Abstract

Using a three stage model of hadron formation we calculate the change of the transverse momentum distribution of hadrons produced in semi-inclusive deep inelastic scattering (SIDIS) on nuclei. In the first stage after its interaction with the virtual photon, the struck quark propagates quasi free in the nuclear environment undergoing multiple collisions with nucleons. During this stage it can acquire transverse momentum. In the second stage a prehadron is formed which has a very small elastic cross section with the nucleons. In the third stage the prehadron turns into a hadron. For HERMES energies, prehadron elastic scatterings contribute little to $p_{\perp}$-broadening. The acquired extra $\Delta p_{\perp}^2$ of hadrons can therefore be deduced entirely from the first stage of quasi free quark propagation with quark-nucleon collisions. We use this model to describe $\pi$-production on Ne, Kr, Xe and compare with the most recent HERMES preliminary data.

1 Introduction

Fragmentation of quarks and gluons into hadrons is a consequence of color confinement and is therefore one of the most-interesting parts of non-perturbative QCD. Use of nuclear targets allows the experimentalist to position detectors...
(nucleon targets) of the hadronization process near the interaction point and probe hadron formation on length scales of a few Fermi.

A struck quark originated from deep-inelastic scattering (DIS) propagates through the nucleus experiencing multiple interactions which cause induced energy loss. In the string model the medium-induced energy loss results from production of new strings in multiple collisions of the quark \[1\]. In a perturbative description the excess of energy loss is related to medium-induced gluon radiation resulting from broadening of the quark transverse momentum \[2\]. In both approaches induced energy loss rises quadratically with the path-length of the quark.

Although induced energy loss contributes to the attenuation of the produced hadron, even stronger suppression may result from absorption of the produced hadron, if the struck quark is neutralized and a colorless dipole is formed inside the nucleus. In the string model, the production of a hadron may be described as a two stage process \[3,4,5\]. First a prehadron (e.g. a dipole of small size) is created, which has a reduced absorption cross section, and later the hadron wave function is formed. These models \[3,4,5\] put the main emphasis on the effects of absorption and neglect the induced energy loss. The attenuation of the produced hadron can be described more consistently within the color-dipole approach employing a path integral technique \[6,7\] which describes the evolution of the dipole propagating through the medium. The induced energy loss evaluated perturbatively in Ref. \[2\] was included in Ref. \[7\].

Some models put the main emphasis on the effects of induced energy loss, assuming that color neutralization of the quark always occurs at long distance from the DIS location, i.e. outside of the nucleus \[8,9\]. Of course, this assumption may only be valid in certain kinematical domains.

A novel mechanism of hadron attenuation was found recently in Ref. \[10\]. It turns out that even if no induced energy loss occurs, and color neutralization happens outside of the nucleus, a significant nuclear suppression can result from quantum-mechanical interferences between different amplitudes.

In this paper we study the recent HERMES preliminary results \[11\] which give preliminary data on \(p_\perp\)-broadening of pions produced in deep inelastic lepton-nucleus scattering (DIS) on Ne, Kr and Xe nuclei. Transverse broadening
\( \Delta p_{\perp}^2 = \langle p_{\perp}^2 \rangle_A - \langle p_{\perp}^2 \rangle_D \) in the nuclear medium of a nucleus with mass number \( A \) has been measured as a function of the hadron fractional momentum \( z_h \), of the virtual photon energy \( \nu \) and its virtuality \( Q^2 \), for different nuclear size and for different hadrons. Here, the index \( D \) in the definition of the transverse broadening refers to a deuteron target.

