Performance analysis on a cooperative transmission scheme of multicast and NOMA in cache-enabled cellular networks

Yue Shan | Qi Zhu | Ying Wang

1 Jiangsu Key Laboratory of Wireless Communications, Nanjing University of Posts and Telecommunications, Nanjing, China
2 Engineering Research Center of Health Service System Based on Ubiquitous Wireless Networks, Ministry of Education, Nanjing University of Posts and Telecommunications, Nanjing, China

Abstract
Caching popular contents at base stations (BSs) can effectively avoid redundant data traffic and improve backhaul capacity. Techniques such as multicast (MC) and non-orthogonal multiple access (NOMA) can significantly improve spectral efficiency by delivering popular contents to multiple users using a single channel. Therefore, we propose a cooperative transmission scheme of MC and NOMA in cache-enabled wireless cellular networks. First, we derive the probability mass function (PMF) of the number of channels in the MC mode as well as the joint PMF of the number of channels and number of NOMA users in the NOMA mode. Second, we analyse the successful transmission probabilities of the two modes by utilising tools from stochastic geometry. Finally, with the derived probabilities using the two modes, we obtain the total successful transmission probability based on probability theory. The quasi-closed form expression of the successful transmission probability is obtained in a special case, where the noise is neglected, and the path-loss exponent is set to be four. Simulation results validate the accuracy of the analysis and demonstrate the performance gain due to the proposed cooperative transmission scheme of MC and NOMA.

1 | INTRODUCTION

According to a recent Cisco report [1], mobile data and video services are rapidly becoming an integral part of the lives of consumers, in addition to essential mobile-based services. Hence, there will be significant bandwidth demands with the dramatic growth in future service requirements. To improve the quality-of-service (QoS) of users and spectral efficiency, some promising techniques, such as cache, multicast (MC), and non-orthogonal multiple access (NOMA), have been employed popularly [2].

Caching popular contents at base stations (BSs) can significantly reduce the use of backhaul networks and eliminate redundant mobile data traffic [3]. Performance analysis in cache-enabled wireless cellular networks is important for next-generation wireless networks. Basic insights into the problem of cache-enabled small cell networks have been provided in [4], where the outage probability and average content delivery rate in a cache-enabled small cell network with limited backhaul capacity are modelled and analysed. The results show that the storage units in BSs can yield significant improvements. Theoretical results on the probability of mobile users successfully downloading content from servers in small-cell base stations (SBSs) have been discussed in [5]. In addition, an analysis of the impact of the SBS parameters on the successful download probability has been presented. Successful content delivery probability for a user located at the cluster centre in a cluster-centric small cell network has been discussed in [6]. The analysis reveals an inherent trade-off between the transmission and content diversities. Caching at BSs improves the probability of successful content delivery by reducing the distance between popular contents and access requests.

Applying MC services at BSs can efficiently utilise the broadcast characteristics of wireless mediums to deliver popular content to multiple users using a single channel. Small cells equipped with caching and MC can ease backhaul burden and substantially reduce transmissions of duplicated content [7]. A paradigm built on caching and MC is proposed in [8] to satisfy the explosive demand for mobile data while minimising energy expenditures. The results demonstrate that combining caching
system model

and MC can improve energy efficiency. Caching and MC have been adopted in [9] to support massive content delivery in a cache-enabled wireless network. The analysis and optimisation have been considered in terms of successful transmission probability performance. In addition, caching and MC have been utilised simultaneously in a large-scale cache-enabled heterogeneous network with backhaul constraints, and an expression for successful transmission probability has been derived [10]. Thus, caching and MC are two efficient methods to support massive content delivery and reduce backhaul load.

NOMA is a promising technique to significantly improve the spectral efficiency and system capacity [11]. It can achieve significant performance gains by allocating the same resource (time, frequency, and spatial) to multiple users [12]. The authors in [13] applied NOMA in the content delivery phase to ensure that more user requests can be served concurrently. The proposed NOMA-assisted caching scheme can push content while simultaneously serving users, which efficiently improves the cache hit probability and reduces the delivery outage probability. By combining NOMA and caching, the coverage performance has been studied to balance the transmission delay of backhaul and spectrum efficiency of access links [14]. The coverage probability analysis of the typical user provides helpful insights on improving the coverage performance. Simulation results demonstrate the performance gain of NOMA in wireless caching networks. The performance of NOMA in a cellular downlink scenario with randomly deployed users has been investigated in [15]. The analytical results show that NOMA is spectrally efficient and can achieve better performance in terms of ergodic sum rates. Content pushing phase and MC phase are considered in [16]. MC is used in the MC phase to deliver the popular content objects requested by multiple users simultaneously. NOMA is used to merge the two phases by transmitting the pushed and the requested contents in the form of a superimposed signal. The results reveal that the combination of these techniques is a promising method for meeting the bandwidth demands of state-of-the-art networks.

The performance gain of cache-enabled wireless communication networks has been analysed in [4–6]. Enabling caching in the BSs can reduce data redundancy and consumption of backhaul. However, the methods to improve the spectral efficiency have yet to be determined. It has been reported that the BS can simultaneously serve multiple users using a single channel by combining caching with MC [8–10] and NOMA [14, 15], respectively. This significantly improves the spectral efficiency. In particular, enabling MC services at BSs is an efficient method to reduce duplicated content transmissions [9] because MC can deliver a file to its multiple requestors simultaneously, using a single channel. However, as the caching capacity of BSs and diversity of user requests increase, many files receive a single request. In such cases, a file only needs to be delivered to one user and using MC becomes redundant. Instead of using NOMA in [16] to deliver different contents to one user, we consider using NOMA to simultaneously deliver different contents to different users. Therefore, we use NOMA to deliver these files with a single request. Applying NOMA as an additional transmission mode reduces the number of channels significantly, and the bandwidth allocated to each user is increased. Under the premise of a pre-specified bandwidth, the introduction of NOMA can increase the channel capacity. Therefore, we propose a cooperative transmission scheme of MC and NOMA in cache-enabled cellular networks. We apply MC to deliver files with multiple requests via an MC transmission. For files with a single request, we apply NOMA to aggregate the requests and transmit files via a superimposed signal. We first analyse the performance in the two modes. Then, with the derived probabilities of using the two modes, we obtain the total probability of successful transmission based on probability theory. The primary contributions of our study are summarised as follows:

