A General Supergravity Formalism for a Naturally Flat Inflaton Potential *

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January 5, 2018

Abstract

A globally supersymmetric model of inflation will not work in a generic supergravity theory because the higher order, nonrenormalisable supergravity corrections destroy the flatness of the inflaton’s potential. In this talk I derive a form for the Kähler potential which eliminates these corrections if $W = W_\varphi = \psi = 0$ during inflation (where $W$ is the superpotential, the inflaton $\in \varphi$, and $W_\psi \neq 0$). I then point out that Kähler potentials of the required form often occur in superstrings and that the target space duality symmetries of superstrings often contain $R$-parities which would make $W = W_\varphi = 0$ automatic for $\psi = 0$.

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*Talk given at the Seventh Marcel Grossmann Meeting on General Relativity, Stanford University, 24-29 July 1994, and based on Ref. 1.
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1 Introduction

The approximate isotropy of the cosmic microwave background radiation implies that the inflation that inflated the observable universe beyond the Hubble radius must have occurred at an energy scale $V^{1/4} \leq 4 \times 10^{16}$ GeV \cite{2}. Thus models of inflation should be constructed in the context of supergravity \cite{3}.

However, this immediately leads to a problem. The positive potential energy $V > 0$ required for inflation spontaneously breaks supersymmetry. This would generally be expected to give an effective mass squared $m^2_{\text{soft}} \sim 8\pi V/m^2_{\text{Pl}} \sim H^2$ to any would-be inflaton. But the effective mass of the inflaton must be much less than the inflationary Hubble parameter $H$.

In this talk I will present a general formalism that solves this problem.

1.1 Notation and basic formulae

I set $m_{\text{Pl}}/\sqrt{8\pi} = 1$ throughout. $\phi$ will represent a vector whose components $\phi^\alpha$ are complex scalar fields, and subscript $\phi$ will denote the derivative with respect to $\phi$, so for example $W_\phi$ represents the vector with components $\partial W/\partial \phi^\alpha$.

The scalar potential in a globally supersymmetric theory is

$$V = |W_\phi|^2 + D-\text{term},$$

where the superpotential $W(\phi)$ is an analytic function of $\phi$. The first term is called the $F$-term. For simplicity I will ignore the $D$-term in this talk.

The $F$-term part of the scalar potential in a supergravity theory is

$$V = e^K \left[(W_\phi + WK_\phi) K^{-1}_{\phi\phi} \left(\bar{W}_\phi + \bar{W} K_{\phi}\right) - 3|W|^2\right],$$

where the Kähler potential $K(\phi, \bar{\phi})$ is a real function of $\phi$ and its hermitian conjugate $\bar{\phi}$.

2 The Problem

At any point in the space of scalar fields $\phi$ we can make a combination of a Kähler transformation and a holomorphic field redefinition such that $\phi = 0$ at that point and, in the neighbourhood of that point, the Kähler potential takes the form

$$K = |\phi|^2 + \ldots,$$

where $\ldots$ stand for higher order terms. Then the scalar kinetic terms will be canonical at $\phi = 0$ and, from Eq. (2), the scalar potential will have the form

$$V = e^{(|\phi|^2 + \ldots)} \left[|W_\phi + W (\phi + \ldots)|^2 (1 + \ldots) \left|\bar{W}_\phi + \bar{W} (\phi + \ldots)\right|^2 - 3|W|^2\right]$$

$$= V|_{\phi=0} + V|_{\phi=0} |\phi|^2 + \text{other terms}.$$
Thus at \( \phi = 0 \) the exponential term gives a contribution \( V \) to the effective mass squared of all scalar fields. Therefore,

\[
\frac{V''}{V} = 1 + \text{other terms},
\]

where the prime denotes the derivative with respect to the canonically normalised inflaton field. But \( |V''/V| \ll 1 \) is necessary for inflation to work. So a successful model of inflation must arrange for a cancellation between the exponential term and the terms inside the curly brackets. This will require fine tuning unless a symmetry is used to enforce it.

