Scaling quark gluon plasma by HBT interferometry with lepton pairs

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We study the intensity interferometry with lepton pairs for nuclear collisions at RHIC and LHC energies. It is argued that the invariant mass dependence of HBT radii extracted from the correlation functions of dilepton pairs can be used as an efficient tool to scale the size and life time of the quark gluon plasma expected to be formed in nuclear collisions at RHIC and LHC. Quantitatively different magnitudes of HBT radii are obtained at RHIC and LHC indicating stronger radial flow at LHC.
1. Introduction

The nuclear collisions at ultra-relativistic energies have been effected at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) to create and study the properties of thermal phase of quarks and gluons which is called quark gluon plasma (QGP). Several observables have been proposed to study the properties of QGP \[1\] - among those the electromagnetic probes - photons and dileptons are advantageous because (i) they are produced at every space-time point of the QGP (ii) they leave the system without rescatterings and hence can deliver the information of the source point without any distortion (\[2\] for review).

It is well known that the electromagnetic probes can be used as thermometer \[3\] as well as a flowmeter too \[4, 5\]. The transverse momentum ($k_T$) distribution of photons and dileptons reflect the temperature of the source. Because their productions from a thermal source depend on the temperature ($T$) of the bath through the thermal phase space factors of the participants of the reaction that produces the photons and dileptons. However, the thermal phase space factor may be changed by several factors - e.g. the transverse kick due to flow received by low $k_T$ photons from the low temperature hadronic phase will mingle with the high $k_T$ photons from the partonic phase, making the task of detecting and characterizing the QGP difficult. For dilepton the situation is, however, different because in this case we have two kinematic variables to describe the spectra - out of these two, the $k_T$ spectra of lepton pairs is affected but the $k_T$ integrated invariant mass ($M$) spectra is unaltered by the flow. Moreover, the $M$ spectra of thermal dileptons is dominated by the late hadronic phase for $M \lesssim m_\rho$ and by the early QGP phase for $M > m_\phi$, where $m_\rho$ and $m_\phi$ are $\rho$ and $\phi$ mesons mass respectively. Therefore, the $M$ spectra of dileptons can be used as a clock for heavy ion collisions - large $M$ corresponds to early time and small $M(\sim m_\rho)$ correlate to large time. This suggests that a judicious choice of $k_T$ and $M$ windows will be very useful to map the temperature and flow of the evolving matter.

2. Bose Einstein Correlation Function for Dileptons

Experimental measurements of two-particle Hanbury-Brown Twiss (HBT) intensity interferometry has been established as an useful tool to study the space-time evolution of the heavy-ion reaction \[6\]. The HBT interferometry with lepton pairs (or with virtual photons which differ form lepton pairs by a known factor) proceeds with the computation of the Bose-Einstein correlation function for two identical lepton pairs defined as,

$$C_2(\vec{k}_1, \vec{k}_2) = \frac{P_2(\vec{k}_1, \vec{k}_2)}{P_1(\vec{k}_1)P_1(\vec{k}_2)} \quad (2.1)$$

where $\vec{k}_i$ is the three momentum of the pair, $i$ and $P_1(\vec{k}_i)$ and $P_2(\vec{k}_1, \vec{k}_2)$ represent the one- and two-particle inclusive lepton pair spectra respectively and is expressed as follows:

$$P_1(\vec{k}) = \int d^4x \, \omega(x,k) \quad (2.2)$$

and

$$P_2(\vec{k}_1, \vec{k}_2) = P_1(\vec{k}_1)P_1(\vec{k}_2) + \frac{\lambda}{3} \int d^4x_1 d^4x_2 \, \omega(x_1,K)\omega(x_2,K) \cos(\Delta x^\mu q_\mu) \quad (2.3)$$
where $K = (k_1 + k_2)/2$, $\Delta k_\mu = k_{1\mu} - k_{2\mu} = q_\mu$, $\Delta x_\mu = x_{1\mu} - x_{2\mu}$. $x_{i\mu}$ and $k_{i\mu}$ are the four co-
ordinates for position and momentum variables respectively. $\omega(x,k)$ is the source function related to the thermal emission of lepton pairs per unit four volume, expressed as follows:

$$\omega(x,k) = \int_{M_1^2}^{M_2^2} dM^2 \frac{dR}{dM^2 d^2 k_T dy}$$  \hfill (2.4)

The inclusion of the spin of the lepton pairs (corresponding to the spin of virtual photon, which is 3) will reduce the value of $C_2 - 1$ by $1/3$. The correlation functions can be evaluated by using Eqs. 2.1, 2.2, 2.3 and 2.4 for different average mass windows, $\langle M \rangle \equiv (M_1 + M_2)/2$. For the low $M$ dilepton production from the hadronic phase the decays of the light vector mesons $\rho$, $\omega$ and $\phi$ have been considered including the continuum [2, 8]. Since the continuum part of the vector meson spectral functions are included, the processes like four pions annihilation [9] are excluded to avoid double counting.

