These are short proceedings of the workshop on “The Standard Model at Low Energies” held at ECT* in Trento, Italy, from April 29 to May 10, 1996. The workshop concentrated on Chiral Perturbation Theory in its various settings. Included are a one page abstract with references per speaker and a listing of some review papers of relevance to the field.

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1 Introduction

The field of Chiral Perturbation Theory is a very active one. It was therefore felt that another topical workshop was needed. This meeting followed the series of workshops in Ringberg (Germany), 1988, Dobog'ok/o (Hungary), 1991, and Karrebæksminde (Denmark), 1993, which were all three of the size of about 50 participants. We were lucky enough to obtain funding from the European Center for Theoretical Studies in Nuclear Physics and related Areas, the ECT*, for this workshop.

The meeting itself took place in Trento in the spring of 1996. There were a lot of discussions and talks. The present abstract booklet is meant as a guide into the literature and provides abstracts and main references of the presentations given. Since most results will be published elsewhere, a full traditional type of proceedings seemed unnecessary.

We would like to take the opportunity to thank the ECT*, its director, Ben Mottelson, and its board of directors for general support and enthusiasm for this meeting. We would also like to thank the secretaries Cristina Costa at ECT* and Anne Lumholdt at NORDITA for taking care of the administrative work involved. Finally we thank the participants for making this a very pleasant and lively meeting.

Unfortunately, we cannot thank the weatherman since it rained most of the workshop, but this way the presence of most participants at the talks was guaranteed.

There exists by now a rather large series of main references related to Chiral Perturbation Theory. We give below a short and subjective list of the most recent ones and those considered classics. Then the list of participants and their electronic mail addresses follows. The program of the meeting and the individual abstracts with their references round off this abstract booklet. The latter are given in the same order as the talks were given. Speakers are indicated in boldface.

Johan Bijnens and Ulf-G. Meißner
2 A short guide to review literature

Chiral Perturbation Theory grew out of current algebra, and it soon was realized that certain terms beyond the lowest order were also uniquely defined. This early work and references to earlier review papers can be found in [1]. Weinberg then proposed a systematic method in [2], later systematized and extended to use the external field method in the classic papers by Gasser and Leutwyler [3,4], which, according to Howard Georgi, everybody should put under his/her pillow before he/she goes to sleep. The field has since then extended a lot and relatively recent review papers are: Ref.[5] with an emphasis on the anomalous sector, Ref.[6] giving a general overview over the vast field of applications in various areas of physics, Ref.[7] on mesons and baryons, and Ref.[8] on baryons and multibaryon processes. In addition there are books by Georgi[9], which, however, does not cover the standard approach, including the terms in the lagrangian at higher order and a more recent one by Donoghue, Golowich and Holstein[10].

There are also the lectures available on the archives by E. de Rafael [11] and A. Pich[12] as well as numerous others. The references to the previous meetings are [13,14,15]. There are also the MIT meeting [16] and the DAΦNE handbook [17] as useful references.

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3 Participants and their email

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4 The program as it finally ended

week 1
Monday 29/4
  morning empty for arrival purposes
  14.30 J. Bijnens Administrative and Other Arrangements
  15.00 U. Meiβner Introductory remarks about CHPT
Tuesday 30/4
  9.15 Paul Büttiker The Chiral coupling constants $\bar{l}_1$ and $\bar{l}_2$ from $\pi\pi$
  10.15 Joseph Schechter Simple description of $\pi\pi$ scattering to 1 GeV
  11.00 Coffee Break
  11.30 Bachir Moussallam Sum rules in $\pi\pi$ scattering
  12.00 Lunch
  15.00 Veronique Bernard $\pi N \rightarrow \pi\pi N$ in CHPT
  15.45 Stefan Scherer Extension of the CHPT Meson Lagrangian to Order $p^6$
  16.30 Coffee Break
  17.00 Eugene Golowich Two-loop analysis of Vector-Current and Axialvector-current Propagator, a progress report.

17.45
Wednesday 1/5
  10.00 Ubirajara van Kolck Isospin as an Accidental Symmetry
  10.45 Coffee Break
  11.15 Christoph Hanhart $\pi$-threshold production in pp-collisions
  11.45 Lunch
  15.00 Frank Meier Quantum Corrections to Baryon Properties in Chiral Soliton Models
  15.30 Andreas Wirzba S-wave pion propagation in dense isosymmetric nuclear matter
  16.15 Coffee Break
  16.45 Jan Stern Experimental signature of quark condensates

Thursday 2/5
  9.15 Jürg Gasser Pion polarizabilities to two loops
  10.15 Coffee Break
  11.00 Tri Nang Pham Chiral Lagrangian with vector and axial vector mesons for $\pi^+ - \pi^0$ mass difference
  11.30 Antonio Perez Electromagnetic mass differences of pions and kaons
  12.00 Lunch
  15.00 Matthias Lutz Chiral symmetry and many-nucleon systems
  15.45 Guido Müller Renormalization of the Pion-Nucleon Lagrangian to order $p^4$
  16.15 Coffee Break
  16.45 Norbert Kaiser Neutral pion photo– and electroproduction

17.30
Friday 3/5
  9.15 Dieter Dreschel Pion Photoproduction of the Nucleon -results from Dispersion Theory
  10.00 Bugra Borasoy Baryon Masses to Second Order in the Quark Masses
10.45 Coffee Break
11.15 James V. Steele       Master Approach in the Nucleon Sector
12.00 Lunch
15.00 A. Smilga          Scalar susceptibility in QCD and in multilavor Schwinger model
15.45 Jan Stern          Quark condensate and density of states
16.15

week 2

Monday 6/5
9.45      Gilberto Colangelo       Elastic $\pi\pi$ scattering to Two Loops
10.30 Coffee Break
11.00 Marc Knecht       The $\pi\pi$ scattering amplitude to two loops
11.45 Dominique Toublan       Low Energy Sum Rules For Pion-Pion Scattering
                               and Threshold Parameters
12.15 Lunch
15.00 Herbert Weigel       Heavy Quark Solitons
15.45 Coffee Break
16.15 Eduardo de Rafael       Low Energy QCD in the large $N_c$ limit
16.45 Coffee Break
17.00

Tuesday 7/5
9.15      Michael Pennington       Dispersive analysis of $\chi$PT predictions
10.00 Christian Wiesendanger       Final State Interactions and Khuri-Treiman Equations
                               in $\eta \rightarrow 3\pi$
10.45 Coffee Break
11.15 Thomas Hemmert       $\Delta(1232)$ in Chiral Perturbation Theory
12.00 Lunch
15.00 Joachim Kambor       Resonance Saturation in the Baryon Sector
15.45 Coffee Break
16.15 Silas Beane       Novel algebraic consequences of chiral symmetry
16.45 Coffee Break
17.00

Wednesday 8/5
9.15      Elisabetta Pallante       Hadronic Contributions to the muon g-2: an updated analysis
10.00 Joaquim Prades       Some Hadronic Matrix Elements in ENJL :
                               $B_K$, Dashen’s Theorem, $\gamma\gamma \rightarrow \pi\pi$
10.45 Coffee Break
11.15 Res Urech             On the corrections to Dashen’s theorem
11.45 Lunch

Thursday 9/5
9.15      Giancarlo D’Ambrosio       Topics in Radiative Non-leptonic Kaon Decays
10.00 Gino Isidori       Radiative Four-Meson Amplitudes
10.45 Coffee Break
11.15 Gerhard Ecker       Aspects of renormalization in CHPT
12.00 Lunch
15.00 Roxanne P. Springer       Chiral Symmetry and Hypernuclei
15.45 Norberto Scoccola       Hyperon Electromagnetic Properties in a Soliton Model
16.15 Coffee Break
16.45 Teruaki Watabe       Strange Contents in Nucleon; Difficulty and Approach
17.30
| Time  | Speaker          | Topic                                                |
|-------|------------------|------------------------------------------------------|
| 9.15  | Joaquim Matias   | CHPT description of the MSM: One and two loop order  |
| 10.00 | Stephan Duerr    | The covariant derivative expansion                   |
| 10.30 | Coffee Break     |                                                       |
| 11.00 | Thomas Waas      | Kaon nucleon interaction and the \( \Lambda(1405) \) in dense matter |
| 11.45 |                  |                                                       |
| 14.00 | Johan Bijnens    | \( \gamma\gamma \rightarrow \pi\pi\pi \) and some comments on \( U(1)_A \). |
| 14.30 | Ulf Meißner      | The organizers have the final word as usual.          |
Remarks on CHPT and EFTs

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In this introductory talk, I discuss certain aspects of chiral perturbation theory (CHPT), which is the effective field theory (EFT) of the standard model. In EFTs, based on the power counting first introduced by Weinberg [1], one considers tree and loop graphs of the light dofs (the heavy ones being integrated out) and the effective Lagrangian is organized according to the chiral dimension (or number of derivatives). Often it is argued that it makes no sense to consider loops in such an approach since the loop momenta can not be considered small as it is the case for the external momenta. Clearly, one could invent a scheme in which one would cut all the pion momenta to be less than the typical scale of the heavy degrees of freedom. Alternatively, using dimensional regularization, one chooses the associated scale to be of the order of the heavy mass scale. This effectively suppresses the high momentum components in the loops. However, this high–energy information is not lost; it is encoded in the values of the associated low energy constants (LECs) appearing to the order one is working.

A second remark concerns the use of dispersion relations to not only extend the range of applicability of the chiral predictions but also to sharpen these at the very low energy end. Having calculated e.g. the imaginary part of the scalar pion form factor and the elastic $\pi\pi$ scattering amplitude to one loop [2] allows one to write a dispersive representation to two loop accuracy for the scalar form factor $\Gamma_\pi$ with a number of subtractions to guarantee convergence [3]. These subtraction constants play a role similar to the LECs in the corresponding “real” two loop calculation. While the analytic structure of the two approaches is identical, the latter one contains more information. First, the LECs can be related to other processes and second, only in certain circumstances the enhancement of certain LECs due to IR logs can be unraveled in the dispersive approach, e.g. the finiteness of the $\Gamma_\pi$ in the chiral limit reveals the $M_\pi^2 \ln M_\pi^2$ dependence of $d_2$. Terms of the type $M_\pi^2 \ln M_\pi^2$ can not be found by such means. If one only considers a certain observable or process, like $\Gamma_\pi$ or $\pi\pi \rightarrow \pi\pi$, a next to next to leading order calculation is certainly much easier done in the dispersive approach. Also, the equivalent of a one loop calculation can be done without ever performing a loop integral. The prize one pays is the loss of information described above.

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Chiral Coupling Constants from $\pi\pi$ Phase Shifts

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ChPT [1] provides the low energy effective theory of the standard model describing interactions involving hadronic degrees of freedom. It is a nonrenormalizable theory; additional coupling constants have to be introduced at each order of the derivative or momentum expansion. At leading order $O(p^2)$ there are two such constants, the pion decay constant $F_\pi$ and the pion mass $m_\pi$. At next to leading order $O(p^4)$ there are ten more constants. Four of them $\bar{l}_1$, $\bar{l}_2$, $\bar{l}_3$, and $\bar{l}_4$ enter the $\pi\pi$ scattering amplitude. As a result, the threshold parameters can be expressed in terms of these as well. In the past the coupling constants $\bar{l}_1$ and $\bar{l}_2$ have been fixed from experimental values for the D-wave scattering lengths [2] or from an analysis of $K_{l4}$ decays [3].

