Quantum tricriticality of chiral-coherent phase in quantum Rabi triangle

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The interplay of strong interactions, symmetries and gauge fields usually intrigues quantum many-body phases. To explore the nature of emerging phases, we study a quantum Rabi triangle system as an elementary building block for synthesizing an artificial magnetic field. Using an exact solution, we find a novel quantum criticality of phase diagram and an interesting chiral-coherent phase, which breaks the $\mathbb{Z}_2$ symmetry and chiral symmetry. In such chiral phase, photons in eigenstates flow unidirectionally and the chirality can be tuned by the gauge field, exhibiting a signature of broken time-reversal symmetry. The finite-frequency scaling exponent analysis further confirms to be in the universality class of the Dicke model. Our work suggests a fundamental unit cell for engineering quantum phases by manipulating photons in an artificial magnetic field.

Introduction.-- The coupling between light and atoms realized in the cavity-QED has brought forth a novel class of quantum many-body systems.1-7 The possibility of quantum phase transition (QPT) of photons has stimulated a lot of discussions in the Jaynes-Cummings (JC) Hubbard lattice 8-10 and the Rabi lattice model 11-13. The basic building block of such systems contains a two-level system and a bosonic field mode, which is the simplest and most fundamental model describing quantum light-matter interactions.1-13 Usually, the QPTs are discussed in the thermodynamical limit.14 However, the quantum Rabi model 17-20 and two-site JC lattice 21 also exhibit the similar scaling behavior of QPTs. Such QPTs in few-body system open a window for investigating related integrability, exotic phases and critical behaviors 17-19, 22.

The intriguing many-body phases generally arise from the interplay of strong interactions, symmetries and external fields. As a representative example, the fractional quantum Hall effect occurs in two dimensional electrons in the presence of a perpendicular magnetic field 23, 24, which breaks the time-reversal symmetry (TRS). For neutral particles, there has been considerable interest recently in seeking to manipulate photons in a manner similar to the manipulation of electrons using a magnetic field 25-30. Due to the charge neutrality of photons, which are not affected by physical magnetic fields, an artificial magnetic field has to be synthesized for quantum platforms with bosonic excitations 31-33. An intriguing phenomenon of fractional quantum Hall (FQHE) physics has predicted in JC Hubbard system by applying an artificial magnetic field, exhibiting the Hofstadter-like fractal spectra 34-36.

In this letter, we study the quantum Rabi triangle (RT), as a fundamental unit cell for synthesizing a magnetic field to manipulate photons in optical cavities, to explore the possibility of phase transitions in few-body system. To study ground-state QPT, mean field approximations are generally performed in many-body systems due to intractable computations. However, this is remarkably not so as we will show in this work. Exact eigenstates and eigen-spectrum can be evaluated analytically to capture the phase diagram in the infinite frequency limit (analogy to thermodynamic limit). We find that the photon hopping with an artificial magnetic field can induce the directional circulation photons due to photon coherence in cavities, breaking the TRS. In such photon coherence regime, we observe a chiral-coherent phase with chiral photon current and the norm-coherent phase without chirality, which provides an intuitive understanding of the response of the gauge field in the ground state. The new chiral eigenstates and novel quantum tricriticality are found, leading to a chiral-symmetry breaking as well as $\mathbb{Z}_2$ parity-symmetry breaking quantum phase transition. The finite-frequency scaling exponents are analyzed and turned out to be in the universality class of the Dicke model.

Model.-- Three cavities are placed in a ring (Fig. 1a), where each cavity is coupled to its neighbours. The
Hamiltonian of the quantum triangle Rabi system is

\[ H_{\text{RT},n} = \sum_{n=1}^{3} H_{\text{R},n} + \sum_{n,n'} J(e^{i\theta} a_{n}^\dagger a_{n'} + e^{-i\theta} a_{n} a_{n'}^\dagger), \]

where each cavity interacting with a two-level atom is described by the quantum Rabi model \( H_{\text{R},n} = \omega a_{n}^\dagger a_{n} + g (a_{n}^\dagger + a_{n}) \sigma_n^+ + \frac{i}{2} \sigma_n^- \). \( a_{n} \) is the atomic annihilation (creation) operator in the \( n \)-th cavity with frequency \( \omega \). \( \sigma_n^\pm = \{ \sigma_n^x, \sigma_n^y, \sigma_n^z \} \) is the Pauli matrix which describes the two-level atom with the energy gap \( \Delta \), \( g \) and \( J \) are the strength of atom-photon coupling and photon hopping.

