Computational Fatigue Life Analysis of Carbon Fiber Laminate

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Abstract: In the present scenario, many traditional materials are being replaced by composite materials for its light weight and high strength properties. Industries like automotive industry, aerospace industry etc., are some of the examples which uses composite materials for most of its components. Replacing of components which are subjected to static load or impact load are less challenging compared to components which are subjected to dynamic loading. Replacing the components made up of composite materials demands many stages of parametric study. One such parametric study is the fatigue analysis of composite material. This paper focuses on the fatigue life analysis of the composite material by using computational techniques. A composite plate is considered for the study which has a hole at the center. The analysis is carried on (0°/90°/90°/90°/90°)s laminate sequence and (45°/−45°)2s laminate sequence by using a computer script. The life cycles for both the lay-up sequence are compared with each other. It is observed that, for the same material and geometry of the component, cross ply laminates show better fatigue life than that of angled ply laminates.

Keywords: Infinite life, Finite life, Computational techniques, Cross-ply laminates, Angled-ply laminates

1. Introduction
In today’s world, many Mechanical systems involve a complex interaction of different types of loads, geometry, environmental conditions, manufacturing processes and type of material. Most of the engineering components undergo failure depending upon the dynamic loads acting on them as well as the type of the material selected.

Fatigue is the process where the material undergoes repeated loading and unloading, due to which the life of the component gradually decreases. The nature of the load is cyclic and the localized damage starts initially at a particular point in the material. Fatigue failure occurs in a component when the component no longer has the ability to withstand high loads. Along with the loading conditions, materials also play an important role in determining the life of the component.

The properties like strength, stiffness, weight etc. decide the behavior of the material for the given application. The advent of composite materials back in 1980’s, led to the increase in its use in various applications like aerospace, automobile sectors, marine applications, railways, turbines, etc. Composite material is defined as the arrangement of two or more materials that results in good physical properties than those of the individual components alone.

In the present paper, the main focus is given on analyzing the fatigue behavior of the composite materials by computational techniques. Experimentation may need many inputs, as well as time and it is more expensive. The computational techniques are used to get the results in short duration of time and is more economical.

Putic et al [1] studied carbon fiber reinforced composites based on high frequency fatigue. Authors have considered ±45° angle ply and 0°/90° cross ply notched and un-notched laminates for the study and the comparison is made. Spearing et al [2] developed new approach for post fatigue modelling for strength of notched composite laminates. In the paper, author have applied tensile cyclic load for the purpose of study from which the notch tip damage zone is analyzed for stable growth. Spearing et al [3] developed a fatigue model to find out the notch tip damage growth of carbon fiber epoxy laminate. In this paper, various forms of damage viz. delamination, transverse ply cracking and splitting in composites is
studied. Spearing et al [4] predicted the post fatigue strength using the proposed model. Using FE techniques representation of notch tip damage is made from which a modified stress distribution for damage is obtained. Spearing et al [5] validated the experimental and theoretical results which were obtained in the previous series of paper. The degraded strength properties are deduced from the results obtained. Fatemi et al [6] investigated and studied thermoplastic composites of two short fibers to find their fatigue behavior and cyclic deformation subjected to various loads. Sosa et al [7] deals with computer methods applied to fiber reinforced composites based on a polymeric matrix and many of them are applicable to a border range of composite materials as well as other anisotropic materials. Hutson [8] has made an attempt to match the results to two of the models. Residual strength and residual stiffness models are tested by making use of the results obtained from repeated tension fatigue tests on unidirectional carbon fiber reinforced epoxy. Lessard et al [9] established the modelling technique for the simulation of fatigue behavior of composite laminates by considering stress concentration and without considering stress concentration called progressive fatigue damage modelling. Mahadevan et al [10] developed a model for damage advancement of composites. The authors have calculated the features of damage advancement in composites and compared them with the damage growth of homogeneous materials. Phillips [11] measured the critical stress intensity factor and fracture surface energies for carbon fibre and glass fibre reinforced composites. Griffith-Irwin criterion are used. Sung et al [12] considered graphite epoxy for the study based on the current Micro Mechanics of Failure (MMF) and both tensile and compressive failure occur through matrix failure. Soutis et al [13] presented an investigation into the compressive fracture properties of carbon fiber epoxy laminate. In this paper, the study is carried out to find the un-notched and notched strengths for different lay-ups. Degriesck et al [14] presents a review of major fatigue models and methodologies for life time prediction for fiber reinforced polymer laminates those are subjected to fatigue loadings. Here fatigue models are categorized into three major models. Min et al [15] developed a micromechanics analysis modelling method for brittle ceramic material to evaluate damage progression of composite structures. A global composite structure is represented by making use of repeated unit cell concept. Brighenti et al [16] has analysed the partially de-bonded fibre as a 3D mixed mode fracture problem for which the assessment is made through fatigue crack propagation laws. Sutcliffe et al [17] showed the theoretical and experimental results for compressive failure for carbon fiber. Spearing et al [18] presented a model for post fatigue strength in notched, cross-ply carbon fiber PEEK laminates. It is found that the behavior of carbon fiber or polyetheretherketone is similar for the reduction of delamination and matrix cracking. Gururaja et al [19] prepared the specimens with different ply orientation of glass/carbon hybrid with epoxy as adhesive. They adopted vacuum bagging technique for fabrication. Genin et al [20] developed plane stress constitutive relations for composite laminates which are undergoing matrix cracking. In this study, the illustration is made to find the effect of non-linear behavior of stress and strain around notches. Shivakumar et al [21] investigated Mode-I fracture behaviour of glass-carbon fibre. In this study, the compact tension (CT) specimen is employed to conduct fracture test. Talreja [22] presented a systematic classification of effects of transverse cracking on the stress-strain data of composite laminate. Using the stiffness damage relationships, stiffness reduction resulting from transverse cracking from crack initiation to crack saturation is predicted. Andre et al [23] used FE method to investigate delamination in composites of notched laminates under flexural loads.

