Probing $Z'$ gauge boson with the spin configuration of top quark pair production at future $e^-e^+$ linear colliders

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Abstract

We explore the effects of extra neutral gauge boson involved in the supersymmetric $E_6$ model on the spin configuration of the top quark pair produced at the polarized $e^-e^+$ collider. Generic mixing terms are considered including kinetic mixing terms as well as mass mixing. In the off-diagonal spin basis of the standard model, we show that the cross sections for the suppressed spin configurations can be enhanced with the effects of the $Z'$ boson through the modification of the spin configuration of produced top quark pair enough to be measured in the Linear Colliders, which provides the way to observe the effects of $Z'$ boson and discriminate the pattern of gauge group decomposition. It is pointed out that the kinetic mixing may dilute the effects of mass mixing terms, and we have to perform the combined analysis.

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I. INTRODUCTION

The existence of extra U(1) gauge group is inevitable in many extended models derived by grand unified theories (GUTs) of higher ranked gauge group and superstring/M theories inspired models [1]. Moreover, the symmetry breaking scale for extra U(1) might be as low as $\mathcal{O}(1)$ TeV in the context of supersymmetry to provide the solution of the $\mu$ problem [2], leading to the possibility to observe the effects of the extra heavy neutral gauge boson, $Z'$ boson at the future collider experiments. The latest bound of the $Z'$ boson mass comes from a direct search at the $p\bar{p}$ collider via Drell–Yan production and subsequent decay to charged leptons [3]. Indirect constraints for the $Z'$ boson mass and the $Z - Z'$ mixing angles are obtained from high precision LEP data at the $Z$ peak energy and from various low energy neutral current experiment data [4–7]. Very recently it is indicated that a missing invisible width in $Z$ decays at LEP 1 and a significantly negative $S$ parameter observed in atomic parity violation of Cs atom can be explained properly if the presence of an $Z'$ boson is assumed [8]. Thus it is timely and interesting to search for an effective way to probe the effects of $Z'$ boson at future colliders.

As a larger class of $Z'$ models are considered from the string perspective, meanwhile, it is natural to introduce kinetic mixing term. The kinetic mixing is a threshold effects of string models at the string scale and can be generated by the renormalization group (RG) evolution from the high energy scale to the scale that we study. Furthermore it may yield significant effects on the phenomenologies of the $Z'$ couplings to the SM sector [9].

In this paper, we study the effects of $Z'$ boson involved in the supersymmetric $E_6$ model in $e^-e^+ \rightarrow t\bar{t}$ process at linear colliders (LC) including the kinetic mixing term. Charges of the standard model (SM) fermions for $Z'$ boson are determined by the gauge group decompositions, along with which the $\psi$-, $\chi$-, and $\eta$-models are defined at low energy scale in the SUSY $E_6$ model framework. Here we take the decoupling limit of the exotic fermions. Since the asymmetry of the left- and right-handed couplings to $Z'$ boson characterizes each model, it is possible to distinguish models using the spin informations of top quark pair with
the polarized initial $e^-e^+$ beams. We can read out the information of the polarization of top quark through the angular distribution of the decay products [10]. The large mass of the top quark prompts itself to decay before hadronization and the information of the top spin is free from the uncertainties of hadronization. The top quark pair is produced in an unique spin configuration at the polarized $e^-e^+$ collision, which reveals remarkable features [11,12] in the off-diagonal basis of spin. Moreover it is interesting to observe the polarization of top quark to probe new physics in this basis, since the off-diagonal basis is model-dependently defined.

The LC with $\sqrt{s} = 500$ GeV is the best testing ground for studying the $t\bar{t}$ production in the off-diagonal basis. If the CM energy is around at threshold of top pair production, the top spins are determined by the electron and positron momentum directions since top quark pair is almost at rest. Then the off-diagonal basis cannot be defined. At high energy, $\sqrt{s} \gg m_t$, the spin basis is close to the usual helicity basis so that the angle $\xi \sim 0$. Thus it is hard to extract the new physics effects from $\xi$ although they exist.

