An analytical method to simulate the HI 21-cm visibility signal for intensity mapping experiments

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ABSTRACT
Simulations play a vital role in testing and validating HI 21-cm power spectrum estimation techniques. Conventional methods use techniques like N-body simulations to simulate the sky signal which is then passed through a model of the instrument. This makes it necessary to simulate the HI distribution in a large cosmological volume, and incorporate both the light-cone effect and the telescope’s chromatic response. The computational requirements may be particularly large if one wishes to simulate many realizations of the signal. In this paper we present an analytical method to simulate the HI visibility signal. This is particularly efficient if one wishes to simulate a large number of realizations of the signal. Our method is based on theoretical predictions of the visibility correlation which incorporate both the light-cone effect and the telescope’s chromatic response. We have demonstrated this method by applying it to simulate the HI visibility signal for the upcoming Ooty Wide Field Array Phase I.

Key words: cosmology: diffuse radiation - large scale structure of the universe; methods: statistical

1 INTRODUCTION
The neutral hydrogen (HI) in the diffuse intergalactic medium (IGM) is ionized at redshifts $z \lesssim 6$ which is referred to as the post-reionization era in the history of the universe. The residual HI is locked within dense pockets which are identified as the Damped Lyman-$\alpha$ systems (DLAs) in quasar observations and have HI column densities $N_{\text{HI}} \gtrsim 2 \times 10^{20}$ atoms/cm$^2$ (Wolfe et al. 2005; Lanzetta et al. 1995; Storrie-Lombardi et al. 1996). HI galaxy surveys (Zwaan et al. 2005; Martin et al. 2010), DLA observations (Rao et al. 2006; Meiring et al. 2011) and HI stacking (Lah et al. 2007; Delhaize et al. 2013; Rhee et al. 2013, 2016) supply measurements of $\Omega_k$ at low redshifts ($z \lesssim 1$), while at high redshifts ($1 < z < 6$) these measurements come from DLA observations (Prochaska & Wolfe 2009; Noterdaeme et al. 2012; Zafar et al. 2013). These measurements suggest $\Omega_k \sim 10^{-3}$ to be almost constant over the redshift range $1 < z < 6$, which implies a neutral hydrogen fraction $x_{\text{HI}} = 0.02$.

Unlike traditional galaxy redshift surveys, HI 21-cm measurements cannot resolve individual HI galaxies at higher redshifts due to the limited angular resolution and sensitivity of present day radio telescopes. The collective emission from the HI sources appears as a diffuse background radiation in low frequency radio observations below 1420 MHz. The angular and frequency domain fluctuations in this background radiation which are, in general, quantified through the HI 21-cm power spectrum have the potential to probe the large scale structure of the universe at high $z$ (Bharadwaj et al. 2001; Bharadwaj & Sethi 2001; Wyithe & Loeb 2008). This technique, widely known as 21-cm intensity mapping (IM), makes it possible to use low frequency radio telescopes to survey larger cosmological volumes (e.g. McQuinn et al. 2006; Seo et al. 2010; Ansari et al. 2012; Bull et al. 2015b).

Measurements of the post-reionization HI 21-cm power spectrum hold the prospect of measuring the baryon acoustic oscillation (BAO), which can be used to constrain models of dark energy (Wyithe et al. 2008; Chang et al. 2008; Seo et al. 2010; Ansari et al. 2011). Measurement of the HI power spectrum can also be used to constrain the background cosmological model without reference to the measurements of the BAO (Bharadwaj et al. 2009; Visbal et al. 2009). The HI 21-cm power spectrum measurement can

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further be used to place constraints on the neutrino mass
(Leeb & Wyithe 2008; Villaescusa-Navarro et al. 2015a; Sarkar et al. 2016). Switzer et al. (2013) have claimed a detec-
tion of the H I 21-cm intensity fluctuations from z ∼ 0.8
using single dish observations with the Green Bank Tele-
scope (GBT). Apart from the H I 21-cm power spectrum,
higher order statistics such as the 21-cm bispectrum hold
the prospect of quantifying the non-Gaussianities in the H i
signal (Ali et al. 2005; Hazra & Sarkar 2012).

A number of radio-interferometric arrays like the Cana-
dian Hydrogen Intensity Mapping Experiment (CHIME1;
Bandura et al. 2014), Baryon Acoustic Oscillation Broad-
band and Broadbeam array (BAOBAB2; Pober et al. 2013),
the Tianlai project3 (Chen 2012; Zhang et al. 2016) and Hy-
drogen Intensity and Real-Time Analysis experiment (HI-
RAX4; Newburgh et al. 2016) are being planned to mea-
sure the BAO using H I 21-cm measurements in the redshift
range 0.5 ≤ z ≤ 2.5. Single dish antennas are also being con-
sidered (BINGO5; Battye et al. 2012, 2016) to measure the
BAO using 21-cm IM observations. The Giant Metrewave
Radio Telescope (GMRT6; Swarup et al. 1991) is capable of
functioning at several different frequencies from 150 MHz to
1420 MHz, which correspond to measuring the H I radiation
from the redshift range 0 ≤ z ≤ 8.5. The prospects of de-
tecting the H I signal using the GMRT have been studied in
earlier works (Bharadwaj & Pandey 2003; Bharadwaj & Ali
2005). Preliminary observations using GMRT have been re-
ported in Ghosh et al. (2011a,b). The GMRT is currently
being upgraded to operate at a larger bandwidth, forecasts
for measuring the H I signal using the upgraded GMRT
(uGMRT) are presented in Chatterjee et al. (in preparation).
LOFAR7 (Yatawatta et al. 2013), MWA8 (Tingay et al.
2013), PAPER9 (Parsons et al. 2010; Aguirre et al. 2014)
and the upcoming HERA10 (DeBoer et al. 2017) all aim
to carry out 21-cm IM observations to measure the Epoch
of Reionization signal. Future radio telescopes like the
SKA1-mid11 and the SKA1-low12 hold promising prospects
for measuring the post-reionization H I 21-cm power spec-
trum at an unprecedented level of precision (Sarkar & Datta
2015; Villaescusa-Navarro et al. 2015b; Bull et al. 2015a;
Santos et al. 2015; Villaescusa-Navarro et al. 2016). IM ex-
periments are also being planned with the CO line (COMAP,
Cleary et al. 2016) and the C ii line (TIME, Crites et al.
2014).

