Scalable Autonomous Vehicle Safety Validation through Dynamic Programming and Scene Decomposition

Anthony Corso¹, Ritchie Lee², and Mykel J. Kochenderfer¹

Abstract—An open question in autonomous driving is how best to use simulation to validate the safety of autonomous vehicles. Existing techniques rely on simulated rollouts, which can be inefficient for finding rare failure events, while other techniques are designed to only discover a single failure. In this work, we present a new safety validation approach that attempts to estimate the distribution over failures of an autonomous policy using approximate dynamic programming. Knowledge of this distribution allows for the efficient discovery of many failure examples. To address the problem of scalability, we decompose complex driving scenarios into subproblems consisting of only the ego vehicle and one other vehicle. These subproblems can be solved with approximate dynamic programming and their solutions are recombined to approximate the solution to the full scenario. We apply our approach to a simple two-vehicle scenario to demonstrate the technique as well as a more complex five-vehicle scenario to demonstrate scalability. In both experiments, we observed an order of magnitude increase in the number of failures discovered compared to baseline approaches.

I. INTRODUCTION

One common practice for automated vehicle (AV) safety validation is to maintain a suite of challenging driving scenarios that the vehicle must successfully navigate after each update to the driving policy. Although useful, this approach will miss any failures that are not already included in the test suite. Automated testing procedures that treat the vehicle as a black box must be developed to catch unknown and unexpected failure modes of the AV which could dramatically decrease testing time and improve the safety of autonomous vehicles.

Much of the literature on black box testing revolves around falsification where inputs are generated that cause a system to violate a specification [1]. Those inputs serve as a counter example to the hypothesis that the system is completely safe. For autonomous driving it is not feasible to create an agent that can avoid all possible accidents [2], so rather than find any failure of an AV, it is preferable to find the most likely failures. Traditional falsification techniques do not consider the probability of the failures they find and are therefore ill-suited to this goal. Adaptive stress testing [3] tries to find the most-likely failure of an autonomous system. This approach can improve the relevance of discovered failures but does not necessarily explore the range of possible failures of the system. The goal of this work is to develop a safety validation approach that can reliably find all of the most relevant failures of an autonomous vehicle.

Our approach attempts to estimate the distribution over failures of an autonomous vehicle operating in a stochastic environment. If we assume that the vehicle’s policy and simulator are Markov then we show that the problem simplifies to estimating the probability of failure at each state, a computation which can be performed using approximate dynamic programming (DP). Approximate dynamic programming is particularly effective at finding failures because it can start at a failure and work backwards to see what led to it. The downside, however, is its inability to scale to large state spaces. For better scalability, we leverage the structure of driving scenarios by decomposing the simulation into pairwise interactions between the ego vehicle and other agents on the road. These subproblems are tractable for approximate dynamic programming and their solutions are recombined to approximate the solution for the full problem. To account for the approximation error due to multi-agent interactions, we apply a global correction learned from rollouts of the full system.

We apply our approach to two driving scenarios: a simple two-vehicle scenario to demonstrate the effectiveness of dynamic programming, and a more complex five-vehicle scenario to demonstrate the favorable scaling of the approach. In both experiments, we observed an order of magnitude increase in the number of failures discovered compared to baseline approaches.

The main contributions of this work are:

- A safety validation approach that estimates the distribution over failures using approximate dynamic programming.
- An algorithm for problem decomposition and reconstruction to scale approximate dynamic programming to complex driving scenarios.
- Demonstration of these techniques on two realistic driving scenarios and observation of a significant increase in rates of discovered failures.

The remainder of the paper is organized as follows: section II gives an overview of related work in the field of black-box validation for autonomous driving, section III describes our proposed technique in detail, section IV outlines the two experiments and describes our results, and section V concludes and discusses future work.

A. Corso and M. J. Kochenderfer are with the Aeronautics and Astronautics Department, Stanford University. E-mail: {acorso,mykel}@stanford.edu
R. Lee is with KBR Inc. at NASA Ames Research Center. E-mail: ritchie.lee@nasa.gov
II. RELATED WORK

Safety validation of autonomous systems has a long history prior to autonomous driving. We first give a brief overview of black-box falsification algorithms and then discuss approaches that were developed specifically for autonomous driving.

