Characterization of the beam loading effects in a laser plasma accelerator

C Rechatin\textsuperscript{1,5}, J Faure\textsuperscript{1}, X Davoine\textsuperscript{2}, O Lundh\textsuperscript{1}, J Lim\textsuperscript{1}, A Ben-Ismail\textsuperscript{1,3}, F Burgy\textsuperscript{1}, A Tafzi\textsuperscript{1}, A Lifschitz\textsuperscript{1,4}, E Lefebvre\textsuperscript{2} and V Malka\textsuperscript{1}

\textsuperscript{1} Laboratoire d’Optique Appliquée, ENSTA Paristech, CNRS, Ecole Polytechnique, UMR 7639, 91761 Palaiseau, France
\textsuperscript{2} CEA, DAM, DIF, Bruyères-le-Châtel, 91297 Arpajon, France
\textsuperscript{3} Laboratoire Leprince Ringuet, Ecole Polytechnique, CNRS-IN2P3, UMR 7638, 91128 Palaiseau, France
\textsuperscript{4} Laboratoire de Physique des Gaz et des Plasmas, CNRS, UMR 8578, Université Paris XI, Bâtiment 210, 91405 Orsay cedex, France
E-mail: clement.rechatin@polytechnique.edu

New Journal of Physics 12 (2010) 045023 (20pp)

Received 9 October 2009
Published 30 April 2010
Online at http://www.njp.org/
doi:10.1088/1367-2630/12/4/045023

Abstract. In this study, electrons were injected into a laser plasma accelerator using colliding laser pulses. By varying the parameters of the injection laser pulse, the amount of charge accelerated in the plasma wave could be controlled. This external control of the injected load was used to investigate beam loading of the accelerating structure and especially its influence on the electron beam energy and energy spread. Information on the accelerating structure and bunch duration was then derived from these experimental observations.

\textsuperscript{5} Author to whom any correspondence should be addressed.
1. Introduction

In the laser–plasma accelerator concept, first proposed by Tajima and Dawson [1], electrons are accelerated in a high-amplitude travelling plasma wave (or wakefield) driven by a laser. The accelerating field, of the order of $E_p(V\,m^{-1}) = 96\sqrt{n_e}(cm^{-3})$, where $n_e$ is the plasma electron density, exceeds 100 GV$m^{-1}$ for typical parameters, and can accelerate electrons to hundreds of MeV in millimetres.

As in any accelerator, the final quality of the accelerated electron bunch critically depends on the injection process. This is all the more true in plasma accelerators since the accelerating–focusing phase of the wakefield, in which the bunch has to be injected, is only a fraction of the plasma wavelength $\lambda_p$, of the order of 10 $\mu$m in current laser plasma accelerators. Furthermore, the high-density accelerated electron bunch can drive its own wake: this effect is referred to as beam loading [2]. It ultimately limits the charge that can be accelerated since for high loads, the superposition of the two wakefields is decelerating for trailing electrons and electrons give their energy back to the wave. Even before reaching this limit, beam loading plays a crucial role in the evolution of the spectrum of the accelerated electron bunch. On one hand, when uncontrolled, it can produce a fast varying field over the bunch length, leading to a fast growth of the energy spread. On the other hand, when the injected electron bunch is carefully shaped, it can flatten the electric field over the bunch length and enable monoenergetic acceleration. These considerations show how important the injection process is.

In most current laser–plasma acceleration experiments, the injected electrons are provided by wave breaking of a highly nonlinear plasma wave. In the blowout or bubble regime [3]–[6], beam loading effects prevent any further wave breaking after the initial injection has occurred, so that injection stays localized. This simple scheme has proved to inject and accelerate
quasi-monoenergetic electrons with final relative energy spreads of 5–10% (full-width half-maximum (FWHM)) [7]–[10]. However, it relies on nonlinear effects and provides neither intrinsic stability nor control over the injected bunch.

To overcome these limitations, controlled injection schemes have been used. As stated above, to inject electrons in a plasma wave, one needs short (⩽ 100 fs) and narrow (⩽ 10 µm) bunches. Such electron bunches are currently unavailable with conventional radio-frequency (RF) technology, even if new injectors and schemes might make this possible in the near future [11]. Therefore, so far, injection mechanisms relying on plasma electrons are the ones that are used the most. For instance, density down ramps that slow down the phase velocity of the wakefield might stabilize and improve the quality of the electron bunch [12]–[14]. Another technique, relying on ionization-induced trapping, in which electrons gain an energy equal to their binding potential to heavy contaminants, might also help us to control the injection [15]–[17], but there is still no experimental evidence of high-quality beams produced in this manner.

Injection of electrons can also be realized by using an additional laser pulse. The first all-optical scheme, proposed by Umstadter et al [18], was based on the momentum boost provided by the transverse ponderomotive force of a second laser propagating perpendicular to the driving laser. Esarey et al [19] suggested that the efficiency could be enhanced by using the standing beatwave at the collision of two counter-propagating laser pulses. In its latest and simplest version [20, 21], this scheme uses two lasers: a driving (or pump) pulse with normalized amplitude $a_0$ and an injection pulse with normalized amplitude $a_1$. According to same authors [19]–[22], the beatwave of the two pulses permits a preacceleration of the electrons of the order of $Δp_z = \sqrt{2a_0a_1m_e}c$, which efficiently injects electrons in the plasma wave. This injection scheme, first demonstrated in [23], has many advantages: (i) the heating of electrons occurs in a very small area, and as the beatwave is linked with the pump pulse, electrons are mostly injected in the first plasma period, resulting in a high-quality electron beam. With this scheme, quasi-monoenergetic electron beams with a 1% FWHM energy spread have been obtained [25]. (ii) Injection occurs for normalized pump amplitudes as low as $a_0 = 1$, which is considerably lower than the requirements for injection in the wave breaking regime, $a_0 \approx 3–4$ [6, 24]. This regime is readily attainable with current laser systems without relying on a cascade of nonlinear effects, which makes the injection process intrinsically more robust. (iii) The injection parameters are easily tunable through the modification of the injection laser while keeping the same accelerating structure, defined by $a_0$ and the plasma parameters [25].

