Possibility of the odderon discovery via observation of charge asymmetry in the diffractive $\pi^+\pi^-$ production at HERA

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Abstract

The interference between the Pomeron and possible odderon mechanisms of diffractive $\pi^+\pi^-$ photoproduction results in charge asymmetry of the produced pions. The observation of charge asymmetry of pions at moderate $M_{\pi^+\pi^-}$ will be an undoubted signal of odderon existence. To make numeral estimates more definite, we limit ourselves by the region $M_{\pi^+\pi^-} = 1.1 \div 1.5$ GeV, where in the odderon mechanism of dipion production, the production via single $f_2(1270)$ resonance is expected to be dominant. We find a very statistically significant effect of the odderon induced charge asymmetry even with very modest estimates for the $f_2$ photoproduction cross section (without referring to any particular model of the odderon).

1 Introduction

Pomeranchuk’s conclusion that the particle and antiparticle cross section differences vanish at asymptotic energies as compared to the cross sections themselves is well known [1]. As early as in 1970 there were debates [2] that certain particle-antiparticle cross section differences might not vanish with energy growth, and properties of amplitudes not satisfying the conditions of the Pomeranchuk theorem have been investigated to much detail by Gribov et al. [3]. Later on, the term odderon, $\mathcal{O}$ has been coined [4] for the singularity with $C = -1$ and the intercept $\alpha_\mathcal{O} \approx 1$. Because the particle-antiparticle cross section difference $\sigma^+ - \sigma^-$ should not exceed the sum $\sigma^+ + \sigma^-$, the intercept of the odderon is expected to be not higher than that of the Pomeron, $\alpha_\mathcal{O} \leq \alpha_{\text{IP}}$. 

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1.1 The theoretical and experimental status of the odderon

The odderon is an integral feature of the QCD motivated picture of high energy scattering, and its experimental discovery is crucial for the QCD description of strong interactions. Within perturbative QCD (pQCD), the Pomeron exchange is naturally modeled by the color-singlet two-gluon exchange in the $t$-channel [5] and at the same very level the odderon is modeled by the $d$-coupled, color-singlet, three-gluon exchange in the $t$-channel [6, 7], which suggests that both Pomeron and odderon intercepts are close to the gluon spin, $\alpha_{\text{IP}} \sim J_g = 1$. The experimental data and the BFKL calculations show that the Pomeron intercept $\alpha_{\text{IP}}(0) > 1$ (for the recent work and references see [9]). The theoretical estimates for $\alpha_\omega$ are not yet conclusive, the published results for the $\alpha_s = \text{const}$ approximation grow gradually with time: $\alpha_\omega(0) = 0.94 \rightarrow 0.96 \rightarrow 1 \rightarrow \ldots$?, see e.g. [10] and references therein. In our discussions we will keep in mind that both $\alpha_{\text{IP}}$ and $\alpha_\omega$ are close to 1, and $\alpha_{\text{IP}} - \alpha_\omega$ is small.

If the Pomeron and the odderon are assumed to be Regge poles, their contributions to the scattering amplitude $AB \rightarrow CD$ have the standard factorized form

$$A_R = \zeta(R) e^{i\pi \alpha_R / 2} \cdot G(ARC) \cdot s^{\alpha_R} \cdot G(BRD),$$

with $R = \text{IP, O}$, $\zeta(\text{IP}) = 1$, $\zeta(\text{O}) = i$. \hspace{1cm} (1)

Here the factors $G(ARC)$ and $G(BRD)$ describe the couplings $ARC$ and $BRD$ respectively. They depend on the particle helicities $\lambda_i$, and at small $t$ one must have

$$G(ARC) \propto |t|^{\lambda_A - \lambda_C} / 2, \quad G(BRD) \propto |t|^{\lambda_B - \lambda_D} / 2.$$ \hspace{1cm} (2)

Since the intercepts $\alpha_{\text{IP}}$ and $\alpha_\omega$ are close to 1, the Pomeron exchange amplitude is predominantly imaginary, while the odderon exchange amplitude is predominantly real.

◊ Although in the pQCD framework the Pomeron and odderon are of a similar status, the experimental quest for the odderon exchange has proven to be a challenging task. The estimates for the cross section difference $\sigma_{pp} - \sigma_{p\bar{p}}$ turned out to be well below the experimental uncertainties. The related studies of $Kp$ scattering are possible only at fixed target in the limited range of $s \lesssim 10^3 \text{ GeV}^2$ and with relatively low statistical accuracy. The diffractive photoproduction of $C = +1$ (pseudo) scalar and tensor mesons $M, \gamma p \rightarrow p'M$ (with $p'$ either proton or its low-mass excitation) suggested a decade ago [11], seems to be a better signature for the odderon exchange. Indeed, the $C = +1$ mesons are excited from $C = -1$ initial photons only via the $C = -1$ exchange in the $t$-channel. For sufficiently large energies the $p$ and $\omega$ exchange contributions die out (see Appendix for details), and such processes will be dominated by the odderon exchange and — for the expected $\alpha_\omega \sim 1$ — have the cross section, which is approximately flat vs. energy. The systematic search for such reactions with flat cross sections is in its formative stage, and they have not yet been observed experimentally. These results at least suggest that the odderon is weakly coupled to the proton as has been argued earlier (see e.g. [8, 12]).

∇ Available theoretical calculations for the soft odderon amplitudes at small $t$ [8, 13, 14] are based on variety of nonperturbative $3$-gluon exchange and nonrelativistic quarks models for mesons and nucleons. Even in hard electroproduction of pion pairs [22] or photoproduction of open charm [18], one cannot eliminate the sensitivity of the odderon amplitude to the soft quark models of the proton, so that, calculations of [18, 22] can be regarded only as crude estimates with large uncertainty. For example, the ratio of absolute values of the forward odderon and Pomeron exchange $NN$ amplitudes varies
from 0 to 0.04 depending on the clustering of quarks in the nucleon [8]. Similar estimates for the exclusive \( \pi^0 \) photoproduction lead to the cross sections varying from 10 to 200 nb [13, 14]. According to ref. [8], in the reggeized 3-gluon exchange quark–diquark model the diagonal \( pO \) vertex disappears in the unrealistic limit of the point-like scalar diquark (and in this limit \( pO \) vertexes with proton excitations \( p' \) become dominant), while the diagonal vertex become essential or even dominant with the growth of the diquark size (unfortunately, this very approximation of the point-like scalar diquark was used in ref. [14]). This type of uncertainty makes all available estimates for odderon rather dubious.

