Antibunching photons in a cavity coupled to an optomechanical system

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Abstract
We study the photon statistics of a cavity linearly coupled to an optomechanical system via second-order correlation functions. Our calculations show that the cavity can exhibit strong photon antibunching even when optomechanical interaction in the optomechanical system is weak. The cooperation between the weak optomechanical interaction and the destructive interference between different paths for two-photon excitation leads to the efficient antibunching effect. Compared with the standard optomechanical system, the coupling between a cavity and an optomechanical system provides a method to relax the constraints to obtain a single photon by optomechanical interaction.

1. Introduction
Photon antibunching is one of the pieces of evidence for the quantum nature of light, and the concept of photon blockade is introduced to explain the strong antibunching of transmitted photons [1]. It is well known that such quantum effects can be observed in strong nonlinear systems, such as an optical cavity strongly coupled to a trapped atom [2, 3], a quantum dot strongly coupled to a photonic crystal resonator [4] and a superconducting qubit coupled to a microwave cavity in both resonant [5] and dispersive regimes [6]. These systems provide a platform to realize non-classical photon states [4], which are of considerable interest for applications in quantum information processing and quantum cryptography [7, 8].

Optomechanics, a system where a mechanical resonator acts as a quantum system coupled to the electromagnetic field via radiation pressure, provides a helpful toolbox for investigating the quantum effects in both optical and mechanical systems (see reviews [9, 10]). In recent years, many great experimental achievements have been obtained in this area, such as the mechanical oscillator being prepared almost in its ground state by sideband cooling, which paves the way for implementing mechanical oscillators into quantum mechanics [11–25]. Additionally, although the coupling constant between the optical and mechanical modes is weak in most standard optomechanical systems, in the recent experiment, strong optomechanical coupling has been obtained by driving the optomechanical system with an extra strong laser field [26]. In the meanwhile, the optical response of optomechanical systems to a signal field is modified by the driving field, leading to effects such as normal-mode splitting [26, 27] and electromagnetically induced transparency (EIT) [27–31].

The statistical properties of photons in optomechanical systems have been theoretically studied in [32, 33]. These studies show that the photon blockade effect can be observed in the optomechanical systems under the strong optomechanical coupling condition. Apart from blockade, a photon can also induce multi-photon tunnelling by the nonlinear interaction in optomechanical systems [34]. Moreover, it has been shown that the nonlinear interactions in two coupled optomechanical systems can be significantly enhanced for mechanical frequencies nearly resonant with the optical level splitting [35, 36]. However, photon blockade effects still only appear in the strong coupling regime, which is beyond the reach of most experiments in the single-photon regime. Thus there is a question of whether single-photon states can be generated using weak optomechanical interaction.

Recently, Liew and Savona analysed the photon statistics of two coupled nonlinear cavities and found that the photons can exhibit strong antibunching in such coupled systems with weak Kerr nonlinearity [37]. Later on, such strong
under the semiclassical approximation, and we analyse the
interaction regime. The paper is organized as follows. In
section 2, the model Hamiltonian is introduced. In section 3,
the analytical expression of the second-order correlation
function is obtained by the quantum Langevin equations
under the semiclassical approximation, and we analyse the
photonic statistical properties of the cavity in section 4. In
section 5, we analyse the second-order correlation function
further by numerical simulation via the master equation and
compare the results with those obtained under the semiclassical
approximation. The summary and conclusions are given in
section 6.

