Distributed Preemption Decisions: Probabilistic Graphical Model, Algorithm and Near-Optimality

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Abstract—
Cooperative decision making is a vision of future network management and control. Distributed connection preemption is an important example where nodes can make intelligent decisions on allocating resources and controlling traffic flows for multi-class service networks. A challenge is that nodal decisions are spatially dependent as traffic flows trespass multiple nodes in a network. Hence the performance-complexity trade-off becomes important, i.e., how accurate decisions are versus how much information is exchanged among nodes. Connection preemption is known to be NP-complete. Centralized preemption is optimal but computationally intractable. Decentralized preemption is computationally efficient but may result in a poor performance. This work investigates distributed preemption where nodes decide whether and which flows to preempt using only local information exchange with neighbors.

In this work, we first model a large number of distributed preemption-decisions using a probabilistic graphical model. We then define the near-optimality of distributed preemption as its approximation to the optimal centralized preemption within a given error bound. We show that a sufficient condition for distributed preemption to be optimal is that local decisions should constitute a Markov Random Field. The decision variables, however, do not possess an exact spatial Markov dependence in reality due to the flows passing through multiple links. Hence we study traffic patterns of flows, and derive sufficient conditions on flows for the distributed preemption to be near-optimal. We develop, based on the probabilistic graphical models, a near-optimal distributed algorithm. The algorithm is used by each node to make collectively near-optimal preemption decisions. We study trade-offs between near-optimal performance and complexity that corresponds to the amount of information-exchange of the distributed algorithm. The algorithm is validated by both analysis and simulation.

Index Terms—Distributed preemption decision, Complexity, probabilistic graphical models, Probabilistic inference.

I. INTRODUCTION

A vision of future network management is to involve nodes to make intelligent decisions on allocating resources and controlling traffic flows. This includes admitting new flows by preempting less important existing flows, which is well studied in the policy based admission control (i.e., admission is based on the priority of flows) [8] [15]. Specifically, preemption is defined at a prioritized multi-class network, where a new call needs to be set up with a high priority between a source (S) and a destination (D) [8] [13] [15] [25] [27]. When the capacity is insufficient at all feasible routes between the source-destination (S-D) pair, some existing flows of the lower priorities need to be forced to reduce their bandwidth, move to the lowest service class (e.g., best-effort-service), or simply preempted to accommodate the new call. Preemption decisions are to decide which lower priority flows to remove to free the reserved bandwidth for the new call at a chosen route [8] [27].

total preempted bandwidth can be minimal under such constraints as bandwidth demand of a new call and available free bandwidth at each link.

The benefit of preemption has been described in the prior works. For example, preemption allows a new high-priority connection to access heavily crowded core networks, e.g., multi-protocol label switched (MPLS) networks [30]. Connection preemption also improves resource utilization by allowing low-priority flows to access unused bandwidths [11] [30]. Preemption sees potential applications in emerging networks. For example, in 802.11e Wireless LAN, delay sensitive IP packets in expedited forwarding (EF) class can be served earlier than the best-effort packets through preemption [26]. Multi-level preemption and precedence (MLPP) is proposed to classify calls by their importance, which can be used for military as well as commercial networks [1].

There are two significant challenges for preemption which are performance and complexity. Performance corresponds to whether right flows are preempted to result in the minimal bandwidth to accommodate a new flow. Complexity corresponds to the amount of information needed for preemption decision. Preemption is known to be NP-complete [8]. The complexity results from a large number of active flows supported by a core network for which preemption decisions need to be made. For example, for a 1Gbps link, if the bandwidth of each flow is in the order of Kbps, there would be thousands of flows supported per link. In addition, a flow generally passes through multiple nodes, making preemption decisions among nodes dependent and thus difficult to be done with local information. Thus preemption is network-centric, and may require a huge amount of information to perform in a large network.

For centralized preemption decisions, a centralized node maintains the routed-path information of active flows, their priorities and bandwidth occupancies at the entire route. The centralized node then decides which active flows to preempt upon the request of a new call. Therefore, centralized preemption can always be optimal, resulting in minimal preempted bandwidth. But the amount of management information needed can be overwhelming at the centralized node. For example, let $F_t$ be the total number of distinct flows per priority class at the route of a new call. Each flow has two states, preempted or not preempted. The total number of possible states is $O(2^{F_t})$ for making a centralized decision. When $F_t$ is in the order of hundreds or thousands [14], centralized preemption becomes computationally intractable. Decentralized preemption is then adopted for reducing the amount of management information [27].

The preempted flows are usually rerouted to other paths. Hence preemption and rerouting can be considered jointly with somewhat different objectives [50]. This work, however, focuses on preemption on a given path without considering rerouting.
Decentralized preemption is done at each node individually, and thus requires a node to maintain its local information, i.e., active flows at the adjacent links, their priorities and bandwidth occupancy. Such information is available locally at nodes. A node then decides, independently from the other nodes, which connections to preempt. This, however, may cause conflicting local decisions on the same flows that pass multiple links on the route, resulting in more preempted bandwidth than necessary. In other words, decentralized preemption decision neglects the spatial dependence for the flows across multiple links, and may perform poorly. But the amount of management information are greatly reduced compared with centralized preemption.

For example, let $F$ be the maximum number of active flows per link. The total number of states is $2^F$ at each link. Since $2^F \ll 2^F$, compared with centralized preemption, decentralized schemes have a much smaller search space for preemption decisions. Therefore, most algorithms in the literature focus on centralized preemption (see [25] [27] and references there in).

This work studies distributed decisions, that take into account spatial dependence among neighboring links through local information exchange. In fact, distributed preemption can be considered as a generalization of centralized and decentralized preemption. Centralized preemption corresponds to one extreme case of distributed preemption that an entire route is the neighborhood for information exchange; whereas, decentralized decisions correspond to another extreme case where the neighborhood size is zero. Therefore, the communication complexity can be characterized in terms of neighborhood size. There is a trade-off between the optimality and the complexity.

In general, it has been shown to be a difficult problem to develop a distributed algorithm whose performance is predictable and within a tolerable degradation (i.e., given error bound) from that of the optimal scheme [31]. Hence, the open issues are: (a) When can distributed decisions collectively result in a near-optimal global preemption? (b) How to model a large number of dependent decision variables and to obtain near-optimal local decisions using distributed algorithms? We apply machine learning to study these issues.

**Machine learning perspective:** A machine learning view of distributed preemption is that individual nodes “learn to make decisions” collectively and iteratively. Ideally, if each node has complete information on all active flows at the route of a new flow, the node will be able to make correct decisions on which flows to preempt. However, at any given time, a node has only partial information on the active flows on the route and its neighbors’ decisions on the flows to preempt. But a node can adapt, i.e., learn to make decisions based on those of its neighbors’. As neighbors learn from neighbors’ neighbors, a node would indirectly learn what farther nodes decide only with a delay. Eventually, all nodes would make local decisions, collectively resulting in a near-optimal preemption at the entire route.

