Baryogenesis and Polyakov line condensate during inflation

As. Abada\textsuperscript{a}, M.B. Gavela\textsuperscript{b} and O. Pène\textsuperscript{a}

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\textsuperscript{a} Laboratoire de Physique Théorique et Hautes Énergies\textsuperscript{[1]}
Université de Paris XI, Bâtiment 211, 91405 Orsay Cedex, France
\textsuperscript{b} Departamento de Física Teórica, Universidad Autónoma de Madrid, Canto Blanco, 28049 Madrid.

Abstract

The thermal average of the Polyakov line is non-zero at high temperature. It has been suggested that its phase could be relevant for baryogenesis within the standard model, providing a source of spontaneously broken CP violation. This scenario suffers from a causal domain problem which cannot be cured by inflation: quantum fluctuations in de Sitter space overwhelm the periodicity of the potential.

\textsuperscript{1}Laboratoire associé au Centre National de la Recherche Scientifique - URA D00063
e-mail: abada@qcd.th.u-psud.fr, gavela@delta.ft.uam.es, pene@qcd.th.u-psud.fr.
1 Introduction

The origin of the absence of primordial antimatter in the universe (or at least in the Virgo cluster of galaxies) remains unclear. The observed baryon to photon density ratio, $n_B/n_\gamma \sim 10^{-9}$, although small, is difficult to explain.

The most promising avenue to address this problem is baryogenesis: starting from a matter-antimatter symmetric universe, the microscopic laws of physics should affect the cosmological evolution so as to result in the above mentioned experimental observation. For this to happen [1], the presence of C, CP and baryon number (B) violating processes during an out-of-equilibrium phase of the expansion is required.

A baryogenesis mechanism based exclusively on the Standard Model is most appealing. It has been argued [2] that an out-of-equilibrium mechanism would be provided by a first order electroweak phase transition, although the pertinent dynamical conditions seem to be excluded by the present bound on the Higgs mass [3]. Assuming such a transition, B-violating transitions stem from the anomaly [4], [2]. The remaining question is whether the Standard Model provides with sufficient strength the rest of the ingredients, that is, C and CP violation, the latter being the most difficult issue.

Two well-known sources of CP violation are potentially present in the standard model, both explicit in character. The QCD $\theta$ parameter, if different from zero, is one of them, albeit a C conserving one. No mechanism based on it has been sucessfully proposed. The other one is of electroweak origin, the phase in the Cabibbo-Kobayashi-Maskawa matrix. This road has been attempted [5], although it has been shown [6] that the thermal noise in the quark-gluon plasma reduces the resulting baryon asymmetry in this scenario to $n_B/n_\gamma \leq 10^{-20}$, less than ten orders of magnitude below the needed figure.

It has been recently realized that two additional sources of CP violation are present in the Standard Model at finite temperature, due to the related breaking of Lorentz invariance. Both are examples of spontaneous breaking of CP violation and will then have to face the issue of uncorrelated CP phases in causally disconnected domains, that is whether the typical size of such domains is larger than the present horizon or, at least, larger than a typical cluster. One of these scenarios is based on $Z$ meson condensates [7], we will not dwell on it here.

The other one [8]-[10] is based on thermal effects of the phase of the Polyakov line:

$$P(\vec{x}) = \frac{1}{n} \text{Tr}\{P e^{i g \int_0^\beta A_0(\vec{x},t) dt}\},$$  \hspace{1cm} (1)$$

where $A_\mu$ is the SU(N) gauge field, $g$ is the coupling constant, $P$ denotes the path ordering and $\beta = 1/T$, $T$ being the temperature.

The thermal average of the Polyakov line, $< P(\vec{x}) >_T$, is a gauge invariant quantity and constitutes an order parameter for confinement in pure Yang-Mills
theory \cite{1}, i.e. when only gauge fields are present. The action presents the well-known $Z(N)$ discrete symmetry whose effect is to multiply the average $<P(\vec{x})>_T$ by one of the numbers: $e^{\frac{2\pi ik}{N}}$, $k = 0, 1, ..., N-1$. At low temperature $<P(\vec{x})>_T = 0$, and the theory confines. At high temperature, the symmetry is spontaneously broken, $<P(\vec{x})>_T \neq 0$, corresponding to the nonconfining phase.

At high temperature it is appropriate to define an effective potential for the quantity $<P(\vec{x})>_T \cite{2}$, which presents $N$ degenerate minima: $<P(\vec{x})>_T = e^{\frac{2\pi ik}{N}}$, $k = 0, 1, ..., N-1$, reflecting the discrete $Z(N)$ symmetry. Those minima with non-trivial phases are putative sources of CP violation.

In the presence of fermions the degeneracy is lifted $\cite{3}$, and $<P(\vec{x})>_T = 1$ ($k = 0$) in the true vacuum. The remaining local minima are metastable. In its cosmological evolution, the universe could have fallen in one of those at some GUT or Planck mass scale $\cite{3}, \cite{10}$. In the standard cosmology these metastable minima decay by tunneling $\cite{13}$. The authors of $\cite{14}$ argue that they are long-lived and have not decayed by the time of the electroweak transition i.e. $10^2$ GeV.

There are some serious objections $\cite{13}-\cite{17}$ to this hypothesis. The domains corresponding to the minima found in the potential of $<P(\vec{x})>_T$, or the barriers (walls) separating them, might not be interpreted as real physical objects in Minkowski space. This issue deserves a careful study not attempted in this work, although it will eventually surface disguised in the form of a “complex chemical potential”.

In this paper we will address the issue of causality. The “choice” between the metastable or stable vacua being spontaneous, one same vacuum cannot cover a region larger than a causally connected domain at the time of spontaneous symmetry breaking, i.e., necessarily before the electroweak phase transition. In a standard expansion of the universe, not inflationary, such a causally connected domain evolves eventually into a region whose size today does not exceed a few million of kilometers, much smaller than the size of a cluster of galaxies, $\sim 25$ Mpc. Such a spontaneously broken CP symmetry could not explain that the baryon number asymmetry keeps the same sign at least all through our cluster of galaxies.

