Comparison of depth-averaged and 3D models for dense granular flows

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Abstract. Debris flows are one of the major threats to mountain communities. They consist of the downslope flow of fine and coarse material, saturated with water, along channelized paths. Due to their high velocity and unpredictability, the evacuation of hit areas may be difficult to execute. To avoid casualties and economic losses, mitigation structures, like filter barriers, are therefore usually adopted. Their primary task is to reduce the flow energy and to retain larger boulders. However, considerable room to improve the design of these structures still exists. In particular, gaining a better understanding of debris flows dynamics is a necessary step to improve the design of barriers. Numerical modelling can contribute to its understanding, and in an effective simulation of the flowing mass dynamics and impact against mitigation barriers. In this frame, the continuum-based Depth-Averaged Modelling (DAM) has been widely used since the 90s. In spite of the good results of this approach, together with the low computational time, the averaging procedure of velocity and pressure along the flow depth causes the loss of crucial information, which is important for correctly simulating the interaction with mitigation structures. A full 3D modelling can overcome this shortcoming by allowing a more complete flow representation, and a more accurate computation of impact forces. However, since debris flow may run for long distances, 3D models would require a large computational time. In this work we aim to study both the shortcomings and the advantages of the DAMs and 3D models. In particular, The DAM model used is DAN-W, while the 3D model is based on the lattice-Boltzmann method. To compare the results from numerical modelling, we use the experimental work performed by Moriguchi et al. (2009) in which a mass of dry sand flows on a steep chute.

1. Introduction
Landslides are harmful natural phenomena, which occur worldwide [1]. In the last decades, several classifications have been proposed. A widely accepted classification has been proposed by Cruden (1991) [2] and Hungr et al. 2014 [3]. They classify landslides by type of material and movement. In this context, debris flows phenomena emerge as extremely dangerous events posing a major threat to mountain communities. They consist of flowing fine and coarse materials saturated with water [4], which propagate in channelized paths [5]. Due to the absence of premonitory signs, and to their high velocity and long runout, the evacuation of local populations is often impractical. Hence, mitigation structures are necessary to protect human life and to prevent economic damages. With this goal, a valid strategy consists in using barriers such as filter barriers [6] and flexible barriers [7]. The main purpose of these structures is to reduce the flow energy [8, 9, 10] and to retain the larger boulder [11, 12, 13]. Currently, the design of these structures still relies on empirical or simplified methods because of the...
debris flows rheology [14] and the flow-structure interaction mechanisms [4]. Nevertheless, reliable rational numerical models can allow gaining fundamental knowledge in the study of flow-structure interaction when a debris flows phenomenon occurs.

In this frame, continuum-based Depth-Averaged Models (DAM) have been widely used since the 90s. They are based on depth-integrated Saint-Venant equations. Their first application for granular flows was conducted by Savage & Hutter (1989) [15], which for the first time depth-averaged the momentum and mass conservation for granular flows. Furthermore, the flowing resistance was taken into account by considering a Coulomb-like basal resistance law. Multiple improvements were proposed after Savage & Hutter (1989) [15]. Gray et al. (1999) performed DAM analyses over complex basal topography [16]. Iverson & Denlinger (2001) discussed a model in which the fluid continuum treated as a mixture of a solid matrix and a liquid fraction [17]. Furthermore, Mangeney-Castelnau et al. (2003) discussed a method in which the depth-averaged equations are referenced in a topography-framework, and solved with the Volume-Finite method [18].

DAMs are particularly reliable to forecast debris flows runout. Moreover, they require little computational time, even if the geometry, or the boundaries conditions, are complex. However, averaging velocity and pressure along the flow depth leads to losing key information regarding the interaction with structures. Loosing this information affects negatively their use in the design of mitigation structures, because the flow-structure interaction is an intrinsically 3D problem. Thus, the design of such structures may be over-simplified.

