Synthesis and sensitivity analysis of the crank-rocker mechanism

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Abstract. Based on the theoretical background, this paper presents a design of a crank-rocker mechanism, which is also known as extreme case of the four-bar linkage. This is very common mechanism used for stone crushing in the practice. Initial functional design is performed by the method of geometric synthesis in combination with analytical calculations. The most common problem associated with the synthesis of the crank-rocker mechanism is that of designing the linkage for a given oscillation angle and a given time ratio. To improve the efficiency of the device is performed the sensitivity analysis in the MSC.ADAMS with the goal to refine the designed geometric parameters.

1. Introduction
The technical progress that came with information technology brought the new possibilities how to analyse and synthesize the construct design, not only by analytical or graphical solutions, or by experimental measurement, but also by simulating a virtual prototype (VP) mathematical model created in an appropriate software [1–11]. This is possible by the fact that current computer programs have implemented the automatic generation and solution of motion equations. Correct design of these VP means determination of values of proposed parameters of the device so that the behaviour of the system is optimal according to pre-selected criteria [12–16]. This paper present both graphical-analytical solution of geometric parameters of crank-rocker mechanism and their solution realized in selected software environment. Create a VP model with all its physical parameters so that it simulates the behaviour of a real body is sometimes difficult. Sometimes a relatively large number of input parameters enter to the simulation, which have a greater or lesser impact on the correctness of the solution. In this paper the design with a solution of kinematic synthesis of crank-rocker mechanism is presented with application of graphical-analytical method of synthesis with addition of sensitivity analysis in software MSC.ADAMS [17–23].

2. Theoretical basis of the crank-rocker mechanism
The aim of the synthesis of the mechanical system is to design a suitable type of structure and its parameters (configuration and dimensions of the links) so that the obtained mechanical system for the prescribed movement of the input drive links generates the required course of motion of the output driven links [17, 18].
2.1. Four-bar mechanism
In figure 1 set of bodies with three movable links that are attached to each other and to the frame by pin supports is shown. The length of the individual links of this four-bar mechanism, their mutual configuration and the force effects have a significant effect on the extent of their mobility [1, 2, 3, 17]. To correct work of the four-bar mechanism, this condition must be apply:

\[ l_{\text{min}} + l_{\text{max}} \leq l' + l'' \]  

(1)

where \( l_{\text{min}} \) and \( l_{\text{max}} \) are the lengths of the shortest and the longest link and \( l' \) and \( l'' \) are the lengths of the other links.

![Figure 1. Kinematic design of a four-bar mechanism.](image)

2.2. The crank-rocker linkage – the rocker amplitude problem
The crank-rocker linkage is the type of linkages where the crank rotates through 360° and the output link or rocker oscillates through an angle \( \theta \). The limiting positions of the rocker occur when the crank and coupler are collinear as shown in the figure 2. In general, the time required for the rocker oscillation in one direction is different from the time required for the other direction. As indicated above, the ratio of the times required for the forward and return motions is called a time ration.

![Figure 2. Crank-rocker mechanism in extreme position.](image)

In the crank-rocker mechanism, the crank moves through the angle \( \psi \) while the rocker moves from \( B_1 \) to \( B_2 \) through the angle \( \theta \). On the return stroke, the crank moves through angle \( 360^\circ - \psi \), and the rocker moves from \( B_2 \) to \( B_1 \) through the angle \( -\theta \). Assuming that the crank moves with constant
angular velocity, the ratio of the times for the forward and reverse strokes of the follower can be related directly to the angles in figure 2. The crank angle for the forward stroke is $\psi$ or $180^\circ + \alpha$. The crank angle for the return stroke is $360^\circ - \psi$ or $180^\circ - \alpha$. Therefore, the time ratio $Q$ can be written as:

$$Q = \frac{180^\circ + \alpha}{180^\circ - \alpha}$$

where $\alpha$ is given in degrees.

The most common problem associated with the synthesis of the crank-rocker mechanism is that of designing the linkage for a given oscillation angle and a given time ratio. The first step in the synthesis is to compute the angle $\alpha$. This can be done by solving equations (1) for $Q$. Then:

$$\alpha = \frac{Q - 1}{Q + 1}180^\circ$$

Once $\alpha$ is known, there are a number of ways to proceed with the design. To calculate the values of the $r_2$ and $r_3$ are used the equations which parameters are in the figure 2:

$$r_2 + r_3 = A \cdot B_1$$

and

$$r_3 - r_2 = A \cdot B_2$$

therefore

$$r_2 = \frac{A\cdot B_1 - A\cdot B_2}{2}$$

and

$$r_3 = \frac{A\cdot B_1 + A\cdot B_2}{2}$$

It should be noted that during the solution procedure, several options are more detail described in [24]. There are an infinite number of options for selecting these parameters, and each selection will give a different link position. It should also be noted that not all solutions are valid [1, 3, 4, 24].

