SPIN AND VELOCITY DEPENDENT CORRECTIONS TO THE INTERQUARK POTENTIAL AND QUARKONIA SPECTRA FROM LATTICE QCD

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We present results on (order $v^2$) QCD interquark potentials from SU(3) lattice simulations on volumes of up to $32^4$ lattice sites at $\beta = 6.2$ and $\beta = 6.0$. Preliminary results on quarkonia spectra as obtained from these potentials are presented.

1 Introduction

Purely phenomenological or QCD inspired potential models have been proven to reproduce the observed charmonium ($J/\psi$) and bottomonium ($\Upsilon$) spectra remarkably well. The success of such models might be understood from QCD as a Schrödinger-Pauli type Hamiltonian with spin dependent (sd) and velocity dependent (vd) contributions — that can be parameterized in terms of seven independent scalar functions of the quark separation (potentials) — has been derived directly from the QCD Lagrangian \[4, 5, 6\]. By computing these potentials nonperturbatively on the lattice, the set of free parameters within the potential picture can be restricted. The present investigations

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correspond to a complete expansion of the QCD Lagrangian up to order \(1/m^2\) in the heavy quark mass \(m\) or, alternatively, order \(v^2\) in the relative velocity between the sources. The form of the Hamiltonian resembles that of NRQCD \([4]\) at this given order. The only additional assumption that enters is the instantaneous approximation, i.e. the potentials are assumed to have no explicit dependence on the interaction time.

In our determination of the central potential as well as the relativistic corrections \([5]\), we find short ranged Coulomb like contributions which had not been expected so far in two of the potentials. The main effect is that the \(J/\psi\) “sees” a stronger effective Coulomb force than the \(\Upsilon\).

2 The Hamiltonian

Starting from a Foldy-Wouthuysen transformation of the Euclidean quark propagator in an external gauge field, the connection between the static interquark potential \(V_0(r)\) and Wilson loops can be derived. By perturbing the propagator in terms of the inverse quark masses \(m_1^{-1}\) and \(m_2^{-1}\) around its static solution, one arrives at the semi-relativistic Hamiltonian (in the CM system, i.e. \(p = p_1 = -p_2\) and \(L = L_1 = L_2\),

\[
H = \sum_{i=1}^{2} \left( m_i + \frac{p_i^2}{2m_i} - \frac{p_i^4}{8m_i^2} \right) + V_0(r) + V_{sd}(r, L, S_1, S_2) + V_{vd}(r, p), \tag{1}
\]

where the potential consists of a central part and sd and vd corrections \([4, 6, 3]\):

\[
V_{sd}(r, L, S_1, S_2) = \frac{LS_1}{m_1^2} + \frac{LS_2}{m_2^2} \left( \frac{V'_0(r) + 2V'_1(r)}{2r} \right) + \frac{L(S_1 + S_2)}{m_1m_2} \frac{V''_1(r)}{r} + \frac{S_i S_j}{m_1m_2} \left( R_{ij} V_3(r) + \frac{\delta_{ij}}{3} V_4(r) \right) \tag{2}
\]

with \(R_{ij} = r_i r_j / r^2 - \delta_{ij} / 3\) and,

\[
V_{vd}(r, p) = \frac{1}{8} \left( \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) \left( \nabla^2 V_0(r) + \nabla^2 V_4(r) \right) \tag{3}
\]

\[
- \frac{1}{m_1m_2} \{p_i, p_j, S_{ij}\}_W + \sum_{k=1}^{2} \frac{1}{m_k^2} \{p_i, p_j, T_{ij}\}_W
\]
with $S_{ij} = \delta_{ij} V_b(r) - R_{ij} V_c(r)$ and $T_{ij} = \delta_{ij} V_d(r) - R_{ij} V_e(r)$. The symbol \( \{\cdot, \cdot, \cdot\}_W = \frac{1}{4}\{\cdot, \cdot\} \) denotes Weyl ordering of the three arguments. \( V'_1 - V'_4 \) are related to spin-orbit and spin-spin interactions, \( V_b - V_c \) to orbit-orbit interactions and the Darwin-like term that incorporates $\nabla^2 V_a$ modifies the central potential. $V'_1 - V'_4$ and $\nabla^2 V_a - V_e$ can be computed from lattice correlation functions of Wilson loop like operators in Euclidean time. Pairs of the potentials are related by Lorentz invariance to the central potential

\[
V'_2 - V'_1 = V'_0, \quad V'_1 - 2V_e = \frac{1}{2} V'_0 - \frac{2}{9} V'_0, \quad V'_1 + 2V_e(r) = -\frac{2}{3} V'_0 \quad [2, 3],
\]

such that only six out of them are truly independent.

