INCOMPRESSIBILITY OF ASYMMETRIC NUCLEAR MATTER

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Using an isospin- and momentum-dependent modified Gogny (MDI) interaction, the Skyrme-Hartree-Fock (SHF) approach, and a phenomenological modified Skyrme-like (MSL) model, we have studied the incompressibility $K_{\text{sat}}(\delta)$ of isospin asymmetric nuclear matter at its saturation density. Our results show that in the expansion of $K_{\text{sat}}(\delta)$ in powers of isospin asymmetry $\delta$, i.e., $K_{\text{sat}}(\delta) = K_0 + K_{\text{sat},2}\delta^2 + K_{\text{sat},4}\delta^4 + O(\delta^6)$, the magnitude of the 4th-order $K_{\text{sat},4}$ parameter is generally small. The 2nd-order $K_{\text{sat},2}$ parameter thus essentially characterizes the isospin dependence of the incompressibility of asymmetric nuclear matter at saturation density. Furthermore, the $K_{\text{sat},2}$ can be expressed as $K_{\text{sat},2} = K_{\text{sym}} - 6L - \frac{J_0}{K_0}L$ in terms of the slope parameter $L$ and the curvature parameter $K_{\text{sym}}$ of the symmetry energy and the third-order derivative parameter $J_0$ of the energy of symmetric nuclear matter at saturation density, and we find the higher order $J_0$ contribution to $K_{\text{sat},2}$ generally cannot be neglected. Also, we have found a linear correlation between $K_{\text{sym}}$ and $L$ as well as between $J_0/K_0$ and $K_0$. Using these correlations together with the empirical constraints on $K_0$ and $L$, the nuclear symmetry energy $E_{\text{sym}}(\rho_0)$ at normal nuclear density, and the nucleon effective mass, we have obtained an estimated value of $K_{\text{sat},2} = -370 \pm 120$ MeV for the 2nd-order parameter in the isospin asymmetry expansion of the incompressibility of asymmetric nuclear matter at its saturation density.

1. Introduction

With the establishment or construction of many radioactive beam facilities around the world, such as the Cooling Storage Ring (CSR) facility at HIRFL in China, the Radioactive Ion Beam (RIB) Factory at RIKEN in Japan, the FAIR/GSI in Germany, SPIRAL2/GANIL in France, and the Facility for Rare Isotope Beams...
(FRIB) in USA, it is possible in terrestrial laboratories to explore the equation of state (EOS) of isospin asymmetric nuclear matter under the extreme condition of large isospin asymmetry and thus to determine the density dependence of the nuclear symmetry energy. This knowledge is important for understanding not only the structure of radioactive nuclei, the reaction dynamics induced by rare isotopes, and the liquid-gas phase transition in asymmetric nuclear matter, but also many critical issues in astrophysics.

For symmetric nuclear matter with equal fractions of neutrons and protons, its EOS is relatively well-determined. In particular, the incompressibility of symmetric nuclear matter at its saturation density $\rho_0$ has been determined to be $240 \pm 20$ MeV from analyses of the nuclear giant monopole resonances (GMR) and its EOS at densities of $2\rho_0 < \rho < 5\rho_0$ has also been constrained by measurements of collective flows and subthreshold kaon production in relativistic nucleus–nucleus collisions. On the other hand, the EOS of asymmetric nuclear matter and the density dependence of the nuclear symmetry energy, is largely unknown except at $\rho_0$ where the nuclear symmetry energy has been determined to be around 30 MeV from the empirical liquid-drop mass formula. The values of the symmetry energy at other densities, particularly at supra-saturation densities, are poorly known.

Heavy-ion collisions, especially those induced by neutron-rich nuclei, provide a unique tool to investigate the EOS of asymmetric nuclear matter, especially the density dependence of the nuclear symmetry energy. During the last decade, significant progress has been made both experimentally and theoretically on constraining the behavior of the symmetry energy at subsaturation density using heavy-ion reactions (See Ref. for the most recent review). More recently, there has also been an attempt to extract the symmetry energy at supersaturation densities from the FOPI data on the $\pi^-/\pi^+$ ratio in central heavy-ion collisions at SIS/GSI. Information on the density dependence of the nuclear symmetry energy has also been obtained from the structure of finite nuclei and their excitations, such as the mass data, neutron skin in heavy nuclei, giant dipole resonances, pygmy dipole resonance, and so on. These studies have significantly improved our understanding of the EOS of isospin asymmetric nuclear matter.

