Transverse Λ polarization in inclusive processes

M. Anselmino¹, D. Boer², U. D’Alesio³, F. Murgia³

¹ Dipartimento di Fisica Teorica, Università di Torino and INFN, Sezione di Torino, Via P. Giuria 1, I-10125 Torino, Italy
² Dept. of Physics and Astronomy, Vrije Universiteit Amsterdam, De Boelelaan 1081, 1081 HV Amsterdam, The Netherlands
³ INFN, Sezione di Cagliari and Dipartimento di Fisica, Università di Cagliari, C.P. 170, I-09042 Monserrato (CA), Italy

Abstract

A formalism proposed to study transverse Λ polarization in unpolarized hadronic processes, based on a generalized pQCD factorization theorem, is extended to semi-inclusive DIS. Analytical expressions and examples of numerical estimates are given.

1. Introduction

Transverse hyperon polarization in high energy, unpolarized hadron-hadron collisions has formed a long-standing challenge for theoretical models of hadronic reactions. The straightforward application of perturbative QCD and collinear factorization in the study of these observables is not successful, giving much too small values compared to the observed data, which may reach well above 20%.

Therefore, we have recently proposed a new approach to this problem based on perturbative QCD and its factorization theorems, and which includes spin and intrinsic transverse momentum, \( k_\perp \), effects. It requires the introduction of a new type of leading twist fragmentation function (FF), which is polarization and \( k_\perp \)-dependent, the so-called polarizing FF. Ideally, our approach could be tested by first extracting these new functions in fitting some available experimental data, and then using the same functions to give consistent predictions for other processes. The problem with such a procedure is the actual availability of “good” experimental data, that is in kinematical regions appropriate to the application of our scheme.

A similar approach, based on new polarization and \( k_\perp \)-dependent distribution and/or fragmentation functions, has already been applied to the study of transverse single spin asymmetries in inclusive pion production, \( p^+p \rightarrow \pi X \), at large \( x_F \) and medium to large \( p_T \).

An alternative, although somewhat related, approach, based on perturbative QCD and its factorization theorems with the inclusion of higher twist functions, has been investigated by Qiu and Sterman and others.

* Talk delivered by U. D’Alesio at the 3rd Circum-Pan-Pacific Symposium on “High Energy Spin Physics”, Beijing, China, October 8-13, 2001.
The main idea behind the polarizing FF is that a transversely polarized hyperon can result from the fragmentation of an unpolarized quark, as long as the hyperon has a nonzero transverse momentum compared to the quark direction (otherwise it would violate rotational invariance). This effect need not be suppressed by inverse powers of a large energy scale, such as the center of mass energy $\sqrt{s}$ in $p$-$p$ scattering or the momentum transfer $Q$ in SIDIS.

In our approach for the $pp \rightarrow \Lambda \uparrow X$ case, the transverse hyperon polarization in unpolarized hadronic reactions at large $p_t$ can be written as follows [1]

$$P_T^\Lambda(x_F, p_T) = \frac{d\sigma^{pp \rightarrow \Lambda \uparrow X} - d\sigma^{pp \rightarrow \Lambda \downarrow X}}{d\sigma^{pp \rightarrow \Lambda \downarrow X} + d\sigma^{pp \rightarrow \Lambda \uparrow X}}$$

$$= \frac{\sum \int dx_a dx_b [d^2k_\perp f_{a/p}(x_a) f_{b/p}(x_b) d\hat{\sigma}(x_a, x_b; k_\perp) \Delta^N D_{\Lambda/c}(z, k_\perp)]}{\sum \int dx_a dx_b [d^2k_\perp f_{a/p}(x_a) f_{b/p}(x_b) d\hat{\sigma}(x_a, x_b; k_\perp) \hat{D}_{\Lambda/c}(z, k_\perp)},$$

