Towards a Unified Description of the Four Interactions in Terms of Dirac-Bergmann Observables.

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Abstract

A review is given of the status and developments of the research program aiming to reformulate the physics of the four interactions at the classical level in a unified way in terms of Dirac-Bergmann observables with special emphasis on the open mathematical, physical and interpretational problems.

At the classical level the accepted mathematical description of the four interactions at the basis of our understanding of nature (gravitational, electromagnetic, weak and strong; without or with the not yet experimentally verified supersymmetry between half-integer and integer spin fields, i.e. between fermions and bosons), is based on action principles which, due to manifest Lorentz invariance, to local gauge invariance (minimal coupling) and/or diffeomorphism invariances, make use of singular Lagrangians implying the Dirac-Bergmann theory of constraints[1, 2] for their Hamiltonian formulation. While behind the gauge freedom of gauge theories proper there are Lie groups acting on some internal space so that the measurable quantities must be gauge invariant, the gauge freedom of theories invariant under diffeomorphism groups of the underlying spacetime (general relativity, string theory and reparametrization invariant systems of relativistic particles) concerns the arbitrariness for the observer in the choice of the definition of “what is space and/or time” (and relative times in the case of particles), i.e. of the definitory properties either of spacetime itself or of the measuring apparatuses. This is the classical mathematical background on which our understanding of the quantum field theory
of electromagnetic, weak and strong interactions in the modern BR S formulation is based. The same is true for our attempts to build quantum gravity notwithstanding our actual incapacity to reconcile the influence of gravitational physics on the existence and formulation of spacetime concepts with the basic ideas of quantum theory, which requires a given absolute background spacetime.

Current research on electromagnetic, weak and strong interactions in special relativity, namely in Minkowski spacetime, has partially bypassed the problem by the covariant approach based on the BRS symmetry which, at least at the level of the algebra of infinitesimal gauge transformations, allows a regularization and renormalization of the relevant theories inside the framework of local quantum field theory (see for instance Ref.[3]). However, problems like the understanding of finite gauge transformations and of the associated moduli spaces, the Gribov ambiguity dependence on the choice of the function space for the fields and the gauge transformations, the confinement of quarks, the definition of relativistic bound states and how to put them among the asymptotic states, the nonlocality of charged states in quantum electrodynamics, not to speak of the foundational and practical problems posed by gravity, suggest that we should revisit the foundations of our theories. It is not yet known whether we can understand which are the physical degrees of freedom hidden behind manifest gauge and/or general covariance and whether we can firstly meaningfully reformulate classical physics in terms of them and secondly to quantize the resulting theories. This will require to abandon local field theory at the nonperturbative level and to understand how to regularize and renormalize the Coulomb gauge of electrodynamics to start with. Moreover, the special relativistic theories will have to be reformulated in such a way to allow a natural transition to the coupling to gravity. Even if usually gravitational contributions are ignored because they are too weak with respect to the other interactions, the existing solution to the ultraviolet divergences of quantum field theory is distributional, so that, at least at the mathematical level, it is not justified to ignore gravity with all its nonlinearities. In turn general relativity must be formulated in a way allowing its deparametrization to recover physics in Minkowski spacetime when the Newton constant is put equal to zero. One also needs a formulation in which some notion of elementary particle exists so to recover Wigner’s definition based on the irreducible representations of the Poincaré group in Minkowski spacetime with the further enrichment of the known good quantum numbers for their classification. Moreover, one needs some way out from the “problem of time”[4, 5, 6], since neither any consistent way to quantize time (is it a necessity?), and generically any timelike variable, nor a control on the associated problem of the relative times of a system of relativistic particles are known. Finally, one has to find a solution to the more basic problem of how to identify physically spacetime points in Einstein’s formulation of general relativity, where general covariance deprives the mathematical points of the underlying 4-manifold of any physical reality[7, 8], while, on the experimental side (space physics, gravitational waves detectors), we are employing a theory of measurements of proper times and spacelike lengths which presuppones the individuation of points.
This problem will appear also in the nowadays most popular program of unification of all the interactions in a supersymmetric way, i.e., in superstring theory and in its searched M-theory extension (see for instance Ref. [9]; string theory will not be touched in this review), when someone will be able to reformulate it in a background independent way.

These motivations induced me to revisit the classical Hamiltonian formulation of theories described by singular Lagrangians trying to choose the mathematical frameworks which at each step looked more natural to clarify the physical interpretational problems by means of the use of suitable adapted coordinates. In particular, after many years of dominance of the point of view privileging manifest Lorentz, gauge and general covariance at the price of losing control on the physical degrees of freedom and on their deterministic evolution (felt as a not necessary luxury only source of difficulties and complications), I went back to the old concept of Dirac observables, namely of those gauge invariant deterministic variables which describe a canonical basis of measurable quantities for the electromagnetic, weak and strong interactions in Minkowski spacetime. Instead, in general relativity, due to the problem of the individuation of the points of spacetime, measurable quantities have a more complex identification, which coincides with Dirac’s observables (in any case indispensable for the treatment of the Cauchy problem) only in a completely fixed gauge (total breaking of general covariance).

In the next Sections I will review the various achievements of the program at the present stage of development (see Refs. [10] for previous reviews). Since there is too vast a bibliography to be covered in this review, I made the choice to concentrate it on my point of view omitting to quote many aspects of the theory and the work of many researchers.

1 Singular Lagrangians, Presymplectic Geometry, the Shanmugadhasan Canonical Transformations and Generalized Coulomb Gauges in Minkowski Spacetime.

A) If a finite-dimensional system with configuration space $Q \quad [q^i, i=1,\ldots,N]$, are local coordinates in a global (assumed to exist for the sake of simplicity) chart of the atlas of $Q$; $(t, q^i(t))$ is a point in $R \times Q$, where $R$ is the time axis; $\dot{q}^i(t) = dq^i(t)/dt$ is described by a singular Lagrangian $L$ [so that the Hessian matrix is degenerate: $\det \left( \frac{\partial^2 L}{\partial \dot{q}^i \partial \dot{q}^j} \right) = 0$], its Euler-Lagrange equations are in general a mixture of equations i) depending only on the $q^i$ (holonomic constraints); ii) depending only on $q^i$ and $\dot{q}^i$ (Lagrangian, in general nonholonomic, constraints and/or intrinsic first order equations of motion violating the so-called second order differential equation (SODE) conditions); iii) depending on $q^i, \dot{q}^i, \ddot{q}^i$ (genuine second order equations of motion, which however cannot be put in normal form, i.e., solved in the $\ddot{q}^i$). More
equations of the types i) and ii) can be deduced from the Euler-Lagrange equations and their time derivatives. The study of this type of degenerate equations can be traced back to Levi-Civita [11]. The solutions of the Euler-Lagrange equations depend on arbitrary functions of time, namely they are not deterministic.

The canonical momenta \( p_i = \frac{\partial L}{\partial \dot{q}_i} \) are not independent: there are relations among them \( \phi_\alpha(q, p) \approx 0 \) called primary Hamiltonian constraints, which define a submanifold \( \gamma \) of the cotangent space \( T^*Q \) [the model is defined only on this submanifold; one uses the Poisson brackets of \( T^*Q \) in a neighbourhood of \( \gamma \) and Dirac’s weak equality \( \approx \) means that the equality sign cannot be used inside Poisson brackets]. The canonical Hamiltonian \( H_c(q, p) \) has to be replaced by the Dirac Hamiltonian \( H_D = H_c + \sum_\alpha \lambda_\alpha(t) \phi_\alpha \), which knows the restriction to the submanifold \( \gamma \) due to the arbitrary Dirac multipliers \( \lambda_\alpha(t) \). The time constancy of the primary constraints, \( \partial_t \phi_\alpha = \{ \phi_\alpha, H_D \} \approx 0 \), either produces secondary Hamiltonian constraints or determines some of the Dirac multipliers. This procedure is repeated for the secondary constraints (this is the Dirac-Bergmann algorithm) and so on. At the end there is a final set of constraints \( \chi_a \approx 0 \) defining the final submanifold \( \bar{\gamma} \) of \( T^*Q \) on which the dynamics is consistently restricted, and a final Dirac Hamiltonian with a reduced set of arbitrary Dirac multipliers describing the remaining indeterminateness of the time evolution. The constraints are divided into two subgroups: i) the first class ones \( \chi^{(1)}_m \approx 0 \), having weakly zero Poisson bracket with all constraints and being the generators of the gauge transformations of the theory (the associated vector fields \( \{., \chi^{(1)}_m \} \) are tangent to \( \bar{\gamma} \)); ii) the second class ones \( \chi^{(2)}_n \approx 0 \) (their number is even) with \( \det \left( \{ \chi^{(2)}_m, \chi^{(2)}_n \} \right) \neq 0 \), corresponding to pairs of inessential eliminable variables (the associated vector fields are normal to \( \bar{\gamma} \)). The solutions of the Hamilton-Dirac equations with the final Dirac Hamiltonian depend on as many arbitrary functions of time as the left Dirac multipliers. The restriction of the symplectic 2-form of \( T^*Q \) to \( \bar{\gamma} \) is a closed degenerate 2-form, which in case of only first class constraints generates a so called presymplectic geometry: \( \bar{\gamma} \) is said to be a presymplectic manifold coisotropically embedded in \( T^*Q \) [see Ref. [12, 13] for what is known on presymplectic structures (they are dual to Poisson structures, but much less studied not being connected with integrable systems) and on the more general ones when second class constraints are present]. When many mathematical conditions are satisfied, the vector fields associated with the first class constraints (they are in the kernel of the degenerate 2-form on \( \bar{\gamma} \)) generate a foliation of the submanifold \( \bar{\gamma} \): each leaf (Hamiltonian gauge orbit) contains all the configurations which are gauge equivalent and which have to be considered as the same physical configuration [11] (equivalence class of gauge equivalent configurations); the canonical Hamiltonian \( H_c \) (if it is not \( H_c \approx 0 \)) generates an evolution which maps one leaf into the others. Therefore, the physical reduced phase space is obtained: i) by eliminating as many pairs of conjugate variables as second class constraints by means of the so called associated Dirac brackets; ii) by going to the quotient with respect to the foliation (a representative of the reduced phase space can be build by adding as many gauge-fixing constraints as first class ones, so to obtain a set of second class constraints). In general this
procedure breaks the original Lorentz invariance.

Let us remark that only the primary first class constraints are associated with arbitrary Dirac multipliers. The secondary, tertiary... first class constraints are, in general, present in the canonical Hamiltonian $H_c$ multiplied by well defined functions of $q^i$, $\dot{q}^i$, which turn out to be arbitrary because they are not determined by the Hamilton-Dirac equations (they are gauge variables). This contradicts the Dirac conjecture\cite{Dirac} that the secondary first class constraints can be added to the Dirac Hamiltonian with extra multipliers (the resulting extended Dirac Hamiltonian would not allow the reconstruction of the original singular Lagrangian by inverse Legendre transformation; since the difference in the dynamics is only off-shell, this explains why the extended Hamiltonian is used in the BFV approach\cite{BFV}). The natural way to add gauge-fixing constraints when there are secondary first class constraints\cite{BFV}, is to start giving the gauge fixings to the secondary constraints. The requirement of time constancy of these gauge fixings will generate the gauge fixings for the primary first class constraints and the time constancy of these new gauge fixings will determine the Dirac multipliers eliminating every residual gauge freedom.

The Dirac observables are the gauge invariant functions on the reduced phase space, on which there is a deterministic evolution generated by the projection of the canonical Hamiltonian. Therefore, the main problem is to find a (possibly global) Darboux coordinate chart of the reduced phase space, namely a canonical basis of Dirac observables (or at least a Poisson algebra of them, according to Ref.\cite{Goldfain}).

One would expect that when this is not possible, the relativistic system is intrinsically ill defined already at the classical level: at the quantum level this should manifest itself with the presence of not curable anomalies (which can be present also for a classically well defined system). Since the mathematical theory of the anomalies relies on cohomological properties of the manifolds (like $Q$ and $\gamma$) relevant to the description of the system, which have to be defined already at the classical level, one expects that a classical background of these properties in the form of obstructions to the determination of the observables should be present in the theory of classical gauge canonical transformations.

When there is reparametrization invariance of the original action $S = \int dt L$, the canonical Hamiltonian vanishes and the reduced phase space is said frozen (like it happens in Hamilton-Jacobi theory). When the canonical Hamiltonian vanishes, both kinematics and dynamics are contained in the first class constraints describing the system: these can be interpreted as generalized Hamilton-Jacobi equations\cite{Goldfain}, so that the Dirac observables turn out to be the Jacobi data. When there is a kinematical symmetry group, like the Galileo or Poincaré groups, an evolution may be reintroduced by using the energy generator as Hamiltonian.

In a series of papers\cite{Goldfain,Goldfain,Goldfain,Goldfain,Goldfain} I made a reformulation of the general theory of singular Lagrangians and Hamiltonian constraints based on an extension of the second Noether theorem\cite{Noether} to include also second class constraints. By means of the resulting Noether identities the Dirac-Bergmann algorithm was reproduced at the Lagrangian level. All the obscure and/or ambiguous points of the theory were
clarified. The understanding of the pathological examples known in the literature led to the discovery of third- and fourth-class constraints [with their associated singularities of the Jacobi equations (linearization of the Euler-Lagrange equations) and their connection with the reject of the Dirac conjecture about adding the secondary first class constraints to the Dirac Hamiltonian with extra Dirac multipliers] and of the phenomena of proliferation of constraints, ramification and joining of chains of constraints. Also the classification of all possible patterns of second class constraints was given. All these phenomena have their counterpart in the study of the Euler-Lagrange equations for a singular Lagrangian in the second-order formalism. In Ref. there is also the status of the art for the much more difficult and still incomplete first-order formulation of the theory on the tangent space $TQ$ or on the first jet bundle $J^1(Q) \approx TQ \times R$, while in Ref. there is the connection with BRS theory.

B) Now I will delineate the main steps for the determination of the Dirac observables for the case in which only primary first class constraints $\phi_\alpha \approx 0$ are present at the Hamiltonian level.

