Pionic decay of a possible $d'$ dibaryon and the short-range NN interaction

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We study the pionic decay of a possible dibaryon $d' \rightarrow N + N + \pi$ in the microscopic quark shell model. The initial $d'$ dibaryon wave function ($J^P=0^-$, $T=0$) consists of one $1h\omega$ six-quark shell-model $s^5p[51]_X$ configuration. The most important final six-quark configurations $s^6[6]_X$, $s^4p^2[42]_X$ and $(s^4p^2 - s^52s)[6]_X$ are properly projected onto the NN channel. The final state NN interaction is investigated by means of two phase-equivalent - but off-shell different - potential models. We demonstrate that the decay width $\Gamma_{d'}$ depends strongly on the short-range behavior of the NN wave function. In addition, the width $\Gamma_{d'}$ is very sensitive to the mass and size of the $d'$ dibaryon. For dibaryon masses slightly above the experimentally suggested value $M_{d'}=2.065$ GeV, we obtain a pionic decay width of $\Gamma_{d'} \approx 0.18$–0.32 MeV close to the experimental value $\Gamma_{d'} \approx 0.5$ MeV.

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I. INTRODUCTION

During the last decade much attention has been devoted to theoretical and experimental investigations of the pionic double charge exchange (πDCX) process on nuclei. Because this reaction $\pi^+ + (A, Z) \rightarrow (A, Z + 2) + \pi^-$ involves (at least) two nucleons in the nucleus, the πDCX cross section depends sensitively on short-range NN-correlations in nuclei. Therefore, it provides a good testing ground for the nucleon-nucleon interaction at short range. Experiments on different nuclear targets have unambiguously confirmed the existence of a narrow resonance-like structure in the πDCX cross-section at small incident pion energies $T_\pi \approx 50$ MeV [1]. The position of this peak turned out to be largely independent of the studied nucleus. The height and width of this peak could not be explained by standard calculations based on the two-step process [2] $(n + n + \pi^+ \rightarrow n + p + \pi^0 \rightarrow p + p + \pi^-)$. So far, these data could only be explained with the assumption of a non-nucleonic reaction mechanism [1,3] proceeding via an intermediate dibaryon resonance, henceforth called $d'$. The quantum numbers of the $d'$ dibaryon candidate were determined as $J^P=0^-, T=0$, and its free mass and hadronic decay width were suggested to be $M_{d'}=2.065$ GeV and $\Gamma_{d'} \approx 0.5$ MeV[1]. More than a decade ago Mulders et al. [5] predicted a dibaryon resonance with quantum numbers $J^P=0^-, T=0$ and a mass $M \approx 2100$ MeV within the MIT bag model. Recently, this dibaryon candidate has been investigated in a series of works [6–8] within the Tübingen chiral constituent quark model. These works emphasize the crucial role of the confinement mechanism for the existence of the $d'$. The quantum numbers $J^P=0^-, T=0$ of the $d'$ resonance prevent the decay into two nucleons and the only allowed hadronic decay channel of the $d'$ is the three-body decay into a $\pi NN$ system with S-waves in each particle-pair [1,8]. Because the $d'$ mass $M_{d'}$ is only $\approx 50$ MeV above the $\pi NN$ threshold, the $d'$ decay width $\Gamma_{d'}$ should be anomalously small owing to a very small phase volume of three-particle final states. We recall that the currently available experimental evidence of dibaryon excitations in nuclei is very limited

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1 This value is uncertain by a factor of two [4].
This is due to very large N-N decay widths of most dibaryon resonances, which renders them undetectable on the background of other hadronic processes at intermediate energy. At present, the experimental evidence for narrow dibaryons is reduced to a single candidate, the \( d' (2065) \). In contrast to the deuteron, which consists of two on the average widely separated nucleons, there are indications \[6,7\], that the \( d' \) is a rather pure, compound six-quark system. Therefore, the dynamics of its hadronic decay into the \( \pi NN \) system should be sensitive to the overlap region of the two outgoing nucleons; a situation that is ideal for understanding the role of quark degrees of freedom in the short-range nucleon-nucleon interaction (see e.g. Ref. \[10\] and references therein).

Starting from this point (for alternative approaches see in Refs. \[2,11,12\]) we consider the \( d' \) decay as a (quark) shell-model transition from one six-quark configuration to another one by emitting a pion. The quark line diagram of the decay is sketched in Figure 1. The calculation of the transition matrix elements \( d' \rightarrow N + N + \pi \) is similar to the calculation of \( \Delta \)-isobar-decay matrix elements \( \Delta \rightarrow N + \pi \) (spin and isospin flip of a quark). In the case of the \( d' \) decay only the initial dibaryon state is a definite six-quark configuration (the lowest shell-model configuration with quantum numbers \( J^P=0^-, T=0 \)), whereas the final state consists of a continuum of NN-states which have to be projected onto a basis of six-quark configurations with quantum numbers \( J^P=0^+, T=1 \) of the NN \( ^1S_0 \) wave. The main difficulty in comparing the calculated width \( \Gamma_{d'} \) with experimental data is its sharp dependence on the energy gap between \( M_{d'} \) and the \( \pi NN \) threshold. A reliable result on \( \Gamma_{d'} \) can be obtained only if the exact mass \( M_{d'} \) in vacuum is known (e.g. from electroexcitation of the \( d' \) on the deuteron at large momentum transfers \[13\]). At present, we have only indirect data in the nuclear medium \[4\]. Due to the absence of vacuum data, we investigate the problem of the \( d' \) decay width starting from theoretical quark-model results \[6,7\] for \( M_{d'} \) and the hadronic \( d' \) size parameter \( b_6 \).

Our first calculation for \( \Gamma_{d'} \) was published in Ref. \[8\]. The aim of the present work is to improve mainly on three important effects which were neglected in Ref. \[8\]: a) antisymmetrization of the final NN-state on the quark level taking into account the effect of quark exchange between the two nucleons at short range, b) insertion of a complete basis of fi-
nal six-quark states including besides the non-excited $s^6$ shell-model state all Pauli-allowed excited configurations $s^4p^2$ and $s^52s$, which have a non-vanishing overlap with the final NN-state and can be populated via the emission of the pion from the initial $d'$ dibaryon, and c) inclusion of the final state interaction (f.s.i.) for the two-nucleon system.

II. DECAY DYNAMICS IN TERMS OF QUARK DEGREES OF FREEDOM

A. Initial state

As in Ref. [8] we consider only the simplest six-quark configuration $s^5p[51]_X$ in the initial state (the energetically lowest $J^P=0^-, T=0$ translationally invariant shell model (TISM) state which satisfies the Pauli exclusion principle). It has been shown in [3,4], that the $d'$ wave function may be considered as a compound six-quark state, for which a single shell model vector provides an adequate description. This state vector is defined by

$$|d'\rangle = |s^5p(b_6)[51]_X, [2^3]_C[3^2]_T([2^21^2]_CT)[42]_S : [21^4]_CTS, LST=110, J^P = 0^- \rangle .$$

The characteristic oscillator parameter in the six-quark wave function $b_6$ may for example be determined from the minimisation of the $d'$ mass for a given microscopic quark-quark Hamiltonian [3,4]. The Young schemes $[f_D]$, $D=X,C,S,T$ in orbital, color, spin and isospin space, as well as for the coupled spaces CT, CTS, are necessary for the unambiguous classification of shell-model basis vectors in terms of irreducible representations (IR) of the following reduction chain for unitary groups:

$$SU(24)_{XCT} \supset SU(2)_X \times SU(12)_{CST} \supset SU(2)_X \times SU(6)_{CT} \times SU(2)_S \supset SU(2)_X \times SU(3)_C \times SU(2)_T \times SU(2)_S$$

The fractional parentage coefficient (f.p.c.) technique [14–17] based on scalar factors (SF) of Clebsch-Gordan coefficients of the above group [14–19], sketched in the following section, is used for the calculation of matrix elements and overlap integrals.
B. Transition operator

