Comparing satellite altimetry and tide gauges observations in the Mediterranean Sea within the framework of singular spectrum analysis

MAHDI HADDAD, HEBIB TAIBI, MOUSTAFA MOKRANE and HOUSSEYN HAMMOUNI

Centre of Space Techniques, Algerian Space Agency. Po. Box 13 Arzew, Oran 31200, Algeria

(Received 26 July 2019, Accepted 27 December 2019)

e mails : mhaddad@cts.asal.dz; haddad_mahdi@yahoo.fr

1. Introduction

Sea level rise is a consequence of global warming through two main processes namely the dilation of sea water and the melting of land ice. Global warming is predicted to cause significant increases in sea level during the 21st century. This sea level rise is a major socio-economic issue as it directly affects the many coastal populations.

From the 20th century, tide gauges were fixed along the coast to inform us about coastal variations in sea level. During the 20th century, these instruments recorded an increase of about 1.5 to 2 mm/year (Douglas, 2001; Church and White, 2006; Holgate, 2007; Domingues et al., 2008). This rise of 15 to 20 cm in a century is greater by a factor of 10 compared to previous centuries; 0.2 mm/year over the last 1000 years (Lambeck and Bard, 2000; Lambeck, 2002).

Since 1993, altimetry satellites have been constantly monitoring sea level variations. The observations recorded are valuable for studies of global variations but also for regional variability in sea level. Observations from altimetry satellites indicate an overall mean sea level rise of 3.07 ± 0.37 mm/year during the period 1993-2017 (Global Sea Level Budget Group et al., 2018). The contribution of the global warming of the oceans in the sea level budget is 42% between 1993 and 2017 (1.3 ± 0.4 mm/year). Glaciers, Greenland and Antarctica...
contribute respectively 21%, 15% and 8% to the global mean sea level over the same period (0.65 ± 0.15 mm/year, 0.48 ± 0.10 mm/year and 0.25 ± 0.10 mm/year) (Global Sea Level Budget Group et al., 2018).

The Mediterranean Sea is a particularly interesting area for studying the variability of the mean sea level and its trends. In fact, when considering a limited area of the world ocean, the average sea level trend is dominated by the transport of water mass flows, which make the regional mean sea level very different from the world level. In the Mediterranean Sea, the transport of water bodies to the Strait of Gibraltar almost equilibrates the flow of surface water, thus changing the nature of the average trend of this sea. Recent studies, based on the analysis of data from satellite altimetry, suggest that the mean level of the Mediterranean has increased by 2.4 ± 0.4 mm/year (altimetry era) (Haddad et al., 2013; Bonaduce et al., 2016).

In this study, sea level time series from satellite altimetry measurements and tide gauge data available in the Mediterranean Sea over a 25-year period (1993-2017) are analyzed using the Singular Spectrum Analysis (SSA). The result of the SSA processing is a decomposition of these time series into several components, which can be identified as a non linear trend, seasonalities and noise components. Thus, the non-linear trend from both altimetry and tide gauge are compared to evaluate how satellite altimetry data solve the near-shore sea level in term of magnitude of change.

The paper is structured as follows. While Section 2 provides a description of the used satellite altimetry and tide gauge datasets; reminders about Singular Spectrum Analysis (SSA) are given in Section 3. Section 4 offer analysis and empirical findings of the study using SSA modeling with concluding remarks on the findings in Section 5.

2. Data

2.1. Satellite altimetry data

Radar altimeters permanently transmit signals to the Earth and receive the reflected echo from the sea surface. The satellite orbit has to be accurately tracked and its position is determined relative to a reference surface (an ellipsoid). The sea surface height (SSH) is calculated by subtracting the measured distance between the satellite and the sea surface from the precise orbit of the satellite. The sea level anomalies (SLA) are defined as variations of the sea surface height (SSH) with respect to an a priori mean sea surface (MSS).

In this study, we also used the monthly gridded maps of Sea Level Anomaly (SLA) data that cover the Mediterranean basin (30° N - 46° N, 6° W - 37° E). These data are produced by AVISO (Archiving, Validation and Interpretation of Satellite Oceanographic data) and are available through the website: http://www.aviso.oceanobs.com/. All of the standard corrections to the altimeter data were applied including tropospheric and ionospheric corrections, removal of ocean tides, sea state bias correction and inverted barometer correction (CLS, 2018). These maps of SLA cover the period from January 1993 to December 2017, with a spatial resolution of 1/8 × 1/8 degrees in latitude and longitude.

