Comparative Analysis of the Results of Aerodynamic Calculation of a Spherical Blunted Cone on a Structured and Unstructured Grid

S T Surzhikov

Ishlinsky Institute for Problems in Mechanics, Russian Academy of Sciences, Vernadsky Street 101(1), Moscow, 119526, Russia

Dukhov Research Institute of Automatics (VNIIA), Suschevskaya Street 22, Moscow 127055, Russia

E-mail: surg@ipmnet.ru

Abstract. The results of numerical simulation and comparison with experimental data of the aerodynamic characteristics of a blunt cone are presented. Two authors' computer codes were used, intended for the numerical integration of the Navier-Stokes and Euler equations on tetrahedral unstructured and multi-block structured computational grids. Alternative numerical models on unstructured grids are analyzed in detail.

1. Introduction

The subject of study of this work are the aerodynamic characteristics of a finite-sized blunt-sphere circular cone, the flow around which was experimentally studied in [1].

This extremely simple configuration of the aerodynamic model allows us to study many of the characteristic features of the flow observed at the surface of high-speed aircraft. Therefore, in the literature one can find a large number of works devoted to this issue (see bibliography in [2, 3]).

Despite the abundance of computational and experimental information on the flow around blunted cones in the development of new, more advanced computer codes, there are often methodological questions, the answers to which are not obvious. In particular, significant problems arise in the numerical simulation of the flow around blunt bodies at hypersonic speeds [4] and, in particular, in the simulation of separated flows [5].

This work has the following objectives:

- comparison of the results of the calculation of the aerodynamic characteristics of a circular cone blunt over the sphere for the experimental conditions [1] obtained using different computer codes on a structured and unstructured grid;
- study of differences in the calculated data on the structure of the separated flow;
- study of differences in the calculated data on the flow around a blunt cone, obtained with different mathematically equivalent formulations of the Navier-Stokes and Euler equations;
- study of the influence of the method of specifying boundary conditions when using insufficiently detailed unstructured computational grids;
- presentation of various formulations of numerical modeling of the Navier-Stokes and Euler equations on unstructured grids.
All these questions, being largely methodical, are important when interpreting the results of numerical simulation of the aerodynamic characteristics of high-speed aircraft when compared with experimental data.

This work consists of three parts. In the first part, the Navier-Stokes system is formulated in a convenient form for numerical integration, and an algorithm for numerical simulation on unstructured tetrahedral grids is presented.

In the second part of the work, the calculated ratios used in numerical integration on tetrahedral grids are given.

In the third part of the work, the obtained calculated data on structured and unstructured calculation grids are analyzed and compared.

2. System of equations and numerical integration algorithm

The system of equations of continuity, Navier-Stokes and conservation of the total energy of a thermally perfect gas will be represented in a vector-matrix form:

\[
\frac{\partial U}{\partial t} + \frac{\partial}{\partial x}\left(F^x_p + F^x - T^x_r\right) + \frac{\partial}{\partial y}\left(F^y_p + F^y - T^y_r\right) + \frac{\partial}{\partial z}\left(F^z_p + F^z - T^z_r\right) = R ,
\]

where:

\[
F^\rho_p = \begin{bmatrix}
\rho u \\
\rho u^2 \\
\rho u w \\
\rho u E
\end{bmatrix},
\]

\[
F^\rho_p = \begin{bmatrix}
\rho v \\
\rho v^2 \\
\rho v w \\
\rho v E
\end{bmatrix},
\]

\[
F^\rho_p = \begin{bmatrix}
\rho w \\
\rho w^2 \\
\rho w E
\end{bmatrix},
\]

\[
F^p_p = \begin{bmatrix}
p \\
p \rho w \\
p w
\end{bmatrix},
\]

\[
F^p_p = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix},
\]

\[
F^p_p = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix},
\]

\[
F^p_p = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix},
\]

\[
T^\rho_r = \begin{bmatrix}
\tau_{xx} \\
\tau_{yx} \\
\tau_{zx}
\end{bmatrix},
\]

\[
T^\rho_r = \begin{bmatrix}
\tau_{xy} \\
\tau_{yy} \\
\tau_{zy}
\end{bmatrix},
\]

\[
T^\rho_r = \begin{bmatrix}
\tau_{xz} \\
\tau_{yz} \\
\tau_{zz}
\end{bmatrix},
\]

\[
T^\rho_r = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix},
\]

\[
T^\rho_r = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix},
\]

\[
T^\rho_r = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix},
\]

where: \( u, v, w \) are the projections of velocity \( V \) on the coordinate axes \( x, y, z \); \( \rho, p \) are the density and pressure; \( E = \rho(e + V^2/2) \) is the total internal energy per unit volume; \( e \) is the specific internal energy; \( f_x, f_y, f_z \) are the mass bulk forces; \( q_x, q_y, q_z \) are the projections of the heat flux.
density vector $\mathbf{q}$ onto the coordinate axes; $Q$ is the volumetric sources of heat; $\tau_{ij}$ are the components of the viscous stress tensor, which are components of the stress tensor components $\Pi_{ij} = -p\delta_{ij} + \tau_{ij}$, $\delta_{ij}$ is the delta Kronecker symbol ($\delta_{ij} = 1$, if $i = j$; $\delta_{ij} = 0$, if $i \neq j$), have the following form:

$$
\tau_{xx} = 2\mu \left[ 2 \frac{\partial u}{\partial x} - \left( \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \right], \quad \tau_{yy} = 2\mu \left[ 2 \frac{\partial v}{\partial y} - \left( \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) \right],
$$

$$
\tau_{zz} = 2\mu \left[ 2 \frac{\partial w}{\partial z} - \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right],
$$

$$
\tau_{xy} = \tau_{yx} = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \quad \tau_{xz} = \tau_{zx} = \mu \left( \frac{\partial w}{\partial z} + \frac{\partial u}{\partial x} \right), \quad \tau_{yz} = \tau_{zy} = \mu \left( \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right).
$$

Thermal and caloric equations of state of a perfect gas are the following:

$$
p = (\gamma - 1) \rho e, \quad e = c_v T
$$

where $T$ is the temperature; $c_v$ is the specific heat at constant volume.

When integrating the equations of motion of an inviscid and non-heat-conducting gas, the components of the viscous stress and heat flux tensor were set equal to zero. The boundary conditions for the velocity components were formulated either in the form of adhesion or in the form of slip (the normal component of the velocity is zero). Next, we set the components of the vector function $\mathbf{R}$ equal to zero.

