Heralded quantum memory for single-photon polarization qubits

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Abstract – We propose a scheme to implement a heralded quantum memory for single-photon polarization qubits with a single atom trapped in an optical cavity. In this scheme, an injected photon only exchanges quantum state with the atom, so that the heralded storage can be achieved by detecting the output photon. We also demonstrate that the scheme can be used for realizing the heralded quantum state transfer, exchange, and entanglement distribution between distant nodes. The ability to detect whether the operation has succeeded or not is crucial for practical application.

Introduction. – A quantum memory, a storage device that can faithfully store and retrieve the quantum state of a traveling light pulse, is a requisite for long-distance quantum communication networks and distributed quantum computers [1–9]. By following the seminal protocol proposed by Duan et al. [3], significant advance has been made in storing single photons using atomic ensembles as storage mediums [10–25]. Early experiments using atomic ensembles mainly focused on the storage and retrieval of single photons with the fixed polarization [12,13]. Recent experiments show that single-photon polarization qubits can be stored in two atomic ensembles [16,17]. However, the above studies are thwarted by photon losses since one cannot determine whether the incoming photon has been stored or lost. Fortunately, such unpredictable failure may be largely remedied by a heralding feature that announces photon arrival and successful storage without destroying the stored quantum state. In ref. [26], a heralded storage has been achieved by means of a spontaneous Raman process that can absorb a single photon with arbitrary polarization and simultaneously emit a photon with fixed polarization. However, due to small spontaneous Raman scattering probability, the heralded storage occurs rarely, while its polarization state is restored with high fidelity. Cavity quantum electrodynamics (QED) provides another ideal interface between atoms and photons [4,27–32]. In the initial proposal for the implementation of quantum networks in cavity QED [4], the quantum information of photons is encoded in the Fock basis, i.e., the zero- and one-photon Fock states [4,30]. Lloyd et al. [32] proposed a robust method for transmitting entangled polarization state over long distances and teleportation of atomic state via measurements of all four Bell states, using a novel method of sequential elimination.

In this paper, we propose a heralded quantum memory for an arbitrary polarization state with a single atom trapped in an optical cavity. Our scheme is based on a quantum-state swap between a single-photon pulse and a trapped atom. The heralded storage can be achieved by detecting the output photon. Numerical simulation results show that our scheme has a high success probability and the retrieved photon has so well-defined waveforms that it is easy to interfere with other photons. We also show that the scheme can allow the heralded quantum state transfer, exchange, and entanglement distribution between distant nodes. The ability to detect whether the operation has succeeded or not is crucial for practical applications [2,26].

The building block and numerical simulations. – As shown in fig. 1(a), our model consists of a single atom inside a one-sided optical cavity. The relevant atomic levels are depicted in fig. 1(b), the states |l⟩ and |r⟩ correspond to two stable ground states, and |e⟩ denotes an excited state. The cavity supports two degenerate cavity modes aʰ and aʰ with different polarizations h and e. We assume that the
transitions $|l\rangle \leftrightarrow |e\rangle$ and $|r\rangle \leftrightarrow |e\rangle$ are resonantly coupled to two cavity modes $a^h$ and $a^v$ with the strength $g_h$ and $g_v$, respectively. The interaction between the atom and cavity modes is described by $H_I = g_h |e\rangle\langle l|a^h + g_v |e\rangle\langle r|a^v + H.c.$. The time evolutions of cavity modes $a^h$ and $a^v$ are given by [4,33,34]

$$\dot{a}^{h(v)}(t) = -i [a^{h(v)}(t), H_I] - \frac{\kappa}{2} a^{h(v)}(t) - \sqrt{\kappa} a^{h(v)}(t), \quad (1)$$

where $H_I = H_I - i \frac{\kappa}{2} |e\rangle\langle e|$ [31,35], $\kappa$ is the cavity decay rate for two cavity modes, and $\gamma_c$ is the spontaneous-emission rate of the excited state. The output operator $a_{out}^{h(v)}(t)$ is connected with the input operator $a_{in}^{h(v)}(t)$ by the input-output relation $a_{out}^{h(v)}(t) = a_{in}^{h(v)}(t) + \sqrt{\kappa} a^{h(v)}(t)$.