The static gauge field \((-\theta)\) for photon hopping (anti-)clockwise is given by an external gauge field \( A_{n,n'} \) as \( \theta = \int_{r,n'}^r A(r)dr \). The effective magnetic flux in the ring is \( \phi = 3\theta \), and is gauge-invariant. To engineer the gauge field, one practical proposal suggests that artificial magnetic field can be created by periodic modulation of the photon hopping strength between cavities. One can also modulate the photon hopping according to \( J \cos(\Omega_{ij} t + \theta) \), and choose \( \Omega_{ij} \) to be the difference between the frequencies of the cavities \([31]\). In the rotating frame, the complex hopping phase between two cavities is given as \( J e^{i\theta} a_{n}^\dagger a_{n'} + h.c. \) by neglecting fast oscillating (details are given in Supplemental Material [37]).

Each cavity possesses Z₂ symmetry, and the time-reversal symmetry (TRS) of the hopping processes among three cavities is artificially broken when the condition \( \theta = m\pi(m \in \mathbb{Z}) \) is not satisfied. For each cavity described by the quantum Rabi model, super-radiant phase transition occurs in the frequency limit \( \Delta/\omega \to \infty \), which is analogous to that in the Dicke model for infinite atoms \([38, 40]\). Considering the gauge field plays a critical role in searching for exotic quantum phase of matter, it may also strongly enrich the QPTs by involving TRS in the RT system. For convenience, we set the scaled coupling strength as \( g_{1} = g/\sqrt{\Delta\omega} \).

**Incoherent phase.** When the strength of photon-atom coupling \( g_{1} \) and photon hopping \( J \) are much smaller than the cavity frequency \( \omega \) and atomic energy gap \( \Delta \), the densities of atoms in excited state and photons excitation are strongly suppressed so that the system stays in the incoherent phase or normal phase. To obtain its energy spectrum, we firstly implement the Schrieffer-Wolff transformation \( S_n = \exp[-ig_{1}\sqrt{\omega/\Delta} \sigma_n^z (a_{n}^\dagger + a_{n})] \) on each cavity. After neglecting higher order terms in the limit \( \Delta/\omega \to \infty \), the Hamiltonian in Eq.\((1)\) becomes

\[ H_{\text{ICP}} = \sum_{n=1}^{3} \omega a_{n}^\dagger a_{n} + \frac{\Delta}{2} \sigma_n^z + \omega g_{1}^2 (a_{n} + a_{n}^\dagger)^2 \sigma_n^z + J(e^{i\theta} a_{n}^\dagger a_{n+1} + h.c.) + O(g_{1}^4 \omega^2 /\Delta^2). \]

Because the transverse operator \( \sigma_n^x \) is eliminated, two levels of each atom are decoupled. Thus, the low-energy Hamiltonian can be obtained by projecting to the subspace of atom \( |1\rangle_n \), i.e., \( H_{\text{ICP}}^{\parallel} = \langle 1| H_{\text{ICP}} |1\rangle \) which is exactly solvable due to its quadratic form of photon operators.

The RT system maintains the C₃ symmetry, so we introduce the discrete Fourier transform \( a_{n}^\dagger = \frac{1}{\sqrt{3}} \sum_{q} e^{imq} a_{q}^\dagger \) with the quasi-momentum \( q \) taking values 0 and ±2π/3. After subtracting the energy constant \( E_0 = -3\Delta/2 - 3\omega g_{1}^2 + 3(\omega + J)g_{1}^2 \omega/\Delta \), the Hamiltonian becomes \( H_{\text{ICP}}^{\parallel} = \sum_{q} \omega a_{q}^\dagger a_{q} + \omega g_{1}^2 (a_{q} + a_{q}^\dagger)^2 + \Im(q/\omega) \). \( \omega \) of \( \omega = -2\omega g_{1}^2 + 2J \cos(q/\theta) \) (see the Supplemental Material [37]). By implementing Bogoliubov transformation \( S_q = \exp(\lambda_q a_{q}^\dagger a_{-q}^\dagger - \lambda_q^* a_{q} a_{-q}) \) with the parameter \( \lambda_q = \frac{1}{\sqrt{3}} \ln \left( \frac{\omega_q + \omega_{q} + 4q \omega_q/\Delta}{\omega_q + \omega_{q} - 4q \omega_q/\Delta} \right) \), we can diagonalize the Hamiltonian as \( H_{\text{ICP}}^{\parallel} = \sum_{q} \varepsilon_q a_{q}^\dagger a_{q} + E_q \), where the ground-state energy is \( E_q = \sum_{q} (\varepsilon_q - q) /2 + E_0 \), and the energy spectra is

\[ \varepsilon_q = \frac{1}{2}\sqrt{(\omega_q + q)^2 - 16\omega^2 g_{1}^4 + \omega_q - q}. \]

Fig.\((2a)\) shows the ground-state energy and the excited-state energies obtained analytically, which agree well with numerical results by exact diagonalization (ED) for finite frequency ratio \( \Delta/\omega = 75 \). It indicates the iCP is a gapped phase and an energy-carrying exists in excited states.

