Penguin-Diagram Induced $B \rightarrow K_X \phi$ Decays  
in the Standard Model 
and 
in the Two-Higgs-Doublet Model

Andrew J. DAVIES$^{(a)}$, Takemi HAYASHI$^{(b)}$, Masahisa MATSUDA$^{(c)}$ and 
Morimitsu TANIMOTO$^{(d)}$

$^{(a)}$ Research School of Physical Sciences and Engineering, 
Australian National University, Canberra ACT 0200, Australia
$^{(b)}$ Kogakkan University, Ise, Mie 516, JAPAN
$^{(c)}$ Department of Physics and Astronomy, Aichi University of Education 
Kariya, Aichi 448, JAPAN
$^{(d)}$ Science Education Laboratory, Ehime University 
Matsuyama, Ehime 790, JAPAN

ABSTRACT

We systematically analyse the gluonic penguin-induced charmless decays $B \rightarrow K_X \phi (K_X$ denotes the meson state $s\bar{q} \ (q = u \ 	ext{or} \ d)$ ), in the standard model and the two-Higgs-doublet model. These processes, being induced at one-loop level, are of great importance in measuring the virtual top quark effect in the standard model, and also in searching for the non-standard signals in the low energy region. It is shown that the QCD effect is also significant in these processes, as in the weak radiative processes $B \rightarrow X_s \gamma$. We also show that the charged Higgs contribution can not provide sizable enhancements for the decays $B \rightarrow K_X \phi$, in contrast to the decays $B \rightarrow K_X \gamma$. It is also found that processes such as $B \rightarrow K_1(1400)\phi$ and $B \rightarrow K(1460)\phi$ have large branching fractions among $B \rightarrow K_X \phi$ decays.

$^1$E-mail: masa@auephys.aichi-edu.ac.jp
1 Introduction

Previously rare $B$ decays such as $b \to s\gamma$, $b \to s\ell\ell$, $b \to s\nu\bar{\nu}$, $b\bar{q}\to \ell\ell$ and $b \to sg$, have been the subject of some interest in the literature. The object of such studies has been to either allow confirmation of the standard model (SM) or to find indications of additional effects beyond the SM. Taking such an approach in our previous paper [1], we analysed the inclusive decays $B \to X_s\gamma$ and exclusive decays $B \to K_X\gamma$ (where $K_X$ denotes the meson state $s\bar{q}$ ($\bar{q} = \bar{u}$ or $\bar{d}$)). In this work we included the nonstandard physical effects due to the charged Higgs contribution in the two-Higgs-doublet model (THDM). We showed that sizable enhancements for these processes are possible compared to the SM. It is well known that, in these processes QCD corrections play an important role [2] in the quantitative discussion, resulting in an enhancement of three to four times in the branching ratio for $b \to s\gamma$.

In the present paper, we discuss another interesting class of processes: $B \to K_X\phi$. There is no previous work with full QCD corrections and including higher $K$-meson resonances. The characteristic feature of these processes is that they are mainly induced through the gluonic penguin interaction via $b \to s\; g^*$, where $g^*$ denotes the virtually emitted gluon. The non-leptonic $B$-decay processes to the final states including $s + s + \bar{s}$ have previously been discussed by three of the present authors and others, with the aim of clarifying the nonstandard physical effects due to the charged Higgs contribution in the THDM [3]. However, QCD corrections were not fully included in these works, and the decays into higher $K$-resonances with the $\phi$-meson were not included.

Now we want to make a more detailed analysis of the processes $B \to K_X\phi$. First we summarize the QCD corrections to the effective Hamiltonian for the process $b \to s + s + \bar{s}$. For this process, the QCD corrections induced by the dominant contribution of the $t$-quark up to the one-loop level essentially require the additional dimension-six local operators of Wilson’s operator-product expansion. These operators have been approximately neglected in the analyses of the other rare $B$ decays [4]. Such consideration of more complete QCD corrections has already been given by Buchalla, Buras and Harlander [5], who provided a detailed renormalization group analysis to investigate the ratio $\epsilon'/\epsilon$ systematically by
including the contributions from the gluonic, photonic, $Z^0$, and neutral Higgs penguin diagrams. They obtained the Wilson coefficient functions with full $O(\alpha_{QED})$ contributions in a compact and transparent form.

The concrete and detailed description of our evaluation of the relevant Wilson functions of the linearly independent dimension six operators is given in section 2, although our evaluation of them is partly the same as that made by Buchalla, Buras and Harlander. Note though that the estimation of the Wilson coefficient functions for full operators in THDM has not been previously given. In section 3, we describe the calculation of the amplitudes and the decay rates for $B \to K\chi\phi$. We emphasize that our calculation of the amplitudes uses the specific form factors given by Isgur, Scora, Grinstein and Wise [6], as well as the factorization assumption as seen in this section. Our results are compared with another model of form factors given by Bauer, Stech and Wirbel [7]. The effect of possible charged Higgs contributions is also studied. Recently, new experimental results, in the form of a limit on and a non-zero value of respectively, for $B(b \to s\gamma)$ and $B(B \to K^*\gamma)$, have been reported by the CLEO experiment [8]. These results lead to new constraints on the values of the relevant parameters occurring in expressions for the amplitudes of these decays. Section 4 is devoted to brief comments on the resultant constraints. In particular, we discuss the constraint on $\cot\beta$ versus $m_H$ in the THDM. In section 5, our summary and conclusions are presented.

