Characteristics of mathematical models of coal slurries processing for the purpose of examining the opportunities for improvement of quality parameters

A Manowska
Faculty of Mining and Geology, Silesian University of Technology, Akademicka 2, 44-100 Gliwice, Poland
E-mail: anna.manowska@polsl.pl

Abstract. Coal enrichment plants are intermediaries between the extraction stage and its distribution. The extracted raw material is a mixture of grains of various sizes, which consist of both organic and mineral substances and are described by a number of quality parameters. In the enrichment process, the raw coal is separated into density fractions and after achieving the appropriate degree of enrichment, the commercial grading stage follows. Considering the environment protection considerations, it is increasingly important to provide products with low emissivity. Coal slurries can also be used for energy purposes. The usefulness of deposited coal slurries in power industry is determined by its quality parameters, such as: ash and sulphur content, calorific value, moisture content, grain-size distribution, etc. Such slurries, in case of low use value, can undergo the enrichment processes. These activities are primarily intended for the improvement of quality parameters of crude slurries and they should be carried out having regard to fight for low emission and keeping in mind procedures aimed at eliminating the sale of low quality coal fuels.

1. Introduction
Analysing changes in the energy market it is possible to notice the growing role of regulations introduced by the European Union which exerts an ever larger influence on functioning of the domestic market. These regulations originate from strategic documents concerning the development of the energy market. The European Union determined the guidelines aimed at orienting the energy and climate policy up to 2050 [1, 2, 3, 4]. These guidelines integrate various political purposes, which include above all: reduction of greenhouse gases emission, safeguarding supplies of fuels and energy, and increase in energy intensity. The European Commission published “Energy Services Action Plan up to 2050”. A communication of the Commission to the European Parliament, European Council, European Economic and Social Committee and Committee of Regions is to hasten the implementation of targets aimed at counteracting climate changes and transformation into low-emission green energy [5]. The communication shows the need for transformation of the present energy production based on burning fossil fuels and it identifies key challenges and technologies as well as it proposes several possible scenarios of development [6]. For over a dozen years it has been possible to notice the decline of coal in the energy mix in the European Union. It has been the result of changes in the reduction of emission and economics of production as well as the growing competition from other technologies [7]. Countries of the European Union mainly import coal, while Poland has rich coal resources. As a consequence as much as 85% of electricity is generated from coal and clean coal technologies enable coal to continually guarantee energy security of the country.
2. Formation and use of coal slurries

The cost of electricity generation in a power plant depends not only on acquisition of fuel to be transferred to the power network, but it is also connected with emission of pollutants. It means that these costs depend on the quality of fuel and they change along with the change of the quality of fuel [8]. Coal output extracted on the mine surface consists of different size grains – from lumps which are several tens of centimeters in size to grains less than 1 mm (and even grains as small as 1 micron). This diversity is the result of the method of mechanical processing of coal seams. During the underground exploitation grains of waste rock get into the excavated coal material. The said waste rock comes from the stone interlayers in the body of coal, and most often from the roof ripping and dinting. The waste rock which gets into the excavated coal material is the impurity that needs to be removed in order to improve the performance of the commercial product sold to the buyers. Removal of impurities takes place in coal preparation plants in the so-called coal cleaning processes [9]. The smallest grains from the mining process get into the water and slurry cycle of a coal preparation plant and they are generally removed from the cleaning process. The slurry grains were treated as waste from the coal preparation processes, but in fact the majority of that waste constitutes energy fuel. Inspection of quality and size analysis of some slurries stored in settling tanks showed that a dozen or so objects inspected in recent years contain low quality coal fuel. For this reason, an interest in using these energy sources has grown in recent years. A lot of settling ponds were exploited and the derived slurries were added to the coal dust in order to make energy mixture [9]. Bearing in mind the fact that the slurry products have certain energy potential it seems reasonable to take any action aimed at its complete recovery.

