Outer synchronization between dynamic varying networks under noisy condition

Hao Liang\textsuperscript{a}, Yumei Sun\textsuperscript{a}, Ronghu Chi\textsuperscript{a,b}, Xinli Fang\textsuperscript{c} and Jiaming Wang\textsuperscript{d}

\textsuperscript{a}Engineering Institute, Yantai Nanshan University, Yantai, People’s Republic of China; \textsuperscript{b}School of Automation & Electronic Engineering, Qingdao University of Science & Technology, Qingdao, People’s Republic of China; \textsuperscript{c}Powerchina Huadong Engineering Corporation Limited, Hangzhou, People’s Republic of China; \textsuperscript{d}Nanshan Electric Power Company, Yantai, People’s Republic of China

\textbf{ABSTRACT}

This paper exploits the outer synchronization between two dynamic networks with diversely varying node behaviours at the presence of noise. Aiming to obtain a generic technical solution for network outer synchronization, the problem is addressed for the dynamic network scenarios with non-identical topological structures, nonlinear inner coupling connections and time-delayed node characteristics. The impact of noise is fully considered in the theoretical and performance analysis which makes the derived synchronization criteria be able to be adopted in practical engineering deployment with minimized hurdles. By using a Lyapunov–Krasovskii energy function, the synchronization criteria and adaptive control solution are presented and the convergence of the system error states is also proved. Finally, the numerical analysis is carried out based on the simulation study and the result verifies the correctness and effectiveness of the suggested control solution.

\textbf{ARTICLE HISTORY}

Received 9 April 2018
Accepted 30 September 2018

\textbf{KEYWORDS}

Dynamic varying network; outer synchronization; nonlinear inner coupling; circumstance noise

\textbf{1. Introduction}

Synchronizing multiple networks to enable the overall system to be operated in a coordinated manner has been investigated as a challenging issue in both research and engineering communities for years. However, most findings and solutions of network synchronization have focused on the inner synchronization, which considers the collective dynamic behaviours of the nodes inside a network (Dai, Chen, Xie, & Jia, 2018; Lee, Park, Ji, Kwon, & Lee, 2012; Sakthivel, Sathishkumar, Kaviarasan, & Marshal Anthoni, 2017; Wang, Li, Yang, & Fei, 2012; Wu & Lu, 2012; Xiang & Zhu, 2011; Zhou, Feng, & Chen, 2011). Unlike inner synchronization, the synchronization problem between two or multiple networks, known as outer synchronization, has been investigated in recent years to look into the interactive behaviours between networks (Fang, Yang, & Yan, 2014; Li, Lü, Yang, Zhou, & Hong, 2018; Liu & Wang, 2017; Wu, Li, Wu, & Kurths, 2012; Wu & Lu, 2012; Yang, Zhang, & Chen, 2012; Zheng, Wang, Dong, & Bi, 2012). Though in (Fang et al., 2014), the outer synchronization problem with nonlinear time-delay characteristics and nonidentical time-varying topological structures has been explored, it should be highlighted that the existing research effort has merely considered the scenario that all the network nodes have the same dynamic characteristics. In fact, the dynamic varying phenomenon often exists among the nodes across the network, i.e. if part of the nodes in the networks is disturbed by some singular influences, the varying rate of different network nodes can be diverse (Zhou, Wang, & Mou, 2012).