Based on the literature survey carried out, it is observed that there are many unsolved difficulties and very few improved life prediction models are currently available. Computational script files are not available abundantly. Time is another main factor which should be considered for the study purpose. It is necessary to develop a computerized script for the easy evaluation of results and to finish the work in minimum time available. Hence, more studies are focused to learn and study the different fatigue life models. This paper focuses on the fatigue life analysis of the composite material i.e., carbon fiber laminate by using computational techniques.
2. Strength Analysis of Composite Materials

The main focus of this section is to predict the material constants of composite material by studying the micromechanics of the problem, i.e. by studying how the matrix and fibres interact. Computing the stresses within the matrix, fibres and fibre-matrix interface is very important for understanding some of the underlying failure mechanisms. It is seen that, variation in fibre volume fraction and matrix volume fraction will contribute to the overall strength of the material. “The ratio of the cross-sectional area of the fibre to the total cross-sectional area of the unit cell is called the fibre volume fraction” [24]. Similarly, “The ratio of the cross-sectional area of the matrix to the total cross-sectional area of the unit cell is called the matrix volume fraction” [24].

2.1. Strength Analysis of Composite Materials based on Fibre Volume Fraction

When the fiber volume fraction is varied, the change in different parameters like Young’s modulus, shear modulus and Poisson’s ratio is studied and the plots are plotted. A carbon fiber ply is considered for the analysis purpose using the relations given in Eqn. (1), Eqn. (2), Eqn. (3) and Eqn. (4). The variation of longitudinal modulus, transverse modulus, shear modulus and Poisson’s ratio is shown in Figure 1, Figure 2, Figure 3 and Figure 4 respectively. A MatLab code is generated to study the variations in different parameters.

\[
E_L = E_f V_f + E_m (1 - V_f) = E_{11} \\
E_T = \frac{E_m V_f + E_f (1 - V_f)}{G_f G_m} = E_{22} \\
G_{12} = G_{13} = \frac{G_f G_m + G_f (1 - V_f)}{G_f G_m} \\
\nu_{12} = \nu_{13} = \nu_f V_f + \nu_m (1 - V_f)
\]

Figure 1. Variation of \(E_1\) vs. \(V_f\)  
Figure 2. Variation of \(E_2\) vs. \(V_f\)

In Figure 1 and 2, the variation of Young’s modulus in longitudinal and transverse direction with fiber volume fraction is shown respectively. It is observed from the graph that, the Young’s modulus in the loading direction increases linearly when the fiber volume fraction increases and the Young’s modulus in the transverse direction increases exponentially with fiber volume fraction.
In Figure 3 and 4, the variation of shear modulus and Poisson’s ratio with fiber volume fraction is shown respectively. It is observed from the graph that, the shear modulus is increasing exponentially with increase in the fiber volume fraction and the Poisson’s ratio is decreasing linearly with increase in fiber volume fraction.