The pionic decay width of the $d'$ is calculated, as in Ref. [8], assuming a direct coupling of constituent quarks with the isotriplet of pion fields $\phi$ through the operator

$$\hat{O}_{\pi q}(k) = \frac{f_{\pi q}}{m_\pi} \sum_{j=1}^{6} (\sigma_j \cdot k)(\tau_j \cdot \phi) \exp\left(-ik \cdot (r_j - R_{\text{c.m.}})\right).$$  \hspace{1cm} (3)

Here, $r_j$, $\sigma_j$ and $\tau_j$ are coordinate, spin and isospin of the $j$-th quark, $k$ is the pion momentum in the c.m.s. of the $d'$, and $E_\pi = \sqrt{m_\pi^2 + k^2}$. $f_{\pi q}$ is the $\pi qq$ coupling constant. Its value is connected with the $\pi NN$ coupling $f_{\pi N}$ (we use $\frac{f_{\pi N}^2}{4\pi} = 0.07491$) through the known relation

$$\langle N(123)|\sum_{j=1}^{3} \sigma_j^{(z)}\tau_j^{(z)}|N(123)\rangle = \frac{5}{3}\langle N|\sigma_N^{(z)}\tau_N^{(z)}|N\rangle,$$

giving $f_{\pi q} = \frac{3}{5}f_{\pi N}$. Because we neglect isospin breaking effects in this work, we chose the average pion mass $m_\pi = 138$ MeV.

C. Final states

In Ref. [8] the wave function of the final NN-state was antisymmetrized and normalized on the nucleon level, assuming a plane wave with wave vector $q$ in the relative coordinate $r$ between the two nucleons. The coordinate representation of the nucleon-nucleon state vector $|\Phi_{NN}(q)\rangle$ was then written as

$$\langle r|\Phi_{NN}(q)\rangle = \Phi_{NN}(q, r) = \frac{1}{(2\pi)^2} \frac{1}{\sqrt{2}} \left[ e^{iqr} - (-1)^{(S+T)}e^{-iqr} \right]; \quad S = 0, T = 1.$$  \hspace{1cm} (4)

The full wave function of the final state respected the three-quark cluster nature of the nucleons

$$\langle r|\tilde{\Psi}_{NN}(q, 123456)\rangle = \Phi_{NN}(q, r)\{N(123)N(456)\}_{ST},$$  \hspace{1cm} (5)

i.e. the nucleon wave function $N(123)$ was given by translationally-invariant shell-model configurations

$$N(123) = |s^3 (b_N) [3]_X, [1^3]_C[21]_T([21]_CT)[21]_S : [1^3]_CTS, LST = 0, 1/2, 1/2\rangle_{T\text{ISM}}$$

$$= \Phi_{N}(123) \cdot \chi^{C=0} \cdot \chi_s^{S=\frac{1}{2}} \cdot \chi_{T_s}^{T=\frac{1}{2}}.$$  \hspace{1cm} (6)

$\chi^{C=0}$, $\chi_s^{S=\frac{1}{2}}$ and $\chi_{T_s}^{T=\frac{1}{2}}$ are color-singlet, spin and isospin three-quark states. $\Phi_{N}(123)$ is the orbital part of the wave function, expressed in terms of the internal Jacobi coordinates

$$\rho = r_1 - r_2$$

and

$$\lambda = r_3 - (r_1 + r_2)/2.$$
\[ \Phi_N(123) = \left( \sqrt{3} \pi b_N^2 \right)^{-3/2} \exp \left[ -\frac{1}{2b_N^2} \left( \frac{1}{2} b^2 + \frac{2}{3} \lambda^2 \right) \right], \]  

(7)

with a characteristic nucleon oscillator parameter \( b_N \). This parameter does not have to be the same as the harmonic oscillator parameter \( b_6 \) for the dibaryon wave function of Eq. (1), as it has been discussed in Refs. [6,7]. The relative Jacobi coordinate between the clusters is given by

\[ r = \frac{r_1 + r_2 + r_3}{3} - \frac{r_4 + r_5 + r_6}{3}. \]  

(8)

Note that the six-quark final state (3) is antisymmetrized automatically when the state vector \( |\tilde{\Psi}_{NN}(q)\rangle \) is substituted into the decay matrix element \( \langle \tilde{\Psi}_{NN}(q); \pi | \hat{O}_{pq} | d' \rangle \), because the initial state of Eq. (1) is fully antisymmetric. However, the antisymmetrizer-projector \( \hat{A} \) \( (\hat{A}^2 = \hat{A}) \) contained in the initial state \( \hat{A} |d' \rangle = |d' \rangle \) reduces considerably the normalization of the final state (it cuts all non-antisymmetrized parts of the cluster function of Eq. (5) which contain about 90% of the wave function – see below). Therefore, it is important to substitute from the beginning a final state wave function which is normalized \( (N) \) and antisymmetrized \( (\hat{A}) \) on the quark level.

\[ |\Psi_{NN}(q, 123456)⟩ = N \hat{A} \{ \Phi_{NN}(q, r) N(123) N(456) \}_{ST}, \quad ST = 01. \]  

(9)

The normalization factor \( N \) is determined by the standard orthonormalization condition

\[ \langle \Psi_{NN}(q') | \Psi_{NN}(q) \rangle = \langle \Phi_{NN}(q') | \Phi_{NN}(q) \rangle = \delta^{(3)}(q' - q), \]  

(10)

which leads to

\[ N^{-2} = \frac{\langle \{ \Phi_{NN}(q') N(123) N(456) \}_{ST} | \hat{A} | \{ \Phi_{NN}(q) N(123) N(456) \}_{ST} \rangle}{\langle \Phi_{NN}(q') | \Phi_{NN}(q) \rangle}. \]  

(11)

The antisymmetrizer-projector \( \hat{A} \) is

\[ \hat{A} = \frac{3!3!2}{6!} \left( 1 - \sum_{i=1}^{3} \sum_{j=4}^{6} P_{ij}^{XCST} \right) = \frac{1}{10} \left( 1 - 9 P_{36}^{XCST} \right), \quad \hat{A}^2 = \hat{A}. \]  

(12)

\( P_{ij}^{XCST} \) is the pair-permutation operator for quarks \( i \) and \( j \) in orbital, color, spin and isospin space. It is instructive to calculate the normalization factor (11) algebraically by factorization.
of the CST and X parts of the pair permutation \( P_{36}^{XST} = P_{36}^{CST} P_{36}^{X} \). The matrix element of \( P_{36}^{CST} \) between two NN-states in the ST=01 (or 10) channel is very well known (see e.g. [18])

\[
\langle \{ N(123)N(456) \}_{ST=01} | P_{36}^{CST} | \{ N(123)N(456) \}_{ST=01} \rangle = \frac{-1}{81} .
\]  

(13)

Inserting this value into Eq. (11) reduces its right-hand-side to

\[
\mathcal{N}^{-2} = \frac{1}{10} + \frac{1}{90} \frac{\langle \Phi_{NN}(q') \Phi_{N}(123) \Phi_{N}(456) | P_{36}^{X} | \Phi_{NN}(q) \Phi_{N}(123) \Phi_{N}(456) \rangle}{\langle \Phi_{NN}(q') | \Phi_{NN}(q) \rangle} ,
\]

(14)

where \( \Phi_{N}(123) \) is the orbital part of the nucleon wave function (3) given in Eq. (7). The numerator in Eq. (14) depends on the form of \( \Phi_{NN}(q, r) \), but for the plane wave in Eq. (4) (or for any continuum wave function including f.s.i.) it is a finite value, i.e. it has to be zero compared with the \( \delta \)-function in the denominator. Therefore, in our case the second term in Eq. (14) vanishes and we obtain

\[
\mathcal{N} = \sqrt{10}
\]

(15)

Note, that the expression \( \langle P_{36}^{X} \rangle \equiv \langle \Phi_{NN} \Phi_{N} | P_{36}^{X} | \Phi_{NN} \Phi_{N} \rangle / \langle \Phi_{NN} | \Phi_{NN} \rangle \) receives its maximal value \( =1 \) in the special case of a Gaussian \( \Phi_{NN}(r) = (2\pi b_{N}^{2}/3)^{-3/4} \exp(-3r^{2}/4b_{N}^{2}) \). Therefore, for any relative NN wave function \( \Phi_{NN} \) we have the following constraints:

\[
0 \leq \langle P_{36}^{X} \rangle \leq 1 , \quad \text{or} \quad 9 \leq \mathcal{N}^{2} \leq 10
\]