A two-step algorithm for integrating the system of equations (1) was used. For convenience of the formulation of the algorithm, we represent (1) in the following symbolic form (in fact, the splitting scheme over physical processes is used), introducing the time integration step $\tau$:

$$
\frac{\mathbf{U}^{\tau+1} - \mathbf{U}}{\tau} + \frac{\mathbf{U} - \mathbf{U}^p}{\tau} + \frac{\partial}{\partial x} (\mathbf{F}_r^{\rho\nu}) + \frac{\partial}{\partial y} (\mathbf{F}_r^{\rho\nu}) + \frac{\partial}{\partial z} (\mathbf{F}_r^{\rho\nu}) +
$$

$$
+ \frac{\partial}{\partial x} (\mathbf{F}^x - T^x) + \frac{\partial}{\partial y} (\mathbf{F}^y - T^y) + \frac{\partial}{\partial z} (\mathbf{F}^z - T^z) = 0
$$

or, successively, in the “predictor” step for the components of velocity and total specific energy:

$$
\frac{\mathbf{U} - \mathbf{U}^p}{\tau} + \frac{\partial}{\partial x} (\mathbf{F}^x - T^x) + \frac{\partial}{\partial y} (\mathbf{F}^y - T^y) + \frac{\partial}{\partial z} (\mathbf{F}^z - T^z) = 0
$$

and on the "corrector" step

$$
\frac{\mathbf{U}^{\tau+1} - \mathbf{U}}{\tau} + \frac{\partial}{\partial x} (\mathbf{F}_r^{\rho\nu}) + \frac{\partial}{\partial y} (\mathbf{F}_r^{\rho\nu}) + \frac{\partial}{\partial z} (\mathbf{F}_r^{\rho\nu}) = 0
$$

Exploded formula formulas for the predictor step are

$$
\mathbf{u} = \mathbf{u}^p - \frac{\tau}{\rho^p} \frac{\partial \rho}{\partial x} + \frac{\tau}{\rho^p} \left( \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} \right),
$$

$$
\mathbf{v} = \mathbf{v}^p - \frac{\tau}{\rho^p} \frac{\partial \rho}{\partial y} + \frac{\tau}{\rho^p} \left( \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} \right),
$$

$$
\mathbf{w} = \mathbf{w}^p - \frac{\tau}{\rho^p} \frac{\partial \rho}{\partial z} + \frac{\tau}{\rho^p} \left( \frac{\partial \tau_{zz}}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} \right),
$$

$$
\frac{\partial}{\partial x} (\mathbf{F}^x - T^x) + \frac{\partial}{\partial y} (\mathbf{F}^y - T^y) + \frac{\partial}{\partial z} (\mathbf{F}^z - T^z) = 0
$$
\[ \tilde{w} = w^p - \frac{\tau}{\rho^p} \frac{\partial p}{\partial z} + \frac{\tau}{\rho^p} \left( \frac{\partial \tau_{x}}{\partial x} + \frac{\partial \tau_{y}}{\partial y} + \frac{\partial \tau_{z}}{\partial z} \right), \]

\[ \tilde{E} = E^p - \frac{\tau}{\rho^p} \left( \frac{\partial p u}{\partial x} + \frac{\partial p v}{\partial y} + \frac{\partial p w}{\partial z} \right) + \frac{\tau}{\rho^p} \left( \frac{\partial T_{x}^{x}}{\partial x} + \frac{\partial T_{y}^{y}}{\partial y} + \frac{\partial T_{z}^{z}}{\partial z} \right), \] (6)

where:

\[ T_{x}^{x} = \mu \tau_{xx} + \nu \tau_{xx} + w \tau_{xx}, \quad T_{y}^{y} = \mu \tau_{yy} + \nu \tau_{yy} + w \tau_{yy}, \quad T_{z}^{z} = \mu \tau_{zz} + \nu \tau_{zz} + w \tau_{zz}. \]

The pressure and components of the viscous stress tensor are calculated on the lower temporal layer "p." Note that at the "predictor" stage, the density does not change, that is, it is assumed that the values of the desired functions (6) change only due to the action of "force" factors. Thus, this first step is interpreted in the method of large particles [6].

At the "corrector" stage, the new density value is first calculated:

\[ \rho^{p+1} = \rho^p - \tau \left( \frac{\partial \rho^p u}{\partial x} + \frac{\partial \rho^p v}{\partial y} + \frac{\partial \rho^p w}{\partial z} \right), \] (7)

and then the remaining functions are calculated

\[ U^{p+1} = \tilde{U} - \tau \left( \frac{\partial F_{x}^{x}}{\partial x} + \frac{\partial F_{y}^{y}}{\partial y} + \frac{\partial F_{z}^{z}}{\partial z} \right). \] (8)

or

\[ u^{p+1} = \frac{\rho^p}{\rho^{p+1}} \tilde{u} - \frac{\tau}{\rho^{p+1}} \left( \frac{\partial \rho^p u \tilde{u}}{\partial x} + \frac{\partial \rho^p v \tilde{v}}{\partial y} + \frac{\partial \rho^p w \tilde{w}}{\partial z} \right), \]

\[ \tilde{v}^{p+1} = \frac{\rho^p}{\rho^{p+1}} \tilde{v} - \frac{\tau}{\rho^{p+1}} \left( \frac{\partial \rho^p u \tilde{u}}{\partial x} + \frac{\partial \rho^p v \tilde{v}}{\partial y} + \frac{\partial \rho^p w \tilde{w}}{\partial z} \right), \]

\[ \tilde{w}^{p+1} = \frac{\rho^p}{\rho^{p+1}} \tilde{w} - \frac{\tau}{\rho^{p+1}} \left( \frac{\partial \rho^p u \tilde{u}}{\partial x} + \frac{\partial \rho^p v \tilde{v}}{\partial y} + \frac{\partial \rho^p w \tilde{w}}{\partial z} \right), \]

\[ E^{p+1} = \frac{\rho^p}{\rho^{p+1}} \tilde{E} - \frac{\tau}{\rho^{p+1}} \left( \frac{\partial \rho^p \tilde{E}}{\partial x} + \frac{\partial \rho^p \tilde{E}}{\partial y} + \frac{\partial \rho^p \tilde{E}}{\partial z} \right). \]

3. Formulation of a finite volume method for a tetrahedral mesh

In this section, the calculated ratios for a finite volume representation of functions and their derivatives as applied to a tetrahedral elementary volume will be presented. The calculated ratios of the finite volume method are obtained by integrating (4) and (5) over the volume of the elementary computational cell selected in space (in our case, the tetrahedron).

Recall that by the Gauss-Ostrogradsky theorem for an arbitrary vector function \( \mathbf{A} \)

\[ \int_{V_{e}} \text{div} \mathbf{A} dW = \int_{S_{e}} \left( \frac{\partial A_{x}}{\partial x} + \frac{\partial A_{y}}{\partial y} + \frac{\partial A_{z}}{\partial z} \right) dW = \]

\[ = \int_{S_{e}} \mathbf{A} dS = \int_{S_{e}} (A_{x} i + A_{y} j + A_{z} k) (dS_{x} + j dS_{y} + k dS_{z}) = \]
\[
\int_{S_L} A_i dS_x + \int_{S_L} A_j dS_y + \int_{S_L} A_k dS_z ,
\]

where: \( W_l, S_L \) – volume and surface of the selected cell \( L \); \( i, j, k \) are the unit orts of a rectangular Cartesian coordinate system.

By virtue of the independence of the functions \( A_i, A_j, A_k \) we can put

\[
\left( \frac{\partial A_i}{\partial x} \right) dW = \int_{S_L} A_i dS_x , \quad \left( \frac{\partial A_j}{\partial y} \right) dW = \int_{S_L} A_j dS_y , \quad \left( \frac{\partial A_k}{\partial z} \right) dW = \int_{S_L} A_k dS_z ,
\]

where: \( W_L, S_L \) are the volume and surface of the selected cell \( L \).