The incompressibility of asymmetric nuclear matter at its saturation density is a basic quantity to characterize its EOS. In principle, this information can be extracted experimentally by measuring the GMR in neutron-rich nuclei. In the present talk, we report our recent work on the incompressibility of isospin asymmetric nuclear matter at saturation density.

2. Incompressibility of isospin asymmetric nuclear matter

The EOS of isospin asymmetric nuclear matter, given by its binding energy per nucleon, can be generally expressed as a power series in the isospin asymmetry $\delta = (\rho_n - \rho_p)/\rho$, where $\rho = \rho_n + \rho_p$ is the baryon density with $\rho_n$ and $\rho_p$ denoting the
neutron and proton densities, respectively. To the 4th-order in isospin asymmetry, it is written as

$$E(\rho, \delta) = E_0(\rho) + E_{\text{sym}}(\rho) \delta^2 + E_{\text{sym},4}(\rho) \delta^4 + O(\delta^6),$$  \hspace{1cm} (1)

where $E_0(\rho) = E(\rho, \delta = 0)$ is the binding energy per nucleon of symmetric nuclear matter, and

$$E_{\text{sym}}(\rho) = \frac{1}{2!} \frac{\partial^2 E(\rho, \delta)}{\partial \delta^2} \bigg|_{\delta = 0}, \quad E_{\text{sym},4}(\rho) = \frac{1}{4!} \frac{\partial^4 E(\rho, \delta)}{\partial \delta^4} \bigg|_{\delta = 0}. \hspace{1cm} (2)$$

In the above, $E_{\text{sym}}(\rho)$ is the so-called nuclear symmetry energy and $E_{\text{sym},4}(\rho)$ is the 4th-order nuclear symmetry energy.

Around the normal density $\rho_0$, the $E_0(\rho)$ can be expanded, e.g., up to 4th-order in density, as

$$E_0(\rho) = E_0(\rho_0) + L_0 \chi + \frac{K_0}{2!} \chi^2 + \frac{J_0}{3!} \chi^3 + \frac{I_0}{4!} \chi^4 + O(\chi^5),$$

where $\chi$ is a dimensionless variable characterizing the deviations of the density from the saturation density $\rho_0$ of symmetric nuclear matter, and it is conventionally defined as $\chi = \frac{\rho - \rho_0}{\rho_0}$. The $E_0(\rho_0)$ is the binding energy per nucleon in symmetric nuclear matter at the saturation density $\rho_0$ and the coefficients of other terms are defined by

$$L_0 = \frac{3 \rho_0}{\rho_0} \frac{dE_0(\rho)}{d\rho} \bigg|_{\rho = \rho_0}, \quad K_0 = \frac{9 \rho_0^2}{\rho_0} \frac{d^2 E_0(\rho)}{d\rho^2} \bigg|_{\rho = \rho_0}, \hspace{1cm} (3)$$

$$J_0 = \frac{27 \rho_0^3}{\rho_0^3} \frac{d^3 E_0(\rho)}{d\rho^3} \bigg|_{\rho = \rho_0}, \quad I_0 = \frac{81 \rho_0^4}{\rho_0^4} \frac{d^4 E_0(\rho)}{d\rho^4} \bigg|_{\rho = \rho_0}. \hspace{1cm} (4)$$

Obviously, we have $L_0 = 0$ according to the definition of the saturation density $\rho_0$ of symmetric nuclear matter. The coefficient $K_0$ is the so-called incompressibility coefficient of symmetric nuclear matter, and it characterizes the curvature of $E_0(\rho)$ at $\rho_0$. The coefficients $J_0$ and $I_0$ correspond, respectively, to 3rd-order and 4th-order incompressibility coefficients of symmetric nuclear matter.

Around $\rho_0$, the $E_{\text{sym}}(\rho)$ can be similarly expanded up to 4th-order in $\chi$ as

$$E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + L \chi + \frac{K_{\text{sym}}}{2!} \chi^2 + \frac{J_{\text{sym}}}{3!} \chi^3 + \frac{I_{\text{sym}}}{4!} \chi^4 + O(\chi^5),$$  \hspace{1cm} (5)

where $L$, $K_{\text{sym}}$, $J_{\text{sym}}$ and $I_{\text{sym}}$ are the slope parameter, curvature parameter, 3rd-order coefficient, and 4th-order coefficient of the nuclear symmetry energy at $\rho_0$.

