Muon anomalous magnetic moment
due to the brane-stretching effect

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Abstract

We investigate the contribution of extra dimensions to the muon anomalous magnetic moment by using an ADD-type 6-dimensional model. This approach analyzes the extent of the influence of classical brane fluctuations on the magnetic moment. When we consider that the brane fluctuations are static in time, they add new potential terms to the Schrödinger equation through the induced vierbein. This paper shows that the brane fluctuation is responsible for the brane-stretching effect. This effect would be capable of reproducing the appropriate order for recent Brookhaven National Laboratory measurements of the muon \((g-2)\) deviation.

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I. INTRODUCTION

The BNL E821 group recently reported the precision measurement of the muon anomalous magnetic moment. Based on their result, a new world average recorded \( a_{\mu}^{(\text{exp})} = 11659208(6) \times 10^{-10}(\pm 0.7\text{ppm}) \) \(^1\), whereas Höcker et al. obtained the Standard Model (SM) prediction \( a_{\mu}^{(\text{SM})} = 11659182(6) \times 10^{-10}(\pm 0.7\text{ppm}) \) \(^2\). The difference in values, \( \Delta a_{\mu} \equiv a_{\mu}^{(\text{exp})} - a_{\mu}^{(\text{SM})} = (26 \pm 9.4) \times 10^{-10} \), suggested that the SM does not strictly hold in the low energy region when the difference exceeds the calculation uncertainties of the Hadronic process and measurement error. This difference has been extensively analyzed by various approaches, such as supersymmetry \(^3\), lepton flavor violation \(^4\), extra dimensions \(^5\), etc. \(^6\). However, there is no conclusive explanation for this observed deviation.

In this context, we attempt to estimate the order of the muon anomalous magnetic moment by using a braneworld model (see \(^7\) for recent reviews). The general formalism using higher dimensional physics was constructed by R. Sundrum \(^8\). This formalism suggests that the SM particles are constrained to live on the world volume of a (3+1)-dimensional hypersurface or a “3-brane,” while only gravity freely propagates in bulk space-time.

The most important aspect of this theory is that the metric and the vierbein are replaced by the induced metric and the induced vierbein, respectively \(^8, 9\). Hence, higher dimensional gravity can be discussed apart from the usual Kaluza-Klein (KK) theory. In general, the KK modes require periodic limits such as the torus structure in extra dimensions. However, the braneworld scenarios need not have these limits because the configuration of extra dimensions is determined by gravity, the position of branes, the cosmological constant, etc. In this paper, we adopt the factorizable 6-dimensional braneworld model of the ADD type (the name is derived from the paper by N. Arkani-Hamed, S. Dimopoulos, and G. Dvali \(^10\)), which has a theoretical motivation that explains the gauge hierarchy problem, i.e., the reason for the scale of electroweak symmetry breaking being so much smaller than the scales of quantum gravity or grand unification. This model has extra compact spaces, and it finds a simple exact solution to the Einstein equation, including explicit brane sources. Applying Gauss’s law to this model, we have

\[
M_{Pl}^2 = M_f^4 V(2),
\]

where \(M_{Pl} \approx 10^{19} \, \text{[GeV]}\), \(M_f\), and \(V(2)\) denote the 4-dimensional Planck mass, 6-dimensional fundamental Planck mass, and volume of the two extra dimensions, respectively. This
relation will be crucial in our discussion because it can give $M_f \approx 1 \text{ [TeV]}$ for $V_{(2)}^{1/2} \approx r = 0.1 \text{ [mm]}^1$. This implies that gravity would be unified with other forces on a TeV scale. On the other hand, there exists an established higher dimensional model, the RS-model \cite{11}, which could possibly resolve the hierarchy problem. However, we shall specifically concentrate on the 6-dimensional ADD-type model \cite{12,13}. This involves the two 3-branes (i.e., our world and another world) and the $U(1)$ gauge field in the bulk. The model can realize a mechanism that does not require any fine-tuning between the brane tension and bulk parameters; this implies that the brane tension can be freely changed. This type of model is referred to as a self-tuning model \cite{13,14}, which is one of the simplest models for exploring the braneworld phenomenology and the effects of extra dimensions.

This paper is organized as follows. Section II introduces some basic notations. Section III comprises a brief explanation of the 6-dimensional model and the scaling property of 4-dimensional physics, section IV focuses on the brane-stretching effect and the estimation of the order of muon $(g-2)$, and section V is the conclusion.

II. SETUP

The effective theory has presented a picture of the low-energy dynamics of a 3-brane universe, i.e., the SM particle is confined to the braneworld-volume topology as $M_4$. Further, only gravity is free to move in bulk space-time with $d > 4$ dimensions, $M_4 \times S^{d-4}$ topology, where $S^d$ denotes the $d$-sphere. The coordinates of bulk space are denoted by $X^M$, where the ones on the brane are denoted by $x^\mu$ and the extra dimensions by $y^m$. The curved bulk coordinate indices, which run over all dimensions, are denoted by uppercase Roman letters beginning from the middle: $M, N \cdots = 0, \cdots d - 1$; the indices denoted by Greek letters run over the first four dimensions: $\mu, \nu, \cdots = 0, 1, 2, 3$; and the indices denoted by lowercase Roman letters run over the remaining $d - 4$ dimensions: $m, n, \cdots = 4, \cdots d - 1$. The local Lorentz indices in the bulk are similarly denoted: the indices denoted by uppercase Roman letters run over all dimensions: $A, B \cdots = 0, \cdots d - 1$; Greek letters run over the first four dimensions: $\alpha, \beta \cdots = 0, 1, 2, 3$; the indices denoted by lowercase Roman letters run over the remaining $d - 4$ dimensions: $a, b \cdots = 4, \cdots d - 1$ (see Table I).

