A THREE-DIMENSIONAL BABCOCK–LEIGHTON SOLAR DYNAMO MODEL

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ABSTRACT

We present a three-dimensional (3D) kinematic solar dynamo model in which poloidal field is generated by the emergence and dispersal of tilted sunspot pairs (more generally bipolar magnetic regions, or BMRs). The axisymmetric component of this model functions similarly to previous 2.5 dimensional (2.5D, axisymmetric) Babcock–Leighton (BL) dynamo models that employ a double-ring prescription for poloidal field generation but we generalize this prescription into a 3D flux emergence algorithm that places BMRs on the surface in response to the dynamo-generated toroidal field. In this way, the model can be regarded as a unification of BL dynamo models (2.5D in radius/latitude) and surface flux transport models (2.5D in latitude/longitude) into a more self-consistent framework that builds on the successes of each while capturing the full 3D structure of the evolving magnetic field. The model reproduces some basic features of the solar cycle including an 11 yr periodicity, equatorward migration of toroidal flux in the deep convection zone, and poleward propagation of poloidal flux at the surface. The poleward-propagating surface flux originates as trailing flux in BMRs, migrates poleward in multiple non-axisymmetric streams (made axisymmetric by differential rotation and turbulent diffusion), and eventually reverses the polar field, thus sustaining the dynamo. In this Letter we briefly describe the model, initial results, and future plans.

Key words: dynamo – Sun: activity – Sun: interior – sunspots

1. INTRODUCTION

Babcock (1961) was the first to describe how the emergence of toroidal magnetic flux through the solar surface and the subsequent evolution of that flux can produce a large-scale poloidal magnetic field. Furthermore, he argued that this process, together with the generation of toroidal field by differential rotation (the $\Omega$-effect) gives rise to the 11 yr solar activity cycle. Later work beginning with Leighton (1964, 1969) fleshed out Babcock’s vision and transformed it into viable numerical dynamo models of the solar cycle.

Though many alternative solar dynamo models have been proposed, the Babcock-Leighton (BL) paradigm has remained compelling because it is firmly grounded in solar observations and provides a robust mechanism for producing cyclic dynamo activity (see reviews by Dikpati & Gilman 2009; Charbonneau 2010). One of the major milestones in model development occurred in the 1990s when meridional circulation was included and was shown to play a crucial role in regulating the cycle period and other cycle features such as the poleward drift of photospheric flux and the phasing of polar field reversals (Wang & Sheeley 1991; Choudhuri et al. 1995; Dikpati & Charbonneau 1999). In recognition of the importance of flux transport (FT) by meridional circulation, these new BL models were christened FT dynamo models.

Though they ostensibly rely on flux emergence and evolution in order to operate, most early BL/FT models did not explicitly include sunspots. Instead, the generation of poloidal field was represented as an idealized poloidal source term in the magnetohydrodynamic (MHD) induction equation. This BL source term is often non-local in the sense that it is confined to the surface layers, but its amplitude is proportional to the strength of the toroidal field near the bottom of the convection zone (CZ). The longitudinally averaged, kinematic MHD induction equation is then solved to follow the evolution mean (axisymmetric) fields.

Another milestone in model development was to replace the non-local $\alpha$-effect with a more phenomenological representation of tilted sunspot pairs. This was originally done in an axisymmetric context through Durney’s (1997) double-ring algorithm which represents a tilted sunspot pair as two overlapping toroidal rings with opposite polarity. This algorithm was later extended and implemented into axisymmetric BL/FT dynamo models by Nandy & Choudhuri (2001), Munoz-Jaramillo et al. (2010) and Guerrero et al. (2012).

A more sophisticated three-dimensional (3D) flux emergence algorithm was recently presented by Yeates & Munoz-Jaramillo (2013; hereafter YM13). To our knowledge, this is the first use of a fully 3D BL source term. In their model, YM13 simulate flux emergence through an imposed helical flow that lifts and twists the dynamo-generated toroidal field such that it emerges through the surface and then evolves according to the action of turbulent diffusion and mean fields.

