The $D$-wave heavy-light mesons from QCD sum rules

Dan Zhou$^1$, Er-Liang Cui$^1$, Hua-Xing Chen$^{1,2}$, Li-Sheng Geng$^1$, Li-Xiang Liu$^{2,3,4}$, and Shi-Lin Zhu$^{4,5,6,7}$

$^1$School of Physics and Nuclear Energy Technology and International Research Center for Nuclear and Particles in the Cosmos, Beihang University, Beijing 100191, China
$^2$School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China
$^3$Research Center for Hadron and CSR Physics, Lanzhou University and Institute of Modern Physics of CAS, Lanzhou 730000, China
$^4$School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China
$^5$Collaborative Innovation Center of Quantum Matter, Beijing 100871, China
$^6$Center of High Energy Physics, Peking University, Beijing 100871, China

We study the $D$-wave $\bar{c}s$ heavy meson doublets ($1^−, 2^−$) and ($2^−, 3^−$) using the method of QCD sum rule in the framework of heavy quark effective theory. Choosing the same threshold values $\omega_{hk}$ around 2.7 GeV, we calculate the masses of the $1^−$ and $3^−$ states. They are $m_{D_{10}} = 2.81 \pm 0.10$ GeV and $m_{D_{30}} = 2.85 \pm 0.08$ GeV, consistent with the newly observed $D_{10}^*(2860)$ and $D_{30}^*(2860)$ states by LHCb. The masses of their $2^−$ partners are calculated to be $2.82 \pm 0.10$ and $2.81 \pm 0.08$ GeV, with large uncertainties. However, the mass splittings within the same doublet are calculated to be $m_{D_{10}} - m_{D_{1}} = 0.016 \pm 0.007$ GeV and $m_{D_{30}} - m_{D_{3}} = 0.039 \pm 0.014$ GeV, with much smaller uncertainties.

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I. INTRODUCTION

Since the observation of $D_{2S}^*(2317)$ in 2013 [1], more and more charmed-strange mesons have been reported experimentally, which include $D_{1S}(2460)$ [2], $D_{1J}(2710)$ [3, 4], $D_{2S}(2860)$ [4, 5], and $D_{3S}(3040)$ [4] (see Ref. [6] for a concise review). Very recently, the LHCb Collaboration announced the observation of two charmed-strange mesons $D_{1S}(2860)$ and $D_{3S}(2860)$ with the resonance parameters [7, 8]:

$$m_{D_{1S}}(2860) = (2859 \pm 12 \pm 6 \pm 23) \text{ MeV},$$

$$m_{D_{3S}}(2860) = (2860.5 \pm 2.6 \pm 2.5 \pm 6.0) \text{ MeV},$$

$$\Gamma_{D_{1S}}(2860) = (159 \pm 23 \pm 27 \pm 72) \text{ MeV},$$

$$\Gamma_{D_{3S}}(2860) = (53 \pm 7 \pm 4 \pm 6) \text{ MeV}.$$

In addition, LHCb specified that it is the first time to identify a spin-3 resonance $D_{3S}^*(2860)$ [7, 8]. At present, the charmed-strange meson family is becoming more and more abundant with the experimental progress.

Until now, there are good candidates of the $1S$ and $1P$ states in the charmed-strange meson family [9]. These newly observed charmed-strange mesons provide a good platform to study the properties of the higher radial and orbital excitations of the charmed-strange meson. For example, in Ref. [10] Sun and Liu suggested that $D_{2S}(3040)$ can be a good candidate of the $2P$ state in the charmed-strange meson family, which is the radial excitation of $D_{1S}(2460)$. The recently reported $D_{1S}(2860)$ and $D_{3S}(2860)$ states stimulated extensive discussions of whether they can be categorized into the $1D$ charmed-strange mesons [11–13]. In Ref. [11], the two-body strong decays of $D_{1S}(2860)$ and $D_{3S}(2860)$ as the $1^D_1$ and $1^3D_3$ states in charmed-strange meson family were studied by the quark pair creation model, which shows that $D_{1S}(2860)$ and $D_{3S}(2860)$ are the $1^D_1$ and $1^3D_3$ states, respectively. Later, Wang studied $D_{1S}(2860)$ and $D_{3S}(2860)$ using the effective Lagrangian approach [12]. Recently, Godfrey and Moats [13] indicated that $D_{1S}(2860)$ and $D_{3S}(2860)$ are the $1^D_1$ and $1^3D_3$ charmed-strange mesons, respectively. Thus, the results in Refs. [12, 13] supports the assignment of $D_{1S}(2860)$ and $D_{3S}(2860)$ proposed in Ref. [11].

