Research on propagation law of one-dimensional stress wave in jointed rock mass under in-situ stress

Qian Dong¹,²
¹Hubei (Wuhan) Institute of Explosion Science and Blasting Technology, Jianghan University, Wuhan, Hubei, 430056, China
²Hubei Key Laboratory of Blasting Engineering, Jianghan University, Wuhan, Hubei, 430056, China
*Corresponding author’s e-mail: dongqian@jhun.edu.cn

Abstract. The propagation law of stress wave in jointed rock mass under in-situ stress has important engineering significance on blasting excavation and protection engineering design of deep rock mass. Based on theoretical analysis, the propagation law of stress wave across multiple nonlinear deformation joints under the initial stress is studied. Firstly, the propagation equation of stress wave across multiple nonlinear joints considering the influence of initial stresses is established, through the discontinuous deformation method (DDM) and the recursive analysis method of stress wave in time domain. On this basis, parametric studies are conducted for the effects of stress wave propagation, such as initial stress, the amplitude of stress wave, the distance and the number of joints. The results show that the existence of initial stress will affect the propagation of stress wave, the geometric distribution of joints, such as spacing and number, can also cause the change of stress wave transmission coefficients, and the amplitude of incident wave varies can also lead to the change of stress wave propagation.

1. Introduction
The in-situ stress environment of deep rock mass engineering can not be ignored. The natural rock mass usually contains a large number of joints, when the stress wave across the joints, the amplitude and velocity of stress wave will be attenuated. And the criterion of the damage degree of rock mass is related to the propagation parameters of stress wave, such as the peak particle velocity (PPV) and the wave velocity[1].

When the stress wave amplitude is large enough, the joint can produce nonlinear deformation. The BB model proposed by Bandis[2] is widely used to describe the nonlinear deformation characteristics of joints in various studies, which is considered that the joint normal stiffness nonlinear increases with the increase of the joint closure. At present, the theory of the propagation of stress waves across multiple joints can be divided into two categories[3]. The first one is based on the theory of discontinuous deformation method (DDM)[4]. It is considered that the stress on both sides of the joint is continuous in the process of stress wave propagating through, but the displacement on both sides of the joint is discontinuous and the reason for displacement discontinuity is due to the nonlinear deformation of the joint. Other scholars combined the discontinuous deformation method (DDM) with different analysis methods, such as the method of characteristic line (MC)[5] and scattering matrix method (SMM)[6]. The second is based on the equivalent medium method (EMM), where effective elastic moduli were calculated and used to represent the jointed medium, such as virtual wave source.
(VWS) method[7]. However, these analytical methods have not considered the influence of initial stress. Fan[8] combined the discontinuous deformation method (DDM) and method of characteristic line (MC), considering the influence of initial stress, to study the stress wave propagation and attenuation through single joint. Li[9] conducted model test to study the propagation and attenuation of elastic waves in deep fractured rock mass.

In this paper, the propagation equation of stress wave vertical across multiple nonlinear parallel joints considering in-situ stress is derived, based on discontinuous deformation method (DDM) and the recursive analysis method of stress wave in time domain. Parameters affecting the stress wave attenuation, such as the in-situ stress, the amplitude of stress wave, the distance between joints and the number of joints are analyzed.

2. Theoretical analysis of stress wave propagation in joints

2.1. Nonlinear mechanics model of joint

In the BB model, the relation between the stress and closure of the joint is shown in equation (1).

\[ d = \frac{\sigma}{k_{ni} + \sigma/d_{ma}} \] (1)

Where, \( k_{ni} \) is the initial normal stiffness of the joint, \( d_{ma} \) is the maximum closure of joint. The relationship between the stress and closure of the joint under the initial stress is shown in figure 1.

![Figure 1. Deformation characteristics of joints under initial stress](image)

It is observed from figure 1 that when the stress wave is propagated in the nonlinear deformation joint with the initial stress the closure of the joint can be divided into two parts, one part is the static closure \( d_s \) generated by the initial stress \( \sigma_i \), and the other is dynamic closure \( d_d \) generated by incident stress wave.