(16)

The value (15) is equal to the usual identity factor \( \sqrt{\frac{6!}{3392}} \) well-known in nuclear cluster physics (see e.g. [24]). It plays an important role for the projection of six-quark configurations onto baryon-baryon channels [21]. From now on we shall omit the antisymmetrizer \( \hat{A} \) in front of the final state in the decay matrix element, but the identity factor (13) may not be omitted:

\[
\langle d' | \hat{O}_{\pi q}(k) | \Psi_{NN}(q), \pi \rangle = \langle d' | \hat{O}_{\pi q}(k) \sqrt{10} \hat{A} | \tilde{\Psi}_{NN}(q), \pi \rangle = \sqrt{10} \langle d' | \hat{O}_{\pi q}(k) | \tilde{\Psi}_{NN}(q), \pi \rangle
\]

(17)

The inclusion of this factor, due to the antisymmetrization of the final two-nucleon wave function on the quark level, improves considerably the agreement of the results obtained in Ref. [8] with the experimentally suggested width.
D. Transition amplitude including intermediate states with up to two harmonic oscillator quanta

As in Ref. [8], we calculate the decay matrix element of Eq. (17) by inserting a complete set of six-quark configurations with quantum numbers of the final \( ^1S_0 \) two-nucleon state (LST=001, \( J^P=0^+ \))

\[
\langle \Psi_{NN}(q), \pi | \hat{O}_{\pi q}(k) | d' \rangle = \sqrt{10} \sum_{(n),(f)} \langle \Phi_{LST=001}^{N(123)N(456)}(n, b_6), \{f\}, LST=001 \rangle \langle (n, b_6), \{f\}, LST=001 | \hat{O}_{\pi q}(k) | s^5p(b_6)[51]_X, [2^21^2]_{CT}LST=110, J^P=0^- \rangle .
\]  

(18)

Here, \( \{f\} = \{[f_X], [f_{CT}]\} \) and \((n)\) defines quark states with \( n \) harmonic oscillator (h.o.) excitation quanta, i.e. \((n) = s^{6-n}p^n, s^{6-2m}(2s)^m, (n = 2m), etc.; \) and \( b_6 \) is the h.o. parameter for the six-quark system. The summation in Eq. (18) extends over a limited set of Young schemes \([f_X]\) and \([f_{CT}]\): the possible representations of \([f_{CT}]\) in the sum in Eq. (18) are given by the series of inner products of the \([2^3]_C\) color and \( T=1 \) [42] isospin Young schemes

\[
[2^3]_C \circ [42]_T = [42]_{CT} + [321]_{CT} + [2^3]_{CT} + [31^3]_{CT} + [21^4]_{CT} .
\]  

(19)

Only two spatial Young schemes \([6]_X\) and \([42]_X\) are compatible with the even-parity (L=0) N-N partial wave. Further constraints follow from the Pauli exclusion principle, i.e. \([f_X] \circ [33]_S \circ [f_{CT}] = [1^6]_{XCSST} \). In the case of full spatial symmetry \([6]_X\), only one color-isospin state \([2^3]_{CT}\) is allowed, but the Young scheme \([42]_X\) of the excited shell-model configurations is compatible with each state from the inner product given in Eq. (19). Our choice of a one-body transition (pion-production) operator defined in Eq. (3) further restricts the number of relevant intermediate states. The one-particle operator (3) can excite (or de-excite) only one quark of the initial \( s^5p \) state. Therefore, the complete set of states in Eq. (18) is reduced to the configurations \( s^6, s^4p^2 \) and \( s^52s \), knowing that higher one-particle excitations can be omitted because of a very small overlap with the final NN-state. Summarizing, the following intermediate states are taken into account in Eq. (18):

- the energetically lowest \((n=0)\) spatially symmetric state \( s^6[6]_X[2^3]_{CT} \),
- the excited \((n=2)\) translationally-invariant (orthogonalized to the 2S excitation of the six-quark c.m.) state \( (s^4p^2 - s^52s) \) with identical Young schemes \([6]_X, [2^3]_{CT}\), and
• five excited (n=2) states $s^4p^2[42]_X[f_{CT}]$ with CT Young schemes from the inner product of Eq. (19).

It is interesting to note that all these configurations are also important for explaining the short-range nucleon-nucleon interaction. This was pointed out almost two decades ago [14,15,21] and thereafter discussed in many papers (see e.g. [28] and references therein). Now we believe that a possible $d'$ dibaryon has much potential for providing additional information on the innermost part of the nucleon-nucleon interaction, i.e. in the region where the nucleons overlap.

E. Final state interaction

To take into account the f.s.i. for the two outgoing nucleons, we consider separable-potential representations of the N-N interaction, namely the phenomenological potential of Tabakin [22], and the separable model of Ueda et al. [23], which is equivalent to the OBEP. The wave functions of the $^1S_0$ NN final states for the Tabakin potential are of the form

$$\Phi^{L=0}_{NN}(q, r) = (2\pi)^{-3/2} \cos \delta_0 \left\{ j_0(qr) - \tan \delta_0 n_0(qr) + A(q) \frac{e^{-\beta r}}{r} + B_1(q) \frac{e^{-\alpha r}}{r} \cos \alpha r + B_2(q) \frac{e^{-\alpha r}}{r} \sin \alpha r \right\},$$

(20)

while the separable potential model of Ueda et al. leads to the $^1S_0$ NN wave function

$$\Phi^{L=0}_{NN}(q, r) = (2\pi)^{-3/2} \cos \delta_0 \left\{ j_0(qr) - \tan \delta_0 n_0(qr) + \tilde{A}(q) \frac{e^{-\gamma r}}{r} - 3 - \sum_{n=1}^{N} \tilde{B}_n(q) \frac{e^{-\beta_n r}}{r} \right\}. \quad (21)$$

Here, $\delta_0(q)$ is the phase shift of NN-scattering in the $^1S_0$ wave, and the functions $A$, $\tilde{A}$, $B_i$ and $\tilde{B}_i$ depend on the choice of parameters $\alpha$, $\beta$, $\gamma$ and $\beta_i$ for the two models (see the Appendix).

We use the non-standard Tabakin potential because at short range, the NN wave functions obtained with this potential differ qualitatively from OBEP wave functions. In Fig. 2, the wave functions (20) and (21) for both models are shown at an NN lab-energy of $E_{NN}=100$ MeV. The relative wave function of Eq. (20) has a node at distances $r \approx 0.4-0.5$ fm (a stable position of the node in a large interval of NN-energies produces the same NN-scattering phase.
shifts as a repulsive core). The two models are phase equivalent, but differ in their off-shell behavior. In the following we will demonstrate that the results for $\Gamma_d$ differ considerably for both models, especially if the dibaryon mass $M_d'$ comes close to the $\pi NN$ threshold.

III. EXPLICIT CALCULATION USING THE FRACTIONAL PARENTAGE COEFFICIENT (F.P.C.) TECHNIQUE

Our approximation for the decay amplitude in Eq. (18) leads to a sum over products of two factors. The first factor is the so-called overlap integral of the intermediate six-quark configuration with the outgoing two-nucleon state. The second factor is a shell-model transition matrix element that describes the production of the pion on a single quark in the dibaryon and the subsequent transition to an intermediate six-quark configuration. Both factors can be calculated with the standard fractional parentage coefficient (f.p.c.) technique, which was developed for quark-model calculations for example in Refs. [14–20].

