Persistent Currents in Mesoscopic Hubbard Rings
with Spin-Orbit Interaction

Satoshi Fujimoto and Norio Kawakami

Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606, Japan

The effect of spin-orbit interaction on persistent currents in mesoscopic Hubbard rings threaded by an Aharonov-Bohm flux is investigated putting stress on the orbital magnetism. The non-perturbative treatment of the spin-orbit interaction developed by Meir et al. is combined with the Bethe ansatz solution to deal with this problem exactly. We find that the interplay of spin-orbit interaction and electron-electron interaction plays a crucial role, bringing about some new effects on the orbital magnetism.

PACS numbers:73.29.Dx, 05.30.-d
Persistently equilibrium current occurring in isolated mesoscopic normal metal rings penetrated by an Aharonov-Bohm (AB) flux is one of the most interesting phenomena in mesoscopic systems [1–9]. It is periodic in the flux with period of a flux quantum (or half a flux quantum) and has either a diamagnetic or paramagnetic sign according to different experimental situations [2,3]. It has been widely accepted that spin-orbit (SO) interaction may play a crucial role in the orbital magnetism, e.g., the sign and the period of currents. Lévy et al. have suggested that persistent currents observed by them may have a diamagnetic sign due to the effect of strong SO interaction (although they pointed out that their determination contains some ambiguity) [2]. On the other hand, Altshuler et al. have claimed from thermodynamical arguments that the current averaged over spatial disorder is always paramagnetic even in the strong SO interaction limit [7]. Extensive studies done subsequently [10,11] have confirmed the result of Altshuler et al., and furthermore have revealed some interesting aspects for SO effects on the orbital magnetism. Particularly in a non-perturbative approach employed in [11] universal and nonuniversal aspects of SO effects have been discussed, such as the reduction factor of currents, etc. The above interesting studies on SO effects, however, have been concerned with a free electron model without electron-electron interaction. It is hence desirable to examine how electron-electron interaction is combined with SO interaction to affect persistent currents in an AB geometry.

In this paper we wish to investigate the effect of SO interaction on persistent currents in mesoscopic rings of mutually interacting electrons for the canonical ensemble. For this purpose, we study the Hubbard ring with SO interaction by combining the non-perturbative treatment of SO interaction of Meir et al. [12] with the Bethe ansatz technique [13,14]. We find that the interplay of SO interaction and electron-electron interaction produce some new effects on the orbital magnetism. We further point out that simple reduction factors of currents due to the SO effect are modified in the presence of electron-electron interaction.

The organization of the paper is as follows. In Sec. II, we briefly depict how to diagonalize
the Hubbard ring with SO interaction by the Bethe ansatz method, and derive the excitation spectrum exactly. The key point is that the non-perturbative treatment of SO interaction for a non-interacting case [12] is still applicable to the Hubbard model because of local SU(2) symmetry of the Hubbard interaction. We then study, in Sec. III, the SO effects on persistent currents for case of the canonical ensemble, putting stress on the sign and the period. As is the case without SO interaction [14,15], the current shows a quite different behavior according to the number of electrons $N_c \mod 4$. We will see that upon averaging over strong SO interaction, some new effects on persistent currents are brought about by the electron-electron interaction. In Sec. IV, a possible extension to more general models including long-range interaction is presented. Summary and conclusion are given in Sec. V.

II. HUBBARD RING WITH SPIN-ORBIT INTERACTION

We consider the mesoscopic Hubbard ring with SO interaction. The Hamiltonian reads

$$H = \sum_{i,\sigma,\sigma'} t_i (S_i)_{\sigma,\sigma'} c_{i,\sigma}^\dagger c_{i+1,\sigma'} + h.c. + U \sum_i c_{i\uparrow}^\dagger c_{i\uparrow} c_{i\downarrow}^\dagger c_{i\downarrow}. \quad (1)$$

where $(S_i)_{\sigma,\sigma'}$ is a matrix of SU(2), and $t_i$ is chosen to be real for the sake of time reversal symmetry. The spin-dependent hopping matrix in (1) reflects the effect of SO interaction. The interplay between the Hubbard interaction and the SO interaction makes it difficult to treat the model directly. It is found, however, that one can still deal with the Hamiltonian exactly by combining the Bethe-ansatz technique with the non-perturbative approach developed for a non-interacting case [12]. The point is that the hopping term in (1) with SO interaction is cast into the diagonal form in spin space by a unitary transformation

$$U^{(i)} = U^{(1)} S_1 \cdots S_{i-1}, \quad (2)$$

with the matrix $U^{(1)}$ being chosen appropriately. The transformation rotates the frame of spin space by a different angle at each site so as to produce the homogeneous hopping matrix. An important point is that the onsite Coulomb term in (1) is invariant under such a local
spin rotation because of its SU(2) symmetry \[16\]. Hence one can formally gauge away SO interaction, and find the conventional Hubbard model in the new spin space,

\[ H = \sum_{i,\tilde{\sigma}} t_{i,\tilde{\sigma}} c_{i,\tilde{\sigma}}^\dagger c_{i+1,\tilde{\sigma}} + h.c. + U \sum_i \tilde{c}_{i,\uparrow}^\dagger \tilde{c}_{i,\uparrow} \tilde{c}_{i,\downarrow}^\dagger \tilde{c}_{i,\downarrow}, \]  

