Spin polarized conductance in ferromagnet / insulator / conventional superconductor junctions

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Abstract. A theory of differential conductance in ferromagnet / insulator / conventional superconductor (F/I/CS) junctions is presented and an experimental conductance is analyzed by the theory. With regard to line shapes of calculated differential conductance, as the magnitude of the exchange interaction in ferromagnets is increased, the differential conductance for the half-metallic state below the energy gap is always reduced in contrast to that for the normal metallic state. This is due to the fact that the retro-reflectivity of the Andreev reflection in the case of CS's is broken by the influence from the exchange interaction. The experimental conductance of ferromagnetic Ru₂₋ₓFeₓCrSi Heusler alloy / superconducting Pb junctions was in good agreement with calculational results of the theory for the F/I/CS junctions.

1. Introduction
Highly spin polarized ferromagnetic materials with high Curie temperature are experimentally required for spintronics device applications, such as tunneling magnetoresistance (TMR) and current-perpendicular-to-plane giant magnetoresistance (CPP-GMR) devices. With regard to theories for ferromagnet / superconductor (F/S) junctions, there are different theoretical models treating F/S contacts in various limits from the ballistic regime to diffusive regime [1–4]. The spin polarization measurements of ferromagnetic materials can be performed by the Andreev reflection technique [5]. In this method, the modified Blonder-Tinkham-Klapwijk (BTK) theories are currently applied to the spin polarization analysis so far [6, 7]. However, in these models, it is simply assumed that spin polarized current is an elementary sum of fully-polarized and non-polarized currents.

In this paper, we present a theory of ferromagnet / insulator / conventional superconductor (F/I/CS) junctions in order to analyze spin polarization of ferromagnet materials, where the effect of spin polarization is introduced as the exchange potential in the ferromagnet. Finally, we discuss analytical results for ferromagnetic Ru₂₋ₓFeₓCrSi Heusler alloy / superconducting Pb junctions.

2. Formulation
For the calculation, a two-dimensional F/I/CS junction with semi-infinite double-layered structures in the clean limit is assumed. A flat interface is perpendicular to the x-axis and
is located at $x = 0$. The insulator is modeled as a $\delta$-function form $H \delta(x)$, where $\delta(x)$ and $H$ are the $\delta$-function and its amplitude, respectively. The Fermi energy $E_F$ and the effective mass $m$ are assumed to be equal both in the ferromagnet and in the superconductor. As a model of the ferromagnetic metal we apply the Stoner model \[1\], using the exchange potential $U(x) = U \Theta(-x)$, where $\Theta(x)$ is the Heaviside step function. The magnitudes of Fermi momentum in the ferromagnet for up and down spins are denoted as $k_{F,\uparrow} = \sqrt{2m(E_F + U)}$ and $k_{F,\downarrow} = \sqrt{(2m/h^2)(E_F - U)}$, respectively. For simplicity, we assume the spatially constant pair potentials and neglect the effects of spin-orbit scattering. The wave functions $\Psi(x)$ are obtained by solving the Bogoliubov-de Gennes (BdG) equation according to the quasiclassical approximation \[8\].

There are four scattering processes for an electron injection from the ferromagnet with up spin and at angle $\theta_F$ with respect to the interface normal \[2\]. These are Andreev reflection (AR) as a hole, normal reflection (NR) as an electron, transmission as an electron-like quasiparticle (ELQ), and transmission as a hole-like quasiparticle (HLQ). The wave vectors of ELQ and HLQ are approximated by $k = |k| \approx \sqrt{2mE_F/h^2}$ in the framework of the quasiclassical approximation \[9\]. Since the translational symmetry holds for the $y$-axis direction, the momenta parallel to the interface are conserved at the interface, $k_{F,\uparrow} \sin \theta_F = k_{F,\downarrow} \sin \theta_F = k_S \sin \theta_S$. In the case of CS’s, the retro-reflectivity of the Andreev reflection is broken due to the difference in the exchange interaction felt by a HLQ. Consequently, $\theta_F$ is not equal to $\theta_F$.

The reflection probabilities of AR ($a_\uparrow$ and $a_\downarrow$) and NR ($b_\uparrow$ and $b_\downarrow$) are determined by solving the BdG equations with the wave function $\Psi(x)$ under the boundary conditions. The normalized differential conductance $\sigma_T(eV)$ in finite temperatures is expressed as

$$
\sigma_T(eV) = \frac{\int_0^\infty dE \int_{-\pi/2}^{\pi/2} d\theta S \cos \theta S (\sigma_{S,\uparrow} + \sigma_{S,\downarrow}) \text{sech}^2 \left( \frac{E - eV}{2k_B T} \right)}{\int_0^\infty dE \int_{-\pi/2}^{\pi/2} d\theta S \cos \theta S (\sigma_{N,\uparrow} + \sigma_{N,\downarrow}) \text{sech}^2 \left( \frac{E - eV}{2k_B T} \right)}, \tag{1}
$$

$$
\sigma_{N,\uparrow} = \frac{4\lambda_+}{(1 + \lambda_+)^2 + Z_{\theta S}^2}, \quad \sigma_{N,\downarrow} = \frac{4\lambda_-}{(1 + \lambda_-)^2 + Z_{\theta S}^2} \Theta(\theta C - |\theta S|), \tag{2}
$$

$$
\lambda_\pm = \sqrt{1 \pm \frac{U}{E_F \cos^2 \theta S}}, \quad Z_{\theta S} = \frac{Z}{\cos \theta S}, \quad Z = \frac{2mH}{h^2 k_F}. \tag{3}
$$

