Muon-proton Colliders: Leptoquarks, Contact Interactions and Extra Dimensions

Kingman Cheung

Department of Physics, University of California at Davis, Davis, CA 95616

Abstract. We discuss the physics potential of the $\mu p$ collider; especially, leptoquarks, leptogluons, $R$-parity violating squarks, contact interactions, and large extra dimensions. We calculate the sensitivity reach for these new physics at $\mu p$ colliders of various energies and luminosities.

INTRODUCTION

The R&D [1,2] of the muon collider is well underway. The First Muon Collider (FMC) will have a 200 GeV muon beam on a 200 GeV anti-muon beam, which could possibly be at the Fermilab [2]. With the existing Tevatron proton beam the muon-proton collision becomes a possible option. It would be a 200 GeV $\otimes$ 1 TeV $\mu p$ collider. The existing lepton-proton collider is the $ep$ collider at HERA. Lepton-proton colliders have been proved to be successful by the physics results from HERA. In this work, we shall discuss the physics potential of the $\mu p$ colliders at various energies and luminosities. Other $\mu p$ colliders that we consider in this study are summarized in Table 1. The nominal yearly luminosity of the 200 GeV $\otimes$ 1 TeV $\mu p$ collider is about 13 fb$^{-1}$. Luminosities for other designs are roughly scaled by one quarter power of the muon beam energy and given in Table 1.

PHYSICS POTENTIAL

The physics opportunities of $\mu p$ colliders are similar to those of $ep$ colliders, but the sensitivity reach might be very different, which depends on how precise the particles can be identified and measured in $ep$ and $\mu p$ environments. Similar to $ep$ colliders the proton structure functions can be measured to very large $Q^2$ and small $x$ in $\mu p$ colliders of higher energies. At the 200 GeV $\otimes$ 1 TeV $\mu p$ collider the

1) Invited talk at the 5th International Conference on the Physics Potential and Development of $\mu^+\mu^-$ Colliders, San Francisco CA, December 1999. Work supported by DOE.
TABLE 1. The center-of-mass energies $\sqrt{s}$ and luminosities $\mathcal{L}$ for various designs of muon-proton colliders.

| $\sqrt{s}$ (GeV) | $\mathcal{L}$ (fb$^{-1}$) |
|------------------|--------------------------|
| 30GeV $\otimes$ 820GeV | 314 | 0.1 |
| 50GeV $\otimes$ 1TeV | 447 | 2 |
| 200GeV $\otimes$ 1TeV | 894 | 13 |
| 1TeV $\otimes$ 1TeV | 2000 | 110 |
| 2TeV $\otimes$ 3TeV | 4899 | 280 |

$Q^2$ can be measured up to $10^6$ GeV$^2$. In addition, QCD studies, search for supersymmetry and other exotic particles can also be carried out. Here we concentrate on leptoquarks, leptogluons, $R$-parity violating squarks, $\mu$-$q$ contact interactions, and the large extra dimensions. The goal here is to estimate the sensitivity reach for these new physics at various energies and luminosities.

**Leptoquarks and Leptogluons**

The second generation leptoquarks made up of a muon and a charm or strange quark are particularly interesting at the $\mu p$ collider because they can be directly produced in the $s$-channel processes, $\mu^\pm c(s) \rightarrow L\mu c$ ($L\mu s$). It is conventional to assume no inter-generational mixing in order to prevent the dangerous flavor-changing neutral currents. The production cross section of the leptoquark in $\mu p$ collisions is

$$\sigma = \frac{\pi \lambda^2}{2s} g(x, Q^2) \times (J + 1),$$

where $\lambda$ is the coupling constant and $J$ is the spin of the leptoquark.

