New physics scales and anomalous magnetic moment

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Abstract

Violation of chiral symmetry together with change of mass sign allows a linear correction in inverse power of new physics scale to the anomalous magnetic moment of muon. In this light we analyse alternative models showing in particular that grand unification, supersymmetry and muon substructure may explain the discrepancy between the experimental value of the muon anomaly and the standard model calculation.

1 Introduction

The recent BNL measurement of the anomalous magnetic moment of the muon $a_\mu = \frac{g-2}{2}$ has confirmed an excess on the calculation of standard model contributions of 
\[ \Delta a_\mu \simeq 2 \times 10^{-9} \]
(1)
at a 2σ level.

This is an indication of possible new physics at an energy scale Λ. It is interesting to estimate the order of the correction of $a_\mu$ in powers of $\frac{m_\mu}{\Lambda}$, where $m_\mu$ is the muon mass. This is related to the validity or breaking of the chiral symmetry of leptons together with the change of sign of $m_\mu$. If this symmetry, which we will call of Weinberg, is respected $\Delta a_\mu \sim (m_\mu/\Lambda)^2$ whereas if it is broken $\Delta a_\mu \sim m_\mu/\Lambda$. This is important because in the latter case the explanation of Eq. (1) may be given by new physics at a relatively high energy whereas in the former it should appear at a scale close to the electroweak one.

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Weinberg symmetry and Standard Model  

Since the mass term breaks chiral symmetry, one can make it invariant changing the mass sign

\[ \mu \rightarrow \gamma_5 \mu, \quad m_\mu \rightarrow -m_\mu. \]  

(2)

The effective interaction for the anomalous magnetic moment has a similar chiral form

\[ \mathcal{L}_{\text{eff}} = a_\mu \frac{e}{4m_\mu} \pi \sigma^{\alpha\beta} \mu F_{\alpha\beta}, \]  

(3)

so that if the symmetry Eq. (2) is valid the corrections to \( a_\mu \) must be of even powers of the ratio of \( m_\mu \) to a larger scale \( \Lambda \)

\[ a_\mu = c_o \left( \frac{m_\mu}{\Lambda} \right)^0 + c_2 \left( \frac{m_\mu}{\Lambda} \right)^2 + \ldots. \]  

(4)

This is what happens in the Standard Model (SM) since, to be invariant in it, Eq. (3) must be written as

\[ \mathcal{L}_{\text{eff}} = a_\mu \frac{e}{4m_\mu} \left( \bar{l}_L \sigma^{\alpha\beta} \mu_R \frac{f \varphi_V}{m_\mu} + \text{h.c.} \right) F_{\alpha\beta}, \]  

(5)

with a doublet Higgs field \( \varphi = \varphi_V + \left( 0, \frac{v}{\sqrt{2}} \right) \) such that

\[ \bar{l}_L = \left( \begin{array}{c} \nu_L \\ \mu_L \end{array} \right), \quad \varphi_V = \left( \begin{array}{c} 0 \\ \frac{v}{\sqrt{2}} \end{array} \right), \quad f \frac{v}{\sqrt{2}} = m_\mu. \]  

(6)

To have the transformations Eq. (2) one must perform

\[ l_L \rightarrow \gamma_5 l_L = -l_L, \quad \mu_R \rightarrow \gamma_5 \mu_R = \mu_R, \quad \varphi \rightarrow -\varphi. \]  

(7)

The Weinberg symmetry (WS) is respected in SM since the charged weak interactions

\[ \bar{\nu}_L \gamma^\alpha \mu_L W_\alpha, \]

neutral and electromagnetic

\[ \bar{\nu}_L \gamma^\alpha \left( g_v + g_a \gamma^5 \right) \mu_Z, \quad \bar{\nu}_L \gamma^\alpha \mu A_\alpha \]

and Yukawa one

\[ \bar{\nu}_\mu h, \]

are invariant under Eq. (7).

