Ultrafast non-destructive measurement of the quantum state of light with free electrons

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Abstract: We demonstrate that free electrons can be used as ultrafast non-destructive photon detectors. Particularly, we show how one can measure photon statistics, temporal coherence, and implement full quantum state tomography using free electrons. © 2021 The Author(s)

Since the birth of quantum optics, the measurement of photon statistics and quantum states of nonclassical light has been of tremendous importance for advancement in the field [1]. To date, all conventional detection schemes at their core rely on light-matter interactions with bound electrons, as in avalanche photodiodes, superconducting nanowires, photomultiplier tubes, and in other platforms [2-4]. Although widely used, the state-of-the-art detectors based on bound electrons still have limited capabilities in simultaneous detection of multiple photon events; they also require long integration times and are generally limited in their response time and electronic bandwidths. Therefore, it is interesting to ask whether fundamentally different types of interactions could be harnessed to improve quantum optical detection.

Here we propose for the first time the concept of utilizing free electrons as a high-precision quantum-optical detector, having a fast temporal response and high sensitivity to the photon number. Our proposal is based on recent measurements of free electrons’ quantum interaction with light, first demonstrated as part of the photon-induced nearfield electron microscopy (PINEM) technique [5-8], wherein free electrons are capable of high-order multi-photon absorption and emission. Using the PINEM interaction, we develop a scheme of free-electron quantum-optical detection (FEQOD), illustrated in Fig. 1a. We show that the electron energy spectrum gives rich information about the photonic state of the quantum light: measure the photon statistics, the degrees of temporal coherence, and implement full quantum state tomography with FEQOD.

![Figure 1](image)

**Fig. 1 Free-electron quantum-optical detection (FEQOD).** (a) The electron energy spectrum measurement after its interaction with a quantum state of light allows us to reconstruct the photon statistics. The experiment can be performed in ultrafast electron microscopes. (b)-(d) The measured electron energy spectra, reconstructed photon statistics, and Wigner functions for coherent, phase-squeezed, and thermal light.

Due to the strong nonlinearity and ultrafast timescale of the PINEM interaction, our technique can reach a sub-attosecond temporal resolution and enables detection of an arbitrary number of photons simultaneously by each electron. Another key feature of our method is the ability to implement non-destructive measurements since the state of light only weakly changes following the interaction with electrons.

The formalism we develop builds on significant recent advances in the theory of PINEM, and especially its description with a second-quantized electromagnetic field [9, 10]. We generalize these recent results and derive a
fully-analytic formula for the electron energy spectrum after the interaction (applicable to any interaction strength \(g_q\) and any number of photons):

\[
P_k = \begin{cases} 
{k^2 |g_q|^2} & \rho_{ph} \frac{1}{\langle k | \rho_{ph} | k \rangle} F_1 \left( 1 + \tilde{n}, 1 + |k|, -|g_q|^2 \right) , \quad k > 0 \end{cases}
\]

\[
\begin{cases} 
{k^2 |g_q|^2} & \rho_{ph} \frac{(\tilde{n}+|k|)+1}{\langle k | \rho_{ph} | k \rangle} F_1 \left( 1 + |k|, 1 + |k|, -|g_q|^2 \right) , \quad k < 0 \end{cases}
\]

where \(P_k\) is the probability for the electron to absorb/emit \(k\) photons; \(\tilde{n}\) is the photon number operator; \(F_1\) is the hypergeometric function; \(\rho_{ph}\) is a trace over the photonic state. Using the same theory but tracing over the electron state, we also quantify the change of the photonic state following the interaction. This way, we analyze the non-destructive nature of FEQOD for both the diagonal and off-diagonal elements of the photonic density matrix.

The examples of different PINEM spectra calculated according to Eq. (1) are shown in (Fig. 1b-d). We show a one-to-one correspondence between the electron energy spectrum described by Eq. (1) and all the photonic number operator moments, which can be used to reconstruct the photon statistics of the light. Specifically, the \(m^n\) moment of the photon number operator \(\langle n^m \rangle = \sum_{n=0}^{\infty} \rho_{ph} n^m\) can be extracted as \(\langle n^m \rangle = \sum_{k=m}^{\infty} d_{mk} P_k\), where the coefficients \(d_{mk}\) are described by

\[
d_{mk} = 2 \frac{-m+1}{2} \frac{-m}{2m} \frac{1}{|g_q|^2} \left( \frac{m+1}{2} \frac{m+1}{2} \right) F_1 \left( m+1, m+1, -|g_q|^2 \right).
\]

We quantify the precision of such measurements as a function of the number of electrons, coupling strength, fluctuations in the coupler’s orientation, and light intensity. FEQOD is ultrafast since the interaction between free electrons and light occurs on femtosecond timescales in regular PINEM experiments and is shown to reach attosecond timescales [11], with prospects for sub-attosecond timescales. Moreover, unlike conventional number-resolving detectors [2-4], a free-electron-based detector does not destroy the quantum state of light – achieving a non-destructive measurement. The full reconstruction of the quantum state of light requires both the photon number and the phase, which can be achieved using a scheme analogous to homodyne tomography [12] – having a second point of interaction with a coherent laser depicted in Fig 2(a). We also propose a scheme (Fig 2(b)) to find the degrees of temporal coherence of light, such as \(g^{(2)}(\tau)\) (similar to the Hanbury-Brown-Twiss experiment).

**Fig. 2** Free-electron-based quantum state reconstruction and measurement of the temporal coherence of light. (a) For quantum tomography of light, the electron interacts first with a laser with phase \(\theta\) and then with the quantum light we want to reconstruct. We depict the electron energy spectrum as a function of the laser phase \(\theta\) and the reconstructed Wigner function for different photonic states. The \(x\)-axis is the electron energy, the \(y\)-axis is the phase \(\theta\), and the probability is denoted by the color bar. (b) For measurement of temporal coherence, the electron interacts with the quantum light twice, with different delays. The electron spectrum is measured and analyzed for each delay. We plot the electron spectrum data and the modified correlation functions \(\tilde{g}_1(\tau)\) and \(\tilde{g}_2(\tau)\) as a function of delay \(\tau\), for different photonic states.

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