Hence follows the method of calculating volume-averaged derivatives:

\[
\left( \frac{\partial f}{\partial x} \right)_{w_L} = \frac{1}{W_L} \int_{S_L} \frac{\partial f}{\partial x} dW = \frac{1}{W_L} \int f_k dS \approx \frac{1}{W_L} \sum_{j=1}^{4} f_j S_{L_j} \omega_{L_j} \approx \frac{1}{W_L} \sum_{j=1}^{4} \frac{1}{2} (f_j + f_{Lj}) S_{L_j} \omega_{L_j} . \quad (9)
\]

Here are used the projection of the elementary area \( dS \) on the axis \( dS_x, dS_y, dS_z \). They are determined by the formulas

\[
dS_x = \omega_{n_x} dS, \quad dS_y = \omega_{n_y} dS, \quad dS_z = \omega_{n_z} dS ,
\]

where: \( \omega_{n_x}, \omega_{n_y}, \omega_{n_z} \) are the direction cosines of external normal to the surface element \( j \) with respect to the coordinate axes \( x, y, z \).

Integration over the surface \( S_L \) of an elementary volume \( L \) is carried out for functions \( f_k \) along the faces of volume \( j \), (by area \( S_{L_j} \)), so the use of a half-sum of values near this \((L)\) and neighboring cell \((LB)\) somewhat increases the accuracy of the calculation.

By the analogy

\[
\left( \frac{\partial f}{\partial y} \right)_{w_L} = \frac{1}{W_L} \int_{S_L} \frac{\partial f}{\partial y} dW = \frac{1}{W_L} \int f_k dS \approx \frac{1}{W_L} \sum_{j=1}^{4} f_j S_{L_j} \omega_{L_j} \approx \frac{1}{W_L} \sum_{j=1}^{4} \frac{1}{2} (f_j + f_{Lj}) S_{L_j} \omega_{L_j} ,
\]

\[
\left( \frac{\partial f}{\partial z} \right)_{w_L} = \frac{1}{W_L} \int_{S_L} \frac{\partial f}{\partial z} dW = \frac{1}{W_L} \int f_k dS \approx \frac{1}{W_L} \sum_{j=1}^{4} f_j S_{L_j} \omega_{L_j} \approx \frac{1}{W_L} \sum_{j=1}^{4} \frac{1}{2} (f_j + f_{Lj}) S_{L_j} \omega_{L_j} .
\]

The volume-averaged derivatives of the components of the viscous stress tensor are determined in a similar way

\[
\left( \frac{\partial \tau_{pq}}{\partial x} \right)_{w_L} \approx \frac{1}{W_L} \sum_{j=1}^{4} \frac{1}{2} \left( \left\langle \tau^{L}_{pq} \right\rangle + \left\langle \tau^{L_{Bj}}_{pq} \right\rangle \right) S_{L_j} \omega_{L_j} , \quad \left( \frac{\partial \tau_{pq}}{\partial y} \right)_{w_L} \approx \frac{1}{W_L} \sum_{j=1}^{4} \frac{1}{2} \left( \left\langle \tau^{L}_{pq} \right\rangle + \left\langle \tau^{L_{Bj}}_{pq} \right\rangle \right) S_{L_j} \omega_{L_j} , \quad \left( \frac{\partial \tau_{pq}}{\partial z} \right)_{w_L} \approx \frac{1}{W_L} \sum_{j=1}^{4} \frac{1}{2} \left( \left\langle \tau^{L}_{pq} \right\rangle + \left\langle \tau^{L_{Bj}}_{pq} \right\rangle \right) S_{L_j} \omega_{L_j} \quad (10)
\]

The averaged components of the viscous stress tensor are calculated using the expressions for the averaged values of the derivatives of the velocity components:

\[
\left\langle \tau^{L}_{i} \right\rangle \approx \frac{2}{3} \mu_L \left[ 2 \left( \frac{\partial u}{\partial x} \right)_{L} - \left( \frac{\partial v}{\partial y} \right)_{L} + \left( \frac{\partial w}{\partial z} \right)_{L} \right] , \quad \left\langle \tau^{L_{Bj}}_{i} \right\rangle \approx \frac{2}{3} \mu_L \left[ 2 \left( \frac{\partial v}{\partial y} \right)_{L} - \left( \frac{\partial w}{\partial z} \right)_{L} + \left( \frac{\partial u}{\partial x} \right)_{L} \right] ,
\]
\[ \langle \tau^L_{\alpha \beta} \rangle \approx \frac{2}{3} \mu_L \left[ 2 \left( \frac{\partial \mathbf{u}}{\partial x} \right)_L - \left( \frac{\partial \mathbf{u}}{\partial x} + \frac{\partial \mathbf{v}}{\partial y} \right)_L \right], \]
\[ \langle \tau^L_{\alpha \beta} \rangle = \mu_L \left[ \frac{\partial \mathbf{u}}{\partial y} + \frac{\partial \mathbf{v}}{\partial z} \right], \]

To complete the calculations, it remains to determine the average derivatives of the velocity components

\[ \left\langle \frac{\partial \mathbf{V}}{\partial x} \right\rangle \approx \frac{1}{W_L} \sum_{j=1}^{4} \left( V_{p, j} + V_{p, LB_j} \right) S_j \omega_j^{L_j}, \]

\[ \text{where: } V_p = u, v, w; \quad p, q = x, y, z. \]

4. Numerical simulation method

Get the calculated ratios to determine the functions \( \bar{u}, \bar{v}, \bar{w}, \bar{E} \).

In order to find the intermediate value in the elementary tetrahedron with the number \( L \), we choose the 2nd line in the function vector \( \mathbf{U} \) and write the scalar equation for it

\[ \rho_L^p \bar{u}_L - u_L^p \tau = - \frac{\partial p}{\partial x} + \frac{\partial \tau_{\alpha \alpha}}{\partial x} + \frac{\partial \tau_{\alpha \beta}}{\partial y} + \frac{\partial \tau_{\alpha \gamma}}{\partial z}, \]

Integrating (13) by volume \( W_L \), we get

\[ W_L \rho_L^p \bar{u}_L - u_L^p \tau = - \int_{w_1}^{w_2} \frac{\partial p}{\partial x} dW + \int_{w_1}^{w_2} \frac{\partial \tau_{\alpha \alpha}}{\partial x} dW + \int_{w_1}^{w_2} \frac{\partial \tau_{\alpha \beta}}{\partial y} dW + \int_{w_1}^{w_2} \frac{\partial \tau_{\alpha \gamma}}{\partial z} dW. \]