Here we construct a 3D kinematic BL/FT model using an alternative flux emergence algorithm that places a spot pair (or, more generally a bipolar magnetic region, or BMR; cf. YM13) localized near the surface above the position where the subsurface toroidal flux peaks. Since the mean-field component of the 3D induction equation is equivalent to a corresponding double-ring algorithm, this approach makes close contact with previous 2.5 dimensional (2.5D, latitude–radius) BL/FT dynamo models. Furthermore, since the emergent field is confined to the surface layers, it makes close contact with surface flux transport (SFT) models that follow the 2.5D (latitude–longitude) evolution of emergent flux in the solar photosphere subject to mean flows and turbulent diffusion (Leighton 1964; Wang & Sheeley 1991; Schrijver 2001; Baumann et al. 2006).

Our model is a unification of BL/FT dynamo models and SFT models. Though it is not the first such unification (see Munoz-Jaramillo et al. 2010; YM13), it is a promising approach that we intend to pursue in the future to study the 3D evolution of the cyclic solar magnetic field and its coupling to the corona
and heliosphere. We describe the basic model components in Section 2 and the flux emergence algorithm in Section 3. We then present preliminary results, conclusions, and future plans in Section 4.

2. DEVELOPMENT OF 3D BABCOCK–LEIGHTON DYNAMO MODEL

Building on the success of 2.5D BL/FT dynamo models, we construct a 3D solar dynamo model by solving the MHD induction equation in the kinematic limit

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta_r \nabla \times \mathbf{B}),$$

(1)

where $\eta_r(r)$ is a turbulent diffusion and solar velocity fields are specified based on photospheric observations and helioseismic inversions. We use spherical polar coordinates $(r, \theta, \phi$) throughout. Unlike many mean-field dynamo models, we do not include an explicit $\alpha$-effect. Instead, the dynamo is sustained by the appearance and evolution of sunspot pairs (BMRs) which are placed on the surface in response to the dynamo-generated field by the “Spotmaker” algorithm described in Section 3.

In this introductory Letter, the velocity field $\mathbf{v}$ is axisymmetric, consisting only of differential rotation and meridional circulation. In this case the evolution of the mean field $\langle \mathbf{B} \rangle$ (brackets denote an average over longitude, $\phi$) is independent of modes with higher azimuthal wavenumbers ($m > 0$). This can be verified by averaging Equation (1) over longitude. Thus, from the perspective of the mean fields, the Spotmaker algorithm is equivalent to the double-ring approach used in previous 2.5D BL/FT dynamo models (Durney 1997; Nandy & Choudhuri 2001; Munoz-Jaramillo et al. 2010; Guerrero et al. 2012), though details such as the spatial profiles and temporal cadence of the spot appearances are different. We view this as beneficial at this stage in the model development because it allows us to make direct contact with existing axisymmetric BL/FT models. Future modeling will incorporate non-axisymmetric flow fields and nonlinear feedbacks which will break this degeneracy with axisymmetric models.

The model is built upon the Anelastic Spherical Harmonic (ASH) code described by Clune et al. (1999) and Brun et al. (2004). ASH is a workhorse code that has been applied extensively to simulate solar and stellar convection (see Miesch 2005; Brun 2010) but here we use it in a kinematic mode to solve only the induction equation. The numerical method is pseudospectral, with a trianglerun truncated spherical harmonic decomposition in the horizontal dimensions, a fourth-order finite difference scheme in the vertical dimension, and mixed semi-implicit/explicit timestepping.

The differential rotation and meridional circulation that comprise $\mathbf{v}$ are the same as used by Dikpati (2011) and $\eta_r$ is given by the two-step profile described by Dikpati & Gilman (2007). The values used for $\eta_r$, ranging from $3 \times 10^{12}$ cm s$^{-1}$ near the surface to $5 \times 10^{10}$ cm s$^{-1}$ in the mid CZ to $10^9$ cm s$^{-1}$ in the radiative zone, place this model in the so-called advection-dominated regime in which the meridional flow dominates FT across the CZ over turbulent diffusion (Jiang et al. 2007).

3. FLUX EMERGENCE ALGORITHM: SPOTMAKER

Our objective is to construct a solar dynamo model that captures both the solar activity cycle and the observed evolution of large-scale magnetic flux on the solar surface. However, capturing the full complexity of active region formation and dispersal through is currently beyond the capability of a single numerical dynamo model. Here we use an idealized flux emergence algorithm to place spots on the solar surface in response to the dynamo-generated toroidal field near the base of the CZ. As mentioned in Section 1 this algorithm can be regarded as a 3D generalization of the axisymmetric double-ring algorithm of Durney (1997) and Munoz-Jaramillo et al. (2010).

