A HERTZSPRUNG-RUSSELL–LIKE DIAGRAM FOR SOLAR/STELLAR FLARES AND CORONA: EMISSION MEASURE VERSUS TEMPERATURE DIAGRAM

Kazunari Shibata and Takaaki Yokoyama

Received 2001 December 13; accepted 2002 May 29

ABSTRACT

In our previous paper, we presented a theory to explain the observed universal correlation between the emission measure (EM = \(n^2V\)) and temperature (\(T\)) for solar/stellar flares on the basis of the magnetic reconnection model with heat conduction and chromospheric evaporation. Here \(n\) is the electron density and \(V\) is the volume. By extending our theory to general situations, we examined the EM-\(T\) diagram in detail and found the following properties: (1) The universal correlation sequence ("main-sequence flares") with EM \(\propto T^{17/2}\) corresponds to the case of constant heating flux or, equivalently, the case of constant magnetic field strength in the reconnection model. (2) The EM-\(T\) diagram has a forbidden region, in which gas pressure of flares exceeds magnetic pressure. (3) There is a coronal branch with EM \(\propto T^{15/2}\) for \(T < 10^7\) K and EM \(\propto T^{13/2}\) for \(T > 10^7\) K. This branch is located on the left side of the main-sequence flares in the EM-\(T\) diagram. (4) There is another forbidden region determined by the length of flare loop; the lower limit of the flare loop is 10\(^7\) cm. Small flares near this limit correspond to nanoflares observed by the Solar and Heliospheric Observatory EUV Imaging Telescope. (5) We can plot the flare evolution track on the EM-\(T\) diagram. A flare evolves from the coronal branch to main-sequence flares, then returns to the coronal branch eventually. These properties of the EM-\(T\) diagram are similar to those of the H-R diagram for stars, and thus we propose that the EM-\(T\) diagram is quite useful for estimating the physical quantities (loop length, heating flux, magnetic field strength, total energy, and so on) of flares and corona when there are no spatially resolved imaging observations.

Subject headings: MHD — stars: coronae — stars: flare — X-rays: stars

1. INTRODUCTION

Recent space observations of solar and stellar flares have revealed that there is a universal correlation between the peak temperature (\(T\)) of a flare and its volume emission measure (EM = \(n^2V\)), such that EM increases with increasing \(T\), where \(n\) is the electron number density and \(V\) is the volume (Watanabe 1983; Stern 1992; Feldman, Laming, & Doschek 1995; Feldman et al. 1996; Yuda et al. 1997). This correlation is extrapolated to not only solar microflares (Shimizu 1995) but protostellar flares (Koyama et al. 1996; Tsuboi et al. 1998; Tsuboi 1999). Figure 1 shows the EM-\(T\) relation for solar and stellar flares, including solar microflares and protostellar flares. It is remarkable that the correlation holds for the wide range of parameter values \(10^{44} \text{ cm}^{-3} < \text{EM} < 10^{56} \text{ cm}^{-3}\) and \(6 \times 10^6 \text{ K} < T < 10^8 \text{ K}\).

Recently, Shibata, & Yokoyama (1999) presented a simple theory to explain this remarkable correlation on the basis of the flare temperature scaling law, which was found from magnetohydrodynamic (MHD) numerical simulations of reconnection coupled with heat conduction and chromospheric evaporation (Yokoyama & Shibata 1998, 2001). Shibata & Yokoyama (1999) derived a theoretical correlation, EM \(\propto B^{-5}T^{17/2}\), which explains the observed correlation if the magnetic field strength \(B\) is nearly constant within a factor of 10 for solar and stellar flares (Fig. 2). In fact, magnetic field strengths of solar and stellar flares are estimated to be 40–300 G (e.g., Rust & Bar 1973; Dulk 1985; Tsuneta 1996; Ohyama & Shibata 1998; Grosso et al. 1997), consistent with the above theory. This means that the observed EM-\(T\) diagram may be useful for estimating the magnetic field strength of stellar flares, which are difficult to observe.

Shibata & Yokoyama (1999) noted also that this theory can be used to estimate the flare loop length. In fact, from the comparison between the theory and the observed EM-\(T\) relation, they estimated that the loop length of solar flares is \(10^9–10^{10}\) cm, and that of stellar flares is \(10^{11}–10^{12}\) cm, which is again consistent with direct observations of solar flare loops and with some indirect estimations of stellar flare loops (Koyama et al. 1996; Hayashi, Shibata, & Matsumoto 1996; Montmerle 1998; see also Haisch, Strong, & Rodono 1991 and Feigelson & Montmerle 1999 for a review of stellar flare observations).

As for estimations of the stellar flare loop length, Reale & Micera (1998) and, more recently, Favata, Micela, & Reale (2001) took into account both the flare impulsive phase and its decay, allowing for continuous energy deposition, and derived the size of the flare loop by calibrating the slope of the light curve in the EM-\(T\) diagram based on the hydrodynamic modeling (Serio et al. 1991). They also took into account the passband of the instrument used to derive the EM and found smaller loop sizes than the \(\sim 10^{11}\) cm obtained in previous papers. Note that the method of estimating the flare loop used in previous papers (e.g., Montmerle 1998; Tsuboi et al. 2000) is based on the assumption that the flare decay can be described through a series of static states (i.e., the quasi-static model; van den Oord & Mewe 1989). On the other hand, our method (Shibata & Yokoyama 1999; this paper) is based only on the peak flare temperature, the peak EM, and the preflare coronal density.

1 Kwasan and Hida Observatories, Kyoto University, Yamashina, Kyoto 607-8471, Japan; shibata@kwasan.kyoto-u.ac.jp.
2 National Astronomical Observatory, Nobeyama, Minamimaki, Minamisaku, Nagano 384-1305, Japan; yokoyama.t@nao.ac.jp.
so that it does not depend on any physical process occurring in the flare decay phase. We discuss this point in more detail in this paper.