In this paper we shall use the method of QCD sum rule to study the $D$-wave heavy meson doublets ($1^−, 2^−$) and ($2^−, 3^−$) containing one heavy anti-quark and one strange quark [14, 15]. We shall work in the framework of the heavy quark effective theory (HQET) [16–18], which has been successful to study heavy hadrons containing a single heavy quark. The mass of the ground state heavy mesons was studied in Refs. [19–24]. The masses of the lowest excited non-strange heavy meson doublets ($0^+, 1^+$) and ($1^+, 2^+$) were studied in Refs. [26–28]. The masses of the lowest excited $\bar{c}s$ heavy mesons in the ($0^+, 1^+$) and ($1^+, 2^+$) doublets were studied in Ref. [29]. There were also some early studies using the method of QCD sum rules but in full QCD [30, 31]. In this paper we shall follow the procedures used in Refs. [25–27, 29], and study the $D$-wave $\bar{c}s$ heavy meson in the ($1^−, 2^−$) and ($2^−, 3^−$) doublets. We shall also follow Refs. [25–27, 29] and consider the $O(1/m_Q)$ corrections, where $m_Q$ is the heavy quark mass.

This paper is organized as follows. In Sec. II, we introduce the interpolating currents for the $D$-wave $\bar{c}s$ heavy meson doublets ($1^−, 2^−$) and ($2^−, 3^−$), and use them to perform QCD sum rule analyses at the leading order. Then in Sec. III we calculate the $O(1/m_Q)$ corrections. The results are summarized and discussed in Sec. IV.
II. THE SUM RULES AT THE LEADING ORDER (IN THE $m_0 \to \infty$ LIMIT)

The interpolating currents for the heavy mesons with arbitrary spin and parity have been studied and given in Refs. [25–27]. Accordingly, we shall use the following interpolating currents to study the D-wave (1, 2) spin doublet:

$$ J_{1-3/2}^{\alpha} = \frac{\sqrt{3}}{4} \bar{h}_t \langle \gamma^0 \rangle \frac{(-i)}{2} \gamma^0 \gamma^0 \gamma^0 \gamma^1 | D_1^a, D_2^b, D_3^c \rangle q, $$

$$ J_{2-5/2}^{\alpha} = \frac{\sqrt{3}}{4} \bar{h}_t \langle \gamma^0 \rangle \frac{(-i)}{2} \gamma^0 \gamma^0 \gamma^0 \gamma^1 | D_1^a, D_2^b, D_3^c \rangle q, \tag{1} $$

and the following interpolating currents to study the D-wave (2, 3) spin doublet:

$$ J_{2-3/2}^{\alpha_2 \alpha_3} = \frac{\sqrt{3}}{4} \bar{h}_t \gamma^0 \frac{(-i)}{2} \gamma^0 \gamma^0 \gamma^0 \gamma^1 | D_1^a, D_2^b, D_3^c \rangle q, $$

$$ J_{3-5/2}^{\alpha_2 \alpha_3} = \frac{\sqrt{3}}{4} \bar{h}_t \gamma^0 \frac{(-i)}{2} \gamma^0 \gamma^0 \gamma^0 \gamma^1 | D_1^a, D_2^b, D_3^c \rangle q, \tag{2} $$

where $\gamma^0 = \gamma^0 - i\gamma^5$ is the gauge-covariant derivative, $h_t$ is the heavy quark field in HQET, $v$ is the velocity of the heavy quark, and $\eta_{\alpha_2 \alpha_3}$ is the transverse metric tensor.

The two currents $J_{1-3/2}$ and $J_{2-3/2}$ give identical sum rules at the leading order, and the sum rules at the $O(1/m_0)$ order can be obtained using either of them [25–27, 29] (ideally the results should be identical, while actually they have small differences but negligible).

