An improved dynamics modeling during milling of the thin-walled parts based on magnetorheological damping fixture

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Abstract
Aerospace thin-walled parts vibrate during the process of milling due to their characteristics of low rigidity, which influences the machining quality of the workpiece. Based on the characteristics of magnetorheological fluid excitation solidification, a magnetorheological damping fixture is designed to semi-actively suppress the vibration generated during the machining process. This paper simplifies the multi-characteristic structure of aerospace thin-walled parts into rectangular thin plates. In accordance with Kirchhoff G.’s small deflection bending theory of elastic thin-plates and Duramberg’s principle, the differential equations for the transverse vibration of the thin-walled parts are established, which aims to obtain the natural frequency as well as vibration mode function of the thin-walled parts. In consideration of damping characteristics and the external dynamic milling force after the magnetorheological fluid excitation solidification, this paper uses the mode superposition method and an improved dynamic response mathematical model of the magnetorheological damping fixture thin-walled parts system of the linear multiple degrees of freedom, which has different volumes of magnetorheological fluids under forced vibration, was established. The maximum error between the predicted and measured values of the fixture-workpiece system dynamic response of displacement and acceleration is 16.2% and 15.5%, respectively. Finally, this paper verifies the feasibility as well as the effectiveness of the model through dynamics experiment and milling machining experiment.

Keywords Thin-walled part · Vibration suppression · Dynamic response · Magnetorheological damping fixture · Milling

1 Introduction

The thin-walled parts, due to the light weight and high strength, is widely used in aerospace. However, machining vibration, cutting force heat, and local stiffness reduction cause poor machining quality of thin-walled parts. Currently, a great number of studies have been carried out on traditional clamping strategies [1–3], but there are few studies on the use of new magnetorheological fixtures to clamp thin-walled parts to suppress milling vibration.

Currently, people have done a great number of studies on the optimization of clamping constraints for thin-walled workpieces. The fixture layout optimization is aimed at making the deformation or vibration of the fixture smaller and optimizing the positioning layout reasonably. It is mainly divided into two categories. One is on the basis of vibration reduction requirements to ensure that the layout plan can constrain the freedom of the workpiece and ensure its stability; the other one is based on the minimum deformation positioning layout so as to ensure that the positioning point is optimized when the positioning is perfect, thereby making the overall plastic deformation decrease. Fritzche et al. [4] have developed a software that automatically analyzes the workpiece geometry at the positioning and clamping points. With target search criteria, the clamping position on the component can be changed so that the clamping contours are geometrically simplified. Thus, a huge reduction in the complexity of the flexible clamping device is possible. Armillott et al. [5] judged the correctness of the workpiece position through judging whether the rank of the position matrix determinant was equal to the number of singular values of the position matrix. Gašpar et al. [6] used a constrained nonlinear optimization method to reconstruct the optimal placement of the fixture for a given set of workpieces, considering the kinematic constraints of...
the fixture. Wan and Zhang [7] optimized the fixture layout on the basis of a novel nonlinear programming problem of frequency sensitivity and maximized the natural frequency of the fixture-workpiece system. To sum up, the research on clamping constraints of thin-walled parts mainly introduces the finite element methods as well as the intelligent algorithms. The research gradually shifts from static modeling to dynamic modeling. However, these work only focus on fixture layout optimization, making fixture-workpiece system vibration become smaller. There is no research on increasing the system damping to suppress the vibration.

