Broadband Omni-resonant Coherent Perfect Absorption in Graphene

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Coherent perfect absorption (CPA) refers to interferometrically induced complete absorption of incident light by a partial absorber independently of its intrinsic absorption (which may be vanishingly small) or its thickness. CPA is typically realized in a resonant device, and thus cannot be achieved over a broad continuous spectrum, which thwarts its applicability to photodetectors and solar cells, for example. Here, we demonstrate broadband omni-resonant CPA by placing a thin weak absorber in a planar cavity and pre-conditioning the incident optical field by introducing judicious angular dispersion. We make use of monolayer graphene embedded in silica as the absorber and boost its optical absorption from \( \approx 1.6\% \) to \( \sim 60\% \) over a bandwidth of \( \sim 70 \) nm in the visible. Crucially, an analytical model demonstrates that placement of the graphene monolayer at a peak in the cavity standing-wave field is not necessary to achieve CPA, contrary to conventional wisdom.

I. INTRODUCTION

Coherent perfect absorption (CPA) refers to the complete absorption of light incident on a partially absorbing structure \([1, 2]\) (see also early premonitions of this concept in [3]). The essence of CPA is that the interaction of an incoming optical field with a partial absorber can be maximized by engineering the field distribution [4]. Indeed, CPA suggests that by arranging for two fields to interfere in a lossy planar structure, one can always guarantee that 100% of the incident radiation is absorbed – independently of the structure’s intrinsic absorption or its thickness. This can be achieved by exploiting two counter-propagating fields incident on the two ports of a planar Fabry-Pérot (FP) resonator containing a partially absorbing layer between two mirrors \( M_1 \) and \( M_2 \) of reflectivities \( R_1 \) and \( R_2 \), respectively [2, 5]. If the intrinsic single-pass absorption in a layer of an optical material is \( A \), then providing symmetric mirrors each having a reflectivity \( R_1 = R_2 = 1 - A \) guarantees that counter-propagating fields with prescribed relative amplitude and phase are completely absorbed [5] – even when \( A \) is vanishingly small. Alternatively, in the case of an ultrathin layer, placing it at a peak in a standing-wave resulting from the interference of two counter-propagating fields can yield complete absorption [6].

A variety of applications in photonics can benefit from maximizing optical absorption in a given structure, ranging from optical detectors [7, 8] to solar cells [9, 10]. Furthermore, a broad range of configurations can exploit the large absorption modulation achievable in CPA, including optical logic gates and switches [6, 11]. Despite the exciting prospects afforded by CPA, several hurdles need to be first overcome, especially in applications where broadband absorption is desired. First, optical absorption in any medium over a finite bandwidth is wavelength dependent \( A(\lambda) \). As such, the cavity mirrors are required to have a wavelength-dependent reflectivity \( R_1(\lambda) = R_2(\lambda) = 1 - A(\lambda) \) that counterbalances the dispersive absorption, which can be realized via aperiodic multilayer dielectric mirrors [18, 19]. Therefore, CPA is in principle materials-agnostic.

Second, practical settings in optical detection or solar cells necessitate single-port incidence rather than a counter-propagating-field configuration. This can be accommodated by placing the absorbing layer in an asymmetric cavity having a back-reflector of reflectivity \( R_2 = 1 \) [5, 18, 20, 21]. In this case, light is incident on a front mirror that must have a reflectivity \( R_1(\lambda) = (1 - A(\lambda))^2 \) to realize complete absorption [5, 18]. Such a configuration is of course reminiscent of critical coupling in micro-cavities [20, 21]. Eliminating the need for counter-propagating fields in turn eliminates the restrictions on the relative amplitude and phase, thus making the arrangement amenable to broadband incoherent light. Indeed, we recently demonstrated CPA in an asymmetric cavity over a bandwidth of \( \sim 60 \) nm utilizing a fluorescein dye [19], and over a full octave of bandwidth \( \sim 800-1600 \) nm using 2-\( \mu \)-in-thick poly-crystalline silicon layer [19].

The third and most serious hurdle is the resonant nature of this CPA configuration. For example, a FP resonator can satisfy the CPA condition over extremely broad bandwidths, but absorption is nevertheless harnessed only at the discrete resonant wavelengths within this spectrum [18, 25]. This is clearly inadequate for enhancing the performance of a photodetector or solar cell, where absorption over a broad continuous bandwidth is imperative. We have recently developed the concept of ‘omni-resonance’, which can help yield continuous-
wavelength CPA in a FP resonator. By introducing a specific angular dispersion into the field such that each wavelength is assigned to a prescribed angle, the entire incident spectrum – which can significantly exceed the cavity free spectral range – resonates with a single longitudinal cavity mode. The bandwidth of the resonant spectrum is thus decoupled from the cavity photon lifetime and quality factor. Combining CPA with omni-resonance in a FP cavity containing a partially absorbing layer can thus yield resonantly enhanced absorption over a broad, continuous spectrum.