In realistic networks, due to the operational environment or the functionality design, the nodes inside the network often exhibit very distinct dynamic properties. Taking the Microgrid (MG) system as an example, which is a controllable unit of the power distribution network spanning across a relatively small geographical area, consisting of various forms of small-scale distributed renewable generation resources (e.g. micro wind turbines, Combined Heat and Power system (CHPS) and solar energy generators etc.) as well as distributed energy storage devices (e.g. Power Electric Vehicles (PEVs) and battery units etc.) (Erol-Kantarci, Kantarci, & Mouftah, 2011; Fang, Yang, & Yan, 2013; Huang, Jiang, & Xu, 2008). In particular, the MGs can operate in an island mode as an autonomous system. Also, multiple MGs can be combined to realize the coordinated power supply to meet the demand upon the detection of failures in the connected power distribution networks, which ensures the reliability and quality of power supply (Lasseter, 2011; Lasseter et al., 2011). During the operation of MGs, the various forms of distributed generators (DGs) in the MGs show different operational paradigm, and hence demonstrate diverse dynamic properties. In order to realize the energy exchange with the utility or the mutual interaction among the MGs, the
network nodes across the different MGs, e.g. DGs, storage units and power loads, need to be synchronized to guarantee the efficient operations and the system stability overall multiple MGs. On the other hand, the renewable energy resources, e.g. the wind turbines, solar energy generators have an intermittent nature induced by environmental factors, the noise factors should be fully considered during the analysis of the performance of MGs (Lidula & Rajapakse, 2011). With such recognition, the synchronization between dynamic varying networks at the presence of noise needs to be well understood and further addressed.

In Zhou et al. (2012), the varying dynamic phenomenon has been included in the network inner synchronization analysis, but its impact to the outer synchronization of networks has not been considered. In Gu (2009) and Li and Cao (2011), an adaptive feedback control strategy and a pinning control strategy are proposed respectively to achieve synchronization of coupled delayed competitive neural networks; in Nagai and Kori (2010), a large population of globally coupled phase oscillators subject to common white Gaussian noise is studied and the relationship between the coupling strength for synchronization and the intensity of common noise is presented. Though the noise factor is involved in the aforementioned analysis, the problem is limited to special network forms. In Sun, Li, and Ruan (2011); Wang, Wang, and Liang (2008), the generalized outer synchronization between two different delay-coupled complex dynamical networks with noise perturbation is investigated and a nonlinear control scheme is developed, but the varying dynamic factor among the network nodes are not included during their analysis process. In this paper, we study the outer synchronization problem for two dynamic varying complex networks with non-identical topological structures and nonlinear inner coupling connections, including the noise factor. It is assumed that the networks have N different dynamic nodes and each node inside the drive and response networks has n-dimensional time-delay states. Considering the following drive-response networks with the noise impact:

\[
\begin{align*}
\dot{x}_i(t) &= [f(x_i(t)) + \sum_{j=1}^{N} c_{ij}\Gamma(x_j(t))] \\
&+ \sum_{j=1}^{N} \left(c_{ij}^t \pi(x_j(t-\tau))\right)dt + \psi_i(t)d\omega_i(t), \\
\eta_i\dot{y}_i(t) &= [f(y_i(t)) + \sum_{j=1}^{N} d_{ij}\Gamma(y_j(t))] \\
&+ \sum_{j=1}^{N} \left(d_{ij}^t \pi(y_j(t-\tau))\right)dt + \Psi_i(t)d\omega_i(t),
\end{align*}
\]

where \( \varepsilon_i \in R^+ \) and \( \eta_i \in R^+ \) are the dynamic-varying factors, \( i = 1, 2, \ldots, N \). \( x_i(t) \) and \( y_i(t) \) denote two n-dimensional state vectors in the drive and response networks, respectively; \( \tau \geq 0 \) is a time-delay constant between the drive and response network; \( f(\cdot): R^n \to R^n \) denotes a continuous integrable function, which represents the self-dynamic functions of individual network nodes; \( \Gamma(\cdot) \) and \( \Pi(\cdot) \) denote the non-delay and time-delay nonlinear inner coupling functions respectively, which describe the relationship of the current and the time-delay nonlinear inner coupling functions respectively, which describe the relationship of the current and the time-delay states of the ith response network node. \( C(t) = (c_{ij})_{N \times N} \) and \( D(t) = (d_{ij})_{N \times N} \) are the network topological matrices which satisfy:

\[
\begin{align*}
\sum_{j=1}^{N} c_{ij} &= -\sum_{j=1, j \neq i}^{N} c_{ij}, & \sum_{j=1}^{N} c_{ij}^t &= -\sum_{j=1, j \neq i}^{N} c_{ij}^t, \\
\sum_{j=1}^{N} d_{ij} &= -\sum_{j=1, j \neq i}^{N} d_{ij}, & \sum_{j=1}^{N} d_{ij}^t &= -\sum_{j=1, j \neq i}^{N} d_{ij}^t.
\end{align*}
\]