It should be noted that the EM and temperature may be a bit ill defined in a nonisothermal plasma such as in a flaring region. Their determination depends strongly on the passband of the instrument used for the observations (Reale & Micera 1998). We note that when we compare our theory and the observed data in this paper, we do not take into account such passband effect. Hence, too detailed comparison between the theory and the observed data, such as within an accuracy of a factor of 2 or 3, may be meaningless, and the correlation between EM and T would be only statistically meaningful. We leave the more accurate comparison between the theory and the observations to future studies, which would incorporate the calibration of the passband of the instrument used for observations. Nevertheless, in this paper we emphasize the importance of the EM-T correlation, because the EM-T diagram for solar and stellar flares is in some sense similar to the H-R diagram for stars. In the case of the H-R diagram, if we plot a star on the H-R diagram, we can infer the stellar radius without direct measurement. We can also discuss several branches of stars, such as main-sequence, giants, white dwarfs, etc., and also their evolution in the H-R diagram. Similarly, we can discuss several branches of flares and flarelike phenomena and their evolution in the EM-T diagram.

In this paper, noting the analogy between the flare EM-T diagram and the H-R diagram, we examine basic physical properties of the EM-T diagram for flares. In § 2, we summarize the derivation of the EM-T correlation (EM \( \propto B^{-5}T^{17/2} \)) by Shibata & Yokoyama (1999), which was based on the reconnection model and the pressure balance of flare loops. We relax this pressure balance assumption and obtain a bit different relation (EM \( \propto T^{15/2} \)) in the case of enthalpy flux-conduction flux balance. The forbidden region is found in the EM-T diagram, which reminds us of Hayashi’s forbidden region in the H-R diagram. In § 3, we investigate a coronal branch in the EM-T diagram, in which the heating is arbitrary and the radiative cooling becomes comparable to conduction flux, which predicts EM \( \propto T^{13/2} \) for solar coronae and EM \( \propto T^{13/2} \) for stellar corona with a temperature higher than 10⁷ K. The coronal branch lies on the left side of the flare branch in the EM-T diagram. In § 4, we discuss the flare evolution track on this EM-T diagram, nanoflare branch, and the estimate of total flare energy using this diagram. Finally, § 5 is devoted to our conclusions.

We emphasize again that the EM-T diagram is useful for estimating currently unobservable physical quantities, such as magnetic field strength, flare loop length, conduction and radiative cooling times, etc., using only three observed quantities: peak EM, peak temperature T, and preflare coronal density \( n_0 \). The detailed time evolution data of stellar flares are not necessary in our method, in contrast to other methods, such as the one based on the flare decay time (e.g., van den Oord & Mewe 1989; Reale & Micera 1998). The useful formulae for estimating various physical quantities using EM, T, and \( n_0 \) are summarized in Appendix A. Limitation of the temperature scaling law (Yokoyama & Shibata 1998) and the condition of chromospheric evaporation are discussed in Appendices B and C, respectively. The effect of the filling factor on the flare EM-T scaling law is discussed in Appendix D.

2. FLARE BRANCH: MAIN SEQUENCE

2.1. The Pressure Balance Scaling Law

The theory of Shibata & Yokoyama (1999) is based on a simple scaling relation for the maximum temperature of
reconnection heated plasma $T_{\text{max}}$ and magnetic field
strength $B$, which is found from MHD simulations by
Yokoyama & Shibata (1998, 2001):

$$T_{\text{max}} \simeq \left( \frac{B^2 \nu_A L}{\kappa_0 2\pi} \right) \simeq 5.3 \times 10^4 B^{6/7} n_0^{-1/7} L^{2/7} K \simeq 3 \times 10^7 \left( \frac{B}{50 G} \right)^{6/7} \left( \frac{n_0}{10^9 \text{ cm}^{-3}} \right)^{-1/7} \times \left( \frac{L}{10^9 \text{ cm}} \right)^{2/7} \text{ K},$$

(1)

where $\nu_A = B/(4\pi \rho)^{1/2}$ is the Alfvén speed, $\rho = mn_0$ is the mass density, $m$ is the proton mass, $n_0$ is the preflare proton number density (= electron density), and $L$ is the characteristic length of the (reconnected) magnetic loop. This relation is derived from the balance between conduction cooling and reconnection heating ($\kappa_0 T^{7/2}/(2L^2)$) (e.g., Fisher & Hawley 1990; Hori et al. 1997) and reconnection heating ($B^2/(4\pi)(\nu_A/L)$). Here, $\kappa_0 \simeq 10^{-6}$ cgs is the thermal conductivity of Spitzer (1956). In this model (see Fig. 2 of Shibata & Yokoyama 1999), after the reconnection occurs, the released energy is transported to the top of the chromosphere by heat conduction and heats the chromospheric plasma suddenly. Then its plasma pressure increases enormously and drives the upward flow back into the (reconnected) coronal magnetic loop, creating hot dense flare loops. (This process is often called “chromospheric evaporation.”) As a result of the chromospheric evaporation, the flare loop density increases to $n$. This evaporated plasma with density $n$ is the source of X-ray emission

$$\text{EM} \simeq n^2 L^3.$$  

(2)

Here, we assumed that $V \sim L^3$ and the filling factor = 1 (see Appendix D for the effect of the filling factor), since Yohkoh/soft X-ray telescope (SXT) observations of microflares and flares (Shimizu 1995; Takahashi 1997) show that the observed aspect ratio of flare loops (loop width/loop length) is 0.1–1 with filling factor 1.0. Furthermore, numerical simulations of Yokoyama & Shibata showed that the temperature of evaporated plasma (i.e., $\sim n^4 L^3$) is a factor of $\sim 3$ lower than the maximum temperature $T_{\text{max}}$:

$$T \simeq \frac{1}{3} T_{\text{max}} \simeq 10^7 \left( \frac{B}{50 G} \right)^{6/7} \left( \frac{n_0}{10^9 \text{ cm}^{-3}} \right)^{-1/7} \left( \frac{L}{10^9 \text{ cm}} \right)^{2/7} \text{ K}.$$  

(3)

