Quantum phases from competing short- and long-range interactions in an optical lattice

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Insights into complex phenomena in quantum matter can be gained from simulation experiments with ultracold atoms, especially in cases where theoretical characterization is challenging. However, these experiments are mostly limited to short-range collisional interactions; recently observed perturbative effects of long-range interactions were too weak to reach new quantum phases1,2. Here we experimentally realize a bosonic lattice model with competing short- and long-range interactions, and observe the appearance of four distinct quantum phases—a superfluid, a supersolid, a Mott insulator and a charge density wave. Our system is based on an atomic quantum gas trapped in an optical lattice inside a high-finesse optical cavity. The strength of the short-range on-site interactions is controlled by means of the optical lattice depth. The long (infinite)-range interaction potential is mediated by a vacuum mode of the cavity3,4 and is independently controlled by tuning the cavity resonance. When probing the phase transition between the Mott insulator and the charge density wave in real time, we observed a behaviour characteristic of a first-order phase transition. Our measurements have accessed a regime for quantum simulation of many-body systems where the physics is determined by the intricate competition between two different types of interactions and the zero point motion of the particles.

Experiments with cold atoms have contributed in many ways to the elucidation of the fundamental behaviour of quantum matter5. An example is the realization of the Bose–Hubbard model, where the balance between the kinetic energy of particles moving in an optical lattice and the on-site collisional interactions drives a quantum phase transition from a superfluid to a Mott insulating phase6,7. While collisions between atoms are naturally present in quantum gases and give rise to short-range interactions8, longer-range interactions are more elusive. To investigate the latter, ultracold gases of particles with large magnetic or electric dipole moments9,10, atoms in Rydberg states11, or cavity-mediated interactions12 have been studied. Indeed, Hubbard models with additional nearest-neighbour interactions are already predicted to show intriguing phases, such as charge and spin density waves, supersolids, topological phases or chequerboard and stripe phases12–18.

In our experiment, we achieve independent control over three energy scales by combining an optical lattice with cavity-mediated interactions (Fig. 1). The underlying static lattices along all three directions are simultaneously ramped up to a certain value direction are simultaneously ramped up to a certain value in the xy–z plane formed by one free space lattice and one intracavity optical standing wave, both at a wavelength of $\lambda = 785.3$ nm. They create periodic optical potentials of equal depths $V_{2D}$ along both directions, which we will specify in units of the recoil energy $E_R = \hbar^2/2m\lambda^2$, where $m$ denotes the mass of $^{87}$Rb. In addition to the lattice potential, the atoms are exposed to an overall harmonic confinement, which results in a maximum density of 2.8 atoms per lattice site at the centre of the trap. The standing wave along the z axis fulfills a second role as it controls the momentum of the particles.

To explore the phase diagram of $\mathcal{H}$, the lattices along the x and z direction are simultaneously ramped up to a certain value $V_{2D}$, keeping the total ramp time constant. This procedure is repeated for different

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Figure 1 | Illustration of the experimental scheme that realizes a lattice model with on-site and infinite-range interactions. Left, a stack of 2D systems along the y axis is loaded into a 2D optical lattice (red arrows) between two mirrors (shown grey). The cavity induces atom–atom interactions of infinite range. Right, illustration of the competing energy scales: tunnelling $t$, on-site interactions $U_s$ and long-range interactions $U_l$.

Relative strengths of short- and long-range interactions $(U_l/U_s)$, controlled via the detuning $\Delta_c$.

To detect a superfluid–insulator phase transition, we probe the spatial coherence of the gas by turning off all confining potentials and taking absorption images of the atomic cloud after ballistic expansion. Figure 2a shows measured projected momentum distributions for four different $V_{2D}$, together with extracted vertical line sums. For small lattice depth $V_{2D}$, spatial coherence can be observed, characterized by a narrow momentum distribution of the cloud and a large BEC fraction $f$, extracted from a bimodal fit to the distribution. When increasing $V_{2D}$, the momentum distributions broaden, indicating a drop of coherence, and $f$ reduces. We observe a kink in $f$ as a function of the interaction strength $U_l/t$ (for details, see Methods and Extended Data Fig. 4), which we associate with the formation of an insulating phase in the cloud and a loss of superfluidity$^{24}$. The extracted transition points are shown as white points in Fig. 2b. We confirmed that coherence between different lattice sites is restored when ramping down the 2D lattice potential again.

