The Origin of the Entropy in the Universe

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Abstract

We discuss the entropy generation in quantum tunneling of a relativistic particle under the influence of a time varying force with the help of squeezing formalism. It is shown that if one associates classical coarse grained entropy to the phase space volume, there is an inevitable entropy increase due to the changes in position and momentum variances. The entropy change can be quantified by a simple expression $\Delta S = \ln \cosh 2r$, where $r$ is the squeeze parameter measuring the “height” and “width” of the potential barrier. We suggest that the universe could have acquired its initial entropy in a quantum squeeze from “nothing” and briefly discuss the implications of our proposal.

One of the major problems of theoretical cosmology is to explain the present day large scale isotropy and homogeneity of the universe. The possible explanation of this puzzle must have an intimate relation to the initial

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conditions at the beginning of expansion. In a beautiful paper [1], Penrose suggested a notion of a geometrical gravitational entropy related to the Weyl tensor in order to explain the “low” matter entropy of the present day universe [2]. In this picture the universe expands from a highly regular initial state characterized by the vanishing Weyl tensor. The irregularities then develop with time due to gravitational clamping and the entropy grows.

To shed more light on the notion of the gravitational entropy related to cosmological expansion two different approaches were adopted. Davies [3] used the analogy of the cosmological horizon to that one of the black holes. In the context of the black hole physics a well-established Bekenstein-Hawking entropy relation exists connecting the area of the event horizon of the black hole to its entropy. The status of the cosmological horizon is different, however, and being an observer dependent quantity its significance as the measure of gravitational entropy is less clear.

In a different approach, a number of authors [4]-[6] have sought the way to relate the entropy of the gravitational field to the capacity of the later to create matter particles.

In this paper we take rather a different view on the origin of the gravitational entropy in the universe. We suggest that the universe could have acquired its initial entropy in the process of quantum tunneling from “nothing” [7]-[10] thus relating the initial entropy of the universe to its quantum origin.

In quantum cosmology one describes the whole spacetime by a wave function rather than by a classical spacetime [11, 12]. The birth of the universe is described as a quantum tunneling process where the small universe nucleates out of “nothing”. This process will be referred to, later on, as to quantum squeezing. While introducing squeeze formalism does not lead to
a new description of quantum tunneling, it makes the relation of tunneling with entropy generation more transparent.

The entropy generation in quantum tunneling process was first discussed by Casher and Englert [13] who have concluded that even in the non-relativistic quantum mechanics, the tunneling can generate entropy, identified with the Legendre transformation of the Euclidean action in the forbidden region, and yield thermal states with negative temperature. The authors appealed in their paper to the apparent unitarity break-down and information loss in the tunneling of a clock. The unitarity break-down is only apparent, as authors pointed out, and is based on the fact that in tunneling one does not account for the “backward” waves generated on the right side of the barrier by tracing them out. There is an information loss about the system, but it is rather an artifact of applying the semiclassical approximation. One could restore the unitarity in principle by trying to keep the “backward” waves in mind.

Does this mean that the entropy generation in a quantum tunneling problem is not real? Not at all. In physics there are many examples where one concentrates on some particular degrees of freedom observing the growth of entropy. For instance, if one focuses on molecular degrees of freedom, entropy does grow in a chemical reaction (the vessel is heating up). The unitarity still strictly holds but is rather confined to same hidden degrees of freedom.

Our first task in this paper will be to show how the entropy generation in quantum tunneling is rather a manifestation of a more general intrinsic property of Quantum Mechanics: the classical entropy growth due to a quantum squeeze.

In an ordinary Quantum Mechanics (see [14], for example), when two Hermitian operators do not commute \([A, B] = iC\), the product of their un-
certainties satisfies the following relation

$$\Delta A \Delta B \geq \frac{1}{2} |\langle C \rangle|.$$  

(1)

The inequality (1) is the manifestation of the essential nature of Quantum Mechanics, the indeterministic character of the theory. The basic principle of Quantum Mechanics states that it is impossible to simultaneously measure, with arbitrary precision, two complementary variables since the product of their uncertainties is bounded from below.

