Inverting a Supernova: Neutrino Mixing, Temperatures and Binding Energy

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We show that the temperatures of the emergent non-electron neutrinos and the binding energy released by a galactic Type II supernova are determinable, assuming the Large Mixing Angle (LMA) solution is correct, from observations at the Sudbury Neutrino Observatory (SNO) and at Super-Kamiokande (SK). If the neutrino mass hierarchy is inverted, either a lower or upper bound can be placed on the neutrino mixing angle θ13, and the hierarchy can be deduced for adiabatic transitions. For the normal hierarchy, neither can θ13 be constrained nor can the hierarchy be determined. Our conclusions are qualitatively unchanged for the proposed Hyper-Kamiokande detector.

Neutrino oscillations convincingly explain the solar and atmospheric neutrino anomalies [1,2]. Atmospheric data and initial K2K data [3] indicate oscillations with |Δm23| = m2 − m3 | ≈ 3 × 10−3 eV2 and sin2 2θ23 ≈ 1 [4] that will be tested to within 10% accuracy at MINOS [5,6], (For the standard parameterization of the neutrino mixing matrix see Ref. [7]). The large mixing angle (LMA) solution (Δm21 ≈ 5 × 10−5 eV2, sin2 2θ12 ≈ 0.8) which is emerging as the solution to the solar neutrino problem [8] will be tested to 10% accuracy [9] at KamLAND [10]. Thus, in the near future, all parameters relevant to neutrino oscillations will be known, except sin2 2θ13, which is bounded above by 0.1 at the 95% C.L. by the CHOOZ experiment [11], the sign of Δm23 and the CP violating phase. In the longer term, long baseline neutrino experiments using upgraded conventional neutrino beams could achieve a sensitivity to sin2 2θ13 of about 10−3 [12], but neutrinos from a galactic supernova can prove that are more than two orders of magnitude smaller.

The objective of this Letter is to determine what the expected neutrino signals at SNO and SK from a Type II galactic supernova can tell us about sin2 2θ13, sgn(Δm23), the neutrino temperatures, and the binding energy released in such an event if the LMA solution is confirmed. Throughout, we assume that solar and atmospheric parameters will be known to within 10% from upcoming experiments. Since low energy νμ and ντ are indistinguishable at SNO and SK, only their transitions with νe can be studied.

Supernova neutrinos: During the early stages of a supernova explosion, as the shock wave rebounds from the dense inner core of the star and crosses the electron neutrinosphere, νe’s from electron capture on protons are released resulting in a breakout or neutronization burst that carries away ≈ 1051 ergs. The duration of this burst lasts only a few milliseconds (no more than 10) and any non-electron neutrino events at SNO during this time are a consequence of νe → νμ,τ oscillations. For progenitor stars of mass ≈ 1.5M⊙, numerical simulations find that following the neutronization burst, 99% of the binding energy released, Eν = 1.5 − 4.5 × 1053 ergs, is roughly equipartitioned in the form of neutrinos and antineutrinos of all flavors [13]. Including effects of nucleon bremsstrahlung and electron neutrino pair annihilation, the luminosities are approximately related by Lνμ ∝ Lνe ∝ (1 − 2)Lντ where x = μ, ν, τ [14,15]. This emission occurs on a timescale of tens of seconds. The mean energies of the different flavors of neutrinos are determined by the strength of their interactions with matter, with the most strongly interacting neutrinos leaving the star with the lowest mean energy i.e., ⟨Eνμ⟩ < ⟨Eντ⟩ < ⟨Eνe⟩. The authors of Ref. [14] (see also Ref. [16]) emphasize that spectral differences are very small, typically ⟨Eνμ⟩ : ⟨Eντ⟩ : ⟨Eνe⟩ ≈ 0.85 : 1 : 1.1. The spectra of neutrinos can be modeled by pinched Fermi-Dirac distributions. We can write the unnormalized differential flux at a distance D from the supernova as

\[ F_α = \frac{L_α}{24\pi D^2 T_α^4 (Li_4(-e^{-\eta_α}))} \frac{E^2}{e^{E/T_α-\eta_α} + 1}, \]  

where α = νe, νμ, ντ, Li_4(z) is the polylogarithm function and η_α is the degeneracy parameter. The temperature of the neutrinos, T_α, is related to ⟨Eα⟩ via

\[ ⟨E_α⟩ = 3\frac{Li_4(-e^{-\eta_α})}{Li_4(-1)} T_α. \]  

We shall use ⟨Eα⟩ and T_α interchangeably since they are equivalent to each other once η_α is specified. Strictly speaking, weak magnetism effects
may result in $T_{\nu_{\mu}}$, being about 7% higher than $T_{\nu_{\mu}}$ [17]. However, we have explicitly checked that the inequality of these temperatures does not affect our results.

