Fuzzy propositional configuration logics

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Abstract
We introduce and investigate a weighted propositional configuration logic over De Morgan algebras. This logic is able to describe software architectures with quantitative features especially the uncertainty of the interactions that occur in the architecture. We deal with the equivalence problem of formulas in our logic by showing that every formula can be written in a specific form. Surprisingly, there are formulas which are equivalent only over specific De Morgan algebras. We provide examples of formulas in our logic which describe well-known software architectures equipped with quantitative features such as the uncertainty and reliability of their interactions.

Keywords: Software architectures, Formal methods, Propositional configuration logics, Fuzzy logic, Quantitative features, Uncertainty.

1 Introduction

Uncertainty is inevitable in software architecture [7]. Software architectures are increasingly composed of many components such as workload and servers. Computations between the components run in environments in which resources may have radical variability [3]. For instance, software architects may be uncertain about the cost and performance impact of a proposed software architecture. They may be aware of the cost and performance of the interactions in the architecture. However, there may be undesirable outcomes such as failure of a component to interact and complete its

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task [7]. Uncertainty may affect functional and non-functional architecture requirements [17]. Hence, it is necessary to consider uncertainty as a basic quantitative characteristic in software architectures. So far, the existing architecture decision-making approaches do not provide a quantitative method of dealing with uncertainty [2]. The motivation of our work is to formally describe and compare software architectures with quantitative features such as the uncertainty. For this, we consider that fuzzy logics rely on the idea that truth comes in degrees. Hence, they constitute a suitable tool in order to deal with uncertainty. Moreover, recently the authors in [16] used fuzzy logic on IoT devices for assisting the blind people for their safe movement. This is a strong indication for the possible future applications of fuzzy logic.

In this paper we extend the work of [6,12,13] by introducing and investigating the fuzzy PCL (fPCL for short) over De Morgan algebras. This work is motivated as follows. In [12,13] we introduced the weighted PCL over commutative semirings (wPCL for short). This logic serves as a specification language for the study of software architectures with quantitative features such as the maximum cost of an architecture or the maximum priority of the involvement of a component. Then in [6], we introduced the weighted PCL over product valuation monoids (wPVM for short) which serves as a specification language for software architectures with quantitative features such as the average of all interactions’ costs of the architecture, and the maximum cost among all costs occurring most frequently within a specific number of components. Those features are not covered in [12,13]. The aforementioned works are not able to model the uncertainty that occurs between the interactions in the architecture. In this paper we deal with this problem by introducing and investigating the fuzzy PCL (fPCL for short) which is a weighted PCL over De Morgan algebras.

The contributions of our work are the following.

- **We introduce the syntax and semantics of fPCL.** The semantics of fPCL formulas are series with values in the De Morgan algebra. This logic is able to describe software architecture with quantitative features such as the uncertainty. Moreover, we are able to compute the weight of an architecture even when unwanted components participate. This is possible since De Morgan algebras are equipped with a complement mapping whereas the algebraic structures in [6,12,13] are not.

- **In the sequel, we construct fPCL formulas which describe the Peer-to-Peer architecture and the Master/Slave architecture for finitely number of components.**
Lastly, we deal with the decidability of equivalence of fPCL formulas. For this, we examine the existence of a normal form. We show that the construction of the normal form of a fPCL formula depends on the properties of the De Morgan algebra. Hence, there may be fPCL formulas which have the same normal form over the fuzzy algebra but different ones over the Boolean algebra. In other words, two fPCL formulas can be equivalent over the fuzzy algebra but not over the Boolean algebra. We give examples to show our point. In our paper, we prove that for every fPCL formula over a set of ports and a Kleene algebra we can effectively construct an equivalent one in normal form. We note that this construction can be easily adapted for fPCL formulas over a Boolean algebra. We conclude that two fPCL formulas are equivalent over a De Morgan algebra if they have the same normal form considering the properties of the aforementioned De Morgan algebra. For this, we give an algorithm which is able to decide the equivalence of two fPCL formulas in normal form, in polynomial time.

2 Related Work

Existing work has investigated the formal description of the qualitative and quantitative properties of software architecture. In particular, the authors in [9] introduced the propositional configuration logic (PCL for short) which was proved sufficient to describe the qualitative properties of software architectures. Later in [12, 13], we introduced and investigated a weighted PCL (wPCL for short) over a commutative semiring which serves as a specification language for the study of software architectures with quantitative features such as the maximum cost of an architecture or the maximum priority of a component. We proved that the equivalence problem of wPCL formulas is decidable. In [6] we extended the work of [12,13] by introducing and investigating weighted PCL over product valuation monoids (w_pvmPCL for short). This logic is proved to be sufficient to serve as a specification language for software architectures with quantitative properties, such as the average of all interactions’ costs of the architecture and the maximum cost among all costs occurring most frequently within a specific number of components in an architecture. However, the aforementioned works do not cover quantitative properties such as the uncertainty and reliability of an architecture.

The authors in [10] address the problem of evaluating the system reliability as a stochastic property of software architectural models in the presence
of uncertainty. Also, the authors in [8] develop a conceptual framework for the management of uncertainty in software architecture in order to reduce its impact during the system’s life cycle. However, the aforementioned works lack of formality of the architecture description, which is crucial since non-formal systems can be unreliable at some point.

3 Preliminaries

3.1 Lattices

Let $K$ be a nonempty set, and $\leq$ a binary relation over $K$ which is reflexive, antisymmetric, and transitive. Then $\leq$ is called a partial order and the pair $(K, \leq)$ a partially ordered set (poset for short). If the partial order $\leq$ is understood, then we shall denote the poset $(K, \leq)$ simply by $K$. For $k, k' \in K$ we denote by $k \lor k'$ (resp. $k \land k'$) the least upper bound or supremum (resp. the greatest lower bound or infimum) of $k$ and $k'$ if it exists in $K$.

A poset $K$ is called a lattice if $k \lor k'$ and $k \land k'$ exist in $K$ for every $k, k' \in K$. A lattice $K$ is called distributive if $k \land (k' \lor k'') = (k \land k') \lor (k \land k'')$ and $(k \lor k') \land k'' = (k \land k') \lor (k' \land k'')$ for every $k, k', k'' \in K$. Moreover, the absorption laws $k \lor (k \land k') = k$ and $k \land (k \lor k') = k$ hold for every $k, k' \in K$.

A poset $K$ is called bounded if there are two elements $0, 1 \in K$ such that $0 \leq k \leq 1$ for every $k \in K$.

A De Morgan algebra is denoted by $(K, \leq, \neg)$, where $K$ is a bounded distributed lattice (bdl for short) with complement mapping $\neg : K \to K$ which satisfies involution and the De Morgan laws $\neg \neg k = k$, $k \land \neg k' = \neg (k \lor \neg k')$, and $\neg (k \land k') = \neg k \lor \neg k'$ for every $k, k' \in K$. A known De Morgan algebra is the structure $([0, 1], \leq, \neg)$ where $\leq$ is the usual order on real numbers and the complement mapping is defined by $\neg k = 1 - k$ for every $k \in [0, 1]$.

The authors in [1, 14] show that a semiring $(K, +, \cdot, 0, 1)$ equipped with a complement mapping $\neg$, which is a monoid morphism from $(K, +, 0)$ to $(K, \cdot, 1)$ and $\neg k = k$ for every $k \in K$, is a De Morgan algebra $(K, \leq, \neg)$. The relation $\leq$ is defined as follows: $k \leq k'$ iff $k + k' = k'$. On the other hand, a De Morgan algebra $(K, \leq, \neg)$ induces a semiring $(K, \lor, \land, 0, 1)$ with a complement mapping $\neg$. In the following, we denote a De Morgan algebra by $(K, \lor, \land, 0, 1, \neg)$. Moreover, a Kleene algebra is a De Morgan algebra that satisfies $k_1 \land k_1 \leq k_2 \lor \neg k_2$, or equivalently, $(k_1 \land \neg k_1) \land (k_2 \lor \neg k_2) = (k_1 \land \neg k_1)$ for every $k_1, k_2 \in K$. A Boolean algebra is a Kleene algebra that satisfies $k \land \neg k = 0$ and $k \lor \neg k = 1$ for every $k \in K$. In the following we present the most
well-known De Morgan algebras. We refer the reader to [4, 11] for further examples of De Morgan algebras.

• The two element Boolean algebra \( 2 = (\{0, 1\}, \lor, \land, 0, 1, -) \), where \( \overline{0} = 1 \) and \( \overline{1} = 0 \).

• The three element Kleene algebra \( 3 = (\{0, u, 1\}, \lor, \land, 0, 1, -) \), where \( \overline{0} = 1 \), \( \overline{1} = 0 \), \( \overline{u} = u \). The operators \( \lor, \land \) are shown in Figure 1a.

• The four element algebra \( 4 = (\{0, u, w, 1\}, \lor, \land, 0, 1, -) \), where \( \overline{u} = u \), \( \overline{w} = w \), \( u \lor w = 1 \) and \( u \land w = 0 \). The operators \( \lor \) and \( \land \) are shown in Figure 1b.

• The fuzzy algebra \( F = ([0, 1], \text{max}, \text{min}, 0, 1, -) \), where for every \( k \in [0, 1] \) the complement mapping is defined by \( \overline{k} = 1 - k \). This algebra is a Kleene algebra. To see this, let \( k, k' \in [0, 1] \) and note that \( \min\{\min\{k, k'\}, \max\{k', \overline{k}\}\} = \min\{k, \overline{k}\} \).

Throughout the paper, \( 3 \) and \( K_3 \) will denote respectively, the three element Kleene algebra and a De Morgan algebra which is a Kleene algebra. Also, by \( 2 \) and \( B \) we will denote respectively, the two element Boolean algebra and a De Morgan algebra which is a Boolean algebra. By \( K \) we will denote an arbitrary De Morgan algebra.

