Internal ballistics of smoothbore guns

K R Crawford¹, N W Mitiukov², *, E L Busygina³, ⁴ and M Y Alies²

¹Gunnery Fire Control Group, 46256 Indianapolis (IN), USA
²Udmurt Federal Research Center of the Ural Branch of the RAS, 426067 Izhevsk, Russia
³International Network Center for Fundamental and Applied Research, 20036 Washington (DC), USA
⁴Kalashnikov’s Izhevsk State Technical University, 426000, Izhevsk, Russia

* nico02@mail.ru

Abstract. The work is devoted to the problem of the formation of a mathematical model of a smoothbore gun. It is shown that due to the low accuracy of the initial data, the high accuracy of the mathematical model is not rational, and the mathematical model in the adiabatic formulation is quite reasonable. On the other hand, simple methods of numerical integration are excluded due to the discontinuity of functions at the beginning of the combustion phase of gunpowder. Accounting for the rotation of the cannonball, necessary for external ballistics, is possible only for a probabilistic task. Therefore, it will be most convenient to evaluate this parameter through the form factor of the external ballistic calculation.

1. Introduction

A few years ago, the authors, with the support of a number of other historians and experts, made an attempt to create a database on the ballistics of rifled large-caliber artillery. This base was in demand for the creators of computer games, but in addition to the entertainment industry, it made it possible to come close to solving some mysteries of history.

It seemed to us that the next logical step would be to identify the parameters of smooth-bore artillery, but the implementation of this project showed that there are many questions, both technical and historical, to overcome which was not at all a trivial task. The fact is that, as a technical system, rifled artillery is certainly more complex than smooth-bore. But at present, a lot of work has been done by many organizations involved in the development of artillery systems to create tested and well-described mathematical models of the process physics. At the same time, there are no such organizations for smoothbore systems. Therefore, ready-made elements of models simply do not exist and they have to be developed independently.

In addition, as we were able to establish, the information available in the literature on the internal ballistics of smoothbore guns was obtained either generally speculatively or based on interpolation of experiments conducted in the 19th century and, despite the fact that it is repeated in many works on smoothbore artillery, correctness is in great doubt.

In addition, we would like to draw attention to one paradox characteristic of modeling in history, not always understood by specialists in mathematical modeling involved in solving modern problems. The fact is that the accuracy of mathematical modeling consists of three components: the accuracy of obtaining the source data, the accuracy of the mathematical method with which the simulations will be
performed and the accuracy of the mathematical model itself. In relation to modern technical tasks, the most modern measuring equipment is used to obtain initial data, which allows to obtain initial data for modeling with very high accuracy. The mathematical model itself is also formed taking into account all the nuances and takes into account the maximum number of factors. Therefore, the question usually stands in the development of mathematical methods adequate to these accuracy (parallel computation methods and other highly efficient methods are obtained). And to verify the resulting forecasts, new experiments are being set up, which give even greater refinement of the initial data.

In the problems of historical modeling, the paradigm is completely different. It should immediately be recognized here that the accuracy of the initial data is very low and is determined not only by measuring equipment two centuries ago, but by the reliability of the historical source itself. Therefore, the high accuracy of the mathematical model simply loses all meaning, as does the high accuracy of the method for solving it. And the data that are obtained as a result of the simulation are not directed outside (for scientific forecasting), but inside, to refine the initial data and to correct or detail the historical source.

2. The mathematical model of a smoothbore gun

The mathematical model of a smoothbore gun was previously discussed by us in a number of works (for example, in [1-3]). The ball located in the bore can be represented by a material point moving with speed $v$, obeying Newton’s law:

$$\frac{dv}{dt} = \frac{(p - p_h)F}{m} - g(sin \alpha - f \cos \alpha),$$

where $p$ is the pressure of the powder gases; $p_h$ is the environmental pressure; $F$ is the area of the midship of the nucleus ($F = \pi d^2 / 4$); $m$ is the mass of the ball; $g$ is the acceleration of gravity; $\alpha$ is the elevation angle of the gun; $f$ is the coefficient of friction.

