Classical Radiation Damping and Emission of Optical Vertex as Microscopic Irreversible Process

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Abstract. Radiation damping process by a charged particle in classical dynamics is discussed in terms of a complex spectral analysis of the Liouville operator in an extended function space outside the Hilbert space. The complex spectral analysis has been introduced to describe irreversible process with a broken time-symmetry on a rigorous dynamical basis starting from the fundamental laws of physics. Thanks to this analysis, we show that the long-standing difficulty of that leads to non-dynamical behaviors such as non-causal behavior and the existence of a runaway solution of the charged particle has been resolve. The propagation of the optical vertex inside a waveguide is also discussed.

Keywords: Radiation damping, Lorentz-Abraham Eq., Broken time-symmetry, Complex spectral analysis

1. Introduction

Recent experimental discovery of optical vertex in a cyclotron radiation of a charged particle has opened new exciting subjects in a wide area of science and technology, including cosmological physics, plasma physics, and technology of accelerator light source [1,2]. The cyclotron radiation is essentially a classical phenomenon described by classical mechanics and classical Maxwell equation. However, theoretical understanding of this damping process has been incomplete, since the well-known Lorentz-Abraham equation that has been introduced for describing this damping process involves a third derivative term with respect to time that is conflict with fundamental laws of physics [3–7]. Specifically, one can enumerate unphysical runaway solution of the Lorentz-Abraham equation for the charged particle, and non-causal initial condition that has been proposed by Dirac to eliminate the runaway solution.
The difficulty to describe this process is attributed to the conflict between the facts that radiation damping is an irreversible process, while fundamental equations of motion in physics are time-symmetric. Without resolving this fundamental conflict, one cannot hope to resolve the difficulty in the radiation damping. The author of this paper has involved the problem of irreversibility for many years with Ilya Prigogine. Then we clarified the essence of the problem stems from the following four reason [8–12]:

1) The solution of the equation of motion for some dynamical system may involve the resonance singularity where the frequency denominator appears in the solution becomes zero. This is the so-called small denominator problem in dynamical system.

2) Nevertheless, the division by zero can be understand as the delta function (i.e., a distribution) without any contradiction to mathematics, by performing the analytic continuation of the frequency denominator to the complex plain for the case of the continuous spectrum of the frequency. Note that appearance of the delta function that is an even function with respect the sign of its argument breaks parity-symmetry, since the frequency denominator is an odd function with respect the sign of the frequency.)

3) One can extend the frame work of the Hilbert space for the state functions in dynamical system to a non-Hilbert space, by extending the frequency into the complex plain.

4) As a result, one can obtain the solution that breaks time-symmetry without contradiction of the basic laws in physics.

As an application of this approach of the irreversible process based on the microscopic laws of physics, we have already found many interesting phenomena, mainly for quantum unstable systems where excited atoms decay due to effect of the resonance singularity that comes from a coupling with a photon field with a continuous spectrum [13–18].

In this paper, we will show that the long-standing problem of the classical radiation damping can be solve without any contradiction of the basic laws of classical dynamics, by replacing the quantum canonical commutation relation to the classical Poisson bracket. So to say, one can solve this long-standing problem by a classicalization of the quantum mechanics instead of the quantization of the classical mechanics [19].

This paper has been prepared to present the recent development for the complex spectral representation for the reader who are interested in the description of the emission process of optical vertex in terms of classical dynamics. Since the derivation of a solution that breaks time-symmetry from the time-symmetric fundamental equation of motion in physics itself is highly nontrivial problem, we will first focus our attention to this problem. Then, we will discuss a propagation of the angular momentum of the optical vertex inside a waveguide.

2. Problem of Lorentz-Abraham Equation

Lorentz-Abraham Equation is given by

\[ m \frac{d^2 x}{dt^2} = F_{\text{ext}}(t) + m \tau \frac{d^3 x}{dt^3}, \]  

(1)
2 \tau e^2 = \left(3 mc^3 \right) \text{ is a passing time scale of light across the classical radius of an electron. This equation has the following solution for the acceleration } a(t) \equiv d^2x/dt^2,

\[ a(t) = \left[ a(0) - \frac{1}{m \tau} \int_0^t dt' F_{\text{ext}}(t') e^{-t'/\tau} \right] e^{t/\tau}. \] 

Hence this equation has an unphysical runaway solution for \( a(0) \neq 0 \), even the case with \( F_{\text{ext}}(t) = 0 \). To solve this difficulty, Dirac has proposed an initial condition for the acceleration with

\[ a(0) = \frac{1}{m \tau} \int_0^\infty dt' F_{\text{ext}}(t') e^{-t'/\tau}. \] 

However, this initial condition violates the causality, since the initial condition is determined by the future of the external force.

