Connecting Dark Gauge Symmetry
to the Standard Model

Ernest Ma

Department of Physics and Astronomy,
University of California, Riverside, California 92521, USA

Abstract

Dark matter is postulated to be a neutral Dirac fermion, charged under a dark $U(1)_D$ gauge symmetry. Scalar partners of the quarks and leptons are also charged under $U(1)_D$. The dark gauge boson $Z_D$ and the dark Higgs boson $h_D$ enable either freeze-out or freeze-in mechanisms to account for the correct dark matter relic abundance. Dark number $D$ is connected to baryon number $B$ and lepton number $L$ through $D = 3B + L - (2j)_{\text{mod } 2}$ where $j$ is the intrinsic spin of the particle.
**Introduction**: The nature of dark matter \[1\] is unknown, but it is likely to be stabilized by an unbroken symmetry, the simplest of which is dark parity, under which it is odd and all particles of the Standard Model (SM) are even. In many proposed models involving Majorana neutrinos, it is derivable \[2\] from lepton parity, i.e. \( \pi_D = \pi_L(-1)^{2j} \), where \( \pi_L = (-1)^L \) and \( j \) is the intrinsic spin of the particle. For Dirac neutrinos with the conservation of lepton number \( L \), the analog connection \[3\] is \( D = L - (2j) \mod 2 \). If baryon number \( B \) is also considered, it may become \( D = 3B + L - (2j) \mod 2 \). In this paper, it is shown how it comes about from a dark gauged \( U(1)_D \) symmetry, which is broken spontaneously by three units \[4\], instead of the customary two.

To support an anomaly-free \( U(1)_D \) gauge symmetry, a neutral Dirac fermion \( N \) is proposed with dark charge \( D = -1 \). It connects to quarks and leptons through their scalar counterparts with \( D = 1 \). The particle content is therefore very similar to that of the supersymmetric standard model. There are however very important differences. Instead of two Higgs superfields, there is only the one SM Higgs doublet, and there are no gauginos. In contrast, there is a dark gauge boson \( Z_D \) and a dark Higgs boson \( h_D \). Together with \( N \) as dark matter, they participate in either freeze-out or freeze-in scenarios. Whereas \( Z_D \) decays immediately to \( N\bar{N} \) if kinematically allowed, \( h_D \) decays mainly through its mixing with the SM Higgs boson. If \( m_{Z_D} < 2m_N \), then \( Z_D \) decays to an SM fermion-antifermion pair, through the latter’s dark magnetic moment. The model and its details are described below.

**Model**: The idea of this model and its implementation are both very simple. All particles of the SM do not transform under \( U(1)_D \). To each quark and lepton, a scalar counterpart is added which is charged +1 under \( U(1)_D \), thereby connecting to the neutral Dirac fermion \( N \) which is charged \( -1 \). The neutral scalar \( \zeta^0 \) breaks \( U(1)_D \) by three units. It cannot couple to \( N_L N_L \) or \( N_R N_R \), hence the global symmetry \( D \) remains, with \( N \) having \( D = -1 \)
and the scalar quarks and leptons having $D = 1$, thereby ensuring the connection $D = 3B + L - (2j)_{\mod 2}$.

**Dark $U(1)_D$ Gauge Symmetry:** Whereas the SM $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry is broken by the one Higgs doublet $\Phi = (\phi^+, \phi^0)$, the dark $U(1)_D$ gauge symmetry is broken by the one Higgs singlet $\zeta^0$. The Higgs potential consisting of $\Phi$ and $\zeta$ is thus very simple, i.e.

$$V = m_1^2 \Phi^\dagger \Phi + m_2^2 \zeta^* \zeta + \frac{1}{2} \lambda_1 (\Phi^\dagger \Phi)^2 + \frac{1}{2} (\zeta^* \zeta)^2 + \lambda_3 (\Phi^\dagger \Phi)(\zeta^* \zeta).$$ (1)

Let $\langle \phi^0 \rangle = v$ and $\langle \zeta \rangle = u$, then

$$v^2 = -\frac{\lambda_2 m_1^2 + \lambda_3 m_2^2}{\lambda_1 \lambda_2 - \lambda_3^2}, \quad u^2 = -\frac{\lambda_1 m_2^2 + \lambda_3 m_1^2}{\lambda_1 \lambda_2 - \lambda_3^2}. \quad (2)$$

The only physical scalars are the SM $h = \sqrt{2} \text{Re}(\phi^0)$ and the new dark Higgs boson $h_D = \sqrt{2} \text{Re}(\zeta^0)$.

