The nucleon and the nuclear force in the context of effective theory and path-integral methods

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Abstract. The nucleon structure and the nuclear force are investigated in the context of the non-perturbative path-integral method of hadronization. Starting from a microscopic quark-diquark model, the nucleon is generated as a relativistic bound state and an effective chiral meson-nucleon Lagrangian is derived. Many of the nucleon physical properties are studied using a theory of at most two free parameters.

INTRODUCTION

The central challenge in nuclear physics remains to understand the origin and nature of the nuclear force due to our inability to solve quantum chromodynamics (QCD), the fundamental theory for the strong interactions. The basic problem of QCD is that its fundamental degrees of freedom, quarks and gluons, are not the observable baryon and meson states. Thus bridging the missing link between the fundamental and observable degrees of freedom stands as one of the stark challenges of nuclear/elementary particle physics today. Although we do have an ab initio approach to solve this problem, that is lattice QCD, this endeavor is still miles away from achieving such a goal. For the time being, we have no alternative but to resort to effective non-perturbative approaches of which this study is one.

This presentation describes our work [1] in addressing this missing link by deriving a chiral meson-nucleon Lagrangian from a microscopic model of quarks and diquarks using the path integral method of hadronization. Chiral symmetry and its spontaneous breaking have proven to be key concepts in understanding meson and baryon structure and many features of the nuclear force. Therefore, we start from a QCD-based chiral effective field theory, the Nambu Jona-Lasinio (NJL) model that accommodates most of QCD symmetries [2]. Then, the nucleon is described as quark-diquark correlations. This assertion is vindicated by a mounting experimental evidence that diquarks play a dynamical role in hadrons [3]. Using the path integral hadronization, we calculate nucleon properties and derive an effective chiral meson-nucleon Lagrangian of the quantum hadrodynamics (QHD) type [4] that describes the rich meson-nucleon interactions in a fully covariant and chirally symmetric formalism.

While this program is applied to the case of nucleons and mesons, it is certainly

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of general nature and can possibly be applied prolifically to yield other baryons and their interactions. Moreover, the idea of using path-integral techniques to transform a Lagrangian from its fundamental to its composite degrees of freedom is a powerful concept in physics of immense impact and utility. As a matter of fact, the authors of Ref. [5] have recently invoked such techniques in their study of high-temperature superconductivity. They succeeded in doing so by converting a model of strongly-correlated electrons into an effective U(1) gauge field theory in terms of composite fields.

The use of hadronization has been introduced in Ref. [6, 7]. Consequently, the authors of Ref. [8] attempted to construct an effective Lagrangian for the nucleon using only scalar diquarks. In the present work, we extend their work by deriving the structure using both axial-vector and scalar diquarks and we employ, as opposed to Ref. [8], a gauge-invariant regularization scheme throughout our analysis. Furthermore, we verify the Ward identity and the Goldberger-Treiman relation and present a full numerical study of various nucleon observables for the case of scalar diquarks drawing special attention to the role of an intrinsic diquark form factor. Hence, this work is the first calculation of an extensive set of nucleon observables using path-integral hadronization since the introduction of the idea more than ten years ago.

**FORMALISM**

We start from an NJL Lagrangian satisfying SU(2)$_L \times$ SU(2)$_R$ chiral symmetry:

$$\mathcal{L}_{NJL} = \bar{q}(i\partial - m_0)q + \frac{G}{2} \left[ (\bar{q}q)^2 + (\bar{q}i\gamma_5 \vec{\tau}q)^2 \right],$$

where $q$ is the current quark field, $\vec{\tau}$ are the Pauli matrices, $G$ is the NJL coupling constant, and $m_0$ is the current quark mass which explicitly breaks chiral symmetry. The color and flavor indices are suppressed for brevity.

