Improving Vertical Positioning Accuracy with the Weighted Multinomial Logistic Regression Classifier

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Abstract In this paper, a method of improving vertical positioning accuracy with the Global Positioning System (GPS) information and barometric pressure values is proposed. Firstly, we clear null values for the raw data collected in various environments, and use the $3\sigma$-rule to identify outliers. Secondly, the Weighted Multinomial Logistic Regression (WMLR) classifier is trained to obtain the predicted altitude of outliers. Finally, in order to verify its effect, we compare the MLR method, the WMLR method, and the Support Vector Machine (SVM) method for the cleaned dataset which is regarded as the test baseline. The numerical results show that the vertical positioning accuracy is improved from 5.9 meters (the MLR method), 5.4 meters (the SVM method) to 5 meters (the WMLR method) for 67% test points.

Keywords vertical positioning · data correction · parameter estimation · multinomial logistic regression · support vector machine · global positioning system

1 Introduction

In recent years, the performance of the Global Positioning System (GPS) is excellent in outdoor environments [21]. When users are outdoors, their locations can be obtained accurately through GPS. However, since the GPS signals are blocked by the buildings and other obstacles, they result in large indoor positioning errors. Thus, the indoor positioning accuracy is often challenged, especially in the vertical direction. In
the meantime, the space we are living in is filled with many high-rise buildings and our most activities are indoors. Considering the practical requirement and the poor indoor positioning performance, researchers have tried many methods to improve the vertical positioning accuracy, such as the radio frequency identification positioning technology [28], the cellular-based positioning technology [24], the ultra-wide bandwidth location technology, the WiFi-based localization technology [8,15,30], the bluetooth positioning technology [25] and the vision-based positioning technology [10].

On the other hand, the GPS chip has been embedded in the most mobile terminals, which provides the location and timing information such as time, latitude, longitude, speed and altitude. Therefore, based on the GPS information, many researchers put forward some effective methods to improve the positioning accuracy of the low-cost GPS about 4 meters to 10 meters in several experiments [13]. Huang and Tsai propose an approach to calibrate the GPS position by using the context awareness technique from the pervasive computing and improve the positioning accuracy of GPS effectively [9]. The machine learning techniques are applied to assess and improve the GPS positioning accuracy under the forest canopy in [19].

In this paper, we provide another machine learning technique [1,2,3,20,22] based on the Multinomial Logistic Regression (MLR) method [11,12,16] for the vertical positioning problem. The research data are measured by many different user equipments and provided by Huawei Technologies Company, some data of which include the GPS three-dimensional information and the barometric pressure values. Some data miss the GPS information or the barometric pressure values. We preprocess the research data firstly. Consequently, we identify the abnormal data with $3\sigma$-rule and clear them. Meanwhile, some noises arises from the inaccurate data records and the different reference standards of different kinds of user equipments. These intrinsic noises lead to a poor distribution law between the air pressure and the corresponding altitude. In order to overcome these noise effects, we convert this vertical positioning problem into a classification problem and revise the weighted MLR method to improve its vertical positioning accuracy. Finally, in order to verify the effect of the WMLR method, we compare the MLR method, the WMLR method, and the Support Vector Machine (SVM) method [5,6,7] for this vertical positioning problem. The numerical results show that the vertical positioning accuracy of the cleaned data is improved from 5.9 meters (the MLR method), 5.4 meters (the SVM method) to 5 meters (the WMLR method) for 67% test points.

The rest of the paper is organized as follows. Section 2 describes the methodology of the data cleaning, the outlier detection and the data correction based on the WMLR classifier. In Section 3, we describe the data source and compare the MLR method, the WMLR method and the SVM method for the cleaned data which is regarded as the test baseline. The promising numerical results are also reported. Finally, some conclusions and the further work are discussed in Section 4.
2 The Methodology

Our positioning method is composed of several stages, including the data cleaning, the outlier detection, the data correction and the prediction of vertical altitude for the test feature vector. We described these procedures in the following subsections.