A. Safety Validation of Black-Box Systems

Falsification of black box systems involves finding inputs to the system that lead to violation of the system specifications. State-of-the-art approaches cast falsification as a global optimization problem over the input space \([1]\) and try to solve it using surrogate models \([3]\), deep reinforcement learning \([5]\), genetic algorithms \([6]\), Monte Carlo tree search \([7]\), or cross-entropy optimization \([8]\). Adaptive stress testing (AST) \([3, 9–11]\) frames the problem of falsification as a Markov decision process and uses reinforcement learning to find the most-likely failures of a system according to a prescribed probability model. The field of statistical model checking \([12]\) deals with estimating the probability of failure, and in doing so will find inputs to the system that cause it to fail.

B. Safety of Autonomous Vehicles

Some work has focused on falsifying components of an autonomous vehicle such as Adaptive Cruise Control \([13]\) or perception systems \([14, 15]\). Other work has focused on the generation of critical test cases. For example Mullins et al. \([16]\) identify regions of the input space that separate distinct types of autonomous behavior, and Althoff and Lutz \([17]\) design adversarial agents to minimize the safe available driving space of the autonomous vehicle.

Several approaches rely on sampling-based methods to discover AV failures. Huang et al. \([18]\) use bootstrapping and importance sampling to obtain a low-variance estimate of the probability of failure. Another approach uses importance sampling via the cross-entropy method to increase the number of failures found in simulation \([19, 20]\). Uesato et al. \([21]\) use previous versions of an autonomous agent to help find failures in the final version, an approach that works when agents have learned behavior.

III. PROPOSED APPROACH

This section describes our approach to the safety validation problem. The first subsection formulates the problem and defines important notation, the next subsection describes our technique for generating failures assuming we know the probability of failure from each state, and the last subsection describes how to compute that probability in a scalable way.

A. Problem Formulation

Suppose we wish to analyze the safety of a black-box autonomous system (system-under-test, or SUT) that operates in a stochastic simulated environment. The state of the SUT and the environment is \(s \in \mathcal{S}\) and the actions \(a \in \mathcal{A}\) are stochastic elements of the environment that influence the behavior of the SUT. A state-action trajectory \(\tau = \{s_0, a_1, s_1, \ldots, a_N, s_N\}\) has a likelihood of occurrence \(p(\tau)\). We define \(E\) as the set of all states that represent a failure of the SUT and the notation \(\tau \in E\) means that \(s_N \in E\) (i.e. the trajectory \(\tau\) ends in a failure of the SUT). Let \(T\) be the set of all terminal states where \(E \subseteq T\).

To fully investigate the space of possible failures of the SUT, we would like to know the distribution over failures

\[
f(\tau) = \frac{\mathbb{1} \{\tau \in E\} p(\tau)}{\mathbb{E}_p[\mathbb{1} \{\tau \in E\}]}\tag{1}\]

where the denominator normalizes the distribution appropriately. Note that \(f(\tau)\) is the minimum-variance importance sampling distribution for estimating the probability of failure.

B. Approach for Generating Failures

The space of all trajectories will be exponentially larger than the state space \(\mathcal{S}\) so it will be challenging to represent the distribution \(f(\tau)\) directly. To reduce the dimensionality of the distribution we assume that the SUT and environment are Markov. The current action \(a\) and next state \(s'\) will only depend on the current state \(s\) such that

\[
p(a, s' | s) = \frac{p(a | s)}{p(s' | s, a)}.	ag{2}\]

If we also assume that the dynamics of the SUT and the environment are deterministic (i.e. all stochasticity is controlled through actions), then

\[
p(a, s' | s) = p(a | s).	ag{3}\]

With these assumptions, the distribution over failures only depends on \(p(a | s)\) and is given by

\[
f(\tau) = \mathbb{1} \{s_N \in E\} \prod_{t=1}^{N} p(a_t | s_{t-1}).\tag{4}\]

The Markov assumption allows us to find a distribution over actions, or stochastic policy, \(\pi\) that generates samples (rollouts) distributed according to \(f\). Let

\[
\pi(a | s) = \frac{p(a | s)v(s')}{\sum_{a'} p(a' | s)v(s')} = \frac{p(a | s)v(s')}{v(s)}.	ag{5}\]

where \(v(s)\) is the probability of failure from state \(s\).