We can also expand the 4th-order nuclear symmetry energy $E_{\text{sym},4}(\rho)$ around $\rho_0$ up to 4th-order in $\chi$ as

$$E_{\text{sym},4}(\rho) = E_{\text{sym},4}(\rho_0) + L_{\text{sym},4} \chi + \frac{K_{\text{sym},4}}{2} \chi^2 + \frac{J_{\text{sym},4}}{3!} \chi^3 + \frac{I_{\text{sym},4}}{4!} \chi^4 + O(\chi^5),$$  \hspace{1cm} (6)

with $L_{\text{sym},4}$, $K_{\text{sym},4}$, $J_{\text{sym},4}$ and $I_{\text{sym},4}$ being the slope parameter, curvature parameter, 3rd-order coefficient, and 4th-order coefficient of the 4th-order nuclear symmetry energy $E_{\text{sym},4}(\rho)$ at $\rho_0$. 
Conventionally, the incompressibility of asymmetric nuclear matter is defined at its saturation density \( \rho_{\text{sat}} \) where \( P(\rho, \delta) = 0 \), thus also called the isobaric incompressibility coefficient \( \kappa_{\text{sat}} \) and is given by

\[
\kappa_{\text{sat}}(\delta) = 9\rho_{\text{sat}}^2 \left. \frac{\partial^2 E(\rho, \delta)}{\partial \rho^2} \right|_{\rho = \rho_{\text{sat}}}. \tag{7}
\]

Its value depends on the isospin asymmetry \( \delta \). Up to 4th-order in \( \delta \), the isobaric incompressibility coefficient \( \kappa_{\text{sat}}(\delta) \) can be expressed as

\[
\kappa_{\text{sat}}(\delta) = K_0 + K_{\text{sat}, 2} \delta^2 + K_{\text{sat}, 4} \delta^4 + O(\delta^6) \tag{8}
\]

with

\[
K_{\text{sat}, 2} = K_{\text{sym}} - 6L - \frac{J_0}{K_0} L, \tag{9}
\]

\[
K_{\text{sat}, 4} = K_{\text{sym}, 4} - 6L_{\text{sym}, 4} - \frac{J_0 L_{\text{sym}, 4}}{K_0} + \frac{9L^2}{K_0} - \frac{J_{\text{sym}} L}{K_0} + \frac{I_0 L^2}{2K_0^2} + \frac{J_0 K_{\text{sym}} L}{K_0^2} + \frac{3J_0 L^2}{2K_0} - \frac{J_0^2 L^2}{2K_0^3}. \tag{10}
\]

It is interesting to see that with precision up to 4th-order in \( \delta \) for the isobaric incompressibility coefficient, we only need to know 8 coefficients \( K_0, J_0, I_0, L, K_{\text{sym}}, J_{\text{sym}}, L_{\text{sym}, 4}, K_{\text{sym}, 4} \) which are defined at the normal nuclear density \( \rho_0 \). Furthermore, the 4 coefficients \( K_0, J_0, L, \) and \( K_{\text{sym}} \) determine completely the isobaric incompressibility coefficient with precision up to 2nd-order in \( \delta \).

It is generally believed that one can extract information on \( K_{\text{sat}, 2} \) by measuring the GMR in neutron-rich nuclei. Usually, one can define a finite nucleus incompressibility \( K_A(N, Z) \) for a nucleus with \( N \) neutrons and \( Z \) protons \( (A = N + Z) \) by the GMR energy \( E_{\text{GMR}} \), i.e.,

\[
E_{\text{GMR}} = \sqrt{\frac{\hbar^2 K_A(N, Z)}{m \langle r^2 \rangle}}, \tag{11}
\]

where \( m \) is the nucleon mass and \( \langle r^2 \rangle \) is the mean square mass radius of the nucleus in the ground state. Similar to the semi-empirical mass formula, the finite nucleus incompressibility \( K_A(N, Z) \) can be expanded as

\[
K_A(N, Z) = K_0 + K_{\text{surf}} A^{-1/3} + K_{\text{curv}} A^{-2/3} + (K_\tau + K_{\text{sat}} A^{-1/3}) \left( \frac{N - Z}{A} \right)^2 + K_{\text{Coul}} \frac{Z^2}{A^{4/3}} + \cdots. \tag{12}
\]