In fact, the equality in the expression (1) only holds when the operators $A$ and $B$ are proportional, so that the states with minimal uncertainty satisfy the following equation

$$(B - \langle B \rangle) \psi = \frac{i}{2} \frac{\langle C \rangle}{(\Delta A)^2} (A - \langle A \rangle) \psi.$$  

(2)

If, for simplicity, the operator $A$ is identified with the position and $B$ with the linear momentum operators, this equation reads

$$\left( \frac{\hbar}{i} \frac{d}{dx} - \langle p_x \rangle \right) \psi = \frac{i\hbar}{2(\Delta x)^2} (x - \langle x \rangle) \psi.$$  

(3)

The solution to the equation (3) gives the following normalized wave function

$$\psi(x) = \left[ 2\pi(\Delta x)^2 \right]^{-1/4} \exp \left[ - \frac{(x - \langle x \rangle)^2}{4(\Delta x)^2} + \frac{i\langle p_x \rangle x}{\hbar} \right],$$  

(4)

for which the minimum uncertainty relation holds:

$$\Delta x \Delta p = \frac{1}{2} \hbar.$$  

(5)

The states of the form (4) minimizing the uncertainty relation are called coherent states in Quantum Optics (see Schumaker [15] and the references there in).
Consider now a typical Gaussian wave function of the form given by (4)

\[ \psi(x) = A e^{-\frac{1}{2} \gamma x^2}. \]  

(6)

This type of wave packets arise naturally in systems governed by quadratic Hamiltonians. It follows [15], that the complex parameter \( \gamma \) is of most importance, since the form of the wavefunction (4) suggests that it is an eigenfunction of the linear combination \( \tilde{x} + \frac{i}{\gamma} \tilde{p} \), so that the parameter \( \gamma \) is related to the variances of position and linear momentum operators.

The linear combination \( \tilde{x} + \frac{i}{\gamma} \tilde{p} \) can be written alternatively with the help of the usual creation and annihilation operators as

\[ \tilde{x} + \frac{i}{\gamma} \tilde{p} = \frac{1}{\sqrt{2}} \left[ (1 + \frac{1}{\gamma}) a + (1 - \frac{1}{\gamma}) a^\dagger \right]. \]  

(7)

Thus, if one is interested in the dynamical evolution of the wavefunction (4) under some quantum fluctuation one may as well study the behavior of the operators \( a \) and \( a^\dagger \) instead.

Suppose now, the evolution of the state is governed by a quadratic Hamiltonian, one then may introduce [13] a unitary operator, the so-called single-mode squeeze operator

\[ S_1(r, \varphi) = \exp \left[ \frac{1}{2} r (e^{-2i\varphi} a^2 - e^{2i\varphi} a^\dagger a^2) \right], \]  

(8)

where \( 0 \leq r < \infty \) and \( -\frac{\pi}{2} \leq \varphi < \frac{\pi}{2} \), are parameters depending on the specific form of the disturbance the initial state is subjected to.

A generic evolution of a typical coherent Gaussian state involves displacement and rotation together with squeeze. For the purposes of our discussion, however, since neither rotation nor displacement produce changes in position and momentum uncertainties, we concentrate here on the action of squeeze operator alone.
The squeeze operator acts on the annihilation operator in the following way

\[ S_1(r, \varphi) a S_1^\dagger(r, \varphi) = a \cosh r + a^\dagger e^{2i\varphi} \sinh r, \]  

(9)

and this implies that the uncertainties in \( a \) and \( a^\dagger \) (\( \tilde{x} \) and \( \tilde{p} \)) are changed under the squeeze, displacing the mean position or linear momentum. If one starts with a coherent state described by the minimal uncertainty relation \( \Delta x \Delta p = 1/2, (\hbar = 1) \) the state will evolve under the squeeze to one with

\[
\langle (\Delta \tilde{x})^2 \rangle = \frac{1}{2} (\cosh 2r - \cos 2\varphi \sinh 2r) 
\]

(10)

\[
\langle (\Delta \tilde{p})^2 \rangle = \frac{1}{2} (\cosh 2r + \cos 2\varphi \sinh 2r) 
\]

(11)

\[
\langle \Delta \tilde{x} \Delta \tilde{p} \rangle = -\frac{1}{2} \sinh 2r \sin 2\varphi. 
\]

(12)