In terms of fixture-workpiece system dynamics modeling and process stability analysis, Wang et al. [8] established a conservatively stable lobe diagram due to the regeneration effect of milling and ruled out instability due to the cumulative effect of successive radial immersion. Meshreki et al. [9] used a semi-analytical method to study the impact of workpiece wall thickness changes on the dynamics of the process system for the processing of panel and frame parts. Li et al. [10] put forward a nonlinear dynamics model for milling titanium alloy thin-walled parts in consideration of process damping. An anti-vibration clearance angle was designed to suppress chatter based on this model. Moussawi et al. [11] developed a substructure-based virtual machining simulation method. It was demonstrated in vibration-prone drilling of thin-walled automotive workpieces. Fixture-affected zones are modeled by a MacNeal-type method. This can address the effects of clamping in dynamic mechanical modeling and create specific models of typical fixture configurations. Liu et al. [12] applied the Hermite difference and put forward an efficient full-discrete method (HFDM) for dealing with the stability of the milling process. Lv and Zhao [13] put forward a complete discretization method (RKCDM) based on Runge–Kutta. A set of time period coefficients and delay differential equations (DDE) were used for describing the delay of change to predict the milling stability with the change of spindle speed. Luo et al. [14] optimized the machining allowance distribution and material removal order in accordance with the machining vibration diagram. Such optimization improved the stability of the machining process and reduced the deformation of the workpiece. Wan et al. [15] put forward an assessment method on the impact of fixture layout on the dynamic response of the machining process for the milling process of frame-like thin-walled parts. Zeng et al. [16] established a tool-fixure-workpiece system model for the vibration problem during the processing of flexible components and optimized the layout of the fixture design, which could effectively suppress vibration and predict vibration displacement response. As for the fact that the existing point contact model cannot accurately describe the dynamic behavior of the fixture-complex surface workpiece system any longer. Maslo et al. [17] used finite element modal analysis to simulate changes in workpiece dynamics with simplified blade geometry. All extracted artifact dynamic states are combined in a simplified LPV model. The LPV model can describe the dynamic behavior of the changing process and guide the selection of favorable spindle speeds to stabilize the machining process. Dang et al. [18] proposed a theoretical method to analyze the dynamic behavior during milling of a bag-like thin-walled workpiece filled with a viscous fluid to suppress flutter. By extending the fluid–structure interaction theory and the principles of structural mechanics, the dynamic models of the spindle-tool system with viscous fluid and the bag-like thin-walled workpiece are derived. Dong et al. [19] adopted a virtual spring-damper model to describe the contact between the positioning element and the workpiece. The static deformation resulted from the clamping force and the dynamic deformation resulted from the milling force were analyzed and predicted on the basis of the finite element method. Li et al. [20] proposed a time-varying dynamics update method for thin-walled parts based on a degree of freedom (DOF) reduction model, which was incorporated into a milling dynamics model to predict chatter. The common point of the studies mentioned above is that the fixture system belongs to the passive mode, but there has been no research on semi-active vibration abatement methods so far.

The semi-active vibration abatement method can quickly change the stiffness and dynamic damping of the system through a small amount of energy input, thereby improving processing stability. Yang et al. [21] put forward a thin-walled workpiece vibration abatement device in accordance with the principle of electromagnetic induction, which could use the generated damping force for suppressing vibration. Pour and Behbahani [22] introduced an integrated mechatronic model for the chatter analysis of a machine tool equipped with MR damper, and the stiffness and damping of the machine tool are varied semi-actively by means of a magnetorheological (MR) damper to suppress chatter. Guo et al. [23] proposed a mechanical/magnetorheological composite clamping method to suppress vibration and improve the milling surface quality of parts, based on the previous magnetorheological fluid (MRF) flexible fixture. Ma et al. [24, 25] designed magnetorheological fixtures and used the Lagrangian method to establish a fixture-workpiece system dynamic equation in consideration of external damping factors, thereby evaluating the stability of the milling process under damping control. Bae et al. [26] developed a magnetically tuned mass damper (mTMD) to suppress the vibration of a cantilever panel similar to satellite solar panels. Xin et al. [27] put forward a shear valve mode MRF damper for pipeline vibration control. A linear quadratic regulator was made use of to study the optimal damping force of the MRF damper. Jiang et al. [28] proposed a MRF composite clamping to suppress milling vibration. And based on the fluid–solid coupling dynamics theory, the dynamics
modal of the MRF clamping was established. The above research work did not take the impact of magnetorheological fluid volumes change on the damping characteristics of the fixture-workpiece system into account, and it did not simplify the thin-walled parts into a more reasonable rectangular thin plate model either. However, the study is the dynamic response of forced vibration under magnetorheological damping.

The organization of this paper is as follows: The second section simplified aerospace thin-walled parts into rectangular thin plates. In accordance with Kirchhoff G.’s small deflection bending theory of elastic thin-plates and Daramberg’s principle and mode superposition principle, the natural frequency and main mode of vibration were calculated, and the dynamic response mathematical model of the magnetorheological damping fixture thin-walled parts system under forced vibration was established so as to evaluate the stability of the fixture-workpiece system. Based on this model, the dynamics experiments of aerospace thin-walled parts and the milling experiments of annular thin-walled parts were designed. The third section discussed and analyzed the experimental results and the machining quality so as to verify the feasibility and effectiveness of the model. The fourth section drew a conclusion of the full paper.

2 Methodology

This paper has developed a magnetorheological damping fixture. Under the action of a magnetic field, the ferromagnetic particles of the magnetorheological fluid in the fixture are rapidly magnetized along the direction of the magnetic field, which gather to form a magnetic chain structure and adhere closely to the surface of the workpiece. Magnetorheological fluid can be used to fill a variety of irregular thin-walled parts. It can play the part of multi-point auxiliary support. As shown in Fig. 1. It is solidified by Newtonian fluid into a solid state. During this process, the shear stress of the magnetorheological fluid increases sharply, which can provide a distributed damping support force for the thin-walled workpiece in contact with the MRF. In addition, the excitation-solidified MRF and the increased MRF mass increase the stiffness of the fixture-workpiece system. In the milling process, the increased damping and stiffness of the fixture-workpiece system suppress the milling vibration and improve the machining stability. When the magnetic field disappears, the magnetorheological fluid is inverted into a liquid, and the clamping is both quick and convenient. This paper studied the effect of restraining flutter of magnetorheological fixture through the theory of semi-active vibration control.

