Maximum brightness temperature for an incoherent synchrotron radio source

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Summary. We discuss here a limit on the maximum brightness temperature achievable for an incoherent synchrotron radio source. This limit, commonly referred to in the literature as an inverse Compton limit, prescribes that the brightness temperature for an incoherent synchrotron radio source may not exceed \( \sim 10^{12} \) K, a fact known from observations. However one gets a somewhat tighter limit on the brightness temperatures, \( T_b < \sim 10^{11.5} \) K, independent of the inverse Compton effects, if one employs the condition of equipartition of energy in magnetic fields and relativistic particles in a synchrotron radio source. Pros and cons of the two brightness temperature limits are discussed.

1 Introduction

From the radio spectra combined with the VLBI (Very large Baseline Interferometry) observations it has been seen that the brightness temperatures for compact self-absorbed radio source do not exceed about \( \sim 10^{11−12} \) K. Kellermann and Pauliny-Toth [1] first time gave an explanation of this in terms of what has since then come to be known in the literature as an Inverse Compton limit. They argued that at brightness temperature \( T_b \geq 10^{12} \) K energy losses of radiating electrons due to inverse Compton effects become so large that these result in a rapid cooling of the system, thereby bringing the synchrotron brightness temperature quickly below this limit. Of course much larger brightness temperatures have been inferred for the variable sources, however this excess in brightness temperatures has been explained in terms of a bulk relativistic motion of the emitting component [2][3][4]. The Doppler factors required to explain the excess in temperatures were initially thought to be \( \sim 5−10 \), similar to the ones required for explaining the superluminal velocities seen in some compact radio sources [5][6][7]. Singal and Gopal-krishna [8] pointed out that under the conditions of equipartition of energy between magnetic fields and relativistic particles in a synchrotron radio source, much higher Doppler factors are needed to successfully explain the variability events.
Singal [9], without taking recourse to any inverse Compton effects, derived a somewhat tighter upper limit $T_b \lesssim 10^{11.5}$ K, first time using the argument that due to the diamagnetic effects the energy in the magnetic fields cannot be less than a certain fraction of that in the relativistic particles and then an upper limit on brightness temperature follows naturally. However, it has to be noted that if one considers the drift currents at the boundaries, which may be present due to the non-uniformities there in the magnetic fields [10], the above limit on the magnetic field energy gets modified. Later similar derivations [11] of the $T_b \lesssim 10^{11.5}$ K limit as well as of large Doppler factors, used essentially the same equipartition arguments as in [8, 9].

Here we first derive the inverse Compton limit on $T_b$ using the approach followed in [1], and then the equipartition limit as done in [9]. In fact we argue that even if we relax the condition of the equipartition of energy between magnetic fields and relativistic particles and let the energy in relativistic particles to be many orders of magnitude larger than that in the magnetic fields we still end up with a rather tight $T_b$ limit. On the other hand if the energy in particles is smaller than that in magnetic fields, then in any case $T_b$ has to be lower than $\sim 10^{11.5}$ K, as pointed out in [9].

2 Inverse Compton limit

We want to study the maximum brightness temperature limit in the rest frame of the source, therefore we assume that all quantities have been transformed to that frame. Hence we will not consider here any effects of the cosmological redshift or of the relativistic beaming due to a bulk motion of the radio source.

In inverse Compton interaction between relativistic electrons and their synchrotron photons, the energy of radio photons could get boosted to X-ray frequencies. The average energy of a photon in an inverse Compton interaction gets boosted by a factor $\gamma_e^2$ where $\gamma_e$ is the Lorentz factor of the interacting relativistic electrons [12]. We confine our discussion to a single scattering case only.

