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Towards forming simulations by means of reduced integration-based solid-shell elements considering gradient-extended damage

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Abstract. The present contribution is concerned with the non-local damage analysis of geometrically non-linear shells. To this end, a low-order displacement-based solid-shell finite element formulation is combined with a gradient-extended damage-plasticity model. Due to a tailored combination of reduced integration with hourglass stabilization, the enhanced assumed strain (EAS) method and the assumed natural strain (ANS) method, the most dominant locking phenomena are eliminated. A polynomial approximation of the strain-like as well as the stress-like quantities within the weak forms enables the definition of a suitable and efficient hourglass stabilization. In this way, the internal element force vectors as well as the element stiffness contributions coming from the hourglass stabilization can be determined analytically. A numerical example of a circumferentially notched cylinder considering plasticity coupled with damage reveals the potential of the proposed methodology. Besides the ability to deliver mesh independent results within the softening regime, the framework is especially suitable for thin-walled structures, in which conventional low-order continuum elements suffer from well-known locking phenomena.

1. Introduction
The numerical simulation of forming processes is a challenging task. Typically, a thin sheet metal is considered as workpiece which is severely bent and consequently formed within a suitable experimental set-up until the desired shape of the sheet is reached. From the numerical point of view, there are many challenges that need to be addressed. First of all, a suitable constitutive model which is able to capture large plastic deformations with combined non-linear isotropic and kinematic hardening has to be considered. The purpose of the latter is to represent the Bauschinger effect which is observed in cyclic loading and which strongly influences the springback behavior (cf. [1, 2]). Furthermore, when forming is limited by fracture rather than necking, an appropriate formulation of (ductile) damage (cf. [3, 4]) needs to be taken into account which captures the degradation phenomena within the workpiece (e.g. coalescence of micro-voids). Damage is usually accompanied by strain localization (cf. [5, 6]). In the context of finite element simulations, it is a well-known problem that the usage of conventional local continuum damage models can lead to severely mesh-dependent results unless a certain regularization technique is introduced (e.g. [7, 8, 9, 10]). At the global level, an efficient and accurate finite element formulation is required. Due to their simplicity and
robustness, low-order continuum elements are widely used in industrial applications. Standard displacement-based low-order formulations, however, suffer from well-known locking effects which is particularly noticeable under the conditions prevailing in forming simulations. These are essentially volumetric locking in metal plasticity as well as (transverse) shear locking in bending dominated situations. Especially for thin-walled structural components, solid-shell formulations (cf. [11, 12, 13, 14]) are well established and have already been successfully applied to forming simulations (cf. [15, 16, 17, 18]). In contrast to classical shell elements, the actual topology is taken into account which enables a simple incorporation of complex three-dimensional constitutive models as well as double-sided contact.

The objective of the present work is to overcome all of the aforementioned difficulties at once by a suitable modeling strategy. For this purpose, the gradient-extended damage-plasticity model for large deformations of [19] is combined with the reduced-integration based solid-shell element of [14]. Due to the gradient extension, and next to the displacements, an additional non-local damage variable emerges as a global unknown of the system, which has to be determined. The focus of this work lies on the incorporation of the additional (non-local) nodal degree of freedom into the framework of the reduced integration-based solid-shell, which requires a most efficient stabilization scheme. While simpler examples are still considered in the current phase of development, the application to real forming processes is also planned in the future, taking into account contact in the simulation.

2. Gradient-extended damage-plasticity model

Within this work, the gradient-extended two-surface damage-plasticity model developed in [19] is utilized. It can be considered as a gradient-extended damage version of the elastoplasticity model for large deformations by [1] which has already been successfully applied to various industrial forming simulations in e.g. [2], [17] and [20]. For this purpose, a short summary of the constitutive model is given below. In order to model elasto-plasticity, the well-established multiplicative split of the deformation gradient $\mathbf{F} = \mathbf{F}_e \mathbf{F}_p$ into elastic and plastic parts is employed. An additional decomposition of $\mathbf{F}_p = \mathbf{F}_{pe} \mathbf{F}_{pi}$ into recoverable and irrecoverable parts is applied to model non-linear kinematic hardening of Armstrong-Frederick type. The Helmholtz free energy is assumed to take the following additive format