2 QCD effects in the rare decay processes of $B$-meson

The rare decay process of the $b$-quark to the final state $s + s + \overline{s}$ is induced at the one-loop level mainly through $b \to s + g^*$.

The fundamental four quark interaction mediated by the process $b \to s + g^*$, i.e. induced by the gluonic penguin diagram, is described by the effective Hamiltonian

$$H_{penguin} = -\frac{\alpha_s G_F V_{ts} V_{tb}}{24\sqrt{2\pi}} V_{ts} V_{tb}(p)\{G_{SM}^\mu + \Gamma_{2H}^\mu\} \frac{\lambda^a}{2} b(p + q)\overline{q}(p_2)\gamma^\mu\frac{\lambda^a}{2} q(p_1),$$

where

$$\Gamma_{SM}^\mu = G_1(x_t)\gamma_\mu(1 - \gamma_5) + 3i\frac{m_b}{q^2} G_2(x_t)\sigma_{\mu\nu} q^\nu(1 + \gamma_5)$$

(1)
and
\[ \Gamma^{2H}_\mu = F_1(y)\gamma_\mu(1 - \gamma_5) + 3\frac{m_b}{q^2} F_2(y)\sigma_{\mu\nu}q^\nu(1 + \gamma_5). \] (3)

The functions \( G_i \) and \( F_i \) are given by

\[
G_1(x_t) = \frac{x_t(1 - x_t)(18 - 11x_t - x_t^2) - 2(4 - 16x_t + 9x_t^2)\ln x_t}{(1 - x_t)^4}
+ 48\left\{ \int_0^1 dt t(1 - t)\ln \frac{m_c^2 - q^2t(1 - t)}{m_W^2(1 - t) + m_t^2t - q^2(1 - t)} - \frac{5}{36} \right\} ,
\]

\[
G_2(x_t) = \frac{x_t(1 - x_t)(2 + 5x_t - x_t^2) + 6x_t\ln x_t}{(1 - x_t)^4} ,
\]

\[
F_1(y) = \frac{y\cot^2 \beta(1 - y)(16 - 29y + 7y^2) + 6(2 - 3y)\ln y}{3(1 - y)^4} ,
\]

\[
F_2(y) = \frac{1}{3}\cot^2 \beta G_2(y) + \frac{2y(1 - y)(-3 + y) - 2\ln y}{(1 - y)^3} ,
\]

where \( x_t = m_t^2/m_W^2 \), \( y = m_H^2/m_t^2 \) and \( \cot \beta = v_d/v_u \) is the usual notation for the ratio of the vacuum expectation values of the neutral sectors of the two-Higgs-doublets. Here, in the first equation for \( G_1(x_t) \), we assume \( V_{tb} = V_{cs} \), \( V_{ts} = V_{cb} \) and \( V_{ub} = 0 \). Note that we have chosen a specific variant of the 2HDM, \textit{i.e.} of the form of the coupling of the Higgs doublets to the fermions. There are four variants of the 2HDM, distinguished by different schemes for the Yukawa couplings to the fermion sector. The reader will find a concise summary of the possible models in reference \[10\]. For what follows we will take model II of that work, in which one doublet gives mass to the up-type quarks, and the other to the down-type quarks and charged leptons. As we will discuss, there are considerable uncertainties in the calculations that follow, and as a consequence, we will content ourselves to investigate the possibility of distinguishing between SM results and those with contributions arising from charged scalars. Distinguishing further between competing models will in general require considerable refinements in the theoretical calculations. We also note that, in the second term of the \textit{r.h.s.}, we should include the charm quark contribution due to the soft GIM cancellation. This term reduces to the familiar result \( 8\ln m_c^2/m_W^2 \) in the limit of the squared momentum of virtual gluon vanishing; \( q^2 = 0 \). Note though that this limit is not realistic for the virtual gluonic penguin induced processes, since the gluon is off mass-shell. In the analysis to follow we take the appropriate nonzero value of \( q^2 \).
The following four-quark operators and the magnetic-transition-type operators are relevant for the processes under consideration. We write the Hamiltonian

\[ H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{8} C_i(\mu) O_i(\mu), \]

where \( \mu \) in the parentheses denotes the energy scale at which the operators are relevant for the decays. The operators \( O_i \) are defined as follows:

\[
\begin{align*}
O_1 &= (\bar{c}_{La} \gamma^\mu b_{L\beta})(\bar{s}_{L\beta} \gamma_\mu c_{La}), \\
O_2 &= (\bar{c}_{La} \gamma^\mu b_{L\alpha})(\bar{s}_{L\beta} \gamma_\mu c_{L\beta}), \\
O_3 &= (\bar{s}_{La} \gamma^\mu b_{L\alpha})(\sum_{5 \text{ quarks}} \bar{q}_{L\beta} \gamma_\mu q_{L\beta}), \\
O_4 &= (\bar{s}_{La} \gamma^\mu b_{L\beta})(\sum_{5 \text{ quarks}} \bar{q}_{L\beta} \gamma_\mu q_{L\alpha}), \\
O_5 &= (\bar{s}_{La} \gamma^\mu b_{L\alpha})(\sum_{5 \text{ quarks}} \bar{q}_{R\beta} \gamma_\mu q_{R\beta}), \\
O_6 &= (\bar{s}_{La} \gamma^\mu b_{L\beta})(\sum_{5 \text{ quarks}} \bar{q}_{R\beta} \gamma_\mu q_{R\alpha}), \\
O_7 &= -\frac{ie}{8\pi^2} m_b \bar{s}_{La} \sigma^{\mu\nu} b_{R\alpha} q_{R\nu}, \\
O_8 &= -\frac{g_c}{8\pi^2} m_b \bar{s}_{La} \sigma^{\mu\nu} T_{\alpha\beta}^{\text{a}} b_{R\beta} q_{\mu} \epsilon_{\nu}.
\end{align*}
\]

Coefficients relevant to our processes are defined at the energy scale \( m_W \) to be

\[
\begin{align*}
C_1(m_W) &= 0, \\
C_2(m_W) &= 1, \\
C_3(m_W) &= -\frac{1}{3} C_4(m_W) = C_5(m_W) = -\frac{1}{3} C_6(m_W) \\
&= -\frac{\alpha_s(m_W)}{288\pi}(G_1(x_t) + F_1(y)), \\
C_7(m_W) &= C_7^{\text{SM}}(x_t) + C_7^{2H}(y), \\
C_8(m_W) &= -\frac{1}{8}(G_2(x_t) + F_2(y)).
\end{align*}
\]

Here \( C_7^{\text{SM}}(x_t) \) and \( C_7^{2H}(y) \) are

\[
C_7^{\text{SM}}(x_t) = \frac{x_t}{24(1-x_t)^3}[8x_t^2 + 5x_t - 7 + \frac{6x_t(3x_t - 2)}{(1-x_t)} \ln x_t]
\]
and

$$C_7^{2H}(y) = \frac{y}{4(1-y)^3} \left[ \frac{5y^2 - 8y + 3}{3} - \frac{2}{3}(3y - 2) \ln y \right]$$

$$- \cot^2 \beta \frac{y}{4(1-y)^4} \left[ \frac{8y^3 - 3y^2 - 12y + 7}{18} + \frac{2}{3}y(1 - \frac{3}{2}y) \ln y \right],$$

(9)

respectively. In Eqs.(7), we neglect the contributions from $Z^0$- and photonic-penguin interactions [11] since these terms are very small numerically due to $\alpha_s \ll \alpha$ and do not affect the present analyses. Also the second term, i.e. the charm contribution, in $G_1(x_t)$ should be taken into account [12] at the scale $\mu = m_b^2$ by setting $q^2 \simeq m_b^2/2$ [13].

Numerically, in the energy region between $m_W$ and $m_b$, we put the flavour number $f = 5$ in the renormalization group equation [14]. We evolve the coefficients $C_i(\mu)$ starting from the scale $m_W$ as given in Eqs.(7) to the scale $\mu = m_b = 4.58 \text{GeV}$, and then we obtain the following numerical coefficients:

$$C_1(m_b) = -0.240, \quad C_2(m_b) = 1.103,$$

$$C_3(m_b) = 0.011 + 1.125C_3(m_W) - 0.121C_4(m_W),$$

$$C_4(m_b) = -0.025 - 0.291C_3(m_W) + 0.824C_4(m_W),$$

$$C_5(m_b) = 0.007 + 0.944C_3(m_W) + 0.083C_4(m_W),$$

$$C_6(m_b) = -0.030 + 0.229C_3(m_W) + 1.465C_4(m_W),$$

$$C_7(m_b) = -0.199 + 0.629C_3(m_W) + 0.931C_4(m_W) + 0.675C_7(m_W) + 0.091C_8(m_W),$$

$$C_8(m_b) = -0.096 - 0.598C_3(m_W) + 1.029C_4(m_W) + 0.709C_8(m_W).$$

(10)