The parameters \( r \) and \( \varphi \) of the single-mode squeeze operator are related to the parameter \( \gamma \) of the Gaussian wave function (6) in the following way

\[
\gamma = \frac{\cosh r + e^{2i\varphi} \sinh r}{\cosh r - e^{2i\varphi} \sinh r}. 
\]

(13)

Many useful properties of the single-mode squeeze operator may be readily found from the properties of the transformation matrix \( C_{r,\varphi} \) associated to it

\[
C_{r,\varphi} = \begin{pmatrix}
\cosh r & e^{2i\varphi} \sinh r \\
e^{-2i\varphi} \sinh r & \cosh r
\end{pmatrix}
\]

(14)

such that

\[
S_1(r, \varphi) \begin{pmatrix} a \\ a^\dagger \end{pmatrix} S_1^\dagger(r, \varphi) \equiv \begin{pmatrix} \alpha(r, \varphi) \\ \alpha^\dagger(r, \varphi) \end{pmatrix} = C_{r,\varphi} \begin{pmatrix} a \\ a^\dagger \end{pmatrix}. 
\]

(15)

Actually the transformation matrices \( C_{r,\varphi} \) can be figured out from the requirement that the unitary transformation on \( \tilde{x}, \tilde{p} \) or \( a \), preserve their commutation relations.
Suppose now one starts with a coherent state and then the state evolves. We assume that the quantum mechanical wave function at large times can be accurately described by classical physics, though instead of classical trajectory, the system would be rather represented by a classical probability distribution \([16, 17]\). Typically, if the evolution is governed by a quadratic Hamiltonian, this will be a Gaussian probability distribution

\[
P(x, p) = A \exp\left\{-\frac{1}{2}[\alpha x^2 + \beta p^2 + 2\gamma xp]\right\},
\]

where \(A\) is fixed by normalizing the probability to unity and \(\alpha, \beta\) and \(\gamma\) are related to the second noise moments of the position and linear momentum operators (the coefficient \(\gamma\) expresses the correlations between the position and the momentum).

For the classical probability distribution one may define entropy as:

\[
S = - \int P(x, p) \ln P(x, p) \, dx \, dp.
\]

Evaluating the integral and discarding a constant contribution one gets

\[
S = -\frac{1}{2} \ln(\alpha \beta - \gamma^2),
\]

or in terms of the second noise moments the expression for the entropy becomes

\[
S = \frac{1}{2} \ln\left(\frac{\langle(\Delta x)^2\rangle\langle(\Delta p)^2\rangle - \langle\Delta x\Delta p\rangle_{sym}^2}{\langle(\Delta x)^2\rangle\langle(\Delta p)^2\rangle}\right).
\]

Since we have started with the coherent state, the initial product of uncertainties is given by \(\Delta x_0 \Delta p_0 = 1/2\). After the state has evolved, the product of uncertainties can be evaluated using the expressions \([10] - [12]\). We thus get

\[
S = \frac{1}{2} \ln\left(\frac{1}{4} \left[\cosh^2 2r - \sinh^2 2r(\cos^2 2\varphi + \sin^2 2\varphi)\right]\right) = \ln \frac{1}{2},
\]
and consequently $\Delta S = 0$.

From a “purist” point of view this is a perfectly expected result, no entropy generation is obtained in a squeeze. After all the evolution is strictly unitary! Note, however, that one starts with a coherent quantum state and then expects to correctly describe the system classically at late times.

In the quantum-to-classical transition the coherence must be lost. To account for the decoherence the system should be coarse-grained. For our example we will define the coarse-grained entropy in the spirit of Brandenberger, Mukhanov and Prokopec [18] by averaging the second noise moments over the whole period of the squeeze angle. The averaged variances are:

$$\langle (\Delta x)^2 \rangle = \frac{1}{2} \cosh 2r$$  \hspace{1cm} (21)

$$\langle (\Delta p)^2 \rangle = \frac{1}{2} \cosh 2r$$  \hspace{1cm} (22)

$$\langle \Delta x \Delta p \rangle_{sym} = 0,$$  \hspace{1cm} (23)

and the “coarse grained” entropy now grows in the squeeze

$$\Delta S = \ln \cosh 2r.$$  \hspace{1cm} (24)

Although the expression (24) was derived for the initially coherent state, one may show that starting with an arbitrary Gaussian state a similar expression for the entropy growth will hold.

