A survey on the applications of the Krein-space theory in signal estimation

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(Received 23 December 2013; final version received 22 January 2014)

In this paper, the applications of theory in the signal estimation problems are surveyed. The signal estimation under consideration includes filtering, prediction, smoothing and fault estimation. We first introduce the basic concepts about Krein spaces and the main principle of the linear estimation theory in Krein spaces. Then, the developments on various Krein-space-based filtering, prediction, smoothing technologies as well as their engineering applications are reviewed in great detail. Subsequently, the recent advances on the Krein-space-based fault estimation approaches are discussed. Finally, conclusions are drawn and several possible future research directions are pointed out.

Keywords: fault estimation; filtering; Krein spaces; prediction; smoothing

1. Introduction

Signal estimation problems have long been a fascinating focus of research attracting constant attention from a variety of engineering areas. The signal estimation concerned here includes the filtering (prediction and smoothing) and fault estimation. Filtering is an algorithm that uses the available measurements which usually contains noises to produce estimates of unknown variables of interest. Prediction and smoothing are in essence special forms of filtering and they are distinguished from the filtering via the time instant of unknown variables to be estimated by the same available measurements. The filter (predictor and smoother) has numerous applications in technology such as guidance, navigation and control of vehicles, particularly aircraft and spacecraft. On the other hand, fault estimation has played an important role and has long been one of the foundational problems in the area of fault diagnosis and fault-tolerant control. For example, in the fault diagnosis problem, the fault estimation often serves a residual generation scheme for the purpose of fault detection, while in fault-tolerant control systems, the fault estimate is usually added into the controller to compensate for the unknown real fault. Therefore, in the past few years, the fault estimation problems have been extensively investigated for various systems and numerous fault estimation schemes have been proposed.

It is worth mentioning that, in the past decade, the Krein-space theory has attracted increasing attention from researchers in both mathematics, control and signal processing communities. Indeed, for those with a performance index described by an indefinite quadratic form, the Krein-space theory has proven to be an effective tool. In a brief, the so-called Krein-space approach is to convert the problem of interest into the minimum issue of a certain indefinite quadratic form and then, by applying innovation analysis and projection theory in Krein spaces, derive the necessary and sufficient condition for the existence of its minimum. Note that such a practice reduces significantly the conservativeness of results and, simultaneously, can lead to some computationally attractive algorithms. Due to these advantages, considerable research efforts have been made on the application of the Krein-space theory in the signal estimation problems and various Krein-space-based estimation schemes have been proposed such as the Krein-space-based filtering, prediction, smoothing and fault estimation technologies.

In this paper, we focus mainly on the applications of the Krein-space theory in the signal estimation problems and aim to give a comprehensive survey on the developments of the Krein-space-based signal estimation technologies. The signal estimation under consideration includes filtering, prediction, smoothing and fault estimation. For the reader’s convenience, we first review some basic concepts about Krein spaces and introduce the main principle about the linear estimation theory in Krein spaces. Then, various filtering, prediction, smoothing technologies based on the Krein-space theory are reviewed in detail and some applications of these technologies in the industrial engineering are given. Subsequently, we pay particular attention on the fault estimation technologies and discuss the recent advances on the fault estimation approaches developed by virtue of the Krein-space theory. Finally, some conclusions are drawn and several possible related research directions are pointed out.

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The remainder of this paper is organized as follows. In Section 2, the basic concepts and main principle on the linear estimation theory in Krein spaces are outlined. Section 3 reviews the developments of the Krein-space-based filtering, prediction and smoothing technologies. Section 4 discusses some recent advances on the Krein-space-based fault estimation approaches. The conclusions and future work are given in Section 5.

2. Preliminaries on Krein spaces

In this section, we briefly review some well established yet important results in Krein spaces that will help readers understand how to deal with the signal estimation problems by employing the Krein-space theory. Here, we only give the most fundamental notions and list those results that are closely related to the topics to be discussed. With respect to the detailed expositions, we refer the reader to Hassibi, Sayed, and Kailath (1996a).

Let us start with the following definition of Krein spaces.

**Definition 1** An abstract vector space \( \{K, \langle \cdot , \cdot \rangle \} \) is called a Krein space if it satisfies the following requirements:

(i) \( K \) is a linear space over the ring \( S \).

(ii) There exists a bilinear form \( \langle \cdot , \cdot \rangle \in S \) on \( K \) such that

\[
\begin{align*}
(a) & \quad \langle y, x \rangle = \langle x, y \rangle^* \\
(b) & \quad \langle ax + by, z \rangle = a\langle x, z \rangle + b\langle y, z \rangle
\end{align*}
\]

for any \( x, y, z \in K \) and \( a, b \in S \), and where the operation * depends on the ring \( S \).