In Table 1 we summarize the numerical values of the above coefficients in the THDM for two values of the parameters $(m_H, \cot \beta)$ [denoted in table by 2HDM(1) and 2HDM(2)], as well as the SM results for typical values of the SM parameters $V_{ij}$ and $m_t$. In order to calculate the enhancement due to QCD corrections, we have compared the coefficients in both cases with and without these corrections. Note that most of the previous predictions for $B \to K\phi$ and $K^*\phi$ decays have been given without the QCD correction.
In Table 1, the heading “Without” QCD means that we include no RG evolution of the coefficients and fix their values at the scale $m_W$, as given in Eq.(7), with $q^2 \simeq m_b^2/2$. We see that the values of $C_i(m_b)$ with the QCD correction are larger than those without by a factor of $1.5 \sim 2$. The values $C_1(m_b) \sim C_6(m_b)$ are almost identical in the SM, the THDM(1) and the THDM(2). It is noteworthy that the differences between the three models exist only for the coefficients $C_7(m_b)$ and $C_8(m_b)$. As mentioned later in section 4, the value of $\cot \beta$ is strongly limited by the recent CLEO experimental search [8] for the $B \rightarrow X_s \gamma$ decay. As a result, here we use the values of $m_H$ and $\cot \beta$ which satisfy the present experimental restrictions. In the next section we apply the above analysis on QCD corrections to the calculation of the decay amplitudes. Note that no clear differences among the SM and the THDMs for the coefficients $C_1(m_b) \sim C_6(m_b)$ are seen in Table 1 even using the QCD corrected coefficients. So we expect that the decay $B \rightarrow X_s \gamma$ will be the best process to search for an effect beyond the SM. However, it is still important to analyse the gluonic penguin effect through the exclusive processes induced by this interaction. This allows us to check the one-loop effects based on the SM, or to study the signals from beyond SM physics. Such an analysis is given in section 3.

3 Exclusive decays $B \rightarrow K_X \phi$ induced by the gluonic penguin interaction

Of the penguin induced charmless $B$ meson decays, the exclusive non-leptonic rare decays $B \rightarrow K_X \phi$ are of special interest, since these processes are the typical of those caused by the gluonic penguin interaction. Analysis of these exclusive decays is also helpful for the future experimental study at $B$-factories. In this section, we calculate these decay rates by including QCD corrections described in section 2. The contribution from new physics is also studied. Since the form factors play an important role in estimating the
branching ratios precisely, we also study the form factor dependence of our results.

In the $s\overline{q}$ system, a rich spectrum of states has been observed. The resonance state $K_X$ is specified by the quantum numbers $n$, $L$, $s$ and $J$, which denote the radial excitation quantum number, the orbital angular momentum, the sum of the spins of the two quarks and the total spin of the meson, respectively. We investigate the following eight states with the notation $n^{2S+1}L_J$: $1^1S_0$, $1^3S_1$, $1^3P_2$, $1^3P_1$, $1^3P_0$, $2^1S_0$ and $2^3S_1$. Physical mesons corresponding to each state are [15]:

1. $1^1S_1$: $K$, $1^3S_1$: $K^*(892)$, $1^3P_2$: $K^*_2(1430)$,
2. $1^3P_1$, $1^1P_1$: $K_1(1270)$, $K_1(1400)$,
3. $1^3P_0$, $2^1S_0$: $K(1460)$, $2^3S_1$: $K^*(1410)$.

The spin 1 mesons $K_1(1270)$ and $K_1(1400)$ are nearly $45^\circ$ mixed states of $1^3P_1$ and $1^1P_1$.

We proceed by showing the formulation to calculate the decay matrix elements. The operators $O_3$, $O_4$, $O_5$, $O_6$ and $O'_8$ are relevant for the decays $B \rightarrow K_X\phi$. Here the operator $O'_8$ is derived from the interaction in which the virtual gluon in the magnetic operator $O_8$ couples with $s + \overline{s}$, to give

$$O'_8 = -i m_b \frac{\alpha_s}{2\pi} \frac{1}{q^2} \bar{s}_L a_t^{\mu \nu} b_R b_\mu \overline{s}_L T^{a_{\mu \nu}}_{\alpha \beta} a_R^{\alpha \beta} \phi^a_s.$$  \hspace{1cm} (12)

By the use of these operators, the decay amplitude may be written as

$$\langle KX\phi \mid H_{eff} \mid B \rangle = \frac{4G_F}{\sqrt{2}} V_{tb} V^*_{ts} \sum_{3,4,5,6,8'} C_i(\mu) \langle K_X\phi \mid O_i(\mu) \mid B \rangle.$$  \hspace{1cm} (13)

In the following analysis we take the value of the coefficient $C'_8(\mu)$ as equal to $C_8(\mu)$. Although $C_8(\mu)$ does not include full the QCD correction of $C'_8(\mu)$ in the leading log approximation, this replacement does not seriously affect the results numerically, as the $O'_8$ term is the next to leading one, compared to the operators $O_3, O_4, O_5$ and $O_6$.

