COSMOLOGICAL TIME DILATION IN GAMMA RAY BURSTS?

David L. Band
CASS, University of California, San Diego, CA 92093
Internet: dlbbat@cass09.ucsd.edu

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ABSTRACT

Norris et al. (1994) report that the temporal structure of faint gamma ray bursts is longer than that of bright bursts, as expected for time dilation in the cosmological models of burst origin. I show that the observed trends can easily be produced by a burst luminosity function and thus may not result from cosmological effects. A cosmological signature may be present, but the tests Norris et al. present are not powerful enough to detect these signatures.

Subject headings: gamma rays: bursts

1. Introduction

Cosmological models for the origin of gamma ray bursts are based solely on the intensity and spatial distributions of the BATSE bursts (e.g., Meegan et al. 1992), and not on any intrinsic burst features. Therefore, Norris and his collaborators (Norris et al. 1993a,b, 1994; Norris 1994; Davis et al. 1994) are searching for evidence of cosmological time dilation in burst time histories. If bursts have a cosmological origin then the most distant (and therefore faintest) bursts BATSE detects originate at $z \sim 1$ (e.g., Wickramasinghe et al. 1993) and their time structure should be stretched by a factor of $\sim 2$ compared to nearby bursts. Norris and collaborators present a series of tests which probe burst temporal structures on different time scales, and report a consistent dilation of $\sim 2$ between the faint and intense bursts. Given the importance of this result, it is crucial that the tests be evaluated; here I focus on the tests presented by Norris et al. (1994; hereafter N94).

N94 assume the peak photon emission rate (i.e., the maximum photon luminosity) is constant, and therefore the peak flux is an exact distance measure (R. Nemiroff 1994,
personal communication). This assumption is overly simplistic. Bursts display a broad range of spectral shapes (Band et al. 1993), and the fraction of the total emission observed within a given spectral band varies. One can argue on physical grounds that the total energy release (e.g., a neutron star binding energy) or peak energy luminosity (e.g., an Eddington luminosity, or a sharply peaked luminosity function such as proposed by Mao & Yi 1994) should be constant; however, with the variety of burst spectra and durations observed the peak photon luminosity in a particular energy band will most likely not be constant. The peak count rate has become the paramount measure of burst intensity not because of burst physics, but because detectors trigger on it (e.g., Paczyński & Long 1988). I will show that when the “standard candle” assumption is relaxed, the burst correlations which N94 state are consistent with a cosmological origin can easily be products of the burst luminosity function.

N94’s search for a cosmological signature entails extensive transformations of the burst time histories. To give all the bursts in their sample the same signal-to-noise ratio and apparent distance, N94 normalize the bursts so that they have the same peak flux, by necessity that of the weakest burst in their sample, and add the appropriate Poisson noise. N94 justify the degradation of the high quality data from bright bursts as necessary to remove biases resulting from differing signal-to-noise ratios. In the various tests they present, N94 use only BATSE bursts with durations greater than 1.5s. They consider averages for three burst groups characterized by peak flux: bright, dim and dimmest. Each sample has approximately 40 bursts. For each burst 65.536s of data was used; the basic data had a time resolution of 64ms, although for some purposes the resolution was reduced.

N94 present three tests which are intended to probe different time scales. I analyze each test in turn (§2-4), and then end with some conclusions (§5).

2. Test 1—Normalized Count Fluences

N94 calculate the average number of counts above background in the normalized time histories. This normalized count fluence is effectively the observed photon fluence $S_N$ divided by peak photon flux $P$, a quantity with units of time which can be interpreted as a duration. Assuming a cosmological burst origin, we expect $S_N/P \propto 1 + z$, and indeed the average normalized photon fluence is about twice as large for the two dim burst samples as for the bright burst group. While this test is physically sensible, operationally it compares a function of the peak flux ($S_N/P$) to the peak flux itself, a procedure which can give a spurious result. For example, if $S_N$ and $P$ are uncorrelated, then clearly $S_N/P$ will be
inversely correlated with \( P \). While \( S_N \) and \( P \) should be correlated because they both decrease with source distance, there are more subtle non-cosmological effects which can produce this apparent signature.