A. Overlap integral of intermediate six-quark configurations with the NN-continuum

In this subsection, we calculate the overlap integral of an intermediate six-quark configuration $(n, b_6)\{f\}$ with the (antisymmetrized and normalized) $^1S_0$ partial wave of the final NN-state introduced in Eq. (9)

$$\langle \Psi_{NN}^{L=0}(q) | (n, b_6)\{f\} \rangle = \sqrt{10} \langle \Phi_{NN}^{L=0}(q) 3\{N((00, b_N)123)N((00, b_N)456)\}_{ST=01} | (n, b_6), [f_X], [f_{CT}], LST=001 \rangle .$$

Beginning with Eq. (22) we denote from now on the nucleon wave function of Eq. (6) of the translationally invariant shell-model as

$$N(123) \equiv N((n'l' = 00, b_N)123) .$$

Here $l'$ is the total orbital angular momentum contained in the internal Jacobi coordinates $\rho, \lambda,$ and $b_N$ is the h.o. size parameter for the three-quark system.

The overlap integral (22) is calculated using the standard f.p.c. technique for the quark shell model [14,17,20]. For this purpose we use a f.p.c. decomposition of the six-quark
configuration \((n, b_6)\{f\}\) into two three-quark clusters with CST quantum numbers of baryons \(\{B_1((n''l'', b_6)123)B_2((n''l'', b_6)456)\}_{\text{CST}}\). Note, that for this procedure, the size parameter in the decomposition \(b_6\) differs from the nucleonic size parameter \(b_N\)

\[
| (n, b_6)[f_X][f_{\text{CT}}]_{\text{LST}=001} \rangle = \sum_{B_1(n''l''')} \sum_{B_2(n''l'')} \sqrt{\frac{n_{f_X} n_{f_X}'}{n_{f_X}}} \ U_{\{f\}}^{B_1B_2} \cdot C_{f_X}^{(n)} (n', n'') \\
\times \left\{ \varphi_{N_L}(r, \sqrt{2/3b_6}) Y_{L_M}(\hat{r}) \right\} \{B_1((n''l'', b_6)123)B_2((n''l'', b_6)456)\}_{\text{ST}=01} \right\}_{L=0} \tag{24}
\]

In expansion \((24)\), \(\varphi_{N_L}(r, \mu r_0)\) is a h.o. wave function in the relative coordinate \(r\) of the two baryons with angular momentum \(\vec{L} = \vec{L} - (\vec{l}'' + \vec{l}''')\) and \(\vec{N} = n - (n'' + n''')\) excitation quanta \((\mu r_0 = \sqrt{2/3b_6}\) is the h.o. size parameter). As usual, \(n_{f_X}\) is the dimension of the IR \([f_X]\) of the permutation symmetry group \(S_6\) for six particles \([25]\). \(n'_{f_X}\) and \(n''_{f_X}\) are the dimensions of IRs \([f'_X]\) and \([f''_X]\) of the subgroups \(S'_3 \) and \(S''_3\) in the reduction \(S_6 \supset S'_3 \times S''_3\).

The coefficients \(U_{\{f\}}^{B_1B_2}\) and \(C_{f_X}^{(n)} (n', n'')\) are f.p.c. in the CST- and X-subspaces respectively.

For simplicity, we omit in Eq. \((24)\) the indices for the dependence of \(U_{\{f\}}^{B_1B_2}\) and \(C_{f_X}^{(n)} (n', n'')\) on the intermediate Young schemes \(f'_{\text{CST}} \equiv f'_X, f''_{\text{CST}} \equiv f''_X, f'_{\text{CT}}, f''_{\text{CT}}, f'_S, f''_S, f'_T, f''_T, f'_C,\) and \(f''_C\) occuring for our chosen reduction chain of Eq. \((24)\).

With the help of \((24)\), we can calculate the overlap \((22)\). The three-quark – three-quark decomposition is of course the most adequate expansion for projecting onto the NN-channel.

The projection for a given intermediate state \((n\{f\})\)

\[
\Phi^{\ell=0}_{(n\{f\})} (r) = \sqrt{10} \{N((00, b_N)123)N((00, b_N)456)\}_{\text{ST}=01} | (n, b_6)[f_X][f_{\text{CT}}]_{\text{LST}=001} \rangle \tag{25}
\]

receives non-vanishing contributions only from NN-components in Eq. \((24)\) because non-nucleonic clusterings, such as \(B_1(123)B_2(456)\), are orthogonal to \(N(123)N(456)\) in CST space. Furthermore, the overlap of excited nucleonic clusters, e.g. \(N((20, b_6)123)\), with the ground state nucleon \(N((00, b_N)123)\) can be neglected if we assume that the size parameter \(b_6\) of the six-quark configuration \((24)\) does not differ considerably from the quark core radius \(b_N\) of the nucleon. In fact, because \(b_6 \neq b_N\), the non-zero overlap integral between excited and non-excited nucleons is

\[
\langle N((20, b_6)123) | N((00, b_N)123) \rangle = \frac{(b_6^2/b_N^2 - 1)}{(1 + b_6^2/b_N^2)} \left( \frac{2b_6/b_N}{1 + b_6^2/b_N^2} \right)^3 . \tag{26}
\]
The sum over all possible terms gives a negligible contribution to the final result because the different terms interfere destructively (see next section). Due to these restrictions we are lead to the expression

\[ \Phi_{(n)}^{L=0}(r) \approx \langle N((00, b_N)123) | N((00, b_6)123) \rangle \langle N((00, b_N)456) | N((00, b_6)456) \rangle \\
\times \sqrt{10} \sqrt{\frac{1}{n_{fx}}} U_{\{J\}}^{NN} C_{fx}^{(n)}(0,0) \varphi_{n0}(r, \sqrt{2/3b_6}) Y_{00}(r) \] (27)

where

\[ C_{fx}^{(n)}(0,0) = \begin{cases} 
1, & \text{if } n = 0, [fx] = [6] \\
-\sqrt{\frac{1}{5}}, & n = 2, [fx] = [6] \\
-\sqrt{\frac{1}{5}}, & n = 2, [fx] = [42] 
\end{cases} \]

The coefficients \( C_{fx}^{(n)}(n',n'') \) are calculated by general methods from the TISM (see, e.g. Ref. [24]). The values of \( U_{\{J\}}^{NN} \) are given in Table I. The general rule for calculating f.p.c. in the CST-subspace is the factorization of the value \( U_{\{J\}}^{B_1B_2} \) [14–18] (symbolically)

\[ U_{\{J\}}^{B_1B_2} = SF_{CT \bullet S} SF_{C \bullet T} \] (28)

in terms of scalar factors \( SF_{CT \bullet S} \) and \( SF_{C \bullet T} \) of Clebsch-Gordan coefficients of the unitary groups \( SU(12)_{CST} \) and \( SU(6)_{CT} \) for the reductions \( SU(12)_{CST} \supset SU(6)_{CT} \times SU(2)_S \) and \( SU(6)_{CT} \supset SU(3)_C \times SU(2)_T \), respectively (which are links of the common reduction chain [4]). The necessary SFs are tabulated in Refs. 13–20. With expression (27), the overlap integral of Eq. (27) reduces to

\[ \langle \Psi_{NN}^{L=0}(q) | (n,b_6)\{J\} \rangle = \langle \Phi_{NN}^{L=0}(q) | \Phi_{(n)}^{L=0}(\{J\}) \rangle \\
\approx \sqrt{10} \left( \frac{2b_6/b_N}{1+b_6^2/b_N^2} \right)^6 \sqrt{\frac{1}{n_{fx}}} U_{\{J\}}^{NN} C_{fx}^{(n)}(0,0) \left( \frac{8\pi b_6^2}{3} \right)^{3/4} I_{NN}^{(n)}(q) . \] (29)

