Exotic Lepton Flavour Violating Processes in the Presence of Nuclei

D.K. Papoulias\(^1\) and T.S. Kosmas\(^2\)
Division of Theoretical Physics, University of Ioannina, GR 45110 Ioannina, Greece
E-mail: \(^1\)dimpap@cc.uoi.gr \(^2\)hkosmas@uoi.gr

Abstract.
The discovery of neutrino oscillations indicates the existence of massive neutrinos in contrast to the massless neutrinos predicted by the Standard Model. One of the simplest extensions of the SM obtained by adding a heavy right-handed neutrino singlet, \(N_R\), per neutrino generation is the Seesaw mechanism. Within the context of this mechanism, flavour changing neutral current neutrino-nucleus reactions of the type \(A_N Z (\nu_\alpha, \nu_\beta) A_N Z^*\) and \(A_N Z (\bar{\nu}_\alpha, \bar{\nu}_\beta) A_N Z^*\), with \(\alpha \neq \beta\), are predicted to occur. In this contribution, motivated by the extensive studies (theoretical and experimental) of the LFV in \(\mu^- \rightarrow e^-\) conversion in nuclei, we investigate FCNC in neutrino-nucleus reactions. From a nuclear theory point of view, the Donnelly-Walecka model for cross sections calculations is employed. To this purpose, the single-particle transition matrix elements are evaluated from a Mathematica code developed in this work. Neutrino-nucleus reactions have important impact in Astrophysics and hence a detailed study of such exotic processes is of significant importance.

1. Introduction
It is well known that neutrinos interact very rarely with matter through the weak interaction of the Standard Model (SM) [1]. Such reactions proceed either by charged-current processes mediated by \(W^\pm\) boson exchange or through neutral-current processes mediated by a neutral \(Z\) boson [1, 2] (see Fig. 1). At nuclear-level, these processes are represented by the following reactions

\[
\nu_\alpha (\bar{\nu}_\alpha) + (A, Z) \rightarrow \alpha^\mp + (A, Z \pm 1),
\]

\[
\nu_\alpha (\bar{\nu}_\alpha) + (A, Z) \rightarrow \nu_\alpha (\bar{\nu}_\alpha) + (A, Z)^*,
\]

where \(\alpha = e, \mu, \tau\), is the flavour index and the asterisk (*) indicates an excited final nuclear state. So far, the \(\nu\)-nucleus reactions (1) and (2) have been extensively studied both theoretically and experimentally, however further research is still needed [3, 4]. Many questions related to the absolute neutrino mass, the total number of neutrino flavours, the Dirac or Majorana nature of neutrinos and others remain still open [4].

Even though neutrinos interact very weakly with matter, they provide a very useful tool for Astrophysical searches since they can escape even from the interior of distant stars [5, 6]. So far, many efforts have been done for a better understanding of the Sun’s nature, the Supernova explosion and other Astrophysical phenomena through \(\nu\)-nucleus interaction probes [1].

Recently, non-standard \(\nu\)-nucleus reactions, that may play significant role to Supernova physics, have been considered [4, 5, 6]. Among these, the flavour changing neutral current

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Figure 1. A set of Feynman diagrams for SM charged-current (left), SM neutral-current (middle) and FCNC (right), processes in the presence of nuclei. The non-standard physics enters in the complicated vertex •.

(FCNC) reactions

\[ \nu_\alpha + (A, Z) \rightarrow \nu_\beta + (A, Z)^*, \]  

with different flavour indices, i.e. \( \alpha \neq \beta \), possess prominent position. Such processes however, have not been studied in detail from a nuclear theory point of view. In addition, the lepton flavour violating (LFV) muon to electron conversion in nuclei is presently intensively investigated [7, 8, 9, 10, 11, 12]. Our aim is to deeper explore the aforementioned exotic reactions, predicted to occur within the context of particle models beyond the SM like the Seesaw mechanism [10], by performing realistic nuclear structure calculations.

2. FCNC neutrino-nucleus and other LFV processes

FCNC \( \nu \)-nucleus processes constitute an attractive scenario to probe new physics both from a theoretical and an experimental nuclear physics point of view. It also provides a new challenge to increase the sensitivity of the relevant branching ratios. Our purpose is to systematically study and understand the unknown new physics entering the complex vertex of Fig. 1.

The quark-level interaction Hamiltonian will be constructed within the context of various models beyond the SM. In the present work, inspired from the methods used in Refs. [7, 10] we adopt the Seesaw mechanism as reference model. The parameters that give rise to LFV entering the quark-level interaction Lagrangian can be constrained by the upper limits on the branching ratio derived from \( \mu^- \rightarrow e^- \) conversion experiments, like the mu2e and Project X at Fermilab, USA [11] and the COMET at J-PARC, Japan [12] at a sensitivity up to \( R_{\mu^- e^-} = 10^{-18} \).