2. Problem formulation and assumptions

This section formulates the problem of network outer synchronization between two nonlinear dynamic varying complex networks with non-identical topological structures and nonlinear inner coupling connections, including the noise factor. The technical contributions made in this work can be summarized as follows: (1) the dynamic varying phenomenon is incorporated in the evaluation and design of network outer synchronization control strategies. To our best knowledge, such studies are insufficient in the literature; (2) noise is considered in the study as it can impose an adverse impact on the system behavior and even destroy the synchronization of networks (Banerjee & Arifin, 2012). Therefore, understanding the network synchronization between dynamic varying networks under noisy condition and designing appropriate control solution are the key issues tackled in this work.

The remainder of this paper is organized as follows: the problem formulation and some preliminaries are described in Section 2; Section 3 presents the outer synchronization criteria along with the proofs and a set of corollaries, followed by Section 4 which carries out the numerical study through an example to verify the correctness of proposed criteria and control solution; finally, Section 5 provides the conclusive remarks and points out the future research work directions.
Let \( \eta_j / e_j = \lambda_i (\lambda_i \in R^+) \) then function (1) can be rewritten as the following form:

\[
\eta_j d\xi_j(t) = [\lambda_i f(\xi_i(t)) + \lambda_i \sum_{j=1}^{N} c_{ij}^\tau \Pi(\xi_j(t-\tau))]dt + \lambda_i \psi(t)d\omega(t)
\]

with initial condition \( \xi \in C_{0}^2([-T, 0]; R^n) \) on \( t \geq 0, w(t) = (w_1(t), \ldots, w_n(t))^T \) is an n-dimensional Brownian motion, which is defined on the complete probability space, \( (\Omega, F, \{F_t\}_{t \geq 0}, P) \), with the filtration \( \{F_t\}_{t \geq 0} \) are both continuous differentiable equations. Then the operator \( LV(t,x,y) \) can be defined as

\[
LV(t,x,y) = V_t(t,x) + V_x(t,x) \Phi(t,x,y)
\]

where \( V_t(t,x) = \frac{\partial V(x,t)}{\partial t}, V_x(t,x) = (\frac{\partial V(x,t)}{\partial x_1}, \ldots, \frac{\partial V(x,t)}{\partial x_n}) \) and

\[
V_{xx} = \left( \frac{\partial^2 V(x,t)}{\partial x_i \partial x_j} \right)_{n \times n}.
\]

A Lasalle-type theorem for stochastic differential delay Equation (6) can be established as follows:

**Lemma 1** (L1 Mao, 1999; Pan & Cao, 2012; Wang, Cao, & Lu, 2010): Assuming that there are functions, \( V \in C^1(\mathbb{R}_+ \times R^n; R_+) \) \( r \in L^1(R_+; R_+) \) and \( w_1, w_2 \in C(\mathbb{R}_+; \mathbb{R}_+) \), such that \( LV(t,x,y) \leq r(t) - w_1(x) + w_2(y) \), \( (t,x,y) \in R^n \times R^n \times R^+ \), \( w_1(x) > w_2(x) \), \( \forall x \neq 0 \): \( \lim_{|x| \to \infty} \inf_{t \to \infty} V(t,x) = \infty \)

then \( \lim_{t \to \infty} x(t, \xi) = 0, a.s. \) for every \( \xi \in C_{0}^2([0,T, 0]; R^n) \).

