GLOBAL DYNAMICS FOR NEWTON AND PLANCK

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We discuss recently introduced scale-free Einstein equations, where the information from their trace part is lost. These equations are classically equivalent to General Relativity, yet the Newton constant becomes a constant of integration or a global dynamical degree of freedom. Thus, from the point of view of standard quantization, this effective Newton constant is susceptible to quantum fluctuations. This is similar to what happens to the cosmological constant in the unimodular gravity where the trace part of the Einstein equations is lost in a different way. Using analogy with the Henneaux-Teitelboim covariant action for the unimodular gravity, we consider different general-covariant actions resulting in these dynamics. This setup allows one to formulate the Heisenberg uncertainty relations for the Newton constant and canonically conjugated quantities. Unexpectedly, one of such theories also promotes the Planck’s quantum constant to a global degree of freedom, which is subject to quantum fluctuations. Following analogy with the unimodular gravity, we discuss non-covariant “unimatter” and “unicurvature” gravities describing the scale-free Einstein equations. Finally, we show that in some limit of the Yang-Mills gauge theory a “frozen” axion-like field can emulate the gravitational Newton constant or even of the quantum Planck constant.

1 Introduction and Main Idea

For the last six billion years our Universe has been expanding with an acceleration. This phenomenon is well described by the gravitational action

$$S[g] = -\frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left(R + 2\Lambda\right),$$

(1)

where the cosmological constant $\Lambda$ has inverse dimensions to the Newton constant $G_N$, while their product is inexplicably tiny:

$$G_N\Lambda \sim 10^{-122}.$$ 

(2)

The origin of this extremely small number remains a mystery. In particular, it is not clear how to keep this product small when taking into account quantum corrections to $\Lambda$ and $G_N$. But

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*We use (+, −, −, −) signature convention and units where $c = 1$, while in most parts of the text $\hbar = 1$ and we restore $\hbar$ only to stress the quantum nature of a formula.*
what if these constants are in fact dynamical quantities? In that case either some dynamics may enforce (2) or quantum cosmology (possibly together with anthropic reasoning) may help to select only those initial values which are close to (2). The minimal number of corresponding degrees of freedom is achieved when \( \Lambda \) and/or \( G_N \) are global degrees of freedom. Such degrees of freedom are space-independent – they remain constant along every Cauchy surface for arbitrary 3+1 foliation of spacetime. The simplest example of a global degree of freedom is provided by the so-called unimodular gravity (UG). Indeed, since Einstein’s paper \(^1\) it is well known that trace-free equations

\[
G_{\mu\nu} - \frac{1}{4} g_{\mu\nu} G = 8\pi G_N \left( T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} T \right),
\]

are equivalent to General Relativity (GR) equipped with \( \Lambda \) being a constant of integration, or a \textit{global} degree of freedom, see e.g.\(^2,3,4,5,6,7\). This can be checked by applying covariant derivative \( \nabla^\mu \) to both sides of this equation and consequently using the Bianchi identity along with the assumed conservation of the total energy-momentum tensor (EMT). These trace-free Einstein equations (3) are invariant under vacuum shifts of the total EMT

\[
T_{\mu\nu} \to T_{\mu\nu} + cg_{\mu\nu},
\]

with \( c = c(x^\mu) \), and in particular with \( c = \text{const.} \)

In \(^8\) we promoted the Newton constant to a similar global degree of freedom. This is achieved again by loosing the information contained in the trace of the Einstein equations. Namely, instead of the trace-free equations (3) one can write \textit{normalized} or \textit{trace-trivial} equations

\[
\frac{G_{\mu\nu}}{G} = \frac{T_{\mu\nu}}{T},
\]

or in a more regular form

\[
G_{\alpha\beta} T_{\mu\nu} = G_{\mu\nu} T_{\alpha\beta}.
\]

These equations are \textit{scale-free} and invariant with respect to rescaling

\[
T_{\mu\nu} \to c T_{\mu\nu},
\]

by a constant, or even an arbitrary function \( c(x^\mu) \). In \(^8\) we demonstrated that equations (5) (or (6)) are equivalent to the usual GR, however, now \( G_N \) becomes a constant of integration. Similarly to the unimodular case, this equivalence follows from the Bianchi identity and assumed EMT conservation.

