Experimental validation of the relationship between internal energy and state of ideal gas

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Abstract

The statement that the internal energy of ideal gas depends only on its temperature but not its volume and pressure in the free expansion process has been a topic of continuous argument in academic community. We report experimental validation of the relationship between internal energy of ideal gas and each state function. The relative internal energy in different pressure systems was determined, resulting the regression equation in P, U coordinate system. The continued equality was derived by comparing the regression equation and the definition of heat capacity at constant volume. The relationship of internal energy with volume and quantity was further derived through the state transformation method. Meanwhile, Meyer’s equation and the equation of enthalpy were also derived. Therefore, the internal energy of ideal gas has interrelationship with each state function; and a close relationship between the continued equality derived herein and current theory of ideal gas was well established.

Introduction

The “ideal gas” is a fundamental theoretical model adopted from the studies of natural principles in thermodynamics. Joule’s Law obtained by free expansion experiment in 1834 states as: “the internal energy of an ideal gas is independent of its volume and pressure; instead it depends only on its temperature”. Joule’s law was then used to define the ideal gas. Many researchers have reported corrections to this experiment [1], and the precise measurements showed that temperature is not a constant in the process of free expansion [2]. This means “internal energy and temperature can be separately considered”, which essentially challenges the “privilege” that internal energy only depends on its temperature. This has also been causing continuous argument [3-6]. Finally someone suggested that the free expansion cannot be used as a verdict whether the internal energy of an ideal gas is independent of its volume [7]. Joule’s Law then relies on the support from Rossini-Frandsen experiment [8]. In recent years, the academic community has been further debating on ideal gas, among which, included the discussion of the relationship between internal energy and state of an ideal gas [9-14]. We reported [15-17] the investigation of the relationship between state and internal energy of ideal gas utilizing the theoretical methods of thermodynamics, statistical physics, analytic geometry and probability theory, and so on. All of these studies generated a consistent conclusion that the internal energy of ideal gas has the same character of dependence with each function of state, and which can also be expressed as a continued equality in mathematics. In this paper we report the experimental data to verify these theoretical results.

Experimental

Experimental principles

For an ideal gas system closed in a fixed container, it can be concluded based on the definition of heat capacity at constant volume and the First Law of Thermodynamics that the temperature increment is directly proportional to its internal energy increment:

\[ dU = C_i dT \]

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The Charles’s Law states that temperature is directly proportional to pressure in this system of ideal gas: $\frac{dP}{dT} = C_3$

So: $dU = C_1 dP = K dP$

If we heat the gas system directly by electrifying the heating wire in a container, the quantity of heat input of the system can be derived as the following equation according to the Law of Electric Conversion: $Q = IU_1 t$

So, $Q = IU_1 dT = dU = C_1 dT = K dP$

The above principles explain that for this ideal gas system with fixed volume in a closed container, the time $t$ for electrifying the heating wire with fixed current and voltage is directly proportional to the internal energy increment $dU$, temperature increment $dT$, and pressure increment $dP$ of the system.

In order to eliminate the effect of heat leakage factor in the experiment, one can record pressure change in the whole process of ohmic heating, and then calculate the slope for time $t = 0$. By combining with the initial voltage $P_0$, the equation used to eliminate the effect of heat leakage can be derived as: $P = k t + P_0$. According to the ideal gas temperature scale experiment, when extrapolating the pressure of this equation to $P \rightarrow 0$, the absolute temperature of this system should also be $T \rightarrow 0$. This indicates that the internal energy difference for the two different determined states ($P$, $T$, $V$) and ($P \rightarrow 0$, $T \rightarrow 0$, $V$) can be derived through a mathematical method. Under the condition of the limit state ($P \rightarrow 0$, $T \rightarrow 0$, $V$), assuming the internal energy value of this system is 0, and then the internal energy difference of these two states is the relative internal energy of the system.