The DAMs shortcoming can be overcome using a full 3D model. Among others [19, 20, 21, 22, 23], Lattice-Boltzmann method (LBM) emerges as a relatively recent fluid solver which can be employed to study the flow-structure interaction rationally. LBM belongs to mesoscopic solvers, which do not solve macroscopic nor microscopic variables directly. LBM links the gap between the macroscopic and microscopic world by considering the behaviour of a group of particles as a unit. A distribution function describes the property of the particle set [24]. The evolution of the distribution function is governed by mesoscopic kinetic equations [25, 26]. In the last decades, LBM has increased in popularity in numerical modelling. In particular, it can simulate multiphase flows such as debris flows [27, 13]. Although 3D models would allow for a more accurate resolution of fluid-structure interaction [7], debris flows may propagate up to kilometres. Hence, a complete 3D analysis would require exceedingly long computational times.

In this paper, we compare weakness and points of strength of both models. To do so, a physical experiment is necessary to compare the outcomes from a DAM model and an LBM 3D model. The experiments conducted by Moriguchi et al. (2009) [28] are used to benchmark the performance of the two approaches. They conducted a physical experiment on a dry granular dense flow. The experiment consisted of releasing dry sand on a steep chute. They recorded the front position during the whole experiment, and they reported the free surface configuration at the end of the experiment. By comparing the results obtained by Moriguchi et al. 2009 [28] we can conclude how suitable are DAM models and LBM model to simulate dry granular flows.

The paper is organized as follow: In Section 2, we focus on a DAM model (DAN-W [29]) and a 3D LBM model (Hybird [30]). In Section 3 and 4, we discuss the experiment setup and compare the experimental and numerical results. Finally, in Section 5 we discuss the results and possible future extension to more complex scenarios.

2. Numerical models

Continuum modelling can best be treated under two headings: DAMs and 3D models. The main difference, consists in the set of governing equations. DAMs rely on the depth-averaged Saint-Venant equation, whereas 3D models solve the full Navier-Stokes equations, and take into account the internal shear deformation of the fluid. Further details concerning DAMs and 3D are provided in the following subsections.
2.1. Depth-Averaged models

Generally, solid materials dislocate along failure surfaces. Whereas, liquids are subjected to continuous deformations [31]. Thus, to model correctly and rationally debris flows (and dense granular flows as well), an idealization must be found. Hungr (1995) [29] tackled the problem by replacing the heterogeneous mixture of solids and liquids with an equivalent fluid, whose features approximate the mixture liquid-solid [29]. The equations available to solve the problem are the depth-averaged momentum conservation and continuity:

\[
\begin{align*}
\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} &= 0 \quad \text{mass conservation}, \\
\frac{\partial (hu)}{\partial t} + \frac{\partial (hu^2)}{\partial x} + \frac{\partial (huv)}{\partial y} + \frac{\partial}{\partial x}\left(\frac{gh^2}{2}\right) &= gh - ghS_f \frac{\bar{u}}{||\bar{u}||} \quad x\text{-momentum conservation}, \\
\frac{\partial (hv)}{\partial t} + \frac{\partial (huv)}{\partial x} + \frac{\partial (hv^2)}{\partial y} + \frac{\partial}{\partial y}\left(\frac{gh^2}{2}\right) &= gh - ghS_f \frac{\bar{y}}{||\bar{y}||} \quad y\text{-momentum conservation}.
\end{align*}
\]

where \( t, h, \bar{u} \) and \( \bar{v} \) represent time, the flow height, the averaged velocity along the \( x \) and \( y \) directions respectively. \( g(x, y, z) = (g_x, g_y, g_z) \) is gravity in its components. Finally, \( S_f \) indicates friction slope acceleration [32]. In DAMs the heterogeneous flow is idealized as an equivalent continuum. Hence, friction due to particle impacts and fluid viscosity must be indirectly computed through rheology. However, the equivalent continuum should not be considered as an ideal fluid. The solution proposed in DAMs is to represent the missing dissipative components due to internal shear as a basal resistance force, whose formulation depends on the rheology. Naef et al. (2006) [33] discuss a complete list of the rheological laws.