(The maximum temperature $T_{\text{max}}$ of the reconnection heated loop would correspond to superhot components of flares [Lin et al. 1981; Masuda 1984; Nitta & Yajii 1997].) Since this evaporated plasma has high gas pressure, we have to assume that the magnetic pressure of the reconnected loop must be larger than the gas pressure of evaporated plasma to maintain stable flare loops. From this, it may be reasonable to assume

$$2nkT \simeq \frac{B^2}{8\pi}$$  

(4)

as a first approximation. (Here $k$ is the Boltzmann’s constant.) In fact, some previous observations of postflare loops showed that the gas pressure of the flare loop is as high as the inferred magnetic pressure (e.g., Tsuneta 1996). Eliminating $n$ and $L$ from equations (2)–(4), we find

$$\text{EM} \simeq 10^{48} \left( \frac{B}{50 G} \right)^{-5} \left( \frac{n_0}{10^9 \text{ cm}^{-3}} \right)^{3/2} \left( \frac{T}{10^7 \text{ K}} \right)^{17/2} \text{ cm}^{-3}.$$  

(5)

This is the EM–$T$ relation derived by Shibata & Yokoyama (1999) and is hereafter called the pressure balance scaling law. We can derive the following equation from the above equations:

$$\text{EM} \simeq 10^{48} \left( \frac{L}{10^9 \text{ cm}} \right)^{5/3} \left( \frac{n_0}{10^9 \text{ cm}^{-3}} \right)^{2/3} \left( \frac{T}{10^7 \text{ K}} \right)^{8/3} \text{ cm}^{-3}.$$  

(6)

The lines of $B = \text{const}$ and $L = \text{const}$ in Figure 2 are drawn based on equations (5) and (6). We can also derive the following two equations:

$$B = 50 \left( \frac{\text{EM}}{10^{48} \text{ cm}^{-3}} \right)^{-1/5} \left( \frac{n_0}{10^9 \text{ cm}^{-3}} \right)^{3/10} \left( \frac{T}{10^7 \text{ K}} \right)^{17/10} \text{ G},$$  

(7a)

$$L = 10^9 \left( \frac{\text{EM}}{10^{48} \text{ cm}^{-3}} \right)^{3/5} \left( \frac{n_0}{10^9 \text{ cm}^{-3}} \right)^{-2/5} \left( \frac{T}{10^7 \text{ K}} \right)^{-8/5} \text{ cm}.$$  

(7b)

These equations will be useful for estimating the magnetic field strength and length of stellar flare loops, which are both difficult to measure with the present observations. Note again that this method depends on only three parameters: the peak temperature ($T$), the peak emission measure (EM), and the preflare electron density ($n_0$), and it is completely independent of the later evolution of flares after the peak of emission measure. This method can estimate $B$ and $L$ with fewer observational information than previous methods (van den Oord & Mewel 1989; Reale & Micela 1998) because we assume magnetic reconnection and confinement of evaporated plasma in a reconnected loop (i.e., the balance between evaporated plasma pressure and magnetic pressure of a reconnected loop).

2.2. The Enthalpy–Conduction Balance Scaling Law

In deriving equation (5), Shibata & Yokoyama (1999) assumed that the density of flare loops is determined by the balance between gas pressure and magnetic pressure: $2nkT \approx B^2/8\pi$. However, as they noted in their Discussion section, the density of flare evaporation flow in the initial phase is determined by the balance between enthalpy flux of evaporation flow and conduction flux

$$5nkTC_s \simeq \kappa_0 T^{7/2}/L,$$  

(8)

where $C_s = [10kT/3m]^{1/2}$ is the sound speed of the evaporation flow. Then we have

$$n \simeq 10^{5} T^2/L,$$  

(9a)

$$\simeq 10^{10} \left( \frac{T}{10^7 \text{ K}} \right)^2 \left( \frac{L}{10^9 \text{ cm}} \right)^{-1} \text{ cm}^{-3}.$$  

(9b)

This density is a minimum density of the evaporating flare loop. Actually, flare loop density gradually increases with time, because evaporated plasma accumulates in the flare loop. If we assume flare loop density is determined by equa-
tion (9a), we then find that the EM becomes

\[ \text{EM} \approx 10^{47} \left( \frac{B}{50 \text{ G}} \right)^{-3} \left( \frac{n_0}{10^9 \text{ cm}^{-3}} \right)^{1/2} \left( \frac{T}{10^7 \text{ K}} \right)^{15/2} \text{ cm}^{-3}. \]  

(10)

Hereafter, we call this equation the enthalpy-conduction balance scaling law. Eliminating \( B \) from equations (3) and (10), we have

\[ \text{EM} \approx 10^{47} \left( \frac{T}{10^7 \text{ K}} \right)^4 \left( \frac{L}{10^9 \text{ cm}} \right) \text{ cm}^{-3}. \]  

(11)

In Figure 3, we plot the enthalpy-conduction balance scaling law (eq. [10]) for \( B = 15, 50, \) and 150 G (dashed line) and compare them with the pressure balance scaling law (eq. [5]; solid lines). We find that the enthalpy-conduction balance scaling law predicts weaker magnetic field strength than the pressure balance scaling law and actual solar observations. Hence, from the viewpoint of magnetic field strength, the pressure balance scaling law fits better with observations. This is also reasonable if we consider the increase in density (beyond the value given by eqs. [9a] and [9b]) in the early phase of evolution of a flare loop, as discussed in §4.

2.3. Forbidden Region

In the case of the enthalpy-conduction balance (minimum density flare loop), there is no guarantee that gas pressure is smaller than magnetic pressure. From equations (3), (9a), (9b), and (10), the plasma beta of the evaporation flow is estimated to be

\[ \beta_{ev} = \frac{2nkT}{B^2/(8\pi)} \]

\[ \approx 0.3 \left( \frac{\text{EM}}{10^{47} \text{ cm}^{-3}} \right)^{-1/3} \left( \frac{n_0}{10^9 \text{ cm}^{-3}} \right)^{-1/3} \]

\[ \times \left( \frac{T}{10^7 \text{ K}} \right)^{2}. \]  

(12)

If the plasma \( \beta \) of evaporated flare loop plasma \( \beta_{ev} \) becomes larger than unity, the plasma cannot be confined by magnetic pressure. Hence, such a case does not exist as a stable flare loop and is forbidden in the EM-T diagram. This forbidden region is defined as the region satisfying the inequality

\[ \text{EM} < 10^{45.5} \left( \frac{n_0}{10^9 \text{ cm}^{-3}} \right)^{-1} \left( \frac{T}{10^7 \text{ K}} \right)^6 \text{ cm}^{-3}, \]  

(13)

which is shown by the hatched area in Figure 4. This explains why there is no flare in the lower right region of the EM-T diagram.

