Manifestation of fundamental quantum complementarities in time-domain interference experiments with quantum dots: a theoretical analysis

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A theoretical analysis is presented showing that fundamental complementarity between the particle-like properties of an exciton confined in a semiconductor quantum dot and the ability of the same system to show interference may be studied in a time domain interference experiment, similar to those currently performed. The feasibility of such an experiment, including required pulse parameters and the dephasing effect of the environment, is studied.

PACS numbers: 78.67.Hc, 03.65.Ta

I. INTRODUCTION

Bohr’s complementarity principle is one of the central points of the quantum theory. It states that laws of nature allow one to describe a quantum system in terms of mutually excluding properties. Such apparently contradictory, complementary descriptions are indispensable for complete understanding of the quantum world but the corresponding properties of a physical system can never fully manifest themselves simultaneously in a single experiment.

The most celebrated complementarity of this kind is the particle–wave duality. The usual starting point for many textbooks is to discuss it in terms of a double-slit (Young) experiment, in which particles form an interference pattern on a screen after passing through a system of two openings. This wave-like behavior disappears if one detects through which of the openings the particle really passed, thus querying a particle-like property (indistinguishability). In fact, visibility reduction results from correlations arising between the particle and its environment, due to which a measurement on the latter might, in principle, yield information on the path chosen by the particle (which path information). Thus, interference is affected no matter whether this measurement is actually done and the effect is the same for controlled measurement (environment being part of a measurement apparatus) and for uncontrolled environment-induced dephasing.

Experimentally pursuing the relation between the available which path information and the ability to show interference presents a twofold difficulty. First, one needs sufficient quantum control over an individual quantum system to observe quantum interference effects in a single-particle experiment. Second, in order to study the whole range of intermediate cases, where some partial information on the particle path is attained, one has to be able to extract information on the quantum state (path) of the system not only in a non-destructive manner but also in a controlled amount, i.e., in a way that allows one to infer the path correctly only with a certain probability $1/2 < p < 1$. Ideally, the extracted information should be experimentally available (which is not the case if the systems is “measured” by the environment in a dephasing process) so that its amount may be independently verified.

In spite of these challenging requirements, the effect of partial which path information on visibility of interference fringes was observed in optical, neutron, and atomic interference experiments. Which path decoherence of electrons has been tested in coherent transport experiments in semiconductor systems. An experiment with a free electron interferometer has also been proposed.

It is natural to investigate quantum complementarity by studying which path decoherence in space domain experiments, where the two system states correspond to real trajectories (paths) of a particle. However, phase-sensitive superpositions and the resulting interference effects are manifested also in time-domain interference experiments, like those performed on excitons (electron-hole pairs) confined in semiconductor quantum dots (QDs). Here, the final exciton occupation (probability of finding the exciton in the QD over a large number of repetitions) after a sequence of two optical pulses oscillates as a function of the relative phase (time delay) between the pulses. Obviously, in spite of the wave-like behavior manifested by this interference effect, a single occupation measurement always yields either 0 or 1, demonstrating the particle-like nature of the exciton. In view of the important role that quantum complementarity plays in the fundamentals of quantum mechanics it seems interesting to take advantage of the recent progress in time domain interference experiments on QDs and to design an experiment in which quantum complementarity manifests itself in these systems.

In this paper it is shown that the existing time-domain interference experiments on QD systems may indeed be extended to test complementarity between the knowledge on the particle-like state of an exciton and its ability to show quantum interference. To this end, one performs a detection of the exciton between the two pulses, when the system is in a superposition state. This is equivalent to determining through which slit a particle passes in a Young setup and corresponds to testing a particle-like property of the confined exciton (is the exciton really there?) treating it as an indivisible entity. Similarly as in the space-domain setup, this must affect the ability of the same system to show wave-like behavior (interfer-
ence), in accordance with the fundamental complementarity principle. A non-destructive method of extracting partial information on the system state is provided in a natural way by a non-projective indirect measurement, via conditional dynamics of a biexciton system leading to a variable degree of correlation with a second exciton (with opposite polarization or confined in a neighboring dot with different transition energy) that plays the role of an ancilla system. Depending on the amount of extracted knowledge on the system state, the visibility of interference fringes is reduced to a various degree, so that the complementarity principle may be studied on the quantitative level. Moreover, the extracted information is accessible via an additional measurement (up to imperfect, but known, detector efficiency) and its amount may be verified so that experimental access to both components of the complementarity principle is possible. In the paper, an appropriate sequence of control pulses for performing the complementarity experiment is discussed. The feasibility of the experiment in terms of spectral selectivity of control pulses and of resilience of the effect against environmental dephasing is also studied.