In the $m_0 \to \infty$ limit we can assume $| j, P, j \rangle$ to be the heavy meson state with the quantum numbers $j, P$, and $j_1$, and the relation between this state and the relevant interpolating field is

$$ \langle 0 | J_{1-3/2}^{\alpha} | j, P, j \rangle = f_{P,j} \delta_{j,j_1} \delta_{P,P} \delta_{j_1,j'_1} \eta_{\alpha}, $$

where $f_{P,j}$ is the decay constant. It has the same value for the two states in the same doublet in the $m_0 \to \infty$ limit. $\eta_{\alpha}$ is the transverse, symmetric, and traceless polarization tensor.

In this paper we need to use $\eta_{\alpha}$ and $\eta_{\alpha_2 \alpha_3}$ which have the following property at the leading order

$$ \eta_{\alpha}^{\alpha} = \delta_{\alpha \alpha}, $$

$$ \eta_{\alpha_2 \alpha_3}^{\alpha} = S_2 \epsilon_{\alpha_2 \alpha_3} g_{\beta_2 \beta_3}^{\alpha} = S_2 \epsilon_{\alpha_2 \alpha_3} g_{\beta_2 \beta_3}^{\alpha}, \tag{6} $$

where $\gamma^\mu = g^{\mu \nu} - q^\mu q^\nu / m^2$ and $S_2$ denotes symmetrization and subtracting the trace terms in the sets ($\alpha_1 \alpha_2 \alpha_3$) and ($\beta_1 \beta_2 \beta_3$).

Using the two interpolating currents $J_{1-3/2}$ and $J_{2-5/2}$, we can construct the two-point correlation function

$$ \Pi_{J_{1-3/2} J_{2-5/2}}(\omega) = i \int d^4x e^{ikx} \langle 0 | T [ J_{1-3/2}^{\alpha} (x) J_{2-5/2}^{\beta} (0)] 0 \rangle $$

$$ = (-1) S_3 \langle g_{\alpha_1 \beta_1} \cdots g_{\alpha_3 \beta_3} \rangle \Pi_{J_{1-3/2} J_{2-5/2}}(\omega), \tag{8} $$

where $\omega = 2v \cdot k$ is twice the external off-shell energy, and $S_3$ denotes symmetrization and subtracting the trace terms in the sets ($\alpha_1 \cdots \alpha_3$) and ($\beta_1 \cdots \beta_3$). At the hadron level, it can be written as

$$ \Pi_{J_{1-3/2} J_{2-5/2}}(\omega) = \frac{f_{P,j}^2}{2 \Lambda_{J_{1-3/2}} - \omega} + \text{higher states}, \tag{9} $$

where $\Lambda_{J_{1-3/2}} = \lim_{m_0 \to \infty} (m_{J_{1-3/2}} - m_Q)$, and $m_{J_{1-3/2}}$ is the mass of the lowest-lying heavy meson state which $J_{1-3/2}(x)$ couples to. At the quark and gluon level, we can calculate the two-point correlation function (8) using the method of QCD sum rule. To do this we follow the approaches used in Refs. [25–27, 29]. After inserting Eq. (1) and (4) into Eq. (8), and performing the Borel transformation, we obtain

$$ \Pi_{J_{1-3/2} J_{2-5/2}}(\omega, T) = \int_{-2\Lambda_{J_{1-3/2}}/T}^{\infty} \frac{d\omega}{2\Lambda_{J_{1-3/2}} - \omega} $$

$$ = \frac{7}{2560\pi^2} \int_{-2m_t}^{\infty} [\omega^6 + 2m_t^2 \alpha^5 - 10m_t^2 \omega^4] e^{-\omega^4/T} d\omega $$

$$ - \frac{1}{8\pi} \langle \sigma, GG \rangle T^3, \tag{10} $$

$$ \Pi_{J_{2-5/2} J_{3-7/2}}(\omega, T) = \int_{-2\Lambda_{J_{2-5/2}}/T}^{\infty} \frac{d\omega}{2\Lambda_{J_{2-5/2}} - \omega} $$

$$ = \frac{1}{640\pi} \int_{-2m_t}^{\infty} [\omega^6 + 2m_t^2 \alpha^5 - 10m_t^2 \omega^4] e^{-\omega^4/T} d\omega $$

$$ - \frac{3}{32\pi} \langle \sigma, GG \rangle T^3. \tag{11} $$

We note that there are $2 \times 2 = 4$ derivatives, and so the calculations are not easy. To deal with them, we have used a software called Mathematica with a package called FeynCalc [32].

