Integration of Variable Acceptance Sampling and Maintenance Policy Based on Process Capability Index

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Abstract. In production process, quality is directly affected by the degradation of the production machine. Information of products quality may be used as feedback to determine the maintenance policy. As far as is known, research on integration of quality and maintenance widely used quality control with 100% inspection. Many literatures mentioned that sampling inspection is cheaper and easier to be performed than 100% inspection. With the same level of protection, the variable acceptance sampling requires fewer samples and provide more information about the manufacturing process than the attribute data. This paper attempt to integrate variable acceptance sampling and maintenance policy. The variable acceptance sampling model developed based on exact sampling distribution approach under Cpk capability index. If the lot is rejected, 100% inspection will be performed and Cpk will be estimated. The estimated Cpk resulted minimum cost rate will be consider as the treshold for conducting PM.

Keywords—Quality based maintenance, process capability index, single acceptance sampling by variable, preventive maintenance.

1. Introduction
The importance of quality and maintenance roles in industry, encourages researchers to determined an optimum strategy to improve production efficiency and productivity. In process production, there are situations where quality is directly affected by the degradation of the production system. Therefore, quality information can be used as feedback to determine maintenance policy. Quality control can monitor the status of the production system that is when the equipment produce defect product, it can be concluded that some damage occurred on the equipment and maintenance measures are required. Therefore, it is possible to link maintenance and quality by determining the tolerance limit. (Lesage & Dehombreux, 2012) By determine the maintenance policy and ensuring product quality simultaneously, it is expected to minimize the cost to be incurred. Tapiero (1986) was the first to formulate a maintenance problem based on product quality control feedback, assuming quality serves to determine the state of engine degradation. Moreover, Mehdi et al. (2010) integrates quality control and preventive maintenance by using a 100% inspection policy to determine the proportion of nonconforming items of each produced lot and then comparing this proportion with multiple thresholds given to make decisions about maintenance actions. Hsu and Kuo (1995) studied the performance of inspection and maintenance policies that initiate 100% inspection of production
quantities after generating a certain quantity of goods and then start a preventive / corrective maintenance activity if the fraction of the damaged part reaches the given threshold. Similarly, Radhoui et al. (2009) also uses a 100% inspection policy to determine the proportion of nonconforming items from each produced lot and then compares this proportion with some of the thresholds given for making decisions regarding PM actions and overhaul actions. Bouslah et al. (2015) has integrated the production, sampling quality control using acceptance sampling by attribute, and maintenance policy. The results show that the economic design of revenue sampling in the context of integration can lead to important cost savings of over 20%. Zhou and Zhu (2008) have developed control charts and maintenance model integration, a grid-search approach is used to find the optimal value of policy variables to minimize cost per hour. As far as is known, research on integration of quality and maintenance widely used quality control with 100% inspection. Many literatures mentioned that sampling inspection is cheaper and easier to perform than 100% inspection. With the same level of protection, the variable acceptance sampling requires fewer samples and the variable data have more information about the manufacturing process or lot than the attribute data. To obtain more accurate results, variable sampling plans are based on the process capability index, which based on several Pearn and Wu studies, has previously proven that the process capability index will require smaller sample sizes and smaller critical acceptance value compared to the approximation approach. With these advantages then by using policy variable sampling plan based on process capability index will be more economical and sampling result obtained more accurate. Therefore, this study will model the integration of maintenance policy and quality control based on variable acceptance sampling plan using process capability index. The paper is organized as follows. Section 2 describes the relevant literature. Methodology and model development are discussed in Section 3. Section 4 presents numerical example followed by conclusion and limitation of results in section 5.

2. Literature Review

2.1 Interaction of Quality control and Inspection

Most of the products and the system will deteriorate with age and usage. Then a maintenance strategy is needed to improve reliability of system, prevent system failure, and reduce maintenance costs. Maintenance activity is an activity that can be used as a control tool in maintaining the process of using the production equipment to avoid the bottleneck that arise due to the damage, so that the production process can run smoothly and effectively. Kurniati et al. (2015) proposes an interaction framework between quality inspection and maintenance. Quality inspection is a way to verify the conformity of the product with the requirements to be the trigger for maintenance. The number of lots rejected becomes symptoms of worsening process. In general, maintenance is categorized into two main classes namely corrective maintenance and preventive maintenance. Corrective maintenance (CM) is a maintenance that occurs when the system fail. According to MIL-STD-721B, CM means all actions are performed as a result of failure, to return the item to a certain condition. Preventive maintenance (PM) is a treatment that occurs when the system operates. According to MIL-STD-721B, PM means all actions taken in order to maintain the item under certain conditions by providing simple systematic checking, detection and prevention (Wang, 2002).