In this paper, we further develop our study described in [26]. After presenting the experimental setup and the main results obtained with this colliding pulse scheme, we will show how this versatile injection mechanism can be used to investigate the beam loading effect. With the help of three-dimensional (3D) self-consistent PIC simulations, we will extract information on the optimal load of our accelerator. Further modelling of the beam loading effect can be used to estimate the electron bunch duration.

### 2. Colliding pulse injection experiment

#### 2.1. Experimental setup

The following experimental results were obtained with the LOA ‘Salle Jaune’ Ti:Sa laser, delivering two laser pulses of 30 fs with a total on-target energy of 1 J. The pump pulse is focused
Figure 1. An overview of the experimental setup. (a) A diagram of the experiment. (b) A typical raw image obtained on the LANEX screen. (c) The corresponding deconvolved spectrum and definition of the beam parameters.

by an $f/16$ spherical mirror to an intensity of $I_0 = 4.6 \times 10^{18}$ W cm$^{-2}$, giving a normalized amplitude of $a_0 = eA_0/m_ec = 1.5$. The injection pulse is focused by an $f/18$ off-axis parabola to a maximal intensity of $I_1 = 4 \times 10^{17}$ W cm$^{-2}$, giving a normalized amplitude of $a_1 = 0.4$. A half-wave plate followed by a polarizer is used to continuously reduce the injection pulse intensity. A third laser pulse (probe) is used to align and synchronize the two main laser beams.

The two pulses collide with an angle of $176^\circ$ as shown in figure 1(a). This slight angle difference from the counter-propagating geometry reduces the risk of laser feedback while keeping the overlap between the two pulses large enough for easy operation. Other experiments in Japan [27], and very recently at LOA, have shown that this scheme also works for a $135^\circ$ colliding angle, which is even more practical for electron beam extraction purposes.

The plasma medium is provided by the ionization of a 3 mm long supersonic helium gas jet. Its density profile, independently characterized by interferometry, exhibits a well-defined plateau over 2.1 mm. The experimental data presented hereafter were obtained with an electron density of $5.7 \times 10^{18}$ cm$^{-3}$.

To extract the spectral information of the electron bunch, a compact magnetic spectrometer was used. It consists of a bending dipole magnet of 1.1 T over 10 cm and a LANEX phosphor screen imaged by a 16-bit camera. A raw image is shown in figure 1(b). The LANEX screen has been absolutely calibrated on a conventional RF accelerator [28]. Therefore, the spectrometer gives access, after deconvolution, to the energy distribution, charge and divergence of the
electron beam. In the following, as the spectra all exhibit well-defined quasi-monoenergetic features, we can consistently use the conventions represented in figure 1(c): the ‘energy’ of the bunch corresponds to the peak energy of the distribution, the ‘energy spread’ to the FWHM energy spread and the ‘charge’ to the charge integrated between the energies whose spectral intensity corresponds to 20% of the maximal value. The energy resolution is ultimately limited by the divergence of the beam. For example, for an electron beam with a typical experimental divergence of 5 mrad, the spectrometer resolution is of the order of 3% at 100 MeV and 5% at 200 MeV.

2.2. Control of injection volume

The colliding pulse injection scheme provides a way to easily change the injection volume by controlling the momentum gained by the electrons in the beatwave. This can be done, for example, by changing the injection laser amplitude. The principle of the method can be seen in figure 2 for a one-dimensional (1D), analytically tractable case. The top panels of this figure represent the wakefield excited in 1D, by a 30 fs, $a_0 = 2$ laser pulse in a plasma of density $n_e = 7 \times 10^{18}$ cm$^{-3}$. The bottom panels represent the orbits of electrons in this wake in the usual phase-longitudinal momentum ($\xi = z - v_p t$, $p_z$) representation, where $v_p$ is the plasma wave phase velocity. The untrapped electrons and trapped electrons are separated by the separatrix represented by the solid red line. The theoretical maximal momentum gain in the beatwave for circularly polarized laser or beatwave separatrix, whose expression is $\sqrt{2a_0(\xi)a_1}$, is also represented. Following the early work of Esarey [19] and Fubiani [20], electrons can be injected

\[ k_p^z = k_p(z - v_p t) \]

**Figure 2.** Theoretical change of injection volume in 1D for circular polarizations: (a) $a_1 = 0.4$ and (b) $a_1 = 0.1$. Top: wakefield (longitudinal electric field). Bottom: orbits in the wakefield and beatwave separatrix. The grey area represents the injection volume. Parameters: $a_0 = 2$, $n_e = 7 \times 10^{18}$ cm$^{-3}$. 
when the separatrices overlap. The overlapping area in phase space, represented in grey, defines the injection volume in which injection can occur [22].

Figures 2(a) and (b) show that a change of injection pulse intensity from \(a_1 = 0.4\) to \(a_1 = 0.1\) reduces this injection volume.

A realistic description of the injection requires taking into account several physical effects neglected in this simple picture, such as stochastic effects due to linear polarizations [29]–[32] and wakefield inhibition [33], but the injection volume representation still holds.

This reduction of injection volume will mainly result in the reduction of the charge injected in the wakefield. Furthermore, in the small load limit the electrons almost behave as test particles and the acceleration can be described by a Hamiltonian. The phase space volume is therefore approximately conserved during the acceleration. That is why, after rotation in phase space, a decrease of injection volume will result in an improvement of the energy spread of the bunch. The effect of beam loading will be discussed in detail in the following section, but for large beam loads, the energy spread is expected to grow with the charge.