The relation (24) is our central result. It shows that generically there is an entropy generation associated with the quantum evolution of a given coherent state. The increase in entropy is related with the increase in the product of uncertainties in position and momentum and is telling one that a certain amount of the information about the state has been irreversibly lost in a quantum-to-classical transition. Quantitatively this increase can be
simply expressed in terms of the squeeze parameter $r$ as given by the relation (24).

The expression (24) is applicable in principle both for small and large squeeze limits. For the large squeeze limit $r \gg 1$ we get

$$\Delta S = 2r - \ln 2,$$

which is identical to the expression given by Gasperini and Giovannini [19] for the entropy generation associated to the particle production re-formulated in squeeze formalism. Also, in the large squeeze limit it is similar to the expression given by Rosu and Reyes [20]. Their expression for the entropy growth associated with squeeze and inferred from the information entropy, also known as the Shannon-Wehrl entropy reads

$$S = 1 + \frac{1}{2} \ln \sinh^2 r.$$ (26)

However, it is easy to see that this expression and those given in [18] as well as in [19] give negative entropy growth for small $r$ which is rather undesirable. Although in the examples [18, 19] the small $r$ limit is physically meaningless, this is not so in the example considered in [20].

We now consider a relativistic massless particle under the action of a time-dependent force $dp(t)/dt$. One may show [21, 22] that the particle may be described by the following wave function

$$\psi(x, t) = \varphi(t) e^{ip(t)x},$$ (27)

where $\varphi(t)$ satisfies the parametric oscillator type equation

$$\frac{d^2 \varphi(t)}{dt^2} + p^2(t) \varphi(t) = 0.$$ (28)

We assume that the force switches off at $t \to -\infty (+\infty)$, and that the corresponding asymptotic values of the momentum are given by $p_- (p_+)$. 
It is well known that one may express the general solution of the equation (28) as a combination of two linearly independent basic solutions

$$\varphi(t) = a\chi(t) + b^\dagger\chi^*(t),$$  \hspace{1cm} (29)

where $a$ and $b^\dagger$ are two arbitrary complex constants.

On the other hand, the general solution $\varphi(t)$ may also be expressed yet in a different base

$$\varphi(t) = c\xi(t) + d^\dagger\xi^*(t),$$  \hspace{1cm} (30)

where $c$ and $d^\dagger$ are different complex constants which are related to the constants $a$ and $b^\dagger$ by the following transformation

$$a = e^{-i\theta} \cosh rc - e^{-i(\theta - 2\varphi)} \sinh rd^\dagger$$  \hspace{1cm} (31)

$$b^\dagger = -e^{i(\theta - 2\varphi)} \sinh rc + e^{i\theta} \cosh rd^\dagger.$$  \hspace{1cm} (32)

If the force vanishes asymptotically at $t \to \pm\infty$, the basic solutions $\chi(t)$ and $\xi(t)$ may be chosen as

$$\chi(t) \rightarrow \frac{1}{\sqrt{2p_-}} e^{-ip_-t}, \quad t \to -\infty$$  \hspace{1cm} (33)

$$\xi(t) \rightarrow \frac{1}{\sqrt{2p_+}} e^{-ip_+t}, \quad t \to \infty.$$  \hspace{1cm} (34)

We assume now, as in [13], that there are no reflected waves in the future region, $d^\dagger d = 0$, this is to say that the particle tunnels in time, then the probability of such a tunneling $T^2$, is given by the ratio

$$T^2 = \frac{c^\dagger c}{a^\dagger a}.$$  \hspace{1cm} (35)

and one may further express it in terms of the parameter $r$ appearing in the transformations (31) and (32)

$$T^2 = \frac{1}{\cosh^2 r}.$$  \hspace{1cm} (36)
Therefore, the parameter $r$ is simply related to the tunneling probability as discussed by Grishchuk and Sidorov [23]. Yet, there is more to it. To shed more light on the physical interpretation of the parameter $r$ we find it necessary to consider further quantization of the theory, i.e. to treat now the wave function $\psi(x,t)$ as a field operator.

