Superembeddings, Non-linear Supersymmetry and 5-branes

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Abstract

We examine general properties of superembeddings, i.e., embeddings of supermanifolds into supermanifolds. The connection between an embedding procedure and the method of non-linearly realised supersymmetry is clarified, and we demonstrate how the latter arises as a special case of the former. As an illustration, the super-5-brane in 7 dimensions, containing a self-dual 3-form world-volume field strength, is formulated in both languages, and provides an example of a model where the embedding condition does not suffice to put the theory on-shell.
1. Introduction

Our understanding of string theory at the non-perturbative level has gone through a dramatic improvement in recent years. Some of the key aspects of this development are connected to the central rôle played by solitonic solutions of the low energy field equations, i.e., various brane configurations that solve the field equations of the supergravity theories. By considering BPS saturated solitonic solutions which preserve e.g. half the supersymmetry of the supergravity theory in question, these supergravity theories can be shown to be related by duality transformations some of which are intrinsically non-perturbative in nature. In fact, (almost) all consistent string and supergravity theories, including 11-dimensional supergravity, are in this way believed to constitute low-energy descriptions of one master theory, the so called M-theory, in either the weak or strong coupling regime of some particular coupling constant in the moduli space of all couplings. An overview of the subject, as well as further references, may be found e.g. in ref. [1].

The known branes come in three main varieties†, p-branes, Dp-branes, and T5-branes, depending on whether the bosonic sector of the field theory on the world-volume of the brane contains only scalars, scalars and vector gauge fields, or scalars together with a third rank anti-symmetric self-dual tensor field strength. (Recently also other types of tensor fields and combination of such have been introduced in these theories to solve certain specific problems [2]. However, this is of no immediate interest for the considerations of this paper). For a review of the different kinds of solitonic branes and their rôles in non-perturbative string theory, see ref. [3]. The scalar fields appearing on the branes are immediately identifiable as Goldstone fields, or collective modes, corresponding to the translation symmetries that are broken when the brane is introduced into target space-time. That is, one obtains one scalar field for each direction transverse to the brane. By checking which supersymmetries get broken, or by viewing the brane as a supersurface embedded in a target superspace, also the number of Goldstone fermions can be deduced. However, when supersymmetry requires the brane supermultiplet to contain also vectors or tensor potentials, there is no analogously simple argument that explains their presence. We will have nothing new to say about this problem in this paper.

From the theory of non-linear realisations (NR) we know that, although the branes fill out multiplets realising all target space symmetries linearly, on the branes the unbroken symmetries are linearly realised while the broken ones are realised non-linearly. In the context of open string theory one knows that the supersymmetric field theory on Dp-branes involve vector multiplets and are highly non-linear Born–Infeld type theories. Using duality arguments similar non-linearities can be seen to arise for T5-branes containing self-dual third rank tensors in \( d = p + 1 = 6 \) brane dimensions [4,5].

Bagger and Galperin [6] have recently verified that the theory of non-linear realisations applied to supermanifolds embedded into target supermanifolds with twice the number of anticommuting coordinates naturally leads to Born–Infeld actions if vector multiplets are involved. This provides

† There are also branes associated with gravitational charges. We will not consider these in the present paper.
a very nice explanation for the rather strange form of the Born–Infeld action as being a direct consequence of the non-linearly realised broken supersymmetries. In this formalism one introduces derivatives that transform in a well-behaved manner under the linearly as well as under the non-linearly realised (super)symmetries. Consistency requirements on the constraints imposed on superfields together with requirement of symmetry under the linearly as well as the non-linearly realised supersymmetry imply the Born–Infeld non-linear action in the case the supermultiplet is chosen to be a Maxwell multiplet in 4 dimensions.

Another recently developed approach giving similar results is the “doubly supersymmetric geometrical approach” \([7,8]\) or the “embedding formalism” \([9]\). In the latter approach one starts from the torsion tensor in target superspace and considers the equation that arises when pulling it back to the super-world-volume. By introducing a particular embedding constraint the torsion pull-back equations can, in the only case analysed explicitly so far namely the T5-brane in 11 dimensions, be seen to give rise to exactly the same non-linear theory as can be argued for from its relation via duality to the Born–Infeld action of a D4-brane. However, in this formalism the non-linearly realised supersymmetry plays no rôle whatsoever, and it is not clear that the non-linearities of the action actually have their origin in some broken symmetries, although this clearly must be the case \([10]\).

It is the purpose of this paper to clarify some aspects of the connection between these two approaches and demonstrate that also for the T5-brane the non-linearities of the action stem from an underlying set of broken symmetries. In section 2 we discuss some basic properties of superembeddings using as an example some results from the theory of non-linear realisations as well as from the theory of superembeddings applied to the T5-brane, with a \((6|8)\) super-world-volume, embedded into a \((7|16)\) target superspace. Here the notation \((m|n)\) refers to a superspace with \(m\) commuting and \(n\) anticommuting coordinates. Section 3 gives the details of this embedding using the theory of non-linear realisations along the lines of Bagger and Galperin \([6]\). This formalism turns out to generate a rather complicated equation that the dimension zero components of the torsion tensor induced on the super-world-volume must satisfy. Although this equation can be solved explicitly, further analysis of the system, e.g. deriving the field equations, seems cumbersome and is not carried out here. Instead we turn in the following sections to an analysis of this T5-brane by means of the embedding formalism. In section 4 we show that the theory of non-linear realisations is just a special case of the embedding formalism, obtained if certain for this formalism unconventional choices of intrinsic torsion components are made. In section 5 we then show that the torsion pull-back equation can be completely analysed and seen to lead to the non-linearities characteristic of T5-brane field theories, as already demonstrated for the T5-brane embedded in 11 dimensions by Howe, Sezgin and West \([4]\). In a final section we summarise our results and present the conclusions.
2. SUPEREMBEDDINGS

In this section we will consider superembeddings \[8,9\] from a general point of view, using some explicit results from subsequent sections to exemplify the ideas but leaving the details of special applications to the later sections. The different parametrisations of the embedding matrix to be used in later sections are introduced, and the geometric properties of the embeddings are analysed, eventually leading to the torsion pullback equation, introduces in ref. [9].

Let us consider an arbitrary embedding \((\mathcal{M},h) \xrightarrow{f} (\mathcal{M},g)\), where the two supermanifolds have dimensions \((m|n)\) and \((m'|n')\) respectively. The signature of the bosonic metric is arbitrary at the moment but later on we will restrict ourselves to \((D-1,1)\) signature. We will use standard notation \[9\] for the local coordinates of the two supermanifolds, i.e., \(z^M = (x^m; \theta^\mu)\) and \(Z^M = (X^m; \Theta^\mu)\).

We now introduce the embedding matrix* \(\mathcal{E}_{\Lambda}^A\), defined in terms of canonical 1-forms \(\theta\) by

\[
\tilde{\theta} := f^* \theta = e^A \mathcal{E}_{\Lambda}^A E_A^\Lambda.
\]

(2.1)

Here, \(e^A\) and \(E_A\) are orthonormal basis vectors on the cotangent space of the world-volume and the tangent of the target space, respectively. We refer to Appendix A for more details of notation. The basis vectors \(E_A := f_* e_A\) span the tangent space of the embedded supermanifold. In order to have a complete basis for the entire tangent space of the target space, we may also introduce normal vectors denoted \(E_A'\). We will use an overlined index representing a composite index for the pair \((A,A')\). We will also introduce a set of dual basis vectors by

\[
< \mathcal{E}_{\Lambda}^A, \mathcal{E}_{\Lambda'}^{\Lambda'} > = \delta_{\Lambda'}^{\Lambda}.
\]

(2.2)

With these objects at hand we have the possibility of splitting the canonical 1-form into tangential and normal terms,

\[
\bar{\theta} = \theta + \theta' = \delta^A \mathcal{E}_{\Lambda}^A + \delta^{\Lambda'} \mathcal{E}_{\Lambda'}^A.
\]

(2.3)

These 1-forms now serve as projectors of vectors down to the tangent and normal parts respectively, i.e., \(X^\| = \theta(X)\), \(X^\perp = \theta'(X)\). By introducing a target space Lorentz matrix \(u_B^A\) relating the basis \(E_A^\Lambda\) to a frame connected to the embedded surface, it is convenient to split the embedding matrix as

\[
\mathcal{E}_{\Lambda}^A = \mathcal{E}_{\Lambda}^A u_B^A.
\]

(2.4)

Concerning the basis \(\mathcal{E}_{\Lambda'}^A\) of normal vectors, the choice is completely arbitrary and physically irrelevant, and it will soon be clear that in explicit parametrisations we can always choose them to be \(\mathcal{E}_{\Lambda'}^A = u_{A'}^A\), i.e., as part of a Lorentz matrix.