Lastly, consider \( K \) be a De Morgan algebra and \( Q \) a set. A formal series (or simply series) over \( Q \) and \( K \) is a mapping \( s : Q \to K \). We denote by \( K \langle\langle Q\rangle\rangle \) the class of all series over \( Q \) and \( K \).

4 Fuzzy Propositional Interaction Logic

In this section we introduce a quantitative version of PIL where the weights are taken in the De Morgan algebra \( K \). Since De Morgan algebras and more generally bdls found applications in fuzzy theory, we call our weighted PIL a fuzzy PIL.
Definition 1 The syntax of formulas of fuzzy PIL (fPIL for short) over $P$ and $K$ is given by the grammar:

$$\varphi ::= \text{true} \mid p \mid !\varphi \mid \varphi \otimes \varphi$$

where $p \in P$ and the operators $!$, $\otimes$ denote the fuzzy negation and the fuzzy disjunction, respectively, among fPIL formulas.

The fuzzy conjunction operator among fPIL formulas $\otimes$ is defined by $\varphi_1 \otimes \varphi_2 := !(!\varphi_1 \otimes !\varphi_2)$.

For the semantics of fPIL formulas over $P$ and $K$ we introduce the notion of a $K$-fuzzy interaction. For this we need to recall the $K$-fuzzy sets from [5]. A $K$-fuzzy set $S$ on a non empty set $X$ is a function $S : X \rightarrow K$. A $K$-fuzzy interaction $\alpha$ on $P$ is a $K$-fuzzy set on $P$ with the restriction that $\alpha(p) \neq 0$ for at least one port $p \in P$. We denote by $fI(P,K)$ the set of $K$-fuzzy interactions $\alpha$ on $P$ and by $fPIL(K,P)$ the set of all fPIL formulas over $P$ and $K$. We interpret fPIL formulas over $P$ and $K$ as series in $K\langle\langle fI(P,K)\rangle\rangle$.

Definition 2 Let $\varphi \in fPIL(K,P)$. The semantics of $\varphi$ is a series $\|\varphi\| \in K\langle\langle fI(P,K)\rangle\rangle$. For every $K$-fuzzy interaction $\alpha \in fI(P,K)$ the value $\|\varphi\| (\alpha)$ is defined inductively on the structure of $\varphi$ as follows:

- $\|\text{true}\| (\alpha) = 1$,
- $\|p\| (\alpha) = a(p)$,
- $\|!\varphi\| (\alpha) = \overline{\|\varphi\| (\alpha)}$,
- $\|\varphi_1 \otimes \varphi_2\| (\alpha) = \|\varphi_1\| (\alpha) \lor \|\varphi_2\| (\alpha)$.

Trivially, we get $\|\varphi_1 \otimes \varphi_2\| (\alpha) = \|\varphi_1\| (\alpha) \land \|\varphi_2\| (\alpha)$ for every $\alpha \in fI(P,K)$. Moreover, we define the fPIL formula $!\text{true} := \text{false}$ and it is valid that $\|\text{false}\| (\alpha) = 0$ for every $\alpha \in fI(P,K)$.

Next, we define the equivalence relation among fPIL formulas. For this, we consider that De Morgan algebras such as the Kleene and the Boolean algebra satisfy some extra properties except from the ones that are valid to every De Morgan algebra by its definition.

Definition 3 Two fPIL formulas $\varphi_1, \varphi_2$ over $P$ and a concrete De Morgan algebra $K_{con}$ are called $K_{con}$-equivalent, and we write $\varphi_1 \equiv_{K_{con}} \varphi_2$, whenever $\|\varphi_1\| (\alpha) = \|\varphi_2\| (\alpha)$ for every $\alpha \in fI(P,K_{con})$.

Two fPIL formulas $\varphi_1, \varphi_2$ over $P$ and an arbitrary De Morgan algebra $K$ are called simply equivalent, and we write $\varphi_1 \equiv \varphi_2$, whenever $\|\varphi_1\| (\alpha) = \|\varphi_2\| (\alpha)$ for every $\alpha \in fI(P,K)$. 


Let $P = \{p, q, r\}$ be a set of ports. Following the previous definition and by the properties of De Morgan algebras, we prove that $p \otimes !p \otimes (q \otimes r) \equiv_2 \text{false}$ and $p \otimes !p \otimes (q \otimes r) \equiv (p \otimes !p \otimes q) \otimes (p \otimes !p \otimes r)$. We proceed with some properties of our fPIL formulas.

**Proposition 4** Let $\varphi_1, \varphi_2$ be fPIL formulas over $P$ and $K$. Then

$$!(\varphi_1 \otimes \varphi_2) \equiv (!\varphi_1) \otimes (!\varphi_2).$$

**Proof.** Let $\alpha \in fI(P,K)$. Then

$$\Vert !(\varphi_1 \otimes \varphi_2) \Vert (\alpha) = \Vert \varphi_1 \otimes \varphi_2 \Vert (\alpha) \equiv \Vert \varphi_1 \Vert (\alpha) \lor \Vert \varphi_2 \Vert (\alpha) \equiv \Vert \varphi_1 \Vert (\alpha) \land \Vert \varphi_2 \Vert (\alpha) \equiv \Vert (!\varphi_1) \otimes (!\varphi_2) \Vert (\alpha).$$

$$\blacksquare$$

**Proposition 5** Let $\varphi$ be a fPIL formula over $P$ and $K$. Then the following hold:

1. $\varphi \otimes \text{true} \equiv \text{true}$,
2. $\varphi \otimes \text{false} \equiv \varphi$,
3. $\varphi \otimes \text{true} \equiv \varphi$,
4. $\varphi \otimes \text{false} \equiv \varphi$.

**Proof.** The proofs are straightforward. $\blacksquare$

**Proposition 6** The operators $\otimes$ and $\otimes$ of the fPIL are associative.

**Proof.** The proposition holds since the operators $\land$ and $\lor$ are associative. $\blacksquare$

**Proposition 7** Let $\varphi, \varphi_1, \varphi_2 \in fPIL(K,P)$. Then

$$\varphi \otimes (\varphi_1 \otimes \varphi_2) \equiv (\varphi \otimes \varphi_1) \otimes (\varphi \otimes \varphi_2).$$

**Proof.** Since $\land$ distributes over $\lor$ we get the proposition. $\blacksquare$

Next, we give the absorption and idempotent laws among fPIL formulas.

**Proposition 8** Let $\varphi, \varphi' \in fPIL(K,P)$. Then

1. $\varphi \otimes (\varphi \otimes \varphi') \equiv \varphi$,
2. $\varphi \otimes (\varphi \otimes \varphi') \equiv \varphi$,
3. $\varphi \otimes \varphi \equiv \varphi$,
4. $\varphi \otimes \varphi \equiv \varphi$.

**Proof.** For the proof of (1) and (2) we apply the absorption laws of De Morgan algebras. The other are valid since $\land$ and $\lor$ are idempotent. $\blacksquare$
5 Fuzzy Propositional Configuration Logic

In this section we introduce and investigate the fuzzy PCL over $P$ and $K$.

**Definition 9** The syntax of formulas of fuzzy PCL ($fPCL$ for short) over $P$ and $K$ is given by the grammar:

$$
ζ ::= ϕ | ¬ζ | ζ ⊕ ζ | ζ ⊎ ζ
$$

where $ϕ$ is a $fPIL$ formula over $P$ and $K$, $¬$, $⊕$ and $⊎$ denote the fuzzy negation, the fuzzy disjunction and the fuzzy coalescing operator, respectively.

Let $ζ, ζ'$ be $fPCL$ formulas over $P$ and $K$. The fuzzy conjunction operator among $ζ$ and $ζ'$ and the closure operator of $ζ$ are defined, respectively, as follows:

1. $ζ ⊗ ζ' := ¬(¬ζ ⊕ ¬ζ')$,
2. $∼ζ := ζ ⊎ \text{true}$.

Next, we denote by $fC(P,K)$ the set of nonempty sets of $K$-fuzzy interactions in $fI(P,K)$, and by $fPCL(K,P)$ the set of $fPCL$ formulas over $P$ and $K$. We define the semantics of $fPCL$ formulas over $P$ and $K$ as series in $K\langle\langle fC(P,K)\rangle\rangle$.

**Definition 10** Let $ζ$ be a $fPCL$ formula over $P$ and $K$. The semantics of $ζ$ is a series $∥ζ∥ ∈ K\langle\langle fC(P,K)\rangle\rangle$. For every set $γ ∈ fC(P,K)$ the value $∥ζ∥(γ)$ is defined inductively on the structure of $ζ$ as follows:

1. $∥ϕ∥(γ) = \bigwedge_{α ∈ γ} ∥ϕ∥(α)$,
2. $∥¬ζ∥(γ) = ∥ζ∥(γ)$,
3. $∥ζ₁ ⊕ ζ₂∥(γ) = ∥ζ₁∥(γ) \lor ∥ζ₂∥(γ)$,
4. $∥ζ₁ ⊎ ζ₂∥(γ) = \bigvee_{γ = γ₁ \cup γ₂} (∥ζ₁∥(γ₁) \land ∥ζ₂∥(γ₂))$.

It is easy to prove that $∥\text{true}∥(γ) = 1$ and $∥\text{false}∥(γ) = 0$ for every $γ ∈ fC(P,K)$.

**Definition 11** Two $fPCL$ formulas $ζ₁, ζ₂$ over $P$ and a concrete De Morgan algebra $K_{\text{con}}$ are called $K_{\text{con}}$-equivalent, and we write $ζ₁ ≡_{K_{\text{con}}} ζ₂$, whenever $∥ζ₁∥(γ) = ∥ζ₂∥(γ)$ for every $γ ∈ fC(P,K_{\text{con}})$.

