A theoretical model for predicting the surface topography of inhomogeneous materials after shot peening

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Received: 28 September 2021 / Accepted: 4 January 2022 / Published online: 26 January 2022
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Abstract
Shot peening is widely used in engineering as a classical strengthening process. Although many studies on shot peening have been done, most have focused on homogeneous target materials. In this paper, a theoretical model is proposed for predicting the surface morphology of inhomogeneous target materials. The topography of target materials after single-shot impact is calculated on the basis of energy conservation and Hertz contact theory, and the final three-dimensional surface topography after multiple-shot impact is obtained through superposition. Single-shot and random multiple-shot finite element models are used to show the advantages of the proposed model over the existing theoretical model for homogeneous target materials. The roughness is found to increase with the shot velocity and shot radius.

Keywords Shot peening · Surface metamorphic layer · Inhomogeneous material · Surface topography · Theoretical model

1 Introduction
Shot peening is a classical processing technology that has been greatly developed in industry owing to its versatility and wide applicability [1, 2]. Compared with other finishing technologies with obvious thermal damage, jet machining technology has major advantages such as very limited heat-affected area [3] and especially high feasibility of machining free-form surfaces. In shot peening, many shots impact the surface of a workpiece with high speed, resulting in grain refinement and work hardening in the near-surface layers. The compressive residual stress introduced by shot peening can also delay crack progression and further improve the fatigue strength of artifacts [4–7].

However, many pits exist on the surface of the workpiece after shot peening, and therefore the surface morphology may be poor. The surface morphology is very important for evaluating the surface quality of the workpiece, and it greatly affects the functions of parts. Such effects include a series of assembly problems such as friction, wear, vibration, and noise of parts [8–11]. The surface morphology also seriously affects the fatigue performance of the workpiece because the rough surface is equivalent to the presence of many notches (or microcracks), which cause stress concentration [12–14]. When the workpiece is under load, the stress concentration causes fatigue sources to form much more easily, leading to fatigue failure. Therefore, it is very important to predict the surface morphology of the workpiece after shot peening.

Numerous experimental and simulation studies have been conducted on shot peening and have obtained valuable results. For example, Beaucamp and Namba [15] found that better surface roughness could be obtained for mold steel using low inlet pressure and a small abrasive after fluid jet polishing. In the polishing of soft metal (such as copper), low jet pressure combined with a large abrasive can reduce the embedding of the abrasive on the target surface, thus reducing surface pollution [16]. Pham et al. [17] and Tsai et al. [18] studied optimal jet parameters for reducing the surface roughness (Ra) of n-BK7 optical glass and SKD61 mold steel workpieces. A multiple-shot finite element model was used by Lin et al. [19] to investigate the effects of shot peening coverage on surface integrity, and it was found that the plastic strain increase with the shot peening coverage, and variations of each roughness parameter with coverage are different.
There have also been several theoretical works on shot peening. For example, a theoretical model [20] of jet machining was proposed for the case in which abrasion by particles does not reach the level of causing brittle fracture on the target material, and compared with the experimental results, this model predicted the roughness of unmasked channels with average errors of about 36%. A theoretical model was proposed [21] for processing quartz glass through air-abrasive jet polishing that considered the randomness of abrasive particle size and the springback of abrasives from the workpiece surface; the model agreed well with experiment, and showed that to obtain a smooth surface, the small-sized abrasives and the low jet airflow pressure were more effective. Wang et al. [22] proposed an analytical model of overlapped footprints in abrasive air jet polishing of optical glass, and they further studied the effects of polishing parameters including the jet pressure, jet angle, and abrasive size on both machined surface roughness and removed material volume. Qu et al. [23] proposed a material removal profile model for obliquely axial ultrasonic vibration-assisted polishing of K9 optical glass, and this model showed that with larger ultrasonic amplitude and larger oblique angle, the material removal capability can be improved significantly. In addition, Zhang et al. [24] theoretically analyzed the residual stress field for different geometrical features after abrasive waterjet peening, the results of residual stress had relatively good agreement with those of finite element model.

Although many studies have been done on shot peening, most have focused on homogeneous target materials. Actually, most workpieces have gone through heat treatment to produce a surface hard enough to give satisfactory resistance [25–27] before the shot peening. This means the material parameters are actually inhomogeneous near the target surface. Therefore, it is necessary to investigate the effect of shot peening on heterogeneous target materials. This study gives a theoretical model for predicting the surface morphology of heterogeneous target materials after shot peening. The model provides an analytical way to understand and optimize shot peening technology.

The paper is organized as follows. Section 2 outlines the theoretical model for predicting surface topography after multiple-shot impact. Then, Sect. 3 presents verification based on the finite element method and discusses the results. Finally, the main conclusions are summarized in Sect. 4.