How would machine learning benefit distributed preemption? The problem of collective learning and decision-making has been a keen interest in machine learning and adaptive control [3] [10], but has just begun to see applications in networking. In particular, [7] proposes using Markov Random Fields as a general model of decision-making in Ad hoc wireless networks. The model is natural to learn, and [22] obtain probabilistic graphical models for ad hoc wireless and wireline networks starting from network properties [22] [23], and the resulting probabilistic models turn out to be multi-layer. This work focuses on distributed decisions on network flows. We view machine learning as a framework in which a large number of decision variables can be treated jointly. Spatial dependence among these variables poses a key challenge to preemption, is an origin of high communication complexity, and has not been dealt with sufficiently in prior works. Machine learning provides feasible approaches for this problem as summarized below.

(a) **Global model of distributed preemption decisions:** We first develop a probabilistic model that represents explicitly the spatial dependence of distributed preemption decisions over a pre-determined preempting route of a new flow. The randomness results from randomly arriving/departing active flows and their locations. The preemption decisions made on flows at each node are also random due to incomplete and inaccurate local information for distributed preemption. We first obtain a cost function for preemption as a “Hamiltonian” (or “system potential energy”) [21]. A Hamiltonian combines local preemption decisions and constraints into a single quantity. The constraints include link capacity, unused bandwidths and bandwidth-demand of a new flow at each link. The Hamiltonian is then used to obtain a spatial probabilistic model as a Gibbs distribution [9].

(b) **Markov Random Field (MRF) and sufficient conditions:** Spatial dependence can be characterized through a probabilistic dependency graph of graphical models [9] [17] [19] in machine learning. A probabilistic dependency graph provides a simple yet explicit representation of the spatial dependence among random variables. We show that if the dependence of decision variables is spatially Markovian, a globally optimal preemption decision can be obtained collectively by iterative local decisions through information exchange only with neighboring nodes. Such a probabilistic model is known as a Markov Random Field [9].

In general, distributed decisions may not be spatially Markov, since the spatial dependence is caused by flows across multiple links. Hence we identify traffic patterns of active flows that result in approximately spatial Markov dependence. We then define the near-optimality of distributed decisions as the difference between the centralized and distributed decisions, measured in the Hamiltonian, and obtain sufficient conditions for the difference to reside within an error bound.

(c) **Distributed Decision Algorithm:** A near-optimal distributed algorithm is derived based on the Markov Random Field. The algorithms can be implemented either message passing [19] or Gibbs sampling [9].

(d) **Trade-offs:** A challenging issue is the performance-complexity trade-off, i.e., “when” and “how” distributed preemption can achieve a near-optimal performance with a moderate complexity. Here the performance measures the optimality of distributed preemption decision relative to that of the centralized optimal decision. The communication complexity of distributed preemption can be characterized by the amount of information used in distributed decision making. Distributed decisions reduce complexity using information exchange only with neighbors, but may deviate from the optimal performance. Hence we study performance and complexity trade-off through both analysis and simulation.

The rest of this paper is organized as follows: Section II presents model and sufficient conditions for the difference to reside within an error bound. Section III obtains near-optimality of distributed decisions as the difference between the centralized and distributed decisions, measured in the Hamiltonian, and obtains sufficient conditions for the difference to reside within an error bound. Section IV presents algorithmic decision.
vides a problem formulation on connection preemption. Section III develops a probabilistic spatial model of distributed preemption, utilizing the graphical models in machine learning and interpreting the derived model in terms of optimality and complexity. Section V proposes a distributed preemption algorithm based on the derived model, using probabilistic inference. Section VI analyzes the performance of distributed preemption. Section VII validates the performance of distributed preemption through simulation. Section VIII provides a further literature review and discussions. Section VIII concludes the paper.

II. DISTRIBUTED PREEMPTION

A. Example

(a) A new call arrival at a network

(b) Preemption decisions at each hop over the route of a new connection

Fig. 1

EXAMPLE OF PREEMPTION

Figure 1(a) shows an example network, and Figure 1(b) illustrates distributed preemption on a given route. Assume that a new call request is made on the route between the SD pair, and for simplicity, all flows have the same bandwidth. To accommodate the new flow, the centralized preemption would preempt two existing flows, i.e., flow 1 and flow 2 that are marked with X. Such a preemption decision is obviously optimal. Now consider distributed preemption. In reality, distributed preemption decisions are made at nodes. For notational convenience, we regard the decision is made at the link which connects the left node. For example, link 2 is the link beginning with node 2. The local information available at node 2 includes the priorities and the bandwidths of flows 1, 3 and 4 that pass through this node. When the bandwidth is the same for all flows, node 2 may decide to preempt flow 4 without knowing that nodes 1 and 3 both decide to preempt flow 1. Such a decision would result in more flows to be preempted than necessary compared with the centralized decisions. In contrast, node 2 would choose to preempt flow 1 if node 2 also has the information on the decisions made by the neighbors (nodes 1 and 3). This example shows the following facts.

2 Note that preemption in a general context can be considered as removing a high priority flow to the best-effort class rather than completely terminating the service for the flow.

3 For example, if flow $k_1$ and $k_2$ belong to class 1 and 2 respectively, $\alpha_{k_1} < \alpha_{k_2}$.

(b) The spatial dependence can be taken into consideration by exchanging local decisions among neighbors. How many nodes should exchange local decisions depend on extents of flows.

(c) The information exchanged would result in consistent decisions across a network, and thus improve the optimality of local preemption done at nodes.

These facts motivate cooperative distributed preemption formulated below.

B. Problem Formulation

Assumptions: We assume that a preempting route $R_p$ is predetermined for a new connection $R_n$, and composed of node 1, $\cdots$, and node $L + 1$. We assume that the traffic flows on the route belong to multiple priority classes 1, $\cdots$, $i_{\text{max}}$, and a new connection belongs to class $i_{\text{new}}$ and demands bandwidth $c_{\text{new}}$.

Variables: Let $S_F$ be a set of all active flows on route $R_p$, where $S_F = \{f^1, \cdots, f^{|S_F|}\}$ with $|S_F|$ being the cardinality of set $S_F$. $f^k$ and $B^k$ denote flow $k$ on the preemting route and its bandwidth. We consider the decisions at links. For notational convenience, without causing any confusion between nodes and links, we denote link $i$ as the link between node $i$ and node $i + 1$ for simplicity for $1 \leq i \leq L$, where $L$ is the number of links on the considering route. Let $f_i$ be the set of all active flows at link $i$ for $1 \leq i \leq L$.

Let $d_{ik}^k$ denote the preemption decision on flow $k$ at link $i$ for $1 \leq i \leq L$. $d_{ik}^k = 1$ if link $i$ decides to preempt the flow; otherwise, $d_{ik}^k = 0$. Let $d_i$ denote the set of local preemption decisions on all active flows at link $i$. Let $d$ denote all local decisions on the route, then $d = \{d^k, 1 \leq k \leq |S_F|\}$, where $d^k$ denotes the preemption decision on flow $k$ over the entire preemting route, $d^k = 1 - \prod_{1 \leq i \leq L} (1 - d_{ik}^k)$. Hence, $d^k = 1$, i.e., flow $k$ is preempted from the given route, if at least one link decides to remove the flow, and $d^k = 0$ if all links decide to keep the flow.

Hence, $d^k = 1$ is a global decision of the entire path, and a local decision $d_{ik}^k (1 \leq i \leq L$ and $k \in f_i)$ impacts the global decision.