This is exactly what can be seen in the experiment when we control the injection volume by changing the injection pulse amplitude. Figures 3(a) and (b) represent raw images and deconvolved spectra for different values of \(a_1\) for an injection position located 400 \(\mu\)m before the centre of the nozzle \((z_{\text{coll}} = -400 \mu\text{m})\). The charge clearly decreases with \(a_1\), \(a_1 = 0.1\) being close to the threshold for injection to occur. As expected, when the charge decreases, the energy distribution becomes narrower. The evolution of the charge and energy spread is given for a full data set in figure 3(c), each point corresponding to an average made over 3–5 shots. It clearly shows that the charge can be tuned continuously from 0 to 30 pC. The energy spread is strongly correlated with the charge in the electron beam.

This control of injection volume provides the perfect test bed for probing beam loading effects: the accelerating structure driven by the pump pulse and the position of injection can be kept the same, while the wakefield can be loaded with various charges. This is what we study in the next section.
3. Observation of beam loading

3.1. Nonlinear model and predicted observables

In the linear theory, the total wakefield of a loaded wake is the superposition of the unloaded wakefield driven by the laser pulse and the electron bunch-driven wakefield. An axisymmetric bunch of density $n_b(r, \xi) = n_e \rho(r) f(\xi)$ drives an electric field [34]:

$$E_z(0, \xi) = E_p k_p^3 \int_0^\infty r \rho(r) K_0(k_p r) dr \int_0^\xi f(\xi') \cos(k_p (\xi - \xi')) d\xi',$$

(1)

where $K_0$ is the modified Bessel function of the second kind. Previous work [35] has shown that beam loading is optimized, i.e. results in a constant electric field over the whole bunch length, for a trapezoidal bunch shape in the longitudinal direction.

However, in our case, we operate in a mildly nonlinear regime ($a_0 \simeq 1.3$). In addition, after some propagation, the laser pulse evolves through self-focusing and self-compression, which causes the laser intensity to further increase. Simulations have shown [36] that in our experimental conditions, $a_0$ reaches values of $a_0 = 2.5 – 3$, which leads to the formation of plasma bubbles where linear theory completely fails.

The beam loading perturbations in the blowout regime (or bubble) have been recently derived by Tzoufras et al [37, 38]. This work is based on the phenomenological description of the plasma bubble by Lu et al [5, 6]. The main benefit of the model is that one can derive all the fields from the shape of the bubble, $r_b(\xi)$, with maximal radius $R_b$. In the very nonlinear regime, $a_0 > 2$, the linear charge density of the bunch (charge density integrated over transverse dimensions) and the shape of the bubble are connected through the differential equation

$$E_z = -\frac{E_p}{2} k_p r_b \frac{d r_b}{d \xi} = \frac{k_p r_b}{2 \sqrt{2}} \sqrt{\frac{16 \int k_p r_b I(\xi) \xi d\xi + C}{(k_p r_b)^4}} - 1,$$

(2)

where $k_p = 2\pi/\lambda_p$, $C$ is a constant determined by continuity and $I(k_p r_b(\xi))$ corresponds to the normalized linear charge density of the bunch, expressed in units of $I_0 = 2\pi m_e c^2/e$, which corresponds, in the laboratory frame, to a normalized current $I_0 = I_0 c = I_{\text{Alfven}}/2 \simeq 8.5 \text{kA}$. Given the definition of $I(r)$, the study is limited to the case where the radius is monotonic with phase in the back end of the bubble. This is not restrictive as a non-monotonic variation will correspond to $E_z \geq 0$, which has to be strictly avoided: in this case, some electrons start to decelerate and give their energy back to the wakefield (previously referred to as the ultimate limit of beam loading).

Integrating equation (2) from the top of the bubble $r_b(\xi) = R_b$ to the back of the bubble gives the loaded wakefield for any load below the ultimate limit. From this electric field, it is then possible to solve the equation of motion of trapped electrons and to compute the self-consistent evolution of an electron bunch in a loaded wakefield: at each time step of integration, the positions of electrons define the longitudinal density profile. This model, albeit very simple compared with the computationally challenging 3D particle-in-cell (PIC) simulations, provides good insight into the evolution of an electron beam in a loaded wakefield.

In the following, we will consider Gaussian bunches of FWHM length $1 \mu m$ (or duration $3.3 \text{fs}$) in a bubble of maximal radius $k_p R_b = 3.4$, which corresponds to the result of simulations in our experimental conditions [36], and an electron density of $n_e = 7 \times 10^{18} \text{cm}^{-3}$. The initial
position of the electron bunch is arbitrarily fixed at $\xi_{\text{inj}} = -2k_p^{-1} \simeq -4 \, \mu m$, where the phase origin is located at the bubble centre ($E_z = 0$). To simulate a realistic initial momentum distribution, electrons are injected with a zero longitudinal emittance $p_z/(m_ec) = 4 + k_p(\xi_{\text{inj}} - \xi)/2$. This small momentum chirp ($\Delta p_z, \text{FWHM} \simeq 100 \, \text{keV}$) corresponds to the fact that all the electrons are not injected at the same time. Figure 4(a) represents the phase space of the bunch after acceleration over $300 \, \mu m$ in the loaded wakefield (also represented by lines) for three different loads, expressed as the initial maximal current of the electron bunch. The resulting spectra are represented in figure 4(b).

The black case corresponds to the unloaded case, for which the orbit description of figure 2 can be applied. Note that in this case, the unloaded wakefield is not constant over the bunch length. Thus, the leading electrons experience a lower accelerating field than the trailing electrons (that are closer to the separatrix). It results in a strong energy chirp after acceleration and in a large energy spread.

