Flux Tubes of Two- and Three-Quark System in Full QCD

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We study the abelian color flux of two- and three-quark systems in the maximally abelian gauge in lattice QCD with dynamical fermions. We find that the abelian flux tube formed between quark and antiquark is very much the same as in quenched QCD up to quark separations of \(R \sim 2\text{fm}\). The profile of the color electric field in three-quark system suggests \(Y\) ansatz, which might be interpreted as the result of the vacuum pressure in the confined phase. In order to clarify the flux structure, we investigate the color electric field of the three-quark system splitting the abelian gauge field into the monopole and photon parts.

1. Introduction

In the dual superconductor picture of confinement, the color flux tube is formed due to the dual Meissner effect caused by monopole condensation \([\square]\). Such a picture is based on the abelian gauge theory obeying Maxwell’s equations, and ’t Hooft built a bridge between the abelian gauge theory and QCD introducing abelian gauge fixing and abelian projection \([\square]\). In the last decade, lattice simulations, especially in the maximally abelian (MA) gauge, presented evidence in favor of this scenario.

The most previous lattice studies, however, have been performed in SU(2) gauge theory without dynamical quarks. We consider the more realistic case of lattice QCD with dynamical quarks. We extend the study of the abelian color flux tube between static quark and antiquark (QQ) to SU(3) gauge group and make first investigation of the abelian color flux tube in the three-quark (3Q) system. In QCD the string formed between static quark and anti-quark breaks and two static-light mesons are created, when the separation is large enough. We measure abelian flux tube with large quark separation and look for effects of the string breaking.

2. Operators and simulation details

The observables describing local structure of the flux tube are determined from the correlation function between an appropriate operator \(O(s)\) and abelian Wilson loop \([\square]\)
The abelian Wilson loop for $\bar{Q}Q$ system is defined using abelian link variables $u_\mu(s)$ as
\[
W_{\text{abel}}^{Q\bar{Q}}(R, T) = \frac{1}{3} \text{Tr} \prod_{s \in C} u_\mu(s) = \frac{1}{3} \text{Tr} e^{i\theta_C},
\]
where $C$ is a rectangular loop of extension $R \times T$, and for 3Q system as [5]
\[
W_{\text{abel}}^{3Q} = \frac{1}{3!} \varepsilon_{abc} u^a_R \cdot u^b_G \cdot u^c_B,
\]
with a path product of abelian link variables along the stapler-type path $\Gamma_C$ shown in Fig.1. Unlike in the nonabelian Wilson loop case, the color of quarks does not change during the propagation, because the off-diagonal components of the gauge field are frozen in the abelian projected theory. Most measurements are done in the plane which is a central time slice of the abelian Wilson loop shown also in Fig. 1.

The simulations were performed using lattice configurations generated with $N_f=2$ non-perturbatively $O(a)$ improved Wilson fermions with at $m_\pi/m_\rho \sim 0.7$ [4]. The link variables are fixed to maximally abelian gauge with a simulated annealing algorithm. For noise reduction, we used a smearing method for spatial links of the abelian Wilson loop.

### 3. Abelian Flux Tube

Fig. 2 shows the structure of $Q\bar{Q}$ abelian flux tube in full lattice QCD for $R \sim 0.5$ fm. Similar to SU(2) [2] and SU(3) [3] gluodynamics, we observe enhancement in the action density. Opposite to this, the monopole density is suppressed in agreement with vanishing of the dual Higgs field in the center region of the flux tube. Furthermore, in the plane perpendicular to the flux tube, the solenoidal monopole current is clearly seen. All these observations are in accordance with the dual superconductor picture of confinement in particular with the assumption that the color electric field is squeezed into flux tubes by the super monopole current.

We expect the flux tube to disappear eventually if the static charges are pulled apart beyond a certain distance. This distance is expected to be around $1.2$ fm [4]. We have studied the distribution of the color electric field for $Q\bar{Q}$ separation up to $\approx 2$ fm. We
observed no sign of string breaking. This is probably because the abelian Wilson loop has only small overlap with the broken string as it is the case for the nonabelian Wilson loop [6].

4. Flux of 3 Quark system

For more than 20 years, the question whether there is a genuine three body force, or the interaction can be described by the sum of the two-body forces is unanswered [7, 8]. In the former case, the flux structure is expected to be of $Y$ shape, which has a junction at the point where the total length of strings connecting every quark to the junction is minimal. In the latter case, the flux structure is expected to be of $\Delta$-shape, i.e. consisting of three sets of two-body interactions.

Fig. 3 shows the abelian action density of the 3Q system in full QCD. One can see a clear junction in the center of the 3 quarks, which suggests $Y$ shape. Fig. 4 shows

Figure 3. Action density in 3 quark system in full QCD.
green component of the color electric field and super monopole current in 3Q system. The electric field flows from the green quark to red and blue quarks. For each leg of the flux, we observe the solenoidal monopole current in the plane perpendicular to the flux. The strength of the monopole current is proportional to the strength of the electric flux. Our results are in qualitative agreement with the dual Ginzburg-Landau theory [9, 10].

In conclusion, we have studied the abelian flux tube in QQ and 3Q systems in full QCD. The abelian flux tube agrees with the dual superconducting scenario, similarly to gluodynamics. We made a first study of the abelian flux in the 3-quark system in lattice QCD with dynamical quarks. Our results support Y shape.

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