In this paper, we demonstrate that broadband CPA can be realized in an ultrathin weakly absorbing layer (for instance, 2D materials such as graphene) once embedded in the appropriate planar photonic environment. Besides their remarkable electronic transport features, 2D materials strongly couple to light, suggesting them as intriguing candidates for optoelectronics. Indeed, monolayer graphene has an intrinsic single-pass absorption in free space of $\mathcal{A}_G \approx 2.3\%$ despite being atomically thin, which can make it potentially visible even to the naked eye. Nevertheless, $\mathcal{A}_G$ is too low for most optoelectronics applications, including optical modulators and photovoltaics. Recent strategies for enhancing optical absorption in graphene include exploiting the extended interaction lengths afforded by waveguides, tuning the absorption via an electrostatic gate (also known as ‘chemical doping’) placing graphene in a standing wave formed by counterpropagating fields, or resonantly enhancing the absorption via guided-mode resonances, attenuated total internal reflection, Salisbury screens, or critical coupling.

Here, we demonstrate resonantly enhanced absorption ($\sim 20-60\%$) in a monolayer of graphene placed within a FP cavity continuously across a bandwidth of $\approx 70$ nm in the visible. The cavity is designed to realize CPA at visible wavelengths and omni-resonance is simultaneously achieved by pre-conditioning the incident broadband incoherent field. Consequently, resonantly enhanced absorption is delivered over a broad, continuous spectrum. We therefore establish a general framework for achieving broadband perfect absorption in high-quality-factor (narrow-linewidth) optical cavities. Our motivation for selecting monolayer graphene is that it is a model for ultrathin absorbers with low intrinsic single-pass absorption. Furthermore, because its absorption is spectrally flat, we can focus on the juxtaposition of CPA with omni-resonance without the complications arising from wavelength-dependent absorption (which we dealt with in previous studies of CPA in planar cavities). Resonantly enhanced absorption because the standing wave cannot be adjusted post-fabrication tuning of the resonant absorption of the absorber within a FP cavity to maximize the absorption. In-
FIG. 1. Achieving broadband, spectrally continuous, omni-resonant CPA in a planar FP resonator by engineering the angular dispersion introduced into the incident field. (a) A collimated broadband field is normally incident on a passive symmetric FP cavity (first row) that transmits light at resonant wavelengths and reflects light otherwise (second and third rows). The resonant wavelengths are those for which the cavity wave vector is an integer multiple of $k_0 = \pi/L$ (fourth row; ignoring the mirror reflection phases). (b) Same as (a) except that the passive FP cavity is asymmetric: it is provided with a back-reflector $R_2 = 1$ (first row); the cavity therefore exhibits a spectrally flat, unity reflectivity (second row) and zero transmission (third row). The resonant wavelengths are the same as those for the symmetric cavity in (a). (c) After inserting a low-absorption material of single-pass absorption $A$ (first row) into the cavity in (b), complete absorption is guaranteed when $R_2 = 1$ and $R_1 = (1 - A)^2$, and CPA is thus realized at the resonant wavelengths, resulting in spectral reflection dips (second row) and absorption peaks (third row). (d) At oblique incidence, the resonances in (c) are blue-shifted. (e) When the incident field is pre-conditioned such that each wavelength $\lambda$ is assigned to an appropriate incidence angle $\theta(\lambda)$ (first row), the wavelengths resonate continuously with the cavity; reflection over the full spectrum is eliminated (second row) and broadband CPA is achieved (third row); and the longitudinal wave-vector components of the incident wavelengths correspond to a single resonant cavity mode (fourth row).

response notably; Fig.1(c). Indeed, if $R_1(\lambda) = (1 - A(\lambda))^2$, then light is fully absorbed on resonance, $A_{\text{tot}}(\lambda) = 1$, whenever $\lambda$ satisfies Eq. 1 -- even when $A$ is vanishingly small [5, 18]. This is a manifestation of CPA, which is analogous here to critical coupling [22–24].