2.1 Data Cleaning

The raw dataset is measured at different places with different user equipments. In the dataset, many data miss the air pressure values due to some mobile devices without the barometers. We delete these data of the missing air pressure values firstly. Additionally, there are some abnormal data which deviate too far from the average value of the dataset and it is shown as follows. Assume that an average sea level pressure is 1013.25 hPa and the corresponding temperature is 15°C, then the air pressure value and its corresponding altitude have the following relationship [27]:

\[ h = 44330.8 - 4946.54p^{0.1902632}, \]  

where the unit of altitude \( h \) is meter, and the unit of the air pressure value \( p \) is hpa. From formula (1), it is not difficult to find that the barometric pressure value and the corresponding altitude are the inverse relationship. However, from Fig. 1, we find that the distribution between the air pressure values and the corresponding altitudes of the given data is irregular. Therefore, we conclude that there exists the data drift in the given real test data. Thus, we use the 3\( \sigma \)-rule to exclude the abnormal data as follows [26]:

\[ X \text{ is thrown away when } |X - \mu| \geq 3\sigma, \]

where the mean \( \mu \) and the variance \( \sigma \) are estimated by the following formula:

\[ \mu = \frac{1}{n} \sum_{i=1}^{n} X_i \text{ and } \sigma = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (X_i - \mu)^2}. \]

After performing the 3\( \sigma \)-rule, we eliminate the large deviation data and the 99.73% data are retained.

2.2 Outlier Detection

In subsection 2.1 we have cleaned away the abnormal data which deviate too much from the dataset. However, there are still some outliers. An outlier is a point which differs significantly from the other points in a subdataset measured by the same device in a short time. We use the spherical distance computed by the haversine formula [23] to identify the outlier. The haversine formula is illustrated by Fig. 2 and calculates...
the spherical distance between the two points \( A(\text{lon}_a, \text{lat}_a) \) and \( B(\text{lon}_b, \text{lat}_b) \) with the coordinate \((\text{longitude}, \text{latitude})\) as follows:

\[

d_{AD} = 2R \sin(\Delta \text{lon}/2) \cos(\text{lat}_a),

d_{CB} = 2R \sin(\Delta \text{lon}/2) \cos(\text{lat}_b),
\]

\[

d_{AB} = 2R \left| \sin^2 \left( \frac{\Delta \text{lat}}{2} \right) + \cos(\text{lat}_a) \cos(\text{lat}_b) \sin^2 \left( \frac{\Delta \text{lon}}{2} \right) \right|^{\frac{1}{2}},
\]

(2)

where \( \Delta \text{lon} = \text{lon}_b - \text{lon}_a \), \( \Delta \text{lat} = \text{lat}_b - \text{lat}_a \), and \( R \) is the radius of the Earth.

Consequently, we estimate the diameter of a subdataset as follows:

\[

d_{\text{max}} = \bar{\nu} \times t,
\]

(3)

where \( \bar{\nu} \) is the mean velocity, and \( t \) is the total measuring time of the subdataset. On the other hand, each point has a distance vector with other points. If over 50% elements of the distance vector are greater than \( d_{\text{max}} \), we regard this point as an outlier.

2.3 Data Correction

In this subsection, we describe the procedure of data correction and it is also the key step of our positioning method. This step is to predict the relatively accurate altitudes of the outliers. As mentioned in section 2.2, the data distribution is roughly similar when the data are measured by the same device. Under this assumption, the altitudes of the subdataset are classified into different classes (labels). Thus, we encounter the multi-class classification problem.
2.3.1 The Multi-class Classification Problem

The outliers of the subdataset have been found with the method in section 2.2. Thus, we select the data except outliers as a training dataset. The input training dataset is composed of \( N \) pairwise points \((X_n, h_n)\) \((n = 1, 2, \ldots, N)\), where \( X_n \) is the feature vector of the \( n \)-th point and \( h_n \) is the corresponding altitude. Denote \( h_{\text{min}} \) and \( h_{\text{max}} \) as the minimum altitude and the maximum altitude, respectively. Parameter \( \delta (h_{\text{min}} < \delta < h_{\text{max}}) \) is the quantization step of altitude. Then, for a given altitude \( h \), its corresponding class \( k \) is computed as follows:

\[
k = \left\lceil \frac{h - h_{\text{min}}}{\delta} \right\rceil + 1,
\]

where \( h_{\text{min}} \leq h \leq h_{\text{max}} \), \( \left\lceil \cdot \right\rceil \) is a function which will round the value toward positive infinity. When the predicted class of a point is obtained, we take the average altitude of its corresponding interval as the predicted altitude and which is computed by the following formula:

\[
h^p_k = \left( k - \frac{1}{2} \right) \delta + h_{\text{min}}, \quad k = 1, 2, \ldots, K.
\]

Thus, after the above transformation procedure, the data correction problem is converted into a multi-class classification problem (see Table 1 where \( K \) represents the number of classes and \( K = \left\lceil (h_{\text{max}} - h_{\text{min}})/\delta \right\rceil \)).

