Interference of Clocks: A Quantum Twin Paradox

Sina Loriani$^1$, Alexander Friedrich$^{1,2}$, Christian Ufrecht$^2$, Fabio Di Pumpo$^2$, Stephan Kleinert$^2$, Sven Abend$^1$, Naceur Gaaloul$^1$, Christian Meiners$^1$, Christian Schubert$^1$, Dorothee Tell$^1$, Étienne Wodey$^1$, Magdalena Zych$^3$, Wolfgang Ertmer$^1$, Albert Roura$^2$, Dennis Schlippert$^1$, Wolfgang P. Schleich$^{1,4}$, Ernst M. Rasel$^1$, and Enno Giese$^2$

$^1$Institut für Quantenoptik, Leibniz Universität Hannover, Welfengarten 1, D-30167 Hannover, Germany
$^2$Institut für Quantenphysik and Center for Integrated Quantum Science and Technology (IQST), Universität Ulm, Albert-Einstein-Allee 11, D-89069 Ulm, Germany
$^3$Centre for Engineered Quantum Systems, School of Mathematics and Physics, The University of Queensland, St Lucia, QLD 4072, Australia
$^4$Hagler Institute for Advanced Study and Department of Physics and Astronomy, Institute for Quantum Science and Engineering (IQSE), Texas A&M AgriLife Research, Texas A&M University, College Station, TX 77843-4242, USA

†These authors contributed equally to this work.

The phase of matter waves depends on proper time and is therefore susceptible to special-relativistic (kinematic) and gravitational time dilation (redshift). Hence, it is conceivable that atom interferometers measure general-relativistic time-dilation effects. In contrast to this intuition, we show that light-pulse interferometers without internal transitions are not sensitive to gravitational time dilation, whereas they can constitute a quantum version of the special-relativistic twin paradox. We propose an interferometer geometry isolating the effect that can be used for quantum-clock interferometry.

INTRODUCTION

Proper time is operationally defined (1) as the quantity measured by an ideal clock (2) moving through spacetime. As the passage of time itself is relative, the comparison of two clocks that travelled along different worldlines gives rise to the twin paradox (3). Whereas this key feature of relativity relies on clocks localised on worldlines, today’s clocks are based on atoms that can be in a superposition of different worldlines. This nature of quantum objects is exploited by matter-wave interferometers which create superpositions at macroscopic spatial separations (4). One can therefore envision a single quantum clock such as a two-level atom in a superposition of two different worldlines, suggesting a twin paradox, in principle susceptible to any form of time dilation (5–7). We demonstrate which atom interferometers implement a quantum twin paradox, how quantum clocks interfere, and their sensitivity to different types of time dilation.

The experimental verification of special-relativistic and gravitational time dilation were milestones in the development of modern physics and have, for instance, been performed by the comparison of two atomic clocks (8–10). Atomic clocks, as used in these experiments, are based on microwave and optical transitions between electronic states and define the state of the art in time keeping (11). On the other hand, light-pulse atom interferometers do not only provide high-precision inertial sensors (12, 13) with applications in tests of the foundations of physics (14–20), but at the same time constitute a powerful technique to manipulate atoms. Atom interferometry in conjunction with atomic clocks has led to the idea of using time dilation between two branches of an atom interferometer as a which-way marker to measure effects like the gravitational redshift through the visibility of the interference signal (5, 6). However, no specific interferometer scheme was proposed. The symmetry of the geometry as well as the mechanism to create it crucially determines if and how the interferometer phase depends on proper time (21). Therefore, the question whether effects connected to time dilation can be observed in atom interferometers is still missing a conclusive answer.

In this work, we study a quantum version of the twin paradox implemented in a light-pulse atom interferometer and show that such a setup is not sensitive to gravitational time dilation such as the gravitational redshift. To this end, we establish a relation between special-relativistic time dilation and kinematic asymmetry of closed atom interferometers, taking the form of recoil measurements (14, 20, 22, 23). For these geometries, a single atomic clock carried by quantum twins in a superposition of two different worldlines undergoes special-relativistic time dilation. The induced distinguishability leads to a loss of visibility upon interference, such that the proposed experiment represents a realisation of the twin paradox in quantum-clock interferometry.

In general relativity the proper time $\tau$ along one particular worldline is invariant under coordinate transformations and can be — after a parametrisation through a coordinate time $t$ — approximately written as

$$\tau = \int \! dt \cong \int \! dt \left[ 1 - (z/c)^2 / 2 + U/c^2 \right]$$  

(1)

after an expansion of the metric tensor to first order in $c^{-2}$, where $c$ denotes the speed of light. Here, $z$ is the velocity of the particle along the worldline, and $U(z)$ is the Newtonian gravitational potential.

This classical quantity is connected to the phase

$$\varphi = -\omega_C \tau + S_{em} / \hbar$$

(2)

acquired by a first quantised matter wave assuming it is sufficiently localised such that it can be associated with this worldline. Here, $\omega_C = mc^2 / \hbar$ denotes the Compton frequency.
of a particle of mass $m$ and

$$S_{em} = -\int dt \, V_{em}$$

is the classical action arising from the interaction of the matter wave with electromagnetic fields described by the potential $V_{em}(z, t)$. This potential can transfer momentum to the matter wave, thus changing its trajectory, which in turn affects proper time.

Atom interferometers use exactly this concept to manipulate matter waves. They consist of a series of light pulses that coherently drive atoms into a superposition of motional states travelling along different branches. The branches are then redirected and finally recombined, such that the probability to find atoms in a specific momentum state displays an interference pattern and depends on the phase difference $\Delta \varphi$ accumulated between both branches. For interferometers closed in phase space (24) and for potentials up to second order in $z$, the phase difference can be calculated from equation (2).

RESULTS

Time dilation and gravito-recoil action

Since the light pulses act differently on the two branches of the interferometer, we add superscripts $\alpha = 1, 2$ to the potential $V_{em}^{(\alpha)}$. Moreover, we separate $V_{em}^{(\alpha)} = V_k^{(\alpha)} + V_p^{(\alpha)}$ into a contribution $V_k^{(\alpha)}$ causing momentum kicks and $V_p^{(\alpha)}$ imprinting the phase of the light pulse without affecting the motional state (25). Consequently, we find that the motion $z^{(\alpha)} = z_g^{(\alpha)} + z_k^{(\alpha)}$ along one branch can also be divided into two contributions: $z_g$ caused by the gravitational potential and $z_k^{(\alpha)}$ determined by the momentum transferred by the light pulses on branch $\alpha$.

For a linear gravitational potential, the proper-time difference between both branches takes the form

$$\Delta \tau = \int dt \left[ \frac{z^{(1)}_k - z^{(2)}_k}{2c^2} \right] \left[ \frac{z^{(1)}_k - z^{(2)}_k}{2c^2} \right]$$

(see Materials and Methods). It is explicitly independent of $z_g$ as well as of the particular interferometer geometry, which is a consequence of the phase of a matter wave being invariant under coordinate transformations. When transforming to a freely falling frame, both trajectories reduce to the kick-dependent contribution $z_k^{(\alpha)}$ and the proper-time difference $\Delta \tau$ is thus independent of gravity (21). Accordingly, closed light-pulse interferometers are insensitive to gravitational time dilation. Our result implies that time dilation in such interferometer configurations constitutes a purely special-relativistic effect caused by the momentum kicks.