The pressure of the powder gases is determined from the equation of state:

$$p = \frac{MRT}{V},$$

where $M$ is the current mass of powder gases; $RT$ is the “power of gunpowder”; $V$ - volume behind the ball: $V = V_b + S\ell + V_s$, $V_b$ - chamber volume; $V_s$ - the volume of burnt powder; $S$ is the area of the bore ($S = 0.25 \pi D^2$); $\ell$ - the current coordinate of the ball when moving along the bore: $\frac{d\ell}{dt} = v$.

Current mass of powder gases:

$$\frac{dM}{dt} = G_1 - G_2,$$

where $G_1$ is the gas inlet (in the case of burning of grain, it is reset to zero); $G_2$ is the gas flow rate.

Gas consumption is determined by the known laws of gas dynamics:

$$G_2 = \begin{cases} \frac{pF_k}{RT} \left( \frac{p}{p_h} \right)^{\frac{k-1}{k}} \sqrt{\frac{2kRT}{k-1}} \left( 1 - \left( \frac{p}{p_h} \right)^{\frac{k+1}{k}} \right), & \text{if } \frac{p}{p_h} < \left( \frac{2}{k+1} \right)^{\frac{k}{k-1}}; \\ \frac{pF_k}{RT} \left( \frac{2}{k+1} \right)^{\frac{k+1}{k-1}}, & \text{if } \frac{p}{p_h} > \left( \frac{2}{k+1} \right)^{\frac{k}{k-1}}. \end{cases}$$

Here $k$ is the adiabatic index of the combustion products; $F_k$ is the area of the passage between the core and the bore: $F_k = \varphi \frac{\pi}{4} (D^2 - d^2)$. Here $\varphi$ is the living section coefficient; in the case of a crescent-shaped gap between a ball and a cylindrical wall, it will be determined by the dependence proposed by us in [3].

The arrival of gas can be determined by the laws of internal ballistics: $G_1 = \frac{de}{dt} S(e)p$, 


where \( \frac{de}{dt} \) is the burning rate; \( S(e) \) is the law of variation of the combustion surface; \( \rho \) is the density of the powder. In this case, the burning rate can be determined by considering the combustion law as artillery: \( \frac{de}{dt} = Ap \), where \( A \) is the constant of the burning rate.

If we take the powder grain in the form of a ball of radius \( R \), then the combustion surface \( S(e) \) as a function of \( e \) is determined as \( S(e) = N4\pi(R-e)^2 \). Here \( N \) is the number of grains that can be determined by knowing the total gunpowder sample \( m_p \): \( N = \frac{m_p}{\frac{4}{3}\pi R^3 \rho} \).

3. **The accuracy of the mathematical model**

As you can see, the model is formulated in an adiabatic formulation, that is, it is believed that there is no heat transfer with the environment. At first glance, the neglect of heat transfer is an extremely gross assumption, in connection with which the installation barrel was calculated in the ANSYS [4] system. For simplicity, an ordinary thick-walled pipe was selected, on one side blocked by a flat wall. The projectile was also presented as a cylinder for simplicity. Ignition of the powder sample occurred, but the powder gases did not pass through the gap between the cylinder and the barrel, i.e. their energy completely transferred to the energy of a moving core and wall heating. In Figure 1 gives the dynamics of temperature changes at the leftmost point of the barrel (near the chamber), which is most exposed to heat. At 3 \( \mu \)s, the main combustion of gunpowder ceased, therefore, at this time a maximum temperature of 370 \(^\circ\)C was recorded, then only small, degressively burning residues that did not increase the temperature burn out. At 11 \( \mu \)s, the cylinder flew out of the barrel, and the inner wall began to cool sharply. The temperature distribution at 3 \( \mu \)s is shown in Fig. 2. It can be seen that, in general, the pipe retains its initial temperature and more or less significant heating occurs only in an extremely limited area in the chamber area.

Comparison of the calculation results for the three-dimensional model taking into account heat transfer and the adiabatic model (zero-dimensional setting) showed that the discrepancy in the velocity of the projectile at the exit from the barrel is not more than 5.5\%, which, given the error in determining the initial data, is a very small value.

