Charged right-handed Higgsino field contribution to the chargino mass spectrum and inverse parameter problem in Left-Right Supersymmetric Models

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Abstract

The contribution of the charged right-handed higgsino fields to the chargino mass spectrum in the context of the Left-Right Supersymmetric Model is studied. Analytical expressions for the chargino masses assuming arbitrary CP-violating phases are given. Also, the corresponding inverse parameter problem is studied. Analytical disentangling of some relevant parameters is presented. A general inversion algorithm, based on suitable measurement of cross-section type observables associated to chargino pair production in electron positron annihilation, is proposed.

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1 INTRODUCTION

In most of the Left-Right Supersymmetric (LRSUSY) model, the right-handed symmetry-breaking energy scale is taken so many order greater than that of its left-handed counterpart due to the great right-handed gauge boson masse requirements and the implementation of the so-called seesaw mechanism [1,2]. Thus at electro-weak scales, the charged right-handed fields commonly appear decoupled from its left-handed counterpart. However, authors studying left-right symmetric and supersymmetric models, have demonstrated that a moderate decoupling limit is also possible, by introducing an intermediate scale or an extra symmetry, and that it could provide testable effects of the remnants of right-handed symmetries in upcoming collider experiments [2,3,4,5,6].

In this article, in the context of on the LRSUSY model, we compute the chargino mass spectrum analytically considering the contributions of the charged right handed Higgsino fields. To determine the chargino masses we must solve a quintic algebraic equation. However, we will see that the analytical resolution of this quintic equation is always possible due to the simple dependence of one of the physical chargino masses with one of the fundamental Higgsino mass parameters. Next, we study the corresponding inverse problem, i.e., how to determinate the fundamental LRSUSY parameters when the physical chargino masses and some measurable physical observables are known. At this stage, the problem is more complex. Nevertheless, as we will see in the last section of this paper, a complete analytical inversion is in principle possible using the projector formalism [7]. The projector method used in this paper constitutes an extension of the formalism used by some authors to compute the neutralino and chargino mass spectrum, to study the corresponding direct and inverse parameter problems, in the context of the Minimal Supersymmetric Standard Model (MSSM) as well as in the context to the LRSUSY model [8,9,10,11].

This paper is organized as follows. In Section 2 we give a brief description of the LRSUSY model focusing our attention on the chargino sector of it. We write the chargino sector Lagrangian density in terms of the charged Higgsino fields, including the right-handed ones, and of the chargino mass matrix. The chargino mass matrix is expressed in terms of the fundamental chargino parameters and an arbitrary set of CP-violating phases. In Section 3 we compute the chargino mass spectrum analytically. We describe the explicit dependence of the chargino masses on the right-handed Higgsino parameters and on the CP-violating phases. In Section 4 we compute the two matrices we need to diagonalize the chargino mass matrix. In Section 5 we generate a novel approach of the projector theory and we connect this theory with the Jarlskog’s projector formalism. In Section 6 we use a system of basic equations derived from the projector theory to disentangle some relevant chargino fundamental parameters from the rest of them. In Section 7 we analyze the inverse parameter problem. A conclusion and some comments are provided in Section 8. Finally, explicit expressions of the entries of the matrices formed from the product between the chargino mass matrix and its corresponding adjoint matrix, and viceversa, on which most of the calculus are based, are written in Appendix A.

2 LRSUSY model, chargino sector, a brief description

The LRSUSY extension of the Standard Model [12,13,14,15,16,17,18,19,20], based on the gauge group $SU(3)_L \times SU(2)_L \times SU(2)_R \times U_{B-L} \times P$ [21], constitutes an alternative to MSSM [22]. In both supersymmetric models, charginos are mixtures of charged gauginos and higgsinos fields. However, the gauge sector of the LRSUSY model differs from gauge sector of the MSSM in an extra neutral $Z^0_R$ and two charged $W^\pm_R$ gauge bosons corresponding to the gauge group $SU(2)_R$. Also, in the Higgs sector both models are different, in the LRSUSY model the Higgs sector contains two bi-doublet fields associated to the quark $u$ and $d$:

$$\phi_{u,d} = \left(\begin{array}{c} \phi^0_1 \\ \phi^0_2 \\ \phi^+_1 \\ \phi^+_2 \end{array}\right)_{u,d}$$
The higgsino fields:

This stage, the gaugino and Higgsino fields mix forming the mass eigenstates of neutralinos and charginos.

Once again, the neutral fields mix forming the massless photon.

The Higgs \( \phi_{u,d} \) transform as \((1/2, 1/2, 0)\), and the Higgs \( \Delta_{L,R} \) transform as \((1, 0, 2)\) and \((0, 1, 2)\), respectively. The triplet Higgs \( \delta_{L,R} \) which transform as \((1, 0, -2)\) and \((0, 1, -2)\), respectively, are introduced to cancel anomalies in the fermionic sector.

The gauge group symmetry must be broken spontaneously in order to generate masses to the quarks, leptons and gauge bosons and to break parity. It can be achieved by choosing the vacuum expectation values (VEV’s) of the Higgs fields in the form \([12]\)

\[ \langle \Delta_L \rangle = \langle \delta_L \rangle = 0, \quad \langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_{\Delta R} & 0 \end{pmatrix}, \quad \langle \delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_{\delta R} & 0 \end{pmatrix}, \quad (2.1) \]

\[ \langle \phi_u \rangle = \begin{pmatrix} \kappa_u & 0 \\ 0 & 0 \end{pmatrix}, \quad \langle \phi_d \rangle = \begin{pmatrix} 0 & 0 \\ 0 & \kappa_d \end{pmatrix}. \quad (2.2) \]

There are breakdowns at three different stages. First parity is breaking, no gauge bosons masses are generated. Next, the spontaneous breaking of \( SU(2)_R \times U(1)_{B-L} \) into \( U(1)_Y \), according to the VEV’s of the \( \Delta_R \) and \( \delta_R \) fields given in Eq. \((2.1)\), generates masses for the gauge fields \( W_R^\pm, W_R^0 \) and \( V_R^0 \). Here, the two neutral states \( W_R^0 \) and \( V_R^0 \) mix yielding the physical field \( Z_R^0 \) and the massless field \( B^0 \). Then, the masses of the weak bosons \( W_L^\pm \) and \( W_L^0 \), as well as of \( B_0 \), are generated at a much lower energy scale by spontaneous breaking of \( SU(2)_L \times U(1)_Y \) into \( U(1)_{em} \), according the VEV’s of \( \phi_{u,d} \) given in Eq. \((2.2)\). Once again, the neutral fields mix forming the massless photon \( A_0 \) and the physical gauge field \( Z_L \). Also at this stage, the gaugino and Higgsino fields mix forming the mass eigenstates of neutralinos and charginos.

The chargino description is determined by the following Lagrangian involving the charged gaugino-higgsino fields:

\[ \mathcal{L}_{\text{chargino}} = -\frac{1}{2} \begin{pmatrix} \psi^+ & \psi^- \end{pmatrix} \begin{pmatrix} 0 & M^T \\ M & 0 \end{pmatrix} \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + H.c, \quad (2.3) \]

with the chargino states given by

\[ \psi^+ = (-i\lambda_L^+ - i\lambda_R^+ \tilde{\phi}_{1u}^+ \tilde{\phi}_{1d}^+ \tilde{\Delta}_R^+) \]

\[ \psi^- = (-i\lambda_L^- - i\lambda_R^- \tilde{\phi}_{2u}^\mp \tilde{\phi}_{2d}^- \tilde{\Delta}_R^-) \]

\[ (2.4) \]

where \( \lambda_{L,R}^\pm \) are the \( SU(2)_{L,R} \) charged gaugino fields, \( \tilde{\phi}_{1u}, \tilde{\phi}_{2u} \) and \( \tilde{\phi}_{1d}, \tilde{\phi}_{2d} \) are the charged higgsino fields associated with the \( u \) and \( d \)-quarks, respectively. The charged right-handed higgsino fields are represented
by $\Delta_R^\kappa$ and $\delta_R^\kappa$. The chargino mass matrix $M$ is given by

$$M = \begin{pmatrix}
M_L & 0 & 0 & \sqrt{2}\tilde{M}_L \cos \theta_\kappa & 0 \\
0 & M_R & 0 & \sqrt{2}\tilde{M}_R \cos \theta_\kappa & \sqrt{2}M_1 \sin \theta_v \\
\sqrt{2}\tilde{M}_L \sin \theta_\kappa & \sqrt{2}\tilde{M}_R \sin \theta_\kappa & 0 & -\mu & 0 \\
0 & 0 & -\mu & 0 & 0 \\
0 & \sqrt{2}M_2 \cos \theta_v & 0 & 0 & \mu_3
\end{pmatrix},$$

where we suppose $M_L = |M|e^{i\Phi_L}$, $\mu = |\mu|e^{i\Phi_\mu}$, $\mu_3 = |\mu_3|e^{i\Phi_3}$,

$$\tilde{M}_L = M_W e^{i\Phi_L}, \quad \tilde{M}_R = \frac{g_R}{g_L}M_W e^{i\Phi_R},$$

(2.6)

$$M_1 = M_{W_1} e^{i\Phi_1}, \quad M_2 = M_{W_2} e^{i\Phi_2},$$

(2.7)

where $M_{W_L}$ denotes the mass of the left-handed gauge boson

$$M_{W_L} = \frac{g_L}{\sqrt{2}}\sqrt{\kappa_u^2 + \kappa_d^2}$$

(2.8)

and $M_{W_R}$ denotes the mass of the right-handed gauge boson

$$M_{W_R} = \frac{g_R}{\sqrt{2}}\sqrt{(v_\Delta_R)^2 + (v_\delta_R)^2}.$$  

(2.9)

Here $g_L$ and $g_R$ are coupling constants associated to the gauge groups $SU(2)_L$ and $SU(2)_R$, respectively; $\mu$ and $\mu_3$, are fundamental Higgsino mass parameters, $M_L, M_R$ are fundamental gaugino mass parameters.

The fundamental parameter $\tan \theta_\kappa = k_u/k_d = \kappa_u/\kappa_d$, represents the ratio between the vacuum expectation values of the Higgs fields which couple to d- and u-type quark respectively. From Eq. (2.8) we deduce that $\kappa_u$ and $\kappa_d$ can be expressed in terms of $M_{W_L}, g_L$ and $\theta_\kappa$ in the form

$$\kappa_u = \sqrt{2}M_{W_L}/g_L \sin \theta_\kappa, \quad \kappa_d = \sqrt{2}M_{W_L}/g_L \cos \theta_\kappa.$$  

(2.10)

In the same way $\tan \theta_v = v_\Delta_R/v_\delta_R$, represents the ratio between the vacuum expectation values of the right-hand Higgs fields. From Eq. (2.9) we deduce that $v_\Delta_R$ and $v_\delta_R$ can be expressed in terms of $M_{W_R}, g_R$ and $\theta_v$ in the form

$$v_\Delta_R = \sqrt{2}M_{W_R}/g_R \sin \theta_v, \quad v_\delta_R = \sqrt{2}M_{W_R}/g_R \cos \theta_v.$$  

(2.11)

For the general CP-violating case, we are assuming that the chargino mass matrix is parameterized by thirteen real parameters, namely, $|M_L|, \Phi_L, |\mu|, \Phi_\mu, M_R, \tilde{\Phi}_L, \tilde{\Phi}_R, \Phi_1, \Phi_2, |\mu_3|, \Phi_3, \tan \theta_\kappa,$ and $\tan \theta_v$.