2. The model

As schematically shown in figure 1(a), the system consists of
two coupled cavities (A and B) with the coupling constant
J. The cavity can be a transmission line resonator, a toroidal
microresonator, a cavity with two mirrors, or a defect cavity
in the photonic crystal. Without loss of generality and for
simplicity, we will focus on the cavity system with two mirrors.
Cavity A is driven by a weak probe field with frequency \( \omega_b \), and
cavity B consists of an oscillating mirror at one end, modelled
as a quantum mechanical harmonic oscillator. In other words,
we study a coupled system, which consists of a driven cavity
and an optomechanical system. The Hamiltonian of the whole
system in the rotating wave approximation is given as

\[
H = \hbar \omega_a a^\dagger a + \hbar \omega_b b^\dagger b + \hbar \Omega b^\dagger b J (a^\dagger b + b^\dagger a) + \hbar g b^\dagger b (c^\dagger + c) + i \hbar \varepsilon_c (a^\dagger e^{-i \varepsilon_c t} - a e^{i \varepsilon_c t}),
\]

(1)

where \( a (a^\dagger) \) is the annihilation (creation) operator for the
light mode of cavity A with frequency \( \omega_a \), \( b (b^\dagger) \) is the
annihilation (creation) operator for the light mode of cavity
B with frequency \( \omega_b \) and \( c (c^\dagger) \) is the phonon annihilation
(creation) operator of the mechanically vibrational mode for
the mirror with frequency \( \omega_m \). The parameter \( g_b \) denotes the
coupling strength between cavity B and the oscillating mirror,
and \( \varepsilon_c \) presents the coupling strength between the driving field
and the cavity field inside cavity A. As \( \omega_a \approx \omega_b \gg \omega_m, J \),
we have dropped the rapidly varying terms \((ab \text{ and } a^\dagger b^\dagger)\)
corresponding to the rotating wave approximation.

![Figure 1.](image.png)

Figure 1. (a) Schematic diagram for an optical cavity (cavity A, driven by a weak coherent laser field) coupled to an optomechanical system (cavity B with a movable right mirror). (b) Energy level diagram for the coupled system. Here, the short black lines denote the energy levels \( |n_a, n_b, n_m\rangle \) for \( \omega'_a = \omega_a \), and four levels are singled out as a reduced diagram (in the green dashed line box). \( \varepsilon_c \) presents the coupling strength between the driving field and cavity field in cavity A. \( J \) is the coupling constant between cavities A and B.

Our calculations (given in the following sections) show
that cavity A can exhibit a strong photon antibunching effect
even when optomechanical interaction in cavity B is weak. For
the physical interpretation of the strong antibunching effect
under the weak coupling condition, we will show the energy
level diagram of the coupled system. It is convenient to change
the Hamiltonian to a displaced oscillator representation \( H_{\text{eff}} = U H U^\dagger \) by the unitary transformation \( U = e^{-\frac{\hbar}{2 \omega_m} b c^\dagger c - c^\dagger b} \); then we obtain

\[
H_{\text{eff}} = \hbar \omega_a a^\dagger a + \hbar \omega_b b^\dagger b - \hbar \frac{g_b^2}{\omega_m} b^\dagger b bb + \hbar \omega_m c^\dagger c + \hbar J [a^\dagger b e^{\frac{\omega_m}{\hbar} (c^\dagger - c)} + ab^\dagger e^{-\frac{\omega_m}{\hbar} (c^\dagger - c)}] + i \hbar \varepsilon_c (a^\dagger e^{-i \varepsilon_c t} - a e^{i \varepsilon_c t}),
\]

(2)

where \( \omega'_a = \omega_a - \frac{g_b^2}{\omega_m} \). In the limit \( J, \varepsilon_c \rightarrow 0 \), the
Hamiltonian is diagonalized and the eigenvalues are

\[
E_{n_a, n_b, n_m} = \hbar \omega_a n_a + \hbar \omega_b n_b - \hbar \frac{g_b^2}{\omega_m} n_b (n_b - 1) + \hbar \omega_m n_m,
\]

