A New Approach to Determine the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ Reaction Rate at Astrophysical Energies

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Abstract. The $\sim 100\%$ uncertainty in the measured rate of the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reaction at low energies ($E_{\alpha} \leq 1 \text{ MeV}$) is the largest source of uncertainty in any stellar evolution model. With development of new, high-current, energy-recovery linear accelerators (ERLs) and high density gas targets, measurement of the $^{16}\text{O}(e,e'\alpha)^{12}\text{C}$ reaction close to threshold using detailed balance opens up a new approach to determine the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reaction rate with increased precision ($< 20\%$). We have considered the optimal design of an experiment where systematic uncertainties are minimized. Once the new ERLs commence operation near their design specifications, an experiment to validate the new approach we propose should be carried out. Our method has broad applicability to radiative capture reactions in astrophysics.

1. Introduction
In stellar nucleosynthesis, at the completion of the hydrogen burning stage, the core of a massive star contracts and heats-up. When the temperature and the density of the core reaches sufficiently high values, the helium starts to burn via the triple-$\alpha \rightarrow ^{12}\text{C}$ process. Subsequently, the $\alpha$ radiative capture reaction, $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$, also becomes possible. The helium burning stage is fully dominated by these two reactions and their rates determine the relative abundance of $^{12}\text{C}$ and $^{16}\text{O}$, after the helium is depleted. At helium burning temperatures, the rate of the triple-$\alpha$ process is known with an uncertainty of about $\pm 10\%$, but the uncertainty of the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reaction rate is much larger. In fact, it is the largest source of uncertainty in any stellar evolution model. Therefore, for many decades it has been the paramount goal of experimental nuclear astrophysics to determine the rate of the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reaction at astrophysical energies with better precision [1].

This task has proven to be very difficult, not withstanding heroic experimental efforts for more than half a century. For the generic radiative capture reaction $A + B \rightarrow C + D + \gamma$, the Coulomb repulsion is characterized by the Gamow factor (or Coulomb barrier penetration factor) between A and B: $P_g = \exp \left( -\sqrt{E_g/E} \right)$, where $E_g \equiv 2m_re^2/\alpha(Z_AZ_B)^2$ is the Gamow energy, $\alpha$ is the fine structure constant and $m_r = m_Am_B/(m_A + m_B)$ is the reduced mass. The cross section, $\sigma$, is then expressed [2] as a product of $P_g$ and the astrophysical S-factor

$$\sigma = \frac{1}{E} \exp \left( -2\pi Z_AZ_B\alpha/v \right) S(E). \quad (1)$$

Furthermore, $\sigma$ must be extrapolated to the Gamow energy.
At the helium burning temperature $\sim 2 \times 10^8$ K and corresponding Gamow energy $E_g \sim 300$ keV, the cross section for the $^{12}$C + $\alpha \rightarrow \gamma + ^{16}$O reaction is $\approx 10^{-5}$ pb, which makes the direct measurement at stellar energies impossible. Unfortunately, the extrapolation is not simple, since the structure of the cross section is complex. For example, it involves interferences of the high-energy tails of subthreshold states in $^{16}$O (see [3]) as well as contributions from higher-energy states.

Through the years, different experimental approaches have been used to determine the rate of the $^{12}$C($\alpha, \gamma$)$^{16}$O reaction. These include measurements of the direct reaction [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17], $\beta-$delayed $\alpha$-decay of $^{16}$N [18, 19, 20], and elastic scattering $^{12}$C($\alpha, \alpha$)$^{12}$C [21, 22]. However, due to the rapid decrease of the cross section in the region where the energy of the final-state $\alpha$-particle in the CM frame ($E_{\alpha}^{CM}$) falls below 2 MeV, the uncertainty in the S-factor experimental determination is increasingly dominated by the large statistical uncertainty. Further, as $E_{\alpha}^{CM}$ decreases, the statistical uncertainties from the different experiments increase dramatically. In recent years, new experimental approaches have been pursued. One novel approach is based on a bubble chamber [23, 24] and another on the optical time projection chamber [25] where the angular distribution of $\alpha$-particles is measured and $S_{E1}$ - and $S_{E2}$ - factors can be determined. A comprehensive review of the experiments and methods developed thus far, and the full list of astrophysical implications of the $^{12}$C($\alpha, \gamma$)$^{16}$O rate can be found in [26].

