3-D Sherlet Transform for Biomedical Applications

KEYWORDS

Wavelet, Surfaselet, Curvelet, Enhancement, Denoising, Video processing, Multi resolution.

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ABSTRACT

It is now widely acknowledged that analyzing the intrinsic geometrical features of an underlying image is essentially needed in image processing. The shearlets are an affine system with a single generating mother shearlet function parameterized by a scaling, shear, and translation parameter the shear parameter capturing the direction of singularities. This transform can even be regarded as matrix coefficients from a group representation of a shearlet group, thereby providing an extensive mathematical framework for its theory. The Shearlet representation is a multi scale pyramid of well-localized waveforms defined at various locations and orientations, which was introduced to overcome the limitations of traditional multi scale systems in dealing with multidimensional data. While the shearlet approach shares the general philosophy of curvelets and surfacellets, it is based on a very different mathematical framework, which is derived from the theory of affine systems and uses shearing matrices rather than rotations.

This allows a natural transition from the continuous setting to the digital setting and a more flexible mathematical structure. The 3-D discrete shearlet transform algorithm proposed in this paper consists in a cascade of multi scale decomposition and a directional filtering stage. The filters employed in this decomposition are implemented as finite-length filters, and this ensures that the transform is local and numerically efficient. To illustrate its performance, the 3-D discrete shearlet transform is applied to problems of video denoising and enhancement, and compared against other state-of-the-art multi scale techniques, including curvelets and surfacellets.

Many important results about the theory and applications of shearlets have been derived during the past 5 years. Yet, there is a need to extend this approach and its applications to higher dimensions, especially 3D, where important problems such as video processing and analysis of biological data in native resolution require the use of 3D representations. The focus of this thesis is the study of shearlet representations in 3D, including their numerical implementation and application to problems of data denoising and enhancement. Compared to other competing methods like 3D curvelet and surfacellet, our numerical experiments show better Peak Signal to Noise Ratio (abbreviated as PSNR) and visual quality.

Introduction

Video processing is a particular case of signal processing, which often employs video filters and where the input and output signals are video files or video streams. Each stream of video is considered as stream of frames which in general terms we refer it as image. Sequence of images or frames constitutes a video. Processing a sequence of digital images refers to the video processing. Digital image processing remains a challenging domain of programming for several reasons.

The aim of video enhancement is to improve the interpretability or perception of information in video for human viewers, or to provide ‘better’ input for other automated video processing techniques.

Video enhancement techniques can be divided into two broad categories:

1. Spatial domain methods, which operate directly on pixels, and
2. Frequency domain methods, which operate on the Fourier transform of an image.

Video Denoising

Video denoising is the process of removing noise from a video signal. It refers to the recovery of a digital video that has been contaminated by noise. Noise here in video is particularly the Additive white Gaussian noise (AWGN). Although other types of noise (e.g., impulse or Poisson noise) have also been studied in the literature of video processing, the term “video denoising” is usually devoted to the problem associated with AWGN. Mathematically, if we use $Y=X+W$ to denote the degradation process ($X$: input video, $Y$: noisy video, $W$: $N(0, \sigma^2)$, the video denoising algorithm attempts to obtain the best estimate of $X$ from $Y$. The optimization criterion can be mean squared error (MSE)-based or perceptual quality driven (though video quality assessment itself is a difficult problem, especially in the absence of an original reference).

Video denoising methods can be divided into:

1. Spatial video denoising methods, where image noise reduction is applied to each frame individually.
2. Temporal video denoising methods, where noise between frames is reduced. Motion compensation may be used to avoid ghosting artifacts when blending together pixels from several frames.

History of Shearlets

Shearlet systems are to date the only representation system, which provide optimally sparse approximations of image class in 2D as well as 3D. Even more, in contrast to all other directional representation systems, a theory for compactly supported Shearlet frames was derived which moreover also satisfy this optimality benchmark. The shearlet transform is a recent sibling in the family of multi resolution representations, and can be seen as an extension of the wavelet 9 transform that combines multi resolution theory with geometric transforms (dilations and shear transforms). This approach gives a lot of flexibility with respect to applications, while still having full control on the mathematical properties of the transform.