The basis for the omni-resonance strategy can be appreciated by examining oblique incidence at an angle $\theta$ with respect to the cavity normal; Fig.1(d). The oblique-incidence resonant condition is determined by the longitudinal component of the cavity wave vector:

$$
\varphi(\lambda, \theta) = 4\pi n \frac{d}{\lambda} \cos \theta' + \alpha_1(\lambda, \theta') + \alpha_2(\lambda, \theta') = 2m\pi, \quad (2)
$$

where $\alpha_1(\lambda, \theta')$ and $\alpha_2(\lambda, \theta')$ are the reflection phases from $M_1$ and $M_2$, respectively, at a free-space wavelength $\lambda$ and an internal angle of incidence $\theta'$, which is related to the external angle of incidence $\theta$ through Snell’s law: $\sin \theta = n \sin \theta'$ (assuming incidence from free space). As a result, all resonances blue-shift: if $\lambda$ corresponds to a particular resonance at normal incidence, then the blue-shifted resonant wavelength $\lambda(\theta)$ at oblique incidence is

$$
\lambda(\theta) = \frac{n}{\sqrt{n^2 - \sin^2 \theta}} \lambda, \quad (3)
$$

assuming that $\alpha_1(\lambda, \theta')$ and $\alpha_2(\lambda, \theta')$ are independent of $\theta'$ in the spectral range of interest, which is typically the case for dielectric multilayer Bragg mirrors within their bandgap.

Omni-resonance relies on arranging for each wavelength $\lambda$ to impinge on the cavity at the appropriate angle $\theta(\lambda)$, such that it satisfies the omni-resonance condition:

$$
\varphi(\lambda) = 4\pi n \frac{d}{\lambda} \cos (\theta'(\lambda)) + \alpha_1(\lambda, \theta'(\lambda)) + \alpha_2(\lambda, \theta'(\lambda)) = 2m\pi. \quad (4)
$$

If this condition is satisfied over a certain bandwidth, it is guaranteed that the incident spectrum will resonate continuously with the cavity. Additionally, if the omni-
resonance condition is met in conjunction with \( R_2 = 1 \) and the proper design of \( R_1(\lambda) \), then the entire spectrum is absorbed and broadband omni-resonant CPA is achieved; Fig. 1(e). This overall strategy is materials-agnostic, and is independent of the physical nature of the absorbing layer. Furthermore, the design can be adjusted in principle for any desired central wavelength or bandwidth. The central challenge is to construct an optical pre-conditioning system that introduces the particular wavelength-dependent angle of incidence \( \theta(\lambda) \) that satisfies the omni-resonance condition, which we present below.

### III. IMPACT OF GRAPHENE LOCATION INSIDE THE CAVITY

Our previous work on CPA using the configuration shown in Fig. 1(c) assumed ‘distributed’ absorption; that is, the single-pass absorption \( A \) results from a layer in which absorption extends uniformly over its thickness \( L \) [5] [15] [19] [20]. This assumption no longer holds for ‘localized’ absorption (such as a 2D material like graphene or a metasurface). It is expected that the location of a deeply ultrathin absorbing layer with respect to the peaks and minima of the cavity standing-wave field structure will impact the net absorption. Intuitively, light will not interact with the absorbing layer placed at a standing-wave null, whereas placing it at a peak will likely maximize absorption. Indeed, previous work on CPA in ultrathin layers has indicated that absorption is achieved only when the layer is placed at a standing-wave peak [11] [12].

In this Section we evaluate the impact of the position of an ultrathin absorbing layer within the cavity on \( A_{\text{tot}} \) and we demonstrate that the cavity shown schematically in Fig. 2(a) modifies previous predictions in a fundamental way. The incident and reflected field amplitudes on the left at normal incidence are \( A_L \) and \( B_L \), respectively, the corresponding quantities on the right are \( A_R \) and \( B_R \), which are related through the cavity transfer matrix \( \mathbf{M} \),

\[
\begin{pmatrix} A_L \\ B_L \end{pmatrix} = \mathbf{M} \begin{pmatrix} A_R \\ B_R \end{pmatrix}.
\]  

(5)

The cavity length is \( L \), and the absorbing layer is placed at a distance \( d \) from mirror \( M_1 \). Therefore, from left to right, the cavity comprises mirror \( M_1 \), propagation a distance \( d \), the absorbing layer, propagation a distance \( L-d \), and mirror \( M_2 \), cascaded according to:

\[
\mathbf{M} = \mathbf{M}_1 \cdot \mathbf{P}(\varphi_1) \cdot \mathbf{M}_G \cdot \mathbf{P}(\varphi_2) \cdot \mathbf{M}_2;
\]  

