Progress with a Universal Theory of Relativity

Sanjay M. Wagh
Central India Research Institute,
Post Box 606, Laxminagar,
Nagpur 440 022, India

(Dated: October 25, 2005)

In this presentation, I will summarize the present status of the developments with a Universal Theory of Relativity. Some general challenges to be overcome will also be discussed.

Presented at the SARS Einstein Centennial Meeting, September 25 - 26, 2005, Durban, South Africa

Newtonian physics is based on four basic conceptions, namely, inertia, force, source of the force and a law of motion. Inertia is the opposition of a physical body to a change in its state of motion as perceived by an observer, the force is the “cause” behind the motion of that physical body and, since only another physical body can provide such a cause of the motion, every physical body is attributed an appropriate “source strength” as per these conceptions. Gravitational mass and electro-static charge are two such source characteristics attributed to physical bodies.

Due to the mutual logical independencies of these conceptions, a law of force has to be postulated in Newton’s theory. Similarly, numerical values for inertia and source strength(s) are also to be postulated or specified by hand and an independent statement for the law of motion is also needed in Newton’s theory. It is only then that the newtonian laws of motion predict the “path” of a physical body that is imagined as a material point of the Euclidean space which underlies the mathematical framework of Newton’s theory. In this way, Newton’s mathematical framework makes a contact with the physical world.

The classical/newtonian field theory too postulates four, mutually logically independent, conceptions, equivalent to the aforementioned four, to account for the physical phenomena from fluid dynamics, electromagnetism etc.

Due to the mutual logical independencies of involved conceptions, newtonian theories cannot, as is well known, explain the origins of inertia and the source characteristics (of forces) attributable to physical matter. It is this same logical independence of underlying conceptions that is also behind “the equality of inertia and gravitational mass of a physical body” being an assumption of Newton’s theory. Still, newtonian theories explain a large body of experimental results.

To explain the experimental results, we then require four fundamental forces, viz, those of gravity, electromagnetism, strong nuclear and weak nuclear interactions. Theoretical physicists, in general, aim to explain these four interactions as arising from a single entity, i.e., to unify these four forces. Clearly, the “unification of the fundamental forces” must of course provide an appropriate universal “explanation” which removes the mutual logical independencies of these involved concepts. Evidently, this aim requires departures from the newtonian framework.

The (orthodox) Quantum Theory replaces the concept of force with that of a potential for the force. (But, the source of potential in quantum theory is the same as the source of the corresponding newtonian force.) Schrödinger’s equation or Heisenberg’s operators of the quantum theory explicitly involve only the potential generated by the “source” of the force. So is the situation with the procedure of second quantization - the creation, the annihilation and the number operators all involve the potential for the force.

The mathematical formalism of quantum theory leads us only to probabilistic predictions about physical phenomena. Issues of whether this probabilistic description is any “complete” description of the physical reality or not are “separate” from those of the mutual logical independencies of the underlying physical concepts.

Regardless of other issues, the inertia, the potential and the “source” for the potential are, once again, mutually independent logical conceptions. Also, an independent statement of the law of potential has to be postulated in quantum theory. Quantum theory too is then based on four mutually logically independent concepts.

Therefore, (methods of) quantum theory also cannot explain the origins of the inertia and the source characteristics (of potentials or forces) attributable to physical matter. Then, faithful applications of mathematical methods of quantum theory cannot lead us to the “true unification” of the four fundamental interactions.
That is to say, quantum theoretical methods “postulate” the potentials corresponding to (basic) forces, but these methods cannot provide a “single” explanation for those potentials, their theoretical framework being too narrow.

Now, Einstein’s principle of general relativity states that the laws of physics must be applicable with respect to all the systems of reference. This principle offers the widest permissible conceptual basis for the entire physics.

To implement and also to test the usefulness of the ideas behind this important principle, Einstein proposed his “preliminary field equations” by replacing only the concept of gravitational force by that of the curvature of the spacetime geometry while treating all the other source characteristics of physical matter (or equivalently, all the non-gravitational forces) as forming the energy-momentum tensor.

Although encouraged by the “initial successes” of his field equations, Einstein was never comfortable with the fact that his field equations of general relativity provided only a theory of gravity. He never stated his (logical) reasons explicitly, but the following illustrates the origins of Einstein’s unease with his field equations.

Consider a statement: Every woman is replaced by a flower. Clearly, it is then logically inconsistent to replace Queen Cleopatra alone by a Ghost. Needless to say, any statements about some actions of Queen’s Ghost and their consequences are logically unacceptable to us, then.

Hence, the statement of a logically consistent theoretical alternative to the newtonian framework must be that the newtonian force is being replaced by such and such conception, universally applicable to every newtonian force.

