About

- This is Part 2 of a two-part review paper titled “Data-driven Decision Making in Power Systems with Probabilistic Guarantees: Theory and Applications of Chance-constrained Optimization” by Xinbo Geng and Le Xie, Annual Reviews in Control (under review).
- Part 1 “Data-driven Decision Making with Probabilistic Guarantees (Part I): A Schematic Overview of Chance-constrained Optimization” is available at arXiv:1903.10621.
- Part 2 “Data-driven Decision Making in with Probabilistic Guarantees (Part II): Applications of Chance-constrained Optimization in Power Systems” is available at arXiv.
- The Matlab Toolbox ConvertChanceConstraint (CCC) is available at https://github.com/xb00dx/ConvertChanceConstraint-ccc.

Please let us know if we missed any critical references or you found any mistakes in the manuscript.

Recent Updates

04/2019 Part II uploaded to Arxiv.
04/2019 More CVaR-based (Convex Approximation) results are added in Part 1.
02/2019 Toolbox published at https://github.com/xb00dx/ConvertChanceConstraint-ccc

We are still working on the toolbox website and documents.
Data-driven Decision Making with Probabilistic Guarantees (Part II):
Applications of Chance-constrained Optimization in Power Systems

Xinbo Geng, Le Xie
Texas A&M University, College Station, TX, USA.

Abstract
Uncertainties from deepening penetration of renewable energy resources have posed critical challenges to the secure and reliable operations of future electric grids. Among various approaches for decision making in uncertain environments, this paper focuses on chance-constrained optimization, which provides explicit probabilistic guarantees on the feasibility of optimal solutions. Although quite a few methods have been proposed to solve chance-constrained optimization problems, there is a lack of comprehensive review and comparative analysis of the proposed methods. Part I of this two-part paper reviews three categories of existing methods to chance-constrained optimization: (1) scenario approach; (2) sample average approximation; and (3) robust optimization based methods. Data-driven methods, which are not constrained by any particular distributions of the underlying uncertainties, are of particular interest. Part II of this two-part paper provides a literature review on the applications of chance-constrained optimization in power systems. Part II also provides a critical comparison of existing methods based on numerical simulations, which are conducted on standard power system test cases.

Keywords: data-driven, power system, chance constraint, probabilistic constraint, stochastic programming, robust optimization, chance-constrained optimization.

1. Introduction
Real-time decision making in the presence of uncertainties is a classical problem that arises in many contexts. In the context of electric energy systems, a pivotal challenge is how to operate a power grid with an increasing amount of supply and demand uncertainties. The unique characteristics of such operational problem include (1) the underlying distribution of uncertainties is largely unknown (e.g. the forecast error of demand response); (2) decisions have to be made in a timely manner (e.g. a dispatch order needs to be given by 5 minutes prior to the real-time); and (3) there is a strong desire to know the risk that the system is exposed to after a decision is made (e.g. the risk of violating transmission constraints after the real-time market clears). In response to these challenges, a class of optimization problems named “chance-constrained optimization” has received increasing attention in both operations research and practical engineering communities.

The objective of this article is to provide a comprehensive and up-to-date literature review on the engineering implications of chance-constrained optimization in the context of electric power systems.

1.1. Contributions of This Paper
The main contributions of this paper are threefold:

1. We provide a detailed tutorial on existing algorithms to solve chance-constrained programs and a survey of major theoretical results. To the best of our knowledge, there is no such review available in the literature;

2. We provide a comprehensive review on the applications of chance-constrained optimization in power systems, with focus on various interpretations of chance constraints in the context of power engineering;

3. We implement all the reviewed methods and develop an open-source Matlab toolbox (ConvertChanceConstraint), which is available on Github. We also provide a critical comparison of existing methods based numerical simulations on IEEE standard test systems.

1.2. Organization of This Paper
The remainder of this paper is organized as follows. Section provides a comprehensive review on applications of CCO in power systems. The structure and usage of the Toolbox ConvertChanceConstraint is in Section. Section also conducts numerical simulations and compares existing approaches to solving CCO problems. Concluding remarks are in Section

1.3. Notations
The notations in this paper are standard. All vectors and matrices are in the real field \( \mathbb{R} \). Sets are in calligraphy fonts, e.g. \( S \). The upper and lower bounds of a variable \( x \) are denoted by \( \bar{x} \) and \( \underline{x} \). The estimation of a random variable \( \epsilon \) is \( \hat{\epsilon} \). We use \( I_n \) to denote an all-one vector in \( \mathbb{R}^n \), the subscript \( n \) is sometimes omitted for simplicity. The absolute value of vector \( x \) is

\[ \|x\| = \sqrt{x^T x} \]
2. Applications in Power Systems

A pivotal task in modern power system operation is to maintain the real-time balance of supply and demand while ensuring the system is low-cost and reliable. This pivotal task, however, faces critical challenges in the presence of rapid growth of renewable energy resources. Chance-constrained optimization, which explicitly models the risk that the system is exposed to, is a suitable conceptual framework to ensure the security and reliability of a power system under uncertainties.

There is a large body of literature adopting CCO for power system applications. Figure 1 presents some existing applications of CCO in power systems. In the following sections, we introduce three important applications of CCO in power systems: security-constrained economic dispatch (SCED) (Section 2.1), security-constrained unit commitment (SCUC) (Section 2.2) and generation and transmission expansion (Section 2.3).

Figure 1 also presents a feed-forward decision making framework for power system operations. The feed-forward framework partitions the overall decision making process into several time segments. The long-term decisions (e.g. generation expansion) are fed into shorter-term decision making processes (e.g. unit commitment). The shorter-term decisions (e.g. generation commitment from SCUC) have direct impacts on real-time operations (e.g. dispatch results in SCED). It is worth noting that as time draws closer to the actual physical operation, information gets much sharper (Xie et al., 2011).

