Comment on Multigraviton Scattering in the Matrix Model

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Abstract

We show by explicit calculation that the matrix model effective action does not contain the term $v_{12}^2 v_{23}^2 v_{13}^2 / R^7 r^7$, in the limit $R \gg r$, contradicting a result reported recently.
1 Introduction

The conjectures of [1] and [2] along with the arguments provided by [3, 4], and numerous other pieces of evidence (for reviews, see [5] and references therein) give one reason to believe that finite $N$ matrix theory describes the discrete light-cone quantization (DLCQ) of M-theory with DLCQ supergravity as its low energy limit. Although there are still many open questions in the matrix formulation of M-theory, we would like to focus on whether the three graviton scattering calculation of [6] shows a discrepancy between the matrix model and supergravity.

It has recently been reported in [7] that supergravity and the matrix model do not disagree on multi-graviton scattering. In this note we will show that the term computed in [7] does indeed have a supersymmetric cancellation and that the matrix model effective action does not contain a term of the form $v_{12}^2 v_{23}^2 v_{13}^2 / R^7 r^7$.

It is worthwhile to review the problems which arise when one tries to compare three graviton scattering in the matrix model picture with supergravity, setting the stage for our notation which will be used in this note. Briefly, the authors of [6] considered the case of three gravitons; two separated a distance $r$ from each other and another a distance $R$ from the other two in the limit $R \gg r$. A term in the supergravity $S$-matrix for three graviton scattering in the small momentum transfer limit was shown to be

$$\frac{(k_1 \cdot k_2)(k_1 \cdot k_3)(k_2 \cdot k_3)}{q_1^2 q_2^2}$$

where $k_i$ are the $i$th graviton momenta and $q_{1,2}$ are the two relevant momenta transfer. In the language of matrix theory, this corresponds to taking the Fourier transform of the two-loop effective potential

$$\frac{v_{12}^2 v_{13}^2 v_{23}^2}{R^7 r^7}$$

where $v_{12} = (v_1 - v_2)$, etc. refer to the relative velocities of the $D0$-branes. The two scales $R$ and $r$ arise from integrating out the massive degrees of freedom introduced by giving the diagonal generators of $SU(3)$ vacuum expectation values:

$$< X_i^a > = r \delta^{a3} \delta_{i1} + R \delta^{a8} \delta_{i2}$$

where $X_i^a$ are the 9 $SU(3)$-valued fields describing the bosonic coordinates. Since $\hat{X}_i = \hat{X}_i^8 T^8 + \hat{X}_i^3 T^3$, one can work out $v_{12}^2$, etc. in terms of $\hat{X}_i^3$ and $\hat{X}_i^8$:

$$v_{23}^2 \sim (\hat{X}_i^3)^2$$

$$v_{13}^2 \sim (\hat{X}_i^3)^2 + (3\hat{X}_i^8)^2 - 6\hat{X}_i^8 \hat{X}_i^3$$

$$v_{12}^2 \sim (\hat{X}_i^3)^2 + (3\hat{X}_i^8)^2 + 6\hat{X}_i^8 \hat{X}_i^3$$

Multiplying these three together yields the expected result for matrix theory

$$v_{12}^2 v_{13}^2 v_{23}^2 \sim (\hat{X}_i^3)^2 (\hat{X}_i^8)^4 + (\hat{X}_i^3)^6 + (\hat{X}_i^3)^4 (\hat{X}_i^8)^2 - (\hat{X}_i^3 \hat{X}_i^8)^2 (\hat{X}_i^3)^2.$$  

\footnote{See note added for the resolution to this discrepancy.}
In [6] it was argued that matrix theory was incapable of reproducing the term,

\[
\frac{(\dot{X}^8)^4(\dot{X}^3)^2}{R^7 r^7}
\]

with the correct powers of \( R \) and \( r \) at two-loops. In [7], it was argued that this term can arise at two-loops from vertices with three massive bosons in the form of the setting-sun diagram, as well as from other two-loop interactions. After describing the background field method used in this note, we go on to show that the one-loop effective operator needed to arrive at the conclusion of [7] does indeed cancel among bosons and fermions. By exploiting the fact that \( \dot{X}_i^8 \) only couples to fields of scale \( R \), we integrate out these most massive modes to find that the first term containing coupling between the heavy and light states without supersymmetric cancellations has the form \( (\dot{X}_i^8)^4(X^a)^2/R^9 \) as described in [6]. Then integrating over the light \( SU(2) \) modes of scale \( r (a=1,2) \), we demonstrate that the term in the matrix model effective action with four powers of \( \dot{X}_i^8 \) and the least suppression in \( R \) is \( (\dot{X}_i^8)^4(X^a)^2/R^9 r^5 \).

