Relativistic short-range correlation effects on the pion dynamics in nuclear matter

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Abstract

Replicated theoretical attempts of relativistic approaches to the pion self-energy in nuclear matter yield unphysical pion spectra. We demonstrate the crucial dependence of the calculated pion spectra on the correct relativistic accounting for the short-range correlation effects on the pion self-energy in the medium. To do this, we simulate the short-range interactions by phenomenological contact terms in the relativistic Lagrangian density, and derive the pion self-energy by carefully taking into account the relativistic kinematics. The obtained spectrum for the pion-like excitations in cold nuclear matter shows physically meaningful branches, in contrast to those obtained before by different authors by the use of simplified relativistic approaches to the short-range correlations.

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1 Introduction

The problem of the pion spectrum in nucleon matter has received renewed interest due to experiments with relativistic heavy-ion collisions. Considerable efforts from many theoretical groups were made to study the relativistic dispersion relation for in-medium pions (see e. g. [1] and references therein). Nevertheless, a complete theoretical treatment of the pion-like excitations in nuclear matter is still not available. Replicated theoretical attempts of relativistic approaches to the pion self-energy in nuclear matter yield the pion spectra with two pion-like branches merging at some point where $d\omega/dk = \infty$ (see e. g. [1], [2], [3]). This kind of spectrum implies that the group velocity of a pion quasi-particle can be larger than that of light, contrarily to canons of the relativistic theory. The situation might be understood in view that the relativistic theory of short-range correlations in nuclear matter remains unsolved, and it forces the authors to use nonrelativistic analogies, although they have never been checked carefully (see e.g. [1], [2], [4], [5]).

To take into account the relativistic kinematics of the correlations, many authors simply replace $-k^2$ with $\omega^2 - k^2$ in the well known non-relativistic expression derived by Migdal [6]. This is actually incorrect and, as we show in our paper, results in the above problems in the pion spectra. The completely relativistic accounting for the short-range correlations in the pion self-energy was suggested recently in [7]. In the following we calculate the relativistic short-range correlations in a different way, and compare our result with that obtained in [7]. The idea of this paper is to show how the relativistically correct treatment of short-range correlation effects prevents the above-mentioned problems in the pion spectra.

Since the short-range correlation distance is small as compared to the Compton wavelength of the pion, we simulate the short-range correlations by phenomenological contact terms in the Lagrangian density with Landau-Migdal parameters $g'_{NN}$, $g'_{N\Delta}$, $g'_{\Delta\Delta}$, and derive the expression for the pion self-energy, which retains the relativistic kinematics.

Since it is not our purpose to construct a fundamental renormalizable theory, we consider the pseudovector coupling of pions with nucleons and deltas, which is more likely to reproduce correctly the in-medium pion mass [8]. We evaluate the pion self-energy by neglecting the vacuum contribution, as widely used in the mean field approximation. In this case, the only strongly model-dependent parameter arising in our calculations is the effective nucleon mass, which, however, can be varied in order to investigate the dependence of pion spectra on this parameter.

The paper is organized as follows. In Section 2 we briefly discuss the model
Lagrangian for the pion field in isosymmetric nucleon matter. In Section 3 we derive the relativistic form of the pion self-energy, including short-range correlations in a nuclear medium, which we simulate by relativistic phenomenological contact term in the Lagrangian. The expression for the pion self-energy is obtained in terms of lowest-order polarization tensors, which we calculate in Appendices A and B. In Sections 4, 5 we discuss the pion dispersion relations. In Section 6 we present the result of our numerical calculations for the pion spectra in isosymmetric nuclear matter. A Summary and conclusions are in Section 7. In what follows, we use the system of units \( \hbar = c = 1 \). Summation over repeated Greek indices is assumed.

2 Formalism

In isotopically symmetric nuclear matter, the dominant contribution to the pion self-energy arises from particle-hole (\( ph \)) and \( \Delta \)-particle - nucleon hole (\( \Delta h \)) excitations. The corresponding Lagrangian density for the pion field can be written in the following form

\[
\mathcal{L}_\pi = \frac{1}{2} \partial_\mu \pi \partial^\mu \varphi - \frac{1}{2} m_\pi^2 \varphi^2 - \frac{f}{m_\pi} \bar{\psi}_N \gamma^\mu \gamma^5 \tau \psi_N \partial_\mu \varphi
\]

\[
+ \frac{f_\Delta}{m_\pi} \bar{\psi}_\Delta \tau^\mu \psi_N \partial_\mu \varphi + \frac{f_\Delta}{m_\pi} \bar{\psi}_N \tau^\mu \psi_\Delta \partial_\mu \varphi. \tag{1}
\]

Here \( \varphi \) is the pseudoscalar isovector pion field, \( m_\pi \) is the bare pion mass, and \( f = 0.988 \) is the pion-nucleon coupling constant. The excitation of the \( \Delta \) in pion-nucleon scattering is described by the last two terms in the Lagrangian with the Chew-Low value of the coupling constant, \( f_{\pi N \Delta} = 2f \). The nucleon field is denoted as \( \psi_N \), and \( \psi_\Delta \) stands for the Rarita-Schwinger spinor of the \( \Delta \)-baryon. Here and below, we denote as \( \tau \) the isospin 1/2 operators, which act on the isobaric doublet \( \psi \) of the nucleon field. The \( \Delta \)-barion is an isospin 3/2 particle represented by a quartet of four states. \( T \) is the isospin transition operator between the isospin 1/2 and 3/2 fields.

The free-pion Green function is given by

\[
D(K) = \frac{1}{\omega^2 - k^2 - m_\pi^2 + i0}, \tag{2}
\]

The in-medium pion propagator obeys the Dyson’s equation

\[
\tilde{D}(K) = D(K) + D(K) \tilde{\Pi}(K) \tilde{D}(K), \tag{3}
\]

\(^1\) The pion propagator is diagonal in isospin space, therefore we omit the isospin indices.
where the self-energy $\tilde{\Pi}(K)$ arises due to the pion interactions with nucleons and delta-resonances. The four-momentum of a pion is denoted as

$$K = (\omega, \mathbf{k}),$$

and we use the notation $K^2_\mu = \omega^2 - k^2$.

3 Short-range correlations

The $ph$ and $\Delta h$ contributions to the pion self-energy can be evaluated via the diagramms in Fig. 1,

$$\tilde{\Pi} = \begin{array}{cc}
\end{array} + \begin{array}{cc}
\end{array}$$

Fig. 1. Diagrammatic representation for the pion self-energy. The first graph corresponds to a virtual nucleon particle–hole pair, and the second one is due to nucleon hole–$\Delta$-isobar excitations. The $\Delta$-isobar is shown by a double-line. The shadowed effective vertices for the pion interaction with nucleons and deltas account for short-range correlations in the medium.