First, we consider both MC and NOMA as transmission modes for users in cache-enabled wireless cellular networks. Applying the proposed scheme can use a single channel simultaneously to deliver a file to its multiple requestors or transmit multiple files to their multiple requestors. This can substantially improve the spectral efficiency. The corresponding transmission mode is determined by the number of users requesting a file. If a file is requested by only one user, it will be transmitted in the form of superimposed signals through NOMA. If a file is requested by multiple users simultaneously, MC will be used to deliver it.

Second, we analyse the number of channels in the two transmission modes. The channel capacity can be improved by increasing the corresponding bandwidth. The bandwidth of each channel is determined by the total bandwidth and number of channels. In particular, the number of channels and NOMA users is not independent in NOMA mode. Therefore, the probability mass function (PMF) of the number of channels in the MC mode as well as the joint PMF of the number of channels and number of NOMA users in the NOMA mode are obtained by the analysis based on the binomial distribution.

Finally, we analyse the performance of the successful transmission probability of the two modes, which can be obtained as closed-form expressions when the noise is neglected, and the path-loss exponent is set to be four. With the derived probabilities of using the two modes, the total successful transmission probability can be obtained by utilising the probability theory. Simulation results validate the analytical results. Moreover, we compare the performance of the MC and NOMA schemes cooperatively and individually. The results show that the proposed cooperative transmission scheme can achieve substantial improvement in terms of the successful transmission probability in most scenarios, compared with schemes using MC and NOMA individually.

2 SYSTEM MODEL

In this study, we consider a downlink transmission scenario in cache-enabled cellular networks. The locations of BSs are modelled as a homogeneous Poisson point process (HPPP) Φ. The intensity of Φ is denoted by $\lambda$. The locations of users are modelled as an independent HPPP with the intensity of $\lambda_u$. The total bandwidth is set to be $W$ Hz. The transmission power is denoted by $P$. We model the propagation channel as the combination
of small-scale fading and large-scale fading. The small-scale fading is modeled as Rayleigh fading with unit power, that is, $|h|^2 \sim \exp(1)$. The large-scale fading is modeled by a standard distance-dependent path-loss attenuation with path-loss exponent $\alpha$. The standard path-loss function is calculated as $d^{-\alpha}$, where $d$ denotes the propagation distance. All the channels are assumed to be independently and identically distributed.

### 2.1 Request distribution and caching policies

We consider a database comprising $K$ different files with the normalised size equal to 1. Let $q_k$ denote the probability that the file $k$ has requested. The PMF of requesting the $K$ files, represented as $\{q_k ; k = 1,2,\ldots , K\}$, is independent and identical among all users. We model the PMF of requesting the files as a Zipf distribution [17], and the request probability $q_k$ for file $k$ is expressed as

$$q_k = \frac{k^{-\gamma}}{\sum_{i=1}^{K} i^{-\gamma}} \quad (1)$$

where $\gamma$ denotes the exponent of the Zipf distribution. A larger value of $\gamma$ implies that the request will be concentrated on several of the most popular files. It is evident that the request probabilities of the files are ordered in decreasing popularity.

In fact, a BS is unable to store the entire database due to the limited storage. Therefore, we assume that the cache capacity of each BS is $N \leq K$. We partition the database into $\frac{K}{N}$ subsets of files, referred to as file groups (FGs) [5]. Each BS can cache one of the $\frac{K}{N}$ FGs. We denote the $i$-th FG by $FG_i$, $i \in \{1,2,\ldots , \frac{K}{N}\}$. We adopt a simple grouping strategy where file $k$ belongs to $FG_i$ if $k \in \{(i-1)N+1, (i-1)N+2, \ldots , iN\} \forall i \in \{1,2,\ldots , \frac{K}{N}\}$. We assume that the caching probability of an FG equals its request probability [18]. Then, the probability $p_{FG_i}$ that a BS caches $FG_i$ can be formulated as

$$p_{FG_i} = \sum_{\forall k \in FG_i} q_k. \quad (2)$$

The probability that a BS caches the file $k$ is equal to the probability that the BS caches the FG containing the file $k$. Thus, the probability $p_k$ that a BS caches the file $k$ is given by $p_k = p_{FG_i}$.

### 2.2 Transmission scheme

Without loss of generality, we proceed with our analysis for a typical user located at the origin, denoted as $u_0$. According to Slivnyak's theorem [19], it is permissible to conduct all analyses on a typical user in an HPPP. In the cache-enabled cellular network, the user will associate with the nearest BS that caches its requested file. We assume that a user can only request one file at a particular instant. As shown in Figure 1, we propose a cooperative transmission scheme of MC and NOMA. In Figure 1(a), the MC mode is chosen because there is another user requesting the same file with $u_0$; in Figure 1(b), the NOMA mode is chosen because no other user requests the same file with $u_0$. Based on the thinning theorem of HPPP [19], the distributions of the BSs and users are modeled as two independent HPPPs with intensities of $\lambda_b$ and $\lambda_u$, respectively. Accordingly, the distribution of BSs that cache file $k$ and the distribution of users that request file $k$ are modeled as thinned HPPP $\Phi_{bk}$ with the intensity of $\lambda_b k$ and independent thinned HPPP with the intensity of $q_k \lambda_u$, respectively.