Our model of atom-light interaction assumes instantaneous momentum transfer and neglects the propagation time of the light pulses. In fact, a potential $V_k$ linear in $z$, where the temporal pulse shape of the light is described by a delta function reflects exactly such momentum kicks. For such a potential, we find the differential action

$$\Delta S_{em} = 2\hbar \omega C \Delta \tau + \Delta S_{pk} + \Delta S_p$$

(see Materials and Methods). The first contribution is caused by the momentum transfer sampling the recoil part of the motion $z_k$ and arises solely from the interaction with the laser. It has the form of the proper-time difference, which highlights that the action of the laser can never be separated from proper time in a phase measurement in the limit given by equation (2). The second contribution in equation (5) is the action that arises from the momentum kicks sampling the gravitational part $z_g$ of the motion and takes the form

$$\Delta S_{pk} = m \int dt \Delta z_k \dot{z}_g,$$  

(6)

where we use the difference $\Delta z_k = z_k^{(1)} - z_k^{(2)}$ between branch-dependent accelerations. We refer to this contribution as gravito-recoil action which is exclusively caused by the interaction with the electromagnetic field. The final contribution imprinted by the lasers is the laser phase action

$$\Delta S_p = -\int dt \Delta V_p,$$  

(7)

with $\Delta V_p = V_p^{(1)}(t) - V_p^{(2)}(t)$.

So far we have not specified the interaction with the light but merely assumed that the potential is linear in $z$. In fact, we will employ the potential $V_k^{(\ell)} = -\sum \hbar k^{(\ell)} z_k^{(\ell)} \delta(t - t_{\ell})$ for the momentum transfers $\hbar k^{(\ell)}$ of the $\ell$th laser pulse at time $t_{\ell}$ and the potential $V_p^{(\alpha)} = -\sum \hbar c(\alpha) \delta(t - t_{\ell})$ to describe the phase $\phi_p^{(\alpha)}$ imprinted by the light pulses (25). Both potentials are branch-dependent. Since the phases imprinted by the lasers can be evaluated trivially and are independent of $z$, we exclude the discussion of $V_p^{(\alpha)}$ from the study of different interferometer geometries and set it to zero in the following.

Atom-interferometric twin paradox

These results are highlighted by a Mach-Zehnder interferometer (MZI) measuring the gravitational acceleration $g$. Its sensitivity with respect to gravity stems entirely from the interaction with the light, i.e. $\Delta S_{pk}$, while the proper-time difference vanishes (25, 26). Indeed, the MZI consists of a sequence of pulses coherently creating, redirecting and finally recombining the two branches. The three pulses are separated by equal time intervals of duration $T$. We show the spacetime diagram of the two branches $z_k^{(\alpha)}$, the momentum kicks $z_k^{(\alpha)}$ as a sequence in time and the gravitationally induced trajectory $z_g$ in Fig. 1 on the left. The momentum kicks are $z_k$ branch-dependent while $z_g$ is common for both arms of the interferometer. From these quantities and with the help of equations (4) and (6), we obtain the phase contributions shown at the bottom of the Fig. 1 (see Materials and Methods). The phase takes the familiar form $\Delta \varphi = -kgT^2$ and has no proper-time contribution, but is solely determined by the gravito-recoil action originating in the electromagnetic interaction.
Fig. 1. Time dilation in different interferometer geometries. Spacetime diagrams for the trajectories $z_k$, momentum kicks $\hat{z}_k$, and gravitationally induced trajectories $z_g$, as well as the proper-time difference $\Delta \tau$, the gravito-recoil action $\Delta S_{gk}$ and the total phase difference $\Delta \varphi$ of an MZI (left), a symmetric RBI (centre) and an asymmetric RBI (right). The first two geometries display a symmetric momentum transfer between the two branches, leading to vanishing proper-time differences. However, the asymmetric RBI features a proper-time difference that has the form of a recoil term. The spacetime diagrams also illustrate the connection to the twin paradox by displaying ticking rates (the dashes) of the two twins travelling along the two branches. Both quantum twins in the MZI and symmetric RBI experience the same time dilation, whereas in the asymmetric RBI one twin stays at rest and the other one leaves and returns so that their proper times are different. The arrows in the plot of $\hat{z}_k$ denote the amplitude of the delta functions that scale with $\pm \hbar k/m$. Due to the instantaneous nature of $\hat{z}_k$, the integration over time in equations (4) and (6) reduces to a sampling of the positions $z_k$ and $z_g$ at the time of the pulses such that the respective phase contributions can be inferred directly from the figure.

| $\Delta \tau$ | 0 | 0 | $-\left(\frac{\hbar k}{mc}\right)^2 T$ |
| $\Delta S_{gk}/\hbar$ | $-kgT^2$ | $-kg(T + T')$ | $-kg(T + T')$ |
| $\Delta \varphi$ | $-kgT^2$ | $-kg(T + T')$ | $-kg(T + T') - \hbar k^2T/m$ |

The vanishing proper-time difference can be explained from the momentum kicks $\hat{z}_k^{(\alpha)}$ that act symmetrically on both branches. We draw on the twin paradox to illustrate the effect: After the first light pulse, the atom moves on two different worldlines like two twins, one having undergone acceleration and therefore experiencing special-relativistic time dilation, as shown by the hypothetical ticking rates in the spacetime diagram (Fig. 1). After a time $T$, the twin with higher momentum stops and the other one starts moving. The momentum of the moving twin corresponds to the one that caused the separation, so that it experiences exactly the same time dilation. Hence, when both twins meet at the final pulse, their clocks are synchronised and no proper-time difference arises.

A similar observation is made for the symmetric Ramsey-Bordé interferometer (RBI), where the atom separates for a time $T$, stops on one branch for a time $T'$, before the other branch is redirected. We show the spacetime diagrams and the momentum kicks in the center of Fig. 1 with the phase contributions below. The two light pulses in the middle of the symmetric RBI are also beam-splitting pulses that introduce a symmetric loss of atoms. As for the MZI, the proper-time difference between both branches vanishes and the phase is determined solely by the laser contribution and the gravitorecoil phase as shown by the ticking rates in the spacetime diagram. The only difference with respect to the MZI is that the two twins travel in parallel for a time $T'$ during which proper time elapses identically for both of them.