This paper is organized as follows. In section II, the extra neutral gauge bosons in the string inspired supersymmetric $E_6$ are briefly reviewed. In section III, we present the formulae of scattering amplitudes of $e^-e^+ \rightarrow t\bar{t}$ process with the $Z'$ bosons in a generic spin basis of the top quark pair. The off-diagonal spin basis is defined and discussed is the way how to probe the $Z'$ boson through the spin configuration of $t\bar{t}$ pair. In section IV, the numerical analysis for each models are performed in the SM off-diagonal basis. Section V is devoted to summary of the paper.

II. $Z'$ BOSON IN SUPERSYMMETRIC $E_6$ MODEL

In the supersymmetric $E_6$ model, there are two additional $U(1)$ factors beyond the SM gauge group since the rank of $E_6$ group is 6. The canonical decompositions of $\psi$ and $\chi$ models are as follows:

$$E_6 \rightarrow SO(10) \times U(1)_\psi,$$
SO(10) → SU(5) × U(1)χ.

After the extra U(1) symmetries are spontaneously broken by the weak iso-singlet Higgs scalar(s), the gauge bosons $Z_ψ$ and $Z_χ$ corresponding to the groups U(1)$_ψ$ and U(1)$_χ$ respectively become massive but are not mass eigenstates in general. We call it $Z'$ a linear combination of $Z_ψ$ and $Z_χ$ parametrized by the mixing angle $θ_E$

$$Z'(θ_E) ≡ Z_χ \cos θ_E + Z_ψ \sin θ_E,$$

which is relatively light enough to mix with the ordinary $Z$ boson and relevant to the low energy phenomenology. The orthogonal mode to $Z'$ boson is assumed to be so massive that its effect is to be decomposed. In the case of $θ_E = 0$, the $Z'$ mode is identified to $Z_ψ$ boson; if $θ_E = π/2$, $Z'$ mode is $Z_χ$ boson. The $η$-model and corresponding $Z_η$ boson is defined by setting $θ_E = \tan^{-1}(-\sqrt{5}/3)$. Here we assume that exotic fermions are heavy enough to be decoupled.

In the effective rank-5 limit with only one extra neutral gauge boson, the interaction Lagrangian is described by

$$- L_{int} = \sum_f \bar{ψ}_f γ^μ \left[ g_3 \lambda^α G^α_μ + g_2 T^a f W^a_μ + g_1 Y_f B_μ + g'_{1/2}(f^f V - f^f A γ^5) Z'_μ \right] ψ_f,$$

where $ψ_f$ is the fermion field with flavour $f$; $λ^α$ and $T^a$ are generators of SU(3)$_C$ and SU(2)$_L$ gauge group respectively. The extra gauge coupling is expressed by $g'_1 = \frac{1}{\sqrt{3}} g_1$ with order 1 parameter $λ$. The exact value of $λ$ depends upon the pattern of symmetry breaking and we set the value 1 in the numerical analysis. The couplings, $f^f V$ and $f^f A$ are the vector and axial vector charges of the fermion for U(1)' group. The U(1)' charge assignment is given in the Table 1 in terms of the vector and axial vector couplings of $Z'$ to fermions.
After the electroweak symmetry breaking, the gauge sector of the Lagrangian with $Z'$ boson is given by

$$\mathcal{L}_{\text{gauge}} = \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{mass}} + \mathcal{L}_{\text{mix}},$$

where the kinetic term and the mass term are written as

$$\mathcal{L}_{\text{kinetic}} = -\frac{1}{4}(\hat{F}^{\mu\nu}\hat{F}_{\mu\nu} + Z^{\mu\nu}Z_{\mu\nu} + Z'^{\mu\nu}Z'^{\mu\nu}),$$