The Ooty Radio Telescope (ORT) (Swarup et al. 1971;
Sarma et al. 1975) is currently being upgraded to oper-
ate as a linear radio-interferometric array the Ooty Wide
Field Array (OWFA) (Prasad & Subrahmanya 2011a,b;
Subrahmanya et al. 2017a). The primary science goals of
OWFA have been outlined in Subrahmanya et al. (2017b),
and the measurement of the z = 3.35 post-reionization
H i 21-cm power spectrum is one of its major objectives.
It has been predicted (Bharadwaj et al. 2015) that a 5σ
detection of the amplitude of the H i 21-cm power spec-
trum is possible with ∼ 150 hrs of observation. Further,
Sarkar et al. (2017) predict that a ∼ 5σ measurement of the
binned H i 21-cm power spectrum is possible in the k-range
0.05 ≤ k ≤ 0.3 Mpc−1 with 1,000 hrs of observation.

The complex visibilities are the primary quantities
which are directly measured in radio-interferometric obser-
vations. The measured visibilities have contributions from
the H i signal, foregrounds and system noise. It is im-
portant and worthwhile to quantify the H i signal in terms
of their expected contribution to the measured visibilities (e.g.
Bharadwaj & Pandey 2003; Bharadwaj & Srikant 2004) De-
tailed predictions for the visibility correlations at different
details and frequency channels for the expected statisti-
cal H i signal, foregrounds and the system noise expected
at OWFA are presented in Ali & Bharadwaj (2014) and
Gehlot & Bagla (2017). Theoretical estimates presented in
these papers, and also direct observations (e.g. Ghosh et al.
2011a,b) predict the foreground contribution to be several
orders of magnitude larger than the H i signal at OWFA.
It is therefore crucial to consider foreground removal for the
H i 21-cm signal. The various astrophysical foregrounds are
expected to have a smooth frequency dependence. In con-
trast to this the expected H i visibility signal at two different
frequencies decorrelates rapidly as the frequency separation
is increased, and the visibility correlation is predicted to
decay to a value very close to zero within a frequency sepa-
ration of ∼ 4 MHz at OWFA (Ali & Bharadwaj 2014). This
is a generic feature of the 21-cm signal (Bharadwaj & Sethi
2001; Bharadwaj & Ali 2005) and most of the foreground
removal techniques (e.g. McGuinn et al. 2006; Jelíč et al.
2008; Gesler et al. 2008; Bowman et al. 2009; Parsons et al.
2012) use this feature to distinguish between the foregrounds
and the H i signal. However, the chromatic response of the
telescope introduces frequency-dependent structures in the
foregrounds, and it is important to model these in order to
quantify these effects in any foreground removal technique.
Simulations incorporating the expected foregrounds, H i sig-
nal and various instrumental and post-processing effects are
an important and worthwhile to quantify the H i signal, foregrounds and system noise expected at OWFA are presented in Ali & Bharadwaj (2014) and Gehlot & Bagla (2017). Theoretical estimates presented in these papers, and also direct observations (e.g. Ghosh et al. 2011a,b) predict the foreground contribution to be several orders of magnitude larger than the H i signal at OWFA. It is therefore crucial to consider foreground removal for the H i 21-cm signal. The various astrophysical foregrounds are expected to have a smooth frequency dependence. In contrast to this the expected H i visibility signal at two different frequencies decorrelates rapidly as the frequency separation is increased, and the visibility correlation is predicted to decay to a value very close to zero within a frequency separation of ∼ 4 MHz at OWFA (Ali & Bharadwaj 2014). This is a generic feature of the 21-cm signal (Bharadwaj & Sethi 2001; Bharadwaj & Ali 2005) and most of the foreground removal techniques (e.g. McGuinn et al. 2006; Jelíč et al. 2008; Gesler et al. 2008; Bowman et al. 2009; Parsons et al. 2012) use this feature to distinguish between the foregrounds and the H i signal. However, the chromatic response of the telescope introduces frequency-dependent structures in the foregrounds, and it is important to model these in order to quantify these effects in any foreground removal technique. Simulations incorporating the expected foregrounds, H i signal and various instrumental and post-processing effects are crucial in validating any measurement technique for the H i 21-cm power spectrum. Marthi (2017) presents a software model for OWFA which has been developed with a view to simulate the visibilities, as well as processing of the visibility data including calibration (Marthi & Chengkapal 2014, for the calibration algorithm) and power spectrum estimation. Simulated foreground predictions for OWFA are presented in Marthi et al. (2017) where, in addition, they present a detailed analysis of the instrument-induced systematics and discuss the multi-frequency angular power spectrum estimator (MAPS) for OWFA. Chatterjee et al. (2017) present simulations of the H i 21-cm visibility signal expected for OWFA.

Numerical simulations of the visibilities would typically start with a simulation of the expected sky signal which would then be passed through a software model of the instrument. While this is relatively straightforward for the foregrounds, it may be computationally challenging for the

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2 http://bao.berkeley.edu/
3 http://tianlai.bao.ac.cn/
4 http://www.energysky.org/hiro
5 http://www.jb.man.ac.uk/research/BINGO/
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infographic-ska1-mid/
12 http://www.skatelescope.org/multimedia/image/ska-
infographic-ska1-low/
redshifted 21-cm signal where it is necessary to simulate the H\textsc{i} distribution in a large cosmological volume. The fact that the H\textsc{i} signal and the cosmological parameters both evolve along the line of sight due to the light-cone effect introduces further computational complications. The difficulty is compounded when it is necessary to simulate several statistically independent realizations of the signal, which an analytical prescription could substantially reduce.

In the present work we provide a prescription for simulating the H\textsc{i} signal visibilities and validate the method for OWFA. Our method, which is described in Section 2 of this paper, uses the analytically predicted visibility correlation as an input. In addition to the chromatic response of the telescope, we incorporate the redshift evolution of both the H\textsc{i} signal and the cosmological parameters within the observing frequency bandwidth. The visibility correlation matrix is decomposed into its eigenbasis or the Koshambi-Karhunen-Loève (KKL)-basis (Kobayashi et al. 2011) which is then used to simulate different random realizations of the H\textsc{i} visibilities. Section 3 gives a brief overview of OWFA and Section 4 presents the H\textsc{i} model used in our analysis. Section 5 presents the results, and we present the summary and discussion in Section 6. We use the Eisenstein & Hu (1998) transfer function to calculate the cosmological matter power spectrum and adopt the PLANCK+WMAP9 best-fit cosmological parameters from Ade et al. (2014).

