Relaxation and a non-local, resistivity boundary layer in superconductors

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Abstract

Superconductors like other solids cannot relax instantaneously from thermally excited (disturbed) states to thermodynamic equilibrium. In this paper, relaxation of a multi-filamentary and of a thin film superconductor from thermal excitations is simulated. Absorption of radiation or, under conductor movement, release and transformation of mechanical tension to thermal energy are examples. The paper applies numerical simulations of superconductor energy states, as many-particle systems, under basic thermodynamic and standard, multi-component heat transfer principles (solid conduction plus radiation in thin films). A recently described microscopic stability model and application of a traditional, continuum cell model allows to explain curvature of the resistance vs. temperature excursion below critical temperature, $T_{\text{Crit}}$, and suggests an alternative to standard explanation of increased electrical conductivity at temperature exceeding $T_{\text{Crit}}$. A non-local, resistivity boundary layer (a temperature uncertainty) is observed near critical temperature within which the resistivity curve smoothly approaches, from the superconducting state, the normal conduction resistivity.

Keywords
Superconductor; phase transition; relaxation; critical current density; critical temperature; thermal fluctuations; boundary layers

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1 Relaxation in a Superconductor
1.1 Overview

The literature reports a large number of magnetic relaxation measurements performed in order to determine pinning potentials in Nb₃Sn, in the BSCCO family and in MgB₂/Fe superconductors. In contrast to these studies, focus of the present paper is on relaxation of the electron system from disturbances.

A catalogue of different kinds of disturbances can be found in Wilson, Chap. 5 [1] and in other, application-oriented standard volumes. Thermal disturbances in superconductors may result from e. g. absorption of radiation or, under conductor movement, release and transformation of mechanical to thermal energy.

Analysis of the relaxation process under such conditions requests careful calculation of relaxation time and relaxation rates. Numerical investigations of relaxation time are reported in this paper. These investigations do not address statistical electron pair decay and recombination in dynamical equilibrium (these relaxations exist without disturbances).

But relaxation (re-organisation) rates of the disturbed (excited) electron system might not be able to fully compensate decay of electron pairs to single electrons. The total, dynamic number of electron pairs might not be able to support zero-loss (critical) current. Critical current density becomes zero and the superconductor quenches, if density of charge carriers (electron pairs) becomes too small. Critical current density thus is correlated with relaxation rate of the electron system.
Relaxation of the excited electron system becomes most interesting when, under continued thermal disturbances, the superconductor has already approached very closely its superconducting to normal conducting phase transition. Superconductor stability against quench thus is an aspect of the relaxation problem. Superconductor stability has been extensively investigated by numerical simulations in [2 - 4]; see also references to our previous papers cited therein.

1.2 Non-uniformity materials and transport properties

Investigation of relaxation in the following is focused on a multi-filamentary (1G) BSCCO 2223 and on a coated (2G) YBaCuO 123, thin film superconductor (Figure 1a,b). These superconductors have been chosen because knowledge of their materials and transport properties is superior, they are thus more suitable for numerical investigations in comparison to other high-T<sub>c</sub> superconductor species.

Numerical analysis reported in [2 - 4] has been realised by combined Finite Element (FE) calculations for conductive and Monte Carlo for radiative transfer simulations of transient temperature fields in the conductor cross sections. The results depend significantly on materials and transport properties.

First, non-uniformity of materials properties is expected from a variety of potential, materials and physical origins. The simulations of both

1 Non-uniformity of materials properties, and materials deficiencies in general, result from preparation (deposition, handling, conductor winding) and operation of a superconductor device. In high-T<sub>c</sub> superconductors, non-local materials properties on a microscopic scale go back to local deviations from perfect stoichiometry, inclusion of foreign atoms, weak links in the grained BSCCO superconductor family, voids, cracks, mechanical tensions arising from conductor movement under Lorentz forces. There is also non-local, magnetic induction induced by currents in neighbouring thin films or filaments, and there are non-local heat transfer mechanisms like scattering of
BSCCO and YBaCuO superconductors therefore apply statistical variations \( \Delta T_{\text{Crit}} \), \( \Delta J_{\text{Crit}} \) and \( \Delta B_{\text{Crit}} \) (the upper critical magnetic field), all as local values. In the present paper, as an additional aspect, also a statistical uncertainty of the thermal diffusivity of the superconductor material is considered. This is important since it is extremely difficult to achieve experimental uncertainty of less than five percent when measuring conductivity or thermal diffusivity of multi-filamentary wires and thin films.

The literature reports that experimental methods have significantly been refined by applying increasingly sharp detection criteria (like electrical field constraints below \( 10^{-6} \) V/cm in transport measurements of \( T_{\text{Crit}} \)). But we frequently find error bars that just reflect rough estimates.

A statistical approach applied in the simulations, like the one reported previous and in the present papers, is a better procedure: It is based, on the microscopic scale, on the unquestionable non-uniformity of materials properties. For this purpose, all tiny Finite elements (a very large number of flat, 4-nodes elements) in the applied Finite Element meshing of the conductor cross sections are assigned statistically distributed materials properties (the properties of each element are different from those of its neighbours).

Figure 2 shows examples of the local, statistical variations \( \Delta T_{\text{Crit}} \) and \( \Delta J_{\text{Crit}} \) of critical temperature and critical current density against mean values.

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radiation in thin films and non-local solid/liquid interactions at the interface to the coolant. All these contribute to materials inhomogeneity and to non-uniform transport properties; they therefore call for using non-uniform input parameters in highly-resolved, numerical simulations.
Simulations using *non-uniform* superconductor critical parameters is contrary to almost all traditional, numerical calculations of temperature fields under disturbances (and also to all standard stability calculations). But facts have to be observed, and available mathematical tools that are suitable for investigation of superconductivity on a *microscopic* level (Finite Element and Monte Carlo methods) should be exploited.