We also use the factorization approximation in order to estimate the hadronic matrix element. The factorization assumption works successfully in $D$ meson and $B$ meson decays [16] within a factor two in the branching ratios. Then, the hadronic matrix elements of
the above operators are given by
\[
\langle K_X \phi | O_3 | B \rangle = \langle K_X \phi | O_4 | B \rangle = \frac{4}{3} \langle K_X \phi | O_5 | B \rangle = 4 \langle K_X \phi | O_6 | B \rangle
\]
which is factorized as follows:
\[
\langle K_X \phi | O_3 | B \rangle = \frac{1}{3} \langle \phi | \bar{s} \gamma_\mu s \varepsilon | 0 \rangle \langle K_X | \bar{s} \gamma^\mu (1 - \gamma_5) b | B \rangle.
\]

Using the Gordon identity and the color algebra relation, the operator \( O'_8 \) reduces to
\[
O'_8 = \frac{i \alpha_s}{4 \pi} m_b \frac{1}{q^2} \left[ i \frac{8}{9} m_b (\bar{s}_L \gamma_\mu b_L) (\bar{s}_L \gamma^\mu s_L) - \frac{4}{9} (\bar{s} \gamma_\mu b) \bar{s} (p')_L (\gamma^\mu p'' - \gamma^\nu p'^\nu) s_L \right]
\]
in the limit \( m_s = 0 \), where a Fierz transformation is performed and the relation \( \langle \phi | \bar{s}_L s_R | 0 \rangle = 0 \) is used. Then, the hadronic matrix element is factorized as follows:
\[
\langle K_X \phi | O'_8 | B \rangle = \frac{i \alpha_s}{4 \pi} m_b \frac{1}{q^2} \left[ i \frac{8}{9} m_b \langle \phi | \bar{s}_L \gamma_\mu s_L | 0 \rangle \langle K_X | \bar{s}_L \gamma^\mu b_L | B \rangle - \frac{4}{9} \langle \phi | \bar{s}_L (\gamma^\mu p'' - \gamma^\nu p'^\nu) s_L | 0 \rangle \langle K_X | \bar{s} \gamma_\mu b | B \rangle \right]
\]
with
\[
\langle \phi | \bar{s}_L \gamma_\mu s_L | 0 \rangle = g_\phi \eta_\mu,
\]
where \( g_\phi = 0.23 \)GeV\(^2\) is taken and \( p'^\mu = p_\phi^\mu / 2 \) is assumed.

The hadronic matrix elements \( \langle K_X \ | \bar{s} \gamma_\mu b | B \rangle \) are given for the \( 1^1 S_0 \) and \( 1^3 S_1 \) states in terms of the form factors by
\[
\langle K(p') \ | \bar{s} \gamma_\mu b | \bar{B}(p) \rangle = i s^T [(p + p')_\mu (p - p')_\nu - (p - p')_\mu (p + p')_\nu],
\]
\[
\langle K^*(p', \epsilon) \ | \bar{s} \gamma_\mu b | \bar{B}(p) \rangle = g^T_+ \epsilon_{\mu\nu} \epsilon^\star \lambda (p + p')^\sigma + g^T_- \epsilon_{\mu\lambda} \epsilon^\star \lambda (p - p')^\sigma
\]
\[
+ h^T \epsilon_{\mu\sigma} (p + p')^\lambda (p - p')^\sigma (\epsilon^\star \epsilon \cdot p).
\]

The form factors appearing in the above equations are easily estimated by using the relations derived from heavy quark symmetry \([17]\);
\[
s^T = \frac{f_+ - f_-}{2 m_b},
\]
\[
h^T = - \frac{g}{m_b} + \frac{a_+ - a_-}{2 m_b},
\]
\[
g^T_+ - g^T_- = -2 m_b g,
\]
\[
g^T_+ + g^T_- = \frac{f}{m_b} + 2 \frac{p \cdot p'}{m_b} g.
\]
where the form factors $f_+,$ $f_-, g,$ $a_+$ and $a_-$ are defined in the hadronic matrix elements of the vector and the axial-vector current \[17\]. For the $1^3P_2$, $2^1S_0$ and $2^3S_1$ states, similar relations are satisfied, while for the $1^3P_1$, $1^1P_1$ and $1^3P_0$ states, the overall signs of r.h.s. in Eq.(19) should be reversed.

These form factors are to be evaluated at $(p - p')^2 = m_\phi^2$ for each final state and they generally depend on the quark potential. Here we use the form factors given by Isgur, Scora, Grinstein and Wise \[3\], which have been successfully applied to the electron energy spectra of the semileptonic $D$ and $B$ meson decays. These are derived using harmonic oscillator type wave functions with a variational method, assuming a Coulomb plus linear potential. This simple model gives quite reasonable spin-averaged spectra of $c\bar{c}$ and $b\bar{b}$ mesons up to $L = 2$. These form factors include the relativistic compensation factor, although their model itself is a nonrelativistic one.

The decay amplitudes are given by

$$
\langle K_X \phi \mid H_{\text{eff}} \mid B \rangle = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \{ A \eta^\mu \langle K_X \mid \bar{s} \gamma_\mu b_L \mid B \rangle 
+ iB(\eta^{\mu} p'_0 - p_0^{\mu} \eta) \langle K_X \mid \bar{s} \sigma_{\mu\nu} b \mid B \rangle \},
$$

with

$$
A \equiv 6\{ C_3(\mu) + C_4(\mu) \} + \frac{3}{2} \{ 3C_5(\mu) + C_6(\mu) \} + \frac{\alpha_s m_b^2}{\pi q^2} C_8(\mu),
$$

$$
B \equiv \frac{\alpha_s m_b}{4\pi q^2} C_8(\mu),
$$

and where $p$ is the three momentum of the $K_X$ meson.

