Distributed Event-Triggered Nonlinear Fusion Estimation Under Resource Constraints

RUSHENG WANG, BO CHEN, Member, IEEE, ZHONGYAO HU, Member, IEEE
LI YU, Member, IEEE
Zhejiang University of Technology, Hangzhou, China
Zhejiang Provincial United Key Laboratory of Embedded Systems, Hangzhou, China

This article studies the event-triggered distributed fusion estimation problems for a class of nonlinear networked multisensor fusion systems without noise statistical properties. In practice, the sensor-to-remote estimator channel and the smart sensor-to-fusion center channel of the communication network will be faced with some resource constraint problems. To meet the finite communication resources during the information transmission, an event-triggered strategy and a dimensionality reduction strategy are introduced in a unified network framework to reduce the communication traffic. Since the reduction of communication information will inevitably degenerate the estimation performance, two kinds of compensation strategies in terms of a unified model are proposed to restructure the untransmitted information. Then, the local fusion estimators are designed based on the compensation information. Moreover, the linearization errors caused by the Taylor expansion are modeled by the state-dependent matrices with uncertain parameters when establishing estimation error systems, and then, different robust recursive optimization problems are established to determine the estimator gains and the fusion criteria. Meanwhile, the stability conditions are also presented such that the square errors of the designed nonlinear estimators are asymptotically bounded.

I. INTRODUCTION

With the development of communication networks and sensor technologies, networked multisensor fusion systems (NMFSs) have attracted extensive attention, where the communication between the sensors, estimators, and fusion center (FC) is connected by the network instead of a dedicated independent connection [1]. Particularly, the NMFSs can be constructed by employing a wireless sensor network (WSN), and the WSN can communicate over a shared wireless channel [2]. In the WSN fusion framework, sensors send information through wireless communication channels, which can overcome the geographical constraints of NMFSs [3], [4], [5]. Since the introduction of the communication networks provides a more flexible structure and easier installation than the conventional multisensor fusion systems, NMFSs have been applied in many areas, such as fault detection [6], target tracking [7], and power systems [8], as well as remote sensing image [9], target detection [10], and integrated navigation [11]. Notice that most information fusion and target information processing systems in real environments are nonlinear, and the development of modern detection methods and monitoring ranges makes the nonlinear problem more prominent. Then, nonlinear filtering techniques, which are based on sensor nonlinear measurements for estimating the target’s state, or FC, which in turn leads to a degradation of the estimation performance. Therefore, it is of great significance to actively reduce the communication traffic to meet the limited communication resources, while a relatively satisfactory estimation performance of the designed estimator should also be guaranteed. Generally, various quantization strategies (QSs) [14], [15], [16], [17] and dimensionality reduction strategies (DRSs) [18], [19], [20], [21], [33], [34], [35] have been developed to reduce the size of data packets before information transmission. For example, the optimal quantization rules and an optimal fusion estimation criterion were established in [14], and a logarithmic quantizer was utilized in [15] to address the $H_\infty$ fusion estimation problem with restricted bandwidth. However, the quantization technique is difficult to deal with high-dimensional signals.

Finally, a vehicle localization system is employed to demonstrate the effectiveness and advantages of the proposed methods.
TABLE I
Comparison of Related Works on State Estimation With Resource Constraints

| Works | System Type | Channel Type | Communication Strategy | Noise Type | Estimation Type |
|-------|-------------|--------------|------------------------|------------|-----------------|
| [14], [17] | Linear | S-RE | QS | Gaussian noises | Fusion estimation |
| [15] | Linear | S-FC | QS | Energy bounded noises | Fusion estimation |
| [16] | Nonlinear | S-FC | QS | Gaussian noises | Fusion estimation |
| [33], [34] | Linear | S-FC | DRS | Gaussian noises | Fusion estimation |
| [18]–[21] | Linear | S-RE | DRS | Gaussian noises | Fusion estimation |
| [22] | Linear | S-RE | ETS | Gaussian noises | State estimation |
| [23] | Linear | S-RE | ETS | Known bounded noises | State estimation |
| [24]–[28], [36], [40] | Linear | S-RE | ETS | Gaussian noises | State estimation |
| [29] | Nonlinear | S-RE | ETS | Energy bounded noises | State estimation |
| [30] | Nonlinear | S-NS | ETS | Gaussian noises | Distributed estimation |
| [31], [32] | Nonlinear | S-RE | ETS | Gaussian noises | Fusion estimation |
| [35] | Linear | S-FC | DRS | Energy bounded noises | Fusion estimation |
| [37]–[39] | Linear | S-NS | ETS | Gaussian noises | Distributed estimation |
| [41] | Nonlinear | S-FC | ETS | Gaussian noises | Fusion estimation |
| [42] | Linear | S-FC | DRS & ETS | Gaussian noises | Fusion estimation |
| [43] | Linear | S-FC | DRS & QS | Unknown bounded noises | Fusion estimation |
| This article | Nonlinear | S-RE & S-FC | DRS & ETS | Unknown bounded noises | Fusion estimation |

and has the limitation of being easily distorted. At the same time, the event-triggered strategies (ETSs) [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32], [36], [37], [38], [39], [40], [41] also provide an effective means to reduce redundant communication traffic and energy consumption. Particularly, a comparison of related works on state estimation with resource constraints is presented in Table I, which are categorized along their main distinctive characteristics. It can be seen that the DRS and the ETS will be employed in this article to deal with the networked nonlinear fusion estimation problems under resource constraints.

A. Related Works

It should be noted that limited communication resources may occur between the sensor and the estimator, or between the sensor and the FC. In practice, some kinds of sensor only have the ability to measure information without data processing, or the measurements need to be sent to the remote estimator for processing. Under this case, the limited resources problem of the sensor-to-remote estimator (S-RE) communication channel should be addressed. Therefore, an optimal compression matrix can be found in [18] by an optimal sensor compression strategy. Then, a dimensionality reduction strategy (DRS), which needs to know the global measurement matrix, has been developed in [19] to compress the measurements. Meanwhile, a preprocessor was designed in [20] for compressing the raw measurements of the corresponding measurement block, while the compression operators were usually difficult to be calculated. Specifically, determining the compression operator requires solving the nonlinear optimization problem that depends on global information, but the solution to this optimization problem cannot be guaranteed. Then, another dimensionality reduction approach was proposed in [21] to meet the finite bandwidth, where only partial components of a group of measurements were chosen to be transmitted to the remote estimator.

Furthermore, the transmission of information in event-based networked systems depends on predefined event-triggered conditions [23]. Specifically, a measurement innovation-based deterministic ETS was designed in [24]. Then, a more general ETS was considered in [25] to address the fusion estimation problem based on the hybrid measurement information, while the conditional distribution of state was assumed to be a Gaussian distribution. Moreover, a kind of stochastic ETS was developed in [26] and [27], which can preserve the Gaussian property of the innovation sequence and the posterior distribution. However, the aforementioned innovation-based ETSs need feedback information from the estimator, which might increase communication costs and be difficult to apply to time-varying systems. Then, an information-based stochastic ETS based on the local observation projection into state space was developed in [28] instead of combining the feedback information from the estimator, while the considered estimation problem was still in the linear multisensor fusion framework. For the nonlinear systems, a measurement-based ETS was introduced in [29] to improve the utilization of network resources, and the T–S fuzzy technology was employed to cope with the nonlinearities. Then, a kind of distributed measurement innovation-based ETS was proposed in [30] to cut down on bandwidth usage, and a distributed recursive estimation algorithm was designed under the stochastic nonlinearities and measurement losses. In [31], the innovation-based deterministic ETS was used for nonlinear NMFSs with random delays, and then, a modified unscented Kalman filter (UKF) and a sequential covariance intersection fusion method were used to address the distributed fusion estimation problem. Meanwhile, a stochastic Send-on-Delta
ETS was introduced in [32] to reduce the redundant information for nonlinear NMFSs subject to jamming attacks. It is worth noting that the preceding works assume that the measurements and/or system noises are Gaussian with known covariance, while the statistical properties of noises are difficult to be accurately obtained.

On the other hand, some smart sensors have the ability to process data locally instead of sending it to remote sensors, and then, local estimates will be further transmitted to the FC through the communication network in this fusion framework. Thus, the problem of resource constraints in the smart sensor-to-fusion center (S-FC) channel should also be considered. Then, a new kind of DRS was developed in [33] to meet the finite bandwidth, where only partial components of local estimates with known global bandwidth constraints were chosen to transmit to the FC. Moreover, another kind of DRS without a fixed structure was developed in [34], in which each component of the local estimate has a selection probability to satisfy the stability conditions, and a compensation strategy was proposed to restructure the untransmitted information. However, the aforementioned methods are all based on the Kalman filter, which needs to know the noise statistical properties. Then, a fusion estimation method was developed in [35] to deal with finite communication resources, while the energy bounded noise was still a kind of special noises.

Moreover, based on the incremental innovative information of the estimates, a Gaussianity-preserving ETS was proposed in [36]. Then, a deterministic ETS and a stochastic ETS, whose triggering conditions were based on the local estimates, were introduced in [37] and [38] for distributed state estimation problems, respectively. Moreover, a new variance-based ETS was developed in [39] for the distributed estimation problem, in which the triggering condition was based on the difference between the estimation error variance and the multistep prediction variance. Particularly, a priori estimate-based ETS was proposed in [40] for nonlinear system without the knowledge of the process noise statistical property, while the covariances of the measurement noise were still required to be known. Notice that, the aforementioned estimate-based ETS considered the distributed estimation problems of NMFSs under resource constraints rather than fusion estimation problems. Then, a fusion estimate-based ETS was developed in [41], in which the raw measurements and the feedback fusion information were employed to derive the local estimation, whereas the fusion feedback might increase the computation cost.

B. Motivations and Contributions

Notice that, only one communication strategy was employed to solve the resource constraint problem of NMFSs in previous works [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], [34], [35], [36], [37], [38], [39], [40], [41], and only one communication channel subject to resource constraints is considered as well. Although a deterministic ETS combined with DRS was proposed in [42] to address the distributed fusion estimation problem of NMFSs under resource constraints, it was still only suitable for linear NMFSs with Gaussian white noise. Meanwhile, the transmitted information must pass through the dimensionality reduction scheduler and the event-triggered scheduler each time, which adds additional energy consumption. Moreover, a DRS and a QS were modeled by a unified framework to address networked fusion estimation problem with bounded noises in [43], while the considered problem was still in the linear distributed fusion framework.

In light of the preceding analysis, the distributed fusion estimation problem for nonlinear NMFSs with unknown statistical properties is still a challenging issue. In this case, we shall study the nonlinear fusion estimation problems under two kinds of networked distributed fusion frameworks, where the noises are without knowledge of statistical properties. The main contributions of this article are listed as follows.

1) In this article, two different means of communication transmission are presented in the nonlinear networked fusion estimation framework. Specifically, both S-RE channel and S-FC channel subject to resource constraints are taken into account, and then, an ETS and a DRS are introduced in a unified framework to meet the finite resources.

2) Two kinds of unified compensation strategies are proposed to restructure the untransmitted information caused by the ETS and the DRS, and then, the corresponding distributed fusion estimation algorithms are developed, which can preserve satisfactory estimation performance.

3) By modeling the linearization errors in terms of the state-dependent matrices and uncertain parameters, a robust recursive optimization approach, which can deal with the unknown but bounded noises in nonlinear NMFSs, is developed such that the square errors (SEs) of the designed nonlinear estimators are asymptotically bounded.