2 SIMULATION OF THE H\textsc{i} VISIBILITIES: ANALYTICAL METHOD

The visibilities V(U, ν) measured at different baselines U and frequency channels ν may be considered to be the sum of three different components

\[ V = S + F + N \]  

namely, the H\textsc{i} signal S, foregrounds F and noise N respectively. The noise in each visibility measurement may be modelled as an independent Gaussian random variable with zero mean and whose variance is known (Chengalur et al. 2007; Thompson et al. 2008). The foregrounds, which largely originate from our Galaxy and external radio galaxies, are found to be 4 to 5 orders of magnitude larger than the H\textsc{i} signal (Ghosh et al. 2012). In this paper we focus on simulating only the H\textsc{i} signal S, and we do not consider the two other components.

The baseline U = d/λ for a fixed antenna spacing d varies with the observing frequency ν. This poses a problem in the notation if we attempt to interpret the visibility V(U, ν) as a function of two independent variables U and ν. We avoid this problem by holding U fixed at the value calculated at the central frequency U = d/λc and expressing d/λ as U/νc. The H\textsc{i} signal contribution to the visibility can then be expressed as

\[ S(U, ν) = \left( \frac{∂B}{∂T} \right)_{νc} \int d^2θ \ A(θ, νc) \ δT_b(θ, νc) e^{2πiθ(U/νc)} \]  

where \( \left( \frac{∂B}{∂T} \right)_{νc} = 2k_B ν^2/c^2 \) is the conversion factor from temperature to specific intensity, \( A(θ, νc) \) is the primary beam pattern of the antennas and \( δT_b(θ, νc) \) is the redshifted H\textsc{i} 21-cm brightness temperature fluctuations which trace the cosmological H\textsc{i} distribution at the redshift z where the 21-cm radiation originated. \( δT_b(θ, νc) \) is usually modelled as a random field which is quantified through various statistical quantities like the two-point correlation function or equivalently the power spectrum. It is also clear that we expect the different visibilities S(U, ν) to be random variables whose statistical properties reflect those of the underlying H\textsc{i} distribution.

The aim of this paper is to present a method to simulate S(U, ν). To this end, we predict the statistical properties of \( S(U, ν) \) and subsequently use these predictions to simulate S(U, ν). Below, we first illustrate the method by applying it to a simplified situation and we subsequently consider a more general situation.

2.1 A simplified analysis

We focus our attention on a fixed baseline U and consider the visibilities S(U, ν) at different frequencies. We make two assumptions which introduce substantial simplifications. First, we assume that the frequency dependence of the visibility S(U, ν) arises entirely from the 21-cm brightness temperature fluctuations \( δT_b(θ, ν) \) and we ignore the frequency dependence of all the other terms in the right hand side of (2) whereby

\[ S(U, ν) = \left( \frac{∂B}{∂T} \right)_{νc} \int d^2θ \ A(θ, νc) \ δT_b(θ, νc) e^{2πiθ(U/νc)} \]  

This assumption essentially ignores the chromatic response of the telescope. Second, we model \( δT_b(θ, ν) \) as a statistically homogeneous Gaussian random field. The statistical homogeneity is with respect to \( x \equiv (x_1, x_2, x_3) = (r_{νc}, θ, r_{νc} ν - νc) \) which denotes the comoving separation between \( (θ, ν) \) and the center of the observational volume which is located at \( (θ = 0, νc) \). Here we use \( r_ν \) to denote the comoving distances corresponding to the redshifted 21cm radiation received at the frequency ν and \( r_{νc} \) and \( r_ν \) respectively refer to \( r_{νc} \) and \( r_ν = dr_ν/dνc \) evaluated at \( ν = νc \). The assumption of statistical homogeneity implies that the different Fourier modes \( δT_b(k) \) are uncorrelated (Peebles 1980). The statistical properties of \( δT_b(x) \) can be completely quantified using the 3D 21-cm brightness temperature power spectrum \( P(k) \), defined through

\[ \langle δT_b(k)δT_b^∗(k') \rangle = (2π)^3 P(k) δ^3_D(k - k') \]  

where \( δ^3_D(k - k') \) is the 3D Dirac delta function. It may be noted that the different Fourier modes will, in general, be correlated if the assumption of statistical homogeneity is broken.

We use the two visibility correlation

\[ S_2(U, ν, νc, ν_2) = \langle S(U, νc) S^∗(U, ν_2) \rangle \]  

to quantify the statistical properties of the visibilities. The two assumptions adopted earlier imply that the visibilities S(U, ν) are statistically homogeneous in frequency, whereby S2(U, ν, νc, ν2) depends only on the frequency separation νc - ν2. Calculating S2(U, νc, ν2) in terms of the power spectrum \( P(k) \) (Bharadwaj & Ali 2005), we have

\[ S_2(U, νc, ν) = \left( \frac{∂B}{∂T} \right)_{νc}^2 \int \frac{d^3k}{(2π)^3} \left| \hat{A} \left( U - \frac{k \cdot r_{νc}}{2π}, νc \right) \right|^2 \]  

\[ P(k) e^{i νc k \cdot (νc - ν)} \]  

\[ \text{MNRAS} \text{ 000, 1–11 (2017)} \]
where $\hat{A}(L, \nu)$ is the Fourier transform of the primary beam pattern $A(\theta, \nu)$ with $L$ being the Fourier conjugate to $\theta$, and $k_ν$ and $k_⊥$ are respectively the components of the wave vector $k$ perpendicular and parallel to the line of sight. We use (6) to predict the statistical properties of the visibilities. The properties of the visibility correlations have been studied in several other works (Bharadwaj & Pandey 2003; Bharadwaj & Ali 2005; Ali & Bharadwaj 2014). We note that since the visibilities at different frequencies are correlated, it is not possible to independently simulate them.

