I. INTRODUCTION

Superheavy elements are highly unstable systems with extremely low production cross sections. As the creation of new ones is very difficult, as a parallel or additional line of study one could try to search for new, long-lived metastable states of already known nuclei. It is well known that an enhanced stability may result from the K-isomerism phenomenon \cite{1,2} which is based mainly on the (partial) conservation of the K-quantum number \cite{11}. Low-lying high-K configurations occur when high-\Omega orbitals lie close to the Fermi energy. When such orbitals are intruders, the resulting unique configuration may have even longer half-life. Possible K values grow with larger-j subshells becoming occupied, that means for larger Z and N.

Currently known are many rotational bands build on 2 q.p. or 4 q.p. K-isomeric band-heads \cite{3-8}. The structure of expected long-lived multi-quasiparticle high-spin isomers in some even-even SH nuclei was analyzed e.g. in \cite{2}. In particular, the assignment of 9^- or 10^- two quasineutron configurations for the 6.0_{+8.2}^{−2.2}ms isomer in 270Ds (the heaviest isomer known) was proposed \cite{3}. Let us stress that the half-life of this isomer is much longer than that of the ground state (100_{+40}^{−10} μs). The same holds for the 8^+ isomer in 256Es, with the half-life of 7.6 h - significantly longer than 25 min of the g.s. Another interesting example is a 16^+ or 14^+ state in 254No, with a half-life of 184 μs, at 2.93 MeV above the g.s \cite{4,5,8}. Other, four-qp isomers in nuclei around 254No were postulated in \cite{11}. The current experimental knowledge on isomers in the heaviest nuclei can be found in \cite{14-16}, while theoretical overview based on the Nilsson - Strutinsky approach was given in \cite{17}. Let us emphasize that all K-isomeric states described above are related to typical prolate equilibria. A quite new possibility for high-K isomers in the superdeformed oblate minima in some SH nuclei was indicated in \cite{9}. We also would like to note that both measurements and predictions are performed mainly for even-even nuclei. Quite recently, we have also predicted high-K ground state configurations in odd and odd-odd systems \cite{10}. In this case, a particular situation occurs above double-closed subshells: N = 162 and Z = 108, where two intruder orbitals: neutron 13/2^- from j_{15/2} and proton 11/2^+ from i_{13/2} spherical subshells are predicted. These orbitals combine to the 12^- g.s. in Z = 109, N = 163.

Although the existence of isomeric states is rather well established, hindrance mechanisms responsible for slowing down their radioactive decay are still poorly understood. In particular, a hindrance of alpha decay, which is the main decay channel in superheavy nuclei with Z = 106 ÷ 118, is not well elucidated. For those nuclei, hindered \alpha transitions would lead to an increase in \alpha-lifetimes and extra stability. This is why, in this letter, we are going to estimate effects protecting high-K nuclear states against this decay mode. Our ultimate goal is to find candidates for long-lived nuclear configurations.

II. THE METHOD

Predictions for high-K multi-quasiparticle nuclear configurations require a model that satisfactorily describes well-known basic nuclear properties as: ground state masses, fission barriers, equilibrium deformations etc. Important is the existence of sufficiently distinct energetic shell gaps: two of them in the proton spectrum, at
around $Z=100$ and $Z=108$, and next two at $N=152$ and $N=162$ in the neutron spectrum. Without this, the spectroscopic studies (especially devoted to high-K-isomers) are just unfeasible in this region of nuclei. The only model available which satisfies both conditions simultaneously is the Microscopic-Macroscopic (MM) approach. Our model is based on the Yukawa-plus-exponential energy in the macroscopic part \[24\], while the single-particle spectrum is obtained from a deformed Woods-Saxon (WS) potential \[20\], diagonalized in the deformed harmonic oscillator basis. Within this model, with parameters adjusted to heavy nuclei in \[22\], it was possible to reproduce data on first \[24\], second \[24, 25\] and third \[26, 27\] fission barriers in actinides, systematically predict ground states \[25, 29\], $Q_a$ values \[30\] and saddle-points \[30\] in even- and odd-Z/N superheavy nuclei up to $Z = 126$ and gain some insight in a SH region beyond $Z = 126$ \[31\].

The blocking method induces a too large reduction in the pairing gap for multi-quasiparticle states. This causes an underestimate in their excitation energies. However, the effects we are going to discuss here are independent of this deficiency: 1) the considered configurations come out as the lowest ones in any method - see e.g. \[11\], so they are anyway the main candidates for isomers, 2) within the accuracy of the current models, even more precisely calculated excitation energies cannot prove/disprove an isomeric character of a many-quasiparticle state, 3) the crucial effect, the reduction of energy of $\alpha$-transitions between the same configurations in parent and daughter, is in a large part independent of the pairing reduction by blocking - the blocking effect in excitation energies in both parent and daughter mostly cancels. Therefore we decided to present the results obtained without any specially adjusted parameters.