The notations and acronyms most frequently used throughout this article are given in Tables II and III.

| Notation | Description |
|----------|-------------|
| $\mathbb{R}^{m \times n}$ | Set of $m \times n$ real matrices |
| $\mathbb{N}_+$ | Set of positive integer |
| $A^T$ | Transpose of matrix $A$ |
| $A < 0$ | Negative definite matrix |
| $I$ | Identity matrix with appropriate dimension |
| $*^*$ | Symmetric term of the symmetric matrix |
| $\| \cdot \|_2$ | 2-norm of the matrix |
| $\mathbb{E}\{ \cdot \}$ | Mathematical expectation |
| $T(\cdot)$ | Trace of the matrix |
| $\text{col}(\cdot)$ | Block column matrix |
| $\text{diag}(\cdot)$ | Block diagonal matrix |
| $\lambda_{\max}(\cdot)$ | Maximum eigenvalue of the matrix |
TABLE III

Description of the Acronyms

| Acronym | Description                  |
|---------|------------------------------|
| NMFSs   | Networked multi-sensor fusion systems |
| WSN     | Wireless sensor network |
| PC      | Fusion center                |
| DRS     | Dimensionality reduction strategy |
| ETS     | Event triggered strategy |
| QF      | Quantization strategy         |
| S-RE    | Sensor-to-remote estimator |
| S-PC    | Sensor-to-fusion center       |
| S-NS    | Sensor-to-neighbor sensor     |
| TSE     | Taylor series expansion       |
| EKF     | Expansion Kalman filter       |
| UKF     | Unscented Kalman filter       |
| CKF     | Cubature Kalman filter        |
| LNS     | Local nonlinear estimator     |
| LRE     | Local remote estimator        |
| LCE     | Local compensation estimator  |
| DFE     | Distributed fusion estimator  |
| DCFE    | Distributed compensation fusion estimator |
| SE      | Square error                 |
| MSE     | Mean square error             |
| RMSE    | Root mean square error        |

II. PROBLEM FORMULATION

Consider a class of NMFSs with $L$ sensor nodes, and the nonlinear state-space model can be modeled by

\[
x(k + 1) = f(x(k)) + \Gamma(k)w(k)
\]

\[
y_i(k) = h_i(x(k)) + D_i(k)v_i(k), \quad i \in \mathcal{L}
\]

where $x(k) \in \mathbb{R}^n$ and $y_i(k) \in \mathbb{R}^{m_i}$ are the system state and measured output of the $i$th sensor, respectively, and $\mathcal{L} = \{1, 2, \ldots, L\}$. $f(x(k)) \in \mathbb{R}^n$ and $h_i(x(k)) \in \mathbb{R}^{m_i}$ are continuously differentiable nonlinear functions, and $\Gamma(k) \in \mathbb{R}^{n \times p}$ and $D_i(k) \in \mathbb{R}^{m_i \times q_i}$ are time-varying matrices. $w(k) \in \mathbb{R}^p$ and $v_i(k) \in \mathbb{R}^{q_i}$ are bounded noises without statistical information, which satisfy the following assumptions:

\[
\|w(k)\|_2 \leq M_w, \quad \|v_i(k)\|_2 \leq M_{v_i}
\]

where $M_w$ and $M_{v_i}$ are unknown. Generally, when designing the estimator based on the raw measurement information $\{y_i(1), y_i(2), \ldots, y_i(k)\}$, this is the network resource constraint problem is not considered in NMFSs. The local nonlinear estimator (LNE) can be given by [44]

\[
\begin{align}
\hat{x}_i^-(k) &= f(\hat{x}_i^-(k-1)) \\
\hat{x}_i^+(k) &= \hat{x}_i^-(k) + \mathcal{K}_i^+(k)y_i(k) - h_i(\hat{x}_i^-(k))
\end{align}
\]

where $\hat{x}_i^-(k)$ and $\hat{x}_i^+(k)$ are one-step prediction and local estimate, respectively, $\mathcal{K}_i^+(k)$ is the estimator gain matrix. Under this case, in order to minimize the upper bound of the estimation error, the $\mathcal{K}_i^+(k)$ and $\hat{x}_i^+(k)$ can be obtained by using a robust design approach [44, Th.1], the detailed deduction is omitted here.

A. ETS and DRS Over S-RE Channel

In NMFSs, the communication network allocating enough bits to transfer the raw measurements from the sensor to the remote estimator is usually impracticable, then the limitation of the network resources in the S-RE channel should be taken into account when designing nonlinear estimators. In this article, an ETS and a DRS are employed to address the fusion estimation problem of the nonlinear NMFSs (1) and (2) under resource constraints. Specifically, a structure of distributed fusion estimation under resource constraints is depicted in Fig. 1.

First, an ETS is adopted to alleviate the network resource burden. Specifically, a predesigned event-triggered condition is given to calculate the decision variable $y^r_m(k) \in [0, 1]$, which decides whether the raw measurement $y_i(k)$ is transmitted to the corresponding remote estimator or not. Then, an event-triggered condition for the $i$th S-RE channel is given by

\[
y^r_m(k) = \begin{cases} 
0, & \text{if } y_i(k) \notin \mathcal{D}^r_m(k) \\
1, & \text{otherwise}
\end{cases}
\]

\[
\mathcal{D}^r_m(k) = \{y_i(k) | \|y_i(k) - \hat{y}_i(k)\|_2 \leq \delta^r_m, \quad i \in \mathcal{L}\}
\]

where $\delta^r_m > 0$ is a predetermined triggering threshold, $\hat{y}_i(k)$ is the measurement for further dimensionality reduction processing at the latest event instant, $\mathcal{D}^r_m(k)$ is a measurement set of each sensor that the event is not occurred. Then, it can be found from the ETS (5)–(6) that the raw measurement $y_i(k)$ will be further processed when $y^r_m(k) = 1$, otherwise there is no measurement information sent to the remote estimator at time $k$. Thus, the event-triggered instants sequence $0 \leq t^r_m(1) \leq \cdots \leq t^r_m(k) \leq \cdots$ is determined by

\[
t^r_m(k + 1) = \min \left\{k \in \mathbb{N}_+ \mid k > t^r_m(k), \quad y_i(k) \notin \mathcal{D}^r_m(k) \right\}.
\]

On the other hand, in order to further reduce redundant communication traffic, a kind of DRS [33] is simultaneously adopted to reduce the size of the measurements from the event-triggered scheduler. Under this DRS, only $s^r_m(1 \leq s^r_m \leq m_i)$ components of measurement $y^r_m(k)y_i(k)$ are allowed to be sent to the remote estimator at each time, rather than all information of $y^r_m(k)y_i(k)$. Notice that, the measurement information does not need to be dimensionally reduced at a certain untriggered moment (i.e., $y^r_m(k) = 0$). Moreover, it is considered that the global bandwidth constraint is
known in advance
\[ \sum_{i=1}^{L} s^i_m = s_m \quad (s_m \in \Omega_+^- , \quad s^i_m \in \Omega_+ , \quad 1 \leq s^i_m \leq m_i) \quad (8) \]
where \( s_m \) is known global constraint, and \( s^i_m \) is the local constraint. Then, if the sensor \( i \) sends dimensionality reduction measurements to the corresponding remote estimator, the measurement received by the estimator will have \( b^i_m \) possible cases
\[ b^i_m = \frac{\prod_{j=0}^{i-1} (m_i - j)}{\prod_{j=1}^{i} I_j} . \]
In other words, only one dimensionality reduction information from the following set will be transmitted to the remote estimator at each instant:
\[ \chi^i_m(k) \triangleq \{ \gamma^i_m(k)\Theta^i_k y_i(k) \mid \kappa_i \in S^i_m \frac{\nu}{\Theta1} \{ 1, 2, \ldots, b^i_m \} \} \quad (10) \]
where \( \Theta^i_k \) is a 0-1 diagonal matrix, whose diagonal terms including \( s^i_m \) elements “1.” In fact, matrix \( \Theta^i_k \) shows the dimensionality reduction status of each measurement. Then, a binary variable \( \sigma^i_k \) is employed to describe the dimensionality reduction matrix in a clear way
\[ \sigma^i_k = \begin{cases} 1, & \text{if } z(k) = y^i_m(k)\Theta^i_k y_i(k) \\ 0, & \text{otherwise} \end{cases} \quad (11) \]
where \( z(k) \) is the transmission measurement based on ETS and DRS, and \( \sigma^i_k \) satisfying
\[ \begin{align*}
\sigma^i_k \sigma^i_m(k) &= 0, \quad \kappa_i \neq \kappa_m, \quad \kappa_m \in S^m_m, \\
\sum_{\kappa_i=1}^{b^i_m} \sigma^i_k &= 1, \quad i \in \mathcal{L}_i, \quad \kappa_i \in S^i_m. 
\end{align*} \]
In this case, the dimensionality reduction matrix \( \Theta^i_m(k) \) can be modeled by
\[ \Theta^i_m(k) = \sum_{\kappa_i=1}^{b^i_m} \sigma^i_k(k)\Theta^i_k \quad (13) \]
where it can be deduced from (13) that \( \Theta^i_m(k) \) is determined by \( \sigma^i_m(k) \). Then, it follows from (10)–(13) that \( z_m(k) \) can be expressed by
\[ z_m(k) = y^i_m(k)\Theta^i_m(k)(k) y_i(k) \quad (14) \]
where \( z_m(k) \) is the measurement information received by each remote estimator.

Since an ETS and a DRS are both employed to address the problem of networked fusion estimation under resource constraints, the estimation performance will be inevitably degraded. To ensure that the designed estimator has a satisfactory estimation performance, a unified compensation model is proposed in this article to compensate the untransmitted measurement information. Specifically, a compensation model is given by
\[ z^i_m(k) = z_m(k) + (I - y^i_m(k)\Theta^i_m(k)) h_i(\hat{x}^i_m(k)) \quad (15) \]
where \( z^i_m(k) \) is the compensatory measurement that can be used in the remote estimator. It can be deduced from the aforementioned compensation model that \( z^i_m(k) \) can be described as following two cases.

1) When the event is occurred (i.e., \( \gamma^i_m(k) = 1 \)), the compensatory measurement reduces to \( z^i_m(k) = \Theta^i_m(k) y_i(k) + (I - \Theta^i_m(k)) h_i(\hat{x}^i_m(k)) \), which implies that the dimensionality reduction measurement \( \Theta^i_m(k) y_i(k) \) can be transmitted to the remote estimator side, and then, partial prediction information \( h_i(\hat{x}^i_m(k)) \) is employed to compensate the dimensionality reduction components of the measurement \( y_i(k) \).

2) When the event is not occurred (i.e., \( \gamma^i_m(k) = 0 \)), the compensatory measurement reduces to \( z^i_m(k) = h_i(\hat{x}^i_m(k)) \), which implies that the remote estimator cannot receive any measurement information, and then, the prediction information \( h_i(\hat{x}^i_m(k)) \) is employed to estimate the system state.

Based on the aforementioned communication strategy and compensation strategy, the event-triggered local remote estimator (LRE) for the systems (1) and (15) is given by
\[ \begin{align*}
\hat{x}^i_m(k) &= f(\hat{x}^m_m(k - 1)) + K^i_m(k)[z^i_m(k) - h_i(\hat{x}^i_m(k))] \\
\hat{x}^m_m(k) &= \hat{x}^m_m(k) + K^m_m[k(\hat{x}^i_m(k) - h_i(\hat{x}^i_m(k)))] \quad (16) 
\end{align*} \]
where \( \hat{x}^i_m(k) \) and \( \hat{x}^m_m(k) \) are one-step prediction and local estimate, respectively, \( K^m_m \) is the estimator gain need to be designed.

Subsequently, on the basis of the LRE (16), the distributed fusion estimator (DFE) when the S-RE channel is subject to resource constraints is given by
\[ \hat{x}^m_m(k) = \sum_{i=1}^{L} W^m_m(k) \hat{x}^m_m(k) \quad (17) \]
where \( \hat{x}^m_m(k) \) is the fusion estimate, and \( W^m_m(k) \) is the distributed weighting fusion matrix.

In conclusion, the case of the S-RE channel subject to resource constraints is formulated in this subsection. Then, based on the aforementioned communication strategy, this article will address the following problems.

1) First, for a given set of binary variables \( \sigma^i_m(k) = \{ \sigma^i_m(k) \mid \kappa_i \in S^i_m , i \in \mathcal{L} \} \) that satisfies (8) and (12), and to determine the local gain \( K^m_m \) such that the SE of the local estimate \( \hat{x}^m_m(k) \) is bounded, i.e.,
\[ \lim_{k \to \infty} (x_i(k) - \hat{x}^m_m(k))^T (x_i(k) - \hat{x}^m_m(k)) < M^i_m \quad (18) \]
where \( M^i_m \) is a positive scalar, and then, to minimize an upper bound of the SE of the corresponding LRE at each instant.

2) Second, based on each LRE (16), to design distributed weighting fusion matrices \( \{ W^m_m(k) \} \sum_{i=1}^{L} \), \( W^m_m(k) = I \) in (17) such that an upper bound of the SE of the DFE is minimal at each instant.

**Remark 1** Generally, the local estimate \( \hat{x}(k) \) without network communication constraints can be calculated by
the LNE (4), while resource constraints are an inevitable problem to be considered in NMFSs. Therefore, the ETS (5)–(6) and the DRS (8)–(14) are adopted in a unified framework to satisfy the finite resources in the S-RE channel for the nonlinear NMFSs (1)–(2). Notice that, the untransmitted measurement information, which may cause a degradation of the estimation performance, are reconstructed by a unified compensation model (15). Then, the compensatory measurement \( z_i^e(k) \) instead of the raw measurement \( y_i(k) \) is used to design the estimators in this article. In this communication strategy, the introduced ETS and DRS provide the possibility to alleviate unnecessary resource consumption, while the compensation strategy provides a desirable estimation performance of the designed estimators.