The paper is organized as follows. In Sec. II, time domain interference experiments on QDs are reviewed. Sec. III describes the experiment in which the complementarity may be tested. The feasibility of the coherent control necessary for realizing the proposed scheme under realistic conditions is studied in Secs. IV and V. The final section contains concluding remarks, including a brief discussion of available experimental techniques.

II. TIME DOMAIN INTERFERENCE

Let us start with a brief review of time domain interference experiments on quantum dots. QDs are artificial atomic-like nanostructures with carrier states quantized due to strong spatial confinement. The fundamental optical transition in a QD consists in promoting an electron from the valence band to the conduction band, thus creating a confined electron-hole pair, referred to as a (confined) exciton, corresponding to a single spectral line. In an ideally circular dot, the confined exciton state is degenerate with respect to the orientation of the spins of the carriers. A single transition out of the degenerate doublet may be addressed by an appropriate choice of circular polarization of the laser beam. In a more general case, breaking of the circular symmetry leads to mixing of the angular momentum eigenstates due to electron-hole exchange interaction and to a fine structure splitting. In the presence of this effect, excitons with definite circular polarization are no longer eigenstates of the system and an oscillation between the two polarization states will appear. Nonetheless, since the fine structure splitting is usually very small (~ 100 µeV or less), the period of these oscillations is relatively long and its effect on the system dynamics may be neglected for sufficiently short pulse sequences.

Time domain interference experiments on QDs are performed with two phase-locked laser pulses selectively tuned to one of the two fundamental optical transitions. The first pulse induces a superposition of no exciton and single exciton states. The second pulse rotates the system state further, with a phase shift depending on the delay between the pulses. The final occupation varies periodically as a function of the phase shift, producing time-domain interference fringes.

We will denote the empty dot state by |0⟩ and the single exciton state by |1⟩. The exciton energy is ε (‘S’ stands for ‘system’ and refers to the exciton transition addressed in the interference experiment). In the rotating basis, |1⟩ = e^{iεsf} |1⟩, |0⟩ = |0⟩, the Hamiltonian for the driven system, upon neglecting the non-resonant terms (rotating wave approximation, RWA), is

\[ H_1 = \frac{1}{2} \mu E_{S1}(t)|0⟩⟨1| + \text{H.c.} \]

\[ + \frac{1}{2} \mu E_{S2}(t - t_{S2})(e^{-iφ}|0⟩⟨1| + \text{H.c.}), \]

where \( μ \) is the inter-band dipole moment, \( E_{S1,S2}(t) \) are the envelopes of the two pulses, and \( φ = ε_{S1}t_{S2}/ℏ \) is the phase shift dependent on the time delay between the pulses. The two terms in Eq. (1) account for the action of the control pulses coupled to a single exciton transition by polarization selectivity: The first pulse (S1) arrives at \( t = 0 \) and prepares the initial superposition state. It corresponds to splitting the particle path in a usual space-domain (two-slit) experiment. The second (S2) pulse arrives at \( t = t_{S2} \). Its arrival time must be tuned with femtosecond accuracy to provide controllable phase-locking of the pulses, as done in QD interference experiments. This pulse plays the role of “beam merger” providing, at the same time, a phase shift between the “paths”.