Problem statement: Assume that the following information is maintained at node $i$ ($1 \leq i \leq L + 1$): (a) complete local information on the active flows at link $i$ which includes flow ID $k$, class priority of flow $k$, bandwidth of flow $k$ $B^k$, for $k \in f_i$ and $1 \leq k \leq |S_F|$; and (b) neighbor information that includes decisions from the neighboring links within $N_d$ hops for $N_d \geq 1$.

Given $\alpha_k (\alpha_k > 0)$ as the priority weight of flow $k$, and $B^0_k \geq 0$ as the amount of unused bandwidth at link $i$, for $1 \leq i \leq L$, preemption is to obtain a set of decisions $d_{ik}^k$’s that

\[
\text{minimize} \quad \sum_{1 \leq k \leq |S_F|} \alpha_k B^k d^k \quad (1)
\]

subject to

\[
c_{\text{new}} \leq A_i, \quad d_{ik}^k \in \{0, 1\}, \quad \text{for } 1 \leq i \leq L,
\]

where $A_i = \sum_k B^k d_{ik}^k + B^0_k$ is the total available bandwidth at link $i$ for the new flow. The constraint requires that the sum of the unused bandwidth before preemption and the preempted bandwidth

\[
1 \leq i \leq L.
\]
at any link \( i \) should be sufficiently large for accommodating the new flow.

**Goal:** The goal of this work is to approximate this global optimization problem through distributed preemption decisions. In particular, the first step is to derive a distributed algorithm that obtains a set of local preemption decisions made at links through information exchange with neighbors. The second step is to obtain near-optimal conditions under which distributed decisions approximate the globally optimal preemption decisions.

Note that the objective function in (1) is the cost corresponding to the total preempted bandwidth. Such an objective function is used by most of the existing works [27] [8] [29]. Since \( d_k^i = 1 - \prod_{i=1}^{L} (1 - d_k^i) \), a flow is preempted if at least one node decides to preempt the flow, i.e., \( d_k^i = 0 \) for at least one \( i \) and a given \( k \).

Hence, the global optimization in (1) requires making consistent decisions on the same flow at all links over the preempting route, i.e., \( d_k^i \)'s are all equal for a given \( k \).

Distributed preemption is to determine \( \{d_k^i\} \) for \( 1 \leq i \leq L \) and \( k \in f \) that minimize (1) using the local and the neighbor information. Hence the objective of global optimization is still global over the entire path. But the management-information exchange is local for making preemption decisions. Hence a key challenge is how to model and coordinate a large number of spatially-dependent local decisions to achieve the global objective of preemption in a fully distributed fashion.

Table 1 summarizes our notations.

| Table 1 | Notations |
|---------|-----------|
| \( d_k^i \) | Local preemption decision made at link \( i \) for flow \( k \) |
| \( d_k \) | Global preemption decision of flow \( k \) of the path |
| \( B_k \) | Bandwidth of flow \( k \) |
| \( L \) | The total number of links at the path |

### III. Probabilistic Spatial Model of Preemption Decisions

We begin by developing a global model to represent the spatial dependence of a large number of distributed preemption decisions. We then derive a local model as an approximation. The global and local models are developed through probabilistic graphical models in machine learning.

#### A. Global Model

A global model should include accurate spatial dependence resulting from flows, objectives and constraints on distributed preemption decisions.

1) Deterministic Flows

We first consider an example with a given set of active flows. Let \( H(d) \) be the cost for setting up a new connection of a high priority. We express \( H(d) \) by expanding the cost from (1) and using the Lagrangian multiplier for the constraints,

\[
H(d) = \sum \alpha_k B^k \left( \sum d_k^i - d_k \right) 
\]

where \( \alpha_k \) is a Lagrangian multiplier, \( 1 \leq i \neq i_2 \leq L \), \( 1 \leq k \leq |S_F| \), \( U(x) \) is an indicator function: \( U(x) = 0 \) if \( x \leq 0 \), \( 1 \), otherwise. The \( \beta \)-term corresponds to the capacity constraint in (1).

Hence minimizing (2) corresponds to deterministic optimization (refer (1)), and conceptually, centralized preemption can always find an optimal set of flows to remove.

Distributed preemption allows each link \( i \) to update its decisions iteratively and asynchronously based on local information \( A_i \) and neighbors’ decisions \( d_j^i, j \in N_i \), where \( N_i \) is a neighborhood of link \( i \). Each link/node can only access limited and initially inaccurate information from near-neighbors and missing information from far-neighbors. But through neighbor’s neighbors, such information would eventually propagate to all nodes, resulting in globally consistent decisions. A difficulty is that deterministic distributed decisions may get stuck at a local optimum [9].

2) Random Flows

What and how many flows are active at which links are related to user behaviors and thus random. Hence active flows and their aggregation at individual links should be regarded as random variables. Preemption decisions made on active flows should be considered as random also. A set of decisions thus form a sample realization of an event. One such sample is shown in the example of Figure [1] where \( d = \{d_1^1, d_1^2, d_2^1, d_2^2, d_3^1, d_3^2, d_4^1, d_4^2, d_4^3\} = \{1, 0, 1, 0, 1, 0, 1, 0, 1\} \). This relates random and deterministic flows and decisions.

To obtain an optimal set of preempted flows, stochastic rather than deterministic optimization should be used, and this requires obtaining a probability distribution of \( d \).

Such a probability distribution can be obtained through graphical models defined on neighborhood systems [9]. A neighborhood system can be characterized by Hamiltonian which is also called system potential energy [21]. The energy of a decision variable corresponds to a per-variable preemption cost, \( \alpha_k B^k d_k^i \), in the first terms of (2). Interactions between decision variables of any two different links result in \( \sum_{i_1 \neq k} \sum_{i_2 \neq k} \alpha_k B^k d_k^i d_k^{i_2} \) as the second terms of (2). The remaining terms correspond to higher-order interactions.

In this context, \( H(d) \) corresponds to a Hamiltonian of \( d \), and results in a Gibbs distribution [9] [21],

\[
P(d) = Z_0^{-1} \exp \left( \frac{-H(d)}{T} \right), \tag{3}
\]

where \( T \) is a parameter (the temperature [9]), and \( Z_0 \) is a normalization constant. The Gibbs distribution is a probability distribution of decision variables, and thus provides a mathematical framework for approximating the globally optimal preemption decisions.
This Gibbs distribution also provides the implementation methodology of near-optimal distributed and iterative preemption decision, which is shown in Section IV. The minimum of the Hamiltonian corresponds to the optimal preemption decisions that maximize the probability.

3) Probabilistic Graphical Models

The spatial dependence among a large number of decision variables can be represented explicitly by probabilistic graphical models. A graphical models relates a probability distribution of random variables with a corresponding dependency graph [9] [17] [19]. A node in the graph represents a random variable and a link between two nodes characterizes their dependence. In particular, a set of random variables \( \mathbf{v} \) forms Gibbs Random Field (GRF) if it obeys a Gibbs distribution [21]. A Gibbs distribution satisfies the positivity condition, meaning that all decisions have a positive probability. One other important property is the spatial Markov dependence defined by the neighborhood system and shown by Hammersley-Clifford theorem.

Hammersley-Clifford Theorem [27]: Let \( S \) be the set of nodes, \( S = \{1, \cdots, N\} \). Let \( \mathbf{v} \) be a set of random variables, \( \mathbf{v} = \{v_1, \cdots, v_N\} \).