These graphs include the nucleon particle–hole pair and the nucleon hole–$\Delta$-isobar pair. The shadowed effective vertices for the pion interaction with nucleons and deltas take into account the short-range correlations in the medium. These vertices are irreducible with respect to pion lines. We perform the relativistic calculation by assuming that the short-range correlation distance is small as compared to the Compton wavelength of the pion. In this case, the correlations can be simulated by phenomenological contact terms in the Lagrangian density of the form

$$\mathcal{L}_{\text{corr}} = \frac{f^2}{m_\pi^2} g'_{NN} \left( \bar{\psi} \gamma_\nu \gamma_5 \tau \psi \right) \left( \bar{\psi} \gamma^\nu \gamma_5 \tau \psi \right) + \frac{f^2}{m_\pi^2} g'_{\Delta \Delta} \left( \bar{\psi}_\Delta^\mu T^+ \psi_N \right) \left( \bar{\psi}_N T \psi_\Delta^\mu \right)$$

$$+ \frac{ff}{m_\pi^2} g'_{N \Delta} \left( \bar{\psi}_\Delta^\mu T^+ \psi_N \right) \left( \bar{\psi}_N \gamma_\mu \gamma_5 \tau \psi_N \right)$$

(5)

With the allowance of a derivative coupling of pions with nucleons and deltas, the pion self-energy, shown in Fig. 1, can be written in the following form

$$\tilde{\Pi} = \tilde{\Pi}^{\mu \nu} K_\mu K_\nu$$

(6)
where $\tilde{\Pi}^{\mu\nu}$ stands for the medium polarization tensor

$$\tilde{\Pi}^{\mu\nu} = \tilde{\Pi}^{\mu\nu}_{\text{ph}} + \tilde{\Pi}^{\mu\nu}_{\Delta h}$$

(7)

which incorporates two terms, $\tilde{\Pi}^{\mu\nu}_{\text{ph}}$ and $\tilde{\Pi}^{\mu\nu}_{\Delta h}$, connected by the Dyson’s equations depicted graphically in Fig. 2. Here, the small rectangular blocks represent the short-range interactions (5).

![Diagram](image)

Fig. 2. The diagrammatic set of the Dyson’s equations connecting the two terms shown in Fig. 1. The small rectangular blocks represent the short-range interactions.

Corresponding to the diagrams of Fig. 2 we have the following equations for the polarization tensors

$$\tilde{\Pi}^{\mu\nu}_{\text{ph}} = \Pi^{\mu\nu}_{\text{ph}} - g'_{NN} \Pi^{\mu\alpha}_{\text{ph}} g_{\alpha\beta} \tilde{\Pi}^{\beta\nu}_{\text{ph}} - g'_{N\Delta} \Pi^{\mu\alpha}_{\text{ph}} g_{\alpha\beta} \tilde{\Pi}^{\beta\nu}_{\Delta h},$$

(8)

$$\tilde{\Pi}^{\mu\nu}_{\Delta h} = \Pi^{\mu\nu}_{\Delta h} - g'_{N\Delta} \Pi^{\mu\alpha}_{\Delta h} g_{\alpha\beta} \tilde{\Pi}^{\beta\nu}_{\text{ph}} - g'_{N\Delta} \Pi^{\mu\alpha}_{\Delta h} g_{\alpha\beta} \tilde{\Pi}^{\beta\nu}_{\Delta h},$$

(9)

where $g^{\rho\mu} = \text{diag}(1, -1, -1, -1)$ is the signature tensor. The lowest-order polarizations $\Pi^{\mu\nu}_{\text{ph}}$ and $\Pi^{\mu\nu}_{\Delta h}$ are defined as the following one-loop integrals

$$\Pi^{\mu\nu}_{\text{ph}}(\omega, k) = -i \frac{2f^{2}}{m^{2}_{\pi}} \text{Tr} \int \frac{d^{4}p}{(2\pi)^{4}} G(p)\gamma^{\mu}\gamma_{5} G(p + K) \gamma^{\nu}\gamma_{5},$$

(10)

$$\Pi^{\mu\nu}_{\Delta h} = -i \frac{f^{2}_{\pi N\Delta}}{m^{2}_{\pi}} \frac{4}{3} \int \frac{d^{4}p}{(2\pi)^{4}} \text{Tr} G(p)S^{\mu\nu}(p_{\Delta} + K)$$

$$-i \frac{f^{2}_{\pi N\Delta}}{m^{2}_{\pi}} \frac{4}{3} \int \frac{d^{4}p}{(2\pi)^{4}} \text{Tr} G(p)S^{\mu\nu}(p_{\Delta} - K),$$

(11)

where $G(p)$ denotes the in-medium nucleon propagator, and $S^{\mu\nu}(p)$ stands for the Rarita-Schwinger propagator for the in-medium $\Delta$ particle.

The set of 32 equations (8), (9) for the components $\tilde{\Pi}^{\mu\nu}_{\text{ph}}$, $\tilde{\Pi}^{\mu\nu}_{\Delta h}$ with $\mu, \nu = 0, 1, 2, 3$ simplifies in the reference frame where $K = (\omega, 0, 0, k)$. With taking also into account that the transverse (with respect to $K$) components of the polarization tensor do not mix to the longitudinal ones, the above equations
reduce to 8 linear algebraic equations for the components $\tilde{\Pi}^{00}_{ph}$, $\tilde{\Pi}^{03}_{ph}$, $\tilde{\Pi}^{30}_{ph}$, $\tilde{\Pi}^{33}_{ph}$, $\tilde{\Pi}^{00}_{\Delta h}$, $\tilde{\Pi}^{03}_{\Delta h}$, $\tilde{\Pi}^{30}_{\Delta h}$, $\tilde{\Pi}^{33}_{\Delta h}$. Solution of the above equations and contraction of the resulting tensor with $K_\mu K_\nu$, as given by Eq. (6), gives the pion self energy in the following form

\[
\tilde{\Pi} = \frac{1}{W} [(1 + (g'_{\Delta \Delta} - g'_{N\Delta}) (A_\Delta - (g'_{\Delta \Delta} - g'_{N\Delta}) B_\Delta)) \Pi_{ph} \\
+ (1 + (g'_{NN} - g'_{N\Delta}) (A_N - (g'_{NN} - g'_{N\Delta}) B_N)) \Pi_{\Delta h} \\
+ K^2 \{ (g'_{N\Delta}^2 - g'_{NN} g'_{\Delta\Delta}) A_N B_\Delta - g'_{\Delta\Delta} B_\Delta \\
+ (g'_{N\Delta}^2 - g'_{NN} g'_{\Delta\Delta}) A_N B_N - g'_{NN} B_N - g'_{N\Delta} F_{N\Delta} \\
- (g'_{NN} + g'_{\Delta\Delta} - 2g'_{N\Delta}) \{ (g'_{N\Delta}^2 - g'_{NN} g'_{\Delta\Delta}) B_N B_\Delta \} \}, \tag{12}
\]