In the experiment, the system is initially in the state |0⟩. The first pulse (S1) is a \( π/2 \) pulse that performs the transformation

\[ U_{S1} = \frac{1}{\sqrt{2}} (I - iσ_x), \]

where \( σ_x = |0⟩⟨1| + |1⟩⟨0| \) is the Pauli matrix. This pulse leaves the system in the equal superposition state

\[ |ψ⟩ = \frac{|0⟩ - i|1⟩}{\sqrt{2}}. \]

The second pulse is again a \( π/2 \) pulse,

\[ U_{S2} = \frac{1}{\sqrt{2}} (I - i\hat{n} · σ), \]

where \( \hat{n} = [\cos φ, \sin φ, 0] \) and \( σ \) is the vector of Pauli matrices in the basis \( |0⟩, |1⟩ \). After this pulse, the average number of excitons in the dot is

\[ N(φ) = |⟨1|U_{S2}|ψ⟩|^2 = \frac{1}{2} (1 - \cos φ), \]
and changes periodically between $N_{\text{min}} = 0$ and $N_{\text{max}} = 1$ as a function of phase shift (or delay time), thus producing an interference pattern.

The quality of the interference pattern is customarily quantified in terms of visibility of interference fringes

$$V = \frac{N_{\text{max}} - N_{\text{min}}}{N_{\text{max}} + N_{\text{min}}}.$$ 

The amplitude (visibility) of the fringes in an ideal experiment is $V = 1$ for $\pi/2$ pulses, i.e., for an equal superposition in between them. Otherwise, some a priori information on the superposition state can be inferred and the visibility is reduced. For simplicity, the present discussion is restricted to the equal superposition case.

### III. COMPLEMENTARITY BETWEEN WHICH PATH INFORMATION AND INTERFERENCE

This Section discusses the essential modification to time domain interference experiments that allows one to attain partial information on the state of the system and to observe the related visibility reduction of the interference pattern. First, however, a measure of the partial information is introduced and the complementarity principle is formulated in a quantitative form.

The notion of “partial information” is understood as follows. The system (S) of interest is coupled to another quantum probe (QP) system and conditional dynamics of the latter is induced, leading to correlations between the states of the systems S and QP. Next, a measurement on QP is performed and its result is used to infer the state of S, i.e., to predict the outcome of a subsequent measurement on S. The probability of a correct prediction ranges from 1/2 (guessing at random in absence of any correlations) to 1 (knowing for sure, when the systems are maximally entangled). From a formal point of view, this procedure is a non-projective, generalized measurement on the exciton system, performed within an indirect measurement scheme.

Quantitatively, an intrinsic measure of information on the system S extracted by QP is provided by the distinguishability of states:

$$\mathcal{D} = 2 \left( p - \frac{1}{2} \right),$$

where $p$ is the probability a correct prediction for the state of S maximized over all possible measurements on QP. In this way, guessing at random and knowing for sure correspond to $\mathcal{D} = 0$ and $\mathcal{D} = 1$, respectively. According to a general theory, the complementarity relation between the knowledge of the system state and the visibility of the fringes may be written, using the distinguishability $\mathcal{D}$ as a measure of information, in the quantitative form,

$$\mathcal{D}^2 + V^2 \leq 1.$$  

The equality holds for systems in pure states.

In a QD, the formal scheme of indirect measurement translates naturally into well known conditional dynamics of a biexcitonic system in which the exciton addressed in the interference experiment described in Sec. III (system S) is coupled to another exciton (QP), localized either in the same or in a neighboring dot. In the former case the subsystems are distinguished by their polarization, while in the latter case they are distinguishable by different excitation energies. The Coulomb (dipole-dipole) interaction between the two excitons shifts the energy of the biexciton state, so that spectrally narrow pulses may induce dynamics of the QP exciton conditional on the state of the other one, as required for the indirect measurement scheme. On the other hand, spectrally broad pulses may be used to perform unconditional rotations. The final measurement is done by detecting photons emitted by recombining excitons, again with polarization or energy resolution. While it is reasonable to neglect the effect of the fine-structure splitting over the relatively short duration of the control sequence, the average time before the photons are emitted (exciton lifetime) is usually longer (hundreds of picoseconds to nanoseconds) and the non-conservation of the exciton polarization may have considerable impact on measurement results in the single-dot scheme based on polarization-selectivity (this restriction may be partly overcome, as explained below).