By introducing composite scalar ($\sigma \sim \bar{q}q$) and pseudoscalar ($\vec{\pi} \sim \bar{q}i\gamma_5 \vec{\tau}q$) fields through the Hubbard-Stratonovich transformation [9], we can rewrite the NJL lagrangian into a semi-bosonized Lagrangian

$$\mathcal{L}'_{NJL} = \bar{q}(i\partial - \sigma - i\gamma_5 \vec{\pi})q - \frac{1}{2G}(\sigma^2 + \vec{\pi}^2),$$

where we have absorbed the bare quark mass $m_0$ into the sigma field $\sigma$. Further, we transform the meson fields according to the non-linear parameterization $[\sigma, \vec{\pi}] \rightarrow [\sigma', \Phi]$:

$$\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi} = (m_q + \sigma') e^{-\frac{1}{F_\pi} \gamma_5 \vec{\tau} \cdot \Phi}.$$ Here, $F_\pi = 93$ MeV is the pion decay constant and $m_q \equiv \langle \sigma \rangle_0$ is the constituent quark mass which is fixed through a gap equation in the meson sector [2].

As a consequence of considering Lorentz structure, there are five types of possible diquark $qq$ correlations. These are the scalar, pseudo-scalar, vector, axial-vector, and tensor diquarks. For each of these Lorentz $qq$ formations, we have also an isoscalar and isovector diquarks. Using permutation symmetry and Fierz transformation, we have verified an earlier assertion [10] that only two diquark formations are independent for the nucleon if its field is to be written as a local operator of three quarks. Hence, we introduce $\vec{D}^\mu$ as an axial-vector isovector diquark field and $D$ as a scalar isoscalar one.

Next we introduce a quark-diquark interaction term in such a manner to generate the nucleon as a linear combination of axial-vector and scalar diquarks. It is convenient
here, considering chiral symmetry, to work with the chirally rotated \( \chi \) “constituent” quark field defined by \( \chi \equiv e^{\frac{i}{2}\gamma_5} \frac{\bar{q}}{q} \). By introducing electromagnetic interactions and batching the semi-bosonized NJL Lagrangian, the diquark contributions and the quark-diquark interaction term, we obtain the following Lagrangian as our microscopic model:

\[
\mathcal{L} = \chi S^{-1} \chi - \frac{1}{2G}(\sigma' + m_q)^2 + \delta \mathcal{L}_{sb} + D^\dagger \Delta^{-1} D + \bar{D}^\dagger \mu \tilde{\Lambda}_{\mu \nu} \bar{D}^\nu +
\]

\[
\tilde{G} \left( \sin \theta \bar{\chi} \gamma^\mu \gamma^5 \tau \cdot \bar{D}^\dagger \mu + \cos \theta \bar{\chi} D^\dagger \right) \left( \sin \theta \bar{D}_v \cdot \tau \gamma^\nu \gamma^5 \chi + \cos \theta D\chi \right). \quad (2)
\]

where

\[
S^{-1} = S_0^{-1} + \mathcal{M}, \quad \text{(3a)}
\]

\[
\mathcal{M} = -\left[ \gamma^\mu \tilde{\gamma}^\pi \frac{\gamma^\mu}{2} + \gamma^\mu \gamma^5 \tilde{\gamma}^\pi \frac{\sigma \gamma^\mu}{2} + \sigma' + \gamma^\mu Q_q A_{EM}^\mu \right], \quad \text{(3b)}
\]

\[
\Delta^{-1} = \Delta_0^{-1} + iQ_S A_{EM}^\mu (\tilde{\gamma}^\mu - \tilde{\sigma}^\mu), \quad \text{(3c)}
\]

\[
\tilde{\Lambda}_{\mu \nu}^{-1} = \tilde{\Lambda}_{0 \mu \nu}^{-1} + iQ_A \left[ (A_{EM}^\mu \delta_{\nu} - A_{EM}^\nu \delta_{\mu}) - g_{\mu \nu} A_{EM} (\tilde{\gamma}^\alpha - \tilde{\sigma}^\alpha) \right]. \quad \text{(3d)}
\]