2 Contributions to the low energy effective action

The matrix model Lagrangian is obtained from the dimensional reduction of \( N = 1 \) supersymmetric Yang-Mills theory in \( D = 9 + 1 \) down to \( D = 0 + 1 \) dimensions [1]. For our purposes it will be useful to initially keep the action in its ten dimensional form expressed as

\[
S = \int d^{10}x \left( \frac{1}{4g} F_{\mu\nu}^a F^{\mu\nu a} + i \frac{1}{2} \bar{\Psi}^a \Gamma^\mu D_\mu \Psi^a \right)
\]

where the field strength is given by

\[
F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc} A_\mu^b A_\nu^c,
\]

and the 32 \( \times \) 32 dimensional Dirac matrices \( \Gamma \) satisfy the usual algebra \( \{\Gamma_\mu, \Gamma_\nu\} = 2g_{\mu\nu} \) with metric \( g_{\mu\nu} = \text{diag}(+1, -1, ..., -1) \). The 32 component Majorana-Weyl adjoint spinor \( \Psi^a \) has only 16 real physical components off mass shell. We should mention that the center of mass motion of the \( D0 \) particles has been removed and we will be considering the \( SU(3) \) theory with the gauge index \( a=1-8 \).

To calculate the one loop contributions to the effective action, we will use the background field method [8] and break the gauge field up into a classical background field and a fluctuating quantum field,

\[
A_\mu^a \rightarrow X_\mu^a + A_\mu^a,
\]

and choose our gauge fixing condition, \( D^\mu A_\mu^a = 0 \), to be covariant with respect to the background field, \( D_\mu = \partial_\mu - it^a X_\mu^a \). By only keeping terms quadratic in the quantum fields, one obtains the gauge-fixed Lagrangian in the Feynman-’t Hooft gauge:

\[
\mathcal{L} = \mathcal{L}_B + \mathcal{L}_A' + \mathcal{L}_\psi + \mathcal{L}_c.
\]
The first piece of the Lagrangian just contains the background gauge field,

\[ \mathcal{L}_B = -\frac{1}{4g} F_{\mu\nu}^a F^{\mu\nu a} \]  

(13)

whereas the other pieces are quadratic in their respective quantum fields and contain the background gauge field in the background covariant derivative squared, \( D^2 \), as well as in the background field strength \( F_{\rho\sigma}^b \):

\[ \mathcal{L}_{A'} = -\frac{1}{2g} \left\{ A'_\mu \left[ -\left( D^2 \right)^{ac} g^{\mu\nu} + \left( F_{\rho\sigma}^b J^{\rho\sigma} \right)^{ac} \delta^{\nu}_b \right] A'_\nu \right\} \]  

(14)

\[ \mathcal{L}_c = \bar{\psi} \left[ -\left( D^2 \right)^{ab} \right] c^b \]  

(15)

where

\[ (J^{\rho\sigma})_{\alpha\beta} = i \left( \delta_\rho^\alpha \delta_\sigma^\beta - \delta_\rho^\beta \delta_\sigma^\alpha \right) \]  

(17)

\[ S^{\mu\nu} = \frac{i}{4} [\Gamma^\mu, \Gamma^\nu]. \]  

(18)

The one loop effective action is obtained by evaluating the functional integral for the quantum fields,

\[ e^{i\Gamma[X]} = \int DA' D\bar{\psi} D\psi Dc \exp \left\{ i \int d^{10}x \left( \mathcal{L}_B + \mathcal{L}_{A'} + \mathcal{L}_\psi + \mathcal{L}_c \right) \right\}, \]  

(19)

giving

\[ \Gamma[X] = \int d^{10}x \left( -\frac{1}{4g} F_{\mu\nu}^a F^{\mu\nu a} \right) + \frac{i}{2} \ln Det \left[ -\left( D^2 \right) g^{\mu\nu} + \left( F_{\rho\sigma}^b J^{\rho\sigma} \right)^{\mu\nu} t^b \right] \]

\[ -\frac{i}{8} \ln Det \left[ -\left( D^2 \right) + \left( F_{\rho\sigma}^b S^{\rho\sigma} \right) t^b \right] - i \ln Det \left[ -(D^2) \right]. \]  

(20)

For the fermion functional integration the extra factor \( \frac{1}{4} \) arises from the fermion field having 16 real components instead of 32 complex ones.