On the other hand $\pi\pi$ scattering has been studied in great detail in axiomatic field theory [4]. Fixed-t dispersion relations have been established in the axiomatic framework and properties of crossing and analyticity have been exploited to establish the Roy equations, a system of integral equations for the partial wave amplitudes [5,6]. Here we report on a direct determination of the coupling constants from the existing phase shift data [7,8] by performing a Roy equation fit to it when $a_0^0$ is restricted to the range predicted by ChPT. Using certain properties of the chiral amplitude [9], we write down a dispersion representation with a certain number of subtractions consistent with $O(p^4)$ accuracy, where the subtraction constants are expressed in terms of the chiral coupling constants. The fixed-t dispersion relations of axiomatic field theory are also rewritten in a form whereby a direct comparison with the chiral dispersive representation can be made while the subtraction constants are now computed in terms of physical partial waves, produced by the Roy equation fit. As an example we cite $\bar{l}_1 = -1.7 \pm 0.15$ and $\bar{l}_2 \approx 5.0$ for the one-loop coupling constants. Our method is powerful enough to be extended in a straightforward manner to determine two-loop coupling constants.

References
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Simple Description of $\pi\pi$ Scattering to One GeV

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In this work, described in detail in [1], we slightly relax the extremely accurate description of the threshold region obtained in chiral perturbation theory in order to describe $\pi\pi$ scattering all the way up to the 1 GeV region.

The present model can be viewed as an attempt to approximate the leading “Born” term of the $1/N_c$ expansion of the $\pi\pi$ amplitude in QCD. It is known that such an amplitude contains contact terms and an infinite number of resonance exchanges. We truncate the resonances to those in the energy region up to about 1.4 GeV. We get the amplitude from a chiral Lagrangian so that crossing symmetry is automatically satisfied. Since the leading $1/N_c$ amplitude contains singularities (zero width resonances) and is otherwise purely real we a) restrict attention to predicting the real part of the amplitude b) regularize the amplitude at the pole positions in such a way that “local unitarity” (near the resonance poles) is maintained.

It is found that the resulting amplitude satisfies the unitarity bounds in addition to crossing symmetry for the $I = l = 0$ channel (the difficult one) up to 1.2 GeV. The components are 1) the “current algebra” contact term, 2) the $\rho$ exchange diagrams, 3) a broad scalar resonance at about 560 MeV, and 4) the $f_0(980)$ with its associated Ramsauer-Townsend effect. The contributions of the “next group” of resonances (comprising the $f_2(1275)$, the $f_0(1300)$ and the $\rho(1450)$) tend to cancel each other and thus do not disturb the nice picture in this energy range. A similar mechanism is observed in the off diagonal process $\pi\pi \rightarrow K\bar{K}$.

References
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Determination of Two-loop $\pi\pi$ Scattering Amplitude Parameters

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The expansion of the $\pi\pi$ amplitude to two-loop chiral order has recently been worked out [1][2], it is expressed in terms of elementary functions of the Mandelstam variables and of six parameters. These involve combinations of $O(p^4)$ as well as $O(p^6)$ low energy constants and chiral logarithms [2]. We have shown that four of these parameters obey sum rules which allow an accurate determination based on existing $\pi\pi$ scattering data at medium energies ($\sqrt{s}$ between 0.5 to 2 GeV)[3]. These sum rules are obtained by matching chiral perturbation theory with dispersion relations, a technique which was used in a variety of applications in recent years. A specific feature of the elastic $\pi\pi$ amplitude, is the invariance under crossing (modulo a crossing matrix). This invariance gives rise to the Roy dispersive representation [4]. We have rederived and improved this representation taking into account the notion of chiral counting, dropping all contributions of chiral order higher or equal to eight. As a consequence, all the necessary and sufficient conditions for crossing symmetry to hold can be explicitated in a simple way (which was not the case in the original formulation): in addition to determining the subtraction functions up to two constants, the so called driving terms are also determined to be polynomials and, finally, a relation is found between three integrals over high-energy data. Equating the scattering function $A(s,t,u)$ from a) the chiral expansion and b) the Roy dispersive representation gives the four sum rules. We have finally shown that it is by no means necessary to solve numerically the Roy equations in order to exploit the sum rules: in the low energy region, where sufficiently precise scattering data is not available, it is as efficient to use the chiral expansion of the amplitude: taking two parameters as input (equivalently, one could use $a_0^0$ and $a_0^2$) the four remaining ones are determined in a self consistent way by the sum rules. One of the two parameters which are left free has a particularly interesting theoretical significance related to the way in which chiral symmetry is spontaneously broken in QCD: the interested reader should consult the contribution of Marc Knecht in these proceedings.

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The reaction $\pi N \to \pi \pi N$ in HBCHPT

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In the framework of heavy baryon chiral perturbation theory (HBCHPT), we give the chiral expansion for the $\pi N \to \pi \pi N$ threshold amplitudes $D_1$ and $D_2$ to linear and to quadratic order in the pion mass. To linear order in the pion mass, we derive low-energy theorems (LETs) for the two threshold amplitudes $D_1$ and $D_2$ which are free of unknown low-energy constants. The numerical predictions of these LETs work well for the reaction $\pi^+ p \to \pi^+ \pi^+ n$ but show some significant deviations for $\pi^- p \to \pi^0 \pi^0 n$ as naively expected [1]. To second order in the pion mass, the theoretical results agree within one standard deviation with the empirical values [2]. We notice that the effect of pion rescattering is efficiently masked by pion–nucleon rescattering and resonance excitation, in particular due to the $N^*(1440)$. We find a novel $N^* \to N(\pi\pi)_S$ coupling which has not been accounted for in previous phenomenological analysis. We also derive a relation between the two threshold amplitudes of the reaction $\pi N \to \pi \pi N$ and the $\pi\pi$ S-wave scattering lengths, $a_0^0$ and $a_0^2$, respectively, to order $O(M^2_\pi)$ [2]. We show that the uncertainties mostly related to resonance excitation make an accurate determination of the $\pi\pi$ scattering length $a_0^0$ from the $\pi\pi N$ threshold amplitudes at present very difficult. From the existing data, we deduce $a_0^0 = 0.21 \pm 0.07$ where the error does not include (presumably large) contributions at $O(M^3_\pi)$. The situation is different in the $\pi\pi$ isospin two final state. Here, the chiral series converges and one finds $a_0^2 = -0.031 \pm 0.007$ somewhat smaller than the two-loop chiral perturbation theory prediction. These results could be used to determine $l_3$ which is the parameter directly related to the size of the condensate. However, at the present state of the art $l_3$ will be given with a rather large error bar. We also point out that previous analysis of the same data using the Olsson–Turner model can not be trusted [3].

References
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Extension of the chiral perturbation theory meson Lagrangian to order $p^6$

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We discuss the most general chirally invariant Lagrangian $L_6$ for the meson sector at order $p^6$ within the framework of standard SU(3) chiral perturbation theory [1]. The result [2] provides an extension of the well-known Gasser-Leutwyler Lagrangian $L_4$ to one higher order, including as well all the odd-intrinsic-parity terms in the Lagrangian. We have developed a systematic strategy so as to get all the independent terms and eliminate the redundant ones in an efficient way. For that purpose we have introduced a twofold hierarchy in terms of a) the number of covariant derivatives and b) the number of traces contained in an expression. This procedure allows to eliminate terms in favor of ones lower in the hierarchy without actually working out the explicit and often extremely complicated relations connecting the corresponding terms. We explain how field transformations can be used to identify redundant terms which are proportional to the lowest-order equation of motion [3]. The claim to have obtained the most general Lagrangian relies on this systematic construction and on the elimination of the redundant quantities using using relations of which we are aware, rather than on a general formal proof of either completeness or independence.

The end result involves more than hundred terms which, under certain assumptions, fall into two distinct classes of interaction terms, according to whether they are even or odd in the number of Goldstone bosons. We have separated the final set of terms into groupings of expressions contributing to increasingly more complicated processes, so that one does not have to deal with the full result when calculating $p^6$ contributions to simple processes.

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Two-loop Analysis of Vector-current and Axialvector-current Propagators in Chiral Perturbation Theory: A Progress Report.

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In this talk, I describe a calculational program by Joachim Kambor and myself for determining the low energy behaviour of propagators $\Delta_{V,A,33}(q^2)$ and $\Delta_{V,A,88}(q^2)$ at two-loop order in ChPT.

The vector-current part of the project was recently brought to a satisfactory conclusion [1,2]. We determined the propagators with straightforward Feynman diagram methods by making appropriate use of external vector sources. At $O(q^4)$ there were three diagrams and at $O(q^6)$ there were ten. To absorb divergences and scale dependence at two-loop level required construction of appropriate counterterms from the $O(q^6)$ chiral lagrangian. Of the four such $O(q^6)$ counterterms found, only three are independent in the vector-current sector. The final results, in finite, covariant, and scale-independent form, yielded a successful fit to data of the two-loop isospin vector spectral function for $E \leq 400$ MeV. [1]

In [2], we used ‘inverse-moment’ sum rules derived in [1] to obtain phenomenological evaluations of two of the three new $O(q^6)$ counterterms. Our analysis also yielded insights on the important but difficult issue regarding contributions of higher orders in the ChPT expansion to the sum rules.

Work continues [3], now on the axialvector propagators. Although the vector and axialvector calculations are similar in overall structure, there are several differences of detail, e.g. two-loop renormalizations of masses and decay constants appear in the axialvector systems via the meson pole. To date, we have determined all one-particle irreducible two-loop diagrams, including the formidable ‘sunset’ diagram. We have also obtained the list of $O(q^6)$ counterterms which contribute to the axialvector sector.

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Isospin as an Accidental Symmetry

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Isospin is shown [1] to be an accidental symmetry, in the sense of being a symmetry that is present in the effective low-energy theory (the general chiral lagrangian involving pions, nucleons, and delta isobars) in lowest order but not in the underlying theory (QC+ED).