Here \( \theta \) is a mixing angle for the two diquark contributions, \( \tilde{G} \) is the quark-diquark coupling constant and \( Q_{q}, Q_{S} \) and \( Q_{A} \) are the quark and diquark charges while \( S_{0} \), \( \Delta_{0} \) and \( \tilde{\Lambda}_{0 \mu \nu} \) are the free quark and diquark propagators (Notice that \( \tilde{\sigma}(Q_{S}^2) \) terms are discarded in Eq. (3)). The \( \mathcal{M} \) matrix contains all interaction vertices of the quark field with meson and electroweak fields (weak part is not shown), where the vector \( \tilde{\gamma}^\pi_\mu \) and the axial vector \( \tilde{\sigma}^\pi_\mu \) fields are defined through the Cartan decomposition (\( \tilde{\xi} \equiv \frac{\bar{\Phi}}{F_{\pi}} \)):

\[
\exp \left( -\frac{i}{2} \gamma^5 \tilde{\xi} \cdot \xi \right) \partial_\mu \exp \left( \frac{i}{2} \gamma^5 \tilde{\xi} \cdot \xi \right) = \frac{i}{2} \gamma^\mu \tilde{\gamma} \cdot \tilde{\sigma}^\pi_\mu (\tilde{\xi}) + \frac{i}{2} \tilde{\gamma} \cdot \tilde{\gamma}^\pi_\mu (\tilde{\xi}).
\]

Subsequently, we introduce collective nucleon fields (\( B \sim \sin \theta \bar{D}_v \cdot \tau \gamma^\nu \gamma^5 \chi + \cos \theta D\chi \)) through another Hubbard-Stratonovich transformation. At this point only quarks and diquarks are dynamical fields with kinetic terms while the meson and nucleon fields are merely auxiliary ones. By integrating over the quark and then over the diquark fields, we obtain a meson-nucleon effective Lagrangian. Accordingly, we have “hadronized” the microscopic theory by producing the dynamical meson (bosonization) and nucleon (fermionization) fields.

Thereupon, we arrive at a compact Lagrangian given by

\[
\mathcal{L}_{eff} = \delta \mathcal{L}_{sb} - \frac{1}{2G}(\sigma' + m_q)^2 - i \text{ tr } \ln S^{-1} - \frac{1}{G} \bar{B}B + i \text{ tr } \ln (1 - \Box) + i \text{ tr } \ln (1 - \Delta_{0} \text{ EM Int}) + i \text{ tr } \ln (1 - \tilde{\Lambda}_{0} \text{ EM Int}) \quad (4)
\]
FIGURE 1. The self-energy diagram which generates the nucleon kinetic and mass terms and produces the mass equation that determines the nucleon mass.

Here the trace is over color, flavor and Lorentz indices while the “EM Int” label stands for the diquark electromagnetic interaction terms. Furthermore,

\[
\begin{align*}
\Box &= \left( \mathcal{A} \mathcal{F}_1 \mathcal{J}^2 \right), \\
\mathcal{A}^\mu_i \nu_j &= \sin^2 \theta \bar{B} \gamma^5 \tau_k \tilde{A}^{\mu k, \mu i} S \tau^i \gamma^5 B, \\
\mathcal{J} &= \cos^2 \theta \bar{B} \Delta S B, \\
(\mathcal{F}_1)^\nu_j &= \sin \theta \cos \theta \bar{B} \Delta S \tau^j \gamma^5 B, \\
(\mathcal{F}_2)^\mu_i &= \sin \theta \cos \theta \bar{B} \Delta \bar{S} \tau^j \gamma^5 B.
\end{align*}
\]

This effective Lagrangian contains plenty of rich physics: kinetic and mass terms for nucleons and mesons together with a multitude of interaction terms of mesons, nucleons, and electroweak gauge bosons. Nonetheless, the most desired part of the Lagrangian is the prized chiral meson-nucleon interaction and nucleon-nucleon vertices which delineates the nuclear force.

