Estimating the Charm Quark Diffusion Coefficient and thermalization time from D meson spectra at RHIC and LHC

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(Dated: July 19, 2017)

We describe the propagation of charm quarks in the quark-gluon plasma (QGP) by means of a Boltzmann transport approach. Non-perturbative interaction between heavy quarks and light quarks have been taken into account through a quasi-particle approach in which light partons are dressed with thermal masses tuned to lQCD thermodynamics. Such a model is able to describe the main feature of the non-perturbative dynamics: the enhancement of the interaction strength near $T_c$. We show that the resulting charm in-medium evolution is able to correctly predict simultaneously the nuclear suppression factor, $R_{AA}$, and the elliptic flow, $v_2$, at both RHIC and LHC energies and at different centralities. The hadronization of charm quarks is described by mean of an hybrid model of fragmentation plus coalescence and plays a key role toward the agreement with experimental data.

We also performed calculations within the Langevin approach which can lead to very similar $R_{AA}(p_T)$ as Boltzmann, but the charm drag coefficient as to be reduced by about a 30% and also generates an elliptic flow $v_2(p_T)$ is about a 15% smaller. We finally compare the space diffusion coefficient $2\pi T D_s$, extracted by our phenomenological approach to lattice QCD results, finding a satisfying agreement within the present systematic uncertainties. Our analysis implies a charm thermalization time, in the $p \rightarrow 0$ limit, of about $4-6\, fm/c$ which is smaller than the QGP lifetime at LHC energy.

PACS: 25.75.-q; 24.85.+p; 05.20.Dd; 12.38.Mh

I. INTRODUCTION

The study of QCD matter under extreme conditions of high temperatures is the primary purpose of ultra-relativistic heavy ion collisions which are being performed at the Relativistic Heavy-ion Collider (RHIC) and at the Large Hadron Collider (LHC). The energy deposited during the collisions produce a medium consisting of deconfined quarks and gluons called Quark Gluon Plasma (QGP)\cite{1,2}. An essential role to characterize the QGP can be played by the hard probes created in the initial stage of the collisions. Among them heavy quarks (HQs), charm and bottom, provide a very promising probe since they travel through the expanding medium interacting with the light particles but their number is expected to be conserved due to the large $M/T$ ratio. Therefore HQ can probe the whole evolution of the QGP and produced out-of-equilibrium are expected to conserve memory of the history of the plasma evolution \cite{3,4}.

Moreover HQ production can be calculated in next to leading order pQCD scheme and before the first experimental results it was expected that their interaction with the medium could be characterized by means of perturbative QCD which led to the expectations of a small suppression of the spectra and a small elliptic flow. However the first observations of non photonic electrons coming from heavy quark decays measured in Au+Au at $\sqrt{s_{NN}} = 200\, GeV$ at RHIC\cite{8,10} shown a surprisingly small $R_{AA}$ and a quite large elliptic flow $v_2$, indicating a quite strong interactions between HQ and the medium which is substantially beyond the expectations from perturbative QCD\cite{11,12}. These observations triggered many studies in which non perturbative approach have been implemented. One of this approach consists of including non-perturbative contributions\cite{13} from the quasi-hadronic bound state with a subsequent hadronization by coalescence and fragmentation \cite{12,14}. Other approaches make use of a pQCD framework supplemented by Hard Thermal Loop (HTL) in order to evaluate Debye mass and running coupling constant \cite{15,16}. Another efficient way is to use a quasi-particle approach in which non perturabative effects are considered by introducing a thermal mass for the particle in the bulk, $m(T) \sim g(T)T$. A fit to lQCD thermodynamics allows to determine $g(T)$\cite{19,20}. All these models are based on collisional energy-loss which should be the dominant mechanism in the low momentum region of charm spectra\cite{21,22}, $p_T \lesssim 3-5M_{HQ}$, while at higher momenta there is a consensus that radiative energy loss becomes dominant even if self-consistently collisional energy loss can never be discarded\cite{23,24}. Furthermore in the high-$p_T$ region pQCD schemes have shown to be able to account for the observed suppression of the spectra\cite{24,25,26} and some group obtained also a satisfying prediction for the elliptic flow\cite{23,24}.
In this present paper we will focus on the results of a quasi-particle model (QPM) for charm quarks. In Ref. [13] it is shown that the quasi particle approach is able to reproduce the lattice QCD equation of state. The extracted coupling $g(T)$ appears to have a significant deviation from pQCD especially a $T \rightarrow T_c$. This leads to a weakly T dependent drag coefficient $\gamma(T)$ at variance with pQCD with a constant coupling $g$ or AdS/CFT where both predict a $T^2$ dependence drag coefficients. Such a feature of QPM has been found by other groups [20, 31] that has also shown that the pattern remains quite similar even when quasi-particle widths (off-shell dynamics) are accounted for. It has been thoroughly studied in Ref. [31] for heavy quarks and also in [32, 33] for the light sector that an interaction increasing as the temperature decreases is one of the key ingredient to generate a larger elliptic flow and thus reducing the tension between the $R_{AA}$ and $v_2$ observed experimentally and calculated theoretically. This along with an hadronization via coalescence is also a main underlying reason of the early T-matrix approach for heavy quark hadronization and the Quasi-Particle approach for HQs. In section II we discuss briefly the Boltzmann transport equation and coalescence to consider the hadronization process of heavy quarks into heavy flavor mesons in QGP. Section IV is devoted to the comparison between the simulation results with experimental results at different colliding energy and different centralities. In section V, we discuss the heavy quark transport coefficient obtained within the present approach. Section V contains a summary and some concluding remarks.