\( \nabla \) Another important limitation of the above cited calculations is that they were performed only in the Born type approximation. In the reggeization program, which is technically carried out by resummation of leading logarithms of energy from loop corrections, the Born terms are just the starting point. It remains just an assumption, although — since the odderon intercept is close to unity — a plausible one, that the Born result sets a reasonable scale for the reggeized physical amplitude.

Still, such estimates must be taken with the grain of salt. Indeed, the trademark of the reggeon amplitude is the factorization (1), with factorized polarization dependence of the form (2). The Born approximation results usually contain both helicity factorized and helicity non-factorized terms. After the resummation, the factorized terms give rise to the Regge behavior, while the contribution from non-factorized terms should vanish at \( s \to \infty \). Consequently, only the factorized components of the Born amplitude can be taken for an estimate of the odderon exchange. This picture is supported by direct calculations in all known cases, for a recent example of the leading log \( s \) resummations see [15]. Such a separation between factorized and non-factorized terms was not performed in refs. [14], as it can be clearly seen from the results for the \( \gamma p \to f_2p' \) photoproduction. Namely, the leading term for the Born amplitude evaluated in [14] corresponds precisely to the non-factorizing amplitude with correlated spin flips in the both vertices \( (\lambda_M - \lambda_\gamma = \lambda_p - \lambda_{p'} = 1) \), so that this amplitude does not vanish at \( t = 0 \), in contrast to (2). According to the above arguments, such a term must decrease with energy after resummation and therefore must be removed from the discussed result. Therefore, two essential conclusions of [14] cannot definitely be related to the odderon contribution:

\begin{enumerate}
\item The values of the cross section estimated;
\item The prediction of the nucleon excitation dominance for the proton vertex.
\end{enumerate}

\( \diamond \) Recently, the H1 collaboration reported first results on the search for the odderon in diffractive photoproduction of \( C = +1 \) mesons. The event selection was inspired by results of [14] and included only events with nucleon excitations. No signal from the odderon was found, and the upper limits on the cross sections were placed at \( \sigma(\gamma p \to \pi^0 X < 39 \text{ nb}[16], \sigma(\gamma p \to f_2X < 16 \text{ nb} \) and \( \sigma(\gamma p \to a_2X < 96 \text{ nb} \) (both in [17]). These bounds are below the correspondent predictions of refs. [14]. In the light of the above discussion, this is hardly surprising. In particular, we see no reason whatsoever to perform any specific selection regarding the nucleon final state.

Below we do not cling to any specific model for the odderon except for very natural assumptions about similarity of the odderon to the other reggeons, and estimate observable effects only by assuming that the odderon mechanism of \( C \)-even dipion production is larger than the non–odderon mechanisms (see Appendix for details). In this respect we use the standard Regge-pole model for the Pomeron and odderon mediated amplitudes that describe the diffractive production of a \( \pi^+\pi^- \) pair in the \( C\)-odd and \( C\)-even states respectively, and assume that these pion pairs are produced via intermediate resonance states.
1.2 Exploiting the charge asymmetry

The chances of discovery of the elusive odderon are arguably enhanced if instead of isolation of the pure odderon exchange reactions one would look for the Pomeron–odderon interference, which is linear in the small odderon amplitude (not quadratic as contribution to cross section). This interference can be observed as the charge asymmetry of diffractively produced particles. The main idea can be formulated as follows. The initial photon has definite $C$-parity, $C = -1$. Since the Pomeron ($I_P$) has vacuum quantum numbers, i.e., $C = +1$, the Pomeron–photon collision produces $C$-odd system. To the contrary, the collision of photon with $C = -1$ odderon ($O$) produces $C$–even systems. Consequently, it is useful to study the production of final system that can be produced both by the Pomeron and the odderon exchanges. The interference of the corresponding $C$-odd and $C$-even amplitudes gives rise to a charge asymmetry in the momentum distribution of produced particles. In the absence of the other $C$-odd exchange mechanisms, it will be an unambiguous signal of the odderon.

The search for the odderon via the charge asymmetry in the photoproduction of $c\bar{c}$ pairs was proposed first in ref. [18]. The obvious disadvantage of this process is the small diffractive cross section, further hampered by a small efficiency of detection of charmed particles (see [19] and references therein). Note that the final estimates obtained in this paper have large uncertainties due to above mentioned uncertainties in the description of odderon–nucleon vertex. Besides, under standard assumptions about the quark–hadron duality for heavy quarks the asymmetry obtained disappears at $\alpha_O \to \alpha_{I_P}$,

$$Re \left( A_{I_P}^\dagger A_O \right) \propto Re \left\{ i \exp \left[ \frac{i\pi(\alpha_{I_P} - \alpha_O)}{2} \right] \right\} = \sin \left[ \frac{\pi(\alpha_{I_P} - \alpha_O)}{2} \right].$$

In sect. 4.1 we show that this conclusion becomes invalid due to the final state strong interaction (FSI).

About two years ago we suggested\(^1\) to look for the charge asymmetries in the much more copious diffractive photoproduction of $\pi^+\pi^-$ pairs at $M_{\pi^+\pi^-} \lesssim 1 \div 1.5$ GeV. The advantages of this process as compared to the $c\bar{c}$ production are

(i) the much higher basic cross sections and high detection efficiency for pions and

(ii) the final state interaction (FSI) is essential and the dipion production amplitudes $F_{I_P}$ and $F_O$ acquire additional phase shifts as compared to (3) (for the early discussion on FSI effects within the Regge formalism see [21]), which are predominantly controlled by the prominent pion-pion resonances. Zooming in on the mass region where the Breit-Wigner phase shifts are such as the small factor $\frac{\alpha_{I_P} - \alpha_O}{2}$ is eliminated, one can gain the charge asymmetry that would persist even if $\alpha_{I_P} = \alpha_O$.

Recently, the idea that the odderon can be discovered via observation of charge asymmetry of pions in diffractive $\pi^+\pi^-$ was extended to hard electroproduction [22]. Certainly, the cross sections calculated in that work are much lower than the photoproduction cross sections discussed here. Besides, the numerical estimates of this paper are unsafe due to the above mentioned uncertainty in the description of $pOp'$ vertex. Last, the statement about the dominance of transverse charge asymmetry is doubtful, since the value of photon virtuality $Q^2 \approx 3$ GeV$^2$ does not seem to be high enough for a definite statement regarding the dominance of the leading twist amplitudes. Note that with growth of the electron scattering angle, the contribution of $Z$ exchange increases.

