Cosmological determination to the values of the prefactors in the corrected entropy-area relation

Nasr Ahmed¹ and Sultan Z. Alamri²

¹ Astronomy Department, National Research Institute of Astronomy and Geophysics, Helwan, Cairo, Egypt
² Mathematics Department, Faculty of Science, Taibah University, Saudi Arabia

Abstract

In this paper, we continue investigating the possible values of the prefactors α and β in the corrected entropy-area relation based on cosmological arguments. In a previous study on the entropy-corrected cosmology using a hyperbolic form of the scale factor, we found that these two prefactors should be zero in order to obtain a stable flat FRW universe with a deceleration-to-acceleration transition and no causality violation through cosmic evolution. In the current work, the entropy-corrected cosmological equations have been solved using a hybrid scale factor and, surprisingly, the same result has been reached for the values of the prefactors. We believe that investigating the corrected entropy-area relation in different gravitational and cosmological contexts can provide an accurate estimation to the correct values of the prefactors. The evolution of the cosmic pressure, energy density, equation of state parameter, jerk parameter and the nonlinear energy conditions has been analyzed.

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1 Introduction and motivation

The discovery of cosmic acceleration [1, 2, 3] has been a major challenge to our understanding of gravity and the way it works on cosmological distances in a flat, highly homogeneous and isotropic universe [6, 7, 8]. An exotic energy with negative pressure (dark energy) acts as a repulsive gravity has been proposed to explain this late-time acceleration. Several theoretical models of this dark energy have been constructed through modifying general relativity [10]-[16] and through scalar fields [5, 17-19, 23-30].

In Hawking radiation [39], the black hole’s entropy $S$ is proportional to its horizon area $A$. This radiation is a quantum phenomenon that represents a connection between gravity and thermodynamics [40]. The first law of black hole thermodynamics $TdS = dE$ connects the horizon entropy $S = \frac{A}{4G}$ with the Hawking temperature $T = \frac{\kappa_{sg}}{2\pi}$, where $\kappa_{sg}$ is the surface gravity and $dE$ is the energy change [46, 47, 48]. In [41], Einstein equations have been derived using Clausius relation $TdS = \delta Q$ and the horizon-entropy area relation, where $\delta Q$ and $T$ are

¹nasr.ahmed@nriag.sci.eg
²szamri@taibahu.edu.sa
the energy flux across the horizon and Unruh temperature respectively. A unified first law of black hole dynamics and relativistic thermodynamics \(dE = TdS + WdV\) has been derived in spherically symmetric space-times [45], where \(W\) is the work density defined by \(-\frac{1}{2}T^{ab}h_{ab}\). Using Clausius relation, Friedmann equations have been derived from the first law of thermodynamics [42, 43, 44]. It has been also indicated that Friedmann equations can be expressed as \(dE = TdS + WdV\) at the apparent horizon [49, 50] where \(E = \rho V\) is the total energy, \(W = \frac{1}{2}(\rho - p)\) is the work density. \(\rho\) and \(p\) are the energy density and pressure of cosmic matter, while \(T\) and \(S\) are temperature and entropy associated with the apparent horizon. The original entropy-area relation is valid only for the case of GR, and it needs corrections when considering higher order curvature terms [51]. Modified Friedmann equations have been derived by applying the corrected entropy-area relation [51]:

\[
S = \frac{A}{4G} + \alpha \ln \frac{A}{4G} + \beta \frac{4G}{A}.
\] (1)

The prefactors \(\alpha\) and \(\beta\) are dimensionless constants whose values are still in debate [52]. The correction terms in (1) appear in loop quantum gravity due to quantum fluctuations (see [67] and references therein). The second correction term also appears in the entropic cosmology introduced in [71]. While positive and negative values for these two prefactors have been suggested before by some authors [57]-[61], it has also been argued that the best guess might simply be zero [53]. A stable flat entropy-corrected FRW universe, with a deceleration-to-acceleration transition, has been reached in [54] for \(\alpha = \beta = 0\). We believe that studying relation (1) in different gravitational and cosmological contexts can provide an accurate estimation to the correct values of the prefactors. In the present work, we aim to find the best values for \(\alpha\) and \(\beta\) required to describe a stable flat universe in a good agreement with observations through a hybrid scale factor. The hybrid scale factor is a specific ansatz describing a deceleration-to-acceleration cosmic transition, and is given as [55]:

\[
a(t) = a_0 \left( \frac{t}{t_0} \right)^{\alpha_1} e^{\beta_1 \left( \frac{t}{t_0} - 1 \right)},
\] (2)

where \(\alpha\) and \(\beta\) are non-negative constants, \(a_0\) and \(t_0\) are the scale factor and age of the present day Universe respectively. Equation (2) can be reduced after suitable transformations to [56]

\[
a(t) = a_1 t^{\alpha_1} e^{\beta_1 t},
\] (3)

where \(a_1 > 0, \alpha_1 \geq 0\) and \(\beta_1 \geq 0\) are constants. This HEL ansatz is a mixture of power-law and exponential-law cosmologies, and can be regarded as a generalization to each of them. The power-law cosmology can be obtained for \(\beta_1 = 0\), and the exponential-law cosmology can be obtained for \(\alpha_1 = 0\). New cosmologies can be explored for \(\alpha_1 > 0\) and \(\beta_1 > 0\).

The paper is organized as follows: The introduction and motivation behind this work is included in section 1. The solution of the cosmological equations with a discussion on the stability and the evolution of different parameters are included in section 2. The final conclusion is included in section 3.
2 Cosmological equations and solutions

Considering the corrected entropy-area relation [1], the modified FRW equations can be written as [51]

\[ H^2 + \frac{k}{a^2} + \frac{\alpha G}{2\pi} \left( H^2 + \frac{k}{a^2} \right)^2 - \frac{\beta G^2}{3\pi^2} \left( H^2 + \frac{k}{a^2} \right)^3 = \frac{8\pi G}{3} \rho. \quad (4) \]

\[ 2 \left( \dot{H} - \frac{k}{a^2} \right) \left( 1 + \frac{\alpha G}{\pi} \left( H^2 + \frac{k}{a^2} \right) - \frac{\beta G^2}{\pi^2} \left( H^2 + \frac{k}{a^2} \right)^2 \right) = -8\pi G (\rho + p). \quad (5) \]

Where \( k = 0, 1, -1 \) for a flat, closed and open universe respectively. Since recent observations indicate that a cosmic deceleration-to-acceleration transition happened [4, 5], new solutions to [4] and [5] can be explored through empirical forms of the scale factor where the deceleration parameter \( q \) changes sign from positive (decelerating universe) to negative (accelerating universe). Taking [3] into account, the deceleration parameter \( q \) is given as

\[ q(t) = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{\alpha_1}{(\beta_1 t + \alpha_1)^2} - 1 \quad (6) \]

The deceleration-to-acceleration transition takes place at \( t = \frac{\sqrt{\alpha - \alpha_1}}{\beta} \) which restricts \( \alpha \) in the range \( 0 < \alpha < 1 \) [56]. Solving (4) and (5) with (3), the cosmic pressure \( p(t) \) and energy density \( \rho(t) \) are expressed as

\[ p(t) = \frac{1}{16\pi^3 t^6} \left( (2\beta_1^6 - 3\pi \alpha \beta_1^4 - 6\pi^2 \beta_1^2) t^6 - 12\beta_1 \alpha_1 (-\beta_1^4 + \pi \alpha \beta_1^2 + \pi^2) t^5 -6\alpha_1 (-5\beta \beta_1^4 + 3\pi \alpha \beta_1^2 + \pi^2) \alpha_1 + \frac{2}{3} \beta \beta_1^4 - \frac{2}{3} \pi \alpha \beta_1^2 - \frac{2}{3} \pi^2 \right) t^4 -12 \left( -\frac{10}{3} \beta \beta_1^2 + \pi \alpha \right) \alpha_1 + \frac{4}{3} \beta \beta_1^2 - \frac{2}{3} \pi \alpha \right) \beta_1 \alpha_1^2 t^3 -3\alpha_1^3 \left( -10\beta \beta_1^2 + \pi \alpha \right) \alpha_1 + 8\beta \beta_1^2 - \frac{4}{3} \pi \alpha \right) t^2 + 12\beta \beta_1 \alpha_1^4 \left( \alpha_1 - \frac{4}{3} \right) t \right) 2\beta \alpha_1^5 (\alpha_1 - 2) \] \( (7) \)

\[ \rho(t) = \frac{1}{16\pi^3 t^6} (\beta_1 t + \alpha_1)^2 \left( (3\pi \alpha \beta_1^2 - 2\beta \beta_1^4 + 6\pi^2) t^4 -\alpha_1 \beta_1 (6\pi \alpha - 8\beta) t^3 + \alpha_1^2 (3\pi \alpha - 12\beta \beta_1^2) t^2 - 8\beta \beta_1 \alpha_1^3 t - 2\beta \alpha_1^4 \right) \] \( (8) \)