[Quantum theory, for example, replaces every force by its potential and is logically consistent in this respect. Hence, methods of quantum theory must be obtainable in a theory that “unifies” the fundamental interactions because the methods of the former are “universal” in replacing the force by the corresponding potential.]

Moreover, since quantum theory makes “probabilistic” predictions, the relation of the quantum theory with the theory that unifies the fundamental interactions can be expected to be very similar to that of the (usual) statistical physics with the classical newtonian theory.

Thus, replacing only one force, say, of gravity, by curvature of geometry while considering all non-gravitational interactions as forming the energy-momentum tensor is logically unacceptable. But, this is what Einstein equations are. Therefore, all solutions of Einstein equations are then like different actions of Queen’s ghost.

Hence, even if certain of the many solutions of Einstein’s equations were to “equationally explain” some observations of the real world, say, the bending of light, precession of the perihelion of Mercury, gravitational redshift, changes in the period of a binary pulsar system, cosmology etc, the “curvature of geometry” is not any “logical reason” for the involved physical phenomena.

To state it bluntly, some action of Queen’s Ghost cannot be any acceptable “reason” for the falling of an apple on Newton’s head even if this action were to be always $1/r^2$. See Fig (1).

Also, if Queen’s ghost were to “dance” rhythmically holding the hand of matter, say, in the LIGO detector then, some “wavy” motion of matter can be “explained” even accurately. See Fig (2). This however cannot form an explanation of the wavy-motion of matter, even if such wavy motion is detected by any detector (8).

Now, the basis of Einstein’s principle of general relativity is the “equality” of all systems of reference, in relative acceleration or not, for the description of all physical phenomenon.

But, changes that can occur to the “physical construction” of reference systems must also be the part of any such description. See Fig (3) and Fig (4). The equality of the description of physical phenomena must then hold incorporating any possible physical changes to the constructions of reference systems. This situation necessitates “changes” also at the conceptual levels.

That is to say, the validity of the principle of general relativity not only demands a mathematical framework that incorporates the physical construction of reference systems but also demands that we “redefine” various physical notions. I will refer to this general theoretical framework as the Universal Theory of Relativity.

In [1], I had discussed a mathematical framework of the theories of measures and dynamical systems for this universal relativity. However, it relied on certain physical observations which did not, quite uniquely, fix the underlying mathematical structures, although the aforementioned logical independencies were removable in it. It could then be considered to be not sufficiently general and, hence, not entirely satisfactory.

What, then, is the appropriate mathematical formalism for the universal theory of relativity? It is the contention here that the mathematical framework of Category Theory is an appropriate basis for the universal relativity.

The rest of this paper constitutes a discussion about the relevant mathematical notions and how these notions could incorporate the ideas of universal relativity. Of course, much more work is still needed on various fronts.
A partially ordered set (poset) $P$ is complete iff every subset of $P$ has a lowest upper bound (lub or sup or join) and a greatest lower bound (glb or inf or meet). A poset $P$ is then complete iff, as a category, $P$ has all limits and all co-limits.

A lattice is a poset having, as a category, all binary products and all binary co-products. A complete lattice with an initial object $0$ and a terminal object $1$ is a complete poset.

A Frame is a complete lattice in which binary meet distributes over arbitrary joins. A Frame homomorphism is a map between Frames preserving finite meets and arbitrary joins. For any topological space $X$, the lattice of its open sets, $\mathcal{O}X$, forms a Frame, ordered by set inclusion.

The correspondence $X \to \mathcal{O}X$ is functorial with the functor $\mathcal{O} : \text{Top} \to \mathcal{Frm}$ being a contravariant functor, with $\text{Top}$ being the category of topological spaces and $\mathcal{Frm}$ being the category of Frames. Any continuous map $f : X \to Y$ determines a Frame homomorphism $\mathcal{O}f : \mathcal{O}Y \to \mathcal{O}X$ with $\mathcal{O}f(U) = f^{-1}(U)$ for all $U \in \mathcal{O}Y$.

The Spectrum functor in the opposite direction $\Sigma : \mathcal{Frm} \to \text{Top}$ assigns to each Frame $L$ its spectrum $\Sigma L$ that is the space of all homomorphisms $\xi : L \to 2$, $2$ being the 2-element lattice, with each homomorphism being called a point of $L$, with open sets $\Sigma a = \{ \xi \in \Sigma L | \xi a = 1 \}$ for any $a \in L$. It also assigns to each of the Frame homomorphisms $h : M \to L$ a continuous map $\Sigma h : \Sigma L \to \Sigma M$ such that $\Sigma h(\xi) = \xi h(\Sigma a)^{-1}(\Sigma a) = \Sigma h(\xi)$ for any $a \in M$.

The two functors $\mathcal{O}$ and $\Sigma$ are adjoint on the right with the unit of this adjunction being denoted by $\eta_L$ and the co-unit by $\varepsilon_X$.