### 3 A Solution

Divide the vector of scalar fields \( \phi \) into two separate vectors, \( \varphi \) and \( \psi \), with the inflaton contained in \( \varphi \):

\[
\phi = (\varphi, \psi), \quad \text{inflaton} \in \varphi.
\]

Assume the \( R \)-parity

\[
\psi \to -\psi, \quad \varphi \to \varphi, \quad W \to -W, \quad K \to K,
\]

and that during inflation

\[
\psi = 0
\]

(a natural value since the necessary condition \( V_\psi = 0 \) is then guaranteed by the \( R \)-parity). Then the \( R \)-parity ensures that during inflation

\[
W = W_\varphi = 0.
\]

Thus the scalar potential Eq. (2) simplifies to

\[
V = e^K W_\psi K^{-1} \bar{\psi} \bar{\psi}.
\]

Now it becomes possible to choose a form for the Kähler potential that cancels the inflaton dependent corrections to the global supersymmetric potential in a natural way.

Expanding the \( R \)-parity invariant Kähler potential about \( \psi = 0 \) gives

\[
K = A(\varphi, \bar{\varphi}) + \bar{\psi} B(\varphi, \bar{\varphi}) \psi + \mathcal{O}(\psi^2, \bar{\psi}^2),
\]

where \( A \) is real and \( B \) is hermitian. Therefore

\[
V = e^A W_\psi B^{-1} \bar{\psi},
\]
and so to eliminate the inflaton dependent corrections to the global supersymmetric potential we require

\[ B^{-1} = f(\varphi, \bar{\varphi}) C^{-1}(\chi, \bar{\chi}) , \]  

(14)

and

\[ A = -\ln f(\varphi, \bar{\varphi}) + g(\chi, \bar{\chi}) , \]  

(15)

where \( f \) and \( g \) are real functions, \( C \) is a hermitian matrix, and \( \chi \) are non-inflaton \( \varphi \) fields. This gives the inflationary potential

\[ V = e^{g(\chi, \bar{\chi})} W \psi C^{-1}(\chi, \bar{\chi}) \bar{W} \bar{\psi} , \]  

(16)

and the Kähler potential is required to have the general form

\[ K = -\ln f(\varphi, \bar{\varphi}) + \frac{\check{\psi} C(\chi, \bar{\chi}) \psi}{f(\varphi, \bar{\varphi})} + g(\chi, \bar{\chi}) + O(\psi^2, \bar{\psi}^2) , \]  

(17)

\[ = -\ln \left[ f(\varphi, \bar{\varphi}) - \check{\psi} C(\chi, \bar{\chi}) \psi \right] + g(\chi, \bar{\chi}) + O(\psi^2, \bar{\psi}^2) . \]  

(18)

4 Superstring Examples

4.1 Orbifold compactifications

The Kähler potential of the untwisted sector of the low-energy effective supergravity theory derived from orbifold compactification of superstrings always contains

\[ K = -\ln \left( S + \bar{S} \right) - \sum_{i=1}^{3} \ln \left( T_i + \bar{T}_i - |\phi_i|^2 \right) , \]  

(19)

where \( S \) is the dilaton, \( T_i \) are the untwisted moduli associated with the radii of compactification, and \( \phi_i \) are the untwisted matter fields associated with \( T_i \). Now if we divide the scalar fields into \( \varphi, \psi \) and \( \chi \) fields as follows

\[
\begin{align*}
T_1 &\in \varphi , \\
\phi_1 &\in \psi , \\
S, T_2, T_3, \phi_2 \text{ and } \phi_3 &\in \chi \subset \varphi ,
\end{align*}
\]  

(20) (21) (22)

then we get a Kähler potential of the required form [Eq. (18)]

\[ K = -\ln \left( \varphi + \bar{\varphi} - |\psi|^2 \right) + g(\chi, \bar{\chi}) , \]  

(23)

and the target space duality symmetries,

\[
T_i \rightarrow \frac{a_i T_i - i b_i}{i c_i T_i + d_i} , \quad \phi_i \rightarrow \frac{\phi_i}{i c_i T_i + d_i} , \quad a_i d_i - b_i c_i = 1 ,
\]  