In order to explore the physics of the nucleon sector, we take the leading term in the loop and derivative expansion of \( i \text{ tr } \ln(1 - \Box) - \frac{1}{G} \bar{B} B \rightarrow - \int d^4 x d^4 y \bar{B}(x) \left[ \Sigma(x, y) + \frac{1}{G} \delta(x - y) \right] B(y), \) which is nothing but the nucleon self-energy (see Fig. 1). The Fourier transform of \( \Sigma \) is then decomposed as \( \Sigma(p^2) = \Sigma_s(p^2) + \frac{1}{G} \Sigma_v(p^2) \). This leading term generates dynamically the nucleon mass \( M_B \) which is extracted as the pole of the propagator:

\[
\frac{1}{G} + \Sigma_s(M_B^2) + M_B \Sigma_v(M_B^2) = 0.
\]

Thus, near the mass shell the inverse nucleon propagator takes the form:

\[
\left[ \Sigma(p^2) + \frac{1}{G} \right] \sim (\not{p} - M_B) Z^{-1},
\]

where \( Z \) is the wave-function renormalization constant (\( B = \sqrt{Z} B_{\text{ren}} \)). Evidently, the nucleon has finally acquired the desired status as a dynamical degree of freedom in the problem.

In computing the various Feynman diagrams in the problem, we encounter divergent integrals that must be regularized. Several regularization schemes were attempted. We
found that the most suitable scheme is the PV technique which we have adopted as the standard method in this work. Accordingly, we verified the Ward-Takahashi identity by computing the electromagnetic vertex shown in Fig. 2. As a matter of principle, the PV mass in the nucleon sector can be different from the NJL cut-off arising in the meson sector [2]. Nonetheless, to minimize the number of free parameters, we elected to equate them. It is noteworthy here that all observables were found to be very insensitive to the value of the PV mass upholding the futility of using it as a free parameter.

We have also verified one of the chiral symmetry relations, the Goldberger-Treiman relation, by computing both the axial-vector coupling constant $g_A$ and the pion nucleon coupling constant $g_{\pi NN}$. In the hadronization formalism, this relation emerges naturally intact as opposed to large violations in the Bethe-Salpeter equation approach [11].

**NUMERICAL STUDY**

Having derived the structure of the Lagrangian, we proceed to generate numerical results using only scalar diquarks ($\theta = 0$ in Eq. (2) and (5)), thereby admitting the possibility of an intrinsic diquark form factor (IDFF). Including only scalar diquarks is not out of place as many recent studies using such diquarks have reported good results for most of the nucleon observables [12, 13, 11]. Moreover, there are strong indications of large scalar diquark dominance in the nucleon [14].

The parameters $G$ and the cut-off $\Lambda$ are fixed to yield the constituent quark mass and the pion decay constant [2]. The diquark masses are also determined in the NJL model [15]. This leaves us with only one free parameter in our model: the quark-diquark coupling constant $\tilde{G}$ which is fixed to determine the nucleon mass of 0.94 GeV through the mass equation (6). Thus the basic quantities in our model are the constituent quark mass $m_q = .390$ GeV, the scalar diquark mass $M_D = .600$ GeV, the quark-diquark coupling constant $\tilde{G} = 159.1$ GeV$^{-1}$, and the Pauli-Villars mass $\Lambda = .600$ GeV. We obtain a binding energy of $\Delta E_{\text{bin}} \equiv m_q + M_D - M_B = 50$ MeV, suggesting a loosely bound state for the nucleon.

Tab. 1 displays our predictions for some of the static properties of the nucleon. For the nucleon magnetic moments, our treatment predicts a number that is two-third of the experimental value for the proton and one-half of that for the neutron. This is not a surprising result as we have not included the axial-vector diquark in the present calculation. The predicted value for the axial-vector coupling $g_A$ of 0.87 is less than the experimental one of 1.26 indicating here also the importance of the axial-vector diquark.