**Lemma 2** (L2 Pan & Cao, 2012): For any vectors \( x, y \in R^n \), there exists scalar \( \varepsilon > 0 \) and positive definite matrix \( P \in R^{n \times n} \), which makes the following inequality holds

\[
2x^T y \leq \varepsilon x^T Px + \varepsilon^{-1} y^T P^{-1} y
\]

**Assumption 1** (A1 Wang & Cao, 2013; Wu & Chua, 1995; Xu, Zhou, & Fang, 2012; Zheng et al., 2012): There is a positive definite diagonal matrix, \( Q = \text{diag}(q_1, \ldots, q_n) \), and a diagonal matrix, \( \Delta = \text{diag}(\delta_1, \ldots, \delta_n) \), such that function \( f(\cdot) \) satisfies the following inequality:

\[
(\varepsilon - \eta)^T Q(f(\varepsilon, t) - f(\eta, t)) - \Delta(\varepsilon - \eta)) \leq -I \cdot (\varepsilon - \eta)^T (\varepsilon - \eta)
\]

where the constant \( I \geq 0 \) \( (I \in R) \).

**Assumption 2:** (A2 Gu, 2009; Zheng et al., 2012): Assuming that function \( \Gamma(\cdot) \) and \( \Pi(\cdot) \) are two Lipschitz continuous function, i.e. there exist two Lipschitz constants \( \alpha \) and \( \beta \) satisfying:

\[
0 < \frac{\Gamma(y) - \Gamma(x)}{y - x} \leq \alpha \quad (\text{10})
\]

\[
0 < \frac{\Pi(y) - \Pi(x)}{y - x} \leq \beta \quad (\text{11})
\]
3. Synchronization criteria and controller design

In this section, by adopting appropriate design of the controller and adaptive update laws, we present the criteria to guarantee the outer synchronization of the dynamic varying networks (1) and (2) at the presence of noise.

Theorem 1 Suppose the assumptions A1 and A2 hold, with the controller (12) and the update laws (13) and (14), the outer synchronization between the dynamic varying networks defined in (1) and (2) can be realized with a sufficient large control parameter matrix $K$.

$$u_i(t) = \sum_{j=1}^{N} p_{ij} \Gamma(x_j(t)) + \sum_{j=1}^{N} q_{ij} \prod(x_j(t - \tau)) - k_i e_i(t)$$

$$\hat{\dot{p}}_j(t) = -e_j^T(t) \cdot \Gamma(x_j(t))$$

$$\hat{\dot{q}}_j(t) = -e_j^T(t) \cdot \prod(x_j(t - \tau))$$

where $T$ is the transpose of a matrix; $p_{ij}$ and $q_{ij}$ are two $n \times 1$ vectors and $k_i$ is the feedback strengths.

**Proof:** Choosing the Lyapunov function as follows:

$$V(t,e(t)) = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \eta_i e_i^T(t) e_i(t) + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} (d_{ij} - \tilde{c}_{ij} + p_{ij})^2$$

$$+ \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} (d_{ij} - \tilde{c}_{ij} + q_{ij})^2$$

Defining $\sigma_i(t) = \psi_i(t) - \tilde{\psi}_i(t)$ and by the differential formula (L1), the operator, $L - V(t,x,y)$, can be obtained as

$$LV(t) = \sum_{i=1}^{N} e_i^T(t) \left[ f(y_i(t)) - f(x_i(t)) \right] + (1 - \lambda_i) f(x_i(t))$$

$$+ \sum_{i=1}^{N} [d_{ij} \Gamma(y_j(t)) - \tilde{c}_{ij} \Gamma(x_j(t))]$$

$$+ \sum_{i=1}^{N} \left[ d_{ij} \prod(y_j(t - \tau)) - \tilde{c}_{ij} \prod(x_j(t - \tau)) \right]$$

$$+ \sum_{i=1}^{N} p_{ij} \Gamma(x_j(t)) + \sum_{j=1}^{N} q_{ij} \prod(x_j(t - \tau)) - k_i e_i(t)$$

$$+ \frac{1}{2} trace[\sigma_i^T(t) \eta_i \sigma_i(t)] + \sum_{i=1}^{N} \sum_{j=1}^{N} (d_{ij} - \tilde{c}_{ij} + p_{ij})$$

$$\cdot \hat{\dot{p}}_j + \sum_{i=1}^{N} \sum_{j=1}^{N} (d_{ij} - \tilde{c}_{ij} + q_{ij}) \cdot \hat{\dot{q}}_j$$