Fig. 1 represents the standard deviation of the SLA maps (over the period 1993-2017). This map shows that some areas are marked by an important variability, such as the Alboran Gyre at the east of Gibraltar, the Algerian Basin and the Ionian Basin. Other areas, such as the northern coasts of the western basin, show a minimum. These results are in good agreement with what is known about variations in the circulation in the Mediterranean.
The map of sea level variation rates (drift or velocity in mm/year), performed from the monthly maps of SLA (1993-2017) by estimating the linear slopes for each cell of this map, show that the Mediterranean Sea level is characterized by an East-West differentiation. While the Eastern Mediterranean basin is subject to a clear increasing trend, an inverse situation occurred at the Ionian Sea where the sea level is falling (Fig. 2).

In order to get an overview of the Mediterranean mean sea level change, a time series of mean sea-level anomalies in the Mediterranean Sea was estimated for the period 1993-2017, by calculating the average values of the cells of each grid (map), which allows us to have an average value of sea level anomalies per month. Figure 3 represents the estimated Mediterranean mean sea level series. To highlight the non-linear trend of the mean sea level series, the Loess smoothing was applied on this series with different smoothing thresholds $\alpha$ (0.15, 0.20 and 0.25) (Fig. 3). The smoothing threshold of 0.25 offers a better estimate of the non-linear trend (no presence of short cyclical variations while keeping the long-term general behaviour of the series). This non-linear trend shows, on one hand, the slow rise in the Mediterranean mean sea level under the combined effect of thermal expansion due to the steric effect, precipitation, etc. and on the other hand, the effect of extreme weather events. The linear regression of this trend shows that the Mediterranean mean sea level is subject to a significant increase of $2.66 \pm 0.07$ mm/year during the period 1993-2017. Note that the realistic error attributed to this quantity is $\pm 0.4$ mm/year. This value is based on comparisons of sea level measurements from the satellite altimetry with those from tide gauges (Mitchum, 2000).

2.2. Tide gauge data

In the framework of this study, we chose to analyze the available monthly averages sea level series from PSMSL RLR (Revised Local Refers) database (http://www.psmsl.org/data/obtaining/). In order to avoid negative numbers in the RLR monthly values, the RLR datum for each station is defined to be approximately 7000 mm below mean sea level. However, corrections for
the vertical movement for the reference, e.g., due to GIA which, on a long-term temporal scale, can have a significant impact, are not applied, because this requires a careful estimation of these movements usually involving a geodynamic model (Holgate et al., 2013).

The selected series with at least 90% monthly data present during the altimetry era (1993-2017) are: Ceuta, Malaga II, Valencia, Barcelona, L’Estartit, Toulon, Trieste, Levkas, Alexandroupolis and Leros. Fig. 4 shows the spatial distribution of these tide gauge stations over the Mediterranean Sea and the corresponding series are represented in Fig. 5.

Table 1 presents the correlation coefficients between the ten tide gauges series classified according to their longitude, from the Strait of Gibraltar towards the East direction. It can be seen, from Table 1, that in the Strait of Gibraltar, the two series relating to the Ceuta and Malaga II stations are strongly correlated (correlation coefficient: $r = 0.90$) and therefore the rates of sea level change should be close at these two stations. Likewise, in the Balearic Sea, the two series of Valencia and Barcelona exhibit a correlation coefficient of 0.96. The Trieste station is the only station considered in the Adriatic Sea which is a semi-enclosed sea characterized by surface currents influenced by its own meteorology. This explains the moderate correlation coefficients of this station compared to the others.

In the Ionian Sea, only the Levkas station is included. The stations of Alexandroupolis and Leros, located in the Aegean Sea, have moderate correlation coefficient ($r = 0.71$), despite their geographical proximity. The rate of sea level change should be different for these two stations.

These suggestions are checked from the magnitude of sea level changes estimated using the classical least squares method that finds the line of best fit for a dataset (Table 2). Columns of Table 2 indicate: country/station PSMSL code, estimated rates in mm / yr and standard error, Vertical Land Motion (VLM) correction based on the most recent model of Peltier (ICE-6G_C) (Peltier, 2015) and the corrected rates from VLM. As can be seen in column 8 of Table 2, all tide gauge series show upward rates of change.