2.4. General EM-T Relation for Arbitrary Heating

The temperature of flares (and also coronae; see §3) is determined by the balance between heating flux (\( F \) ergs cm\(^{-2}\) s\(^{-1}\)) and conduction cooling flux (\( \sim \kappa_0 F^{7/2}/L \)),

\[ F \approx \frac{\kappa_0 T^{7/2}}{L}. \]  

(14)

where \( \kappa_0 (\approx 10^{-6} \text{ cgs}) \) is the Spitzer’s thermal conductivity. From this, we obtain

\[ T \approx \left( \frac{F L}{\kappa_0} \right)^{2/7} \]

\[ \approx 10^6 \left( \frac{F}{10^{10} \text{ ergs cm}^{-2} \text{ s}^{-1}} \right)^{2/7} \left( \frac{L}{10^9 \text{ cm}} \right)^{2/7} \text{ K}. \]  

(15)

If the heating is due to magnetic reconnection, the heating flux is

\[ F \approx (B^2/4\pi)V_A \propto B^3. \]  

(16)

Then, we recover the temperature scaling law (eq. [1])

\[ T \propto B^{6/7} L^{2/7}. \]  

(17)
Using equation (15), the EM is written as
\[
\text{EM} \propto B^4 F^{-3} T^{17/2} \quad \text{for pressure balance scaling law},
\]
\[
\text{EM} \propto F^{-1} T^{15/2} \quad \text{for enthalpy-conduction scaling law}.
\]

It is now clear from the above argument that the observed flare EM-T relation corresponds to the relation that heating flux is constant; this is exactly true in the case of enthalpy-conduction balance. In the case of pressure balance, it corresponds to \(B^4 F^{-3}\), or both heating flux and magnetic field strength are constant. In the reconnection heating model, however, heating flux depends only on \(B\), so that the \(B = \text{constant}\) line is the same as the heating flux = constant line.

3. CORONAL BRANCH

3.1. Coronal EM-T Scaling Law

If the heating time becomes sufficiently long, the coronal loop becomes dense enough so that the radiative cooling balances with conduction flux. At that time, we have a steady coronal loop, in which (unknown) heating \(F\), conduction flux \(\kappa_0 T^{7/2}/L\), and radiative cooling \(n^2 \Lambda(T) L\) are all comparable:
\[
F \approx \kappa_0 T^{7/2}/L \approx n^2 \Lambda(T) L.
\]

Here the radiative loss function \(\Lambda(T)\) is approximated to
\[
\Lambda(T) \approx 3 \times 10^{-23}(T/10^7 \text{ K})^{-1/2} \quad \text{for } T < 10^7 \text{ K},
\]
\[
\Lambda(T) \approx 3 \times 10^{-23}(T/10^7 \text{ K})^{1/2} \quad \text{for } T > 10^7 \text{ K}.
\]

Equations (19), (20a), and (20b) are reduced to
\[
n \approx 10^{6.5} T^{2}/L \quad \text{for } T < 10^7 \text{ K},
\]
\[
n \approx 10^{10} T^{1.5}/L \quad \text{for } T > 10^7 \text{ K}.
\]

Note that equation (21a) is basically the same as the scaling law of Rosner-Tucker-Vaiana (1978) \([T \propto (pL)^{1/4}]\). It is interesting to note that equation (21a) has the same dependence on \(T\) and \(L\) with equation (9a) for the enthalpy flux-conduction balance.

Then, the EM-T relation becomes
\[
\text{EM} \approx 10^{47} \left( \frac{T}{10^6 \text{ K}} \right)^{15/2} \left( \frac{F}{10^5 \text{ ergs cm}^{-2} \text{ s}^{-1}} \right)^{-1} \text{ cm}^{-3}
\]
for \(T < 10^7 \text{ K}\),
\[
\text{EM} \approx 10^{52.5} \left( \frac{T}{10^7 \text{ K}} \right)^{13/2} \left( \frac{F}{10^7 \text{ ergs cm}^{-2} \text{ s}^{-1}} \right)^{-1} \text{ cm}^{-3}
\]
for \(T > 10^7 \text{ K}\).

Eliminating \(F\) from equation (22a) using equation (19), we obtain
\[
\text{EM} \approx 10^{47} \left( \frac{T}{10^6 \text{ K}} \right)^4 \left( \frac{L}{10^{10} \text{ cm}} \right) \text{ cm}^{-3} \quad \text{for } T < 10^7 \text{ K},
\]
\[
\text{EM} \approx 10^{51} \left( \frac{T}{10^7 \text{ K}} \right)^3 \left( \frac{L}{10^{10} \text{ cm}} \right) \text{ cm}^{-3} \quad \text{for } T > 10^7 \text{ K}.
\]

3.2. Comparison with Observations and Effect of Filling Factor

Figure 5 shows this coronal branch (eqs. [23a] and [23b]) in the EM-T diagram, as well as observed EM-T relations of solar-active regions (Yashiro 2000). We see that the theoretical EM-T relation for the solar corona fits well with observations. However, the heating flux necessary for the heating of the active region corona becomes \(10^7-10^8 \text{ ergs cm}^{-2} \text{ s}^{-1}\), which is larger than the heating flux required for the heating of the average active-region corona \((\sim 10^6-10^7 \text{ ergs cm}^{-2} \text{ s}^{-1})\); e.g., Withbroe & Noyes (1977). This is probably due to the effect of the filling factor. In deriving the above relations, we implicitly assumed that the filling factor of the corona with \(n\) and \(T\) in the volume of \(L^3\) is unity. If we take into account the effect of the filling factor \(f\), the EM is written as
\[
\text{EM} = \frac{fn^2 L^3}{C_0}
\]
so that the coronal branch scaling law becomes
\[
\text{EM} \approx 10^{46} \left( \frac{f}{0.1} \right) \left( \frac{T}{10^6 \text{ K}} \right)^{15/2} \left( \frac{F}{10^5 \text{ ergs cm}^{-2} \text{ s}^{-1}} \right)^{-1} \text{ cm}^{-3}
\]
for \(T < 10^7 \text{ K}\),
\[
\text{EM} \approx 10^{51.5} \left( \frac{f}{0.1} \right) \left( \frac{T}{10^7 \text{ K}} \right)^{13/2} \left( \frac{F}{10^7 \text{ ergs cm}^{-2} \text{ s}^{-1}} \right)^{-1} \text{ cm}^{-3}
\]
for \(T > 10^7 \text{ K}\).