When $u_0$ requests file $k$, the serving BS of $u_0$ is denoted as $b_{k,0}$. We use $N_k$ to represent the number of residual users simultaneously requesting file $k$ at $b_{k,0}$, except the typical user $u_0$.

**Lemma 1.** The PMF of $N_k$ is given by

$$\Pr(N_k = n) = \frac{\Gamma(4.5+n)}{n! \Gamma(4.5)} \left(3.5 p_b \lambda_b\right)^{4.5} \left(q_k \lambda_u + 3.5 p_b \lambda_b\right)^{4.5+n}. \quad (3)$$

**Proof.** Please refer to Appendix 1. \hfill \Box

We first consider the transmission mode for delivering file $k$ to $u_0$. We denote $p_{mk,k}$ and $p_{mc,k}$ as the probability that $u_0$ receives file $k$ through NOMA and MC, respectively. If no other users requests file $k$ at $b_{k,0}$, except user $u_0$, then the file $k$ is delivered to $u_0$ through NOMA. That is, the probability that $u_0$ receives its requested file through NOMA is $\Pr(N_k = 0)$. In contrast, the probability that $u_0$ receives its requested file through MC is $1 - \Pr(N_k = 0)$. According to Lemma 1, we can obtain the expressions of $p_{mk,k}$ and $p_{mc,k}$ as

$$p_{mk,k} = \left(3.5 p_b \lambda_b\right)^{4.5} \left(q_k \lambda_u + 3.5 p_b \lambda_b\right)^{4.5} \quad (4)$$

$$p_{mc,k} = 1 - \left(3.5 p_b \lambda_b\right)^{4.5} \left(q_k \lambda_u + 3.5 p_b \lambda_b\right)^{4.5}. \quad (5)$$

![FIGURE 1 Cooperative transmission scheme of multicast (MC) and non-orthogonal multiple access (NOMA). There are three files ($N = 3$) cached at the base station (BS), represented by three numbers. (a) MC mode, (b) NOMA mode.](image-url)
3 | PERFORMANCE ANALYSIS

3.1 PMF analyses in different modes

To analyse the channel capacity, the bandwidth of each channel must be known. However, the bandwidth of each channel is determined by the number of channels, which is a random variable in the proposed cooperative transmission scheme. Hence, we first derive the PMF of the number of channels.

We define the FG that contains file $k$ with $N$ elements as $F_k$ and that without file $k$ as $F_{-k}$ to denote a subset of $F_k$, thus, $F_{-k} = F_k \setminus \{k\}$. Each of the $n-1$ different files in $F_{-k}$ forms a new combination. Accordingly, there are $J_{n-1} = C_{N-1}^{n-1}$ different combinations of $n-1$ different files in total. The set of $J_{n-1}$ combinations is denoted by $\theta_{n-1} = \{1, 2, ..., J_{n-1}\}$. We characterise combination $j \in \theta_{n-1}$ as an $N-1$ dimensional vector $x_j = (x_{j,m})_{m \in F_{-k}}$, where $x_{j,m} \equiv 1$ indicates that file $m$ is included in the combination $j$, and $x_{j,m} \equiv 0$ means file $m$ is excluded. We denote $M_j = \{m: x_{j,m} \equiv 1\}$ as the set of $n-1$ files contained in combination $j$, and $G_j = F_{-k} \setminus M_j$ as the complement of the subset $M_j$ of $F_{-k}$. When $n-2$ different files form a combination, the same definition is represented as $J_{n-2} = C_{N-2}^{n-2}$.

The corresponding set of $J_{n-2}$ combinations yields $\theta_{n-2} = \{1, 2, ..., J_{n-2}\}$. Further, $M_j' = \{m: x_{j,m} \equiv 1\}$ is the set of $n-2$ files contained in combination $j$, and $G_j' = F_{-k} \setminus M_j'$.

For $m \in F_{-k}$, $Pr(N_m = 0)$ means that file $m$ has no requestor, and $Pr(N_m = 1)$ means that file $m$ has one requestor and the delivery to its requestor will be executed through NOMA. Further, $1 - Pr(N_m = 0) - Pr(N_m = 1)$ means that file $m$ has more than one requestor and the delivery to its requestors will be performed through MC. We denote $p_{m}^0$, $p_{m}^w$, and $p_{m}^m$ as the probabilities of file $m$ delivered through MC and NOMA, respectively. The probability that there is no user request for file $m$ is denoted as $p_{m}^0$. According to Equation (3), the expressions of $p_{m}^0$, $p_{m}^w$, and $p_{m}^m$ are obtained as

$$p_{m}^0 = \left(\frac{3.5p_A q_\lambda^2}{q_\lambda^2 + 3.5p_A q_\lambda^2}\right)^{4.5}$$

$$p_{m}^w = \left(\frac{4.5p_A q_\lambda}{q_\lambda^2 + 3.5p_A q_\lambda^2}\right)^{4.5}$$

$$p_{m}^m = 1 - p_{m}^0 - p_{m}^w$$

3.1.1 MC mode

When $u_0$ receives file $k$ through MC, we denote $N_{mc} \in \{1, ..., N\}$ as the number of channels in the MC mode. Note that $N_{mc}$ is a discrete random variable for which the PMF needs to be analysed. The number of channels occupied by MC transmission is the same as the number of files transmitted by MC. All files that need to be delivered by NOMA are transmitted via one channel in the form of superimposed signals. Therefore, the number of channels is determined by the number of files delivered by MC and whether or not there are NOMA users. The PMF of $N_{mc}$ is given by

$$Pr(N_{mc} = n) = \begin{cases} \sum_{j=1}^{J_{n-1}} \left(\prod_{i \in M_j} \frac{p_{i}^0}{p_{i}^0 + p_{i}^m}\right) \left(\prod_{i \in G_j} \frac{p_{i}^m}{p_{i}^0 + p_{i}^m}\right), & n = 1 \\ \sum_{j=1}^{J_{n-2}} \left(\prod_{i \in M_j'} \frac{p_{i}^0}{p_{i}^0 + p_{i}^m}\right) \left(\prod_{i \in G_j'} \frac{p_{i}^m}{p_{i}^0 + p_{i}^m}\right), & 2 \leq n \leq N \end{cases}$$