The cosmological effects result from time dilation and spectral redshifting which are continuous functions of the redshift, affecting even the bright bursts. If bursts have a power law spectrum \( N(E) \propto E^\alpha \) over the energy band of interest, then the redshift dependence is \( P \propto (1+z)^\alpha/d_m^2 \) and \( S_N \propto (1+z)^{1+\alpha}/d_m^2 \), where \( d_m \) is the proper motion distance. Thus \( S_N/P \propto 1+z \), as expected. Assuming \( q = 1/2 \) (i.e., an \( \Omega = 1 \) cosmology) and \( \alpha = -1 \) then

\[
P = \left[ \frac{\sqrt{1+z_m} - 1}{\sqrt{1+z} - 1} \right]^2, \quad \text{where } P \text{ is scaled by its value at } z_m \sim 1.
\]

Figure 1 shows \( \langle S_N/P \rangle \) as a function of \( P \) for both cosmological bursts with constant peak photon luminosity \( L \) (related to \( P \)) and for sources distributed uniformly in Euclidean space out to a maximum radius. For the Euclidean case I use \( n(L) \propto L \) for \( L \leq L_0 \) and \( n(L) \propto L^{-3} \) for \( L \geq L_0 \) as the luminosity function, chosen so that the number of bursts converges. Note that 85% of the bursts fall within a luminosity range of 6.5, consistent with the constraints of Horack et al. (1994). As long as the total number of photons \( N \) emitted (related to \( S_N \)) is uncorrelated with \( L \), its distribution is irrelevant to relative values of \( \langle S_N/P \rangle \). The maximum \( z_m = 1.25 \) used by N94 and a burst spectral index \( \alpha = -1 \) are assumed. For the Euclidean model I normalize \( \langle S_N/P \rangle \) to 1 at \( P = 100 \). In the cosmological model the bright bursts show some time dilation and the difference in dilation between the dim and dimmest bursts is significant. In the Euclidean model the bursts at any given value of \( P \) originate from a range of distances (unless \( L \) is a standard candle, in which case a single distance is singled out), with the more distant bursts providing, on average, smaller values of \( S_N \). However, if the source density decreases beyond some distance, there are fewer low \( S_N \) bursts as \( P \) decreases, raising the average \( S_N \). Thus a Euclidean model can have correlations similar to cosmological time dilation.

N94 assume that the photon luminosity \( L \) is a standard candle, while in the example in Figure 1 I assume that \( L \) is not constant and that the photon emission \( N \) is uncorrelated with \( L \). These two cases are drawn from a continuum of possible distributions of \( L \) and \( N \). Both the joint distribution of \( L \) and \( N \) (as also shown by Brainerd 1994) and the geometry of the source distribution affect the observed correlation of \( \langle S_N/P \rangle \) with \( P \). In a Euclidean source geometry the dependence of \( \langle S_N/P \rangle \) on \( P \) is determined by the relation between \( L \) and \( \langle N \rangle_L \) (\( N \) averaged at constant \( L \)): if flatter than \( \langle N \rangle_L \propto L \) then \( \langle S_N/P \rangle \) increases at small \( P \). The extent of the change of \( \langle S_N/P \rangle \) at small \( P \) is a function of the dynamic range of \( L \) and the abruptness of the source distribution’s spatial cutoff. Consequently the distributions of \( N \) and \( P \) in Euclidean space can very easily mimic a cosmological signature, and thus this test is inconclusive. The second and third tests in Davis et al. (1994) are
essentially the same as the first test in N94.
3. Test 2—Wavelets

Wavelets find a signal’s frequency content as a function of time. N94 use the Haar transform which, while not optimal in temporally resolving the frequency content (Daubechies 1992, pp. 10-13), can nonetheless be calculated easily. N94 calculate Haar transforms on time scales ranging from 512ms to 65.536s, a total of 8 time scales. They then average the absolute value of the transform amplitudes on each time scale, producing an “activity” at discrete time scales which is akin to the power spectrum for the traditional Fourier transform. The activities for all the bursts of a brightness sample are then averaged. For time scales less than \( \sim 2s \) the average activity is flat at the same value for each burst sample, as expected for Poisson noise (Poisson noise is “white”). On longer time scales the average activity curve increases with time scale. However, the curves get progressively flatter as the sample gets brighter; thus the dimmest sample lies above, and the bright sample below, the dim sample. N94 compare simulations of dilated and undilated burst distributions and find that the undilated average activity curve lies below the dilated curve; once again the observations appear to be consistent with cosmological bursts. However, N94 do not develop a detailed quantitative or qualitative understanding of how cosmological time dilation produces the activity curves, e.g., in terms of the widths of the spikes within the bursts, the separation between spikes, the number of counts in a burst, etc. In addition, the simulations do not calibrate the magnitude of the cosmological effect.

