Merger events of close double neutron stars (DNS) lie at the basis of a number of current issues in relativistic astrophysics, such as the indirect and possible direct detection of gravitational waves, the production of gamma-ray bursts at cosmological distances, and the origin of r-process elements in the universe. In assessing the importance or relevance of DNS coalescence to these issues, knowledge of the rate of coalescence in our Galaxy is required. In this paper, I review the current estimates of the DNS merger rate (theoretical and empirical) and discuss new ways to obtain limits on this rate using all information available at present.

1 Introduction

Double neutron star (DNS) systems observed as binary radio pulsars have provided striking empirical confirmation of general relativity with the measurement of orbital decay accompanied with gravitational-wave emission in the prototypical system PSR B1913+16. This orbital period decrease is expected to lead eventually to the merger of the two neutron stars. DNS coalescence events represent one of the most promising sources for the direct detection of gravitational waves by the new generation of laser-interferometer detectors currently under construction (e.g., LIGO, VIRGO). The final coalescence of DNS has also been discussed as a possible central engine of gamma-ray bursts and has been suggested as the possibly dominant source of r-process elements in the universe.

Estimates of the rate of DNS coalescence in our Galaxy are crucial for assessing the prospects for gravitational-wave detection and the possible connection of DNS mergers to gamma-ray bursts and r-process elements. Formation rates of coalescing DNS (systems with tight enough orbits that merge within a Hubble time) have been calculated so far either theoretically, based on evolutionary models of DNS formation, or empirically, based on the observed DNS sample. In this paper I present a critical review of the current coalescence-rate estimates addressing the uncertainties involved and I discuss new ways of constraining these estimates using all available
observational information and current theoretical understanding. It is useful to provide a scale for the various estimates in the context of gravitational-wave detection: based on the expected performance of the second-generation LIGO observatories, for a detection rate of one merger event per year, a Galactic DNS coalescence rate of $\sim 10^{-5}$ yr$^{-1}$ is required. For comparison, the estimated rate of gamma-ray bursts per galaxy is $\sim 10^{-7}$ yr$^{-1}$.

2 Theoretical Estimates

Theoretical calculations of the formation rate of coalescing DNS are possible, given a sequence of evolutionary stages that leads from primordial binaries to DNS formation. Over the years a relatively standard picture has been formed describing the birth of DNS, although more recently variations of it have also been discussed. In all versions of the DNS formation path the main picture remains the same. The initial binary progenitor consists of two binary members massive enough to eventually collapse into neutron stars. Its evolution involves multiple phases of stable or unstable mass transfer, common-envelope evolution, and accretion onto neutron stars, as well as two supernova explosions.

Such theoretical modeling of DNS formation has been undertaken by various authors by means of population syntheses. This provides us with ab initio predictions of the coalescence rate. The evolution of an ensemble of primordial binaries with assumed initial properties is followed through specific evolutionary stages until a coalescing DNS is formed. The changes in the properties of the binaries at the end of each stage are calculated based on our current understanding of the various processes involved: wind mass loss from massive hydrogen- and helium-rich stars, mass and angular-momentum losses during mass transfer phases, dynamically unstable mass transfer and common-envelope evolution, effects of highly super-Eddington accretion onto neutron stars, and supernova explosions with kicks imparted to newborn neutron stars. Given that several of these phases are not very well understood, the results of population synthesis are expected to depend on the assumptions made in the treatment of the various processes. Therefore, exhaustive parameter studies are required by the nature of the problem.

Recent studies of DNS formation and calculations of coalescence rates have explored the input parameter space and the robustness of the results at different levels of (in)completeness. Almost all have studied the sensitivity of the coalescence rate to the average magnitude of the kicks imparted to newborn neutron stars. The range of predicted Galactic rates from all these studies obtained by varying the kick magnitude is $5 \times 10^{-7} - 5 \times 10^{-4}$ yr$^{-1}$. This large range indicates the importance of supernovae (two in this case) in binaries. Variations in the assumed mass-ratio distribution for the primordial binaries can further change the predicted rate by about a factor of 10, while assumptions of the common-envelope phase add another factor of about 10 – 100. Variation in other parameters typically affects the results by factors of two or less.