2.1. Security-Constrained Economic Dispatch

2.1.1. Deterministic SCED

Security-constrained Economic Dispatch (SCED) lies at the center of modern electricity markets and short-term power system operations. It determines the most cost-efficient output levels of generators while keeping the real-time balance between supply and demand. Different variations of the SCED problem are all based on the direct current optimal power flow (DCOPF) problem. We present a typical form of DCOPF with wind generation.

\[
\text{(det-DCOPF): } \min_{\mathbf{g}} c(\mathbf{g}) \tag{1a}
\]

s.t.
\[
\mathbf{1}^\top \mathbf{g} = \mathbf{1}^\top \mathbf{d} - \mathbf{1}^\top \hat{\mathbf{w}} \tag{1b}
\]

\[
f(\hat{\mathbf{w}}, \mathbf{\varphi}) = H_\mathbf{d} (\mathbf{g} - \mathbf{1}^\top \hat{\mathbf{w}} \mathbf{\eta}) - H_\mathbf{d} \hat{\mathbf{w}}\mathbf{\eta} + H_\mathbf{d} \hat{\mathbf{w}} \tag{1c}
\]

\[
\mathbb{P} \left( \mathbb{P} \left( \sum_{i=1}^{\mathbf{g}} f_i \right) \leq \tilde{f} \right) \tag{1d}
\]

\[
\mathbb{P} \left( \mathbb{P} \left( \sum_{i=1}^{\mathbf{g}} f_i \right) \leq \tilde{f} \right) \tag{1e}
\]

The decision variables are generation output levels \( \mathbf{g} \in \mathbb{R}^n \). The objective of (det-DCOPF) is to minimize total generation cost \( c(\mathbf{g}) \), while ensuring total generation equates total net demand \( \mathbf{1}^\top \mathbf{d} \) and generation capacity limits \( \mathbf{1}^\top \mathbf{g} \leq \mathbf{1}^\top \hat{\mathbf{w}} \). Constraints include transmission line flow limits \( \mathbf{1}^\top \mathbf{g} \leq \mathbf{1}^\top \hat{\mathbf{w}} \) and generation capacity limits \( \mathbf{1}^\top \mathbf{g} \leq \mathbf{1}^\top \hat{\mathbf{w}} \). Transmission line flows \( f \) are calculated using (1c), in which \( H_\mathbf{d} \) is the power transfer distribution factor (PTDF) matrix, and \( H_\mathbf{d} \in \mathbb{R}^{n \times n} \) denotes the submatrix formed by the columns of \( H \) corresponding to generators (loads, wind farms). (1) utilizes the expected wind generation or wind forecast \( \hat{\mathbf{w}} \), we refer to (1) as deterministic DCOPF (det-DCOPF) since no uncertainties are being considered.

2.1.2. Chance-constrained SCED

Many researchers advance (det-DCOPF) towards a chance-constrained formulation with wind uncertainties. A representative formulation is (2), which appears in a majority of the existing literatures, e.g. (Bienstock et al., 2014; Vrakopoulou et al., 2013a).

\[
\text{(cc-DCOPF): } \min_{\mathbf{g}} c(\mathbf{g}) \tag{2a}
\]

s.t.
\[
\mathbf{1}^\top \mathbf{g} = \mathbf{1}^\top \mathbf{d} - \mathbf{1}^\top \hat{\mathbf{w}} \tag{2b}
\]

\[
f(\hat{\mathbf{w}}, \mathbf{\varphi}) = H_\mathbf{d} (\mathbf{g} - \mathbf{1}^\top \hat{\mathbf{w}} \mathbf{\eta}) - H_\mathbf{d} \hat{\mathbf{w}}\mathbf{\eta} + H_\mathbf{d} \hat{\mathbf{w}} \tag{2c}
\]

\[
\mathbb{P} \left( \mathbb{P} \left( \sum_{i=1}^{\mathbf{g}} f_i \right) \leq \tilde{f} \right) \tag{2d}
\]

\[
\mathbb{P} \left( \mathbb{P} \left( \sum_{i=1}^{\mathbf{g}} f_i \right) \leq \tilde{f} \right) \tag{2e}
\]

Unlike (det-DCOPF) using wind forecast \( \hat{\mathbf{w}} \), chance-constrained SCED (cc-DCOPF) explicitly models wind generation as a random vector \( \mathbf{w} \in \mathbb{R}^n \). The wind generation \( \mathbf{w} = \hat{\mathbf{w}} + \tilde{\mathbf{w}} \) is decomposed into two components: the deterministic wind forecast value \( \hat{\mathbf{w}} \in \mathbb{R}^n \) and the uncertain forecast error \( \tilde{\mathbf{w}} \in \mathbb{R}^n \). To guarantee the real-time balance of supply and demand, (cc-DCOPF) introduces an affine control policy \( \mathbf{\eta} \in [-1, 1]^n \) to proportionally allocate total wind fluctuations \( \mathbf{1}^\top \tilde{\mathbf{w}} \) to each generator. It is easy to verify that constraints (2b) and (2c) imply the supply-demand balance in the presence of

---

\(^2\)Wind generation is treated as negative loads.
wind uncertainties, i.e.

\[ \mathbf{1}^\top (g - \mathbf{1}^\top \tilde{\mathbf{w}}) = \mathbf{1}^\top d - \mathbf{1}^\top (\tilde{\mathbf{w}} + \tilde{\mathbf{w}}), \]  

(3)

The affine policy vector \( \eta \in \mathbb{R}^n \) is sometimes referred as participation factor or distribution vector. \( \text{Vrakopoulou et al.} \, 2013a \). The (joint) chance constraint \( \Omega(z) \) constrains the transmission flow and generation within their capacities with high probability \( 1 - \epsilon \) in the presence of wind uncertainties.

