Two-loop finiteness of self-energies in higher-derivative SQED$_3$

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Abstract

In the $\mathcal{N}=1$ superfield formalism, two higher-derivative kinetic operators (Lee-Wick operators) are implemented into the standard three dimensional supersymmetric quantum electrodynamics (SQED$_3$) for improving its ultraviolet behavior. It is shown in particular that the ghosts associated with these Lee-Wick operators allow to remove all ultraviolet divergences in the scalar and gauge self-energies at two-loop level.

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I. INTRODUCTION

Higher-derivative supersymmetric theories bring together two entirely distinct mechanisms of removing ultraviolet (UV) divergences in the perturbation formulation. In higher-derivative theories the UV divergences are removed by an exchange of positive- and negative-metric (ghosts) states, while in supersymmetric theories the UV cancellation occurs by an exchange of virtual particles with opposite statistics. The ghost stumbling blocks (lost of unitarity and Lorentz violation), on the other hand, can be avoided for instance by applying the Lee-Wick’s ideas [1] and by adopting the Feynman contour prescription of Cutkosky et al. [2].

One of us and coworkers [3] has recently investigated the classical and quantum effects of higher derivative operators in the three dimensional Wess-Zumino (WZ) model within the superfield approach. In that work, aside from the study of the one-loop effective potential, it was shown that the two-loop scalar self-energy becomes to be finite by introducing a single higher-derivative operator in the kinetic part of the usual WZ action. Since the analytical continuation from the Minkowski space to the Euclidean space is lost in higher-derivative theories due to the presence of complex poles in the complex energy plane, the finiteness of the two-loop self-energy of the Wess-Zumino model rests on the assumption that all residues of the complex poles involved in the Cauchy’s theorem for performing the “Wick rotation” are finite. As argued in [3], the assumption about the finiteness of the residues, just like at one-loop level in four dimensions [4], is valid at two-loop level in three dimensions. Therefore as far as the analysis of ultraviolet divergences is concerned the choice of Feynman contours in momentum integrals is irrelevant.

In this paper we continue with our study of higher-derivative operators in the $\mathcal{N} = 1$ superfield approach at three dimensions. Specifically, we deform the standard supersymmetric quantum electrodynamics (SQED$_3$) with scalar self-interaction by implementing two higher-derivative operators in its kinetic action. One of them is the susy extension of the Lee-Wick operator $\partial_\mu F^{\mu\nu} \partial_\rho F_{\rho\nu}$ and the other is the gauge extension of the Lee-Wick-WZ operator $D^2 \Phi D^2 \Phi$. We show that the implementation of these two higher-derivative operators in standard SQED$_3$ is enough to remove all residual susy divergences in the scalar and gauge self-energies at two-loop level.

Our paper is planned as follows. In Sec. [II] we discuss in general terms the construction
of supersymmetric gauge theories in the $\mathcal{N} = 1$ superfield formalism at three dimensions by imposing the symmetry and renormalization requirements. It is shown that the most general renormalizable superspace Lagrangian is set up of operators with mass dimension less than or equal to 2. We emphasize also that this result depends on the definition of the kinetic part of the action which describes the theory. To close this section, we focus our attention on the SQED$_3$ with scalar self-coupling and identify its power counting divergences in the scalar and gauge self-energies at two-loop order. In Sec. [III] as commented above, we implement two higher-derivative operators in SQED$_3$. These operators on incorporating in the kinetic part of the SQED$_3$ action improve the convergence of the propagators in such a way that all usual two-loop divergences in the scalar and gauge self-energies are removed. Our results are summarized in Sec. [IV].

II. $\mathcal{N} = 1$ SUPERSYMMETRIC GAUGE THEORIES IN THREE DIMENSIONS

In this section we construct the most general (lower-derivative) Abelian gauge theory in the $\mathcal{N} = 1$ superfield formalism at three dimensions by using the symmetry and renormalization requirements. As in the next section we shall extend the standard SQED$_3$ by introducing higher-derivative operators in its action, we emphasize the importance of the definition of the kinetic part of the action in the renormalization procedure. Finally, we classify the sorts of UV divergences which are present in the standard SQED$_3$ with scalar self-coupling.