It is evident that recent theoretical predictions for the DNS coalescence rate cover a disappointingly wide range of values (typically 3-4 orders of magnitude), which actually includes the “nominal” value of $\sim 10^{-5}$ yr$^{-1}$. Note that DNS properties other than the coalescence rate, such as orbital sizes, eccentricities, center-of-mass velocities, are much less sensitive to the various input parameters and assumptions; the latter affect more severely the absolute normalization (birth rate) of the population. Given these results it seems fair to say that population synthesis calculations have very limited predictive power and provide fairly loose constraints on the DNS coalescence rate. Overall, we cannot use them to make a robust statement about the prospects for detection of merger events by the upcoming gravitational-wave observatories.
3 Empirical Estimates

Another way to estimate the coalescence rate is to use the properties of the observed coalescing DNS (only two systems: PSR B1913+16 and PSR B1534+12) combined with models of selection effects in radio pulsar surveys. For each observed object, a scale factor is calculated based on the fraction of the Galactic volume within which pulsars with properties identical to those of the observed pulsar could be detected, in principle, by any of the radio pulsar surveys, given their detection thresholds. This scale factor is a measure of how many more pulsars like those detected in the coalescing DNS systems exist in our galaxy. The coalescence rate can then be calculated based on the scale factors and estimates of detection lifetimes summed up for all the observed systems. This basic method was first used by Phinney[15] and Narayan et al. [16] who estimated the Galactic rate to be $\sim 1 - 3 \times 10^{-5} \, \text{yr}^{-1}$.

Since then, estimates of the coalescence rate have decreased significantly primarily because of (i) the increase of the Galactic volume covered by radio pulsar surveys with no additional coalescing DNS being discovered [17], (ii) the increase of the distance estimate for PSR B1534+12 based on measurements of post-Newtonian parameters [18] (see also contribution by I. Stairs, this volume), (iii) changes in the lifetime estimates for the observed systems [19, 20]. The most recently published study [20] gives a lower limit of $2 \times 10^{-7} \, \text{yr}^{-1}$ and a “best” estimate of $\sim 6 - 10 \times 10^{-7} \, \text{yr}^{-1}$.

Some of the assumptions made in obtaining the above estimates, are not clearly justifiable or testable. In particular, one assumes that the sample of the two observed coalescing DNS is representative of the total Galactic population [21], that the detection volume for each object and its lifetime are independent and separable (see contribution by T.A. Prince, this volume, for arguments against this in the presence of pulsar luminosity evolution and subsequent further reduction of the estimated rate), and that the DNS pulsar luminosity function is similar to that of young, non-recycled pulsars. Additional uncertainties arise from estimates of pulsar ages and distances, the pulsar beaming fraction, the spatial distribution of DNS in the Galaxy, and the number of undetectable pulsars with luminosities below the detection limits of surveys.

Despite all these uncertainties the empirical estimates of the DNS coalescence rate appear to span a range of $\sim 1 - 2$ orders of magnitude, which is relatively narrow compared to the range covered by the theoretical estimates.

3.1 Small-number Sample

One important limitation of empirical estimates of the coalescence rates is that they are derived based on only two observed DNS systems, under the assumption that the observed sample is representative of the true population. Therefore, assessing the effect of small-number statistics on the results of the above studies is necessary. Assuming that DNS pulsars follow the radio luminosity function of young pulsars and that therefore their population is dominated in number by low-luminosity pulsars, it can be shown that the current empirical estimates underestimate the true coalescence rate. If a small-number sample is drawn from a parent population dominated by low-luminosity (hence hard to detect) objects, it is statistically more probable that the sample will actually be dominated by objects from the high-luminosity end of the population. Consequently, the empirical estimates based on such a sample will tend to overestimate the detection volume for each observed system, and therefore underestimate the scale factors and the resulting coalescence rate (this effect is partly related to the Malmquist bias).

This effect can be clearly demonstrated with a Monte Carlo experiment (R. Narayan, private communication) with the use of simple models for the pulsar luminosity function and the survey selection effects. As a first step, the average observed number of pulsars is calculated given a known true total number of pulsars in the Galaxy (thick-solid line in Figure 1). As a second step, a large number of sets of a given number of ‘observed’ (simulated) pulsars with assigned
luminosities according to the assumed luminosity function are realized using Monte Carlo methods. Based on each of these sets, one can estimate the total number of pulsars in the Galaxy using empirical scale factors, as is done for the real observed sample. The many (simulated) ‘observed’ samples can then be used to obtain the distribution of the estimated total Galactic numbers of pulsars. The median and 25% and 75% percentiles of this distribution are plotted as a function of the assumed number of systems in the (fake) ‘observed’ samples in Figure 1 (thin-solid and dashed lines, respectively).

It is evident from Figure 1 that, in the case of small-number observed samples (less than \( \sim 10 \) objects), it is highly probably that the estimated total number, and hence the estimated coalescence rate, is underestimated by a significant factor. For a two-object sample, for example, the true rate maybe higher by more than a factor of ten.