2 Theoretical model

Let us consider a shot impacting on a target material. When the shot velocity is lower than the elastic critical value $v_{ela}$, the maximum stress inside the target is less than the yield stress, so there is only elastic deformation on the target material (see Fig. 1a). When the shot velocity is higher than $v_{ela}$, there will be residual plastic deformation on the target material after the shot rebounds (see Fig. 1b).

Fig. 1 Schematics of shot impacts on the target material for (a) $v \leq v_{ela}$ and (b) $v > v_{ela}$

### 2.1 Calculating the elastic critical velocity and maximum depth

For the case shown in Fig. 1a, we ignore energy loss in the form of friction. The kinetic energy of the shot is converted into the work done by the contact force $F_{ela}$ exerted by the target material as the velocity decreases to zero:

$$\frac{1}{2}mv^2 = \int_0^{\lambda_{ela}} F_{ela} d\lambda ,$$

where $m$ and $v$ are the mass and velocity of the shot, $F_{ela}$ is the contact force, and $\lambda_{ela}$ is the critical depth. The relationship between the elastic contact force and the impact depth can be obtained according to Hertz contact theory [28–30]. When the maximum stress in the vertical contact force system is equal to the yield strength of the target material, the shot peening process enters the critical stage of shaping, and the critical depth $\lambda_{ela}$ can be obtained from

$$\lambda_{ela} = \frac{1}{4\sigma_y^2} \left( \frac{1}{E_1} + \frac{1}{E_2} \right) \frac{1 - \nu_1^2}{E_1} \frac{1 - \nu_2^2}{E_2} R ,$$

where $\sigma_y$ is the yield strength of the surface layer; $E_1$ and $E_2$, are the elastic moduli of the shot and target material, respectively; $\nu_1$ and $\nu_2$ are the Poisson ratios of the shot and target material, respectively; and $R$ is the radius of the shot. In our study, $E_1$ and $E_2$ are set equal and constant because the elastic modulus in the near-surface layers is unchanged [31].
2.2 Calculating the residual depth of the dent

When the shot velocity is greater than \( v_{\text{ela}} \), there is a dent on the target material caused by residual plastic deformation after the shot rebounds, as shown in Fig. 1b. For this case, the kinetic energy of the shot is converted into the work of the contact forces \( F_{\text{ela}} \) and \( F_{\text{pla}} \) exerted by the target material. If we also ignore the energy loss of friction and other forms, then

\[
\frac{1}{2} m (v^2 - v_{\text{ela}}^2) = \int_{\lambda_{\text{ela}}}^{\lambda_{\text{pla}}} F_{\text{pla}} d\lambda ,
\]

(3)

where \( \lambda_{\text{pla}} \) is the maximum depth of impact. For homogeneous target materials, the plastic contact force \( F_{\text{pla}} \) can be evaluated according to full plasticity theory as

\[
F_{\text{pla}} = k \times 2\pi \int_0^{a_p} \sigma_y r dr ,
\]

(4)

where \( k = 3 \) is a correction coefficient [32], and \( a_p \) is the radius of the dent. For a material with an inhomogeneous metamorphic layer, the yield stress \( \sigma_y \) in the metamorphic layer can be expressed as

\[
\sigma_y = f(h), \quad h \in (0, h_i) ,
\]

(5)

where \( h_i \) is the thickness of the metamorphic layer. This paper assumes a perfectly elastoplastic model.

As shown in Fig. 2a, \( \sigma_y \) can also be written in terms of the radius \( r \) at depth \( h \) as

\[
\sigma_y = g(r), r \in (0, a_p) .
\]

(6)

According to the geometric relationship between the pits shown in Fig. 2b, the relationship between \( g(r) \) and \( f(h) \) can be obtained as

\[
g(r) = f(\sqrt{R^2 - r^2} - R + \lambda ) .
\]

(7)

To introduce the inhomogeneity of the target material, let us consider a circle ring with a width \( dr \) (see Fig. 3).

![Fig. 3 The inside of the ring is regarded as a homogeneous material](image)

The plastic contact force provided by the ring is \( dF_{\text{pla}} = k \sigma_y 2\pi rdr \). Clearly, the direction of the plastic contact force \( F_{\text{pla}} \) is vertical and upward.

