

\section*{Abstract}
We employ one-particle-exchange method to study $D \to PV$ decays in $D \to K\rho$, $\pi K^*$, $\pi \rho$ processes. Taking into account a strong phase and considering nonfactorizable effect, we can get good results consistent with the experimental data. Nonfactorizable effect is not always large, but in some cases, the nonfactorizable effect is necessary to accommodate the experimental data. Strong phase is approximately $SU(3)$ flavor symmetric.

\section{Introduction}

To understand the quark mixing sector of the standard model (SM) and search for new physics beyond the SM, one needs to study the decays of heavy mesons and calculate precisely the transition matrix elements of the heavy mesons decays. The short distance effects due to hard gluon exchange can be calculated reliably and the effective hamiltonian and factorization approach has been constructed \cite{1,2}, thus a lot of results which fit the experimental data well have been obtained by factorization approach. However, there are still many decay modes which cannot be accommodated by factorization approach. In fact, the quarks in heavy mesons are bound by strong interaction which is described by nonperturbative QCD.

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After weak decays of heavy mesons, the final particles can rescat-ter into other particle states through nonperturbative strong interaction [3, 4], this is called final state interaction (FSI). Many authors have studied FSI effects and found that FSI effects may play a crucial role in some decay modes [5, 6]. Therefore it is necessary to study heavy meson two-body weak decays beyond the factorization approach. Since the FSI process refers to the soft rescattering process which is controlled by nonperturbative QCD and can not be reliably evaluated with well-established theoretical frame, we have to rely on phenomenological models to analyze the FSI effects in certain processes. One can model this rescattering effect as one-particle exchange process [7, 8]. There are also other ways to treat the nonperturbative and FSI effects in $D$ decays, the readers can refer to Ref.[9]. In this paper, we study some channels of $D \to PV$ decays. We use the one-particle-exchange method to study the final state interactions in these decays. The magnitudes of hadronic couplings needed here are extracted from experimental data on the measured branching fractions of resonance decays. In addition, we consider a strong phase for the hadronic coupling [10] which is important for obtaining the correct branching ratios of $D \to PV$ decays. We also take into account some possible nonfactorizable effect [11, 12], which is needed for some decay mode from the phenomenological point of view. The coupling constants extracted from experimental data are small for $s$-channel contribution and large for $t$-channel contribution. Therefore the $s$-channel contribution is numerically negligible in $D \to PV$ decays. We can safely drop the $s$-channel contribution in this paper.

The paper is organized as follows. Section II presents the calculation in naive factorization approach. Section III gives the main scheme of one-particle-exchange method. Section IV is devoted to the numerical calculation and discussions. Finally a brief summary is given.

2 Calculations in the factorization approach

The low energy effective Hamiltonian for charm decays is given by [13]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \{ V_{us} V_{cs}^* [ C_1 (\bar{s}c)_{V-A} (\bar{u}s)_{V-A} + C_2 (\bar{s}s)_{V-A} (\bar{u}c)_{V-A} ] 
+ V_{ud} V_{cd}^* [ C_1 (\bar{d}c)_{V-A} (\bar{u}d)_{V-A} + C_2 (\bar{d}d)_{V-A} (\bar{u}c)_{V-A} ] 
+ V_{ud} V_{cs}^* [ C_1 (\bar{s}c)_{V-A} (\bar{u}d)_{V-A} + C_2 (\bar{s}d)_{V-A} (\bar{u}c)_{V-A} ] \} + \text{h.c.} , \quad (1)$$

where $C_1$ and $C_2$ are the Wilson coefficients at $m_c$ scale. We need not consider the contributions of the QCD and electroweak penguin operators in the decays of $D \to PV$, since
their effects are small in $D$ decays. The values of $C_1$ and $C_2$ at $m_c$ scale are taken to be $C_1 = 1.216$, $C_2 = -0.415$ \[13]\)