Table 1: Values of the various parameters used in the relativistic hydrodynamical calculations.

| Input | RHIC | LHC |
|-------|------|-----|
| $N_f$ | 2.5  | 3   |
| $T_i$ | 290 MeV | 640 MeV |
| $\tau_i$ | 0.6 fm | 0.1 fm |
| $T_{ch}$ | 170 MeV | 170 MeV |
| $T_{fo}$ | 120 MeV | 120 MeV |
| EoS | 2+1 Lattice QCD | 2+1 Lattice QCD |

For the space time evolution of the system relativistic hydrodynamical model with cylindrical symmetry [10] and boost invariance along the longitudinal direction [11] has been used. The initial temperature ($T_i$) and proper thermalization time ($\tau_i$) of the system is constrained by the hadronic multiplicity ($dN/dy$) through the relation $dN/dy \sim T_i^3 \tau_i$. The equation of state (EoS) which governs the rate of expansion/cooling has been taken from the lattice QCD calculations [12]. The chemical ($T_{ch}$) and kinetic ($T_{fo}$) freeze-out temperatures are fixed by the particle ratios and the slope of the $k_T$ spectra of hadron [13]. The values of these parameters are displayed in Table 1.

With all the ingredients mentioned above we evaluate the correlation function $C_2$ for different invariant mass windows for 0-5% Au+Au collisions with 0-5% centrality at $\sqrt{s_{NN}} = 200$ GeV and Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV as a function of $q_{side}$ and $q_{out}$ which are related to transverse momentum of individual pair as follows (see [4] for details):

$$q_{out} = \frac{(k_{1T}^2 - k_{2T}^2) / f(k_{1T}, k_{2T})}{f(k_{1T}, k_{2T})}$$

$$q_{side} = \frac{(2k_{1T}k_{2T} \sqrt{1 - \cos^2(\psi_1 - \psi_2)}) / f(k_{1T}, k_{2T})}{f(k_{1T}, k_{2T})}$$ \hfill (2.5)

where $f(k_{1T}, k_{2T}) = \sqrt{k_{1T}^2 + k_{2T}^2 + 2k_{1T}k_{2T} \cos(\psi_1 - \psi_2)}$. 


3. The HBT Radii

The dimension of the source can be obtained by parameterizing the calculated correlation function ($C_2$) of the lepton pairs by the empirical (Gaussian) form:

$$C_2 = 1 + \frac{\lambda}{3} \exp(-R_i^2 q_i^2).$$

where the subscript $i$ stand for out and side and $\lambda$ represents the degree of chaoticity of the source. The deviation of $\lambda$ from 1 will indicate the presence of non-chaotic sources. We have evaluated the $C_2$ for $\langle M \rangle = 0.3, 0.5, 0.7, 1.2, 1.6$ and $2.5$ GeV and the HBT radii are extracted by using Eq. 3.1. The $R_{\text{side}}$ scales the transverse dimension and the $R_{\text{out}}$ measures both the transverse size and duration of particle emission [6, 14]. The extracted $R_{\text{side}}$ and $R_{\text{out}}$ (using Eq. 3.1) for different $\langle M \rangle$ are shown in Fig. 1 both for RHIC and LHC energies.

![Figure 1](image_url)

**Figure 1:** The variation of $R_{\text{side}}$ (left panel) and $R_{\text{out}}$ (right panel) with $\langle M \rangle$ for RHIC (dashed line) and LHC (solid line) energies. $R_{\text{side}}$ is evaluated with $k_1T = k_2T = 1$ GeV and $\psi_2 = 0$ and $R_{\text{out}}$ for $\psi_1 = \psi_2 = 0$ and $k_1T = 1$ GeV.