On the other hand, a leptogluon has a spin of either $1/2$ or $3/2$, a lepton quantum number (in this case it is the muon), and a color quantum number (the same as gluon.) The interaction for a spin $1/2$ leptogluon is given by

$$\mathcal{L} = g_s \frac{M_{L\mu g}}{2\Lambda_{\mu g}^2} L_{\mu g}^a \sigma^{\mu\nu} \mu \sigma_{\mu\nu}^g \delta_{ab} + h.c.,$$

where $\Lambda_{\mu g}$ is the scale that determines the strength of the interaction. The leptogluon can also be produced in the $s$-channel and the production cross section is

$$\sigma = \frac{4\pi^2 \alpha_s}{s} \left( \frac{M_{L\mu g}^2}{\Lambda_{\mu g}^2} \right)^2 g(x, Q^2),$$

where $g(x, Q^2)$ is the gluon luminosity.

The $R$-parity violating squarks can be considered special scalar leptoquarks that are the SUSY partners of quarks. The cross section for $\mu^+ p \rightarrow \tilde{t}_L$ is given by
\[ \sigma_{\ell L} = \frac{\pi |\lambda_{231}|^2}{4s} \frac{d}{s} \left( \frac{m_{\ell L}^2}{s}, Q^2 = m_{\ell L}^2 \right), \]  

(4)

where \( d \) is the down-quark luminosity. The above formula can be easily modified to the production of other squarks with the corresponding subscripts in \( \lambda' \) and parton functions.

If kinematically allowed the leptoquarks, leptogluons, and the \( R \)-parity violating squarks are produced in the \( s \)-channel and thus give rise to a spectacular enhancement in a single bin of the invariant mass \( M \) distribution or the \( x = M^2/s \) distribution.

## Contact Interactions

The effective four-fermion contact interactions can arise from fermion compositeness or exchanges of heavy particles like heavy \( Z' \), heavy leptoquarks, or other exotic particles. The conventional Lagrangian for \( llqq \) \((l = e, \mu)\) contact interactions has the form [3]

\[
L_{NC} = \sum_q \left[ \eta_{LL} \left( \overline{\ell}_L \gamma_\mu l_L \right) \left( \overline{q}_L \gamma^\mu q_L \right) + \eta_{RR} \left( \overline{\ell}_R \gamma_\mu l_R \right) \left( \overline{q}_R \gamma^\mu q_R \right) \right.
\]

\[
+ \left. \eta_{LR} \left( \overline{\ell}_L \gamma_\mu l_R \right) \left( \overline{q}_R \gamma^\mu q_L \right) + \eta_{RL} \left( \overline{\ell}_R \gamma_\mu l_L \right) \left( \overline{q}_L \gamma^\mu q_R \right) \right],
\]

(5)

where \( \eta_{\alpha\beta} = e \frac{4\pi}{\Lambda_{\alpha\beta}^2} \). We introduce the reduced amplitudes \( M_{\alpha\beta}^{\mu\nu} \), where the subscripts label the chiralities of the initial lepton (\( \alpha \)) and quark (\( \beta \)). The SM tree-level reduced amplitudes for \( \mu q \rightarrow \mu q \) are

\[
M_{\alpha\beta}^{\mu\nu}(\hat{s}) = -\frac{e^2 Q_q}{t} + \frac{e^2}{\sin^2 \theta_w \cos^2 \theta_w t - m_Z^2} g_{\alpha}^{\mu} g_{\beta}^{\nu}, \quad \alpha, \beta = L, R.
\]

(6)

The new physics contributions to \( M_{\alpha\beta}^{\mu\nu} \) from the \( \muqq \) contact interactions are \( \Delta M_{\alpha\beta}^{\mu\nu} = \eta_{\alpha\beta}^{\mu\nu} \). The differential cross sections are given by [3]

\[
\frac{d\sigma(\mu^+)}{dx \, dy} = \frac{sx}{16\pi} \left\{ u(x, Q^2) \left[ |M_{\mu LR}^{\mu\nu}|^2 + |M_{\mu RL}^{\mu\nu}|^2 + (1 - y)^2 \left( |M_{\mu LL}^{\mu\nu}|^2 + |M_{\mu RR}^{\mu\nu}|^2 \right) \right] \right.
\]

\[
+ \left. d(x, Q^2) \left[ |M_{\mu LR}^{\mu\nu}|^2 + |M_{\mu RL}^{\mu\nu}|^2 + (1 - y)^2 \left( |M_{\mu LL}^{\mu\nu}|^2 + |M_{\mu RR}^{\mu\nu}|^2 \right) \right] \right\},
\]