where the following notations are used

\[
W = 1 + (g'_{NN} A_N - g'_{N\Delta} A_\Delta - g'_{\Delta\Delta} B_\Delta + g'_{NN} g'_{\Delta\Delta} A_N A_\Delta \\
+ (g'_{N\Delta}^2 - g'_{NN} g'_{\Delta\Delta}) (g'_{\Delta\Delta} A_N B_\Delta + g'_{NN} A_N B_N) \\
+ (g'_{N\Delta}^2 - g'_{NN} g'_{\Delta\Delta})^2 B_N B_\Delta - g'_{N\Delta}^2 \Phi_{N\Delta} \tag{13}
\]

\[
\Pi_{ph} = K_\mu \Pi_{ph}^{\mu\nu} K_\nu, \quad \Pi_{\Delta h} = K_\mu \Pi_{\Delta h}^{\mu\nu} K_\nu \tag{14}
\]

\[
A_N = \Pi^{00}_{ph} - \Pi^{33}_{ph}, \quad B_N = \Pi^{00}_{ph} \Pi^{33}_{ph} - \Pi^{03}_{ph} \Pi^{30}_{ph} \tag{15}
\]

\[
A_\Delta = \Pi^{00}_{\Delta h} - \Pi^{33}_{\Delta h}, \quad B_\Delta = \Pi^{00}_{\Delta h} \Pi^{33}_{\Delta h} - \Pi^{03}_{\Delta h} \Pi^{30}_{\Delta h} \tag{16}
\]

\[
F_{N\Delta} = \Pi^{00}_{ph} \Pi^{33}_{\Delta h} - \Pi^{03}_{ph} \Pi^{30}_{\Delta h} - \Pi^{30}_{ph} \Pi^{03}_{\Delta h} + \Pi^{03}_{ph} \Pi^{33}_{\Delta h} \tag{17}
\]

\[
\Phi_{N\Delta} = \Pi^{00}_{ph} \Pi^{33}_{\Delta h} - \Pi^{03}_{ph} \Pi^{30}_{\Delta h} - \Pi^{30}_{ph} \Pi^{03}_{\Delta h} + \Pi^{33}_{ph} \Pi^{33}_{ph} \tag{18}
\]

A standard calculation of the lowest order polarization tensors is simple but results in cumbersome expressions, therefore we preferred to show them in the Appendices A, B. The explicit relativistic expressions for the real and imaginary part of the polarizations are given by Eqs. (A.6) - (A.8), (A.11) and (B.6), (B.11).

It is interesting to compare our Eq. (12) with the corresponding expression derived recently in [7]. As can be seen, the pion self-energy given in [7] by Eq. (9), in the limit of vanishing Landau-Migdal couplings, does not reproduce the lowest-order self-energy given by Eq. (3) of the same work, having instead the opposite sign. Using standard definitions for the polarization tensor, all the minus signs in Eq. (9) of the work [7] should be replaced with plus. After
these modifications the result obtained in [7], for the case of nuclear matter at rest, becomes identical to our Eq. (12).

It is instructive to show how the relativistic expression (12) transforms into the known non-relativistic form in the limit \( p_n, p_p \ll M^* \). The latter conditions assume also \( k \ll M \) and \( \omega \lesssim p^2_n / (2M^*) \ll k \). In this case, the non-relativistic reduction of the pion-nucleon and pion-delta coupling leads to an effective interaction Hamiltonian of the form

\[
H_{\text{int}} = \frac{f}{m_\pi} (\sigma \cdot \nabla) (\tau \cdot \varphi) + \frac{f_\Delta}{m_\pi} (S^+ \cdot \nabla) (T^+ \cdot \varphi) + \text{h.c.}
\]  

where \( S^+ \) and \( T^+ \) are the transition spin and isospin operators, respectively, connecting spin 1/2 and 3/2 states. Thus, in the non-relativistic limit only the \( \Pi_{33}^{ph} \) and \( \Pi_{33}^{\Delta h} \) components of the lowest-order polarization tensor contribute to the pion self-energy, and the rest of components can be neglected. This yields

\[
\tilde{\Pi}_{nr} = k^2 \left[ \left( \Pi_{33}^{ph} + \Pi_{33}^{\Delta h} \right) - \left( g'_{NN} + g'_{\Delta \Delta} - 2g'_{N\Delta} \right) \Pi_{33}^{ph} \Pi_{33}^{\Delta h} \right] \frac{1}{1 - g'_{NN} \Pi_{33}^{ph} - g'_{\Delta \Delta} \Pi_{33}^{\Delta h} + \left( g'_{NN} g'_{\Delta \Delta} - g'^2_{N\Delta} \right) \Pi_{33}^{ph} \Pi_{33}^{\Delta h}}.
\]  

If, for the moment, one assumes the universal coupling for nucleons and deltas, i.e. \( g'_{NN} = g'_{N\Delta} = g'_{\Delta \Delta} = g' \), then

\[
\tilde{\Pi}_{nr} \simeq \frac{k^2 \Pi_{33}^{33}}{1 - g' \Pi_{33}^{33}}
\]

The \( \Pi_{33} \) component can be identified with the non-relativistic pion susceptibility \( \chi = -\Pi_{33} \). Then we arrive to the well-known non-relativistic form for the pion self-energy [10]:

\[
\tilde{\Pi}_{nr} \simeq \frac{-k^2 \chi}{1 + g' \chi}
\]  

where the non-relativistic pion susceptibility is given by (see Appendix A and B)

\[
\chi = \frac{4f^2}{m_\pi^2} \left( \Phi_0 (\omega, k, p_F) + \Phi_0 (-\omega, k, p_F) \right)
- \frac{f^2}{9m_\pi^2} \left( \Phi_0 (\omega - \omega_R, k; p_F) - \Phi_0 (-\omega - \omega_R, k; p_F) \right)
\]  

Here \( \Phi_0 (\omega, k; p_F) \) is the Migdal function Eq. (A.21), and the \( \Delta \)-resonant frequency is given by

\[
\omega_R = \frac{M_{\Delta}^2 - M^* 2}{2M^*}.
\]  

In the static limit, \( M_{\Delta}^*, M^* \rightarrow \infty \), this expression can be reduced to the widely
used form [10]:

\[
\chi = \frac{f^2 n}{m^2} \left( \frac{1}{k^2/2M^* - \omega} + \frac{1}{k^2/2M^* + \omega} \right) + \frac{8 f_{\pi N\Delta}^2}{9 m^2} \frac{n\omega_R}{\omega^2_R - \omega^2},
\]

(24)

with \( n = n_n + n_p \) being the total number density of nucleons.