I start by constructing the operators involving the low-energy degrees of freedom that break chiral symmetry in the same way as quark mass and charge difference terms in QC+ED; such operators appear in the chiral lagrangian with coefficients proportional to powers of the up-down mass difference and the fine structure constant. I use naive dimensional power counting to show that there are no isospin violating operators in lowest order: in most processes isospin violation will therefore be down compared to isospin conserving terms not only by a ratio of the quark mass difference to the sum, but also by additional powers of the ratio between the low energy of interest ($Q \sim$ pion mass) and the QCD scale ($M \sim$ rho mass). The operators are next used to study simple processes in leading orders. It is easy to see that isospin violation in pion-pion scattering should come predominantly from explicit photon loops. Pion-nucleon scattering, on the other hand, could in principle show “large” (i.e., not suppressed by extra powers of $Q/M$) isospin violation related to quark mass effects in the $t$-channel isoscalar amplitude, but this is hard to measure. Turning to nuclear systems, I find that $i$) the leading breaking of isospin comes from the (predominantly electromagnetic) pion mass difference in the one-pion-exchange two-nucleon potential, an effect that still preserves charge symmetry; $ii$) charge symmetry breaking is $O(Q/M)$ smaller, and arises from an isospin-violating pion-nucleon coupling, and two short-range interactions, all mainly quark mass effects. (From the viewpoint of meson-exchange models, they might come respectively from $\pi-\eta$ mixing, and $\rho-\omega$ and $a_1-f_1$ mixings [2].) Finally, the use of such a chiral lagrangian is illustrated by a computation of the pion-range, isospin-violating two-nucleon potential to third order in the chiral expansion [3]. This includes, besides the tree graphs with isospin dependent pion masses and couplings mentioned above, also one-loop diagrams with pion [2] and photon dressings at vertices and propagators, and with simultaneous pion and photon exchange.

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Threshold pion production in nucleon-nucleon collisions

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We performed a momentum-space calculation of the reaction $NN \rightarrow NN\pi$ near threshold, extending our earlier study [1]. The following pion production mechanisms are considered: (i) Direct emission of the pion form one of the nucleons. (ii) (s-wave) rescattering where the produced pion first scatters off the other nucleon before its emission. (iii) Contributions from meson-exchange currents due to the exchange of heavy mesons ($\sigma$, $\omega$) connected to intermediate $N\bar{N}$ pairs[2]. The Bonn OBEPT potential [3] is used for the distortions in the initial and final $NN$ states. For the evaluation of the rescattering contribution a microscopic meson–exchange model of $\pi N$ interaction developed recently by the Jülich group [4] is utilized. A soft form factor (of monopole type) with a cut–off mass $\Lambda_{\pi NN} = 800$ MeV, as suggested, e.g., by recent QCD lattice calculations [5], is employed at the pion production vertex. In the calculation of the heavy-meson-exchange contributions we use the same $\omega$ vertex parameters as in OBEPT. The $\sigma$, however, is an effective parameterization of correlated $2\pi$exchange (and other processes) and its strength should be different in the NN interaction and in the present case, where it couples to $N\bar{N}$ pairs. Therefore the coupling constant of the $\sigma$ is treated as free parameter. We achieve a quantitative description of the measured total cross section for the reaction $pp \rightarrow pp\pi^0$ near threshold. With the same model (and the same parameter set) we are also able to reproduce the $pp \rightarrow d\pi^+$ cross section near threshold with similar quality. Note, however, that in the latter reaction the pion production via a $\Delta$-isobar excitation could be important [6]. This process is so far neglected in our study. The reaction $NN \rightarrow NN\pi$ close to threshold promises to give a deeper insight into the off–shell properties of the $NN$ interaction as well as into the short range correlations of the NN force. It might also be sensitive to additional constrains from chiral symmetry as suggested by recent investigations using chiral perturbation theory[7].

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S-wave pion propagation in dense isosymmetric nuclear matter

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The starting point of the talk is the isoscalar S-wave interaction between pions and nucleons. The corresponding $\mathcal{O}(Q^2)$-heavy-baryon lagrangian of ref. [1] (in the mean-field approximation for the nucleons) is then applied to finite baryon densities $\rho$. Note that nuclear correlations are thus neglected. Within the generating functional formalism of ref. [2] the effective mass of the pion in isosymmetric homogeneous nuclear matter is derived [3,4] and shown to be independent of the various off-mass-shell extension schemes (as e.g. PCAC) [3].

With the help of the corresponding generating functional the density-dependent quark condensate $\langle \bar{u}u + \bar{d}d \rangle_\rho$ as well as the time-like pion decay constant $F_π^t(\rho)$ (which in the matter background is bigger than its space-like counterpart $F_π^s(\rho)$) and the pseudoscalar coupling constant $G_\pi^*(\rho)$ are deduced [3,4]. These quantities combine to satisfy the in-medium extension of the Gell-Mann-Oakes-Renner relation,

$$F_π^t(\rho)^2 m_π^*(\rho)^2 = -\hat{m}_q \langle \bar{u}u + \bar{d}d \rangle_\rho + \mathcal{O}(Q^3, \rho^2),$$

and the density-dependent PCAC relation,

$$F_π^t(\rho)m_π^*(\rho)^2 = \hat{m}_q G_\pi^*(\rho) + \mathcal{O}(Q^3, \rho^2),$$

where $\hat{m}_q$ is the SU(2)-averaged current quark mass. Furthermore the results are compatible with Migdal’s approach to finite Fermi systems on the composite hadron level provided the Migdal propagator has been identified correctly [5].

Finally, the mean-field lagrangian sets constraints for the in-medium extension of chiral perturbation theory [4]. As the matter background selects a special Lorentz-frame, the in-medium version of ChPT cannot satisfy Lorentz invariance, but only the left-over Euclidean rotational invariance. It should therefore be classified as a non-relativistic ChPT of ref. [6]. Indeed, the dispersion of the S-wave pion-propagation in isosymmetric nuclear matter is to order $\mathcal{O}(Q^2)$ the same as for the corresponding Goldstone ($\pi$) bosons of an antiferromagnet, where the spacelike “$F_\pi^s$” is smaller than the time-like “$F_\pi^t$” [6] as well. However, the fact that the $\pi N$ S-wave lagrangian predicts, in the mean-field approximation, the first corrections to the isoscalar channel to be of order $\mathcal{O}(Q^3)$ is incompatible with standard ChPT (where they are of order $\mathcal{O}(Q^4)$ whether in the relativistic or non-relativistic version). Thus the in-medium (non-relativistic) ChPT has to be of the generalized form of ref. [7]. This is supported by the possibility that, with increasing baryon density, the in-medium quark condensate can potentially become so small, that the higher in-medium quark condensates cannot be neglected any longer as it is the case in standard ChPT in the vacuum.

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Experimental Signature of Quark-Antiquark Condensation in the QCD Vacuum

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It is generally believed that in QCD, the spontaneous breakdown of chiral symmetry is a consequence of a strong quark-antiquark condensation in the vacuum. The condensate parameter $B_0 = -\bar{q}q/F^2$ is expected to be sufficiently large to insure that, for actual values of running quark masses, the expansion of the square of the Goldstone boson masses is dominated by the first Gell-Mann–Oakes–Renner term. This assumption is crucial in the standard formulation [1] of CHPT, but so far, it has not been tested experimentally. Moreover, a sound theoretical alternative [2] in which the condensate $B_0$ would be marginal (typically 10 times smaller than believed) or even vanishing, could naturally arise in QCD and it is not excluded by any existing data. For these reasons an experimental probe of quark condensation becomes of fundamental importance. The best evidence in favor of or against a strong $\bar{q}q$ condensation would be provided by new high precision low energy $\pi\pi$ scattering experiments [3]. (The actual uncertainty, c.f. $a_0 = 0.26 \pm 0.05$ encompasses both alternatives of a strong and weak quark condensation.) Additional signature could emerge from the analysis of deviations from the Goldberger Treimann relations [4], $\eta \to 3\pi$ decays [5], $\gamma\gamma \to \pi^0\pi^0$ near threshold, provided corresponding experimental data become more accurate. The question of the strength of quark condensation could influence on various (not yet tested) predictions: estimates of light quark masses [6], estimates of $\varepsilon'/\varepsilon$ and some issues in the B-physics, among others.

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Pion polarizabilities to two loops

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I have discussed in my talk the evaluation of the pion electric ($\alpha_\pi$) and magnetic ($\beta_\pi$) polarizabilities in the framework of chiral perturbation theory. The polarizabilities are obtained by expanding the Compton amplitude near threshold in powers of the photon momenta. The leading term in this momentum expansion is proportional to the square of the charge of the pion, whereas the coefficients of the next-to-leading-order term are determined by $\alpha_\pi$ and $\beta_\pi$. In the chiral expansion, the polarizabilities receive their leading order contribution from terms at order $p^4$ [1-3]. At this order, one has $\alpha_\pi + \beta_\pi = 0$ ($D$-wave term). Therefore, to determine the first nonvanishing contribution to $\alpha_\pi + \beta_\pi$, a two-loop evaluation of the Compton amplitude is needed.

The result of this calculation in the neutral channel has been published some time ago [4], whereas the evaluation in the charged channel (which involves considerably more diagrams, because there is a tree graph contribution) has been completed recently [5]. The two low-energy constants that enter the polarizabilities at two-loop order have been determined by resonance saturation in these references. Chiral logarithms contribute in both channels in a nonnegligible manner. Bürgi has also investigated the importance of various graphs in the $\overline{\text{MS}}$ scheme [5], using the sigmamodel parametrization of the $U$–matrix. He finds that the acnode and the box graphs generate substantial contributions to the polarizabilities.

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In this talk, I report on a recent work [1] on a simple derivation of the forward virtual Compton scattering off a soft pion target for use in the $\pi^+ - \pi^0$ electromagnetic mass difference calculation. References to previous works can be found in this paper.

By using a nonlinear chiral Lagrangian with vector and axial vector meson incorporated in a modified gauged chiral model, it is shown that the simple expression for the forward virtual Compton scattering on a soft pion usually obtained from Current Algebra can be derived in a simple manner. This shows also that the absence of the double pole behaviour for the Born terms is a consequence of chiral symmetry. Though this result has also been obtained previously by Ecker et al. and also more recently by Donoghue et al. in which the vector and axial vector meson fields are treated as antisymmetric tensor representation instead of the usual four-vector field operator, we show that one can also derive the Current Algebra result in a simple manner with the conventional four-vector representation for vector and axial vector meson. We note also that the unsubtracted dispersion relation for the $\Delta I = 2$ amplitude can be made consistent with the soft pion result by including also the contact term from the vector meson pole term in a modified Born term. Then it would be more convenient to use the dispersion relation approach to calculate the $\pi^+ - \pi^0$ mass difference since terms of $O(p^2)$ can also be analysed in a straightforward manner.

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The Electromagnetic Mass Differences of Pions and Kaons

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We use the Cottingham method to calculate the pion and kaon electromagnetic mass differences with as few model dependent inputs as possible. The constraints of chiral symmetry at low energy, QCD at high energy and experimental data in between are used in the dispersion relation. We find excellent agreement with experiment for the pion mass difference. The kaon mass difference exhibits a strong violation of the lowest order predictions via Dashen’s theorem, in qualitative agreement with several other recent calculations.

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Weinberg [1] and van Kolck [2] introduced a nonrelativistic perturbative scheme with consistent chiral counting rules for the nucleon-nucleon potential. In this talk we present a relativistic chiral expansion scheme for the nucleon-nucleon scattering amplitude. It is advantageous to work with the manifestly Lorenz covariant chiral Lagrangian where we achieve the desired nonrelativistic $1/m$ expansion by a proper regrouping of the interaction terms in the Lagrangian. The $1/m$ expansion can then be performed at the level of each individual Feynman diagram. We find that our relativistic scheme naturally sums the non-polynomial terms in $1/m$ needed to reconcile proper dispersion relations and threshold behaviour.