* Note the difference in notation compared to refs. [9,4], where the matrix \(\mathcal{E}\) does not denote the embedding matrix, the latter being denoted \(E_{\Lambda}^A\).
As a starting point for a general superembedding, the orientation in target superspace of the super-world-volume tangent space is parametrised by a point in the super-grassmanian

$$\text{SGr}[(m|n); (\underline{m}|\underline{n})] := \frac{\text{OSp}(m|n)}{\text{OSp}(m|n) \times \text{OSp}(\underline{m} - m|\underline{n} - n)} ,$$

(2.5)
i.e., there are $m(m - m) + n(n - n)$ bosonic parameters and $m(\underline{n} - n) + n(m - m)$ fermionic ones.

One way of representing these degrees of freedom is to introduce the four fields

$$m_a^{b'} \leftrightarrow m(m - m) ,$$
$$h_\alpha^{\beta'} \leftrightarrow n(\underline{n} - n) ,$$
$$\chi_a^{\beta'} \leftrightarrow m(\underline{n} - n) ,$$
$$e_\alpha^{b'} \leftrightarrow n(m - m) ,$$

(2.6)

and locally represent the embedding by

$$e_A^B = e_A^{\underline{B}} u_B^B = \left( \begin{array}{ccc} u_a^a & m_a^{b'} u_b^a & \chi_a^{\alpha'} u_\alpha^\alpha \\ e_a^{b'} u_b^a & e_\alpha^{\beta'} & h_\alpha^{\beta'} u_\beta^\beta \end{array} \right) .$$

(2.7)

If we put this together with the normal vectors we get

$$e_A^B = e_A^{\underline{B}} u_B^B = \left( \begin{array}{ccc} \delta_a^b & m_a^{b'} & 0 \\ 0 & e_a^{b'} & \chi_a^{\alpha'} \\ 0 & 0 & \delta_\alpha^{\beta'} \end{array} \right) ,$$

(2.8)

with the inverse

$$(e^{-1})_A^{\underline{B}} = \left( \begin{array}{ccc} \delta_a^b & -m_a^{b'} & 0 \\ 0 & \delta_\alpha^{\beta'} & -\chi_\alpha^{\beta'} \\ 0 & 0 & \delta_\alpha^{\beta'} \end{array} \right) .$$

(2.9)

We notice that the information of the embedding lies entirely in the matter fields, and that $u_A^B$ can be chosen arbitrarily. As we will see in the case of non-linear realisations in sections 3 and 4, they may for example be chosen to be just $\delta_A^B$.

In all applications we will choose the part of the embedding matrix not containing the fields of (2.6), i.e., the $u$’s, to be part of a Lorentz matrix. This choice is always possible, recalling that the essential property of the embedding matrix is that it defines the orientation of the embedded hypersurface, so that different embedding matrices with identical span of the vectors $e_A$ represent the same point in the grassmannian (2.5), and thus the same embedding. To put it concretely, this
degree of arbitrariness in the embedding matrix is identified with the invariance of its definition (2.1) under
\[ e^A \rightarrow e^B M^A_B, \]
\[ e^A_A \rightarrow (M^{-1})_A^B e^B_B, \]
allowing us to go to a representation (2.7) with lorentzian u's.

The canonical 1-forms are now expressed in terms of the matter fields in the following way:
\[ \theta_\chi = \theta_0 + m' + \epsilon + \chi + h, \]
\[ \theta'_\chi = \theta'_0 - m' - \epsilon - \chi - h, \]
and if we define new vielbeins by \( E_A^\alpha := u^\alpha_B E^B_B, \) we see that
\[ m' = E^a m_a^b E^b_b, \]
\[ \epsilon' = E^a \epsilon_a^b E^b_b, \]
\[ \chi' = E^a \chi_a^b E^b_b, \]
\[ h' = E^a h_a^b E^b_b. \]

An example of the present parametrisation of the embedding matrix is given by the NR case (section 4), where we work in a supersymmetric supermanifold with \( n = n/2. \) There we will see that the fields of (2.6) are simply
\[ m_a^b = \nabla_a \phi^b, \]
\[ \epsilon_a^b = \nabla_a \phi^b - i(\Gamma^b \psi)_a, \]
\[ \chi_a^b = \nabla_a \psi^b, \]
\[ h_a^b = \nabla_a \psi^b, \]
where the bosonic matter fields \( \phi^b \) are shifted [9] as
\[ \phi^b = x^b + \frac{i}{2} \Gamma^b \psi. \]

We also see that on imposing the embedding condition [8,9]
\[ \epsilon_a^b = 0, \]
(this condition, which is a basic geometric relation reducing the number of field components in the embedding formalism, will be more closely examined in section 4) we get a relation [9]
\[ \psi^{b\alpha} = -\frac{i}{m-m}(\Gamma_c^c)^{b\alpha} \nabla_{a\psi^c}. \]
between the bosonic fields $\phi^{b'}$ and the fermionic $\psi^{\beta'}$, which are the matter fields containing the independent degrees of freedom of the embedding $((m-m)$ and $(n-n)$ respectively). Of course, since these fields are both superfields, they contain in general too many physical degrees of freedom. This problem will be eliminated by analysing the torsion equation together with the embedding condition.

Returning to our study of the embedding matrix, we note that with the above parametrisation it is only lorentzian for all matter fields equal to zero. It is easy to convince oneself that the field $m^a_{\, \, b'}$ can always be rotated away by a (target space) Lorentz transformation:

$$m^a_{\, \, b'} \hat{u}^b_{\, \, c} := u^c_{\, \, a} + m^a_{\, \, b'} u^b_{\, \, c}, \quad (2.17)$$

so that the $m(m-m)$ parameters of the orientation of the bosonic embedding are absorbed into $\hat{u}$. The price to be paid for this change of frame is that the fermions rotate accordingly, and the lower right hand corner of (2.7) changes. Again, it is possible to retain the form $u^\alpha_{\, \, \alpha} + h^\alpha_{\, \, \beta'} u^\beta_{\, \, \alpha}$ by utilising the invariance (2.10) with a non-lorentzian matrix $M$. The embedding matrix then takes the form

$$e_A^a = \left( \begin{array}{c} m^b_{\, \, a} u^a_{\, \, b} \\ u^a_{\, \, b} \\ 0 \\ \chi^a_{\, \, \alpha'} u^\alpha_{\, \, \alpha} \\ 0 \\ u^\alpha_{\, \, \alpha} + h^\alpha_{\, \, \beta'} u^\beta_{\, \, \alpha} \\ u^\alpha_{\, \, \alpha} \end{array} \right), \quad (2.18)$$

where the $u$'s are again lorentzian (the tilde is dropped). This Lorentz matrix should of course not be identified with the one in (2.7), neither should the fields denoted by identical symbols. We have also dropped the $e_A^a$ term as it will vanish due to the embedding condition. The new parametrisation also involves a new choice of basis for the normal vectors.

Equation (2.18) is the form of the embedding matrix to be used in the rest of the present section, and in section 5. The invariance (2.10) used to move between the two versions (2.7) and (2.18) of the embedding matrix involves a redefinition of the intrinsic vielbeins $e^A$, and we may expect the torsion tensors in the two versions of the theory to exhibit differences, which is what we will see in the following sections. It is striking that the seemingly different theories, from a geometric point of view, are related by a transformation that modifies the intrinsic world-volume geometry by matter fields. We will not analyse the transformations in detail, but note that they may be worth further study.

The inverse of the modified embedding matrix is

$$e_A^a = \left( \begin{array}{c} u_a^{\, \, b} (n^{-})^b_{\, \, a} \\ 0 \end{array} \right), \quad (2.19)$$
and the canonical 1-forms therefore take the form

\[
\begin{align*}
\theta_\chi &= \theta_0 + \chi + h, \\
\theta'_\chi &= \theta'_0 - \chi - h,
\end{align*}
\] (2.20)

where

\[
\begin{align*}
\chi &= E^a(m^{-1})_{ab}^\gamma E_{\gamma'}, \\
h &= E^a h_{\alpha \beta} E_{\beta'}.
\end{align*}
\] (2.21)

Here one should also mention that none of the free parameters in \(m_{ab}^{\prime}\) ends up in \(m_{ab}\); the latter becomes determined completely in terms of \(h\).

This is all we will say at this point about the parametrisation of the embedding matrix. We will now discuss the origin of the torsion pull-back equation and in later sections look at some of its solutions. To facilitate the understanding of the torsion equation we will point out some conceptual difficulties that may appear in connection with it. One problem is that when working with an embedding of the type \((\mathcal{M}, h) \overset{f}{\rightarrow} (\mathcal{N}, g)\) we have to consider two different metrics on \(\mathcal{N}\): on the one hand the a priori (intrinsic) metric on the world-volume \(h\) and on the other the metric induced by the embedding, \(g = f^* g\). The problem is that we no longer have one connection on \(\mathcal{M}\) but two, each compatible with one distinct metric. We will use the notation \(\mathcal{D}\) and \(\nabla\), schematically fulfilling

\[
\begin{align*}
\mathcal{D} h &= 0, \\
\nabla g &= 0.
\end{align*}
\] (2.22)

Another upcoming problem is connected to the fact that the embedding is not Lorentz, unless the matter fields vanish. In order to distinguish the situations, we will denote the matter fields collectively by \(\chi\), and let a lorentzian embedding correspond to \(\chi \rightarrow 0\).