2 Unimodular Gravity

Let us first briefly recall UG. A simple covariant action\(^7\) for UG was introduced by Henneaux and Teitelboim in \(^7\)

\[
S[g,W,\Lambda] = -\frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left[ R + 2\Lambda \left( 1 - \nabla_\mu W^\mu \right) \right].
\]

For every foliation of spacetime, variation with respect to the spatial components \( W^i \) yields a constraint \( \partial_i \Lambda = 0 \). Hence, \( \Lambda \) is a global quantity – it is space independent. Following “quantization without tears” by Faddeev-Jackiw\(^10\), one can use this constraint in the action to infer that the energy density \( \varepsilon_\Lambda = \Lambda / (8\pi G_N) \) is a canonical momentum conjugated to another global quantity often called cosmic time

\[
\tau(t) = \int_{\Sigma} d^4x \sqrt{-g} W^t(t,x).
\]

\(^8\)For a higher-derivative and Weyl-invariant formulation of UG see\(^9\).
Variation of (8) with respect to \( \Lambda \) gives \( \nabla_\mu W^\mu = 1 \). For \( W \) vanishing at the boundary (e.g. at the spatial infinity) the last equation implies that \( \tau (t) \) measures four-volume \( \Omega \) of space-time between Cauchy hypersurfaces \( \Sigma_2 \) and \( \Sigma_1 \):

\[
\tau (t_2) - \tau (t_1) = \int_{\Omega} d^4 x \sqrt{-g} .
\]  

(10)

As \( \varepsilon_\Lambda \) and \( \tau \) are canonically conjugated, one can use the correspondence principle of quantization and find that they are subject to the Heisenberg uncertainty relation

\[
\delta \varepsilon_\Lambda \times \delta \tau \geq \frac{\hbar}{2} .
\]  

(11)

similar to the energy-time uncertainty relation. This uncertainty relation can be also written as

\[
\delta \Lambda \times \delta \int_{\Omega} d^4 x \sqrt{-g} \geq 4 \pi \ell_{Pl} .
\]  

(12)

where \( \ell_{Pl} = \sqrt{\hbar G N} \) is the Planck length. These unavoidable quantum fluctuations are the main difference of UG from the usual GR. This property is insensitive to higher order curvature invariants appearing in effective action for gravity. These fluctuations may be relevant close to singularities. Except of this phenomenon, perturbative “unimodular” gravity is equivalent to GR also in quantum realm, for recent discussions see e.g. 11,12,13,14,15,16,17,18,19,20,21,22,23,24. There are only two ways to claim that (12) is not applicable. Namely, either \( \int_{\Omega} d^4 x \sqrt{-g} \) is not well defined, say it is divergent, or one is ready to violate the main postulate of the canonical quantization that the Poisson bracket for the canonically conjugated quantities are directly mapped into commutators of the corresponding operators. Another conceptual issue with a potential to undermine the usefulness of (12) is the following. It may be difficult for a local observer to measure a total space-time volume between Cauchy hypersurfaces, as this involves events with space-like separation. In this case, four volume could have arbitrary large fluctuations, provided they are due to causally disconnected regions. These arbitrary large fluctuations of four volume would imply that the observer can be at an eigenstate of \( \Lambda \).

To illustrate the importance of the uncertainty relation let us take closed radiation-dominated Friedmann Universe

\[
ds^2 = a^2 (\eta) \left[ d\eta^2 - d\chi^2 + \sin^2 \chi (d\theta^2 + \sin^2 \theta d\phi^2) \right] ,
\]  

(13)

where \( a (\eta) = a_m \sin \eta \), so that conformal time \( \eta \) and radial coordinate \( \chi \) both belong to the interval \((0, \pi)\). The total four volume for this universe is

\[
\Omega_{tot} = \int_0^\pi d\eta \int d^3 x \sqrt{-g} = 4 \pi a_m^4 \int_0^\pi \sin^2 \eta d\eta \int_0^\pi \sin^2 \chi d\chi = \frac{3 \pi^3}{4} a_m^4 .
\]  

(14)

Quasiclassical approximation requires that at least \( \delta \tau < \Omega_{tot} \). Hence, under the assumption of quasiclassical evolution, equation (11) yields the bound

\[
\varepsilon_\Lambda > \delta \varepsilon_\Lambda > \frac{4}{3 \pi^3} \frac{\hbar}{a_m^4} .
\]  

(15)