$$\mathcal{L}_{\text{mass}} = \frac{1}{2}(m_{Z}^2Z^{\mu}Z_{\mu} + m_{Z'}^2Z'^{\mu}Z'^{\mu}),$$

where $\hat{F}$, $Z^{\mu\nu}$, and $Z'^{\mu\nu}$ are the usual field strength tensor for the fields $\hat{A}_\mu$, $Z_\mu$ and $Z'_\mu$ respectively. The fields $\hat{A}_\mu$ and $Z_\mu$ are defined by

$$Z = c_W W_3 - s_W B,$$

$$\hat{A} = s_W W_3 + c_W B,$$

where the shortened notation $s_W = \sin \theta_W$ and $c_W = \cos \theta_W$ with the weak mixing angle $\theta_W$. We write $\mathcal{L}_{\text{mix}}$ including the gauge invariant kinetic mixing term,

$$\mathcal{L}_{\text{mix}} = -\frac{\sin \chi}{2}Z'^{\mu}B^{\mu\nu} + \delta M^2 Z'^{\mu}Z^{\mu},$$

where $B^{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ is the field strength tensor of $U(1)_Y$ gauge boson.

The mass eigenstates ($A, Z_1, Z_2$) are obtained by diagonalizing the mass terms and kinetic terms with the transformation

| Models | $\psi$ | $\chi$ | $\eta$ |
|--------|-------|-------|-------|
| particles | $f_V/\sqrt{5/72}$ | $2\sqrt{6}f_V$ | $12f_V$ |
| $\nu$ | 0 | 1 | 6 |
| $e$ | 0 | 1 | 3 |
| $u$ | 0 | 1 | 3 |
| $d$ | 0 | 1 | 3 |

Table 1: $U(1)'$ charge assignment for the standard model fermions
\[
\begin{pmatrix}
A \\
Z_1 \\
Z_2
\end{pmatrix} =
\begin{pmatrix}
1 & 0 & c_w s_x \\
0 & c_\zeta - c_\zeta s_w s_x + s_\zeta c_x \\
0 & -s_\zeta & c_\zeta c_x + s_\zeta s_w s_x
\end{pmatrix}
\begin{pmatrix}
\hat{A} \\
Z \\
Z'
\end{pmatrix}
\]

where

\[
\tan 2\zeta \equiv \frac{-2 c_\chi (\delta M^2 + m_Z^2 s_w s_x)}{m_{Z'}^2 - m_Z^2 s_w^2 s_x^2 + 2 \delta M^2 s_w s_x},
\]

where \( s_\chi = \sin \chi, \ c_\chi = \cos \chi, \) and \( s_\zeta = \sin \zeta, \ c_\zeta = \cos \zeta. \) The lighter \( Z_1 \) boson is identified to the ordinary \( Z \) boson. We recast the Lagrangian in terms of the mass eigenstates \( Z_1 \) and \( Z_2 \) to obtain the interaction terms of \( Z_i \bar{f} f, \) \( i = 1, 2 \) as

\[
- \mathcal{L}_{Z_i \bar{f} f} = \frac{e}{2 s_w c_w} \left( 1 + \frac{\alpha_T}{2} \right) \sum_f \tilde{\Psi}_f \gamma^\mu [(g_f^V + \zeta \tilde{f}_f^V) - (g_f^A + \zeta \tilde{f}_f^A) \gamma_5] \Psi_f Z_{1\mu}
\]

\[
+ \sum_f \tilde{\Psi}_f \gamma^\mu [(h_f^V - \zeta g_f^V) - (h_f^A - \zeta g_f^A) \gamma_5] \Psi_f Z_{2\mu}
\]

\[
\equiv \sum_f \sum_i \tilde{\Psi}_f \gamma^\mu [V_{if} - A_{if} \gamma_5] \Psi_f Z_{i\mu},
\]

