= \Gamma \int d^Dx (s^n)^{IJ} \text{tr} \phi_J^T \Phi_n^{-1}[\tilde{G}(\tilde{\partial} c \phi_J)],$$

(A.11)

where a useful relation $\int d^Dx A^T \Phi_n[\tilde{G}(B)] = \int d^Dx \Phi^m[\tilde{G}(A)^T B$ is used. This implies that $\alpha_1^n$ is given as (A.8). Finally, we derive $\alpha_0^n$ from (2.36). It is straightforward to obtain

$$- \Delta \alpha_0^n - \frac{1}{2} \sum_{k=1}^{n-1} (\alpha_2^k, \alpha_0^{n-k}) - \frac{1}{2} \sum_{k=1}^{n-1} (\alpha_k^0, \alpha_1^{n-k}) = \Gamma \int d^Dx (s^n)^{IJ} \frac{1}{2} \text{tr} \phi_J^T \Phi_n[\tilde{G}(\phi_J)].$$

(A.12)

This implies that $\alpha_0^n$ is given as (A.9). Summarizing the above results, we have shown that $S^n$ is definitely given as (A.6) with (A.7)-(A.9).
We have derived $S^k$ ($k = 1, 2, \cdots$). The free action is given in (A.1). Gathering these together we obtain the total action

$$S = S^0 + \sum_{k=1}^{\infty} g^k S^k$$

$$= \int d^D x \text{tr} \left[ \frac{1}{2} \phi^T G(\phi_I) + \phi^* T \partial c_I + \frac{1}{2} \phi^T G(\phi) + \phi^* T \partial c + \frac{1}{2} \phi^T \sum_{k=1}^{\infty} g^k(s^k)^I J \Phi^k[G(\phi_J)] 
+ \phi^T \sum_{k=1}^{\infty} g^k(s^k)^{I J} \Phi^{k-1}[G(\partial_c \phi_I^*)] + \frac{1}{2} \phi^T \partial c [\sum_{k=2}^{\infty} g^k(s^k)^{I J} \Phi^{k-2}[G(\partial_c \phi_I^*)]] \right].$$

(A.13)

We find that the action turns into the form expanded in $agh$ as

$$S = S_0 + S_1 + S_2,$$

(A.14)

$$S_0 = \int d^D x \text{tr} \left[ \frac{1}{2} \phi^T \sum_{k=0}^{\infty} [(gs)^k]^I J \Phi^k[G(\phi_J)] + \frac{1}{2} \phi^T G(\phi) \right],$$

(A.15)

$$S_1 = \int d^D x \text{tr} \left[ \phi^* T \partial c_I + \phi^* T \partial c + \phi^T \sum_{k=0}^{\infty} [(gs)^{k+1}]^I J \Phi^k[G(\partial_c \phi_I^*)] \right],$$

(A.16)

$$S_2 = \int d^D x \text{tr} \left[ \frac{1}{2} \partial_c \phi_I^* T \sum_{k=0}^{\infty} [(gs)^{k+2}]^I J \Phi^k[G(\partial_c \phi_I^*)] \right],$$

(A.17)

where $agh(S_i) = i$.

We have obtained BRST-invariant deformed action of gauge fields by using the BRST-antifield formalism. Here we examine the gauge-invariant action $S_0$ in (A.15). First of all we derive equations of motion. Varying $S_0$ with respect to $\phi$ and $\phi_I$, we obtain

$$G(\phi) - \frac{1}{2} g s^I J \sum_{k=0}^{\infty} [(gs)^k]^I K \Phi^k[G(\phi_K)] \left( \sum_{l=0}^{\infty} [(gs)^l]^J L \Phi^l[G(\phi_L)] \right)^T = 0,$$

(A.18)

$$\sum_{k=0}^{\infty} [(gs)^k]^I J \Phi^k[G(\phi_J)] = 0.$$  

(A.19)

Substituting (A.19) into the first equation leads to $G(\phi) = 0$. Here we examine the gauge invariance of the action $S_0$ in (A.15). The gauge transformation can be read off from the BRST transformation. We find that the gauge transformation with a rank-$(n_I - 1)$ tensor parameter $\xi_I$ remains unchanged

$$\delta \phi = 0, \quad \delta \phi_I = \partial \xi_I,$$

(A.20)

while the gauge transformation with a rank-$(n - 1)$ tensor parameter $\xi$ turns to

$$\delta \phi = \partial \xi, \quad \delta \phi_I = -\partial_c \partial_c \phi^* T \sum_{k=0}^{\infty} [(gs)^k]^I J \Phi^k[G(\phi_J)].$$