Two $fPCL$ formulas $ζ₁, ζ₂$ over $P$ and an arbitrary De Morgan algebra $K$ are called simply equivalent, and we write $ζ₁ ≡ ζ₂$, whenever $∥ζ₁∥(γ) = ∥ζ₂∥(γ)$ for every $γ ∈ fC(P,K)$. 

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In the following, we examine the relation between the fPIL and fPCL operators on fPIL formulas. Firstly, we show that the application of negation operators $!$ and $\neg$ on a fPIL formula derive in general non equivalent fPCL formulas. Indeed, let $p \in P$ and $\gamma = \{a_1, a_2\} \in fC(P, K)$. Then we have

$$\|!p\|(\gamma) = \bigwedge_{a \in \gamma} \|!p\|(a) = \|p\|(a_1) \land \|p\|(a_2) = a_1(p) \land a_2(p)$$

and

$$\|\neg p\|(\gamma) = \|p\|(\gamma) = \bigvee_{a \in \gamma} \|p\|(a) = \|p\|(a_1) \lor \|p\|(a_2) = a_1(p) \lor a_2(p)$$

which implies that $!p \not\equiv \neg p$.

Similarly, we show that in general $\varphi \otimes \varphi' \not\equiv \varphi \oplus \varphi'$ where $\varphi, \varphi'$ are fPIL formulas. For this, let $P$ be a set of ports, $\varphi = p \in P$ and $\varphi' = p' \in P$, where $p \neq p'$. If $\gamma = \{\alpha_1, \alpha_2\} \in fC(P, K)$, then we get $\|p \otimes p'\|(\gamma) \neq \|p \oplus p'\|(\gamma)$ and so $p \otimes p' \neq p \oplus p'$.

However, as we show in the next proposition, the application of the operators $\land$ and $\otimes$ on fPIL formulas produce equivalent fPCL formulas.

**Proposition 12** Let $\varphi_1, \varphi_2$ be fPIL formulas over $P$ and $K$. Then

$$\varphi_1 \otimes \varphi_2 \equiv \varphi_1 \otimes \varphi_2.$$  

**Proof.** For every $\gamma \in fC(P, K)$ we compute

$$\|\varphi_1 \otimes \varphi_2\|(\gamma) = \bigwedge_{a \in \gamma} \|\varphi_1 \otimes \varphi_2\|(a)$$

$$= \bigwedge_{a \in \gamma} (\|\varphi_1\|(a) \land \|\varphi_2\|(a))$$

$$= \left(\bigwedge_{a \in \gamma} \|\varphi_1\|(a)\right) \land \left(\bigwedge_{a \in \gamma} \|\varphi_2\|(a)\right)$$

$$= \|\varphi_1\|(\gamma) \land \|\varphi_2\|(\gamma)$$

$$= \|\neg (\neg \varphi_1 \oplus \neg \varphi_2)\|(\gamma)$$

$$= \|\varphi_1 \otimes \varphi_2\|(\gamma),$$

where the third equality holds by the commutativity and associativity of $\land$. 

In the sequel, we prove several properties of our fPCL formulas.
Proposition 13 The fPCL operators $\oplus$, $\otimes$ and $\uplus$ are associative and commutative.

Proof. We prove only the associativity of the $\uplus$ operator. The rest are analogously proved. Let $\zeta_1, \zeta_2, \zeta_3 \in fPCL(K, P)$ and $\gamma \in fC(P, K)$. Then

$\|\zeta_1 \uplus (\zeta_2 \uplus \zeta_3)\|(\gamma) = \bigvee_{\gamma'=\gamma_1 \cup \gamma'} (\|\zeta_1\|(\gamma_1) \land \|\zeta_2 \uplus \zeta_3\| (\gamma'))$

$= \bigvee_{\gamma'=\gamma_1 \cup \gamma'} \left(\|\zeta_1\|(\gamma_1) \land \left(\bigvee_{\gamma''=\gamma_2 \cup \gamma_3} (\|\zeta_2\|(\gamma_2) \land \|\zeta_3\|(\gamma_3))\right)\right)$

$= \bigvee_{\gamma'=\gamma_1 \cup \gamma_2 \gamma_3} (\|\zeta_1\|(\gamma_1) \land (\|\zeta_2\|(\gamma_2) \land \|\zeta_3\|(\gamma_3)))$

$= \bigvee_{\gamma'=\gamma_1 \cup \gamma_2 \gamma_3} \left(\left(\bigvee_{\gamma''=\gamma_1 \cup \gamma_3} (\|\zeta_1\|(\gamma_1) \land \|\zeta_2\|(\gamma_2))\right) \land \|\zeta_3\|(\gamma_3)\right)$

$= \bigvee_{\gamma'=\gamma_1 \cup \gamma_2 \gamma_3} (\|\zeta_1 \uplus \zeta_2\| (\gamma') \land \|\zeta_3\| (\gamma))$

where the third and fifth equalities hold since $\land$ distributes over $\lor$ and the fourth one by the associativity of the $\land$ operator.

Proposition 14 Let $\zeta \in fPCL(K, P)$. Then

$\|\sim \zeta\|(\gamma) = \bigvee_{\gamma' \leq \gamma} \|\zeta\|(\gamma')$

for every $\gamma \in fC(P, K)$.

Proof. For every $\gamma \in fC(P, K)$ we have

$\|\zeta\|(\gamma) = \bigvee_{\gamma'=\gamma \cup \gamma''} (\|\zeta\|(\gamma') \land \|true\|(\gamma''))$

$= \bigvee_{\gamma' \leq \gamma} \|\zeta\|(\gamma')$. 

\vspace{1cm}

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Proposition 15 Let \( \zeta, \zeta_1, \zeta_2 \in fPCL(K, P) \). Then
\[
\zeta \uplus (\zeta_1 \oplus \zeta_2) \equiv (\zeta \uplus \zeta_1) \oplus (\zeta \uplus \zeta_2).
\]

Proof. For every \( \gamma \in fC(P, K) \) we have
\[
\| \zeta \uplus (\zeta_1 \oplus \zeta_2) \| (\gamma) = \bigvee_{\gamma = \gamma_1 \cup \gamma_2} (\| \zeta_1 \| (\gamma_1) \land (\zeta \uplus \zeta_2) (\gamma_2))
\]
\[
= \bigvee_{\gamma = \gamma_1 \cup \gamma_2} (\| \zeta \| (\gamma_1) \land (\| \zeta \| (\gamma_2) \lor (\zeta \uplus \zeta_2) (\gamma_2)))
\]
\[
= \bigvee_{\gamma = \gamma_1 \cup \gamma_2} (\| \zeta \| (\gamma_1) \lor (\| \zeta \| (\gamma_2) \lor (\zeta \uplus \zeta_2) (\gamma_2)))
\]
\[
= \bigvee_{\gamma = \gamma_1 \cup \gamma_2} (\| \zeta \uplus \zeta_1 \| (\gamma_1) \lor (\| \zeta \uplus \zeta_2 \| (\gamma_2))
\]
where the third equality holds since \( \land \) distributes over \( \lor \) and the fourth one
by the associativity of \( \lor \).

Proposition 16 Let \( \zeta, \zeta_1, \zeta_2 \in fPCL(K, P) \). Then
\[
\zeta \otimes (\zeta_1 \oplus \zeta_2) \equiv (\zeta \otimes \zeta_1) \oplus (\zeta \otimes \zeta_2).
\]

Proof. By the distributivity of \( \land \) over \( \lor \) we get the subsequent proposition.

Proposition 17 Let \( \zeta \in fPCL(K, P) \). Then
\[
\begin{align*}
(1) & \quad \neg \neg \zeta \equiv \zeta. \\
(2) & \quad \zeta \oplus \zeta \equiv \zeta. \\
(3) & \quad \zeta \otimes \zeta \equiv \zeta. \\
(4) & \quad \zeta \oplus true \equiv true. \\
(5) & \quad \zeta \oplus false \equiv \zeta. \\
(6) & \quad \zeta \otimes true \equiv \zeta. \\
(7) & \quad \zeta \otimes false \equiv false \equiv \zeta \uplus false.
\end{align*}
\]

Proof. By the properties of De Morgan algebras we can prove the above
properties.

Proposition 18 Let \( \zeta_1, \zeta_2 \in fPCL(K, P) \). Then
\[
\begin{align*}
(1) & \quad \neg (\zeta_1 \oplus \zeta_2) \equiv (\neg \zeta_1) \otimes (\neg \zeta_2). \\
(2) & \quad \neg (\zeta_1 \otimes \zeta_2) \equiv (\neg \zeta_1) \oplus (\neg \zeta_2).
\end{align*}
\]
Proof. This proposition holds since $k \lor k' = k \land k'$ for every $k, k' \in K$. ■

In the following proposition, we present the absorbing laws of our fPCL formulas.

**Proposition 19** Let $\zeta, \zeta' \in fPCL(K, P)$. Then

1. $\zeta \otimes (\zeta \oplus \zeta') \equiv \zeta$.
2. $\zeta \oplus (\zeta \otimes \zeta') \equiv \zeta$.