There are two possible ways out:

1. A mechanism of explicit breaking of the CP symmetry which tilts the spontaneous one, as for instance advocated in $\cite{7}$ for the $Z$ condensate. We know of no such mechanism in the scenario under discussion.

2. Inflation. If the domains have formed long enough before the end of an inflationary period, one might expect them to grow into a region much larger today than a cluster of galaxies$\cite{4}$. The purpose of this paper is to

\footnote{The group constituted by the elements: $e^{\frac{2\pi ik}{N}} \mathbf{1}$, $k = 0, 1, ..., N-1$, is the center of SU(N).}

\footnote{During inflation, a small and causally connected patch of size $\leq H^{-1}$ grows exponentially to encompass the entire today’s universe.}
decide whether this might have happened or not.

A priori, as the temperature falls exponentially during inflation, it can be expected that the effective potential for the condensate flattens. Thermal fluctuations should become negligible as well. Quantum fluctuations in de Sitter space may allow the growth of the condensate, which could then get arbitrarily large values by the end of inflation. Upon reheating, the effective potential starts to wiggle again, and the condensate could fall into the nearest minimum.

The question to decide is whether the ensemble of then causally disconnected domains which forms our universe today (or our cluster) has fallen into the same minimum, or else have a random distribution, incompatible with observation. Those domains, disconnected at the end of inflation, have certainly belonged to a common causally connected one, some time in the past, during inflation. The issue is whether this common memory forces them to fall into the same minimum of the effective potential during reheating.

This type of problem has been extensively studied for scalar field potentials. The new elements to take into account are:

1) The fact that the condensate, although described by an effective potential, is not alike to a simple dynamical field, such as the inflaton.

2) The periodicity in the phase of the effective potential, and the dependence of the period on temperature, which will play a major role.

The scenario uses the imaginary time thermofield formalism in which the real time is complexified: $t \rightarrow i\tau$. The “evolution” in the imaginary time direction describes the Boltzmann distribution $e^{-\beta H}$ ($\beta = 1/T$), and the real time disappears since one describes a stationary thermal equilibrium.

During inflation, in order to consider the interplay of thermal effects with quantum ones, it is necessary to develop some tools for treating gauge fields at finite temperature in de Sitter space. Even when the temperature becomes negligible, the thermal density matrix is relevant to understand the beginning and the end of inflation. More important, a redshifted density matrix remains during inflation as the relic from the pre-inflationary thermal one. A real time variable has to be kept as a record of the cosmological time. The choice of a complex time variable, for temperature purposes, is non trivial as many possible choices are possible in cosmology, which would lead to very different results after complexification. We will have to find our way through these meanders.

We solve this puzzle starting from basics. We derive the thermal density matrix that generalizes the Boltzmann distribution. We then investigate whether any time variable has an evolution operator that allows to describe the latter density matrix via complexification, and we determine it.

The complete scenario requires to consider the effective potential for $SU(3) \times SU(2) \times U(1)$ with fermions. Our physical results are general enough so as

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4 This is in fact a delicate point, as the dynamics of reheating is a difficult problem which remains to be explored. The conclusions of our paper make this issue irrelevant.
to be present in a simpler field theory. For this reason we will perform the computations for a pure gauge $SU(2)$ Lagrangian. All throughout the paper, the gravitational effects of the gauge fields are neglected, that is, the vacuum energy is approximated by the inflaton contribution and taken as constant.

In section 2, we derive the field equations for the gauge fields in de Sitter space, and build the corresponding Fock space. In section 3, the free energy both with real and imaginary time is computed, and an effective potential with imaginary time follows. In section 4, we study the stability of the metastable vacua for thermal and quantum fluctuations and finally we conclude.

2 Fock expansion of the gauge fields

2.1 Metric

During inflation, the scale factor $a(t)$ grows exponentially and the metric can be written in the form:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dt^2 - e^{2Ht}(dx^2 + dy^2 + dz^2).$$  \hfill (2)

where $t$ is the proper time measured in the comoving frame where the spatial variables are constant, and therefore denoted comoving variables, which means that a massive object at rest at the position $\vec{x}$ will keep the coordinate $\vec{x}$ in later times. $H$ is the expansion rate $\dot{a}(t)/a(t)$ of the universe, that is, the Hubble“constant”, and the metric tensor $g_{\mu\nu}$ is given by

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -e^{2Ht} & 0 & 0 \\ 0 & 0 & -e^{2Ht} & 0 \\ 0 & 0 & 0 & -e^{2Ht} \end{pmatrix}; \quad g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -e^{-2Ht} & 0 & 0 \\ 0 & 0 & -e^{-2Ht} & 0 \\ 0 & 0 & 0 & -e^{-2Ht} \end{pmatrix}. \quad (3)$$

Its determinant is

$$\text{det}\{g_{\mu\nu}\} = \sqrt{-g} = e^{3Ht}. \quad (4)$$

It is convenient to express the metric in terms of the “cosmic time” defined by $d\eta = dt/a(t) + b$, where the scale factor has the form $a(t) = a_0 e^{Ht}$. The origin for $t$ and $\eta$ is chosen so that $b = 0$ and $H\eta = -1$ at the beginning of inflation. The cosmic time,

$$\eta = -H^{-1} e^{-Ht}, \quad (5)$$

is such that the metric takes the form

$$ds^2 = a(\eta)^2(d\eta^2 - (dx^2 + dy^2 + dz^2)), \quad (6)$$
which explains why \( \eta \) is often dubbed “conformal time”.

It is sometimes useful to use the so-called physical variables, where the proper time is kept, and \( d\vec{x}_{\text{phys}} = e^{Ht} d\vec{x} \). All computations will be performed in the comoving variables, the others will be referred to in physical discussions.