It is convenient to decompose the visibilities into delay channels (Morales 2005)

$$S(U, \nu) = \int s(U, \tau) e^{-2\pi i \nu \tau} d\tau.$$  \hfill (7)

The statistical homogeneity of $S(U, \nu)$ along frequency implies that the different delay (or Fourier) modes $s(U, \tau)$ are uncorrelated

$$\langle s(U, \tau) s^*(U, \tau') \rangle = \delta_{\tau \tau'} \langle p(U, \tau) \rangle$$  \hfill (8)

where $p(U, \tau)$ is the power spectrum of $s(U, \tau)$. Using (7) to calculate the visibility correlation we have

$$S_2(U, \nu_1, \nu_2) = \int p(U, \tau) e^{2\pi i (\nu_1 - \nu_2) \tau} d\tau.$$  \hfill (9)

Comparing (6) with (9), it is possible to identify $\tau = k_veralic / (2\pi)$ and

$$p(U, \tau) = \left( \frac{\partial B}{\partial T} \right)^2 \frac{1}{2\nu_0} \int \frac{d^2k_{\parallel}}{(2\pi)^2} \hat{A} \left( U - \frac{k_\perp r_{\nu_0}}{2\pi}, \nu_0 \right)^2 P(k).$$  \hfill (10)

The different $s(U, \tau)$ are independent, and have amplitude $\sqrt{p(U, \tau)}$. It is straightforward to simulate $s(U, \tau)$ and use these to simulate the visibilities

$$S(U, \nu) = \sqrt{\frac{\int p(U, \tau) [x(\tau) + iy(\tau)] e^{-2\pi i \nu \tau} d\tau}{2}}$$  \hfill (11)

where $x(\tau)$ and $y(\tau)$ are two independent Gaussian random fields with zero mean and unit variance, that is

$$\langle x(\tau) y(\tau) \rangle = \delta_{\tau \tau'}.$$  \hfill (12)

In summary, given a telescope we can use the input 21-cm power spectrum $P(k)$ along with eqs. (10) and (11) to simulate the visibilities at different frequencies for a fixed baseline.

### 2.2 A generalized analysis

A radio-interferometric array typically has many different baselines $U_a$ and its observing band is split into frequency channels $\nu_a$, with the possibility that the signal in several of the baselines and across frequency channels may be correlated (Ali & Bharadwaj 2014). It is therefore necessary to simultaneously consider all the visibilities that will be observed by the array. Here we use $S_a = S(U_a, \nu_a)$ to denote the different data elements where the index $a$ refers to a combination of baseline and frequency channel $(U_a, \nu_a)$ and $a = 1, 2, ..., N$ spans the entire visibility data observed by the array. We now use the two-visibility correlation

$$S_{a\beta} = \langle S_a S_\beta \rangle$$  \hfill (13)

to quantify the statistical properties of the visibility data.

The present analysis incorporates the chromatic response of the telescope, and we use (2) to calculate the visibilities. We also incorporate the Light Cone (LC) effect which essentially implies that the comoving distance $r_\nu$ and the look back time both vary with the observational frequency $\nu$. This modifies the mapping from $\theta, \nu)$ to $x$, and we now have $(x_{\perp}, x_\parallel) = (r_\nu \theta, r_\nu - r_{\nu_0})$ which is a non-linear relation compared to the simple linear approximation used in 2.1, breaking the statistical homogeneity of $\delta T_b(x)$ along the line of sight direction $x_\parallel$. Here we model $\delta T_b(x)$ by assuming that $\delta T_b(x)$ is linearly related to $\delta(x)$ which is the density contrast of the underlying matter distribution normalized at the present epoch, the relation between $\delta T_b(x)$ and $\delta(x)$ containing factors which incorporate the cosmological evolution and redshift space distortion. The fluctuations $\delta(x)$ may be assumed to be statistically homogeneous and isotropic, and it is convenient to consider their Fourier transform $\Delta(k)$. Statistical homogeneity implies that the different Fourier modes $\Delta(k)$ is $\Delta(k)$ is uncorrelated, and we use $P(k)$ to denote the power spectrum of the underlying matter fluctuations $\Delta(k)$. Note that $P(k)$ is normalized at the present epoch. This allows us to write

$$\delta T_b(\theta, \nu) = Q_\nu \int \frac{d^2k}{(2\pi)^3} [1 + \beta\nu\mu^2] \Delta(k) e^{-i[k_\perp r_\nu + k_\parallel (r_\nu - r_{\nu_0})]}$$  \hfill (14)

where $Q_\nu = [Tb_{HI}\tilde{X}_{HI}D_\nu]$ refers to the product of a set of quantities namely $T$ the mean 21-cm brightness temperature (Bharadwaj & Ali 2005), $\tilde{X}_{HI}$ the linear H i bias, $\tilde{X}_{HI}$ the mean Hydrogen neutral fraction and $D$ the growing mode of linear density perturbations, all of which evolve with redshift $z$ or equivalently vary with the observational frequency $\nu$. The factor $[1 + \beta\nu\mu^2]$ incorporates the effect of redshift space distortion (Kaiser 1987), where $\mu = k_\parallel / k$ and $\beta$ is the redshift distortion parameter which also varies with $\nu$.

We can now use (2) to calculate the visibilities as

$$S(U, \nu) = Q_\nu \int \frac{d^2k}{(2\pi)^3} \hat{A} \left( U_\nu \nu, \frac{k_\parallel r_\nu}{2\pi}, \nu \right) [1 + \beta\nu\mu^2] \Delta(k) e^{-i[k_\perp r_\nu + k_\parallel (r_\nu - r_{\nu_0})]}$$  \hfill (15)

and the visibility correlation as

$$S_{a\beta} = Q_{a\beta} Q_{\nu a} \int \frac{d^2k}{(2\pi)^3} \hat{A} \left( U_\nu \nu, \frac{k_\parallel r_\nu}{2\pi}, \nu \right) \hat{A}^* \left( U_\nu \nu, \frac{k_\perp r_\nu}{2\pi}, \nu \right) \times P_{rad}(k) e^{-i[k_\perp (r_\nu - r_{\nu_0})]},$$  \hfill (16)

where $P_{rad}(k) = [1 + \beta \nu_0 \mu_0^2][1 + \beta \nu_0 \mu_0^2] P(k)$ is the effective power spectrum considering the redshift space distortion. The fluctuations $\delta(x)$ are here assumed to be a Gaussian random field, and the statistical properties of the visibilities are entirely predicted by (15).