The equation (28), then, becomes an operator-valued equation and may be obtained from the following Hamiltonian

$$
\mathcal{H} = \hat{\pi}^* \hat{\pi} + F(t)(\hat{\pi}^* \hat{\phi} + \hat{\phi}^* \hat{\pi} + \hat{\pi}^* \hat{\phi} + \hat{\phi}^* \hat{\pi}) + G(t) \hat{\phi}^* \hat{\phi},
$$

(37)

where $F(t)$ and $G(t)$ are functions determined by the form of the external force and $\hat{\pi}$ is the canonical momentum of the field $\hat{\phi}$.

The *quadratic Hamiltonian* (37) determines the time evolution of the quantum state. In Schrödinger picture, this evolution is given by the following equation

$$
i\hbar \frac{d}{dt} |\phi\rangle = \mathcal{H} |\phi\rangle,
$$

(38)

with Gaussian functions been its solutions.

On transforming the equation (28) into an operator-valued equation, the $c$-numbers $a, b^\dagger, c$ and $d^\dagger$ become operators themselves at the same time satisfying an operator-valued analogue of the transformations (31) and (32).

These operator equations may be obtained, on the other hand, by considering a unitary evolution in time of the field under the following operator

$$
U(t) = R(\theta) S(r, \varphi)
$$

(39)

where the operators $R(\theta)$ and $S(r, \varphi)$ are [23, 24]

$$
R(\theta) = \exp \{-i \theta(t) (c^\dagger c + d^\dagger d)\}
$$

(40)

$$
S(r, \varphi) = \exp \{r(t)(e^{-2i\varphi(t)} cd - e^{2i\varphi(t)} c^\dagger d^\dagger)\},
$$

(41)
and the evolution of the $a$ and $b^\dagger$ is determined by

\begin{align}
    a & = R^\dagger S^\dagger cSR \\
    b^\dagger & = R^\dagger S^\dagger d^\dagger SR.
\end{align}

(42) (43)

We thus see that the parameter $r$ in the equations (31) and (32) can be physically interpreted as the squeeze parameter of the second quantized theory and is related to the entropy definition (24) on one hand, but it is also associated to the tunneling probability in the first quantization by equation (36) on the other.

Thus, to calculate the squeeze parameter the first quantization is sufficient. However, to understand its physical connection to squeeze operator, one must consider the further quantization of the theory.

Now, the tunneling probability is completely determined by the squeeze parameter $r$ and is independent of the squeezing angle $\varphi$. In the classical region at late times, the knowledge of $\varphi$ is useless, thus one may justify averaging over all possible values of $\varphi$.

Consequently, using Eq. (24), one expects that a typical coherent state will be squeezed on passing through a potential barrier with the entropy gain of

$$\Delta S = \ln \cosh 2r.$$  

(44)

For large $r \gg 1$, one readily gets

$$\Delta S = -2 \ln T + \ln 2,$$

(45)

where $T$ is the transmission amplitude.

Casher and Englert [13] obtain a similar result, the difference being, however, in the physical origin of the “tunneling entropy”. While Casher and
Englert argue that the entropy has a thermal character, we hold that the entropy growth is produced due to a quantum-to-classical transition decoherence. Quantitatively the information loss may be evaluated by considering the growth of the phase space volume of the coarse grained system.

We now turn to cosmology. In quantum cosmology the metric is substituted by a purely quantum quantity, the wave function of the universe, which in turn contains all the information and the answers to the question one could ask about the state of the universe. The wave function of the universe is a solution to the Wheeler-DeWitt equation, which is an analog of the Schrödinger equation. While in general it looks impossible to deal with this equation, one may reduce the degrees of freedom of the wave function just to two (the scale factor of the universe and some matter field) defining the solutions of the Wheeler-DeWitt equation on the so-called minisuperspace. Even this simplifying procedure would not leave one with the unique wave function for the universe. One must still supply the Wheeler-DeWitt equation with the appropriate boundary conditions. This is one of the main difficulties with the approach of quantum cosmology, for the boundary conditions must be based on some physical experience or intuition which we rather lack. The simplest and probably the most natural boundary condition for the wave function of the universe was proposed by Vilenkin [10]. He suggested to impose the boundary conditions directly on the superspace by requiring that the wave function of the universe should contain only the outgoing waves on the boundaries of the superspace. This is the so-called tunneling boundary condition.