where the SM couplings are modified by \( Z-Z' \) mixing effects

\[
g_f^A = T_{3f}^f, \quad g_f^V = T_{3f}^f - 2 Q_f s^2,
\]

and the extra U(1) couplings

\[
\tilde{f}_{fV,A}^f \equiv \frac{g_f^V \tilde{f}_{V,A}^f}{g \cos \chi},
\]

\[
h_{fV}^V = \tilde{f}_{fV}^V + \tilde{s}(T_{3f}^f - 2 Q_f) \tan \chi,
\]

\[
h_{fA}^V = \tilde{f}_{fA}^V + \tilde{s} T_{3f}^f \tan \chi.
\]

Effective weak mixing angles are defined by

\[
s^2_s = s^2_w + \zeta c^2_w s_w \tan \chi - \zeta^2 \frac{c^2_w s^2_w}{c^2_w - s^2_w} \left( \frac{M^2_2}{M^2_1} - 1 \right),
\]

\[
\tilde{s} = s_w + \frac{s^3_w}{c^2_w - s^2_w} \left( \frac{\alpha S}{4 c^2_w} - \frac{\alpha T}{2} \right),
\]

where the Peskin-Takeuchi variable \( S \) and \( T \) [13] are given by
\[ \alpha S = 4 \zeta_c \cos^2 \theta \tan \chi, \]
\[ \alpha T = \zeta \frac{M^2}{M_T^2} - 1 + 2 \zeta s_W \tan \chi, \]
up to the leading order of \( \zeta \).

### III. TOP QUARK PAIR PRODUCTION IN THE OFF-DIAGONAL BASIS

For the process
\[ e^- (p_1, s_1) \rightarrow p_2 (s_2) \rightarrow t(k_1, r_1) \bar{t}(k_2, r_2), \]  
we have s-channel Feynman diagrams mediated by photon, Z and \( Z' \) boson exchanges depicted in Fig. 1. In Eq. (13), \( p_i \) and \( k_i \) denote the momenta and \( s_i \) and \( r_i \) the polarizations of electrons and top quarks respectively. In the center of momentum (CM) frame, we write the momenta as
\[ p_1 = (E, E\hat{n}), \quad p_2 = (E, -E\hat{n}), \quad k_1 = (E, 0, 0, |k|), \quad k_2 = (E, 0, 0, -|k|), \]
where the unit vector \( \hat{n} = (-\sin \theta, 0, \cos \theta) \) indicates the spatial direction of the electron beam. We assume that the whole process is confined on the \( xz \)-plane when taking the direction of the produced top quark to be \( z \)-axis.

We study the spin configuration of top quark pair in a generic spin basis suggested in Ref. [11]. The spin states of the top quark and top anti-quark are defined in their own rest-frame by decomposing their spins along reference axes. The reference axis for top quark is expressed by an angle \( \xi \) between the axis and the top anti-quark momentum in the rest frame of the top quark as depicted in Fig. 2. The usual helicity basis is obtained by taking \( \xi = \pi \). In this general spin basis, the explicit expression for spin four-vectors of the \( t\bar{t} \) is given by:
\[ r_1 = (-\frac{|k|}{m} \cos \xi, \sin \xi, 0, -\frac{E}{m} \cos \xi), \]
\[ r_2 = (-\frac{|k|}{m} \cos \xi, -\sin \xi, 0, \frac{E}{m} \cos \xi), \]  
(17)
in the CM frame. It is to be notified that the spin vectors of the produced top quark pair
die in the production plane at tree level if the CP-invariance of the scattering amplitude is
preserved.