For example, in 3D, the redundancy factor of the transform (i.e. the ratio of the number of transform coefficients and
This representation is part of a new class of multi scale methods introduced during the last ten years with the goal of overcoming the limitations of wavelets and other traditional methods through a framework that combines the Standard multi scale decomposition and the ability to efficiently capture anisotropic features. Other notable such methods include the curvelets and the contourlets. Indeed, both curvelets and shearlets have been shown to form Parseval frames of, L2 (R2) which are (nearly) optimally sparse in the class of cartoonlike images, which is a standard model for images with edges. Specifically, if FM is the M term approximation obtained by selecting the M largest coefficients in the shearlet or curvelet expansion of a cartoonlike image, then the approximation error satisfies the asymptotic estimate, i.e.,

\[ \| f - \sum_{i=1}^{M} c_i \psi_{j_i,k_i} \|_2 \leq C \frac{1}{M^{1/2}} \]

Up to the log like factor, this is the optimal approximation rate, in the sense that no other orthonormal systems or even frames can achieve a rate better than M^{-2}. By contrast, wavelet approximations can only achieve rate M^{-1} for functions in this class. Scientists face a rapidly growing deluge of data, which requires highly sophisticated methodologies for analysis and compression. Simultaneously, the complexity of the data is increasing, evidenced in particular by the observation that data becomes increasingly high-dimensional. One of the most prominent features of data are singularities which is justified, for instance, by the observation from computer visionists that the human eye is most sensitive to smooth geometric areas divided by 10 sharp edges. Intriguingly, already the step from univariate to multivariate data causes a significant change in the behavior of singularities. Whereas one-dimensional (1D) functions can only exhibit point singularities, singularities of two-dimensional (2D) functions can already be of both point as well as curve-linear type.

Thus, in contrast to isotropic features point singularities, suddenly anisotropic features curvilinear singularities are possible. And, in fact, multivariate functions are typically governed by anisotropic phenomena. Think, for instance, of edges in digital images or evolving shock fronts in solutions of transport-dominated equations. These two exemplary situations also show that such phenomena occur even for both explicitly as well as implicitly given data. One major goal both for the purpose of compression as well as for an efficient analysis is the introduction of representation systems for „good“ approximation of anisotropic phenomena, more precisely, of multivariate functions governed by anisotropic features.

Model of 3-D Shearlet in pyramid representation

The shearlet transform is found to be the best in the family of multi resolution representations, and can be seen as an extension of the wavelet transform that combines multi resolution theory with geometric transforms (dilations and shear transforms).

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**Sparse Multidimensional Analysis**

The notion of efficient representation of data plays an increasingly important role in areas across applied mathematics, science and engineering. Over the past few years, there has been a rapidly increasing pressure to handle ever larger and higher dimensional data sets, with the challenge of providing representations of these data that are sparse (that is, “very” few terms of the representation are sufficient to accurately approximate the data) and computationally fast. Sparse representation has implications reaching beyond data compression. Understanding the compression problem for a given data type entails a precise knowledge of the modeling and approximation of that data type. This in turn is useful for many other important tasks, including classification, denoising, interpolation, and segmentation.

Multiscale techniques based on wavelets have emerged over the last 2 decades as the most successful methods for the efficient representation of data, as testified, for example, by their use in the new FBI fingerprint database and in JPEG2000, the new standard for image compression. Indeed, wavelets are optimally efficient in representing functions with point wise singularities. More specifically, consider the wavelet representation (using a “nice” wavelet basis) of a function f of a single variable that is smooth apart from a point discontinuity. Because the elements of the wavelet basis are well-localized (i.e., they have very fast decay both in the spatial and in the frequency domain), very few of them interact significantly with the singularity, and, thus, very few elements of the wavelet expansion are sufficient to provide an accurate approximation.

This contrasts sharply with the Fourier representation, for which the discontinuity interacts extensively with the elements of the Fourier basis. Denoting by FM the approximation obtained by using the largest M coefficients in the wavelet expansion, the asymptotic approximation error satisfies

\[ \| f - \sum_{i=1}^{M} c_i \psi_{j_i,k_i} \|_2 \leq C \frac{1}{M^{1/2}} \]

This is the optimal approximation rate for this type of functions, and outperforms the corresponding fourier approximation error rate M^{-1}. In addition, the Multiresolution Analysis (MRA) associated with wavelets provides very fast numerical algorithms for computing the wavelet coefficients. However, despite their remarkable success in applications, wavelets are far from optimal in dimensions larger than one. Indeed wavelets are very efficient in dealing with point wise singularities only. In higher dimensions other types of singularities are usually present or even dominant, and wavelets are unable to handle them very efficiently. Consider, for example, the wavelet representation of a 2-D function that is smooth away from a discontinuity along a curve of finite length (a reasonable model for an image containing an edge).

