Eikonal reaction theory for two-neutron removal reactions

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The eikonal reaction theory (ERT) proposed lately is a method of calculating one-neutron removal reactions at intermediate incident energies in which Coulomb breakup is treated accurately with the continuum discretized coupled-channels method. ERT is extended to two-neutron removal reactions. ERT reproduces measured one- and two-neutron removal cross sections for 6He scattering on 12C and 208Pb targets at 240 MeV/nucleon and also on a 28Si target at 52 MeV/nucleon. For the heavier target in which Coulomb breakup is important, ERT yields much better agreement with the measured cross sections than the Glauber model.

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Introduction. Removal reactions are a quite useful tool for investigating structure of valence nucleons in weakly-bound nuclei such as one- and two-neutron halo nuclei. Spectroscopic factors and orbital angular momenta of valence nucleons in incident nuclei can be deduced from the removal reactions; see for example Ref. [1]. In particular, two-neutron removal reactions are crucial for analyzing two-neutron correlations between valence neutrons. The two-neutron removal reactions were investigated for light targets [2–4] with the Glauber model [5].

The Glauber model is based on the eikonal and adiabatic approximations. The theoretical foundation of the model is shown in Ref. [6]. Once Coulomb breakup is taken into account in the Glauber model, the calculated removal cross sections diverge because of the adiabatic approximation. For this reason, the model has been applied to light targets. Lately a way of making Coulomb corrections to Glauber-model calculations was proposed [7, 8]; the divergent dipole component of the eikonal Coulomb phase is replaced by that estimated by the first-order perturbation.

The Coulomb problem is solved by the eikonal reaction theory (ERT) [17, 18] in which Coulomb breakup is treated accurately with the continuum discretized coupled-channels method (CDCC) [19–21]. For one-proton and -neutron removal cross sections of deuteron scattering at 200 MeV/nucleon, the Glauber-model results are found to be largely deviated from the ERT results for heavier targets [18]. In Ref. [17], ERT was applied to recently measured one-neutron removal cross sections for 31Ne scattering from 12C and 208Pb targets at 230 MeV/nucleon [22]. Spectroscopic factors and asymptotic normalization coefficients for the 30Ne + n bound system were consistently deduced from the measured cross sections for both the light and heavy targets. The analysis for both light and heavy thus makes it possible to determine spectroscopic factors and asymptotic normalization coefficients of valence neutrons definitely.

Scattering of three-body projectiles such as 6He can be described by the four-body model composed of three constituents of projectile and a target. Four-body CDCC [9–13] is a method of treating projectile breakup in the four-body scattering. Four-body CDCC was applied so far to elastic scattering and exclusive breakup reactions of 6He from 12C and 208Pb targets [9–13], 6Li elastic scattering from a 209Bi target [14], 11Li elastic scattering from a 209Bi target [15] and 16C elastic scattering from a 12C target [16] with success in reproducing the experimental data, and importance of projectile breakup was shown for the three-body projectiles.

In this Brief Report, we extend ERT to two-neutron removal reactions. As a test calculation to show the validity of ERT, we analyze measured one- and two-neutron removal cross sections for 6He scattering on 12C and 208Pb targets at 240 MeV/nucleon and also on a 28Si target at 52 MeV/nucleon. Here 6He is described by the n + n + α three-body model, and 6He removal reactions from a target (T) is analyzed by four-body CDCC based on the n + n + α + T model. Since 6He is well described by the three-body model [13], the spectroscopic factor is assumed to be 1. Hence the four-body CDCC calculations has no free parameter and 6He removal reaction is a good case to show the validity of ERT. The validity of the Glauber model is also discussed.

Formulation. We start with the n + n + α + T four-body system to analyze scattering of 6He on T; for later convenience, the two neutrons are labeled by n1 and n2. The Schrödinger equation for the four-body system can be written by

$$\left[ K_{R} + U + h - E \right] \Psi = 0 \quad (1)$$

with

$$U = U^{(Nucl)}_{n1} + U^{(Nucl)}_{n2} + U^{(Nucl)}_{\alpha} + U^{(Coul)}_{\alpha} \quad (2)$$

where $K_{R}$ is the kinetic energy operator with respect to the relative coordinate $R = (b, Z)$ between 6He and T, $h$ the internal Hamiltonian of 6He, $E$ the total energy of this system, and $\Psi$ is the total wave function. Here $U^{(Nucl)}_{\alpha}$ for $x = n_1, n_2$ and $\alpha$ stands for the nuclear part of the potential $U_{\alpha}$ between $x$ and T, whereas $U^{(Coul)}_{\alpha}$ denotes the Coulomb part of the potential $U_{\alpha}$ between $\alpha$ and T.