We use the absorption model [3,5] to calculate the nuclear modifications of hadron production in DIS. The hadron formation time is computed analytically in the framework of the LUND string fragmentation model as a three-step process. In the first stage the quark (or antiquark) ejected from the nucleon propagates and undergoes multiple collisions in the nucleus. In the second stage color neutralization takes place and a prehadron is formed. Inelastic interactions of the prehadron or hadron result in a considerable shift of the final (detected) hadron towards smaller \( z_h \). We treat this process as absorption. Only elastic rescatterings preserve the (pre-)hadron and contribute to broadening. However, the elastic cross sections of hadrons and prehadrons are very small compared to the inelastic cross sections. In the third stage the final state hadron is formed from the surviving prehadrons. With an inelastic prehadron nucleon cross section which is reduced compared to the hadron-nucleon cross section, the model showed rather good agreement with the available HERMES data [12,13,14] for pions and kaons. In the LUND model, the hadron is formed at the formation length \( l_h \)

\[
l_h = l_p + z_h \frac{\nu}{\kappa} .
\]  

(1)

Here, \( l_p \) denotes the prehadron formation or production length. For relativistic quarks confined in one dimension the only scale setting parameter is the string tension \( \kappa \). Therefore, in the conventional estimate of the Lund model for the formation length enters the photon energy divided by the string tension

\[
L = \frac{\nu}{\kappa} , \quad \kappa = 1 \text{GeV} / \text{fm} .
\]

(2)

We have fitted the prehadron cross section to the pion-data for the multiplicity ratios as a function of \( z_h \) and \( \nu \) and found an optimal fit with a prehadron cross section equal to \((2/3)\) of the hadron cross section in the extended modelling
\[ \sigma_{\text{prehadron}} \approx \frac{2}{3} \sigma_{\text{hadron}}. \]  

(3)

This value of the prehadronic cross section is in agreement with Ref. [4].

Such a reduction may be partially because the prehadron does not yet have the full size of the hadron and therefore interacts with a smaller cross section due to color transparency. Besides, a considerable reduction should be also expected, since the exponential attenuation of (pre)hadrons used in the fit (see below Eq. (6)) misses the Gribov inelastic corrections [15], which make nuclei much more transparent [16,17,18].

Further on, we have considered in [5] the production process for particles which cannot be formed from a valence quark in the proton which is knocked out by the photon. For example, negative kaons as well as antiprotons cannot be formed by a struck valence quark picking up an antiquark from the string break-up. They can only be formed from struck sea quarks, which are subdominant at HERMES, or from \( q \bar{q} \) pairs formed inside the colour string. This different mechanism implies a flavour-dependent formation time from the LUND string fragmentation model.

2 Hadronic \( p_\perp \)-broadening as a function of \( z_h \) and \( \nu \)

In this paper we want to test the three stage model further by concentrating on quark propagation through the nucleus where most of the transverse momentum is acquired. The theoretical calculation is very similar to reference [3]. The length \( l_p \) after which the prehadron is formed [3,5] depends on the energy \( \nu \) transferred to the quark, the string tension \( \kappa \) and the energy fraction \( z_h \) of the produced hadron. From energy conservation already follows that if the hadron has a very large \( z_h \) the quark cannot have radiated very much energy. Therefore the formation length of the colour neutral state must have been very short [1,19,20]. The prehadron formation length is computed analytically in the framework of the LUND string fragmentation model. If one assumes that the prehadrons can be formed directly from the struck quark by picking up
an antiquark from the string break up, then the prehadron formation length reads

\[ l_p = \frac{\nu}{\kappa} z_h (1 - z_h) \times \left[ 1 + \frac{1 + D_q 1 - z_h}{2 + D_q z_h} 2F_1 \left( 2 + D_q, 2 + D_q, 3 + D_q, \frac{z_h - 1}{z_h} \right) \right]. \quad (4) \]

Here the parameter \( D_q \) is equal to \( D_q = 0.3 \) and \( 2F_1 \) is the Gauss hypergeometric function. The corrections to the simple \( z_h(1 - z_h) \)-behavior of the prehadronic formation length \( l_p \) given by the Gauss hypergeometric function can be recast into effective powers of \( z_h \) and \( 1 - z_h \) normalized by an appropriate prefactor. One can obtain an excellent fit to the "exact" prehadron formation length by

\[ l_p \simeq 1.19 \frac{\nu}{\kappa} z_h^{0.61} (1 - z_h)^{1.09}. \quad (5) \]