(A.21)

We note that the gauge algebra is abelian on-shell.
A.1 Higher-spin bosonic gauge fields on AdS spaces

We derive a higher-spin gauge model on AdS spaces by applying the BRST deformation scheme built on AdS spaces. The Fronsdal tensor for a rank-\(n\) tensor gauge field on AdS spaces is

\[
\mathcal{F}_{\mu_1 \cdots \mu_n}(\phi) = \Box \phi_{\mu_1 \cdots \mu_n} - \nabla_{(\mu_1} \nabla^{\sigma} \phi_{\mu_2 \cdots \mu_n)\sigma} + \frac{1}{2} \nabla_{(\mu_1} \nabla_{\mu_2} \phi_{\mu_3 \cdots \mu_n)} - \frac{m^2}{l^2} \phi_{\mu_1 \cdots \mu_n} - \frac{2}{l^2} g_{(\mu_1 \mu_2} \phi'_{\mu_3 \cdots \mu_n)},
\]

(A.22)

where \(\Box = g_{\mu \nu} \nabla_{\mu} \nabla_{\nu}\) and \(m^2 = n^2 + n(D - 6) - 2(D - 3)\). \(\mathcal{F}\) reduces to \(F\) in (2.1) in the flat limit. The covariant derivative satisfies

\[
\delta \phi_{\mu_1 \cdots \mu_n} = \nabla_{(\mu_1} \xi_{\mu_2 \cdots \mu_n)},
\]

(A.24)

the Fronsdal tensor changes to

\[
\delta \mathcal{F} = \frac{1}{2} \nabla_{(\mu_1} \nabla_{\mu_2} \nabla_{\mu_3} \xi'_{\mu_4 \cdots \mu_n)} - \frac{4}{l^2} g_{(\mu_1 \mu_2} \nabla_{\mu_3} \xi'_{\mu_4 \cdots \mu_n)},
\]

(A.25)

which implies \(\delta \mathcal{F} = 0\) due to \(\xi' = 0\). By introducing a rank-(\(n - 1\)) tensor ghost \(c_{\mu_2 \cdots \mu_n}\) and antifields \(\{\phi^*, c^*\}\), the gauge symmetry may be lifted to the BRST symmetry\(^{18}\)

\[
\delta_B \phi = \nabla c, \quad \delta_B c = 0, \quad \delta_B \phi^* = -\mathcal{G}(\phi), \quad \delta_B c^* = -n \nabla \cdot c + \frac{n}{D + 2n - 6} g \nabla \cdot c^*,
\]

(A.26)

where \(\mathcal{G}\) is defined as

\[
\mathcal{G}_{\mu_1 \cdots \mu_n}(A) = \mathcal{F}_{\mu_1 \cdots \mu_n}(A) - \frac{1}{2} g_{(\mu_1 \mu_2} \mathcal{F}'_{\mu_3 \cdots \mu_n)(A)}
\]

(A.27)

where \(A\) is a symmetric rank-\(n\) tensor. The last term on the right-hand side of the last equation in (A.26) is required for the nilpotency \(\delta_B c^* = 0\). A key relation for the nilpotency

\[
\nabla \cdot \mathcal{G} - \frac{1}{D + 2n - 6} g \nabla \cdot \mathcal{G}' = 0
\]

(A.28)

follows from the double-traceless condition \(\phi'' = 0\). The free action \(S^0\) which reproduces the BRST transformation (A.26) by \(\delta_B = (X, S^0)\), is

\[
S^0 = \int d^D x \sqrt{-g} \left[ \sum_{l=1,2} \left( \frac{1}{2} \phi^{\mu_1 \cdots \mu_{l \cdot n_1}} \mathcal{G}_{\mu_1 \cdots \mu_{n_1}}(\phi_I) + \phi^*_{\mu_1 \cdots \mu_{l \cdot n_1}} \nabla_{(\mu_1} c_{\mu_{l+2} \cdots \mu_{n_1})} \right) + \frac{1}{2} \phi^{\mu_1 \cdots \mu_n} \mathcal{G}_{\mu_1 \cdots \mu_n}(\phi) + \phi^*_{\mu_1 \cdots \mu_n} \nabla_{(\mu_1} c_{\mu_{2} \cdots \mu_n)} \right].
\]