3. Methodology - Reminders about Singular Spectrum Analysis (SSA)

This Section introduces the algorithm of Singular Spectrum Analysis (SSA) for the analysis of time series following the approach described in (Golyandina et al., 2001; 2015). SSA is a nonparametric technique that works with arbitrary statistical processes, whether linear or nonlinear, stationary or non-stationary, Gaussian or non-Gaussian. The main idea of SSA is performing a Singular Value Decomposition (SVD) of the trajectory matrix obtained from the original time series with a subsequent reconstruction of the series.

The basic version of SSA consists of four steps, which are performed as follows:

**Step 1. Embedding** : The first step consists in the construction of the so-called trajectory matrix ($\mathcal{T}_{SSA}$) by associating a time series $x = (x_1, \ldots, x_N)$ to $(K = N \cdot L+1)$
Fig. 5. Sea level series from tide gauges (black line) and satellite altimetry (red line). From top to bottom and from left to right: Ceuta, Malaga II, Valencia, Barcelona, L’Estartit, Toulon, Trieste, Levkas, Alexandroupolis and Leros.
column-vectors of dimension \( L \), where \( L \) is the window length:

\[
\mathcal{T}_{\text{SSA}} = \begin{bmatrix}
X_1 & X_2 & X_3 & \ldots & X_L \\
X_2 & X_3 & X_4 & \ldots & X_{L+1} \\
X_3 & X_4 & X_5 & \ldots & X_{L+2} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
X_L & X_{L+1} & X_{L+2} & \ldots & X_N
\end{bmatrix}
\]

(1)

Note that the trajectory matrix is a Hankel matrix, which means that all the elements along the diagonal \( i + j = \text{const} \) are equal.

**Step 2. Singular Value Decomposition**: Compute the eigenvalues and eigenvectors of the matrix \( S \), \( d = \max \{ j : \lambda_j > 0 \} \), \((U_1, V_1) ; \ldots ; (U_d, V_d)\) be eigen values of the matrix \( S \), \( d = \max \{ j : \lambda_j > 0 \} \), \((U_1, V_1) ; \ldots ; (U_d, V_d)\), the associated singular vectors left and right. The trajectory matrix \( X \) can be written as (Golyandina et al., 2001; 2015):

\[
X = \sum_{j=1}^{d} X_j, \text{ with } X_j = \sqrt{\lambda_j} U_j V_j^T
\]

(2)

The triple \( \sqrt{\lambda_j} U_j V_j^T \) is called the \( j \)th eigentriple.

**Step 3. Grouping**: This step partitions the \( X_i \) set of indices \( \{1, \ldots, d\} \) into \( m \) disjoint subsets \( I_1, \ldots, I_m \). For a subset \( \{i_1, \ldots, i_p\} \), the matrix \( X_I \) corresponding to the group \( I \) is defined as:

\[
X_I = X_{i_1} + \ldots + X_{i_p}
\]

(3)

**Step 4. (Reconstruction of the one-dimensional series)**: At this final step, each matrix of the decomposition “(3)” is transferred back to the form of the input object \( x \). Let the Hankelization
for ~ (4), the \( \mathcal{T} \) harmonic & residual components using the SSA technique. The diagonal of the matrix 
operator \( \Pi^{(i)} \) be averaging the corresponding diagonals of the matrix \( \widetilde{X}_k = X_i \) for \( i = 1, ..., m \). Thus, denote the reconstructed matrices \( \widetilde{X} = \Pi^{(i)} \widetilde{X}_k \), the trajectory matrices of the \( \widetilde{X}_k = \mathcal{T}^{-1} X_k \) the reconstructed data and the reconstructed data themselves. Then, the resultant decomposition of the initial data has the form:

\[
x = \widetilde{x}_1 + \ldots + \widetilde{x}_m
\]

(4)

4. Empirical results and discussion

At the location of each selected tide gauge station, a time series of sea level was constructed from the maps of sea-level anomalies at the Mediterranean scale, using the nearest neighbor interpolation method. Fig. 5 shows the selected tide gauge series and the interpolated satellite altimetry series with a length of \( N = 300 \) (25 years). We analyze separately each sea level time series (satellite altimetry and tide gauge) to determine its non-linear trend, harmonic & residual components using the SSA technique.