### 3. Classical Friedrichs Model

In order to solve this difficulty without contradiction with the fundamental equation of motion in physics, let us consider the following Hamiltonian [19],

\[ H = \omega_1 q_1^{cc} q_1 + \int_{-\infty}^{+\infty} dk \omega_k q_k^{cc} q_k + \lambda \int_{-\infty}^{+\infty} dk V_k (q_1^{cc} q_k + q_k^{cc} q_1), \]

where \( cc \) means complex conjugate, \( q_1 \) is a normal mode of a harmonic oscillator with a characteristic frequency \( \omega_1 \), and \( q_k \) is a normal mode of the light with a wave number \( k \). Moreover, \( V_k \) is a form factor that characterizes the interaction between the harmonic oscillator 1 and the light. We assume that \( V_k \) is a real function of \( k \). The form of \( V_k \) is determined by the Lorentz equation of a charged particle and by the Maxwell equation. For example, this Hamiltonian is obtained for a charged particle with a cyclotron motion with a frequency \( \omega_1 \). We can obtained the propagation of the electro-magnetic wave emitted from the particle through the time evolution \( q_k \). If the cyclotron motion is achieved inside an infinitely long waveguide, the wave vector \( k \) is one-dimensional continuous variable parallel to the axial direction of the waveguide. For this case, the dispersion relation a normal mode of light with an discrete index \( \nu = (m, n) \) that is parallel to the cross section of the waveguide is given by \( \omega_{\nu} = \sqrt{c^2 k^2 + \omega^2_{\nu}}, \) where we consider a single mode of the light specified by \( \nu \) in the cross section [4]. The frequency \( \omega_{\nu} \) is a cut-off frequency of the mode of light. Hence, the light behaves as if it has a mass corresponding to the cut-off frequency. If the light is not confined by any boundary, we have \( \omega_\nu = 0 \).

In order to obtain this form of Hamiltonian, we have performed the dipole approximation and the rotating approximation. It is not important to a detail structure of \( V_k \) for discussing the essential mechanism of the classical radiation damping. The interaction part of the Hamiltonian without the rotating approximation has the form with \((q_1^{cc} + q_1)(q_k^{cc} + q_k)\) in the last term of (4), which contains the so-called the virtual process. The virtual process is necessary to treat the causality based on the relativity. However, since the Hamiltonian is bilinear in the normal modes in both cases with and without the virtual process, one can exactly diagonalize the Hamiltonian by the Bogolubov transformation. Moreover, since the
essential element to obtain a damping solution that brakes time symmetry is the resonance singularity in both case of the Hamiltonians, we will focus our argument to the simpler Hamiltonian (4) without the virtual process to avoid technical complexity in mathematical manipulations.

The Poisson brackets of the classical normal modes are given by

\[
\{f, g\}_{\text{PB}} = -i \sum_{\alpha} \left( \frac{\partial f}{\partial q_{\alpha}} \frac{\partial g}{\partial q_{\alpha}^c} - \frac{\partial f}{\partial q_{\alpha}^c} \frac{\partial g}{\partial q_{\alpha}} \right),
\]

where and hereafter we will use a conventional discrete notation for the case of continuous valuable \(\alpha\), such as the well-known box normalization. We should take the continuous limit for a suitable situation in the manipulation to evaluate the case with continuous spectrum. The normal modes satisfy the canonical relations,

\[
\{q_1, q_{1}^c\}_{\text{PB}} = -i, \\
\{q_k, q_{k'}^c\}_{\text{PB}} = -i\delta(k - k'), \\
\{q_{\alpha}, q_{\beta}\}_{\text{PB}} = 0,
\]

and so on, where \(\delta(k)\) is the Dirac delta function for the continuous variable \(k\). If we replace them as \(q_{\alpha}^c \rightarrow \sqrt{\hbar}a_{\alpha}^c\) and with the commutation relation as \(\{a, b\}_{\text{PB}} = -i\hbar[a, b]\), then we obtain a second quantized Hamiltonian. This quantized Hamiltonian is called as the Hamiltonian for the Friedrichs model. For this quantum Hamiltonian, the damping process of an harmonic oscillator by emitting a photon has been obtained as an exactly solution of the Schrödinger equation [20]. This solution is important as the Bohr’s phenomenological hypothesis on the quantum jump was first time obtained as an exact solution of the Schrödinger equation. (However, the original prove by Friedrichs has been achieved for a Hamiltonian system without the second quantization.)