4 Limits on Coalescence Rates

One way to circumvent the uncertainties involved in the estimates of the DNS coalescence rate is to focus on obtaining upper or lower limits to this rate. Depending on how their value compares to the value of \( \sim 10^{-5} \text{ yr}^{-1} \) needed for one LIGO II event per year, such limits can provide us with valuable information about the prospects of gravitational-wave detection.

Bailes\textsuperscript{22} used the absence of any young pulsars detected in DNS systems and obtained a rough upper limit to the rate of \( \sim 10^{-5} \text{ yr}^{-1} \), while recently Arzoumanian \textit{et al}\textsuperscript{20} reexamined this in more detail and claimed a more robust upper limit of \( \sim 10^{-4} \text{ yr}^{-1} \).

An upper bound to the rate can also be obtained by combining our theoretical understanding of orbital dynamics (for supernovae with neutron-star kicks occurring in binaries) with empirical estimates of the birth rates of other types of pulsars related to DNS formation\textsuperscript{24}. Binary progenitors of DNS systems experience two supernova explosions when the neutron stars are formed. The second supernova explosion (forming the neutron star that is not observed as a pulsar) provides a unique tool for the study of DNS formation, since the post-supernova evolution
of the system is simple, driven only by gravitational-wave radiation.

There are three possible outcomes after the second supernova: (i) a coalescing DNS is formed (CB), (ii) a wide DNS (with a coalescence time longer than the Hubble time) is formed (WB), or (iii) the binary is disrupted (D) and a single pulsar similar to the ones seen in DNS (DNS-like) is ejected. Based on supernova orbital dynamics we can calculate the probability branching ratios for these three outcomes, $P_{CB}$, $P_{WB}$, and $P_D$. For a given kick magnitude, we can calculate the maximum ratio $(P_{CB}/P_D)_{\text{max}}$ for the complete range of pre-supernova parameters defined by the necessary constraint $P_{CB} \neq 0$ (Figure 2). Given that the two types of systems have a common parent progenitor population, the ratio of probabilities is equal to the ratio of the birth rates $(BR_{CB}/BR_{D})$.

We can then use (i) the absolute maximum of the probability ratio ($\approx 0.26$ from Figure 2) and (ii) an empirical estimate of the birth rate of single DNS-like pulsars based on the current observed sample to obtain an upper limit to the DNS coalescence rate. The selection of this small-number sample involves some subtleties, but a preliminary analysis shows $BR_{CB} > 0.5 - 3 \times 10^{-5} \text{ yr}^{-1}$. Note that this number could be increased because of the small-number sample bias affecting this time the empirical estimate of $BR_D$ (see §3.1).

This is an example of how we can use observed systems other than DNS to improve our understanding of their coalescence rate. A similar calculation can be done using the wide DNS systems instead of the single DNS-like pulsars.

5 Conclusions

A comparison of the various results on the DNS coalescence rate indicates that theoretical estimates based on modeling of DNS formation have a quite limited predictive power (range of at least 3-4 orders of magnitude), whereas empirical estimates based on the observed DNS sample appear to be more robust (within a factor of $\sim 100$). Nevertheless, current estimates, which fall right around the range of interest for evaluating the prospects of gravitational-wave detection, still suffer from uncertainties and systematic effects (e.g., small-number statistics,
distances, luminosity function, beaming). Therefore, the need for additional constraints and alternative methods for estimates is clear. As an example, a fairly robust upper bound to the rate can be obtained making use all of the available information for other systems evolutionarily linked to DNS.

Apart from DNS systems, binaries with two black-holes or a black-hole and a neutron star are also of interest in the context of gravitational-wave detection and possibly gamma-ray bursts. Since at present there are no observed systems of this type, we have to rely on theoretically predicted formation rates for them, keeping in mind the normalization uncertainties associated with them. We note, however, that the absence of such systems from the observed samples combined with some basic understanding of their formation relative to DNS could provide valuable information for the frequency of their formation in our Galaxy.

Acknowledgments

I would like to thank T.A. Prince for discussions, R. Narayan for discussions and for providing me with the results of his Monte Carlo simulations, and D.R. Lorimer for providing me with an empirical estimate of the birth rate of single DNS-like pulsars. I would also like to thank D. Psaltis and F. Rasio for comments on an initial manuscript. I am grateful to the organizers for inviting me to the meeting and acknowledge full support by the Smithsonian Institute in the form of a CfA Post-doctoral Fellowship.

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