TECHNICOLOR ENHANCEMENT OF $t\bar{t}$ PRODUCTION AT TeV-COLLIDERS

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January 8, 2018

Abstract

It is shown that a technicolor theory containing a color-octet technipion, usually denoted by $P_8^{0'}$, will give rise to an enhancement of $t\bar{t}$ production at the Tevatron, LHC and SSC, via the process $gg \rightarrow P_8^{0'} \rightarrow t\bar{t}$. The relevant cross-sections are computed taking into account the large lower bound on the top mass coming from the "top search" experiments at LEP and CDF. At the LHC and SSC, the signal is found to be comparable to the QCD background, making the process quite accessible.
Technicolor theories typically contain pseudo-Goldstone bosons (technipions), which arise from the breakdown of global chiral symmetries. Here, the production and subsequent decay of a color-octet technipion at the Tevatron, LHC and SSC is studied. The sub-process considered is $\text{gluon} - \text{gluon} \rightarrow P^{0'}_8 \rightarrow t\bar{t}$, where $P^{0'}_8$ is the technipion. The subscript "8" denotes that it is a QCD octet, the superscript "0" that it is neutral, and the prime that it is a singlet under the weak $SU(2)_L$. This particle is expected to be light compared to most technihadrons, and therefore more accessible to the TeV-colliders. However, it is assumed throughout this paper that its mass is above the $t\bar{t}$ production threshold.

The effective ETC couplings of the technipions to the quarks are proportional to $m_q/F_\pi$. Therefore, $q\bar{q}$ fusion as a production mechanism is not considered, nor is the decay of the technipions into quarks lighter than the top or the bottom. The decay of the technipions to two bottom quarks or to two gluons is not considered either, even though it gives non-negligible cross-sections, since the background to these processes makes them difficult to observe. Moreover, we consider only the color-octet $P^{0'}_8$ as the intermediate technipion, because color counting factors make its cross-section approximately eight times larger than the color-singlet one. Also, because the color-singlet technipion mass does not receive a QCD contribution, it could lie below the $t\bar{t}$ production threshold. The results of this paper could, however, also be applied to a color-singlet technipion, if corrected by the relevant counting factors.

Color-octet technipions exist only in models with at least one colored doublet of technifermions. A one family model will be employed here. It has a global $SU(8)_L \times SU(8)_R$ chiral symmetry that breaks down to an $SU(8)_{L+R}$ and thus
produces 63 Goldstone bosons, 3 of which are "eaten" by the $W$'s and the $Z^0$, and 60 of which acquire masses depending on their quantum numbers. Even though present experimental constraints on the $S$ parameter do not favor a large technicolor sector \[1\], a theory with a complete techni-family is viable if the technicolor group is not too large \[2\]. In the following, the technicolor gauge group $SU(N_{TC})$ with $N_{TC} = 2$ will be used.

The process in question attracted the interest of several authors some years ago \[4-7\]. However, the top mass used in those computations never exceeded 70 GeV \[5\]. Given the dependance of the technicolor signal and the QCD background on the quark and technipion masses, the current lower bound on $m_t$ ( $m_t > 91$ GeV \[7\]) and the emergence of new classes of technicolor theories (the so-called walking theories \[8\]) that lead to larger technipion masses make it important to recompute the relevant cross-sections.

The mass of the color-octet technipion is estimated first. It receives contributions mainly from QCD and ETC interactions. The QCD contribution, according to a previous estimate \[9\], gives

$$M_P^2 \approx \frac{9 \ln 2}{2 \pi} \alpha_s(M_P^2) M_V^2 ,$$

(1)

where $M_V$ is the mass of the lightest technivector resonance. To find $M_V$, the following scaling relation is used:

$$M_V = (3/N_{TC})^{1/2} (F_\pi/f_\pi) m_\rho ,$$

(2)

where $f_\pi$ is the pion decay constant, and $F_\pi = \frac{246 \text{ GeV}}{\sqrt{N_D}}$, with $N_D$ the number of technidoublets. For the one family model, $N_D = 4$, so $F_\pi = 123$ GeV. Inserting the expression for $M_V$ into Eq.\[1\] gives $M_P \approx 400$ GeV .
There is also an ETC contribution to the $P^0_8$ mass. A previous estimate gives roughly