**Quantum tricriticality.** When energy gap is closed, the quantum phase transitions happen. It is worth to notice that the spectra \( \varepsilon_q \) in Eq.\((3)\) shows the first-excited...
state taking different momentum \( q \) at different \( \theta \). Thus, the gap closing of the first-excited state may induce transition of states with different momentum. The vanishing of \( \varepsilon_q \) gives the critical scaled coupling strength

\[
g_{1c}(q) = \sqrt{\frac{1 + \frac{4J^2}{\omega} \cos \theta \cos q + \frac{4J^2}{\omega^2} \cos(\theta + q) \cos(\theta - q)}{4(1 + \frac{J^2}{\omega^2} \cos \theta \cos q)}}. \tag{4}
\]

As shown in the phase diagram, the critical lines of \( q = 0 \) and \( q = \pm 2\pi/3 \) discontinuously join together. By solve equation \( g_{1c}(0) = g_{1c}(\pm 2\pi/3) \), we obtain the joint point

\[
\theta_c = \pm \text{ArcCos} \left( -\frac{2J}{\sqrt{8J^2 + \omega^2}} \right), \tag{5}
\]

\[
g_{1c} = \frac{1}{2} \left\{ \frac{3}{2} - \sqrt{8J^2 + \omega^2} \right\} / \omega. \tag{6}
\]

When \( J/\omega \to 0 \), the discontinuous point tends to \( \theta_c = \pm \pi/2 \) and \( g_{1c} = 1/2 \).

From the spectra, we can conclude the closing of gap resulting in the QPT from the incoherent phase. When the coupling strength approaches to \( g_{1c}(q) \), the excited energy \( \varepsilon_q \) tends to be zero. It leads to the excitation of photons with different momentum \( q = 0, \pm 2\pi/3 \), and the second-order phase transition is expected. In the presence of the gauge field, the excited photons become to move unidirectionally among cavities. Consequence, the system enters into a coherent phase with strongly interacting photons among cavities. The discontinuous point \( \{ \theta_c, g_{1c} \} \) hints a first-order QPT from \( q = 0 \) coherent phase to \( q = \pm 2\pi/3 \) coherent phase. Thus, a novel quantum tricritical point may emerge.

Coherent phases.– As the coupling strength increases and exceeds the critical value \( g_{1c} \), the number of photons becomes proportional to \( \Delta/\omega \). The cavity field \( a_n^\dagger \) is expected to be shifted as \( a_n^\dagger \rightarrow a_n^\dagger + \alpha_n^* \) with the complex displacement \( \alpha_n = A_n + iB_n \), where \( A_n, B_n \in \mathbb{R} \). The RL Hamiltonian \( H_{RT} \) in Eq. (1) is given by

\[
H_{CP} = \sum_n \omega_n^* a_n^\dagger + \frac{\Delta_n^\prime}{2} \tau_n^z + g' (a_n^\dagger + a_n) \tau_n^z + J_a^\dagger(e^{i\theta} a_{n+1} + e^{-i\theta} a_{n-1}) + V_{off} + E_0, \tag{7}
\]

where \( \tau_n^z = \Delta/\Delta_n^\prime \sigma_n^z + 4gA_n/\Delta_n^\prime \sigma_n^x \) is the transformed Pauli matrix, the renormalized energy gap is \( \Delta_n^\prime = \sqrt{\Delta^2 + 16g^2 A_n^2} \), and the effective coupling strength is \( g' = g\Delta/\Delta_n^\prime \). The off-diagonal term \( V_{off} \) and the energy constant \( E_0 \) are given in the Supplemental Material [37]. Vanishing the imaginary and real parts of \( V_{off} \) leads to two equations

\[
A_n = -\frac{J \cos \theta + J^2 \sin^2 \theta}{\omega - J^2 \cos \theta} (A_{n+1} + A_{n-1}), \tag{8}
\]

\[
B_n = -\frac{J \sin \theta}{\omega - J \cos \theta} (A_{n+1} - A_{n-1}).
\]