2.2. Propagation model of stress wave across multiple joints under initial stress

The mechanical model of stress wave vertically incident a set of parallel joints under the normal stress is shown in figure 2. It is assumed that the rock on both sides of the joint are homogeneous, isotropic and linearly elastic material. From the figure 2, it can be found that the reflected wave is generated in the left side of the first joint, and the transmitted wave is generated on the right side of the last joint in the process when stress wave passing through the parallel joints. When the stress wave length is greater than the thickness of joint, the effect of joint thickness on stress wave propagation can be neglected.

In the whole process of stress wave propagation in the parallel joints, the initial stress on both sides of each joint is kept as a constant, multiple reflection will occur among the joints, and the both sides of the joint will exist the left and right running stress wave, as shown in figure 3.
2.3. Propagation model of stress wave across multiple joints under initial stress

Based on the theory of wave front momentum conservation and the interaction between stress wave and joint, when the stress wave is vertically incident on parallel joints, the dynamic stress at the two sides of the $J$th joint can be rewritten as followed equation (2) and (3).

$$\sigma_d^- = z_p \cdot v_{R_p} + z_p \cdot v_{L_p}$$  \hspace{1cm} (2)

$$\sigma_d^+ = z_p \cdot v_{R_p}^+ + z_p \cdot v_{L_p}^+$$  \hspace{1cm} (3)

The velocities on the two interfaces of the joint are as follows:

$$v_n = v_{R_p} - v_{L_p}$$  \hspace{1cm} (4)

$$v_d^+ = v_{R_p}^+ - v_{L_p}^+$$  \hspace{1cm} (5)

Where, the symbol “+” and “-” represent the right and left side of joint respectively, $v_{R_p}$ and $v_{L_p}$ are the particle velocities of right and left running stress wave on the left side of the joint, $v_{R_p}^+$ and $v_{L_p}^+$ are the particle velocities of right and left running stress wave on the right side of the joint, The wave impedance $z_p$ of the rock is calculated by the following equation.

$$z_p = \rho C_p$$  \hspace{1cm} (6)

Where, $C_p$ is the stress wave propagation velocity, $\rho$ is the rock density. For the $J$th joint, the stresses and the displacements before and after the two sides of the joint should satisfy the displacement discontinuous boundary condition shown in the equation (7) and (8).

$$\sigma_s + \sigma_d^+ = \sigma_s^- + \sigma_d^- = \sigma$$  \hspace{1cm} (7)

$$u_a^--u_a^+ = \frac{\sigma}{k_m + \sigma/d_m}$$  \hspace{1cm} (8)

When equation (8) is differential with respect to time $t$, and generalized stiffness $k_g$ is defined as follows:

$$k_g = \frac{k_m + (\sigma_s + \sigma_d^+(i))/d_m}{k_m}$$  \hspace{1cm} (9)

$$v_a^- - v_a^+ = \frac{1}{k_g} \cdot \frac{\sigma_s^+(i+1) - \sigma_s^+(i)}{\Delta t}$$  \hspace{1cm} (10)
Where, $v^+_{ip}(i)$ and $v^-_{ip}(i)$ denote particle vibration velocity on both sides of the joint at time $i$, $\Delta t$ is a small time interval. When equation (9) and (10) are combined with equation (3) and (8), that is:

$$v^+_p(i) = v^-_p(i) + v^-_{ip}(i) - v^-_{ip}(i)$$

$$v^-_{np}(i+1) = -v^+_p(i+1) + (1-A)v^-_p(i) + (1+A)v^-_{ip}(i) + Av^-_p(i) - Av^-_{ip}(i)$$

In the process of stress wave propagation in the parallel joints, the rock is assumed to be an isotropic and linearly elastic material, so the relations for the right and left running stress wave between two adjacent joints must satisfy the time-shifting function [10] as shown in following equations.