The quantities $\sigma_{S,\uparrow}$ and $\sigma_{S,\downarrow}$ are the differential conductance for the up and down spin electron injections in the superconducting state. In the F/I/CS junctions, we note that the Fermi surface effect largely influences the reflection process. The retro-reflectivity of AR is broken due to the influence of the exchange interaction in the ferromagnet. In the following, we will consider the situation where $k_{F,\downarrow} < k_S < k_{F,\uparrow}$ is satisfied \[10\]. For $|\theta_S| > \cos^{-1} \sqrt{U/E_F} \equiv \theta_C$, the Andreev reflection does not exist as a propagating wave. This intrinsic property in the F/I/CS junction is caused by the fact that the Andreev reflected hole has an antiparallel spin to that of the injection electron. Based on the calculation of $d$-wave superconductors \[2\], differential conductance $\sigma_{S,\uparrow}$ and $\sigma_{S,\downarrow}$ are given by

$$
\sigma_{S,\uparrow} = \sigma_{N,\uparrow} \frac{1 - |\Gamma|^2 (1 - \sigma_{N,\downarrow} + \sigma_{N,\downarrow} |\Gamma|^2)}{1 - |\Gamma|^2 \sqrt{1 - \sigma_{N,\downarrow} + \sigma_{N,\downarrow} |\Gamma|^2} \left(1 - \Theta(\theta C - |\theta S|) \Theta(\theta C - |\theta S|) \right)} \tag{4}
$$

$$
\sigma_{S,\downarrow} = \sigma_{N,\downarrow} \frac{1 - |\Gamma|^2 (1 - \sigma_{N,\uparrow} + \sigma_{N,\uparrow} |\Gamma|^2)}{1 - |\Gamma|^2 \sqrt{1 - \sigma_{N,\downarrow} + \sigma_{N,\downarrow} |\Gamma|^2} \left(1 - \Theta(\theta C - |\theta S|) \Theta(\theta C - |\theta S|) \right)} \tag{5}
$$

$$
\sigma_{N,\uparrow} = \sigma_{N,\downarrow} = \frac{4\lambda_+}{(1 + \lambda_+)^2 + Z_{\theta S}^2} \Theta(\theta C - |\theta S|), \tag{3}
$$

$$
\lambda_\pm = \sqrt{1 \pm \frac{U}{E_F \cos^2 \theta S}}, \quad Z_{\theta S} = \frac{Z}{\cos \theta S}, \quad Z = \frac{2mH}{h^2 k_F}. \tag{3}
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$$
\sigma_{S,\uparrow} = \sigma_{N,\uparrow} \frac{1 - |\Gamma|^2 (1 - \sigma_{N,\downarrow} + \sigma_{N,\downarrow} |\Gamma|^2)}{1 - |\Gamma|^2 \sqrt{1 - \sigma_{N,\downarrow} + \sigma_{N,\downarrow} |\Gamma|^2} \left(1 - \Theta(\theta C - |\theta S|) \Theta(\theta C - |\theta S|) \right)} \tag{4}
$$

$$
\sigma_{S,\downarrow} = \sigma_{N,\downarrow} \frac{1 - |\Gamma|^2 (1 - \sigma_{N,\uparrow} + \sigma_{N,\uparrow} |\Gamma|^2)}{1 - |\Gamma|^2 \sqrt{1 - \sigma_{N,\downarrow} + \sigma_{N,\downarrow} |\Gamma|^2} \left(1 - \Theta(\theta C - |\theta S|) \Theta(\theta C - |\theta S|) \right)} \tag{5}
$$

$$
\sigma_{N,\uparrow} = \sigma_{N,\downarrow} = \frac{4\lambda_+}{(1 + \lambda_+)^2 + Z_{\theta S}^2} \Theta(\theta C - |\theta S|), \tag{3}
$$

$$
\lambda_\pm = \sqrt{1 \pm \frac{U}{E_F \cos^2 \theta S}}, \quad Z_{\theta S} = \frac{Z}{\cos \theta S}, \quad Z = \frac{2mH}{h^2 k_F}. \tag{3}
$$
\[
\exp(i\varphi) = \frac{1 - \lambda_- + i\theta_s}{\sqrt{1 - \sigma_{N,\downarrow}(1 + \lambda_- - i\theta_s)}}, \quad \exp(-i\varphi_T) = \frac{1 - \lambda_- - i\theta_s}{\sqrt{1 - \sigma_{N,\uparrow}(1 + \lambda_- + i\theta_s)}},
\]
\[
\Gamma = \frac{\Delta_0}{E + \sqrt{E^2 - |\Delta_0|^2}}.
\]

In the above formulations, when the ferromagnet changes to a normal metal (i.e., \(U = 0\)), \(\sigma_T(eV)\) in the BTK formula [11] is completely reproduced. On the other hand, the magnitude of the Fermi momentum becomes zero for down spin electrons in the half-metallic ferromagnet limit (i.e., \(U = E_F\)), then \(\sigma_{N,\downarrow} = 0\). The wave function of the reflected hole by AR becomes evanescent wave, and hence does not contribute to the net current. In this case, since the numerator of the conductance formula in equation (1) vanishes at \(|E| < \Delta_0\), the \(\sigma_T(eV) = 0\) for \(|E| < \Delta_0\).