Although DAMs have shown some remarkable results [18, 32, 19, 34], they might have difficulties in representing flow-structure in its entirety. DAM assumptions idealize the problem as a 2D flow. Nevertheless, the fluid-structure interaction is intrinsically a 3D problem. Hence, a full 3D model can overcome this shortcoming by allowing a more complete flow representation, and a more accurate computation of impact forces.

2.2. 3D models

As stated in Sec. 2.1, complete 3D models are fundamental to study the flow-structure interaction problem. The Lattice-Boltzmann Method (LBM) is a valid candidate to simulate 3D flows. LBM is a mesoscopic method, which relies on kinetic theory and Boltzmann equations rather than discretizing the Navier-Strokes equations directly. It aims to describe macroscopic variables, such as velocity and pressure, through mesoscopic quantities. A complete description would be beyond the scope of this work. The interested reader is redirected towards Succi (2001) [25], or to Mohamad (2011) [24] for a more practical approach.

LBM relies on a probability density function \( f(x, t, c) \), representing the probability of finding fluid particles with speed \( c \) at location \( x \) and time \( t \). In LBM, space is discretized using a regular grid, or lattice, with unitary spacing. In this paper, we employ a 3D lattice with 19 allowed velocities \( c_i \), the so-called D3Q19 lattice. Time is discretized by unit as well. With the \( f(x, t, c) \) definition, LBM can easily compute the macroscopic variables the following equations:

\[
\begin{align*}
\rho &= \sum_i f_i \quad \text{(2)} \\
\mathbf{u} &= \sum_i f_i c_i \quad \text{p} = c_s^2 \cdot \rho \quad \text{(3)}
\end{align*}
\]

where \( c_s^2 = 1/3 \) is the lattice speed of sound, \( \rho \) is the bulk density, \( p \) is the pressure.
Figure 1: Laboratory flume (a) assembled by Moriguchi et al. (2009) and its scheme implemented in the simulations (b). The sand is placed at the top of the flume with slope angle $\theta = 45^\circ$. A barrier is placed 1.8 m from the gate.

The equation governing the evolution of $f$ is a discretized form of the Boltzmann equation and leads to thermodynamic equilibrium:

$$f_i(x + c_i, t + 1) = f_i(x, t) + \Omega_{\text{coll}}(x, t)$$

Where $\Omega_{\text{coll}}$ is an operator reproducing the effect of microscopic collisions.

3. Numerical examples

An implementation of DAM and a 3D model are here applied to simulate the experiment carried out by Moriguchi et al. (2009) [28]. They conducted a series of experiments using dry fine Toyoura sand. The tested sand was rather uniform, with the grain size of approximately 0.2 mm. Figure 1 shows the flume assembled by Moriguchi et al. (2009) [28] and sketches the configuration of the flume and sand initial position. One side wall of the flume was made of acrylic to allow detailed observations. The flume length is 2.5 m. Originally, Moriguchi et al. (2009) [28] conducted tests with several slope angles. However, for this paper purpose, we carry out numerical analyses with 45° slope angle, since DAMs might lose accuracy for steeper angles. A container filled with sand was placed at the flume top. The container had a side door (gate) that could be open instantaneously to initiate the flow. The experiment conducted by Moriguchi et al. (2009) is particularly suitable for studying the advantages of DAMs and 3D models. Indeed, a dam break problem at the laboratory scale allows observing the phenomena evolution i.e., the front position and the free surface configuration at the equilibrium.

In the following pages, we discuss the results. However, before proceeding to examine the results, it is important to highlight the numerical input for the analyses. Since the two models used here are a DAM and a 3D model, the input of information for each model is different. DAM requires the topography configuration and the rheological parameters. In this paper, we apply the frictional rheology [17], whose only parameter is the frictional angle $\phi$. Whereas, the 3D LBM solver requires a larger number of information than DAM. The space and time discretization are 2.5 mm and $1 \times 10^5$ s respectively. Moreover, the geometrical configuration and the rheological parameters are required. In this scenario, we apply the $\mu(I)$ rheology [35]. Table 1 and Table 2 show the input parameters of DAN-W and Hybird. Although Hybird is a 3D solver, and assuming the flow to be self-similar in the
transversal direction we carry out (as in Ref.[35]) a 2-dimensional analysis by imposing periodic condition in the third dimension for computational convenience. In table 1 $\varphi$, is the frictional angle.