Our solutions show that there are two different phases in the phase diagram versus the phase \( \theta \) in the region \( g > g_{1c} \) (see Fig. 1(a)): (i) normal-coherent phase (nCP). When the gauge phase enters into the regime \( |\theta| \leq |\theta_c| \), the excitation spectrum \( \varepsilon_q \) in Eq. (3) becomes to zero with the momentum \( q = 0 \). It indicates that the excited photons in the ground state take zero momentum. It is reasonable to set the displacement \( \alpha_n \) real and identical, giving \( B_n = 0 \). By solving Eq. (8), one obtains \( A_n = \sqrt{\frac{\omega + 2J^2 \cos \theta \theta^2}{16 \omega^2 g^4 (\Delta/\Delta_n^\prime)^4}} \) explicitly. The excited spectrum in the nCP is

\[
\varepsilon_q = \frac{1}{2} (\omega_q' - \omega_q^*) + \sqrt{(\omega_q^* + \omega_q^{'*})^2 - 16\omega^2 g^4 (\Delta/\Delta_n^\prime)^4}, \tag{9}
\]

where \( \omega_q^* = \omega - 2g^2/\Delta^\prime + 2J \cos(\theta - q) \).

(ii) chiral-coherent phase (cCP). When the momentum taken by photons is nonzero, \( q = \pm 2\pi/3 \), in the regime \( -\theta_c < \theta < \theta_c \), the displacement \( \alpha_n \) should be complex and different for each cavity. \( B_n \) and \( A_n \) can be obtained by numerically solving the Eq. (8). Then, by using generalized Bogoliubov transformation, the energy spectrum can also be calculated (see the Supplemental Material [37]).

Fig. (2b) shows the behavior of the ground-state energy \( E_q \) obtained by our solution dependent on the hopping phase \( \theta \) at a large coupling \( g > g_{1c} \), displaying the two representative cases for the nCP and cCP respectively. The analytical ones fit well with numerical results by using ED with the truncated photon number \( N_{tr} = 35 \). The discontinuity can be clearly observed at analytic critical point \( \theta_c/\pi = \pm 5.16 \), exhibiting the first-order phase transition. Thus, \( \{ \theta_c, g_{1c} \} \) is an exotic quantum tricritical point.

The first-order transition is characterized by the changing of the photon momentum from \( q = 0 \) in the nCP to \( \pm 2\pi/3 \) in the cCP. To give the critical line, we obtain the displacement in the cCP satisfying a critical condition \( A_1 = A_2 = a \) and \( A_3 = -a \) by solving Eq. (8). It requires \( J \cos \theta + J^2 \sin^2 \theta/(\omega - J \cos \theta) = 0 \), resulting in the solution \( \theta = \theta_c \) (details are given in Supplemental Material [37]). And the energy in the cCP along the line \( \theta = \theta_c \) is verified to be the same as that in the nCP. Thus, the exact first-order critical line between cCP and nCP locates at \( \theta = \theta_c \).

Chiral eigenstates.– Inspired by the momentum taken by photons, the chirality of the photon propagation in the closed loop is valuable, which provides an intuitive understanding how eigenstates responds to the gauge field. By using the exact solution of the Hamiltonian, the photon current in various eigenstates can be evaluated analytically. Analogous to the continuity equation in classical systems, the chiral current operator can be explicitly defined as \( I_{ph} = \frac{\partial \rho_{ph}}{\partial t} + \nabla \cdot \mathbf{j}_{ph} \).
excitation energy and level-crossing occurs in Fig. 2(a). It ascribes to the iCP phase in Fig. 3(c). In the regime such coherent phases, the photon current of the ground state \( I_\text{g} \) as a function of the phase \( \theta \) for \( g_1 = 0.1 \). (c) Photon current and (d) chirality operator in the coherent phase for the ground state \( I_{\text{ph},0} \) and \( C_{\text{ph},0} \) (black square) for \( g_1 = 0.7 \). The analytical results (red solid line) are listed. The parameters are \( \Delta/\omega = 50 \) and \( J/\omega = 0.05 \) with \( \omega = 0.2 \).

\[
i [a_1^d a_2 + a_2^d a_3 + a_3^d a_1]e^{i\theta} - h.c.\]

In the momentum space, one obtains \( I_{\text{ph}} = -\sum_q a_q^d a_q \sin(\theta - q) \), which predicts the smooth function of the photon current. On the other hand, in order to quantitatively analyze the chirality of the quantum states, we define the chiral operator of photon as \( C_{\text{ph}} = -2i \sum_{\langle i,j,k \rangle} \varepsilon_{ijk} a_i^d a_j^+ n_k \), which is similar to the chiral operator in the spin system

\[
C = \sum_{\langle i,j,k \rangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k).
\]