The situation changes significantly when we consider an asymmetric RBI, where one branch is completely unaffected by the two central pulses as shown on the right of Fig. 1. In fact, the twin that moved away from its initial position experiences a second time dilation on its way back so that there is a proper-time difference when both twins meet at the final pulse. It is therefore the kinematic asymmetry that causes a non-vanishing proper-time difference, as indicated in the figure by the ticking rates. In fact, the proper-time difference

$$\Delta \tau_{\text{RBI}} = -(\hbar k/mc)^2 T$$

is proportional to a kinetic term that depends on the momentum kicks (22), as already implied by equation (4). With the momentum kicks $\hat{z}_k^{(\alpha)}$ as well as the gravitationally induced trajectory $z_g$ also shown on the right of the figure, we find the same contribution for $\Delta S_{gk}$ given in Fig. 1 as for the symmetric RBI. The other contribution of $\Delta S_{em}/\hbar$ has the form $2\omega_C \Delta \tau_{\text{RBI}}$ and all of them together contribute to the phase difference $\Delta \varphi$.

Clocks carried by quantum twins

While the twin paradox is helpful in gaining intuitive understanding and insight into the phase contributions, the dashing of the world lines in Fig. 1 only indicates the ticking rate of a hypothetical co-moving clock. In fact, the atoms are in a
stationary internal state during propagation, whereas the concept of a clock requires a periodic evolution between two states. As a consequence, the atom interferometer can be sensitive to special-relativistic time dilation but lacks the notion of a clock. In a debate (27) about whether the latter is accounted for by the Compton frequency \( \omega_C \) as the pre-factor to the proper-time difference in equation (2), an additional superposition of internal states (5, 6) was proposed. This idea leads to an experiment where each branch corresponds to a quantum twin that measures proper time by carrying a clock in the conventional sense.

To illustrate the effect, we introduce an effective model for an atomic clock which moves along branch \( \alpha = 1, 2 \) in an interferometer. In this framework (7, 28), the Hamiltonian

\[
\hat{H}_j^{(\alpha)} = m_j c^2 + \frac{p_j^2}{2m_j} + m_j g \hat{z} + V_{\text{em}}^{(\alpha)}(\hat{z}, t) \quad \text{with} \quad j \in \{a, b\} \quad (9)
\]

describes a single internal state of energy \( E_j = m_j c^2 \) with an effective potential \( V_{\text{em}}^{(\alpha)} \) which models the momentum transfer (see Materials and Methods). This Hamiltonian includes relativistic contributions associated with its internal energies as indicated by the different masses \( m_j \) of the individual internal states. This is a direct post-Newtonian consequence of mass-energy equivalence which associates proper time with an internal state according to equation (1). To connect with our previous discussion, we take the limit of instantaneous pulses and again decompose the interaction into \( V_{\text{em}}^{(\alpha)}(\hat{z}, t) = V_{\text{e}}^{(\alpha)} + V_{\text{p}}^{(\alpha)} \). To this end, we use the quantised version of the expressions for the effective potentials introduced above and replace \( z \) by the position operator \( \hat{z} \). Assuming instantaneous pulses corresponds to neglecting the delay of the light front propagating from the laser to the atoms. A perturbative treatment shows that this approximation is valid if the interaction time of the laser pulses with the atoms is sufficiently short compared to the duration of the interferometer. With these considerations, the Hamiltonian for a clock consisting of an excited state \( |a\rangle \) and a ground state \( |b\rangle \), both forced by Bragg pulses that do not change the internal state (29) on two branches \( \alpha = 1, 2 \), reads

\[
\hat{H}^{(\alpha)} = \hat{H}_a^{(\alpha)} |a\rangle \langle a| + \hat{H}_b^{(\alpha)} |b\rangle \langle b| . \quad (10)
\]

Since the Hamiltonian is diagonal in the internal states, we write the time evolution along branch \( \alpha \) as \( \hat{U}^{(\alpha)} = U_{a}^{(\alpha)} |a\rangle \langle a| + U_{b}^{(\alpha)} |b\rangle \langle b| \), where \( U_{j}^{(\alpha)} \) is the time-evolution operator that arises from the Hamiltonian \( \hat{H}^{(\alpha)} \). For an atom initially in a state \( |j\rangle \) with \( j = a, b \), the output state is determined by the superposition \( U_{j}^{(1)} + U_{j}^{(2)} \) and leads to an interference pattern \( P_j = (1 + \cos \Delta \varphi_j)/2 \), where the phase difference \( \Delta \varphi_j \) depends on the internal state.

In the case of quantum-clock interferometry the initial state for the interferometer is a superposition of both internal states \( (|a\rangle + |b\rangle)/\sqrt{2} \), which form a clock that moves along both branches in superposition. The outlined formalism, shows that such a superposition leads to the sum of two interference patterns, that is \( P = (P_a + P_b)/2 \). The sum of the probabilities \( P_{a/b} \) with slightly different phases, which corresponds to the concurrent operation of two independent interferometers for the individual states, leads to a beating of the total signal and an apparent modulation of the visibility. Expressing the masses of the individual states by their mass difference \( \Delta m \), i.e. \( m_{a/b} = m \pm \Delta m/2 \) and identifying the energy difference \( \Delta E = \Delta mc^2 = \hbar \Omega \) leads to the interference pattern

\[
P = \frac{1}{2} \left[ 1 + \cos \left( \eta \frac{\Delta \tau}{2} \right) \cos \left( \eta \omega_C \Delta \tau + \frac{\Delta S_{gk} + \Delta S_p}{\hbar} \right) \right] , \quad (11)
\]

where the scaling factor \( \eta = 1/[1 - \Delta m^2/(2mc^2)] \) depends on the energy difference of the two states. In this form, the first cosine can be interpreted as a slow but periodic change of the effective visibility of the signal. In fact, to first order in \( \Delta m/m \) we find \( \eta = 1 \), so that the effective visibility \( \cos (\Delta \Omega \Delta \tau/2) \) corresponds to the signal of a clock measuring the proper-time difference. In this picture, the loss of contrast can be seen as a consequence of distinguishability (6): Since a superposition of internal states travels along each branch, the system can be viewed as a clock with frequency \( \Omega \) travelling in a spatial superposition. On each branch, the clock measures proper time and by that contains which-way information, leading to a loss of visibility as a direct consequence of complementarity.

We illustrate this effect in Fig. 2a) using an asymmetric double-loop RBI, where the gravito-recoil action vanishes, i.e. \( \Delta S_{gk} = 0 \), because it is insensitive to linear accelerations like other symmetric double-loop geometries that are routinely used to measure rotations and gravity gradients (30). The measured phase takes the form \( \Delta \varphi = 2\omega_C \tau_{\text{RBI}} = -2\hbar k^2 T/m \). Even though this expression is proportional to a term that has the form of proper time, it also comprises contributions from the interaction with the laser pulses, see the first term of equation (5). The two internal states, denoted by the blue and red ticking rates, travel along both branches such that each twin carries its own clock, leading to a distinguishability when they meet. This distinguishability depends on the frequency \( \Omega \) of the clock and implies a loss of visibility, as shown in Fig. 2b). However, since each internal state experiences a slightly different recoil velocity \( \hbar k/m_{a/b} \), it can be associated with a slightly different worldline, displayed in red and blue in Fig. 2a). The interpretation as a clock travelling along one particular branch is therefore only valid to lowest order in \( \Delta m/m \).

