SPIN-DEPENDENT FRAGMENTATION FUNCTIONS FOR BARYONS IN A DIQUARK MODEL

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The perturbative QCD calculation of heavy quark fragmentation into a heavy meson is extended to predict fragmentation into heavy flavor baryons. This is accomplished by implementing the quark-diquark model of the baryons. Several diquark form factors are used to enable the integration over the virtual heavy quark momentum. The resulting spin independent functions for charmed quarks to fragment into charmed baryons with spin 1/2 and 3/2 are compared with recent data. Predictions are made for the spin dependent fragmentation functions as well, particularly for the functions $\hat{g}_1$ in the case of spin 1/2 baryons.

1 Introduction

Fragmentation functions have received considerable attention in recent years. While experimental information beyond the pion distribution (presumably from light quarks) has been slow in accumulating, theoretical interest has been growing. The particular functional form for heavy flavored quarks to fragment into heavy mesons has been studied using Operator Product Expansion techniques, light cone quantization, QCD perturbation theory, Heavy Quark Effective Theory, and other methods. Some of these methods yield general properties that reflect the overall structure of QCD, as it is currently understood. Other approaches take particular models of the low energy behavior expected from QCD, but in regions that are not perturbatively calculable. The particulars of the various approaches will be subject to some experimental scrutiny in the future, but are not yet put to the test. One general feature is known - the peak of the hadron distribution moves toward higher momenta as the quark mass increases. This feature is a result of the kinematics implicit in most models of the non-perturbative process, and is incorporated in the phenomenological Peterson function, which is used to fit the sparse data on heavy quark fragmentation.

Even more difficult to test experimentally are the spin dependences of the fragmentation processes. Yet these dependences are important to know. They reflect the details of the primarily non-perturbative mechanism by which parton polarization is passed on to the hadrons. In this sense, the spin-dependent fragmentation involves the reverse of the process by which the nucleon spin is shared by its partons (the “spin crisis”), and may reveal a similarly myste-
rious decoupling of valence quark spin and hadron spin for some regions of kinematics.

Over the last several years a number of theorists have noticed that for fragmentation of a heavy flavor quark into “doubly heavy” mesons, like the c-quark into the $J/\Psi$ or the b-quark into the $B_c$, perturbative QCD may be applicable. If this is the case, the fragmentation functions are calculable, at an appropriate scale, and QCD radiative corrections can be obtained from the Renormalization Group or the Altarelli-Parisi equations. Such calculations have been performed and scrutinized. It has been shown that in the heavy quark limit (i.e. the mass goes to infinity) the functions have the form expected from more general considerations. This corresponds to the heavy meson taking all of the heavy quark’s momentum; the distribution becomes a delta function at $z = 1$. The $1/m_Q$ corrections are calculated also. In any case, this approach can predict the spin-dependent fragmentation functions along with their momentum and mass dependences. In the heavy mass limit, of course, the spin of the heavy quark is conserved, so the spin dependence is simple. What is of phenomenological interest is the next order correction, at least, since that has non-trivial spin dependence.

In some circumstances the spin dependence of fragmentation is most readily studied experimentally by observing baryons rather than mesons. This is true for the production of hyperons or heavy hyperons ($\Lambda_c$, $\Lambda_b$, etc.), wherein the weak, parity violating decays provide polarization analyses. To consider fragmentation into baryons in this perturbative scheme, the three quark system has to be confronted. A simple alternative is to consider the baryons as quark-diquark bound states, and to use the same perturbative method as for the mesons. In order for the perturbative calculation to be useful the creation of a heavy pair of quarks or diquarks must be an intermediate step. Hence, doubly heavy baryon fragmentation is an appropriate testing ground for these ideas. It is not expected that sufficient data to study this process will be available in the near future, however.