In Fig 1 (left panel) we display the variation of $R_{\text{side}}$ with $M$. It can be shown that the $R_{\text{side}}$ is related to the collective motion of the system through the relation: $R_{\text{side}} \sim 1/(1 + E_{\text{collective}}/E_{\text{thermal}})$. With the onset of transverse expansion a rarefaction wave moves toward the center of the cylindrical geometry - as a consequence the transverse dimension of the emission zone reduces with time. Therefore, the size of the emission region is larger at early times and smaller at late times. The high $\langle M \rangle$ regions are dominated by the early partonic phase where the collective flow [5] is not developed fully i.e. the ratio of collective to thermal energy is small hence show larger $R_{\text{side}}$ of the source. The lepton pairs with $M \sim m_\rho$ are emitted from the late hadronic phase where the size of the emission zone is smaller due to larger collective flow effects giving rise to a smaller $R_{\text{side}}$. The ratio, $E_{\text{collective}}/E_{\text{thermal}}$ is quite large for $M \sim m_\rho$ and hence $R_{\text{side}}$ is small which is reflected as a dip in the variation of $R_{\text{side}}$ with $\langle M \rangle$ around the $\rho$-mass region (Fig. 1 left panel). Thus the variation of $R_{\text{side}}$ with $M$ can be used as a yardstick to scale the size of the evolving matter. The change of $R_{\text{side}}$ with $\langle M \rangle$ for RHIC and LHC is qualitatively similar but quantitatively different. The smaller values of $R_{\text{side}}$ for LHC is due to the larger radial expansion which can be understood from the fact that the quantity $E_{\text{collective}}/E_{\text{thermal}}$ is larger at LHC than RHIC.
The $R_{out}$ probes both the transverse size and the duration of emission. The large $M$ regions are populated by lepton pairs from the partonic phase where the effect of flow is low (size is large) but the duration of emission is small - resulting in a small values of $R_{out}$. Lepton pair with $M \sim m_p$ suffers from larger flow effects which should have resulted in a minimum as in $R_{side}$ in this $M$ region. However, $R_{out}$ probes the duration of emission too, which is large for hadronic phase because of the slower expansion due to softer EoS used in the present work for the hadronic phase. The larger duration compensates the reduction of $R_{out}$ due to flow resulting is a bump in $R_{out}$ for $M \sim m_p$ (Fig. 1 right panel). The $R_{out}$ at LHC is smaller than RHIC because the larger flow (corresponds to smaller size) at LHC compensates other factor (like duration of emission) which has an enhancing effect on $R_{out}$.

The $R_{out}$ and $R_{side}$ are the measures of the average size of the system \[?] and has some dependence on space-time evolution models that is used. However, in the ratio, $R_{out}/R_{side}$ some of the uncertainties associated with this modeling get canceled out. The ratio, $R_{out}/R_{side}$ gives the duration of particle emission \[15, 16, 17\] for various domains of $\langle M \rangle$ corresponding to different time slices.

![Figure 2](image-url)

**Figure 2:** The ratio $R_{out}/R_{side}$ (left panel) and the difference $\sqrt{R_{out}^2 - R_{side}^2}$ (right panel) as a function of $\langle M \rangle$ for RHIC (dashed line) and LHC (solid line) energies is shown.

Fig. 2 depicts change of the $R_{out}/R_{side}$ and $R_{diff}(=\sqrt{R_{out}^2 - R_{side}^2})$ as a function of $\langle M \rangle$ for RHIC as well as LHC energies. Both show a non-monotonic dependence on $\langle M \rangle$. The results reveal that both the ratio and the difference of HBT radii for LHC at low $M$ are larger than the corresponding quantities at RHIC.

Now we indicate below the experimental challenges in performing the HBT interferometry with lepton pairs at LHC (for RHIC see [4]). We compute the number of events from the luminosity ($\mathcal{L}$), the $pp$ in-elastic cross section ($\sigma$) and the run time ($\mathcal{T}$) of the LHC as:

$$N_{\text{event}} = \mathcal{L} \times \sigma \times \mathcal{T}$$  (3.2)

For $\mathcal{T} = 12$ weeks, $\mathcal{L} = 50 \times 10^{27}$(cm$^2$sec) and $\sigma = 60$ mb we get $N_{\text{event}} \sim 2 \times 10^{10}$. As an example for $\langle M \rangle = 500$ MeV and $k_T = 1$ GeV, the value of $(dN/d^2k_Tdy)$ for Pb+Pb collision at $\sqrt{s_{NN}}=2.76$ TeV is $\sim 0.138 \times 10^{-3}$. Therefore, the total (for $2 \times 10^{10}$ number of events) differential number of pairs in the above range of $k_T$ and $M$ is $\sim 2 \times 10^{10} \times 0.138 \times 10^{-3} \sim 2.7 \times 10^6$. 


Similarly for the $\langle M \rangle = 1.02$ GeV and $k_T = 1$ GeV, the total (differential) number of pairs is $2 \times 10^6$. In this domain of $k_T$ and $M$ the number of pairs produced per event is $\sim 10^{-4}$. Therefore, the probability to get two pairs is $10^{-8}$. Therefore, roughly $10^8$ events will be necessary to perform the interferometry with lepton pairs in this region of $k_T$ and $M$.

4. Summary

In summary, the correlation functions for dilepton pairs have been evaluated and the HBT radii have been extracted for both Au+Au collisions at RHIC and Pb+Pb collision at LHC energies for different $\langle M \rangle$ windows. We argue that the variation of HBT radii with $M$ for dilepton pairs can be used as an efficient tool to follow the change of the dimension of the evolving system with time. The quantitative difference in the HBT radii at RHIC and LHC indicate the development of larger flow at LHC compared to RHIC.

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