### 2.2 Field equations

We assume that the inflationary process is driven by a scalar “inflaton” field. Within this space, it is possible to study the field equations for gauge and fermion fields, and it is consistent to simultaneously neglect the effect of those fields on the inflation rate. The latter is dominated by the “inflaton” ones and will be taken as constant during inflation.

In flat space, the pure \( SU(2) \) Lagrangian density is

\[
\mathcal{L} = -\frac{1}{4} G^a_{\mu
u} G^{a\mu\nu}, \tag{7}
\]

where \( a = 1, 2, 3 \) labels the isospin. In this paper we shall restrict ourselves to the \( SU(2) \) gauge group, although a generalization to \( SU(N) \) is straightforward.

In de Sitter space it is convenient to define as well the Lagrangian

\[
\mathcal{L}^{\text{DS}} = \sqrt{-g} \mathcal{L} = -\frac{1}{4} e^{3Ht} G_{\mu\sigma} G^\sigma_{\nu} g^{\nu\mu}, \tag{8}
\]

so that the action is given by

\[
S = \int d^4x \mathcal{L}^{\text{DS}}. \tag{9}
\]

\( \mathcal{L}^{\text{DS}} \) can be expressed in a non-covariant way,

\[
\mathcal{L}^{\text{DS}} = \frac{\sqrt{-g}}{2} \sum_{a=1}^{3} \left[ e^{-2Ht} \sum_{l=1}^{3} (G^a_{0l}^2) - e^{-4Ht} \sum_{j<k}^{3} (G^a_{jk}^2) \right]. \tag{10}
\]

Developing eq. (10) up to terms quadratic in the fields,

\[
\mathcal{L}^{\text{DS}} = \frac{\sqrt{-g}}{2} \left\{ e^{-2Ht} \sum_{i=1}^{3} \left\{ \sum_{a=1}^{3} \left( (\partial_0 A^a_i - \partial_i A^a_0 + (A_0 \wedge A_i)^a)^2 \right) \right\} \right. \\
- \frac{1}{2} e^{-4Ht} \sum_{j,k}^{3} \left\{ \sum_{a=1}^{3} (\partial_j A^a_k - \partial_k A^a_j)^2 \right\} \left. + \ldots \right\}. \tag{11}
\]

Higher order terms are neglected as, at high temperature, the interaction is assumed to be weak. We will work in the temporal gauge \( A^a_0 = 0, a = 1, 2, 3 \).

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5 Indeed, it will be shown that the gauge field contribution to the energy-momentum tensor vanishes as \( e^{-4Ht} \).
In terms of the conformal time, eq. (5), the de Sitter Lagrangian writes:

\[
L_{DS} = \frac{e^{-Ht}}{2} \left\{ \sum_{i=1}^{3} \left\{ \sum_{a=1}^{3} (\partial_\eta A_i^a)^2 \right\} - \frac{1}{2} \sum_{j,k} \left\{ \sum_{a=1}^{3} (\partial_j A_k^a - \partial_k A_j^a)^2 \right\} + \ldots \right\}. \tag{12}
\]

It is noticeable that this Lagrangian is identical to the flat space one up to the global factor \(e^{-Ht}\). For later use we have chosen a privileged direction in isospin space, \(\hat{3}\), and the following convention for the transverse gauge fields:

\[
A_i^\pm = \frac{1}{\sqrt{2}} (A_i^1 \pm i A_i^2). \tag{13}
\]

The equations of motion for the fields, \(A_i^a\) and \(A_i^\pm\), can be obtained either by varying the action \((S = \int d^4x \sqrt{-g}L)\), or using the conservation of the energy-momentum tensor, \(T_{\mu\nu}\).

For \(A_i^a\) \((a = +, -, 3)\) the evolution equation is

\[
\ddot{A}_i^a + H \dot{A}_i^a - e^{-2Ht}(\Delta A_i^a - \partial_i (\vec{\nabla} \cdot \vec{A}^3)) = 0. \tag{14}
\]

The transversally polarized components of \(A_i^a\), verify

\[
\ddot{A}_i^a + H \dot{A}_i^a - e^{-2Ht} \Delta A_i^a = 0, \tag{15}
\]

and we will restrict ourselves to their study from now on, since only radiation modes are relevant in this problem. In terms of the conformal time, eq. (15) reduces to

\[
\frac{\partial^2 A_i^a}{\partial \eta^2} + \Delta A_i^a = 0. \tag{16}
\]

Before expanding the gauge fields in momentum space, it is necessary to determine the conjugate fields in order to fix the normalization of the solutions.

\[
\pi^{\mu(a)} = \frac{\partial L_{DS}}{\partial (\partial_\mu A_i^a)}; \quad \left\{ \begin{array}{l} \pi^{0(a)} = 0 \\ \pi^i = E^i(a) \end{array} \right. \tag{17}
\]

In analogy with \(A_i^1\) and \(A_i^2\), the transverse conjugate fields, \(E_i^1\) and \(E_i^2\), are defined by

\[
E_i^\pm = \frac{1}{\sqrt{2}} (E_i^1 \pm i E_i^2). \tag{18}
\]

\footnote{It is necessary to pay careful attention to the position of the spatial index. Because of the form of \(g^{\mu\nu}\), eq. (8), \(f_i\) corresponds to \(\frac{d}{dt}f_i\) only when the index \(i\) is down, and \(\vec{\nabla} \cdot \vec{A}^3 = \sum_j \partial_j A_j^3\).}
and the corresponding conjugate fields, $E_i^a$, are

$$ E_i^a = -e^{3Ht}(\dot{A}_i^a) = -\frac{1}{\eta^2 H^2} \frac{\partial}{\partial \eta} A_i^a. \quad (19) $$

The standard canonical commutation relations are still valid in de Sitter space,

$$ [A_i^a, E_j^b] = ig_{ij} \delta^3(\vec{x} - \vec{y}) \delta_{ab} = -i \delta_{ij} \frac{1}{\eta^2 H^2} \delta^3(\vec{x} - \vec{y}) \delta_{ab}. \quad (20) $$