(7)

\[
\frac{d\sigma(\mu^-)}{dx \, dy} = \frac{sx}{16\pi} \left\{ u(x, Q^2) \left[ |M_{\mu LL}^{\mu\nu}|^2 + |M_{\mu RR}^{\mu\nu}|^2 + (1 - y)^2 \left( |M_{\mu LR}^{\mu\nu}|^2 + |M_{\mu RL}^{\mu\nu}|^2 \right) \right] \right.
\]

\[
+ \left. d(x, Q^2) \left[ |M_{\mu LL}^{\mu\nu}|^2 + |M_{\mu RR}^{\mu\nu}|^2 + (1 - y)^2 \left( |M_{\mu LR}^{\mu\nu}|^2 + |M_{\mu RL}^{\mu\nu}|^2 \right) \right] \right\}. \tag{8}
\]
Model of extra dimensions

Arkani-Hamed, Dimopoulos and Dvali [4] proposed that in the extra dimensions gravity is free to propagate while the SM particles are restricted to a 3-D-brane. The size of the extra dimensions is postulated to be as large as mm to solve the hierarchy problem by bringing the effective Planck scale $M_S$ down to TeV. It implies a new gravity interaction for the graviton in the bulk. In our 3 + 1 dimensional point of view, the graviton behaves as a tower of closely-spaced Kaluza-Klein states. Each state still couples to the SM particles with a normal gravitational strength of order of $1/M_{Pl}$ but, however, there are a huge number of such states. Collectively, the overall coupling strength becomes of order of $1/M_S$. In the presence of the new interaction the double differential cross section is given by [5].

$$\frac{d^2\sigma(\mu^+p)}{dx dy} = \frac{sx}{16\pi} \sum_q f_q(x) \left\{ (1-y)^2(|M_{LL}|^2 + |M_{RR}|^2) + |M_{LR}|^2 + |M_{RL}|^2 ight.$$  

$$+ \frac{\pi^2}{2} (sx)^2 \left( \frac{F}{M^4_s} \right)^2 \left( 32 - 64y + 42y^2 - 10y^3 + y^4 \right)$$  

$$+ 2\pi e^2 Q_e Q_q \left( \frac{F}{M^4_S} \right) \left( \frac{2-y}{y} \right)^3$$  

$$+ \frac{2\pi e^2}{\sin^2 \theta_w \cos^2 \theta_w} \left( \frac{F}{M^4_s} \right) (sx) \frac{1}{Q^2 - M^2_Z} \left[ g_a g_a (6y - 6y^2 + y^3) + g_v g_v (y - 2)^3 \right] \right\}$$  

$$+ \frac{\pi}{2} f_g(x) (sx)^3 \left( \frac{F}{M^4_s} \right)^2 (1-y)(y^2 - 2y + 2). \tag{9}$$

Unlike the leptoquarks, the contact interactions and the extra dimensions do not enhance the cross section in a single invariant mass bin, instead, they enhance the cross section at large $Q^2$.

SENSITIVITY REACH

The 95% sensitivity reach on the contact interactions and extra dimensions are calculated as follows. We use the the 2-dimensional $x$-$y$ distribution to calculate the sensitivity to these new interactions, so as to maximize the sensitivity [6]. We divide the $x$-$y$ plane ($0.05 < x < 0.95$ and $0.05 < y < 0.95$) into a grid. We calculate the number of events predicted by the standard model in each bin with an efficiency of 0.8. We then follow the Monte Carlo approach in Ref. [6].

