An Interference of P-waves from Weakly Conducting and Transparent Dielectric Films

A A Yushkanov and N V Zverev
Moscow Region State University,
Very Voloshinoi str. 24, 141014 Mytishchi, Moscow Region, Russian Federation
E-mail: yushkanov@inbox.ru, zverev_nv@mail.ru

Abstract. An influence of kinetic and quantum properties of non-degenerate electron plasma on the interference of P-waves from films of the similar thickness made of weakly conducting substance and of transparent dielectric one, is investigated numerically. The reflection and transmission coefficients of interfering P-waves as well as their phase differences are studied. It is found that for the frequencies of the order of plasma frequency in case of small film thicknesses, the results for quantum non-degenerate electron plasma differ from the ones for classical non-degenerate electron plasma and for classical electron gas. Here one observes a temperature dependence of the values in the cases of quantum and classical non-degenerate electron plasma.

1. Introduction
At present, a study of values of the low size conducting matter as functions of the temperature, external fields and other parameters, shows a large interest [1]. Such an interest is enforced by the development of nanotechnologies as well as a construction of the controlling optical devices (wave guides, optical gates etc.) having a fine band of transmission or reflection radiation.

At the same time, the conducting substances with evident temperature behavior of a series of parameters have low carrier electron number density order $10^{-5} - 10^{-2}$ of atom number density. Hence, the substances possess a weak conductivity. The electron gas of the weakly conducting matter is almost non-degenerate and therefore, is described by the classical Maxwell distribution law. However, in cases of the low sizes such as nanometer ones as well as of the high frequencies like terahertz or more, one should take into account both the kinetic effects and the quantum wave properties of the carrier electrons.

It the paper, one investigates an influence of the kinetic and the quantum wave properties of the non-degenerate electron plasma of carrier electrons on the interference of the P-waves reflected from and transmitter throw the weakly conducting matter and the transparent dielectric substance. Here one studies the optical power coefficients and phase differences of the interfering waves and compares the results for the quantum non-degenerate electron plasma with the results for the classical non-degenerate electron plasma as well as for the classical electron gas. And also, one considers the temperature dependence of the optical coefficients and the phase differences in cases of the quantum and the classical non-degenerate electron plasma.
2. The model and the dielectric functions of non-degenerate electron plasma

We consider two parallel films of the same width $d$ made from weakly conducting matter and from transparent substance with permittivity $\varepsilon_3$. These films are localized between two transparent media with the permittivities $\varepsilon_1$ and $\varepsilon_2$.

Let the electromagnetic $\mathbf{P}$-wave (the $\mathbf{E}$ vector of the wave lies the incidence plane) is incident on the films under the incidence angle $\theta$ from the medium with $\varepsilon_1$. Then the waves reflected from and transmitted through the films are interfered. And hence, the optical power coefficients of the interfering waves just the reflectance $R_{ulf}$ and the transmittance $T_{ulf}$ as well as the phase differences of the reflected and transmitted waves $\Delta \phi_R$ and $\Delta \phi_T$, look as [2, 3]

$$R_{ulf} = \frac{1}{4} |r_1 + r_2|^2, \quad T_{ulf} = \frac{1}{4} |t_1 + t_2|^2 \text{Re} \left( \frac{\cos \theta'}{\cos \theta} \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \right),$$

$$\Delta \phi_R = \phi_{r1} - \phi_{r2}, \quad \Delta \phi_T = \phi_{t1} - \phi_{t2}.$$

(1)

Here the coefficients $r_j$, $t_j$ and the phases $\phi_{rj}$, $\phi_{tj}$ ($j = 1, 2$) are defined by the equations

$$r_1 = \frac{U^{(1)} + U^{(2)}}{V^{(1)} + V^{(2)}}, \quad t_1 = \frac{U^{(1)}V^{(2)} - U^{(2)}V^{(1)}}{V^{(1)} + V^{(2)}},$$

$$r_2 = \frac{\tilde{U}^{(1)} + \tilde{U}^{(2)}}{V^{(1)} + V^{(2)}}, \quad t_2 = \frac{\tilde{U}^{(1)}V^{(2)} - \tilde{U}^{(2)}V^{(1)}}{V^{(1)} + V^{(2)}},$$

$$r_j = |r_j| \exp(i\phi_{rj}), \quad t_j = |t_j| \exp(i\phi_{tj}).$$

(3)

(4)

(5)