In deriving the torsion equation, we start from the Gauss–Weingarten equations\(^\dagger\) [11]

\[
\begin{align*}
\sum \chi Y &= \nabla \chi Y + \mathcal{K}'(X, Y), \\
\sum \chi Y' &= \nabla \chi Y' + \mathcal{K}(X, Y'),
\end{align*}
\] (2.23)

where we have used the notation \(X\) for tangential vectors and \(X'\) for normal vectors. We see from these equations that the covariant derivative splits into a tangential derivative, a normal derivative and two tensors which are the so called extrinsic curvatures of the embedding, also known as the second fundamental form\(^\ast\). These equations are purely tensorial and independent of the form of the embedding. They are also independent of the intrinsic metric \(h\) on the world-volume. If we now set \(Y = E_A\) we get

\[
\nabla E_A = \Omega_A^B E_B = \left(\begin{array}{c}
\Omega_A^B \\
K_{AB}' \\
\Omega_A^{B'}
\end{array}\right) \left(\begin{array}{c}
E_B \\
K_{AB}' \\
\Omega_A^{B'}
\end{array}\right).
\] (2.24)

The reason for taking \(\mathcal{D}\) here instead of \(\nabla\) is that we need to make a distinction between whether the embedding is Lorentz or not. If the embedding is Lorentz then all quantities in these equations

\(^\dagger\) Ref. [9] gives similar equations, that in addition to our terms on the right hand side also contains the entities \(\mathcal{L}, \mathcal{L}'\) which will soon be defined. The difference, as will be clear from the following discussion, resides entirely in the use of induced contra intrinsic connection in the derivative.

\(^\ast\) The term is reserved for \(\mathcal{K}'\), but \(\mathcal{K}\) is determined from it by \(g(\mathcal{K}(X, Y), Z') + g(\mathcal{K}'(X, Y), Z) = 0\).
will lie in the algebra \( \text{spin}(m) \) but not otherwise. We will therefore make a distinction between
the extrinsic curvatures of the two types of embeddings by denoting the extrinsic curvature of a
Lorentz embedding by roman letters and a matter triggered embedding by calligraphic ones. From
the Gauss–Weingarten equations it follows that

\[
\mathcal{K}_{AB}^{C'} = \langle \nabla_A (E_B), E^{C'} \rangle .
\]

This means that, as the \( \mathcal{E} \) tend to \( E \) as \( \chi \to 0 \), the extrinsic curvature will tend to the Lorentz
one, i.e., \( \mathcal{K}_{AB}^{C'}|_{\chi=0} = K_{AB}^{C'} \). Of course

\[
K_{AB}^{C'} = \langle \nabla_A (E_B), E^{C'} \rangle = \langle \nabla_A (u_B), u^{C'} \rangle ,
\]

where \( u_B = u_A E_A^B \) and \( u^{A'} = E_A^{A'} \). Now since we will use the intrinsic world-volume metric
\( h \) as an auxiliary field in the forthcoming torsion equation, we need a relation between the two
connections on \( \mathcal{M} \). Let us define a difference operator of the two of them by

\[
\mathcal{L} := \nabla - \mathcal{D} .
\]

This operator is of course a tensor. Proceeding as for the extrinsic curvature we let \( \mathcal{L}|_{\chi=0} =: L \).
We will also extend our covariant derivatives on \( \mathcal{M} \) to act on world-volume vectors as well as
target space vectors and denote them as \( \nabla \) and \( \mathcal{D} \). This enables us to note the following important
relations

\[
\nabla (\tilde{\theta}) = \mathcal{K}' , \\
\mathcal{D} (\tilde{\theta}) = \mathcal{L} + \mathcal{K}'.
\]

(These are tensor equations, so there is no wedge product involved), from which we see that the
tensors can be written

\[
\mathcal{L}_{AB}^{C} = \mathcal{D}_{A} (\mathcal{E}_{BC}^{-}) \mathcal{E}_{C}^{+} , \\
\mathcal{K}_{AB}^{C'} = \mathcal{D}_{A} (\mathcal{E}_{BC}^{-}) \mathcal{E}_{C'}^{+} ,
\]

and consequently

\[
L_{AB}^{C} = \mathcal{D}_{A} (u_B^{C}) u_{C}^{+} , \\
K_{AB}^{C'} = \mathcal{D}_{A} (u_B^{C'}) u_{C'}^{+} .
\]

Let us introduce yet another covariant derivative in order to get relations between these fields:

\[
\widetilde{\mathcal{D}} = \mathcal{D}_{\text{diag}} + \hat{X} ,
\]

where

\[
\hat{X}_{\mathcal{A}}^{\mathcal{B}} := \begin{pmatrix}
L_A^B & 0 \\
0 & L_A^{B'}
\end{pmatrix} ,
\]
and where the connection in the first term on the right hand side contains the target space connection projected on the part not mixing tangential and normal directions. Let us also define

\[
\mathcal{Y}_A^B := \mathcal{D}(\mathcal{E}_A^C)\mathcal{E}_B^C := \left( \mathcal{L}_{A B}^{A' B'} \mathcal{X}_{A B}^{A' B'} \right). \tag{2.33}
\]

This notion is natural because it will tend to \( L \) and \( K \) as \( \chi \to 0 \). We now get the relation between the fields

\[
\mathcal{Y}_A^B = \mathcal{X}_A^B + \mathcal{D}(\mathcal{E}_A^C)(\mathcal{E}^C_{\gamma \delta})^{\gamma \delta}_B + \mathcal{D}^C_{\gamma \delta}K_{C D'}^{D'}(\mathcal{E}^C_{\gamma \delta})^{D'}_B + \mathcal{D}^C_{\gamma \delta}C_{\gamma \delta}^{D'} K_{D C'}^{D'}(\mathcal{E}^C_{\gamma \delta})^{D'}_B, \tag{2.34}
\]

from which, if we look at our parametrisation of the embedding matrix in particular, we now get the following relations

\[
\begin{align*}
\mathcal{L}_b^c &= L_b^c + (\mathcal{D}m_b^d)(\mathcal{E}^{\gamma \delta})_{\delta d}^c, \\
\mathcal{L}_b^{\gamma} &= \chi_b^{\beta \gamma} K_{\beta \gamma}, \\
\mathcal{L}_b^{\beta} &= 0, \\
\mathcal{X}_b^{\gamma} &= L_b^{\gamma} + h_b^{\beta \gamma} K_{\beta \gamma}, \\
\mathcal{X}_b^{\gamma} &= m_b^c K_{c}^{\gamma}, \\
\mathcal{X}_b^{\gamma} &= \mathcal{D}X_b^{\gamma} - (\mathcal{D}m_b^c)(\mathcal{E}^{\gamma \delta})_{\delta d}^c - \chi_b^{\beta \gamma} K_{\beta \gamma} h_{\gamma \gamma}, \\
\mathcal{X}_b^{\gamma} &= 0, \\
\mathcal{X}_b^{\gamma} &= K_{\beta \gamma} + \mathcal{D}h_{\beta \gamma} - h_{\beta \gamma} K_{\beta \gamma} h_{\gamma \gamma}, \\
\mathcal{X}_b^{\gamma} &= K_{d}^{\gamma}, \\
\mathcal{X}_b^{\gamma} &= 0, \\
\mathcal{X}_b^{\gamma} &= 0, \\
\mathcal{X}_b^{\gamma} &= K_{d}^{\gamma}, \\
\mathcal{X}_b^{\gamma} &= L_b^{\gamma}, \\
\mathcal{X}_b^{\gamma} &= -K_{d}^{\gamma} h_{\gamma \gamma}, \\
\mathcal{X}_b^{\gamma} &= 0, \\
\mathcal{X}_b^{\gamma} &= L_{d}^{\gamma} - K_{d}^{\gamma} h_{\gamma \gamma}.
\end{align*} \tag{2.35}
\]

Some of the zeroes are directly related to the embedding condition (2.15). The virtue of these relations is that they display explicitly which properties of the geometry are induced by matter fields. They are important because we will use them in the process of solving the torsion equation. We now turn to the issue of deriving the torsion equation, which is the final subject of this section.

