Does the $J^{PC} = 1^{+-}$ counterpart of the $X(3872)$ exist?

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(Dated: April 5, 2022)

We explore the possible existence of the $J^{PC} = 1^{+-}$ counterpart of the $X(3872)$ state in a coupled-channels calculation within a constituent quark model, with the aim of confirming the existence of the so-called $X(3872)$ state observed by the COMPASS Collaboration. Two states are found in the energy region of the $X(3872)$ signal, both with almost equal mixture of $car{c} \, 2^1 P_1$ state and $D^* \bar{D}^{(*)}$ channels: One that can be identified as the dressed $car{c} \, 2^1 P_1$ and a bound state below the $Dar{D}^*$ threshold. We provide predictions of strong and radiative decays that could help to clarify the existence of such structures.

PACS numbers: 12.39.Pn, 14.40.Lb, 14.40.Rt
Keywords: Potential models, Charmed mesons, Exotic mesons

Since the discovery, in 2003, by the Belle Collaboration [1] of the exotic hadron $X(3872)$, observed as a narrow peak in the $J/\psi \pi^+ \pi^-$ mass spectrum from the decay $B^+ \rightarrow K^+ J/\psi \pi^+ \pi^-$, there has been plenty of states in this charmonium mass range, either found experimentally or predicted theoretically, that did not fit into the scheme predicted by the quark model.

On the theoretical side, a useful tool used to predict new hadronic states with heavy quarks is the Heavy-Quark Spin Symmetry (HQSS). This symmetry is based on the fact that, in the limit of infinite heavy quark mass, the strong interaction in the system is independent of the heavy quark spin. Using this symmetry, Baru et al. [2] predicted the existence of three degenerate spin partners of the $X(3872)$ with quantum numbers $0^{++}$, $1^{+-}$ and $2^{++}$. A recent systematic study of heavy-antihybrid hadronic molecules has been performed by Dong et al. [3], leading to similar conclusions and predicting more than 200 new molecules.

The $1^{+-}$ state may coincide with the signal found by the COMPASS Collaboration [4] in a search for muon production of the $X(3872)$ through the reaction

$$\mu^+ N \rightarrow \mu^+ X_0 \pi^\pm N' \rightarrow \mu^+ (J/\psi \pi^+ \pi^-) \pi^\pm N'$$

where $X_0$ is produced by virtual photons

$$\gamma^* N \rightarrow X_0 \pi^\pm N'$$

Here $N$ denotes the target nucleon, $X_0$ is an intermediate states decaying to $J/\psi \pi^+ \pi^-$ and $N'$ the unobserved recoil system.

The resulting $J/\psi \pi^+ \pi^-$ invariant mass distribution shows two peaks with positions and widths compatible with the $\psi(2S)$ and the $X(3872)$. However, the shape of the second peak disagrees with previous observations of the $X(3872)$. The mass spectrum of the two pions from the decay of the $X(3872)$ shows a preference for the $X(3872) \rightarrow J/\psi \rho^0$ decay mode, while the shape of the $\pi^+ \pi^-$ invariant mass distribution of the $X_0$ appears very different and it is inconsistent with the quantum numbers $J^{PC} = 1^{+-}$. Due to these differences, COMPASS Collaboration concluded that the observed signal is an evidence for a new charmonium-like state, dubbed the $X(3872)$, with quantum numbers $J^{PC} = 1^{+-}$ and a significance of 4.1$\sigma$. The measured mass and width of the $X(3872)$ are respectively $M = 3860 \pm 10.4$ MeV/c$^2$ and $\Gamma < 51$ MeV.

To assess the possibility of the existence of this predicted state, we perform a calculation similar to the one done in previous studies for the $X(3872)$ [5, 6], but in the channel $J^{PC} = 1^{+-}$. The same model and parametrization of the aforementioned study are employed: a coupled-channels calculation of two and four quark sectors in the framework of a widely tested constituent quark model [7, 8]. The model details can be found in Refs. [5, 6] and references therein but, in the following, we briefly describe its main aspects.