4 Pion-like excitations in nuclear matter

The Dyson equation Eq. (3) for the pion propagator can be solved to give

\[
\tilde{D}^{-1}(\omega, k) = \omega^2 - m^2_\pi - k^2 - \tilde{\Pi}(\omega, k)
\]

(25)

where the pion self-energy is given by Eq. (12). The eigen-modes of the pion field in the medium are found from the poles of the pion propagator. In general, the pion self-energy has both a real and an imaginary part. From the poles of the propagator (25) we obtain the following dispersion equation

\[
\left[ \omega^2 - m^2_\pi - k^2 - \text{Re} \tilde{\Pi}(\omega, k) \right]^2 + \left[ \text{Im} \tilde{\Pi}(\omega, k) \right]^2 = 0,
\]

(26)

which has no real solutions \( \omega(k) \) if \( \text{Im} \tilde{\Pi}(\omega, k) \neq 0 \). In this case the system has no stationary excitations, although it possesses resonant properties at some frequencies. The retarded pion propagator \( \tilde{D}^R_{ab}(\omega, k) = \delta_{ab}\tilde{D}^R(\omega, k) \) represents a generalized susceptibility for the components \( \varphi_a \) of the pion field. According to the fluctuation-dissipation theorem, the spectral distribution of the pion field fluctuations can be expressed as

\[
(\varphi^2_a)_{\omega, k} = \frac{i}{2} \coth \left( \frac{\omega}{2T} \right) \left[ \tilde{D}^R(\omega, k) - \tilde{D}^{R\ast}(\omega, k) \right]
\]

(27)

The imaginary part of the retarded pion propagator is connected to that of the time-ordered (causal) propagator as

\[
\text{Im} \tilde{D}^R(\omega, k) = \tanh \left( \frac{\omega}{2T} \right) \text{Im} \tilde{D}(\omega, k),
\]

(28)

which results in

\[
(\varphi^2_a)_{\omega, k} = -\text{Im} \tilde{D}(\omega, k)
\]

(29)

Thus, the function

\[
-\text{Im} \tilde{D}(\omega, k) = \frac{\text{Im} \tilde{\Pi}(\omega, k)}{\left[ \omega^2 - m^2_\pi - k^2 - \text{Re} \tilde{\Pi}(\omega, k) \right]^2 + \left[ \text{Im} \tilde{\Pi}(\omega, k) \right]^2}.
\]

(30)
contains a complete information about the pion field fluctuations. It possesses maxima\(^2\) at the frequencies corresponding to the pion-like eigen modes in the system. Indeed, in the limiting case of \(\text{Im} \tilde{\Pi} (\omega, k) \rightarrow 0\) one has \(-\text{Im} \tilde{D} (\omega, k) \rightarrow \pi \delta (\omega^2 - m^2 - k^2 - \text{Re} \tilde{\Pi} (\omega, k))\), so that it differs from zero only if the frequency \(\omega (k)\) is a solution to the above dispersion equation. If the imaginary part of the pion self-energy is a small, but finite, value the function \(-\text{Im} \tilde{D} (\omega, k)\) is sharply peaked along a line \(\omega (k)\) where
\[
\omega^2 - m^2 - k^2 - \text{Re} \tilde{\Pi} (\omega, k) = 0
\]
and the width \(\gamma (k)\) of the resonance is connected to the momentum-dependent decay rate of the excitation \(\omega (k)\). If the condition \(\gamma (k) \ll \omega (k)\) is not fulfilled, any small perturbation of frequency \(\omega (k)\) undergoes aperiodic damping.

5 Results

To illuminate the problem of the relativistic pion spectra mentioned in the Introduction we, at first, reproduce the widely used calculations, where the "relativistic" corrections to the short-range correlation are included by the simple replacement of \(-k^2\) with \(\omega^2 - k^2\) in the non-relativistic Eq. (21). As it has been made by many authors, we perform this calculation by assuming the universality of the Landau-Migdal coupling, \(g'_{NN} = g'_{N\Delta} = g'_{\Delta\Delta} = g'\).

![Graph](image)

**Fig. 3.** Pion spectra obtained with universal coupling \(g' = 0.6\) for isosymmetric nuclear matter at saturation density \(n = n_0\). (a) The spectrum obtained with incorrect inclusion of the short-range correlations. (b) Same but now with inclusion of the correlation effects as given by Eq. (12)

When the "relativistic" corrections to the short-range correlation are included\(^2\) for bosons, the imaginary part of the causal propagator is negative.
by the simple replacement of $-k^2$ with $\omega^2 - k^2$ in the non-relativistic Eq. (21), we obtain the pion spectrum as shown in Fig. 3a. This spectrum, obtained with $g' = 0.6$ at the saturation density $n_0 = 0.17 \, fm^{-3}$, possesses an unphysical behaviour, with two pion-like branches merging at some point where the quasiparticle velocity, $d\omega/dk$, becomes infinite. In Fig. 3b we show the result of the same calculation, but now incorporating the pion self-energy as given by Eq.(12). This spectrum of pion-like excitations drastically differs from the spectrum of Fig. 3a. Here the solid lines are solutions to Eq. (26) where the imaginary part of the self-energy vanishes. In the same picture, the dashed lines show the solutions to Eq. (31) in the domains with a non-zero imaginary part of the pion self-energy. The lower dashed line represents the spectrum of the spin-isospin sound, which undergoes a strong Landau damping. The above relativistic corrections to this low-frequency mode are negligible, therefore it is of the same form as in Fig. 3a. The upper dashed line is the $\Delta$-resonant mode.

As one can see, the obtained spectrum does not contain any pion branches where the pion velocity becomes infinite, in contrast to the ones obtained before in relativistic calculations of many authors. We claim that this result is due to the correct relativistic incorporation of the short-range correlations.

The experimental information about the Landau-Migdal parameters is very limited. Previous theories basically used the universal ansatz, $g'_{NN} = g'_{N\Delta} = g'_{\Delta\Delta} = g'$, with $g' = 0.6 \div 0.7$. However, modern experiments and theoretical estimates [12], [9], [13] point out that $g'_{N\Delta}$ must be essentially smaller than $g'_{NN}$ and $g'_{\Delta\Delta}$. The most recent analysis, reported in [14], suggest $g'_{NN} = 0.6$, $g'_{N\Delta} = 0.24 \pm 0.10$, $g'_{\Delta\Delta} = 0.6$. While we will do not discuss possible deviations from this set of Landau-Migdal parameters, let us investigate the behaviour of the pion spectra in this case. First we calculate the pion spectrum for symmetric nuclear matter at normal density by the use of the expression for the pion self energy derived in [4], [5], [2] from the non-relativistic form of the contact interaction, Eq. (19). We use the notation $\Pi$ for this incorrect form of the pion self-energy:

\[
\Pi = K^2_{\mu} \left[ \Pi_{ph} \left( K^2_{\mu} + g'_{\Delta\Delta} \Pi_{h} \right) + \Pi_{\Delta h} \left( K^2_{\mu} + g'_{NN} \Pi_{ph} \right) - 2g'_{N\Delta} \Pi_{ph} \Pi_{h} \right] \left( K^2_{\mu} + g'_{NN} \Pi_{ph} \right) \left( K^2_{\mu} + g'_{\Delta\Delta} \Pi_{h} \right) - g^2_{N\Delta} \Pi_{ph} \Pi_{h} (32)
\]

The result of this calculation is shown in Fig. 4a, which explicitly demonstrates the above-mentioned problem.
Two pion-like branches merge at some point where the quasi-particle velocity, $d\omega/dk$, becomes infinite.