Particularly, the quark condensate $\langle \bar{q}q \rangle$ and the mixed condensate $\langle \bar{q}q \sigma Gq \rangle$ both vanish in this case. This is much different from those sum rules for $(0^+, 1^+)$ and $(1^+, 2^+)$ doubllets [25–27, 29], and it makes the convergence of Eq. (10) and (11) very good. To clearly see this, we show the convergence of Eq. (11) in Fig. 1, where $\omega_\pi$ is taken to be 2.7 GeV, and the following values for the condensates and other parameters are used [25–27, 29, 33]:

$$ \langle \bar{q}q \rangle = -0.24 \text{ GeV}^3, $$

$$ \langle \sigma, GG \rangle = 0.005 \pm 0.004 \text{ GeV}^4, $$

$$ m_\pi = 0.15 \text{ GeV}, $$

$$ \langle \bar{q}q \sigma Gq \rangle = M_0^2 \times \langle \bar{q}q \rangle, \tag{12} $$

$$ M_0^2 = 0.8 \text{ GeV}^2. $$
Finally, we differentiate Eqs. (10) and (11) with respect to $-2/T$, divide the results by themselves, and obtain

$$\Lambda_{j,P/j}(\omega_c, T) = \frac{\partial}{\partial \omega - 2/T} \Pi_{j,P/j}(\omega_c, T).$$

(13)

The results can be furtherly used to evaluate $f_{P,j}$:

$$f_{P,j}(\omega_c, T) = \sqrt{\Pi_{j,P/j}(\omega_c, T) \times e^{2\Lambda_{j,P/j}(\omega_c, T)/T}}.$$

(14)

Here we note again that the sum rule obtained by using the high-order power corrections be less than $30\%$ of the pole contribution for $J_{3/2}$ and continuum contributions for $J_{5/2}$. We obtain the maximum value $\omega_c$ in the region $0.25\, \text{GeV} < T < 0.55\, \text{GeV}$, but we find that their dependence on the Borel mass $T$ becomes weaker in our working region $0.35\, \text{GeV} < T < 0.48\, \text{GeV}$. We obtain the following numerical results:

$$\Lambda_{-3/2} = 1.10 \pm 0.06\, \text{GeV},$$

(16)

$$f_{-3/2} = 0.19 \pm 0.05\, \text{GeV}^{7/2},$$

(17)

where the central value corresponds to $T = 0.42\, \text{GeV}$ and $\omega_c = 2.7\, \text{GeV}\). We obtain the following numerical results:

$$\Lambda_{-5/2} = 1.14 \pm 0.05\, \text{GeV},$$

(18)

$$f_{-5/2} = 0.15 \pm 0.04\, \text{GeV}^{7/2},$$

(19)

where the central value corresponds to $T = 0.43\, \text{GeV}$ and $\omega_c = 2.7\, \text{GeV}\). We obtain the following numerical results:

These figures are shown in the region $0.25\, \text{GeV} < T < 0.55\, \text{GeV}$, but we find that their dependence on the Borel mass $T$ becomes weaker in our working region $0.35\, \text{GeV} < T < 0.48\, \text{GeV}$. We obtain the following numerical results:

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Finally, we solve Eq. (13) and Eq. (14), and evaluate $\Lambda_{-j_{hi}}$ and $f_{-j_{hi}}$. We show the variations of $\Lambda_{-3/2}$ and $f_{-3/2}$ with respect to the Borel mass $T$ and the threshold value $\omega_c$ in Fig. 3.