### 2.3 Gauge field expansion into creator and annihilators

It will be shown that the solutions of the field equations are analogous to those in flat space, with the replacement of $t$ by the cosmological time $\eta$.

$$ A_i^3(\vec{x}, t) = \int d^3k \sum_{\lambda=1,2} [A_k^{\lambda 3}(\eta) \varepsilon_i^{\vec{k}, \lambda} a^{\lambda}(\vec{k}, \lambda) e^{i\vec{k} \cdot \vec{x}} + A_k^{* \lambda 3}(\eta) \varepsilon_i^{\vec{k}, \lambda} a^{\lambda \dagger}(\vec{k}, \lambda) e^{-i\vec{k} \cdot \vec{x}}] \quad (21) $$

$$ A_i^\pm(\vec{x}, t) = \int d^3k \sum_{\lambda=1,2} [A_k^{\lambda \pm}(\eta) \varepsilon_i^{\vec{k}, \lambda} a^{\lambda \mp}(\vec{k}, \lambda) e^{i\vec{k} \cdot \vec{x}} + A_k^{* \lambda \mp}(\eta) \varepsilon_i^{\vec{k}, \lambda} a^{\lambda \mp \dagger}(\vec{k}, \lambda) e^{-i\vec{k} \cdot \vec{x}}] \quad (22) $$

In eqs. (21) and (22), the $\sum$ extends upon the transversal polarization modes, with the corresponding vectors, $\varepsilon_i^{\vec{k}, \lambda}$, chosen to be real and normed, $\varepsilon_i^{\vec{k}, \lambda} \varepsilon_j^{\vec{k}', \lambda'} = \delta_{ij}$, for $\lambda = 1, 2$.

The annihilation and creation operators satisfy the commutation relations

$$ \begin{align*}
[a^{b}(\vec{k}, \lambda), a^{c \dagger}(\vec{k}', \lambda')] &= \delta_{bc} \delta_{\lambda \lambda'} \delta^3(\vec{k} - \vec{k}') \\
[a^{b \dagger}(\vec{k}, \lambda), a^{c \dagger}(\vec{k}', \lambda')] &= [a^{b}(\vec{k}, \lambda), a^{c}(\vec{k}', \lambda')] = 0.
\end{align*} \quad (23) $$

From eq. (14),

$$ \frac{d^2}{d\eta^2} A_k^{\lambda a}(\eta) + k^2 A_k^{\lambda a}(\eta) = 0, \quad (24) $$

with solution

$$ A_k^{\lambda a}(\eta) = ae^{ik\eta} + be^{-ik\eta}, \quad (25) $$

where $k = |\vec{k}|$. For $Ht \to 0$, the matching with the flat space solution, $\propto e^{-ikt}$, requires $a = 0$. Imposing finally the canonical commutation relations, eq. (20), it results

$$ A_k^{\lambda a}(\eta) = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2k}} e^{-ik\eta}. \quad (26) $$
3 The effective potential for the Polyakov line

Our goal in this section is to determine the thermodynamical potential corresponding to the gauge fields, in order to derive the effective potential for the Polyakov line,

\[ V_{\text{eff}}(\langle P(\vec{x}) \rangle) = \frac{F(\mu, V, T)}{V}. \]  

(27)

For this, it suffices to compute the energy-momentum tensor. We recall that the “thermodynamical potential” of the Grand-Canonical ensemble is given by

\[ F(\mu, V, T) = -\beta^{-1} \log Z_G, \]  

(28)

where \( Z_G \) is the partition function of the ensemble, \( \mu \) is the chemical potential, \( V \) the volume and \( T \) the temperature. From now on the parameters will be implicit, and \( \mu = 0 \) unless otherwise stated. Recalling that

\[ dF = -Nd\mu - pdV - SdT, \]  

(29)

where \( N, p, S \) denote the average number of modes, the pressure and the entropy respectively, it results

\[ p = -\frac{\partial F}{\partial V}, \]  

(30)

and in the thermodynamical limit, \( N \to \infty, V \to \infty \), the function \( F \) becomes the free energy.

The pressure may be deduced from the energy-momentum tensor through the relation

\[ \langle T_{\mu}^\nu \rangle = \text{diag}(\rho, -p, -p, -p), \]  

(31)

\( \rho \) being the energy density and \( \langle T_{\mu}^\nu \rangle \) the thermal average of \( T_{\mu}^\nu \). In writing (31), the isotropy in the comoving frames, in which the fluid is instantaneously at rest, has been used.

Furthermore, from the equation of state for radiation\footnote{We shall derive this equation in de Sitter space in the next subsection.},

\[ p = \frac{1}{3}\rho, \]  

(32)

it results for the free energy

\[ F = -\frac{1}{3}\rho V, \]  

(33)

and we just need to compute \( \rho = \langle T_{00} \rangle \). In the following subsection we will then compute the energy-momentum tensor.
3.1 Gauge field energy-momentum tensor and the effective potential

The energy-momentum tensor is defined by:

\[ T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\partial(\sqrt{-g} \mathcal{L})}{\partial g^{\mu\nu}} - \frac{\partial}{\partial \rho} \frac{\partial(\sqrt{-g} \mathcal{L})}{\partial (\partial_{\rho} g^{\mu\nu})}. \] (34)

Using

\[ \frac{\partial \sqrt{-g}}{\partial g^{\mu\nu}} = -\frac{1}{2} \sqrt{-g} g_{\mu\nu}, \] (35)

we get

\[ T_{\mu\nu} = -G_{\mu\alpha} G_{\nu\beta} g^{\alpha\beta} - g_{\mu\nu} L, \] (36)

where the sum over color or weak isospin indices is understood. More explicitly:

\[ T_{00} = -\frac{1}{2} \left[ \sum_i G_{\alpha i}^2 g^{ii} - \sum_{i<k} g^{ii} g^{kk} G_{ik}^2 \right], \] (37)

\[ T_{i\bar{i}} = -G_{0i}^2 + \frac{1}{2} g_{ii} \left( \sum_l g^{ll} G_{0l}^2 \right) - \sum_k G_{ik}^2 g^{kk} + \frac{1}{2} g_{ii} \sum_{j<k} G_{jk}^2 g^{kk} g^{jj}, \] (38)

\[ \sum_i T_{i\bar{i}} = \frac{1}{2} \left[ \sum_i G_{\alpha i}^2 - \sum_{i<k} g^{kk} G_{ik}^2 \right]. \] (39)

From eq. (31),

\[ p = -\sum_i \frac{1}{3 g_{ii}} < T_{i\bar{i}} > \] (40)

and

\[ -\sum_i \frac{1}{3 g_{ii}} T_{i\bar{i}} = \frac{1}{3} T_{00}, \] (41)

verifying the equation of state, eq. (32).