3 Chargino mass spectrum

The physical chargino mass eigenstates are related to the states given by the Eqs. (2.4) and (2.5) as

$$\psi_1^+ = \sum_{j=1}^5 V_{ij} \chi_j^+, \quad i = 1, \ldots, 5,$$

(3.1)
\[ \psi_i^- = \sum_{j=1,5} U_{ij} \chi_j^- \quad i = 1, \ldots, 5. \] (3.2)

The unitary matrices \( U \) and \( V \) satisfy

\[
M_D = U^T M V,
\]

\[
\equiv \sum_{j=1}^4 m_{\chi_j^\pm} E_j, \tag{3.3}
\]

and

\[
M_D^2 = V^{-1} M^\dagger M V = U^T M M^\dagger U^* \equiv \sum_{j=1}^5 m_{\chi_j^\pm}^2 E_j, \tag{3.4}
\]

where \((E_j)_{5 \times 5}\) are the basic matrices defined by

\[
(E_j)_{ik} = \delta_{ji} \delta_{jk}. \tag{3.5}
\]

Here, we suppose that the real eigenvalues of \( M_D \) are ordered in the following way

\[ m_{\chi_1^\pm} < m_{\chi_2^\pm} < m_{\chi_3^\pm} < m_{\chi_4^\pm} < m_{\chi_5^\pm}. \tag{3.6} \]

The chargino masses, at the tree level, are given by the positive roots of the eigenvalues associated to either the Hermitian matrix \( H \equiv M^\dagger M \) or the Hermitian matrix \( \tilde{H} \equiv M M^\dagger \). These eigenvalues can be obtained by solving the characteristic equation associated to these matrices. In this particular case, according to either Eq. (A.129) or Eq. (A.130), we can show that the characteristic equation can be factorized in the form:

\[
(m_{\chi_j^\pm}^2 - |\mu|^2) \left( (m_{\chi_j^\pm}^2)^4 - a (m_{\chi_j^\pm}^2)^3 + b (m_{\chi_j^\pm}^2)^2 - c m_{\chi_j^\pm}^2 + d \right) = 0, \tag{3.7}
\]

where

\[ a = |\mu|^2 + |M_L|^2 + 2 |\tilde{M}_L|^2 + 2 |M_R|^2 + |\mu_3|^2 + 2 M_W^2 + M_R^2, \tag{3.8} \]

\[ b = |\mu|^2 |\mu_3|^2 + 2 |\tilde{M}_R|^2 |\mu|^2 + 2 |\tilde{M}_L|^2 |\mu|^2 |\tilde{M}_L|^2 |\mu|^2 + 4 \cos^2(\kappa) |\tilde{M}_R|^4 \sin^2(\kappa) + 4 \cos^2(\kappa) |\tilde{M}_L|^4 \sin^2(\kappa) \]

\[ + 2 \cos(\Theta_3 - \Theta_2) |\mu| M_L |\tilde{M}_L| \sin(2\theta_k) + 2 \left(|\mu|^2 + 2 |\tilde{M}_R|^2 \cos^2(\theta_k) \right) M_W^2 \sin^2(\theta_c) \]

\[ + 2 \left(|\mu|^2 + 2 |\tilde{M}_R|^2 \cos^2(\theta_k) \right) M_W^2 \cos^2(\theta_c) + M_R^2, \]

\[ + 2 \cos(\Theta_2) |\mu| M_R |\tilde{M}_R|^2 \sin(2\theta_k) - 2 \cos(\Theta_1) |\mu_3| M_R M_W^2 \sin(2\theta_c) \]

\[ + |\mu|^2 M_R^2 + |\mu_3|^2 M_R^2 + |M_L|^2 \left(|\mu|^2 + 2 |\tilde{M}_R|^2 + |\mu_3|^2 + 2 M_W^2 + M_R^2 \right) \]

\[ + 2 |\tilde{M}_L|^2 \left(|\mu_3|^2 + |\tilde{M}_R|^2 \sin^2(2\theta_k) + 2 M_W^2 + M_R^2 \right), \tag{3.9} \]
Here, we solve Eq. (3.7) to get the analytic expressions for the chargino masses:

\[
c = 2 |\mu| M_R^2 (|\mu_3| \sin(2 \theta_k) [|\mu_3| M_R \cos(\Theta_2) - M_{WR}^2 \cos(\Theta_1 - \Theta_2) \sin(2 \theta_v)]
+ (|\mu|^2 + 2 |M_L|^2) [M_{WR}^4 \sin^2(2 \theta_v) - 2 |\mu_3| M_R M_{WR}^2 \cos(\Theta_1) \sin(2 \theta_v) + |\mu_3|^2 M_R^2]\n+ |M_L|^2 (|\tilde{M}_R|^4 \sin^2(2 \theta_k) + 2 |\mu|^2 M_{WR}^2 + M_{WR}^4 \sin^2(2 \theta_v))
- 2 |\mu_3| M_R M_{WR}^2 \cos(\Theta_1) \sin(2 \theta_v) + |\mu|^2 M_R^2
+ 2 |\tilde{M}_R|^2 [|\mu|^2 + 2 M_{WR}^2 (\cos^2(\theta_k) \cos^2(\theta_v) + \sin^2(\theta_k) \sin^2(\theta_v)) + \cos(\Theta_2) |\mu| \sin(2 \theta_k) M_R]\n+ |\mu|^2 \left(|\mu| + M_R^2 \right)\]
\times \left[|M_L|^2 + |\tilde{M}_R|^2 \right]\left|\mu_3|^2 + |M_L|^2 \left(2 |\tilde{M}_R|^2 M_{WR}^2 + |M_L|^2 (2 M_{WR}^2 + M_R^2)\right)\right],
\]

\[
d = \left(|\mu|^2 |M_L|^2 + 4 \cos^2(\theta_k) |\tilde{M}_L|^4 \sin^2(\theta_k)\right)
\times \left[M_{WR}^4 \sin^2(2 \theta_v) - 2 |\mu_3| M_R M_{WR}^2 \cos(\Theta_1) \sin(2 \theta_v) + |\mu_3|^2 M_R^2\right]
+ |M_L| |\tilde{M}_R|^2 |\mu_3|^2 \sin^2(2 \theta_k) \left[M_L| |\tilde{M}_R|^2 |\mu_3|^2 + |M_L|^2 \left(2 |\mu_3| M_R \cos(\Theta_3)\right)\right]
- 2 M_{WR}^2 \cos(\Theta_1 - \Theta_3) \sin(2 \theta_v)\right)\right] + 2 |\mu| |M_L| \sin(2 \theta_k)
\times \left[M_L| |\tilde{M}_R|^2 |\mu_3|^2 \left(|\mu_3| M_R \cos(\Theta_2) - M_{WR}^2 \cos(\Theta_1 - \Theta_2) \sin(2 \theta_v)\right)\right]
+ \cos(\Theta_3 - \Theta_2) |\tilde{M}_L|^2 \left(M_{WR}^4 \sin^2(2 \theta_v) - 2 |\mu_3| M_R M_{WR}^2 \cos(\Theta_1) \sin(2 \theta_v) + |\mu_3|^2 M_R^2 \right)\right].
\]

(3.10)

Here, \(\Theta_1 = \Phi_1 + \Phi_2 - \Phi_3\), \(\Theta_2 = 2 \Phi_R - \Phi_\mu\) (3.11) (3.12)

and \(\Theta_3 = \Phi_L - 2 \Phi_R + 2 \Phi_R\). (3.13)

Solving Eq. (3.7), we get the analytic expressions for the chargino masses:

\[
m_{\chi_1^\pm}^2, m_{\chi_2^\pm}^2 = \frac{a}{4} - \frac{c}{2} \pm \frac{1}{2} \sqrt{\theta - \infty - \frac{\lambda}{4\xi}},
\]

(3.14)

\[
m_{\chi_3^\pm}^2 = |\mu|^2
\]

(3.15)

\[
m_{\chi_4^\pm}^2, m_{\chi_5^\pm}^2 = \frac{a}{4} + \frac{c}{2} \pm \frac{1}{2} \sqrt{\theta - \infty + \frac{\lambda}{4\xi}},
\]

(3.16)
where

\[
\varsigma = \sqrt{\frac{\varrho}{2} + \varpi},
\]

\[
\varpi = \frac{\epsilon}{3 \, 2^{\frac{3}{2}}} + \left(2^{\frac{3}{2}} \gamma\right),
\]

\[
\epsilon = \frac{\delta + \sqrt{\delta^2 - 4 \gamma^2}}{3 \epsilon},
\]

\[
\varrho = \frac{a^2 - 4b}{2} - \frac{3}{3},
\]

\[
\lambda = a^3 - 4ab + 8c,
\]

\[
\gamma = b^2 - 3ac + 12d,
\]

\[
\delta = 2b^3 - 9abc + 27c^2 + 27a^2d - 72bd.
\]

(3.17)

According to the characteristic equation (3.7), it is always possible to find a neighborhood in the fundamental parameter space where one of the chargino masses takes the value $|\mu|$. Assuming the neighborhood of the fundamental parameter space for $m_{\tilde{\chi}_i^\pm} = |\mu|$, the physical masses given in Eqs. (3.14-3.16) are automatically arranged according to their magnitude, from the lightest to the heaviest. In regions where a new chargino mass different from $m_{\tilde{\chi}_i^\pm}$ takes the value $|\mu|$, we must simply redefine the masses’ suffixes given in Eqs. (3.14) and (3.16), without altering the increasing order, taking into account the suffix of this new chargino mass.

Notice that, the expressions for the chargino masses given in Eqs. (3.14-3.16) include the impact of the given CP-violating phases. The chargino masses only depend on the phase combinations (3.11-3.13) which describe the influence of the charged right-handed higgsino fields upon the chargino masses. Thus, they constitutes a generalization of some results found in the literature [11, 23, 24].

4 Diagonalizing matrices $V$ and $U^*$

From Eq. (3.4), we can show that the entries of the diagonalizing matrix $V$, for a fixed $\ell$ ($\ell = 1, 2, \ldots, 4$ or 5), are given by

\[
V_{ij} = \frac{\Delta^{(\ell)}_{ij}}{\Delta^{(\ell)}_{\ell j}} \frac{|\Delta^{(\ell)}_{\ell j}| e^{-i\theta_{\ell j}}}{\sum_{k=1}^{5} |\Delta^{(\ell)}_{kj}|^2}, \quad i, j = 1, \ldots, 5,
\]

(4.18)

where

\[
\Delta^{(\ell)}_{ij}(H) = \text{Det} \left( H^{(\ell,\ell)} - m_{\tilde{\chi}_i^\pm}^2 I_4 \right)
\]

(4.19)

and $\Delta^{(\ell)}_{ij}(H)$, $i = 1, \ldots, 5$, $i \neq \ell$, are formed from $\Delta^{(\ell)}_{ij}(H)$ by substituting the 1, \ldots, 4-th columns by the 4 × 1 matrices obtained from $\left(-H_{1\ell}^\dagger; \ldots; -H_{5\ell}^\dagger\right)$ by eliminating the $\ell$-th row, respectively. $H^{(\ell,\ell)}$ is the minor matrix formed from $H$ if we eliminate the $\ell$-th row and the $\ell$-th column. $I_4$ is the $4 \times 4$ identity matrix.

In the same way we can write the matrix elements of $U^*$, we get

\[
U_{ij}^* = \frac{\tilde{\Delta}^{(\ell)}_{ij}}{\Delta^{(\ell)}_{ij}} \frac{|\Delta^{(\ell)}_{\ell j}| e^{-i\tilde{\theta}_{\ell j}}}{\sum_{k=1}^{5} |\tilde{\Delta}^{(\ell)}_{kj}|^2}, \quad i, j = 1, \ldots, 5,
\]

(4.20)

where $\tilde{\Delta}^{(\ell)}_{ij}(\tilde{H}) \equiv \Delta^{(\ell)}_{ij}(\tilde{H})$. 

7
Thus, to know the $V$-type and $U_\ast$-type diagonalizing matrices we only need to compute the $\Delta_{\alpha j}^{(\ell)}$’s and \( \hat{\Delta}_{\alpha j}^{(\ell)} \)’s basic quantities, respectively. From the definition for these quantities, assuming that \( m_{\chi_3} = |\mu| \), and taking \( \ell = 1 \), we get \((j = 1, 2, 4, 5)\)

\[
\begin{align*}
\Delta_{1j} &= \left( m_{\chi_3}^2 - |\mu|^2 \right) \left\{ m_{\chi_3}^6 - m_{\chi_3}^4 \left[ M_R^2 + 2 \cos^2(\theta_\kappa) |\bar{M}_L|^2 + 2 |\bar{M}_R|^2 + g_R^2 \left( (v_{\Delta R})^2 + (v_{\delta R})^2 \right) \right] + |\mu|^2 + |\mu_3|^2 \left[ g_R^4 (v_{\Delta R})^2 (v_{\delta R})^2 + M_R^2 |\mu|^2 + g_R^2 \left( (v_{\Delta R})^2 + (v_{\delta R})^2 \right) |\mu|^2 - 2 g_R^2 M_R \right. \\
& \quad \left. \times \cos(\Theta_1) v_{\Delta R} v_{\delta R} |\mu_3| + \left( M_R^2 + |\mu|^2 \right) |\mu_3|^2 + 2 \cos^2(\theta_\kappa) |\bar{M}_L|^2 \left( M_R^2 + g_R^2 \right) \right) \left( (v_{\Delta R})^2 + (v_{\delta R})^2 \right) + |\mu_3|^2 + 2 |M_R|^2 \sin^2(\theta_\kappa) + |M_R|^4 \sin^2(2 \theta_\kappa) + 2 |M_R|^2 \left( |\mu_3|^2 + g_R^2 \right) \\
& \quad \times \left( \cos^2(\theta_\kappa) (v_{\delta R})^2 + (v_{\Delta R})^2 \sin^2(\theta_\kappa) + M_R \cos(\Theta_2) |\mu| \sin(2 \theta_\kappa) \right) \bigg| - |\mu|^2 \\
& \quad \bigg( g_R^4 (v_{\Delta R})^2 (v_{\delta R})^2 - 2 g_R^2 M_R \cos(\Theta_1) v_{\Delta R} v_{\delta R} |\mu_3| + M_R^2 |\mu_3|^2 \bigg) - 2 \cos^2(\theta_\kappa) \\
& \quad \left| \bar{M}_L \right|^2 \left( g_R^4 (v_{\Delta R})^2 (v_{\delta R})^2 - 2 g_R^2 M_R \cos(\Theta_1) v_{\Delta R} v_{\delta R} |\mu_3| + M_R^2 |\mu_3|^2 + 2 |M_R|^2 \right) \\
& \quad \left( g_R^2 (v_{\Delta R})^2 + |\mu_3|^2 \right) \sin^2(2 \theta_\kappa) + 2 |M_R|^2 |\mu| |\mu_3| \sin(2 \theta_\kappa) \\
& \quad \left( g_R^2 \cos(\Theta_1 - \Theta_2) v_{\Delta R} v_{\delta R} - M_R \cos(\Theta_2) |\mu_3| \right), \tag{4.21} \end{align*}
\]