### 3 Results

We find the sd potentials $V'_2, V_3, V_4$ to be short ranged and to agree with tree level lattice perturbation theory. The agreement is even better when a running coupling in momentum space is allowed. Apart from a constant part, which accounts for the string tension, the spin-orbit potential $V'_1$ contains an attractive Coulomb-like piece which is not expected from perturbation theory and makes up about 20% of the Coulomb part of the central potential. To leading order minimal surface approximation, $V_a$ does not contain a long range component while the short range part is expected to vanish from weak coupling perturbation theory. We, however, find it to behave like $-1/r$ which means that the Coulomb force within $c\bar{c}$ systems is enhanced by about 20% in comparison to the $b\bar{b}$ case. $V_b - V_e$ are found to agree qualitatively with the combined long and short distance expectations. By neglecting the very short range behaviour ($r < 0.15$ fm), which is affected by running coupling effects, the data can be parameterized as follows:

\[
\begin{align*}
V_0 &= \kappa r - \frac{e}{r}, \\
\nabla^2 V_a &= -\frac{h}{r}, \\
V'_1 &= -\kappa - \frac{d}{r^2},
\end{align*}
\]

\[
\begin{align*}
V'_2 &= \frac{a}{r^2}, \\
V_3 &= \frac{3a}{r^3}, \\
V_4 &= 8\pi a \delta^3(r),
\end{align*}
\]

\[
\begin{align*}
V_b &= \frac{2e}{3r} - \frac{\kappa r}{9}, \\
V_c &= \frac{e}{2r} - \frac{\kappa r}{6}, \\
V_d &= -\frac{\kappa r}{9}, \\
V_e &= -\frac{\kappa r}{6}
\end{align*}
\]

with $a = e - d$. $V_0, V_a, V_b$ and $V_d$ contain unphysical self energy constants which have to be subtracted. The parameter values are: $e = 0.321(7), d = 0.065(11)$ and $h = 3.75(31)\kappa$. 

\[3\]
Figure 1: The bottomonium spectrum (in GeV). Horizontal lines are experimental results.

4 Spectroscopy

Two free QCD parameters have to be determined from experiment, i.e. from a fit to the $\Upsilon$ and $J/\psi$ spectra, namely the heavy quark mass, $m_b$ or $m_c$, and a scale which sets the initial condition for the running of the QCD coupling, e.g. the parameter $\kappa$. Since we have not included sea quark effects into the central potential is unbounded, the zero point energy is in principle arbitrary. However, it turns out that a zero point energy $C$ in $V_0$ will give rise to counter terms in $V_b$ and $V_d$ which can be absorbed into a redefinition of the quark mass $\frac{C}{2}$: $m \rightarrow m + \frac{C}{2}$. We choose $C = 0$ by demanding $V_d(0) = 0$. Apart from this, the constituent quark masses of Eq. 1 can differ from the kinetic masses appearing in the dispersion relation to this order in $1/m$. We find that allowing for such a difference does not substantially improve the agreement with experiment.
our lattice simulation yet, for the time being, we allow for a free Coulomb coefficient \( e \). The optimal values of the fit parameters turn out to be \( e = 0.495, \sqrt{k} \approx 430 \text{ MeV}, m_b \approx 4.79 \text{ GeV} \) and \( m_c \approx 1.41 \text{ GeV} \) with an error of about 0.1 on \( e \). Simulations of full QCD [6] suggest that values \( e \approx 0.40 \) are realistic in a world with three light sea quarks. In Fig. 1 the \( \bar{b}b \) spectrum from our best fit \( (e = 0.495) \) is compared to experiment (horizontal lines) as well as the result with \( e = 0.4 \). Fits to improved parametrizations that incorporate the QCD running coupling are in preparation.

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References

[1] E. Eichten and F. Feinberg, Phys. Rev. D 23, 2724 (1981).

[2] D. Gromes, Z. Phys. C 22, 265 (1984).

[3] A. Barchielli, E. Montaldi, G.M. Prosperi, Nucl. Phys. B 296, 625 (1988); Nucl. Phys. B 303, 752 (1988); A. Barchielli, N. Brambilla, G. Prosperi, Nuovo Cimento 103, 59 (1990).

[4] B.A. Thacker and G.P. Lepage, Phys. Rev. D 43, 196 (1991).

[5] G.S. Bali, K. Schilling, A. Wachter, in preparation.

[6] SESAM collaboration: U. Glässner et al., Phys. Lett. B 383, 98 (1996).