If we look at the Gauss–Weingarten equations we see that

\[
\mathcal{T}(X,Y) := \mathcal{D}X Y - \mathcal{D}Y X - [X,Y] = T(X,Y) + T'(X,Y), \tag{2.36}
\]
where $T(X,Y)$ is the induced torsion inherited from the connection on $\mathcal{M}$ and

$$T'(X,Y) := \mathcal{H}'(X,Y) - \mathcal{H}'(Y,X) \quad (2.37)$$

is called the extrinsic torsion of the embedding. But we know that we have a relation between the induced torsion and the intrinsic torsion denoted $\mathcal{T}$ from the relation of the two connections on $\mathcal{M}$. This relation is

$$T(X,Y) = \mathcal{T}(X,Y) + \mathcal{L}(X,Y) - \mathcal{L}(Y,X), \quad (2.38)$$

which together with the relation

$$\mathcal{D} \wedge \tilde{\theta} = \wedge \mathcal{L} + T' = -\mathcal{T} + T + T' \quad (2.39)$$

(the notation $\wedge \mathcal{L}$ meaning the antisymmetric part) finally yields the torsion equation in the form

$$\mathcal{D} \wedge \tilde{\theta}(X,Y) + \mathcal{T}(X,Y) = T(X,Y), \quad (2.40)$$

where of course $X,Y$ everywhere are super-world-volume tangent vectors. This is nothing but the usual torsion equation that figures in the physics literature \[8,9\]. Putting $X = \mathcal{E}_A$ and $Y = \mathcal{E}_B$ and contracting with $\mathcal{E}_C$ we get it in the more transparent form

$$\mathcal{D}_A \mathcal{E}_B \mathcal{E}_C - (-1)^{AB} \mathcal{D}_B \mathcal{E}_A \mathcal{E}_C + \mathcal{T}_{AB} \mathcal{E}_C \mathcal{E}_C = (-)^{A(B+E)} \mathcal{E}_B \mathcal{E}_A \mathcal{T}_{AB} \mathcal{E}_C \quad (2.41)$$

In order to solve this equation we will project it onto the tangent and the normal directions, respectively, giving

$$2\mathcal{L}_{(AB)}^C + \mathcal{T}_{AB}^C = T_{AB}^C \quad (2.42)$$

and

$$2\mathcal{H}_{(AB)}^C' = T_{AB}^C' \quad (2.43)$$

where the graded anti-symmetrisation is defined by $V_{(AB)} := \frac{1}{2}(V_{AB} - (-1)^{AB}V_{BA})$. Now for the parametrisation in eq. (2.18) we have the following induced torsion components (if the target space
is flat):

\[
T_{ab}^c = i[\chi_a (\Gamma^d) \chi_b] (m^{-1}) d^c,
\]
\[
T_{ab}^\gamma = 0,
\]
\[
T_{a\beta}^c = -i[\chi_a (\Gamma^d) h_\beta] (m^{-1}) d^c,
\]
\[
T_{a\beta}^\gamma = 0,
\]
\[
T_{\alpha\beta}^c = -i[(\Gamma^d)_{\alpha\beta} + h_\alpha (\Gamma^d) h_\beta] (m^{-1}) d^c,
\]
\[
T_{\alpha\beta}^\gamma = 0,
\]
\[
T_{ab}^{c'} = 0,
\]
\[
T_{ab}^{\gamma'} = -i[\chi_a (\Gamma^d) \chi_b] (m^{-1}) d^c \chi^{c'} \gamma',
\]
\[
T_{a\beta}^{c'} = -i[\chi_a (\Gamma^d) h_\beta],
\]
\[
T_{a\beta}^{\gamma'} = i[(\Gamma^d)_{a\beta} + h_\alpha (\Gamma^d) h_\beta] (m^{-1}) d^c \chi^{c'} \gamma',
\]
\[
T_{\alpha\beta}^{c'} = -i[2h_\alpha (\Gamma^d) h_\beta],
\]
\[
T_{\alpha\beta}^{\gamma'} = i[(\Gamma^d)_{\alpha\beta} + h_\alpha (\Gamma^d) h_\beta] (m^{-1}) d^c \chi^{c'} \gamma'.
\]

The \( \Gamma \) matrices have been split according to appendix B, and summed \( \alpha' \) indices are suppressed, e.g. \( h_\alpha (\Gamma^d) h_\beta \equiv h_\alpha (\Gamma^{C^d})_{\alpha'\beta'} \). Together with the expressions for the fields \( \mathcal{X} \), \( \mathcal{L} \) and of course \( \mathcal{J} \) it is just to begin solving for the matter fields. We already here see that the solutions will depend on the chosen intrinsic world-volume torsion \( \mathcal{J} \), but we will come back to this in later sections. If we instead look at the case of our first parametrisation, given in eq. (2.7), where we had a direct coupling to the NR case, we get

\[
T_{ab}^c = i\chi_a (\Gamma^c) \chi_b ,
\]
\[
T_{ab}^\gamma = 0 ,
\]
\[
T_{a\beta}^c = -i\chi_a (\Gamma^c) h_\beta ,
\]
\[
T_{a\beta}^\gamma = 0 ,
\]
\[
T_{\alpha\beta}^c = -i[\Gamma^c]_{\alpha\beta} + h_\alpha (\Gamma^c) h_\beta ,
\]
\[
T_{\alpha\beta}^\gamma = 0
\]

(again, although the fields denoted by the same letters in (2.44) and (2.45) are related by field redefinitions, they should by no means be identified), which we will see in later sections is nothing but the relations for the torsion derived from the algebra of the induced covariant derivatives.

3. The \( D = 6 \) tensor multiplet and non-linear realisations

In this section we will review the basic steps of the theory of non-linear realisations [12], which is a systematic way of studying the properties of Goldstone fields. It is well-known that the spontaneous breaking of supersymmetry gives rise to a massless spin-\( \frac{1}{2} \) Goldstone fermion [13]. This fermion then belongs to the massless multiplet of the residual unbroken supersymmetry. However, the choice of the Goldstone multiplet is not unique. The partial breaking of \( N = 2 \) supersymmetry to \( N = 1 \) in four dimensions was studied in [6], for three different multiplets. We will use non-linear
realisations to describe the spontaneous breaking of \( N = 1 \) supersymmetry in \( D = 7 \) to \( N = (1, 0) \) in \( D = 6 \) and pick the self-dual tensor multiplet in 6 dimensions \([14,15]\) as the Goldstone multiplet.

Let \( \mathcal{M}^{(7|16)} \) be a flat \( N = 1 \) target superspace with local coordinates \( Z_M = (X_m, \Theta^\mu) \). Our starting point is the 7-dimensional \( N = 1 \) supersymmetry algebra \( \{Q_\alpha^i, Q_\beta^j\} = (\Gamma^a)_{\alpha\beta} P^a \). \( \text{(3.1)} \)

Making the \( 7 \rightarrow 6 + 1 \) split, using the conventions of appendix B, this algebra reads:

\[
\begin{align*}
\{Q_\alpha^i, Q_\beta^j\} &= \epsilon^{ij}(\gamma^a)_{\alpha\beta} P_a , \\
\{Q_\alpha^i, S_\beta^j\} &= \delta^i_\alpha \delta^j_\beta Z , \\
\{S_\alpha^i, S_\beta^j\} &= \epsilon^{ij}(\gamma^a)_{\alpha\beta} P_a .
\end{align*}
\( \text{(3.2)} \)

Here \( \epsilon^{ij} \) is the invariant tensor of the SU(2) automorphism group. From a 6-dimensional point of view, this is an \( N = (1, 1) \) algebra with a central charge \( Z \), the momentum in the seventh direction. We now consider the partial breaking of this \( N = (1, 1) \) algebra down to \( N = (1, 0) \). Let \( Q_\alpha^i \) be the unbroken \( N = 1 \) supersymmetry generator and \( S_\alpha^i \) its broken counterpart. A parametrisation of the \( N = 1 \) target superspace \( \mathcal{M}^{(7|16)} \) suitable for our problem is

\[
\Omega = \exp[i(x^a P_a + \theta^i_\alpha Q_\alpha^i)] \exp[i(y Z + \psi^i_\alpha S_\alpha^i)] .
\( \text{(3.3)} \)

Now the spinor \( \psi^i_\alpha = \psi^i_\alpha(x, \theta) \) is the Goldstone superfield associated with the broken generator \( S_\alpha^i \), and the scalar \( y = y(x, \theta) \) is the Goldstone superfield associated with the central charge \( Z \). Here we have employed the "static gauge" for the splitting of target space coordinates:

\[
\begin{align*}
X^m &= x^m , \\
\Theta^\mu &= \theta^i_\alpha , \\
X^6 &= y(x, \theta) , \\
\Theta^\mu' &= \psi^i_\alpha(x, \theta) .
\end{align*}
\( \text{(3.4)} \)

Note that this construction naturally corresponds to the embedding \( \mathcal{M}^{(6|8)} \hookrightarrow \mathcal{M}^{(7|16)} \), where the Goldstone fields are bosonic and fermionic coordinates describing the shape of the supersurface \( \mathcal{M}^{(6|8)} \), which automatically breaks half of the supersymmetry.

