THE POMERON BEYOND BFKL

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Abstract

Conformally invariant reggeon interactions derived from $t$-channel unitarity are discussed and progress towards understanding the “physical Pomeron”, via massless quark reggeon interactions, is briefly outlined.

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1. INTRODUCTION

I will discuss two topics that go beyond the BFKL Pomeron in QCD. The first is the derivation of conformally invariant reggeon interactions from $t$-channel unitarity. In particular $\ln^4[\rho_{11}' \rho_{22}' / \rho_{12}' \rho_{12}']$ should appear\[1], in some approximation, in the NLO BFKL kernel and higher powers of the same logarithm can be expected in higher orders.

Secondly I will briefly outline my work in progress\[2] that is aimed at understanding the “physical Pomeron” via the reggeon interactions due to massless quarks. I discuss how triple-Regge vertices contain an infra-red anomaly which produces reggeon diagram infra-red divergences at zero quark mass when the SU(3) gauge symmetry is broken to SU(2). The resulting divergent amplitudes contain a “single gluon” SuperCritical Pomeron and, I hope to prove, a hadron spectrum with confinement and chiral symmetry breaking. I begin by briefly listing the elements of the formalism I use\[3].

2. ANALYTIC MULTI-REGGE THEORY

The key ingredients are

i) Angular Variables - For an N-particle amplitude $M_N(P_1, \ldots, P_N) \equiv M_N(t_1, \ldots, t_{N-3}, g_1, \ldots, g_{N-3})$ where $t_j = Q_j^2$ and $g_j \in$ the little group of $Q_j$, i.e. $g_j \in SO(3)$ or $g_j \in SO(2,1)$ for $t_j < 0$. There are (N-3) $t_i$ variables, (N-3) $z_j(\equiv \cos \theta_j)$ variables and (N-4) $u_{jk}(\equiv e^{i(\mu_j - \nu_k)})$ variables.

ii) Multi-Regge Limits - all $z_j \to \infty$; Helicity-Pole Limits - some $u_{jk} \to \infty$.

iii) Partial-wave Expansions - $f(g) = \sum_{J=0}^\infty \sum_{|n|,|n'|<J} D_{J_{nn}}(g) a_{J_{nn}}$, $g \in SO(3)$

$\to M_N(t, g) = \sum \prod_i D_{J_{nn_i}}(g_i) a_{J_{nn_i}(t)}$

iv) Asymptotic Dispersion Relations - $M_N(p_1, \ldots, p_N) = \sum_C M^C_N(p_1, \ldots, p_N) + M^0$

$M^C_N(p_1, \ldots, p_N) = \frac{1}{(2\pi)^N} \int \Delta^C(\ldots t_{i-1}, u_{jk}, \ldots) D_{J_{nn_i}}(g_i) a_{J_{nn_i}(\hat{t})}$

and $\sum_C$ is over all sets of (N-3) Regge limit asymptotic cuts.

v) Sommerfeld-Watson Representations of Spectral Components e.g.

$M^C_4 = \frac{1}{8} \sum_{N_1, N_2} \int \frac{dn_1dn_2dJ_1 u_{n_1}^{n_1} u_{n_2}^{n_2} \phi_{n_1}^{n_1} \phi_{n_2}^{n_2}(z_1)d_{n_1,n_2}^{n_1+n_2}(z_2)d_{n_2,n_0}^{n_2+n_0}(z_3)\sin \pi n_2 \sin \pi (n_1-n_2) \sin \pi (J_1-n_1) a_{N_2N_3}^C(J_1, n_1, n_2, t)}$
from which the form of multi-Regge behaviour in any limit can be extracted.

vi) **t-channel Unitarity in the *J*-plane**  
Multiparticle unitarity can be partial-wave projected, diagonalized, and continued to complex *J* in the form

\[ a_+^J - a_-^J = i \int d\rho \sum N \frac{d\rho_{12}}{\sin \pi (J - n_1 - n_2)} \frac{d\rho_{34}}{\sin \pi (n_1 - n_3 - n_4)} \cdots a_+^J a_-^J \]