(3)

corresponding to the eigenstates \( |n_a, n_b, n_m\rangle \), where \( |n_a, n_b, n_m\rangle \equiv U |n_a, n_b, n_m\rangle \) and \( |n_a, n_b, n_m\rangle \) represents that there are \( n_a \) \( (n_b) \) photons in cavity A (B) and \( n_m \) phonons in the mechanical resonator. The energy levels are shown by short black lines in figure 1(b) according to equation (3) by setting \( \omega'_a = \omega_a \), and the terms for external driven \( (\varepsilon_c) \) and tunnelling between the two cavities \( (J) \) are added and represented by lines with arrows in the diagram.
The optomechanical interaction in cavity $B$ and the quantum interference effect between the two cavities (cavities $A$ and $B$) are responsible for the photon antibunching effect [38]. As shown in the reduced diagram in figure 1(b), the interference is between two paths for two-photon excitation in cavity $A$: (i) the direct excitation from one photon to two photons in cavity $A$, and (ii) one photon tunnelling from cavity $A$ to cavity $B$, then exciting another photon in cavity $A$, and finally the photon inside cavity $B$ tunnelling back to cavity $A$. The destructive interference between the two paths reduces the probability of two-photon excitation in cavity $A$.

In order to analyse this phenomenon more precisely, the second-order correlation function is calculated by the quantum Langevin equations under the semiclassical approximation and by numerical simulation via the master equation in the following sections. To remove the time-dependent factor, let us transform the Hamiltonian in equation (1) into the rotating reference frame through a unitary operator $R(t) = \exp[-i\omega_1 t(a^d a + b^d b)]$, and thus equation (1) becomes

$$
\tilde{H} = h\Delta_a a^d + h\Delta_b b^d + h\omega_0 c^d c + hJ(a^d b + b^d a)
+ h\kappa_b b(c^d + c) + h\xi_c (a^d - a),
$$

(4)

where $\Delta_a = \omega_a - \omega_c$ and $\Delta_b = \omega_b - \omega_c$ are the frequency detunings of the cavity fields from that of the driving field.

3. Langevin equations and second-order correlation functions

The dynamics of the cavity fields and mechanical oscillator can be described by quantum Langevin equations. By considering the dissipation and fluctuation of the light fields and mechanical mode, we can write a set of nonlinear quantum Langevin equations as follows:

$$
\frac{d}{dt} a = -\left(\frac{\kappa_a}{2} + i\Delta_a\right) a - iJ b + \epsilon_c + \sqrt{\kappa_a} a_m,
$$

(5)

$$
\frac{d}{dt} b = -\left[\frac{\kappa_b}{2} + i(\Delta_b + g_b q)\right] b - iJ a + \sqrt{\kappa_b} b_m,
$$

(6)

$$
\frac{d}{dt} q = \omega_m p,
$$

(7)

$$
\frac{d}{dt} p = -\omega_m q - g_b b^d b - \frac{\gamma_m}{2} p + \xi,
$$

(8)

where $\kappa_a$, $\kappa_b$ and $\gamma_m$ are the damping rates of cavity $A$, cavity $B$ and the moving mirror, respectively, $q = (c + c^d)/\sqrt{2}$, $p = (c - c^d)/(i\sqrt{2})$, and $g_b = \sqrt{2}\gamma_b$. $\xi$ is a Brownian stochastic force with zero mean value, i.e. $\langle \xi(t) \rangle = 0$, which comes from the coupling of the oscillating mechanical resonator to its thermal environment and satisfies correlation

$$
\langle \xi(t)\xi(t') \rangle = \frac{\gamma_m}{2\omega_m} \int \frac{d\omega}{2\pi} e^{-i\omega(t-t')} \left[ 1 + \coth\left( \frac{\hbar\omega}{2k_B T} \right) \right].
$$

(9)

where $k_B$ is the Boltzmann constant and $T$ is the effective temperature of the environment of the mechanical resonator. $a_m$ and $b_m$ represent the vacuum radiation noise inputs to cavities $A$ and $B$ with $\langle a_m(t) \rangle = \langle b_m(t) \rangle = 0$, and they obey the following correlation functions [44]:

$$
\langle a_m(t)a_m(t') \rangle = 0,
$$

(10)

$$
\langle a_m(t)a_m(t') \rangle = \delta(t-t'),
$$

(11)

$$
\langle b_m(t)b_m(t') \rangle = 0,
$$

(12)

$$
\langle b_m(t)b_m(t') \rangle = \delta(t-t').
$$

(13)

Here, we have assumed that the whole system is in a low-temperature environment, and therefore the equilibrium mean thermal photon numbers in two cavities at optical frequencies have been neglected.

