Probing the cosmological viability of non-gaussian statistics

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Received May 27, 2016
Revised August 9, 2016
Accepted August 15, 2016
Published August 23, 2016

Abstract. Based on the relationship between thermodynamics and gravity we propose, with the aid of Verlinde’s formalism, an alternative interpretation of the dynamical evolution of the Friedmann-Robertson-Walker Universe. This description takes into account the entropy and temperature intrinsic to the horizon of the universe due to the information holographically stored there through non-gaussian statistical theories proposed by Tsallis and Kaniadakis. The effect of these non-gaussian statistics in the cosmological context is to change the strength of the gravitational constant. In this paper, we consider the $\omega$CDM model modified by the non-gaussian statistics and investigate the compatibility of these non-gaussian modification with the cosmological observations. In order to analyze in which extend the cosmological data constrain these non-extensive statistics, we will use type Ia supernovae, baryon acoustic oscillations, Hubble expansion rate function and the linear growth of matter density perturbations data. We show that Tsallis’ statistics is favored at $1\sigma$ confidence level.

Keywords: dark energy theory, dark matter theory

ArXiv ePrint: 1509.05059v2

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1 Introduction

There are theoretical evidences that the understanding of gravity has been greatly benefited from a possible connection to thermodynamics. Pioneering works of Bekenstein [1] and Hawking [2] have described this issue. For example, quantities as area and mass of black holes are associated with entropy and temperature respectively. Working on this subject, Jacobson [3] interpreted Einstein field equations as a thermodynamic identity. Padmanabhan [4] gave an interpretation of gravity as an equipartition theorem. Recently, Verlinde [5, 6] brought an heuristic derivation of gravity, both Newtonian and relativistic, at least for static spacetime. The equipartition law of energy has also played an important role. The analysis of the dynamics of an inflationary universe ruled out by the entropic gravity concept was investigated in [7, 8]. On the other hand, one can ask: what is the point of view of the gravitational models coupled with thermostatistical theories and vice-versa?

The concept introduced by Verlinde is analogous to Jacobson’s [3] one, who proposed a thermodynamic derivation of Einstein’s equations. The result has shown that the gravitation law derived by Newton can be interpreted as an entropic force originated from perturbations in the information “manifold” caused by the motion of a massive body when it moves away from the holographic screen. An holographic screen can be understood as a storage device for information which is constituted by bits. Bits are the smallest units of information. Verlinde has used this idea together with the Unruh result [9] and consequently he obtained Newton’s second law. The idea of an entropic gravity/cosmology has been extensively investigated in different contexts. See [10–19] for recent results.

Moreover, assuming the holographic principle together with the equipartition law of energy, the Newton law of gravitation could be derived. The connection between nonextensive (NE) statistical theory and the entropic gravity models [23, 24] makes us to realize...
an arguably bridge between nonextensivity and gravity theories. In this paper we propose an alternative interpretation of the dynamical evolution of the Friedmann-Robertson-Walker Universe (FRW) through non-gaussian statistical theories. We have used the most recent observational data of Supernovae of Type Ia (SN Ia), Baryon Acoustic Oscillation (BAO), Hubble parameter and the growth function to investigate the cosmological consequences of such modifications through dark energy (DE) models.

This paper is organized as follows. In section II we will make a brief review of the formulations concerning both non-gaussian Tsallis’ and Kaniadakis’ statistics. In section III we will present the formalism of Verlinde and its consequences for the gravitational framework. In section VI we will introduce the modified dynamic FRW universe. In section V we will use SN Ia, BAO, \( H(z) \) and \( f(z) \) data to constrain the non-gaussian statistics modifications on the \( w_{\text{CDM}} \) modified model. Lastly, section VI briefly delivers our main conclusions and offers some final remarks. As usual, a zero subscript means the present value of the corresponding quantity.