An even–odd imbalance causes a $\lambda$-periodic density modulation that acts as a Bragg grating, off which photons from the $z$-lattice beam are scattered into the cavity mode and vice versa. The amplitude of the scattered light field adiabatically follows the atomic density distribution$^5$ and is continuously monitored using a heterodyne detection (see Methods). Figure 2c displays mean intracavity photon numbers $\bar{n}_{ph}$ measured as a function of $V_{2D}$. The onset of a cavity field is clearly visible and is taken as the transition point to a phase with even–odd imbalance $\Theta$, marked with black points in Fig. 2d (for details, see Methods and Extended Data Fig. 4). The imbalance $\Theta$ can be quantified using equation (2) (see Methods):

$$\Theta = \frac{\sum_{\xi} \langle \hat{n}_{\xi} \rangle - \sum_{\eta} \langle \hat{n}_{\eta} \rangle}{\sum_{\xi} \langle \hat{n}_{\xi} \rangle + \sum_{\eta} \langle \hat{n}_{\eta} \rangle} \approx \frac{1}{N} \sqrt{\frac{n_{ph}}{\eta^2}} \Delta_c^2$$

Here, $\eta$ is the two-photon Rabi frequency of the scattering process and $N$ is the total atom number.

To establish a phase diagram, we combine all determined transition points in Fig. 3. We identify four phases that arise from the competition of the three energy scales: a superfluid (SF), a supersolid (SS), a Mott insulator (MI) and a charge density wave (CDW) phase. Far away from cavity resonance, that is, $\Delta_c/2\pi \approx -52$ MHz, $U_l$ becomes small and the system undergoes, for large enough $V_{2D}$, a transition from an SF to an MI phase. The latter is characterized by a loss of coherence, the stability of the fit (see Methods), $c$. Scattered photons $\bar{n}_{ph}$ of single repetitions as a function of $V_{2D}$ for pump–cavity detunings $\Delta_c/2\pi$ of $-12$ MHz (i), $-22$ MHz (ii) and $-32$ MHz (iii). $d$. Imbalance $\Theta$ mapped as a function of $\Delta_c$ and $V_{2D}$. We assign the onset of a scattered cavity light field (black points) to the formation of a phase with even–odd imbalance. In the region indicated by the three dotted lines at values $\Delta_c/2\pi = [-47, -49.5, -52]$ MHz, the onset of the cavity light field showed a large variation. Error bars indicate the s.d. of the fit; an additional systematic error of 0.2$E_k$ stems from the data analysis. The detection background is growing with decreasing $V_{2D}$ and increasing detuning from cavity resonance (see Methods). Grey areas were not recorded.

**Figure 2 | Characterization of the phases.** a–d. Characterization via spatial coherence (a, b) and via even–odd imbalance (c, d). a, Absorption images in the $x$–$z$ plane (upper panels), and the same signal integrated along the cavity axis (lower panels, red), taken after a ballistic expansion for lattice depths $V_{2D}$ of $2E_k$ (I), $6.5E_k$ (II), $11E_k$ (III) and $16E_k$ (IV) at $\Delta_c/2\pi = -22$ MHz. Black lines show fits with a bimodal distribution including higher momentum peaks. Owing to the cavity mirrors, the field of view along the $x$ direction is restricted. b, Extracted BEC fraction $f$ as a function of $V_{2D}$ and $\Delta_c$. White points mark the transition from a superfluid to an insulating phase and are obtained from a piecewise linear fit to the BEC fraction (see Extended Data Fig. 4). Error bars indicate fit uncertainties and contain contributions from the s.d. and from the stability of the fit (see Methods). $c$, Scattered photons $\bar{n}_{ph}$ of single repetitions as a function of $V_{2D}$ for pump–cavity detunings $\Delta_c/2\pi$ of $-12$ MHz (i), $-22$ MHz (ii) and $-32$ MHz (iii). $d$. Imbalance $\Theta$ mapped as a function of $\Delta_c$ and $V_{2D}$. We assign the onset of a scattered cavity light field (black points) to the formation of a phase with even–odd imbalance. In the region indicated by the three dotted lines at values $\Delta_c/2\pi = [-47, -49.5, -52]$ MHz, the onset of the cavity light field showed a large variation. Error bars indicate the s.d. of the fit; an additional systematic error of 0.2$E_k$ stems from the data analysis. The detection background is growing with decreasing $V_{2D}$ and increasing detuning from cavity resonance (see Methods). Grey areas were not recorded.
Increasing the 2D lattice depth