We have the scattering amplitudes for each spin configurations of top quark pair produced
by the left-handed polarized electron and right-handed polarized positron beams, including
$Z'$ effects,

\[ M(LR, \uparrow \uparrow) = -C_1 s(\cos \theta \sin \xi - 1/\gamma \sin \theta \cos \xi) + C_2 s \beta \sin \xi = -M(LR, \downarrow \downarrow), \]
\[ M(LR, \downarrow \uparrow) = C_1 s(\cos \theta \cos \xi + 1 + 1/\gamma \sin \theta \sin \xi) - C_2 s \beta (\cos \theta + \cos \xi) \]
\[ M(LR, \uparrow \downarrow) = C_1 s(\cos \theta \cos \xi - 1 + 1/\gamma \sin \theta \sin \xi) + C_2 s \beta (\cos \theta - \cos \xi). \]  

(18)

where $\beta \equiv \sqrt{1 - 4m_t^2/s}$ and $\gamma \equiv 1/\sqrt{1 - \beta^2}$. The coefficients $C_1$ and $C_2$ are defined by

$C_1 = P_{VV} + P_{VA}$ and $C_2 = P_{AV} + P_{AA}$, and the effective coupling strength of current-current
interactions $P_{\alpha \beta}$ are given by

\[ P_{VV}/\sqrt{N_c} \equiv e^2 Q_t Q_e D_0(s) + V_1^i V_1^e D_1(s) + V_2^i V_2^e D_2(s) \]
\[ P_{VA}/\sqrt{N_c} \equiv -V_1^i A_1^e D_1(s) - V_2^i A_2^e D_2(s) \]
\[ P_{AV}/\sqrt{N_c} \equiv -A_1^i V_1^e D_1(s) - A_2^i V_2^e D_2(s) \]
\[ P_{AA}/\sqrt{N_c} \equiv A_1^i A_1^e D_1(s) + A_2^i A_2^e D_2(s), \]  

(19)

where $N_c$ is the number of colors and $Q_{t(e)}$ is the electric charge for the top quark (electron).

$D_0$, $D_1$ and $D_2$ are the propagation factors for the photon, $Z_1$ and $Z_2$ bosons respectively;

\[ D_0(s) \equiv \frac{1}{s}, \quad D_1(s) \equiv \frac{1}{s - m_1^2}, \quad D_2(s) \equiv \frac{1}{s - m_2^2}, \]  

(20)

while $V_i^f$ and $A_i^f$ are the model-dependent vector and axial vector couplings for fermion $f$
and gauge boson $i = \text{photon, Z}_1, Z_2$, defined in Eq. (10).

The scattering amplitudes for right-handed polarized electron and left-handed polarized
positron are obtained in the similar manner

\[ M(RL, \uparrow \uparrow) = C_1 s(\cos \theta \sin \xi - 1/\gamma \sin \theta \cos \xi) + C_2 s \beta \sin \xi = -M(RL, \downarrow \downarrow), \]
\[ M(\RL, \uparrow\uparrow) = -C_1 s(\cos \theta \cos \xi - 1 + 1/\gamma \sin \theta \sin \xi) + C_2 s\beta(\cos \theta - \cos \xi), \]
\[ M(\RL, \uparrow\downarrow) = -C_1 s(\cos \theta \cos \xi + 1 + 1/\gamma \sin \theta \sin \xi) - C_2 s\beta(\cos \theta + \cos \xi), \] (21)

There exist the angles \( \xi_L \) and \( \xi_R \) such that the scattering amplitudes for the like-spin states of top quark pair, \( (\uparrow, \uparrow) \) and \( (\downarrow, \downarrow) \) vanish for the left- and right-handed electron beam, respectively. From the Eq. (18) and (21), we find the angle \( \xi_L \) and \( \xi_R \)

\[
\xi_L(s, \theta) \equiv \arctan \left( \frac{\tan \theta}{\gamma(1 - (C_2/C_1)\beta \sec \theta)} \right), \\
\xi_R(s, \theta) \equiv \arctan \left( \frac{\tan \theta}{\gamma(1 + (C_2/C_1)\beta \sec \theta)} \right),
\] (22)

which is always defined in terms of the scattering angle \( \theta \). It is called the off-diagonal basis since only the scattering amplitudes for off-diagonal spin states are non-zero [11]. For given kinematics, the angles \( \xi_{L,R} \) are determined by the model-dependent ratio \( (C_2/C_1) \), so \( \xi_{L,R} \) depend upon the existence of new physics.