**Proposition 1:** Trajectories generated from rollouts of the policy \(\pi\) will be distributed according to \(f\).

**Proof:** Let \(f^*(\tau)\) be the distribution induced by rollouts of the policy \(\pi\). We will show that for any \(\tau\), \(f(\tau) = f^*(\tau)\). First, we define the Bellman equation that describes the probability of failure of a Markov system as

\[
v(s) = \begin{cases} 1 & \text{if } s \in E \\ 0 & \text{if } s \notin E, s \in T \\ \sum_a p(a | s)v(s') & \text{otherwise} \end{cases}\tag{6}\]

Then consider an arbitrary trajectory \(\tau\) that has probability

\[
f^*(\tau) = \prod_{t=1}^{N} \pi(a_t | s_{t-1}) = \prod_{t=1}^{N} \frac{p(a_t | s_{t-1})v(s_t)}{v(s_{t-1})}.	ag{7}\]
\[ f(\tau) = \mathbb{1}\{s_N \in E\} \prod_{t=1}^{N} p(a_t \mid s_{t-1}) \quad (8) \]

**Case 1:** \( \mathbb{1}\{\tau \notin E\} \)

By definition \( f(\tau) = 0 \). With final state \( s_N \in T \) and \( s_N \notin E \), we have \( v(s_N) = 0 \). The last term in the product in eq. (7) contains \( v(s_N) \), making \( f^{*}(\tau) = 0 \).

**Case 2:** \( \mathbb{1}\{\tau \in E\} \)

Considering the definition of \( f^{*}(\tau) \), we have

\[ f^{*}(\tau) = \prod_{t=1}^{N} p(a_t \mid s_{t-1}) v(s_t) \]

\[ = \frac{\prod v(s_t)}{v(s_0)} \prod_{t=1}^{N} p(a_t \mid s_{t-1}) \]

\[ = f(\tau) \quad (11) \]

where, in the second line, all of the \( v(s_t) \) terms were canceled except \( v(s_0) \) and \( v(s_N) \), and eq. (6) was used to let \( v(s_N) = 1 \). Thus, in all cases, \( f(\tau) = f^{*}(\tau) \).

Assuming that the distribution over actions \( p(a \mid s) \) is provided (either from domain knowledge or data) then the problem of computing the distribution over failures amounts to computing the probability of failure at each state \( v(s) \). Additionally, once \( v(s) \) is known, failures can be generated from any initial condition where a failure can be reached.

### C. Computing the Probability of Failure

The feasibility of computing the probability of failure \( v(s) \) largely depends on the size of the state and action spaces. If those spaces are discrete and relatively small, then dynamic programming can be used to compute \( v(s) \) to any desired level of accuracy. If the state space is continuous, but is small enough to be discretized, then local approximation dynamic programming can be used to estimate \( v(s) \) [22]. As will be demonstrated by our experiments, this approach is feasible for interactions between two vehicles on the road. For more vehicles, discretizing the state space becomes intractable and we must rely on further approximation.