3. Modelling of the coal cleaning operations

The process of quality improvement of coal slurries can be carried out by means of applying the selected methods of gravity separation, i.e. separation on concentrating tables, spiral separators or enrichment by floatation. The enrichment products are: concentrate containing coal grains, a semi-finished product consisting of grains of coal interlayers and wastes – containing grains of minerals. Having regard to economic factors of the course of an enrichment process it seems reasonable to formulate mathematical models for processing operations in order to use these models to carry out simulations which would allow to determine the best possible technological parameters of the processes. A convenient method applied to analyse coal preparation processes is a computer simulation. It is possible to analyse various variants of technological system solutions quickly and at low cost, and to try to find optimal conditions for coal preparation processes [10]. Formulation of mathematical models for coal preparation processes should be considered in probabilistic terms, in which the measured yields and quality parameters are random variables. Instantaneous and mean values of these random variables are assessed based on measurements of appropriate quantities in a sample of coal.

3.1. Analytic and graphic determination of the yields of coal enrichment products

The task of the separators is to separate the starting material according to specific gravity. At perfect operation of these machines the yield and the ash content in the obtained products should be up to the values determined from the specific gravity curve drawn on the basis of distribution in suspension liquids (figure1).
In practice, however, separation in industrial machines is carried out not only according to specific gravity, but the size and shape of grains have also an effect on that separation. The mentioned factors contribute to the fact that the quality of products separated by the machines differs from the quality, which results from the densimetric analysis of the tested material. Separation does not occur according to the specific gravity and some part of grains get into the light material – concentrate. During the three-material separation these grains, according to their specific gravity, should be found in the intermediate product and the waste. That phenomenon occurs also in the opposite direction, namely, a part of grains, which should be in the light product due to their specific gravity, get into the heavy or average product during the enrichment process. The same happens in case of grains which at the three-product enrichment should constitute a component part of the average product due to their specific gravity – they will go to both heavy and light product. But some part of grains, which due to their specific gravity should be found in the light and heavy product, will go to the average product [11].

3.2. Analytical method of two-product enrichment

Illustrating the graphic image of the results of separation in liquid suspension figure 1 presents washability curves of the raw material and two products drawn up on the basis of densimetric analysis. Figure 2 illustrates close relationship between ordinates of the curves $\delta_B$, $\delta_L$ and $\delta_S$, and the ordinates $\lambda_B$, $\lambda_L$, $\lambda_S$ resulting from the fact that during the enrichment process of individual grains their ash content does not change. This implies that the loss of grains of a given specific gravity and of a given ash content in one product brings about simultaneous increase of identical grains by the same amount in the second product. Therefore, in accordance with the observed properties there is dependence [11]:

$$AB = CD \text{ i } HG = FE$$

(1)

from which following equation is obtained [6]:

$$F_B = L \cdot F_L + (1 - L) F_S$$

(2)

where:

$L$ – yield of the light product, concentrate,

$S$ – yield of the heavy product, waste material,

$F_B$ – portion of considered fraction with specific gravity less than $\delta r$ in raw material,

$F_L$ – portion of considered fraction with specific gravity less than $\delta r$ in light product - concentrate,
$F_S$ – portion of considered fraction with specific gravity less than $\delta r$ in heavy product – waste material, $\delta r$ – specific gravity.

After conversion of the equation 2 the light product yield is [11]:

$$L = \frac{F_B - F_S}{F_L - F_S} \times 100\%$$  \hspace{1cm} (3)

Formula 3 enables percentage calculation of the light product yields in proportion to the whole raw material based on the defined percentage content of fraction with specific gravity less than $\delta r$ in raw material and in the light and heavy product.

**Figure 2.** Washability curves which present graphic results of coal separation, source [11].

The following dependences can occur between specific gravity $\delta$ and limiting specific gravity for separation $\delta r$.

