Numerical simulation of the relationship between resistivity and microscopic pore structure of sandstone

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Abstract: The microscopic pore structure of the sandstone rock layer determines the water richness and permeability of the rock layer. Mastering the relationship between the resistivity of the sandstone rock layer and the microscopic pore structure is an important way to evaluate the water richness of the water-bearing rock layer from qualitative analysis to quantitative calculation. Using the finite element method to study the basic single and double modal structures in the sandstone layer of constant current field, the matrix resistivity, pore throat water resistivity, pore shape and difference in single sand model, pore throat model and capillary model are studied. The relationship between the direction of the current and the resistivity of the rock layer. The results show that the resistivity of the sandstone rock layer changes with the change of the matrix resistivity and has a linear relationship. When the contrast ratio between the sand and the matrix resistivity is between 10^-1 and 10, the resistivity of the rock layer changes linearly with the resistivity of the sand; when the contrast ratio is greater than 10, the influence of the resistivity of the matrix is smaller. And when the contrast is greater than 10^3, the resistivity of the rock layer tends to be constant; when the contrast is less than 10^-2, the smaller the resistivity of the sand grains, the smaller the effect on the resistivity of the rock layer, and when the contrast is less than 10^-3, the resistivity of the rock layer tends to constant. Since the water in the sandstone rock layer is the main conductive medium, when the water resistivity is fixed, the smaller the water saturation or the smaller the porosity, the smaller the pores through which water can flow and the greater the resistivity of the sandstone formation. When the microstructures of the horizontal and vertical rock layers are different, different transmitting and receiving electrode layouts have a greater impact on the resistivity of the rock layer, and the different microstructures lead to electrical anisotropy of the rock layer. The research results of this paper lay the foundation for the analysis of the relationship between the more complex sandstone rock model resistivity and micro-pore structure, and the quantitative calculation of the water richness of the sandstone rock layer.

1. Introduction
The heterogeneity of rocks is universal, and there are many factors that affect the heterogeneity. The difference in microscopic pore structure is one of the important reasons for the heterogeneity of rocks[1-4]. With the development of geophysical exploration technology, fine coal mine exploration has
gradually become the mainstream of exploration. Electrical exploration technology is an important part of mine geophysical exploration method, which is a detection method based on the difference of rock electrical properties[5-7]. Electrical exploration is often used to detect the geological anomaly of low resistance water disaster in mine, and the microscopic pore structure of rock directly affects the electrical characteristics of rock. Therefore, mastering the relationship between the microscopic pore structure of rock and resistivity is an important way to improve the accuracy of electrical exploration in mine, and it is also an important way to make electrical exploration from qualitative interpretation to quantitative evaluation. This paper aims at the typical aquifer sandstone in coal measure strata, and explores the relationship between sandstone resistivity and micro pore structure through three-dimensional numerical calculation of sandstone resistivity in constant electric field.

2. Theory and method

2.1. Calculation of apparent resistivity of rock formation in constant current field

Maxwell

The differential equation about electric field and magnetic field is

\[
\begin{align*}
\nabla \times \vec{H} &= \vec{j} + \frac{\partial \vec{B}}{\partial t} \\
\nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t}
\end{align*}
\]

In a constant current field, equation (1) becomes

\[
\begin{align*}
\nabla \times \vec{E} &= 0 \\
\n\nabla \cdot \vec{E} &= 0
\end{align*}
\]

The symbols \( \vec{H}, \vec{B}, \vec{E}, \) and \( \vec{j} \) denote the magnetic field intensity, the magnetic induction intensity, the electric field intensity, and the current density[8].