**Remark 2** Although there are several dimensionality reduction methods have been developed in [18], [19], [20] to meet the finite communication resources, the calculation of the compression operator is complicated of these methods. Nevertheless, this article adopts another kind of DRS that directly selects partial components of measurement rather than using the data compression approach. In addition, it can be found from (8) that the global bandwidth constraint is assumed to be known in advance, thus a group \( \sigma_i^m(k) \) can be given to satisfy (12) by adjusting the local constraint \( \zeta_m^i(k) \) to satisfy (8), and then, the dimensionality reduction matrix \( \Theta_m^i(k) \) can be determined. For instance, if \( m_i = 4 \), \( i = 1, 2 \), and consider that the global bandwidth constraint is \( \zeta_m = 5 \), then the local bandwidth constraint \( \zeta_m^i(i = 1, 2) \) can be selected from the following set to satisfy (8):

\[
Q_m = \{ (\zeta_m^i, \zeta_m^j) : (1, 4), (2, 3), (3, 2), (4, 1) \}. \tag{19}
\]

If a bandwidth combination (2,3) from the set \( Q_m \) is chosen, then \( h_6^i = 6, h_5^i = 4 \) can be calculated by (9), and then, \( \Theta_m^i \) and \( \sigma_i^m(k) \) are determined by (10)–(12). It can be calculated from (13) that

\[
\begin{align*}
\Theta_m^i(k) = \text{diag}\{ \sigma_1^i(k) + \sigma_2^i(k) + \sigma_3^i(k), \sigma_4^i(k) + \sigma_5^i(k) + \sigma_6^i(k) \\ + \sigma_7^i(k) + \sigma_8^i(k) + \sigma_9^i(k), \sigma_1^i(k) + \sigma_2^i(k) + \sigma_3^i(k) \\ + \sigma_4^i(k) + \sigma_5^i(k) + \sigma_6^i(k) + \sigma_7^i(k) + \sigma_8^i(k) + \sigma_9^i(k) \} \\
\sigma_m^i(k) = \text{diag}\{ \sigma_1^i(k) + \sigma_2^i(k) + \sigma_3^i(k), \sigma_4^i(k) + \sigma_5^i(k) + \sigma_6^i(k) \\ + \sigma_7^i(k) + \sigma_8^i(k) + \sigma_9^i(k), \sigma_1^i(k) + \sigma_2^i(k) + \sigma_3^i(k) \\ + \sigma_4^i(k) + \sigma_5^i(k) + \sigma_6^i(k) + \sigma_7^i(k) + \sigma_8^i(k) + \sigma_9^i(k) \}.
\end{align*}
\]

Therefore, it can be concluded from (10)–(13) that \( \Theta_m^i(k) \) is decided by \( \sigma_i^m(k) \).

**Remark 3** Notice that, the innovation-based deterministic and stochastic ETSs have been well developed in [24], [25], [26], and [27] for networked time-invariant systems, and the stochastic ETS indeed provides an important advantage by preserving the Gaussian property of the innovation sequence. However, the stochastic ETS either requires feedback from the estimator or becomes irrelevant to the sensor model, which may increase the communication consumption and be problematic in applications of time-varying systems. In this article, the considered event-based problem is in the framework of nonlinear NMFSs without a Gaussian assumption, and thus, an effective deterministic ETS (5)–(6) is proposed. Generally, the estimation performance will be improved with the increase in the communication rate, but it may not be significantly improved after the communication rate reaches a certain level. In this case, the predefined threshold \( \delta_m^i \) should be suitably selected to meet a trade-off between the estimation performance and the resource constraints.

**B. ETS and DRS Over S-FC Channel**

It is worth noting that smart sensors are capable of calculating local estimates in some practical applications. Then, the local estimate from each smart sensor is sent to the FC through the communication network in another kind of networked distributed fusion framework. Hence, it is essential to address the distributed fusion estimation problem of the S-FC channel under restricted resources as well. In particular, a specific distributed compensation fusion estimation structure is depicted in Fig. 2.

![Fig. 2. Distributed fusion estimation for a class of nonlinear NMFSs when the S-FC channel is subject to resource constraints.](image-url)

Similarly, an estimate-based ETS with respect to local estimation is installed in each smart sensor to schedule the communication rates. In this scheme, the decision variable \( \gamma_i^e(k) (\in [0, 1]) \) is determined based on the following event-triggered condition:

\[
\gamma_i^e(k) = \begin{cases} 
0, & \text{if } x_i^e(k) \in \mathcal{D}_i^0(k) \\
1, & \text{otherwise}
\end{cases}
\]

\[
\mathcal{D}_i^0(k) = \{ x_i^e(k) | \| x_i^e(k) - \hat{x}_i^e(k) \|_2 \leq \delta_i^e, i \in \mathcal{I} \} \tag{22}
\]

where \( \delta_i^e > 0 \) is a predefined threshold, \( \hat{x}_i^e(k) \) is the latest transmitted estimate for further dimensionality reduction processing, and \( \mathcal{D}_i^0(k) \) denotes the local estimate set of each smart sensor at untriggered moments. Specifically, \( \gamma_i^e(k) = 1 \) indicates that the \( x_i^e(k) \) does not belong to \( \mathcal{D}_i^0(k) \) and will be processed further; otherwise, no estimate will be sent to the FC at time \( k \). Then, the event-triggered instants sequence \( 0 \leq t_1^e(k) \leq \cdots \leq t_i^e(k) \leq \cdots \) is determined by

\[
t_i^e(k+1) = \min \{ k \in \mathbb{N} \mid k > t_i^e(k), \ x_i^e(k) \notin \mathcal{D}_i^0(k) \}. \tag{23}
\]
Notice that, another kind of DRS [34] is introduced to lower estimation packets to meet the finite communication bandwidth in this subsection. Similarly, only partial estimates of $y_j^c(k)\tilde{x}_j^c(k)$ are allowed to be transmitted in this DRS. Namely, only $y_j^c(\varsigma_j^c \in \Omega_1, 1 \leq \varsigma_j^c \leq n)$ components of $\tilde{x}_j^c(k)$ have the opportunity to be transmitted to the FC at each instant. It is obvious that there is no estimate transmitted to the FC when $y_j^c(k) = 0$. Since the partial description of this DRS is similar to (9)–(13), the detailed description is omitted here. Then, in terms of the mathematical description, the transmitted state estimate $\tilde{x}_j^c(k)$ can only take one element from the following set:

$$\tilde{x}_j^c(k) \triangleq \left\{ y_j^c(\varsigma_j^c)\Theta_{h_j^c}^c\tilde{x}_j^c(\varsigma_j^c) \mid h_j \in \mathcal{S}_j^c \triangleq \left\{ 1, 2, \ldots, b_j^c \right\} \right\}$$ (24)

where $\Theta_{h_j}^c$ is also a 0-1 diagonal matrix with $\varsigma_j^c$ 1-elements, and $b_j^c = \prod_{i=0}^{\varsigma_j^c-1} (n-i)/\prod_{j=1}^{\varsigma_j^c}$. Moreover, the transmitted estimate $\tilde{x}_j^c(k)$ of each smart sensor can be formulated by

$$\tilde{x}_j^c(k) = y_j^c(\varsigma_j^c)\sum_{h_j=1}^{b_j^c} \sigma_j^c \Theta_{h_j}^c\tilde{x}_j^c(\varsigma_j^c) = y_j^c(\varsigma_j^c)\Theta_j^c(k)\tilde{x}_j^c(k)$$ (25)

where $\sigma_j^c \in \{0, 1\}$ is a binary variable for describing the dimensionality status, and the $\sum_{h_j=1}^{b_j^c} \sigma_j^c = 1$ should be held. In particular, $\Theta_j^c(k)$ is one of elements in the set $\{\Theta_{h_j}^c \mid h_j \in \mathcal{S}_j^c\}$, which depend on the choice of corresponding $\sigma_j^c(k)$. Furthermore, it is concluded from the aforementioned DRS that the practical communication status is related to the variable $\sigma_j^c(k)$ at each instant. In this case, the stochastic process $\sigma_j^c(k)$ is assumed to be independent and identically distributed (i.i.d.) [43], i.e.,

$$\mathbb{E} \left\{ \sigma_j^c(k)\sigma_j^c(k_0) \right\} = \begin{cases} 0, & i = j, k = k_0, h_i \neq h_i^0 \\ \mathbb{E} \{ \sigma_j^c(k) \}, & i = j, k = k_0, h_i = h_i^0 \\ \mathbb{E} \{ \sigma_j^c(k) \} \mathbb{E} \{ \sigma_{j_i}^c(k_0) \} & \forall i, j, k, h_i, h_i^0 \end{cases}$$ (26)

where $\mathbb{Pr}[\sigma_j^c(k) = 1] = \pi_{h_i}^c$ is the previously determined probability and satisfying

$$\sum_{h_i=1}^{b_j^c} \pi_{h_i}^c = 1.$$ (27)

It should be noted that the local bandwidth constraint $\varsigma_j^c$ is known instead of global bandwidth constraints in this communication strategy. Then, a group of $\sigma_j^c(k) = \{\sigma_j^c(h_i) \mid h_i \in \mathcal{S}_j^c, i \in \Omega_1\}$ will be generated based on the selection probabilities $\pi_{h_i}^c(h_i \in \mathcal{S}_j^c)$, which gives a different way for determining dimensionality matrix $\Theta_j^c(k)$.

At the same time, to improve the accuracy of the fusion estimation, a unified compensation model with respect to local estimation is also developed to address the fusion estimation problem. Then, the local compensation estimator (LCE) is modeled by

$$\tilde{x}_j^c(k) = \tilde{x}_j^c(k) + (I - \gamma_j^c(k)\Theta_j^c(k))f(\tilde{x}_j^c(k - 1))$$ (28)

where $\tilde{x}_j^c(k)$ is the compensatory estimate, and $f(\tilde{x}_j^c(k - 1))$ is the one-step prediction of the local compensation estimation. For the compensation model (28), when $\gamma_j^c(k) = 1$, the dimensionality reduction estimate $\tilde{x}_j^c(k)$ will be transmitted to the FC, and $(I - \Theta_j^c(k))f(\tilde{x}_j^c(k - 1))$ is utilized to compensate the untransmitted components of $\tilde{x}_j^c(k)$; otherwise, the FC will not receive any local estimation at time $k$, and then, the prediction $f(\tilde{x}_j^c(k - 1))$ is employed to compensate the untransmitted local estimation directly.

Next, when the S-FC channel is subject to resource constraints, the distributed compensation fusion estimator (DCFE) is given by

$$\hat{x}_c(k) = \sum_{i=1}^{L} W_i^c(k)\tilde{x}_i^c(k)$$ (29)

where $\hat{x}_c(k)$ is the compensatory fusion estimate, and $W_i^c(k)$ is the weighting fusion matrix to be determined.

In conclusion, the case of the S-FC channel subject to resource constraints is considered in this subsection. For a different communication strategy mentioned previously, the following problems in this article will be addressed:

1) First, the selection probabilities $\pi_{h_i}^c \prod_{y=1}^{b_i^c} \pi_{h_i}^c = 1$ in (27) should be determined such that the mean square error (MSE) of the LCE (28) is bounded, i.e.,

$$\lim_{k \to \infty} \mathbb{E} \left( (x(k) - \hat{x}_c(k))^T (x(k) - \hat{x}_c(k)) \right) < \mathcal{M}_i^c$$ (30)

where $\mathcal{M}_i^c$ is a positive scalar.

2) Second, based on each LCE (28), the distributed weighting fusion matrices $W_i^c(k) \sum_{i=1}^{L} W_i^c(k) = I$ in (29) should be determined such that an upper bound of the MSE of the DCFE is minimal at each instant.