In the naive factorization approach, the decay amplitude can be generally factorized into a product of two current matrix elements and can be obtained from eq.(1)

$$
\begin{align*}
A(D^0 \to \bar{K}^0 \rho^0) &= \sqrt{2} G_F V_{ud} V_{cs}^* a_2 m_{\rho} f_\pi A^{D\rho}_{0} \epsilon_\rho \cdot P_{\bar{K}^0}, \\
A(D^0 \to K^- \rho^+) &= \sqrt{2} G_F V_{ud} V_{cs}^* a_1 m_{\rho} f_\rho F^{Dk} \epsilon_{\rho^+} \cdot P_{D^0}, \\
A(D^+ \to \bar{K}^0 \rho^+) &= \sqrt{2} G_F V_{ud} V_{cs}^* m_{\rho^+} (a_1 F^{D\rho}_{1} \epsilon_{\rho^+} \cdot P_{D^+} + a_2 f_K A^{D\rho}_{0} \epsilon_{\rho^+} \cdot P_{K^0}), \\
A(D^0 \to \pi^0 K^*) &= \sqrt{2} G_F V_{ud} V_{cs}^* m_{K^*} f_{K^*} F^D_{1} \epsilon_{K^*} \cdot P_{D^0}, \\
A(D^0 \to \pi^+ K^{*-}) &= \sqrt{2} G_F V_{ud} V_{cs}^* a_1 m_{K^*} f_{\pi} A^{DK}_{0} \epsilon_{K^{*-}} \cdot P_{\pi^+}, \\
A(D^+ \to \pi^+ K^{*-}) &= \sqrt{2} G_F V_{ud} V_{cs}^* m_{K^*} f_{\pi} A^{DK}_{0} \epsilon_{K^{*-}} \cdot P_{\pi^+}, \\
A(D^0 \to \pi^0 K^0) &= \sqrt{2} G_F V_{ud} V_{cs}^* m_{K^0} (a_1 F^{D\rho}_{1} \epsilon_{K^0} \cdot P_{\pi^0} + a_2 f_{K} A^{D\rho}_{0} \epsilon_{K^0} \cdot P_{D^0}), \\
A(D^0 \to \pi^0 \rho^0) &= \sqrt{2} G_F V_{ud} V_{cs}^* a_1 m_{\rho} f_\pi A^{D\rho}_{0} \epsilon_\rho \cdot P_{\pi}, \\
A(D^0 \to \pi^- \rho^-) &= \sqrt{2} G_F V_{ud} V_{cs}^* m_{\rho^-} f_\pi A^{D\rho}_{0} \epsilon_\rho \cdot P_{\pi}, \\
A(D^0 \to \pi^- \rho^0) &= \sqrt{2} G_F V_{ud} V_{cs}^* a_2 m_{\rho} f_\rho F^D_{1} \epsilon_\rho \cdot P_{D^0}, \\
A(D^+ \to \pi^0 \rho^0) &= \frac{G_F}{2} V_{ud} V_{cs}^* (2a_1 f_{\rho} F_{1}^{D\rho} \epsilon_{\rho^+} \cdot P_{D^0} - \sqrt{2} a_2 f_{\pi} A^{D\rho} \epsilon_\rho \cdot P_{\pi^0}),
\end{align*}
$$

where the parameters $a_1$ and $a_2$ are taken as \[12]\)

$$
\begin{align*}
a_1 &= c_1(\mu) + c_2(\mu) \left( \frac{1}{N_c} + \chi(\mu) \right), \\
a_2 &= c_2(\mu) + c_1(\mu) \left( \frac{1}{N_c} + \chi(\mu) \right),
\end{align*}
$$

with the color number $N_c = 3$, and $\chi(\mu)$ is the phenomenological parameter introduced for taking care of nonfactorizable effects. The parameters in calculation are: 1) the form factors, $F_{1}^{D\pi}(0) = 0.69$, $F_{1}^{DK}(0) = 0.76$, $A^{D\rho}_{0}(0) = 0.67$, $A^{DK\ast}_{0}(0) = 0.73$ \[2]; 2) the decay constants, $f_{\pi} = 0.133GeV$, $f_{K} = 0.158GeV$, $f_{\rho} = 0.2GeV$, and $f_{K^\ast} = 0.221GeV$.