$$\psi = f_{dam}(D)(\psi_e(C_e) + \psi_p(C_{pe}, \xi_p)) + \psi_d(\xi_d) + \psi_{\bar{d}}(D - \bar{D}, \text{Grad}(\bar{D})) , \quad (1)$$

where $\psi_e$ denotes the elastic part of the energy which is a function of the elastic right Cauchy-Green deformation tensor $C_e = \mathbf{F}_e^T \mathbf{F}_e$. The plastic part $\psi_p$ depends on $C_{pe} = \mathbf{F}_{pe}^T \mathbf{F}_{pe}$ and the isotropic hardening variable $\xi_p$. Damage hardening is considered in $\psi_d$, which is a function of the damage hardening variable $\xi_d$. The energy related to the gradient-extension is considered in $\psi_{\bar{d}}$ which is a function of the difference between the local damage variable $D$ and the micromorphic (or non-local) damage variable $\bar{D}$ as well as its gradient Grad $(\bar{D})$ with respect to the reference configuration, respectively. Moreover, it is assumed that the scalar-valued damage degradation function $f_{dam}(D)$ (which is further specified in Sec. 4) acts on the elastic as well as the plastic part of the free energy function.

The individual energies can be freely chosen and in principle be adapted to any kind of material behavior. In the present work, the same choice is made as in [19]. For the interested reader, the final model equations are summarized below
State laws

\[ S = f_{\text{dam}}(D) 2 F_p^{-1} \frac{\partial \psi_p}{\partial C_e} F_p^{-T} = f_{\text{dam}}(D) \left( \mu (C_p^{-1} - C_e^{-1}) + \frac{\lambda}{2} \left( \frac{\det C_p}{\det C_p - 1} \right) C_e^{-1} \right) \]  

(2)

\[ a_0 = \frac{\partial \psi_d}{\partial D} = -H (D - \bar{D}) \]  

(3)

\[ b_0 = \frac{\partial \psi_d}{\partial \text{Grad } D} = A \text{Grad } (\bar{D}) \]  

(4)

- Thermodynamic conjugate forces – plasticity and damage

\[ X = f_{\text{dam}}(D) 2 F_p^{-1} \frac{\partial \psi_p}{\partial C_p} F_p^{-T} = f_{\text{dam}}(D) a \left( C_p^{-1} - C_e^{-1} \right) \]  

(5)

\[ q_p = f_{\text{dam}}(D) \frac{\partial \psi_p}{\partial \xi_p} = f_{\text{dam}}(D) e (1 - \exp(-f \xi_p)) \]  

(6)

\[ Y = -\frac{df_{\text{dam}}(D)}{dD} (\psi_e + \psi_p) - \frac{\partial \psi_d}{\partial D} = -\frac{df_{\text{dam}}(D)}{dD} (\psi_e + \psi_p) - H (D - \bar{D}) \]  

(7)

\[ q_d = \frac{\partial \psi_d}{\partial \xi_d} = r (1 - \exp(-s \xi_d)) \]  

(8)

- Evolution equations

\[ \dot{C}_p = 2 \dot{\lambda}_p \frac{3^{3/2}}{f_{\text{dam}}(D)} \frac{\dot{Y}' C_p}{\sqrt{\dot{Y}' (\dot{Y}')^T}}, \quad \dot{C}_{p_i} = 2 \dot{\lambda}_p \frac{b/a}{f_{\text{dam}}(D)} \dot{Y}_i \text{kin} C_{p_i} \]  

(9)

\[ \dot{\xi}_p = \frac{\dot{\lambda}_p}{f_{\text{dam}}(D)}, \quad \dot{D} = \dot{\lambda}_d, \quad \dot{\xi}_d = \dot{\lambda}_d \]  

(10)

- Yield and damage loading function

\[ \Phi_p = \sqrt{\frac{3}{2}} \frac{\sqrt{\dot{Y}' (\dot{Y}')^T} - (\sigma_0 + \bar{q}_p)}{\Phi_d = Y - (\dot{Y}_0 + q_d) \]  

