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Tachyonic AdS/QCD, Determining the Strong Running Coupling and $\beta$-function in both UV and IR Regions of AdS Space

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In this paper, we investigate QCD-like running coupling $\alpha_s^{AdS}(Q^2)$ and its associated $\beta$-function $\beta(Q^2)$ in the spirit of tachyonic AdS/QCD. We distort the bulk AdS space using color dielectric function $G(\phi(z))$, with $\phi(z)$ the tachyon field. The function presents different properties of $\alpha_s^{AdS}(Q^2)$ at small and large values of the fifth-dimensional holographic variable $z$. The function distorts the AdS space, giving $\alpha_s^{AdS}(Q^2)$ and its $\beta$-function $\beta(Q^2)$ at any $Q^2$ scale, with $Q^2$ the space-like momentum. The result obtained for a large value of $z$ is expected to show characteristics similar to nonperturbative QCD. On the other hand, the result obtained for a small value of $z$ is expected to show characteristics similar to perturbative QCQ. The presence of free tachyons leads to distortion of the AdS space at a small $z$, that notwithstanding, condensed tachyon states also lead to large $z$ distortion. This provides a unified background for determining $\alpha_s^{AdS}(Q^2)$ and its $\beta(Q^2)$ in both the ultraviolet (UV) and infrared (IR) regions using a single function in the framework of tachyonic AdS/QCD.

I. INTRODUCTION

AdS/CFT correspondence [1, 2] has been receiving some attention among experts these days. This duality scenario enable us to establish relation between quantum gravity on $d + 1$-dimensional Anti-de Sitter (AdS) space and $d$-dimensional conformal field theory (CFT). In principle, we can calculate physical observables in strongly coupled gauge theory in terms of weakly coupled classical gravity theory. One of the most intriguing examples [3] of the duality; is the correspondence between SU($N$) $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory with gravity. The correspondence is the basis for initiating holographic principles by ’t Hooft [4]. The gravitational dual is type IIB supergravity or string theory [5]. This correspondence has proven to give a supergravity explanation to field theories that display confinement and chiral symmetry breaking characteristics [6, 7]. The gauge theory dual to the AdS space has shown that the potential between two static color point particles has Coulombic potential contribution to the net potential behavior [8, 9] indicating deconfinement phase transition. Also, investigations by [10] suggest that considering compact boundary, AdS corresponds to the confinement phase while the black hole resulting from the AdS phase corresponds to the deconfinement phase. This interesting outcome is associated with Hawking and Page, transition [11], where they considered two separate solutions to the Einstein equation, the AdS, and the black hole AdS spaces. Also, QCD phenomenology derived from holographic models has been applied successfully in studying several strong interacting characteristics such as; light flavor mesons [12] and glueball phenomenology [13] using a more robust approach referred to as the configurational entropy [14–16].

In the AdS/CFT correspondence, the effective tension $T_{\text{string}}$ of the string and the string coupling $g_s$ can be related to the ’t Hooft coupling $\lambda := g^2 N$, as $g_s = \lambda/4\pi N$ and $T_{\text{string}} = \sqrt{\lambda}/2\pi$. Thus, $g_s = g^2/4\pi$, where $g$ is the gauge coupling of the SYM theory. In the limit $N \to \infty$: $g_s \to 0$, and perturbative string theory is applicable. On the other hand, in the limit, $\lambda \to 0$ (weak gauge coupling), perturbation theory in terms of Feynman diagrams apply to the gauge theory [17, 18]. In the ’t Hooft limit $N \to \infty$, $\lambda = \text{fixed}$, spacetime geometry has a curvature, $\mathcal{R}$, much smaller compared to the string scale $1/g_s$, so gravity can be a good approximation of the gauge theory. Otherwise, both theories are highly complex, making their equivalence non-trivial. Also, small curvature $\mathcal{R}$ means large AdS
radius \( R \), since \( R \sim 1/R^2 \), with \( R = (4\pi g_s N)^{1/4} l_s \), where \( l_s \) is the string length [3]. The gauge and the string couplings are related \( g_s \approx g^2 \), and in the 't Hooft limit, the string coupling becomes small such that the stringy effect decouples. In this limit, nonplanar interactions are removed, so we consider free strings on the AdS5 × S5 background. Additionally, large \( N \) is not the same as QCD; however, in many instances, the results for large \( N \) are equivalent to \( N_c = 3 \) theories, with \( N_c \), QCD color number. The 't Hooft parameter, \( \lambda \), divides large \( N \) QCD into two regions. In the limit \( \lambda \ll 1 \), we have a perturbative theory where Feynman diagrams are used to calculate amplitudes. In the limit \( \lambda \gg 1 \), we have strong coupling with nonplanar diagrams [19]. Also, Light-front holography provides means for establishing a relationship between the boost-invariant light-front wavefunctions and bound-state amplitudes in AdS space [20]. Holographic QCD provides a suitable description of hadrons with known spectroscopic dynamical characteristics [21, 22].

In this work, we deform the AdS space [23–28] with a Higgs-like dimensionless color dielectric function \( G(\phi) \) associated with tachyonic potential [29–32]. The color dielectric function deforms the AdS space in the UV region in the presence of free tachyons (active tachyons) whilst tachyon condensed color dielectric function \( G(\eta) \), where \( \eta \) is the condensed tachyon field, also deforms the AdS space in the IR region similar to the positive sign dilaton profile used in determining strong couplings in AdS/QCD. We show that the tachyon field \( \phi \) is associated with glueball field \( \varphi \), and the color dielectric function is also associated with higher dimensional operator \( H_{\mu\nu}H^{\mu\nu} \) coupled to a Standard Model (SM) gauge field \( \mathcal{O}_{SM} \) that leads to strongly interacting light glueballs [33]. The AdS action intended for this study will be formulated from the Dirac-Born-Infeld (DBI) action modified at a tachyonic vacuum similar to Sen’s AdS5 tachyonic action [34]. Firstly, we investigate the characteristics of both the strong running coupling \( \alpha_s(Q^2) \), and the associated \( \beta \)-function \( \beta(Q^2) \), by distorting the AdS space with tachyonic \( G(\phi) \). Secondly, we examine \( \alpha_s(Q^2) \) and \( \beta(Q^2) \) by distorting the AdS space with \( G(\eta) \). We expect \( \alpha_s(Q^2) \) and \( \beta(Q^2) \) to behave similarly to the perturbative QCD (pQCD) for the UV deformation of the AdS space. On the other hand, we expect AdS/QCD or nonperturbative QCD-like behavior for the IR deformation of the AdS space. The results obtained from these regions will be compared with effective couplings determined from different observables, such as: lattice QCD results, QCD phenomenology, and \( g_1 \) scheme where \( \alpha_s(Q^2) \) is extracted from the well-measured Bjorken sum rule [35–40]. This approach will bring a new perspective to AdS/QCD, where pQCD coupling characteristics can be determined through direct UV deformation of the AdS space instead of extrapolation from the IR deformation of the AdS space. Also, we will study the parameter that controls the transition from the pQCD to nonperturbative QCD.