For $q^2$ dependence of the form factors, we take the BSW model \[2\] i.e., the monopole dominance assumption:

$$
\begin{align*}
F_{1}(q^2) &= \frac{F_{1}(0)}{1 - q^2/m_{1^-}^2}, \\
A_{0}(q^2) &= \frac{A_{0}(0)}{1 - q^2/m_{0^-}^2},
\end{align*}
$$

where $m_{1^-}, m_{0^-}$ is the relevant pole mass.

The decay width of a $D$ meson at rest decaying into $PV$ is

$$
\Gamma(D \to PV) = \frac{1}{8\pi} |A(D \to PV)|^2 \frac{|\vec{p}|}{m_D^2},
$$

(5)
where the momentum of the final state particle is given by

$$|\vec{p}| = \frac{[(m_D^2 - (m_1 + m_2)^2))(m_D^2 - (m_1 - m_2)^2)]^{1/2}}{2m_D}, \quad (6)$$

where $m_1, m_2$ are the masses of final state particles. The corresponding branching ratio is

$$Br(D \rightarrow PV) = \frac{\Gamma(D \rightarrow PV)}{\Gamma_{tot}}. \quad (7)$$

Table 1: The branching ratios of $D \rightarrow PV$ obtained in the naive factorization approach and compared with the experimental results.

| Decay mode | Br (Theory) $(\chi = 0)$ | Br (Theory) $(a_1 = 1.26, a_2 = -0.51)$ | Br (Experiment) |
|------------|-------------------------|------------------------------------------|-----------------|
| $D^0 \rightarrow K^0\rho^0$ | $3.92 \times 10^{-3}$ | $5.92 \times 10^{-3}$ | $(1.21 \pm 0.17) \times 10^{-2}$ |
| $D^0 \rightarrow K^-\rho^+$ | $10.66 \times 10^{-2}$ | $11.45 \times 10^{-2}$ | $(10.8 \pm 0.9) \times 10^{-2}$ |
| $D^+ \rightarrow K^0\rho^0$ | $17.35 \times 10^{-2}$ | $16.91 \times 10^{-2}$ | $(6.6 \pm 2.5) \times 10^{-2}$ |
| $D^0 \rightarrow \pi^0 K^*_0$ | $1.37 \times 10^{-2}$ | $2.08 \times 10^{-2}$ | $(3.1 \pm 0.4) \times 10^{-2}$ |
| $D^0 \rightarrow \pi^+ K^-$ | $3.06 \times 10^{-2}$ | $3.29 \times 10^{-2}$ | $(5.0 \pm 0.4) \times 10^{-2}$ |
| $D^+ \rightarrow \pi^+ K^*_0$ | $8.72 \times 10^{-3}$ | $3.66 \times 10^{-2}$ | $(1.90 \pm 0.19) \times 10^{-2}$ |
| $D^+ \rightarrow \pi^+ \rho^0$ | $8.12 \times 10^{-3}$ | $9.33 \times 10^{-3}$ | $(1.05 \pm 0.31) \times 10^{-3}$ |
| $D^0 \rightarrow \pi^+ \rho^-$ | $1.36 \times 10^{-3}$ | $1.46 \times 10^{-3}$ | – |
| $D^0 \rightarrow \pi^- \rho^0$ | $4.48 \times 10^{-3}$ | $5.89 \times 10^{-3}$ | – |
| $D^+ \rightarrow \pi^- \rho^+$ | $1.78 \times 10^{-2}$ | $1.98 \times 10^{-2}$ | – |

The numerical results of the branch ratios of $D$ decays are given in Table 1. The second column is for the case $\chi(\mu) = 0$ which means there is no nonfactorizable contribution. When $\chi(\mu) = 0$, the parameters $a_1 = 1.216$ and $a_2 = -0.415$. The parameters in the third column $a_1 = 1.26, a_2 = -0.51$ are phenomenologically used in many references [14], which is relevant to taking non-zero parameter $\chi(\mu)$.

Comparing the results of the naive factorization in the second and third column of Table 1 with the experimental data, one can notice that, even considering some nonfactorizable contribution, some of the results from the naive factorization approach deviate significantly from the experimental data.