### Table 1: Fermions and scalars in the dark $U(1)_D$ model.

| fermion/scalar | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ | $U(1)_D$ | $B$ | $L$ |
|---------------|-----------|-----------|----------|---------|-----|-----|
| $(u, d)_L$    | 3         | 2         | 1/6      | 0       | 1/3 | 0   |
| $u_R$         | 3         | 1         | 2/3      | 0       | 1/3 | 0   |
| $d_R$         | 3         | 1         | −1/3     | 0       | 1/3 | 0   |
| $(\nu, l)_L$  | 1         | 2         | −1/2     | 0       | 0   | 1   |
| $\nu_R$       | 1         | 1         | 0        | 0       | 0   | 1   |
| $l_R$         | 1         | 1         | −1       | 0       | 0   | 1   |
| $(\phi^+, \phi^0)$ | 1     | 2         | 1/2      | 0       | 0   | 0   |
| $N_{L,R}$     | 1         | 1         | 0        | −1      | 0   | 0   |
| $(\bar{u}_L, d_L)$ | 3     | 2         | 1/6      | 1       | 1/3 | 0   |
| $\bar{u}_R$   | 3         | 1         | 2/3      | 1       | 1/3 | 0   |
| $\bar{d}_R$   | 3         | 1         | −1/3     | 1       | 1/3 | 0   |
| $(\bar{\nu}_L, \bar{l}_L)$ | 1   | 2         | −1/2     | 1       | 0   | 1   |
| $\bar{\nu}_R$ | 1         | 1         | 0        | 1       | 0   | 1   |
| $\bar{l}_R$   | 1         | 1         | −1       | 1       | 0   | 1   |
| $\zeta^0$     | 1         | 1         | 0        | 3       | 0   | 0   |
\[ V = \lambda_1 v^2 h^2 + \lambda_2 u^2 h_D^2 + 2\lambda_3 v u h_D h + \frac{1}{\sqrt{2}} \lambda_1 v h^3 + \frac{1}{\sqrt{2}} \lambda_2 u h_D^3 \]
\[ + \frac{1}{\sqrt{2}} \lambda_3 v u h_D^2 + \frac{1}{\sqrt{2}} \lambda_3 u h_D h^2 + \frac{1}{8} \lambda_1 h^4 + \frac{1}{8} \lambda_2 h_D^4 + \frac{1}{4} \lambda_3 h^2 h_D^2. \]  
\[ \text{(3)} \]

Hence \( h_D \) mixes with \( h \) according to

\[ M_{hh_D}^2 = \begin{pmatrix} 2\lambda_1 v^2 & 2\lambda_3 vu \\ 2\lambda_3 vu & 2\lambda_2 u^2 \end{pmatrix}, \]  
\[ \text{(4)} \]

and decays to SM fermions through the \( h \) Yukawa couplings. Note that \( \zeta^0 \) has \( D = 3 \), hence \( h_D \) does not couple to \( N_L N_L \) or \( N_R N_R \).

As for \( Z_D \), the gauge invariant term \( |(\partial^\mu - 3i g_D Z_D^\mu) \zeta|^2 \) yields \( m_{Z_D} = 3\sqrt{2} g_D u \), together with the interaction

\[ \mathcal{L}_{\text{int}} = 9\sqrt{2} g_D^2 u h_D Z_D^\mu Z_D^\mu + 9 g_D^2 h_D^2 Z_D^\mu Z_D^\mu. \]  
\[ \text{(5)} \]

This means that \( h_D \) may decay into \( Z_D Z_D \) if kinematically allowed. It also couples to \( N \bar{N} \) indirectly through \( Z_D \) in one loop.