\( \mathbf{v} \) is said to be a Markov Random Field if (i) \( P(\mathbf{v}) > 0 \) for every \( \mathbf{v} \) in sample space; (ii) \( P(v_i|v_j, j \in S\backslash\{i\}) = P(v_i|v_j, j \in N_i) \), where \( N_i \) is a neighborhood of node \( i \) for \( i \in S \).

The random field \( \mathbf{v} \) is also said to be a Gibbs Random Field if its probabilistic distribution can be written in the form

\[
P(\mathbf{v}) = \prod_{c \in C} V_c(\mathbf{v}),
\]

where \( c \) is a clique, \( C \) is the set of all feasible cliques, and \( V_c(\mathbf{v}) \) is a general positive function called a clique potential function.

There is an equivalence between a Gibbs Random Field and a Markov Random Field if and only if the Gibbs distribution \( P(\mathbf{v}) \) possesses the spatial Markov property.

Markov Random Fields correspond to an interesting type of probabilistic graphical models where a random variable is conditionally independent of the other nodes given its neighbors. The conditional independence is spatially nested, i.e., a node depends on its far neighbors through neighbors’ neighbors. Such nested dependence can be observed explicitly through local connections among nodes in a dependency graph. The corresponding Gibbs distribution is thus factorizable over clique potentials [9].

An important implication to distributed preemption is that if distributed decisions result in an MRF, local decisions using neighbor information are collectively optimal. But do preemption decisions \( \mathbf{d} \) form a Markov Random Field in the first place? We plot the dependency graph for the Gibbs distribution with the Hamiltonian in (2). In particular, a factor graph [19] in Figure 2 is used to draw the dependency graph of the decision variables for the set of flows shown in Figure 1.

A factor graph is a bipartite graph that expresses the spatial dependence between the variable nodes and the function nodes [19]. In Figure 2 circles represent the decisions on the active flows at links. Squares denote the link-functions, corresponding to the local potentials at individual links. Specifically, \( g_i(\mathbf{d}) \) is a local potential that encompasses the flows passing through link \( i \). A connection between a circle and a square indicates a functional relationship.

The corresponding Gibbs distribution is

\[
P(\mathbf{d}) = Z_0^{-1} \exp \left( -H(\mathbf{d}) \right).
\]
$P^i(d)$ is an approximated likelihood function,

$$P^i(d) = Z_0^{-1} \prod_{i=1}^{L} \exp \left( -\frac{g_i(d)}{T} \right),$$  \hspace{2cm} (5)

where $\exp \left( -\frac{g_i(d)}{T} \right)$ is a local likelihood function for the connections at link $i$, and can be further decomposed into all clique potentials associated with connections at link $i$:

$$\exp \left( -\frac{g_i(d)}{T} \right) = \exp \left( -\sum_{c \in C_i} \psi_c(d) \right),$$  \hspace{2cm} (6)

where $C_i$ is the set of all cliques of link $i$, and $\psi_c(d)$ is a potential function of clique $c$.

For example, if the neighborhood size $N_d = 1$ for all links, the corresponding factor graph has only nearest neighbor connections as shown in Figure 3, where the dash lines denote the neglected dependency links.

![Fig. 3](https://example.com/fig3.png)

**LOCALIZED SPATIAL DEPENDENCE OF d WITH FACTOR GRAPH**

IV. DISTRIBUTED PREEMPTION ALGORITHMS

We now assume that a local model is obtained as a good approximation of the global model. The spatial Markov local model then can be used to derive a distributed algorithm where nodes can make local decisions on connection preemptions through information exchange with neighbors.

A. Distributed Algorithm

The distributed algorithm obtains a set of local decisions that maximizes the approximated likelihood function, which is equivalent to minimizing the cost function,

$$\hat{d} = \arg \max_d P^i(d)$$

$$\hat{d} = \arg \min_d H^i(d).$$ \hspace{2cm} (7)

Since $P^i(d)$ is factorizable, maximizing the global likelihood function reduces to maximizing the local likelihood function at cliques, i.e., $P^i(d_l|d_{N_l})$ for $1 \leq i \leq L$, where $d_{N_l}$ is the set of decision variables of neighboring links. As these local likelihoods are functions of the decision variables of neighboring links, the decisions can be updated locally. In addition, the local maximizations result in coupled equations due to the nested Markov dependence, which shows that information exchange is needed only when the neighborhood size $N_d = 1$. That is, a node only exchanges information with its nearest neighbors.

Maximizing local likelihood functions can be implemented as local learning algorithms at individual nodes. The learning algorithms perform probabilistic inference using either approximated sum product algorithm [19] or stochastic relaxation [9]. The sum product algorithm can be applied to the factor graph in Figure 3. This algorithm produces an exact solution for a graph that has no loops. However, the factor graph [19] of preemption problem is usually loopy, resulting in approximated (non-optimal) decisions.

Stochastic relaxation can be applied for each link to make local preemption decisions. Let $d_{\text{pre}}^i \setminus \{k\}$ be a set of decisions on active flows at link $i$, excluding the decision on flow $k$. Here $S_F \setminus \{k\}$ denotes a set operation, which excludes $k$ from $S_F$. Now we add time variable $t$ to the decisions, and let $d_{\text{pre}}^i(t + 1)$ be an updated decision on flow $f^k$ at the $(t + 1)$th iteration and at link $i$. Then,

$$d_{\text{pre}}^i(t + 1) = 1,$$ \hspace{2cm} (8)

with the probability

$$P^i \left( d_{\text{pre}}^i(t + 1) = 1 | d_{\text{pre}}^i \setminus \{k\}(t), d_{N_l}(t) \right) =$$

$$\frac{\exp \left( -\psi_i(d_{\text{pre}}^i(t + 1) = 1)/T(t + 1) \right)}{\sum_{d_{\text{pre}}^i(t + 1) \in \{-1, 1\}} \exp \left( -\psi_i(d_{\text{pre}}^i(t + 1))/T(t + 1) \right)},$$

where

$$\psi_i(d_{\text{pre}}^i(t + 1) = 1) = \alpha_k B^k - \sum_{i_2 \in N_l} \alpha_k B^k d_{i_2}(t + 1) + \beta U(c_{\text{nect}} - A_i).$$

This means that a random decision at time epoch $t + 1$ is made based on local information $d_{\text{pre}}^i \setminus \{k\}(t)$ and neighbor information $d_{N_l}(t)$ at the previous time epoch $t$. A cooling schedule is applied to the temperature $T(t) = T_0/\log(1 + t)$ with $T_0=3.0$. This results in an almost-sure convergence of the algorithm to the global minimum Hamiltonian (i.e., optimal decisions) [9]. That is, with the iterative and distributed updates, the global minimum of the approximated Hamiltonian $H^i(d)$ can be reached asymptotically with probability one.

B. Example

We now revisit Figure 1 to show an example of the distributed algorithm. Consider links 1 and 2, and assume that the neighbor-neighbor size $N_d=1$. That is, a node only exchanges information with its nearest neighbors.