- $\delta < \delta r$: $AB=CD$ – case 1
- $\delta = \delta r$: $AB=CD$ – case 2
- $\delta > \delta r$: $AB=CD$ – case 3

For case no. 1 it is [11]:

$$L \cdot F_L - (F_B - S) = S \cdot (1 - F_S)$$  \hspace{1cm} (4)

$$S = 1 - L$$  \hspace{1cm} (5)

$$L = \frac{F_B - F_S}{F_L - F_S} \times 100\%$$  \hspace{1cm} (6)

For case no. 2 it is [11]:

$$L \cdot F_L = S \cdot (1 - F_S) = S - S \cdot F_S$$  \hspace{1cm} (7)

and

$$S = F_B$$  \hspace{1cm} (8)
This leads to:

\[ L \cdot F_L = F_B - F_S + L \cdot F_S \]  \hspace{1cm} (9)

After conversion the dependence 3 was obtained.

For case no. 3 it is [11]:

\[ L \cdot F_L = F_B - S \cdot F_S \]  \hspace{1cm} (10)

Using the relationship:

\[ S = 1 - L \]  \hspace{1cm} (11)

You will obtain:

\[ L \cdot F_L = F_B - F_S + L \cdot F_S \]  \hspace{1cm} (12)

Thus the dependence 3 was also obtained.

In each considered case, the formula 3 is used for calculating the quantity L. The yield of the heavy product is calculated on the basis of the dependence [11]:

\[ S = 100 - L \% \]  \hspace{1cm} (13)

and when you have derived the formulas 3 and 13 it is possible to analytically determine yields of products of the enrichment process. In order to efficiently make analytical calculations, a tool was implemented in the Visual Basic environment, which allows to quickly define theoretical yields of the coal slurry enrichment products.

Table 1 contains the results of separation of coal cleaning products in heavy liquids in a two-product jig.

| δ  | F_B [kg] | F_B [%] | Σ F_B [%] | F_L [kg] | F_L [%] | Σ F_L [%] | F_S [kg] | F_S [%] | Σ F_S [%] |
|----|----------|---------|-----------|----------|---------|-----------|----------|---------|-----------|
| <1,3 | 7,06    | 40,90  | 40,90     | 7,06    | 56,51  | 56,51    | 1,78     | 7,96   | 7,96     |
| 1,3÷1,4 | 3,66   | 21,20  | 62,10     | 6,22    | 35,84  | 92,35    | 1,84     | 8,24   | 16,20    |
| 1,4 ÷1,5 | 1,31   | 7,60  | 69,70     | 0,66    | 4,60  | 96,95    | 1,40     | 6,26   | 22,46    |
| 1,5 ÷1,6 | 0,98   | 5,68  | 75,38     | 0,29    | 2,02  | 98,97    | 1,04     | 4,65   | 27,11    |
| 1,6 ÷1,7 | 0,91   | 5,27  | 80,65     | 0,08    | 0,55  | 99,52    | 0,67     | 3,00   | 30,11    |
| 1,7 ÷1,8 | 0,78   | 4,52  | 85,17     | 0,05    | 0,34  | 99,86    | 0,45     | 2,01   | 32,12    |
| 1,8 ÷1,9 | 0,12   | 0,70  | 85,87     | 0,02    | 0,14  | 100,00   | 0,60     | 2,68   | 34,80    |
| 1,9÷2,0 | 0,14   | 0,81  | 86,68     |        |       |          | 0,78     | 3,49   | 38,29    |
| Razem | 17,26  | 100   | 14,38     | 100     | 22,36 | 100,00   | -        | -      | -         |

Source: [11]

If the portion of the light product in raw material is used as the basis for calculation, then according to the dependence 3 the product yield L will be:

For the light fraction < 1,5

\[ L = \frac{69.70-22.46}{96.95-22.46} \cdot 100\% = 63,42\% \]  \hspace{1cm} (14)

For the light fraction < 1,8

\[ L = \frac{85.17-32.12}{99.86-32.12} \cdot 100\% = 78,31\% \]  \hspace{1cm} (15)