Concretely, we will construct a theory (with lower-derivatives) which describes the evolution of the superfields

$$\Phi (z) = \varphi (x) + \theta^\alpha \psi_\alpha (x) - \theta^2 F (x) ,$$

and

$$A_\alpha (z) = \chi_\alpha (x) - \theta_\alpha B (x) + i \theta^\beta V_{\alpha\beta} (x) + \theta^2 \left( -i \partial_{\alpha\beta} \chi^\beta - 2 \lambda_\alpha \right) .$$

Here $\Phi (z)$ and $A_\alpha (z)$ stand for a scalar superfield and a spinor superfield, respectively. In this paper we adopt the notation of [5].

This theory in addition has to be supersymmetric, Lorentz invariant and to satisfy the following (gauge) symmetry:

$$\Phi' (z) = e^{-i e K} \Phi (z) \quad A'_\alpha (z) = A_\alpha (z) + D_\alpha K (z) ,$$

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where $D_\alpha = \partial_\alpha + i\theta^\beta \partial_{\alpha\beta}$ is the susy-covariant derivative and $K(z)$ denotes an arbitrary scalar superfield.

In order to be able to make use of the renormalization requirement, we will need to compute the superficial degree of divergence and to this end we must first define the kinetic part of action.

For reasons to be explained below, we start with the action

$$S = \int d^5z \left\{ \frac{1}{2} W^\alpha W_\alpha + \bar{\Phi} D^2 \Phi + U(A_\alpha, \Phi) \right\},$$

where $W_\alpha$ is the field strength and is defined as $W_\alpha = \frac{1}{2} D_\beta D_\alpha A_\beta$. By using the identity $D_\alpha D_\beta D_\alpha = 0$, it is easy to prove that $W_\alpha$ is invariant under the gauge transformations (3). $U(A_\alpha, \Phi)$, on the other hand, denotes all the other operators (vertices) which are compatible with the symmetry and renormalization requirements. To respect the supersymmetry, $U(A_\alpha, \Phi)$ must be constructed from covariant objects (superfields and susy-covariant derivatives), and so its most general form has to be

$$U \sim (D_\alpha)^{N_D} (A_\alpha)^{N_A} (\bar{\Phi}\Phi)^{N_\Phi/2},$$

where $N_D$ represents the number of spinor susy-covariant derivatives, $N_A$ the number of spinor superfields, and $N_\Phi$ the number of scalar superfields. Even though $i\partial_{\alpha\beta}$ is also a (spacetime) susy-covariant derivative, it has not been included in (5) because it can be eliminated by means of the identity $i\partial_{\alpha\beta} = \frac{1}{2} \{D_\alpha, D_\beta\}$. Note also that the symmetries of the theory restrict strongly the values of $N_D$, $N_A$ and $N_\Phi$. In particular, rotational symmetry implies that $N_\Phi$ must take only even values ($N_\Phi = 0, 2, \cdots$), while Lorentz symmetry demands a complete spinor contraction as well as that $N_D + N_A = \text{even number}$.

Before computing the propagators of the theory and analyzing its renormalization, some comments are in order with respect to the action (4). First of all, it should be noted that one only needs to know the asymptotic behavior of the propagators (i.e. their behavior when $k^2 \to \infty$) for determining the superficial degree of divergence. Since these propagators are obtained by inverting the kernels of the quadratic parts in the fields, they will depend on the definition of the kinetic part of the action. The most general kinetic action (without considering the gauge symmetry) has the form

$$S_0 = \int d^5z \left\{ \frac{1}{2} A^\alpha \sigma_{\alpha\beta} A^\beta + \bar{\Phi} \partial \Phi \right\},$$
where

\[ O_{\alpha\beta} = \sum_{i=0}^{1} [r_i R_{i,\alpha\beta} + s_i S_{i,\alpha\beta}] \]

\[ = (r_0 + r_1 D^2) i \partial_{\alpha\beta} + (s_0 + s_1 D^2) C_{\alpha\beta} \]  

(7)

\[ O = c_0 + c_1 D^2. \]  

(8)