A relativistic electron of Lorentz factor $\gamma_e$ gyrating in a magnetic field $B$ emits most of its radiation in a frequency band near its characteristic synchrotron frequency [13] [14]

$$\nu_c = 0.29 \frac{3}{4\pi} \frac{e B}{m_0 c} \gamma_e^2$$

which gives us $\gamma_e^2 = 8.2 \times 10^2 \nu_c / B$ for $\nu_c$ in GHz and $B$ in Gauss. From this we find that for $B \sim 10^{-3}$ Gauss as inferred in compact radio sources, $\nu_c = 0.1$ to 10 GHz yields $\gamma_e^2 \sim 10^5$ to $\sim 10^7$. Thus during an inverse Compton interaction while at the lower end the synchrotron photons can get boosted to infrared frequencies ($\sim 10^{13}$) Hz, at the higher end the synchrotron photons of radio frequencies $\sim 10^{10}$ Hz could get boosted to X-ray frequencies ($\sim 10^{17}$) Hz.
The power radiated by the inverse Compton process, $P_c$, as compared to that radiated by synchrotron mechanism, $P_s$, is given as,

$$\frac{P_c}{P_s} = \frac{W_p}{W_b}$$  \hspace{1cm} (2)

where $W_p$ and $W_b$ respectively are the photon energy density and magnetic field energy density within the source. This relation is true in the case where Thomson scattering cross-section is valid \cite{12}, which is true in our case as we have $\gamma_e h \nu << m_0 c^2$, for $\nu < 100$ GHz.

The photon energy density is related to the radiation intensity as \cite{14}

$$W_p = \frac{4\pi}{c} \int_{\nu_1}^{\nu_2} I_\nu \, d\nu$$ \hspace{1cm} (3)

where the specific intensity $I_\nu$ is defined as the flux density per unit solid angle, at frequency $\nu$. In radio sources, the observed flux density in the optically thin part of the spectrum usually follows a power law, i.e., $I_\nu \propto \nu^{-\alpha}$, between the lower and upper cut off frequencies $\nu_1$ and $\nu_2$. In synchrotron theory this spectrum results from a power law energy distribution of radiating electrons $N(E) \propto E^{-\gamma}$ within some range $E_1$ and $E_2$, with $\gamma = 2\alpha + 1$ and $E_1, E_2$ related to $\nu_1, \nu_2$ by Eq. (1). In compact radio sources the source may become self-absorbed with flux density $\propto \nu^2$ below a turnover frequency $\nu_m$. From Eq.(3) we can get

$$W_p = 2.27 \times 10^{-7} \frac{f(\alpha) F_m \nu_m^\alpha}{\Theta_x \Theta_y} \left[ \frac{\nu_2^{-\alpha} - \nu_1^{-\alpha}}{1 - \alpha} \right] \text{erg cm}^{-3}$$ \hspace{1cm} (4)

the expression to be evaluated in the ‘limit’ for $\alpha = 1$. Here $F_m$ (Jy) is the flux density at frequency $\nu_m$ (GHz) corresponding to the point of spectral turnover, while $\Theta_x$ and $\Theta_y$ (mas) represent the angular size along the major and minor radio axes of the source component, assumed to be an ellipse. For calculating the photon energy density it may be appropriate to take the lower limit of the spectrum at $\nu_m$ itself, therefore we can put $\nu_1 = \nu_m$ in the above expression. Here it should be noted that $F_\nu$ is the flux density in the optically thin part of the synchrotron spectrum, accordingly $F_{\nu_m}$ obtained from an extrapolation up to $\nu = \nu_m$ using the straight slope $\alpha$, is not the same as the actual peak flux density $F_m$ at the turnover bend (see e.g., \cite{15}). In Table 1 values of $f(\alpha) = F_{\nu_m}/F_m$ are listed, which as we see are of the order of unity.

We can express $W_p$ in terms of the brightness temperature at the peak of the spectrum

$$T_m = 1.763 \times 10^{12} F_m \Theta_x^{-1} \Theta_y^{-1} \nu_m^{-2} \text{K}$$ \hspace{1cm} (5)

to get

$$W_p = 1.29 \times 10^{-7} \frac{f(\alpha)}{1 - \alpha} \left[ \left( \frac{\nu_2}{\nu_m} \right)^{1-\alpha} - 1 \right] \nu_m^3 \left( \frac{T_m}{10^{12}} \right) \text{erg cm}^{-3}.$$ \hspace{1cm} (6)
On the other hand, from the synchrotron self-absorption we get,
\[ B = 10^{-5} b(\alpha) F_{m}^{-2} \Theta_{x}^{2} \Theta_{s}^{2} \nu_{m}^{5}, \] (7)
values of \( b(\alpha) \) are given in Table 1.