where \( \tilde{\sigma} \) denotes a transformed spin variable, and \( \uparrow \) and \( \downarrow \) label “up” and “down” spins respectively in the new spin frame defined differently on each site. The effect of SO interaction is now incorporated into spin-dependent twisted boundary conditions for the eigenfunctions \[12\],

\[ \psi_{\tilde{\sigma}}(N + 1) = \exp[i(\Phi + \tilde{\sigma}\delta)]\psi_{\tilde{\sigma}}(1), \]

where \( \exp(i\tilde{\sigma}\delta) \) is the eigen value of \( S_1 \cdots S_N \), \( \tilde{\sigma} = \pm 1 \) corresponds to “up” and “down” spins in the new spin space, and \( \Phi \) is an AB flux in unit of \( \Phi_0/(2\pi) \) with the flux quantum \( \Phi_0 = h/e \). The electron wave function acquires an additional phase shift \( \pm \delta \) due to SO interaction after transversing the ring. We will refer to \( \delta \) as the SO phase shift hereinafter. It is noteworthy that the above trick to simplify the model is not specific to the Hubbard-type interaction, but is also applicable to any interaction, including long-range type, which has local SU(2) symmetry. Such extensions will be discussed later in this paper.

In the following, \( t_n \) is assumed to be site-independent so that the lattice is regular without spatial disorder. Thus we put \( t_i = -t \). It is now straightforward to obtain the Bethe ansatz solution to the Hamiltonian \[3\] with twisted boundary conditions \[4\]. Following a standard method, two kinds of rapidities are necessarily introduced to diagonalize the Hamiltonian. For the ring system with \( N \) sites, one thus gets the coupled transcendental equations for charge \( (p_j) \) and spin \( (\lambda_\alpha) \) rapidities \[18,17\],

\[ p_j N = 2\pi I_j + (\Phi + \delta) + \sum_{\beta=1}^{N_c} \eta(p_j - \lambda_\beta), \]

\[ \sum_{j=1}^{N_c} \eta(p_j - \lambda_\alpha) = 2\pi J_\alpha - 2\delta - \sum_{\beta=1}^{N_c} \eta((\lambda_\beta - \lambda_\alpha)/2), \]
with \( \eta(p) = -2\tan^{-1}(4tp/U) \), where the number of total electrons (down-spin electrons) is \( N_c \) \((N_s)\). The quantum numbers \( I_j \) and \( J_\alpha \) are integers (or half integers), which specify charge and spin excitations. The total energy is given in terms of the charge rapidity, \( E = -2t \sum_{j=1}^{N_c} \cos p_j \). Note that the effect of SO interaction effectively shifts the quantum numbers \( I_j \) and \( J_\alpha \) whereas that of Coulomb interaction appears via the two-body phase shift function \( \eta(k) \).

Let us now consider the effect of SO interaction on the energy spectrum. Applying a machinery developed by Woynarovich \[13\] to Eqs.(5) and (6), one can readily classify the excitation spectrum including the finite-size corrections which are important for the mesoscopic Hubbard ring. For the fixed number of electrons, low-lying excitations are specified by two kinds of quantum numbers \( D_c \) and \( D_s \), which respectively carry the momentum \( 4k_FD_c \) (charge current) and \( 2k_FD_s \) (spin current), where \( k_F \) is the Fermi momentum. The excitation spectrum including SO interaction is written down as \[13,19\],

\[
E(\Phi, \delta) - E_0 = \frac{4\pi v_c}{N} K_\rho \left(D_c + \frac{D_s}{2} + \frac{\Phi}{2\pi}\right)^2 + \frac{\pi v_s}{N} \left(D_s - \frac{\delta}{\pi}\right)^2,
\]

where \( v_c \) and \( v_s \) are the velocities of charge and spin excitations respectively, and \( K_\rho \) is the critical exponent for the \( 4k_F \) oscillating piece of charge correlation functions \[13\]. For the Hubbard model \( 1/2 \leq K_\rho \leq 1 \) \[13,21\]. These three fundamental quantities characterize the Luttinger liquid properties of interacting electrons completely \[22\], which can be straightforwardly evaluated using the Bethe-ansatz integral equation resulting from (5) and (6) \[13,19,21\]. For a given number of electrons it is necessary to find the lowest energy state correctly in order to derive the expression for currents. It is crucial for this purpose to notice that the quantum numbers \( D_c \) and \( D_s \) respect the following selection rule reflecting the Fermi statistics \[13\],

\[
D_c = \frac{N_c + N_s + 1}{2} \quad (\text{mod } 1),
\]

\[
D_s = \frac{N_c}{2} \quad (\text{mod } 1).
\]
We note that in the absence of SO interaction, the above spectrum has been analyzed by Yu and Fowler to study persistent currents [14].