The choice of a proper deformation space is important. Recently, an effect of the deformation $\beta_{90}$ on high-K isomer properties in superheavy nuclei has been discussed by Liu et. al. \[22\]. The authors noted a wider shell gap around $Z=100$ and $N=150$ after the inclusion of $\beta_{90}$, which gave much better agreement with existing experimental data. This conclusion is compatible with the previous one, given by Patyk and Sobiczewski in \[12, 13\]. In this work, the shape of a nucleus is parameterized via spherical harmonics $Y_{lm}(\theta, \phi)$ in a four-dimensional deformation space: $R(\theta, \phi) = c(\{\beta\}) R_0 \{1 + \beta_{20} Y_{20} + \beta_{30} Y_{30} + \beta_{40} Y_{40} + \beta_{50} Y_{50} + \beta_{60} Y_{60} + \beta_{70} Y_{70} + \beta_{80} Y_{80}\}$, where $c(\{\beta\})$ is the volume-fixing factor and $R_0$ is the radius of a spherical nucleus. One can see that the used parameterization still contains one more dimension compared to \[32\]. In the considered region, $106 \leq Z \leq 118$, nuclei are expected to be reflection-symmetric in the ground- and excited states, so we do not need odd multipole deformations here and the intrinsic parity of states is well-defined. Mother, as well as daughter nuclei, are also assumed axially-symmetric, what gives $K$ as a good quantum number (for a search of more exotic nuclear shapes in SH nuclei see \[29\]). Admittedly, this assumption cannot be exact for high-K states, in which the time-reversal breaking effects are expected to break axial symmetry to some degree. To obtain excitation energies, after blocking of a chosen configuration, a four-dimensional minimization over $\beta_{20} - \beta_{80}$ was performed.

![FIG. 1: Excitation energy of the two-proton configuration in the isotopic chains for Sg, Hs, Ds and Cn.](image)

### III. RESULTS AND DISCUSSION

We will consider here three characteristic multi-quasiparticle configurations, namely: i) two-neutron (2 q.p.): $\nu\pi = 10^- \nu: \{9/2^+[615], 11/2^-[725]\}$; ii) two-proton (2 q.p.): $\pi\pi = 10^- \pi: \{9/2^-[505], 11/2^+[615]\}$; iii) two-proton - two-neutron (4 q.p.): $\nu\pi = 20^+ \nu\pi: \{10^- \nu: \{9/2^+[615], 11/2^-[725]\}\} \otimes 10^- \pi: \{9/2^-[505], 11/2^+[615]\}$. The results obtained for another low-lying configuration, two-neutron (2 q.p.) $\nu\pi = 9^- \nu: \{7/2^+[613], 11/2^-[725]\}$, considered e.g. in \[11\], are similar to those for the above configuration i).

We begin our analysis with excitation energies in isotopic chains. Energies of two-proton quasiparticle states are shown in Fig. 1. As can be seen, those excited configurations are very low-lying for Ds and Cn nuclei what makes them promising candidates for (2 q.p.) isomers. One can also see that the lowest energies are obtained in Darmstadtium and that in both, Ds and Cn nuclei, excitation energies show very weak isotopic dependence.

From Fig. 2 one can choose the best candidates for two quasi-neutron isomers. Among all considered nuclei, the lowest energies occur for $N = 154 - 160$. Figures 1 & 2 allow a prediction of the most favorable four quasiparticle isomers. As can be seen in Fig. 3, 4 q.p. high-K isomeric states can appear most likely in $^{264-270}$Ds and $^{266-272}$Cn because of the smallest excitation energy. In particular, this energy amounts to 1.4 MeV, which means...
that it is the sum of excitations of the individual s.p. states, with nearly vanishing pairing gap. It is much less than 2.41 MeV given by Liu et. al., who used the more realistic Lipkin-Nogami procedure.

Stability of high-spin isomers against alpha decay is determined mainly by three factors: i) the overlap between final and initial states wherein a similar structure of states favors the transition between them; ii) a change in angular momentum - a significant change is associated with a large centrifugal barrier which blocks a decay; iii) transition energy, which we shall also call \( Q_\alpha \) for a given decay, that follows from the \( Q_\alpha \) value for the g.s. \( \to \) g.s. transition and the difference in the excitation energies of the initial and final state in, respectively, mother and daughter nucleus. We intend to analyze each of them.