2 Non-gaussian statistics

The objective of this section is to provide the reader with the main tools that will be used in the next sections. Although both formalisms are well known in the literature, these brief reviews can emphasize precisely that there is a connection between both ideas and the one that was established recently [23]. The study of entropy has been an interesting task through recent years thanks to the fact that it can be understood as a measure of information loss concerning the microscopic degrees of freedom of a physical system, when describing it in terms of macroscopic variables. Appearing in different scenarios, we can conclude that entropy can be considered as a consequence of the gravitational framework [1, 2]. These issues motivated some of us to consider other alternatives to the standard Boltzmann-Gibbs (BG) theory to work with Verlinde’s ideas together with other subjects [24].

2.1 Tsallis’ statistics

An important formulation of a NE BG thermostatistics has been proposed by Tsallis [25] in which the entropy is given by the formulation

\[
S_q = k_B \left( 1 - \frac{\sum_{i=1}^{W} p_i^q}{q - 1} \right), \quad \left( \sum_{i=1}^{W} p_i = 1 \right), \tag{2.1}
\]

where \( p_i \) is the probability of the system to be in a microstate, \( W \) is the total number of configurations and \( q \), known in the current literature as Tsallis’s parameter or the NE parameter, is a real parameter\(^1\) quantifying the degree of nonextensivity. The definition of entropy (2.1) has, as motivation, to analyze multifractal systems and it also has the usual properties of positivity, equiprobability, concavity and irreversibility. It is important to note that Tsallis’s formalism contains the BG statistics as a particular case in the limit \( q \to 1 \) where the usual additivity of entropy is recovered. Plastino and Lima [26] used a generalized velocity distribution for free particles [27]

\[
f_q(v) = B_q \left[ 1 - (1 - q) \frac{mv^2}{2k_B T} \right]^{1-\frac{1}{q}}, \tag{2.2}
\]

\(^1\)Some authors have shown that the Tsallis’s parameter not is necessarily real [20–22].
where $B_q$ is a dependent normalization constant, $m$ and $v$ are the mass and velocity of the particle, respectively. They have derived a NE equipartition law of energy whose expression is given by
\[
E_q = \frac{1}{5 - 3q} N k_B T, \tag{2.3}
\]
where the range of $q$ is $0 \leq q < 5/3$. For $q = 5/3$ (critical value) the expression of the equipartition law of energy, eq. (2.3), diverges. It is easy to observe that for $q = 1$, the classical equipartition theorem for each microscopic degrees of freedom can be recovered.

### 2.2 Kaniadakis’ statistics

Kaniadakis’ statistics [28], also called $\kappa$-statistics, similarly to the Tsallis formalism, generalizes the standard BG statistics initially by the introduction of $\kappa$-exponential and $\kappa$-logarithm defined by
\[
\exp_\kappa(f) = \left(\sqrt{1 + \kappa^2 f^2} + \kappa f\right)^{1/\kappa}, \tag{2.4}
\]
\[
\ln_\kappa(f) = \frac{f^\kappa - f^{-\kappa}}{2\kappa}, \tag{2.5}
\]
where the following operation is satisfied
\[
\ln_\kappa(\exp_\kappa(f)) = \exp_\kappa(\ln_\kappa(f)) = f. \tag{2.6}
\]

By eqs. (2.4) and (2.5) we can observe that the $\kappa$-parameter deforms the usual definitions of both exponential and logarithm functions.

The $\kappa$-entropy associated with this $\kappa$-framework is given by
\[
S_\kappa(f) = -\int d^3p f^{\kappa} \frac{f^\kappa - f^{-\kappa}}{2\kappa}, \tag{2.7}
\]
which recovers the BG entropy in the limit $\kappa \to 0$. It is important to mention here that the $\kappa$-entropy satisfied the concavity, additivity and extensivity properties. The Tsallis entropy satisfies the property of concavity and extensivity but not additivity. This property is not fundamental, in principle. The $\kappa$-statistics has been successfully applied in many experimental fronts. As examples we can mention cosmic rays [29, 30], quark-gluon plasma [31], kinetic models describing a gas of interacting atoms and photons [32] and financial models [33].