Here \( R_{i,\alpha\beta} \) and \( S_{i,\alpha\beta} \) constitute a basis in the gauge sector [6], while \( r_i, s_i, \) and \( c_i \) are functions of the d’Alembertian operator \( \Box = \partial^\mu \partial_\mu \). Clearly, if one wants a local theory these functions have to be polynomials in \( \Box \). We have chosen (4) as the starting action because the gauge invariant operator \( W^\alpha W_\alpha \) contains after carrying out the integration over \( \theta \) the Maxwell Lagrangian,

\[ D^2 (W^\alpha W_\alpha) \bigg| = -\frac{1}{8} F^{\mu\nu} F_{\mu\nu} + \cdots, \]  

(9)

where \( F_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu \). Second, since \( \bar{\Phi} D^2 \Phi \) is not gauge invariant, other operators must be included in the action (4) to restore its gauge invariance. In fact these operators are encoded in \( U \) and will be found by means of the renormalization condition.

As in conventional gauge theories, to determine the propagator of the gauge superfield it is necessary to fix the gauge (i.e. to eliminate the redundant degrees of freedom). For simplicity, we choose the gauge fixing term

\[ S_{gf} = -\frac{1}{4\alpha} \int d^5 z D^\alpha A_\alpha D^2 D^\beta A_\beta. \]  

(10)

This gauge fixing term has the advantage of uncoupling the physical superfields from the (anti-)ghost superfields in the perturbation approach.

Inserting (10) into (4), we easily calculate with the help of the techniques described in [6] the propagators for the scalar and gauge superfields. They are given by

\[ \langle \Phi (k, \theta) \Phi (-k, \theta') \rangle = -i \frac{D^2}{k^2} \delta^2 (\theta - \theta') \]  

(11)

and

\[ \langle A_\alpha (k, \theta) A_\beta (-k, \theta') \rangle = -\frac{i}{2k^2} \left\{ (\alpha + 1) C_{\alpha\beta} + \frac{(\alpha - 1)}{k^2} k_{\alpha\beta} D^2 \right\} \delta^2 (\theta - \theta'). \]  

(12)

By virtue of these results, the superficial degree of divergence \( \omega \) for any arbitrary Feynman diagram of the model reads

\[ \omega = IV + 2 - \frac{E_\Phi}{2} - \frac{n_D}{2}, \]

(13)
where \( I \) represents the index of divergence corresponding to the generic vertex \((5)\) and is given by

\[
I = \frac{N_\Phi + N_D}{2} - 2,
\]

(14)

\( V \) is the number of vertices, \( E_\Phi \) is the number of external \( \Phi \) lines, and \( n_D \) is the number of susy-covariant derivatives \((D_\alpha)\) transferred to the external lines during the Grassmann reduction procedure (D-algebra) \([5]\).

From (13) we can see that our theory will be renormalizable if and only if the condition \( I \leq 0 \) is satisfied. That is, if \( N_\Phi + N_D \leq 4 \). Despite the fact that \( I \) does not depend explicitly on \( N_A \) so that any vertex with \( I \leq 0 \) regardless of the gauge lines is acceptable, we shall see below that the symmetries of the theory limit strongly the form of the vertices, in particular, the number of the gauge lines.

Consider first the gauge sector, i.e., set \( N_\Phi = 0 \) in (14). This condition implies that any renormalizable operator has to be made up of at most four susy-covariant derivatives \((N_D \leq 4)\).

Using the Lorentz condition \((N_D + N_A = \text{even})\) and the gauge symmetry, it is possible to show that there are only two gauge operators which obey all physical requirements. They are given by

\[
D^\beta D^\alpha A_\beta A_\alpha \sim W^\alpha A_\alpha \quad (N_D = 2) \\
D^\beta D^\alpha A_\beta D^\gamma D_\alpha A_\gamma \sim W^\alpha W_\alpha \quad (N_D = 4).
\]

(15)

The former is gauge invariant only within the superspace integral and gives rise after performing the Grassmann integral to the well-known Chern-Simons term,

\[
D^2 (W^\alpha A_\alpha) \big| = -\frac{1}{2} \epsilon^{\mu
\rho\nu} v_\mu \partial_\nu v_\rho + \cdots ,
\]