2.3.2 The Weighted Multinomial Logistic Regression Model

Logistic Regression (LR) is a machine learning method and widely used to the binary classification problem [4]. The MLR method extends the binary LR method to
Table 1 The class, its corresponding interval and predicted altitude.

| Class | Interval (meter) | Predicted altitude (meter) |
|-------|------------------|----------------------------|
| 1     | $h_{	ext{min}} \sim h_{	ext{min}}$ | $\frac{1}{2}h_{	ext{min}}$ |
| 2     | $h_{	ext{min}} + h_{	ext{min}} \sim 2h_{	ext{min}}$ | $\frac{3}{2}h_{	ext{min}}$ |
| ...   | ...              | ...                        |
| $k$   | $(k-1)h_{	ext{min}} + h_{	ext{min}} \sim kh_{	ext{min}}$ | $(k - \frac{1}{2})h_{	ext{min}}$ |
| ...   | ...              | ...                        |
| $K$   | $(K-1)h_{	ext{min}} + h_{	ext{min}} \sim Kh_{	ext{min}}$ | $(K - \frac{1}{2})h_{	ext{min}}$ |

the multiple classification problem. For the MLR model, each class has its parameter vector. According to the parameter vector and the data feature vector, the MLR method determines the classification of the data. In the positioning application scenario, every feature vector consists of time, longitude, latitude, air pressure value and altitude.

The training process of the MLR model needs to obtain the parameter $\omega_k$ of the $k$-th class via solving the the maximum likelihood function [29], where $k = 1, 2, \cdots, K$. The conditional probability of the feature vector $X$ belonging to the class $Y$ is given by the following formula:

$$P(Y = k | X = x) = \frac{e^{\omega_k^T x}}{\sum_{i=1}^{K} e^{\omega_i^T x}}, \quad k = 1, 2, \cdots, K.$$  \hspace{1cm} (5)

Then, the MLR method predicts the data category $k^*$ via solving the following maximum problem:

$$k^* \in \arg\max_{k \in \{1, 2, \cdots, K\}} P(Y = k | X = x).$$  \hspace{1cm} (6)

After the data preprocessing of the previous steps, we obtain the training dataset, which consists of $N$ pairwise points $(X_n, Y_n) (n = 1, 2, \ldots, N)$, where $X_n$ represents the data feature vector and $Y_n$ represents its corresponding data class. According to formula (5) and the independent assumption of the multivariate distribution, we obtain the likelihood function as follows:

$$\prod_{n=1}^{N} P(Y = Y_n | X = X_n) = \prod_{n=1}^{N} \left( \frac{e^{\omega_{Y_n}^T X_n}}{\sum_{k=1}^{K} e^{\omega_k^T X_n}} \right).$$  \hspace{1cm} (7)

Taking the logarithm of the two sides of formula (7), we obtain the following log-likelihood function:

$$\log \left( \prod_{n=1}^{N} P(Y = Y_n | X = X_n) \right) = \sum_{n=1}^{N} \left( \omega_{Y_n}^T X_n - \log \left( \sum_{k=1}^{K} e^{\omega_k^T X_n} \right) \right).$$  \hspace{1cm} (8)
Since the value of expression (8) is less than zero, we define function $f(\Omega)$ as

$$f(\Omega) = \sum_{n=1}^{N} \left( -\omega Y_n T X_n + \log \left( \sum_{k=1}^{K} e^{\omega Y_n T X_k} \right) \right),$$

(9)

where $\Omega = [\omega_1, \omega_2, \ldots, \omega_K]$. Then, we obtain the maximum likelihood estimation $\Omega^*$ of parameter matrix $\Omega$ via solving the following optimization problem:

$$\Omega^* = \arg\min_{\Omega} f(\Omega).$$

(10)

Since the training dataset is separable, the value of function $f(\Omega)$ can be made arbitrarily close to zero via multiplying $\Omega$ by a large value [12]. In order to maintain the finiteness of $\Omega$, we obtain the parameter matrix $\Omega^*$ by solving its regularized problem of problem (9) as follows:

$$\Omega^* = \arg\min_{\Omega} (f(\Omega) + \lambda \eta(\Omega)),$$

(11)

where $\lambda > 0$ is the regularized parameter and the regularized function $\eta(\Omega)$ is convex and non-smooth. For this convex optimization problem, there are many efficient optimization methods to tackle it such as the quasi-Newton BFGS method (p. 198, [18]). Once the MLR model has been trained, we can predict the data category via solving the maximum problem (6).

