Kwon, Y. S.; Mednykh, A. D.; Mednykh, I. A.
On Jacobian group and complexity of the generalized Petersen graph $GP(n, k)$ through Chebyshev polynomials. (English) [Zbl 1365.05135]
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Summary: In the present paper we give a new method for calculating Jacobian group $\text{Jac}(GP(n, k))$ of the generalized Petersen graph $GP(n, k)$. We show that the minimum number of generators of $\text{Jac}(GP(n, k))$ is at least two and at most $2k + 1$. Both estimates are sharp. Also, we obtain a closed formula for the number of spanning trees of $GP(n, k)$ in terms of Chebyshev polynomials and investigate some arithmetical properties of this number.

MSC:
05C30 Enumeration in graph theory
39A10 Additive difference equations

Keywords:
spanning tree; Jacobian group; Petersen graph; Laplacian matrix; Chebyshev polynomial

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References:
[1] Arnaud, D.; Fiorenzi, P.; Francini, P., Sandpile group on the graph $S_{D_n}$ of the dihedral group, European J. Combin., 24, 7, 815-824, (2003) · Zbl 1026.05055
[2] Bacher, R.; de la Harpe, P.; Nagnibeda, T., The lattice of integral flows and the lattice of integral cuts on a finite graph, Bull. Soc. Math. France, 125, 167-198, (1997) · Zbl 0891.05062
[3] Baker, B.; Norine, S., Harmonic morphisms and hyperelliptic graphs, Int. Math. Res. Not., 15, 2914-2955, (2009) · Zbl 1186.05003
[4] Biggs, N. L., Chip-firing and the critical group of a graph, J. Algebraic Combin., 9, 1, 25-45, (1999) · Zbl 0947.05028
[5] Boesch, F. T.; Prodinger, H., Spanning tree formulas and Chebyshev polynomials, Graphs Combin., 2, 1, 191-200, (1986) · Zbl 0651.05028
[6] Chang, S. C.; Chen, L. C.; Yang, W. S., Spanning trees on the sierpinski gasket, J. Stat. Phys., 126, 649-667, (2007) · Zbl 1137.60010
[7] Cori, R.; Rossin, D., On the sandpile group of dual graphs, European J. Combin., 21, 4, 447-459, (2000) · Zbl 0969.05034
[8] D'Angeli, D.; Donno, A., Weighted spanning trees on some self-similar graphs, Electron. J. Combin., 181, 16-43, (2011)
[9] Davis, P. J., Circulant matrices, (1994), AMS Chelsea Publishing · Zbl 0898.15021
[10] Deryagina, M.; Mednykh, I., On the Jacobian group for Möbius ladder and prism graphs, (Proceedings of the Fifteenth International Conference on Geometry, Integrability and Quantization, (2014), Avangard Prima Sofia, Bulgaria), 117-126 · Zbl 1314.05094
[11] Dhar, D.; Ruelle, P.; Sen, S.; Verma, D.-N., Algebraic aspects of abelian sandpile models, J. Phys. A, 28, 805-831, (1995) · Zbl 0848.60062
[12] Frucht, R.; Graver, J. E.; Watkins, M. E., The groups of the generalized Petersen graphs, Proc. Cambridge Philos. Soc., 70, 211-218, (1971) · Zbl 0222.05069
[13] Gera, R.; Stanič, P., The spectrum of generalized Petersen graphs, Australas. J. Combin., 49, 39-45, (2011) · Zbl 1228.05206
[14] Kotani, M.; Sunada, T., Jacobian tori associated with a finite graph and its abelian covering graphs, Adv. in Appl. Math., 24, 89-110, (2000) · Zbl 1017.05038
[15] Lorenzini, D., Smith normal form and Laplacians, J. Combin. Theory Ser. B, 98, 6, 1271-1300, (2008) · Zbl 1175.05008
[16] Mednykh, I. A.; Zinidinota, M. A., On the structure of Picard group for moebius ladder, Siberian Elektron. Mat. Izv., 8, 54-61, (2011) · Zbl 1329.05147
[17] Mednykh, A. D.; Mednykh, I. A., On the structure of the Jacobian group for circulant graphs, Dokl. Math., 94, 1, 445-449, (2016) · Zbl 1350.05061
[18] Nikolopoulos, S. D.; Papadopoulos, C., The number of spanning trees in $S_{K_n^{\infty}}$-complements of quasi-threshold graphs, Graphs Combin., 20, 383-397, (2004) · Zbl 1054.05058
[19] Shrock, R.; Wu, F. Y., Spanning trees on graphs and lattices in d-dimensions, J. Phys. A, 33, 3881-3902, (2000) · Zbl 0949.05041
[20] Steimle, A.; Staton, W., The isomorphism classes of the generalized Petersen graphs, Discrete Math., 309, 1, 231-237, (2009) · Zbl 1219.05098

[21] Sun, W.; Wang, S.; Zhang, J., Counting spanning trees in prism and anti-prism graphs, J. Appl. Anal. Comput., 6, 65-75, (2016)

[22] Wang, J.; Pan, Y. L.; Xu, J. M., The critical group of $K_m \times C_n$, Acta Math. Sin. (Engl. Ser.), 27, 1, 169-184, (2011) · Zbl 1222.05181

[23] Xiebin, C.; Lin, Qiuying; Zhang, Fuji, The number of spanning trees in odd valent circulant graphs, Discrete Math., 282, 1, 69-79, (2004) · Zbl 1042.05051

[24] Yaoping, Hou; Woo, Chingwah; Pingge, Chen, On the sandpile group of the square cycle, Linear Algebra Appl., 418, 457-467, (2006) · Zbl 1104.05052

[25] Yaoping, Hou; Pingge, Chen, On the critical group of the mobius ladder graph, Australas. J. Combin., 36, 133-142, (2006) · Zbl 1104.05032

[26] Yuanping, Zhang; Xuerong, Yong; Golin, M. J., The number of spanning trees in circulant graphs, Discrete Math., 223, 1, 337-350, (2000) · Zbl 0969.05036

[27] Yuanping, Zhang; Yong, Xuerong; Golin, M. J., Chebyshev polynomials and spanning tree formulas for circulant and related graphs, Discrete Math., 298, 1, 334-364, (2005) · Zbl 1070.05029

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