**Remark 4** When considering that the S-FC channel is subject to resource constraints, an unfixed structure DRS [34] is adopted in this subsection. Since the $\Theta_j^c(k)$ in (25) is decided by the $\sigma_j^c(k)$ and the $\Theta_j^c(k)$ is an i.i.d. stochastic process, the $\Theta_j^c(k)$ is a random matrix. In this case, a distributed fusion Kalman filter was proposed in [34] for linear NMFS under communication constraints, while the system noise covariances need to be known a priori. To overcome this limitation, a bounded recursive optimization scheme has been developed in [43] to deal with the bounded noises, in which a QS and a DRS were utilized to alleviate the communication burden. However, the design problem of event-triggered estimators for distributed nonlinear NMFSs with unknown noise statistical properties is still a challenging issue. Therefore, the ETS (21)–(22) and the DRS (24)–(27) are proposed in this article with a unified framework for the nonlinear NMFSs without noise statistical properties, and then, the nonlinear compensation estimators
Remark 5 Although the DRS and the ETS were used in [42] to address the networked fusion estimation problems under resource constraints, it was still in a linear framework with the Gaussian noise assumption. Moreover, the original local estimate was directly utilized to compensate for the untransmitted estimate, while it seems unrealistic to have a complete local estimate in the FC under this communication strategy. In contrast, the one-step prediction \( f(x_i^*(k - 1)) \) of the local compensation estimate \( x_i^*(k - 1) \) is ideal as the compensation information, which can be obtained in the FC. Meanwhile, the ETS in [42] was based on the dimensionality reduction estimation, while the ETS in this article is based on the original local estimation. Under the proposed communication strategy, the DRS (24)–(25) is not required to work when \( \gamma_i^0(k) = 0 \) (i.e., the event is not occurred at time \( k \)), which can save unnecessary resource consumption.

III. MAIN RESULTS

Before deriving the main results, a useful Lemma is introduced as follows.

Lemma 1 [47] Let \( S_1 = S_1, S_2 \) and \( S_3 \) be real matrices of appropriate dimensions with \( P(k) \) satisfying \( P^{T}(k)P(k) \leq I \). Then,

\[
S_1 + S_2P(k)S_2 + S_2^TP(k)S_1^T < 0
\]

If and only if there exists a positive scalar \( \varepsilon > 0 \) such that

\[
\begin{bmatrix}
-\varepsilon I & \varepsilon S_2 & 0 \\
* & S_1 & S_3 \\
* & * & -\varepsilon I
\end{bmatrix} < 0.
\]

A. Distributed Fusion Estimation Based on Compensation Measurement for Nonlinear NMFSs

When considering the estimation problems of the S-RE channel under resource constraints, a deterministic ETS and a directly DRS are adopted in Section II-A to meet the finite communication resources. Then, according to the designed LRE (16) and DFE (17), the problems of determining the local estimator gain \( K_i^m(k) \) and the distributed weighting fusion matrix \( W_i^m(k) \) for nonlinear NMFSs (1)–(2) will be solved in Theorem 1.

Theorem 1 For a predefined threshold \( \delta_i^m \) satisfying (5), (6), and a given global bandwidth constraint \( \zeta_m \) and a group of binary variables \( \sigma_i^m(k) \) satisfying (8)–(14), the following convex optimization problem is developed to determine each LRE gain \( K_i^m(k) \):

\[
\begin{aligned}
\min_{\Psi_i(k), \Theta_i(k), \Phi_i(k), \Gamma_i(k), \Theta_i^m(k)} & \quad \Psi_i(k) + \Gamma_i(k) + \Phi_i(k) + \Theta_i^m(k) \\
\text{s.t. :} & \quad \begin{bmatrix}
-\varepsilon_1(k)I & O_1^m(k) & 0 \\
* & \Delta_m(k) & K_i^W(k) \\
* & * & -\varepsilon_1(k)I
\end{bmatrix} < 0 \\
& \quad \begin{bmatrix}
-\varepsilon_2(k)I & O_2^m(k) & 0 \\
* & \Delta_m(k) & K_i^W(k) \\
* & * & -\varepsilon_2(k)I
\end{bmatrix} < 0 \\
& \quad \begin{bmatrix}
-\varepsilon_3(k)I & O_3^m(k) & 0 \\
* & \Delta_m(k) & K_i^W(k) \\
* & * & -\varepsilon_3(k)I
\end{bmatrix} < 0
\end{aligned}
\]

where

\[
\begin{aligned}
K_i^m(k) & \triangleq \gamma_i^0(k)K_i^m(k) - O_i^m(k)C_i^m(k)C_i^m(k) \backslash \Theta_i(k) \\
K_i^c(k) & \triangleq \gamma_i^0(k)K_i^c(k) - O_i^c(k)C_i^c(k)C_i^c(k) \backslash \Theta_i(k) \\
K_i^L(k) & \triangleq \gamma_i^0(k)K_i^L(k) - O_i^L(k)C_i^L(k)C_i^L(k) \backslash \Theta_i(k) \\
K_i^M(k) & \triangleq \gamma_i^0(k)K_i^M(k) - O_i^M(k)C_i^M(k)C_i^M(k) \backslash \Theta_i(k) \\
K_i^F(k) & \triangleq \gamma_i^0(k)K_i^F(k) - O_i^F(k)C_i^F(k)C_i^F(k) \backslash \Theta_i(k) \\
O_i^L(k) & \triangleq \begin{bmatrix}
I & (K_i^L(k)M_i^L(k) - \gamma_i^0(k)M_i^L(k)) \Theta_i(k) \\
-\gamma_i^0(k)M_i^L(k) \Theta_i(k) & I
\end{bmatrix} \\
O_i^M(k) & \triangleq \begin{bmatrix}
I & (K_i^M(k)M_i^M(k) - \gamma_i^0(k)M_i^M(k)) \Theta_i(k) \\
-\gamma_i^0(k)M_i^M(k) \Theta_i(k) & I
\end{bmatrix} \\
O_i^F(k) & \triangleq \begin{bmatrix}
I & (K_i^F(k)M_i^F(k) - \gamma_i^0(k)M_i^F(k)) \Theta_i(k) \\
-\gamma_i^0(k)M_i^F(k) \Theta_i(k) & I
\end{bmatrix}
\end{aligned}
\]

Then, the SE of the local estimate \( \tilde{x}_i^m(k) \) in (16) will be bounded, i.e.,

\[
\lim_{k \to \infty} (\tilde{x}_i^m(k))^T \tilde{x}_i^m(k) < \mathcal{M}_i^m
\]

where \( \mathcal{M}_i^m \) is a positive scalar. Moreover, a convex optimization problem is established to obtain distributed weighting fusion matrix \( W_i^m(k) \) as follows:

\[
\begin{aligned}
\min_{\Psi_i(k), \Theta_i(k), \Phi_i(k), \Gamma_i(k), \Theta_i^m(k)} & \quad \Psi_i(k) + \Gamma_i(k) + \Phi_i(k) + \Theta_i^m(k) \\
\text{s.t. :} & \quad \begin{bmatrix}
-\varepsilon_1(k)I & O_1^m(k) & 0 \\
* & \Delta_m(k) & K_i^W(k) \\
* & * & -\varepsilon_1(k)I
\end{bmatrix} < 0 \\
& \quad \begin{bmatrix}
-\varepsilon_2(k)I & O_2^m(k) & 0 \\
* & \Delta_m(k) & K_i^W(k) \\
* & * & -\varepsilon_2(k)I
\end{bmatrix} < 0 \\
& \quad \begin{bmatrix}
-\varepsilon_3(k)I & O_3^m(k) & 0 \\
* & \Delta_m(k) & K_i^W(k) \\
* & * & -\varepsilon_3(k)I
\end{bmatrix} < 0
\end{aligned}
\]
Algorithm 1: Distributed Networked Fusion Estimation under S-RE Channel Resource Constraints.

1: Initialization states \( x(0) \) and \( \hat{x}_m(0) (i \in I) \);
2: for \( i := 1 \) to \( L \) do
3: Given a predefined threshold \( \delta_m \) to calculate \( y_m(k) \);
4: Given a known global bandwidth constraint \( z_m \), and choose a group of binary variables \( \sigma_m(k) \) to determine \( \Theta_m(k) \);
5: Solve the convex optimization problem (31) by using the ‘mincx’ function of MATLAB LMI Toolbox, then determine LRE gain \( K_m(k) \);
6: Calculate \( \hat{x}_m(k) \) by (16);
7: end for
8: Based on each \( \hat{x}_m(k) \), determine weighting fusion matrices \( W_m^k(k) (i \in I) \) by solving (34);
9: Calculate fusion estimate \( \bar{x}_m(k) \) by (17);
10: Return to step 2 and repeat steps 2-9 to calculate \( \hat{x}_m(k+1) \) and \( \bar{x}_m(k+1) \).

where

\[
\begin{align*}
A_m^I(k) & \triangleq \text{diag}[A_m^I(k), \ldots, A_m^N(k)] \\
\Gamma_0(k) & \triangleq \text{col}[\Gamma(k), \ldots, \Gamma(k)] \\
\sigma_m^I(k) & \triangleq \text{diag}[\sigma_m^1(k) I, \ldots, \sigma_m^L(k) I] \\
K_m^m(k) & \triangleq \text{diag}[K_m^m_1(k) M^I_1(k), \ldots, K_m^m_N(k) M^I_N(k)] \\
K_m^W(k) & \triangleq W_m(k) \text{diag}[K_m^W_1(k), \ldots, K_m^W_N(k)] \\
K_m^W(k) & \triangleq \text{diag}[D_k^1(k), \ldots, D_k^N(k)] \\
K_m^W(k) & \triangleq [W_m(k) K_m^W(k)]^T \\
O_m^I(k) & \triangleq [0 \quad \varepsilon_1(k) I \quad 0] \\
O_m^A(k) & \triangleq [0 \quad \varepsilon_3(k) \sigma_m^I(k) \quad 0] \\
O_m^A(k) & \triangleq [0 \quad \varepsilon_3(k) A_m^I(k-1) \quad \varepsilon_3(k) \Gamma_0(k-1) \quad 0] \\
\Delta_m(k) & \triangleq \frac{1}{3} \begin{bmatrix}
-I & K_m^W(k) & K_m^W(k) \\
* & -\Psi(k) & -\Psi(k) \\
* & -\Phi(k) & -\Phi(k) \\
* & * & -\gamma(k)
\end{bmatrix}
\end{align*}
\]

\( (35) \)

**Proof:** See the proof in Appendix A.

Based on Theorem 1, the fusion estimation problem of the NMFSs can be addressed in this subsection when the S-RE communication channel is under resource constraints. In summary, the local estimate \( \hat{x}_m(k) \) in (16) and the fusion estimate \( \bar{x}_m(k) \) in (17) can be obtained by implementing the Algorithm 1.

**Remark 6** For nonlinear cyber-physical systems with bounded noises, a security fusion estimation problem subject to DoS attacks has been investigated in [45]. Since the linearization errors caused by the TSE were modeled as bounded noises, then the stability of the designed estimator has not been addressed in [45]. Indeed, the neglected linearization errors can have an influence on the stability analysis and the estimation precision of the estimator. Therefore, this article introduces the state-dependent matrices and the unknown bounded parameters to model the high-order terms of the TSE. In this case, the stability of the designed nonlinear estimator (12) can be further analyzed, and then, the stability conditions such that the SE of the local/fusion estimators is asymptotically bounded is presented in Theorem 1.

B. Distributed Fusion Estimation Based on Local Compensation Estimate for Nonlinear NMFSs

Since the networked fusion estimation problem of the S-FC channel under resource constraints is also taken into account in this article, the ETS (21)–(22) and the DRS (24)–(27) are proposed in Section II-B. In this subsection, the process for determining the distributed weighted fusion matrix \( W_m^k(k) \) is presented in Theorem 2.