The constituent quark model we use is based on the assumption that light quarks (that is, $\{u, d, s\}$ quarks) acquire a dynamical mass as a consequence of the chiral symmetry breaking at some momentum scale. The breaking of the chiral symmetry entails the appearance of Goldstone boson exchanges between quarks. In the heavy sector (when a $c$ or $b$ quark is involved), chiral symmetry is explicitly broken by the heavy quark mass and this type of interaction does not act for $QQ$ and $Qq$ quark pairs, being $Q = \{c, b\}$ and $q = \{u, d, s\}$. However, in the proximity of the $D^* \bar{D}^{(*)}$ thresholds, it provides a natural way to incorporate pion exchanges into the $D^* \bar{D}^{(*)}$ dynamics through the light quarks.

Beyond the scale of the chiral symmetry breaking, quark-quark dynamics is governed by QCD perturbative
The cut-off $\Lambda$ fixes the scale of the chiral symmetry breaking.

The Goldstone boson matrix $U^{\gamma_5}$ can be expanded in terms of the boson fields

$$U^{\gamma_5} = 1 + \frac{i}{f_\pi} \gamma^a \pi^a - \frac{1}{2f_\pi^2} \sigma^a \pi^a + \ldots$$  \hspace{1cm} (7)

The first term of the expansion generates the constituent quark mass, the second one gives rise to the pseudoscalar-exchange interaction among quarks and the main contribution of the third term is on two-pion exchanges, which is modeled by means of a scalar-meson exchange potential. Explicit expressions for the $\pi$ and $\sigma$ exchange potentials can be found in Ref. [7].

The final piece of the quark-quark interaction is the confinement potential, which prevents the existence of colored hadrons. This potential shows a linear behavior but, at a certain distance, the link between the quarks breaks up, giving rise to the creation of light quark–antiquark pairs. This dynamics can be translated into a screened potential [11] as

$$V_{\text{CON}}(\vec{r}) = [-a_c(1 - e^{-\mu_c r}) + \Delta] \langle \vec{X}_q \cdot \vec{X}_\bar{q} \rangle,$$  \hspace{1cm} (8)

where $a_c$, $\mu_c$, and $\Delta$ are model parameters (see Tab. I). At short distances, this potential presents a linear behavior with an effective confinement strength, $\sigma = -a_c \mu_c (\vec{X}_q \cdot \vec{X}_\bar{q})$, while it becomes constant at large distances, with a plateau at $V_{\text{CON}}(r \to \infty) = (\Delta - a_c) \langle \vec{X}_q \cdot \vec{X}_\bar{q} \rangle$.

To model the $J^{PC} = 1^{++}$ charmonium sector we follow the steps done in Ref. [5] for the $1^{++}$ sector. The full hadronic state is, hence, assumed to be given by

$$|\Psi\rangle = \sum_\alpha c_\alpha |\psi_\alpha\rangle + \sum_\beta \chi_\beta(P)|\phi_M,\phi_M,\beta\rangle$$  \hspace{1cm} (9)

where $|\psi_\alpha\rangle$ are $c\bar{c}$ eigenstates of the two body Hamiltonian, $\phi_M, \phi_M, \beta$ are $c\bar{c}$ eigenstates describing the $D$ ($D$) mesons, $|\phi_M,\phi_M,\beta\rangle$ is the two meson state with $\beta$ quantum numbers coupled to total $J^{PC}$ quantum numbers and $\chi_\beta(P)$ is the relative wave function between the two mesons in the molecule. As we always work with eigenstates of the $C$-parity operator we use the usual notation in which $DD^*$ is the right combination of $DD^*$ and $D^* D$.