(16)

whereas the latter is a bona fide operator in the sense that is gauge invariant out of the superspace integral. We stress that any other combination of gauge fields and susy-covariant derivatives is forbidden either by renormalization or by gauge invariance. For instance, an operator of the fashion \( W^\alpha A_\alpha W^\beta A_\beta \) is renormalizable but not gauge invariant, despite the fact that the object \( W^\alpha A_\alpha \) is both renormalizable and gauge invariant. In this case the gauge invariance is destroyed by the presence of a number of gauge superfields larger than two in the vertex. It is important to point out that the gauge symmetry in this renormalization analysis must be imposed by hand since the superfield perturbation formalism is not manifestly gauge invariant.
The scalar and gauge-scalar sectors are particularly interesting from the renormalization point of view. Note first that the renormalization condition \((N_\Phi + N_D \leq 4)\) implies that any operator constructed with more than four scalar superfields is immediately non-renormalizable. Hence \(N_\Phi\) may solely take three values: \(N_\Phi = 0, 2, 4\). Since the \(N_\Phi = 0\) case (gauge sector) has been already studied, we are going to proceed with the \(N_\Phi = 2\) case. Here any renormalizable operator must contain at most two susy-covariant derivatives \((N_D \leq 2)\). Thus as workable operators one has

\[
\Phi \Phi, \quad D^\alpha \Phi D_\alpha \Phi, \quad A^\alpha D_\alpha \Phi, \quad A^\alpha \Phi D_\alpha \Phi, \quad A^\alpha A_\alpha \Phi \Phi, \quad \cdots
\]

where the ellipsis stands for other operators with \(N_D + N_A > 2\). Note that the object \(D^\alpha A_\alpha \Phi \Phi\) is not included above because this is a linear combination of the third and fourth operators after integration by parts. Similarly \(D^\alpha \Phi D_\alpha \Phi\) is equivalent by by-part integration to \(\Phi D^2 \Phi\).

Making use of the gauge transformations (3) we may easily verify that the first operator in (17), i.e. \(\Phi \Phi\), is gauge invariant, while the others are not (at least independently). Note however that since the transformation of the gauge field \(A_\alpha\) involves a susy-covariant derivative, we might expect that the linear combination of the four remaining operators in (17) turns out to be gauge invariant. After a dimensional analysis of the operators in (17) and invoking the Hermiticity property of the action as a whole, it is easy to verify that such linear combination has to be of the form

\[
D^\alpha \Phi D_\alpha \Phi + i e u \left( A^\alpha D_\alpha \Phi \Phi - A^\alpha \Phi D_\alpha \Phi \right) + e^2 v A^2 \Phi \Phi,
\]

where \(A^2 \equiv A^\alpha A_\alpha / 2\) and \(u\) and \(v\) are dimensionless constants. Requiring the invariance of this expression under the gauge transformations (3), we straightforwardly identify that \(u = -1\) and \(v = 2\).

By introducing the object \(\nabla_\alpha \equiv D_\alpha - i e A_\alpha\), we can encapsulate the four operators in (18) into a single operator. That this,

\[
\nabla^\alpha \Phi \nabla_\alpha = D^\alpha \Phi D_\alpha \Phi - i e \left( A^\alpha D_\alpha \Phi \Phi - A^\alpha \Phi D_\alpha \Phi \right) + 2 e^2 A^2 \Phi \Phi.
\]

The object \(\nabla_\alpha\) is called gauge covariant derivative due to its transformation property under (3): \(\nabla_\alpha \rightarrow e^{i e K} \nabla_\alpha e^{-i e K}\). In terms of this gauge-covariant derivative we can easily construct other gauge invariant operators. However, as we have seen, the only operator which satisfies
all the physical requirements is that shown in (19). Note that any other operator with more than two covariant derivatives $\nabla_\alpha$ turns out non-renormalizable. Finally, for $N_\Phi = 4$, which in turn implies that $N_D = 0$, one finds the scalar self-coupling $(\bar{\Phi}\Phi)^2$ as the only physically acceptable operator.

In sum, we have shown that the most general renormalizable supersymmetric gauge theory is described by the action

$$S = \int d^5z \left\{ \frac{1}{2} W^\alpha W_\alpha + m W^\alpha A_\alpha - \frac{1}{2} \nabla^\alpha \bar{\Phi} \nabla_\alpha \Phi + M \bar{\Phi} \Phi + \lambda (\bar{\Phi}\Phi)^2 \right\}.$$  \hspace{1cm} (20)

This action (with lower-derivative operators) is the supersymmetric version of the usual Maxwell-Chern-Simons theory coupled to matter superfield.