\(^1\) At different stages, the preliminary results were published in ref. [20] and have been repeatedly reported during last two years [25].
The axial component of \( Z \) current can produce \( C \)-even dipions in fusion with Pomeron as well. This effect can overshoot the small odderon effect at large enough \( Q^2 \).

Our approach is similar in some respect to that used for the description of charge asymmetry in the process \( e^+e^- \rightarrow e^+e^-\pi^+\pi^- \) \[26,27\], which is suitable for the study of low energy phenomena and resonances in pion and kaon physics.

The paper is organized as follows. In Sect. 2 we introduce notation, define the forward-backward (FB) and the transverse (T) asymmetries. In Sect. 3 we describe the Pomeron and odderon helicity amplitudes for dipion production via intermediate resonance state. The charge asymmetry due to Pomeron–odderon interference is calculated in Sect. 4. In Sect. 5 we present numerical estimates which appear very promising. Discussion of the results obtained and the conclusions are presented in Sect. 6. Last, in the Appendix we discuss non-odderon mechanisms of \( C = \pm 1 \) meson production and find the lowest value of odderon mediated cross section that — provided natural cuts are applied — cannot be mimicked by non-odderon mechanisms.

\section{2 Kinematics}

We focus on the real photoproduction reaction \( \gamma p \rightarrow \pi^+\pi^-p' \) with energies lying in the HERA range. The pion system with a small to moderate invariant mass \( M \) is produced with a large rapidity gap from the recoil proton \( p' \). The initial momenta of the photon and proton are \( q \) and \( P \) respectively, \( s = (q + P)^2 \), initial photon polarization is \( \vec{e} \). With \( k_+ \) and \( k_- \) being the momenta of the charged pions, we consider

\[ r^\mu = k_+^\mu - k_-^\mu, \quad \Delta^\mu = k_+^\mu + k_-^\mu, \quad M^2 = \Delta^2. \tag{4} \]

The discussed charge asymmetry must appear precisely as the antisymmetry of the differential cross sections under replacement \( r^\mu \rightarrow -r^\mu \).

We perform calculations in the helicity basis where \( \lambda_\gamma \) and \( \lambda_R \) are the helicities of photon and produced dipion respectively, while \( \lambda_p \) and \( \lambda_{p'} \) are the helicities of incident and scattered proton respectively. The final results are averaged over initial photon polarizations.

We define the \( z \)-axis as the \( \gamma p \) collision axis. Throughout the paper 2D transverse vectors will be marked with the vector sign, while the 3-vectors will be given in bold. We direct the \( x \)-axis along vector \( \vec{\Delta} \) and define by \( \psi \) the azimuthal angle of the vector \( \vec{\Delta} \), i.e., the production plane, with respect to the fixed lab frame of reference, in which the helicity states of the incoming photon are defined. For instance, for the tagged photons in electroproduction \( ep \rightarrow e\pi^+\pi^-p' \), this frame of reference can be related to the electron scattering plane. Then the polarization vector of the initial photon with helicity \( \lambda_\gamma = \pm 1 \) can be written as \( \vec{e}^\lambda = -\frac{1}{\sqrt{2}} e^{i\lambda_\gamma \psi} (\lambda_\gamma, i) \). Hereafter we neglect the pion mass compared to the mass of dipion \( M \).

To define the appropriate independent charge-asymmetric observables, let us first denote by \( z_+ \) and \( z_- \) the standard light cone variables for each charged pion, \( z_{\pm} = (e_{\pm} + p_{\pm})/(2E_{\gamma}) = (k_{\pm} P)/(qP) \) (for the considered diffractive type processes \( z_+ + z_- = 1 \)). Then we define appropriate variables for the description of charge asymmetry

\[ \xi = \frac{z_+ - z_-}{z_+ + z_-}, \quad v = \frac{2(k_+^2 - k_-^2 - \xi \Delta^2)}{M|\Delta|} \equiv \frac{2(\vec{\rho} \vec{\Delta})}{M|\Delta|} \text{ with } \vec{\rho} = \vec{r} - \xi \vec{\Delta}. \tag{5} \]

(Note that the transverse momentum of each charged pion is split in two parts as \( \hat{k}_\pm = \pm(\vec{\rho}/2) + z_{\pm}\vec{\Delta} \). Here \( \vec{\rho}/2 \) is relative transverse momentum of \( \pi^+ \) in respect to
the total transverse momentum of the dipion.) With such definition, the variables $\xi$ and $v$ describe the forward-backward (FB) and transverse (T) asymmetries of the charged pions respectively. In terms of these variables the discussed charge asymmetry is non-invariance of the differential cross section under the transformation $\xi \to -\xi$ and $v \to -v$. For example, positive transverse asymmetry means that the number of events with $v > 0$ exceeds that with $v < 0$.

For the pion pair production the above observables $\xi$ and $v$ are related simply to the polar and azimuthal angles $\theta$ and $\phi$ of the 3-momentum $k_+$ in the dipion rest frame $R$

$$k_+|_R = \frac{1}{2}(\vec{p}, M\xi) = \frac{1}{2}M(\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta). \quad (6)$$

In this frame for $k_-$ we have the same equations with $(\theta, \phi) \to (\pi - \theta, \pi + \phi)$. The charge asymmetric variables (5) are related to these angles as

$$\xi = \cos \theta, \quad v = \sin \theta \cos \phi. \quad (7)$$

In the following we shall discuss the five-fold differential cross section of the reaction $\gamma p \to \pi^+ \pi^- p'$

$$2\pi \frac{d\sigma}{dM^2 d\Delta^2 d\cos \theta d\phi d\psi} \equiv 2\pi \frac{d\sigma}{dM^2 d\Delta^2 d\xi dv d\psi} \frac{1}{2} \sqrt{1 - \xi^2 - v^2}. \quad (8)$$

Factor 1/2 reflects degeneracy of $v$ in respect to replacement $\phi \to 2\pi - \phi$. The integration measure in terms of $\xi, v$ is

$$\int d\Omega \equiv \int_1^{-1} d\cos \theta \int_0^{2\pi} d\phi \quad \rightarrow \quad 2 \int \int \frac{d\xi dv}{\sqrt{1 - \xi^2 - v^2}} \theta(1 - \xi^2 - v^2). \quad (9)$$

3 The Pomeron and odderon helicity amplitudes

In this section we consider the main features of the high energy diffractive $\gamma p \to \pi^+ \pi^- p'$ amplitudes from the Pomeron and the odderon exchange.

