E-SUPER EDGE MAGIC LABELING ON SOME CLASSES OF GRAPHS

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Abstract

A (p,q) graph G with p vertices and q edges, a bijection \( f: V(G) \cup E(G) \rightarrow \{1,2, \ldots , p + q\} \) is called edge magic labeling of G if \( f(u) + f(uv) + f(v) = k \), a constant for any edge \( uv \) of G.G is known as E-super edge magic if \( f(E(G)) = \{1,2, \ldots , q\} \).Herein, we explore some classes of E-super edge-magic graphs.

Keywords: Edge magic labeling, E-super edge magic labeling, E-super edge magic graphs.

INTRODUCTION

In this whole paper we deal with only a non-trivial simple undirected graphs.

Let G be a graph with vertex set \( V(G) \) and the edge set \( E(G) \) such that the order of G = \( |V(G)| = p \) and the size of G = \( |E(G)| = q \). Assigning of integers to vertices(edges) into a set of numbers is known as Graph Labeling.

Different kinds of labelings have been examined by several experts and an eminent survey of graph labelings can be found in[3]. In 1963, Sedláček[5] identified the concept of magic labeling in graphs. A graph G is magic if the edges of G can be labeled by a set of numbers\{1,2, \ldots , q\} so that the sum of labels of all the edges incident with any vertex is the same. In 1966, Stewart also worked on the concept of magic labeling[6].

In 1970, Kotzig and Rosa [4] defined a magic labeling of a graph G as a bijection \( f: V(G) \cup E(G) \rightarrow \{1,2, \ldots , p + q\} \) such that for all edges \( uv, f(u) + f(uv) + f(v) = k \), is constant. Enomoto[2] and Wallis[7] call an edge magic total labeling as super edge magic if set of vertex label is \{1,2, \ldots , p\}. R.M.Figueroa-Centeno et al[1] discussed the concept of super edge-magic labelings among other classes of labelings.
In 2018, U.VijayaNarayanan and P.Parthiban [8] discussed about Some classes of Super edge magic graphs. By using the definition of super edge magic labeling, we define a new labeling called E-super edge magic labeling. A (p,q) graph G with p vertices and q edges, a bijection \( f: V(G) \cup E(G) \rightarrow \{1,2,\ldots,p+q\} \) is called edge magic labeling of G if \( f(u) + f(uv) + f(v) = k \), a constant for any edge \( uv \) of G. G is said to be E-super edge magic if \( f(E(G)) = \{1,2,\ldots,q\} \). Here we observe that \( p = q \).

In this paper we find some graphs that admit E-super edge magic labeling.

**E-SUPER EDGE MAGIC GRAPHS**

Let \( C_y \) be a cycle with vertices \( v_1, v_2, \ldots, v_y \), where \( y \geq 3 \) is odd. The Graph \( C_y(1) \) is obtained from the cycle \( C_y \) by attaching a path of length 1 at the vertex \( v_1 \).

**THEOREM 1**

The graph \( C_y(1) \) is E-Super edge magic.

**Proof:**

We label the vertices of \( C_y(1) \) as follows:

Let the vertices of the graph \( v_1, v_2, \ldots, v_p \).

Define \( f: V(C_y(1)) \rightarrow \{p + 1, p + 2, \ldots, p + q\} \) by

\[
f(v_i) = \begin{cases} 
\frac{(2p+1)+i}{2}, & \text{if } i \text{ is odd} \\
\frac{3p+i}{2}, & \text{if } i \text{ is even}
\end{cases}
\]

And \( f \left( E(C_y(1)) \right) = \{1,2,\ldots,q\} \)

Obviously \( \{f(u) + f(v): uv \in E(G)\} \) contains \( q \)-consecutive integers and \( f(u) + f(uv) + f(v) = k \), a constant. Hence \( f \) admits E-super edge magic labeling.

**EXAMPLE 1**

Consider the graph \( C_5(1) \). The vertices of the graph are \( v_1, v_2, \ldots, v_6 \).

Define \( f: V(C_5(1)) \rightarrow \{7,8,\ldots,12\} \) by
Let $C_y$ be a cycle with vertices $v_1, v_2, ..., v_y$ where $y \geq 3$ is odd. The graph $C_y(n, n)$ is obtained from the cycle $C_y$ by attaching a path of length $n - 1$ at the vertices $v\left(\frac{y+1}{2}\right)$ and $v\left(\frac{y+3}{2}\right)$.