In this paper, two cases are considered: i) the atom is prepared in the state $|l\rangle$ (or $|r\rangle$), and a $v$-polarized (h-polarized) photon $|v\rangle$ ($|h\rangle$) is injected. In this case, the Hamiltonian $H_I$ does not work and the injected photon sees an empty cavity. Thus, the polarization of the input photon is not changed and we have [33,34]

$$a_{out}^{v} (\omega) = -\frac{\kappa}{2} - i \frac{\delta}{2} a_{in}^{v} (\omega), \quad (2)$$

where we assume that the center frequency of input photon pulse $\omega_0$ is resonant with the cavity mode and $\delta = \omega - \omega_0$. If $\kappa \gg \delta$, we have $a_{out}^{v}(\omega) \approx -a_{in}^{v}(\omega)$. Next, we consider the case ii): the atom is prepared in the state $|l\rangle$ (or $|r\rangle$), and a $h$-polarized ($v$-polarized) photon $|h\rangle$ ($|v\rangle$) is injected. In this case, taking the adiabatic limit [31,35], we can obtain the input-output relation

$$a_{out}^{h} (\omega) = a_{out}^{h(v)} (\omega) + a_{out}^{h(v)} (\omega), \quad (3)$$

with

$$a_{out}^{h(v)} (\omega) = \left[ 1 - \frac{\kappa}{2} - i \delta + 2g_h^2 (\omega)/\kappa (4g_v^2 (\omega) + i \kappa \gamma_c) \right] a_{in}^{h(v)} (\omega). \quad (4)$$

It is seen from eqs. (3) to (5) that if conditions $\kappa \gg \delta$, $4g_v^2 (\omega) \gg \kappa \gamma_c$, and $g_h \approx g_v$ are satisfied, we have $a_{out}^{h(v)} (\omega) \approx a_{in}^{h(v)} (\omega)$. Therefore, the polarization of the input photon is changed and we realize the state flip operation $|l\rangle |h\rangle \leftrightarrow |r\rangle |v\rangle$.

Now we give a detailed analysis of this building block through numerical simulation with the Hamiltonian [31,35]

$$H = H_I + \sum_{f=h,v} \int_{-\omega_h}^{\omega_h} d\omega \Theta_f^{\dagger} (\omega) \Theta_f (\omega) + i \sum_{f=h,v} \int_{-\omega_h}^{\omega_h} d\omega [a_f^{\dagger} \Theta_f^{\dagger} (\omega) - a_f^{\dagger} \Theta_f (\omega)], \quad (6)$$

where $\Theta_f (\omega)$ with the standard relation $[\Theta_f (\omega), \Theta_f^{\dagger} (\omega)] = \delta (\omega - \omega')$ denotes the one-dimensional free-space mode coupled to the cavity modes.

Assuming that the input single-photon pulse is Gaussian pulse of the form $f(t) \propto \exp \left[ -(t - T/2)^2 / (\frac{T}{4})^2 \right]$, where $T$ is the pulse duration. We first consider the above-mentioned case ii) that a polarized photon $|h\rangle$ ($|v\rangle$) is injected into cavity and the atom is prepared in the state $|l\rangle$ ($|r\rangle$). Let $P$ be the success probability for the flip $|l\rangle |h\rangle \rightarrow |r\rangle |v\rangle$. In fig. 2, we plot $P$ vs. the normalized cavity coupling rate $g/\kappa$, assuming $\gamma_c = \kappa$, with different pulse width: $\kappa T = \{10, 20, 30, 40, 50, 60, 120\}$. From fig. 2, we see that when $g \geq 2 \kappa$ and $\kappa T \geq 60$ the success probability can be up to 90%. Figure 3 shows that when $\kappa T \geq 60$ the output pulse shapes $|f(t)|$ almost completely overlap with the input pulse shapes. Then, we consider the case i) that a polarized photon $|h\rangle$ ($|v\rangle$) is injected into cavity and the atom is in the state $|r\rangle$ ($|l\rangle$). In this case, the atom is decoupled
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Fig. 3: (Color online) The shape functions $f(t)$ for the input pulse (solid curve) and the output pulse (dashed curve) for the case $\kappa T = (a) 10$, (b) 30, (c) 60, and (d) 90. Other common parameters: $\gamma_e = \kappa$, $g_0 = g_e = g$, and $g = 2\kappa$.