At initial stage, no flows are preempted, i.e. $\{d_1^1(0)=0, d_2^1(0)=0, d_2^2(0)=0, d_1^2(0)=0\}$. When $t = 1$, the decision variables are updated,

$$d_1^1(1) = \arg \max_{d_1^1(0 \cup 1)} P(d_1^1(1) = d_1^1(0), d_{N_1}(0)),$$

$$d_2^2(2) = \arg \max_{d_2^2(0 \cup 1)} P(d_2^2(2) = d_2^2(1), d_{N_2}(1)),$$

where $d_{N_1}(0) = \{d_2^1(0), d_2^2(0)\}$. The updated decision $d_1^1(1)$ is sent to the neighboring links. This process is applied similarly to the other decision variables. At the second time epoch ($t = 2$),

$$d_1^2(2) = \arg \max_{d_1^2(0 \cup 1)} P(d_1^2(2) = d_1^2(1), d_{N_2}(1)),$$

where $d_{N_2}(1) = \{d_1^2(1), d_2^1(1), d_2^2(1), d_3^2(1)\}$.

The process is repeated until an equilibrium state (i.e., of no more changes) is reached.
C. Information Exchange

The distributed preemption decisions require information exchange with neighbors. The clique structure of the Markov Random Field determines the range of information exchange, which is the neighborhood size \( N_d \). The type of the information exchanged is binary, i.e., \( d_{N_d}(t) \), as in the conditional probability in (8). The amount of information used at a decision making characterizes the communication/computation complexity. The information exchange is per-flow based but moderate when limited to neighbors.

V. Near-Optimality and Complexity

In this section, we conduct analytical studies to identify sufficient conditions for the near-optimality of the distributed preemption, the communication/computation complexity, and the optimality-complexity trade-off.

A. Short-Range Dependent Decision Variables

The near-optimality is in regard to the question when distributed preemption decisions are nearly optimal. To answer this question, we need to consider how well a Markov Random Field approximates the global model. This should be done by studying the traffic patterns of active flows since the flows across multiple links over the preemption route and the limited link-capacity constraints at links are the origins of spatial dependence of distributed decisions.

1) Bounded-Length Flows

Traffic patterns of active flows result in spatial dependence among distributed decision variables. Consider simplified traffic patterns where the hop-count of each active flow is bounded by \( h \) for \( h \geq 1 \). Then the set of distributed preemption decision variables are strictly Markov as shown below.

**Lemma 1:** Assume that the hop-count of each active flow is bounded by \( h \) \( (h \geq 1) \). Let \( N_i^h \) be a set of neighborhood of link \( i \) and include all links within \( h \) hops from \((i, j)\). Let \( d_{N_i^h} = \{d_m\} \) for \( \forall m \in N_i^h \) denote a set of decisions in the neighborhood, and \( d \setminus d_i \) be all decision variables except \( d_i \). Then, \( P(d_i | d \setminus d_i) = P(d_i | d_{N_i^h}) \).

The proof is provided in Appendix II. Lemma 1 shows that the set of decision variables on active flows of a limited span forms a Markov Random Field (MRF), where \( h \) corresponds to an upper bound of the neighborhood size of the MRF. This is intuitive as the active flows of a bounded length would only introduce short-range spatial dependence.

2) Shortest-Path Flows

In reality, however, the hop-count of active flows is a variable and cannot be assumed to be bounded with a meaningfully small value (e.g., 1 or 2 hops). Thus, we study the spatial dependence of decision variables for shortest-path flows that constitute more realistic traffic patterns. In particular, we consider shortest-path flows with the following assumptions for analytical convenience:

1. A network is planar and homogeneous where each node (except the source and destination nodes) has the same nodal degree \( \pi \).
2. A source-destination pair is chosen randomly from all pairs in the network.
3. A preemption route is a shortest-path between the source and the destination of a new connection.
4. Active flows are assumed to take shortest routes from randomly-chosen source-destination pairs whose paths may partially coincide with the route of the new connection.
5. The route length of a new flow is assumed to be \( L \) hops for analytical simplicity, where \( L \geq 1 \).

We now define a measure of spatial dependence of two links on the preemption route (e.g., link \((i - 1, i)\) and \((j, j + 1)\) for \( 2 \leq i \), and \( i \leq j \leq L \)).

**Definition 1. Link-Dependency Probability \( P_{ij} \):** \( P_{ij} \) denotes the probability that a flow uses both link \((i - 1, i)\) and \((j, j + 1)\) that are separated by \(|j - i|\) hops on the preemption route.

The link-dependency probability \( P_{ij} \) then characterizes the spatial dependence of any two flows at these two links. \( P_{ij} \) is difficult to obtain exactly, and thus bounded as follows.

**Lemma 2:** Let \( P_{ij}^l \) be a lower bound of \( P_{ij} \), i.e., \( P_{ij} \geq P_{ij}^l \). For shortest-path flows under assumptions (1) through (5), \( P_{ij}^l = \frac{(L - |j - i|)}{L} \left( \frac{1}{d_{ij}} \right) |j - i| \).

The proof is provided in Appendix II. This lemma suggests that the length of a shortest-path flow over the preemption route follows at least a geometric probability, where \( \frac{1}{d_{ij}} \) is the lower bound of the probability for such a flow to continue at the next hop.

**Lemma 3:** Let \( P_{ij}^u \) be an upper bound of \( P_{ij} \), i.e., \( P_{ij} \leq P_{ij}^u \). Consider a network topology of a regular lattice with nodal degree \( 4 \). For shortest-path flows under assumptions (1) through (5), \( P_{ij}^u \approx \frac{1}{3 \sqrt{2} |j - i|} \).

The proofs can be obtained by counting the number of shortest paths between node \( i \) and \( j \), as is given in Appendix III.

Figure IV depicts both the upper and lower bound as well as an empirical probability \( P_{ij} \). The probability \( P_{ij} \) is estimated on a regular lattice network with 250 nodes, where active flows are routed onto the shortest paths between randomly chosen source-destination pairs. 10 runs are conducted and the results are averaged to obtain the empirical probability. As shown in the figure, \( P_{ij} \) decays rapidly close to the exponential decreasing rate of the lower bound \( P_{ij}^l \).

**Lemma 2, 3 and the empirical result suggest that on the average, shortest-path flows share only few hops with the preemption route.** Thus, as we shall soon see, Markov Random Fields is a good approximation to a set of decision variables \( d \) for a mesh topology.

B. Sufficient Conditions for Near-Optimality

We now define the near-optimality of distributed preemption.
near-optimality needs to specify when the effects of aggregated long flows are negligible in the truncated Hamiltonian. For feasibility of analysis, we consider a simplified scenario that the bandwidths of active flows are bounded.

Theorem 1: Let $B_0 > 0$ be a constant bandwidth. Consider a straight route of a new flow. For given $\epsilon_B (0 < \epsilon_B < 1)$, assume that bandwidth $B^k$ of flow $k$ satisfies $\frac{B^k}{B_0} \leq \epsilon_B$ for all $k$. Then $E(\Delta) \leq 2c_{\text{new}} \cdot \frac{1}{1 - \epsilon_B} \cdot L \cdot [(1 + p_c^{N_d})^{L - N_d} - 1]$, where $N_d$ is the neighborhood size for information exchange in distributed preemption decisions. When $p_c^{N_d} L = o(1)$, the upper bound of $2c_{\text{new}} \cdot \frac{1}{1 - \epsilon_B} \cdot L \cdot [(1 + p_c^{N_d})^{L - N_d} - 1] = 2c_{\text{new}} \cdot \frac{1 + \epsilon_B}{1 - \epsilon_B} \cdot L \cdot (L - N_d)p_c^{N_d} + o(Lp_c^{N_d})$.