The values $U^{(j)}$, $V^{(j)}$ and $\tilde{U}^{(j)}$, $\tilde{V}^{(j)}$ ($j = 1, 2$) look as

$$U^{(j)} = \frac{\cos \theta - Z_P^{(j)} \sqrt{\varepsilon_1}}{\cos \theta' + Z_P^{(j)} \sqrt{\varepsilon_2}}, \quad V^{(j)} = \frac{\cos \theta + Z_P^{(j)} \sqrt{\varepsilon_1}}{\cos \theta' + Z_P^{(j)} \sqrt{\varepsilon_2}},$$

$$\tilde{U}^{(j)} = U^{(j)}|Z_P = \tilde{Z}_P|, \quad \tilde{V}^{(j)} = V^{(j)}|Z_P = \tilde{Z}_P|.$$

(6)

(7)

Here $\theta'$ is the refractive angle to the medium with $\varepsilon_2$ evaluated according to the refraction law

$$\sqrt{\varepsilon_1} \sin \theta = \sqrt{\varepsilon_2} \sin \theta',$$

(8)

and $Z_P^{(j)}$ and $\tilde{Z}_P^{(j)}$ ($j = 1, 2$) are dimensionless surface impedances for the conducting [4] and for dielectric films, respectively:

$$Z_P^{(j)} = \frac{2 \text{Re} \omega}{d} \sum_n \frac{1}{k_n^2} \left( \frac{k_n^2}{\omega^2 \varepsilon_I(\omega, k_n)} + \frac{(\pi n/d)^2}{\omega^2 \varepsilon_{tr}(\omega, k_n) - (\epsilon k_n)^2} \right),$$

$$\tilde{Z}_P^{(j)} = Z_P^{(j)}|\varepsilon_I = \varepsilon_{tr} = \varepsilon_3|,$$

(9)

(10)

where $k_n = [k_n^2 + (\pi n/d)^2]^{1/2}$, $k_x = \omega \sqrt{\varepsilon_1} \sin \theta$, and summation by $n$ is performed over all odd values $n = \pm 1, \pm 3, \pm 5, \ldots$ at $j = 1$, and over all even $n = 0, \pm 2, \pm 4, \ldots$ at $j = 2$.

The surface impedance $Z_P^{(j)}$ was evaluated in the case of mirror electron reflections of electron plasma from the film borders [4]. In the equation (9), $\varepsilon_I(\omega, k)$ and $\varepsilon_{tr}(\omega, k)$ are the longitudinal and transverse dielectric functions (permittivities) of the conducting electrons, respectively. We consider the dielectric functions of quantum non-degenerate electron plasma which look as follows [5, 6]:

$$\varepsilon_I^{(qm)}(\omega, k) = 1 - \frac{2}{Q^2} \frac{(\Omega + i\gamma)G(\Omega + i\gamma, Q)G(0, Q)}{\Omega G(0, Q) + i\gamma G(\Omega + i\gamma, Q)},$$

(11)
The plasma frequency \( \Omega \) of the conducting electron plasma in the classical limit \( h \to 0 \) go over to the dielectric functions of the classical non-degenerate electron plasma \[7\]:

\[
\varepsilon_{tr}(\omega, k) = 1 - \frac{1}{Q^2} \left( 1 + \frac{\Omega G(\Omega + i\gamma, Q) + i\gamma G(0, Q)}{\Omega + i\gamma} \right).
\]

Here

\[
G(\Omega + i\gamma, Q) = \frac{Q^2}{\sqrt{\pi}} \int_0^{+\infty} \frac{[(\Omega_+ + i\gamma)(\Omega_- + i\gamma) + (Q\xi)^2] \exp(-\xi^2)}{[(\Omega_+ + i\gamma)^2 - (Q\xi)^2][((\Omega_- + i\gamma)^2 - (Q\xi)^2]} \, d\xi,
\]

and the values

\[
\Omega = \frac{\omega}{\omega_p}, \quad \Omega_\pm = \Omega \pm \frac{1}{2m_e\omega_p} k^2, \quad \gamma = \frac{1}{\omega_p \tau}, \quad Q = \frac{v_T k}{\omega_p}, \quad v_T = \sqrt{\frac{2k_B T}{m_e}}.
\]

In these equations, \( \omega \) is the wave frequency, \( c \) is the speed of light, \( \omega_p \) is the plasma frequency, \( k \) is the wave number, \( m_e \) is the effective mass and \( \tau \) is the relaxation time of the conducting electrons, \( h \) is the Planck constant, \( k_B \) is the Boltzmann constant, \( T \) is the absolute temperature. And \( v_T \) is the thermal velocity of the conducting electrons in the non-degenerate electron plasma.