Regge poles at \( n_i = \alpha_i \), together with “nonsense poles” at \( J = n_1 + n_2 - 1, n_1 = n_3 + n_4 - 1, \ldots \) generate Regge cuts. The *J*-plane regge cut discontinuity due to *M* Regge poles \( \alpha_\sim = (\alpha_1, \alpha_2, \cdots, \alpha_M) \)

\[ \text{disc}_{J=\alpha_M(t)} a_\sim N_J(J) = \xi_M \int d\hat{\rho} a_\sim(J^+) a_\sim(J^-) \frac{\delta(J - 1 - \sum_{k=1}^M (n_k - 1))}{\sin \frac{\pi}{2}(\alpha_1 - \tau_1) \cdots \sin \frac{\pi}{2}(\alpha_M - \tau_M)} \]

is referred to as *reggeon unitarity*. Because the gluon “reggeizes”, reggeon unitarity is a strong constraint on multigluon exchange amplitudes.

### 3. NLO CONFORMAL SYMMETRY.

The *J*-plane unitarity equations can also be used \([4]\) to study the *t*-channel thresholds of reggeon (reggeized gluon) interactions due to nonsense gluon (particle) states. The gauge group is inserted via Regge pole vertices. Known leading log results can be easily rederived. In particular gluon reggeization is due to two-gluon nonsense states \( (\otimes \otimes \otimes \otimes \otimes \otimes) \) while the \( O(g^2) \) BFKL kernel is given by the three-gluon nonsense state in the two-reggeon interaction \( (\otimes \otimes \otimes \otimes) \). The four-gluon nonsense state gives a NLO \( O(g^4) \) contribution \( K^{(4)} \) to the BFKL kernel \( (\otimes \otimes \otimes \otimes) \).

The phase-space integration \( \int d\rho \) gives two-dimensional \( k_\perp \) integrals via \( \int dt_1 dt_2 \chi^{-1/2}(t, t_1, t_2) = 2 \int d^2k_\perp \). Using transverse momentum diagrams \([4]\)

**BFKL kernel** \( \leftrightarrow \sum \left( -\frac{1}{2} \begin{array}{c} \overline{\otimes} \\ \overline{\otimes} \end{array} + \begin{array}{c} \overline{\otimes} \\ \overline{\otimes} \end{array} - \frac{1}{2} \begin{array}{c} \overline{\otimes} \\ \overline{\otimes} \end{array} \right) \)

Since the unitarity analysis has no scale, if the results obtained are infra-red finite they are automatically scale-invariant.

**\( K^{(4)} \) kernel** \( \leftrightarrow \sum \left( -\frac{2}{3} \begin{array}{c} \overline{\otimes} \\ \overline{\otimes} \end{array} - \begin{array}{c} \overline{\otimes} \\ \overline{\otimes} \end{array} - \begin{array}{c} \overline{\otimes} \\ \overline{\otimes} \end{array} \right) \)

Using eigenfunctions \( \phi_{\nu,n}(k) = |k|^\nu e^{i\frac{\pi}{2} \theta} \), the spectrum of \( K^{(4)} \) has the form \([4]\)

\[ \mathcal{E}(\nu, n) = \frac{1}{4} |\chi(\nu, n)|^2 - \Lambda(\nu, n) \]

where \( \chi(\nu, n) \) are the eigenvalues of the BFKL kernel and \( \Lambda(\nu, n) = -\frac{1}{4\pi} \left( \beta'\left(\frac{|n|+1}{2} + i\nu\right) + \beta'\left(\frac{|n|+1}{2} - i\nu\right) \right) \)

with \( \beta(x) = \int_0^1 dy y^{x-1}[1 + y]^{-1} \). The holomorphic separability of \( \Lambda(\nu, n) \), together with BFKL conformal invariance, suggests that the impact parameter space kernel \( \tilde{K}^{(4)}(\rho_1, \rho_2, \rho_{\nu}, \rho_{\nu'}) \) should have a conformal invariance property.
Gauge invariance allows terms independent of any of $\rho_1, \rho_2, \rho_1', \rho_2'$ to be added to $\tilde{K}^{(4)}$. Using $\int d^2k \ e^{ik\cdot\rho}/(k^2 + m^2) = -\ln|m\rho|/2 + \psi(1) + O(m)$ the two gluon propagator $\prod_{j=1}^{J} \delta^2(k_1-k_{1j})\delta^2(k_2-k_{2j})$ gives the $\rho$-space analog of $\frac{4}{(2\pi)^4} \ln |\rho_1\rho_2|$ (where $\rho_{1'} = \rho_1 - \rho_1'$ etc.) which, after symmetrizing under $1 \leftrightarrow 2$, is equivalent to $\frac{4}{(2\pi)^4} \ln^2 R$, with $R = |\rho_1\rho_2'/\rho_1\rho_2|$. Evaluating the $k_\perp$ diagrams of $K^{(4)}$ gives that, up to terms that can be dropped because of gauge invariance, each diagram also has a $\rho$-space analog with $\ln |\rho_{ij}|$ “propagators”, e.g. $\ln |\rho_1\rho_2'| \ln |\rho_{21}'| \ln |\rho_1\rho_2| + \ldots$. The sum of all diagrams gives the remarkably simple, manifestly conformally invariant, representation