The dynamics of the system is determined by the small fluctuations when the system reaches the steady state. Thus, let us now apply the semiclassical approximation to solve the steady state with small quantum fluctuations. That is, we assume $a = a_0 + \delta a$, $b = b_0 + \delta b$, $q = q_0 + \delta q$, $\kappa_0$ and $\beta_0$ are the mean values of the cavity fields and mechanical mode when the system reaches the steady state, and operators $\delta a, \delta b$ and $\delta q$ describe the small fluctuations around the steady state with a zero mean value, $\langle \delta a \rangle = 0, \langle \delta b \rangle = 0$ and $\langle \delta q \rangle = 0$.

The steady-state values satisfy the following equations:

$$
\left[\frac{\kappa_a}{2} + i\Delta_a\right] a_0 + iJ b_0 = \epsilon_c,
$$

(14)

$$
\left[\frac{\kappa_b}{2} + i(\Delta_b + g_b q_0)\right] b_0 + iJ a_0 = 0,
$$

(15)

$$
\omega_m q_0 = -g_b |b_0|^2.
$$

(16)

Here, we have used the factorization assumption, e.g., $\langle q b \rangle = \langle q \rangle \langle b \rangle$. The dynamics of small fluctuations around the steady state can be obtained by linearizing equations (5)–(8) as

$$
\frac{d}{dt} \delta a = -\left(\frac{\kappa_a}{2} + i\Delta_a\right) \delta a - iJ \delta b + \sqrt{\kappa_a} a_m,
$$

(17)

$$
\frac{d}{dt} \delta b = -\left[\frac{\kappa_b}{2} + i(\Delta_b + g_b q_0)\right] \delta b - iJ \delta a + \sqrt{\kappa_b} b_m,
$$

(18)

$$
\frac{d}{dt} \delta q = \omega_m \delta p,
$$

(19)

$$
\frac{d}{dt} \delta p = -\omega_m \delta q - g_b (b_m^* \delta b + \delta b^* b_m) - \frac{\gamma_m}{2} \delta p + \xi,
$$

(20)

here, the high-order terms of small fluctuations, e.g., $\delta q \delta b$, have been neglected. The system is stable only if all the eigenvalues of the coefficient matrix of the above differential equations have negative real parts, and the stability condition can be given explicitly by using the Routh–Hurwitz criterion [45]. However, it is too cumbersome to be given here. All of the parameters we will use satisfy the stability condition, and it is easy to fulfill because the driving field in our system is weak.

By applying the Fourier transform and solving dynamical equations in the frequency domain, we obtain

$$
\delta a(\omega) = E(\omega)a_m(\omega) + F(\omega)a_m^d(\omega)
+ G(\omega)b_m(\omega) + H(\omega)b_m^d(\omega) + Q(\omega)\xi(\omega),
$$