![Figure 5](image_url)

Figure 5.—Coronal branch (eqs. [23a] and [23b]) is shown with dash-dotted lines in the EM-T diagram for three cases: heating flux \(F = 10^6, 10^7,\) and \(10^8 \text{ ergs cm}^{-2} \text{ s}^{-1}\). The filling factor \(f = 0.1\) is assumed. The data for solar corona (active region; Yashiro 2000) are indicated with hatched area around \(T \approx 2-3 \times 10^6 \text{ K}\). Coronal loop length \(L_c\) is denoted with solid lines for \(L_c = 10^6, 10^7, 10^8, 10^9, 10^{10}\), and \(10^{11} \text{ cm}\) on the coronal EM-T scaling law. Recent observations of nonflare data with ASCA (crosses: Herbig Be [Hamaguchi et al. 2000]; triangle: T Tauri [Ozawa 2000]) and Chandra (squares; Class I, II, and III [Imanishi et al. 2001]) are also plotted. The thick solid line is the theoretical correlation line for flare sequence (EM \(\propto T^{17/2}\) for \(B = 50 \text{ G}\); see Fig. 2).
If we assume that $f = 0.1$, the heating flux necessary for the heating of average active-region coronae becomes consistent with observations (see Fig. 5). It is also interesting to note that the theory predicts that the coronal branch for protostars is closer to the flare branch than in the solar case.

Figure 5 shows curves of constant flare loop length and comparison with stellar observations (Hamaguchi et al. 2000; Hamaguchi 2001; Ozawa et al. 1999; Ozawa 2000; Imanishi, Koyama, & Tsuboi 2001) for $f = 0.1$. Stellar coronal data (nonflare data) are distributed in a wider area compared with solar active-region data. This might mean that the coronal heating flux in the stellar corona has a wider distribution than in the solar corona. Alternatively, this may simply be a result of “stellar microflares.” That is, even in the nonflare stellar corona, there would be a lot of “microflares” as in the solar corona. Such stellar microflares can be seen in these data. In fact, in the case of the solar coronal data analysis, microflare data are carefully removed from the “solar corona” data (Yashiro 2000), and the combination of the solar corona and stellar microflares shows wide distribution similar to that of stellar nonflare data. If this interpretation is right, the data in the leftmost side of stellar nonflare data (with EM $\sim 10^{31}$–$10^{33}$ cm$^{-3}$, $T \sim 1$–$2 \times 10^{7}$ K) correspond to true (nonmicroflare) stellar coronae and suggest that the heating flux $\sim 10^{7}$–$10^{9}$ ergs cm$^{-2}$ s$^{-1}$ is an order of magnitude larger than the solar coronal heating flux, and the stellar coronal loop length $\sim 10^{11}$–$10^{12}$ cm is also 1–2 orders of magnitude larger than the size of the solar-active region corona if the filling factor $= 0.1$.

4. DISCUSSION

4.1. Flare Evolution Track

In § 1, we noted that the EM-T diagram is similar to the H-R diagram for stars. In the case of the H-R diagram, we can plot a stellar evolution track on it. What about flare evolution in the EM-T diagram? We can plot a flare evolution track in the EM-T diagram, as discussed below.

The flare evolution is often discussed in the n-T diagram (Fig. 6), which is obtained from one-dimensional hydrodynamic numerical simulations of flare loops (e.g., Jakimiec, Sylwester, & Sylwester 1992, Sylwester 1996, for a review). In this case, when a sudden flare energy release occurs, the temperature of flare plasma increases until the conduction front reaches the top of the chromosphere. After that, the temperature becomes constant, $T_{\text{max}}$ (eq. [1]), and the evaporation starts with density $n_{\text{ev}}$ (eqs. [9a] and [9b]) determined by the enthalpy-conduction balance. The flare loop density gradually increases owing to evaporation as long as the flare heating continues. If the heating time is long enough, the loop density becomes comparable to the density ($n_{\text{RTV}}$, eqs. [21a] and [21b]) determined by the radiation-conduction balance. (This is equivalent to the scaling law of Rosner-Tucker-Vaiana’s 1978 steady coronal value, in which conduction time ($\tau_c$) is comparable to radiative cooling time ($\tau_r$). After that, the temperature and density gradually decrease and eventually return to the initial preflare coronal value.

(Fig. 7) From the above discussion, we see that one reason for the scatter of various flares (“main-sequence” flares) on the EM-T diagram may be the difference of flare heating time, which leads to different flare density (thus EM).
4.2. Nanoflare

If the flare loop length is smaller than \( \sim 10^7 \text{ to } 10^8 \text{ cm} \), the top of the flare loop is lower than the height of the transition region. In this case, the loop density is so large that it is difficult to produce high-temperature plasmas, i.e., flares. Hence, for loops shorter than \( 10^7 \text{ cm} \), there are no flares (i.e., we have another forbidden region). Along \( L = 10^7 \text{ cm} \), \( EM = 10^{41} \text{ to } 10^{44} \text{ cm}^{-3} \text{ for } T \sim 10^6 \text{ to } 10^7 \text{ K} \). These flares correspond to nanoflares observed with an EUV imaging telescope such as the Solar and Heliospheric Observatory (SOHO) EUV Imaging Telescope (EIT) and the Transition Region and Coronal Explorer (TRACE).

Using EUV data of nanoflares observed with SOHO/EIT and TRACE and soft X-ray data of microflares and flares observed with Yohkoh/SXT, Aschwanden (2000) empirically obtained \( EM \propto T^7 \). We plotted his data in Figure 4. We see that the nanoflare is on the \( L = \text{constant} = 10^7 \text{ cm} \) line (\( EM \propto T^{8/3} \)), and microflares-flares are on the \( EM \propto T^{15/2} \) or \( T^{17/2} \) line. Hence, we suggest that Aschwanden’s \( EM \propto T^7 \) is explained by a combination of \( EM \propto T^{8/3} \) line for nanoflares \((T < 10^6 \text{ K})\) and \( EM \propto T^{15/2} \) or \( T^{17/2} \) line for microflares \((T > 10^6 \text{ K})\).