The decay widths of the possible modes are given in terms of the form factors by

$$
\Gamma(B \to K_X \phi) = \frac{G_F^2 g_\phi^2}{81\pi m_B^2} | V_{tb} V_{ts}^* |^2 p( A^2 X_A + B^2 X_B + AB X_{AB} ).
$$

The quantities $X_A$, $X_B$ and $X_{AB}$ for each state are given in terms of the form factors as follows;

for the $0^-(1^1S_0)$ state,

$$
X_A = \frac{m_B^2}{m_\phi^2} f_+ p^2, \quad X_B = 16m_B m_\phi^2 s T^2 p^2, \quad X_{AB} = -8m_B^2 f_+ s T p^2,
$$

\[23\]
for the $1^-(1^3S_1)$ state,
\[
X_A = \frac{1}{2}(f^2 + 4m_B^2g_2^2p^2) + \frac{1}{4m_K^2m_\phi^2}\{(p_{K+} \cdot p_\phi)f + 2m_B^2a_\perp p^2\}^2, \\
X_B = 32m_B^2T_2^2p^2, \quad X_{AB} = 8m_B^2gg_\perp^2p^2, \quad (24)
\]

for the $2^+(1^3P_2)$ state,
\[
X_A = \frac{m_B^2}{4m_K^2}(k^2 + 4m_B^2h_2^2p^2) + \frac{m_B^2}{6m_K^2m_\phi^2}\{(p_{K+} \cdot p_\phi)k + 2m_B^2b_\perp p^2\}^2, \\
X_B = 16\frac{m_B^4}{m_K^2}g_2^2p^4, \quad X_{AB} = 4\frac{m_B^4}{m_K^2}hg_2^2p^4, \quad (25)
\]

for the $1^+(1^1P_1)$ or the $1^+(1^3P_1)$ states,
\[
X_A = \frac{1}{2}\{(\ell^2 + 4m_B^2g_2^2p^2) + \frac{1}{4m_K^2m_\phi^2}\{(p_{K+} \cdot p_\phi)\ell + 2m_B^2c_\perp p^2\}^2, \\
X_B = 8\{m_\phi^2(g_{1+}^T - g_{1-}^T) - 2(p_{K+} \cdot p_\phi)g_{1+}^T\}^2 + \frac{4}{m_K^2m_\phi^2}\{[m_\phi^2(g_{1+}^T - g_{1-}^T) - 2(p_{K+} \cdot p_\phi)g_{1+}^T](p_{K+} \cdot p_\phi) \\
+ 2m_B^2(g_{1+}^T - m_\phi^2h_{1+}^T)p^2\}^2, \\
X_{AB} = - \frac{2}{m_K^2m_\phi^2}\{(p_{K+} \cdot p_\phi)\ell + 2m_B^2c_\perp p^2\} \times \\
[[(g_{1+}^T - g_{1-}^T)m_\phi^2 - 2(p_{K+} \cdot p_\phi)g_{1+}^T](p_{K+} \cdot p_\phi) + 2m_B^2(g_{1+}^T - m_\phi^2h_{1+}^T)p^2] \\
- 4\{(m_\phi^2(g_{1+}^T - g_{1-}^T) - 2(p_{K+} \cdot p_\phi)g_{1+}^T\}, \quad (26)
\]

and for the $0^+(1^3P_0)$ state,
\[
X_A = \frac{m_B^2}{m_\phi^2}u_\perp^2p^2, \quad X_B = 0, \quad X_{AB} = 0. \quad (27)
\]

For the $0^-(2^1S_0)$ and $1^-(2^3S_1)$ states, similar forms of $X_A$, $X_B$ and $X_{AB}$ as for $0^-(1^1S_0)$ and $1^-(1^3S_1)$ apply, respectively. The explicit forms of the form factors $f_+, f, g, a_+, k, h, b_+, \ell, q, c_+$ and $u_+$ are given in the Appendix of Ref.[6]. The form factors $s^T, g_{+}^T, g_{1+}^T, g_{2+}^T$ and $h_{1+}^T$ are calculated by using Eqs.(19) and the form factors referred to above.

The decay branching ratios are shown in Table 2 in the SM, where $C_i(\mu)$ are evaluated at $\mu = m_b = 4.58$GeV.

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**Table 2**
It is found that the decays $B \to K_1(1400)\phi$ and $B \to K(1460)\phi$ dominate the total decay rate of $B \to K_X\phi$. We also show the predicted values “without” QCD corrections, which cases were investigated by two of the present authors previously in the second reference of Refs. [3]. It should be remarked that the QCD effects significantly increase the branching ratios by almost a factor of three. This is to be compared to the process $b \to s\gamma$, in which the QCD corrections enhance the decay ratio by one order as discussed in paper I. Since the predicted values depend on the values of the form factors, we compare our predicted branching ratios for $K\phi$ and $K^*(890)\phi$ final states with the ones calculated using another model of the form factors given by Bauer, Stech and Wirbel [7]. As shown in Table 3, the predicted values in the latter model are almost three times larger than the ones in the GSIW model. Thus, the predicted values depend significantly on the model of the form factors, and it would obviously be of great value to improve the reliability of choosing these form factors.

| Table 3 |
| --- |

These form factors will possibly be tested by the radiative rare decay $B \to K^*(890)\gamma$, which is the process discussed in the next section.

