Longitudinal, transverse-plus and transverse-minus \(W\)-bosons in unpolarized top quark decays at \(O(\alpha_s)\)

M. Fischer, S. Groote, J.G. Körner and M.C. Mauser
Institut für Physik der Johannes-Gutenberg-Universität, Staudinger Weg 7, D–55099 Mainz, Germany

We consider the \(O(\alpha_s)\) radiative corrections to the decay of an unpolarized top quark into a bottom quark and a \(W\)-gauge boson where the helicities of the \(W\) are specified as longitudinal, transverse-plus and transverse-minus. The \(O(\alpha_s)\) radiative corrections lower the normalized longitudinal rate \(\Gamma_L/\Gamma\) by 1.06\% and increase the normalized transverse-minus rate \(\Gamma_{-}/\Gamma\) by 2.17\%. We find that the normalized transverse-plus rate \(\Gamma_{+}/\Gamma\), which vanishes at the Born term level for \(m_b \to 0\), receives radiative correction contributions at the sub-percent level. We discuss \(m_b \neq 0\) effects for the Born term and the \(\alpha_s\)-contributions but find these to be small. Our results are discussed in the light of recent measurements of the helicity content of the \(W\) in top quark decays by the CDF Collaboration.

I. INTRODUCTION

The CDF Collaboration has recently published the results of a first measurement of the helicity content of the \(W\) gauge boson in top quark decays [1]. Their results are

\[
\begin{align*}
\Gamma_L/\Gamma &= 0.91 \pm 0.37 \text{(stat)} \pm 0.13 \text{(syst)} \quad (1) \\
\Gamma_{+}/\Gamma &= 0.11 \pm 0.15 \quad (2)
\end{align*}
\]

where \(\Gamma_L\) and \(\Gamma_{+}\) denote the rates into the longitudinal and transverse-plus polarization state of the \(W\)-boson and \(\Gamma\) is the total rate.

The errors on this measurement are still rather large but will be much reduced when larger data samples become available in the future from TEVATRON RUN II, and, at a later stage, from the LHC. Optimistically the measurement errors can eventually be reduced to the \((1-2)%\) level [2]. If such a level of accuracy can in fact be reached it is important to discuss the radiative corrections to the different helicity rates [3], [4] considering the fact that the \(O(\alpha_s)\) radiative corrections to the total width \(\Gamma\) are rather large (\(\approx -8.5\%\)) [3], [4].

The transverse-plus rate \(\Gamma_{+}\) is particularly interesting in this regard. Simple helicity considerations show that \(\Gamma_{+}\) vanishes at the Born term level in the \(m_b = 0\) limit. A nonvanishing transverse-plus rate could arise from i) \(m_b \neq 0\) effects, ii) \(O(\alpha_s)\) radiative corrections due to gluon emission, or from iii) non-SM \(t \to b\) currents. As we shall show the \(O(\alpha_s)\) and the \(m_b \neq 0\) corrections to the transverse-plus rate occur only at the sub-percent level. It is safe to say that, if top quark decays reveal a violation of the Standard Model (SM) \((V-A)\) current structure that exceeds the 1\% level, the violations must have a non-SM origin. In this context we mention that a possible \((V+A)\) admixture to the \(t \to b\) current is already severely bounded indirectly by existing data on \(b \to s + \gamma\) decays [5], [6].

The results of the radiative correction calculation have already been published before by some of us [7]. However, in Ref. [8] the emphasis was on polarized top decay. Besides, in Ref. [8] the results on the transverse components of the \(W\) were given for the “unpolarized-transverse” and the “forward-backward” components which differ from those used in the CDF analysis. We thought it would be useful to collect together in one place all formulae relevant for an understanding of the new CDF measurement. This includes also a discussion of \(m_b \neq 0\) effects for the Born term and for the \(\alpha_s\) radiative corrections, which is new.