where $d\sigma^{pp \rightarrow \Lambda X}$ stands for $E_{\Lambda} d\sigma^{pp \rightarrow \Lambda X} / d^2p_{\Lambda}$; $f_{a/p}(x_a)$ and $f_{b/p}(x_b)$ are the usual unpolarized parton densities; $d\hat{\sigma}(x_a, x_b; k_\perp)$ stands for $d\hat{\sigma}_{ab \rightarrow ccd} / dt$ and is the lowest order partonic cross-section with the inclusion of $k_\perp$ effects; the $\sum$ takes into account all possible elementary interactions; $\hat{D}_{\Lambda/c}(z, k_\perp)$ and $\Delta^N D_{\Lambda/c}(z, k_\perp)$ are respectively the unpolarized and the polarizing FF [2] for the process $c \rightarrow \Lambda + X$. The polarizing FF is defined as:

$$\Delta^N D_{h^+/a}(z, k_\perp) \equiv \hat{D}_{h^+/a}(z, k_\perp) - \hat{D}_{h^-/a}(z, k_\perp)$$

$$= \hat{D}_{h^+/a}(z, k_\perp) - \hat{D}_{h^+/a}(z, -k_\perp),$$

and denotes the difference between the density numbers $\hat{D}_{h^+/a}(z, k_\perp)$ and $\hat{D}_{h^-/a}(z, k_\perp)$ of spin 1/2 hadrons $h$, with longitudinal momentum fraction $z$, transverse momentum $k_\perp$ and transverse polarization $\uparrow$ or $\downarrow$, inside a jet originated from the fragmentation of an unpolarized parton $a$. From the above definition it is clear that the $k_\perp$ integral of the function vanishes and that, due to parity invariance, the function itself vanishes in case the transverse momentum and transverse spin are parallel. Conversely for a hadron transversely polarized along $\hat{P}_h$, one can write

$$\hat{D}_{h^+/q}(z, k_\perp) = \frac{1}{2} \hat{D}_{h^+/q}(z, k_\perp) + \frac{1}{2} \Delta^N D_{h^+/q}(z, k_\perp) \frac{\hat{P}_h \cdot (p_q \times k_\perp)}{|p_q \times k_\perp|}$$

where $\hat{D}_{h^+/q}(z, k_\perp) = \hat{D}_{h^+/q}(z, k_\perp) + \hat{D}_{h^-/q}(z, k_\perp)$ is the $k_\perp$-dependent unpolarized fragmentation function and $k_\perp = |k_\perp|$. We will adopt also the following notations:

$$\Delta^N D_{h^+/q}(z, k_\perp) \equiv \Delta^N D_{h^+/q}(z, k_\perp) \frac{\hat{P}_h \cdot (p_q \times k_\perp)}{|p_q \times k_\perp|} = \Delta^N D_{h^+/q}(z, k_\perp) \sin \phi,$$

where $\phi$ is the angle between $k_\perp$ and $P_h$, which, in our configuration (as explained later), is the difference between the azimuthal angles of $P_h$ and $k_\perp$, $\phi = \phi_{P_h} - \phi_{k_\perp}$.

Eq. (1) is based on some simplifying assumptions (for a detailed discussion we refer to [1]): 1) The $\Lambda$ polarization is assumed to be generated in the fragmentation process; 2) We neglect the intrinsic $k_\perp$ effects in the unpolarized initial nucleons; 3) The $\Lambda$ FF’s also include $\Lambda$’s coming from decays of other hyperon resonances. Only valence quarks in the polarized fragmentation process are considered.
Eq. (1) was used in Ref. [1], together with a very simple parameterization of the polarizing FF, to fit the available existing data on $P_{T}^{Λ}$. We consider now the same problem in SIDIS processes, $ℓ p → ℓ' Λ↑ X$.

2. A Gaussian model for $k_⊥$-dependent fragmentation functions

We take into account only leading twist and leading order contributions, looking at the process in the virtual boson-target nucleon c.m. reference frame (VN frame). Under these conditions the elementary virtual boson-quark scattering is collinear (and along the direction of motion of the virtual boson) and the transverse momentum of the final hadron with respect to the fragmenting quark, $k_⊥$, coincides with the hadron transverse momentum, $p_T$, as measured in the VN frame.

