Chaotic inflation with running nonminimal coupling

Toshifumi Futamase
Astronomical Institute, Tohoku University, Sendai, 980-77 Japan
Masahiro Tanaka
Department of Physics, Ochanomizu University, Tokyo, 112 Japan

Abstract

We have found a successful model of chaotic inflation with an inflaton coupled nonminimally with gravity. The nonminimal coupling constant $\xi$ runs with the evolution of the inflaton. The running nature of the coupling leads naturally to the situations where the coupling becomes small enough to have sufficient period of the inflation to resolve the cosmological puzzles.

1 Introduction

Chaotic inflation\cite{1,2} is known to be a successful scenario of inflation. Although its setup is regarded as simpler than other scenarios, it still needs fine-tuning of some parameters as others do. The major two are the inflaton self-coupling constant\cite{2}, and the nonminimal coupling constant to gravity\cite{3,4,5,6}.

As far as the former is concerned, we may identify the direction which contains only quadratic terms, e.g. mass term, with inflaton and then we can avoid the fine-tuning\cite{7}. The direction is called flat direction\cite{1} which frequently appears in supersymmetric (SUSY) models. Interactions still exist in the theory but such terms do not appear along flat direction.

Unfortunately so far no mechanism is known to avoid the latter fine-tuning. Namely we have the freedom to specify how the inflaton couples with gravity, namely how to choose the nonminimal coupling constant. The nonminimal coupling with gravity leads to a soft SUSY breaking\cite{7} and is needed for renormalization. In the case of positive nonminimal coupling constant it has been shown that the chaotic

\footnote{We mean the direction without interaction terms by flat direction. We assume that non-renormalizable terms are also vanishing because the inflaton will be the scalar field with the most flat potential.}
inflationary scenario does not work unless it is sufficiently small[3, 4, 5]. On the other hand, the scenario does work in the case of negative coupling constant. In particular Fakir and Unruh[6] show that a model with a large negative nonminimal coupling constant gives the appropriate amplitude of density perturbation without making any fine-tuning for the self-coupling.

However, the previous studies on the chaotic inflation scenario with nonminimal coupling are restricted to the classical treatment. The aim of this paper is to take into account of the quantum effect on the coupling which makes the coupling constant running with the inflaton field. The running nature of the coupling constant may allow us to have new possibility which is not available in classical treatments. Namely, there are always some regions in spacetime where the coupling constant becomes small enough to have sufficient period of inflation. We shall study this possibility in SUSY minded models and find that this is the case with reasonable range of parameters.

2 Chaotic inflation with nonminimal coupling to gravity

According to the above argument, we shall take the following action for our model of the nonminimally coupled inflaton in Einstein gravity[9, 10]:

\[
S = -\frac{1}{2} \int d^4x \sqrt{-g} \left[ \frac{M_{Pl}^2}{8\pi} \mathcal{R} + \partial_{\mu} \phi \cdot \partial^{\mu} \phi + m^2 \phi^2 - \xi \mathcal{R} \phi^2 \right],
\]

where \( M_{Pl} = 1/\sqrt{G} \) and \( \xi \) is the nonminimal coupling constant. Conformal invariance yields \( \xi = 1/6 \). The above action is considered to be the bosonic sector of a Wess-Zumino model in curved spacetime and the Einstein term. The nonminimal coupling is a soft SUSY breaking term as mass term. The equations of motion are

\[
(\Box - m^2 + \xi \mathcal{R})\phi = 0
\]

\[
\mathcal{R}_{\mu\nu} - \frac{1}{2} \mathcal{R} g_{\mu\nu} = \frac{8\pi}{M_{Pl}^2} T^{\text{eff}}_{\mu\nu},
\]

where

\[
T^{\text{eff}}_{\mu\nu} = \frac{M_{Pl}^2}{M_{Pl}^2 - 8\pi \xi \phi^2} \left[ (1 - 2\xi) \partial_\mu \phi \cdot \partial_\nu \phi + \left( 2\xi - \frac{1}{2} \right) \partial_\mu \phi \cdot \partial^\rho \phi g_{\mu\nu} - 2\xi \nabla_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 g_{\mu\nu} - 2\xi g_{\mu\nu} \partial^\rho \phi \cdot \partial_\rho \phi \right].
\]