Fig. 4: (Color online) The shape functions $f(t)$ for the input pulse (solid curve) and the output pulse (dashed curve) for the case $\kappa T = (a) 30$, (b) 60, (c) 90, and (d) 120.

from the cavity field, thus the influence of the atomic spontaneous emission can be ignored and the dominant noise is the distortion between the output and the input pulse. In fig. 4, we plot the output and the input pulse shapes with different pulse width: $\kappa T = \{30, 60, 90, 120\}$.

Quantum swap gate and quantum memory. – Assume that the injected photon and the atom are initially in arbitrary superposition of two polarized states and two ground states, respectively, i.e., $(\alpha | h \rangle + \beta | v \rangle) \otimes (\zeta | l \rangle + \eta | r \rangle)$, where coefficients $\alpha$, $\beta$, $\zeta$, and $\eta$ satisfy the relations $|\alpha|^2 + |\beta|^2 = 1$ and $|\zeta|^2 + |\eta|^2 = 1$. After the photon pulse is reflected by the cavity, the state of the total system in the ideal case will evolve into $(-\eta | h \rangle + \zeta | v \rangle) \otimes (-\beta | l \rangle + \alpha | r \rangle)$, which corresponds to a quantum state exchange operation between photon and atom, apart from phase factors that can be eliminated by the appropriate subsequent logic operations [36]. We quantify the quality of the swap gate through a numerical simulation, and the parameters are referred to ref. [37], i.e., $(g_0, \kappa, \gamma_e)/2\pi = (27, 4.8, 6)$ MHz. Suppose that the initial state of the system is given by $|\Psi(0)\rangle = (| h \rangle + | v \rangle) \otimes (| l \rangle + | r \rangle)/2$, fig. 5 shows that the swap gate has a high fidelity and the variation of fidelity is about 0.01 for $g$ varying from $g_0$ to $g_0/2$.

Based on the above-mentioned swap operation, we consider the heralded quantum storage. Suppose that the photon is initially in the arbitrary polarized state $(\alpha | h \rangle + \beta | v \rangle)$ and the atom is prepared in the state $| r \rangle$. After the photon pulse is reflected by the cavity, the state of the system becomes

$$(\alpha | h \rangle + \beta | v \rangle) \otimes | r \rangle \rightarrow | h \rangle \otimes (\beta | l \rangle - \alpha | r \rangle).$$

The detection of the reflected photon in the state $| h \rangle$ heralds the mapping of the input polarization state onto the atom. We note that Fleischhauer et al. have proposed an interesting scheme for storing single-photon quantum states by adiabatic evolution of dark states, which has a high success probability, but has not been heralded [31]. For transmitting an entangled polarization state over long distances in realistic noisy channels, Lloyd et al. [32] proposed a robust method for capturing photon in optical cavities, and storing it in atoms, in which they detected if the atom has jumped out of the initial state, i.e., absorbed a photon, using the fluorescence technique by driving a cycling transition from the initial state of the atom to an accessorial excited state.

The approach for retrieving photonic states is similar to that for the storing process. A photon pulse in the state $| h \rangle$ is reflected by the cavity, the state evolution of the system can be represented by

$$| h \rangle \otimes (\beta | l \rangle - \alpha | r \rangle) \rightarrow (\alpha | h \rangle + \beta | v \rangle) \otimes | r \rangle.$$

After exchanging quantum states, the photonic qubit was successfully retrieved, and the atom is left in the initial state $| r \rangle$ for storage of another photon.

Quantum communication protocols. – One application of our scheme is the heralded quantum state
transfer and exchange between two atomic memories, as shown in fig. 6(a). Suppose that the initial state of a single-photon pulse and two atomic memories is \( |\Phi(0)\rangle = |h\rangle \otimes (\alpha|l\rangle_A + \beta|r\rangle_A) \otimes |l\rangle_B \), where \( |\theta\rangle_\chi = (\theta = l, r, \chi = A, B) \) denotes the quantum state of the atom trapped in cavity \( \chi \). The photon pulse is sequentially reflected by the cavities \( A \) and \( B \), the state of the combined system in the ideal case will evolve into \(|v\rangle \otimes |r\rangle_A \otimes (\alpha|l\rangle_B + \beta|r\rangle_B)\). The detection of the photon in the state \(|v\rangle\) heralds that the quantum state transfer from cavity \( A \) to cavity \( B \) has been performed successfully. If the initial state of the photon-atom system is prepared in \( |\Phi(0)\rangle = |h\rangle \otimes (\alpha|l\rangle_A + \beta|r\rangle_A) \otimes (\zeta|l\rangle_B + \eta|r\rangle_B) \), the photon pulse is sequentially reflected by the cavities \( A \) and \( B \), then reflected by the cavity \( A \) again. The state of the combined system will evolve into \(|h\rangle \otimes (\zeta|l\rangle_A + \eta|r\rangle_A) \otimes (\alpha|l\rangle_B + \beta|r\rangle_B)\). After the detection of the photon in the state \(|h\rangle\), one can know that the quantum states of the atoms trapped in cavities \( A \) and \( B \) have been directly exchanged.

