Research Article

Gauge Boson Mixing in the 3-3-1 Models with Discrete Symmetries

P. V. Dong, V. T. N. Huyen, H. N. Long, and H. V. Thuy

Institute of Physics, VAST, 10 Dao Tan, Ba Dinh, Hanoi 1000, Vietnam

Correspondence should be addressed to P. V. Dong, pvdong@iop.vast.ac.vn

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The mixing among gauge bosons in the 3-3-1 models with the discrete symmetries is investigated. To get tribimaximal neutrino mixing, we have to introduce sextets containing neutral scalar components with lepton number $L = 1, 2$. Assignment of VEVs to these fields leads to the mixing of the new gauge bosons and those in the standard model. The mixing in the charged gauge bosons leads to the lepton number violating interactions of the $W$ boson. The same situation happens in the neutral gauge boson sector.

1. Introduction

The experimental evidences of nonzero neutrino masses and mixing [1] have shown that the standard model of fundamental particles and interactions must be extended. Among many extensions of the standard model known today, the models based on gauge symmetry $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (called 3-3-1 models) [2–9] have interesting features. First, $[SU(3)_L]^3$ anomaly cancelation requires that the number of $SU(3)_L$ fermion triplets must be equal to that of antitriplets. If these multiplets are respectively enlarged from those of the standard model, the fermion family number is deduced to be a multiple of the fundamental color number, which is three, coinciding with the observation (see Frampton in [2]). In addition, one family of quarks has to transform under $SU(3)_L$ differently from the other two. This can lead to an explanation why the top quark is characteristically heavy (see, e.g., [10]). To complete the fundamental representations for leptons, the right-handed neutrinos or neutral fermions can be imposed which imply natural seesaw mechanisms for the neutrino small masses [11]. The 3-3-1 models can also provide a solution of electric charge quantization observed in the nature [12–16].
Table 1: Character table of $S_3$, where $\chi$ stands for character of representation and $C$ for class.

| Class | $n$ | $h$ | $\chi_1$ | $\chi_1'$ | $\chi_2$ |
|-------|-----|-----|-----------|-----------|---------|
| $C_1$ | 1   | 1   | 1         | 1         | 2       |
| $C_2$ | 2   | 3   | 1         | 1         | -1      |
| $C_3$ | 3   | 2   | 1         | -1        | 0       |

There are two typical versions of the 3-3-1 models concerning respective lepton contents. In the minimal 3-3-1 model [2–4] the lepton triplets include ordinary leptons of the standard model such as $(\nu_L, l_L, l_R^c)$. The 3-3-1 model with right-handed neutrinos [5–9] introduces right-handed neutrinos into the lepton sector, that is, $(\nu_L, l_L, \nu_R^c)$ and $l_R$. In the framework of 3-3-1 models, to explain the smallness of neutrino masses and the tribimaximal mixing [17–20]

$$U_{HPS}^{\text{left}} = \begin{pmatrix}
\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 1 \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}
\end{pmatrix},$$

we should propose another variant of the lepton sector such as $(\nu_L, l_L, N_R^c)$ and $l_R$ where $N_R$ are neutral chiral fermions carrying no lepton number (called 3-3-1 model with neutral fermions), and including discrete symmetries either $A_4$ or $S_4$ [21, 22]. The 3-3-1 model with neutral fermions based on $S_3$ flavor symmetry instead of $A_4$, $S_4$ has been studied in [23].

One of the most important ingredients is the sextets in which neutral scalar fields carrying lepton number $L = 1$ or 2. Assignment of VEVs to these fields leads to the mixing among new gauge bosons and that of the SM similarly in the economical 3-3-1 model [24–26], and such mixing leads to the lepton violating interactions. In this work we will pay attention to gauge bosons in the mentioned 3-3-1 models and give some phenomenological consequences.

The rest of this work is follows. In Section 2 we give a review of the 3-3-1 model with neutral fermions-based $S_3$ flavor symmetry. The other models with $A_4$ and $S_4$ can be done similarly, thus should be skip. Section 3 identifies gauge bosons and obtained the mixings among the standard model gauge bosons and the new ones. Section 4 is devoted to charged currents and give a constraint on the charged gauge boson mixing-angle. Finally we make conclusions in Section 5.

2. Brief Review of the Model

Before looking into the model, we provide a sketch of $S_3$ group theory [27, 28]. The $S_3$ that is a permutation group of three objects has six elements divided into three conjugacy classes. It possesses three nonequivalent irreducible representations $\mathbf{1}, \mathbf{1}'$ of one dimension, and $\mathbf{2}$ of two dimensions. Denoting $n$ and $h$ as the order of class and the order of elements within each class, respectively, the character table is given by Table 1.
We will work in the basis that \( \mathbb{2} \) is complex (see, e.g. [27]). Decomposition rules are

\[
\begin{align*}
\mathbb{1} \otimes \mathbb{1} &= \mathbb{1}(11), \\
\mathbb{1}' \otimes \mathbb{1}' &= \mathbb{1}(11), \\
\mathbb{1} \otimes \mathbb{1}' &= \mathbb{1}'(11), \\
\mathbb{1} \otimes \mathbb{2} &= \mathbb{2}(11,12), \\
\mathbb{1}' \otimes \mathbb{2} &= \mathbb{2}'(11,-12), \\
\mathbb{2} \otimes \mathbb{2} &= \mathbb{1}(12+21) \oplus \mathbb{1}'(12-21) \oplus \mathbb{2}(22,11).
\end{align*}
\]  

(2.1)