III. EFFECTS OF SPIN-ORBIT INTERACTION ON PERSISTENT CURRENTS

We now study the effects of SO interaction on persistent equilibrium currents in the Hubbard ring. Since we wish to discuss the magnetism for the canonical ensemble with the fixed number of electrons, let us briefly summarize some results known for the canonical ensemble. The property of the orbital magnetism is sensitive to the number of fermions carrying the currents [23,24]. For example, a free electron model without SO interaction the orbital magnetism depends on the total number of electrons $N_c$ modulo 4; i.e., the ground state is diamagnetic for $N_c = 4n + 2$, paramagnetic for $N_c = 4n$ with period of a flux quantum, and paramagnetic with period of half a flux quantum for $N_c = 4n + 1, 4n + 3$ [15]. The results are slightly modified in the presence of electron-electron interaction as shown for the 1D Hubbard model [14,25]. For instance the paramagnetic state for $N_c = 4n$ is altered to a diamagnetic one by electron-electron interaction except near half filling (one electron per lattice site). In the presence of SO interaction, further modifications are expected to occur in the orbital magnetism due to the interplay of SO interaction and electron-electron interaction. In particular we will see below that for a certain parameter regime of the interaction strength, the paramagnetic state is stabilized for $N_c = 4n + 2$ in contrast to the case without SO interaction for which the ground state is always diamagnetic.

In order to clearly see what is going on, we first study the case of $4n + 2$ in detail, and mention the other cases later in this section. It may be plausible to introduce here an important key quantity $v_c K_\rho / v_s$, which will be helpful for following arguments. For noninteracting electrons, $v_c K_\rho / v_s = 1$ for any electron concentrations because $v_c = v_s$ and $K_\rho = 1$, whereas in correlated cases of $U \neq 0$ the value of $v_c K_\rho / v_s$ ranges from 0 to $\infty$ depending on the interaction strength as well as electron concentrations. We show $v_c K_\rho / v_s$ as a function of electron concentrations in Fig. 1. Roughly speaking, one can see from
this quantity whether the SO effect may be enhanced or suppressed by the electron-electron interaction. For example, the SO effect is suppressed for $v_c K_\rho / v_s > 1$, whereas it is enhanced for $v_c K_\rho / v_s < 1$.

**A. Persistent currents for $N_c = 4n + 2$ ($N_s = 2n + 1$)**

It is seen from Eqs. (8) and (9) that the selection rule for the quantum numbers in this case is $D_c = 0 \pmod{1}$ and $D_s = 0 \pmod{1}$, which implies that the ground state in the absence of SO interaction is diamagnetic around $\Phi = 0$ \cite{20,23,20}. Turning on an AB flux, there occur two different situations according to the magnitude of $v_c K_\rho / v_s$. It is straightforward to derive the persistent current $I = -\partial E / \partial \Phi$ from Eqs. (7). For $v_c K_\rho / v_s \geq 1$, the current takes the form \cite{14}

$$I = \begin{cases} 
-\frac{v_c K_\rho}{\pi N} (\Phi + \pi) & -\pi \leq \Phi \leq -\Phi_c \\
-\frac{v_c K_\rho}{\pi N} \Phi & -\Phi_c < \Phi < \Phi_c \\
-\frac{v_c K_\rho}{\pi N} (\Phi - \pi) & \Phi_c \leq \Phi \leq \pi 
\end{cases} \quad (10)$$

where

$$\Phi_c = \frac{\pi}{2} + \frac{\pi v_s}{2 K_\rho v_c} - \frac{v_s \delta}{K_\rho v_c}. \quad (11)$$

One can see that the SO phase shift $\delta$ simply alters the critical value $\Phi_c$ of the AB flux, and the diamagnetic nature around $\Phi = 0$ is not modified by SO interaction. From Eq. (11) it is explicitly seen that the effect of SO interaction is suppressed as the value of $v_c K_\rho / v_s$ increases. Such cases with $v_c K_\rho / v_s \geq 1$ realize at lower electron concentrations for $U \neq 0$, as seen from Fig. 1. It is instructive to point out here that for $\delta = \pi/2$ the effects of electron-electron interaction disappear in Eq. (11), and hence the period is halved as has been known for a free electron model \cite{12}. An alternative expression of Eq. (10) in Fourier series expansion is found to be more convenient for following discussions,

$$I(\Phi) = \sum_n \frac{v_c K_\rho (-1)^n}{\pi N} \cos \left[ n \left( \frac{\pi}{2} - \frac{\pi v_s}{2 K_\rho v_c} + \frac{v_s \delta}{K_\rho v_c} \right) \right] \sin(n \Phi). \quad (12)$$
In contrast to the above case, the expression for currents in the case of \( v_c K / v_s < 1 \) depends on the value of the SO phase shift \( \delta \). For \( 0 \leq \delta < \pi K v_c / 2 v_s + \pi / 2 \) the current is given by the same expression as Eq. (10), whereas for \( \pi K v_c / 2 v_s + \pi / 2 \leq \delta \leq \pi \) it is changed to

\[
I = \begin{cases} 
\frac{-v_c K}{\pi N} (\Phi + \pi) & -\pi \leq \Phi \leq 0 \\
\frac{-v_c K}{\pi N} (\Phi - \pi) & 0 < \Phi \leq \pi.
\end{cases} \tag{13}
\]

This is cast into an alternative formula in the Fourier series expansion as

\[
I(\Phi) = \sum_n \frac{v_c K}{\pi N n} \sin(n\Phi). \tag{14}
\]

One can see from Eq. (13) that the current has a paramagnetic sign around \( \Phi = 0 \). This change in sign is due to the interplay of the SO interaction and electron-electron interaction, which still plays a crucial role upon averaging over the strong SO interaction (see below).