For a large Universe this is a tiny bound independent of the composition and structure of the radiation. It is useful to compare (15) with the minimal energy density \( \varepsilon_m = 3 / (8 \pi G N a_m^2) \). Hence, \( \varepsilon_m \gg \delta \varepsilon_\Lambda \) provided \( a_m \gg \ell_{Pl} \).
3 Action for Scale-Free Gravity, Changing Gravity

Following the Henneaux-Teitelboim formulation of the UG (8) discussed above, it is easy to write a similar action to promote the Newton constant to a global degree of freedom:

$$S[g, C, \alpha] = \frac{1}{2} \int d^4 x \sqrt{-g} (\nabla_{\mu} C^\mu - R) \alpha.$$ (16)

Variation with respect to $C^\mu$ yields $\partial_{\mu} \alpha = 0$ while variation with respect to the metric gives

$$\alpha G_{\mu\nu} = T_{\mu\nu}.$$ (17)

Thus $G_N = (8\pi \alpha)^{-1}$ becomes a constant of integration. In complete analogy with Henneaux-Teitelboim formulation of UG discussed in the previous section, we obtain that $\alpha$ is a canonical momentum conjugated to the global quantity

$$\varrho(t) = \frac{1}{2} \int_{\Sigma} d^3 x \sqrt{-g} C^t(t, x).$$ (18)

For appropriate boundary conditions on $C^i$, due to the constraint, $\nabla_\mu C^\mu = R$, this global quantity measures the integrated Ricci scalar

$$\varrho(t_2) - \varrho(t_1) = \frac{1}{2} \int_{\Omega} d^4 x \sqrt{-g} R.$$ (19)

For canonically conjugated pair $(\varrho, \alpha)$ the Heisenberg uncertainty relation reads $\delta \varrho \times \delta \alpha \geq \hbar/2$. One can write it using observable quantities and the Planck length $\ell_{Pl} = \sqrt{\hbar G_N}$ as

$$\frac{\delta G_N}{G_N} \times \frac{\delta \int_{\Omega} d^4 x \sqrt{-g} R}{\ell_{Pl}^2} \geq 8\pi.$$ (20)

Again this inequality should have rather nontrivial consequences close to singularities. Quasi-classical description implies that

$$8\pi G_N \left| \int_{\Omega} d^4 x \sqrt{-g} T \right| \gg \delta \int_{\Omega} d^4 x \sqrt{-g} R.$$ (21)

Thus, there is a lower bound on fluctuations of the Newton constant

$$\frac{\delta G_N}{G_N} \gg \hbar \left| \int_{\Omega} d^4 x \sqrt{-g} T \right|^{-1}.$$ (22)

Hence, conformal anomaly plays a crucial role for the magnitude of fluctuations of $\delta G_N$. On the other hand, most of the fields in standard model are conformal for high temperatures. But on top of conformal anomaly there can be vacuum energy $\epsilon_\Lambda$ as the source of $T$. In that case (22) implies that

$$\frac{\delta G_N}{G_N} \gg \frac{\hbar}{4 \epsilon_\Lambda} \left( \int_{\Omega} d^4 x \sqrt{-g} \right)^{-1}.$$ (23)

The rather weak lower bounds (22) and (23) is a novel extension of material presented in\textsuperscript{8}.

\textsuperscript{c}cf. 25,26 written for local version of the vacuum energy sequester 27
4 Action for Scale-Free Gravity, Changing Matter

Interestingly there is another way to write an action for the Newton constant as a global degree of freedom. Namely, one can preserve the Einstein-Hilbert action, but change the usual action for matter fields $\Phi_m$,

$$S_0 [g, \Phi_m] = \int d^4 x \sqrt{-g} \mathcal{L}_m \,,$$

(24)

to one (c.f. 28) similar to (16):

$$S [g, \beta, L, \Phi_m] = \int d^4 x \sqrt{-g} \beta (\mathcal{L}_m - \nabla_\mu L^\mu) \,.$$

(25)

In this formulation of the theory, on the right hand side of the Einstein equations one obtains a rescaled EMT for matter fields $\Phi_m$

$$T_{\mu\nu} = 2 \sqrt{-g} \delta S / \delta g_{\mu\nu} = \beta T_{\mu\nu}^{(m)} \,,$$

(26)

where

$$T_{\mu\nu}^{(m)} = 2 \sqrt{-g} \delta S_0 / \delta g_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta (\sqrt{-g} \mathcal{L}_m)}{\delta g_{\mu\nu}} \,.$$ 

(27)