(6)

here \( \mathbf{M}_1 \) and \( \mathbf{M}_2 \) represent mirrors \( M_1 \) and \( M_2 \), respectively, which have the form given in Eq. (2), \( \mathbf{P}(\varphi_1) \) and \( \mathbf{P}(\varphi_2) \) represent propagation for distances \( d \) and \( L-d \), respectively, where \( \varphi_1 = nkd \), \( \varphi_2 = nk(L-d) \), \( k = \frac{2\pi}{\lambda} \) is the free-space wavelength, and \( \mathbf{P}(\varphi) \) has the form given in Eq. (3) and \( \mathbf{M}_G \) represents the ultrathin absorbing layer, taken to be monolayer graphene for concreteness [56]:

\[
\mathbf{M}_G = \begin{pmatrix} 1+\delta & \delta \\ -\delta & 1-\delta \end{pmatrix},
\]  

(7)

where \( \delta = \frac{\sigma \mu c}{2\pi} \), \( \mu_o \) is the free-space magnetic permeability, \( c \) is the light speed in vacuum, \( n \) is the refractive index of the surrounding medium, and the optical conductivity of monolayer graphene is \( \sigma = 6.1 \times 10^{-5} \Omega^{-1} \) [57]. Accordingly, the single-pass absorption is \( A_G = \frac{2\delta}{1+\delta} \approx 2\delta = 1.6\% \) \((\delta \ll 1)\).

The cavity reflection amplitude \( r_L = \frac{\tilde{r}_L}{\tilde{n}_L} \) is given by:

\[
r_L = -e^{i(2\beta_1 - \alpha_1)} \frac{r_1 \delta_+ e^{-i\psi_o} - \delta_- e^{i\psi_o} + \delta e^{-i\Delta \psi} + r_1 \delta e^{i\Delta \psi}}{r_1 \delta e^{-i\psi_o} - r_1 \delta e^{i\psi_o} + r_1 \delta e^{-i\Delta \psi} + \delta e^{i\Delta \psi}},
\]  

(8)

where \( \delta_+ = 1 + \delta \), \( \delta_- = 1 - \delta \), \( \psi_o \equiv nkL = \varphi_1 + \varphi_2 \), and \( \Delta \psi \equiv 2\varphi_o - \alpha_1 + \alpha_2 = \text{the cavity round-trip phase,} \) \( \Delta \varphi \equiv \varphi_2 - \varphi_1, \beta_1 \) and \( \alpha_2 \) are the transmission and reflection phases for \( M_1 \), respectively, \( \alpha_2 \) is the reflection phase for \( M_2 \), and \( r_1 \) is the reflection amplitude from \( M_1 \) \((r_1 = |r_1|^2)\) for incidence from within the cavity (from a medium of refractive index \( n \)). The net cavity absorption is \( A_{\text{tot}} = 1 - |r_L|^2 \), and we maximize the resonant absorption by finding \( r_1 \) that minimizes \( |r_L| \).

On resonance \( \psi_o = m\pi \), by setting \( \Re(r_L) = 0 \) \((3\{r_L \} \ll 1)\) we find the optimal \( R_1 \) to realize \( A_{\text{tot}} \approx 1 \) is:

\[
R_1 \approx 1 - 2A_G \sin^2 \left( \frac{m\pi d}{L} - \frac{\alpha_1 + \alpha_2}{4} \right).
\]  

(9)

We can now establish a connection between the location of the absorber and the total absorption by noting that the intensity of the cavity standing-wave in absence of the absorber is \( I(d) \propto \sin^2(\Delta \varphi / 2) \). The phase \( \Delta \varphi \) therefore determines the location of the thin absorber with respect to the cavity standing-wave: \( \Delta \varphi = 0 \) corresponds to a null and \( \Delta \varphi = m\pi \) to a peak. When the absorber is placed at a null, Eq. (9) indicates that the optimal reflectivity is \( R_1 \to 1 \), and absorption is eliminated, as one expects; Fig. 2(b). Surprisingly, however, the absorber need not be placed at a peak to realize CPA. Indeed, if the absorber is placed anywhere except for the null \( \Delta \varphi \neq 0 \), then \( A_{\text{tot}} \approx 1 \) can always be maximized by selecting the reflectivity of \( M_1 \) according to Eq. (9). For example, if the absorber is placed midway between a null and a peak \( \Delta \varphi = \frac{\pi}{2} \), then selecting \( R_1 = (1 - A_G)^2 \) yields complete absorption; Fig. 2(c). Note that this is the optimal reflectivity for \( M_1 \) when employing a distributed absorber having single-pass absorption \( A_G \). If one makes use of this same mirror, but now places the absorber at a peak in the standing wave, as shown in Fig. 2(d), then maximum absorption is not attained; indeed, here \( A_{\text{tot}} \approx 90\% \). Placing the absorber at this position requires instead a reflectivity \( R_1 = (1 - 2A_G)^2 \) to achieve maximum absorption.
FIG. 2. Impact of the position of an ultrathin absorbing layer on the net cavity absorption $A_{\text{tot}}$. (a) Schematic depiction of a planar FP resonator containing a localized absorber having single-pass absorption $A_G$. (b–d) Three scenarios in a FP cavity having $R_1 = (1 - A_G)^2 \approx 96.8\%$, which corresponds to the CPA condition for a distributed absorber with $A = A_{\text{tot}}$. The thin absorber, which is represented by a thick black line, is considered at three distinct locations within the cavity standing-wave. (b) When the absorber is placed at the standing-wave null $z_1 = 0$, $A_{\text{tot}} = 0$. (c) Placing the absorber midway between a null and peak $z_2 = \lambda'/8$ results in perfect absorption $A_{\text{tot}} = 1$; $\lambda'$ is the wavelength inside the cavity. (d) When the absorber is placed at a standing-wave peak $z_3 = \lambda'/4$, the net absorption is sub-optimal $A_{\text{tot}} \approx 0.9$. (e) Calculated $A_{\text{tot}}$ as a function of the absorbing layer position $z$ and the front-mirror reflectivity $R_1$. The three red dots correspond to the positions $z_1$, $z_2$, and $z_3$ in (b), (c) and (d), respectively. The solid curve is the CPA contour $A_{\text{tot}} = 1$. The dashed curves are the contours for $A_{\text{tot}} \approx 0.9$, and the dotted curves are for $A_{\text{tot}} \approx 0.5$.