**B. Strong SO interaction limit**

We have seen that the SO phase shift \( \delta \) crucially modifies the characteristic behavior of the orbital magnetism in correlated electrons. The quantity \( \delta \) is to be determined in the range \([0, \pi]\) for a given scattering process by SO interaction. For example when the scattering length of SO interaction would be comparable to the length of the ring, the value of \( \delta \) could be around \( \pi / 2 \), which results in the period of currents halved by the effect of the SO interaction as seen from (12). In more general cases when the length of the ring may be much larger than the scattering length, one has to take into account all possible rotations in spin space due to SO scattering. Such a limit is referred to as the strong SO interaction limit [11,12]. Since the SO phase shift \( \delta \) is directly related to the rotation angle in spin space, one should average currents over \( \delta \) from 0 to \( \pi \) with the weight \( \sin^2 \delta \) [11,27]. The Fourier transformed formulae of currents are more convenient to carry out the average. Using Eqs. (12) and (14), we thus derive the expression for persistent currents in the strong SO interaction limit. In the case of \( v_c K / v_s \geq 1 \) (low electron densities), we get
\[ I(\Phi) = \sum_m \frac{v_c K_\rho}{2\pi^2 N} \left( \frac{v_c K_\rho}{v_s m} + \frac{v_s v_c K_\rho m}{4v_c^2 K_\rho^2 - 4v_s^2 m^2} \right) (-1)^m \sin \left( \frac{m v_s \pi}{v_c K_\rho} \right) \sin(2m\Phi). \] (15)

It is seen that all the odd harmonics are dropped, and hence the period is reduced to half a flux quantum. In contrast to a free electron case in which only harmonics \( n = 0, 2 \) remain and the harmonics \( n = 2 \) gives a diamagnetic sign change with a reduction factor \( 1/2 \) [11], all higher even harmonics survive in the correlated case. However it is found that the ground state is still diamagnetic around \( \Phi = 0 \). To see this more explicitly we evaluate the sum of the coefficient by Fourier transformation:

\[ \sum_m \left( \frac{v_c K_\rho}{mv_s} + \frac{v_s v_c K_\rho m}{4v_c^2 K_\rho^2 - 4v_s^2 m^2} \right) (-1)^m \sin \left( \frac{v_s m \pi}{v_c K_\rho} \right) = -\frac{1}{2}, \] (16)

which results in the diamagnetic current \( I(\Phi) = -(v_c K_\rho/\pi N)\Phi \). Note that this is exactly the same as the current without SO interaction. Hence we can conclude that in case of \( v_c K_\rho/v_s \geq 1 \), the behavior of currents around \( \Phi = 0 \) is not modified by the SO effects even in the strong interaction limit. This is because the SO interaction can affect the occupation of energy levels only when there exists a finite AB flux in the system.

The situations are somewhat different for \( v_c K_\rho/v_s < 1 \) (close to half filling). There are both diamagnetic and paramagnetic contributions to currents depending on \( \delta \). The current average over the strong SO interaction is thus given by

\[ I(\Phi) = \sum_n \frac{v_c K_\rho}{2\pi^2 N n} \left[ \frac{v_s^2 n^2}{4v_c^2 K_\rho^2 - v_s^2 n^2} \sin \left( \frac{\pi v_c K_\rho}{v_s} \right) + \frac{8v_c^3 K_\rho^3}{v_s n(4v_c^2 K_\rho^2 - v_s^2 n^2)} \sin \left( \frac{n\pi}{2} + \frac{n\pi v_s}{2v_c K_\rho} \right) \right], \] (17)

The period is not halved upon averaging over \( \delta \) in this case. For small positive \( \Phi \) with \( 1/3 < v_c K_\rho/v_s < 1 \), one can perform the inverse Fourier transformation to get for \( \Phi \to 0 \),

\[ I(\Phi) = -\frac{2v_c K_\rho}{\pi N} \left\{ \left[ -\frac{v_c K_\rho}{v_s} \cos \left( \frac{\pi v_c K_\rho}{v_s} \right) + \frac{v_s - v_c K_\rho}{v_s} \right] \Phi - \frac{1}{2} \left[ \pi - \frac{\pi v_c K_\rho}{v_s} - \sin \left( \frac{\pi v_c K_\rho}{v_s} \right) \right] \right\}, \] (18)

which has a paramagnetic sign. Since paramagnetic contributions to the current averaged over \( \delta \) increase compared with diamagnetic ones when \( v_c K_\rho/v_s \) is decreased, we can say that
the current for \(0 \leq v_c K_\rho / v_s < 1\) always shows a paramagnetic sign around \(\Phi = 0\). We would like to stress here that this paramagnetic state realizes as a consequence of the interplay of SO interaction and electron-electron interaction, and should not appear if either of the two is absent.

