The Double Quantum Dot Feline Cousin of Schrödinger’s Cat: An Experimental Testbed for a Discourse on Quantum Measurement Dichotomies

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Abstract

Quantum measurement theory is a perplexing discipline fraught with paradoxes and dichotomies. Here we discuss a gedanken experiment that uses a popular testbed - namely, a coupled double quantum dot system - to revisit intriguing questions about the collapse of wavefunctions, irreversibility, objective reality and the actualization of a measurement outcome.

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Quantum measurement theory is a sub-discipline replete with many subtleties of quantum mechanics. Its basic underpinning can be summarized by a fundamental and yet profound question: when and how does a pure state, descriptive of a quantum system entangled with a measuring apparatus (also a quantum system), evolve into a mixed state that results in distinguishable outcomes of the measurement. Since in standard quantum mechanics, no unitary time evolution can cause a pure state to evolve into a mixed state there is essentially no cookbook “quantum recipe” to forge distinguishable outcomes in quantum measurement.

A number of formalisms that augment the standard mathematical framework of quantum mechanics provide a dynamical description of the measurement process in terms of an actual transition of a pure state into a mixed state. This has been termed “collapse of a wave function”. However, even if we accept the augmented mathematical framework, some mysteries still remain. How does the collapse occur? Is it a discrete event in time or is it a continuous process? Is the collapse observer-dependent (i.e. it happens only when an observer decides to look at the outcome of a quantum measurement) or does the outcome materialize at some time independent of the observer? In this short communication, we re-visit these issues in the context of a popular quantum system that illustrates many of the subtleties in quantum measurement theory.

Consider a double quantum dot system coupled by a translucent tunnel barrier. The conduction band diagram is shown in Fig. 1(a). The two quantum dot materials are identical in all respects except in their elastic constants. That is, electrons cannot distinguish between them, but phonons can. An electron is introduced into the ground state of the system and exists in a coherent superposition of two states $|1\rangle$ and $|2\rangle$

$$\psi = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)$$

where $|1\rangle$ is a semi-localized wave function in the left dot and $|2\rangle$ is a semi-localized wave function in the right dot. A weakly coupled point detector in the vicinity of one of the dots can tell whether that dot is occupied by the electron or the other one is. This experimentally realizable system has been studied in the context of the quantum measurement problem by a number of authors recently.

We now summarize three different viewpoints regarding the quantum measurement problem.

The orthodox viewpoint associated with the Copenhagen interpretation is epitomized by Von-Neumann: the wave function collapses when an observer chooses to look at the detector and gain knowledge about where the electron is. This is an observer-dependent reality and has been much discussed in the context of the Schrödinger cat paradox. A different viewpoint espoused by a
Figure 1: (a) The conduction band profile of two semiconductor quantum dots with an intervening tunnel barrier. All subbands are aligned in energy to allow resonant tunneling of electrons between the two dots. The only difference in the material of the two dots is in their elastic constants. (b) The experimental set-up.

The number of researchers [9, 10, 11] is predicated on objective reality. It can be briefly stated as follows: once a measurement outcome is actualized, it remains “out there” forever to be inspected by an observer at any subsequent time without changing the outcome. The outcome does not depend on when, or if at all, the observer inspects it, and does not change once actualized. Home and Chattopadhyay [12] have suggested an experiment involving UV-exposed DNA molecules to empirically determine at what instant an outcome is actualized and the result recorded in a stable and discernible form for perpetuity. A third viewpoint [5] claims that there may be no such precise instant. The pure state may gradually evolve towards a mixed state and concomitantly decoherence begins to set in, but the system may never quite completely decohere in a finite time (we define complete decoherence as the state in which the off-diagonal terms of the $2 \times 2$ density matrix associated with Equation (1) vanish). The off-diagonal terms may decay with time owing to the interaction with the detector (and this may slow down the wiederkehr quantum oscillation between the states $|1> \rangle$ and $|2> \rangle$ - the so-called quantum Zeno effect) but the off-diagonal terms need not ever vanish completely. This has been termed a “continuous collapse”. Korotkov [3] claims that continuous measurement need not cause any decoherence or collapse (i.e, the off-diagonal terms need not decay at all because of the interaction with the detector) if continuous knowledge of the measurement result at all stages of detection is used.
to faithfully reconstruct the pure state. These three viewpoints are quite disparate and cannot be reconciled easily.