**Proof.** This proof is done analogously to the proof of Proposition 8. ■

**Proposition 20** Let $\varphi \in fPIL(K, P)$ and $\zeta_1, \zeta_2 \in fPCL(K, P)$. Then

$$\varphi \otimes (\zeta_1 \uplus \zeta_2) \equiv (\varphi \otimes \zeta_1) \uplus (\varphi \otimes \zeta_2).$$

**Proof.** For every $\gamma \in fC(P, K)$ we compute

$$\|\varphi \otimes (\zeta_1 \uplus \zeta_2)\| (\gamma)$$
$$= \|\varphi\| (\gamma) \land \|\zeta_1 \uplus \zeta_2\| (\gamma)$$
$$= \|\varphi\| (\gamma) \land \bigvee_{\gamma = \gamma_1 \cup \gamma_2} (\|\zeta_1\| (\gamma_1) \land \|\zeta_2\| (\gamma_2))$$
$$= \bigvee_{\gamma = \gamma_1 \cup \gamma_2} \left( \bigwedge_{\alpha \in \gamma} \|\varphi\| (\alpha) \land \|\zeta_1\| (\gamma_1) \land \|\zeta_2\| (\gamma_2) \right)$$
$$= \bigvee_{\gamma = \gamma_1 \cup \gamma_2} \left( \bigwedge_{\alpha_1 \in \gamma_1} \|\varphi\| (\alpha_1) \land \bigwedge_{\alpha_2 \in \gamma_2} \|\varphi\| (\alpha_2) \land \|\zeta_1\| (\gamma_1) \land \|\zeta_2\| (\gamma_2) \right)$$
$$= \bigvee_{\gamma = \gamma_1 \cup \gamma_2} (\|\varphi\| (\gamma_1) \land \|\varphi\| (\gamma_2) \land \|\zeta_1\| (\gamma_1) \land \|\zeta_2\| (\gamma_2))$$
$$= \bigvee_{\gamma = \gamma_1 \cup \gamma_2} (\|\varphi \otimes \zeta_1\| (\gamma_1) \land \|\varphi \otimes \zeta_2\| (\gamma_2))$$
$$= \|((\varphi \otimes \zeta_1) \uplus (\varphi \otimes \zeta_2))\| (\gamma)$$

where the third equality holds since $\land$ distributes over $\lor$ and the fifth one by the idempotency and associativity of the $\land$ operator. ■

**Proposition 21** Let $\varphi \in fPIL(K, P)$. Then

1. $\varphi \uplus \varphi \equiv \varphi$.
2. $\neg \neg \varphi \equiv \neg \varphi$.
3. $\neg \varphi \equiv \neg \neg \varphi$. 

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Proof. For every $\gamma \in fC(P, K)$ we have

(1)

\[ \| \varphi \uplus \varphi \| (\gamma) = \bigvee_{\gamma = \gamma_1 \uplus \gamma_2} \left( \bigwedge_{\alpha_1 \in \gamma_1} \| \varphi \| (\alpha_1) \wedge \bigwedge_{\alpha_2 \in \gamma_2} \| \varphi \| (\alpha_2) \right) \]

\[ = \bigvee_{\gamma = \gamma_1 \uplus \gamma_2} \left( \bigwedge_{\alpha \in \gamma_1 \uplus \gamma_2} \| \varphi \| (\alpha) \right) \]

\[ = \bigwedge_{\alpha \in \gamma} \| \varphi \| (\alpha) = \| \varphi \| (\gamma) \]

where the third and fourth equalities hold since the operators $\wedge$ and $\vee$ are idempotent.

(2)

\[ \| \neg \sim \varphi \| (\gamma) = \| \neg (\varphi \uplus \text{true}) \| (\gamma) \]

\[ = \| \varphi \uplus \text{true} \| (\gamma) \]

\[ = \bigvee_{\gamma = \gamma_1 \uplus \gamma_2} \left( \| \varphi \| (\gamma_1) \wedge \| \text{true} \| (\gamma_2) \right) \]

\[ = \bigvee_{\gamma' \subseteq \gamma} \| \varphi \| (\gamma') \]

\[ = \bigvee_{\gamma' \subseteq \gamma} \bigwedge_{\alpha \in \gamma'} \| \varphi \| (\alpha) \]

\[ = \bigwedge_{\alpha \in \gamma} \bigvee_{\gamma' \subseteq \gamma \alpha \in \gamma'} \| \varphi \| (\alpha). \]

Let $\gamma = \{\alpha_1, \ldots, \alpha_n\}$ and $\{\alpha_1\}, \ldots, \{\alpha_n\}, \gamma_1, \ldots, \gamma_k$ be all possible subsets of $\gamma$, where $k = 2^n - (n + 1)$ and $|\gamma_i| > 1$ for every $i \in \{1, \ldots, k\}$. Therefore, we get

\[ \| \neg (\varphi \uplus 1) \| (\gamma) = \bigwedge_{\gamma' \subseteq \gamma} \bigvee_{\alpha \in \gamma'} \| \varphi \| (\alpha) \]

\[ = \left( \bigwedge_{i=1}^{n} \bigvee_{\alpha \in \{\alpha_i\}} \| \varphi \| (\alpha) \right) \wedge \left( \bigwedge_{j=1}^{k} \bigvee_{\alpha \in \gamma_j} \| \varphi \| (\alpha) \right) \]

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\[
\|\varphi\| (\alpha_1) \land \cdots \land \|\varphi\| (\alpha_n) \land \left( \bigwedge_{j=1}^{k} \bigvee_{\alpha \in \gamma_j} \|\varphi\| (\alpha) \right)
\]

\[
= \left( \bigwedge_{\alpha \in \gamma} \|\varphi\| (\alpha) \right) \land \left( \bigwedge_{j=1}^{k} \bigvee_{\alpha' \in \gamma_j} \|\varphi\| (\alpha') \right)
\]

\[
= \left( \bigwedge_{\alpha \in \gamma} \|\varphi\| (\alpha) \right) \land \left( \bigwedge_{j=1}^{k} \bigvee_{\alpha' \in \gamma_j} \|\varphi\| (\alpha') \right)
\]

\[
= \bigwedge_{j=1}^{k} \bigvee_{\alpha' \in \gamma_j} \left( \bigwedge_{\alpha \in \gamma} \|\varphi\| (\alpha) \land \|\varphi\| (\alpha') \right)
\]

\[
= \bigwedge_{\alpha \in \gamma} \|\varphi\| (\alpha)
\]

\[
= \|\varphi\| (\gamma)
\]

where for the validity of the last equality we give the following explanation. For every \( j \in \{1, \ldots, k\} \) and for every \( \alpha' \in \gamma_j \) we have \( \bigwedge_{\alpha \in \gamma} \|\varphi\| (\alpha) \land \|\varphi\| (\alpha') = \bigwedge_{\alpha \in \gamma} \|\varphi\| (\alpha) \) since \( \alpha' \in \gamma \) and \( \land \) is idempotent. Lastly, by the idempotency of \( \lor \) we get the last equality.

(3)

\[
\|\sim \varphi\| (\gamma) = \|\neg (\neg \sim \varphi)\| (\gamma)
\]

\[
= \|\neg \neg \varphi\| (\gamma)
\]

\[
= \|\neg \varphi\| (\gamma)
\]

where the second equality holds by Proposition 21(2).

\begin{prop}
Let \( J \) be a finite index set and \( \varphi_j \) a fPIL formula over \( P \) and \( K \) for every \( j \in J \). Let \( \gamma \in \mathcal{F}(P, K) \). Then

\[
\|\bigvee_{j \in J} \varphi_j\| (\gamma) = \bigvee_{j' \subseteq J} \bigvee_{\gamma_{j'} = \gamma} \left( \bigwedge_{j' \in J'} \|\varphi_{j'}\| (\gamma_{j'}) \right)
\]

where \( \cup \) denotes the disjoint union of sets.
\end{prop}
Proof. Let $\gamma = \{\alpha_1, \ldots, \alpha_n\} \in fC(P, K)$ and a finite index set $J$. Then

$$\left\| \bigvee_{j \in J} \varphi_j \right\| (\gamma) = \bigwedge_{\alpha \in \gamma} \left\| \bigvee_{j \in J} \varphi_j \right\| (\alpha)$$

$$= \bigwedge_{\alpha \in \gamma} \left( \bigvee_{j \in J} \| \varphi_j \| (\alpha) \right)$$

$$= \bigvee_{(j_1, \ldots, j_n) \in J^n} \left( \| \varphi_{j_1} \| (\alpha_1) \land \| \varphi_{j_2} \| (\alpha_2) \land \cdots \land \| \varphi_{j_n} \| (\alpha_n) \right)$$

$$= \bigvee_{J' \subseteq J} \bigvee_{j' \in J'} \left( \bigwedge_{j' \in J'} \| \varphi_{j'} \| (\gamma_{j'}) \right)$$

where for the validity of the last equality we give the following explanation. In the third equality we have all possible $n$-tuples with elements from $J$. Hence, there are cases where we have repetitions of some $j_i$’s. Moreover, in every parenthesis in the third equality, each $\alpha_j$ appears exactly once and therefore we get the disjoint unions of sets that are equal to $\gamma$ in the last equality. Lastly, considering the above, the commutativity of the operators $\land$, $\lor$ and the idempotency of the $\lor$ operator, we get the last equality.