The EoS parameter \( \omega(t) = \frac{p(t)}{\rho(t)} \) is simply the division of (7) and (8). Determining the value of the EoS parameter is essential to investigate the nature of dark energy. The value of this parameter is 0 for dust, 1/3 for radiation and −1 for the current cosmological constant (dark energy) epoch. \( \omega \leq -1 \) for phantom scalar field and \( -1 \leq \omega \leq 1 \) for quintessence scalar field. It can also evolve across the phantom divide line \( \omega = -1 \) for quintom field. \( \omega = 1 \) is the largest value of this parameter consistent with causality and is supposed to happen for some exotic
type of matter called stiff matter \cite{72} where the sound speed equals the speed of light. The jerk parameter is defined as \cite{68,69}

\[ j = \frac{\ddot{a}}{a H^3} = q + 2q^2 - \frac{\dot{q}}{H} \]  

(9)

where \( q \) is the deceleration parameter. Since \( j = 1 \) for flat \( \Lambda CDM \) models \cite{70}, this parameter helps to describe models close to \( \Lambda CDM \). The value of \( j \) for the current model is

\[ j = \frac{\beta_1 t + \alpha_1}{t} \]  

(10)

Fig. 1(a) shows that the deceleration parameter varies in the range \(-1 \leq q \leq 1\). It starts at \( q = 1 \) (a decelerating radiation-dominated era), crosses the decelerating matter-dominated era at \( q = \frac{1}{2} \), changes sign to negative (accelerating era) and ends at \( q = -1 \) (de Sitter universe). The evolution of the jerk parameter shows that it tends to 1 at late-times where the current model becomes in a good agreement with the flat \( \Lambda CDM \) model. The cosmic pressure \( p \) also changes sign from positive in early decelerating time where attractive gravity dominates, to negative in late accelerating time where repulsive gravity (represented in dark energy) dominates. We have tried all possible values of the four basic parameters in the current model, namely \( \alpha, \beta, \alpha_1 \) and \( \beta_1 \). The possibility of a causality violation where the EoS parameter exceeds unity (\( \omega(t) > 1 \)) exists for all values of \( \alpha \) and \( \beta \) except when both of them are zero where we obtain \(-1 \leq \omega(t) \lesssim \frac{1}{3}\). Consequently, the evolution of the EoS parameter in the current model strongly supports the zero values of the prefactors. It has also been shown in \cite{53} that the zero value is the unique choice consistent with both the holographic principle and statistical mechanics. Table (1) shows the behaviour of the cosmic pressure, energy density, EoS parameter and the new nonlinear energy conditions for different positive, negative and zero values of \( \alpha, \beta, \alpha_1 \) and \( \beta_1 \). We have found that some choices of \( \alpha \) and \( \beta \) are not allowed where the energy density \( \rho(t) \) shows a wrong behavior and goes to \(-\infty\) as \( t \to 0 \). We can also see from the table that the most stable and physically acceptable solution happens when the prefactors \( \alpha \) and \( \beta \) take zero values. The evolution of the EoS parameter shows no quintom behavior (no cosmological constant boundary crossing) as the lower bound is \(-1\) for all possible values of \( \alpha \) and \( \beta \).