For simplicity, we only account for the major source of uncertainties (i.e. wind) in the real-time. Many references provide more complicated formulation of (cc-DCOPF), e.g. considering joint uncertainties from load and wind \( \text{Doostizadeh et al.} \, 2016 \), \( \text{Hlipfordt et al.} \, 2017 \), and contingencies of potential generator or transmission line outages. \( \text{Roald et al.} \, 2015a \).

There exist a few different but similar formulations of (cc-DCOPF). In general, policies of any form could help balance supply with demand under uncertainties. The affine policy in (cc-DCOPF) is the simplest choice and lead to optimization problems that are easy to solve. There are other papers applying different forms of policies, e.g. \( \text{Jabr} \, 2013 \) introduces a matrix form of the affine policy \( Y \in \mathbb{R}^{\eta \times m} \), which specifies the corrective control of each generator on each wind farm. (cc-DCOPF) is a single snapshot dispatch problem, it is straightforward to extend it to a multi-period or look-ahead dispatch problem \( \text{Modarresi et al.} \, 2018 \), \( \text{Vrakopoulou et al.} \, 2013a \). Many papers evaluate the impacts of new elements in modern power systems, such as demand response \( \text{Ming et al.} \, 2017 \), \( \text{Zhang et al.} \, 2017a \), ambient temperatures and meteorological quantities \( \text{Bucher et al.} \, 2013 \), and frequency control \( \text{Li and Mathieu} \, 2015 \), \( \text{Zhang et al.} \, 2017a \).

2.1.3. Solving cc-DCOPF

Table 1 summarizes various methods to solve (cc-DCOPF). The most popular one consists of two steps: (i) decomposing the joint chance constraint \( \Omega(z) \) into individual ones \( \Pi_i(f_i(x, \xi) \leq 0) \geq 1 - \epsilon_i, i = 1, 2, \ldots, m \); (ii) deriving the deterministic equivalent form of each individual chance constraint by making the Gaussian assumption. More technical details of this method are in Section 3.2 of (Geng and Xie 2019a). This method is taken by many researchers for its simplicity and computationally tractable reformulation. Although the Gaussian assumption enjoys the law of large numbers, it is often an approximation or even doubtful assumption. For example, (Hodge and Milligan 2011) shows that the wind forecast error is better represented by Cauchy distributions instead of Gaussian ones. The first step of this method is to decompose a joint chance constraint \( \Omega_\epsilon(f(x, \xi) \leq 0) \geq 1 - \epsilon \) into individual ones. As discussed in Section 2.2 and 7.4.1 of (Geng and Xie 2019a), this step often introduces conservativeness because of the limitation of Bonferroni inequality. The level of conservativeness could be significant when the number of constraints \( m \) is large, which is typically the case in power systems.

The scenario approach is another commonly-accepted method. It provides rigorous guarantees on the quality of the solution and does not assume the distribution is Gaussian or any particular type. Most papers adopting the scenario approach apply the a-priori guarantees (e.g. Theorem 5 and 6 in (Geng and Xie 2019a) on (cc-DCOPF) and verify the a-posteriori feasibility of solutions through Monte-Carlo simulations. One common observation is that the solution \( x^* \) is often quite conservative, i.e. \( \forall (x^*_i) \ll \epsilon \). One major source of conservativeness is the loose sample complexity bounds \( N \). Since (cc-DCOPF) is convex, Theorem 4 in (Geng and Xie 2019a) states that the number of decision variables \( n \) is an upper bound of the number of support scenarios \( |S| \) or Helly’s dimension \( h \). This upper bound, as pointed out in (Modarresi et al. 2018), is indeed very loose. (Modarresi et al. 2018) reported only \( \sim 5 \) support scenarios for a chance-constrained look-ahead SCED problem with thousands of decision variables. By exploiting the structural features of (cc-DCOPF), the sample complexity bound \( N \) can be significantly improved. Unfortunately, only (Modarresi et al. 2018) and (Ming et al. 2017) followed this path to reduce conservativeness.

There are also many papers utilizing the robust optimization related methods to solve (cc-DCOPF). (Jiang et al. 2012) constructs uncertainty sets with the help of probabilistic guarantees in (Bertsimas and Sim 2004). References (Summers et al. 2014) incorporate the convex approximation framework and compare different choices of generating functions \( \phi(z) \) on (cc-DCOPF). Although there are no explicit forms of chance

\[ \text{Geng and Xie}, 2019a, p. } 4 \]
the most important procedures in power system day-ahead or contingency planning. In this paper, we focus on deterministic unit commitment (SCUC) except the chance constraint (5b)-(5d). In (cc-SCUC), the joint chance constraint (5b) is sometimes written as two (joint) chance constraints:

\[
P\left(1^T g^{ij} \geq 1^T \tilde{d} + \tilde{d} - 1^T \tilde{w} + \tilde{w}^*\right) 
\leq \epsilon, \quad k \in [0,n], t \in [1,n] 
\]

An important metric to evaluate power system reliability is the loss of load probability (LOLP), which is defined as the probability that the total demand is not met by the total generation. This can be found in [Allan and others, 2013; Qiu et al., 2016]. It can...
be seen that (6a) is essentially ensuring the value of LOLP will not exceed a desired level \( \epsilon \). Similarly, we could define the concept of transmission line overload probability (TLOP) \( \text{TLOP} \) (Wu et al., 2014). Then (6b) is the same as \( TLOP \leq \epsilon \) for each time period \( t \).