The \( S \)-supersymmetry acts with \( g = \exp(i \eta S) \) on \( \Omega \) by left multiplication, \( g \Omega = \Omega' \), which induces a transformation on the bosonic coordinates

\[
\delta_\eta x^a = -\frac{i}{2} \gamma^a \psi .
\( \text{(3.5)} \)
This in turn makes the transformations of the Goldstone fields contain non-linear terms, in addition to the usual shifts:
\[
\delta_\eta \psi^i_\alpha = \eta^i_\alpha + \frac{i}{2} \eta \bar{\gamma}^a \psi \partial_a \psi^i_\alpha , \\
\delta_\eta y = -\frac{i}{2} \eta \theta + \frac{i}{2} \eta \bar{\gamma}^a \psi \partial_a y .
\] (3.6)

Since the Cartan 1-form \( \Omega^{-1} d\Omega \) takes its value in the supersymmetry algebra, we can parametrise it in the following way
\[
\Omega^{-1} d\Omega = i [E^a P_a + E^6 Z + E^i_\alpha Q^i_\alpha + E^i_\alpha S^a_\alpha] .
\] (3.7)

This expansion gives the covariant world-volume Goldstone 1-forms:
\[
E^a = dx^a - \frac{i}{2} [d\theta \bar{\gamma}^a \theta + d\psi \bar{\gamma}^a \psi] , \\
E^6 = dy - \frac{i}{2} [d\theta \psi + d\psi \theta] , \\
E^i_\alpha = d\theta^i_\alpha , \\
E^a_\alpha = d\psi^i_\alpha .
\] (3.8)

Here we use the notation \( \bar{\gamma}^a := \epsilon^{ij} (\gamma^a)_{\alpha\beta} \) and \( \bar{\gamma}^a := \epsilon_{ij} (\gamma^a)^{\alpha\beta} \). The vielbein matrix \( E_M^A \) is found from the expansion of the world-volume 1-form \( E^a = (E^a, E^6) \) with respect to the coordinate differential \( dz^M = (dz^m, d\theta^i_\alpha) \) of the world-volume, \( E^A = dz^M E_M^A \). The \( N = 2 \) derivatives induced by the Goldstone superfields are then given by†
\[
\nabla_A = (E^{-1})_A^M \partial_M .
\] (3.9)

These covariant derivatives can be explicitly written as:
\[
\nabla_a = (E^{-1})_a^m \partial_m , \\
\nabla^i_\alpha = D^i_\alpha + \frac{i}{2} (D^i_\alpha \psi) \bar{\gamma}^a \psi \nabla_a .
\] (3.10)

It is interesting to note that the covariant derivative \( \nabla_a \) satisfies the implicit relation
\[
\nabla_a = D_a + \frac{i}{2} (D_a \psi) \bar{\gamma}^a \psi \nabla_a ,
\] (3.11)

which simply follows from solving for \( \partial_m \) above. This expression then most easily gives the expression for \( \nabla^i_\alpha \) above, which otherwise, when directly solved for as the dual of (3.8), is expressed in terms of the bare derivatives \( \partial_\alpha \). Here \( D^i_\alpha \) and \( D_a \) are the ordinary flat \( N = 1 \) covariant derivatives.

It is then straightforward to calculate the algebra of the \( N = 2 \) covariant derivatives [6]:
\[
[\nabla_a, \nabla_b] = -i (\nabla_a \psi) \bar{\gamma}^c (\nabla_b \psi) \nabla_c , \\
[\nabla_a, \nabla^i_\alpha] = i (\nabla^i_\alpha \psi) \bar{\gamma}^b (\nabla_a \psi) \nabla_b , \\
\{ \nabla^i_\alpha, \nabla^j_\beta \} = i \epsilon^{ij} \gamma^a_{\alpha\beta} \nabla_a + i (\nabla^i_\alpha \psi) \bar{\gamma}^a (\nabla^j_\beta \psi) \nabla_a ,
\] (3.12)

† These induced covariant derivatives, denoted \( \nabla \) in the present paper (see appendix A) equal those denoted \( \mathscr{D} \) in ref. [6].
in accordance with eq. (2.45).

It is convenient to introduce the scalar superfield

$$\Phi := \frac{1}{2} \theta \psi - iy \ ,$$  \hspace{1cm} (3.13)

which, in particular, implies:

$$E^6 = i d \Phi - i d \theta \psi \ .$$  \hspace{1cm} (3.14)

This shift, anticipated in section 2, is necessary in order to obtain a scalar superfield under the 6-dimensional supersymmetry algebra. Note that at this stage there is no relation between the Goldstone fields. We now impose the irreducibility condition

$$F^{i6} = 0 \ ,$$  \hspace{1cm} (3.15)

or equivalently

$$\psi^i_\alpha = \nabla^i_\alpha \Phi \ .$$  \hspace{1cm} (3.16)

In the next section we will see that this constraint is inherent in the embedding formalism, where it is part of the embedding condition $E^i_\alpha = 0$. In the present treatment its remaining components $E^a_\alpha$ vanish trivially.

The on-shell self-dual tensor multiplet in 6 dimensions is given by

$$(1, 0) \oplus 2(\frac{1}{2}, 0) \oplus (0, 0) \leftrightarrow A^{ia}_a \oplus \psi^i_\alpha \oplus \phi \ ,$$  \hspace{1cm} (3.17)

where $\psi^i_\alpha$ and $\phi$ are the leading components of the spinor Goldstone superfield and the shifted scalar superfield, respectively, and where we have used the standard labeling of the massless particles by the helicity states of the little group $\text{Spin}(4) \approx \text{SU}(2) \times \text{SU}(2)$. The minimal $N = (1, 0)$ supersymmetry in 6 dimensions does indeed admit this tensor multiplet [14]. Here $A$ is a 2-form potential coming from the symmetric bispinor superfield [15]

$$F_{\alpha\beta} := \frac{1}{2} \nabla_{(\alpha i} \nabla_{\beta)} \Phi := \nabla_{\alpha\beta} \Phi \ ,$$  \hspace{1cm} (3.18)

which corresponds to a self-dual field strength $F_{\alpha\beta} = \frac{1}{6} (\Gamma^{abc})_{\alpha\beta} F_{abc}$. It has been suggested [6] that there might be an extension of the $N = 2$ supersymmetry which associates a Goldstone-like symmetry with this field and the tensor gauge field might itself be a Goldstone field.
To describe the on-shell self-dual tensor multiplet, the superfield $\Phi$ has to be further constrained. This constraint is most easily expressed in terms of the $N = 2$ covariant derivatives. An appropriate constraint can be found from the decomposition

$$\nabla_a \nabla_\beta \equiv -\frac{1}{2} T^{ij}_{a\beta} \nabla_a + \epsilon^{ij} \nabla_{a\beta} + \nabla^{(i}_{[a} \nabla^{j]}_{\beta]} . \tag{3.19}$$

Let us first consider the linear case. This decomposition then reads

$$D^i_a D^j_{\beta} \equiv \frac{i}{2} \epsilon^{ij}(\gamma^a)_{a\beta} \partial_a + \epsilon^{ij} D_{a\beta} + D^{[i}_{(a} D^{j]}_{\beta]} , \tag{3.20}$$

since the representation $(10,3)$ vanishes, $D^{(i}_{(a} D^{j]}_{\beta)} \equiv 0$. It is easily shown [15] that the constraint

$$D^{(i}_{[a} D^{j]}_{\beta]} \Phi = 0 \tag{3.21}$$

postulating the absence of fields in the representation $(6,3)$, describes the on-shell self-dual tensor multiplet.

Turning to the full non-linear case again, we make the assumption, later to be verified, that the constraint generalises as

$$\nabla^{(i}_{[a} \nabla^{j]}_{\beta]} \Phi = 0 . \tag{3.22}$$

The world-volume torsion is given by the implicit equation

$$\{\nabla_a, \nabla_\beta\} = -T^{ij}_{a\beta} \nabla_a = i \epsilon^{ij}(\gamma^a)_{a\beta} \nabla_a + i(\nabla_i \psi)\gamma^{a}_{a} (\nabla_j \psi) \nabla_a . \tag{3.23}$$

Note that this is a highly non-linear equation, since the fact that $\psi^i_a = \nabla_i \Phi$ implies that also the right hand side contains torsion. We now proceed to give an explicit expression for this component of the induced torsion on-shell. Using the constraint above and acting on the scalar superfield $\Phi$, we get the torsion equation on the form

$$2T^{ij}_{a\beta} = \gamma^{ij}_{a\beta} + (T^{ik}_{a\gamma} + \epsilon^{ik} F_{a\gamma}) \gamma^{j}_{k\delta} (T^{j}_{\beta\delta} + \epsilon^{jl} F_{\beta\delta}) , \tag{3.24}$$

where $T^{ij}_{a\beta} := -\frac{1}{2} T^{ij}_{a\beta} \nabla_a \Phi$ and $\gamma^{ij}_{a\beta} := i \epsilon^{ij}(\gamma^a)_{a\beta} \nabla_a \Phi$. The crucial point is that the totally symmetric representation $(10,3)$ drops out of the torsion after the on-shell constraint is imposed, and therefore

$$T^{ij}_{a\beta} = \epsilon^{ij} T^{a}_{a\beta} . \tag{3.25}$$

* We label the irreducible parts of the decomposition as $(4,2) \oplus (6,1) \oplus (10,1) \oplus (6,3) \oplus (10,3)$, reflecting the group structure $\text{Spin}(1,5) \times \text{SU}(2)$. 
The torsion equation can then be written as the matrix equation

\[ 2T = \gamma + (T + F)\gamma(F - T), \quad (3.26) \]

by extracting an overall \( \epsilon^{ij} \). It is convenient to introduce a matrix \( A \) such that \( A^2 := \gamma \). Then let \( B := AFA \) and \( X := ATA \). The torsion equation now reads

\[ 2X = A^4 + (X + B)(B - X), \quad (3.27) \]

with the solution

\[ X = -1 + \sqrt{(1 + A^4 + B^2)}. \quad (3.28) \]