Here a plausible assumption has been made that the direction of the magnetic field vector, with respect to the line of sight, changes randomly over regions small compared to a unit optical depth. For a uniform magnetic field direction throughout the source region, the co-efficients \( a(\alpha) \) (see next section) and \( b(\alpha) \) would be modified by factors of the order of unity.

Then the magnetic field energy density \( W_{b} = B^{2}/8\pi \) can be written as,
\[ W_{b} = 3.84 \times 10^{-11} b^{2}(\alpha) \nu_{m}^{2} \left( \frac{T_{m}}{10^{12}} \right)^{-4} \text{ erg cm}^{-3}. \] (8)
Equations (2), (6) and (8) lead us to,
\[ \frac{P_{c}}{P_{s}} = \nu_{m} \left( \frac{T_{m}}{10^{11.3} p(\alpha)} \right)^{5} \] (9)
where the function
\[ p(\alpha) = \left[ \frac{f(\alpha)}{b^{2}(\alpha)(1-\alpha)} \left\{ \left( \frac{\nu_{2}}{\nu_{m}} \right)^{1-\alpha} - 1 \right\} \right]^{-1/5} \] (10)
is of the order of unity (Table 1\(^{2}\)).

3 Equipartition temperature limit

Energy density of the relativistic electrons in a synchrotron radio source component is given by \[ W_{e} = 8.22 \times 10^{-9} \frac{F_{\nu}}{a(\alpha) (\alpha - 0.5)} B^{-1.5} \left( \frac{\nu_{1}(\alpha)}{\nu_{1}} \right)^{\alpha-0.5} - \left( \frac{\nu_{2}(\alpha)}{\nu_{2}} \right)^{\alpha-0.5} \] (11)
this expression to be evaluated in the ‘limit’ for \( \alpha = 0.5 \). Here \( W_{e} \) (erg cm\(^{-3}\)) is the energy density of radiating electrons; \( F_{\nu} \) (Jy) is the flux density at frequency \( \nu \) (GHz), with \( \nu_{1} < \nu < \nu_{2} \); and \( s(\text{pc}) \) is the characteristic depth of the component along the line of sight.

\(^{1}\) Calculated from the tabulated functions in \[13]. \(^{2}\) In Table 1 and 2 we have taken \( \nu_{1} \) and \( \nu_{2} \) to be 0.01 and 100 GHz, and the turnover frequency \( \nu_{m} \) is taken to be 1 GHz. \(^{3}\) Values of \( a(\alpha) \) in Table 1, calculated by us from the tabulated functions given in \[14], appear slightly different from the ones in \[13\] but are in agreement with those in \[17\].
Table 1. Various functions of the spectral index $\alpha$

| $\alpha$ | $\gamma$ | $a(\alpha)$ | $b(\alpha)$ | $f(\alpha)$ | $p(\alpha)$ | $t(\alpha)$ | $y_1(\alpha)$ | $y_2(\alpha)$ |
|----------|----------|-------------|-------------|-------------|-------------|-------------|--------------|--------------|
| 0.25     | 1.5      | 0.149       | 2.07        | 1.10        | 0.66        | 0.68        | 1.3          | 0.011        |
| 0.5      | 2.0      | 0.103       | 2.91        | 1.19        | 0.83        | 0.85        | 1.8          | 0.032        |
| 0.75     | 2.5      | 0.0831      | 2.85        | 1.27        | 0.94        | 0.80        | 2.2          | 0.10         |
| 1.0      | 3.0      | 0.0741      | 2.52        | 1.35        | 1.00        | 0.67        | 2.7          | 0.18         |
| 1.5      | 4.0      | 0.0726      | 1.79        | 1.50        | 1.03        | 0.43        | 3.4          | 0.38         |

In terms of the brightness temperature we can write,

$$W_e = 4.66 \times 10^{-9} f(\alpha) \left[ \frac{y_1(\alpha)}{\nu_1/\nu_m} \right]^{\alpha - 0.5} - \left( \frac{y_2(\alpha)}{\nu_2/\nu_m} \right)^{\alpha - 0.5} \left[ \frac{\nu_2^{2.5}}{\nu_m^{1.5}} \left( \frac{T_m}{10^{12}} \right) \right]^{1/8}.$$

(12)

From the overall charge neutrality of the plasma we expect the electrons to be accompanied by an equal number of positive charges. Any positrons would have already been accounted for in our equation above, however presence of heavy particles will contribute additionally to the total particle energy density. Let the energy in the heavy particles be $\xi$ times that in the lighter particles, then the total particle energy density is $W_k = (1 + \xi) W_e$.