The most interesting question, however, still has to be answered: It is not whether critical temperature can be *measured* precisely. On the contrary, in a superconductor it is the uniquely defined *existence* of critical temperature that has to be verified. Existence of a uniquely defined $T_{\text{Crit}}$ depends on the thermodynamic state of the superconductor electron system\(^2\).

This is a fundamental problem that shall be discussed in the following Section. It is correlated with relaxation of the electron system and is undoubtedly a matter and a consequence of hard, thermodynamic facts.

Second, for simulation of the relaxation process, we need more than before reliable criteria, and have to safely identify constraints, to successfully obtain numerical results during simulation of transient current transport and of transient temperature fields. This concerns decisions on

\(^2\) Critical temperature like critical current density is a property of solely the electron system. It is an interesting question which temperature is measured, electron or phonon (lattice) temperature, when sensors attached to a superconductor solid or thin film, or remotely by radiation detectors. It is not clear that both temperatures would be identical, at least because of weak-coupling between electrons and the lattice, and it becomes the more significant, the more closely the superconductor has approached a phase transition.
(i) how to calculate local critical current density, \( J_{\text{Crit}} \), with simple models that are suitable for numerical investigations,

(ii) how to treat critical temperature, \( T_{\text{Crit}} \), in the simulations as uniform or as non-uniform, local values?

Explanations how to proceed with item (i) are given in the Appendix; reference is made to [5 - 8]. By lack of better suited expressions for \( J_{\text{Crit}}(T) \), we continue with the standard procedure, \( J_{\text{Crit}}(T) = J_{\text{Crit}0} \left(1 - \frac{T}{T_{\text{Crit}}}\right)^n \), using the Ginzburg-Landau values of the exponent \( n = 3/2 \), and with no additional terms in this equation. It is important that in this equation, local values of \( J_{\text{Crit}}(T) \) are applied for the simulations.

For item (ii), again local values of \( T_{\text{Crit}} \) and of the ratio \( T/T_{\text{Crit}} \) have to be chosen. Uniform or non-uniform, local values exert strong impacts on calculated temperature excursions, in particular when their temperature, \( T \), is close to the local \( T_{\text{Crit}} \).

In Figures 5a,b, 6, 9a,b and 10 in [7], a significant impact of these, and of other parameter variations, on temperature excursions, current transport and superconductor stability is reported (the stability function is defined by Eq. (8) of the same reference).

Emphasis in this paper is on "local" properties, parameters, solutions and results.

1.3 Organisation of the paper
Temperature distributions in the conductor cross sections, \( T(x,y,t) \), obtained in [2 - 4], are not uniform, even within the minute, filament cross sections of 1G multi-filamentary superconductors. Under thermal
disturbances, or if transport current exceeds critical current density (a particular aspect of disturbances), the resulting temperature excursion, dT/dt, and temperature gradients can be very large.

On the basis of these results, the paper is organised in two parts.

In part I, consequences for recovery to, and existence of, equilibrium states from relaxation rates and relaxation time are explained. Calculation of relaxation time in high-$T_c$ superconductors has never been thoroughly analyzed in the existing physical and engineering literature. A dynamic (microscopic) stability model [9] is used for the analysis of the relaxation process.

In part II, tentative application of this model to the thermal fluctuations state (well-known in the literature) would strongly support its validity if this approach is successful.

The same model serves for explaining existence of a non-local, resistivity boundary layer (a temperature uncertainty, $dT_\rho$ near critical temperature), as an analogue to well-known local temperature or local liquid flow boundary layers in thermal, in particular radiative transfer, and in fluid dynamics.

**Part I**

2 Uncertainty of $T_{\text{Crit}}$ from relaxation time

Relaxation of the superconductor electron system from an excited state usually is expected to follow an exponential decay law.
Assume, as a thermal excitation, a sudden temperature increase originating from e. g. absorption of a radiative pulse. The excitation disturbs the dynamic equilibrium between statistical (but continued) single electron (quasi-particle) generation (decay of electron pairs) and their recombination. The excitation decays proportional to $\exp(-t/\tau)$, with $\tau$ the relaxation time that traditionally is assumed as constant. But this is not clear: The density, $n_S(T)$, of electron pairs strongly depends on temperature that in turn, under a disturbance, is a function of time, $t$. Relaxation time, $\tau$, is hardly a constant.

The point is: Relaxation cannot be completed instantaneously. This follows already from the above mentioned, rough exponential decay law, $(\exp(-t/\tau))$. Trivially, relaxation time is correlated with relaxation rates. Spontaneously, one would assume that relaxation time increases the stronger, the smaller the relaxation rate (the longer it takes the superconductor to relax to a new thermodynamic equilibrium) - provided the new equilibrium exists at all.

An approach to calculate relaxation time, $\tau$, as a time-dependent quantity, because of its dependence on temperature, has been suggested by the recently reported, "microscopic" stability model [9].

To give a short description, this model is a multi-physics approach using four formal analogues from multi-particle physics to describe the temporal aspect of re-organisation of the total electron system after a disturbance. Relaxation not only means recombination of a limited number of single electrons (as a finite number of decay products under a disturbance) to electron pairs but re-organisation of the total electron body, i. e. of all electrons as far as they are available in dynamic
equilibrium. Available means: The percentage of electrons that is ready to contribute to thermal and electrical transport.

The model [9] includes

(a) Calculation of “coefficients of fractional parentage” (cfps), a concept applied in atomic and nuclear physics. Here, this concept applies to the total number of electron states involved in (available for) the re-organisation of the electron body and, after completion of the relaxation, allows to calculate the total time needed to arrive at the new, thermodynamic equilibrium. This requests information how long it takes a single (of a very large number) of decay and recombination processes to be completed; this is provided by

(b) a “time of flight”-concept with a mediating Boson. In nuclear physics, the Yukawa-model, as one of possible analogues, a pion, π, mediates coupling (binding) of two nucleons, like the n and p in a Deuteron.

(c) and (d) the Pauli selection rule, and the uncertainty principle.