Above, the integration formulas for the elementary tetrahedral volume were determined, according to which:

\[ \bar{u}_L = u_L^p - \frac{r}{\rho^p W_L} \sum_{j=1}^{4} S_j \omega_j^{L_j} \frac{1}{2} \left( p_L + p_{LB_j} \right) - S_j \omega_j^{L_j} \left\langle \tau_L^{\alpha \alpha} \right\rangle - S_j \omega_j^{L_j} \left\langle \tau_L^{\alpha \beta} \right\rangle - S_j \omega_j^{L_j} \left\langle \tau_L^{\alpha \gamma} \right\rangle, \]

where: \( S_j \) is the area of the \( j \)-th face of the element \( L \); \( \left\langle \tau_L^{\alpha \beta} \right\rangle \) is the averaged values of the components of the stress tensor in the element \( L \)

\[ \bar{u}_L = u_L^p - \frac{r}{\rho^p W_L} \sum_{j=1}^{4} S_j \omega_j^{L_j} \frac{1}{2} \left( p_L + p_{LB_j} \right) - S_j \omega_j^{L_j} \left\langle \tau_L^{\alpha \alpha} \right\rangle - S_j \omega_j^{L_j} \left\langle \tau_L^{\alpha \beta} \right\rangle - S_j \omega_j^{L_j} \left\langle \tau_L^{\alpha \gamma} \right\rangle, \]

\[ \bar{v}_L = v_L^p - \frac{r}{\rho^p W_L} \sum_{j=1}^{4} S_j \omega_j^{L_j} \frac{1}{2} \left( p_L + p_{LB_j} \right) - S_j \omega_j^{L_j} \left\langle \tau_L^{\alpha \alpha} \right\rangle - S_j \omega_j^{L_j} \left\langle \tau_L^{\alpha \beta} \right\rangle - S_j \omega_j^{L_j} \left\langle \tau_L^{\alpha \gamma} \right\rangle, \]

\[ \bar{w}_L = w_L^p - \frac{r}{\rho^p W_L} \sum_{j=1}^{4} S_j \omega_j^{L_j} \frac{1}{2} \left( p_L + p_{LB_j} \right) - S_j \omega_j^{L_j} \left\langle \tau_L^{\alpha \alpha} \right\rangle - S_j \omega_j^{L_j} \left\langle \tau_L^{\alpha \beta} \right\rangle - S_j \omega_j^{L_j} \left\langle \tau_L^{\alpha \gamma} \right\rangle, \]

\[ \bar{E} = E^p - \frac{r}{\rho^p W_L} \sum_{j=1}^{4} S_j \omega_j^{L_j} \frac{1}{2} \left( p_L + p_{LB_j} \right) + S_j \omega_j^{L_j} \frac{1}{2} \left( p_L + p_{LB_j} \right), \]
\[ + S_{L_j} \omega_j \frac{1}{2} \left( p_{L_j} v_{L_j} + p_{L_{LB_j}} v_{L_{LB_j}} \right) \] 

\[ + \frac{\tau}{\rho^p W_L} \sum_{j=1}^4 S_{L_j} \omega_j \frac{1}{2} \left( T_{t,5,L_j}^x + T_{t,5,L_{LB_j}}^x \right) + S_{L_j} \omega_j \frac{1}{2} \left( T_{t,5,L_j}^y + T_{t,5,L_{LB_j}}^y \right) + \] 

\[ + S_{L_j} \omega_j \frac{1}{2} \left( T_{t,5,L_j}^z + T_{t,5,L_{LB_j}}^z \right) \] 

(14)

where the functions \( T_{t,5,L_j}^p \) are calculated by the formulas

\[ T_{t,5,L_j}^x = \bar{u}_L \left( \tau_{x_L}^t \right) + \bar{v}_L \left( \tau_{x_L}^t \right) + \bar{w}_L \left( \tau_{x_L}^t \right), \]

\[ T_{t,5,L_j}^y = \bar{u}_L \left( \tau_{y_L}^t \right) + \bar{v}_L \left( \tau_{y_L}^t \right) + \bar{w}_L \left( \tau_{y_L}^t \right), \]

\[ T_{t,5,L_j}^z = \bar{u}_L \left( \tau_{x_L}^t \right) + \bar{v}_L \left( \tau_{y_L}^t \right) + \bar{w}_L \left( \tau_{z_L}^t \right), \]

After determining the intermediate values of the velocity components, you can refer to the preliminary solution of the continuity equation

\[ \int_{W_L} \frac{\partial \rho}{\partial t} dW + \int_{W_L} \text{div} (\rho \vec{V}) dW = 0, \] 

(15)

or

\[ \frac{\rho_{L_j}^{p+1} - \rho_{L_j}^p}{\tau} W_L + \int_{S_L} \rho \vec{V} dS = 0, \]

or

\[ \frac{\rho_{L_j}^{p+1} - \rho_{L_j}^p}{\tau} W_L + \sum_{j=1}^4 S_{L_j} G_{L_j} = 0, \] 

(16)

If \( \Omega_{S_j} \) is the unit vector of the external normal to the surface element \( dS \), then with a positive value of the flux density through the surface element \( dS \)

\[ G_{L,dS} = \rho \left( \vec{V}_{S_L} \cdot \Omega_{S_L} \right) > 0, \]

gas flows out of the volume, and when

\[ G_{L,dS} < 0, \]

gas flows into the volume.

With a finite-difference representation:

\[ \rho_{L_j}^{p+1} = \rho_{L_j}^p - \frac{\tau}{W_L} \sum_{j=1}^4 S_{L_j} \rho_{L_j} \vec{V}_{S_{L_j}}, \] 

(17)

\[ \rho_{L_j} = \begin{cases} \rho_{L_j}^p, & \vec{V}_{S_{L_j}} > 0 \\ \rho_{L_{LB_j}}^p, & \vec{V}_{S_{L_j}} < 0 \end{cases} \]
where $V_{s_{ij}}$ is the normal velocity to $j$ face.

The calculation of the value $V_{s_{ij}}$ requires the use of some additional considerations. One of the easiest ways is this. Considering that the velocity components averaged over the elementary volumes $u_L$, $v_L$, $w_L$ were found at the predictor step, using them we find the desired normal velocity component

$$V_{s_{ij}} = u_{L_{s_{ij}}} + v_{L_{s_{ij}}} + w_{L_{s_{ij}}}.$$  \hspace{1cm} (18)

By analogy, for the normal velocity component on the same face of the adjacent volume:

$$V'_{s_{ij}} = -u_{L_{s_{ij}}} - v_{L_{s_{ij}}} - w_{L_{s_{ij}}}.$$  \hspace{1cm} (19)

Note that the direction cosines in this case still define the outer normal to the faces of the volume $L$.