$$\tilde{K}^{(4)} \leftrightarrow \frac{1}{24} \ln^4 R$$

This representation was initially found via the Feynman diagram NNLO calculation of the large rapidity scattering of two virtual photons.

Note that $\ln^3 R$ contains the diagrams that are naturally associated with the BFKL kernel. In fact the spectrum result for $K^{(4)}$ implies that (formally) we can write $K_{BFKL} = c_1 \ln^3 R + c_2 [\ln^4 R - K_2]^\frac{1}{2}$, where $\ln^3 R$ is antisymmetric under $1 \leftrightarrow 2$ and $[\ln^4 R - K_2]^\frac{1}{2}$ is symmetric. $K_2$ is defined by the eigenvalue spectrum $\Lambda(\nu,n)$. Possibly $K_{BFKL}$ can be usefully defined as a holomorphic extension of $\ln^3 R$.

$\ln^m R$ is directly related to the diagrams involving two pairs of points joined by $m$ propagators and would naturally appear in a high-order conformal approximation to $K_{BFKL}$. An all-orders sum might then produce powers of $R$. Whether $t$-channel unitarity can be used to discuss such contributions is, at present, an open question. Understanding the relationship between $K^{(4)}$ and the exact NLO kernel should help. Wüsthoff is currently investigating whether the spectra of $\ln^m R$ can be obtained via the generating function $\mathcal{G}(R, \delta) = R^\delta$.

4. MASSLESS QUARK REGGEON INTERACTIONS

In the last Section I described the use of Multi-Regge theory to obtain perturbative Regge results. At a more ambitious level, I would like to use the same formalism to study the full dynamical Pomeron and include (de-derive?) confinement and chiral symmetry breaking. The essential idea is that massless quarks give anomalous triple-Regge interactions (directly related to the triangle anomaly) producing the axial charge violation normally associated with non-perturbative instanton interactions. These interactions produce zero quark-mass reggeon diagram infra-red divergences. The divergent amplitudes contain both hadrons (as multi-quark reggeon states) and the dynamical
Pomeron. To carry through a full analysis of the divergences and diagrams involved requires maximal use of all the Multi-Regge theory ingredients listed above together with the necessary QCD calculations. In the following I will be able to provide little more than a glimpse of what is involved.

For multiparticle amplitudes, reggeon unitarity, is very powerful. For example, consider the eight-particle amplitude in the "helicity-pole limit" \( u_1, u_2, u_3, u_4 \to \infty \).

The S-W representation determines that only one (analytically-continued) partial-wave amplitude is involved. Since this amplitude satisfies reggeon unitarity in all \( t \)-channels, the asymptotic behavior can be completely represented by transverse momentum integrals of the form

\[
\Gamma^{T_L, T_R} \left[ \frac{d^4 k}{k^2 - m^2} \right] \sim \frac{Q^2}{m^2} \int \frac{d^4 k}{k^2 - m^2} \frac{1}{[(q_1 + k)^2 - m^2][(q_2 + k)^2 - m^2][(q_3 + k)^2 - m^2]}
\]

where \( \xi \) contains all elastic scattering reggeon diagrams. \( T^L, T^R \) contain connected and disconnected interactions that involve both elastic scattering (helicity non-flip) reggeon vertices and also new "helicity-flip" vertices. The new vertices can be studied in a "non-planar" triple-regge limit. Consider three quarks scattering via gluon exchange with the triple-gluon coupling given by a quark loop and take the limit

\[
P_1 \sim (p_1, p_1, 0, 0), \ P_2 \sim (p_2, 0, p_2, 0), \ P_3 \sim (p_3, 0, 0, p_3), \ \forall p_i \to \infty.
\]