To begin to see the structure it will be worthwhile to stretch the region of applicability to the “singly” heavy baryons. We have been carrying out this program to see the expected spin and kinematic dependences, with the hope of providing an experimentally testable model. Of immediate interest is the question of whether the baryon fragmentation functions have the same kinematic dependence as the meson case. In general the answer is no in this model, but the details have to be studied. Furthermore, the spin dependent

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*a*It is not clear to the present authors that all sources for such terms have been considered, particularly corrections coming from the treatment of the Bethe-Salpeter bound state wavefunction for the meson.
fragmentation is interestingly distinct from the naive heavy quark limit. The perturbative calculation and its results will be presented below, along with a comparison with some recent data.

2 Perturbative calculation

The first calculations of the fragmentation functions in the perturbative scheme were applied to some of the inclusive heavy flavor meson decays of the $Z^0$, as produced at LEP. The partial width for this inclusive process can be written in general for a hadron $H$ as

$$d\Gamma(Z^0 \to H(E) + X) = \sum_i \int_0^1 dz \, d\hat{\Gamma}(Z^0 \to i(E/z) + X, \mu) \, D_{i \to H}(z, \mu), \quad (1)$$

where $H$ is the hadron of energy $E$ and longitudinal momentum fraction $z$ relative to the parton $i$, while $\mu$ is the arbitrary scale whose value will be chosen to avoid large logarithms. The fragmentation function enters here in a factorized form (that can be maintained through the evolution equations).

Now, consider the final state with one heavy flavor meson, say the $B_c$ for definiteness. To leading order the $B_c$ meson arises from the production of a pair of $b$-quarks, in which one of the quarks fragments into the meson. As Fig.1 illustrates, with $Q = b, Q' = c, \bar{Q}' = \bar{c}$, the perturbative contribution involves the virtual $b$-quark radiating a hard gluon (in axial gauge, so that there is no contribution from the opposite quark). The hard gluon produces a heavy flavor pair of $c$-quarks. The gluon must have energy at least twice the charm mass, so that the coupling $\alpha_s(Q^2)$ is small. For matching momenta (or relative 3-velocity zero) the $b$ and $\bar{c}$ form the $B_c$ meson with probability given by the square of the Bethe-Salpeter wave function. Since the doubly heavy mesons are weakly bound objects, the wave function at the origin is known from non-relativistic quark models for the heavy-heavy system.

Figure 1: The amplitude for $Z^0 \to Meson(Q\bar{Q}') + X$ or Baryon(QD)+X.
The amplitude for Fig.1, $A_1$, can be evaluated explicitly from perturbation theory. The decay rate for unpolarized $Z^0 \to B_c + \bar{c} + b$ can be written

$$\Gamma_1 = \frac{1}{2M_Z} \int [d\bar{q}] [dp] [dp'] (2\pi)^4 \delta^4(Z - \bar{q} - p - p') \frac{1}{3} \sum |A_1|^2,$$

where $\bar{q}$, $p$, and $p'$ are the 4-momenta of the $\bar{b}$, $B_c$, and $c$, respectively.

To obtain the full inclusive width the unobserved quarks must be integrated over. The phase space integration can be simplified considerably when the limiting case of $M_Z \to \infty$ is approached by taking leading order in $m_b/M_Z$ (with $m_c < m_b$). The two body phase space for $p$ and $p'$ can be written as an integration over $z$ and $s$, with transverse momentum fixed for each such pair. In the large $M_Z$ approximation the transverse momentum of the hadron is small and $p = zq$. Once the square of the amplitude $A_1$ is summed over spins and simplified by dropping non-leading contributions, the width for $Z^0 \to \bar{c}c$ can be factored out of the expression Eq. (2) leaving an integral over the fragmentation function. Then

$$\int_0^1 dz D_{c\to H}(z) = \frac{8\alpha_s^2 |R(0)|^2}{27\pi m_c} \int_0^\infty ds \int_0^1 dz \Theta(s - \frac{4m_c^2}{z} - \frac{m_b^2}{1 - z}) F(s, z),$$

where $R(0)$ is the Bethe-Salpeter wavefunction at the origin, and $F$ is the remaining integrand, which depends on $s = q^2$, $z$ and the quark masses. The upper limit on the $s$ integration appears as the $M_Z \to \infty$ limit. So the partial width for $Z^0 \to H + X$ is given by an integral over the virtuality of the heavy quark and the phase space of the unobserved degrees of freedom.