(iii) The vector space \( K \) admits a direct orthogonal sum decomposition

\[
K = K_+ \oplus K_-
\]

such that \( \{K_+, \langle \cdot , \cdot \rangle \} \) and \( \{K_-, \langle \cdot , \cdot \rangle \} \) are modules, and \( \langle x, y \rangle = 0 \) for any \( x \in K_+ \) and \( y \in K_- \).

From the above definition, it can be seen that the Krein space does not necessarily satisfy the condition that \( \langle x, x \rangle > 0 \) when \( x \neq 0 \), which means the Krein space is a special kind of indefinite metric space. Nevertheless, Krein spaces still share many characteristics of the conventional Hilbert spaces, and the projection also plays an important role in the linear estimation theory in Krein spaces. The notion of projection is defined as follows.

**Definition 2** Given the elements \( \{y_0, y_1, \ldots, y_N\} \) in \( K \), the projection of a vector \( z \in K_n \) onto \( L^2(y_0, y_1, \ldots, y_N) \) is defined as an element \( \hat{z} \in L^2(y_0, y_1, \ldots, y_N) \) such that

\[
\hat{z} = ky, \quad k \in C^{n \times (N+1)},
\]

where \( y = \text{col} \{y_0, y_1, \ldots, y_N\} \) and \( k \) satisfies \( \langle z - ky, y \rangle = 0 \).

As is known to all, in the Hilbert space setting, projections always exist and are unique. However, this is not the case in the Krein space setting. In order to ensure the existence and uniqueness of the Krein-space projections, we need a standing assumption that the Gramian \( R_y = \langle y, y \rangle \) is nonsingular. Under this assumption, the Krein-space projection is given in the following theorem.

**Theorem 1** In Krein spaces, if \( R_y \) is nonsingular, then the unique projection of \( z \) onto \( L^2(y) \) is given by

\[
\hat{z} = k_0^* y, \quad k_0 = R_y^{-1} R_{yz},
\]

where \( R_{yz} = \langle y, z \rangle \). This projection yields the unique stationary point of the error Gramian \( P(k) = \langle z - k^* y, z - k^* y \rangle \) over all \( k \in C^{n \times (N+1)} \), and the value of \( P(k) \) at the stationary point is given by

\[
P(k_0) = R_z - R_{yz} R_y^{-1} R_{yz}.
\]

Furthermore, the projection \( \hat{z} \) minimizes the above \( P(k) \) if and only if \( R_y > 0 \).

Theorem 1 provides an explicit expression of the Krein-space projection that standardizes a certain quadratic form, i.e. the error Gramian \( P(k) \) and, under some additional conditions, the stationary points can be minima. This is a stochastic minimization problem in Krein spaces. Let’s now examine the deterministic minimization problem for a certain scalar quadratic form over ordinary complex variables. To this end, we consider the following scalar second-order form:

\[
J(z, y) = [z^* \quad y^*] \left[ \begin{array}{cc} R_z & R_{zy} \\ R_{yz} & R_y \end{array} \right]^{-1} \left[ \begin{array}{c} z \\ y \end{array} \right],
\]

(1)

where \( R_z \) and the block matrix are assumed to be nonsingular and \( z \) and \( y \) are regarded as ordinary vectors of complex numbers.

Noting that

\[
\left[ \begin{array}{cc} R_z & R_{zy} \\ R_{yz} & R_y \end{array} \right]^{-1} = \left[ \begin{array}{cc} I & 0 \\ -R_{yz}^{-1} R_z & I \end{array} \right] \left[ \begin{array}{cc} R_z - R_{yz} R_y^{-1} R_{yz} & 0 \\ 0 & R_y \end{array} \right]^{-1} \left[ \begin{array}{cc} I & 0 \\ -R_{yz}^{-1} R_y^{-1} & I \end{array} \right],
\]

the scalar quadratic form Equation (1) can be further given by

\[
J(z, y) = [z^* - y^* R_{yz}^{-1} R_y \quad y^*] \left[ \begin{array}{cc} R_z - R_{yz} R_y^{-1} R_{yz} & 0 \\ 0 & R_y \end{array} \right]^{-1} \left[ \begin{array}{c} z - R_y R_{yz}^{-1} y \\ y \end{array} \right].
\]

(2)