The kinetic foundations of $\kappa$-statistics lead to a velocity distribution for free particles given by [34]
\[
f_\kappa(v) = \left(\sqrt{1 + \kappa^2 \frac{mv^2}{2k_B T}} - \kappa \frac{mv^2}{2k_B T}\right)^{1/\kappa}. \tag{2.8}
\]

The expectation value of $v^2$ is given by
\[
\langle v^2 \rangle_\kappa = \frac{\int_0^\infty f_\kappa v^2 dv}{\int_0^\infty f_\kappa dv}. \tag{2.9}
\]

Using the integral relation [35]
\[
\int_0^\infty dx x^{r-1} \exp_\kappa(-x) = \frac{2|\kappa|^{-r}}{1+r} \Gamma\left(\frac{1}{2|\kappa|} - \frac{r}{2}\right) \Gamma\left(\frac{1}{2|\kappa|} + \frac{r}{2}\right), \tag{2.10}
\]

we have that
\[ <v^2> \kappa = \frac{2k_B T}{m} \frac{1 + \frac{1}{2} \kappa \Gamma \left( \frac{1}{2\kappa} - \frac{3}{4} \right) \Gamma \left( \frac{1}{2\kappa} + \frac{1}{4} \right)}{1 + \frac{1}{2} \kappa \Gamma \left( \frac{1}{2\kappa} + \frac{3}{4} \right) \Gamma \left( \frac{1}{2\kappa} - \frac{1}{4} \right)}, \]  
\tag{2.11}

The \( \kappa \)-equipartition theorem is then obtained as
\[ E_\kappa = \frac{1}{2} N \frac{1 + \frac{1}{2} \kappa \Gamma \left( \frac{1}{2\kappa} - \frac{3}{4} \right) \Gamma \left( \frac{1}{2\kappa} + \frac{1}{4} \right)}{1 + \frac{1}{2} \kappa \Gamma \left( \frac{1}{2\kappa} + \frac{3}{4} \right) \Gamma \left( \frac{1}{2\kappa} - \frac{1}{4} \right)} k_B T. \]  
\tag{2.12}

The range of \( \kappa \) is \( 0 \leq \kappa < 2/3 \). For \( \kappa = 2/3 \) (critical value) the expression of the equipartition law of energy, eq. (2.12), diverges. For \( \kappa = 0 \), the classical equipartition theorem for each microscopic degrees of freedom can be recovered.

3 Verlinde’s formalism and the modified gravitational constant

The formalism proposed by E. Verlinde [5, 6] derives the gravitational acceleration by using, basically, the holographic principle and the equipartition law of energy. This model considers a spherical surface as the holographic screen, with a particle of mass \( M \) positioned in its center. A holographic screen can be imagined as a storage device for information. The number of bits (the term bit means the smallest unit of information in the holographic screen) is assumed to be proportional to the area \( A \) of the holographic screen
\[ N = \frac{A}{l_p^2}, \]  
\tag{3.1}

where \( A = 4\pi r^2 \) and \( l_p = \sqrt{\frac{G\hbar}{c^3}} \). In Verlinde’s formalism we assume that the total energy of the bits on the screen is given by the equipartition law of energy
\[ E = \frac{1}{2} N k_B T. \]  
\tag{3.2}

It is important to mention here that the usual equipartition theorem, eq. (3.2), is derived from the usual BG thermostatistics. In a NE thermostastistics scenario, the equipartition law of energy will be modified in a sense that a NE parameter \( q \) will be introduced in its expression. Considering that the energy of the particle inside the holographic screen is equally divided through all bits, then we can write the equation
\[ M c^2 = \frac{1}{2} N k_B T. \]  
\tag{3.3}