The Euler-Lagrange equations associated with a singular Lagrangian do not determine the gauge part of the extremals. However it cannot be totally arbitrary, but must be compatible with the algebraic properties of the Noether gauge transformations induced by the first class constraints under which the action is either invariant or quasi-invariant as implied by the second Noether theorem. In the Hamiltonian formulation these properties are contained in the structure constants, or functions, of the Poisson brackets of the first-class constraints among themselves $[\{\phi_\alpha, \phi_\beta\} = C_{\alpha\beta\gamma}\phi_\gamma, \{\phi_\alpha, H_c\} = C_{\alpha\beta}\phi_\beta]$ and the gauge arbitrariness of the trajectories is described by the Dirac multipliers appearing in the Dirac Hamiltonian. In both formulations one has to add extra equations, the either Lagrangian or Hamiltonian multitemporal equations, to have a consistent determination of the gauge part of the trajectory (see the generalized Lie equations of Ref.). These equations are obtained by rewriting the variables $q^i(t), p_i(t)$ in the form $q^i(t, \tau_\alpha), p_i(t, \tau_\alpha)$, and by assuming that the original t-evolution generated by the Dirac Hamiltonian $H_D = H_c + \sum_\alpha \lambda_\alpha(t)\phi_\alpha$ is replaced by: i) a deterministic t-evolution generated by $H_c$; ii) a $\tau_\alpha$-evolution (reassorbing the arbitrary Dirac multipliers $\lambda_\alpha(t)$), for each $\alpha$, generated in a suitable way by the first class constraints $\phi_\alpha$. The $\tau_\alpha$-dependence of $q^i, p_i$ determined by these multitemporal (or better multiparametric) equations, which are integrable due to the first-class property of the constraints, describes their dependence on the gauge orbit containing the given Cauchy data for the Hamilton-Dirac equations. From the point of view of the study of the multitemporal equations, the secondary first class constraints are treated like the primary ones, namely as if there would be associated extra Dirac multipliers, and one should use as canonical Hamiltonian $H_c$ restricted to zero value of the secondary constraints.

When the Poisson brackets of the Hamiltonian first class constraints imply a canonical realization of a Lie algebra, the extra Hamiltonian multitemporal equations have the first class constraints as Hamiltonians (so that the Dirac Hamiltonian

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is reduced to the canonical Hamiltonian) and the time parameters (replacing the Dirac multipliers) are the coordinates of a group manifold for a Lie group whose algebra is the given Lie algebra: they enter in the multitemporal equations via a set of left invariant vector fields \( Y_\alpha \) on the group manifold \( [Y_\alpha A(q,p) = \{A(q,p),\phi_\alpha\}] \). In the ideal case in which the gauge foliation of \( \bar{\gamma} \) is nice, all the leaves (or gauge orbits) are diffeomorphic and in the simplest case all of them are diffeomorphic to the group manifold of a Lie group. In this ideal case to rebuild a gauge orbit from one of its points (and therefore to determine the gauge part of the trajectories passing through that point) one needs the Lie equations associated with the given Lie group: the Hamiltonian multitemporal equations are generalized Lie equations describing all the gauge orbits simultaneously. In a generic case this description holds only locally for a set of diffeomorphic orbits, also in the case of systems invariant under diffeomorphisms.

Once one has solved the multitemporal equations, the next step is the determination of a Shanmugadhasan canonical transformation[26]. In the finite dimensional case general theorems[27] connected with the Lie theory of function groups[28] ensure the existence of local canonical transformations from the original canonical variables \( q^i, p_i \), in terms of which the first class constraints (assumed globally defined) have the form \( \phi_\alpha(q,p) \approx 0 \), to canonical bases \( P_\alpha, Q_\alpha, P_A, Q_A \), such that the equations \( P_\alpha \approx 0 \) locally define the same original constraint manifold (the \( P_\alpha \) are an Abelianization of the first class constraints); the \( Q_\alpha \) are the adapted Abelian gauge variables describing the gauge orbits (they are a realization of the times \( \tau_\alpha \) of the multitemporal equations in terms of variables \( q^i, p_i \)); the \( Q_A, P_A \) are an adapted canonical basis of Dirac observables. These canonical transformations are the basis of the Hamiltonian definition of the Faddeev-Popov measure of the path integral[29] and give a trivialization of the BRS construction of observables (the BRS method works when the first class constraints may be Abelianized[30]). Therefore the problem of the search of the Dirac observables becomes the problem of finding Shanmugadhasan canonical transformations. The strategy is to find abelianizations \( P_\alpha \) of the original constraints, to solve the multitemporal equations for \( q^i, p_i \) associated with the \( P_\alpha \), to determine the multitimes \( Q_\alpha = \tau_\alpha \) and to identify the Dirac observables \( P_A, Q_A \) from the remaining original variables, i.e. from those their combinations independent from \( P_\alpha \) and \( Q_\alpha \). Second class constraints, when present, are also taken into account by the Shanmugadhasan canonical transformation[26].

Putting equal to zero the Abelianized gauge variables one defines a local gauge of the model. If a system with constraints admits one (or more) global Shanmugadhasan canonical transformations, one obtains one (or more) privileged global gauges in which the physical Dirac observables are globally defined and globally separated from the gauge degrees of freedom [for systems with a compact configuration space \( Q \) this is impossible]. These privileged gauges (when they exist) can be called generalized Coulomb gauges. When the system under investigation has some global symmetry group, the associated theory of the momentum map[31] is a source of globality.
C) Now all the physical systems defined in the flat Minkowski spacetime, have the global Poincare’ symmetry. This suggests to study the structure of the constraint manifold $\tilde{\gamma}$ from the point of view of the orbits of the Poincare’ group. If $p^\mu$ is the total momentum of the system, the constraint manifold has to be divided in four strata (some of them may be absent for certain systems) according to whether $p^2 > 0$, $p^2 = 0$, $p^2 < 0$ or $p^\mu = 0$. Due to the different little groups of the various Poincare’ orbits, the gauge orbits of different sectors will not be diffeomorphic. Therefore the manifold $\tilde{\gamma}$ is a stratified manifold and the gauge foliations of relativistic systems are nearly never nice, but rather one has to do with singular foliations.

For an acceptable relativistic system the stratum $p^2 < 0$ has to be absent to avoid tachyons. To study the strata $p^2 = 0$ and $p^\mu = 0$ one has to add these relations as extra constraints. For all the strata the next step (see however the next Section) is to do a canonical transformation from the original variables to a new set consisting of center-of-mass variables $x^\mu$, $p^\mu$ and of variables relative to the center of mass. Let us now consider the stratum $p^2 > 0$. By using the standard Wigner boost $L^\mu_\nu(p, \hat{p}) (p^\mu = L^\mu_\nu(p, \hat{p}) \hat{p}^\nu, \hat{p}^\mu = \eta \sqrt{p^2}(1; \vec{0}), \eta = \text{sign } p^0)$, one boosts the relative variables at rest. The new variables are still canonical and the base is completed by $p^\mu$ and by a new center-of-mass coordinate $\tilde{x}^\mu$, differing from $x^\mu$ for spin terms. The variable $\tilde{x}^\mu$ has complicated covariance properties; instead the new relative variables are either Poincare’ scalars or Wigner spin-1 vectors, transforming under the group $O(3)(p)$ of the Wigner rotations induced by the Lorentz transformations. A final canonical transformation, leaving fixed the relative variables, sends the center-of-mass coordinates $\tilde{x}^\mu$, $p^\mu$ in the new set $p \cdot \tilde{x}/\eta \sqrt{p^2} = p \cdot x/\eta \sqrt{p^2}$ (the time in the rest frame), $\eta \sqrt{p^2}$ (the total mass), $\vec{k} = \hat{p}/\eta \sqrt{p^2}$ (the spatial components of the 4-velocity $k^\mu = p^\mu/\eta \sqrt{p^2}$, $k^2 = 1$), $\vec{z} = \eta \sqrt{p^2}(\tilde{x} - \tilde{x}^0 \hat{p}/p^0)$. $\vec{z}$ is a noncovariant center-of-mass canonical 3-coordinate multiplied by the total mass: it is the classical analog of the Newton-Wigner position operator (like it, $\vec{z}$ is covariant only under the little group $O(3)(p)$ of the timelike Poincare’ orbits). Analoguous considerations could be done for the other sectors. In Ref. [33] there is the definition of other canonical bases, the spin bases, adapted to the spin Casimir of the Poincare’ group.

The nature of the relative variables depends on the system. The first class constraints, once rewritten in terms of the new variables, can be manipulated to find suitable global and Lorentz scalar Abelianizations. Usually there is a combination of the constraints which determines $\eta \sqrt{p^2}$, i.e. the mass spectrum, so that the time in the rest frame $p \cdot x/\eta \sqrt{p^2}$ is the conjugated Lorentz scalar gauge variable. The other constraints eliminate some of the relative variables (in particular the relative energies for systems of interacting relativistic particles and the string): their conjugated coordinates (the relative times) are the other gauge variables: they are identified with a possible set of time parameters by the multitemporal equations. The Dirac observables (apart from the center-of-mass ones $\vec{k}$ and $\vec{z}$) have to be extracted from the remaining relative variables and the construction shows that they will be either Poincare’ scalars or Wigner covariant objects. In this way in each stratum preferred global Shanmugadhasan canonical transformations are identified, when no other
kind of obstruction to globality is present inside the various strata.

D) In gauge field theories the situation is more complicated, because the theorems ensuring the existence of the Shanmugadhasan canonical transformation have not been extended to the infinite-dimensional case. One of the reasons is that some of the constraints can now be interpreted as elliptic equations and they can have zero modes. Let us consider the stratum \( p^2 > 0 \) of free Yang-Mills theory as a prototype and its first class constraints, given by the Gauss laws and by the vanishing of the time components of the canonical momenta. The problem of the zero modes will appear as a singularity structure of the gauge foliation of the allowed strata, in particular of the stratum \( p^2 > 0 \). This phenomenon was discovered in Ref.[34] by studying the space of solutions of Yang-Mills and Einstein equations, which can be mapped onto the constraint manifold of these theories in their Hamiltonian description. It turns out that the space of solutions has a "cone over cone" structure of singularities: if we have a line of solutions with a certain number of symmetries, in each point of this line there is a cone of solutions with one less symmetry. In the Yang-Mills case the “gauge symmetries” of a gauge potential are connected with the generators of its stability group, i.e. with the subgroup of those special gauge transformations which leave invariant that gauge potential (this is the Gribov ambiguity for gauge potentials; there is also a more general Gribov ambiguity for field strengths, the “gauge copies” problem). Since the Gauss laws are the generators of the gauge transformations (and depend on the chosen gauge potential through the covariant derivative), this means that for a gauge potential with non trivial stability group those combinations of the Gauss laws corresponding to the generators of the stability group cannot be any more first class constraints, since they do not generate effective gauge transformations but special symmetry transformations. This problematics has still to be clarified, but it seems that in this case these components of the Gauss laws become third class constraints, which are not generators of true gauge transformations. This new kind of constraints was introduced in Refs.[19, 22] in the finite dimensional case as a result of the study of some examples, in which the Jacobi equations (the linearization of the Euler-Lagrange equations) are singular, i.e. some of their solutions are not infinitesimal deviations between two neighbouring extremals of the Euler-Lagrange equations. This interpretation seems to be confirmed by the fact that the singularity structure discovered in Ref.[34] follows from the existence of singularities of the linearized Yang-Mills and Einstein equations. These problems are part of the Gribov ambiguity, which, as a consequence, induces an extremely complicated stratification and also singularities in each Poincaré stratum of \( \bar{\gamma} \).

Other possible sources of singularities of the gauge foliation of Yang-Mills theory in the stratum \( p^2 > 0 \) may be: i) different classes of gauge potentials identified by different values of the field invariants; ii) the orbit structure of the rest frame (or Thomas) spin \( \vec{S} \), identified by the Pauli-Lubanski Casimir \( W^2 = -p^2 \vec{S}^2 \) of the Poincaré group.

The final outcome of this structure of singularities is that the reduced phase-
where the space of the gauge orbits, is in general a stratified manifold with singularities [16]. In the stratum $p^2 > 0$ of the Yang-Mills theory these singularities survive the Wick rotation to the Euclidean formulation and it is not clear how the ordinary path integral approach and the associated BRS method can take them into account. The search of a global canonical basis of Dirac observables for each stratum of the space of the gauge orbits can give a definition of the measure of the phase space path integral, but at the price of a non polynomial Hamiltonian. Therefore, if it is not possible to eliminate the Gribov ambiguity (assuming that it is only a mathematical obstruction without any hidden physics), the existence of global Dirac observables for Yang-Mills theory is very problematic.

E) Firstly, inspired by Ref. [37] where a canonical basis of Dirac observables was found for the electromagnetic field interacting with a fermion field (whose Dirac observable is a fermion field dressed with a Coulomb cloud), the canonical reduction to noncovariant generalized Coulomb gauges, with the determination of the physical Hamiltonian as a function of a canonical basis of Dirac’s observables, has been achieved for the following isolated systems (for them one asks that the 10 conserved generators of the Poincaré algebra are finite so to be able to use group theory; theories with external fields can only be recovered as limits in some parameter of a subsystem of the isolated system):

1) Relativistic particle mechanics. Its importance stems from the fact that quantum field theory has no particle interpretation: this is forced on it by means of the asymptotic states of the reduction formalism which correspond to the quantization of independent one-body systems described by relativistic mechanics [or relativistic pseudoclassical mechanics [36], when one adds Grassmann variables to describe the intrinsic spin]. Besides the scalar particle ($p^2 - m^2 \approx 0$ or $p^2 \approx 0$), one has control on: i) the pseudoclassical electron ($p_\mu \xi^\mu - m \xi_5 \approx 0$ or $p_\mu \xi^\mu \approx 0$, where $\xi^\mu, \xi_5$ are Grassmann variables; $p^2 - m^2 \approx 0$ or $p^2 \approx 0$ are implied; after quantization the Dirac equation is reproduced); ii) the pseudoclassical neutrino ($p_\mu \xi^\mu + i \xi_5 \epsilon^{\mu \nu \rho \sigma} p_\nu \xi_\rho \xi_\sigma \approx 0$, $p^2 \approx 0$, giving the Weyl particle wave equation $p_\mu \gamma^\mu (1 - \gamma_5) \psi(x) = 0$ after quantization); iii) the pseudoclassical photon ($p^2 \approx 0, p_\mu \theta^\mu \approx 0, p_\mu \theta^{\star \mu} \approx 0, \theta^\star \theta^\mu \approx 0$, where $\theta^\mu, \theta^{\star \mu}$ are a pair of complex Grassmann four-vectors to describe helicity $\pm 1$; after quantization one obtains the photon wave equations $\Box A^\mu(x) = 0, \partial_\mu A^\mu(x) = 0$; the Berezin-Marino Grassmann distribution function allows to recover the classical polarization matrix of classical light and, in quantization, the quantum polarization matrix with the Stokes parameters); iv) the vector particle or pseudoclassical massive photon ($p^2 - m^2 + (1 - \lambda)p_\mu \theta^{\star \mu} p_\nu \theta^\nu \approx 0, \theta^\mu \theta^{\star \mu} \approx 0$, which, after quantization, reproduce the Proca-like wave equation $\Box + \mu^2) A^\mu(x) = 0, \partial_\mu A^\mu(x) = 0$.