\[
\begin{align*}
\Delta_{2j} &= 2 |\bar{M}_L| |\bar{M}_R| e^{-i (\Phi_1 + \Phi_2 + \Phi_L + \Phi_R + \Phi_\mu)} \left( m_{\chi_3}^2 - |\mu|^2 \right) \left\{ e^{i(\Phi_1 + \Phi_2 + 2 \Phi_L + \Phi_R)} m_{\chi_3}^4 \sin^2(\theta_\kappa) \\
& \quad - e^{i(\Phi_1 + \Phi_2)} m_{\chi_3}^2 \left[ e^{i(\Phi_L + \Phi_\mu)} \cos(\theta_\kappa) |\bar{M}_L| \left( e^{i \Phi_\mu} |\mu| \sin(\theta_\kappa) - e^{2i \Phi_R} M_R \cos(\theta_\kappa) \right) \right. \\
& \quad + e^{2i \Phi_L} \sin(\theta_\kappa) \left( 2 \cos(\theta_\kappa)^2 |\bar{M}_L|^2 + 2 \cos^2(\theta_\kappa) \right) \left( M_R^2 + g_R^2 (v_{\Delta R})^2 + 2 |\mu_3|^2 \right) \bigg| + \cos(\theta_\kappa) e^{i(\Phi_L + \Phi_\mu)} |\bar{M}_L| \left( e^{i(\Phi_1 + \Phi_2 + 2 \Phi_R)} g_R^2 \cos(\theta_\kappa) \right) \bigg| \\
& \quad v_{\Delta R} v_{\delta R} |\mu_3| - e^{i(\Phi_1 + \Phi_2 + 2 \Phi_R)} M_R \cos(\theta_\kappa) \left| \mu_3 \right|^2 + e^{i(\Phi_1 + \Phi_2 + \Phi_\mu)} g_R^2 (v_{\Delta R})^2 |\mu| \sin(\theta_\kappa) \\
& \quad + e^{i(\Phi_1 + \Phi_2 + \Phi_\mu)} |\mu| |\mu_3|^2 \sin(2 \theta_\kappa) \bigg| \right) + e^{2i \Phi_L} \sin(\theta_\kappa) \left( e^{i(\Phi_1 + \Phi_2 + \Phi_\mu)} g_R^2 |\bar{M}_L|^2 \left( v_{\Delta R} \right)^2 2 \sin(2 \theta_\kappa) \\
& \quad - e^{i(\Phi_2 + 2 \Phi_R)} g_R^2 v_{\Delta R} v_{\delta R} |\mu| |\mu_3| + e^{i(\Phi_1 + \Phi_2)} |\mu_3|^2 \left[ e^{2i \Phi_R} M_R |\mu| + e^{i \Phi_\mu} \sin(2 \theta_\kappa) \right] \\
& \quad \left( \left| \bar{M}_L \right|^2 + |\bar{M}_R|^2 \right) \bigg| \bigg), \tag{4.22} \end{align*}
\]

\[
\Delta_{3j} = 0, \tag{4.23}
\]
\[
\Delta_{4j} = -e^{-i(\Phi_L + 2\Phi_R + \Phi_\mu)} \sqrt{2} |M_L| \left( \frac{m_2^{\beta}}{\chi_j} - |\mu|^2 \right) \left\{ \left( \frac{m_2^{\beta}}{\chi_j} - g_R^2 (v_{\Delta_R})^2 - |\mu|^2 \right) \right.
\]
\[
\times \left[ e^i(\Phi_L + 2\Phi_R + \Phi_\mu) \cos(\theta_\kappa) |M_L| \left( M_R^2 - \frac{m_2^{\beta}}{\chi_j} + g_R^2 (v_{\delta_R})^2 \right) \right.
\]
\[
- e^{-2i\Phi_L} \sin(\theta_\kappa) \left( e^{-2i\Phi_R} \left( M_R^2 - \frac{m_2^{\beta}}{\chi_j} + g_R^2 (v_{\delta_R})^2 \right) |\mu| + e^{i\Phi_\mu} M_R |M_R|^2 \sin(2\theta_\kappa) \right)
\]
\[
+ e^{-i(\Phi_L + 2\Phi_R + \Phi_\mu)} g_R^2 \left( e^{i\Phi_3} M_R |v_{\Delta_R} + e^{i(\Phi_1 + \Phi_2)} v_{\delta_R} |\mu_3| \right) - e^{-2i\Phi_L} \sin(\theta_\kappa) \left( e^{i(\Phi_1 + \Phi_2 + \Phi_R)} M_R \right)
\]
\[
\times |M_L| \left( e^{i(\Phi_1 + \Phi_2)} M_R |v_{\Delta_R} + e^{i(\Phi_3 + \Phi_R)} v_{\delta_R} |\mu_3| \right) - e^{-2i\Phi_L} \sin(\theta_\kappa) \left( e^{i(\Phi_1 + \Phi_2 + \Phi_R)} M_R \right)
\]
\[
\times |v_{\Delta_R} |\mu| + e^{i(\Phi_1 + \Phi_2 + \Phi_R)} v_{\delta_R} |\mu| |\mu_3| + e^{i(\Phi_1 + \Phi_2 + \Phi_R)} |M_R|^2 \sin(2\theta_\kappa) \right]\},
\]
(4.24)

\[
\Delta_{5j} = 2gR |\tilde{M}_L| |\tilde{M}_R| \left( \frac{m_2^{\beta}}{\chi_j} - |\mu|^2 \right) \left\{ e^{-1(\Phi_1 + \Phi_3 + \Phi_L + 2\Phi_R + \Phi_\mu)} \right.
\]
\[
\times \cos(\theta_\kappa)^2 \left| M_L \right| v_{\Delta_R} + e^{2i\Phi_L} \sin(\theta_\kappa) \left( e^{i(\Phi_3 + \Phi_\mu)} M_R |v_{\Delta_R} \sin(\theta_\kappa) |\mu| + e^{i(\Phi_1 + \Phi_2 + \Phi_\mu)} \right)
\]
\[
\times v_{\delta_R} |\mu_3| \sin(\theta_\kappa) - e^{i(\Phi_3 + 2\Phi_R)} \cos(\theta_\kappa) v_{\Delta_R} |\mu| \right] - \cos(\theta_\kappa) \left[ e^{2i\Phi_L} \sin(\theta_\kappa) \right.
\]
\[
\times \left[ e^{i(\Phi_1 + \Phi_2)} v_{\delta_R} |\mu_3| \left( e^{2i\Phi_R} M_R |\mu| + e^{i\Phi_\mu} \left( |M_L|^2 + |M_R|^2 \right) \sin(2\theta_\kappa) \right) \right.
\]
\[
- e^{2i\Phi_R} g_R^2 \left( |\mu| - e^{i\Phi_\mu} M_R |M_L|^2 \sin(2\theta_\kappa) \right) + e^{i(\Phi_L + \Phi_\mu)} |M_L| \left[ e^{i\Phi_3} v_{\Delta_R} \right.
\]
\[
\times \left( e^{2i\Phi_R} g_R^2 \cos(\theta_\kappa) (v_{\delta_R})^2 + e^{i\Phi_\mu} M_R |\mu| \sin(\theta_\kappa) + e^{2i\Phi_R} |M_R|^2 \sin(2\theta_\kappa) \sin(2\theta_\kappa) \right)
\]
\[
- e^{i(\Phi_1 + \Phi_2)} v_{\delta_R} |\mu_3| \left( e^{2i\Phi_R} M_R \cos(\theta_\kappa) - e^{i\Phi_\mu} |\mu| \sin(\theta_\kappa) \right) \right] \},
\]
(4.25)

where \(v_{\Delta_R}\) and \(v_{\delta_R}\) are given by Eq. (2.11). The expressions for \(\tilde{\Delta}_{kj}, k = 1, \ldots, 5\), are obtained by interchanging \(v_{\Delta_R} \leftrightarrow v_{\delta_R}, \Phi_1 \leftrightarrow \Phi_2\) and \(\sin \theta_\kappa \leftrightarrow \cos \theta_\kappa\) in the equations \(\Delta_{1j}, \Delta_{2j}, \Delta_{3j}, \Delta_{4j}, \) and \(\Delta_{5j}\) given above, respectively.

The remaining \(\Delta_{ij}\) and \(\tilde{\Delta}_{ij}\) factors can be deduced taking into account the properties of \(V\) and \(U^*\). Equation (4.23) implies that all the entries of the third row of the \(V\) matrix, except \(V_{33}\), are zero. Since \(V\) is an invertible matrix, \(V_{33}\) must be different from zero. In addition, \(V\) is unitary, then all the entries of the third column of this matrix, except \(V_{33}\), must be equal to zero. Consequently, the norm of \(V_{33}\) is equal to 1. From Eq. (4.18) we deduce \(\Delta_{a3} = 0, a = 1, 2, 4, 5\). Similarly, using the same arguments, for the matrix \(U^*\), we have \(U_{a3}^* = 0, a = 1, 2, 3, 5\) and \(U_{43}^* = 0, \beta = 1, 2, 4, 5\). Also, the norm of \(U_{43}\) must be equal to 1. Finally, from Eq. (4.20) we deduce \(\tilde{\Delta}_{43} = 0, \beta = 1, 2, 4, 5\).

5 Generalized projector formalism

Based on the diagonalization process of the chargino mass matrix studied in the previous section, we implement a projector method which can be easily generalized to deal with any symmetric or non-symmetric mass matrix. This method provides both a system of basic equations connecting the fundamental parameters with the physical chargino masses, the reduced projectors, and the eigenphases and a set of equivalences (when
we combine with the Jarlskog’s projector formulation). These equations and equivalences, as we will see in the next sections, are useful for both to disentangle some relevant fundamental parameters from the rest of the parameters (see Section 6) and to obtain a systematic inversion process based on the measurement of some suitable set of physical observables. (see Section 7).

5.1 Reduced projectors

Defining the type-$V$ reduced as projectors

$$p^{(\ell)}_{j\alpha} = \frac{(\Delta^{(\ell)}_{\alpha j})^*}{(\Delta^{(\ell)}_{\ell j})^*}$$

(5.26)

we can express the entries of the diagonalizing matrix $V$ given in Eq. (4.18) in the compact form

$$V_{\alpha j} = \left[ \frac{P^{(\ell)}_{j\ell\ell} (p^{(\ell)}_{j\alpha})^*}{\eta_j} \right]^{1/2},$$

(5.27)

where $\eta_j^{(\ell)} \equiv e^{2i\vartheta_j^{(\ell)}}$ stands for the type-$V$ eigenphases and

$$P^{V}_{j\alpha\beta} = P^{V}_{j\ell\ell} (p^{(\ell)}_{j\alpha})^* p^{(\ell)}_{j\beta} = \frac{(p^{(\ell)}_{j\alpha})^* p^{(\ell)}_{j\beta}}{\sum_{k=1}^{5} |p^{(\ell)}_{jk}|^2},$$

(5.28)

stands for the type-$V$ projectors (note that in general $p^{(\ell)}_{j\ell} = 1$ whatever $\Delta^{(\ell)}_{\ell j} \neq 0$).

Similarly, by defining the type-$U^*$ reduced projectors

$$\tilde{p}^{(\ell)}_{j\alpha} = \frac{(\tilde{\Delta}^{(\ell)}_{\alpha j})^*}{(\tilde{\Delta}^{(\ell)}_{\ell j})^*},$$

(5.29)

we can express the entries of the diagonalizing matrix $U^*$ given in Eq. (4.20) in the compact form

$$U_{\alpha j}^* = \left[ \frac{P^{U^*}_{j\ell\ell} (\tilde{p}^{(\ell)}_{j\alpha})^*}{\tilde{\eta}_j^{(\ell)}} \right]^{1/2},$$

(5.30)

where $\tilde{\eta}_j^{(\ell)} \equiv e^{-2i\tilde{\vartheta}_j^{(\ell)}}$ stands for the type-$U^*$ CP eigenphases and

$$P^{U^*}_{j\alpha\beta} = P^{U^*}_{j\ell\ell} (\tilde{p}^{(\ell)}_{j\alpha})^* \tilde{p}^{(\ell)}_{j\beta} = \frac{(\tilde{p}^{(\ell)}_{j\alpha})^* \tilde{p}^{(\ell)}_{j\beta}}{\sum_{k=1}^{5} |	ilde{p}^{(\ell)}_{jk}|^2},$$

(5.31)

stands for the type-$U^*$ projectors.

Note that the reduced projectors are not all independent. Since $V$ and $U^*$ are unitary matrices, from (5.27) and (5.30), we get the constraints

$$P^{V}_{i\ell\ell} \sum_{k=1}^{5} p^{(\ell)}_{ik} (p^{(\ell)}_{jk})^* = \delta_{ij}$$

(5.32)

and

$$P^{U^*}_{i\ell\ell} \sum_{k=1}^{5} \tilde{p}^{(\ell)}_{ik} (\tilde{p}^{(\ell)}_{jk})^* = \delta_{ij}.$$

(5.33)
For $i = j$, Eqs. (5.32) and (5.33) verify identically. For $i, j = 1, \ldots, 5, \ | \ i > j$, each one represents a system of ten complex algebraic equations which can be used to reduce up to 20 the number of real independent parameters on each set of reduced projectors.