We will use a tensor product notation with $|0\rangle$ and $|1\rangle$ denoting the absence and presence of the respective exciton, as previously, with the interfering system (S) always to the left. In the rotating basis with respect to both subsystems, the RWA Hamiltonian for the biexciton system is

$$H = H_1 \otimes I + \Delta |1\rangle\langle 1| \otimes |1\rangle\langle 1| + \frac{1}{2} \mu E_{QP} (t - t_{QP}) I \otimes |0\rangle\langle 1| + H.c.$$  

where $\Delta$ is the bi-exciton energy shift and $E_{QP}(t)$ is the envelope of the pulse coupled to the QP exciton. Here the first term contains the Hamiltonian and corresponds to the pulse sequence of the interference experiment described in the previous Section, the second one accounts for the bi-excitonic energy shift and the third term describes the action of the pulse coupled to the second (QP) exciton and spectrally tuned to the exciton-biexciton transition. This pulse arrives at $t = t_{QP}$, between the other two pulses (that is, $0 < t_{QP} < t_{S2}$), and will induce the conditional dynamics necessary for the indirect measurement. Its phase is irrelevant and will be assumed 0. The structure of system excitations, the sequence of pulses and the corresponding quantum-logic diagram are shown in Fig. 4.

Assume that the exciton (system S) is in the equal superposition state $|\psi\rangle$. The probability of correctly guessing the result of a measurement in the $|0\rangle, |1\rangle$ basis without any additional information is obviously $1/2$. Now, we can correlate this excitonic system with the other one.
It performs the conditional transformation a selective (spectrally narrow) pulse such that taking the state $|\psi\rangle$ into

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left| 0 \right\rangle \otimes \left| 0 \right\rangle - \frac{i}{\sqrt{2}} \left| 1 \right\rangle \otimes \left( \cos \frac{\alpha}{2} \left| 0 \right\rangle + \sin \frac{\alpha}{2} \left| 1 \right\rangle \right).$$

For $\alpha = \pi$, this pulse performs a CNOT-like transformation on the biexcitonic system. As a result, the total system is in the maximally entangled state $(|0\rangle|0\rangle - i|1\rangle|1\rangle)/2$ and a measurement on the QP system uniquely determines the state of the system S. Hence, due to quantum correlations between the systems, complete information on the state of S has been extracted to QP. On the other hand, if the biexciton is excited with a pulse with area $\alpha < \pi$ the correlation between the subsystems is weaker and a measurement on QP cannot fully determine the state of S, although the attained information may increase the probability for correctly predicting the result of a subsequent measurement on S. According to the discussion above, this means that partial information on the state of S is available.

In order to find the distinguishability measure in the biexciton scheme discussed here, we write the density matrix of the total system corresponding to the state $|\psi\rangle$,

$$\varrho = \frac{1}{2} \sum_{nm} |n\rangle \langle m| \otimes \rho_{nm},$$

with

$$\rho_{00} = |0\rangle \langle 0|,$$

$$\rho_{11} = \frac{1}{2} (\mathbb{1} + \cos \alpha \sigma_z - \sin \alpha \sigma_y),$$

$$\rho_{01} = \rho_{10}^\dagger = i \cos \frac{\alpha}{2} |0\rangle \langle 0| - \sin \frac{\alpha}{2} |0\rangle \langle 1|,$$

where $\sigma_z$ and $\sigma_y$ are Pauli matrices. Note that $\rho_{00}$ and $\rho_{11}$ (but not $\rho_{01}$) are density matrices.

According to the general theory\textsuperscript{21,22}, the best chance for correctly guessing the state of S results from the measurement of the observable $\rho_{00} - \rho_{11}$ and the probability of the correct prediction is then

$$p = \frac{1}{2} \left( \frac{1}{4} \right) \text{Tr} |\rho_{00} - \rho_{11}|,$$

where $|\rho|$ is the modulus of the operator $\rho$. Using the explicit forms of the density matrices $\rho_{00}, \rho_{11}$ and the definition\textsuperscript{5} one finds for the distinguishability in our case

$$D = \frac{1}{2} \text{Tr} |\rho_{00} - \rho_{11}| = \left| \sin \frac{\alpha}{2} \right|. \quad (9)$$

Thus, the amount of information on the system S accessible via a measurement on QP increases from 0 (no QP pulse at all) to 1 (for a $\pi$ pulse).

