Free convection boundary layer flow over a horizontal circular cylinder in $\text{Al}_2\text{O}_3$-Ag/water hybrid nanofluid with viscous dissipation

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Abstract

In this paper, the mathematical model of free convection boundary layer flow of horizontal circular cylinder immersed in $\text{Ag}/\text{Water}$ nanofluid and $\text{Al}_2\text{O}_3$-$\text{Ag}/\text{Water}$ hybrid nanofluid are considered. The governing non-linear partial differential equations are first transformed to a more convenient way before being solved numerically using the Keller-box method. The numerical values for the reduced Nusselt number and the reduced skin friction coefficient are obtained and illustrated graphically as well as temperature profiles and velocity profiles. Effects of the Prandtl number, Eckert number and nanoparticle volume fraction are analyzed and discussed. It is found that the Nusselt number for $\text{Al}_2\text{O}_3$-$\text{Ag}/\text{Water}$ hybrid nanofluid is comparable with $\text{Ag}/\text{Water}$ nanofluid with a reduction in skin friction coefficient. The preliminary results reports here are important as a reference in exploring the potential of hybrid nanofluid to reduce the production cost compared to the used of metal nanofluid.

Keywords: Free convection, hybrid nanofluid, circular cylinder, viscous dissipation

INTRODUCTION

Recent eras of technology saw the widely used of nanofluid as a cooling or heat transfer medium in many industrial, automotive and electrical devices. Nanofluid has a potential in the thermal management system such as liquid submerged cooling for transformer and electronic circuit board after the successful involvement in engine downsizing in automotive segment (Mohamed et al., 2018). Better performance in thermal conductivity, viscosity, thermal diffusivity and convective heat transfer as well as no clogging are the reason for the employment of nanofluid compared to base fluid (Wong and De Leon, 2010).

It is known that the metal nanoparticles like copper $\text{Cu}$ and silver $\text{Ag}$ performed better in heat transfer capabilities compared to oxide nanoparticles, thanks to the nanoparticle’s higher thermal conductivity. Unfortunately, this type of nanomaterial is expensive and not economical in mass production (Devi and Devi, 2017).

Present study investigates the flow and heat transfer on horizontal circular cylinder immersed in blended metal and oxide nanofluid called as hybrid nanofluid. Considering the flow on a circular cylinder, there are many investigations have been done in the past decade, for example Salleh and Nazar (2010) and Sheikhholeslami et al. (2012) who studied the Newtonian heating boundary conditions and magnetic effects on flow towards the horizontal circular cylinder. It is found that the skin friction coefficient is increasing at the middle of the cylinder before decreases to the end of the cylinder surface. Other researchers considered this topic with a Newtonian and non-Newtonian industrial fluid like power-law fluids by Chandra and Chhabra (2012), Bingham plastic fluid by Nirnmalark et al. (2014) and Bose et al. (2015), Casson fluid by Makanda et al. (2015) and nanofluid by Tham et al. (2012) and Mohamed et al. (2016). The nanofluid Buongiorno-Darcy model is employed and concluded that the increase in Brownian motion parameter and thermophoresis parameter has reduced the values of Nusselt number. Next, the flow and heat transfer immersed in a viscoelastic fluid, Jeffrey fluid, and viscoelastic nanofluid have been observed by Widodo et al. (2016), Zokri et al. (2017; 2018) and Mahat et al. (2018).

Recent studies on fluid flow on horizontal circular cylinder included the works from Gaffar et al. (2019), Yasin et al. (2020) and Alwawi et al. (2020) who investigate the heat transfer analysis of Casson nanofluid, ferrofluid, and third-grade fluid with magnetohydrodynamic effect. It is worth mentioning here that the ferrofluid is a nanofluid with ferrite nanoparticles. The ferrofluid heat transfer is dominance with the presence of a magnetic effect.