As you will see in this paper, Friedrichs’ solution contains nontrivial solution with the resonance singularity. Thanks to this singularity, one can obtain a solution that breaks time-symmetry from the time-symmetric equation without any mathematical contradiction. Our main idea to solve the difficulty in the classical radiation damping is that we focus to the fact that the algebra of the Poisson bracket is isomorphic to the algebra of the commutation relation. Using this fact, one can obtain the exact solution of the classical damping for the Poisson bracket, so to say, by a “classicalization” of the quantum mechanics, instead of the quantization of the classical mechanics.

Before we consider the irreversible case with a radiation damping, we first consider the case where the system is reversible since there is no resonance singularity because \(\omega_1\) is smaller enough than \(\omega_0\) that the frequency denominator \(\omega_1 - \omega_k\) cannot have a value which is close to zero. For this case, one can diagonalize the Hamiltonian (4) by the following classical Bogolubov transformation and its inverse transformation,

\[
Q_1 = N_1^{1/2} \left[ q_1 + \int_{-\infty}^{+\infty} dk \frac{\lambda V_k q_k}{\Omega_1 - \omega_k} \right] ,
\]

\[
Q_k = q_k + \frac{\lambda V_k}{\eta^* (\omega_k)} \left[ q_1 + \int_{-\infty}^{+\infty} dk' \frac{\lambda V_{k'} q_{k'}}{\omega_k - \omega_{k'} - i\epsilon} \right].
\]
Then we have
\[ H = \Omega_1 Q_1^e Q_1 + \int_{-\infty}^{+\infty} dk \omega_k Q_k^e Q_k, \tag{9} \]

where \( N_1 \) is a normalization constant, \( \epsilon > 0 \) is a positive infinitesimal, and
\[ \eta(z) \equiv z - \omega_1 - \int_{-\infty}^{+\infty} dk \frac{\lambda^2 V_k^2}{z - \omega_k}. \tag{10} \]

The function \( \eta(z) \) is a two-valued function in a complex plane of \( z \) with cut starting from a blanch point for a physically natural form factor \( V_k \). We denote \( \eta^-(z) \) (and \( \eta^+(z) \)) for an analytically continued function from the lower-half plane to the upper-half plane (and from the upper-half plane to the lower-half plane). If we chose the blanch with \( \eta^-(z) \) as shown in (7), the mathematical manipulation is simpler for \( t > 0 \) than the choice of \( \eta^+(z) \). The new normal mode \( Q_1 \) are renormalized dressed modes. The value \( \Omega_1 \) is determined by the dispersion equation
\[ \Omega_1 = 0. \]

For the case without the resonance singularity, which we consider now, \( \Omega_1 \) is a real number.

The Liouvillian \( L_H \) that is a generating operator for the classical evolution of the motion is given by \( L_H f \equiv -i[H, f]_{PB} \) for the Hamiltonian \( H \). For the diagonalized representation with (9), it is given by
\[ L_H = \sum_{a} \tilde{\omega}_a \left( Q_a^e \frac{\partial}{\partial Q_a^e} - Q_a \frac{\partial}{\partial Q_a} \right), \tag{11} \]

where \( \tilde{\omega}_1 \equiv \Omega_1 \), and \( \tilde{\omega}_k \equiv \omega_k \). The Liouvillian is a Hermitian operator in the usual Hilbert space. Using the form (9) for the Hamiltonian, the equation of motion for the renormalized mode \( Q_1 \) for the charged particle is obtained as
\[ i\frac{dQ_1}{dt} = -L_H Q_1 = \Omega_1 Q_1. \tag{12} \]

This gives us \( Q_1(t) = \exp[-i\Omega_1 t]Q_1(0) \). Moreover, this relation shows that \( -Q_1 \) is an eigenfunction of the Liouvillian, and its eigenvalue is \( \Omega_1 \).