The sensitivity reach on the contact interaction scales $\Lambda_{\alpha_3}^{\mu q}$ is tabulated in Table 2. The maximum reach of $\Lambda$ at each center-of-mass energy roughly scales as $\Lambda \sim 40\sqrt{s}$. The effect of luminosity on $\Lambda$ is rather small: $\Lambda$ only scales as the 1/4th power of
TABLE 2. The 95% sensitivity reach on $\Lambda_{\mu q}^{\alpha \beta}$, ($\alpha, \beta = L, R$; $q = u, d$) at various $\mu^+(\mu^-)p$ colliders.

| $\sqrt{s}(\text{GeV})$ | $\mathcal{L}$ (fb$^{-1}$) | 30GeV $\otimes$ 820GeV | 50GeV $\otimes$ 1TeV | 200GeV $\otimes$ 1TeV | 1TeV $\otimes$ 1TeV | 2TeV $\otimes$ 3TeV |
|-------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| $\mu^+p$                |                          |                           |                           |                           |                           |                           |
| $\Lambda_{LL}^{\mu u}$  | 3.4                      | 4.7                      | 4.4                      | 3.2                      | 1.9                      | 2.1                      |
| $\Lambda_{LR}^{\mu u}$  | 3.4                      | 4.2                      | 3.4                      | 3.0                      | 2.9                      | 2.2                      |
| $\Lambda_{RL}^{\mu u}$  | 3.4                      | 4.7                      | 9.7                      | 9.0                      | 7.0                      | 5.7                      |
| $\Lambda_{RR}^{\mu u}$  | 3.4                      | 4.7                      | 9.7                      | 9.0                      | 7.0                      | 5.7                      |
| $\Lambda_{LL}^{\mu d}$  | 1.9                      | 2.1                      | 2.1                      | 1.4                      | 1.6                      | 1.6                      |
| $\Lambda_{LR}^{\mu d}$  | 1.9                      | 2.1                      | 2.1                      | 1.4                      | 1.6                      | 1.6                      |
| $\Lambda_{RL}^{\mu d}$  | 1.9                      | 2.1                      | 2.1                      | 1.4                      | 1.6                      | 1.6                      |
| $\Lambda_{RR}^{\mu d}$  | 1.9                      | 2.1                      | 2.1                      | 1.4                      | 1.6                      | 1.6                      |
| $\Lambda_{VV}$          | 6.5                      | 6.5                      | 6.5                      | 6.5                      | 6.5                      | 6.5                      |
| $\Lambda_{AA}$          | 2.8                      | 2.8                      | 2.8                      | 2.8                      | 2.8                      | 2.8                      |

The sensitivity reach on the effective Planck scale $M_S$ for the model of large extra dimensions is tabulated in Table 3.

To estimate the sensitivity reach for $R$-parity violating squarks, leptoquarks and leptogluons with a mass $m$, we assume the enhancement in cross section is in the mass bin of $(0, 0.9m, 1.1m)$. We calculate the number of events predicted by the standard model in this bin with an efficiency of 0.8, call it $n^{\text{sm}}$. Then we use the poisson statistics to estimate the $n^{th}$ that $n^{\text{sm}}$ can fluctuate to at the 95% CL. Once the $n^{th}$ is obtained the coupling constant $\lambda$ or the leptogluon scale $\Lambda_{\mu g}$ can be determined. These results are tabulated in Tables 4 to 6.

REFERENCES

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TABLE 3. The 95% sensitivity reach on $\eta = F / M_\|$ and the corresponding $M_S$ for $n = 3 - 6$ at various $\mu^+ (\mu^-) p$ colliders.

| $\mu^+ p$ | 30GeV $\otimes$ 820GeV | 50GeV $\otimes$ 1TeV | 200GeV $\otimes$ 1TeV | 1TeV $\otimes$ 1TeV | 2TeV $\otimes$ 3TeV |
|------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\sqrt{s}$(GeV) | 314 | 447 | 894 | 2000 | 4899 |
| $\mathcal{L}$ (fb$^{-1}$) | 0.1 | 2 | 13 | 110 | 280 |
| $\eta$ (TeV$^{-4}$) | 2.24 | 1.69 $\cdot$ 10$^{-1}$ | 9.35 $\cdot$ 10$^{-3}$ | 3.01 $\cdot$ 10$^{-4}$ | 1.38 $\cdot$ 10$^{-5}$ |
| $M_S$ (TeV) | | | | | |
| $n = 3$ | 0.97 | 1.86 | 3.82 | 9.03 | 19.5 |
| $n = 4$ | 0.82 | 1.56 | 3.22 | 7.59 | 16.4 |
| $n = 5$ | 0.74 | 1.41 | 2.91 | 6.86 | 14.8 |
| $n = 6$ | 0.69 | 1.31 | 2.70 | 6.38 | 13.8 |