**Figure 3.** Variation of $G12$ vs. $V_f$

**Figure 4.** Variation of $NU12$ vs. $V_f$

### 2.2. Strength Analysis of Composite Materials based on Fibre Orientation

Apart from the variation in fiber volume fraction, the fiber orientation will play a vital role and contributes to different parameters of the composite material and hence to the strength of the material. Previously, the evaluation of engineering constants based on the local co-ordinate system is presented. It is possible to define elastic constants with respect to global co-ordinate system also. When the fiber orientation is varied, it contributes to the change in various material properties like, Young’s modulus and shear modulus. A carbon fiber ply is considered for the analysis purpose using the relations given in Eqn. (5), Eqn. (6), Eqn. (7) and Eqn. (8). The variation of longitudinal modulus, transverse modulus, shear modulus and Poisson’s ratio is shown in Figure 5, Figure 6, Figure 7 and Figure 8 respectively.

\[
E_x = \frac{E_1}{m^4 + \left(\frac{E_1}{G_{12}} - 2\theta_{12}\right)n^2m^2 + \frac{E_1}{E_2}n^4} \\
E_y = \frac{E_2}{m^4 + \left(\frac{E_2}{G_{12}} - 2\theta_{21}\right)n^2m^2 + \frac{E_2}{E_1}n^4} \\
G_{xy} = \frac{G_{12}}{n^4 + m^4 + 2\left(\frac{2G_{12}}{E_1}(1 + 2\theta_{12}) + \frac{2G_{12}}{E_2} - 1\right)n^2m^2}
\]

Figure 5 and 6 shows the variation of Young’s modulus in longitudinal and transverse direction with fiber orientation respectively. It is observed that, Young’s modulus in the longitudinal direction is maximum for $0^\circ$ fiber orientation whereas it is minimum for $+90^\circ$ and $-90^\circ$ fiber orientation and Young’s modulus in the transverse direction is maximum for $+90^\circ$ and $-90^\circ$ fiber orientation whereas it is minimum for $0^\circ$ fiber orientation in both positive and negative direction.
Figure 7 and 8 show the variation of shear modulus and Poisson’s ratio with fiber orientation angle respectively. It is observed that, shear modulus is gradually increasing from minimum value for 0° fiber orientation and reaching maximum for +45° and -45° fiber orientation. It is again decreasing to a minimum value for +90° and -90° fiber orientation. On the other hand, it is noted from the graph that the value of $\nu_{yx}$ is maximum at 0° fiber angle and is minimum at ±90° fiber orientation.

![Variation of Ex vs. $\theta$](image1)

![Variation of Ey vs. $\theta$](image2)

![Variation of Gxy vs. $\theta$](image3)

![Variation of NUxy vs. $\theta$](image4)

3. Results and Discussion
The significance of stress concentrators, such as notches on crack propagation has long been recognized in the design of metallic components. In the monotonic loading of laminated composites, notches have a greater effect on the growth of damage zones. In this section, the fatigue life of a carbon fiber plate with dimensions $(200 \times 100 \times 0.2\,\text{mm})$ is analyzed using numerical integration method. The plate has a hole at the center of 40 mm diameter. The fatigue crack growth curve is extracted [1]. This curve is used to find out the values of constant in the Paris’s equation. Regression analysis is carried out to find the exact values of constants. Two composite lay-up sequences are used for the study. One is $(0°/90°/90°/90°/90°)$$_s$ laminate sequence and the other one is $(45°/-45°)_{2s}$ laminate sequence. The crack growth is studied for both the lay-up sequences and the life prediction is made for the same. The obtained results for two laminate sequences are compared with each other. Fibre fraction of 65% is
considered for each ply. A ply of thickness 0.2 mm each is taken for the study. A tensile monotonic load is applied on both the sides of the laminate and the fatigue crack growth is plotted for both the lay-up sequence and the life is compared.

3.1. Crack Propagation and Fatigue Life Prediction in the Model
In the model shown in Figure 9, the crack propagation rate depends on stress intensity factor $\Delta k$ and stress ratio $R$.

Paris’s law is generally used for the purpose of predicting the life of the component and is stated as,

$$\frac{da}{dN} = c(\Delta k)^m$$

where, $c$ and $m$ are material constants. Further, the crack propagation life may be expressed by substituting the value of $\Delta k$ in the Paris’s equation.

The relation obtained is,

$$N = \frac{a_o\left(\frac{m}{2}+1\right) - a_n\left(\frac{m}{2}+1\right)}{(m/2-1)\times c \times f\left(\frac{a_o}{W}\right)\left(\Delta k\right)^m\pi \frac{m}{2}}$$

Eqn. (9) is the final expression to calculate the value of life of the component for different stress levels. The constant values $m$ and $c$ are obtained from Paris’s equation.

3.2. Regression Analysis
To find out the values of constants $m$ and $c$, regression analysis is done by considering the concentrated points of Figure 10. Regression analysis is a process of estimating relationships between variables. It is used to predict the value of target variable. Regression analysis is also used to understand which among the independent variables are related to the dependent variable and to explore the forms of these relationships.