Since \( N \) is charged under \( U(1)_D \), as are the scalar quarks and leptons, they are decay products of \( Z_D \) if kinematically allowed. If not, then \( Z_D \) may decay into an SM fermion-antifermion pair. The \( \gamma^\mu \) coupling is zero from dark gauge invariance. The \( \sigma^{\mu\nu} \) coupling is obtained as shown in Fig. 1, where \( \tilde{f}_{1,2} \) are the mass eigenstates from \( \tilde{f}_L - \tilde{f}_R \) mixing. Let
\[ f_L = \cos \theta \tilde{f}_1 - \sin \theta \tilde{f}_2 \text{ and } f_R = \sin \theta \tilde{f}_1 + \cos \theta \tilde{f}_2, \text{ with } m_{1,2}^2 >> m_N^2 >> m_f^2, \text{ then the dark}
\]

magnetic moment of \( f \) is [5]
\[
a_f = \frac{g_f^2 \sin \theta \cos \theta}{4\pi^2} m_f m_N \left[ \frac{1}{m_1^2} \left( \ln \frac{m_1^2}{m_N^2} - \frac{7}{4} \right) - \frac{1}{m_2^2} \left( \ln \frac{m_2^2}{m_N^2} - \frac{7}{4} \right) \right], \quad (6)
\]

where \( g_f^L = g_f^R \) has been assumed. The corresponding interaction is
\[
\mathcal{L}_{\text{int}} = \frac{ig_D a_f}{2m_f} Z_D^\mu \nu \bar{f} \sigma_{\mu \nu} f. \quad (7)
\]

Scottogenic Fermion Masses: According to Table 1, quarks and leptons obtain masses as in the SM. However, it is a simple matter to allow some light fermions, such as the Dirac neutrinos, to acquire radiative masses through dark matter [6], by the implementation of a softly broken \( Z_2 \) symmetry. For example, let \( \nu_R \) and \( \tilde{\nu}_R \) be odd under this \( Z_2 \), then the dimension-four Yukawa coupling \( \bar{\nu}_R (\nu_L \phi^0 - l_L \phi^+) \) is forbidden, but \( \bar{\nu}_R N_L \tilde{\nu}_R \) is allowed. Hence the one-loop generation of Dirac neutrino mass is possible as shown in Fig. 2, where the soft

breaking of \( Z_2 \) occurs with the scalar trilinear \( \bar{\nu}_R^* (\phi^0 \nu_L - \phi^+ \tilde{l}_L) \) term. Analogous constructions are possible for the electron and muon masses if desired. Note that \( U(1)_D \) is also applicable to the dark sector of the recently proposed [7] scotogenic \( A_5 \rightarrow A_4 \) model of Dirac neutrino masses.

Freeze-Out Scenario: The neutral dark fermion \( N \) is a candidate for the dark matter of the Universe from thermal freeze-out. Assuming that \( m_N > m_{Z_D} \), the annihilation of
$N\bar{N} \rightarrow Z_DZ_D$ is shown in Fig. 3. The cross section $\times$ relative velocity is given by

$$\sigma v_{rel} = \frac{g_D^4}{16\pi m_N^2} \left(1 - \frac{m_{Z_D}^2}{m_N^2}\right)^{\frac{3}{2}} \left(1 - \frac{m_{Z_D}^2}{2m_N^2}\right)^{-2}.$$  \hspace{1cm} (8)

Setting this value to the canonical $6 \times 10^{-26}$ cm$^3$/s for a Dirac fermion, and assuming $m_N = 1$ TeV and $m_{Z_D} = 800$ GeV, it is satisfied for $g_D = 0.86$. Once produced, $Z_D$ thermalizes with the SM particles through $h_D$ and its interactions listed in Eqs. (3) and (5).

As for direct detection, $N$ interacts with quarks through their scalar counterparts. For simplicity, let $g_u^L = g_u^R = g_d^L = g_d^R = g_0$ and $m_{\tilde{u}_L} = m_{\tilde{u}_R} = m_{\tilde{d}_L} = m_{\tilde{d}_R} = \tilde{m}_0$, then the elastic scattering of $N$ off a Xenon nucleus per nucleon is given by

$$\sigma_0 = \frac{g_0^4\mu^2}{64\pi(\tilde{m}_0^2 - m_N^2)^2},$$  \hspace{1cm} (9)

where $\mu = m_Nm_{Xe}/(m_N + m_{Xe})$ is the reduced mass of $N$. For $m_N = 1$ TeV, and using $m_{Xe} = 122.3$ GeV with $\sigma_0 < 10^{-45}$ cm$^2$ \cite{9}, the lower limit on $\tilde{m}_0/g_0$ is about 70 TeV. Note that this severe constraint rules out the possibility that the relic abundance of $N$ comes from this interaction. Without the dark gauge $U(1)_D$ symmetry, $N$ would not be a dark matter candidate in this case.