As for now, the experimental study regarding this topic is expensive and difficult to be realized. Some of the nanoparticles combinations in hybrid nanofluid are not yet able to being synthesizing, thus provided limited findings and knowledge. The approached from a numerical analysis based on a mathematical model is the alternative and relevant way to be considered. It is cheap, fast, and provided the theoretical knowledge for the hybrid nanofluid, therefore proposed an early idea about the fluid flow and heat transfer characteristics. Based on the literature studies, a study of free convection of hybrid nanofluid on a horizontal circular cylinder with the presence of viscous dissipation
effect is never been done before, so the reported results in this study are new.

**MATHEMATICAL FORMULATIONS**

The horizontal circular cylinder with radius \( a \), which is heated to a constant temperature \( T_w \) embedded in a hybrid nanofluid with ambient temperature \( T_e \) is considered. The physical model is shown in Fig 1. The orthogonal coordinates of \( \tau \) is measured along the cylinder surface, starting from the lower stagnation point \( \tau = 0 \), and \( \psi \) measures the distance normal from the surface. Under the assumptions that the boundary layer approximation is valid, the dimensional governing equations of steady free convection boundary layer flow are (Mohamed et al., 2016; Devi and Devi, 2017):

\[
\frac{\partial \bar{u}}{\partial \tau} + \frac{\partial \bar{u}}{\partial \psi} = 0,
\]

subject to the boundary conditions

\[
\bar{u}(\tau,0) = \tau(\tau,0) = 0, \quad T(\tau,0) = T_w,
\]

where \( \bar{u} \) and \( \bar{v} \) are the velocity components along the \( \tau \) and \( \psi \) axes, respectively. \( \bar{u}_{\text{hf}} \) is the hybrid nanofluid dynamic viscosity, \( \rho_{\text{hf}} \) is the hybrid nanofluid density, \( g \) is the gravity acceleration, \( \beta_{\text{hf}} \) is the hybrid nanofluid thermal expansion, \( r \) is local temperature, \( \rho C_p \) is the heat capacity of hybrid nanofluid, \( \nu_{\text{hf}} \) is the kinematic viscosity of hybrid nanofluid and \( k_{\text{hf}} \) is the thermal conductivity of hybrid nanofluid which can be expressed as follows (Devi and Devi, 2017):

\[
\nu_{\text{hf}} = \frac{\mu_{\text{hf}}}{\rho_{\text{hf}}}, \quad \rho_{\text{hf}} = \frac{\mu_{f}}{(1-\phi_1)^2(1-\phi_2)^2},
\]

\[
(\rho C_p)_{\text{hf}} = (1-\phi_1)(1-\phi_2)(\rho C_p)_{f} + \phi_1(\rho C_p)_{s1} + \phi_2(\rho C_p)_{s2},
\]

\[
(\nu C_p)_{\text{hf}} = (1-\phi_1)(1-\phi_2)(\nu C_p)_{f} + \phi_1(\nu C_p)_{s1} + \phi_2(\nu C_p)_{s2},
\]

\[
k_{\text{hf}} = k_f + 2k_f(k_f - k_{s1}) - 2\phi_1(k_f - k_{s2}) - 2k_f(k_f - k_{s1}),
\]

Note that the subscript \( \text{hf}, f, s1 \) and \( s2 \) represent the physical properties of hybrid nanofluid, base fluid, alumina \( Al_2O_3 \), nanoparticle, and silver \( Ag \) nanoparticle, respectively.

In this study, initially 0.06 vol. solid nanoparticle of \( Al_2O_3 (\phi_1 = 0.1) \) is added into \( Ag/Water \) nanofluid. Next, 0.1 vol. solid nanoparticle of \( Ag/Water \) nanofluid to form the \( Al_2O_3-Ag/Water \) hybrid nanofluid namely. Next, it is introduced the governing non-dimensional variables:

\[
x = \frac{\tau}{a}, \quad y = Gr^{1/4} \frac{\psi}{a}, \quad u = \frac{a}{v_f} \bar{u}, \quad v = \frac{a}{v_f} Gr^{-1/4} \bar{v},
\]

\[
\theta(\eta) = \frac{T - T_e}{T_w - T_e},
\]

where \( \theta \) are the rescaled dimensionless temperature of the fluid and \( Gr = \frac{g\beta_f(T_w - T_e) a^3}{\nu_f^2} \) is a Grashof number. Using (6), (1)-(4) becomes

\[
\frac{\partial \bar{u}}{\partial \tau} + \frac{\partial \bar{v}}{\partial \psi} = 0,
\]

subject to the boundary conditions

\[
u(x,0) = 0, \quad \nu(x,0) = 0, \quad \theta(x,0) = 1, \quad u(x,\infty) = 0, \quad \theta(x,\infty) = 0.
\]