It is assumed that the length of the rock with regular shape is \( L \), and the cross-sectional area is \( S \). It is known from physics that the relationship between rock resistivity \( R \) and rock resistance \( r \) can be calculated by equation (3)

\[
r = R \cdot \frac{L}{S}
\]

Add a potential difference to both ends and measure the current flowing, then its resistance can be calculated by equation (4)

\[
r = \frac{\Delta U}{I}
\]

When the shape and size are determined, the cross-sectional area and length of the rock remain unchanged, and the ratio of the two is defined as equation (5)

\[
K = \frac{S}{L}
\]

The symbol \( K \) in equation (5) refers to the device coefficient, and its value is only related to the size of the electrode in the model. The model constructed in this paper is cubic sandstone, and it can be seen that its installation coefficient is only related to the edge length of the sandstone model. And the rock resistivity can be transformed into the following form equation (6)

\[
R = r \cdot \frac{S}{L} = K \cdot \frac{\Delta U}{I}
\]

In the numerical simulation calculation, the current can be obtained by follow equation

\[
I = \iint_f \vec{j}_n \cdot d\vec{S}
\]

The rock resistivity can be obtained by bringing the current calculated by equation (7) into equation (6)[9].

2.2. The relationship between rock resistivity and water saturation

There is a direct relationship between rock formation water richness and water saturation. The Archie formula describes in detail the relationship between sandstone resistivity \( R_f \) and formation water resistivity \( R_w \), water saturation \( S \), and porosity \( \varphi \). Its common form is equation (8).
\[ R_f = \frac{ab\rho_w}{S_\phi \rho_m} \]  

\(a\) and \(b\) are constants, and \(m\) is the cementation index, and \(n\) is the saturation index\([10-12]\).

Therefore, if the porosity, cementation index, saturation index, and conductivity of pore water can be grasped, the water saturation can be obtained first based on the measured resistivity, and then the water richness of the formation can be evaluated.

3. Relationship between resistivity and pore structure of monomodal sandstone formation

3.1. Single sand model

The physical model of single sand particle is shown in Figure 1. The cube rock layer is wrapped with single sand particle. The edge length of the cube is 1 mm and the radius of the sand particle is 0.4 mm. The upper surface of the rock layer is the emitter and the lower surface is the receiver. The mesh of the model is shown in Figure 1 (b).

![Figure 1](image)

(a) Geometric model of a single sand-grained rock layer.  
(b) Model after meshing.

Figure 1. Model of spherical single sand-grained rock layer.

3.1.1. The influence of matrix resistivity on the resistivity of sandstone formation

The resistivity of sand is \(10\ \Omega\cdot\text{m}\) and \(10^3\ \Omega\cdot\text{m}\), the matrix resistivity and the formation resistivity are shown in Figure 2, and the cross-sectional current distribution law is shown in Figure 3.

![Figure 2](image)

Figure 2. The influence of matrix on low resistivity of rock formation.

It can be seen from Figure 2, that the resistivity of sand particles is \(10\ \Omega\cdot\text{m}\) and \(10^3\ \Omega\cdot\text{m}\), and the resistivity of sandstone formation changes with the change of matrix resistivity, and the relationship is linear.
In Figure 3(a), the sand resistivity is greater than the matrix resistivity, and it is difficult for the current to pass through the sand. The current lines mainly reach the loop end from the matrix path. In Figure 3(b), since the sand resistivity is less than the matrix resistivity, the current is mainly followed by the sand path to the end of the loop.

3.1.2. The influence of sand resistivity on the resistivity of sandstone formation

When the matrix resistivity is set to $10^3 \Omega \cdot m$ and $10^5 \Omega \cdot m$, the resistivity of the sandstone layer changes with the resistivity of sand particles as shown in Figure 4.

When the matrix resistivity is $10^3 \Omega \cdot m$ and the sand resistivity value is in the range of $10^2$ to $10^4 \Omega \cdot m$, the formation resistivity changes linearly with the sand resistivity; when the sand resistivity is much greater than $10^4 \Omega \cdot m$, the resistivity of the formation tends to be constant; when the resistivity of the sand is much less than $10^5 \Omega \cdot m$ the current is difficult to pass through the sand, and mainly affected by the resistivity of the matrix, the sand resistivity has less and less influence on the resistivity of the formation, and when the resistivity of the sand is greater than $10^6 \Omega \cdot m$, the resistivity of the formation tends to be constant. Similar conclusions can be drawn when the substrate resistivity is $10^4 \Omega \cdot m$. It can be seen from the figure that the resistivity of the sandstone formation has a nonlinear relationship with the resistivity of sand particles and the resistivity of the background matrix.