3 The one particle exchange method for FSI

From Table 1, we can see that the calculation from naive factorization approach is in disagreement with the experimental results for the branching ratios of $D \rightarrow PV$ decays. The
reason is that the physical picture of naive factorization is too simple, in which nonperturbative strong interaction is restricted in single hadrons, or between the initial and final hadrons which share the same spectator quark. If the mass of the initial particle is large, such as the case of $B$ meson decay, the effect of nonperturbative strong interaction between the final hadrons most probably is small because the momentum transfer is large. However, in the case of $D$ meson, its mass is not so large. The energy scale of $D$ decays is not very high. Nonperturbative effect may give large contribution. According to the one-particle exchange method, there are $s$-channel and $t$-channel contribution to the final state interaction \cite{4, 8}. The diagrams of these nonperturbative rescattering effects can be depicted in Figs\textasciitilde{}1 and \textasciitilde{}2. The first part $D \rightarrow P_3V_2$ or $D \rightarrow V_1P_2$ represents the direct decay where the decay amplitudes can be obtained by using naive factorization method. The second part represents rescattering process where the effective hadronic couplings are needed in numerical calculation, which can be extracted from experimental data on the relevant resonance decays.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{s-channel contributions to final-state interaction in $D \rightarrow PV$ decays.}
\end{figure}

There are many resonances near the mass scale of $D$ meson, it is possible that nonperturbative interaction is propagated by these resonance states, such as, $K^*(892)$, $K^*(1430)$, $f_0(1710)$, $K^*(1680)$, $K^*(1020)$, $\phi(1680)$, $\pi(1300)$, etc. For $s$-channel the correct quantum number of the resonance should be $J^P = 0^-$ (in charged $D$ decays). In neutral decay modes, the resonance should be with quantum number $J^{PC} = 0^{--}$. For $D^0 \rightarrow \pi^0 \rho^0$, only $\pi(1300)$ has the correct quantum number \cite{15}. Fig\textasciitilde{}1 is the $s$-channel contribution to the final state interaction in $D^0 \rightarrow \pi^0 \rho^0$. Here $V_1$ and $P_2$ are the intermediate mesons. Because the coupling of $\pi(1300)$ with $\pi^0 \rho^0$ is too small \cite{10}, we can ignore the $s$-channel contribution in the numerical analysis. Fig\textasciitilde{}2 shows the $t$-channel contribution to the final state interaction. $P_1$, $V_2$ and $V_1$, $P_2$ are the intermediate states from direct weak decays. They resscatter into the
Figure 2: $t$-channel contributions to final-state interaction in $D \to PV$ due to one particle exchange. (a) $D \to P_1 V_2 \to PV$, (b) $D \to V_1 P_2 \to PV$.

final states by exchanging one resonance state $P$. In this paper the intermediate states are treated to be on their mass shell, because their off-shell contribution can be attributed to the quark level. We assume the on-shell contribution dominates in the final state interaction. The exchanged resonances are treated as a virtual particle. Their propagators are taken as Breit-Wigner form,

$$\frac{i}{k^2 - m^2 + i m \Gamma_{tot}},$$

where $\Gamma_{tot}$ is the total decay width of the exchanged resonance.

We consider the $t$ channel contribution. For the $t$-channel contribution, the concerned effective vertex in Fig.[3] is $VPP$, which can be related to the $V$ decay amplitude. Explicitly the amplitude of $V \to PP$ can be written as

$$T_{VPP} = g_{VPP} \epsilon \cdot (p_1 - p_2),$$

where $p_1$ and $p_2$ are the four-momentum of the two pseudoscalars, respectively. To extract $g_{VPP}$ from experiment, one should square eq.(9) to get the decay widths,

$$\Gamma(V \to PP) = \frac{1}{3} \frac{1}{8\pi} |g_{VPP}|^2 \left[ m_V^2 - 2m_1^2 - 2m_2^2 + \left( \frac{m_1^2 - m_2^2}{m_V^2} \right)^2 \right] \frac{|\vec{p}|}{m_V^2},$$

where $m_1$ and $m_2$ are the masses of the two final particles $PP$, respectively, and $|\vec{p}|$ is the momentum of one of the final particle $P$ in the rest frame of $V$. From the above equations, one can see that only the magnitudes of the effective couplings $|g_{VPP}|$ can be extracted from experiment. On the quark level, the effective vertex should be controlled by nonperturbative
Figure 3: The effective coupling vertex on the hadronic level

QCD. It is reasonable that a strong phase can appear in the effective coupling, which is contributed by strong interaction. Therefore we can take a strong phase for each hadronic effective coupling \([10]\). In the following, the symbol \(g\) will only be used to represent the magnitude of the relevant effective coupling. The total one should be \(ge^{i\theta}\), where \(\theta\) is the strong phase. For example, the effective couplings will be written in the form of \(gVPPe^{i\theta_{VP}}\).