According to the ideal gas temperature scale experiment, for the gas system that sealed up in the same container, no matter how its initial pressure (or gas quantity) changes, the states of the system have the same characteristic when $P \rightarrow 0$ ($P \rightarrow 0$, $T \rightarrow 0$, $V$). In this experiment, setting the internal energy of this limit state $U = 0$ as the reference criterion, and making the electrifying time $t$ as a measure unit of internal energy, the internal energy of the system under the initial condition of $P$ can be measured uniformly when “volume and temperature are constant”. Finally, we can then study the relationship between pressure and internal energy via the regression analysis of their corresponding values.

**Experimental devices**

The experimental devices are depicted in Figure 1. The calorimeter used in this experiment has an interlayer, which ensures the environment temperature stable in the experimental gas system by a flow of water with constant temperature. There is a heating wire in the calorimeter, which provides energy to the system by utilizing constant current and voltage. The Precision thermometer in the calorimeter is used for testing the linear relationship between temperature and pressure in the system of a uniform state. Calorimeter and mercury pressure gauge are connected, and video equipment is used to record the time for electrifying the heating wire and the pressure change in the whole process.

**Main experimental instruments**

Calorimeter volume is about 800 ml, with an interlayer for water circulation.

Material: thick wall and horniness glass (accuracy of pressure instruction: 1 mmHg)
experiment with hot water in the interlayer. In the changing ranges of temperature increasing and pressure increasing during the electrifying process, read the corresponding values of temperature and pressure for five balanced systems to verify the linear relationship between pressure and temperature. All other experiments were carried out in the same calorimeter and at the same temperature by the same procedure as described above except the varying pressure to examine the effect of pressure.

Experimental observation: 1) Pressure had no obvious change in the first second since electrifying, and it then began to increase steadily, however, the pressure oscillated irregularly until the end of electrifying. The delay at the initial stage is due to the internal energy increment was tiny. In the process of overcoming the mercury inertia in the pressure gauge and converting internal energy to kinetic energy, the time is required for gradually accumulating energy. This is one of the error sources. Therefore, the error has to be removed during data processing. The oscillation occurred later can be considered as the alternative conversion of the internal energy of the system with the kinetic energy and the potential energy of mercury while the sum of all the energies for each oscillation is constant. The pressure of the system observed at the equilibrium point in oscillation should be correct. The mercury in the capillary of the pressure gauge rose about 20 mm with ohmic heating because the internal energy loss and volume change caused are so small that it can be neglected; 2) A good linear relationship between pressure and temperature of the system was observed. However, in the process of electrifying, the change of temperature reflected by mercury thermometer was obviously lagged behind the change of pressure, and their changes are not at the same speed until the late stage of the experiment. This indicated that there is a dynamic temperature field in the system during the electrifying process. The results obtained from any one temperature point cannot be used to describe the state of the system. The temperature results obtained only through pressure conversion can be considered as the overall representative results. It corresponds to the finally balanced temperature value after the system is fully “isolated”; 3) The experimental results under different pressure generated the heating curves with extremely similar form, characteristics, and phenomena.

Results and discussion

Experimental determination of the system internal energy

Here, we take a specific experimental example to describe the process for the determination of system internal energy. The experimental conditions are: natural atmosphere pressure $P = 992.5$ hPa, initial system temperature $t = 30 °C$, initial system pressure $P = 627.2$ hPa, current of heating wire $I = 1$ A, voltage $Ue = 6$ V, and the electrifying time $t = 30$ minutes.

Verification of linear relationship between temperature and pressure: The experiment results in Figure 2 shows the linear relationship between temperature and pressure when the device is in the equilibrium state.

The following results were generated from above experimental data (Figure 2): 1). The regression equation of the linear relationship between pressure and temperature measured by this device is:

$$ T = 15.0946 + 0.45923 P, \ r = 0.9996 $$

The confidence rate is over 99%, which indicates the linearity of this pressure thermometer itself is very high.

2). When extrapolating the pressure to 0 Pa, the limit value of temperature equals to 15.0946 K, and the absolute temperature of ideal gas should be 0 K theoretically. In addition, the extrapolation error is about 5 K per 100 K, which indicates the experimental error is in the reasonable error range of the U-tube mercury pressure gauge.