2.2 Process Capability Index

Process capability index is known as an index to establish the relationship between actual process performance and manufacturing specifications (including target values and specification limits). Statistical Process Control (SPC) is unable to quantitatively analyze an ongoing process, because the SPC only monitors the running processes. The function of process capability index is to know a process running in a capable or incapable process (Pearn, et al., 1998). The process capability index, \( C_{pk} \), has been popularly used in the manufacturing industry to measure whether a process is capable of producing product items within a particular manufacturing tolerance. There are two equivalent forms of the \( C_{pk} \) index. The \( C_{pk} \) index is designed for processes with two sides specification limits namely LSL and USL to measure the magnitude of the variability of the entire process. The first formulation takes into account LSL and USL separately and the second
formulation uses the average deviation of the process from the midpoint of the specification (Kane, 1986).

\[
C_{pk} = \min \left\{ \frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma} \right\} = \frac{d - |\mu - M|}{3\sigma} \tag{1}
\]

Pearn & Wu (2007) obtained a precise form of the cumulative distribution of natural estimator functions \( C_{pk} \) by using a similar integration technique in Vänmann (1997), which is expressed in terms of a mixture of chi-square and normal distribution. The CDF equation is:

\[
FC_{pk}(y) = 1 - b \int_0^\infty \left( \frac{(n-1)(b\sqrt{n} - t)^2}{9nc^2} \right) \times \left( \Phi(t + \xi \sqrt{n}) + \Phi(t - \xi \sqrt{n}) \right) dt \tag{2}
\]

Where for \( y > 0, b = \frac{d}{\sigma} \) and \( \xi = \frac{\mu - M}{\sigma} \), \( G(.) \) Is the cumulative distribution function of the chi-square distribution with the degree of freedom \( n-1, \chi^2_{n-1} \) and \( \Phi(.) \) is the probability density function of the normal distribution of \( N(0, 1) \).

3. Methodology and Model Development

Mathematical notation to be used in this research model is as follows:

- \( E(H) \): Total Cost per unit time
- \( E(T) \): Expectation cycle time
- \( E(C) \): Cost Expectation
- \( C_{ins} \): Inspection Cost per unit
- \( C_{det} \): Detecting damage cost
- \( C_{PM} \): PM cost
- \( T_i \): inspection time
- \( T_{det} \): Machine damage identification time
- \( T_{PM} \): Time to perform PM
- \( Q \): lot size (unit / batch)

Optimum value:

- \( n \): number of samples (units)
- \( C_0 \): The critical value acceptance sample
- \( C_x \): threshold determination of PM

An event that shows symptoms of damage to equipment is usually described by equipment failure to operate or in situations where defects in quality increase. An intensive corrective maintenance action was performed to restore the situation to original state. However, with preventive maintenance policy, equipment failure can be prevented and the risk of increased defective product production can be minimized. So, this research tries to integrate of quality control and maintenance policy, which based on quality control information will be used as a reference to determine the preventive maintenance policy to prevent the occurrence of failure of the production system.

The quality control policy in this study refers to Pearn and Wu (2007) research using single variable acceptance sampling plan based on exact sampling distribution by using Cpk process capability index approach. Acceptance sampling plan with approximation approach and exact sampling distribution basically consists of determining the number of sample size and acceptance criteria. Well designed sampling planning should give at least 1-\( \alpha \) probability of acceptance lots on AQL and the probability of acceptance no more than \( \beta \) in RQL. Therefore, the OC curve of the acceptance sampling plan must pass through two points (AQL, 1-\( \alpha \)) and (RQL, \( \beta \)). Appropriate numbers and alternative hypotheses can be expressed as:

- \( H_0 : p = AQL \) (process capable)
- \( H_1 : p = RQL \) (process incapable)
Thus the required sample size \( n \) and the critical acceptance value \( C_0 \) for the sampling plan can be solved according to the following two equations:

\[
\Pr \left\{ \text{Accepting the lot} \mid C_{pk} = C_{AQL} \right\} \geq 1 - \alpha 
\]

\[
\Pr \left\{ \text{Accepting the lot} \mid C_{pk} = C_{RQL} \right\} \leq \beta
\]