Next, we study the effect of extracting the which path information on the interference fringes. In the state\textsuperscript{35}, the reduced density matrix of the subsystem S is

$$\rho_S = \text{Tr}_{QP} \varrho = \frac{1}{2} \left( \mathbb{1} - \cos \frac{\alpha}{2} \sigma_y \right).$$

Upon applying the second pulse of the interference experiment scheme, namely the unconditional $\pi/2$ pulse [Eq. (4)], the average number of excitons in the dot is

$$N(\phi) = \langle 1 | U_{S2} \rho_S U_{S2}^\dagger | 1 \rangle = \frac{1}{2} \left( 1 - \cos \frac{\alpha}{2} \cos \phi \right).$$

Now, the average occupation oscillates between the limiting values $(1 \pm |\cos \alpha|)/2$ and the visibility of the fringes is

$$V = |\cos(\alpha/2)|. \quad (10)$$

Comparing Eq. (10) with Eq. (6) it is clear that the more certain one is whether the exciton is there or not ($D$ increases), the less clear the interference fringes become ($V$ decreases). Quantitatively, the relation $D^2 + V^2 = 1$ holds which is consistent with the complementarity relation\textsuperscript{36}. As we will see below, in the presence of coupling to the environment, where both subsystems are in mixed states.
states due to dephasing, this relation will turn into inequality.

Let us notice that the impact of the which path information on interference fringes is the same no matter whether the QP subsystem is measured before or after generating and detecting the interference fringes and even whether it is measured at all. In fact, the result of any such measurement (assuming perfect detectors) is known in advance since the state of this subsystem is completely determined by the area $\alpha$ of the QP pulse. This allows the essential part of the experiment to be performed even without polarization-selective measurement (e.g., using the current measurement in a photo-diode structure\cite{28}), by subtracting the known quantum probe exciton contribution from the detected signal. The same procedure may be applied if the exciton polarization is not conserved over the exciton lifetime which precludes polarization-resolved optical measurement in the single-dot setup.

With polarization-resolved detection or in a two-dot setup with spectral resolution, it is possible to demonstrate that information contained in the quantum probe system indeed increases the chances of correctly guessing the state of the first excitonic system to the extent predicted by the theory. To this end, one measures directly both exciton occupations, without applying the third pulse (S2), by registering both emitted photons with time-tagging enabling the identification of coincidences. The optimal measurement of the second exciton state should be done in the basis of eigenstates of $\rho_{00} - \rho_{11}$, which are

$$|\pm\rangle = \frac{\sqrt{1 \pm \sin(\alpha/2)}}{\sqrt{2}} |0\rangle \mp i \frac{\sqrt{1 \pm \sin(\alpha/2)}}{\sqrt{2}} |1\rangle.$$  

Since detection of photons emitted during exciton recombination corresponds to measurement in the $|0\rangle, |1\rangle$ basis, unconditional rotation to the optimal basis is necessary. This is done by a pulse with area $\beta$ such that $\cos \beta = \sqrt{1 + \sin(\alpha/2)}/\sqrt{2}$ and phase exactly opposite to that of the QP pulse. Upon detecting a photon from the quantum probe exciton one “predicts” that the state of the first subsystem should be $|1\rangle$ and vice versa. The fraction of correct predictions is equal to the fraction of coinciding measurements (detection or no detection on both polarizations). With a perfect detector, it equals to

$$p_c = \frac{1}{2} (1 + D) = \frac{1}{2} \left(1 + \sin \frac{\alpha}{2}\right)$$

which can also be found directly from the state $|\psi'\rangle$. If the detectors have efficiency $f$ and negligible dark and background count rate then the actual coincidence rate may still be recovered from the detection coincidence rate $p'_c$ upon removal of detection error asymmetry by preceding half of the measurements with an unconditional $\pi$ rotation on both subsystems. A simple analysis of the relevant conditional probabilities then yields $p_c = (p'_c + f - 1)/f^2$.