Regression analysis results as shown in figure 3 from the whole process of ohmic heating can be generated as follow:

The physical equation: $p = t / (0.904009 + 0.026636 t) + P_{30}$ ($P_{30}$ refers to the pressure of initial state system when temperature is 30 °C)

The statistics test of the regression equation: $r = 0.993354$, the linearity is over 99%.

The regression equation for the time of ohmic heating and the pressure change in the whole process: $p = \frac{t}{b + ct} + p_{30}$

When the ohmic heating time $t = 0$, the system can be considered as no heat leaking, the slope of this point is: $lim_{\omega \rightarrow 0} \frac{dp}{dt} = \frac{1}{b}$

Then, under the no-leaking condition, the linear equation of pressure with time can be derived as $p = \frac{1}{b}t + p_{30}$

Extrapolating the linear relationship of pressure $P$ and time $t$ to limit of $P \rightarrow 0$, the internal energy of this system can be calculated as follow:

The slope obtained according to the result of the experiment is:

$$ k = 1.106184 $$

The pressure equation of the system with no heat leaking is:

$$ p = 1.106184t + 627.2 \ (initial \ pressure \ p_{30} = 627.2 \ hPa). $$

The pressure difference between initial state and the state of $P \rightarrow 0$ is converted to ohmic heating time: $t = 627.2 + 1.106184 = 566.992$ seconds.
The internal energy of initial state in this system is: \(1 \times 6 \times 566.992 \text{ seconds} = 3401.954 \text{ J}\).

**Experimental result**

This experiment is conducted by the same calorimeter in six different pressure systems, respectively. Initial temperature is controlled at 30 °C. The internal energy results can be obtained from the six experimental systems according to the process described above. The experimental result is summarized in Table 1.

We can conclude from the results in Table 1:

**Conclusion 1:** Internal energy is directly proportional to pressure in the system (see figure 4).

The regression equation of initial pressure and internal energy of the system is:

\[
U = 1447.581 + 3.309104P_{30}; r= 0.985939
\]

This result shows that in the gas system with “the same volume and the same temperature”, the probability that internal energy is directly proportional to pressure or gas quantity of the system is above 99%.

**Conclusion 2:** The slope \(k\) of the internal energy changing with pressure has a linear relationship with the initial pressure (corresponding data in the first column and last column are in a linear relationship in Table 1) (Figure 5).

The regression equation of slope and the initial pressure is:

\[
k = 0.55432+0.000767P_{30}; r = 0.958943
\]

The probability that the slope of the internal energy changing with pressure is in proportion to the initial pressure or gas quantity is above 99%.

**Continued equality derived according to experimental result**

According to the conditions of “standard state of ideal gas”, all the measurements resulted from this experiment using the air at 30 °C and under less than 1atm pressure should be considered as the ideal gas of thermodynamics theory. The results stated above at Conclusion 1 above: \(U = 1447.581+3.31P_{30}\), indicates that the internal energy is directly proportional to pressure (gas quantity). After differentiation, substituting it by equation of ideal gas and comparing to the definition of heat capacity at constant volume, we can then get:

\[
dU = 3.31dp_{30} = \frac{nR}{V}dT = n\frac{\partial Q}{\partial V}dT
\]

So, under the current experimental condition, \(3.31 = \frac{\partial Q}{\partial V}\).

Then \(dU = \frac{\partial Q}{R}V dp_{30} = n\frac{\partial Q}{\partial V}dT\)

The experimental result verified that the “continued equality” is tenable to ideal gas in the thermodynamics theory system.

Based on the theory of statistical physics, only under the condition of pressure \(p_{30} \to 0\) for the initial state during experiment, it meets the requirement of the “ideal gas”:

\[
\]
The result from Conclusion 1 is: $U = 1447.581 + 3.31\, p_{30}$, when $p_{30} \to 0$, $U = 1447.6$.