II. TRANSPORT EQUATION FOR CHARM QUARKS IN THE QGP

The evolution of the charm quark distribution function is obtained solving the relativistic Boltzmann transport equations for charm quarks scattering in a bulk medium of quarks and gluons:

$$p^{\mu} \partial_{\mu} f_Q(x, p) = C[f_q, f_g, f_Q](x, p)$$

$$p^{\mu}_{x} \partial_{\mu_{x}} f_q(x, p) = C[f_Q, f_g](x_q, p_{g})$$

$$p^{\mu}_{g} \partial_{\mu_{g}} f_g(x, p) = C[f_Q, f_q](x_g, p_{q})$$

where $f_k(x, p)$ is the on-shell phase space one-body distribution function for the $k$ parton and $C[f_q, f_g, f_Q](x, p)$ is the relativistic Boltzmann-like collision integral and the phase-space distribution function of the bulk medium consists of quark and gluons entering the equation for charm quarks as an external quantities in $C[f_q, f_g, f_Q]$. We assume that the evolution of $f_q$ and $f_g$ are independent of $f_Q(x, p)$ and discard collisions between heavy quarks which is by far a solid approximation. We are interested in the evolution of the HQ distribution function $f_Q(x, p)$. The evolution of the bulk of quark and gluons is instead given by the solution of the other two transport equations where the $C[f_q, f_g]$ is tuned to a fixed $\eta/s(T)$, as discussed in detail in ref. [37]. This is however quite equivalent to a modeling where the bulk is given by viscous hydrodynamics.

The collision integral for heavy quarks is given by

$$C[f_Q] = \frac{1}{2E_1} \int \frac{d^3p_2}{2E_2(2\pi)^3} \int \frac{d^3p'_1}{2E'_1(2\pi)^3}$$

$$\times [f_Q(p_1') f_{g,q}(p_2') - f_Q(p_1) f_{g,q}(p_2)]$$

$$\times |\mathcal{M}_{(q,g)\rightarrow Q}(p_1 p_2) \rightarrow p_1' p_2'||^2$$

$$\times (2\pi)^4 \delta^4(p_1 + p_2 - p_1' - p_2),$$

where $\mathcal{M}_{(q,g)\rightarrow Q+(q,g)+Q}$ corresponds to the transition amplitude of the HQ scatterings. In order to solve the collision integral it is necessary to evaluate the scattering matrix of the microscopical process. In the present paper this is done in the framework of a Quasi-Particle model as described in the following.

The evolution of the QGP bulk given by an approach in which we gauge the collision integral to the wanted $\eta/s$ as described in [37, 40]. In this way we are able to simulate the dynamical evolution of a fluid with specified $\eta/s$ by means of the Boltzmann equation. In the case considered here we have, more specifically employed a bulk with massive quarks and gluons that provide the possibility to have a softening of the equation of state with a decreasing speed of sound when the cross over region is approached. Within this approach we describe the evolution of a system that dynamically has approximatively the...
I/QCD equation of state \[^{[41]}\]. As shown in \[^{[42]}\] within this approach we recover universal features of hydrodynamics and it permits to study the impact of \(\eta/s(T)\) on observables like \(v_n(p_T)\) in analogy to what is done within hydrodynamical simulations \[^{[43\text{-}46]}\].

The numerical solution of the Boltzmann equation is obtained by means of the test particle method to map the one body distribution and we divide the space in a three-dimensional grid. For being in a regime of convergency we employ a number of test particle per real particle of 400, which we have verified allow to give a good convergency also for differential observables like \(v_2(p_T)\). More generally it has been checked that the numerical solution of the Boltzmann equation for HQs leads to the Boltzmann-Juttner equilibrium distribution function in all the relevant momentum range.

