A DEMOCRATIC RESUMMATION PROCEDURE OF SOFT GLUON EMISSION FOR HADRONIC INELASTIC CROSS-SECTIONS AND SURVIVAL PROBABILITIES

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Abstract
We discuss a model for soft gluon re-summation based on a statistical description of independent emissions during inelastic collisions. The model is applied to estimate Survival Probabilities at the LHC. A comparison with other models and experimental data is presented.

1 Introduction
Survival probabilities at LHC energies are of special interest when looking for hard scattering events which need to be selected from the large hadronic background accompanying them. The concept was introduced in [1], later defined and discussed in [2]. We recently presented our estimates and discussed them in comparison with other models in [3]. In this contribution, we shall summarize our findings and the particulars of the model we use for calculating the total and the inelastic cross-sections.

As discussed in [3], the probability to find events devoid of hadronic background in the central region can be obtained in its simplest form as:

$$S^2(s) = \int d^2b A(b, s) P^{ND}_{no-hadr-bckg}(b, s)$$

where $P^{ND}_{no-hadr-bckg}(b, s)$ represents the probability of events without activity in the central rapidity region, which can be approximated as the non-diffractive (ND) region of phase space. This is clearly an approximation. However, our aim, as it was in [3], is to give an order of magnitude estimate of the Survival probabilities, and compare it with other existing predictions. The quantity $A(b, s)$ refers to the normalized distribution of such events in impact parameter space, and the problem is to calculate the function $A(b, s)$ appropriate to those events excluded by $P^{ND}_{no-hadr-bckg}(b, s)$.

In the sections to follow, we shall describe our model for these two quantities and present our phenomenological analysis for $S^2(s)$. To estimate survival probabilities following [2] we shall use the model for the total cross-section we developed in [1][5]. This model is based on i) single channel eikonal formalism, ii) QCD mini-jets to drive the rise of the total cross-section, iii) soft gluon emission to tame the rise that leads to a high energy behaviour consistent with the Froissart bound.
2 Mini-jets vs total cross-sections

Our suggestion is to extract the quantities, \( A(b,s) \) and \( P_{\text{ND-hadr-bckg}}(b,s) \), from single channel mini-jet models 6). We start with the following expressions for the total cross-section:

\[
\sigma_{\text{total}} = 2 \int d^2b \, \Im F_{el}(b,s) = 2 \int d^2b \left[ 1 - \exp(-\chi_I(b,s)) \right] = \int d^2b \left[ 1 - \exp(-\bar{n}(b,s)/2) \right] \tag{2}
\]

where the imaginary part of the eikonal function is obtained from the average number of inelastic hadronic collisions. In this approximation, the inelastic total cross-section obtains as

\[
\sigma_{\text{inel}} = \int d^2b \left[ 1 - \exp(-\bar{n}(b,s)) \right] = \int d^2b \left[ 1 - \exp(-A_{FF}(b,s)\sigma_{\text{soft}}(s) - A_{\text{mini-jets}}(b,s)\sigma_{\text{mini-jets}}(s)) \right] \tag{3}
\]

with \( \sigma_{\text{soft}}(s) \) either a constant or a slowly decreasing function of energy, and \( \sigma_{\text{mini-jets}}(s) \) is calculated from perturbative QCD, i.e. using Parton Distribution Functions (PDFs) DGLAP evolved and folded with parton-parton cross-sections. Our mini-jet calculation uses the asymptotic freedom expression for the strong coupling constant and thus implies using a lower cut-off for outgoing partons, \( p_{\text{tmin}} \), which effectively separates perturbative and non-perturbative collisions. We show in the left panel of Fig. 1 the behaviour of \( \sigma_{\text{mini-jets}}(s) \) when calculated for different LO PDFs, and different values of \( p_{\text{tmin}} \). The comparison with the total cross-section shown in the same figure, indicates that a mechanism to slow down the excessive growth of the mini-jet cross-section at high energy must be present.

In our model, such taming of the mini-jet cross-section is obtained through the average parton distribution function in impact parameter space \( A_{\text{mini-jets}}(b,s) \), for which a distinctive choice based on soft gluon emission processes is made, as we shall describe in the next section. As for the \( b \)-distribution of non-mini-jet events \( A_{FF}(b) \), the present version of the model is obtained from the Fourier transform of the proton e.m. form factor.

3 Soft Gluon Re-summation : a democratic pathway through confinement

In the right hand panel of Fig. 1 we show the mechanism which we propose to be responsible for the taming of the mini-jet effect, soft gluon re-summation (SRG). To tackle SRG, we proceed with the following guiding ideas:

- if the total cross-section has to follow the limitations of the Froissart bound, hadronic interactions must exhibit a large distance cut-off,
- the large distance behaviour (Froissart bound) is controlled by contributions from very low momentum gluons, i.e. gluons with momentum lower than the pQCD cut-off \( \Lambda_{QCD} \),
- since very soft emitted gluons are not individually counted, only missing energy-momentum is the observed quantity, and the development of a formalism for infrared gluons requires energy momentum balance to be enforced on the soft gluon sea.