And the result from Conclusion 2 is: $k = 0.55 + 0.000767\, p_{30}$, when $p_{30} \to 0$, $k = 0.55$.

So, the gas in the closed calorimeter is in the extremely low pressure limit:

$$U = 0.55\, p + 1447.6 \quad (p \to 0)$$

It then indicates that internal energy is also proportional to pressure in the gas system with low pressure limit. The same continued equality can also be obtained using the same mathematical method mentioned above.

$$U = \frac{Q}{R} \, pV = nC_pT \quad \lim_{t \to 0} U = 0$$

The value became very small in the experimental process, which exhibited the conceptual difference of the ideal gas between statistical physics and thermodynamics. With these series of experimental data, we have successfully verified "continued equality."

Further verification of experiment

Whether this new standpoint Continue Equality stands mainly depends on whether it can well connect to all the current theory systems. Therefore, we derived the same results from the internal energy vs pressure and transformed internal energy vs temperature by utilizing the experimental data described above. The reliability of the experimental condition and resulted relationships obtained were further verified by micro-disturbance of the experimental results.

Methods for analysis

A coordinate system including the results of extrapolation has to be established first. Y-coordinate denotes pressure $P$ of the system; make the point $P = 0$ on the pressure axis as the origin O of the coordinate system; x-coordinate denotes the time $t$ (i.e. function of internal energy) for electrifying the heating wire. Setting the origin by this way means the internal energy difference of initial state pressure $P_0$ and $P = 0$ in the experimental system can also be converted to electrifying time of the system (see Figure 6).

Dotted Lines bcd and all other similar curves are recorded from the six experiments described thus far (see Figure 3 as an example). When the electrifying wire releases heat in the closed system that has constant volume, the temperature and pressure of the system will rise. Because of the possible heat leakage of calorimeter itself, the processes of temperature and pressure increment present as curves. The system keeps stable in the new thermal equilibrium state when the speeds of hot leakage and heat release of heating wire are equal.

Line abO and similar straight lines are those represented by the equations in the column of "pressure equations of no hot leaking" in Table 1. Each line is the tangent of corresponding experimental curve when the ohmic heating time $t = 0$. Point b is not only the initial state point of the experimental system, but also the tangent point of the pressure curve at the place of $t = 0$.

Obviously, as the initial state of the system has been known, one degree of freedom $t$ is still required if we want to determine the point b in the coordinate system shown in Figure 6, and it must be determined through experiment. This also demonstrated the validation process highly depends on the experiment. Section 1.5.2 of this paper gave the method for the determination of point b.

Line ebf is the straight line in Figure 4 obtained by connecting initial state point (or every tangent point) of every experiment. The equation thus obtained is $U (f) = 1447.581 + 3.309104 \, p_{30}$. Because this line reflects the corresponding relationship between pressure and internal energy in this system, the line ebf is also the verification line of the thermodynamics. In this experiment, the same volume and temperature are used, therefore the pressure differences can only be obtained through adjusting gas quantity in every experiment. As we know based on the extensive property of internal energy that "internal energy is directly proportional to gas quantity", the straight line ebf is obtained.

2. Influence of temperature and volume on results in the coordinate system of internal energy and pressure

The influence of temperature on line ebf was examined first: the equation for line ebf is: $U (f) = 1447.581 + 3.309104 \, p_{30}$. Assuming the experimental temperature was decreased by 1/10 (about 0 °C = 273.15 K) from 30 °C (303.15 K), under the condition of same volume and same gas quantity, the corresponding pressure obtained from the ideal gas equations is $P = \frac{np^0}{V} = 0.9\, p_{30}$. According to the definition of heat capacity, then its corresponding internal energy is $U = nC_p \cdot 0.9\, T_0 = 0.9\, U_{30}$. The corresponding converted result is shown in Table 2.