Finally, we solve Eq. (13) and Eq. (14), and evaluate $\Lambda_{-j_{hi}}$ and $f_{-j_{hi}}$. We show the variations of $\Lambda_{-3/2}$ and $f_{-3/2}$ with respect to the Borel mass $T$ and the threshold value $\omega_c$ in Fig. 3.

III. THE SUM RULES AT THE $O(1/m_Q)$ ORDER

The Lagrangian of HQET, up to the $O(1/m_Q)$ order, can be written as [27, 29]

$$\mathcal{L}_{\text{eff}} = \bar{h}_c i\gamma_5 Dm_c h_c + \frac{1}{2m_Q} K + \frac{1}{2m_Q} S,$$

(20)

where $K$ is the operator of the nonrelativistic kinetic energy with a negative sign:

$$K = \bar{h}_c (iD_+)^2 h_c,$$

(21)

and $S$ is the Pauli term to describe the chromomagnetic interaction:

$$S = \frac{g}{2} C_{\text{mag}}(m_Q/\mu) \bar{h}_c \sigma_{\mu\nu} G^{\mu\nu} h_c,$$

(22)
where \( C_{mag}(m_\Omega/\mu) = [\alpha_s(m_\Omega)/\alpha_s(\mu)]^{3/5} \) and \( \beta_0 = 11 - 2n_f/3 \).

We use \( \delta m \) and \( \delta f \) to denote the corrections to the mass \( m_{j,P_{ji}} \) and the coupling constant \( f_{P_{ji}} \) at the \( O(1/m_\Omega) \) order. The pole term on the hadron side, Eq. (9), can be written as:

\[
\Pi(\omega)_{\text{pole}} = \frac{(f + \delta f)^2}{2(\Lambda + \delta m) - \omega} = \frac{f^2}{2\Lambda - \omega} - \frac{2\delta mf^2}{(2\Lambda - \omega)^2} + \frac{2f\delta f}{2\Lambda - \omega}.
\]

In this paper we shall only evaluate \( \delta m \). To do this, we use the Lagrangian (20) defined at the \( O(1/m_\Omega) \) order, and consider the following three-point correlation functions

\[
\delta \Pi_{j,P_{ji}}^{\alpha_i\beta_i\gamma}(\omega,\omega') = \int d^4x d^4y e^{ik\cdot x - ik'\cdot y} \langle 0|T[J_{j,P_{ji}}^{\alpha_i\beta_i\gamma}(x)O(0)J_{j,P_{ji}}^{\alpha_i\beta_i\gamma}(y)]|0\rangle
\]

\[
= (-1)^j S_2[g_2^{\alpha_i\beta_i} \cdots g_2^{\alpha_i\beta_i}] \delta \Pi_{j,P_{ji}}(\omega),
\]

where \( O = K \) or \( S \). At the hadron level, we can pick their pole parts

\[
\delta \Pi(\omega,\omega')_{j,P_{ji}} = \frac{f^2 K_{P_{ji}}}{(2\Lambda - \omega)(2\Lambda - \omega')} + \frac{f^2 G_K(\omega')}{2\Lambda - \omega} + \frac{f^2 G_K(\omega)}{2\Lambda - \omega'},
\]

\[
\delta S(\omega,\omega')_{j,P_{ji}} = \frac{d_M f^2 S_{P_{ji}}}{(2\Lambda - \omega)(2\Lambda - \omega')} + \frac{d_M f^2 G_S(\omega')}{2\Lambda - \omega} + \frac{d_M f^2 G_S(\omega)}{2\Lambda - \omega'},
\]

where

\[
K_{P_{ji}} = \langle j, P, j| h_\pi(iD_\perp)^2 h_\pi| j, P, j \rangle,
\]

\[
2d_M S_{P_{ji}} = \langle j, P, j| g_{\pi\rho}^a \sigma_{\mu
u} G_{\rho\nu}^a h_\pi| j, P, j \rangle,
\]

\[
d_M = d_{j,i},
\]

\[
d_{j-1/2,i} = 2j_i + 2,
\]

\[
d_{j+1/2,i} = -2j_i.
\]

From these equations we know that the term \( S \) causes a mass splitting within the same doublet, while the term \( K \) does not. Moreover, the term \( S \) can also cause a mixing of states with the same \( j, P \) but different \( j_i \), such as a mass splitting between \( |2, -3/2 \rangle \) and \( |2, -5/2 \rangle \). This effect has been studied.
in Ref. [34], where its corrections are found to be negligible. Hence, we do not consider this effect in this paper.