The presence of a cavity therefore modifies the dependence of absorption on the location of the absorber in a fundamental way. Specifically, we find – counter-intuitively – that complete absorption can be reached for any position of the thin absorber (as long as it is not at a null) if the appropriate reflectivity for $M_1$ is utilized. This is emphasized in Fig. 2(e) where we plot $A_{\text{tot}}$ as a function of the position $z$ of the absorber over an optical wave-length ($\Delta \psi = 2\pi z / \lambda$) and $R_1$. From Fig. 2(e) it is clear that substantial tolerance is afforded by the cavity design ($R_1$ and the absorber position) to approach $A_{\text{tot}} \approx 100\%$ for an ultrathin absorber. The region between the two dashed contours in the parameter space in Fig. 2(e) corresponds to $A_{\text{tot}} < 90\%$ (an enhancement of $\approx 55\times$ over $A_G$), whereas between the dotted contours $A_{\text{tot}} < 50\%$ (an enhancement of $\approx 30\times$). In other words, large resonant absorption enhancement can be achieved even when the absorber position deviates from the standing-wave peak, or $R_1$ does not match the targeted reflectivity. For example, taking $R_1 = 99.5\%$ results in $A_{\text{tot}} > 50\%$ for $\approx 90\%$ of the positions of the graphene monolayer within the cavity standing-wave. Indeed, Fig. 2(c,d) in which $R_1 = 96.8\%$ indicate that $A_{\text{tot}} > 90$ is reached for $\approx 50\%$ of the locations.

FIG. 3. Calculated cavity net absorption $A_{\text{tot}}$ as of function of the graphene position $z$ and the graphene single-pass absorption $A_G$ for a given mirror reflectivity, $R_1 = 0.95$. Compared to Fig. 2(e), this is more applicable plot since the mirror reflectivities are not changeable after the device fabrication, while the graphene absorption can be still varied in a range via electrical injection. Depending on the graphene position, the graph shows the cavity absorption can be quite modulated via tuning the graphene single-pass absorption. The white and the yellow dashed curves determine the areas within which the cavity absorption is higher than 50% and 70%, respectively.
well for constructing planar devices that resonantly enhance the absorption in 2D materials without exorbitant fabrication tolerances.

IV. DESIGN AND CHARACTERIZATION

We now proceed to our experiments validating resonantly enhanced absorption in a graphene monolayer. The FP cavity we utilize comprises two multilayer dielectric Bragg mirrors sandwiching a silica spacer in which a graphene monolayer is embedded. In this Section we describe the components of the CPA cavity and the performance of the cavity as a whole on resonance, and in the next Section we will combine it with a light pre-conditioning system that establishes omni-resonance over a broad continuous bandwidth.

Because the graphene’s intrinsic single-pass spectral absorption $A(\lambda) \approx A_G \approx 1.6\%$ in silica is flat over a broad spectrum as shown in Fig. 4(a), the mirror reflectivity required to realize CPA at any wavelength within this range is also spectrally flat $R_1(\lambda) \approx R_1 = (1 - A_G)^2 \approx 96.8\%$, which can be realized with periodic dielectric Bragg mirrors. The light source used in all our measurements is a halogen lamp (Thorlabs, QTH10/M), and the spectrum is recorded with an optical spectrum analyzer (OSA; Advantest Q8383). Light from the halogen lamp is spatially filtered by coupling it into a 1-mm-diameter 50-µm-diameter optical fiber terminated with a fiber collimator.