| $\mu^- p$ | 30GeV $\otimes$ 820GeV | 50GeV $\otimes$ 1TeV | 200GeV $\otimes$ 1TeV | 1TeV $\otimes$ 1TeV | 2TeV $\otimes$ 3TeV |
|------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\sqrt{s}$(GeV) | 314 | 447 | 894 | 2000 | 4899 |
| $\mathcal{L}$ (fb$^{-1}$) | 0.1 | 2 | 13 | 110 | 280 |
| $\eta$ (TeV$^{-4}$) | 2.14 | 1.72 $\cdot$ 10$^{-1}$ | 8.95 $\cdot$ 10$^{-3}$ | 2.99 $\cdot$ 10$^{-4}$ | 1.38 $\cdot$ 10$^{-5}$ |
| $M_S$ (TeV) | | | | | |
| $n = 3$ | 0.98 | 1.85 | 3.87 | 9.05 | 19.5 |
| $n = 4$ | 0.83 | 1.55 | 3.25 | 7.61 | 16.4 |
| $n = 5$ | 0.75 | 1.40 | 2.94 | 6.87 | 14.8 |
| $n = 6$ | 0.70 | 1.31 | 2.73 | 6.40 | 13.8 |

TABLE 4. 95% sensitivity reach on $x'_{231}$ ($x'_{213}$) for a few choices of $m_{\tilde{t}_L}$ ($m_{\tilde{b}_R}$) at various $\mu^+ p$ ($\mu^- p$) colliders. The subprocess is $\mu^+ d \rightarrow \tilde{t}_L$ ($\mu^- u \rightarrow \tilde{b}_R$).

| $m_{\tilde{t}_L, \tilde{b}_R}$ (GeV) | 30GeV $\otimes$ 820GeV | 50GeV $\otimes$ 1TeV | 200GeV $\otimes$ 1TeV | 1TeV $\otimes$ 1TeV | 2TeV $\otimes$ 3TeV |
|---------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\sqrt{s}$(GeV) | 314 | 447 | 894 | 2000 | 4899 |
| $\mathcal{L}$ (fb$^{-1}$) | 0.1 | 2 | 13 | 110 | 280 |
| $m_{\tilde{t}_L, \tilde{b}_R}$ (GeV) | | | | | |
| 200 | 0.014 (0.0097) | 0.0043 (0.0036) | 0.0025 (0.0023) | 0.0015 (0.0014) | 0.0010 (0.0010) |
| 300 | $\infty$ (0.23) | 0.0091 (0.0062) | 0.0031 (0.0028) | 0.0019 (0.0018) | 0.0014 (0.0014) |
| 400 | - | 0.062 (0.029) | 0.0039 (0.0034) | 0.0021 (0.0020) | 0.0017 (0.0016) |
| 500 | - | - | 0.0054 (0.0042) | 0.0024 (0.0022) | 0.0019 (0.0019) |
| 600 | - | - | 0.0083 (0.0057) | 0.0026 (0.0024) | 0.0021 (0.0020) |
| 700 | - | - | 0.016 (0.0095) | 0.0029 (0.0026) | 0.0023 (0.0022) |
| 800 | - | - | 0.067 (0.027) | 0.0032 (0.0028) | 0.0025 (0.0023) |
| 900 | - | - | - | 0.0036 (0.0031) | 0.0026 (0.0024) |
| 1000 | - | - | - | 0.0041 (0.0034) | 0.0027 (0.0026) |
| 1500 | - | - | - | 0.012 (0.0071) | 0.0033 (0.0030) |
| 2000 | - | - | - | - | 0.0041 (0.0036) |
| 2500 | - | - | - | - | 0.0053 (0.0043) |
| 3000 | - | - | - | - | 0.0075 (0.0055) |
| 3500 | - | - | - | - | 0.012 (0.0078) |
| 4000 | - | - | - | - | 0.027 (0.014) |
| 4500 | - | - | - | - | 0.12 (0.052) |
TABLE 5.  95% sensitivity reach on $\lambda^0 \lambda^1$ for a few choices of $m_{L^0,L^1}$ at various $\mu^- p$ colliders. The subprocess is $\mu^- (c,s) \rightarrow L^{0,1}$.