As the neutrinos leave the star, they encounter a density profile that falls like $1/r^3$ [18]. If the mass hierarchy is normal, i.e. $\Delta m^2_{31} > 0$, (inverted, i.e. $\Delta m^2_{31} < 0$), neutrinos (antineutrinos) pass through a resonance at high densities ($10^7 - 10^8$ g/cm$^3$) which is characterized by $(\Delta m^2_{31}, \sin^2 2\theta_{13})$ and the neutrinos pass through a second resonance at low densities ($\sim 20$ g/cm$^3$ for the LMA solution) that is determined by $(\Delta m^2_{21}, \sin^2 2\theta_{12})$ [19]. Transitions in the latter resonance are almost adiabatic, with an essentially zero probability of level crossing. We denote the jumping probability in the high density resonance by $P_H$ and adopt the potential, $V_0(R/R_\odot)^3$ with $V_0 = 1.25 \times 10^{-14}$ eV and the solar radius, $R_\odot = 6.96 \times 10^{10}$ cm. Note that $P_H$ is the same for both neutrinos and antineutrinos [20] and has an $e^{-\sin^2 \theta_{13} (|\Delta m^2_{31}|/E)\nu^{3/2}}$ dependence [21]. Thus, even an order of magnitude uncertainty in $V_0$ does not have a qualitatively significant effect on $P_H$.

The integrated spectra at SNO and SK: Information on the neutrinos emerging from the supernova after the neutronization burst will be contained in $\nu_e$ and $\bar{\nu}_e$ spectra observed at SNO and SK.

For the normal hierarchy, the $\nu_e$ flux will be partially or completely converted into $\nu_{\mu}$ and $\nu_{\tau}$ with the survival probability given by [19]

$$P = P_H P_{2e} + (1 - P_H) \sin^2 \theta_{13}. \quad (2)$$

The sensitivity of the signal depends on $\sin^2 2\theta_{13}$ both explicitly and implicitly through $P_H$. The survival probability for electron antineutrinos is $\bar{P} = \bar{P}_{1e}$ [19], which is the probability that an antineutrino reaching the earth in the $\nu_1$ mass eigenstate interacts as a $\bar{\nu}_e$.

In the case of the inverted hierarchy, the $\nu_e$ survival probability is $P = P_{2e}$ [19], which is the probability that a neutrino reaching the earth in the $\nu_2$ mass eigenstate will interact in a detector as $\nu_e$. The $\bar{\nu}_e$ survival probability is [19]

$$\bar{P} = P_H \bar{P}_{1e} + (1 - P_H) \sin^2 \theta_{13}. \quad (3)$$

Since $P_{1e}$ and $P_{2e}$ depend only on oscillation parameters at the solar scale (and the supernova’s zenith angle $\theta_Z$), nothing can be learned about $\sin^2 2\theta_{13}$ from the $\bar{\nu}_e$ ($\nu_e$) flux if the hierarchy is normal (inverted).

For either hierarchy, we expect little sensitivity to $T_{\nu_e}$ because the survival probability of electron neutrinos is no more than about $P_{2e} \sim \sin^2 \theta_{12} \sim 0.3$.

For the 32 kton fiducial volume of the reinstrumented SK detector (with a 7.5 MeV threshold), and the 1.4 kton fiducial volume of the light water tank at SNO (with a threshold of 5 MeV), we only consider events that are isotropic and indistinguishable from each other; they are

$$\bar{\nu}_e + p \rightarrow n + e^+, \quad (4)$$
$$\nu_e + O \rightarrow F + e^-, \quad \bar{\nu}_e + O \rightarrow N + e^+. \quad (5)$$

We do not consider electron scattering events. The good directional capability on these events allows their separation, and they play an important role in the reconstruction of the direction of the supernova that in turn determines the extent to which earth matter effects may be important [22].

Neutrinos will interact with deuterium in the 1 kton fiducial volume of the heavy water tank at SNO (with a 5 MeV threshold) via the charged current (CC) reactions,

$$\nu_e + d \rightarrow p + p + e^{-}, \quad (6)$$
$$\bar{\nu}_e + d \rightarrow n + n + e^{+}. \quad (7)$$

In addition we include the reactions of Eq. (5). Two neutron captures in addition to a Cherenkov light cone.
can distinguish $\bar{\nu}_e$-$d$ events from the other charged current scattering events on deuterium or oxygen. All NC events (whose signal is a single neutron capture and no electron), and electron scattering events are neglected.