In another interpretation, the quantum twin experiment is performed for each state independently. The worldlines are different for each state and the proper-time difference as well as the Compton frequency are mass-dependent, so that the interferometer phase depends explicitly on the mass. The loss of visibility can therefore be explained by the beating of the two different interference signals, which is caused by the mass difference \( \Delta m = \hbar \Omega /c^2 \). In the spacetime diagram of Fig. 2a) the finite speed of light pulses causing the momentum transfers is not taken into account. However, to illustrate the neglected effects induced by the propagation time, Fig. 2c) magnifies such an interaction and showcases the assumption we made in our calculation: both internal states interact simultaneously and
instantaneously with the light pulse, even though they might be spatially separated. For feasible recoil velocities, as well as interferometer and pulse durations, this approximation is reasonable.

**DISCUSSION**

An experimental realisation requires atomic species that feature a large internal energy splitting, suggesting typical clock atoms like strontium (Sr) with optical frequencies $\Omega$ in the order of hundreds of THz. The proper-time difference is a property of the interferometer geometry and is enhanced for large splitting times $T$ and effective momentum transfers $hk$. Besides large-momentum-transfer techniques (4, 20), this calls for atomic fountains in the order of meters (19, 31, 32) or the operation in microgravity (17, 33).

To observe a full drop in visibility and its revival, the accumulated time dilation in the experiment needs to be in the order of femtoseconds. In the example of Sr, this can be achieved for $T = 325$ ms and $k = 1200$ $k_m$, where $k_m = 1.5 \times 10^7$ m$^{-1}$ is the effective wave number of the magic two-photon Bragg transition. At the magic wavelength (34) of 813 nm, the differential ac-Stark shift of the two clock states vanishes to first order, such that the beam splitters act equally on the two internal states and hence leave the clock unaffected. Increasing $T$ and by that $\Delta_T$, one should observe a quadratic loss of visibility as a signature of which-path information, assuming this loss can be distinguished from other deleterious effects. Indeed, times up to $T = 350$ ms and $k = 580$ $k_m$ induce a visibility reduction of 10%.

Although they do not use the same species, large atomic fountains (19) already realise long free evolution times and large momentum transfer with hundreds of recoil momenta has been demonstrated (4, 20). Techniques to compensate the impact of gravity gradients (24) and rotations (35) have already proven successful (19, 31, 36). The main challenge in implementing quantum-clock interferometers as described above lies in the concurrent manipulation of the two clock states, requiring a transfer of concepts and technologies well established for alkaline atoms to alkaline earth species. Besides magic Bragg diffraction, other mechanisms like simultaneous single-photon transitions between the clock states (37) are also conceivable and relax the requirements on laser power. In view of possible applications to gravitational wave detection (38), atom interferometry based on single-photon transitions is already becoming a major line of research. To this end, first steps towards quantum-clock interferometry have been demonstrated in Sr (28).

Because the effect can be interpreted as a beating of the signal of two atomic species (defined through their internal state), one can also determine the phase for each state independently and infer their difference in the data analysis. A differential phase of 1 mrad assuming $T = 60$ ms and $k = 70$ $k_m$ may already be resolved in a table-top setup in a few hundred shots with $10^6$ atoms, supposing shot-noise limited measurements of the two internal states. Equation (4) shows that proper-time differences in our setting arises only from special-relativistic effects caused by the momentum kicks. Such an experiment is equivalent to the comparison of two recoil measurements (14, 20) performed independently but simultaneously to suppress common-mode noise. Beyond recoil spectroscopy, state resolving measurements can be of particular interest for a doubly differential measurement scheme that, in contrast to the setup discussed above, does not rely on an initial superposition of two internal states. Instead, the superposition of internal states is generated during the interferometer (37), such that these setups can be used to measure the time dilation caused by a gravitational redshift. In contrast, our discussion highlights the relevance of special-relativistic time dilation for the interference of quantum clocks in conventional interferometers without internal transitions.

In summary, we have shown that for an interferometer that does not change the internal state during the sequence, the measured proper-time difference is in lowest order independent of
gravity and is non-vanishing only in recoil measurements, connecting matter-wave interferometry to the special-relativistic twin paradox. As a consequence of this independence, such light-pulse atom interferometers are insensitive to gravitational time dilation.

The light pulses creating the interferometer cause a contribution to its phase that is of the same form as the special-relativistic proper-time difference and depends on the position of the branches in a freely falling frame, which can be associated with the worldline of a quantum twin. Since these worldlines and by that proper time depends on the recoil velocity that is slightly different for different internal states, an initial superposition causes a beating of two interference patterns. In such a quantum version of the twin-paradox, each twin carries a clock leading to a genuine implementation of quantum-clock interferometry but based on special-relativistic time dilation only.

MATERIALS AND METHODS

Recoil terms and proper time

In this section, we show that for light pulses acting instantaneously on both branches and gravitational potentials up to linear order, the proper time consists only of recoil terms. We provide the explicit expressions for the proper-time difference and find a compact form for the action of the electromagnetic potential that contributes to the phase of the atom interferometer.

As already implied by the decomposition from equation (3), the interaction of an atom with a light pulse transfers momentum and imprints a phase on the atom (25). Since the latter contribution does not modify the motion of the atom, we find $\delta \varphi_\alpha / \partial z = 0$. Consequently, the classical equations of motion can be written as $m^2 \ddot{z} = -\partial (mU) / \partial z = -\ddot{V}_z \dot{z} - m \ddot{z}_g + \ddot{m} \dot{z}_g$. The integration of these equations leads to the trajectory $z = z_g + z_k$, where we collect the initial conditions in $z_g$.

Proper time takes in lowest order expansion in $c^{-2}$, i.e. for weak fields and low velocities, according to equation (1) for a linear gravitational potential the form

$$c^2 \tau = \int \left( \frac{d^2 - c^2 / 2 - \dot{z}_g \dot{z}}{2} \right).$$

We simplify this expression by integrating the kinetic term $\dot{z}_g \dot{z}$ by parts and make the substitution $z = z_g + z_k$ in the remaining integral, so that we find

$$c^2 \tau = \frac{\dot{z}_g \dot{z}_k}{2} + \int \left( \frac{d^2 + \dot{z}_g \dot{z}_k - \frac{\dot{z}_g \dot{z}_k}{2} + \frac{\ddot{z}_g \dot{z}_k}{2}}{2} \right),$$

for the proper time. Partial integration of the last term in the integral leads to the compact form

$$c^2 \tau = \frac{\dot{z}_g \dot{z}_k - \frac{\ddot{z}_g \dot{z}_k}{2}}{2} + \int \left( \frac{d^2 - \frac{\ddot{z}_g \dot{z}_k}{2}}{2} \right),$$

that explicitly depends on the initial and final positions and velocities.