Eq. (29) contains a simple radial integral

\[ \left( \frac{8\pi b_6^2}{3} \right)^{3/4} I_{NN}^{(n)}(q) = \sqrt{\frac{4\pi}{3}} \int_0^\infty r^2 dr \Phi_{NN}^{L=0}(q,r) \varphi_{n0}(r, \sqrt{2/3b_6}) , \] (30)

which can be calculated analytically for a plane wave \( \Phi_{NN}^{L=0}(q,r) = (2\pi)^{-3/2} j_0(qr) \), as well as for the Tabakin f.s.i. wave functions of Eq. (20) and for the Ueda f.s.i. wave functions given in Eq. (24). Results for \( I_{NN}^{(n)}(q) \) are listed in the appendix. The large bracket in Eq. (29) involving the ratio of the two h.o. size parameters \( b_N/b_6 \) comes from the overlap of the two nucleon clusters \( \langle N((00, b_N)123) | N((00, b_6)123) \rangle \langle N((00, b_N)456) | N((00, b_6)456) \rangle \).
The shell-model matrix element of the pion-production operator $\hat{O}_{\pi q}$ defined in Eq. (8) (the second factor in the decay amplitude introduced in Eq. (18)) is proportional to the one-particle matrix element of the spin-isospin-flip operator $\sigma_6^{(\mu)} \tau_6^{(\kappa)}$. The remaining five quarks act as spectators for the transition. Choosing the sixth quark, we write the matrix element for the emission of a $\pi^-$:

$$\langle (n, b) | \tilde{L} \tilde{S} \tilde{T} = 001, T_z = 1; \pi^- | \hat{O}_{\pi q}(k) | d' \rangle$$

$$= \frac{6 f_{\pi q}}{m_\pi \sqrt{2E_\pi(2\pi)^3}} \langle (n, b) | \tilde{L} \tilde{S} \tilde{T} = 001, T_z = 1 | (\sigma_6 \cdot \hat{k}) \tau_6^{(+)} e^{-\frac{i}{6}\hat{k} \cdot \rho_6} | d' \rangle$$

(31)

Here, $\rho_6 = r_6 - \frac{1}{5} \sum_{i=1}^{5} r_i$, is the Jacobi coordinate, and $\sigma_6$ and $\tau_6$ are spin and isospin of the sixth quark. The momentum of the pion is $k = k \hat{k}$, and the factor 6 in front of the one-particle matrix element contains the summation over all six quarks. The natural choice for a f.p.c. decomposition is clearly the separation of the last quark $q^6 \rightarrow q^5 \times q$ (one-particle f.p.c.), which allows to exploit the orthogonality constraints for the five spectator quarks. With the one-particle f.p.c. expansion of the shell-model states, the right-hand side of Eq. (31) reduces to a sum of one-particle spin-isospin-flip amplitudes with algebraic coefficients:

$$\langle (n, b) | \tilde{L} \tilde{S} \tilde{T} = 001, T_z = 1 | (\sigma_6 \cdot \hat{k}) \tau_6^{(+)} e^{-\frac{i}{6}\hat{k} \cdot \rho_6} | d' \rangle = \sum_{f_S} \sum_{f_T} \frac{n_{f_S}}{n_{f_X}} \frac{n_{f_X}}{n_{f_{T'}}} \left\{ \delta_{n_0} C_{[51]}^{(1)}(s^4 p, s) C_{f_X}^{(2)}(s^4 p, p) X_6(k; 11 M, 00) \right. $$

$$+ \delta_{n_2} C_{[51]}^{(1)}(s^5 p, s) C_{f_X}^{(2)}(s^5 p, s) + \delta_{n_2} C_{[51]}^{(1)}(s^5 p, s) X_6(k; 11 M, 00) \left\} \times 3i \langle \tilde{S}', S_6 : S = 0 | (\sigma_6 \cdot \hat{k}) | S', S_6 : S = 1, M | \tilde{T}', T_6 : T = T_z = 1 | \tau_6^{(+)} | T', T_6 : T = 0 \rangle$$

(32)

where the functions $X_6(k; 11 M, n0)$ are spatial integrals:

$$X_6(k; 11 M, n0) = \int_{4\pi} (\hat{k} \cdot \hat{\rho}_6) Y_{1-M}(\hat{\rho}_6) d^2 \hat{\rho}_6$$

$$\times \int_{0}^{\infty} \rho_6^2 d\rho_6 \varphi_{00}(\rho_6, \sqrt{6/5b_6}) \varphi_{11}(\rho_6, \sqrt{6/5b_6}) j_1(5/6k\rho_6)$$

(33)

Here $S'$ and $T'$ ($\tilde{S}'$ and $\tilde{T}'$) are spin and isospin of the five spectator quarks after the separation of the sixth quark. The first term in curly brackets on the right-hand-side of
Eq. (32) corresponds to the quark transition from an initial s state to a final p state. The second term corresponds to the quark transition from an initial p state to a final s or 2s state. In Eq. (32), we use almost the same notation for one-particle f.p.c. $U_{\{f\}}(S'T', S_6 T_6)$ and $C_{fX}^{(n)}((n'), (n''))$ as before for the three-particle f.p.c.’s in Eq. (24). Note, that the one-particle f.p.c. $U_{\{f\}}(S'T', S_6 T_6)$ in the CST-subspace can be calculated with one-particle scalar factors $\text{SF}$ (cf. Eq. (28)), which can be found for example in Ref. [19].

Due to the orthogonality restrictions for the five spectator quarks, the summations over $\bar{f}'_X, f'_X, S', \bar{T}', S', T'$ collapse to $\delta_{f'_X f'_X} \delta_{S', S'} \delta_{\bar{T}', \bar{T}'} \delta_{T', T'}$, and the only non-vanishing elementary spin- and isospin-flip amplitudes in Eq. (32) are

\[
\langle \bar{S}' = S_6 = 1/2 : \bar{S} = 0 \mid \sigma_6^{(\mu)} \mid S' = S_6 = 1/2 : S = 1, -M \rangle = -(-1)^\mu \delta_{\mu M} \\
\langle \bar{T}' = T_6 = 1/2 : \bar{T} = 1, \bar{T}_z \mid \tau_6^{(\kappa)} \mid T' = T_6 = 1/2 : T = 0 \rangle = (-1)^\kappa \delta_{\kappa \bar{T}_z}
\]  

(34)

C. Decay amplitude after summation over allowed intermediate states

Collecting the shell-model matrix element of Eqs. (31)-(32) for pion production and the overlap integrals of Eq. (29) for all intermediate six-quark configurations, and performing the remaining radial integrals in Eqs. (30) and (33), leads to the following result for the full decay amplitude defined in Eq. (18)

\[
\langle \Psi_{NN}(q), \pi^- \mid \hat{O}_{\pi q}(k) \mid d' \rangle = -\frac{10i}{27(2\pi)^{3/2}} \left(\frac{2}{3\pi}\right)^{3/4} \frac{f_{\pi q}}{m_{\pi}} \sqrt{b_6} \frac{b_6}{E_\pi} \left(\frac{2b_6/b_N}{1 + b_6^2/b_N^2}\right)^6 (kb_6)^2 \exp\left(-\frac{5}{24}k^2b_6^2\right) \\
\times \left[I_{NN}^{(0)}(q) + \frac{2}{27} \left(1 - \frac{k^2b_6^2}{24}\right) I_{NN}^{(2)}(q) \right].
\]  

(35)

The overlap integrals $I_{NN}^{(0)}(q)$ and $I_{NN}^{(2)}(q)$ can be found in the appendix. Note that the inclusion of overlap terms with excited nucleon configurations in the intermediate six-quark states, originating from the different harmonic oscillator parameters $b_6 \neq b_N$, given in Eq. (27) leads to a non-essential renormalization factor of the decay amplitude (38)

\[
1 - \frac{1}{5\sqrt{6}} \frac{(b_6^2/b_N^2 - 1)}{(1 + b_6^2/b_N^2)} \left(1 - \frac{5}{6}k^2b_6^2\right)
\]  

(36)

in front of the term $I_{NN}^{(0)}(q)$ on the right hand side of Eq. (35). This factor (36) can be omitted for small $kb_6$. The $k^2$ behavior of the decay amplitude is due to (i) a factor $k$ in the
transition operator of eq. (3), (ii) due to the fact that a p-wave quark is involved either in the initial or the final state of the one-particle transition matrix element on the right-hand-side of the Eq. (37). We recall that in the case of the $\Delta$-isobar decay into the $\pi N$ channel, the transition matrix element is proportional only to $k_1$, corresponding to the $\sigma_j \cdot k$ term in Eq. (3). The $k^2$ behavior of the $d'$ decay amplitude (35) leads to a very strong dependence of the $d'$ decay width on the value of $M_{d'}$, as we will see in the next section.