**FIGURE 2.** The Feynman diagrams for the electromagnetic coupling which generate the electromagnetic vertex of the nucleon and subsequently are used to test the validity of the Ward identity.
TABLE 1. Some of the nucleon static properties as predicted in the present calculation using the intrinsic diquark form factor (IDFF) or without it. Experimental values are taken from Ref. [16, 17].

|       | $\mu_p$ | $\mu_n$ | $g_A$ | $<r^2>_E^p$ (fm$^2$) | $<r^2>_E^n$ (fm$^2$) | $<r^2>_M^p$ (fm$^2$) | $<r^2>_M^n$ (fm$^2$) |
|-------|---------|---------|-------|----------------------|----------------------|----------------------|----------------------|
| Theory with IDFF | 1.57 | -0.75 | 0.87 | 0.77 | -0.11 | 0.82 | 0.84 |
| Theory without IDFF | 1.57 | -0.75 | 0.87 | 0.68 | -0.19 | 0.82 | 0.85 |
| Experiment | 2.79 | -1.91 | 1.26 | 0.74 | -0.12 | 0.74 | 0.77 |

FIGURE 3. The nucleon electric form factors with and without the intrinsic diquark form factor (IDFF) along with its quark and diquark contribution. The left panel shows the proton results while the right panel displays those for the neutron. Experimental data can be found in Ref. [19, 20].

The nucleon size is nicely well-produced in our model as the electric and magnetic radii for the nucleon are very close to the experimental measurements. The negative charge radius of the neutron has been suggested as an indication of a scalar diquark clustering in the nucleon [18] and our treatment dynamically manifests this assertion. These numbers point to a physical picture of a “heavy” diquark at the center with a quark rotating around it. The extended size of the diquark contributes a positive value of about 0.10 fm$^2$ for the nucleon electric radii. As expected, the IDFF has virtually no effect on the magnetic radii as the scalar diquark has a negligible contribution to the magnetic form factors.
Next we calculate the nucleon form factors. The left panel of Fig. 3 displays the proton form factor with and without the IDFF along with its quark and diquark contribution. Our treatment produces beautifully this observable. It is evident here that the IDFF [21] plays an important role specially at large values of momentum transfer ($Q^2 \equiv -q^2$ where $q^\mu$ is the momentum transfer). The neutron electric form factor tells a similar story (right panel). Clearly, the quark contribution is negative in value (d-quark) and thus cancels much of the diquark contribution leading to a small form factor. It is noteworthy here that the neutron form factor is a potent test of any treatment as it is a delicate cancellation of two large contributions [13]. Saliently, the cancellation is naturally produced in our study.

In Fig. 4 we present the nucleon magnetic form factor as calculated with or without the intrinsic diquark form factor. Unmistakably, the scalar diquark contribution is virtually vanishing due to the lack of an intrinsic spin. Nevertheless, there is a very small contribution due to a small orbital angular-momentum effect in the bound quark-diquark system. A comparison with experimental data suggests the need for the axial-vector diquark, which does have an intrinsic spin, to supplement the quark contributions and to provide the missing strengths for the magnetic form factors.
CONCLUSIONS

In conclusion, we have tackled the nucleon structure and the challenging problem of understanding the origin and nature of the nuclear force by deriving a meson-nucleon Lagrangian using the path-integral method of hadronization. The treatment produced a remarkable agreement with experimental data for the nucleon size and its electric form factors, while our calculations show missing strengths for the magnetic properties and $g_A$. The discrepancy is likely due to the absence of the axial-vector diquark in the present numerical study. This presentation describes the first work in our program of using path-integral hadronization to study baryon structure and the nuclear force. Deriving this force provides nuclear physics with a solution to the stigma of no fundamental foundation that has tarnished its image for decades.

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