3.1 General properties

- The properties of the Pomeron amplitude are well constrained by the experimental studies at HERA:
  - The main contribution to vertex $p p' p'$ comes from the proton-elastic scattering, $p' = p$ (the admixture from proton dissociation to excited states with masses $M' \lesssim 2$ GeV is well known to be below 25% [28, 29]). To a good approximation, the s-channel helicity conservation (SCHC) holds for this vertex [28].
  - The main contribution to the cross section is given by amplitudes with production of two pions in the (C-odd) $\rho$-meson state. At higher effective masses of dipion other $\rho$ type resonances should also be accounted for. Besides, the SCHC takes place at small $t$, i.e. the $\rho$-meson is produced mainly with the same helicity as the helicity of the initial photon.
  - There is no experimental information about the odderon amplitude and here one is bound to the theoretical estimates.

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\( \diamond \) In the reggeized 3-gluon exchange quark–diquark model the spin properties of the odderon coupling to nucleons can be summarized as follows [8]:

(i) odderon exchange satisfies SCHC and the spin-flip \( pO\rho \) amplitude vanishes in the 
SU(6) nonrelativistic quark model, 

(ii) the relative strength of the spin-flip \( pO\rho \) vertex depends on the size of the scalar 
diquark clustering, it becomes the dominant one in the unrealistic limit of the point-like 
scalar diquark (unfortunately, this very approximation of the point-like scalar diquark 
has been used in ref. [14]).

Therefore, we assume that the properties of \( pO\rho' \) vertex are roughly similar to those 
for Pomeron, i.e., the SCHC elastic transition with \( p' = p \) is not suppressed.

\( \diamond \) The vertex \( \gamma O\pi^+\pi^- \) is of main interest to us. We assume that — as it is customary 
for other phenomena at \( M \lesssim 1.5 \text{ GeV} \) — the pion pairs are produced mainly via resonance 
states \( (C\text{-even } f_0 \text{ and } f_2 \text{ mesons in our case}). \) Below we discuss how shape of the \( C\text{-even } \pi^+\pi^- \) spectrum helps us separate out the odderon signal, and for numerical estimates 
we take the \( f_2(1270) \) meson. Since the helicity structure of \( \gamma O \to f_2 \to \pi^+\pi^- \) vertex is 
not known, we will present results based on several limiting cases.

### 3.2 Detailed description

The amplitudes of dipion production via production of resonances \( R \) of spin \( J \) and helicity 
\( \lambda_R \) (with SCHC in the proton vertex) can be conveniently written in a factorized form:

\[
A_{\text{IP}} = A^{\lambda_R\lambda_\gamma}_{\text{IP}} \cdot D_J(M^2) \cdot \mathcal{E}_{\lambda_\gamma}^{J,\lambda_R}, \quad A_O = iA^{\lambda_R\lambda_\gamma}_{\text{O}} \cdot D_J(M^2) \cdot \mathcal{E}_{\lambda_\gamma}^{J,\lambda_R}.
\]  

(10)

The additional factor \( i \) for odderon amplitude is related to the opposite signature of the 
oderon as compared with the Pomeron.

In these equations the first factor describes the Regge amplitude of production of the 
resonance \( R \) with helicity \( \lambda_R \) (the energy dependence is included in the quantity \( \sigma_R \))

\[
A^{\lambda_R\lambda_\gamma}_{\text{Regge}} = g^{\lambda_R} \sqrt{\sigma_R B_R} e^{i\alpha_R} e^{-\frac{i}{2} B_R \Delta^2} \left( \frac{\sqrt{B_R} \Delta}{|\lambda_\gamma - \lambda_R|} \right)^{|\lambda_\gamma - \lambda_R|} \left( \frac{\sqrt{B_R} \Delta}{|\lambda_\gamma - \lambda_R|} \right)^{-|\lambda_\gamma - \lambda_R|} \left( \frac{\sqrt{B_R} \Delta}{|\lambda_\gamma - \lambda_R|} \right).
\]  

(Regge = \( \text{IP}, O \)).

(11)

Here \( (g^{\lambda_R})^2 \) is the fraction of total cross section of the production of resonance \( R \) with 
helicity \( \lambda_R \) by the photon with helicity \( +1 \). Since only helicity difference is essential, this 
very quantity describes the transition of photon with helicity \( -1 \) to dipion with helicity 
\( -\lambda_R \) as well. Since SCHC approximately holds for the Pomeron, we have \( g_0^1 \approx 1 \) and 
\( |g_0^0| \ll g_0^1 \). The \( t \) dependence of this amplitude comes mostly from the vertex factors [2].

It can be parameterized by \( \exp(-B_R \Delta^2/2) \) modulo to helicity-flip factors \( |t|^{\Delta^2/2} \). 
For the sake of simplicity, we suppress the \( M \) dependence of the diffraction slopes \( B_\rho \) and 
\( B_J \) and the \( t \)-dependence of trajectories \( \alpha_R \).

The second factor in eq. (10) describes the \( \pi^+\pi^- \) invariant mass dependence of the 
production amplitude for the dipion state with angular momentum (spin) \( J \) and directly 
reflects the shape of the corresponding \( C\text{-even or } C\text{-odd } \pi^+\pi^- \) spectrum. In a simple case 
of single-resonance dominance, it can be reasonably well approximated by the standard 
Breit-Wigner factor for this resonance together with its coupling to pions. Thus, for the 
(\( C\text{-odd) } P\text{-wave } \pi^+\pi^- \) spectrum, where the dominance of the \( \rho \) meson is established, 
and for the (\( C\text{-even) } D\text{-wave } \pi^+\pi^- \) spectrum, where the \( f_2(1270) \) meson is expected to 
be the dominating feature, near the resonance peak

\[
D_J(M^2) = \frac{\sqrt{m_R^2 \Gamma_R} B_R (R \to \pi^+\pi^-) / \pi}{M^2 - m_R^2 + i m_R \Gamma_R}.
\]  

(12)
For numerical estimates we use this form even away from the resonance peaks, $|M^2 - m_R^2| > M_R \Gamma_R$. The shape of the $S$-wave $\pi^+\pi^-$ spectrum is expected to be more complicated.