When scaling to much larger state-spaces, we can leverage the structure of the problem to improve scalability. We propose to decompose a complicated driving scenario into pairwise interactions between the ego vehicle and other agents on the road, similar to the decomposition approach used by Bouton et al. [23]. Each subproblem can then be solved for the probability of failure between the ith vehicle and the ego vehicle yielding \( v_i(s^{(i)}) \), where \( s^{(i)} \) is the subset of the state representing only those vehicles. To combine the probability of failure from each of \( m \) subproblems, we use a fusion function (e.g. mean, max, or min), \( \ell \) such that

\[ v(s) \approx \bar{v}(s) = \mathbb{1}(v_1(s^{(1)}), \ldots, v_m(s^{(m)})) \]

(12)

There is no guarantee that the subproblem fusion will provide a good estimate of the true probability of failure for each state due to the complex interactions that emerge in the full system. We can correct for errors in the estimate using a parameterized global correction such that

\[ v(s) \approx \bar{v}(s) + \delta \nu_{\theta}(s) \]

(13)

The global correction parameters \( \theta \) are learned from rollouts of the full simulator and the representation of \( \delta \nu_{\theta} \) could be any function. This work uses a linear model due to its simplicity, stability, and effectiveness.

The global correction can be learned with a variety of approximate dynamic programming and Monte Carlo (MC) techniques. The approach that provided the best stability and performance was Monte Carlo policy evaluation with function approximation [24], described in Algorithm 1. The rollout policy \( \pi \) returns the trajectory \( \tau \) and the accumulated returns for each state defined as

\[ G(s_i) = \mathbb{1}\{s_N \in E\} \prod_{t=1}^{N} \frac{p(a_t \mid s_{t-1})}{\pi(a_t \mid s_{t-1})} \quad (14) \]

The function \( \bar{v} \) provides an estimate for the probability of failure of state \( s \). The algorithm runs for \( N_{\text{iter}} \) iterations and samples \( N_{\text{eps}} \) episodes per iteration. The parameters \( \theta \) are updated after each iteration so that more informative samples can be obtained on the next iteration.

Approximate dynamic programming and scene decomposition have some limitations. First, the discretization and the linear interpolation introduce errors into the estimate of \( v(s) \). During the experimentation we discovered that if the magnitude of these errors was on the same order or larger than the probability of failure, then there was no way to get an accurate value for \( v(s) \). To mitigate this problem, we used ideas from importance sampling and used a new action distribution \( q(a \mid s) \) that increased the probability of failure. Another challenge for this approach is the exponential scaling of the action space of the full system. If there are \( m \) subproblems each with an action space of size \( A \) then the full problem must consider \( A^m \) actions per step. To mitigate this problem, we only let one agent act at each timestep, reducing the action space to \( A \cdot m \).

### Algorithm 1 MC evaluation with function approximation

**procedure** MCPolicyEval(\( \tau, \bar{v}, N_{\text{iter}}, N_{\text{eps}} \))

\[ \theta \leftarrow 0 \]

for \( i \in 1 : N_{\text{iter}} \)

\[ \tau, G \leftarrow \text{Rollout}(\pi, N_{\text{eps}}) \]

\[ y \leftarrow G - \bar{v}(s \in \tau) \]

\[ \theta \leftarrow \text{fit}(\theta, \tau, y) \]

return \( \theta \)

### IV. EXPERIMENTS

This section describes two experimental driving scenarios, a simple scenario with two vehicles, and a more complex scenario with five vehicles. The simulations were designed with AutomotiveDrivingModels.jl, an open-source julia package. Both simulations rely on the same road geometry and autonomous driving policy. The system under test is an autonomous vehicle referred to as the ego vehicle.

The road geometry and initial vehicle configurations are pictured in fig. 7. The roadway models an unprotected left turn onto a two-lane road. The ego vehicle (in blue) attempts to make a left turn from the vertical road segment onto the horizontal road segment. Other vehicles (referred to
adversarial vehicles) are initialized on the horizontal road and can either continue straight or turn onto the vertical roadway (the yellow dot represents a turn signal). The right-of-way rules are:

- Vehicles on the through street have right-of-way over vehicles turning on to the through street.
- Vehicles turning right have right-of-way over vehicles turning left.

The state of each vehicle can be described with four variables

- $s$: Position along the lane (Continuous)
- $v$: Velocity along the lane (Continuous)
- $B$: Whether the blinker is on (Boolean)
- $L$: Integer indicating lane (Boolean)

For approximate dynamic programming, the position and velocity were each discretized into 20 values so each vehicle had a total of 1600 states.