Since we focus on small regions in the spacetime in which the inflaton distributes homogeneously and examine if such a region will undergo sufficient period of the
inflation, we shall assume that the region is homogenous and isotropic, and thus can be described by Robertson-Walker metric. Then we have the Friedmann and Raychaudhuri equation for such a region:

\[ H^2 + \frac{k}{a^2} = \frac{\dot{\phi}^2 + 12\xi H\dot{\phi} + m^2\phi^2}{6(M_{Pl}^2/8\pi - \xi\phi^2)}, \]  

\[ \dot{H} + H^2 = \frac{(6\xi - 2)\dot{\phi}^2 + m^2\phi^2 + 6\xi\ddot{\phi} + 6H\dot{\phi}}{6(M_{Pl}^2/8\pi - \xi\phi^2)}. \]  

where \( H = \dot{a}/a \) is the Hubble parameter of the region, with \( a \) the scale factor. Inserting the Eqs. (5) and (6) into Eq. (2), we obtain the Klein-Gordon type equation for the inflaton:

\[ \ddot{\phi} + 3H\dot{\phi} + \left[ m^2 + \frac{(6\xi - 2)\dot{\phi}^2 + 6\xi(\ddot{\phi} + 3H\dot{\phi})}{M_{Pl}^2/8\pi - \xi\phi^2} \right] \phi = 0. \]  

For a negative \( \xi \) it is known that there are the two saddle points in the phase space, where \( \phi_{cr} = \pm M_{Pl}/\sqrt{8\pi|\xi|} \) and \( \dot{\phi} = 0 \). The origin is the unique stable point in the phase space. If the inflaton initially satisfy the condition

\[ - \frac{M_{Pl}}{\sqrt{8\pi|\xi|}} < \phi < \frac{M_{Pl}}{\sqrt{8\pi|\xi|}}, \]  

chaotic inflation may occur. Otherwise, the scalar field keeps growing exponentially: inflation occurs but never terminates.

On the other hand, for a positive \( \xi \) the anisotropic shear diverges as the inflaton approaches at the following points without bound:

\[ \phi_{cr} = \pm \frac{M_{Pl}}{\sqrt{8\pi\xi}}. \]  

The evolution of the region with \( \phi \) larger than \( \phi_{cr} \) will terminate at \( \phi_{cr} \), and such regions do not evolve into our present Universe. We shall call this points as the critical points. Thus we shall only pay attention to the region with \( \phi \) lying between the two critical points.

It has been known that the minimal initial value of the inflaton is about \( 5M_{Pl} \) to realize sufficient period of inflation in the framework of chaotic inflationary scenario. Then the above condition (8) and (9) with \( \phi \sim 5M_{Pl} \) give us the condition \( |\xi| \sim 10^{-3} \) for successful chaotic inflationary senario with nonminimal coupling. In Ref. [3] they demand also more severe constraint for natural realization of inflation, because they consider that inflaton probably have Planck energy density initially. We do not adopt such a view, instead we assume that the initial value is distributed randomly below Planck energy density. Therefore we just need \( |\phi| \gtrsim 5M_{Pl} \) for the initial condition.
3 Running nonminimal coupling

We adopt the one-loop effective action along a flat direction of the Wess-Zumino model [12] as the Lagrangian (1). The superpotential is

$$W = g \Phi_1 \Phi_2 \Phi_2 / 2.$$ 

The action is written as

$$e^{-1} \mathcal{L} = -(\partial_{\mu} \phi_i)^* (\partial^\mu \phi_i) - V(\phi_1, \phi_2)$$

$$+ \frac{1}{2} (\bar{\psi}_1 \bar{\psi}_2) [i \Phi + M(\phi_1, \phi_2)] \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix},$$