Additionally, we show that \( \alpha_s(Q^2) \) and \( \beta(Q^2) \) are related to strongly interacting scalar glueballs with mass \( m_g \) and discuss its effect. We will discuss any Landau singularity that may be observed in the UV region and propose how to deal with it in the model framework. Strong running coupling is a subject of active research due to its limited understanding in the low momentum transfer region. Good knowledge of \( \alpha_s(Q^2) \) at \( Q \to \infty \) is necessary to match for the growing precision of hadron scattering experiments and enhance the understanding of high energy unification of strongly interacting and electroweak theories. On the other hand, a precise understanding of \( \alpha_s(Q^2) \) at \( Q \to 0 \) on the scale of proton mass enables us to understand hadron structure, confinement, and hadronization [41–44] (and references therein).

This paper is organized such that in Sec. II we review the well-known holographic QCD in two subsections: In Sec. II A we review the Light-Front Holography, and in Sec. II B we review the AdS/CFT and Holographic QCD. We set the basis for the study in Sec. III with Strong Interacting Glueballs and Association With Tachyons. Under this section, we discuss Strong Interacting Glueballs and SM Particles in Sec. III A, Confinement at Tachyonic Vacuum in Sec. III B and Tachyonic AdS/QCD Action in Sec. III C. We proceed to present Deformation of the AdS Space in the UV Region in Sec. IV under this section; we discuss the Strong Running Coupling in Sec. IV A and the \( \beta \)-function in Sec. IV B. In Sec. V, we study Deformation of the AdS Space in the IR region, which leads to the study of the associated Strong Running Coupling in Sec. V A and \( \beta \)-function in Sec. V B. We finalize by presenting our Analyses and Conclusion in Sec. VI; with the Analysis in Sec. VIA, and the Conclusion in Sec. VIB.

II. REVIEW OF HOLOGRAPHIC QCD

A. Light-Front Holography

Light-front (LF) quantization [45, 46] provides suitable grounds for investigating the structure of the hadrons concerning quark and gluon degrees of freedom. The LF wavefunctions (LFWFs) are the relativistic generalization of the Schrödinger wavefunctions. They are determined at fixed time \( \tau = x^+ = x^0 + x^3 \) referred to as the LF time [47] instead of the ‘usual’ fixed time \( t \). LF holography [22, 48–51] relates the equations of motion in AdS space to QCD Hamiltonian formalism in physical spacetime quantized on LF at \( \tau \). LFWFs in hadron physics are analogous to Schrödinger wavefunctions in atomic physics [45]. The correspondence allows for a direct relationship between hadronic amplitude \( \Phi(z) \) in AdS space and LFVF \( \phi_L(\xi) \) describing constituent quarks and gluons that form the
hadron, with \( \zeta \) the invariant impact LF variable. The connection is a consequence of AdS/CFT correspondence, a theory established to be scale independent [3]. The term correspondence is applied because we can map a weakly interacting gravity-related theory in \( d + 1 \)-dimensional AdS space onto any strongly interacting CFT (such as SYM theory) in \( d \)-dimensions, giving rise to the name gauge/gravity duality. This mapping generates a relation between the \( 5 \)-dimensional variable \( z \) of AdS space and \( \zeta \) in physical spacetime.

The mapping between \( z \) and \( \zeta \) was originally obtained by relating the electromagnetic current matrix element in AdS space [24] to the corresponding current matrix element in LF theory [22, 52]. Also, we can establish a relationship between their energy-momentum matrix elements [50]; this demonstrates the consistency of holographic mapping to physical observables in LF. The resulting equation for meson bound states \( qq \) at \( \tau \) has the form of single-variable relativistic Lorentz invariant Schrödinger equation

\[
\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right)\phi_L(\zeta) = M^2\phi_L(\zeta),
\]

where \( M \) is the mass, \( L \) relative orbital angular momentum of \( q \) (quark) and \( \bar{q} \) (antiquark), \( U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S + 1) \) is the confining potential in the soft-wall model with mass-scale \( \kappa \) and spin \( S \). Therefore, when the key dynamical impact variable \( \zeta \); is correctly identified, we can reduce the theory to a semi-classical approximation. In this case, the properties of the strong interacting dynamics; are centered on an effective potential \( U(\zeta) \). It is striking that, in the semi-classical approximation, the LF Hamiltonian has a structure that exactly agrees with the eigenvalue equations in the \( 5 \)-dimensional AdS space. That permits us to relate the AdS wave function \( \Phi(\zeta) \) to the internal constituent structure of the hadron. This characteristic yields a relation between the coordinate \( z \) in the AdS space with \( \zeta \) [45].

B. AdS/CFT and Holographic QCD

In this section, we review QCD running coupling \( \alpha_s \) and its associated \( \beta \)-function in AdS\(_5\) space-time obtained from conformal field theory. QCD is neither conformal nor supersymmetric, even though it shows minimal conformal behavior in both at far ultraviolet (UV) [53–55] and deep infrared (IR) regions [56, 57] with quark-gluon and hadron degrees of freedom respectively. Therefore, to obtain color confinement, the conformal symmetry in AdS\(_5\) space needs to be broken. This can be accomplished in many different ways. The general procedure governing the AdS/CFT approach is the isomorphism of the Poincaré group and conformal transformation \( SO(4, 2) \) to the group of isometrics of AdS\(_5\) space, the group then transforms giving us the AdS\(_5\) metric

\[
ds^2 = \frac{R^2}{z^2} \left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right),
\]

where \( R \) is the AdS radius. The metric is invariant under scale changes in the fifth-dimensional variable \( z \to \tilde{\lambda}z \) and the spacetime variable \( x^\mu \to \tilde{\lambda}x^\mu \). The \( z \) coordinate acts like a scaling variable in the Minkowski space. Different values of \( z \) correspond to a different measure of 4-momentum for which the hadron can be studied [58]. Some well-known functions that are introduced to break the AdS\(_5\) space are; dilaton profiles \( e^{\phi(z)} \) with \( \phi(z) \) the dilaton field, [21, 59] or a warp factor \( e^{A(z)} \), where \( A(z) \) is any suitable function, commonly, logarithmic functions are used i.e., \( A(z) \sim \log(z) \). Whichever function is chosen to distort the AdS\(_5\) space leads to confinement in uniquely different form [60]. A more successful form of this study has been achieved through LF Holographic mapping to classical gravity when the AdS\(_5\) space is modified by a positive-sign dilaton background. These changes induce an analytically color confinement leading to nonperturbative AdS/QCD running coupling, \( \alpha_s^{AdS}(Q^2) \). It is important to note that the understanding of running coupling for pQCD is limited to short distances or high \( Q^2 \) regimes. In a similar sense, the AdS/QCD with positive dilaton background is based on long-distance interactions and a small \( Q^2 \) regime [58].