Neglecting the \( K_{\text{curv}} \) term, the \( K_{\text{sat}} \) term and other higher-order terms in Eq. (12), one can express the finite nucleus incompressibility \( K_A(N, Z) \) as

\[
K_A(N, Z) = K_0 + K_{\text{surf}} A^{-1/3} + K_\tau \left( \frac{N - Z}{A} \right)^2 + K_{\text{Coul}} \frac{Z^2}{A^{4/3}}, \tag{13}
\]

where \( K_0, K_{\text{surf}}, K_\tau, \) and \( K_{\text{Coul}} \) represent the volume, surface, symmetry, and Coulomb terms, respectively. The \( K_\tau \) parameter is usually thought to be equivalent to the \( K_{\text{sat}, 2} \) parameter. However, we would like to stress here that the \( K_{\text{sat}, 2} \)
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The incompressibility parameter is a theoretically well-defined physical property of asymmetric nuclear matter as shown previously while the value of the $K_\tau$ parameter may depend on the details of the truncation scheme in Eq. (12). As shown in Ref. [47], $K_\tau$ may be related to the isospin-dependent part of the surface properties of finite nuclei, especially the surface symmetry energy. Therefore, cautions are needed to interpret the $K_\tau$ parameter as the $K_{\text{sat,2}}$ parameter.

3. Results and discussions

In the following, we show our results based on three theoretical models, namely, the modified finite-range Gogny effective interaction (MDI), the Skyrme-Hartree-Fock (SHF) approach, and a phenomenological modified Skyrme-like (MSL) model. For the details of these models, we refer readers to Ref. [49]. A very useful feature of these models is that analytical expressions for many interesting physical quantities in asymmetric nuclear matter at zero temperature can be obtained, and this makes the study of the properties of asymmetric nuclear matter more transparent.

3.1. Incompressibility of asymmetric nuclear matter at saturation density

![Diagram](image)

Fig. 1. (Color online) The incompressibility at saturation density $K_{\text{sat}}$ as a function of $\delta^2$ for the MDI interaction with $x = 1$, 0, and $-1$. Corresponding results up to $\delta^2$ and $\delta^4$ are also included for comparison. The insets display corresponding results at smaller isospin asymmetries with $\delta^2 \leq 0.1$. Taken from Ref. [49].

Shown in Fig. 1 is the incompressibility at saturation density $K_{\text{sat}}(\delta)$ as a function of $\delta^2$ for the MDI interaction with $x = 1$, 0, and $-1$. It is seen that $K_{\text{sat}}(\delta)$ generally decreases with increasing isospin asymmetry. This feature is consistent
with earlier calculations based on microscopic many-body approaches. Corresponding results at smaller isospin asymmetries with $\delta^2 \leq 0.1$ are given in the insets of Fig. 1, and it shows that the result from keeping up to the $\delta^2$ term approximates very well the exact $K_{\text{sat}}(\delta)$ and the contribution of $K_{\text{sat},4}$ term is not important as $K_{\text{sat}}(\delta)$ displays a good linear correlation with $\delta^2$. This feature implies that the absolute value of $K_{\text{sat},2}$ is much larger than that of $K_{\text{sat},4}$. Our results are also consistent with the very recent study based on the RMF model.

### 3.2. Correlations of $J_0$-$K_0$ and $L$-$K_{\text{sym}}$

As shown in Eq. (16), the $K_{\text{sat},2}$ parameter is completely determined by the 4 characteristic parameters $K_0$, $J_0$, $L$, and $K_{\text{sym}}$ at the normal nuclear density. While the $K_0$ parameter has been relatively well determined to be $240 \pm 20$ MeV from the nuclear GMR data, the $J_0$ parameter is poorly known, and there is actually no experimental information on the $J_0$ parameter.