For convenience, we introduce two auxiliary quantities:

$$\left( \begin{array}{c} V_{s_{ij}} \\ V'_{s_{ij}} \end{array} \right) = \left( \begin{array}{c} V_{s_{ij}} + |V_{s_{ij}}| \\ V_{s_{ij}} - |V_{s_{ij}}| \end{array} \right).$$

Then the relation (17) can be written in the form:

$$\rho_{L}^{p+1} = \rho_{L}^{p} - \frac{\tau}{W_{p+1}} \sum_{j} S_{L_{j}} \left( \rho_{L_{j}} V_{s_{ij}}^{+} + \rho_{L_{j}} V'_{s_{ij}}^{-} \right).$$  \hspace{1cm} (20)

The components of velocity and total energy are calculated according to the same scheme at the final stage of the corrector:

$$u_{L}^{p+1} = \rho_{L}^{p} \frac{\tau}{\rho_{L}^{p+1}} \bar{u}_{L} - \frac{\tau}{\rho_{L}^{p+1}} \sum_{j} S_{L_{j}} \left( \rho_{L_{j}} \bar{u}_{L} V_{s_{ij}}^{+} + \rho_{L_{j}} \bar{u}_{L} V'_{s_{ij}}^{-} \right)$$

$$v_{L}^{p+1} = \rho_{L}^{p} \frac{\tau}{\rho_{L}^{p+1}} \bar{v}_{L} - \frac{\tau}{\rho_{L}^{p+1}} \sum_{j} S_{L_{j}} \left( \rho_{L_{j}} \bar{v}_{L} V_{s_{ij}}^{+} + \rho_{L_{j}} \bar{v}_{L} V'_{s_{ij}}^{-} \right)$$

$$E_{L}^{p+1} = \rho_{L}^{p} \frac{\tau}{\rho_{L}^{p+1}} \bar{E}_{L} - \frac{\tau}{\rho_{L}^{p+1}} \sum_{j} S_{L_{j}} \left( \rho_{L_{j}} \bar{E}_{L} V_{s_{ij}}^{+} + \rho_{L_{j}} \bar{E}_{L} V'_{s_{ij}}^{-} \right).$$  \hspace{1cm} (21)

5. Using the equation of conservation of internal energy

The main advantage of the formulated system of equations is complete conservatism in the sense of defining [4]. The correct solution of this system of equations for the total specific energy $E = e + 0.5(u^2 + v^2 + w^2)$ ensures the conservation of this quantity.

However, this formulation has a known drawback associated with the possibility of obtaining non-physical negative specific internal energy, which is very likely, for example, in separated flows of supersonic flows.

This, most often, is related to the error accumulated in the uncoordinated solving of the equations of motion and the conservation of total energy.

There are a number of ways to prevent the generation of negative specific internal energy. One of them is the complete or partial rejection of the use of the equation for the specific total energy. The loss of the conservative property of total energy is not always fatal with the careful self-consistent solution of the complete system of equations.

Consider the features of the integration of the system of equations of motion using the equation of conservation of specific internal energy, written in vector-matrix form:
\[
\frac{\partial W}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} + \frac{\partial G}{\partial z} = R
\]  \quad (23)

\[
W = \begin{bmatrix}
\rho \\
\rho u \\
\rho v \\
\rho w \\
\rho e
\end{bmatrix}, \quad R = \begin{bmatrix}
0 \\
\rho f_x \\
\rho f_y \\
\rho f_z \\
\Phi_\mu - p \cdot \text{div} \mathbf{V} + Q_z
\end{bmatrix},
\]

\[
E = \begin{bmatrix}
\rho u \\
\rho u^2 + p \\
\rho w \\
\rho u w + q_x \\
\rho u + q_x
\end{bmatrix}, \quad F = \begin{bmatrix}
\rho v \\
\rho v^2 + p \\
\rho w \\
\rho v w + q_y \\
\rho v + q_y
\end{bmatrix}, \quad G = \begin{bmatrix}
\rho w \\
\rho w^2 + p \\
\rho v w + q_z \\
\rho w e + q_z
\end{bmatrix}
\]

\[
\Phi_\mu = \tau_{ij} \frac{\partial u_i}{\partial x_j} = \nabla \cdot (\tau_{ij} \cdot \mathbf{V}) - (\nabla \cdot \tau_{ij}) \cdot \mathbf{V}
\]

is the dissipative function or

\[
\Phi_\mu = \mu \left[ 2 \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 + 2 \left( \frac{\partial w}{\partial z} \right)^2 + \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} + \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial z} + \frac{\partial w}{\partial z} \right)^2 \right]
\]

The method of transition from equation (1) to the equation for specific internal energy (23) is described in [4].

This equation has the following vector form

\[
\frac{\partial \rho e}{\partial t} + \text{div} (\rho e \mathbf{V}) = -\text{div} (\mathbf{q}) + \Phi_\mu - p \cdot \text{div} \mathbf{V}.
\]

Note that the last term in this equation is of fundamental importance in supersonic flows, especially in the absence of noticeable heat fluxes and small dissipation. Another mathematically equivalent formulation can be proposed

\[
\frac{\partial \rho e}{\partial t} + \text{div} (\rho e \mathbf{V}) = -\text{div} (\mathbf{q}) + \Phi_\mu - \rho \text{div} \left( p \mathbf{V} \right) + \mathbf{V} \text{grad} (p).
\]

For a perfect gas with a thermal equation of state, we get another equivalent formulation

\[
\frac{\partial \rho e}{\partial t} + \text{div} (\rho e \mathbf{V}) = -\text{div} (\mathbf{q}) + \Phi_\mu - (\gamma - 1)(\rho e) \text{div} \mathbf{V},
\]

or

\[
\frac{\partial \rho e}{\partial t} + \gamma \text{div} (\rho e \mathbf{V}) = -\text{div} (\mathbf{q}) + \Phi_\mu - (\gamma - 1) \mathbf{V} \text{grad} (\rho e),
\]

And, finally, using the thermodynamic ratio that determines the specific enthalpy
\[ h = e + \frac{p}{\rho}, \]

one can get

\[ \frac{\partial \rho h}{\partial t} + \text{div}(\rho h \mathbf{V}) = -\text{div}(\mathbf{q}) + \Phi_\mu + \mathbf{V} \text{grad}(p), \]

Using the thermal equation of state of a perfect gas, we rewrite (23) in the form:

\[ p = \rho h - \rho e, \]

\[ \rho h = \rho e + (\gamma - 1)\rho e = \gamma \rho e, \quad \rho e = \rho h / \gamma, \quad p = \rho h / \gamma = \frac{\gamma - 1}{\gamma} \rho h, \]

\[ \frac{\partial \rho h}{\partial t} + \text{div}(\rho h \mathbf{V}) = -\text{div}(\mathbf{q}) + \Phi_\mu + \frac{\gamma - 1}{\gamma} \mathbf{V} \text{grad}(\rho h). \]

Thus, we have the following mathematically equivalent equations for the conservation of specific internal energy:

\[ \frac{\partial \rho e}{\partial t} + \text{div}(\rho e \mathbf{V}) = -\text{div}(\mathbf{q}) + \Phi_\mu - p \text{div}(\mathbf{V}), \quad (24) \]

\[ \frac{\partial \rho e}{\partial t} + \text{div}(\rho e \mathbf{V}) = -\text{div}(\mathbf{q}) + \Phi_\mu - \text{div}(p \mathbf{V}) + \mathbf{V} \text{grad}(p), \quad (25) \]

\[ \frac{\partial \rho e}{\partial t} + \text{div}(\rho e \mathbf{V}) = -\text{div}(\mathbf{q}) + \Phi_\mu - (\gamma - 1)(\rho e) \text{div}(\mathbf{V}), \quad (26) \]

\[ \frac{\partial \rho e}{\partial t} + \gamma \text{div}(\rho e \mathbf{V}) = -\text{div}(\mathbf{q}) + \Phi_\mu - (\gamma - 1) \mathbf{V} \text{grad}(\rho e), \quad (27) \]

\[ \frac{\partial \rho h}{\partial t} + \text{div}(\rho h \mathbf{V}) = -\text{div}(\mathbf{q}) + \Phi_\mu + \mathbf{V} \text{grad}(p), \quad (28) \]

\[ \frac{\partial \rho h}{\partial t} + \text{div}(\rho h \mathbf{V}) = -\text{div}(\mathbf{q}) + \Phi_\mu + \frac{\gamma - 1}{\gamma} \mathbf{V} \text{grad}(\rho h). \quad (29) \]

