SMALL X BEHAVIOR OF PARTON DISTRIBUTIONS WITH FLAT INITIAL CONDITIONS.
A STUDY OF HIGHER-TWIST EFFECTS

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Abstract
We study the $Q^2$ evolution of parton distributions at small $x$ values, obtained in the case of flat initial conditions. The contributions of twist-two and (renormalon-type) higher-twist operators of the Wilson operator product expansion are taken into account. The results are in good agreement with deep inelastic scattering experimental data from HERA.

1 Introduction

The measurements of the deep-inelastic scattering structure function (SF) $F_2$ in HERA [1, 2] have permitted the access to a very interesting kinematical range for testing the theoretical ideas on the behavior of quarks and gluons carrying a very low fraction of momentum of the proton, the so-called small $x$ region. In this limit one expects that non-perturbative effects may give essential contributions. However, the reasonable agreement between HERA data and the next-to-leading order (NLO) approximation of perturbative QCD that has been observed for $Q^2 > 1\text{GeV}^2$ (see the review in Ref. [3]) indicates that perturbative QCD could describe the SF evolution up to very low $Q^2$ values, traditionally explained by soft processes $^1$. It is of fundamental importance to find out the kinematical region where the well-established perturbative QCD formalism can be safely applied at small $x$.

The standard program to study the small $x$ behavior of quarks and gluons is carried out by comparison of data with the numerical solution of the DGLAP equations by fitting the parameters of the $x$ profile of partons at some initial $Q^2_0$ and the QCD energy scale $\Lambda$ (see, for example, [6, 7]). However, if one is interested in analyzing exclusively the small $x$ region ($x \leq 0.01$), there is the alternative of doing a simpler analysis by using some of the existing analytical solutions of DGLAP in the small $x$ limit (see, for example, Ref. [3] for review). This was done so in Refs. [8]-[12] where it was pointed out that the HERA small $x$ data can be interpreted in terms of the so-called doubled asymptotic scaling phenomenon related to the asymptotic behavior of the DGLAP evolution discovered in [13] many years ago.

Here we compile results obtained in [9]-[11], where the contributions of higher-twist (HT) operators (i.e. twist-four ones and twist-six ones) of the Wilson operator product

$^1$The agreement can be explained [4] by an effective coupling constant scale essentially higher than $Q^2$ (see review [5] and references therein).
expansion are taken into account. The importance of the contributions of HT operators at small-$x$ has been demonstrated in many studies (see [15]).

We would like to note that the results of [9] are the extension to the NLO QCD approximation of previous leading order (LO) studies [13, 8]. The main ingredients are:

1. Both, the gluon and quark singlet densities are presented in terms of two components (‘+’ and ‘−’) which are obtained from the analytical $Q^2$ dependent expressions of the corresponding (‘+’ and ‘−’) parton distributions moments.
2. The ‘−’ component is constant at small $x$, whereas the ‘+’ component grows at $Q^2 \geq Q_0^2$ as $\sim \exp(\sigma_{NLO})$, where

$$
\sigma_{NLO} = 2\sqrt{(d_s + D_p) \ln x},
$$

and the LO term $d_+ = -12/\beta_0$ and the NLO one $\hat{D}_+ = \hat{d}_{++} + \hat{d}_+\beta_1/\beta_0$ with $\hat{d}_{++} = 412f/(27\beta_0)$. Here the coupling constant $a_s = \alpha_s/(4\pi)$, $s = \ln(a_s(Q_0^2)/a_s(Q^2))$ and $p = a_s(Q_0^2) - a_s(Q^2)$, $\beta_0$ and $\beta_1$ are the first two coefficients of QCD $\beta$-function and $f$ is the number of active flavors.

## 2 Basical formulae

Thus, our purpose is to demonstrate the small $x$ asymptotic form of parton distributions in the framework of the DGLAP equation starting at some $Q_0^2$ with the flat function:

$$
f_a^{r2}(Q_0^2) = A_a \quad (\text{hereafter } a = q, g), \quad (1)
$$

where $f_a^{r2}$ are the leading-twist (LT) parts of parton (quark and gluon) distributions multiplied by $x$ and $A_a$ are unknown parameters that have to be determined from data. Through this work at small $x$ we neglect the non-singlet quark component.

We would like to note that new HERA data [2] demonstrate a rise of SF $F_2$ at low $Q^2$ values ($Q^2 < 1 \text{GeV}^2$) when $x \to 0$ (see Fig.2, for example). The rise can be explained in natural way by incorporation of HT terms in our analysis (see section 2.2).