Frames for which $\eta_L$ is an isomorphism are called spatial. $\mathcal{O}X$ is trivially a spatial Frame.

Spaces for which $\varepsilon_X$ is a homomorphism are called the Sober Spaces. Category of Sober spaces $\mathcal{Sob}$ is dually equivalent to the full subcategory of $\mathcal{Sp Frm}$ of spatial Frames.

Category opposite to $\mathcal{Frm}$ is called as the category $\mathcal{Loc}$ of locales and it contains $\mathcal{Sob}$ as a full subcategory. Category $\mathcal{Loc}$ is often used in the relevant mathematical literature on pointless topology which provides a wider basis than the usual topological structure.

Next, a sheaf is a “continuous set-valued” function. A sub-sheaf of a sheaf $F$ over a set $X$ is a sub-functor of $F$ which is itself a sheaf. Sheaves over a set $X$ form the category $\mathcal{Sh}(X)$. Categorical co-limits of finite limits in $\mathcal{Sh}(X)$ provide geometric constructions which are point-wise and providing sheaves. Category $\mathcal{Sh}(X)$ of sheaves over $X$ then contains an object - called the generic set - from which every other object of $\mathcal{Sh}(X)$ can be geometrically constructed.

When $X$ is a topological space, sub-sheaves of the terminal object of $\mathcal{Sh}(X)$ provide the open subsets of $X$. When $X$ are Sober spaces, continuous maps between them are functors between the corresponding categories of sheaves preserving the geometric constructions.

This above provides a generalization of a topological space - the generalized space of sets - because the generic set is, in general, not isomorphic to the initial object of $\mathcal{Sh}(X)$, and the sub-objects of the terminal object of $\mathcal{Sh}(X)$ not necessarily its elements. Consequently, the notion of “continuity” acquires, for example, a new meaning in terms of the generic set.

Topos Theory was then developed based on a complete Cartesian closed category with a sub-object classifier - a Topos. Also, the differential structure built by giving up the Law of Excluded Middle in the Euclidean setup leads to Synthetic Differential Geometry.

Using a topos equipped with the notion of an infinitesimal time-interval, it is then possible to recast the newtonian framework into the topos theoretic language. In a wave equation is also discussed from this point of view.

Physically, the construction of a reference system must necessarily use physical bodies. For its mathematical expression, we may consider a physical body as a collection of open subsets of some topological space $X$. Then, the physical construction of a reference system is mathematically representable as a Frame.

With the above identification, the category $\mathcal{Frm}$ of Frames or, equivalently, the category $\mathcal{Loc}$ of locales could form the underlying mathematical basis for Universal Relativity. The category $\mathcal{Sh}(X)$ would then be equally relevant. Issues regarding the physical characteristics of material bodies will then involve suitable measures over Frames. These issues have not been addressed as yet.

Furthermore, the category $\mathcal{Frm}$ is not a Topos. (There exist many other categories which are not toposes.) Consequently, it is unclear whether the Topos Theory could form a mathematical basis for the universal relativity as it did for the newtonian theoretical framework.

However, the notion of a category is very general indeed and, hence, could form the basis for universal relativity. An important unanswered mathematical issue is then of defining an appropriate notion of (generalized) measures in the general setting of the category theory.

Notably, very general ideas from Physics and Mathematics appear to become quite identical here. Starting with the limited formalism in $\mathcal{Frm}$, this is certainly a significant progress.
Acknowledgments

Discussions with Partha Ghosh as well as with Gareth Amery, Sunil Maharaj and many others helped me focus my own ideas. I thank them as well as all the organizers of the SARS Einstein Centennial Meeting for excellent arrangements and for an opportunity to present this work. I also wish to thank Dr T Smith of IOP for pointing out to me the exact reference for the work in [3].

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[17] I will reserve the word “Frame” for the mathematical notion to be defined later and will use “system of reference” or “reference system” to imply the use of a physical body as a reference for the purpose of a physical measurement.

[18] Changes in the curvature of 4-geometry affecting LIGO matter are, then, like Queen’s ghost changing the location of LIGO matter in an undulatory fashion. We are then led to challenge the “physical existence of gravitational radiation” as ripples of curvature propagating on the spacetime fabric.

[19] Objection that the coordinate axes (intervals) be closed, compact and bounded, since such is the case with the real line $\mathbb{R}$, does not apply here because physically interesting quantity is only the measurable or observable “distance” between bodies and is definable for Frames as “distance between sets”. I thank Prof B Banaschewski for raising this issue during a short discussion.
FIG. 1: Ghost causing physical motion

FIG. 2: Ghost causing Wavy Motion

FIG. 3: Physical construction of reference system

FIG. 4: Change to the physical construction of reference system as a physical phenomenon