The key role is certainly played by the scattering matrix \(\mathcal{M}_{(g,g)+Q+(g,g)+Q}\) that is the kernel of the interaction that allows also to calculate the drag and diffusion transport coefficients. The ingredient of the Quasi Particle model are the thermal masses: \(m_{g}^2(T) = 3/4 g^2(T) T^2\), \(m_{u,d}^2(T) = 1/3 g^2(T) T^2\) and \(m_{u,d}^2(T) - m_{s}^2 = 1/3 g^2(T) T^2\). The parametrized form of the strong coupling constant \(g(T)\), is evaluated by making a fit of the energy density obtained by lattice QCD calculations and in our case has been parametrized as:

\[
g^2(T) = \left(\frac{48\pi^2}{(11N_c-2N_f)\ln(\frac{T}{T_C})}\right)^2 \tag{3}\]

where \(N_c = N_f = 3\), \(\lambda = 2.6\) and \(T_s/T_c = 0.57\). It has been shown in \[^{[19]}\] that QPM is able to reproduce with good accuracy the lattice QCD pressure and interaction measure \(T^\mu_\mu = \epsilon - 3P\). The main feature of this approach is that the resulting coupling is significantly stronger than the one coming from pQCD running coupling, particularly as \(T \to T_c\). The evaluation of the scattering matrix \(\mathcal{M}_{(g,g)+Q+(g,g)+Q}\) is then performed considering the leading-order diagram with the effective coupling \(g(T)\) that leads to effective vertices and a dressed massive gluon propagator for \(gQ \leftrightarrow Q\) and massive quark propagator for \(gQ \leftrightarrow gQ\) scatterings. The detail of the calculations for all \(u,t,s\) channels and their interferences is quite long even if proceed along a standard procedure and can be found in Ref. \[^{[47]}\], where also a comparison with the massless case and the massive including collisional widths is presented.

In Fig. 1 we show the behavior of the drag coefficient \(\gamma\) with temperature in QPM by solid line. Such a drag is evaluated with the same scattering matrix \(\mathcal{M}_{(g,g)+Q+(g,g)+Q}\) driving the Boltzmann transport. By dotted line we show the same coefficient that is rescaled to describe the same \(R_{AA}(p_T)\), as the one obtained with Boltzmann dynamics but with the Langevin dynamics, see Fig. 1 and the related discussion. In fact as first observed in ref \[^{[32]}\], we need a smaller drag coefficient in Langevin dynamics to describe nearly the same experimental results than the Boltzmann transport approach. In the same figure we have also included for comparison a standard LO-pQCD calculation with a constant coupling. This allows us to have an indication of the enhancement for the drag coefficient w.r.t. the LO pQCD to describe the experimental data. Moreover we notice that QPM has a weaker temperature dependence of the drag coefficient which is one of the key ingredient for a simultaneous description of heavy quark \(R_{AA}(p_T)\) and \(v_2\). This has been discussed in Ref. \[^{[30]}\], but within a Langevin and not with a Boltzmann approach and not including the impact of coalescence, see Section IV.

The physics behind the different temperature dependence of QPM w.r.t. pQCD is in the increase of the non perturbative dynamics as the temperature decrease that in a QPM is induced by the fit to the QCD thermodynamics. The former implies a coupling \(g(T)\) which increase as \(T \to T_c\) in a way that nearly compensates the decrease of the density resulting in a quite weak temperature dependence of the drag coefficient.

For a constant coupling and massless particles the drag \(\gamma\) would go like \(1/T^2\), solid black line in Fig. 1 with the strong decrease mainly driven by the decrease of the bulk density scatterers that is proportional to \(1/T^3\). The same \(T\) dependence appears in AdS/CFT because also in such a case the strength of the interaction, the coupling to the medium, is not temperature dependence. Of course, however, in such a case the absolute value is much larger of
about one order of magnitude with respect to the pQCD one, see also Fig. [12]

### III. HADRONIZATION FOR CHARM QUARKS VIA COALESCENCE AND FRAGMENTATION

Hadronization dynamics plays an important role in determining the final spectra and therefore the \( R_{AA}(p_T) \) and \( v_2(p_T) \) in both the light and heavy quark sector [13, 15, 16, 48]. In particular for heavy quarks it is generally expected that a coalescence mechanism is in action especially at low and intermediate \( p_T \). We consider here a hybrid model of coalescence plus fragmentation discussing in detail its impact on both \( R_{AA} \) and \( v_2 \).

In our approach the hadronization hypersurface is determined by the isothermal surface of the bulk dynamics, which means that is determined stopping collisions between the light particles and the heavy quarks when the temperature of a cell drops below the critical temperature that has been fixed to \( T=155 \) MeV.

The contribution to hadronization due to coalescence is evaluated according to:

\[
\frac{d^2 N_M}{dP_T^2} = g_M \int \prod_{i=1}^{2} \frac{d^3 p_i}{(2\pi)^3 E_i} p_i \cdot d\sigma_i f_q(x_i, p_i) \times f_M(x_1 - x_2, p_1 - p_2) \times \delta^{(2)}(P_T - p_{T,1} - p_{T,2})
\]

where \( d\sigma_i \) denotes an element of a space-like hypersurface, \( g_M \) is the statistical factor to form a colorless hadron from quark and antiquark with spin 1/2, \( f_q \) are the quark (anti-quark) distribution in phase space. \( f_M \) is the Wigner function and describes the spatial and momentum distribution of quarks in the \( D \) meson.