Given the well-known simplicity of the thermal equation of state, it is easy to obtain similar equations for temperature:

\[ \frac{\partial \rho T}{\partial t} + \text{div}(\rho T \mathbf{V}) = -\frac{1}{c_v} \text{div}(\mathbf{q}) + \frac{1}{c_v} \Phi_\mu + \frac{(\gamma - 1)}{\gamma} \mathbf{V} \text{grad}(\rho T), \quad (30) \]

\[ \frac{\partial \rho T}{\partial t} + \text{div}(\rho T \mathbf{V}) = -\frac{1}{c_v} \text{div}(\mathbf{q}) + \frac{1}{c_v} \Phi_\mu - (\gamma - 1) \rho T \text{div}(\mathbf{V}), \quad (31) \]

\[ \frac{\partial \rho T}{\partial t} + \gamma \text{div}(\rho T \mathbf{V}) = -\frac{1}{c_v} \text{div}(\mathbf{q}) + \frac{1}{c_v} \Phi_\mu - (\gamma - 1) \mathbf{V} \text{grad}(\rho T), \quad (32) \]

and finally, using the continuity equation, we get:

\[ \frac{\partial T}{\partial t} + \frac{1}{\gamma} \mathbf{V} \text{grad}T = -\frac{1}{\rho c_p} \text{div}(\mathbf{q}) + \frac{1}{\rho c_p} \Phi_\mu + \frac{\gamma - 1}{\gamma} \mathbf{V} \text{grad}(\ln \rho) \quad (33) \]
\[ \frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T = - \frac{1}{\rho c_v} \text{div}(\mathbf{q}) + \frac{1}{\rho c_v} \Phi_\mu - (\gamma - 1) \text{div}(\mathbf{V}) \]  \hspace{1cm} (34)

\[ \frac{\partial T}{\partial t} + \gamma \mathbf{V} \cdot \nabla T = - \frac{1}{\rho c_v} \text{div}(\mathbf{q}) + \frac{1}{\rho c_v} \Phi_\mu + \]  
\[ + (\gamma - 1) \frac{T}{\rho} \text{div}((\gamma - 1) \frac{T}{\rho} \text{div}(\mathbf{V})) \]  \hspace{1cm} (35)

\[ \frac{\partial p}{\partial t} + \text{div}(\rho \mathbf{pV}) = - \frac{1}{c_p} \text{div}(\mathbf{q}) + \frac{1}{c_p} \Phi_\mu + \frac{(\gamma - 1)}{\gamma} \mathbf{V} \cdot \nabla (p). \]  \hspace{1cm} (36)

Note that the use of approximate finite-volume solution methods can lead to some differences obtained by the solutions presented by the relations. The analysis of these differences for some equations with examples of the relatively simple problem of flow around a blunt cone at different angles of attack is the subject of research of this work.

The presented formulations allow a significant variety of computational algorithms. In particular, from (23) at the predictor stage:

\[ \tilde{e}_L = e_{L} + \frac{\tau}{\rho L \mathbf{V}_L} \left[ u_L \sum_{j=1}^{4} S_L \omega_L^j \frac{1}{2} \left( p_L + p_{LB} \right) + ight. \]  
\[ \left. + v_L \sum_{j=1}^{4} S_L \omega_L^j \frac{1}{2} \left( p_L + p_{LB} \right) + w_L \sum_{j=1}^{4} S_L \omega_L^j \frac{1}{2} \left( p_L + p_{LB} \right) \right] \]

or, for example:

\[ \tilde{e}_L = e_{L} + \frac{\tau}{\rho L \mathbf{V}_L} \sum_{j=1}^{4} S_L \omega_L^j \frac{1}{2} \left( u_L - u_{LB} \right) + \]  
\[ + \frac{\tau}{\rho L \mathbf{V}_L} \sum_{j=1}^{4} S_L \omega_L^j \frac{1}{2} \left( v_L - v_{LB} \right) + \frac{\tau}{\rho L \mathbf{V}_L} \sum_{j=1}^{4} S_L \omega_L^j \frac{1}{2} \left( w_L - w_{LB} \right) \]  \hspace{1cm} (37)

For the numerical integration of the system of equations (23), we formulate the following boundary conditions.

\[\textbf{Figure 1. Definition of boundary values}\]
For each tetrahedral element adjacent to one of the faces of the streamlined surface, one can determine the local outward normal to the specified face. The cosine guides of this normal \( \omega_{LB}^x, \omega_{LB}^y, \omega_{LB}^z \) are already defined in the grid preprocessor, therefore the normal velocity component to the specified face is determined by the formula

\[
V_{n,LB} = u_n \omega_{LB}^x + v_n \omega_{LB}^y + w_n \omega_{LB}^z
\]

The no-flow condition of the streamlined surface is physically equivalent to the fact that the normal component of velocity in the fictitious cell is equal in magnitude and opposite to the direction of the normal velocity component found above.

Then the velocity projections in the dummy cell must satisfy the condition:

\[
V_{n,x,LB}^{'} = u_{n,LB}^{'} = -V_{n, LB} \omega_{LB}^x
\]

\[
V_{n,y,LB}^{'} = v_{n, LB}^{'} = -V_{n, LB} \omega_{LB}^y
\]

\[
V_{n,z,LB}^{'} = w_{n, LB}^{'} = -V_{n, LB} \omega_{LB}^z
\]

In the case of sticking conditions (when solving the Navier-Stokes equations), all components of the velocity on the surface should be zero, therefore:

\[
u_n^{'} = -u_{n, LB}
\]

\[
v_n^{'} = -v_{n, LB}
\]

\[
w_n^{'} = -w_{n, LB}
\]

6. Numerical simulation results
Numerical simulation of the aerodynamic characteristics of a blunt cone was carried out using the example of experimental data [1]. The asymmetric flow around at the angle of attack \( \alpha = 0 \) is considered. The conditions in the incident flow are presented in Table 1. The results of numerical simulations obtained using the NERAT-2D [8] computer code were used as reference solutions.

| \( p_\infty, \text{erg/sm}^3 \) | 188 000 |
| \( T_o, \text{K} \) | 56.9 |
| \( \gamma \) | 1.4 |
| \( M_\infty \) | 6.77 |
| \( T_w, \text{K} \) | 297 |
To begin, a study was conducted on the effect of the details of the computation grids on the flow field when using structured 4 block computation grids. This structured grid is shown in figure 2. Numbers of the grids nodes are presented in table 2.

| Table 2. Parameters of the calculation grids |
|---------------------------------------------|
|   | a | b | c |
| 1-st block | 51×45 | 101×89 | 201×177 |
| 2-nd block | 51×65 | 101×129 | 201×257 |
| 3-rd block | 51×45 | 101×89 | 201×177 |
| 4-th block | 45×45 | 89×89 | 177×177 |

From a comparison of calculated data in figures 3 and 4, it is concluded that the effects of the grids are rather weak. Moreover, among the analyzed features of the flow, we note the existence of a relatively high-temperature compressed layer near to the frontal surface. The axial temperature distribution along the front crucial line (see figure 5) shows that the calculation gives a good match of the temperature in the compressed layer with the theoretically predicted stagnation temperature.

\[ T_0 = T_\infty + \frac{1}{2} \frac{V_\infty^2}{\rho_\infty} = 580 \text{ K} \]

**Figure 2.** Structured 4-blocks grid at \( z = 0 \)

**Figure 3.a.** Temperature (in K) and longitudinal velocity \( V_x = u/V_\infty \) for the first structured grid

**Figure 3.b.** Temperature (in K) and longitudinal velocity \( V_x = u/V_\infty \) for the second structured grid
Figure 3, c. Temperature (in K) and longitude velocity $V_x = u/V_\infty$ for the third structured grid.