In order to compute \( \rho = < T_{00} > \), we first expand \( T_{00} \) over the fields \( A_i^a \):

\[ T_{00} = -\frac{1}{2} \sum_i g^{ii} \left\{ [\dot{A}_i^+ \dot{A}_i^- + \dot{A}_i^- \dot{A}_i^+] + (\dot{A}_i^3)^2 \right\} 
- \frac{1}{2} \sum_k g^{kk} \left[ (\partial_i A_k^+ - \partial_k A_i^+) (\partial_i A_k^- - \partial_k A_i^-) 
+ (\partial_i A_k^+ - \partial_k A_i^-) (\partial_i A_k^- - \partial_k A_i^+) + (\dot{A}_i^3 - \dot{A}_k^3)^2 \right]. \] (42)
As interaction terms have been neglected, the density matrix is diagonal in the Fock basis. The thermal average of creation and annihilation operators is then \( \langle a^b a^{b \dagger} \rangle = \langle a^b a^b \rangle = 0 \). \( \langle T_{00} \rangle \) is thus expressed in terms of \( \langle a^{b \dagger} a^b \rangle \) and \( \langle a^b a^{b \dagger} \rangle \).

Denoting by \( T_0 = 1/\beta_0 \) the temperature at the beginning of the de Sitter phase, it results

\[
\begin{align*}
\langle T_{00} \rangle &= e^{-4Ht} \int_0^\infty dk k^3 \sum_{b, \lambda} \left[ n_k^{b \lambda}(\beta_0) + \frac{1}{2} \right],
\end{align*}
\]

where \( n_k^{b \lambda} \) are the “Bose-Einstein” distribution functions (we are in the “perfect gaz approximation”):

\[
\begin{align*}
n_k^{b \lambda}(\beta_0) &= \frac{1}{e^{\beta_0 k} - 1}. 
\end{align*}
\]

In terms of these distribution functions, \( \langle T_{00} \rangle \) writes:

\[
\langle T_{00} \rangle = \frac{3 e^{-4Ht}}{\pi^2} \int_0^\infty dx x^2 \log \left( 1 - e^{-x} \right).
\]

It turns out that the energy density simply scales as \( T^4 = T_0^4 \exp[-4Ht] \), as naively expected. This result is reassuring, though, as an unexpected time behaviour of thermal effects has been observed in another problem [22] for the scalar field condensates in de Sitter space.

Using the following relation,

\[
\begin{align*}
n_k^{b \lambda}(\beta_0) &= \frac{1}{\beta_0} \frac{\partial}{\partial k} \log Z_k^{b \lambda} = \frac{1}{\beta_0} \frac{\partial}{\partial k} \log \left( 1 - e^{-\beta_0 k} \right),
\end{align*}
\]

where \( Z_k^{b \lambda} \) is the contribution of the mode \((k, b, \lambda)\) to the partition function, it results

\[
\begin{align*}
\langle T_{00} \rangle &= -\frac{9}{\pi^2} \frac{e^{-4Ht}}{\beta_0} \int_0^\infty dk k^2 \log \left( 1 - e^{-\beta_0 k} \right) \\
&= -\frac{9}{\pi^2} \left( \frac{e^{-Ht}}{\beta_0} \right)^4 \int_0^\infty dx x^2 \log \left( 1 - e^{-x} \right).
\end{align*}
\]

From eq. (B3), the free energy at a given time \( t \), integrated over a comoving patch which had volume \( V_0 \) at the beginning of inflation, is given by

\[
\begin{align*}
F(\beta) &= -\frac{1}{\beta_0} \log Z(\beta_0) = -\frac{1}{3} \rho V = -\frac{1}{3} \rho a(t)^3 = -\frac{1}{3} \langle T_{00} \rangle V_0 e^{3Ht} \\
&= \frac{3}{\pi^2} V_0 T_0^4 e^{-Ht} \int_0^\infty dx x^2 \log \left( 1 - e^{-x} \right),
\end{align*}
\]

where \( \beta = \beta_0 e^{Ht} = 1/T \), and \( V \) is the volume of the patch at time \( t \).
3.2 Computation of the effective potential with an imaginary conformal time

In flat space, thermal averages of Green functions can be obtained as vacuum expectation values of the corresponding operators in a field theory with an imaginary (Euclidian) time, \( \tau \in [0, \beta = 1/T] \), with periodic boundary conditions on the fields, \( A_\mu^a(t = 0) = A_\mu^a(t = \beta) \) [18].

In de Sitter space, the free energy, eq. (49), can be recovered from the field theory eq. (7), by analytically continuing the action, in the complex plane of the conformal time \( \eta \), eq. (5).

Let us define \( \gamma = \text{Im} \eta \). The gauge field equations will be expressed in terms of \( \gamma \), while we will keep the real time \( t \) (or equivalently \( \text{Re} \eta \)) to describe the gravitational expansion. All fields are taken as periodic functions of \( \gamma \) with period \( \beta_0 \).