We use the length of the quark trajectory Eq. (4) to calculate with the dipole model the acquired \( \Delta p_{t}^2 \) of the quark under the constraint that the subsequent prehadron is not absorbed on its way through the nucleus, i.e. that it can be finally detected as a hadron. This is necessary, since the information about the acquired transversal momentum of the struck quark is encoded in the detected hadrons only.

\[
\langle \Delta p_{t}^2 \rangle_q = \frac{\langle \sigma q_{t}^2 \rangle}{\langle S_s \rangle} \int_{-\infty}^{\infty} d^2 b d z \rho_A (\vec{b}, z) \int_{z}^{z+l_p} d z' \rho_A (\vec{b}, z') \exp \left( -\sigma_* \int_{z+l_p}^{z+\infty} d z'' \rho_A (\vec{b}, z'') \right),
\]

\[
\langle S_s \rangle = \int_{-\infty}^{\infty} d^2 b d z \rho_A (\vec{b}, z) \exp \left( -\sigma_* \int_{z+l_p}^{\infty} d z' \rho_A (\vec{b}, z') \right). \quad (6)
\]

In this equation, the quantity \( \langle \sigma q_{t}^2 \rangle \) is the mean transverse momentum squared \( q_{t}^2 \) acquired by the quark in one collision multiplied with the corresponding quark nucleon cross section. This quantity is related to the dipole nucleon cross-section \cite{21,22} as shown below. Furthermore, we assume a sharp distribution of prehadron formation points, namely the prehadron is produced
after travelling a distance \( l_p \) through the nucleus. Hence, the final induced momentum broadening calculated in Eq. (6) can be read as the mean transverse momentum \( q_{\perp}^2 \) acquired by the ejected quark in a single collision multiplied with the average number of collisions in the nucleus. The resulting transverse momentum is averaged over all virtual photon interaction points and weighted by the prehadron survival probability. To be precise, the first integral over the nuclear density \( \rho_A \) averages over all primary interaction points in which a quark is ejected from a nucleon. The second integral multiplied with the cross section \( \sigma \) yields the number of collisions suffered by the ejected quark. The exponential factor at the end represents the prehadron survival probability \( S_* \) \[5\]. It is dictated by the longitudinal thickness of the nucleus at a given impact parameter \( b \) and by the mean free path of the prehadron in the nucleus \( \lambda_*^{-1} = \sigma_* \rho_A \), where the prehadron nucleon cross-section \( \sigma_* = 2/3 \sigma_{\pi N} \) (see Ref. \[5\]). The mean free path \( \lambda_* \) is of the same magnitude as \( l_p \) and \( R_A \). Hence, more weight is given to production points close to the back-surface of the nucleus which have large prehadron survival probabilities. In order to normalize this expression to the actual number of detected hadrons, we divide by the \( z \)-integrated prehadron survival factor \( \langle S_* \rangle \).

The mean transverse momentum squared times the cross-section, i.e. \( \langle \sigma q_{\perp}^2 \rangle \), can be derived from the dipole nucleon cross section as follows. In the eikonal approximation, the ejected high momentum parton moves on a classical trajectory with impact parameter \( \vec{b} \) and picks up a non-abelian phase factor \( V(\vec{b}) \) in the background gauge field generated by the nucleon
\[
V(\vec{b}) = \mathcal{P} \exp \left[ i g \int_{-\infty}^{+\infty} dx^\mu A_\mu(x) \right].
\] (7)