Variation with respect to $L^\mu$ yields $\partial_\mu \beta = 0$. Hence, the effective Newton constant, $\bar{G}_N$, is just a rescaling of the constant from the Einstein-Hilbert action

$$\bar{G}_N = G_N \beta \,.$$ 

(28)

Following the same path as in (9) and (18), the global dynamical degree of freedom canonically conjugated to $\beta$ is

$$I (t) = - \int_{\Sigma} d^3 x \sqrt{-g} L^t (t, x) \,.$$ 

(29)

Under appropriate boundary conditions, due to $\nabla_\mu L^\mu = \mathcal{L}_m$, this variable measures the matter action between the Cauchy hypersurfaces $\Sigma_2$ and $\Sigma_1$

$$I (t_2) - I (t_1) = - \int_{\Omega} d^4 x \sqrt{-g} \mathcal{L}_m \,.$$ 

(30)

Applying again the Heisenberg uncertainty relation to the canonical pair $(I, \beta)$ one obtains $\delta I \times \delta \beta \geq \hbar/2$, which can be written in terms of observables as

$$\delta \bar{G}_N \times \delta \left( \int_{\Omega} d^4 x \sqrt{-g} \mathcal{L}_m \right) \geq \frac{1}{2} \ell_{Pl}^2 \,.$$ 

(31)

where the Planck length $\ell_{Pl} = \sqrt{\hbar G_N}$ corresponds to $G_N$ from the Einstein-Hilbert action.

More interestingly, the introduction of $\beta$ also rescales the commutation relations for usual matter. For instance, for the usual scalar field $\phi$ the canonical momentum gets rescaled similarly to the EMT:

$$\pi = \beta \pi^{(m)} = \beta \sqrt{-g} \frac{\partial \mathcal{L}_m}{\partial \dot{\phi}} \,.$$ 

(32)

Thus, the canonical commutator relation, $[\phi (x), \pi (y)] = i \hbar \delta (x - y)$, implies that usual commutator gets rescaled as well

$$[\phi (x), \pi^{(m)} (y)] = \frac{i \hbar}{\beta} \delta (x - y) \,.$$ 

(33)

Hence, the effective Planck constant is

$$\bar{\hbar} = \frac{\hbar}{\beta} \,.$$ 

(34)
Interestingly, the Planck length (and time) $\ell_{Pl} = \sqrt{\hbar G_N}$ remains invariant under this rescaling, as $\hbar G_N = \hbar G_N$. Using this invariance one can write (31) as

$$\frac{\delta G_N}{G_N} \times \delta \int_{\Omega} d^4x \sqrt{-g} L_m \geq \frac{1}{2} \bar{\hbar}.$$  \hspace{1cm} (35)

Following the same logic as in (22) one obtains a new lower bound

$$\frac{\delta G_N}{G_N} \geq \frac{1}{2} \bar{\hbar} \left| \int_{\Omega} d^4x \sqrt{-g} L_m \right|^{-1}. \hspace{1cm} (36)$$

Even more unorthodox, one can interpret the uncertainty relation $\delta I \times \delta \beta \geq \hbar/2$ as uncertainty in the effective Planck constant. Indeed, through (34) fluctuations $\delta \beta$ correspond to $\delta \bar{\hbar} = \hbar \delta \beta / \beta^2$ so that

$$\delta \bar{\hbar} \times \delta \int_{\Omega} d^4x \sqrt{-g} L_m \geq \frac{1}{2} \bar{\hbar}^2, \hspace{1cm} (37)$$

with the corresponding lower bound on fluctuations

$$\frac{\delta \bar{\hbar}}{\bar{\hbar}} \geq \frac{1}{2} \bar{\hbar} \left| \int_{\Omega} d^4x \sqrt{-g} L_m \right|^{-1}. \hspace{1cm} (38)$$

For many theories the Lagrangian density is vanishing on equations of motion. However, similarly to (23) one can include vacuum energy $\varepsilon_\Lambda$ into $L_m$.

5 Unimodular, Unicurvature and Unimatter

Fixing a gauge and a class of coordinates in the UG action (8) such that

$$W^\mu = \delta^\mu_t \frac{t}{\sqrt{-g}}, \hspace{1cm} (39)$$

before performing the variation ensues

$$\int d^4x \Lambda \left[ 1 - \sqrt{-g} \right]. \hspace{1cm} (40)$$

This is a usual non-covariant formulation of the UG $^{29,30,31,32}$. Interestingly, this formulation still results in the seemingly covariant traceless Einstein equations (3). In $^8$ we demonstrated that one can do the same to obtain non-covariant formulations for theories with the globally dynamical Newton (16) and Planck constants (25).