Chiral counting rules, predicting the leading chiral power for the two-nucleon irreducible Feynman diagrams of the Bethe-Salpeter kernel, are in full analogy to Weinberg’s counting rules for the nucleon-nucleon potential. To further derive chiral power counting rules for the $NN$-scattering amplitude we introduce a subtraction scheme (for a given finite cutoff $\Lambda$) at the level of the Bethe-Salpeter equation such that its solution, the $NN$-scattering amplitude, is independent of the subtraction scheme with its characteristic scale $\mu$. A small subtraction scale $\mu$, of the order of the pion mass, renders the unitary iterations of the properly subtracted pion exchanges perturbative with a well defined chiral counting rule: each intermediate two-nucleon state generates a chiral enhancement power -1. The subtraction scale determines the relative importance of the unitary iterations of the 2-nucleon vertices as compared to the strength provided by unitary iterations of pion exchanges. We find that a small subtraction scale $\mu$ causes a strong renormalization of the local s-wave nucleon-nucleon interaction vertices at given physical cutoff $\Lambda$. In this case the natural s-wave bare couplings mutate into large couplings, which acquire the anomalous chiral power $-1$. The thus renormalized local s-wave interaction vertices pick up sufficient strength to generate naturally the deuteron bound state and the pseudo-bound state in the nucleon-nucleon scattering amplitude upon unitary iterations.

Our scattering amplitude exhibits simple complex pole terms, reflecting the presence of the pseudo bound state and the deuteron bound state, and a remainder which comprises the proper cut structure from multiple pion exchanges. To leading orders the free parameters of our scheme are in one-to-one correspondence to s-wave scattering lengths and ranges and p-wave scattering volumes.

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Renormalisation of the SU(3) chiral meson-baryon lagrangian to order $q^3$

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Three-flavor chiral perturbation theory with baryons is a topic of current interest. The dynamics of kaon-nucleon interactions or kaon photo(electro)production are based on the SU(3) extension of chiral effective lagrangians. We remind that for the isospin-odd pion-nucleon scattering length the loop correction only can fill the gap between the Weinberg-Tomozawa prediction and the empirical value. Loops produce in general ultraviolet divergences, which can be absorbed by introducing counterterms [1]. The knowledge of the full divergence structure allows to control these calculations. We perform the complete regularisation of all Green functions with a single incoming and outgoing baryon to order $q^3$ in the chiral SU(3) meson-baryon system. The method is based on the work of Ecker who performed the complete renormalisation of Green functions of the pion-nucleon interaction in the heavy baryon formalism [2]. This allows for a consistent chiral power counting. The divergences can be extracted in a chiral invariant manner by making use of the heat kernel representation of the propagators in d-dimensional Euclidean space. The main difference between the two calculations lies in the fact that the nucleons are in the fundamental representation of SU(2), while the baryons are in the adjoint representation of SU(3). This leads to some algebraic consequences for the construction of the one-loop generating functional [3].

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Neutral Pion Photoproduction off Protons

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I analyse the new threshold data for neutral pion photoproduction off protons in the threshold region [1,2] within the framework of heavy baryon chiral perturbation theory at order $q^4$. It is shown that indeed large loop corrections as predicted by CHPT at order $q^4$ are needed to understand the small value of the S-wave multipole $E_{0+}$ at threshold [3]. Due to the rather slow convergence of this quantity in powers of the pion mass it does not provide anymore a good testing ground of chiral dynamics. However, there are new and rapidly converging low energy theorems for two combinations of the P-wave multipoles [3]. The one for $P_1$ holds at the few percent level when compared to the new TAPS and SAL data [1,2]. The low energy theorem for $P_2$ can be tested soon with polarized photon data taken at MAMI. The few low energy constants entering the calculation can be well understood from resonance saturation [4].

Furthermore, I discuss double $\pi^0$ photoproduction off protons close to threshold. The chiral loops enter here already at leading nonvanishing order and considerably enhance the near threshold total cross section. This feature remains in a full order $q^4$ calculation of all next-to-leading order corrections [5].

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Photoproduction of Pions off the Nucleon - Results from Dispersion Theory

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Dispersion relations at constant $t$ [1] have been used to analyze the recent precision experiments at MAMI (Mainz) and ELSA (Bonn). The partial wave amplitudes fulfill the phase relations required by the Watson theorem at the lower energies and some approximate relation at the higher energies. The contributions of the dispersion integral above $2\text{GeV}$ are replaced by a fraction of the vector meson exchange representing the Regge behaviour of the amplitudes expected at very high energies. This unknown high-energy behaviour and the necessity to add multipoles of the homogeneous equations to the solutions of the coupled system of multipole amplitudes, leads to the introduction of 9 free parameters determined by a fit to the data in the energy region of $160\text{MeV} \leq E_\gamma \leq 450\text{MeV}$.

The threshold region has not been included in our fit because isospin symmetry breaking effects play an important role in that region due to the different pion masses. However, the threshold predictions obtained from our analysis are in very good agreement with the results from chiral perturbation theory (ChPT) [2]. In particular, we find $E_{0+}(n\pi^+)=28.4\cdot10^{-3}/m_\pi$ and $E_{0+}(p\pi^-)=-31.9\cdot10^{-3}/m_\pi$. The latter value is consistent with the angular distribution and the total cross section of a recent TRIUMF experiment [3] whose analysis had led to a threshold amplitude of $E_{0+}(p\pi^-)=-34.7\cdot10^{-3}/m_\pi$, in disagreement with ChPT and low energy theorems. It is also interesting that the data at the higher energies, via dispersion relations, lead to a prediction of $E_{0+}(p\pi^0)=-0.4\cdot10^{-3}/m_\pi$ at $\pi^+$-threshold. This is in good agreement with both the data [4] and ChPT, and a consequence of the importance of loop corrections to the "old" low energy theorem.

In the region of the $\Delta(1232)$ isobar, we have decomposed the $E_{1+}^{(3/2)}$ and $M_{1+}^{(3/2)}$ multipoles into resonance and background contributions using the speed-plot technique [5]. In this way we are able to determine the position of the resonance pole in the complex plane at $W = M_R - i\Gamma_R/2$ with $M_R = (1211 \pm 1)\text{MeV}$ and $\Gamma_R = (100 \pm 2)\text{MeV}$ [6], in excellent agreement with results from pion-nucleon scattering. The resonant contributions to the two multipoles are then determined as the complex residues at the resonance pole, $R_{E/M}\exp(i\phi_{E/M})$. While the Watson theorem requires that the physical amplitudes $E_{1+}^{(3/2)}$ and $M_{1+}^{(3/2)}$ have the same phase, the corresponding ratio for the resonant amplitude is a complex number, $R_{E}\exp(i\phi_E)/R_{M}\exp(i\phi_M) = (-0.35, -0.46)$. Recent experiments on angular distributions and photon asymmetries [7] have found an $E/M$ ratio of $(-2.4 \pm 0.2)\%$ at $W = 1232\text{MeV}$. Since the experiment is sensitive to the ratio $Re\{E_{1+}/M_{1+}\}/|M_{1+}|^2$, our analysis predicts that the resonant contribution to the $E/M$ ratio is $-3.5\%$.

In conclusion, dispersion relations at constant $t$ give a possibility to analyze and to interpret the new precision data on pion photoproduction. Further improvements are expected to come from subtracted dispersion relations, by using the threshold results from ChPT as input, with the consequence of reducing the sensitivity on the unknown behaviour of the dispersion integrals at the higher energies.

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Baryon Masses and $\sigma$–terms
to second order in the quark masses

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We analyze the octet baryon masses and the pion/kaon–nucleon $\sigma$–terms in the framework of heavy baryon chiral perturbation theory, [1]–[4]. We include all terms up-to-and-including quadratic order in the light quark masses, $m_q$. The pertinent low–energy constants are fixed from resonance exchange within the one–loop approximation. This includes contribution from loop graphs with intermediate the spin–3/2 decuplet and the spin–1/2 octet states and from tree graphs including scalar mesons. We demonstrate that two–loop corrections indeed modify the leading one–loop results for some of these coefficients. Retaining only the contributions to the low–energy constants to one–loop order, the only free parameter is the baryon mass in the chiral limit, $\hat{m}$. We find $\hat{m}= 840 \pm 100$ MeV, [5],[6]. While the corrections of order $m_q^2$ are small for the nucleon and the $\Lambda$, they are still large for the $\Sigma$ and the $\Xi$. Therefore a definitive statement about the convergence of three–flavor baryon chiral perturbation can not yet be made. The pion–nucleon $\sigma$–term is given parameter–free, we get $\sigma_{\pi N}(0) = 43 \pm 10$ MeV, which is in good agreement with dispersion-theoretical determinations, together with the strangeness content of the nucleon, $y = 0.08 \pm 0.12$. We also estimate the kaon–nucleon $\sigma$–terms, the shifts to the respective Chang–Dashen points and some two–loops contributions to the nucleon mass.

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The Master Formula Approach in the Pion and Nucleon Sector

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The master formula approach uses an on-shell chiral reduction scheme in order to make predictions for meson-nucleon dynamics [1]. It relates scattering amplitudes to measurable vacuum correlation functions hence allowing the inclusion of resonances. In particular, it has been applied to $\pi\pi$ scattering where a model independent verification of the rho dominance in the vector channel was shown [2].

In addition, a semi-classical expansion in $\hbar \sim 1/f_\pi^2$ may be done. This is equivalent to a momentum expansion in the pion sector. Two solutions of the master formula result: one reproduces CHPT and the other allows for an additional relation between the divergences, reducing the number of parameters to two at one-loop [1,2].

The extension of MF to the nucleon sector was also reported on at this conference. The $1/f_\pi$ expansion is no longer equivalent to CHPT since there are many mass scales involved. The $\pi$-$N$ sigma term is found to be given by the Goldberger-Treiman discrepancy to tree level [3]. Using this one finds naturally that every time $g_A$ appears it is replaced by $g_{\pi NN}$.

A comparison with all form factors and many scattering amplitudes calculated in the relativistic formulation of chiral perturbation theory [4,5] shows additional differences. The scalar form factor and $\pi N \rightarrow \pi \pi N$ scattering amplitude have additional momentum dependent terms proportional to the $\pi$-$N$ sigma term at one-loop. Further analysis is in progress [6].