Let us now consider our main theme that is the case where the classical radiation damping occurs because of the situation \( \omega_1 > \omega_\mu \). For this case, the solution of the dispersion equation \( \eta^+(z) = 0 \) becomes a complex number with \( z = z_1 \equiv \Omega_1 - iy \) with \( \gamma > 0 \) for a suitable form factor \( V_k \) due to the resonance singularity. Friedrichs showed that there was no transformation corresponding to \( Q_1 \) in the above Bogolubov transformation (7). In addition, he showed that the complete set of the normal modes consisted of only \( Q_k \) in (8). This argument holds for our classical case by replacing the commutation relation for the canonical variables by the Poisson bracket. Hence, we have the new Hamiltonian for the resonance case as
\[ H = \int_{-\infty}^{+\infty} dk \omega_k Q_k^e Q_k, \tag{13} \]

instead of (9). This is the solution that we call the “Friedrichs’ solution” for the classical radiation damping.
For this case, one can obtain the exact solution for the mode with the continuous variable \( k \) as \( Q_k(t) = Q_k(0) \exp[-i\omega_k t] \). Then, one can obtain the time evolution of the original normal mode \( q_1(t) \) by substituting \( Q_k(t) \) in the inverse transformation of the classical Bogolubov transformation (7) with \( Q_1 = 0 \). In this calculation, we see that the contour deformation of the integration over \( k \) in the complex plane has a pole contribution from the factor \( 1/\eta(\omega_k) \) in (7) at \( \omega_k = z_1 = \Omega_1 - i\gamma \), that leads to damping oscillation factor \( \exp[-iz_1 t] \) in the time evolution for \( q_1(t) \). In this exact solution there is no unphysical runaway solution as in the Lorentz-Abraham equation. As a result, one obtains the damping solution as far as for the original normal mode \( q_1(t) \) without any contradiction to the fundamental principle of classical dynamics.

4. Complex Normal Modes and Broken-time Symmetry

In Friedrichs solution, there is no renormalized mode associated to the unstable charge particle when we have a resonance singularity. This fact leads to a new question that if there is a decaying renormalized mode in such a way that the Louvillian with the resonance singularity may have an eigenstate that has complex eigenvalue \( z_1 \). We have obtained the positive answer for this question. However, it is necessary to extend the function space that describes the time evolution of the system outside the Hilbert space, in order to have a complex eigenvalue for the Hermitian operator. In the following part of the paper, we will present a brief introduction of the complex normal modes. The reader should see the reference [19] for more detail.

A dual set of the renormalized normal modes for unstable particle is given by

\[
Q_k^{cc} = q_k^{cc} + \int_{-\infty}^{+\infty} dk' \frac{\lambda V_k q_k^{cc}}{(z_1 - \omega_k')^+},
\]

\[
\tilde{Q}_k = q_k + \int_{-\infty}^{+\infty} dk' \frac{\lambda V_k q_k'}{(z_1 - \omega_k')^+},
\]

where \( + \) on the parentheses of the frequency denominator denote that function that is analytically continued from the upper-half plane to the lower-half plane as in the case of \( \eta(z) \).

Note that \( \tilde{Q}_1 \neq Q_1 \). For this case, the renormalization constant \( N_1 \) is essentially complex number apart from the phase factor, which is the essential different from the eigenstates in the Hilbert space.

For dual sets of the new renormalized normal modes for the light for the unstable case are given by

\[
Q_k^{cc} = q_k^{cc} + \frac{\lambda V_k}{\eta^+(\omega_k)} \left[ q_1^{cc} + \int_{-\infty}^{+\infty} dk' \frac{\lambda V_k q_k'}{(\omega_k - \omega_k')^+ + i\epsilon} \right],
\]

\[
\tilde{Q}_k = q_k + \frac{\lambda V_k}{\eta^+(\omega_k)} \left[ q_1 + \int_{-\infty}^{+\infty} dk' \frac{\lambda V_k q_k'}{(\omega_k - \omega_k')^+ - i\epsilon} \right],
\]