Investigating the Haar transform analytically provides greater insight into the activity curves. The Haar wavelet transform of the function \( f(x) \), using a time scale hierarchy separated by factors of 2, is

\[
T_{m,n} = \int dx f(x)2^{-m/2}\Psi\left(2^{-m}x - n\right), \quad m \geq 0, \quad n \geq 0
\]  

(1)

where \( \Psi(y) = 1 \) for \( 0 \leq y \leq 1/2 \), -1 for \( 1/2 < y \leq 1 \), and 0 otherwise. The time variable \( x \) is in units of the shortest time scale, and thus data points are every half unit. Finally, \( m \) indicates the time scale and \( n \) the time interval. N94 define the activity as the average transform for a given time scale, or

\[
A_m = \langle T_m \rangle = \frac{1}{J} \sum_{n=0}^{J-1} |T_{m,n}|
\]  

(2)

where there are \( J \) time intervals characterized by the time scale indexed by \( m \). The activity is defined at only a small number of discrete time scales; line segments are drawn in to guide the eye. If \( f(x) = a \) for half the basic time scale (i.e., a single data point), and zero otherwise (i.e., effectively a \( \delta \)-function for the time scales under consideration), the activity
for a data window whose length is $2^N$ of the shortest time scales (with $2^{N+1}$ data points) is

$$A_m = 2^{1/2-N+m/2}a \quad , \quad m \geq 0. \quad (3)$$

The dependence on $N$ results from averaging the single nonzero element over the entire data window. Thus on successively longer time scales (each twice the length of the preceding time scale) the activity increases by a factor of $2^{1/2}$ (as seen from the $m$ dependence in eqn. [3]): the activity curve (joining the small number of points where the activity is actually calculated) will be a power law with index $1/2$. Eqn. (3) can be generalized to signals greater than half the basic time scale; signals with a duration less than half a given time scale will appear to be constant on half that time scale in calculating the activity for longer time scales. The activity of a $\delta$-function is not flat because the activity was defined as the average of $|T_{m,n}|$ and not of $T_{m,n}^2$. On the other hand, white noise has a flat activity curve. Each amplitude $T_{m,n}$ for the transform of noise has approximately the same magnitude, so the averages of its absolute value and its square are both constant.

Therefore the burst activity curve has three regions: a flat, noise-dominated region at short time scales; intermediate times scales in which burst structures are evident; and time scales longer than twice the burst duration where the activity curve is a power law with index $1/2$ and a normalization proportional to the total number of counts in the burst.

Figure 2 shows the activity of GB910717 observed by BATSE (trigger #543—see Fishman et al. 1994 for the time history). The signal-to-noise ratio is sufficiently large such that noise may dominate only the shortest time scales (i.e., only for $m = 0$ and 1 does the activity curve appear to be flat). The burst duration is less than $7s$, and thus on time scales of $16.384s$ (more than twice the duration) and greater the activity is a power law with index $1/2$ (the linear segment for $m \geq 7$).

The activity for a burst which has undergone cosmological time dilation keeping the peak flux constant (N94 normalize all time histories to the same peak flux) follows from the relation $F(x) = f(x/(1+z))$ between $f(x)$ and $F(x)$, the time histories in the rest and observer’s frames, respectively:

$$A_{m,\text{obs}} = (1+z)^{3/2}A_{[m-\ln_2(1+z)]\text{rest}} \quad (4)$$

where $\ln_2$ is the base 2 logarithm (i.e., $y = \ln_2(2^y)$). Since structures are shifted to longer time scales, activity on one time scale in the rest frame is translated to activity on a longer time scale in the observer’s frame. On this longer time scale the activity, which averages over the burst duration and post-burst background, is diluted by less background, an effect which contributes a factor of $1+z$. Finally, the transforms have a different normalization on the new time scale, which provides the last factor of $(1+z)^{1/2}$. Thus we expect the time dilated activity curve to be shifted to longer time scales and up relative to the undilated
curve. Figure 2 also shows the activity for a time dilated burst at \( z = 1 \) where the photon rate has been kept constant. A complication arises when the dilated burst is longer than the data window since then part of the dilated time history is not included in the calculation of the activity, and the dilution of burst activity by post-burst background is reduced. Because of the reduced dilution, the leading factor in eqn. (4) ranges from \( (1 + z)^{1/2} \) up to the maximum of \( (1 + z)^{3/2} \). In addition, the character of the temporal structure may vary across a burst. Barring such systematic trends, we expect the dilated activity curve to have the same shape as the undilated curve, albeit shifted to longer time scales; the relative normalization depends on the data window.