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Scalar susceptibility and critical behavior in QCD and Schwinger model

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We evaluate the leading infrared behavior of the scalar susceptibility

\[
\chi = \int d^4x \langle \sum_{i=1}^{N_f} \bar{q}_i(x) q_i(x) \rangle - V \langle \sum_{i=1}^{N_f} \bar{q}_i q_i(0) \rangle = \frac{1}{V} \partial_m^2 \log Z \bigg|_{m=0},
\]

(1)
in QCD and in the multiflavor Schwinger model for small non-zero fermion mass \(m\) and/or small nonzero temperature as well as the scalar susceptibility for QCD at finite volume. In QCD, it is determined by one-loop chiral perturbation theory, with the result that the leading infrared singularity behaves as \(\sim \log m\) at zero temperature:

\[
\chi_{IR}^T = \frac{N_f^2 - 1}{8\pi^2} \left( \frac{\sum F^2}{F^2} \right)^2 \log \frac{\Lambda^2}{M^2},
\]

(2)

and as \(\sim T/\sqrt{m}\) at finite temperature:

\[
\chi_{IR}^T = \frac{(N_f^2 - 1)}{4\pi} \frac{T}{\sqrt{2m}} \left( \frac{\sum F^2}{F^2} \right)^{3/2} \left( 1 + \frac{1}{8N_f F^2} \right).
\]

(3)

(where also two loop chiral graphs are taken into account). These are exact results to be checked in lattice and/or instanton model numerical calculations.

In the Schwinger model with several flavors we use exact results for the scalar correlation function. We find that the Schwinger model has a phase transition at \(T = 0\) with critical exponents that satisfy the standard scaling relations and do not coincide with the mean field theory predictions. The singular behavior of this model depends on the number of flavors with a scalar susceptibility that behaves as \(\sim m^{-2/(N_f+1)}\). At finite volume \(V\) we show that the scalar susceptibility is proportional to \(1/m^2 V\). Recent lattice calculations of this quantity by Karsch and Laermann [1] and the related lattice work by Kocic and Kogut [2] are discussed.

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Finite Volume Analysis of Chiral Symmetry Breaking in QCD

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It is argued [1] that in QCD, there exists a natural possibility of spontaneous breakdown of chiral symmetry (SBCHS) without quark-antiquark condensation in the vacuum. In contrast to the Nambu Jona-Lassinio model, the ground state of QCD might be characterized by a non Gaussian distribution of small eigenvalues of the (Euclidean) Dirac operator. This could lead to the large volume behaviour $V^{-1/2}$ of the averaged lowest Dirac levels. The corresponding level density would not be sufficient to trigger $\bar{q}q$ condensation [2,3] (the large volume behaviour $V^{-1}$ is necessary), although it could be sufficient to make appear Goldstone bosons coupled to the conserved axial-vector currents. New sum rules for inverse powers of the Dirac eigenvalues are derived [3,1] which could be suitable for a numerical study of the mechanism of SBCHS not suffering from the usual drawbacks of lattice simulation near the chiral limit.

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Elastic $\pi\pi$ scattering to two loops

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I have presented the calculation of the $\pi\pi$ scattering amplitude to two loops in Chiral Perturbation Theory. The calculation has been done without any approximations and the result is given in analytical form [1].

A thorough numerical analysis is still in progress. However, if we use the current values for the $O(p^4)$ constants, and estimate with resonance saturation the new $O(p^6)$ constants, we get small corrections to the quantities of direct experimental interest, like $a_0^0$, $a_0^0 - a_2^0$ and $\delta_0^0 - \delta_1^0$. As an example, $a_0^0$, which at one loop is 0.201 [2], becomes 0.217. The two loop calculation confirms that CHPT can yield very sharp predictions for $\pi\pi$ scattering at low energy, as stressed in [2]. A clear discrepancy with experimental data would then require a significant revision of our picture of the vacuum structure of QCD. As shown in [3], a value of $a_0^0$ much larger than the CHPT predictions would be the signal for a quark–antiquark condensate much smaller than what is usually assumed.

Finally I have compared the CHPT predictions to a recent lattice calculation of the two $S$–wave scattering lengths [4]. Despite the systematic effects, such as quenching, the agreement is quite impressive. It would be interesting to improve the study of these scattering lengths on the lattice to clarify whether the agreement is accidental or not.

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The amplitude for elastic $\pi\pi$ scattering at low energy has been computed to two-loop accuracy in the chiral expansion [1]. As shown previously [2], it is determined by the Goldstone nature of the pion, combined with the general S-matrix properties of analyticity, crossing symmetry and unitarity, up to six independent combinations of low energy constants, denoted by $\alpha$, $\beta$, $\lambda_i$, $i = 1, 2, 3, 4$, and which are not fixed by chiral symmetry. The four constants $\lambda_i$ were determined via sum rules, evaluated using available data on $\pi\pi$ interaction at medium and high energies [3]. The remaining two parameters, $\alpha$ and $\beta$, have to be determined from low energy $\pi\pi$ data. An experimental determination of $\alpha$ is of particular importance. The value of this parameter is intimately correlated to the ratio $x = -2\hat{m} \langle 0 | \bar{q}q | 0 \rangle / F^2 \pi M^2$, or equivalently, to the quark mass ratio $m_s/\hat{m}$. The commonly accepted picture that spontaneous breakdown of chiral symmetry results from a strong condensation of $q - \bar{q}$ pairs in the QCD vacuum requires that $x \sim 1$ (or $r \sim 25$), and it is only compatible with values of $\alpha$ and $\beta$ close to unity, as predicted by standard $\chi$PT [4,5]. Unfortunately, a fit to the presently available $\pi\pi$ data obtained from $K_{\ell 4}$ decays gives $\alpha = 2.16 \pm 0.86$, $\beta = 1.074 \pm 0.053$, and remains thus inconclusive in this respect. Clearly, additional information, which would make such fits more accurate, is needed. This may be provided by new high statistics $K_{\ell 4}$ experiments, from e. g. KLOE at DAΦNE, or by a precise measurement of the lifetime of $\pi^+\pi^-$ atoms, as planed by the DIRAC experiment at CERN.

On the other hand, varying $\alpha$ and $\beta$ in the above ranges leads to values of the threshold parameters which are in perfect agreement with the results obtained from Roy equation analyses of available $K_{\ell 4}$ data. In the range of energies accessible in $K_{\ell 4}$ decays, the two-loop chiral expression of the $\pi\pi$ amplitude together with the determination of the four constants $\lambda_i$ thus contains all the relevant information on the $\pi\pi$ interaction which is already encoded in the Roy equation.

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Constraints on $\pi-\pi$ scattering threshold parameters from low energy sum rules

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In ChPT, the effective theory of QCD at low energy, the $\pi-\pi$ scattering threshold parameters play a central role [1,2]. Despite the available phase shift analysis of the $S$- and $P$-waves, the experimental information on the threshold parameters is not very accurate [3,4]. $\pi-\pi$ scattering being a fundamental strong interaction process well suited for theoretical investigations, the principles of axiomatic field theory lead to a wealth of rigorous results [5]. Using analyticity, unitarity and crossing symmetry, and with the help of the homogeneous variables [6] three sum rules involving dispersion integrals dominated by low-energy $S$- and $P$-waves can be constructed from amplitudes which are completely symmetric in the Mandelstam variables [7].

The dispersion integrals depend significantly on poorly known threshold parameters. This lead us to a parametrization of the $S$- and $P$-wave absorptive parts occurring in the integrands reproducing the main features of the cross sections above threshold whereas the scattering lengths and effective ranges remain free parameters [8]. The sum rules are turned into nonlinear equations for the $S$- and $P$-wave threshold parameters and a combination of $D$-wave scattering lengths. We show that the solutions of these equations which are compatible with the data are confined to a rather small portion of the experimentally allowed domain and enforce a strong correlation between them [7]. This is our main result and it establishes the relevance of our sum rules. Furthermore both Standard and Generalized ChPT satisfy these sum rules constraints at the one-loop level.

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A generalization of the effective meson Lagrangian possessing the heavy quark symmetry (HQS) \[1\] to finite meson masses is employed to study the meson mass dependence of the spectrum of S– and P wave baryons containing one heavy quark or anti–quark. These baryons are described as respectively heavy mesons or anti–mesons bound in the background of a soliton of the light meson fields \[2\]. No further approximation is made to solve the bound state equations for S– and P wave heavy baryons. It is observed that the HQS–prediction for the binding energies of these baryons is approached only very slowly as the mass \(M\) of the heavy meson increases \[3\]. On the other hand the bound state wave–functions satisfy the HQS–relations reasonably well at masses as low as \(M \approx 5\text{GeV}\) \[3\].

For physically relevant mass parameters associated with the charm and bottom sectors, two types of models supporting soliton solutions for the light mesons are considered: the Skyrme model of pseudoscalars only as well as an extension containing also light vector mesons. It has been found that only the Skyrme model with vector mesons provides a reasonable description of the spectrum of both light and heavy baryons \[3\]. It furthermore turns out that the anti–quarks are unbound in the charm sector and only weakly bound, if at all, in the bottom sector \[3\]. Subsequently the system consisting of the vector meson soliton and the heavy meson bound state is projected onto states with good spin and isospin. As consistency check it has been shown that the mass gap between heavy baryons with spin \(\frac{1}{2}\) and \(\frac{3}{2}\) decreases as \(1/M\). Turning again to the physical parameters the model predicts the following mass differences: \(M(\Sigma_C) - M(\Lambda_C) = 178\text{MeV}, M(\Lambda_C) - M(N) = 1.321\text{GeV}\) and \(M(\Lambda_B) - M(N) = 4.495\text{GeV}\). These compare reasonably well with the empirical values \(165\text{MeV}, 1.345\text{GeV}\) and \((4.701 \pm 0.050)\text{GeV}\), respectively.

In the heavy quark limit the coupling between mesons containing a heavy quark and the soliton of the Nambu–Jona–Lasinio model is studied in addition. As this soliton configuration contains quark fields with non–vanishing grand spin conceptually different coupling schemes between the heavy meson field and the soliton are discovered \[4\]. These new schemes appear in addition to those which are already present \[2,3\] in Skyrme type models and may yield a larger binding of the baryon with a heavy quark \[4\].

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Low Energy QCD in the large $N_c$ Limit

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My talk starts with a quick historical review of developments of QCD in large $N_c$. I also compare the observed hadronic spectrum with the one expected from large $N_c$. I use this as a motivation to introduce and discuss the ENJL-model (see Ref.[1] and references therein,) as a low energy model of QCD at large $N_c$. The successes and drawbacks of this model are reviewed.

I emphasize the need to develop good models for the low energy QCD effective action. This is most dramatic in the non–leptonic sector of the Standard Model. I show examples of low–energy observables which require the knowledge of euclidean Green’s functions at all values of the euclidean momenta: the hadronic contributions to the muon $g-2$; and the electromagnetic $\pi^+ - \pi^0$ mass difference are two examples I discuss in the light of the ENJL–model. I show how the problem of matching long– and short–distances can be successfully solved in these two cases. I also discuss how the combined hypothesis: large $N_c$ and $\langle \bar{\psi}\psi \rangle \to 0$, require at least two more Weinberg sum rules which implies severe phenomenological constraints (see Ref.[2].)

I next discuss, within the example of the Adler’s function, the general question of matching short distance QCD behavior obtained within the QCD sum rules à la SVZ, with the long distance hadronic behavior as predicted by the ENJL–model. (See Ref.[3].) This clearly shows no overlap of the two regimes, and the need for a $G_V \neq 0$. I then discuss the possibility of matching the two regimes using the QCD–Hadronic Duality arguments developed in Ref.[4], and suggest a similar approach to solve the problem of matching encountered in the recent calculations of the light–by–light hadronic contributions to the muon $g-2$ and the $B_K$–factor discussed in this Workshop, (see these proceedings.)