Each vehicle on the road including the ego vehicle, follows a modified version of the intelligent driver model (IDM) \cite{Treiber2000}. The IDM is a vehicle-following algorithm that tries to drive at a specified velocity while avoiding collisions with leading vehicles. In our experiments, the IDM is parameterized by a desired velocity of 29 m/s, a minimum spacing of 5 m, a maximum acceleration of 3 m/s$^2$ and a comfortable braking deceleration of $-2$ m/s$^2$, and a simulation timestep of $\Delta t = 0.18$s. The IDM was modified with a rule-based algorithm (algorithm \ref{alg:intersection}) for navigating the T-intersection. Each vehicle reasons about right-of-way and turning intention of other vehicles based on the state of their blinker, and uses current vehicle speeds to calculate if the intersection is safe to cross.

The actions of the environment correspond to disturbances to the deterministic actions of all adversarial vehicles. The disturbances and their corresponding probabilities are shown in table \ref{tab:actions}. The first action produces no disturbance, so the vehicle accelerates by $a_{\text{IDM}}$, the acceleration computed by the modified IDM. The next four actions perturb the vehicle's acceleration by an amount $\delta a \in [-3 \text{m/s}^2, 3 \text{m/s}^2]$ so that the actual acceleration of the vehicle is $a_{\text{IDM}} + \delta a$. The sixth action toggles the vehicle's turn signal which is observed by other vehicles and used to determine the vehicle's intention. The final action changes the hidden vehicle intention as to whether or not it will turn.

The choice of a disturbance probability model should be driven by real-world driving data. In absence of that data, we chose a simple probability model that made disturbances rare according to their magnitude (see MC Probability in table \ref{tab:actions}). Medium slowdowns and speedups were given a probability of $1 \times 10^{-2}$ per timestep while major slowdowns and speedups, toggling the blinker, and toggling turn intention had a per-timestep probability of occurrence of $1 \times 10^{-3}$. With this model, the probability of failure was smaller than the approximation error due to the discretization. We therefore applied a uniform importance sampling distribution for the approximate dynamic programming approach (see IS Probability in table \ref{tab:actions}).

The metric we chose to evaluate our approach is the number of failures found after 100 simulations. The experiment was repeated 5 times for each approach and the sample mean and standard deviations are reported. We compare with two baselines:

1) Monte Carlo rollouts: Random simulations with the true probability distribution over actions.
2) Uniform importance sampling sampling rollouts: Random simulations done with a uniform distribution over actions.

\begin{algorithm}
\caption{Intersection navigation algorithm}
\begin{algorithmic}
\Function{ComputeAcceleration}{veh, scene}
\State $v \leftarrow \text{velocity}(veh)$
\State $v_{\text{lead}}$, $\Delta s_{\text{lead}} \leftarrow \text{leading}_\text{vehicle}(veh, scene)$
\State $\text{acc} \leftarrow a_{\text{IDM}}(\Delta s_{\text{lead}}, v, v_{\text{lead}})$
\If {$veh$ does not have right of way}
\State $\text{ttc} \leftarrow \text{time_to_cross_intersection}(veh)$
\State $\text{acc} \leftarrow -\text{distance_to_intersection}(veh) / \text{ttc}$
\EndIf
\For {$agent$ in scene}
\State $\text{tenter} \leftarrow \text{time_to_enter_intersection}(agent)$
\State $\text{ttexit} \leftarrow \text{time_to_exit_intersection}(agent)$
\If {$\text{tenter} < \text{ttc}$ and $\text{ttexit} + \delta t > \text{ttc}$}
\State $\text{acc} \leftarrow a_{\text{IDM}}(\Delta s_{\text{int}}, v, 0)$
\State \text{break}
\EndIf
\EndFor
\State \text{return acc}
\EndFunction
\end{algorithmic}
\end{algorithm}