Some papers (e.g., Wu et al., 2014) further break down the joint chance constraint (6a)–(6b) into individual chance constraints (7a)–(7b), which can be interpreted as constraints on LOLP or TLOP for each time period \( t \).

\[
\begin{align*}
\mathbb{P}\{1^T g^k k \geq &\ 1^T (d^f t + d^g t + \hat{w} t + \hat{v} t) \} \geq 1 - \epsilon_{LOLP} t, \\
&k \in [0, n_k], \ t \in [1, n_t]. \quad (7a)\\
\mathbb{P}\{f^k k \leq &\ H^k g^k k - H^k g^k k (d^f t + d^g t) + H^k g^k k (\hat{w} t + \hat{v} t) \leq \tilde{T} \} \geq 1 - \epsilon_{TLOP} t, \\
&k \in [0, n_k], \ t \in [1, n_t]. \quad (7b)
\end{align*}
\]

Another interesting set of chance constraints in cc-SCUC guarantees the utilization ratio of wind generation greater than a desired threshold with high probability \( 1 - \epsilon \) (Wang et al., 2012, 2013; Zhao et al., 2014). Different variations of the chance constraint on wind utilization ratios can be found in (Wang et al., 2012).

### 2.2.3. Solving Chance-constrained SCUC

As mentioned in Section 2.2.2, there is no uniform formulation of chance-constrained SCUC. Many references in Table 1 concentrate on exploring alternative formulations of cc-SCUC. Therefore theoretical guarantees or quality solution is not a major concern.

Among all the reviewed methods, sample average approximation is commonly used when solving chance-constrained SCUC (Wang et al., 2012, 2013; Zhao et al., 2014; Tan and Shaaban, 2016; Bagheri et al., 2017; Zhang et al., 2017c). Section 3.2 of (Geng and Xie, 2019a) shows that SAA reformulates (CCO) to a mixed integer program, which is difficult to solve in general. Many references apply various techniques from integer programming to speed up the computation, e.g. (Zhao et al., 2014; Jiang et al., 2016).

Section 5.2 of (Geng and Xie, 2019a) shows that there is no upper bound on the number of support scenarios for non-convex problems in general. Thus, a majority of results of the scenario approach cannot be directly applied on cc-SCUC. Recently, (Campi et al., 2018) extends the a-posteriori guarantees of the scenario approach towards non-convex problems. (Geng et al., 2019) adopts the approach in (Campi et al., 2018) and shows the possibility to apply the theoretical results of the scenario approach on (cc-SCUC). It is worth mentioning that some theoretical results in robust optimization still apply in spite of the non-convexity of SCUC from integer variables \((\cdot', \cdot', \cdot')\), e.g. (Bertsimas et al., 2018). This could be an interesting direction to explore.

### 2.3. Generation and Transmission Expansion

Generation and transmission expansion (the expansion problem in short) is a critical component in long-term power system planning exercises. The expansion problem answers the following critical questions: (i) when to invest on new elements such as transmission lines and generators in the system; (ii) what types of new elements are necessary; and (iii) how much capacity is needed and where the best locations would be for those new elements. A typical objective of the expansion problem is to minimize (i) total cost of investment in new generators and transmission line; (ii) environmental impacts; and (iii) cost of generation. Constraints of the expansion problem often include total or individual costs within budget, capacity constraint, reliability requirement, supply-demand balance, power flow equations, and operation requirements such as generation or transmission limits.

The expansion problem typically needs to deal with uncertainties from demand, generation and transmission outages, and renewables. Chance constraints often appear as requirements on reliability metrics such as LOLP (6a) and TLOP (6b).

Among all the papers incorporating chance constraints in the expansion problem, a majority of them assume the underlying distribution is Gaussian and derive the second order cone equivalent form (Section 3.2, (Geng and Xie, 2019a)), e.g. (Sanghvi et al., 1982; Lopez et al., 2007; Mazadi et al., 2009; Manickavasagam et al., 2015). A few papers design its own simulation-based iterative algorithms because of complicated problem formulations, e.g. (Yang and Wen, 2005; Qiu et al., 2016). Although Monte-Carlo simulation is typically performed to evaluate the actual feasibility, there is no rigorous guarantees on these results.

Similar to the chance constrained DCOPF problem, deriving deterministic equivalent forms is the most popular choice. Considering the expansion problem is usually ultra-large-scale and involves lots of integer variables, the simplicity of deterministic equivalent form becomes particularly attractive. Additional pros and cons of this approach are analyzed in Section 2.1.3.

Similar to chance-constrained SCUC, the expansion problem includes many integer variables and is non-convex in nature. As discussed in Section 2.2.3, the scenario approach and sample average approximation can still be applied on the expansion problem. Because of the size of the expansion problem, the required sample complexity could be astronomical, which lead to major computational issues. Although the scenario approach and sample average approximation could provide better theoretical guarantees, it is essential to overcome the major obstacles in computation to apply some better methods on the expansion problem.

### 3. Numerical Simulations

#### 3.1. Simulation Settings

Chance-constrained DCOPF (2) serves as a benchmark problem for a critical comparison of solutions to (CCO). We provide numerical solutions of cc-DCOPF on two test systems: a 3-bus system and the IEEE 24-bus RTS test system.