$$= \sum_{i=1}^{N} e_i^T(t) \left[ f(y_i(t)) - f(x_i(t)) \right]$$

$$+ \sum_{i=1}^{N} e_i^T(t) (1 - \lambda_i) f(x_i(t))$$

$$+ \sum_{i=1}^{N} \sum_{j=1}^{N} e_i^T(t) d_{ij} \Gamma(y_j(t)) - \sum_{i=1}^{N} \sum_{j=1}^{N} e_i^T(t) \tilde{c}_{ij} \Gamma(x_j(t))$$

$$+ \sum_{i=1}^{N} \sum_{j=1}^{N} e_i^T(t) d_{ij} \prod(y_j(t - \tau))$$

$$+ \sum_{i=1}^{N} \sum_{j=1}^{N} e_i^T(t) \tilde{c}_{ij} \prod(x_j(t - \tau))$$

$$+ \sum_{i=1}^{N} \sum_{j=1}^{N} e_i^T(t) p_{ij} \Gamma(x_j(t))$$

$$+ \sum_{i=1}^{N} \sum_{j=1}^{N} e_i^T(t) q_{ij} \prod(x_j(t - \tau))$$

$$+ \frac{1}{2} trace[\sigma_i^T(t) \eta_i \sigma_i(t)] + \sum_{i=1}^{N} \sum_{j=1}^{N} (d_{ij} - \tilde{c}_{ij} + p_{ij})$$

$$\cdot \hat{\dot{p}}_j + \sum_{i=1}^{N} \sum_{j=1}^{N} (d_{ij} - \tilde{c}_{ij} + q_{ij}) \cdot \hat{\dot{q}}_j$$

$$\leq \sum_{i=1}^{N} e_i^T(t) \cdot I_i \cdot e_i(t) + \sum_{i=1}^{N} e_i^T(t) \delta_i e_i(t)$$
$\sum_{i=1}^{N} \| e_i^T(t)(1 - \lambda_i)f(x_i(t)) \|$ 

$- \sum_{i=1}^{N} k_i e_i^T(t) e_i(t) + \frac{1}{2} \sum_{j=1}^{N} \sigma_j^T(t) \eta_j \sigma_j(t)$ 

$+ \sum_{i=1}^{N} \sum_{j=1}^{N} e_i^T(t) d_{ij} \cdot \alpha_i e_j(t)$ 

$+ \sum_{i=1}^{N} \sum_{j=1}^{N} e_i^T(t) d_{ij}^T \cdot \beta_j e_j(t - \tau)$ 

$\leq e^T(t) \cdot L \cdot e(t) + e^T(t) \cdot \Delta \cdot e(t)$ 

$+ \sum_{i=1}^{N} \| e_i^T(t)(1 - \lambda_i)f(x_i(t)) \| - e^T(t) \cdot K \cdot e(t)$ 

$+ \frac{1}{2} \sum_{j=1}^{N} \sigma_j^T(t) \eta_j \sigma_j(t)$ 

$+ e^T(t)(D \cdot A)e(t) + e^T(t)(D^T \cdot B)e(t - \tau),$ \hspace{1cm} (16)

where \( L = diag(l_1, l_2, \ldots, l_N) \), \( \Delta = diag(\delta_1, \delta_2, \ldots, \delta_N) \), \( A = diag(\alpha_1, \alpha_2, \ldots, \alpha_N) \), \( B = diag(\beta_1, \beta_2, \ldots, \beta_N) \), \( K = diag(k_1, k_2, \ldots, k_N) \), \( e(t - \tau) = [e_1(t - \tau), e_2(t - \tau), \ldots, e_N(t - \tau)]^T \).