Alternatively, an expression for the superconducting electron density, \( n_S(T)/n_0 = 1 - (T/T_{\text{Crit}})^4 \), can be found in Eq. (8) of [10], with \( n_0 \) the total number of electrons at \( T = 0 \). But this expression neglects (because does not explicitly describe) the dynamic aspects of the problem, namely a sequence of large number of \emph{single}, statistical, quasi-particle generation and recombination processes.

Yet comparison of the curves \( n_S(T)/n_0 \) vs. temperature obtained from both [9] and [10] shows at least qualitative agreement (yellow and green
diamonds in Figure 6a in the Appendix). Differences between both result, apart from involved different procedures in both models, from numerical values of electron pair density, \( n_0 = n_S(T_0) \).

Both the direct (microscopic) method [9] and, with some additional assumptions, the possibly heuristic Eq. (8) in [10], allow calculation of relaxation rates and relaxation time. Regardless how relaxation rates are calculated, like in [9] or [10], the rates converge to zero when the system during warm-up very closely approaches its phase transition.

But the far-reaching conclusion, from relaxation time onto existence of a uniquely defined critical temperature, can be drawn from only [9].

This is because [10] apparently assumes simultaneous re-organisation of the decay products to a new complete, recombined set of electron pairs. This violates the Pauli principle. Instead, re-organisation has to include, for each new electron pair, \( n \), the calculation of its own cfps (coefficients of fractional parentage) from the previous \( N - 1 \) results. It requests expanding the wave function of the new recombination state \( N \) in terms of the foregoing expansion \( (N - 1) \) and the state, \( n \) (the new pair), with a check of the Pauli principle in each step of the sequence. This assumes the new wave function can be described as being composed of single-particle states.

Prediction of the relaxation time obtained with application of [9] to YBaCuO 123 and LHe-cooled superconductors, and is divergence at \( T \) near \( T_{\text{Crit}} \) is shown in Figure 6b in the Appendix. An application of this model to a (2G) thin film superconductor has recently been described in [11].
The following conclusion, to be drawn from this principle, is inevitable, for simply (and solely) thermodynamic reasons:

If $T_{\text{Crit}}$ exists, it can be understood as being either

- a rough description of a thermodynamic non-equilibrium electron state (this is a standard but only approximate interpretation), which means: From purely thermodynamic standpoints, $T_{\text{Crit}}$ cannot be explained as a thermodynamic temperature of uniquely defined value,
- or $T_{\text{Crit}}$ correlates, as a thermodynamic equilibrium variable, with a corresponding, completed (which means, really existing, final) equilibrium electron state. The prediction from [9] (Figure 6b of the present paper) is that in this case the new equilibrium electron state cannot be reached within finite (reasonably extended) process or simulation time.

Relaxation time, as is shown in Figure 6b, increases the more the closer the electron system during warm-up approaches its phase transition.

This conclusion is not in conflict with Buckel and Kleiner [12], Chap. 4, p. 262: The authors state: "In conventional superconductors the concentration of unpaired electrons decreases exponentially with decreasing temperature, and hence the probability that an unpaired electron finds a suitable partner for recombination to form a Cooper pair also decreases." So far this is acceptable, but the present situation really to be modelled is strongly different:
Under increasing temperature, the number of single electrons increases (these quasi-particles are thermally initiated decay products; there is no fixed, permanent, static distribution). Starting from an original dynamic equilibrium state at a temperature, T, followed by a disturbance at this temperature, increasingly more single particles have to recombined to pairs in order to generate, by reorganisation of the total number, \( N_{\text{Total}} \), the new dynamic equilibrium, at the new equilibrium temperature, \( T' \). There, again a statistical, un-displaced dynamic equilibrium, between generation and recombination processes, would be obtained only when relaxation is completed.

During warm-up, conservation of energy requests \( T' > T \). This temperature increase, from \( T \) to \( T' \), concerns all electrons that as a fraction of the total electron body are concerned ("available") in this process. The fraction depends on materials properties of the superconductor, temperature range (high-\( T_c \) or LHe-cooled?), interactions with the lattice and, most importantly, binding and energy states. The fractions may be quite different: In low-\( T_c \) and high-\( T_c \) superconductors, in the literature they are roughly estimated as 0.1 and 10 percent, respectively. Applicable data hardly can be found, but it is the principle that is in the foreground: Reorganisation of this fraction of electrons, regardless of the reaô percentage of the whole body, and not only a limited number available from e. g. injection, see below.

The increased number, \( N_{\text{Total}}(T') \), of single electrons to be reorganised to pairs accordingly increases relaxation time (while, reversely, relaxation time decreases with decreasing temperature). It is thus the fractional parentage principle that causes the enormous increase of \( \tau \), as a chain of events, observed near \( T_{\text{Crit}} \) in Figure 6b.
The explanation suggested in [12] and original papers by Gray et al. [13, 14] does not refer to a temperature increase from absorption of a radiation pulse or from other thermal disturbances (this would affect the *totally* available body of electrons, by conservation of energy). Contrary to the situation described in [9], the papers by Gray [13,14] describe injection experiments to create an *additional* number ΔN of quasi-particles above the "core" (N) of bound electrons in the undisturbed superconductor. The excursion with time of the number ΔN, the relaxation time, τ, is described by equations of the type ∂ΔN/∂t = A + B, with sources, A, and sinks, B. The contribution A contains the injection rate, B comprises different types of quasi-particle losses resulting from e. g. escape processes and by interaction with lattice phonons. These equations do not take into account the *whole*, *available* electron body (N) but only the additional, injected number ΔN of particles.

Therefore, relaxation times reported in [12] and in [13, 14] naturally must be strongly different from the predictions made in [9]. Relaxation time obtained from quasi-particle injection has little relevance for relaxation from a thermal excitation (the thermal disturbance comprises all single and paired electrons, not only those that are injected). Accordingly, the model [9] neither contradicts [12] (or *is* contradicted by this reference, because the balance to yield N in [12] is not complete), nor does it contradict [13, 14] (or *is* contradicted by these references, because [9] cannot be applied to injection experiments).