Figure 4, a. Pressure $P = p/\rho_\infty V_\infty^2$ and Mach number for the first calculation grid.

Figure 4, b. Pressure $P = p/\rho_\infty V_\infty^2$ and Mach number for the second calculation grid.

Figure 4, c. Pressure $P = p/\rho_\infty V_\infty^2$ and Mach number for the third calculation grid.

Here the influence of the details of the computational grid is negligible. It should be borne in mind the temperature drop in the boundary layer. It can be seen from figure 5 that in the conditions under consideration the thickness of the boundary layer does not exceed 0.002 cm.

Figure 6 shows the temperature distribution along the back of the critical line. Here a somewhat more noticeable influence of the computational grid is observed, which is not surprising, since a very complex flow structure is observed in the zone of separated flow, and the computational grid here is rather coarse.
Figure 7. Flow field and pressure $p/p_\infty V_\infty^2$ in the bottom area

Figure 8. Density $\text{Ro} = \rho/\rho_\infty$ and Mach numbers

We draw attention to the peculiarity of the temperature distribution in this flow region. Near the surface, there is a strong increase in temperature to almost 400 K. This increase is due to the heating of the gas moving to the surface with the surface itself. The flow structure is clearly visible in figure 7. The return flow region with the number $M > 2$ (see figure 5) is formed near the axis of symmetry inside the region of the separated flow beyond the rear cone boundary. At the same time, the pressure in the return flow area is very low, less than $0.001 \times p_\text{din} = \rho_\infty V_\infty^2$ (see figure 6). An approximately 5-fold increase in pressure is observed near the surface. However, even here, the pressure is significantly lower (about 3 times) than in the near wake (at a distance of about 15 cm from the rear section of the cone).

It is noteworthy that the low pressure and density areas located in the middle of the vortex flow zone (see figure 8). It follows that in order to correctly calculate the pressure distribution in a given region a very detailed grid resolution is required.

Axial distributions along the back crucial line for three calculation grids are shown for the Mach numbers, the longitudinal velocity and pressure in figures 9–11. Same as for the temperature, the influence of calculation grids is more pronounced for flow field in near and far wake. Distribution of pressure along the surface of the cone, which is shown in figure 12, shows that the influence of the calculation grids is only noticeable for the backside surface. Also noteworthy is the local increase in pressure in the area of flow separation near the axis (figure 11).
Figures 11–16 show the radial distributions of Mach numbers, pressure, temperature, and longitudinal velocity at different distances from the back surface of the cone. With increasing distance from the surface, a pronounced current in the wake forms, which, as is well known, is well described by the boundary layer marching method.

In figures 17, a, b an image of an unstructured grid in the plane of symmetry is given. For comparison, figure 17, b shows the zoomed element of the grid.

The results of the numerical simulation of the flow around a cone on unstructured grids are shown in figures 18–23. These data are obtained using the Navier-Stokes system of equations together with the equation for the conservation of total energy. Note that these calculations were performed on very coarse grids. Figures 17 show that the smallest linear size of the tetrahedral cell at the surface is approximately 0.1 cm, which exceeds the thickness of the boundary layer at the stagnation point by more than an order of magnitude.
Figure 15. Radial distribution of temperature at different distances along back crucial line

Figure 16. Radial distribution of longitudinal velocity at different distances along back crucial line

Figure 17, a. Unstructured mesh in the plane of symmetry z = 0. General view. The number of tetrahedral elements $N_{el} = 686,145$ ($N_{pt} = 140,889$).

Figure 17, b. Unstructured mesh in the plane of symmetry z = 0 near the cone.

Figure 18. Mach number. The Navier-Stokes equations with the full energy conservation equation.

Figure 19. Pressure $P_{res} = \rho/\rho_{din}$. The Navier-Stokes equations with the full energy conservation equation.
**Figure 20.** Density $\text{Ro} = \rho / \rho_c$.
The Navier-Stokes equations with the full energy conservation equation.

**Figure 21.** Temperature (in K).
The Navier-Stokes equations with the full energy conservation equation.

**Figure 22.** Longitudinal velocity $V_x = u / V_\infty$.
The Navier-Stokes equations with the full energy conservation equation.

**Figure 23.** Distribution of pressure along surface versus axial coordinate $x$.
The Euler equations with the full energy conservation.

**Figure 24.** Distribution of pressure along surface versus axial coordinate $x$.
The Navier-Stokes equations with the full energy conservation.
Nevertheless, as the practice of calculations shows, including the calculations of this work, these grids can be used to determine the aerodynamic characteristics of supersonic flow around bodies of complex shape. Therefore, in this case, we will be interested only in the values of the drag coefficient and the general pattern of flow around the blunted cone.

Table 3 presents the values of the drag coefficient

\[
C_d = \frac{1}{S_{mid}} \left( \frac{1}{2} \rho \bar{V}_{\infty} \right) \int_{S} \left[ \left( \rho - \rho_s \right) \cdot \omega_n^\tau + \tau \omega_r^\tau \right] \, ds
\]

obtained in this paper by different methods. Here \( \tau \) is the friction force at the surface; \( \omega_n^\tau, \omega_r^\tau \) direction cosines of the normal and tangent to the surface.

With high accuracy, these values coincide for all the methods used and modifications of the computational algorithms, as it is seen from table 3.

| Calculation method, algorithm | \( C_p \) |
|------------------------------|-----------|
| Calculation on a structured grid | 0.160 |
| I | 0.160 |
| II | 0.160 |
| III | 0.160 |
| Unstructured grid. The Navier-Stokes equation with the total energy conservation equation | 0.163 |
| Unstructured grid. The Navier-Stokes equation with the specific internal energy conservation equation | 0.163 |

The overall configuration of the flow field is similar to that obtained on structured grids. In particular, the same size of the disturbed flow region is observed for the distributions of Mach numbers and pressure in figures 18, 19 and figure 4. The configuration of the flow field in the region of detached flow and the near wake differs from the presented in figures 2–4 with the lower number of details of the flow field, which is quite natural. Particular, the three-dimensional calculation shows the absence of symmetry in the resulting solution due to unsymmetrical calculation mesh. However, the main features of the flow, such as a drop in temperature and pressure in the region of reciprocating vortex motion, acceleration of flow behind the trailing edge of the cone, a local increase in pressure in the region of flow separation in the near flow wake, are remained in force.