Because the discontinuity is spatially distributed, it interacts extensively with the elements of the wavelet basis. As a consequence, the wavelet coefficients have a slow decay, and

Ahmad, I. (1995). Antibody titer and hematology following vaccination and immunopotentiation of Sahiwal cows in last trimester of pregnancy. PhD Thesis, Univ. Agri., Faisalabad, Pakistan. | Duncan, D. B. (1955). Multiple range and Multiple F test: Biometrics, 11: 1-42. | Dutta, J. C., Baruah, R. N., Dutta, L. and Bakul, S. C. (1988). Blood biochemical changes in anaesthous and normal cyclic cattle. Indian Vet. J., 65: 239-241 | Esievo, K. A. N. and Moore, W. E. (1979). Effect of dietary protein and stage of lactation on the haematological and erythrocyte enzymes activities of high producing dairy cattle. Res. Vet. Sci., 23: 75-79.
the approximation error \( f \rightarrow \infty \) as \( M \rightarrow \infty \)

Therefore, there is, large room for improvements, and several attempts have been made in this direction both in the mathematical and engineering communities, in recent years. Those include contourlets [7], complex wavelets and other ‘directional wavelets’ in the filter bank literature as well as brushlets, ridgelets, curvelets and bandelets. The most successful approach so far are the curvelets of Candès and Donoho. This is the first and so far the only construction providing an essentially optimal approximation property for 2D piecewise smooth functions with discontinuities along \( C^2 \) curves. The main idea in the curvelet approach is that, in order to approximate functions with edges accurately, one has to exploit their geometric regularity much more efficiently than traditional wavelets.

This is achieved by constructing an appropriate tight frame[1] of well-localized functions at various scales, positions and directions. We refer to for more details about this construction. The main goal of this paper is to show that the shearlets, a construction based on the theory of composite wavelets, also provides an essentially optimal approximation property for 2D piecewise smooth functions with discontinuities along \( C^2 \) curves. We will show that the approximation error associated with the \( M \) term reconstruction obtained by taking the \( M \) largest coefficients in the shearlet expansion satisfies

\[
\| f - M -2(\log M) \|_3 \rightarrow \infty \quad 3.3
\]

This is exactly the approximation rate obtained using curvelets. The proof of our result adapts and simplifies several ideas from the corresponding sparsity result of the curvelets), but does not follows directly from the curvelets construction. Indeed, as we will argue in the following, our 34 alternative approach has some mathematical advantages with respect to curvelets, including a simplified construction that provides the framework for a simpler mathematical analysis and fast algorithmic implementation. The theory of composite wavelets, recently proposed by the authors and their collaborators, provides a most general setting for the construction of truly multidimensional, efficient, multistage representations. Unlike the curve let s, this approach takes full advantage of the theory of affine systems on \( R^n \). Specifically, the affine systems with composite dilatons are the systems.

Algorithm for 3D Shearlet Transform

The practical algorithm for the discrete shearlet transform is applied to a 3D data set. Because the algorithm is in fact a straightforward extension to 3D of our 2D algorithm presented in ref. [7] we will only present a brief overview of our approach here. For the forward transform, we use a recursive algorithm that is based on pyramid decomposition and a multi directional filter bank. Because the shearlet filters will be defined in Fourier space (similar to the shearlet basis functions) and because the filters are band limited (hence do not have a compact support in spatial domain), the whole algorithm is implemented in the Discrete Fourier transform (DFT) domain. A schematic overview of the forward transform is given in Figure 4.3. The new implementation can be described as the cascade of multi scale decomposition, based on a version of the Laplacian pyramid filter, followed by a stage of directional filtering. The main novelty of the 3D-D approach consists in the design of the directional filtering stage, which attempts to faithfully reproduce the frequency decomposition provided by the corresponding mathematical transform by using a method based on the pseudo spherical Fourier transform.

Algorithm

The algorithm steps which are demonstrated in figure 4.3 are as follows:

1. Computation of the odd-frequency DFT (OF-DFT) of the input image, using a fast Fourier transform (FFT) algorithm.
2. Analysis of the OF-DFT of the input image using a Laplacian pyramid-like filter bank (with sub sampling).
3. Application of a multidirectional filter bank to the resulting sub bands.
4. (Optionally) Shear the sub bands such that the spectral content of each band lies in a ‘central’ box in frequency space.
5. Sub sampling in two dimensions to remove the remaining zero-DFT coefficients.
6. Computation of the inverse OF-DFT of every resulting sub band, using the IFFT.