The 3-bus system is a modified version of the 3-bus system in (Lesieutre et al., 2011). The major difference is the removal of the load at bus 2 and the synchronous condensor at bus 3 in order to visualize the feasible region and the space of uncertainties. The original 3-bus system “case3sc.m” is available in

---

**References**

1. Bertsimas, D., Lyar, J., & Teboulle, M. (2018).
2. Sanghvi, J. V., et al. (1982).
3. Lopez, C. J., et al. (2007).
4. Mazadi, A. R., et al. (2009).
5. Manickavasagam, R., et al. (2015).
6. Yang, Y., & Wen, P. (2005).
7. Qiu, J., et al. (2016).
8. Wang, L., et al. (2012, 2013).
9. Zhao, X., et al. (2014).
10. Geng, Y., & Xie, G. (2019a).
11. Zhao, X., et al. (2014).
12. Geng, Y., & Xie, G. (2019a).
13. Ben-Tal, A., et al. (2018).
14. Bertsimas, D., Lyar, J., & Teboulle, M. (2018).
15. Lesieutre, B. C., et al. (2011).
the Matpower toolbox [Zimmerman et al. (2011)]. The modified system in this paper can be found in the examples of CCC [github.com/xb00dx/ConvertChanceConstraint-ccc/tree/master/examples]. For simplicity, we only consider uncertainties of loads, which is modeled as Gaussian variables with 5% standard variation.

The 24-bus system in this paper is a modified version of the IEEE 24-bus RTS benchmark system [Grigg et al. (1999)]. The transmission line capacities are set to be 60% of the original capacities. We conduct two sets of simulations on the 24-bus system with different distributions of uncertainties. The first one is similar with the 3-bus case, nodal loads are modeled as independent Gaussian variables with 5% standard deviation. The second one models the errors of nodal load forecasts as independent beta-distributed random variables, with parameters $\alpha = 25.2414$ and $\beta = 25.2692$.

Ten Monte-Carlo simulations are conducted on every method to examine the randomness of solutions. For the 3-bus case, each Monte-Carlo simulation uses 100 i.i.d samples to solve cc-DCOPF. 2048 points are used in each run to solve (cc-DCOPF) of the 24-bus system. The returned solutions are evaluated on an independent set of 10^4 points.

We use Gurobi 7.10 ([Gurobi Optimization] [2016]) to get results of scenario approach and sample average approximation. Cplex 12.8 is used to solve (CCO) with robust counterpart and convex approximation.

3.2. Feasible Region

Although there are four variables ($g_1, g_2, \eta_1, \eta_2$) in (cc-DCOPF) for the 3-bus system, only two of them (e.g. $g_1$ and $\eta_1$) are free variables because of constraints (2b) and (2e). Thanks to this, we could visualize the violation probability function $V(g_1, \eta_1)$ in the 3D space. The function $V(g_1, \eta_1)$ is evaluated over a grid of 65536 points in the $(g_1, \eta_1)$ space, the value of $V(g_1, \eta_1)$ at every point is estimated using 1000 i.i.d realizations of uncertainties $d$. Figure 2 shows $V(g_1, \eta_1)$. Based on the estimation of $V(g_1, \eta_1)$, we could also visualize the feasible region $F_\epsilon = \{x : V(x) \leq \epsilon\}$, which is shown in Figure 3.

3.3. Simulation Results

We solve cc-DCOPF on the 3-bus system with eight different methods: (1) scenario approach with prior guarantees, (SA:prior, Corollary 1 in (Geng and Xie 2019a)); (2) scenario approach with posterior guarantees (SA:posterior, Theorem 7 in (Geng and Xie 2019a)); (3) sample average approximation, where $N$ and $\epsilon$ are chosen based on the sampling and discarding Theorem (SAA:s&d, Theorem 9 in (Geng and Xie 2019a)); (4-7) Robust counterfeit with different uncertainty sets specified in Theorem 13 in (Geng and Xie 2019a): box (RC:box), ball (RC:ball), ball-box (RC:ball-box) and budget (RC:budget) uncertainty sets; (8) convex approximation with Markov bound (CA:markov, Theorem 11 and Proposition 5 in (Geng and Xie 2019a)).

We first examine the feasibility of the returned solutions from eight algorithms. Figure 5 and 6 show the out-of-sample violation probabilities $\hat{\epsilon}$ versus desired $\epsilon$ in the setting. The green dashed lines in Figure 5 and 6 denote the ideal case where $\hat{\epsilon} = \epsilon$. Any points above the green dashed line indicate infeasible solutions that $V(x) > \epsilon$. Clearly all methods return feasible solutions (with high probability) to (CCO). From Figure 5, sample average approximation and convex approximation are less conservative than other methods. However, it is worth noting that when $\epsilon$ is small (e.g. $10^{-2}$), the data-driven approximation of CVaR (Proposition 5 in (Geng and Xie 2019a)) does not necessarily give a safe approximation to (CCO) [Chen et al. 2010]. The robust counterpart methods are typically 10 ~ 100 times more conservative than other methods, as illustrated in the comparison of Figure 6a with Figure 6b. The conservativeness could be significantly reduced by better construction.
of uncertainty sets, e.g. Chen et al. (2010); Bertsimas et al. (2018). Among four different choices of uncertainty sets, the ball-box set is the least conservative one, which combines the advantages of the ball and box uncertainty sets.

Figure 5 present the results of the 24-bus system with Gaussian distributions. Simulation results of the beta distribution are in Figure 8. Observations from Figure 8 are similar with the case of Gaussian distributions. Every method behaves more conservative in the case of beta distributions than the case of Gaussian distributions. It is worth noting that the RO-based methods (RC:box, RC:ball, RC:ball-box in Figure 9) are so conservative that lead to zero empirical violation probability \( \hat{\epsilon} \).