Analytically, the AdS\(_5\) action bears resemblance to the general relativity action \( S_{GR} \propto \int d^4x\sqrt{|det g_{\mu\nu}|}R/G_N \), where \( R \) is the Ricci scalar and \( G_N \) is the Newton’s constant. In AdS/QCD we replace Ricci scalar with the gauge field as \( \sqrt{R} \to F_{\mu\nu} \), \( G_N \) is replaced by the gauge coupling \( \sqrt{G_N} \to g_s \), and the modulus of the metric determinant \( |det g_{\mu\nu}| \) is replaced by the modulus of the AdS\(_5\) metric determinant \( \sqrt{|det g_{\mu\nu}|} \to \sqrt{|g_{AdS}|}e^{\kappa^2z^2} \) which includes the dilaton profile \( e^{\kappa^2z^2} \) to distort the AdS\(_5\) space, here, \( \phi_d(z) = \kappa^2z^2 \). Therefore, the AdS\(_5\) action takes the form

\[
S_{AdS} = -\frac{1}{4} \int d^4x dz \sqrt{|g_{AdS}|}e^{\kappa^2z^2} \frac{1}{g_s^2} F_{\mu\nu} F^{\mu\nu}.
\]

From Eq. (2) the determinant of the metric is \( \sqrt{|g_{AdS}|} = (R/z)^5 \) and from the above expression, after introducing the dilaton profile, it turns into \( \sqrt{|g_{AdS}|} = (R/z)^5 e^{\kappa^2z^2} \). The prefactor \( g_s^{-2}(z) = g_s^{-2} e^{\kappa^2z^2} \),
restores the conformal symmetry in Eq. (3) and leads to color confining properties, in this case, $g_5^2$ has length dimension. Now, comparing $g_5$ to the coupling in the $\mathcal{N} = 4$ SYM theory $g$, and mapping $z \rightarrow \zeta$, we can calculate the AdS/QCD $\alpha_s^{AdS}(Q^2)$ and the associated $\beta(Q^2)$ [44].

III. STRONG INTERACTING GLUEBALLS AND ASSOCIATION WITH TACHYONS

In this section, we intend to establish the relationship between the hidden scalar glueball state and the tachyonic field used in this paper. Glueballs are candidates for dark matter particles with significant implications in cosmology and particle physics. Also, some glueballs may be self-interacting through strong force with crucial astrophysical implications [33, 61–63]. These particles may be observed in particle physics experiments when established that they interact with SM particles through high dimensional operators. We will establish a relationship between glueballs and tachyons and calculate the QCD-like strong running coupling $\alpha_s$ and the associated $\beta$-function in a QCD-like fashion using Higgs-like tachyonic potential.

A. Strong Interacting Glueballs and SM Particles

It has been demonstrated that glueballs interact with the SM sector through higher dimensional operators, as will be discussed below. That means glueballs may be observed through particle physics experiments instead of the overlap of gravitational means that have proven difficult over the years. In Ref. [33], the authors investigated this phenomenon using a simple setup to establish the interaction between the glueballs and the SM particles through higher dimensional operators. In their model, they derived

$$\mathcal{L} = \frac{1}{M^n} H_{\mu\nu} H^{\mu\nu}\mathcal{O}_{SM},$$

(5)

where $M$ is the cutoff scale, $n$ is a positive constant related to the chosen spacetime dimension, and the choice of the SM interaction part $\mathcal{O}_{SM}$ is open. Consequently, we can express

$$\mathcal{L} = \frac{1}{M^{4n}} H_{\mu\nu} H^{\mu\nu}(F_{\alpha\beta} F^{\alpha\beta}) \rightarrow \frac{N m^3}{M^4} \varphi F_{\alpha\beta} F^{\alpha\beta},$$

(6)

where $F_{\alpha\beta}$ is the abelian field strength, $m$ is the glueball mass, $H_{\mu\nu} H^{\mu\nu}$ is the higher dimensional operator, $N$ is unspecified number of colors from SU($N$) gauge group and $\varphi$ is the lightest glueball field. Also, $N$ is chosen to make $\varphi$ self-interacting. Detailed discussion on this subject is contained in [33] and the references therein.

B. Confinement at Tachyonic Vacuum

To derive an equivalence relation for the tachyonic fields for comparison with Eq. (6); we start with the Dirac-Born-Infeld (DBI) action [64–68] and subsequently, we will modify it with the tachyonic field at tachyonic vacuum. The interesting feature about bosonic opened strings at tachyonic vacuum is that they behave like closed strings with no Dp-branes. In that view, the endpoints of the opened strings are connected by a flux line on the Dp-brane worldvolume to form closed strings [69–81]. Also, in this region, the negative tachyon energy density $V(\phi)$ exactly cancels the Dp-brane tension $\varepsilon_p$, i.e., $V(\phi) + \varepsilon_p = 0$ [74, 75, 77, 78, 82]. Therefore, there is no energy cost in rearranging the strings on the remaining lower-dimensional branes in the region, and there are no opened string excitations as well [83, 84]. Thus, the remaining lower-dimensional branes connecting the endpoints of the strings; serve as the source and sink of the color charges. In the current study, the particles involved are valence gluons. As a result, the closed strings formed at the tachyonic vacuum; are treated in the spirit of color flux-tube picture in chromoelectric flux confinement associated with QCD theory. Here, the opened strings considered are initially living on nonperturbative Dp-branes [85]. That makes them suitable for applications in nonperturbative QCD. We will focus on the gauge field for the current analyses. Thus all the other fields are decoupled for convenience. Therefore, the DBI action takes the form

$$S_{DBI} = -T_p \int d^{p+1} \xi e^{-\phi_4} \sqrt{-\det(\eta_{\mu\nu} + (2\pi \alpha') F_{\mu\nu})},$$

(7)

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the abelian gauge field strength living on the worldvolume of the Dp-brane, Dp-brane tension

$$\tau_p = \frac{T_p}{g_s} = \frac{1}{g_s \sqrt{\alpha'} (2\pi \sqrt{\alpha'})^p},$$

(8)
string tension
\[ T_{\text{string}} = \frac{1}{2\pi \alpha'}, \]  
and string coupling \( g_s = e^{(\phi_d)} \). To understand the strong interaction dynamics of the opened strings on the Dp-brane worldvolume, we derive the dimensional reduced U(1) Yang-Mills theory \([86, 87]\) by small perturbation in Eq.\((7)\), which yields
\[ S_{YM} = -\tau_p V_p - \frac{1}{4 g_{YM}^2} \int d^{p+1} \xi F_{\mu\nu} F^{\mu\nu} + \mathcal{O}(F^4). \]  
Here, \( V_p \) is the p-brane worldvolume, which can be ignored subsequently, and \( g_{YM} \) is the Yang-Mills coupling given by,
\[ g_{YM}^2 = \frac{1}{(2\pi \alpha')^2 \tau_p} = \frac{g_s}{\sqrt{\alpha'}} \left( \frac{2\pi \sqrt{\alpha'}}{\alpha'} \right)^{p-2}, \]  
hence, the relation \( g_{YM}^2 \sim g_s \), that forms the basis for the gauge/gravity duality; is established. This expression reduces to the 10 d \( N = 1 \) Super Yang-Mills action. Moreover, since the mass of the string is proportional to its length, the closer the branes, the less massive the attached strings may be \([68, 88]\). Thus, for stacks of \( N \) Dp-branes, we have U(1)\(^N\) gauge groups on its worldvolume which can be decomposed as U(\(N\)) = SU(\(N\)) \(\times\) U(1). That can further be decomposed into standard model symmetry depending on the number of Dp-branes crossing each other \([89]\). Now we modify Eq.\((7)\) with a tachyon potential \( V(\phi) \)[90], with \( \phi \) the tachyon field
\[ S = -T_p \int d^{p+1} \xi e^{-\phi_d} V(\phi) \sqrt{-\det(\eta_{\mu\nu} + (2\pi \alpha') F_{\mu\nu})} \]
\[ = -\tau_p \int d^{p+1} \xi V(\phi) \left[ 1 + \frac{1}{4} (2\pi \alpha')^2 F_{\mu\nu} F^{\mu\nu} + \mathcal{O}(F^4) \right], \]  
in the last step, we considered slowly varying tachyons with a constant dilaton field \( \langle \phi_d \rangle \). At the minimum of the potential \( V(\phi_0) \) the action disappears \([69, 72, 83]\). Also, the potential is symmetric under \( \phi_0 \rightarrow -\phi \), with maximum at \( \phi = 0 \) and minimum at \( \phi_0 = \pm a \). We will disregard the first term and consider only the gauge fields propagating on the Dp worldvolume for the analysis because the gauge field is enough for the current study. Additionally, we introduce a dimensionless function \( G(\phi) = (2\pi \alpha')^2 V(\phi) \), which will be referred to as the color dielectric function that drives the confinement and deconfinement transitions. Consequently,
\[ \mathcal{L} = -\frac{1}{4 g_s} G(\phi) F_{\mu\nu} F^{\mu\nu}, \]  
for \( T_p = 1 \). Comparing Eqs.\((6)\) and \((13)\), we can identify the higher dimensional operator, \( H_{\mu\nu} H^{\mu\nu} \) with \( G(\phi) \) and the glueball field \( \varphi \) with the tachyonic field \( \phi \). Introducing the kinetic energy component of the tachyon field into the last step of Eq.\((12)\), we studied the phenomenology of confinement in references \([29\text{--}32, 91]\).