Finally, we study the effect of new physics on these decays. As a typical example of such new physics, the effect of charged Higgs bosons is investigated. Then, $C_i(\mu)$ are modified as discussed in section 2. We show the predicted branching ratios for the typical case where $m_H = 100\text{GeV}$ and $\cot \beta = 1$ in Table 2, where we see that the effect of the charged Higgs boson increases the decay rate at most by $10 \sim 18\%$. This is the extreme case to see the Higgs effect and seems to be excluded by recent CLEO experiment [8] as discussed in the next section. Then the charged Higgs boson appears to have a minor contribution to these decays. Thus, it may be difficult to observe the evidence of the new physics in these decays if we take into account the large ambiguity in the choice of the form factors, as well as the assumption of factorization. However, the radiative exclusive rare
decays such as $B \to K^*(890)\gamma$ may help to choose between models of the form factors, and
the factorization assumption will be studied precisely in the normal non-leptonic decays
of the B meson. Then the theoretical calculations of these rare decays will become more
reliable, and the contribution of effects due to new physics will be able to be discussed
quantitatively.

4 The renewed analysis based on the new CLEO experimental results

Recently, the new experimental limit on $B(b \to s\gamma)$ and the new value of $B(B \to K^*\gamma)$
have been reported by the CLEO group\cite{8} as follows;

$$B(B \to X_s\gamma) < 5.4 \times 10^{-4},$$

(28)

$$B(B \to K^*\gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}.$$ (29)

The experimental data on these processes improve the previous constraints \cite{18} on the
charged Higgs mass and the parameter $\cot \beta$ in the THDM. In Fig.1, we show the modified
allowed parameter region of $\cot \beta$ versus $m_H$ obtained from the new upper limit of $B(B \to
X_s\gamma)$, which replaces that of the Fig.2 of paper I.

The value of $m_H$ is restricted to be larger than about 220Gev which is for $\cot \beta = 0$, $m_t =
140\text{GeV}$ and $m_b = 4.58\text{GeV}$. It is of interest to note that the new experimental upper
bound $B(B \to X_s\gamma) < 5.4 \times 10^{-4}$ is very close to the QCD corrected predicted value of
$3.3 \times 10^{-4}$(for $m_t = 140\text{GeV}$) in the SM.
The predicted branching fractions of $B \to K^*(892)\gamma$ given in the paper I are

$$B(B \to K^*(892)\gamma) = \begin{cases} 3.0 \times 10^{-5} \\ (5.3 \sim 6.9) \times 10^{-5} \quad (\text{THDM: } m_H = 200\text{GeV}, \cot \beta = 0.0 \sim 2.0) \end{cases}$$

(30)

at $m_t = 140\text{GeV}$. These values are consistent with the recent CLEO experimental results given in Eq.(29). At present it is hard to distinguish the effect of the charged Higgs boson, because of large experimental errors. In our calculation, for both cases, the ratio of the exclusive decay rate of $B \to K^*(892)\gamma$ to the inclusive decay rate of $B \to X_s\gamma$ is 7%.

Further experimental progress on both the inclusive and the exclusive processes is eagerly anticipated, to help settle whether the SM correctly describes these decays, or whether we should take into account the possibility of physics beyond the SM.

5 Summary and conclusion

We have analysed the decays $B \to K_X\phi$ in the SM, and also studied the effect of inclusion of the charged Higgs contribution.

First we presented the QCD effects for the effective Hamiltonian relevant for $b \to s + s + \bar{s}$, which is mainly induced at the one-loop level through the gluonic penguin, where the approximately neglected operators in the previous analyses of the other rare $B$ decays are properly included. Then the numerical values of the relevant Wilson coefficient functions $C_i(m_b)$ in the SM as well as in the THDM are summarized, where we find that the noteworthy differences among the SM and the THDMs are seen only for the two coefficients $C_7(m_b)$ and $C_8(m_b)$.

In estimating the hadronic matrix elements of the exclusive decays $B \to K_X\phi$, factorization and a specific model for the form factors are assumed, the latter being those given in Ref. [3], based on the harmonic oscillator type wave functions with variational method using the Coulomb plus linear potential. As well, these form factors are based on relations derived from heavy quark symmetry.

We obtained the branching ratios of these processes in the standard model and then we found that the decays $B \to K_1(1400)\phi$ and $B \to K(1460)\phi$ are the dominant modes.
of $B \to K_X \phi$. We also found that, as in the weak radiative processes $B \to X_s \gamma$, the QCD effect significantly increases the branching ratios by a factor of about three. However, the predicted values depend significantly on the model of the form factors, which we have shown by comparing the results obtained by using two sets of form factors, those of Ref.\cite{6} and Ref.\cite{7}. Accurate measurements of decays such as $B \to K^*(890) \gamma$ may be helpful to select the most reliable model of the form factors. We also studied the effect of the possible existence of charged Higgs boson contributions to these decays. However, according to our calculation, it gives only a minor contribution, increasing the decay rates at most by 10-18%. Due to the theoretical ambiguities of the prediction, such as its dependence on the model of form factors and the factorization assumption, such minor contribution makes the clear observation of evidence for the existence of the charged Higgs contribution difficult. So weak radiative decays may be more favourable to search for charged Higgs effects than the processes we consider here, $B \to K_X \phi$.