4. Concluding Remarks

This paper consists of two parts. The first part presents a comprehensive review on the fundamental properties, key theoretical results, and three classes of algorithms for chance-constrained optimization. An open-source MATLAB toolbox ConvertChanceConstraint is developed to automate the process of translating chance constraints to compatible forms for mainstream optimization solvers. The second part of this paper presents three major applications of chance-constrained optimization in power systems. We also present a critical comparison of existing algorithms to solve chance-constrained programs on IEEE benchmark systems.

Many interesting directions are open for future research. More thorough and detailed comparisons of solutions to (CCO) on various problems with realistic datasets is needed. In terms of theoretical investigation, an analytical comparison of existing solutions to chance-constrained optimization is necessary to substantiate the fundamental insights obtained from numerical simulations. In terms of applications, many existing results can be improved by exploiting the structural properties of the problem to be solved. The application of chance-constrained optimization in electric energy systems could go beyond operational planning practices. For example, it would be worth investigating into the economic interpretation of market power issues through the lens of chance-constrained optimization.

References

Allan, R.N., others, 2013. Reliability evaluation of power systems. Springer Science & Business Media.

Bagheri, A., Zhao, C., Guo, Y., 2017. Data-driven chance-constrained stochastic unit commitment under wind power uncertainty, in: Power & Energy Society General Meeting. 2017 IEEE, IEEE. pp. 1–5.

Bent, R., Bienstock, D., Cherkov, M., 2013. Synchronization-aware and algorithm-efficient chance constrained optimal power flow, in: Bulk Power System Dynamics and Control-IX Optimization, Security and Control of the Emerging Power Grid (IREP), 2013 IREP Symposium, IEEE, pp. 1–11.

Bertsimas, D., Gupta, V., Kallus, N., 2018. Data-driven robust optimization. Mathematical Programming.

Bertsimas, D., Litvinov, E., Sun, X.A., Zhao, J., Zheng, T., 2013. Adaptive robust optimization for the security constrained unit commitment problem. IEEE Transactions on Power Systems 28, 52–63.

Bertsimas, D., Sim, M., 2004. The price of robustness. Operations research 52, 35–53.

Bienstock, D., Cherkov, M., Harnett, S., 2013. Robust modeling of probabilistic uncertainty in smart grids: Data ambiguous chance constrained optimum power flow, in: Decision and Control (CDC), 2013 IEEE 52nd Annual Conference on, IEEE. pp. 4335–4340.

Bienstock, D., Cherkov, M., Harnett, S., 2014. Chance-constrained optimal power flow: Risk-aware network control under uncertainty. SIAM Review 56, 461–495.

Bucher, M.A., Vrakopoulou, M., Andersson, G., 2013. Probabilistic N-1 security assessment incorporating dynamic line ratings, in: Power and Energy Society General Meeting (PES), 2013 IEEE, IEEE. pp. 1–5.

Calafiore, G., Campi, M.C., 2005. Uncertain convex programs: randomized solutions and confidence levels. Mathematical Programming 102, 25–46.

Calafiore, G.C., 2010. Random convex programs. SIAM Journal on Optimization 20, 3427–3464.

Campi, M.C., Garatti, S., 2008. The exact feasibility of randomized solutions of uncertain convex programs. SIAM Journal on Optimization 19, 1211–1230.

Campi, M.C., Garatti, S., Ramponi, F.A., 2018. A general scenario theory for non-convex optimization and decision making. IEEE Transactions on Automatic Control.

Chen, W., Sim, M., Sun, J., Teo, C.P., 2010. From CVaR to uncertainty set: Implications in joint chance-constrained optimization. Operations research 58, 470–485.

Ding, X., Lee, W.J., Jianxue, W., Liu, L., 2010. Studies on stochastic unit commitment formulation with flexible generating units. Electric Power Systems Research 80, 130–141.

DoostiZadeh, M., Aminifar, F., Ghosami, H., Lesani, H., 2016. Energy and reserve scheduling under wind power uncertainty: An adjustable interval approach. IEEE Transactions on Smart Grid 7, 2943–2952.

Franco, J.F., Rider, M.J., Romero, R., 2016. Robust multi-stage substitution expansion planning considering stochastic demand. IEEE Transactions on Power Systems 31, 2125–2134.

Geng, X., Modarress, S., Xie, L., 2019. Security-constrained Unit Commitment with Probabilistic Guarantees. IEEE Transactions on Power Systems (submitted).

Geng, X., Xie, L., 2019a. Data-driven Decision Making with Probabilistic Guarantees (Part 1): A Schematic Overview of Chance-constrained Optimization. arXiv:1903.10621.

Geng, X., Xie, L., 2019b. Data-driven Decision Making with Probabilistic Guarantees (Part 2): Applications of Chance-constrained Optimization in Power Systems. arXiv.

Grigg, C., Wong, P., Billinton, R., others, 1999. The IEEE reliability test system-1996. A report prepared by the reliability test system task force of the application of probability methods subcommittee. IEEE Transactions on Power Systems 14, 1010–1020.

Gurobi Optimization, L., 2016. Gurobi Optimizer Reference Manual. URL: http://www.gurobi.com
Figure 5: Violation Probabilities (cc-DCOPF of the 3-bus System)

(a) with logarithmic y-axis

(b) with error bars showing standard deviations

Figure 6: Violation Probabilities (cc-DCOPF of the 24-bus System, Gaussian Distributions)

(a) robust counterpart methods

(b) other methods

Figure 7: Objective Values (cc-DCOPF of the 24-bus System, Gaussian Distributions)

(a) robust counterpart methods

(b) other methods
strained optimal power flow with renewable generation. IEEE Transactions on Power Systems 31, 3840–3849.