Proposition 23 Let $J$ be a finite index set and $\varphi_j$ a $\Pi$PIL formula over $P$ and $K$ for every $j \in J$. Then

$$\biguplus_{j \in J} \varphi_j \equiv \bigotimes_{j \in J} (\sim \varphi_j) \otimes \left( \bigvee_{j \in J} \varphi_j \right).$$

Proof. Let $\gamma \in fC(P, K)$ and $J = \{1, \ldots, n\}$ a finite index set. Then we get

$$\left\| \bigotimes_{j \in J} (\sim \varphi_j) \otimes \left( \bigvee_{j \in J} \varphi_j \right) \right\| (\gamma)$$

$$= \bigwedge_{j \in J} \left( \bigvee_{\gamma_j \subseteq \gamma} \| \varphi_j \| (\gamma_j) \right) \land \left( \bigvee_{J' \subseteq J} \bigcup_{j' \in J'} \bigwedge_{\gamma_{j'} \subseteq \gamma} \left( \bigvee_{j' \in J'} \| \varphi_{j'} \| (\gamma_{j'}) \right) \right)$$

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\[
\begin{align*}
= & \bigvee_{\gamma_{\subseteq} \gamma} \ldots \bigvee_{\gamma_{\subseteq} \gamma} \left( \bigwedge_{j \in J} \| \varphi_j \| (\gamma_j) \right) \land \left( \bigvee_{J' \subseteq J} \bigvee_{j' \in J'} \left( \bigwedge_{j'' \in J'} \| \varphi_{j''} \| (\gamma_{j''}) \right) \right) \\
= & \bigvee_{\gamma_{\subseteq} \gamma} \ldots \bigvee_{\gamma_{\subseteq} \gamma} \bigwedge_{J' \subseteq J} \bigvee_{j' \in J'} \left( \| \varphi_1 \| (\gamma_1) \land \ldots \land \| \varphi_n \| (\gamma_n) \land \bigwedge_{j \in J'} \| \varphi_j \| (\gamma_j) \right) \\
= & \bigvee_{\gamma_{\subseteq} \gamma} \ldots \bigvee_{\gamma_{\subseteq} \gamma} \bigwedge_{J' \subseteq J} \bigvee_{j' \in J'} \left( \| \varphi_{j'} \| (\gamma_{j'} \cup \gamma_j) \land \bigwedge_{j \in J \setminus J'} \| \varphi_j \| (\gamma_j) \right)
\end{align*}
\]

where the first equality holds by Proposition 22 and the fourth one by the idempotency of \( \land \). We observe that the sets \( \gamma_{j'} \cup \gamma_j \) and \( \gamma_j \) for every \( j' \in J' \) and \( j \in J \setminus J' \), consist all possible subsets of \( \gamma \) where the union of them is equal to \( \gamma \). So, by the idempotency of \( \land \) and \( \lor \) we get

\[
\| \bigotimes_{j \in J} (\sim \varphi_j) \otimes \left( \bigvee_{j \in J} \varphi_j \right) \| (\gamma) = \bigvee_{\gamma'_1 \cup \ldots \cup \gamma'_n = \gamma} \left( \| \varphi_1 \| (\gamma''_1) \land \ldots \land \| \varphi_n \| (\gamma''_n) \right)
\]

\[
= \| \varphi_1 \lor \ldots \lor \varphi_n \| (\gamma).
\]

\textbf{Proposition 24} Let \( J \) be a finite index set and \( \varphi_j \) a fPIL formula for every \( j \in J \). Then

\[
\neg \biguplus_{j \in J} \varphi_j \equiv \bigoplus_{j \in J} (! \varphi_j) \oplus \sim \left( \bigotimes_{j \in J} ! \varphi_j \right)
\]

\textbf{Proof.} We get

\[
\neg \biguplus_{j \in J} \varphi_j \equiv \neg \left( \bigotimes_{j \in J} (\sim \varphi_j) \otimes \left( \bigvee_{j \in J} \varphi_j \right) \right)
\]

\[
\equiv \bigoplus_{j \in J} \left( \sim \varphi_j \right) \oplus \sim \left( \bigvee_{j \in J} \varphi_j \right)
\]

\[
\equiv \bigoplus_{j \in J} (\varphi_j) \oplus \sim \left( \bigvee_{j \in J} \varphi_j \right)
\]

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\[= \bigoplus_{j \in J} (!\varphi_j) \oplus \bigotimes_{j \in J} (!\varphi_j)\]

where the third equivalence holds by Proposition 21. ■

**Proposition 25** Let \( J \) be a finite index set and \( \varphi_j \) a fPIL formula for every \( j \in J \). Then

\[\bigotimes_{j \in J} (\sim \varphi_j) \equiv \sum_{j \in J} \varphi_j.\]

**Proof.** Let \( \gamma \in fC(P, K) \). Then

\[
\left\| \bigotimes_{j \in J} (\sim \varphi_j) \right\| (\gamma) = \bigwedge_{j \in J} \left( \bigvee_{\gamma_j \leq \gamma} \| \varphi_j \| (\gamma_j) \right)
\]

\[
= \bigvee_{U_{j \in J} \gamma_j \leq \gamma} \left( \bigwedge_{j \in J} \| \varphi_j \| (\gamma_j) \right)
\]

\[
= \bigvee_{\gamma' \leq \gamma} \left( \bigvee_{\gamma_j = \gamma'} \left( \bigwedge_{j \in J} \| \varphi_j \| (\gamma_j) \right) \right)
\]

\[
= \bigvee_{\gamma' \leq \gamma} \left\| \bigoplus_{j \in J} \varphi_j \right\| (\gamma')
\]

\[
= \left\| \sim \bigoplus_{j \in J} \varphi_j \right\| (\gamma)
\]

where the second equality holds since \( \wedge \) distributes over \( \vee \). ■

**Proposition 26** Let \( J \) and \( K \) be finite index sets and \( \varphi_j, \varphi'_k \) fPIL formulas for every \( j \in J \) and \( k \in K \). Then

\[
\left( \bigoplus_{j \in J} \varphi_j \right) \otimes \left( \bigoplus_{k \in K} \varphi'_k \right) \equiv \left( \bigoplus_{j \in J} \varphi_j \uplus \bigoplus_{k \in K} \varphi'_k \right) \otimes \left( \bigvee_{(j,k) \in J \times K} (\varphi_j \odot \varphi'_k) \right)
\]

**Proof.** By Proposition 17(1) we get

\[
\left( \bigoplus_{j \in J} \varphi_j \right) \otimes \left( \bigoplus_{k \in K} \varphi'_k \right) \equiv \neg \left( \left( \bigoplus_{j \in J} \varphi_j \right) \otimes \left( \bigoplus_{k \in K} \varphi'_k \right) \right).
\]
By Proposition 24 it is valid that

\[- \left( \biguplus_{j \in J} \varphi_j \otimes \biguplus_{k \in K} \varphi'_k \right) \]

\[\equiv - \left( \biguplus_{j \in J} \varphi_j \right) \oplus - \left( \biguplus_{k \in K} \varphi'_k \right) \]

\[\equiv \bigoplus_{j \in J} (!\varphi_j) \oplus \bigoplus_{k \in K} (!\varphi'_k) \equiv \bigoplus_{j \in J} (!\varphi_j) \oplus \bigoplus_{k \in K} (!\varphi'_k) \]

and so

\[- \left( \biguplus_{j \in J} \varphi_j \otimes \biguplus_{k \in K} \varphi'_k \right) \]

\[\equiv - \left( \bigoplus_{j \in J} (!\varphi_j) \oplus \bigoplus_{k \in K} (!\varphi'_k) \right) \]

\[\equiv \bigotimes_{j \in J} (\sim \varphi_j) \otimes \bigotimes_{k \in K} (\sim \varphi'_k) \]

where the third equivalence holds by Proposition 21 and the last one by Proposition 25.

We proceed with an important property of fPCL formulas over $P$ and a Kleene algebra.
Proposition 27 Let $P$ be a set of ports and $K_3$ a Kleene algebra. Then
\[(p \otimes !p) \otimes (q \otimes !q) \equiv_{K_3} p \otimes !p\]
where $p, q \in P$.

Proof. Let $\gamma \in fC(P, K_3)$. Then
\[
\| (p \otimes !p) \otimes (q \otimes !q) \| (\gamma) = \bigwedge_{\alpha \in \gamma} \| (p \otimes !p) \otimes (q \otimes !q) \| (\alpha)
\]
\[
= \bigwedge_{\alpha \in \gamma} \left( (\alpha(p) \land \overline{\alpha(p)}) \land (\alpha(q) \lor \overline{\alpha(q)}) \right)
\]
\[
= \bigwedge_{\alpha \in \gamma} (\alpha(p) \land \overline{\alpha(p)})
\]
\[
= \| p \otimes !p \| (\gamma)
\]
where the third equality holds since $(k \land \overline{k}) \land (k' \lor \overline{k'}) = k \land \overline{k}$ for every $k, k' \in K_3$.

Next, we give properties of $fPCL$ formulas over $P$ and a Boolean algebra.

Proposition 28 Let $\varphi$ and $\zeta$ be a $fPIL$ and a $fPCL$ formula, respectively, over $P$ and a Boolean algebra. Then
\begin{enumerate}
\item $\varphi \otimes !\varphi \equiv_B false.$
\item $\varphi \otimes !\varphi \equiv_B true.$
\item $\zeta \otimes \neg\zeta \equiv_B false.$
\item $\zeta \otimes \neg\zeta \equiv_B true.$
\end{enumerate}

Proof. By the properties of the Boolean algebra we get the proposition.

Let $P$ be a finite set of ports. By the previous results we get that $p \otimes !p \equiv_2 false$ but $p \otimes !p \not\equiv_3 false$ for every $p \in P$.

6 Examples

Example 29 (Peer-to-Peer architecture) Peer-to-peer architecture (P2P for short) is a commonly used computer networking architecture. All peers in the architecture have some available resources, such as processing power and network bandwidth (cf. [15]). Those resources are available to the other peers that participate in the architecture without the need of a central component to coordinate their interactions. This is not the case in Request/Response architecture where a coordinator is needed (cf. [9]). All peers in the architecture are both suppliers and consumers of their resources and so there is no distinction between them.
In our example, we consider four components $C_1, C_2, C_3$ and $C_4$ (Figure 2). Every component has two ports denoted by $r$ and $s$ which represent, respectively, the functions receive and send. Let $J = \{1, 2, 3, 4\}$. So, the set of ports is $P = \bigcup_{j \in J} \{r_j, s_j\}$. Each component can receive and send information to as many other components in the architecture except from itself. One possible architecture scheme is shown in Figure 2. In the sequel we construct a fPCL formula which describes the P2P architecture with four components with a fPCL formula.