A load of 25 kA, represented in blue, corresponds to the optimal load case: the wakefield is flattened by the load and most of the electrons in the bunch experience the same accelerating field. This results in a quasi-monoenergetic acceleration with a relative FWHM energy spread $\Delta E/E = 0.4\%$. From [38], the current that locally flattens the electric field at the front of a top hat bunch can be expressed as $I = k_p^2\left(R_b^4/8r_{\text{inj}}^2 + r_{\text{inj}}^2/8\right)I_0$ corresponding to an initial current of $I \simeq 30 \, \text{kA}$. The small difference here is due to the Gaussian shape of the bunch.

When the load is twice as large as the optimal load (red curve in figure 4) the accelerating field is no longer monotonic and trailing electrons are less accelerated than leading electrons. The charge accumulates at the electric field extrema, explaining the double peak structure in the spectrum.

If the energy spread variation is the most noticeable feature of figure 4, it is not the best observable to study the effect of beam loading in our experiment. The reason is linked with the fact that when we change the load, we also change the injection volume. As we have seen in the previous section, this change directly impacts the energy spread, through
Figure 5. Experimental results. (a) Electron spectra obtained for three different injection laser amplitudes (from left to right): $a_1 = 0.1$ (8 pC), 0.24 (21 pC) and 0.4 (38 pC). (b) Bunch peak energy versus charge trapped in the peak. Red squares correspond to the varying $a_1$ data set and blue circles to 30 consecutive shots with laser fluctuations.

As can be seen in figure 4(b), another consequence of beam loading is the decrease of peak energy from 90 MeV for the unloaded case to 73 MeV in the overloaded case. This is due to the fact that trailing electrons experience the field perturbation of the leading electrons and are less accelerated. We will see that, even though the variation of the injection volume might also change the peak energy, the two effects are easily distinguishable. Furthermore, this observable is robust against pump laser intensity fluctuations. If beam loading were not playing any role, an increase of charge should be linked to an increase of energy since a stronger pulse will not only inject more electrons but also drive a stronger wakefield. Therefore the decrease of energy with increasing trapped charge cannot be related to laser fluctuations.

Another possible way to diagnose beam loading is to monitor the charge trapped in the trailing plasma buckets. Those electrons, also heated in the beatwave, are not trapped in the first bucket mainly because of wakefield inhibition [33], but they can be trapped in the following periods of the plasma wave and they are accelerated to lower energies. However, as the load of the first bucket also damps the field in the trailing plasma buckets, beam loading should prevent the trapping of a large charge after the first bucket and therefore reduce the background current of the accelerator.

3.2. Experimental results

Figure 5(a) represents the electron spectra obtained with three different injection pulse amplitudes. It clearly shows a decrease of the peak energy with increasing injection amplitude and beam charge. Here, the peak energy of the quasi-monoenergetic component goes from 197 MeV, for an injected charge of 8 pC ($a_1 = 0.1$), to 151 MeV, for a charge of 38 pC ($a_1 = 0.4$).

New Journal of Physics 12 (2010) 045023 (http://www.njp.org/)
Figure 6. Experimental results: (a) electron spectra for different charge loads; (b) evolution of the ratio between the charge trapped in the trailing buckets (above 45 MeV) and in the first peak as a function of charge trapped in the first peak. Red squares correspond to the varying $a_1$ data set and blue circles to 30 consecutive shots with laser fluctuations.

To represent more data and give statistically clearer results, we represent in figure 5(b) the peak energy versus the charge of the quasi-monoenergetic peak for two complete data sets. One set of data (circles) is obtained by looking at the fluctuations of charge and spectra over 30 consecutive shots, for which the only variations are the laser intensity and pointing fluctuations at the collision position of 400 $\mu$m before the centre of the nozzle ($z_{\text{coll}} = -400 \mu m$). The other set of data (squares) is the same as presented in figure 3, i.e. obtained for injection at $z_{\text{coll}} = -250 \mu m$ by varying $a_1$, and thus forcing a change of trapped charge in the first bucket over a wider range. Those curves clearly confirm the strong correlation between trapped charge and energy. These data points also exhibit, for small charges ($<20$ pC), a linear slope (dashed lines). When normalized by the acceleration lengths (1.45 and 1.3 mm respectively), those slopes are similar: $1.6 \text{GV m}^{-1} \text{pC}^{-1}$ for injection at $z_{\text{coll}} = -400 \mu m$ (circles) and $1.55 \text{GV m}^{-1} \text{pC}^{-1}$ for injection at $z_{\text{coll}} = -250 \mu m$ (squares).

Figure 6(a) presents spectra with low injected charges. Whereas for a peak charge of 13 pC most electrons are contained in the high energy peak, the background current increases when the peak charge is smaller. Figure 6(b) represents the ratio $Q_{\text{trail}}/Q_{\text{first}}$ versus $Q_{\text{first}}$, where $Q_{\text{first}}$ is the charge in the high-energy peak and $Q_{\text{trail}}$ is the charge in the rest of the distribution (above 45 MeV). This ratio represents a measurement of relative background current and clearly decreases with the charge trapped in the first peak, as expected from the beam loading effects.

3.3. Comparison with 3D PIC simulations

These experimental observations reveal the effects of beam loading but simulations are needed to fully confirm this interpretation and exactly understand the role of the variation of the injection volume, which might also change the energy of the electron bunch. To model the experiment, 3D PIC simulations have been performed with the code CALDER [39].
Figure 7. Spectra obtained in the test particle simulations (without beam loading) for $a_1 = 0.05, 0.1, 0.2, 0.3$ and $0.4$.

physical parameters were similar to the experimental ones: a normalized amplitude of $a_0 = 1.3$ and an electron density $n_e = 7.5 \times 10^{18} \text{ cm}^{-3}$ for a collision position $z_{\text{coll}} = -575 \mu\text{m}$, which gives a trapped charge similar to the experimental results [36]. Simulations are performed for different values of the injection pulse intensity but, to limit the computational time, the simulations are stopped only $300 \mu\text{m}$ after injection, resulting in limited acceleration, typically to $70 \text{ MeV}$. To disentangle the effects of varying the injection volume from beam loading, we also run simulations in which beam loading is artificially removed. These simulations are similar to the usual 3D PIC simulations, except that all electrons with longitudinal momentum above $12m_ec$ are treated as test particles, i.e. do not generate electric fields. Those simulations, which model realistically the injection process, can be used to determine the effect of the change of injection volume when $a_1$ is increased. Resulting spectra are represented in figure 7 for $a_1 = 0.05, 0.1, 0.2, 0.3$ and $0.4$. They first confirm the experimental increase of charge with $a_1$ seen in figure 3.