The window length \( l \) is the only parameter in the decomposition stage. Selection of the proper window length depends on the problem in hand and on preliminarily information about the time series. Theoretical results tell us that should be large enough but not greater than \( N/2 \). Following this approach and assuming that there is a dominant annual periodicity in the series (Fig. 5), the value of \( l \) is set at 144 (close to \( N/2 \)).

The third step of SSA or MSSA technique demands the grouping to make subgroups of the decomposed trajectory matrix and diagonal averaging to reconstruct the new time series from the subgroups. In practice, the singular values of the two eigentriples of a harmonic series are often very close to each other and this fact simplifies the visual identification of the harmonic components. Therefore, explicit plateau in the eigenvalue spectra prompts the ordinal numbers of the paired eigentriples of harmonic components. Another way to identify the harmonic components of the series is to examine the pair wise scatterplots of the singular vectors. Pair wise scatterplots like spiral circles, spiral regular polygons or stars determine periodic components of the time series provided these components are separable from the residual component.

Trend is usually defined as a smooth component containing information about time series global change. Thus the grouping of slowly varying eigenvectors corresponds to the trend. The residuals are obtained from grouping the eigentriples, which do not contain elements of trend and oscillations.

Tables 3 and 4 give the identified eigenvalues for the trend and seasonality (annual and semi-annual) components related to the decomposition of the sea level series from both techniques, as well as the contribution of the trend and of seasonalties in the original series. Figs. 6-8 represent the reconstructed non-linear trends, the seasonalties and the residuals components.
Fig. 6. Non-linear trends of satellite altimetry (in dashed red line) and tide gauges (in black line) observations
Fig. 7. Seasonality components of satellite altimetry (in dashed red line) and tide gauges (in black line) observations.
Fig. 8. Residual components of satellite altimetry (in dashed red line) and tide gauges (in black line) observations.
The strong correlations between the original sea level series (altimetry and tide gauges) and their components revealed by SSA, given in Table 5, show the potentiality of satellite altimetry to represent faithfully the local sea level. We note also that the correlation between the extracted trends is relatively better compared to the correlation between the original series.

The differences between the linear changes in sea level at the ten stations, estimated by applying the linear regression on the extracted trends from the sea level series of the two measurement techniques, are given Table 6. These differences vary between 0.18 and 4.29 mm/year, in absolute values, with an average of 1.55 mm/year. Nevertheless, all local trends show positive rates. These small differences show the corroboration and complementarily of the two measurement techniques (satellite altimetry and tide gauging).

The highest trend estimated from in-situ measurements of 7.21 mm/year is observed at the Levkas tide station on the west coast of Greece. While the absolute trend estimated from the altimetry measurements is of 2.92 mm/year, a difference of 4.29 mm/year suggesting a high subsidence rate at the Levkas station.

It should be noted that the differences between the rates estimated using the sea level series from the two techniques are mainly due to the lack of altimetry data in the coastal areas, since observations are usually made every 10 days along the ground tracks and on the other hand to the reduced accuracy of the corrective terms in this dynamic environment between land and sea. The tide gauges are fundamentally different and measure with precision the in situ sea level at a rate of one second to one hour.

The strong correlations between the original sea level series (altimetry and tide gauges) and their components revealed by SSA, given in Table 5, show the potentiality of satellite altimetry to represent faithfully the local sea level. We note also that the correlation between the extracted trends is relatively better compared to the correlation between the original series.
5. Conclusions

The analysis of time series from satellite altimetry and tide gauges observations using the Singular Spectrum Analysis (SSA) allowed us to identify and compare non-linear trends and seasonal patterns of these sea level series along the Mediterranean coasts.

The non-linear trends of measurements from both techniques are in good agreement and state that the sea level near the coasts of the Mediterranean Sea is object to an increasing behavior. The differences between the estimated rates of change from satellite altimetry and in-situ measurements range from -1.94 to +4.29 mm/year. One of the reasons for these differences can be attributed to the fact that altimetry measurements are not as accurate near the coast. Nevertheless, all local trends show positive rates. The maximum difference of +4.29 mm/year related to the Levkas station suggests a high rate of subsidence.