In the last part of my talk I review some recent exact results for low energy observables which, following Ref.[5], have been obtained within the framework of a simultaneous expansion in the large $N_c$ limit and $U(3)_L \times U(3)_R$ chiral perturbation theory. See in particular Refs.[6] and [7].

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Dispersive Analysis of the Predictions of Chiral Perturbation Theory for $\pi\pi$ Scattering

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A way of testing the $\pi\pi$ predictions of Chiral Perturbation Theory against experimental data is to use dispersion relations to continue experimental information into the subthreshold region where the theory should unambiguously apply. Chell and Olsson [1] have proposed a test of the subthreshold behaviour of chiral expansions which highlights potential differences between the Standard [2] and the Generalized [3] forms of the theory. We illustrate how, with current experimental uncertainties, data cannot distinguish between these particular discriminatory coefficients despite their sensitivity [4]. Nevertheless, the Chell-Olsson test does provide a consistency check of the chiral expansion, requiring that the $O(p^6)$ corrections to the discriminatory coefficients in the Standard theory must be $\sim 100\%$. Indeed, some of these have been deduced [5] from the new $O(p^6)$ computations [6] and found to give such large corrections. One can then check that the $O(p^8)$ corrections must be much smaller.

We conclude that this test, like others, cannot distinguish between the different forms of Chiral Symmetry Breaking embodied in the alternative versions of Chiral Perturbation Theory without much more precise experimental information near threshold.

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FSI in $\eta \to 3\pi$ and the quark mass ratio $Q^2$

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To leading order the mass ratios of the three light quark flavours $u, d, s$ are easily accessible and known for a long time. The next-to-leading order analysis has been performed by Gasser and Leutwyler [1]. They have shown that the quantity $Q^2 = \frac{m_s^2 - \hat{m}_s^2}{m_d^2 - m_u^2} = \frac{M_K^2 - M_{K^0}^2 - M_{K^+}^2}{M_{\pi}^2 (1 + O(m^2))}$ is given by the above ratio of pure QCD meson masses, up to corrections of second order. To use the experimental mass values for the mesons one has to correct for the e.m. mass contributions. This is highly controversial as Dashen’s theorem may receive large corrections [2].

An independent way to measure $Q^2$ is provided by the isospin-violating decay $\eta \to 3\pi$ as the corresponding rate is proportional to $Q^{-4}$ [3]. Sutherland’s theorem proves to be stable [4] and the main uncertainties in obtaining a reliable rate come from the strong FSI of the $\pi$’s. To evaluate those Kambor, Wiesendanger and Wyler [5] use extended Khuri-Treiman equations. The subtraction to the dispersion relation may then be fixed by the one-loop amplitude of Gasser and Leutwyler [3]. The FSI corrections are moderate and enhance the amplitude by 14% at the center of the Dalitz plot. This reduces the usual value for $Q^2 = 24.1$ obtained with Dashen to $Q^2 = 22.4 \pm 0.9$. In agreement with this result Anisovich and Leutwyler [6] have obtained $Q^2 = 22.7 \pm 0.8$ in their dispersive analysis.

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Heavy Baryon ChPT with Light Deltas

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In recent years baryon chiral perturbation theory has matured into a systematic field theory. Starting from work in the fully relativistic framework [1], the introduction of the heavy mass formalism [2] led to the development of the so called Heavy Baryon Chiral Perturbation Theory (HBChPT) which allowed a consistent chiral power counting [3] to all orders. Once one goes beyond the leading order lagrangian in the baryon sector, one encounters two different classes of vertices: One class is accompanied by unknown counterterms analogous to the the meson sector whereas the other class corresponds to the so called 1/m corrections (relativistic corrections) to vertices of lower order in the chiral expansion. Both classes of vertices and the interplay between them are now well understood for the case of spin 1/2 nucleons up to chiral order $O(p^3)$ [4].

In the scheme of HBChPT described so far, all baryon resonances are treated as being infinitely heavy and decoupled from the theory [5]. Therefore they only contribute to higher order counterterms in the chiral lagrangian through effective contact interactions. However, it is well known from phenomenology that the first nucleon resonance, $\Delta(1232)$, plays a strong role in low energy baryon processes. It has therefore been advocated for quite a while [6] that one should keep the lowest lying spin 3/2 baryon resonances as explicit degrees of freedom in the chiral lagrangian. Several calculations along this line of thinking exist in the literature. However, many of these calculations are incomplete. In particular, the construction of the above mentioned 1/m corrected vertices involving spin 3/2 fields has been missing in the literature.

We report on recent work [5,7] in SU(2) HBChPT that allows a systematic treatment of spin 1/2 nucleon and explicit $\Delta(1232)$ degrees of freedom. Following the approach of [3], we start from the most general relativistic spin 3/2 lagrangian, explicitly keeping "point-transformation" invariance and all possible "off-shell" coupling structures. After having separated the spin 3/2 and the spurious spin 1/2 components of the Rarita-Schwinger spinor via a projector formalism, we make the transition to the heavy mass formalism. To leading order, we reproduce the results of [3] (NN-sector) and [6] ($\Delta\Delta$, $\Delta N$-sector). In next-to-leading order ($O(p^2)$), we explicitly construct all 1/m corrected vertices for the $NN$, $N\Delta$ and $\Delta\Delta$ lagrangians. We also discuss how the $O(p^2)$ $NN$ lagrangian of [3] has to be changed, once one allows for explicit $\Delta(1232)$ degrees of freedom in the theory. This leads us to a new understanding of "resonance saturation" in the baryon sector (see [5] for details). Furthermore, we discuss the $O(p^2)$ vertices of the $\Delta\Delta$ and $NN$ lagrangians accompanied by counterterms and show how our methods can be generalised to obtain the corresponding lagrangians beyond $O(p^2)$.

As a specific example we discuss the effect of $\Delta(1232)$ in the process of $\pi^0$ photoproduction at threshold. Keeping the delta-resonance in the theory introduces a new mass scale $\Delta = M_\Delta - M_N \approx 300$MeV, which is non-vanishing in the chiral limit, nevertheless small compared with the chiral symmetry breaking scale $\Lambda_\chi \approx 1$GeV. We therefore organise the calculation into a $\delta$-expansion, where $\delta$ corresponds to any of the small quantities $p, m_\pi, \Delta$. Calculating up to order $\delta^3$, we find that the leading order contribution is given by a diagram involving one of the new $O(p^2)$ 1/m corrected $N\Delta\pi$-vertices. We compare this result with a standard HBChPT calculation [8] that has the deltas "frozen out" and close with a numerical estimate.

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Resonance Saturation in the Baryonic Sector of Chiral Perturbation Theory

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Heavy baryon chiral perturbation theory (HBChPT) [1,2] including spin 3/2 delta-resonance degrees of freedom [3] has recently been reformulated by making use of a 1/m-expansion, m being the nucleon mass [4,5,6]. The theory admits a systematic expansion in the small scale $\delta$, where $\delta$ collectively denotes soft momenta, the pion mass or the delta-nucleon mass difference. It is pointed out that valuable information about HBChPT can be obtained by comparing the chiral expansion with the $\delta$-expansion. Large corrections originating from intermediate deltas can be identified and the convergence of the chiral expansion with respect to these effects can be studied. Renormalization as well as the different meaning of counterterms in HBChPT and the $\delta$-expansion, respectively, is discussed in detail. This leads directly to a reformulation of resonance saturation in the baryonic sector of ChPT [6]. As an explicit example, the scalar sector of one-nucleon processes in chiral SU(2) is worked through. In particular, it is shown that the shift of the scalar form factor of the nucleon between the Cheng-Dashen point and zero, $\sigma(2m^2_\pi) - \sigma(0) \approx 15$ MeV [7], has a natural explanation in the $\delta$-expansion.

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The empirical success of the Ademollo-Veneziano-Weinberg mass relation provides an example of regularity in the hadronic spectrum that remains unexplained by the symmetries of QCD [1]. We provide an explanation for the success of this relation based on the premise that all hadrons fill out —in general— reducible representations of $SU(2) \times SU(2)$ [2]. Mass-squared matrix elements of heavy hadrons and light hadrons are related using heavy quark and chiral symmetries [3]. Our result suggests that hadrons might be profitably viewed as bound states of weakly interacting, parity-doubled constituent quarks. We illustrate the essence of our result using a simple effective lagrangian model.

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Hadronic contributions to the muon g-2: an updated analysis
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The anomalous magnetic moment of the muon is one of the best candidates to probe the electroweak sector of the Standard Model. For a review of the theoretical aspects see e.g. [1]. Three types of contributions are present: the pure QED contributions, the hadronic contributions and the weak contributions. Pure QED contributions are largely dominant \( a_\mu^{\text{QED}} = 11658.4706(0.2) \cdot 10^{-10} \) [1], but the leading hadronic vacuum polarization contribution is sizable \( a_\mu^{\text{h.v.p.}} = 725.04(15.76) \cdot 10^{-10} \) [2] and theoretically predicted with an uncertainty of the same size of the weak contributions \( a_\mu^{\text{EW}} = 151.0(4) \cdot 10^{-10} \) [3]. To disentangle weak contributions we need to further reduce the theoretical uncertainty which affects the hadronic sector. A new BNL experiment is planning to reach an accuracy of \( \pm 40 \cdot 10^{-11} \) in the determination of the muon \( a_\mu = (g - 2)/2 \), more than a factor of twenty of reduction respect to the latest determination at the Cern Storage Ring \( a_\mu^{\text{exp}} = 11659.230(84) \cdot 10^{-10} \). This motivated the recently raised interest in the theoretical determination of this observable with an improved accuracy. At present hadronic contributions are the main source of uncertainty in the theoretical prediction. We distinguish three classes of hadronic contributions to \( a_\mu \): a) hadronic vacuum polarization (h.v.p.) contribution which appears at order \( (\alpha/\pi)^2 \) b) higher order corrections to the hadronic vacuum polarization diagram [4] and c) light-by-light scattering contributions which start at order \( (\alpha/\pi)^3 \).

The h.v.p. contribution can be extracted via phenomenological dispersive analysis from the total \( e^+ e^- \to \text{hadrons} \) cross section. This is at present the most accurate way of determination [2]. To further reduce its uncertainty new more precise data of \( e^+ e^- \to \text{hadrons} \) cross section are needed. Alternatively a low energy effective model of QCD can be used. In spite of the lack of confinement and the theoretical debated connection with QCD the Extended Nambu-Jona Lasinio (ENJL) model [5] does satisfy few phenomenological constraints (e.g. Weinberg Sum Rules) which are necessary conditions to guarantee a good matching with PQCD and provides a systematic treatment of observables dominated by long distance dynamics. Its prediction of the h.v.p. contribution [6] is in good agreement with phenomenological determinations.