A version of the phase diagram in Hamiltonian parameters is shown in Figure 3. Black data points indicate the onset of an even–odd imbalance, while white data points depict where spatial coherence is lost. Increasing the 2D lattice depth \( V_{2D} \) simultaneously increases short- and long-range interactions. The detuning \( \Delta \) changes only the strength of the long-range interactions. The slanted lines indicate the region where CDW and MI phases may coexist. At detuning \( \Delta/2\pi = +8 \) MHz, \( U_1 \) becomes negative and favours zero imbalance, thus only SF and MI phases appear. No data were taken at detunings indicated by the grey bar. A version of the phase diagram in Hamiltonian parameters is shown in Extended Data Fig. 2.

as well as the absence of an even-odd imbalance. The observed SF to MI transition line is shifted to larger values of \( V_{2D} \) than theoretically expected for a homogeneous systemootnote{Yan, B. et al. Observation of dipolar spin-exchange interactions with lattice-confined polar molecules. Nature 501, 521–525 (2013).}, which we attribute to the harmonic confinement of our 2D systemsootnote{Baier, S. et al. Extended Bose-Hubbard models with ultracold magnetic atoms. Preprint at http://arxiv.org/abs/1507.03500 (2015).}. Approaching cavity resonance increases \( U_1 \). Above \( \Delta/2\pi \approx -52 \) MHz, this leads to the formation of a structured phase with even-odd imbalance, heralded by the onset of a light field scattered into the cavity. Depending on the relative strength of tunnelling and short-range interactions, the structured phase can either be an SS phase, where superfluidity is supported, or a CDW phase, where spatial coherence is lost. The identification of the SS phase is further supported by the observation of additional interference peaks corresponding to a \( \lambda \)-periodic density modulation (see Extended Data Fig. 5)ootnote{Baier, S. et al. Extended Bose-Hubbard models with ultracold magnetic atoms. Preprint at http://arxiv.org/abs/1507.03500 (2015).}. The SF to SS phase boundary shifts to smaller \( V_{2D} \) when approaching the cavity resonanceootnote{Yan, B. et al. Observation of dipolar spin-exchange interactions with lattice-confined polar molecules. Nature 501, 521–525 (2013).}.

The transition line from an SS to a CDW follows the same trend. The transition line from an SS to a CDW simultaneously increases short- and long-range interactions, the detuning \( \Delta \) and then continuously vary \( U_1 \) by changing \( \Delta \), before returning to the initial value of \( \Delta \) (see Methods). The cavity output field tracks the instantaneous even–odd imbalance \( \Theta \) in real time. Figure 4a shows the evolution of the imbalance when decreasing \( \Delta \) from an initial value in the CDW phase. The data show a hysteretic behaviour with a lower imbalance on return. The imbalance evolution for a starting value of \( \Delta \) in the transition region between CDW and MI phases shows a similar behaviour when decreasing \( \Delta \) and the opposite behaviour when increasing \( \Delta \). In Fig. 4c, where we started in the MI phase, the imbalance remains low throughout the measurement. We measured the hysteretic behaviour to be insensitive to the ramp speed (Extended Data Fig. 7).

This hysteretic behaviour of the system points towards a first-order phase transition between CDW and MI phases. When starting in an MI phase and increasing \( U_1/U_2 \) beyond a certain point, the CDW phase will become energetically favourable, but cannot be reached because of an energy barrier between the two phases. Further increasing \( U_1/U_2 \) lowers this energy barrier until the system is driven out of the metastable state. We suggest that this is activated in our inhomogeneous system by \( \lambda \)-periodic density–density correlations that are created in residual compressible regions, or superfluid shells, acting as impurities. In the opposite direction, moving from a CDW to an MI phase, the energy offset between even and odd lattice sites stabilizes the CDW phase beyond the point where the MI phase becomes energetically favourable, which results in the observed hysteretic behaviour.
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Author Information Reprints and permissions information is available at www.nature.com/reprints. The authors declare no competing financial interests. Readers are welcome to comment on the online version of the paper. Correspondence and requests for materials should be addressed to T.D. (donner@phys.ethz.ch).
METHODS

Preparation of a BEC in 2D layers. We produce a Bose–Einstein condensate (BEC) of 4.2(4) × 10^12 67Rb atoms at a temperature of 42(2) nK in the [F, m_F] = [1, −1] hyperfine state where F and m_F are respectively the total angular momentum and the corresponding magnetic quantum number. The quantization axis is defined by a magnetic field pointing along the z direction. The BEC is confined to the centre of a TEM_00 mode of the cavity by an optical dipole trap at a wavelength of 852 nm, with trap frequencies of ω_{x,z}/2π = [70.6(3), 31.4(5), 29.4(2)] Hz. Further details of the cavity set-up can be found in ref. 3.