\section{IV. FEASIBILITY ANALYSIS}

So far, the basic scheme for testing the relation \cite{40} in an optical interference experiment on a QD was discussed in an idealized case, assuming perfectly selective or non-selective pulses, as required, and neglecting dephasing. In this Section we study the feasibility of coherent control required for demonstration of the complementarity principle from the point of view of spectral selectivity of the pulses.

The short pulses must provide for unconditional excitation of the first subsystem, so their spectral width must be larger than the bi-excitonic shift. Otherwise, the dynamics of the interfering system is conditioned on the detection (QP) subsystem and the results cannot be interpreted in terms of quantum complementarity. On the contrary, the QP pulse must distinguish between the single exciton and bi-exciton transition in order to act as an efficient detector. If this distinguishability fails, the actual amount of gained information is lower than the \textit{a priori} value of $\sin(\alpha/2)$ and the fringe visibility remains higher than expected. Finally, the overall duration of the pulse sequence should not be too long because of the competition of the dephasing processes (even on 10 ps time scales in some QD systems\cite{25,29,30}) and of the polarization rotation due to electron-hole exchange interaction.

First, we simulate the system behavior without dephasing in order to verify that the desired spectral selectivity may be achieved for typical system parameters. The simplest way to assure spectral selectivity of the QP pulse is to make it long and therefore spectrally narrow. It turns out that the degree of selectivity is affected by the duration of this pulse but also depends periodically on the time delay between the pulses. In the numerical simulation the bi-excitonic shift of $\Delta = -2.0$ meV was assumed. For this value, very good agreement with the idealized prediction is found for a Gaussian pulse with full width at half-maximum (FWHM) of the pulse amplitude of 2.5 ps and 0.15 ps for the long (QP) and short (S) pulses, respectively, and the delay times between the pulses $t_S = 10.3$ ps, $t_{QP} = t_S/2$. The central frequency of the short pulses is tuned half-way in between the exciton and bi-exciton transitions.

The resulting visibility of the occupation interference fringes as a function of the \textit{a priori} distinguishability of the exciton states (determined by the area of the QP pulse) is shown with a solid line in Fig. 2. For all values of $\alpha$, the squares of the visibility and distinguishability add exactly to 1 (dashed line), so that the relation \cite{40} becomes an equality, as expected for the pure-state case. This shows that the dynamics for the selected pulse parameters satisfies the pulse selectivity conditions required for a complementarity experiment.

A disadvantage of such a realization is the long overall duration of the pulse sequence. A way to reduce this duration is to use a simple pulse-shaping technique and to replace the long pulse with two short ones, separated by $\tau = \pi \hbar/\Delta$. With the FWHM of each of these pulses

\begin{equation}
\begin{aligned}
\rho_{\text{QP}} &= \frac{1}{2} \left(1 + \sin \frac{\alpha}{2}\right) \\
&= \frac{1}{2} \left(1 + \sin \frac{\alpha}{2}\right)
\end{aligned}
\end{equation}
Another important factor leading to reduction of visibility are decoherence processes that take place between the pulses. Such processes build up correlations between the confined carriers and their environment so that the environment has some which path knowledge on the carrier state and the visibility is decreased, even though no useful knowledge on the system state is available, turning Eq. (6) into inequality. In this Section, the pulse parameters established above will be used to simulate the system evolution in the presence of dephasing resulting from coupling with the environment. Here we choose a simple, Markovian model of dephasing that was successfully used to explain observed properties of interface fluctuation QDs.

Dephasing may be understood by treating the environment as a third party that continuously extracts information on both subsystems during the experimental sequence. The fringe contrast is decreased not only because some information has been extracted by the occupation measurement but also due to the portion of information transferred to the environment. On the other hand, the dephasing of the second subsystem amounts to sharing the information between the latter and the environment. This does not affect the fringe visibility but does affect the availability of information stored in the second subsystem, so that experimentally accessible information that may be used to predict the outcome of a measurement on the first subsystem is now lower than the a priori amount of $\sin(\alpha/2)$.