Then, the following result is obtained immediately.
Theorem 2 Suppose that both \( R_y \) and the block matrix in Equation (1) are nonsingular. Then the stationary point \( z_0 \) of \( J(z,y) \) over \( z \) is given by

\[
z_0 = R_y R^{-1} y,
\]
and the value of \( J(z,y) \) at the stationary point is

\[
J(z_0, y) = y^T R^{-1} y.
\]
Furthermore, the stationary point \( z_0 \) minimizes the scalar second-order form Equation (1) if and only if

\[
R_x - R_z R^{-1} R_{z^T} > 0.
\]

By comparing the results of Theorems 1 and 2, it can be observed that the stationary point \( z_0 \) is the stationary point for a certain quadratic form. Actually, for a given quadratic form, we can always rewrite it into a form as shown in Equation (1) and then view the coefficient matrices \( R_x, R_z, R_{y^T} \) and \( R_{y^T} \) as the Gramians and cross-Gramians of stochastic vectors \( z \) and \( y \) in Krein spaces. Subsequently, the desired stationary point can be derived by calculating the Krein-space projection of \( z \) onto \( L^2[y] \). However, it should be pointed out that the condition for a minimum given in Theorem 2 is different from that for the Krein-space projection. Therefore, the deterministic minimization problem mentioned above is only “partially” equivalent to the stochastic minimization one in Krein spaces.

In the following sections, all the signal estimation technologies to be discussed are developed based on the main principle given above.

3. Krein-space theory-based filtering, prediction and smoothing

Filtering, prediction and smoothing problems are the most foundational problems in signal processing. The Krein-space approach, due to its advantages in estimation accuracy and computation complexity, has attracted increasing attention from researchers in the signal processing community, and the Krein-space-based filtering, prediction and smoothing technologies have been developed well in the past decade, see, e.g. Zhao, Zhang, Wang, and Guo (2011), Feng, Wen, and Xu (2013), Feng, Yu, Yang, and Gao (2012), Arov and Staffans (2007), Bolzern Colaneri, De Nicolao, and Shaked (2002), Xu, Wang, and Qi (2011), Lu, Zhang, and Yan (2010), Ra, Jin, and Park (2004), Lee, Ra, Yoon, and Park (2004), Jin, Park, Kim, and Yoon (2001) and Kim and Chun (2000). In this section, we shall review and discuss the classical filtering, prediction and smoothing technologies which are developed based on the Krein-space theory and introduce some successful applications of such signal processing technologies.

In 1996, a self-contained theory for linear estimation in Krein spaces has been developed in Hassibi et al. (1996a) which has been published as Regular Papers in the IEEE Transactions on Automatic Control. This paper has discussed in detail the connection between Krein-space projection and the stationary points of certain quadratic forms, and showed that the stationary points can be obtained by the same expression as the projection of one suitably defined Krein-space vector. Then, these results have been further specialized to the case of state-space constraint, and the corresponding stationary points have been recursively computed by a Krein-space-Kalman filter. Based on the developed Krein-space estimation theory, in Hassibi, Sayed, and Kailath (1996b), several applications have been described in great details including the \( H_\infty \) filtering, quadratic game theory and sensitive control and estimation problems.

Note that, while the Krein-space estimation framework was being established, some possible research topics have also been raised in Hassibi et al. (1996a). For example, in design of \( H_\infty \) filters, how can we obtain more attractive algorithms for the actual implementations of filters and how can we build the positive condition required for the existence of the \( H_\infty \) filters into the algorithms? Some efforts have been made on these issues. In Hassibi, Kailath, and Sayed (2000), an array algorithms for \( H_\infty \) filtering has been developed that can be regarded as the Krein-space generalizations of \( H_\infty \) array algorithms. These algorithms involve propagating the indefinite square-roots of the quantities of interest and have the interesting property that the appropriate inertia of these quantities is preserved. Moreover, the conditions for the existence of the \( H_\infty \) filters have been built into the algorithms so that filter solutions will exist if and only if the algorithms can be executed. In Nishiyama (2000), a forgetting factor has been introduced to improve the \( H_\infty \) filtering performance for time-varying systems, and the forgetting factor has been optimized through a process noise that is determined by the Riccati equation. The \( H_\infty \) filter derived in Nishiyama (2000) has been further explained and a fast algorithm of the \( H_\infty \) filter has been presented in Nishiyama (2004) for the case that the observation matrix has a shifting property. It has also been shown that the fast \( H_\infty \) filter outperforms the least-mean-squared algorithm and the fast Kalman filter in convergence rate.