### C. Tachyonic AdS/QCD Action

Considering 5D holographic QCD models, the background flavor branes are D4-D4 system \([92, 93]\). However, in this study, we consider opened strings with their endpoints on the same brane, so we consider a background D4-brane. Hence, we can express AdS\(_5 \) action similar to the one proposed in \([34]\) adopted in \([94]\) for studying hadronic properties in the light of AdS/QCD, from Eq.\((12)\)
\[ S = -\frac{1}{4 g_5} \int d^4 x dz G(\phi(z)) F^{\mu\nu} F_{\mu\nu}, \]  
where \( G(\phi(z)) \) is only a function of \( z \) and \( g_s \rightarrow g_5^2 \) has the dimension of length. Additionally, tachyonic corrections were introduced in Ref.\([95]\) to show equivalence in AdS/QCD and tachyonic AdS/QCD \([96]\) in 4D holographic dual to 5D weakly coupled gravity using similar action. The \( G(\phi) \) will be carefully defined to achieve the phenomenon of confinement and asymptotic freedom in separate energy regimes. Again, we demonstrate that the tachyonic color dielectric function, \( G(\phi) \), deforms the AdS\(_5 \) metric at the asymptotic AdS\(_5 \) bulk in the UV limit \( (z \rightarrow 0) \) corresponding
to deconfinement transition associated with AdS$_5$ black holes [11]. On the other hand, deforming the AdS$_5$ metric with condensed tachyonic color dielectric function $G(\eta)$ leads to IR deformation of the AdS$_5$ bulk leading to the phenomenon of color confinement in the IR regime ($z \to \infty$), similar to deforming the ‘usual’ AdS$_5$ space with positive dilaton background [28]. Here, the confining transition corresponds to color singlet glueball states with $N^3$ degree of freedom whilst the deconfinement transition corresponds to the state where the gluons are free with $N^2$ degrees of freedom [10, 19].

### IV. DEFORMATION OF THE ADS SPACE IN THE UV REGION

#### A. Strong Running Coupling

The strong running coupling $\alpha_s^{AdS}(Q^2)$ can be determined by defining a new coupling $g_5(z)$ with dependence on $z$, that restores the conformal symmetry in Eq. (14),

$$g_5^{-2}(z) = g_s^{-2}G(\phi(z)).$$

The deformation of the conformal symmetry in Eq. (14) is relevant for obtaining running coupling and a nonzero $\beta$-function because conformal symmetry leads to constant coupling and zero $\beta$-function. The scale-dependent gauge field depicted by the expression above should be regarded as a consequence of gauge field strength renormalization, $F^{\mu\nu} \rightarrow G^{1/2}F^{\mu\nu}$. In ‘regular’ QCD theory, we can rescale the gluon field as $A^{\mu} \rightarrow \tilde{A}^{\mu}$ which leads to $G^{\mu\nu} \rightarrow \tilde{G}^{\mu\nu}$, with $G^{\mu\nu}$ the non-abelian field strength, in the QCD Lagrangian density, $L_{\text{QCD}}$, this is compensated with rescaling the coupling strength $g \rightarrow \tilde{\lambda}^{-1}g$. Thus, the renormalization of the physical QCD coupling $g_{\text{phys}} = Z_3^{-1/2}g_0$ [97], with $Z_3$ the gluon propagator renormalization factor and $g_0$ the bare coupling in UV-regulated theory similar to renormalization of gauge field and gauge field strength: $A^{\mu}_{\text{ren}} = Z_3^{-1/2}A_0^{\mu}$, $G^{\mu\nu}_{\text{ren}} = Z_3^{-1/2}G_0^{\mu\nu}$ resulting into a rescaled Lagrangian $\mathcal{L}_{\text{QCD}}^{\text{ren}} = Z_3^{-1}\mathcal{L}_{\text{QCD}} = (g_{\text{phys}}/g_0)^{-2}\mathcal{L}_{\text{QCD}}^{0}$. All quantities with subscript or superscript ‘0’ are bare and the ones with ‘ren’ are their renormalized counterparts.

Moreover, we can relate the length scale $z$ with the invariant LF impact variable $\zeta$, which appears in the LF Hamiltonian. The nonconformal dynamics, confinement, and deconfinement transitions associated with QCD will now be investigated through $g_5(z)$ where, $g_5(z) \rightarrow g_{YM}(\zeta)$ — see Eq. (10) for the comparison with YM theory. Therefore, we define AdS$_5$ coupling constant, $\alpha_s^{AdS} \equiv g_5^2/4\pi$, which requires $Q^2$-dependence to fully describe the dynamics of the particles in both UV and the IR domains, corresponding to short and long-distance interparticale separations, respectively. What makes the $G(\phi)$ interesting is that it distorts the AdS$_5$ space curvature in two different forms. One in the UV regime and the other in the IR regime, so perturbation and nonperturbative characteristics of the QCD-like coupling can be investigated in a single framework [58]. We can relate the LF strong coupling with $G(\phi(\zeta))$ as

$$\alpha_s^{AdS}(\zeta) = \frac{g_{YM}^2}{4\pi} \propto G(\phi(\zeta))^{-1}. \quad (16)$$

It is necessary to identify the nature of $G(\phi(\zeta))$ in order to compute $\alpha_s^{AdS}(Q^2)$ and $\beta(Q^2)$. The form of $G(\phi(\zeta))$ has been proposed in Refs. [29–32] and used in investigating the phenomenon of (de)confinement of glueballs and glueball-fermion mix states in $1+3$-dimensions. As we have demonstrated in Sec. III B, the dimensionless $G(\phi(\zeta))$ is directly associated with the tachyon potential $V(\phi)$ for strings with $T_{\text{strings}} = 1$ corresponding to $(2\pi\alpha')^2 = 1$, thus, we adopt

$$V(\phi(\zeta)) = G(\phi(\zeta)) = \frac{1}{4}[\phi(\zeta)^2 - a^2]^2. \quad (17)$$