Here \( V(\vec{b}) \) is the Wilson line of the parton with impact parameter \( \vec{b} \) relative to the proton. We use the notation \( A_\mu \equiv A_\mu^a t^a \), where \( t^a \)'s are the generators of the group \( SU(N_c) \) in the fundamental representation. The differential cross section to produce a parton with transverse momentum \( \vec{q}_{\perp} \) is given by projecting the eikonal phase onto \( \vec{q}_{\perp} \) and by taking the modulus of the amplitude integrated over all possible impact parameters
\[
\frac{d\sigma}{d^2q_{\perp}} = \frac{1}{(2\pi)^2} \int d^2b d^2b' e^{i \vec{q}_{\perp}(\vec{b}-\vec{b}')} \frac{1}{N_c} \left\langle \text{Tr} \left[ V^\dagger(\vec{b}) V(\vec{b}) \right] \right\rangle.
\] (8)
Hence, a fake dipole of size $\vec{r}_\perp = \vec{b} - \vec{b}'$ is constructed from the ejected parton in the $V$-amplitude and in the $V^\dagger$-amplitude. Their trajectories are displaced from each other by the distance $r_\perp$. The expectation values of the Wilson lines have to be evaluated with respect to the target ground state. In the dipole model, the total cross-section for the interaction of a dipole of size $\vec{r}_\perp$ with a target nucleon is given by

$$\sigma_{dN}(\vec{r}_\perp) = 2 \int d^2b \left( 1 - \frac{1}{N_c} \langle \text{Tr} \left[V^\dagger(\vec{b} + \vec{r}_\perp) V(\vec{b})\right] \rangle \right). \quad (9)$$

We define the quantity $\langle \sigma q^2_{\perp} \rangle$ as the integral over transverse momentum $d^2q_\perp$ of the differential cross section given in Eq. (8) multiplied by $q^2_{\perp}$. Differentiating the phase factor appearing in Eq. (8) twice with respect to the transversal separation and performing the integral over $d^2q_\perp$ one sees that $\langle \sigma q^2_{\perp} \rangle$ is related to the dipole nucleon cross section.

$$\langle \sigma q^2_{\perp} \rangle \equiv \int d^2q_\perp \frac{d\sigma}{d^2q_\perp} q^2_{\perp} = \frac{1}{(2\pi)^2} \int d^2q_\perp \int d^2b d^2r_\perp \left(-\nabla^2_{\perp} e^{i\vec{q}_\perp \cdot \vec{r}_\perp}\right) \frac{1}{N_c} \langle \text{Tr} \left[V^\dagger(\vec{b} + \vec{r}_\perp) V(\vec{b})\right] \rangle = \frac{1}{2} \nabla^2_{\perp} \sigma_{dN}(\vec{r}_\perp) \bigg|_{r_\perp=0}. \quad (10)$$

This expression confirms the result derived in [21]. Because of the $q_{\perp}$-integration only the second derivative of the $r^2_{\perp} = 0$-part in the dipole cross section is relevant for $p_{\perp}$-broadening. This derivative is a constant due to the color transparency behavior of the dipole cross section $\sigma_{dN}(\vec{r}_\perp)_{r_\perp \to 0} \propto r^2_{\perp}$ [16]. We use a form of the $x$-dependent phenomenological dipole nucleon cross section of Ref. [23], which has been adjusted to include soft interactions in Ref. [24]

$$\sigma_{dN}(\vec{r}_\perp) = \sigma_0(s) \left[ 1 - \exp \left(-\frac{\vec{r}_\perp^2}{r^2_0(s)}\right) \right], \quad (11)$$

where $r_0(s) = 0.88 (s/s_0)^{-0.14}$ fm, $s_0 = 1000$ GeV$^2$ and

$$\sigma_0(s) = 23.6 \left( \frac{s}{s_0} \right)^{0.08} \left( 1 + \frac{3}{8} \frac{r^2_0(s)}{0.44 \text{ fm}^2} \right) \text{ mb}. \quad (12)$$