We emphasize that due to the limited computational grid, in this case, it is not possible to talk about solving the Navier-Stokes equations, at least in the region of large function gradients near the surface. Obviously, in this case, it is more correct to analyze the results of the integration of the Euler equations (with the corresponding boundary conditions). The results of such calculations for the pressure distribution along the surface are presented in figure 23. Comparison of figures 23 and 24 show the closeness of the spatial distributions of the pressure. It is noteworthy that the coefficients of resistance of the blunted cone being calculated for these two models are almost equal to each other.

Basically, the use of the Navier-Stokes equations on coarse unstructured grids could not be justified at all (due to the incorrectness of such a formulation). But as the practice of calculations has shown, the use of sticking conditions makes the computational procedure more stable. Therefore, when establishing the fact of a weak influence of the choice of the method on the solution of the problem of determining aerodynamic coefficients, preference can be given to the formulation of Navier-Stokes. We note that a partial justification for this is the solution of the problem of determining the resistance coefficients on structured grids, where it was shown that the contribution of surface friction stresses to
the value of $c_p$ in the case under consideration is less than 0.1%. The analogous conclusion can be made concerning the use of the equation of conservation of internal energy.

Figures 25–29 show distribution of gas-dynamic functions calculated with the use of the Navier-Stokes equations and equation for conservation of internal energy. Let's pay attention to the fact that despite the closeness of the results of the calculation of the resistance coefficients (see table 3) the fields of the dynamic functions differ markedly from those calculated using the total energy conservation equation (figures 18–22).

![Figure 25](image)

**Figure 25.** Mach number. Equations Navier-Stokes with the equation for conservation of the internal energy.

![Figure 26](image)

**Figure 26.** Pressure $P_{res} = \frac{p}{\rho_\infty} V^2$. The Navier-Stokes equations with the equation for conservation of the internal energy.

![Figure 27](image)

**Figure 27.** Density $Ro = \frac{\rho}{\rho_\infty}$. The Navier-Stokes equations with the equation for conservation of the internal energy.
Finally, we will consider some results of calculations of gas-dynamic functions obtained when solving the problem on unstructured grids with the use of some alternative formulations.

Figures 30, 31 show the pressure distribution along the surface in calculations based on the Euler and Navier-Stokes models with the use of the equation of conservation of total energy. The identity of the results is obvious. Some slight difference in pressure distribution is observed in figures 32, 33. In this case, the Navier-Stokes and Euler models were used in conjunction with the equation for the conservation of internal energy in the form (24). In figures 34 and 35 similar data are shown but obtained using equations (25) and (26). Some non-monotonous, observed in figures 33 and 34, correspond to the established solution. Note that in figures 23, 24, 30–35 there are two almost coinciding curves corresponding to the pressure distribution on the upper and lower parts of the surface.

In general, we can state the closeness of the pressure distribution for all the calculated cases, which leads to the closeness of the results for the integral resistance coefficient (see table 3). In figures 36, 37 shows the temperature distribution along the surface of the cone. In this case equations of Euler and Navier-Stokes were used with the equation for conservation full energy.

It should be noted that in the considered case of a very coarse unstructured mesh, it is more correct to speak about the average temperature in tetrahedral elements, one face of which lies on the surface. At this value have a fundamental attention a number of factors:

- The details of the computational grid near the surface;
- The boundary conditions on the surface.

In the case shown in figures 36, 37 the boundary condition of the 1-st kind was used, that is, the surface temperature Tw was set. Figure 41 shows the temperature distributions along the surface for the two kinds of boundary conditions.

From the analysis of data in figures 36, 37 can be seen, that temperature distributions are close when using the total energy conservation equation. Moreover, even the stagnation temperature at the crucial point is close to the theoretical value. The use of the internal energy conservation equation (figures 38–41) leads to a noticeable difference in the temperature distribution at the surface. But use the Euler equations gives better agreement on the stagnation temperature (figure 38).
Figure 30. Distribution pressure along the surface. The Euler equations with the internal energy conservation equation.

Figure 31. Distribution pressure along the surface. The Navier-Stokes with the internal energy conservation equation.

Figure 32. Distribution of pressure along the surface. The Navier-Stokes equations with internal energy conservation equation in form (25).

Figure 33. Distribution of pressure along the surface. The Navier-Stokes equations with internal energy conservation equation in form (26).

Figure 34. Distribution of pressure along the surface. The Navier-Stokes equations with internal energy conservation equation in form (27).

Figure 35. Distribution of pressure along the surface. The Navier-Stokes equations with internal energy conservation equation in form (28).
**Figure 36.** Distribution of temperature along the surface. The Euler equations with the full energy conservation equation.

**Figure 37.** Distribution of temperature along the surface. The Navier-Stokes equations with full energy conservation equation.

**Figure 38.** Distribution of temperature along the surface. The Euler equations with internal energy conservation equation.

**Figure 39.** Distribution of temperature along the surface. The Navier-Stokes equations with internal energy conservation equation.

**Figure 40.** Influence of boundary conditions on temperature distribution along surface. The Euler equations with full energy conservation equation.

**Figure 41.** Influence of boundary conditions for temperature distribution along surface. The Euler equations with internal energy conservation equation.
7. Conclusion

Using the example of experimental data on the flow around a blunt cone, a comparative numerical study of the Navier-Stokes and Euler computational models on structured and unstructured tetrahedral nets was performed.

A detailed derivation of the calculated ratios of the numerical splitting scheme over physical processes for unstructured computational grids is presented.

A number of alternative equations have been proposed and investigated using the equation for the conservation of specific internal energy.

The data on the structure of the flow in the vicinity of the blunt cone at $M = 6.7$ are obtained by calculation and showed good agreement with available experimental data.

Acknowledgments

The work was supported by the Russian Foundation for Basic Research grant # 19-01-00515.

References

[1] Neal L Jr 1963 Aerodynamic characteristics at a Mach number of 6.77 of a 90 cone configuration, with and without spherical afterbodies, at angles of attack up to 1800 with various degrees of nose blunting NASA TN D-1606
[2] Bertin J J 1994 Hypersonic aerothermodynamics AIAA Education Series 608 p
[3] Anderson J D 1989 Hypersonic and High Temperature Gas Dynamics McGraw-Hill, Inc. 690 p.
[4] Tannehill J C, Anderson D A, Pletcher R H 1997 Computational Fluid Vtchanics and Heat Transfer Taylor & Francis 792 p.
[5] Chang P K Separation of Flow Pergamon Press Inc 771 p.
[6] Ewans M V and Harlow F H 1957 The particle-in-cell method for hydrodinamic calculations Los Alamos Scientific Lab Rept NLA–2139 Los Alamos
[7] Belotserkovsky O M and Davydov Yu M 1982 Method of the Large Particles for Gas Dynamics Moscow: Nauka p 391 (in Russian)
[8] Surzhikov S T and Shang J S 2012 Convective heating of hemisphere cylinder at hypersonic velocities. AIAA 2012-0364
[9] Surzhikov S T 2017 Validation of computational code UST3D by the example of experimental aerodynamic data J.Phys.: Conf. Ser. 815 012023