In a light-pulse atom interferometer, light pulses act independently through the potentials $V^{(\alpha)}_k$ on the two branches $\alpha = 1, 2$ and give rise to the trajectories $z^{(\alpha)}_k$. In turn, these branch-dependent potentials lead to a proper-time difference $\Delta \tau = \tau^{(1)} - \tau^{(2)}$ between the upper and lower branch of the interferometer and cause a phase contribution to the interference pattern. In an interferometer closed in phase space, the initial and final positions as well as velocities are the same for both branches and thus the first term in equation (1) vanishes. Since $z_g$ is branch-independent, the first two terms in the integral cancel as well and we are left with

$$\Delta \tau = \int \left[ \frac{d^{(1)}_k - d^{(2)}_k}{2} \right] / \left( 2c^2 \right),$$

for the lowest order of the proper-time difference of an atom interferometer in a linear gravitational potential. In fact, the proper-time difference in a closed interferometer is independent of gravity and constitutes a special-relativistic effect. This result can also be derived for a time-dependent gravitational acceleration $g(t)$.

Since proper time is invariant under coordinate transformations, the proper-time difference of a closed atom interferometer is independent of the gravitational acceleration by considering the common freely falling frame. In this frame, the trajectories are straight lines and correspond to $z^{(\alpha)}_k$, as implied by Fig. 1, so that the proper-time difference is of special relativistic origin. Hence, equation (15) can be also interpreted as a direct consequence of transforming to a freely falling frame in a homogeneous gravitational field.

The laser contribution to the phase can be calculated from equation (3) and we write the classical action in the form of

$$S_{em} = -\int dt (V_\ell + V_\nu) = m \int dt \dot{z}_k - \int dt V_\nu,$$

where we assumed that $V_\ell = -m \ddot{z}_g \dot{z}$ is linear in $z$. When we again use the decomposition of the position $z = z_g + z_k$ into a part that is caused by gravity and a part that is caused by the momentum kicks, we find

$$S_{em} = m \int dt \dot{z}_g z_k + m \int dt \dot{z}_k \dot{z}_g - \int dt V_\nu$$

for the action. Since $z_g$ is branch-independent in contrast to $z_k$, the difference

$$\Delta S_{em} = m \int dt \left[ \frac{d^{(1)}_k - d^{(2)}_k}{2} \right] + m \int dt \ddot{z}_g \dot{z}_k - \int dt V_\nu$$

between upper and lower branch depends on $\Delta S_{g\nu} = \Delta S_{g\nu}^{(1)} - \Delta S_{g\nu}^{(2)}$ and $\Delta V_\nu = V_\nu^{(1)} - V_\nu^{(2)}$. With the expression for the proper time from equation (4), the gravito-recoil action from equation (6) and the laser phase action from equation (7) we arrive at the form of equation (5) for the action of the interaction with the electromagnetic field.

For the specific form of the phase contributions and proper time, we first calculate the trajectory that arises from $\dot{z}_g = -g$ and find by simple integration $z_g(t) = z(0) + \dot{z}(0) t - \frac{1}{2} g t^2$, which is branch-independent. For the specific form of the phase contributions and proper time, we calculate the trajectories that arise from the atom-light interaction. To this end, we assume the potential $V^{(\alpha)}_k = -\sum \ell h_k^{(\alpha)} \delta(t - t_{\ell})$ that causes the momentum kicks $m \dot{z}_k^{(\alpha)} = \sum \ell h_k^{(\alpha)} \delta(t - t_{\ell})$. Integrating the acceleration leads to the two branches of the interferometer given by the two trajectories

$$z_k^{(\alpha)}(t) = \sum_{\ell=1}^{n} \left( \frac{2}{\ell} - t_{\ell} \right) h_k^{(\alpha)} / m$$

for $t_{n+1} > t > t_n$. Using the expression for $z_k^{(\alpha)}$, the gravito-recoil phase from equation (6) takes the explicit form

$$\Delta S_{g\nu} / h = \sum_{\ell} \Delta k_\ell \dot{z}_g(t_{\ell}),$$

where we evaluate the gravitationally induced trajectory from equation (19) at the times of the pulses. Note that we defined the differential momentum transfer $\Delta k_\ell = k^{(1)}_\ell - k^{(2)}_\ell$ of the $\ell$th laser pulse.

With the branch-dependent trajectory from equation (20) and $z_k^{(\alpha)}$, we perform the integration in equation (15) to arrive at the expression

$$\omega \tau = \frac{\hbar}{2m} \sum_{n=1}^{n} \int \left[ d^{(1)}_k + d^{(2)}_k - \frac{d^{(1)}_k - d^{(2)}_k}{2} \right] / (n_{a} - t_{a}),$$

where $M$ is the total number of light-matter interaction points. This phase difference is proportional to $h/m$ and includes a combination of the transferred momenta and separation between the laser pulses.

Post-Galilean bound systems in a Newtonian gravitational field

We consider a static spacetime with a line element of the form

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu = -N(x)(c dt)^2 + B_{ij}(x)dx^i dx^j,$$

where $g_{\mu\nu}$ is the metric with Greek indices running from 0 to 3, $N$ is the lapse function and $B_{ij}$ is the three-metric with Latin indices running from 1 to 3. In the limit
of Newtonian gravity the lapse function becomes $N(x) = 1 + 2U(x)/c^2$ with $U$ being the Newtonian gravitational potential while $B_{ij} = [1 - 2U(x)/c^2] \delta_{ij}$, where $\delta_{ij}$ is the Kronecker symbol. The post-Newtonian correction to the spatial part of the metric decomposition only needs to be considered for the electromagnetic field but not for the atoms inside a light-pulse atom interferometer.

To model atomic multi-level systems inside such a background metric including effects that arise from special-relativistic and general-relativistic corrections due to the lapse function of the metric to the bound state energies, one can resort to a quantum field theoretical treatment (39) and perform the appropriate limit to a first-quantised theory afterwards. In this approach one first performs the second quantisation of the respective interacting field theory in the classical background metric provided by equation (23) and derives the bound state energies as well as possible spin states of e.g. hydrogen-like systems. For each pair of energy and corresponding internal state, one can take the limit of a first-quantised theory and Newtonian gravity. Expanding the Newtonian gravitational potential up to second order leads to a Hamiltonian

$$\hat{H}_{0,j} = m_j c^2 + \frac{\hat{p}_j^2}{2m_j} + m_j \left(g_j^2 \hat{x} + \frac{1}{2} \hat{t} \hat{S}_i \right).$$  \hspace{1cm} (24)

Here, $E_j = m_j c^2$ is the energy corresponding to the energy eigenstate $|j\rangle$, $\hat{p}$ and $\hat{x}$ are the momentum and position operator, respectively, $g$ is the (local) gravitational acceleration vector and $\Gamma$ is the (local) gravity gradient tensor. This Hamiltonian includes special-relativistic and possibly post-Newtonian contributions to its internal energies as indicated by the different masses $m_j$ of the individual internal states, which is a direct manifestation of the mass-energy equivalence. In principle, terms proportional to $\hat{p}^2$ and $\hat{p}(g_j^2 \hat{x} + \frac{1}{2} \hat{t} \hat{S}_i)$ appear as a correction to the centre-of-mass Hamiltonian. However, since these states are state independent to order $1/c^2$, they leave the beating in equation (11) unaffected and will therefore be disregarded. The third addend in equation (24) is the Newtonian gravitational potential energy which we denote by $V_{\text{G}}(\hat{x})$. The fact that each state couples separately to the (expanded) gravitational potential with its respective mass $m_j$ directly highlights the weak equivalence principle. The full Hamiltonian of an atomic system with multiple internal states labelled by the index $j$ is thus $\hat{H}_0 = \sum_j \hat{H}_{0,j} |j\rangle \langle j|$. This Hamiltonian is diagonal with respect to different internal states and gravity induces no cross-coupling between the states of freely-moving atoms.