$$v^+_{ip}(i) = v^+_{ip}(i-n_p)_{j-1}$$

$$v^-_{ip}(i) = v^-_{ip}(i-n_p)_{j+1}$$

$$v^+_{ip}(i+1) = v^+_{ip}(i+1-n_p)_{j+1}$$

Where, $n_p$ is the shifting times for the stress waves between two adjacent joints, and is calculated by the following equation (16).

$$n_p = \text{int}[S/(\Delta t \cdot C_p)]$$

Where, $S$ is the joint spacing, $\Delta t$ is a small time interval. Combining the equation (12)–(16) and the initial and boundary conditions, transmission coefficient $T_p$ of stress wave can be derived shown in following equation (17).

$$T_p = \frac{\max|v^+_{np}(i)_{N}|}{\max|v^-_{np}(i)_{H}|}$$

3. Parametric analysis

The incident stress wave is assumed to be a half cycle sine wave as shown in following equation (18).

$$v = \begin{cases} A \sin(2\pi ft) & 0 \leq t \leq 1/2f \\ 0 & t < 0, t > 1/2f \end{cases}$$

Where $A$ and $f$ denote the amplitude and frequency respectively of the incident wave, and the frequency is 100 Hz in this section. The mechanical parameters of rock and joint are assumed as: the rock density is 2,930 kg/m$^3$, the stress wave velocity in the rock is 5,000 m/s, the initial normal stiffness and the maximum closure of the joint is 3.5 GPa/m and 1 mm. Nondimensional fracture spacing, $\xi$, defined as the ratio of fracture spacing to incident wavelength, is adopted to study the effects of joint spacing.

3.1. Effects of stress wave amplitude on wave propagation

For the joint with linear deformation, the variation of the incident wave amplitude can not led to the change of joint stiffness, for the nonlinear deformation of the joint, its stiffness is related to the in-situ stress and the amplitude of incident wave. The transmission coefficient $T_2$ of different amplitude stress wave across two parallel nonlinear joints under different magnitude of in-situ stress is shown in figure 4. In the calculation process, the nondimensional fracture spacing is 0.01, the in-situ stress are 0, 1, 3, 5, 10, 15 MPa, respectively, and the variation range of stress wave amplitude is 0.01~1 m/s.

Figure 4 shows that the transmission coefficient $T_2$ increases with the increase of the amplitude of the incident wave under different in-situ stress, and finally approaches to 1. The reason is that the nonlinear stiffness of joint enlarge with the increase of stress wave amplitude when the stress amplitude is large enough, the stiffness of the joint and rock is more and more close, which can led to...
the increase the transmission coefficient. Meanwhile, it is observed that when in-situ stress is less than 5 MPa, the transmission coefficient increases sharply with the increase of initial stress, when the in-situ stress is 15 MPa, the transmission coefficient is almost close to 1. The effect of the in-situ stress can also lead to the increase of the stiffness of the nonlinear joint, so that the transmission coefficient increases correspondingly. When the in-situ stress is large enough, will lead the stiffness of the joint is infinitely close to the intact rock, the stress wave will be completely transmitted, and the mechanical effect of the joint in rock mass disappears.

Figure 4. The relationship between the amplitude of the incident wave and the transmission coefficient under different in-situ stresses

3.2. Effects of joints spacing on wave propagation

The influence of the joints spacing under different in-situ stresses on the transmission coefficient of stress wave is considered, the relationship between the transmission coefficient $T_2$ and the joints spacing under different in-situ stresses is shown in the figure 5. The range of nondimensional fracture spacing is 0.01~0.5, the amplitude of the incident wave is 0.1 m/s.