In the Greco-Ko-Levai (GKL) approach [13] for a heavy meson the Wigner function is taken as a Gaussian of radius \( \Delta_x \) in the coordinate and \( \Delta_p \) in the momentum space, these two parameters are related by the uncertainty principle \( \Delta_x \Delta_p = 1 \),

\[
f_M(x_1, x_2; p_1, p_2) = 8 \exp(x_r^2/2\Delta_x^2) \exp(p_r^2/2\Delta_p^2)
\]

where the relative coordinates \( x_r = x_1 - x_2 \) and \( p_r = p_1 - p_2 \) are the quadri-vectors for the relative coordinates. A pattern confirmed by all the groups, despite differences in the details, is that an hadronization by coalescence is dominant at low momenta [17, 23, 31, 34], since the early work in Ref.s 13, 13. We determine the width parameter \( \Delta_p \) by requiring that the mean square charge radius of \( D^+ \) meson is \( < r^2 >_{ch} = 0.43 \) fm according to quark model. Given that for our wave function:

\[
< r^2 >_{ch} = \frac{3}{2} \frac{Q_1 m_1^2 + Q_2 m_2^2}{(m_1 + m_2)^2} \frac{1}{\Delta_p^2}
\]

with \( Q_1 = +2/3 \) and \( Q_2 = +1/3 \) we find \( \Delta_p = 0.283 \) GeV. We also include the \( D^* \) resonant states suppressed according to the statistical thermal weight with respect to the ground state.

We compute the coalescence probability for each charm quark in the phase space point \( (\vec{x}, \tau, \vec{p}) \) and then assign a probability of fragmentation as \( P_{\text{frag}}(\vec{x}, \tau, \vec{p}) = 1 - P_{\text{coal}}(\vec{x}, \tau, \vec{p}) \). Therefore the charm distribution function undergoing fragmentation is evaluated convoluting the momentum of heavy quarks which do not undergone to coalescence with the Peterson fragmentation function [49]:

\[
f(z) \propto \frac{1}{|z|^{1 - \frac{2}{\epsilon_c} - \frac{2}{12} - \frac{1}{12} + x}}
\]

where \( z = p_D/p_c \) is the momentum fraction of the heavy meson fragmented from the heavy quark and \( \epsilon_c \) is a free parameter to fix the shape of the fragmentation function. As discussed in the next Section the \( \epsilon_c \) parameter will be determined assuring that the available data on \( D \) meson production in \( pp \) collisions are well described by a fragmentation hadronization mechanism. Finally, given the momentum distribution of the charm quarks obtained solving the Boltzmann equation the momentum distribution of \( D \) meson is calculated summing up the \( D \) meson spectrum obtained via coalescence with the one from fragmentation.

### IV. COMPARISON TO THE EXPERIMENTAL OBSERVABLES

We present in this section the comparison of the results we get for the nuclear modification factor \( R_{AA} \) and for the elliptic flow \( v_2 \) with the experimental data. We calculate the nuclear suppression factor, \( R_{AA} \), as the ratio of our initial heavy meson distribution at \( t = \tau_i \) and final heavy meson distribution at \( t = \tau_f \) as:

\[
R_{AA}^{c,D}(p_T) = \frac{f_{c,D}(p_T, \tau_f)}{f_{c,D}(p_T, \tau_i)}
\]

where \( f_{c,D}(p_T, \tau_f) \) indicates the momentum distribution already integrated in the \( r \)-space and in the rapidity range \( |y| < 0.5 \) for charm or \( D \) mesons. The \( f_{c,D}(p_T, \tau_i) \) is the same distribution we employ for \( pp \) collisions, and that is shown in Fig. [2] and [3] see discussion below. However we note that at LHC where the shadowing effect is expected to be large [50], we have also considered the case where in AA
we start from an initial distribution function that is not the $f_c(p_T, \tau_i)$ that goes in the denominator of Eq.8 but is given by:

$$f_c^{SW}(p_T, \tau_i) = f_c(p_T, \tau_i) * S(p_T) \quad (9)$$

where the parametrization of $S(p_T)$ is a shadowing function $\epsilon_c$ in the fragmentation function, in Eq. 7, has been fixed by comparison to the $D^0$ meson production in $p+p$ collisions at RHIC energy as measured by STAR [52]. With $\epsilon_c=0.006$ we obtain the spectrum shown in Fig.2 by solid line.

In Fig. 3 we show $D^0$ and $D^+$ meson spectra in $p+p$ collisions at LHC energy by solid black and light red lines, obtained with $\epsilon_c=0.02$, and compare to the experimental results [52] that is an extrapolation from 7 TeV to 2.76 TeV. With this initial conditions for charm distribution function and their fragmentation function we proceed to evaluate the D mesons spectra in heavy-ion collisions at RHIC and LHC.