As in [12], we choose a gauge,

\[
A_\gamma^a(\gamma, \vec{x}) = \delta_{a3} \left( C_0 + \phi(\vec{x}) \right)
\]

(50)

where the spatial average of \( \phi(\vec{x}) \), is \( \int d\vec{x} \phi(\vec{x}) = 0 \). A derivation analogous to the one developped by N. Weiss [12], results in the partition function:

\[
Z(\beta) = N \int \mathcal{D}(g\beta_0 A_\gamma) dA_\gamma^a(\vec{x}, \gamma) e^{-\frac{1}{2} \int_0^{\beta_0} d\gamma \int d^3x [ (\partial_\gamma A_i - \partial_i A_\gamma + gA_i \times A_i)^2 + B^2]}
\]

(51)

where

\[
B^2 = \sum_{i<j} G_{ij}^2
\]

(52)

The calculation then proceeds exactly as in [12] leading to

\[
\langle T_{00} \rangle = -\frac{3}{\pi^2} \frac{e^{-Ht}}{\beta_0^4} \int_0^\infty dx \left\{ x^2 \log \left( 1 + e^{-2x} - 2e^{-x} \cos(\beta_0 C_0) \right) + x^2 \log \left( 1 - e^{-x} \right) \right\}
\]

(53)

and to the free energy

\[
F(\beta) = -\frac{1}{\beta} \log Z(\beta) = -\frac{1}{3} \langle T_{00} \rangle V_0 e^{3Ht}
\]

\[
= \frac{1}{\pi^2} V_0 T_0^4 e^{-Ht} \int_0^\infty dx \left\{ x^2 \log \left( 1 + e^{-2x} - 2e^{-x} \cos(\beta_0 C_0) \right) + x^2 \log \left( 1 - e^{-x} \right) \right\}
\]

\[
= \frac{1}{\pi^2} V T^4 \int_0^\infty dx \left\{ x^2 \log \left( 1 + e^{-2x} - 2e^{-x} \cos(\beta_0 C_0) \right) \right\}
\]

13
where the last contribution in eqs. (53) and (54) correspond to the excitations along the $\hat{3}$ direction, and where $V = V_0 \exp[3Ht]$.

Notice that taking $C_0 = 0$ in eq. (54) gives the relation (49), which was derived in Minkowski space with a density matrix formalism in real time. This agreement indicates that it is possible to use the imaginary time formalism in de Sitter space.

Using the relations:

$$\log\left(1 + e^{-2x} - 2e^{-x} \cos(\beta_0 C_0)\right) = 2\text{Re} \log\left(1 - e^{-x} e^{i\beta_0 C_0}\right),$$

and

$$\log\left(1 - e^{-x} e^{i\beta_0 C_0}\right) = -\sum_{n=1}^{\infty} \frac{(e^{-x} e^{i\beta_0 C_0})^n}{n},$$

it results

$$\left\langle T_{00}\right\rangle = +6\pi^2 \left(\frac{e^{-Ht}}{\beta_0}\right)^4 \left\{\frac{1}{45}(1 + \frac{1}{2}) - \frac{1}{24}\left(1 - \left(\frac{\beta_0 C_0}{\pi}\right)_{\text{Mod}2} - 1\right)^2\right\}^2.$$  (58)

Finally, the effective potential of the system is

$$V_{\text{eff}}\left\langle P(\vec{x})\right\rangle = -\frac{1}{3}\rho = -2\pi^2 \left(\frac{e^{-Ht}}{\beta_0}\right)^4 \left\{\frac{1}{45}(1 + \frac{1}{2}) - \frac{1}{24}\left(1 - \left(\frac{\beta_0 C_0}{\pi}\right)_{\text{Mod}2} - 1\right)^2\right\}^2.$$  (59)

The effective potential in de Sitter space, eq. (59), is strikingly similar to the flat space one, eq. (24) in ref. [12]. Up to a global factor $e^{-4Ht}$, it does not depend on the real time $t$. Notice also that the integration over momentum $k$ has been taken from 0 to $\infty$ including the wavelengths larger than the size of the horizon, $H^{-1}e^{-Ht}$. A discussion on this issue will appear in subsection 4.1.

### 3.3 Back to Minkowski space

Is it possible to understand the physical meaning of the $\cos(\beta C_0)$ appearing in eqs. (53)-(59)? The strange thermodynamical properties of domains with a non-vanishing $C_0$ have been analysed in the case of a flat space in [13]. One amusing exercise consists in introducing chemical potentials $\mu^+$ and $\mu^-$ related to the charges + and - respectively, in equations eqs. (44)-(49). We will proceed
briefly along these lines just for illustrative purposes, as such a choice is void of physical meaning: there is no thermodynamical stability unless $\mu^+ = \mu^-$. It leads to the Bose-Einstein distribution functions

$$n_k^{(\pm)}(\beta_0) = \frac{1}{e^{\beta_0 (k-\mu^\pm)} - 1},$$
$$n_k^{(3)}(\beta_0) = \frac{1}{e^{\beta_0 k} - 1}.$$  \hspace{1cm} (60)

Considering imaginary chemical potentials, $\mu^+ = -\mu^- = iC_0$,

$$n_k^{(+)} + n_k^{(-)} = \frac{1}{\beta_0} \frac{\partial}{\partial k} \log \left( 1 + e^{-2\beta_0 k} - 2e^{-\beta_0 k} \cos(\beta_0 C_0) \right),$$  \hspace{1cm} (61)

and the effective potential for the $+, -$ components writes\(^8\):

$$V_{\text{eff}}(\langle P(\vec{x}) \rangle) = \frac{1}{\pi^2} T_0 e^{-4Ht} \int_0^\infty x^2 \log \left( 1 + e^{-2x} - 2e^{-x} \cos(\beta_0 C_0) \right) dx + ....$$  \hspace{1cm} (62)

in agreement with (54) using (27). The meaning of a partition function with an imaginary chemical potential for the total charge of the system remains to be understood and, on this item, we have nothing to add to the previous literature [15]-[17].