Figure 5. The relationship between joints spacing and transmission coefficient under different in-situ stresses

According to the calculation results, it is shows that when the in-situ stress is relatively small (less than 5 MPa), and transmission coefficient firstly increases, then decreases with the increment of joints spacing. And when the stress is greater than 10MPa, the transmission coefficient basically kept as a constant close to 1, with the increase of the joint spacing, in other words, nondimensional fracture
spacing $\xi$ decreases, the transmission coefficient is even greater than 1. The phenomenon is caused by the multiple transmission and reflection of stress wave propagation in joints, and the superposition phenomenon occurred in the transmission wave. When the joint space is located in a certain range, the superposition effect becomes very strong, resulting in the transmission coefficient is greater than 1.

With the increase of joint spacing coefficient, the transmission coefficient finally close to a constant, this is because when the nondimensional joint spacing exceeds a certain value $\xi_{\text{thr}}$, the multiple transmission and reflection in joints lose effect on transmission coefficient. Fig.5 shows that the value $\xi_{\text{thr}}$ decreases with the increase of in-situ stress.

3.3. Effects of the number of joints on wave propagation

The number of nonlinear joints considering in the above two sections is two, and the number of joints also influence the propagation of stress wave, the transmission coefficient $T_N$ of stress wave across different number of parallel nonlinear joints under different in-situ stresses is shown in Figure.6(a) In the calculation process, the joint spacing coefficient $\xi = 0.01$, which corresponds to the small joint spacing, and the stress amplitude is 0.01 m/s.

![Figure 6. The relationship between joints numbers and transmission coefficient under different in-situ stress](image)

Figure 6(a) shows that under the same in-situ stress, the transmission coefficient $T_N$ decreases with the increase of the number of joints, and finally tends to a constant value, and the value increases with the increase of in-situ stress. When the joint spacing is small, jointed rock mass can be treated as the equivalent medium[11], the multiple transmission and reflection tends to be stable, as well as when the joint number exceeds the threshold $N_{\text{thr}}$, the number of joints has no effect on the transmission coefficient.

When the joint spacing coefficient $\xi = 0.3$, the relationship between the transmission coefficient $T_N$ and the number of joints is shown in Figure 6(b), which shows that when the joint spacing is larger, the variation law of the transmission coefficient $T_N$ is the same as the small joint spacing without in-situ stress, and the transmission coefficient is increased in the presence of in-situ stress. Different from the small joint spacing condition, the transmission coefficient $T_N$ decreases with the increase of the number of joints, but no longer tends to be a constant, and the higher the in-situ stress is, the lower the reducing rate is.

4. Conclusion

The propagation equation of stress wave across multiple nonlinear joints considering the influence of initial stresses is established, and parametric studies are conducted. The following conclusions are obtained:
1. When the in-situ stress is small, the transmission coefficient increases sharply with the increase of initial stress. If the in-situ stress or the amplitude of stress wave is large enough, will lead the stiffness of the joints is infinitely close to the intact rock, the stress wave will be completely transmitted and the mechanical effect of the joints in rock mass disappears.

2. The transmission coefficient firstly increases and then decreases with the increment of joints spacing. When the in-situ stress is relatively high and the joint space is located in a certain range, the transmission coefficient is even greater than 1. The phenomenon is caused by the multiple transmission and reflection while stress wave propagation in joints, and the transmitted wave superposition phenomenon occurred. Meanwhile, the nondimensional joint spacing $\xi$ has a threshold value $\xi_{thr}$, and the transmission coefficient is kept constant when the nondimensional joint spacing $\xi$ exceeds the threshold value $\xi_{thr}$, the value $\xi_{thr}$ decreases with the increase of in-situ stress.

3. For the case when joint spacing is small, the transmission coefficient decreases with the increase of the number of joints, and finally tends to a constant value when the number of joints exceeds the threshold value $N_{thr}$, the value $\xi_{thr}$ enlarges with the increase of in-situ stress. For the larger joint spacing case, the transmission coefficient increase in the presence of in-situ stress, the transmission coefficient $T_N$ decreases with the increase of the number of joints, but no longer tends to be a constant, and the higher the in-situ stress is, the lower the reducing rate is.

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