Automated Systematic Generation of Flat Directions in Free Fermionic Heterotic Strings

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Abstract. It has been shown that string moduli are not completely stabilized in traditional weakly coupled free fermionic heterotic string flat direction phenomenology. This necessitates systematic investigation of the VEV moduli space in order to compare with reality. We summarize current efforts at Baylor University to enhance the efficiency and depth of the current systematic free fermionic heterotic string investigations to include the calculation of flat direction derived phenomenological properties through supercomputing. We focus on the necessary assumptions, complexity and open questions that arise while making a fully-automated flat direction analysis program.

1. Introduction
As a leading candidate for the unification of gravity and gauge theories, string theory has revealed the landscape, a moduli space which yields some $10^{100}$ to $10^{1000}$ models. Sifting through the various models to search for the Standard Model (SM) or its Grand Unified embedding necessitates choosing a particular low-energy string construction method. Heterotic string theory received a lot of early attention due to its ease of generating GUT models [1]. Particular success was found among the free fermionic constructions since its naturally generated $SO(10)$ GUT group can be broken to a variety of phenomenologically realistic models [2] - [9].

In the weakly-coupled free fermionic (WCFF) approach, the NAHE basis vector set is a natural start to model building [7]. These vectors, and subsequent additions, represent the boundary conditions of fermions propagating around the string worldsheet. Hence, the vectors represent the compactification process. One shortcoming to the WCFF construction is the inability to map a given model, as generated by the addition of more vectors to the NAHE set, to its low energy effective field theory (LEEFT). Thus, some groups prefer to utilize string theory constructions where it is easier to, a priori, generate a desirable model. This set-back is compensated by the ease in which the WCFF landscape subregions can be computationally analyzed. It is in this vein that we proceed to discuss some of the advantages and difficulties of automating WCFF heterotic sting (WCFFHS) searches.

The model construction consists of two parts: model generation and flat direction analysis. Early work done in these two areas was quite successful using a combination of computational (FORTRAN) and analytic techniques [10, 11]. However, a fully automated search was not possible until recent advances in supercomputing technology. Most recently, the development of the C++ based FF and Gauge Frameworks at Baylor University have allowed for the generation
of large sets of WCFFHS models. The second aspect of model building, flat direction analysis, has yet to be fully automated. This is an area of active research at Baylor University and the remainder of this article will focus on flat direction calculations.

The importance of flat directions should not be overlooked as all of the interaction and mass information is revealed only through these calculations. Therefore, a ‘true’ string model can exist only after the flat directions have been investigated. Often the term ‘model’ is used loosely among string phenomenologists, so context must guide the reader as to the intended definition. Here, we will use the term to mean a set of models specified by a given collection of gauge groups which have the same charges. In other words, a model is a charge matrix. This matrix can yield many actual models due to the freedom allowed by post-compactification moduli stabilization.

2. Charge Matrix
To study flat directions, analysis of a charge matrix, $Q$, is required. This encodes the actual gauge symmetry information of the model where the columns are the various quantum fields which are charged under the gauge groups of the theory (the rows). The value for the charge is the dimension of the specific representation that field is charged under. Therefore, $Q_{ij} \in \mathbb{Z}$.

2.1. General Structure
An $M \times N$ charge matrix will consist of $N$ fields charged under $M$ gauge groups. Since $M < N$, $Q$ will usually be singular. Typically, a model will consist of 50 to 100 fields charged under 5 to 22 gauge groups. This yields: $M \in [50, 100]$, $N \in [5, 22]$, and $M - N \in [28, 95]$.

Some fields have charges that can be paired. These paired fields are called “vector-like”. For Abelian groups or complex NA representations, fields are pairable if they have opposite (non-trivial, non-singlet) charges. For real reps of NA groups, the charges must be the same for both fields. Let the number of paired fields in a given model be: $N_p$, with $N_s$ and $N_{NA}$ representing the unpaired NA singlet and non-singlet fields, respectively. It is convention to rearrange and relabel the fields as:

$$\Phi_1, \ldots, \Phi_{N_p}, \Phi_{N_p+1}, \ldots, \Phi_{N_p+N_s}, \Phi_{N_p+N_s+1}, \ldots, \Phi_M$$

3. Flat Directions
In particle physics, the Vacuum Expectation Value (VEV) of a field, denoted as $\langle \Phi \rangle$, is a parameter of the model. As VEV’s are specified, a surface or point in the parameter space governed by the model’s scalar potential is chosen. This then dictates the physics (particle masses, interaction types, strengths of the forces etc . . . ) of the model.