Therefore the corrections to \( a_\mu \) are of the type of Eq. (4) with the electroweak scale \( \Lambda_{\text{EW}} \), corresponding the first term to the electromagnetic contributions \( c_o = \frac{\alpha}{2\pi} + \ldots \) and the second to the weak ones \(~ 10^{-9} \).
3 Grand Unification Theories

In principle, if there is a second Higgs doublet \( \varphi' \) which breaks at a scale \( \Lambda \) higher than that of SM and is not directly related to the muon mass, it is possible to have an additional effective interaction of new physics

\[
L_{\text{eff}} \sim \frac{1}{\Lambda^2} (\bar{L}_L \sigma^{\alpha\beta} \mu_R \varphi' V + \text{h.c.}) F_{\alpha\beta}.
\]

(8)

Being \( \varphi' \sim \Lambda \), Eq. (8) is of the form

\[
L_{\text{eff}} = \Delta a_\mu \frac{e}{4m_\mu} \bar{\mu} \sigma^{\alpha\beta} \mu F_{\alpha\beta}
\]

(9)

with \( \Delta a_\mu = O \left( \frac{m_\mu}{\Lambda} \right) \) which breaks the Weinberg symmetry.

Regarding Grand Unification Theories(GUT) it is not obvious that a particular model has interactions which violate this symmetry and therefore produce an effective interaction like Eq. (8).

E.g. starting with SU(5), the only new interaction which contributes to \( a_\mu \) is that due to leptoquarks \( X^\alpha \) i.e. \( \bar{\nu}\gamma^\alpha q X_\alpha \) which changes muon into quarks but respects chiral symmetry \( \mu \rightarrow \gamma_5 \mu \), \( q \rightarrow \gamma_5 q \). Since the Yukawa interaction is unchanged because the Higgs which breaks SU(5) does not affect the muon mass, the WS is still valid, the effective interaction is of the type of Eq. (5) and

\[
\Delta a_{\mu}^{SU(5)} \sim \left( \frac{m_\mu}{\Lambda} \right)^2,
\]

(10)

which is negligible for the GUT scale \( \sim 10^{15} \text{GeV} \).

In the case of SO(10), the only change is that the high-scale Higgs \( \varphi' \) gives mass to \( \nu_R \) with a Majorana term. Then the low scale Higgs \( \varphi \) gives also to \( \nu \) a Dirac mass. The diagonalization of the mass matrix produces the see-saw mechanism which involves a mixture of \( \nu_L \) and \( \nu_R \). But this only means that also the charged weak interactions have a small right-lepton contribution. Since \( \bar{\nu}\gamma^\alpha \mu W_\alpha \) is chiral invariant and the Yukawa muon term \( \bar{\mu} \mu h \) is unchanged, the WS is preserved and again

\[
\Delta a_{\mu}^{SO(10)} \sim \left( \frac{m_\mu}{\Lambda} \right)^2.
\]

The substantial difference of the exceptional group \( E_6 \) is that it has 11 additional superheavy fermions among which a charged lepton \( M \) that can mix with \( \mu \).
If the breakings of symmetry are due to a 351 of $E_6$, when GUT is broken the mass eigenstates $\mu_o$ (massless) and $\hat{M}$ (superheavy) are determined by the expectation values of the ($SO(10), SU(5)$) multiplets $\varphi'$ (54, 24) and $\varphi''$(144, 24) through a large mixture of left components

$$
\mu^o_L = \mu_L \cos \theta_L + M_L \sin \theta_L, \quad \hat{M}_L = M_L \cos \theta_L - \mu_L \sin \theta_L
$$

with the same mixing of $\nu_L$ and the neutral exotic lepton $N_L$.

The small mass of ordinary muon is due to the appearance of an expectation value of a Higgs $H(10, 5)$ which in terms of the doublet $\varphi$ gives

$$
\mathcal{T}_{LR} f \varphi + h.c. = \frac{f}{\sqrt{2}} (v + h) (\overline{\varphi_R} \mu_L + h.c.).
$$

(13)

Since the right components were not mixed at the GUT scale, the mass term from Eq. (13) is

$$
\frac{f}{\sqrt{2}} v (\overline{\varphi_R} \hat{M}_L \sin \theta_L + h.c.).
$$

Diagonalizing the whole mass matrix to give the physical state $\hat{\mu}$ there will be also a mixture of $\mu^o_R$ with $\hat{M}_R$ which will be small due to the very different GUT and EW scales, but with a relevant contribution to the muon mass. Therefore it will not be possible to argue that the transformation $\varphi \rightarrow -\varphi$ will assure the change $m_\mu \rightarrow -m_\mu$.