Here the first and second factors of the terms appearing in the parentheses indicate to the multiplet components of the first and second representations given in l.h.s, respectively. In this basis, the conjugation rules are given by

\[
\mathbb{2}^* (1^*, 2^*) = \mathbb{2}(2^*, 1^*), \\
\mathbb{1}' (1^*) = \mathbb{1}(1^*), \\
\mathbb{1}'' (1^*) = \mathbb{1}'(1^*).
\]  

(2.2)

The lepton number in the 3-3-1 model with \( S_3 \) symmetry [23] does not commute with the gauge symmetry. It is thus better to work with a new lepton charge \( \mathcal{L} \) related to the lepton number \( L \) by diagonal matrices \( L = xT_3 + yT_8 + \mathcal{L} \). Applying \( L \) to the lepton triplet with the notation that \( L(N_R) = 0 \), the coefficients are defined as \( x = 0, y = 2/\sqrt{3} \), and thus \( L = (2/\sqrt{3})T_8 + \mathcal{L} \) [29]. The leptons and quarks under \([SU(3)_L U(1)_X U(1)_{\mu} \mathbb{2}_3]\) symmetries correspondingly transform as follows:

\[
\begin{align*}
\psi_{1L} &= (\nu_{1L}, l_{1L}, N_{1R}^c)^T \sim \left[ 3, -\frac{2}{3}, \frac{1}{3}, 1 \right], \quad l_{1R} \sim [1, -1, 1, 1], \\
\psi_{aL} &= (\nu_{aL}, l_{aL}, N_{aR}^c)^T \sim \left[ 3, -\frac{1}{3}, \frac{2}{3}, 2 \right], \quad l_{aR} \sim [1, -1, 1, 2], \\
Q_{1L} &= (u_{1L}, d_{1L}, U_L)^T \sim \left[ 3, \frac{1}{3}, \frac{1}{3}, 1 \right], \\
Q_{aL} &= (d_{aL}, -u_{aL}, D_{aL})^T \sim \left[ 3^*, 0, \frac{1}{3}, 2 \right], \\
u_{1R} &= \left[ 1, \frac{2}{3}, 0, 1 \right], \quad d_{1R} = \left[ 1, \frac{1}{3}, 0, 1 \right], \quad U_R \sim \left[ 1, \frac{2}{3}, -1, 1 \right], \\
u_{aR} = \left[ 1, \frac{2}{3}, 0, 2 \right], \quad d_{aR} = \left[ 1, \frac{1}{3}, 0, 2 \right], \quad D_{aR} \sim \left[ 1, -\frac{1}{3}, 1, 2 \right],
\end{align*}
\]

(2.3)

where \( \alpha = 2, 3 \) is a family index of the last two lepton and quark families, which are in order defined as the components of \( \mathbb{2} \) representations.

To generate masses for the charged leptons, we need two scalar multiplets:

\[
\phi = \begin{pmatrix}
\phi_1^+ \\
\phi_2^0 \\
\phi_3^-
\end{pmatrix} \sim \left[ 3, \frac{2}{3}, -\frac{1}{3}, 1 \right], \quad \phi' = \begin{pmatrix}
\phi_1'^+ \\
\phi_2'^0 \\
\phi_3'^-
\end{pmatrix} \sim \left[ 3, \frac{2}{3}, -\frac{1}{3}, 1' \right].
\]  

(2.4)
with VEVs $\langle \phi \rangle = (0, v, 0)^T$ and $\langle \phi' \rangle = (0, v', 0)^T$. To generate masses for quarks, we additionally acquire the following scalar multiplets:

$$\chi = \left( \chi_1^0, \chi_2^0, \chi_3^0 \right)^T \sim \left[ 3, \frac{1}{3}, -\frac{2}{3}, 1 \right],$$

$$\eta = \left( \eta_1^0, \eta_2^0, \eta_3^0 \right)^T \sim \left[ 3, \frac{1}{3}, -\frac{1}{3}, 1 \right],$$

$$\eta' = \left( \eta_1^0, \eta_2^0, \eta_3^0 \right)^T \sim \left[ 3, \frac{1}{3}, -\frac{1}{3}, 1' \right].$$

(2.5)

Suppose that the VEVs of $\eta$, $\eta'$, and $\chi$ are $u$, $u'$, and $w$, where $u = \langle \eta_1^0 \rangle$, $u' = \langle \eta_1^0 \rangle$, $w = \langle \chi_3^0 \rangle$, and $\langle \eta_2^0 \rangle$, $\langle \eta_3^0 \rangle$, and $\langle \chi_1^0 \rangle$ vanish. The exotic quarks get masses $m_D = f_1 w$ and $m_{D,3} = f w$. In addition, $w$ has to be much larger than those of $\phi$ and $\eta$. Notice that the numbered subscripts are the indices of SU(3)$_L$.

Because of the $L$-symmetry, the couplings $\overline{q}_L q_L \phi$ and $\overline{q}_L q_L \phi'$ are suppressed. We therefore propose a new SU(3)$_L$ antisextet instead coupling to $\overline{q}_L q_L$ responsible for neutrino masses. The antisextet transforms as

$$s = \begin{pmatrix} s_{11}^0 & s_{12}^+ & s_{13}^0 \\ s_{12}^+ & s_{22}^+ & s_{23}^+ \\ s_{13}^0 & s_{23}^+ & s_{33}^0 \end{pmatrix} \sim \left[ 6^*, \frac{2}{3}, -\frac{4}{3}, 2 \right],$$

(2.6)

where the numbered subscripts are the SU(3)$_L$ indices. Henceforth the indices of $S_3$ on scalar fields will be kept and should be understood. The VEVs of $s$ is set as $(\langle s_1 \rangle, \langle s_2 \rangle)$ under $S_3$, where

$$\langle s_i \rangle = \begin{pmatrix} \lambda_i & 0 & v_i \\ 0 & 0 & 0 \\ v_i & 0 & \Lambda_i \end{pmatrix} \quad (i = 1, 2).$$

(2.7)

Due to the $S_3$ symmetry, all these VEVs are equal to each others, that is, $\lambda_1 = \lambda_2$, $v_1 = v_2$ and $\Lambda_1 = \Lambda_2$, which can be found from the potential minimization.