The targeted reflectivities for $M_1$ and $M_2$ are shown in Fig. 4(b): $R_2 = 1$ for $M_2$, and $R_2 \approx 96.8\%$ for $M_1$. This reflectivity is selected assuming the graphene monolayer is placed midway between the peak and null of the cavity standing-wave field distribution. Here $R_1$ is the reflectivity of $M_1$ for incidence from the silica spacer within the cavity. In Fig. 4(b) we plot the corresponding reflectivity for incidence from air to facilitate comparison with measurements. Both $M_1$ and $M_2$ are constructed of bilayers of 94-nm-thick SiO$_2$ and 61-nm-thick Ti$_2$O$_3$ that form a broad reflection bandwidth $\sim 100$ nm centered at $\sim 550$ nm. The mirrors $M_1$ and $M_2$ are formed of 5 and 10 bilayers, respectively, and their measured normal-incidence reflectivities from free space are plotted in Fig. 4(c), displaying excellent agreement with their theoretical counterparts in Fig. 4(b).

The overall FP cavity structure for CPA is depicted schematically in Fig. 4(a). Its construction starts with depositing the back mirror $M_2$ onto a 1-mm-thick, 25-mm-diameter glass slide, followed by a 4-µm-thick silica spacer. The graphene monolayer (of nominal thickness 0.35 nm [59]) is deposited on the spacer, and is then secured in place with a 100-nm-thick silica layer, before depositing the front mirror $M_1$. All deposition steps for the dielectric layers are carried out via e-beam evaporation (Appendix A). Using the transfer-matrix method, we calculate the net spectral absorption $A_{\text{tot}} = 1 - |R_1|^2 - |R_2|^2$ (throughout we have $t_2 \approx 0$) in the cavity as a function of the angle of incidence $\theta$. We plot $A_{\text{tot}}(\lambda, \theta)$ in Fig. 4(b) where it is clear that absorption enhancement is achieved at the cavity resonances. However, 100% absorption is not reached here because of the finite steps in reflectivity achieved by adding each bilayer to $M_1$. Indeed, 4 bilayers yield $R_1 = 89\%$ for incidence from air and 5 bilayers yield $R_1 = 96\%$, whereas the target reflectivity is $R_1 = 94\%$ (for incidence from free space). The dashed curve in Fig. 4(b) is the theoretical limit on $A_{\text{tot}}$ given our particular cavity design. The corresponding measured absorption spectra are plotted in Fig. 4(c) for a broadband field of diameter $\approx 10$ mm incident at angles in the range $0^\circ$ to $60^\circ$ with respect to the cavity normal. The absorption resonances do not reach the theoretical limit due to the imperfections in the deposited structure (departures from the targeted layer thicknesses and from an exact planar condition), and due to uncertainty in the exact location of the graphene monolayer within the silica spacer as discussed above. The absorption peaks extending over a bandwidth of 100 nm reach a maximum
FIG. 5. Characterization of the CPA cavity containing the graphene monolayer. The is practically no transmitted field through the back-reflector $M_2$. (a) Schematic of the CPA. (b) Calculated net cavity spectral absorption for normally incident light is shown on the left, and as a function of the angle of incidence $A_{\text{tot}}(\lambda, \theta)$ on the right. (c) Measured absorption spectra corresponding to (b). The dashed curves in the left panels in (b) and (c) represent the theoretical limit on absorption given the experimentally realized value of the reflectivity $R_1$ for the front mirror $M_1$.

of $\approx 60\%$.

V. REALIZATION OF BROADBAND COHERENTLY ENHANCED OMNI-RESONANT ABSORPTION

It is clear from Fig. 5(b,c) that the spectral absorption in the cavity at any incidence angle is resonantly enhanced only at discrete resonances rather than across a spectrally continuous band. The latter is realized by exploiting the concept of omni-resonance as illustrated in Fig. 1(e). This requires introducing angular dispersion into the incident field to associate each wavelength $\lambda$ with a specific angle of incidence $\theta(\lambda)$ via an optical system that pre-conditions the incident field. The experimental arrangement for demonstrating omni-resonance is shown in Fig. 6(a). Broadband light from the halogen lamp is directed through a 1-mm-wide slit onto a reflective diffraction grating (Thorlabs GR25-1850; 1800 lines/mm, area $25 \times 25$ mm$^2$), which is oriented at $50^\circ$ with respect to the incident light. The spectrally resolved wave front in the first diffraction order is focused onto the cavity by a lens $L_1$, which is an aspheric condenser (Thorlabs, ACL50832U) placed at a distance 12 cm from the grating. The cavity is mounted on a rotational stage near the focal plane of $L_1$, and is tilted an angle $\psi$ with respect to an optical axis defined by a central wavelength $\lambda_c = 550$ nm. This combination of grating and lens $L_1$ provides the requisite linear angular dispersion over the wavelength range of interest [19], see Appendix B for further details. Light reflected from the cavity is colli-
nated by a 25-mm-focal-length lens $L_2$ and directed to a monochrome CCD sensor (Thorlabs DCC1545M). After calibration with a tunable narrowband spectral filter ($\approx 3$-nm bandwidth), the spatial positions on the CCD correspond to wavelengths, and the reflected spectrum is captured as an image on the sensor for each $\psi$.