In Fig. 4b we show the pion spectrum obtained under the same conditions, but with inclusion of the short-range correlations as given by our Eq. (12). As in Fig. 3b, this spectrum does not contain any pion branches where the pion velocity becomes infinite, in contrast to Fig. 4a. In order to investigate the dependence of our results on the effective nucleon mass, in Fig 5a, we show the curves obtained for the cases $M^* = 0.7$ and $M^* = 0.9$. 

Fig. 5. Pion spectra in symmetric nuclear matter at $n = n_0$ obtained with $g'_{NN} = g'_{\Delta\Delta} = 0.6$ and $g'_{N\Delta} = 0.24$. (a) The curves obtained for the cases $M^* = 0.7$ and $M^* = 0.9$. In both cases the short-range correlations are included, as given by Eq. (12). (b) The thick solid line and long-dash lines correspond to the relativistic calculation where the short-range correlations are included as in Eq. (12). The thin solid line and short-dashed lines are obtained with the
non-relativistic accounting for the short-range correlations, as given by Eq. (20).

As one can see, the spectra obtained for these two cases are very close to each other. The obtained curves are almost indistinguishable.

It is interesting to compare the pion spectrum shown in Fig. 4b with that one obtained by the use of a simple non-relativistic expression, as given by Eq. (20). In Fig. 5b we show the corresponding pion spectra. As can be seen, the relativistic accounting for short-range correlations results in an increasing in the effective pion mass, defined as \( m_\pi^* = \sqrt{m_\pi^2 + \Pi(m_\pi^*,0)} \), by about 20%. This is caused by the delta-resonant contribution to the pion optical potential, which becomes repulsive when \( \omega > k \), due to relativistic kinematics.

6 Summary and Conclusion

The purpose of this work was to demonstrate the crucial dependence of the calculated pion spectra on the correct relativistic accounting for the short-range correlation effects on the pion self-energy in the medium. We have derived the pion self-energy in a nucleon medium with allowance for the relativistic kinematics of short-range correlations. Since the short-range correlation distance is small as compared to the Compton wavelength of the pion, we simulated the short-range interactions by phenomenological contact terms in the Lagrangian density with the Landau-Migdal parameters \( g'_{NN}, g'_{N\Delta}, g'_{\Delta\Delta} \). The analytic expression for the pion self-energy is obtained in terms of the lowest-order polarization tensors, which we have calculated on the basis of the mean field ground state of the nucleon system.

In order to illuminate the effects of relativistic short-range correlations we do not consider in our calculations any additional broadening like the off-shell correction for the vertex form factors or the decay width for the \( \Delta \)-baryon in the medium.

We used the pion self-energy obtained in this way to solve the dispersion equation for the pion-like excitations in cold symmetric nuclear matter at the saturation density, and compare the obtained pion spectra with those calculated by the use of the simplified expressions, which have been employed by many authors.

As is clearly shown in Fig. 3a and in Fig. 4a, the pion spectra calculated by the use of the simplified expressions possess an unphysical behaviour. Similar spectra, which show the group velocity of a pion quasi-particle larger than that of light, were obtained in relativistic calculations of many authors, where
the short-range correlations in nuclear matter are incorporated in a simplified way. In Figs. 3b and 4b we show the pion spectra calculated with the correct inclusion of relativistic effects, as given by Eq. (12). In this case the calculated pion branches drastically differ from those of Figs. 3a and 4a and have a physically meaningful behaviour. In order to investigate the dependence of the calculated pion spectra on the model of nuclear matter, we repeated the calculations by varying the effective nucleon mass. As shown in Fig. 5a, the pion spectra, obtained with $M^* = 0.7$ and 0.9, are almost indistinguishable.

We have also found that, due to relativistic kinematics, the delta-resonant contribution to the pion optical potential is repulsive near the threshold. This results in an increasing of the effective pion mass by about 20%.

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A Lowest-order ph polarization

In terms of the particle-hole propagators, the nucleon contribution to the lowest-order polarization tensor consists on the direct and crossed terms, as shown by loops in Fig. 6.

![Fig. 6. The direct and crossed contribution to the ph polarization tensor.](image)

In the case of symmetric nuclear matter, the analytic expression for polarization tensors are identical for the three pion species:

$$\Pi^\mu_{ph} (\omega, \mathbf{k}) = -\frac{2f^2}{m^2_\pi} \text{Tr} \int \frac{d^4p}{(2\pi)^4} \frac{G(p)\gamma^\mu\gamma_5 G(p + q)\gamma^\nu\gamma_5}{}. \quad (A.1)$$
Here the nucleon propagator at finite density is given by

\[ G(p) = (P + M^*) \left[ \frac{1}{p^2 - M^*^2 + i\eta} + \frac{i\pi}{\varepsilon(p)} \delta \left( p^0 - \varepsilon(p) \right) f(p) \right] \]  

(A.2)

with

\[ P = \left( p^0 - U_N, p \right), \]  

(A.3)

\[ U_N \] is the nucleon mean-field potential. For isosymmetric nuclear matter, one has \( U_p = U_n \). Hereinafter we use the notation

\[ \varepsilon(p) = \sqrt{M^*^2 + p^2}. \]  

(A.4)

The last term in Eq. (A.2), where \( f(p) \) is the Fermi distribution for nucleons, \( N = (n, p) \), corrects the free nucleon propagator (Feynman piece) to account for the presence of the Fermi sea. To obtain the \( ph \) contribution to the polarization we omit the product of two Feynman pieces when inserting the nucleon propagators into Eq. (A.1). This is consistent with the mean-field approximation for the ground state. After shifting the integration variable \( p_0 \rightarrow p_0 + U_N \) the density dependent part of the polarization tensor takes the following form:

\[
\Pi_{\mu\nu}^{ph}(K_0, k) = \frac{2 f^2}{m^2} \int \frac{d^4p}{(2\pi)^4} \text{Tr} \left( g^\mu g^5 \right) \frac{\gamma^\mu \gamma_5 (p + K + M^*) \gamma^\nu \gamma_5}{\varepsilon(p) \varepsilon(p + k)} \left[ i\pi \delta(p^0 - \varepsilon(p)) f(p) \right] \\
+ \frac{\pi \delta(p^0 - \varepsilon(p)) f(p)}{\varepsilon(p) \varepsilon(p + k) (p^2 - M^*^2 + i\eta) (p + K)^2 - M^*^2 + i\eta)} \\
+ \frac{\pi \delta(p^0 + \omega - \varepsilon(p + k)) f(p + k)}{\varepsilon(p + k) (p^2 - M^*^2 + i\eta)} \right].
\]  

(A.5)

As it is apparent from the above equation, the polarization does not depend on the value of the mean field potential.