Regarding the interaction with the light Higgs $h$, there will be a “flavour-changing” term approximately equal to

$$
\mathcal{L}_{eff} = -\frac{f h}{\sqrt{2}} \left(\overline{\mu_R} \hat{M}_L \sin \theta_L + h.c.\right),
$$

(14)

due to the slight difference between $\mu^o$ and the physical state $\hat{\mu}$. This interaction does not respect WS because even though it is invariant under

$$
\hat{\mu} \rightarrow \gamma_5 \hat{\mu}, \quad \hat{M} \rightarrow \gamma_5 \hat{M}, \quad h \rightarrow -h,
$$

(15)
as said above the last transformation does not imply $m_\mu \rightarrow -m_\mu$. As a consequence one may expect a linear correction of the muon magnetic moment. In fact the explicit calculation of the one-loop contribution caused by Eq. (14) gives

$$
\Delta a^{FCh}_\mu \simeq \frac{1}{16\pi^2} \kappa^2 \frac{m_\mu}{M_M},
$$

(16)
where, being \( \kappa = \sqrt{2} f \sin \theta_L \lesssim 1 \) and here \( f \) not necessarily as small as \( \sqrt{2 m_\mu} \),
to explain the discrepancy Eq. (1) one needs \( M_M \lesssim 10^6 \text{ GeV} \). Even though this mass value seems small for a GUT particle, it is not unreasonable considering the strong mixture of exotic and ordinary fermions. The interaction Eq.(14) we have deduced from a particular scheme of mixture of muon with exotic lepton of \( E_6 \) is equivalent to the one of a singlet charged heavy lepton with the muonic and light Higgs doublets \( \tilde{H} \).

The mixture of \( \mu \) and \( M \) produces additional corrections to \( a_\mu \) of electroweak type. If the scheme of breakings is based on the multiplet 351 as said above, the equal mixings for \( \mu_L \) and \( \nu_L \) avoid any correction in the charged current interaction. The same happens for the neutral charge interaction with \( Z \) if only the left mixtures are considered. But if the small right mixture is included, a coupling \( \bar{\mu} \gamma^a \tilde{M} Z \alpha \) appears which is however chiral invariant and gives

\[
\Delta a_\mu^{FCZ} \simeq \frac{\alpha}{\pi} (\sin \theta_R \cos \theta_R)^2 \left( \frac{m_\mu}{M_Z} \right)^2,
\]

where \( (\sin \theta_R \cos \theta_R)^2 < 10^{-2} \) not to spoil the experimental \( \mu \mu Z \) coupling \( \tilde{F} \), so that \( \Delta a_\mu^{FCZ} \lesssim 10^{-11} \) i.e. irrelevant.

For the correction given by a loop with exchange of a heavy neutral boson \( Z^\prime \), with different charges of the corresponding abelian group for \( \mu \) and \( M \), there will be a “flavour changing” contribution

\[
\Delta a_\mu^{FCZ'} \simeq \frac{\alpha}{\pi} (\sin \theta_L \cos \theta_L)^2 \left( \frac{m_\mu}{M_{Z'}} \right)^2 + \frac{\alpha}{\pi} \sin \theta_L \cos \theta_L \sin \theta_R \cos \theta_R \frac{m_\mu}{M_{Z'}} \frac{m_\mu}{M_{Z'}} \tag{18}
\]

where the first chiral conserving term depends only on the large left mixture and is quadratic in the new physics scale, whereas the second chiral interference term needs the small right mixture reflecting the two-scales building of mass states and is linear in \( m_\mu/M_{Z'} \).

But, compared with Eq. (16), even if \( M_{Z'} \cdot \) is so low as \( 10^6 \text{GeV} \) this linear term gives again \( \Delta a_\mu \lesssim 10^{-11} \) i.e. negligible.

### 4 Other theories beyond the Standard Model

One of the most obvious is the Minimal Supersymmetric Standard Model (MSSM) which has two Higgs doublets with a ratio of expectation values \( \tan \beta = \frac{\langle H \rangle}{\langle H' \rangle} \).
where $H$ gives mass to electron and quark $d$, and $H'$ to quark $u$.