As a consequence, even if it in a warm-up experiment the temperature of the electron system might become infinitely close to the traditional $T_{Crit}$, re-organisation of the corresponding, total available electron state will not be completed within reasonable time scales (Figure 6b). Critical
temperature, if understood as a thermodynamic equilibrium quantity, after a disturbance close to the phase transition, accordingly is not uniquely defined.

Since under disturbances, temperature evolution in a superconductor is not uniform, as has been demonstrated in our previous papers, the phase transition can be completed neither spatial uniformly nor will this final state be attained simultaneously in the superconductor cross section or volume.

The consequence to be taken from [9] concerning non-existence of a uniquely defined $T_{\text{Crit}}$ is absolutely contrary to standard understanding of critical temperature. But $T_{\text{Crit}}$ may be hidden, be out of sight during finite, experimental or simulated periods of time, from experimental observation.

Critical temperature, if it is understood as a *sharply defined quantity*, therefore is a fiction. It is not *an observable* that exactly describes a phase transition occurring simultaneously and uniformly in the superconductor volume. Rather, it is a very small temperature *interval within which* events and their distance from the phase transition can be studied.

This conclusion will be supported from results obtained in Sect. 3, as a by-product of an alternative explanation of thermal fluctuations effects. There a uncertainty temperature interval is reflected by a non-local layer, $\delta T_{\rho}$ (Figure 4a,b).
This conclusion from [9] also specifies a note that Annett [15], p. 52, added to the left column of his book. There we have: "The word superconductor is used only to mean a material with a definite phase transition and critical temperature." (correction by the present author: "definite" probably means "completed"). Yes, but the phase transition, here the completion of relaxation after a disturbance, takes finite, possibly diverging time. Critical temperature is the dynamic equilibrium of the electron state when no more disturbances, besides the statistical fluctuations in dynamic equilibrium, have to be compensated.

All these conclusions are based on hard thermodynamic facts and result from properties of many-particle systems (here an analogy to nuclear physics), basic thermodynamic considerations (temperature uniquely defined under solely thermodynamic equilibrium) and from standard, multi-component heat transfer principles (solid conduction plus radiation in thin films).

Consequences from this conclusion might also concern levitation experiments: Levitation height would not converge, within reasonable periods of time, to a final, equilibrium vertical co-ordinate if a disturbance occurs that might drive the sample close to its phase transition.

Again inevitable under reasonably limited experimental efforts, also results of standard measurements of critical current density in dependence of temperature might not reflect completion to equilibrium. it would be interesting to see long-time measurements of $J_{\text{Crit}}$, with no temperature increase during finite, possibly diverging time, at exactly stable experimental conditions.
The results reported in the following Sections and the numerical method how to achieve convergence, as described in the Appendix, therefore are provisional. Strictly speaking, simulations can safely be performed only if a convergence limes of a series would exist within which $T$ steadily approaches $T_{\text{Crit}}$. Practically, all decisions related to the superconductor/normal conductor phase transition are uncertain within a very small temperature interval, the interval, $\delta T_p$, that will be explained in the next Section.

The question comes up whether the extend of $\delta T_p$ can be reduced so that definition of $T_{\text{Crit}}$ finally might become sharp.

**Part II**

3 Thermal fluctuations

In the literature, increased electrical conductivity of metallic superconductors, at $T > T_{\text{Crit}}$, is considered as an intrinsic materials property, a "sort of fluctuating superconductivity", see Glover III [16]. This reference explains the observations as originating from superconducting carriers of finite lifetime when they are formed in a homogeneous superconductor material. Glover III claims evidence that observed rounding of the transition curve in thin samples with short electron mean free paths relies on this effect.

Against this traditional explanation, the increased conductivity, as an alternative explanation instead, might be correlated with variations of the superconductor order parameter. The extent of this contribution is unknown, but an attempt is made in the following to at least qualitatively confirm this hypothesis, namely whether rounding of the transition curve
can be explained in this way. Rounding is observed at temperature close to $T_{\text{Crit}}$, above and below this value.

If this hypothesis can be confirmed, it also would support the model described in [9] and its application in simulations of superconductor relaxation.

### 3.1 A two-fluid model for electrical resistivity near critical temperature

In the following, this observation shall tentatively be explained by a two-fluid model consisting of two phases (residual normal conducting electrons and superconducting electron pairs), all in dynamic equilibrium after a completed relaxation.

A cell model is in the following applied to derive an effective, total electrical resistivity or conductivity of this situation. This does not address random distribution of superconducting and normal conducting volume elements, a mixture that could be explained from local variations of $T_{\text{Crit}}$ (like those assigned to the Finite Elements of a mesh, consider Figure 2). Such distributions, too, would allow definition of an effective conductivity or resistivity. But it is a distribution of super- and normal conducting phases (instead of a distribution of elements), not between volume elements of arbitrary size.

To be successful with the application of a cell model, an internal "porosity", $\pi$ must be defined that in the present case describes the dominating, normal conducting contribution to the total electrical conductivity, or the part $1 - \pi$ of superconducting "inclusions".
Explanation of porosity usually is related to geometrical or materials issues (e.g. a distribution of solid spheres in a liquid). Here it will be applied not with respect to geometrical or materials property issues but on an atomistic scale, namely ratios by which electrons, in their two phases, contribute to total thermal resistivity. This is no too far away from Glover III's idea of a fluctuating superconductivity, with the exception that we do not consider fluctuating phases. Contribution of the phases is solely related to temperature.

The contribution $1 - \pi$ is given by the Ginzburg-Landau order parameter, $f_S = n_S(T)/n_S(T=4K)$, with $n_S$ the density of electron pairs. Since $f_S$ strongly depends on temperature (Figure 6a in the Appendix), the fraction $\pi = 1 - f_S$ cannot be constant. For an explanation of the order parameter with respect to resistance of superconductors see Sect. 6.3 in [17].