(A.29)

\(^{18}\)We comment on the case \(D + 2n - 6 = 0\), namely \((D, n) = (6, 0)\) and \((4, 1)\). When \((D, n) = (6, 0)\), gauge transformation (A.24) becomes trivial. When \((D, n) = (4, 1)\), \(\mathcal{G} = \mathcal{F}\) as \(\mathcal{F}' = 0\). Since \(\nabla \cdot \mathcal{G} = \nabla \cdot (\Box \phi - \nabla \nabla \cdot \phi - \frac{m^2}{l^2} \phi) = 0\), the second term on the right-hand side of the last equation in (A.26) is not needed.
By applying the BRST deformation scheme, we can obtain vertices in the manner similar to one explained above. The action of the BRST differentials $\Delta$ and $\Gamma$ is summarized in Table 4. It is straightforward to see that
\[
\int d^D x \sqrt{-g} \, \text{tr} \varphi_1^T \mathcal{G}(\varphi_2) = \int d^D x \sqrt{-g} \, \text{tr} \, \mathcal{G}(\varphi_1)^T \varphi_2
\]  
(A.30)
where $\varphi_i$ ($i = 1, 2$) are totally-symmetric tensors. We have introduced
\[
\mathcal{G}_{\mu_1...\mu_n}(A) = \mathcal{G}_{\mu_1...\mu_n}(A) + \frac{1}{4} g_{(\mu_1\mu_2} \nabla_{\mu_3} \nabla_{\mu_4} A_{\mu_5...\mu_n)}. 
\]  
(A.31)
Using this relation (A.30), we obtain the action for three bosonic higher-spin gauge fields on AdS spaces
\[
S = S_0 + S_1 + S_2,
\]  
(A.32)
\[
S_0 = \int d^D x \sqrt{-g} \, \text{tr} \left[ \frac{1}{2} \phi_1^T \sum_{k=0}^{\infty} [(gs)^k]^{IJ} \phi^k \mathcal{G}(\phi_J) + \frac{1}{2} \phi^T \mathcal{G}(\phi) \right],
\]  
(A.33)
\[
S_1 = \int d^D x \sqrt{-g} \, \text{tr} \left[ \phi_1^T \nabla c_1 + \phi^T \nabla c_1 + \phi_1^T \sum_{k=0}^{\infty} [(gs)^k]^{IJ} \phi^k \mathcal{G}(\nabla c_1^*) \right],
\]  
(A.34)
\[
S_2 = \int d^D x \sqrt{-g} \, \text{tr} \left[ \nabla c_1^* \phi_1^T \sum_{k=0}^{\infty} [(gs)^k]^{IJ} \phi^k \mathcal{G}(\nabla c_1^*) \right].
\]  
(A.35)
$\Phi$ is introduced as $\Phi[A] = A$, $\Phi[A] = \mathcal{G}(\phi A)$, $\Phi^2[A] = \mathcal{G}(\phi \mathcal{G}(\phi A))$ and so on.

The action $S_0$ in (A.33) is found to be invariant under the gauge transformation with a rank-$(n_I - 1)$ tensor parameter $\xi_I$
\[
\delta \phi = 0, \quad \delta \phi_I = \nabla \xi_I,
\]  
(A.36)
and the gauge transformation with a rank-$(n - 1)$ tensor parameter $\xi$
\[
\delta \phi = \nabla \xi, \quad \delta \phi_I = -\nabla \xi T g \sum_{k=0}^{\infty} [(gs)^k]^{IJ} \phi^k \mathcal{G}(\nabla c_1^*) \right].
\]  
(A.37)

