A Developed Approach to Estimate the Reliability of Atomium Bridge

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Abstract:
In this paper special attention was focused on a system that cyclic paths and one terminal, this paper produces a new technique to evaluate the reliability system and how to make a special kind of equations to arrive to the reliability of each part of the system network.

Key word: Reliability, nodes probability, edge probability, shortest path, cyclic path.

I. Introduction:
Reliability network is one of the main concerns of the early designers of computer circuits it products a nice attention for the design, like communications, electrical circuits, transport telematics[1],[2]. Reliability can be loosely defined as the ability of the system to remain operational for a periodic time.

In this setting we can state the problem related to network reliability.

The Atomium was supposed to be a temporary monument but its success made it the symbol of Brussels and a popular tourist attraction. It was constructed in 1958, which was hosted in Brussels.

The Atomium is a giant 165 billion times enlarged model representing an elementary iron crystal cell and it can be considered as a mixture between sculpture and architecture.

The Atomium was designed by the Belgian engineer Andre Waterkeyn. He entrusted this work to two architects – brothers Ander and Jean Polka, who were Waterkeyn's brothers in law.

The structure itself consist of nine stainless steel clad spheres connected by 20 tubes. According to the records form 1958, the object weighs as much as 2400 tons. In total it is 102 meters high, the tubes between two spheres have diameter 3 m and length 23 m and each spheres has a diameter 18 m and weight 250 tons.
To study such a model is a NP-hard problem, this is because most of their studies were done on models with two or more sources[3].

In Atomium bridge there is one source the same is the director(source/sink), this means cycle paths will be formed.

It is not easy to find the minimal cycle path in such situation, it is not easy to calculate the reliability of the bridge.

There are several stages in order to calculate the reliability of the Atomium bridge, it depends on the movement of people and their movements between the balls.

We rely on reliability calculation based on (input=output) the people who enter all balls are the same ones who leave it. we will explain it in the next section.

II. Reliability of Atomium bridge

In this work, we created two types of equations to calculate the probability of each ball. The first one is about the amount of people visiting the bridge, and the second one is about which ball is the most important in the tower.

The probability of edges are represented by the number of edges outside each node[4], for example, node 2 contains two edges out of it, so the probability of each visit is the half, as shown in the figure(1).
Figure (1): Represent the system of Atomium bridge and the probability of the edges.

It's same way for other node(ball), now to find the probability of each node depends on the summation of the number of edges entered for each node multiplied by the probability of each edges outside of it.

So we will get the following equations:

\[
P(2) = P(3).1 + P(6).1 + P(1).1/2 \quad \text{... (1)}
\]

\[
P(3) = P(4).1/3 + P(1).1/3 + P(7).1/3 \quad \text{... (2)}
\]

\[
P(4) = P(8).1/3 \quad \text{... (3)}
\]

\[
P(5) = P(2).1/2 + P(4).1/3 + P(9).1/3 \quad \text{... (4)}
\]

\[
P(6) = P(9).1/3 + P(1).1/3 + P(7).1/3 \quad \text{... (5)}
\]

\[
P(7) = P(8).1/3 \quad \text{... (6)}
\]

\[
P(8) = P(1).1/3 \quad \text{... (7)}
\]

\[
P(9) = P(8).1/3 \quad \text{... (8)}
\]

\[
P(1) = P(2).1/2 + P(7).1/3 + P(4).1/3 + P(5).1 + P(8).1/3 + P(9).1/3 \quad \text{... (9)}
\]

Depending on the principle of the nouns entered are the same as those who departed. We have a second set of equations:

\[
P_{62} + P_{12} + P_{32} = P_{21} + P_{25} \quad \text{... (9)}
\]

\[
P_{89} = P_{45} + P_{45} + P_{45} \quad \text{... (10)}
\]

\[
P_{51} + P_{41} + P_{71} + P_{21} + P_{91} + P_{81} = P_{12} + P_{13} + P_{16} + P_{1} \quad \text{... (11)}
\]

\[
P_{13} + P_{46} + P_{73} = P_{32} \quad \text{... (12)}
\]

\[
P_{25} + P_{45} + P_{95} = P_{15} \quad \text{... (13)}
\]
P_{87} = P_{76} + P_{71} + P_{73} \quad \ldots \ (14)

P_{76} + P_{16} + P_{96} = P_{62} \quad \ldots \ (15)

P_{18} = P_{89} + P_{87} + P_{84} + P_{81} \quad \ldots \ (16)

P_{89} = P_{96} + P_{91} + P_{95} \quad \ldots \ (17)

Such that express the probability P_{ij} by

P_{ij} = P(i) \cdot (i-j) ; i=1,2,\ldots,9 ,such that (i-j) is the probability of the edge from node i to node j, so P_{ij} = P(i) \cdot 1 ,

thus we have the following equations:

P(6) + P(1) \cdot 1/2 + P(3) = P(2) \quad \ldots \ (18)

P(8) \cdot 1/3 = P(4) \quad \ldots \ (19)

P(5) + P(4) \cdot 1/3 + P(7).1/3 + P(2).1/3 + P(9).1/3 = P(1).9/6 \quad \ldots \ (20)

P(1).1/3 + P(4).1/3 + P(7).1/3 = P(3) \quad \ldots (21)

P(2).1/2 + P(4).1/3 + P(9).1/3 = P(5) \quad \ldots (22)

P(8).1/3 = P(7) \quad \ldots (23)

P(7).1/3 + P(1).1/3 + P(9).1/3 = P(6) \quad \ldots (24)

P(1).1/3 = P(8).4/3 \quad \ldots (25)

P(8).1/3 = P(9) \quad \ldots (26)

And now there are two model of equations, after solving equations (1)-(26) we get:

P(1) = 9/13 , \ p(2)=1 , \ p(3)=11/39 , \ p(4)=1/13 , \ p(5)=3/18 , \ p(6)=11/39 , \ p(7)=1/13 , \ p(8)=3/13 , \ p(9)=1/13 .

Therefore , the reliability of Atomium bridge calculated by sacking on minimal path who passes through all the nodes[5], this process is also NP-hard problem , because the bridge contains many interfering paths and it must pass through one entrance and exit.

To search for all minimal path depended on two steps ,first step is to find all minimal path of Atomium by an algorithm will be shown later, second step is to find all minimal path who passes through all the nodes.

After display two step we a four minimal paths from node 2:

1) 2-5-1-8-7-1-8-9-6-2-1-8-4-3-2
2) 2-5-1-8-7-3-2-1-8-9-6-2-1-8-4-3-2
3) 2-5-1-8-4-3-2-1-8-7-6-2-1-8-9-6-2
4) 2-5-1-8-9-6-2-1-8-4-3-2-1-8-7-6-2

We can here draw every path as follow:

Fig(2) represent path(1) ,
Figure (2) : Represent path(1) ,the first path that pass through all nodes of Atomium.

In path(1) there is a component in series form and another in parallel form but another neither series nor parallel, to find reliability of path(1) we follow the following steps,

\[ R_{S3} = R_1 R_{18} R_8 R_{87} \]
\[ R_{S4} = R_2 R_6 R_{69} R_9 R_{98} \]
\[ R_{S2} = R_7 R_7 \]
\[ R_{S5} = R_3 R_4 R_3 R_{34} R_4 R_{84} \]
\[ R_{S1} = 1 - (1 - R_{21})(1 - R_{25} R_5) \]
\[ R_{S6} = 1 - (1 - R_{S4})(1 - R_{S5}) \]

Heir is the reliability of the series and the parallel component such that if the components in series form the reliability of them are
\[ R_S = \prod_{i=1}^{n} R_i \] where \( i = 1 \ldots n \), if the components in parallel form the reliability are
\[ R_P = 1 - \prod_{i=1}^{n} (1 - R_i) \] where \( i = 1 \ldots n \).

Fig(3) explains it ,

Figure (3) : Represent the evaluation reliability of the components of path(1).
The component $R_{S2}, R_{S3}$ is not in the form series or parallel, it's not use the law of the series or parallel, we have another Technique to solve this problem, the technique is Chapman-Kolmogorov equation\[6\] in mathematics, specifically in the theory of Markovian stochastic processes in probability theory, the Chapman-Kolmogorov equation is an identity relating the joint probability of different sets of coordinates on a stochastic process.

The Chapman-Kolmogorov equations can be expressed in terms of (possibly infinite dimensional) matrix multiplication, thus:

$$P(t+s) = P(t)P(s)$$

Where $p(t)$ is the transition matrix of jump $t$, i.e., $p(t)$ is the matrix such that entry $(i,j)$ contains the probability of the chain moving from state $i$ to state $j$ in $t$ steps.

Here we can benefit from it to find the reliability of fig(3)

So in the case of the component $R_{S2}$ and $R_{S3}$, let $u, d$ are tow node and $P(u), P(d)$ are the probability that the node $u$ and $d$ visited, let $R_{S2}$ be the way from $d$ to $u$ and $R_{S3}$ be the way from $u$ to $d$, so by Chapman-Kolmogorov equation we get

$$P(u)R_{S3} = P(d)R_{S2} \quad \ldots (1)$$

$$P(u) + p(d) = 1 \quad \ldots (2)$$

$$P(d) = 1 - p(u)$$

By substitute $p(d)$ in equation(1) we get

$$P(u) = R_{S2}(R_{S3} + R_{S2})$$

so the reliability of the fig3 was found such that

$$R_{S7} = R_{S2}(R_{S3} + R_{S2})$$

So fig (4) explained it

Figure (4) : In figure(4) compute reliability of the component of path(1).