The Russell cell model [18] is a standard means to calculate materials and transport properties of two-component systems. It was originally derived for thermal transport. The model is easy to handle: In its original version, it just contains porosity and resistivity of both, solid and porous phases, either for normal conduction, electrical or thermal transport. The model thus is flexible (the role of particles and voids without much effort can be interchanged). Importantly, it is a continuum model. For its apparently first application to a superconductor, in the sense explained above, see Sect. 3.5 and Figure 6b of [2].

The Russell and other continuous cell models apply conductivity or resistivity of the corresponding phases. Electron pair, zero resistance therefore has to be represented by a finite, pseudo non-zero resistance. This resistance is, by many orders, at least 20, of magnitude, smaller
than the resistance of the normal conducting phase. Without a finite, non-zero resistance, the cell model does not work.

Predictions of the cell model may diverge at very small and very large values of the porosity (this is a weak point of all cell models). Application of this model thus becomes critical when under warm-up the number of electron pairs near $T_{\text{Crit}}$ goes to zero (the porosity then approaches 1).

Figure 3 schematically shows the simulated, overall two-component resistive network for current transport. See Figure captions for detailed explanations.

A finite non-zero resistance in any case originates from weak links that strongly limit critical current density in high-$T_c$ superconductors. Separate presentation of this contribution (another rectangle in Figure 3) is neglected (this resistance is enclosed in the blue rectangles).

3.2 Results
Total specific resistivity, $\rho_{\text{El}}(T)$, an effective value, of the BSCCO 2223 and YBaCuO 123 samples calculated using the microscopic stability and Russell cell models, is shown in Figure 4a,b. Detailed inspection of the result demonstrates their deviation from the expected, sharp jump of $\rho_{\text{El}}(T)$ at $T_{\text{Crit}}$ (sharp transitions between different phases are not observed in classical physics).

Up to present, explanation and, in particular, quantification of this phenomenon in superconductors has remained the subject of controversial discussions. But the behaviour of the calculated $\rho_{\text{El}}(T)$, as indicated in Figure 4a,b, the smooth convergence of curves $\rho(T)$, to the
normal conduction value within a *finite* temperature interval, confirms the
findings of standard experiments (rounded curves) to determine critical
temperature.

In a parameter test, results $\rho_{\text{El}}(T)$, have been obtained for different $\rho_{\text{el,SC}}$, the pseudo-resistivity of the superconducting phase ($\rho_{\text{el,SC}}$ has to be considered, as is explained above, as of finite value and as independent of temperature). The question is whether, at $T$ significantly smaller temperature than $T_{\text{Crit}}$, the curvature would depend on the values of $\rho_{\text{el,SC}}$. The total specific resistivity, curves $\rho_{\text{El}}(T)$, of course reflect the values of $\rho_{\text{el,SC}}$, but *curvature* of $\rho_{\text{El}}(T)$ near $T_{\text{Crit}}$ is clearly maintained under all variations of $\rho_{\text{el,SC}}$ (between $1\times10^{-16}$ and $1\times10^{-32}$ $\Omega$ m) that were applied in the calculations.

The curvature of $\rho_{\text{El}}(T)$ in Figure 4b below $T_{\text{Crit}}$, suggests definition of a *non-local* "boundary layer" (in the $\rho_{\text{El}}(T)$-diagram a temperature uncertainty $\delta T_{\rho}$) like the *local* temperature boundary layers (jumps, $\delta T$) well-known from thermal, in particular radiative transfer and from local $\delta v$ in fluid dynamics. Within this layer (the temperature uncertainty, $\delta T_{\rho}$), the curves $\rho_{\text{El}}(T)$ in Figure 4a,b smoothly approach, under increasing temperature, the value of the normal conducting state (blue diamonds in Figure 4b).

Existence of this layer, like in thermal or radiative transfer and in fluid dynamics, relies on the assumption of a conductive *continuum*. *This is* fulfilled by the Russell cell model, and the continuum is a substantial property also of the microscopic stability model. It is thus within the temperature interval, $\delta T_{\rho}$, in Figure 4a,b that $T_{\text{Crit}}$ cannot be defined sharply: the resistivity $\rho_{\text{El}}(T)$ is differentiable with respect to $T$. 
Finally, how can we reduce the non-local width $\delta T_\rho$? Can materials development correlated with this uncertainty?

Reduction of the width $\delta T_\rho$ could be possible on the basis of correlations between $T_{\text{Crit}}$, $J_{\text{Crit}}$ and relaxation. Such correlations do exist, as is indicated in Figure 5a,b (copied from Figure 8a,b in [7]; this is a main result of correlations reported in this reference). We should find out whether, and to which extent, reduction of the width $\delta T_\rho$, too, can be correlated to relaxation. According to present experience, this investigation might involve serious numerical convergence problems. More investigations will become necessary.

4 Summary and Outlook

During warm-up, critical temperature exists only as a convergence $\text{limes}$ of a series of excited and sequentially re-organised superconductor states.

The $\text{limes}$ cannot be obtained within reasonable times in standard experiments or in standard simulations if a disturbance is experienced at temperature already close to the superconducting/normal conducting phase transition.

Critical temperature, if it is understood as a sharply, with zero tolerance $\text{defined}$ quantity, therefore would be a fiction.

The curvature of the specific resistivity near critical temperature suggests definition of a resistivity "boundary layer" expressed as a temperature
uncertainty interval $\delta T_\rho$. It is within this layer that critical temperature cannot uniquely be defined.

**Final remark by the author**

Thank you for your patience. Experiments to verify all these predictions would be difficult to perform. But this is essentially not new physics. This paper results from my lectures "Applied Superconductivity" given in the Department of Physics at the University of Wuerzburg, Germany. Comments and nominations of potential referees would be highly welcome.