With the probability of accepting lot can be expressed as:

\[
\pi_A(C_{pk}) = \int_0^{b_2} G\left\{ \frac{(n-1)(b_2 \sqrt{\sigma^2 - t})^2}{9n_c^2} \right\} \times \left( \Phi(t + \xi \sqrt{\sigma}) + \Phi(t - \xi \sqrt{\sigma}) \right) dt
\]

then equations (1) and (2) can be written to be:

\[
1 - \alpha \leq \int_0^{b_1} G\left\{ \frac{(n-1)(b_1 \sqrt{\sigma^2 - t})^2}{9n_c^2} \right\} \times \left( \Phi(t + \xi \sqrt{\sigma}) + \Phi(t - \xi \sqrt{\sigma}) \right) dt
\]

\[
\beta \geq \int_0^{b_2} G\left\{ \frac{(n-1)(b_2 \sqrt{\sigma^2 - t})^2}{9n_c^2} \right\} \times \left( \Phi(t + \xi \sqrt{\sigma}) + \Phi(t - \xi \sqrt{\sigma}) \right) dt
\]

By determining the value of \( \alpha \)-risk (producer risk), \( \beta \)-risk (consumer risk), \( C_{AQL} \) (Acceptable Quality Level for \( C_{pk} \) Index) and \( C_{RQL} \) (Rejectable Quality Level for \( C_{pk} \) index ) are required, it can be determined the number of samples \( n \) and the critical value of acceptance \( C_0 \) by solving equations (6) and (7) simultaneously. As noted by Pearn and Wu (2007) that the process parameters \( \mu \) and \( \sigma \) are unknown, then the parameter \( \xi = (\mu - M)/\sigma \) is also unknown which has to be estimated in real applications. Such approach introduces additional sampling errors from estimating \( \xi \) in finding the critical acceptance values and the required sample sizes. To eliminate the need for estimating \( n \), they performed extensive calculations to investigate the behavior of the critical acceptance value and sample size for various parameters and found that the required sample size and the critical acceptance value will be conservative by setting \( \xi = 1.00 \). Thus, we calculate the parameters \( (n, C_0) \) with the condition \( \xi = 1.00 \) to ensure that the decisions made are reliable.

Based on the number of samples and the critical acceptance value which obtained according to the capability process requirements in the company, and consider of the probability of accepting the product will be determined threshold for maintenance policy using the following equation.

\[
E(H) = \frac{E(C)}{E(T)}
\]

Where, \( E(T) = (T_{ins} \times n) + (T_{ins} \times (1 - P_a) \times (Q - n)) + \gamma_{det} T_{det} + \gamma_{PM} T_{PM} \)

\[
E(C) = (C_{ins} \times n) + (C_{ins} \times (1 - P_a) \times (Q - n)) + \gamma_{det} C_{det} + \gamma_{PM} C_{PM}
\]

The cycle time and cost of the integration model consist of the time and cost to perform quality control policy and the time and cost to implement preventive maintenance policy. Time and cost of quality control performing consists of acceptance sampling and 100% inspection when the result of lot decision is rejected \( (C_{pk} < C_0) \). Furthermore, when \( C_{pk} \) is less than the threshold of minimum acceptable process level \( (C_x) \), preventive maintenance will be performed. Value of \( \gamma_{det} \) and \( \gamma_{PM} \) is 1 if identification and PM is applied and is 0 if no PM identification and implementation is performed.

The operating procedure of design integration model can be seen in Figure 1.
4. Numerical Example

To show the applicability of the proposed methodology, we present a case taken from Pearn and Wu (2007) for example i.e. case study on the liquid-crystal module (LCM) manufacturing process. The bonding precision is an essential process parameter. The specification limits are $T = M = 0$, $USL = 15\mu$, and $LSL = -15\mu$. If the characteristic data do not fall within the tolerance (LSL, USL), the lifetime or reliability of the LCM will be discounted. In the contract, the $C_{AQL}$ and $C_{LTPD}$ are set to 1.33 and 1.00 with $\alpha$-risk=0.05 and $\beta$-risk=0.05. Based on the above specified values in the contract, we could find the critical acceptance value and inspected sample size of the sampling plan $(n, c) = (80, 1.1669)$ by solving equations (6) and (7) simultaneously. Furthermore, to determine of threshold for PM, the data simulation will be done using experimental on equation (8) for varied $C_{pk}$ values. Table 1 presents the effect of cost per period on the $C_{pk}$ value of the simulated data.
Table 1 Numerical Experimental Simulation Results

| No | Nilai Cpk | E(H)  |
|----|-----------|-------|
| 1  | 1.6       | 8.562 |
| 2  | 1.5       | 8.215 |
| 3  | 1.4       | 6.153 |
| 4  | 1.3       | 5.187 |
| 5  | 1.2       | 5.048 |
| 6  | 1.1       | 5.023 |
| 7  | 1         | 5.018 |
| 8  | 0.9       | 5.017 |
| 9  | 0.8       | 5.017 |
| 10 | 0.7       | 5.017 |
| 11 | 0.6       | 5.017 |