Indeed, similarly to (39) one can fix

$$C^\mu = \pm \delta^\mu_t \frac{t}{\sqrt{-g}} M_\alpha^2, \hspace{1cm} (41)$$

in the action, before variation. Here, " + " corresponds to the positive Ricci scalar, while " - " to the negative one, while $M_\alpha$ is some mass scale introduced for dimensional reasons. Then, in units where $M_\alpha = 1$, the action takes a non-covariant form

$$S \left[ g, \alpha \right] = \frac{1}{2} \int d^4x \left( \pm 1 - \sqrt{-g} R \right) \alpha. \hspace{1cm} (42)$$

Thus instead of the “unimodular” constraint $\sqrt{-g} = 1$ we have a “unicurvature” condition $\sqrt{-g} R = \pm 1$. In $^8$ we provided a proof that the action (42) together with the usual matter action (24) describe same trace-trivial equations (5). Thus, along with the well-known unimodular gravity one can study the novel “unicurvature” gravity (42). Here it was crucial that the gauge fixing does not affect the matter sector so that the total EMT is conserved.
Finally, one can fix
\[ L^\mu = \pm \delta^\mu_\nu \frac{t}{\sqrt{-g}} M_\beta^4 , \] 
before varying the action (25). Here \( M_\beta \) is again some mass scale in units of which the action takes an unusual non-covariant form
\[ S \left[ g, \beta, \Phi_m \right] = \int d^4x \beta \left( \sqrt{-g} \mathcal{L}_m \mp 1 \right) . \] 

Variation with respect to \( \beta \) results in \( \sqrt{-g} L_m = \pm 1 \) which by analogy with the unimodular constraint \( \sqrt{-g} = 1 \) can be called “unimatter” condition. In \(^8\) we provided a proof that action (44) accompanied with the usual Einstein-Hilbert action again describe the same trace-trivial equations (5). Thus, this construction can be called “unimatter” gravity. The key point of the proof in \(^8\) is that the transformation properties of (44) with respect to diffeomorphisms imply that
\[ \nabla^\nu \left( \beta T_{\mu\nu}^{(m)} \right) = -\mathcal{L}_m \partial_\mu \beta . \] 
Then the constancy of \( \beta \) ensues from the Bianchi identity.

### 6 Frozen Axions

In \(^3^3\) it was showed\(^4\), that vector field \( W^\mu \) in UG can be exchanged in (8) with a more convenient Chern-Simons current of a (non-abelian or abelian) gauge field \( A_\mu \). In this way instead of \( \nabla_\mu W^\mu \) one can plug in \( F_{\alpha\beta} \hat{F}_{\alpha\beta} \), where the gauge field strength is \( F_{\mu\nu} = D_{\mu} A_\nu - D_{\nu} A_\mu \) with the covariant derivative \( D_\mu = \nabla_\mu + iA_\mu \), while the Hodge dual is defined as usual
\[ F_{\star\alpha\beta} = \frac{1}{2} \varepsilon_{\alpha\beta\mu\nu} \sqrt{-g} F_{\mu\nu} \equiv \frac{1}{2} F^{\alpha\beta\mu\nu} . \]

In the absence of the usual kinetic term \(-F_{\alpha\beta} F^{\alpha\beta}/4g^2\), variation of the action with respect to \( A_\mu \) forces \( \Lambda \) to be constant. It is easy to further extend \(^3^3\) this action to resemble the one of the usual axion, cf. \(^3^5\)
\[ S \left[ g, A, \theta \right] = \int d^4x \sqrt{-g} \left[ -\frac{R}{16\pi G_N} + \frac{1}{2} (\partial \theta)^2 + \frac{\theta}{f_\Lambda} F_{\alpha\beta} F^{\star\alpha\beta} - V_\Lambda (\theta) \right] , \]

where now \( \theta \) is a canonically normalized pseudoscalar, while \( f_\Lambda \) is some mass scale emulating axion decay constant and \( V_\Lambda (\theta) \) is a “potential”. We would like to stress again, that it is the absence of the usual kinetic term for the gauge field \(-F_{\alpha\beta} F^{\alpha\beta}/4g^2\) which freezes \( \theta \) to be constant. This gives hopes to embed unimodular gravity in some more usual high energy system. Indeed, this action can appear from the usual axion and Yang-Mills construction in the dynamical regime when one can neglect the usual kinetic term \(-F_{\alpha\beta} F^{\alpha\beta}/4g^2\) for the gauge field. Naively, this setup corresponds to a confinement or an infinitely strong coupling \( g \rightarrow \infty \) in IR.