### 4 Instability of the metastable vacua

#### 4.1 Absence of thermal decay of metastable vacua

The authors of ref. [14] have estimated the interface tension between the $Z(N)$ different vacua in flat space. Their method consisted in combining in one effective Lagrangian the effective potential (at one loop level) $V_{\text{eff}}(\langle P(\vec{x}) \rangle) = F(\beta)/V$ with the kinetic energy (tree level). They estimated then the nucleation rate following the method proposed by Coleman [13]. Using these results, the authors of [10] have estimated the nucleation rate of the true vacuum within a metastable vacuum: the decay rate per unit space time volume is $\Gamma \sim T^4 e^{-S_{\text{eff}}}$, where $S_{\text{eff}}$ turns out to be much larger than $10^3$ in any case. Their conclusion for flat space was that thermal nucleation had no chance to take place. As previously mentioned, the nucleation rate estimated in ref. [14] is the subject of controversy [13]-[17]. However, accepting their conclusion for the time being, it is straightforward to extend it to de Sitter space, given the similarity between the effective action in de Sitter and flat space. Indeed, thermal nucleation is even less likely to happen in inflation. At the very beginning of inflation, $T/H \sim 10^5$, the nucleation

\(^8\)In a background field, $A_0^a = i\frac{\delta}{g} e^{-Ht} C_0$, and with a vanishing chemical potential, we would also had obtained eq.(60)-(62) by performing formally the same steps as performed above in the temporal gauge. However, such a choice seems to be void of physical meaning.
probability in a space time volume, of size $\sim H^{-4}$, is $\Gamma / H^4 \sim (T/H)^4 e^{-S_{\text{eff}}} \ll 1$. The temperature then falls exponentially and thermal fluctuations are drastically damped. In short, the relevant space-time volume in de Sitter space is smaller than in flat space, and the nucleation probability is even more negligible.

4.2 Quantum decay of metastable vacua

Up to this point we have not considered the quantum fluctuations related to inflation. We recall that the set of spatial variables $\vec{x}$ we use is the set of comoving ones. From the metric, eq. (2), $|dx/dt| = \exp[-Ht]$ for light, and two points with coordinates $\vec{x}$ and $\vec{y}$ are within one’s other horizon only if $|\vec{x} - \vec{y}| < H^{-1}e^{-Ht}$. At a given time $t$, only wave lengths smaller than $H^{-1}e^{-Ht}$ may be generated by quantum fluctuations. Longer wave lengths are only fossil remains of quantum fluctuations that took place earlier. In other words, at a time $t$ the field modes with wave lengths larger than $H^{-1}e^{-Ht}$ can be considered as classical: they are constant (not fluctuating) in the whole causally connected domain.

In eq. (50), $\phi(\vec{x})$ was defined such that $\int d\vec{x} \phi(\vec{x}) = 0$. However, at a time $t$, and considering a causal domain around $\vec{x}_0$, the modes $|\vec{k}| < H e^{Ht}$ have been “frozen” out by inflation, and the average within this horizon is $\int_{|\vec{x} - \vec{x}_0| < H^{-1}e^{-Ht}} d\vec{x} \phi(\vec{x}) \neq 0$. It is convenient to redefine the constant $C_0$ in eq. (50) in order to incorporate these “frozen” modes into a time and spatial domain dependent $C_0(t, \vec{x}_0)$. We decompose the gauge field

$$A^3_\gamma(\gamma, \vec{x}) = \frac{C_0(t, \vec{x}_0)}{g} + \phi_t(\vec{x}_0)(\vec{x}),$$

with

$$\int_{|\vec{x} - \vec{x}_0| < H^{-1}e^{-Ht}} d\vec{x} \phi_t(\vec{x}_0)(\vec{x}) = 0,$$

where $\gamma$ is again the complexified conformal time and $t$ the real time. This implies

$$C_0(t, \vec{x}_0) = C_0 + g \int_{|\vec{k}| < H e^{Ht}} \frac{d\vec{k}}{(2\pi)^3} \bar{\phi}(\vec{k}) e^{i\vec{k}.\vec{x}_0},$$

where $\bar{\phi}(\vec{k})$ is defined from

$$\phi(\vec{x}) = \int \frac{d\vec{k}}{(2\pi)^3} \bar{\phi}(\vec{k}) e^{i\vec{k}.\vec{x}}.$$  

We will now consider the change in $C_0(t, \vec{x}_0)$ during what is commonly called an e-folding (a time range of length $H^{-1}$):

$$\delta C_0 = C_0(t + H^{-1}, \vec{x}_0) - C_0(t, \vec{x}_0),$$

We recall that this phenomenon is at the origin of the standard understanding of the classical fluctuations observed in the cosmic background radiation.
where from now on, we keep implicit the dependence on $t$ and $\vec{x}_0$. It follows that

$$
\delta C_0 = g \int_{H^t e^{Ht} < |\vec{k}| < H e^{H(t+1)}} \frac{d\vec{k}}{(2\pi)^3} \tilde{\phi}(\vec{k}) e^{i\vec{k}.\vec{x}_0} .
$$

(68)

The value of $\tilde{\phi}(\vec{k})$ is undetermined, but its probability distribution is known. In some sense, inflation is “measuring” these modes.