The same procedure can be applied directly to the baryons, if the quark-diquark model of the baryons is used. The hard gluon in the process must produce a diquark–anti-diquark pair, and the diquark (color anti-triplet) combines with the quark to form the baryon. Note that an alternative scenario has the heavy quark fragment into a diquark first, and then the diquark dresses itself to form the baryon. This leads to very different results, as pointed out in Ref. 10, and is not justifiable herein, where the diquark is not necessarily heavy flavor. These latter authors have performed a calculation that is similar in spirit to part of the spin independent procedure we follow below.

We now proceed with the calculation of fragmentation functions for (singly) heavy flavor baryons. The basic covariant coupling of diquarks to gluons was written long ago. There is one coupling constant for the scalar diquark color octet vector current coupling to the gluon field—a color charge strength, along with a possible form factor $F_s$. The momentum space color octet current (which couples to the gluon field vector) is

$$J_{\mu}^{A(S)} = g_s F_s(k^2)(p + p')_{\mu} S^{\alpha\beta} \lambda^{A}_{\alpha\beta} S^\beta,$$
where $p$ and $p'$ are the scalar diquark 4-momenta and $k = p' - p$. For the vector diquark there are three constants - color charge, anomalous chromomagnetic dipole moment $\kappa$, and chromoelectric quadrupole moment $\lambda$, along with the corresponding form factors, $F_E$, $F_M$, and $F_Q$.

$$J^\mu_{A(V)} = g_A(\lambda A)_{\beta\alpha} \left[ F_E(k^2)[\epsilon^\alpha(p) \cdot \epsilon^\beta(p')](p + p')_{\mu} 
+ (1 + \kappa)F_M(k^2)[\epsilon^\alpha(p)p \cdot \epsilon^\beta(p') + \epsilon^\beta(p')p' \cdot \epsilon^\alpha(p)] 
+ \frac{\lambda}{m_5^2}F_Q(k^2)[\epsilon^\alpha(p)\epsilon^\beta(p') + \frac{1}{2}g_{\rho\nu}\epsilon^\alpha(p) \cdot \epsilon^\beta(p')k^\rho k^\nu(p + p')_{\mu}] \right] ,$$

(5)

where $A$ is the color octet index, $\alpha, \beta, \ldots$ are color anti-triplet indices, the $\epsilon$'s are polarization 4-vectors for the diquarks.

In the perturbative diagrams involved here, the virtual heavy quark emits a time-like off-shell gluon, that produces a diquark-antidiquark pair, while attaining nearly on-shell 4-momentum. The diquark combines with the heavy quark to form a heavy flavor baryon, whose amplitude for formation is related to the Bethe-Salpeter wavefunction for the diquark-quark system. As in the meson production calculations, it is assumed that the constituents are heavy enough so that the binding is relatively weak, i.e. the quark and diquark are both on-shell and the binding energy is negligibly small. This is expected to be true for constituents with masses well above $\Lambda_{QCD}$, and even the light flavor diquarks almost satisfy this constraint. The basic perturbative amplitude is shown in Fig.1 with the $Q'$-quark line replaced by an (anti-)diquark D line.

It should be realized that the integration (over $s$, the square of the virtual heavy quark mass) involved in the calculation would diverge for point-like vector diquarks, since the gluon coupling to a pair, Eq. 5 carries momentum factors. The virtual mass in the integration, $\sqrt{s}$, is passed on to the gluon and, subsequently, to the gluon-diquark vertex. Hence it is essential to regulate the integrand by some means. This is best accomplished via the chromoelectromagnetic form factors for the gluon coupling to the diquark. The form factor approach makes physical sense - it is a result of the compositeness of the diquarks. And for consistency, once the vector has form factors, the scalar diquark must have one also.