From now on, we confine ourselves to study the ultraviolet behavior of the massless supersymmetric quantum electrodynamics (SQED$_3$). In particular, we study the possibility of removing the residual susy divergences in the two-loop scalar and gauge self-energies at two-loop level by introducing appropriate higher derivative operators in the kinetic part of the action.

Setting $m = 0 = M$ in (20) and using the gauge fixing term (10), the action for the SQED$_3$ with scalar self-coupling reads

$$S = \int d^5z \left\{ \frac{1}{2} W^\alpha W_\alpha - \frac{1}{2} \nabla^\alpha \bar{\Phi} \nabla_\alpha \Phi + \lambda (\bar{\Phi}\Phi)^2 - \frac{1}{4\alpha} D^\alpha A_\alpha D^\beta A_\beta + \bar{C} D^2 C \right\}. \hspace{1cm} (21)$$

Here $C$ and $\bar{C}$ are respectively the ghost and anti-ghost superfields. This action is invariant under the following BRST transformations

$$\delta \Phi = ie\Lambda C \Phi, \quad \delta A_\alpha = -\Lambda D_\alpha C, \quad \delta C = 0, \quad \delta \bar{C} = \frac{1}{\alpha} \Lambda D^2 D^\alpha A_\alpha, \hspace{1cm} (22)$$

where $\Lambda$ is a Grassmann parameter. Notice that as in usual field theories these transformations are nilpotent, i.e. $\delta^2 = 0$. On the other hand, since the ghost and anti-ghost do not couple with the other fields, they can be ignored in the perturbative analysis.

Using the propagators in (11-12) and identifying all kinds of vertices in the action (21),

$$S_{int} = \int d^5z \left\{ \frac{ie}{2} \left( A^\alpha D_\alpha \Phi \Phi - A^\alpha \Phi D_\alpha \Phi \right) - e^2 A^2 \Phi \Phi + \lambda (\Phi \Phi)^2 \right\}, \hspace{1cm} (23)$$

we compute easily the superficial degree of divergence for the SQED$_3$,

$$\omega = 2 - \frac{1}{2} V_1^{(3)} - V_0^{(4)} - \frac{1}{2} E_\Phi - \frac{1}{2} n_D, \hspace{1cm} (24)$$

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Figure 1: Divergent radiative corrections to the scalar and gauge self-energies for the SQED$_3$ with scalar self-interaction. Solid lines represent scalar propagators, while wavy lines represent gauge propagators.

where $V_{N_l}^{(N_D)}$ denotes the number of scalar-gauge vertices with $N_l$ lines and $N_D$ susy-covariant derivatives ($D_\alpha$), while $E_\Phi$ and $n_D$ have the same meanings as in (13).

If we rewrite (24) in the way $\omega = 2 - x$, with $x$ representing all the other terms, and spotting that $x$ is strictly an integer positive number ($x \geq 1$), it is easy to realize that the (lower-derivative) SQED$_3$ with scalar self-coupling possesses just logarithmic ($\omega = 0$) and linear ($\omega = 1$) divergences. These divergences ($1/\epsilon$) will appear from the two-loop diagrams, for in planar physics and within the dimensional reduction (DReD$_3$) scheme [7] all one-loop diagrams are finite [8]. In Fig. 1 we display all two-loop contributions to the self-energy of the scalar and gauge superfields which are divergent on power counting grounds. It is important to point out however that diagrams (c), (d), (e), (k) and (l) in Fig. 1 turn out to be finite after performing the D-algebra. The reason is that each two-loop momentum integral can be expressed as the product of two independent finite one-loop integrals. The remaining infinities in the scalar and gauge self-energies at two-loop level must be removed by the usual renormalization procedure. In this respect, it has been shown in [9] that there is an unusual gauge in which the SQED$_3$ (without scalar self-interaction) turns out finite. In the next section we introduce appropriate higher-derivative operators in the standard SQED$_3$ action to remove these infinities and so improve the ultraviolet behavior of the usual theory.
III. HIGHER-DERIVATIVE SQED$_3$ AND TWO-LOOP FINITENESS OF THE SELF-ENERGIES

This section is devoted to implement two higher derivative operators in the lower-derivative SQED$_3$ with scalar self-coupling. These operators are judiciously chosen and inserted into the kinetic part of the action for improving the asymptotic behavior of the propagators, so that to get rid of all two-loop divergences in the scalar and gauge self-energies. Clearly, according to the preceding discussion, these sorts of operators are non-renormalizable in the usual theory.