To compute the determinants for the different fields, it is useful to expand \( D^2 \),

\[- D^2 = -\partial^2 + \triangle_1 + \triangle_2 \]  

(21)

where

\[ \triangle_1 = it^a \left( \partial_\mu X^{\mu a} + X^{\mu a}_\mu \right) \]  

(22)

\[ \triangle_2 = X^{\mu a}_\mu X^{\nu b}_\mu t^b. \]  

(23)

At this point it is convenient to dimensionally reduce to 1-D while choosing \( X_0^a = 0 \), so \( \triangle_1 = 0 \).

By letting \( X^{\mu}_\mu \rightarrow r \delta_3^\delta \delta^1_\mu \) and \( r \delta_8^\delta \delta^2_\mu \), we can break \( SU(3) \rightarrow U(1) \times U(1) \) giving

\[ \triangle_2 = -r^2 t^3 - 2rt^a t^3 - R^2 t^8 - 2RX^a t^a X^{ib} t^b \]  

(24)
with the Latin index going 1–9 and fields \(X_i^a\) depending only on time. It is important to note that in 1-D, \(r\) and \(R\) are dynamical variables and we are holding them fixed in the spirit of doing a Born-Oppenheimer approximation. The magnetic moment interaction for the bosons

\[
\Delta B^j = \left( F^b_{\rho\sigma} J^{\rho\sigma} \right)^{\mu\nu} t^b
\]

dimensionally reduced becomes

\[
\Delta B^j = 2 \left( \partial_0 X_i^b X^{0i} \right)^{\mu\nu} t^b
\]

since we will be working in a flat direction. Similarly for the fermions one has

\[
\Delta \psi^j = 2 \left( \partial_0 X_i^b S^{0i} \right) t^b
\]

The general form of a determinant in (20) can be written

\[
Tr \ln(-\partial_0^2 + \Delta_2 + \Delta_J)
\]

Because we are interested in the limit \(R \gg r\) and will be letting only the most massive modes (scale \(R\)) run in the loop (gauge index \(a=4-7\)) then \(t^3 t^3 r^2 = \frac{1}{4} r^2\) and \(t^8 t^8 R^2 = \frac{3}{4} R^2\). It is convenient to rescale, \(r \to 2r\), \(R \to \frac{2}{\sqrt{3}} R\) and define

\[
\Delta_F = \frac{1}{-\partial_0^2 - R^2 - r^2}
\]

in addition to

\[
\Delta_r = -4 t^3 t^i X^a_i t^a
\]

\[
\Delta_R = -\frac{4}{\sqrt{3}} R t^a X^a_i t^i
\]

\[
\Delta'_2 = -X_i^a t^a X^{ib} t^b
\]

then the trace becomes

\[
Tr \ln(-\partial_0^2 - R^2 - r^2) + Tr \ln[1 + \Delta_F (\Delta'_2 + \Delta_r + \Delta_R + \Delta_J)].
\]

The first piece involving \(-\partial_0^2 - R^2 - r^2\) is a constant and the second contains the one loop quantum corrections to the effective action which we will evaluate below by expanding the logarithm for various numbers of external background fields. We will find that the first non-zero terms contain four derivatives even if one just integrates over the most massive modes, \(R\).

### 2.1 Terms with no derivatives

We will display in this section a supersymmetric cancellation between bosons and fermions for all operators which can be constructed from \(-D^2\). Even before considering the expansion of
$-D^2$ in (21), it is straightforward to see that all terms in the one loop effective action with no derivatives cancel. This is because the determinants of the bosons and fermions differ only by derivative terms, and there are an equal number of bosonic and fermionic factors in the determinant. Given that a non-derivative operator is particularly important in the analysis of [7], we show explicitly in this section how non-derivative operators are cancelled.

The operator in question has the form

$$\delta L_{eff}^{(1)} = \frac{r^2}{R^8} x_1^b x_1^b$$

(34)

where the gauge index, $b=1-2$, for the small mass $SU(2)$ subgroup (scale $r$). Such a term arises from expanding the logarithm in (33) and is given by

$$- \frac{1}{2} Tr [\Delta F \Delta r \Delta F \Delta r]$$

(35)

or in frequency space

$$-8 Tr [t^3 t^a t^3 t^b] r^2 \int \frac{dw_1}{2\pi} x_1^a (w_1) x_1^b (-w_1) \int \frac{dw}{2\pi} \frac{1}{w^2 - R^2} \frac{1}{(w - w_1)^2 - R^2}$$

(36)

where we have dropped $r^2$ in $\Delta F$ for the leading $1/R$ behavior. Integrating (36) in the limit $w_1 \to 0$ and then Fourier transforming gives (34). Now the important point to notice is that $\Delta r$ arises from $-D^2$ which occurs in each determinant for the gauge, fermion and ghost fields (20). However, they each give a different contribution to $Tr [t^3 t^a t^3 t^b] \sim \delta^{ab} d(j)$, where $d(j)$ is the number of components for the various fields

$$d(j)^\psi = 32 \quad d(j)^{A'} = 10 \quad d(j)^c = 1.$$  (37)