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Figure 1a Simulation scheme for the multi-filamentary (1G) BSCCO 2223 superconductor; the Figure shows part of the wire cross section. Conductor architecture and dimensions are standard. Superconductor filaments (black) are embedded in a Ag-matrix. The simulations include weak-link material. The Figure is a slightly modified part of Figure 6a of [2]; see this reference for more details.
Figure 1b Simulation scheme of coil and of conductor geometry (detail of Figure 1 of [11]), here showing turns 96 and 97 of a coil (schematic, not to scale). Coil winding (100 turns) applies an (2G) coated, YBaCuO 123, thin film superconductor. Crystallographic c-axis of the YBaCuO-layers is parallel to y-axis of the co-ordinate system. Conductor architecture and dimensions again are standard. Superconductor (SC) layer thickness (red rectangles) is 2 μm, width 6 mm; thickness and width of the Ag elements (lilac) is the same, width of the interfacial layers (IFL, light green) is 40 nm (the IFL are included to simulate surface roughness and diffusion of species between the SC and its neighbouring Ag- and MgO-layers, respectively). The thick, dashed-dotted line at the bottom of this diagram indicates the artificial axis of vertical symmetry introduced for support of the Finite Element part of the simulations, compare text for explanation.
Figure 2 Random variations (distributions within the Finite mesh) of $\Delta J_{\text{Crit}}$ of critical current density (upper diagram) and of $\Delta T_{\text{Crit}}$ of critical temperature (below). The variations $\Delta J_{\text{Crit}}$ are within 1 percent around the mean (thin film) value $J_{\text{Crit}} = 3 \times 10^{10} \text{ A/m}^2$ of YBaCuO 123 in zero magnetic field and at $T = 77 \text{ K}$, the variations $\Delta T_{\text{Crit}}$ are within 1 K around the mean value $T_{\text{Crit}} = 92 \text{ K}$ in zero magnetic field. In the lower diagram, the inset shows element temperatures in the immediate neighbourhood of the centroid of turn 96 (orange rectangle). Solid green and red lines indicate mean values, blue and red, dashed-dotted lines are mean-square deviations.
Figure 3. Resistance networks: (a) thermal and (b) electrical resistances, all at \( T < T_{\text{Crit}} \). The diagrams are used to specify application of the Russell cell model [18] to thermal and electrical conduction transport in single volume elements of a grained or thin film superconductor (schematic, strongly simplified). The percentage of electrons (ready for thermal and electrical transport) from the total body applies to the composite of resistances 2 and 3 in both diagrams (a) and (b).

(a) Conduction heat transfer at different temperature. Dark brown and blue thermal rectangles indicate phonon (non-zero) and single electron thermal resistances, \( R_{\text{Ph}} \) and \( R_{\text{El}} \), respectively. The black resistance, \( R_{\infty} \), applies to electron pairs and illustrates infinitely large thermal resistance,
which means, regardless of the number of electron pairs, zero contribution to conduction heat transfer by vanishing collisions with the lattice (vertical length of the black rectangles schematically indicates increasing number of electron pairs). Total thermal conduction heat transfer results from series or parallel combinations of the interwoven resistances (transport channels) 1 to 3: Zero with combinations (1,3), (2,3) or with the single channel 3; nonzero with combinations (1,2). At very low temperature, heat transfer represented by the sum of the dark brown and blue (and black) rectangles, approaches the small thermal conductivity of an electrically insulating crystal (nearly complete correlation of free electrons to electron pairs is the cause for their zero contribution to total thermal conductivity of the scheme). In both Figures, $R_{\text{Ph}} > R_{\text{El}}$, which means the thermal conductivity of the superconductor to the most part is by single electrons (channel 3 and its combinations with 1 and 2 is completely closed). Under given temperatures, $T_1$ and $T_2$, solution of Fourier's differential equation yields the phonon temperature within resistance 1 that, at any co-ordinate, is different from electron temperature in resistance 2, in particular if coupling between both transport channels is weak. Temperature of channel 3 is below $T_{\text{Crit}}$ but otherwise undetermined.

(b) Current transport, like in (a) at different temperature (again schematic, strongly simplified). Dark brown and blue rectangles indicate phonon (totally insulating) and single electron electrical resistances, $R_{\text{Ph}}$ and $R_{\text{El}}$, respectively. The black section (resistance, $R_o$) illustrates zero resistance, which means, regardless of the number of pairs, total contribution to current transport is by electron pairs only. However, to make the Russell cell (resistance) model applicable, an at least 20 orders of magnitude smaller electrical resistance, in relation to normal electrical conduction, has to be assumed (this is schematically indicated by the small, non-zero vertical length of the black rectangles).
Figure 4a Total specific, electrical resistivity, $\rho_{El}(T)$, of BSCCO 2223 (above) and YBaCuO 123 samples (below) near their phase transitions. The results are calculated using the Russell cell model [18], with porosity $\pi$ from application of the microscopic stability model [9] and, as a parameter, using a finite (non-zero) pseudo-resistivity, $\rho_{El,SC}$ of the superconducting phase. In the upper and lower diagrams, the value $\rho_{El,SC}$ amounts to 1e-16 or 1e-32 $\Omega$ m, respectively. Curvature of $\rho_{El}$ near $T_{Crit}$ and the interval $\delta T_{\rho}$ are observed for any reasonable variations of $\rho_{El,SC}$. The interval denotes a non-local, resistivity boundary layer (a temperature uncertainty) the existence of which sharply questions sharply defined critical temperature.
Figure 4b Detail of Figure 4a (upper diagram, strongly magnified) showing, very close to $T_{\text{Crit}}$, the non-local, resistivity boundary layer (the temperature uncertainty, $\delta T_{\rho}$) in the BSCCO 2223 sample. As a worst case condition, like in the upper part of Figure 4a, the value $\rho_{E,\text{SC}} = 1e-16 \ \Omega \text{m}$ has been applied in this calculation. It would be desirable to have data points also between $T_{\text{Crit}} - 1e-6 \ \text{K}$ and exactly $T_{\text{Crit}} = 108 \ \text{K}$ to complete a curvature, but severe numerical problems are experienced within this temperature interval. Future simulations are planned to close this gap.
Figure 5a Correlations between different superconductor parameters. Results apply to the YBaCuO 123 thin film superconductor. Upper diagram: Differential critical current density, \( J_{\text{Crit}} \), per relaxation time, \( \tau \) and per relaxation rate, \( \zeta \), calculated with \( \Delta E_0 = 60 \text{ meV} \). Below: \( d\Delta J_{\text{Crit}}/d\Delta \zeta \) calculated with different \( \Delta E_0 \); in both diagrams, results are plotted vs. \( J_{\text{Crit}} \). See [7] for more explanations.
Figure 5b Relaxation time $\tau$, relaxation rate $\zeta$, difference $\Delta T = T_{\text{Crit}} - T(t)$ and $J_{\text{Crit}}$ vs. energy gap $\Delta E(T)$ near critical temperature. The $\tau$- and $\zeta$-curves are calculated from [9] and $J_{\text{Crit}}$ from Eq. (1a,b) using $n = 3/2$. See again [7] for more explanations. Arrows indicate quantitative correlations and causal relations (the latter are unidirectional) 1: temperature (here the difference $\Delta T$) $\rightarrow$ energy gap (causal), 2: energy gap $\rightarrow$ $J_{\text{Crit}}$ (causal), 3 (in parallel to 2): energy gap $\rightarrow$ relaxation rate (causal), 4: relaxation rate $\rightarrow$ $J_{\text{Crit}}$. Accordingly, arrow 4 indicates, as the final result of this paper, the causally but unidirectionally oriented correlation between relaxation rate and critical current density.
Figures to the Appendix