The effects of dephasing are included within a Markovian model, assuming that the two excitons are coupled to independent reservoirs, both characterized by an occupation relaxation time $T_1 = 100$ ps and a dephasing time $T_2 = (\gamma_1 + \gamma_2)^{-1}$, where $\gamma_1 = 1/(2T_1)$ and $\gamma_2$ accounts for pure dephasing effects resulting, e.g., from fluctuating electrostatic environment. The relevant Master equation reads

$$\dot{\rho} = -i[H, \rho] + D[\rho]$$

with the Hamiltonian $H$ and the dissipator $D[\rho] = D_1[\rho] + D_2[\rho]$ consisting of two parts. The first one accounts for the radiative decay of the individual excitons,

$$D_1[\rho] = \gamma_1 \sum_{i=1}^{2} \left[ \Sigma_{-}^{(i)} \rho \Sigma_{+}^{(i)} - \frac{1}{2} \{ \Sigma_{+}^{(i)} \Sigma_{-}^{(i)}, \rho \} \right],$$

where $\Sigma_{-}^{(1)} = \sigma_{z} \otimes I$, $\Sigma_{-}^{(2)} = I \otimes \sigma_{z}$. The second contribution describes the additional pure dephasing

$$D_2[\rho] = \gamma_2 \sum_{i=1}^{2} \left[ \Sigma_{+}^{(i)} \rho \Sigma_{-}^{(i)} - \frac{1}{2} \{ \Sigma_{+}^{(i)} \Sigma_{-}^{(i)}, \rho \} \right].$$

The evolution equation (11) is solved numerically for a range of pure dephasing rates $\gamma_2$. Based on the results of the simulations, visibility of interference fringes is calculated and plotted in Fig. 3 both for long pulse and shaped pulse case (left and right, respectively). Upper panels show the visibility as a function of the QP pulse area (determining the certainty with which the presence of the exciton is detected). Dashed line: plotted values are obtained from a simulation of system dynamics, for pulse parameters as discussed in the text, without dephasing.

V. EFFECT OF ENVIRONMENT-INDUCED DEPHASING

VI. CONCLUSION

It has been shown that the fundamental complementarity between the wave-like properties of a quantum system (interference effects) and its particle-like characteristics (the presence or absence of an exciton, treated as an indivisible entity) may be quantitatively tested in a time-domain interference experiment on semiconductor quantum dots. The feasibility of such an experiment has been confirmed by numerical simulation of system dynamics with realistic parameters and under typical environmental dephasing, as known from experiments.
exciton decay via the two nearly-degenerate single-exciton states\textsuperscript{36,37}, where information transfer due to entanglement with an emitted photon destroys coherence of the remaining exciton state. In contrast to these works, the demonstration of quantum complementarity requires that the correlations be induced in a controlled way by exciting the system with a particular pulse sequence. Hence, in this case correlations between excitons appear due to external optical driving, while photon emission serves only as an indication of the exciton presence.

The time-domain manifestation of quantum complementarity discussed here not only broadens the class of experiments in which fundamental aspects of the quantum world may be tested but also has the advantage of being independent of the position-momentum (Heisenberg’s) uncertainty that has been historically tied to the space-domain discussions of complementarity\textsuperscript{38}. In fact, it is independent of any uncertainty principles whatsoever. Indeed, the only two quantities which are measured in the time-domain interference experiment are the occupations of two different excitons. These quantities refer to different subsystems and, therefore, are obviously commuting and simultaneously measurable. Although the quantum probe exciton is created in a way that correlates it with the presence of the other exciton (S), this cannot be interpreted as a (projective) measurement on S, since the QP exciton is definitely a quantum (microscopic) system and not a classical, macroscopic measurement device. Thus, the experimental procedure described in this paper demonstrates quantum complementarity in its pure form, involving only the notion of information on the system state and independent of any uncertainty relations between non-commuting observables.

Acknowledgement

The author is grateful to the Alexander von Humboldt Foundation for support. This work was supported by the Polish MNI under Grant No. PBZ-MIN-008/P03/2003.
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