Using L2, the inequality (16) can be further rewritten as

\[ LV(t) \leq r(t) + e^T(t) \cdot L \cdot e(t) + e^T(t) \cdot \Delta \cdot e(t) \]

\[ + e^T(t)(D \cdot A)e(t) - e^T(t) \cdot K \cdot e(t) \]

\[ + \frac{1}{2} e^T(t)(D^T \cdot B)^T(D^T \cdot B)e(t) \]

\[ + \frac{1}{2} e^T(t - \tau) \cdot I \cdot e(t - \tau) \]

\[ = r(t) + e^T(t) \left[ L + \Delta + (D \cdot A) \right] \]

\[ + \frac{(D^T \cdot B)^T(D^T \cdot B) - K}{2} \cdot e(t) \]

\[ + \frac{1}{2} e^T(t - \tau) \cdot I \cdot e(t - \tau) \]

\[ = r(t) - e^T(t) \cdot [K - \Delta - L \cdot (D \cdot A) - ((D^T \cdot B)^T \times (D^T \cdot B))/2] \cdot e(t) + \frac{1}{2} e^T(t - \tau) \cdot I \cdot e(t - \tau) \]

\[ = r(t) - \omega_1(e(t)) + \omega_2(e(t - \tau)), \hspace{1cm} (17) \]

where \( r(t) = \sum_{i=1}^{N} \| e_i^T(t)(1 - \lambda_i)f(x_i(t)) \| + \sum_{j=1}^{N} \| \sigma_j^T(t) \eta_j \sigma_j(t) \| \)

\( w_1(e(t)) = e^T(t) \cdot [K - \Delta - L \cdot (D \cdot A) - ((D^T \cdot B)^T(D^T \cdot B))/2] \cdot e(t) \)

\( w_2(e(t - \tau)) = \frac{1}{2} e^T(t - \tau) \cdot I \cdot e(t - \tau) \) and \( I \) is an identity matrix. It can be seen that with a sufficient large control parameter matrix \( K \), the inequality (17) and

$w_1(x) > w_2(x)$ holds. Moreover, when \( ||e(t)|| \to \infty \), the \( \lim V(e(t), t) = \infty \) can be easily obtained, so from L1, we have \( \lim_{t \to \infty} e_i(t) = 0. \)

4. Numerical simulation and result

In this section, we present a numerical simulation to verify the correctness of the proposed outer synchronization criteria. It is well known that the Lorenz system is a typical chaotic system, which can be described as follows:

\[ \dot{x} = \begin{cases} 
10(x_2 - x_1), \\
28x_1 - x_2 - x_1x_3, \\
(-\omega_0 + 8)/3x_3 + x_1x_2, 
\end{cases} \]

(18)

where \( \dot{x} = [x_1, x_2, x_3]^T \) and when \( \omega_0 \in [0, 1] \) the above system can be guaranteed to be chaotic. In our simulation, for simplicity, we choose \( \omega_0 = 0 \) and the time-delay constant \( \tau = 1 \). The nonlinear inner coupling activation functions are chosen as \( \Gamma(x) = 0.5 \sin x \) and \( \Pi(x) = -0.5 \cos y \), which apparently meet A2. In addition, to ensure the generality of the experiment, the initial states of the drive network and response network are configured with random values in the range of (0,1) and (0,10), respectively. Based on the modelling approach presented in (Fang et al., 2013), and to analyse the topological characteristics, the following topological matrices are generated for the numerical study:

\[ c = \begin{bmatrix} 
-3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \]

\[ c' = \begin{bmatrix} 
-4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \]
In the simulation, the time-varying noise intensity functions are

\[ \varphi(t) = 0.1x(t) + 0.02 \sum_{1}^{10} x(t - 1) \]

and

\[ \psi(t) = 0.1y(t) + 0.02 \sum_{1}^{10} y(t - 1) \].

To obtain the steady state statistics, the simulation is repeated 10 times and the dynamic varying factors \( \xi_i \) and \( \eta_i \) are selected in a random manner at each experiment. Based on the aforementioned assumptions and experimental parameters, the numerical simulation is carried out by the use of MATLAB through a comparative study to evaluate the system error performance without and with the control actions, and the key results are illustrated in Figures 1 and 2 (where \( i = 1, \ldots, 10 \)).