Finally, the decay factor $\mathcal{E}_{\chi_\gamma}^{J,\lambda R}$ describes the angular part of the helicity amplitude. Because the pions are spinless, it takes a particularly simple form,

$$\mathcal{E}_{\chi_\gamma}^{J,\lambda R} = Y_{J\lambda R}(\theta, \phi)e^{i\lambda R \psi},$$

where $J = 1, 2$ for the $\rho_0$ and $f_2$, respectively, $Y_{lm}(\theta, \phi)$ is the standard angular momentum wave function normalized as $\int |Y_{lm}(\theta, \phi)|^2 d\Omega d\psi/(2\pi) = 1$.

4 Charge asymmetry

4.1 Invariant mass dependence

The charge asymmetry effect is given by the interference of suitable Pomeron and odderon amplitudes integrated over the redundant phase space variables

$$d\sigma_{\text{asym}} = \sum_{\lambda R, \lambda \rho} 2Re \left( A_{\text{IP}}^{\lambda R} A_{\mathcal{O}}^{\lambda \rho} \right) d\Gamma.$$  (14)

The pattern of $M$-dependence of the $\text{IP} - \mathcal{O}$ interference is controlled by the $D_J(M)$ factors in the above amplitudes in the form of overlap functions

$$I_{12}(M^2) = Re \left[ D_1(iD_2)^\dagger e^{i\delta_{\text{IP}}} \right], \quad I_{10}(M^2) = Re \left[ D_1(iD_0)^\dagger e^{i\delta_{\text{IP}}} \right].$$  (15)

The exact form of these overlap functions is given by the precise form of different $D_J$ functions. Below, we will consider mainly the region $M > 1100$ MeV, where there are no $S$-wave dipion resonances and, consequently, only the overlap function $I_{12}$ is of interest (see Figure 2 and related discussion). For estimates here we use Breit–Wigner form of $D_1$, which is given by $\rho$-meson contribution from Pomeron, and $D_2$, which is given by $f_2$-meson contribution from odderon

$$I_{12}(M^2) = Im \left( \frac{e^{i\delta_{\mathcal{O}}} \sqrt{m_\rho m_f \Gamma_\rho \Gamma_f Br(f_2 \rightarrow \pi^+\pi^-)Br(\rho \rightarrow \pi^+\pi^-)}}{\pi(M^2 - m_\rho^2 + im_\rho \Gamma_\rho)(M^2 - m_f^2 - im_f \Gamma_f)} \right)$$

$$\text{with } \delta_{\text{IP}} = \frac{\pi}{2}(\alpha_{\text{IP}} - \alpha_{\mathcal{O}}).$$  (16)

In our analysis we neglect the $M$ dependence of diffraction parameters, such as $B_\rho$, $B_\rho$, $g_R^{\lambda R}$, etc. In this approximation, the above overlap function is the only factor that contains $M$ and it is universal for all asymmetries and $t$ dependencies.

Because the difference between Pomeron and odderon intercepts $\delta_{\text{IP}}$ is small, the overlap function is large only when the phase shift between the two Breit-Wigner factors is close to $\pi/2$. This will be the case in the vicinity of the resonance peaks, where for the one resonance the $D_{J_1}$ is almost real while for the other one the $D_{J_2}$ is almost imaginary, which compensates the additional factor $i$ in the odderon amplitude. The mass-dependence of the charge asymmetric overlap function is shown in Fig. 1. It exhibits only weak sensitivity to the poorly known $\alpha_{\text{IP}} - \alpha_{\mathcal{O}}$. It is precisely the strong FSI that lifts the suppression, which would come into play if FSI were neglected.
4.2 Final expressions

We consider the differential cross sections averaged over initial photon spin states and integrated over $\psi$, which is relevant to $ep$ experiments with untagged scattered electrons. The integration over $\psi$ leaves in the result only terms with identical $\lambda_\gamma$. Besides, it is well known that for the real photons the other factors depend only on $|\lambda_\gamma - \lambda_R|$, not on the value of helicity itself. Therefore, the observed interference effects will be proportional to sums over opposite initial photon helicities with simultaneous change of the sign of the produced resonance helicities, and will have the azimuthal and polar angle dependence of the form

$$E^{*J_\rho,\lambda_\rho} E^{J_R,\lambda_R} + E^{*J_\rho,-\lambda_\rho} E^{-J_R,-\lambda_R}. \quad \text{At odd } J_\rho - J_R, \text{ this changes sign under replacement } k_\pm \to k_\mp (\theta \to \pi - \theta, \phi \to \pi + \phi), \text{ i.e. exhibits charge asymmetry, either forward-backward or transverse.}

Using the well known azimuthal dependence of spherical harmonics, we have

$$E^{*J_\rho,\lambda_\rho} E^{J_R,\lambda_R} + E^{*J_\rho,-\lambda_\rho} E^{-J_R,-\lambda_R} \propto \cos[(\lambda_\rho - \lambda_R)\phi].$$

This dependence give us the key to the type of charge asymmetry that takes place for different helicities of the $\rho$ and $C$-even resonance $R$. The terms with odd $\lambda_\rho - \lambda_R$ change sign under $\phi \to \pi + \phi$, i.e. under $v \to -v$. They are responsible for the T asymmetry. The terms with even $\lambda_\rho - \lambda_R$ remain invariant under $\phi \to \pi + \phi$. Therefore, they must change sign under $\theta \to \pi - \theta$, i.e. they are responsible for the FB asymmetry. In other words, the interference of amplitudes with even $|\lambda_\rho - \lambda_R|$ ($0$ or $2$ in our examples) generates the FB asymmetry, $\propto \xi P(\xi^2, v^2)$, whereas the interference of amplitudes with odd $|\lambda_\rho - \lambda_R|$ ($1$ or $3$) generates the T asymmetry, $\propto vP_1(\xi^2, v^2)$.

Starting from now, we will give expressions for the case of $J = 1$ and $J = 2$ interference ($\rho - f_2$) only. Similar formulas for $J = 1$ and $J = 0$ interference (which are essential for the analysis below $1.1 \text{ GeV}$) can be immediately obtained in the same manner.