The data in Table 2 gives the new equation of internal energy: $U = 1302.823 + 3.309104 \, p_{30}$. Comparing to the original equation, only intercept changes by 1447.581–1302.823 = 144.758, which means the

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**Table 1. Experimental Result at 30 °C and 303.15 K**

| Exp. No. | Initial pressure $p_{30}$ | Internal energy $U_{30}$ | Pressure equation of no heat leaking | $r$ | Slope of pressure change(k) |
|----------|---------------------------|--------------------------|-------------------------------------|-----|---------------------------|
| 1        | 353.5544                  | 2495.737                 | $P = 0.84999 + 353.5544$           | 0.9829 | 0.8499                  |
| 2        | 479.0103                  | 3192.628                 | $P = 0.9002 + 479.0103$           | 0.9889 | 0.9002                  |
| 3        | 501.0084                  | 3171.532                 | $P = 0.8782 + 501.0084$           | 0.9938 | 0.8782                  |
| 4        | 627.1977                  | 3401.954                 | $P = 1.1062 + 627.1977$           | 0.9933 | 1.1062                  |
| 5        | 745.5214                  | 3953.161                 | $P = 1.1131 + 745.5214$           | 0.9907 | 1.1131                  |
| 6        | 869.8417                  | 4393.619                 | $P = 1.2247 + 869.8417$           | 0.9871 | 1.2247                  |

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**Figure 6. Principle Chart for Result Analysis**

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difference is 1/10. This explains that the intercept of line ebf is the direct function of temperature. But the slope keeps the same at 3.309104, indicating that the slope of line ebf is independent of temperature.

Therefore, the equation obtained above indicated that the slope of line ebf is the function of volume in the linear relationship between pressure and internal energy.

Attempting to change the volume of the experimental system with the methods described above: according to Boyle’s Law and under the constant temperature and gas quantity of the experimental system, when volume of calomel meter increased 10%, the corresponding pressure is \( P = \frac{P_0}{1.1} \). In this case, \( U = nC_i \), the internal energy of this system, is not changed according to the definition of heat capacity. The result in Table 3 was then derived.

According to the results in Table 3, the data regression analysis shows: \( U = 1447.581 + 3.640015V \), and \( U = \frac{\dot{Q}}{nC_i} + \frac{\dot{W}}{nC_i} \) shows: \( U = 1447.581 + 3.640015 \).

The expression of general function form is:

\[
U = f(V)P + g(T) = \left(k_1V + k_2\right)P + \left(k_3T + k_4\right)
\]

Since we set the reference criteria \( U = 0 \) according to the ideal gas temperature scale experiment in classical theory, so when \( T \to 0, \) we will surely get \( g(T) \to 0 \), therefore \( k_3 = 0 \) can be determined. In addition, the ideal gas temperature scale under the same pressure can also be expressed as: \( \lim_{V \to 0} U = 0 \), so, when \( V \to 0, \) then, \( k_4 = 0 \). The formula above can be simplified as: \( U = k_1V + k_2T \)

Transforming the formula above through differential and the ideal gas equation into the function form of temperature, and then comparing with the definition of heat capacity, the following equation is derived:

\[
dU = k_1d(PV) + k_2d(T) = k_1d(PV) + k_2\frac{d(PV)}{nR} = n\tilde{C}_i dT
\]

The other types of expression for heat capacity can be obtained:

\[
\tilde{C}_i = \frac{Rk_1}{n} + \frac{k_2}{n}
\]

The result can be transformed into the function form of pressure and volume using the ideal gas equation:

\[
dU = k_1d(PV) + k_2d(T) = k_1d(PV) + k_2\frac{d(PV)}{nR} = n\tilde{C}_i dT
\]

After rearrangement and appropriate substitution, the equation of heat capacity is:

\[
dU = \frac{Rk_1}{n}d(PV) = \frac{Rk_1}{n}d(PV) = nC_i dT
\]

So far, continued equality is derived again.

If the state of this closed system changes, the First Law of Thermodynamics can be used to verify that the continued equality is tenable.