$$M^2_P \approx g^2_{ETC} < \psi \bar{\psi}>^2 / M_{ETC}^2 F^2\pi,$$

where $M_{ETC}$ can range from 10 to 1000 TeV, and $g^2_{ETC}/4\pi^2$ is expected to be $O(1)$. In conventional technicolor theories, the chiral condensate $<\psi \bar{\psi}> \approx \Lambda_{TC}^2 / 4\pi^2$, where $\Lambda_{TC} \approx M_V \Lambda_{QCD}$ denotes the technicolor confinement scale. In these theories, therefore, the ETC contribution to the technipion mass can be at most about 100 GeV. Walking technicolor models, however, can give larger masses to the technipions via high-momentum enhancement. The upper limit of the technicolor condensate $<\psi \bar{\psi}>$ in these models is of order $M_{ETC} \Lambda_{TC}^2$, so the ETC contribution could even approach 1 TeV. The $SU(N_{TC} = 2)$ technicolor model with $N_D = 4$ gives a somewhat enhanced condensate, but well below the upper limit. Masses in the range 350 – 550 GeV will be considered here.

Next, in order to determine the cross-section for the sub-process $gg \rightarrow P^0_8 \rightarrow t \bar{t}$, the partial decay widths of the $P^0_8$ to two gluons and to a quark pair are needed. They are

$$\Gamma(P^0_8 \rightarrow gg) = \frac{5 N_{TC}^2}{384\pi^3} \alpha_s^2(M^2_P) \frac{M_P^3}{F^2\pi},$$

and

$$\Gamma(P^0_8 \rightarrow t \bar{t}) \approx \frac{m_t^2 M_P}{4\pi F^2\pi} \left(1 - 4 \frac{m_t^2}{M_P^2}\right)^{1/2},$$

where $M_P$ is the technipion mass, and $\alpha_s^2(M^2_P)$ is the QCD coupling evaluated at $M_P^2$. The width for $t \bar{t}$ decay depends on the specific assumption made for the ETC-induced coupling of the technipion to $t \bar{t}$. Here, a CP-conserving coupling of strength $2m_t/F\pi$ is used. A typical choice for the top mass, $m_t = 120$ GeV, and
the technipion mass, $M_P = 350$ GeV, gives 0.05 GeV and roughly 20 GeV for the decay widths of the technipion to $gg$ and $t\bar{t}$ respectively. The latter could be smaller if the technipion mass is closer to the $t\bar{t}$ production threshold.

To estimate the total decay width of the technipion, denoted here by $\Gamma_{tot}$, we note that it decays predominantly into a $t\bar{t}$ pair. This is the case as long as $m_t > \sim 2 \times 10^{-2} M_P$. Thus, $\Gamma_{tot} \approx \Gamma(P_8^{0'} \to t\bar{t})$. This estimate is valid as long as the technipion mass does not become very close to the $t\bar{t}$ production threshold, i.e. as long as $1 - 4m_t^2/M_P^2 > \sim 10^{-6}$.

Using the relativistic Breit-Wigner formula, the sub-process cross-section, for $N_{TC} = 2$, then reads

$$\hat{\sigma}_{TC}(gg \to P_8^{0'} \to t\bar{t}) = \frac{\pi \Gamma(P_8^{0'} \to gg) \Gamma(P_8^{0'} \to t\bar{t})}{2 (\hat{s} - M_P^2)^2 + M_P^2 \Gamma_{tot}^2}$$

$$= \frac{10}{3(8\pi)^3} \frac{\alpha_s^2(M_P^2) M_P^4 m_t^2 (1 - 4m_t^2/M_P^2)^{1/2}}{F_\pi^4 ( (\hat{s} - M_P^2)^2 + M_P^2 \Gamma_{tot}^2 )}. \quad (5)$$

The QCD final-state corrections are neglected in this expression. The decay widths of Eq.4 should actually be corrected before being inserted in the cross-section for $gg \to P_8^{0'} \to t\bar{t}$ since the relevant quantity entering these widths is not the mass of the technipion, but the invariant mass $\sqrt{\hat{s}}$ of the process. These corrections are not taken into account, partly because the use of strict invariant mass cuts makes their effects negligible, and partly because the uncertainty introduced by the technipion and top-quark masses, which are still experimentally unknown, makes such an analysis too detailed. It should be noted in particular that Eq.5 cannot be applied for technipion masses, or $\sqrt{\hat{s}}$ values too close to the $t\bar{t}$ production threshold.