The trapped BEC is loaded into a blue-detuned optical lattice of wavelength λ_0 = 670 nm oriented along the y direction. This is done by implementing a smooth amplitude ramp (S-ramp) in time t which is of the form: V(t) = V_0 [3(t/t_0)^2 − 2(t/t_0)^3], where V_0 is the final lattice depth and t_0 is the total duration of the ramp. The lattice depth is increased to a final value of 24.9(1)E_R in 100 ms where E_R^0 = h^2/2mλ_0^2 is the atomic recoil energy with m being the mass of a 67Rb atom. The trap frequencies are kept constant during the loading by increasing the dipole trap depth simultaneously with the blue-detuned lattice. In this way, the whole BEC is cut into roughly 60 2D layers with about 1,300 atoms in the central layer.

Loading into the square lattice. After the preparation of 2D layers, the BEC is exposed to a 2D optical lattice in the x–z plane at a wavelength of λ = 785.3 nm. The lattice along the z direction is formed by a free space retro-reflected standing wave laser field which is linearly polarized along the y direction. The lattice direction along the x direction is created by pumping the TEM_00 mode of the cavity with linear polarization along the z direction. The effect of interference between the x and z lattices on atoms is minimized by introducing a frequency offset of at least 5 MHz between the two laser frequencies. Both lattices are ramped simultaneously within a fixed time of 50 ms to a variable lattice depth V and two 2D layers again using the S-ramp. The lattice potential seen by the atoms is of the form: V(x,z) = V_{2D}[cos^2(kx) + cos^2(kz)] where k = 2π/λ is the wave number and V_{2D} is the depth of the lattice in units of the corresponding recoil energy E_R = h^2/2mλ_0^2.

Characterization of the optical lattices. The lattice depth along the x and z directions are calibrated using Raman–Nath diffraction23, whereas the lattice depth along the x direction is calibrated using amplitude modulation spectroscopy between the lowest and the first two excited Bloch bands24. We estimate the calibration uncertainties on all lattice depths to be smaller than 4%. The uncertainty in the intracavity optical lattice depth is enlarged to about 10% by shifts of the cavity resonance frequency due to atomic redistribution during the V_{2D} ramp and residual drifts of the input coupling into the resonator.

The heating effect of the near-resonant x–z optical lattices on the BEC is characterized by ramping back down the lattices after reaching the insulating regime. We recover a BEC fraction larger than 0.45 and observe an atom loss of 5–10%. Loading of lattices in the x–z plane also increases the overall confinement. The trap frequencies are ω_{x,z}/2π = [170, 165] Hz at a typical lattice depth of 10E_R.

Lattice model with long-range interactions. The single-particle Hamiltonian \( \hat{H}_0 \) describing the dynamics of an atom strongly coupled to a single cavity mode and moving in a 2D layer in the presence of static optical lattices, is given as3,20:

\[ \hat{H}_0 = \hat{H}_0 + V_{\text{trap}}(x,z) + \hbar \alpha (\hat{a}^\dagger \hat{a}) \cos(kx) \cos(kz) - \hbar(\Delta_c - U_c \cos^2(kx))\hat{a}^\dagger \hat{a} \]

\( \hat{H}_0 \) consists of the kinetic energy of the particle and the potential seen due to the optical lattices in the x–z plane:

\[ \hat{H}_0 = \frac{\hat{p}_x^2}{2m} + \frac{\hat{p}_z^2}{2m} + V_{\text{trap}}(\cos^2(kx) + \cos^2(kz)) \]

\( V_{\text{trap}}(x,z) \) incorporates the inhomogeneous confining potential seen by the atoms. \( \hat{a} (\hat{a}^\dagger) \) annihilates (creates) a photon in the cavity mode. Scattering of the light field from the lattice into the cavity mode at a two-photon Rabi frequency \( \gamma \) creates a self-consistent checkerboard lattice for the atoms and is represented by the third term in \( \hat{H}_0 \). This term describes how the atomic motion self-consistently determines the occupation of the cavity field mode inducing infinite-range interactions between the atoms. The last term in \( \hat{H}_0 \) represents the cavity field in the rotating frame of the lattice with \( \Delta_c = \omega_c - \omega_0 \). The effect of the dispersive shift of the cavity resonance frequency is also included with \( U_0 \) being the maximum light shift per atom.