\subsection{A. Two-Vehicle Interaction}

The first scenario is an interaction between the ego vehicle and one adversarial vehicle (see fig. \ref{fig:2v} left). Figure \ref{fig:2v} shows the normal behavior of the scenario: the ego vehicle correctly arrives so it proceeds with the left turn. Table \ref{tab:2v} shows the number of failures observed with each sampling method. The Monte Carlo rollouts show no failures observed, while the importance sampling rollouts average 0.2 failures per 100 trials. Our approximate dynamic programming approach generates an order of magnitude more failures with an average of 10.2. A sample failure is shown in fig. \ref{fig:2v-fail}. We can see that the adversary had to accelerate early in the simulation to cause a collision with the ego vehicle, which did not predict that an acceleration would occur.

\begin{table}[h]
\centering
\begin{tabular}{|l|l|l|l|}
\hline
Action & Acceleration & MC Probability & IS Probability \\
\hline
No Disturbance & $0$ m/s$^2$ & 0.976 & 0.143 \\
Medium slowdown & $-1.5$ m/s$^2$ & $1 \times 10^{-2}$ & 0.143 \\
Major slowdown & $-3$ m/s$^2$ & $1 \times 10^{-3}$ & 0.143 \\
Medium speedup & $1.5$ m/s$^2$ & $1 \times 10^{-2}$ & 0.143 \\
Major speedup & $3$ m/s$^2$ & $1 \times 10^{-3}$ & 0.143 \\
Toggle blinker & N/A & $1 \times 10^{-3}$ & 0.143 \\
Toggle turn intent & N/A & $1 \times 10^{-3}$ & 0.143 \\
\hline
\end{tabular}
\caption{Action space for adversarial vehicles}
\end{table}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig1.png}
\caption{Initial conditions of the two driving scenarios.}
\end{figure}
is an order of magnitude increase in the number of failures found (11.6), while fusion with min and max generate similar results to the importance sampling baseline. The next entry shows that the global linear model performs well (36.2 failures) without any subproblem estimation. The final three rows show the performance of the subproblem estimation with the global linear correction and the three fusion functions, of which the mean fusion performs best with 41.4 failures.

Based on these results we can conclude that using the mean of the subproblems is the most effective fusion technique with and without the correction. We also note that a global linear model performed well on its own, suggesting that simple representations may still be useful for AV validation. The highest performing approach was the subproblem estimate with mean fusion and a global linear correction.

V. CONCLUSIONS

In this work we have made progress toward the goal of automated testing of autonomous vehicles. We introduced a safety validation formulation that uses a approximate dynamic programming to estimate the distribution over failures and create sequences of disturbances that cause an autonomous system to fail. The problem of scalability was addressed by decomposing the driving scenario into pairwise interactions between the ego vehicles and other agents on the road. These subproblems were solved and fused back together to estimate the probability of failure of the full system. To correct for errors in this estimate, we used a linear global correction that was trained using Monte Carlo policy evaluation. We observed an order of magnitude increase in the number of failures found when compared to an importance sampling approach in a two-vehicle driving scenario and a more complex five-vehicle driving scenario, demonstrating the benefit of this approach.

There are several directions this work could go in the future: 1) use the calculated policy to obtain a low-variance estimate of the probability of failure from desired initial conditions, 2) observe performance on more complicated driving scenarios with many agents, and 3) attempt to interpret the learned parameters to understand causes of failures.

### TABLE III: 5-Car Scenario Failure Rates

| Method                        | No. Failures (per 100) |
|-------------------------------|------------------------|
| Monte Carlo                   | 0.0 ± 0.0              |
| Importance Sampling           | 1.6 ± 0.5              |
| Mean Utility Fusion           | 11.4 ± 4.4             |
| Max Utility Fusion            | 1.8 ± 2.2              |
| Min Utility Fusion            | 1.6 ± 1.3              |
| Linear Global Correction (LGC)| 36.2 ± 3.3             |
| Mean Utility Fusion + LGC     | 41.4 ± 4.4             |
| Max Utility Fusion + LGC      | 24.4 ± 4.7             |
| Min Utility Fusion + LGC      | 32.8 ± 4.0             |