To simulate the energy spectra for the channels under consideration at the two experiments we assume a typical supernova [14] at a distance of 10 kpc: $E_o = 3 \times 10^{53}$ ergs, $\langle E_{\nu_e} \rangle : \langle E_{\bar{\nu}_e} \rangle : \langle E_{\nu_x} \rangle :: 0.85 : 1 : 1.1$ with $\langle E_{\nu_e} \rangle = 15$ MeV, $\eta_{\nu_e} = 2$ ($T_{\nu_e} = \langle E_{\nu_e} \rangle / 3.61$), $\eta_{\bar{\nu}_e} = 3$ ($T_{\bar{\nu}_e} = \langle E_{\bar{\nu}_e} \rangle / 3.99$), $\eta_{\nu_x} = 1.5$ ($T_{\nu_x} = \langle E_{\nu_x} \rangle / 3.45$), $L_{\nu_e} = L_{\bar{\nu}_e}$ and $L_{\nu_x} = 1.5 L_{\nu_e}$. We fix $\sin^2 \theta_{12} = 0.81$, $\Delta m^2_{31} = 5.6 \times 10^{-5}$ eV$^2$ and $|\Delta m^2_{31}| = 3 \times 10^{-3}$ eV$^2$, since variation of these parameters within their future bounds has very little effect on the analysis. We generate $\nu_e$ and $\bar{\nu}_e$ spectra [22] and then simulate data by choosing a point from a Gaussian distribution centered at the expectation for the bin and of width equal to its square root. In all, there are four spectra; one for SK, one for the light water tank at SNO, one for processes (5) and (6), and one for process (7). The SK spectrum is simulated with 18 bins and the SNO spectra have 13 bins each. We simulate four datasets, two for each type of hierarchy and for two values of $\sin^2 2\theta_{13}$ that correspond to adiabatic ($P_H = 0$) and non-adiabatic ($P_H = 1$) oscillation transitions. We perform a $\chi^2$-analysis, freely varying $E_o$, $\langle E_{\nu_e} \rangle$, $\langle E_{\bar{\nu}_e} \rangle$, $\langle E_{\nu_x} \rangle$, and $\sin^2 2\theta_{13}$ to find the 90% ($\Delta \chi^2 < 7.78$) and 99% C.L. ($\Delta \chi^2 < 13.3$) allowed regions in $E_o$, $\langle E_{\nu_e} \rangle$, $\langle E_{\nu_x} \rangle$ and $\sin^2 2\theta_{13}$. Although we scan in $\langle E_{\nu_x} \rangle$, we do not count it as a free parameter since we do not attempt to determine it (knowing a priori of the limited sensitivity to this parameter). We allow the ratio $L_{\nu_e}/L_{\nu_x}$ to vary between 1 and 2 to accommodate both perfect equipartitioning and large departures from it. We fix $L_{\bar{\nu}_e} = L_{\bar{\nu}_x}$.

Figure 1 shows the results of this fit for the normal hierarchy. The left-hand and right-hand panels correspond to data simulated at $\sin^2 2\theta_{13} = 10^{-5}$ (for which $P_H = 1$) and $\sin^2 2\theta_{13} = 10^{-2}$ (for which $P_H = 0$), respectively. In either case, we see that the supernova parameters can be determined with high precision, but that $\sin^2 2\theta_{13}$ is unconstrained. Since the overall normalization of the neutrino fluxes depends critically on $E_o$, it is determined with good accuracy. The values of $T_{\bar{\nu}_e}$ and $T_{\nu_x}$ are also determined precisely since these parameters control the $\bar{\nu}_e$ spectral distortion (which is independent of $\sin^2 2\theta_{13}$) obtained from thousands of events at SK and hundreds more at SNO. The experiments are not sensitive to $\sin^2 2\theta_{13}$ because the $\nu_e$-$d$ events at SNO and $\nu_e$-$O$ events at SK and SNO are statistically insufficient.