For the light fraction < 2,0

\[ L = \frac{86.68-38.29}{F_L-F_S100-38.29} \cdot 100\% = 78,41\% \]  \hspace{1cm} (16)
It results from the calculations that there are big differences between the yields of the light product for individual specific gravities of separation, at small specific gravity of the light fraction the relative error of measurement will be much bigger than at the high specific gravity of fraction, irrespective of that the errors can be positive for some yields and negative for the others. Therefore, it seems appropriate to apply the summary fractions because the errors will compensate and the \( L \) value will be close to the real value.

The yield \( L \) is usually determined by calculating the yields for the series of the light fraction and then the arithmetic average is calculated from them. On the basis of the data contained in table 1 the yield values are as follows:

For the light fraction \(< 1,3\)

\[
L = \frac{40.90-7.96}{56.51-7.96} \quad 100\% = 67.84\% \quad (17)
\]

For the light fraction \(< 1,4\)

\[
L = \frac{62.10-16.20}{92.35-16.20} \quad 100\% = 60.27\% \quad (18)
\]

For the light fraction \(< 1,5\)

\[
L = \frac{69.70-22.46}{96.95-22.46} \quad 100\% = 63.42\% \quad (19)
\]

For the light fraction \(< 1,6\)

\[
L = \frac{75.38-27.11}{98.97-27.11} \quad 100\% = 67.17\% \quad (20)
\]

For the light fraction \(< 1,7\)

\[
L = \frac{80.65-30.11}{99.52-30.11} \quad 100\% = 72.81\% \quad (21)
\]

For the light fraction \(< 1,8\)

\[
L = \frac{85.17-32.12}{99.86-32.12} \quad 100\% = 78.31\% \quad (22)
\]

For the light fraction \(< 1,9\)

\[
L = \frac{85.87-34.80}{100.00-34.80} \quad 100\%=78,30\% \quad (23)
\]

For the light fraction \(< 2,0\)

\[
L = \frac{86.68-38.29}{100.00-38.29} \quad 100\% = 78.41\% \quad (24)
\]

The average value will be:

\[
L = \frac{67.84+60.27+63.42+67.17+72.81+78.31+78.30+78.41}{8} \quad 100\% = 70.82\% \quad (25)
\]

Heavy product yield on the basis of the formula 13 will be:

\[
S = (100 - 70.82) = 29,18\% \quad (26)
\]

Calculating the \( F_L \) and \( F_S \) values relative to the content in raw material you can obtain values of the so-called apparent yield of the raw material, which was marked \( F_B \).

The content of particular fractions in concentrate and waste material are not qual. The error values marked with \( \varepsilon \), depend on the accuracy of sampling and weighing, and also on the specific gravity of the separation.
4. Triangle of error method
Knowledge of the enrichment results obtained in practice, i.e. yields and the ash content present in them, allow to compare their deviations from theoretical results applying the triangle of error method. Table 2 contains results of the analysis of the fraction of output, concentrate and waste material.