The quantum dielectric functions \(11\), \(12\) contain both the kinetic and quantum wave of the non-degenerate electron plasma. These functions in the classical limit \( h \to 0 \) go over to the dielectric functions of the classical non-degenerate electron plasma \[7\]:

\[
\varepsilon_t^{(cn)}(\omega, k) = 1 - \frac{4(\Omega + i\gamma) F_2(\Omega + i\gamma, Q)}{\Omega - 2i\gamma Q^2 F_2(\Omega + i\gamma)},
\]

\[
\varepsilon_{tr}^{(cn)}(\omega, k) = 1 - \frac{2(\Omega + i\gamma)}{\Omega} F_0(\Omega + i\gamma, Q),
\]

where

\[
F_\alpha(\Omega + i\gamma, Q) = \frac{1}{\sqrt{\pi}} \int_0^{+\infty} \frac{\xi^\alpha \exp(-\xi^2)}{(\Omega + i\gamma)^2 - (Q\xi)^2} \, d\xi, \quad (\alpha = 0, 2).
\]

And in the long wave limit \( k \to 0 \) when the kinetic properties are disregarded, both the quantum and the classical dielectric functions \(11\), \(12\) and \(15\), \(16\) go over to the dielectric functions of the classical electron gas in the Drude – Lorentz approach \[4, 7\]:

\[
\varepsilon_t^{(DL)}(\omega) = \varepsilon_{tr}^{(DL)}(\omega) = 1 - \frac{1}{\Omega(\Omega + i\gamma)}.
\]

3. Results and discussion

The plasma frequency \( \omega_p \) and the relaxation time \( \tau \) of the conducting electron plasma in the conducting substance are evaluated according to the equations \[7\]:

\[
\omega_p = \sqrt{\frac{e^2 n_e}{\varepsilon_0 m_e}}, \quad \tau = \frac{m_e}{e^2 n_e \rho_0}.
\]

Here \( e \) is the elementary electrical charge, \( n_e \) is the number density of conducting electrons, \( \varepsilon_0 \) is the electrical constant in the SI units and \( \rho_0 \) is the static specific resistance of the substance.

For numerical simulations, we took the data for the graphite as a weakly conducting matter \[8, 9\]: \( n_e = 2 \cdot 10^{25} \, m^{-3} \), \( \rho_0 = 4 \cdot 10^{-7} \) Ohm · m, \( m_e = 9 \cdot 10^{-31} \) kg. Then using the equations \(19\), one evaluates \( \omega_p = 2.54 \cdot 10^{14} \) s\(^{-1}\) and \( \tau = 4.4 \cdot 10^{-12} \) s.

Numerical simulations with use of equations \(1\) – \(18\) have shown that the optical power coefficients of the interfering waves and their phase differences evaluated for quantum non-degenerate electron plasma with dielectric functions \(11\) and \(12\), differ from those both in the...
The $T_{if}$ (left plot) and $\cos \Delta \phi_R$ (right plot) values as functions of $\omega$ at $d = 25$ nm, $\theta = 75^\circ$, $\omega_p = 2.54 \cdot 10^{14}$ s$^{-1}$, $T = 294$ K, $\varepsilon_1 = 1$ (air), $\varepsilon_2 = 2$ (quartz), $\varepsilon_3 = 1.6$ (gel): 1 – quantum non-degenerate electron plasma (solid line), 2 – classical non-degenerate electron plasma (dashed line), 3 – classical electron gas (dotted line).

This result means an influence both the kinetic and the quantum wave properties of the conducting electrons on the optical values. The difference is observed near regions of the resonant behavior of the values for the frequencies $\omega \gtrsim \omega_p$ at the large enough incidence angles $\theta$ and the film widths $d \ll c/\omega_p$ (the skin depth). Such a behavior is caused by an influence of the longitudinal electron plasma oscillations inside the weak conductor film that contributes to the $\varepsilon_l(\omega, k)$ [2, 4].

The $R_{if}$, classical non-degenerate electron plasma (left plot), and $\cos \Delta \phi_R$, quantum non-degenerate electron plasma (right plot) values as functions of $\omega$ at $d = 50$ nm, $\theta = 60^\circ$, $\omega_p = 2.54 \cdot 10^{14}$ s$^{-1}$, $\varepsilon_1 = 2$ (quartz), $\varepsilon_2 = 1$ (air), $\varepsilon_3 = 1.6$ (gel): 1 – $T = 265$ K (solid line), 2 – $T = 294$ K (dashed line), 3 – $T = 323$ K (dotted line).

It was found also a temperature dependence of the power coefficients and the phase differences in the resonant regions of both the quantum and the classical non-degenerate electron plasma. The resonant frequencies increase with the temperature growth (see figure 2).

The obtained results should be used for creation and utilization of optical devices using wave interference from thin films made of weakly conducting and transparent dielectric materials.
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