**Freeze-In Scenario:** An alternative for $N$ to be dark matter is the freeze-in mechanism \cite{10}. Here $N$ is assumed to be light, and its only production is through Higgs decay \cite{11} as shown in Fig. 4. This scenario works if the reheat temperature of the Universe is 1 to 10 TeV, and...
the thermal production of $N$ is very much suppressed, so that its relic abundance comes only from Higgs decay until the latter goes out of thermal equilibrium with the other SM particles. The effective $h$ coupling to $\bar{N}N$ is

$$g_h = \frac{g_f^2 \mu_f m_f}{8\pi^2} F(\tilde{m}_f^2, m_f^2),$$

where $F(x, y) = (x - y)^{-1} - y \ln(x/y)(x - y)^{-2}$, and $\mu_f$ is the $h\tilde{f}_L\tilde{f}_R$ coupling. The decay rate of $h \to N\bar{N}$ is then

$$\Gamma_h = \frac{g_h^2 m_h}{8\pi} \sqrt{1 - 4r^2(1 - 2r^2)},$$

where $r = m_N/m_h$. The correct relic abundance is possible if $g_h$ is very small. Hence $N$ could be FIMP (Feebly Interacting Massive Particle) dark matter [10], and for $r << 1$, the right number density is obtained for [12]

$$g_h \sim 10^{-12} r^{-1/2}.$$  

Assuming $m_N = 2$ GeV and $\tilde{m}_f/g_f = 10^5$ GeV [13], this would require $\mu_f m_f \sim 6$ GeV$^2$. In this scenario, both $Z_D$ and $h_D$ should also be much heavier than $h$.

**Possible Light $Z_D$ and $h_D$:** In the freeze-out scenario, it is possible to have light $Z_D$ and $h_D$, such that $h$ decays to $h_D h_D$, then $h_D$ decays to $Z_D Z_D$, and $Z_D$ decays to an SM fermion-antifermion pair. This would result in multi-lepton final states of the SM Higgs boson decay,
as pointed out previously \[14\]. The respective decay rates are

\[
\Gamma(h \to h_D h_D) = \frac{\lambda_h^2 v^2}{16 \pi m_h} \sqrt{1 - \frac{4m_{h_D}^2}{m_h^2}}, \quad (13)
\]

\[
\Gamma(h_D \to Z_D Z_D) = \frac{9g_{h_D}^2}{16 \pi m_{h_D}} (m_{h_D}^2 + 2m_{Z_D}^2) \sqrt{1 - \frac{4m_{Z_D}^2}{m_{h_D}^2}}, \quad (14)
\]

\[
\Gamma(Z_D \to f \bar{f}) = \frac{g_{Z_D}^2 a_f^2}{96 \pi m_{Z_D} m_f^2} (m_{Z_D}^2 + 2m_f^2) (m_{Z_D}^2 + 8m_f^2) \sqrt{1 - \frac{4m_f^2}{m_{Z_D}^2}}. \quad (15)
\]

**Concluding Remarks** : In the SM, fermions dominate in the form of quarks and leptons. Bosons appear as the necessary vector gauge particles of the SM $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry, and only one physical scalar Higgs boson remains. It is postulated here that in the dark sector, the opposite occurs, with one neutral Dirac fermion $N$ and many scalars, i.e. the scalar partners of the SM quarks and leptons. They are charged under a dark $U(1)_D$ gauge symmetry, which is broken spontaneously through a neutral scalar by 3 units, resulting in one dark gauge boson $Z_D$ and one dark Higgs boson $h_D$. Consequently, a residual global dark number is identified as $D = 3B + L - (2j)_{[mod \ 2]}$, where $j$ is the intrinsic spin of the particle. The combined SM and dark sectors are also very suitable for obtaining scotogenic Dirac neutrino masses with the imposition of a softly broken $Z_2$ symmetry.

The stable dark matter is $N$. It may be thermally produced and annihilates to $Z_D$ in the freeze-out mechanism, or through the rare decay of the SM Higgs boson $h$ in the freeze-in mechanism. In the former case, $h$ may decay to $h_D h_D$, then $h_D$ to $Z_D Z_D$, then $Z_D$ to an SM fermion-antifermion pair, resulting in multilepton final states.

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