Figure 6a Relative density (the order parameter), $f_S = n_S(T)/n_S(T=4K)$, in dependence of temperature, calculated for the thin film, YBaCuO 123 superconductor (dark-blue diamonds). Dark-yellow diamonds indicate the minimum relative density of electron pairs that would be necessary to generate a critical current density of $3 \times 10^{10}$ A/m$^2$ (YBaCuO 123 at 77 K), in zero magnetic field. The diagram compares predictions of the microscopic stability model [9] with analytical results (light-green) calculated from Eq. (8) in [10]. The Figure is copied from [17] (this Figure and the following Figures of the Appendix are re-plotted only for convenience of the reader):

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Figure 6b Relaxation time of the superconductor electron system after a disturbance (here the sudden increase of transport current in the windings to a large multiple of the nominal value); results are given vs. element temperature. All solid, dark-brown, circles in this Figure are recalculated and extended against the original curves to element temperatures still closer approaching $T_{\text{Crit}}$. Computation time becomes enormous. For just the uppermost data point, at 91.999 K, on a standard, 4-core PC with 3.4 GHz and 16 GB workspace, the calculation under Windows 7 without using any approach and without any interruption may take several 24 hrs. The Figure is re-plotted from [17], again for convenience of the reader:

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Figure 7 Initial and mean decay or relaxation rates (per unit volume) of thermally excited electron states calculated using a screening factor, $\chi = 0.01$, to the Coulomb potential, in a virtual conductor volume, $V_C$, of the superconductor YBaCuO 123 (see [9] for detailed explanations). If the system continuously creates new (intermediate), dynamic equilibrium states, decay rates in this Figure are equal to relaxation rates. The area below the curves corresponds to the ordered phase (electrons condensed to electron pairs) that is separated, from the thermally disordered phase (electrons from decayed electron pairs), by the classical critical, finite temperature boundary (here the dashed dark-blue and light-green lines). The Figure is re-plotted from [17]:

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Figure 8 Convergence of calculated superconductor element temperature (centroid of turn 96) of the YBaCuO 123 thin film superconductor; results are plotted vs. simulation time. Results are obtained using the simulation scheme including a flow chart (Figure 11a,b of [7]). Note the saw-tooth behaviour of T(t) that results from this procedure and which explains convergence of T(t). Arrows indicate results obtained during the iterations (sub-steps, i) within a particular load step, j, before convergence is finally achieved: These are the values enclosed within the black convergence circles. The saw-tooth behaviour of T(t) beginning at t = 4.1 and 4.2 ms of conductor temperature in the present Figure is shown for different values of the exponent, n, used in the relation for the YBaCuO 123 thin film superconductor $J_{\text{Crit}}(x,y,t) = J_{\text{Crit}}(x,y,t_0) [1 - T(x,y,t)/T_{\text{Crit}}(x,y,t))]^n$ (Eq. 1b), and for constant anisotropy ratio, $D_{ab}/D_c = 5$, of the thermal diffusivity, $D$, $I_{\text{Transp}}/I_{\text{Crit}} = 1$ (no fault, just nominal transport current). Results are calculated with the random variations $\Delta J_{\text{Crit0}}$, $\Delta T_{\text{Crit0}}$ and $\Delta B_{\text{Crit20}}$ (Figure 2) of the critical parameters. The local temperature run-away at t = 4.1 ms (and also at t = 4.2 ms) in reality predicts onset of a total quench. Differences resulting from the exponents n in this region of temperature are significant (but the stability function, trivially, is not affected in this region since $T > T_{\text{Crit}}$).
Appendix (Supporting Information)

Critical current density, $J_{\text{Crit}}$, depends on temperature; under transient conditions, $T = T(t)$, we have the standard expression

$$J_{\text{Crit}}([T(t), B(t)]) = \frac{J_{\text{Crit}}[T(t)]}{B_0 + B(t)}$$  \hspace{1cm} (1a)

using for the temperature dependency of $J_{\text{Crit}}$,

$$J_{\text{Crit}}(T) = J_{\text{Crit0}} \left( 1 - \frac{T}{T_{\text{Crit}}} \right)^n$$  \hspace{1cm} (1b)

with $T_{\text{Crit}}$ the critical temperature. In a coil, $B(t)$ in Eq. (1a) describes magnetic induction, like from neighbouring windings (but $B_0$ in the following is considered as constant). The exponent $n$ in Eq. (1b) has to be chosen with respect to materials and current transport properties.