It is quite obvious from (13) that the visibilities $S(U, \nu)$ are not statistically homogeneous along frequency. In this case, the Fourier transform can not be considered the ideal basis. The Fourier basis used in 2.1 also does not incorporate the fact that the visibilities at different baselines may be correlated. What then are the correct basis vectors which can be used to decomposes the visibil-
ity signal into component which are statistically independent? Kosambi (Kosambi 1943), and subsequently Karhunen (Karhunen 1947) and Loève (Loève 1955), showed that the correct basis is provided by the eigenvectors \( \tilde{e}_\alpha \) of the data covariance matrix (the visibility correlation)

\[
S_{2ab} = \sum_{\alpha=1}^{N} \lambda_\alpha \left[ \tilde{e}_\alpha^a \right] \left[ \tilde{e}_\alpha^b \right] \left. \right| .
\]  

(17)

where the index \( \alpha \) refers to the different eigenvalues \( \lambda_\alpha \) and \( \alpha \) runs from 1 to \( N \) which is the total number of eigenvalues as well as the total number of visibility data.

It is convenient to decompose the visibility data \( S_a \) in this “KKL basis” \( \tilde{e}_\alpha^a \) using

\[
S_a = \sum_{\alpha=1}^{N} s_\alpha \tilde{e}_\alpha^a
\]

(18)

which is in exact analogy with (7). Here \( s_\alpha \) are the components of the data vector \( S_a \) in the \( \tilde{e}_\alpha \) basis. The different components \( s_\alpha \) are uncorrelated

\[
\langle s_\alpha s_\beta^* \rangle = \delta_{\alpha,\beta} \lambda_\alpha ,
\]

(19)

and have amplitude \( \sqrt{\lambda_\alpha} \), in exact analogy with (9). Note that \( \delta_{\alpha,\beta} \) here refers to the Kronecker delta.

It is now straightforward to simulate a random realization of the visibility data. We proceed by first generating a random realization of the \( N \) components \( s_\alpha \) using

\[
s_\alpha = \sqrt{\frac{\lambda_\alpha}{2}} (x_\alpha + iy_\alpha).
\]

(20)

Here \( x_\alpha \) and \( y_\alpha \) are \( 2N \) independent Gaussian random variables of unit variance \( \langle x_\alpha x_\beta \rangle = \delta_{\alpha,\beta} \). These components \( s_\alpha \) are then used in (18) to generate a random realization of the visibility data. This is in exact analogy with (11) which has been discussed earlier. The same process (20 and 18) can be repeated with different sets of the \( 2N \) Gaussian random numbers \( x_\alpha \) and \( y_\alpha \) to simulate different random realizations of the visibility data \( S_a \).

In summary, \( S_{2ab} \) (15) quantifies the statistical properties of the entire visibility data. In order to simulate the visibility data, it is necessary to numerically determine the eigenvalues and eigenvectors of \( S_{2ab} \). Random realizations of the eigenvalues obtained through 20 can hence be used to generate as many distinct realizations of the visibilities.

3 THE OOTY WIDE FIELD ARRAY (OWFA)

Efforts are currently underway to upgrade the Ooty Radio Telescope (ORT) into a radio-linear-interferometric array (Prasad & Subrahmanya 2011a,b; Subrahmanya et al. 2017b; Subrahmanya et al. 2017a), the Ooty Wide Field Array (OWFA). With a nominal frequency of \( \nu_0 = 326.5 \text{ MHz} \), this directly corresponds to measuring the H I radiation from the redshift \( z = 3.35 \). The telescope, situated on the Nilgiri hills, is a paraboloidal cylindrical reflector of length 530 m and width 30 m. The long axis of the cylinder is tilted by 11° which matches the telescope latitude of 11° North. This makes the telescope’s long axis parallel to the Earth’s rotation axis (Swarup et al. 1971; Sarma et al. 1975) and enables the telescope to observe the same part of the sky with a single rotation. The telescope has a feed system that consists of 1056 half-wavelength dipoles which are placed almost end-to-end along the length of the cylinder. The entire feed is placed off-axis to avoid maximal obstruction to the incoming radiation.

An upgrade to the OWFA will allow operation in two concurrent interferometric modes - Phase I and Phase II (Prasad & Subrahmanya 2011a,b; Ali & Bharadwaj 2014). This study focuses on Phase I which has a total of \( N_A = 40 \) antennas arranged linearly along the length of the cylindrical reflector. Each antenna is effectively a small parabolic cylindrical reflector of width 30 m and length 11.5 m containing 24 dipole elements along the focal line. The antennas have a rectangular aperture of dimension \( b \times d \), where \( b = 30 \text{ m} \) and \( d = 11.5 \text{ m} \). We consider the telescope aperture to lie on the \( x - y \) plane with the \( x \) axis along the length of the cylinder.

The aperture power pattern of OWFA can approximately be written as (Ali & Bharadwaj 2014),

\[
\tilde{\Lambda}(L, \nu) = \frac{\lambda^2}{(b \nu d)} A \left( \frac{L \lambda}{d} \right) \lambda \left( \frac{L \lambda}{b} \right)
\]

(21)

where \( \lambda = c/\nu \) is the observing wavelength, \( L = (L_x, L_y) \) and the triangular function \( A(\nu) \) has been defined in the usual manner as, \( A(\nu) = (1 - |w|) \) for \( |w| < 1 \) and zero elsewhere. We note that aperture power pattern \( \tilde{\Lambda}(L, \nu) \) peaks at \( L = (0, 0) \) and extends over the range \( -d/\lambda \leq L_y \leq d/\lambda \) along \( L_y \) and the range \( -b/\lambda \leq L_x \leq b/\lambda \) along \( L_x \), and is zero beyond, and has as such been used to calculate the visibility covariance for OWFA.

OWFA is a linear interferometric array with all the antennas equally spaced along the length of the ORT cylinder. This allows us to write the OWFA baselines \( U_n \) as

\[
U_n = n \left( \frac{d}{\lambda} \right)
\]

(22)

where \( \lambda_c \) is the central observing wavelength and \( n \) is the baseline index which can have values in the range \( 1 \leq n \leq N_A - 1 \) where \( N_A \) is the total number of antennas in the array. The smallest baseline measures \( |d| = 11.5 \text{ m} \) which is the spacing between two adjacent antennas in the array. We have a total of \( (N_A - 1) = 39 \) baselines for Phase I of OWFA.