(21)
where
\[
E(\omega) = \sqrt{\kappa_0} A_{11}(\omega), \tag{22}
\]
\[
F(\omega) = -\sqrt{\kappa_0} A_{22}(\omega), \tag{23}
\]
\[
G(\omega) = -\sqrt{\kappa_0} A_{33}(\omega) \tag{24}
\]
\[
H(\omega) = -\sqrt{\kappa_0} A_{44}(\omega), \tag{25}
\]
\[
Q(\omega) = -i g_0 \chi(\omega)(\beta_0 A_{33}(\omega) + \beta_0^* A_{44}(\omega)), \tag{26}
\]
and
\[
A_{11}(\omega) = \left[ \frac{\kappa_0}{2} - i(\Delta_0 + \omega) \right] \left[ \frac{\kappa_0}{2} - i(\Delta_0 + \omega)^2 + \Delta_0^2 \right] - \left[ \frac{\kappa_0}{2} - i(\Delta_0 + \omega) \right] g_0^2 \beta_0 \chi(\omega) \omega_m \omega_m^2 \tag{27}
\]
\[
A_{22}(\omega) = -i J^2 g_0^2 (\beta_0)^2 \frac{\chi(\omega)}{\omega_m} \tag{28}
\]
\[
A_{33}(\omega) = -i J^2 \left[ \frac{\kappa_0}{2} - i(\Delta_0 + \omega) \right] \left[ \frac{\kappa_0}{2} - i(\Delta_0 + \omega) \right] \tag{29}
\]
\[
A_{44}(\omega) = -J^2 g_0^2 (\beta_0)^2 \frac{\chi(\omega)}{\omega_m} \left[ \frac{\kappa_0}{2} - i(\Delta_0 + \omega) \right] \tag{30}
\]
\[
D(\omega) = \left[ \frac{\kappa_0}{2} + i(\Delta_0 - \omega) \right] A_{11}(\omega) + i A_{33}(\omega). \tag{31}
\]
Here, we introduce \( \Delta_0^2 = \Delta_0^2 + g_0^2 (\beta_0)^2 \frac{\chi(\omega)}{\omega_m} \), and the dynamical response function of the mirror \([46]\)
\[
\chi(\omega) = \frac{\omega_m}{\omega_m^2 - \omega^2 - i\omega \gamma_m / 2} \tag{32}
\]
with \( \chi^*(\omega) = \chi(\omega) \).

The second-order correlation functions that can be measured outside the cavity have the structure of a time-antiordered product followed by a time-ordered product, which is obtained by the properties of the Gaussian process \([40,41]\). Using the expression of \( \delta a(t) \), equations (21) and the correlations (9)–(13), the correlations of \( \delta a(t) \) and \( \delta a'(t) \) are given by
\[
\langle \delta a^\dagger(t) \delta a(t + \tau) \rangle = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X_{a,a}(\omega) e^{i\omega \tau} d\omega, \tag{37}
\]
\[
\langle T [ \delta a(t + \tau) \delta a(t) ] \rangle = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X_{a,a}(\omega) e^{-i\omega \tau} d\omega, \tag{38}
\]
where
\[
X_{a,a}(\omega) = |Q(\omega)|^2 \frac{\gamma_m}{2\omega_m} \left[ 1 + \coth \left( \frac{\hbar \omega}{2k_B T} \right) \right] + |F(\omega)|^2 + |H(\omega)|^2,
\]
\[
X_{a,a}(\omega) = Q(\omega)(\omega_m^2 - \omega^2 - i\omega \gamma_m / 2) \tag{39}
\]
\[
X_{a,a}(\omega) = X_{a,a}(\omega) Q(\omega) \frac{\gamma_m}{2\omega_m} \left[ 1 + \coth \left( \frac{\hbar \omega}{2k_B T} \right) \right] + E(\omega) F(\omega) - G(\omega) H(\omega). \tag{40}
\]

4. Statistical properties of the field in cavity A

The standard optomechanical system consists of a cavity with an oscillating mirror at one end \([32,33]\), and as shown in \([32]\), the strong optomechanical coupling \( (g_0 > \kappa) \) is the necessary condition for obtaining efficient antibunching photons in the standard optomechanical system. In this section, we will show that the cavity can exhibit strong photon antibunching if it is coupled to an optomechanical system even when optomechanical interaction in the system is weak.