The result of Eq.5 can be carried one step further, in order to compute the
differential cross-section with respect to the scattering angle. Since the technipion has zero spin, \( \frac{d\hat{\sigma}_{TC}}{d\cos \theta} = \hat{\sigma}_{TC}/2 \), where \( \theta \) is the scattering angle measured in the gluon-gluon center-of-mass frame.

The ingredients are now in hand to compute the integrated cross-section for the process \( pp \to P_8^0 \to t\bar{t} \). Denoting by \( s \) the total C.M. energy of the \( pp \) beams, we have \( \hat{s} = sx_ax_b = s\tau \), where \( \tau \equiv x_ax_b \), and \( x_a \) and \( x_b \) are the momentum fractions of the two interacting partons \( a \) and \( b \), considered to be massless. In our case, they are gluons. The parton distribution functions for gluons, denoted by \( f_g(x_i, \hat{s}) \), \( i = a, b \), can now be used, to write

\[
\sigma_{TC}(pp \to P_8^0 \to t\bar{t}) = \int_{\tau_-}^{\tau_+} d\tau \int_{-Y}^{Y} dy_1 \int_{y_-}^{y_+} dy_2 \frac{f_g(x_a, \hat{s}) f_g(x_b, \hat{s})}{2 \sqrt{1 - 4m_t^2/\hat{s} \cosh^2(\frac{y_1 - y_2}{2})}} \frac{d\hat{\sigma}_{TC}}{d\cos \theta}(gg \to P_8^0 \to t\bar{t}) ,
\]

where the \( q\bar{q} \) contribution to the process has been neglected. Here \( y_{1,2} \) are the rapidities of the two top quarks measured in the laboratory frame and \( \tau \) is defined as above. According to the above definitions, \( x_a^b = \sqrt{\tau} \exp(\pm y_+ - y_-) \). The quantities \( y_\pm \) are defined as \( y_\pm = \min_{\max}(\pm Y, \pm \ln \tau - y_1) \). The integration limits \( \tau_\pm, Y, \) and \( y_\pm \) correspond to experimental cuts, and will be discussed shortly. The integrated cross-section \( \sigma_{TC} \) is then computed for various top and technipion masses. For the gluon distribution functions \( f_g \), the HMRS(B) functions are used, as described in Ref.[11]. The integrals in Eq.(6) are done numerically by a Monte-Carlo algorithm [12]. The program is stopped as soon as \( 10^4 \) Monte-Carlo-generated events have passed all the cuts. This gives sufficiently small statistical errors.

The form of Eq.(5) implies that, while the cross-section can increase quadratically with \( m_t \), this only happens for \( m_t \) small enough so that the total width is
dominated by decays other than the decay $P^0_8 \rightarrow t\bar{t}$. For the allowed $m_t$ range, however, $\Gamma(P^0_8 \rightarrow t\bar{t})$ dominates, making the integrated cross-section (Eq.6) roughly independent of the top mass, when the $\sqrt{s}$ integration range is larger than the largest technipion width considered. If, however, the integration is done over a fixed invariant mass range smaller than, or comparable to, the width to the lightest $t\bar{t}$ pair considered, the integrated cross-section decreases with the top mass, since the portion of the width falling inside the integration bin decreases.

It is essential to estimate the background for this process, in order to see how clear the signal will be experimentally. The quantities $\hat{\sigma}_{QCD}(gg \rightarrow t\bar{t})$ and $\hat{\sigma}_{QCD}(q\bar{q} \rightarrow t\bar{t})$ are considered here, as they are expected to give the main contribution to the background. It should be noted that the technicolor process has a substantial interference with the QCD process $gg \rightarrow t\bar{t}$. This interference is neglected here, however, since the cuts applied on the invariant mass (see discussion on cuts below) reduce its effects to less than 10%. The QCD cross-sections for $q\bar{q}$ and $gg$ fusion are taken from Ref.[13], and they are then inserted into the numerical algorithm, in order to compute the cross-section $\sigma_{bgd}(pp \rightarrow t\bar{t})$. For the $q\bar{q}$ processes, the up, down, and sea-quark distribution functions are used, again according to the analysis of Ref.[11]. The computation for the Tevatron takes into account that it is a $p\bar{p}$ collider, unlike the SSC and LHC, which will be $pp$ colliders. Finally, there is an ambiguity as to which scale $\mu$ should be used for the calculation of the QCD coupling for the background processes, having $s$-channel amplitudes on one hand, and $t$- and $u$-channel on the other, but this does not affect our results by more than 10%. The value $\mu = \sqrt{s}$ is used here.