Using eq. (3.1) and the Unruh temperature formula [9]
\[ k_B T = \frac{1}{2\pi} \frac{\hbar a}{c}, \]  
\tag{3.4}

we are in a position to derive the (absolute) gravitational acceleration formula
\[ a = \frac{l_p^2 c^3 M}{\hbar r^2} = G \frac{M}{r^2}. \]  
\tag{3.5}

We can observe that from eq. (3.5) the Newton constant \( G \) is just written in terms of the fundamental constants, \( G = l_p^2 c^3 / \hbar. \)
As an application of NE equipartition theorem in Verlinde’s formalism we can use the NE equipartition formula, i.e., eq. (2.3). Hence, we can obtain a modified acceleration formula given by \[24\]
\[
a = G_q \frac{M}{r^2}, \tag{3.6}
\]
where \(G_q\) is an effective gravitational constant which is written as \[
G_q = \frac{5 - 3q}{2} G. \tag{3.7}
\]

From result (3.7) we can observe that the effective gravitational constant depends on the NE parameter \(q\). For example, when \(q = 1\) we have \(G_q = G\) (BG scenario) and for \(q = 5/3\) we have the curious and hypothetical result which is \(G_q = 0\). This result shows us that \(q = 5/3\) is an upper bound limit when we are dealing with the holographic screen. Notice that this approach is different from the one demonstrated in \[36\], where the authors considered in their model that the number of states is proportional to the volume and not to the area of the holographic screen.

On the other hand, if we use the Kaniadakis equipartition theorem, eq. (2.12), in Verlinde’s formalism, the modified acceleration formula is given by \[
a = G_\kappa \frac{M}{r^2}, \tag{3.8}
\]
where \(G_\kappa\) is an effective gravitational constant which is written as \[
G_\kappa = 2\kappa \left( 1 + \frac{3}{2} \kappa \Gamma \left( \frac{1}{2\kappa} + \frac{3}{4} \right) \Gamma \left( \frac{1}{2\kappa} - \frac{1}{4} \right) \right) \Gamma \left( \frac{1}{2\kappa} + \frac{1}{4} \right) G. \tag{3.9}
\]

From result (3.9) we can observe that the effective gravitational constant depends on the \(\kappa\) parameter. For example, from the Gamma functions properties we have that \[
\lim_{\kappa \to 0} \frac{\Gamma \left( \frac{1}{2\kappa} - \frac{3}{4} \right) \Gamma \left( \frac{1}{2\kappa} + \frac{1}{4} \right)}{\Gamma \left( \frac{1}{2\kappa} + \frac{3}{4} \right) \Gamma \left( \frac{1}{2\kappa} - \frac{1}{4} \right)} = 2\kappa. \tag{3.10}
\]

Then, using eq. (3.9) we obtain for \(\kappa = 0\) that \(G_\kappa = G\) (BG scenario).

4 Dark energy models through non-gaussian statistics

It was demonstrated in \[24\] that one modification in the dynamics of the FRW universe in NE Tsallis’ statistics can be obtained simply by making the prescription \(G \to G_q = (5 - 3q)G/2\) in the standard field equations. From this proposal in \[19\] new cosmological constraints on the parameter \(q\) were obtained. Analogously to Tsallis statistics, we can modify the Friedmann equation in the Kaniadakis framework by making the prescription \(G \to G_\kappa\), where \(G_\kappa\) is given by (3.9). Thus, for a homogeneous and isotropic universe filled with perfect fluids, the equations of motion for non-gaussian statistics can be written as \[
H^2 + \frac{k}{a^2} = \frac{8\pi}{3} G_q(\kappa)\rho, \tag{4.1}
\]
and \[
\dot{H} + H^2 = -\frac{4\pi}{3} G_q(\kappa)(\rho + 3p), \tag{4.2}
\]
where \( H = \dot{a}/a \) is the Hubble function, \( G_{q(\kappa)} \) denotes the effective gravitational constant in the Tsallis (Kaniadakis) formalism where \( \rho \) and \( p \) are, respectively, the total density and pressure of the fluid. These equations can be combined to obtain the conservation equation,

\[
\dot{\rho} + 3H(1 + w)\rho = 0. \tag{4.3}
\]