After differentiating of experimental equation \( U = k_1V + k_2T \), and comparing it with \( AU = Q-W \), the following equation is derived:

\[
dU = \delta Q-\delta W = d(k_1VP) + d(k_2T)
\]

In the isobaric change, \( dP = 0 \), substituting \( \delta Q = n\tilde{C}_i dT, \delta W = PVdV \) into the equation above resulted in:

\[
dU = n\tilde{C}_i dT - PVdV = k_1PV + k_2dT
\]

So \( k_1 = -1, k_2 = n\tilde{C}_i \)

Substituting the results into the equation of heat capacity derived above gives: \( \tilde{C}_i = \frac{Rk_1}{n} + \frac{k_2}{n} \)

So, \( \tilde{C}_i = \tilde{C}_i - R \), this result is the Meyer’s Equation routinely used in thermodynamics.

Substituting \( k_1 = -1, k_2 = n\tilde{C}_i \) into the equation \( U = k_1V + k_2T \) obtained from this experiment resulted in:

\[
U = n\tilde{C}_i dT - VP
\]

The enthalpy is expressed as \( H = n\tilde{C}_i T \) under constant pressure. Further re-arrangement and substitution resulted in \( H = U + VP \), which is the definition of the enthalpy. That is, the line ebf obtained from current experiment represents the enthalpy concept in the P, U coordinate system. The following two equivalent forms of equations can thus be obtained after simplified by the ideal gas equation:

Equation 1: \( U = n\tilde{C}_i T - VP = n\tilde{C}_i T + nRT - VP = n\tilde{C}_i T \)

Equation 2: \( U = n\tilde{C}_i T - VP = \frac{\tilde{C}_i}{R}PV + nRT - VP = \frac{\tilde{C}_i}{R}PV \)

Connecting these two equations resulted in: \( \tilde{C}_i = \frac{\tilde{C}_i}{R}PV = n\tilde{C}_i T \)

Comparison with the Rossini- Frandsen results by experiment

In 1932, Rossini and Frandsen concluded that the internal energy and pressure of a system are in a linear relationship utilizing high-pressure gas expanding toward environment by keeping temperature constant. The result is shown in Figure 7. The straight line in Figure 7 is expressed as: \( U(P, T) = f(T)P + g(T) \)

When \( P \to 0 \), then \( U(P, T) \to g(T) \). It, therefore, allowed the researchers concluding that “the internal energy of an ideal gas depends only on its temperature, but not its pressure “.

Our experimental results shown in Figure 5 can be generally expressed as:

\[
U(P, V, T) = f(V)P + g(T)
\]

Similarly, internal energy also has linear relationship with pressure in P, U coordinate system when \( P \to 0, U(P, V, T) \to \tilde{g}(T) \). However, we cannot simply understand as “the internal energy is independent of pressure” because we can study it in the T, U coordinate system through the state transformation.

| Exp. No. | 0.9P | 0.9U |
|----------|-------|-------|
| 1        | 318.1990 | 2246.163 |
| 2        | 431.1093 | 2873.365 |
| 3        | 450.9076 | 2854.379 |
| 4        | 564.4779 | 3061.759 |
| 5        | 670.9693 | 3557.845 |
| 6        | 807.1575 | 3954.257 |

| Table 3. | Pressure vs Internal Energy after the Volume Increased by 10% |
|----------|------------------|
| Exp. No. | 0.9P | 0.9U |
| 1        | 321.4131 | 2495.737 |
| 2        | 435.4639 | 3192.628 |
| 3        | 455.4622 | 3171.532 |
| 4        | 570.1797 | 3401.954 |
| 5        | 677.7467 | 3953.161 |
| 6        | 815.3106 | 4393.619 |
Because the device in this experiment adopts the principle of "pressure thermometer under constant volume of gas", therefore, in the process of ohmic heating, internal energy can be measured not only by the change of pressure, but also by the change of temperature. The same set of data obtained from the experiment can be transferred to the coordinate system of temperature T and internal energy U. It is helpful to fully understand the experiment results if one studies it from different angles.