Actually, it was from the Krein-space estimation theory developed in Hassibi et al. (1996a, 1996b) that the research on the Krein-space approach has stirred a great deal of research interests for the signal estimation purpose. For example, in Nishiyama (1999), in order to extract a single complex sinusoid from measurements corrupted by white noise, a sinusoidal model with the state-space representation has been first established and then, by using the Krein-space theory, an \( H_\infty \) sinusoidal estimator has been obtained that is more robust than the Kalman sinusoidal estimator developed previously. In Zhang, Xie, and Soh (2001), the Krein-space approach has been applied to solve...
the $J$-spectral factorization problem for general discrete rational matrices. By constructing a stochastic state space filtering model in Krein space, it has been shown that the spectral matrix of the output is equal to the rational matrix to be factorized and, based on it, the spectral factor has been derived. In Zhang, Xie, and Soh (2000), a simple frequency domain approach in Krein space has been developed for the infinite horizon $H_{\infty}$ deconvolution problems including the deconvolution filtering, prediction, and fixed-lag smoothing problems. Note that these problems have been only considered for continuous-time case. In Zhang, Xie, and Soh (2003), the steady-state risk-sensitive filtering, prediction and smoothing problems have been investigated for a class of discrete-time singular systems. By using a simple derivation technique based on a Krein space signal model and an autoregressive moving average innovation model, the risk-sensitive estimator has been designed for the considered discrete-time singular systems.

We would like to make a special note of the reorganized innovation analysis approach in the Krein space which plays an important role in the designs of $H_{\infty}$ filters, smoothers as well as predictors with various forms of time-delays. This novel approach was originally proposed in Zhang, Xie, Wang, and Lu (2004) where the finite horizon $H_{\infty}$ fixed-lag smoothing problem has been investigated for linear continuous time-varying systems. By using the proposed reorganized innovation approach, a necessary and sufficient condition for the existence of an $H_{\infty}$ fixed-lag smoother has been given in terms of the solutions to two Riccati differential equations (RDEs) including one standard $H_{\infty}$ filtering RDEs and one $H_2$ type of RDEs. In Zhang, Xie, Soh, and Zhang (2005), the reorganized innovation analysis approach has been developed for the discrete-time case and the same problem has been discussed for the discrete linear time-varying systems. Following it, the results have been further extended to discrete-time descriptor systems in Wang, Dai, Zhang, and Liu (2006). It is worth mentioning that it has been shown in Zhang, Feng, and Lu (2007) that the $H_{\infty}$ white noise central smoother is actually an $H_2$ fixed-lag smoothing with delayed measurements. That is, the developed reorganized innovation analysis approach is also in effect to deal with the problems of $H_{\infty}$ filtering, prediction and smoothing with delayed measurements.

It should be pointed out that, in all the problems mentioned above, the measurements are assumed to be instantaneous. However, perfect communication is not always possible in many engineering systems especially in a networked environment. For example, due to the limit of the communication capacity, the measurement delays may inevitably occur and even the measurements will be missing totally at some time instants. For the case of delayed measurements, the reorganized innovation analysis approach has proven to be an effective technique in the analysis and design of the $H_{\infty}$ filters, predictors and smoothers. Some representative examples are introduced as follows. In Zhang, Zhang, and Xie (2004), by reorganizing the innovation, the $H_{\infty}$ prediction problem has been studied for linear continuous-time systems with both instantaneous and delayed measurements, and the desired $H_{\infty}$ predictor has been designed by solving a RDE and a matrix differential equation. Note that, in Zhang et al. (2004), only the single measurement delay has been taken into account. A more complicated form of measurements, i.e. the measurements with the multiple delays, has been considered in Zhang, Feng, Duan, and Lu (2006) where the $H_{\infty}$ filtering problem has been studied in the framework of Krein spaces by employing the same reorganized innovation technique. As an extension of the results in Zhang et al. (2004), the $H_{\infty}$ filtering problem has been investigated in Wang, Zhang, and Han (2010) for the case when the measurements are subject to time-varying delays. With respect to the uncertain observations (also called missing measurements), the corresponding treatment is slightly different from the one mentioned above. For example, in Zhao and Zhang (2012), in order to deal with the uncertain observations, a certain indefinite quadratic form in the stochastic sense has been introduced, and then the design problem of the $H_{\infty}$ filter amounts to the minimum problem of the introduce indefinite quadratic form. It should be mentioned that there has appeared some work on the $H_{\infty}$ filtering with uncertain observations in the framework of Krein spaces, but the corresponding research is still in its early stages.