In the following we need only the \( \Pi_{00}^{ph}, \Pi_{03}^{ph}, \Pi_{30}^{ph}, \Pi_{33}^{ph} \) components of the polarization tensor. The zero-temperature Fermi distribution for nucleons, \( f(p) = \theta(p_F - |p|) \), is given by the Heaviside’s step-function, where \( p_n = p_p = p_F \) stands for the nucleon Fermi momentum. A standard calculation gives:

\[
\text{Re} \Pi_{00}^{ph} = -\frac{f^2}{4\pi^2 m^2} \int_0^{p_F} \frac{dp}{\varepsilon} \left[ 4p^2 + K_\mu^2 \right] \ln \left( \frac{(K_\mu^2 - 2kp)^2 - 4\omega^2 \varepsilon^2}{(K_\mu^2 + 2kp)^2 - 4\omega^2 \varepsilon^2} \right) \\
+ 8kp + 4\omega \varepsilon \ln \left( \frac{(K_\mu^2)^2 - (2kp - 2\omega \varepsilon)^2}{(K_\mu^2)^2 - (2kp + 2\omega \varepsilon)^2} \right)
\]  

(A.6)
\[
\text{Re} \, \Pi_{ph}^{03} = \text{Re} \, \Pi_{ph}^{30}
= -\frac{f^2}{4\pi^2} \omega \int_0^{\mu} \frac{dp}{\varepsilon} \left[ (4\varepsilon^2 + K_\mu^2) \ln \left( \frac{(K_\mu^2 - 2kp)^2 - 4\omega^2\varepsilon^2}{(K_\mu^2 + 2kp)^2 - 4\omega^2\varepsilon^2} \right) + 8kp + 4\omega \varepsilon \ln \left( \frac{(K_\mu^2)^2 - (2kp - 2\omega\varepsilon)^2}{(K_\mu^2)^2 - (2kp + 2\omega\varepsilon)^2} \right) \right], \quad (A.7)
\]

\[
\text{Re} \, \Pi_{ph}^{33} = -\frac{f^2}{4\pi^2} \omega^2 \int_0^{\mu} \frac{dp}{\varepsilon} \left[ (4\varepsilon^2 + K_\mu^2) + 4k^2 M^2 \right] \ln \left( \frac{(K_\mu^2 - 2kp)^2 - 4\omega^2\varepsilon^2}{(K_\mu^2 + 2kp)^2 - 4\omega^2\varepsilon^2} \right)
+ 8pk + 4\omega \varepsilon \ln \left( \frac{(K_\mu^2)^2 - (2kp - 2\omega\varepsilon)^2}{(K_\mu^2)^2 - (2kp + 2\omega\varepsilon)^2} \right). \quad (A.8)
\]

The imaginary part of (A.5) is evaluated as

\[
\text{Im} \, \Pi_{ph}^{\mu \nu} = -\frac{f^2}{16\pi^2} \int d^4p \text{Tr} [(\not p + M^*) \gamma^\mu \gamma_5 (\not p + \not K + M^*) \gamma^\nu \gamma_5]
\times \frac{\Theta (\varepsilon (\not p), \omega)}{\varepsilon (\not p) \varepsilon (\not p + \not k)} \delta (p^0 - \varepsilon (\not p)) \delta (p^0 + \omega - \varepsilon (\not p + \not k)) \quad (A.9)
\]

where

\[
\Theta (\varepsilon, \omega) \equiv f (\varepsilon) [1 - f (\varepsilon + \omega)] + f (\varepsilon + \omega) [1 - f (\varepsilon)] \quad (A.10)
\]

At zero temperature, for the case \( K_\mu^2 < 4M^2 \) of our interest, the imaginary part of the tensor is found to be

\[
\text{Im} \, \Pi_{ph}^{\mu \nu} = -\frac{f^2}{12\pi} \frac{1}{m_\pi^2} \frac{1}{k} \theta (-K_\mu^2)
\times \left[ \Theta (\omega) \theta (\varepsilon_F - \varepsilon_0) J^{\mu \nu} (\text{max} (\varepsilon_F - \omega, \varepsilon_0), \varepsilon_F)
+ \Theta (-\omega) \theta (\varepsilon_F - \omega - \varepsilon_0) J^{\mu \nu} (\text{max} (\varepsilon_F, \varepsilon_0), \varepsilon_F - \omega) \right] \quad (A.11)
\]

with

\[
J^{00} (\varepsilon_1, \varepsilon_2) = (\varepsilon_2 - \varepsilon_1) \left[ 4 \left( \varepsilon_1^2 + \varepsilon_2 \varepsilon_1 + \varepsilon_2^2 \right)
+ 6\omega (\varepsilon_1 + \varepsilon_2) + 3K_\mu^2 - 12M^2 \right], \quad (A.12)
\]
\[ J^{03}(\varepsilon_1, \varepsilon_2) = J^{30}(\varepsilon_1, \varepsilon_2) = \frac{\omega}{k}(\varepsilon_2 - \varepsilon_1) \left[ 4 \left( \varepsilon_1^2 + \varepsilon_2 \varepsilon_1 + \varepsilon_2^2 \right) + 6\omega (\varepsilon_1 + \varepsilon_2) + 3K_\mu^2 \right] , \]  
(A.13)

\[ J^{33}(\varepsilon_1, \varepsilon_2) = \frac{\omega^2}{k^2}(\varepsilon_2 - \varepsilon_1) \left[ 4 \left( \varepsilon_1^2 + \varepsilon_2 \varepsilon_1 + \varepsilon_2^2 \right) + 6\omega (\varepsilon_1 + \varepsilon_2) + 3K_\mu^2 + 12M^*k^2/\omega^2 \right] . \]  
(A.14)

In the above \( \varepsilon_0 = \sqrt{p_{\text{min}}^2 + M^*^2} \), where

\[
p_{\text{min}} = \left| \frac{\omega}{2} \sqrt{1 - \frac{4M^*^2}{K_\mu^2} - \frac{k}{2}} \right| , \]  
(A.15)

is the minimal space-momentum of a nucleon hole arising from the kinematical restrictions of the process.