In the process of machining, the height of thin-walled parts is generally much smaller than the size of the bottom surface, and this forms a thin flat elastic body. Although the thin-walled parts are very thin, they still have a certain bending rigidity and the deflection is much smaller than its thickness. Thus, the thin-walled parts are simplified to a more reasonable rectangular thin plate. The study on its dynamic response during milling can provide methods and strategies for suppressing vibration.

2.1 Natural frequency and main vibration mode of thin-walled parts

The aerospace thin-walled parts that were used in the experiment are shown in Fig. 2. There are upper and lower ends, and there are many grooves on the side. The thinnest part of
the wall is only 1 mm. This paper simplified the model as a rectangular thin-walled plate of equal thickness with length $a$, width $b$, thickness $h$, and density $\rho$, as shown in Fig. 3, which replaced the micro-element body $hdxdy$ with any rectangular micro-element body $dxdy$ on the neutral surface and expressed the load and the internal force on the cross section on the neutral plane, as shown in Fig. 4. In Fig. 4, $Q_x, Q_y, Q_z$ are the transverse shear forces per unit width, and $M_x, M_y, M_z$ are the upward bending per unit width bending moment, $M_{xy}, M_{yx}, M_{yz}, M_{zy}$ are the torque per unit width, the exciting force distributed on the unit area is $p(x, y, t)$, the inertial force per unit area of the thin plate is,

\[
\nabla^4 w + \frac{\rho h}{D_0} \frac{\partial^2 w}{\partial t^2} = p(x, y, t)
\] (1)

In the formula, $D_0 = \frac{Eh^3}{12(1-\nu^2)}$ is the bending stiffness of the thin plate, $E$ is the elastic modulus of the thin plate, $\nu$ is the Poisson’s ratio, and is the reharmonization operator, $\nabla^4 = \frac{\partial^4}{\partial x^4} + 2 \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}$.

In the differential equation of the lateral bending vibration of the thin plate, set the excitation force $p(x, y, t) = 0$, then the differential equation of the free vibration of the thin plate can be obtained as follows:

\[
\nabla^4 w + \frac{\rho h}{D_0} \frac{\partial^2 w}{\partial t^2} = 0
\] (2)

Using the semi-inverse solution method, it is assumed that the free vibration displacement response of the thin plate is as follows:

\[
w(x, y, t) = W(x, y) \sin (\omega t + \varphi)
\] (3)

In the formula, $W(x, y)$ is the main vibration mode of the thin plate. In accordance with the boundary conditions, the following formula can be obtained:

\[
\nabla^4 W - \rho^4 W = 0
\] (4)

In the formula,

\[
\rho^4 = \frac{\rho h}{D_0} \omega^2
\] (5)

![Fig. 3 Simplified rectangular thin-walled plate](image)

![Fig. 4 The rectangular micro-element body on the neutral surface of the rectangular thin-walled plate](image)
The main mode of Euler Bernoulli beam is, \(i = 1, 2, \ldots\). In accordance with this formula, the main mode of rectangular thin plate can be assumed as follows:

\[
W_{ij}(x, y) = A_{ij} \sin \frac{i \pi x}{a} \sin \frac{j \pi y}{b} \tag{6}
\]

In the formula, \(A_{ij}\) are related to the initial conditions. Through substituting formula (5) into formula (4), the following can be obtained:

\[
\left[ \left( \frac{i \pi}{a} \right)^2 + \left( \frac{j \pi}{a} \right)^2 \right]^2 - \beta^4 \left( \frac{i \pi}{a} \sin \frac{i \pi x}{a} \sin \frac{j \pi y}{b} \right) = 0 \tag{7}
\]

In order to make this condition meet all points on the neutral surface of the rectangular thin plate, that is, both \(x\) and \(y\) can be met, so the frequency equation must meet the following relationship:

\[
\left[ \left( \frac{i \pi}{a} \right)^2 + \left( \frac{j \pi}{a} \right)^2 \right]^2 - \beta^4 = 0 \tag{8}
\]

Through substituting Eq. (5) into Eq. (8), the natural frequencies of each order of the rectangular thin plate can be solved as follows:

\[
\omega_{ij} = \pi \left( \frac{i^2}{a^2} + \frac{j^2}{b^2} \right) \sqrt{\frac{D_0}{\rho h}} \quad i = 1, 2, \ldots \quad j = 1, 2, \ldots \tag{9}
\]

2.2 Dynamic response of magnetorheological damping fixture thin-walled parts system with different volumes of magnetorheological fluids under forced vibration