Here reliability of the nod $2, R_{S4}, R_{S7}$ calculate as

$$R_{S8} = R_{S2}R_{S3}R_{S7}$$

scountly a new figure ,fig(5)
Figure (5): Represent the last step to evaluate the reliability of path(1)

Then,

\[ R_{sys} = R_{ss}(R_{66} + R_{88}) \]

Similarly, path 2, 3, and 4 we follow the same steps to find the reliability.

Fig (6) described path(2),

Path(2) = 2-5-1-8-7-3-2-1-8-9-6-2-1-8-4-3-2

Figure (6): Represent path(2) that pass from each nodes of Atomium bridge.

Here

\[ R_{S1} = R_{25}R_{51} \]
\[ R_{S2} = 1 - (1 - R_{S1})(1 - R_{21}) \]
\[ R_{S3} = R_{52}R_{1} \]
\[ R_{S4} = R_{23}R_{51}R_{13}R_{3} \]
\[ R_{S5} = R_{89}R_{90}R_{96}R_{62} \]
\[ R_{S6} = R_{84}R_{43} \]
\[ R_{S7} = R_{67}R_{77} \]
\[ R_{38} = 1 - (1 - R_{36})(1 - R_{37}) \]

\[ R_{39} = R_9R_3R_{23} \]

\[ R_{310} = 1 - (1 - R_{35})(1 - R_{39}) \]

\[ R_{sys} = R_{310}(R_{34} + R_{310}) \], fig(7) and fig(8) explain it.

Figure (7) and figure(8): Represent the steps of evaluate the reliability of component of path(2).

In path(3) gotten:

Path(3) = 2-5-1-8-4-3-2-1-8-7-6-2-1-8-9-6-2 , fig(10) describe it.

Figure (10): Represent path(3) that pass from all the nodes of Atomium bridges

\[ R_{s1} = R_2R_3R_{31} \]

\[ R_{s2} = 1 - (1 - R_{s1})(1 - R_{21}) \]

\[ R_{s3} = R_2R_{32}R_1R_{18}R_8 \]

\[ R_{s4} = R_{32}R_4R_{43}R_3R_{32} \]

\[ R_{s5} = R_8R_9R_{97} \]

\[ R_{s6} = R_9R_9R_{96} \]

\[ R_{s7} = 1 - (1 - R_{s2})(1 - R_{s6}) \]

\[ R_{s8} = R_{35}R_4R_{42} \]
RS9 = 1 - (1 - RS4)(1 - RS8)

So the reliability system is:

\[ R_{sys} = R_{S3}(R_{S9} + R_{S3}) \]

Figure (11) and Figure (12) explained it.

**Path(4):**

2-5-1-8-9-6-2-1-8-4-3-2-1-8-7-6-2

Figure (13): Represent path(4) that pass through all nodes of Atomium bridge.

So the reliability is:

- \[ R_{S1} = R_{25}R_{5}R_{51} \]
- \[ R_{S2} = 1 - (1 - R_{S1})R_{21} \]
- \[ R_{S3} = R_{2}R_{S2}R_{1}R_{18}R_{8} \]
- \[ R_{S4} = R_{48}R_{4}R_{34}R_{3}R_{32} \]
- \[ R_{S5} = R_{89}R_{9}R_{69} \]
- \[ R_{S6} = R_{87}R_{7}R_{67} \]
- \[ R_{S7} = 1 - (1 - R_{S5})(1 - R_{S6}) \]
- \[ R_{S8} = R_{S3}R_{9}R_{S7} \]
- \[ R_{S9} = 1 - (1 - R_{S4})(1 - R_{S8}) \]
- \[ R_{sys} = R_{S3}(R_{S9} + R_{S3}) \]

Fig(14), fig(15) explained it.
Algorithm of minimal-path is

**Input:** Connection Matrix (CM)

**Output:** Sumation of all paths (Sum path)

for i=1 to size(CM)-1 do

1. Change the value of remain element of CM according to equation (2)
   \[ a'_{ij} = a_{ij} + a_{il}a_{lj} \]

2. If node l is removed, where \( i \neq j \), \( i \neq l \), \( j \neq l \), \( 1 \leq i \leq n \), \( 1 < j \leq n \) for \( i=1,2,...,n \).

3. Eliminate the second node by deleting the second row and column of CM.

   \[ CM(i=1:size(CM)-1,j=1:size(CM)-1) = CM(i\neq2,j\neq2) \]

Return the eliminated CM which equal to Sum path.

III. Conclusion

This paper presented a new idea for finding/evaluating the reliability network that are of the type one terminal network. This technique can be applied for all network of the type one and two terminal.

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