The many-body description of the system is obtained by introducing the bosonic field operator \( \hat{\psi}(r) \) which annihilates (creates) a particle at position \( r = (x, z) \) and satisfies bosonic commutation relations. In the framework of second quantization, the many-body Hamiltonian \( \hat{H}_2^{\text{ind}} \) reads:

\[ \hat{H}_2^{\text{ind}} = \int \mathbf{d}r \ \hat{\psi}^\dagger(r) [\hat{H}_0 + g_{\text{2D}} \hat{\psi}^\dagger(r) \hat{\psi}(r) - \mu] \hat{\psi}(r) \]

where \( \mu \) is the chemical potential and \( g_{\text{2D}} \) is the modified short-range interaction strength in a 2D layer66. We expand \( \hat{\psi}(x,z) \) in the basis of Wannier functions localized on different lattice sites which are obtained from the lowest Bloch band defined by \( \hat{H}_0 \):

\[ \hat{\psi} = \sum_m W_m(x,z) \hat{b}_m \]

where \( \hat{b}_m (\hat{b}_m^\dagger) \) represent the annihilation (creation) operators of a single particle at site \( m = (m_x, m_z) \lambda/2 \) and \( W_m(x,z) \) is the Wannier function localized on site \( m \).

A site is referred to as even (odd) if \( m_x + m_z \) is even (odd). The Wannier functions localized on neighbouring lattice sites are related to each other by a translation of the lattice constant. Keeping interactions only up to the nearest neighbouring sites, we obtain the Bose–Hubbard model with additional terms21:

\[ \hat{H}_2^{\text{ind}} = -\sum_{\langle i,j \rangle} \left( \hat{b}_i^\dagger \hat{b}_j + \text{h.c.} \right) + \frac{U_2}{2} \sum_{i \in \varepsilon} \hat{n}_i(\hat{n}_i - 1) + g_{\text{2D}} \int d^2x d^2z \cos(\Phi(x,z)) \int d^2x d^2z \cos(\Phi(x,z)) + \mu \hat{n} \]

where \( \mu = \mu_e - \Delta_c \) describes the local chemical potential at site \( i \) incorporating the effect of \( V_{\text{trap}} \) and \( \hat{n}_i = \hat{b}_i^\dagger \hat{b}_i \) counts the number of particles on site \( i \). Indices \( e \) and \( o \) refer to all even and odd lattice sites, respectively. \( \delta = \mu_e \Delta_c/N \) is the dispersive shift of the cavity due to the BEC with \( N \) the total number of atoms. The two overlap integrals \( M_0, M_1 \) are defined as:

\[ M_0 = \int d^2x d^2z \cos(\Phi(x,z)) \cos(\Phi(x,z)) \]

\[ M_1 = \int d^2x d^2z \cos^2(\Phi(x,z)) \]

A higher-order correction to the tunnelling along the x direction by the self-consistent cavity lattice is neglected. The cavity decay rate \( \kappa \) is large compared to the atomic recoil frequency which allows us to adiabatically eliminate the cavity field1. Its steady state value is given by:

\[ \kappa = g_{\text{2D}} \frac{\Delta_c}{\delta - \Delta_c} \approx K \frac{\Delta_c}{\delta - \Delta_c} \]

Therefore, the light leaking out of the cavity will be proportional to the imbalance of the number of atoms on the two kind of sites and it can herald the presence of a phase with broken Z_2-symmetry of the underlying static lattice. Inserting equation (10) into equation (7), we recover equation (1) of the main text, with cavity-mediated long-range interaction strength \( U_0 \) given by:

\[ U_0 = -K \frac{\Delta_c}{\delta - \Delta_c} \]

To describe our stack of 2D layers within this theoretical framework, we assume that a system of many 2D layers can be combined to form one 2D layer with an accordingly larger number of lattice sites, containing all atoms. Validity of the theoretical model. In the derivation of equation (1), we assume the validity of the single-band approximation. To deduce the experimental parameter space where this assumption holds, we compare the strength of all Hamiltonian...
parameters with the excitation energy to the next higher Bloch band, as shown in Extended Data Fig. 1.

Phase diagram in terms of Hamiltonian parameters. We convert the phase diagram from Fig. 3 into Hamiltonian parameters (see Extended Data Fig. 2) using equations (8), (9) and (11), taking into account the effect of two nearly-degenerate polarization modes of the cavity in the definition of $U_t$. Starting in an SF phase and increasing $U_t$ takes the system into an SS phase and eventually into a CDW phase. At the transition from the SF to the SS phase, the system needs to overcome additional short-range interaction energy. As a result, an increasingly larger critical long-range interaction strength is required to enter the SS phase for increasing $U_t$. A similar effect is seen for the transition from an SS to a CDW phase.