II. ANGULAR DECAY DISTRIBUTION

Let us begin by writing down the angular decay distribution for the decay process \(t \to X_b + W^+\) followed by \(W^+ \to l^+ + \nu_l\) (or by \(W^+\) \(\to q + q\)). For unpolarized top decay the angular decay distribution is determined by two transverse components (transverse-plus and transverse-minus) and the longitudinal component of the \(W\)-boson. One has [8]

\[
\frac{d\Gamma}{d\cos\theta} = \frac{3}{8}(1 + \cos\theta)^2 \Gamma_{+} + \frac{3}{8}(1 - \cos\theta)^2 \Gamma_{-} + \frac{3}{4}\sin^2\theta \Gamma_L.
\]

Integrating over \(\cos\theta\) one recovers the total rate

\[
\Gamma = \Gamma_{+} + \Gamma_{-} + \Gamma_L.
\]

We describe the angular decay distribution in cascade fashion, i.e. the polar angle \(\theta\) is measured in the \(W\) rest frame where the lepton pair or the quark pair emerges back-to-back. The angle \(\theta\) denotes the polar angle between the \(W^+\) momentum direction and the antilepton

![FIG. 1. Definition of the polar angle \(\theta\)](image-url)
term rate. One has

\[ \Gamma_+ = \frac{G_F m_b^2 m_t}{8 \sqrt{2} \pi} |V_{tb}|^2 \sqrt{\lambda} \times \]

\[ \frac{(1 - y^2)^2 + x^2(1 - 2x^2 + y^2)}{x^2}. \]

The partial helicity rates are given in terms of the Born term rate. One has

\[ \Gamma_+/\Gamma_0 = \frac{(1 - y^2)^2 - x^2(1 + y^2)}{(1 - y^2)^2 + x^2(1 - 2x^2 + y^2)} \]

\[ = \frac{1}{1 + 2x^2} + \ldots \]

\[ \Gamma_-/\Gamma_0 = \frac{x^2(1 - x^2 + y^2 + \sqrt{\lambda})}{(1 - y^2)^2 + x^2(1 - 2x^2 + y^2)} \]

\[ = \frac{2x^2}{1 + 2x^2} + \ldots \]

In Eqs. (1) we have also listed the leading components of the small \( y^2 \) expansion of the Born term rate ratios. As the second equation of (1) shows and as already remarked on before, the transverse-plus Born term contribution vanishes in the \( m_b = 0 \) limit. Note that the leading contribution to the transverse-plus rate is proportional to \( (m_b/m_t)^2 \) and not proportional to \( (m_b/m_W)^2 \) as stated in Ref. [1].

The \( m_b \neq 0 \) effects are quite small. Using \( m_t = 175 \text{ GeV}, m_W = 80.419 \text{ GeV} \), and a pole mass of \( m_b = 4.8 \text{ GeV} \), one finds that \( \Gamma_0, \Gamma_L/\Gamma_0, \) and \( \Gamma_-/\Gamma_0 \) decrease by 0.27\%, 0.091\%, and 0.095\%, resp. when going from \( m_b = 0 \) to \( m_b = 4.8 \text{ GeV} \). The leakage into the transverse-plus rate ratio \( \Gamma_+/\Gamma_0 \) from bottom mass effects is a mere 0.036\%.

IV. \( O(\alpha_s) \) RADIATIVE CORRECTIONS

The \( O(\alpha_s) \) corrections are determined by the one-loop vertex correction shown in Fig. 2a and the gluon emission graphs shown in Figs. 2b, 2c, and 2d. The one-loop results have already been known for quite some time [13,14] and will not be discussed any further.

We do want to make a few technical remarks about how the tree-graph integration was done. We use a gluon mass to regularize the IR singularity. Concerning the collinear singularity we have kept the full bottom mass dependence in our calculation and have only set the bottom mass to zero at the very end. We have thus effectively used a mass regulator to regularize the collinear singularity.