In the context of the tachyon potential; $a$ and $\phi$ are dimensionful constants with energy dimension and $\Lambda$ is a dimensionless constant. However, in the context of the $G(\phi)$; these quantities are dimensionless due to $(2\pi\alpha')^2$ multiplying the potential. We use a linear ansatz $\phi(z) = \alpha z$, where $\alpha$ is the tachyon decay scale and $f_\alpha = 1/\alpha$ is the decay constant, hence,

$$G(\phi) = \frac{\Lambda}{4}[((\alpha \zeta)^2 - a^2)^2], \quad (18)$$

where $\alpha$ is a dimensionful constant with the dimension of energy. The scale anomaly in QCD induces a nonvanishing energy-momentum trace tensor, given as

$$\langle \Theta^{\mu}_\nu \rangle = \frac{9}{8} \left\langle \frac{\alpha_s}{\pi} G^{\mu\nu} G_{\mu\nu} \right\rangle, \quad (19)$$
where $\alpha_s$ is the strong coupling and $G^{\mu\nu}$ is the non-abelian gauge field strength. Using parton-hadron duality principle [98–101]

$$\langle \mathcal{O}_\mu^\nu \rangle = \left( 4V(\phi) - \phi \frac{dV}{d\phi} \right)_{\phi_0=0} = 4V(\phi_0) = \Lambda a^4. \quad (20)$$

Substituting Eq.(18) into the above expression, we obtain the bag constant, $-B_0 = \Lambda a^4$. Equating the Bag constant to the gluon condensate yields,

$$\Lambda = \frac{9(\alpha_s G^{\mu\nu} G_{\mu\nu})}{8\pi a^4}, \quad (21)$$

hence, the coupling $\Lambda$, is proportional to the gluon condensate [102, 103]. Calculating the physical running coupling measured on $Q^2$-scale, we use Mellin transform,

$$\alpha_s^{AdS}(Q^2) \sim \int_0^{\infty} \alpha_s^{AdS}(\zeta) \zeta^{Q^2/m_5^2 - 1} d\zeta$$

$$= \frac{\pi}{\Lambda a^4 m_5^2} \left( -\frac{\alpha^2}{a^2} \right)^{-Q^2/2m_5^2} \left( 2m_5^2 - Q^2 \right) \csc \left( \frac{\pi Q^2}{2m_5^2} \right)$$

$$= \frac{\sigma \Lambda \pi}{m_5^2} \left( \frac{m_5^2}{\Lambda f_\alpha^2} \right)^{Q^2/2m_5^2} \left( 2m_5^2 - Q^2 \right) \csc \left( \frac{\pi Q^2}{2m_5^2} \right). \quad (22)$$

We substituted Eq.(16) and introduced $m_5^2$, the glueball mass into the integral to obtain the above expression. We will demonstrate below that, the glueball mass is related to the scale $Q_\Lambda$ at which pQCD breaks down in the model framework. Also, $\sigma$ is a dimensionful proportionality constant associated with the Regge slope, $\sigma \sim 1/(2\pi a')^2$ with dimension, $[\sigma] = \text{energy}$. We showed in Eq.(12) and stated below it that, $G(\phi)$ is a dimensionless function associated with the tachyon potential through the Regge slope, so the proportionality constant serves as a dimensional correction to the equation since dimensionful quantities such as the glueball mass are introduced. It should be noted that in the tachyon potential $a$ and $\phi$ have dimensions of energy but in $G(\phi)$ these quantities are dimensionless. Hence, the proportionality constant $\sigma$ is introduced to make $\alpha$ and $a$ dimensionful. In the last step, we substituted $-m_5^2 = V''(\phi)|_{\phi_0=0} = a^2 \Lambda$, obtained from Eq.(17) around its 'false' vacuum; here, $m_5^2 > 0$ and $a^2 \Lambda < 0$. For mathematical convenience, we set the dimensionless coupling constant, $\Lambda$, to unity $\Lambda = 1$, and also recall that $(2\pi a')^2$ has also been set to unity below Eq.(12), consequently; $\sigma \sim 1$, but we shall keep it for dimensional consistency. Therefore, Eq.(22) simplifies to

$$\alpha_s^{AdS}(Q^2) = \frac{\sigma \pi}{m_5^2} \left( m_5^2 f_\alpha^2 \right)^{Q^2/2m_5^2} \left( 2m_5^2 - Q^2 \right) \csc \left( \frac{\pi Q^2}{2m_5^2} \right). \quad (23)$$

The negative mass square represents instability in this region due to free tachyons. Since glueballs are bound states of pure gluons with gluon degrees of freedom, and $\Lambda$ is proportional to the gluon condensate, negative $m_5^2$ implies a change in the sign of the gluon condensate, a phenomenon expected in the quark-gluon-plasma (QGP) regime. It has been shown in Ref.[30] that in the perturbative UV region, the square of the glueball mass $m_5^2$ changes sign, i.e., $m_5^2 \rightarrow -m_5^2$. These glueball masses' presence reflects the QGP’s nonperturbative properties beyond the transition temperature $T > T_C$: Indeed, the isoscalar glueball mass, and the gluon condensate, are expected to change sign at $T > T_C$ [104, 105]. Lattice QCD calculations for pure SU(3)$_c$ affirms this sign changes [106]. Additionally, since the four-momentum of tachyons is necessarily space-like due to their imaginary mass, they can be seen with negative energy in some inertial frames. In this case, we can reverse the sign of the time intervals. Thus, negative tachyon energy moving backward in time can be thought of as a positive energy tachyon moving forward in time. As a result, the sign of the tachyon decay constant $f_\alpha$, which can be associated with glueball decay, can be altered $f_\alpha \rightarrow -f_\alpha$ without affecting the physical properties of the particles [107, 108].
FIG. 1: A graph of $\alpha_s^{AdS}(Q^2)/\pi$ against $Q$ for $m_\phi = 0.86$ GeV (left) and $m_\phi \to \infty$ (right).

(a) Left Panel

(b) Right Panel

We compare the graph (left) with the ones contained in Refs. [27, 44] for $\alpha_g(Q)/\pi$ (pQCD) and other experimental data in [110], and we observe the expected decrease of $\alpha_s$ at large $Q$. We present results for both; the lightest glueball mass (left) and the heaviest possible glueball mass (right). However, the theory governing the study is consistent with light glueball masses, so the graph in the right panel is for analytical purposes and may not have any physical consequences in the current study. We observe that $\alpha_s^{AdS}(Q^2)$ decreases sharply with increasing $Q$; on the other hand, an increase in $m_\phi \to \infty$ moves the graph more towards a small $Q$ region beyond the Landau pole, $Q_{\Lambda}$, (vertical blue line) and also falls quickly at large $Q$. Thus, heavy glueballs are more likely to be confined than lighter ones.

We plot the leading order of the perturbative result of the QCD running coupling given by

$$\alpha_s^{pQCD}(Q^2) = \frac{4\pi}{\beta_0 \ln \left( \frac{Q^2}{\Lambda^2_{QCD}} \right)}$$

where $\Lambda_{QCD}$ is the QCD scale,

$$\beta_0 = 11 - \frac{2}{3} n_f,$$

is the first term of the $\beta$-series, with $n_f$ the number of quark flavors active at scale $Q^2$ [44] for comparison with our results.

FIG. 2: A graph of $\alpha_s^{pQCD}(Q^2)/\pi$ (blue dashed) and $\alpha_s^{AdS}(Q^2)/\pi$ (red solid line) against $Q$ for $\Lambda_{QCD} = 0.58$ GeV.