A novel determination of the light-by-light scattering contribution has been proposed in [7] within the ENJL framework (see also [8] for an alternative derivation). The dominant contribution is the twice anomalous pseudoscalar exchange diagram. The final result we get is \( a_\mu^{\text{light-by-light}} = (-9.2 \pm 3.2) \cdot 10^{-10} \). This is between two and three times the expected experimental uncertainty at the forthcoming BNL muon \( g - 2 \) experiment. Adding the other Standard Model contributions to \( a_\mu \) the present theoretical estimate for the muon \( g - 2 \) is \( a_\mu^{\text{th}} = 11659.182(16) \cdot 10^{-10} \).

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Some Hadronic Matrix Elements within the Extended NJL Model

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The test of the Standard Model at low energy is generally affected by the uncertainty associated with the calculation of hadronic matrix elements at low energy. Even at low energies, the calculation of hadronic matrix elements requires, in general, the knowledge of the strong interactions at all scales. This is for instance what happens in the $B_K$ parameter [1] or in the corrections to the Dashen’s theorem [2]. There, a virtual boson ($W$ or photon) is integrated out making the internal scale to run from zero up to $M_W(\infty)$. There are other hadronic matrix elements (like $\gamma\gamma \to \pi^0\pi^0$) [3] that start at high order within CHPT and therefore are more sensible to the high energy behaviour of QCD. Also in this type of hadronic matrix elements one would like to obtain some matching with QCD. We have attacked the problem of calculating hadronic matrix elements using the Extended NJL model version in Refs. [4,5,6,7] as a good hadronic model at low energies and imposing short distance QCD behaviour at high energies. Though the matching obtained is not very good and more work to improve the intermediate energy region is needed, we have already obtained interesting results [1,2,3]. Work in the same direction is in progress for $\Delta S = 1$ decays like $K \to \pi, 2\pi$. For some work to improve the matching between the low-energy contributions and the short distance for two point functions, see the contribution by Eduardo de Rafael to this Workshop and references therein.

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On the Corrections to Dashen’s Theorem

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The electromagnetic corrections to the masses of the pseudoscalar mesons $\pi$ and $K$ are considered. At order $O(e^2)$ in the chiral limit Dashen’s theorem [1] is given by the relation

$$\Delta M_K^2 - \Delta M_\pi^2 = 0,$$

where $\Delta M_P = M_P^2 - M_{P0}^2$. At order $O(e^2 m_q)$ this relation is subject to corrections, which are probably large [2]. We calculate the contributions at order $O(e^2 m_q)$ that arise from resonances within a photon loop in the framework of chiral perturbation theory [3]. Within this approach we find rather moderate deviations to Dashen’s theorem [4].

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$K \rightarrow \pi \gamma \gamma$ decays : unitarity corrections and vector meson contributions

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$K \rightarrow \pi \gamma \gamma$ are interesting processes by themselves as ChPT tests and $K_L \rightarrow \pi^0 \gamma \gamma$ in particular might have an important rôle as a CP conserving amplitude contributing to $K_L \rightarrow \pi^0 e^+e^-$. Two different helicity amplitudes contribute to $K_L \rightarrow \pi^0 \gamma \gamma$: $A$ and $B$. The first appears at $\mathcal{O}(p^4)$ [1], it is vanishing for small diphoton invariant mass and generates a suppressed amplitude for $K_L \rightarrow \pi^0 e^+e^-$. The second amplitude $B$ appears at $\mathcal{O}(p^6)$, it is non–vanishing for small diphoton invariant mass and generates an unsuppressed amplitude for $K_L \rightarrow \pi^0 e^+e^-$ [2]. Though the experimental spectrum for $K_L \rightarrow \pi^0 \gamma \gamma$ seems very well reproduced by the $\mathcal{O}(p^4)$ leading contribution the rate is not. This has lead several authors to consider some $\mathcal{O}(p^6)$ contributions (see references quoted in [2]): i) unitarity corrections from physical $K_L \rightarrow \pi^0 \pi^+ \pi^-$ amplitude give a $20 – 30\%$ increase in the amplitude with a slight deformation of the $\mathcal{O}(p^4)$ spectrum, ii) an appropriate choice of the $B$ amplitude generated by vector meson contributions can accomodate width and spectrum, and also iii) unitarization of the $\pi \pi$ intermediate states amplitude with inclusion of the experimental $\gamma \gamma \rightarrow \pi^0 \pi^0$ amplitude should help.

$K^+ \rightarrow \pi^+ \gamma \gamma$ is also an appealing channel which will be measured soon. The leading contribution is $\mathcal{O}(p^4)$ with loops and local contributions which size is an interesting test of weak hadron dynamics [3].

We show [4] that unitarity corrections to $K^+ \rightarrow \pi^+ \gamma \gamma$ are important and generate also a $20 – 30\%$ increase in the $B$ amplitude. We then study [5] $\mathcal{O}(p^6)$ vector meson models contributing to this channel and to $K_L \rightarrow \pi^0 \gamma \gamma$ showing that local contributions generated by vector meson exchange in the charged channel are likely to be negligible contrarily to the neutral channel. This can be studied in the spectrum for small diphoton invariant mass, while the $\mathcal{O}(p^4)$ unknown local contribution can be determined from the rest of the kinematical region, or from the rate.

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Radiative Four–Meson Amplitudes in CHPT

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Chiral perturbation theory (CHPT) [1,2,3] naturally incorporates electromagnetic gauge invariance. To lowest order in the derivative expansion, $O(p^2)$ in the meson sector, amplitudes for radiative transitions are completely determined by the corresponding non–radiative amplitudes. Direct emission (DE), carrying genuinely new information, appears only at $O(p^4)$. In the case of $K \to 2\pi \gamma$ and $K \to 3\pi \gamma$ decays, the study of these information is of great interest to understand the structure of the $O(p^4)$ nonleptonic weak lagrangian [4,5].

The fact that $O(p^2)$ radiative amplitudes are completely determined by the corresponding non–radiative ones is a consequence of Low’s theorem [6]. In the case of radiative three–meson processes, like $K \to 2\pi \gamma$ decays, where the on–shell non–radiative amplitude is constant, it is straightforward to extend the relation between radiative and non–radiative amplitudes to higher orders in the chiral expansion [7,8,9,10]. On the other hand, in the case of radiative four–meson processes, the dependence from kinematical variables of the non–radiative amplitudes makes this extension less trivial. It has been shown in Ref. [11] how to extend Low’s theorem by means of second derivatives of the non–radiative amplitudes to define a “generalized bremsstrahlung” (GB). This amplitude include all the contributions to the radiative process generated by local $O(p^4)$ counterterms that contribute to the non–radiative one. By this way, the remaining part of the radiative amplitude receives $O(p^4)$ contributions only form genuine radiative counterterms (operators with an explicit electromagnetic strength tensor) and from loop diagrams.

In principle, the GB can be calculated using the experimental information on the non radiative process, minimizing the uncertainties related to higher order effects in CHPT. On the other hand, the remaining contributions must be computed using $O(p^4)$ CHPT predictions. The only loop diagrams that contribute to the DE, i.e. which are not included in the GB, are the so–called “fish–diagrams”. In Ref. [11] a compact but completely general expression for these loop amplitudes has been presented. Using the most general parametrization of the $O(p^2)$ four–meson vertices, the loop amplitudes of Ref. [11] can be applied to any radiative four–meson process, both in the strong and in the weak sector (known results for $K \to 2\pi \gamma$ decays [7,8,9,10] are recovered as a particular case).

Detailed numerical analysis for $K \to 3\pi \gamma$ and $\eta \to 3\pi \gamma$ transitions are in progress [12].

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Aspects of Renormalization in Chiral Perturbation Theory

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In a short introduction to the loop expansion in CHPT, I discussed the advantages for a consistent chiral power counting of using the lowest–order mesonic chiral Lagrangian of $O(p^2)$ to define the classical solution as the starting point for the loop expansion to any chiral order. One important implication is that the lowest–order equation of motion can be used in the generating functional for any chiral order. The renormalization procedure for the $\pi\pi$ scattering amplitude as well as for $M_\pi$ and $F_\pi$ to $O(p^6)$ [1] was the main topic of my presentation at the Workshop. Two–loop, one–loop and tree–level diagrams add up to the final renormalized quantities. Various consistency checks for the calculation were discussed that are essentially due to the proper handling of subdivergences of $O(p^4)$. One of these conditions allows for the calculation of the leading squares of chiral logs appearing at $O(p^6)$ in terms of one–loop diagrams only (with a single $O(p^4)$ vertex) [2,3]. In the last part, I analysed the mesonic generating functional of $O(p^4)$ with one off–shell meson line to arrive at the following general conclusions:

1. In the calculation of the meson–baryon functional of $O(p^3)$ [4,5] in heavy–baryon CHPT, the relative contributions of the local meson–baryon action and of the reducible tree–level diagrams with one vertex from the mesonic Lagrangian of $O(p^4)$ depend on the choice of the latter Lagrangian, i.e. on the choice of meson fields. The sum is of course independent of the chosen convention.

2. Expanding the mesonic low–energy constants of $O(p^4)$ in a Laurent expansion around $d = 4$, the coefficients linear in $d – 4$ appear in general in mesonic amplitudes of $O(p^6)$ due to two–loop diagrams. These terms can always be absorbed in the coupling constants of $O(p^6)$ in a process independent fashion. In other words, those coefficients are not measurable quantities independent of the low–energy constants of $O(p^6)$.

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Hypernuclei provide another laboratory for testing predictions of heavy baryon chiral perturbation theory[1]. In large-A nuclei, the free $\Lambda \to N\pi$ mesonic decay is Pauli blocked. Instead, the hypernucleus decays through the (nonmesonic) reaction $\Lambda N \to NN$. The meson exchange contribution to this process is dominated by pion exchange, where both the weak and strong vertices required can be found experimentally. The total nonmesonic decay widths are well reproduced using a variety of models, while the ratio of proton induced to neutron induced decays is much more difficult to understand. Shell model calculations on $^{12}\Lambda C$[2] indicate that the kaon meson exchange plays an important role in such ratios. We calculate the leading SU(3) breaking one-loop corrections to the weak KNN couplings relevant for this decay [3]. One of the motivations for this calculation is to further investigate the hyperon p-wave problem. For many years it has been known that the chiral coefficients dictated by the s-wave hyperon decays reproduce the p-wave data very poorly. Further, the leading logarithmic loop calculations for this process are found to be large, yet still in severe disagreement with the data [4,5]. This finding led to concerns about the validity of chiral perturbation theory for this process [6]. The p-wave KNN couplings that we calculate arise from the same set of diagrams which correct the p-wave hyperon decays. Therefore, a comparison of this calculation with data extracted from hypernuclear decays may lead to a better understanding of what should be expected from chiral perturbation theory in this sector. We find that the leading logarithmic corrections to KNN couplings are well behaved. This supports the suggestion that the problem in hyperon p-wave predictions comes from accidental cancellations of tree level diagrams rather than problems inherent in the theory[5]. The values for the weak KNN couplings that we find are smaller than the tree-level values. These couplings will now be used in a shell model calculation to test agreement with experimental observables[7].