Note that \( A^4 = (\nabla \Phi)^2 \). In the weak-field expansion we get

\[ X = \sum_{n=1}^{\infty} \left( \frac{1}{n} \right) A^{4n} + \sum_{n=1}^{\infty} \left( \frac{1}{n} \right) (1 + A^4)^n - 2n B^{2n}. \quad (3.29) \]

The torsion is then explicitly given by

\[ T = \frac{1}{2} \gamma + \sum_{n=1}^{\infty} \left( \frac{1}{n + 1} \right) (\nabla \Phi)^2 \gamma + \sum_{n=1}^{\infty} \left( \frac{1}{n} \right) (1 + (\nabla \Phi)^2)^{2n-1} F(\gamma F)^{2n-1}. \quad (3.30) \]

It is essential for obtaining \( T_{\alpha \beta} \) that it is possible to extract a factor \( \nabla \Phi \). To check that the supersymmetry algebra closes on the self-dual tensor multiplet it is sufficient to calculate \( \nabla_\alpha \nabla_\beta \nabla_\gamma \Phi \). This check is cumbersome due to the fact that \( T_{\alpha \beta} \) is given by a linear equation which in turn depends on \( T_{\alpha \beta} \). From the solution (3.30), we see that the supersymmetry transformations of the component fields will be extremely non-linear. However, no more fields are generated. Hence our \( N = 2 \) covariant constraint, eq. (3.22) is correct, and puts the theory on-shell.

We conclude that the \( N = (1,0) \) self-dual tensor multiplet in 6 dimensions can indeed be given an interpretation as a Goldstone multiplet for the chirally broken \( N = (1,1) \) (or, actually 7-dimensional) supersymmetry, which is natural from a brane viewpoint. Since there is no lagrangian formulation of the theory (without the introduction of auxiliary fields [16], which however do not seem to have any natural interpretation in the present framework), the program pursued for e.g. the Maxwell multiplet in ref. [6], where a lagrangian formulation was derived, has no counterpart for this supermultiplet. The constraint (3.22), which is the most naive covariantisation of the irreducibility constraint of the linear theory, turns out to be consistent, and encodes the full non-linear field equations. Due to the complicated nature of the torsion, given by an implicit relation
(3.24) solved as (3.30), the derivation of the field equations for the component fields becomes cumbersome, and will not be performed here. We note that the explicit form of the torsion may be summarised as a formal square root, an observation that probably is connected to the relation with Born–Infeld theory.

4. Non-linear realisations in the embedding formalism

In this section we review some of the salient features of the embedding formalism, as applied to the superembedding of the 5-brane in \( D = 7 \). The “embedding formalism” \([9]\) or the “doubly supersymmetric approach” \([7,8]\) to describe \( p \)-brane dynamics\(^\dagger\) are based on a geometrical condition specifying the superembedding of a world-volume into target space. This condition can furthermore be obtained from a ”generalised geometrical action principle” \([7]\). The power of the formalism was demonstrated in \([4]\) for the T5-brane in 11 dimensions, where the embedding condition was postulated and supersymmetric equations of motion obtained before a complete supersymmetric action for them was constructed \([5]\).

Consider the flat target superspace \( \mathcal{M}(7\mid 16) \) locally parametrised with coordinates \( Z_M = (X^m, \Theta^\mu) \), and introduce the supersymmetric cotangent basis 1-forms in target space

\[
\Pi^m = dX^m - \tfrac{i}{2} d\Theta^m \Omega^\alpha, \\
\Xi^\mu = d\Theta^\mu.
\]

An arbitrary frame is obtained by SO(1,6) rotations

\[
E_a^m = \Pi^m u_m a, \\
E_\alpha^\mu = \Xi^\mu u_\mu \alpha.
\]

Here \( u_m a \) and \( u_\mu \alpha \) are the “Lorentz harmonics”. The embedding matrix \( \mathcal{E}_A^\Delta \) is defined as the pullback of the target space 1-form \( E_\Delta \) onto the world-volume:

\[
\mathcal{E}_A^\Delta := E_A(f^* E_\Delta) = E_A^M (\partial_M Z^a) E_M^\Delta = (\nabla_A Z^a) E_M^\Delta.
\]

Here \( \nabla_A \) is the induced covariant derivative on the world-volume. The essential ingredient of the doubly supersymmetric approach is the ”geometro-dynamical condition” \([7,8]\), or the embedding condition \([9]\)

\[
\mathcal{E}_\alpha^\Delta = 0.
\]

Geometrically, this is simply the requirement that, at any point of \( \mathcal{M} \), the odd tangent space to \( \mathcal{M} \) lies entirely within the odd tangent space to \( \mathcal{M} \). In a number of interesting cases \([9]\), the

\(^\dagger\) We do not strictly want to call these separate formalisms; rather we would like to reserve the former term for the specific procedure of extracting information about the dynamics from the torsion equation.
The integrability condition for this constraint is so strong that it reproduces all the equations of motion for the extended object. This happens e.g. for the T5-brane in $D=11$ [9, 4]. In the next section, however, we show that the embedding condition alone is not sufficient to put the $D=7$ 5-brane multiplet on-shell. It has to be augmented by a suitable constraint, as conjectured in ref. [4].

The embedding matrices can be read off from the induced vielbeins on the world volume. Expressed in terms of the Goldstone fields, they are, as mentioned in section 2:

$$
\delta_{\alpha} = \delta_{\alpha}^{\alpha} + i(\nabla_{\alpha} \Phi)\delta_{\alpha}^{6},
$$

$$
\delta_{\alpha}^{\alpha} = \delta_{\alpha}^{\alpha} + (\nabla_{\alpha} \Theta^{\alpha})\delta_{\alpha}^{\alpha},
$$

and

$$
\delta_{\alpha}^{\alpha} = (\nabla_{\alpha} \Theta^{\alpha})\delta_{\alpha}^{\alpha}.
$$

The embedding condition reads explicitly

$$
\delta_{\alpha} = \nabla_{\alpha} X^{\alpha} - \frac{1}{2}(\nabla_{\alpha} \Theta)\Gamma_{\alpha} = 0.
$$

In particular, $\delta_{i} = 0$ gives the $D=7$ non-linear "master constraint" of [9]:

$$
\psi_{i} = \nabla_{i} \Phi,
$$

($\psi$ being the normal spinor coordinate as in eq. (3.4)) as advertised in section 2. We know that the linearised version of the above constraint is not sufficient to put our theory on-shell. In the next section we show that this is also true at the non-linear level, without using a particular gauge, e.g. the static gauge.

Turning now to the induced world-volume torsion, it can be calculated from the integrability condition for the embedding matrix, $\nabla_{(\alpha} \delta_{\beta)} = 0$, which gives

$$
i T_{\alpha \beta} \epsilon_{\alpha} = \epsilon_{\alpha} \delta_{\alpha}^{\alpha} \epsilon_{\alpha} = (\nabla_{\alpha} \Theta)\Gamma_{\alpha} = 0.
$$

This is also known as the "twistor constraint" since $\delta_{\alpha} = \nabla_{\alpha} \Theta$ is a twistor-like bosonic superfield. The world-volume torsion is then given by the equation

$$
i T_{\alpha}^{ij} = \epsilon^{ij}(\gamma^{\alpha})_{\alpha \beta} + \epsilon_{kl}(\nabla_{\alpha} \nabla_{\gamma} \Phi)\gamma^{\alpha}(\nabla_{\beta} \nabla_{\delta} \Phi)
$$

(see eq. (2.45)), which is identical to the one obtained in the non-linear realisation formalism.
5. The equations of motion

We are now going to derive the equations of motion for the super-5-brane in 7 dimensions. As target space we will choose a flat $D = 7$ superspace, i.e., all torsion components vanish except for

$$T_{\alpha\beta} = -i(\Gamma^c)_{\alpha\beta}. \tag{5.1}$$

The intrinsic world-volume geometry is chosen to be $N = 1, d = 6$ conformal supergravity [17] and the constraints that we will need in order to obtain the equations of motion are

$$\mathcal{I}_{\alpha\beta} = -i(\Gamma^c)_{\alpha\beta} \tag{5.2}$$

and

$$\mathcal{I}_{\alpha\beta} = \mathcal{I}_{ab} = \mathcal{I}_{ab} = 0. \tag{5.3}$$