Now if we assume the energy in particles to be related to that in magnetic fields by

$$W_k = \eta W_h,$$

then we get

$$B^{7/2} = 1.17 \times 10^{-7} f(\alpha) \left[ \left( \frac{y_1(\alpha)}{\nu_1/\nu_m} \right)^{\alpha - 0.5} - \left( \frac{y_2(\alpha)}{\nu_2/\nu_m} \right)^{\alpha - 0.5} \right] \times \left( \frac{1 + \xi}{\eta} \right) \frac{\nu_2^{2.5}}{\nu_m^{1.5}} \left( \frac{T_m}{10^{12}} \right).$$

(14)

Now substituting for magnetic field from Eq.(7) we get,

$$\eta \left( \frac{1 + \xi}{1 + \xi} \right) = \frac{1}{s \nu_m} \left( \frac{T_m}{10^{10.9} t(\alpha)} \right)^8$$

(15)

where the function

$$t(\alpha) = \left[ \frac{f(\alpha)}{(\alpha - 0.5) a(\alpha) b^{3.5}(\alpha)} \left\{ \left( \frac{y_1(\alpha)}{\nu_1/\nu_m} \right)^{\alpha - 0.5} - \left( \frac{y_2(\alpha)}{\nu_2/\nu_m} \right)^{\alpha - 0.5} \right\} \right]^{-1/8}$$

(16)

is of the order of unity (Table 1).
4 A Correction to the derived $T_m$ values

Actually $T_m$ values have been calculated (both here as well as in the literature) for the turnover point in the synchrotron spectrum where the flux density peaks. However, the definition of the brightness temperature (Eq. 5) also involves $\nu^{-2}$. Therefore a maxima of flux density is not necessarily a maxima for the brightness temperature also. In fact a zero slope for the flux density with respect to $\nu$ would imply for the brightness temperature a slope of $-2$. Therefore the peak of the brightness temperature will be at a point where flux density $\propto \nu^{-2}$, so that $T_b \propto F \nu^{-2}$ has a zero slope with respect to $\nu$. The peak of $F_\nu \nu^{-2}$, can be determined in the following manner. The specific intensity in a synchrotron self-absorbed source is given by \[ I_\nu = \frac{c_5(\alpha)}{c_6(\alpha)} \left( \frac{\nu}{2c_1} \right)^{2.5} \left[ 1 - \exp \left\{ - \left( \frac{\nu}{\nu_1} \right)^{-(\alpha+2.5)} \right\} \right] B_{\perp}^{-0.5} \] where $c_1, c_5(\alpha), c_6(\alpha)$ are tabulated in [14]. The optical depth varies with frequency as $\tau = (\nu/\nu_1)^{-(\alpha+2.5)}$, $\nu_1$ being the frequency at which $\tau$ is unity. The equivalent brightness temperature (in Rayleigh-Jeans limit) is then given by \[ T_\nu = \frac{c^2 c_5(\alpha)}{8 k c_1^2 c_6(\alpha)} \left( \frac{\nu}{2c_1} \right)^{0.5} \left[ 1 - \exp \left\{ - \left( \frac{\nu}{\nu_1} \right)^{-(\alpha+2.5)} \right\} \right] B_{\perp}^{-0.5} \] where $k$ is the Boltzmann constant and $c$ is the speed of light. We can maximize $T_\nu$ by differentiating it with $\nu$ and equating the result to zero. This way we get an equation for the optical depth $\tau_o$, corresponding to the peak brightness temperature $T_o$, which is different from the one that is available in the literature for the optical depth $\tau_m$ at the peak of the spectrum. The equation that we get for $\tau_o$ is \[ \exp(\tau_o) = 1 + (2\alpha + 5) \tau_o, \] solutions of this transcendental equation for different $\alpha$ values are given in Table 2. It is interesting to note that while the peak of the spectrum for the typical $\alpha$ values usually lies in the optically thin part of the spectrum ($\tau_m \lesssim 1$; Table 2), peak of the brightness temperature lies deep within the optically thick region ($\tau_o \sim 3$). Both the frequency and the intensity have to be calculated for $\tau_o$ to get the maximum brightness temperature values. The correction factors are then given by, \[ \frac{\nu_o}{\nu_m} = \left( \frac{\tau_m}{\tau_o} \right)^{1/(\alpha+2.5)} \] \[ \frac{T_o}{T_m} = \left( \frac{\nu_o}{\nu_m} \right)^{0.5} \left[ 1 - \exp(-\tau_o) \right] \left[ 1 - \exp(-\tau_m) \right]^{-0.5} \]
Table 2. Values for the temperature limits