The meson eigenstates $\phi_C(\vec{p}_C)$ are calculated by means of the two-body Schrödinger equation, using the Gaussian Expansion Method [12]. This method provides enough numerical accuracy for the solution of the Schrödinger equation and simplifies the subsequent evaluation of the needed matrix elements. With the aim of optimizing the Gaussian ranges employing a reduced number of free parameters, we use Gaussian trial functions whose ranges are given by a geometrical progression [12]. This choice produces a dense distribution at short distances, enabling a better description of the dynamics mediated by short range potentials.

effects. They are usually taken into account through the one-gluon exchange interaction [9] obtained from the Lagrangian,

$$\mathcal{L}_{qqg} = i\sqrt{4\pi\alpha_s} \bar{\psi} \gamma_\mu G^\mu_\lambda \lambda^\mu \psi,$$  \hspace{1cm} (3)

where $\alpha_s$ is the strong coupling constant, $\lambda^\mu$ are the $SU(3)$ color matrices and $G^\mu_\lambda$ is the gluon field. The strong coupling constant, $\alpha_s$, has a scale dependence which allows a consistent description of light, strange and heavy mesons, given by

$$\alpha_s(\mu) = \frac{\alpha_0}{\ln \left( \frac{\mu^2 + m^2}{\Lambda_0^2} \right)}$$  \hspace{1cm} (4)

where $\mu$ is the reduced mass of the quark pair and $\mu_0$ and $\Lambda_0$ are model parameters, which can be found in Table I.

Below the chiral symmetry scale, the simplest Lagrangian is provided by the Instanton Liquid Model (ILM) [10]

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu - M(q^2)U^{\gamma_5}) \psi,$$  \hspace{1cm} (5)

being $U^{\gamma_5} = e^{i\lambda_\mu \phi^{\mu,\gamma_5} / f_\pi}$ the matrix of Goldstone-boson fields, $\pi^a$ the nine pseudoscalar fields $\{\eta_8, \pi, K_1, K_8\}$ with $i = 1, \ldots, 4$ and $M(q^2)$ is the dynamical mass.

An expression of this dynamical mass can be obtained from the ILM theory [10], but its behavior can be simulated by the simple parametrization $M(q^2) = m_q F(q^2)$, where $m_q$ is a parameter that corresponds to the constituent quark mass and

$$F(q^2) = \frac{\Lambda^2}{\Lambda^2 + q^2}$$  \hspace{1cm} (6)

| TABLE I. Quark model parameters. |
|-----------------------------------|
| Quark masses | $m_\alpha$ (MeV) | 313 |
| $m_s$ (MeV) | 555 |
| $m_c$ (MeV) | 1763 |
| Goldstone bosons | $m_\sigma$ (fm$^{-1}$) | 0.70 |
| $m_\omega$ (fm$^{-1}$) | 3.42 |
| $\Lambda$ (fm$^{-1}$) | 4.20 |
| $g_{\sigma h}^2/(4\pi)$ | 0.54 |
| Confinement | $a_c$ (MeV) | 507.4 |
| $\mu_c$ (fm$^{-1}$) | 0.576 |
| $\Delta$ (MeV) | 184.432 |
| $a_s$ | 0.81 |
| OGE | $\alpha_0$ | 2.118 |
| $\Lambda_0$ (fm$^{-1}$) | 0.113 |
| $\mu_0$ (MeV) | 36.976 |
| $\hat{r}_0$ (fm) | 0.181 |
| $\hat{r}_g$ (fm) | 0.259 |

The cut-off $\Lambda$ fixes the scale of the chiral symmetry breaking.

The Goldstone boson matrix $U^{\gamma_5}$ can be expanded in terms of the boson fields

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The coupling between the two and four quark sectors requires the creation of a light-quark pair \( n\bar{n} \). Similar to the strong decay process, this coupling should be in principle driven by the same interquark Hamiltonian which determines the spectrum. However, Ackleh et al. [13] have shown that the quark pair creation \( 3P_0 \) model [14] gives similar results to the microscopic calculation. The model assumes that the pair creation Hamiltonian is

\[
\mathcal{H} = g \int d^3x \bar{\psi}(x)\psi(x)
\]

which in the non-relativistic reduction is equivalent to the transition operator [15]

\[
T = -3\sqrt{2} \gamma' \sum_\mu \int d^3p d^3p' \delta^{(3)}(p + p') \times \left[ \mathcal{V}_1 \left( \frac{p - p'}{2} \right) b_\mu(p) b_\mu^\dagger(p') \right]^{C=1,I=0,S=1,J=0}
\]