2.3. Transmission angle

Based on figure 2, the crank-rocker problem is characterized by seven variables ($\alpha$, $\phi$, 0, $r_1$, $r_2$, $r_3$, $r_4$). Before we can solve the problem, we need to specify five of the seven variables. For each value chosen for one of the input variables, we will compute different values $r_2$ and $r_3$. Therefore, we need to establish criteria by which we can compare different design with the goal of identifying the optimum design. A common criterion to use is the transmission angle. For a crank-driven linkage, the transmission angle ($\eta$) is the internal angle between the coupler and the output link. When $\eta = \pm\pi/2$ a force from the coupler on the link 4 will produce the maximum output torque. When $\eta = 0^\circ$ or $\eta = \pi$, the coupler force will produce zero torque on link 4 regardless of the size of the force from the coupler. Therefore for optimum force transmissibility, we would like the transmission angle to be as close to $\pi/2$ as possible for all positions of the linkage. The positions for the linkage when the maximum ($\eta_{\text{max}}$) and minimum ($\eta_{\text{min}}$) transmission angles occur are shown in figure 3. Typically, a poor transmission angle corresponds to a large value of $[\pi/2 - \eta_{\text{max/min}}]$. Note that the maximum and minimum values for the transmission angle do not occur at the extreme positions of $r_4$, but $\eta_{\text{max}}$ and $\eta_{\text{min}}$ can be easily computed using the geometry in figure 3.
Figure 3. Positions corresponding to the maximum and minimum transmission angles.

The equations are:

$$
\eta_{\text{max}} = \cos^{-1}\left[\frac{r_4^2 - (r_1 + r_2)^2 + r_3^2}{2r_3r_4}\right] \tag{8}
$$

$$
\eta_{\text{min}} = \cos^{-1}\left[\frac{r_4^2 - (r_1 - r_2)^2 + r_3^2}{2r_3r_4}\right] \tag{9}
$$

If $\eta_{\text{max}}$ is negative, then $\eta_{\text{max}} = \pi + \eta_{\text{max}}$. Otherwise, $\eta_{\text{max}} = \eta_{\text{max}}$. Similar conditions apply to $\eta_{\text{min}}$.

3. Synthesis of the rock-crusher mechanism

The task is to propose a rock-crusher mechanism as shown in figure 4, which is to determine the proportions and configurations of the individual links so that the values of the input parameters are observed: vibration angle $\theta = 80^\circ$, time ratio $Q = 1.1$, distance of the ground pivots $A^*$ and $B^*$, i.e. $r_1 = 3000$ mm. The task is using the method of graphic-analytical synthesis and sensitivity analysis, to determine the lengths of links $r_2$, $r_3$ a $r_4$ so that:

- the proposed mechanism must acted as a crank-rocker mechanism,
- the transmission angle $\eta$ must be during it motion cycle, as much as possible around $90^\circ$, because by observing this condition the maximum torque per working link of the mechanism will be deduced.
3.1. Geometric synthesis using the Hall method

To solve this problem is chosen Allen Hall's method [24, 25–30].

The input parameters are: vibration angle $\theta$, time ratio $Q$, distance of links with frame $r_1$. This procedure consists in determining the geometric position for all possible positions of the B link at the first and second extreme deflection positions. We proceed as shown in figure 5 as follows [4]:

1. From the equation (3) the angle $\alpha$ is calculated:

$$\alpha = \frac{Q - 1}{Q + 1} \times 180^\circ = \frac{1.1 - 1}{1.1 + 1} \times 180^\circ = 8.57^\circ$$ (10)

2. Distance $|A^*B^*| = r_1 = 3000$ mm.

3. Draw the line $a$ through the point $A^*$ at an angle $\theta/2 - \alpha$ (positive clockwise) relative to $A^*B^*$.

4. Draw the line $b$ through the point $B^*$ at an angle $\theta/2$ (positive counter clockwise) relative to $A^*B^*$.

At the intersection of lines $a$, $b$ we get a point $G$.

5. Point $G'$ is axially symmetric with point $G$ along the line segment $A^*B^*$.

6. Draw the circle arc $k_2$ of radius $|GA^*|$ centred at $G$ and the second circle arc $k_1$ of radius $|G'A^*|$ centred at $G'$.

7. Draw the line $c$ through the point $B^*$ at an angle $\theta$ (positive counter clockwise) to the $A^*B^*$ and $B_{2\text{max}}$ is the intersection of the circular arc $k_2$ with the line $c$.

8. Draw the line $d$ through the point $B^*$ at an angle $\theta$ (positive clockwise) to the $A^*B^*$ and $B_{1\text{max}}$ is the intersection of the circular arc $k_1$ with the line $d$.