**Theorem 2** For a given triggering threshold \( \delta_m > 0 \), if there exist integer \( N_i \geq 0 \) and \( \rho_m(k) > 0 \) such that the selection probability \( \pi_m(k) \) in (27) satisfying

\[
\begin{bmatrix}
-\rho_m(k) I & 0 & \rho_m(k) I & 0 \\
* & -I & \Theta_m^A(k-1) & \Theta_m^I(k) \\
* & * & -I & 0 \\
* & * & * & -\rho_m(k) I
\end{bmatrix} < 0
\]

\( (36) \)

\[
\|\bar{R}_m(k - N_i, \bar{R}_m(k - N_i - 1, \bar{R}_m(\ldots, \bar{R}_m(k, I)))).\|_2 < 1.
\]

\( (37) \)

Then, the MSE of the LCE (28) will be asymptotically bounded. Moreover, a convex optimization problem is constructed to calculate distributed weighted fusion matrix \( W_m^k(k) \) as follows:

\[
\min_{\Delta(k) \geq 0, \Theta_m(k) \geq 0, \Theta_m^I(k) \geq 0, \bar{R}_m(\ldots, \bar{R}_m(k, I))} \text{Tr} \{ \Xi(k) \} + \text{Tr} \{ \Lambda(k) \} + \text{Tr} \{ \Sigma(k) \}
\]

\[
\begin{bmatrix}
-\rho_m(k) I & O_m^I(k) & 0 \\
* & \Delta_m(k) & L_m^W(k) \\
* & * & -\rho_m(k) I
\end{bmatrix} < 0
\]

\( (38) \)
where
\[
\begin{align*}
A_T^j(k) &\triangleq \text{diag}(A_{11}^j(k), \ldots, A_{22}^j(k)) \\
A_L^j(k) &\triangleq \text{diag}(A_{11}^j(k), \ldots, A_{22}^j(k)) \\
L_T^j(k) &\triangleq \text{diag}(L_{11}^j(k), \ldots, L_{22}^j(k)) \\
\alpha_T^j(k) &\triangleq \text{diag}(\alpha_{11}^j(k), \ldots, \alpha_{22}^j(k)) \\
L_L^j(k) &\triangleq \text{diag}(L_{11}^j(k), \ldots, L_{22}^j(k)) \\
K_T^j(k) &\triangleq \text{diag}(K_{11}^j(k), \ldots, K_{22}^j(k)) \\
K_L^j(k) &\triangleq \text{diag}(K_{11}^j(k), \ldots, K_{22}^j(k)) \\
\Theta_T^j(k) &\triangleq \text{diag}(\Theta_{11}^j(k), \ldots, \Theta_{22}^j(k)) \\
\Theta_L^j(k) &\triangleq \text{diag}(\Theta_{11}^j(k), \ldots, \Theta_{22}^j(k)) \\
\end{align*}
\]

**Algorithm 2: Distributed Compensation Fusion Estimation under S-FC Channel Resource Constraints.**

1: Initialization states \(x(0)\) and \(\hat{x}_c(0)\);
2: for \(i := 1 \text{ to } L\) do
3: Calculate LNE gain \(K_i^j(k)\) and each local estimate \(\hat{x}_i^j(k)\) by [44, Th.1];
4: Given a predefined threshold \(\delta_i^j\) to calculate \(\gamma_i^j(k)\);
5: Select probabilities \(\pi_i^j(h_i \in \mathcal{D}_i^j)\) to satisfy (36) and (37), and then generate a group of binary variables \(\alpha_i^j(k)\) to determine \(\Theta_i^j(k)\);
6: Calculate local compensatory estimate \(\hat{x}_i^j(k)\) by (28);
7: Solve the convex optimization problem (38) to determine weighting fusion matrices \(W_i^j(k)(i \in \mathcal{L})\);
8: Calculate fusion estimate \(\hat{x}_c(k)\) by (29);
9: end for

9: Return to step 2 and repeat steps 2-9 to calculate \(\hat{x}_c(k+1)\).

**Proof:** See the proof in Appendix B.

Based on Theorem 2, the fusion estimation problem of the NMFSs can be addressed in this subsection when the S-FC communication channel is under resource constraints, and the compensatory fusion estimate \(\hat{x}_c(k)\) can be calculated by implementing the Algorithm 2.

**Remark 7** According to the DRS (24)–(27), each smart sensor can generate a group of stochastic variables \(\sigma_i^j(h_i \in \mathcal{D}_i^j)\) for the given selection probabilities \(\pi_i^j(h_i \in \mathcal{D}_i^j)\). Therefore, when \(\pi_i^j(h_i \in \mathcal{D}_i^j)\) is selected from the developed stability conditions (36)–(37), the information transmission matrix \(\Theta_i^j(k)\) in (25) can be determined because it is decided by \(\sigma_i^j(h_i \in \mathcal{D}_i^j)\). Then, the LCE (28) can be calculated based on the decision variable \(\gamma_i^j(k)\) and dimensionality matrix \(\Theta_i^j(k)\). In this case, under the proposed communication strategy (21)–(27), the probability-dependent selection criterion (36)–(37) provides a pledge such that the MSEs of the designed LCE (28) and DCFE (29) are asymptotically bounded in this article.

**IV. SIMULATION EXAMPLE**

Consider a vehicle localization system in the 2-D horizontal space, and the vehicle’s motion state is defined by
\[
x(k) = \text{col}(p_x, p_y, \theta(k)),\quad \text{where} \quad p_x \text{ and } p_y \text{ are the position of vehicle along } X- \text{ and } Y- \text{axes, respectively, and } \theta(k) \text{ denotes the heading angle. Then, the motion model of the moving vehicle can be given by [45]}
\]
\[
\begin{align*}
\dot{p}_x(k+1) &= \dot{p}_y(k+1) = p_x(k) + C_r^0 \cos \left( \theta(k) + \frac{hd_0^j(k)}{2} \right) \\
\dot{\theta}(k+1) &= \dot{\theta}(k) + \dot{\theta}(k) \sin \left( \theta(k) + \frac{hd_0^j(k)}{2} \right) \\
\end{align*}
\]
\[
\begin{align*}
(40)
\end{align*}
\]
where \(C_r\) and \(C_t\) are the motion commands to control the translational velocity and rotational velocity, respectively, \(w_t(k)\) and \(w_r(k)\) are bounded disturbances, and \(h_0\) is the sampling period.

From (40), the nonlinear function and process noise can be expressed by
\[
\begin{align*}
\begin{bmatrix}
p_x(k+1) \\
p_y(k+1) \\
\theta(k+1)
\end{bmatrix}
\end{bmatrix} \\ w(k) \triangleq \text{col}\left( \frac{\dot{\theta}(k) \cos \left( \theta(k) + \frac{hd_0^j(k)}{2} \right)}{C_r}, \frac{\dot{\theta}(k) \sin \left( \theta(k) + \frac{hd_0^j(k)}{2} \right)}{C_r}, k_0w_r(k) \right) \\
\Gamma(k) = \text{diag}[1, 1, k_0].
\end{align*}
\]
\[
\begin{align*}
(41)
\end{align*}
\]
In addition, six distance sensors divided into two groups are set to track the moving vehicle, and the distance between each sensor and the vehicle can be calculated by
\[
d_i^j(k) = \sqrt{\left( p_x(k) - p_x^j \right)^2 + \left( p_y(k) - p_y^j \right)^2}
\]
\[
(42)
\]
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where \((p^1_{x_s}, p^2_{x_s})(i = 1, 2; j = 1, 2, 3)\) are the positions of the sensors in the \(X-Y\) plane. When considering the measurement noises, the measurement information can be obtained by

\[
y_i(k) = h_i(x(k)) + D_i v_i(k)
\]

(43)

where \(h_i(x(k)) = \text{col} \{d^1_i(k), d^2_i(k), d^3_i(k)\} \). Thus, the motion vehicle model \((40)-(43)\) can be formulated as \((1)-(2)\).

In this example, some common parameters are taken as \(k_0 = 1.8, c_t = 0.7, c_r = 0.8\); the measurement noise matrices \(D_1 = \text{diag} \{0.7, 0.6, 0.5\}, D_2 = \text{diag} \{0.6, 0.7, 0.8\}\), and the system noises \(w_i(k), w_i(k)\), and \(v_i(k)(i = 1, 2)\) are given by

\[
\begin{align*}
\begin{cases}
w_1(k) = 0.3 \beta_1(k) - 0.1; w_2(k) = 0.2 \beta_1(k) - 0.1 \\
v_1(k) = \text{col} \{0.3 \beta_1(k) - 0.2, 0.2 \beta_1(k) - 0.1 \} \\
v_2(k) = \text{col} \{0.2 \beta_21(k) - 0.1, 0.5 \beta_22(k) - 0.3 \}
\end{cases}
\end{align*}
\]

(44)

where \(\beta_1(k), \beta_1(k), \) and \(\beta_2(j)(i = 1, 2; j = 1, 2, 3)\) are uniformly distributed random variables over \([0, 1]\). Meanwhile, the coordinates of six distance sensors are \((p^1_{x_1}, p^2_{x_1}) = (-25, -5), (p^1_{x_2}, p^2_{x_2}) = (-30, 15), (p^1_{x_3}, p^2_{x_3}) = (-25, 35), (p^1_{x_4}, p^2_{x_4}) = (25, -5), (p^2_{x_4}, p^2_{x_4}) = (30, 15),\) and \((p^2_{x_5}, p^2_{x_5}) = (25, 35)\), respectively.

A. ETS and DRS Based on Measurement Information

When the S-RE channel is suffering from resource constraints, the compensation measurement \((15)\) is used to estimate the system state \((40)\) by the theoretical analysis in Section II-A. Based on the ETS (5)-(6), the triggering thresholds in Case A are set to \(\delta_{m} = \delta_{m} = 1.2\), respectively. Since there are three bandwidth channels in the measurement model \((43)\), assuming the global bandwidth constraint is \(\tilde{s}_m = 4\) in this example, and then, the local bandwidth constraint \(\tilde{s}_m(i = 1, 2)\) can be chosen from the following set to satisfy \((8)\):

\[
\mathcal{Q}_m = \left \{ (\tilde{s}_m, \tilde{s}_m^2) | (2, 2), (1, 3), (3, 1) \right \}.
\]

(45)

If the bandwidth group \((2, 2)\) is chosen, each measurement group has \(h_1^1 = h_1^2 = 3\) different transmission statuses. In particular, it can be concluded from Remark 2 that the dimensionality reduction matrix \(\Theta_m^1(k)\) is determined by \(\sigma_{\delta_m}(k)\), i.e.,

\[
\begin{align*}
\Theta_m^1(k) &= \text{diag} \{\sigma_1^1(k) + \sigma_2^1(k), \sigma_3^1(k) + \sigma_4^1(k) \} \\
\Theta_m^2(k) &= \text{diag} \{\sigma_1^2(k) + \sigma_2^2(k), \sigma_3^2(k) + \sigma_4^2(k) \}
\end{align*}
\]

(46)

Thus, the matrix \(\Theta_m^1(k)\) can be determined at each time. Similarly, by selecting the local bandwidth constraints from the \(\mathcal{Q}_m\), the corresponding \(\Theta_m^1(k)\) can be determined.

Moreover, by using the Taylor series expansion, the linearized model for the nonlinear system \((40)-(43)\) are given by

\[
\begin{align*}
x(k) &= f_i(x_m(k - 1)) + A^{p}_m(k - 1) x_m(k - 1) + M^{p}_m(k - 1) \tilde{x}_m(k - 1) + \Gamma(k - 1) w(k - 1) \\
y_i(k) &= h_i(x_m(k)) + C^{m}_m(k) \tilde{x}_m(k) + M^{p}_m(k) N^{p}_m(k) \tilde{x}_m(k) + D_i v_i(k)
\end{align*}
\]

(47)

where \(\Gamma(k), w(k),\) and \(v_i(k)\) have been given in \((43)\) and \((44)\), and

\[
\begin{align*}
A^{p}_m(k) &= \begin{bmatrix} 0 & -\frac{\omega}{c_r}\sin \left(\theta(k) + \frac{\theta}{c_r}\right) \\
0 & \frac{\omega}{c_r}\cos \left(\theta(k) + \frac{\theta}{c_r}\right) \\
0 & 1 \\
0 & 1 \\
\end{bmatrix} \\
C^{m}_m(k) &= \begin{bmatrix} \frac{p^2_{x_1}(k)}{\sqrt{p^2_{x_1}(k) + p^2_{x_1}(k)}} & \frac{p^2_{x_2}(k)}{\sqrt{p^2_{x_2}(k) + p^2_{x_2}(k)}} \\
\frac{p^2_{x_3}(k)}{\sqrt{p^2_{x_3}(k) + p^2_{x_3}(k)}} & \frac{p^2_{x_4}(k)}{\sqrt{p^2_{x_4}(k) + p^2_{x_4}(k)}} \\
\frac{p^2_{x_5}(k)}{\sqrt{p^2_{x_5}(k) + p^2_{x_5}(k)}} & \frac{p^2_{x_6}(k)}{\sqrt{p^2_{x_6}(k) + p^2_{x_6}(k)}} \\
\end{bmatrix} \\
\end{align*}
\]

(48)

Here, \(\tilde{p}_c(k) \triangleq p_c(k) - p^1_{x_s}(k),\) and \(\tilde{p}_c(i)(i = 1, 2; j = 1, 2, 3)\). Moreover, the state-dependent matrices in \((51)\) are taken as \(M^1 f = \text{diag} \{0.02, 0.01, 0.03\}, M^2 f = \text{diag} \{0.03, 0.01, 0.02\}, M^1 p = \text{diag} \{0.03, 0.03, 0.02\}, M^2 p = \text{diag} \{0.03, 0.02, 0.02\},\) and \(\alpha^2 c = \alpha^2 c = 1\).