where \( \mu (\bar{\mu}) \) are the quark (antiquark) quantum numbers and \( \gamma' = 2^{5/2}\pi^{1/2} \gamma \) with \( \gamma = \frac{\beta}{2\sqrt{\pi}} \) is a dimensionless constant that gives the strength of the \( n\bar{n} \) pair creation from the vacuum. From this operator we define the transition potential \( V_{\beta\alpha}(P) \) within the \( 3P_0 \) model as [16]

\[
\langle \phi_M, \phi_M | T | \psi_{\alpha} \rangle = PV_{\beta\alpha}(P) \delta^{(3)}(\vec{P}_{CM})
\]

where \( P \) is the relative three-momentum of the two meson state.

Using the wave function from Eq. (9) and the coupling Eq. (12), we arrive to the coupled equations

\[
M_\alpha c_\alpha + \sum_\beta \int V_{\alpha\beta}(P) \chi_\beta(P) P^2 dP = E c_\alpha
\]

\[
\sum_\beta \int H_{\beta\beta}(P', P) \chi_\beta(P) P^2 dP + \sum_\alpha V_{\beta\alpha}(P') c_\alpha = E \chi_{\beta'}(P')
\]

where \( M_\alpha \) are the masses of the bare \( c\bar{c} \) mesons and \( H_{\beta\beta} \) is the RGM Hamiltonian for the two meson states obtained from the \( q\bar{q} \) interaction [17, 18]. This includes direct diagrams that have contributions from one-pion exchange (OPE), one-sigma exchange (OSE) and the annihilation through a gluon, and also rearrangement diagrams [5] that have contributions from OPE, OSE, confinement and one-gluon exchange (see Fig. 1).

Solving these coupled equations, we can describe both the renormalization of the bare \( c\bar{c} \) states due to the presence of nearby meson-meson thresholds and the generation of new states through the underlying \( q\bar{q} \) interaction that generates the residual meson-meson interaction and the additional interaction from the coupling with intermediate \( c\bar{c} \), as it is the case for the \( X(3872) \) in our model [5].

![Diagrams for the quark-quark interactions considered in the RGM Hamiltonian \( H_{\beta\beta} \) of Eq. (13): a) Direct exchange of \( \pi \) and \( \sigma \), b) and c) annihilation diagrams through a gluon and d) quark rearrangement diagrams, where the gray band represents the sum of interactions between quarks of different clusters and the dotted line represents contributions from \( \pi \), \( \sigma \), confinement and one-gluon-exchange potentials.

The present calculation of the \( J^{PC} = 1^{--} \) sector includes the \( 2^1P_1 \) \( c\bar{c} \) state corresponding to the \( h_c(2P) \) meson with bare mass of 3955.7 MeV/\( c^2 \), coupled to the \( I = 0 \) \( D\bar{D}^* \) and \( D^*\bar{D}^* \) molecular states in \( 3S_1 \) and \( 3D_1 \) partial waves. The \( D^*\bar{D}^* \) is negligible for the \( J^{PC} = 1^{++} \) sector as it is only allowed in a relative \( 5D_1 \) partial wave but, as we will see, this channel has a sizable effect for the \( J^{PC} = 1^{--} \) sector. All the parameters of the model are constrained from previous analysis of the heavy meson phenomenology, and the value of the \( \beta \) parameter is taken from previous studies of the charmonium 3.9 GeV energy region [5, 6, 19], thus, in that sense, it is a parameter-free calculation. However, being a phenomenological model, predictions have to be taken with care, since systematic uncertainties cannot be evaluated, which could be significant for this specific problem. Here, we hope this is not the case and just give the predictions of our model as a possible indication of the existence of the \( X \) state.

We find two states which, provisionally, we will call state \( A \) and state \( B \). The mass and width of both states are shown in Table II.