| $\sqrt{s}$ (GeV) | 30GeV⊙ | 50GeV⊙ | 200GeV⊙ | 1TeV⊙ | 1TeV⊙ | 2TeV⊙ |
|------------------|--------|--------|--------|------|------|------|
| 820GeV           | 314    | 447    | 894    | 2000 | 2000 | 4899 |
| $\mathcal{L}$ (fb⁻¹) | 0.1    | 2      | 13     | 110  | 280  |      |

| $m_{L^0}$ (GeV) | $\mu^- c(s) \rightarrow L^0$ | $\mu^- c(s) \rightarrow L^1$ |
|-----------------|-------------------------------|-------------------------------|
| 200             | 0.097 (0.072) | 0.068 (0.051) |
| 300             | 2.3 (2.3)    | 1.6 (1.6)    |
| 400             | 0.43 (0.43)  | 0.30 (0.30)  |
| 500             | -              | 0.30 (0.30)  |
| 600             | -              | 0.27 (0.27)  |
| 700             | -              | 0.27 (0.27)  |
| 800             | -              | 0.27 (0.27)  |
| 900             | -              | 0.27 (0.27)  |
| 1000            | -              | 0.27 (0.27)  |
| 1500            | -              | 0.27 (0.27)  |
| 2000            | -              | 0.27 (0.27)  |
| 2500            | -              | 0.27 (0.27)  |
| 3000            | -              | 0.27 (0.27)  |
| 3500            | -              | 0.27 (0.27)  |
| 4000            | -              | 0.27 (0.27)  |
| 4500            | -              | 0.27 (0.27)  |

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TABLE 6. 95% sensitivity reach on $\Lambda_{\mu g}$ for a few choices of $m_{L_{\mu g}}$ at various $\mu^- p$ colliders. The subprocess is $\mu^- g \rightarrow L_{\mu g}$.

\begin{tabular}{|c|c|c|c|c|c|}
\hline
$\sqrt{s}$ (GeV) & 30GeV $\otimes$ 820GeV & 50GeV $\otimes$ 1TeV & 200GeV $\otimes$ 1TeV & 1TeV $\otimes$ 1TeV & 2TeV $\otimes$ 3TeV \\
\hline
$\mathcal{L}$ (fb$^{-1}$) & 0.1 & 2 & 13 & 110 & 280 \\
\hline
$\mu^- g \rightarrow L_{\mu g}$ ($\Lambda_{\mu g}$) in TeV & & & & & \\
\hline
$m_{L_{\mu g}}$ (GeV) & 200 & 1.9 & 4.3 & 8.5 & 14.4 & 19.5 \\
200 & 300 & 0.2 & 3.3 & 9.0 & 17.0 & 23.9 \\
300 & 400 & - & 1.2 & 8.6 & 18.6 & 27.4 \\
400 & 500 & - & - & 7.8 & 19.6 & 30.4 \\
500 & 600 & - & - & 6.5 & 20.1 & 32.8 \\
600 & 700 & - & - & 4.7 & 20.2 & 35.0 \\
700 & 800 & - & - & 2.3 & 20.0 & 36.7 \\
800 & 900 & - & - & - & 19.5 & 38.2 \\
900 & 1000 & - & - & - & 18.7 & 39.4 \\
1000 & 1500 & - & - & - & 11.9 & 42.4 \\
1500 & 2000 & - & - & - & - & 41.8 \\
2000 & 2500 & - & - & - & - & 38.8 \\
2500 & 3000 & - & - & - & - & 34.0 \\
3000 & 3500 & - & - & - & - & 27.2 \\
3500 & 4000 & - & - & - & - & 18.3 \\
4000 & 4500 & - & - & - & - & 7.7 \\
4500 & & & & & & \\
\hline
\end{tabular}

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