Firstly, let two distinct components $C_j$ and $C_{j'}$ where $j, j' \in J$. The interaction between $C_j$ and $C_{j'}$ where $C_j$ receives information from $C_{j'}$, is characterised by the following fPIL formula

$$\varphi_{j, j'} = r_j \otimes s_{j'} \otimes !s_j \otimes !r_{j'} \otimes \bigotimes_{j'' \in J \setminus \{j, j'\}} (!r_{j''} \otimes !s_{j''})$$

Next, as it was mentioned above, $C_j$ can receive information from more than one components. Let $J' \subseteq J \setminus \{j\}$. The interactions between $C_j$ and $C_{j'}$, where $j' \in J'$ are characterized by $\zeta_{j, J'} = \biguplus_{j' \in J'} \varphi_{j, j'}$. However, $J'$ can be any non empty subset of $J \setminus \{j\}$. So the fPCL formula

$$\zeta_j = \biguplus_{J' \in \mathcal{P}(J \setminus \{j\}) \setminus \{\emptyset\}} \zeta_{j, J'}$$

, where $\mathcal{P}(J \setminus \{j\})$ denotes the power set of $J \setminus \{j\}$, describes all possible architecture schemes between $C_j$ and the rest components in the architecture. Lastly, some components may not interact at all with the others. Therefore, the fPCL formula

$$\zeta = \biguplus_{J'' \in \mathcal{P}(J) \setminus \{\emptyset\}} \biguplus_{j \in J''} \zeta_j$$



Figure 2: Peer-to-peer architecture.
describes all possible architecture schemes of the P2P architecture with four components.

In our example, we consider that every port has a degree of uncertainty. Let the fuzzy algebra and a configuration set $\gamma \in fC(P,F)$. For every $\alpha \in fI(P,F)$ the value $\alpha(p)$ represents the degree of uncertainty of the port $p \in P$. If $\alpha(p) = 0$ then the port has an absolute uncertain behavior. If $\alpha(p) = 1$ the port will participate with no uncertainty, i.e., it will participate with no fault in its behavior. Then the value $\|\sim \zeta\| (\gamma)$ gives the maximum uncertainty that can occur in the architecture considering the given interactions of $\gamma$.

**Example 30** We recall from [9] the Master/Slave architecture for two masters $M_1, M_2$ and two slaves $S_1, S_2$ with ports $m_1, m_2$ and $s_1, s_2$, respectively. Masters can interact only with slaves, and vice versa. Each slave can interact with only one master.

As it was mentioned in the Introduction, software architectures have a degree of uncertainty. We show how we can compute the uncertainty of the Master/Slave architecture over a finite number of components. We consider the fuzzy algebra and the set of ports $P = \{s_1, m_1, s_2, m_2\}$.

Next, we construct the fPCL formula which describes the architecture. The interaction between a master $m \in \{m_1, m_2\}$ and a slave $s \in \{s_1, s_2\}$ is described by the fPIL formula

$$\varphi_{s,m} = s \otimes m \otimes !s' \otimes !m'$$

where $s' \neq s$ and $m' \neq m$. Moreover, as it was mentioned above, every master can interact with only one slave, and vice versa. Hence, the fPCL formula

$$\zeta = (\varphi_{s_1,m_1} \oplus \varphi_{s_2,m_1}) \uplus (\varphi_{s_1,m_2} \oplus \varphi_{s_2,m_2})$$

describes the Master/Slave architecture with two masters and two slaves. Let $\gamma \in fC(P,K_F)$ be a set of estimations of uncertainty of the ports in the architecture and the fPCL formula $\sim \zeta$. The value $\|\sim \zeta\| (\gamma)$ gives the maximum value among the values that represent the maximum uncertainty among the architecture patterns.

### 7 Normal Form and Decidability of Equivalence

In this section we examine the decidability of equivalence of fPCL formulas. Let $\zeta$ and $\zeta'$ be fPCL formulas over the set of ports $P = \{p,q,r\}$ and the fuzzy algebra $F$. By Definition 11, $\zeta \equiv_F \zeta'$ if $\|\zeta\| (\gamma) = \|\zeta'\| (\gamma)$ for every $\gamma \in fC(P,F)$. However, the set $fC(P,F)$ is infinite and so it is impossible
to check the equivalence by the previous way. This is not the case for fPCL formulas over De Morgan algebras with finite set $K$ such as the two element Boolean algebra and the three element Kleene algebra. However, if we prove that two fPCL formulas have the same normal form, then they are equivalent.

In the sequel, we show that every fPCL formula over $P$ and a Kleene algebra $K_3$, can be equivalently written in a normal form. Consequently, we show that the equivalence problem for fPCL formulas over $P$ and a Kleene algebra $K_3$ is decidable. In the following, we give some useful definitions for the definition of the normal form of our fPCL formulas.

**Definition 31** Let $P$ be a set of ports. A fPIL formula $\varphi$ is called f-monomial if it is of the form

$$\varphi = \bigwedge_{p_1 \in P_1} p_1 \otimes \bigwedge_{p_2 \in P_2} \neg p_2.$$  

where $P_1, P_2 \subseteq P$ and $P_1 \cup P_2 \neq \emptyset$.

Following the previous definition, $P_1 \cap P_2$ can be either empty or not. Consider $P = \{p, q, r\}$ be a set of ports. The fPIL formulas $p \otimes \neg p \otimes \neg q$ and $p \otimes r$ are f-monomials.

**Definition 32** Let $P$ be a set of ports and $K$ a De Morgan algebra. A fPIL formula $\varphi$ is said to be in fpil-normal form if it is of the form

(1) $\varphi \equiv \bigvee_{i \in I} \varphi_i$, where $I$ is a finite index set, $\varphi_i$ is a f-monomial for every $i \in I$ and $\varphi_i \neq \varphi_{i'}$ for every $i, i' \in I$ with $i \neq i'$, or

(2) $\varphi \equiv \text{true}$, or

(3) $\varphi \equiv \text{false}$.

**Definition 33** Let $P$ be a finite set of ports and $K$ a De Morgan algebra. A fPCL formula $\zeta$ over $P$ and $K$ is said to be in normal form if it is of the following form:

(1) $\zeta = \bigoplus_{i \in I} \bigcup_{j \in J_i} \varphi_{i,j}$, where $I, J_i$ are finite index sets for every $i \in I$ and $\varphi_{i,j} \neq \text{false}$ is in fpil-normal form for every $i \in I$ and $j \in J_i$, or

(2) $\zeta = \text{true}$, or

(3) $\zeta = \text{false}$.
By Propositions 8, 17, 21, for every $\text{fPCL}$ formula in normal form we can construct its equivalent one in normal form satisfying the following statements:

1. Let $i \in I$. Then $\varphi_{i,j} \not\equiv \varphi_{i,j'}$ for every $j \neq j'$.
2. Let $i, i' \in I$ with $i \neq i'$. Then $\biguplus_{j \in I_i} \varphi_{i,j} \not\equiv \biguplus_{j \in I_{i'}} \varphi_{i',j}$.

In the sequel, every $\text{fPCL}$ formula in normal form is considered to satisfy the above statements.

Next, we present our results on the existence and the construction of the normal form $\text{fPCL}$ formulas. But first, we need to note a very important observation. For this we give the following example.

**Example 34** Let $P = \{p, q\}$ be a set of ports and the $\text{fPIL}$ formulas $\varphi = p \otimes \neg p$ and $\varphi' = (p \otimes \neg p \otimes q) \otimes (p \otimes \neg p \otimes \neg q)$. Those two formulas are in normal form. Considering an arbitrary De Morgan algebra we get that $\varphi \not\equiv \varphi'$ since their normal forms are not equivalent. However, we prove they are equivalent over the fuzzy algebra. For this

$$
\varphi' = (p \otimes \neg p \otimes q) \otimes (p \otimes \neg p \otimes \neg q)
\equiv_F (p \otimes \neg p) \otimes (q \otimes \neg q)
\equiv_F p \otimes \neg p = \varphi
$$

where the first equivalence holds since $\otimes$ distributes over $\otimes$ and the second one by Proposition 27. We conclude that $\varphi \equiv_F \varphi'$. Analogously, we prove that $\varphi \equiv_{K_3} \varphi'$. Also, $\varphi \equiv_B \varphi'$ since both formulas are equivalent to false over a Boolean algebra. However, $\varphi$ and $\varphi'$ are not equivalent if we consider the four element algebra 4. Let $\gamma = \{\alpha\} \in fC(P, 4)$, where $\alpha(p) = u$ and $\alpha(q) = w$. Then $\|\varphi\|_\gamma(u) = u \neq 0 = \|\varphi'\|_\gamma$. So $\varphi \not\equiv_4 \varphi'$.

By Example 34, we observe that for the construction of the normal form of a $\text{fPCL}$ formula we need to take into account the properties of the De Morgan algebra. In the following, we show that for every $\text{fPCL}$ formula over $P$ and a Kleene algebra, we can effectively construct its equivalent $\text{fPCL}$ formula in normal form.

**Theorem 35** Let $P$ be a set of ports and $K$ an arbitrary De Morgan algebra. Then for every $\text{fPCL}$ formula $\zeta_1 \in f\text{PCL}(K, P)$, $\zeta_2 \in f\text{PCL}(K_3, P)$ and $\zeta_3 \in f\text{PCL}(B, P)$, we can effectively construct an equivalent $\text{fPCL}$ formula $\zeta'_1 \in f\text{PCL}(K, P)$, $\zeta'_2 \in f\text{PCL}(K_3, P)$ and $\zeta'_3 \in f\text{PCL}(B, P)$, respectively, in normal form. The time complexity of the construction is polynomial.
Proof. We prove our theorem by induction on the structure of fPCL formulas over $P$ and a Kleene algebra $K_3$. We deal with the other cases at the end of this proof.