In particular for the case of \(U = E_F\), since the \(\Gamma^2 = 1\) is satisfied independent of \(\theta_s\), for \(|E| < \Delta_0\), the numerator of equation (4) is zero and the resulting conductance \(\sigma_{S,\uparrow}(eV) = 0\).

3. Results and discussion

We choose \(\Delta_0 = 1.0\) meV and \(T = 1.0\) K for calculations of differential conductance in F/I/CS junctions because using equations (1)–(7) we will analyze our experimental data, which was obtained under the similar experimental condition. Figure 1 shows the line shapes of \(\sigma_T(eV)\) in the F/I/CS junction with various \(X = U/E_F\) for high transparency of the potential barrier, \(Z = 0.0\). In this case, since the probability of the Andreev reflection is suppressed due to finite \(U\), the differential conductance inside the energy gap (\(|E| < \Delta_0\)) is drastically reduced with the increase of \(X\), similar to the case of \(d\)-wave superconductors [2]. Figure 2 represents the line shapes of \(\sigma_T(eV)\) for \(Z = 1.0\), corresponding to middle transparency of the potential barrier. In the lower \(X\), the dip around zero bias in the line shapes of \(\sigma_T(eV)\) indicates the reduction of Andreev reflection inside the energy gap. Figure 3 shows the line shapes of \(\sigma_T(eV)\) for relatively low transparency of the potential barrier, \(Z = 5.0\). Contrastively, the behavior of the line shapes of \(\sigma_T(eV)\) for \(Z = 5.0\) becomes quite different from high transparency cases and is less sensitive to \(X\), due to the absence of Andreev reflection inside the energy gap. As shown in figures 1–3, the results of the BTK limit [11] are reproduced for the line shapes of \(\sigma_T(eV)\) for \(X = 0.0\).

We have measured differential conductance of Ru\(_{2-x}\)Fe\(_x\)CrSi/Pb planar junctions by the Andreev reflection technique [5], where a superconducting Pb thin film was attached to an Fe-rich compound for Ru\(_{2-x}\)Fe\(_x\)CrSi Heusler alloys. The experimental details are given in elsewhere [12]. Based on the theory in section 2, we have analyzed spin polarized conductance

![Figure 1](image1.png)  
**Figure 1.** Normalized conductance spectra with various \(X\) for \(Z = 0.0\), \(\Delta_0 = 1.0\) meV and \(T = 1.0\) K.

![Figure 2](image2.png)  
**Figure 2.** Normalized conductance spectra with various \(X\) for \(Z = 1.0\), \(\Delta_0 = 1.0\) meV and \(T = 1.0\) K.
Figure 3. Normalized conductance spectra with various $X$ for $Z = 5.0$, $\Delta_0 = 1.0$ meV and $T = 1.0$ K.

Figure 4. Fitting result of the ferromagnetic Ru$_{0.5}$Fe$_{1.5}$CrSi Heusler alloy / superconducting Pb planar junction by the theory.

of Fe-rich Ru$_{2-x}$Fe$_x$CrSi Heusler alloys, which are ferromagnets and are candidates of half-metals [13]. Figure 4 represents an analytical result for the normalized differential conductance $\sigma_T(eV)$ of a Ru$_{0.5}$Fe$_{1.5}$CrSi/Pb planar junction at $T = 1.5$ K. The analysis was successful to obtain the good fitting result, as shown in figure 4. The resulting values of fitting parameters are also presented in figure 4. With regard to the energy gap, $\Delta_0 = 0.65$ meV is smaller than $\Delta_0 = 1.36$ meV in the previous reports [14]. The reduction in $\Delta_0$ is able to be comprehended by the proximity effect at the F/CS junction interface. We have obtained good agreement between the experimental data and theoretical calculation. Hence, we were able to verify that the theory is useful to analyze an experimentally differential conductance in actual F/I/CS junctions.

4. Conclusions

We have calculated line shapes of $\sigma_T(eV)$ in F/I/CS junctions. In the case of relatively high transparency, $\sigma_T(eV)$ inside the energy gap was reduced drastically as the magnitude of the exchange interaction was increased. This was due to the break down of the retro-reflectivity of the Andreev reflection. The experimental conductance of Ru$_{2-x}$Fe$_x$CrSi/Pb planar junctions was analyzed by the theory. We have obtained good agreement between the experimental data and theoretical calculation. Hence, we emphasize that the theory is useful to analyze an experimentally differential conductance in actual F/I/CS junctions.

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