With the scalar multiplets as defined, the Yukawa lagrangian is given by

$$\mathcal{L}_Y = h_1 \overline{q}_{1L} \phi_1^R + h (\overline{q}_{2L} l_{2R} + \overline{q}_{3L} l_{3R}) \phi + h' (\overline{q}_{3L} l_{3R} - \overline{q}_{2L} l_{2R}) \phi'$$

$$+ f_1 \overline{q}_{1L} \chi l_R + f \overline{q}_{L} \chi^* D_R + h_1' \overline{Q}_L \eta u_{1R} + h' \overline{Q}_L \eta^* d_R$$

$$+ h'' \overline{Q}_L \eta'' d_R + h'' \overline{Q}_L \phi d_{1R} + h'' \overline{Q}_L \phi^* u_R + h'' \overline{Q}_L \phi'' u_R$$

$$+ \frac{1}{2} \chi (\overline{q}_{2L} q_{2L} s_1 + \overline{q}_{3L} q_{3L} s_2) + \frac{1}{2} \chi' (q_{2L} q_{2L} + q_{3L} q_{3L})$$

$$+ H.c.$$
It is easily shown that the charged leptons and ordinary quarks get consistent masses \[ \frac{2}{3} \]. However, this case does not lead to neutrino masses and mixing consistent with the experimental data. The analysis in [21, 22] shows that (i) a “perturbation” is required:

\[
\lambda_1 \approx \lambda_2, \quad v_1 \approx v_2, \quad \Lambda_1 \approx \Lambda_2. \tag{2.9}
\]

A possibility to derive this is to impose another antisextet \( s' \) but with the VEVs being very smaller than those of \( s \), respectively. Thus, in the followings the \( s' \) should be skipped since it does not contribute at the first order. Otherwise, the \( s' \) contributions start from the second order in similarity to those of \( s \) which are easily included. (ii) A scalar triplet \( \rho \) similar to \( \phi' \) must be imposed. The \( \rho \) is also skip for the same reason as \( s' \), that is, its contribution is similar to that of \( \phi' \). Let us emphasis that our conclusions remain unchanged if \( s' \) and \( \rho \) present.

The hierarchies in the VEVs were given in [23]:

\[
\lambda_1, \lambda_2 < u_1, \quad u_2 < v, v', \quad u, u' < \omega, \quad \Lambda_1, \Lambda_2. \tag{2.10}
\]

In the following, the two limits are often taken into account: (i) the lepton-number violating parameters tend to zero, that is, \( \lambda_{1,2}, u_{1,2} \to 0 \), and (ii) the large scales of SU\(_3\)L symmetry break down to that of the standard model approx infinity, that is, \( \omega, \Lambda_{1,2} \to \infty \). Let us note also that \( v, v', u, \) and \( u' \) are in the electroweak scale as well as the large scales all conserving the lepton number.

### 3. Gauge Bosons

The covariant derivative of a general triplet \( \Phi \) is given by

\[
D_\mu = \partial_\mu + igT_aW_{a\mu} + ig_XT_XB_\mu \\
\equiv \partial_\mu - iP_\mu, \tag{3.1}
\]

where the gauge fields \( W_a \) and \( B \) transform as the adjoint representations of SU\(_3\)L and U(1)\(_X\), respectively, and the corresponding gauge coupling constants \( g \) and \( g_X \). The \( T_9 = \text{diag}(1,1,1)/\sqrt{6} \) is chosen so that \( \text{Tr}(T_aT_b) = \delta_{ab}/2 \) with \( a,b = 1,2,...,9 \). The neutral gauge bosons of the theory get masses from the triplet as follows:

\[
\mathcal{L}_{\text{mass}}^\Phi = \left( D^\mu_{\text{H}}(\Phi) \right)^+ \left( D^{\mu}_{\text{H}}(\Phi) \right), \tag{3.2}
\]

where the subscript \( H \) denotes diagonal part of the covariant derivative:

\[
D^\mu_{\text{H}} = \partial_\mu + igT_3W^\mu_3 + igT_8W^\mu_8 + ig_XT_\chi B_\mu. \tag{3.3}
\]
The covariant derivative for an antisextet with the VEV part is [30]

\[ D_\mu(s_i) = -\frac{ig}{2} \left\{ A_\mu^a A^*_a(s_i) + \langle s_i \rangle A_\mu^a T^a \right\} + igx T_9 X B_\mu(s). \] (3.4)

Let us denote the antisextet in term of the SU(3)_L indices by \( \Gamma_{ij} \). Then, the mass Lagrangian due to the antisextet's contribution is given by

\[ \mathcal{L}_{\text{mass}}^\Gamma = \left( D_\mu^H (\Gamma)_{ij} \right)^\dagger \left( D^{H\mu} (\Gamma)_{ij} \right). \] (3.5)