**Light-matter interaction and total Hamiltonian**

In typical light-pulse atom interferometers the light-matter interaction is only switched on during the beam-splitting pulses. Hence, the propagation of the atoms through an interferometer can be partitioned into periods of free propagation and periods where the lasers are acting on the atoms. In particular, the light-matter interaction including post-Newtonian corrections to the atoms' bound state energies (40) in the low-velocity, dipole approximation limit reduces to

$$\hat{V}_{\text{em}}(\hat{x}, t) = -\hat{\Phi} \hat{E}(\hat{x}, t) + \hat{V}_G(\hat{x}, t; \hat{\Phi})$$  \hspace{1cm} (25)

where $\hat{\Phi}$ is the electric dipole moment operator, $\hat{E}$ is the external electric field and $\hat{V}_G$ is the Röntgen contribution to the interaction Hamiltonian. The information about special-relativistic corrections to the energies is included in the definition of the dipole moments. Moreover, since the light-matter coupling is of the usual form we can apply the standard framework of quantum optics to derive effective models for the interaction.

However, before proceeding we simplify our model by only considering unidirectional motion in the $z$-direction and an acceleration $g$ anti-parallel to it, so that the Hamiltonian for the full interferometer becomes

$$\hat{H}(\hat{z}, t) = \sum_j \left[ m_j c^2 + \frac{\hat{p}_j^2}{2m_j} + m_j \left(g_j^2 \hat{z} + \frac{1}{2} \hat{t} \hat{S}_i \right) \right] |j\rangle \langle j|$$

$$+ \sum_{i,j} \hat{V}_{\text{em},ij}(\hat{z}, t) |i\rangle \langle j|$$  \hspace{1cm} (26)

where $\hat{V}_{\text{em},ij}$ are the time-dependent matrix elements of the light-matter coupling which include the switch-on/off of the lasers. Based on this model we can derive an effective potential description for e.g. two-photon Raman or Bragg transitions inside an interferometer. In particular, for magic Bragg diffraction we consider pairs of one relevant state $|j\rangle$ and one ancilla state out of the atomic state manifold, each interacting with two light fields. After applying the rotating wave approximation, adiabatic elimination, two-photon resonance conditions (29) and taking the limit of instantaneous pulses, we can replace the electromagnetic interaction by the effective potential

$$\hat{V}_{\text{em}} = -\hbar \sum_j \left( k \hat{z} + \phi_j \right) \delta(t - t_j) |j\rangle \langle j|. $$  \hspace{1cm} (27)

Here we evaluate the effective momentum transfer $\hbar k \delta \tau$ as well as the phase $\phi_j$ of the electromagnetic field at time $t_j$ of the pulse. Although we have written the state dependence explicitly in equation (27), the effective interaction $\hat{V}_{\text{em}}$ becomes state independent since the sum over the relevant states $|j\rangle$ corresponds to unity in case of magic Bragg diffraction.

During a typical light-pulse atom interferometer sequence one usually has multiple wave-packet components centred on different trajectories, each of which constitute an individual branch of the interferometer. In this case, the previously defined interaction can be applied (24) on each branch individually. Hence, the effective interaction Hamiltonian in the case of instantaneous Bragg pulses becomes

$$\hat{V}_{\text{em}}^{(\alpha)} = -\hbar \sum_j \left( k^{(\alpha)} \hat{z} + \phi_j^{(\alpha)} \right) \delta(t - t_j),$$  \hspace{1cm} (28)

where the superscript $\alpha$ labels the individual branch.

**Branch-dependent light-pulse atom interferometry**

As shown above, the momentum transfer caused by light pulses can be described by an effective potential that in general depends on the classical trajectory of the particle. In our limit this dependence reduces to a mere dependence on the two branches of an atom interferometer. Since our description is diagonal in the different internal states and we assume that throughout the free propagation inside the interferometer the internal state of the atoms does not change, the time evolution along branch $\alpha = 1, 2$ takes the form

$$\hat{U}^{(\alpha)} = \hat{U}_0^{(\alpha)} |a\rangle \langle a| + \hat{U}_b^{(\alpha)} |b\rangle \langle b|,$$  \hspace{1cm} (29)

where $\hat{U}^{(\alpha)}$ is the time-evolution operator for state $|j\rangle$ along path $\alpha$ ending in one particular exit port. We limit our discussion to one excited state and one ground state, hence we use the labels $a, b$, respectively. If $|\psi_j(0)\rangle$ describes the initial external degree of freedom of the atoms in state $|j\rangle$ and we project onto the internal state when we perform the measurement, the postselected state in one of the output ports of the interferometer is a superposition of the two branches, i.e.

$$|\psi_j(\tau)\rangle = (\hat{U}^{(1)} + \hat{U}^{(2)}) |\psi_j(0)\rangle /2,$$  \hspace{1cm} (30)

leading to the interference pattern

$$P_j = \langle \psi_j | \langle \psi_j | = \frac{1}{2} \left[ 1 + \frac{1}{2} \left( \psi_j(0) |\hat{U}_0^{(1)} |\psi_j(0)\rangle + \psi_j(0) |\hat{U}_0^{(2)} |\psi_j(0)\rangle \right) + \text{c.c.} \right].$$

The calculation of the inner product can now be performed using the explicit form of the branch-dependent potentials. For a closed geometry and potentials up to linear order, the calculation reduces to the description outlined in the main part of the article (25). This treatment is also exact in the presence of gravity gradients and rotations but will lead in general to open geometries, which can be closed through suitable techniques (24). When we introduce the state-dependent Compton frequency $\omega_j = m_j c^2 / \hbar$ and proper-time difference $\Delta t_j$, where $m$ has to be replaced by $m_j$, we find $P_j = (1 + \cos \Delta \phi_j) / 2$ and the phase difference

$$\Delta \phi_j = -\omega_j \Delta t_j + 2\omega_j \Delta t_j + \Delta \phi_k / \hbar + \Delta \phi_p / \hbar.$$  \hspace{1cm} (31)

Here, we used the fact that both $\Delta \phi_k$ and $\Delta \phi_p$ do not depend on the internal states in agreement with the weak equivalence principle. Moreover the phases are degenerate if the proper-time difference vanishes.