**B Higher-spin gauge model of two bosons**

We present a deformed higher-spin gauge model of two bosonic tensors, which is a generalization of the model given in [37]. We introduce two gauge fields, rank-$n_1$ tensor $\phi_1$ and a rank-$n$ tensor $\phi$. We also introduce ghosts $c_1$ and $c$, and antifields $\{\phi_1^*, \phi^*, c_1^*, c^*\}$. The free action is
\[
S^0 = \int d^D x \left[ \frac{1}{2} \phi_1^T \mu_1...\mu_{n_1} G_{\mu_1...\mu_{n_1}}(\phi_1) + \phi_1^T \mu_1...\mu_{n_1} \partial(\mu_1 c_1 \mu_2...\mu_{n_1}) \\
+ \frac{1}{2} \phi^T \mu_1...\mu_{n} G_{\mu_1...\mu_{n}}(\phi) + \phi^T \mu_1...\mu_{n} \partial(\mu_1 c_2...\mu_{n}) \right].
\]  
(B.1)
In the manner similar to one explained in appendix A, we can show that the higher order term is given as
\[ S_n = \alpha_n^2 + \alpha_n^1 + \alpha_n^0 \]
with
\[ \alpha_n^2 = \int d^D x \frac{1}{2} \text{tr} (\tilde{\partial} c \phi^*_n)^T \Phi^{n-2} [\tilde{G}(\tilde{\partial} c \phi^*_n)], \]
\[ \alpha_n^1 = \int d^D x \text{tr} (\tilde{\partial} c \phi^*_n)^T \Phi^{n-1} [\tilde{G}(\phi_1)], \]
\[ \alpha_n^0 = \int d^D x \frac{1}{2} \text{tr} \phi^*_1 \Phi^n [\tilde{G}(\phi_1)]. \]

We find that the action turns to the form expanded in \(agh\) as
\[ S = S_0 + S_1 + S_2, \]
\[ S_0 = \int d^D x \left[ \frac{1}{2} \phi^*_1 \sum_{k=0}^{\infty} g^k \Phi^k [G(\phi_1)] + \frac{1}{2} \phi^T G(\phi) \right], \]
\[ S_1 = \int d^D x \left[ \phi^*_1 \tilde{\partial} c + \phi^*_1 \tilde{\partial} c + \phi^T \sum_{k=0}^{\infty} g^{k+1} \Phi^k [\tilde{G}(\tilde{\partial} c \phi^*_1)] \right], \]
\[ S_2 = \int d^D x \frac{1}{2} (\tilde{\partial} c \phi^*_1)^T \sum_{k=0}^{\infty} g^{k+2} \Phi^k [\tilde{G}(\tilde{\partial} c \phi^*_1)]. \]

The action \(S_0\) is invariant under the gauge transformation with a rank-(\(n_1 - 1\)) tensor parameter \(\xi_1\)
\[ \delta \phi = 0, \quad \delta \phi_1 = \partial \xi_1, \]
and the gauge transformation with a rank-(\(n - 1\)) tensor parameter \(\xi\)
\[ \delta \phi = \partial \xi, \quad \delta \phi_1 = -\tilde{\partial} ??? \sum_{k=0}^{\infty} g^{k+1} \Phi^k [\tilde{G}(\tilde{\partial} c \phi^*_1)]. \]

It is straightforward to derive the corresponding action on AdS spaces
\[ S = S_0 + S_1 + S_2, \]
\[ S_0 = \int d^D x \sqrt{-g} \left[ \frac{1}{2} \phi^*_1 \sum_{k=0}^{\infty} g^k \Phi^k [G(\phi_1)] + \frac{1}{2} \phi^T G(\phi) \right], \]
\[ S_1 = \int d^D x \sqrt{-g} \left[ \phi^*_1 \tilde{\partial} c + \phi^*_1 \tilde{\partial} c + \phi^T \sum_{k=0}^{\infty} g^{k+1} \Phi^k [\tilde{G}(\tilde{\partial} c \phi^*_1)] \right], \]
\[ S_2 = \int d^D x \sqrt{-g} \frac{1}{2} (\tilde{\partial} c \phi^*_1)^T \sum_{k=0}^{\infty} g^{k+2} \Phi^k [\tilde{G}(\tilde{\partial} c \phi^*_1)]. \]

The action \(S_0\) in (B.12) is invariant under the gauge transformation with a rank-(\(n_1 - 1\)) tensor parameter \(\xi_1\)
\[ \delta \phi = 0, \quad \delta \phi_1 = \nabla \xi_1, \]
and the gauge transformation with a rank-$(n-1)$ tensor parameter $\xi$ turns to

$$\delta \phi = \nabla \xi, \quad \delta \phi_1 = -\tilde{\nabla} \xi^T \sum_{k=0}^{\infty} g^{k+1} \Phi^k [G(\phi_1)].$$

(B.16)

The gauge algebra remains abelian on-shell.

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