By integrating along the surface of the dent, we can express the plastic contact force \( F_{\text{pla}} \) as

\[
F_{\text{pla}} = k \times 2\pi \int_0^{a_p} g(r) r dr ,
\]

(8)

where \( a_p \) is a geometrical parameter given by the geometrical relationship \( a_p^2 = 2R\lambda_{\text{pla}} \) [32], we can obtain \( \lambda_{\text{pla}} \) by solving Eq. (3) numerically. After that, the corresponding permanent deformation depth can be obtained via

\[
\lambda_{\perp} = \lambda_{\text{pla}} - \lambda_{\text{ela}} .
\]

(9)

2.3 Calculating the morphologies after shot peening

Following Ref. [21], the morphology generated by a single shot is assumed to be part of a sphere. When the shot velocity is lower than the elastic critical value \( v_{\text{ela}} \), the \( i \)-th shot has no effect on the surface morphology of the target material. When the shot velocity is higher than \( v_{\text{ela}} \), as shown in Fig. 4, the morphology caused by a single shot can be calculated [21] via

\[
y_i = \begin{cases} 
\frac{R^{(i)2} - (x - x_i')^2 - (z - z_i')^2}{2} \bigg|_{\lambda_{i-1} (\lambda \geq \lambda_{\text{ela}})}^{{\frac{1}{2}}} + y_{0,i}' (\lambda \geq \lambda_{\text{ela}}), \\
y_i = R^{(i)} - \lambda_{\perp}^{(i)}, \quad \lambda < \lambda_{\text{ela}}
\end{cases}
\]

(10)

where \( y_{0,i}' = R^{(i)} - \lambda_{\perp}^{(i)} \), and \( R^{(i)} \) is the radius of the \( i \)-th shot.

In this study, we calculate the surface morphology of the target after multi-shot peening via superposition of the pit morphology formed by each single shot. When the distance between two dents is greater than the sum of the radii of the two dents, the dents do not overlap and the depth of each dent is not affected [21]. The depth coordinate of a dent is \( y_i' = R^{(i)} - \lambda_{\perp}^{(i)} \). When the distance between two dents is less than the sum of the dent radii but greater than either dent
radius, there is a slight overlap between the dents. In this case, the depth of a dent is not affected, and the depth coordinates are the same as in the previous case. When the distance between the two dents is less than either dent radius, the dents overlap significantly, and the dent depth increases to some extent. The increase [21] in depth is

\[
\lambda_{l_{i}}^{(i-1)} = \lambda_{l_{\text{per}}}^{(i-1)} - \frac{l^2_{\text{min}i}}{2R^{(i)}},
\]

(11)

where \(l^2_{\text{min}i}\) is the minimum distance between the \(i\)-th dent and other dents (minimum dent spacing), and the depth coordinate in this case is \(y_{0}^{i} = R^{(i)} - \lambda_{l_{\text{per}}}^{(i-1)} - \lambda_{l_{i}}^{(i-1)}\).

It should be mentioned that the method of superposition employed in this paper is only an approximate method. This is because that the deformation process of the target under multi-shot peening is a nonlinear, therefore it is almost impossible to obtain the surface morphology of the target after multi-shot peening accurately by superposition of the pit morphology formed by single shot.

3 Verification and discussion

3.1 Verification of the depth caused by a single shot

To validate the proposed theoretical model, we used a two-dimensional axisymmetric single-shot finite element model based on the Abaqus (see Fig. 5). We considered a target material with a metamorphic layer in which the yield strength is inhomogeneous (varying with depth). The constitutive model was an ideal elastoplastic model. The size of the target material was 3 mm × 3 mm. The size of the strengthening area was 1 mm × 1 mm with a minimum mesh size of 10 μm × 10 μm. The thickness of the metamorphic layer was 1 mm and was discretized with five layers. The element type of the target body was CAX4R. The shot was constrained as a rigid body and set to surface contact with the target. To verify the applicability of the method, a variety of shot velocities and radii were selected in the finite element model.

Both a linear distribution and an exponential distribution of yield strength along with the vertical direction were used to verify the theoretical model. The detail forms of yield strength are listed in captions of Figs. 6 and 7, respectively. This study used the yield strengths
of the base material and the surface material to calculate the impact depth $\lambda_{\text{per}}$ of a single shot according to the homogeneous theory [21], the results of the homogeneous model with the yield strength of the surface material and base material are denoted by $H\text{-}S$ and $H\text{-}B$, respectively. Figures 6 and 7 show that the results of base yield strength calculated via the homogeneous algorithm is higher than the numerical result of the heterogeneous. Furthermore, the results of surface yield strength calculated via the homogeneous algorithm is lower than the numerical result of the heterogeneous. The theoretical results considering heterogeneity are closer to the numerical results, verifying the accuracy of the heterogeneous model. Therefore, it is necessary to include heterogeneity when building a three-dimensional surface model.

