Numerical Simulation of Fractured Reservoir with Full Coupling Matrix Finite Element Method

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Abstract. Fractured reservoir is an important part of China's oil and gas resources, which is a phenomenon that will be encountered in the exploitation of most oil and gas resources. However, there are many difficulties in the exploitation of fractured reservoirs, such as internal complexity, basic assumptions of models, fracture identification technology, computer hardware and so on. Therefore, we must consider the full coupling matrix finite element analysis, which will overcome the limitations of the traditional dual medium model and the discrete fracture network model. Through the full coupled matrix finite element analysis, we can establish a new numerical simulation of fractured reservoir. First, this paper analyzes the basic types of fractured reservoirs. Then, this paper analyzes the full coupling matrix finite element method.

Keywords: Fractured Reservoir, Numerical Simulation, Finite Element, Full Coupling

1. Introduction

With the rapid development of science and technology, there are many methods to simulate fractured reservoirs in the world, which are mainly the different decomposition of the control equation. The discrete principle can be divided into finite difference method, finite volume method and finite element method, which will be applied to different kinds of numerical simulation. Among them, the finite element method is more suitable for the simulation of complex boundary[1]. Through higher order differential equations, we can overcome the problem of weak grid orientation, which will improve the accuracy of numerical simulation. Therefore, through the equivalent medium model, the irregular grid can be discretized in this paper, which will avoid the grid tendency of regular grid[2]. Then, through the finite element method, we can simulate the fractured reservoir.

2. Basic concept of fractured reservoir

2.1. Identification of natural fractures

We need to observe the core fracture first, which will distinguish natural fracture or artificial fracture, which is the key to analyze the development of underground fracture[3]. By determining the formation process of the fracture, we can determine the opening state of the fracture. The general recognition characteristics of natural fractures are as follows First, the fracture is filled with cement, mineral
crystal and mineral film, Second, the fracture is contained in the core and does not extend to the core edge. Thirdly, the fractures occur as a series, which accords with the law of tectonic stress. Fourth, the crack has a scratch surface and steps, which will indicate that the direction of movement is consistent with the direction of regional stress. Fifth, feather marks are often developed on the fracture surface, as shown in Figure 1.

Figure 1. FMI Imaging Log of natural fracture

2.2. Classification of crack action
The spatial structure of fractured reservoir is complex and abrupt, and its classification is shown in Figure 2.

Figure 2. Classification of crack action

3. Numerical simulation of fractured reservoir with full coupling matrix finite element

3.1. Mathematical model
In this paper, the integral form of flow equation in fracture area is developed based on DFN model without considering the influence of cross flow, as shown in Formula 1.

\[
\int \frac{Fd\Omega}{\Omega} = \int \frac{Fd\Omega m + a \times Fd\Omega f}{\Omega} \tag{1}
\]

Among them, \( m \) is the matrix, \( f \) is the fracture, and \( \Omega \) is the reservoir area.

3.2. Discrete crack matrix form
In this paper, the basic assumptions of fractured reservoir are as follows: First, the reservoir is oil-water two-phase, which conforms to Darcy flow. Second, the rock and fluid are incompressible in the matrix fractures[4]. Thirdly, the fluid flow is isothermal. Fourth, ignore the influence of gravity[5-6].

Based on the above assumptions, the mathematical model of reservoir seepage in matrix can be
obtained in this paper, as shown in formula 2.

\[
\frac{\partial (\rho_s \phi S_s)}{\partial t} + \nabla (\rho_s \phi v_s) = q_s, \quad v_s = -\frac{k_k}{\mu_s} \nabla p_s, S_s = 1, p_s = p_s - p_s
\]

(2)

Therefore, we can get the pressure equation of the fluid in the matrix, as shown in formula 3.

\[
\nabla (\lambda_s \nabla p_s) - \nabla (\lambda_s + \lambda_s) \nabla p_s = q_s + q_s
\]

(3)

Therefore, the saturation equation is shown in equation 4.

\[
\phi \frac{\partial S_s}{\partial t} = q_s
\]

(4)

Therefore, we can get the matrix form of fluid flow in the matrix, as shown in formula 5.

\[
\begin{bmatrix} 0 & 0 \\ 0 & \phi \end{bmatrix} \frac{\partial}{\partial t} \begin{bmatrix} p_s \\ S_s \end{bmatrix} + \nabla \left[-(\lambda_s + \lambda_s) \ nabla p_s \right] = \begin{bmatrix} q_s + q_s \\ 0 \end{bmatrix}
\]

\[
\lambda_s = \frac{k_k}{\mu_s}
\]

(5)

The direction of fluid flow is mainly controlled by large fractures. When there are large cracks, we consider the large cracks, which is mainly because the capillary force of small cracks can be ignored. Therefore, the pressure equation of the fluid in the fracture is shown in formula 6.