We shortly compile below the main results found in [9, 12] at the LO approximation (the LT results at the NLO approximation may be found in [9]). The full small $x$ asymptotic results for parton distributions and SF $F_2$ at LO of perturbation theory is:

$$
F_2(x, Q^2) = e \cdot \left[ f_q^{r2}(x, Q^2) + f_q^{h\tau}(x, Q^2) + f_g^{h\tau}(x, Q^2) \right], \quad (2)
$$

where $e = (\sum_i f_i^2)/f$ is the average charge square. The LT parts $f_a^{r2}(x, Q^2)$ and the HT ones $f_a^{h\tau}(x, Q^2)$ can be represented as sums of the ‘+’ and ‘−’ components

$$
\begin{align*}
  f_a^{r2}(x, Q^2) &= f_a^{r2,+}(x, Q^2) + f_a^{r2,-}(x, Q^2), \\
  f_a^{h\tau}(x, Q^2) &= f_a^{h\tau,+}(x, Q^2) + f_a^{h\tau,-}(x, Q^2).
\end{align*} \quad (3)
$$

## 2.1 The LT contribution

The small $x$ asymptotic results for parton distributions have the form

$$
f_g^{r2,+}(x, Q^2) = \left( A_g + \frac{4}{9} A_q \right) \tilde{I}_0(\sigma) e^{-\gamma_{4,(1)}s} + O(\rho), \quad (4)
$$

\footnote{Refs. [10, 11] and the present note deal with the contributions of HT operators only in the renormalon model approach (see [14]). The complete analysis of the HT contributions to the SF $F_2$, $dF_2/d\ln(Q^2)$ and $F_L$ will be presented in [12].}
where $\tilde{d}_+(1) = 1 + 20 f/(27 \beta_0)$ and $d_-(1) = 16 f/(27 \beta_0)$ are the regular parts of $d_+$ and $d_-$ anomalous dimensions, respectively, in the limit $n \to 1^3$. The functions $\tilde{I}_\nu (\nu = 0, 1)$ are related to the modified Bessel function $I_\nu$ and to the Bessel function $J_\nu$ by:

$$\rho^\nu \tilde{I}_\nu (\sigma) = \begin{cases} 
\rho^\nu I_\nu (\sigma), & \text{if } s \geq 0 \\
(-\overline{\sigma})^\nu J_\nu (\sigma), & \text{if } s < 0
\end{cases} \quad (8)$$

The variables $\sigma$ and $\rho$ are given by

$$\sigma = 2 \sqrt{d_+ s \ln(x)}, \quad \rho = \sqrt{\frac{d_+ s \ln(x)}{2 \ln(1/x)}} \quad (9)$$

### 2.2 The HT contribution

Using the results in [10] [11] (which are based on calculations [14] [16]), we demonstrate the effect of HT corrections in the renormalon model case (see recent review of renormalon models in [17]). We sketch below the basic results making the following substitutions in the corresponding twist-two results presented in Eqs.(4)-(7) (complete formulae can be found in [12]):

- $f^{g^2, +}(x, Q^2)$ and $f^{q^2, +}(x, Q^2)$ (see Eqs.(4) and (5)) → $f^{hr, +}(x, Q^2)$ and $f^{q^2, +}(x, Q^2)$ by

$$A_a \tilde{I}_0 (\sigma) \rightarrow A_a \frac{16 f}{15 \beta_0^2} \sum_{m=1}^2 k_m (A_{m,a})^{2m} \left( \frac{2}{\rho} \tilde{I}_1 (\sigma) - \ln \left( \frac{\Lambda_{m,a}^2}{Q^2} \right) \right), \quad (10)$$

$$A_a \rho \tilde{I}_1 (\sigma) \rightarrow A_a \frac{128 f}{45 \beta_0^2} \sum_{m=1}^2 k_m (A_{m,a})^{2m} \left( \frac{2}{\rho} \tilde{I}_1 (\sigma) - \ln \left( \frac{\Lambda_{m,a}^2}{Q^2} \right) \right), \quad (11)$$

where $k_1 = 1$, $k_2 = -8/7$ and $A_{1,a}$ and $A_{2,a}$ are magnitudes of twist-four and twist-six corrections.

