Computational analysis of hydromagnetic boundary layer stagnation point flow of nano liquid by a stretched heated surface with convective conditions and radiation effect

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Abstract
In this study, the boundary layer phenomena for stagnation point flow of water-based nanofluids is being observed with the upshot of MHD and convective heating on a nonlinear stretching surface. To develop a fundamental flow model, a boundary layer approximation is done, which signifies time-dependent momentum, energy, and concentration expressions. Through a proper transformation framework, the modeled boundary layer partial differential equations (PDEs) have been diminished to a dimensionless system of nonlinear ordinary differential equations (ODEs). With the assistance of a built-in algorithm in Mathematica software, the fundamental flow equations are analyzed numerically by imposing a shooting technique explicitly. A stability and convergence analysis were also unveiled, and the ongoing investigation was found to have converged. The effect of mathematical abstractions on velocity, energy, and concentration is plotted and discussed. The influence of skin-friction and Nusselt number on the sheet are debated for the various values of important parameters.

Keywords
Numerical solution, MHD, nanofluid, Brownian motion, thermophoresis, stagnation point flow

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Introduction
Numerous magnetohydrodynamic (MHD) flow studies have been done due to its importance in numerous practical applications in modern industry, such as MHD power generators, the petroleum industry, liquid metals systems of fusion reactors, earth’s core motion, and so on. The MHD boundary layer flow of an electrically conducting fluid was first studied by Pavlov¹ with a stretched plane elastic surface in the presence of a...
uniform transverse magnetic field. Then, Chakrabarti and Gupta extended this study to include the temperature distribution over a stretching sheet in the presence of a uniform suction. Later, this problem was extended further to a power-law fluid over a stretching sheet by Andersson et al., recently to Eyring-Powell fluid by Sher Akbar et al., and to a nanofluid by Ibrahim et al. The latest study reported the MHD flow over both stretched and shrinking sheets with the effect of radiation taken into consideration. They found that radiation decreases the heat transfer rate at the surface. The MHD flow of an electrically conducting fluid is important in modern metallurgy and metalworking processes, such as the process of fusing of metals in an electrical furnace by applying a magnetic field and the process of cooling of the first wall inside a nuclear reactor containment vessel where the hot plasma is isolated from the wall. Choi was the first to introduce the theory of nanofluid with its application to non-Newtonian fluids. The nanofluids that are commonly used are toluene, water, oil, etc. Choi showed that the addition of a tiny number of nanoparticles into the liquid increased the thermal conductivity of the fluid. On the other hand, Nandy and Pop have investigated the MHD nanofluid flow at a stagnation point in the presence of a magnetic field and thermal radiation. Sheikholeslami and Houman studied the effect of Lorentz forces on nanofluid flow in a porous complex shaped enclosure by means of a non-equilibrium model using the control volume based finite element method (CVFEM). Stagnation point flow has various practical applications. These applications include the cooling of electronic devices, the cooling of nuclear reactors during emergency shutdown, and hydrodynamic processes in engineering applications. Again, the study of magnetohydrodynamic (MHD) flow of an electrically conducting fluid is of considerable interest in metallurgical and metalworking processes due to the fact that the rate of cooling can be controlled by the application of a magnetic field. Hydromagnetic stagnation point flow and heat transfer find applications in boundary layers along material handling conveyors, in aerodynamic extrusion of plastic sheets and in blood flow problems. Hiemenz considered a two-dimensional stagnation flow problem on a stationary plate and used similarity transformations to reduce the Navier Stokes equations to non-linear ordinary differential equations. Akbar et al. investigated radiation effects on MHD stagnation point flow of nanofluid toward a stretching surface with convective boundary condition. Bhatti et al. studied a robust numerical method for solving stagnation point flow over a permeable shrinking sheet under the influence of MHD. Sheikholeslami and Houman analyzed magnetic nanofluid flow and convective heat transfer in a porous cavity considering Brownian motion effects. explained the effects of slip on nonlinear convection in nanofluid flow on stretching surfaces. Stagnation electrical MHD nanofluid mixed convection with slip boundary on a stretching sheet was studied by Hsiao. Heat transfer improvement and pressure drop during condensation of refrigerant-based nanofluid was studied experimentally by Sheikholeslami et al. Khan et al. studied convective heat flow features of stagnation point flow of MHD over a nonlinear stretching surface with slip velocity and variable heat reservoir source. Several other studies have addressed various aspects of nanofluids with stretching sheets. Gangaiith et al. described the effects of thermal radiation and heat source/sink parameters on the mixed convective MHD flow of a Casson nanofluid with zero normal flux of nanoparticles over an exponentially stretching sheet along with convective boundary conditions. Ghozatloo et al. studied the chemical vapor deposition (CVD) method at atmospheric pressure using synthesis of Graphene. Ramya et al. studied the steady 2D flow of a viscous nanofluid in magnetohydrodynamic (MHD) flow and heat transfer characteristics for the boundary layer flow over a nonlinear stretching sheet are considered. Nadeem et al. discussed water-base hybrid nanofluid flow over an exponentially curved permeable surface. The hybrid nanofluid comprises two types of nanoparticles along with the base fluid, which gains a larger rate of heat transfer compared to simple nanofluid. Abbas et al. carried out steady state flow of micropolar hybrid nanofluid over a stretched Riga plate with slip condition and radiation effect. Abbas et al. explored MHD stagnation point flow and heat transfer due to hybrid nanofluid over a stretching cylinder with a uniform magnetic field.