Phase I has an operating bandwidth of \( B = 39 \text{ MHz} \). For the present analysis we consider a smaller bandwidth \( B = 16 \text{ MHz} \) and \( N_c = 128 \) frequency channels of width \( \Delta \nu_c = 0.125 \text{ MHz} \) each to reduce the computation time. Phase-I provides 4992 (39 baselines \( \times 128 \) channels) instantaneous visibility measurements; the visibility correlation matrix \( S_{2ab} \) (15) is hence Hermitian. The correlation matrix

\[
S_{2ab} = \langle S(U_a, \nu_a) S^\dagger(U_b, \nu_b) \rangle
\]

(23)

may be thought of as a combination of three different kinds of blocks. We first consider \( S_2(U_a, \nu_a, \nu_b) \) (6) which is the correlation between visibilities measured at the same baseline \( U_a = U_b = u_a \) but the two frequencies \( \nu_a \) and \( \nu_b \) can differ. There are 39 such blocks each of which has 128 \( \times 128 \) elements. Next, there are baseline pairs \( U_a = U_m \) and \( U_b = U_n \) with \( m \neq n \). The correlation \( S_{2ab} \) is non-zero if there is a range of \( U \) values where the aperture power patterns \( \tilde{\Lambda}(U - \nu, \nu) \) and \( \tilde{\Lambda}(U - U, \nu) \) have an overlap. For OWFA there is an overlap only for the adjacent baselines \( m = n \pm 1 \) (see Fig. 2 of Ali & Bharadwaj 2014),

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and the correlation

$$S_2(U_m, \nu_p; U_n, \nu_q) = \langle S(U_m, \nu_p)S^*(U_n, \nu_q) \rangle$$  \hspace{1cm} (24)

has a non-zero value only when $m = n$ or $m = n \pm 1$. The correlations between the adjacent baselines are approximately one-fourth of those between the same baselines (Bharadwaj et al. 2015). There are 38 such blocks, each with $128 \times 128$ elements. The complex conjugate of these blocks also appears in $S_{2ab}$. The third kind of blocks arise from visibility correlations beyond the adjacent baselines. The visibilities in these baselines are uncorrelated, and all the elements of the blocks are zero.

4 HI MODEL

We now discuss the parameters of the input HI model (13) which has been used to compute the visibility covariance (15) for OWFA. Our HI model is completely described by the parameter $Q_\nu = [\tilde{B}_{HI} \tilde{x}_{HI}]_{\nu}$, the redshift distortion parameter $\beta$ and the matter power spectrum $P(k)$. We have used the fit to the baryonic matter power spectrum provided by Eisenstein & Hu (1998) throughout our analysis. The bandwidth $B = 16$ MHz used in our analysis corresponds to a redshift range $3.24 \leq z \leq 3.46$. The measurement $\Omega_{HI} = 10^{-3}$ from DLA observations (e.g. Zafar et al. 2013) combined with $\Omega_0 = 0.048$ (Ade et al. 2014) together imply $\tilde{x}_{HI} = 0.02$. Results from simulations (e.g. Sarkar et al. 2016) and analytic modeling (Marin et al. 2010) indicate a scale independent HI bias $b_{HI} = 2.0$ at large scales $k \lesssim 1$ Mpc$^{-1}$. We assume that the values $\tilde{x}_{HI} = 0.02$ and $b_{HI} = 2.0$ do not evolve significantly over the relevant redshift range, and we have held these fixed for our analysis. The values of $\tilde{T}$ and $D(z)$ respectively vary across the range $17.41 \geq \tilde{T} \geq 16.95$ and $2.81 \times 10^{-4} \leq D(z) \leq 2.95 \times 10^{-4}$ over the relevant redshift range, and the parameter $Q_\nu$ varies from $0.195$ to $0.200$.

The redshift distortion parameter is defined as $\beta = f(\Omega)/b_{HI}$ where $f(\Omega)$ is the growth factor of the underlying matter density fluctuations. Although accounting for the redshift evolution of $f(\Omega)$, the resulting variation in our simulation is less than $0.5\%$.

We note that the redshift evolution of $Q_\nu$ and $\beta$ is not very significant in our analysis as the fractional bandwidth is quite small, in contrast with observations which span a large redshift range. Although accounting for $\beta$ is very important for the other baseline pairs shown in Fig. 1, we expect it to work well for such scenarios.

5 RESULTS

The first step in our simulation is to calculate the expected visibility correlation matrix $S_{2ab}$ (e.g. 12 and 15) of dimensions $4992 \times 4992$. As discussed earlier, it is convenient to express the OWFA visibility covariance as $S_2(U_m, \nu_p; U_n, \nu_q)$ (24) for which the non-zero elements can be decomposed into blocks. We have a non-zero correlation only when $U_m = U_n$ (same baseline) or $U_m = U_{n \pm 1}$ (adjacent baselines), all the other correlations are zero. The upper row of Fig. 1 shows the predicted visibility correlations, the left panel shows $S_2(U_1, \nu_p; U_1, \nu_q)$ which is the correlation between the visibilities measured at different frequencies for the fixed baseline $U_1$, the right panel shows the same for $U_2$ whereas the middle panel shows the cross-correlation between the visibilities measured at $U_1$ and $U_2$. Each panel here is a $128 \times 128$ block and there are a total of 39 such blocks where the two baselines are the same ($U_m = U_n$) and 38 blocks for the adjacent baseline pairs ($U_m = U_{n \pm 1}$). The main features highlighted in the subsequent discussion are common to all the non-zero blocks including those not shown here.

We see that for every baseline pair the visibility correlation peaks along the diagonal ($\nu_p = \nu_q$). The visibility correlation is also seen to increase with frequency $\nu_p$ along the diagonal. The visibility correlation goes down as we move away from the diagonal ($|\nu_p - \nu_q| > 0$), and the signal decorrelates within a frequency separation of $\sim 4$ MHz. As expected, the visibility correlation for the adjacent baseline pair ($U_1$, $U_2$) is considerably lower compared to that for the same baselines ($U_1$, $U_1$) and ($U_2$, $U_2$). We further observe that the magnitude of the visibility correlation decreases as the baseline is increased from ($U_1$, $U_1$) to ($U_2$, $U_2$), and it decreases even further for the larger baselines.