4.3. Total Energy Released by Flares and Comparison with Recent Stellar Flare Observations

As emphasized in previous sections, the EM-T diagram is useful in estimating unobservable physical quantities such as magnetic field strength, flare loop length, etc., from the observed three parameters \( EM, T, \) and pref flare density \( n_0 \) (see Appendix A). Among these three quantities, it may be difficult to estimate the pref flare density. In that case, we assume \( n_0 = 10^9 \text{ cm}^{-3} \), which is a typical value of the solar active-region coronal density (Yashiro 2000). Then we can estimate the total energy released by a flare, which is simply assumed to be the total magnetic energy included in a flare loop

\[
E_{\text{mag}} = \frac{B^2}{8\pi} L^3,
\]

which is comparable to the total thermal energy content of a flare loop

\[
E_{\text{th}} = 3nkTL^3 = \frac{3}{2} E_{\text{mag}}
\]

by our pressure balance assumption. Using equation (26) and those in Appendix A, we find

\[
EM = 10^{46} \left( \frac{E_{\text{mag}}}{10^{29} \text{ ergs}} \right)^{5/7} \left( \frac{T}{10^7 \text{ K}} \right) \left( \frac{n_0}{10^9 \text{ cm}^{-3}} \right)^{3/7} \text{ cm}^{-3}
\]

(28)

This relation is plotted in Figure 8 for \( 10^{26}, 10^{29}, 10^{32}, 10^{35}, \) and \( 10^{38} \) ergs. We immediately find that energy ranges of solar microflares and solar flares are \( 10^{26} \text{ to } 10^{29} \) and \( 10^{29} \text{ to } 10^{32} \) ergs, respectively, both of which are consistent with the previous estimate. We also plotted new observational data of stellar flares from ASCA (Hamaguchi 2001; Ozawa 2000) and Chandra (Imanishi et al. 2001). It is found that stellar flare energy ranges from \( 10^{34} \) to \( 10^{38} \) ergs, if the pref flare coronal density is \( \sim 10^7 \text{ cm}^{-3} \). These values are again comparable to previous estimates of stellar flare energy using other methods.

Finally, we should emphasize that our method is based only on observed peak temperature, peak EM, and pref flare coronal density. We do not have to measure the detailed time evolution of flares. This is in contrast with other methods, such as the method using the flare decay time (e.g., van der Oord & Mewe 1989; Reale & Micela 1998).

4.4. Comment on the Stellar Flare Loop Length

Recently, using a hydrodynamic model (Reale & Micela 1998), Favata et al. (2001) reanalyzed large flaring events on pre-main-sequence stars and found that the size of the flaring regions is much smaller \((L < R_*)\) than previous estimates \((L > R_*)\) based on the quasi-static model (van der Oord & Mewe 1989). Here \( R_* \) is the stellar radius of the order of \( 10^{11} \text{ cm} \). Our results in Figure 2 predict that many stellar flares have loop length larger than \( 10^{11} \text{ cm} \). Are these results completely inconsistent with the results of Favata et al.? Two comments should be made for this kind of comparison. First, Figure 2 is based on the assumption that the pref flare coronal density is \( 10^9 \text{ cm}^{-3} \). If \( n_0 = 10^{10} \text{ cm}^{-3} \), which can often occur in very active solar active regions, the predicted loop length based on equation (7b) would decrease by a factor of \( 10^{2/5} \sim 2.5 \). Second, if the flare temperature is a factor of 2–2.5 larger than the original estimate, as discussed in Favata et al. (2001), the predicted loop length would be smaller by a factor of \( 2^{-8/5} \sim 2.5 \sim 3–4 \) (see eq. [7b]). In fact, in the case of faint stellar flares, the long exposure time may reduce the observed peak temperature compared with the real one. In either case, the loop length can decrease by a factor of \( \sim 3 \), which may fit with the results of Favata et al. (2001). In order to clarify whether these effects (pref flare density and true flare temperature) are really important or not, more detailed study will be necessary that takes into account the passband effect of observing instruments.3

3 For more detailed studies, the effects of the filling factor and unseen (out of band) EM should also be considered, although both effects would increase the predicted flare loop lengths (see eq. [D3]).
5. CONCLUSIONS

In this paper, we propose that the EM-\(T\) diagram for solar/stellar flares is similar to the H-R diagram for stars. The EM-\(T\) diagram shows the following properties:

1. It shows the universal correlation EM \(\propto T^{17/2}\) (for pressure balance loops) or \(T^{15/2}\) (for enthalpy-conduction balance loops) for the flare peak EM and \(T\). The former agrees with observations better than the latter. These may be called main-sequence flares. Physically, the main-sequence flare corresponds to the flare with magnetic field strength = constant or equivalently, heating flux = constant.

2. It has the forbidden region, which corresponds to the region in which the gas pressure of evaporated plasma exceeds the magnetic pressure of a flare loop so that a stable flare loop cannot exist.

3. It shows the coronal branch with EM \(\propto T^{15/2}\) for \(T < 10^7\) K and EM \(\propto T^{13/2}\) for \(T > 10^7\) K. This corresponds to the scaling law of Rosner, Tucker, & Vaiana (1978).

4. There is another forbidden region determined by the length of a flare loop; the lower limit of the flare loop is \(10^7\) cm. Those small flares on the \(L = \text{constant}\) line have a low temperature \((T < 10^6\) K\), thus explaining EUV nanoflares. This may be called the nanoflare branch.

5. We can plot the flare evolution track on the EM-\(T\) diagram. A flare evolves from the coronal branch to main-sequence flares and then eventually returns to the coronal branch.

6. Using the EM-\(T\) diagram, we can estimate unobservable physical quantities, such as the magnetic field strength, flare loop length, total flare energy, etc., using only three observed quantities: the peak temperature \(T\), peak emission measure EM, and preflare coronal density \(n_0\). We do not need observations of detailed flare evolution. Hence, even with limited data around the flare peak intensity and preflare phase, we can estimate the magnetic field strength, flare loop length, total flare energy, etc.