In previous comprehensive \( \mu^- \rightarrow e^- \) conversion studies, it has been claimed that the reformulation of the quark-level effective Lagrangian yields the effective nucleon-level Lagrangian written in terms of the nucleon isospin operators as [7, 8, 9, 10]

\[
L_{\text{eff}}^N = G_\alpha \left[ \sum_{A,B} j^A \left( \alpha_{A,B} (0) J^A_{\mu} + \alpha_{A,B} (3) J^A_{\mu} \right) + \sum_{C,D} j^C \left( \alpha_{C,D} (0) J^C_{\mu} + \alpha_{C,D} (3) J^C_{\mu} \right) \right] + j_{\mu\nu} \left[ \alpha_T (0) J^T_{\mu\nu} + \alpha_T (3) J^T_{\mu\nu} \right], \quad \alpha = \text{ph, nph}. 
\]  

Here, the index \( \alpha \) refers to the photonic (\( \alpha = \text{ph} \)) and non-photonic (\( \alpha = \text{nph} \)) contribution [10]. The components of the isoscalar, \( J_{(0)} \), and isovector, \( J_{(3)} \), nucleon currents are defined as

\[
J^V_{\mu (k)} = \bar{N} \gamma^\mu \tau_k N, \quad J^{A\mu}_{(k)} = \bar{N} \gamma^\mu \gamma_5 \tau_k N, \quad J^S_{(k)} = \bar{N} \tau_k N, \quad J^P_{(k)} = \bar{N} \gamma_5 \tau_k N, \quad J^{\mu\nu}_{(k)} = \bar{N} \sigma^{\mu\nu} \tau_k N,
\]
with $k = 0, 3$. Each component is separately treated, however not all of them should be taken into account. In fact, the pseudoscalar and tensor nucleon components are negligible. Therefore, at nuclear-level the effective interaction Hamiltonian $H_{\text{eff}}$, takes the well-known current-current form [7, 10].

The nuclear calculations for partial transition rates are based on the matrix elements (ME) of the interaction Hamiltonian between the initial and any final nuclear states, i.e.

$$
\langle f | H_{\text{eff}} | i \rangle = \frac{G_F}{\sqrt{2}} \int d^3x e^{-i\mathbf{q} \cdot \mathbf{x}} (f_{\text{nuc}})(\mathbf{x}) | i_{\text{nuc}} \rangle,
$$

(5)

($G_F$ is the weak interaction coupling constant). The magnitude of the three momentum transfer $|\mathbf{q}|$, is obtained from the kinematics of the reaction [1, 13] and the ME of the leptonic current has been written as

$$
\langle l_f | j_{\mu} | l_i \rangle = l_\mu e^{-i\mathbf{q} \cdot \mathbf{x}}.
$$

(6)

Hence, Eq. (5) implies that one needs to compute the ME of the hadronic current density.

FCNC in the $\nu$-nucleus reactions (3) may be predicted by the same particle physics theories beyond the SM that predict the exotic $\mu^- \rightarrow e^-$ conversion in nuclei. The same holds also for their description within the framework of a nuclear structure method (Shell-model, RPA, QRPA, Fermi-Gas models, etc.) [7].

3. Brief Description of the Basic Nuclear Operators

In the present work, we employ the Donnelly-Walecka method, that describes in a unified way any semi-leptonic process in the presence of nuclei [1, 2]. It has been proved, that the multipole expansion of the hadronic current density ME, leads to eight irreducible tensor operators written in terms of the weak nucleon form factors $F_x$, $x = 1, 2, A, P$, as

$$
M_{jM}^{\text{coul}}(q) = F_1^{J}(q)M_J^{jM}(q), \quad iM_{jM}^{\text{coul}5}(q) = \frac{q}{M} \left[ F_A^{\Omega_M} M_J^{jM}(q) + \frac{1}{2} (F_A + \omega F_P) \Sigma_{M}^{nJ}(q) \right],
$$

(7)

where

$$
L_{JM}(q) = \frac{\omega}{q} M_{JM}^{\text{coul}}(q), \quad -iL_{JM}^{55}(q) = \left[ F_A - \frac{q^2}{2M} F_P \right] \Sigma_{M}^{nJ}(q),
$$

(8)

$$
T_{JM}^{55}(q) = \frac{q}{M} \left[ F_1^{\Delta_J} \Delta_{JM}(q) + \frac{1}{2} (F_1 + 2FM_2) \Sigma_{M}^{J}(q) \right], \quad -iT_{JM}^{55}(q) = F_A \Sigma_{M}^{J}(q),
$$

(9)

$$
iT_{JM}^{\text{mag}5}(q) = \frac{q}{M} \left[ F_1^{\Delta_J} \Delta_{JM}(q) - \frac{1}{2} (F_1 + 2FM_2) \Sigma_{M}^{J}(q) \right], \quad T_{JM}^{\text{mag}5}(q) = F_A \Sigma_{M}^{J}(q).
$$