- $f^{g^2, -}(x, Q^2)$ and $f^{q^2, -}(x, Q^2)$ (see Eqs.(4) and (7)) → $f^{hr, -}(x, Q^2)$ and $f^{hr, -}(x, Q^2)$ by

$$A_g \rightarrow A_g \frac{16 f}{15 \beta_0^2} \sum_{m=1}^2 k_m (A_{m,g})^{2m} \ln \left( \frac{Q^2}{x^2 A_{m,g}^2} \right), \quad (12)$$

$$A_q \rightarrow A_q \frac{128 f}{45 \beta_0^2} \sum_{m=1}^2 k_m (A_{m,q})^{2m} \left( \ln \left( \frac{Q^2}{x A_{m,q}^2} \right) - \frac{11}{3} \right) \ln \left( \frac{1}{x} \right). \quad (13)$$

From Eqs.(10) - (13) one can notice that the higher-twist terms modify the flat condition Eq.(11). They lead to a rise of parton distributions and, thus, SF $F_2$ at low value $Q^2$, when $x \to 0$. This is in excellent agreement with the recent low $Q^2$ HERA data [2].

\[ ^3 \text{From now on, for a quantity } k(n) \text{ we use the notation } \hat{k}(n) \text{ for the singular part when } n \to 1 \text{ and } \overline{k}(n) \text{ for the corresponding regular part.} \]
Figure 1: The SF $F_2$ as a function of $x$ for different $Q^2$ bins. The experimental points are from H1 [1]. The inner error bars are statistic while the outer bars represent statistic and systematic errors added in quadrature. The dashed and dot-dashed curves are obtained from fits (based on LT formulae) at LO and NLO respectively with fixed $Q^2_0 = 1 \text{ GeV}^2$. The solid line is from the fit at NLO giving $Q^2_0 = 0.55 \text{ GeV}^2$.

3 Results of the fits

With the help of the results obtained in the previous section we have analyzed $F_2$ HERA data at small $x$ from the H1 and ZEUS collaborations [1, 2]. In order to keep the analysis as simple as possible we have fixed the number of active flavors $f=4$ and $\Lambda_{\text{MS}}(n_f = 4) = 250 \text{ MeV}$, which is a reasonable value extracted from the traditional (higher $x$) experiments. Moreover, we put $\Lambda_{1,\alpha} = \Lambda_{2,\alpha}$ in agreement with [18]. The initial scale of the parton densities was also fixed into the fits to $Q^2_0 = 1 \text{ GeV}^2$, although later it was released to study the sensitivity of the fit to the variation of this parameter. The analyzed data region was restricted to $x < 0.01$ to remain within the kinematical range where our results are accurate.

Fig. 1 shows $F_2$ calculated from the fit (based only on LT formulae) with $Q^2 > 1 \text{ GeV}^2$ in comparison with 1994 H1 data (first article in [1]). Only the lower $Q^2$ bins are shown. One can observe that the NLO result (dot-dashed line) lies closer to the data than the LO curve (dashed line). The lack of agreement between data and lines observed at the lowest $x$ and $Q^2$ bins suggests that the flat behavior should occur at $Q^2$ lower than 1 GeV$^2$. In order to study this point we have done the analysis considering $Q^2_0$ as a free parameter. Comparing the results of the fits (see [9]) one can notice the better agreement with the experiment at fitted $Q^2_0 = 0.55 \text{ GeV}^2$ (solid curve) is apparent at the lowest kinematical bins.

Fig. 2 shows $F_2$ calculated from the fit at LO (based on LT & HT formulae) in comparison with 1995 H1 and ZEUS data [2]. One can observe that these results (solid line) lies closer to the data than the twist-two results (dashed line). We have done the analysis considering $Q^2_0$ as a free parameter. Comparing the results of the fits (see [12]) one can notice the better agreement with the experiment at fitted $Q^2_0 = 0.61 \text{ GeV}^2$, which
Figure 2: The SF $F_2$ as a function of $x$ for different $Q^2$ bins. The experimental points are from H1 and ZEUS [2]. The inner error bars are statistic while the outer bars represent statistic and systematic errors added in quadrature. The solid curves are obtained from fits at LO, when contributions of HT terms have been incorporated. The dashed curves show only twist-two contributions.

is close to $Q^2_0$ in the analysis of 1994 H1 data (see Fig. 1).

4 Conclusions

We have shown that the results developed recently in [9]-[12] have relatively quite simple form and reproduce many properties of parton distributions at small $x$, that have been known from global fits.

We found very good agreement between our approach based on QCD and HERA data, as it has been observed earlier with other approaches (see the review [3]). The (renormalon-type) higher-twist terms lead to the natural explanation of the rise of $F_2$ at low $x$ for the lowest values of $Q^2$ ($\leq 1$ GeV$^2$). The rise has been discovered in recent HERA experiments [2].

As next step of our investigations, we plan to study contributions of HT operators to relations between parton distributions and deep inelastic structure functions, observed, for example, in [19 20].

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