Fixing $\omega = \omega'$ and comparing Eq. (23), Eq. (25) and Eq. (26), we obtain

$$\delta m_{P,j} = -\frac{1}{4m_Q} (K_{P,j} + d_M C_{\text{max}} \Sigma_{P,j}).$$ (28)

At the quark and gluon level, we can calculate Eqs. (24) using the method of QCD sum rule, and evaluate $K_{P,j}$ and $\Sigma_{P,j}$. To do this, again we follow the approaches used in Refs. [27, 29]: after inserting Eq. (1) and (4) into Eq. (24), we make a double Borel transformation for both $\omega$ and $\omega'$, and obtain two Borel parameters $T_1$ and $T_2$. Then we take these two Borel parameters to be equal, and obtain the following two sum rules for $K_{-3/2}$ and $\Sigma_{-3/2}$:

$$f_{-3/2}^2 K_{-3/2} e^{-2\Lambda_{-3/2}/T} = \frac{11}{7168\pi} \int^{\omega_c} \omega^8 e^{-\omega/T} d\omega + \frac{91}{64\pi} (\alpha_s G G)' T^5,$$ (29)

$$f_{-3/2}^2 \Sigma_{-3/2} e^{-2\Lambda_{-3/2}/T} = \frac{7}{240\pi} (\alpha_s G G)' T^5,$$ (30)

and the following two sum rules for $K_{-5/2}$ and $\Sigma_{-5/2}$:

$$f_{-5/2}^2 K_{-5/2} e^{-2\Lambda_{-5/2}/T} = -\frac{1}{1280\pi} \int^{\omega_c} \omega^8 e^{-\omega/T} d\omega + \frac{71}{96\pi} (\alpha_s G G)' T^5,$$ (31)

$$f_{-5/2}^2 \Sigma_{-5/2} e^{-2\Lambda_{-5/2}/T} = \frac{1}{40\pi} (\alpha_s G G)' T^5.$$ (32)

We note that in these sum rules the $m_s$ corrections are neglected.

Finally, we obtain $K_{-3/2}$ and $\Sigma_{-3/2}$ by simply dividing Eq. (29) and (30) by the sum rule (10), and $K_{-5/2}$ and $\Sigma_{-5/2}$ by simply dividing Eq. (31) and (32) by the sum rule (11). We show the variations of $K_{-3/2}$ and $\Sigma_{-3/2}$ with respect to the Borel mass $T$ and the threshold value $\omega_c$ in Fig. 5 in the region $0.25 \text{ GeV} < T < 0.55 \text{ GeV}$, and their dependence on the Borel mass $T$ becomes weaker in our working region $0.35 \text{ GeV} < T < 0.48 \text{ GeV}$. We obtain the following numerical results:

$$K_{-3/2} = -2.25 \pm 0.36 \text{ GeV}^2,$$ (33)

$$\Sigma_{-3/2} = 0.010 \pm 0.004 \text{ GeV}^2,$$ (34)

where the central value corresponds to $T = 0.42 \text{ GeV}$ and $\omega_c = 2.7 \text{ GeV}$.

Similarly we show the variations of $K_{-5/2}$ and $\Sigma_{-5/2}$ in Fig. 6 in the region $0.30 \text{ GeV} < T < 0.60 \text{ GeV}$, and their dependence on the Borel mass $T$ becomes weaker in our working region $0.39 \text{ GeV} < T < 0.47 \text{ GeV}$. We obtain the following numerical results:

$$K_{-5/2} = -2.16 \pm 0.28 \text{ GeV}^2,$$ (35)

$$\Sigma_{-5/2} = 0.017 \pm 0.006 \text{ GeV}^2,$$ (36)

where the central value corresponds to $T = 0.43 \text{ GeV}$ and $\omega_c = 2.7 \text{ GeV}$.