Following Ref. [17], we assume the product form $\Psi = \hat{O} \psi$ for $\Psi$. Here the operator $\hat{O}$ is defined as

$$\hat{O} = \frac{1}{\sqrt{\hbar v}} e^{iKZ} \quad (3)$$

with the velocity operator $\hat{v} = \sqrt{2(E - h)/\mu}$ and the reduced mass $\mu$ between 6He and T. Applying the eikonal approxima-
hence the operators numbers as approximation is good for the short-range nuclear interactions where approximation, full-quantum calculations with eikonal ones.

The S-matrix elements in the Glauber model are obtained by applying the adiabatic approximation to Eq. (5). In the approximation, \( h \) is replaced by the ground-state energy \( \nu_0 \), and hence the operators \( P \) and \( \hat{O} \hat{U} \hat{O} \) are replaced by classical numbers as \( P \to 1 \) and \( \hat{O} \hat{U} \hat{O} \) in Eq. (5), where \( \nu_0 \) is the velocity of \( ^6 \)He in the ground state relative to \( T \).

At intermediate energies of our interest, the adiabatic approximation is good for the short-range nuclear interactions \( U^{(\text{Nucl})}_x \) but not for the long-range Coulomb interaction \( U^{(\text{Coul})}_x \). In ERT, the adiabatic approximation is thus made to \( U^{(\text{Nucl})}_x \) only. This leads to the following replacement:

\[
\hat{O} \hat{U}^{(\text{Nucl})}_2 \hat{O} \to \frac{U^{(\text{Nucl})}_2}{\hbar \nu_0}.
\]

In other words, \( U^{(\text{Nucl})}_2 \) is commutative with \( \hat{O} \). Using this property, we can separate \( S \) as

\[
S = S_{n_1} S_{n_2} S_{\alpha}
\]

with

\[
S_{n_1} = \exp \left[ -i \frac{1}{\hbar \nu_0} \int_{-\infty}^{\infty} dZ T^{(\text{Nucl})}_{n_1} \right],
\]

\[
S_{n_2} = \exp \left[ -i \frac{1}{\hbar \nu_0} \int_{-\infty}^{\infty} dZ T^{(\text{Nucl})}_{n_2} \right],
\]

\[
S_{\alpha} = \exp \left[ -i P \int_{-\infty}^{\infty} dZ \hat{O} \hat{U} \hat{O} \right],
\]

where \( U_{\alpha} = U^{(\text{Nucl})}_\alpha + U^{(\text{Coul})}_\alpha \). The operator \( S_{\alpha} \) is the formal solution to the eikonal equation (4) with \( U_{\alpha} \) instead of \( U \). One can then get the S-matrix elements, \( \langle \varphi_0 | S_{\alpha} | \varphi_0 \rangle \) and \( \langle \varphi_k | S_{\alpha} | \varphi_0 \rangle \), by solving Eq. (4) with four-body CDCC. The method of solving the eikonal equation (4) with CDCC was already formulated in Ref. [29] and is called eikonal-CDCC. The same procedure can be taken for \( S_{n_1} \) and \( S_{n_2} \). The validity of the approximation for \( S_{n_1} \) and \( S_{n_2} \) is directly confirmed by comparing four-body CDCC and adiabatic-approximation solutions to the Schrödinger equation (1) with no \( U^{(\text{Coul})}_\alpha \) as shown latter.