![Figure 1. The dynamics of the heating of the wall: the upper curve is the inner wall of the barrel, the lower curve is the outer wall of the barrel.](image1.png)

![Figure 2. The distribution of temperature in the pipe at 3 \( \mu \)s.](image2.png)

4. **The accuracy of the mathematical method**

The numerical solution of the proposed model of smooth-bore guns made it possible to find yet another complexity related to the accuracy of the mathematical method. As can be seen from the
model, at the first moment of the onset of combustion, the surface of combustion from zero immediately becomes maximum, i.e. there is a gap in the function. Obviously, the maximum arrivals of gas, surface, etc. in combination with minimum speeds, coordinates, pressures, etc. when dividing one quantity by another, they give mathematical uncertainty, which ultimately lays the error in the final result. The paradox is that the smaller the step at the beginning of the solution, the more the error is set at these steps.

Usually, an increase in the accuracy of calculations is associated with a decrease in the step of numerical integration. As practice shows, when the integration step is reduced, the accuracy of the result is corrected (Fig. 3, curve 1). If the step decreases below a certain limit $h_K$, the result is stabilized. However, as the calculations showed, the result of solving the internal ballistics problem for a smoothbore gun behaves as curve 2. On the entire range of integration steps, the final result changes significantly, often when the step decreases below a certain limit, giving completely unphysical results.

A similar picture is observed during integration by Euler, Runge-Kutta, and other methods with a constant step. It is possible to overcome this crisis, as calculations have shown, only by using methods of a variable integration step, for example, Runge-Kutta-Merson [5].

![Figure 3. The dependence of the result $R$ on the integration step $h$.](image)

5. The accuracy of the source data

Thus, a mathematical model of a smoothbore gun was formulated, the accuracy of which, taking into account various assumptions, was predicted to be within 10%, which, under rather rough conditions for obtaining the initial data, seemed quite satisfactory. However, when comparing with published data, we discovered another paradox. In fig. 4 shows the data for the pressure in the barrel channel depending on the position of the core and the diameter of the gunpowder grain, supposedly obtained experimentally [6]. In fig. 5 shows the calculation data for approximately the same conditions (there were no complete source data in the article).

As you can see, the discrepancies in the maximum pressure values are almost 10 times, with the predicted accuracy of our model in the region of 10%. Thus, this paradox cannot be explained by the low accuracy of the mathematical model.

A series of experiments was carried out to identify the mathematical model based on the material base of the Izhevsk Pyrotechnic Laboratory. For this, pyrotechnic bombs with a caliber of 35 mm were used, loaded into the barrel with a gap of 1 mm. The trunks stood upright. According to the documentation of the manufacturers and the measurements taken, the height to which the bombs were raised averaged 35 m. Model calculations performed with the same initial conditions gave a result of 37.2 m, or 6% differential in accuracy.

6. Necessary complication of the mathematical model

From a formal point of view, the proposed model takes into account all the necessary factors that are obtained on the basis of modern ideas formed by research on rifled artillery. However, as our work showed, for the cannonball there is one parameter that in our model was not defined. The external
ballistics of the cannonball is largely dependent on the speed of its rotation. This can explain the discrepancy between the aerodynamic characteristics of the sphere obtained during modern blowing in wind tunnels and the characteristics of the cannonball obtained during firing in the XIX-th century. Moreover, the discrepancy can reach 20% or more [7]. In this regard, it is necessary to know the speed of rotation of the ball at the exit from the bore. Two extreme cases are possible. If the friction force between the cannonball and the bore is absent, then the rotation of the cannonball will be $\omega = 0$. If the adhesion between the cannonball and the barrel is perfect, similar to the toothed plate and gear, then if $v$ is the speed of the ball, and $r$ – its radius, then the angular velocity will be $\omega = v / r$.

Figure 4. The dependence of the pressure in the bore from the coordinate of the projectile [6] (pressure 1 t/inch$^2$ = 13.79 MPa = 152.4 atm).

Figure 5. The calculated dependence of the pressure in the bore from the coordinate of the projectile.