![Fig. 5: The variation of $K_{-3/2}$ and $\Sigma_{-3/2}$ with respect to the Borel mass $T$ and the threshold value $\omega_c$. The short-dashed, solid and long-dashed curves are obtained by fixing $\omega_c = 2.5$, 2.7 and 2.9 GeV, respectively. Our working region is $0.35 \text{ GeV} < T < 0.48 \text{ GeV}$.](image)

**IV. NUMERICAL RESULTS AND DISCUSSIONS**

Combining the results obtained in Sec. II and Sec. III, we arrive at the following weighted average mass for the $D$-wave heavy meson doublet ($1^-, 2^-$):

$$m_c (3m_{D_{st}}, 5m_{D_{sl}}) = m_c + (1.10 \pm 0.06) \text{ GeV}$$

$$+ \frac{1}{m_c} [(0.56 \pm 0.09) \text{ GeV}^2],$$ (37)

where $D_{sl}$ is used to denote the $2^-$ partner of $D_{st}^*$. Their mass splitting is:

$$m_{D_{st}} - m_{D_{sl}} = \frac{1}{m_c} [(0.021 \pm 0.008) \text{ GeV}^2].$$ (38)

From these values, we find that the $O(1/m_Q)$ corrections are important and can not be neglected. Moreover, they have large uncertainties. So do the results at the leading order, $\Lambda_{-3/2}$. This makes our results also have large uncertainties, except their mass splitting:

$$m_{D_{st}} = 2.81 \pm 0.10 \text{ GeV},$$

$$m_{D_{sl}} = 2.82 \pm 0.10 \text{ GeV},$$ (39)

$$m_{D_{sl}} - m_{D_{st}} = 0.016 \pm 0.007 \text{ GeV},$$

$$m_{D_{st}} - m_{D_{sl}} = 0.008 \pm 0.007 \text{ GeV}.$$
where we have used the PDG value \( m_c = 1.275 \pm 0.025 \text{ GeV} \) [35]. The mass of \( 1^- \) state is \( 2.81 \pm 0.10 \text{ GeV} \), consistent with the \( D_{s1}^* (2860) \) newly observed by LHCb, \( m_{Ds1}^{\exp} = 2859 \pm 12 \pm 6 \pm 23 \text{ MeV} \) [7].

Similarly, we obtain the following weighted average mass for the \( D^- \)-wave \( (2^- , 3^-) \) heavy meson doublet:

$$
\frac{1}{12} \left( 5m_{D_{s2}} + 7m_{D_{s3}} \right) = m_c + (1.14 \pm 0.05) \text{ GeV} + \frac{1}{m_c} \left[ (0.54 \pm 0.07) \text{ GeV}^2 \right],
$$

(40)

where \( D_{s2}^- \) is used to denote the \( 2^- \) partner of \( D_{s3}^- \). Their mass splitting is:

$$
m_{D_{s3}^-} - m_{D_{s2}^-} = \frac{1}{m_c} \left[ (0.050 \pm 0.018) \text{ GeV}^2 \right].
$$

(41)

Again we find that the \( O(1/m_c) \) corrections are important and have large uncertainties. Our results are:

$$
m_{D_{s2}^-} = 2.81 \pm 0.08 \text{ GeV},
\quad
m_{D_{s3}^-} = 2.85 \pm 0.08 \text{ GeV},
\quad
m_{D_{s3}^-} - m_{D_{s2}^-} = 0.039 \pm 0.014 \text{ GeV}.
$$

The mass of the \( 3^- \) state is \( 2.85 \pm 0.08 \text{ GeV} \), also consistent with the \( D_{s3}^*(2860) \) newly observed by LHCb, \( m_{Ds3}^{\exp} = 2860.5 \pm 2.6 \pm 2.5 \pm 6.0 \text{ MeV} \) [7].