![Figure 1: a) The mini-jet proton-proton cross section for inelastic events compared with the total cross-section as a function of c.m. energy b) the soft gluon emission mechanism proposed to tame the fast mini-jet rise.](image)

- Figure 1: a) The mini-jet proton-proton cross section for inelastic events compared with the total cross-section as a function of c.m. energy b) the soft gluon emission mechanism proposed to tame the fast mini-jet rise.
We propose to use a semiclassical re-summation procedure, inspired by what was originally proposed in [8] for soft photons. This approach is based on a *democratic* treatment, a term we shall render more explicit below, and which is represented graphically on the left-hand side in Fig. 2. Let us start with a discrete description of the process of emission. Additional details about the soft-gluon re-summation model can be found in our review [7].

Let \( n_k \) be the number of gluons emitted with a given momentum value \( k \). If these gluons are *soft*, they are by definition indistinguishable, and independent from the source. Hence, the first assumption: these \( n_k \) gluons, all having exactly the same momentum \( k < \Lambda_{QCD} \), are all emitted independently from each other (and from the source).

In analogy to what Bloch and Nordsieck demonstrated [9] for the case of QED, for each value of the momentum \( k \) the number of soft gluons \( n_k \) is taken as being distributed according to a Poisson distribution around an average value \( \bar{n}_k \). The next step in the derivation of the expression we propose, is to consider all possible values of the soft gluon momentum \( k \), each contributing equally to the final energy momentum imbalance. Thus we obtain an overall probability for emission as the product over \( k \) of the individual Poisson distributions, i.e.

\[
P\{\{n_k\}\} = \prod_k \frac{[\bar{n}_k]^{n_k}}{n_k!} e^{-\bar{n}_k}
\]

The next three steps are:
1. for each possible number of gluons, \( n_k \), impose energy-momentum conservation, i.e. \( K_\mu = \sum_k n_k k_\mu \),
2. considering the distribution in transverse momentum, sum on all the distributions giving the observed missing transverse momentum \( K_t \),
3. exchange the product with the sum,
4. take the continuum limit.

Explicitly, from

\[
d^2P(K_t) = \sum_{n_k} P\{\{n_k\}\}d^2K_td^2(K_t - \sum_k k_nk) = \sum_{n_k} \prod_k \frac{[\bar{n}_k]^{n_k}}{n_k!} e^{-\bar{n}_k} d^2K_td^2(K_t - \sum_k k_nk)
\]

and using the integral representation of the delta-function, one exchanges the sum with the product obtaining

\[
d^2P(K_t) = \frac{d^2K_t}{(2\pi)^2} \int \int d^2b e^{-iK_t \cdot b} \exp\{- \sum_k \bar{n}_k[1 - e^{iK_t \cdot b}]\}
\]

Going to the continuum, brings

\[
d^2P(K_t) = \frac{d^2K_t}{(2\pi)^2} \int d^2b e^{-iK_t \cdot b} \exp\{- \int d^3\bar{n}_k[1 - e^{iK_t \cdot b}]\}
\]

Taking then the Fourier transform of Eq. (7), we obtain the impact parameter distribution as an input into the eikonal formalism for the inelastic hadronic cross-section, namely

\[
A_{\text{mini-jets}} \equiv A_{B\text{N}}(b, s) = N(s)e^{-h(b, s)}
\]
with $\mathcal{N}(s)$ the normalisation factor, required for dimensional reasons. Following derivations in our previous publications, we have

$$h(b, s) = \frac{8}{3\pi^2} \int_0^{q_{\text{max}}(s)} d^2k_t [1 - e^{-\frac{k_t}{b} \alpha_s(k_t^2)}] \frac{\ln(2q_{\text{max}}/k_t)}{k_t}$$

(9)

Of notice are the two limits of integration, the upper limit which is chosen from the kinematics of single gluon emission, and the lower limit, which, in our model, we put to zero. Thus, a specification for the coupling of soft gluons to their source is needed, since the asymptotic freedom expression of pQCD cannot be used for $k_t \leq \Lambda_{\text{QCD}}$. For such $k_t$ values, we propose a phenomenological ansatz of a singular but integrable behaviour, namely $\alpha_s(k_t^2) \rightarrow (\Lambda_{\text{QCD}}/k_t)^2p$, with the condition $1/2 < p < 1$. Then the integral in Eq. (9) is finite, and the total cross-section is found to behave asymptotically as $\sigma_{\text{tot}} \simeq (\ln s)^{1/p}$.