Since the field strength $W_\alpha$ is invariant ($W'_\alpha = W_\alpha$) under gauge transformations, there is an infinity of possible higher derivative operators in the gauge sector which obey all symmetries of the standard theory. Nevertheless there is only one operator which removes partly the divergences, but that is the supersymmetric extension of the operator $\partial_\mu F^{\mu\nu} \partial_\rho F^\rho_\nu$ which epitomizes the four dimensional Lee-Wick quantum electrodynamics (Lee-Wick-QED$_4$). It is not hard to see that this higher derivative operator is given by $D^2 W^\alpha D^2 W_\alpha$. Indeed after performing the integral over $\theta$ it exhibits the following content

$$D^2 \left( D^2 W^\alpha D^2 W_\alpha \right) = \frac{1}{2} \partial_\mu F^{\mu\nu} \partial_\rho F^\rho_\nu + 2 \lambda^\alpha i \partial_\alpha \Box \lambda_\beta,$$

(25)

where the spinor field $\lambda_\alpha$ is the susy partner (photino) of the photon $\nu^\mu$ field.

On the other hand, an examination of the superficial degree of divergence of the theory, which results of adding this operator in (21), reveals that it is necessary to introduce one more operator in this scalar sector to remove successfully all divergences in the diagrams displayed in Fig. 1. Needless to say, this additional operator must be constructed by using the gauge covariant derivative $\nabla_\alpha = D_\alpha - i A_\alpha$ in order to respect the gauge symmetry. This higher derivative operator becomes to be $\nabla^2 \bar{\Phi} \nabla^2 \Phi$, where $\nabla^2 = \nabla^\alpha \nabla_\alpha / 2$, and gives rise to new interaction vertices with five and six lines on incorporating in the standard SQED$_3$. The higher-derivative vertices are given by

$$S'_{\text{int}} = b \int d^5 z \left( \nabla^2 \bar{\Phi} \nabla^2 \Phi - D^2 \bar{\Phi} D^2 \Phi \right) = b \int d^5 z \left[ \frac{ie}{2} \left( \Sigma D^2 \Phi - \Sigma D^2 \bar{\Phi} \right) - e^2 A^2 \left( \bar{\Phi} D^2 \Phi + \Phi D^2 \bar{\Phi} \right) + \frac{ie^3}{2} A^2 \left( \bar{\Phi} \Sigma - \Phi \bar{\Sigma} \right) + \frac{e^2}{4} \Sigma \bar{\Sigma} + e^4 A^4 \bar{\Phi} \Phi \right].$$

(26)

Here for simplicity we have introduced the superfield $\Sigma (A, \Phi) \doteq D^\alpha (A_\alpha \Phi) + A^\alpha D_\alpha \Phi$. 

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Taking into account these two operators, our higher-derivative supersymmetric quantum electrodynamics in three dimensions (HSQED\(_3\)) is defined by

\[
S = \int d^5z \left\{ \frac{1}{2} W^\alpha W_\alpha + \frac{a}{2} D^2 W^\alpha D^2 W_\alpha - \frac{1}{2} \bar{\nabla}^\alpha \Phi \nabla_\alpha \Phi + b \bar{\nabla}^2 \Phi \nabla^2 \Phi + \lambda (\bar{\Phi} \Phi)^2 \right\} \quad (27)
\]

Note that the negative dimensions of the higher derivative coefficients ([a] = −2, [b] = −1) mirror the non-renormalizability of their corresponding operators, as discussed in the foregoing section.