We have computed persistent currents using the formulae (15) and (17) together with the Bethe equations for \(v_c, v_s\), and \(K_\rho\). In Figs. 2 and 3, persistent currents in the strong SO limit are shown as a function of the AB flux \(\Phi\) for two different cases.

C. Reduction factors in currents

Here we make a brief comment on reduction factors in persistent currents by the SO effects. According to Meir et al. [12] and Entin-Wohlman et al. [11], reduction factors due to the SO interaction are written in a simple and universal form for a noninteracting model with even number of electrons. An essence of their idea is that the persistent current \(I(\Phi)\) in the presence of SO interaction can be expressed in the form

\[
I(\Phi) = I_0(\Phi + \delta) + I_0(\Phi - \delta) \tag{19}
\]

where \(I_0\) is the current per each spin component without SO interaction. This leads to universal reduction factors in the form of Fourier expansion [11,12],

\[
I(\Phi) = \sum_n \cos(n\delta)a_n \sin(n\Phi) \tag{20}
\]

with \(a_n\) being the Fourier coefficients for the case without SO interaction. One can see that the reduction factor \(\cos(n\delta)\) depends only on the SO phase shift. This expression leads to a rather simple result that upon averaging in the strong SO limit, the current is reduced by a factor of \(1/2\), and the fluctuations by a factor \(1/4\) [11,12]. For interacting electrons, however, it is seen that the SO phase shift should not appear in a simple form of \(\Phi \pm \delta\). It is instead combined with a factor of \(v_c K_\rho / v_s\) reflecting the electron correlation effects. Thus in the presence of electron-electron interaction, the reduction factors follow from Eqs.(12),
$$\cos \left[ n \left( \frac{\pi}{2} - \frac{\pi v_s}{2v_c K_p} + \frac{v_s \delta}{v_c K_p} \right) \right] / \cos \left[ n \left( \frac{\pi}{2} - \frac{\pi v_s}{2v_c K_p} \right) \right],$$

(21)

for $v_c K_p / v_s \geq 1$. For $v_c K_p / v_s < 1$ we can not define the reduction factor in such a simple form, as seen from Eqs.(17). So, the above remarkable properties obtained for the SO effects based on a free electron model are changed when the electron-electron interaction is introduced. To avoid confusions we would like to mention that the present nonuniversal results may not be contradicted with the universal reduction factor of currents expected in disordered system [10,11,27–30]. In such cases the average over disorder may play an essential role, which has not been taken into account in the present calculation.

**D. Electron-number dependence**

We have been concerned so far with the case of $N_c = 4n + 2$. As mentioned before the results are sensitively dependent on the number of electrons. Here we summarize the results for other cases. The calculation can be performed in parallel to the above example of $N_c = 4n + 2$.

(a) $N_c = 4n$ ($N_s = 2n$). In this case the selection rule for the quantum numbers reads, $D_c = 1/2$ (mod 1) and $D_s = 0$ (mod 1). For $v_c K_p / v_s \geq 1$ the persistent current in the Fourier transformed form is then given by

$$I(\Phi) = \sum_n \frac{v_c K_p}{\pi N n} \cos \left[ n \left( \frac{\pi}{2} - \frac{v_s \pi}{2v_c K_p} \right) \right] \sin(n \Phi),$$

(22)

which shows a diamagnetic sign around $\Phi = 0$. In the case of $v_c K_p / v_s < 1$, where only the paramagnetic state realizes without SO interaction, the expression is the same as Eq.(22) for $\pi / 2 - v_c K_p \pi / 2v_s < \delta \leq \pi$, while Eq.(14) for $0 \leq \delta \leq \pi / 2 - v_c K_p \pi / 2v_s$. Thus the paramagnetic state near the metal-insulator transition (half-filling) is changed to a diamagnetic one by SO interaction for $\pi / 2 - v_c K_p \pi / 2v_s < \delta \leq \pi$. It is remarkable, however, that in the strong SO interaction limit we obtain exactly the same results in currents as for the case of $N_c = 4n + 2$ upon averaging over all possible phases $\delta$ (see Figs. 2 and 3).
We can hence summarize the results for the even number of electrons as follows; the ground state is always diamagnetic for $v_cK/p > 1$ (low electron densities) in the strong SO limit, whereas for $v_cK/p < 1$ (near the metal-insulator transition point) the paramagnetic state is stabilized not only for $N_c = 4n$ but also for $N_c = 4n + 2$ as a result of the SO effects.

(b) Odd case ($N_c = 4n + 1, 4n + 3$). We first consider the case for $N_c = 4n + 3$. Since Eq. (7) holds only for the case with $N^\uparrow \geq N^\downarrow$, the roles of “up” spin and “down” spin for $\Phi > 0$ are interchanged for $\Phi < 0$ in Eq. (7). Thus the result for $\Phi < 0$ in the case of odd number of electrons can be deduced from that for $\Phi > 0$ by changing $\delta \rightarrow -\delta$. Noting the selection rule $D_c = 1/2$ (mod 1) and $D_s = 1/2$ (mod 1) for $\Phi > 0$, the current in the ground state is now given by

$$I = \begin{cases} -\frac{v_cK}{\pi N} \left( \Phi + \frac{\pi}{2} \right) & -\pi \leq \Phi < 0 \\ -\frac{v_cK}{\pi N} \left( \Phi - \frac{\pi}{2} \right) & 0 \leq \Phi \leq \pi \end{cases}$$

which has a period of half a flux quantum, and exhibits a paramagnetic sign around $\Phi = 0$.