We shall now look at how the visibility correlation $S_2(U_m, \nu_p; U_n, \nu_q)$ varies with frequency in more detail. We first consider the variation with frequency separation $\Delta\nu = \nu_p - \nu_q$. The curves in the three panels of Fig. 2 show the values of $S_2(U_m, \nu_p; U_n, \nu_q)$ along different horizontal sections through the corresponding panels of Fig. 2. Each curve corresponds to a fixed value of $\nu_q$ and it shows how the correlation changes as $\nu_p$ is varied. We see that the correlation peaks when $\nu_p = \nu_q$ and falls sharply on either side as $|\nu_p - \nu_q|$ increases. For the baseline pair ($U_1$, $U_1$) the visibility correlation drops to 0.5 times the peak value at a frequency separation of $\Delta\nu_{1/2} \sim 1.1$ MHz, and drops to zero at a frequency separation of $\sim 4$ MHz. The correlation shows a small amplitude oscillation around zero at larger frequency separations. The variation with $\nu_p = \nu_q$ is very similar for the other baseline pairs shown in Fig. 2. It is however important to note that the $\Delta\nu$ dependence changes at the larger baselines ($U > 30$) where $S_2(U_m, \nu_p; U_n, \nu_q)$ decorrelates considerably faster as $\Delta\nu$ is increased (Fig. 2 of Ali & Bharadwaj 2014). As noted earlier, the amplitude of the correlation drops as the baseline pair gets longer from ($U_1$, $U_1$) to ($U_2$, $U_2$) and it drops by a factor of $\sim 4$ for ($U_3$, $U_3$).

The correlations $S_2(U_m, \nu_q; U_n, \nu_q)$ shown in Fig. 2 all peak when $\nu_q = \nu_p$, we now discuss how the peak value varies with the frequency $\nu_p$. Note that in all the panels of Fig. 2, the peak value of the correlation increases as $\nu_p$ is increased. This is further illustrated in the left panel of Fig. 3 which shows how the peak correlation changes with $\nu_p$ for the baseline pair ($U_1$, $U_1$). We see that the predicted $S_2(U_1, \nu_p; U_1, \nu_p)$ increases linearly by $\sim 20\%$ as $\nu_p$ is varied across the $B = 16$ MHz OWFA bandwidth. This variation of $S_2(U_1, \nu_p; U_1, \nu_p)$ is caused by a combination of (i) the redshift dependence of the HI model as quantified by the parameters $Q_\nu$ and $\beta$ (ii) the redshift dependence of the comoving distance $r_\nu$, and (iii) the chromatic response of the
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Figure 1. The visibility correlation $S_2(U_m, \nu_p; U_n, \nu_q)$ as a function of the frequencies $\nu_p$ and $\nu_q$ for the baseline pairs $(U_m, U_n) = (U_1, U_1)$ (left), $(U_1, U_2)$ (middle) and $(U_2, U_2)$ (right). The upper row shows the theoretical predictions which were used as inputs for the simulation and the lower row shows its difference with the results averaged over $10^4$ realizations (color online).

Figure 2. $S_2(U_m, \nu_p; U_n, \nu_q)$ at a few fixed values of $\nu_q$, shown as solid line plots here, are rows from the matrices of the upper panel of Fig. 1. The points are obtained from averaging $10^4$ simulations. The correlation peaks when $\nu_p = \nu_q$ (color online).
telescope as introduced through the variation of the beam pattern and the baseline with frequency. We note that $Q_{\nu}$ and $\beta$ change by $\sim 2.5\%$ and $\sim 0.5\%$ respectively whereas $r_{\nu}$ varies by $\sim 2\%$ over the OWFA bandwidth. This indicates that the frequency dependence seen in the left panel of Fig. 3 is a consequence of the telescope’s chromatic response.

The right panel of Fig. 3 shows how the visibility correlation $S_2(U_m, \nu_p; U_n, \nu_p)$ varies with baseline $U_m$ for $n = m$ (self) and $n = m + 1$ (adjacent) with the frequency fixed at a value $\nu_p = 326.5$ MHz. The other frequencies not shown here exhibit a similar behavior. We find that both the self and adjacent correlation decrease with increasing baseline $U_m$. This decrease is relatively slow for small baselines $U < 100$ beyond which it falls faster as $|U|^{-1.2}$. The slope of the visibility correlation is related to that of the power spectrum $P(k)$. We note the baseline $U = 100$ corresponds to the Fourier mode $k_\parallel = 2\pi U / r \sim 0.1$ Mpc$^{-1}$ where the power spectrum has a slope $P(k) \sim k^{-1.94}$. We see that the self and adjacent correlations show a very similar $U_m$ dependence, the adjacent correlations are a factor of $\sim 4.3$ smaller than the self correlation at the smallest baselines and this factor changes to $\sim 4$ at the largest baselines.

In the next step, we have determined the eigenvalues and eigenvectors of the predicted visibility correlation $S_{2ab}$ to simulate (20) different realizations of the visibilities. For three different baselines, Fig. 4 shows the real and imaginary parts for a single realization of the simulated visibilities. As expected, the visibilities are random fluctuating quantities with zero mean. We see that the visibilities at the smallest baseline $U_1$ fluctuate relatively slowly with varying frequency as compared to the larger baselines where the signal fluctuates more rapidly with varying frequency. This is consistent with the theoretical input that the visibilities at the smaller baselines remain correlated over a larger frequency separation as compared to the visibilities at the larger baselines.

A total of $10^4$ Gaussian random realizations of the visibilities $S_{ab}$ were simulated. As mentioned earlier, a single random realization of the visibilities can be simulated (eqs. 20 and 18) using a set of $2N$ Gaussian random variables $(x_a, y_a)$. We then averaged over these simulations to estimate the mean visibility correlation for OWFA. The lower row of Fig. 1 shows the difference between the simulated and the predicted visibility correlations for the same baseline pairs as those for which the theoretical predictions have been shown in the upper row. We find that difference between the simulated and the theoretical predictions are quite small, which indicates that our simulation faithfully reproduces the theoretically predicted visibility covariance. The points in Fig. 2 and Fig. 3 show the visibility correlation values obtained from the simulations; the results from simulations, shown as the points, are found to be in excellent agreement with the predictions shown as the solid curves. The deviations between the simulation and the theory seen in the left panel of Fig. 3 can be attributed to the statistical fluctuations inherent to the signal. The coherence scale in frequency in Fig. 3 reflects the frequency correlation seen in Fig. 2.