A convenient choice of cuts, $\tau_\pm$ for $\tau$, $\pm Y$ for $y_1$, and $y_\pm$ for $y_2$ is needed
in order to reduce the background as much as possible. The minimum possible 
\( \tau (= \hat{s}/s) \) cut will be determined by the experimental resolution \( \delta \sqrt{s} \) for the invariant mass. We will assume that a resolution of roughly 20 GeV - approximately 5% of the \( \sqrt{s} \) values studied here and comparable to the natural line width of the technipion - is possible. This of course depends on the details of detection of the hadronic jets and charged leptons after both \( t \)'s undergo the decay \( t \rightarrow Wb \). In Tables 1-3, numerical results are presented for integrated cross sections for both the technicolor process and the background. The results are for a 20-GeV \( \sqrt{s} \) bin centered around the mass of the technipion. Thus, for this bin,

\[
\tau_{\pm} = \frac{(M_P \text{ (GeV)} \pm 10 \text{ GeV})^2}{s \text{ (GeV}^2)}.
\]  
(7)

This choice has also the advantage of reducing the interference effects of the technicolor and QCD processes to less than 10%, which is satisfactory for our purposes.

Rapidity cuts are also placed, constraining the longitudinal momenta of \( t \) and \( \bar{t} \) to lie between the two wings of a momentum-space hyperboloid around the beam axis. This reduces the QCD background, which is larger along the beam axis, and it also removes from the integration region of Eq.6 part of the phase space that is experimentally inaccessible, due to its proximity to the beam axis. The rapidities \( y_1 \) and \( y_2 \), measured in the laboratory frame, are constrained by \( Y \) and \( y_{\pm} \) respectively. A specific value for \( Y \), for a given \( \tau \), automatically determines \( y_{\pm} \).

The cut \( Y = 2.5 \) is chosen for the results presented in Tables 1-3. This is not very restrictive, since, even for \( p_{\perp} = 0 \), it allows \( p_{\parallel} \) to be roughly as large as \( 6m_t \). The signal-to-background ratio does not vary substantially with the choice of \( Y \). This is due to the heaviness of the top quark, which makes the QCD background less anisotropic than it is for lighter quarks.
Table 1: Numerical results for the different cross-sections in picobarns, for $M_P = 350$ GeV. The numbers given are rounded up to contain only two significant figures. The invariant mass bin width is 20 GeV. The results are for $N_{TC} = 2$; the technicolor cross-section increases quadratically with $N_{TC} = 2$. Various choices for the top mass are made. The three different choices for the total C. M. energy correspond to the energies of the Tevatron, LHC, and SSC respectively. The Monte-Carlo relative statistical errors are typically on the order of 1%.

From the tables, it is apparent that colliders with larger center-of-mass energies lead to larger production cross-sections. This is because, for a given invariant mass $\sqrt{s}$, they probe regions of smaller $\tau$’s, where the parton, and especially the gluon, distribution functions $f(\sqrt{\tau}, \hat{s})$ are larger. Another manifestation of the decrease of $f(\sqrt{\tau}, \hat{s})$ with increasing $\tau$ is the fact that, for a given $\sqrt{s}$, larger technipion masses, corresponding to larger $\sqrt{\hat{s}}$ values, give smaller cross-sections.

For the range of technipion and top masses chosen, the background gg-fusion process has a cross-section that decreases with increasing top mass, unlike the $q\bar{q}$ process. Therefore, since the total C.M. energies considered here correspond to small $\tau$ values, where the gluon distribution functions dominate over the quark ones, the total background decreases with increasing top mass. The technicolor cross-sections also decrease with increasing top-mass, a behavior in agreement with the rough theoretical expectations based on Eq.5.