One more interesting feature of the off-diagonal basis is that the process into the \( (\uparrow\downarrow) \) state for the left-handed electron beam and the \( (\downarrow\uparrow) \) state for the right-handed one is dominant. This pure dominance is very stable under the one-loop QCD corrections where the soft gluon emissions dominate so that the QCD corrections are factored out. At high energy, the degree of this dominance is close to 100% [15].

**IV. ANALYSIS**

In the off-diagonal basis, the scattering amplitudes of the like-spin states are identical to zero and so are the corresponding cross sections. Including new physics effects, the basis does not remain as the off-diagonal basis any more and the characteristic features of the off-diagonal basis is modified through the model-dependence of the angle \( \xi_{L,R} \). The \( Z' \) boson exchange diagrams yield the deviation of the cross sections for like-spin states from zero. Therefore observation of sizable cross sections for like-spin states can be a smoking-gun signals of new physics.
In Fig. 3−5, we plot the differential cross sections for left-handed polarized electron beam with respect to the scattering angle in the $\psi$-, $\chi$- and $\eta$-models respectively. The SM predictions are denoted by solid lines. The dashed lines denote the model predictions with no kinetic mixing terms, the dotted lines the predictions with the kinetic mixing $\tan \chi = 0.2$ and the dash-dotted lines with $\tan \chi = -0.2$. For the numerical analysis, we take $m_{Z'} = 600$ GeV, the lower bound from the direct search by CDF [3], and the mixing angles to be the latest bounds in Ref. [4] to maximize the new contributions. We have 2 lines for each predictions corresponding to the upper and lower limits of the $Z−Z'$ mixing angle $\zeta$ respectively.

It is apparent from the figures that the cross sections $\sigma(\uparrow\uparrow)$ and $\sigma(\downarrow\downarrow)$ are nonzero with $Z'$ boson effects, which can be as large as $10^{-2}$ pb, of order 1% of the total cross section of $t\bar{t}$ production. With the expected integrated luminosity $\int L > 50$ fb$^{-1}$ for energy at $\sqrt{s} = 500$ GeV, we will have more than 500 events for like-spin states, which is sufficient to examine the nonzero cross section. We also find that the pure dominance of $(\uparrow\downarrow)$ state is contaminated with $Z'$ boson effects from the figures. Cross sections for states other than $(\uparrow\downarrow)$ state increases with $Z'$ boson effects in general. However the pure dominance is essentially affected by the alteration of $\sigma(\uparrow\downarrow)$, since actually the total cross section is still dominated by $(\uparrow\downarrow)$ state. We present the ratios of $\sigma(\uparrow\downarrow)/\sigma_{total}$ for each model in Table 2.

Since the asymmetry of vector and axial vector charges to $Z'$ boson features the models, the forward-backward asymmetry of $t\bar{t}$ production can be an useful observable to discriminate models. The forward-backward asymmetry $A_{FB}$ defined by

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \quad (23)$$

increases with $Z'$ boson effects in the $\psi$-model while it decreases in the $\chi$- and $\eta$-model from the figures. The $\eta$-model is a mixture of $\psi$- and $\chi$-models and behaves about halfway. The smallness of $A_{FB}$ is a characteristic feature of $\chi$-model. The asymmetry $A_{FB}$ for each model is listed in Table 2.