The \(t\)-channel contribution in Fig. 2(a) is

\[
A_{P_1,V_2}^{FSI} = \frac{1}{2} \int \frac{d^3\vec{p}_1}{(2\pi)^32E_1} \int \frac{d^3\vec{p}_2}{(2\pi)^32E_2} (2\pi)^4\delta^4(p_D - p_1 - p_2) \times g_1 \epsilon_3 \cdot (p_1 + k) \frac{i \epsilon^{i(\theta_1 + \theta_2)}}{k^2 - m^2 + im\Gamma_{tot}} F(k^2)^2 g_2 \epsilon_2 \cdot (p_4 + k), \tag{11}
\]

where \(F(k^2) = (\Lambda^2 - m^2)/(\Lambda^2 - k^2)\) is the form factor which is introduced to compensate the off-shell effect of the exchanged particle at the vertices \([16]\). We choose the lightest resonance state as the exchanged particle that gives the largest contribution to the decay amplitude. After a few steps of integration to the above equation, we get

\[
A_{P_1,V_2}^{FSI} = \int_{-1}^{1} \frac{d(\cos \theta)}{2\pi m_D} |\vec{p}_1|X_1 \frac{i \epsilon^{i(\theta_1 + \theta_2)}}{k^2 - m^2 + im\Gamma_{tot}} F(k^2)^2 g_2 H_1, \tag{12}
\]

where

\[
H_1 = m_2 f_1 A_0 [-(E_1E_4 + |\vec{p}_4| |\vec{p}_1| \cos \theta)] + \frac{1}{2m_2^2} (M_D^2 - m_1^2 - m_2^2) (E_2E_4 - |\vec{p}_2| |\vec{p}_4| \cos \theta)]
\times \left[\frac{1}{m_3}(|\vec{p}_3|E_1 - E_3 |\vec{p}_1|)\right], \tag{13}
\]

and \(X_1\) represents the relevant direct decay amplitude of \(D\) decaying to the intermediate pair \(P_1\) and \(V_2\) divided by \(|\langle P_1|(V - A)_{\mu}|0\rangle\langle V_2|(V - A)_{\mu}|D\rangle|.

\[
X_1 = \frac{A(D \to P_1V_2)}{|\langle P_1|(V - A)_{\mu}|0\rangle\langle V_2|(V - A)_{\mu}|D\rangle|.}
\]
The \(t\)-channel contribution in Fig.2(b) is

\[
A_{F_{SI}}^{V_1, P_2} = \frac{1}{2} \int \frac{d^3 \vec{p}_1}{(2\pi)^3 2E_1} \int \frac{d^3 \vec{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_D - p_1 - p_2) A(D \to V_1 P_2)
\]

\[
\times g_1 \epsilon_1 \cdot (p_3 - k) \frac{i e^{i(\theta_1 + \theta_2)}}{k^2 - m^2 + im\Gamma_{tot}} \int \frac{d^3 \vec{p}_1}{(2\pi)^3} E_1 \int \frac{d^3 \vec{p}_2}{(2\pi)^3} E_2 F(k^2)^2 g_2 \epsilon_4 \cdot (p_2 - k),
\]

(14)

and we obtain

\[
A_{F_{SI}}^{V_1, P_2} = \int_{-1}^{1} \frac{d(\cos \theta)}{2\pi m_D} |\vec{p}_1| \frac{i e^{i(\theta_1 + \theta_2)}}{k^2 - m^2 + im\Gamma_{tot}} X_2 g_1 g_2 F(k^2)^2 H_2 ,
\]