For $Au+Au$ collisions at $\sqrt{s} = 200$ AGeV, the initial conditions for the bulk in the $r$-space are given by the standard Glauber condition, while in the $p$-space we use a Boltzmann-Juttner distribution function up to a transverse momentum $p_T = 2$ GeV and at larger momenta mini-jet distributions as calculated by pQCD at NLO order [16]. The initial maximum temperature at the center of the fireball is $T_0 = 345$ MeV and the initial time for the simulations is $\tau_0 = 0.6$ fm/c, as commonly assumed in hydrodynamical simulations [44, 46, 56], which is about corresponding to the $T_0 \cdot \tau_0 \sim 1$ criterium. In our calculation quarks and gluons are massive in order to reproduce the lattice QCD equation of state, as mentioned in Section II. In the $p$-space the charm quarks are distributed according to the Fixed Order + Next-to-Leading Log (FONLL) calculations, taken from Ref. [54, 55]. In the coordinate space HQ are distributed according to number of binary nucleon-nucleon collisions ($N_{coll}$), corresponding to a constant cross section $\sigma_{NN} = 40 mb$ at RHIC and $\alpha\sigma_{NN} = 72 mb$ at LHC.

The dynamical evolution of the bulk is constrained by an $\eta/s = 1/4\pi$, as discussed in section II, in such way that the model reproduces the experimental data on the bulk spectra and elliptic flow [37, 40]. When the system reaches locally the critical temperature the one body distribution functions of heavy quark are frozen and used to get the momentum distribution. This allows to evaluate the nuclear modification factor and the elliptic flow, of the D mesons by means of the hadronization model described in the previous section.

In Fig. 4 the $R_{AA}(p_T)$ as a function of $p_T$ in $Au+Au$ collisions at $\sqrt{s} = 200$ AGeV for centralities $0 - 10\%$ that we obtained within our model calculation is depicted and compared with the experimental data measured at RHIC energy [57]. In this figure we indicate the impact of coalescence on $R_{AA}$ showing
the $R_{AA}$ we obtain considering only fragmentation (dashed line) along with the results obtained including the coalescence mechanism plus fragmentation (green solid line). We observe that the coalescence implies an increasing of the $R_{AA}$ for momenta larger than 1 GeV thus a reduction of the suppression. This is due to the hadronization mechanism which implies that a D mesons from coalescence of one light quark and a charm quark get a momentum kick with respect to the D mesons obtained from fragmentation that on the contrary has a reduced momentum w.r.t. the original quark according to $\epsilon_c$ in Eq\[7\]. This along with the fact that charm spectrum decreases with $p_T$ implies that the final spectrum of D meson does not scale with the spectrum of the original charm. An increasing in the number of particle in the region of $p_T > 1$ GeV is observed. At larger momenta, see Fig.4, fragmentation becomes anyway the dominant mechanism of hadronization. Such a decrease of coalescence impact that appears naturally in our model seems to be necessary to describe the $p_T$ dependence observed experimentally. It has to be mentioned here that the trend of the experimental data at low $p_T$ supports also the coalescence as the mechanism of heavy quark hadronization. Heavy quark hadronization only fragmentation could not describe the marked low $p_T$ bump of the experimental data.

It is shown in ref.\[35\] that a non negligible difference arises between the Langevin and the Boltzmann approach to describes the HQ momentum evolution in QGP. In the present study at RHIC we evaluate the $R_{AA}(p_T)$ in $Au + Au$ collisions at $\sqrt{s} = 200$ AGeV within the Langevin dynamics for the centralities 0 – 10% and compare the results obtained within the Boltzmann transport approach. For the details of the Langevin simulations of HQ dynamics in QGP, we refer to Ref.\[30, 35, 58\]. In Fig. 4 we show the variation of $R_{AA}(p_T)$ obtained within the Langevin dynamics using only fragmentation as the hadronization mechanism and compare the results obtained within the Boltzmann transport approach. As shown in Fig. 4, we can obtain a very similar $R_{AA}(p_T)$ within both the Langevin dynamics as well using Boltzmann transport approach. However, the drag coefficient needed to predict a similar $R_{AA}(p_T)$ within both the approach has to be rescaled down by about 30% as shown in Fig.1. On the other hand it is noteworthy that once the drag coefficient is rescaled by a constant (momentum independent factor) the prediction for $R_{AA}(p_T)$ is nearly identical in quite a large range of $p_T$. An additional comment is however necessary. As discussed in Ref.\[35\] for some ideal case, the comparison of a Langevin dynamics with a Boltzmann one depends also on the way fluctuation-dissipation theorem (FDT) is implemented. We report here the case in which Langevin and Boltzmann are more similar. This occurs when the drag $\gamma(T, p_T)$ is evaluated from the scattering matrix while the diffusion coefficient are determined as $B_T = B_T = T E \gamma$. As known there are other possible choice which would lead in general to larger differences w.r.t. the Boltzmann dynamics, see \[35\].