The existing investigation aims to explore the MHD stagnation point flow of nanofluid toward a stretching surface. The effect of thermophoretic force, Brownian movement, and concentration of nanoparticles on the thermal boundary layer with heat transfer due to nanofluid. The governing boundary layer partial differential equations have been transformed into ODEs via similarity solutions. The nonlinear system has been tackled numerically through shooting technique. The consequences of pertinent constraints on flow fields have been analyzed through plotted graphs and tables.

**Mathematical formulation**

Consider a steady MHD two-dimensional boundary layer flow of a nanofluid over a stretching surface with the velocity \( u_m(x) = ax^n \), where \( a \) is the constant. The flow takes place at \( y>0 \), where \( y \) is the coordinate measured normal to the stretching surface as shown in Figure 1. It is presumed that the lowest surface of the sheet is heated by convection from a hot fluid at constant temperature \( T_f \), which offers a constant heat...
transfer. The constant surface temperature and concentration of the sheet are and \( C_w \), whereas the ambient fluid is \( T_\infty \) and \( C_\infty \), respectively. The flow is subjected to a constant transverse magnetic field of strength \( B_0 \), which is employed in the \( y \) direction, normal to the surface. The induced magnetic field is assumed to be smaller compared to the applied magnetic field and is neglected. Under these assumptions, the leading flow equations can be written in the Cartesian Coordinates:

**Governing equations**

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0, \\
\frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{\partial^2 u}{\partial y^2} + U_\infty \frac{\partial u_\infty}{\partial x} + \frac{\sigma B_0^2}{\rho_f} (U_\infty - u), \\
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \alpha \left( \frac{\partial^2 T}{\partial y^2} \right) + \Gamma \left( D_B \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right), \\
\frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} &= D_B \left( \frac{\partial^2 C}{\partial y^2} \right) + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right) \, .
\end{align*}
\]

Extreme conditions are:

\[
\begin{align*}
u &= u_w(x) = ax, v = 0, -k \frac{\partial T}{\partial y} = h_f (T_f - T), \\
D_B \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right) &= 0, y = 0, \\
u \rightarrow U_\infty &= bx, v = 0, T \rightarrow T_\infty, C \rightarrow C_\infty, y = \infty.
\end{align*}
\]

Where \((u, v)\) denotes velocity components in Coordinates axes, \( \nu \) is the kinematics viscosity, \( T \) fluid temperature, \((pc)_p\) denote effective heat capacity \((pc)_p\), \((pc)_f\) denote fluid thermal capacity, \( \rho \) represent fluid density, \( \Gamma = \frac{(pc)_p}{(pc)_f} \) constant parameter, \( T_\infty \) denote ambient temperature.

**Similarity transformation**

Considering the transformation\(^5\,34\) to diminish the system of (PDEs) to the system of (ODEs):

\[
\begin{align*}
\zeta = \sqrt{\frac{a}{v}} y, \psi = \sqrt{av} x f(\zeta), \Theta(\zeta) &= \frac{T - T_\infty}{T_f - T_\infty}, \phi(\zeta) = \frac{C - C_\infty}{C_\infty},
\end{align*}
\]

Stream function \( \xi(x, y) \) is defined as:

\[
u = \frac{\partial \xi}{\partial y}, v = -\frac{\partial \xi}{\partial x}.
\]