In order to show the effectiveness of the proposed nonlinear fusion estimation method that is summarized in Algorithm 1, the vehicle’s motion trajectory and the fusion estimation trajectory are depicted in Fig. 3(a). It is seen from this subfigure that the introduced ETS (5)-(6) and DRS (8)-(14) are such that the proposed nonlinear DFE algorithm can track the vehicle’s motion trajectory well. Moreover, the estimation performance is evaluated in terms
of the root mean square error (RMSE), and the RMSEs of the LREs (16) and the DFE (17) with 100 Monte Carlo runs are shown in Fig. 3(b). It can be seen that the estimation error of the nonlinear fusion estimator is less than those of each local estimator, which implies that the estimation precision can be further improved by the developed nonlinear fusion estimation algorithm under the communication strategy presented in Section II-A. However, it should be pointed out that the superiority of fusion estimation over local estimation in Fig. 3(b) is not as pronounced as it would be without resource constraints (see Fig. 6), this is primarily due to the fact that more redundant information can lead to better fusion performance, while the introduced ETS and DRS eliminate a lot of redundant information. At the same time, the triggering status of two group measurements is plotted in Fig. 3(c), where the value “1” indicates that the event is occurred and the raw measurements can be further dimensionality reduced; otherwise, the corresponding remote estimator cannot receive any measurements from the sensor node. As shown in this subfigure, there are some untriggered instants in the corresponding time interval, which implies that the measurements can be effectively reduced based on the proposed ETS to meet the finite resources. On the other hand, the sequences of the decision variables $\sigma_k^i$ in (46) are plotted in Fig. 4. It can be seen that $\sigma_m^i = \sum_{i=1}^{3} \sigma_k^i(k) = 2$ and $\sigma_m = 4$ satisfy the bandwidth constraint (8), and then, the corresponding dimensionality matrix $\Theta_m^i(k)$ in (46) is also determined based on each $\sigma_k^i(k)$. Therefore, it can be summarized from Figs. 3(a)–(c) and 4 that Algorithm 1 is effective in the reduction of information transmission while preserving satisfactory estimation performance.

To show the impact of the event-triggered threshold $\delta_m$ on the estimation performance, the RMSEs of LRE (16) and DFE (17) with different communication rates (caused by ETS) is depicted in Fig. 5(a). It is seen from this subfigure that the higher communication rates present a better estimation performance, this is because the higher communication rates can provide more measurements transmitted to the corresponding remote estimators. Notice that, when the communication rate is increased to 40%, the estimation performance is not significantly improved. Therefore, the predefined triggering threshold can be appropriately adjusted to achieve lower communication rates while ensuring satisfactory estimation performance. Moreover, the superiority of the proposed compensation strategy (15) is presented in Fig. 5(b). It shows the performance of the fusion estimator compensated by the last transmission measurement $z_{k-1}$ and the prediction information $h_k(x_m^k)$, respectively. Obviously, the proposed compensation strategy in this article has a better estimation performance. Meanwhile, the compensation strategy [45] by using the last transmission measurement, which caused the system to augment when designing the estimator, will increase the extra computational burden.

Subsequently, on the basis of multisensor fusion systems without resource constraints, the comparisons of fusion estimation performance for different communication strategies are presented in Fig. 6. Since Gaussian noise is a kind of bounded noise in most practical applications, the proposed Algorithm 1 can also be used. Then, the noise covariances are set as $Q_w = \text{diag}[1, 1, 1] \times 10^{-3}$, $Q_v = \text{diag}[1, 1, 1] \times 10^{-3}$ in this example. Fig. 6(a) shows the fusion estimation performance under different communication strategies, it can be seen that the estimation accuracy of NMFSs without resource constraints (i.e., WRC, the red line) is the highest as compared with several other communication strategies. Obviously, this is because local estimators without resource constraints can provide more information. Moreover, if only DRS is used to deal with the bandwidth constraints, it cause a degradation in estimation performance (the green line). This is because some communication bandwidth was abandoned to transmit the measurement information, while partial measurements will still be transmitted to the remote estimator each time. If only using ETS to meet the resource constraints, the estimation performance is also degraded.
Fig. 6. (a) Effect of fusion estimation under different communication strategies with Gaussian noises. (b) Effect of fusion estimation under different communication strategies with bounded noises.

relative to without resource constraints (the blue line). This is because the ETS will cause the entire measurement information to be discarded at an untriggered instant. In comparison, when considering ETS and DRS in a unified framework, the estimation performance is the worst among several communication strategies (the magenta line), but more communication traffic can be reduced to solve the estimation problem in NMFSs under resource constraints. Meanwhile, the statistical properties of system noise are difficult to obtain accurately in most actual environments. When considering the bounded noises, the estimation performance under different communication strategies is presented in Fig. 6(b). It has a similar comparison result as the Gaussian noise case. Most importantly, with the compensation model proposed in this article, the estimation performance of other communication strategies is not significantly degraded compared to the case without resource constraints.

To further show the advantages of the proposed method, the comparison of estimation performance between the proposed method and other classical nonlinear filters (such as EKF [49], UKF [50], and CKF [51]) is presented in Fig. 7. First, when considering multisensor fusion systems without resource constraints, Fig. 7(a)–(b) shows the estimation effect of these methods under Gaussian noises and bounded noises, respectively. It can be seen that these nonlinear methods have similar estimation performances under Gaussian noise, where the EKF, UKF, and CKF are usually required to know the accurate statistical characteristics of noise. However, when the statistical information of the system noises cannot be obtained accurately [such as (44)], then the RMSE of the proposed method is the lowest, and the RMSEs of the EKF, UKF, and CKF methods are very similar, which indicates that the estimation accuracy of the proposed method is better than other methods in the case of unknown bounded noises. Notice that, this article focuses on the fusion estimation problem of NMFSs under resource constraints. Thus, the comparisons of estimation performance for these methods under Gaussian noises and bounded noises are depicted in Fig. 7(c) and (d), respectively. It is seen from these subfigures that the proposed nonlinear estimation method based on ETS and DRS has better estimation accuracy in both noise cases. In fact, these classical nonlinear estimation methods and various extension methods are usually designed to deal with Gaussian noises with known covariances, while the bounded noises with unknown statistical information are addressed in this article for designing nonlinear estimators. Therefore, in the case of bounded noises without accurate covariances in practical applications, the designed nonlinear estimator has a wider range of applications.

B. ETS and DRS Based on Local Nonlinear Estimation

In this example, when the S-FC channel is under limited resources, the motion trajectory (40) of the vehicle can be tracked by implementing Algorithm 2. Specifically, the triggering thresholds in ETS (21)–(22) are set as \( \delta_1 = \delta_2 = 1.0 \). Then, considering there are two components of the local estimate \( \hat{x}^L_i(k) \) that can be transmitted to the FC, \( b_1 = b_2 = 3 \) is calculated by (9). Therefore, \( \Theta^L_i(k) \) has a similar form as \( \Theta^L_m(k) \) in (46). It can be seen from the analysis in Section II-B that \( \Theta^L_i(k) \)
is decided by $\sigma_{h_i}^j(k)$, and the stochastic process $\{\sigma_{h_i}^j(k)\}$ in (26) obeys i.i.d.. Then, according to $\sum_{h_i=1}^{3} \pi_{h_i}^j = 1$, the selection probabilities are given as $\pi_1^j = 0.3$, $\pi_2^j = 0.2$, $\pi_3^j = 0.5$, $\pi_2^j = 0.3$, $\pi_3^j = 0.3$, and $\pi_3^j = 0.4$ to satisfy (36)–(37). Moreover, the linearized matrices $L_{f_i}(k)$, $A_{c_i}(k)$, and $C_{h_i}(k)$ can be calculated in a same way as (43) but with different expansion points, and the sate-dependent matrices are taken as $L_{f_1}^j = \text{diag}(0.03, 0.01, 0.02)$, $L_{f_2}^j = \text{diag}(0.02, 0.01, 0.03)$, $L_{f_3}^j = \text{diag}(0.03, 0.01, 0.03)$, $L_{c_1}^j = \text{diag}(0.03, 0.02, 0.02)$, $L_{c_2}^j = \text{diag}(0.03, 0.02, 0.03)$, and $A_{c_1}^j = A_{c_2}^j = 1$.

To show the effectiveness of the proposed nonlinear fusion estimation Algorithm 2, the actual vehicle’s trajectory and the DCFE trajectory are presented in Fig. 8(a), which shows that the proposed nonlinear compensation fusion estimator can track the vehicle’s position well with incomplete information. Then, the RMESs of the LCEs (28) and the DCFE (29) with 100 Monte Carlo runs are shown in Fig. 8(b), respectively. It can be clearly seen from this subfigure that even under the ETS (21)–(22) and DRS (24)–(27) presented in Section II-B, the performance of the DCFE is still better than that of each LCE based on the action of the compensation strategy. Meanwhile, the triggering status of two smart sensors is plotted in Fig. 8(c), where some untriggered instants indicate the communication traffic of the local estimation can really be reduced. Furthermore, the estimation performance of the LCEs (28) and the DCFE (29) with different communication rates is presented in Fig. 10(a). Similarly, it can be seen that the estimation performance has been improved with the increase in communication rates. Meanwhile, when the communication rate is reduced to 50%, the estimation performance is still in a relatively ideal range. Therefore, as stated in Remark 3, the triggering threshold $\delta_i^j$ can be set such that the communication rate is approximately within this range to meet the limited resources. At the same time, Fig. 10(b) shows the estimation performance of the fusion estimator compensated by the last compensation estimation $\hat{x}_i(k - 1)$ and the prediction estimation $f(\hat{x}_i(k - 1))$, respectively. It can be clearly seen from this subfigure that the compensation strategy used in this article has a better estimation performance.

Finally, the comparisons of the fusion estimation performance for different communication strategies are presented in Fig. 11. On the basis of local bandwidth constraints

![Fig. 8. (a) Vehicle’s true motion trajectory and fusion estimation trajectory. (b) Estimation performance comparison between the LCEs and DCFE. (c) Triggering status of two smart estimators.](image1)

![Fig. 9. Sequences of the variables $\sigma_{h_i}^j(k)(i = 1, 2; h_i = 1, 2, 3)$.](image2)

![Fig. 10. (a) Performance of fusion estimation under different communication rates. (b) Performance of fusion estimation under different compensation strategies.](image3)
\[ s_1^1 = s_2^2 = 2 \] selected in this example, Fig. 11(a) shows the RMESs for different dimensionality reduction status (i.e., \( s_1^1 = s_1^2 = 1, s_2^1 = 1, s_2^2 = 2 \), respectively). It is seen from this subfigure that the more components of the local estimate that are transmitted to the FC under the same triggering thresholds, the better the estimation accuracy is. Indeed, this is because the capacity of the bandwidth channel limits the transmission of local estimates. Moreover, compared with the fusion estimation performance of NMFSs without resource constraints, the RMESs of different communication strategies under resource constraints are plotted in Fig. 11(b). Similarly, the NMFSs without resource constraints have the best fusion estimation accuracy. As the analysis in the aforementioned example shows, only using ETS or DRS may reduce the burden of communication resources to a certain extent, and the estimation performance will be degraded as well. In comparison, although the ETS and DRS in a unified framework considered in this article have the worst estimation accuracy among these communication strategies, they can reduce more communication traffic to meet the requirements of the resource constraints, and the fusion estimation performance is still maintained to an acceptable extent.

V. CONCLUSION

In this article, two different communication frameworks for the nonlinear NMFSs have been considered, where the S-RE channel and the S-FC channel were subject to resource constraints, respectively. Specifically, an ETS and a DRS were employed to alleviate the communication burden, which could meet the finite communication resources. Meanwhile, to preserve a certain fusion estimation performance, a unified compensation model was proposed to restructure the reduced information. Then, the nonlinear local/fusion estimators were designed based on the compensation information, and the uncertain parameters together with the state-dependent matrices were introduced to model the linearization errors. In this case, different convex optimization problems were established to solve the estimator gains based on the idea of bounded recursive optimization. Moreover, the proposed robust design approach can be such that the SEs of the designed compensation estimators asymptotically bounded. Finally, two simulation cases were presented to show the effectiveness and advantages of the proposed methods.

Furthermore, the time-delay, asynchronous sampling, and out of order will cause an asynchronous fusion problem in the nonlinear networked fusion structure, which has also attracted significant attention. In particular, some neural networks [52], [53] and robust methods [54], [55] give us inspiration for future work dealing with unknown perturbations, linearization errors, and uncertainty problems in the field of asynchronous multisensor fusion systems. Therefore, how to design stable estimators for nonlinear asynchronous NMFSs will be one of our future tasks.