Hence, one determines the parameter $\langle \sigma q^2_{\perp} \rangle \simeq 4.6$ for $\sqrt{s} \simeq 5$ GeV, which is the typical $\sqrt{s}$ for the quark nucleon scattering at HERMES energies.
Notice that the quark-nucleon differential cross section $\sigma$ is infrared divergent. For this reason the mean momentum transfer squared $\langle q^2 \rangle = 0$. Nevertheless, the broadening, $\langle \sigma q^2 \rangle$, is nonzero and finite. It results from infinitely many soft rescatterings with vanishingly small momentum transfers. One can introduce an ad hoc infra-red cutoff and regularize the problem. However, this cutoff does not affect the final result Eq. (10) [21]. The new scale controlling broadening is called saturation momentum $Q_s$. For a quark propagating a path-length $L$ in nuclear medium of density $\rho_0$ the saturation momentum reads

$$
(Q_s^A)^2 = \frac{1}{2} \rho_0 L \sigma_0 (Q_s^N)^2,
$$

(13)

where $\sigma_0$ and $(Q_s^N)^2 = 4/r_0^2$ are defined in terms of the saturated form [23,24] of the dipole cross section, Eq. (11). This increase in saturation scale is identical to the broadening of the mean transverse momentum squared of the quark. One should mention that $Q_s^A$ is the saturation scale for quarks. For gluons, the saturation scale squared is $9/4$ times larger. A review of other approaches to nuclear broadening can be found in Ref. [25]. In this reference, the relevant quantity is $\hat{q}_F = \langle \sigma q^2 \rangle \rho_0$ which is given as $\hat{q}_F = 0.035$ GeV$^2$/fm compared with our determination $\hat{q}_F = 0.032$ GeV$^2$/fm at this dipole energy.

The acquired transverse momentum of the quark given in Eq. (6) can be computed analytically if one uses a hard sphere approximation for the target nucleus which has a homogeneous nuclear density $\rho_0 = (4\pi/3 r_0^3)^{-1}$ with $r_0 = 1.2$ fm

$$
\rho_A(b, z) = \rho_0 \Theta(R_A - b) \Theta(R(b) - |z|), \quad R(b) = \sqrt{R_A^2 - b^2}.
$$

(14)

Cylindrical coordinates $(b, z)$ are favorable due to rotational invariance in the impact parameter plane. Here, the impact parameter of the initial virtual photon is $b$ and $R_A = r_0 A^{1/3}$ denotes the nuclear radius. The hard sphere approximation is reasonable for large nuclei in which the thickness of the boundary of the nucleus is small in comparison to its extension. This approximation gives us some insights into the underlying physics of the $p_\perp$ broadening.
\[ \langle \Delta p^2 \rangle_q = \langle \sigma q^2 \rangle \rho_0 \left\{ l_p \left[ 1 - \frac{1}{\langle S^* \rangle} \cdot \left( \frac{3}{8} \frac{l_p}{R_A} - \frac{1}{64} \left( \frac{l_p}{R_A} \right)^3 \right) \right] \Theta(2R_A - l_p) \right. \\
+ \left. \frac{3}{4} R_A \Theta(l_p - 2R_A) \right\}. \quad (15) \]

The \( p^2 \)-broadening is given by the acquired mean momentum squared per unit length \( \langle \sigma q^2 \rangle \rho_0 \) times the in-medium propagation length of the quark multiplied with the normalized survival probability of the prehadron. This product is represented by the expression in square brackets.

The in-medium propagation length of the quark has two contributions which differ depending on the relation between the prehadron formation length \( l_p \) and the nuclear diameter \( 2R_A \). If the prehadron formation length is larger than the nuclear diameter, then the prehadron is formed outside of the nucleus and the in-medium propagation length of the quark is given by the average nuclear thickness \( 3/4 R_A \) seen by the quark. The prehadron survival probability is equal to one in this case. For prehadron formation lengths which are smaller than the nuclear diameter, the in-medium propagation length of the quark equals the prehadron formation length \( l_p \) plus some higher order corrections in \( l_p/R_A \) which are due to the finite size of the nucleus. These corrections account for the possibility that the prehadron is formed outside of the nucleus such that not the entire prehadron formation length \( l_p \) contributes. The finite size corrections are normalized by the prehadron survival probability, whereas the expression which one would expect for an infinitely extended cold nuclear medium (the term with \( l_p \)) remains unchanged. One should remark, that the corrections due to the finite survival probability of the prehadron are numerically very small in general. If one assumes big homogeneous nuclei for which \( R_A \gg l_p \), the finite size effects and the survival probability become negligible and the acquired \( \Delta p^2 \) is given by

\[ \langle \Delta p^2 \rangle_q = \langle \sigma q^2 \rangle \rho_0. \quad (16) \]