Because our model for re-summation was inspired by the Bloch and Nordsieck theorem, we refer to it as the BN model.

4 The inelastic cross-section and survival probabilities

We now apply the above model to describe proton data for the total and inelastic hadronic cross-section in the available energy c.m. range. Applying Eqs. (2) and (3), we obtain the curves shown in Fig. 3, with the blue band indicating the uncertainty arising, at very high energies, from the different low-x- behaviour of the proton PDFs used in the mini-jet calculation. Fig. 3 summarises the results obtained with the mini-jet model described in the previous sections, and compares them with results from an empirical model, which was fashioned after [11], and which provides a fit to the total, the elastic and, by subtraction, to the inelastic cross-section. The empirical model is shown by the dotted lines,
which are obtained using an elastic scattering amplitude parametrised as
\[ A(s, t) = i[F_p^2(t)] \frac{\sqrt{A(s)} e^{B(s)t/2}}{e^{i\phi(s)}} \frac{\sqrt{C(s)} e^{D(s)t/2}}{e^{i\phi(s)}} \]  
(10)

with \( F_p(t) \) the e.m. proton form factor.

Comparing the fit with the blue band, confirms that the mini-jet model used here for \( \sigma_{\text{inel}} \) does non include diffractive events. This was discussed in our previous publications where we also noted that Single Diffractive (SD) events constitute about 10% of the full inelastic cross-section (approximately indicated by the yellow band in Fig. 3). Their origin can be connected to hadronic products from single hard QCD bremsstrahlung from the quarks in one of colliding protons, but are not described by a single-channel eikonal model, with only two components in the eikonal, a non perturbative one, and one from mini-jets, calculable from semi-hard gluon-gluon scattering. However, the model we have presented can be used in the calculation of the survival probability when searching for events unaccompanied by hadronic semi-hard activity in the central region.

In Fig. 4 we show results for the survival probability, estimated using two different models. For the curves shown at left, we use Eq. (1), with the probability of events with no hadronic background in the central region given as
\[ P_{\text{ND no-hadr-bckg}}(b, s) = \exp\{ -\bar{n}(b, s) \} \]
with the \( \bar{n}(b, s) \) function determined through our description of the inelastic cross-section, as described in the previous section. As for the impact parameter distribution, we have used \( A(b) = A_{\text{FF}}(b) \), namely the Fourier transform of the electromagnetic form factor. This follows previous estimates, from Bloch, Durand, Ha and Halzen and our BN model as well (BN-2008 model). Our improved proposal is shown in panel b) of Fig. 4, where dotted and full curves correspond to different PDFs in the mini-jet calculation. These curves are obtained using the results of the previously described BN model into an expression for the survival probability, where soft and a mini-jet contributions are estimated according to their overall weight, as follows:
\[ S^2_{\text{soft}}(s) = S^2_{\text{soft}}(s) + S^2_{\text{mini-jets}}(s) \equiv w_{\text{soft}}(s) < |S(b)|^2 >_{\text{soft}} + w_{\text{mini-jets}}(s) < |S(b)|^2 >_{\text{mini-jets}} \]  
(11)
with

\[ < |S(b)|^2 >_{\text{soft}} = \int d^2 b A_{FF}(b,s) e^{-\bar{n}_{\text{soft}}(b,s)} \]  

\[ < |S(b)|^2 >_{\text{mini-jets}} = \int d^2 b A_{BN}(b,s) e^{-\bar{n}_{\text{mini-jets}}(b,s)} \]  

\[ \bar{n}_{\text{soft}}(b,s) = A_{FF}(b) \sigma_{\text{soft}}(s), \quad \bar{n}_{\text{mini-jets}}(b,s) = A_{BN}(b,s) \sigma_{\text{mini-jets}}(s) \]  

\[ w_{\text{soft/mini-jets}}(s) \equiv \frac{\sigma_{\text{soft/mini-jets}}(s)}{\sigma_{\text{soft}}(s) + \sigma_{\text{mini-jets}}(s)} \]  

Our proposed additive model is compared in panel b) of Fig. 4 with other model predictions \cite{13,16,15}, as well as with CMS data for the survival probability associated to diffractive jet production at LHC \cite{17}.

Comparing results between the two panels and within each figure, we see very large differences, of almost one order of magnitude, and also large uncertainties, between the various estimates. Following our present picture of how mini-jet events populate the central region, we propose Eq. (11) as an adequate way to develop a realistic approximation of survival probabilities in the central region.

This contribution is based on recent joint work with our collaborators, Agnes Grau, Daniel A. Fagundes and Olga Shekhovtsova. YS would like to thank the Department of Physics & Geology at the University of Perugia for their hospitality.

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