Figure 7 also clearly shows that the injection volume variation only affects the low energy cutoff of the spectrum. This can be understood with simple arguments. As can be seen in figure 2, the main consequence of an increase of injection volume is that electrons can be trapped further away from the separatrix where they are accelerated by lower electric fields to lower final energies. In contrast, the high-energy cutoff of the spectrum, corresponding to electrons trapped near the separatrix, is unchanged when the injection volume increases. An interesting consequence of this observation is that any change of the high-energy cutoff of the spectra is therefore exclusively linked with beam loading effects. This is especially true for low accelerated charges since, as can be seen in figure 4, the higher energy electrons will be the trailing electrons, most subjected to beam loading.

When taking the beam loading effect into account, 3D PIC simulations give results in very good agreement with the nonlinear theory. Figure 8(a), presenting the phase space of electrons as well as the on-axis longitudinal electric field for different loads, is indeed highly reminiscent of figure 4. The pale grey colour map and dotted black curve represent the test particle case (no beam loading) with $a_1 = 0.3$. Trailing electrons see a stronger accelerating field than leading electrons, which induces a large energy spread in the spectrum, represented in dotted black.
in figure 8(b). When beam loading is included, trailing electrons are less accelerated and the energy spread can be reduced. This is the case with $a_1 = 0.3$ represented in solid blue, which is near optimal since it almost flattens the bunch distribution in phase space. The comparison of the two spectra shows that beam loading clearly affects the high-energy cutoff, whereas the low-energy cutoff is almost unchanged in this case.

These simulations thereby confirm the fact that any change of the high-energy cutoff for small loads is linked with beam loading effects. The change of high-energy cutoff, which can be clearly seen in figure 5(b), is therefore an unambiguous observation of beam loading in our experiment.

The case of $a_1 = 0.4$, represented in red in figure 8, for which more charge is injected, corresponds to an overloaded case. Trailing electrons are less accelerated than leading electrons and end up with a lower energy. The acceleration process finally results in a spectrum that is no longer monoenergetic but displays peaked structures, corresponding to local extrema of the electric field. These structures, which we might already see at $Q = 38 \text{ pC}$ in figure 5(b), will be helpful to determine the optimal load of the accelerator.

Figure 9(a) (respectively (b)) represents the evolution of the peak energy (respectively the ratio of charge trapped in the following buckets of the plasma wave over charge trapped in the first bucket) with charge trapped in the first bucket. Figure 9(b) confirms the reduction of the background current with increasing charge as observed in the experiment (see figure 6). Figure 9(a) exhibits a decrease of peak energy with charge, and when divided by the acceleration length (300 $\mu$m), shows a linear slope of 2 $\text{GV m}^{-1} \text{pC}^{-1}$ for small charges, in very good agreement with the experimentally measured 1.6 $\text{GV m}^{-1} \text{pC}^{-1}$. The simulations can also be used to determine the influence of beam loading on the peak energy, which is the most easily measurable feature of our electron beams. They show that approximately half the decrease of the peak energy is due to the decrease of the high energy cut-off and beam loading, while the other half originates from the variation of the injection volume, as can be inferred from...
Figure 9. Simulation results. (a) Evolution of the bunch peak energy with charge, (b) evolution of the ratio between the charge trapped in the trailing buckets (above 10 MeV) and in the first bucket as a function of charge trapped in the first bucket.

The simulations have now confirmed the validity of the experimental observables of beam loading. The reduction of background current and the decrease of energy with charge can now be interpreted as solid observations of beam loading.

The simulations also confirm the validity of the nonlinear model: even if the latter does not take into account the laser evolution during its propagation, it gives a good approximation of the evolution of the electron bunch in a loaded wakefield.

4. Discussion

Using these experimental observations, one can try to deduce the properties of the accelerating structure and the electron bunch. In the following, we will estimate the optimal load of our accelerator and the duration of the electron bunch.

4.1. Optimal load

The first possible signature of the optimal load is also linked with the energy variation. For over-optimal load, the electrons corresponding to the highest peak energy are sitting at the local minimum of the electric field, at the front of the bunch (see figures 4(a) and 8(a)). As beam loading can only have an effect on trailing electrons, the decrease of peak energy of the bunch with charge should be smaller for overloaded cases. This trend appears clearly in the nonlinear simulations, as well as in the 3D PIC simulations. As a reminder, figure 10 represents the energy evolution with charge in the experiment and in the 3D PIC simulations.

The change of slope appearing on the experimental data around 20 pC can indeed be compared with the change of slope appearing around 40 pC in the simulations, which corresponds to the case closest to the optimal load. The difference of optimal loads in both cases...
could be explained by a small mismatch of simulation parameters with experimental parameters or by the fact that the simulations are stopped only 300 µm after the collision. Dephasing and evolution of the accelerating structure might indeed also have an influence on the optimal load.

This estimate of optimal charge is confirmed by the second possible signature of the optimal load, the variation of energy spread. For under optimal loads, the energy spread will grow when the injection volume (and hence charge) increases. This growth of energy spread will be counterbalanced by the beneficial effect of beam loading, which flattens the electric field. For over-optimal loads, beam loading will also be deleterious for the energy spread of the bunch so that both effects tend to increase the energy spread. We therefore expect a fast increase of energy spread with charge above a given value corresponding to the optimal load.