$N_{mc}$ is equal to 1 when the users aggregated at $b_{k,i}$ all request for file $k$. If $n$ is greater than 2, it is necessary to analyse whether there are NOMA users, therefore, we divide the analysis into two parts. The first part of the expression represents the situation where there are no NOMA users. In this case, the number of channels is equal to the number of files delivered by MC. The second part of the expression represents the situation where there are NOMA users. Because NOMA users only occupy one channel, the number of channels is equal to the number of files delivered by MC plus one.

3.1.2 NOMA mode

When $u_0$ receives file $k$ through NOMA, we need to analyse the joint PMF of the number of channels and number of NOMA users because they are not independent in the NOMA mode. We denote the number of the channels and number of NOMA users as $N_{nmc}$ and $N_{nno}$, respectively. There will be $V_{n-1} = C_{N-1-(n-1)}^{n-1}$ different combinations by extracting $i-1$ files of $G_j$ to form a combination. The set of $V_{n-1}$ combinations is denoted by $\zeta_{n-1} = \{1, 2, ..., V_{n-1}\}$. We characterise combination $\nu \in \zeta_{n-1}$ as a vector $Y_{\nu} = (y_{\nu,m})_{m \in F_{-k}}$, where $y_{\nu,m} \equiv 1$ indicates that file $m$ is included in the combination $\nu$, and $y_{\nu,m} \equiv 0$ means that file $m$ is excluded. We denote $M_{\nu} = \{m: y_{\nu,m} \equiv 1\}$ as the set of $i-1$ files contained in combination $\nu$. Therefore, the joint PMF of $N_{nmc}$ and $N_{nno}$ is expressed as

$$Pr\left(N_{nmc} = n, N_{nno} = \ell\right) = \sum_{j=1}^{J_{n-1}} \left(\prod_{i \in M_j} \frac{p_{i}^0}{p_{i}^0 + p_{i}^m}\right) \left(\prod_{i \in G_j} \frac{p_{i}^m}{p_{i}^0 + p_{i}^m}\right)$$

$$\left(\prod_{i \in M_{\nu}} \frac{p_{i}^0}{p_{i}^0 + p_{i}^m}\right) \left(\prod_{i \in G_{\nu}} \frac{p_{i}^m}{p_{i}^0 + p_{i}^m}\right), 1 \leq n \leq N_i 1 \leq \ell \leq N, n + \ell \leq N + 1$$

Note that the cache capacity of a BS is limited to $N$. When there are $\ell$ NOMA users, the number of files delivered through MC is at most $N - \ell$ and the number of channels is
at most \(N - i + 1\). Conditioned on the typical user receiving its requested file by NOMA, the number of channels is equal to the number of files delivered by MC plus one.

### 3.2 General case and main results

Successful transmission probability is an important performance metric of networks. It denotes the probability that a file is successfully downloaded by its requester, which is possible only when the requested file is cached in the BS and is successfully delivered. Using the proposed cooperative transmission scheme of MC and NOMA, each channel can simultaneously accommodate multiple users, which significantly reduces the number of channels and improves the corresponding bandwidth of each user. Therefore, we consider the channel capacity as a performance indicator. The transmission is successful when the corresponding channel capacity exceeds a given threshold. Next, we conduct the performance analysis in MC and NOMA modes, respectively.

#### 3.2.1 MC mode

When the requested file \(k\) is delivered to \(u_0\) through MC, we formulate the received signal-to-interference plus noise ratio (SINR) from \(b_{k,0}\) of \(u_0\) as

\[
\text{SINR}^\text{mc}_k = \frac{P|b_0|^2 d_0^{-\alpha}}{I + \sigma^2} \tag{11}
\]

where \(|b_0|^2 \sim \exp(1)\) is the small-scale Rayleigh fading channel gain between the typical user \(u_0\) and its associated BS \(b_{k,0}\); \(d_0\) is the distance between \(u\) and \(b_{k,0}\), \(\sigma^2\) denotes the Gaussian noise power. Additionally, \(I = \sum_{i \in \Phi \backslash k} P|\varrho_i|^2 d_i^{-\alpha}\) is the aggregated interference from all BSs except \(b_{k,0}\), \(d_i\) is the distance between the \(i\)-th interference BS and \(u_0\), and \(|\varrho_i|^2 \sim \exp(1)\) is the small-scale Rayleigh fading channel gain between the \(i\)-th interference BS and \(u_0\). According to [20], the probability density function of \(d_i\) is expressed as

\[
f_{d_i}^\text{mc}(d) = 2\pi P \lambda_i d^2 \exp\left(-\pi P \lambda_i d^2\right) \tag{12}
\]

The corresponding channel capacity of \(u_0\) is given by

\[
C^\text{mc}_k = \frac{W}{N_{\text{sub}}} \log_2 \left(1 + \text{SINR}^\text{mc}_k\right) \tag{13}
\]

We consider channel capacity as the metric of successful reception and let \(\theta\) be the minimum channel capacity required for successful transmission. Then, the successful transmission probability of file \(k\) requested by \(u_0\) is formulated by

\[
P^\text{mc}_{u_0,k} = \Pr(C^\text{mc}_k \geq \theta) \tag{14}
\]