The computed absorption spectrum as we vary the cavity tilt angle $\psi$ is plotted in Fig. 6(b). It is clear that at certain values of $\psi$ the absorption is spectrally flat; i.e., broadband omni-resonance has been achieved, and the attendant CPA is realized over a continuous broad spectrum. Examples of omni-resonant CPA occur at $\psi \approx 50^\circ$ and $60^\circ$ in Fig. 6(b). Each of the angular settings for $\psi$ identifies an ‘achromatic resonance’. This is a single resonance of the bare cavity that is now spread over an extended, continuous spectral range rather than the narrow linewidth at a fixed wavelength as shown in Fig. 5(b). The re-orientation of the resonance trajectory in wavelength-angle $(\lambda, \psi)$ space is a consequence of the angular dispersion introduced into the incident field. Moreover, note that each achromatic resonance extends in bandwidth across multiple free spectral ranges of the bare cavity $[49–51]$.

The measurement results for the spectral absorption while varying $\psi$ are plotted in Fig. 6(c). Because of the physical constraints in the measurement apparatus resulting from the size of the cavity and the focal lengths of the lenses $L_1$ and $L_2$, measurements were collected for cavity tilt angles over the range $45^\circ < \psi < 60^\circ$. The measurements indicate that a prominent wideband absorption occurs at the tilt angle $\psi = 57^\circ$ associated with an achromatic resonance. The absorption spectrum at this tilt angle is plotted in Fig. 6(d), where it is compared to the corresponding spectra of the graphene monolayer and that of the bare cavity. We observe that the resonant enhancement in absorption is no longer confined to the resonant linewidths separated by the FSR. Rather, a continuous broadband spectrum of resonant enhancement is now realized as a result of the juxtaposition of CPA with omni-resonance. Two points must be noted here. First, the observed continuous absorption spectrum is not a consequence of ‘filling in’ the intra-resonance gaps. Instead, the absorption spectrum is associated with a single achromatic resonance. Second, this achromatic resonance does not correspond to any of the bare cavity resonances shown in Fig. 5(b,c). Instead, it corresponds to a higher-order bare-cavity resonance that becomes spectrally flat in our wavelength range of interest; see Fig. 6(b).

VI. CONCLUSION

We have combined in the same device the physically independent phenomena of CPA (resonantly enhanced complete absorption in a planar cavity containing a partially absorbing layer) and omni-resonance (the continuous, broad resonance spectrum realized when light incident on the cavity is pre-conditioned by introducing the appropriate angular dispersion). By embedding a graphene monolayer of a single-pass absorption $A_G$ in a planar cavity provided with a back-reflector and an appropriately designed front mirror, light on resonance can be fully absorbed, despite the low intrinsic absorption of graphene. By adding a light pre-conditioning system, the resonantly enhanced absorption is now realized over a broad continuous spectrum and is associated with a single achromatic resonance. Our measurements reveal at one such achromatic resonance enhanced absorption to $\sim 60\%$ in monolayer graphene observed over a continuous bandwidth of $\approx 70$ nm.

We have also elucidated the impact of the position of an ultrathin absorber such as monolayer graphene on the net cavity absorption. We have shown that it is no longer necessary to place the absorbing layer at a peak in the standing-wave pattern within the cavity as is commonly understood to be the case in free space. Instead, maximum absorption is guaranteed to reach its maximum value if the absorber is placed anywhere except at a field null. This requires only adjusting the reflectivity of the cavity front mirror. Furthermore, for fixed front-mirror reflectivity, adjusting the intrinsic absorption in the ultrathin layer (e.g., via an electric potential for graphene $[58]$) can maximize the net cavity absorption.