(11)

- Loading / unloading conditions

\[ \dot{\lambda}_p \geq 0, \quad \Phi_p \leq 0, \quad \dot{\lambda}_p \Phi_p = 0 \]  

(12)

\[ \dot{\lambda}_d \geq 0, \quad \Phi_d \leq 0, \quad \dot{\lambda}_d \Phi_d = 0 \]  

(13)

In Eq. (9) and (11), the auxiliary stress tensors are defined as \( Y = CS - C_p X \) and \( Y_{\text{kin}} = C_p X \) and the deviatoric part of a tensorial quantity is indicated by \( (\bullet)' \). Furthermore, since plasticity solely acts on the undamaged part of the material, the evolution equations and the yield function are formulated in terms of effective quantities \( (\bullet) := (\bullet)/f_{\text{dam}}(D) \). Altogether, the material model includes twelve parameters \( (\lambda, \mu, \sigma_0, a, b, e, f, Y_0, r, s, H, A) \). Except for the penalty parameter \( H \), they need to be identified by suitable experiments. Any further details are omitted at this point for brevity, but can be found in [19].
3. Reduced integration-based solid-shell formulation

The departure of the present solid-shell formulation is the three field functional

\[
\begin{align*}
g_u(u, w, \bar{D}) &= \int_{\Omega_0} S(E, \bar{D}) \cdot \delta E_c \, dV - g_u^{\text{ext}} = 0 , \\
g_w(u, w, \bar{D}) &= \int_{\Omega_0} S(E, \bar{D}) \cdot \delta E_c \, dV = 0 , \\
g_d(u, w, \bar{D}) &= \int_{\Omega_0} (a_0(E, \bar{D}) \delta \bar{D} + b_0(E, \bar{D}) \cdot \text{Grad} (\delta \bar{D})) \, dV = 0 ,
\end{align*}
\]

which is here expressed with respect to the reference configuration \( \Omega_0 \) of a body under consideration. The weak form consists of the balance of linear momentum \( g_u \), the so-called orthogonality condition \( g_w \) (cf. [21], [14]) as well as the micromorphic balance equation \( g_d \) (cf. [10], [19]). The displacement vector \( u \), the incompatible displacement vector \( w \) and the non-local damage variable \( \bar{D} \) represent the independent field quantities. The 2nd Piola-Kirchhoff stress tensor \( S \) and the generalized stresses \( a_0 \) and \( b_0 \) are implicit functions of the total Green-Lagrange strain tensor \( E = E_c + E_e \) as well as the non-local damage variable \( D \). The compatible strain \( E_c \) is related to the displacement field \( u \) and the enhanced strain \( E_e \) stems from the incompatible displacement field \( w \). Furthermore, the external virtual work is denoted by \( g_u^{\text{ext}} \).

Regarding the interpolation, the isoparametric concept for an eight-node continuum element is considered, in which the same tri-linear Lagrange shape functions are used for the position vector of the reference configuration \( X \), the displacement vector \( u \) and the non-local damage variable \( D \). Locking pathologies are eliminated due to a tailored combination of the assumed natural strain method (ANS), the enhanced assumed strain method (EAS), as well as reduced integration. For further details concerning the ANS and the EAS within the present formulation, the interested reader is referred to [14, 22]. Regarding the method of reduced integration, an appropriate (hourglass) stabilization scheme needs to be considered in order to avoid spurious and possibly singular eigenvalues of the resulting coupled element stiffness matrix. Here, similar considerations as already presented in [13, 23, 24, 25, 26] are taken into account. The key idea is to represent the integral equations (14)-(16) and therefore also all integrands within these equations as polynomials with respect to Cartesian coordinates. Note that this is generally not the case for arbitrary element shapes that deviate from the shape of a parallelepiped. These polynomials can further be separated into physically relevant parts denoted by \( (\bullet)^\ast \) which belong to the integration points and a remainder, the so-called hourglass parts denoted by \( (\bullet)^{hg} \) which represent the stabilization terms of the present formulation. The polynomial split is achieved by means of a series of Taylor expansions. In the following, \( (\bullet) \) indicates a symmetric second order tensor in vector (Voigt) notation. First, a Taylor expansion of the strain-like quantities \( \alpha = (E_c, \text{Grad}(D))^T \) with respect to the element center \( \xi^0 = (0,0,0)^T \) up to bilinear terms leads to