A FRW universe is pervaded by radiation, non-relativistic matter (baryonic plus dark matter) and some sort of dark energy. To take into account the late time cosmic acceleration, the Friedmann equation (4.1) becomes

\[
\frac{H^2(a)}{H_0^2} = \frac{G_{q(\kappa)}}{G} \left( \Omega_{r,0}a^{-4} + \Omega_{m,0}a^{-3} + \Omega_{k,0}a^{-2} + \Omega_{x,0}f(a) \right), \tag{4.4}
\]

where \( \Omega_{i,0} = 8\pi G \rho_{i,0}/(3H_0^2) \) is the density parameter of the \( i \)-th component (\( i = r, m, \) and \( x \) for radiation, matter (baryonic more dark) and dark energy, respectively), \( \Omega_{k,0} = -k/H_0^2 \) is the curvature density parameter and

\[
f(a) = \frac{\rho_x}{\rho_{x,0}} = a^{-3} \exp \left( -3 \int_1^a \frac{w(a') da'}{a'} \right), \tag{4.5}
\]

is the density ratio for a dark energy fluid with a generic equation of state parameter \( w(a) \equiv p_x/\rho_x \). In the above equations the subscript 0 denotes the observable at present time.

In the non-gaussian scenario, the normalization condition reads as

\[
\Omega_{r,0} + \Omega_{m,0} + \Omega_{k,0} + \Omega_{x,0} = \frac{G}{G_{q(\kappa)}}, \tag{4.6}
\]

which is an interesting result since it can show us, one more time, that the value \( q = 1 \) (\( \kappa = 0 \)) recovers the standard normalization condition. Values such as \( q > 5/3 \) and \( \kappa < 0 \), which bring a negative normalization condition make no sense.

In this paper we have assumed spatial flatness and we will restrict ourselves to the case \( w = \text{const} \) so that eq. (4.4) becomes

\[
\frac{H^2(a)}{H_0^2} = \frac{G_{q(\kappa)}}{G} \left[ \Omega_{r,0}a^{-4} + \Omega_{m,0}a^{-3} + \left( \frac{G}{G_{q(\kappa)}} - \Omega_{r,0} - \Omega_{m,0} \right) (1 + z)^3(1+w) \right]. \tag{4.7}
\]

From eqs. (3.7), (3.9) and (4.6) it is possible to note that the parameters \( q \) and \( \kappa \) affects the energy balance of the universe. If \( q \) is greater (smaller) than one, the effective gravitational constant is smaller (grater) than \( G \) so that more (less) dark matter will be required to provide the observed late time universe acceleration. By its turn, the Kaniadakis framework, if \( \kappa > 0 \), the gravitational field is weaker than in the gravitational field in the standard BG scenario so that we will need more dark matter to accommodate the cosmic acceleration. Since \( \kappa \geq 0 \), the Kaniadakis statistics is more restrictive than Tsallis’ statistics.

In the next section, we will use some of the available cosmological observations to obtain new constraints on the non-gaussian statistical parameters \( q \) and \( \kappa \).

## 5 Observational constraints

In order to constrain the parameters \( (q, \kappa \text{ and } w) \) we perform a joint analysis involving the 580 SNe Ia distance measurements of the Union 2.1 data set [37], the 30 measurements of the Hubble parameter \( H(z) \) given in table 4 of ref. [39], 17 measurements of the growth function \( f(z) \) listed in table 5.2 and the six estimates of the BAO parameter given in table 3 of ref. [40].
5.1 Type Ia Supernovae