The proof is provided in Appendix IV. Theorem 1 provides the following observations when active flows follow a geometric distribution.

(a) For a given $p_c$ and $c_{\text{new}}$, the larger the neighborhood size $N_d$ in the Markov Random Field, the smaller the upper bound, and the better the performance may be for distributed preemption. In fact, the error bound decreases exponentially with respect to $N_d$.

(b) The upper bound increases linearly with respect to the bandwidth demand of a new flow $c_{\text{new}}$ as $\frac{c_{\text{new}}}{B_0(1 - \epsilon_B)}$ characterizes the maximum number of active flows to accommodate the new flow with $c_{\text{new}}$ at a link. Thus, the larger the $c_{\text{new}}$, the more existing flows may need to be preempted, the higher the probability for the distributed algorithm to make inconsistent decisions at links. That is, the performance of distributed preemption may degrade when the bandwidth demand of a new flow increases.

(c) The upper bound also increases with respect to the route length $L$, since a longer route consists of more links and thus a higher probability for distributed decisions made at links to be inconsistent.

It should be noted that the above studies of the optimality assume that the stochastic relaxation is capable of obtaining a global minimum of the global and local models. This holds true as the convergence of the algorithm occurs almost surely.

C. Complexity

A key advantage of distributed preemption is the reduced complexity, i.e., the information exchange is limited to only neighbors.

Definition 4. Communication Complexity (CC): Let $N_d$ denote the neighborhood size for exchanging binary information (bits) in distributed preemption. Let $f_{\text{max}}$ denote the maximum number of active flows at a link. Let $t_{\text{iter}}$ denote the total number of iterations needed for the distributed algorithm to converge. Communication complexity (CC) of a node is defined as the total amount of information exchanged for a link to make a decision using the distributed algorithm, i.e., $CC = O(N_d f_{\text{max}} t_{\text{iter}})$.

Note that $O(N_d t_{\text{iter}})$ is the bits of information exchanged for making a preemption decision on one flow. There are at most $f_{\text{max}}$ flows at a node. This results in $CC = O(N_d f_{\text{max}} t_{\text{iter}})$. Hence, if $t_{\text{iter}}$ can be bounded by a moderate value, as shall be shown in Section VI, $CC$ would be $O(N_d f_{\text{max}})$ which grows linearly with the neighborhood size.
We now compare qualitatively CC with decentralized preemptions. Min-Conn and Min-BW [27] are the representatives of the existing decentralized algorithms that minimize the number of preempted flows and the amount of preempted bandwidth at each hop, respectively, without information exchange. The complexity of Min-Conn and Min-BW are $O(f_{\text{max}}^2)$ and $O(f_{\text{max}}2^{f_{\text{max}}})$, respectively.

Hence by bounding $i_{\text{ter}}$ with a moderate value, we can obtain a globally near-optimal decision that is obtained with a smaller complexity than that of decentralized algorithms. We shall show this in the next section.

We now compare qualitatively CC with centralized preemptions. The communication/computation complexity for a centralized scheme increases linearly with the number of hops on the preempting route $L$. Hence, if $N_d << L$, the complexity for distributed preemption is much smaller than that of the centralized preemption due to local information exchange among neighbors.

D. Optimality and Complexity Trade-off

How large should $N_d$ be for a given $L$, traffic pattern ($p_c$) and other parameters? Theorem 1 shows that reducing $N_d$, i.e., the communication/computation complexity, results in a simpler local model but a larger error bound. Therefore, a trade-off between the optimality and complexity needs to be explored.

**Corollary 1:** For a given performance $\epsilon$, if $N_d \geq \frac{\log \left( \frac{p_{\text{new}} \Delta_{\text{new}} p_{\text{old}} \Delta_{\text{old}}}{\epsilon} \right)}{\log \left( \frac{L_{\text{new}}}{L_{\text{old}}} \right)}$, then $E(\Delta) \leq \epsilon$. For $L$ large, $L \gg N_d$ and $\epsilon_B > 0$ small, the condition reduces to $N_d \geq \Omega \left( \frac{\log \left( \frac{\epsilon}{\log \left( \frac{L}{p_c} \right)} \right)}{\log \left( \frac{L}{p_c} \right)} \right)$.

The proof of Corollary 1 can be obtained directly from Theorem 1 by letting $E[\Delta] < \epsilon$ and simple algebraic manipulations, and thus omitted.

The corollary implies that a sufficient condition for distributed preemption decisions to be near optimal is for $N_d$ to be of an order $\log L$, when the flow continuity probability decays exponentially with the number of hops. Meanwhile, the larger $p_c$ and the smaller $\epsilon$ are, the larger $N_d$ is. This shows clearly a trade-off between the performance and complexity.

VI. SIMULATIONS

We now study further how the performance of distributed decisions varies with respect to neighborhood size and traffic patterns through simulation. We also compare the distributed preemption with the methods used in the prior work.

A. Performance Metrics and Simulation Setting

We use the average preempted bandwidth per link at a chosen path, $\frac{1}{2} \sum L_k d_k^k$, as the performance metric in our simulation. The metric is used to quantify the effectiveness of distributed preemption in bandwidth savings.

Our simulation generates a network topology, a certain traffic pattern, and a chosen route. To be specific, our simulation studies consider a network with two service-classes. The capacity $C$ of each link is $2$ Mbps. The source and destination of a new connection are chosen at random in a network. The resulting route has about average for the S-D pair of each active flow.

B. Performance and Neighborhood Size

A new connection setup assumes bandwidth demand $c_{\text{new}}=20$ Mbps and class $i_{\text{new}}=2$. Distributed preemption decisions are made by changing neighborhood size $1 \leq N_d \leq 2$. Decentralized decisions are implemented using Min-Conn algorithm [27] for comparisons. There, each node makes decentralized preemption decisions independently without any cooperation with neighbors. The performance in the average preempted bandwidth per link is shown in Table III for distributed preemption of different neighborhood sizes and decentralized preemption. For both topologies, the preemption costs are reduced sharply with the cooperation with neighbors.

For planar lattice topologies, the nodal degree $d_0=4$ results in the flow-continuity probability $p_c=1/3$. The link-dependency probability $P_d$ thus decreases in $O(p_c^{1/d})$. As shown in Table III, even with the smallest neighborhood $N_d = 1$, the preemption cost of distributed decision can be reduced by $50\%$ compared to the decentralized preemption algorithm.

For power-law topologies, nodes have different degrees, and the path-length of a connection is around 2 or 3 hops on the average. Thus, the link-dependency probability $P_d$ also decays sharply. The effectiveness of the distributed preemption is similar to that of planar lattice topologies.

C. Neighborhood Size and Traffic Patterns

We now study how the performance of the distributed preemption algorithm varies with different flows in the network. We use the average preempted bandwidth at a chosen path, $\frac{1}{2} \sum L_k d_k^k$, as the performance metric in our simulation. The metric is used to quantify the effectiveness of distributed preemption in bandwidth savings.

Our simulation generates a network topology, a certain traffic pattern, and a chosen route. To be specific, our simulation studies consider a network with two service-classes. The capacity $C$ of each link is $2$ Mbps. The source and destination of a new connection are chosen at random in a network. The resulting route has about average for the S-D pair of each active flow.

