Spinor model of a perfect fluid and their applications in Bianchi type-I and FRW models

Bijan Saha

Abstract Different characteristic of matter influencing the evolution of the Universe has been simulated by means of a nonlinear spinor field. Exploiting the spinor description of perfect fluid and dark energy evolution of the Universe given by an anisotropic Bianchi type-I (BI) or isotropic Friedmann-Robertson-Walker (FRW) one has been studied.

Keywords Spinor field · Perfect fluid · Dark energy

1 Introduction

Detection and further experimental reconfirmation of current cosmic acceleration pose to cosmology a fundamental task of identifying and revealing the cause of such phenomenon. This fact can be reconciled with the theory if one assumes that the Universe is mostly filled with so-called dark energy. This form of matter (energy) is not observable in laboratory and it does not interact with electromagnetic radiation. These facts played decisive role in naming this object. In contrast to dark matter, dark energy is uniformly distributed over the space, does not intertwine under the influence of gravity in all scales and it has a strong negative pressure of the order of energy density. Based on these properties, cosmologists have suggested a number of dark energy models, those are able to explain the current accelerated phase of expansion of the Universe. Here we list the main tendencies:

The $\Lambda$ term: The $\Lambda$ term or cosmological constant was first introduced by Einstein (1917, 1919) in order to ensure the static cosmological model of the Universe. Though Friedmann (1922, 1924) proposed the model of expanding Universe in 1922, only after the experimental confirmation by Hubble of this fact Einstein returned to his original form of equations. In the second half of 1960’s the $\Lambda$ term temporarily reemerged again. Finally after the hallmark paper by Guth (1981) on the inflationary cosmology, researchers began to study the models with cosmological constant. When in 1998 it was found that our Universe is undergoing an accelerated expansion, a positive $\Lambda$ term, which corresponds to a repulsive force, was used to explain this new founded phenomenon of the evolution of the Universe. It should be noted that in order to overcome the problem of eternal acceleration related to accelerated mode of expansion many authors use a negative $\Lambda$ term corresponding to an additional gravitational force. Models with $\Lambda$ terms of different signs were considered in the papers (Saha 2001a, 2006a, Saha and Boyadjiev 2004).

Quintessence: This is the most widely encountered dark energy type with the equation of state (Caldwell et al. 1998; Sahni and Starobinsky 2000; Zlatev et al. 1999; Saha 2006b)

$$w = \frac{p_q}{\varepsilon_q}.$$  \hspace{1cm} (1.1)

$w$ is considered to be constant. The above equation of state is well known, namely, for $w \in [0, 1]$ it describes a perfect fluid. One of its specific terms is the cosmological constant (the $\Lambda$ term) for $w = -1$ (Saha 2006a; Padmanabhan 2003; Sahni 2004). Note that, in order for the expansion of a Universe filled predominantly with such a substance to take place at an accelerating rate the conditions $w < -1/3$ must hold. Usually the value of the constant $w$ varies between $-1$ and $-1/3$, that is, $w \in [-1, -1/3]$. This bound is connected with the following fact. The rigorous definition of $w$ (both for the equilibrium state and for small perturbations) implies...
that when $w < -1$ the propagation velocity of small perturbations (for instance, the sound) in quintessence exceeds the speed of light and, hence, the inequality leads to violation of the causality principle.

The Chaplygin Gas: In order to combine two different physical concepts as dark matter and dark energy, and, hence, to reduce two physical parameters to a single one, a rather exotic equation of state was proposed, (see Kamenshchik et al. 2001). In this paper the authors describe a transformation of a Universe filled with dust into a Universe expanding at an accelerating rate. The model proposed in Kamenshchik et al. (2001) was generalized in the papers (Bilic et al. 2002; Sen et al. 2002a, 2002b) defined by the following equation of state

$$p_{ch} = -\frac{A}{\epsilon_{ch}}$$

(1.2)

where $A$ is some positive constant and $0 < \alpha \leq 1$. The standard model proposed in Kamenshchik et al. (2001) corresponds to the case $\alpha = 1$. Note that the model with Chaplygin gas in anisotropic cosmology was studied in Saha (2005).