The SNe Ia sample of the Union 2.1 is given in terms of distance modulus $\mu$. Theoretically, the distance modulus is given by

$$\mu_{th}(z_i) = 5 \log H_0 d_L(z_i) + 5 \log(3/h) + 40,$$

(5.1)

where $h = H_0/100 \text{Km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ and

$$d_L(z) = (1 + z) \int_0^z \frac{dz'}{H(z')} ,$$

(5.2)

is the luminosity distance for a spatially flat universe. The usual $\chi^2$ function is calculated as

$$\chi^2_{SN} = \sum_i \frac{(\mu_{th} - \mu_{i}^{obs})^2}{\sigma_{\mu_i}^2} ,$$

(5.3)

where $\mu_{i}^{obs}$ is the observed value of the distance modulus at redshift $z_i$ and $\sigma_{\mu_i}$ its uncertainty. In our analysis we treat the Hubble constant $H_0$ as a nuisance parameter and marginalize over $H_0$ so that the $\chi^2$ function to be minimized is

$$\chi^2_{SN} = A - \frac{B^2}{C} ,$$

(5.4)

where the quantities $A$, $B$, $C$ are given by:

$$A = (\mu_{i}^{th} - \mu_{i}^{obs}) (C_{SN}^{-1})_{ij} (\mu_{j}^{th} - \mu_{j}^{obs}) ,$$

(5.5)

$$B = \sum_i (C_{SN}^{-1})_{ij} (\mu_{j}^{th} - \mu_{j}^{obs}) ,$$

(5.6)

and

$$C = \sum_{ij} (C_{SN}^{-1})_{ij} ,$$

(5.7)

and $(C_{SN}^{-1})_{ij}$ is the inverse covariance matrix.

5.2 Growth function

In the linear regime the matter density perturbations $\delta = \delta \rho_m/\rho_m$ satisfy

$$\ddot{\delta} + 2H \dot{\delta} - 4\pi G_{\text{eff}} \rho_m \delta = 0 ,$$

(5.8)

where $G_{\text{eff}}$ is the effective gravitational constant for a given theory of gravity. For the models studied in this paper, $G_{\text{eff}}$ is given by (3.7) for Tsallis’ statistics and by (3.9) for Kaniadakis’ statistics. By defining the growth factor $f \equiv d \ln \delta/\ln a$, this second order time differential equation is reduced to

$$f'' + f^2 + \left(2 - \frac{3}{2} \Omega_m\right) f - \frac{3}{2} \Omega_m^{\text{eff}} = 0 ,$$

(5.9)

where $f' = df/da$ and

$$\Omega_m^{\text{eff}} = \frac{8\pi G_{\text{eff}} \rho_m}{3H^2} = \frac{\Omega_{m,0} a^{-3}}{\Omega_{r,0} a^{-4} + \Omega_{m,0} a^{-3} + \left(\frac{G}{r_{Q}(c)} - \Omega_{r,0} - \Omega_{m,0}\right) a^{-3(1+w)}} ,$$

(5.10)
Table 1. Currently available data for growth rates used here.

| \(z\) | \(f\)    | \(\sigma_f\) | ref. |
|-------|---------|--------------|------|
| 0.02  | 0.482   | 0.09         | 43   |
| 0.067 | 0.56    | 0.11         | 44   |
| 0.11  | 0.54    | 0.21         | 45   |
| 0.15  | 0.49    | 0.14         | 46   |
| 0.15  | 0.51    | 0.11         | 47, 48 |
| 0.22  | 0.60    | 0.10         | 49   |
| 0.32  | 0.654   | 0.18         | 50   |
| 0.34  | 0.64    | 0.09         | 51   |
| 0.35  | 0.70    | 0.18         | 52   |
| 0.41  | 0.70    | 0.07         | 49   |
| 0.42  | 0.73    | 0.09         | 53   |
| 0.55  | 0.75    | 0.18         | 54   |
| 0.59  | 0.75    | 0.09         | 53   |
| 0.60  | 0.73    | 0.07         | 49   |
| 0.77  | 0.91    | 0.36         | 46   |
| 0.78  | 0.70    | 0.08         | 49   |
| 1.4   | 0.90    | 0.24         | 55   |
| 2.125 | 0.78    | 0.24         | 56   |
| 2.72  | 0.78    | 0.24         | 56   |
| 3.0   | 0.99    | 0.24         | 45   |