We have reviewed in detail the development of the Krein-space-based filtering, prediction and smoothing technologies, and discussed the advantage of these technologies from the algorithm analysis. It is worth noting that these Krein-space-based technologies have also found a wide application in industrial engineering. We only mention here some representative applications. The first application to be discussed is the electrocardiogram (ECG) artefacts. Motivated by the fact that the $H_{\infty}$ techniques developed in Hassibi et al. (1996a) are robust to the model uncertainties and are not required to know the statistical information about noises, two adaptive algorithms based on the $H_{\infty}$ techniques have been proposed in Puthusserypady and Ratnarajah (2006) to minimize the EOG artefacts from the corrupted electroencephalographic (EEG) signals. Studies performed on the real recorded signal have shown that the proposed $H_{\infty}$ techniques based algorithms work slightly better than the recursive least-squared algorithm (especially when the input signal-to-noise ratio is very low) and always outperform the least-mean-squared algorithm in minimizing the EOG artefacts from corrupted EEG signals. The second application comes from Cao, Xie, and Zhang (2008) where the robust channel estimation problem has been studied for direct-sequence code-division multiple-access systems with time-varying multipath fading channels. The main approach used is based on the Krein space polynomial approach proposed in Zhang et al. (2000). The developed $H_{\infty}$ estimator is of a better estimation performance than other algorithms, such as the restricted least-squares and zero-forcing estimators. Another example is the application
of the Krein-space Kalman filters in the sensor networks observing a physical process with parametric uncertainty (Ahmad, Gani, & Yang, 2008). In this work, the considered heterogeneous sensor network consists of two types of nodes (type A and type B) and central base station. The information form of robust Kalman filter by using the Krein space approach has proven to be useful to fuse the cluster state estimates, and the performance of the centralized state estimate is comparable to the performance of the global state estimate. The last application to be mentioned here is the face recognition problem. In Liwicky, Zafeiriou, Tzimiropoulos, and Pantic (2012), the kernel principal component analysis (KPCA) has been extended from a reproducing kernel Hilbert spaces to Krein spaces and an exact framework for online learning with a family of indefinite kernels has been established. The established kernel framework has been applied to deal with the face recognition problem and the performance of direct KPCA has been evaluated for face recognition with illumination changes and pose variations.

4. Krein-space theory-based fault estimation

Due to the increasing complexity of modern technical processes, faults or abnormal changes may occur in the individual parts, such as actuators, sensors and components. The occurring faults can severely affect system safety and reliability. Therefore, the past few decades have witnessed constant research interests on various aspects of fault diagnosis and fault-tolerant control, see, e.g. Brahim, Bouattour, Mehdi, Chaabane, and Hashim (2013), Li and Zhong (2010), Wang, Pei, and Wang (2013) and Zhang, Zhang, Luo, and Guan (2013). Fault estimation has played an important role and has long been one of the foundational problems in this area. Among the various fault estimation schemes, \( H_\infty \) fault estimation is an effective robust fault estimation technology that has recently received considerable attention. Similar to the \( H_\infty \) filtering (prediction and smoothing) technologies, the performance index of \( H_\infty \) fault estimation can also be described by an indefinite quadratic form. Therefore, it seems to be natural to utilize the Krein-space approach to design the \( H_\infty \) fault estimator. In this section, we shall review some recent advances on the \( H_\infty \) fault estimation schemes developed by using the Krein-space theory.

The pioneering work on the Krein-space \( H_\infty \) fault estimation has been reported in Zhong, Liu, and Zhao (2008). In this paper, the Krein-space approach has been originally applied to the \( H_\infty \) fault estimation problem for a class of linear discrete time-varying systems. The fault vector considered has been dealt with via the augmentation together with the state vector. Then, the \( H_\infty \) fault estimation problem addressed has been related to a minimum problem of a scalar quadratic form and, by using the innovation analysis in Krein spaces, a sufficient and necessary condition for the existence of an \( H_\infty \) fault estimator has been derived in terms of a matrix Riccati equation. Note that the approach proposed in Zhong et al. (2008) is not applicable for a fault uncorrelated with current measurement output. In order to overcome this shortcoming, in Zhong, Zhou, and Ding (2010), an observer-based fault estimator with residual feedback is proposed as a residual generator and, based on this, a modified finite horizon \( H_\infty \) smoothing formulation of FDI has been presented. Then by using the innovation analysis similar to that in Zhong et al. (2008), the observer-based \( H_\infty \) fault estimator has been computed recursively via Riccati recursions.