Using the same interaction, as of Fig. 4, within Boltzmann transport approach, we proceed to compare the results at RHIC for a different centrality of 10 – 40% compared to STAR data. Experimental data has been taken from Ref.\[57\].
at RHIC energy [57]. By black dashed line the results obtain within only fragmentation and the green solid line obtained with fragmentation plus coalescence. In this centrality also we are getting reasonable agreement with the experimental data again, once the coalescence is included along with the fragmentation as the hadronization mechanism.

In Fig. 6 the probability of a charm hadronizing to a D mesons ($D^0, D^+, D_s^0, D_s^{+*}$) through coalescence is depicted as a function of $p_T$. The charm quark hadronization probability to all hadrons, i.e. including charm baryons, is set to one in the $p \rightarrow 0$ limit, as usually done by several groups [21, 31, 34, 59]. The hadronization probability decreases with momentum as the coalescence probability involves the product of two distribution functions that are decreasing with $p_T$. We found at LHC energy the coalescence probability is only marginally smaller than RHIC due to the harder charm quark distribution at LHC than RHIC.

In Fig. 7 are depicted the results for the elliptic flow as a function of momentum in $Au+Au$ collisions at $\sqrt{s} = 200$ AGeV and centrality 0 – 80% compared to STAR data. Experimental data has been taken from Ref [60].

If coalescence plus fragmentation mechanism is included for the hadronization, the $v_2$ of the D-mesons increases with respect to the elliptic flow of charm quarks by about a 30%, solid green line in Fig. 7. This is because the D meson is the result of the coalescence of a charm quark and a light quark and thus the D mesons anisotropy in momentum space reflects both the heavy quark and light quark anisotropies in momentum space. The solid red line shows the $v_2$ of D mesons produced only via coalescence. As

![Diagram](image_url)
expected, the $v_2$ developed only coalescence is larger than the $v_2$ developed due to coalescence plus fragmentation. It can even lead to an increase of about a factor of two at $p_T > 2$ GeV. This is due mainly to the large $v_2$ of light quarks w.r.t. charm. The solid magenta line indicates the elliptic flow produced by the fragmentation of charm once also coalescence has been switched on. In this case the elliptic flow is smaller with respect to that obtained when fragmentation is the only hadronization mechanism, indicated by dashed double dotted line. This last result is an indirect consequence of the phase space selection implicit in the coalescence mechanism that favors the quark pairs that are more correlated and hence those having momenta closer to the collective flow direction have a large coalescence probability. The ensemble of charm quarks left over from the coalescence process have a smaller $v_2(p_T)$ than the one of all the charm quark before hadronization. This is the reason why the $v_2$ is small for D mesons fragmented after coalescence than the D meson formed when only fragmentation is considered.

We have seen in Fig.4 and 5 that rescaling the interaction by about a 30% Langevin gives results very similar to a Boltzmann dynamics. We extend here the comparison to elliptic flow showing in Fig.8 a comparison with the $v_2(p_T)$ obtained in Langevin simulations. Langevin dynamics generates about a 15% smaller elliptic flow at charm level which propagates also to the elliptic flow of single electrons from D mesons decay, shown in Fig.9 by orange solid line along with the one of the Boltzmann case in black solid line. We mention that if for the Langevin case the interaction is not scaled down by 30% then the elliptic flow would be quite similar to the Boltzmann case, but the corresponding $R_{AA}(p_T)$ would be quite smaller. We also show in Fig.8 by solid thick red line our prediction for the single $e^\pm$ with the Boltzmann dynamics and hadronization by coalescence plus fragmentation that indeed is in good agreement with the experimental data from PHENIX. We just remind experimental single electrons come also from B meson decay and are expected to be significant for $p_T > 2$ GeV.

Using the same QPM drag coefficient as at RHIC, we have carried out a simulation of $Pb+Pb$ collisions at $\sqrt{s} = 2.76$ ATeV for centralities $0 - 10\%$ and $30 - 50\%$. In this case the initial maximum temperature in the center of the fireball is $T_0 = 490$ MeV and the initial time for the simulations is $\tau_0 \sim 1/T_0 = 0.3$ fm/c. In Fig.9 the results for the $R_{AA}$ at $0 - 10\%$ centrality are depicted with only fragmentation and fragmentation plus coalescence. In LHC, as of RHIC, we observe that the coalescence implies an increasing of the $R_{AA}$ for momenta larger than 1GeV. As evident, the effect of coalescence is less significant at LHC energy than at RHIC energy. This is because the effect of coalescence depends on the slope of the charm quark momentum distribution. For a harder charm quark distribution the gain in momentum reflects in a smaller increase of the slope spectrum, instead if the charm quark distribution decreases faster in momentum then the same momentum gain due to coalescence will result in a stronger increase of the spectrum. For a harder charm quark distribution like at LHC energy the impact of coalescence is therefore less pronounced, despite still we see that it leads to a better agreement with the experimental data also at LHC.