Now it becomes clear that all terms coming from $-D^2$ in each of the three determinants appearing in (20) will cancel. To be explicit one gets

$$[\frac{i}{2}(10) - \frac{i}{8}(32) - i(1)][\frac{r^2}{R^8} x_1^b x_1^b] = 0.$$  (38)

A similar result holds for any number of external fields without derivatives involving $\Delta_r, \Delta_R, \Delta'_{2}$.

### 2.2 Cancellation of $(F^a_{0i})^2$ or $(\dot{X}^a_i)^2$

In this section, we show that terms with two derivatives cancel as well. This result is familiar in higher dimensions, where it is well known that the kinetic terms of the fields are not renormalized.

Based on the arguments given above the only possible non-vanishing term with two external fields contains two derivatives and is given by

$$-\frac{1}{2} Tr [\Delta F \Delta J \Delta F \Delta J].$$

(39)
The supersymmetric cancellation of (39) between bosons and fermions requires the determination of $\text{Tr}[S^0_i S^0_j] = 8g^{ij}$ for fermions and $\text{Tr}[J^0_i J^0_j] = 2g^{ij}$ for bosons. Putting the term into (20) gives
\[
\frac{i}{2}(2) - \frac{i}{8}(8) - i(0)[\text{Tr}[t^a t^b] \int \frac{dw_1}{2\pi} w_1^2 X^a_i(w_1)X^b_i(-w_1) \int \frac{dw}{2\pi (w^2 - R^2)} \frac{1}{((w - w_1)^2 - R^2)} = 0
\]
which shows that the 2-point contribution to the effective action at one-loop is zero. We can also generalize this result to show that all possible non-derivative insertions on a loop with two derivatives will not give a contribution to the effective action.

2.3 $V^4/R^7$

Since all terms with two derivatives, no derivatives, or a mixture cancel by the arguments given above, the only possible non-vanishing term with four external fields is the four derivative term given by
\[
-\frac{1}{4} \text{Tr}[(\Delta_F \Delta_J)^4]
\]
or in frequency space
\[
\int \frac{dw_2 dw_3 dw_4}{(2\pi)^4} \frac{-(w_2 + w_3 + w_4)X^8_i[(-w_2 + w_3 + w_4)]w_2 X^8_j(w_2)w_3 X^8_k(w_3)w_4 X^8_l(w_4)}{[(w + w_2)^2 - R^2][(w + w_2 + w_3)^2 - R^2][(w + w_2 + w_3 + w_4)^2 - R^2][w^2 - R^2]}
\]
with the prefactor
\[
-\frac{1}{4} 2^4 \text{Tr}[(t^8)^4] \text{Tr}[(J^0)^4]
\]
for the gauge boson case. An identical result holds for fermions if one replaces the Lorentz generator trace with
\[
\text{Tr}[S^0_i S^0_j S^0_k S^0_l] = 2(g^{ij}g^{kl} - g^{ik}g^{jl} + g^{il}g^{jk})
\]
whereas for the gauge bosons one finds
\[
\text{Tr}[J^0_i J^0_j J^0_k J^0_l] = (g^{ij}g^{kl} + g^{il}g^{jk}).
\]
Now using (20) and the low energy approximation $w_1, w_2, w_3, w_4 \to 0$, we get
\[
-\frac{27i}{4} [\langle F^8_{0i} \rangle^2] \int \frac{dw}{2\pi (w^2 - R^2)^4}
\]
The integral can be performed in the complex plane using the usual $+i\epsilon$ prescription for handling the poles. Defining $(\hat{X}^8)^4 = (F^8_{0i})^4 \equiv V^4$, one is left with the result that the first non-vanishing contribution to the effective potential has four derivatives,
\[
\delta L^{(1)}_{\text{eff}} \sim \frac{V^4}{R^7},
\]
even when the gauge group experiences multiple levels of breaking.
2.4 $V^4x^2/R^9$ and $V^4v^2/R^9r^5$

Looking at possible insertions with two background fields on a massive loop with four derivatives gives terms of the form,

$$\text{Tr}[(\triangle F \triangle J)^4 \triangle J']$$

$$-\frac{5}{2}\text{Tr}[(\triangle F \triangle J)^4(\triangle F \triangle R)^2]$$

$$-\frac{5}{2}\text{Tr}[(\triangle F \triangle J)^4(\triangle F \triangle r)^2]$$

$$-5\text{Tr}[(\triangle F \triangle J)^4(\triangle F \triangle R)(\triangle F \triangle r)] \tag{51}$$