where

\[
\frac{1}{\eta^+(\omega_k)} \equiv \frac{1}{\eta^+(\omega_k) (z_1 - \omega_k)^+},
\]
that is defined in such a way that the effect of the pole on the lower-half plane of \( \omega_k \) in
\( 1/\pi^+(\omega_k) \) is offset. This analytic continuation is the “delayed analytic continuation” [8].
These renormalized modes satisfy
\[
\{ \tilde{Q}_1, Q_1^c \}_{PB} = -i,
\{ \tilde{Q}_k, Q_k^c \}_{PB} = -i\delta(k - k'),
\{ \tilde{Q}_a, \tilde{Q}_b \}_{PB} = 0,
\]
and so on, corresponding to the relation (6).
These new dual modes \( \tilde{Q}_1 \) and \( Q_1^c \) are obtained through the residue at the pole \( \omega_k = z_1 \) in
the complex plane in the contour deformation of the integration in the Hamiltonian (13).
The rest of the contour integral defines \( \tilde{Q}_k \) and \( Q_k^c \). Using these new complex normal modes,
the Hamiltonian (13) is now diagonalized in a dual form as by
\[
H = z_1 Q_1^c \tilde{Q}_1 + \int_{-\infty}^{+\infty} dk \omega_k Q_k^c \tilde{Q}_k.
\]
The Liouvillan is now written by
\[
L_H = \sum_{\alpha} z_{\alpha} \left( Q_{\alpha}^c \frac{\partial}{\partial Q^c_{\alpha}} - Q_{\alpha} \frac{\partial}{\partial Q_{\alpha}} \right),
\]
where \( z_k \equiv \omega_k \). Then we have
\[
i \frac{d\tilde{Q}_1}{dt} = -L_H \tilde{Q}_1 = z_1 \tilde{Q}_1.
\]
This leads to the exact solution \( \tilde{Q}_1(t) = \exp[-iz_1t] \tilde{Q}_1(0) \). This relation also shows that \( \tilde{Q}_1 \) is
an eigenstate of the Liouvillian with the complex eigenvalue \( -z_1 \).

5. Propagation of Optical Vertex inside a Waveguide

Let us now focus our attention to the emitted light. As an illustration of the optical vertex,
we show the case of TE mode propagating inside the waveguide with the rectangular cross
section with the length \( a \) in \( x \) direction with \(-a/2 \leq x \leq +a/2 \), and \( b \) in \( y \) direction with
\(-b/2 \leq y \leq +b/2 \). We assume that the waveguide is infinitely long in \( z \) direction. For the
TE mode, we have \( E_z(r, t) = 0 \) for the \( z \)-component of the electric field, and \( r = (x, y, z) \).
All other components of the electric field \( E \) and the magnetic field \( H \) are functionals of the
\( z \)-component the magnetic field \( H_z \). The solution of \( H_z \) for the Maxwell equation gives us
\[
H_z(r, t) = \frac{1}{\sqrt{2\pi ab}} \sum_{m' = 0}^{\infty} \sum_{n' = 0}^{\infty} \int_{-\infty}^{\infty} dk' \hat{h}_z(k') \cos\left( \frac{m'\pi x}{a} \right) \cos\left( \frac{n'\pi y}{b} \right) e^{ik'z} e^{-i\Omega_k t},
\]
where \( k = (m\pi/a, n\pi/b, k_z) \), \( \Omega_k \equiv ck \) with \( k = |k| \), and \( h_z(k) \) is determined from \( H_z(r, 0) \).
The optical vertex is characterized by the angular momentum carrying with the electromagnetic wave. The angular momentum density \( J(r, t) \) of the electro-magnetic field is given by

\[
J(r, t) = r \times S(r, t),
\]

(24)

where \( S(r, t) \) is the momentum density of the electro-magnetic field, called as the Poynting vector. This is given by

\[
S(r, t) = \frac{1}{c^2} E(r, t) \times H(r, t).
\]

(25)