**Table 2. Enrichment results.**

| Feed | Concentrate | Waste | \( \Sigma \gamma \ast \gamma_0 \) + |
|------|-------------|-------|-------------------------------------|
| \( \gamma \) | \( \Sigma \gamma \) | \( \lambda \gamma \) | \( \lambda \gamma \) | \( \gamma \ast \gamma \) | \( \Sigma \gamma \ast \gamma \) | \( \gamma \) | \( \gamma \ast \gamma \) | \( \Sigma \gamma \ast \gamma \) | \( \gamma \ast \gamma \) |
| 1.25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1.3 | 50.09 | 50.09 | 1.7 | 2.8 | 58.53 | 50.09 | 0 | 0.00 | 0.00 | 85.58 |
| 1.4 | 20.83 | 70.92 | 5.3 | 11.2 | 24.34 | 20.83 | 70.92 | 0 | 0.00 | 0.00 | 85.58 |
| 1.5 | 5.02 | 75.94 | 15.6 | 21.4 | 5.87 | 5.02 | 75.95 | 0 | 0.00 | 0.00 | 85.58 |
| 1.6 | 2.27 | 78.21 | 26.8 | 32.1 | 2.65 | 2.27 | 78.21 | 0 | 0.00 | 0.00 | 85.58 |
| 1.8 | 3.94 | 82.15 | 39.5 | 50.3 | 4.47 | 3.83 | 82.04 | 0.77 | 0.11 | 0.11 | 85.69 |
| 2 | 2.58 | 84.73 | 59.8 | 66.2 | 2.13 | 1.82 | 83.86 | 5.29 | 0.76 | 0.87 | 86.46 |
| 2.2 | 2.91 | 87.64 | 71.3 | 75.6 | 1.17 | 1.00 | 84.86 | 13.25 | 1.91 | 2.78 | 88.37 |
| 2.5 | 12.36 | 100 | 79.3 | 80.5 | 0.84 | 0.72 | 85.58 | 80.69 | 11.63 | 14.42 | 100.00 |

Source: author’s own elaboration [11]

It shows that the practical yield of concentrate \( L=85.51 \), and the yield of waste \( S=14.49 \). The table presents also data concerning the content of ash in the output fraction. In figure 3 one co-ordinate system was applied to draw specific gravity curves defining the mine output, concentrate and waste. The results which were obtained should be consistent with theoretical results, but due to inaccuracy of the operation of separators a part of heavy fraction passes to the concentrate, and a part of light fraction gets to the waste. It is particularly visible in the analysis of coal slurries. That is why the curves which illustrate the dependence of specific gravity on the yield of product and they are deviated from the basic curve of the output. The difference between ash content in theoretical waste and the real one is presented by the field of the triangle of concentrate and waste, and the difference between these fields is equal to the area of the triangle of error for the coal cleaning process.

In practice, the area of the triangles of error can be determined be means of the procedure written in Visual Basic, saving time needed for drawing curves in the system of coordinates and calculating the area of the triangles of error.

The procedure uses the method of calculating the area under the curve - the rectangles method. All you need is to divide the products according to two specific gravities 1.5 and 2.0, and to determine weight quantities of incidental fractions in products and the ash content present in them. In order to calculate them it is necessary to determine the weigh percentage of incidental fractions in the products of beneficiation and the ash content present in them in relation to the output at the equivalent specific gravity of separation. The results of the procedure operation are shown in figure 3, and the triangle fields are 0.20%, 0.13% and 0.50%.
Applying that method of determining the areas of the triangles of error it is necessary to accurately establish in the products of beneficiation the quantity of the incidental fractions and the ash content present in them according to the equivalent specific gravities of separation. It is often not possible, in production conditions, to draw the weight curves and that is why the approximate method of the triangle of error is applied. This method is used to determine the triangle of error based on the incidental fractions in the products of beneficiation laid down in layers in accordance with the control specific gravities.

5. Summary
The current state of scientific knowledge as regards minerals engineering allows to use coal slurries very effectively in the process of combustion for energy generation purposes. After suitable preparation the deposited coal slurries can become power generating raw material of full value. After accession of Poland to the European Union, according to its coal related quality requirements, the aspect of coal processing which develops the quality of commercial coal became particularly important. The article presented a group of mathematical methods which can be applied to examine the efficiency of the coal beneficiation process. The degree of their accuracy of application was assumed as the efficiency criterion, i.e. placing such products one the market, which are not contaminated with incidental fractions. For this purpose, the method of the triangle of error was applied as the measure for evaluating the effectiveness of work and in order to improve complicated calculations, a function in the Visual Basic environment was implemented which is used to determine the areas below the curve: for specific gravity, ash content and yield, and it allows to establish the real number of incidental fractions in the product.

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