9. The circular arc $k_1$ indicates the geometric location of the point $B_1$ in the first extreme position of the rocker and the circular arc $k_2$ indicates the geometric location of the point $B_2$ in the second extreme position of the rocker.

10. Through the point $B_1$ draw the line segment $B^*B_1$. From line segment $B^*B_1$ draw the line $e$ at an angle $\theta$ (counter clockwise). The intersection of line $e$ and circle arc $k_2$ results in $B_2$, i.e. the second extreme position of the rover.

11. The lengths of the links ($r_2$ a $r_3$) are calculated from the equations (3) a (4).

![Figure 5. Circular arcs defining geometric points of B in extreme positions.](image)
From the picture it is clear that the position of the rocker is chosen and there are an infinite number of options for the selection of other parameters, and from each choice there will be a different configuration and proportion of the mechanism [12, 24].

3.2. Analytical solution of geometric parameters
After defining point B₁ and solving the position of the point B₂ that is B₁B₂ = B₂B₃ = r₄, which is a constant. From the equations (3) a (4) are calculated the distances of the links r₂ a r₃, which are a function of the location of the points B₁ a B₂:

\[ r_2 = \frac{A'B_1 - A'B_2}{2} = \frac{3857 - 2456}{2} = 700.5\text{mm} \quad (11) \]
\[ r_3 = \frac{A'B_1 + A'B_2}{2} = \frac{3857 + 2456}{2} = 3156.5\text{mm} \quad (12) \]

Based on the described methodology of graphical-analytical solution of the crank-rocker mechanism synthesis, the links lengths for six alternatives are presented in table 1.

| Parameter | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 |
|-----------|---------|---------|---------|---------|---------|---------|
| r₁ (mm)   | 3000    | 3000    | 3000    | 3000    | 3000    | 3000    |
| r₂ (mm)   | 700.5   | 838     | 940     | 1040    | 1140    | 1273.5  |
| r₃ (mm)   | 3156.5  | 3091    | 3034    | 2969.7  | 2898    | 2788.5  |
| r₄ (mm)   | 1147    | 1350    | 1500    | 1650    | 1800    | 2000    |

3.3. Sensitive analysis of geometric parameters in the software ADAMS/View
The solution of the length of links of crank-rocker mechanism for rock crushing was carried out in the software MSC.ADAMS. For the solution was created a parametric model of the crank-rocker mechanism. The sensitivity analysis method was used, and the six different designs of mechanism were simulated. The dimensions of the links are presented in table 1. The input angular velocity was applied to link 2 and had a value 0.52 rad s⁻¹. The transmission angle values η as a function of time were recorded. The end time of the simulation was set to 12 s, i.e. the crank rotation was 360°. The 200 steps were chosen in the simulation process. On the graphs in the figure 6, the working part of the movement of the mechanism is from 0–3 seconds. After that is opens and then crushed material is compressed again from 9 to 12 seconds.

The best model is that which has a transmission angle that oscillates as much as possible around 90 degrees during working movement. From all of the six models meet this condition the models 4, 5 and 6. In the next step the new dimensions of the links were designed and the procedure of the simulation was repeated again. The dimensions of the links are presented in table 2. In this way was achieved a greater approximation to the required condition for the size of the transmission angle.

Figure 7 shows that the change in waveforms of individual models are insignificant, so there is no need to further narrow the range of dimensions.
Figure 6. Waveforms of the transmissions angles.

Table 2. Improved dimensions of the links for models.

| Parameter | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 |
|-----------|---------|---------|---------|---------|---------|---------|
| \( r_1 \) (mm) | 3000 | 3000 | 3000 | 3000 | 3000 | 3000 |
| \( r_2 \) (mm) | 1040 | 1087 | 1132.5 | 1180.5 | 1226.5 | 1273.5 |
| \( r_3 \) (mm) | 2969.7 | 2937 | 2903.5 | 2866.5 | 2828.5 | 2788.5 |
| \( r_4 \) (mm) | 1650 | 1720 | 1790 | 1860 | 1930 | 2000 |

Figure 7. Waveforms of the transmissions angles (the improved dimensions of the links).

4. Conclusion
The aim of the article was a presentation for the synthesis and sensitivity analysis of the crank-rocker mechanism installed in a rock crushing plant. The first part of the article was focused on the geometrical-numerical synthesis of the mechanism, which was based on the prescribed mechanical properties such as the configuration of selected pin supports, the transmission angle and the time ratio. Based on theoretical knowledge, we know that the moment from the force acting on the rocker from the sub-member may be affected by the transmission angle. Therefore, we focused on changing the size of the transmission angle during the working part of the crank-rocker mechanism and in the
sensitivity analysis process the geometric parameter of the mechanism was selected so that the load on the working part was maximally based on the given conditions.

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