\[
-\nabla (\lambda' + \lambda') \nabla p_s = q_s + q_s
\]

(6)

The saturation equation is shown in equation 7.

\[
\phi \frac{\partial S_s}{\partial t} = q_s
\]

(7)

The full coupling equation in the fracture is shown in equation 8.

\[
\begin{bmatrix} 0 & 0 \\ 0 & \phi' \end{bmatrix} \frac{\partial}{\partial t} \begin{bmatrix} p_s' \\ S_s' \end{bmatrix} + \nabla \left[-(\lambda' + \lambda') \ nabla p_s' \right] = \begin{bmatrix} q_s' + q_s' \\ 0 \end{bmatrix}
\]

(8)

The initial conditions are shown in equation 9. The boundary condition is Neuman boundary condition, as shown in formula 10.

\[
\begin{cases} p_s \big|_a = p_s (0) \\ S_s \big|_a = S_s (0) \\ v_s \times n = 0, \nabla S_s \times n = 0 \end{cases}
\]

(9)

(10)

3.3. Finite element solution

According to the principle of virtual displacement, we can change the differential equation into equation 11.

\[
\int_{\Omega} \nabla (\lambda_s \nabla p_s) \delta p_s \, d\Omega - \int_{\Omega} \nabla (\lambda_s + \lambda_s) \nabla p_s \delta p_s \, d\Omega = \int_{\Omega} (q_s + q_s) \delta p_s \, d\Omega
\]

\[
\int_{\Omega} \phi \frac{\partial S_s}{\partial t} \delta S_s \, d\Omega - \int_{\Omega} \nabla (\lambda_s \nabla p_s) \delta S_s \, d\Omega = \int_{\Omega} q_s \delta S_s \, d\Omega
\]

(11)

According to Green's theory, under the boundary condition, we can get formula 12. According to the Garlerkin method, we can get formula 13.

\[
\int_{\Omega} (\lambda_s \nabla p_s) \nabla (\delta p_s) \, d\Omega - \int_{\Omega} (\lambda_s + \lambda_s) \nabla p_s \nabla (\delta p_s) \, d\Omega = \int_{\Omega} (q_s + q_s) \delta p_s \, d\Omega
\]

\[
\int_{\Omega} \phi \frac{\partial S_s}{\partial t} \delta S_s \, d\Omega - \int_{\Omega} (\lambda_s \nabla p_s) \delta S_s \, d\Omega = \int_{\Omega} q_s \delta S_s \, d\Omega
\]

(12)
\[ p'_o = \sum_{j=1}^{N} N_j p'_o = N p'_o , S'_o = \sum_{j=1}^{N} N_j S'_o = N S'_o , \]

Among them, \( M \) is the number of unit nodes, \( p'_o \) is the tentative solution of pressure, \( m \) is the number of unit nodes, \( N \)-unit shape function, \( p'_o \) is the value of oil phase flow potential of unit nodes, and \( S'_o \) is the value of water phase saturation of unit nodes. We can get the solution form of matrix discrete matrix, as shown in formula 14.

\[
\begin{bmatrix}
0 & 0 \\
A_{22} & \begin{bmatrix}
B_{11} & B_{12} \\
B_{21} & 0
\end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
p_o' \\
S'_o
\end{bmatrix}
=
\begin{bmatrix}
Q_1 \\
Q_2
\end{bmatrix}
\]

Among them,

\[ A_{22} = \int_{\Omega} N^T \phi N d\Omega B_{11} + \int_{\Omega} \int_{\Omega} \int_{\Omega} N^T \phi N d\Omega d\Omega B_{12} = \int_{\Omega} \int_{\Omega} \int_{\Omega} \int_{\Omega} \phi N d\Omega d\Omega d\Omega d\Omega \]

We do not consider the effect of capillary forces in cracks. Therefore, the saturation pressure at the interface of fracture and matrix is continuous, as shown in formula 15.

\[
\begin{bmatrix}
0 & 0 \\
C_{22} & \begin{bmatrix}
D_{11} & D_{12} \\
D_{21} & 0
\end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
p_o' \\
S'_o
\end{bmatrix}
=
\begin{bmatrix}
Q_1 \\
Q_2
\end{bmatrix}
\]

Among them,

\[ C_{22} = \int_{\Omega} N^T \phi N d\Omega D_{11} \]

The matrix and fracture equation are assembled as a whole matrix, and the time is solved by backward difference. We can get the finite element solution.

4. Conclusions

Through the equivalent medium model, we can simulate the authenticity and continuity of reservoir fracture. The traditional numerical simulation method can’t solve the numerical simulation problem of discrete fractured reservoir, which is the factor that the direction of oil-water flow is difficult to control. However, the finite element method needs a lot of calculation, which requires the computer to have a strong processing ability.

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