The interaction of muon with the chargino $\tilde{W}$, $\mu\tilde{W}\tilde{\nu}$, in a way similar to Eq. (18), gives the correction of $a_\mu$

$$\Delta a_\mu^{\tilde{W}} \sim \frac{\alpha}{\pi} \left( \frac{m_\mu}{M_W} \right)^2 + \frac{\alpha}{\pi} \frac{m_\mu}{M_W} O \left( \frac{m_\mu \tan \beta}{M_W} \right),$$

(19)

for large $\tan \beta$, where the first quadratic term corresponds to conservation of chirality and the second linear one is due to the chiral violation in internal line caused by the coupling of the two Higgs.

An analogous correction $\Delta a_\mu^{\tilde{Z}}$ comes from the coupling $\mu\tilde{Z}\tilde{\mu}$ replacing in Eq. (19) $M_W$ by $M_Z$. Since one expects $M_W \sim M_Z \sim 1$ TeV, it is possible that these two chiral violating contributions add to $\Delta a_\mu^{SUSY} \sim 10^{-9}$.

The hypothesis of a large extra dimension, and a simple Higgs doublet, gives way to a strong gravitation exchange producing for a relevant number of Kaluza-Klein states

$$\mathcal{L}_{\text{eff}}^{ED} \sim \left( \frac{m_\mu}{\Lambda} \right)^2 \frac{e}{4m_\mu} \sigma^{\alpha\beta} F_{\alpha\beta},$$

(20)

that to satisfy the discrepancy Eq. (1) would require $\Lambda \sim$ TeV which seems excluded by astrophysical observations. An analogous contribution may come from the exchange of an antisymmetric tensor field encountered in string theory.

It is interesting that a model of breaking of SUSY through boundary conditions in a 5th dimension of radius $R \sim$ TeV$^{-1}$ allows to use only one Higgs doublet which would modify the contribution to $a_\mu$ given by Eq. (19).

The possibility of a new abelian gauge symmetry felt by muon and tauon but not by electron would give an additional quadratic contribution to $a_\mu$ similar to that of the exchange of $Z$ of the second term of Eq. (4) which might be $\sim 10^{-9}$. Mixing of the two neutral gauge bosons may produce a linear correction of $a_\mu$.

Finally, the models of substructure of muons are different in the sense that they do not correspond simply to add new particles to those of the SM. Their effect to $a_\mu$ depends on the model assumptions.

One reasoning is that since the terms of mass and anomalous magnetic moment of muon have the same chiral structure, the scale of substructure should enter into them as $\Lambda \mu\mu$ and $\frac{\epsilon}{\Lambda} \sigma^{\alpha\beta} F_{\alpha\beta}$, respectively. One would expect that in the same way as a decreasing factor $\frac{m_\mu}{\Lambda}$ perhaps related to a chiral
symmetry must be introduced in the former, a similar one in the latter should give 
\( \Delta a_\mu \sim \left( \frac{m_\mu}{\Lambda} \right)^2 \).

However, a specific model \[16\] which assumes the appearance at a scale \( \Lambda \) of a Yukawa-type vertex \( \mu^- \rightarrow \pi^- N \), with a heavy neutrino \( N \), gives a linear correction 
\( \Delta a_\mu \sim \frac{m_\mu}{\Lambda} \) since the WS is violated because the pion field has nothing to do with \( m_\mu \).

5 Conclusion

We have seen that interactions which break chiral symmetry together with change of sign of muon mass favour the contribution of new physics to the anomalous magnetic moment \( a_\mu \). In this way, parameter regions of MSSM, additional neutral gauge interactions, particular models of muon substructure, or even a GUT alternative of strong mixing of ordinary left lepton doublet with the exotic one of \( E_6 \) together with a small mixture of the corresponding right charged component, may explain the discrepancy between the recent experimental measurement of \( a_\mu \) and its calculation within Standard Model. It is obviously too soon to say that the need of physics beyond the SM has been proved since the above discrepancy is so far only a \( 2\sigma \) effect.

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