On the other hand, from the particular structure of the $H$ and $\tilde{H}$ matrices, we see that the entries of the $V$ and $U^*$ matrices are related as follows

\[
U^*_{ij} = \frac{1}{\sqrt{\chi_{\pm j}}} \sum_{k=1}^{5} M_{ik} V_{kj} \tag{5.34}
\]

and

\[
V_{ij} = \frac{1}{\sqrt{\chi_{\pm j}}} \sum_{k=1}^{5} M_{ik}^* U_{kj}. \tag{5.35}
\]

Hence, inserting Eqs. (5.27) and (5.30) into Eq. (5.34) and Eq. (5.35) we deduce the fundamental basic equations ($\alpha = 1, \ldots, 5$)

\[
m_{\chi_{\pm j}}^{(\ell)} \zeta_{j}^{(\ell)} \left( \frac{P_{\ell \ell} U^{*}}{P_{\ell \ell} V} \right)^{\ast} \left( \bar{p}^{(\ell)} \right)^{\ast} = \sum_{\beta=1}^{5} M_{\alpha \beta} \left( \bar{p}_{j}^{(\ell)} \right)^{\ast} \tag{5.36}
\]

and

\[
m_{\chi_{\pm j}}^{(\ell)} \zeta_{j}^{(\ell)} \sqrt{\frac{P_{\ell \ell} V}{P_{\ell \ell} U}} \left( p_{j}^{(\ell)} \right) = \sum_{\beta=1}^{5} M_{\beta \alpha} \bar{p}_{j}^{(\ell)}, \tag{5.37}
\]

respectively. Here, $\zeta_{j}^{(\ell)} \equiv \sqrt{\frac{\eta_{j}^{(\ell)}}{\bar{\eta}_{j}^{(\ell)}}}$ stands for the global eigenphases.

Equations (5.36) and (5.37) represent, for fixed $j$, a system of ten complex algebraic equations serving to determine the fundamental parameters of the model in terms of the reduced projectors, the chargino physical masses $m_{\chi_{\pm j}}$ and the eigenphases, and vice versa.

Note that, in Eqs. (5.27) and (5.30) as well as in Eqs. (5.36) and (5.37), without any loss of generality, we could choose the $U^*$-type eigenphases either $\eta_{j}^{(\ell)} = 1, j = 1, \ldots, 5$, such that $\zeta_{j}^{(\ell)} = \sqrt{\eta_{j}^{(\ell)}}$; $\left( \eta_{j}^{(\ell)} \right)^{\ast} = \eta_{j}^{(\ell)}$, $j = 1, 2, \ldots, 5$, such $\zeta_{j}^{(\ell)} = \eta_{j}^{(\ell)}$, or any other suitable choice allowing us to eliminate five superfluous parameters.

Note that the generalization of the projector formalism to any number of charginos is direct, for instance, for $n$ charginos, $n = 2, \ldots$, we only need to consider $\ell$ a fixed number between 1 and $n$, $I_{\ell-1}$ in place of $I_4$ and the subindex $\alpha, \beta, j$ running from 1 to $n$.

### 5.2 Reduced projectors in terms of the fundamental parameters and eigenphases

From Eqs. (5.36) and (5.37), we can express the reduced projectors in terms of the fundamental parameters and eigenphases without any dependence on the parameters $|M_L|$ and $\Phi_L$. Indeed, choosing $\ell = 1$, and combining the corresponding equations for $j = 1, 2, 4, 5$, and $\alpha = 2, 3, 4, 5$, we get the non trivial reduced
projectors:

\[
p_{j2} = \frac{2 e^{-i(\Phi + \Phi_\mu)} \tilde{M}_L^{*} \tilde{M}_R^{*}}{D_j} \left\{ e^{2i \Phi_L} \sqrt{\frac{P_{j11}^{U^*}}{P_{j11}}} \xi_j^* \cos(\theta_\kappa) m_{\chi_j^\pm}^2 \left[ e^{i(\Phi_3 + 2 \hat{\Phi}_R)} |\mu| \sin(\theta_\kappa) \right] \right. \\
\times \left( g^2_R (v_{\Delta_R})^2 + |\mu_3|^2 - m_{\chi_j^\pm}^2 \right) + e^{i \Phi_\mu} \cos(\theta_\kappa) \left( e^{i(\Phi_1 + \Phi_2)} g^2_R v_{\Delta_R} v_{\delta_R} |\mu_3| \right) \\
+ e^{i \Phi_3} M_R \left( m_{\chi_j^\pm}^2 - |\mu_3|^2 \right) \right] - e^{i \Phi_\mu} \sin(\theta_\kappa) \left[ e^{i \Phi_\mu} \cos(\theta_\kappa) |\mu| \left( e^{i \Phi_3} M_R \right) \right. \\
\times \left( m_{\chi_j^\pm}^2 - |\mu_3|^2 \right) + e^{i(\Phi_1 + \Phi_2)} g^2_R v_{\Delta_R} v_{\delta_R} |\mu_3| \right) - e^{i(\Phi_3 + 2 \hat{\Phi}_R)} \\
\times \left( m_{\chi_j^\pm}^4 \sin(\theta_\kappa) - m_{\chi_j^\pm}^2 \left( 2 \cos(\theta_\kappa) |\tilde{M}_R|^2 + g^2_R (v_{\Delta_R})^2 + |\mu_3|^2 \right) \sin(\theta_\kappa) \right) \\
+ \cos(\theta_\kappa) |\tilde{M}_R|^2 |\mu_3|^2 \sin(2 \theta_\kappa) \right) \right] \right) (5.38)
\]

\[
p_{j4} = \frac{\sqrt{2} \tilde{M}_L^{*}}{D_j} \left\{ e^{2i \Phi_L} \sqrt{\frac{P_{j11}^{U^*}}{P_{j11}}} \xi_j^* \cos(\theta_\kappa) m_{\chi_j^\pm}^2 \left[ m_{\chi_j^\pm}^4 + g^2_R (v_{\Delta_R})^2 (v_{\delta_R})^2 + M_R^2 |\mu_3|^2 \right] \\
- 2 g^2_R M_R \cos(\Phi_1 + \Phi_2 - \Phi_3) |v_{\Delta_R}| |v_{\delta_R}| |\mu_3| + 2 |\tilde{M}_R|^2 \left( g^2_R (v_{\Delta_R})^2 + |\mu_3|^2 \right) \sin(\theta_\kappa) \\
- m_{\chi_j^\pm}^2 \left( M_R^2 + g^2_R ((v_{\Delta_R})^2 + (v_{\delta_R})^2) + |\mu_3|^2 \right) + 2 |\tilde{M}_R|^2 \sin(\theta_\kappa) \right] \\
\times \left[ \sin(2 \theta_\kappa) \left( e^{i \Phi_3} g^2_R |v_{\Delta_R}| |v_{\delta_R}| |\mu_3| + e^{i(\Phi_1 + \Phi_2)} M_R \left( m_{\chi_j^\pm}^2 - |\mu_3|^2 \right) \right) \right] \right\} (5.39)
\]

\[
p_{j5} = -2 g R \tilde{M}_L^{*} \tilde{M}_R^{*} e^{-i(\Phi_2 + \Phi_\mu)} \frac{e^{2i \Phi_L}}{D_j} \left\{ e^{2i \Phi_L} \sqrt{\frac{P_{j11}^{U^*}}{P_{j11}}} \xi_j^* \cos(\theta_\kappa) m_{\chi_j^\pm}^2 \left[ e^{2i \Phi_R} |\mu| \sin(\theta_\kappa) \right] \right. \\
\times \left( e^{i(\Phi_1 + \Phi_2)} M_R v_{\Delta_R} + e^{i \Phi_3} v_{\delta_R} |\mu_3| \right) - e^{i \Phi_\mu} \cos(\theta_\kappa) \left( e^{i \Phi_3} M_R v_{\delta_R} |\mu_3| \right) \\
+ e^{i(\Phi_1 + \Phi_2)} v_{\Delta_R} \left( m_{\chi_j^\pm}^2 - g^2_R (v_{\delta_R})^2 - 2 |\tilde{M}_R|^2 \sin(\theta_\kappa) |\mu_3| \right) \right] \\
+ e^{i \Phi_\mu} \cos(\theta_\kappa) |\mu| \left( e^{i(\Phi_1 + \Phi_2)} v_{\Delta_R} \left( m_{\chi_j^\pm}^2 - g^2_R (v_{\delta_R})^2 \right) + e^{i \Phi_3} M_R v_{\delta_R} |\mu_3| \right) \\
e^{2i \Phi_R} \sin(\theta_\kappa) \left( e^{i(\Phi_1 + \Phi_2)} M_R m_{\chi_j^\pm}^2 v_{\Delta_R} + e^{i \Phi_3} v_{\delta_R} |\mu_3| \right) \\
\times \left( m_{\chi_j^\pm}^2 - 2 \cos(\theta_\kappa) |\tilde{M}_R|^2 \right) \right] \right) \right\} (5.40)
\]

The reduced projectors \( \tilde{p}_{j2}, \tilde{p}_{j3} \) and \( \tilde{p}_{j5} \) are obtained by interchanging \( v_{\Delta_R} \leftrightarrow v_{\delta_R}, \Phi_1 \leftrightarrow \Phi_2 \) and \( \sin \theta_\kappa \leftrightarrow \cos \theta_\kappa \) in \( p_{j2}^{*}, p_{j4}^{*} \) and \( p_{j5}^{*} \), respectively. On the other hand, taking into account the analysis at the end of Section 1, we deduce that \( p_{33} = p_{33}^{\alpha} = 0, \alpha = 1, 2, 4, 5 \). Similarly, \( p_{34} = 0, \alpha = 1, 2, 4, 5, \) and \( \tilde{p}_{33}^{\beta}, \beta = 1, 2, 3, 5 \). Moreover, without any loss of generality we can choose \( p_{33} = \tilde{p}_{34} = 1 \).
5.3 Connection with the Jarlskog’s formulation

Combining Eq. (5.28) with Eq. (5.32) and Eq. (5.31) with (5.33), it is easy to verify that the $V$-type and $U^*$-type projectors satisfies the standard projector relations

$$P_i P_j = P_j P_i = P_j \delta_{ij}, \quad \text{Tr} P_j = 1, \quad \sum_{j=1}^{5} P_j = I_5, \quad P_{j\alpha\beta} P_{j\beta\alpha} = |P_{j\alpha\beta}|^2. \quad (5.41)$$

In general, in the Jarlskog’s formulation [7], the $P^V_j$ projectors writes ($j = 1, \ldots, 5$)

$$P^V_j = \prod_{k=1; k \neq j}^{5} \frac{m^2_{\chi^\pm_k} - H}{m^2_{\chi^\pm_k} - m^2_{\chi^\pm_j}}. \quad (5.42)$$

We can show that they also can be written in the form

$$P^V_j = \frac{\tilde{P}^V_j}{\Delta_j}, \quad (5.43)$$

where

$$\tilde{\Delta}_j = 4(m^2_{\chi^\pm_j})^5 - 3\tilde{\alpha}(m^2_{\chi^\pm_j})^4 + 2\tilde{b}(m^2_{\chi^\pm_j})^3 - \tilde{c}(m^2_{\chi^\pm_j})^2 + \tilde{e} \quad (5.44)$$

and

$$\tilde{P}^V_{j\alpha\beta} = (m^2_{\chi^\pm_j})^4 H_{\alpha\beta} + (m^2_{\chi^\pm_j})^3 (H_{\alpha\beta}^2 - \tilde{a} H_{\alpha\beta}) + (m^2_{\chi^\pm_j})^2 (H_{\alpha\beta}^3 - \tilde{a} H_{\alpha\beta}^2 + \tilde{b} H_{\alpha\beta}) + m^2_{\chi^\pm_j} (H_{\alpha\beta}^4 - \tilde{a} H_{\alpha\beta}^3 + \tilde{b} H_{\alpha\beta}^2 - \tilde{c} H_{\alpha\beta}) + \tilde{e} \delta_{\alpha\beta}. \quad (5.45)$$

Here, $\tilde{\alpha}, \ldots, \tilde{e}$ are the coefficients of the characteristic polynomial which determine the chargino masses

$$\text{Det} \left( m^2_{\chi^\pm_j} I_5 - H \right) = (m^2_{\chi^\pm_j})^5 - \tilde{a}(m^2_{\chi^\pm_j})^4 + \tilde{b}(m^2_{\chi^\pm_j})^3 - \tilde{c}(m^2_{\chi^\pm_j})^2 + \tilde{d}(m^2_{\chi^\pm_j}) - \tilde{e}, \quad (5.46)$$

i.e., comparing Eq. (5.7) with Eq. (5.46), the coefficients are given by

$$\tilde{a} = a + |\mu|^2, \quad \tilde{b} = b + a |\mu|^2, \quad \tilde{c} = c + b |\mu|^2, \quad \tilde{d} = d + c |\mu|^2, \quad \tilde{e} = d |\mu|^2. \quad (5.47)$$

Now, inserting (5.43) into (5.28) and using the definition (5.26) for the type-$V$ reduced projectors, we can show that ($\alpha = 1, \ldots, 5$)

$$\tilde{P}^V_{j\ell\alpha} = m^2_{\chi^\pm_j} \left( \Delta^{(\ell)}_{\alpha\jmath} \right)^*. \quad (5.48)$$

Similarly, for the $U^*$-type projectors we have ($\alpha = 1, \ldots, 5$)

$$\tilde{P}^{U^*_j\ell\alpha} = m^2_{\chi^\pm_j} \left( \Delta^{(\ell)}_{\alpha\jmath} \right)^*. \quad (5.49)$$

where the quantities $\tilde{P}^{U^*_j\ell\alpha}$ are given by (5.45) but with $\tilde{H}$ in the place of $H$.