IV. NUMERICAL RESULTS AND DISCUSSION

The total hadronic decay width of the possible $d'$ dibaryon $\Gamma_{d'}$ contains three partial widths

$$\Gamma_{d'} = \Gamma_{\pi^- pp} + \Gamma_{\pi^0 pn} + \Gamma_{\pi^+ nn} = 3 \Gamma_{\pi^- pp}$$

which are equal to each other $\Gamma_{\pi^- pp} = \Gamma_{\pi^0 pn} = \Gamma_{\pi^+ nn}$, when we neglect isospin breaking effects. The partial $\pi^- pp$ decay width $\Gamma_{\pi^- pp}$ is defined by the standard expression [8]

$$\Gamma_{\pi^- pp} = 2\pi \int d^3q \int d^3k \delta \left(M_{d'} - 2M_N - \frac{k^2}{4M_N} - \frac{q^2}{M_N} - \sqrt{m_\pi^2 + k^2}\right) \times \left|\langle \Psi_{NN}(q), \pi^- | \hat{O}_{\pi q}(k) | d' \rangle\right|^2,$$

where $q = \frac{q_1 - q_2}{2}$ is the relative momentum of the two final protons and $k$ is the momentum of emitted pion in the c.m. of the $d'$ dibaryon. The $\delta$-function conserves the energy in the decay, while the integration over the momentum conserving $\delta(3)(q_1 + q_2 + k)$ has already been exploited [8] in Eq. (38). The integration over three-particle phase space leads to the following result for the partial $d' \to \pi^- pp$ decay width

$$\Gamma_{\pi^- pp} = \frac{2^5 10^2 \sqrt{6}}{3^8 \sqrt{\pi}} \frac{f_{\pi q}^2}{4\pi} \frac{1}{m_\pi^2} \left(\frac{2b_6/b_N}{1 + b_6^2/b_N^2}\right)^{12} \times \int_{q_{\text{max}}} \frac{2M_N(k_0 b_6)^5}{2M_N + \sqrt{m_\pi^2 + k_0^2}} \exp \left(-\frac{5}{12} \frac{k_0^2 b_6^2}{b_N^2}\right) \left[I_{NN}(q) + \sqrt{\frac{2}{27}} \left(1 - \frac{k_0^2 b_6^2}{b_N^2}\right) I_{NN}^{(2)}(q)\right]^2 q^2 dq.$$

Here, energy conservation relates the pion momentum $k_0$ to the NN relative momentum $q$ via

$$k_0(q) = \left\{4M_N \left[\left(M_{d'} - \frac{q^2}{M_N}\right) - \sqrt{\left(M_{d'} - \frac{q^2}{M_N}\right)^2 - \left(M_{d'} - 2M_N - \frac{q^2}{M_N}\right)^2 + m_\pi^2}\right]\right\}^{1/2}.$$
and for \( q_{\text{max}} = \sqrt{M_N(M_{d'} - 2M_N - m_\pi)} \), all available decay energy is converted to kinetic energy in the relative NN-system, and none to the pion \( E_\pi = m_\pi, k_0 = 0 \).

The calculated decay widths are shown in Table I, where we have introduced the abbreviations p.w., T and U. Here, p.w. refers to a calculation employing a plane-wave final N-N state (4), while T and U refer to calculations using the Tabakin [22] (T) and Ueda et al. [23] (U) separable NN potentials for the final state interaction.

In parentheses we give the results obtained in the approximation of using only one intermediate six-quark configuration \( s^6 \) (\( n=0 \)). With the exception of the results for the Ueda NN-potential for sets 1, 2 and 4 (for which the \( d' \) mass is \( 400 – 650 \) MeV above the \( \pi NN \) threshold), the inclusion of all Pauli principle allowed intermediate 2\( \hbar \omega \) shell-model configurations tends to increase the decay width by some \( 20 – 30 \% \). The largest effect is obtained for \( d' \) masses rather close to threshold, exemplarily shown for sets 3 and 5.

It can be seen from Table I and Fig. 3 that the pionic decay width of the \( d' \) is very sensitive to the dibaryon mass \( M_{d'} \), which determines the available phase space of the three-body \( \pi NN \) decay. The sensitivity grows dramatically near the \( \pi NN \) threshold (2016 MeV). If we extrapolate the results of Table I to the experimental value of \( M_{d'} = 2065 \) MeV, we obtain a very strong reduction of \( \Gamma_{d'} \) as compared with the quite realistic variants (sets 3 and 5) in Table I:

\[
\begin{align*}
\Gamma_{d'}^{\text{p.w.}} & = 0.032 \text{ MeV}, \quad \Gamma_{d'}^{T} = 0.046 \text{ MeV}, \quad \Gamma_{d'}^{U} = 0.083 \text{ MeV}, \quad \text{if } b_N=0.595 \text{ fm and } b_6=0.95 \text{ fm} \\
\Gamma_{d'}^{\text{p.w.}} & = 0.018 \text{ MeV}, \quad \Gamma_{d'}^{T} = 0.045 \text{ MeV}, \quad \Gamma_{d'}^{U} = 0.040 \text{ MeV}, \quad \text{if } b_N=0.6 \text{ fm and } b_6=1.24 \text{ fm}.
\end{align*}
\]

This strong dependence of \( \Gamma_{d'} \) on the value of \( M_{d'} \) is a consequence of the high power of \( (k_0 b_6)^5 \) in the integrand of Eq. (39). The origin of this \( k_0^5 \) behavior (compared with a \( k_0^3 \) behavior in case of the \( \Delta \)-isobar decay) comes, as explained above, from the necessity to excite (or de-excite) a p-wave quark for the production of a pion. Note that for small \( q_{\text{max}} \) (when \( M_{d'} \) is close to the \( \pi NN \) threshold) the function \( k_0(q) \) is linear in the factor \( \sqrt{q_{\text{max}}^2 - q^2} \) and can be written as \( k_0(q) \approx q_{\text{max}} \sqrt{4m_\pi(1 - q^2/q_{\text{max}}^2)}/M_{d'} \). Therefore, for small \( q_{\text{max}} \) the integral in Eq. (39) behaves as \( q_{\text{max}}^8 \). The second high-power factor in Eq. (39) is the scale factor

\[
\left( \frac{2b_6/b_N}{1 + b_6^2/b_N^2} \right)^{12}
\]
which depends sensitively on the ratio $b_6/b_N$. However, this sensitivity is considerably reduced by the factor $b_6^5$ in the integrand. The product

$$b_6^5 \left( \frac{2b_6/b_N}{1 + b_6^2/b_N^2} \right)^{12}$$

is a quite smooth function of $b_N/b_6$. For $b_N=0.6$ fm this product varies from 0.078 fm$^5$ to 0.158 fm$^5$, if $b_6$ varies from 0.6 fm to 1.24 fm.

For small $q_{\text{max}}$, f.s.i. make an important contribution to the $d'$ decay width because of the large scattering length in the $^1S_0$ wave $a_s = -23.7$ fm. The f.s.i. enhances the decay width for example by about 85% for set 5 in Table II. At the experimental mass $M_{d'}=2065$ MeV, the hadronic decay width is more than doubled by the final state interaction. It is interesting that in the case of the Tabakin model with a nodal NN wave function at short range, the contribution from f.s.i. is smaller than for the Ueda model and can even decrease the width compared to the plane wave result (cf. set 4). This is a direct consequence of an approximate orthogonality of the nodal wave function of the Tabakin model to the projection of the intermediate $s^6$ configuration (i.e. the h.o. function $\varphi_{00}$) of Eq. (27) onto the NN channel. This can easily be seen from Fig. 2, where both wave functions are shown. The approximate orthogonality of the functions $\varphi_{00}$ and $\Phi_{\text{Tabakin}}^{NN}$ in the integrand of Eq. (29) reduces considerably the overlap factor $I_{NN}^0(q)$, which gives the dominant contribution to the $d'$ decay width (see values in parenthesis in Table I). As it can be seen in Fig. 3, the disagreement between the Tabakin and Ueda models grows with increasing dibaryon mass $M_{d'}$ (the influence of the large scattering length $a_s$, which is common for both models, becomes negligible compared to the effect of the larger phase space). For sets 1, 2 and 4 in Table I, the Tabakin model leads again to values of $\Gamma_{d'}$, which are even smaller than $\Gamma_{d'}$ in the plane wave approximation neglecting f.s.i.