The notion of omni-resonance is a subclass of the more general framework of classical entanglement, that refers to classical optical fields in which non-separability is created by introducing deterministic correlations between physically independent degrees of freedom $[60–65]$. Here, classical entanglement is introduced between two continuous degrees of freedom: the wavelengths and their associated angles of incidence, whereby each wavelength is assigned to a single prescribed angle $[60]$, resulting thereby in omni-resonance. In addition to omni-resonance, a similar conception of classical entanglement has been instrumental in developing a new class of propagation-invariant pulsed beams of wave packets that we have coined the name ‘space-time’ (ST) wave packets $[57–70]$. Moreover, we have recently established the connection between omni-resonance and ST wave packets and have demonstrated that a family of such pulsed beams are omni-resonant and can thus traverse planar FP cavities without change even when the pulse bandwidth exceeds the cavity resonant linewidth $[52–53]$.

The effect demonstrated here is an important step towards the utilization of CPA in energy harvesting technologies despite the inherently resonant nature of CPA. It is an open question whether the free-space arrangement we made use of here (a diffraction grating and a lens) can be replaced by a single element (such as a metasurface) that may be integrated with the multilayer cavity structure into a single device. This may pave the way to future highly-efficient ultrathin solar cell devices and photodetectors.
The grating is directed to the FP cavity via a lens \( L_1 \). We take \( \lambda = 550 \text{ nm} \) as the central wavelength that defines the optical axis. The tilt angle \( \psi \) of the cavity is measured with respect to this optical axis. We define the angle \( \gamma (\lambda) \), which is the diffraction angle of wavelength \( \lambda \) with respect to the grating normal. The central wavelength \( \lambda_c = 550 \text{ nm} \) is diffracted at \( \gamma (\lambda_c = 550 \text{ nm}) = \gamma_0 \) and coincides with the optical axis, and any other wavelength \( \lambda \) makes an angle \( \gamma (\lambda) - \gamma_0 \) with respect to the optical axis. This angle is boosted via the lens \( L_1 \) by a ratio \( d_1/d_2 \), where \( d_1 \) and \( d_2 \) are the distances from the grating to \( L_1 \) and from \( L_1 \) to the cavity, respectively. The incidence angle with respect to the optical axis after \( L_1 \) is

\[
\varphi (\lambda) = \tan^{-1} \left\{ \frac{d_1}{d_2} \tan[\gamma (\lambda) - \gamma_0] \right\},
\]  

with \( \varphi_o = \varphi (\lambda_c = 550 \text{ nm}) = 0 \). The distances \( d_1 \) and \( d_2 \) are selected such that the illuminated spot on the grating is imaged onto the cavity. If the focal length of \( L_1 \) is \( f \), then \( d_2 = \frac{fd_1}{f-d_1} \). When the cavity is oriented such that it is perpendicular to the optical axis, the angle of incidence of each wavelength is \( \varphi (\lambda) \). Upon tilting the cavity by \( \psi \), the angle of incidence with respect to the normal to the cavity is \( \theta (\lambda) = \varphi (\lambda) + \psi \). By adjusting \( \psi \), one can provide the optimal angular dispersion required to achieve the omni-resonance condition.

Appendix C: Transfer matrix analysis for the CPA cavity

The scattering matrices associated with the front and back mirrors (\( S_1 \) and \( S_2 \), respectively) take the form:

\[
S_j = \left( \begin{array}{cc}
       t_j e^{i \beta_j} & r_j e^{i \alpha_j} \\
       -r_j e^{-i (\beta_j - \alpha_j)} & t_j e^{i \beta_j}
\end{array} \right),
\]  

where \( t_j \) and \( r_j \) (\( j = 1, 2 \)) are the transmission and reflection amplitudes of the mirrors, respectively, and \( \beta_j \) and \( \alpha_j \) are the associated phases. The scattering matrix relates the right and left outgoing waves (\( A_R \) and \( B_L \), respectively) to the left and right incoming waves (\( A_L \) and \( B_R \), respectively). The transfer matrix \( M \) corresponding to \( S \) is:

\[
M_j = \frac{1}{t_j} \left( \begin{array}{cc}
       e^{-i \beta_j} & -r_j e^{-i (\beta_j - \alpha_j)} \\
       -r_j e^{i (\beta_j - \alpha_j)} & e^{i \beta_j}
\end{array} \right).
\]  

Propagation for a distance \( d \) in a medium of refractive index \( n \) is represented by a transfer matrix:

\[
M(\varphi) = \left( \begin{array}{cc}
       e^{-i \varphi} & 0 \\
       0 & e^{i \varphi}
\end{array} \right)
\]  

where \( \varphi = nkd \) is the optical phase. This transfer matrix applies to propagation for a distance \( d \) from \( M_1 \) to the graphene monolayer, and then a distance \( L - d \) to \( M_2 \).
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