(10)

where $\mathcal{D}_\mu$ is covariant derivative and the potential term $V$ is defined as

$$V(\phi_1, \phi_2) = (m_1^2 - \xi_1 \mathcal{R}) \phi_1^* \phi_1 + \frac{1}{4} |g|^2 (\phi_2^* \phi_2)^2$$

$$+ (m_2^2 - \xi_2 \mathcal{R}) \phi_2^* \phi_2 + |g|^2 (\phi_1^* \phi_1) (\phi_2^* \phi_2),$$

(11)

and the mass matrix $M(\phi_1, \phi_2)$ is given by

$$M(\phi_1, \phi_2) = \begin{pmatrix} 0 & \text{Re}(g \phi_2) - \text{Im}(g \phi_2) \gamma^5 \\ \text{Re}(g \phi_2) - \text{Im}(g \phi_2) \gamma^5 & \text{Re}(g \phi_1) - \text{Im}(g \phi_1) \gamma^5 \end{pmatrix}. $$

(12)

$\mathcal{R}$ is the scalar curvature and $\mathcal{R} = 12 H^2$ in de Sitter spacetime. In the scalar potential (11), we have introduced the soft SUSY breaking mass terms $m_i^2 \phi_i^* \phi_i$ and the nonminimal curvature couplings $\xi_i \mathcal{R} \phi_i^* \phi_i$, in addition to the minimal extension of the Wess-Zumino model in curved spaces. Since the nonminimal coupling receives the renormalization as will be seen below, the bare term $\xi_i \mathcal{R} \phi_i^* \phi_i$ is necessary for this model to be renormalizable. From the tree potential (11), we see $\phi_2 = 0$ is actually a flat direction in this model; namely the potential energy remains flat for any values of $\phi_1$ except for the SUSY breaking mass term and the nonminimal coupling term.

Now let us consider the one-loop effective potential. We decompose the scalar fields as

$$\phi_1 = \frac{\phi_1}{\sqrt{2}} + \varphi_1 + i \varphi_2, \quad \phi_2 = \frac{\phi_2}{\sqrt{2}} + \varphi_3 + i \varphi_4, $$

(13)

where all the fields are real and $\phi_i$ are the classical fields.

Using the DeWitt-Schwinger technique [13] we obtain

$$\mathcal{V}_{\text{eff}}(\phi_1, \phi_2) = \frac{1}{2} (m_1^2 - \xi_1 \mathcal{R}) \phi_1^2 + \frac{1}{2} (m_2^2 - \xi_2 \mathcal{R}) \phi_2^2 + \frac{1}{16} g^2 \phi_2^4 + \frac{1}{4} g^2 \phi_1^2 \phi_2^2$$

$$+ \frac{g^2}{32\pi^2} \ln \frac{\phi_1^2 + \cdots}{\Lambda^2} \left[ \left\{ m_1^2 + \left( \xi_1 - \frac{1}{4} \right) \right\} \phi_1^2 + \phi_2^2 \right]$$

$$+ \frac{1}{4} g^2 \phi_1^2 \phi_2^2 + \frac{1}{16} g^2 \phi_2^4 + \frac{g^2}{32\pi^2} \ln \frac{\phi_2^2 + \cdots}{\Lambda^2} \left[ \left\{ m_2^2 + \left( \xi_2 - \frac{1}{4} \right) \right\} \phi_2^2 + \phi_1^2 \right]$$

$$+ \left( \xi_1 - \frac{1}{4} \right) \phi_2^2 + \frac{1}{4} g^2 \phi_1^2 \phi_2^2.$$ 

(14)
in the flat limit, where we have omitted the $\phi_i$-independent convergent terms which are irrelevant in our present analysis. ($\cdots$) do not contain $\phi_1$-dependent term. Likewise\[14, 15\], the renormalized wave functions are defined as

$$\phi_{1R}^2 = \left(1 + \frac{g^2}{32\pi^2} \ln \frac{\Lambda^2}{\phi_1^2 + \cdots}\right)\phi_1^2,$$

(15)

$$\phi_{2R}^2 = \left(1 + \frac{g^2}{32\pi^2} \ln \frac{\Lambda^2}{\phi_1^2 + \cdots} + \frac{g^2}{32\pi^2} \ln \frac{\Lambda^2}{\phi_2^2 + \cdots}\right)\phi_2^2.$$  