Both, (5.48) and Eq. (5.49) satisfy identically when $\alpha = 1, \ldots, 5, \alpha \neq \ell$, whereas when $\alpha = \ell$, they are useful equivalences.
5.4 Explicit form of some relevant projectors

As an example, let us compute the projectors $\tilde{P}_{j\ell}^V$ and $\tilde{P}_{j\ell}^{U^*}$, when $\ell = 1$. From equation (5.55) with $\alpha = \beta = 1$, we can show that the $V$-type quantities $\tilde{P}_{j11}^V$ can be written in the form

$$\tilde{P}_{j11}^V = \left( m_{\chi_j}^2 - |\mu|^2 \right) \left[ \tilde{D}_j |M_L|^2 + \tilde{B}_j |M_L| + \tilde{C}_j^V \right],$$

(5.50)

where

$$\tilde{D}_j = m_{\chi_j}^2 - g_R^4 (v_{\Delta R})^2 (v_{\Delta R})^2 |\mu|^2 - m_{\chi_j}^2 \left( M_R^2 + 2 |M_R|^2 + g_R^2 \left( (v_{\Delta R})^2 + (v_{\Delta R})^2 \right) + |\mu|^2 + |\mu_3|^2 \right)$$

$$+ 2 g_R^2 v_{\Delta R} v_{\delta R} |\mu| |\mu_3| \left( M_R \cos(\Theta_1) |\mu| + \cos(\Theta_1 - \Theta_2) |M_R|^2 \sin(2\theta_\kappa) \right) - |\mu_3|^2 \left( M_R^2 |\mu|^2 + 2 M_R \cos(\Theta_2) |M_R|^2 |\mu| \sin(2\theta_\kappa) + |M_R|^4 \sin(2\theta_\kappa)^2 \right)$$

$$+ m_{\chi_j}^2 \left( g_R^4 M_R^2 |\mu|^2 + g_R^2 \left( (v_{\Delta R})^2 + (v_{\Delta R})^2 \right) |\mu|^2 - 2 g_R^2 M_R \cos(\Theta_1) v_{\Delta R} v_{\delta R} |\mu_3| \right) + \left( M_R^2 + |\mu|^2 \right) |\mu_3|^2 + |\tilde{M}_R|^2 |\mu_3|^2 + |\tilde{M}_R|^2 \sin(2\theta_\kappa)^2$$

$$+ 2 |\tilde{M}_R|^2 \left( |\mu_3|^2 + g_R^2 \left( \cos(\Theta_\kappa) \left( (v_{\Delta R})^2 + (v_{\Delta R})^2 \sin^2(\Theta_\kappa) \right) + M_R \cos(\Theta_2) |\mu| \sin(2\theta_\kappa) \right) \right) \right),$$

(5.51)

$$\tilde{B}_j = 2 |\tilde{M}_L|^2 \sin(2\theta_\kappa) \left[ \cos(\Theta_3 - \Theta_2) |\mu| \right. $$

$$\times \left( -m_{\chi_j}^4 - g_R^4 (v_{\Delta R})^2 (v_{\Delta R})^2 + 2 g_R^2 M_R \cos(\Theta_1) v_{\Delta R} v_{\delta R} |\mu_3| - M_R^2 |\mu_3|^2 $$

$$+ m_{\chi_j}^2 \left( M_R^2 + g_R^2 \left( (v_{\Delta R})^2 + (v_{\Delta R})^2 \right) + |\mu_3|^2 \right) \right) $$

$$+ |\tilde{M}_R|^2 \left( g_R \cos(\Theta_1 - \Theta_3) v_{\Delta R} v_{\delta R} |\mu_3| \right) $$

$$+ M_R \cos(\Theta_3) \left( m_{\chi_j}^2 - |\mu_3|^2 \right) \sin(2\theta_\kappa) \right],$$

(5.52)

$$\tilde{C}_j^V = |\tilde{M}_L|^2 \left[ -2 m_{\chi_j}^2 \left( -m_{\chi_j}^4 - g_R^4 (v_{\Delta R})^2 (v_{\Delta R})^2 + 2 g_R^2 M_R \cos(\Theta_1) v_{\Delta R} v_{\delta R} |\mu_3| \right) 

- M_R^2 |\mu_3|^2 - 2 \cos(\Theta_\kappa)^2 |\tilde{M}_R|^2 \left( g_R^2 (v_{\delta R})^2 + |\mu_3|^2 \right) 

+ m_{\chi_j}^2 \left( M_R^2 + 2 \cos(\Theta_\kappa)^2 \left( |\tilde{M}_L|^2 + |\tilde{M}_R|^2 \right) + g_R^2 \left( (v_{\Delta R})^2 + (v_{\Delta R})^2 \right) + |\mu_3|^2 \right) \right) \sin(\theta_\kappa)^2 

+ |\tilde{M}_L|^2 \left( - \left( g_R^4 (v_{\Delta R})^2 (v_{\Delta R})^2 \right) + 2 g_R^2 M_R \cos(\Theta_1) v_{\Delta R} v_{\delta R} |\mu_3| - M_R^2 |\mu_3|^2 \right) 

+ m_{\chi_j}^2 \left( M_R^2 + g_R^2 \left( (v_{\Delta R})^2 + (v_{\Delta R})^2 \right) + |\mu_3|^2 \right) \sin(2\theta_\kappa)^2 \right].$$

(5.53)

Similarly, the $U^*$-type quantities $\tilde{P}_{j11}^{U^*}$ can be written in the form

$$\tilde{P}_{j11}^{U^*} = \left( m_{\chi_j}^2 - |\mu|^2 \right) \left[ \tilde{D}_j |M_L|^2 + \tilde{B}_j |M_L| + \tilde{C}_j^{U^*} \right],$$

(5.54)

where the coefficients $\tilde{C}_j^{U^*}$ are obtained by interchanging $v_{\Delta R} \leftrightarrow v_{\delta R}$ and $\sin \theta_\kappa \leftrightarrow \cos \theta_\kappa$ in $\tilde{C}_j^V$. 

14
6 Parameter problem determination

In this section we implement a procedure to express the parameter $M_L$ and the phase angle $\Theta_3$ in terms of chargino masses, eigenphases, and complementary set of fundamental parameters. In fact, each of the fundamental parameters which are located on the diagonal of the chargino mass matrix $M$ can be easily disentangled from the rest the parameters using the generalized projector formalism. In the $M$ matrix diagonal we have the complex parameter $M_L$, the real parameter $M_R$ and the complex parameter $\mu_3$. To disentangle $M_L$ we have to use the generalized projector formulation with $\ell = 1$, whereas to disentangle $M_R$ and $\mu_3$ we have to take $\ell = 2$ and $\ell = 5$, respectively.

6.1 $M_L$ in terms of $m_{\tilde{\chi}_{j}^{\pm}}$, $\zeta_j$, and the complementary parameters

To express the fundamental parameter $M_L$ in terms of the physical chargino masses, the eigenphases, and the remaining parameters we can use either Eq. (5.36) or Eq. (5.37), with $\alpha = \ell = 1$. Thus, for instance, from Eq. (5.36) for $\alpha = 1$, considering that $\bar{p}_{j1} = 1, j = 1, 2, 4, 5$ and using (2.6), we get

$$m_{\tilde{\chi}_{j}^{\pm}} \zeta_j \sqrt{\frac{\tilde{P}^{11}}{P^{11}}} = M_L + \sqrt{2} \tilde{M}_L \cos \theta_k p^*_j 4.$$  \hfill (6.55)

Now we have two ways to continue. We can either substitute Eq. (5.39) into (6.55) or express $p^*_j 4$ in terms of $\Delta_{1j}$ and $\Delta_{4j}$, according to Eq. (5.26). Then using the equivalences (5.48) and (5.49), we get

$$M_L = \tilde{A}_j \zeta_j + \tilde{B}_j, \quad j = 1, 2, 4, 5$$  \hfill (6.56)

where

$$\tilde{A}_j = \frac{\sqrt{\Delta_{1j} \Delta_{4j}}}{(m_{\tilde{\chi}_{j}^{\pm}}^2 - |\mu|^2) D_j} m_{\tilde{\chi}_{j}^{\pm}},$$  \hfill (6.57)

and

$$\tilde{B}_j = \frac{\tilde{M}_L^2 \sin(2\theta_k)}{D_j} \left[ \mu \left( 2 g_R^2 M_R \cos(\Phi_1 + \Phi_2 - \Phi_3) |v_{\Delta_2}| |v_{\delta R}| |\mu_3| - m_{\tilde{\chi}_{j}^{\pm}}^4 - g_R^4 (v_{\Delta R})^2 (v_{\delta R})^2 \right) - M_R^2 |\mu_3|^2 + m_{\tilde{\chi}_{j}^{\pm}}^2 \left( M_R^2 + g_R^2 \left( (v_{\Delta R})^2 + (v_{\delta R})^2 + |\mu_3|^2 \right) \right) + e^{-i(\Phi_1 + \Phi_2)} M_R^2 \right]$$

$$\times \sin(2 \theta_k) \left( e^{i \Phi_3} g_R^2 |v_{\Delta R}| |v_{\delta R}| |\mu_3| + e^{i(\Phi_1 + \Phi_2)} M_R \left( m_{\tilde{\chi}_{j}^{\pm}}^2 - |\mu_3|^2 \right) \right),$$  \hfill (6.58)

where $\tilde{D}_j$ is given by Eq. (5.51).

Equation (6.56) allows us to determinate the behaviour of $|M_L|$ and $\Phi_L$ in terms of the eigenphases $\zeta_j$ and the physical masses $m_{\tilde{\chi}_{j}^{\pm}}$, when the rest of fundamental parameters are known. This is a general equation which take into account the contribution of the right handed Higgsino fields.

6.2 Disentangling $|M_L|$

It is useful to express the norm of $M_L$ in terms of the physical masses and remaining parameters, without any explicit dependence on the eigenphases. We combine Eq. (5.48), when $\alpha = \ell = 1$, with Eqs. (4.21) and (5.50) to get the quadratic equation

$$\tilde{D}_j |M_L|^2 + \tilde{B}_j |M_L| + \tilde{C}_j = 0, \quad j = 1, 2, 4, 5,$$  \hfill (6.59)
where \( \tilde{C}_j = \tilde{C}_j^V - \Delta_{1j}/(m_{\tilde{\chi}^\pm_j}^2 - |\mu|^2) \). Solving for \( |M_L| \) we get

\[
|M_L| = \frac{-\tilde{B}_j \pm \sqrt{\tilde{B}_j^2 - 4\tilde{C}_j \tilde{D}_j}}{2 \tilde{D}_j}.
\] (6.60)

Equation (6.60) expresses the norm of \( M_L \) in terms of the physical chargino masses, the CP-violating phases and the remaining parameters. Since \( |M_L| \geq 0 \), the following constraints must be satisfied:

\[
\tilde{B}_j^2 - 4\tilde{C}_j \tilde{D}_j \geq 0, \quad \frac{\tilde{B}_j}{\tilde{D}_j} < 0.
\] (6.61)

These results include the contribution of the right handed Higgino parameters.