For the data presented in figure 5, the trapped charge stays smaller or close to the supposed optimal charge of 20 pC. In order to observe the effect of large loads, we use data obtained in the same laser and plasma conditions but with different injection positions. We indeed know that some nonlinear effects during the propagation might help trap more electrons when injection occurs later in the gas jet [36, 40]. At inner positions of injection, we have been able to inject more than 100 pC.

Figure 11(a) shows raw and deconvolved spectra obtained at a position of injection $z_{\text{coll}} = -50 \mu\text{m}$. When 23 pC are trapped, the energy spread of the bunch is of the order of 10%, close to the spectrometer resolution. When 100 pC are trapped, the spectrum exhibit a long tail extending to low energies and peaked structures, as expected from the simulations in the over-optimal cases.

The evolution of the energy spread of this low-energy tail with charge is represented in figure 11(b). For this figure, as different shots might correspond to different collision positions [$z_{\text{coll}} \in (-450 \mu\text{m} : 850 \mu\text{m})$] and different acceleration lengths, we represent the relative energy spread. Simulations based on the nonlinear model of beam loading tend to show that the shape of the accelerating field does not significantly evolve for an acceleration distance smaller than the dephasing length and thus justify this representation. Figure 11(b) clearly shows two different types of behaviour: (i) for low trapped charges, the relative energy spread is kept small, close
to the spectrometer resolution and (ii) for high trapped charges, the energy spread increases at a fast pace. The change of behaviour, corresponding theoretically to the optimal load case, occurs again around 20 pC.

Those two independent observations, represented in figures 10 and 11, both show that the optimal load of our accelerator is around 20 pC. This does not give further information on the accelerating structure since, as we have seen previously, the optimal load is not defined by charge but by longitudinal charge density or current. Unfortunately, there is currently no direct measurement of the bunch at the fs level. However, it is possible to use the experimental observations on beam loading to indirectly infer the bunch duration.

4.2. Indirect duration measurement

In linear theory, the beam loading field is intrinsically related to the bunch shape and duration through equation (1) and as the relation is linear with charge it is possible to write the dependence of the decelerating field per charge with bunch duration and diameter. This approach cannot be used in our conditions since we operate in the nonlinear regime. As the effect of beam loading in the linear and nonlinear cases is different [37] (a linear wake is much more affected by beam loading effects), accurate estimations of the bunch shape can only be made in the frame of the nonlinear theory.

A first rough estimate of the bunch duration relying on the optimal current given by the nonlinear theory ($\simeq 25$ kA) yields a bunch duration of approximately 1 fs, but this method is not reliable since it depends strongly on the bubble shape, that can only be known through complex simulations.

However, we can develop a simple model to determine the bunch duration, using the two independent observations of the decelerating field due to beam loading and the optimal load of
the accelerator. This model is based on the assumption that, in the unloaded case, the electron bunch is accelerated by a linearly varying wakefield: \( E_z = \alpha (\xi - \xi_{E_z=0}) \). This feature, related to the spherical structure of the wake, seems to be verified in our case (see figure 8). In the following calculations, we will choose the phase origin such as \( \xi_{E_z=0} = 0 \).

An electron located at the front of the bunch, initially at phase \( \xi_2 \), will gain

\[
\gamma_2 = \alpha \left( 2\xi_2 + \frac{l_{acc}}{2\gamma_p^2} \right) \frac{l_{acc}}{4\gamma_p^2},
\]

where \( l_{acc} \) is the acceleration length.

As the electron at the front is not influenced by beam loading effects it will always gain \( \gamma_2 \) regardless of the load of the accelerator.

The situation is different for a trailing electron, initially located at \( \xi_1 \). In the case where there is no beam loading it gains

\[
\gamma_1 = \alpha \left( 2\xi_1 + \frac{l_{acc}}{2\gamma_p^2} \right) \frac{l_{acc}}{4\gamma_p^2}.
\]

However, for an optimal load, the electric field is flat between the leading and the trailing electrons. Therefore \( \gamma'_1 \), the energy gain of the trailing electron in the optimal load case, will be the same as the energy gain of the leading electron \( \gamma_2 \).

The relative energy difference between the two cases can therefore be written as

\[
\frac{\gamma_1 - \gamma'_1}{\gamma_1} = -2 \frac{\Delta \xi}{2\xi_1 + l_{acc}/2\gamma_p^2}.
\]

For low injection energies, electrons are initially located at the rear of the bubble (\( \xi_1 \approx -\lambda_p/2 \)) and for acceleration lengths small compared to the dephasing length (\( l_{acc} < \gamma_p^2\lambda_p \approx 4 \text{ mm in our conditions} \)), this relation can be further simplified to

\[
\frac{\gamma_1 - \gamma'_1}{\gamma_1} = 2 \frac{\Delta \xi}{\lambda_p},
\]

where \( \gamma_1 \) is the energy of the trailing electron without beam loading (low charge bunch) and \( \gamma'_1 \) the energy of the same electron for the optimal load (flat \( E_z \)). When dephasing is considered, the bunch length can be expressed as \( \Delta \xi = (\frac{\gamma_2}{2} - \frac{l_{acc}}{4\gamma_p^2}) \frac{\gamma_2 - \gamma'_1}{\gamma_1} \).