The operators in (48) and (49) lead to terms of the form $V^4x^2/R^9$ with $x$ being a light field (scale $r$) in agreement with [6], whereas the operators in (50) and (51) give terms with more powers of $R$ in the denominator. At this point in our analysis, one might worry that we have thrown out the vertices coupling three quantum fields (two of mass $R$ and one of mass $r$) with one background field which was found to be important in the result of [7]. However, by considering the $x$’s as background plus quantum fields, the effective operator $V^4x^2/R^9$ contains the sum of all non-vanishing vertices with up to four derivatives constructable from such a vertex. We can now use $V^4x^2/R^9$ in the path integral (19) and integrate over the light modes $x'$ to generate

$$\frac{V^4v^2}{R^9r^5} \tag{52}$$

where $v^2 \equiv (\dot{X}_i^3)^2$. Clearly (52) has the wrong dependence on $R$ and $r$ to reproduce the term of interest in the supergravity scattering amplitude.

3 Comment on the Eikonal approximation

When analyzing D0-brane scattering most authors (see e.g. [3, 9] and references therein) have chosen to use an explicit background given by $x = vt + b$ where $v$ is a relative velocity of the D0-branes and $b$ an impact parameter. Such an approach allows one to construct the exact propagator as a power series in $b$, $v$, and $t$. By organizing the calculation along the lines suggested by our analysis above, we can exhibit the cancellation of all $V^4v^2/R^7r^7$ contributions to the effective action. The point, again, is to take advantage of the large $R$ limit. In the functional integral, one first does the integration over the fields with mass of order $R$. As explained in section 2.1 terms involving only $D^2$ cancel, allowing one to write a simplified expression for the effective action which only depends on the difference of the derivative terms between bosons and fermions

$$\Gamma[X] = \frac{i}{2}\text{Trln}[1 + \triangle F \triangle J'] - \frac{i}{8}\text{Trln}[1 + \triangle F \triangle J'] \tag{53}$$
where $\Delta_F \equiv -D^{-2}$ is the propagator for the heavy fields and is a function of the background and the light fields. Again, terms with two derivatives of the background or light fields cancel as in (44). Terms with four derivatives and factors of $r^2$ expanded up from the heavy propagator yield precisely the structure $V^4 x^2/R^9$. So again, there are no terms of the form $V^4 v^2/R^7 r^7$ in the effective action.

This of course does not mean that there are not individual diagrams with the behavior $V^4 v^2/R^7 r^7$. However, we see explicitly from this analysis that there are cancellations between bosons and fermions. In [7], a particular diagram with this behavior was exhibited. But we see that this contribution is cancelled by diagrams involving fermions.

4 Discussion

We have shown by explicit calculation in an arbitrary background that the operators one first encounters in the matrix model effective action after integrating out just the most massive modes contain four derivatives and are of the form $V^4 x^2/R^9$ as discussed in [6]. When we use such an operator to construct $V^4 v^2/R^7 r^5$ at two loops by integrating over the modes of scale $r$, one finds the wrong scaling with $R$ and $r$ to correspond with the term $V^4 v^2/R^7 r^7$ in the supergravity $S$-matrix. In our analysis, we found no velocity independent terms indicating that fermionic contributions were missed in the work of [7]. In fact, in [12] the missing fermionic piece was identified.

What does one conclude about the correspondence between the matrix model and supergravity for three graviton scattering? It is possible that a proper treatment of various subtleties of DLCQ supergravity will show complete agreement with the finite $N$ matrix model [12]. It is likely that at large $N$, the supergravity prediction is recovered. The recent work of [10] showing that there is a non-renormalization theorem for $v^6$ in $SU(2)$ indicates that the matrix model-DLCQ supergravity correspondence is working for the $v^6$ terms, but using reasoning similar to [11] it is not hard to show that some $v^6$ terms are renormalized in $SU(N)$ for $N \geq 4$. For now, we will have to wait and see how the issue of three graviton scattering is resolved.

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Note added: Concurrent with the appearance of this note on hep-th, the work of [13] showed

\footnote{While this work was being completed we received word of this result.}
conclusively that supergravity and the matrix model do agree for 3-graviton scattering (including the effect of recoil [14]). The results reported in this note coincide with these findings since [13] have no terms of the form $v_{12}^2 v_{13}^2 v_{23}^2 / R^7 r$ in the effective action of supergravity or the matrix model. The source of the error in [13] occurs in extracting the S-matrix from the matrix model effective action [17].

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