Based on the results reported in the above papers, some efforts have also been made on the \( H_\infty \) fault estimation problem for time-delayed systems. For example, in Zhao, Zhong, and Zhang (2010), by using the same methods as ones in Zhong et al. (2010), a one-step smoother has been proposed as a residual generator and the observer based \( H_\infty \) fault detection filter has been designed for linear discrete time-varying systems with delayed state and thereby the result derived in Zhong et al. (2010) has been successfully extended to the time-delayed systems. In Shen, Ding, and Wang (2013a), the finite-horizon \( H_\infty \) fault estimation problem has been investigated for a class of linear discrete time-varying systems with both instantaneous and delayed measurements. By using the reorganized innovation approach proposed in Zhang et al. (2004), the considered measurements are reorganized into a tractable form, based on which an associated stochastic system in a Krein space has been introduced. Then, by applying the projection theory in the Krein space, a fault estimator has been designed to achieve the specified \( H_\infty \) performance criterion.

In practice, uncertainties usually enter systems in an unknown way and may deteriorate the pre-specified estimation performance. Therefore, it is both theoretically important and practically necessary to develop a so-called robust technology to eliminate the effects of such uncertainties. In Shen, Ding, and Wang (2013b), the finite-horizon \( H_\infty \) fault estimation problem has been investigated for a class of uncertain linear discrete time-varying systems with known inputs. A new \( H_\infty \) performance index including the known inputs has been put forward in order to better reflect the effect of the known input on the whole fault estimation systems. To cope with the uncertainties, an auxiliary system has been constructed with a certain indefinite quadratic form. By recurring to Krein-space theory, the optimization problem of the associated indefinite quadratic form has been solved and a sufficient condition with much less conservativeness has been established for the existence of the desired fault estimator. Then, all the estimator parameters have been derived simultaneously in terms of an explicit solution to a matrix equation.

5. Conclusions and future work

In this paper, we have surveyed the applications of the Krein-space theory in the signal estimation area. The basic
concepts about Krein spaces have been reviewed and the main results on the linear estimation theory in Krein spaces have been listed. Then, we have introduced the developments of the Krein-space-based filtering, prediction and smoothing technologies and their applications in practical engineering. Subsequently, we have discussed some recent advances on the fault estimation approaches developed with the help of the Krein-space theory. Related topics for the future research work are listed below:

- It can be seen from the above discussion that the Krein-space-based filtering (prediction and smoothing) technologies have been well studied for various linear systems. As for the fault estimation problem, however, the corresponding research is still in its early stages. Therefore, an interesting future research topic would be to consider the Krein-space-based fault estimation problems for more complicated systems such as time-delayed systems, singular systems and uncertain systems.

- The measurement information incompleteness is often encountered in practical engineering, especially in the networked environment. Therefore, the problems of filtering, prediction, smoothing and fault estimation with the incomplete measurement information are of engineering significance. This would be another interesting research topic.

- Nonlinear systems have long been one of the important research topics in the research community since nonlinearity is arguably one of the main causes in reality that has resulted in considerable system complexity. However, the existing Krein-space filtering or fault estimation technologies are only developed for the linear systems and there are few results available for the nonlinear case. Therefore, the Krein-space-based filtering or fault estimation problem for nonlinear systems is a challenging topic that deserves further investigation. This research work may start from some special classes of nonlinear systems, e.g., the systems with nonlinearities bounded by a linearity-like form.

- From the above survey, it is known that the Krein-space-based filtering technologies have been well studied. As a duality of filtering problem, the Krein-space-based control approach should also be paid attention. Therefore, the control problem in the framework of Krein spaces such as Nash dynamic game control problem would be another challenging topic.

- As shown in this paper, the Krein-space-based filtering technologies have been successfully applied in some industrial engineering problems. However, it should be pointed out that, comparing with the well-developed theoretical studies, the application research is still relatively few and, therefore, the applications of the existing theories and methodologies to more practical engineering problems would be another one of the future work.

Acknowledgement
This work was supported in part by the National Natural Science Foundation of China under Grants 61104125, 61134009 and 61273156, the Shanghai Rising-Star Program of China under Grant 13QA1400100, the Fundamental Research Funds for the Central Universities, and DHU Distinguished Young Professor Program.

Note
1. \( \{\mathcal{K}, \langle \cdot, \cdot \rangle\} \) is referred to as a module if the inner product \( \langle \cdot, \cdot \rangle \in S \) is positive.

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