Now, let us focus on the statistical properties of light field in cavity A. From now on, we assume that the decay rates \( \kappa_0 = \kappa_b = \kappa \), the detunings \( \Delta_0 = \Delta_0 = \gamma_m^2 / (2\omega_m) = \Delta \), and we normalize all parameters to \( \kappa \). The equal-time second-order correlation function of the photons inside cavity A, \( g_{aa}^{(2)}(0) \), is given as functions of \( \Delta / \kappa \) and \( g_0 / \kappa \) in figure 2(a) for \( J = 30 \kappa \). We can see that there is an optimal point for the field in cavity A showing strong antibunching at \( \Delta = -0.29 \kappa \) and \( g_0 = 0.2 \kappa \) for \( J = 30 \kappa \). The result shows that the photons in cavity A can exhibit strong antibunching when it is coupled to an optomechanical system under the weak coupling condition \( \Delta_0 < \kappa \) in the resolved sideband regime \( (\omega_m \gg \kappa) \).

As given in section 2, the strong antibunching comes from the cooperation between the weak optomechanical interaction and the destructive interference between different paths for two-photon excitation as shown in the reduced diagram in figure 1(b).

A two-dimensional plot of the equal-time second-order correlation function of cavity A, \( g_{aa}^{(2)}(0) \), as functions of both \( g_0 / \kappa \) and \( J / \kappa \) is shown in figure 2(b) for \( \Delta = -0.29 \kappa \). With increasing \( J \), the value of \( g_0 \) for getting the strong antibunching descends gradually. This implies that the linear coupling
between the cavity and the optomechanical system can be used to lower the strength of the optomechanical interaction that is required to achieve strong antibunching.

In order to understand the optimal conditions for the strong antibunching, we will find the optimal parameters for the system in the steady state. As $\Delta_a = \Delta_b - \frac{g_0^2}{\omega_m} = \Delta$, $J(a^\dagger b - b^\dagger a) + \text{H.c.}$ approximately equals $J(a^\dagger b^\dagger b^\dagger b) + \text{H.c.}$ under the conditions that $\Delta \ll \omega_m$, $g_0/\omega_m \ll 1$ and $J < \omega_m/2$. Then in the frame rotating with frequency $\omega_c$, the effective Hamiltonian (equation (2)) can be written approximately as

$$\tilde{H}_{\text{eff}} \approx \hbar \Delta_a a + \hbar \Delta b^\dagger b - \hbar \frac{g_0^2}{\omega_m} b^\dagger b^\dagger b^\dagger b + \hbar \omega_m c^\dagger c + \hbar J(ab^\dagger a^\dagger b + \text{H.c.})$$

Due to the fact that the phonon states are decoupled from the photon states, the state of the system can be written as $|\Psi\rangle = |\phi\rangle_m |\varphi\rangle$, where $|\varphi\rangle$ is the photon state and $|\phi\rangle_m$ is the phonon state. Under the weak pumping conditions, using the ansatz

$$|\phi\rangle = C_{00} |0, 0\rangle + C_{10} |1, 0\rangle + C_{01} |0, 1\rangle + C_{20} |2, 0\rangle + C_{11} |1, 1\rangle + C_{02} |0, 2\rangle,$$

and $C_{00} \gg C_{10}, C_{01} \gg C_{20}, C_{11}, C_{02}$, we can obtain the optimal conditions for $C_{20} = 0$ as given in [38] as follows:

$$\Delta_{\text{opt}} \approx - \frac{1}{2} \sqrt{9J^2 + 8\Delta^2 - 3J^2 - \kappa^2},$$

$$g_{0,\text{opt}} \approx \frac{\omega_m \Delta (5\omega^2 + 4\Delta^2)}{2(3J^2 - \kappa^2)}.$$  

For the two optical cavities under the strong coupling condition $J \gg \kappa$, the optimal conditions are simplified as

$$\frac{\Delta_{\text{opt}}}{\kappa} \approx - \frac{1}{2\sqrt{3}} \approx -0.29,$$  

which perfectly agree with the results shown in figures 2(a) and (b). The terms on the right-hand side of equation (46) are a factor, a ratio greater than $\sqrt{2}$ and the square root of the cavity decay rate to the coupling between the two cavities. It can be clearly seen that in the resolved sideband limit, the optomechanical coupling constant can be pushed below the single-photon strong coupling limit provided that the coupling between the two cavities is much larger than the optical linewidth of each cavity, but still less than half of the mechanical frequency.