For this purpose, we define a hindrance factor as a ratio of half-lives for two transitions: between the excited states with the same fixed structure and between the ground states: \( HF = [T_{1/2}] / [T_{1/2}^{gs-gs}] \).

Appropriate hindrance factors are collected in Table I for two-neutron and two-proton configurations: \( K^\pi = 10^{-\nu} : \{9/2^+[615], 11/2^-[725]\}; \ K^\pi = 10^{-\pi} : \{9/2^-[505], 11/2^+[615]\} \).

The influence of the centrifugal term can be estimated by evaluating the WKB integral. Since for a proton and neutron configuration a change in the angular momentum \( \Delta L = 10\hbar \) is the same, the centrifugal barrier \( V_L = L(L+1)/(2J_z) = 1.2 \text{ MeV} \) increases the half-life of both by about four orders of magnitude. Moment of inertia can be found in [33] of about \( J_z = 47\hbar^2/\text{MeV} \). An insignificant difference between them results from a slightly different turning points, calculated for slightly different \( Q_\alpha \).

Structural hindrance factors \( Log_{10}[HF]_S \) have been calculated by Delion et. al. in [34]. For our nonaligned states, the authors of [34] gave: \( Log_{10}[HF]_S = 4.74 \) and 4.08, for \( K^\pi = 10^{-\nu} \) and \( K^\pi = 10^{-\pi} \), respectively [22]. So for both proton and neutron excited states the size of the structural hindrance is again similar.

Hindrances corresponding to changes in \( Q_\alpha \) (\( \Delta Q_\alpha \equiv Q^g_{ex} - Q^gs_{gs} \)) are completely different though. While for \( K^\pi = 10^{-\nu} \) there is no hindrance (adequate hindrance factor is zero), for \( K^\pi = 10^{-\pi} \) quite a strong
hindrance is calculated $-\log_{10}[HF]_Q = 5.44$. This fact is connected with an opposite excitation scheme and follows from the deformed single-particle Woods-Saxon spectrum. In $^{270}$Ds, the neutron state: $11/2^+ - [725]$ lies $\approx 0.2$ MeV below the neutron Fermi level, while $9/2^- - [734]$ lies only $\approx 0.5$ MeV deeper. This gives the excitation energy of about 0.7 MeV, visible in Fig[4]. As mentioned in the introduction, this two-neutron configuration, $K^\pi = 10^- \nu\{[9/2^+ [615], 11/2^- [725]\}$, was assigned in [3] to the 1.13 MeV isomer in $^{270}$Ds. In protons, the $11/2^+$, dominantly $[615]$ state, lies at the Fermi level. Thus the excitation of the two-proton configuration equals roughly the excitation of the $9/2^-$[734] state, $\approx 0.5$ MeV below the Fermi level, see Fig. 2. So, the excitation energies of the proton and neutron configurations in the parent nucleus are similar. In the daughter, the energy of the two quasi-proton excitation is nearly 5 times larger ($\approx 2.5$ MeV) than that of the two quasi-neutron one. As a result, the same two-proton configuration lies at much higher energy in the daughter nucleus. This leads to a significant reduction in the energy of the alpha transition and an increase in the $\alpha$-half-life.

A very similar scheme of excitations occurs also in $^{268}$Ds and $^{266}$Ds. A slightly different situation one can observe in the alpha decay of $^{264}$Ds. The $K^\pi = 20^+ (19^+)$ state in $^{260}$Hs has a sizable excitation built to an equal degree by a two quasi-proton and two quasi-neutron components. This leads to a reduction of the transition energy and a greater stability of this state.

It should be emphasized that in our model there is no reason for a hindrance of the alpha decay between the same neutron $K^\pi = 10^- \nu$ configurations: there is no spin or structure change, and no change in $Q_\alpha$ with respect to the g.s. However, a transition from this state to the ground state is significantly hindered ($HF_{TOT} \approx 10^7$). The alpha decay from a two quasi-proton state has a hindrance factor $HF_{TOT} = 10^{5.44}$ for the configuration-preserving transitions ($ex \to ex$) and $HF_{TOT} \approx 10^7$ for the $ex \to g.s.$ transitions.

As follows from the above estimates, crucial is the hindrance in the fastest channel, between two identical configurations. This is especially true for four quasi-particle states, for which one expects a significant increase in the centrifugal barrier. With $\Delta L = 20\hbar$, the centrifugal barrier $V_L = 4.5$ MeV gives a huge $HF_L = 10^{12}$. A similar magnitude of the centrifugal barrier effect, $\approx 10^{13}$, was estimated by Karamian et. al. [35] for the $\alpha$ decay of the 4 q.p. $16^+$ isomer in $^{178}$Hf, the $^{178}HF^{16}$ state.