The quantum average can be computed using the partition function. From eq. (51) we extract the part that depends on $\phi(\vec{x})$:

$$
Z_\phi(\beta) = \int D\phi e^{-\frac{1}{2} \int_0^\beta d\gamma \int d\vec{x} (\vec{\nabla} \phi)^2} .
$$

(69)

In order to compute this integral, we discretize the momentum space in a finite volume $V$:

$$
\int \frac{d\vec{k}}{(2\pi)^3} \to \frac{1}{V} \sum_k .
$$

In this way, the partition function becomes

$$
Z_\phi(\beta) = \prod_k \int d\tilde{\phi}_k e^{-\frac{1}{2} \phi_k^2 |\tilde{\phi}_k|^2} ,
$$

(70)

and

$$
\delta C_0 = \frac{g}{V} \sum_{\vec{k} \in K_t} \tilde{\phi}_k e^{i\vec{k}.\vec{x}_0} .
$$

(71)

where $K_t$ corresponds to $\vec{k}$ such that $H e^{Ht} < |\vec{k}| < H e^{H(t+1)}$. The quantum average $< \delta C_0 >$ is then

$$
< \delta C_0 > = \frac{g}{V} \sum_{\vec{k} \in K_t} e^{i\vec{k}.\vec{x}_0} \int d\tilde{\phi}_k \tilde{\phi}_k e^{-\frac{1}{2} \phi_k^2 |\tilde{\phi}_k|^2} = 0 .
$$

(72)

The typical value of $|\delta C_0|$ can be estimated from the squared average $< \delta C_0^2 >$.

$$
\delta C_0^2 = \frac{g^2}{V^2} \sum_{\vec{k}, \vec{k}' \in K_t} \tilde{\phi}_k \tilde{\phi}_{k'} e^{i(\vec{k}-\vec{k}').\vec{x}_0} .
$$

(73)

Given the form of $Z_\phi(\beta)$, only the terms with $\vec{k} = \vec{k}'$ contribute to the average, leading to

$$
< \delta C_0^2 > = \frac{g^2}{V^2} \sum_{\vec{k} \in K_t} \int d\tilde{\phi}_k |\tilde{\phi}_k|^2 e^{-\frac{1}{2} \phi_k^2 |\tilde{\phi}_k|^2} = \frac{g^2}{V^2} \beta_0 \sum_{\vec{k} \in K_t} \frac{1}{|\vec{k}|^2} .
$$

(74)
Taking the continuum limit, we get

\[ < \delta C_0^2 > = T_0 \frac{4\pi}{(2\pi)^2} g^2 \int_{H e^{H t}}^{H e^{H t+1}} dk = \frac{T_0}{g^2} \frac{2\pi^2}{2\pi^2} H e^{H t} (e - 1) , \]  

(75)

from which it follows that

\[ |\delta C_0| \sim \sqrt{T_0 H e^{H t}} \sqrt{\frac{e - 1}{2\pi^2}} \gg (2\pi) T_0 , \]  

(76)

as soon as \( H e^{H t} \gg T_0 \).

Since \( C_0 \) is a periodic variable with a period of \( (2\pi) T_0 \), it results that soon after inflation begins, the value of \( C_0(t, \vec{x}_0) \) is changed at random after every e-folding by an amount larger than its period. Consequently, the value of \( C_0(t, \vec{x}_0) \) will keep no memory of its value one e-folding before and will be distributed at random in the interval \( \in [0, 2\pi T_0] \). Finally, the values \( C_0(t, \vec{x}_0) \) and \( C_0(t, \vec{x}_0 + \delta \vec{x}_0) \) in the neighbouring domains are totally uncorrelated since both are uncorrelated from their common parent domain (the domain where they were both located one e-folding before).

Further physical insight in the above result can be gained using the set of variables \( \vec{x}_{\text{phys}} = e^{H t} \vec{x} \). In this parametrization the size of a causal domain remains constant, \( \sim H^{-1} \), while the period of the variable \( C(t, \vec{x}_0) \equiv C_0(t, \vec{x}_0) e^{-H t} \) becomes \( 2\pi T \), which shrinks to zero as \( T \) falls exponentially \( T = T_0 e^{-H t} \). The typical size of quantum fluctuations overwhelms this period. In these variables, and defining \( \delta C \equiv \delta C_0 \exp[-H t] \),

\[ < \delta C^2 > = T \frac{2\pi^2}{2\pi^2} H (e - 1) \gg 4\pi^2 T^2 , \]  

(77)

for \( T \ll H \). Recall that the present universe comes out from the last \( \simeq 53 \) e-foldings during inflation, when the inequality \( T \ll H \) is certainly valid.

4.3 The end of inflation and the reheating.

At the end of inflation, the temperature raises suddenly by the creation of particles to a reheating temperature \( T_{\text{RH}} \). The effective potential \( V_{\text{eff}}(C) \) computed in \[12\] becomes valid with a periodicity \( 2\pi T_{\text{RH}} \). It is difficult to figure out the evolution of \( C \), a variable defined in thermal equilibrium, during this out-of-equilibrium reheating. No mechanism is known, however, to justify the alignment of \( C \) on distances larger than the distance of the causal horizon. In each connected domain, the value of \( C \) might fall into the closest minimum, the latter being uncorrelated with the minimum in the next causal domain, though. Then the causal problem will jump in again, as the CP phases will be randomly distributed in domains much smaller than the present cluster of galaxies.
5 Conclusion

At the onset of an inflationary era, the putative temperature-dependent effective potential for the Polyakov line quickly flattens. The value of the condensate can change and grow, though, through random quantum fluctuations in de Sitter space, and can thus be non-zero in a given causal domain when the end of inflation approaches.

Before reheating, the typical difference in the value of the condensate between two domains which came out of the horizon in two consecutive e-foldings is of order $\sim \sqrt{T_0 H \exp \frac{H t}{2}}$, where $T_0$ was the temperature at the beginning of inflation.

Upon reheating, the potential starts to sensibly wiggle again and the value of the condensate in a given causal domain might fall into the nearest vacuum, which may have a non-trivial CP phase for large enough gauge groups, such as the standard model one. The trouble comes from the periodicity of the potential, $\sim 2\pi T_0$. This is much smaller than the random quantum change mentioned above, which characterizes for instance the $\sim 53$ last e-foldings of inflation which gave birth to the present universe. The thermal effects produce a potential which wraps around itself once in a period, while every quantum fluctuation makes the field wrap around this “dormant” potential many times during one e-folding. Two neighbouring domains of our present universe are totally uncorrelated in what concerns the value of the condensate, as they are uncorrelated with the common parent domain produced some e-foldings before, as soon as the temperature starts to rise again. Our cluster of galaxies would result of many domains with opposite CP eigenvalues for the effective field, which would lead to the random presence of lumps of matter and antimatter, an untenable result.

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