The most important two-body system is the DrozVincent-Todorov-Komar model [11] with an arbitrary action-at-a-distance interaction instantaneous in the rest frame as shown by its energy-momentum tensor [12] $p_i^2 - m_i^2 + V(r_i^2) \approx 0, i=1,2, r_1^\mu = (p^{\mu \nu} - p^\mu p^\nu/p^2) r_\nu, r^\mu = x_1^\mu - x_2^\mu, p_\mu = p_1\mu + p_2\mu$. This model has been completely understood both at the classical and quantum level [32]. Its study led to the identification of a class of canonical transformations (utilizing the standard
Wigner boost for timelike Poincaré orbits) which allowed to understand how to define suitable center-of-mass and relative variables (in particular a suitable relative energy is determined by a combination of the two first class constraints, so that the relative time variable is a gauge variable), how to find a quasi-Shanmugadhasan canonical transformation adapted to the constraint determining the relative energy, how to separate the four, topologically disjoined, branches of the mass spectrum (it is determined by the other independent combination of the constraints; therefore, there is a distinct Shanmugadhasan canonical transformation for each branch). At the quantum level it was possible to find four physical scalar products, compatible with both the resulting coupled wave equations (i.e. independent from the relative and the absolute rest-frame times): they have been found as generalization of the two existing scalar products of the Klein-Gordon equation: all of them are non-local even in the limiting free case and differ among themselves for the sign of the norm of states on different mass-branches. This example shows that the physical scalar product knows the functional form of the constraints.

The connection with the Bethe-Salpeter equation of the quantized model has been studied in Ref.[43], where it is shown that the constraint wave function can be obtained from the Bethe-Salpeter one by multiplication for a delta function containing the relative energy to exclude its spurious solutions (non physical excitations in the relative energy). The extension of the model to two pseudoclassical electrons and to an electron and a scalar has been done in Ref.[44], and the first was used to get good fits to meson spectra.

The previous canonical transformations were then extended to N free particles described by N mass-shell first class constraints $p_i^2 - m_i^2 \approx 0$[45]: N-1 suitable relative energies are determined by N-1 combinations of the constraints (so that the conjugate N-1 relative times are gauge variables), while the remaining combination determines the $2^N$ branches of the mass spectrum. The N gauge freedoms associated with these N combinations of the first class constraints are the freedom of the observer: i) in the choice of the time parameter to be used for the overall evolution of the isolated system; ii) in the choice of the description of the relative motions with any given delay among the pairs of particles.

In Ref.[46] 2- and N-body Newton mechanics was reformulated in a multitemporal way in terms of N first class constraints obtained from the relativistic ones in the limit $c \to \infty$. After a comparison with predictive mechanics, it was shown that the “no-interaction-theorem” (namely that the multitemporal configurational and canonical position coordinates of a particle coincide only in absence of interactions) exists also at the nonrelativistic level, being a property of the multitemporal description of particles and not of the kinematical symmetry group.

2) Both the open and closed Nambu string, after an initial study with lightcone coordinates, have been treated[47] along the lines of the two-body model in the stratum $p^2 > 0$. Both Abelian Lorentz scalar constraints and gauge variables have been found and globally decoupled, and a redundant set of Dirac’s observables $[\vec{z}, \vec{k}, \vec{a}_n]$ has been found. It remains an open problem whether one can extract a
global canonical basis of Dirac's observables from the Wigner spin 1 vectors $\tilde{a}_n$, which satisfy sigma-model-like constraints; if this basis exists, it would define the Liouville integrability of the Nambu string and would clarify whether there is any way to quantize it in four dimensions.

3) Yang-Mills theory with Grassmann-valued fermion fields \[48\] in the case of a trivial principal bundle over a fixed-$x^0$ $R^3$ slice of Minkowski spacetime with suitable Hamiltonian-oriented boundary conditions; this excludes monopole solutions (to have them, even if they have been not yet found experimentally, one needs a nontrivial bundle and a variational principle formulated on the bundle \[49\], because the gauge potentials on Minkowski spacetime are not globally defined) and, since $R^3$ is not compactified, one has only winding number and no instanton number. After a discussion of the Hamiltonian formulation of Yang-Mills theory, of its group of gauge transformations and of the Gribov ambiguity, the theory has been studied in suitable weighted Sobolev spaces where the Gribov ambiguity is absent \[50\] and the global color charges are well defined. The global Dirac observables are the transverse quantities $\tilde{A}_{\perp}(\tilde{x}, x^0)$, $\tilde{E}_{\perp}(\tilde{x}, x^0)$ and fermion fields dressed with Yang-Mills (gluonic) clouds. The nonlocal and nonpolynomial (due to the presence of classical Wilson lines along flat geodesics) physical Hamiltonian has been obtained: it is nonlocal but without any kind of singularities, it has the correct Abelian limit if the structure constants are turned off, and it contains the explicit realization of the abstract Mitter-Viallet metric.

4) The Abelian and non-Abelian SU(2) Higgs models with fermion fields \[51\], where the symplectic decoupling is a refinement of the concept of unitary gauge. There is an ambiguity in the solutions of the Gauss law constraints, which reflects the existence of disjoint sectors of solutions of the Euler-Lagrange equations of Higgs models. The physical Hamiltonian and Lagrangian of the Higgs phase have been found; the self-energy turns out to be local and contains a local four-fermion interaction.

5) The standard SU(3)xSU(2)xU(1) model of elementary particles \[52\] with Grassmann-valued fermion fields. The final reduced Hamiltonian contains nonlocal self-energies for the electromagnetic and color interactions, but “local ones” for the weak interactions implying the nonperturbative emergence of 4-fermions interactions.

F) When a good description of the system in terms of Dirac observables exists, one is going to face the problem of quantizing only the true physical degrees of freedom, which generically are nonlinear and nonlocal functions or functionals of the original variables. When a quantization is possible, there is a high probability to get a quantum theory inequivalent to that obtained by first quantizing the original variables and then making the reduction to the physical degrees of freedom at the quantum level (see for instance the BRS method).

With regards to field theory, this method has the drawback that generically the physical Hamiltonian, and therefore also the Lagrangian, is non polynomial in the physical degrees of freedom. Power counting methods cannot be used when looking
for regularizations and renormalizations of the theory, and the advantages of a global control of the dynamics of physical quantities and of the possibility to check whether a model is classically well defined are destroyed by our present inabilty to solve these problems. The question, which puzzled both Dirac and Yukawa, reappears, whether it is possible to define an intrinsic ultraviolet cutoff and a regularization scheme independent from the power counting.

2 The Separation of the Center of Mass in Special Relativity, the Rest-Frame Instant Form of Dynamics and Wigner-Covariant Generalized Coulomb Gauges.

The next problem is how to covariantize these results valid in Minkowski spacetime with Cartesian coordinates. Again the starting point was given by Dirac[1] with his reformulation of classical field theory on spacelike hypersurfaces foliating Minkowski spacetime $M^4$ [the foliation is defined by an embedding $R \times \Sigma \rightarrow M^4$, $(\tau, \vec{\sigma}) \mapsto z^{(\mu)}(\tau, \vec{\sigma}) \in \Sigma_\tau$, with $\Sigma$ an abstract 3-surface diffeomorphic to $R^3$, with $\Sigma_\tau$ its copy embedded in $M^4$ labelled by the value $\tau$ (the Minkowski flat indices are $(\mu)$; the scalar “time” parameter $\tau$ labels the leaves of the foliation, $\vec{\sigma}$ are curvilinear coordinates on $\Sigma_\tau$ and $(\tau, \vec{\sigma})$ are $\Sigma_\tau$-adapted holonomic coordinates for $M^4$); this is the classical basis of Tomonaga-Schwinger quantum field theory]. In this way one gets a parametrized field theory with a covariant 3+1 splitting of Minkowski spacetime and already in a form suited to the transition to general relativity in its ADM canonical formulation (see also Ref.[53], where a theoretical study of this problem is done in curved spacetimes). The price is that one has to add as new independent configuration variables the embedding coordinates $z^{(\mu)}(\tau, \vec{\sigma})$ of the points of the spacelike hypersurface $\Sigma_\tau$ [the only ones carrying Lorentz indices] and then to define the fields on $\Sigma_\tau$ so that they know the hypersurface $\Sigma_\tau$ of $\tau$-simultaneity [for a Klein-Gordon field $\phi(x)$, this new field is $\tilde{\phi}(\tau, \vec{\sigma}) = \phi(z(\tau, \vec{\sigma}))$: it contains the nonlocal information about the embedding]. Then one rewrites the Lagrangian of the given isolated system in the form required by the coupling to an external gravitational field, makes the previous 3+1 splitting of Minkowski spacetime and interprets all the fields of the system as the new fields on $\Sigma_\tau$ (they are Lorentz scalars, having only surface indices). Instead of considering the 4-metric as describing a gravitational field (and therefore as an independent field as it is done in metric gravity, where one adds the Hilbert action to the action for the matter fields), here one replaces the 4-metric with the the induced metric $g_{AB}[z] = z_A^{(\mu)} \eta_{(\mu)(\nu)} z_B^{(\nu)}$ on $\Sigma_\tau$ [a functional of $z^{(\mu)}$; here we use the notation $\sigma^A = (\tau, \sigma^r)$; $z_A^{(\mu)} = \partial z^{(\mu)} / \partial \sigma^A$ are flat tetrad fields on Minkowski spacetime with the $z_A^{(\mu)}$’s tangent to $\Sigma_\tau$] and considers the embedding coordinates $z^{(\mu)}(\tau, \vec{\sigma})$ as independent fields [this is not possible in
metric gravity, because in curved spacetimes \( z_A^\mu \neq \partial z^\mu / \partial \sigma^A \) are not tetrad fields so that holonomic coordinates \( z^\mu (\tau, \vec{\sigma}) \) do not exist. From this Lagrangian, besides a Lorentz-scalar form of the constraints of the given system, we get four extra primary first class constraints

\[
\mathcal{H}(\mu)(\tau, \vec{\sigma}) = \rho(\mu)(\tau, \vec{\sigma}) - l(\mu)(\tau, \vec{\sigma}) T_{\text{sys}}^{\tau\tau}(\tau, \vec{\sigma}) - z(\mu)(\tau, \vec{\sigma}) T_{\text{sys}}^{\tau\nu}(\tau, \vec{\sigma}) \approx 0
\]

[here \( T_{\text{sys}}^{\tau\tau}(\tau, \vec{\sigma}), T_{\text{sys}}^{\tau\nu}(\tau, \vec{\sigma}) \), are the components of the energy-momentum tensor in the holonomic coordinate system, corresponding to the energy- and momentum-density of the isolated system; one has \( \{\mathcal{H}(\mu)(\tau, \vec{\sigma}), \mathcal{H}(\nu)(\tau, \vec{\sigma})\} = 0 \) implying the independence of the description from the choice of the 3+1 splitting, i.e. from the choice of the foliation with spacelike hypersurfaces. The evolution vector is given by \( z_\mu(\tau) = N_{[z]}(\text{flat}) l(\mu) + N_{[z]}^r(\text{flat}) z^\mu(\tau) \), where \( l(\mu)(\tau, \vec{\sigma}) \) is the normal to \( \Sigma_\tau \) in \( z^\mu(\tau, \vec{\sigma}) \) and \( N_{[z]}(\text{flat})(\tau, \vec{\sigma}), N_{[z]}^r(\text{flat})(\tau, \vec{\sigma}) \) are the flat lapse and shift functions defined through the metric like in general relativity: however, now they are not independent variables but functionals of \( z^\mu(\tau, \vec{\sigma}) \).

The Dirac Hamiltonian contains the piece \( \int d^3 \sigma \lambda(\mu)(\tau, \vec{\sigma}) \mathcal{H}(\mu)(\tau, \vec{\sigma}) \) with \( \lambda(\mu)(\tau, \vec{\sigma}) \) Dirac multipliers. It is possible to rewrite the integrand in the form \( [3g^{rr} \text{ is the inverse of } g_{rs}] \)

\[
\lambda(\mu)(\tau, \vec{\sigma}) \mathcal{H}(\mu)(\tau, \vec{\sigma}) = [(\lambda(\mu)(l(\nu)))(l(\nu)\mathcal{H}(\nu)) - (\lambda(\mu)z^\mu(\tau))(3g^{rr}z_{a(\nu)}\mathcal{H}(\nu))](\tau, \vec{\sigma})
\]

\[
\equiv N_{(\text{flat})}(\tau, \vec{\sigma})(l(\mu)\mathcal{H}(\mu))(\tau, \vec{\sigma}) - N_{(\text{flat})}^r(\tau, \vec{\sigma})(3g^{rr}z_{a(\nu)}\mathcal{H}(\nu))(\tau, \vec{\sigma})
\]

with the (nonholonomic form of the) constraints \( (l(\mu)\mathcal{H}(\mu))(\tau, \vec{\sigma}) \approx 0, (3g^{rr}z_{a(\nu)}\mathcal{H}(\mu))(\tau, \vec{\sigma}) \approx 0 \), satisfying the universal Dirac algebra of the ADM constraints. In this way we have defined new flat lapse and shift functions

\[
N_{(\text{flat})}(\tau, \vec{\sigma}) = \lambda(\mu)(\tau, \vec{\sigma}) l(\mu)(\tau, \vec{\sigma}),
N_{(\text{flat})}^r(\tau, \vec{\sigma}) = \lambda(\mu)(\tau, \vec{\sigma}) z^\mu(\tau, \vec{\sigma}),
\]

which have the same content of the arbitrary Dirac multipliers \( \lambda(\mu)(\tau, \vec{\sigma}) \), namely they multiply primary first class constraints satisfying the Dirac algebra. In Minkowski spacetime they are quite distinct from the previous lapse and shift functions \( N_{[z]}(\text{flat}), N_{[z]}^r(\text{flat}) \), defined starting from the metric. Instead in general relativity the lapse and shift functions defined starting from the 4-metric are the coefficients (in the canonical part \( H_c \) of the Hamiltonian) of secondary first class constraints satisfying the Dirac algebra.