The first step in the algorithm is to define a spot-producing toroidal flux near the base of the CZ as follows

$$\hat{B}_{t0}(\theta, \phi, t) = \int_{r_a}^{r_b} h(r) \hat{B}_t(r, \theta, \phi, t) \, dr,$$

(2)

where $h(r) = h_0 (r - r_a) (r_b - r)$ and $h_0$ is defined such that $\int_{r_a}^{r_b} h(r) \, dr = 1$. This is similar to analogous expressions used by Rempel (2006), but unlike previous models, the flux $\hat{B}_t(\theta, \phi, t)$ is not necessarily axisymmetric; longitudinal structure is permitted in the toroidal bands that give rise to active regions. Here we use $r_a = 0.70 \, R$ and $r_b = 0.71 \, R$.

The next step is to suppress sunspot formation at high latitudes. This is empirically motivated but may have a dynamical explanation in terms of the disruption of high-latitude toroidal flux systems by the magneto-rotational instability (Parfrey & Menou 2007). We accomplish this by applying a mask to $\hat{B}_t(\theta, \phi, t)$ such that

$$B^*(\theta, \phi, t) = \frac{2 g_0 |\sin \theta \cos \theta|}{1 + \exp(-\gamma \theta')} \hat{B}(\theta, \phi, t),$$

(3)

where $\theta' = \theta - \pi/4$ in the northern hemisphere (NH) and $3\pi/4 - \theta$ in the southern hemisphere (SH). Here we use $\gamma = 30$ and choose the normalization $g_0$ such that the maximum value of the masking function is unity (see Dikpati et al. 2004).

The placement of a spot pair in latitude and longitude is given by the location where the amplitude of $B^*(\theta, \phi, t)$ is maximal. If this occurs over a broad range of longitude (as from axisymmetric initial conditions), then a longitude is chosen at random from those locations where $B^*(\theta, \phi, t)$ is within 0.1% of its peak value. For the simulation shown in Section 4 the toroidal field at the base of the CZ is nearly axisymmetric, with small departures arising from the 3D nature of the poloidal field. We expect more substantial departures when we consider nonlinear feedbacks and non-axisymmetric flow fields.

A spot is placed if the maximum amplitude of $B^*$ exceeds a threshold value $B_t$. However, in order to avoid introducing overlapping spots at every time step, a time delay is also required. This can be loosely regarded as a dynamical adjustment time between flux emergence events. Here we use a cumulative lognormal distribution function defined as

$$C(\Delta) = \frac{1}{2} \left[ 1 - \operatorname{erf} \left( \frac{-\ln \Delta - \mu}{\sigma \sqrt{2}} \right) \right],$$

(4)

where $\Delta = t - t_i$ is the time lag since the last appearance of a spot, $r_i$, $\sigma^2 = (2/3)[\ln(r_i) - \ln(r_p)]$, $\mu = \ln r_p + \sigma^2$, $r_p$ is the mean time between spots, and $r_p$ is the mode of the distribution. Spots are placed if $\max(B^*) > B_t$ and $C(\Delta) \geq z$, where $z$ is a random number chosen every time step. Only one spot can emerge at a time in each hemisphere but simultaneous emergence in the NH and SH is permitted.

The two-dimensional (latitude, longitude) profile of the spot pair on the solar surface is expressed as

$$B_k(\theta, \phi) = S B_t [g_T(\theta, \phi) - g_L(\theta, \phi)],$$

(5)
where \( S \) is the sign of \( B^* \) at the (co)latitude and longitude of the spot pair, \( \theta_s \) and \( \phi_s \). The functions \( g_L(\theta, \phi) \) and \( g_T(\theta, \phi) \) are Gaussian or polynomial profiles defining the leading and trailing spots. For example, \( g_L(\theta, \phi) = 1 - 3s^2 + 2s^3 \) for \( s \leq 1 \) where \( s^2r_0^2 = (\theta - \theta_0)^2 + (\phi - \phi_0)^2 \) and \( r_0 \) is the angular radius of each spot (see below). A similar expression holds for \( g_T(\theta, \phi) \).