(15)

where

\[
H_2 = m_1 f_1 F_1[-M_D E_3 + \frac{1}{m_1^2} E_1 M_D (E_1 E_3 - |\vec{p}_1||\vec{p}_3| \cos \theta)]
\]

\[
\times \frac{1}{m_4} (|\vec{p}_4| E_2 - E_4 |\vec{p}_2| \cos \theta),
\]

(16)

and \(X_2\) represents the relevant direct decay amplitude of \(D\) decaying to the intermediate pair \(V_1\) and \(P_2\) divided by \(\langle V_1|(V - A)_\mu|0\rangle \langle P_2|(V - A)^\mu|D\rangle\),

\[
X_2 \equiv \frac{A(D \to V_1 P_2)}{\langle V_1|(V - A)_\mu|0\rangle \langle P_2|(V - A)^\mu|D\rangle}.
\]

4 Numerical calculation and discussions

To calculate \(FSI\) contribution of \(D\) decays with the eq.(12) and eq.(13), we need to know which channels can rescatter into the final states. For \(D \to K\rho, \pi K^*, \pi \rho\), from Figs.4~6 one can see that \(D \to \pi K^* \to K\rho, D \to K\rho \to K\rho, D \to \pi K^* \to \pi K^*, D \to \rho K \to \pi K^*, D \to \pi \rho \to \pi \rho, D \to KK^* \to \pi \rho\) can give the largest contributions, because these intermediate states have the largest couplings with the final states and the masses of exchanged meson are small which give the largest \(t\)-channel contributions. When calculating the contribution of each diagram in Figs.4~6, we should, at first, consider all the possible iso-spin structure for each diagram in Figs.4~6, and draw all the possible sub-diagrams on the quark level. Second, write down the iso-spin factor for each sub-diagram. For example, the \(u\bar{u}\) component in one final meson \(\rho^0\) contributes an isospin factor \(\frac{1}{\sqrt{2}}\), and the \(d\bar{d}\) component contributes \(-\frac{1}{\sqrt{2}}\). For the intermediate state \(\rho^0\), the factor \(\frac{1}{\sqrt{2}}\) and \(-\frac{1}{\sqrt{2}}\) should be dropped \([15]\). Third, sum the factors of all the possible sub-diagrams of each diagram to get the iso-spin factor for each diagram on the hadronic level. For example, in the diagram (a) of \(D^0 \to \bar{K}^0 \rho^0\), the
Figure 4: Intermediate states in rescattering process for $D \to K\rho$ decays.

iso-spin factor of one sub-diagram is $\frac{1}{\sqrt{2}}$ and is $-\frac{1}{\sqrt{2}}$ in another sub-diagram, so the factor of the diagram (a) of $D^0 \to \bar{K}^0\rho^0$ is zero.