1 The conversion factor is $1[\text{GeV}^{-1}] = 2 \cdot 10^{-13} \text{[mm]}$. 

The bulk metric $G_{MN}$ describes the fundamental gravitational degrees of freedom. The Lorentz metric in the bulk is $\eta_{AB}$, and vielbein is $E^A_M(X)$. The bulk and Lorentz metrics are related by the following equation:

$$E^A_M(X)\eta_{AB}E^B_N(X) = G_{MN}(X)$$

$$E^A_M(X)G^{MN}(X)E^B_N(X) = \eta^{AB}.$$ 

The bulk coordinates occupied by a point $x$ on the brane are denoted by $Y^M(x)$. However, since the theory has reparametrization invariance, a different parametrization of the surface describing the brane $x \rightarrow x'(x)$ would lead to the same physics. Therefore, it is necessary to identify the coordinates spanned by the brane with the first four bulk components in order to eliminate the non-physical components from $Y^M(x)$. Hence, we choose the gauge fixing condition

$$Y^\mu(x) = x^\mu.$$ 

### III. SCALING PROPERTY

We review the 6-dimensional model with two brane sources in two extra dimensions, where the brane and the extra space have an $M_4$ and an $S^2$ topology, respectively [13, 14, 15, 16, 17, 18, 19, 20, 21]. The total action consists of the 6-dimensional Einstein-Maxwell action and the two brane actions with negative tension. In this model, the stability of bulk geometry and brane fluctuations requires the negative tension brane.

First, in order to obtain a background solution, we discuss the description that does not consider the localized fields on brane and brane fluctuations. The effective action is shown
to be as follows:

\[ S_{\text{total}} = S_{\text{branes}} + S_6 \]

\[ = -T_0 \int d^4x \sqrt{-g} - T_1 \int d^4x \sqrt{-g} + \int d^6x \sqrt{-G} \left[ M_f^4 R_6 - \Lambda_6 - \frac{1}{4} F_{MN}^2 \right]. \] (5)

Here, \( T_i \) \((i = 0, 1)\) denotes the brane tension, \( M_f \) is the 6-dimensional Planck mass, \( \Lambda_6 \) is the 6-dimensional cosmological constant, and \( F_{MN} \) is the 6-dimensional 2-form field strength.

We can obtain the 6-dimensional Einstein equation including brane sources by varying the action with respect to the 6-dimensional metric. We consider that a brane is located on a conical singularity in the extra dimensions. Fortunately, in this scenario, we can easily obtain a solution that maintains a 4-dimensional Minkowski space-time, because the equation can split into 4- and 2-dimensional components. Thus, we obtain the solution

\[ ds_6^2 = \eta_{\mu \nu}dx^\mu dx^\nu + \gamma_{mn}(y)dy^m dy^n, \] (6)

and the solution for the equation of motion for \( F_{MN} \) as

\[ F_{mn} = \sqrt{\gamma} B_0 \epsilon_{mn}, \] (7)

where \( B_0 \) is a constant, \( \gamma \) is the determinant of \( \gamma_{mn} \), and \( \epsilon_{mn} \) is a completely antisymmetric tensor, i.e., \( \epsilon_{45} = -\epsilon_{54} = 1 \). Solution (7) denotes a magnetic flux through the compactified two extra dimensions.

In this background, the simplest technique to realize the stabilized bulk geometry would be to locate two fixed branes having identical tensions, \( T_0 = T_1 \), at opposite poles of the spherical two extra dimensions. This condition can be ensured by imposing a \( \mathbb{Z}_2 \) symmetry at the equator [13]. Further, using the conformal symmetry, we can then obtain the solution

\[ ds_6^2 = \eta_{\mu \nu}dx^\mu dx^\nu + a_0^2 (d\theta^2 + \alpha^2 \sin^2 \theta d\phi^2), \] (8)

if the parameters \( B_0 \) and \( \lambda_6 \) satisfy

\[ \frac{1}{a_0^2} = \frac{B_0^2}{2M_f^4}, \quad \lambda_6 = \frac{B_0^2}{2}. \] (9)

These relations are necessary to maintain a 4-dimensional Minkowski space-time and spherical two extra dimensions. The solution has the following relation on the conical singularity;

\[ \delta = 2\pi(1 - \alpha) = \frac{T_0}{2M_f^2}. \] (10)

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2 The metric signature is diag \((+, -, -, - ,-, -)\).
where $\delta$ is the deficit angle of the two extra dimensional sphere, and $\alpha$ is a dimensionless fixed parameter, $0 < \alpha < 1$. On the basis of a property in 2-dimensional gravity\textsuperscript{22}, the Einstein equation for the extra dimensional component presents a solution that removes a wedge from the sphere and was identified with opposite sides of the wedge. Thus, the 4-dimensional component remains exactly Lorentz invariant because the change in the tension affects only the geometry of the extra dimensions. This means that the tension can be freely changed since there is no fine tuning between bulk parameters and brane tension. This type of model is referred to as a self-tuning model.