The one- and two-neutron removal cross sections, \( \sigma_{-1n} \) and \( \sigma_{-2n} \), are described by

\[
\sigma_{-1n} = \sigma_{br} + \sigma_{1n \text{ str}}, \quad (11)
\]

\[
\sigma_{-2n} = \sigma_{br} + \sigma_{1n \text{ str}} + \sigma_{2n \text{ str}}, \quad (12)
\]

with the elastic breakup cross section \( \sigma_{br} \), the one-neutron stripping cross section \( \sigma_{1n \text{ str}} \) and the two-neutron stripping cross section \( \sigma_{2n \text{ str}} \), defined by

\[
\sigma_{br} = \int d^2 b [\langle \varphi_0 | S_{\alpha} S_{n_1} S_{n_2} | \varphi_0 \rangle^2] - [\langle \varphi_0 | S_{\alpha} S_{n_1} S_{n_2} | \varphi_0 \rangle]^2, \quad (13)
\]

\[
\sigma_{1n \text{ str}} = 2 \int d^2 b [\langle \varphi_0 | S_{n_1}^2 | 1 - | S_{n_1}^2 | \varphi_0 \rangle] - 2 | \langle \varphi_0 | S_{\alpha} S_{n_1} S_{n_2} | \varphi_0 \rangle |^2,
\]

\[
\sigma_{2n \text{ str}} = 2 \int d^2 b [\langle \varphi_0 | S_{n_2}^2 | 1 - | S_{n_2}^2 | \varphi_0 \rangle] - 2 | \langle \varphi_0 | S_{\alpha} S_{n_1} S_{n_2} | \varphi_0 \rangle |^2.
\]

When \( U^{(\text{Coul})}_\alpha = 0 \), these cross sections agree with those in the Glauber model [2]; when both the eikonal and adiabatic approximations are taken in model calculations, we call the model the Glauber model for simplicity in this paper, even if the phenomenological optical potentials are used as \( U^{(\text{Nucl})}_x \). Here the total reaction cross section \( \sigma_R \) is defined by

\[
\sigma_R = \int d^2 b [1 - | \langle \varphi_0 | S_{\alpha} S_{n_1} S_{n_2} | \varphi_0 \rangle |^2]
\]

and, \( \sigma_R \) and \( \sigma_{br} \) are obtained by solving the eikonal equation (4) with four-body CDCC. The elastic breakup and total reaction cross sections, \( \sigma_{br}(-1n) \) and \( \sigma_R(-1n) \), are defined by Eqs. (13) and (16) in which \( S_{\alpha} S_{n_1} S_{n_2} \) is replaced by \( S_{\alpha} S_{n_1} S_{n_2} \) and \( S_{\alpha} S_{n_1} S_{n_2} \). Hence \( \sigma_{br}(-1n) \) and \( \sigma_R(-1n) \) are obtained by solving the eikonal equation (4) with \( U_{\alpha} + U_{n_1} \) instead of \( U \) by using four-body CDCC. Similarly, the elastic breakup and total reaction cross sections, \( \sigma_{br}(-2n) \) and \( \sigma_R(-2n) \), are obtained by solving the eikonal equation (4) with \( U_{\alpha} \) instead of \( U \) by using four-body CDCC. All of \( \sigma_{br}, \sigma_{1n \text{ str}}, \sigma_{2n \text{ str}} \) and \( \sigma_{-2n} \) are thus obtainable with four-body CDCC.

In actual four-body CDCC calculations, we take the same modelspace and internal Hamiltonian for \(^4\)He as in Ref. [13]. The calculated S-matrix elements are well converged with respect to increasing the modelspace. Since the experimental data for high-energy \(^4\)He and neutron scattering are not available, one cannot construct any phenomenological optical potentials. In this work, the optical potentials \( U^{(\text{Nucl})}_x \) for the \( x-A \) subsystems are obtained by folding the Melbourne nucleon-nucleon \( g \)-matrix interaction [23] with target densities in which the proton density is determined from the electron scattering and the neutron distribution is assumed to have the same geometry as the proton one.