**THEOREM 2**

The graph $C_y(n, n)$ is E-super edge magic, for all $n \geq 2$.

**PROOF:**

We label the vertices of the graph $C_y(n, n)$ as $v_1, v_2, ..., v_p$.

Define an onto map 

$$f: V\left(C_y(n, n)\right) \to \{p + 1, p + 2, ..., p + q\}$$

$$f(v_i) = \begin{cases} 
\frac{(2p+1)+i}{2}, & \text{if } i \text{ is odd} \\
\frac{(3p+1)+i}{2}, & \text{if } i \text{ is even}
\end{cases}$$

And $f\left(E(C_y(n, n))\right) = \{1, 2, ..., q\}$. 

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Fig 1: E-Super Edge magic labeling of $C_5(1)$ with $k = 23$. 

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Obviously \( \{f(u) + f(v) : uv \in E(G)\} \) contains \( q \)-consecutive integers and 
\[
f(u) + f(uv) + f(v) = k,
\]
a constant.
Hence \( f \) is E-super edge magic labeling.

**EXAMPLE 2**

Consider the graph \( C_5(3,3) \). The vertices of the graph are \( v_1, v_2, \ldots, v_9 \).
Define \( f : V(C_5(3,3)) \rightarrow \{10, 11, \ldots, 18\} \) by
\[
f(v_i) = \begin{cases} 
\frac{19+i}{2}, & \text{if } i \text{ is odd} \\
\frac{28+i}{2}, & \text{if } i \text{ is even}
\end{cases}
\]
And \( f(E(C_5(3,3))) = \{1, 2, \ldots, 9\} \)

Obviously \( \{f(u) + f(v) : uv \in E(G)\} \) contains \( q \)-consecutive integers and 
\[
f(u) + f(uv) + f(v) = 33.
\]
Hence \( G \) is E-super edge magic.

![Fig 2: E-Super Edge magic labeling of \( C_5(3,3) \) with \( k = 33 \)](image-url)
Let $C_y$ be a cycle with vertices $v_1, v_2, \ldots, v_y$ where $y \geq 3$ is odd. The graph $C_y(1, n, n)$ is obtained from the cycle $C_y$ by attaching a path of length 1 at the vertex $v_1$, and a path of length $n - 1$ at the vertices $v_{\left(\frac{y+1}{2}\right)}$ and $v_{\left(\frac{y+3}{2}\right)}$.

**THEOREM 3**

The graph $C_y(1, n, n)$ is E-super edge magic, for all $n \geq 2$.

**PROOF**

We label the vertices of the graph $C_y(1, n, n)$ as $v_1, v_2, \ldots, v_p$.

Define an onto map

$$f: V(C_y(1, n, n)) \rightarrow \{p + 1, p + 2, \ldots, p + q\}$$

$$f(v_i) = \begin{cases} 
\frac{(2p+1)+i}{2}, & \text{if } i \text{ is odd} \\
\frac{(3p)+i}{2}, & \text{if } i \text{ is even}
\end{cases}$$

And $f(E(C_y(1, n, n))) = \{1, 2, \ldots, q\}$

Obviously $\{f(u) + f(v): uv \in E(G)\}$ contains $q$-consecutive integers and $f(u) + f(uv) + f(v) = k$, a constant.

Hence $f$ is E-super edge magic labeling.

**EXAMPLE 3**

Consider the graph $C_5(1, 4, 4)$. The vertices of the graph are $v_1, v_2, \ldots, v_{12}$.

Define $f: V(C_5(1, 4, 4)) \rightarrow \{13, 14, \ldots, 24\}$ by

$$f(v_i) = \begin{cases} 
\frac{25+i}{2}, & \text{if } i \text{ is odd} \\
\frac{36+i}{2}, & \text{if } i \text{ is even}
\end{cases}$$

And $f(E(C_5(1, 4, 4))) = \{1, 2, \ldots, 12\}$

Obviously $\{f(u) + f(v): uv \in E(G)\}$ contains $q$-consecutive integers and $f(u) + f(uv) + f(v) = 44$.

Hence $G$ is E-super edge magic.
CONCLUSION

In this paper, we have discussed some classes of graphs that admit E-Super Edge Magic Labeling. In future, we can prove different classes of graphs which satisfy E-Super Edge Magic Labeling.
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