The time evolution of the second-order correlation function of cavity $A$, $g_2^{(2)}(\tau)$, is shown in figure 2(c). As reported in [37, 38], $g_2^{(2)}(\tau)$ oscillates with the period $2\pi/J$. This oscillation comes from the probability oscillation between the photon states $|1, 0\rangle$ and $|0, 1\rangle$. Besides, the timescale of the antibunching is about $2\pi/\kappa$, which is the lifetime of the photon states.

The second-order correlation functions for different temperatures are shown in figure 2(d). From the figure, we can see that the increase of the temperature will suppress the exhibition of the antibunching effect, because the phonons in the environment may disturb the quantum statistics of the system. In order to get a strong antibunching effect, keeping the mechanical resonator in the optomechanical system in a low-temperature environment is one of the necessary conditions.

In addition, we show $G_1(0)$, $G_2(0)$ and $g_2^{(2)}(0)$ as functions of $\Delta/\kappa$ in figure 3(a), as functions of $J/\kappa$ in (b) and as functions of $g_0/\kappa$ in (c). In [50], the second-order correlation function is approximately replaced by $G_1(0)$ with $g_2^{(2)}(0)$ dropped, under the condition that $|\alpha|^2 \gg |\delta(t)\delta(t')|^2$. From figure 3, we can see that if the driving field is weak, then $G_2(0)$ plays an important role and should be considered here, otherwise $g_2^{(2)}(0)$ becomes negative under some condition.

\[ \Delta_{\text{opt}} \approx - \frac{1}{2\sqrt{3}} \approx -0.29, \]

\[ g_{0,\text{opt}} \approx \frac{\omega_m \Delta (5\omega^2 + 4\Delta^2)}{2(3J^2 - \kappa^2)}. \]
As the noises in the system are Gaussian [40, 41], the four-operator correlations are equal to the sum of products of pair correlation functions as shown in equation (36).

5. Numerical solution by the master equation

We have obtained the analytical expression for the second-order correlation function by solving the Langevin equations, but we have also performed many approximations, such as the semiclassical approximation, factorization assumption and ignored the high-order terms of small fluctuations. In this section, we calculate the second-order correlation function by numerically solving the master equation of the density matrix, and compare the results to the predictions given by the analytical solution derived above. The master equation of the coupled system is given as [51]

\[
\frac{\partial \rho}{\partial t} = \frac{-i}{\hbar} \{ \hat{H}, \rho \} + \frac{\kappa_b}{2} (2a \rho^a - a^+ a \rho - \rho a^+ a) \\
+ \frac{\kappa_b}{2} (2b \rho^b - b^+ b \rho - \rho b^+ b) \\
+ \gamma_m \frac{\nu}{2} (2p \rho c^+ c - \rho c^+ c) \\
+ \gamma_m \frac{\nu}{2} (c \rho c^+ + c^+ \rho c - \rho c c^+),
\]

(47)

where \( \tilde{n}_m \) is the mean thermal phonon number of the moving mirror given by the Bose–Einstein statistics \( \tilde{n}_m = [\exp(\hbar \omega_0 / k_B T) - 1]^{-1} \), \( k_B \) is the Boltzmann constant and \( T \) is the effective temperature of the moving mirror. The master equation can be solved on the basis of the photon and phonon number states \( |n_a, n_b, n_m \rangle \), and \( \rho \) can be written as the density matrix

\[
\rho = \rho_{n_a,n_b,n_m} |n_a, n_b, n_m \rangle \langle n_a, n_b, n_m|.
\]