Let us denote the following combinations:

\[ W^\pm_\mu = \frac{W_{1\mu} \mp iW_{2\mu}}{\sqrt{2}}, \]
\[ Y^\mu = \frac{W_{6\mu} \mp iW_{7\mu}}{\sqrt{2}}, \]
\[ X^{0*}_\mu = \frac{W_{4\mu} - iW_{5\mu}}{\sqrt{2}} \]

having defined charges under the generators of the SU(3)_L group. For the sake of convenience in further reading, we note that \( W_4 \) and \( W_5 \) are pure real and imaginary parts of \( X^{0*}_\mu \) and \( X^{0*}_\mu \), respectively:

\[ W_{4\mu} = \frac{1}{\sqrt{2}} \left( X^{0}_\mu + X^{0*}_\mu \right), \]
\[ W_{5\mu} = \frac{i}{\sqrt{2}} \left( X^{0}_\mu - X^{0*}_\mu \right). \] (3.7)

Then \( P_\mu \) is rewritten in a convenient form:

\[ \frac{\sqrt{2}}{g^2} \left( \begin{array}{ccc}
W_{3\mu} + \frac{1}{\sqrt{3}} W_{8\mu} + t \sqrt{\frac{2}{3}} X B_\mu & \sqrt{2} W^+_\mu & \sqrt{2} X^{0}_\mu \\
\sqrt{2} W^-_\mu & -W_{3\mu} + \frac{1}{\sqrt{3}} W_{8\mu} + t \sqrt{\frac{2}{3}} X B_\mu & \sqrt{2} Y^-_\mu \\
\sqrt{2} X^{0*}_\mu & \sqrt{2} Y^+_\mu & -\frac{2}{\sqrt{3}} W_{8\mu} + t \sqrt{\frac{2}{3}} X B_\mu
\end{array} \right). \] (3.8)

with \( t = g_x/g \).
The covariant derivative acting on the antisextet VEV is given by

\[ D_\mu(s_i)_{11} = -ig \left( \lambda_i W_{3\mu} + \lambda_{i1} \frac{1}{\sqrt{3}} W_{8\mu} - t\lambda_{i1} \sqrt{\frac{2}{3}} B_\mu + \sqrt{2} u_i X_\mu^0 \right), \]

\[ D_\mu(s_i)_{12} = -\frac{ig}{\sqrt{2}} (\lambda_i W^-_\mu + u_i Y^-_\mu), \]

\[ D_\mu(s_i)_{13} = -\frac{ig}{2} \left( u_i W^-_{3\mu} - u_i \frac{1}{\sqrt{3}} W_{8\mu} - t u_i \sqrt{\frac{2}{3}} B_\mu + \sqrt{2} \Lambda_i X^0_\mu + \sqrt{2} \Lambda_i X^{d_1}_\mu \right), \]

\[ D_\mu(s_i)_{22} = 0, \]

\[ D_\mu(s_i)_{23} = -\frac{ig}{\sqrt{2}} (u_i W^-_\mu + \Lambda_i Y^-_\mu), \]

\[ D_\mu(s_i)_{33} = ig \left( 2 \Lambda_i \frac{1}{\sqrt{3}} W_{8\mu} + t\lambda_{i1} \sqrt{\frac{2}{3}} B_\mu - \sqrt{2} u_i X^{d_1}_\mu \right), \]

\[ D_\mu(s_i)_{12} = D_\mu(s_i)_{21}, \]

\[ D_\mu(s_i)_{13} = D_\mu(s_i)_{31}, \]

\[ D_\mu(s_i)_{23} = D_\mu(s_i)_{32}. \]

The masses of gauge bosons in this model are followed from

\[ \mathcal{L}_{\text{mass}}^{\text{GB}} = (D_\mu(\phi))^+ D^\mu(\phi) + (D_\mu(\phi'))^+ D^\mu(\phi') + (D_\mu(\chi))^+ D^\mu(\chi) \]

\[ + (D_\mu(\eta))^+ D^\mu(\eta) + (D_\mu(\eta'))^+ D^\mu(\eta') + \text{Tr} \left[ (D_\mu(s_i))^+ D^\mu(s_i) \right] \]

\[ + \text{Tr} \left[ (D_\mu(s_2))^+ D^\mu(s_2) \right]. \]  

In the following, we notice that \( s_1 = s_2 \); namely, \( u_1 = u_2, \lambda_1 = \lambda_2, \) and \( \Lambda_1 = \Lambda_2 \) are taken into account.

From (3.10), the imaginary part \( W_5 \) is decoupled with mass given by

\[ M_{W_5}^2 = \frac{g^2}{2} \left( 16u_1^2 + 4\lambda_1^2 - 8\lambda_1 \lambda_2 + 16\Lambda_1^2 + \omega^2 + u^2 + u^2 \right). \]  

In the limit \( \lambda_1, u_1 \to 0, \)

\[ M_{W_5}^2 = \frac{g^2}{2} \left( u^2 + u^2 + \omega^2 + 4\Lambda_1^2 \right). \]
The charged gauge bosons $W$ and $Y$ mix via

$$L_{\text{CG mass}}^\mu = \frac{g^2}{24} \left( W^-_{\mu}, Y^-_{\mu} \right) M_{WY}^2 \left( W^+_{\mu}, Y^+_{\mu} \right)^T,$$

where

$$M_{WY}^2 = 2 \begin{pmatrix} v^2 + v'^2 + u^2 + u'^2 + 4u_1^2 + 4\lambda_1^2 & 4(\Lambda_1 u_1 + \lambda_1 u_1) \\ 4(\Lambda_1 u_1 + \lambda_1 u_1) & v^2 + v'^2 + \omega^2 + 4\Lambda_1^2 + 4u_1^2 \end{pmatrix}.$$  \hspace{1cm} (3.13)

Diagonalizing this mass matrix, we get physical charged gauge bosons

$$W^-_{\mu} = \cos \theta W_{\mu}^- + \sin \theta Y_{\mu}^-,$$

$$Y^-_{\mu} = -\sin \theta W_{\mu}^- + \cos \theta Y_{\mu}^-.$$  \hspace{1cm} (3.15)