2. A New Approach

Here, we present a new approach to the determination of the $^{12}$C($\alpha, \gamma$)$^{16}$O reaction at stellar energies, which is described in detail in [27]. We consider the inverse reaction initiated by an electron beam, where a virtual photon $\gamma$ is exchanged, by contrast with the direct reaction which uses a real photon beam, where $q = E_{\gamma}$. The theoretical formalism to relate electro- and photo-disintegration has been developed [28]. The idea has been previously proposed for a storage ring [29], but was not carried out, and has been more recently discussed by [30, 31]. Most importantly, a new generation of high intensity ($\approx 10$ mA) low-energy ($\approx 100$ MeV) energy-recovery linear (ERL) electron accelerators is under development [32, 33] which, when used with state-of-the-art gas targets [34], can deliver luminosities of $\approx 10^{36}$ cm$^{-2}$ s$^{-1}$. In this way, the weakness of the electromagnetic force can be overcome. By knowing the electron scattering kinematics it is possible to focus on a specific value of the excitation energy of the final-state $\alpha + ^{12}$C system, for instance quite close to threshold, but to vary the three-momentum transfer $|q|$ for any value that keeps the exchanged virtual photon spacelike. Of course, the real-photon result is recovered by taking the limit $|q| \rightarrow \omega$.

The differential, semi-inclusive electrodisintegration cross section for the reaction $^{16}$O($e,e^{'\alpha}$)$^{12}$C in the laboratory frame takes the form [27]

$$\frac{d\sigma}{d\omega d\Omega_{e} d\Omega_{\alpha}}(e,e^{'\alpha}) = \frac{M_{\alpha} M_{12C} p_{\alpha} f_{rec}^{-1} \sigma_{Mott}}{8\pi^{3} M_{16O} (\hbar c)^{3}}$$

$$\times \left[ v_{L} R_{L} + v_{T} R_{T} + v_{TL} R_{TL} + v_{TT} R_{TT} \right],$$

where $\sigma_{Mott}$ is the pointlike cross section. For unpolarized exclusive electron scattering we have four nuclear response functions $R_{K}$: the longitudinal $R_{L}$ and transverse $R_{T}$ nuclear electromagnetic current components (L and T with respect to the direction of the virtual photon $q$), and two interference responses, namely transverse-longitudinal $R_{TL}$ and transverse-transverse $R_{TT}$. The functions $v_{K}$ are electron kinematic factors [28].

A sketch of the general kinematic landscape is given in Fig. 1, which illustrates the $R_{T}$ response as a function of $q$ and $\omega$ together with the real-$\gamma$ line; here $\omega_{T}$ is the threshold value of $\omega$. 

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[Note: The document contains references to figures and equations that are not visible in the text but are likely to be present in the full document. The text is structured to maintain clarity and coherence.]
Figure 1. Transverse response function \( R_T \) as function of the photon energy \( \omega \) and the three-momentum transfer \( q \), for the real-photon case \( q = \omega \) (solid line) and virtual photon case \( q > \omega \) (surface plot), where the \( \omega_T \)-point denotes the value of the threshold photon energy for the reaction.

for the reaction. The strategy in photodisintegration studies is to perform experiments at values of \( \omega = E_\gamma \) where the cross section is large enough to be measured and then extrapolate along the real-\( \gamma \) line to the very low energies of interest for astrophysics. The electrodisintegration reaction extends these ideas: now one can focus on small values of \( \omega \) but have \( q \) large enough to yield measurable cross sections. The extended strategy is then to extrapolate in both dimensions, namely, for the responses as functions of \( q \) to approach the real-\( \gamma \) line and as functions of \( \omega \) to reach the interesting low-energy region. The advantage of having \( q \) large enough is that one may work near threshold but have sufficient three-momentum imparted to the final-state \( \alpha \)-particles that they can emerge from the target and be detected.