![Figure 2](image)

Fig. 2. (Color online) Left panels: The $J_0$ and the ratio $J_0/K_0$ as functions of $K_0$ from the MSL model, the MDI interaction and the SHF prediction with the 63 Skyrme interactions. Right panels: Correlation between $K_{\text{sym}}$ and $L$ from the MSL model with $\gamma_{\text{sym}} = 4/3$ and $5/3$, the MDI interaction and the SHF prediction with the 63 Skyrme interactions. The results from two simple one-parameter symmetry energies are also shown for comparison. Taken from Ref. 49.

Shown in the left panels of Fig. 2 are $J_0$ and $J_0/K_0$ as functions of $K_0$ in the MSL model for different values of the nucleon effective mass $m^*_0 = m$, 0.9$m$, 0.8$m$ and 0.7$m$. Also included in the figure are corresponding results from the MDI interaction and the SHF prediction with 63 Skyrme interactions (see Ref. 49 for the details). One can see the $J_0/K_0$ value deviates significantly from zero for a reasonable $K_0$ value, which implies that the higher order $J_0$ contribution to $K_{\text{sat},2}$ generally cannot be neglected. It is further seen that the correlation between $J_0$ and $K_0$ is similar among these three different models. Also, all three models
show an approximately linear correlation between $J_0/K_0$ and $K_0$. We note that the correlation between $J_0$ and $K_0$ obtained in the present work is consistent with the early finding by Pearson. While there do not exist any empirical constraints on the $J_0$ parameter, we assume here the correlation between $J_0$ and $K_0$ from the MSL model is valid and then determine $J_0/K_0$ from the experimental constraint on $K_0$.

Shown in the right panels of Fig. 2 are the correlation between $K_{\text{sym}}$ and $L$ from the MSL model with $\gamma_{\text{sym}} = 4/3$ and $5/3$, the MDI interaction, the SHF prediction with the 63 Skyrme interactions, and two one-parameter parameterizations on the symmetry energies. It is very interesting to see that for larger $L$ values ($L \geq 45$ MeV which is consistent with the constraint from heavy-ion collision data), all above symmetry energy functionals from different models and parameterizations give consistent predictions for the $K_{\text{sym}}-L$ correlation. This nice feature implies that using these different models and parameterizations for the symmetry energy will not influence significantly the determination of the $K_{\text{sat},2}$ parameter. On the other hand, the $K_{\text{sym}}-L$ correlation from the two one-parameter parameterizations on the symmetry energies deviate significantly from the MDI, MSL and SHF predictions for small $L$ values. Actually, the two forms of one-parameter parametrization for the symmetry energy may be too simple to describe a softer symmetry energy. As shown in Ref. 33, the symmetry energy form $E_{\text{sym}}(\rho) \approx 31.6(\rho/\rho_0)^{\gamma}$ cannot describe correctly the density dependence of the symmetry energy from the MDI interaction with $x = 1$ (the Gogny interaction). Although there is no direct empirical information on the $K_{\text{sym}}$ parameter and some uncertainty on the $K_{\text{sym}}-L$ correlation still exist, we assume here that the correlation between $K_{\text{sym}}$ and $L$ from the MSL model with $\gamma_{\text{sym}} = 4/3$ and $5/3$ is valid and then use the experimental constraint on $L$ to extract the value of $K_{\text{sym}}$.

