The Gravity dual of the Non-Perturbative $N = 2$ SUSY Yang-Mills Theory

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Abstract

The anomalous Ward identity is derived for $N = 2$ SUSY Yang-Mills theories, which is resulted out of Wrapping of $D_5$ branes on Supersymmetric two cycles. From the Ward identity One obtains the Witten-Dijkgraaf-Verlinde-Verlinde equation and hence can solve for the pre-potential. This way one avoids the problem of enhancon which maligns the non-perturbative behaviour of the Yang-Mills theory resulted out of Wrapped branes.
1. Introduction

Recently the gauge theory/gravity duality, which is commonly known as Ads/CFT duality is extended to non-conformal pure $\mathcal{N} = 1$ or $\mathcal{N} = 2$ supersymmetric Yang-Mills (SYM) theories [1]. So far the perturbative behaviour of the $\mathcal{N} = 2$ SYM is produced by this duality. The conventional folklore is that the instantons which are responsible for the non-perturbative part of the pre-potential are suppressed in the large N limit (since the gauge gravity duality is valid only in the large N limit). Also the non-perturbative strong coupling behaviour of SYM concerns the dilaton which is plagued with the singularity of "enhancon" [2]. Here we establish the anomalous super conformal Ward identity for $\mathcal{N} = 2$ SYM in the gravity dual picture. Further the super conformal Ward identity is written as Witten-Dijkgraaf-Verlinde-Verlinde (WDVV)[3] equation from which one can obtain the exact pre-potential.

2. The Strategy

We start with type IIB little string theory e la’ a collection of a large number of $NS_5$ brane in the vanishing string coupling limit which gives rise to $D = 6$ SYM [4]. Then we dimensionally reduce two of its spatial world volume in such a way that we retain $\mathcal{N} = 2$ SYM in the low energy limit. The $NS_5$ brane has $SO(4)$ R-symmetry as the normal bundle. When one identifies the $U(1)$ subgroup of the $SO(4)$ R-symmetry with the $U(1)$ spin connection of the two cycle which is compactified, one gets a covariant constant spinor and SUSY is retained, which is commonly known as twisting [1]. This is called wrapping of $NS_5$ brane on a supersymmetric two cycles. If the compact space is a two-sphere, then there will be no extra hyper- multiplet and in the low energy limit i.e. in the scale much lower than the radius of the sphere we will get pure $\mathcal{N} = 2$ SYM. Thus it amounts to consider a gauged $D = 7$ supergravity solution and then lift it to get the solutions in ten dimensions. We use here the results of [5, 6] classical solutions of $D = 7$ gauged supergravity which is amenable to ten dimensional string theory.

\[
\begin{align*}
\text{\textit{ds}}_{10}^2 &= e^\Phi \left[ dx_{1,3}^2 + \frac{z}{x^2} \left( d\theta^2 + \sin^2 \theta \, d\varphi^2 \right) + \frac{1}{x^2} e^{2z} \, dz^2 \\
&\quad + \frac{1}{x^2} \left( d\theta_1^2 + \frac{e^{-x}}{f(x)} \cos^2 \theta_1 (d\theta_2 + \cos \theta \, d\varphi)^2 + \frac{e^x}{f(x)} \sin^2 \theta_1 \, d\theta_2^2 \right) \right], \quad (1)
\end{align*}
\]

where the dilaton is

\[
e^{2\Phi} = e^{2z} \left[ 1 - \sin^2 \theta_1 \, \frac{1 + e^{-2z}}{2z} \right] \quad (2)
\]

and

\[
f(x) = e^x \cos^2 \theta_1 + e^{-x} \sin^2 \theta_1, \quad (3)
\]
also
\[ e^{-2x} = 1 - \frac{1 + c e^{-2z}}{2z} \quad (4) \]

where \( \lambda \) is the gauge coupling constant of seven dimensional gauged supergravity and \( c \) is a parameter as the integration constant of the classical solution. For \( c \geq -1 \) the range of the radial variable is \( z_0 \leq z \leq \infty \) where \( z_0 \) is the solution for \( e^{-2x(z_0)} = 0 \).

Here \( \theta \) and \( \varphi \) are the angles of compact two-sphere with radius of compactification as \( \frac{1}{\lambda^2} \) and \( \theta_1, \theta_2 \) and \( \theta_3 \) are angles of transverse three-sphere. The conservation of the RR-charge on the transverse sphere \( S_3 \) fixes \( \frac{1}{\lambda^2} = N g_s \alpha' \) for \( N \) number of \( D_5 \) branes with string coupling \( g_s \). The \( D_5 \) brane action is given by
\[
S = -\tau_5 \int d^6 \xi \ e^{-\Phi} \sqrt{-\det (G + 2\pi \alpha' F)} + \tau_5 \int \left( \sum_n C^{(n)} \wedge e^{2\pi \alpha' F} \right) \quad (5)
\]

where \( F \) is the world volume gauge field and \( \tau_5 \) is the brane tension. The BPS condition is fixed from the condition of the vanishing of the potential between two branes which gives \( \theta_1 \) to be \( \pi \). This condition makes the transverse boundary of the \( D \) brane to be a two dimensional space consisting \( z \) and \( \theta_3 \) which will eventually the moduli space of \( \mathcal{N} = 2 \) SYM.