Due to the different kinematical scales involved in SIDIS [3] we consider here a more realistic form of the polarizing FF than in Ref. [1], using for the unpolarized and polarizing FF the following general form:

$$\hat{D}_{Λ/q}(z, k_⊥) = \hat{D}_{Λ/q}(z, k_⊥) = \frac{d(z)}{M^2} \exp \left[ -\frac{k_⊥^2}{M^2 f(z)} \right], \quad (5)$$

$$\Delta^N D_{Λ/q}(z, k_⊥) = \frac{\delta(z)}{M^2} \frac{k_⊥}{M} \exp \left[ -\frac{k_⊥^2}{M^2 \varphi(z)} \right], \quad (6)$$

where $M = 1 \text{ GeV}/c$ is a typical hadronic momentum scale and $f(z), \varphi(z)$ are generic functions of $z$, which we choose to be of the form $Nz^a(1-z)^b$.

Eqs. (5) and (6) must satisfy the positivity bound

$$\frac{|\Delta^N D_{Λ/q}(z, k_⊥)|}{\hat{D}_{Λ/q}(z, k_⊥)} = \frac{|\delta(z)|}{d(z)} \frac{k_⊥}{M} \exp \left[ -\frac{k_⊥^2}{M^2 \left( \frac{1}{\varphi} - \frac{1}{f} \right)} \right] \leq 1, \quad (7)$$

which, with $\varphi(z) = rf(z)$, implies $r < 1$ and

$$\frac{|\delta(z)|}{d(z)} \leq \left[ \frac{2 e(1-r)}{r} \right]^{1/2}. \quad (8)$$

The functions $d(z), f(z)$ in Eq. (5) are simply related to the usual, unpolarized and $k_⊥$-integrated FF and to the hadron mean squared transverse momentum inside the observed fragmentation jet, $\langle k_⊥^2(z) \rangle$, giving for the unpolarized FF:

$$\hat{D}_{Λ/q}(z, k_⊥) = \frac{D_{Λ/q}(z)}{\pi \langle k_⊥^2(z) \rangle} \exp \left[ -\frac{k_⊥^2}{M^2 \langle k_⊥^2(z) \rangle} \right]. \quad (9)$$

A comparison between Eqs. (5) and (6) yields the expressions of $d(z)$ and $f(z)$ in terms of $D(z)$ and $\langle k_⊥^2(z) \rangle$.

Some experimental information on $\langle k_⊥^2(z) \rangle$ is available for pions but not yet for $Λ$ particles. To obey Eq.(8) in a most natural and simple way we can write

$$\frac{\delta(z)}{M^3} = \left[ N_q \frac{z^α(1-z)^β}{\alpha^α \beta^β / (\alpha + \beta)^{(\alpha+\beta)}} \right] \frac{D_{Λ/q}(z)}{\pi \langle k_⊥^2(z) \rangle^{3/2}} \left[ 2e(1-r)/r \right]^{1/2} \quad (10)$$
with $\alpha, \beta > 0$, $|N_q| \leq 1$.

In this approach $\delta(z)$ is an unknown function depending on the parameters $\alpha, \beta, N_q$ and $r$, while $\varphi(z)$ is fixed by $r$, if we assume to know the functions $\langle k^2_{\perp}(z) \rangle$ and $D_{\Lambda/q}$.

If one were to demand consistency with the results of Ref. [1] $\delta(z)$ and $\varphi(z)$ could be fixed, leading us to give predictions for $P_{\Lambda}^T$ in SIDIS. However, this would be an optimistic procedure, which could be viewed only as a consistent example of using information from one set of data, in order to give predictions for other processes; it should be based on the assumption that all $p-p$ data originate from kinematical regions where pQCD and factorization scheme hold, which is doubtful for the presently available data.