3.1.3. The influence of sand size on the resistivity of sandstone formation

Controlling the size of the sandstone layer remains unchanged, the influence of the change in sand size on the resistivity of the rock formation is shown in Figure 5. The matrix resistivity in the figure is $10^5 \Omega \cdot m$, and the sand particle resistivity varies from $10^{-2}$ to $10^3 \Omega \cdot m$. In Figure 5, when the sand particle resistivity is less than the matrix resistivity $10^5 \Omega \cdot m$, the larger the sand particle radius, the smaller the rock layer resistivity. When the sandstone resistivity is greater than the matrix resistivity $10^5 \Omega \cdot m$, the larger the sand particle radius, the larger the rock layer resistivity. The analysis suggests that when the radius of sand grains gradually becomes larger, the space occupied by sand grains in the rock formation becomes larger and larger. When the sand radius is very small relative to the rock formation, the influence of sand grains on the resistivity of the rock formation will be weakened and tends to be constant.
3.2. The influence of formation resistivity under the pore throat model
The pores in the monomodal sandstone layer are equivalent to spherical pores, and the throat is equivalent to columnar pores. The model is shown in Figure 6.

3.2.1. The influence of simple pore throat model resistivity

The simple pore throat model of single-modal sandstone established is shown in Figure 7. The model size of the sandstone rock layer in the model is 1mm, the equivalent spherical pore radius is 0.25mm, the matrix resistivity is $10^4\,\Omega\cdot\text{m}$, and the pore throats are completely filled with water and which resistivity is $1\,\Omega\cdot\text{m}$. The relationship between the resistivity of the sandstone layer and the radius of the roar is shown in Figure 8.

In Figure 8, when the pore throat is saturated with low-resistance water, the resistivity of the formation is obviously affected by the change of the throat radius. When the radius of the throat increases, the pore water increases and the resistivity of the formation decreases rapidly. When the radius of the throat continues to increase, the decline of the resistivity of the formation slows down. After the radius of the throat increases to 0.08mm, the resistivity of the formation tends to be constant.
Figure 8. The influence of throat radius on the resistivity of sandstone rock.

Figure 9. The influence of pore throat water resistivity changes on sandstone rock resistivity.

The equivalent pore radius is 0.25mm, the throat radius is 0.1mm, the matrix resistivity is $10^4\ \Omega\cdot m$, and the pore throat water resistivity varies from $10^{-2}$ to $10^4\ \Omega\cdot m$, as shown in Figure 9. When the resistivity of the pore throat water increases, the resistivity of the formation shows an upward trend, and the resistivity of the formation changes linearly with the resistivity of the pore throat water in the range of $10^{-2}$ to $10^2\ \Omega\cdot m$. When the resistivity of pore throat water exceeds $10^2\ \Omega\cdot m$, the influence on the resistivity of sandstone formation gradually weakens, and the resistivity of pore throat water continues to increase and the formation resistivity tends to be constant.

Figure 10. The influence of different sandstone matrix resistivity on sandstone layer resistivity.

Figure 11. Resistivity of rock formations with different pore-throat ratios when saturated with water.

Set the throat water resistivity to 1\Omega\cdot m, the sandstone matrix resistivity varies from 10 to $10^4\ \Omega\cdot m$, and the influence of matrix resistivity on the resistivity of sandstone formations is shown in Figure 10. When the resistivity of the sandstone matrix is less than $1\ \Omega\cdot m$, the resistivity of the rock formation is less affected by the matrix. In the range of 0 to $10^2\ \Omega\cdot m$, the resistivity of the rock formation increases linearly with the increase of the matrix resistivity. When the matrix resistivity is greater than $10^3\ \Omega\cdot m$, the resistivity of the formation is affected by the resistivity of the matrix and begins to weaken and eventually tends to be constant.