It should be noted that the above expression is independent of $\delta$, and hence there are not any modifications due to SO interaction. The effect of SO interaction indeed appears in the next order corrections in $1/N_c$, which are not taken into account in our formulation. For a non-interacting case the next-order corrections have been evaluated by Entin-Wohlman et al., resulting in a small shift of energy minima from $\Phi = \pm\pi/2$ to $\Phi = \pm [(1 + 1/N_c)\pi/2 - \delta/N_c]$ \cite{11,31}, which may be actually neglected in mesoscopic rings. We note that for another odd case of $N_c = 4n + 1$, we get the exactly same results of currents as for $N_c = 4n + 3$.

In summary we can say that characteristic properties of persistent currents in the strong SO limit are classified by the parity of the electron number even for correlated electron systems.

**IV. EXTENSION TO MORE GENERAL MODELS**

In the Hubbard model the electron-electron interaction is assumed to be short-ranged (on-site), which in turn enables us to treat the model exactly. In more general cases, however,
effective interaction would be of long-range type, since the screening effect of the Coulomb interaction may become less effective for mesoscopic metallic rings. In such cases it is quite difficult to get the energy spectrum exactly. So it is desirable to find a possible way to extend our analysis to more general cases including long-range interaction. We wish to briefly depict a simple idea how to treat such cases.

We recall here a trick used to simplify the Hubbard model with SO interaction, i.e. a unitary transformation which incorporates the SO interaction into the boundary effects (2). Note that this technique is still applicable to more general long-range interactions so long as they retain local SU(2) symmetry. Such a local SU(2) symmetry for interaction may be expected to hold in ordinary cases, such as partially screened Coulomb interaction, etc. The remaining problem is then how to obtain the expression for the low-energy spectrum like Eq.(7) including the SO effect and the AB flux. To this end the bosonization technique may be useful [32,33,21,22] because it can formally describe low-energy states even for non-integrable systems. The bosonization scheme has been previously used by Loss to discuss persistent currents for a spinless fermion system [24].

Following a standard technique in bosonization [32,33], we now discuss how the SO effect on currents can be treated in nonintegrable systems. We do not have to specify an explicit form of interaction here, but only assume the interaction $V(r)$ to be invariant under local SU(2) transformations. In general, low-energy gapless states of 1D metallic electron systems compose of two independent Luttinger liquids [22] corresponding to charge and spin degrees of freedom [19,21]. Hence the system is described by the sum of two Gaussian models with conformal charge $c = 1$.

Let us now introduce the boson fields for spin and charge degrees of freedom. It is found that the spin-dependent twisted boundary conditions (2) due to an AB flux and SO interaction are incorporated into boson phase fields as [22,24]

$$
\phi_\rho = \sum_{k \neq 0} \left| \frac{\pi}{2Lk} \right|^\frac{1}{2} e^{ikx} [a_{k,\rho}^\dagger + a_{-k,\rho}] + \phi_{0,\rho} + M_\rho \frac{\pi x}{L}, \quad (24)
$$
\[ \phi_\sigma = \sum_{k \neq 0} \left| \frac{\pi}{2Lk} \right|^2 e^{ikx} [a_{k,\sigma}^\dagger + a_{-k,\sigma}] + \phi_0 + M_\sigma \frac{\pi x}{L}, \]  

\[ \theta_\rho = \sum_{k \neq 0} \left| \frac{\pi}{2Lk} \right|^2 \text{sign}(k) e^{ikx} [a_{k,\rho}^\dagger - a_{-k,\rho}] + \theta_{0,\rho} + \left( J_\rho + \sqrt{2} \frac{\Phi}{\pi} \right) \frac{\pi}{L} \left( x + \frac{L}{2} \right), \]  

\[ \theta_\sigma = \sum_{k \neq 0} \left| \frac{\pi}{2Lk} \right|^2 \text{sign}(k) e^{ikx} [a_{k,\sigma}^\dagger - a_{-k,\sigma}] + \theta_{0,\sigma} + \left( J_\sigma + \sqrt{2} \frac{\delta}{\pi} \right) \frac{\pi}{L} \left( x + \frac{L}{2} \right), \]  

where \( a_{k,\rho}(a_{k,\rho}^\dagger) \) and \( a_{k,\sigma}(a_{k,\sigma}^\dagger) \) are boson annihilation (creation) operators for charge and spin densities, respectively, and \( M_\rho(M_\sigma) \) and \( J_\rho(J_\sigma) \) are the charge (spin) number and the charge (spin) current, respectively, which are defined by \( M_\rho = (M_\uparrow + M_\downarrow)/\sqrt{2} \), \( M_\sigma = (M_\uparrow - M_\downarrow)/\sqrt{2} \), \( J_\rho = (J_\uparrow + J_\downarrow)/\sqrt{2} \), and \( J_\sigma = (J_\uparrow - J_\downarrow)/\sqrt{2} \). We note here that \( M_{\uparrow(\downarrow)} \) and \( J_{\uparrow(\downarrow)} \) describe topological excitations introduced by Haldane [22]. They satisfy the selection rules, \((-1)^{N_\sigma +1} = (-1)^{M_\sigma + J_\sigma} \), where \( s = \uparrow, \downarrow \) and \( N_0(\uparrow) \) is the total number of up (down) spins. \( \phi_{0,\nu} \) and \( \theta_{0,\nu} \) are conjugate variables of \( J_\nu \) and \( M_\nu \), respectively.