Note that \( Ec = \frac{v_f^2 Gr}{a^2 (C_p)_{f} (T_w - T_e)} \) is an Eckert number. In order to solve equations (7) to (9), the following functions are introduced:

\[
\psi = x f(x, y), \quad \theta = \theta(x, y),
\]

where \( \psi \) is the stream function defined as \( u = \frac{\partial \psi}{\partial \tau} \) and \( v = \frac{\partial \psi}{\partial \psi} \) which identically satisfies Eq. (7). Substituting Eq. (11) into Eqs. (7)-(9), the following partial differential equations are obtained:

\[
v_{\text{hf}} \frac{\partial^2 \psi}{\partial \psi^2} + f \frac{\partial^2 \psi}{\partial y^2} = \frac{\partial^2 \theta}{\partial \psi^2} + \frac{\partial \theta}{\partial \tau} + \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial \psi} + \frac{\partial^2 \theta}{\partial \psi^2} \sin x \theta = 0,
\]

\[
s + \frac{\partial \psi}{\partial \tau} + \frac{\partial \theta}{\partial \psi} - x Ec \frac{\partial \psi}{\partial \psi} \frac{\partial \theta}{\partial \psi} \frac{\partial^2 \theta}{\partial \psi^2} \sin x \theta = 0,
\]

where \( Pr = \frac{v_f (C_p)_{f}}{k_f} \) is the Prandtl number. Other quantities are detailed as follows:
parameter $t$ method is taking part and linearized by reducing the equations to blend Keller conditions (1)

Furthermore, the velocity profiles and temperature distributions can be obtained from the following relations:

The physical quantities of interest are the skin friction coefficient $C_f$ and the local Nusselt number $Nu$ which given by

The surface shear stress $\tau_v$ and the surface heat flux $q_u$ are given by

Using variables in Eq. (6) and Eq. (11) give

Furthermore, the velocity profiles and temperature distributions can be obtained from the following relations:

**NUMERICAL METHOD**

The partial differential equations (12) and (13) subject to boundary conditions (14) are solved numerically using the Keller-box method. Keller-box method actually is an implicit finite difference method blend with Newton’s method for linearization. Details regarding this method are clearly described by Na (1979), Cebeci and Cousteix (2005) and recently by Mohamed (2018). Keller-box method starts with reducing the equations to a first-order system. The finite difference method is taking part and linearized by using Newtons method. Then, the resulting algebraic equations are written in matrix-vector the form and lastly, being solved by the block tridiagonal elimination technique.

The algorithm of the Keller-box method is coded into MATLAB software to numerically compute. It is worth to noticed that the boundary layer thickness from $7$ to $10$ with step size $\Delta x = 0.02$, $\Delta y = 0.01$ are used in obtaining the converging and precise numerical results. The computation focused on the effects of a pertinent parameter which is the Prandtl number $Pr$, the Eckert number $Ec$ and nanoparticle volume fraction for alumina $Al_2O_3(\phi_1)$ and silver $Ag(\phi_2)$. The values of thermophysical properties of water and nanoparticles consider are tabulated in Table 1. The calculation is obtained from the stagnation region $(x = 0 rad)$ until to the end of the cylinder surface $(x = \pi rad)$.

For comparison purposes, Tables 2 and 3 show the comparison values with previously published results. From both tables, it is found that the results agreed and in a good agreement, hence it is believed that whole results presents in this study are precise in computing numerically.