There is no direct information about the chromoelectromagnetic form factors. We may expect that the ordinary electromagnetic form factors will have the same functional form as their QCD counterparts—the source of both sets of form factors is the matrix element of a conserved vector current operator. In the relevant case here, though, the vector operator is the gluon field — a color octet. Also, what is of concern here is the time-like region of the form factor. For diquarks, of course, there is not any direct empirical evidence about their electromagnetic form factors, but diquark-quark models of the nu-
cleon have constrained the parameterization of the form factors. Dimensional counting rules require that asymptotically, $F_S \sim 1/|q|^2$, $F_{E, M} \sim 1/|q|^4$, and $F_Q \sim 1/|q|^6$. Jacob, et al., have obtained electromagnetic form factors for the diquarks (as has a recent study of higher twist contributions to the nucleon structure functions). The diquark form factors are assumed to have simple pole or dipole forms, and the resulting pole positions appear near 1 GeV. If we make the assumption that the color form factors have the same functional form as these empirical electromagnetic form factors, we can proceed. In the integration that will be done here, the time-like $q^2$ region begins at $4m_D^2$ (below the $NN$ threshold) for the value $z = 1/(1 + m_D/m_B)$, and at higher values for other choices of $z$. This implies that the integration region either overlaps or comes near to overlapping the pole positions. We treat the poles as real resonance positions, by including a sizeable imaginary part (of 1 GeV). This is sensible physically, since the color octet form factor would be dominated by color octet vector mesons, and the latter are not expected to be strongly bound or narrow. Hence we have the Breit–Wigner forms and their squares, with pole positions as given by Jacob, et al.. This choice hides our ignorance and provides an interpolation between the space-like and time-like asymptotic regions.

The amplitudes for the baryon production can now be calculated. The spin 1/2 ground state baryons are composed of a scalar diquark and a heavy quark in an s-state. There is only one coupling, and it involves the $F_S$. The amplitude is

$$A_{S \frac{1}{2}} = -\frac{\psi(0)}{\sqrt{2m_d}}F_S(k^2)\bar{U}_BG_5[k\lambda - 2m_d\gamma_\lambda]P^\lambda,$$  

(6)

where

$$P^\lambda = \Lambda^\lambda_{\gamma_5\rho}m_Q(1 + \gamma_\rho) + k_{\rho} - \frac{m_Q(1 + \gamma_\rho)}{(s - m_Q^2)}\Gamma.$$  

(7)

For the vector diquark baryons, there are two form factors (we take the quadrupole to be zero – it falls as $1/|q|^6$ asymptotically). The chromomagnetic coupling involves a parameter $\kappa$, the “anomalous chromomagnetic moment”. This is taken as 1.39. The s-state baryons are spin 3/2 and 1/2, which we will refer to as 1/2'. The 1/2' lies between the 3/2 and the ground state 1/2 baryon. The amplitude for vector diquarks to be produced, along with the heavy quark, contributes to both 3/2 and 1/2' states. The amplitude is conveniently divided into a chromoelectric and chromomagnetic part, involving the two distinct form factors. The chromoelectric part contributing to the spin
The chromomagnetic contribution to the spin $1/2'$ baryon is

$$A_{E1/2} = -\frac{\psi(0)}{\sqrt{3m_d}} F_E(k^2) \bar{U}_B \gamma^\mu \gamma^5 \frac{1 + v}{2} g_s \bar{\epsilon}_\mu^* [k_\lambda - 2m_d v_\lambda] P^\lambda. \quad (8)$$

The chromomagnetic contribution to the spin $1/2'$ baryon is

$$A_{M1/2} = \frac{\psi(0)}{\sqrt{3m_d}} F_M(k^2)(1 + \kappa) \bar{U}_B \gamma^\mu \frac{1 + v}{2} g_s[g_{\mu\lambda}(\bar{\epsilon}^* v)m_d - \bar{\epsilon}_\lambda^* k_\mu] P^\lambda. \quad (9)$$