It is interesting to see how these higher derivatives are distributed among the component fields. For simplicity let us consider merely the quadratic part of the Lagrangian. Carrying out the \(\theta\)-integral in (27), using the \(\theta\)-Taylor expansions (1-2), it is easy to show that the quadratic Lagrangian is given by

\[
\mathcal{L}_0 = -\frac{1}{8} F_{\mu\nu} F^{\mu\nu} + \lambda i \partial \lambda + \frac{a}{4} \partial_\mu F^{\mu\nu} \partial_\nu F_{\nu}^\rho + a \lambda i \partial \Box \lambda + \bar{\varphi} \Box \varphi + \bar{\psi} i \partial \psi \\
+ \bar{F} F + b \left( \bar{\varphi} \Box F + \bar{\varphi} \Box \bar{F} \right) + b \bar{\psi} \Box \psi. \quad (28)
\]

Here the spinor indices are contracted by following the north-west rule (\(\nwarrow\)) and the square of a spinor includes a factor of 1/2 in its definition. Notice that the scalar field \(\varphi\) acquires a higher-derivative operator only in the on-shell formulation, i.e. after eliminating the auxiliary field \(F\) through its equation of motion \((F = -b \Box \varphi)\).

These higher-derivative operators do not modified the classical potential. This may be seen by setting \(\Phi = \varphi - \theta^2 F\) and \(A_\alpha = 0\) into (27), with \(\varphi\) and \(F\) labeling constants. Doing this and ignoring an over-all space-time integral \(\int d^3x\), we get for the classical potential

\[
V_{cl} = -\int d^2 \theta \left\{ \bar{\Phi} D^2 \Phi + \lambda (\bar{\Phi} \Phi)^2 \right\} = -\bar{F} F - 2\lambda \bar{\varphi} \varphi \left( \bar{\varphi} F + \varphi \bar{F} \right). \quad (29)
\]

If we eliminate the auxiliary field \(F\), writing \(\varphi = \frac{1}{\sqrt{2}} (\varphi_r + i \varphi_i)\), the classical potential can be written as \(V_{cl} = \frac{\lambda^2}{2} (\varphi_r^2 + \varphi_i^2)^3\). Thus at classical level supersymmetry remains intact. At this point it is worth mentioning that the Kählerian potential at one-loop level was recently calculated for a family of three-dimensional superfield Abelian gauge theories with higher-derivative operators in the gauge sector [10].

In what follows we shall show that the higher-derivative kinetic operators introduced in the standard SQED\(_3\) improve the asymptotic behavior of the propagators in such a way that the scalar and gauge self-energies at two-loop order become to be finite.
The propagators for the scalar and gauge fields associated with the HSQED\(_3\) action are given by

\[
\langle \bar{\Phi}(k, \theta) \Phi(-k, \theta) \rangle = -i \frac{(D^2 + bk^2)}{k^2 (1 + b^2 k^2)} \delta^2(\theta - \theta') \tag{30}
\]

and

\[
\langle A_\alpha(k, \theta) A_\beta(-k, \theta) \rangle = \frac{i}{2 (k^2)^2} \left[ \frac{1}{(1 - a k^2)} D_\beta D_\alpha - \alpha D_\alpha D_\beta \right] D^2 \delta^2(\theta - \theta'). \tag{31}
\]

Here once again we have fixed the gauge through (10). The asymptotic behavior of these propagators by virtue of the relation \(\{D_\alpha, D_\beta\} = 2 k^{\alpha \beta}\) is

\[
\langle \bar{\Phi} \Phi \rangle \sim \frac{1}{b^2 k^3} + \frac{1}{b k^2} \quad \langle AA \rangle \sim \frac{1}{a k^4} - \frac{\alpha}{k^2} \sim \frac{1}{k^4} \quad \text{(Landau gauge).} \tag{32}
\]

Note that these propagators are more convergent than their counterparts (11-12), whichever value for the gauge parameter \(\alpha\) is chosen.

To work out the superficial degree of divergence for the HSQED\(_3\), we regard the less convergent part (i.e. \(1/k^2\)) of the scalar propagator and choose the Landau gauge (\(\alpha = 0\)) in order to obtain a well-defined asymptotic gauge propagator. In this way we get

\[
\omega = 2 - 2 P_A - \frac{3}{2} V^{(3)}_1 - \frac{1}{2} V^{(3)}_3 - 2 V^{(4)}_0 - V^{(4)}_2 - 2 V^{(5)}_0 - \frac{3}{2} V^{(5)}_1 - 2 V^{(6)}_0 - \frac{1}{2} n_D \tag{33}
\]

where \(V^{(\phi)}_0\) and \(P_A\) denote the number of pure scalar vertices and the number of gauge propagators, respectively. The other variables were defined in (24). An immediate consequence of this result is that any Feynman diagrams with one or more internal gauge propagators turns out to be convergent on power counting grounds.