Because the correct values of the prefactors in the corrected entropy-area relation \cite{1} are still in debate, studying this relation in different contexts, gravity theories and different setups is very helpful in determining the correct values. It is interesting to note that the same result we have obtained here on the zero values of \( \alpha \) and \( \beta \) using the hybrid law, has been also reached using the hyperbolic law \( a(t) = A (\sin(\zeta t))^{\frac{1}{2}} \) when solving the cosmological equations \cite{4} and \cite{5} \cite{54}. As we have indicated in \cite{54}, the hyperbolic scale factor \( a(t) = A (\sin(\zeta t))^{\frac{1}{2}} \) also allows a deceleration-to-acceleration transition, its jerk parameter tends to a flat \( \Lambda CDM \) (\( j = 1 \)) at late-times, and it appears in many contexts of cosmology (see \cite{54} and references therein). We have also found the following for the hyperbolic ansatz: (1) The pressure is positive during the early-time decelerating era and negative during the late-time accelerating era. (2) The possibility of a causality violation exists for all values of \( \alpha \) and \( \beta \) except for \( \alpha = \beta = 0 \) where \(-1 \leq \omega(t) \lesssim \frac{1}{3}\). (3) The most stable solution is obtained for the flat universe \((k = 0)\) with zero values of the prefactors \( \alpha \) and \( \beta \). We have got the same results in the current work using the hybrid expansion law.

Because of the presence of semiclassical quantum effects in the current model, we have tested the physical acceptability of the model through the new nonlinear energy conditions
which are: (i) The flux energy condition (FEC): \( \rho^2 \geq p_i^2 \) [62, 63]. (ii) The determinant energy condition (DETEC): \( \rho \Pi p_i \geq 0 \) [64]. (ii) The trace-of-square energy condition (TOSEC): \( \rho^2 + \sum p_i^2 \geq 0 \) [64]. All of them are satisfied for the current entropy-corrected model.

3 Conclusion

The values of the prefactors in the corrected entropy-area relation have been cosmologically investigated using a hybrid ansatz of the scale factor. The results obtained in the current study using the hybrid ansatz have been compared to the results of a previous study using a hyperbolic ansatz and they both found to be the same. The main common points are

- The best values for \( \alpha \) and \( \beta \) required to describe a stable flat universe with a deceleration-to-acceleration transition and no causality violation are the zero values.
- The cosmic pressure is positive during the early-time decelerating epoch and negative during the late-time accelerating epoch.
- The violation of causality is possible for all values of \( \alpha \) and \( \beta \) except for \( \alpha = \beta = 0 \) where \( -1 \leq \omega(t) \lesssim \frac{1}{3} \).

Predicting zero values of \( \alpha \) and \( \beta \) in two different cosmological solutions represents a strong support for the zero values of the two prefactors.

| \( \alpha \) | 0 | 0.2, 0.3, 0.4 | 0.5 | 0.2, 0.3, 0.4, 0.5 | 0 | 0 |
| \( \beta \) | 0 | 0.5 | 0.5 | 0.5 | 0.1 | 0.4 |
| \( \alpha_1 \) | 0.5 | 0.5 | 0.5 | 0.5 | 0.1 | 0.4 |
| \( \beta_1 \) | 0.5 | 0.5 | 0.5 | 0.5 | 0.1 | 0.4 |
| \( \rho \to \infty \) as \( t \to 0 \) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| \( \rho \to -\infty \) as \( t \to 0 \) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| \( \omega(t) \) | \(-1 \leq \omega(t) \leq \frac{1}{3}\) | \(-1 \leq \omega(t) \leq \frac{1}{2}\) | \(-1 \leq \omega(t) \leq 1\) | \(-1 \leq \omega(t) \leq 1.7\) | \(-1 \leq \omega(t) \leq 5.7\) | \(-1 \leq \omega(t) \leq 0.7\) |
| PEC | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| DETC | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| TOSEC | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |

Table 1: The behavior of \( p(t) \), \( \rho(t) \), \( \omega(t) \) and the nonlinear energy conditions for different values of \( \alpha, \beta, \alpha_1 \) and \( \beta_1 \).
Figure 1: Fig. 1(a) The deceleration parameter varies in the range $-1 \leq q \leq 1$. (b) The jerk parameter $j = 1$ at late-times where the current model tends to a flat $\Lambda CDM$ model. (c) The cosmic pressure shows a sign flipping from positive to negative. (d) The energy density is always positive. (e) $\omega(t)$ varies in the range $-1 \leq q \leq \frac{1}{3}$. (f), (g) and (H) show the validity of the three nonlinear energy conditions. Here $\alpha = \beta = 0$ and $\alpha_1 = \beta_1 = 0.5$.

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