In Fig.10 we present $R_{AA}$ with respect to $p_T$ for more peripheral collisions at $30 - 50\%$. We see a sim-
show a shadowing effect in agreement with EPS09 
Indeed the data and our calculations seem to clearly 
Experimental data has been taken from Ref. [62].

FIG. 11: D meson elliptic flow in Pb + Pb collisions at 
\( \sqrt{s} = 2.76 \text{ ATeV} \) and centrality 30 – 50% compared to 
ALICE data. Experimental data has been taken from 
Ref. [62] are an average of the measurements in 
only fragmentation. We notice the very recent data 
after coalescence than the D meson produced due to 
coalescence cannot be mimicked by heavy quark diffusion, 
contributing to the data, towards a simultaneous description of 
heavy meson \( R_{AA} \) and \( v_2 \). At LHC energy also, the 
\( v_2 \) is significantly smaller for D mesons fragmented 
after coalescence than the D meson produced due to 
only fragmentation. We notice the very recent data 
is an average of the measurements in Pb + Pb 
at \( \sqrt{s} = 5.02 \text{ ATeV} \) that we include here because are 
the only available data on \( v_2 \) with a not too larger 
uncertainty. On the other hand in our modeling the results 
increasing the beam energy up to 5.02 ATeV 
would only affect the elliptic flow by few percent 
which is quite negligible with respect to the present 
uncertainties.

It is important to note that the impact of coalescence cannot be mimicked by heavy quark diffusion, 
in fact at variance with it coalescence leads to an 
enhancement of both \( R_{AA}(p_T) \) and \( v_2(p_T) \). At RHIC energy, 
considering coalescence plus fragmentation as the charm quark hadronization mechanism, the 
D meson get about 30% of \( v_2 \) from the light partons 
as a consequence of coalescence. But still the major part of the D meson \( v_2 \) (about 65-70%) is coming 
from the heavy quark diffusion within QGP (heavy quark-bulk interaction). On the other at LHC energy 
the D meson get about 20-25% \( v_2 \) from the 
light partons as a consequence of coalescence where 
as about 75-80% is coming from the heavy quark dif-
time, $\tau_{th} \equiv \gamma^{-1}(p \to 0)$, for charm quarks:

$$\tau_{th} = \frac{M}{2\pi T^2} (2\pi TD_s) \approx 1.8 \frac{2\pi TD_s}{(T/T_c)^2} \text{ fm/c} \quad (10)$$

where we have substituted a charm quark mass $M = 1.4$ GeV and we have written $\tau_{th}$ in terms of a dimensional quantities to facilitate a direct evaluation. For example at $T \approx 2T_c$ the dashed orange line as the central value of IQCD gives $2\pi TD_s = 10$ which means a $\tau_{th} \approx 4.5$ fm/c as indicated directly in the figure. Similarly one can easily derive the values in the other points of the plot, in the QGP branch. Notice that none of the behavior shown corresponds to a constant thermalization time which would imply a drag $\gamma$ completely $T$-independent, i.e. $2\pi TD_s \propto T^2$. In particular the flat $2\pi TD_s$ corresponds to a thermalization time proportional to $1/T^2$ which is the case of AdS/CFT and pQCD with constant coupling.

The predictions relative to RAA and $v_2$ obtained by means of the Boltzmann equation with the QPM model indicate a thermalization time that stays in the the range $3.5 - 6$ fm/c which at LHC energy is smaller then the lifetime of the QGP, especially in central collisions. However it has been noticed in Ref. [71] that the $R_{AA}(p_T)$ still significantly deviates, also at low $p_T$ smaller than 3-4 GeV, from the one expected in case of full thermalization. This is an aspect that has to be investigated in more details to spot the origin of such a deviation. However it has to be considered that our estimate of thermalization time is done in the $p \to 0$ limit, should take into account for the decreasing of the interaction with the increasing of the momentum which implies a thermalization time of about a 50% larger or more, once average over momentum is done.

We also mention that the $2\pi TD_s$ estimated in the Langevin case, orange dashed line, is nearly identical to the one of the $T$-matrix approach [31], while the Boltzmann one is quite close to the one from the PHSD transport approach which is based indeed on a dynamical quasi-particle model that includes also finite widths [31].

It may be mentioned that our $D_s$ is marginally below the lattice QCD data point near $T_c$. A final assessment requests in principle the inclusions of initial state fluctuations [41, 72], which helps to develop a large suppression than the averaged one, but such an effect is in practice quite nominal pointing to a somewhat smaller drag coefficient, hence, a larger $D_s$. Such an effect is expected to be within a 10%. Also as shown in Ref. [74, 72], the pre-equilibrium phase may affect the heavy quark suppression pointing also to a somewhat larger $D_s$ for a better agreement with data from the experiments. However the impact of these further aspects is in general less relevant with respect to the differences between Langevin and Boltzmann approaches and quite smaller than current experimental error bars and the systematic uncertainties in lattice QCD calculations. Certainly we are reaching a stage where it will become relevant to include them, especially with the upcoming experimental data expected with much smaller error bars and the new data on the triangular flow $v_3$ [76, 77] that however is beyond the scope of the present paper. Certainly our work gives a further contribution in showing that the extraction of the diffusion coefficient from the phenomenology of heavy-ion collisions is possible and reliable.