We suggest a simple gedanken experiment to resolve some of these conflicting viewpoints. Consider the situation when we have two independent detectors capable of detecting which dot is occupied by the electron in Fig. 1. The detectors are independent in the sense that they are located vast distances apart and initially there is no coupling between them. One detector is the weakly coupled point detector (see Fig. 1b) in the vicinity of a dot capable of fairly non-invasive measurement which causes at most gradual collapse a lá Gurvitz. The other detector is a phonon detector located far away. Suppose that when the electron is in the right dot it emits a zero energy acoustic phonon which has a finite wave vector and hence a finite momentum. It also has a finite group velocity. Such phonons do not typically exist in bulk materials, but exist in quantum confined structures like wires [13] and dots. The emitted phonon has different wave vectors depending on whether the emission took place in the left dot or the right dot because elastic constants (and hence the phonon dispersion relations) in the two dots are different. When the phonon arrives at the detector, it is absorbed by an electron and by measuring the momentum imparted to the electron (or equivalently the associated current), one can tell whether the phonon came from the left dot or the right dot. Thus, monitoring the current in the phonon detector will constitute a “measurement”. Let us say that the phonon was emitted at time $t = 0$\footnote{It may bother the reader that Heisenberg’s Uncertainty Principle is being violated in this thought experiment. If the phonon has precisely zero energy, how can we say that it is emitted at exactly time $t=0$? The answer is that at time $t=0$, we are not measuring the energy. If we ever wanted to measure the phonon’s energy, we could take forever. If indeed Heisenberg’s Uncertainty Principle were relevant here, then all elastic collisions (e.g. electron-impurity collision) will take forever. Yet we can calculate an effective scattering time for an electron impurity collision from Fermi’s Golden Rule.} and it arrives at the phonon detector at time $t = t_1$. The detector finds that the phonon came from the right dot.

If the viewpoint of objective reality [12, 9, 10, 11, 3] is correct, then the actualization of the outcome took place at time $t = 0$. Thereafter, the electron will be always found in the right dot. We can empirically pinpoint this instant at a later time $t > 0$ (actually at $t \geq t_1$) since we can determine $t_1$, the time of flight of the phonon between the dot and the phonon detector. We simply have to know the distance between the dot and the detector and the phonon group velocity to know $t_1$. Thus when the phonon detector registers the phonon, we will know that the actualization took place $t_1$ units of time prior to the registration event. Additionally, if we know...
the time $t = -t_2$ when the electron was injected into the double dot system, then we can find out how long thereafter the actualization of the outcome took place (this time is simply $t_2$). This is similar to what Home and Chattopadhyay had proposed to achieve in their UV-exposed DNA system [12].

We now come to the central issue. Between the time $t = 0$ and $t = t_1$ (i.e. while the phonon is in flight), the observer (phonon detector) is still ignorant of the outcome, but the actualization of the measurement [12] has supposedly already taken place. During this critical time period, the weakly coupled point detector tries to continuously determine which dot is occupied. If the observer-independent viewpoint is correct, then the electron will be always found in the right dot. But, if the observer-dependent viewpoint is correct [8], then the Schrödinger cat is in suspended animation between $t = 0$ and $t = t_1$ since the observer (phonon detector) has not registered any phonon yet. Consequently, the almost non-invasive point detector (which takes a very long time to destroy the superposition acting alone) should have a non-zero probability of finding the electron in the left dot. To ensure that these are the only two possible scenarios, we will allow the maximum latitude. For instance, we will assume: (i) the quantum oscillation period between the two dots (wiederkehr) is much smaller than the time of flight $t_1$ and the Zeno effect [4] is negligible because of the weak coupling with the non-invasive point detector, (ii) the emission of zero energy phonon does not alter the electron’s energy and hence does not subsequently disallow resonant tunneling between the quantum dots, and (iii) the remote phonon detector is unaware of the set-up before time $t = t_1$ and hence cannot influence events before time $t = t_1$ (causality). Thus, if the point detector ever finds the electron in the left dot between $t = 0$ and $t = t_1$, the objective reality (observer-independent) viewpoint will be suspect. In this pathological example, the difference between the observer-dependent and observer-independent viewpoint can be simply stated thus: in the first viewpoint, the collapse took place at $t = t_1$ and in the second viewpoint, it took place at $t = 0$. As long as any non-invasive detector in the timeframe $t = 0$ till $t = t_1$ finds the electron in the left dot and the phonon detector at time $t_1$ finds the electron to have emitted the phonon in the right dot, we will know that the “collapse” did not take place at $t = 0$ which would then contradict the observer independent viewpoint. We will then be forced to admit that perhaps collapse ultimately takes place in the sensory perception of the observer [15]. This is currently a contentious topic.