3.1. SUSY and Anomalous $U(1)$’s
Supersymmetry (SUSY) is a pivotal aspect of the SM. Since SUSY particles are as yet undetected, SUSY must be broken at or above 1 TeV. The choice of SUSY-breaking scale determines how the coupling constants of the forces run and meet. SUSY breaking can occur due to the anomalous $U(1)$ charges, $U(1)_A$, which are unbalanced charges that give rise to the VEV of the dilaton field. The D-flat directions must cancel the anomalous charge.

3.2. D-flatness
For the case of $U(1)$-charged fields, D-flatness requires: $\langle D_A \rangle = \langle D_\alpha \rangle = 0$ where:
The $\Phi_k$ are the fields which acquire VEVs of order $\sqrt{\xi}$. The $Q_Ak$ and $Q_\alpha k$ denote the anomalous and non-anomalous charges and $M_{Pl} \approx 2 \times 10^{15}$ TeV denotes the reduced Planck mass. With the $\{Q_{ij}\}$ constituting the charge matrix, $Q$, we can re-express the above as a system of linear equations where the solution space is constrained according to:

$$|\langle \Phi_i \rangle|^2 \in \mathbb{Z}, \ 1 \leq i \leq N_p$$

and

$$|\langle \Phi_j \rangle|^2 \in \mathbb{N}_0, \ N_p < j \leq M.$$
It is readily apparent that the coefficient space will likely be unbounded. Therefore, fully automated searches will be done in a piecewise manner. Additional constraints may come from specifying allowable VEV ranges for the fields before D-flat directions are even calculated. These constraints would be nicely applied to visible sectors but could be relaxed, in varying degrees, to the hidden sector. In fact, by controlling these ranges we may be able to tune the order at which SUSY is broken. Investigations into methods to \textit{a priori} constraining the coefficient space (for example, using Linear/Integer Programming (LP)) are on-going.

3.2.2. \textit{NA D-flatness}  
It should be noted that our lack of attention to NA D-flatness constraints is intentional. In fact, it is not necessary to perform the full D-flatness calculation before moving onto the F-terms. In [12] and elsewhere, it was claimed that the space of gauge invariant monomials is in one-to-one correspondence to the space of D-flat stabilizing moduli. This was shown to be too strong of a condition in [13]. There it was shown that this mapping is, in fact, surjective and that there exist D-flat solutions which do not correspond to gauge invariant monomials. In our current build approach, we take the middle ground by requiring D-flatness, through the $U(1)$ charges, and will ensure that superpotential terms are gauge invariant. This means that our searches will not be exhaustive for the moment. Extension of the framework is certainly possible and searching for gauge variant terms could then be performed in future runs.

3.3. \textit{F-flatness}  
The F-flatness constraints are imposed on the superpotential, $W$. The $n$-th order term of $W$ is a sum of all possible terms whose powers add up to $n$:

$$W_n = \sum \Phi_1^{r_1} \ldots \Phi_M^{r_M},$$

$$n = \sum_{i=1}^{M} r_i.$$  

The requirements for F-flatness are:

$$\langle F_i \equiv \frac{\partial W}{\partial \Phi_i} \rangle = 0;$$  

$$\langle W \rangle = 0.$$  

General superpotential terms begin at order one and range through infinity. In stringy potentials, however, first and second orders do not exist. Additionally, the order at which the superpotential fails to be F-flat (i.e. does not satisfy Eq. 8) corresponds to the scale at which SUSY is broken. At $3^{rd}$ through $4^{th}$ order SUSY is broken at about the Planck scale: $10^{15}$ TeV and for each superpotential order increase, the energy scale decreases by a factor of 10. SUSY breaking at 1 TeV then corresponds to $n = 17$. This is a desirable condition although having the superpotential flat to all orders is also acceptable since there are other mechanisms that allow for SUSY breaking.

3.3.1. \textit{Stringent Flat Directions}  
The difficulty in calculating F-flatness is the sheer number of terms that need to be calculated if one were to perform an order-by-order analysis and unfortunately, flatness at one order does not ensure it will remain at higher orders. Another difficulty arises in the allowance for cross-cancellations. Eq. 8 produces a sum of many terms which, when VEV’d, can only be equal to zero if: 1) the individual terms are zero, 2) there is cancellation between them or 3) some combination of the two. One way to circumvent a truncated order-by-order analysis is to search for stringent flat directions. These are sets of
flat solutions where no cross-cancellation is allowed. While not exhaustive, it is believed that all other solutions branch off or lie near these ‘root’-like stringent flat directions. This has been observed in previous calculations. If possible, though, non-stringent solutions should be calculated despite their inherit ‘fine-tuning’ problem. Additionally, all-order results can be calculated by investigating potentially dangerous superpotential terms. These are terms which contain less than two factors of non-VEV’ed fields. This can be realized by having, at minimum, two unique non-VEV’ed fields or one non-VEV’ed field raised to a power greater than one.