The authors would like to thank S. Yashiro for allowing us to use his solar coronal data on the EM-\(T\) relation before publication. We also thank K. Hamaguchi, K. Imanishi, K. Koyama, F. Nagase, H. Ozawa, Y. Tsuboi, and T. Watanabe for useful discussion on stellar flares and coronae. This work is supported in part by the JSPS grant for Japan-US Collaboration in Scientific Research.

APPENDIX A

USEFUL FORMULA

Using the EM-\(T\) diagram, we can estimate various physical quantities of solar and stellar flares. Here we summarize a useful formula for estimating various physical quantities on the basis of three observables (EM, \(T\), and \(n_0\)). Here, it should be noted that \(n_0\) is the electron density in preflare coronae (i.e., preevaporation corona) and is not equal to the electron density of a flare loop. It is also noted that \(n_0\) is not necessarily well observed. In the case of the preflare solar corona in active regions, \textit{Yohkoh} observations revealed that \(n_0 \sim 10^9\) cm\(^{-3}\) on average (e.g., Yashiro 2000; Yashiro & Shibata 2001). Hence, we often assume that \(n_0 = 10^9\) cm\(^{-3}\) when discussing actual application in this paper, although we leave the dependence on \(n_0\) in many formulas.

The basic quantities—magnetic field strength \(B\), flare loop length \(L\), and flare loop electron density \(n\) (after evaporation)—can be all derived from (EM, \(T\), and \(n_0\))

\[
B_{50} = \text{EM}_{48}^{1/5} T_7^{17/10} n_{09}^{3/10}, \quad L_9 = \text{EM}_{48}^{3/5} T_7^{-8/5} n_{09}^{-2/5}, \quad n_9 = 10^{1.5} \text{EM}_{48}^{-2/5} T_7^{12/5} n_{09}^{3/5}.
\]

Here, \(B_{50} = B/ (50\) G\), \(\text{EM}_{48} = \text{EM}/(10^{48}\) cm\(^{-3}\)), \(T_7 = T/(10^7\) K\), \(n_{09} = n_0/(10^9\) cm\(^{-3}\)), \(L_9 = L/(10^9\) cm\), and \(n_9 = n/(10^9\) cm\(^{-3}\)).

Using the above quantities, for preevaporation corona, we can calculate the Alfvén time (= reconnection heating time) \(\tau_A\), conduction cooling time \(\tau_c\), and radiative cooling time \(\tau_r\):

\[
\tau_A = L/V_A = L/(4\pi nm)^{1/2}/B \simeq 3 T_7^{-33/10} \text{EM}_{48}^{4/5} n_{09}^{-1/5}\text{ s}, \quad \tau_c = \frac{3nkT}{\kappa_0 T^{1/2}/L^2} \simeq 1.4 T_7^{-57/10} \text{EM}_{48}^{6/5} n_{09}^{1/5} \text{ s}, \quad \tau_r = \frac{3nkT}{n\Lambda(T)} \simeq 1.4 \times 10^5 T_7^{3/2} n_{09}^{-1}\text{ s for } T < 10^7\text{ K,}\]

\[
\tau_r \simeq 1.4 \times 10^5 T_7^{1/2} n_{09}^{-1}\text{ s for } T > 10^7\text{ K}. \quad (A6a) \quad (A6b)
\]

Here, \(\Lambda(T)\) is given by equations (20a) and (20b).
APPENDIX B

LIMIT OF APPLICATION FOR FLARE TEMPERATURE FORMULA

In deriving the flare temperature formula (eq. [1]), we implicitly assumed that the conduction cooling time \( \tau_c \) (eq. [A5]) is shorter than the reconnection heating time \( \tau_A \) (eq. [A4]). Here we shall derive the condition that the inequality

\[
\tau_c < \tau_A
\]

(B1)
is satisfied. Using equations (A4) and (A5), we find that the inequality becomes

\[
\text{EM}_{48} < 10^{5/2} T_r^{6} n_{09}^{-1} .
\]

(B2)

It is seen from Figure 9 that most of the data (for \( B = 50-150 \text{ G} \)) for solar and stellar flares satisfy this condition. In the case of stellar flares, some of the data lie above the critical line. If the preflare coronal density is higher, say, \( 10^{10} \text{ cm}^{-3} \), more data become marginal.

Above this line (Fig. 9), the flare maximum temperature is determined in situ by slow shock heating or ohmic heating in the diffusion region, if the radiative cooling time \( \tau_r \) is longer than the conduction time \( \tau_c \). That is, in an order-of-magnitude estimate, the flare temperature is determined by the energy (pressure) balance \( p \sim B^2/8\pi \) and is written as

\[
T_{\text{flare}} \sim B^2/(16\pi nk) .
\]

(B3)

APPENDIX C

CONDITION FOR OCCURRENCE OF CHROMOSPHERIC EVAPORATION

If the radiation cooling time \( \tau_r \) becomes comparable to or shorter than the conduction cooling time \( \tau_c \) in the reconnection-heated (but preevaporation) flare plasmas, the evaporation cannot occur any more. In this case, heating flux is balanced with in situ radiative cooling. Hence, the condition for chromospheric evaporation is

\[
\tau_c < \tau_r .
\]

(C1)

Using equations (A5), (A6a), and (A6b), we find that this inequality is rewritten as

\[
\text{EM}_{48} < 10^{4.2} T_r^{6} n_{09}^{-1} \quad \text{for} \quad T < 10^7 \text{ K} ,
\]

(C2a)

\[
\text{EM}_{48} < 10^{4.2} T_r^{31/6} n_{09}^{-1} \quad \text{for} \quad T > 10^7 \text{ K} .
\]

(C2b)

The line \( \tau_c = \tau_r \) is shown in Figure 10. It is seen from Figure 10 that the main flare sequence (for \( B = 50-150 \text{ G} \)) for solar and stellar flares satisfies this condition.