An interesting question is whether the phonon emission is a collapse event. There is no energy dissipation involved in emitting a zero-energy phonon, but energy dissipation is not necessarily for col-
lapse since elastic interaction of an electron with a magnetic impurity that causes a change in the internal degree of freedom of the scatterer (say, spin flip) constitutes effective collapse. “Creation” of a phonon is certainly changing its internal degrees of freedom in a major way and therefore should be viewed as a collapse event within the framework of standard models.

But what if the point detector will find the electron in the left dot after time $t = t_1$ when the phonon detector has already determined that the electron collapsed in the right dot. This will make standard collapse models suspect \[16\] since we must then admit that the phonon emission did not cause a collapse. Complete collapse is an irreversible event (equivalent to saying that the Zeno time is infinite). However the third viewpoint of Gurvitz \[5\] guarantees that the electron will be ultimately delocalized (and hence found in the left dot with a non-zero probability) if we make a continuous measurement with the point detector. In contrast, if frequent repeated measurements are made, then the Zeno effect guarantees that the opposite will happen; the electron will become more localized in one dot as the frequency of observation is increased. Thus, there is an essential dichotomy when one considers the fact that a continuous measurement is really the ultimate limit of frequent repeated measurements and yet they make opposite predictions. It is not clear how this dichotomy will be ultimately resolved.

In this communication, we have proposed a gedanken experiment to resolve some of the dichotomies between the myriad viewpoints permeating quantum measurement theory. Experiments such as the one proposed here will soon be within the reach of modern technology. Hopefully, they will shed new light on this fascinating topic.

References

[1] D. Home, *Conceptual Foundations of Quantum Physics - An Overview From Modern Perspectives* (Plenum, New York, 1997).

[2] S. Weinberg, *Dreams of a Final Theory* (Vintage, London, 1993).

[3] G. C. Ghiradi, A. Rimini and T. Weber, *Phys. Rev. D*, **34**, 470 (1986); G. C. Ghiradi, R. Grassi and A. Rimini, *Phys. Rev. A*, **42**, 1057 (1990)

[4] R. Penrose, *Gen. Rel. and Gravit.*, **28**, 581 (1996).

[5] S. A. Gurvitz, *Phys. Rev. B*, **56**, 15215 (1997); e-print quant-ph 9806050

[6] A. N. Korotkov, e-print quant-ph 9808026

[7] L. Stodolsky, e-print quant-ph 9805081
[8] J. Von Neumann, *Mathematische Grundlagen der Quantenthorie* (Springer, Berlin, 1931).

[9] D. Bohm and B. J. Hiley, *The Undivided Universe* (Routledge, London, 1993).

[10] N. Gisin in *Fundamental Problems in Quantum Theory*, Ed. D. M. Greenberger and A. Zeilinger (Annals of the New York Academy of Sciences, New York, 1995), 524.

[11] R. Omnes, *The Interpretation of Quantum Mechanics*, (Princeton University Press, Princeton, 1994).

[12] D. Home and R. Chattopadhyay, *Phys. Rev. Lett.*, **76**, 2836 (1996).

[13] A. Svizhenko, A. Balandin, S. Bandyopadhyay and M. A. Stroscio, *Phys. Rev. B*, **57**, 4687 (1998).

[14] B. Misra and E. C. G. Sudarshan, *J. Math. Phys.*, **18**, 756 (1977); R. A. Harris and L. Stodolsky, *Phys. Lett.*, **B 116**, 464 (1982); C. Priscilla, R. Onofrio and U. Tambini, *Ann. Phys.*, **248**, 95 (1996).

[15] F. Aicardi, F. Borsellino, G. C. Ghirardi and R. Grassi, *Found. Phys. Lett.*, **4**, 109 (1991).

[16] A. J. Leggett, in *Nanostructure Physics and Fabrication*, Eds. M. A. Reed and W. P. Kirk (Academic Press, Boston, 1989).