Since stringent F-flatness is solely concerned with the presence of the VEV’ed and non-VEV’ed fields the solutions space can be represented in terms of powers of the individual F-terms:

$$
\Phi_{r_1} F_1 \ldots \Phi_{r_M} F_M \rightarrow \vec{R} = \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_M \end{pmatrix}
$$

(10)

The mapping we need to investigate is:

$$
Q \cdot \vec{R} = \vec{0}.
$$

(11)

In other words we need to consider the nullspace of Q. The values of the components of \( \vec{R} \) are of no consequence except for those \( r_i \) that correspond to solutions of the D-flat equation (Eq. 4) where \( x_i = 0 \). Let the columns of \( Ns(Q) \) be the basis vectors that span the nullspace(\( Q \)). The investigation of potentially dangerous F-terms can then be recast into two possibilities. The first can be seen as satisfying Eq. 8 and is represented as:

$$
Ns(Q)_{red} \cdot P = I
$$

(12)

where \( Ns(Q)_{red} \) is the reduced nullspace basis for the charge matrix constructed by keeping the rows of \( Ns(Q) \) which correspond to \( x_i = 0 \) and removing all others. The columns of \( P, col(P) \), will be the coefficients to be used with the \( Ns(Q) \) basis vectors to see if that combination stays in the nullspace:

$$
Q \cdot \sum_i P_{ij} \ col(Ns(Q))_i = \vec{0}
$$

(13)

where \( j \) is equal to the number of \( x_k \) that are equal to zero and \( i \in [0, n - l] \) as dictated by Eq. 5.

The second F-term constraint that must be tested is the null case, which is related to Eq. 9 (the supergravity constraint):

$$
Ns(Q)_{red} \cdot \vec{p} = \vec{0}.
$$

(14)

This case is more difficult since it may be severely underdetermined. If there were a finite number of solutions, they could be assembled into the matrix \( P \) as before and tested as in Eq. 13. In general, there will be zero, one or infinite solutions to Eq. 14. However, after integerizing the solutions, only a finite number should exist within the physically realizable order range of the superpotential: 3 - 17. In the case of all order infinite solutions existing beyond order seventeen, we find ourselves severely handicapped at how to approach SUSY breaking without relying on some other mechanism such as, gravity-mediation, kinetic mixing or shadow charges [14, 15].

For either case (Eqs. 12,14), if a particular set of nullspace basis vector coefficients, \( col(P) \), is shown to be mapped outside of the nullspace then we have found a potentially dangerous term which will then be tested against worldsheet selection rules [16]. If it survives, the vector with the smallest value of \( \sum_i |r_i| \) is the energy scale of SUSY breaking closest to 1 TeV.
4. Conclusion

Fully automated flatness analysis is computationally feasible and within reach. Great strides have been made at Baylor University in creating a C++ framework which will perform stringent searches of the WCFFHS models. As way of generating flat directions, we have seen that SVD is a fast and reliable method to solving the $U(1)$ D-flatness constraints. Additionally, SVD plays a larger role in the C++ framework in directly producing D-flat solutions than in the previous FORTRAN code. Due to the unboundedness of the D-flat solutions, a form of ‘rough tuning’ for allowable VEV ranges will need to be initially applied until a more effective method can be constructed and coded. One viable option is Linear/Integer Programming which investigates the population densities for the constrained coefficient space.

The primary goal for these flat direction searches will be to investigate phenomenologically viable models. Masses (and mass hierarchies), interactions strengths, proton stability and SUSY-breaking scales will be some of the first calculations performed on each model generated. The modularity and popularity afforded by C++ will allow for easy extension of the Flat Direction Framework. Future additions will likely include non-stringy model capabilities, a LP coefficient generator, Kähler potential calculations, NA D-flat constraints and Non-stringent F-flatness. The latter three will probably require a more robust numerical solver likely supplied through numerical polynomial homotopy continuation algorithms. This is an avenue also under current investigation.

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