A simple analysis of the superficial degree of divergence (33) shows that all Feynman diagrams in Fig. 1, which are divergent within the standard SQED\(_3\), become now finite. In this case there are however other contributions to the scalar and gauge self-energies that we must regard. These two-loop contributions correspond to the extra higher-derivative interactions (26) and are depicted in Fig. 2. By invoking once again the power counting criterion, it is easy to show that these extra diagrams are also convergent. In this way we have shown that the introduction of two higher-derivative operators in the kinetic part of the SQED\(_3\) action improves the ultraviolet behavior of the theory, allowing in particular the elimination of all two-loop divergences in the scalar and gauge self-energies.

The key role of the ghosts in removing UV divergences is hidden in the super-power counting analysis. In the component formalism the identification of the ghosts is relatively
simple. They become evident either by reformulating the higher-derivative theory in terms of lower-derivative operators (a theory with indefinite metric), i.e. by eliminating the higher-derivative operators by means of auxiliary fields (ghosts), or by splitting the higher-derivative propagators, using the expression

$$\frac{1}{(k^2 + m^2_1)(k^2 + m^2_2)} = \frac{1}{m^2_2 - m^2_1} \left[ \frac{1}{k^2 + m^2_1} - \frac{1}{k^2 + m^2_2} \right], \quad (34)$$

into elementary propagators, where the propagators with “wrong sign” correspond to the ghosts. In the superfield formalism, by contrast, the ghosts become visible only after performing the Grassmann reduction procedure (D-algebra) in the Feynman amplitudes so that the formula (34) can be used to identify the ghost contributions. In our previous work [3] we have shown, by evaluating explicitly the diagram (a) in Fig. 1, how the mutual cancellation between ghost divergences and “normal” ones happens. Note that in this kind of theory the “normal” divergences are indeed the residual divergences of the susy mechanism of removing UV infinities. Finally, it is important to mention that the two-loop momentum integrals in [3] were calculated by using the master integral [11]

$$\int_{k,q} \frac{1}{(k^2 + x^2)(q^2 + y^2)[(k+q)^2 + z^2]} = \frac{\mu^{-2}\epsilon}{32\pi^2} \left\{ \frac{1}{\epsilon} - \gamma + 1 - \ln \left[ \frac{(x+y+z)^2}{4\pi^2\mu^2} \right] \right\} + B_{res} \quad (35)$$

where $\epsilon = 3 - D$, with $D$ denoting the spacetime dimension, $\gamma$ is the Euler’s constant, and $B_{res}$ stands for the total residue contribution of the complex poles inside an energy contour appropriate for performing the “Wick-rotation”. As commented in the introduction, in the ultraviolet analysis, we are assuming that the residue contributions are finite.
IV. CONCLUSIONS

Within the $\mathcal{N} = 1$ superfield formalism, we have deformed the standard supersymmetric quantum electrodynamics in three dimensions (SQED$_3$) by introducing two higher-derivative operators in its kinetic action. One of them is the susy extension of the higher-derivative operator $\partial_\mu F^{\mu\nu} \partial_\rho F^\rho_\nu$ in Lee-Wick-QED in four dimensions and the other is the gauge extension of the operator $D^2 \bar{\Phi} D^2 \Phi$ in the HWZ$_3$ model. These operators respect all of the symmetries of the original model, but are non-renormalizable on power counting grounds. In the on-shell component formulation, these operators introduce higher derivatives for all component fields. Here the auxiliary field $F$ plays the role of a Lee-Wick field by introducing the higher-derivative operator for the the scalar field $\varphi$ only in the on-shell case. At the quantum level, we show, under a rather general assumption about the complex poles, that the ghosts associated with these two higher-derivative operators cancel out all residual susy divergences in the scalar and gauge self-energies at two-loop level.

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