### TABLE II: 2-Car Scenario Failure Rates

| Method                        | No. Failures (out of 100) |
|-------------------------------|--------------------------|
| Monte Carlo                   | 0.0 ± 0.0                |
| Importance Sampling           | 0.2 ± 0.4                |
| Local Approximation DP        | 10.2 ± 1.3               |

Fig. 2: Normal 2-car scenario at \( t = (1.26\, s, 2.88\, s) \)

Fig. 3: Collision in 2-car scenario at \( t = (1.26\, s, 1.80\, s) \)

B. 5 Vehicle Interaction

The second scenario involves the interaction of the ego vehicle with four adversarial drivers (see fig. 1, right). The normal behavior of the scenario is shown in fig. 2 where the cars on the left and the trailing car on the right go straight, while the leading car on the right turns onto the vertical road segment. The ego vehicle gives way to all four vehicles and completes the left turn after they have passed.

The driving scenario was broken into four subproblems, one for each adversarial vehicle. The probability of failure was computed for each driving scenario using approximate dynamic programming, as in the previous experiment. To find failures of the five-car system, the probability of failure was estimated from the subproblem values using three different fusion functions: mean, max and min. Then, to improve the estimate, rollouts of the five-car scenario were used to train a linear global correction \( \delta v_{\theta} = s_{\theta}^T \) where \( s_{\theta} \) is a featurized state space defined for vehicle \( i \) as

\[
s_{\text{feat},i} = [l_i, s_i \odot l, v_i \odot l, v_i^2 \odot l, B_i \odot l]^T \tag{15}
\]

where \( l_i \) is a one-hot encoding of the vehicle lane and \( \odot \) represents element-wise multiplication. The linear model was trained over 25 iterations, each with 100 rollouts.

The results are shown in table III. The first two entries show that the baselines, Monte Carlo rollouts and importance sampling rollouts, respectively produce 0 and 1.6 failures per 100. The next three rows show the failure rates using the subproblem probability of failure estimate without any correction for the three different fusion functions. When averaging is used to fuse the subproblem probabilities, there
Fig. 4: Normal 5-car scenario at $t = (0.9 \text{ s}, 4.32 \text{ s}, 7.02 \text{ s})$

Fig. 5: Collision in 5-car scenario at $t = (0.9 \text{ s}, 2.34 \text{ s}, 3.42 \text{ s})$