The angular distribution of the \( \alpha \)-particles in the final state can be measured for both photo- and electro-disintegration reactions. This yields information on the various multipoles that contribute to the process. We assume that \( \omega \) is always quite small compared with a typical energy scale; in addition, for the electrodisintegration reaction we assume that \( q \) is smaller than a typical scale for nuclear momenta, \( q_0 \), taken to be roughly of order 200–250 MeV/c. Given this, it is possible to limit the multipoles to a relatively small number. This is commonly done for the photodisintegration reaction near threshold where only E1 (electric dipole) and E2 (electric quadrupole) multipoles are assumed [4], although the electric octupole E3 multipoles may be non-negligible. Since the nuclear ground states involved are all 0\(^+\) states only electric multipoles can occur, and magnetic multipoles are absent. Here we have assumed that only the ground states of \(^4\)He and \(^{12}\)C are involved and that any excited states can be ignored by using the over-determined kinematics of the reaction. In the electrodisintegration reaction, both Coulomb and electric multipoles contribute: here we consider C0, C1/E1 and C2/E2 multipoles.
At low values of the momentum transfer, \( q \ll q_0 \), each multipole is dominated by its low-\( q \) behavior which enters as a specific power of \( q \), e.g., the CJ multipole matrix elements go as \( (q/q_0)^j \) at low \( q \) [27]. Thus, another advantage of electron scattering, where \( q \) may be varied while keeping \( \omega \) fixed, is that the balance of the multipole contributions can be varied. An example of this could, for instance, be the potential C3/E3 contributions: by increasing \( q \) (still, of course, staying in the region where \( q \ll q_0 \)) one may increase the relative importance of the octupole effects over the monopole, dipole, and quadrupole effects to explore whether or not the former need to be taken into account.

Not only is there a richer set of multipoles involved in the electron scattering case, but there are more response functions to be exploited. For real photons one has the transverse response \( R_T \) at \( q = \omega \) and potentially the transverse interference response \( R_{TT} \) also at \( q = \omega \) if linearly polarized real photons are involved (see Sect. III of [27] for more discussion). In the T and TT responses only EJ multipoles enter, not simply squared but through interferences. The L response contains only CJ multipoles, again with interferences, while the TL response has interferences between CJ and EJ multipoles. All of this means that potentially one has more information with which to disentangle the various contributions. The angular distributions as functions of the \( \alpha \)-particle angles \( \theta_\alpha \) and \( \phi_\alpha \) can be determined in detail [27].

Having developed general expressions for the cross sections and for the leading contributions to the angular distributions as functions of \( \theta_\alpha \) (the angle between the \( \alpha \) momentum and \( q \)) [27], we have developed a model for the electroproduction reaction. We do this in two steps: first, we use the present knowledge of the real-\( \gamma \) cross sections to constrain the leading-order behavior (i.e., as functions of \( q \)) of the E1 and E2 multipoles, by fitting the second order polynomial to all existing \( S_{E1} \) and \( S_{E2} \) data having \( E_{cm} \leq 1.7 \text{ MeV} \). In the low-\( q \) limit, current conservation then yields the leading-order behavior of the C1 and C2 multipoles. Second, we invoke “naturalness” (see [27]) to model the next-to-leading order (NLO) dependences on \( q \) in the C1/E1 and C2/E2 multipoles, which are not simply related by current conservation, as well as make an assumption concerning the behavior of the C0 multipole. Our goal is to develop a “reasonable” model and, using this model, to explore the feasibility of making electrodisintegration measurements in the interesting low-\( \omega \)/low-\( q \) region. We emphasize that the model is used only to determine the feasibility of such experiments; in undertaking them the actual higher-order \( q \)-dependences will be measured and the region where the parametrizations are operative will be determined.

### 3. Projected Results

We have carried out a Monte-Carlo simulation using the model described and assuming the experimental parameters of Table 1. For our purposes, we assume an oxygen cluster-jet target [34] capable of achieving an areal thickness of \( 5 \times 10^{18} \text{ atoms/cm}^2 \), which for a 2 mm wide jet corresponds to a density of \( 6.65 \times 10^{-4} \text{ g/cm}^3 \). We also require an electron accelerator which can deliver a beam energy of about 100 MeV and a beam current of order 40 mA. Two suitable electron accelerators are currently being constructed. MESA, which should deliver a beam current of 10 mA [32] and CBETA which should be able to reach 40 mA [35] for beam energies of 42, 78, 114 and 150 MeV. In what follows, we assume a beam current of 40 mA and a jet target, as described above, which is equivalent to a luminosity of \( 1.25 \times 10^{36} \text{ cm}^{-2} \text{s}^{-1} \).