In the following, we will briefly describe the manner in which this mechanism affects 4-dimensional physics. The change in $T_0$ retains the regular part of the geometry and modifies only the singular part of the geometry, i.e., the deficit angle $\delta$ given by (10). This results in a change in the bulk volume related to $M_{Pl}$ by (11). Hence, the change in $T_0$ signifies a change in $M_{Pl}$. Interestingly, a self-tuning model of this type can be constructed only in six dimensions\textsuperscript{14}.

Subsequently, we focus on fermion $\psi(x)$ and gauge field $A_\mu(x)$ and ignore scalar field on brane. However, prior to the discussing these behaviors, we should elaborate on a covariant derivative for the fermion. It behaves as a spin 1/2-spinor under the local Lorentz group. Lorentz generators of $n$-dimensional spinor representation are usually denoted as:

$$\sigma_{(\alpha\beta)} = \frac{1}{4}[\gamma_\alpha, \gamma_\beta],$$

where $\gamma_\alpha$ represents a set of Dirac matrices satisfying the following condition:

$$\{\gamma_\alpha, \gamma_\beta\} = 2\eta_{\alpha\beta}.$$\textsuperscript{12}

The local Lorentz group on the brane is regarded as an internal $SO(3,1)$ group, which connects the Minkowski space with the curved space through the vielbein that satisfy Eqs. (2) and (3). The covariant derivative that maintains the Lorentz and gauge symmetry for $\psi$ is

$$D_\mu = \partial_\mu - ieA_\mu - \frac{1}{2}\omega^{\alpha\beta}_\mu \sigma_{(\alpha\beta)},$$

$$\omega^{\alpha\beta}_\mu = \frac{1}{2}e^{\alpha\nu}(\partial_\mu e^{\beta}_\nu - \partial_\nu e^{\beta}_\mu) + \frac{1}{4}e^{\alpha\nu}e^{\beta\sigma}(\partial_\sigma e^{\gamma}_\nu - \partial_\nu e^{\gamma}_\sigma)e_{\gamma\mu} - (\alpha \leftrightarrow \beta),$$\textsuperscript{13}

Thus, the effective brane action is as follows:

$$S_{brane} = \int d^4x\sqrt{-g}\left[-T_0 + i\bar{\psi}\gamma_\mu D_\mu \psi - m_f \bar{\psi}\psi - \frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} + \cdots\right],$$\textsuperscript{15}
where the ellipsis represents the higher dimensional interactions that can be constructed with coefficients given by powers of $1/M_f$, and $m_f$ is the mass parameter of the fermion in fundamental gravity.

In the following, we discuss only the effect of the brane tension on 4-dimensional physics (see [14] for details). The higher dimensional theory that results in a change in $M_{Pl}$ generates an effective theory depending on the change in $M_{Pl}$. Thus, the 4-dimensional effective action consists of

$$S_{\text{eff}} = M_{Pl}^2(T_0) \int d^4x \sqrt{-g}R_4 + \int d^4x \sqrt{-g}\mathcal{L}_4,$$

where $M_{Pl}$ is dependent on $T_0$ as follows:

$$M_{Pl}^2(T_0) = \left[1 - \frac{T_0}{4\pi M_f^4}\right] M_{Pl}^2(0),$$

where $M_{Pl}(0)$ represents the Planck mass in the absence of branes. When we rescale $g_{\mu\nu} = \tilde{g}_{\mu\nu}/\alpha$, we obtain

$$S_{\text{eff}} = M_{Pl}^2(0) \int d^4x \sqrt{-\tilde{g}}\tilde{R}_4 + \int d^4x \sqrt{-\tilde{g}}\tilde{\mathcal{L}}_4(\alpha).$$

It is obvious that the $\alpha$ dependence shifts from the Planck mass to the fields localized on the brane. Hence, after rescaling the fermions as $\psi = \alpha^{\frac{3}{4}}\tilde{\psi}$ on the basis of (15), we obtain

$$S_4 = \int d^4x \sqrt{-\tilde{g}} \left[i\bar{\tilde{\psi}}\gamma^\alpha \left(\partial_\mu - ieA_\mu + \frac{1}{2}\omega^\beta_\mu \sigma(\beta\gamma)\right)\tilde{\psi}ight.$$

$$- \frac{m_f}{\sqrt{\alpha}}\bar{\tilde{\psi}}\tilde{\psi} - \frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}\left].
$$

Based on the redefinition $\psi = \tilde{\psi}$ and $g_{\mu\nu} = \tilde{g}_{\mu\nu}$, we recognize the action as invariant, except for the mass term. Thus, since $\alpha$ is the fixed parameter, we can regard $m_f/\sqrt{\alpha}$ as a physical mass $m$. As a result, the effect of bulk gravity does not become apparent in the 4-dimensional world. This implies that if fermion is massless, the action becomes scale-invariant, i.e., the scale invariance is broken by fermion mass. The usual field theory also maintains this property. In the next section, we consider the effect of the brane tension and brane fluctuations. The 4-dimensional field theory should be extended to a brane world that maintains this property. Further, we show that the scale transformation is instrumental in restricting the form of the induced metric.
IV. APPLICATION TO MUON (G-2)