REFERENCES

[1] A. Donz and O. Maler, “Robust satisfaction of temporal logic over real-valued signals,” in International Conference on Formal Modeling and Analysis of Timed Systems (FORMATS), 2010.
[2] S. Shalev-Shwartz, S. Shammah, and A. Shashua, “On a formal model of safe and scalable self-driving cars,” ARXIV, no. 1708.03742, 2017.
[3] R. Lee, M. J. Kochenderfer, O. J. Mengshoel, G. P. Brat, and M. P. Owen, “Adaptive stress testing of airborne collision avoidance systems,” in Digital Avionics Systems Conference (DASC), 2015.
[4] L. Mathesen, S. Yaghoubi, G. Pedrielli, and G. Fainekos, “Falsification of cyber-physical systems with robustness uncertainty quantification through stochastic optimization with adaptive restart,” English (US), in IEEE Conference on Automation Science and Engineering (CASE), Aug. 2019.
[5] T. Akazaki, S. Liu, Y. Yamagata, Y. Duan, and J. Hao, “Falsification of cyber-physical systems using deep reinforcement learning,” in International Symposium on Formal Methods, 2018.
[6] Q. Zhao, B. H. Krogh, and P. Hubbard, “Generating test inputs for embedded control systems,” IEEE Control Systems Magazine, vol. 23, no. 4, pp. 49–57, 2003.
[7] Z. Zhang, G. Ernst, S. Sedwards, P. Arcaini, and I. Hasuo, “Two-layered falsification of hybrid systems guided by Monte Carlo tree search,” IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems (TCAD), vol. 37, no. 11, pp. 2894–2905, 2018.
[8] S. Sankaranarayanan and G. Fainekos, “Falsification of temporal properties of hybrid systems using the cross-entropy method,” in ACM international conference on Hybrid Systems: Computation and Control (HSCC), 2012.
[9] M. Koren, S. Alsaif, R. Lee, and M. J. Kochenderfer, “Adaptive stress testing for autonomous vehicles,” in IEEE Intelligent Vehicles Symposium (IV), 2018.
[10] A. Corso, P. Du, K. Driggs-Campbell, and M. J. Kochenderfer, “Adaptive stress testing with reward augmentation for autonomous vehicle validation,” in IEEE International Conference on Intelligent Transportation Systems (ITSC), 2019.
[11] M. Koren and M. Kochenderfer, “Efficient autonomy validation in simulation with adaptive stress testing,” in IEEE International Conference on Intelligent Transportation Systems (ITSC), Oct. 2019.
[12] G. Agha and K. Palmskog, “A survey of statistical model checking,” ACM Transactions on Modeling and Computer Simulation (TOMACS), vol. 28, no. 1, pp. 1–39, 2018.
[13] M. Koschi, C. Pek, S. Maierhofer, and M. Althoff, “Computationally efficient safety falsification of adaptive cruise control systems,” in IEEE International Conference on Intelligent Transportation Systems (ITSC), 2019.
[14] Y. Cao, C. Xiao, B. Cyr, Y. Zhou, W. Park, S. Rampazzi, Q. A. Chen, K. Fu, and Z. M. Mao, “Adversarial sensor attack on lidar-based perception in autonomous driving,” in ACM SIGSAC Conference on Computer and Communications Security, 2019.
[15] A. Balakrishnan, A. Puranic, X. Qin, A. Dokhanchi, J. Deshmukh, H. Ben Amor, and G. Fainekos, “Specifying and evaluating quality metrics for vision-based perception systems,” English (US), in Design, Automation and Test in Europe (DATE), May 2019.
[16] G. E. Mullins, P. G. Stankiewicz, R. C. Hawthorne, and S. K. Gupta, “Adaptive generation of challenging scenarios for testing and evaluation of autonomous vehicles,” Journal of Systems and Software, vol. 137, pp. 197–215, 2018.
[17] M. Althoff and S. Lutz, “Automatic generation of safety-critical test scenarios for collision avoidance of road vehicles,” in IEEE Intelligent Vehicles Symposium (IV), 2018.
[18] Z. Huang, M. Arief, H. Lam, and D. Zhao, “Evaluation uncertainty in data-driven self-driving testing,” in IEEE International Conference on Intelligent Transportation Systems (ITSC), 2019.
[19] Y. Kim and M. J. Kochenderfer, “Improving aircraft collision risk estimation using the cross-entropy method,” Journal of Air Transportation, vol. 24, no. 2, pp. 55–62, 2016.
[20] M. O’Kelly, A. Sinha, H. Namkoong, R. Tedrake, and J. C. Duchi, “Scalable end-to-end autonomous vehicle testing via rare-event simulation,” in Advances in Neural Information Processing Systems (NIPS), 2018.
[21] J. Uesato, A. Kumar, C. Szepesvari, T. Erez, A. Ruderman, K. Anderson, N. Heess, P. Kohli, et al., “Rigorous agent evaluation: An adversarial approach to uncover catastrophic failures,” ARXIV, no. 1812.01647, 2018.
[22] M. J. Kochenderfer, Decision making under uncertainty: Theory and application, MIT Press, 2015.
[23] M. Bouton, K. D. Julian, A. Nakhaei, K. Fujimura, and M. J. Kochenderfer, “Decomposition methods with deep corrections for reinforcement learning,” International Conference on Autonomous Agents and Multiagent Systems (AAMAS), vol. 33, no. 3, pp. 330–352, 2019.
[24] R. S. Sutton and A. G. Barto, Reinforcement learning: An introduction, MIT Press, 2018.
[25] M. Treiber, A. Hennecke, and D. Helbing, “Congested traffic states in empirical observations and microscopic simulations,” Physical Review E, vol. 62, pp. 1805–1824, 2000.