Each spot pair is given a tilt in accordance with Joy’s law, as seen in solar observations; \( \delta = \delta_0 \cos \theta \) where \( \delta_0 = 32.1 \pm 0.7 \) (Stenflo & Kosovichev 2012). The angular distance between spots is an input parameter (here equal to \( 3r_s \)). The resulting surface field is illustrated in Figure 1(a).

The field strength \( B_s \) and radius \( r_s \) of the spot pair are determined by the flux content

\[
\Phi_s = 2\Phi_0 \frac{10^{23}}{1 + (\hat{B}(\theta_s, \phi_s, t)/B_q)^2} \text{Mx} = B_s r_s^2.
\]  

(6)

Here \( B_q \) is a quenching field strength (here \( 10^3 \) G) and \( \Phi_0 \) is an amplification factor that can be adjusted to promote supercritical dynamo action. Solar observations suggest \( \Phi_0 \sim 1 \), implying a flux of \( 10^{23} \) Mx in the strongest active regions but for the preliminary proof-of-concept models presented here, we use fewer, stronger spots, with \( \Phi_0 = 200 \), \( r_p = 400 \) days, and \( \tau_s = 600 \) days. Typically we specify the spot strength as an input parameter \( B_s = 3000\Phi_0 \) G and set \( r_s = (\Phi_s/B_s)^1/2 \). However, we often find it practical to set minimum and maximum values for \( r_s \), (here \( 8–43 \) Mm), and then adjust \( B_s \) accordingly to give the desired flux. For \( \Phi_0 = 200 \), this preliminary procedure yields artificially strong spots of \( 600–1500 \) kG. Future models will incorporate more realistic spot/BMR distributions.

The Spotmaker algorithm does not currently take into account the depletion of toroidal flux in the tachocline during the creation of a spot pair as described, for example by Nandy & Choudhuri (2001). This is an important development that we will include in the near future in order to improve the self-consistency of the model and to investigate its role in dynamo saturation. However, in this Letter we focus on achieving self-sustained cycles with the poloidal field generated by a resolved BL mechanism. We do not expect this aspect of the model to depend sensitively on flux depletion.

The 3D structure of the field in a given spot pair is computed by doing a potential field extrapolation below the surface, \( B_{\text{spot}}(r, \theta, \phi) = \nabla \Gamma \) where

\[
\Gamma(r, \theta, \phi) = \sum_{\ell m}(a_{\ell m}r^{\ell + 1} + b_{\ell m}r^{-(\ell + 1)})Y_{\ell m}(\theta, \phi).
\]  

(7)

The coefficients \( a_{\ell m} \) and \( b_{\ell m} \) are chosen such that \( B_r(r, \theta, \phi) = \partial \Gamma/\partial r = B_R(\theta, \phi) \) at \( r = R \) and \( B_r = 0 \) for \( r \leq r_p \), where \( r_p \) is an input parameter representing the initial penetration depth of active regions (here we use \( r_p = 0.95 R \)). This means that the spot field does not necessarily satisfy the outer boundary condition, often taken to either radial field or a match to an external potential field that vanishes at infinity. Essentially, there is an unresolved current sheet at the surface that dissipates quickly due to resolved and numerical diffusion. This adjustment mimics the observed tendency for spot fields to become nearly vertical as they fragment and become part of the magnetic network. Figure 1(b) illustrates the potential field boundary condition whereas the radial field boundary condition is used for the example simulation in Section 4. In the future we will also consider current-free boundary conditions that are consistent with the Spotmaker algorithm.

We do not expect the subsurface field structure of actual sunspots to be potential. However, Equation (7) is easy to implement and it makes close contact with previous axisymmetric BL solar dynamo models in which the BL source term is assumed to be confined to the surface layers. This is justified by solar observations and modeling efforts that suggest active regions decouple from their roots within a few days after emergence (Schüssler & Rempel 2005), a time short compared to the 11 yr solar activity cycle. The other limit, in which active regions remain anchored to progenitor fields in the lower CZ and tachocline after emergence, will be considered in future work (see also YM13).

4. RESULTS AND DISCUSSIONS

Figure 2 highlights magnetic cycles achieved in a solar dynamo simulation represented in terms of butterfly diagrams. The numerical resolution of this simulation is \( 300 \times 512 \times 1024 \) in \( r, \theta, \phi \) and the computation domain extends from \( 0.69 R \) to \( R \), with an electrically conducting inner boundary and a radial field imposed on the outer surface.