The components of both states are shown in Table III.
The first striking result is that both states share almost the same proportion of \( h_c(2P) \) and \( D^*\bar{D}^{(*)} \) components which makes its interpretation challenging. In the calculation of the \( J^{PC} = 1^{++} \) sector done in Ref. [6] we obtained two states: the first one with 87\% of \( D^0\bar{D}^{*0} \) component and a second state with more than 60\% of the \( 2^3P_1 \bar{c}\bar{c} \) state. Thus, in that case, the coupling of \( \bar{c}\bar{c} \) and \( DD^* \) channels produced a clear extra state, the \( X\) (3872), and a renormalized \( \bar{c}\bar{c} \) state, assigned to the \( X(3940) \) resonance, whose dressing evolution with increasing \( \gamma \) is shown in the Fig. 2.

The case of the \( J^{PC} = 1^{+-} \) is different. Besides an extra bound state, called state \( A \), the \( 2^1P_1 \bar{c}\bar{c} \) moves towards the \( DD^* \) threshold. The \( D^*\bar{D}^* \) channel, although it is 280 MeV above, produces a sizable attraction effect, which is not present in the \( J^{PC} = 1^{++} \) sector as there only the relative \( \tilde{5}D_1 \) partial wave is opened. In Fig. 2, we show the evolution of the dressed \( 2^1P_1 \bar{c}\bar{c} \) state with increasing \( \gamma \) values to make clearer the dressing mechanism in comparison with the \( J^{PC} = 1^{++} \). With the aim of showing the effect of the \( D^*\bar{D}^* \) channel in the dynamics of the \( 1^{+-} \) sector, we also add the evolution of the dressed \( 2^1P_1 \bar{c}\bar{c} \) state coupled solely to the \( DD^* \).

Our results show that the coupling with the bare \( \bar{c}\bar{c} \) states modifies the consequences of HQSS [20]. This symmetry predicts that the \( DD^* \) and the \( D^*\bar{D}^* \) channels includes a mix of spin-singlet and spin-triplet states with equal weights. However, the \( h_c(2^1P_1) \) \( \bar{c}\bar{c} \) state only couples, via the \( 3^3P_0 \) mechanism, with the spin-singlet part of those channels and, therefore, it enhances this component (see Table IV). This spin structure is important to understand the decay channels of both states.

### Table II. Masses (in MeV/\( c^2 \)) and decay widths (in MeV) of the states \( A \) and \( B \).

| State | Mass | Width |
|---|---|---|
| \( A \) | 3868 | 1.35 |
| \( A \) | 0.00 | 0.64 |
| \( A \) | 0.65 | 0.56 |
| \( A \) | 0.017 | 0.169 |
| \( A \) | 0.069 | 0.062 |
| \( B \) | 3877 | 45.07 |
| \( B \) | 43.2 | 0.09 |
| \( B \) | 0.81 | 0.026 |
| \( B \) | 0.064 | 0.062 |

### Table III. Probabilities (in \% ) of the three coupled channels for the two different states described in the text.

| State | \( h_c(2P) \) | \( DD^* \) | \( D^*\bar{D}^* \) |
|---|---|---|---|
| \( A \) | 49.7 | 45.5 | 4.8 |
| \( B \) | 44.7 | 50.0 | 5.3 |

In Table II we also show the widths of both states \( A \) and \( B \) corresponding to the decay channels discussed below.

The state \( B \) is above the \( DD^* \) threshold and, therefore, its main decay channel is \( DD^* \). The state \( A \) is below the \( DD^* \) threshold. Then, its decays to \( DD^* \) or \( D^*\bar{D}^* \) channels are forbidden.

On top of that, some relevant decay channels of both states are the \( \omega \eta_c \) and the \( \eta J/\psi \). These decays go through exchange diagrams where the quarks are rearranged inside the \( D^*\bar{D}^{(*)} \) channels (see Ref. [5] for details). Even if exchanges diagrams are usually small compared to the direct diagrams in coupled-channels calculations, the decay width may be sufficiently large to be measured. The \( \eta J/\psi \) decay goes through the spin-triplet component, whereas the \( \omega \eta_c \) through the spin-singlet one. However, the widths corresponding to both decays are of the same order of magnitude, because the phase space of the \( \eta J/\psi \) channel is larger than the \( \omega \eta_c \) one.