\[
\alpha \approx \alpha|_{\xi^0} + \sum_{i=1}^{3} \frac{\partial \alpha}{\partial \xi_i} |_{\xi^0} \xi_i + \frac{1}{2} \sum_{i=1}^{3} \sum_{j \neq i=1}^{3} \frac{\partial^2 \alpha}{\partial \xi_i \partial \xi_j} |_{\xi^0} \xi_i \xi_j := \alpha^\ast + \alpha^{hg}.
\]

Second, a Taylor expansion of the stress-like quantities \( \beta = (\hat{S}, a_0, b_0)^T \) with respect to the normal of the element center \( \xi^* = (0,0,\zeta)^T \) up to linear terms leads to

\[
\beta \approx \beta|_{\xi^*} + \sum_{i=1}^{3} \frac{\partial \beta}{\partial \xi_i} |_{\xi^*} (\xi_i - \xi^*_i) := \beta^\ast + \beta^{hg}.
\]
The purpose of the latter is to capture potential material non-linearities over the thickness of the element. Due to the fact that the hourglass parts are solely needed for stabilization purposes, they should be modeled as efficiently as possible. Therefore, an analytical integration of the resulting residual and stiffness contributions coming from these terms is sought. This is generally not possible, since \( S^{hg} \) and \( a^{hg}_b \) still involve certain material sensitivities (tangents) which are non-linear and thus hinder the aforementioned goal. The problem is solved by adjusting these sensitivities adaptively such that they are constant within each global Newton iteration. Furthermore, they are chosen to be positive (definite) which guarantees a robust element stability. This means that potential instabilities coming for instance from damage softening or bifurcation may only occur due to the (●)-part which is physically reasonable, since no further modifications are involved in these terms. More details can be found in [14, 26].

Due to the missing continuity requirement for the incompatible displacements \( w \), the latter are eliminated from the set of independent fields by means of static condensation. The resulting 32×1 internal element force vector reads

\[
R_e = \left( \begin{array}{c}
R_{e, u} - K_{e, uu} K_{e, u}^{-1} K_{e, u} + R^{hg}_{e, u} \\
R_{e, d} - K_{e, du} K_{e, u}^{-1} K_{e, d} + R^{hg}_{e, d}
\end{array} \right)
\]

The linearization of Eq. (19) finally leads to the 32×32 element stiffness matrix

\[
K_e = \left( \begin{array}{cc}
K_{e, uu} - K_{e, uu} K_{e, u}^{-1} K_{e, u} + K_{e, uu}^{hg} & K_{e, ud} - K_{e, uu} K_{e, w}^{-1} K_{e, d} + K_{e, ud}^{hg} \\
K_{e, du} - K_{e, du} K_{e, w}^{-1} K_{e, u} & K_{e, dd} - K_{e, du} K_{e, w}^{-1} K_{e, d} + K_{e, dd}^{hg}
\end{array} \right).
\]

Inspecting the latter equations, a strong coupling between \( u, w \) and \( \bar{D} \) can be recognized. Furthermore, as already mentioned, the hourglass contributions within Eqs. (19) and (20) can be determined analytically which makes the formulation particularly efficient in comparison to approaches based on full integration. Finally, by applying the conventional assembling procedure and taking the boundary conditions into account, the global set of finite element equations \( K \Delta V = -R \) is solved monolithically for the increments \( \Delta V = (\Delta u, \Delta D)^T \).