In fact, recently it has become apparent that the formalism employed in our $p-p$ paper results in at most a few percents of the unpolarized cross-section, at least in the kinematical regions where both polarization and cross-section data are available (which is only a subset of the total region where data on $P_{\Lambda}^T$ have been published and used). This casts doubts on the obtained polarizing FF and therefore these should not be used to make predictions. A full assessment will be published elsewhere.

We also notice that a similar situation holds for pion or photon productions, in that the pQCD calculations, even in the central rapidity region and at large $p_T$ values, can be up to a factor 100 smaller than data [1, 2, 3, 4]; in those cases the discrepancy is explained by the introduction of $k_{\perp}$ effects in the distribution functions: these give a large, spin independent, enhancing factor, which brings the cross-sections in agreement with data. Such factors would not alter the calculation of the $\Lambda$ polarization in our approach, where spin effects are present only in the fragmentation process, as they cancel in the ratio of cross-sections of Eq. (1). However, it is too early to draw a definite conclusion, and a more detailed study is in progress.

Here we only present the analytical formalism, which is meaningful and valid in the appropriate regions, and show some numerical results based on simple guesses for $N_q, \alpha, \beta, r$ on the basis of a qualitative analysis of hadronic data and adopting the expression of $\langle k^2_{\perp}(z) \rangle$ valid for pions at LEP energies.

3. Analytical results

We consider transverse $\Lambda, \bar{\Lambda}$ polarization in semi-inclusive DIS, both for neutral and charged current interactions (but neglecting electro-weak interference effects). We give the expression of the polarization as a function of the usual SIDIS variables, $x, y, z_h$, in the current fragmentation region ($x_F > 0$).

As already discussed, we present our results in the virtual boson-proton c.m. reference frame (VN frame). More specifically, we take the $\hat{z}$-axis of our frame along the direction of motion of the exchanged virtual boson; the $\hat{x}$-axis is chosen along the transverse momentum $p_T = k_{\perp}$ of the observed hadron with respect to the $\hat{z}$-axis. In this configuration, the transverse polarization is measured along the +$\hat{y}$-axis, and the angle $\phi$ is fixed at $\phi = \pi/2$.

The transverse $\Lambda$ polarization is given by

$$P_{\Lambda}^T(x, y, z_h, p_T) = \frac{d\sigma^{\Lambda^+} - d\sigma^{\Lambda^-}}{d\sigma^{\Lambda^+} + d\sigma^{\Lambda^-}}. \quad (11)$$

---

1This procedure has been adopted in Ref. [3], which is presently under revision; we thank A. Efremov, J. Soffer and W. Vogelsang for emphasizing this point to us.
with
\[ d\sigma^{\Lambda^\uparrow(i)} = \frac{d\sigma^{\ell^+ p \rightarrow \ell^+ \Lambda^\uparrow X}}{dxdydz_hdp_T} = \sum_{i,j} f_{q_i/p}(x) \frac{d\hat{\sigma}_{\ell q_i \rightarrow \ell q_j}}{dy} \hat{D}_{\Lambda^\uparrow(i)/q_j}(z_h,p_T), \]

where \( i,j \) indicate different quark flavors and the sum includes both quark and antiquark contributions.

We consider separately different processes.

\textbf{a) } \ell^+ p \rightarrow \ell^+ \Lambda^\uparrow X

This case is of interest for several experiments, \textit{e.g.} HERMES, H1 and ZEUS at DESY, COMPASS at CERN, E665 at SLAC. One gets
\[ P^\Lambda_T(x, y, z_h, p_T) = \frac{\sum_q e_q^2 f_{q/p}(x) \left( d\hat{\sigma}^{\ell q}/dy \right) \Delta^N D_{\Lambda^\uparrow/q}(z_h, p_T)}{\sum_q e_q^2 f_{q/p}(x) \left( d\hat{\sigma}^{\ell q}/dy \right) \hat{D}_{\Lambda^\uparrow/q}(z_h, p_T)} \approx \frac{(4u + d) \Delta^N D_{\Lambda^\uparrow/u} + s \Delta^N D_{\Lambda^\uparrow/s}}{(4u + d) D_{\Lambda/u} + s D_{\Lambda/s}}, \]

where we have switched to the notation \( f_{q/p}(x) \rightarrow q(x) \); the second line is obtained by neglecting terms containing non-leading quark contributions both in the partonic distribution and fragmentation functions.