While in table 2 $\mu_s$ and $\mu_d$ are the static and dynamic friction angle [35, 36]. $I_0$, $d_p$ and $\rho_p$ are a dimensionless material parameter, the particle diameter and the particle density, respectively. These rheological parameters are in agreement with studies performed with similar materials [35, 37, 38].

### Table 1: DAM input parameters.

| Event duration (s) | $\varphi$ |
|--------------------|-----------|
| 1.6 s              | 18°       |

### Table 2: LBM input parameters.

| N° points | Event duration | $\mu_s$ | $\mu_d$ | $I_0$ | $d_p$ | $\rho_p$ |
|-----------|----------------|---------|---------|-------|-------|----------|
| $2 \cdot 10^5$ | 1.6 s         | 18°      | 51°     | 0.279 | 0.2 mm | 2300 kg/m$^3$ |

![Figure 2](image.png)

**Figure 2:** Numeric results: (a) front position, (b) mass final configuration.

### 4. Results

We observe the front position and the free surface final configuration. figure 2 shows on the left the front position detected with DAN-W (DAM) and Hybird (3D LBM). The mass impacts the barrier at the bottom of the flume at the time $t = 0.65$ s (see figure 1). Whereas, on the right, the final free-surface configuration has been plotted. The front position is successfully with DAN-W and Hybird. However, in figure 2(a) a few differences between DAN-W and Hybird results models can be seen. In DAN-W, the mass seems to accelerate along the flume and once the bottom is reached no further advancement can occur. No deceleration can be observed due to the barrier. In the Hybird simulation, the mass accelerates along the flume. However, in this case at $t = 0.65$ s a sudden deceleration takes places. The deceleration is caused by the presence of the barrier, which halts the flow and is eventually overflown. Comparing the two results, it can be seen that as long as the flow does not interact with an obstacle, both DAN-W and Hybird are very similar to the experimental results. Nevertheless, when the flow reaches
the obstacle, DAN-W cannot replicate the interaction equally well. On the contrary Hybrid can capture accurately the flow-structure interaction. Considering the final free-surface configuration, figure 2(b) shows that DAN-W and Hybrid produce different results. DAN-W seems to neglect the barrier. Furthermore, the free surface configuration appears as if no obstacle is present on the flow path. By contrast, Hybrid results are more similar to the experimental results. This suggests that DAMs and 3D models are an useful tool to predict the runout and that the front position is well represented as long as no obstacle is present. However, if the flow impinges on a barrier, DAM appears to reconstruct the flow-structure interaction with lower accuracy. While the 3D model can represent the process accurately independently from the geometry.

5. Conclusions
In this paper, two numerical models have been presented, and their application to the dam-break of a dense granular flow has been shown. Both models are based on the continuum approach. This study has shown that the front position can be predicted rather well by either the 3D models and by DAMs, as long as obstacles are not present. However, the presence of an obstacle can create some unrealistic results if DAMs are employed. This is probably due to the depth averaging process that DAMs compute. The second major finding is that 3D models can predict the free surface final configuration accurately. Whereas DAMs models, might have some difficulties to interpret the flow-structure due to the depth-averaging process.

The major limitation of this study is the size of the flume. It does not allow to study of the scale effect of the phenomena. Moreover, the absence of fluids in the flowing mass and the rather uniform sand do not allow studying the importance of multiple phases and of a non-uniform grain-size distribution, which leads to the emergence of segregation phenomena.

It would be interesting to repeat the study with a non-uniform sand. A larger flume model could also highlight the presence of scale effects. Further study will be devoted to developing a coupling between DAMs and 3D models through domain decomposition.

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