A hadron with momentum fraction \( z_h \) has a \( \langle \Delta p^2 \rangle_h \) reduced by \( z_h^2 \) compared to the quark \( \langle \Delta p^2 \rangle_q \). This is a purely kinematical factor and accounts for the fact that the average \( \langle \Delta p \rangle_q \) is shared among the produced hadrons according to their energy fractions (c.f. also Ref. [26]).

\[ \langle \Delta p^2 \rangle_h = z_h^2 \langle \Delta p^2 \rangle_q. \quad (17) \]
Fig. 1. $p_{\perp}$-broadening as a function of $z_h$ for pions in Ne, Kr and Xe.

For a more realistic computation of the transversal momentum broadening of the quark, we use a Woods-Saxon distribution for the nuclear density in the following. In Fig. 1 we compare our results for $p_{\perp}$ broadening with the HERMES preliminary data [11] for $\pi^+$ and $\pi^-$ for the three nuclei $^{20}$Ne, $^{84}$Kr and $^{132}$Xe as a function of $z_h$. Since the prehadron formation length entering the calculation is a function of $z_h$ and $\nu$, we take for the value of $\nu$ the experimental average in the given $z_h$-bin. As one can see in the plots for the dependence on $z_h$ there is qualitative agreement between the calculation and the preliminary experimental data. The general shape of the $z_h$-dependence has deficiencies: For intermediate $z_h$ the agreement is good in all three cases, but in the small $z_h$ bin the theoretical $\Delta p_{\perp}^2$ for Kr and Xe is too small and in the large $z_h$ bins it is too large. Furthermore, our model does not differentiate between $\pi^+$ and $\pi^-$. In order to do so, one would need to employ a more sophisticated model which allows for flavor dependent prehadron formation lengths (see Ref. [5]). Within the error bars, the preliminary data do not discriminate between $\pi^+$ and $\pi^-$ and the expected effect seems to be small.

In Fig. 2 we display the dependence of $\Delta p_{\perp}^2$ on the photon energy $\nu$. Similar to the $z_h$ plot, we use in the computation of the prehadron formation length
the experimental mean value of $z_h$ in the given $\nu$-bin. Because of the increase of formation time with $\nu$ one would expect that the $p_\perp$-broadening increases with photon energy. In the preliminary data and in the calculation, however, the broadening $\Delta p_\perp^2$ decreases with $\nu$. We think that this is due to the experimental constraints on the kinematics. With increasing $\nu$ the experimental $\langle z_h \rangle$ (typically $z_h \approx 0.35 - 0.45$ here) decreases which lowers the resulting $\Delta p_\perp^2$. We remark that $\Delta p_\perp^2$ at constant $z_h$ increases with $\nu$ in the preliminary CLAS data at J-Lab [27]. The much lower energy of DIS in this experiment, however, may also allow hadron elastic scattering as an important source of nuclear broadening.

Fig. 2. $p_\perp$-broadening of pions in Ne, Kr and Xe as a function of the photon energy $\nu$.

3 Hadronic $p_\perp$-broadening as function of $Q^2$

The variation of $p_\perp$-broadening with the photon virtuality is the third piece of information we have from the preliminary data. Recently an evolution equation has been constructed which includes not only the splitting terms in the evolution of the parton distributions but also a scattering term for the in-
teractions of the parton with the hot medium, i.e. the quark-gluon plasma [28]. We will use the equivalent equation in cold nuclear matter. Soft collisions with the nucleons do not change the virtuality of the parton, only increase its transverse momentum. The “fragmentation functions” $D^h_q(z_h, Q^2, \vec{p}_\perp)$ give the probability for an initial quark $q$ to convert into hadron $h$ with momentum fraction $z_h$, virtuality $Q^2$ and transverse momentum $\vec{p}_\perp$ in the course of the cascade. The evolution equation for this multiple differential function has the following form in vacuum [29]:

$$\frac{\partial D^h_q(z_h, Q^2, \vec{p}_\perp)}{\partial \log(Q^2)} = \frac{\alpha_s(Q^2)}{2\pi} \int \frac{dy}{z_h} P^r_q(y, \alpha_s(Q^2)) \times$$

$$\int \frac{d^2\vec{q}_\perp}{\pi} \delta \left(y(1-y)Q^2 - \frac{Q^2_0}{4} + q^2_\perp\right) D^h_r \left(\frac{z_h}{y}, Q^2, \vec{p}_\perp - \frac{z_h}{y} \vec{q}_\perp\right).$$

(18)

The above equation takes care of the mass constraint $y(1-y)Q^2 = Q^2_0/4 + q^2_\perp$ arising in the splitting with momentum fractions $y$ and $1-y$. The transverse momenta appear together with longitudinal momentum fractions to guarantee boost invariance. After integration over transverse momentum one obtains the standard DGLAP equation. For electron-nucleus collisions we consider the shower inside nuclear matter with a homogeneous nuclear density $\rho_0$. We assume that the medium nucleons change the transverse momentum of the quark by giving $\vec{q}_\perp$ kicks, but they do not change its mass scale or virtuality. Strictly speaking, this is only true for small momentum transfers i.e. small angle scattering. In nuclear matter radiation is interleaved with scattering. Therefore a scattering term $S(z_h, Q^2, \vec{p}_\perp)$ has to be added on the right-hand side of Eq. (18). It has two parts, a gain term for scattering into the given $z_h$-bin under consideration and a loss term.

$$S(z_h, Q^2, \vec{p}_\perp) = \nu Q^2 \rho_0 \int \frac{1}{z_h} \int \frac{d^2\vec{q}_\perp}{\pi} \frac{d\sigma}{d^2\vec{q}_\perp}$$

$$\times \left( D^h_q(w, Q^2, \vec{p}_\perp - w\vec{q}_\perp) - D^h_q(z_h, Q^2, \vec{p}_\perp) \right) \delta \left(w - z_h - \frac{q^2_\perp}{2m_b\nu}\right).$$

(19)

The evolution allows to calculate two different $\langle p^2_\perp \rangle$ from the respective frag-
mentation functions in the nucleus and in the vacuum Eq. (18)

\[ \langle p_{\perp}^2 \rangle = \frac{\int d^2 p_{\perp} D(z, Q^2, p_{\perp})}{\int d^2 p_{\perp} D(z, Q^2, p_{\perp})}. \]

However, the \( \langle p_{\perp}^2 \rangle \) defined in this equation is solely coming from the evolution. The medium modification of the DGLAP evolution gives the difference of mean transverse momentum generated in the evolution in the nucleus and in the vacuum. This piece is an additional contribution to the multiple scattering contribution \( (\Delta p_{\perp}^2)_h(Q^2) \) which we fix at \( \bar{Q}^2 = 2.5 \) GeV\(^2\) to the preliminary data. In the calculation of the mean transverse momentum broadening of the hadron the same averaged \( \langle \sigma q_{\perp}^2 \rangle \) of the quark and the factor \( z_h^2 \) converting quark to hadron transversal momentum squared appears naturally together with transverse momentum integrated fragmentation function of this specific hadron divided by this hadron multiplicity.

\[ (\Delta p_{\perp}^2)_h(Q^2) = (\Delta p_{\perp}^2)_h(Q^2) + z_h^2 \nu \rho_0 \langle \sigma q_{\perp}^2 \rangle \left( \frac{1}{Q^2} - \frac{1}{\bar{Q}^2} \right). \]

To lowest order, \( \Delta p_{\perp}^2 \) is generated by the scattering term which gives a higher twist contribution to the evolution from \( \bar{Q}^2 \) to \( Q^2 \). For \( Q^2 > \bar{Q}^2 \), the evolution enhances the mean \( p_{\perp}^2 \) and for \( Q^2 < \bar{Q}^2 \) the devolution decreases the mean \( p_{\perp}^2 \). Although HERMES preliminary data are for relatively small photon virtualities of \( Q^2 = 1.5 - 4.5 \) GeV\(^2\), this yields a sizeable effect for \( \Delta p_{\perp}^2 \). As one sees in Fig. 3 the calculated \( Q^2 \)-dependence is in good qualitative agreement with the preliminary data. It is very encouraging to see that the formalism of modified evolution equations which we proposed for the quark-gluon plasma can also be related to deep inelastic scattering on nuclei.