The half-period of the magnetic cycle is roughly 11–12 yr, comparable to the solar cycle. As in other advection-dominated axisymmetric BL/FT models, this is regulated largely by the imposed meridional flow, which has an amplitude of about 14 m s\(^{-1}\) in the upper CZ and 1.5 m s\(^{-1}\) in the lower CZ respectively. To our knowledge this is the first demonstration of a self-sustained, cyclic solar dynamo model that incorporates a 3D flux emergence algorithm for the generation of poloidal field. The dynamo not only includes sunspots (BMRs), but as a BL model, it relies on them for its operation.
Figure 2. Butterfly diagram for a representative solar dynamo simulation. (a) Mean radial field $⟨B_\text{r}\rangle$ at the surface ($r = R$) as a function of latitude and time. Blue and red denote inward and outward polarity respectively and the saturation level of the color table is $±100$ G. (b) Mean toroidal field $⟨B_\phi⟩$ near the base of the convection zone ($r = 0.71 R$; blue westward, red eastward, saturation $±30 kG$). Vertical lines denote times of 58.57, 58.83, and 64.94 yr represented in Figure 4.

Figure 3. Zoom-in of Figure 2(a) highlighting the time interval shown in Figure 4.

Figures 3 and 4 show an illustrative example of surface flux evolution. The sequence begins at $t = 58.57$ yr (a) when a new sunspot pair has just emerged in the NH amid remnant flux from previous emergence events. Note also the slightly older spot pair in the SH at a latitude of about $−45°$ and longitude near $180°$. About three months later (b), trailing flux from the northern spot (blue) has begun to disperse and merge with a growing axisymmetric band of negative flux at a latitude of roughly $65°$. Similarly, trailing flux from the southern spot (red) contributes to a positive-polarity band of flux at a latitude of about $−67°$. About six years later (c), these bands (red in the north and blue in the south) have migrated toward higher latitudes and have begun to reverse the polar fields (see also Figure 2(a)). Meanwhile, the next generation of sunspots has already begun to build opposite-polarity bands equatorward of these. Gibbs ringing is clearly visible in the fresh spots in Figures 4(a) and (c) but this is quickly diffused away and subsumed into more diffuse flux patches as can be seen by following the evolution of the northern spot in frames a and b. This effectively diminishes the flux in the spot by a small amount (less than 1%).

This surface flux evolution is similar to that seen in photospheric magnetograms and SFT models and it demonstrates that the BL mechanism does indeed operate in a 3D context as originally envisioned by Babcock (1961) and Leighton (1964, 1969). The dispersal of tilted sunspot pairs due to differential rotation, meridional circulation, and turbulent diffusion generates a mean poloidal field that sustains the dynamo. Trailing flux migrates toward the poles in a series of streams (Figure 3) while leading flux cancels across the equator. Note that this cancellation occurs only in a time-integrated sense, since the randomness of spot appearances essentially guarantees that the 3D field distribution at any instant is not symmetric about the equator.

The mean toroidal field near the base of the CZ, often taken as a proxy for the sunspot number, exhibits systematic equatorward propagation at low latitudes similar to the solar butterfly diagram (Figure 2(b)). However, the explicit behavior of the spots themselves as seen in Figure 2(a) does not agree as well with solar observations, showing a tendency to linger at mid-latitudes before a rapid rush toward the equator near the end of a cycle. This can largely be attributed to the masking function in Equation (3) which favors mid-latitudes. Since this masking function was originally designed to mimic Joy’s-law tilts that we capture explicitly, it will be justified to replace it with a more uniform low-latitude profile that may be calibrated to more closely match solar observations. Similarly, the timing of the polar field reversals relative to toroidal field reversals and sunspot maximum is sensitive to the details of the Spotmaker algorithm (including spot distribution and emergence latitudes), which we have not yet attempted to calibrate.
In summary, the main result of this Letter is the construction of a viable 3D BL/FT solar dynamo model using a novel algorithm for flux emergence. We focused here on the kinematic regime with imposed mean flows but the real promise of this model will be realized when we consider nonlinear feedbacks and non-axisymmetric flows.

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