**RESULTS AND DISCUSSION**

The Figs. 1-8 are illustrated in order to get a clear view regarding the pertinent parameter effects on hybrid nanofluid flow and heat transfer. Figs. 2-5 show the variation of the reduced skin friction coefficient $C_{f,Gr^{1/4}}$, reduced Nusselt number $Nu_{Gr^{1/4}}$, temperature profiles $\Theta(y)$ and velocity profiles $f'(y)$ for various values of $\phi_1$ and $\phi_2$, respectively. From Fig. 1, it was found that values of the $C_{f,Gr^{1/4}}$ is unique at the stagnation region $(x = 0 rad)$. At this point, the presents of nanoparticles gave no effect on friction. As flow passes through the cylinder body, the $C_{f,Gr^{1/4}}$ increases when 0.06 vol. of silver Ag nanoparticles is added up into water-based fluid to form the $Ag/Water(\phi_1 = 0.0, \phi_2 = 0.06)$ nanofluid. The values of $C_{f,Gr^{1/4}}$ then increase again with the adding 0.1 vol. of alumina $Al_2O_3$ nanoparticles into $Ag/Water$ to form the $Al_2O_3/Ag/Water(\phi_1 = 0.1, \phi_2 = 0.06)$ hybrid nanofluid. Further, the $Al_2O_3/Ag/Water(\phi_1 = 0.1, \phi_2 = 0.06)$ hybrid nanofluid is compare with $Ag/Water(\phi_1 = 0.0, \phi_2 = 0.16)$ nanofluid and it is found that the $Ag/Water(\phi_1 = 0.0, \phi_2 = 0.16)$ nanofluid have higher friction compared to $Al_2O_3/Ag/Water(\phi_1 = 0.1, \phi_2 = 0.06)$ hybrid nanofluid. Physically, the $Ag$ has high density, hence score more friction in skin friction.

Fig. 2 shows the variation of the reduced Nusselt number $Nu_{Gr^{1/4}}$ for various values of $\phi_1$ and $\phi_2$. It was observed that the effects of changes in $\phi_1$ and $\phi_2$ are more significantly at the stagnation region. Further, it is noticed that the $Nu_{Gr^{1/4}}$ is decreasing along a cylinder surface. From Fig. 2, the $Al_2O_3/Ag/Water(\phi_1 = 0.1, \phi_2 = 0.06)$ hybrid nanofluid score highest values in $Nu_{Gr^{1/4}}$ compared to water-based fluid and $Ag/Water(\phi_1 = 0.0, \phi_2 = 0.16)$ nanofluid. It is clearly shown that the hybrid nanofluid which consists of the combination metal and low-cost oxide nanoparticles generate a comparable heat transfer capabilities with premium metal nanofluid.

| $C_{f,Gr^{1/4}}$ | $Nu_{Gr^{1/4}}$ |
|-----------------|-----------------|
| $1$             | $k_f / k_f$     |
| $(1 - \phi_2)\rho_f + \phi_1 (\rho C_p)_f$ | $(1 - \phi_2)\rho_f + \phi_1 (\rho C_p)_f$ |
| $(1 - \phi_2)\rho_f + \phi_1 (\rho C_p)_f$ | $(1 - \phi_2)\rho_f + \phi_1 (\rho C_p)_f$ |
| $(1 - \phi_2)\rho_f + \phi_1 (\rho C_p)_f$ | $(1 - \phi_2)\rho_f + \phi_1 (\rho C_p)_f$ |
| $(1 - \phi_2)\rho_f + \phi_1 (\rho C_p)_f$ | $(1 - \phi_2)\rho_f + \phi_1 (\rho C_p)_f$ |
| $(1 - \phi_2)\rho_f + \phi_1 (\rho C_p)_f$ | $(1 - \phi_2)\rho_f + \phi_1 (\rho C_p)_f$ |
| $(1 - \phi_2)\rho_f + \phi_1 (\rho C_p)_f$ | $(1 - \phi_2)\rho_f + \phi_1 (\rho C_p)_f$ |
| $(1 - \phi_2)\rho_f + \phi_1 (\rho C_p)_f$ | $(1 - \phi_2)\rho_f + \phi_1 (\rho C_p)_f$ |
| $(1 - \phi_2)\rho_f + \phi_1 (\rho C_p)_f$ | $(1 - \phi_2)\rho_f + \phi_1 (\rho C_p)_f$ |