Magnetorheological fluid damping effect: Under the action of a magnetic field, the ferromagnetic particles in the magnetorheological fluid are magnetized and aggregated into a chain-like particle molecular structure under the action of the magnetic field. When the system is in an excitation state, the magnetic chain generates dislocation motion. Internal molecular friction dissipates the energy input to the system. Thus, the damping produced

\[
D_0 \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \eta_{ij}(t) \int_{\Omega} \left( \nabla^4 \psi_{ij} \right) + c_e \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \left( \nabla^2 \psi_{ij} \right) \eta_{ij}(t) + \rho h \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \psi_{ij} \dot{\eta}_{ij}(t) = F(x, y, t) \tag{15}
\]
By multiplying both sides of the above equation by another regular mode shape $\psi_{r,s}(x, y)$ and integrating $x$ and $y$ on the area domain $\Omega$ of the rectangular thin plate, the following formulation can be obtained:

$$
\sum_{j=1}^{\infty} \sum_{i=1}^{\infty} \bar{q}_{s,i}(t) \iint_{\Omega} D_0 \nabla^4 \psi_{r,s} \psi_{r,s} dxdy 
+ \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} \bar{\psi}_{s,i}(t) \iint_{\Omega} c_r (\nabla \psi_{r,s} \psi_{r,s}) dxdy 
+ \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} \bar{\psi}_{s,i}(t) \iint_{\Omega} \rho \psi_{r,s} \psi_{r,s} dxdy 
= \iint_{\Omega} F(x, y, t) \psi_{r,s} dxdy
$$

In accordance with the orthogonalization of the canonical mode, the vibration equation under the decoupled canonical coordinates is shown as follows:

$$
\ddot{\eta}_{r,s}(t) + 2\xi_{r,s} \omega_{r,s} \dot{\eta}_{r,s}(t) + \omega_{r,s}^2 \eta_{r,s}(t) = q_{r,s}(t) 
$$

In the formula, $\xi_{r,s}$ is the regular milling force, and $q_{r,s}(t) = \iint_{\Omega} F(x, y, t) \psi_{r,s} dxdy$. 

Assume that the initial conditions of the magnetorheological damping thin plate system are as follows:

$$
\psi(x, y, 0) = g_1(x, y), \left. \frac{\partial \psi}{\partial t} \right|_{t=0} = g_2(x, y)
$$

Substituting formula (18) into formula (14), the following formulation can be obtained:

$$
\psi(x, y, 0) = g_1(x, y) = \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} \psi_{i,j}(x, y) \bar{\eta}_{i,j}(0) 
$$

$$
\left. \frac{\partial \psi}{\partial t} \right|_{t=0} = g_2(x, y) = \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} \psi_{i,j}(x, y) \dot{\bar{\eta}}_{i,j}(0)
$$

Through multiplying both sides of the above two equations by $\rho \psi_{r,s}(x, y)$ and integrating them on the area $\Omega$ of the thin plate, and using the orthogonalization condition of the canonical mode, the initial conditions of the canonical coordinates can be obtained as follows:

$$
\eta_{r,s}(0) = \iint_{\Omega} \rho g_1(x, y) \psi_{r,s}(x, y) dxdy 
$$

$$
\dot{\eta}_{r,s}(0) = \iint_{\Omega} \rho g_2(x, y) \psi_{r,s}(x, y) dxdy
$$

In accordance with the principle of mode superposition, the forced vibration response of a multi-degree-of-freedom thin-walled workpiece system constrained by magnetorheological damping can be considered as the result of the superposition of multiple single-degree-of-freedom vibration responses. In accordance with Duhamel's integral, the response of the system to any excitation force can be considered as the sum of the response to a series of impulses...
in the corresponding time interval, and the regular response
of the system can be obtained from Eqs. (17), (21) and (22)
as follows:

\[ \eta_{r,s}(t) = e^{-\xi_{r,s} \omega_{r,s}'t} \left[ \eta_{r,s}(0) \cos \omega_{r,s}'t + \frac{\dot{\eta}_{r,s}(0) + \xi_{r,s} \omega_{r,s} \eta_{r,s}(0)}{\omega_{r,s}'} \sin \omega_{r,s}'t \right] 
+ \frac{1}{\omega_{r,s}'} \int_{0}^{t} q_{r,s}(\tau) e^{-\xi_{r,s} \omega_{r,s}'(t-\tau)} \sin \omega_{r,s}'(t-\tau) d\tau \]

(23)

Through the normalized condition, convert the main
mode of vibration to the regular mode of vibration:

\[ w(x, y, t) = \sqrt{\frac{4}{abh\rho}} \sin \frac{i\pi x}{a} \sin \frac{j\pi y}{b} \sum_{r=1}^{\infty} \sum_{s=1}^{\infty} e^{-\xi_{r,s} \omega_{r,s}'t} \left( \eta_{r,s}(0) \cos \omega_{r,s}'t + \frac{\dot{\eta}_{r,s}(0) + \xi_{r,s} \omega_{r,s} \eta_{r,s}(0)}{\omega_{r,s}'} \sin \omega_{r,s}'t \right) 
+ \frac{1}{\omega_{r,s}'} \int_{0}^{t} q_{r,s}(\tau) e^{-\xi_{r,s} \omega_{r,s}'(t-\tau)} \sin \omega_{r,s}'(t-\tau) d\tau \]