These results show that, on one hand, the sea level trends computed at coastal stations from satellite altimetry measurements are consistent for the period 1993-2017 and, on the other, the agreement of the two measurement techniques (spatial altimetry and tide gauges) to cover the ocean surfaces of the Earth just as well in the open ocean as in coastal areas.

Acknowledgements

The tide gauge data that support this study are available from the Permanent Service of Mean Sea Level (PSMSL) Website: http://www.psmsl.org. The altimetry data that support this study are available from AVISO (Archiving, Validation and Interpretation of Satellite Oceanographic data) through the website: http://www.aviso.oceanobs.com/.

The authors are enormously grateful to PSMSL for providing sea level time series and to AVISO for providing altimetry data. The authors greatly thank the anonymous reviewers for their valuable and constructive comments.

The contents and views expressed in this research paper are the views of the authors and do not necessarily reflect the views of the organizations they belong to.

References

Bonaduce, A., Pinardi, N., Oddo, P., Spada, G. and Larnicol, G., 2016, “Sea - level variability in the Mediterranean Sea from altimetry and tide gauges”, Climate Dynamics, 47, 9-10, 2851-2866.

Church, J. A. and White, N. J., 2006, “A 20th century acceleration in global sea-level rise”, Geophysical Research Letters, 33, L01602, 1-4.

CLS, 2018, “SSALTO/DUACS Experimental Product Handbook”, SALP-MU-P-EA-23172-CLS, 49 pp. https://www.aviso.altimetry.fr/fileadmin/documents/data/tools/hdbk_duacs_experimental.pdf. Accessed on 24 Sep., 2018.

Domingues, C. M., Church, J. A., White, N. J., Gleckler, P. J., Wijffels, S. E., Barker, P. M. and Dunn, J. R., 2008, “Improved estimates of upper-ocean warming and multi-decadal sea-level rise”, Nature, 453, 1090-1093.

Douglas, B. C., 2001, “Sea level change in the era of the recording tide gauge”, in Sea Level Rise: History and Consequences, edited by B. Douglas, et al., I. G. Ser., 37-64, Academic, San Diego.

Global Sea Level Budget Group, W. C. R. P., Cazenave, A. and Merchant, C., 2018, “Global sea level budget 1993-present”, Earth System Science Data, 10, 3, 1551-1590.

Golyandina, N., Korobeynikov, A., Shlemov, A. and Usevich, K., 2015, “Multivariate and 2D Extensions of Singular Spectrum Analysis with the Rssa Package”, Journal of Statistical Software, 67, 2, p78.

Golyandina, N., Nekrutkin, V. and Zhigljavsky, A., 2001, “Analysis of Time Series Structure: SSA and Related Techniques”, Edited by Chapman & Hall/CRC, p309.

Haddad, M., Hassan, H. and Taibi H., 2013, “Sea level in the Mediterranean Sea: seasonal adjustment and trend extraction within the frame-work of SSA”, Earth Science Informatics, 6, 2, 99-111.

Holgate, S. J., 2007, “On the decadal rates of sea level change during the twentieth century”. Geophysical Research Letters, 34, L01602, 1-4.

Holgate, S. J., Matthews, A., Woodworth, P. L., Rickards, L. J., Tamisiea, M. E., Brading, E., Foden, P. R., Gordon, K. M., Jevrejeva, S. and Pugh, J., 2013, “New Data Systems and Products at the Permanent Service for Mean Sea Level”, Journal of Coastal Research, 29, 3, 493-504.

Lambeck, K. and Bard, E., 2000, “Sea level change along the French Mediterranean coast since the time of the last Glacial Maximum”, Earth Planetary Science Letters, 175, 203-222.

Lambeck, K., 2002, “Sea level change from mid-Holocene to recent times : An Australian example with global implications, Ice Sheets, Sea Level and the Dynamic Earth”, edited by J.X. Mitrovica and B.L.A. Vermeersen, Eds., Geodynamics Series, 29, 33-50.

Mitchum, G., 2000, “An improved calibration of satellite altimetric heights using tide gauge sea levels with adjustment for land motion”, Marine Geodesy, 23, 3, 145-166.

Peltier, W. R., Argus, D. F. and Drummond, R., 2015, “Space geodesy constrains ice-age terminal deglaciation: The global ICE-6G.C (VM5a) model”, Journal of Geophysical Research - Solid Earth, 120, 450-487.