The mixing angle is given by

$$\tan \theta = \frac{4(\Lambda_1 u_1 + \lambda_1 u_1)}{\omega^2 + 4\Lambda_1^2 - u^2 - u'^2 - 4\lambda_1^2} \sim \frac{u_1}{\Lambda_1},$$

provided that $\omega^2 \sim \Lambda_1^2 \gg u^2, u'^2, \lambda_1^2, \Lambda_1^2$. The mass eigenvalues are

$$M_W^2 = \frac{g^2}{4} \left\{ v^2 + v'^2 + 2u^2 + 2u'^2 + \omega^2 + 4\lambda_1^2 + 4\Lambda_1^2 - \left[ \left( v^2 + v'^2 - \omega^2 \right)^2 + 16\lambda_1^4 + 16\Lambda_1^4 + 128\lambda_1 \Lambda_1 u_1^2 \right. \right. + 8\Lambda_1^2 \left. \left( \omega^2 - v^2 - v'^2 + 8u_1^2 \right) \right. - \left. \left. 8\lambda_1^2 \left( 4\Lambda_1^2 \omega^2 - v^2 - v'^2 - 8u_1^2 \right) \right]^{1/2} \right\},$$

$$M_Y^2 = \frac{g^2}{4} \left\{ v^2 + v'^2 + 2u^2 + 2u'^2 + \omega^2 + 4\lambda_1^2 + 4\Lambda_1^2 + \left[ \left( v^2 + v'^2 - \omega^2 \right)^2 + 16\lambda_1^4 + 16\Lambda_1^4 + 128\lambda_1 \Lambda_1 u_1^2 \right. \right. + 8\Lambda_1^2 \left. \left( \omega^2 - v^2 - v'^2 + 8u_1^2 \right) \right. - \left. \left. 8\lambda_1^2 \left( 4\Lambda_1^2 \omega^2 - v^2 - v'^2 - 8u_1^2 \right) \right]^{1/2} \right\}. \hspace{1cm} (3.17)$$

Note that, in the limit $\lambda_1, u_1 \rightarrow 0$, the mixing angle tends to zero and the mass eigenvalues are

$$M_W^2 = \frac{g^2}{2} \left( v^2 + v'^2 + u^2 + u'^2 \right),$$

$$M_Y^2 = \frac{g^2}{2} \left( v^2 + v'^2 + \omega^2 + 4\Lambda_1^2 \right). \hspace{1cm} (3.18)$$
There is a mixing among the neutral gauge bosons \( W_3, W_8, B, \) and \( W_4 \). The mass Lagrangian in this case has the form

\[
\mathcal{L}_{\text{mass}}^{NG} = \frac{1}{2} V^T \mathcal{M}^2 V,
\]

\( V^T \equiv (W_3, W_8, B, W_4). \)

In the basis of these elements, the mass matrix is given by

\[
\mathcal{M}^2 = \frac{g^2}{4} \begin{pmatrix}
\mathcal{M}_{11}^2 & \mathcal{M}_{12}^2 & \mathcal{M}_{13}^2 & \mathcal{M}_{14}^2 \\
\mathcal{M}_{21}^2 & \mathcal{M}_{22}^2 & \mathcal{M}_{23}^2 & \mathcal{M}_{24}^2 \\
\mathcal{M}_{31}^2 & \mathcal{M}_{32}^2 & \mathcal{M}_{33}^2 & \mathcal{M}_{34}^2 \\
\mathcal{M}_{41}^2 & \mathcal{M}_{42}^2 & \mathcal{M}_{43}^2 & \mathcal{M}_{44}^2
\end{pmatrix},
\]

where

\[
\begin{align*}
\mathcal{M}_{11}^2 &= 2 \left( 4u_1^2 + 8\lambda_1^2 + u^2 + u'^2 + v^2 + v'^2 \right), \\
\mathcal{M}_{22}^2 &= \frac{2}{3} \left( 4u_1^2 + 8\lambda_2^2 + u^2 + u'^2 + 32\lambda_1^2 + v^2 + v'^2 + 4\omega^2 \right), \\
\mathcal{M}_{33}^2 &= \frac{4\mu^2}{27} \left( 16u_1^2 + 8\lambda_1^2 + u^2 + u'^2 + 8\lambda_1^2 + 4v^2 + 4v'^2 + \omega^2 \right), \\
\mathcal{M}_{44}^2 &= 2 \left( \omega^2 + u^2 + u'^2 + 16u_1^2 + 4\lambda_1^2 + 4\lambda_1^2 + 8\Lambda_1 \Lambda_1 \right), \\
\mathcal{M}_{12}^2 &= \frac{2}{\sqrt{3}} \left( -4u_1^2 + 8\lambda_1^2 + u^2 + u'^2 - v^2 - v'^2 \right), \\
\mathcal{M}_{13}^2 &= -\frac{2}{3} \sqrt{\frac{2}{3}} t \left( 8u_1^2 + 8\lambda_1^2 + u^2 + u'^2 + 2v^2 + 2v'^2 \right), \\
\mathcal{M}_{14}^2 &= 8(3u_1 \lambda_1 + u_1 \Lambda_1), \\
\mathcal{M}_{23}^2 &= \frac{2\sqrt{7}}{9} t \left( 8u_1^2 + 8\lambda_1^2 - u^2 - u'^2 + 16\Lambda_1^2 + 2v^2 + 2v'^2 + 2\omega^2 \right), \\
\mathcal{M}_{24}^2 &= \frac{8}{\sqrt{3}} (u_1 \lambda_1 - 5u_1 \Lambda_1), \\
\mathcal{M}_{34}^2 &= -\frac{32}{3} \sqrt{\frac{2}{3}} t (u_1 \lambda_1 + u_1 \Lambda_1).
\end{align*}
\]