This relation, extracted from a simple two-particle model, can be tested in the nonlinear model, for Gaussian bunches. Figure 12(b) shows the peak energy difference (after 300 \( \mu \text{m} \) acceleration) between the unloaded case and optimal case as a function of the FWHM bunch duration. The obvious linear correlation confirms the validity of the two-particle model estimate. It also gives a numerical relation, for Gaussian bunches, between the FWHM bunch length and the relative peak energy difference in the unloaded and optimally loaded cases

\[
\frac{\gamma_1 - \gamma'_1}{\gamma_1} = 1.6 \frac{\Delta \xi}{\lambda_p}.
\] (3)

In 3D PIC simulations, the unloaded case beam energy is 95 MeV while the beam energy in the optimal case is 65 MeV, and we know that half of this decrease is effectively due to beam
loading so that $\gamma_1 - \gamma_1' \simeq 15 \text{ MeV}$. Using relationship (3), a bunch duration of $\tau_{\text{bunch}} = 4 \text{ fs}$ can be inferred. This value is very close to the simulated value of $4.4 \text{ fs}$ (for $a_1 = 0.3$).

In the experiment, the 40 MeV energy difference between the low charge case ($E = 205 \text{ MeV}$) and the optimal load case ($E = 165 \text{ MeV}$) is also assumed to be half due to beam loading effects. In that case, $(\gamma_1 - \gamma_1')/\gamma_1 \simeq 10\%$, which gives, following (3), a bunch duration of $3 \text{ fs}$ (or $2 \text{ fs}$ when dephasing is taken into account ($l_{\text{acc}} = 1.35 \text{ mm}$)).

The optimal peak current will therefore be around $10 \text{ kA}$, which is reasonably close to the estimate of $25 \text{ kA}$ obtained with the nonlinear model.

4.3. Efficiency

Efficiency might become a key issue for future applications and we should rightfully wonder if a load of $20 \text{ pC}$, giving a disappointing laser-to-electron efficiency of $0.5\%$, is a fundamental limit of laser plasma accelerators. To answer this question we have to consider, on the one hand, the laser-to-plasma efficiency and, on the other hand, the plasma-to-electron efficiency. The first can be optimized by choosing an acceleration length close to the depletion length of the laser. In that case, according to Lu et al [6], laser-to-plasma efficiency can be as high as $50\%$ for $a_0 = 2$. The latter is exactly the problem of beam loading and has been addressed in [37, 38]. The optimal load is defined in terms of longitudinal density and not charge. In our experiment the electron beam is ultrashort, which limits the extraction of energy from the wake ($\simeq 10\%$). However, if we were able to load the wake continuously until the rear of the bubble, with the appropriate ramp profile, a high-charge, low-energy spread bunch could still be accelerated. In that case, a near $100\%$ plasma-to-electron efficiency could be achieved. For example, following the scaling given in [38], for our experimental conditions, $200 \text{ pC}$ could be accelerated while keeping the same final energy. Whether this tailored profile could be injected or not with an optical scheme is still an open question but preliminary results of simulations based on the cold injection concept [41] tend to show that some control of the profile is achievable.
4.4. Implications for future applications

The effect of beam loading is often overlooked in many works relying on the fluid approximation. We show here that this might not be legitimate since even for modest charges \( Q \approx 10 \) pC, and in a nonlinear regime, beam loading can have a drastic effect on the acceleration process and especially on the energy spread and beam energy. As high-current applications are the main prospect of laser-accelerated electron beams, beam loading is clearly the limiting factor for many experiments. For example, for the free electron laser in the x-ray range, very low energy spreads are required (typically less than 0.1%). Thus, it will be absolutely necessary to consider and optimize beam loading, which will also put a strong limitation on the current. Following Tzoufras et al \[37, 38\] the optimal current will scale as \( a_0 \). The fact that injection will need to be precisely controlled to reach low-energy spreads and therefore that \( a_0 \) will stay under the self-injection threshold (\( a_0 < 4 \)) might ultimately limit the current to tens of kiloamperes.

Those observations of beam loading also stress why the control of the load is important in a laser plasma accelerator. The simplified nonlinear model indeed shows that the charge has to be controlled with more than 10% accuracy to obtain an energy spread below 1%. In that respect, controlled injection schemes might have an important role to play in the further development and improvement of laser-plasma-based electron sources.

An energy spread of 1% has been obtained with an optical injection scheme \[25\], presumably by reducing the injection volume and optimizing the beam loading. Methods to further reduce the energy spread might be (i) operating at lower densities, (ii) optical injection in density down-ramps \[42\] or (iii) cold injection \[41\], to reduce the initial spread of the electron bunch.

5. Conclusion

In this paper, we have shown that optical injection based on colliding pulses permits us to control the injection volume and therefore the charge injected in a plasma wakefield. This control of the charge injected in a given accelerating structure is used to investigate beam loading. It is shown experimentally that, in our conditions, beam loading is the source of a decelerating field per charge of approximately \( 0.8 \) GV m\(^{-1}\) pC\(^{-1}\) and reduces the amount of charge trapped in the following buckets, thus improving the background current of our accelerator. It has been further shown, using indirect observations, that the optimal load of our accelerator, minimizing the energy spread, is around 20 pC. Finally, using simple arguments, these observations indicate that the bunch duration is ultrashort, of the order of 3 fs. All those observations are consistent with 3D PIC simulations and with the theoretical model of beam loading in the nonlinear regime.