#### 3.2.2 NOMA mode

When the requested file \(k\) is delivered to \(u_0\) through NOMA, successive interference cancellation (SIC) is performed at receivers to subtract the signals from users with weaker channel conditions before decoding their own received signal. Without loss of generality, we assume that the user index \(n\) determines the detection order. Because the large-scale path-loss has a greater impact on channel gain than the small-scale Rayleigh fading, we establish the detection order by employing the path loss. We sort the path loss of each NOMA user associated with \(b_{k,0}\) in ascending order [14] and denote the sorted path loss as \(d_1^{-\alpha} \leq d_2^{-\alpha} \leq \cdots \leq d_{N_{\text{noue}}}^{-\alpha}\). The probability that \(u_0\) becomes the \(n\)-th user is \(1/N_{\text{noue}}\). For the \(n\)-th user \((1 \leq n' \leq u \leq N_{\text{noue}})\), the received SINR of user \(n'\) at user \(u\) can be written as

\[
\text{SINR}^\text{no}_u = \frac{P|b_{n'}|^2 d_{n'}^{-\alpha} + I + \sigma^2}{\sum_{i=n'+1}^{N_{\text{noue}}} P|b_i|^2 d_i^{-\alpha} + I + \sigma^2} \tag{16}
\]

where the first term in the denominator indicates the interference from other NOMA users with stronger channel conditions. This part of the interference is equal to 0 while \(n' = N_{\text{noue}}\). \(I\) is defined in the same way as in Equation (11) to represent the aggregated interference from other BSs. \(P_i\) denotes the allocated transmit power for the \(i\)-th user, and the corresponding channel capacity is given by

\[
C^\text{no}_{u,n'} = \frac{W}{N_{\text{sub}}} \log_2 \left(1 + \text{SINR}^\text{no}_{u,n'}\right) \tag{17}
\]

Note that \(d_1, d_2, \ldots, d_{N_{\text{noue}}}\) represent a group of order statistics [22] with identical distributions, \(d_i\), the distance between \(u_0\) and its serving BS, becomes the \((N_{\text{noue}} + 1 - n)\)-th order statistic as \(u_0\) becomes the \(n\)-th user with probability \(1/N_{\text{noue}}\). Therefore, the

**Theorem 1.** The successful transmission probability of file \(k\) delivered to \(u_0\) in the MC mode is given by

\[
P^\text{mc}_{u_0,k} = \sum_{n=1}^{N_{\text{sub}}} \Pr(N_{\text{sub}} = n) \int_0^{\infty} 2\pi P \lambda \exp\left(-\frac{T\text{mc} \sigma^2 \lambda^2}{\rho}\right) \exp\left(-\pi P \lambda \left(\frac{2(1 - P_k) T\text{mc}^2}{\rho} \exp(1) B\left(\frac{2}{\alpha}, \frac{\alpha - 2}{\alpha}\right) + \frac{2 P_k T\text{mc}}{\alpha - 2} \text{F}_1\left(1, \frac{\alpha - 2}{\alpha}, \frac{2\alpha - 2}{\alpha}; -T\text{mc}\right)\right) dx \tag{15}
\]

where \(T\text{mc} = 2\sigma^2 - 1\). Furthermore, \(\text{F}_1()\) represents the Gauss hypergeometric function, and \(B()\) denotes the beta function [21].

**Proof.** Please refer to Appendix 2.
probability density function of $d_0$ can be expressed as

$$f_{d_0}(d) = \frac{N_0^{\alpha_0}}{(N_0^{\alpha_0} - u)!} 2\pi p_0 \lambda_0 d \left[1 - \exp\left(-\pi p_0 \lambda_0 d^2\right)\right]^{N_0^{\alpha_0} - u} \left[\exp\left(-\pi p_0 \lambda_0 d^2\right)\right]^u.$$

(18)

In the NOMA mode, due to the implementation of SIC, it is necessary to ensure that previous users can be detected successfully at $u_0$. Thus, the successful transmission probability of file $k$ requested by $u_0$ (the $u$-th NOMA user) can be formulated as

$$P_{\text{succ}}^{\text{no}} = \Pr\left(C_{u,0} \geq \theta, \ldots, C_{n,0} \geq \theta\right) \quad (19)$$

**Theorem 2.** The successful transmission probability of file $k$ delivered to $u_0$ in the NOMA mode is given by

$$P_{\text{succ}}^{\text{no}} = \sum_{n=1}^{N} \sum_{i=1}^{N} \Pr\left(N_{i,0}^{\text{no}} = n, N_{n,0}^{\text{no}} = i\right) \left(\frac{(i-1)!}{(i-u)!(u-1)!}\right) \int_0^\infty 2\pi p_i \lambda_i d \left[1 - \exp\left(-\pi p_i \lambda_i d^2\right)\right]^{i-u} \exp\left(-\pi \sigma^2 \lambda_i d^2\right)$$

$$\times \left[\exp\left(-\pi \sigma^2 \lambda_i d^2\right)\right]^{i-u} \exp\left(-\pi \sigma^2 \lambda_i d^2\right) \left[\exp\left(-\pi \sigma^2 \lambda_i d^2\right)\right]^{i-u}$$

$$\times \left[\exp\left(-\pi \sigma^2 \lambda_i d^2\right)\right]^{i-u} \exp\left(-\pi \sigma^2 \lambda_i d^2\right) \left[\exp\left(-\pi \sigma^2 \lambda_i d^2\right)\right]^{i-u}$$

(20)

where

$$T^{\text{no}} = \max \left\{ \frac{\phi_0}{2 - 1}, \ldots, \frac{\phi_0}{2 - 1} \right\}.$$

**Proof.** Please refer to Appendix 3.

Therefore, the complete successful transmission probability of $u_0$ is formulated as

$$P_{\text{succ}}^{\text{no}} = \sum_{k=1}^{K} q_k \left(P_{\text{mc},k}^{\text{no},k} + P_{\text{mc},k}^{\text{no},k}\right) \quad (21)$$

### 3.3 Special case and main results

Since the interference power easily dominates thermal noise, the impact of the noise can be neglected, that is, $\sigma^2 = 0$. Furthermore, $\alpha = 4$ is a typical and practical value of the path-loss exponent. Then, we simplify the derivation of the successful transmission probability in the special case of $\sigma^2 = 0$ and $\alpha = 4$ as follows.