The fields occurring in the following equations are those found in the parametrisation (2.18) of the embedding matrix. We start by extracting the information hidden in (2.41) using the constraints (2.35) and (2.44). We thus obtain

(i) \[ \mathcal{G}[\alpha \beta] = \chi([\alpha \beta]) \]

(ii) \[ m_a^b K^c_{\beta} \]

(iii) \[ m_a^b K^c_{\beta} \]

(iv) \[ \mathcal{G}[\alpha \beta] \]

(v) \[ m_a^b K^c_{\beta} \]

(vi) \[ m_a^b K^c_{\beta} \]

(vii) \[ \mathcal{G}[\alpha \beta] \]

(viii) \[ \mathcal{G}[\alpha \beta] \]

(ix) \[ (\Gamma^d)_{\alpha \beta} m^c_{\gamma} = (\Gamma^c)_{\alpha \beta} + (h_{\alpha \beta} \Gamma^c) \]

(x) \[ 0 = h_{\alpha \beta} \Gamma^c \]

(xi) \[ L_{\alpha \beta} + h_{\alpha \beta} K_{\alpha \beta} \]

(xii) \[ \mathcal{G}[\alpha \beta] \gamma \]

If we go through these equations we see that (i), (ii) and (iv) contain no information for the fields but simply describe parts of the torsion in the connection. Equations (iii) and (vii) determine the remaining world-volume torsion components in terms of the fields. Equation (v) does not generate
any new fields and thus becomes an algebraic identity for the next-to-leading term in the superfield \( m_{ab} \). Two, more manifest, algebraic identities are (vi) and (xi). From (ix) and (x) we get

\[
h_{\alpha \beta}' = \frac{1}{6} (\Gamma^{abc})_{\alpha \beta} h_{abc} \tag{5.5}
\]

and

\[
m_{ab} = \delta_{ab} - 2k_{ab} \tag{5.6}
\]

where \( k_{ab} = h_{acd}h^{bcd} \). We note that putting \( T_{\alpha \beta c} = -i (\Gamma^c)_{\alpha \beta} \) implies that

\[
h^{(ij)} = 0 \tag{5.7}
\]

which is identical to the on-shell constraint imposed in the NR formalism of the previous sections.

In order to get the Dirac equation we take (xii):

\[
\varepsilon(\alpha \beta)_{\gamma'} = \frac{i}{2} (\Gamma^c)_{\alpha \beta} \chi^c_{\gamma'} \tag{5.8}
\]

and trace the three free spinor indices in different ways to extract the information. By applying \( (\Gamma_d)^{\alpha \beta} \) and \( (\Gamma^d)_{\gamma' \beta} \) plus noting that

\[
\varepsilon(\alpha \beta \gamma') = \frac{1}{2} (\Gamma^b c)_{\beta \gamma'} \varepsilon(\beta \gamma' \alpha) = -i (\Gamma^b c)_{\beta \gamma'} \varepsilon(\beta \gamma' \alpha) \tag{5.9}
\]

we get

\[
\chi^a_{\gamma'} = -\frac{1}{2} (\Gamma_a)_{\alpha \beta} \varepsilon(\alpha \beta \gamma') \tag{5.10}
\]

and

\[
i \left( (\chi^c_{\gamma'} (\Gamma^c d)_{\gamma' \alpha} + \chi^d_{\gamma' \alpha}) = \frac{1}{2} (\Gamma_a b \Gamma_d)_{\alpha \beta} \varepsilon(\beta \alpha \gamma') - \frac{1}{6} (\Gamma_{abc} \Gamma^d)_{\alpha \beta} \varepsilon(\beta \alpha \gamma') \tag{5.11}
\]

respectively. Now multiplying (5.11) by \( (\Gamma_d)_{\gamma' \alpha} \), in order to get rid of \( h_{abc} \), gives

\[
(\Gamma^c)_{\beta \gamma'} \chi^c_{\gamma'} = \frac{i}{2} (\Gamma_a b \Gamma_d)_{\alpha \beta} \varepsilon(\beta \alpha \gamma') \tag{5.12}
\]

and if we use (5.10) in (5.12) we get

\[
(\Gamma_a b \Gamma_d)_{\beta \gamma'} = 0 \tag{5.13}
\]
By comparing the two last equations we see that

\[(\Gamma^a)_{\alpha'\gamma'}\chi_a^{\gamma'} = 0, \quad (5.14)\]

which is the Dirac equation.

In order to get the scalar and tensor equations of motion we take (viii):

\[\mathcal{D}_\beta \chi_a^{\gamma'} + Z_{a\beta}^{\gamma'} = \mathcal{K}_{a\beta}^{\gamma'}, \quad (5.15)\]

where

\[Z_{a\beta}^{\gamma'} := L_{\beta a}^d \chi_d^{\gamma'} + \chi_a^{\beta\delta} K_{\beta\delta}^{\gamma} h_{\gamma'}^{\gamma'}. \quad (5.16)\]

By using (5.9) in (5.15) we get

\[\mathcal{D}_\beta \chi_a^{\gamma'} + Z_{a\beta}^{\gamma'} = (\frac{1}{6}\Gamma^{bcd} \mathcal{D}_a h_{bcd} - \frac{1}{2}\Gamma^b \Gamma_c \mathcal{K}_{ab}^{\gamma'})_{\beta}^{\gamma'}. \quad (5.17)\]

We now multiply (5.17) by \((\Gamma^{a'c'})_{\gamma'\beta}\) and use the Dirac equation, which gives us the scalar equation

\[\eta^{ab} \mathcal{K}_{ab}^{c'} = \frac{1}{4}(\Gamma^{ac'})_{\gamma'\beta} Z_{a\beta}^{\gamma'}. \quad (5.18)\]

If we instead multiply (5.17) by \((\Gamma^a \Gamma_{ef})_{\gamma'\beta}\) (and again use the Dirac equation) we get the tensor equation

\[\mathcal{D}_c h_{abc} = \frac{1}{8}(\Gamma^b \Gamma_{ab})_{\gamma'\beta} Z_{c\beta}^{\gamma'}. \quad (5.19)\]

These equations of motion are analogous to the ones derived in ref. [4], and contain non-linearities of the same kind.

6. Summary and conclusions

We have given a detailed account of the geometry involved in embeddings of supermanifolds into supermanifolds. Special emphasis is put on the distinction between the different geometric objects encountered, since confusing e.g. intrinsic and induced geometry obscures the understanding of the formalism. Two preferred parametrisations of the embedding matrix in terms of matter fields, equations (2.7) and (2.18) have been presented, aiming towards distinct formulations of the world-volume field theory, each one emphasising different properties of the theory.
The second of these, referred to as the “embedding formalism”, investigated by Howe, Sezgin and West \[9\], uses the torsion equation (2.41) together with the geometric “embedding condition” \(\varepsilon_{\alpha\beta\gamma} = 0\) in order to derive equations of motion for the fields parametrising the embedding matrix (2.18). The second formulation occurs in the theory of non-linear realisations applied to the second supersymmetry (and the broken translations), as advocated by Bagger and Galperin [6]. By using the second parametrisation (2.7) of the embedding matrix, that formalism is rederived.

We also described briefly the transformations involved in going from one parametrisation to the other. Although it was straightforward to show that these transformations exist, we did not examine them in detail. As commented on in section 2, the transformations, eq. (2.10), represent a kind of local symmetry inherent in the definition of the embedding matrix, and it may be interesting to pursue the investigation further in order to extract information from the field redefinitions. We remind that the transformations involve not only the matter fields, but also the world-volume geometry.

We see two valuable aspects of this exercise. On one hand, the equivalence of two seemingly different starting points is established, and it becomes clear why they yield the same results (e.g. Born–Infeld dynamics). On the second hand it casts some light on the embedding procedure in explaining clearly why the obtained theory is one whose non-linearities stem from the (non-linearly realised) symmetry under the target space supersymmetry generators broken by the embedding.

The two parametrisations have been applied to a concrete case, namely the 5-brane of 7-dimensional supergravity. Here it was shown (in the first of the parametrisations) that the embedding condition alone did not provide enough information to put the theory on-shell. An additional irreducibility constraint, completely analogous to the one in the linear theory, had to be imposed. While this “algebraic” consideration became transparent in the language of non-linear realisations, the torsion components here become so complicated that we find the extraction of the field equations, though in principle possible, quite non-transparent. The second of the parametrisations, on the other hand, is quite suited for finding the field equations (section 5). In this case we did not need to impose any additional constraint after the world-volume torsion was chosen to be that of conformal 6-dimensional supergravity. Since we could associate the irreducibility constraint of our second formulation with the vanishing of a specific torsion component, we conjecture that the choice of torsion in the second case was more than a conventional one, so that the irreducibility constraint is hidden in the vanishing of the \(\gamma_{\alpha\beta\epsilon}\) part of the dimension-0 torsion component.