In special relativity, it is convenient to restrict ourselves to arbitrary spacelike hyperplanes \( z^\mu(\tau, \vec{\sigma}) = x^\mu_s(\tau) + b^\mu_f(\tau) \sigma^r \). Since they are described by only 10 variables, after this restriction we remain only with 10 first class constraints determining the 10 variables conjugate to the hyperplane in terms of the variables of the system:
\[ H^{(\mu)}(\tau) = p^{(\mu)}_s - p^{(\mu)}_{(sys)} \approx 0, \quad H^{(\nu)}(\tau) = S^{(\mu)}_s - S^{(\mu)}_{(sys)} \approx 0. \]

After the restriction to spacelike hyperplanes the previous piece of the Dirac Hamiltonian is reduced to \( \tilde{H}^{(\mu)}(\tau) = H^{(\mu)}(\tau) - \frac{1}{\gamma} \tilde{x}^{(\mu)}(\tau) H^{(\nu)}(\tau) \). Since at this stage we have \( \tau^{(\mu)}(\tau, \tilde{\sigma}) \approx b^{(\mu)}(\tau) \), so that \( \tilde{z}^{(\mu)}(\tau, \tilde{\sigma}) \approx N_{[z]}(\text{flat})(\tau, \tilde{\sigma}) l^{(\mu)}(\tau, \tilde{\sigma}) + N_{[\tilde{z}]}(\text{flat})(\tau, \tilde{\sigma}) \) \( b^{(\mu)}(\tau, \tilde{\sigma}) \approx \tilde{x}^{(\mu)}(\tau) + b^{(\mu)}(\tau) \sigma^\tau = -\tilde{\lambda}^{(\mu)}(\tau) - \tilde{\bar{\lambda}}^{(\mu)}(\tau) b^{(\nu)}(\tau) \sigma^\tau \), it is only now that we get the coincidence of the two definitions of flat lapse and shift functions (this point was missed in the older treatments of parametrized Minkowski theories):

\[
N_{[z]}(\text{flat})(\tau, \tilde{\sigma}) \approx N(\text{flat})(\tau, \tilde{\sigma}) = -\tilde{\lambda}^{(\mu)}(\tau) l^{(\mu)} - l^{(\mu)} \bar{\lambda}^{(\mu)}(\tau) b^{(\nu)}(\tau) \sigma^\tau,
N_{[\tilde{z}]}(\text{flat})(\tau, \tilde{\sigma}) \approx N(\text{flat})(\tau, \tilde{\sigma}) = -\tilde{\lambda}^{(\mu)}(\tau) b^{(\nu)}(\tau) - b^{(\nu)}(\tau) \tilde{\lambda}^{(\mu)}(\tau) b^{(\nu)}(\tau) \sigma^\tau.
\]

The 20 variables for the phase space description of a hyperplane are:

i) \( x^{(\mu)}(\tau) \), \( p^{(\mu)}_s \), parametrizing the origin of the coordinates on the family of spacelike hyperplanes. The four constraints \( H^{(\mu)}(\tau) \approx 0 \) say that \( p^{(\mu)}_s \) is determined by the 4-momentum of the isolated system.

ii) \( b_A^{(\mu)}(\tau) \) (with the \( b^{(\mu)}(\tau) \)'s being three orthogonal spacelike unit vectors generating the fixed \( \tau \)-independent timelike unit normal \( b^{(\mu)}_r \) to the hyperplanes) and \( S^{(\mu)}_s - S^{(\mu)}_{(sys)} \) with the orthonormality constraints \( b_A^{(\mu)} b_B^{(\nu)} = 4 \eta_{AB} \) [enforced by assuming the Dirac brackets \( \{ S^{(\mu)}_s, b_A^{(\nu)} \} = 4 \eta^{(\mu)(\nu)} b_A^{(\mu)} - 4 \eta^{(\mu)(\nu)} b_A^{(\nu)} \), \( \{ S^{(\mu)}_s, S^{(\alpha)(\beta)}_s \} = C^{(\mu)(\nu)(\alpha)(\beta)} S^{(\gamma)}_s \delta^{(\gamma)(\delta)} \) with \( C^{(\mu)(\nu)(\alpha)(\beta)} \) the structure constants of the Lorentz algebra]. In these variables there are hidden six independent pairs of degrees of freedom. The six constraints \( H^{(\nu)}(\tau) \approx 0 \) say that \( S^{(\mu)}_s \) coincides the spin tensor of the isolated system. Then one has that \( p^{(\mu)}_s, J^{(\nu)}_s = x^{(\mu)}_s p^{(\nu)}_s - x^{(\nu)}_s p^{(\mu)}_s + S^{(\mu)(\nu)}_s \), satisfy the algebra of the Poincaré group.

Let us remark that, for each configuration of an isolated system there is a privileged family of hyperplanes (the Wigner hyperplanes orthogonal to \( p^{(\mu)}_s \), existing when \( p^2_s > 0 \) corresponding to the intrinsic rest-frame of the isolated system. If we choose these hyperplanes with suitable gauge fixings, we remain with only the four constraints \( H^{(\mu)}(\tau) \approx 0 \), which can be rewritten as

\[
\sqrt{p^2_s} \approx [\text{invariant mass of the isolated system under investigation}] = M_{sys}; \quad \vec{p}_{sys} = [\text{3 - momentum of the isolated system inside the Wigner hyperplane}] \approx 0.
\]

There is no more a restriction on \( p^{(\mu)}_s \), because \( u^{(\mu)}_s(p_s) = p^{(\mu)}_s / p^2_s \) gives the orientation of the Wigner hyperplanes containing the isolated system with respect to an arbitrary given external observer.

In this special gauge we have \( b_A^{(\mu)} \equiv L^{(\mu)} A(p_s, \vec{p}_s) \) (the standard Wigner boost for timelike Poincaré orbits), \( S^{(\mu)}_s \equiv S^{(\mu)}_{sys} \), and the only remaining canonical variables are the noncovariant Newton-Wigner-like canonical “external” center-of-mass coordinate \( \bar{x}^{(\mu)}(\tau) \) (living on the Wigner hyperplanes) and \( p^{(\mu)}_s \). Now 3 degrees of freedom of the isolated system [an “internal” center-of-mass 3-variable \( \bar{\sigma}_{sys} \) defined
inside the Wigner hyperplane and conjugate to \( \vec{p}_{\text{sys}} \) become gauge variables [the natural gauge fixing is \( \vec{\sigma}_{\text{sys}} \approx 0 \), so that it coincides with the origin \( x_s^{(\mu)}(\tau) = z^{(\mu)}(\tau, \vec{\sigma} = 0) \) of the Wigner hyperplane], while the \( \vec{x}^{(\mu)} \) is playing the role of a kinematical external center of mass for the isolated system and may be interpreted as a decoupled observer with his parametrized clock (point particle clock). All the fields living on the Wigner hyperplane are now either Lorentz scalar or with their 3-indices transformaing under Wigner rotations (induced by Lorentz transformations in Minkowski spacetime) as any Wigner spin 1 index.

One obtains in this way a new kind of instant form of the dynamics (see Ref.[54]), the “Wigner-covariant 1-time rest-frame instant form”[55] with a universal breaking of Lorentz covariance. It is the special relativistic generalization of the nonrelativistic separation of the center of mass from the relative motion \( [H = \frac{\vec{p}^2}{2M} + H_{\text{rel}}] \). The role of the center of mass is taken by the Wigner hyperplane, identified by the point \( \vec{x}^{(\mu)}(\tau) \) and by its normal \( p^{(\mu)}_{\text{sys}} \). The invariant mass \( M_{\text{sys}} \) of the system replaces the nonrelativistic Hamiltonian \( H_{\text{rel}} \) for the relative degrees of freedom, after the addition of the gauge-fixing \( T_s - \tau \approx 0 \) [identifying the time parameter \( \tau \), labelling the leaves of the foliation, with the Lorentz scalar time of the center of mass in the rest frame, \( T_s = p_{\text{sys}} \cdot \vec{x}_{\text{sys}}/M_{\text{sys}} \); \( M_{\text{sys}} \) generates the evolution in this time].

The determination of \( \vec{\sigma}_{\text{sys}} \) may be done with the group theoretical methods of Ref.[56]: given a realization on the phase space of a given system of the ten Poincaré generators one can build three 3-position variables only in terms of them, which in our case of a system on the Wigner hyperplane with \( \vec{p}_{\text{sys}} \approx 0 \) are: i) a canonical center of mass (the “internal” center of mass \( \vec{\sigma}_{\text{sys}} \); ii) a noncanonical Møller center of energy \( \vec{\sigma}^{(E)}_{\text{sys}} \); iii) a noncanonical Fokker-Pryce center of inertia \( \vec{\sigma}^{(FP)}_{\text{sys}} \). Due to \( \vec{p}_{\text{sys}} \approx 0 \), we have \( \vec{\sigma}_{\text{sys}} \approx \vec{\sigma}^{(E)}_{\text{sys}} \approx \vec{\sigma}^{(FP)}_{\text{sys}} \). By adding the gauge fixings \( \vec{\sigma}_{\text{sys}} \approx 0 \) one can show that the origin \( x_s^{(\mu)}(\tau) \) becomes simultaneously the Dixon center of mass of an extended object and both the Pirani and Tulczyjew centroids (see Ref.[57] for the application of these methods to find the center of mass of a configuration of the Klein-Gordon field after the preliminary work of Ref.[58]). With similar methods one can construct three “external” collective positions (all located on the Wigner hyperplane): i) the “external” canonical noncovariant center of mass \( \vec{x}_{\text{sys}}^{(\mu)} \); ii) the “external” noncanonical and noncovariant Møller center of energy \( R^{(\mu)}_{\text{sys}} \); iii) the “external” covariant noncanonical Fokker-Pryce center of inertia \( \vec{Y}^{(\mu)}_{\text{sys}} \) (when there are the gauge fixings \( \vec{\sigma}_{\text{sys}} \approx 0 \) it also coincides with the origin \( x_s^{(\mu)} \)).

It turns out that the Wigner hyperplane is the natural setting for the study of the Dixon multipoles of extended relativistic systems[54] and for defining the canonical relative variables with respect to the center of mass. After having put control on the relativistic definitions of center of mass of an extended system, the lacking kinematics of relativistic rotations in now under investigation. The Wigner hyperplane with its natural Euclidean metric structure offers a natural solution to the problem of boost for lattice gauge theories and realizes explicitly the machian aspect of dynamics that only relative motions are relevant.

The isolated systems till now analyzed to get their rest-frame Wigner-covariant

\[ 15 \]
generalized Coulomb gauges [i.e. the subset of global Shanmugadhasan canonical bases, which, for each Poincaré stratum, are also adapted to the geometry of the corresponding Poincaré orbits with their little groups; these special bases can be named Poincaré-Shanmugadhasan bases for the given Poincaré stratum of the presymplectic constraint manifold (every stratum requires an independent canonical reduction); till now only the main stratum with \( p^2 \) timelike and \( W^2 \neq 0 \) has been investigated]

are:

a) The system of \( N \) scalar particles with Grassmann electric charges plus the electromagnetic field \([55]\). The starting configuration variables are a 3-vector \( \vec{\eta}_i(\tau) \) for each particle \([x_i^{(\mu)}(\tau) = z^{(\mu)}(\tau, \vec{\eta}_i(\tau))]\) and the electromagnetic gauge potentials \( A_A(\tau, \vec{\sigma}) = \frac{\partial z^{(\mu)}(\tau, \vec{\sigma})}{\partial \sigma^A} A_{(\mu)}(z(\tau, \vec{\sigma})) \), which know the embedding of \( \Sigma_\tau \) into \( M^4 \). One has to choose the sign of the energy of each particle, because there are no mass-shell constraints (like \( p_i^2 - m_i^2 \approx 0 \)) among the constraints of this formulation, due to the fact that one has only three degrees of freedom for particle, determining the intersection of a timelike trajectory and of the spacelike hypersurface \( \Sigma_\tau \). For each choice of the sign of the energy of the \( N \) particles, one describes only one of the \( 2^N \) branches of the mass spectrum of the manifestly covariant approach based on the coordinates \( x_i^{(\mu)}(\tau), p_i^{(\mu)}(\tau), i=1,...,N \), and on the constraints \( p_i^2 - m_i^2 \approx 0 \) (in the free case). In this way, one gets a description of relativistic particles with a given sign of the energy with consistent couplings to fields and valid independently from the quantum effect of pair production [in the manifestly covariant approach, containing all possible branches of the particle mass spectrum, the classical counterpart of pair production is the intersection of different branches deformed by the presence of interactions]. The final Dirac’s observables are: i) the transverse radiation field variables \( \vec{A}_\perp, \vec{E}_\perp \); ii) the particle canonical variables \( \vec{\eta}_i(\tau), \vec{\kappa}_i(\tau) \), dressed with a Coulomb cloud. The physical Hamiltonian contains the mutual instantaneous Coulomb potentials extracted from field theory and there is a regularization of the Coulomb self-energies due to the Grassmann character of the electric charges \( Q_i, [Q_i^2 = 0] \). In Ref.\([60]\) there is the study of the Lienard-Wiechert potentials and of Abraham-Lorentz-Dirac equations in this rest-frame Coulomb gauge and also scalar electrodynamics is reformulated in it. Also the rest-frame 1-time relativistic statistical mechanics has been developed \([55]\).

b) The system of \( N \) scalar particles with Grassmann-valued color charges plus the color SU(3) Yang-Mills field \([61]\): it gives the pseudoclassical description of the relativistic scalar-quark model, deduced from the classical QCD Lagrangian and with the color field present. The physical invariant mass of the system is given in terms of the Dirac observables. From the reduced Hamilton equations the second order equations of motion both for the reduced transverse color field and the particles are extracted. Then, one studies the \( N=2 \) (meson) case. A special form of the requirement of having only color singlets, suited for a field-independent quark model, produces a “pseudoclassical asymptotic freedom” and a regularization of the quark self-energy. With these results one can covariantize the bosonic part of the standard
model given in Ref. [52].

c) The system of N spinning particles of definite energy [(1/2, 0) or (0, 1/2) representation of SL(2,C)] with Grassmann electric charges plus the electromagnetic field [52] and that of a Grassmann-valued Dirac field plus the electromagnetic field (the pseudoclassical basis of QED) [63]. In both cases there are geometrical complications connected with the spacetime description of the path of electric currents and not only of their spin structure, suggesting a reinterpretation of the supersymmetric scalar multiplet as a spin fibration with the Dirac field in the fiber and the Klein-Gordon field in the base; a new canonical decomposition of the Klein-Gordon field into center-of-mass and relative variables [58, 57] will be helpful to clarify these problems. After their solution and after having obtained the description of Grassmann-valued chiral fields [this will require the transcription of the front form of the dynamics in the instant one for the Poincaré strata with \( P^2 = 0 \)] the rest-frame form of the full standard \( SU(3) \times SU(2) \times U(1) \) model can be achieved.

The rest-frame description of the relativistic perfect gas is now under investigation.

All these new pieces of information will allow, after quantization of this new consistent relativistic mechanics without the classical problems connected with pair production, to find the asymptotic states of the covariant Tomonaga-Schwinger formulation of quantum field theory on spacelike hypersurfaces (to be obtained by quantizing the fields on \( \Sigma_T \)): these states are needed for the theory of quantum bound states [since Fock states do not constitute a Cauchy problem for the field equations, because an in (or out) particle can be in the absolute future of another one due to the tensor product nature of these asymptotic states, bound state equations like the Bethe-Salpeter one have spurious solutions which are excitations in relative energies, the variables conjugate to relative times]. Moreover, it will be possible to include bound states among the asymptotic states.