Figure 11 shows the changing law of resistivity of water-bearing formations with different pore-throat ratios. In this figure, the pore-throat ratio (the ratio of the pores to the throat radius) varies in the range of 2 to 8. When the throat radius is $4*10^{-5}m$, the resistivity of the sandstone layer gradually
decreases with the increase of the pore-throat ratio, and the resistivity of the sandstone layer decreases linearly with the increase of the pore-throat ratio.

3.2.2. The influence of resistivity of complex pore throat model

![Complex pore throat geometry model](image1)  
![After meshing](image2)

Figure 12. Complex pore-throat model.

In the model shown in Figure 12, the equivalent pore radius is 0.2mm, the throat radius varies from $10^{-5}$ m to $10^{-4}$m, the matrix resistivity is $10^4 \Omega \cdot m$, and the water resistivity in the pore throat is $1 \Omega \cdot m$.

In the complex pore throat model, when the pore throat is saturated with water, the resistivity of the formation decreases rapidly as the diameter of the throat increases. By comparison, it can be seen that when the pore throat is saturated with low water resistance, the throat radius has a greater influence on the resistivity of the formation. This conclusion is similar to the conclusion in the single pore throat model.

![Rock layer resistivity corresponding to the change in throat radius](image3)  
![Rock formation resistivity corresponding to changes in pore throat water resistivity](image4)

Figure 13. Rock layer resistivity corresponding to the change in throat radius  
Figure 14. Rock formation resistivity corresponding to changes in pore throat water resistivity

The equivalent pore radius is 0.2mm, the throat radius is 0.025mm, the sandstone matrix resistivity is $10^4 \Omega \cdot m$, and the pore throat water resistivity varies from $10^{-2}$ to $10^4 \Omega \cdot m$. The resistivity of the sandstone formation increases with the increase of the pore throat water resistivity, and when the pore throat water resistivity increases in the range of $0.01$ to $10^3 \Omega \cdot m$, the resistivity of the sandstone formation increases linearly, and the pore throat water resistance increases. When the rate continues to increase beyond $10^2 \Omega \cdot m$, the influence of the pore throat water resistivity on the resistivity of the sandstone layer gradually weakens.

The equivalent pore radius is 0.2mm, and the throat radius is 0.025mm. The resistivity of sandstone matrix varies from $10^{-2}$ to $10^4 \Omega \cdot m$. When the pore throat is saturated with water, when the sandstone matrix resistivity exceeds $10^2 \Omega \cdot m$, the influence of the matrix resistivity on the formation resistivity...
begins to decrease. When the sandstone matrix resistivity exceeds $10^3\,\Omega\,\text{m}$, the formation resistivity is approximately equal to a constant.

![Figure15. Rock layer resistivity corresponding to the change of rock matrix resistivity.](image)

![Figure16. Rock formation resistivity corresponding to different pore-throat ratios in the complex pore-throat model.](image)

In Figure 16, when the pore throat is saturated with water, the resistivity of the formation shows a downward trend when the porosity becomes larger, and when the throat radius is relatively large, the pore throat ratio has a greater influence on the pore volume. When the throat radius is 0.04mm, the increase of the pore throat radius than the throat radius causes the pores inside the rock to increase rapidly. Affected by the low water resistance, the resistivity of the rock formation decreases obviously.

4. The relationship between resistivity and pore structure of dual-modal sandstone formations

4.1. The influence of porosity in dual-modal sandstone formation on formation resistivity

![Figure17. Bimodal sandstone stratum model.](image)

Take the first-level sand particles and arrange them closely in a cube, and the radius of the second-level sand particles is smaller than the minimum radius of the cube surface, as shown in Figure 17. The radius of the first grade sand grain is 0.5mm, and the radius of the second grade sand grain is less than 0.2mm according to the petrological geometric characteristics. When the radius of the first grade sand grain is 0.5mm and the radius of the second grade sand grain is 0.2mm, the porosity takes the minimum value [13- 14]. In the model, it is assumed that the sand particle resistivity is $10^3\,\Omega\,\text{m}$, and the pore water resistivity ranges from $10^{-2}$ to $10^4\,\Omega\,\text{m}$.
Figure 18. The influence of porosity on rock resistivity.