Neglecting contributions with higher helicity flip, $|\lambda_R - \lambda_\gamma| > 1$, we obtain the $C$-odd
interference cross section of the form

\[
\frac{d\sigma_{\text{interf}}}{dM^2 d\Delta^2 d\xi dv} = \frac{3\sqrt{5}}{2\pi \sqrt{1-\xi^2-v^2}} I_{12}(M^2) \sigma_p \sigma f B_p B_f \exp\left(-\frac{B_p + B_f}{2}|t|\right) \otimes T
\]

\[
T = g_p^0 g_f^0 (1-\xi^2)\xi + v g_p^1 \left[\frac{1}{2} g_f^2 (1-\xi^2) + \frac{1}{\sqrt{6}} g_f^0 (3\xi^2 - 1)\right] \sqrt{B_f |t|} + g_p^0 g_f^1 \sqrt{2 v \xi^2 (B_p - t) + \xi g_p^0 \left[\frac{1}{\sqrt{2}} g_f^2 (2v^2 + \xi^2 - 1) + \frac{1}{\sqrt{3}} g_f^0 (3\xi^2 - 1)\right] \sqrt{B_f B_p |t|}.
\]

(17)

- **The forward–backward asymmetry** is obtained from here by integration over \(v\) (relative transverse motion of pions):

\[
\frac{d\sigma_{FB}}{dM^2 d\Delta^2 d\xi dv} = \frac{3\sqrt{5}}{2} I_{12}(M^2) \sigma_p \sigma f B_p B_f \exp\left(-\frac{B_p + B_f}{2}|t|\right) \otimes \xi T_\xi,
\]

\[
T_\xi = g_p^0 g_f^0 (1-\xi^2) + \frac{1}{\sqrt{3}} g_p^0 g_f^0 (3\xi^2 - 1) \sqrt{B_f B_p |t|}.
\]

(18)

If the SCHC holds for the odderon, then the principal effect would be the FB asymmetry dominated by the first term in this equation. If the mechanism of the \(f_2\) production significantly violates SCHC, then the first term is dominant only at small \(t\). With the growth of \(|t|\), the terms with helicity flip for both the Pomeron and odderon become essential, and generally, not small. Note that upon the azimuthal integration the contribution from production of \(f_2\) in the state with helicity 2 vanishes because \(\int \cos 2\phi d\phi = 0\). Of course, one can isolate this contribution to the FB asymmetry looking at the differential dependence on the azimuthal variable \(v\) or with integration over some region of \(v\), e.g. \(v^2 > v_0^2\).

- **The transverse asymmetry** is obtained from (17) by integration over \(\xi\) (relative longitudinal motion of pions) at fixed \(v\):

\[
\frac{d\sigma_T}{dM^2 d\Delta^2 dv} = \frac{3\sqrt{5}}{4} I(M^2) \sqrt{\sigma_p \sigma f B_p B_f |t|} \exp\left(-\frac{B_p + B_f}{2}|t|\right) \otimes v T_v,
\]

\[
T_v = g_p^1 g_f^2 \sqrt{B_f} \frac{1 + v^2}{2} + g_p^1 g_f^0 \frac{1 - 3v^2}{2} + g_p^0 g_f^1 \sqrt{B_f} \sqrt{\frac{2}{3}} (1 - v^2).
\]

(19)

Due to its kinematical \(t\)-dependence, the transverse asymmetry becomes naturally small at small \(t\) while the background is high here. Therefore, imposing cuts from below in \(|t|\) might improve the signal to background ratio.

The transverse asymmetry is the dominant one in the case of SCHNC for odderon, in particular, if the \(f_2\) meson is produced mainly in the state with maximal helicity \(\lambda_f = \pm 2\). Evidently, the similar transverse asymmetry is generated always when either the SCHC odderon exchange interferes with the SCHNC helicity-flip Pomeron exchange or vice versa. Certainly, one cannot exclude accidental compensations among coefficients of these amplitudes.

The above analysis is similar to the well known partial wave analysis (PWA). The eq. (17) can be written in terms of angular variables \(\theta\), \(\phi\), and above description can be continued with PWA to a more detailed analysis of helicity structure of the odderon amplitude (with known helicity structure of the Pomeron amplitude) neglecting terms with large helicity flips. To find all helicity flip amplitudes, one should consider additionally dependence on initial azimuthal angle \(\psi\), which is measurable in the ep experiments (as it was done for the charge symmetric contributions in ref. [33]). Note that the analysis of charge asymmetry in terms of asymmetry in \(\xi\) and \(v\) is suitable also to multipion final states where PWA is very complicated.
5 Numerical estimates

The main background to the charge asymmetry is given by the Pomeron–photon (Pomeron–Primakoff) interference which is predominantly transverse one since Primakoff mechanism produces $f_2$ only in the states with helicity 2 and 0. To suppress this background we suggest impose different cuts for the FB and T asymmetries:

$$|t_{FB}| = \Delta^2 \geq 0.1 B_{p}^{-1} \approx 0.01 \text{ GeV}^2, \quad |t_T| = \Delta^2 \geq B_{p}^{-1} \approx 0.1 \text{ GeV}^2.$$ \hspace{1cm} (20)

Below we use parameters of resonances from ref. \[31\] and well known quantities for the $\rho$ meson photo–production, $\sigma_{\rho} \approx 12 \mu b$ (for the diagonal in proton case, $p' = p$), $B_{\rho} \approx 10 \text{ GeV}^{-2}$, $g_{\rho}^1 \approx 1$, $g_{\rho}^0 \approx 0.10$ (see refs. \[34\], \[35\] and \[33\] for the data and their analysis). For the odderon contribution we have no data. The estimates in Appendix show that at HERA the odderon contribution would definitely dominate over the other mechanisms if $\sigma_f \geq 1 \text{ nb}$. Therefore, in order to be able to make as strong conclusions as possible, we will take the value $\sigma_f = 1 \text{ nb}$ for the numerical estimates. Note that this number is more than one order of magnitude smaller than the currently discussed upper bound on the $\gamma p \rightarrow f_2 p$ cross section. We hope that the real cross section is higher than our very cautious estimate. We also take the slope parameter for the $f_2$–meson $B_{f} = B_{\rho}$.

We checked that the sensitivity of the results to reasonable variations of $B_{f}/B_{\rho}$ is weak.

Obviously, the charge asymmetric contribution vanishes upon the angular integration.

We quantify the charge asymmetry by

$$\Delta \sigma_{FB} = \int d\sigma(\xi > 0) - \int d\sigma(\xi < 0), \quad \Delta \sigma_T = \int d\sigma(v > 0) - \int d\sigma(v < 0).$$ \hspace{1cm} (21)

We now focus on the two limiting cases for the helicity structure of the odderon amplitude.