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Hyperon Electromagnetic Properties in a Soliton Model

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The predictions obtained within the bound state soliton model[1] for the electromagnetic decay widths of the decuplet hyperons, the electromagnetic decay widths of the Λ(1405) resonance and the electric and magnetic static polarizabilities of the octet hyperons are discussed. Details of this work are given in Refs.[2].

Our results for the radiative decay widths of the decuplet hyperons are in good agreement with those obtained using the non-relativistic quark model (NRQM), the bag model, heavy baryon chiral perturbation theory (HBChPT) and quenched lattice QCD. This overall agreement between different models contrasts with the situation for the Λ(1405) decay widths. There, our predictions agree rather well with the results of the cloudy bag model but are, however, much smaller than those of the NRQM. Concerning the static electric polarizabilities we obtain rather small splittings between the values corresponding to the different hyperons. Moreover, they are dominated by the seagull terms which are basically given by the non-strange contributions. The structure is richer in the magnetic case because of the interplay between a large (negative) seagull part with the relevant dispersive contribution. Although some of our results are in agreement with those of the NRQM, in general this is not the case. In addition, the calculations performed in the framework of HBChPT lead to still different predictions. In this situation, it is clear that the future experimental data from CEBAF and FNAL could be of great help to discriminate among the different existing models of hyperons.

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Strange contents in the nucleon  
– The effects of kaonic cloud –  
– on the neutron electric form factor –

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As well known the nucleon is constructed by three quarks. For instance, the neutron has one up quark, which has a charge of \( +\frac{2}{3} \), and two down quarks, with a charge of \( -\frac{1}{3} \). The experiments indicate that the nucleon has an abundant structure measuring its form factor and magnetic moment. The electric charge distribution of the nucleon comes from the bare valence quarks \( (qqq) \) and valence quarks with the mesonic excitation of the vacuum \( (qqq' + \bar{q}'q) \). The total charge of neutron is zero, therefore the bare valence quark contribution to the electric charge distribution is nearly zero and the neutron electric charge distribution is dominated by the mesonic excitation of the vacuum. Hence one can expect that mesonic clouds play quite important role on the electric properties of the neutron. We investigate them employing the chiral quark soliton model. The chiral quark soliton (\( \chi QS \)) model provides a well-defined and reliable framework in studying effects of mesonic clouds on the nucleon properties. The \( \chi QS \) model is derived based on the instanton picture of the QCD vacuum [1] and described by a very simple QCD effective action, in which quarks interact via Goldstone bosons. The mesonic clouds in the \( \chi QS \) model are quite distinguished from other hedgehog models, since they are generated by the Dirac-sea quark polarization via one-quark loops.

The naive evaluation of the neutron electric form factor with the hedgehog pion which has the Yukawa tail behavior characterized by the pion mass gives a serious underestimation, because the kaon field arises as the rotational excitation of the hedgehog pion field which has the same tail behavior as the pion field. We have solved this problem using the hybrid method of treating the mesonic clouds [2]. The neutron electric form factor is quite sensitive to the mesonic clouds and the result of hybrid method fairly agrees with the experiments. We have shown also the strange electric form factor and the square radius using the hybrid method and obtained remarkably smaller results than those appearing in the previous works done in the same model framework. We have investigated also the proton electric properties, however they are rather insensitive to the mesonic clouds.

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χPT description of the MSM: One and two loop order

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χPT provide us with a general parametrization, in terms of the coefficients of the chiral lagrangian, of the symmetry breaking sector of the SM at low energies. The coefficients of the operators compatibles with the symmetries[1] (usually called chiral coefficients) can be fixed either from the experiment, in that case we will end up with a model independent description for the dynamics of the light fields (W and Z), or from the different possible candidates as underlying theory. In the latter case the chiral coefficients will encode the non-decoupling effects[2] of the heavy particle/s that have been integrated out in each particular model. Due to the good agreement of the LEP data with the MSM it seems reasonable to start by deriving the values of the chiral coefficients $a_i^{\text{MSM}}[3]$ corresponding to this model. The difference often minute between these $a_i^{\text{MSM}}$ and the contribution to the chiral coefficients corresponding to the other alternative models is where the clues of what lies beyond the SM are. We consider a scenario with a Higgs large enough to allow for a mass gap in between the light and heavy particle but sufficiently light to be able to define a perturbative series. The technique used to derive the chiral coefficients are the matching conditions between transverse connected Green functions. They have been extended to include the two next-to-leading corrections[4], the one-loop 1/$M_H^2$ order and the two-loop $M_H^2$ contribution, to the LEP1 relevant coefficients. It is proposed a new formulation of the matching conditions at higher loop orders that solves automatically the subtleties concerning gauge invariance and gives information on the scheme dependence of the chiral coefficients. As an outcome of the computation[4,5], it is shown how χPT combined with the properties of the on-shell scheme and the screening theorem of Veltman provide us with a powerful tool. It allows to improve the usual power counting estimation of the Higgs contribution[6] to the renormalized self-energies at nth-loop order by one power of $M_H^2$ less in the Z and W renormalized self-energies and two powers less in the photon self-energy. $\Delta \rho$, $\Delta r$ and $\Delta \kappa$ are also computed. All conclusions can be made extensible to any other perturbative underlying theory (MSSM, multiHiggs models, new gauge extensions, …).

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The Covariant Derivative Expansion Method (Generalized Euler Heisenberg Lagrangians)

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The Covariant Derivative Expansion Method (CDEM) is a powerful tool to calculate oneloop effective actions arising from a heavy particle which is integrated out in a given gauge field background - for a minimal coupling in the original theory as well as for a nonminimal Pauli term interaction.

In this talk emphasis is put both on the principles which make this method so efficient for its specific purpose and on one typical situation where it is a useful tool inside a more general effective field theory approach to a phenomenological problem of general interest. It is organized as follows:

In the first section, the idea of Euler Heisenberg Lagrangians is reviewed by looking at the situation in QED/QCD. It is shown that the virtual photon-photon / photon-gluon / gluon-gluon scattering effects due to the box diagram in lowest order result in an infinite tower of higherdimensional mixed photonic and gluonic operators which are suppressed by increasing powers of the heavy particle mass and give the best approximation of the original nonlocal vertex by a series of local ones. Up to operators of dimension 10 and higher the additional part in the effective action consists of one dimension 6 (GGG) and several dimension 8 (FFFF,FFGG,FGGG,GGGG) operators whose coefficients are finite. For more preliminaries see e.g. ref. [1].

In the second section an exposition of the CDEM as one of the powerful tools to calculate those (finite) coefficients is given. The CDEM was developped and promoted in ref. [2] and later improved by several autors [3]. Its main virtue is the fact that it maintains gauge invariance at every step.

The third section raises the problem of how to determine the phenomenological impact of additional CP-violation of SM-extensions (left-right-symmetric models, multi-higgs-generalizations) generated at $\Lambda \geq m_t$ by CP-violating sunset diagrams (with external gluons attached to it) at the much lower hadronic scale $\mu \sim m_s$. A step-by-step calculation is advocated which forces one to calculate the effective action stemming from integrating out the bottom quark coupled to the gluonic background through an additional CP-violating $\sigma_{\mu\nu}G^{\mu\nu}$ term induced in the previous step.

In the fourth section it is shown how the CDEM can be generalized to this situation as well. In addition the receipe for including interactions with an electromagnetic background is given. See ref. [5] for more information.

In the fifth section an RG analysis is performed which turns the present experimental bound on the neutron electric dipole moment into an upper bound on the bottom chromoelectric dipole moment at the scale where it arises. This is intended to be part of the answer to the question wether the previously mentioned high energy theories are already in conflict with present data. See ref. [4] for more information.

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Kaon-nucleon interaction and $\Lambda(1405)$ in dense nuclear matter

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We examine the free meson-baryon interaction in the strangeness $S = -1$ sector using an effective chiral Lagrangian [1]. Potentials are derived from this Lagrangian and used in a coupled-channel calculation of the low-energy observables. The potentials are constructed such that in Born approximation the s-wave scattering length is the same as that given by the effective chiral Lagrangian, up to order $q^2$. A comparison is made with the available low-energy hadronic data of the coupled $K^-p$, $\Sigma\pi$, $\Lambda\pi$ system, which includes the $\Lambda(1405)$ resonance. Good fits to the experimental data and estimates of previously unknown Lagrangian parameters are obtained. The $\Lambda(1405)$ emerges in this approach as a quasi-bound state between an antikaon and a nucleon.

Including Pauli blocking, Fermi motion and binding of the nucleons we find that the binding forces between the antikaon and the nucleon are reduced inside nuclear matter [2]. Therefore the $\Lambda(1405)$ dissolves inside nuclear matter at higher densities. Connected with this dynamics of the $\Lambda(1405)$ is a strong non-linear density dependence of the $K^-p$ scattering amplitude in nuclear matter. The real part of the $K^-p$ scattering length changes sign already at a small fraction of nuclear matter density, less than $0.2\rho_0$. This may explain the striking behavior of the $K^-$-nuclear optical potential found in the analysis of kaonic atom data.

Solving the in-medium kaon dispersion relation [3], we find a strong non-linear density dependence of the $K^-$ effective mass and decay width in symmetric nuclear matter at densities around 0.1 times normal nuclear matter density $\rho_0$. At higher densities the $K^-$ effective mass decreases slowly but stays above $0.5m_K$ at least up to densities below $3\rho_0$. In neutron matter the $K^-$ effective mass decreases almost linearly with increasing density but remains relatively large ($m_K^* > 0.65m_K$) for $\rho_n \lesssim 3\rho_0$. The $K^+$ effective mass turns out to increase very slowly with rising density. The different behavior of $K^+$ and $K^-$ effective mass in matter lead to observable consequences for $K^\pm$ production rates in heavy ion collisions, especially for sub-threshold kinematics. Recent data taken at GSI are consistent with a lowering of $K^-$ versus $K^+$ in-medium masses [4].

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\[ \gamma \gamma \rightarrow \pi \pi \pi \text{ and some comments on } U(1)_A \]

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My talk concentrated on three points:

1. The process \[ \gamma \gamma \rightarrow \pi \pi \pi \] is remarkable in the sense that the the next-to-leading order is more than order of magnitude larger than the tree level prediction s. Still we expect the next-to-leading order result to be reasonably accurate.

2. It has been generally believed that the presence of the \( U(1)_A \) as broken by the anomaly, makes chiral Lagrangians with a ninth Goldstone Boson rather unpredictive. Here we show that the functions of \( \phi_0 + \theta \) that occur naively can always be reduced by field redefinitions to just a few constants. As an example, to order \( p^2 \) the presence of the \( \eta' \) introduces at most 4 new constants. This observation generalizes to higher order making a proper extension of CHPT to the \( U(1) \) sector in principle feasible without recourse to \( 1/N_c \) counting.

3. Another problem involving \( U(1)_A \) is the fact that in the quenched approximation the \( \eta' \) has a double pole. As a result quenched QCD is not a field theory. This leads to counterintuitive results and makes the chiral limit sick. This part was to make some propaganda for the work of Bernard, Golterman, Sharpe and collaborators. The topics I discussed are in [3,4].

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