#### 3.3.1 MC mode

Substituting $\sigma^2 = 0$ and $\alpha = 4$ into Equation (15), the quasi-closed form expression of the successful transmission probability with MC mode is expressed as

$$P_{\text{succ}}^{\text{mc}} = \sum_{n=1}^{N} \Pr\left(N_{i,0}^{\text{mc}} = n\right) \int_0^\infty 2\pi p_i \lambda_i (\frac{1 - p_k}{2} + \frac{1}{2} - p_k) \exp\left(-\pi \lambda_i (\frac{1 - p_k}{2} + \frac{1}{2} - p_k)\right)$$

$$\times \exp\left(-\pi \lambda_i (\frac{1 - p_k}{2} + \frac{1}{2} - p_k)\right) \left[\exp\left(-\pi \lambda_i (\frac{1 - p_k}{2} + \frac{1}{2} - p_k)\right)\right]^{i-u}$$

(22)

#### 3.3.2 NOMA mode

Substituting $\sigma^2 = 0$ and $\alpha = 4$ into Equation (20), the quasi-closed form expression of the successful transmission probability with NOMA mode is given in Equation (23).

$$P_{\text{succ}}^{\text{no}} = \sum_{n=1}^{N} \sum_{i=1}^{N} \Pr\left(N_{i,0}^{\text{no}} = n, N_{n,0}^{\text{no}} = i\right) \sum_{m=1}^{t} C_m \int_0^\infty 2\pi p_m \lambda_m \left[1 - \exp\left(-\pi p_m \lambda_m d^2\right)\right]^{i-u}$$

$$\times \exp\left(-\pi p_m \lambda_m d^2\right) \left[\exp\left(-\pi p_m \lambda_m d^2\right)\right]^{i-u} \exp\left(-\pi p_m \lambda_m d^2\right) \left[\exp\left(-\pi p_m \lambda_m d^2\right)\right]^{i-u}$$

$$\times \left[\exp\left(-\pi p_m \lambda_m d^2\right)\right]^{i-u} \exp\left(-\pi p_m \lambda_m d^2\right) \left[\exp\left(-\pi p_m \lambda_m d^2\right)\right]^{i-u}$$

(23)
where
\[ \exp(-\pi \lambda_r x^2 (p_k + C_2)) \, dx \]
\[ = \sum_{m=1}^{N \lambda} \sum_{i=1}^{N \lambda} \Pr(N_{serv}^m = n, N_{nove}^m = i) \sum_{i=1}^{C_1 p_k} \left( \frac{1}{C_2 + \frac{i}{n}} \right) \sum_{t=1}^{\infty} \left[ (t-n) (t-n-1) \cdots (t-n-i+1) \right] \prod_{l=1}^{t} \left( C_2 + ip_k \right) \]
(23)

where \( C_1 = \frac{(t-1)!}{(t-n)!(t-n-1)!} \) and \( C_2 = np_k + \frac{\pi (1-p_k) \sqrt{T \alpha P}}{2} \), \( p_k \sqrt{T \alpha P} \arctan(\sqrt{T \alpha P}) \).

(a) follows from the power series expansion formula: \((1 + x)^2 = 1 + \sum_{i=1}^{\infty} a_i \frac{x^i}{i!} \).

Therefore, the complete successful transmission probability in the special case can be written as
\[ p = \sum_{k=1}^{K} q_k \left( p_{n,k} p_{c,n,k} + p_{m,k} p_{c,n,k} \right) \]
(24)

4 NUMERICAL RESULTS

In this section, we first present our numerical and simulation results of the successful transmission probability. Then, we compare the proposed cooperative transmission scheme with the scheme only using MC in [9] and the only NOMA scheme. Applying the only MC scheme, one file is delivered to one or more requestors using a single channel. Applying the only NOMA scheme, all the files are delivered to their requestors in the form of a superimposed signal by occupying one channel.

In the simulation, the BSs and users are randomly distributed in an area of 5 × 5 km according to two independent HPPPs. Without loss of generality, we assume the path-loss exponent as equal to 4. The transmission power is 30 dBm, the noise power is 104 dBm and the total bandwidth is 10 MHz. The power allocation method for NOMA users [15] is as follows: \( P_1 = P \) for \( N_{serv}^m = 1 \); \( P_1 = 0.8 \times 2P \) and \( P_2 = 0.2 \times 2P \) for \( N_{serv}^m = 2 \);

\( P_1 = \frac{2(N_{serv}^m - i + 1)}{N_{serv}^m + 1} P, 1 \leq i \leq N_{serv}^m \) for \( N_{serv}^m \geq 3 \).

Figure 2 illustrates the probabilities of using the two modes. The greater the probability of using NOMA, the bandwidth allocated to each user will increase because of the reduction of the number of channels. When the BS density is constant, we can observe that the probability of using NOMA increases as the user density decreases. On the contrary, the probability of using MC increases with the user density. The reason for this observation is that the probability that the requests are duplicated increases with the user-to-BS density ratio. Thus, with a higher user density, the probability of using MC becomes larger and the probability of using NOMA conversely becomes smaller.