The resulting amplitude has the form

\[
g^6 \frac{p_1 p_2 p_3}{Q_1 Q_2 Q_3} \Gamma^{1+2+3+} (q_1, q_2, q_3)
\]

where

\[
\Gamma_{\mu_1 \mu_2 \mu_3} = i \int \frac{d^4 k}{k^2 - m^2} \frac{Tr(\gamma_{\mu_1}(q_1 + k + m)\gamma_{\mu_2}(q_1 + k + m)\gamma_{\mu_3}(q_1 + k + m))}{[(q_1 + k)^2 - m^2][(q_2 + k)^2 - m^2][(q_3 + k)^2 - m^2]}
\]

is the usual quark triangle diagram amplitude and \( \gamma_{i+} = \gamma_0 + \gamma_i \). Because of the triangle singularity, there is an "infra-red anomaly" in that the limits \( q_1, q_2, q_3 \sim Q \to 0, \ m \to 0 \) do not commute for the \( O(m^2) \) part of \( \Gamma^{1+2+3+} \), i.e.

\[
\Gamma^{1+2+3+, m^2} \sim Q \ i \ m^2 \int \frac{d^k}{k^2 - m^2} \to R Q
\]

Including color factors and summing diagrams we find that \( \Gamma^{1+2+3+, m^2} \) appears only in those triple-regge vertices where all reggeon states have "anomalous color parity" (i.e. color parity \( \neq \) signature) e.g.
In this diagram each three reggeon state has odd signature but the color factor must contain a product of $f$ and $d$-tensors that makes the color parity even (c.f. the winding-number current $K_\mu^i = \epsilon^{\mu
u\gamma\delta} f_{ijk} d_{jrs} A_{r}^k A_{s}^i A_{\delta}^\nu$).

In such vertices the “anomalous” survival of $O(m^2)$ quark helicity-flip processes, when $Q_i \to 0$ before $m \to 0$, potentially reproduces the axial charge violation of instanton interactions. The presence of $\Gamma_{1+2+3-,m^2}$ leads to the breakdown of the usual Ward identity cancellations when $m \to 0$ in (non-planar) multi-regge diagrams where $Q_1 \sim Q_2 \sim Q_3 \sim 0$ is part of the integration region. As a result an infra-red divergence appears, e.g. in the diagram

A divergence will occur for $Q, Q_1, Q_1' \sim 0$ as $m \to 0$ if $V$ is anomalous, i.e. if $\quad$ is an anomalous color parity reggeon state. To show that this divergence is not cancelled by other diagrams requires a systematic analysis in which the SU(3) gauge symmetry is initially broken to SU(2) (c.f. an instanton interaction is always associated with SU(2) subgroup). With the SU(3) symmetry broken, a divergence occurs when $\quad$ is an SU(2) singlet state containing one or more massive gluons (or quarks) and $\quad$ carries anomalous color parity.

Divergent higher-order diagrams involve $V$ vertices in a similar manner e.g.

An overall logarithmic divergence occurs when all $Q_i$ entering each $V$ vertex vanish. The coefficient of the divergence gives “physical amplitudes” in which all anomalous reggeon states carry zero $k_{\perp}$. Effectively physical reggeon states are SU(2) singlet “parton” states in a background “reggeon condensate”. Both hadrons and the Pomeron can be described this way. I will not give details here. However, I am optimistic that I have finally resolved how the “Super-Critical Pomeron” of Reggeon Field Theory is realized in QCD and that many other attractive results follow.

In first approximation, (in the condensate background) the Pomeron is a reggeized gluon and hadrons are “constituent quark” reggeon states with the confinement and chiral symmetry breaking spectrum. The Critical Pomeron is inter-related with the restoration of SU(3) gauge symmetry ...
References

[1] C. Corianò, A. R. White and M. Wüsthoff, *Nucl. Phys. B* - in press.

[2] Recent experimental results from H1, indicating that at intermediate $Q^2$ the Pomeron behaves like a single gluon, have encouraged me to try to put my earlier work [*Int. J. Mod. Phys. A*8, 4755, (1993); *Phys. Rev. D*29, 1435 (1984)] on firmer ground. The results will be published soon. As outlined above, the same picture re-emerges, but the details are different in important ways.

[3] A. R. White, *Int. J. Mod. Phys. A*6, 1859 (1991).

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