We denote $I = \{1, 2, \ldots, I\}$ as the index set of the feature vector $X$, where $I$ represents the dimension of the feature vector $X$. Select randomly $r$ features from $I$ features and record the index of selected features as the subset $S$ of the index set $I$. Since the $\ell_1$ regularizer is easier to obtain a sparse solution than the $\ell_2$ regularizer, we define a group-$\ell_1$-regularizer as

$$\eta_S(\Omega) = \sum_{i \in S} \| [\Omega]_{ik} \|_1,$$

(12)

where $[\Omega]_{ik}$ is the $i$-th row of parameter matrix $\Omega$, and $\|x\|_1 = \sum_{i=1}^{m} |x_i|$ for vector $x \in \mathbb{R}^m$. Thus, the problem (11) is written as the following group-sparse problem:

$$\min_{\Omega} (f(\Omega) + \lambda \eta_S(\Omega)).$$

(13)

If the parameter $\lambda$ is suitably selected, the solution $\Omega^*$ of problem (13) will be group-row-sparse [14].

After $L$ operations as the procedure above, we obtain $L$ parameter matrices $\Omega_1^*, \Omega_2^*, \ldots, \Omega_L^*$. Multiply the $L$ parameter matrices $\Omega^*_l (l = 1, 2, \ldots, L)$ by their corresponding sub-features, then we obtain the predicted categories $k^*_l (l = 1, 2, \ldots, L)$ with formulas (5)-(6) and its predicted altitudes $h^*_l (l = 1, 2, \ldots, L)$ with formula (4) as follows:

$$k^*_l = \arg\max_{k \in \{1, 2, \ldots, K\}} (\omega_k T)^T [X]_{S_l}, \text{ and } h^*_l = (k^*_l - \frac{1}{2}) \delta + h_{\min}, \quad l = 1, 2, \ldots, L,$$

(14)
where \([X]_{S_l}\) represents the sub-features selected from the feature vector \(X\) and the \(i\)-th element of \([X]_{S_l}\) equals \(X(S_l(i))\), \(\omega^*_{k_l}\) is the \(k\)-th element of matrix \(\Omega_l\).

Compute \(L\) absolute errors between the original altitude \(h\) and the \(l\)-th predicted altitude \(h^p_l\) \((l = 1, 2, \ldots, L)\) as follows:

\[
Err_l = |h - h^p_l|, \ l = 1, 2, \ldots, L.
\]

Then, we obtain the weighted predicted altitude of the feature vector as follows:

\[
h^* = \sum_{l=1}^{L} w_l h^p_l,
\]
where the weighted coefficients \(w_l (l = 1, 2, \ldots, L)\) are computed by the following formula:

\[
w_l = \frac{Err_l}{\sum_{l=1}^{L} Err_l}, \ l = 1, 2, \ldots, L.
\]

According to the above discussions, we give the weighted multinomial logistic regression method for the vertical position problem in Algorithm 1.

**Algorithm 1** The WMLR method for the vertical positioning problem

**Input:** The training data \((X_n, h_n); n = 1, 2, \ldots, N\);

The test feature vector \(X\) and its corresponding altitude \(h\).

**Output:** The predicted altitude \(h^*\) of the feature vector \(X\).

1: Given the regularized parameter \(\lambda\), the dimension \(r\) of the sub-feature vector, the quantization step \(\delta\) of altitude, the number of the group-sparse operations \(L\).
2: for \(l = 1, 2, \ldots, L\) do
3: Select randomly \(r\) features from every feature vector of the training dataset and denote its corresponding index set of \(r\) features as \(S_l\).
4: Obtain the \(l\)-th regression coefficient matrix \(\Omega^*_{l}\) via solving the optimization problem \(\Omega^*_{l} = \text{argmin} (f(\Omega) + \lambda \eta_{S_l}(\Omega))\), where \(f(\Omega)\) is defined by equation (11) and \(\eta_{S_l}(\Omega)\) is defined by equation (12).
5: Obtain the predicted category \(k^*_l\) and the \(l\)-th predicted altitude \(h^p_l\) of the feature vector \(X\) via solving problem (14).
6: Compute the absolute error \(Err_l\) between the original altitude and the predicted altitude of the feature vector \(X\) from equation (15).
7: end for
8: Compute \(L\) weighted coefficients \(w_l (l = 1, 2, \ldots, L)\) from equation (17).
9: Obtain the weighted predicted altitude \(h^*\) of the feature vector \(X\) from equation (16).