Figure 1 shows the chaotic effects of the system errors of the drive-response network with the environmental noise. It can be seen that the fluctuations of error states are significant due to the impacts of noise, and the error evolving dynamics along with the simulation time demonstrate unpredictable and irregular phenomenon without any trend of convergence. Figure 2 shows the control effect by adopting the proposed control schemes and update laws and the subgraph shows the detailed evolutionary process of the system error. It clearly indicates that the synchronization error states can converge along with simulation time, and the fluctuation ranges are significantly constrained in a small range. This result confirms our expectation and validates the correctness of the suggested control approach.

5. Conclusions and future work

The mechanisms and control schemes for outer synchronization between complex networks are considered challenging and exploited for years. However, most of the existing control solutions are obtained based on a strong assumption that the networks are without dynamic varying characteristics, which significantly limit their applicability and effectiveness in practical deployment. This paper looked into the outer synchronization issue between the dynamic varying networks with nonlinear inner coupling characteristics and non-identical topological structures, and in particular at the presence of noise. Based on the Lyapunov stability theory, we derived the synchronization criteria and presented an adaptive controller design by adopting appropriate Lyapunov–Krasovskii energy function. The convergence of the synchronization error states and the impacts of the control solution are well validated and evaluated through
The performance of error convergence (with control).

Figure 2. The performance of error convergence (with control).

the numerical experiment. It is also worth noting that the existing results of network outer synchronization in the literature can be effectively derived from such generic synchronization criteria and control solution by given certain conditions.

Disclosure statement
No potential conflict of interest was reported by the authors.

References
Banerjee, S., & Ariffin, M. (2012). Noise induced synchronization of time-delayed semiconductor lasers and authentication based asymmetric encryption. Optics and Laser Technology, 45, 435–442.

Dai, H., Chen, W., Xie, J., & Jia, J. (2018). Exponential synchronization for second-order nonlinear systems in complex dynamical networks with time-varying inner coupling via distributed event-triggered transmission strategy. Nonlinear Dynamics, 92(3), 853–867.

Erol-Kantarci, M., Kantarci, B., & Mouftah, H. T. (2011). Reliable overlay topology design for the smart microgrid network. IEEE Network, 25(5), 38–43.

Fang, X., Yang, Q., & Yan, W. (2013). Topological characterization and modeling of dynamic evolving power distribution networks. Simulation Modeling Practice and Theory, 31, 186–196.

Fang, X., Yang, Q., & Yan, W. (2014). Outer synchronization between complex networks with nonlinear time-delay characteristics and time-varying topological structures. Mathematical Problems in Engineering, 1–6. ID:437673.

Gu, H. (2009). Adaptive synchronization for competitive neural networks with different time scales and stochastic perturbation. Neurocomputing, 73(1), 350–356.

Huang, J., Jiang, C., & Xu, R. (2008). A review on distributed energy resources and Microgrid. Renewable and Sustainable Energy Reviews, 12(9), 2472–2483.

Lasseter, R. (2011). Smart distribution: Coupled microgrids. In Proceedings of the IEEE (pp. 1074–1082). Madison: University of Wisconsin.

Lasseter, R., Eto, J., Schenkman, B., Stevens, J., Vollkommer, H., Klapp, D., . . . Roy, J. (2011). CERTS microgrid laboratory test bed. IEEE Transactions on Power Delivery, 26(1), 325–332.

Lee, T., Park, J., Ji, D., Kwon, O., & Lee, S. (2012). Guaranteed cost synchronization of a complex dynamical network via dynamic feedback control. Applied Mathematics & Computation, 218(11), 6469–6481.

Li, L., & Cao, J. (2011). Cluster synchronization in an array of coupled stochastic delayed neural networks via pinning control. Neurocomputing, 74(5), 846–856.

Li, C., Lu, L., Yang, M., Zhou, S., & Hong, Y. (2018). Research on outer synchronization between uncertain time-varying networks with different node number. Statistical Mechanics and its Applications, 492(0), 2301–2309.