| α   | γ   | τ_m | τ_0 | ν_0/ν_m | T_0/T_m | log (T_{ic}) | log (T_{eq}) |
|-----|-----|-----|-----|---------|---------|--------------|--------------|
| 0.25| 1.5 | 0.19| 2.80| 0.37    | 3.5     | 11.7         | 11.3         |
| 0.5 | 2.0 | 0.35| 2.92| 0.50    | 2.2     | 11.6         | 11.2         |
| 0.75| 2.5 | 0.50| 3.03| 0.58    | 1.8     | 11.5         | 11.1         |
| 1.0 | 3.0 | 0.64| 3.13| 0.64    | 1.6     | 11.5         | 10.9         |
| 1.5 | 4.0 | 0.88| 3.32| 0.72    | 1.4     | 11.5         | 10.7         |

In Table 2 we have listed ν_0/ν_m and T_0/T_m, for different α values. Further we have also tabulated values of maximum brightness temperatures log (T_{ic}) for W_p = W_b and log (T_{eq}) for η = 1 (with ξ = 0, s = 1pc) calculated from equations (9) and (15) respectively for different α values, incorporating the above corrections.

### 5 Discussion and Conclusions

From Table 2 we see that, depending upon the spectral index value α, theoretical maximum brightness temperature values for the inverse Compton limits are around T_{ic} ∼ 10^{11.6±0.1} K while for equipartition conditions the limits are lower by a factor of ∼ 3 (T_{eq} ∼ 10^{11.0±0.3} K). From the observational data [18] it seems that the intrinsic T_b is ≲ 10^{11.3} K, which is broadly consistent with the either interpretation. It should be noted that though inverse Compton scattering increases the photon energy density, yet it does not increase the radio brightness as the scattered photons get boosted to much higher frequency bands. If anything, some photons get removed from the radio window, but the change in radio brightness due to that may not be very large. What could be important is the large energy losses by electrons which may cool the system rapidly. However, these inverse Compton losses become important only when T_b > 10^{11.5} K, but the equipartition conditions may keep the temperatures well below this limit. This is not to say that inverse Compton effects cannot occur, it is only that conditions in synchrotron radio sources may not arise for inverse Compton losses to become very effective. Since we are considering the brightness temperature limit in the radio-band (after all that is where observationally such limits have been seen), then variations in ν_m that we may consider would at most be about an order of magnitude around say, 1 GHz. With the reasonable assumption that a self-absorbed radio source size may not be much larger than ∼ a pc, from Eq. (15) it follows that an order of magnitude higher T_m values would require η to increase by about a factor ∼ 10^8, that is departure from equipartition will go up by about eight orders of magnitude. Actually for a given ν_m, the magnetic field energy density will go down by a factor ∼ 10^4 (Eq. 8), while that in the relativistic particles will go up by a similar factor (Eq. 12, note the presence of B^{1.5} in the denomi-
nator). This also implies that the total energy budget of the source will be higher by about $\sim 10^4$ than from the already stretched equipartition energy values. Further, it is not the photon energy density that really goes up drastically with higher brightness temperatures (as $W_p \propto T_m^4$, Eq. 6), rather it is the drastic fall in the magnetic field energy ($W_b \propto T_m^{-4}$, Eq. 8) that increases the ratio $P_c/P_s$ to go up as $T_m^5$. Therefore, it is still not clear whether the inverse Compton effects actually do play a significant role in maintaining the maximum brightness temperature limit in incoherent synchrotron radio sources.

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