When the porosity is 0.35, 0.4 and 0.45, the formation resistivity changes with the pore water resistivity as shown in Figure 18. The resistivity of pore water is the same. When the porosity is 0.35, the resistivity of the formation is the largest, when the porosity is 0.4, the resistivity of the formation is second, and when the porosity is 0.45, the resistivity of the formation is the smallest. The analysis believes that the model is controlled and the size of the sand particles is unchanged, and the radius of the secondary sand particles is adjusted to control the porosity. When the porosity is larger, the current passes through the pores in the rock formation with less hindrance, so the resistivity is smaller.

4.2. Resistivity characteristics of dual-modal porous sandstone capillary model

The pores and throat shapes of the sandstone layer under the accumulation of dual-modal sandstone are irregular, and the pore structure of dual-modal sandstone is more complicated. The capillary model is the most commonly used and most effective model for studying porous media [18]. It can describe the characteristics of the more complex pores and throats in dual-modal sandstone reservoirs. The rock matrix resistivity is $10^4\,\Omega\cdot m$, and the capillary dielectric resistivity ranges from $10^{-2}$ to $10^3\,\Omega\cdot m$. $R_h$ represents the resistivity of the formation along the horizontal arrangement of the capillary, and $R_v$ represents the resistivity of the formation along the vertical arrangement of the capillary. The four capillary models established are shown in Figure 19.

Figure 19. Schematic diagram of capillary models with different shapes.
Figure 20. Current resistivity of the formation along the direction of the capillary. At this time, the resistivity of the pore-throat capillary formation is relatively large, and the resistivity increases as the resistivity of the capillary water becomes larger, in a linear relationship.

Figure 21. Current resistivity of rock formation in vertical capillary direction. When the capillary water resistivity is less than $10^3 \Omega \cdot m$, the formation resistivity is less affected by the capillary water resistivity change. When the capillary water resistivity is greater than $10^3 \Omega \cdot m$, the formation resistivity increases sharply as the capillary water resistivity increases. The non-equal diameter ordinary capillary model has the largest resistivity and the curved capillary has the smallest resistivity.

Figure 22. Current distribution law of curved tube. When the current follows the direction of the capillary, it is easier for the current to pass through the capillary. Most of the current avoids the sandstone matrix with high resistivity and reaches the loop end through the capillary. Therefore, the resistivity in this direction is obviously affected by the capillary water resistivity. When the current passes through the capillary vertically, the current first passes through the high-resistivity sandstone matrix to reach the capillary, and then through the high-resistivity sandstone matrix. Most of
the current passes through the matrix, so the resistivity of the formation is basically less affected by the capillary water.

5. Conclusion
(1) Matrix resistivity and formation resistivity Sandstone formation resistivity changes with the matrix resistivity, and shows a linear relationship.

(2) When the resistivity contrast between sand and matrix is between 10^{-1} to 10, the resistivity of the formation changes linearly with the resistivity of the sand; when the contrast is greater than 10, it is affected by the resistivity of the matrix, and the resistivity of the sand affects the resistivity of the formation. When the contrast ratio is greater than 10^{3}, the resistivity of the rock formation tends to be constant; when the contrast is less than 10^{-2}, because the sand grains are wrapped by the matrix, the current loop must pass through the matrix, and the resistivity of the sand grains is affected by the matrix resistivity. Smaller has less influence on the resistivity of the formation, and when the contrast is less than 10^{-3}, the resistivity of the formation tends to be constant.

(3) The resistivity of the formation in the single-mode and dual-mode models is affected by the pore-throat ratio, pore-throat water and matrix resistivity. Since sandstone resistivity and matrix resistivity are both very large, almost non-conductive, and rock formation water is the main conductive medium. When the water resistivity is constant, the smaller the water saturation or the smaller the porosity, the pores through which water can flow. The smaller the value, the greater the resistivity of the sandstone formation.

(4) When the horizontal and vertical rock formations have different microstructures, different transmitter and receiving electrode layouts have a greater impact on the resistivity of the formations, and the different microstructures lead to the electrical anisotropy of the formations.

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