The basic idea of this approach is that the universe is created from nothing, and is based on the fact that the probability of quantum creation of a closed spacetime is nonzero. The simplest model to consider would be the
closed model with a cosmological constant $\Lambda$.

The Wheeler-DeWitt equation for the wave function $\Psi(a)$ reads \[25\]
\[
\frac{d^2 \Psi(a)}{da^2} + \omega^2(a) \Psi(a) = 0,
\]
(46)
where $\omega^2(a) = -a^2(1 - \Lambda a^2)$ and $a$ represents the scale factor of the universe.

One can easily see that this equation is the same as the equation (28), with scale factor playing the role of time.

In the classically allowed region $a \geq \Lambda^{-1/2}$ the solution to the equation (46) is
\[
\Psi(a) = B_1 \sqrt{\omega(a)} \exp\left\{i \int_{\Lambda^{-1/2}}^{a} \omega(a')da'\right\} + B_2 \sqrt{\omega(a)} \exp\left\{-i \int_{\Lambda^{-1/2}}^{a} \omega(a')da'\right\}.
\]
(47)

The condition that at $a \to \infty$ one has only expanding universes leads to the following solution [10] for $a > \Lambda^{-1/2}$,
\[
\Psi(a) = \frac{1}{\sqrt{\omega(a)}} \exp\left\{i \int_{\Lambda^{-1/2}}^{a} \omega(a')da' - i\pi/4\right\},
\]
(48)
note that this condition is equivalent to the absence of reflected waves to the right of the barrier in our previous example.

The squeeze parameter $r$ for this tunneling may be evaluated as
\[
R = \ln 2 + \frac{1}{3\Lambda} = \ln 2 + \frac{3}{16G^2\rho_v},
\]
(49)
where we have substituted the vacuum energy density for $\Lambda$.

Thus the entropy gained by the universe in the course of the quantum squeeze will be
\[
\Delta S = \frac{3}{8G^2\rho_v} + \ln 2.
\]
(50)

After the tunneling the universe is described by the classical solution to the Einstein equations with a positive cosmological constant, the de Sitter
solution. Note, that the entropy generated in a quantum squeeze from “nothing” agrees with the entropy associated with the de Sitter solution obtained previously by various authors. Arguing in a somewhat different way, G. Horwitz \[26\] comes to a similar idea of interconnecting the tunneling entropy to the very interesting question of the origin of time in quantum cosmology.

One would certainly like to start with a universe having a rather small tunneling entropy. This implies that the initial vacuum energy density should be as large as possible. At any rate, one may try to put a lower bound for the vacuum energy density by arguing that the universe should have had acquired less entropy in the squeeze than the present day entropy of the universe. Estimating the present day entropy in the universe as $10^{100}$ \[3\], one may conclude that $\rho_v$ should have been larger than $10^{-24}GeV^4$ initially. The value $10^{-24}GeV^4$ is the lowest possible value the $\rho_v$ could have ever had.

For a typical SUSY theory, $\rho_v$ is of order $10^{12}GeV^4$ which would leave one with an appreciable entropy of $10^{64}$. This is rather a large entropy to start with, however, larger values of $\rho_v$ would give a lower value.

One could try to put the lower limit on the initial radius of the universe by using the Bekenstein entropy bound for a closed system \[27\], however, it is clear that unless we know more about fundamental interactions it is difficult to make much practical use, in the context of quantum cosmology, of the entropy value.

Nevertheless, our main point here was to show that quantum systems which behave almost classically at large times, may be characterized by an entropy increase. The entropy acquired has quite a universal character. Either it is a particle creation, universe nucleation, or any other different form of a quantum squeeze, it is accompanied by the entropy generation due

\[1\] We are very grateful to Professor Horwitz for communicating his results to us.
to the increase in uncertainty or in the volume of the phase space necessary to describe the system in the classical regime. The existence of such a classical regime is a must for our conclusions to be meaningful because the classical entropy notion introduced here does not apply for the systems which can not be described quasi-classically. For those systems, one should probably use different entropy measures [28, 29]. Yet, since many physical systems of interest do behave classically or almost classically, the entropy generation process as described here, could shed more light on the physics of these systems.

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