As one can easily check, the imaginary part of the polarization functions equals to zero at \( \omega = 0 \), and near this point behave as

\[
\text{Im} \Pi_{ph}^{00} \approx -\frac{f^2}{4\pi m_\pi^2} \left( 4p_F^2 - k^2 \right) \frac{\omega}{k} \text{sign} (\omega)
\]

\[
\text{Im} \Pi_{ph}^{03} \approx \text{Im} \Pi_{NN}^{30} = -\frac{f^2}{4\pi m_\pi^2} \frac{\omega^2}{k^2} \left( 4\varepsilon_F^2 - k^2 \right) \text{sign} (\omega)
\]

\[
\text{Im} \Pi_{ph}^{33} \approx -\frac{f^2}{4\pi m_\pi^2} \frac{4M^*^2\omega}{k} \text{sign} (\omega)
\]  
(A.16)

By contracting the tensor \( \Pi_{\mu\nu}^{ph} \) with \( K_\mu K_\nu \) we obtain the lowest-order pion self-energy caused by the nucleon-particle \( \rightarrow \) nucleon-hole excitations in the medium

\[
\text{Re} \Pi_{ph} (K) = \frac{f^2}{\pi^2} \frac{K_\mu^2 M^*^2}{m_\pi^2 k} \int_0^{p_F} \frac{dpp}{\varepsilon (p)} \ln \left| \frac{(K_\mu^2 - 2kp)^2 - 4\omega^2\varepsilon^2}{(K_\mu^2 + 2kp)^2 - 4\omega^2\varepsilon^2} \right| \]  
(A.17)

This expression coincides with that obtained in [2]. For the imaginary part we have

\[
\text{Im} \Pi_{ph} = \frac{f^2}{\pi} \frac{M^*^2 K_\mu^2}{m_\pi^2} \theta (-K_\mu^2) \times \left[ \theta (\omega) \theta (\varepsilon_F - \varepsilon_0) (\varepsilon_F - \max (\varepsilon_F - \omega, \varepsilon_0)) + \theta (-\omega) \theta (\varepsilon_F - \omega - \varepsilon_0) (\varepsilon_F - \omega - \max (\varepsilon_F, \varepsilon_0)) \right] \]  
(A.18)
As it follows from Eq. (A.16), in the vicinity of $\omega = 0$ one has

$$\text{Im } \Pi_{ph} \simeq -\frac{f^2}{\pi \frac{M^*}{m^*_\pi}} k \omega \text{sign} (\omega) \quad (A.19)$$

It is instructive to consider the low-density limit of the function (A.17) in order to compare it with the well-known non-relativistic expression. At low density of the nucleons, $p_{n,p} / M^* \ll 1$ one has $\varepsilon (p) \simeq M^*$. With this replacement, the integration can be performed to give

$$\text{Re } \Pi_{ph} (\omega, k) = \frac{4f^2}{m^*_\pi^2} K^2 (\Phi_0 (\omega, k; p_F) + \Phi_0 (\omega, k; p_F)) \quad (A.20)$$

where

$$\Phi_0 (\omega, k; p_F) = \frac{1}{4\pi^2} \frac{M^*^3}{k^3} \left( \frac{1}{2} \left( a^2 - k^2 V_F^2 \right) \ln \frac{a + k V_F}{a - k V_F} - a k V_F \right) \quad (A.21)$$

is the Migdal function, with

$$a = \omega + \frac{K^2}{2M^*}, \quad V_F = p_F / M^* \quad (A.22)$$

This non-relativistic limit of the particle-hole self-energy has been obtained from relativistic kinematics. If one neglects the relativistic kinematics, i.e. $K^2 \rightarrow -k^2$, it reduces to the standard non-relativistic formula stemming from the particle-hole excitation [11], [10].

**B Lowest order $\Delta h$ polarization**

The lowest order pion self energy involves also the terms caused by the intrinsic excitation of a nucleon from below the Fermi surface into a $\Delta (1232)$, as shown in Fig. 7.

![Fig. 7. The direct and crossed contribution to the $\Delta h$ polarization tensor.](image)

Analytically we obtain
\begin{align}
\Pi^{\mu\nu}_{\Delta h} &= -i \frac{f_{\pi N \Delta}^2}{m_\pi^2} \frac{4}{3} \int \frac{d^4p}{(2\pi)^4} \ Tr G_D(P_n) S^{\mu\nu}(P_\Delta + K) \\
&\quad - i \frac{f_{\pi N \Delta}^2}{m_\pi^2} \frac{4}{3} \int \frac{d^4p}{(2\pi)^4} \ Tr G_D(P_n) S^{\mu\nu}(P_\Delta - K) \tag{B.1}
\end{align}

where

\begin{equation}
S^{\mu\nu} = \frac{(P_\Delta + M_\Delta^*)}{P_\Delta^2 - M_\Delta^2 + i0} \left[ g_{\mu\nu} - \frac{1}{3} \gamma^\mu \gamma^\nu - \frac{2}{3M_\Delta^2} P^\mu P^\nu - \frac{1}{3M_\Delta^*} \left( \gamma^\mu P_\Delta^\nu - \gamma^\nu P_\Delta^\mu \right) \right],
\end{equation}

with \( P_\Delta = (p_0 - U_\Delta, p) \), denotes the Rarita-Schwinger propagator for a \( \Delta \) particle of effective mass \( M_\Delta^* \). As widely used we employ the universal coupling of mesons with nucleons and deltas, which is compatible with the data on electromagnetic excitations of the \( \Delta \) in nuclei [15]. In this case the mean-field potentials for nucleons and deltas coincide, \( U_\Delta = U_N \), and the effective mass of the \( \Delta \) particle is shifted by the scalar \( \sigma \)-field in the same way as that for nucleons (see e.g. [2]). We have

\begin{equation}
M_\Delta^* = M_\Delta - M + M^*.
\end{equation}

This allows us to make a shift on the variable \( p_0 \to p_0 + U_N \) to recast Eq. (B.1) in the following form

\begin{align}
\Pi^{\mu\nu}_{\Delta h} &= \frac{f_{\pi N \Delta}^2}{m_\pi^2} \frac{4}{3} \int \frac{d^4p}{16\pi^3} \ Tr \left[ (\not{p} + M^*) S^{\mu\nu}(p + K) \right] \frac{\delta (p_0 - \varepsilon(p)) f(p)}{\varepsilon(p)} \\
&\quad + \frac{f_{\pi N \Delta}^2}{m_\pi^2} \frac{4}{3} \int \frac{d^4p}{16\pi^3} \ Tr \left[ (\not{p} + M^*) S^{\mu\nu}(p - K) \right] \frac{\delta (p_0 - \varepsilon(p)) f(p)}{\varepsilon(p)}. \tag{B.4}
\end{align}