As said in Ref. [60, 61], the quantization of these rest-frame models has to overcome two problems. On the particle side, the complication is the quantization of the square roots associated with the relativistic kinetic energy terms: in the free case this has been done in Ref. [64] [see Refs. 65 for the complications induced by the Coulomb potential]. On the field side (all physical Hamiltonian are nonlocal and, with the exception of the Abelian case, nonpolynomial, but quadratic in the momenta), the obstacle is the absence (notwithstanding there is no no-go theorem) of a complete regularization and renormalization procedure of electrodynamics (to start with) in the Coulomb gauge: see Ref. 66 (and its bibliography) for the existing results for QED.

However, as shown in Refs. [55, 48], the rest-frame instant form of dynamics automatically gives a physical ultraviolet cutoff in the spirit of Dirac and Yukawa: it is the Möller radius [57] \( \rho = \sqrt{-W^2/p^2} = |\vec{S}|/\sqrt{p^2} \) (\( W^2 = -p^2\vec{S}^2 \) is the Pauli-Lubanski Casimir when \( p^2 > 0 \)), namely the classical intrinsic radius of the worldtube, around the covariant noncanonical Fokker-Pryce center of inertia \( Y(\mu) \), inside which the noncovariance of the canonical center of mass \( \vec{x}^\mu \) is concentrated. At the quantum level
$\rho$ becomes the Compton wavelength of the isolated system multiplied its spin eigenvalue $\sqrt{s(s+1)}$, $\rho \mapsto \hat{\rho} = \sqrt{s(s+1)}\hbar/M = \sqrt{s(s+1)}\lambda_M$ with $M = \sqrt{p^2}$ the invariant mass and $\lambda_M = \hbar/M$ its Compton wavelength. Therefore, the criticism to classical relativistic physics, based on quantum pair production, concerns the testing of distances where, due to the Lorentz signature of spacetime, one has intrinsic classical covariance problems: it is impossible to localize the canonical center of mass $\vec{x}^\mu$ adapted to the first class constraints of the system (also named Pryce center of mass and having the same covariance of the Newton-Wigner position operator) in a frame independent way.

Let us remember [55] that $\rho$ is also a remnant in flat Minkowski spacetime of the energy conditions of general relativity: since the Møller noncanonical, noncovariant center of energy $R^{(\mu)}$ has its noncovariance localized inside the same worldtube with radius $\rho$ (it was discovered in this way) [67], it turns out that for an extended relativistic system with the material radius smaller of its intrinsic radius $\rho$ one has: i) its peripheral rotation velocity can exceed the velocity of light; ii) its classical energy density cannot be positive definite everywhere in every frame.

Now, the real relevant point is that this ultraviolet cutoff determined by $\rho$ exists also in Einstein’s general relativity (which is not power counting renormalizable) in the case of asymptotically flat spacetimes, taking into account the Poincaré Casimirs of its asymptotic ADM Poincaré charges (when supertranslations are eliminated with suitable boundary conditions). The generalization of the worldtube of radius $\rho$ to asymptotically flat general relativity with matter, could also be connected with the unproved cosmic censorship hypothesis.

Moreover, the extended Heisenberg relations of string theory [55], i.e. $\Delta x = \frac{\hbar}{\Delta p} + \frac{\Delta p}{\Delta x} = \frac{\hbar}{\Delta p} + \frac{\hbar \Delta p}{L_{cs}^2}$, implying the lower bound $\Delta x > L_{cs} = \sqrt{\hbar/T_{cs}}$ due to the $y+1/y$ structure, have a counterpart in the quantization of the Møller radius [77]: if we ask that, also at the quantum level, one cannot test the inside of the worldtube, we must ask $\Delta x > \hat{\rho}$ which is the lower bound implied by the modified uncertainty relation $\Delta x = \frac{\hbar}{\Delta p} + \frac{\hbar \Delta p}{\rho^2}$. This could imply that the center-of-mass canonical noncovariant 3-coordinate $\vec{z} = \sqrt{P^2(\vec{x} - \vec{P})}$ [55] cannot become a self-adjoint operator. See Hegerfeldt’s theorems (quoted in Refs. [18, 55]) and his interpretation pointing at the impossibility of a good localization of relativistic particles (experimentally one determines only a worldtube in spacetime emerging from the interaction region).

Since the eigenfunctions of the canonical center-of-mass operator are playing the role of the wave function of the universe, one could also say that the center-of-mass variable has not to be quantized, because it lies on the classical macroscopic side of Copenhagen’s interpretation and, moreover, because, in the spirit of Mach’s principle that only relative motions can be observed, no one can observe it (it is only used to define a decoupled “point particle clock”). On the other hand, if one rejects the canonical noncovariant center of mass in favor of the covariant noncanonical Fokker-Pryce center of inertia $Y^\mu$, $\{Y^\mu, Y^\nu\} \neq 0$, one could invoke the philosophy of quantum groups to quantize $Y^\mu$ to get some kind of quantum plane for the
center-of-mass description. Let us remark that the quantization of the square root Hamiltonian done in Ref. [64] is consistent with this problematic.

In conclusion, the best set of canonical coordinates adapted to the constraints and to the geometry of Poincaré orbits in Minkowski spacetime and naturally predisposed to the coupling to canonical tetrad gravity is emerging for the electromagnetic, weak and strong interactions with matter described either by fermion fields or by relativistic particles with a definite sign of the energy.

3 Tetrad Gravity, Physical Hamiltonian Degrees of Freedom of the Gravitational Field and the Deparametrization of General Relativity.

Tetrad gravity is the formulation of general relativity natural for the coupling to the fermion fields of the standard model. However, we need a formulation of it, which allows to solve its constraints for doing the canonical reduction and to solve the deparametrization problem of general relativity (how to recover the rest-frame instant form when the Newton constant is put equal to zero, $G=0$). Since neither a complete reduction of gravity with an identification of the physical canonical degrees of freedom of the gravitational field nor a detailed study of its Hamiltonian group of gauge transformations (whose infinitesimal generators are the first class constraints) has ever been pushed till the end in an explicit way, a new formulation of tetrad gravity [69, 70, 71, 72] was developed.

To implement this program we shall restrict ourselves to the simplest class of spacetimes [time-oriented pseudo-Riemannian or Lorentzian 4-manifold $(M^4,\text{^4}g)$ with signature $\epsilon (+ - - -) (\epsilon = \pm 1$ according to either particle physics or general relativity convention) and with a choice of time orientation], assumed to be:

i) Globally hyperbolic 4-manifolds, i.e. topologically they are $M^4 = R \times \Sigma$, so to have a well posed Cauchy problem [with $\Sigma$ the abstract model of Cauchy surface] at least till when no singularity develops in $M^4$ [see the singularity theorems]. Therefore, these spacetimes admit regular foliations with orientable, complete, non-intersecting spacelike 3-manifolds $\Sigma_\tau \: \left[ \tau: M^4 \rightarrow R, \ z^\mu \mapsto \tau(z^\mu) \right.$, is a global timelike future-oriented function labelling the leaves (surfaces of simultaneity)]. In this way, one obtains 3+1 splittings of $M^4$ and the possibility of a Hamiltonian formulation.

ii) Asymptotically flat at spatial infinity, so to have the possibility to define asymptotic Poincaré charges [3, 4, 3]: they allow the definition of a Møller radius also in general relativity and are a bridge towards a future soldering with the theory of elementary particles in Minkowski spacetime defined as irreducible representation of its kinematical, globally implemented Poincaré group according to Wigner. This excludes Einstein-Wheeler closed universes without boundaries (no asymptotic Poincaré charges), which were introduced to eliminate boundary conditions at spatial infinity to make the theory as machian as possible.
iii) Admitting a spinor (or spin) structure for the coupling to fermion fields. Since we consider noncompact space- and time-orientable spacetimes, spinors can be defined if and only if they are “parallelizable”, like in our case. This implies that the orthonormal frame principal SO(3)-bundle over \( \Sigma \) (whose connections are the spin connections determined by the cotriads) is trivial.

iv) The noncompact parallelizable simultaneity 3-manifolds (the Cauchy surfaces) \( \Sigma \) are assumed to be topologically trivial, geodesically complete and, finally, diffeomorphic to \( R^3 \). These 3-manifolds have the same manifold structure as Euclidean spaces: a) the geodesic exponential map \( \exp_p : T_p \Sigma \to \Sigma \) is a diffeomorphism; b) the sectional curvature is less or equal zero everywhere; c) they have no “conjugate locus” [i.e. there are no pairs of conjugate Jacobi points (intersection points of distinct geodesics through them) on any geodesic] and no “cut locus” [i.e. no closed geodesics through any point].

v) Like in Yang-Mills case, the 3-spin-connection on the orthogonal frame SO(3)-bundle (and therefore cotriads) will have to be restricted to suited weighted Sobolev spaces to avoid Gribov ambiguities. In turn, this implies the absence of isometries of the noncompact Riemannian 3-manifold \( (\Sigma, g) \) [see for instance the review paper in Ref. [79]].

Diffeomorphisms on \( \Sigma \) are interpreted in the passive way, following Ref. [80], in accord with the Hamiltonian point of view that infinitesimal diffeomorphisms are generated by taking the Poisson bracket with the 1st class supermomentum constraints [passive diffeomorphisms are also named ‘pseudodiffeomorphisms’].

The new formulation of tetrad gravity [see Refs. [81] for the existing versions of the theory] utilizes the ADM action of metric gravity with the 4-metric expressed in terms of arbitrary cotetrad. Let us remark that both in the ADM metric and tetrad formulation one has to introduce the extra ingredient of the 3+1 splittings of \( M^4 \) with foliations whose leaves \( \Sigma \) are spacelike 3-hypersurfaces. However, their points \( z^\mu (\tau, \vec{\sigma}) \) are not tetrads when \( M^4 \) is not Minkowski spacetime with Cartesian coordinates, because
\[
4 g^{AB} \frac{\partial z^\mu}{\partial \sigma^A} \frac{\partial z^\nu}{\partial \sigma^B} = 4 g^{\mu\nu} \neq 4 \eta^{(\mu)}(\nu).
\]

By using \( \Sigma \)-adapted holonomic coordinates for \( M^4 \), one has found a new parametrization of arbitrary tetrads and cotetrad on \( M^4 \) in terms of cotriads on \( \Sigma \) \( [3 e^{(a) r}(\tau, \vec{\sigma})] \), of lapse \( [N(\tau, \vec{\sigma})] \) and shift \( [N(\alpha)(\tau, \vec{\sigma}) = [3 e^{(a) r} N^r](\tau, \vec{\sigma})] \) functions and of 3 parameters \( [\varphi(a)(\tau, \vec{\sigma})] \) parametrizing point-dependent Wigner boosts for timelike Poincaré orbits. Putting these variables in the ADM action for metric gravity \( \mathcal{L} \) (with the 3-metric on \( \Sigma \) expressed in terms of cotriads: \( 3 g_{rs} = 3 e^{(a) r} e^{(a) s} \)), one gets a new action depending only on lapse, shifts and cotriads, but not on the boost parameters (therefore, there is no need to use Schwinger’s time gauge). There are 10 primary and 4 secondary first class constraints and a weakly vanishing canonical Hamiltonian containing the secondary constraints like in ADM metric gravity \( \mathcal{L} \). Besides the 3 constraints associated with the vanishing Lorentz boost momenta (Abelianization of boosts), there are 4 constraints saying
that the momenta associated with lapse and shifts vanish, 3 constraints describing rotations, 3 constraints generating space-diffeomorphisms on the cotriads induced by those \((Diff \Sigma)\) on \(\Sigma\) (a linear combination of supermomentum constraints and of the rotation ones; a different combination of these constraints generates \(SO(3)\) Gauss law constraints for the momenta \(\pi^a\) conjugated to cotriads with the covariant derivative built with the spin connection) and one superhamiltonian constraint. The six constraints connected with Lorentz boosts and rotations replace the constraints satisfying the Lorentz algebra in the older formulations. The boost parameters \(\varphi(a)(\tau, \bar{\sigma})\) and the three angles \(\alpha(a)(\tau, \bar{\sigma})\) hidden in the cotriads are the extra variables of tetrad gravity with respect to metric gravity: they allow a Hamiltonian description of the congruences of timelike accelerated observers used in the formulation of gravitomagnetism\[82, 83\].

It turns out that with the technology developed for Yang-Mills theory, one can Abelianize the 3 rotation constraints and then also the space-diffeomorphism constraints so that we can arrive at a total of 13 Abelianized first class constraints. In the Abelianization of the rotation constraints one needs the Green function of the 3-dimensional covariant derivative containing the spin connection, well defined only if there is no Gribov ambiguity in the SO(3)-frame bundle and no isometry of the Riemannian 3-manifold \((\Sigma, g)\). The Green function is similar to the Yang-Mills one for a principal SO(3)-bundle \[48\], but, instead of the Dirac distribution for the Green function of the flat divergence, it contains the Synge-DeWitt bitensor \[84\] defining the tangent in one endpoint of the geodesic arc connecting two points (which reduces to the Dirac distribution only locally in normal coordinates). Moreover, the definition of the Green function now requires the geodesic exponential map.

In the resulting quasi-Shanmugadhasan canonical basis, the original cotriad can be expressed in closed form in terms of 3 rotation angles, 3 diffeomorphism-parameters and a reduced cotriad depending only on 3 independent variables (they are Dirac’s observables with respect to 13 of the 14 first class constraints) and with their conjugate momenta, still subject to the reduced form of the superhamiltonian constraint: this is the phase space over the superspace of 3-geometries\[85\].

Till now no coordinate condition\[86\] has been imposed. It turns out that these conditions are hidden in the choice of how to parametrize the reduced cotriads in terms of three independent functions. The simplest parametrization (the only one studied till now) corresponds to choose a system of global 3-orthogonal coordinates on \(\Sigma\), in which the 3-metric is diagonal. With a further canonical transformation on the reduced cotriads and conjugate momenta, one arrives at a canonical basis containing the conformal factor \(\phi(\tau, \bar{\sigma}) = e^{\varphi(\tau, \bar{\sigma})/2}\) of the 3-geometry and its conjugate momentum \(\rho(\tau, \bar{\sigma})\) plus two other pairs of conjugate canonical variables \(r_{\bar{a}}(\tau, \bar{\sigma}), \pi_{\bar{a}}(\tau, \bar{\sigma})\), \(\bar{a} = 1, 2\). The reduced superhamiltonian constraint, expressed in terms of these variables, turns out to be an integro-differential equation for the conformal factor (reduced Lichnerowicz equation) whose conjugate momentum is, therefore, the last gauge variable. If we replace the gauge fixing of the Lichnerowicz\[87\] and York\[88, 89, 83\] approach [namely the vanishing of the trace of the extrinsic curva-
ture of $\Sigma$, $K(\tau, \vec{\sigma}) \approx 0$, also named the internal extrinsic York time\cite{90} with the natural one $\rho(\tau, \vec{\sigma}) \approx 0$ and we go to Dirac brackets, we find that $r_\alpha(\tau, \vec{\sigma})$, $\pi_\alpha(\tau, \vec{\sigma})$ are the canonical basis for the physical degrees of freedom or Dirac’s observables of the gravitational field in the 3-orthogonal gauges. Let us remark that the functional form of the non-tensorial objects $r_\alpha$, $\pi_\alpha$, depends on the chosen coordinate condition.