We set $n_f = 0$ in this plot to consider pure gluon states for the comparison. The dashed vertical blue line and the faint solid blue line superimposed on each other are the positions of the Landau pole for both $\alpha_s^{pQCD}(Q^2)$ and $\alpha_s^{AdS}(Q^2)$ respectively. We chose the position of the Landau poles for $\alpha_s^{pQCD}(Q^2)$ and $\alpha_s^{AdS}(Q^2)$ to coincide with each other for easy comparison.
B. Beta-Function

The $\beta$-function is calculated using the renormalization group theory,

$$
\beta(\alpha^{AdS}(Q^2)) = Q^2 \frac{d \alpha^{AdS}(Q^2)}{dQ^2}
= -Q^2 \frac{2 \pi \sigma}{m_\phi^8} \left( \frac{f_\alpha^2 m_\phi^2}{2 m_\phi^2} \right) Q^2 / 2 m_\phi^2 \csc \left( \frac{\pi Q^2}{2 m_\phi^2} \right) \left( (2 m_\phi^2 - Q^2) \left[ \pi \cot \left( \frac{\pi Q^2}{2 m_\phi^2} \right) - \log [f_\alpha^2 m_\phi^2] \right] + 2 m_\phi^2 \right).
$$

The "Principle of Conformality" is satisfied, i.e.,

$$
\beta(Q \to 0) = \beta(Q \to \infty) = 0.
$$

(27)

In the classical approximation to QCD without hadronization, $\beta$-function vanishes, and the corresponding theory is conformal [111]. This behavior is maintained when quantum corrections are included for large $Q^2$ due to asymptotic freedom and attain a finite value close to $Q \to 0$ due to the infrared fixed point. Infrared fixed point translates naturally into color confinement [112] with hadron degrees of freedom. Because the propagator of the colored particles has their maximum wavelength in the IR regime, all the loop integrals for the self-interacting gluon energy decouples at $Q \to 0$ [113]. The other condition is,

$$
\beta(Q) < 0 \quad \text{for} \quad Q > 0,
$$

(28)

where $Q_\Lambda$ is the point at which pQCD fails. This condition reflects the anti-screening behavior of QCD theory at large $Q$ where its running coupling, $\alpha_s$, vanishes due to asymptotic freedom with quark-gluon degrees of freedom. The QCD $\beta$-function is generally negative, and its value increases from the UV regime and assumes its maximum value in the IR domain.

FIG. 3: A graph of $\beta(Q^2)$ against $Q$ for $m_\phi = 0.86$ GeV (left) and $m_\phi \to \infty$ (right).

(a) Left Panel

(b) Right Panel

The graph (left) is compared with $\beta(Q)$ (pQCD) reproduced in Refs. [27, 58]. The graph in the right panel shows the behaviour of the $\beta(Q^2)$ for $m \to \infty$.

We expect the $\beta$-function to have a usual analytical behavior in both the UV and the IR regimes with or without the Landau pole in $\alpha_s$. The 'usual' behavior of $\beta(Q^2)$ is due to its conformal invariant characteristics leading to the well-behaved cut-off in both the IR and the UV regimes. Regardless of the singularity exhibited by $\alpha_s$.

The result in Eq. (23) displays the expected decrease in $\alpha_s$ at large $Q^2$ similar to the running coupling displayed for pQCD in Fig. 2. We determine the position of the Landau pole $Q_\Lambda$ in Eq. (23) using the relation for determining the transition point,

$$
\frac{d \beta}{dQ} = 0.
$$

(29)

However, the results obtained from the above operation are not analytically solvable for $Q_\Lambda$ because of the trigonometric functions involved. So we expand it for a small $Q$ leading to

$$
\frac{8}{m_\phi^8 Q^2} + \left( \frac{\pi^2 - 6 \log [f_\alpha^2 m_\phi^2]}{3 m_\phi^5} \right) \frac{Q}{m_\phi^8} + O(Q^3) = 0,
$$

(30)
\[ Q_\Lambda = \left( - \pi^2 + 6 \log[f_\phi^2 m_\phi^2] - 3 \log[f_\phi^2 m_\phi^2] \right)^{1/4} m_\phi. \] (31)

Using the same parameters used in plotting Figs. 1, 2 and 3, i.e., \( m_\phi = 0.86 \text{ GeV} \) and \( f_\phi = 0.50 \text{ GeV}^{-1} \) we obtain \( Q_\Lambda \approx 0.58 \text{ GeV} \) as the position of the Landau pole. The position of the pole is denoted by a solid blue line in Figs. 1, 2, and 3 above. The Landau singularity signals the scale at which pQCD breaks down hence, it is often related to the confinement scale or hadronic mass scale. The singularity is also attributed to the scale dependence of \( \alpha_s \). Hence, it serves as an expansion parameter for other quantities \(^{109}\) such as the \( \beta \)-function.

V. DEFORMATION OF THE ADS SPACE IN THE IR REGIME

For nonperturbative glueballs, we perturb the tachyon fields about its ‘true’ vacuum \( \phi_0 = \pm a \) by introducing a new field \( \eta \) through \( \phi \rightarrow \eta + \phi_0 \). This stabilizes the tachyons and the new field acquires a positive mass \( V''(\phi)|_{\phi_0=\pm a} = 2a^2 \lambda = m_\phi^2 \) consequently, Eq.(17) become

\[ V(\eta) = V(\phi)|_{\phi_0=\pm a} + V'(\phi)|_{\phi_0=\pm a} \eta + \frac{1}{2} V''(\phi)|_{\phi_0=\pm a} \eta^2 \]

\[ = \frac{1}{2} m_\phi^2 \eta^2, \] (32)

this process coincides with tachyon condensation. The new field \( \eta \) is identified with a stable glueball field \(^{30, 32}\) in the IR regime with a solution

\[ \eta(\zeta) = \frac{e^{m_\phi \zeta}}{\zeta}, \] (33)

which produces confinement of glueballs in the limit of low energy or large particle separation distance where color confinement is expected, \( \zeta \rightarrow \zeta_* \) \(^{29}\). Here, \( \zeta_* \) is the separation distance within which confinement occurs. The choice of Eq.(33) was selected, guided by the introduction of the kinetic energy term of \( \phi \) into Eq.(12) and solving the resulting equations of motion in the static sector for the new field \( \eta \) in spherical coordinates that leads to phenomenological Cornell-like confining potential for heavy quarks in 3D. We have carried out this studies comprehensively in our previous papers \(^{29–32}\), and established that in the limit of large spatial separation distance \( r \rightarrow r_* \) the solution Eq.(33) is dominant.

A. Strong Running Coupling

Using the color dielectric function for the condensed tachyons, Eq.(16) alters to

\[ \alpha_s^{AdS}(\zeta) \propto G(\eta(\zeta))^{-1}. \] (34)

Using Laplace transform

\[ \alpha_s^{AdS}(Q^2) \sim \int_0^\infty \alpha_s^{AdS}(\zeta)e^{-Q^2\zeta/m_\phi}d\zeta \]

\[ = \frac{4m_\phi \rho}{(2m_\phi^2 + Q^2)^3}, \] (35)

where \( \rho \) is a proportionality constant. At \( Q = 0 \), \( \rho = 2m_\phi^3 \alpha_s^{AdS}(0) \), and the above expression transforms into

\[ \alpha_s^{AdS}(Q^2) = \frac{(2m_\phi^2)^3}{(2m_\phi^2 + Q^2)^3} \alpha_s^{AdS}(0). \] (36)
FIG. 4: A graph of $\alpha^{AdS}(Q^2)/\pi$ against $Q$ for $m_\phi = 1.73$ GeV (left) and $m_\phi \to \infty$ (right).