4. Numerical example

In the following numerical example, a circumferentially notched cylinder subjected to axial tension is investigated. The considered geometry with boundary conditions inspired by [27] and [28] is depicted in Fig. 1. The left end of the cylinder is fixed (i.e. \( u_x = u_y = u_z = 0 \)) and an axial loading is applied to the right edge of the structure. Due to symmetry, only one half of the whole cylinder needs to be considered in the simulation as indicated in grey color. The material parameters are given in Table 1, where the elastic material parameters are adopted from [27]. The plastic material parameters are taken from [17] for an 6111-T4 aluminum alloy. The degradation function introduced in Eq. (1) is specified as \( f_{dam}(D) = (1 - D)^2 \). For simplicity, kinematic hardening as well as damage hardening are not considered within this study. In order to show mesh convergence, different mesh sizes with 1800, 3960 and 12168 elements are considered in the simulation. Here, mesh refinement is mainly performed in the region where damage is expected to occur during the deformation process, see Fig. 2 for the coarsest and the finest discretization, respectively. Furthermore, only two Gaussian points over the thickness of the sheet are considered, since test computations with several integration points in this direction did not show any influence on the global structural response. Figure 3 shows the global load-displacement curves for the considered meshes. Depicted is the axial displacement \( u_x \) at the right cylinder end over the retention force at the clamping. For convenience, all curves are normalized by the maximum force \( F_{max} = 317.39 \text{ N} \) obtained in the simulation with 12168 elements.
Figure 1. Geometry with dimensions in [mm] and boundary conditions

| Symbol | Material parameter                           | Value   | Unit   |
|--------|---------------------------------------------|---------|--------|
| $\lambda$ | first Lamé parameter                       | 48889   | MPa    |
| $\mu$   | second Lamé parameter                       | 27500   | MPa    |
| $\sigma_0$ | yield stress                               | 161     | MPa    |
| $a$     | first kinematic hardening parameter         | $10^{-8}$ | MPa    |
| $b$     | second kinematic hardening parameter        | 0       | -      |
| $e$     | first isotropic hardening parameter         | 207     | MPa    |
| $f$     | second isotropic hardening parameter        | 9.74    | -      |
| $Y_0$   | damage threshold                            | 1       | MPa    |
| $r$     | first damage hardening parameter            | 0       | MPa    |
| $s$     | second damage hardening parameter           | 0       | MPa    |
| $A$     | internal length scale parameter             | 1       | MPa mm$^2$ |
| $H$     | penalty parameter                           | $10^5$  | MPa    |

Table 1. Material parameters

Figure 2. Mesh with 1800 elements (left) and 12168 elements (right)

elements. Mesh convergence is clearly demonstrated. An illustration of the deformation process is given in Fig. 4. Depicted are contour snapshots of the local damage variable $D$ as well as the accumulated plastic strain $\kappa$ at three different stages. Damage initiates at the notch tip of the cylinder and continuously propagates in circumferential direction around the cylinder until a fully ‘broken’ state is reached, where no further loading can be applied. The distribution of the damaged region corresponds qualitatively very well with the ‘crack’ patterns reported in [27]
Figure 3. Normalized load-axial displacement curves

and [28]. Therefore, the present framework is able to capture ductile damage until final failure within curved structures. For further details, amongst others the comparison to conventional formulations and computational efficiency, the interested reader is referred to [26] and [29].

Figure 4. Contours of the damage variable $D$ (a1)-(a3) and the accumulated plastic strain $\kappa$ (b1)-(b3) at three different stages ($u_z = 0.075$ mm, 0.125 mm, 0.23 mm) during the deformation process (12168 elements)

5. Conclusion
In this work, a new methodology for the non-local damage analysis of geometrically nonlinear shells was presented. For this purpose, the low-order solid-shell formulation for large deformations of [14] was combined with the finite strain gradient-extended two-surface damage-plasticity model of [19]. The solid-shell element which is locking-free due to a tailored combination of ANS, EAS as well as reduced integration with hourglass stabilization, was
extended by an additional non-local nodal degree of freedom. The numerical example of an elasto-plastic circumferentially notched cylinder revealed that the novel framework is able to overcome the severe drawback of mesh dependence of conventional shell formulations involving ‘local’ damage and, therefore, to accurately predict the degradation processes within thin-walled structures. Future work will focus on its application to real forming processes. For this, a suitable contact formulation has to be considered in the simulation.

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