In Figure 3, we compare the simulation and the numerical results concerning the successful transmission probability of the proposed cooperative transmission scheme. It can be seen that the numerical results closely match the simulation results, which proves the correctness of our theoretical derivation. Meanwhile, it is clearly observed that the successful transmission probability increases with the Zipf exponent. With a larger Zipf exponent, fewer popular contents hold a majority of the content requests. In such cases, the files requested by the users and cached by the BSs will be more centralised, so that the successful transmission probability will become sufficiently high, as the BSs have a higher probability of hitting users’ requests. Besides, the figure shows that when the user density is constant, the higher the BS density is, the higher the successful transmission probability will be. Conversely, when the BS density is constant, the higher the user density is, the lower the successful transmission probability will be. This is because the probability of using NOMA increases as the user-to-BS density ratio decreases. Fewer users
and the use of NOMA increase channel capacity by broadening the allocated bandwidth for each user. As a result, the corresponding channel capacity increases, and the successful transmission probability is improved.

Figure 4 illustrates that the successful transmission probability will be higher if the value of the threshold becomes lower. Obviously, the probability that the corresponding channel capacity exceeds the threshold will increase as the threshold decreases. This figure also shows that the successful transmission probability increases as the cache capacity of BSs increases. With a larger cache capacity, the BS can cache more files and better hit users’ requests. Hence, the successful transmission probability is increased. Therefore, increasing the cache capacity of BSs can bring the improvement of the successful transmission probability.

In Figure 5, we compare the successful transmission probability of the three transmission schemes versus the threshold $\theta$. The three schemes are the cooperative transmission scheme of MC and NOMA proposed in this study, the only MC scheme of \cite{9}, and the only NOMA scheme. We can see that the performance of all the schemes decreases as $\theta$ increases. This is because when the threshold is increased, the probability in which the corresponding channel capacity exceeds the threshold is reduced. In the following, it can be obviously seen that, compared with the only MC scheme in \cite{9}, the scheme we proposed improves the successful transmission probability in all scenarios. This improvement is achieved by introducing NOMA as another transmission mode to improve spectral efficiency. When the threshold is small, the only NOMA scheme has a higher successful transmission probability than the other two schemes. When the threshold is large, our proposed scheme outperforms the other two schemes. This result illustrates that the scheme proposed in this study has a great advantage when users have high requirements for the QoS.

Figure 6 plots the successful transmission probability versus the Zipf exponent $\gamma$ to compare the performances of the proposed cooperative transmission scheme, the only MC scheme of \cite{9} and the only NOMA scheme. We can see that the proposed cooperative scheme outperforms the only MC scheme in all scenarios since the use of NOMA can greatly improve the channel capacity. It also can be seen that the only NOMA scheme is slightly better than the other two schemes when $\gamma$ is small. As $\gamma$ increases, the trend becomes the opposite, and the gap becomes wider. The reason behind this trend is that for a large $\gamma$, users’ requests become more concentrated. Applying MC can greatly reduce the duplicate content transmission. Thus, the proposed cooperative scheme exhibits a significantly better performance than the only NOMA scheme. In conclusion, the combination of MC and NOMA can make full use of their respective advantages to improve channel capacity.
We compare the successful transmission probability of the three transmission schemes versus the user density $\lambda_u$ in Figure 7. We can see that the performance of all the schemes decreases with $\lambda_u$. The reason is that for a high $\lambda_u$, the bandwidth that can be allocated to each user would be narrowed, and the corresponding channel capacity would be smaller. However, the decline of the only NOMA scheme is particularly marked. The reason behind this trend is that SIC is used at the receiver to cancel multi-user interference, and the interference gets larger as the NOMA users increases. In fact, the receiver has to decode other users’ information prior to decoding its own information, and signal decoding by using SIC requires additional implementation complexity. Besides, the imperfect SIC causes error propagation in subsequent decoding of the signals of NOMA users. The complexity caused by SIC increases as the number of users in the cell of interest increases [23]. As a result, the implementation of the only NOMA scheme is not only complex but also extremely low in terms of the successful transmission probability with a large number of users.

In Figure 8, simulations have been provided to validate analytical results in the special case with $\sigma^2 = 0$ and $\alpha = 4$. This figure demonstrates that the quasi-closed expression we obtained under the special case is consistent with the simulation results. By comparison with Figure 3, it is found that the increase of the successful transmission probability by neglecting noise is almost negligible. This result suggests that the influence of noise on the successful transmission probability is very weak, and the main influencing factor is the interference from other BSs.

5 | CONCLUSION

In this study, we propose a cooperative transmission scheme of MC and NOMA in cache-enabled wireless cellular networks. By utilising tools from probability theory and stochastic geometry, we first derive the PMF of the number of channels in the MC mode and the joint PMF of the number of channels and number of NOMA users in the NOMA mode. Then, the performance of successful transmission probability is analysed in the MC and NOMA modes, respectively. The closed-form expression of the successful transmission probability is obtained in the special case with $\sigma^2 = 0$ and $\alpha = 4$. Finally, we obtain the total successful transmission probability based on the formula of total probability. Our simulation results show that, compared with schemes using MC and NOMA individually, the proposed cooperative transmission scheme of MC and NOMA enhances the successful transmission probability significantly in most scenarios. In future work, we will consider evaluating an optimal power allocation law for NOMA users and an optimal caching policy for BSs to maximise the successful transmission probability.

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ORCID

Yue Shan  https://orcid.org/0000-0003-0276-3792
Qi Zhu  https://orcid.org/0000-0003-2524-9796

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APPENDICES

Appendix 1

We denote $A$ as the cell size of the Voronoi cell to which a randomly chosen user belongs. According to [24], the probability density function of $A$ is given as

$$f_A(\alpha) = \frac{(3.5\nu \lambda_s)^4.5}{\Gamma(4.5)} x^{-3.5} e^{-3.5\nu \lambda_s x}$$

(A1)

where $\Gamma(\cdot)$ is the gamma function.