(26)

\[ \int_{\Omega} \rho h W_{ij}^2(x, y) dx dy = \int_{0}^{a} \int_{0}^{b} \rho h \left( A_{ij} \sin \frac{i\pi x}{a} \sin \frac{j\pi y}{b} \right)^2 dx dy = 1 
(24) \]

The solution is \( A_{ij} = \sqrt{\frac{4}{abh\rho}} \), so the regular mode shape
can be solved as:

\[ \psi_{ij}(x, y) = \sqrt{\frac{4}{abh\rho}} \sin \frac{i\pi x}{a} \sin \frac{j\pi y}{b} \]

(25)

Through substituting formulas (25) and (23) into for-
mula (14), the displacement response of forced vibration
of the magnetorheological damping fixture thin-walled
parts system related to the magnetorheological fluid vol-
umes under generalized coordinates can be obtained as
follows:

Fig. 7 Schematic diagram of
ring-shaped thin-walled parts

(a) Three-dimensional drawing of annular thin-walled parts
(b) The dimensions of annular thin-walled part
The second-order derivative of the displacement response in formula (26) can be used to obtain the acceleration response:

\[
A(x, y, t) = \ddot{w}(x, y, t) = \sum_{r=1}^{\infty} \sum_{s=1}^{\infty} \sqrt{\frac{4}{ab^2}} \sin \frac{i\pi x}{a} \sin \frac{j\pi y}{b} \left\{ e^{-\xi_r\omega_{rs}t} \left( \eta_{r,s}(0) \cos \omega_{rs}t + \frac{\eta_{r,s}(0) + \xi_r\omega_{rs}\eta_{r,s}(0)}{\omega_{rs}^2} \sin \omega_{rs}t \right) \right. \\
+ \frac{1}{\omega_{rs}^2} \int_0^t q_{rs}(\tau) e^{-\xi_r\omega_{rs}(t-\tau)} \sin \omega_{rs}(t-\tau)d\tau \\
- \frac{q_{rs}(t)}{\omega_{rs}^2} \left[ e^{-\xi_r\omega_{rs}t} \left( -\omega_{rs}^2 \eta_{r,s}(0) \sin \omega_{rs}t + \xi_r\omega_{rs}\eta_{r,s}(0) \cos \omega_{rs}t \right) \\
+ e^{-\xi_r\omega_{rs}t} \left( -\omega_{rs}^2 \eta_{r,s}(0) \cos \omega_{rs}t - \omega_{rs}^2 \eta_{r,s}(0) \sin \omega_{rs}t + \xi_r\omega_{rs}\eta_{r,s}(0) \sin \omega_{rs}t \right) \\
+ \frac{q_{rs}(t)}{\omega_{rs}^2} \left[ e^{-\xi_r\omega_{rs}t} \left( -\omega_{rs}^2 \eta_{r,s}(0) \sin \omega_{rs}t + \omega_{rs}^2 \eta_{r,s}(0) \cos \omega_{rs}t \right) \right] \int_0^t q_{rs}(\tau) e^{\xi_r\omega_{rs}\tau} \sin \omega_{rs}\tau d\tau \\
- \frac{q_{rs}(t)}{\omega_{rs}^2} \left[ e^{-\xi_r\omega_{rs}t} \left( \omega_{rs}^2 \eta_{r,s}(0) \cos \omega_{rs}t + \omega_{rs}^2 \eta_{r,s}(0) \sin \omega_{rs}t \right) \right] \int_0^t q_{rs}(\tau) e^{\xi_r\omega_{rs}\tau} \sin \omega_{rs}\tau d\tau \left\} \right.
\]

(27)

Before using the magnetorheological fixture for milling, the required volume of magnetorheological fluid and milling force are substituted into the dynamic model, and response prediction and stability analysis can be performed to guide the relevant settings of the fixture before machining.