\textit{note added}: After most of this work was completed, we have received the related paper by R. Fleischer\cite{12}.

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Table Captions

Table 1: Coefficients $C_3(m_b)$ to $C_8(m_b)$ of the interactions Eqs.(6) in the SM and THDM “without” and “with” the QCD effect. $C_1(m_b)$ and $C_2(m_b)$ are given in Eq.(10). The mass of top quark is fixed to be 140Gev and $m_b=4.58$GeV. The THDM(1) and THDM(2) results correspond to the case $m_H=220$GeV, cot $\beta = 0$ and $m_H=300$GeV, cot $\beta = 1$, respectively.

Table 2: Branching ratios in the standard model “with” and “without” QCD effect. The predictions including the contribution from the charged Higgs bosons ($m_H = 100$GeV,cot $\beta = 1$) “with” QCD are also shown in the last column.

Table 3: Predicted branching ratios using the ISGW form factor model and the BSW form factor model in the SM including QCD corrections.

Figure Captions

Fig. 1 The allowed region of parameters $m_H$ and cot $\beta$ in the THDM. The allowed region is to the right hand side of the corresponding line. The thick and the thin lines correspond to the cases $m_b = 4.58$GeV and $m_b = 5.00$GeV, respectively. The three cases $m_t = 110$GeV, 140GeV and 170GeV are shown as indicated in the figure.
| Coefficient | “Without” QCD | “With” QCD(at $m_b$) |
|-------------|---------------|---------------------|
|             | SM            | THDM(1)             | SM | THDM(1) | THDM(2) |
| $C_3$       | 0.008-0.003i  | 0.008-0.003i        | 0.015-0.006i |
|             | 0.008-0.003i  | 0.008-0.003i        | 0.015-0.006i |
| $C_4$       | -0.024+0.009i | -0.024+0.009i       | -0.038+0.017i |
|             | -0.024+0.009i | -0.024+0.009i       | -0.038+0.017i |
| $C_5$       | 0.008-0.003i  | 0.008-0.003i        | 0.012-0.006i |
|             | 0.008-0.003i  | 0.008-0.003i        | 0.012-0.006i |
| $C_6$       | -0.024+0.009i | -0.024+0.009i       | -0.043+0.017i |
|             | -0.024+0.009i | -0.024+0.009i       | -0.043+0.017i |
| $C_7$       | -0.170        | -0.304              | -0.423        |
|             | -0.282        | -0.320              | -0.423        |
| $C_8$       | -0.089        | -0.157              | -0.234        |
|             | -0.216        | -0.247              | -0.234        |

Table 1
Table 2

| Process | ISGW model | BSW model |
|---------|------------|-----------|
| $B \rightarrow K\phi$ | $0.24 \times 10^{-5}$ | $0.80 \times 10^{-5}$ |
| $B \rightarrow K^*(890)\phi$ | $0.27 \times 10^{-5}$ | $0.92 \times 10^{-5}$ |

Table 3

| Process | SM “with” QCD | SM “without” QCD | Including Charged Higgs “with” QCD |
|---------|---------------|-----------------|----------------------------------|
| $B \rightarrow K\phi$ | $0.24 \times 10^{-5}$ | $0.08 \times 10^{-5}$ | $0.27 \times 10^{-5}$ |
| $B \rightarrow K^*(890)\phi$ | $0.27 \times 10^{-5}$ | $0.10 \times 10^{-5}$ | $0.31 \times 10^{-5}$ |
| $B \rightarrow K_2^*(1430)\phi$ | $0.07 \times 10^{-5}$ | $0.03 \times 10^{-5}$ | $0.08 \times 10^{-5}$ |
| $B \rightarrow K_1(1270)\phi$ | $0.57 \times 10^{-5}$ | $0.16 \times 10^{-5}$ | $0.65 \times 10^{-5}$ |
| $B \rightarrow K_1(1400)\phi$ | $2.05 \times 10^{-5}$ | $0.74 \times 10^{-5}$ | $2.41 \times 10^{-5}$ |
| $B \rightarrow K_0(1430)\phi$ | $0.08 \times 10^{-5}$ | $0.03 \times 10^{-5}$ | $0.10 \times 10^{-5}$ |
| $B \rightarrow K(1460)\phi$ | $1.21 \times 10^{-5}$ | $0.42 \times 10^{-5}$ | $1.38 \times 10^{-5}$ |
| $B \rightarrow K^*(1410)\phi$ | $0.25 \times 10^{-5}$ | $0.09 \times 10^{-5}$ | $0.29 \times 10^{-5}$ |