We estimate the muon (g-2) deviation by assuming that brane fluctuations are static in time. The new compensation terms occur through the induced vierbein. This would lead to the possibility of compensating the magnetic moment which has a static property. Under gauge fixing condition (11), the induced metric is as follows:

\[ g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{mn} \partial_\mu Y^m \partial_\nu Y^n. \]  

(20)

For simplicity, we suppose that off-diagonal components of the 6-bein are zero, as shown below:

\[ E^A_M(X) = \begin{pmatrix} \delta_\alpha^\mu & 0 \\ 0 & E_\alpha^m \end{pmatrix}. \]  

(21)

In order to obtain the induced vierbein on the brane, we use the following definition [8]:

\[ e_\alpha^\mu \equiv R_\alpha^A E^A_M(X) \partial_\mu Y^M. \]  

(22)

Thus, the induced vierbein obtains, up to the second order;

\[ e_\alpha^\mu = \delta_\alpha^\mu + \frac{1}{2} \gamma_{mn} \partial^\alpha Y^m \partial_\mu Y^n + O(\epsilon^4). \]  

(23)

The expansion of \( \sqrt{-g} \) of induced metric [20] becomes

\[ \sqrt{-g} = 1 - \frac{1}{2} \partial^\mu Y^m \partial_\mu Y^m + \cdots. \]  

(24)

The ellipsis consists of higher dimension terms of \( \partial_\mu Y^m \) in pairs. When the above expansion is substituted into the minimal brane action

\[ S_{\text{brane}} = \int d^4 x \sqrt{-g} \left[ -T_0 + \mathcal{L}_4(g_{\mu\nu}) \right], \]  

(25)

we obtain

\[ S_{\text{brane}} = S^{(0)}_{\text{eff}} + S^{(2)}_{\text{eff}} + \cdots, \]  

(26)

\[ S^{(0)}_{\text{eff}} = \int d^4 x \left[ -T_0 + \mathcal{L}_4(\eta_{\mu\nu}) \right], \]  

(27)

\[ S^{(2)}_{\text{eff}} = \int d^4 x \left[ \frac{T_0}{2} \partial_\mu Y^m \partial^\mu Y^m + \frac{1}{2} \partial_\mu Y^m \partial_\nu Y^m T_{4 \mu \nu} \right], \]  

(28)
where $T_{4}^{\mu\nu}$ is the conserved energy-momentum tensor of matter fields evaluated in the 4-dimensional Minkowski space-time. Considering the canonically normalized condition for $\partial_{\mu}Y^{m}$ in (28), we can put

$$\partial_{\mu}Y^{m}\partial^{\mu}Y^{m} \rightarrow \frac{1}{T_{0}}\partial_{\mu}Y^{m}\partial^{\mu}Y^{m}. \quad (29)$$

$Y^{m}$ is considered as the Nambu-Goldstone mode associated with the spontaneous isometry breaking due to the presence of the brane in bulk [8][12][24, 25, 26, 27]. Before discussing muon (g-2), the relation between the brane fluctuations and the scaling property mentioned in section III should be noted. On the assumption that the change in $Y^{m}$ is static in time, the induced metric becomes

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & \eta_{ij} + \frac{1}{T_{0}}\gamma_{mn}\partial_{i}Y^{m}\partial_{j}Y^{n} \end{pmatrix},$$

(30)

where $i, j = 1, 2, 3$: indices are raised and lowered by the Euclidean metric $\delta_{ij} = -\eta_{ij}$. The 4-dimensional field theory is scale-invariant for massless fermions and gauge fields, but not for massive fermions. We consider that the braneworld would preserve this property. Thus, induced metric (30) requires the following rescaling for $\eta_{\mu\nu} \Rightarrow \eta_{\mu\nu}/\alpha$:

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & \eta_{ij} + \frac{1}{T_{0}}\gamma_{mn}\partial_{i}Y^{m}\partial_{j}Y^{n} \end{pmatrix} \Rightarrow \frac{1}{\alpha}\begin{pmatrix} 1 & 0 \\ 0 & \eta_{ij} + \frac{1}{T_{0}}\gamma_{mn}\partial_{i}Y^{m}\partial_{j}Y^{n} \end{pmatrix}.$$ 

(31)

However, the existence of the brane breaks the isometry symmetry. It denotes that the 6-dimensional bulk is separated into the 4-dimensional branes and 2-dimensional extra dimensions. This implies that $\gamma_{mn}$ does not have the abovementioned transformation because we can rescale $G_{MN} \Rightarrow G_{MN}/\alpha$ if and only if no brane exists in the bulk. Therefore, in order to recover the scaling property, we restrict the form of $g_{ij}$:

$$g_{ij} = \eta_{ij} + \eta_{ij} \frac{1}{T_{0}}H^{2}M_{f}^{2}, \quad (32)$$

where $H$ has a mass dimension of 1. This form guarantees that the $\alpha$ dependence changes from $M_{Pl}(\alpha)$ into the fermion mass in the same way as action (19). Subsequently, we present
a solution \( Y^m \) that satisfies (32). The \( Y^m \) equation of motion derived from effective action (28) is written as
\[
\partial_\mu \left[ \partial^\mu Y^m + \frac{1}{T_0} \partial_\nu Y^m T^{\mu\nu}_4 \right] = 0. \tag{33}
\]
When we introduce the dimensionless coordinate \( Ex^i \) that characterizes the physical process at energy \( E \), we parametrize \( Y^m \) as follows (see Fig.1):
\[
Y^m(x) = Y_0^m + M_f \tilde{e}_i^m Ex^i, \tag{34}
\]
where \( Y_0^m \) is a constant and the basis vectors
\[
\frac{\partial Y^m}{\partial x^i} = M_f E \tilde{e}_i^m \tag{35}
\]
satisfy the completeness relation
\[
\gamma_{mn} \frac{\partial Y^m}{\partial x^i} \frac{\partial Y^n}{\partial x^j} = M_f^2 E^2 \eta_{ij}, \tag{36}
\]
i.e.,
\[
\gamma_{mn} \tilde{e}_i^m \tilde{e}_j^n = \eta_{ij}. \tag{37}
\]
The coordinate \( Y^m \) (34) satisfies (33) and maintains (32) as \( H = E \). This implies that the spatial part of the brane is stretched due to brane fluctuations, whose magnitude depends on the energy scale of the physical process. This is physically plausible because under general relativity, space-time is not rigid but dynamical. In addition, solution (34) is consistent with the general covariance of general relativity. Substituting (34) into spin connection (14) via induced vierbein (23), we can directly obtain
\[
\omega^\alpha_{\mu\beta} = - \frac{1}{2} \partial_\mu \partial^\beta Y^m \partial^\alpha Y^m - \frac{1}{2} \partial_\mu \partial^\alpha Y^m \partial^\beta Y^m \tag{38}
\]
\[
= 0.
\]
The vanishing of the spin connection denotes that the equation of motion for a fermion agrees with laws of special relativity. Therefore, solution (34) supports Lorentz symmetry.

In the following, we will see that the brane-stretching effect generates the suitable order for muon (g-2). The variation of action (25) with respect to \( \bar{\psi} \) yields the equation of motion:
\[
\left[ i e^\alpha_\mu \gamma^\alpha \left( \partial_\mu - i e A_\mu - \frac{1}{2} \omega_\mu^{\beta\gamma} \sigma_{(\beta\gamma)} \right) - m \right] \psi = 0. \tag{39}
\]
FIG. 1: The existence of brane separates bulk space-time into 4D-space-time and extra space.

(a) The static brane fluctuation generally allows the extra dimensional coordinate \( y^m \) to acquire a dependence of spatial coordinates \( x^i \) \( (i = 1, 2, 3) \), \( y^m = Y^m(x) \). This implies the existence of a local frame given by the set of three basis vectors \( \tilde{e}^m_i(x) \) which are tangent to the spatial part of 3-brane. (b) However, on a flat brane, we consider the basis vectors that do not depend on the local coordinate. Using these considerations, we can parametrize the coordinate \( Y^m \) as \( Y^m = Y_0^m + M_f \tilde{e}^m_i E x^i \) based on the dimensional analysis; this is valid at the lower scale \( E \ll M_f \).

Then, we perform a nonrelativistic approximation, i.e., the Schrödinger approximation. This is demonstrated in Appendix by using the static induced metric (30) and solution (34). Since the two extra dimensions give the relation

\[
M_{pl}^2 = 4\pi a_0^2 \alpha M_f^4
\]  

(40)

by (11), we obtain the anomalous magnetic moment:

\[
a_{\mu} = \frac{1}{T_0} E^2 M_f^2
\]

= \frac{1}{4\pi M_f^2 (1 - \alpha)} E^2 M_f^2

(42)

= \frac{a_0^2 \alpha}{M_{pl}^2 (1 - \alpha)} E^2 M_f^2.

(43)

Finally, since we are interested in the physics at the muon scale \( E \approx 106[\text{MeV}] \) and
\( M_f \approx 1[\text{TeV}] \) for \( a_0 \approx 0.1 \, [\text{mm}] \), we obtain the following:

\[
a_\mu \approx \frac{\alpha}{1 - \alpha} \left( \frac{0.1[\text{mm}]}{10^{19}[\text{GeV}]} \right)^2 \times \left( 106[\text{MeV}] \cdot 1[\text{TeV}] \right)^2
\]

\[
= \frac{\alpha}{1 - \alpha} \times 10^{-10}.
\]

This result almost reproduces the deviation of the muon \( (g-2) \) measurement, except for the previous dimensionless factor. \( \alpha \) may be determined by future studies on the self-tuning mechanism [14, 15, 16, 17, 18, 19, 20, 21]. However, it is important that we consider its behavior in the bound \( 0 < \alpha < 1 \) because it is possible that \( \alpha \) has an extreme value. If \( \alpha \to 1 \), muon \( (g-2) \) has a value greater than the experimental result. On the contrary, if \( \alpha \to 0 \), muon \( (g-2) \) has a small value. Moreover, it generates a large hierarchy between the fundamental parameter \( M_f \) and \( m_f \). Consequently, when \( \alpha \) has a moderate value, this model would be capable of reproducing \( \Delta a_\mu \equiv a_\mu^{(\text{exp})} - a_\mu^{(\text{SM})} = (26 \pm 9.4) \times 10^{-10} \).

As a side remark, from recent astrophysical research, it is known that the bounds on the mass of KK-gravitons [28] impose much tighter constraints on the radius of Large extra dimension. These suggest the exclusion of the TeV scale gravity. This indicates that we need to consider much more than the TeV scale. However, even in this case, if the order of \( a_0 \) is smaller than 0.1 \([\text{mm}]\), the region \( \alpha \to 1 \) can give the appropriate \( (g-2) \) value if \( \alpha \) is suitably selected.