2.3 Dynamics experiment setup for aerospace thin-walled parts

In order to verify the validity of the model and analyze the influence of different magnetorheological fluid volumes on the stiffness and damping characteristics of the fixture workpiece system, the relevant dynamics experiments are designed in this paper. As shown in Fig. 5, when fixing the aerospace thin-walled parts in the magnetorheological damping fixture and adding 0L, 3L, 5L, 7L magnetorheological fluids into the fixture box respectively, the 7L magnetorheological fluid can basically reach the state of fully covering the thin-walled parts. As shown in Fig. 6, the modal hammer percussion experiment uses a laser vibrometer (PSV-400) to test the dynamic response of adding different volumes of magnetorheological fluid and analyzes the impact of magnetorheological fluid volumes change on suppressing flutter of magnetorheological damping fixture.
2.4 Experiment setup for milling of annular thin-walled parts

In order to verify the impact of the magnetorheological damping fixture filled with different volumes of magnetorheological fluid on the processing of different types of thin-walled parts, a magnetorheological damping composite fixture for clamping annular thin-walled parts was designed to study the change of milling force and milling vibration acceleration. As shown in Fig. 7, for the existing annular aluminum alloy blanks, the inner wall is reamed from 26 to 27 mm, and the thinnest wall thickness is 1.5 mm. As shown in Fig. 8, by adding 0.1L, 0.2L, 0.3L magnetorheological fluid to the magnetorheological damping fixture and installing a dynamometer (Kistler, 9139AA) at the bottom of the fixture, the milling force can be measured. Beside it is possible to choose 4mm4-edge cemented carbide milling cutter, the process parameters are spindle speed 12,000 r/min, cutting depth 1 mm, feed rate 100 mm/min, and milling on a three-axis milling machine (Carver S600).

3 Results and discussion

This section mainly verifies the dynamic model that is proposed in the previous section. It also analyzes the impact of different magnetorheological fluid volumes on the stiffness and damping characteristics of the fixture-workpiece system. Besides, it studies the milling characteristics of different magnetorheological fluid volumes, including the impact of milling force and milling vibration acceleration. And it also discusses the processing quality of thin-walled parts after milling.

![Graphs showing displacement response with different MRF volumes](image_url)

**Fig. 10** The displacement response with different magnetorheological fluid volumes added
3.1 Verification of dynamic model of the magnetorheological damping fixture-workpiece system

In order to accurately analyze the difference in the dynamic characteristics of the magnetorheological damping fixture-workpiece system resulted from the change of the magnetorheological fluid volume, the dynamics experiment results of aerospace thin-walled parts were discussed. Figure 9 plots the frequency response function of the fixture-workpiece system. The results have shown that, except for the addition of 7L magnetorheological fluid, in the first-order mode, the amplitude of the frequency response function is greatly reduced with the increase of the volume of the added magnetorheological fluid and the natural frequency gradually increases; the natural frequency value of the system is the smallest when no magnetorheological fluid is added, which is 441.5 Hz, and the natural frequency when 5L of magnetorheological fluid is added is the largest, which is about 580.6 Hz, with an increase of 31.5%. At a certain magnetic field strength, when the magnetization saturation is reached, the magnetic field cannot completely magnetize the ferromagnetic particles inside the MRF. At this time, the MRF is a solid–liquid mixed state, so the damping and stiffness will be slightly lower. At the magnetic field strength set in this experiment, when about 5L MRF is added to the magnetorheological damping fixture, the magnetization saturation is reached. Too much MRF cannot be completely magnetized, so the natural frequency of the system is higher than that when 7L MRF is added to fixture. The whole experimental results have shown that the magnetorheological damping fixture improves the rigidity of the fixture-workpiece system effectively than that of the non-magnetorheological fixture. When the magnetorheological fluid excitation was not saturated, as the magnetorheological fluid volume increased, the greater the rigidity of the system, the better the vibration abatement effect.

Figure 10 shows the displacement response of the fixture-workpiece system without magnetorheological fluid and adding different volumes of magnetorheological fluid. Table 1 shows the peak displacement and deformation corresponding to different situations and the time it takes for the vibration of the workpiece starting from excitation to the steady state. It is found that as the volume of the magnetorheological fluid added increased, the vibration deformation displacement and the time-consuming to stabilize state gradually decreased, except for the case when 7-L magnetorheological fluid was added. Since the solidified magnetorheological fluid has the damping characteristics, the energy applied to the system by external excitation was gradually dissipated through magnetorheological damping. The more magnetorheological fluid, the faster the energy dissipation, and as the system vibration

| MRF volume | Without MRF | With 3L MRF | With 5L MRF | With 7L MRF |
|------------|-------------|-------------|-------------|-------------|
| Response maximum displacement (μm) | 43.5 | 42.4 | 40.5 | 41.5 |
| Excitation to Stability time-consuming (s) | 0.35 | 0.28 | 0.15 | 0.26 |

Fig. 11 Comparison of predicted and measured displacement response with different magnetorheological fluid volumes
tended to stabilize, it became faster. Thus, it took the longest time for the non-magnetorheological fluid fixture to stabilize, which was 0.35 s, and the damping fixture with 5-L magnetorheological fluid took the shortest time, which was 0.15 s. The time has been decreased by 57.1%. The measured data was abnormal when 7-L magnetorheological fluid was added because of the phenomenon of magnetic saturation. The entire experimental results have verified that the magnetorheological damping fixture dissipated the excitation energy faster than the non-magnetorheological fixture. When the solidification of magnetorheological fluid was in unsaturation condition, as the volumes of the magnetorheological fluid increased, the more prominent the system damping characteristics, the better the vibration abatement effect.