Using the above phase fields we can write down the low-energy effective Hamiltonian, from which the persistent current directly follows, as long as the system belongs to the universality class of Luttinger liquids. The energy spectrum of the effective Hamiltonian with finite-size correction terms reads [22]

\[ E(\Phi) - E_0 = \frac{\pi}{2L} \left[ v_{\rho M} M_\rho^2 + v_{\rho J} \left( J_\rho + \sqrt{2} \frac{\Phi}{\pi} \right)^2 + v_{\sigma M} M_\sigma^2 + v_{\sigma J} \left( J_\sigma + \sqrt{2} \frac{\delta}{\pi} \right)^2 \right], \]  

where \( v_{\nu M} \) and \( v_{\nu J} \) (\( \nu = \rho, \sigma \)) are Luttinger liquid parameters describing the velocity of excitations. It is to be noted that the effect of electron-electron interaction \( V(r) \) is only to renormalize these parameters. The ground state is given by the condition, \( M_\rho = M_\sigma = 0 \), i.e. \( M_\uparrow = M_\downarrow = 0 \). Thus using the selection rules mentioned above we obtain the topological constraints for \( J_\uparrow \) and \( J_\downarrow \): (1) \( J_\uparrow \) even, \( J_\downarrow \) even for \( N_c = 4n + 2 \), (2) \( J_\uparrow \) odd, \( J_\downarrow \) odd for \( N_c = 4n \), (3) \( J_\uparrow \) odd, \( J_\downarrow \) even (and vice versa) for \( N_c = 4n + 3 \), (4) \( J_\uparrow \) even, \( J_\downarrow \) odd (and vice versa) for \( 4n + 1 \). Consequently, if we put \( v_{\rho J} = K_\rho v_c \) and \( v_{\sigma J} = v_s \), we find that the energy (28) with these topological constraints is equivalent to Eq.(11), and \( D_c \) and \( D_s \) correspond to \( J_\uparrow/2 \) and \( (J_\downarrow - J_\uparrow)/2 \), respectively.
Using Eq. (28) and the above topological constraints, we can obtain the persistent current, and discuss its sign and period quite similarly to the case for the Hubbard ring. Therefore all the expressions derived in the previous section can be directly applied to the present case by regarding \( v_c K_\rho / v_s \) as a free parameter to be determined. In order to determine the parameter, we generally need input data from another microscopic calculation, e.g., numerical diagonalization. We note that several elegant techniques to get the Luttinger liquid parameters numerically for nonintegrable systems have been already developed [22,20], which will be helpful for us to evaluate the persistent currents explicitly.

V. SUMMARY

In this paper we have discussed the effects of SO interaction on persistent currents in the mesoscopic Hubbard ring. We have investigated the problem exactly combining the Bethe-ansatz solution with a unitary transformation which incorporates the SO effects into spin-dependent twisted boundary conditions. It has been shown that characteristic properties of the orbital magnetism in the Hubbard ring are classified according to the value of \( v_c K_\rho / v_s \) and the number of electrons \( \text{modulo} \ 4 \). In particular, we have demonstrated that \( v_c K_\rho / v_s \) is an important key quantity to see whether the SO effects are enhanced or suppressed by the electron-electron interaction.

In the strong SO interaction limit it has been found that the formula obtained for currents is classified by the parity of the electron number. For the even number of electrons, the ground state is diamagnetic with period of half a flux quantum for \( v_c K_\rho / v_s \geq 1 \) (low electron densities), and paramagnetic with period of a flux quantum for \( v_c K_\rho / v_s < 1 \) (close to half filling). In particular the paramagnetic state for \( v_c K_\rho / v_s < 1 \) is realized by a combined effect arising from the interplay of SO interaction and electron-electron interaction. In the Hubbard model, the condition \( v_c K_\rho / v_s < 1 \) is satisfied near half filling which implies that the system would be close to the Mott insulator. Therefore such a novel phenomenon for \( v_c K_\rho / v_s < 1 \) is expected in general to occur for interacting electrons in a metallic phase close
to the Mott insulator. In contrast to the even case, the ground state for the odd number of electrons is found to be always paramagnetic with period of half a flux quantum, which is not affected by SO interaction as long as the corrections up to the order of $O(1/N)$ are concerned.