(16)

Then we define the renormalized parameters

$$g_R^2 = \left(1 + \frac{3g^2}{32\pi^2} \ln \frac{\phi_1^2 + \cdots}{\Lambda^2}\right)g^2,$$

(17)

$$m_{1R}^2 = m_1^2 + \frac{g^2}{32\pi^2} \left(\ln \frac{\phi_1^2 + \cdots}{\Lambda^2}\right) \left(m_1^2 + 2m_2^2\right),$$

(18)

$$m_{2R}^2 = m_2^2 + \frac{3g^2}{32\pi^2} \left(\ln \frac{\phi_1^2 + \cdots}{\Lambda^2}\right) m_2^2,$$

(19)

$$\xi_{1R} = \xi_1 + \frac{g^2}{32\pi^2} \left(\ln \frac{\phi_1^2 + \cdots}{\Lambda^2}\right) \left(\xi_1 + 2\xi_2 - \frac{1}{2}\right),$$

(20)

$$\xi_{2R} = \xi_2 + \frac{g^2}{32\pi^2} \left(\ln \frac{\phi_1^2 + \cdots}{\Lambda^2}\right) \left(3\xi_2 - \frac{1}{2}\right),$$

(21)

where we present only correction with $\phi_1$ (the flat direction). The renormalization groups are

$$\frac{d}{d\phi} \frac{g^2}{16\pi^2},$$

(22)

$$\frac{d}{d\phi} \frac{m_1^2}{16\pi^2} = \frac{g^2}{16\pi^2} (m_1^2 + 2m_2^2),$$

(23)

$$\frac{d}{d\phi} \frac{m_2^2}{16\pi^2} = \frac{3g^2}{16\pi^2} m_2^2,$$

(24)

$$\frac{d}{d\phi} \frac{\xi_1}{16\pi^2} = \frac{g^2}{16\pi^2} \left(\xi_1 + 2\xi_2 - \frac{1}{2}\right),$$

(25)

$$\frac{d}{d\phi} \frac{\xi_2}{16\pi^2} = \frac{3g^2}{16\pi^2} \left(\xi_2 - \frac{1}{6}\right),$$

(26)

where we simply denote $\phi_1$ by $\phi$ and hereafter all the quantities are renormalized. The renormalization group analysis of the nonminimal coupling says

$$\xi_2(\phi) = \frac{g_2^2(\phi)}{g_2^2(M)} \left[\xi_2(M) - \frac{1}{6}\right] + \frac{1}{6}.$$  

(27)
where
\[ g^2(\phi) = g^2(M) \left[ 1 - \frac{3g^2(M)}{32\pi^2} \ln \frac{\phi^2}{M^2} \right]^{-1}. \] (28)
Eqs. (25) and (26) allow us to assume \( \xi_1 = \xi_2 \equiv \xi \). This theory is free and conformal invariant in the infrared limit; \( \phi \rightarrow 0 \). In other words, in this limit the free scalar field propagates along the light cone.\[5\] The form of Eq. (27) is universal no matter if its conformal invariance appears in the ultraviolet or infrared region.\[16, 17\]

As far as \( g^2(\phi) < 1 \) and \( 3g^2(M) \ln(\phi^2/M^2)/32\pi^2 < 1 \), one-loop calculation is reliable.

Contrary to \( m \), identification of the inflaton with some elementary particle cannot constrain \( \xi \). If we define the renormalization point \( M \) as
\[ \xi(M) = 0, \] (29)
then Eq. (28) becomes
\[ \xi(\phi) = \frac{1}{6} - \frac{1}{6} \left[ 1 - \frac{3g^2(M)}{32\pi^2} \ln \frac{\phi^2}{M^2} \right]^{-1}. \] (30)
We use the Eq. (30)\[4\] for the later argument, where we assume Eq. (30) is applicable up to the Planck energy density: \( \phi \simeq M_{Pl}^2/m \).