Figure 11 shows the comparison of the fixture-workpiece system displacement response predictions as well as the measured values with different magnetorheological fluid volumes added. The maximum error was 16.2%, which is compared with the previous modeling methodology [29], and the prediction accuracy of the model is improved by 10.2%. The predicted displacement response was consistent with the measured value trend. The results have verified that the dynamic model of the magnetorheological damping fixture-workpiece system could effectively predict the displacement response of the thin-walled workpieces.

### 3.2 Effects of the magnetorheological fluid volumes on the milling characteristics

In order to further analyze the impact of the magnetorheological damping fixture filled with different volumes of magnetorheological fluid on the actual milling conditions, the milling force and milling vibration acceleration are discussed. They were measured in Sect. 2.4 for the machining of annular thin-walled parts under certain milling process parameters. The measured results of the milling force experiment are shown in Fig. 12. In Fig. 12, the X direction is the tool feed direction and the Y direction is the tool width direction, so the maximum milling force in the X direction is greater than the Y direction; the Z axis is the tool spindle, which undertakes the main cutting action, so the maximum milling force in the Z direction is much larger than in the X and Y directions. As the volumes of the magnetorheological fluid in the fixture increases, the maximum milling force in the three directions gradually decreases. However, the maximum milling force in the Z-axis direction does not change so much. The milling force of the fixture without magnetorheological fluid is the largest. When 0.1-L and 0.3-L magnetorheological fluid are added, the maximum milling force in the X, Y, and Z directions of the workpiece is 52.49 N, 43.56 N, 58.89 N, 41.38 N, 31.21 N,
and 56.16 N, respectively, the maximum milling force is decreased by 21.2%, 28.4%, and 4.6% respectively. The results show that the damping force provided by the solidified magnetorheological fluid can offset part of the milling force effectively. The Y direction has the best milling force control effect, followed by the X direction.

By substituting the maximum milling force measured in Fig. 13 into formula (27), the maximum milling vibration acceleration can be calculated, as shown in Fig. 14b. By comparing the predicted value with the measured value, the predicted value of the maximum milling vibration acceleration is generally smaller than the measured value and the maximum error is 15.5%. In many previous model prediction studies, and considering uncontrollable factors such as external environmental vibration and measurement error, it is often used as a method to evaluate the validity of the model based on whether the prediction accuracy is less than 20%. Therefore, the prediction results have verified that the dynamics model of the magnetorheological damping fixture-workpiece system could effectively predict the acceleration response of the thin-walled parts under the milling force.

### 3.3 Effects of the magnetorheological fluid volumes on machining quality

Compared with the results of dynamic experiments and milling experiments, the processing quality of thin-walled parts...
can directly reflect the difference in vibration suppression of different volumes of magnetorheological fluids. This section carries out comparative analysis of vibration suppression effects according to the processing quality results in three aspects: roughness, coaxiality, and cylindricity. Roughness is a significant factor for measuring the accuracy of the surface of the workpiece. It is also the standard for judging the quality of processing. This section made use of the Form Talysurf200 surface roughness meter to test the inner surface roughness value of the milled annular thin-walled parts in the previous section. Besides, a marker was used to mark 4 points to be measured (1–4) at the same height on the surface of the inner hole (1–4). The 4 points were tested respectively and the average value was calculated. These not only reduced the error resulted from the experimental measurement, but also could judge the processing stability, as shown in Fig. 15.

In accordance with the roughness test results in Table 2, it is concluded that the use of magnetorheological damping fixtures can greatly reduce the surface roughness of
the workpiece compared with non-magnetorheological fixtures. The inner hole of the workpiece is rougher after adding 0.1-L and 0.3-L magnetorheological fluid. The degree values Ra, Rz, and Rq are 0.25, 1.36, 0.33, 0.15, 0.83, and 0.21, respectively, which are decreased by 40%, 39.0%, and 36.4%, respectively.

Coaxiality theory considers the reference axis as an ideal straight line, and the measured axis bends, tilts, and shifts. This section used the Talyrond 565 cylindricity meter and the PMM-C 8.10.6 ultra-high-precision three-coordinate measuring machine to test the cylindricity as well as the coaxiality of the milled workpiece. Several test points were taken on the inner surface of the workpiece so as to obtain the cylindricity as well as the position of each axis based on one of the axes, and the deviation of the two axes was compared. The field test is shown in Fig. 16.