Acknowledgments

One of the authors (CR) acknowledges useful discussions with Drs W Lu and M Tzoufras. We acknowledge the support of the European Community—New and Emerging Science and Technology Activity under the FP6 ‘Structuring the European Research Area’ programme (project EuroLEAP, contract number 028514), the European Research Council (project ‘PARIS’, grant number 226424), the French National Agency (ANR-05-NT05-2-41699, ‘ACCEL1’) and the RTRA through the project APPEAL.
References

[1] Tajima T and Dawson J M 1979 Phys. Rev. Lett. 43 267
[2] Katsouleas T, Wilks S, Chen P, Dawson J M and Su J J 1987 Particle Accelerators vol 22 (London: Gordon and Breach) p 81
[3] Pukhov A and Meyer-ter-Vehn J 2002 Appl. Phys. B 74 355–61
[4] Gordienko S and Pukhov A 2005 Phys. Plasmas 12 043109
[5] Lu W, Huang C, Zhou M, Mori W B and Katsouleas T 2006 Phys. Rev. Lett. 96 165002
[6] Lu W, Tzoufras M, Joshi C, Tsung F S, Mori W B, Vieira J, Fonseca R A and Silva L O 2007 Phys. Rev. ST Accel. Beams 10 061301
[7] Hafz N et al 2008 Nat. Photon. 2 571–7
[8] Geddes C G R, Tóth C, van Tilborg J, Esarey E, Schroeder C B, Bruhwiler D, Nieter C, Cary J and Leemans W P 2004 Nature 431 538–41
[9] Mangles S P D et al 2004 Nature 431 535–8
[10] Faure J, Glinec Y, Pukhov A, Kiselev S, Gordienko S, Lefebvre E, Rousseau J P, Burgy F and Malka V 2004 Nature 431 541–4
[11] Khachatryan A 2002 Phys. Rev. E 65 46504–4
[12] Geddes C G R, Nakamura K, Plateau G R, Tóth C, Cormier-Michel E, Esarey E, Schroeder C B, Cary J R and Leemans W P 2008 Phys. Rev. Lett. 100 215004
[13] Suk H, Barov N, Rosenzweig J B and Esarey E 2001 Phys. Rev. Lett. 86 1011
[14] Bulanov S, Naumova N, Pegoraro F and Sakai J 1998 Phys. Rev. E 58 R5257–60
[15] Pak A, Marsh K A, Martins S F, Lu W, Mori W B and Joshi C 2010 Phys. Rev. Lett. 104 025003
[16] McGuffey C, Thomas A G R, Schumaker W, Matsuoka T, Chvykov V, Dollar F J, Kalintchenko G, Yanovsky V, Maksimchuk A and Krushelnick K 2010 Phys. Rev. Lett. 104 025004
[17] Rowlands-Rees T P et al 2008 Phys. Rev. Lett. 100 105005
[18] Oz E et al 2007 Phys. Rev. Lett. 98 084801
[19] Umstadter D, Kim J K and Dodd E 1996 Phys. Rev. Lett. 76 2073
[20] Esarey E, Hubbard R F, Leemans W P, Ting A and Sprangle P 1997 Phys. Rev. Lett. 79 2682
[21] Kotaki H, Masuda S, Kando M, Koga J and Nakajima K 2004 Phys. Plasmas 11 3596
[22] Schroeder C B, Lee P B, Wurtele J S, Esarey E and Leemans W P 1999 Phys. Rev. E 59 6037
[23] Faure J, Rechatin C, Norlin A, Lifschitz A, Glinec Y and Malka V 2006 Nature 444 737–9
[24] Mangles S P D, Thomas A G R, Lundh O, Lindau F, Kaluza M C, Persson A, Wahlström C G, Krushelnick K and Najmudin Z 2007 Phys. Plasmas 14 056702
[25] Rechatin C, Faure J, Ben-Ismail A, Lim J, Fitour R, Specka A, Videau H, Tafzi A, Burgy F and Malka V 2009 Phys. Rev. Lett. 102 164801
[26] Rechatin C, Davoine X, Lifschitz A, Ismail A B, Lim J, Lefebvre E, Faure J and Malka V 2009 Phys. Plasmas 16 056702
[27] Kotozaki H et al 2008 IEEE Trans. Plasma Sci. 36 1760–4
[28] Glinec Y, Faure J, Guenmin-Tafao A, Monard V M H, Larbre J P, Waele V D, Marignier J L and Mostafavi M 2006 Rev. Sci. Instrum. 77 103301
[29] Sheng Z M, Mima K, Sentoku Y, Jovanović M S, Taguchi T, Zhang J and ter Vehn J M 2002 Phys. Rev. Lett. 88 055004
[30] Mendonça J T 1983 Phys. Rev. A 28 3592
[31] Malka V, Faure J, Rechatin C, Ben-Ismail A, Lim J K, Davoine X and Lefebvre E 2009 Phys. Plasmas 16 056703
[32] Rechatin C, Faure J, Lifschitz A, Davoine X, Lefebvre E and Malka V 2009 New J. Phys. 11 013011
[33] Rechatin C, Faure J, Lifschitz A, Malka V and Lefebvre E 2007 Phys. Plasmas 14 060702
[34] Esarey E, Sprangle P, Krall J and Ting A 1996 IEEE Trans. Plasma Sci. 24 252–88
[35] Wilks S C, Katsouleas T, Dawson J M, Chen P and Su J 1987 IEEE Trans. Plasma Sci. 15 210–7
[36] Davoine X, Lefebvre E, Faure J, Rechatin C, Lifschitz A and Malka V 2008 Phys. Plasmas 15 113102
[37] Tzoufras M, Lu W, Tsung F S, Huang C, Mori W B, Katsouleas T, Vieira J, Fonseca R A and Silva L O 2009 Phys. Plasmas 16 056705
[38] Tzoufras M, Lu W, Tsung F S, Huang C, Mori W B, Katsouleas T, Vieira J, Fonseca R A and Silva L O 2008 Phys. Rev. Lett. 101 145002
[39] Lefebvre E et al 2003 Nucl. Fusion 43 629–33
[40] Faure J, Rechatin C, Ben-Ismail A, Lim J, Davoine X, Lefebvre E and Malka V 2009 C. R. Phys. 10 148–158
[41] Davoine X, Lefebvre E, Rechatin C, Faure J and Malka V 2009 Phys. Rev. Lett. 102 065001
[42] Fubiani G, Esarey E, Schroeder C B and Leemans W P 2006 Phys. Rev. E 73 026402