The next step is to find the physical Hamiltonian for them and to solve the deparametrization problem. If we wish to arrive at the soldering of tetrad gravity with matter and parametrized Minkowski formulation for the same matter, we must require that the lapse and shift functions of tetrad gravity [which must grow linearly in $\vec{\sigma}$, in suitable asymptotic Minkowski coordinates, according to the existing literature on asymptotic Poincaré charges at spatial infinity \cite{74} must agree asymptotically with the flat lapse and shift functions, which, however, are unambiguously defined only on Minkowski spacelike hyperplanes as we have seen.

In metric ADM gravity the canonical Hamiltonian is $H^{ADM}_{(c)} = \int d^3\sigma [N\tilde{H} + N_r\tilde{H}'](\tau, \vec{\sigma}) \approx 0$, where $\tilde{H}(\tau, \vec{\sigma}) \approx 0$ and $\tilde{H}'(\tau, \vec{\sigma}) \approx 0$ are the superhamiltonian and supermomentum constraints. It is differentiable and finite only for suitable $N(\tau, \vec{\sigma}) \rightarrow |\vec{\sigma}| \rightarrow \infty 0$, $N_r(\tau, \vec{\sigma}) \rightarrow |\vec{\sigma}| \rightarrow \infty 0$ defined by Beig and Ó’Murchadha\cite{74} in suitable asymptotic coordinate systems. For more general lapse and shift functions one must add a surface term $\mathcal{S}$ to $H^{ADM}_{(c)}$, which contains the “strong” Poincaré charges $P^A_{ADM}$, $J^{AB}_{ADM}$ [they are conserved and gauge invariant surface integrals]. To have well defined asymptotic Poincaré charges at spatial infinity\cite{74, 73} one needs: i) the selection of a class of coordinates systems for $\Sigma_\tau$ asymptotic to flat coordinates; ii) the choice of a class of Hamiltonian boundary conditions for the fields in these coordinate systems [all the fields must belong to some functional space of the type of the weighted Sobolev spaces]; iii) a definition of the Hamiltonian group $\mathcal{G}$ of gauge transformations (and in particular of proper gauge transformations) with a well defined limit at spatial infinity so to respect i) and ii). The scheme is the same needed to define the non-Abelian charges in Yang-Mills theory\cite{48}. The delicate point is to be able to exclude supertranslations\cite{76}, because the presence of these extra asymptotic charges leads to the replacement of the asymptotic Poincaré group with the infinite-dimensional spin group of asymptotic symmetries, which does not allow the definition of the Poincaré spin due to the absence of the Pauli-Lubanski Casimir. This can be done with suitable boundary conditions (in particular all the fields and gauge transformations must have direction independent limits at spatial infinity) respecting the “parity conditions” of Beig and Ó’Murchadha\cite{74}.

Let us then remark that in Ref.\cite{91} and in the book in Ref.\cite{1} (see also Ref.\cite{74}), Dirac introduced asymptotic Minkowski rectangular coordinates

$$z^{(\mu)}(\tau, \vec{\sigma}) = x^{(\mu)}(\tau) + b^{(\mu)}(\tau)'r \sigma^r$$

in $M^4$ at spatial infinity $S_\infty = \cup_\tau S^2_{\tau, \infty}$ For each value of $\tau$, the coordinates $x^{(\mu)}(\infty)(\tau)$ labels a point, near spatial infinity chosen as origin of $\Sigma_{\tau}$. On it there is a flat tetrad
\[ b_{(\infty)}^{(\mu)} A(\tau) = \{ l_{(\infty)}^{(\mu)} = b_{(\infty)}^{(\mu)} \}, \quad b_{(\infty)}^{(\mu)}(\tau) = e^{(\mu)}_{(\alpha)(\beta)(\gamma)} l_{(\infty)}^{(\alpha)}(\tau) b_{(\infty)}^{(\beta)} b_{(\infty)}^{(\gamma)}(\tau); \]

\[ b_{(\infty)}^{(\mu)} A(\tau) \}, \quad \text{with } l_{(\infty)}^{(\mu)} \tau \text{-independent, satisfying } b_{(\infty)}^{(\mu)} A 4 \eta_{(\mu)(\nu)} b_{(\infty)}^{(\nu)} B = 4 \eta_{AB} \text{ for every } \tau \text{ [at this level we do not assume that } l_{(\infty)}^{(\mu)} \text{ is tangent to } S_{(\infty)}, \text{ as the normal } l_{(\infty)}^{(\mu)} \text{ to } \Sigma_{\tau}. \]

There will be transformation coefficients \( b_{(\infty)}^{(\mu)} A(\tau, \vec{\sigma}) \) from the holonomic adapted coordinates \( \sigma^A = (\tau, \sigma^r) \) to coordinates \( x^\mu = \eta_{\mu \sigma} \sigma^\sigma(\tau) \) in an atlas of \( M^4 \), such that in a chart at spatial infinity one has \( z^\mu(\tau, \vec{\sigma}) = \delta^\mu_\mu z^\mu(\tau, \vec{\sigma}) \) and \( b_{(\infty)}^{(\mu)} A(\tau, \vec{\sigma}) = \delta^\mu_\mu b_{(\infty)}^{(\mu)} A(\tau) \) [for \( \tau \to \infty \) one has \( 4 g_{\mu \nu} \to \delta^\mu_\mu \delta^\nu_\nu 4 \eta_{(\mu)(\nu)} \) and \( 4 g_{AB} = b_{(\infty)}^{(\mu)} A 4 g_{\mu \nu} b_{(\infty)}^{(\nu)} B = b_{(\infty)}^{(\mu)} A 4 \eta_{(\mu)(\nu)} b_{(\infty)}^{(\nu)} B = 4 \eta_{AB} \).]

Dirac, and, then, Regge and Teitelboim proposed that the asymptotic Minkowski rectangular coordinates \( z_{(\infty)}^{(\mu)}(\tau, \vec{\sigma}) = x_{(\infty)}^{(\mu)}(\tau) + b_{(\infty)}^{(\mu)}(\tau) \sigma^r \) should define 10 new independent degrees of freedom at the spatial boundary \( S_{(\infty)} \), as it happens for Minkowski parametrized theories when restricted to spacelike hyperplanes [defined by \( z_{(\infty)}^{(\mu)}(\tau, \vec{\sigma}) \approx z_{(\infty)}^{(\mu)}(\tau) + b_{(\infty)}^{(\mu)}(\tau) \sigma^r \); then, 10 conjugate momenta should exist. These 20 extra variables of the Dirac proposal can be put in the form: \( x_{(\infty)}^{(\mu)}(\tau), p_{(\infty)}^{(\mu)}(\tau) \) with \( b_{(\infty)}^{(\mu)} A(\tau) \) [with \( b_{(\infty)}^{(\mu)} A(\tau) = b_{(\infty)}^{(\mu)}(\tau) \tau \)-independent], \( S_{(\infty)}^{(\mu)(\nu)}(\tau) \), with Dirac brackets implying the orthonormality constraints \( b_{(\infty)}^{(\mu)} A 4 \eta_{(\mu)(\nu)} b_{(\infty)}^{(\nu)} B = 4 \eta_{AB} \) [so that \( p_{(\infty)}^{(\mu)} \) and \( J_{(\infty)}^{(\mu)(\nu)} = x_{(\infty)}^{(\mu)} p_{(\infty)}^{(\nu)} - x_{(\infty)}^{(\nu)} p_{(\infty)}^{(\mu)} + S_{(\infty)}^{(\mu)(\nu)} \) satisfy a Poincaré algebra]. In analogy with Minkowski parametrized theories restricted to spacelike hyperplanes, one expects to have 10 extra first class constraints of the type

\[ p_{(\infty)}^{(\mu)} - P_{(\infty)}^{(\mu)} \approx 0, \quad S_{(\infty)}^{(\mu)(\nu)} - S_{(\infty)}^{(\mu)(\nu)} \approx 0 \]

with \( P_{(\infty)}^{(\mu)} \), \( S_{(\infty)}^{(\mu)(\nu)} \) related to the ADM Poincaré charges \( P_{(\infty)}^{A(\mu)} \), \( J_{(\infty)}^{A B(\mu)(\nu)} \). The origin \( x_{(\infty)}^{(\mu)} \) is going to play the role of a decoupled observer with his parametrized clock.

Let us remark that if we replace \( p_{(\infty)}^{(\mu)} \) and \( S_{(\infty)}^{(\mu)(\nu)} \), whose Poisson algebra is the direct sum of an Abelian algebra of translations and of a Lorentz algebra, with the new variables (with holonomic indices with respect to \( \Sigma_{\tau} \)) \( P_{(\infty)}^{A(\mu)} = b_{(\infty)}^{(\mu)} A p_{(\infty)}^{(\mu)}(\tau) \), \( J_{(\infty)}^{A B(\mu)(\nu)} = b_{(\infty)}^{(\mu)} A b_{(\infty)}^{B(\mu)(\nu)} S_{(\infty)}^{(\mu)(\nu)}(\tau) \) [\( \neq b_{(\infty)}^{(\mu)} A b_{(\infty)}^{B(\mu)(\nu)} J_{(\infty)}^{(\mu)(\nu)}(\tau) \)], the Poisson brackets for \( p_{(\infty)}^{(\mu)} \), \( b_{(\infty)}^{(\mu)} A, S_{(\infty)}^{(\mu)(\nu)} \) imply that \( p_{(\infty)}^{(\mu)} \), \( J_{(\infty)}^{AB(\mu)(\nu)} \) satisfy a Poincaré algebra. This implies that the Poincaré generators \( P_{(\infty)}^{A(\mu)} \), \( J_{(\infty)}^{A B(\mu)(\nu)} \) define in the asymptotic Dirac rectangular coordinates a momentum \( P_{(\infty)}^{A(\mu)} \) and only an ADM spin tensor \( S_{(\infty)}^{(\mu)(\nu)} \) [to define an angular momentum tensor \( J_{(\infty)}^{A B(\mu)(\nu)} \) one should find a “center of mass of the gravitational field” \( X_{(\infty)}^{A(\mu)}[3, 3\Pi] \) (see Ref. for the Klein-Gordon case) conjugate to \( P_{(\infty)}^{A(\mu)} \), so that \( J_{(\infty)}^{A B(\mu)(\nu)} = X_{(\infty)}^{A(\mu)} P_{(\infty)}^{B(\mu)} - X_{(\infty)}^{B(\mu)} P_{(\infty)}^{A(\mu)} + S_{(\infty)}^{(\mu)(\nu)} \).

The following splitting of the lapse and shift functions and the following set of boundary conditions fulfill all the previous requirements [soldering with the lapse and shift functions on Minkowski hyperplanes; absence of supertranslations [strictly speaking one gets \( P_{(\infty)}^{A(\mu)} = 0 \) due to the parity conditions; \( r = |\vec{\sigma}|]
\[3g_{rs}(\tau, \vec{\sigma}) \rightarrow r \rightarrow \infty (1 + \frac{M}{r})\delta_{rs} + 3h_{rs}(\tau, \vec{\sigma}) = (1 + \frac{M}{r})\delta_{rs} + o_4(r^{-3/2}),\]

\[3\Pi^{rs}(\tau, \vec{\sigma}) \rightarrow r \rightarrow \infty 3k^{rs}(\tau, \vec{\sigma}) = o_3(r^{-5/2}),\]

\[N(\tau, \vec{\sigma}) = N_{(as)}(\tau, \vec{\sigma}) + n(\tau, \vec{\sigma}), \quad n(\tau, \vec{\sigma}) = O(r^{-3+\epsilon}),\]

\[N_r(\tau, \vec{\sigma}) = N_{(as)r}(\tau, \vec{\sigma}) + n_r(\tau, \vec{\sigma}), \quad n_r(\tau, \vec{\sigma}) = O(r^{-\epsilon}),\]

\[N_{(as)A}(\tau, \vec{\sigma}) \overset{\text{def}}{=} (N_{(as)}; N_{(as)r})(\tau, \vec{\sigma}) = -\tilde{\lambda}_A(\tau) - \frac{1}{2}\tilde{\lambda}_{As}(\tau)\sigma^s,\]

\[\Rightarrow 3e_{(a)r}(\tau, \vec{\sigma}) = (1 + \frac{M}{r^2})\delta_{(a)r} + o_4(r^{-3/2}),\]

with \(3h_{rs}(\tau, -\vec{\sigma}) = 3h_{rs}(\tau, \vec{\sigma}), \ 3k^{rs}(\tau, -\vec{\sigma}) = -3k^{rs}(\tau, \vec{\sigma})\); here \(3\Pi^{rs}(\tau, \vec{\sigma})\) is the momentum conjugate to the 3-metric \(3g_{rs}(\tau, \vec{\sigma})\) in ADM metric gravity.

These boundary conditions identify the class of spacetimes of Christodoulou and Klainermann \[92\] (they are near to Minkowski spacetime in a norm sense, contain gravitational radiation but evade the singularity theorems, because they do not satisfy the hypothesis of conformal completion to get the possibility to put control on the large time development of the solutions of Einstein’s equations). These spacetimes also satisfy the rest-frame condition \(P^{r}_{\text{ADM}} = 0\) (this requires \(\tilde{\lambda}_A(\tau) = 0\) like for Wigner hyperplanes in parametrized Minkowski theories) and have vanishing shift functions (but non trivial lapse function).