Thus time dilation should shift the average activity curve to longer time scales. The Haar transform used here is calculated for time scales separated by factors of 2, and therefore the dilation for bursts at \( z = 1 \) shifts the activity by one time scale. The bursts in the simulations of N94 are at \( z = 1.25 \), and therefore the dilated and undilated activities cannot be related cleanly. The simulations in Norris (1994) use a redshift of \( z = 1 \), and we find that the predicted relationship between activity curves holds approximately at the large time scales: the dilated activity is shifted to the next longest time and up by a factor of \( \sim 2.1 \). Since the dilated and undilated samples use different simulated bursts the exact translation predicted by eqn. (4) is not expected. However, the simulated dilated and undilated curves clearly diverge, whereas the observed activity curves in N94 are not so cleanly separated. In particular, on the 32.768s time scale the activity curves for the dim and bright samples intersect. Therefore the consistency of the observed activity curves with the simulated curves in N94 is not overwhelmingly convincing.

The first test shows that the average count fluence in the normalized time histories is twice as large for the dim and dimmest burst samples as for the bright sample. Therefore we expect that on the longest time scales, which will usually be longer than the burst duration, the activities of the dim and dimmer samples will be larger than for the bright sample: in eqn. (3), which is applicable to time scales longer than twice the burst duration, the activity is proportional to number of counts. Thus the separation between dim and bright bursts may be a natural consequence of the difference in the normalized photon fluence and not of dilation. Thus this test may be no more than the first test in a different form.

4. **Test 3—Mitrofanov Peak Alignment**

Pioneered by Mitrofanov et al. (1993), this test aligns the highest peaks of each normalized burst, and considers the peak width of the average of the aligned time histories.
The average peak should be dilated for the more distant bursts if they are at cosmological distances, and indeed N94 find that the peaks get successively broader as the bright, dim and dimmest samples are compared. Simulations of cosmological time dilation produce broadening similar to that observed in the data, indicating the observations are consistent with \( z \sim 1 \) for BATSE’s most distant bursts.

Note that Mitrofanov et al. (1994) perform the same test on bursts selected from the first BATSE catalog (Fishman et al. 1994) with durations \( T_{90} \) greater than 1s, and do not find a difference between the dim and bright events. However, Mitrofanov et al. divide the bursts into broader intensity groups and include shorter bursts than N94, making time dilation, if present, more difficult to detect. The authors of both papers have additional unpublished results (personal communication from both J. Norris and I. Mitrofanov, 1994), and we must await future resolution of this observational issue.

I am concerned that in implementing this test N94 may have inadvertently broadened some burst structures by filtering their time histories to make the peaks more evident. The time histories’ wavelet transforms for each time scale are reduced by the amplitude expected for noise, and then the time histories are reconstituted by taking the inverse transform. Consequently both noise and signal are filtered out on the shortest time scales. The wavelet activities used in the second test indicate that Poisson noise dominates for time scales shorter than \( \sim 2s \), of order of the peak widths in this peak alignment test. The same filtering was used for the normalized time histories (with added Poisson noise) of all three flux groups, and thus all three should be affected similarly. However, N94 do not provide the extent of the filtering on each time scale (i.e., the reduction in the wavelet transform for each sample). Note that this filtering conserves the number of counts in the time history since the wavelet basis functions have zero mean.

The peaks of the dim bursts may be broader than the bright bursts because the faint bursts have more normalized counts, the phenomenon found by the first test. Since the peaks of the normalized time histories have the same value, the dim bursts have more counts by having more emission surrounding the peak; the averaged time history will most likely have a broader peak. I showed in §2 that low peak flux bursts can have more normalized counts if the intrinsic peak photon luminosity function is uncorrelated with the total photon emission; a distribution of peak FWHMs at constant photon emission will produce such a luminosity function. Once again, this test’s results may be a consequence of the effect found by the first test.