APPENDIX A

PROOF OF THE THEOREM 1

Let \( \hat{x}_m^i(k) \triangleq x(k) - \hat{x}_m^i(k) \) and \( \hat{x}_m^i(k) \triangleq x(k) - \hat{x}_m^i(k) \) denote the prediction error and local estimation error, respectively. Then, it follows from (1) and (16) that

\[
\begin{align*}
\hat{x}_m^i(k) &= f(x(k-1)) - f_i(\hat{x}_m^i(k-1)) + \Gamma(k-1)w(k-1) \\
\hat{x}_m^i(k) &= x_m(k) - \hat{x}_m^i(k) - K_m^i[k] \left[ s_i^1 - h_k(\hat{x}_m^i(k-1)) \right].
\end{align*}
\]

To further analyze the aforementioned nonlinear estimation error, the nonlinear functions \( f(x(k-1)) \) and \( h_k(x(k)) \) in (49) are linearized by using the first-order Taylor series expansion (TSE), and then, the higher order terms are modeled by state-dependent matrices with uncertain parameters [46], one has

\[
\begin{align*}
f(x(k-1)) &= f_i(\hat{x}_m^i(k-1)) \\
+ & \left( A_m^i(k-1) + M_j^i(k)N_j^i(k) \right) \hat{x}_m^i(k-1) \\
h_k(x(k)) &= h_k(\hat{x}_m^i(k)) + C_m^i(k) + M_j^i(k)N_j^i(k) \hat{x}_m^i(k)
\end{align*}
\]

where

\[
\begin{align*}
A_m^i(k) &= \frac{\partial f(x(k))}{\partial x(k)} \big|_{x(k)=\hat{x}_m^i(k)} \\
C_m^i(k) &= \frac{\partial h_k(x(k))}{\partial x(k)} \big|_{x(k)=\hat{x}_m^i(k)} \\
O_m^i((\hat{x}_m^i(k-1))^2) &= M_j^i(k)N_j^i(k) \hat{x}_m^i(k-1) \\
O_m^i((\hat{x}_m^i(k))^2) &= M_j^i(k)N_j^i(k) \hat{x}_m^i(k)
\end{align*}
\]

Here, \( M_j^i(k) > 0 \) and \( M_j^i(k) > 0 \) are state-dependent scaling matrices, \( N_j^i(k) \) and \( N_j^i(k) \) are unknown bounded matrices that satisfy \( \|N_j^i(k)\|_2 \leq 1 \) and \( \|N_j^i(k)\|_2 \leq 1 \), respectively.

Then, substituting (50) into (49), the local estimation error \( \hat{x}_m^i(k) \) can be rewritten as

\[
\begin{align*}
\hat{x}_m^i(k) &= A_k^i(k)\hat{x}_m^i(k-1) + \Gamma_k^i(k)w(k-1) + \Delta_k^i(k)w(k)
\end{align*}
\]

Fig. 11. (a) Effect of fusion estimation under different bandwidth constraints. (b) Effect of fusion estimation under different communication strategies.
where
\[
\begin{align*}
A_k^m(k) &\equiv K_L^m(k) + K_H^m(k)M_f^m(k)N_i^m(k) - K_H^m(k)M_f^m(k) \\
&\quad \times N_i^m(k)A_i^m(k) - \alpha_m^k A_i^m(k) \Gamma_H^m(k)N_i^m(k) \\
\Gamma_k^m &\equiv \begin{bmatrix} K_L^m(k) - K_H^m(k) & M_f^m(k)N_i^m(k) - \alpha_m^k M_f^m(k) \\
\end{bmatrix} \\
D_k^m &\equiv -\gamma_m^k A_i^m(k)M_f^m(k)D_i^m(k) \\
N_i^m(k) &\equiv (\alpha_m^k)^{-1}N_i^m(k)M_f^m(k)N_i^m(k).
\end{align*}
\]
(53)

Here, \( \alpha_m^k(k) \) is a scalar that is equal or greater than the max element of \( M_f^m(k) \), and \( K_L^m(k) \), \( K_H^m(k) \), and \( M_f^m(k) \) have been defined in (32).

To construct an upper bound of the SE of the LRE (16), and then, a performance index [48] is introduced as follows:
\[
J_m^p(k) \triangleq (\hat{\xi}_m^p(k))^T\hat{\xi}_m^p(k) - (\bar{\xi}_m^p(k) - 1)^T\Psi(k)\bar{\xi}_m^p(k) - (w(k-1))^T\Phi_1(k)w(k-1) - v_i^T(k)\Psi_i(k)v_i(k)
\]
(54)

where \( \Psi_i(k) > 0, \Phi_1(k) > 0 \) and \( \Psi_i(k) > 0 \) are introduced unknown positive definite matrices. In fact, it can be deduced from (54) that \( J_m^p(k) < 0 \) holds when the following inequality is held:
\[
\begin{bmatrix}
-I & A^m_i(k) & \Gamma_i^m(k) & D^m_i(k) \\
* & -\Psi(k) & 0 & 0 \\
* & * & -\Phi_1(k) & 0 \\
* & * & * & -\Psi_i(k)
\end{bmatrix} < 0.
\]
(55)

It can be seen from (53) that the aforementioned matrix inequality contains several uncertain matrices, which are introduced by the linearized process (50). In this case, the inequality (55) cannot be directly solved by an optimization problem. However, the uncertainties in the (55) can be addressed by using Lemma 1, and then, the inequality (55) is converted to the first three inequalities in the optimization problem (31).

Furthermore, the bounded stability conditions of the designed LRE (16) can be obtained from the similar analysis of [44, Th.1]. That is, the inequality (33) holds when the last two inequalities in (31) are held. The detailed derivation is omitted here. Then, from (54) and \( J_m^p(k) < 0 \), the upper bound of the SE of each LRE can be constructed as
\[
(\hat{\xi}_m^p(k))^T\hat{\xi}_m^p(k) < \xi_i(k)(\bar{\xi}_m^p(k) - 1)^T\bar{\xi}_m^p(k) - 1
\]
\[
+ \lambda_{\text{max}} \begin{bmatrix} w(k-1) \\
v_i(k) \end{bmatrix}^T \begin{bmatrix} w(k-1) \\
v_i(k) \end{bmatrix}
\times (\text{Tr}(\Phi_1(k)) + \text{Tr}(\Psi_i(k)))
\]
(56)

where \( \xi_i(k) \) has been defined in (31). Notice that, in order to minimize the upper bound of \( (\hat{\xi}_m^p(k))^T\hat{\xi}_m^p(k) \), \( \min(\text{Tr}(\Sigma_i(k)) + \text{Tr}(\Psi_i(k))) \) was selected as the optimization objective to determine the LRE gain \( K_H^m(k) \), then the convex optimization problem (31) in terms of linear matrix inequalities (LMIs) was established.

Next, the distributed fusion matrix \( W_i^m(k) \) will be determined by the following analysis. Let \( \tilde{x}_m(k) \triangleq x(k) - \hat{x}_m(k) \) denote the fusion estimation error, it follows from (1) and (17) that
\[
\tilde{x}_m(k) = \sum_{i=1}^{L} W_i^m(k)\tilde{x}_i^m(k).
\]
(57)

Combining (52) and (57), the fusion estimation error can be formulated as
\[
\tilde{x}_m(k) = W_m(k)\left[A_k^m(k)\tilde{x}_M(k) - 1\right] + \Gamma_k^m(k)w(k-1) + D_k^m(k)v(k)
\]
(58)

where
\[
\begin{bmatrix}
A_k^m(k) & \text{diag}(A_1^k(k), \ldots, A_L^k(k)) \\
\Gamma_k^m(k) & \text{col}(\Gamma_1^k(k), \ldots, \Gamma_L^k(k)) \\
D_k^m(k) & \text{diag}(D_1^k(k), \ldots, D_L^k(k)) \\
\bar{\xi}_M^L(k) & \text{col}(\bar{\xi}_M^1(k), \ldots, \bar{\xi}_M^L(k)) \\
v(k) & \text{col}(v_1(k), \ldots, v_L(k))
\end{bmatrix}
\]
\[
W_m(k) \equiv \begin{bmatrix} W_i^m(k) \end{bmatrix}, \quad \sum_{i=1}^{L} W_i^m(k) = I - \sum_{i=1}^{L} W_i^m(k).
\]
(59)

Similarly, introducing some unknown matrices \( \Psi(k) > 0, \Phi(k) > 0, \Psi(k) > 0, \Psi_i(k), \Phi_1(k), \) and \( \Psi_i(k) \) to construct an upper bound of the SE of the DFE (17). From (58), one has
\[
\bar{\xi}_m^T(k)\tilde{x}_m(k) < \begin{bmatrix} \bar{\xi}_M^L(k) - 1 \end{bmatrix}^T \Delta(k) \begin{bmatrix} \bar{\xi}_M^L(k) - 1 \end{bmatrix}
\]
\[
\begin{bmatrix} w(k-1) \\
v_i(k) \end{bmatrix}^T \begin{bmatrix} w(k-1) \\
v_i(k) \end{bmatrix}
\]
\[
\Delta(k) \equiv \begin{bmatrix} \Psi(k) & \Psi_1(k) & \Psi_2(k) \\
* & \Phi(k) & \Phi_1(k) \\
* & * & \Psi_i(k) 
\end{bmatrix}.
\]
(60)

Subsequently, using a derivation process similar to that in [44, Th.2], the inequality (60) can be converted to the LMIs in (34). Moreover, it is seen from Section II-A that minimizing the constructed upper bound of \( \tilde{x}_m^T(k)\tilde{x}_m(k) \) is one of the aims of this article. Then, \( \min(\text{Tr}(\Psi(k)) + \text{Tr}(\Phi(k)) + \text{Tr}(\Psi_i(k))) \) is chosen as the optimization objective to construct the convex optimization problem (34) to calculate distributed weighting fusion matrices \( \{W_i^m(k)\} \sum_{i=1}^{L} W_i^m(k) = I, i \in \mathcal{L} \). The detailed derivation is omitted here.

**APPENDIX B**

**PROOF OF THE THEOREM 2**

Notice that, each local estimate \( \hat{x}_i^m(k) \) can be calculated by using [44, Th.1] based on LNE (4). Then, the local estimation error \( \hat{x}_i^m(k) = x(k) - \hat{x}_i^m(k) \) is given by
\[
\hat{x}_i^m(k) = A_i^m(k)\hat{x}_i^m(k) - 1
\]
\[
+ \Gamma_i^m(k)w(k-1) - K_i^m(k)D_i^m(k)v_i(k).
\]
(61)

Meanwhile, let \( \tilde{x}_i^m(k) = x(k) - \hat{x}_i^m(k) \) denote local compensation estimation error, one has from (1), (28), and (62) that
\[
\tilde{x}_i^m(k) = A_i^m(k)\tilde{x}_i^m(k) - 1 + \gamma_i^m(k)\Theta_i^m(k)A_i^m(k)\tilde{x}_i^m(k) - 1
\]
\[
+ \Gamma_i^m(k)w(k-1) - \gamma_i^m(k)\Theta_i^m(k)K_i^m(k)D_i^m(k)v_i(k).
\]
(63)
where

\[
\begin{align*}
A^j_i(k) & \triangleq K^j_i(k)A^j_i(k-1) + K^j_i(k)L^j_i(k)P^j_i(k) \\
& \quad - K^j_i(k)L^j_i(k)P^j_i(k)A^j_i(k-1) \\
& \quad - \alpha^j_i(k)K^j_i(k)L^j_i(k)P^j_i(k) \\
\Gamma^j_i(k) & \triangleq (K^j_i(k) - K^j_i(k)L^j_i(k)P^j_i(k))\Gamma(k-1) \\
A^j_i(k) & \triangleq (I - \gamma^j_i(k)\Theta^j_i(k))A^j_i(k-1) + L^j_i(k)P^j_i(k)) \\
\Gamma^j_i(k) & \triangleq (I - \gamma^j_i(k)\Theta^j_i(k)K^j_i(k)L^j_i(k)P^j_i(k))\Gamma(k-1) \\
K^j_i(k) & \triangleq I - K^j_i(k)C^j_i(h) \\
P^j_i(k) & \triangleq (\alpha^j_i(k)^{-1})P^j_i(k)I^j_i(k)P^j_i(k) \\
A^j_i(k) & \triangleq \frac{\partial f(x(k))}{\partial x(k)}_{x(k)=\bar{x}_i(k)} \\
C^j_i(h) & \triangleq \frac{\partial h_i(x(k))}{\partial x(k)}_{x(k)=\bar{x}_i(k)} \\
A^j_i(k) & \triangleq \frac{\partial f(x(k))}{\partial x(k)}_{x(k)=\bar{x}_i(k)}.
\end{align*}
\]  

(64)

Similarly, the nonlinear functions in the derivation process of (62) and (63) are also addressed by model-free methods with uncertain matrices. Specifically, the introduced matrices \(L^j_i(k), L^j_i(k), \) and \(L^j_i(k)\) are positive state-dependent matrices, the uncertain matrices \(P^j_i(k), P^j_i(k), \) and \(P^j_i(k)\) are assumed to satisfy \(\|P^j_i(k)\|_2 \leq 1, \|P^j_i(k)\|_2 \leq 1, \) and \(\|P^j_i(k)\|_2 \leq 1,\) respectively. Meanwhile, \(\alpha^j_i(k)\) is a scalar that is equal or greater than the max element of \(L^j_i(k)\).