For negligible tunnelling, a direct transition from an MI to a CDW phase is found at a relative strength of $U_t/U_1 = 0.66(4)$. In the absence of tunnelling and trapping potential the Hamiltonian (equation (1)) supports a stable MI only for commensurate filling. In this case, the phase boundary between MI and CDW lies at a relative strength of $U_t/U_1 = 0.3$. Deviations from this value can be attributed to the presence of the trap, incommensurate filling and the non-local nature of the long-range interactions.

For negligible $U_t$, the transition from SF to MI is observed at a relative strength of $U_t/U_1 = 28(4)$. The value is larger than the theoretically predicted value of $U_t/U_1 = 16$ for a homogeneous system with unity filling$^{25}$, as discussed in the main text.

Effect of the trapping potential. The harmonic trapping potential experienced by the atoms has a stabilizing effect on the MI phase in the presence of long-range interactions. Extended Data Fig. 3 illustrates, for fixed atom number and zero tunnelling, the effect of the trapping potential in the presence of the self-consistent lattice potential. For any non-zero $U_1$ and for non-integer filling, the homogeneous system will arrange itself in a structured phase with even–odd imbalance. However, the presence of a trapping potential can favour the coexistence of MI and CDW phases or of an MI phase alone, since the system has to pay additional energy for arranging atoms away from the trap centre. This energy cost has to be compared with the gain in energy due to the formation of a CDW phase. For larger fillings, we expect that the system will develop a ‘wedding cake’ like structure, similar to experimental realizations of MI phases. In our system, the plateaus can also host partially modulated and fully modulated CDW phases. The presence of any CDW in the system will be signalled by a finite light field scattered into the cavity. We do not expect a qualitative change of the phase diagram that depends on the steepness of the trap.

Extraction of the BEC fraction. We take an absorption image of the atomic distribution in the $x$–$z$ plane after 15 ms of ballistic expansion. The obtained momentum distribution is integrated over the cavity direction. We perform a bimodal fit to the resulting distribution, in which we distinguish two contributions. The first component represents coherent atoms diffracted by the lattice potential, captured by a Thomas–Fermi profile plus two Gaussian interference peaks at $\pm 2hkt$. The second component is a broad Gaussian distribution resulting from the incoherent addition of atomic signals from the insulating part of the cloud. The BEC fraction $f$ is finally extracted from the ratio $f = N_{\text{BEC}}/N$, where $N_{\text{BEC}}$ is the integrated atom number in the coherent part and $N$ is the total atom number.

$N_{\text{BEC}}$ is obtained from the mean total atom number of all experimental data at low lattice depths, $V_{2D} < 2E_h$. For deeper lattices, the growing spatial extent of the incoherent background is affected by inhomogeneities in the imaging and by the cropped field of view.

To constrain the number of free fit parameters, we fix the position of the interference peaks with respect to the central peak. Furthermore, their widths are linearly correlated to the width of the central peak$^{21}$, see Fig. 2a. We double count the interference peaks to correct for the non-visible peaks along the $z$ direction, where the field of view is cropped by the cavity mirrors. The contribution of each peak to the total atom number is of the order of a few percent at most. The chequerboard lattice in the supersonic phase leads to extra interference peaks, which lie outside the field of view (see Extended Data Fig. 5). Their contribution to the overall atom number is even lower than the one from the $\pm 2hkt$ peaks in most parts of the phase diagram and is therefore neglected.

Extraction of the even–odd imbalance. During each experimental sequence, the Bragg scattered light leaking out of the cavity is detected with a heterodyne set-up having a sensitivity of $0.67(1)\sqrt{V_{ph}}$ per intracavity photon. The heterodyne detection is insensitive to the laser field creating the static lattice along the cavity axis by the choice of orthogonal polarizations and a minimum frequency difference of 5 MHz. Both phase and magnitude of the light field are recorded. To separate out the coherent part of the light field, we apply a low pass filter to the quadratures before taking the absolute square to obtain an intracavity photon number $n_{ph}$. It is mapped to an even–odd particle imbalance obtained from equation (10) with $n_{ph} = \langle \hat{a}_e^\dagger \hat{a}_o \rangle$. We define the effective even–odd imbalance $\Theta$ under the assumption of completely localized atoms on either even or odd sites ($M_0 = 1$):

$$
\Theta = \frac{\sum_e (\hat{a}_e - \sum_{\omega} \hat{a}_\omega) + \sum_{\omega} (\hat{a}_\omega - \sum_{\omega} \hat{a}_\omega)}{N_{ph} |\langle \hat{a}_e^\dagger \hat{a}_o \rangle|^2} \frac{1}{1 + F(\Delta_k)}
$$

with $F(\Delta_k) = \frac{\Delta_k^2 \cos^2(\alpha) + \Delta_k^2 \sin^2(\alpha)}{(\Delta_k^2 - \delta - \theta/2)^2 + \kappa^2}$ for $|\Delta_k| >> \delta/\theta$. Their contribution to the overall atom number is even lower than the one from $-l$-lattice photons. This effect is seen for the transition from an MI to a CDW phase.