We now make a quantitative comparison between the simulated and the predicted visibility correlations, for which we use the ratio

$$r = \frac{\Delta S_2}{\sigma S_2} \times \sqrt{N}$$

where $\Delta S_2$ is the difference between the simulated and the
predicted visibility correlation \( S_2(U_m, \nu_p; U_n, \nu_q) \). \( N_r \) is the number of realizations of the simulations and \( \sigma_v^2 \) is the predicted variance for the visibility correlation. Under the assumption that the visibilities are Gaussian random variables, we have calculated the variance to be

\[
\sigma_v^2 = \frac{1}{2} [S_2(U_m, \nu_p; U_n, \nu_q) + S_2(U_m, \nu_q; U_m, \nu_p)] 
\times S_2(U_n, \nu_q; U_n, \nu_q)].
\]

The dimensionless ratio \( r \) quantifies the difference between the simulated and predicted visibility correlations. We expect the values of \( r \) to have a Gaussian distribution of unit variance and zero mean. Fig. 5 shows \( r \) for the baselines pairs of Fig. 1, while Fig. 6 shows the statistics of the values of \( r \). We see that this is in reasonably good agreement with a Gaussian distribution of unit variance. The deviations seen can be attributed to the inherent random fluctuations of the signal. The deviations between the predicted and the simulated distribution are of the order of a few percent, which is expected with 10\(^4\) Gaussian random realizations.

6 DISCUSSION AND SUMMARY

The 21-cm intensity signal is buried in foregrounds which are several orders of magnitude larger. Precision of calibration, the telescope’s chromatic response, beam and other instrumental systematics pose additional challenges. Simulations play a vital role in testing and validating \( {\text{H}}^1 \) 21-cm power spectrum estimation techniques. Simulations are particularly important in quantifying the impact of foreground removal, calibration, and various instrumental and post-processing effects. Conventional methods for simulating the expected 21-cm visibility signal requires simulating the sky signal which is then passed through a software model of the instrument to generate the visibilities. This may pose a computational challenge in that it is necessary to simulate the \( {\text{H}}^1 \) distribution in a large computational volume. Further, the \( {\text{H}}^1 \) signal and the cosmological parameters both vary with line of sight distance within the observational volume due to the light cone effect. The computational requirements scale in such cases with the number of independent realizations of the signal sought. In this paper, we present an analytical method to simulate the \( {\text{H}}^1 \) visibility signal and have demonstrated applying it to simulate the \( {\text{H}}^1 \) visibility signal for the upcoming OWFA Phase I.

The first step in our method is to compute the expected visibility correlation for the signal at different baselines and frequency channels. Our approach for calculating the visibility correlation differs from the previous studies on two accounts. First, we have incorporated the light-cone effect whereby the statistical properties of the \( {\text{H}}^1 \) signal and cosmological parameters both evolve with redshift or equivalently, with observing frequency. Second, we have also accounted for the fact the telescope has a chromatic response, that is the telescope parameters and baselines vary with observing frequency. The combined effect of these two features makes the \( {\text{H}}^1 \) visibility signal non-ergodic in frequency, which implies that the correlation between the visibilities at two different frequencies \( \nu_p \) and \( \nu_q \) is no longer a function of the frequency separation \( \nu_p - \nu_q \). The Fourier basis or the delay channel \( \tau \) cease to be the appropriate choice once this ergodicity is broken. We note that the general formalism adopted here does not assume the \( {\text{H}}^1 \) visibility signal to be ergodic in frequency. We find that the deviations from ergodicity are \( \sim 20\% \) for the limited bandwidth of 16 MHz considered here, however these effects are expected to be important for observations that span a larger bandwidth.

As was shown by Kosambi (Kosambi 1943) and subsequently Karhunen (Karhunen 1947) and Loève (Loève 1955), the eigenvectors of the visibility correlation matrix provide the correct basis for analyzing the \( {\text{H}}^1 \) visibility signal even if the ergodicity is broken. We have used the eigenvalues and eigenvectors of the predicted visibility correlation matrix to simulate the \( {\text{H}}^1 \) visibility signal. The computational effort here goes into calculating the expected visibility correlation matrix, and finding its eigenvalues and eigenvectors. Once the eigenvalues and eigenvectors are known, it is only necessary to generate a set of random numbers (20) to simulate an entire set of the \( {\text{H}}^1 \) visibility signal. Multiple random realizations of the \( {\text{H}}^1 \) visibility signal can be simulated by using different realizations of the random numbers. This feature makes this method particularly efficient if one wishes to simulate many random realizations of the \( {\text{H}}^1 \) visibility signal. In this paper we have simulated \( 10^4 \) random realizations of the \( {\text{H}}^1 \) visibility signal and used these to calculate the visibility correlation matrix. We find that the simulations are in very good agreement with the theoretical predictions which have been used as input.

The simulation method presented here is particularly well suited for OWFA which has only a few independent baselines that do not change with the rotation of the Earth. The situation could become significantly more complex for an array like the GMRT which has many different baselines that change with Earth rotation. In such cases, conventional techniques which involve simulating the sky signal may be more efficient. However, we note that this limitation could

**Figure 4.** The real and imaginary parts of a single realization of the simulated visibilities for the fixed baselines \( U_n \) with \( n = 1, 16, 32 \) respectively. The dotted line in each panel is the expectation of the visibilities, \( \langle S(U_n, \nu) \rangle = 0 \) (color online).

\( \nu \) MHz

\( n = 32 \)

\( n = 16 \)

\( n = 1 \)

\( S(U_n, \nu) \times 10^{-5} \) Jy

\( 320 \) 326 332

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Figure 5. Each panel shows $r$ (defined in 25), which is a dimensionless ratio that quantifies the difference between the simulated and the predicted visibility correlations, for each baseline pair of Fig. 1 (color online).

Figure 6. The points show the distribution of the values of $r$ from Fig. 5. The $r$ values have been binned with an interval $\Delta r = 0.04$ and the solid line shows the prediction for a Gaussian of unit variance. Only every other point in the binned histogram has been shown for clarity (color online).

be overcome by gridding the baselines to reduce the complexity of the problem. Another limitation of the present technique is that it assumes the 21-cm signal to be a Gaussian random field ignoring the non-Gaussianity which would arise due to the non-linear evolution of the underlying matter perturbations. This aspect of the 21-cm signal would be naturally incorporated in a N-body simulation. It is useful to note that the method presented here can be extended to handle a H$_i$ signal field that is not purely Gaussian: non-Gaussianity can be incorporated by modifying the statistics of the random variables used to simulate the H$_i$ visibility signal, which we plan to address in the future.

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