A structural hindrance for 4 q.p. isomers is also substantial. If one assumes that it is a product of the hindrance factors for protons and neutrons, one obtains $HF_L = 10^9$. Taken together, this leads to the conclusion that transitions $ex \to g.s.$ or $g.s. \to ex$ are excluded. Therefore, we concentrate in a further discussion (in Fig. [5] and [6]) on configuration-preserving transitions: $g.s. \to g.s., 10^-_\pi \to 10^-_\pi, 10^-_\nu \to 10^-_\nu$ and $20^+_\nu \to 20^+_\nu$. In such transitions, a hindrance follows from differences in alpha-decay energy $Q_\alpha$.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
$Q_\alpha$ & $\Delta Q_\alpha$ & $\log^{HF}\log^{TOT}$ & $\log^{ROY}\log^{TOT}$ & $\log^{PS}\log^{TOT}$ \\
\hline
$^{270}$Cn & 13.06 & 0.48 & -0.87 & -0.92 & -0.88 \\
$^{270}$Ds & 9.36 & -2.02 & 6.75 & 5.42 & 5.13 \\
\hline
\end{tabular}
\caption{$Q_\alpha$-values (in MeV) and hindrance factors corresponding to the change $\Delta Q_\alpha = Q_\alpha - Q_\alpha^{\text{dec}}$ for the $K^\pi = 20^+ \nu\pi : (10^- \nu : \{[9/2^+ [615], 11/2^- [725]\} \otimes 10^+ \pi : \{[9/2^- [505], 11/2^+ [615]\})$ configuration in $^{270}$Cn and $^{270}$Ds, calculated using: WKB method (WKB) [37], the formula of Royer [38] (ROY), and the Viola-Saxonberg-type formula by Parkhomenko and Sobiczewski (PS) [39].}
\end{table}

Let us examine the properties of 4 q.p. states in two isobaric nuclei $^{270}$Cn and $^{270}$Ds. Due to a difference in excitation energy in the mother and daughter, in $^{270}$Ds (the right part of Fig. 4) the $\alpha$-transition energy is much smaller than between the ground states. Thus, it seems that the $\alpha$-decay will be correspondingly slower. In the neighbouring $^{270}$Cn, an opposite energy relation will make the 4 q.p. $\to$ 4 q.p. transition about two orders of magnitude faster than the $gs \to gs$ one. In both cases, 2 q.p. proton excitations are responsible for this scenario, see Fig. 1. Hindrance factors for decays of $K^\pi = 20^+ \nu\pi$ states in $^{270}$Ds and $^{270}$Cn are shown in Table II. One can see that the results are actually independent of the method used to convert the energy difference to the alpha half-life. In a further analysis we use the recipee given in [38].

[Fig. 4: Decay scheme of $^{270}$Cn and $^{270}$Ds shown relative to the daughter g.s.]

[Table II: $Q_\alpha$-values (in MeV) and hindrance factors corresponding to the change $\Delta Q_\alpha = Q_\alpha - Q_\alpha^{\text{dec}}$ for the $K^\pi = 20^+ \nu\pi : (10^- \nu : \{[9/2^+ [615], 11/2^- [725]\} \otimes 10^+ \pi : \{[9/2^- [505], 11/2^+ [615]\})$ configuration in $^{270}$Cn and $^{270}$Ds, calculated using: WKB method (WKB) [37], the formula of Royer [38] (ROY), and the Viola-Saxonberg-type formula by Parkhomenko and Sobiczewski (PS) [39].]