After the addition of the surface term, the resulting canonical and Dirac Hamiltonians of ADM metric gravity are

\[
H_{(c)\text{ADM}} = \int d^3\sigma [(N_{(as)} + n)\tilde{H} + (N_{(as)r} + n_r)\tilde{H}'(\tau, \vec{\sigma})] \rightarrow \\
\rightarrow H'_{(c)\text{ADM}} = \int d^3\sigma [(N_{(as)} + n)\tilde{H} + (N_{(as)r} + n_r)\tilde{H}'(\tau, \vec{\sigma})] + \\
+ \tilde{\lambda}_A(\tau)\hat{P}^A_{\text{ADM}} + \tilde{\lambda}_{AB}(\tau)\hat{J}^{AB}_{\text{ADM}} = \\
= \int d^3\sigma [n\tilde{H} + n_r\tilde{H}'(\tau, \vec{\sigma}) + \tilde{\lambda}_A(\tau)\hat{P}^A_{\text{ADM}} + \tilde{\lambda}_{AB}(\tau)\hat{J}^{AB}_{\text{ADM}}] + \\
\approx \tilde{\lambda}_A(\tau)\hat{P}^A_{\text{ADM}} + \tilde{\lambda}_{AB}(\tau)\hat{J}^{AB}_{\text{ADM}}.
\]

with the “weak conserved improper charges” \(\hat{P}^A_{\text{ADM}}, \hat{J}^{AB}_{\text{ADM}}\) [they are volume integrals differing from the weak charges by terms proportional to integrals of the constraints]. The previous splitting implies to replace the variables \(N(\tau, \vec{\sigma}), N_r(\tau, \vec{\sigma})\) with the ones \(\tilde{\lambda}_A(\tau), \tilde{\lambda}_{AB}(\tau) = -\tilde{\lambda}_{BA}(\tau), n(\tau, \vec{\sigma}), n_r(\tau, \vec{\sigma})\) [with conjugate momenta \(\tilde{\pi}^A(\tau), \tilde{\pi}^{AB}(\tau) = -\tilde{\pi}^{BA}(\tau), \tilde{\pi}^a(\tau, \vec{\sigma}), \tilde{\pi}_{ab}(\tau, \vec{\sigma})\) in the ADM theory.

With these assumptions one has the following form of the line element (also its form in tetrad gravity is given)

\[ds^2 = \epsilon([N_{(as)} + n]^2 - [N_{(as)r} + n_r]^3 e^e_{(a)} \tilde{\epsilon}_{(a)s} [N_{(as)s} + n_s])(d\tau)^2 - \\
- 2\epsilon [N_{(as)r} + n_r]d\tau d\sigma^r - \epsilon^3 e^e_{(a)r} \tilde{\epsilon}_{(a)s} d\sigma^r d\sigma^s.
\]

The final suggestion of Dirac is to modify ADM metric gravity in the following way:

i) add the 10 new primary constraints \(p^A_{(\infty)} - \hat{P}^A_{\text{ADM}} \approx 0, J^{AB}_{(\infty)} - \hat{J}^{AB}_{\text{ADM}} \approx 0\), where
Let us consider the Sen-Witten connection \( J_{(\infty)}^{AB} = b_{(\infty)(\mu)}^A b_{(\infty)(\nu)}^B S_{(\infty)}^{(\mu)(\nu)} \) [remember that \( p_{(\infty)}^A \) and \( J_{(\infty)}^{AB} \) satisfy a Poincaré algebra];

ii) consider \( \tilde{\lambda}_A(\tau), \lambda_{AB}(\tau) \), as Dirac multipliers for these 10 new primary constraints, and not as configurational (arbitrary gauge) variables coming from the lapse and shift functions [so that there are no conjugate (vanishing) momenta \( \tilde{\pi}^A(\tau), \tilde{\pi}^{AB}(\tau) \) and no associated Dirac multipliers \( \zeta_A(\tau), \zeta_{AB}(\tau) \), in the assumed Dirac Hamiltonian [it is finite and differentiable]

\[
H_{(D)ADM} = \int d^3 \sigma [n \tilde{\mathcal{H}} + n_r \tilde{\mathcal{H}}^r + \lambda_n \tilde{\pi}^n + \lambda_{\tilde{\pi}_{\tilde{\pi}}}^\tau(\tau, \tilde{\sigma}) - \lambda_A(\tau)[p_{(\infty)}^A - \hat{P}_{(\infty)}^A] - \tilde{\lambda}_{AB}(\tau)[J_{(\infty)}^{AB} - \hat{J}_{(\infty)}^{AB}] \approx 0,
\]

The reduced phase space is still the ADM one: on the ADM variables there are only the secondary first class constraints \( \mathcal{H}(\tau, \tilde{\sigma}) \approx 0, \mathcal{H}^r(\tau, \tilde{\sigma}) \approx 0 \) [generators of proper gauge transformations], because the other first class constraints \( p_{(\infty)}^A - \hat{P}_{(\infty)}^A \approx 0, J_{(\infty)}^{AB} - \hat{J}_{(\infty)}^{AB} \approx 0 \) do not generate improper gauge transformations but eliminate 10 of the extra 20 variables.

In this modified ADM metric gravity, one has restricted the 3+1 splittings of \( M^4 \) to foliations whose leaves \( \Sigma_{\tau} \) tend to Minkowski spacelike hyperplanes asymptotically at spatial infinity in a direction independent way. Therefore, these \( \Sigma_{\tau} \) should be determined by the 10 degrees of freedom \( x_{(\infty)}^{(\mu)}(\tau), \lambda_{(\infty)}^{(\mu)}(\tau) \), like it happens for flat spacelike hyperplanes: this means that it must be possible to define a “parallel transport” of the asymptotic tetrads \( b_{(\infty)}^{(\mu)}(\tau) \) to get well defined tetrads in each point of \( \Sigma_{\tau} \). While it is not yet clear whether this can be done for \( \tilde{\lambda}_{AB}(\tau) \neq 0 \), there is a solution for \( \lambda_{AB}(\tau) = 0 \). This case corresponds to go to the Wigner-like hypersurfaces [the analogue of the Minkowski Wigner hyperplanes with the asymptotic normal \( l_{(\infty)}^{(\mu)} = l_{(\infty)}^{(\mu)}(\tau) \parallel \hat{P}_{(\infty)}^{(\mu)} \)] following the same procedure defined for Minkowski spacetime, one gets \( S_{(\infty)}^{rs} = J_{(\infty)}^{rs} \) [see Ref. [55] for the definition of \( S_{(\infty)}^{AB} \)], \( \tilde{\lambda}_{AB}(\tau) = 0 \) and

\[-\tilde{\lambda}_{A}(\tau)[p_{(\infty)}^A - \hat{P}_{(\infty)}^A] = -d_{(\infty)}(\tau)[\epsilon_{(\infty)} - \hat{P}_{(\infty)}^r] + \tilde{\lambda}_{r}(\tau)P_{(\infty)}^r \]

so that the final form of these four surviving constraints is \( (P_{(\infty)}^r = 0 \) implies \( P_{(\infty)}^r = 0) \), \( M_{(\infty)} = \sqrt{P_{(\infty)}^2} \approx \hat{P}_{(\infty)}^r \) is the ADM mass of the universe

\[\epsilon_{(\infty)} - \hat{P}_{(\infty)}^r \approx 0, \quad P_{(\infty)}^r \approx 0.\]

On this subclass of foliations [whose leaves \( \Sigma_{\tau}^{[WSW]} \) will be called Wigner-Sen-Witten hypersurfaces; they define the intrinsic asymptotic rest frame of the gravitational field] one can introduce a parallel transport by using the interpretation of Ref. [96] of the Witten spinorial method of demonstrating the positivity of the ADM energy field [48]. Let us consider the Sen-Witten connection [95, 94] restricted to \( \Sigma_{\tau}^{[WSW]} \) (it depends on the trace of the extrinsic curvature of \( \Sigma_{\tau}^{[WSW]} \)) and the spinorial Sen-Witten equation associated with it. As shown in Ref. [96], this spinorial equation can be rephrased as an equation whose solution determines (in a surface dependent
dynamical way) a tetrad in each point of $\Sigma_{\tau}^{W^{SW}}$ once it is given at spatial infinity (again this requires a direction independent limit). Therefore, at spatial infinity there is a privileged congruence of timelike observers, which replaces the concept of “fixed stars” in the study of the precessional effects of gravitomagnetism on gyroscopes and whose connection with the definition of post-Newtonian coordinates has still to be explored.

On the Wigner-Sen-Witten hypersurfaces the spatial indices have become spin-1 Wigner indices [they transform with Wigner rotations under asymptotic Lorentz transformations]. As said for parametrized theories in Minkowski spacetime, in this special gauge 3 degrees of freedom of the gravitational field [an internal 3-center-of-mass variable $\hat{\sigma}_{ADM}[^3g,^3\Pi]$ inside the Wigner-Sen-Witten hypersurface] become gauge variables, while $\tilde{x}^{(\mu)}$ [the canonical non-covariant variable replacing $x^{(\mu)}_{(\infty)}$] becomes a decoupled observer with his “point particle clock” $[\bar{t},\bar{\Pi}]$ near spatial infinity. Since the positivity theorems for the ADM energy imply that one has only timelike or lightlike orbits of the asymptotic Poincaré group, the restriction to universes with timelike ADM 4-momentum allows to define the Möller radius $\rho_{ADM} = \sqrt{-\hat{W}_{ADM}^2/\hat{P}_{ADM}^2}$ from the asymptotic Poincaré Casimirs $\hat{P}_{ADM}^2, \hat{W}_{ADM}^2$.

By going from $\tilde{z}^{(\mu)}_{(\infty)}, \tilde{u}^{(\mu)}_{(\infty)}$ to the canonical basis $T_{(\infty)} = p_{(\infty)(\mu)} z^{(\mu)}_{(\infty)}/\epsilon_{(\infty)} = p_{(\infty)(\mu)} x^{(\mu)}_{(\infty)}/\epsilon_{(\infty)}, \epsilon_{(\infty)}, \tilde{z}^{(i)}_{(\infty)} = \epsilon_{(\infty)}(\tilde{x}^{(i)}_{(\infty)} - p^{(i)}_{(\infty)} \tilde{x}^{(0)}_{(\infty)}/p^{(0)}_{(\infty)}), k^{(i)}_{(\infty)} = p^{(i)}_{(\infty)}/\epsilon_{(\infty)} = u^{(i)}(p^{(\mu)}_{(\infty)}), \tau = \epsilon_{(\infty)}\epsilon_{(\infty)}\epsilon_{(\infty)}\epsilon_{(\infty)}\epsilon_{(\infty)}\epsilon_{(\infty)}\epsilon_{(\infty)}\epsilon_{(\infty)}\epsilon_{(\infty)}$, like in the flat case one finds that the final reduction requires the gauge-fixings $T_{(\infty)} = \tau \approx 0$ and $\sigma^{\tau}_{ADM} \approx 0$, where $\tau = \sigma^{\tau}_{ADM}$ is a variable representing the “internal center of mass” of the 3-metric of the slice $\Sigma_{\tau}$ of the asymptotically flat spacetime $M^4$. Since $\{T_{(\infty)}, \epsilon_{(\infty)}\} = -\epsilon$, with the gauge fixing $T_{(\infty)} = \tau \approx 0$ one gets $\tilde{\lambda}_{\tau}(\tau) \approx \epsilon$, and the final Dirac Hamiltonian is $H_D = M_{ADM} + \tilde{\lambda}_{\tau}(\tau) \hat{P}_{ADM}^\tau$ with $M_{ADM}$ the natural physical Hamiltonian to reintroduce an evolution in the “mathematical” $T_{(\infty)} \equiv \tau$: namely in the rest-frame time identified with the parameter $\tau$ labelling the leaves $\Sigma_{\tau}^{W^{SW}}$ of the foliation of $M^4$. Physical times (atomic clocks, ephemeris time...) must be put in a local 1-1 correspondence with this “mathematical” time. This point of view excludes any Wheeler-DeWitt interpretation of an internal time (like the extrinsic York one or the WKB times), which is used in closed universes of the Einstein-Wheeler type.

All this construction holds also in our formulation of tetrad gravity (since it uses the ADM action) and in its canonically reduced form in the 3-orthogonal gauges. The final physical Hamiltonian of tetrad gravity for the physical gravitational field is the reduced volume form of the ADM energy $\hat{P}_{ADM}^\tau[r_{\bar{a}}., \pi_{\bar{a}}, \phi(r_{\bar{a}}, \pi_{\bar{a}})]$ with the conformal factor $\phi$ solution of the reduced Lichnerowicz equation in the 3-orthogonal gauge with $\rho(\tau, \sigma) \approx 0$. The Hamilton-Dirac equations generated by this Hamiltonian for $r_{\bar{a}}, \pi_{\bar{a}}$ generate the pair of second order equations in normal form for $r_{\bar{a}}$ hidden in the Einstein equations in this particular gauge.

Let us compare the standard generally covariant formulation of gravity based on the Hilbert action with its invariance under $Diff M^4$ with the ADM Hamiltonian formulation.
Regarding the 10 Einstein equations of the standard approach, the Bianchi identities imply that four equations are linearly dependent on the other six ones and their gradients. Moreover, the four combinations of Einstein’s equations projectable to phase space (where they become the secondary first class superhamiltonian and supermomentum constraints of canonical metric gravity) are independent from the accelerations being restrictions on the Cauchy data. As a consequence the Einstein equations have solutions, in which the ten components $^4g_{\mu\nu}$ of the 4-metric depend on only two truly dynamical degrees of freedom (defining the physical gravitational field) and on eight undetermined degrees of freedom. This transition from the ten components $^4g_{\mu\nu}$ of the tensor $^4g$ in some atlas of $M^4$ to the 2 (deterministic)+8 (undetermined) degrees of freedom breaks general covariance, because these quantities are neither tensors nor invariants under diffeomorphisms (their functional form is atlas dependent).

Since the Hilbert action is invariant under $Diff M^4$, one usually says that a “dynamical gravitational field” is a 4-geometry over $M^4$, namely an equivalence class of spacetimes $(M^4,^4g)$, solution of Einstein’s equations, modulo $Diff M^4$. See, however, the interpretational problems about what is observable in general relativity for instance in Refs.[7, 8], in particular the facts that at least before the restriction to the solutions of Einstein’s equations i) scalars under $Diff M^4$, like $^4R$, are not Dirac’s observables but gauge dependent quantities; ii) the functional form of $^4g_{\mu\nu}$ in terms of the physical gravitational field and, therefore, the angle and distance properties of material bodies and the standard procedures of defining measures of length and time based on the line element $ds^2$, are gauge dependent.

Instead in the ADM formalism with the extra notion of 3+1 splittings of $M^4$, the (tetrad) metric ADM action (differing from the Hilbert one by a surface term) is quasi-invariant under the (14) 8 types of gauge transformations which are the pull-back of the Hamiltonian group $\mathcal{G}$ of gauge transformations, whose generators are the first class constraints of the theory. The Hamiltonian group $\mathcal{G}$ has a subgroup (whose generators are the supermomentum and superhamiltonian constraints) formed by the diffeomorphisms of $M^4$ adapted to its 3+1 splittings, $Diff M^{3+1}$ [it is different from $Diff M^4$]. Moreover, the Poisson algebra of the supermomentum and superhamiltonian constraints reflects the embeddability in $M^4$ of the foliation associated with the 3+1 splitting [7].