During each experimental sequence, the system will develop a ‘wedding cake’ like structure, similar to experimental realizations of MI phases. In our system, the plateaus can also host partially modulated and fully modulated CDW phases. The presence of any CDW in the system will be signalled by a finite light field scattered into the cavity. We do not expect a qualitative change of the phase diagram that depends on the steepness of the trap.

Sample size. No statistical methods were used to predetermine sample size.

Phase boundaries. Coherence. We convert the 2D lattice depth $V_{2D}$ to the corresponding ratio $U_t/U_1$ of short-range interaction strength and nearest neighbour tunnelling by using the Wannier functions obtained from the lowest Bloch band of the applied static lattices. In this way, we obtain a BEC fraction $f$ as a function of $U_t/U_1$ (see Extended Data Fig. 4), which we fit with a piecewise linear function. The first kink in the fit is associated with the transition point to an insulating phase$^{21}$. By analysing the stability of the fit with respect to initial parameters, we deduce an additional uncertainty on top of the s.d. and include it in the error bar displayed for each transition point in Figs 2b and 3.

Even–odd imbalance. From each experimental repetition, we obtain a time trace of the light field scattered into the cavity. The maximum photon number $n_{ph,max}$ at the end of the trace and the corresponding lattice depth $V_{2D}$ are averaged in a time window of 10 ms (spanning 7.8% of $V_{2D}$), resulting in one data point extracted per time trace. For each detuning, $n_{ph,max}(V_{2D})$ is fitted with a piecewise linear and power law function (see Extended Data Fig. 4) to determine the point where the intracavity light field starts building up. This method largely increases the signal to noise ratio while keeping systematic shifts of the onset point below 0.2E_h. In the region of $\pm 52$ MHz $\leq \Delta_k \leq -47$ MHz, the intracavity field becomes very small and fluctuates strongly from shot to shot (see Extended Data Fig. 4c). We therefore indicate a region for the transition to a phase with $\lambda$-periodic density modulation by dashed lines in Figs 2d and 3. The starting point of these dashed lines indicates the earliest onset of an even–odd imbalance including the s.d. of the fit. Owing to the fixed time of the lattice ramp, we cross the transition to an even–odd imbalanced phase non-adiabatically when ramping into deep lattices. The non-adiabaticity leads to a small shift of the onset point towards higher lattice depths, which can be seen in Fig. 2c. This behaviour was studied previously and explained with Kibble–Zurek theory$^{21}$. The described method of discretizing the data is intrinsically less sensitive to this type of shift compared to fitting a single time trace to extract the transition point.

Self-consistent chequerboard lattice. The SS and CDW phases give rise to a light field inside the cavity due to the Bragg scattering of $z$-lattice photons. This cavity field is self-consistent as it depends on the strength of the $\lambda$-periodic density modulation in the atomic cloud and has a depth $V_{2D} = n_{ph} \times 12.3 \times 10^{-2}\text{E}_h$. Interference of this self-consistent $z$ lattice with the field of the $z$ lattice produces a chequerboard lattice potential of depth $V_{\text{CB}} = \sqrt{2V_{2D}}V_{\text{in}}$, displayed in Extended Data Fig. 6. The line of constant $V_{\text{CB}}$ bends towards smaller values of $V_{2D}$ when approaching cavity resonance, substantiating the assumption that this energy offset causes the observed behaviour of the SS to CDW boundary line. When the energy offset between even and odd sites due to $V_{\text{CB}}$ becomes comparable to the tunnelling energy, the effective tunnelling strength between nearest-neighbours reduces and higher-order tunnelling processes begin to play a significant role (U. Bissbort, personal communication).
Hysteresis measurements. We initialize the system in the insulating region at either $V_{2D}=14E_R$ or $18E_R$ using a 50-ms-long S-shaped amplitude ramp. The detuning $\Delta/2\pi$ is then changed with an S-shaped frequency ramp at an average speed of 0.67 MHz ms$^{-1}$ reaching a different detuning value. After holding for 10 ms, we scan back to the initial detuning. Residual atom loss continuously reduces the measured mean intracavity photon number $n_{ph}$, which we take into account by rescaling the data before converting it into an imbalance $\Theta$. The scaling factor is extracted from reference measurements, where we hold at different $\Delta_c$ for 50 ms. We deduce a linear decrease in $n_{ph}$ by 48(4)% (41(4)%) for lattice depths of $V_{2D}=14E_R$ (18$E_R$). After rescaling the data, we observe a remaining relative drift of the imbalance level of 8(4)% during the hold time. Extended Data Fig. 7 shows detuning scans performed at $V_{2D}=18E_R$, where a similar hysteretic behaviour is observed as in Fig. 4 of the main text. To test the sensitivity of the hysteretic behaviour on the ramp speed, we slow down the frequency ramp by a factor of two and observe a comparable evolution of the even–odd imbalance; see Extended Data Fig. 7.