In the iCP for \( g_1 < g_{1c} \), there is no photon current \( I_{\text{ph},0} \) in the ground state, and the chirality \( C_{\text{ph},0} \) is also zero in Fig. 3(a–b). For the first-excited state, photons can be triggered to propagate in three cavities by the gauge field. Both of \( I_{\text{ph},1} \) and \( C_{\text{ph},1} \) exhibit discontinuous jumps at the hooping phase \( \theta = 0, \pm 2\pi/3 \), where the energy level-crossing occurs in Fig. 2(a). It ascribes to the excitation energy \( \varepsilon_\parallel \) with different momentum dependent on \( \theta \). The first-excited state with one-photon excitation can be approximately written as \( |\varphi\rangle_1, q = \frac{1}{\sqrt{3}} (e^{i\varphi}|100\rangle + e^{i2\varphi}|010\rangle + e^{i3\varphi}|001\rangle)(ggg) \). The corresponding expected value of photon chirality is \( C_{\text{ph},1} = \pm \sqrt{3} \) for \( q = \pm 2\pi/3 \), and \( C_{\text{ph},1} = 0 \) for \( q = 0 \), which is very close to the value in Fig. 3(b).

As \( g_1 \) increases to exceed the critical value \( g_{1c} \), the photons in each cavity are macroscopically excited. In such coherent phases, the photon current of the ground state is a hundred order of magnitude larger than that in the iCP phase Fig. 3(c). In the regime \( \theta_c \leq |\theta| \leq \pi \), the photon current in the nCP flows towards the opposite direction of the phase \( \theta \), and the chirality is zero due to zero momentum. Thus, the nCP has finite photon current without breaking the chiral symmetry. In contrast, the photon current in eCP follows the same direction as the phase \( \theta \), and the chirality is nonzero at \( 0 \leq |\theta| \leq \theta_c \) in Fig. 3(d), which indicates the chiral symmetry is broken. Especially, both current and chirality are zero at \( \theta = 0 \) or \( \pm \pi \), because the Hamiltonian is back to real and the TRS is reserved. Our analytical results agree well with numerical computations. It reveals how chiral ground state respond to the gauge field in the iCP and eCP phases.

**Universal scaling.** The universal scaling of the QPT can be characterized by the critical exponents for finite values of \( \eta = \Delta/\omega \). Fig. 4 illustrates the finite-\( \eta \) scaling of the ground-state energy and the average photon number obtained by numerical diagonalization in the critical regime. In the limit \( \eta \to \infty \), the scaled ground-state energy \( E_g/\eta \) obtained analytically at the critical point approaches to \( c_0 = -3\omega/2 \). To show the leading finite-\( \eta \) corrections, we calculate \( E_g/\eta - c_0 \) versus \( \eta \) for different \( \theta \), on a log-log scale in Fig. 4(a). The corresponding slope of the curves in the large-\( \eta \) regime gives a universal exponent \(-1\). Meanwhile, a power-law behavior of the photon number \( N_p = \sum_n (a_n^d a_n) \) exists at large \( \eta \) in Fig. 4(b). The corresponding finite-\( \eta \) exponents extracted from the curve converges to be \(-0.667 \) in the inset. We find that the scaling exponents for the ground-state energy and the average photons number are universal, giving two power law expressions as \( E_g/\eta - c_0 \propto \eta^{-1} \) and \( N_p/\eta \propto \eta^{-2/3} \). Consequently, the QPT between coherent phase and incoherent phase in RT system belongs to the same universality of the Dicke model 33 and the single-site Rabi model in the limit 17, 18.

**Conclusion.** We present an exact solution to the quantum Rabi triangle system as a basic building block for exploring emerging phases induced by a gauge field. The rich phase diagram with quantum tricriticality is ob-
tained explicitly, and a novel chiral-coherent phase induced by the gauge field is found, which predicts chiral photon currents in eigenstates. It exhibits an analytical excitation spectrum to explore the nature of the second-order and first-order quantum phase transition. Studying the quantum phases in this few-body system under the introduction of an artificial magnetic field would open intriguing avenues for exploring their connection to strongly correlated photons in two-dimension lattices system, such as Rabi lattice [11-13]. One could hope to realize new many-body phenomena in strongly interacting photons system, where the existence of rich phase diagram has been predicted. In future works it would be intriguing to engineer many-body phases in such fundamental building block of three-cavity interacting system by synthesizing magnetic fields. This would enable one to not only directly detect rich phases, but also modulate the chirality of photon current in eigenstates. An implementation of the system considered in this letter will provide crucial insight into the physics of quantum Hall effect in two-dimensional optical cavities.

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