If a relation is considered as given in Eqn. (10),

$$\log\left(\frac{da}{dN}\right) = \log(c) + m \times \log(\Delta k)$$

Then the formulae to find the values of two constants are,

$$\log(c) = \frac{(\Sigma da/dN)(\Sigma(\Delta k)^2) - (\Sigma(\Delta k))(\Sigma(\Delta k \times da/dN))}{n(\Sigma(\Delta k)^2) - (\Sigma(\Delta k))^2}$$

$$m = \frac{n(\Sigma(\Delta k \times da/dN)) - (\Sigma(\Delta k))(\Sigma da/dN)}{n(\Sigma(\Delta k)^2) - (\Sigma(\Delta k))^2}$$

Figure 9. Plate with central hole
For \((0°/90°/90°/90°)\) laminate sequence, the evaluated values of \(m\) and \(c\) are,
\[
m = 4.867 \text{ and } c = 2.29 \times 10^{-13}
\]

Figure 10. Fatigue Crack Growth Curve [1]

Similarly, for \((45°/−45°)_{2S}\) laminate sequence, the evaluated values of \(m\) and \(c\) are,
\[
m = 5.84 \text{ and } c = 3.55 \times 10^{-13}
\]

3.3. Fatigue Life of \((0°/90°/90°/90°/90°)\)_s and \((45°/−45°)_{2S}\) laminate
10 plies of 0.2 mm thickness each are stacked up together and the computational analysis is carried out. When the crack growth increases, the number of cycles to failure also increases correspondingly. Figure 11 shows the graph of number of cycles to failure verses crack growth for \((0°/90°/90°/90°/90°)\)_s laminate sequence for the stress levels varying from 20 MPa to 100 MPa. It is observed that, for the maximum value of stress i.e., at 100 MPa, the life of the component is \(4.7 \times 10^3\) cycles. Similarly, for the minimum value of stress i.e., at 20 MPa, the life of the component is \(1.2 \times 10^7\) cycles.

Figure 11. Fatigue crack growth verses Number of cycles

It is clearly noticed that at higher stress level, the number of cycles to failure is very less i.e., carbon fibre material will fail very early, at approximately \(10^3\) cycles. At lower stress levels the carbon fibre material will fail after large number of cycles i.e., at approximately \(10^7\) cycles. It is inferred that carbon fibre has infinite life at lower stress amplitudes for lay-up sequence of \((0°/90°/90°/90°/90°)\)_s.
Eight plies of 0.2 mm thickness are stacked up together to form a laminate for $(45^\circ/-45^\circ)_{2s}$ laminate and MatLab codes are generated to carry out the computational analysis. Figure 12 shows the graph of number of cycles to failure verses crack growth for $(45^\circ/-45^\circ)_{2s}$ laminate sequence for the stress levels varying from 20 MPa to 100 MPa. It is observed that, for the maximum value of stress i.e., at 100 MPa, the life of the component is $1.3 \times 10^2$ cycles. Similarly, for the minimum value of stress i.e., at 20 MPa, the life of the component is $1.3 \times 10^6$ cycles.

![Figure 12. Fatigue crack growth verses Number of cycles](image)

It is clearly noticed that at higher stress level, the number of cycles to failure is very less i.e., carbon fibre material will fail very early, at approximately $10^2$ cycles. At lower stress levels the carbon fibre material will fail after large number of cycles i.e., at approximately $10^6$ cycles. It is inferred that carbon fibre has finite life at lower stress amplitudes for the lay-up sequence of $(45^\circ/-45^\circ)_{2s}$.

![Figure 13. Life of Carbon Fibre for varied Stress Amplitudes](image)

Comparison is made between carbon fibre composites of $(0^\circ/90^\circ/90^\circ/90^\circ/90^\circ)$ lay-up and $(45^\circ/-45^\circ)_{2s}$ lay-up. Figure 11 and Figure 12 shows that, cross-ply laminate possesses infinite life whereas angled ply laminate possesses finite life. It is inferred that cross-ply laminate sequence has higher fatigue life than that of angled ply laminate sequence. In Figure 13, clear comparison is shown between angled ply laminate and cross-ply laminate for various stress levels.
4. Conclusion
Carbon fiber is the material used for the study where two lay-up sequences are used and compared with each other. Fibre fraction of 65% and a ply of thickness 0.2 mm each is considered for the purpose of study. The final life of carbon fibre laminate is determined and the curve is plotted for varied stress levels. It is inferred that, for lower stress value of 20 MPa, the cross-ply laminate fails at $1.2 \times 10^7$ cycles and angle-ply laminate fails at $1.2 \times 10^6$ cycles. For higher stress value of 100 MPa, the cross-ply laminate fails at $4.7 \times 10^3$ cycles and angle-ply laminate fails at $1.3 \times 10^2$ cycles. The time consumption was very less for the simulation done.

5. References
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