(24)

contain the desired \( R \)-parity [Eq. (8)] on setting \( b_i = c_i = 0, a_1 = d_1 = -1 \) and \( a_2 = a_3 = d_2 = d_3 = 1 \).
4.2 Fermionic four-dimensional string models

The Kähler potential of the untwisted sector of the revamped flipped SU(5) model [4] is [5]

\[ K = -\ln \left( 1 - |\Phi_1|^2 - |\Phi_{23}|^2 - |\Phi_{23}|^2 - |h_1|^2 - |h_T|^2 + \frac{1}{4} |\Phi_1^2 + 2\Phi_{23}\Phi_{23} + 2h_1h_T|^2 \right) 
- \ln \left( 1 - |\Phi_2|^2 - |\Phi_{31}|^2 - |\Phi_{31}|^2 - |h_2|^2 - |h_T|^2 + \frac{1}{4} |\Phi_2^2 + 2\Phi_{31}\Phi_{31} + 2h_2h_T|^2 \right) 
- \ln \left( 1 - |\Phi_4|^2 - |\Phi_5|^2 - |\Phi_3|^2 - |\Phi_{12}|^2 - |\Phi_{12}|^2 - |h_3|^2 - |h_T|^2 
+ \frac{1}{4} |\Phi_4^2 + \Phi_5^2 + \Phi_3^2 + 2\Phi_{12}\Phi_{12} + 2h_3h_T|^2 \right). \] (25)

Now if we divide the fields as follows

\[ \Phi_4 \text{ and } \Phi_5 \in \varphi, \] (26)
\[ \Phi_3, \Phi_2, \Phi_{23}, h_3 \text{ and } h_T \in \psi, \] (27)
\[ \Phi_1, \Phi_2, \Phi_{23}, \Phi_{31}, \Phi_{31}, h_1, h_T, h_2 \text{ and } h_T \in \chi \subset \varphi, \] (28)

then we get a Kähler potential of the required form [Eq. (18)]

\[ K = -\ln \left( 1 - |\varphi|^2 + \frac{1}{4} |\varphi^T \varphi|^2 - |\psi|^2 \right) + g(\chi, \bar{\chi}) + O \left( \psi^2, \bar{\psi}^2 \right), \] (29)

and the target space duality symmetries [3] contain the desired R-parity [Eq. (8)].

4.3 More orbifold compactifications

A Kähler potential of the form

\[ K = -\ln \left[ (A_1 + \bar{A}_1) (A_2 + \bar{A}_2) - (A_3 + \bar{A}_4) (A_4 + \bar{A}_3) \right] \] (30)

often occurs in orbifold compactifications [4, 5, 6] and is of the required form.

4.4 Calabi-Yau compactifications

Kähler potentials of the form

\[ K = -\ln \left( 1 - |N|^2 - |C|^2 \right) + O \left( C^2, \bar{C}^2 \right) \] (31)

occur for subspaces of enhanced symmetry of the moduli space of a simple Calabi-Yau manifold [5]. They are of the required form [Eq. (18)].
5 Summary

A globally supersymmetric model of inflation will not work in a generic supergravity theory because the higher order, nonrenormalisable supergravity corrections destroy the flatness of the inflaton’s potential. In this talk I have derived a form for the Kähler potential which eliminates these corrections if $W = W_\varphi = \psi = 0$ during inflation (where $W$ is the superpotential, the inflaton $\in \varphi$, and $W_\psi \neq 0$). It is encouraging that Kähler potentials of the required form often occur in superstrings and that the target space duality symmetries of superstrings often contain $R$-parities which would make $W = W_\varphi = 0$ automatic for $\psi = 0$.

Acknowledgements

I thank D. H. Lyth for his help in simplifying the presentation of this work. I am supported by a JSPS Postdoctoral Fellowship and this work was supported by Monbusho Grant-in-Aid for Encouragement of Young Scientists No. 92062.

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