In \(^8\) we proposed that the same procedure can be done for theories with the globally dynamical Newton (16) and Planck constants (25). Namely (16) can be extended to
\[ S \left[ g, A, \nu \right] = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} \nu^2 R + \frac{1}{2} (\partial \nu)^2 + \nu \frac{\nu}{f_\alpha} F_{\gamma\sigma} F^{\gamma\sigma} - V_\alpha (\nu) \right] , \]

where now \( \nu \) is another frozen “axion”, which is constant on all solutions, while \( f_\alpha \) is some mass scale and \( V_\alpha (\nu) \) is a corresponding potential. The presence of non-minimal coupling to gravity breaks shift-symmetry. Thus, potential and the ensuing cosmological constant appear naturally in this setup. This action is written in the Jordan frame and we remind the reader that the standard Einstein-Hilbert term is absent. Connecting to (16) one notices that \( \alpha = \nu^2 \), so that the effective Newton constant is given by \( G_N = 1/(8\pi \nu^2) \).

\(^4\)cf. \(^3^4\)
Completely analogously to the previous cases, one can extend (25) to

\[
S[g, \eta, A, \Phi_m] = \int d^4x \sqrt{-g} \left[ \frac{1}{2} (\partial \eta)^2 - V_\beta(\eta) + \frac{\eta^2}{M^2_m} \mathcal{L}_m - \frac{\eta}{f_\beta} F_\mu\nu F^{*\mu\nu} \right],
\]

(48)

where \( \eta \) is again a frozen “axion”, which is constant on all solutions. This frozen “axion” universally couples to the Lagrangian \( \mathcal{L}_m \) of all other matter fields \( \Phi_m \). This coupling is quadratic, as this is the lowest possible order which does not introduce ghosts. Due to this coupling the shift-symmetry is broken. Hence, potential \( V_\beta(\eta) \) with the corresponding cosmological constant are naturally incorporated in this construction. The usual dimensions of \( \eta \) are restored by introducing two mass scales \( M_m \) and \( f_\beta \). Here gravity is described by the standard Einstein-Hilbert action. Connecting to (25) one notices that \( \beta = \eta^2/M^2_m \), so that the effective Newton constant (28) is given by \( \bar{G}_N = G_N \eta^2/M^2_m \) while the effective Planck constant (34) is \( \bar{h} = \hbar M^2_m/\eta^2 \).

Finally, it is worth mentioning that combining (48) or (47) with the (46) provides an “axionic” setup for the vacuum energy sequestering\(^{27,25}\) and brings it closer to the usual particle physics models.

7 Conclusions

In\(^8\) we proposed theories where the effective Newton constant and Planck constants are global dynamical degrees of freedom. In this proceeding we first presented scale-free or scale trivial Einstein equations where the Newton constant is the constant of integration. In Section II we recall covariant formulation of the unimodular gravity (UG) due to Henneaux and Teitelboim.\(^7\) In particular we stressed the presence of the unavoidable uncertainty relation which distinguishes quasiclassical UG from GR. Then, in Section III and IV we presented two distinct extensions of Henneaux and Teitelboim construction to describe global dynamics of \( G_N \) and \( \bar{h} \), (16) and (25) respectively. Extending\(^8\), we derived new lower bounds on quantum fluctuations of effective \( G_N \), see (22), (36) and (38) for effective \( \bar{h} \). Then, in Section V we discussed non-covariant formulations of such theories: so-called “unimatter” (44) and “unicurvature” (42) gravities. Finally, in Section VI we considered how one can embed covariant formulations of such theories into axion and a confined Yang-Mills gauge theory.

There remain a lot of issues to understand better in such theories. In particular one can mention: minisuperspace quantum cosmology; canonical analysis and quantization for axionic construction; boundary terms and matching conditions; and finally global degrees of freedom in the presence of horizons and more importantly close to the end-of-time singularities of GR. It would be exciting, if quantum mechanics of such global degrees of freedom could solve the global end/beginning of time problems in GR.

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