Let $\zeta = \varphi \in fPIL(K_3, P)$. If $\zeta$ is equal to true or false, then we are done. Otherwise, by Propositions 4, 5, 6, 7 and 8 we get its equivalent formula in fPIL-normal form. Then we go to Step 2(2) and by Propositions 7 and 8 we get its equivalent formula of $\zeta$ in normal form.

Now, let $\zeta_1, \zeta_2$ be fPCL formulas and assume that both $\zeta_1$ and $\zeta_2$ are not equivalent to true or false. Those cases can be treated analogously to the cases we show below and the properties of the De Morgan algebra. Consider $\zeta'_1 = \bigoplus_{i_1 \in I_1} \biguplus_{j_1 \in J_{i_1}} \varphi_{i_1,j_1}$ and $\zeta'_2 = \bigoplus_{i_2 \in I_2} \biguplus_{j_2 \in J_{i_2}} \varphi_{i_2,j_2}$ be their equivalent normal forms, respectively. Then we go to Step 1.

Step 1

(1) Firstly, let $\zeta = \zeta_1 \oplus \zeta_2$. The formula $\zeta$ is equivalent to $\zeta'_1 \oplus \zeta'_2$ which is of the form $\bigoplus_{i \in I} \biguplus_{j \in J_i} \varphi_{i,j}$ where $\varphi_{i,j}$ is in fPIL-normal form for every $j \in J_i$. Then we go to Step 2.

(2) Next, let $\zeta = \zeta_1 \uplus \zeta_2$. Then

$$\zeta \equiv \zeta'_1 \uplus \zeta'_2$$

$$\equiv \left( \bigoplus_{i_1 \in I_1} \biguplus_{j_1 \in J_{i_1}} \varphi_{i_1,j_1} \right) \uplus \left( \bigoplus_{i_2 \in I_2} \biguplus_{j_2 \in J_{i_2}} \varphi_{i_2,j_2} \right)$$

$$\equiv \bigoplus_{i_1 \in I_1} \bigoplus_{i_2 \in I_2} \left( \biguplus_{j_1 \in J_{i_1}} \varphi_{i_1,j_1} \uplus \biguplus_{j_2 \in J_{i_2}} \varphi_{i_2,j_2} \right)$$

where the last equivalence holds by Proposition 15. Then we go to Step 2.

(3) Now, let $\zeta = \zeta_1 \otimes \zeta_2$. Then we get

$$\zeta \equiv \zeta'_1 \otimes \zeta'_2$$

$$\equiv \left( \bigoplus_{i_1 \in I_1} \biguplus_{j_1 \in J_{i_1}} \varphi_{i_1,j_1} \right) \otimes \left( \bigoplus_{i_2 \in I_2} \biguplus_{j_2 \in J_{i_2}} \varphi_{i_2,j_2} \right)$$

$$\equiv \bigoplus_{(i_1,i_2) \in I_1 \times I_2} \left( \biguplus_{j_1 \in J_{i_1}} \varphi_{i_1,j_1} \otimes \biguplus_{j_2 \in J_{i_2}} \varphi_{i_2,j_2} \right)$$
\[
\equiv \bigoplus_{(i_1,i_2) \in I_1 \times I_2} \left( \bigoplus_{j_1 \in J_{i_1}} \varphi_{i_1,j_1} \bigoplus_{j_2 \in J_{i_2}} \varphi_{i_2,j_2} \right) \otimes \\
\bigvee_{(j_1,j_2) \in J_{i_1} \times J_{i_2}} (\varphi_{i_1,j_1} \otimes \varphi_{i_2,j_2})
\]

where the third equivalence holds by Proposition 16 and the fourth one by Proposition 26. Consider the fPIL formula \(\varphi_{(i_1,i_2)} = (\varphi_{i_1,j_1} \otimes \varphi_{i_2,j_2})\) for every \((i_1,i_2) \in I_1 \times I_2\). Then by Propositions 17 and 20 we get

\[
\zeta \equiv \bigoplus_{(i_1,i_2) \in I_1 \times I_2} \left( \bigoplus_{j_1 \in J_{i_1}} (\varphi_{i_1,j_1} \otimes \varphi_{(i_1,i_2)}) \bigoplus_{j_2 \in J_{i_2}} (\varphi_{i_2,j_2} \otimes \varphi_{(i_1,i_2)}) \bigoplus (\varphi_{(i_1,i_2)} \otimes \text{true}) \right)
\]

\[
\equiv \bigoplus_{i_1 \in I_1} \left( \bigoplus_{j_1 \in J_{i_1}} (\varphi_{i_1,j_1}) \right) \bigoplus \bigoplus_{j_1 \in J_{i_1}} \left( \bigoplus_{i_2 \in I_2} (\varphi_{i_2,j_2} \otimes \varphi_{(i_1,i_2)}) \right) \bigoplus (\varphi_{(i_1,i_2)} \otimes \text{true})
\]

Then we go to Step 2.

(4) Let us assume that \(\zeta = \neg \zeta_1\). Then

\[
\zeta \equiv \neg \zeta_1
\]

\[
\equiv \neg \left( \bigoplus_{j_1 \in J_{i_1}} \bigoplus_{j_1 \in J_{i_1}} \varphi_{i_1,j_1} \right)
\]

\[
\equiv \bigotimes_{i_1 \in I_1} \left( \neg \left( \bigoplus_{j_1 \in J_{i_1}} \varphi_{i_1,j_1} \right) \right)
\]

\[
\equiv \bigotimes_{i_1 \in I_1} \left( \bigoplus_{j_1 \in J_{i_1}} (\neg \varphi_{i_1,j_1}) \oplus \sim (\bigoplus_{j_1 \in J_{i_1}} \varphi_{i_1,j_1}) \right)
\]

where the third equivalence holds by Proposition 18 and the fourth one by Proposition 24. In the sequel by applying Propositions 4, 7, 16, 20 and 26 we get a formula of the form \(\bigoplus_{i \in I} \bigoplus_{j \in J_{i_1}} \varphi_{i,j} \otimes \varphi_{i,j,k}\) where \(\varphi_{i,j} = \bigvee_{k \in K_{i,j}} \varphi_{i,j,k}\) and \(\varphi_{i,j,k}\) is a f-monomial for every \(k \in K_{i,j}\) and \(j \in J_{i}\).

Next, we proceed to Step 2.
**Step 2**

Let a formula of the form \( \bigoplus_{i \in I} \bigcup_{j \in J} \varphi_{i,j} \) where the formula \( \varphi_{i,j} = \bigvee_{k \in K_{i,j}} \varphi_{i,j,k} \) and \( \varphi_{i,j,k} \) is a f-monomial for every \( k \in K_{i,j} \) and \( j \in J_i \). In order to get its equivalent formula in normal form we apply the following.

1. Firstly, we apply Propositions 5, 8, 17 and 21(1) in order to discard any repetitions when the operations allow it. So we get a formula of the form \( \bigoplus_{i' \in I'} \bigcup_{j' \in J'} \varphi_{i',j'} \) where \( \varphi_{i',j'} = \bigoplus_{k' \in K'_{i',j'}} \varphi_{i',j',k'} \) is in \( fPil \)-normal form for every \( (i', j') \in I' \times J'_i \).

2. Next, since we consider a Kleene algebra, we apply Proposition 27. Let for instance a f-monomial \( \varphi \) of the following form:

\[
\varphi = \bigotimes_{p_1 \in P_1} (p_1 \otimes !p_1) \otimes \bigotimes_{p_2 \in P_2} p_2 \otimes \bigotimes_{p_3 \in P_3} !p_3
\]

where the sets \( P_1, P_2, P_3 \subseteq P \) are pairwise disjoint. Then we consider the set \( P' = P \setminus (P_1 \cup P_2 \cup P_3) \) and by Proposition 27 we get the following:

\[
\varphi \equiv \bigotimes_{p_1 \in P_1} (p_1 \otimes !p_1) \otimes \bigotimes_{p_2 \in P_2} p_2 \otimes \bigotimes_{p_3 \in P_3} !p_3 \otimes \bigotimes_{p \in P'} (p \otimes !p).
\]

We follow the above procedure for every f-monomial \( \varphi \) of the form \( \bigotimes_{p_1 \in P_1} (p_1 \otimes !p_1) \otimes \bigotimes_{p_2 \in P_2} p_2 \otimes \bigotimes_{p_3 \in P_3} !p_3 \). By this step we “appear” the ports that get eliminated by the property of the Kleene algebra.

3. Lastly, we apply Propositions 5, 8, 17 and 21(1) to discard again any repetitions created by the previous step.

By following the steps given above, we get an equivalent formula in normal form. In order to prove our claim for the time complexity of the algorithm presented above. In every step of our construction, we applied the distribution and idempotency properties of our logic, which are done in polynomial time. This concludes our proof.

Consider a Boolean algebra and a \( fPCL \) formula \( \zeta \in fPCL(B, P) \). For the construction of its equivalent \( fPCL \) formula over \( P \) and \( B \) in normal form, we follow the above proof where we replace Step 2(2) with the application of Proposition 28. If \( K \) is an arbitrary De Morgan algebra and \( \zeta \in fPCL(K, P) \), then for the construction of its equivalent \( fPCL \) formula over \( P \) and \( K \) in normal form, we follow the above proof without Steps 2(2) and 2(3). The complexity of those constructions is again polynomial.
Next, we prove that the equivalence problem for $fPCL$ formulas is decidable.