For the spin $3/2$ baryon the corresponding amplitudes are

$$A_{E3/2} = -\frac{\psi(0)}{\sqrt{2m_d}} F_E(k^2) \bar{\Psi}_B^\mu g_s \bar{\epsilon}_\mu^* [k_\lambda - 2m_d v_\lambda] P^\lambda, \quad (10)$$

and

$$A_{M3/2} = \frac{\psi(0)}{\sqrt{2m_d}} F_M(k^2)(1 + \kappa) \bar{\Psi}_B^\mu g_s[g_{\mu\lambda}(\bar{\epsilon}^* v)m_d - \bar{\epsilon}_\lambda^* k_\mu] P^\lambda. \quad (11)$$

In these amplitudes, $\psi(0)$ is the Bethe-Salpeter wavefunction at the origin (for the s-state Q-diquark system), $m_d$ is the appropriate diquark mass, $U_B$ is a spin $1/2$ Dirac spinor for the baryon, $\Psi_B^\mu$ is the Rarita-Schwinger spinor for the spin $3/2$ baryon, $v = p/M$ for the heavy baryon of mass $M$, $k$ is the 4-momentum of the gluon and $\Delta_\lambda^\mu$ is the corresponding propagator in axial gauge, $\Gamma$ is the production vertex for the heavy quark–antiquark pair. Each amplitude should be multiplied by the color factor $4/3\sqrt{3}$.

Considerable simplification of these amplitudes follows. There are 3 cases to consider—3 final state baryons. For the two states resulting from the vector diquark, the electric and magnetic amplitudes must be added together. Then for each baryon, the amplitude is squared and a trace is taken to sum over spins.
(including spin projection operators for the spin dependent cases). The analog of Eq. 3 is obtained for each baryon. By carefully organizing the terms in the integrand, the width for the inclusive production of the virtual heavy quark can be divided out to yield the analog of Eq. 3 for each baryon. Finally the integration over \( s = q^2 \) can be performed numerically—the form factors make it difficult to write an analytic expression for each case. The resulting \( z \) dependent fragmentation functions are the “boundary” functions, obtained at a scale \( \mu^2 \) at the threshold \( 4m_D^2 \). To consider higher momentum scales, the Altarelli-Parisi evolution equations are used.

We have taken some particular cases to illustrate the results. For the c(su) or c(sd) baryons, the \( \Xi_c \) states, the diquark is given a mass of 0.95 GeV/c\(^2\) and the ratio of diquark to hadron mass is \( r = 0.33 \). The resulting function, \( \hat{f}_1(z,Q^2) \) is shown in Fig.2 for the boundary value at \( \mu = 2m_d \) and for \( Q = 5.5 \) GeV. The three \( s \)-states lead to different behavior and overall probability. It is particularly noteworthy that the 1/2 ground state is produced far less frequently than the 3/2 state or the 1/2’ state from the vector diquark. As we will see, the observed 1/2 ground states are primarily from the decays of these vector diquark states.

In Fig.3 the corresponding longitudinal fragmentation function \( \hat{g}_1(z,Q^2) \) is plotted for the two spin 1/2 states. Note that it is very similar in shape to the spin averaged case. Lastly, the behavior of the transversity function \( \hat{h}_1(z,Q^2) \) remains to be determined.