From eq.(12), eq.(15) and considering Figs.4-6, we can calculate the amplitudes of $D \to PV$ decays. In this paper we consider $D \to K\rho$, $\pi K^*$ and $\pi\rho$ decays. There should be some input parameters in our calculation, such as, the transition form factors for $D$ decays, decay constants of the final mesons, the phenomenological nonfactorizable parameter $\chi(\mu)$, the off-shell compensating parameter $\Lambda$ in function $F(k^2)$ introduced in eq.(11), the effective couplings of relevant hadronic states and the relevant strong phases for these effective couplings. For the transition form factors and decay constants, we take 1) the form factors, $F_1^{D\pi}(0) = 0.69$, $F_1^{DK}(0) = 0.76$, $A_0^{D\rho}(0) = 0.67$, $A_0^{DK^*}(0) = 0.73$. 2) the decay constants, $f_\pi = 0.133GeV$, $f_K = 0.158GeV$, $f_\rho = 0.2GeV$, and $f_{K^*} = 0.221GeV$. We should be careful for these parameters, because except for the decay constants $f_\pi$ and $f_K$, etc., the values of the transition form factors have not been known exactly yet. We have to take them from model-dependent calculations. For the phenomenological nonfactorizable parameter $\chi(\mu)$, at first, we tried to proceed by taking $\chi(\mu) = 0$, which means that nonfactorizable contribution is neglected. We find that if nonfactorizable contribution is neglected, no matter how the other parameters (the strong phases and $\Lambda$) are tuned, we can not reproduce the experimental data for all the $D \to PV$ decays simultaneously. So we have to keep it as an phenomenological parameter which will be determined later. The hadronic effective cou-
The parameters involved in this study are $g_{\rho \pi \pi}$ and $g_{K^* K \pi}$, which can be determined from the center value of the measured decay width of $\rho \to \pi \pi$ and $K^* \to K \pi$ \cite{13}. We obtain $g_{\rho \pi \pi} = 6.0$, $g_{K^* K \pi} = 4.6$. The parameter $\Lambda$ in the off-shellness compensating function $F(k^2)$ introduced in eq. (11) is not an universal parameter. It is process-dependent in general. However, in this paper we use one value for $\Lambda$ in all the possible channels of $D \to PV$ decays. We assume $\Lambda$ is in the range from 0.5$GeV$ to 1.0$GeV$, which is the range of the masses of the final state particles, $\rho$ and $K^*$, etc.. We scanned all the possible value for $\chi(\mu)$ and $\Lambda$, and find that if we take $\chi(\mu) = 0.16$ and $\Lambda = 0.7$ GeV, we can reproduce the experimental data of all the detected $D \to PV$ decay modes well. $\chi(\mu) = 0.16$ means the nonfactorizable contribution is not large. $\Lambda = 0.7$ GeV is in the mass range of the final state particles. In the following we give the decay amplitudes of some $D \to PV$ decay modes as function of the strong phases $\theta_{K^* K \pi}$, $\theta_{\rho \pi \pi}$ and $\theta_{\rho KK}$ by taking $\chi(\mu) = 0.16$ and $\Lambda = 0.7$ GeV,

\[
\begin{align*}
A(D^0 \to \bar K^0 \rho^0) &= -6.768 \times 10^{-7} + 1.40508 \times 10^{-6} i e^{i2\theta_{\rho KK}}, \\
A(D^0 \to K^- \rho^+) &= 4.09 \times 10^{-6} - 2.2512 \times 10^{-7} i e^{i2\theta_{\rho KK}} - 6.739 \times 10^{-7} i e^{i(\theta_{K^* K \pi} + \theta_{\rho \pi \pi})}, \\
A(D^+ \to \bar K^0 \rho^+) &= 3.2579 \times 10^{-6} + 1.14838 \times 10^{-6} i e^{i2\theta_{\rho KK}} + 6.114 \times 10^{-7} i e^{i(\theta_{K^* K \pi} + \theta_{\rho \pi \pi})}, \\
A(D^0 \to \pi^0 K^*_0) &= -1.1837 \times 10^{-6} + 1.128 \times 10^{-6} i e^{i2\theta_{K^* K \pi}}, \\
A(D^0 \to \pi^+ K^{*-}) &= 2.352 \times 10^{-6} - 5.501 \times 10^{-7} i e^{i2\theta_{K^* K \pi}} - 4.51 \times 10^{-7} i e^{i(\theta_{K^* K \pi} + \theta_{\rho \pi \pi})}, \\
A(D^+ \to \pi^+ K^*_0) &= 1.2056 \times 10^{-6} + 3.7368 \times 10^{-7} i e^{i2\theta_{K^* K \pi}} + 1.81603 \times 10^{-6} i e^{i(\theta_{K^* K \pi} + \theta_{\rho \pi \pi})},
\end{align*}
\]
Figure 6: Intermediate states in rescattering process for \( D \rightarrow \pi \rho \) decays.

\[
\begin{align*}
A(D^+ \rightarrow \pi^+ \rho^0) &= -6.728887 \times 10^{-7} - 9.61 \times 10^{-7} i e^{2\theta_{\rho K K}} + 2.2334286 \times 10^{-7} i e^{i(\theta_{K*K\pi} + \theta_{\rho K K})}, \\
A(D^0 \rightarrow \pi^0 \rho^0) &= -4.8216 \times 10^{-7} + 1.137 \times 10^{-7} i e^{i(\theta_{K*K\pi} + \theta_{\rho K K})} + 2.1 \times 10^{-7} i e^{2\theta_{\rho \pi \pi}}, \\
A(D^0 \rightarrow \pi^+ \rho^-) &= -2.608 \times 10^{-7} + 1.821 \times 10^{-7} i e^{i(\theta_{K*K\pi} + \theta_{\rho \pi \pi})}, \\
A(D^0 \rightarrow \pi^- \rho^+) &= -8.736 \times 10^{-7} + 6.879 \times 10^{-8} i e^{i(\theta_{K*K\pi} + \theta_{\rho K K})} + 2.08 \times 10^{-7} i e^{2\theta_{\rho \pi \pi}}, \\
A(D^+ \rightarrow \pi^0 \rho^+) &= -1.0597 \times 10^{-6} - 9.6106 \times 10^{-7} i e^{2\theta_{\rho \pi \pi}} + 2.21 \times 10^{-7} i e^{i(\theta_{K*K\pi} + \theta_{\rho \pi \pi})}. 
\end{align*}
\]