The tree-graph integration has to be done over two-dimensional phase space. As phase space variables we use the gluon energy \( k_0 \) and the W energy \( q_0 \). The IR behaviour of the hadronic tree-graph matrix element \( W_{\mu\nu}(q_0, k_0) \) was improved by subtracting from it the soft-gluon contribution \( G_{\mu\nu}(q_0, k_0) \) which was then added again according to the prescription

\[ W_{\mu\nu}(q_0, k_0) = (W_{\mu\nu}(q_0, k_0) - G_{\mu\nu}(q_0, k_0)) + G_{\mu\nu}(q_0, k_0) \]

(7)

The first piece \( (W_{\mu\nu}(q_0, k_0) - G_{\mu\nu}(q_0, k_0)) \) has thereby been rendered IR finite and can be integrated without a gluon mass regulator which considerably simplifies the phase space integration. The IR singularity resides in the soft gluon piece \( G_{\mu\nu}(q_0, k_0) \) which is, however, simple and universal and can be easily integrated. In fact the soft gluon contribution factorizes into the Born term contribution \( B_{\mu\nu} \) and a universal soft gluon factor \( S(q_0, k_0) \) according to

\[ G_{\mu\nu}(q_0, k_0) = B_{\mu\nu} \cdot S(q_0, k_0) \cdot \alpha_s. \]

(8)
The Born term contribution $B_{\mu\nu}$ is given by
\[ B^{\mu\nu} = 8(p_t^a p_t^b + p_t^b p_b^a - g^{\mu\nu} p_t \cdot p_b + i\epsilon^{\mu\nu\alpha\beta} p_{b,\alpha} p_{t,\beta}), \] (9) while the soft-gluon factor in FIG has the standard form
\[ S(q_0, k_0) = \frac{m_t^2}{(p_t k)^2} - \frac{2p_t p_b}{(p_t k)(p_b k)} + \frac{m_b^2}{(p_b k)^2}. \] (10)

Note that the tensor structure carrying the spin information of the produced W-boson has been factored out and is now entirely contained in the Born term factor $B_{\mu\nu}$. Since the Born term factor does not depend on the phase space variables, the phase space integration needs to be done only with respect to the soft gluon state of the W-boson, and thus needs to be done only once irrespective of the polarisation of the W-boson. Needless to say that this is a very welcome simplifying feature of the above subtraction procedure.

This is different for the phase space integration of the IR-finite piece $(W_{\mu\nu}(q_0, k_0) - G_{\mu\nu}(q_0, k_0))$ where the integration has to be done separately for each polarization state of the W-boson. To do the necessary two-dimensional phase space integrations in analytical form is somewhat involved. In particular the integrations are more difficult than those needed for the total rate calculation which has already been done some time ago [5–9]. More difficult than those needed for the total rate calculation since new classes of phase space integrals appear.

For our final results are presented for the $m_b = 0$ limit where the rate expressions reduce to a rather compact form. We add together the Born term, the one-loop and the tree-graph contribution. The results are taken from Ref. [3]. We divide out the total rate $\Gamma$ and denote the scaled rates $\hat{\Gamma}_i = \Gamma_i/\Gamma_0$ ($i = U$, $L$, $L,$ $+$, $-$) by a hat symbol. The two transverse helicity rates $\dot{i} = +, -$ are obtained from the $i = U, F$ rates given in FIG by taking the linear combinations $\Gamma_\pm = \frac{1}{2}(\Gamma_U \pm \Gamma_F)$ as discussed before. One has ($C_F = 4/3$)

\[
\hat{\Gamma} = 1 + \frac{\alpha_s}{2\pi} C_F \frac{x^2}{(1 - x)^2(1 + 2x^2)} \times \left\{ (1 - x)^2 \left( \frac{5}{2} + 9x^2 - 6x^4 \right) \right.
- \frac{(1 - x)^2(5 + 4x^2)}{2x^2} \ln(1 - x^2)
- \frac{4(1 - x^2)^2(1 + 2x^2)}{x^2} \left( \ln(x) \ln(1 - x^2) + \frac{\pi^2}{6} \right)
- \frac{8(1 - x^2)^2(1 + 2x^2)}{x^2} \ln(1 - x^2) \left. \right\}
\] (11)