The Tevatron cross-sections are quite small, and no more than a few signal
Table 2: Results of the same computation as in Table 1, but with $M_P = 450$ GeV.
The increased technipion mass allows larger top masses to be considered. However, results for a top quark heavier than 180 GeV are not given, since present constraints on the $\rho$ parameter [2] indicate that it is unlikely to be heavier than that.

| $m_t$ (GeV) | $\sigma_{TC}$ (pb) | $\sigma_{bgd}$ (pb) | $\sigma_{TC}$ (pb) | $\sigma_{bgd}$ (pb) | $\sigma_{TC}$ (pb) | $\sigma_{bgd}$ (pb) |
|------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| 90         | 0.041              | 0.57               | 100                | 300                | 490                | 1300               |
| 120        | 0.028              | 0.52               | 73                 | 210                | 350                | 960                |
| 150        | 0.022              | 0.46               | 54                 | 140                | 260                | 640                |
| 180        | 0.019              | 0.39               | 44                 | 80                 | 210                | 370                |

events per year should be expected. The signal-to-background ratio is also worse for the Tevatron than for the LHC and SSC. This is due to the fact that the $q\bar{q}$ contribution to the background is larger at the Tevatron, where the $\hat{s}/s$ values considered correspond to larger quark distribution functions. Another reason why the contribution of $q\bar{q}$ fusion to the background is larger is that the Tevatron is a $p\bar{p}$ collider. At the LHC and SSC, the technicolor and background cross-sections are comparable to each other, indicating that a substantial signal could be observed in future experiments for the range of top masses studied here, if the $P_8^{0'}$ exists.

It is also useful to plot the corresponding differential cross-sections in the invariant mass. The computation of $\frac{d\sigma_{TC}}{d\sqrt{s}}$ and $\frac{d\sigma_{bgd}}{d\sqrt{s}}$ can be done by a Monte-Carlo algorithm similar to the one used for Eq. 6. A plot of the differential cross-sections $\frac{d\sigma_{TC}}{dp_{\perp}}$ and $\frac{d\sigma_{bgd}}{dp_{\perp}}$ could be even more useful, as $p_{\perp}$ may be easier to determine experimentally than $\sqrt{s}$ [13]. Both of these plots will appear in a future publication.

It is worth pointing out that the signal coming from a technipion $P_8^{0'}$ can be
Table 3: Results for \( M_P = 550 \) GeV.

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
M_P = 550 \text{ GeV} & \sqrt{s} = 1.8 \text{ TeV} & \sqrt{s} = 16 \text{ TeV} & \sqrt{s} = 40 \text{ TeV} \\
\hline
m_t (\text{GeV}) & \sigma_{TC} (\text{pb}) & \sigma_{bgd} (\text{pb}) & \sigma_{TC} (\text{pb}) & \sigma_{bgd} (\text{pb}) & \sigma_{TC} (\text{pb}) & \sigma_{bgd} (\text{pb}) \\
\hline
90 & 0.78 \times 10^{-2} & 0.14 & 64 & 120 & 310 & 620 \\
120 & 0.52 \times 10^{-2} & 0.13 & 41 & 96 & 200 & 480 \\
150 & 0.37 \times 10^{-2} & 0.12 & 29 & 72 & 140 & 360 \\
180 & 0.29 \times 10^{-2} & 0.12 & 22 & 52 & 110 & 260 \\
\hline
\end{array}
\]

distinguished from a Higgs particle \( H^0 \), since the latter has a much smaller cross-section for the same detection channel. Indeed, the fact that the technipion is a color octet, and that its decay width \( \Gamma(P_{S}^{0} \rightarrow gg) \) depends quadratically on the number of technicolors, makes the technipion production rate roughly \( 8N_{TC}^2 \) times larger than the corresponding Higgs process [3].

To conclude, our results show that presently allowed values for the top mass are such that a considerable enhancement of \( t\bar{t} \) pairs at LHC and SSC energies can be expected from color-octet technipion production and decay. A similar, but somewhat smaller, enhancement can be expected from the color-singlet technipion, if it lies above the \( t\bar{t} \) threshold. Therefore, the process considered here can be used as a direct test of a large class of technicolor models. Finally, it is worth noting that the enhancement of \( b\bar{b} \) production from technipion decay could also be interesting, especially if the technipion masses are below the \( t\bar{t} \) production threshold.

**Acknowledgements**

We thank Charles Baltay, Ken Lane, Steven Manly and Torbjorn Sjostrand for very helpful discussions.
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