---

Fig. 9.—Critical line (\( \tau_A = \tau_c \); eq. [B2]) above which the temperature scaling law (eq. [1]) is no longer valid. The observed data for solar and stellar flares are also shown.
In our previous paper, as well as in §2 of this paper, we implicitly assumed that the flare loop with volume $L^3$ is filled with hot dense plasma with single temperature $T$ and density $n$. However, actually this may not be true, and we have to consider the effect of the filling factor $f$. In this case, we should write the EM including the effect of the filling factor

$$\text{EM} = fn^2L^3,$$

which is the same as equation (24). Then the flare EM-$T$ relation for the pressure balance scaling law becomes

$$\text{EM} \approx 10^{47} \left( \frac{f}{0.1} \right) \left( \frac{B}{50 \text{ G}} \right)^{-5} \left( \frac{n_0}{10^9 \text{ cm}^{-3}} \right)^{3/2} \left( \frac{T}{10^7 \text{ K}} \right)^{17/2} \text{ cm}^{-3}.$$  \hspace{1cm} (D1)

Hence, the effect of the filling factor is only to decrease the EM by that factor. The physical quantities derived from observed (EM, $T$, and $n_0$) with the effect of the filling factor are now written as

$$B_{50} = f_{0.1}^{-1/5} \text{EM}_{47}^{-1/5} T_{7}^{17/10} n_{09}^{3/10},$$  \hspace{1cm} (D2)

$$L_0 = f_{0.1}^{-3/5} \text{EM}_{47}^{3/5} T_{7}^{-8/5} n_{09}^{-2/5},$$  \hspace{1cm} (D3)

$$n_0 = 10^{1.5} f_{0.1}^{-2/5} \text{EM}_{47}^{2/5} T_{7}^{12/5} n_{09}^{-3/5},$$  \hspace{1cm} (D4)

where $f_{0.1} = f/(0.1)$. Unlike the coronal case (§3.2), there is no strong reason that the filling factor must be $\sim 0.1$ for solar and stellar flares.

REFERENCES

Aschwanden, M. 1999, Sol. Phys., 190, 233
Dere, K. P., Bartoe, J.-D. F., & Brueckner, G. E. 1989, Sol. Phys., 123, 41
Dulk, G. A. 1985, ARA&A, 23, 169
Favata, F., Micela, G., & Reale, F. 2001, A&A, 375, 485
Feigelson, E. D., & Montmerle, T. 1999, ARA&A, 37, 363
Feldman, U., Doschek, G. A., Behring, W. E., & Phillips, K. J. H. 1996, ApJ, 460, 1034
Feldman, U., Laming, J. M., & Doschek, G. A. 1995, ApJ, 451, L79
Fisher, G. H., & Hawley, S. L. 1990, ApJ, 357, 243
Grosso, N., Montmerle, T., Feigelson, E. D., Andre, P., Casanova, S., & Gregorio-Hetem, J. 1997, Nature, 387, 56
Haisch, B. M., Strong, K. T., & Rodono, M. 1991, ARA&A, 29, 275
Hamaguchi, K. 2001, Ph.D. thesis, Kyoto Univ.
Hamaguchi, K., Terada, H., Bamba, A., & Koyama, K. 2000, ApJ, 532, 1111
Hayashi, M. R., Shibata, K., & Matsumoto, R. 1996, ApJ, 468, L37
Hori, K., Yokoyama, T., Kosugi, T., & Shibata, K. 1997, ApJ, 489, 426
Imanishi, K., Koyama, K., & Tsuibo, Y. 2001, ApJ, 557, 747
Jakimiec, J., Sylvester, B., & Sylvester, J. 1992, A&A, 253, 269
Koyama, K., Ueno, S., Kobayashi, N., & Feigelson, E. 1996, PASJ, 48, L87
Lin, R. P., Schwartz, R. A., Pelling, R. M., & Hurley, K. C. 1981, ApJ, 251, L109
Masuda, S. 1994, Ph.D. thesis, Univ. Tokyo
Montmerle, T. 1998, in Proc. IAU Symp. 188, Hot Universe, ed. K. Koyama et al. (Dordrecht: Kluwer), 17
Nitta, N., & Yama, K. 1997, ApJ, 484, 927
Ohbaya, M., & Shibata, K. 1998, ApJ, 499, 934
Ozawa, M. 2000, Ph.D. thesis, ISAS
Ozawa, H., Nagase, F., Ueda, Y., Dotani, T., & Ishida, M. 1999, ApJ, 523, L81
Pallavicini, R. 2001, Adv. Space Res., 26, 1713
Reale, F., & Micela, G. 1998, A&A, 334, 1028
Rosner, R., Tucker, W. H., & Vaiana, G. S. 1978, ApJ, 220, 643
Rust, D. M., & Bar, V. 1973, Sol. Phys., 33, 445
Serio, S., Reale, F., Jakimiec, J., Sylwester, B., & Sylwester, J. 1991, A&A, 241, 197
Shibata, K., & Yokoyama, T. 1999, ApJ, 526, L49
Shimizu, T. 1995, PASJ, 47, 251
Spitzer, L., Jr. 1956, Physics of Fully Ionized Gases (New York: Interscience)
Stern, R. A. 1992, in Frontiers of X-Ray Astronomy, ed. Y. Tanaka & K. Koyama (Tokyo: Univ. Academy Press), 259
Sylwester, B. 1996, Space Sci. Rev., 76, 319
Takahashi, M. 1997, Ph.D. thesis, Graduate Univ. Advanced Study (National Astron. Obs. Japan)
Tsuboi, Y. 1999, Ph.D. thesis, Kyoto Univ.
Tsuboi, Y., Imanishi, K., Koyama, K., Grosso, N., & Montmerle, T. 2000, ApJ, 532, 1089

Tsuboi, Y., Koyama, K., Murakami, H., Hayashi, M., Skinner, S., & Ueno, S. 1998, ApJ, 503, 894
Tsuneta, S. 1996, ApJ, 456, 840
van den Oord, G. H. J., & Mewe, R. 1989, A&A, 213, 245
Watanabe, T. 1983, in Proc. of the Japan-France Seminar on Active Phenomena in the Outer Atmosphere of the Sun and Stars, ed. J.-C. Pecker & Y. Uchida (CNRA & Paris Obs.), 289
Withbroe, G. L., & Noyes, R. W. 1977, ARA&A, 15, 363
Yashiro, S. 2000, Ph.D. thesis, Univ. Tokyo
Yashiro, S., & Shibata, K., 2001, ApJ, 550, L113
Yokoyama, T., & Shibata, K., 1998, ApJ, 494, L113
———. 2001, ApJ, 549, 1160
Yuda, S., Hiei, E., Takahashi, M., & Watanabe, T. 1997, PASJ, 49, 115