(10)

On the right-hand side of Eqs. (7)-(10), we notice that seven (due to CVC theory) linearly independent new operators, namely $M_{JM}^{\text{coul}}, \Sigma_{M}^{J}, \Sigma_{M}^{nJ}, \Delta_{JM}, \Delta_{JM}^{55}$ and $\Omega_{M}^{J}$, that do not carry form-factor dependence, may be defined (see Ref. [2]). For the reduced ME of the latter operators (they enter the description of the cross section for any semi-leptonic process in the presence of nuclei), a compact formalism has been developed, which reads [2]

$$
\langle j_1 || T_1^{\mu} || j_2 \rangle = e^{-\beta y_2/2} \sum_{\mu=0}^{n_{\text{max}}} \mathbb{P}_{\mu} j_1^{\mu} j_2^{\mu}, \quad i = 1, \ldots, 7.
$$

(11)

$T_i^{\mu}$, $i = 1, \ldots, 7$, represent any of the above operators, $n_{\text{max}} = (N_1 + N_2 - \beta)/2$ and $N_i = 2n_i + \ell_i$ indicates the harmonic oscillator quanta of the $i$-th level. The well-defined quantum numbers in our notation are $|n_1\ell_1 j_1\rangle \equiv |j_1\rangle$. For an analytical description of the aforementioned formalism the reader is referred to [2].
4. Results and Discussion

As a first step of our calculational procedure, we have constructed an efficient modern Mathematica code to compute the reduced ME for all basic operators \( T_i^J \) and for any configuration \((j_1, j_2)J\), using the analytical expressions of Eq. (11). As expected, the coefficients obtained in Ref [2, 13], coincide with those of the present work. As an example, the coefficients \( P_{\mu}^{6,J} \) for the reduced ME of the operator \( T_6^J \equiv \Delta M_J^J \) are tabulated in Table 1. We mention however, that in Ref. [13], only configurations for which \( N_1 - N_2 \leq 3 \) are included, while in our code there is not any similar restriction. We conclude that all geometrical coefficients \( P_{\mu}^{J} \) are simple rational numbers for the diagonal elements (not shown here), or square roots of rational numbers for the non-diagonal elements. The double differential cross sections of processes (3) from which the nuclear calculations start, includes reduced ME of the form \(|\langle j_1|T_i^J||j_2\rangle|^2\), hence their evaluation is crucial. We currently perform realistic nuclear structure calculations for coherent cross sections that depend on the nuclear form factors for protons and neutrons. The appropriate code, provides improved results, by a refined version of the fractional occupation probabilities method of Ref. [14]. These calculations are in progress and will appear soon.

Table 1. The coefficients \( P_{\mu}^{6,J} \) for the \(|\langle j_1|\Delta M_J^J||j_2\rangle|\).

| \((n_1\ell_1)_{j_1} - (n_2\ell_2)_{j_2}\) | \(J\) | \(\mu = 0\) | \(\mu = 1\) | \(\mu = 2\) | \(\mu = 3\) | \(\mu = 4\) |
|---|---|---|---|---|---|---|
| \(0p_{1/2} - 0s_{1/2}\) | \(1\) | \(\sqrt{\frac{1}{6}}\) |  |  |  |  |
| \(0p_{3/2} - 0s_{1/2}\) | \(1\) | \(-\sqrt{\frac{1}{3}}\) | 0 |  |  |  |
| \(0d_{5/2} - 0s_{1/2}\) | \(2\) | \(-\sqrt{\frac{3}{5}}\) | 0 |  |  |  |
| \(0f_{5/2} - 0p_{1/2}\) | \(2\) | \(-\sqrt{\frac{7}{5}}\) | \(\sqrt{\frac{20}{63}}\) | 0 |  |  |
| \(0d_{7/2} - 0p_{1/2}\) | \(4\) | \(\sqrt{\frac{8}{63}}\) | 0 |  |  |  |
| \(1p_{3/2} - 1s_{1/2}\) | \(1\) | \(-\sqrt{\frac{5}{9}}\) | \(\sqrt{\frac{4}{45}}\) | \(-\sqrt{\frac{4}{45}}\) | \(0\) |  |
| \(0d_{3/2} - 0f_{7/2}\) | \(3\) | \(-\sqrt{\frac{16}{175}}\) | \(\sqrt{\frac{4}{45}}\) | \(-\sqrt{\frac{16}{175}}\) | \(0\) |  |
| \(1f_{5/2} - 2p_{3/2}\) | \(2\) | 0 | \(\sqrt{\frac{8}{7}}\) | \(-\sqrt{\frac{288}{343}}\) | \(\sqrt{\frac{2888}{27783}}\) | \(-\sqrt{\frac{32}{27783}}\) |

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