**Theorem 36** Let $K$ be a De Morgan algebra and $P$ a set of ports. Then, for every $\zeta_1, \zeta_2 \in fPCL(K, P)$ the equivalence $\zeta_1 \equiv \zeta_2$ is decidable. The run time is polynomial.

**Proof.** By Theorem 35 we can effectively construct $fPCL$ formulas $\zeta'_1, \zeta'_2$ in normal form such that $\zeta_1 \equiv \zeta'_1$ and $\zeta_2 \equiv \zeta'_2$. In order to prove whether $\zeta_1$ and $\zeta_2$ are equivalent or not, we need to examine if $\zeta'_1 \equiv \zeta'_2$ or not. For this, we write our formulas in a form of sets which we compare using Algorithm 1 given in A.

Firstly, we consider that $\zeta'_1 = \bigoplus_{i \in I} \bigcup_{j \in J_i} \varphi_{i,j}$ where $\varphi_{i,j} \neq \text{false}$ is in $fpi\ell$-normal form for every $i \in I$ and $j \in J_i$. Hence, $\zeta'_1 = \bigoplus_{i \in I} \bigcup_{j \in J_i} \bigvee_{k \in K_{i,j}} \varphi_{i,j,k}$ where $\varphi_{i,j,k}$ is a $f$-monomial for every $k \in K_{i,j}$. If there exists $i \in I$ and $j \in J_i$ such that $\varphi_{i,j} \equiv \text{true}$, then $\varphi_{i,j}$ can be written as $\bigvee_{k \in K_{i,j}} \text{true}$ where $K_{i,j} = \{1\}$. Analogously, $\zeta'_2 \equiv \bigoplus_{m \in M} \bigcup_{n \in N_m} \bigvee_{l \in L_{m,n}} \varphi'_{m,n,l}$.

Next, for every $f$-monomial $\varphi'_{i,j,k} = \bigcap_{p \in P_{i,j,k}} p \bigcap \bigcup_{p' \in P'_{i,j,k}} \neg p$ in $\zeta'_1$, we let the set $S_{i,j,k} = \bigcup_{p \in P_{i,j,k}} \{p\} \bigcup \bigcup_{p' \in P'_{i,j,k}} \{\neg p\}$. If $\varphi'_{i,j,k} = \text{true}$ then $S_{i,j,k} = \{\text{true}\}$.

Then the following set

\[ S'_{\zeta'_1} = \bigcup_{i \in I} \bigcup_{j \in J_i} \bigcup_{k \in K_{i,j}} \{S_{i,j,k}\} \]

represents $\zeta'_1$ in the form of sets. If $\zeta'_1 = \text{true}$, then $S'_{\zeta'_1} = \{\{\{\text{true}\}\}\}$ since $I = \{1\}$, $J_1 = \{1\}$, $K_{1,1} = \{1\}$ and $S_{1,1,1} = \{\text{true}\}$. Analogously, if $\zeta'_1 = \text{false}$, then $S'_{\zeta'_1} = \{\{\{\text{false}\}\}\}$. Next, we compute the set $S'_{\zeta'_2}$ which represents $\zeta'_2$ in the form of sets. We need to note that the representation of a $fPCL$ formula $\zeta = \bigoplus_{i \in I} \bigcup_{j \in J_i} \bigvee_{k \in K_{i,j}} \varphi_{i,j,k}$, which is in normal form, in the form of sets is possible since

1. $\bigcup_{j \in J_i} \bigvee_{k \in K_{i,j}} \varphi_{i,j,k} \neq \bigcup_{j \in J_i} \bigvee_{k \in K_{i,j'}} \varphi'_{i,j,k}$ for every $i, i' \in I$ with $i \neq i'$,

2. $\bigvee_{k \in K_{i,j}} \varphi_{i,j,k} \neq \bigvee_{k \in K_{i,j'}} \varphi'_{i,j,k}$ for every $j, j' \in J_i$ with $j \neq j'$, and

3. $\varphi_{i,j,k} \neq \varphi_{i,j,k'}$ for every $k, k' \in K$ with $k \neq k'$.

In order to prove if $\zeta'_1$ and $\zeta'_2$ are equivalent, we need to examine if the sets $S'_{\zeta'_1}$ and $S'_{\zeta'_2}$ are equal. For this we give Algorithm 1 given in A. Given the sets $S'_{\zeta'_1}$ and $S'_{\zeta'_2}$ as inputs for Algorithm 1, we can decide whether $S'_{\zeta'_1} = S'_{\zeta'_2}$ or not.
As for the time complexity of the algorithm, we prove that is polynomial. The construction of the sets $S_{\zeta_1}$ and $S_{\zeta_2}$ is done in polynomial time. Let that the variable of the $i$-th for loop, where $i \in \{1, \ldots, 8\}$, ranges from 1 to $n_i \in \mathbb{N}^*$. So, the number of computations in total are $n_1 \cdot \ldots \cdot n_8$. Let $n = \max\{n_1, \ldots, n_8\}$. Then $n_1 \cdot \ldots \cdot n_8 \leq n^8$ and the complexity is $O(n_1 \cdot \ldots \cdot n_8) = O(n^8)$. Hence, considering the complexity of the construction of the normal forms of $\zeta_1$ and $\zeta_2$, we conclude that the run time of the equivalence problem is polynomial. ■

**Remark 37** Let $\zeta_1, \zeta_2 \in fPCL(K,P)$. By Theorem 36 we can decide whether $\zeta_1 \equiv \zeta_2$ or not. If $\zeta_1 \equiv \zeta_2$ then $\zeta_1 \equiv_{K_{con}} \zeta_2$ for every $K_{con}$ De Morgan algebra. However, as shown in Example 34, it is possible that $\zeta_1 \equiv_{K_3} \zeta_2$ but $\zeta_1 \not\equiv \zeta_2$. Hence, if $\zeta_1 \not\equiv \zeta_2$, then we can examine whether $\zeta_1 \equiv_{K_3} \zeta_2$ and/or $\zeta_1 \equiv_{B} \zeta_2$ following the constructions in Theorems 35 and 36.

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A Algorithm for Decidability of Equivalence

Let \( \zeta_1 \) and \( \zeta_2 \) be fPCL formulas over \( P \) and \( K \). By Theorem 35 we can effectively construct their equivalent fPCL formulas \( \zeta'_1 \) and \( \zeta'_2 \), respectively, in normal form. By Theorem 36, if \( S_{\zeta'_1} = S_{\zeta'_2} \), then \( \zeta'_1 \equiv \zeta'_2 \). In order to show whether \( S_{\zeta'_1} \) and \( S_{\zeta'_2} \) are equal or not, we give Algorithm 1 where as input we have the sets \( S_{\zeta'_1} \) and \( S_{\zeta'_2} \).

Let \( P = \{ p, q, r \} \) be a set of ports. For better understanding, the reader can apply the algorithm for the sets

- \( S_{\zeta'_1} = \{\{\{p, q\}, \{r\}\}, \{\{p\}, \{\{true\}\}\}\} \)
- \( S_{\zeta'_2} = \{\{\{p, q\}, \{\{true\}\}\}\} \)

which represent the fPCL formulas

- \( \zeta'_1 = (((p \otimes q) \oplus (p \otimes !r)) \oplus ((!p \otimes !r) \odot (p \otimes r)) \oplus p \oplus true) \)
- \( \zeta'_2 = (((p \otimes q) \oplus !r) \oplus q) \oplus (!q \oplus p \oplus r) \),

respectively.
Algorithm 1 Main
Input : $S_{\zeta_1}, S_{\zeta_2}$

if $\text{card}(S_{\zeta_1}) = \text{card}(S_{\zeta_2})$ then
  \( k \leftarrow 0 \)
  for \( i \) in range \((1, \text{card}(S_{\zeta_1}))\) do
    for \( j \) in range \((1, \text{card}(S_{\zeta_2}))\) do
      if SetEq1($S_{\zeta_1}[i], S_{\zeta_2}[j]$) = true
        then
          \( k \leftarrow k + 1 \)
      end if
    end for
  end for
if \( k = \text{card}(S_{\zeta_1}) \) then
  “Equivalent”
else
  “Not equivalent”
end if
else
  “Not equivalent”
end if

Algorithm 2 SetEq1
Input : A, B
Output: E

if $\text{card}(A) = \text{card}(B)$ then
  \( k \leftarrow 0 \)
  for \( i \) in range \((1, \text{card}(A))\) do
    for \( j \) in range \((1, \text{card}(B))\) do
      if SetEq3(A[i], B[j]) = true
        then
          \( k \leftarrow k + 1 \)
      end if
    end for
  end for
if \( k = \text{card}(A) \) then
  E ← true
else
  E ← false
end if
end if
return E

Algorithm 3 SetEq2
Input : A, B
Output: E

if $\text{card}(A) = \text{card}(B)$ then
  \( k \leftarrow 0 \)
  for \( i \) in range \((1, \text{card}(A))\) do
    for \( j \) in range \((1, \text{card}(B))\) do
      if SetEq3(A[i], B[j]) = true
        then
          \( k \leftarrow k + 1 \)
      end if
    end for
  end for
if \( k = \text{card}(A) \) then
  E ← true
else
  E ← false
end if
end if
return E

Algorithm 4 SetEq3
Input : A, B
Output: E

if $\text{card}(A) = \text{card}(B)$ then
  \( k \leftarrow 0 \)
  for \( i \) in range \((1, \text{card}(A))\) do
    for \( j \) in range \((1, \text{card}(B))\) do
      if SetEq3(A[i], B[j]) = true
        then
          \( k \leftarrow k + 1 \)
      end if
    end for
  end for
if \( k = \text{card}(A) \) then
  E ← true
else
  E ← false
end if
end if
return E