(48)

If the elements of the steady-state density matrix, \( \rho_{n_a,n_b,n_c} \), are given, then the equal-time second-order correlation function can be easily calculated by

\[
g^{(2)}_{ab}(0) = \frac{\text{Tr}(\rho a^+ b^+ a b)}{\text{Tr}(\rho a a^+ b b)},
\]

(49)

\[
g^{(2)}_{bb}(0) = \frac{\text{Tr}(\rho b^+ b^+ b b)}{\text{Tr}(\rho b b b b)},
\]

(50)

\[
g^{(2)}_{ab}(0) = \frac{\text{Tr}(\rho a^+ b^+ b a)}{\text{Tr}(\rho a a^+ b b)},
\]

(51)

where \( g^{(2)}_{ab}(0) \) is the cross correlation between the photons in cavities \( A \) and \( B \).

For comparison, the second-order correlation functions calculated by the master equation and quantum Langevin equations are shown in the same figure as functions of \( \Delta/k \) in figure 4(a), as functions of \( J/k \) in (b) and as functions of \( g_0/k \) in (c) and (d). From figures 4(a)–(c), we can see that the results obtained by the two methods match quantitatively. As shown in figure 4(d), for \( g_0 < 0.65k \), the predictions of the two methods agree with each other. But with further increasing of \( g_0 \), the difference between them becomes significant gradually, and the linearized quantum Langevin equations method can only describe this qualitatively.

Finally, let us look at the statistical properties of photons in the entire system. The equal-time second-order correlation functions \( g^{(2)}_{ij}(0) \) can be calculated by using equations (49)–(51) and the results are shown in figure 5. From figure 5(a) we can see that, under the weak optomechanical interaction condition, there is strong antibunching in cavity \( A \) around \( g_0 = 0.66k \), while weak antibunching in cavity \( B \) and bunching for the photons between the two cavities.

Dependence of \( g^{(2)}_{ij}(0) \) on \( \Delta/k \) is drawn in figure 5(b). Figure 5(b) shows us two interesting phenomena as \( \Delta \) is in different domains: for \( \Delta < -0.1k \), there is strong antibunching in cavity \( A \) and weak antibunching in cavity \( B \), while the cross correlation between the modes in the two cavities exhibits bunching. In contrast, when \( \Delta > 0.05k \), there is bunching in cavities \( A \) and \( B \), while the cross correlation between the modes in the two cavities exhibits weak antibunching \( g^{(2)}_{ab}(0) < 1 \). Under the weakly driven condition [34], \( g^{(2)}_{ab}(0) < 1 \) and \( g^{(2)}_{bb}(0) < 1 \) indicate that there is no more than one photon in each cavity, and \( g^{(2)}_{ab}(0) > 1 \) shows that there is a large chance that each cavity has one photon simultaneously. \( g^{(2)}_{ab}(0) > 1 \) and \( g^{(2)}_{bb}(0) > 1 \) indicate that there is a large chance for more than one photon present in each cavity, while \( g^{(2)}_{ab}(0) < 1 \) shows that the probability
that each cavity has one photon simultaneously is low. In other words, when the system is driven weakly, and there are two photons in the coupled system, if $\Delta < -0.1\kappa$, then they are likely to be in the state that each cavity has one photon simultaneously, and if $\Delta > 0.05\kappa$, then they prefer to stay in one of the cavities together at the same time. Similar phenomena have been reported in the system with Kerr nonlinearity [38].

6. Conclusions

We have studied the photon statistics of a cavity linearly coupled to an optomechanical system. Due to the destructive quantum interference effect between different paths for two-photon excitation, the cavity can exhibit strong photon antibunching with weak optomechanical interaction in the optomechanical system. Both analytical and numerical methods are employed to figure out our results. The results allow us to be hopeful that the photon blockade effect will be observed with the current experimental parameters of optomechanics.

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