A calculation of the traces gives

\begin{equation}
\frac{\text{Tr} (\not{p} + M^*) S^{\mu\nu}(p \pm K)}{3M_\Delta^2 (p \pm K)^2 - M_\Delta^2 + i0} \left( M_\Delta^2 g^{\mu\nu} - (p^\mu \pm K^\mu)(p^\nu \pm K^\nu) \right).
\end{equation}

Here one can make the following remark. The problem of describing a spin 3/2 particle in relativistic field theory remains unsolved. Strictly speaking, the expression (B.2) is valid only for "on-shell" particles. Fortunately, due to the large effective mass of the \( \Delta \)-baryon, we do not need the totally relativistic form of Eq. (B.5). This can be substantially simplified if we are interested in
values of $|\omega|, k \lesssim m_\pi$. By neglecting the small terms $kp \lesssim p_\pi^2 \sim p_F^2 \ll M_\Delta^2$ in the numerator of Eq. (B.5) we have the simpler form, which however retains the relativistic kinematics. In this way we obtain

$$\text{Re } \Pi_{33}^{\Delta h} (\omega, k) = \frac{4 f_{\pi N}^2}{18} \frac{(M_\Delta^2 + M^*)}{m_\pi^2 M^*} F_\Delta^{(1)} (\omega, k; p_F) \tag{B.6}$$

$$\text{Re } \Pi_{03}^{\Delta h} (\omega, k) = \text{Re } \Pi_{30}^{\Delta h} (\omega, k) = \frac{4 f_{\pi N}^2}{18} \frac{k M^* M_\Delta^2 + M^*}{m_\pi^2 M^*} F_\Delta^{(1)} (\omega, k; p_F) \tag{B.7}$$

$$\text{Re } \Pi_{00}^{\Delta h} (\omega, k) = \frac{4 f_{\pi N}^2}{18} \frac{(M^* - M_\Delta^2) (M_\Delta^2 + M^*)^2}{m_\pi^2 M^* M_\Delta^2} F_\Delta^{(1)} (\omega, k; p_F)$$

$$+ \frac{4 f_{\pi N}^2}{18} \frac{\omega (M^* + M_\Delta^2)^2}{m_\pi^2 M^* M_\Delta^2} F_\Delta^{(2)} (\omega, k; p_F) \tag{B.8}$$

where the functions $F_\Delta^{(1,2)}$ are defined as

$$F_\Delta^{(1)} (\omega, k; p_F) = \frac{M^*}{\pi^2 k} \int_0^{p_F} dp \frac{p}{\varepsilon} \ln \left\{ \frac{(M^* \omega_R - kp)^2 - \omega^2 \varepsilon^2}{(M^* \omega_R + kp)^2 - \omega^2 \varepsilon^2} \right\} \tag{B.9}$$

$$F_\Delta^{(2)} (\omega, k; p_F) = \frac{M^*}{\pi^2 k} \int_0^{p_F} dp \frac{p}{\varepsilon} \ln \left\{ \frac{(\omega \varepsilon - kp)^2 - M^* \omega_R}{(\omega \varepsilon + kp)^2 - M^* \omega_R} \right\}$$

and the $\Delta$-resonant frequency is given by

$$\omega_R = \frac{M_\Delta^2 - M^2 - K_2^2}{2M^*}. \tag{B.10}$$

The imaginary part of the polarization arises from the poles of the integrand of (B.4). It is non-vanishing for $0 < K_2^2 \leq \omega_R^2$. For example, $\text{Im } \Pi_{33}^{\Delta h}$ is given by

$$\text{Im } \Pi_{33}^{\Delta h} (\omega, k) = -\frac{2 f_{\pi N}^2}{9 \pi} \frac{M^* M_\Delta^2 + M^*}{m_\pi^2 k} \theta (\varepsilon_2 (\omega, k) - \varepsilon_1 (\omega, k))$$

$$\times (\varepsilon_2 (\omega, k) - \varepsilon_1 (\omega, k) + \varepsilon_2 (-\omega, k) - \varepsilon_1 (-\omega, k)) \text{sign } (\omega) \tag{B.11}$$

where

$$\varepsilon_1 (\omega, k) = \max \left\{ M^*, \frac{M^*}{K_2^\mu} \left( \omega \omega_R - \sqrt{k^2 \left( \omega_R^2 - K_2^\mu \right)} \right) \right\} \tag{B.12}$$

and

$$\varepsilon_2 (\omega, k) = \min \left\{ \varepsilon_F, \frac{M^*}{K_2^\mu} \left( \omega \omega_R + \sqrt{k^2 \left( \omega_R^2 - K_2^\mu \right)} \right) \right\} \tag{B.13}$$
are the limiting energies of a nucleon hole, coming from the kinematics of the process. In the above \( p_F \) and \( \varepsilon_F \) stand for the Fermi momenta and kinetic Fermi energies of neutrons and protons.

One can easily find that

\[
\text{Im} \Pi_{\Delta h}^{33} (0, k) = 0
\]  

(B.14)

because \( \theta \left( \varepsilon_2 (0, k) - \varepsilon_1 (0, k) \right) = 0 \).

The lowest-order contribution to the pion self-energy can be obtained by contraction of the above tensor with \( K_\mu K_\nu \). The lowest-order self-energy has the correct non-relativistic limit. Indeed, in the non-relativistic limit only the \( \text{Im} \Pi_{\Delta h}^{33} \) component contributes. This gives

\[
\text{Re} \Pi_{\Delta h} (K) = \frac{4 f_{\pi N \Delta}^2 M^* M_*^*}{9 m_\pi^2} \frac{M_*^2 + M^*^2}{2 M^*^2} k^2 F_\Delta^{(1)} (\omega, k; p_F)  
\]  

(B.15)

To the lowest order in \( p_n/M^* \) one can replace \( \varepsilon (p) \to M^* \), then the integral over the nucleon momentum can be performed to give

\[
F_\Delta^{(1)} (\omega, k; p_F) \to \frac{1}{4} \left( \Phi_0 (\omega - \omega_R, k; p_F) + \Phi_0 (-\omega - \omega_R, k; p_F) \right) 
\]  

(B.16)

where \( \Phi_0 (\omega, k; p_F) \) is the Migdal function (A.21). By considering the static limit, \( M_*^*, M^* \to \infty \), of Eqs. (B.15) and (B.16) we find the following expression

\[
\Pi_{\Delta h}^{\text{st}} (K_\mu) = -\frac{8 f_{\pi N \Delta}^2}{9 m_\pi^2} \frac{n \omega_R k^2}{\omega_R^2 - \omega^2},  
\]  

(B.17)

in agreement with the well-established static form [10].

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