The multi scale and multidirectional filter bank

An overview of the multi scale and multidirectional filter bank is shown in Figure 4.4. The multiscale decomposition is implemented as a Laplacian pyramid [8] with orthogonal filters to have a tight frame [9]. The Laplacian pyramid will be free of aliasing if the wavelet is compactly supported in frequency domain and if the decimation factors are adapted to the bandwidths of the filters. The Meyer wavelet is one example of such a wavelet.

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but this does not define what “black” or “white” is in terms of colorimeter.

Another convention is to employ percentages, so the scale is then from 0% to 100%. This is used for a more intuitive approach, but if only integer values are used, the range encompasses a total of only 101 intensities, which are insufficient to represent a broad gradient of grays. Also, the percentile notation is used in printing to denote how much ink is employed in halftoning, but then the scale is reversed, being 0% the paper white (no ink) and 100% a solid black (full ink).

In computing, although the grayscale can be computed through rational numbers, image pixels are stored in binary, quantized form. Some early grayscale monitors can only show up to sixteen (4-bit) different shades, but today grayscale images (as photographs) intended for visual display (both on screen and printed) are commonly stored with 8 bits per sampled pixel, which allows 256 different intensities (i.e., shades of gray) to be recorded, typically on a non-linear scale. The precision provided by this format is barely sufficient to avoid visible banding artifacts, but very convenient for programming due to the fact that a single pixel then occupies a single byte.

Once the RGB video is converted into the gray scale video. Our next motive is to divide the video into number of frames as the standard frames division may be 30 frames per second, 50 frames per second or as we wish. Here we are considering the division of 10 frames per second. We are considering a video of the building and converting it into a 256 gray scale video. The same converted gray scale video is divided into 10 numbers of frames and those frames are shown in the below figure.

In order to illustrate the potential of the Shearlet Transform we study additive white Gaussian noise removal by means of hard thresholding estimator. We want to estimate signals that are considered to be realization of real random vector f. The noisy measurements are

\[ g = f + W \]

Where W is a zero-mean Gaussian white noise of variance \( \sigma^2 \).

For our numerical tests. We compared the performance of the Shearlet Transform to other transforms such as Wavelet Transform. To highlight the performance of the Shearlet Transform relative to other transforms, we apply hard threshold on the coefficients of various transforms. We choose the threshold

\[ T_{j,l} = K_{j,l} \]

here \( j \) is the index of decomposition level, \( K_j \) are constants experimentally determined so that they can produce overall optimal performance for each transform and is the variance of noise. The frames with salt and pepper noise as discussed are shown in below figure.

The application of salt and pepper noise degrades the frames and once the denoised frames are obtained as shown in the above figure the next step is retrieving the enhanced frames of the above frames through the next step of the algorithm.
form and proved that the Shearlet transform is far better than the wavelet transform. Such a comparison is shown in figure 5.6 with the measuring factors indicated. The measuring factors are the Peak signal to noise ratio (PSNR) and the redundancy with the coefficients.

Fig: Comparison of Shearlet Transform with Wavelets as per the measuring factors PSNR and Redundancy

CONCLUSION AND FUTURE SCOPE
A new class of multidimensional representations obtained from the action of translations, dilations, and shear transformations on a finite set of generators in L2(R3). One advantage of this approach is that these systems can be constructed using a generalized multiresolution analysis and implemented efficiently using an appropriate version of the classical cascade algorithm. This proposal faithfully reproduces the frequency decomposition of the 3-D shearlet transform, which has been recently shown to optimally provide sparse approximations for a large class of 3-D data. This transform can even be regarded as matrix coefficients from a group representation of a special non-abelian group, the shearlet group, thereby providing an extensive mathematical framework for its theory. Another benefit of this approach is that, again thanks to their mathematical structure, these systems provide a Multiresolution analysis similar to the one associated with classical wavelets, which is very useful for the development of fast algorithmic implementations. To illustrate the benefits derived from the sparsity of the shearlet representation, our 3-D algorithm has been tested on problems of video denoising and video enhancement.

Future Scope can be seen in the area of processing time reduction while dividing frames and converting it into the denoised and enhanced frames. Another aspect is providing the video to be compatible to play in universal video players.

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