Eq. (1b) reflects Ginzburg-Landau theory, as reported in Eq. (4-35) of [5]. This result follows from $\partial J_s/\partial v_s = 0$, to find the maximum of $J_s$, a supercurrent (and with $v_s$ the velocity of its constituents). In Eq. (4-36) of this reference, $J_s$ is identified as the critical current, $J_{\text{Crit}}$.

This result of the Ginzburg-Landau equation for $J_{\text{Crit}}$ applies to a continuum by assuming the amplitude of the order parameter is uniform in the sample. We thus will find its applicability mostly in thin metallic films and alloys (Sect. 5.6 of [6]), at temperature near critical temperature.

But the Ginzburg-Landau theory does not take into account local superconductor parameters and temperature, does not consider
anisotropic materials and transport properties of type II, hard superconductors and thus in total might not be applicable in these cases.

We have discussed this problem and its pros and contras in previous papers and again in [7] to apply the simple Eq. (1a,b). Certainly, we cannot apply Eq. (1b) with inclusion of an exponential damping factor, $J_{\text{Crit}}(T,B) = J_{\text{Crit}}(T=0,B) (1 - T/T_{\text{Crit}}) \exp(-T/T_{\text{Crit}})$, as was suggested in [8] for investigation of coated superconductors. While this would fit the architecture of the superconductor investigated in the present paper, application of the exponential term in the numerical simulations would cause serious convergence problems (thermal runaway and too strong reduction of $J_{\text{Crit}}$).

Therefore, the traditional Eq. (1b), without an additional, exponential term has been applied in the present simulations, using the interval $0.5 \leq n \leq 2$. After a variety of calculations, we returned to the Ginzburg-Landau value $n = 3/2$

For item (ii) of Sect. 1.2, the most probable, realistic approach that should be used in numerical simulations is to apply, in both $T - T_{\text{Crit}}$ and $T/T_{\text{Crit}}$, local values of conductor temperature and of critical temperature. The obvious reasons to do so is given, on the microscopic scale, by the non-uniformity of materials and transport properties of grained superconductors. Instead of the standard $\Delta T = T - T_{\text{Crit0}}$ and ratio $T/T_{\text{Crit0}}$ (using uniform $T_{\text{Crit0}} = 92$ K), we have applied in [7] and in the present paper

\begin{align}
\Delta T(x,y,t) &= T(x,y,t) - T_{\text{Crit}}(x,y,t) \\
J_{\text{Crit}}[T(x,y,t)] &= J_{\text{Crit0}} [1 - T(x,y,t)/T_{\text{Crit}}(x,y,t)]^n
\end{align}
provided $T(x,y,t) < T_{\text{Crit}}(x,y,t)$ at all local co-ordinates $(x,y)$ in the conductor cross section, and all transient times, $t$.

Our previous simulations have definitely shown that under thermal disturbances and from the ratio of transport to critical current, in particular in situations close to the superconducting/normal conducting phase transition, uniform distribution of temperature and of critical and transport current in the superconductor cross section cannot be expected. Assumption of perfectly, uniform materials properties is too optimistic and may serve, at best, for worst case estimates.

We have applied the numerical simulations to a (1G) multi-filamentary superconductor and to a superconductor coil to obtain transient temperature fields, $T(x,y,t)$, developing from initial values, $T(x,y,t_0)$. The coil integrates 100 turns prepared from a (2G) coated, YBaCuO 123 thin film superconductor (a detail of the corresponding, standard conductor architectures is shown in Figure 1a,b, for more information on simulations of the coil see [3]). The transient temperature distributions under disturbances in this paper are obtained from 2D solutions of Fourier’s differential equation.

This start of the simulations is chosen because temperature, like magnetic field, is a thermodynamic variable that deserves primary attention, while current is not. We do not apply multi-physics approaches like H- or T/A formulations that beyond doubt are very suitable for large-scale superconductor applications.

Instead, what is needed in the present paper, in order to describe physics on a microscopic scale, is to consider, in each of the Finite
Elements, the Meissner effect, weak links and viscous, thermally assisted flux flow resistivity, flux creep and contact resistances between super- and normal conducting layers.

We also do not apply constant temperature approximations but describe local interplay of temperature-, field- and current-dependent variables with transient temperature fields. It is questionable that standard Finite Element codes providing multi-physics element options (like electric field, heat transfer, structural mechanics couplings in the FE code Ansys) would be suitable to safely achieve convergence in the neighbourhood of a phase transition.

Therefore, we first calculate the solution, \( T(x,y,t) \), by application of the FE code to highly differentiated, geometrical and materials boundary conditions. From this solution, all physical variables that depend on temperature are calculated. In successive load-steps, the Finite Element integration is restarted with the new variables. A bi-directional interconnection between electric/magnetic and thermal interplay is realised by this sequential procedure, and the convergence problem is reduced to that of solely the temperature field (of course, this procedure requires considerable computational efforts).

The FE procedure for this purpose is embedded in a comprehensive, "master scheme". This scheme, a flow chart, is described in Figure 11a,b of [7]. Step by step iterative repetitions of the proper (inner) Finite Element integration cycles are intersected by intermediate, analytic calculations of electrical and magnetic variables (penetration of magnetic field, Meissner effect, local resistances, local critical and transport current distribution and local losses like flux flow, Ohmic or inductive).
In case of divergences, this FE procedure, during its many iterative repetitions, allows to introduce corrections (adjustments) to physically reasonable values like the 77 K coolant temperature constraint. This is realised without restarts of the simulations, a rather exceptional exploitation of simulation potentials provided by Ansys.