However, not all is so simple. Known as ‘balloting,’ Lt. Col. C.H. Owen, R.A, described the phenomenon as pertained to smooth bore guns in 1873. “Irregularity in the flight of the projectile, in consequence of the windage, arises from the fact that the center of the ball is below the axis of the piece, and therefore, the elastic gas in the first instance upon the upper portion of the projectile, driving it against the bottom of the bore; the shot re-acts at the same time that it is impelled forward by the charge, and strikes the upper surface of the bore some distance down, and so on, by a succession of rebounds, until it leaves the bore in an accidental direction, and with uncertain rotation, depending chiefly upon the last impact…The bores…are more or less injured by the rebounds…these rebounds increase materially with the windage” [8, p 8-9].

The Colonel illustrates the balloting with some exaggeration for effect (Fig. 6). What can be inferred from the Colonel’s explanation is that there seems to be three determinative factors associated with balloting, the amount of windage, the mass of the projectile and the amount of pressure acting upon the projectile. It also follows that the friction of each rebound would reduce the acceleration of the projectile slightly and increased the speed of rotation.

Figure 6. The motion scheme of the cannonball in the bore [8].

The problem for determining the rotation of the cannonball is that the task becomes a probabilistic task. It can be assumed that the conditions of the shot and spin of the cannonball were largely
determined by the design of the gun and the insignificant loading conditions of the cannonball. It turns out a problem whose solution is similar to that for rifled artillery, although the physical background is completely different.

In rifled artillery, the design and shape of the projectile varies greatly from one artillery system to another. Therefore, an approach was developed according to which the function of the aerodynamic drag of a projectile is considered standard, and the specific design is distinguished by correcting and multiplying it by a form factor. The aerodynamic drag function of the sphere is almost constant and independent of caliber. But this function will have to be corrected by multiplying by the form factor to take into account the uncertainty of the conditions of the shot when determining the rotation of the cannonball.

7. Conclusions
Thus, it turns out that our ideas about the ballistics of medieval smoothbore guns need a radical revision. Currently, the team of authors is trying to clarify a number of blocks of our mathematical model.

An accurate mathematical model loses its meaning due to the uncertainty of the source data. And the adiabatic formulation of the model will be quite satisfactory. But the low accuracy of the method of solving the mathematical model is excluded, since at the initial moment of ignition there are risks to obtain serious uncertainty in the result.

The addition of a mathematical model to the unit for recording the angular velocity of rotation of the cannonball is not desirable, since it will obviously have a probabilistic nature. Identification of the degree of influence of various factors is possible only by determining the form factor for the entire known data set of historical sources for smoothbore weapons.

Acknowledgment
The authors express their gratitude to a student of Kalashnikov’s Izhevsk State Technical University Nikolay Solomennikov [4, 5, 9] for the calculations. The article was prepared with the support of the Integrated Program for Basic Research Ural Branch of Russian Academy of Science, № 0427-2019-0019.

References
[1] Eliseev V V, Piskunov V A 2016 Sovremennoe mashinostroenie. Nauka i obrazovanie No 5 pp 421-430
[2] Eliseev V V, Piskunov V A 2016 Izvestiya Samarskogo nauchnogo centra RAS Vol. 18 No 1-2 pp 205-208
[3] Crawford K R, Busygina E L, Mitiukov N W 2019 IEEE Xplore. № 8933821. DOI: 10.1109/FarEastCon.2019.8934356
[4] Solomennikov N N, Mitiukov N W, Busygina E L 2015 Novy universitet. Serija: Tehnicheskie nauki No 11–12 pp 11–16. DOI: 10.15350/2221-9552.2015.11-12
[5] Solomennikov N N 2013 Novy universitet. Serija: Tehnicheskie nauki No 8–9 pp 110–118
[6] Guilmartin J F, Jr. 1989 British Naval Armaments. Royal Armouries Conference Proceedings I (London) pp 73-98
[7] Crawford K R, Mitiukov N W, Busygina E L 2020 IOP Conference Series: Materials Science and Engineering No 709 pp 022101. DOI: 10.1088/1757-899X/709/2/022101
[8] Owen C H 1873 The Principle and Practice of Modern Artillery (London)
[9] Solomennikov N N 2014 Vestnik KIGIT No 2 (44) pp 77-80