Phantom Type Dark Energy: The phantom is dark energy with a strong negative pressure, and it can be modeled by a scalar field, with a negative kinetic energy in the Lagrangian

$$L = \frac{l}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi),$$

(1.3)

where $l = -1$ corresponds to the phantom, while $l = 1$ to the standard scalar field. Here, $V(\phi)$ is a potential. The most striking result that is attributed to the phantom is that the energy density grows proportionally to the scale factor. Thus, in contrast to standard sources when the increase of the energy density corresponds to a reduction of the scale factor, in this case, the energy density’s increase is accompanied by Universe expansion. This leads to the appearance of singularities in the future, which is called the Big Rip because of some special properties. In this case the size of the Universe becomes infinite during a finite time interval (Diego et al. 2003; Dabrowski 2007). Note that for $w < -1$ the equation of state (1.1) also yields a phantom.

Oscillating Dark Energy: A lot of problems emerged after the discovery of a positive acceleration in the Universe expansion. One of the most baffling of those is the problem of eternal acceleration Rubano et al. (2003). A positive value of the cosmological term (Saha 2006a), as well as most of the model proposed in the literature, leads to an eternally accelerating rate. There are many different approaches to the elimination of these problems. A cosmological model of a cyclic Universe experiencing periodic expansions and contractions was proposed in the paper (Steinhardt and Turok 2002). In this case the Universe emerges from a Big Bang and vanishes into a Big Crunch only to reemerge again from a Big Bang. The expansion phase of every cycle contains radiation, matter, and quintessence eras; the latter corresponds to the current accelerated expansion. The paper (Felder et al. 2002) investigates cosmological models where the effective potential $V(\phi)$ can be negative for some values of $\phi$. In this case a cyclic model of a Universe is realized. One of the simplest ways to obtain a cyclic model is to introduce a negative $\Lambda$ term together with some potential (Saha 2001a; Saha and Boyadjiev 2004; Cardenas et al. 2003).

Models with interaction between dark energy and dark matter: Experimental checks conducted within the solar system impose strict constraints on the possibility of non-minimal interaction between dark energy and usual (background) matter (Will 2001). Nevertheless, a possibility of additional (non-gravitational) interaction between dark energy and dark matter, without a contradiction with the experimental data, appears due to the unknown nature of the dark matter as the main fraction of that background. Moreover, it was shown in the papers (Olivares et al. 2005; Pavon et al. 2004) that the models with interacting dark energy are in good agreement with the modern observational data. A lot of papers proposing models with interacting dark energy and dark matter appeared in this connection (see Gonzalez and Quiros 2007; Chimento et al. 2003).

Scalar-tensor models of dark energy: Scalar tensor theory of gravitation is an alternative to or a generalization of Einstein’s theory of gravitation, where a scalar field is present in addition to the tensor field. It was proposed almost half a century ago in a series of papers (Fierz 1937; Jordan 1959; Brans and Dicke 1961), and even at the present time it remains important for explaining the accelerating expansion phase, especially in the inflational and quintessence scenarios. The main assumption of this theory is the connection between the matter, and the scalar and gravitational fields $\phi$ and $g_{\mu \nu}$ via some effective metrics $\tilde{g}_{\mu \nu} = A(\phi)g_{\mu \nu}$. The paper (Gannouji et al. 2007) considers a scalar-tensor model of dark energy with a new physical degree of freedom, namely, the scalar field $\phi$ of the graviton is responsible for variations in gravity. Scalar-tensor models of the usual and phantom dark energy were investigated in Gannouji et al. (2006). Similar models for the case of the Bianchi type-I spaces were considered in Fay (2000). This paper considers the dynamic behavior of the metric functions for three different interactions.

Models with Tachyon Matter: The tachyon idea is not new, and after the series of papers (Sen 2002a, 2002b) these particles found applications in cosmology again. They still have not been observed experimentally, and some of their types (the rolling tachyon) possess a very interesting equation of state, where the tachyon parameters exhibit smooth variations within the interval $(-1, 0)$. These properties make the tachyon one of the candidates for modeling dark energy (Srivastava 2004; Shao and Gui 2007; Copeland et al. 2005;