Now in tetrad gravity the interpretation of the 14 gauge transformations and of their gauge fixings (it is independent from the presence of matter) is the following [a tetrad in a point of $\Sigma_\tau$ is a local observer]:

i) the gauge fixings of the gauge boost parameters associated with the 3 boost constraints and of the gauge angles associated with the 3 rotation constraints are equivalent to choose the congruence of timelike observers to be used as a standard of non rotation;

ii) the gauge fixings of the 3 gauge parameters associated with the passive space diffeomorphisms [$Diff \Sigma_\tau$; change of coordinates charts] are equivalent to a fixation of 3 standards of length by means of a choice of a coordinate system on $\Sigma_\tau$ [the
measuring apparatus (the “rods”) should be defined in terms of Dirac’s observables for some kind of matter, after its introduction into the theory];

iii) according to constraint theory the choice of 3-coordinates on Στ induces the gauge fixings of the 3 shift functions \( [\text{i.e. of } ^4g_{\alpha}] \), whose gauge nature is connected with the “conventionality of simultaneity” \( [98] \) [therefore, the gauge fixings are equivalent to a choice of synchronization of clocks and, as a consequence, to a statement about the isotropy or anisotropy of the velocity of light in that gauge];

iv) the gauge fixing on the the momentum \( \rho(\tau, \vec{\sigma}) \) conjugate to the conformal factor of the 3-metric [this gauge variable is the source of the gauge dependence of 4-tensors and of the scalars under \( DiffM^4 \), together with the gradients of the lapse and shift functions] is a nonlocal statement about the extrinsic curvature of the leaves Στ of the given 3+1 splitting of \( M^4 \); since the superhamiltonian constraint produces normal deformations of Στ \( [97] \) and, therefore, transforms a 3+1 splitting of \( M^4 \) into another one (the ADM formulation is independent from the choice of the 3+1 splitting), this gauge fixing is equivalent to the choice of a particular 3+1 splitting; v) the previous gauge fixing induces the gauge fixing of the lapse function (which determines the packing of the leaves Στ in the chosen 3+1 splitting) and, therefore, is equivalent to the fixation of a standard of proper time [again “clocks” should be built with the Dirac’s observables of some kind of matter].

In the Hamiltonian formalism it is natural to define a “Hamiltonian kinematical gravitational field” as the equivalence class of spacetimes modulo the Hamiltonian group \( \mathcal{G} \), and different members of the equivalence class have in general different 4-Riemann tensors [these equivalence classes are connected with the conformal 3-geometries of the Lichnerowicz-York approach and contain different gauge-related 4-geometries]. Then, a “Hamiltonian dynamical gravitational field” is defined as a Hamiltonian kinematical gravitational fields which is solution of the Hamilton-Dirac equations generated by the weak ADM energy \( \hat{P}_{ADM} \). Since the Hilbert and ADM actions, even if they have different local symmetries and invariances, both generate the same Einstein equations, the equivalence classes of the “Hamiltonian dynamical gravitational fields” and of the standard “dynamical gravitational fields” (a 4-geometry solution of Einstein’s equations) coincide. Indeed, on the solutions of Einstein’s equations the gauge transformations generated by the superhamiltonian constraint (normal deformations of Στ) and those generated by the canonical momenta of the lapse and shift functions together with the Στ diffeomorphisms generated by the supermomentum constraints are restricted by the Jacobi equations associated to Einstein’s equations to be those Noether symmetries of the ADM action which are also dynamical symmetries of the Hamilton equations and therefore they are a subset of the spacetime diffeomorphisms \( DiffM^4 \) (all of which are dynamical symmetries of Einstein’s equations).

The 3-orthogonal gauges of tetrad gravity are the equivalent of the Coulomb gauge in classical electrodynamics (like the harmonic gauge is the equivalent of the Lorentz gauge). Only after a complete gauge fixing the 4-tensors and the scalars under \( DiffM^4 \) become measurable quantities (like the electromagnetic vector po-
tential in the Coulomb gauge): an experimental laboratory does correspond by definition to a completely fixed gauge. At this stage it becomes acceptable the proposal of Komar\cite{99} and Bergmann\cite{80} of identifying the points of a spacetime \((M^4, ^4g)\), solution of the Einstein’s equations in absence of matter, in a way invariant under spacetime diffeomorphisms, by using four bilinears and trilinears in the Weyl tensors, scalar under \(DiffM^4\) and called “individuating fields” (see also Refs.\cite{7, 8}), which do not depend on the lapse and shift functions (but only on the gauge variables corresponding to the 3-coordinates on \(\Sigma_\tau\) and to the momentum conjugate to the conformal factor of the 3-metric, so that these fields carry the information on the choice of the 3-coordinates and of a generalized extrinsic time), to build “physical 4-coordinates” (in each completely fixed gauge they depend only on the two canonical pairs of Dirac’s observables of the gravitational field), justifying a posteriori the standard measurement theory presented in all textbooks on general relativity, which presupposes the individuation of spacetime points.

Our approach breaks the general covariance of general relativity completely by going to the special 3-orthogonal gauges. But this is done in a way naturally associated with theories with first class constraints: the global Shanmugadhasan canonical transformations (when they exist) correspond to privileged Darboux charts for presymplectic manifolds defined by the first class constraints. Therefore, the gauges identified by these canonical transformations should have a special (till now unexplored) role also also in generally covariant theories, in which traditionally one looks for observables invariant under diffeomorphisms and not for not generally covariant Dirac observables.

Let us remember that Bergmann\cite{80} made the following critique of general covariance: it would be desirable to restrict the group of coordinate transformations (spacetime diffeomorphisms) in such a way that it could contain an invariant subgroup describing the coordinate transformations that change the frame of reference of an outside observer (these transformations could be called Lorentz transformations; see also the comments in Ref.\cite{100} on the asymptotic behaviour of coordinate transformations); the remaining coordinate transformations would be like the gauge transformations of electromagnetism. This is what we have done. In this way “preferred” coordinate systems will emerge (the WSW hypersurfaces with their preferred congruences of timelike observers whose 4-velocity becomes asymptotically normal to \(\Sigma_r^{(WSW)}\) at spatial infinity), which, as said by Bergmann, are not “flat”: while the inertial coordinates are determined experimentally by the observation of trajectories of force-free bodies, these intrinsic coordinates can be determined only by much more elaborate experiments (probably with gyroscopes), since they depend, at least, on the inhomogeneities of the ambient gravitational fields. See also Ref.\cite{101} for other critics to general covariance: very often to get physical results one uses preferred coordinates not merely for calculational convenience, but also for understanding (this fact has been formalized as the “principle of restricted covariance”).

Since in the 3-orthogonal gauges we have the physical canonical basis \(r_a, \pi_a\), it is possible, but only in absence of matter, to define “void spacetimes” as the
equivalence class of spacetimes “without gravitational field”, whose members in the
3-orthogonal gauges are obtained by adding by hand the second class constraints
\( r_{a}(\tau, \vec{\sigma}) \approx 0, \pi_{a}(\tau, \vec{\sigma}) \approx 0 \) [one gets \( \phi(\tau, \vec{\sigma}) = 1 \) as the relevant solution of the reduced Lichnerowicz equation] and, in particular, their Poincaré charges vanish (this corresponds to the exceptional \( p(\mu) = 0 \) orbit of the Poincaré group and shows the peculiarity of these solutions with zero ADM mass). It is expected that the void spacetimes can be defined in a gauge-independent way by adding to the ADM action the requirement that the leaves \( \Sigma_{\tau} \) of the 3+1 splitting be 3-conformally flat, namely that the Cotton-York 3-conformal tensor vanishes. The members of this equivalence class (the extension to general relativity of the Galilean non inertial coordinate systems with their Newtonian inertial forces) are gauge equivalent to Minkowski spacetime with Cartesian coordinates and it is expected that they describe pure acceleration effects without physical gravitational field (no tidal effects).

See Ref.[102] for the \( c \to \infty \) contraction of the ADM action of metric gravity: a theory with 26 independent fields (most of them describe inertial forces) and with general Galileo covariance has been obtained. This formulation of Newton gravity should be the natural nonrelativistic limit of Einstein’s general relativity in the framework of singular Lagrangians; however, its connection with the post-Newtonian approximations has still to be explored.

If we add \([72]\) to the tetrad ADM action the action for N scalar particles with positive energy in the form of Ref.[52] [where it was given on arbitrary Minkowski spacelike hypersurfaces], the only constraints which are modified are the superhamiltonian one, which gets a dependence on the matter energy density \( \mathcal{M}(\tau, \vec{\sigma}) \), and the 3 space diffeomorphism ones, which get a dependence on the matter momentum density \( \mathcal{M}_{r}(\tau, \vec{\sigma}) \). The canonical reduction and the determination of the Dirac observables can be done like in absence of matter. However, the reduced Lichnerowicz equation for the conformal factor of the 3-metric in the 3-orthogonal gauge and with \( \rho(\tau, \vec{\sigma}) \approx 0 \) acquires now an extra dependence on \( \mathcal{M}(\tau, \vec{\sigma}) \) and \( \mathcal{M}_{r}(\tau, \vec{\sigma}) \).

Since, as a preliminary result, we are interested in identifying explicitly the instantaneous action-at-a-distance (Newton-like and gravitomagnetic) potentials among particles hidden in tetrad gravity (like the Coulomb potential is hidden in the electromagnetic gauge potential), we shall make the strong approximation of neglecting the (tidal) effects of the physical gravitational field by putting \( r_{a}(\tau, \vec{\sigma}) \approx 0, \pi_{a}(\tau, \vec{\sigma}) \approx 0 \), even if it is not strictly consistent with the Hamilton-Dirac equation (extremely weak gravitational fields). If, furthermore, we develop the conformal factor \( \phi(\tau, \vec{\sigma}) \) in a formal series in the Newton constant \( G \) \( \left[ \phi = 1 + \sum_{n=1}^{\infty} G^{n} \phi_{n} \right] \), one can find a solution \( \phi = 1 + G \phi_{1} \) at order \( G \) (post-Minkowskian approximation) of the reduced Lichnerowicz equation where we put \( r_{a} = \pi_{a} = 0 \). However, due to a self-energy divergence in \( \phi \) evaluated at the positions \( \vec{n}_{i}(\tau) \) of the particles, one needs to rescale the bare masses to physical ones, \( m_{i} \mapsto m_{i}^{(\text{phys})} \) \( \phi^{-2}(\tau, \vec{n}_{i}(\tau)) m_{i}^{(\text{phys})} \), and to make a regularization of the type defined in Refs. [103]. Then, the regularized solution for \( \phi \) can be put in the reduced form of the ADM energy, which becomes \( \vec{\kappa}_{i}(\tau) \) are the particle momenta conjugate to \( \vec{n}_{i}(\tau) \); \( \vec{n}_{ij} = [\vec{n}_{i} - \vec{n}_{j}]/|\vec{n}_{i} - \vec{n}_{j}| \)
\[ \hat{P}_{ADM}^\tau = \sum_{i=1}^{N} c \sqrt{m_i^{(phys)}^2 c^2 + \kappa_i^2(\tau)} - \frac{G}{c^2} \sum_{i \neq j} \frac{m_i^{(phys)}}{|\vec{\eta}_i(\tau) - \vec{\eta}_j(\tau)|} \left( \frac{\kappa_i^2(\tau)}{2 m_i^{(phys)}} \right) - \frac{G}{8c^4} \sum_{i \neq j} \frac{3 \kappa_i(\tau) \kappa_j(\tau) - 5 \kappa_i(\tau) \bar{\kappa}_j(\tau) \kappa_j(\tau) - \bar{\kappa}_i(\tau) \bar{\kappa}_j(\tau)}{|\vec{\eta}_i(\tau) - \vec{\eta}_j(\tau)|} + O(G^2, r_\alpha, \pi_\alpha). \]

One sees the Newton-like and the gravitomagnetic (in the sense of York) potentials (both of them need regularization) at the post-Minkowskian level (order G but exact in c) emerging from the tetrad ADM version of Einstein general relativity when we ignore the tidal effects. For G=0 we recover N free scalar particles on the Wigner hyperplane in Minkowski spacetime, as required by deparametrization. For \( c \rightarrow \infty \), we get the post-Newtonian Hamiltonian

\[ H_{PN} = \sum_{i=1}^{N} \frac{\kappa_i^2(\tau)}{2 m_i^{(phys)}} (1 - \frac{2G}{c^2} \sum_{j \neq i} \frac{m_j^{(phys)}}{|\vec{\eta}_i(\tau) - \vec{\eta}_j(\tau)|} ) - \frac{G}{2} \sum_{i \neq j} \frac{m_i^{(phys)} m_j^{(phys)}}{|\vec{\eta}_i(\tau) - \vec{\eta}_j(\tau)|} - \frac{G}{8c^4} \sum_{i \neq j} \frac{3 \kappa_i(\tau) \kappa_j(\tau) - 5 \kappa_i(\tau) \bar{\kappa}_j(\tau) \kappa_j(\tau) - \bar{\kappa}_i(\tau) \bar{\kappa}_j(\tau)}{|\vec{\eta}_i(\tau) - \vec{\eta}_j(\tau)|} + O(G^2, r_\alpha, \pi_\alpha), \]

which is of the type of the ones implied by the results of Refs. \[103, 104\] [the differences are probably connected with the use of different coordinate systems and with the fact that one has essential singularities on the particle worldlines and the need of regularization].

The main open problems now under investigation are: i) the linearization of the theory in the 3-orthogonal gauges in presence of matter to find the 3-orthogonal gauge description of gravitational waves and to go beyond the previous instantaneous post-Minkowskian approximation at least in the 2-body case relevant for the motion of binaries; ii) the replacement of scalar particles with spinning ones to identify the precessional effects (like the Lense-Thirring one) of gravitomagnetism; iii) the coupling to perfect fluids for the simulation of rotating stars and for the comparison with the post-Newtonian approximations; iv) the coupling of tetrad gravity to the electromagnetic field, to fermion fields and then to the standard model, trying to make to reduction to Dirac’s observables in all these cases and to study their post-Minkowskian approximations; v) the quantization of tetrad gravity in the 3-orthogonal gauge with \( \rho(\tau, \vec{\sigma}) \approx 0 \) (namely after a complete breaking of general covariance): for each perturbative (in G) solution of the reduced Lichnerowicz equation one defines a Schroedinger equation in \( \tau \) for a wave functional \( \Psi[\tau; r_\alpha] \) with the associated quantized ADM energy \( \hat{P}_{ADM}^\tau[r_\alpha, i \frac{\partial}{\partial r_\alpha}] \) as Hamiltonian; no problem of physical scalar product is present, but only ordering problems in the Hamiltonian; moreover, one has the Møller radius as a ultraviolet cutoff. Also a comparison with “loop quantum gravity” \[102\], which respects general covariance but only for fixed lapse and shift functions, has still to be done.

Therefore, a well defined classical stage for a unified description of the four interactions is emerging, even if many aspects have only been clarified at a heuristic level so that a big effort from both mathematical and theoretical physicists is still needed.
It will be exciting to see whether in the next years some reasonable quantization picture will develop from this classical framework.

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