### 3.3. Constraining the $K_{\text{sat},2}$ parameter

From above analyses, we can now extract information on the $K_{\text{sat},2}$ parameter from the experimental constraints on the $K_0$ parameter and the $L$ parameter. Results on $K_{\text{sat},2}$ as a function of $L$ are shown in Fig. 3 for $\gamma_{\text{sym}} = 4/3$ (panel (a)) and $5/3$ (panel (b)) and with $K_0 = 220, 240, \text{ and } 260$ MeV. In the MSL model, the correlation between $K_{\text{sym}}$ and $L$ also depends on $E_{\text{sym}}(\rho_0)$. To take into consideration of the uncertainty in the value of $E_{\text{sym}}(\rho_0)$, we also include in Fig. 3 the results with $K_0 = 220$ MeV and $E_{\text{sym}}(\rho_0) = 25$ MeV as well as $K_0 = 260$ MeV and $E_{\text{sym}}(\rho_0) = 35$ MeV, which represent, respectively, the upper and lower bounds on $K_{\text{sat},2}$ for a fixed value of $L$. The shaded region in Fig. 3 further shows the constrained $L$ values from heavy-ion collision data, namely, $46 \text{ MeV} \leq L \leq 111 \text{ MeV}$. The lower limit of $L = 46$ MeV is obtained from the lower bound in the ImQMD analyses of the isospin diffusion data and the double neutron/proton ratio while the upper limit of $L = 111$ MeV corresponds to the upper bound of $L$ from the IBUU04 transport model analysis of the isospin diffusion data. We note that the constraint $46 \text{ MeV} \leq L \leq 111 \text{ MeV}$ obtained from heavy-ion...
collisions is consistent with the analyses of the pygmy dipole resonances \(^4\) the giant
dipole resonance (GDR) of \(^{208}\)Pb analyzed with Skyrme interactions \(^5\) the very
precise Thomas-Fermi model fit to the binding energies of 1654 nuclei \(^6\) and the
recent neutron-skin analysis \(^7\). These empirically extracted values of \(L\) represent
the best and most stringent phenomenological constraints available so far on the
nuclear symmetry energy at sub-saturation densities. It should be mentioned that
the proposed experiment of parity-violating electron scattering from \(^{208}\)Pb at the
Jefferson Laboratory is expected to give an independent and accurate measurement
of its neutron skin thickness (within 0.05 fm) \(^8\) and thus to impose a stringent
constraint on the slope parameter \(L\) in future.

From the shaded region indicated in Fig. \(\text{F}3\) we find that for \(\gamma_{\text{sym}} = 4/3\), we have
\(-437 \text{ MeV} \leq K_{\text{sat},2} \leq -292 \text{ MeV} \) for \(L = 46 \text{ MeV}\) and \(-487 \text{ MeV} \leq K_{\text{sat},2} \leq -306
\text{ MeV} \) for \(L = 111 \text{ MeV}\). These values are changed to \(-477 \text{ MeV} \leq K_{\text{sat},2} \leq -302
\text{ MeV} \) for \(L = 46 \text{ MeV}\) and \(-461 \text{ MeV} \leq K_{\text{sat},2} \leq -251 \text{ MeV} \) for \(L = 111 \text{ MeV}\) when
\(\gamma_{\text{sym}} = 5/3\) is used. Our results thus indicate that based on the MSL model with
\(4/3 \leq \gamma_{\text{sym}} \leq 5/3\), \(K_0 = 240 \pm 20 \text{ MeV}, 25 \text{ MeV} \leq E_{\text{sym}}(\rho_0) \leq 35 \text{ MeV}, \) and \(46
\text{ MeV} \leq L \leq 111 \text{ MeV}\), the \(K_{\text{sat},2}\) parameter can vary from \(-251 \text{ MeV} \) to \(-487 \text{ MeV} \).

The results shown in Fig. \(\text{F}3\) is obtained from a \(J_0/K_0\) value that is evaluated with
the default value \(m_{s,0}^* = 0.8m\) in the MSL model. Similar analyses indicate that

\[\text{Fig. 3. (Color online) } K_{\text{sat},2} \text{ as a function of } L \text{ from the MSL model with } \gamma_{\text{sym}} = 4/3 \text{ (a) and } 5/3 \text{ (b) and } m_{s,0}^* = 0.8m \text{ and } m_{s,0}^* = 0.7m \text{ for different values of } K_0 \text{ and } E_{\text{sym}}(\rho_0). \text{ The shaded region indicates the constraints within the MSL model with } K_0 = 240 \pm 20 \text{ MeV, } E_{\text{sym}}(\rho_0) = 30 \pm 5 \text{ MeV, and } 46 \text{ MeV} \leq L \leq 111 \text{ MeV from the heavy-ion collision data. The results from the SHF approach with 63 Skyrme interactions are also included for comparison. In addition, the constraint of } K_{\gamma} = -550 \pm 100 \text{ MeV obtained in Ref. } 22\text{ from measurements of the isotopic dependence of the GMR in even-A Sn isotopes is also indicated. Taken from Ref. } 19\text{.} \]
the $K_{\text{sat},2}$ parameter varies from $-261$ MeV to $-489$ MeV if we use $m_{s,0}^* = 0.7m$ while it varies from $-245$ MeV to $-485$ MeV if $m_{s,0}^* = 0.9m$ is used. Therefore, the extracted value for $K_{\text{sat},2}$ is not sensitive to the variation of the nucleon effective mass. The MSL model analyses with 4 lead to an estimate value of $E_{\text{sym}}(\rho_0) = 30 \pm 5$ MeV, $46$ MeV $\leq L \leq 111$ MeV, and $m_{s,0}^* = 0.8 \pm 0.1m$ thus lead to an estimate value of $K_{\text{sat},2} = -370 \pm 120$ MeV for the 2nd-order parameter in the isospin asymmetry expansion of the incompressibility of asymmetric nuclear matter at its saturation density.