6.3 \( |M_L| \) in terms of the two lightest chargino masses

Writing \( \tilde{B}_j \) in the form

\[
\tilde{B}_j = \frac{\tilde{P}_j + \tilde{Q}_j \tan(\Theta_3)}{\sqrt{1 + \tan^2(\Theta_3)}}, \quad j = 1, 2,
\] (6.62)

with

\[
\tilde{P}_j = 2 |\tilde{M}_L|^2 \sin(2\theta_\kappa) \left[ |\mu| \cos(\Theta_2) \left( m_{\tilde{\chi}^\pm_j}^2 \left( M_R^2 + g_R^2 (v_{\Delta_R})^2 + (v_{\delta_R})^2 + |\mu|^2 \right) - m_{\tilde{\chi}^\pm_j}^4 - g_R^4 (v_{\Delta_R})^2 (v_{\delta_R})^2 + 2 g_R^2 M_R \cos(\Theta_1) v_{\Delta_R} v_{\delta_R} |\mu_3| - M_R^2 |\mu_3|^2 \right) \right.
\]

\[
+ \left. |\tilde{M}_R|^2 \sin(2\theta_\kappa) \left( g_R^2 \cos(\Theta_1) v_{\Delta_R} v_{\delta_R} |\mu_3| + M_R (m_{\tilde{\chi}^\pm_j}^2 - |\mu|^2) \right) \right]
\] (6.63)

and

\[
\tilde{Q}_j = 2 |\tilde{M}_L|^2 \sin(2\theta_\kappa) \left[ |\mu| \sin(\Theta_2) \left( m_{\tilde{\chi}^\pm_j}^2 \left( M_R^2 + g_R^2 (v_{\Delta_R})^2 + (v_{\delta_R})^2 + |\mu|^2 \right) - m_{\tilde{\chi}^\pm_j}^4 - g_R^4 (v_{\Delta_R})^2 (v_{\delta_R})^2 + 2 g_R^2 M_R \cos(\Theta_1) v_{\Delta_R} v_{\delta_R} |\mu_3| - M_R^2 |\mu_3|^2 \right) \right.
\]

\[
+ \left. |\tilde{M}_R|^2 \sin(2\theta_\kappa) g_R^2 \sin(\Theta_1) v_{\Delta_R} v_{\delta_R} |\mu_3| \right],
\] (6.64)

and inserting it into Eq. (6.60), after some algebraic manipulations we get

\[
\tan(\Theta_3) = \tilde{\Theta} \equiv \frac{-\tilde{B} + \tilde{\epsilon} \sqrt{\tilde{B}^2 - 4\tilde{A} \tilde{C}}}{2\tilde{A}},
\] (6.65)

where \( \tilde{B}^2 - 4\tilde{A} \tilde{C} \geq 0 \),

\[
\tilde{A} = \frac{1}{2} F(\tilde{\Omega}_1, \tilde{\Omega}_2, \tilde{\Phi}_1, \tilde{\Phi}_2) - (\tilde{D}_1 \tilde{C}_2 - \tilde{D}_2 \tilde{C}_1)^2,
\] (6.66)

\[
\tilde{B} = F(\tilde{\Omega}_1, \tilde{\Phi}_2, \tilde{\Phi}_1, \tilde{\Phi}_2)
\] (6.67)

and

\[
\tilde{C} = \frac{1}{2} F(\tilde{\Omega}_1, \tilde{\Phi}_2, \tilde{\Phi}_1, \tilde{\Phi}_2) - (\tilde{D}_1 \tilde{C}_2 - \tilde{D}_2 \tilde{C}_1)^2,
\] (6.68)
with

\[
F(\tilde{P}_1, \tilde{P}_2, \tilde{Q}_1, \tilde{Q}_2) = (\tilde{D}_1 \tilde{C}_2 + \tilde{D}_2 \tilde{C}_1)(\tilde{P}_1 \tilde{Q}_2 + \tilde{P}_2 \tilde{Q}_1) - 2(\tilde{D}_1 \tilde{C}_2 \tilde{P}_2 + \tilde{D}_2 \tilde{C}_1 \tilde{P}_1),
\]

(6.69)

where \( \tilde{\epsilon} = \pm 1 \).

Moreover, combining Eq. (6.60) for \( j = 1 \) with Eq. (6.60) for \( j = 2 \), and then using Eq. (6.62) for \( j = 1 \),

\[
|M_L| = \frac{(\tilde{D}_1 \tilde{C}_2 - \tilde{D}_2 \tilde{C}_1) \sqrt{1 + \tilde{R}^2}}{(\tilde{D}_2 \tilde{P}_1 - \tilde{D}_1 \tilde{P}_2) + (\tilde{D}_2 \tilde{Q}_1 - \tilde{D}_1 \tilde{Q}_2)\tilde{R}}.
\]

(6.70)

Equations (6.65) and (6.70) allow us to determine the phase \( \Theta_3 \) and the norm \( |M_L| \), respectively, up to a
twofold discrete ambiguity, in terms of the two lightest chargino masses and the remaining fundamental parameters.

7 Complete parameter inversion

The fundamental parameter reconstruction from measurements of some suitable physical observables is a
non trivial problem in many SUSY theories beyond the Standard Model. In the context of the MSSM,
some important techniques have been introduced in the literature to obtain the chargino and neutralino
parameters. For instance, measurements of some cross section type observables, involving the neutralino
and chargino pair production in electron-positron annihilation \[25, 26, 27, 28, 29, 30, 31, 32\]. Using this
principle and the projector technique, it is possible to implement a systematic method to reconstruct the
fundamental neutralino and chargino parameters from the experiments, in the context of either the MSSM
or the LRSUSY model \[8, 9, 10, 11\].

Let us consider some class of cross section-type observables associated with the chargino pair production
\( \tilde{\chi}_i^\pm \tilde{\chi}_j^\pm \), \( i, j = 1, 2, \ldots, 5 \), from the \( e^+e^- \) annihilation, at the future Linear Collider. For instance, total
cross section with polarized or unpolarized beams, angular chargino distribution, forward and backward
asymmetries. These type of observables depend, in addition to the chargino masses and leptons masses,
on the entries of the \( V \) and \( U^* \), \[24, 33\]. Thus, in principle, if we are able to measure a set of appropriate
cross section-type observables and to determine the entries of \( V \) and \( U \), then we could invert Eqs. (4.18) and
(4.20) to find the fundamental LRSUSY parameters. However, at this stage, the inversion process is very
difficult because the high complexity of these relations. It is necessary to find out a new parametrization
of the cross section-type observables. A more suitable procedure is parameterizing these observables in
terms of the fundamental reduced projectors and eigenphases. It can be done easily by using Eqs. (5.27)
and (5.30). Then, if we are able to determine the value of the reduced projectors and eigenphases from the
experiments, with the help of the Eqs. (5.36) and (5.37), we can find appropriate expressions to reconstruct
the fundamental parameters.

7.1 Fundamental parameter inversion equations

Using the basic Eqs. (5.36) and (5.37), we can express the fundamental parameters in terms of the reduced
projectors, the physical masses and the eigenphases. We get (\( j=1,2,4,5 \))

\[
M_L = m_{\tilde{\chi}_j^\pm} \zeta_j \frac{\sqrt{p_{j1}^{*\prime} p_{j3} \tan \theta_k} - \sqrt{p_{j1}^{\prime\prime} p_{j4}^*}}{p_{j3} \tan \theta_k - p_{j4}^*},
\]

(7.71)
\[
\tilde{M}_L = -\frac{m_{\tilde{\chi}_j^\pm}}{\sqrt{2}} \frac{\sqrt{p_{j1}^{11} - p_{j1}^{11}}}{p_{j3} \sin \theta_k - p_{j4}^* \cos \theta_k}, (7.72)
\]

\[
\tilde{M}_R = -\frac{m_{\tilde{\chi}_j^\pm}}{\sqrt{2}} \frac{\sqrt{p_{j1}^{11} - p_{j1}^{11}} (|\tilde{p}_{j2}|^2 + |\tilde{p}_{j5}|^2) - \sqrt{p_{j1}^{11} - p_{j1}^{11}} (|p_{j2}|^2 + |p_{j5}|^2)}{p_{j3}^* \tilde{p}_{j2} \cos \theta_k - p_{j2}^* \tilde{p}_{j3} \sin \theta_k}, (7.73)
\]

\[
\mu = \frac{m_{\tilde{\chi}_j^\pm}}{\sqrt{2}} \frac{\sqrt{p_{j1}^{11} - p_{j1}^{11}} (|\tilde{p}_{j2}|^2 + |\tilde{p}_{j5}|^2) - \sqrt{p_{j1}^{11} - p_{j1}^{11}} (|p_{j2}|^2 + |p_{j5}|^2)}{(p_{j4}^* - \tilde{p}_{j3} p_{j2}^* \tan \theta_k - \tilde{p}_{j2} p_{j4}^*)}
\times \left\{ \frac{P_{j1}^{11}}{P_{j1}^{11}} \left[ \tilde{p}_{j2}^* p_{j3} p_{j4}^* - \left( (p_{j2}^* - \tilde{p}_{j2})(|\tilde{p}_{j5}|^2) + p_{j2}^* |\tilde{p}_{j3}|^2 \right) \tan \theta_k \right] + \frac{P_{j1}^{11}}{P_{j1}^{11}} \tan \theta_k \left[ p_{j2}^* \tilde{p}_{j3} p_{j4} \tan \theta_k - \left( (\tilde{p}_{j2} - p_{j2}^*)(|p_{j5}|^2) + \tilde{p}_{j2} |p_{j4}|^2 \right) \right] \right\}, (7.74)
\]

\[
\sqrt{2} M_1 \sin \theta_v = g_R v_{\Delta_R} e^{i\Phi_1} = \frac{m_{\tilde{\chi}_j^\pm} \zeta_j}{\tilde{p}_{j2}} \sqrt{\frac{P_{j1}^{11}}{P_{j1}^{11}} p_{j5}^* - \mu_3 \tilde{p}_{j5}}, (7.75)
\]

\[
\sqrt{2} M_2 \cos \theta_v = g_R v_{\delta_R} e^{i\Phi_2} = \frac{m_{\tilde{\chi}_j^\pm} \zeta_j}{p_{j2}^*} \sqrt{\frac{P_{j1}^{11}}{P_{j1}^{11}} \tilde{p}_{j5}^* - \mu_3 p_{j5}^*}, (7.76)
\]

and

\[
M_R = \mu_3 \frac{p_{j5}^* \tilde{p}_{j5}}{p_{j2}^* \tilde{p}_{j2}} + \frac{m_{\tilde{\chi}_j^\pm}}{p_{j2}^* \tilde{p}_{j3} \tan \theta_k - \tilde{p}_{j2} p_{j4}^*} \left[ \frac{P_{j1}^{11}}{P_{j1}^{11}} \tilde{p}_{j2} \tilde{p}_{j3} \tan \theta_k - \frac{P_{j1}^{11}}{P_{j1}^{11}} \tilde{p}_{j2} \tilde{p}_{j4}^* \right]
\times \left\{ \frac{P_{j1}^{11}}{P_{j1}^{11}} \tilde{p}_{j2} \tilde{p}_{j3} |\tilde{p}_{j5}|^2 \tan \theta_k \right\}, (7.77)
\]

When \( j = 3 \), we only have

\[
\zeta_3 = e^{i\theta_3} = -\frac{\mu}{|\mu|}, (7.78)
\]

Note that in Eqs. (7.75) and (7.76) we have considered the possibility that the fundamental parameters \( v_{\Delta_R} \) and \( v_{\delta_R} \) vary independently. At this stage, the set of fundamental parameters is expressed in addition to the chargino masses, reduced projectors and eigenphases in terms of \( \tan \theta_k \) and \( \mu_3 \). However, in the CP-violating case, only the phases of \( M_L \) and \( M_R \) have been considered as unknown, so from Eqs. (7.72) and (7.73) we get two additional equations allowing us to express \( \tan \theta_k \) in terms of the reduced projectors and eigenphases. Since \( M_R \) has been chosen real, then inserting the above mentioned values of \( \tan \theta_k \) in Eq. (7.77) we get two additional equations which can be used to determine \( \mu_3 \) in terms of the reduced projectors and eigenphases. Thus, we are able to determine all the fundamental parameters in terms of the chargino masses, the reduced projectors and the eigenphases.

Note that in the CP-conserving case \( \zeta_j = \pm 1, \forall j = 1, \ldots, 5, \Phi_\mu = 0, \pi \) and all the remaining phases are equal to zero. For the CP-conserving case, there are not additional constraints to determine \( \mu_3 \) and \( \tan \theta_k \) in terms of the eigenphases and reduced projectors, so in order to express the fundamental parameters in
terms of them, we must know \( \mu_3 \) and \( \tan \theta_k \) from other ways. In the CP-conserving case, the role of the eigenphases is to remedy the sign ambiguity of the physical chargino masses represented by the eigenvalues of the Hermitian matrix \( H \) or \( \tilde{H} \), which can be either positive or negative \cite{8, 9}. In addition to that, in the CP-violating case, the eigenphases contain information about the complex phases introduced in the chargino mass matrix.