Lidula, N., & Rajapakse, A. (2011). Microgrids research: A review of experimental microgrids and test systems. Renewable and Sustainable Energy Reviews, 15(1), 186–202.

Liu, S., & Wang, Q. (2017). Outer synchronization of small-world networks by a second-order sliding mode controller. Nonlinear Dynamics, 89(3), 1817–1826.

Mao, X. (1999). LaSalle-type theorems for stochastic differential delay equations. Journal of Mathematical Analysis and Applications, 236(2), 350–369.

Nagai, K., & Kori, H. (2010). Noise-induced synchronization of a large population of globally coupled nonidentical oscillators. Physics Review E, 81(6), 1–4.

Pan, L., & Cao, J. (2012). Stochastic quasi-synchronization for delayed dynamical networks with intermittent control. Communications in Nonlinear Science and Numerical Simulation, 17(3), 1332–1343.

Sakthivel, R., Sathishkumar, M., Kaviarasan, B., & Marshal Anthoni, S. (2017). Synchronization and state estimation for stochastic complex networks with uncertain inner coupling. Neurocomputing, 238(0), 44–55.

Sun, L., Li, W., & Ruan, J. (2011). Generalized outer synchronization between complex dynamical networks with time delay and noise perturbation. Communications in Nonlinear Science and Numerical Simulation, 83(4), 989–998.
systems. *Nonlinear Analysis: Real World Applications*, 14(1), 842–851.

Wang, G., Cao, J., & Lu, J. (2010). Outer synchronization between two nonidentical networks with circumstance noise. *Physica A: Statistical Mechanics and its Applications*, 389(7), 1480–1488.

Wang, T., Li, T., Yang, X., & Fei, S. (2012). Cluster synchronization for delayed Lure dynamical networks based on pinning control. *Neurocomputing*, 83(15), 72–82.

Wang, Y., Wang, Z., & Liang, J. (2008). A delay fractioning approach to global synchronization of delay complex networks with stochastic disturbances. *Physics Letters A*, 372(39), 6066–6073.

Wu, C., & Chua, L. (1995). Synchronization in an array of linearly coupled dynamical systems. *IEEE Transactions on Circuits and Systems I*, 42(8), 430–447.

Wu, Y., Li, C., Wu, Y., & Kurths, J. (2012). Generalized synchronization between two different complex networks. *Communications in Nonlinear Science and Numerical Simulation*, 17(1), 349–355.

Wu, X., & Lu, H. (2012). Generalized function projective (lag, anticipated and complete) synchronization between two different complex networks with nonidentical nodes. *Communications in Nonlinear Science and Numerical Simulation*, 17(7), 3005–3021.

Wu, X., & Lu, H. (2012). Hybrid synchronization of the general delayed and non-delayed complex dynamical networks via pinning control. *Neurocomputing*, 89(15), 168–177.

Xiang, L., & Zhu, J. (2011). On pinning synchronization of general coupled networks. *Nonlinear Dynamics*, 64(4), 339–348.

Xu, Y., Zhou, W., & Fang, J. (2012). Adaptive synchronization of the complex dynamical network with double non-delayed and double delayed coupling. *International Journal of Control, Automation and Systems*, 10(2), 415–420.

Yang, Z., Zhang, Q., & Chen, Z. (2012). Adaptive linear generalized synchronization between two nonidentical networks. *Communications in Nonlinear Science and Numerical Simulation*, 17(1), 2628–2636.

Zheng, S., Wang, S., Dong, G., & Bi, Q. (2012). Adaptive synchronization of two nonlinearly coupled complex dynamical networks with delayed coupling. *Communications in Nonlinear Science and Numerical Simulation*, 17(1), 284–291.

Zhou, X., Feng, H., & Chen, S. (2011). The effect of control strength on the synchronization in pinning control questions. *Computers & Mathematics with Applications*, 61(2), 2014–2018.

Zhou, W., Wang, T., & Mou, J. (2012). Synchronization control for the competitive complex networks with time delay and stochastic effects. *Communications in Nonlinear Science and Numerical Simulation*, 17(8), 3417–3426.