\[
\hat{\Gamma}_L = \frac{1}{1 + 2x^2} + \frac{\alpha_s}{2\pi} C_F (1 - x^2)^2(1 + 2x^2) \times \left\{ (1 - x)^2 \left( \frac{5}{2} + 47x^2 - 4x^4 \right) \right.
- \frac{(1 + 5x^2 + 2x^4)}{x^2} \frac{2\pi^2}{3}
+ 16(1 + 2x^2) \ln(x) - \frac{3(1 - x^2)^2}{x^2} \ln(1 - x^2)
- 2(1 - x)^2 \frac{2x + 6x^2 + 3x^3}{x^2} \ln(1 - x) \ln(x)
- 2(1 + x)^2 \frac{2x + 6x^2 - 3x^3}{x^2} \ln(x) \ln(1 + x)
- 2(1 - x)^2 \frac{4 + 3x + 8x^2 + 3x^3}{x^2} \ln(1 + x)
- 2(1 + x)^2 \frac{4 - 3x + 8x^2 - 3x^3}{x^2} \ln(1 - x) \left. \right\}
\] (12)

\[
\hat{\Gamma}_U = \frac{\alpha_s}{2\pi} C_F \frac{x^2}{(1 - x)^2(1 + 2x^2)} \times \left\{ -\frac{1}{2}(1 - x)(25 + 5x + 9x^2 + 3x^3)
+ (7 + 6x^2 - 2x^4) \frac{\pi^2}{3} - 2(5 + 7x^2 - 2x^4) \ln(x) \right. \left. \right\}
\] (13)
\[-2(1-x^2)(5-2x^2)\ln(1+x)\]
\[\begin{align*}
-\frac{(1-x)^2}{x}(5+7x^2+4x^3)\ln(x)\ln(1-x) \\
+\frac{(1+x)^2}{x}(5+7x^2-4x^3)\ln(x)\ln(1+x) \\
-\frac{(1-x)^2}{x}(5+7x^2+4x^3)\text{Li}_2(x) \\
+\frac{1}{x}(5+10x+12x^2+30x^3-x^4-12x^5)\text{Li}_2(-x) \end{align*}\]

\(\dot{\Gamma}_- = \frac{2x^2}{1+2x^2} + \frac{\alpha_s^2}{2\pi} \frac{x^2}{(1-x)^2(1+2x^2)} \times \left\{ -\frac{1}{2}(1-x)(13+33x-7x^2+x^3) \\
+(3+4x^2-2x^4)\frac{x^2}{3} - 2(5+7x^2-2x^4)\ln(x) \\
-2\frac{(1-x^2)(1+2x^2)}{x^2}\ln(1-x) \\
-\frac{(1-x)^2}{x^2}(5+7x^2+4x^3)\ln(x)\ln(1-x) \\
+\frac{(1+x)^2}{x}(5+7x^2-4x^3)\ln(x)\ln(1+x) \\
-\frac{(1-x)^2}{x}(5+3x)(1+x+4x^2)\text{Li}_2(x) \\
+\frac{1}{x}(5+2x+12x^2+6x^3-x^4-4x^5)\text{Li}_2(-x) \right\} \)

The radiative corrections to the longitudinal and transverse-minus rates are sizeable where the radiative correction to the longitudinal rate is largest. The radiative corrections lower the normalized longitudinal rate \(\Gamma_L/\Gamma\) by 1.06% and increase the normalized transverse-minus rate \(\Gamma_-/\Gamma\) by 2.17%. The radiative correction to the transverse-plus rate is quite small. For the normalized transverse-plus rate \(\Gamma_+ / \Gamma\) we obtain a mere 0.10% which is only marginally larger than the value of 0.036% obtained from the Born term level \(m_b \neq 0\) effects discussed in Sec. 3.