This mass matrix contains one exact eigenvalue:

\[
\mathcal{M}_1^2 = 0.
\]
The associated eigenvector is

\[
A_\mu = \frac{1}{\sqrt{18 + 4t^2}} \begin{pmatrix} \sqrt{3}t \\ -t \\ 3\sqrt{2} \\ 0 \end{pmatrix}.
\] (3.23)

Using continuation of the gauge coupling constant \(g\) of the SU(3) \(_L\) at the spontaneous symmetry breaking point, we have [2–9]

\[
t = \frac{3\sqrt{2}s_W}{\sqrt{3 - 4s_w^2}}
\] (3.24)

In order to diagonalize the mass matrix, we choose the base of \((A_\mu, Z_\mu, Z'_\mu, W_{4\mu})\), with

\[
Z_\mu = c_W W_{3\mu} - s_W \left( -\frac{t_W}{\sqrt{3}} W_{9\mu} + \frac{1}{3} \sqrt{1 - \frac{t_W^2}{3}} B_\mu \right),
\] (3.25)

\[
Z'_\mu = \sqrt{1 - \frac{t_W^2}{3}} W_{8\mu} + \frac{t_W}{\sqrt{3}} B_\mu.
\]

The new base is changed from the old by unitary matrix:

\[
U = \begin{pmatrix}
s_W & c_W & 0 & 0 \\
-\frac{c_W t_W}{\sqrt{3}} & \frac{s_W t_W}{\sqrt{3}} & \sqrt{1 - \frac{t_W^2}{3}} & 0 \\
c_W \sqrt{1 - \frac{t_W^2}{3}} & -s_W \sqrt{1 - \frac{t_W^2}{3}} & \frac{t_W}{\sqrt{3}} & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}.
\] (3.26)

In this basis, the mass matrix \(M^2\) becomes

\[
M^2 = U^+ M^2 U = \frac{\alpha^2}{4} \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & M_{22}^2 & M_{23}^2 & M_{24}^2 \\
0 & M_{23}^2 & M_{33}^2 & M_{34}^2 \\
0 & M_{24}^2 & M_{34}^2 & M_{44}^2
\end{pmatrix}.
\] (3.27)
In the approximation $\lambda_i^2, u_i^2 \ll \Lambda_i^2$, we have

\[ M^2_{22} = 2 \left( u^2 + u'^2 + v^2 + v'^2 \right) \frac{1}{c^2_{W}}, \]

\[ M^2_{23} = \frac{-2 \left[ 2 s^2_W (u^2 + u'^2) - (v^2 + v'^2) \right] \sqrt{\alpha_0}}{c^2_{W}}, \]

\[ M^2_{24} = \frac{8 u_1 \lambda_1}{c_W}, \]

\[ M^2_{33} = \frac{2 \left( u^2 + u'^2 \right)}{c^2_{W} \alpha_0} - \frac{2 \left( v^2 + v'^2 \right) \alpha_0}{c^2_{W}} + 8 \omega^2 c^2_W \alpha_0 + 64 \lambda_1^2 c^2_W \alpha_0, \]

\[ M^2_{34} = \frac{-8 x_0 u_1 \lambda_1}{c_W \sqrt{\alpha_0}}, \]

\[ M^2_{44} = 2 \left( \omega^2 + u^2 + u'^2 + 4 \lambda_1 \lambda_1 + 4 \lambda_1^2 \right), \]

with

\[ x_0 = \left( 4 c^2_W + 1 \right), \]

\[ \alpha_0 = \frac{1}{\left( 4 c^2_W - 1 \right)}. \]

It is noteworthy that in the limit $u_1 = 0$, the elements $M^2_{24}$ and $M^2_{34}$ (or equivalently $M^2_{14}$, $M^2_{24}$, $M^2_{34}$ in the old base) vanish. In this case, the mixing between $W_4$ and $Z, Z'$ disappears. Three bosons gain masses via seesaw mechanism:

\[ M^2_{Z} = M^2_{22} - \left( M^{\text{off}} \right)^T \left( M^2_{2X2} \right)^{-1} M^{\text{off}}, \]

\[ M^2_{2X2} \approx M^2_{2X2}, \]

where

\[ M^{\text{off}} = \begin{pmatrix} M^2_{23} \\ M^2_{24} \end{pmatrix}, \]

\[ M^2_{2X2} = \begin{pmatrix} M^2_{33} & M^2_{34} \\ M^2_{43} & M^2_{44} \end{pmatrix}. \]
We have then

$$M_Z^2 = \frac{g^2}{4} \left( \frac{M_{22}^2 - \left( M_{23}^2 \right)^2 M^4_{44} - 2M^2_{23}M^2_{24}M^2_{34} + \left( M_{23}^2 \right)^2 M^2_{33}}{M^2_{33}M^2_{44} - \left( M^2_{34} \right)^2} \right) $$

(3.32)

where

$$\Delta M^2_{22} = \frac{g^2}{4} \left[ \frac{32(2x_0x_1 + x_3)u_1^2 \Lambda^2_1 + x_1^2 x_2}{x_2 x_3 - 32x_0^2 u_1^2 \Lambda^2_1} \right],$$

(3.33)

$$x_1 = c_2 W \left( u^2 + u' \right) - \left( v^2 + v' \right),$$

$$x_2 = \left( \omega^2 + u^2 + u'^2 + 4\Lambda_1 \lambda_1 + 4\Lambda^2_2 \right),$$

$$x_3 = c_2 W \left( u^2 + u'^2 \right) + v^2 + v'^2 + 4c_2^4 \omega^2 + 32c_2^4 \Lambda^2_1.$$
Hence

\[ 0.8403 \text{ GeV}^2 \leq \Delta_{M^2_{22}} \leq 21.0065 \text{ GeV}^2, \]  

(3.38)

or

\[ 0.239 \text{ GeV} \leq u_1 \leq 1.197 \text{ GeV}, \]  

(3.39)

where we have used \( c_W^2 = 0.769 \) and \( M_W = 80.384 \text{ GeV} \).