Each mode of the magnetic field \( H_z(r, t) \) in (23) is given by putting

\[ h_z(k'_z) = h_0 \delta_{m',m} \delta_{n',n} (k'_z - k_z) \]

with an amplitude \( h_0 \) associated to the mode \((m, n, k_z)\). Then, one can obtain the each component of the angular momentum density \( J(r, t) \) from (23) after the straight forward but tedious calculation. Since expression is long, we present here only the \( z \)-component of \( J(r, t) \) coming from the single mode:

\[
J_z(r, t) = xS_y(r, t) - yS_x(r, t) = \frac{\mu_0 h_0^2}{2c^2 \sqrt{2\pi ab}} \frac{\Omega k}{k - k_c} \times \left[ x \cos^2\left(\frac{m\pi x}{a}\right) \sin\left(\frac{2\pi y}{b}\right) - \sin\left(\frac{2m\pi x}{a}\right) y \cos^2\left(\frac{n\pi y}{b}\right) \right] e^{2ik_z(z - \Omega t)},
\]

(26)

where \( \mu_0 \) is the magnetic permeability. Similarly, one can obtain non-vanishing contributions for \( J_x(r, t) \) and \( J_y(r, t) \).

Because \( J_z(r, t) \) is an odd function of \( x \) and \( y \), respectively, the \( z \)-component of the total angular momentum of the electro-magnetic field (i.e., the integration over the cross section of the waveguide) vanishes. However, the waveguide can carry the angular momentum in a subdomain of the cross section. Hence, one can detect the optical vertex propagating inside the waveguide. The result (26) show also that smaller the values \( m \) and \( n \), larger the total angular momentum in the subdomain, because the larger mode of \( m \) and \( n \) has a rapid oscillation in \( x \) and \( y \) that gives a smaller contribution in integration over \( x \) and \( y \) in the subdomain.

6. Concluding Remarks

We have shown that one can construct the complex mode that are eigenstates of the Liouvillian with the complex eigenvalue for the unstable case with the resonance singularity in the dual space in spite of the Liouvillian being an Hermitian operator in the Hilbert space. The dual space is an extension of the Hilbert space. As a result, we have shown that one can construct the renormalized normal mode associated to the unstable charged particle that brakes time-symmetry without contradiction to the fundamental laws of classical dynamics.

We have also shown the explicit form of the angular momentum of the electro-magnetic field propagating inside the waveguide with the rectangular cross section. The result shows
that the propagation of the angular momentum of the optical vertex can observe in a subdomain of the cross section, in spite of the fact that its total angular momentum vanishes in the entire domain of the cross section. Moreover, we have shown that lower the mode of $m$ and $n$, larger the angular momentum in the subdomain. In addition to the rectangular waveguide we have already obtain a similar result for the cylindrical waveguide. We are planing to present the result of the cylindrical waveguide, elsewhere.

Using this new complex normal modes, one can describe the emission process of the light from the unstable charged particle in detail based on the fundamental laws of physics. Hence, this description gives us a theoretical foundation to study a detailed property of the optical vertex emitted from the cyclotron motion of a charged particle.

As shown in this paper, emitting process of light in terms of the normal modes in classical mechanics has a lot of common features with quantum mechanics because of the isomorphism in algebra between the Poisson bracket and the commutation relation. However, there is of course an essential difference in classical mechanics from quantum mechanics, since there is no vacuum fluctuation in classical mechanics. For example, the cyclotron motion will eventually completely stop after a total emission of light, which is not the case in quantum mechanics.

As an example of the application of the complex normal modes, we have obtained some preliminary result of the emitted light for the case where the cyclotron motion inside a rectangular waveguide has a characteristic frequency near the cut-off frequency. For this case we have found that the decay rate of the cyclotron motion is much bigger than the one obtained by the well-known Fermi golden rule. For example the decay rate is $10^4$ times bigger than the one bigger for the electron, and $10^7$ times bigger than the one bigger than the one for the proton, respectively, obtained by the well-known Fermi golden rule for the proton. This is due to the so-called Van Hove singularity in the density of states of the light [17, 18]. This phenomena cannot be predicted by the existing Lorentz-Abraham equation. We are expecting that the effect of the classical Van Hove singularity to the optical vertex near the cut-off frequency has a unique property different from the case the singularity is negligible. We hope to present the result, elsewhere.

It is known that we need a higher harmonic emission of light to have the optical vertex [2]. A good candidate to have the higher harmonics is the system with a time periodic external force that is treated in terms of the Froque Hamiltonian [13].

From the aspect of irreversible process due to the resonance singularity, we are only just beginning to understand the emission of the optical vertex based on the fundamental laws of classical dynamics. We believe that this problem is an interesting problem that will lead to many new discoveries in many different fields in physics.

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