Manickavasagam, M., Anjos, M.F., Rosehart, W.D., 2015. Sensitivity-based chance-constrained generation expansion planning. Electric Power Systems Research 127, 32–40.

Margellos, K., Rostampour, V., Vrakopoulou, M., Prandini, M., Andersson, G., Lygeros, J., 2013. Stochastic unit commitment and reserve scheduling: A tractable formulation with probabilistic certificates, in: Control Conference (ECC), 2013 European, IEEE. pp. 2513–2518.

Martinez, G., Anderson, L., 2015. A risk-averse optimization model for unit commitment problems, in: System Sciences (HICSS), 2015 48th Hawaii International Conference on, IEEE. pp. 2577–2585.

Mazidi, M., Rosehart, W., Malik, O., Aguado, J., 2009. Modified chance-constrained optimization applied to the generation expansion problem. IEEE Transactions on Power Systems 24, 1635–1636.

Mhlfordt, T., Faulwasser, T., Roald, L., Hagenmeyer, V., 2017. Solving optimal power flow with non-Gaussian uncertainties via polynomial chaos expansion, in: Decision and Control (CDC), 2017 IEEE 56th Annual Conference on, IEEE. pp. 4490–4496.

Ming, H., Xie, L., Campi, M., Garatti, S., Kumar, P., 2017. Scenario-based Economic Dispatch with Uncertain Demand Response. IEEE Transactions on Smart Grid.

Moradresi, M.S., Xie, L., Campi, M., Garatti, S., Car, A., Thatte, A., Kumar, P., 2018. Scenario-based Economic Dispatch with Tunable Risk Levels in High-renewable Power Systems. IEEE Transactions on Power Systems.

Pozo, D., Conteras, J., 2013. A chance-constrained unit commitment with an Sm-k5 security criterion and significant wind generation. IEEE Transactions on Power Systems 28, 2842–2851.

Qu, J., Dong, Z.Y., Zhao, J., Xu, Y., Luo, F., Yang, J., 2016. A risk-based approach to multi-stage probabilistic transmission network planning. IEEE Transactions on Power Systems 31, 4867–4876.

Roald, L., Misra, S., Chertkov, M., Andersson, G., 2015a. Optimal power flow with weighted chance constraints and general policies for generation control, in: Decision and Control (CDC), 2015 IEEE 54th Annual Conference on, IEEE. pp. 6927–6933.

Roald, L., Misra, S., Krause, T., Andersson, G., 2017a. Corrective control to handle forecast uncertainty: A chance constrained optimal power flow. IEEE Transactions on Power Systems 32, 1626–1637.

Roald, L., Misra, S., Morrison, A., Andersson, G., 2017b. Optimized risk limits for stochastic optimal power flow, in: Decision and Control (CDC), 2017 IEEE 56th Annual Conference on, IEEE. pp. 4476–4483.

Roald, L., Oldewurtel, F., Krause, T., Andersson, G., 2013. Analytical reformulation of security constrained optimal power flow with probabilistic constraints, in: PowerTech (POWERTECH), 2013 IEEE Grenoble, IEEE. pp. 1–6.

Roald, L., Vrakopoulou, M., Oldewurtel, F., Andersson, G., 2014. Risk-constrained optimal power flow with probabilistic guarantees, in: Power Systems Computation Conference (PSCC), 2014, IEEE. pp. 1–7.

Roald, L., Oldewurtel, F., Andersson, G., 2015b. Risk-based optimal power flow with probabilistic guarantees. International Journal of Electrical Power & Energy Systems 72, 66–74.

Sanghvi, A.P., Shavel, I.H., Spann, R.M., 1982. Strategic planning for power system reliability and vulnerability: an optimization model for resource planning under uncertainty. IEEE Transactions on Power Apparatus and Systems, 1420–1429.

Summers, T., Warrington, J., Morari, M., Lygeros, J., 2013. Stochastic optimal power flow based on convex approximations of chance constraints, in: Power Systems Computation Conference (PSCC), IEEE.

Summers, T., Warrington, J., Morari, M., Lygeros, J., 2015. Stochastic optimal power flow based on convex approximations of chance constraints, in: Power Systems Computation Conference (PSCC), IEEE.

Takriti, S., Birge, J.R., Long, E., 1996. A stochastic model for the unit commitment problem. IEEE Transactions on Power Systems 11, 1497–1508.

Tan, W.S., Shaaban, M., 2016. A hybrid stochastic/deterministic unit commitment based on projected disjunctive milp reformulation. IEEE Transactions on Power Systems 31, 5290–5291.

Vrakopoulou, M., Li, B., Mathieu, J.L., 2019. Chance Constrained Reserve Scheduling Using Uncertain Controllable Loads Part I: Formulation and Scenario-based Analysis. IEEE Transactions on Smart Grid.

Vrakopoulou, M., Margellos, K., Lygeros, J., Andersson, G., 2013a. A probabilistic framework for reserve scheduling and security assessment of systems...
with high wind power penetration. IEEE Transactions on Power Systems 28, 3885–3896.

Vrakopoulou, M., Margellos, K., Lygeros, J., Andersson, G., 2013b. Probabilistic guarantees for the N-1 security of systems with wind power generation, in: Reliability and Risk Evaluation of Wind Integrated Power Systems. Springer, pp. 59–73.

Wang, Q., Guan, Y., Wang, J., 2012. A chance-constrained two-stage stochastic program for unit commitment with uncertain wind power output. IEEE Transactions on Power Systems 27, 206–215.

Wang, Q., Wang, J., Guan, Y., 2013. Price-based unit commitment with wind power utilization constraints. IEEE Transactions on Power Systems 28, 2718–2726.