If the pressure of every experiment was transferred to \( P = 896.8417 \) hPa (the pressure level of the sixth experiment) utilizing the pressure equations with no heat leaking obtained from every experiment in Table 1 of 1.5.3, the corresponding temperature of the system can be calculated according to the Law of Charles: \( T = \frac{T_0}{P_0} \cdot P \). All data from these six experiments can be converted to the results in the system with constant volume and pressure while at different temperature, and then the relationship between temperature and internal energy can be examined.

If the pressure of every experiment is changed to \( P = 896.8417 \) hPa, then the required electrifying time for heating is \( t = \frac{896.8417 - P}{k} \). After conversion, the corresponding internal energy of the system is \( U = U_0 + IU_t \). All the property parameters obtained through such a transforming process are listed in Table 4.

Regression analysis of "temperature K after conversion" and "system internal energy J after conversion" listed in the table resulted in the equation: \( U = 4.494617 \cdot T + 3133.579, r = 0.950455, \) which illustrates the probability for this series of data having linear relationship greater than 99%. Comparing to the equation \( U = 4.494617 \cdot T + 3133.579 \) derived before gas quantity changes, the intercept keeps constant as 3133.579; the difference of slope values is 4.494018–4.49617 = 0.00461, that is 10%. This indicates that the slope of this straight line is directly proportional to the internal energy.

Studies on the intercept of the straight line in Figure 7: According to the data in Table 4, the pressure of the system decreases 10%, then \( P = 0.9P_0 \). Keeping the gas quantity and volume constant, temperature can be determined by the ideal gas equation: \( T = \frac{PV}{nR} = 0.9T_0 \). According to the definition of heat capacity, the internal energy of system can be calculated as \( U = nC_v 0.9T_0 = 0.9U_0 \). The calculated result is listed in Table 6.

Comparing to the equation \( U = 4.494617 \cdot T + 3133.579 \) derived before the gas quantity change, both slopes are the same as 4.494617; the difference of intercepts is 3133.579–2820.221 = 313.3579, reducing by 10%, again. It is concluded that the intercept of this straight line is directly proportional to the pressure of this system. Therefore, the general equation for the temperature being directly proportional to the internal energy in the T, U coordinate system should be expressed as:

\[
U = 4.494617 \cdot T + 3133.579, r = 0.950455, \]

In addition, the change of internal energy can be determined as \( U = 1.1nC_v \cdot T_0 = nC_v T_0 \) by the definition of heat capacity, which indicates that the internal energy value of each system is unchanged. Therefore, the data in Table 5 can be derived.

Regression of temperature and internal energy data in Table 5 resulted in the equation \( U = 4.994018 \cdot T + 3133.579, r = 0.950455, \) which illustrates the probability for this series of data having linear relationship greater than 99%. Comparing to the equation \( U = 4.494617 \cdot T + 3133.579 \) derived before gas quantity changes, the intercept keeps constant as 3133.579; the difference of slope values is 4.994018–4.49617 = 0.497851, that is 10%. This indicates that the slope of this straight line is directly proportional to the gas quantity.

Studies on the slope of the straight line in Figure 7: In order to keep the system pressure constant, the temperature will definitely have corresponding change if the gas quantity in the system increases 10% based on the data in Table 4. According to the ideal gas equation, the resulted temperature after the change of gas quantity can be determined as \( T = \frac{PV}{nR} = \frac{T_0}{1.1} \).

Studies on the intercept of the straight line in Figure 7: According to the data in Table 4, the pressure of the system decreases 10%, then \( P = 0.9P_0 \). Keeping the gas quantity and volume constant, temperature can be determined by the ideal gas equation: \( T = \frac{PV}{nR} = 0.9T_0 \). According to the definition of heat capacity, the internal energy of system can be calculated as \( U = nC_v 0.9T_0 = 0.9U_0 \). The calculated result is listed in Table 6.