We find that the kinetic mixing derives much shift on the observables. Here the kinetic mixing is taken to be $\tan \chi = \pm 0.2$, which is the bound obtained in Ref. [14]. It is to be noti-
fied that the effects of kinetic mixing term act on the effects of $Z'$ boson both constructively and destructively with respect to the sign of $\tan \chi$. The effects of $Z'$ boson may be diluted and even canceled by the kinetic mixing effects. For instance, the pure dominance of $(\uparrow \downarrow)$ final state is almost recovered in the $\psi$-model when $\tan \chi = -0.2$. In this case, the precise measurement of $A_{FB}$ can still be an evidence of $Z'$ boson. Hence it is essential to perform the analysis with more than 2 observables to probe the $Z'$ effects and to discriminate the models.

| Models $\tan \chi$ | $\sigma(e_L^- e_R^+ \to t\bar{t}_\psi)/\sigma_{total}$ | $A_{FB}$ |
|-------------------|---------------------------------|---------|
| SM               | 99.3%                           | 0.4046  |
| 0                | 98.5%                           | 0.5539  |
| $\psi$           | 0.2                             | 95.6%   | 0.6796  |
| -0.2             | 99.2%                           | 0.4703  |
| $\psi$           | 0.2                             | 95.6%   | 0.6796  |
| -0.2             | 99.2%                           | 0.4703  |
| $\chi$           | 0.2                             | 95.4%   | 0.0886  |
| -0.2             | 93.6%                           | 0.0056  |
| $\eta$           | 0.2                             | 97.5%   | 0.2036  |
| -0.2             | 98.9%                           | 0.3279  |

Table 2: The ratios of the cross section for $(\uparrow \downarrow)$ spin state of top quark pair to the total cross section of $e_L^- e_R^+ \to t\bar{t}$ production in the SM off-diagonal basis and the forward-backward asymmetries are presented for the standard model, $\psi$-, $\chi$-, and $\eta$-models.

V. SUMMARY AND CONCLUSIONS

We have explored the effects of $Z'$ boson arising in the supersymmetric $E_6$ model framework at $e^- e^+ \to t\bar{t}$ process, including the kinetic mixing terms. Considering the spin configuration of produced top quark pair, we propose useful probes not only to search for the
$Z'$ boson but also to discriminate the models corresponding to the pattern of gauge group decomposition. Provided that we take the off-diagonal spin basis of the SM, the existence of nonzero cross sections for diagonal spin states, $t_{\uparrow}\bar{t}_{\uparrow}$ and $t_{\downarrow}\bar{t}_{\downarrow}$ can be a direct evidence of new physics. As a matter of fact, only one spin configuration is appreciable for top quark pair in this basis and violation of such a pure dominance of a peculiar spin state is a signature of $Z'$ gauge boson, which is almost free from loop corrections. Alternatively, $A_{FB}$ is an effectual observable to probe $Z'$ boson due to the asymmetry of left- and right-handed couplings of $Z'$ boson to fermions. Meanwhile it is shown that the kinetic mixing effects results in substantial shift on the observables discussed here. Moreover changing the sign of kinetic mixing term, its effects can be additive or subtractive to the mass mixing effects, by which it is possible that the $Z'$ effects is wiped out. As a consequence, we conclude that combined analysis with more than one observable is indispensable to study the structure of $Z'$ gauge boson.

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REFERENCES

[1] see J. Hewett and T. Rizzo, Phys. Rep. 183, 193 (1989); A. Leike, Phys. Rep. 317, 143 (1999) and references therein.

[2] M. Cvetic and P. Langacker, hep-ph/9707451 in Perspectives in Supersymmetry, edited by G.L. Kane, (World Scientific, Singapore, 1998) p. 312; P. Langacker, hep-ph/9805486.

[3] F. Abe et al., CDF Collaboration, Phys. Rev. Lett. 79, 2192 (1997).

[4] J. Erler and P. Langacker, Phys. Lett. B 456, 68 (1999).

[5] J. Chay, K.Y. Lee and S.-h. Nam, Phys. Rev. D 61, 035002 (2000).