Diagonalizing the mass matrix \( M^2_{2 \times 2} \), we get new gauge bosons:

\[ Z'_\mu = \cos \phi Z''_\mu + \sin \phi W'_{4\mu}, \]  

\[ W'_{4\mu} = -\sin \phi Z''_\mu + \cos \phi W'_{4\mu}. \]  

(3.40)

The mixing angle is defined by

\[ \tan \phi = \frac{2M^2_{44}}{M^2_{44} - M^2_{33} + \sqrt{(M^2_{44} - M^2_{33})^2 + 4(M^2_{54})^2}}. \]  

(3.41)

Substituting (3.28) into (3.41), we get

\[ \tan \phi = -2\sqrt{a_1u_1\Lambda_1} \left\{ a_2(u^2 + u'^2) + a_3(v^2 + v'^2) - 2a_0\omega^2 + 8\lambda_1 \Lambda_1 - 8x_0\alpha_0\Lambda_1^2 \right. \]

\[ \left. + \left[ a_2(u^2 + u'^2) + a_3(v^2 + v'^2) - 2a_0\omega^2 + 8\lambda_1 \Lambda_1 - 8x_0\alpha_0\Lambda_1^2 \right]^2 + 4a_1u_1^2\Lambda_1^2 \right\}^{-1/2} \]

\[ \approx \frac{\sqrt{4c_W^2 - 1}}{c_W} \frac{u_1}{\Lambda_1}, \]  

(3.42)

provided that \( \omega^2 \sim \Lambda_1^2 \gg u^2, u'^2, v^2, v'^2, u_1^2, \lambda_1^2 \), where

\[ a_1 = \frac{64x_0^2\alpha_0}{c_W^2}, \]  

\[ a_2 = \frac{-2 + 6c_W^2\alpha_0}{c_W^2}, \]  

\[ a_3 = \frac{-2}{c_W^2}\alpha_0. \]  

(3.43)
The physical mass eigenvalues are defined by

\[ M^2_{Z',W_{3\mu}} = \frac{g^2}{4} M^2_{44} + M^2_{33} \mp \sqrt{(M^2_{44} - M^2_{33})^2 + 4(M^2_{34})^2} \]

\[ = \frac{g^2}{8} \left( \alpha_4 \left( u^2 + u'^2 \right) - \alpha_3 \left( v^2 + v'^2 \right) + \alpha_5 w^2 + 8\lambda_1 \Lambda_1 + \alpha_6 \Lambda_1^2 \right) \]

\[ \pm \sqrt{ \left( \alpha_2 \left( u^2 + u'^2 \right) + \alpha_3 \left( v^2 + v'^2 \right) - 2\alpha_6 w^2 + 8\lambda_1 \Lambda_1 - 8\alpha_0 \alpha_0 \Lambda_1^2 \right)^2 + 4\alpha_1 u^2 \Lambda_1^2} \]

(3.44)

with

\[ \alpha_4 = \frac{2 - 10c_w^2 + 16c_w^4}{c_w^2} \alpha_0, \]

\[ \alpha_5 = \left( 16c_w^2 - 2 \right) \alpha_0, \]

\[ \alpha_6 = 8 \left( 12c_w^2 - 1 \right) \alpha_0. \]

In the limit \( \lambda_1, u_1 \to 0 \), we have

\[ M^2_{Z'} = \frac{g^2}{2} \left[ \frac{c_w^2 \left( u^2 + u'^2 \right) + v^2 + v'^2 + 4c_w^4 \omega^2 + 32c_w^4 \Lambda_1^2}{2c_w^2} \right] \]

\[ M^2_{W_{3\mu}} = \frac{g^2}{2} \left( u^2 + u'^2 + \omega^2 + 4\Lambda_1^2 \right). \]

Thus the \( W'_4 \) and \( W_5 \) components have the same mass. With this result, we should identify the combination of \( W'_4 \) and \( W_5 \)

\[ \sqrt{2} X^0_{\mu} = W'_{4\mu} - iW_{5\mu} \]

(3.47)

as physical neutral non-Hermitian gauge boson. The subscript 0 denotes neutrality of gauge boson \( X \). However, to get tribimaximal mixing, the previous limit is not valid [21, 22]. This means that neutrino tribimaximal mixing leads to the masses of \( X^0 \) and \( X_0 \) to be different. Consequence of this fact is that there is CPT violation \[1, 31\] in the model under consideration. We will return to this problem in the future work.