In the plasma, the corresponding scattering term was causing a suppression of the fragmentation function in the medium i.e. jet quenching [28]. However, the situation in cold nuclear matter is new: The scattering term is much smaller and the medium fragmentation functions \( D(z, Q^2) \) are almost unchanged. To lowest order, the medium-induced \( (\Delta p_{\perp}^2)_h(Q^2) \) is originating only from the higher twist scattering term \( \propto 1/Q^2 \). Scaling violations due to higher twist effects have been considered in [30,31] for processes with a large \( z_h \).
Fig. 3. $p_{\perp}$-broadening of pions in Ne, Kr and Xe as a function of the photon virtuality $Q^2$.

The dependence of $\Delta p^2_{\perp}$ on $Q^2$ is also discussed in Ref. [32]. This reference also suggests two additional mechanisms as possible sources for the $Q^2$-dependence, namely NLO processes like photon-gluon fusion and possible colored prehadrons which lose energy by gluon bremsstrahlung.

4 Discussion

We have calculated $p_{\perp}$-broadening in transverse momentum distributions from the dipole model and compared it to recent HERMES preliminary data. We find qualitative agreement with the $\nu h^{-}, \nu^{-}$ and $Q^2$-dependences of $p_{\perp}$-broadening. The dependence on the photon virtuality has been calculated with a modified DGLAP evolution equation. Finally we have estimated the effect of absorption of the prehadronic state for $p_{\perp}$-broadening.

In a recent paper [32] the nuclear multiplicity ratio $R_M$ has been related to $p_{\perp}$ broadening, using a similar picture of hadronization [35]. In this paper, the prehadron formation time is extracted from the multiplicity ratio to $t_p \equiv$
$l_p \propto 0.8 \kappa^{0.5} \nu^{0.5}(1 - z_h)$. This formation time is similar to Eq. (1) but up to 30% smaller at mid $z_h$. On the other hand the difference between hadronic and partonic $\Delta p^2_\perp$ is not spelled out.

There is another question which needs to be addressed: How do the HERMES preliminary data [11] match with the preliminary CLAS data [27]? The main difference between CLAS and HERMES is the beam energy, which is 2–5 GeV in CLAS in contrast to 7 – 23 GeV at HERMES. Therefore one expects that the prehadron formation time $l_p$ is smaller by a factor $\gtrsim 3$ at CLAS and consequently also the resulting hadronic broadening $\Delta p^2_\perp$ would be much smaller. In the CLAS experiment, however, effects for hadronic $\Delta p^2_\perp$ are of similar magnitude as in the HERMES experiment. A possible explanation can be that the prehadron stage contributes to the hadronic broadening. At these low energies the pion-nucleon elastic cross section is of the same magnitude as the inelastic cross section. Therefore elastic scattering competes with absorption for the outgoing prehadron. Since the angular distribution of pion-nucleon scattering has still sizeable contributions from $u$-channel exchange, large transverse momentum exchanges are possible. A good check is possible when the whole angular distribution of the produced hadron is measured. Another important feature of the preliminary CLAS data is the linear rise with $\nu$ which possibly saturates at $\nu \simeq 4$ GeV. This linear rise is consistent with $\Delta p^2_\perp \propto l_p \propto \nu$ as proposed in Eq. (10). As discussed, at CLAS hadronization may set in inside the nucleus in contrast to HERMES. This does require also a careful analysis of the energy dependence of elastic prehadronic scatterings. Therefore in our three stage model of hadronization, the second and third step play a more important role.

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