**TABLE II**

| Decentralized Preemption | $N_d=1$ | $N_d=2$ |
|--------------------------|---------|---------|
|                          | 16.7    | 7.6     | 6.3     |
TABLE III

PREEMPTION COSTS ON A POWER-LAW TOPOLOGY

| Decentralized Preemption | $N_d=1$ | $N_d=2$ |
|--------------------------|---------|---------|
|                          | 17.2    | 8.8     | 7.5     |

each experiment, active flows are generated randomly for each flow-continuity probability $p_c$ and neighborhood size $N_d$. This is repeated for a wide range of $p_c$ and $N_d$ values. The preempting route has $L = 10$ hops, and a new connection has bandwidth demand $c_{new}=20$ Mbps.

Figure 5 (a) shows that the preempted bandwidth decreases sharply by including the information only from the nearest neighbors. This is especially significant for a small $p_c$ (e.g., $p_c=0.3$), which corresponds to short flows. $N_d=0$ corresponds to decentralized decisions where there is no information exchange with neighbors. Hence the figure shows that the cooperation with the nearest neighbors (i.e., $N_d=1$) can improve the performance by 53%.

The cooperation with farther neighbors (e.g., $N_d=4$) results in another 3.3% bandwidth saving for $p_c = 0.3$. But the improvement is not significant given the increase of communication complexity. Hence, for short flows, the information exchange between the nearest neighbors seems to be sufficient to achieve the near-optimality.

As $p_c$ increases, the dependence among decision variables on different links increases, and the performance gains are more pronounced with a larger neighborhood size. Figure 5 (b) shows that the preempted bandwidth decreases linearly with an increase of $p_c$ for a given $N_d$. This is because that the dependence of two links increases along with $p_c$. Thus, for a given $N_d$, the amount of preempted bandwidth decreases with an increase of $p_c$.

Figure 5 (b) also shows that the preempted bandwidth of the distributed preemption is smaller than that of Min-Conn [27] algorithm. The complexity of Min-BW algorithm is $O(f_{\text{max}} \cdot f_{\text{max}}^2)$, which is computationally intractable for $f_{\text{max}}$ large. Thus, Min-Conn algorithm (whose complexity is $O(f_{\text{max}})$) is used for comparison.

D. Path Length

Now we consider the impact of path length $L$ for fixed $p_c=0.4$. Other parameters used are $c_{new}=20$ Mbps, and $C = 100$ Mbps. For a given $N_d$, as $L$ increases, the preemption cost decreases.

Figure 6 (a) shows that for all $N_d$ values, the corresponding preempted bandwidth decreases as $L$ increases. However, the decrease of preempted bandwidth is lower bounded for $L > 30$ hops, such as Figure 6 (b).

E. Bandwidth Demand

Now we consider the impact of bandwidth demand $c_{new}$ of a new connection together with the neighborhood size. Other parameters are fixed and chosen as $p_c=0.4$, and $C = 100$ Mbps.

Figure 7 shows that for all $N_d$ values, the corresponding preempted bandwidth increases with $c_{new}$ linearly. Moreover, the gain of the distributed preemption is significant compared to decentralized preemption.

VII. RELATED WORK AND DISCUSSIONS

Connection preemption: Garay and Gopal [8] show that centralized connection preemption is NP-complete, and thus develop heuristic algorithms. The algorithms consider all connections on a preemption route but are not optimal in preemption decisions.

Peyravian and Kshemkalyani [27] propose two practical decentralized algorithms. The algorithms incorporate multiple factors such as the priority of each connection, the bandwidth and the number of connections to be preempted. The algorithms are locally optimal at each link but not globally optimal at the entire route since there is no information exchange among links. Oliveira [25] formulates connection preemption through linear programming and proposes adaptive heuristic algorithms. The lower-priority connections are then preempted to minimize the impact to the active bandwidth. The optimality is not considered here.

Stanisic [29] develop two randomized preemption algorithms whose complexity is linear in the number of lower priority flows.
et.al. [24] considers soft preemption that utilizes a preemption pending flag to mitigate gracefully the re-route process of preempted LSP.

As the existing algorithms provide important empirical results, the issues of optimality and complexity have been studied insufficiently. Most of these algorithms do not consider information exchange among neighbors.

**Distributed Management:** Flow preemption can be cast in a general context of distributed management. Distributed algorithms and protocols have been designed using local information on configuration management (see [32] and references therein). Self-organizing protocols are proposed for sensor networks and developed for p2p self-stabilizing using graph coloring (see [18] and references therein). The distributed management, however, has been traditionally done based on heuristic local rules. Near-optimal distributed algorithms remain to be an open problem when spatial dependence is non-Markovian.

**Probabilistic Graphical Models:** Probabilistic graphical models have been used to represent the spatial dependence in complex systems. Markov Random Fields have been used to model the cooperation of mobile agents [3] in addition to their wide applications in image processing [9]. Factor graphs are used to represent the dependence and as computationally efficient distributed algorithms [19].

One related work is a cross-layer graphical model developed for optical networks [22], where a Markov Random Field models the spatial dependence of routes at the network layer. The other related work is our prior work where the probabilistic graphical models are used for the distributed management of wireless ad hoc networks [16]. The work presented in this paper, however, considers a different problem of flows in a different setting of wireline networks.

**Optimality and Complexity:** Optimality in terms of network capacity has been investigated for wireless ad hoc and sensor networks [28]. However, near-optimal algorithms have been studied little for controlling flows.

**VIII. Conclusions**

In this work, we have studied distributed connection preemption in multi-class networks. The work is motivated by the fact that connection preemption is known to be NP-complete. Centralized preemption can achieve an optimal performance but has an intractable communication complexity. Decentralized preemption is computationally feasible but lack of a good performance. This work has focused on whether a near-optimal performance can be achieved by distributed preemption at a moderate communication complexity. We have developed a distributed framework, where nodes make local preemption decisions through cooperation with neighbors. The framework treats distributed preemption as a machine learning problem where a large number of statistically dependent decisions can be treated jointly.

Specifically, we have developed a probabilistic spatial model of distributed preemption decisions. We have shown that a sufficient condition for distributed preemption to be near-optimal is that the spatial model is a Markov Random Field. We have shown that the sufficient condition is satisfied by our framework.
identified a cause of spatial dependence which is due to flows trespassing multiple links. We have examined commonly-used traffic patterns including short-range dependence flows and shortest-path flows. This results in a certain sufficient conditions on the near-optimality. In particular, the sufficient conditions quantify joint impacts of the flow-continuity probability, the bandwidth demand of a new flow, the communication complexity of distributed algorithms, and route lengths for short-range dependent active flows.

We have shown that the spatial dependence can be characterized by probabilistic graphical models. The graphical models allow us to apply distributed algorithms based on stochastic relaxation. We have shown through analysis and simulations that for short-range dependent flows, information exchange with only the nearest neighbors can significantly improve the performance of preemption. The use of more neighbors result in further improvements but not as pronounced given an increase in the communication complexity.