A similar expression holds also for the \( \ell^+ p \rightarrow \ell^+ \bar{\Lambda}^\uparrow X \) processes, with the exchange \( q(x) \leftrightarrow \bar{q}(x) \).

\textbf{b) } \nu p \rightarrow \nu \Lambda^\uparrow X

This process is of interest for the planned neutrino factories, and is currently under investigation by the NOMAD Collaboration at CERN. We get:
\[ P^\Lambda_T(x, y, z_h, p_T) \approx \frac{(Ku + d) \Delta^N D_{\Lambda^\uparrow/u} + s \Delta^N D_{\Lambda^\uparrow/s}}{(Ku + d) D_{\Lambda/u} + s D_{\Lambda/s}}, \]

where \( K = (1 - 8C)/(1 - 4C) \simeq 0.55 \) (with \( C = \sin^2 \theta_w/3 \simeq 0.077 \); terms quadratic in \( C \) and non-leading quark contributions have been neglected.

An analogous expression holds for \( \bar{\nu} p \rightarrow \bar{\nu} \Lambda^\uparrow X \) processes, with a factor \( K \) ranging now between 0.78 and 4 for \( y = 0 \) and \( y = 1 \) respectively.

\textbf{c) } \nu p \rightarrow \ell^- \Lambda^\uparrow X

This case is again of interest for neutrino factories and for the NOMAD experiment, which recently have published results for \( \Lambda \) and \( \bar{\Lambda} \) polarization \cite{10, 11}. One finds
\[ P^\Lambda_T(x, y, z_h, p_T) = \frac{(d + Rs) \Delta^N D_{\Lambda^\uparrow/u} + \bar{u} (\Delta^N D_{\Lambda^\uparrow/d} + R \Delta^N D_{\Lambda^\uparrow/s}) (1 - y)^2}{(d + Rs) \bar{D}_{\Lambda/u} + \bar{u} (\bar{D}_{\Lambda/d} + R \bar{D}_{\Lambda/s}) (1 - y)^2}, \]

where \( R = \tan^2 \theta_C \simeq 0.056 \). Neglecting sea-quark contributions leads to
\[ P^\Lambda_T(x, y, z_h, p_T) \simeq \frac{\Delta^N D_{\Lambda^\uparrow/u}}{D_{\Lambda/u}}. \]

The same expression is true for the case \( \ell^+ p \rightarrow \bar{\nu} \Lambda^\uparrow X \).
d) $\bar{\nu} p \rightarrow \ell^+ \Lambda^+ X$

This case is very close to the previous one, with obvious modifications:

$$P_T^\Lambda(x, y, z_h, p_T) = \frac{(1 - y)^2 u(\Delta^N D_{\Lambda^+/d} + R \Delta^N D_{\Lambda^+/s}) + (\bar{d} + R \bar{s}) \Delta^N D_{\Lambda^+/\bar{u}}}{(1 - y)^2 u(D_{\Lambda^+/d} + R D_{\Lambda^+/s}) + (d + R s) D_{\Lambda^+/\bar{u}}}.$$  \hspace{1cm} (17)

Again, when sea quark contributions are neglected and isospin symmetry is invoked, we find the very simple expression

$$P_T^\Lambda(x, y, z_h, p_T) \simeq \Delta^N D_{\Lambda^+/u} + R \Delta^N D_{\Lambda^+/s}.$$ \hspace{1cm} (18)

The same expression holds for the $\ell^- p \rightarrow \nu \Lambda^+ X$ case.