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APPENDIX A: Notation and conventions

Since a number of different geometric objects referring to different structures are encountered in this paper, we try to summarise the notation in the following table*.

|                  | world-volume | world-volume | Extrinsic | Normal | Target space |
|------------------|--------------|--------------|-----------|--------|--------------|
|                  | Intrinsic    | Induced      |           |        |              |
| Metric           | $h$          | $g$          | $g'$      | $g$    |              |
| Vielbein         | $e_A$        | $E_A$        | $E_A'$    | $E_A$  |              |
| Connection       | $\omega_{AB}^*$ | $\Omega_{AB}^*$ | $\Omega_{AB}'^*$ | $\Omega_{AB}$ |              |
| Torsion          | $\mathcal{A}$ | $T^A$        | $T^A'$    | $T^A$  |              |
| Curvature        | $\mathcal{K}_{AB}^*$ | $R_{AB}^*$   | $R_{AB}'^*$ | $R_{AB}$ |              |
| Exterior derivative | $d$         | $d$          | $d'$      | $d$    |              |
| Canonical 1-form | $\theta$     | $\theta$     | $\theta'$ | $\theta$ |              |
| Covariant derivative | $\partial$   | $\nabla$     | $\nabla'$ | $\nabla$ |              |

APPENDIX B: Spinors in 6 and 7 dimensions

The $D=7$ $\Gamma$-matrices decompose as

$$
(\Gamma_{a})_{\alpha\beta} = 
\begin{pmatrix}
(\Gamma_{a})_{\alpha\beta} & 0 \\
0 & (\Gamma_{a})_{\alpha'\beta'}
\end{pmatrix}
$$

(B.1)

and

$$
(\Gamma_{a'}')_{\alpha\beta} = 
\begin{pmatrix}
0 & (\Gamma_{a'}')_{\alpha\beta'} \\
(\Gamma_{a'}')_{\alpha'\beta} & 0
\end{pmatrix}
$$

(B.2)

with respect to the tangential and normal directions and they satisfy

$$
(\Gamma_{a})_{\alpha\beta} = (\Gamma_{a})_{\beta\alpha}.
$$

(B.3)

To raise and lower composite indices we use

$$
C_{\alpha\beta} = C_{\alpha\beta} = 
\begin{pmatrix}
0 & 1 \\
-1 & 0
\end{pmatrix}
= 
\begin{pmatrix}
0 & \delta_{\alpha\beta'} \\
-\delta_{\alpha'\beta} & 0
\end{pmatrix},
$$

(B.4)

* Like the authors of ref. [7] we use the term “intrinsic” for the a priori defined world-volume entities, but note that there is an unfortunate disagreement on terminology. Mathematical literature may use the term for what we call “induced”.
with the convention that
\[ \psi^\alpha = C^{\alpha\beta} \psi_\beta, \]
\[ \psi_\beta = \psi^\alpha C_{\alpha\beta}. \] (B.5)

The algebra is
\[ \{ \Gamma^a, \Gamma^b \} = 2\eta^{ab}, \] (B.6)
which implies that
\[ \{ \Gamma^a, \bar{\Gamma}^b \} := \Gamma^a \bar{\Gamma}^b + \bar{\Gamma}^b \Gamma^a = -2\eta^{ab} \delta_\alpha^\beta, \]
\[ \{ \bar{\Gamma}^a, \Gamma^b \} := \bar{\Gamma}^a \Gamma^b + \Gamma^b \bar{\Gamma}^a = -2\eta^{ab} \delta_\alpha^\beta', \]
\[ \{ \Gamma^{a'}, \Gamma^{b'} \} := \Gamma^{a'} \Gamma^{b'} + \Gamma^{b'} \Gamma^{a'} = 2\eta^{a'b'} \delta_{\alpha'}^\beta', \]
\[ \{ \Gamma^{a'}, \bar{\Gamma}^b \} := \Gamma^{a'} \bar{\Gamma}^b + \bar{\Gamma}^b \Gamma^{a'} = 0. \] (B.7)

We split the 16 component indices according to
\[ \psi_\alpha \rightarrow \psi_i^\alpha, \]
\[ \psi_\alpha' \rightarrow \psi_i'^\alpha, \] (B.8)
where after the split \( \alpha \) is a Spin(1,5) index and \( i \) is a SU(2) index. For the \( \Gamma \)-matrices this implies
\[ (\Gamma^a)^\alpha_\beta = \begin{pmatrix} 0 & -\delta^{ij}(\gamma^a)_{ij} \\ 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & -\varepsilon_{ij}(\gamma^a)_{ij} \\ 0 & 0 \end{pmatrix} \] (B.9)
and
\[ (\Gamma^{a'})^\alpha_{\beta'} = \begin{pmatrix} 0 & \delta_{ij}(\gamma^a)_{ij} \\ 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & \delta_{ij}(\gamma^a)_{ij} \\ 0 & 0 \end{pmatrix}, \] (B.10)
where \( \gamma^a \) are the 6-dimensional gamma matrices [18]. They satisfy
\[ (\gamma^a)_{\alpha\beta} = -(\gamma^a)_{\beta\alpha}, \] (B.11)
\[ (\gamma_a)_{\alpha\beta}(\gamma^b)_{\alpha\beta} = -4\delta_a^b \]
and
\[ (\gamma^a)_{\alpha\beta}(\gamma_a)_{\gamma\delta} = -2\varepsilon_{\alpha\beta\gamma\delta}. \] (B.12)

Indices are raised and lowered according to
\[ \psi^i = \varepsilon^{ij} \psi_j, \]
\[ \psi_i = \psi^j \varepsilon_{ji}, \] (B.13)
and

\[ \psi^{\alpha \beta} = \frac{1}{2} \varepsilon^{\alpha \beta \gamma \delta} \psi_{\gamma \delta}, \]
\[ \psi_{\alpha \beta} = \frac{1}{2} \varepsilon_{\alpha \beta \gamma \delta} \psi^{\gamma \delta}. \]  

(B.14)

Notice that we can only raise and lower Spin(1,5) indices in pairs.

**Appendix C: Some useful relations**

In order to transform between vector and spinor indices we need the following relations, following from the lorentzian property of the \( u \) matrices:

\[
(\mathcal{D} u^{\alpha \beta}) u^{-}_{\alpha \beta} = -\frac{1}{n} (\Gamma_{\alpha}^{\beta})_{\alpha}^{\beta} (\mathcal{D} u^{\gamma \delta}) u^{-}_{\gamma \delta},
\]
\[
(\mathcal{D} u_{\alpha \beta}) u^{\gamma \beta} = \frac{2}{n} (\Gamma^{\beta}_{\alpha})^{\beta} (\mathcal{D} u_{\gamma \delta}) u^{\gamma \delta}, \]  

(C.1)

where \( n \) is the dimension of the target space spinor representation. If we take into account the split into tangential and normal indices we get

\[
(\mathcal{D} u_{\alpha \beta}) u^{\gamma \beta} = -\frac{1}{4} \left( (\Gamma_{\alpha}^{\beta})_{\alpha}^{\beta} (\mathcal{D} u^{\gamma \delta}) u^{\gamma \delta} + (\Gamma^{\gamma \delta})_{\alpha}^{\beta} (\mathcal{D} u_{\gamma \delta}) u^{\gamma \delta} \right),
\]
\[
(\mathcal{D} u_{\alpha \beta}) u^{\gamma \beta} = -\frac{1}{4} \left( (\Gamma_{\alpha}^{\beta})_{\alpha}^{\beta} (\mathcal{D} u^{\gamma \delta}) u^{\gamma \delta} + (\Gamma^{\gamma \delta})_{\alpha}^{\beta} (\mathcal{D} u_{\gamma \delta}) u^{\gamma \delta} \right),
\]
\[
(\mathcal{D} u_{\alpha \beta}) u^{\gamma \beta} = -\frac{1}{4} \left( (\Gamma_{\alpha}^{\gamma \beta})_{\alpha}^{\gamma \beta} (\mathcal{D} u^{\gamma \delta}) u^{\gamma \delta} + (\Gamma^{\gamma \delta})_{\alpha}^{\gamma \beta} (\mathcal{D} u_{\gamma \delta}) u^{\gamma \delta} \right), \]  

(C.2)

and

\[
(\mathcal{D} u_{\alpha \beta}) u^{\gamma \beta} = \frac{4}{n} (\Gamma^{\beta}_{\alpha})^{\beta} (\mathcal{D} u_{\gamma \delta}) u^{\gamma \delta} = \frac{4}{n} (\Gamma^{\gamma \delta})_{\alpha}^{\beta} (\mathcal{D} u_{\gamma \delta}) u^{\gamma \delta},
\]
\[
(\mathcal{D} u_{\alpha \beta}) u^{\gamma \beta} = \frac{4}{n} (\Gamma^{\beta}_{\alpha})^{\beta} (\mathcal{D} u_{\gamma \delta}) u^{\gamma \delta} = \frac{4}{n} (\Gamma^{\gamma \delta})_{\alpha}^{\beta} (\mathcal{D} u_{\gamma \delta}) u^{\gamma \delta}, \]  

(C.3)
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