Another immediate application is the heralded distribution of entanglement between two atomic memories, which is shown in fig. 6(b). A single-photon pulse in the state \(|v\rangle\) is divided into two paths using a 50/50 beam splitter (BS), one is reflected by a one-sided optical cavity trapped by a single atom with the quantum state \(|r\rangle\) and the other is reflected by a mirror. The state evolution of this process can be written as \(|v\rangle|r\rangle \rightarrow (|v\rangle|r\rangle + |h\rangle|l\rangle) / \sqrt{2} \), which is an atom-photon maximal entangled state. Then, the state of the photon is stored in another atomic memory by the detection of the reflected photon. After the successful generation of entanglement between two atomic memories within the attenuation length, one wants to extend the quantum communication distance. This is done through entanglement swapping [2,3,8,39]. Suppose that we start with two pairs of entangled memories. First, one of each entangled pair is mapped into a single photon with the retrieval operation. Second, two photons from two pairs of entangled ensembles are mapped using the Bell-state measurement with two-photon interference [40]. After an entangled state has been established between two distant memories, we can use it in entanglement-based communication protocols, such as quantum teleportation, cryptography, and Bell inequality detection with linear optics elements [3], since the quantum states between atomic memories and photons can be transferred in a reversible manner [1].

Next, we briefly address the experiment feasibility of the proposed schemes. We consider a \(^{87}\)Rb atom trapped in a one-sided Fabry-Perot cavity [37]. The states \(|l\rangle\) and \(|r\rangle\) correspond to \(|F = 1, m = -1\rangle\) and \(|F = 1, m = 1\rangle\) of \(5S_{1/2}\) ground levels, respectively, while \(|e\rangle\) corresponds to \(|F = 1, m = 0\rangle\) of \(5P_{3/2}\) excited level. The relevant cavity QED parameters for this system are assumed as \((g_0, \kappa, \gamma_c)/2\pi = (27, 4.8, 6)\) MHz [37], which fit well the condition \(4g_0^2/\kappa \gamma_c \approx 22\gg 1\). Suppose that the average photon absorption rate in a fiber is \(1 - e^{-L/L_{\text{att}}}\), where \(L\) is the length of the fiber and the channel attenuation length \(L_{\text{att}} \sim 22\) km [39]. If the single-photon detection efficiency is \(\eta_d \sim 0.95\) [39], the success probability for the heralded quantum state or the generation of entanglement between two atomic memories with the distance \(L \sim 10\) km is \(P_1 \sim P_2 \eta_d e^{-L/L_{\text{att}}} \sim 0.55\). For quantum communication over a long distance \(L_m = 2^mL\), one needs \(m\)-th \((m = 1, 2, \ldots, N)\) entanglement connection. The total probability to create entanglement across two communication nodes at a distance of 1280 km is about \(P_3 \prod_{n=1}^{N} P_n \sim P_3 \prod_{n=1}^{N} (\frac{1}{2} P_2^2 \eta_d^2 e^{-L/L_{\text{att}}} \eta_d)^n \sim 0.0002\).

**Conclusion.** - In summary, we have analyzed the heralded quantum memory for a single-photon polarization state with a single atom trapped in an optical cavity [37], and demonstrated its applications in the quantum communication network [1]. In our scheme, storage and retrieval of photon polarization states have a high probability and fidelity. Heralded storage, entanglement distribution, and quantum communication are achieved by detecting the reflected photon. Thus, in a realistic application operation errors due to all sources of photon loss, including atomic spontaneous emission, cavity mirror absorption and scattering, and photon collection, are always signaled by the absence of a photon count. As a result, the photon loss only decreases the success probability but has no contribution to the gate infidelity if the operation succeeds (i.e., if a photon count is registered). The ability to detect whether the operation has succeeded or not, is crucial for practical application.

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