In table 5.2 we have listed the 20 measurements of \(f\). The usual \(\chi^2\) function is calculated as

\[
\chi_f^2 = \sum_{i=1}^{20} \frac{(f_{\text{obs}}^i - f_{\text{theo}}^i)^2}{\sigma_f^2^i}, \tag{5.11}
\]

where \(f_{\text{obs}}^i\) is the observed value of the growth function at redshift \(z_i\), \(\sigma_f^i\) its uncertainty and \(f_{\text{theo}}^i\) the value of \(f(z_i)\) provided theoretically. In order to obtain \(f_{\text{theo}}^i\) we have used the approximation \(f(z) \approx \left[\Omega_{\text{eff}}^m(z)\right]^\gamma\) [42], where \(\gamma\) is the growth index which depends on the underlying cosmological model. For the \(w\)CDM model, \(\gamma = 3(w-1)/(6w-5)\).

### 5.3 Hubble parameter

In ref. [57], Jimenez has developed a method to use the relative age of old and passive galaxies, \(dz/dt\), to infer the Hubble parameter as a function of the redshift,

\[
H(z) = \frac{\dot{a}}{a} = -\frac{1}{1+z} \frac{dz}{dt}. \tag{5.12}
\]

The cosmic chronometers method to measure \(H(z)\) does not depend on any integrated distance measurement over redshifts and is independent of cosmological models. We have used 30 \(H(z)\) data obtained from the cosmic chronometers method listed in table 4 of [39]. The usual \(\chi^2\) function is calculated as

\[
\chi_H^2 = \sum_{i=1}^{30} \frac{(H_{\text{obs}}^i - H_{\text{theo}}^i)^2}{\sigma_H^2^i}, \tag{5.13}
\]

where \(H_{\text{obs}}^i\) is the observed value of the Hubble parameter at redshift \(z_i\), \(\sigma_H^i\) its uncertainty and \(H_{\text{theo}}^i\) the value of the Hubble parameter at \(z_i\) provided theoretically.
Our BAO analysis is based on the BAO parameter:

\[
A(z) = D_V(z) \sqrt{\Omega_{m,0}^{(k)} H_0^2},
\]

where

\[
D_V(z) = \left[ \frac{1}{H(z)} \left( \frac{1}{z} \int_0^z \frac{dz'}{H(z')} \right)^2 \right]^{1/3},
\]

is the so-called dilation scale and \(\Omega_{m,0}^{(k)} = G_{q(k)0} / G\) is the matter density modified to take
into account the effects of the non-gaussian statistics. Here we have used the six estimates
of the BAO parameter given in table 3 of ref. [40]. The usual \(\chi^2\) function is calculated as

\[
\chi^2_{\text{BAO}} = \sum_{i=1}^{6} \frac{(A_i^{\text{obs}} - A_i^{\text{theo}})^2}{\sigma_{A_i}^2},
\]

where \(A_i^{\text{obs}}\) is the observed value of the BAO parameter at redshift \(z_i\), \(\sigma_{A_i}\) its uncertainty
and \(A_i^{\text{theo}}\) the theoretical value of the BAO parameter at redshift \(z_i\).

As the likelihood function is defined by \(L \propto \exp(-\chi^2/2)\) the said values follow after we
minimize the quantity

\[
\chi^2_{\text{total}} = \chi^2_{\text{SNIa}} + \chi^2_{H} + \chi^2_{f} + \chi^2_{\text{BAO}}.
\]

In order to break the parameters degeneracy, we have used the Planck collaboration
results [41] to fix \(y_{m,0}^{(k)} h^2\) at 0.1415 and left \(\Omega_{m,0}\) free. Given that the contribution of radiation
is negligible in the redshift range covered by the data used in our analysis, we set \(\Omega_{r,0} = 0\).