If the atoms are initially in a superposition of the two internal states, i.e. $|a\rangle + |b\rangle) / \sqrt{2}$, the exit port probability without postselection on one internal state is $P = (P_a + P_b) / 2$, which corresponds to the sum of the two interference patterns. After some trigonometry we find

$$P = \frac{1}{2} \left[ 1 + \cos \left( \frac{\Delta \phi_a - \Delta \phi_b}{2} \right) \cos \left( \frac{\Delta \phi_a + \Delta \phi_b}{2} \right) \right]$$  \hspace{1cm} (32)

so that the two interference patterns beat. The first term, i.e. the difference of the phases of the individual states, can be interpreted as a visibility modulation of the concurrent measurement. Because the two masses $m_{a/b} = m \pm \Delta m / 2$ are connected to the energy difference $\Delta E = \hbar \Omega = \hbar m c^2$ between excited and ground state, the frequency $\Omega$ determines the beating.
Connection to clock Hamiltonians. We discuss in the main body of the article that in an expansion of the phase difference in orders of $\Delta m/m$, the beating effect can be interpreted as a loss of contrast due to the distinguishability of two internal clock states. In this section we show the connection of the Hamiltonian from equation (26) to a clock Hamiltonian (6)

$$\hat{H}_{\text{int}} = \left( mc^2 - \frac{\hbar \Omega}{2} \right) |b\rangle \langle b| + \left( mc^2 + \frac{\hbar \Omega}{2} \right) |a\rangle \langle a|. \quad (33)$$

In fact, expanding equation (10) up to linear order of $\Delta m/m$, we find the expression

$$\hat{H}^{(\alpha)} = \hat{H}_{\text{int}} + \frac{\hbar^2}{2m} + mg\xi + V^{(\alpha)} + \left( -\frac{\hbar^2}{2m} + mg\xi \right) \hat{H}_{\text{int}} \frac{mc^2}{\hbar^2} \quad (34)$$

with the help of equation (33). In this form, the coupling of the internal dynamics to the external degrees of freedom is prominent and leads to the interference signal from equation (11) with $\eta = 1$. Hence, the Hamiltonian describes a moving clock experiencing time dilation (6).

REFERENCES AND NOTES

1. A. Einstein, “Zur Elektrodynamik bewegter Körper,” Ann. Phys. 322, 891–921 (1905).
2. Ch. Møller, “The Ideal Standard Clocks in the General Theory of Relativity,” Helv. Phys. Acta, Suppl. 4, 54–57 (1956).
3. A. Einstein, “Dialog über Einwände gegen die Relativitätstheorie,” Naturwissenschaften 6, 697–702 (1918).
4. T. Kovachy, P. Asenbaum, C. Overstreet, C. A. Donnelly, S. M. Dickerson, A. Sugarbaker, J. M. Hogan, and M. A. Kasevich, “Quantum superposition at the half-metre scale,” Nature (London) 528, 530–533 (2015).
5. S. Sinha and J. Samuel, “Atom interferometry and the gravitational redshift,” Class. Quantum Grav. 28, 145018 (2011).
6. M. Zych, F. Costa, I. Pikovsky, and Č. Brukner, “Quantum interferometer visibility as a witness of general relativistic proper time,” Nat. Commun. 2, 505 (2011).
7. Magdalena Zych, Quantum systems under gravitational time dilation, Springer Theses (Springer, 2017).
8. J. C. Hafele and R. E. Keating, “Around-the-World Atomic Clocks: Predicted Relativistic Time Gains,” Science 177, 166–168 (1972); “Around-the-World Atomic Clocks: Observed Relativistic Time Gains,” Science 177, 168–170 (1972).
9. R. F. C. Vessot, M. W. Levine, E. M. Mattison, E. L. Blomberg, T. E. Hoffman, G. U. Nystrom, B. F. Farrel, R. Decher, P. B. Eby, C. R. Baugher, J. W. Watts, D. L. Teuber, and F. D. Wills, “Test of Relativistic Gravitation with a Space-Borne Hydrogen Maser,” Phys. Rev. Lett. 45, 2081–2084 (1980).
10. C. W. Chou, D. B. Hume, T. Rosenband, and D. J. Wineland, “Optical Clocks and Relativity,” Science 329, 1630–1633 (2010).
11. T. L. Nicholson, S. L. Campbell, R. B. Hutson, G. E. Marti, B. J. Bloom, R. L. McNally, W. Zhang, M. D. Barrett, M. S. Sanfronova, G. F. Strouse, W. L. Tew, and J. Ye, “Systematic evaluation of an atomic clock at $2 \times 10^{18}$ total uncertainty,” Nat. Commun. 6, 6896 (2015).
12. C. Freier, M. Hauth, V. Schkolnik, B. Leykauf, M. Schilling, H. Wzietek, H. Scheirner, J. Müller, and A. Peters, “Mobile quantum gravity sensor with unprecedented stability,” J. Phys.: Conf. Ser. 723 (2015).
13. D. Savoie, M. Altorio, B. Fang, L. A. Sidorenkov, R. Geiger, and A. Landragin, “Interleaved atom interferometry for high-sensitivity inertial measurements,” Sci. Adv. 4 (2018).
14. R. Bouchendira, P. Cladé, S. Guellati-Khélifa, F. Nez, and F. Biraben, “New Determination of the Fine Structure Constant and Test of the Quantum Electrodynamics,” Phys. Rev. Lett. 106, 080801 (2011).
15. D. Schlipphert, J. Hartwig, H. Albers, L. L. Richardson, C. Schubert, A. Roura, W. P. Schleich, W. Ertmer, and E. M. Rasel, “Quantum Test of the Universality of Free Fall,” Phys. Rev. Lett. 112, 203002 (2014).
16. L. Zhou, S. Long, B. Tang, X. Chen, F. Gao, W. Peng, W. Duan, J. Zhong, Z. Xiong, J. Wang, Y. Zhang, and M. Zhan, “Test of Equivalence Principle at $10^{-8}$ Level by a Dual-Species Double-Diffraction Raman Atom Interferometer,” Phys. Rev. Lett. 115, 013004 (2015).
17. B. Barrett, L. Antoni-Micollier, L. Chichet, B. Battelier, T. Lévêque, A. Landragin, and P. Bouyer, “Dual-matter-wave inertial sensors in weightlessness,” Nat. Commun. 7, 13786 (2016).
18. G. Rosi, F. Sorrentino, L. Cacciapuoti, M. Prevedelli, and G. M. Tino, “Precision measurement of the Newtonian gravitational constant using cold atoms,” Nature (London) 510, 518–521 (2014).
19. C. Overstreet, P. Asenbaum, T. Kovachy, R. Notermans, J. M. Hogan, and M. A. Kasevich, “Effective Inertial Frame in an Atom Interferometric Test of the Equivalence Principle,” Phys. Rev. Lett. 120, 183604 (2018).
20. R. H. Parker, C. Yu, W. Zhong, B. Estey, and H. Müller, “Measurement of the fine-structure constant as a test of the Standard Model,” Science 360, 191–195 (2018).
21. E. Giese, A. Friedrich, F. Di Pumpo, A. Roura, W. P. Schleich, D. M. Greenberger, and E. M. Rasel, “Proper time in atom interferometers: Diffractive versus specular mirrors,” Phys. Rev. A 99, 013627 (2019).
22. S.-Y. Lan, P.-C. Kuan, B. Estey, D. English, J. M. Brown, M. A. Hohensee, and H. Müller, “A Clock Directly Linking Time to a Particle’s Mass,” Science 339, 554–557 (2013).
23. Ch. J. Bordé, M. Weitz, and T. W. Hänsch, “New optical atomic interferometers for precise measurements of recoil shifts. Applications to atomic hydrogen,” in AIP Conf. Proc. (AIP, 1993).
24. A. Roura, “Circumventing Heisenberg’s Uncertainty Principle in Atom Interferometry Tests of the Equivalence Principle,” Phys. Rev. Lett. 118, 160401 (2017).
25. W. P. Schleich, D. M. Greenberger, and E. M. Rasel, “Redshift Controversy in Atom Interferometry: Representation Dependence of the Origin of Phase Shift,” Phys. Rev. Lett. 110, 010401 (2013).
26. P. Wolf, L. Blanchet, Ch. J. Bordé, S. Reynaud, C. Salomon, and C. Cohen-Tannoudji, “Does an atom interferometer test the gravitational redshift at the Compton frequency?” Class. Quantum Grav. 28, 145017 (2011).
27. H. Müller, A. Peters, and S. Chu, “A precision measurement of the gravitational redshift by the interference of matter waves,” Nature (London) 463, 926–929 (2010).
28. L. Hu, N. Poli, L. Salvi, and G. M. Tino, “Atom Interferometer with the Sr Optical Clock Transition,” Phys. Rev. Lett. 119, 263601 (2017).
29. E. Giese, A. Roura, G. Tackmann, E. M. Rasel, and W. P. Schleich, “Double Bragg diffraction: A tool for atom optics,” Phys. Rev. A 88, 053608 (2013).
30. K.-P. Marzlin and J. Audretsch, “State independence in atom interferometry and insensitivity to acceleration and rotation,” Phys. Rev. A 53, 312–318 (1996).
31. S. M. Dickerson, J. M. Hogan, A. Sugarbaker, D. M. S. Johnson, and M. A. Kasevich, “Multiaxis Inertial Sensing with Long-Time Point Source Atom Interferometry,” Phys. Rev. Lett. 111, 083001 (2013).
32. J. Hartwig, S. Abend, C. Schubert, D. Schlipphert, H. Ahlers, K. Posso-Trujillo, N. Gaulou, W. Ertmer, and E. M. Rasel,
“Testing the universality of free fall with rubidium and ytterbium in a very large baseline atom interferometer,” New J. Phys. 17, 035011 (2015).