Taking into account Eq. (2.11) and Eqs. (7.75) and (7.76), we can express the phases \( \Phi_1, \Phi_2 \) and the complex parameter \( \mu_3 \) in terms of the chargino masses, the reduced projectors, the eigenphases, and the fundamental parameter \( \tan \theta_v \). Indeed, let us write

\[
p_{ij} = |p_{ij}|e^{i\beta_{ij}}
\]

(7.79)

and

\[
\tilde{p}_{ij} = |\tilde{p}_{ij}|e^{i\tilde{\beta}_{ij}},
\]

(7.80)

respectively, where \( \beta_{ij} \) and \( \tilde{\beta}_{ij} \) are real phases. Eliminating \( \mu_3 \) from Eqs. (7.75) and (7.76) we get

\[
\Omega_j \cos \omega_{1j} - \Gamma_j \cos \gamma_{2j} - \Lambda_j \cos \vartheta_j = 0,
\]

(7.81)

\[
\Omega_j \sin \omega_{1j} - \Gamma_j \sin \gamma_{2j} - \Lambda_j \sin \vartheta_j = 0,
\]

(7.82)

where

\[
\Omega_j = \sqrt{\frac{2}{|p_{j5}|}}|\tilde{p}_{j5}|M_W \sin \theta_v
\]

(7.83)

\[
\Gamma_j = \sqrt{\frac{2}{|p_{j2}|}}|\tilde{p}_{j2}|M_W \cos \theta_v,
\]

(7.84)

\[
\Lambda_j = m_{\tilde{\chi}^{\pm}_j} \left[ \sqrt{\frac{P_{j11}}{P_{j11}^*}}|p_{j5}|^2 - \sqrt{\frac{P_{j11}^*}{P_{j11}^*}}|\tilde{p}_{j5}|^2 \right],
\]

(7.85)

\[
\omega_{1j} = \Phi_1 + \tilde{\beta}_{j2} - \beta_{j5},
\]

(7.86)

\[
\gamma_{2j} = \Phi_2 + \beta_{j5} - \tilde{\beta}_{j2}.
\]

(7.87)

Solving the system (7.81-7.82), we get

\[
\omega_{1j} = \pm \arccos[\tau_j + \pi_j], \quad \gamma_{2j} = \pm \arccos[\nu_j + \sigma_j]
\]

(7.88)

or

\[
\omega_{1j} = \pm \arccos[\tau_j - \pi_j], \quad \gamma_{2j} = \pm \arccos[\nu_j - \sigma_j],
\]

(7.89)

where

\[
\tau_j = \left[ \frac{\Omega_j^2 - \Gamma_j^2 - \Lambda_j^2}{2\Gamma_j \Lambda_j} \right] \cos \vartheta_j,
\]

(7.90)

\[
\nu_j = \left[ \frac{\Omega_j^2 - \Gamma_j^2 + \Lambda_j^2}{2\Omega_j \Lambda_j} \right] \cos \vartheta_j,
\]

(7.91)

\[
\pi_j = \left[ \frac{\sqrt{\Gamma_j^2 \Lambda_j^2 (\Omega_j + \Gamma_j + \Lambda_j) (\Omega_j + \Gamma_j - \Lambda_j) (\Omega_j - \Gamma_j - \Lambda_j) \cos^2(\vartheta_j)}}{2 \Gamma_j^2 \Lambda_j} \right] \tan \vartheta_j
\]

(7.92)
\[
\sigma_j = \left[ \frac{\Gamma_j^2 \Lambda_j^2 (\Omega_j + \Gamma_j + \Lambda_j) (\Omega_j + \Gamma_j - \Lambda_j) (\Omega_j - \Gamma_j - \Lambda_j) \cos^2(\theta_j)}{2 \Omega_j \Gamma_j \Lambda_j^2} \right] \tan \theta_j. \tag{7.93}
\]

By comparison of Eqs. (7.86) for \(\omega_{1j}\) and (7.87) for \(\gamma_{2j}\), with Eqs. (7.88) and (7.89), respectively, we can get the phases \(\Phi_1\) and \(\Phi_2\) in terms of the chargino masses, the projector-type parameters, and \(\tan \theta_v\), up to a eighth fold ambiguity. Finally, inserting these results into Eqs. (7.75) and (7.76), and regrouping the terms suitably, we get

\[
\mu_3 = \frac{m_{x_j} \xi_j}{\left| p_{j5} \right|^2 + \left| \tilde{p}_{j5} \right|^2} \left[ \sqrt{\frac{\rho_{j1}^V}{p_{j11}^V}} + \sqrt{\frac{\rho_{j5}^V}{p_{j11}^V}} - \frac{\Omega_j e^{i\theta_{1j}}}{\left| p_{j5} \right|^2} + \frac{\Gamma_j e^{i\gamma_{2j}}}{\left| \tilde{p}_{j5} \right|^2} \right]. \tag{7.94}
\]

In this way, the fundamental parameter \(\mu_3\) can also be expressed in terms of the chargino masses, the reduced projectors, the eigenphases, and \(\tan \theta_v\).

In sum, we have shown that all the fundamental parameters can be expressed in terms of the chargino masses, the projector-type parameters, \(\tan \theta_v\), and \(\tan \theta_u\). Moreover, the considerations that follow Eq. (7.78), about the additional constraints in the case of the CP-violating are also valid in this case, i.e., a complete disentanglement of the fundamental parameters in terms of the chargino masses and projector-type parameters can be reached.

### 7.2 Additional set of constrains, independent reduced projector-type parameters

The set of not trivial \(V\)-type reduced projectors is given by \(\{p_{j2}, p_{j4}, p_{j5}\}\) and the set of \(U^*\)-type reduced projectors is given by \(\{\tilde{p}_{j2}, \tilde{p}_{j3}, \tilde{p}_{j5}\}\), \(j = 1, 2, 4, 5\). In the CP-violation case, the number of \(V\)-type reduced projector real parameters is 24 and the number of \(U^*\)-type reduced projector real parameters is also 24, but, as we have seen in Section 5.1, the reduced projectors are not all independent, they relate each other by Eqs. (5.32) and (5.33).

For instance, inserting (7.79) into (5.32), and splitting the real and imaginary part, we get the set of 12 constraints

\[
1 + a_{ij}^{(2)} \cos(\beta_{i2} - \beta_{j2}) + a_{ij}^{(4)} \cos(\beta_{i4} - \beta_{j4}) + a_{ij}^{(5)} \cos(\beta_{i5} - \beta_{j5}) = 0 \tag{7.95}
\]

and

\[
a_{ij}^{(2)} \sin(\beta_{i2} - \beta_{j2}) + a_{ij}^{(4)} \sin(\beta_{i4} - \beta_{j4}) + a_{ij}^{(5)} \sin(\beta_{i5} - \beta_{j5}) = 0, \tag{7.96}
\]

where \(a_{ij}^{(k)} = |p_{ik}| |p_{jk}|, k = 2, 4, 5, i, j = 1, \ldots, 5, (i > j \text{ or } j > i, j \neq 3)\). This means that the number of real independent parameters describing the \(V\)-type reduced projectors is equal to 12.

From Eq. (5.32) we also have the following identities

\[
P_{j11}^V (1 + |p_{j2}|^2 + |p_{j4}|^2 + |p_{j5}|^2) = 1, \quad j = 1, 2, 4, 5. \tag{7.97}
\]

Then, we can parameterize the norm of the \(V\)-type reduced projectors in terms of hyper-spherical angles. Indeed, we can choose

\[
|p_{j2}| = \tan \psi^{(j)} \sin \phi^{(j)} \cos \theta^{(j)}, \tag{7.98}
\]

\[
|p_{j4}| = \tan \psi^{(j)} \sin \phi^{(j)} \sin \theta^{(j)}, \tag{7.99}
\]

\[
|p_{j5}| = \tan \psi^{(j)} \cos \phi^{(j)}, \tag{7.100}
\]

with

\[
\sqrt{P_{j11}^V} = \cos \psi^{(j)}. \tag{7.101}
\]
In the same way, inserting (7.80) into (5.33), and splitting the real and imaginary part, we get the set of 12 constraints

\[ 1 + \tilde{a}_{ij}^{(2)} \cos(\tilde{\beta}_i - \tilde{\beta}_j) + \tilde{a}_{ij}^{(3)} \cos(\tilde{\beta}_i - \tilde{\beta}_j) + \tilde{a}_{ij}^{(5)} \cos(\tilde{\beta}_i - \tilde{\beta}_j) = 0, \tag{7.102} \]

and

\[ \tilde{a}_{ij}^{(2)} \sin(\tilde{\beta}_i - \tilde{\beta}_j) + \tilde{a}_{ij}^{(3)} \sin(\tilde{\beta}_i - \tilde{\beta}_j) + \tilde{a}_{ij}^{(5)} \sin(\tilde{\beta}_i - \tilde{\beta}_j) = 0, \tag{7.103} \]

where \( \tilde{a}_{ij}^{(k)} = |\tilde{p}_{ik}| |\tilde{p}_{jk}|, k = 2, 3, 5, i, j = 1, \ldots, 5, (i > j \perp i, j \neq 3) \). This means that the number of real independent parameters describing the \( U^* \)-type reduced projectors is also equal to 12.

Now, from Eq. (5.33) we have the following identities

\[ P_{11}^{U^*}(1 + |\tilde{p}_{j2}|^2 + |\tilde{p}_{j3}|^2 + |\tilde{p}_{j5}|^2) = 1, \quad j = 1, 2, 4, 5. \tag{7.104} \]

Then, we can parameterize the norm of the \( U^* \)-type reduced projectors in terms of hyper-spherical angles. Indeed, we can choose

\[ |\tilde{p}_{j2}| = \tan \tilde{\phi}^{(j)} \sin \tilde{\theta}^{(j)} \cos \tilde{\theta}^{(j)}, \tag{7.105} \]

\[ |\tilde{p}_{j3}| = \tan \tilde{\phi}^{(j)} \sin \tilde{\theta}^{(j)} \sin \tilde{\theta}^{(j)}, \tag{7.106} \]

\[ |\tilde{p}_{j5}| = \tan \tilde{\phi}^{(j)} \cos \tilde{\theta}^{(j)}, \tag{7.107} \]

with

\[ \sqrt{P_{11}^{U^*}} = \cos \tilde{\psi}^{(j)}. \tag{7.108} \]

At this stage, there are several ways of choosing the set of independent reduced projector-type parameters. The better choice depends on the type of problem we are analyzing and on the experimental data. Also, it is important to consider the most adequate set of independent parameters providing some regularities or symmetries, allowing to solve (7.96, 7.97, 7.103, 7.102) in an easier way.

### 7.2.1 Standard set of hyper-spherical independent parameters

Assuming that we are able to measure the mass of the lightest charginos and some of its cross-section related quantities, it can be better to use the most lower index reduced projectors as independent variables. A good choice could be the sets \{ \|p_{j2}|, |p_{j4}|, |p_{j5}|, \beta_{12}, \beta_{14}, \beta_{15} \}, j = 1, 2, 4 \] and \{ \|p_{j2}|, |p_{j3}|, |p_{j5}|, \beta_{12}, \beta_{13}, \beta_{15} \}, j = 1, 2, 3.

Solving the system (7.96, 7.97) for the unknown \( \beta_{ij} \) variables, we get

\[ X_{ij}^{(2)} = \frac{1}{2a_{ij}^{(2)} Z_{ij}^{(5)}} \left\{ - \left( 1 + X_{ij}^{(5)} a_{ij}^{(5)} \right) \left[ Z_{ij}^{(5)} + (a_{ij}^{(2)} - a_{ij}^{(4)}) (a_{ij}^{(2)} + a_{ij}^{(4)}) \right] \right. \]

\[ \pm \left. a_{ij}^{(5)} \sqrt{\left( X_{ij}^{(5)} \right)^2 - 1} \left[ Z_{ij}^{(5)} - (a_{ij}^{(2)} - a_{ij}^{(4)})^2 \right] \left[ Z_{ij}^{(5)} - (a_{ij}^{(2)} + a_{ij}^{(4)})^2 \right] \right\}, \tag{7.109} \]

\[ X_{ij}^{(4)} = \frac{1}{2a_{ij}^{(4)} Z_{ij}^{(5)}} \left\{ - \left( 1 + X_{ij}^{(5)} a_{ij}^{(5)} \right) \left[ Z_{ij}^{(5)} + (a_{ij}^{(4)} - a_{ij}^{(2)}) (a_{ij}^{(4)} + a_{ij}^{(2)}) \right] \right. \]

\[ \pm \left. a_{ij}^{(5)} \sqrt{\left( X_{ij}^{(5)} \right)^2 - 1} \left[ Z_{ij}^{(5)} - (a_{ij}^{(4)} - a_{ij}^{(2)})^2 \right] \left[ Z_{ij}^{(5)} - (a_{ij}^{(4)} + a_{ij}^{(2)})^2 \right] \right\}, \tag{7.110} \]

where

\[ X_{ij}^{(k)} = \cos(\beta_{ik} - \beta_{jk}), \tag{7.111} \]
and

\[ Z_{ij}^{(5)} = \left( 1 + 2 X_{ij}^{(5)} a_{ij}^{(5)} + (a_{ij}^{(5)})^2 \right). \quad (7.112) \]

Thus from Eqs. (7.109) and (7.110) we get

\[ \beta_{ik} = \beta_{1k} + \arccos(X_{i1}^{(k)}), \quad i = 2, 4, 5, \quad (7.113) \]

and the constrains

\begin{align*}
\arccos(X_{21}^{(k)}) + \arccos(X_{42}^{(k)}) - \arccos(X_{41}^{(k)}) & = 0, \quad (7.114) \\
\arccos(X_{21}^{(k)}) + \arccos(X_{52}^{(k)}) - \arccos(X_{51}^{(k)}) & = 0, \quad (7.115) \\
\arccos(X_{42}^{(k)}) + \arccos(X_{54}^{(k)}) - \arccos(X_{52}^{(k)}) & = 0, \quad (7.116)
\end{align*}

when \( k = 2, 4. \)

Equation (7.113) allows us to express the phases \( \beta_{ik}, i = 2, 4, 5; k = 2, 4 \) in terms of the phases \( \beta_{12}, \beta_{14} \) and \( \beta_{5}, i = 1, 2, 4, 5 \), and the norm of the reduced projectors. On the other hand, Eq. (7.114) can be used to determine the phases \( \beta_{25} \) and \( \beta_{45} \) in terms of \( \beta_{5} \) and the set of independent reduced projector norms \( \{|p_{2}|, |p_{4}|, |p_{5}|\}, i = 1, 2, 4 \). Finally, inserting the above results into Eqs. (7.115) and (7.116), we get a system of equations to determine the phase \( \beta_{55} \) and the norms \( \{|p_{52}|, |p_{54}|, |p_{55}|\} \). This can be obtained in terms of the independent reduced projector phases and norms. The same proceeding could be applied to the treatment of the \( U^* \)-type independent reduced projector parameters.