The above results, Eqs. (13)-(18), relate measurable polarizations to different combinations of (known) distribution functions, (less known) unpolarized and (unknown) polarizing FF; the different terms have relative coefficients which depend on the dynamics of the elementary partonic process and/or on the relevance of $s$ quark contributions in the partonic distribution functions.

This large variety of possibilities gives a good opportunity to investigate and test the relevant properties of the unpolarized and polarizing $\Lambda$ FF, by measuring the hyperon transverse polarization. In some special cases, Eq. (16), experiment offers direct information on these new functions.

Analogous results hold for the production of $\bar{\Lambda}$.

4. Examples of numerical estimates

Let us consider the parameters $\alpha, \beta, N_q, r$ appearing in Eqs. (8) and (11), and their possible values. Looking at the data on transverse $\Lambda$ polarization in hadronic reactions we expect: negative contributions for up and down quarks ($N_{u,d} < 0$) and positive for strange quarks ($N_s > 0$), in order to have $P^\Lambda < 0$ and $P^{\bar{\Lambda}} \simeq 0$; a polarizing FF peaked at large $z$ to explain the increasing in magnitude of the polarization with $x_F$ at fixed $p_T$ (thus implying large values of $\alpha$, while $\beta \simeq O(1)$); a Gaussian shape similar for unpolarized and polarizing FF to explain the large values of the polarization (which means $r \simeq O(1)$).

We then fix $\beta = 1$, $r = 0.7$; $\sqrt{\langle k_T^2(z) \rangle} = 0.61 z^{0.27}(1 - z)^{0.2}$ GeV/c (as used in Ref. [4]). We adopt for the unpolarized, $k_T$-integrated $\Lambda$ FF, the $SU(3)$ symmetric parameterizations of Ref. [12], and consider for the polarizing FF: 1) a scenario with almost the same weight for up and strange quarks, with $N_u = -0.8$ and $N_s = 1 (\alpha \simeq 6)$ and 2) a scenario similar to the model of Burkardt and Jaffe [13] for the longitudinally polarized FF $\Delta D_{\Lambda/q}$, with $N_u = -0.3$ and $N_s = 1 (\alpha \simeq 4)$.

We consider kinematical configurations typical of running experiments (HERMES at DESY, NOMAD at CERN, E665 at SLAC).

Since the $Q^2$ evolution of the polarizing FF is not under control at present, and the HERMES and NOMAD experiments involve a relatively limited range of $Q^2$ values, in our numerical calculations we have chosen a fixed scale, $Q^2 = 2$ (GeV/c)$^2$. For the unpolarized distribution functions we have adopted the MRST99 set [4].

\hspace{1cm} $^2$Isospin symmetry is assumed to hold, that is we take $(\Delta^N)D_{\Lambda^+/d} = (\Delta^N)D_{\Lambda^+/u}$. 

6
Our results are shown in Figs. 1-2. Fig. 1 shows $P_\Lambda^T$ as a function of $z_h$, after $p_T$ average, for all the cases a)-d) considered above and for kinematical configurations typical of the corresponding relevant experiments (as indicated in the legends); the two plots (left and right) differ by the scenario of the polarizing FF.

The polarization is in general large in magnitude and negative: contributions from $\Delta N_D/\Lambda^\uparrow/s$ are always suppressed, either by the $s$ quark distribution [via factors like $s/(K u + d)$] or by the Standard Model factor $R$, see Eqs. (13)-(18). Thus, the strange quark contribution is suppressed, unless one uses a $SU(3)$ asymmetric FF set for which $|\Delta N_D/\Lambda^\uparrow/u,d| \gg |\Delta N_D/\Lambda^\uparrow/u,d|$. This is reflected by the fact that in Fig. 1 all processes have similar polarizations, approximately given by the $p_T$-averaged ratio $\Delta N_D/\Lambda^\uparrow/u,\bar{u}$. The polarization is smaller in the right plot (scenario 2) simply because $|\Delta N_D/\Lambda^\uparrow/u|$ is smaller.