Figure 2 shows the results for the inverted hierarchy. Again, the left-hand and right-hand columns correspond to data simulated at $\sin^2 2\theta_{13} = 10^{-5}$ and $\sin^2 2\theta_{13} = 10^{-2}$, respectively. In the case of non-adiabatic transitions, an upper bound on $\sin^2 2\theta_{13}$ can be placed. For adiabatic transitions, $\sin^2 2\theta_{13}$ and $\langle E_{\nu_x} \rangle$ cannot be simultaneously bounded if both are left free. When we restrict $\langle E_{\nu_x} \rangle$ to lie between 10.5 MeV and 19 MeV and $\langle E_{\nu_x} \rangle / \langle E_{\nu_e} \rangle$ to be larger than about 0.7 (recall that $\langle E_{\nu_x} \rangle / \langle E_{\nu_e} \rangle$ is expected to be larger than unity), a lower bound on $\sin^2 2\theta_{13}$ is obtained. For values of $\sin^2 2\theta_{13}$ between $10^{-4}$ and $10^{-3}$, an upper or lower bound can be placed on $\sin^2 2\theta_{13}$ depending on whether it is closer to $10^{-4}$ or to $10^{-3}$. In the inverted hierarchy, we have a lesser sensitivity to $T_{\bar{\nu}_e}$ and $T_{\nu_x}$ since the $\bar{\nu}_e$ flux is also sensitive to $\sin^2 2\theta_{13}$ leading to competition between these parameters. Bounds on $\sin^2 2\theta_{13}$ can be placed because the $\nu_e$ spectrum is more sensitive to $\sin^2 2\theta_{13}$ in the inverted hierarchy and the $\bar{\nu}_e$ signal at SK is huge.

Although the regions in Figs. 1 and 2 are calculated assuming that the neutrinos detected at SK crossed both the mantle and core of the earth ($\cos \theta_Z = -0.93$), and those at SNO crossed the mantle only ($\cos \theta_Z = -0.1$), we have established that the bounds placed are largely independent of the supernova’s zenith angles at the two experiments.

If the mass hierarchy is unknown at the time of a supernova signal, it can be deduced provided $\sin^2 2\theta_{13} \gtrsim 10^{-3}$ [19,23], and the hierarchy is inverted. For values of $\sin^2 2\theta_{13}$ smaller than $\approx 10^{-4}$, $P_H$ is not close to zero and the survival probabilities are similar for the two hierar-
chies rendering them indistinguishable [19]. In the case of a normal hierarchy, we see from Fig. 1 that $\sin^2 2\theta_{13}$ is unconstrained even for adiabatic transitions, thereby indicating a lack of discriminatory power between $P_H = 0$ and $P_H = 1$ or equivalently between the mass hierarchies. On the other hand, for the inverted hierarchy and adiabatic transitions, a lower bound on $\sin^2 2\theta_{13}$ can be placed which in turn means that the inverted hierarchy can be selected over the normal hierarchy.

**Future prospects:** The next generation of proton decay experiments such as Hyper-Kamiokande (HK) [24] and UNO [25] are expected to offer a new level of sensitivity to the physics of supernovae and neutrino mixing. We consider the proposed 1 Mton HK detector. With no specific information about the detector, we treat it as a scaled-up version of SK. We assume a fiducial volume for supernova neutrinos of 890 kt, which is consistent with the fiducial volume to total volume ratio expected for the proposed UNO detector [26].

We find that our qualitative conclusions for SK and SNO continue to hold for HK. The quantitative differences are easily anticipated as a result of its larger volume. The supernova parameters can be determined with greater accuracy although $T_{\nu_e}$ will remain unknown. In the case of the inverted hierarchy and adiabatic transitions, while $T_{\nu_e}$ can be determined without theoretical prejudice, a plausible window has to be chosen for $T_{\nu_e}$ to constrain $\sin^2 2\theta_{13}$. Also, tighter upper or lower bounds can be placed on $\sin^2 2\theta_{13}$. We emphasize that in the case of a normal hierarchy, both $\sin^2 2\theta_{13}$ and the hierarchy remain unknown.

**Summary:** We have considered what information can be extracted from neutrinos detected at SNO and SK from a galactic supernova. The information they carry is of major importance in understanding the astrophysics of supernovae. The binding energy released in the supernova and the temperatures of the non-electron neutrinos expelled may be determined with good precision for most values of $\sin^2 2\theta_{13}$. Bounds on $\sin^2 2\theta_{13}$ can be placed if the neutrino mass hierarchy is inverted. In this case the hierarchy can be determined if $\sin^2 2\theta_{13} \gtrsim 10^{-3}$.

The above conclusions apply to Hyper-Kamiokande as well.

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