We want to establish here the anomalous super conformal Ward identity. In the presence of gravity the trace anomaly
\[
\langle \rho^\mu \rangle = \frac{1}{2} \frac{\beta(g)}{g^3} \left( F^a_{\mu\nu} \right)^2 + \frac{c(g^2)}{16\pi^2} \left( W_{\mu\nu\rho\sigma} \right)^2 - \frac{a(g^2)}{16\pi^2} \left( \tilde{R}_{\mu\nu\rho\sigma} \right)^2 \quad (6)
\]

where \( \beta(g) \) is the beta function of SYM, \( a(g) \) and \( c(g) \) are central functions near the criticality, \( W_{\mu\nu\rho\sigma} \) is the Weyl tensor and \( \tilde{R}_{\mu\nu\rho\sigma} \) is the dual of the curvature tensor.

However this relation can be extracted from the two point functions of the energy momentum tensors[7]. In Ref.[8], it is shown how to extract these functions from the absorption cross-section of soft dilatons or gravitons by the \( D \) branes. The probability of absorption is taken as ratio of the flux near \( z_0 \) to the in coming flux at very large \( z \). This gives
\[
\sigma = \frac{N^4}{128\pi^3} (z - z_0)^2 \omega^3. \quad (7)
\]

where \( \omega \) is the frequency of the soft gluon. Here we see the presence of \( (z - z_0)^2 \) which if we write as in Ref.[6] \( e^z = \rho \) we get a term \( \left( \log \frac{\rho}{\rho_0} \right)^2 \) in the cross-section signaling the asymptotic freedom or logarithmic coupling. Also we see the presence of enhancon when \( z_0 \) is zero. This gives \( \beta(g) = -\frac{N}{8\pi^2} g^3 \). Similarly one can also calculate from \( U(1) \) R-current the chiral anomaly. Combining trace anomaly, chiral
anomaly and supertrace anomaly one can write the Ward identity as [9]

$$2F - F(A)'A = \frac{N}{8\pi^2} (tr\psi^2),$$

(8)

where $F$ is the effective potential or the pre-potential, $A$ is the chiral multiplet coupled to vector multiplet in the $\mathcal{N} = 2$ SYM and $tr\psi^2$ is the anomaly multiplet for example $trF^2$ will correspond to $\theta_\mu^\nu$. Here the bosonic component of $A$ corresponds to $z - i\theta_3$ or in radial coordinate $ve^{i\gamma}$. In the broken phase the branes will be distributed on a circle or $A_i$ the eigen values of $A$ which are $U(N)$ matrices will be distributed on a circle. The second part of eq.(8) will read as $\sum_i \frac{\partial F}{\partial A_i} A_i$. This equation one can in principle solve in the large N limit and obtain the exact pre-potential in this limit [10].

References

[1] J. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231, J.Maldacena and C. Nunez, Phys. Rev.Lett. 86,(2001)588.

[2] C.V.Johnson,A.Peet and J.Polchinski, Phys. Rev. D 61(2000)086001

[3] E. Witten, Nuc.Phys. B 340(1990)281, R. Dijkgraaf,E.Verlinde and H. Verlinde, Nuc.Phys. B 352(1991)59, B. Dubrovin, Lect. notes. hep-th-9407018.

[4] N. Seiberg, Phys. Lett. B 408(1997)98

[5] J. Gauntlet,N.Kim,D.Martelli , D. Waldram, Phys.Rev. D 64 (2001) 106008, F.Bigazzi,A.Cotrone, A. Zaffaroni, Phys.Lett.B 519(2001)269.

[6] M.Bertolini,P. Di Vecchia, M. Frau, A.Lerda, R. Marotta, I. Pesando, JHEP 0102 (2001)014, P. Di Vecchia, A.Lerda and P. Merlatti, Nucl.Phys.B646 (2002) 43, P. Di Vecchia, hep-th-0212162.

[7] D. Anselmi, JHEP 9805(1998)005, D. Anselmi,D.Z. Freedman, M. Grisaru, A. Johansen, Nucl. Phys. B 526(1998) 543.

[8] S.S. Gubser and I.R. Klebanov, Phys.Lett. B 413(1997)41.

[9] Marco Matone, Phys. Lett. B 357 (1995)342,G.Bonelli and M. Matone Phys. Rev. Lett. 77 (1996)4712, F.Fucito and G. Travaglini,Phys.Rev. D 55(1997)1099, N.Dorey,V.Khoze,M.Mattis, Phys.Lett. B390 (1997)205, P.S. Howe and P.C.West, Phys.Lett.B 400(1997)30.
[10] M.R. Douglas and S. Shenker, Nucl.Phys.B447 (1995) 271, F. Ferrari, Nucl.Phys.B 612 (2001) 151.