Table 1. Thermophysical properties of water and nanoparticles.
Table 2. Comparison values of \( Nu Gr^{1/4} \) with previous published results for various values of \( x \) when \( Pr = 1 \) and \( \phi_1 = \phi_2 = E \epsilon = 0 \).

| \( x \) | Merkin (1976) | Nazar et al. (2002) | Molla et al. (2006) | Azim (2014) | Present |
|-------|----------------|---------------------|---------------------|-------------|---------|
| \( \pi / 6 \) | 0.4214 | 0.4214 | 0.4214 | 0.4216 | 0.4213 |
| \( \pi / 3 \) | 0.4161 | 0.4161 | 0.4161 | 0.4163 | 0.4161 |
| \( \pi / 2 \) | 0.3745 | 0.3741 | 0.3740 | 0.3742 | 0.3743 |
| 2\( \pi / 3 \) | 0.3364 | 0.3355 | 0.3355 | 0.3356 | 0.3359 |
| 5\( \pi / 6 \) | 0.2825 | 0.2811 | 0.2812 | 0.2811 | 0.2815 |
| \( \pi \) | 0.1945 | 0.1916 | 0.1917 | 0.1912 | 0.1934 |

Table 3. Comparison values of \( C_p Gr^{1/4} \) with previous published results for various values of \( x \) when \( Pr = 1 \) and \( \phi_1 = \phi_2 = E \epsilon = 0 \).

| \( x \) | Merkin (1976) | Nazar et al. (2002) | Molla et al. (2006) | Azim (2014) | Present |
|-------|----------------|---------------------|---------------------|-------------|---------|
| \( 0 \) | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| \( \pi / 6 \) | 0.4151 | 0.4148 | 0.4145 | 0.4139 | 0.4156 |
| \( \pi / 3 \) | 0.7558 | 0.7542 | 0.7539 | 0.7528 | 0.7534 |
| \( \pi / 2 \) | 0.9579 | 0.9545 | 0.9541 | 0.9526 | 0.9557 |
| 2\( \pi / 3 \) | 0.9756 | 0.9698 | 0.9696 | 0.9678 | 0.9726 |
| 5\( \pi / 6 \) | 0.7822 | 0.7740 | 0.7739 | 0.7718 | 0.7773 |
| \( \pi \) | 0.3391 | 0.3265 | 0.3264 | 0.3239 | 0.3356 |

Fig. 1 Variation of \( C_p Gr^{1/4} \) against \( x \) for various values of \( \phi_1 \) and \( \phi_2 \) when \( Pr = 7 \) and \( E \epsilon = 0.1 \).

Fig. 2 Variation of \( Nu Gr^{1/4} \) against \( x \) for various values of \( \phi_1 \) and \( \phi_2 \) when \( Pr = 7 \) and \( E \epsilon = 0.1 \).

Fig. 3 Temperature profiles \( \theta(y) \) against \( y \) for various values of \( \phi_1 \) and \( \phi_2 \) when \( Pr = 7 \) and \( E \epsilon = 0.1 \).
The temperature profiles \( \theta(x) \) and velocity profiles \( f'(\eta) \) at a stagnation region \( (x = 0) \) for various values of \( \phi_1 \) and \( \phi_2 \) are illustrated in Figs. 3 and 4, respectively. It is shown the increase of nanoparticles has increased the thermal boundary layer thickness and the fluid velocity while reduced the velocity boundary layer thicknesses. The increase of nanoparticles in hybrid nanofluid has added up an extra thermal conductivity in fluid thus raised the thermal diffusivity and increase the thermal boundary layer thickness. In Figure 4, the presence of nanoparticles in fluid raised the fluid momentum which translates to the increasing in fluid velocity. This is realistic especially for nanofluids like silver \( Ag \) in \( Ag/Water (\phi_1 = 0, \phi_2 = 0.16) \) nanofluid. The higher density nanofluid or hybrid nanofluid decelerate rapidly compared to water-based fluid due to friction between fluid and cylinder surface, thus lead to a reduction in velocity boundary layer thickness.