- Let SCHC hold for the $f_2$ meson production, i.e. the dominant final state is with helicity 1 and $g_f^1 \approx 1$. In this case the main effect will be the FB asymmetry \[18\].

  The phase space integration subject to the cut $t_{FB}$ \[20\] gives

  $$\frac{d\Delta \sigma_{FB}}{dM^2} = 0.9 \cdot \frac{3\sqrt{5}}{4} \sqrt{\sigma_{\rho} \sigma_f} \cdot I_{12}(M^2) \approx 1.5 \sqrt{\sigma_{\rho} \sigma_f} \cdot I_{12}(M^2).$$ \hspace{1cm} (22)

- Let the $f_2$ meson be produced in the SCHNC state with helicity 2, i.e. $g_f^2 \approx 1$. In this case the main effect will be the transverse asymmetry \[19\]. The phase space integration subject to the cut $t_T$ \[20\] yields

  $$\frac{d\Delta \sigma_T}{dM^2} = 0.507 \cdot \frac{9\sqrt{5}}{16} \sqrt{\sigma_{\rho} \sigma_f} \cdot I_{12}(M^2) \approx 0.64 \sqrt{\sigma_{\rho} \sigma_f} \cdot I_{12}(M^2).$$ \hspace{1cm} (23)

The charge symmetric background is a sum of the Pomeron and the odderon cross sections. Since the odderon amplitude is considered to be very small, it contributes negligibly to the background even far from the $p$ peak, and the charge symmetric background can be approximated by the $\rho$ contribution

$$\frac{d\sigma_{bkgd}}{dM^2} = \sigma_{\rho} |D_1(M^2)|^2 \times \left\{ \begin{array}{ll} 0.9 & \text{for } FB \quad (|t| > 0.1 B_{p}^{-1}), \\ 0.367 & \text{for } T \quad (|t| > B_{p}^{-1}). \end{array} \right.$$ \hspace{1cm} (24)

Below we report $\Delta \sigma_{FB}$ and $\Delta \sigma_T$ integrated over the suitable $M^2$ region.
We present our results in the form of the statistical significance of asymmetries, which
is defined as the ratio of the signal to statistical fluctuations of background:

\[ SS_a = \frac{N_S}{\sqrt{N_B}} = \frac{\mathcal{L} \Delta \sigma_a}{\sqrt{\mathcal{L} \sigma_B}} \quad (a = FB \text{ or } T), \]

where \( \mathcal{L} \) is the effective luminosity integral for the \( \gamma p \) collisions. We see no reasons for
the recording of scattered electrons or protons. Therefore, for the sake of definiteness,
we take the luminosity \( \mathcal{L} = 0.1 \text{ pb}^{-1} \), implying that various detection efficiencies are
absorbed into this quantity. (This \( \mathcal{L} \) corresponds approximately to one million detected
events under the \( \rho \) peak.)

Let us discuss first values of statistical significance for cross sections averaged over
small interval of \( M^2 \pm \Delta M^2 \), \( SS(M^2) \) (note that \( SS \) under interest does not coincide
with \( \int SS(M^2) dM^2 \)). According to eqs. (16) and (24) for both FB and T asymmetries,

\[ SS_a(M^2) \propto \frac{I_{12}(M^2)}{|D_1(M^2)|} = \frac{\text{Im}(D_2^* D_1 e^{i\delta_{\text{PO}}})}{|D_1|} \leq |D_2|. \]

Therefore the largest values of this \( SS(M^2) \) are located within the \( f_2 \) peak. It is il-
lustrated by Fig. 2, where local values of these \( SS_a(M^2) \) are shown in arbitrary units. Hence, to obtain the best value of \( SS \), we consider signals and background integrated

\[ M_f - \Gamma_f < M < M_f + \Gamma_f. \]

Estimate (26) shows that the influence of nonresonant background as well as tails of other
resonances in the Pomeron channel changes our estimates of \( SS \) only weakly. Certainly,
such very estimate for interference with \( S \)-wave \( \pi^+\pi^- \) final states produced by odderon
will show that the corresponding signals are located near \( f_0(600) \) and \( f_0(980) \) peaks, and
they are negligible at \( M > 1100 \text{ MeV} \).

\[ ^2 \text{ The bump at } M \approx m_\rho \text{ is not very useful, since the asymmetry in this region is also affected by the}\]
\[ \text{interference with scalar resonances.} \]
With $\sigma_f = 1$ nb, integrating the asymmetries within the region yields

$$\Delta\sigma_{FB} = 15.7 \text{ nb}; \quad \Delta\sigma_T = 6.6 \text{ nb},$$

which must be compared to the background cross section

$$\sigma_B = 428 \text{ nb for } FB, \quad \sigma_B = 174 \text{ nb for } T.$$  \hspace{1cm} (28)

The statistical significance of the asymmetries for the luminosity $L = 0.1 \text{ pb}^{-1}$ equals

$$SS_{FB} \approx 7.5; \quad SS_T \approx 5.0.$$ \hspace{1cm} (30)

These numbers are still very promising, despite the fact that we used very cautious estimates both for $f_2$ photoproduction cross section and for the integrated luminosity. This offers certain confidence that the odderon signal is indeed within the reach of the current experiments even with very low value for the odderon induced cross section and luminosity used here for the estimates.

6 Discussion

The charge asymmetry of dipions in diffractive photoproduction $\gamma + p \rightarrow \pi^+\pi^- + p'$ emerges as a very attractive signature for the odderon exchange. In the absence of competitive mechanisms, an observation of such an asymmetry will be an unambiguous discovery of the odderon.

As far as the discovery potential of the HERA experiments on $ep$ collisions is concerned, the main contribution to the dipion comes from the quasireal photons. We see no reasons for either tagged photons or going to deep inelastic regime with strongly suppressed cross section. One must simply focus on the dipion final states separated by a large rapidity gap from the scattered proton or its low mass excitations.

With modest estimates for the $f_2$ production cross section ($\sigma(\gamma p \rightarrow f_2 p) \geq 1$ nb), the statistical significance still turns out high ($SS \geq 5 \div 8$) with very cautious estimate for effective $\gamma p$ luminosity, $L_{\gamma p} = 0.1 \text{ pb}^{-1}$. This suggests that the odderon signal is definitely within the reach of the HERA experiments.