Fig. 2 shows the corresponding results for the case of $\bar{\Lambda}$ SIDIS production; here the effect of cancellations between up (down) and strange contributions is more significant: notice, for instance, how $P_\Lambda^T(\nu p \to \nu \bar{\Lambda}^\uparrow X)$ is suppressed compared to the analogous process for $\Lambda$.

In conclusion, single spin effects, suppressed in leading twist collinear applications of pQCD factorization theorems, may instead reveal new interesting aspects of non-perturbative QCD. Here we have considered the long-standing problem of the transverse polarization of hyperons produced from unpolarized initial nucleons. Our project is based on the use of a QCD factorization scheme, generalized to include intrinsic $k_{\perp}$ in the fragmentation process: this allows to introduce new spin dependences in the fragmentation functions of unpolarized quarks.

These new functions, the polarizing FF, are supposed to describe universal features of the hadronization process, which is factorized in a similar way as the usual $k_{\perp}$-integrated FF. If correct, this idea should allow a consistent phenomenological description of hyperon polarization in different processes, provided data are in kinematical regions where a hard scattering approach can be relevant.
Figure 2: Transverse $\bar{\Lambda}$ polarization, $P_{\bar{\Lambda}}$, vs. $z_h$ and averaged over $p_T$, for several SIDIS production processes, with scenario 1 (on the left) and scenario 2 (on the right) for the polarizing FF.

Acknowledgements

M.A. and U.D. would like to thank the organizers for their kind invitation to such an extremely interesting Conference. U.D. and F.M. are grateful to COFINANZIAMENTO MURST-PRIN for partial support.

References

[1] M. Anselmino, D. Boer, U. D’Alesio and F. Murgia, *Phys. Rev.* **D63**, 054029 (2001).
[2] P.J. Mulders and R.D. Tangerman, *Nucl. Phys.* **B461**, 197 (1996); *Nucl. Phys.* **B484**, 538 (E) (1997).
[3] M. Anselmino, D. Boer, U. D’Alesio and F. Murgia, e-Print Archive: [hep-ph/0109186](https://arxiv.org/abs/hep-ph/0109186).
[4] M. Anselmino, M. Boglione and F. Murgia, *Phys. Lett.* **B362**, 164 (1995); *Phys. Rev.* **D60**, 054027 (1999).
[5] J. Qiu and G. Sterman, *Phys. Rev. Lett.* **67**, 2264 (1991); *Nucl. Phys.* **B378**, 52 (1992).
[6] E706 Collaboration, L. Apanasevich *et al., Phys. Rev. Lett.* **81**, 2642 (1998).
[7] C.-Y. Wong and H. Wang, *Phys. Rev.* **C58**, 376 (1998).
[8] X.-N. Wang, *Phys. Rev.* **C61**, 064910 (2000).
[9] Y. Zhang, G. Fai, G. Papp, G.G. Barnaföldi and P. Lévai, e-Print Archive: [hep-ph/0109233](https://arxiv.org/abs/hep-ph/0109233).
[10] NOMAD Collaboration, P. Astier *et al., Nucl. Phys.* **B588**, 3 (2000); *Nucl. Phys.* **B605**, 3 (2001).
[11] D.V. Naumov, for the NOMAD Collaboration, AIP Conference Proceedings **570**, 489 (2001), e-print Archive: [hep-ph/0101325](https://arxiv.org/abs/hep-ph/0101325).
[12] C. Boros, J.T. Londergan and A.W. Thomas, *Phys. Rev.* **D61**, 014007 (2000).
[13] M. Burkardt and R.L. Jaffe, *Phys. Rev. Lett.* **70**, 2537 (1993); *Eur. Phys. J.* **C21**, 501 (2001).
[14] A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne, *Eur. Phys. J.* **C14**, 133 (2000).