### 4 Condition on Yukawa coupling

In this section we clarify the condition on the Yukawa coupling constant for the realization of a successful inflationary scenario using the above results. First of all we assume that the inflaton must have the initial value\[1, 2, 3\]:
\[ \phi > \sim 5M_{Pl} \] (31)
As discussed in section 2 this is needed for a sufficient period of inflation. It is convenient to devide the range of \( M \) as
\[ 5M_{Pl} < M \lesssim 5M_{Pl}, \] which we call them case (I) and case (II), respectively.

\[\text{2} \text{The running of the mass parameter is } m_2^2(\phi) = \left\{ g^2(\phi)/g^2(M) \right\} m_2^2(M). \text{ If we choose a right-handed sneutrino as the inflaton, it is plausible to assume } m(M_{Pl}) \simeq 10^{13} \text{GeV. } m(\phi) \text{ does not change its order of magnitude drastically in the relevant scale.} \]

\[\text{3} \text{What we actually do is called renormalization group improvement of effective potential.} \]

\[\text{4} \text{In other words we choose the flow which decreases monotonically to the negative infinity as } 3g^2(M) \ln(\phi^2/M^2)/32\pi^2 \rightarrow 1. \]

\[\text{5} \text{Hereafter we concentrate on the positive side of } \phi. \]
4.1 case (I)

In this case $\xi(5M_{Pl})$ is positive. Then during evolution of inflaton $\xi(\phi)$ is always positive. Since we must avoid the critical point, we obtain the condition:

$$\frac{M_{Pl}}{\sqrt{8\pi\xi(\phi)}} > \phi$$

(32)
during the whole evolution of $\phi$, $\phi \lesssim 5M_{Pl}$.

The above condition is rewritten as (See Figure 1)

$$\frac{M_{Pl}}{2} \sqrt{\frac{3}{\pi} - \frac{32\pi}{g^2(M)\ln(\phi/M)^2}} > \phi$$

(33)

for all $\phi \lesssim 5M_{Pl}$.

Numerical results say that for $10 \lesssim M/5M_{Pl} < 10^2$ we need $g^2(M) \lesssim 1$ and for $10^2 \lesssim M/5M_{Pl} \lesssim 10^6$ we need $g^2(M) \lesssim 10^{-1}$. We will not consider the range $M/5M_{Pl} > 10^6$ because the energy density becomes larger than Planck energy density for such ranges.

4.2 case (II)

In this case $\xi(5M_{Pl})$ is negative. $5M_{Pl}$ should be smaller than the unstable point and during the subsequent evolution $\phi$ must avoid the singularity:

$$\frac{M_{Pl}}{\sqrt{8\pi|\xi(5M_{Pl})|}} > 5M_{Pl}$$

(34)

$$\frac{M_{Pl}}{\sqrt{8\pi\xi(\phi)}} > \phi$$

(35)

for all $\phi < M$. Eq. (34) is sufficient for $\phi > M$ since the value of the unstable point $(M_{Pl}/\sqrt{8\pi|\xi(\phi)|})$ monotonically decreses in such a region.

Eq. (34) gives the condition on $\xi$ as

$$|\xi(5M_{Pl})| \lesssim 10^{-3},$$

(36)

which is the same as the condition for the constant $\xi$ [3]. Roughly this gives us the following condition on $g(M)$ (See Figure 2)

$$g^2(M) \lesssim \frac{1}{\ln(5M_{Pl}/M)^2}.\quad (37)$$

This gives us the condition on $g(M)$. For example, $g^2(M) \sim 10^{-1}$ for $M/5M_{Pl} \sim 10^{-2}$, $g^2(M) \sim 10^{-2}$ for $M/5M_{Pl} \sim 10^{-22}$ and so on. We can see the condition is not unreasonable at all in a SUSY model [7].

The second condition (35) is automatically satisfied if perturbation is reliable: $g(\phi) < 1$(See Figure 2).
5 Conclusions and Discussion

We have found that chaotic inflation by nonminimal coupled inflaton is naturally realized by taking into account of the running nature of the coupling constant. The condition to have a successful inflation is just $g(M) \lesssim 10^{-1}$ around a wide range of $M$ where $\xi(M) = 0$ which is not unreasonable in some models. It also implies that efficient reheating may be possible. Nonminimal coupling constant becomes to be small enough ($|\xi| \lesssim 10^{-3}$) when inflation starts and evolves to the conformal value, where the universe is radiation dominant.