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Extended Data Figure 1 | Validity of the single-band approximation. The energy scales of the Hamiltonian are plotted in units of the minimum gap $\Delta_{ex}$ between the lowest and the first excited Bloch band. The single-band approximation is assumed to be valid if all the energy scales ($U_s$, $U_l$ and $t$) are at least 5 times smaller than $\Delta_{ex}$, that is, if they lie below the black dashed line. This criterion is fulfilled for $\Delta_c/2\pi < -18.3$ MHz and $18E_R > V_{2D} > 3E_R$. For detunings in the interval $-18.3$ MHz $< \Delta_c/2\pi < -10.9$ MHz, the approximation is only partially valid, depending on $V_{2D}$. We use this information to illustrate the region of validity in Extended Data Fig. 2.
Extended Data Figure 2 | Phase diagram plotted as a function of Hamiltonian parameters $U_s/t$ and $U_l/t$. The experimental parameters in Fig. 3 have been converted to Hamiltonian parameters: the region of validity for this conversion lies to the right of the solid black line, grey areas were not recorded. The white data points indicate where spatial coherence is lost, and the black data points depict the onset of an even–odd imbalance. The white shaded regions around the data points represent the respective converted error bars. The dotted black lines show, as in Fig. 3, the region where the onset of the cavity light field showed a large variation.
Extended Data Figure 3 | Influence of the trapping potential on the long-range interacting system. Shown are sketches of a 1D slice through a 2D layer, displaying the ground state configurations of 13 particles that depend on the relative influence of trapping potential and long-range interaction. a, Results for a homogeneous system with finite $U_1$. b–d, The state of the system for increasing but finite $U_1$, starting with small but finite $U_1$ (see legend at right).
Extended Data Figure 4 | Determination of the phase boundaries. Shown are the BEC fraction \( f \) (averaged into 100 equally spaced bins) and maximum photon number \( n_{\text{ph, max}} \) (filled and open symbols, respectively) as a function of \( U_s/t \) for detunings \( \Delta_c/2\pi \) of \(-12\) MHz (a), \(-22\) MHz (b) and \(-47\) MHz (c). The red curve in each panel shows the result of a piecewise linear fit to \( f \). We confirmed that the initial BEC fraction has no systematic dependence on \( \Delta_c \). The blue curve displays a power law fit to \( n_{\text{ph, max}} \).
Extended Data Figure 5 | Momentum distribution in the SS phase.
Absorption image from a calibration measurement taken after a short ballistic expansion of 7 ms at a detuning of \( \Delta_c/2\pi = -23 \text{ MHz} \) and a lattice depth \( V_{2D} \) 39% above the onset of an even–odd imbalance in the SS phase. We observe interference peaks at \( p_z = \pm 2\hbar k \). Additional interference peaks resulting from the emerging chequerboard lattice appear at \((p_x, p_z) = (\pm \hbar k, \pm \hbar k)\). This observation indicates an SS phase. These additional momentum peaks lie outside the field of view for the longer ballistic expansion time of 15 ms.
Extended Data Figure 6 | Strength of the self-consistent chequerboard lattice. The chequerboard (CB) lattice depth extracted from the measured mean intracavity photon number $n_{ph}$ is shown as a function of the applied lattice depth $V_{2D}$ and detuning $\Delta_c$. The CB lattice depth becomes comparable to the depth of the static lattices close to cavity resonance, but drops rapidly when moving away due to its detuning dependence. Exemplary equipotential lines at $0.05E_R$, $1E_R$ and $3E_R$ are shown.
Extended Data Figure 7 | Sensitivity to the ramp speed. The hysteretic behaviour in the insulating regime at $V_{2D} = 18E_R$ is shown. The detuning $\Delta/2\pi$ is ramped at two speeds, 0.67 MHz ms$^{-1}$ (blue) and 0.33 MHz ms$^{-1}$ (orange). Lines result from an average of two to five measurements, using 400 $\mu$s time bins. Stars signify starting points, arrows show the scan direction and dashed lines indicate the return to the starting point.