Next, Figs. 6 and 7 show the variation of \( C_fGr^{1/4} \) and \( Nu_{Gr}Gr^{-1/4} \) for various values of \( Pr \) and \( Ec \), respectively. Both Figs. 4 and 5 agreed that the viscous dissipation effect \( Ec \) is negligible at stagnation region \( (x = 0) \). This is unsurprisingly and clearly stated in Eq. (13). The viscous dissipation effect is more pronounced in the middle of the cylinder body. It shows that the presence of viscous dissipation has promoted the skin friction effects as in Fig. 5 but reduced the convective heat transfer capability which represents by \( Nu_{Gr}Gr^{-1/4} \) as shown in Fig. 6. Physically, the reduction in convective heat capabilities enhances the convective heat transfer properties. Meanwhile, the increase of \( Pr \) results to the increase in \( Nu_{Gr}Gr^{-1/4} \).

Lastly, the variations of \( C_fGr^{1/4} \) and \( Nu_{Gr}Gr^{-1/4} \) for the various concentration of hybrid nanofluid are illustrated in Figs. 8 and 9, respectively. The flow and heat transfer performance of \( Al_2O_3:Ag/Water (\phi_1 = 0.1, \phi_2 = 0.06) \) hybrid nanofluid is compared with another hybrid nanofluid consist of \( TiO_2 \) and \( Cu \) nanoparticles. From both figures, it is learned that the different and appropriate combination nanoparticles in hybrid nanofluid provided desired flow and heat transfer characteristics (Devi and Devi, 2017). From Fig. 8, it is observed the values of \( C_fGr^{1/4} \) are not affected with the combination nanoparticles at the stagnation region \( (x = 0) \). It is more pronounced at the middle of the cylinder, different with \( Nu_{Gr}Gr^{-1/4} \) which more significantly affected at the stagnation region as shown in Fig. 8. Further, it is noticed that the \( Al_2O_3:Ag/Water \) hybrid nanofluid score highest in \( Nu_{Gr}Gr^{-1/4} \) followed by \( Al_2O_3-Cu/Water \) hybrid nanofluid. Specifically, \( Ag \) has the highest values of thermal conductivity compared \( Cu, TiO_2 \) and \( Al_2O_3 \). Hence, any nanoparticle combination with \( Ag \) will provide high in heat transfer capability.

**CONCLUSION**

In this paper, the problem of free convection boundary layer flow over a horizontal circular cylinder in hybrid nanofluid was numerically studied. It was shown the effects of Prandtl number \( Pr \), the Eckert number \( Ec \) and alumina \( Al_2O_3 (\phi_1) \) as well as silver \( Ag (\phi_2) \) nanoparticles volume fraction for hybrid nanofluid.

As a conclusion, it is found that the increase of nanoparticles in nanofluid has increased the values of skin friction coefficient. The high-density nanoparticles like silver in nanofluid also contribute to a high in friction between fluid and cylinder surface. In industrial case, this situation damaging the component surface. From numerical investigation, the appropriate nanoparticles combination like alumina-silver in hybrid nanofluid may reduce these skin friction phenomena but yet still gave the heat transfer capabilities comparable to silver nanofluid. Noticed that alumina-silver hybrid nanofluid is cheaper to produce than silver nanofluid.

Next, it is observed that the Nusselt number shows a reducing variation along with the cylinder while skin friction experienced high in friction in the middle of the cylinder surface. The increase in Prandtl number results to the increase in Nusselt number while Eckert number which represented the viscous dissipation effects does contrary. Further, it is noticed that the high values of thermal conductivity nanoparticles like silver will provided high in heat transfer capability.
Fig. 7 Variation of $C_f$ against $Gr^{1/4}$ for various concentration of hybrid nanofluid when $Pr = 7$ and $Ec = 0.1$.

Fig. 8 Variation of $Nu_x Gr^{1/4}$ against $x$ for various concentration of hybrid nanofluid when $Pr = 7$ and $Ec = 0.1$.

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