The graph shows that when a high Cpk value (in the capable range), probability of 100% inspection is very small, so that if the condition is PM and 100% inspection are applied, it will only result in greater cost per period. Therefore, the application of PM will be more effective if done at Cpk < 1.00 because at that point the value of cost expectation per period is minimum and then constant, so to prevent deterioration of quality and fatal machine damage then in this condition PM will be applied. Hence, the inspected samples are taken from the lot randomly and the observed measurements are displayed at Table 2.

Table 2 Sample data of 80 observations (unit: µm)

|     |       |       |     |       |       |       |       |     |       |
|-----|-------|-------|-----|-------|-------|-------|-------|-----|-------|
| 1.28| -5.12 | 6.75  | -7.34| 9.50  | 5.70  | 9.40  | 1.09  | 1.32| -5.59 |
| -4.73| 3.14  | 0.38  | 8.36 | -6.88 | -7.06 | 3.47  | -4.42| 3.34 | 4.55 |
| 2.84| 10.25 | 5.72  | -0.11| 6.59  | -3.31 | -8.18 | 3.71  | 4.38 | 3.25 |
| -4.70| -3.45 | 1.07  | -1.58| 2.45  | 7.02  | -7.28 | 4.48  | 1.28 | -2.54|
| 2.58| -5.98 | 4.50  | 4.66 | -6.75 | 1.19  | -2.11 | -2.34 | -7.46| 5.92 |
| 2.93| -2.44 | -5.51 | 2.63 | 2.04  | -2.19 | 1.40  | -2.53 | -4.14| -1.93|
| 4.93| -0.17 | 9.70  | 3.47 | 4.86  | 1.02  | -2.06 | 2.90  | 5.50 | 1.06 |
| -4.86| 4.75  | 8.25  | 6.12 | 4.63  | -5.15 | 4.11  | 4.90  | -4.74| 4.03 |

From the sample data obtained the average sample of 0.9594 and standard deviation of 4.8352, so it can be calculated Cpk value of:

\[
\hat{C}_{pk} = \frac{d - \left| \mu - M \right|}{3\sigma} = \frac{15 - 0.9594}{3(4.8352)} = 0.9679
\]

Cpk value less than the value of Cpk is 1.1669, it can be concluded that the lot is rejected. Since the lot is rejected, 100% inspection is performed to replace products that are out of specification limits with good products. In the sample data presented in Table 1 it can be seen that as much as 33% of samples are outside the specification limit (marked with red sample data). In addition, the value of Cpk estimates shows less than 1.00. In this case the PM policy will be applied to minimize production of products beyond the specification limits and prevent the occurrence of machine damage.

5. Conclusion and Limitation

In this paper, we have proposed a new approach to the joint of the variable acceptance sampling and the preventive maintenance scheduling, considering an capability process index. The critical value of the acceptance and sample size of a single variable sampling plan based on the process capability index is determined in accordance with process capability requirements, ie producer risk (α-risk), consumer risk (β-risk), C\text{AQL} and C\text{RQL}. Suppose that by using α-risk of 0.05, β-risk of 0.05 and
capability requirement $C_{AQL} = 1.33$ and $C_{RQL} = 1.00$ then 80 sample data will be taken with the critical value of $C_o$ receipt is 1.1669. When the $C_{pk}$ sample value is $< C_o$ then the lot is rejected. Maintenance policy will be taken to avoid the occurrence of engine damage and minimize the production of defective products. From the simulation results, the $C_{pk}$ threshold $<1.00$ is obtained then PM will be applied.

One limitation of our model is we don’t attention to the effect of age and machine realibility level in integrating quality control and maintenance policy. Future research could be conducted to Future research could be conducted to taking into account the influence of age and machine realibility level in integrating quality control and maintenance policy. Moreover, the effect of defective product manufacturing processes needs to be considered and re-identified for further research, both in cost and in terms of inventory.

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