The phases of the effective hadronic couplings \( \theta_{K*K\pi}, \theta_{\rho \pi \pi} \) and \( \theta_{\rho \pi \pi} \) can not be known from direct experimental measurement or from any nonperturbative calculations because there are no any such kind of calculations yet. We only know that the values of \( \theta_{K*K\pi}, \theta_{\rho \pi \pi} \) and \( \theta_{\rho \pi \pi} \) should not differ too much according to \( SU(3) \) flavor symmetry. We tried some values for these phase parameters, and find that the ranges which can reproduce the experimental data of the measured \( D \rightarrow PV \) decays are not very narrow. To show the situation that the experimental data are accommodated, we give the numerical results for \( \theta_{K*K\pi} = 51.0^\circ, \theta_{\rho \pi \pi} = 51.0^\circ \) and \( \theta_{\rho \pi \pi} = 57.3^\circ \) in table 2.

Table 2 shows that the contribution of FSI is strongly channel dependent. For example, For \( D^0 \rightarrow \bar{K}^0 \rho^0 \), the braching ratio in naive factorization is \( 3.922 \times 10^{-3} \), while the braching ratio including FSI is \( 1.25 \times 10^{-2} \). we can see that FSI contribution in \( D^0 \rightarrow \bar{K}^0 \rho^0 \) is large, but FSI contribution in \( D^0 \rightarrow K^- \rho^+ \) is small. The reason for the difference is that the external rescattering diagrams for \( D^0 \rightarrow \bar{K}^0 \rho^0 \) and \( D^0 \rightarrow K^- \rho^+ \) are different. Without the contribution of FSI, predictions of naive factorization for most detected \( D \rightarrow PV \) decays
are seriously in disagreement with the experimental results. After including FSI, the results can accommodate the experimental data well. For the other decay modes $D^0 \to \pi^+ \rho^-$, $\pi^0 \rho^0$, $\pi^- \rho^+$ and $D^+ \to \pi^0 \rho^+$, their branching ratios have not been detected in experiment yet. In our model, they are all predicted to be at the order of $O(10^{-3})$. For $D^0 \to \pi^+ \rho^-$ and $\pi^- \rho^+$, the effect of FSI is enhancement. While for $D^0 \to \pi^0 \rho^0$ and $D^+ \to \pi^0 \rho^+$, FSI suppresses the prediction of naive factorization.

Before the end of this section, some comments should be given: there are many uncertainties in the input parameters which may change the above result numerically, such as, the $D$ decay transition form factors and some decay constants which have not been known exactly yet, they need to be measured from leptonic and semileptonic decays of $D$ mesons which are quite possible in CLEO-C program in the near future. The other sources which may cause uncertainties are the shape of the off-shell compensating function $F(k^2)$, or in more general the effective hadronic couplings in the off-shell region, the strong phases of the effective couplings, and the nonfactorization parameter $\chi(\mu)$, both of which are needed to be studied in some nonperturbative methods based on QCD in the future.

### 5 Summary

We have studied some channels of $D \to PV$ decays. The total decay amplitude includes direct weak decays and final state rescattering effects. The direct weak decays are calculated in factorization approach, and the final state interaction effects are studied in one-particle-
exchange method. The prediction of naive factorization is far from the experimental data. After including the contribution of final state interaction, the theoretical prediction can accommodate the experimental data well. The strong phases of the effective hadronic couplings are necessary to reproduce experimental data.

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