(a) Left Panel

(b) Right Panel

The graph in the left panel has a similar behavior with the effective coupling for $\alpha_{s,g,1}(Q)$ in Refs. [27, 44, 58] normalized at $\alpha^{AdS}(0) = \pi$. In the right panel, we observe that increasing $m_\phi \to \infty$ reduces the deconfinement energy threshold and strengthens confinement.

B. Beta-Function

We can also calculate the $\beta$-function from Eq.(26) which leads to the expression

$$\beta(Q^2) = -\frac{3(2m_\phi^2)^3Q^2}{(2m_\phi^2 + Q^2)^4} \alpha^{AdS}(0).$$

(37)

This expression satisfies the restrictions

$$\beta(Q \to 0) = \beta(Q \to \infty) = 0 \quad \text{and} \quad \beta(Q) < 0 \quad \text{for} \quad Q > 0.$$  

(38)

The first condition satisfies the restrictions that QCD approximates conformal theory in both the far UV and the deep IR regimes, known as "the principle of maximum conformality": meanwhile, the second condition is a consequence of the asymptotic freedom properties of QCD theory. Hence, $\alpha^{AdS}(Q^2)$ can be normalized to $\alpha_{s,g,1}$ scheme. A well-measured effective charge obtained from the Bjorken sum rule [35], for polarized deep inelastic lepton-proton scattering, by exploring the similarities in the behavior of the $\beta$-functions. It vanishes in the deep infrared domain, rises to a higher negative intermediately, and fall-off to zero in the far UV domain. We take advantage of these similar features and normalize $\alpha^{AdS}(Q^2 = 0) = \pi$ for comparison. Taking the derivative with respect to $Q$,

$$\frac{d\beta}{dQ} = -\frac{6(2m_\phi^2)^3Q\pi}{(2m_\phi^2 + Q^2)^4} \left[1 - \frac{4Q^2}{2m_\phi^2 + Q^2}\right],$$

(39)

consequently, there is a transition at $Q_0 = \sqrt{(2/3)m_\phi}$. Perturbation theory is applicable in the regime $Q_0 > \sqrt{(2/3)m_\phi}$ whilst non-perturbation theory is applicable in the regime $Q_0 < \sqrt{(2/3)m_\phi}$. Also, the conditions

$$\frac{d\beta}{dQ} \bigg|_{Q=Q_0} = 0, \quad \frac{d\beta}{dQ} \bigg|_{Q_0} < 0 \quad \text{for} \quad Q < Q_0 \quad \text{and} \quad \frac{d\beta}{dQ} \bigg|_{Q_0} > 0 \quad \text{for} \quad Q > Q_0$$

(40)

are all satisfied [27, 28].
The graphs in the left panel (red, blue, and black) for different $m_\phi$ are compared with the results for $\beta(Q)$ for AdS and Modified AdS presented in Refs. [27, 58] however, they agree better with the Modified AdS. The transition varies depending on the magnitude of the glueball mass present. For $m_\phi = 1.73\text{ GeV}$, the transition is at $Q_0 \approx 1.41\text{ GeV}$, for $m_\phi = 0.98\text{ GeV}$; $Q_0 \approx 0.80\text{ GeV}$ and $m_\phi = 1.37\text{ GeV}$; $Q_0 \approx 1.12\text{ GeV}$. After $Q_0$ is reached, the graph starts decreasing in magnitude with increasing $Q$, representing asymptotic freedom. The graph in the right panel shows an increase in the transition momentum $Q_0$ for increasing $m_\phi \to \infty$.

A schematic diagram showing the behavior of the strong running coupling in both the UV (dashed blue) and IR (red) regions in the model framework. This diagram shows the transition point at 0.89 GeV (the two graphs 1 (left) and 4 (left) intersect at this point), corresponding to $m_\phi \approx 1.09\text{ GeV}$.

VI. ANALYSIS AND CONCLUSION

A. Analysis

The graphs plotted for the couplings determined from the UV region were for $f_\alpha = 0.50\text{ GeV}$ and $m_\phi = 0.86\text{ GeV}$. On the other hand, the couplings determined from the IR region were plotted for $m_\phi = 1.73\text{ GeV}$ Generally, we considered the lightest scalar glueball mass consistent with the background of the study presented in Sec. III. From QCD lattice simulations results, QCD sum rule and QCD phenomenology, the lightest scalar glueball mass with quantum number $J^{PC} = 0^{++}$ and frequency $f_0(1710)$, has been determined to be $m_\phi = 1.73\text{ GeV}$ [30–32](and references therein). Other glueball masses with the same quantum number and frequencies $f_0(500)$, $f_0(980)$, $f_0(1370)$ have also been identified [114, 115]. Despite the negative sign of $m_\phi^2$ determined in the UV region, its magnitude is half the one determined in the IR region — see below Eq.(22), and above Eq.(32) — this accounts for the discrepancies in the magnitude of $m_\phi$ in the UV and the IR regions. Studies in Refs. [104–106] shows that the magnitude of the scalar glueball mass determined in the QGP region is less than the one determined in the IR region. We established connection between the couplings ($\alpha_s^{\text{AdS}}(Q^2)$ and $\beta(Q^2)$) and $m_\phi$ in both regions. Additionally, we determined the position of the Landau pole to be $Q_L \approx 0.58\text{ GeV}$, Eq.(31), it is also related to the glueball mass present. Again, we established a relationship between the transition momentum $Q_0$ and $m_\phi$, $Q_0 = \sqrt{(2/3)m_\phi}$, for the couplings.
determined in the IR region. For \( m_\phi = 1.73 \text{GeV} \) we determined, \( Q_0 \approx 1.41 \text{GeV} \), so glueballs with larger masses are more likely to be confined than the lighter ones. Although the couplings do not directly involve quark masses, it is known that; heavy quarks are more likely to confine than the lighter ones, similar to the behavior of the glueballs identified here. That implies that relatively heavier glueballs may be involved in confining heavy quarks and vice versa. We intend to investigate this phenomenon by fitting a potential model with dependence on glueballs into the heavy quarkonium spectroscopy to determine the involvement of glueballs in quark confinement at various masses.

This paper introduces a new perspective to determining the AdS/QCD strong running coupling and the associated \( \beta \)-function. Instead of determining \( \alpha_\text{AdS}^V(Q^2) \) in the less understood IR region by IR distortion of the AdS space and extrapolating the results to the relatively well-understood UV region. We introduced a \( G(\phi) \) associated with tachyons that distort the AdS space in two different regions, i.e. when the tachyons are in their free state and when they are in a condensed state corresponding to UV and IR deformations of the AdS space respectively. The results obtained, compared to the well-measured effective charge \( \alpha_{s,g_1} \) determined from the Bjorken sum rule for pQCD and AdS/QCD couplings. We established that the coupling determined from the UV region has a sinusoidal fall off \( \alpha_\text{AdS}^V(Q^2) \sim 1/\sin(\pi Q^2/2m_\phi^2) \), compared to the pQCD with logarithmic fall off \( \alpha_\text{AdS}^V(Q^2) \sim 1/\ln(Q^2/A_{\text{QCD}}^2) \). On the other hand, the AdS/QCD model with positive dilaton background leads to exponential fall off \( \alpha_\text{AdS}^V(Q^2) \sim e^{-Q^2/4m^2_\phi} \); in our case, the fall-off is \( 1/(2m_\phi^2 + Q^2)^3 \). Indeed, there seems to be an agreement that the behavior of the QCD running coupling in the IR domain is not well-defined because different experiments formulated on momentum space yield different outcomes [38, 110]. Thus, computations of hadronic characteristics remain the only viable approach for comparing lattice simulation schemes [116]. We established that the \( \beta \)-functions determined from the model framework satisfy "the principle of maximum conformality" in both UV and IR regions.