In the limit \( \omega^2 - \Lambda_1^2 \gg u^2, u'^2, v^2, v'^2, u_{1,2}, \lambda_1 \) (or \( \omega, \Lambda_1 \to \infty \)), the mixings between the charged gauge bosons \( W - Y \) and the neutral ones \( W'_4 - Z' \) are in the same order since from (3.16) and (3.42) they are proportional to \( u_1/\Lambda_1 \). In addition, from (3.46), \( M^2_{Z'} \approx 2g^2(\omega^2 + 16c_w^2 \Lambda_1^2) \) is bigger than \( M^2_{W_4} \approx (g^2/2)(\omega^2 + 4\Lambda_1^2) \) (or \( M^2_{X_0} \)). It is also verified that \( |M^2_{Z'} - M^2_{X_0}| < M^2_W \). In that limit, the masses of \( X^0 \) and \( Y \) degenerate.
Note that the formulas for masses and mixing of gauge bosons previously presented, are common for the 3-3-1 models with more complicated Higgs sector such as with $A_4$ or $S_4$ discrete symmetries.

### 4. Charged Currents

The interaction among fermions with gauge bosons arises from part

$$i \bar{q} \gamma_\mu D^\mu q = \text{kinematic terms} + H^{CC} + H^{NC}. \quad (4.1)$$

Similarly in the economical 3-3-1 model, despite neutrality, the gauge bosons $X^0$ and $X^{0s}$ belong to this section by their nature. Because of the mixing among the SM $W$ boson and the charged bilepton $Y$ as well as among $X^0$ and $X^{0s}$ with $(W_3, W_8, B)$, the new terms exist the same as the economical 3-3-1 model [25, 26]:

$$H^{CC} = \frac{g}{\sqrt{2}} \left( J^{\mu W^-} W^\mu_{\mu} + J^{\mu Y} Y^\mu_{\mu} + J^{\mu X_0} X^\mu_{\mu} + \text{H.c.} \right), \quad (4.2)$$

where

$$J^{\mu W^-} = c_\theta \left( \bar{v}_{iL} Y^\mu e_{iL} + \bar{\nu}_{iL} Y^\mu d_{iL} \right) + s_\theta \left( \bar{\nu}_{iL} Y^\mu e_{iL} + \bar{U}_{L} Y^\mu d_{1L} + \bar{d}_{aL} Y^\mu D_{aL} \right),$$

$$J^{\mu Y} = c_\theta \left( \bar{v}_{iL} Y^\mu e_{iL} + \bar{U}_{L} Y^\mu d_{1L} + \bar{d}_{aL} Y^\mu D_{aL} \right) - s_\theta \left( \bar{v}_{iL} Y^\mu e_{iL} + \bar{d}_{aL} Y^\mu d_{1L} \right),$$

$$J^{\mu X_0} = \left( 1 - t_{2\theta}^2 \right) \left( \bar{v}_{iL} Y^\mu v_{iL} + \bar{u}_{1L} Y^\mu U_{L} - \bar{d}_{aL} Y^\mu d_{aL} \right) - t_{2\theta}^2 \left( \bar{v}_{iL} Y^\mu u_{1L} + \bar{U}_{L} Y^\mu u_{1L} - \bar{d}_{aL} Y^\mu D_{aL} \right)$$

$$+ \frac{t_{2\theta}}{1 + 4t_{2\theta}^2} \left( \bar{v}_{iL} Y^\mu v_{iL} + \bar{u}_{1L} Y^\mu U_{L} - \bar{d}_{aL} Y^\mu d_{al} + \bar{D}_{aL} Y^\mu D_{aL} \right). \quad (4.3)$$

All aforementioned interactions are lepton-number violating and weak (proportional to $\sin \theta$ or its square $\sin^2 \theta$). However, these couplings lead to lepton-number violations only in the neutrino sector.

Let us consider some constraints on the parameters of the model; one of the ways to do that is the consideration for $W$ decay. In our model, the $W$ boson has the following normal main decay modes:

$$W^- \rightarrow l \bar{\nu}_l \quad (l = e, \mu, \tau), \quad (4.4)$$

$$\downarrow u^c d, u^c s, u^c b, \quad (u \rightarrow c),$$

which are the same as in the SM and in the 331 with right-handed neutrinos. Beside the aforementioned modes, there are additional ones which are lepton-number violating ($\Delta L = 2$) the model’s specific feature:

$$W^- \rightarrow l \nu_l \quad (l = e, \mu, \tau). \quad (4.5)$$
The total decay width of $W$ [GeV] is given by [25, 26]

$$\Gamma_W = 1.039 \frac{\alpha M_W}{2 s_W^2} \left(1 - s_W^2\right) + \frac{\alpha M_W}{4 s_W} \left(1 - s_W^2\right) + \frac{\alpha M_W}{4 s_W^4} s_W^4,$$

where the first term is due to the quark productions (with $\alpha_s = 0.1184$ chosen for the QCD radiative corrections), the second term comes from the normal modes with leptons, and the last one is for the unnormal modes. Let us choose $\alpha(M_Z) = 1/128$, $M_W = 80.399$ GeV, and $\Gamma_W = 2.085 \pm 0.042$ GeV [1]. The total decay width is plotted in Figure 1. From the figure, we get an upper limit on the $\sin \theta$ in the model:

$$\sin \theta \leq 0.15,$$

which is bigger than that given in [25, 26].

There are lepton number violating interactions in the neutral Gauge boson sector, we refer interested reader to [25, 26].
5. Conclusions

In this paper, we have investigated Gauge boson sector: their mixing and masses. The vacuum expectation values $u_i$ and $\lambda_i$ are a source of lepton-number violations and a reason for the mixing between the charged Gauge bosons—the standard model $W$ and the singly-charged bilepton Gauge bosons, as well as between neutral non-Hermitian $X^0$ and neutral Gauge bosons: the photon, the $Z$, and the new exotic $Z'$. The interesting new physics compared with 3-3-1 models is the neutrino physics. Due to lepton-number violating couplings, we have many interesting consequences. We have shown that the neutrino tribimaximal mixing leads to the CPT violation. This feature will be considered in the future publication.

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