VI. SUMMARY AND CONCLUSIONS

We have studied the heavy quark propagation in QGP at RHIC and LHC within a Boltzmann transport approach. We start with a charm quark initialization describing the $D$ mesons production in $p + p$ collisions reasonable well at both RHIC and LHC energies within an hadronization by independent fragmentation. The heavy quark and the bulk interaction has been taken into account within a
QPM which is able to reproduce the lattice QCD equation of state and their in-medium evolution has been treated with a Boltzmann transport equation with some comparison to the Langevin one. The hadronization of heavy quark in AA collisions is described by means of a hybrid model of fragmentation plus coalescence. Using the same interaction we have calculated the $R_{AA}$ and $v_2$ of heavy meson at different centrality class as well as different colliding energies. Our model gives a good description for D meson $R_{AA}$ and $v_2$ both at RHIC and LHC within the still significantly large systematic and statistical uncertainties. There are three main ingredients that we identify as playing a key role toward the agreement to the experimental data. The first one is the QPM that enhances the heavy quark bulk interaction near $T_c$ with respect to a mere decrease of the drag coefficient $\gamma$ with $1/T^2$. This non-perturbative behavior caught by a QPM modeling fitted to the lattice QCD thermodynamics, was identified as the key ingredient for the build-up of a large $v_2$ in a Langevin approach and is confirmed within the Boltzmann approach mainly discussed in the present paper.

Implementation of heavy quark hadronization by means of a hybrid model of fragmentation plus coalescence help to increase both the $R_{AA}$ and $v_2$ close to the data, if the coalescence is regulated to exhaust hadronization in the zero momentum region, the effect is quite large and essential. Finally as discussed in ref. 32 for test calculations and ideal cases, once the interaction is tuned to very similar $R_{AA}(p_T)$ the Boltzmann approach is more efficient in reproducing the elliptic flow w.r.t. Langevin an effect that for the QPM model is of about a 15%.

The underlying $2\pi T D_s$ diffusion coefficient rises about linearly with temperature $T$ and leads to an initial thermalization time of about 3 fm/$c$ at the maximum initial temperature reached at LHC energy ($T \simeq 3 T_c$) increasing only to about a 6 fm/$c$ around $T_c$. This would suggest that the core of charm quarks produced may be essentially kinetically thermalized at the time where most of hadronization occurs. A finding that was certainly unexpected before starting the endeavor to study heavy quark diffusion in the quark gluon plasma and that can have important implications for the understanding also of the quarkonia production. It may be mentioned here that the hadronic rescattering, while generally not affecting $R_{AA}$, give a further contribution to D meson $v_2$ that is in the range of 10-20% depending on the $T_c$ assumed that is generally in the range 155 – 175 MeV. Considering the fact that we use $T_c=155$ MeV, our results will not be affected significantly by the hadronic rescattering. In any case hadronic rescattering would lead to an improvement of the agreement between the experimental data and the present modeling.

Our result and the estimated $D_s(T)$ shows nice agreement with lattice QCD data within the still significantly large uncertainties. This feature is shared nowadays by most of the modelings, as discussed in 7, in particular our estimate of the value and $T$ dependence of $D_s(T)$ is quite close to 31 54. Even if it should be quantified if also the agreement to the data is quantitatively similar. Other approaches can lead to a current estimate that indicates similar values but with a weaker temperature dependence. In general this can be traced back to some difference in the evolution of the bulk matter and/or to a different impact of coalescence. However especially for this last point upcoming data on $\Lambda_c/D$ and $D_s/D$ will shed a new light and allow for more stringent constraints on the hadronization mechanism. Therefore in the next future it would be important that all the phenomenological approaches aiming at the evaluation of the heavy transport coefficient provide their predictions also for such ratios as a function of transverse momentum. Furthermore we think that we are reaching a stage where starts, to be appropriate, to have a comparison to several experimental data as a function of energy and centrality that is statistical quantified. This effort has already been started by some groups 79, but it will also be more meaningful and powerful with the new upcoming data that are expected to carry much smaller error bars and to be extended to lower $p_T$, the region more relevant for the transport coefficient determination. Given that current uncertainties in the phenomenological approach is comparable if not smaller than the current lattice QCD data it is desirable that in the following years the latter will be able to reduce the present systematic uncertainties. Certainly Open Heavy Flavor physics in ultra-relativistic heavy-ion collisions is showing to really have the potential to link the phenomenology to lattice QCD for studying the transport properties of the Hot QCD matter.

Acknowledgments

We acknowledge the support by the ERC StG under the QGPDyn Grant n. 259684. V.G. acknowledge also the warm hospitality at the UCAS in Beijing under the President International Fellowship Ini-ziative (2016VMA063).
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