In the test results of cylindricity and concentricity of annular thin-walled parts in Table 3, NOMINAL represents the ideal value of concentricity of annular thin-walled parts, TOL (tolerance) means the set tolerance value of concentricity, MEAS (measurement) represents the actual measured value of the coaxiality, DEV (deviation) means the difference between the measured value of the coaxiality and the ideal value, and OUTTOL (out tolerance) represents the difference between the measured value of the coaxiality and the set tolerance value, the out tolerance is 0 when the measured value is within the set tolerance range. In accordance with the actual measured value of cylindricity and coaxiality of annular thin-walled parts, it was found that when the volume of magnetorheological fluid increased with the addition of 0.1L and 0.3L magnetorheological fluid, the concentricity value decreased, the cylindricity value increased, and the coaxiality increased greatly, reaching 54.5%. The inner cylindricity increased by 42.0%.

From the above data, it can be seen that through adding a suitable volume of magnetorheological fluid, it was possible to provide damping force after excitation solidification. This could improve the stability of the inner hole machining of thin-walled parts and effectively suppress the milling vibration as well as the quality of surface machining.

**Table 2** Results of roughness test of annular thin-walled parts

| MRF volume | Test point | 1 | 2 | 3 | 4 | Average value |
|------------|------------|---|---|---|---|---------------|
| Without MRF | Ra(μm) | 0.33 | 0.29 | 0.29 | 0.29 | 0.30 |
| | Rz(μm) | 1.57 | 1.27 | 1.37 | 1.33 | 1.39 |
| | Rq(μm) | 0.49 | 0.38 | 0.38 | 0.37 | 0.41 |
| With 0.1L MRF | Ra(μm) | 0.25 | 0.25 | 0.24 | 0.26 | 0.25 |
| | Rz(μm) | 1.42 | 1.38 | 1.30 | 1.35 | 1.36 |
| | Rq(μm) | 0.35 | 0.31 | 0.33 | 0.32 | 0.33 |
| With 0.2L MRF | Ra(μm) | 0.21 | 0.18 | 0.19 | 0.19 | 0.19 |
| | Rz(μm) | 1.18 | 1.12 | 1.08 | 1.09 | 1.12 |
| | Rq(μm) | 0.25 | 0.27 | 0.26 | 0.21 | 0.25 |
| With 0.3L MRF | Ra(μm) | 0.14 | 0.15 | 0.14 | 0.16 | 0.15 |
| | Rz(μm) | 0.79 | 0.85 | 0.81 | 0.85 | 0.83 |
| | Rq(μm) | 0.19 | 0.20 | 0.19 | 0.25 | 0.21 |

**Fig. 16** Scheme of cylindricity and concentricity measurement
### 4 Conclusion

The magnetorheological damping fixture proposed in this paper can effectively suppress the vibration of the workpiece. An improved dynamic response mathematical model of the magnetorheological damping fixture thin-walled parts system of the linear multiple degrees of freedom under forced vibration was established, which provides theoretical and technical basis for the subsequent magnetorheological fluid volumes matching milling quality of workpiece. The results provide a new idea for the use of magnetorheological damping fixtures so as to control the machining quality of thin-walled parts in aerospace. The conclusions are summarized as follows:

1. Through simplifying aerospace thin-walled parts into rectangular thin plates, in accordance with Kirchhoff G.’s small deflection bending theory of elastic thin-plates, Daramberg’s principle and the mode superposition principle, an improved dynamic response mathematical model of the magnetorheological damping fixture thin-walled parts system of the linear multiple degrees of freedom, which has different volumes of magnetorheological fluids under forced vibration, was established in this paper. The maximum errors of displacement and acceleration responses of the theoretical calculation value and the experimental value were 16.2% and 15.5%, respectively. The results verified the validity of the model.

2. The dynamics experiment as well as the milling experiment proved that as the increase of the magnetorheological fluid volumes, the natural frequency as well as the damping of the magnetorheological damping fixture-workpiece system increased, and the system’s dynamic rigidity and damping characteristics improved. When 5-L magnetorheological fluid was added, the vibration abatement was the best.

3. Through comparing the milling quality of thin-walled parts with different volumes of magnetorheological fluids, it was found in this paper that the maximum reduction of roughness was 40%, the coaxiality increased by 54.5%, and the inner cylindricity increased by 42.0%, which effectively and directly verified that through adding a suitable volume of magnetorheological fluid, it was possible to suppress vibration.

### Author contribution
Xiaohui Jiang constructed the idea. Ning Yang performed the experiments and wrote the manuscript, while Yong Zhang and Shan Gao contributed to the conception of the study and helped perform the analysis with constructive discussions.

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### Data availability
The data used to support the findings of this study are included within the article.

### Declarations

**Ethics approval** This work does not include human and animal; hence, ethical approval from any committee is not required.

**Consent to participate** This work does not include human and animal; hence, ethical approval from any committee is not required.

**Consent to publication** The participants have consented to the submission of the case report to the journal.

**Conflict of interest** The authors declare no competing interests.

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