V. CONCLUSION

This paper has presented a new approach according to which brane fluctuations compensate for the muon anomalous magnetic moment. The most important fact to be considered is that we have obtained a new potential term for the magnetic moment based on the assumption that brane fluctuations are static in time. This method reflects the effect of a novel classical contribution, namely, brane-stretching effect due to brane fluctuations, which is not based on the previously studied KK-gravitons [3]. In particular, we would obtain a suitable order for \( a_\mu \) in the 6-dimensional model. This implies that the SM is consistently extended to the braneworld model that maintains the usual scaling property and Lorentz invariance for fermion. In future research, we should promote the investigation of \( a_\mu \) by using the metric constructed by other higher dimensional models. Moreover, we can expect that the brane-stretching effect will evolve into different configurations in a very high energy.
This may be related to the Lorentz violation \cite{29}. Since our study leaves a lot of issues to be discussed further, we are confident that this will be a crucial subject on which further research should be conducted.

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APPENDIX: THE SCHÖDINGER APPROXIMATION

In this appendix, we demonstrate the non-relativistic approximation for fermion in the action \cite{23}, and drive the magnetic moment. Varying the action with respect to $\bar{\psi}$, we obtain the equation of motion:

$$\left[ie^\mu_\alpha \gamma^\alpha \left( \partial_\mu - ieA_\mu - \frac{1}{2}\omega^\beta_\mu \sigma(\beta\gamma) \right) - m \right] \psi = 0, \quad (A.1)$$

where $e^\mu_\alpha$ is represented by \cite{23}. Operating on

$$\left[ie^\mu_\alpha \gamma^\alpha \left( \partial_\mu - ieA_\mu - \frac{1}{2}\omega^\beta_\mu \sigma(\beta\gamma) \right) + m \right] \quad (A.2)$$

from the left, we get

$$\left[ g^{\mu\nu} \left( \partial_\mu - ieA_\mu - \frac{1}{2}\omega^\beta_\mu \sigma(\beta\gamma) \right) \left( \partial_\nu - ieA_\nu - \frac{1}{2}\omega^\delta_\nu \sigma(\delta\gamma) \right) + ie\sigma^{(\dot{\alpha}\alpha)} e^\mu_{\dot{\alpha}} F_{\mu\dot{\mu}} + \frac{1}{2}\sigma^{(\dot{\alpha}\alpha)} e^\mu_{\dot{\alpha}} \Omega_{\mu\dot{\mu}} \sigma_{\beta\gamma} + \gamma^{\dot{\alpha}\gamma\alpha} e^\mu_{\dot{\alpha}} \partial_{\mu} e^\mu_{\alpha} \left( \partial_\mu - ieA_\mu - \frac{1}{2}\omega^\beta_\mu \sigma(\beta\gamma) \right) + m^2 \right] \psi = 0 \quad (A.3)$$

by using the formula

$$\gamma^{\dot{\alpha}\gamma\alpha} = \eta^{\dot{\alpha}\alpha} + 2\sigma^{(\dot{\alpha}\alpha)}, \quad (A.4)$$

and

$$\sigma^{(\dot{\alpha}\alpha)} e^\mu_{\dot{\alpha}} e^\mu_{\alpha} \left( \partial_\mu - ieA_\mu - \frac{1}{2}\omega^\beta_\mu \sigma(\beta\gamma) \right) \left( \partial_\mu - ieA_\mu - \frac{1}{2}\omega^\beta_\mu \sigma(\beta\gamma) \right)$$

$$= \frac{1}{2}\sigma^{(\dot{\alpha}\alpha)} e^\mu_{\dot{\alpha}} e^\mu_{\alpha} \left( ieF_{\mu\dot{\mu}} + \frac{1}{2}\Omega_{\mu\dot{\mu}} \sigma_{\beta\gamma} \right) \quad \text{(A.5)}$$
where $F_{\mu\nu} \equiv \partial_{[\mu}A_{\nu]}$ and $\Omega^{\beta\gamma}_{\mu\nu} \equiv \partial_{[\mu}\omega^{\beta\gamma}_{\nu]}$. Further, given the assumption that the change in $Y^m(x)$ is static in time, we obtain the induced metric

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & \eta_{ij} + \gamma_{mn}\partial_i Y^m \partial_j Y^n \end{pmatrix}.$$  \hfill (A.6)

Thus, rewriting

$$p^\mu = i\partial^\mu, \quad A_\mu = (\phi, \vec{A}),$$  \hfill (A.7)

the (A.3) transforms into

$$- \left( iE + ie\phi - \frac{1}{2}\omega^\beta_0 \sigma_0^{\beta\gamma} \right) \psi = \left[ g^{ij} \left( -ip_i - ieA_i - \frac{1}{2}\omega_i^{(\beta\gamma)} \sigma_0^{(\beta\gamma)} \right) \left( -ip_j - ieA_j - \frac{1}{2}\omega_j^{(\beta\gamma)} \sigma_0^{(\beta\gamma)} \right) \\
- ie\sigma^{ij}e^k_i e^l_j F_{kl} - \frac{1}{2}\sigma^{ij}e^k_i e^l_j \Omega^{\beta\gamma}_{kl} \sigma_0^{\beta\gamma} \\
+ \gamma^i \gamma^j e^k_i \partial_k e^l_j \left( -ip_l - ieA_l - \frac{1}{2}\omega_l^{(\beta\gamma)} \sigma_0^{(\beta\gamma)} \right) + m^2 \right] \psi$$  \hfill (A.8)