More general traffic patterns require a further study. It remains open how to quantify the validity-invalidity of distributed preemption for long-range dependent flows. For example, the long-range spatial dependence cannot be ignored in linear or ring topologies, and thus the neighborhood for distributed preemption is not localized. Hence, distributed algorithms need to be extended to multiple routes and other traffic patterns in a more realistic network setting where the impact of topologies may become significant. From a computational standpoint, one disadvantage of distributed preemption using stochastic relaxation is a slow convergence. Hence future work also involves a study of computation time in terms of delays.

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APPENDIX

I. PROOF OF LEMMA 1

Consider two links $i$ and $m$. Let $|i - m|$ denote the distance of these two links. The distance of two links indicates the hop-counts of the shortest path between them.

When $|i - m| \leq h_0$, there can be a flow that shares both links. Otherwise, i.e., $|i - m| > h_0$, there cannot be any flow that shares both links. Therefore, the decisions at link $i$ are conditionally independent of the other links beyond $h_0$ hops.

II. PROOF OF LEMMA 2

As given in assumption (1), there are multiple routes from node $i$ to node $j$ on the planar regular topology, one of which is the preemption route. Since each node has nodal degree $d_0$, the probability is $\left(\frac{1}{d_0-1}\right)$ for a connection that passes link $(i-1)$ on the preemption route to trespasses through the next link $i$. Thus, the probability that an active flow trespasses links $i-1, \cdots, j$ on the preemption route is $\left(\frac{1}{d_0-1}\right)^{|j-i|}$. Now consider $|j-i| > h_0$. Using Stirling formula, $n! \approx \sqrt{2\pi n} (n/e)^n$. Thus, $C \left( |j-i|, \frac{1}{2} \right) \approx \frac{2^{|j-i|+1}}{2\pi |j-i|}$, and $P_{ij} \approx \left(\frac{|j-i|}{L}\right) \frac{1}{2\pi |j-i|}$.

IV. PROOF OF THEOREM 1

Consider a set of randomly generated active flows and a randomly chosen preemption route of $L$ hops.

Then, $\Delta = |H(d^\ast) - H(\hat{d})|$. To find an upper bound of $\Delta$, we have $|H(d^\ast) - H(\hat{d})| = |H(d^\ast) - H(d^\ast)| + |H(d^\ast) - H(\hat{d})|$. Therefore, a lower bound of $P_{ij}$ is $\left(\frac{L-|j-i|}{L}\right)\left(\frac{1}{d_0-1}\right)^{|j-i|}$. Thus, the probability that an active flow meet on link $i-1$ at the preemption route proceeds to link $j$ is $\left(\frac{L-|j-i|}{L}\right)\left(\frac{1}{d_0-1}\right)^{|j-i|}$, where $|j-i|$ is the number of hops between link $i-1$ and link $j$. Therefore, a lower-bound of $P_{ij}$ is $\left(\frac{L-|j-i|}{L}\right)\left(\frac{1}{d_0-1}\right)^{|j-i|}$.
Assume that for the global and local optimal decisions, $d^*$ and $d^i$, the second-order consistency is achieved as shown in the constraint in $H^i()$. That is, $d^k_i = d^i_k$ for $|i - j| \leq N_d$, and for $1 \leq k \leq |S_p|$. Since $d^k_i$’s are binary, the second-order consistency implies all orders up to $N_d$ consistency. That is, for a given link $i$. Consider a set $S_i = i_1, \ldots, i_l$ that contains indices of any other links within $N_d$ hops of link $i$. $d^{i_1}_i = \ldots = d^{i_l}_i$.

Thus,

$$E([H(d) - H(d^*)] + E([H(d^*) - H^i(d^*)]) \leq I_2 + I_3 + \ldots + I_L,$$

where

$$I_2 = 2 \sum_k \left( E \left[ \sum_{i_1, i_2 \neq i_1} d^{i_1}_i d^{i_2}_i \right] - \sum_{i_1, i_2 \neq i_1, |i_1 - i_2| \leq N_d} d^{i_1}_i d^{i_2}_i \right),$$

and

$$I_L = 2 \sum_k \left( E \left[ \prod_{i=1}^{L-1} d^{i_L-1}_i \right] - E \left[ \prod_{i=1}^{L-2} d^{i_L-1}_i \right] \right)$$

for $|i_1 - i_2| \leq N_d$, for $l = 2, \ldots, L$.

Hence, within a neighborhood $N_d$ of any given link, all links make consistent decisions. Beyond such a neighborhood, links can make different and thus incorrect preemption decisions. To bound the error, we let $d^i_k$ be a feasible preemption decision without distinguishing whether it is globally or locally optimal. Consider an active flow $k$ on link $i$ of a preempting route. For the local model $H^i(d^i)$ of neighborhood size $N_d$, the continuity of the active flow is neglected beyond the neighborhood $N_d$, i.e., on the links $(i + N_d + m)$ or $(i - N_d - m)$ for $m \geq 1$. Hence, the error caused by using the local model can be counted by the neglected active flows beyond the neighborhood.

Specifically, from $I_2$, the error caused by a flow $k$ that leaves at link $i + N_d + m$ corresponds to the bandwidth $B^k$ of flow $k$, and the probability that the flow leaves at this portion of the path is $p_{c}^{N_d+m-1}(1 - p_c)$, for $m \geq 1$. Therefore, for one active flow on the preempting route, the total expected error of ignoring the second-order terms for $\forall m \geq 1$ is less than $B^k \sum_{m=1}^{\infty} (L - N_d) p_{c}^{N_d+m-1}(1 - p_c) \leq B^k p_{c}^{N_d}(L - N_d)$.

Thus, for a feasible configuration $d^i$, the expected error caused by neglecting the second-order terms (i.e., $I_2$) for all flows is upper bounded by $2 \frac{c_{max}}{b_0(1 - \epsilon_B)} p_{c}^{N_d} B_1(1 + \epsilon_B)(L - N_d)$, where $\frac{c_{max}}{b_0(1 - \epsilon_B)}$ denotes the maximum number of flows feasible to be preempted at a link.

Similarly, from the third term (i.e., $I_3$), the expected error caused by $l$ with the preempting route is $B^k(1 - p_c)^2 p_c |j - i| + |l - i| - 2$ for $|j - i| > N_d$ and $|l - i| > N_d$, where $(1 - p_c) p_c |j - i| - 1$ is the probability a flow continues to the link $j$ from link $i$ and then exists the route. Since there are at most $(L - N_d)$ such terms for each flow in the third term of $I_3$, this error is upper bounded by $2 \frac{c_{max}}{b_0(1 - \epsilon_B)} p_{c}^{N_d} (L - N_d)$. A similar bound can be obtained for the $m$-th-order term of $I_3$, for $m = 2, \ldots, L$. Thus, let $A = 2 \frac{c_{max}}{b_0(1 - \epsilon_B)}$, then

$$E(\Delta) \leq E([H(d) - H^i(d^*)]) + E([H(d^*) - H(d^*)]) \leq A(L - N_d) \left( \frac{(L - N_d)}{1} p_{c}^{N_d} + \frac{(L - N_d)}{2} p_{c}^{2N_d} + \ldots + p_{c}^{N_d}(L - N_d) \right) \leq A \left( (1 + p_{c}^{N_d})(L - N_d - 1) \right).$$

For $p_{c}^{N_d} = o(1)$, the bound is $A(L - N_d)p_{c}^{N_d} + o(Lp_{c}^{N_d})$.

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