33. D. Becker, M. D. Lachmann, S. T. Seidel, H. Ahlers, A. N. Dinkelaker, J. Grosse, O. Hellmig, H. Müntinga, V. Schkolnik, T. Wendrich, A. Wenzlaws, B. Wesps, R. Corgier, T. Franz, N. Gaaloul, W. Herr, D. Lüdtke, M. Popp, S. Amri, H. Duncker, M. Erbe, A. Koffeldt, A. Kubelka-Lange, C. Braxmaier, E. Charron, W. Ertmer, M. Krutzik, C. Lämmerzahl, A. Peters, W. P. Schleich, K. Sengstock, A. Wicht, H. Müntinga, T. Wendrich, A. Wenzlawski, B. Weps, R. Corgier, T. Franz, N. Gaaloul, W. Herr, D. Lüdtke, M. Popp, S. Amri, H. Duncker, M. Erbe, A. Koffeldt, A. Kubelka-Lange, C. Braxmaier, E. Charron, W. Ertmer, M. Krutzik, C. Lämmerzahl, A. Peters, W. P. Schleich, K. Sengstock, A. Wicht, P. Windpassinger, and E. M. Rasel, “Space-borne Bose–Einstein condensation for precision interferometry,” Nature (London) 562, 391 (2018).

34. H. Katori, M. Takamoto, V. G. Pal’chikov, and V. D. Ovsiannikov, “Ultrastable Optical Clock with Neutral Atoms in an Engineered Light Shift Trap,” Phys. Rev. Lett. 91, 173005 (2003).

35. J. M. Hogan, D. M. S. Johnson, and M. A. Kasevich, Atom Optics and Space Physics: Proceedings of the International School of Physics “Enrico Fermi”, Course CLXVIII, edited by E. Arimondo, W. Ertmer, E. M. Rasel, and W. P. Schleich (IOS Press, Amsterdam, 2008) Chap. Light-pulse atom interferometry.

36. S.-Y. Lan, P.-C. Kuan, B. Estey, P. Haslinger, and H. Müller, “Influence of the Coriolis Force in Atom Interferometry,” Phys. Rev. Lett. 108, 090402 (2012).

37. J. M. Hogan, D. M. S. Johnson, and M. A. Kasevich, Atom Optics and Space Physics: Proceedings of the International School of Physics “Enrico Fermi”, Course CLXVIII, edited by E. Arimondo, W. Ertmer, E. M. Rasel, and W. P. Schleich (IOS Press, Amsterdam, 2008) Chap. Light-pulse atom interferometry.

38. P. W. Graham, J. M. Hogan, M. A. Kasevich, and S. Rajendran, “Resonant mode for gravitational wave detectors based on atom interferometry,” Phys. Rev. D 94, 104022 (2016).

39. C. Anastopoulos and B. L. Hu, “Equivalence principle for quantum systems: dephasing and phase shift of free-falling particles,” Class. Quantum Grav. 35, 035011 (2018).

40. M. Sonnleitner and S. M. Barnett, “Mass-energy and anomalous friction in quantum optics,” Phys. Rev. A 98, 042106 (2018).

ACKNOWLEDGEMENTS

Funding We acknowledge financial support from DFG through CRC 1227 (DQ-mat), project B07. The presented work is furthermore supported by CRC 1128 geo-Q, the German Space Agency (DLR) with funds provided by the Federal Ministry of Economic Affairs and Energy (BMWi) due to an enactment of the German Bundestag under Grant No. 50WM1641 and 50WM0837, and by “Niedersächsisches Vorab” through the “Quantum- and Nano- Metrology (QUANOMET)” initiative within the project QT3 and through “Förderung von Wissenschaft und Technik in Forschung und Lehre” for the initial funding of research in the new DLR-SI Institute. The work of IQST is financially supported by the Ministry of Science, Research and Arts Baden-Württemberg. DS gratefully acknowledges funding by the Federal Ministry of Education and Research (BMBF) through the funding program Photonics Research Germany under contract number 13N14875. MZ acknowledges support through ARC DECRA grant no. DE180101443 and ARC Centre EQuS CE170100009. WPS thanks Texas A&M University for a Faculty Fellowship at the Hagler Institute for Advanced Study at Texas A&M University and Texas A&M AgriLife for the support of this work.

Author contributions All authors contributed to scientific discussions, the execution of the study, and the interpretation of the results. SL and AF contributed equally to this work. SL, AF, CU, FDP and EG prepared the manuscript with input from all other authors. EMR, WPS and EG supervised the project.

Competing interests All authors declare that they have no competing financial and nonfinancial interests.

Data and materials availability All data generated or analyzed during this study are included in this published article or are available from the corresponding author on reasonable request.