### 3.2 Analysis of 3D morphological model

In actual shot peening, the surface morphology of the target material is the superposition of many random dents. Therefore, we used a 3D finite element model with random shots to validate the 3D morphology predicted by the proposed model. As shown in Fig. 8, the $y$ coordinate of the $i$-th shot particle was $i \times 2R$, while the $x$ and $z$ coordinates of each shot were generated by a random function.

The size of the target material was $3 \text{ mm} \times 3 \text{ mm} \times 3 \text{ mm}$, with a $1 \text{ mm} \times 1 \text{ mm} \times 1 \text{ mm}$ strengthening area, as shown in Fig. 9a. Eight-node elements with reduced integration (C3D8R) were used, and the minimum mesh size was $20 \mu\text{m} \times 20 \mu\text{m} \times 20 \mu\text{m}$. An infinite element layer (CIN3D8) was used to avoid stress wave reflection on the boundaries (see Fig. 9b).

The Avrami equation [33–35] for the coverage rate and shot number is

$$C\% = 100 \times \left[1 - e^{-\frac{N \times \pi R^2}{S}}\right], \quad (12)$$

where $C\%$ is the expected coverage rate, $N$ is the number of shots, and $S$ is the area of the region to be strengthened.

Figure 10 shows the surface morphologies predicted by the proposed model and the numerical calculation when the
Fig. 10 Numerical and theoretical results for 3D surface topography and 2D sectional contours when \( v = 80 \text{ m/s} \) and (a) \( R = 0.2 \text{ mm} \), (b) \( R = 0.4 \text{ mm} \), (c) \( R = 0.6 \text{ mm} \), and (d) \( R = 0.8 \text{ mm} \).
shot velocity is 80 m/s and the shot radii are 0.2 mm, 0.4 mm, 0.6 mm, and 0.8 mm. The contours generated by the theoretical model are mostly consistent with the numerical results, and the numbers of peaks and troughs in the theoretical model are almost the same as those of the numerical results. Table 1 lists the values of the roughness $R_a$ for different radii.

**Table 1** Roughness $R_a$ for different radii when $v = 80$ m/s

| Shot radii (mm) | 0.2   | 0.4   | 0.6   | 0.8   |
|-----------------|-------|-------|-------|-------|
| $R_a$ (μm)      | Numerical | 7.178 | 10.658| 12.283| 13.968|
|                 | Theoretical | 5.414 | 10.823| 13.150| 13.639|
| Error(%)        | 24.58 | 1.55  | 7.06  | 2.36  |

The theoretical results agree well with the numerical results, and $R_a$ increases with the shot radius.

**Table 2** Roughness $R_a$ for different velocities when $R = 0.8$ mm

| Shot speed (m/s) | 20   | 40   | 60   | 80   |
|------------------|------|------|------|------|
| $R_a$ (μm)       | Numerical | 5.178 | 8.202| 11.451| 13.968|
|                 | Theoretical | 4.511 | 8.027| 10.762| 13.639|
| Error(%)         | 12.88 | 2.13 | 6.02 | 2.36 |

Figure 11 shows the surface morphologies for the shot velocities 20 m/s, 40 m/s, and 60 m/s for a shot radius of 0.8 mm, and Table 2 lists the values of the roughness $R_a$ for different velocities for this radius. As in Fig. 10 and Table 1, the theoretical results agree well with the numerical results, and $R_a$ increases with the shot radius.

**Fig. 11** Numerically and theoretically obtained 3D surface topographies and 2D sectional contours for $R = 0.8$ mm and (a) $v = 20$ m/s, (b) $v = 40$ m/s, and (c) $v = 60$ m/s.
results agree well with the numerical results for different velocities, which further confirms the proposed model.

Figure 12 shows the roughness versus shot radius and velocity in the proposed theoretical model. Clearly, $Ra$ increases with both the radius and velocity of the shot. However, $Ra$ varies almost linearly with shot velocity but nonlinearly with the shot radius.

4 Conclusion

In this paper, a theoretical model was proposed for predicting the surface topography of an inhomogeneous target material after shot peening. Finite element models were used to validate the proposed model. The main conclusions drawn from the results are:

1. For the residual depth after one shot, the proposed inhomogeneous theoretical model agrees better with the finite element models than the existing theoretical model for homogeneous target materials.
2. The proposed inhomogeneous theoretical model effectively predicts the 3D surface topography and roughness $Ra$ after multiple-shot impact. The relative errors of $Ra$ between theoretical and numerical results are in the range of 1.55% to 25.51% for different radii when $v=80$ m/s; and for different velocities when $R=0.8$ mm, the relative errors are in the range of 2.13% to 12.88%.
3. The roughness $Ra$ increases almost linearly with shot velocity while increasing nonlinearly with shot radius.
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