3 Numerical Experiments

In this section, we compare the MLR method, the WMLR method (Algorithm 1) and the SVM method (coded by C. Chang and C. Lin. [6]) for the vertical positioning problem. The programs are performed on the MATLAB environment [17].
Fig. 3 The data volume of the corresponding userID of the raw dataset

The raw dataset is provided by Huawei Technologies Company and collected by different user equipments. From Fig. 3, we find that there are 12796 UserIds and the number of data collected by each UserId is different. In the dataset, each piece of data includes time, longitude, latitude, speed, altitude and some data also contain barometric pressure value. The measurement time of the experiment dataset spans almost three months from October 5 to December 25, 2018. The air pressure is relatively high because the temperature is relatively low in that season. Except for null values, the data type is numeric.

Since the raw dataset contains many null and abnormal values, we exclude those null and abnormal values with the method in subsection 2.1. Table 2 presents the statistical results of the cleaned data. From Table 2, we find that the distribution of data is not Gaussian. Thus, we standardize and normalize the data. After the data cleaning and normalization, we obtain a training set, every data element of which includes time, speed, longitude, latitude, pressure. We divide the dataset into two parts, i.e. 70% data for training and 30% data for testing.

Then, in order to verify the effect of Algorithm 1 (the WMLR method), we compare the performance of the MLR method, Algorithm 1 and the SVM method for the cleaned data. For Algorithm 1, we set the regularized parameter $\lambda = 10^{-3}$, the length of the group-sparse feature $r = 4$ and $L = C_4^5 = 5$. The numerical results are put in Table 3 and Fig. 4. From Table 3, we find that the vertical positioning accuracy is improved from 5.9 meters (the MLR method), 5.4 meters (the SVM method) to 5 meters (the WMLR method) for 67% test points. Therefore, the WMLR method has some improvements of the positioning accuracy for this vertical positioning problem.
Table 2  The statistical results of the cleaned data.

|        | longitude | latitude | speed   | pressure | label | altitude |
|--------|-----------|----------|---------|----------|-------|----------|
| mean   | 121.5767  | 31.2595  | 5.8808  | 1021.3788| 0.9181| 22.9314  |
| std    | 0.0030    | 0.0020   | 6.7051  | 1.2559   | 0.2742| 10.9594  |
| min    | 121.5708  | 31.2566  | 0.0000  | 1017.1787| 0.0000| 0.0534   |
| 25%    | 121.5742  | 31.2579  | 1.0000  | 1020.5680| 1.0000| 15.7657  |
| 50%    | 121.5765  | 31.2590  | 3.0000  | 1021.3281| 1.0000| 20.1303  |
| 75%    | 121.5792  | 31.2610  | 10.0000 | 1022.3744| 1.0000| 28.5893  |
| max    | 121.5820  | 31.2653  | 26.0000 | 1024.0759| 1.0000| 78.1991  |

Table 3  Vertical positioning accuracies (m) of the MLR, WMLR and SVM methods.

|       | Min   | Max   | Mean  | Median | Std   | 67%   | 90%   |
|-------|-------|-------|-------|--------|-------|-------|-------|
| MLR   | 0.0211| 48.8268| 5.9795| 4.4133 | 6.7941| 5.9705| 11.7054|
| WMLR  | 0.0211| 31.9072| 4.6628| 3.2539 | 3.2539| 5.0216| 10.1085|
| SVM   | 0.0211| 25.2855| 4.9508| 3.9297 | 4.0743| 5.4383| 10.3968|

Fig. 4  The comparison of three different methods.

4 Conclusion and Future Work

In this paper, a vertical positioning method with GPS information and the air pressure values is proposed. Firstly, we clean the missing and abnormal data. Then, according to the spherical distance matrix between points, we identify and exclude outliers. Consequently, we divide the cleaned data into two parts, i.e. 70% data for training and 30% data for testing. Based on the cleaned data, we compare the performances of the MLR method, the WMLR method (Algorithm 1), and the SVM method for this
vertical positioning problem. The numerical results show that the vertical positioning accuracy is improved from 5.9 meters (the MLR method), 5.4 meters (the SVM method) to 5 meters (the WMLR method). Therefore, the WMLR method has some improvements of the positioning accuracy for this vertical positioning problem and can be used as a workhorse in practical applications. However, due to the heterogeneity of user equipments and the complexity of the real environment, there are some room of improvement on the vertical positioning accuracy of the WMLR method.

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