where $i, j, k, l = 1, 2, 3$ and $E$ represents the energy eigenvalue. Putting $E = m + W$ where $m$ is the rest energy, the L.H.S of (A.8) is as follows:

$$\text{L.H.S} = \left[ m^2 + 2m \left( W + e\phi + \frac{i}{2}\omega_0^{\beta\gamma} \sigma_0^{(\beta\gamma)} \right) \\
+ \left( W + e\phi + \frac{i}{2}\omega_0^{\beta\gamma} \sigma_0^{(\beta\gamma)} \right)^2 \right] \psi.$$  \hfill (A.9)

In addition, we assume that $W \ll m$, i.e., the energy due to a magnetic field is extremely small. In this case, dividing both the L.H.S and R.H.S of (A.8) by $2m$ so as to ignore the last term in (A.9), we obtain

$$W\psi = \frac{1}{2m} \left[ g^{ij} \left( -ip_i - ieA_i - \frac{1}{2}\omega_i^{(\beta\gamma)} \sigma_0^{(\beta\gamma)} \right) \left( -ip_j - ieA_j - \frac{1}{2}\omega_j^{(\beta\gamma)} \sigma_0^{(\beta\gamma)} \right) \\
- ie\sigma^{ij}e^k_i e^l_j F_{kl} - \frac{1}{2}\sigma^{ij}e^k_i e^l_j \Omega^{\beta\gamma}_{kl} \sigma_0^{\beta\gamma} \right] \psi \\
- \left( e\phi + \frac{i}{2}\omega_0^{\beta\gamma} \sigma_0^{(\beta\gamma)} \right) \psi.$$  \hfill (A.10)

This is the eigenvalue equation for a charged particle in a magnetic field and gravity. From this equation, we can ascertain the energy shift term, which is produced by the following
interaction:

\[
\frac{\partial W}{\partial H_i} H_i = -\frac{ie}{2m} \sigma^{ij} \epsilon^k_\ell e^\ell_j F_{kl}. \tag{A.11}
\]

Hence, when evaluating Eq. (A.11) by using \(e^\ell_k\) which is the inverse of Eq. (23) and the solution (34), we obtain

\[
\frac{\partial W}{\partial H_i} H_i = -\frac{e}{2m} \left[ \left( 2 + 2 \frac{E^2 M^2}{T_0} \right) \vec{\sigma} \cdot \vec{H} \right] \tag{A.12}
\]

where \(F_{23} = -F_{32} = H_1, F_{31} = -F_{13} = H_2,\) and \(F_{12} = -F_{21} = H_3.\) The parenthesis of the term proportional to \(\vec{\sigma}/2 \cdot \vec{H}\) represents the magnetic moment.

[1] H. N. Brown et al. [Muon (g-2) Collaboration], Phys. Rev. D 62, 091101 (2000); H. N. Brown et al. [Muon g-2 Collaboration], Phys. Rev. Lett. 86, 2227 (2001); G. W. Bennett et al. [Muon g-2 Collaboration], Phys. Rev. Lett. 89, 101804 (2002) [Erratum-ibid. 89, 129903 (2002)]; G. W. Bennett et al. [Muon g-2 Collaboration], Phys. Rev. Lett. 92, 161802 (2004).

[2] A. Hocker, arXiv:hep-ph/0410081; T. Kinoshita and M. Nio, Phys. Rev. D 70, 113001 (2004); M. Davier and W. J. Marciano, Ann. Rev. Nucl. Part. Sci. 54, 115 (2004); M. Davier, S. Eidelman, A. Hocker and Z. Zhang, Eur. Phys. J. C 31, 503 (2003); K. Hagiwara, A. D. Martin, D. Nomura and T. Teubner, Phys. Rev. D 69, 093003 (2004).

[3] H. Baer, C. Balazs, J. Ferrandis and X. Tata, Phys. Rev. D 64, 035004 (2001); K. Choi, K. Hwang, S. K. Kang, K. Y. Lee and W. Y. Song, Phys. Rev. D 64, 055001 (2001); L. L. Everett, G. L. Kane, S. Rigolin and L. T. Wang, Phys. Rev. Lett. 86, 3484 (2001); J. L. Feng and K. T. Matchev, Phys. Rev. Lett. 86, 3480 (2001); E. A. Baltz and P. Gondolo, Phys. Rev. Lett. 86, 5004 (2001); U. Chattopadhyay and P. Nath, Phys. Rev. Lett. 86, 5854 (2001); S. Komine, T. Moroi and M. Yamaguchi, Phys. Lett. B 506, 93 (2001); R. Arnowitt, B. Dutta, B. Hu and Y. Santoso, Phys. Lett. B 505, 177 (2001); J. Hisano and K. Tobe, Phys. Lett. B 510, 197 (2001).

[4] E. Ma and M. Raidal, Phys. Rev. Lett. 87, 011802 (2001) [Erratum-ibid. 87, 159901 (2001)]; T. Huang, Z. H. Lin, L. Y. Shan and X. Zhang, Phys. Rev. D 64, 071301 (2001); S. K. Kang and K. Y. Lee, Phys. Lett. B 521, 61 (2001); Z. z. Xing, Phys. Rev. D 64, 017304 (2001).