The spin dependent fragmentation functions for the spin 3/2 baryons are even richer in complexity. There are seven such functions at leading twist, many of which will be accessible from the decay distributions of these states into the 1/2 state plus a pion.
3 Comparison with some data and summary

The production of charmed baryons is becoming sizeable at CESR and data now exist from the CLEO collaboration for some of the Ξ⁺ states. In particular, spin independent fragmentation functions have been determined for the lowest mass spin 1/2 and 3/2 states. The lowest 1/2⁺ states are the c + [u, s] and c + [d, s] states, Ξ⁺ and Ξ⁰, involving the antisymmetric, spin 0 diquarks. The spin 3/2⁺ states are c + {u, s} and c + {d, s} baryons, Ξ⁺⁺ and Ξ⁰⁺⁺, involving the symmetric, spin 1 diquarks. The spin 1/2⁺ partners, Ξ⁺⁺, of 3/2⁺ states have not been seen yet. They are presumed to have a mass below the Ξ⁺ + π threshold, so must be seen in radiative decay channels. Note that these latter Ξ⁺⁺ states have the same isospin as the lower lying ground states 1/2⁺ Ξ⁺ and could mix with them, in principle. In any case, the measured fragmentation functions provide a crude test of the model. The data are fit by the experimenters with a common parameterization of the Peterson function.

It is easy to see in Fig.2 that the data fall nicely on \( \hat{f}_1 \) of our model (with arbitrary normalization), evolved to \( Q = 5.5 \) GeV, but are not sufficiently accurate to be a crucial test of the model. Note that the experimental variable \( x_p \) does not correspond exactly to our \( z \), the light cone variable.

The ratio of the 3/2 to 1/2 production can be extracted from the data with some uncertainty. The percentage of all Ξ⁺ states that arose from decays Ξ⁺⁺ \( \rightarrow \Xi_c^+ + \pi^- \) is given as \((27 \pm 8)\%\) and the percentage of all Ξ⁰ states that arose from decays Ξ⁺⁺ \( \rightarrow \Xi_c^0 + \pi^+ \) is given as \((17 \pm 6)\%\). (Note that we have combined the statistical and systematic errors here.)

The experimenters do not see the π⁰ channels, Ξ⁺⁺ \( \rightarrow \Xi_c^+ + \pi^0 \) and Ξ⁺⁺ \( \rightarrow \Xi_c^0 + \pi^0 \). From isospin conservation these channels account for 1/3 of the decays into Ξ⁺ + π, while the reported charged π channels constitute 2/3. Suppose \( N \) Ξ⁺ states of both charges are produced. Then 2/3 \( N \) will be seen in the charged π decay mode. The total number of Ξ⁺⁺ states seen will be \( N_{+,0} = \frac{2}{3} N \) (supressing errors until the end). The number of Ξ⁺⁺ states not coming from the decays of the 3/2 states will be \( N_{+,0} - N \). Assume that \( n_{+,0} \) of the Ξ⁺⁺ states come from other fragmented states’ decays. Then \( N_{+,0} - N - n_{+,0} \) is the number of direct fragmentation products of the charmed quark. The ratio \( R(0) \) of directly fragmented Ξ⁺⁺ to Ξ⁺⁺ is given thereby as \( R(0) = 1.5 \pm 0.7 - n_{+,0}/N : 1 \) and \( R(+) = 2.9 \pm 1.4 - n_0/N : 1 \).

The numbers \( n_{+,0} \) will come from the radiative decays of the heavier 1/2 states, as well as higher Ξ⁺ states (radial and orbital excitations of the c + (su) and c + (sd) systems). We have calculated the fragmentation functions for the spin 1/2 quark–vector-diquark states and hence the number of Ξ⁺ spin 1/2⁺ states vs. Ξ⁺⁺ spin 3/2⁺ states. That is 1.7:1 for the parameterization used in...
Fig.2. Assuming $n_{+0}$ is due entirely to these $1/2'$ states decaying 100% into the ground state $\Xi^{+0}$, we have for the different charge states $R(+) = -0.2 \pm 0.7$ and $R(0) = 1.2 \pm 1.4$, both of which are consistent with the small ratio of 1/9 predicted by the same model calculation.

Hence an interesting feature of our model is that the directly fragmented ground state baryon is very rare compared to the vector diquark states. In the calculations for heavy-heavy baryons by Marteynenko and Saleev[10], a ratio closer to unity was obtained. It will be interesting to see whether the small ratio, as suggested by our model, persists as more data are obtained.

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