In conclusion the effect of SO interaction together with electron-electron interaction gives rise to a novel and qualitative change in the orbital magnetism for 1D interacting electron systems. There remain several important problems to be investigated. For example we have not considered the effect of disorder in this paper, which would induce interesting phenomena together with the SO effects as well as with the correlation effects. Furthermore an extension of the theory to multichannel cases is desirable to confront the results with various experiments. These problems are now under consideration.

ACKNOWLEDGMENTS

This work is partly supported by the Grant-in-Aid from the Ministry of Education, Science and Culture.
REFERENCES

1 For a recent review, see, e.g. B. L. Altshuler, in Nanostructures and Mesoscopic Phenomena, edited by W. P. Kirk and M. A. Reed (Academic, San Diego, 1992).

2 L. P. Lévy, G. Dolan, J. Dunsmuir, and H. Bouchiat, Phy. Rev. Lett. 64, 2074 (1990).

3 V. Chandrasekhar, R. A. Webb, M. J. Brady, M. B. Ketchen, W. J. Gallagher, and A. Kleinsasser, Phys. Rev. Lett. 67, 3578 (1991).

4 H. F. Cheung, Y. Gefen, E. K. Riedel, and W. H. Shih, Phys. Rev. B 37, 6050 (1988)

5 H. Bouchiat and G. Montambaux, J. Phys. (Paris) 50, 2695 (1989).

6 V. Ambegaokar and U. Eckern, Phys. Rev. Lett. 65, 381 (1990).

7 B. L. Altshuler, Y. Gefen, and Y. Imry, Phys. Rev. Lett. 66, 88 (1991).

8 A. Schmid, Phys. Rev. Lett. 66, 80 (1991).

9 F. von Oppen and E. K. Riedel, Phys. Rev. Lett. 66, 84 (1991).

10 H. Mathur and A. D. Stone, Phys. Rev. B 44, 10957 (1991).

11 O. Entin-Wohlman, Y. Gefen, Y. Meir, and Y. Oreg, Phys. Rev. B 45, 11890 (1992).

12 Y. Meir, Y. Gefen, O. Entin-Wohlman, Phys. Rev. Lett. 63, 798 (1989).

13 F. Woynarovich, J. Phys. A 22, 4243 (1989).

14 N. Yu and M. Fowler, Phys. Rev. B 45, 11795 (1992).

15 D. Loss and P. Goldbart, Phys. Rev. B 43, 13762 (1991).

16 see, e.g., E. Fradkin, Field Theories of Condensed Matter Systems (Addison-Wesley, 1991).

17 B. S. Shastry and B. Sutherland, Phys. Rev. Lett. 65, 243 (1990).

18 E. H. Lieb and F. Y. Wu, Phys. Rev. Lett. 20, 1445 (1968).
19 H. Frahm and V. E. Korepin: Phys. Rev. B 42, 10553 (1990).

20 H. J. Schulz, Phys. Rev. Lett. 64, 2831 (1990); Int. J. Mod. Phys. 5 57 (1991).

21 N. Kawakami and S. K. Yang, Phys. Rev. Lett. 65, 2309 (1990).

22 F. D. M. Haldane, J. Phys. C, 14, 2585 (1981); Phys. Rev. Lett. 47, 1840 (1981); ibid 45, 1358 (1980).

23 N. Byers and C. N. Yang. Phys. Rev. Lett. 7, 46 (1961).

24 D. Loss, Phys. Rev. Lett. 69, 343 (1992). This paper also corrects some minor errors in the expressions of the boson phase fields in ref. [22].

25 R. M. Fye, M. J. Martins, D. J. Scalapino, J. Wagner, and W. Hanke, Phys. Rev. B 44, 6909 (1991).

26 N. Kawakami and S. K. Yang, Phys. Rev. B 44, 7844 (1991).

27 G. Bergman, Solid State Commun. 42, 815 (1982).

28 S. Hikami, A. I. Larkin, and Y. Nagaoka, Prog. Theor. Phys. 63, 707 (1980).

29 B. L. Altshuler and B. I. Shklovskii, Zh. Eksp. Teor. Fiz. 91, 220 (1986) [Sov. Phys. JETP 64, 127 (1986)].

30 P. A. Lee, A. D. Stone, and H. Fukuyama, Phys. Rev. B 35, 1039 (1987).

31 The results obtained in ref. [14] for odd number of electrons also drop the $1/N$ correction obtained by ref. [15] for non-interacting case.

32 V. J. Emery, Highly Conducting One-Dimensional Solids (Plenum, New York, 1979).

33 J. Sólyom, Adv. Phys. 28, 209 (1979).
FIGURES

FIG. 1.

Plots of $v_c K_p / v_s$ as a function of electron densities $n$ for the Hubbard model. The half filling corresponds to $n = 1$.

FIG. 2.

Persistent currents plotted against $\Phi / 2\pi$ for $U/t = 4$ and $n = 0.65$ ($v_c K_p / v_s = 1.32$) in the case of even number of electrons. The current normalized by $t/N$ is shown.

FIG. 3.

Persistent currents plotted against $\Phi / 2\pi$ for $U/t = 4$ and $n = 0.95$ ($v_c K_p / v_s = 0.58$) in the case of even number of electrons. The current normalized by $t/N$ is shown.