The \(m_b \neq 0\) corrections to the \(\alpha_s\)-contributions can be obtained by using the results given in Ref. [4] where the full \(m_b\) dependence was included in the radiative correction calculation. Taking again a pole mass \(m_b = 4.8\text{GeV}[12]\) we find that the full rate \(\Gamma\) and the helicity rates \(\Gamma_L, \Gamma_+\) and \(\Gamma_-\) change by \(-0.16\%, -0.21\%, +19.91\%\) and \(-0.10\%,\) respectively. The corresponding numbers for the Born term alone for \(\Gamma, \Gamma_L\) and \(\Gamma_-\) are \(-0.27\%, -0.35\%\) and \(-0.17\%.\) The leakage into the Born term rate \(\Gamma_+\) through \(m_b \neq 0\) effects was given in Sec. 3. It is interesting to note that the \(m_b \neq 0\) corrections to the \(\alpha_s\) contributions are larger than those for the Born terms. This can be understood in part by noting that the latter contain contributions proportional to \((m_b^2/m_W^2)\ln(m_b^2/m_t^2)\) = \(-0.026\) which is not a very small number.

VI. SUMMARY AND CONCLUSIONS

We have presented results on the \(O(\alpha_s)\) radiative corrections to the three helicity rates in unpolarized top quark decays which can be determined from doing an angular analysis on the decay products or from an analysis of the shape of the lepton spectrum. While the radiative corrections to the unnormalized transverse-minus and longitudinal rate are sizable (\(\approx 6-10\%\)), the radiative corrections to the normalized helicity rates are smaller (\(\approx 1-2\%\)). The radiative correction to the transverse-plus rate is very small. The measurements of the helicity rates by the CDF Collaboration can be seen to be fully compatible with the predictions of the Standard Model. The errors on these measurements are, however, too large to allow one to meaningfully compare the present measurements with quantum effects brought in by QCD radiative corrections. There is hope that this will change in the future.

V. NUMERICAL RESULTS

We are now in a position to discuss our numerical results. Our input values are \(m_t = 175\text{GeV}\) and \(m_W = 80.419\text{GeV},\) as before. For the strong coupling constant we use \(\alpha_s(m_t) = 0.107\) which was evolved downward from \(\alpha_s(m_Z) = 0.1175.\) Our numerical results are presented in terms of the hatted helicity rates \(\dot{\Gamma}_i = \Gamma_i / \Gamma_0\) \((i = U, L, L, +, -)\) introduced in Sec. 4. In order to be able to quickly assess the percentage changes induced by the \(O(\alpha_s)\) corrections, we have factored out the Born term helicity rates (when applicable) from the \(O(\alpha_s)\) results. One has

\[\dot{\Gamma} = 1 - 0.0854,\]  
\[\dot{\Gamma}_L = 0.703(1 - 0.095),\]  
\[\dot{\Gamma}_+ = 0.000927,\]  
\[\dot{\Gamma}_- = 0.297(1 - 0.0656),\]
[1] CDF Collaboration, T. Affolder et al., Phys. Rev. Lett. 84 (2000) 216
[2] S. Willenbrock, “Studying the top quark” [hep-ph/0008189], and M. Narain (private communication).
[3] M. Fischer, S. Groote, J.G. Körner, B. Lampe and M.C. Mauser, Phys. Lett. 451 B (1999) 406
[4] M. Fischer, S. Groote, J.G. Körner and M.C. Mauser, “Complete angular analysis of polarized top decay at O(α_s)”, to be published.
[5] A. Denner and T. Sack, Nucl. Phys. B358 (1991) 46
[6] J. Liu and Y.-P. Yao, Int. J. Mod. Phys. 6 (1991) 4925
[7] A. Czarnecki, Phys. Lett. 252 B (1990) 467
[8] C.S. Li, R.J. Oakes and T.C. Yuan, Phys. Rev. D43 (1991) 3759
[9] M. Jezabek and J.H. Kühn, Nucl. Phys. B314 (1989) 1
[10] K. Fujikawa and A. Yamada, Phys. Rev. D49 (1994) 5890
[11] P. Cho and M. Misiak, Phys. Rev. D49 (1994) 5894
[12] A.A. Penin and A.A. Pivovarov, Nucl. Phys. B549 (1999) 217; Phys. Lett. 443 B (1998) 264
[13] K. Schilcher, M.D. Tran and N.F. Nasrallah, Nucl. Phys. B181 (1981) 91; Erratum ibid. B187 (1981) 594
[14] G.J. Gounaris and J.E. Paschalis, Nucl. Phys. B222 (1983) 473