4. Summary and conclusions

We have studied the incompressibility of an asymmetric nuclear matter at its saturation density. We find that in its power series expansion in isospin asymmetry the magnitude of the 4th-order $K_{\text{sat},4}$ parameter is generally small compared to that of the 2nd-order $K_{\text{sat},2}$ parameter, so the latter essentially characterizes the isospin dependence of the incompressibility of asymmetric nuclear matter at saturation density. Furthermore, the $K_{\text{sat},2}$ parameter can be uniquely determined by the slope $L$ and the curvature $K_{\text{sym}}$ of nuclear symmetry energy at normal nuclear matter density and the ratio $J_0/K_0$ of the 3rd-order and incompressibility coefficients of symmetric nuclear matter at saturation density, and we find the higher order $J_0$ contribution to $K_{\text{sat},2}$ generally cannot be neglected. Since there is no experimental information on the $J_0$ parameter and the $K_{\text{sym}}$ parameter, we have thus used the MSL model, which can reasonably describe the general properties of symmetric nuclear matter and the symmetry energy predicted by both the MDI model and the SHF approach, to estimate the value of $K_{\text{sat},2}$. Interestingly, we find that there exists a nicely linear correlation between $K_{\text{sym}}$ and $L$ as well as between $J_0/K_0$ and $K_0$ for the three different models used here, i.e., the MDI interaction, the MSL interaction, and the SHF approach with 63 Skyrme interactions. These correlations have allowed us to extract the values of the $J_0$ parameter and the $K_{\text{sym}}$ parameter from the empirical information on $K_0$, $L$ and $E_{\text{sym}}(\rho_0)$. In particular, using the empirical constraints of $K_0 = 240 \pm 20$ MeV, $E_{\text{sym}}(\rho_0) = 30 \pm 5$ MeV, $46$ MeV $\leq L \leq 111$ MeV and a nucleon effective mass $m_{s,0}^* = 0.8 \pm 0.1m$ in the MSL model leads to an estimate of $K_{\text{sat},2} = -370 \pm 120$ MeV.

While the estimated value of $K_{\text{sat},2} = -370 \pm 120$ MeV in the present work has small overlap with the constraint of $K_\gamma = -550\pm100$ MeV for the symmetry term in a semi-empirical mass formula-like expansion of the incompressibility of finite nuclei obtained in Refs. [22,23] from recent measurements of the isotopic dependence of the GMR in even-A Sn isotopes, its magnitude is significantly smaller than that of the constrained $K_\gamma$. Recently, there are several studies [57,58,59] on extracting the value of the $K_{\text{sat},2}$ parameter based on the idea initiated by Blaizot and collaborators that the values of both $K_0$ and $K_{\text{sat},2}$ should be extracted from the same consistent theoretical model that successfully reproduces the experimental GMR energies of a variety of nuclei. These studies show that no single model (interaction) can simul-
taneously describe the recent measurements of the isotopic dependence of the GMR in even-A Sn isotopes and the GMR data of $^{90}$Zr and $^{208}$Pb nuclei, and this makes it difficult to accurately determine the value of $K_{\text{sat,2}}$ from these data. Also, a very recent study 59 indicates that the effect due to the nuclear superfluidity may also affect the extraction of the $K_{\text{sat,2}}$ parameter from the nuclear GMR. As pointed out in Ref. 52, these features suggest that the $K_{\tau} = -550 \pm 100$ MeV obtained in Ref. 22 may suffer from the same ambiguities already encountered in earlier attempts to extract the $K_0$ and $K_{\text{sat,2}}$ of infinite nuclear matter from finite-nuclei extrapolations. This problem remains an open challenge, and both experimental and theoretical insights are needed in future studies.

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