Certainly, the observation of charge asymmetry of pions in the dipion mass region below 1.1 GeV will be also unambiguous signal of the odderon, and it can be even larger than that estimated above. However, a detailed calculation in this mass region demands knowledge of specific models e.g. for the interaction of odderon to different $f_0$ resonances, which cannot be developed unambiguously now (for example, this coupling can be reduced strongly if some of these resonances have large admixture of gluonium). Therefore, in this paper we only suggest to look for charge asymmetry for the discovery of the odderon. The subsequent detailed analysis of the charge asymmetry at $M < 1.1$ GeV (predominantly transverse) can be used for the study of the coupling of different $f_0$'s to the odderon. It can be the subject of separate paper(s).

The analysis of charge asymmetries proposed here is a very general tool for extracting new information whenever the production mechanism involves the both $C = +1$ and $C = -1$ exchanges (see 24 for other problems).

- **Photon polarization dependence.** If the photons are supposed to arise from electrons in $ep$ collider, then, in the case of unpolarized initial electrons, the photons will be linearly polarized in the electron scattering plane. This modification changes neither
the value nor the shape of the charge asymmetry, and only introduces an overall factor that depends on $\psi$.

For the longitudinally polarized electrons, the photons acquire circular polarization. In this case the effective overlap function acquires additional $\phi$ dependence, which leads to non-universal helicity-sensitive azimuthal-charge asymmetry.

• **A brief comment on the breaking of quark–hadron duality.** Our optimism on the $\pi^+\pi^-$ production is based on the observation that at the hadronic level the Pomeron–odderon interference is strongly enhanced by the final-state interaction. The study of the charge asymmetry in the $c\bar{c}$ final state for the odderon discovery was proposed by [18]. Certainly, the observation of this asymmetry requires observation of both $c$ quarks (i.e. $D$ or $D^*$ mesons and pions). Near the open charm threshold the essential part of these final states is given by $DD$ or $DD^*$ or $D^*D^*$ states with well defined effective mass, which allows one to discuss charge asymmetry in a definite form. In this region, such final states come from decays of a number of known C-odd and yet undiscovered C–even $c\bar{c}$ resonances (effect of FSI). The overlapping of these resonances should enhance the effect essentially as compared to that at the partonic level at least near the open charm threshold. This is the same FSI effect that was discussed above for the dipions, and it exemplifies a general statement that in the charge asymmetry the quark–hadron duality can be badly broken even for heavy quarks.

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**Appendix: Non-odderon contributions**

The two sources of the non-odderon $C$-even diffractive dipion production are the $\omega$ and $\rho$ reggeon exchanges, and the photon exchange — the Primakoff effect. The estimates below show that the odderon induced effect with $\sigma_f \geq 1$ nb with cuts [20] cannot be mimicked by these non-odderon mechanisms.

• **The $\rho/\omega$ reggeon exchange contributions** are estimated via the $\gamma p \rightarrow f_2 p$ at a fixed target photon energy $\nu_0 = 6$ GeV ($s_0 = 12.1$ GeV$^2$) equals $\sigma_{\rho/\omega}(\gamma p \rightarrow f_2 p) \approx 120$ nb (see [37]). Taking the $\rho/\omega$ reggeon trajectory from [31], this cross section can be extrapolated to higher energies as $\sigma_{\rho/\omega}(s) \approx \sigma_{\rho/\omega}(s_0) \cdot (s_0/s)^{0.9}$. At a typical HERA energy $\sqrt{s} = 200$ GeV this yields

$$\sigma_{\rho/\omega}(\gamma p \rightarrow f_2 p)_{\text{HERA}} \approx 0.15 \text{ nb}. \quad (31)$$

• **For the Primakoff one-photon exchange $f_2$ production** the cross section can be estimated reliably in the equivalent photon approximation in terms of the two-photon width of resonance $\Gamma_{\gamma\gamma}$ [38]:

$$d\sigma_{Pr} = \frac{8\pi \alpha \Gamma_{\gamma\gamma}(2J+1)}{M^3} \cdot \frac{\Delta^2 d\Delta}{(\Delta^2 + Q_m^2)^2} \quad \text{with} \quad Q_m^2 = \left(\frac{m_p M^2}{s}\right)^2. \quad (32)$$

The integration over $\Delta^2$ is effectively limited from above by the proton form-factor at $\Delta^2 \approx m_p^2$. It leads to a large logarithm $L = 2 \ln(m_p s/m_p M^2) \approx 15$ in the total cross section, $\sigma_{tot}^{Pr} \approx 8$ nb. This large cross section is concentrated strongly near forward direction. If we impose the lower cut of $|\Delta|^2 = B_{\rho}^{-1} \approx 0.1$ GeV$^2$, then the logarithmic
enhancement goes down more than one order in the magnitude, yielding the Primakoff background cross section \( \lesssim 0.5 \) nb in the observation region.

It is useful to note that Primakoff effect can produce \( f_2 \) meson only in the states \( \lambda_f = 2, 0 \) (and it is experimentally confirmed that the \( \lambda_f = 2 \) dominates). Since SCHC holds for \( \rho \) meson production, it means, according to the above analysis, that the main charge asymmetry from Primakoff–Pomeron (\( \text{PIP} \)) interference will be transverse (T) while the \( \text{PIP} \) FB asymmetry is low, especially at small \(|\Delta|\). Therefore, in the study of FB asymmetry under interest one can use much lower cut in \( \Delta \) \( (20) \).

This estimate together with \( (31) \) sets an approximate lower limit of the odderon cross section, for which the observed charge asymmetry must be regarded as an unambiguous evidence for the odderon.

- As we see, in the region we are interested \( (20) \), the Primakoff contribution is expected to be smaller than the odderon signal. However, at very low momentum transfer, \(|\Delta| < 50 \div 100 \text{ MeV}\), the Primakoff contribution will dominate over the odderon one. Thus, the data from this region can give us information about the phase of the forward \( \gamma p \to \rho p \) amplitude ("Pomeron phase") \( (32) \).

- Finally, we mention ref. \( (40) \) which discusses charge asymmetry in electroproduction of dipions. Authors consider the C–even dipion production only via Primakoff mechanism and the C–odd dipion production only via the bremsstrahlung mechanism. The latter is negligible compared to the dominant Pomeron exchange completely overlooked in \( (30) \).

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