$N_k$ follows the Poisson distribution [25]:

$$Pr(N_k = n) = \frac{(\rho \lambda_s)^n}{n!} e^{-\rho \lambda_s x}.$$  (A2)

Then, the distribution of the average number of $N_k$ is given by

$$Pr(N_k = n) = \int_0^\infty Pr(N_k = n|A = x)f_A(x) \, dx$$  (A3)

Combining Equations (A3) with (A1) and (A2), we can obtain the explicit expression for the probability mass function of $N_k$. Therefore, we complete the proof.

Appendix 2

When the requested file $k$ is delivered to $n_t$ through multicast, the successful transmission probability can be expressed as

$$p^{suc}_{mc,k} = \Pr\left(\frac{W}{N_{mc}} \log_2 \left(1 + SINR_{k,c}^{mc}\right) \geq \theta\right)$$

$$= \sum_{n=1}^{N} \Pr(N_{mc} = n) \Pr\left(\frac{W}{n} \log_2 \left(1 + \frac{P|h|^2 d_t^{-\alpha}}{\xi + \sigma^2}\right) \geq \theta\right)$$

$$(a) = \sum_{n=1}^{N} \Pr(N_{mc} = n) \int_0^\infty p^{suc}_{mc,k} f_{\lambda_1} (\alpha) d\alpha$$

$$(b) = \sum_{n=1}^{N} \Pr(N_{mc} = n) \int_0^\infty p^{suc}_{mc,k} (\alpha) d\alpha$$

$$(A4)$$

where ($a$) is obtained by denoting $T_{mc} = \frac{W}{n} \log_2 \left(1 + \frac{P|h|^2 d_t^{-\alpha}}{\xi + \sigma^2}\right) \geq \theta$, (b) follows from $|h|^2 \sim \exp(1)$ and $s = \frac{T_{mc}}{p}$, $\mathcal{L}_1(\alpha) = \mathcal{E}_1(\alpha^{-\alpha})$ is the Laplace transform of random variable $I$ evaluated on $s$, the interference $I$ contains two independent parts: $I_1 = \sum_{i \in \Phi_{k-i} \setminus \Phi_{k-c}} p|l_i|^2 l_{1-z}$ and $I_2 = \sum_{i \in \Phi_{k-c}} p|l_i|^2 l_{1-z}$. $I_1$ is generated by the BSs that cache file $k$, their distances from $n_t$ is greater than $d_t$. $I_2$ is produced by the BSs that do not cache the file $k$, these BSs are dispersed throughout the entire area of the network, and $\Phi_{k-c}$ denotes the point process generated by these BSs. Due to the independence of $I_1$ and $I_2$, we obtain $\mathcal{L}_1(\alpha) = \mathcal{L}_{I_1}(\alpha) \cdot \mathcal{L}_{I_2}(\alpha)$. Similar to [9], the expressions of $\mathcal{L}_{I_1}(\alpha)$
and \( \mathcal{L}_{h_i}(s) \) are given as

\[
\mathcal{L}_{h_i}(s) = \exp \left( -\frac{2\pi p_i \lambda_i s \nu_r^{\alpha-2}}{\alpha-2} - 2 \tilde{F_1} \left( 1, \frac{\alpha-2}{\alpha}, \frac{2\alpha-2}{\alpha}; -s\right) \right)
\]

(A5)

\[
\mathcal{L}_{h_i}(s) = \exp \left( -\frac{2\pi (1-p_i) \lambda_i s \nu_r^{\alpha-2}}{\alpha} s^2 \right)
\]

(A6)

Substituting \( s = \frac{T_{no} x^2}{\alpha} \) into Equations (A5) and (A6),

\[
\mathcal{L}_{h_i} \left( \frac{T_{no} x^2}{\alpha} \right) = \exp \left( -\frac{2\pi (1-p_i) \lambda_i s \nu_r^{\alpha-2}}{\alpha} - 2 \tilde{F_1} \left( 1, \frac{\alpha-2}{\alpha}, \frac{2\alpha-2}{\alpha}; -T_{no} x^2 \right) \right)
\]

(A7)

\[
\mathcal{L}_{h_i} \left( \frac{T_{no} x^2}{\alpha} \right) = \exp \left( -\frac{2\pi (1-p_i) \lambda_i s \nu_r^{\alpha-2}}{\alpha} - 2 \tilde{F_1} \left( 1, \frac{\alpha-2}{\alpha}, \frac{2\alpha-2}{\alpha}; -T_{no} x^2 \right) \right)
\]

(A8)

Combining Equations (A4) with (A7) and (A8), we can obtain the complete expression of \( p_{mc,k}^{no} \). Therefore, we complete the proof.

**Appendix 3**

In the NOMA mode, the successful transmission probability of file \( k \) requested by \( h_0 \) can be formulated as

\[
p_{mc,k}^{no} = \sum_{t=1}^{N_{mc}} \sum_{i=1}^{N_{mc}} \Pr \left( N_{mc}^{no} = t \right) \sum_{t=1}^{1} \frac{1}{t} \log_2 \left( 1 + \min \left\{ S/N\nu_{mc1}^{no}, \ldots, S/N\nu_{mcN}^{no} \right\} \right) \geq \theta \)
\]

(A9)

where

\[
\Pr \left[ \frac{W}{n} \log_2 \left( 1 + \min \left\{ S/N\nu_{mc1}^{no}, \ldots, S/N\nu_{mcN}^{no} \right\} \right) \geq \theta \right] = \int_0^\infty \frac{W}{n} \log_2 \left( 1 + \min \left\{ \sum_{i=1}^{t} \frac{P_i |b_i|^2 \chi^{-\alpha}}{\sum_{i=1}^{t} P_i |b_i|^2 \chi^{-\alpha} + I + \sigma^2} \right) \right) \frac{d\chi}{\chi}
\]

The expression of the \( p_{mc,k}^{no} \) can be obtained by combining Equations (A9), (A10), (A11) and (A12). Therefore, we complete the proof.