[5] M. L. Graesser, Phys. Rev. D 61, 074019 (2000); K. Agashe, N. G. Deshpande and G. H. Wu, Phys. Lett. B 511, 85 (2001); C. S. Kim, J. D. Kim and J. H. Song, Phys. Lett. B 511, 251
(2001); S. C. Park and H. S. Song, Phys. Lett. B 506, 99 (2001); S. C. Park and H. S. Song, Phys. Lett. B 523, 161 (2001); R. Casadio, A. Gruppuso and G. Venturi, Phys. Lett. B 495, 378 (2000).

[6] J. R. Ellis, D. V. Nanopoulos and K. A. Olive, Phys. Lett. B 508, 65 (2001); P. Das, S. Kumar Rai and S. Raychaudhuri, arXiv:hep-ph/0102242; D. Choudhury, B. Mukhopadhyaya and S. Rakshit, Phys. Lett. B 507, 219 (2001); K. m. Cheung, Phys. Rev. D 64, 033001 (2001); U. Mahanta, Eur. Phys. J. C 21, 171 (2001).

[7] V. A. Rubakov, Phys. Usp. 44, 871 (2001) Y. A. Kubyshin, arXiv:hep-ph/0111027; C. Csaki, arXiv:hep-ph/0404096.

[8] R. Sundrum, Phys. Rev. D 59, 085009 (1999).

[9] K. Akama, Lect. Notes Phys. 176, 267 (1982), arXiv:hep-th/0001113.

[10] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B 429, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B 436, 257 (1998); N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Rev. D 59, 086004 (1999).

[11] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999); L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 4690 (1999).

[12] R. Sundrum, Phys. Rev. D 59, 085010 (1999).

[13] S. M. Carroll and M. M. Guica, arXiv:hep-th/0302067.

[14] H. P. Nilles, A. Papazoglou and G. Tasinato, Nucl. Phys. B 677, 405 (2004).

[15] Y. Aghababaie, C. P. Burgess, S. L. Parameswaran and F. Quevedo, Nucl. Phys. B 680, 389 (2004).

[16] J. W. Chen, M. A. Luty and E. Ponton, JHEP 0009, 012 (2000).

[17] I. Navarro, JCAP 0309, 004 (2003); I. Navarro, Class. Quant. Grav. 20, 3603 (2003).

[18] J. Vinet and J. M. Cline, Phys. Rev. D 70, 083514 (2004); J. M. Cline, J. Descheneau, M. Giovannini and J. Vinet, JHEP 0306, 048 (2003)

[19] J. Garriga and M. Porrati, JHEP 0408, 028 (2004).

[20] H. M. Lee and A. Papazoglou, Nucl. Phys. B 705, 152 (2005).

[21] S. Mukohyama, Y. Sendouda, H. Yoshiguchi and S. Kinoshita, JCAP 0507, 013 (2005)

[22] S. Deser, R. Jackiw and G. ’t Hooft, Annals Phys. 152, 220 (1984); S. Deser and R. Jackiw, Annals Phys. 153, 405 (1984).

[23] M. Veltman, Methods in Field Theory, Proceedings of the Les Houches Summer School, Les
Houches, France, 1975, edited by R. Bailian and J. Zinn-Justin, Les Houches Summer School Proceedings Vol. XXXVIII (North-Holland, Amsterdam, 1976), p.265.

[24] J. Hisano and N. Okada, Phys. Rev. D 61, 106003 (2000).

[25] M. Bando, T. Kugo, T. Noguchi and K. Yoshioka, Phys. Rev. Lett. 83, 3601 (1999).

[26] T. Kugo and K. Yoshioka, Nucl. Phys. B 594, 301 (2001).

[27] A. Dobado and A. L. Maroto, Nucl. Phys. B 592, 203 (2001); J. Alcaraz, J. A. R. Cembranos, A. Dobado and A. L. Maroto, Phys. Rev. D 67, 075010 (2003).

[28] L. J. Hall and D. R. Smith, Phys. Rev. D 60, 085008 (1999); S. Cullen, M. Perelstein and M. E. Peskin, Phys. Rev. D 62, 055012 (2000); V. D. Barger, T. Han, C. Kao and R. J. Zhang, Phys. Lett. B 461, 34 (1999); C. Hanhart, J. A. Pons, D. R. Phillips and S. Reddy, Phys. Lett. B 509, 1 (2001); C. Hanhart, D. R. Phillips, S. Reddy and M. J. Savage, Nucl. Phys. B 595, 335 (2001); R. Allahverdi, C. Bird, S. Groot Nibbelink and M. Pospelov, Phys. Rev. D 69, 045004 (2004); S. Hannestad and G. G. Raffelt, Phys. Rev. D 67, 125008 (2003) [Erratum-ibid. D 69, 029901 (2004)]; S. Hannestad and G. G. Raffelt, Phys. Rev. Lett. 88, 071301 (2002); S. Hannestad and G. Raffelt, Phys. Rev. Lett. 87, 051301 (2001); S. Hannestad, Phys. Rev. D 64, 023515 (2001).

[29] V. A. Kostelecky and R. Lehnert, Phys. Rev. D 63, 065008 (2001); V. A. Kostelecky, Phys. Rev. D 69, 105009 (2004).