[6] J. Erler and P. Langacker, Eur. Phys. J. C 3, 90 (1998); P. Langacker and M. Luo, Phys. Rev. D 45, 278 (1992); P. Langacker, M. Luo and A. Mann, Rev. Mod. Phys., 64, 87 (1992).

[7] G.-C. Cho, K. Hagiwara and Y. Umeda, Nucl. Phys. B531, 65 (1998); Y. Umeda, G.-C. Cho and K. Hagiwara, Phys. Rev. D 58, 115008 (1998); G. Altarelli, et al., Phys. Lett. B 263, 459 (1991); Mod. Phys. Lett. A 5, 495 (1990).

[8] J. Erler and P. Langacker, Phys. Rev. Lett. 84, 212 (2000); G.-C. Cho, Mod. Phys. Lett. A 15, 311 (2000).

[9] K. S. Babu, C. Kolda and J. March-Russell, Phys. Rev. D 57, 6788 (1998); ibid D 54, 4635 (1996).

[10] G. Mahlon and S. Parke, Phys. Rev. D 53, 4886 (1996); T. Arens and L.M. Sehgal, Phys. Rev. D 50, 4372 (1994); Nucl. Phys. B393, 46 (1993).

[11] S. Parke and Y. Shadmi, Phys. Lett. B 387, 199 (1996).

[12] K.Y. Lee, S.C. Park, H.S. Song, J. Song, C. Yu, Phys. Rev. D 61, 074005 (2000); K.Y. Lee, H.S. Song, J. Song, C. Yu, Phys. Rev. D 60, 093002 (1999).
[13] M.E. Peskin and T. Takeuchi, Phys. Rev. Lett. 65, 964 (1990).

[14] T.G. Rizzo, Phys. Rev. D 59, 015020 (1999).

[15] S. Parke, hep-ph/9807573.
Figure Captions

Fig. 1 : Feynman diagrams for the $e^-e^+ \rightarrow t\bar{t}$ process (a) in the standard model, (b) with the $Z'$ boson.

Fig. 2 : Definition of generic top spin basis (a) in the top quark rest frame, (b) in the anti-top quark rest frame.

Fig. 3 : The differential cross sections in the $\psi$-model for the spin configuration of the top quark pair, drawn with respect to the scattering angle of the top quark at $\sqrt{s} = 500$ GeV with the left-handed electron beam. The solid line denotes the SM prediction; the dashed line includes the $Z'$ boson effects with the kinetic mixing $\tan\chi = 0$, the dotted line with $\tan\chi = 0.2$, the dash-dotted line with $\tan\chi = -0.2$.

Fig. 4 : The differential cross sections in the $\chi$-model for the spin configuration of the top quark pair, drawn with respect to the scattering angle of the top quark at $\sqrt{s} = 500$ GeV with the left-handed electron beam. The solid line denotes the SM prediction; the dashed line includes the $Z'$ boson effects with the kinetic mixing $\tan\chi = 0$, the dotted line with $\tan\chi = 0.2$, the dash-dotted line with $\tan\chi = -0.2$.

Fig. 5 : The differential cross sections in the $\eta$-model for the spin configuration of the top quark pair, drawn with respect to the scattering angle of the top quark at $\sqrt{s} = 500$ GeV with the left-handed electron beam. The solid line denotes the SM prediction; the dashed line includes the $Z'$ boson effects with the kinetic mixing $\tan\chi = 0$, the dotted line with $\tan\chi = 0.2$, the dash-dotted line with $\tan\chi = -0.2$. 
FIG. 1.
FIG. 2.
FIG. 3.

\( \psi \)-model

\[ \frac{d\sigma(\ell^+\ell^-)}{d\cos \delta} \text{ (pb/GeV)} \]

\( \cos \delta \)

\( \tan \chi = 0.2 \)
\( \tan \chi = 0 \)
\( \tan \chi = -0.2 \)
\(\chi\)-model

\[\text{FIG. 4.}\]
FIG. 5.