V. SUMMARY

In this work we have studied the pionic decay of a possible $d'$ dibaryon within the microscopic quark shell-model. We use a single-quark transition operator which describes the production of the pion on a single quark. The dibaryon wave function is given as a single
six-quark translationally invariant shell model configuration, which has been found to pro-
vide an adequate description of the $d'$ \cite{4,5}. Previous results from \cite{8} have been improved
mainly in three points, leading to a complete calculation in the sense, that i) the calculation
is performed consistently on the quark level, i.e. the final two-nucleon state is normalized
and antisymmetrized on the quark level, ii) all important intermediate six-quark states with
nonvanishing overlaps with the final two nucleons are included, and iii) the strong final state
interaction for the two nucleons is taken into account on the basis of the separable Tabakin
\cite{22} and Ueda et al. \cite{23} NN potentials.

Not surprisingly, the small available phase space in the three-body decay is the dominant
mechanism for the narrow width of the $d'$. The large identity factor (15), on the other
hand, enhances the results of previous evaluations disregarding the identity of quarks from
different nucleons. The inclusion of all Pauli principle allowed intermediate $2\hbar\omega$ shell-model
configurations tends to increase the decay width by some $20 - 30\%$. Furthermore, final
state interaction for the two outgoing nucleons also increases the decay width considerably,
if the $d'$ mass is close to the $\pi NN$ threshold. Due to these three effects, the calculated pionic
decay widths lie between $\Gamma_{d'} = 0.18 - 0.32$ MeV for the most realistic set 5, having a $d'$ mass
close to the experimentally suggested one and a characteristic hadronic size of the dibaryon
of $b_0 \approx 1$ fm. This qualitatively agrees with the experimentally suggested value $\Gamma_{d'} = 0.5$
MeV.

Despite the fact that both, the Tabakin and Ueda f.s.i. models, are unsuitable for large NN
energies (as in parameter sets 1, 2 and 4), the two models demonstrate the strong influence of
the short-range behavior of the NN wave function on the $d'$ decay width. (See e. g. Figs. 2 and
3). Recall that these two models are typical representatives of qualitatively different classes
of NN phenomenology. Whereas the Ueda separable potential is an approximation of the
OBEP, i.e. a model with short-range repulsion, the Tabakin potential can be considered as
a unitary-pole approximation (UPA) \cite{20} of a Moscow-type potential model \cite{27} with short-
range attraction and forbidden states. The Moscow model proceeds from the assumption
of a six-quark origin of the short-range NN interaction and pretends to give an adequate
description of the non-local character of the NN force. The main conclusion to be drawn
here is, that these two models, which are phase equivalent, differ considerably in their effect on the \( d' \) decay width. Therefore, a possible \( d' \) dibaryon would provide a natural laboratory for detailed studies of the short-range NN interaction.

An interesting continuation of this work would be to go beyond phenomenological NN potential models and use a completely microscopic quark model approach (see e.g. [28] and references therein). For example, one could calculate the pionic decay of the \( d' \) dibaryon using a final \( ^1S_0 \) NN-scattering wave function that is based on the same microscopic quark Hamiltonian which simultaneously describes the mass \( M_{d'} \) and structure of the \( d' \) dibaryon. However, such a calculation is complicated by the fact that we have used two different Hamiltonians, i.e. two different confinement strengths, for three-quark baryons and the six-quark \( d' \) dibaryon [7]. Thus, the \( d' \) dibaryon could not be explained in terms of the standard constituent quark model, using a common Hamiltonian for any number of quarks. On the other hand, if the \( d' \) really exists, this may be taken as an indication that the effective (nonperturbative) quark-quark interaction depends on the state of the system.

**APPENDIX A:**

In this appendix we present the analytical expressions for the radial integrals \( I_{NN}^{(n)}(q) \) defined in Eq. (30), which are needed to calculate the overlap integral of Eq. (22) between different intermediate six-quark shell-model configurations and the two outgoing nucleons. We recall, that the relative NN-wave function may be described by a simple plane wave (p.w.) \( \Phi_{NN}^{L=0}(q, r) = (2\pi)^{-3/2}j_0(qr) \) or by f.s.i. wave functions resulting, for example, from separable potential representations of the NN-interaction. We use the results for the NN-wave function obtained by Tabakin [22] given in Eq. (20) and the result obtained by Ueda and co-workers [23] given in Eq. (21). Note that coefficients \( \tilde{A} \) and \( \tilde{B}_n \) in Eq. (21) depend on parameters of the one-term separable potential of Ueda et al. [23] \( V(q, q') = -M_{11} g(q) g(q') \), \( g(q) = \sum_n \frac{c_n(q^2 - q_n^2)}{(q^2 + \beta_n^2)(q^2 + \gamma^2)} \). For \( \tilde{A} \) and \( \tilde{B}_n \) we use the following expressions:

\[
\tilde{A}(q) = \alpha^2 \frac{g(q)}{1 - G(q)} \sum_n \frac{c_n}{\gamma^2 - \beta_n^2} \left( \frac{\gamma^2 + q_n^2}{\gamma^2 + q^2} \right),
\]
\[ \tilde{B}_n(q) = \alpha^2 \frac{g(q)}{1 - G(q)} \left( \frac{c_n}{\gamma^2 - \beta_n^2} \right) \left( \frac{\beta_n^2 + q_n^2}{\beta_n^2 + q^2} \right), \]

where \( \alpha^2 = 2\pi^2 m_N M_{11}/h^2 \), \( G(q) = \frac{2}{\pi} \alpha^2 \int_0^\infty [(g^2(k)k^2 - g^2(q)q^2)/(k^2 - q^2)]dk \) and a value of \( \gamma \), which is not fixed in the ref. [23], is fitted to the singlet scattering length \( (a_s=-23.7 \text{ fm}) \), \( \gamma = 11.114 \text{ fm}^{-1} \).

In Table III we introduced the following abbreviations:

\[ f^{(0)}(x) = e^{-x^2/3}, \]
\[ f^{(2)}(x) = -\frac{3}{2} \left( 1 - \frac{4}{9} x^2 \right) e^{-x^2/3}, \]
\[ g^{(0)}(x) = \frac{\sqrt{3}}{x \sqrt{\pi}} e^{-x^2/3} \text{Im} \Phi \left( \frac{ix}{\sqrt{3}} \right), \]
\[ g^{(2)}(x) = -\frac{3}{2} \sqrt{\frac{3}{x \sqrt{\pi}}} \left( \frac{1}{3} - \frac{4}{9} x^2 \right) - \left( 1 - \frac{4}{9} x^2 \right) e^{-x^2/3} \text{Im} \Phi \left( \frac{ix}{\sqrt{3}} \right), \]
\[ F^{(0)}(x) = \frac{\sqrt{3}}{x \sqrt{\pi}} e^{-x^2/3} \left[ 1 - \Phi(x/3) \right], \]
\[ F^{(2)}(x) = -\frac{3}{2} \left\{ \frac{\sqrt{3}}{x \sqrt{\pi}} \left( \frac{1}{3} + \frac{4}{9} x^2 \right) - \left( 1 + \frac{4}{9} x^2 \right) e^{x^2/3} \left[ 1 - \Phi(x/3) \right] \right\}, \]
\[ G^{(0)}_1 = \frac{\sqrt{3}}{x \sqrt{\pi}} \left( \sin x^2/3 - \cos x^2/3 \right) \left[ 1 - \text{Re} \Phi \left( (1 + i)x/\sqrt{3} \right) \right] - \left( \sin x^2/3 + \cos x^2/3 \right) \text{Im} \Phi \left( (1 + i)x/\sqrt{3} \right), \]
\[ G^{(0)}_2 = \left( \sin x^2/3 + \cos x^2/3 \right) \left[ 1 - \text{Re} \Phi \left( (1 + i)x/\sqrt{3} \right) \right] + \left( \sin x^2/3 - \cos x^2/3 \right) \text{Im} \Phi \left( (1 + i)x/\sqrt{3} \right), \]
\[ G^{(2)}_1 = -\frac{3}{2} \left\{ \frac{\sqrt{3}}{x \sqrt{\pi}} \left( \frac{1}{3} - \frac{4}{9} x^2 \right) \sin x^2/3 - \left( \frac{1}{3} - \frac{4}{9} x^2 \right) \cos x^2/3 \right\} \left[ 1 - \text{Re} \Phi \left( (1 + i)x/\sqrt{3} \right) \right] - \left( \frac{1}{3} - \frac{4}{9} x^2 \right) \sin x^2/3 + \left( \frac{1}{3} + \frac{4}{9} x^2 \right) \cos x^2/3 \right\} \text{Im} \Phi \left( (1 + i)x/\sqrt{3} \right), \]
\[ G^{(2)}_2 = -\frac{3}{2} \left\{ \frac{4}{9} x^2 \frac{\sqrt{3}}{x \sqrt{\pi}} + \left( \frac{1}{3} - \frac{4}{9} x^2 \right) \sin x^2/3 + \left( \frac{1}{3} - \frac{4}{9} x^2 \right) \cos x^2/3 \right\} \left[ 1 - \text{Re} \Phi \left( (1 + i)x/\sqrt{3} \right) \right] - \left( \frac{1}{3} + \frac{4}{9} x^2 \right) \sin x^2/3 - \left( \frac{1}{3} - \frac{4}{9} x^2 \right) \cos x^2/3 \right\} \text{Im} \Phi \left( (1 + i)x/\sqrt{3} \right). \]