Thus, according to the \( V \)-type Eqs. (7.98-7.100), with \( j = 1, 2, 4, \) and the analogous ones for the \( U^* \)-type, we are able to parameterize the complete problem with six sets of hyper-spherical angles, three \( V \)-type reduced projector phases and three \( U^* \)-type reduced projector phases.

### 7.2.2 Right-handed parameters in terms of the lightest chargino parameters

Other useful choice is to consider a set of parameters \( |p_{j5}|, \beta_{j5} \) when \( j = 1, 2, 4, \) and \( |p_{5j}|, \beta_{5j} \), when \( j = 2, 4, 5 \), expressed in terms of the reduced projector associated to the lightest charginos, i.e., in terms of \( |p_{j2}|, |p_{j4}|, \beta_{2}, \beta_{4} \), when \( j = 1, 2, 4 \). Actually, it is not possible, but we can obtain a similar result except by an independent right-handed reduced projector phase and a dependent lightest chargino reduced projector-type parameter.

Combining Eqs. (7.95) and (7.96), we get \( (i, j = 1, 2, 4, 5; i > j) \)

\[ \beta_{i5} - \beta_{j5} = \gamma_{ij} = \arctan(n_{ij}/d_{ij}) \quad (7.117) \]

and

\[ a_{ij}^{(5)} = \sqrt{(n_{ij})^2 + (d_{ij})^2}, \quad (7.118) \]

where

\[ n_{ij} = a_{ij}^{(2)} \sin(\beta_{i2} - \beta_{j2}) + a_{ij}^{(4)} \sin(\beta_{i4} - \beta_{j4}) \quad (7.119) \]

and

\[ d_{ij} = 1 + a_{ij}^{(2)} \cos(\beta_{i2} - \beta_{j2}) + a_{ij}^{(4)} \cos(\beta_{i4} - \beta_{j4}). \quad (7.120) \]

Inserting Eqs. (7.119) and (7.120) into Eq. (7.118), we obtain

\[ a_{ij}^{(5)} = \left\{ 1 + \left( a_{ij}^{(2)} \right)^2 + \left( a_{ij}^{(4)} \right)^2 + 2 a_{ij}^{(2)} a_{ij}^{(4)} \cos[(\beta_{i2} - \beta_{j2}) - (\beta_{i4} - \beta_{j4})] \
+ 2 a_{ij}^{(2)} \cos(\beta_{i2} - \beta_{j2}) + 2 a_{ij}^{(4)} \cos(\beta_{i4} - \beta_{j4}) \right\}^{1/2} \quad (7.121) \]
From Eq. (7.117) we get
\[ \beta_{i5} = \beta_{15} + \Upsilon_{i1}, \quad i = 2, 4, 5, \tag{7.122} \]
and the constraints
\[ \Upsilon_{21} - \Upsilon_{41} + \Upsilon_{42} = 0, \tag{7.123} \]
\[ \Upsilon_{42} - \Upsilon_{52} + \Upsilon_{54} = 0, \tag{7.124} \]
\[ \Upsilon_{41} - \Upsilon_{51} + \Upsilon_{52} = 0. \tag{7.125} \]

On the other hand, combining Eqs. (7.118) for the different values of \( i, j \), we get
\[
|p_{15}| = \sqrt{\frac{a_{51}^{(5)} a_{41}^{(5)}}{a_{54}^{(5)}}}, \quad |p_{25}| = \sqrt{\frac{a_{52}^{(5)} a_{42}^{(5)}}{a_{54}^{(5)}}}, \tag{7.126}
\]
\[
|p_{45}| = \sqrt{\frac{a_{54}^{(5)} a_{41}^{(5)}}{a_{51}^{(5)}}}, \quad |p_{55}| = \sqrt{\frac{a_{52}^{(5)} a_{42}^{(5)}}{a_{21}^{(5)}}}. \tag{7.127}
\]
and the constraints
\[ a_{51}^{(5)} a_{42}^{(5)} = a_{54}^{(5)} a_{21}^{(5)}, \quad a_{51}^{(5)} a_{42}^{(5)} = a_{52}^{(5)} a_{41}^{(5)} \tag{7.128} \]
where the \( a_{ij}^{(5)} \) factors are given by the right side member of Eq. (7.121).

In Eq. (7.122) the phases \( \beta_{i5}, i = 2, 4, 5 \), are expressed in terms of \( \beta_{15} \) and the parameters \( |p_{i2}|, |p_{i4}|, \beta_{i2}, \beta_{i4}, i = 1, 2, 4, 5 \). Similarly, Eqs. (7.126) and (7.127) imply that \( |p_{i5}|, i = 1, 2, 4, 5 \), can be expressed in terms of the parameters \( |p_{i2}|, |p_{i4}|, \beta_{i2}, \beta_{i4}, i = 1, 2, 4, 5 \). However, Eqs. (7.124) and (7.125) together with the constraints (7.128), constitute a system of four algebraic equations relating the parameters \( |p_{i2}|, |p_{i4}|, \beta_{i2}, \beta_{i4}, \) to \( |p_{j2}|, |p_{j4}|, \beta_{j2}, \beta_{j4}, \) when \( j = 1, 2, 4 \). Thus, in principle, we can use these equations to express the set of four parameters in terms of the last set of parameters. Moreover, the constraint given in Eq. (7.123), only involves the \( |p_{j2}|, |p_{j4}|, \beta_{j2}, \beta_{j4}, \) when \( j = 1, 2, 4 \). Thus, that equation can be used to express one of the parameters of the set in terms of the rest of the parameters of the same set. In sum, the set of 12 independent parameters can be chosen to be \( \beta_{15} \) plus 11 parameters taken from \( |p_{j2}|, |p_{j4}|, \beta_{j2}, \beta_{j4}, \) when \( j = 1, 2, 4 \).

A similar way can be used for \( U^* \)-type parameters.

8 Conclusion

In this paper we have computed analytically the chargino mass spectrum, at the tree level, in the context of the LRSUSY model, including a general set of CP-violating phases, \( \Phi_L, \Phi_R, \Phi_\mu, \Phi_1, \Phi_2, \Phi_3 \).

We have shown that there is always a neighborhood in the fundamental parameter space where one of the chargino masses relates in a simple manner with the parameters \( |\mu| \). This fact allows us to factorize the quintic polynomial representing the characteristic equation, which is used to determine the chargino mass spectrum, and to arrange the charginos in a determined neighborhood according to the size of their masses. We have also shown that in the most general CP-violating scenario the chargino masses depend only on three global phases \( \Theta_1, \Theta_2, \) and \( \Theta_3 \).

We have computed analytically the diagonalizing matrices \( V \) and \( U^* \). The entries of these matrices can be expressed in terms of the more fundamental quantities \( \Delta_{ij}^{(5)} \) and \( \Delta_{ij}^{(5)} \), respectively. Then, with the help of these fundamental quantities we have implemented a generalized projector formalism which provides a system of basic equations connecting the reduced projectors, the eigenphases and the chargino masses with the chargino parameters. These equations constitute the keystone on which the parameter inversion process
is based. Some connections with the Jarlskog’s formulation allows us to disentangle in a direct manner some relevant parameters, specially those ones lying on the diagonal of the chargino mass matrix.

We have shown that a systematic reconstruction of the fundamental parameter is possible if we are able to measure an adequate set of observables, for instance, some cross-section type observables derived from the chargino pair production in electron-positron annihilation.

Concerning this last point, we have shown that it is possible to re-parameterize the cross-section type observables in terms of the chargino masses, the reduced projectors, and the eigenphases. The minimal number of reduced projector-type parameters that we can use to parameterize this kind of observables is 24, that is, 12 for $V$-type and 12 for $U^*$-type. We have seen that there are many ways to choose the set of independent reduced projector-type parameters, we have analyzed two of them. The first one consists of six sets of hyper-spherical angles, three $V$-type reduced projector phases and three $U^*$-type reduced projector phases. The second one involves eleven $V$-type and eleven $U^*$-type reduced projector parameters associated to the lightest charginos, and one $V$-type and one $U^*$-type reduced projector phases associated to the right-handed contribution.

The systematic inversion method used in this paper to determine the fundamental chargino parameters, based on measurements of physical observables, can be applied to any number of charginos and to any number of neutralinos, no matter the particular model we used to describe them.

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**A $H$ and $\tilde{H}$’s entries**

The matrices $H$ and $\tilde{H}$ in terms of the entries of the mixing matrix $M$ are given by $H_{ij} = \sum_{k=1}^{5} M_{ki}^{*} M_{kj}$ and $\tilde{H}_{ij} = \sum_{k=1}^{5} M_{ik} M_{jk}^{*}$, respectively. We get that

\begin{align}
H_{11} &= |M_L|^2 + 2|\tilde{M}_L|^2 \sin^2 \theta_\kappa, \\
H_{22} &= M_R^2 + 2|\tilde{M}_R|^2 \sin^2 \theta_\kappa + g_R^2 (\delta_R^2)^2, \\
H_{33} &= |\mu|^2, \\
H_{44} &= |\mu|^2 + 2(|\tilde{M}_L|^2 + |\tilde{M}_R|^2) \cos^2 \theta_\kappa, \\
H_{55} &= g_R^2 (\delta_R^2)^2 + |\mu_3|^2, \\
H_{12} &= H_{21}^* = 2|\tilde{M}_L||M_R|e^{i(\tilde{\Phi}_R - \Phi_L)} \sin^2 \theta_\kappa, \\
H_{13} &= H_{31}^* = 0, \\
H_{14} &= H_{41}^* = \sqrt{2}|\tilde{M}_L| [e^{i(\tilde{\Phi}_L - \Phi_L)}|M_L| \cos \theta_\kappa - e^{i(\tilde{\Phi}_R - \Phi_R)}|\mu| \sin \theta_\kappa], \\
H_{15} &= H_{51}^* = 0, \\
H_{23} &= H_{32}^* = 0, \\
H_{24} &= H_{42}^* = \sqrt{2}|\tilde{M}_L| [e^{i\tilde{\Phi}_R} M_R \cos \theta_\kappa - e^{i(\tilde{\Phi}_R - \Phi_R)}|\mu| \sin \theta_\kappa], \\
H_{25} &= H_{52}^* = g_R |M_R| v_{\delta_R} e^{i\Phi_1} + v_{\delta_R} |\mu_3| e^{i(\Phi_3 - \Phi_2)}, \\
H_{34} &= H_{43}^* = 0, \\
H_{35} &= H_{53}^* = 0, \\
H_{45} &= H_{54}^* = \sqrt{2} g_R |\tilde{M}_R| v_{\delta_R} e^{i(\Phi_1 - \Phi_R)} \cos \theta_\kappa,
\end{align}

(A.129)
$$\tilde{H}_{ij} = \sum_{k=1}^{5} M_{ik} M_{jk}^* :$$

\[\begin{align*}
\tilde{H}_{11} &= |M_L|^2 + 2|\tilde{M}_L|^2 \cos^2 \theta_{\kappa}, \\
\tilde{H}_{22} &= M_R^2 + 2|\tilde{M}_R|^2 \cos^2 \theta_{\kappa} + g_R^2 (v_{\Delta R})^2, \\
\tilde{H}_{33} &= |\mu|^2 + 2(|\tilde{M}_L|^2 + |\tilde{M}_R|^2) \sin^2 \theta_{\kappa}, \\
\tilde{H}_{44} &= |\mu|^2, \\
\tilde{H}_{55} &= g_R^2 (v_{\Delta R})^2 + |\mu|^2, \\
\tilde{H}_{12} &= \tilde{H}_{21} = 2|\tilde{M}_L| |\tilde{M}_R| e^{i(\tilde{\Phi}_L - \tilde{\Phi}_R)} \cos^2 \theta_{\kappa}, \\
\tilde{H}_{13} &= \tilde{H}_{31} = \sqrt{2}|\tilde{M}_L| [e^{i(\Phi_L - \tilde{\Phi}_L)} |M_L| \sin \theta_{\kappa} - e^{i(\tilde{\Phi}_L - \Phi_\mu)} |\mu| \cos \theta_{\kappa}], \\
\tilde{H}_{14} &= \tilde{H}_{41} = 0, \\
\tilde{H}_{15} &= \tilde{H}_{51} = 0, \\
\tilde{H}_{23} &= \tilde{H}_{32} = \sqrt{2}|\tilde{M}_R| [e^{-i\tilde{\Phi}_R} M_R \sin \theta_{\kappa} - e^{i(\tilde{\Phi}_R - \Phi_\mu)} |\mu| \cos \theta_{\kappa}], \\
\tilde{H}_{24} &= \tilde{H}_{42} = 0, \\
\tilde{H}_{25} &= \tilde{H}_{52} = g_R [v_{\Delta R} |\mu|^3 e^{i(\Phi_1 - \Phi_3)} + M_R v_{\delta R} e^{-i\Phi_2}], \\
\tilde{H}_{34} &= \tilde{H}_{43} = 0, \\
\tilde{H}_{35} &= \tilde{H}_{53} = \sqrt{2} g_R |\tilde{M}_R| v_{\delta R} e^{i(\tilde{\Phi}_R - \Phi_2)} \sin \theta_{\kappa}, \\
\tilde{H}_{45} &= \tilde{H}_{54} = 0,
\end{align*}\]

(A.130)

where $v_{\Delta R}$ and $v_{\delta R}$ are defined in Eq. (2.11).

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