To identify events belonging to the \( ^{16}\text{O} (e,e'\alpha)^{12}\text{C} \) reaction we need to detect the scattered electron in coincidence with the produced \( \alpha \)-particle. Fig. 2 shows a schematic layout of a possible experiment. A high precision magnetic spectrometer is suitable for detection of the scattered electron. For the purpose of defining electrons accepted by the electron spectrometer, we will assume that the spectrometer has an in-plane acceptance of \( \pm 2.08^\circ \) and out-of-plane acceptance of \( \pm 4.16^\circ \), which corresponds to a solid angle of 10.5 mrad. For the accepted \( \alpha \)-particles, we will assume that the low-energy ion detectors cover enough solid angle to accept all \( \alpha \)-particles having the in-plane scattering angle \( \theta_{cm}^\alpha \) in range from \( 0^\circ \) to \( 60^\circ \) and to have the
Table 1. Summary of the experimental parameters for the rate calculations used in [27].

| Oxygen Target Thickness  | 5 × 10^{18} \text{ atoms/cm}^2 |
|-------------------------|---------------------------------|
| Density                 | 6.65 × 10^{-4} \text{ g/cm}^3  |
| Electron Beam Current   | 40 mA                           |
| Energies (E_e)          | 78, 114, 150 MeV                |
| Electron Accept.        | In-plane ±2.08°                  |
|                         | Out-of-plane ±4.16°              |
|                         | Solid angle 10.5 msr             |
| α-particle Accept.      | In-plane 60°                     |
|                         | Out-of-plane 360°                |
|                         | Solid angle 3.14 sr              |
| Luminosity              | 1.25 × 10^{36} \text{ cm}^{-2} \text{s}^{-1} |
| Integrated Luminosity (100 days) | 1.08 × 10^7 \text{ pb}^{-1} |
| Central electron scatt. angles (θ_e) | 15°, 25°, 35° |
| \(E_{\alpha}^{cm}\)-range of interest | 0.7 \leq E_{\alpha}^{cm} \leq 1.7 \text{ MeV} |

Figure 2. Schematic layout of our proposed \(^{16}\text{O}(e,e'\alpha)^{12}\text{C}\) experiment: \(^{16}\text{O}\), inside a gas cluster-jet target, is disintegrated by the electron beam into the α and \(^{12}\text{C}\) nuclei. The scattered electron is detected in an electron spectrometer and the produced α-particle in a large acceptance array of ion detectors.
full acceptance for the out-of-plane angle $\phi_\alpha$ from $0^\circ$ to $360^\circ$.

Figure 3. Reconstructed astrophysical $S_{E1}$- and $S_{E2}$-factors with statistical error bars (represented by solid circles) from our calculation for $E_e = 114$ MeV and $\theta_e = 15^\circ$ from [27], integrated luminosity of $1.08 \times 10^7$ pb$^{-1}$, and experimental data from [4, 5, 6, 7, 8, 10, 11, 9, 12, 14, 15, 17]. The solid line represents the AZURE2 [36] R-Matrix fit of the world data set.

Figure 3 shows the projected statistical uncertainties in the $S_{E1}$- and $S_{E2}$-factors, after 100 days of running, for $E_e = 114$ MeV and $\theta_e = 15^\circ$ from [27], as well as data from past experiments. Compared with the most accurate measurements from [12] and [15], the uncertainties in the determination of $S_{E1}$ and $S_{E2}$ at a given energy above threshold are improved by at least $\times 5.6$ and $\times 23.9$, respectively. The significantly smaller projected statistical uncertainty for $S_{E2}$ comes from both the enhancement of a factor of $q/\omega$ in the response functions as well as the choice of $\theta_\alpha^{cm}$ angular range [27]. Further, we have considered [27] systematic uncertainties such as both
isotopic and chemical contamination of the oxygen gas; energy, angle and timing constraints of the final-state particles; energy loss in the gas jet and radiative corrections. A combination of measurements of the scattered electron momentum and angle, and the recoil $\alpha$-particle’s energy and timing with sufficient resolution is adequate to reduce these uncertainties below the projected statistical value.

4. Conclusions

In summary, we present a new approach that has the potential to determine the $^{12}$C($\alpha$, $\gamma$)$^{16}$O reaction with unprecedented precision. We have considered the optimal experimental kinematics in terms of the incident electron energy, the oxygen gas target, the scattered electron spectrometer, and the final-state, low-energy $\alpha$-particle detection. Operating low-energy $\alpha$-detectors in a Megawatt electron beam environment is challenging but ERL beams of this type have been shown to be bright with minimal halo [37]. We propose an initial measurement of $^{16}$O(e, $e^\prime$ $\alpha$)$^{12}$C using an ERL with incident energy of order 100 MeV. This experiment would take data at higher $E_{cm}^\alpha$ where the reaction rates are relatively high and the running time is of order a month and would aim to validate the extrapolation to photo-production and determine the contributions of the different multipoles. If successful, it would set the stage for a longer experiment (of order 6 months) with the highest electron intensity available to determine the $^{12}$C($\alpha$, $\gamma$)$^{16}$O reaction rate with unprecedented precision in the astrophysical region.

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