The above argument is true to all the models which have flat directions since the form of the renormalization group equation is universal.

A model with $-\xi \gtrsim 10^4$ is considered to circumvent fine-tuning of the self-coupling constant in $\lambda\phi^4$ theory \[1\]. But such a large $|\xi|$ is realized by a large $\phi$ which causes very high energy density larger than the Planck energy density. Accordingly, such a model seems to be not so feasible. However, the more detailed investigation is necessary to draw any definite conclusion for such cases.

Acknowledgement

The authors thank to M. Hotta and K. Kumekawa for valuable comments.

References

[1] A. Linde, Phys. Lett. 129B, 177 (1983).
[2] For a review, see A. Linde, Particle Physics and Inflationary Cosmology, (harwood academic publishers, 1990).
[3] T. Futamase and K. Maeda, Phys. Rev. D 39, 399 (1989).
[4] T. Futamase, T. Rothman, and R. Matzner, Phys. Rev. D 39, 405 (1989).
[5] V. Faraoni, Phys. Rev. D 53, 6813 (1996).
[6] R. Fakir and W.G. Unruh, Phys. Rev. D 41, 1783 (1990); N. Makino and M. Sasaki, Prog. Theor. Phys. 86, 103 (1991).
[7] H. Murayama, H. Suzuki, T. Yanagida, and J. Yokoyama, Phys. Rev. Lett. 70, 1912 (1993).
[8] M. Tanaka, [hep-th/9701063].
[9] L. Amendola, M. Litterio, and F. Occhionero, Int. J. Mod. Phys. A 5, 3861 (1990).

[10] A. Barroso, J. Casasayas, P. Crawford, P. Moniz, and A. Nunes, Phys. Lett. 275B, 264 (1992).

[11] A.A. Starobinsky, Pis’ma Astron. Zh. 7, 67 (1981)[Sov. Astron. Lett. 7, 36 (1981)].

[12] H. Suzuki and M. Tanaka, Phys. Rev. D 49, 6692 (1994).

[13] N.D. Birrell and P.C.W. Davies, Quantum fields in curved spacetime (Cambridge, 1982).

[14] R.D. Ball, Phys. Rep. 182, 1 (1989).

[15] M. Tanaka, Prog. Theor. Phys. 92, 1105 (1994).

[16] I.L. Buchbinder and S.D. Odintsov, Yad. Fiz. (Sov. J. Nucl. Phys.) 40, 1338 (1984); Lett. Nuovo Cim. 42, 379 (1985).

[17] For a review, see I.L. Buchbinder, S.D. Odintsov, and I.L. Shapiro, Effective Action in Quantum Gravity, (IOP, 1992).

[18] S. Coleman and E. Weinberg, Phys. Rev. D7, 1888 (1973).

[19] I.L. Buchbinder and S.D. Odintsov, Class. Quant. Grav. 2, 721 (1985).
Figure 1: $x = \phi/M_{Pl}$, $x_c = \phi_{cr}/M_{Pl}$, and $M/M_{Pl} = 50$. The upper line corresponds to the case $g^2(M) = 10^{-2}$, the middle line to the case $g^2(M) = 10^{-1}$, and the lower to the case $g^2(M) = 1$. The straight line indicates $x_c = x$: $\phi_{cr} = \phi$. The former two lead to a successful inflation because $\phi_{cr} > \phi$ for $\phi < 5M_{Pl}$.
Figure 2: $x = \phi/M_{Pl}$, $x_c = \phi_{cr}/M_{Pl}$, and $M/M_{Pl} = 2$. The upper line corresponds to the case $g^2(M) = 1/3$ and the lower does to the case $g^2(M) = 1$. The straight line indicates $x_c = x$: $\phi_{cr} = \phi$. The former leads to a successful inflation because $\phi_{cr} > \phi$ for $\phi < 5M_{Pl}$. From Eq.(37) we can find with this value of $M$ we need $g^2(M) < 0.54$ approximately.