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Table 4. Conversion Results from P, U to T, U Coordinates

| Exp. No. | Equation of no heat leaking for every experiment | Pressure after conversion (hPa) | Temperature after conversion (K) | Heating time for conversion (s) | Internal energy after conversion (J) |
|----------|-----------------------------------------------|-------------------------------|---------------------------------|--------------------------------|----------------------------------|
| 1        | 0.84999t+353.5544                             | 896.8417                      | 768.88                          | 639.1766                      | 6330.796                        |
| 2        | 0.90021t+479.0103                             | 896.8417                      | 567.58                          | 464.1447                      | 5977.496                        |
| 3        | 0.87821t+501.0084                             | 896.8417                      | 542.66                          | 450.7233                      | 5875.872                        |
| 4        | 1.1062t+627.1977                              | 896.8417                      | 433.48                          | 243.7605                      | 4864.517                        |
| 5        | 1.1315t+745.5214                              | 896.8417                      | 364.68                          | 131.7305                      | 4755.544                        |
| 6        | 1.2247t+896.8417                              | 896.8417                      | 303.15                          | 0                             | 4393.619                        |

Table 5. Corresponding Temperature and Internal Energy of the System after Gas Quantity is Increased by 10% (derived from the data obtained when P = 896.846 hPa)

| Exp. No. | 0.9P | 0.9U |
|----------|------|------|
| 1        | 692.0855 | 6330.796 |
| 2        | 510.8237 | 5977.496 |
| 3        | 488.3946 | 5875.872 |
| 4        | 390.1319 | 4864.517 |
| 5        | 328.2130 | 4755.544 |
| 6        | 272.8350 | 4393.619 |

Table 6. Corresponding Temperature and Internal Energy of the System after Pressure Reduced by 10% (derived from the data obtained when P = 896.846 hPa)

| Exp. No. | 0.9P | 0.9U |
|----------|------|------|
| 1        | 692.0855 | 5697.717 |
| 2        | 510.8237 | 5379.746 |
| 3        | 488.3946 | 5288.285 |
| 4        | 390.1319 | 4378.065 |
| 5        | 328.2130 | 4279.989 |
| 6        | 272.8350 | 4171.257 |

$U = f(nT + gt(P))$ Namely, when $T \to 0$, then $U \to g(P)$.

If we understand it in the same way, we would conclude that “internal energy is independent of temperature, but depends only on pressure”. Therefore, the two sets of results obtained from the same experiment are conflicting, which indicates that the understanding and analysis of the experiment are insufficient for the conclusion drawn. Therefore, the results “when $P \to 0$, then $U (P, T) \to g(T)$”, obtained by Rossini and Frandsen, only shows that the internal energy of gas is related to temperature, but it does not mean that internal energy is not related to pressure. The final comprehensive results of our experiment can be expressed as $U = k_1VP + k_2nT$. This is the result merged from the two expressions of internal energy in different coordinate systems $U (P, V, T)$ and $U (P, T, n)$ that involve every state function of gas system. Therefore, any one state function can’t be excluded to have influence on the internal energy of the ideal gas. And therefore the privilege that “the internal energy of an ideal gas depends only on its temperature” does not stand.

The phenomena observed from our experiment are the overall results combined from all the experimental facts of three classical gas laws, electro-thermal equivalent, ideal gas temperature scale, Fourier heat conduction, and others. The phenomena of this experiment and causality of the known facts form an inseparable “fact community” of each other. The continued equality finally derived according to the experimental phenomena includes not only the First Law of Thermodynamics, the ideal gas equation, and the concept of heat capacity; it also includes Meyer equation, concept of enthalpy, the ideal gas temperature scale, free expansion, the Joule law, and others. The continued equality described above constitutes an indivisible “logic community” together with almost all basic content related to ideal gas.

Conclusions

The phenomenon observed and the results derived from our experiments realized widely-connected “fact community” and “logic community” with the known facts and the current theories, which means the conclusions obtained can be merged with all the current accepted view regarding “phenomenon” and “understanding”. Therefore, it can be concluded that the internal energy of the ideal gas has interrelationship with each state function, and the derived result is mathematically expressed as a continued equality: $U = \frac{\partial U}{\partial V} PV + \frac{\partial U}{\partial T} TV \quad (\text{lim} \Delta U = 0)$.

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