Motivated by the stability analysis of the compensation estimators in [43, Th.2], the stability conditions of the LCE (28) and the DCFE (29) will be analyzed. First, it follows from (62) and (63) that

\[
\dot{\bar{x}}^j_i(k) = (I - \gamma^j_i(k)\Theta^j_i(k))A^j_i(k-1) + L^j_i(k)P^j_i(k)) \\
\times \Xi^j_i(k-1) + \sigma_i(k)
\]  

(65)

where \(\sigma_i(k) \triangleq \gamma^j_i(k)\Theta^j_i(k)\Xi^j_i(k) + (I - \gamma^j_i(k)\Theta^j_i(k))\Gamma(k-1)w(k-1)\). Then, it is derived from the aforementioned equation that

\[
\mathbb{E}[\dot{\bar{x}}^j_i(k)] = \Theta^j_i(A^j_i(k-1) + L^j_i(k)P^j_i(k)) \\
\times \mathbb{E}[\dot{\bar{x}}^j_i(k-1)] + \mathbb{E}[\sigma_i(k)]
\]  

(66)

where \(\Theta^j_i = \mathbb{E}[I - \gamma^j_i(k)\Theta^j_i(k)]\). Meanwhile, it can be deduced from (21) and (27) that

\[
0 \leq \mathbb{E}[\gamma^j_i(k)] \leq 1, 0 \leq \mathbb{E}[\Theta^j_i(k)] \leq 1, 0 \leq \Theta^j_i \leq 1.
\]  

(67)

Since \(w(k-1)\) is assumed bounded in (1), and the SE of the \(\bar{x}_i^j(k)\) is asymptotically bounded from the analysis in [44, Th.1], thus, \(\mathbb{E}[\sigma_i(k)]\) in (66) will be asymptotically bounded as well. Then, it follows from (66) that

\[
\|\mathbb{E}[\dot{\bar{x}}^j_i(k)]\|_2 \leq \|\Theta^j_i(A^j_i(k-1) + L^j_i(k)P^j_i(k))\|_2 \\
\times \|\mathbb{E}[\dot{\bar{x}}^j_i(k-1)]\|_2 + \|\mathbb{E}[\sigma_i(k)]\|_2.
\]  

(68)

In this case, when considering that there exist a positive scalar \(\mathcal{M}_c\) to satisfy

\[
\lim_{k \to \infty} \|\mathbb{E}[\dot{\bar{x}}^j_i(k)]\|_2 < \mathcal{M}_c
\]  

(69)

the following condition should be held:

\[
\|\Theta^j_i(A^j_i(k-1) + L^j_i(k)P^j_i(k))\|_2 < 1.
\]  

(70)

Then, by the means of the Schur complement lemma [47] and Lemma 1, the inequality (70) holds equivalent to (36) holding.

Furthermore, it follows from (65) that

\[
\dot{\bar{x}}^j_i(k+1) = \Pi^j_i(k, N_i)\bar{x}^j_i(k) - \bar{\theta}_i(k)
\]  

(71)

where

\[
\Pi^j_i(k, N_i) \triangleq \sum_{i=0}^{N_i}(I - \gamma^j_i(k-t+1)\Theta^j_i(k-t+1)) \\
\times (A^j_i(k-t) + L^j_i(k-t+1)P^j_i(k-t+1)) \\
\times \Xi^j_i(k-t+1) + (I - \gamma^j_i(k-t+1)\Theta^j_i(k-t+1)) \\
\times \Theta^j_i(k-t+1)\bar{\theta}_i(k-t+1)w(k-t)).
\]  

(72)

notice that \(w(k-1)\) is bounded noise, and from (26) and (71), one has

\[
\begin{align*}
\mathbb{E}[(\bar{x}_i^j(k+1))^T\bar{x}_i^j(k-1)] \\
= \mathbb{E}[(\bar{x}_i^j(k-N_i))^T\bar{I}_c(k)\bar{x}_i^j(k-N_i)] + \mathcal{O}_i^j(k)
\end{align*}
\]  

(73)

Then, it follows from (71) and (73) that

\[
\begin{align*}
\mathbb{E}[\bar{x}_i^j(k+1)] \\
\quad \mathbb{E}[(\bar{x}_i^j(k-N_i))^T\bar{I}_c(k)\bar{x}_i^j(k-N_i)] + \mathcal{O}_i^j(k)
\end{align*}
\]  

(74)

where

\[
\bar{I}_c(k) \triangleq (\Pi^j_i(k, N_i))^T\Pi^j_i(k, N_i) \\
\mathcal{O}_i^j(k) \triangleq \mathbb{E}[\bar{\theta}_i^j(k)\bar{\theta}_i(k)].
\]  

(75)

From (72), one has

\[
\begin{align*}
\bar{I}_c(k) = \left( \prod_{t=k-N_i}^{k} (A^j_i(k-t) + L^j_i(k-t+1)P^j_i(k-t+1)) \\
\times (I - \gamma^j_i(k-t+1)\Theta^j_i(k-t+1)) \right)^T \\
\times \prod_{t=0}^{k-N_i}(I - \gamma^j_i(k-t+1)\Theta^j_i(k-t+1)) \\
\times (A^j_i(k-t) + L^j_i(k-t+1)P^j_i(k-t+1)) \\
= \mathcal{R}_i(k-N_i, \mathcal{R}_i(k-N_i-1, \mathcal{R}_i(\ldots, \mathcal{R}_i(k, I)))
\end{align*}
\]  

(76)

where \(\mathcal{R}_i(k, Q) \triangleq (A^j_i(k) + L^j_i(k+1)P^j_i(k+1))T(I - \gamma^j_i(k+1)\Theta^j_i(k+1))Q(I - \gamma^j_i(k+1)\Theta^j_i(k+1))A^j_i(k) + L^j_i(k+1)P^j_i(k+1))\). In this case, it is deduced from (26) and (76) that

\[
\mathbb{E}[\bar{I}_c(k)] = \mathcal{R}_i(k-N_i, \mathcal{R}_i(k-N_i-1, \mathcal{R}_i(\ldots, \mathcal{R}_i(k, I))))
\]  

(77)



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where  \( \mathcal{R}_c(k,Q) = \mathbb{E}[\mathcal{R}_c(k,Q)] \). Moreover, it follows from (74) and (76) that
\[
\mathbb{E}\{\hat{x}_c^T(k+1)\hat{x}_c(k+1)\} \leq \lambda_{\text{max}}(\mathbb{E}\{T_c^T(k)\}) \mathbb{E}\{\hat{x}_c(k-N_i)^T\hat{x}_c(k-N_i)\} + O_c(k).
\]
(78)

Then, it is concluded from (3), (69), and (72) that \( O_c(k) \) is bounded. By using a similar derivation of (69), \( \gamma_{\text{max}}(\mathbb{E}\{T_c^T(\cdot)\}) < 1 \) holds equivalent to inequality (37) holding, and \( \lim_{k \to \infty} \mathbb{E}\{\hat{x}_c^T(k)\hat{x}_c(k)\} \) will be bounded.

On the other hand, the distributed weighting fusion matrix \( W_i(k) \) will be determined. Specifically, define \( \tilde{x}_c(k) \triangleq \text{col}[^\hat{x}_c^T(k), \ldots, \hat{x}_c^T(k)] \), \( \tilde{x}_s(k) \triangleq \text{col}[\tilde{x}_c(k), \ldots, \tilde{x}_c(k), \ldots, \tilde{x}_c(k)] \), and \( \tilde{x}_c(k) \triangleq \text{col}[(\tilde{x}_c(k), \tilde{x}_s(k)) : \tilde{x}_c(k)] \). Then, combining (62) and (63), one has
\[
\tilde{x}_s(k) = A_s^T(k)\tilde{x}_s(k-1) + F_s(k)u(k-1) + D_s^T(k)v(k)
\]
(79)

where
\[
\begin{align*}
A_s^T(k) &\triangleq \begin{bmatrix} A_L^T(k) & 0 \\ \Theta_r(k)A_L^T(k) & A_L^T(k) \end{bmatrix} \\
\Gamma_s(k) &\triangleq \begin{bmatrix} (T_L^T(k))^T & (T_L^T(k))^T \end{bmatrix}^T \\
D_s^T(k) &\triangleq \begin{bmatrix} D_L^T(k) & (\Theta_r(k)D_L^T(k))^T \end{bmatrix}^T \\
A_L^T(k) &\triangleq \text{diag}(A_L^T(k_1), \ldots, A_L^T(k)) \\
\Gamma_L^T(k) &\triangleq \text{diag}(\Gamma_L^T(k_1), \ldots, \Gamma_L^T(k)) \\
\Theta_r(k) &\triangleq \text{diag}(\gamma_r^T(k_1)\Theta_r(k_1), \ldots, \gamma_r^T(k_1)\Theta_r(k_1)) \\
D_L^T(k) &\triangleq \text{diag}(-K_L^T(k)D_L(k_1), \ldots, -K_L^T(k)D_L(k)).
\end{align*}
\]
(80)

Subsequently, the fusion error system can be constructed by (1), (29), and (79) as follows:
\[
\tilde{x}_c(k) = W_o^c(k)\tilde{x}_s(k)
\]
(81)

where \( W_o^c(k) = [0 \ W_o(k)] \). Then, the upper bounded of the MSE of the \( \tilde{x}_c(k) \) can be determined by \( \mathbb{E}\{[\tilde{x}_c(k) - \hat{x}_c(k)]^2\} < \lambda_{\text{max}}(\mathbb{E}\{T_c^T(k)W_o(k)\}) \mathbb{E}\{[\hat{x}_c^T(k)\hat{x}_c(k)]\} \). Thus, when the conditions (36) and (37) are satisfied, the MSE of the DCFE (29) is asymptotically bounded. Moreover, by using a similar analysis as Theorem 1, the convex optimization problem (38) is established to determine \( W_o(k) \). The detailed proof is omitted here.

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Rusheng Wang received the B.S. degree in mathematics and applied mathematics from Huainan Normal University, Huainan, China, in 2013, and the M.S. degree in applied mathematics from Wenzhou University, Wenzhou, China, in 2016. He is currently working toward the Ph.D. degree in control science and engineering, Zhejiang University of Technology, Hangzhou, China. His current research interests include multisensor information fusion, distributed state estimation, and networked fusion systems.

Bo Chen (Member, IEEE) received the B.S. degree in information and computing science from the Jiangxi University of Science and Technology, Ganzhou, China, in 2008, and the Ph.D. degree in control theory and control engineering from the Zhejiang University of Technology, Hangzhou, China, in 2014. He joined the Department of Automation, Zhejiang University of Technology, in 2018, where he is currently a Professor. He was a Research Fellow with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore, from 2014 to 2015 and from 2017 to 2018. He was also a Postdoctoral Research Fellow with the Department of Mathematics, City University of Hong Kong, Hong Kong, from 2015 to 2017. His current research interests include information fusion, distributed estimation and control, networked fusion systems, and secure estimation of cyber-physical systems. Dr. Chen was the recipient of the Outstanding Thesis Award of Chinese Association of Automation in 2015 and also was the recipient of the First Prize of Natural Science of Ministry of Education in 2020. He serves as an Associate Editor for *IET Control Theory and Applications, Journal of the Franklin Institute, and Frontiers in Control Engineering*. 

Authorized licensed use limited to the terms of the applicable license agreement with IEEE. Restrictions apply.
Zhongyao Hu received the B.E. degree in automation from Wuhan Polytechnic University, Wuhan, China, in 2019. He is currently working toward the Ph.D. degree in control science and engineering with the Zhejiang University of Technology, Hangzhou, China.

His current research interests include multi-sensor fusion and state estimation for networked systems.

Li Yu (Member, IEEE) received the B.S. degree in control theory from Nankai University, Tianjin, China, in 1982, and the M.S. and Ph.D. degrees in control theory and control engineering from Zhejiang University, Hangzhou, China, in 1988 and 1999, respectively.

He is currently a Professor with the College of Information Engineering, Zhejiang University of Technology, Hangzhou. He has successively presided over 20 research projects. He has published five academic monographs, one textbook, and more than 300 journal papers. His current research interests include robust control, networked control systems, cyber-physical systems security, and information fusion.