In Fig. 5 we have shown calculated $Q_\alpha$ values for Durmasidium isotopes. In the $gs \to gs$ transition, a semi-magic gap predicted at N=162 is clearly visible. A very different behavior can be seen for $10^-_\pi \to 10^-_\pi$ and $10^-_\nu \to 10^-_\nu$ transitions. The first show an even deeper minimum in $Q_\alpha$ than the one for the g.s. and it would signal then an extra stable nuclear state; the second show an opposite situation. Unfortunately, the excitation energy of both types of states is quite high and rather does not suggest their isomeric character.
The Fig. 5 confirms the proton character of states with delayed alpha-decay, and the effect of the proton configuration on the total isometric half-life in transitions of the type $20^+_\pi ightarrow 20^+_{\pi}$, for $^{266-270}\text{Ds}$. A very interesting situation can be observed in $^{264}\text{Ds}$ where $Q_\alpha$ values for two-neutron and two-proton configurations are similar and both lie significantly below the energy for the $g.s. ightarrow g.s.$ transition. These transitions will be therefore much slower ($10^{-4}$) than that between the ground states. The alpha half-lives corresponding to the discussed transition energies are shown in Fig 6. One can see, for example, that according to our calculation, a two-neutron isomeric state in $^{270}\text{Ds}$ does not live longer than the ground state, as it was found experimentally for the isomer. Recently, Clark and Rudolph [40] obtained the proper half-life (3.9 ms) of the isomer in the frame of the superfluid tunnelling model (by assuming two-neutron q.p. character of the isomeric state, taking the experimentally measured $Q_\alpha$ value and arbitrarily weakening the pairing gap in the isomeric $10^-\pi$ state). One can see in Fig. 6 that in $^{264}\text{Ds}$, as well as in $^{270}\text{Ds}$, the g.s. to g.s. decay occurs with a half life of $0.1\mu$s, while for the 4 q.p. state, the predicted half life is of the order of seconds. This allows to think about chemical studies of isomeric states instead of ground states.

The final results – the hindrance factors for Ds isotopic chain - are shown in Fig. 7. One can see that, contrary to the suggestion from the experimental paper on $^{270}\text{Ds}$ [3], the decay of the two-neutron quasi-particle $(10^-\pi)$ state is not at all hindered, while the decay of the proton two quasi-particle state $(10^-\pi)$ is strongly forbidden: $\log_{10}[\text{HF}]|_Q = 5.42$. The most prominent hindrance of the alpha decay among the four quasi-particle $(K^\pi = 20^+)$ states - $10^-\pi$ - is predicted for $^{264}\text{Ds}$. However, due to the short g.s. half life, the total half life for this particular isomer will be practically on the same level as for $^{270}\text{Ds}$. Thus, in addition to two nuclei, $^{266}\text{Ds}$ and $^{268}\text{Ds}$, proposed by Liu [11] as candidates for long-lived configurations, even more pronounced effect of an enhanced stability against alpha decay from the $K^\pi = 20^+$ state is predicted in $^{264}\text{Ds}$ and $^{270}\text{Ds}$. Our calculations also indicate that the decay of the 4q.p. state will not be hindered at all in $^{268}\text{Cn}$ and $^{270}\text{Cn}$, contrary to what was suggested in the same paper [11]. Admittedly, for these nuclei, the energies of these high-K states are sufficiently low to make them candidates for high-K isomers, but as follows from the discussion of excitations in $^{270}\text{Cn}$ (Fig. 4), we do not expect any hindrance here. On the contrary, a decay from the isomeric states should be faster.
than from the ground states. This happens in all Cn nuclei in which the excitation energy of the 2 or 4 q.p. state is low, as shown in Fig. [8]. From this figure one can see the only one exception, namely $^{266}Cn$, in which we expect some hindrance of the alpha decay from the isomer built on the neutron excitation. However, the predicted hindrance is not very large ($\approx 10^3$).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure8}
\caption{Hindrance factors in alpha decay of Cn.}
\end{figure}

IV. CONCLUSIONS

To summarize our investigations concerning the nature and behaviour of multi-quasiparticle states in the super-heavy nuclei in the context of their alpha-decay process:

i) We have found a quite strong hindrance against alpha decay for four quasi-particle states: $K^\pi = 20^+$ and/or $19^+$. This, together with their relatively low excitation suggests a possibility that they could be isomers with an extra stability - five and more orders of magnitude longer-lived than the ground states. This would mean that chemical studies of such exotic high-K states would be more likely than for quite unstable ground states. Among all tested nuclei, the best candidates for long-lived high-K isomers are predicted in $^{264-270}Ds$.

ii) Except a moderate (about 3 orders of magnitude) alpha-decay hindrance in $^{266}Cn$ for a 2 q.p. neutron state, there are no more candidates for an enhanced stability against alpha decay in Cn nuclei.

iii) Contrary to what has been recognized so far, our analysis indicates that the alpha-decay hindrance results mainly from the proton 2q.p. component.

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[41] K (Ω for a s.p. state) is a total angular momentum along the symmetry axis and is a "good" quantum number in the case of axial symmetry.
[42] These authors’ definition of the HF is, from a formal point of view, different from ours: "Hindrance factor is a measure of whether the parent nucleus would prefer to decay from the exited state instead of the ground state, or perhaps more realistically to which extent the parent nucleus would prefer to decay from the g.s."