The \( X(3872) \) particle was spotted in the \( J/\psi\pi^+\pi^- \) invariant mass spectrum, so it is worth discussing the details of this decay channel. Thinking naively, in view of the dominance of the spin-singlet components, one could expect that the states found must decay through their \( h_c(2^1P_1) \) component. However, the \( h_c(2^1P_1) \rightarrow J/\psi\pi^+\pi^- \) decay is a spin-flip hadronic transition and, therefore, its probability should be low. Alternatively, the existence of the spin-triplet component may enhance the \( J/\psi\pi^+\pi^- \) branching through the molecular transition \( D^+\bar{D}^{(*)} \rightarrow J/\psi f_0 \) in P wave, \( f_0 \) decaying into \( \pi^+\pi^- \) where \( f_0 \) is a \( 0^{++} \) meson with a width of 455 MeV.

Regarding radiative decays, in this work we consider them given by the \( \bar{c}\bar{c} \) components of the states, and so we don’t consider the decay through the molecular
component. Since the $h_c(2^1P_1)$ has negative C parity, it is very likely that it decays into a photon plus a pseudoscalar meson such as $\eta_c$ and $\eta'_c$. Looking at the PDG data [21] of the $h_c(1^1P_1)$ state, for comparison, the widths of these decays may be competitive with the hadronic one.

As seen in Table II, the widths of the $J/\psi\pi^+\pi^-$ hadronic decay and the radiative decays $\gamma\eta_c$ and $\gamma\eta'_c$ are of the same order of magnitude, at keV range. However, in the case of the state $B$, it represents $\sim 1\%$ of the total width, whereas the contribution to the total width is negligible for the state $B$.

Of course, the way to distinguish between the two states, is that state $B$ can decay into $DD^*$ which is responsible of almost all the decay width, and should be more accessible in this channel. However, if the production of the $X$ is not significative, its signal may be hidden in the $X(3872)$ threshold enhancement in the $DD^*$ channel, so the radiative or strong decays mentioned earlier could be more promising. Taken into account our predictions at Table II, we encourage experimentalists to search for such resonances in the $\omega\eta_c$ and $\eta J/\psi$ channels.

As summary, we have performed a coupled channels calculation within the constituent quark model of the charmonium $J^{PC}=1^{++}$ sector in order to confirm the possible existence of the partner of the $X(3872)$ named $\tilde{X}(3872)$.

We found in this energy region two almost degenerate states with masses $M_A = 3868$ MeV/$c^2$ and $M_B = 3877$ MeV/$c^2$. Considering strong and radiative decays, the width of the $A$ state is 1.35 MeV whereas the $B$ state has a width of 45.07 MeV.

Our results are in line with previous studies, which usually employ HQSS to predict partners of the $X(3872)$. We have already cited Ref. [3] where the authors predict a $1^{+-}$ bound state with a mass between 3874.4 and 3839.8 MeV. Similarly, Ref. [22] finds a negative C-parity virtual state at 3860.0±10.4 MeV, Ref. [23] predicts a $1^{+-}$ state at a mass of 3840.69 MeV, while Ref. [24], using QCD sum rules, finds a $1^{+-}$ state at 3.89 ± 0.09 GeV together with the $1^{++}$ state. The difference with our calculation is that we include the coupling with $cc$ bare states which, together with the molecular state, leads to the emergence of a second pole due to the dressing of the $2^1P_1$ $cc$ state in the same energy region.

Both state are mainly a composite of $h_c(2^1P_1)$ $cc$ states and $D^*\bar{D}^*$(s) molecule with almost the same probability. With the scarce data available, it is difficult to experimentally identify these states as a renormalized $h_c(2^1P_1)$ and an extra state $\tilde{X}(3872)$. We suggest possible ways to obtain more precise measurements of their properties via radiative and strong decays.

ACKNOWLEDGMENTS

The authors would like to thank Prof. F.-K. Guo for fruitful discussions and useful comments.

This work has been partially funded by Ministerio de Ciencia, Innovación y Universidades under Contract No. PID2019-105439GB-C22/AEI/10.13039/501100011033, and by the EU Horizon 2020 research and innovation program, STRONG-2020 project, under grant agreement No. 824093.
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