Results. Figure 1 shows \( \sigma_{br}, \sigma_{1n \text{ str}} \) and \( \sigma_{2n \text{ str}}, \sigma_R \) for \(^6\)He scattering from \(^{12}\)C and \(^{208}\)Pb targets at 240
MeV/nucleon. The Glauber model calculations are done by switching on the adiabatic approximation and off the Coulomb interaction $U_\alpha^{(\text{Coul})}$ in the ERT calculations. For the light target, the ERT results (solid circles) reproduce the experimental data [25] with no free parameter, and the Glauber model results (solid triangles) are close to the ERT results. For the heavy target in which Coulomb breakup is important, the ERT results yield much better agreement with the experimental data than the Glauber-model results for $\sigma_{1_{\text{br}}}$ and $\sigma_{2_{\text{br}}}$. As for $\sigma_{1_{\text{str}}}$ and $\sigma_{2_{\text{str}}}$, the Glauber model results are close to the ERT results even for the heavy target, since the cross sections are determined by the absolute values of $S_\alpha$, $S_{1_{\text{str}}}$, and $S_{2_{\text{str}}}$ and hence mainly by the imaginary part of $U$. For the elastic breakup and two-neutron removal cross sections, meanwhile, the Glauber-model results underestimate the ERT ones, because Coulomb breakup is not included in the Glauber model. As a reasonable approximation, we can therefore propose the hybrid calculation in which $\sigma_{1_{\text{str}}}$ and $\sigma_{2_{\text{str}}}$ are calculated with the Glauber model and $\sigma_{\text{br}}$ with CDCC. The present results for $^{12}\text{C}$-target are consistent with the previous Glauber-model results in Ref. [2].

Similar analyses are made in Fig. 2 for $^6\text{He}$ scattering from $^{28}\text{Si}$ at 52 MeV/nucleon. In the analyses, the optical potential $U_\alpha^{(\text{Nucl})}$ is determined so as to reproduce the measured differential elastic cross section for $^4\text{He} + ^{28}\text{Si}$ scattering at 60 MeV/nucleon [26] and the measured total reaction cross section at 48.1 MeV/nucleon [27] by multiplying the real and imaginary parts of folding potential by 0.91 and 1.39, respectively. The ERT results are consistent with the experimental data for both $\sigma_{2_{\text{br}}}$ and $\sigma_{\text{R}}$, whereas the Glauber model slightly underestimates the experimental data for $\sigma_{\text{R}}$. For this incident energy, the deviation of the Glauber-model results from the ERT ones for $\sigma_{2_{\text{br}}}$ and $\sigma_{\text{R}}$ are about 10%, whereas the error of the adiabatic approximation itself is 3% for $\sigma_{2_{\text{br}}}$ and $\sigma_{\text{R}}$. The 10% deviation is due to Coulomb breakup and its interference with nuclear breakup. The Coulomb breakup effects are more important for $\sigma_{\text{br}}$, as expected.

**Summary.** In this BriefReport, we extended ERT to two-neutron removal reactions. The method was successful in reproducing measured one- and two-neutron removal cross sections for $^6\text{He}$ scattering from $^{12}\text{C}$ and $^{208}\text{Pb}$ targets at 240 MeV/nucleon and also on a $^{28}\text{Si}$ target at 52 MeV/nucleon, with no free parameter. Particularly for the heavier target, ERT yields much better agreement with the measured cross sections than the Glauber model. As a reasonable approximation, we propose the hybrid calculation in which $\sigma_{1_{\text{str}}}$ and $\sigma_{2_{\text{str}}}$ are calculated with the Glauber model and $\sigma_{\text{br}}$ with CDCC.

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**FIG. 1:** (Color online) Elastic breakup ($\sigma_{1_{\text{br}}}$), one-neutron stripping ($\sigma_{1_{\text{str}}}$), two-neutron stripping ($\sigma_{2_{\text{str}}}$), two-neutron removal ($\sigma_{2_{\text{br}}}$), and total reaction cross sections ($\sigma_{\text{R}}$) for $^6\text{He}$ scattering from $^{12}\text{C}$ (lower panel) and $^{208}\text{Pb}$ (upper panel) at 240 MeV/nucleon. The right vertical axis stands for ($\text{mb}$), whereas the left one does for $\sigma_{1_{\text{br}}}$, $\sigma_{1_{\text{str}}}$, $\sigma_{2_{\text{str}}}$, and $\sigma_{2_{\text{br}}}$. The ERT and Glauber-model results are shown by the circles and triangles, respectively. Experimental data are taken from Ref. [25].

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**FIG. 2:** (Color online) The same as Fig. 1 but for $^6\text{He}$ scattering from a $^{28}\text{Si}$ target at 52 MeV/nucleon. Experimental data are taken from Ref. [28].
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