The UV deformation of the AdS space studied in Sec. IV shows the appearance of nonphysical Landau singularity in \( \alpha_\text{AdS}^V(Q^2) \), with the Landau pole at \( Q_\Lambda \). On the contrary, the IR deformation, Sec. V, shows a 'regular' behavior deep in the IR and far UV regions, as expected. The 'regular' behavior of \( \alpha_\text{s} \) is a result of hadron degrees of freedom. In that view, at \( Q^2 \to 0 \) the gluons dynamically acquire mass \( Q^2 = -q^2 \equiv m_\Lambda^2 \) [120, 121] which increases \( \alpha_\text{s} \) infinitely. Thus, we can find the IR freezing point [122, 123] by introducing \( m_\Lambda^2 \) into \( Q^2 \) such that its effect is negligible at higher energies \( Q^2 \to \infty \) but has dominant effect at \( Q^2 \to 0 \) [112]. That will regulate the IR singularity and enhance the qualitative understanding of color confinement. It is important to note that; the dynamically generated gluon masses have no association with spontaneous chiral symmetry breaking. The forces between gluons are; sufficiently strong enough to generate bound states, especially \( J^{PC} = 0^+ \) color singlet glueballs described by scalar fields. Their vacuum expectation values are associated with the gluon mass and gluon condensate \( \langle G^{\mu\nu}G_{\mu\nu} \rangle \). Similar to the approach adopted in this paper. The gluon mass \( m_A \) has been determined from the leading order of Yang-Mills theory with auxiliary field \( \phi \) to have a direct relation with the scalar glueball mass \( m_\phi \) through, \( m_A = m(0^{++})/\sqrt{6} \) [133, 134], where \( m(0^{++}) \) is the fluctuations around \( \phi \). Similar relations have been reached in Refs. [30, 32] to be \( m_A = m_\phi/2 \), using models with color dielectric function. In this case, \( \alpha_\text{AdS}^V(Q^2) \) determined from the UV region of the AdS space takes the form

\[
\alpha_\text{AdS}^V(Q^2 + m_\Lambda^2) = \frac{\pi}{m_\phi^2} \left( \frac{m_\phi^2 f_\phi^2}{m_\Lambda^2} \right) \frac{(Q^2 + m_\Lambda^2)/2m_\phi^2}{(2m_\phi^2 - (Q^2 + m_\Lambda^2)) \csc \left( \frac{\pi(Q^2 + m_\Lambda^2)}{2m_\phi^2} \right)}. \tag{41}
\]

For \( m_\phi = 0.86 \) we get \( m_A = 0.43 \) which leads to a finite IR freezing point as presented in Fig. 7. On the other hand, the IR model self-regulates at low \( Q^2 \) with a finite freezing point normalized at \( \alpha_\text{AdS}^V(0) = \pi \), on \( g_1 \) scheme [44].
FIG. 7: A graph of $\alpha_{s}^{AdS}(Q^2 + m_A^2)/\pi$ (dashed blue) and $\alpha_{s}^{AdS}(Q^2)/\pi$ (red) against $Q$ for $m_A = 0.43\,\text{GeV}$ and $m_\phi = 0.86\,\text{GeV}$.

The dashed blue graph is well-behaved in the IR region due to the IR freezing point while the red graph shows the singularity in UV region. However, in the limit of high $Q$ they both fall off in a similar manner.

FIG. 8: A graph of $\beta(Q^2 + m_A^2)$ (dashed blue) and $\beta(Q^2)$ (red) against $Q$ for $m_A = 0.43\,\text{GeV}$ and $m_\phi = 0.86\,\text{GeV}$.

We compare $\beta$-function with IR freezing to the one without IR freezing.

B. Conclusion

We established a connection between tachyonic field $\phi$ and strong interacting glueball field $\varphi$ in Sec. III and developed; an action for tachyonic AdS/QCD from DBI action in the subsequent subsections to set the grounds for the investigation. We determined AdS/QCD running coupling $\alpha_{s}^{AdS}(Q^2)$ and the associated $\beta$-function $\beta(Q^2)$ by distorting the AdS space in both UV and IR regions using color dielectric function. The free tachyon $G(\phi)$ deforms the AdS space in the UV region, while the condensed tachyon $G(\eta)$ distorts the AdS space in the IR region. In Sec. IV we determined $\alpha_{s}^{AdS}(Q^2)$ and $\beta(Q^2)$ through the UV distortion of the AdS space and found a good agreement between the analytical behaviors of our result with both the pQCD and $\alpha_{s,g_1}$ effective charge. The results are presented in Figs. 1, 2 and 3 and compared. We determined that increasing the glueball mass $m_\phi$ goes into strengthening color confinement and increases the transitional momentum. In Sec. V, we introduced condensed tachyon color dielectric function $G(\eta)$ and used it to deform the AdS space in the IR region. We determined $\alpha_{s}^{AdS}(Q^2)$ and $\beta(Q^2)$ in this
region and compared the results with the one obtained from AdS/QCD through IR distortion of the AdS space with a positive dilaton profile. The results displayed in Figs. 4 and 5. The behavior of $\alpha_s^{\text{AdS}}(Q^2)$ and $\beta(Q^2)$; at large glueball mass limits were studied. We established that glueballs with larger masses are more likely to confine than lighter ones. A diagram showing a unified $\alpha_s^{\text{AdS}}(Q^2)$ for both IR and UV regions displayed in Fig. 6. This diagram shows a transition at 0.89 GeV corresponding to a glueball mass $m_\phi \approx 1.09$ GeV. We also observed Landau singularity in $\alpha_s^{\text{AdS}}(Q^2)$ in the UV region with its pole at $Q_\Lambda$ and proposed how to fix it. We introduced dynamically generated gluon mass $m_A^2$ into $Q^2$ to correct the nonphysical behavior at $Q \to 0$ limit, and the results presented in Figs. 7 and 8. Other important results obtained from the model framework are; relationship between $\alpha_s^{\text{AdS}}(Q^2)$, $\beta(Q^2)$ and $m_\phi$ in both regions. Subsequently, we determined that the transition momentum $Q_0$ is directly related to $m_\phi$ as well. We determined that larger glueball masses are more likely to confine than lighter ones in relatively high momentum transfer regions. We have also shown that with the appropriate function, particularly, Higgs-like tachyon potential, one can deform the AdS space in the UV region with similar characteristics as pQCD. Finally, we intend to extend these investigations to tachyonic AdS/QCD couplings in both UV and IR regions at a finite temperature. Again, we intend to study the hadronic spectra and the hadron form factor using the $G(\eta)$ ansatz since it falls within the IR region.

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