5.5 Results

Figure 1 shows the 68% and 95% confidence regions in the \(w - q\) plane, in the \(w - \kappa\) plane
and in the \(w - \Omega_{m,0}\) plane for the modified \(w\)CDM model. The left panels stands for Tsallis’
statistics and the right panels stands for Kaniadakis’ statistics. At 2σ C. L, the best fit points are:
\(q = 0.907^{+0.093}_{-0.137}, w = -1.021^{+0.089}_{-0.099}, \Omega_{m,0} = 0.270^{+0.038}_{-0.041}\) with \(\chi^2_{\text{min}} = 590.55\) for Tsallis’
statistics and \(\kappa = 0.026^{+0.146}_{-0.026}, w = -0.995^{+0.085}_{-0.091}, \Omega_{m,0} = 0.308^{+0.018}_{-0.009}\) with \(\chi^2_{\text{min}} = 594.40\)
Kaniadakis’ statistics. Quintessential \((w > -1)\) and phantom \((w < -1)\) fluids are compatible
with both, Tsallis’ and Kaniadakis’ statistics. Tsallis’ statistics provides much better fit to
the data than Kaniadakis’ statistics. As we can see, Tsallis’ statistics is favored at 1σ C. L
while Kaniadakis’ statistics is entirely compatible with the standard BG statistics at 1σ C. L.
However, it is important to note that, at 2σ C. L, these data do not exclude the BG statistics,
\(\kappa = 1\) and \(\kappa = 0\). For Tsallis’ statistics \(0 \leq G_{\text{eff}} / G \leq 2.5\) while for Kaniadakis’ statistics
\(0 \leq G_{\text{eff}} / G \leq 1.\) Thus, we can attribute this result to the additional freedom allowed by
Tsallis’ statistics since the data seems to choose a slightly greater gravitational constant.
Since the gravitational field is ever weaker than in the Kaniadakis scenario, we need more
dark matter considering Kaniadakis’ statistics than for Tsallis’ statistics.

6 Conclusion

Currently, two extensions of standard statistical mechanics, known as Kaniadakis’ and Tsallis’
statistics, have been used to explain a very large class of phenomena observed experimentally
in different areas, e.g., in low and high energy physics, astrophysics, econophysics, biology, etc.
In this paper, we have explored the possibility of one modification in the dynamics of the FRW universe obtained through an entropic force theory generalized for the Kaniadakis and Tsallis statistics.

From the combination SNIa+BAO+$H(z)+f(z)$ datasets we have obtained new cosmological constraints over $q$ and $\kappa$ parameters.

To sum up, in this paper, we have obtained new values for the free parameters that characterizes the non-gaussian statistical theory proposed by Kaniadakis and Tsallis. Based on the data used in this paper, we have noted that non-gaussian statistics can not be ruled out by cosmological observations although the BG statistics remains in fully agreement with the data. From the results obtained here we can conclude that the parameters $q$ and $\kappa$ affects the energy balance of the universe. The gravitational field is weaker for $q > 1$ and $\kappa > 0$ so that we need more dark matter than the amount we would have if we consider the standard BG scenario. For $0 \leq q < 1$, the strength of the gravitational field is greater than the one inside the standard BG scenario. Besides, less dark matter is required to explain the cosmological observations. The results obtained in this paper favor the Tsallis' statistics and a stronger gravitational field at $1\sigma$ C.L.
Acknowledgments

The authors thank CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico), Brazilian scientific support federal agency, for partial financial support, Grants numbers 302155/2015-5, 302156/2015-1 and 442369/2014-0 and E.M.C.A. thanks the hospitality of Theoretical Physics Department at Federal University of Rio de Janeiro (UFRJ), where part of this work was carried out.

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