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I. T. Obukhovsky et al., Figure 1,

"Pionic decay of a possible $d'$ dibaryon and ..."

FIG. 1. Quark line diagram of the pionic dibaryon decay. The elementary pion is produced on a single quark, leaving the remaining six quarks in a relative $^{1}S_0$ nucleon-nucleon scattering state.
I. T. Obukhovsky et al., Figure 2, 
"Pionic decay of a possible $d'$ dibaryon and ..."

FIG. 2. Wave functions of the final $^1S_0$ state for two NN interaction models [22,23] at fixed lab-energy $E_{NN}=100$ MeV. The projection of the $s^6$ six-quark configuration onto the NN channel is also shown.
I. T. Obukhovsky et al., Figure 3,

"Pionic decay of a possible $d'$ dibaryon and ..."

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**FIG. 3.** Pionic decay width $\Gamma_{d'}$ of the $d'$ as a function of the dibaryon mass $M_{d'}$ for different final state interactions (f.s.i.) between the outgoing nucleons: (i) plane wave (p.w., dotted curve), (ii) with f.s.i. using the Tabakin [22] potential (dashed curve), (iii) with f.s.i. using the Ueda et al. [23] potential (plain curve). The harmonic oscillator parameters $b_N=0.595$ fm and $b_6=0.95$ fm are those of set 5 in Table 2. The $\pi NN$ threshold for the decay is at 2016 MeV, while the experimentally suggested resonance position of the $d'$ is at 2065 MeV.
Tables:

| $f_{CTS}$ | $[^1]_{CTS}$ | $[^212]_{CTS}$ |
|-----------|-------------|----------------|
| $f_{CT}$  | $[^2]_{CT}$ | $[^42]_{CT}$  |
|           | $[^3]_{CT}$ | $[^321]_{CT}$ |
|           | $[^3]_{CT}$ | $[^31]_{CT}$  |
|           | $[^3]_{CT}$ | $[^21]_{CT}$  |
| $U_{(f)}^{NN}$ | $\sqrt{\frac{1}{19}}$ | $-\sqrt{\frac{19}{15}}$ |
|           | $\sqrt{\frac{16}{15}}$ | $\sqrt{\frac{1}{18}}$ |
|           | $-\sqrt{\frac{1}{18}}$ | 0               |

TABLE I. The CST part of the f.p.c. three-quark – three-quark decomposition for the projection onto the NN-channel $U_{(f)}^{NN}$, $\{f\} = \{[f_{CTS}], [f_{CT}]\}$

| set | $b_{N}[fm]$ | $b_{6}[fm]$ | $M_{d'}[MeV]$ | $\Gamma_{\pi^-pp}[MeV]$ | $\Gamma_{d'}[MeV]$ |
|-----|--------------|-------------|--------------|----------------|----------------|
|     |              |             | p.w.         | T            | U            |
| 1   | 0.45         | 0.59        | 2705         | 56.8(44.1)   | 7.8(5.2)     | 41.2(46.0)   | 170.5 | 23.3 | 123.6 |
| 2   | 0.47         | 0.65        | 2680         | 44.2(35.7)   | 8.3(7.0)     | 32.6(36.5)   | 132.5 | 24.9 | 97.8  |
| 3   | 0.6          | 1.24        | 2162         | 0.22(0.17)   | 0.27(0.18)   | 0.25(0.22)   | 0.67  | 0.81 | 0.76  |
| 4   | 0.595        | 0.78        | 2484         | 28.3(22.6)   | 10.9(8.9)    | 21.8(21.9)   | 84.8  | 32.6 | 65.4  |
| 5   | 0.595        | 0.95        | 2092         | 0.058(0.036) | 0.061(0.049) | 0.107(0.071) | 0.173 | 0.183 | 0.321 |

TABLE II. Calculated $\pi^-$ decay width $\Gamma_{\pi^-pp}$ and the total hadronic decay width $\Gamma_{d'}$ of the $d'$ dibaryon for five different $d'$ masses and wave functions ($b_{6}$ is the characteristic $d'$ size parameter, and $b_{N}$ is the quark core radius of the nucleon). Masses and wave functions of the $d'$ were obtained in Refs. [6,7] within different models for the microscopic q-q interaction.
| Model | $I^{(0)}_{NN}(q) =$ | $I^{(2)}_{NN}(q) =$ |
|-------|-----------------|-----------------|
| p.w.  | $f^{(0)}(qb_6)$ | $f^{(2)}(qb_6)$ |
|       | $\cos\delta_0 \left[ f^{(0)}(qb_6) + \tan\delta_0 g^{(0)}(qb_6) \right]$ | $\cos\delta_0 \left[ f^{(2)}(qb_6) + \tan\delta_0 g^{(2)}(qb_6) \right]$ |
|       | $A(q)\alpha F^{(0)}(\alpha b_6) + B_1(q)\beta G_1^{(0)}(\beta b_6) + B_2(q)\beta G_2^{(0)}(\beta b_6)$ | $A(q)\alpha F^{(2)}(\alpha b_6) + B_1(q)\beta G_1^{(2)}(\beta b_6) + B_2(q)\beta G_2^{(2)}(\beta b_6)$ |
| Eq. (20) | $\tilde{A}(q)\gamma F^{(0)}(\gamma b_6) - \sum_n \tilde{B}_n(q)\beta_n F^{(0)}(\beta_n b_6)$ | $\tilde{A}(q)\gamma F^{(2)}(\gamma b_6) - \sum_n \tilde{B}_n(q)\beta_n F^{(2)}(\beta_n b_6)$ |
| Ueda  | $\cos\delta_0 \left[ f^{(0)}(qb_6) + \tan\delta_0 g^{(0)}(qb_6) \right]$ | $\cos\delta_0 \left[ f^{(2)}(qb_6) + \tan\delta_0 g^{(2)}(qb_6) \right]$ |
| Eq. (21) | $\tilde{A}(q)\gamma F^{(0)}(\gamma b_6) - \sum_n \tilde{B}_n(q)\beta_n F^{(0)}(\beta_n b_6)$ | $\tilde{A}(q)\gamma F^{(2)}(\gamma b_6) - \sum_n \tilde{B}_n(q)\beta_n F^{(2)}(\beta_n b_6)$ |

**TABLE III.** Radial integrals $I^{(n)}_{NN}(q)$ for plane waves (p.w.) and f.s.i. functions given in Eqs. (20) and (21) for the Tabakin [22] and Ueda [23] separable potential formulation of the relative NN-wave function.