Wang, Z., Shen, C., Liu, F., Wu, X., Liu, C.C., Gao, F., 2017. Chance-constrained economic dispatch with non-Gaussian correlated wind power uncertainty. IEEE Transactions on Power Systems 32, 4880–4893.

Wu, H., Shahidehpour, M., Li, Z., Tian, W., 2014. Chance-constrained day-ahead scheduling in stochastic power system operation. IEEE Transactions on Power Systems 29, 1583–1591.

Wu, L., Shahidehpour, M., Li, T., 2007. Stochastic Security-Constrained Unit Commitment. IEEE Transactions on Power Systems 22, 800–811. doi: 10.1109/TPWRS.2007.894843.

Wu, Z., Zeng, F., Zhang, X.P., Zhou, Q., 2016. A solution to the chance-constrained two-stage stochastic program for unit commitment with wind energy integration. IEEE Transactions on Power Systems 31, 4185–4196.

Xie, L., Carvalho, P.M., Ferreira, L.A., Liu, J., Krogh, B.H., Popli, N., Ilic, M.D., 2011. Wind integration in power systems: Operational challenges and possible solutions. Proceedings of the IEEE 99, 214–232.

Yang, N., Wen, F., 2005. A chance constrained programming approach to transmission system expansion planning. Electric Power Systems Research 75, 171–177.

Yang, P., Nehorai, A., 2014. Joint optimization of hybrid energy storage and generation capacity with renewable energy. IEEE Transactions on Smart Grid 5, 1566–1574.

Zhang, Y., Giannakis, G.B., 2013. Robust optimal power flow with wind integration using conditional value-at-risk, in: Smart Grid Communications (SmartGridComm), 2013 IEEE International Conference on, IEEE, pp. 654–659.

Zhang, Y., Shen, S., Mathieu, J., 2017a. Distributionally Robust Chance-Constrained Optimal Power Flow with Uncertain Renewables and Uncertain Reserves Provided by Loads. IEEE Transactions on Power Systems.

Zhang, Y., Shen, S., Mathieu, J.L., 2015. Data-driven optimization approaches for optimal power flow with uncertain reserves from load control, in: American Control Conference (ACC), 2015, IEEE, pp. 3013–3018.

Zhang, Y., Wang, J., Li, Y., Cao, X., 2017b. Chance-constrained Transmission Expansion Planning with Guaranteed Wind Power Utilization, Chicago, IL.

Zhao, C., Wang, J., Zeng, B., Hu, Z., 2017c. Chance-Constrained Two-Stage Unit Commitment under Uncertain Load and Wind Power Output Using Bilinear Benders Decomposition. IEEE Trans on Power Systems.

Zhao, C., Wang, Q., Wang, J., Guan, Y., 2014. Expected value and chance constrained stochastic unit commitment ensuring wind power utilization. IEEE Transactions on Power Systems 29, 2696–2705.

Zheng, Q.P., Wang, J., Liu, A.L., 2015. Stochastic optimization for unit commitment review. IEEE Trans. Power Syst 30, 1913–1924.

Zimmerman, R.D., Murillo-Snchez, C.E., Thomas, R.J., 2011. MATPOWER: Steady-state operations, planning, and analysis tools for power systems research and education. IEEE Transactions on power systems 26, 12–19.
| Methods                  | Expansion                | SCUC          | SCED          | Other Applications                  |
|--------------------------|--------------------------|---------------|---------------|--------------------------------------|
| Deterministic Equivalent | Gaussian (Sanghvi et al., 1982) [Lopez et al., 2007]; Mazadi et al., 2009; Manickavasagam et al., 2015] | (Ding et al., 2010) | (Bent et al., 2013); Bienstock et al., 2013; [Roald et al., 2013]; [Lopez et al., 2015]; [Lopez et al., 2015]; [Franco et al., 2016] | |
|                          |                          | (Pozo and Contreras, 2013; Wu et al., 2014) | (Bent et al., 2013); [Roald et al., 2014]; Li and Mathieu et al., 2015; Roald et al., 2015a; Zhang et al., 2015; Lubin et al., 2016; Doostizadeh et al., 2016; Roald et al., 2017a,b); Wang et al., 2017; Vrakopoulou et al., 2019); Li et al., 2019) | |
| Scenario Approach | a-priori | - | (Geng et al., 2019); (Bucher et al., 2013); Vrakopoulou et al., 2013a,b); Roald et al., 2014, 2015b); Zhang et al., 2015); Ming et al., 2017); Modarresi et al., 2018); Geng and Xie, 2019b) | |
|                          |                          | | | (Yang and Nehorai, 2014) |
|                          | a-posteriori | - | (Margellos et al., 2013); Geng et al., 2019); Hreinsson et al., 2015) | (Modarresi et al., 2018); Geng and Xie, 2019b) |
| Sample Average Approximation | - | (Zhang et al., 2017b) | (Wang et al., 2012); Zhao et al., 2013; Tan and Shaaban, 2016; Bagheri et al., 2017; Zhang et al., 2017c) | (Geng and Xie, 2019b) |
| RO-based Approach | RLO | - | Jiang et al., 2012) | (Geng and Xie, 2019b) |
|                          | Convex Approximation | - | - | (Zhang and Giannakis, 2013); Summers et al., 2014, 2015); Geng and Xie, 2019b) |
|                          | Others | - | (Yang and Wen, 2005; Qiu et al., 2016); Anderson et al., 2015; Ke et al., 2016); Mhiqordi et al., 2017); Wang et al., 2017) | |

Table 1: Power System Applications of Chance-constrained Optimization