The \( \bar{b}s \) system can be similarly studied by replacing \( m_b \) by \( m_b \) and multiplying \( \Sigma_{-i} \) by \( C_{mag} \approx 0.8 \) [27, 29]. Here we only give their mass differences within the same doublet because their mass depends much on the bottom quark mass \( m_b \), whose value has large uncertainties. Using the same threshold values \( \omega_k \) around 3.3 GeV and assuming \( 4 \text{ GeV} < m_b < 5 \text{ GeV} \), we obtain the mass differences within the same doublet:

$$
m_{B_{s1}^-} - m_{B_{s2}^-} = 0.004 \pm 0.002 \text{ GeV},
\quad
m_{B_{s3}^-} - m_{B_{s2}^-} = 0.009 \pm 0.004 \text{ GeV}.
$$

(43)

We can similarly replace the strange quark by up and down quarks and extract the masses of the non-strange \( D^- \)-wave heavy mesons. To do this we use slightly smaller threshold values \( \omega_k \sim 2.5 \text{ GeV} \), and obtain the working region \( 0.39 \text{ GeV} < T < 0.43 \text{ GeV} \) for \( (1^- , 2^-) \) doublet. However, there is no stability window for \( (2^- , 3^-) \) doublet, unless we require the pole contribution to be greater than 20% only, and now the working region is \( 0.46 \text{ GeV} < T < 0.49 \text{ GeV} \). The numerical results are:

$$
m_{D_{s2}^{'}} = 2.75 \pm 0.09 \text{ GeV},
\quad
m_{D_{s3}^{'}} = 2.78 \pm 0.09 \text{ GeV},
\quad
m_{D_{s2}^{'}} - m_{D_{s3}^{'}} = 0.02 \pm 0.01 \text{ GeV},
\quad
m_{D_{s2}^{'}} - m_{D_{s3}^{'}} = 0.02 \pm 0.01 \text{ GeV},
\quad
m_{D_{s3}^{'}} - m_{D_{s2}^{'}} = 0.06 \pm 0.03 \text{ GeV}.
$$

(44)

Again we note that the masses have large uncertainties, but their differences within the same doublet are produced quite well.

In summary, we have studied the \( D^- \)-wave \( (1^- , 2^-) \) and \( (2^- , 3^-) \) heavy meson doublets and calculated their masses up to the \( O(1/m_Q) \) order using the method of QCD sum rule in the framework of HQET. The masses of \( 1^- \) and \( 2^- \) states are calculated to be \( m_{D_{s2}} = 2.81 \pm 0.10 \text{ GeV} \) and \( m_{D_{s3}} = 2.85 \pm 0.08 \text{ GeV} \), consistent with the newly observed \( D_{s1}^*(2860) \) and \( D_{s3}^*(2860) \) states by LHCb [7]. In our calculations we have chosen the same threshold value \( \omega \approx 2.7 \text{ GeV} \) for both of them, and obtained a mass difference between \( D_{s1}^* \) and \( D_{s3}^* \) to be 0.04 GeV. Considering the mass uncertainties are about 0.1 GeV, our results are consistent with the experimental data [7]. The masses of their \( 2^- \) partners are calculated to be \( 2.82 \pm 0.10 \text{ GeV} \) and \( 2.81 \pm 0.08 \text{ GeV} \), with large uncertainties. However, the mass splittings within the same doublet are reproduced quite well, i.e., \( m_{D_{s2}} - m_{D_{s3}} = 0.016 \pm 0.007 \text{ GeV} \) and \( m_{D_{s3}} - m_{D_{s2}} = 0.039 \pm 0.014 \text{ GeV} \). We have also estimated their decay constants at the leading order (in the \( m_Q \rightarrow \infty \) limit), that is \( f_{-3/2} = 0.19 \pm 0.05 \text{ GeV}^{-1/2} \) and \( f_{-5/2} = 0.15 \pm 0.04 \text{ GeV}^{-1/2} \).

At present, the two \( 2^- \) charmed-strange mesons are still missing. The predicted masses of these two \( 2^- \) charmed-strange mesons in this work can be further tested by future experiments. We also expect more experimental progresses on higher radial and orbital excitations in the charmed-strange meson family. We also obtained the two decay constants \( f_{-3/2} \) and \( f_{-5/2} \), both of which are important input parameters.
when performing the dynamical study relevant to the $D$-wave charmed-strange mesons.

With the running of the LHCb experiment, it is an exciting time to explore the higher charmed-strange mesons. The experimental and theoretical efforts will establish the charmed-strange meson family step by step, which is a research area full of challenges and opportunities.

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