5. Conclusions
The results of the second and third tests in N94 are consequences of the first test: if the normalized photon fluence of the dim bursts is greater than that of the bright bursts, the wavelet “activity” on long time scales (which is proportional to the number of counts) will also be greater, and the averaged normalized time profiles should be broader (to get more counts under the time history curve with the same peak flux). The effect observed in the first test—dim bursts have more counts when normalized by the peak flux—can very easily be a spurious correlation if there is a distribution of peak photon luminosities. Consequently N94 must demonstrate that the peak photon luminosity is a standard candle for their tests to be convincing.

It should be noted that N94 do not compare the observations to quantitative predictions of the cosmological model. Only local bursts and bursts at the maximum redshift, $z_m = 1.25$, are simulated. Yet the redshift varies across each of the peak flux bins, reducing the observed cosmological signature. Also, it is not clear how sensitive the observations are to the parameters of the cosmological model, e.g., to the value of $z_m$.

Given the importance of determining the burst distance scale, we need tests of the cosmological hypothesis. Unfortunately, many of these tests are easily confounded by very plausible relations among burst properties or by broad intrinsic distributions. Thus comparing the observed burst durations and the peak flux $P$ is more robust than using $S_N/P$ and $P$. However, the duration and the peak photon luminosity $L$ could be inversely related, if for example the total photon emission $N$ is constant. Also, the cosmological signature is diluted by a broad peak photon luminosity function. In addition, an accurate determination of the duration is more difficult for fainter bursts; detailed study of the measured duration as a function of burst intensity is required for this test (as well as for any systematic use of the duration as a burst property). Similarly, searches for the redshifting of burst spectra (Paczyński 1992; Mao & Paczyński 1992; Dermer 1992; Brainerd 1993) are complicated by the diversity of burst spectra (Band et al. 1993) and the unknown photon luminosity function. Nonetheless, additional tests of the cosmological hypothesis should be sought; sophisticated techniques such as those discussed by Loredo & Wasserman (1994) are designed to compare different model distributions to the observations.

As N94 point out, observing the expected cosmological time dilation does not prove that the burst sources are at cosmological distances since burst physics may conspire to produce the same properties. Conversely, showing that the apparent temporal broadening in faint bursts can easily be a product of burst property distributions does not prove that the effect is not time dilation. Observing a theory’s predicted consequence helps validate that theory. However, the argument in a theory’s favor is less convincing if the predicted effect can have other plausible causes. In this study I showed that the tests presented by
N94 could be misleading, and therefore these tests should not increase our confidence in the validity of the cosmological hypothesis.
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Figures

Figure 1. Model relationships between normalized counts \( (S_N/P) \) and peak photon flux \( P \). The peak flux is normalized by \( P_t \), the faintest flux BATSE can detect. The solid curve is for a cosmological model where BATSE detects bursts to \( z_m = 1.25 \), and the peak photon luminosity is constant. The dashed curve is \( \langle S_N/P \rangle \) for a homogeneous source population (in Euclidean space) out to a finite radius \( R_M \); the bursts in this scenario have a broken power law luminosity function \( \phi(L) \propto L \) for \( L < L_0 \) and \( \phi(L) \propto L^{-3} \) for \( L \geq L_0 \) where \( L_0 = 0.75 \times 4\pi R_M^2 P_t \). For the Euclidean case \( \langle S_N/P \rangle \) is normalized to 1 at \( P = 100 \), while for the cosmological case \( \langle S_N/P \rangle \) is normalized by its asymptotic large \( P \) value. The vertical lines indicate the sample bins used by N94: dimmest \( (P = 1 \) to \( 1.71 \)); dim \( (P = 1.71 \) to \( 3.21 \)); and bright \( (P = 12.9 \) to \( 178 \)).

Figure 2. Wavelet activity for burst GB910717 (solid curve), assumed to be local, and for the same burst dilated to \( z = 1 \) maintaining the same count rate (dashed curve). The activity is the wavelet analog to a Fourier power spectrum; it is defined at a small number of discrete time scales. Note that the dashed curve is the same as the solid curve shifted to one larger time scale and up by \( 2^{3/2} \).