The following system of ODEs is obtained by using equation (6) in equations (1)–(5):

\[
\begin{align*}
\frac{d^3 F}{d\xi^3} + F \frac{d^2 F}{d\xi^2} - \left( \frac{dF}{d\xi} \right)^2 - M \frac{dF}{d\xi} + A(1 + M) &= 0, \\
\frac{d^2 \Theta}{d\xi^2} + Pr \left( \frac{d\Theta}{d\xi} + Nb \left( \frac{d\phi}{d\xi} \right) \left( \frac{d\Theta}{d\xi} \right) + Nt \frac{d\Theta}{d\xi} \right) &= 0, \\
\frac{d^2 \phi}{d\xi^2} + PrLeF \frac{d\phi}{d\xi} + \frac{Nt d^2 \Theta}{Nb d\xi^2} &= 0.
\end{align*}
\]

the transforms extreme conditions are:

\[
\begin{align*}
F &= 0, \frac{dF}{d\xi} = 1, \frac{d\Theta}{d\xi} = Bi(\Theta - 1), Nb \frac{d\phi}{d\xi} + Nt \frac{d\Theta}{d\xi} = 0, \\
\frac{dF}{d\xi} &= A, \Theta = 0, \phi = 0 \text{ at } \xi = \infty.
\end{align*}
\]

Variables appearing in equations (8)–(11) are defined and label as:

\[
\begin{align*}
M = \frac{\sigma B_0^2}{\rho_f a}, Nb = \frac{(pc)_p D_B C_\infty}{(pc)_f}, Nt = \frac{(pc)_p D_T (T_f - T_\infty)}{(pc)_f v T_\infty}, \\
Bi = \sqrt{\frac{v h_f}{a k}}, A = \frac{b}{a}, Pr = \frac{\nu}{\alpha}, Le = \frac{\alpha}{D_B}
\end{align*}
\]

magnetic parameter, Brownian parameter, thermophoretic parameter, Biot number, velocity ratio parameter, Prandtl number, and Lewis number.

Expressions of engineering importance are \((Cf, Nu_\infty)\) defined by:
\[ C_f = \frac{\tau_w}{\mu u'_{\infty}} \]

\[ N_{u_t} = \frac{x q_w}{k(T_f - T_w)} \]  

(12)

(13)

The drag force and heat transfer rate parameter can be labeled as:

\[ \sqrt{Re_x}C_f = -\frac{d^2F}{d\xi^2}_{|\alpha\xi = 0} \]  

(14)

\[ \frac{N_{u_t}}{\sqrt{Re_x}} = -\frac{d\Theta}{d\xi}_{|\alpha\xi = 0} \]  

(15)

**Solution methodology**

For numerical results the first order (ODEs) is recruited from equations (8)–(10) by introducing transformation variables. Let the transformations variables are defined as:

\[ F = x_1, \frac{dF}{d\xi} = x_2, \frac{d^2F}{d\xi^2} = x_3, \Theta = x_4, \frac{d\Theta}{d\xi} = x_5, \]

\[ \varphi = x_6, \frac{d\varphi}{d\xi} = x_7 \]

The linear system of (ODEs) thus, generated by

\[ x'_1 = x_2 \]  

(16)

\[ x'_2 = x_3 \]  

(17)

\[ x'_3 = [x_2^2 - x_1 x_5 - M x_2 + A(A - M)] \]  

(18)

\[ x'_4 = x_5 \]  

(19)

\[ x'_5 = - Pr [x_1 x_5 + N_{b}x_7 x_5 + N_{t}x_5^2] \]  

(20)

\[ x'_6 = x_7 \]  

(21)

\[ x'_7 = - \left[ Le Pr x_1 x_7 + \frac{N_t}{N_b} x_5 \right] \]  

(22)

We need seven initial conditions to solve these seven unknowns first order ODEs numerically. In this case, \( F \) has two initial conditions, while \( \Theta \) and \( \varphi \) each have single condition. The three end conditions are used to acquire the remaining three unknown initial conditions.

\[ x_1(0) = 0, x_2(0) = 1, x_3(0) = Bi(x_4(0) - 1), \]

\[ N_{b}x_7(0) + N_{t}x_5(0) = 0 \]  

(23)

\[ x_2(\infty) = A, x_4(\infty) = 0, x_6(\infty) = 0. \]

We select a variable step size of \( \Delta \xi = 0.001 \) and repeated the process until the result met asymptotically at the tolerance level of \( 10^{-8} \). The routine numerical algorithm is disclosed in Figure 2.

**Graphical results and analysis**

The dimensionless governing flow equations (8)–(10) subject to extreme conditions in equation (11) are solved via the shooting method. The diagram of numerous parameters, such as magnetic parameter, ratio parameter, Brownian parameter, thermophoretic parameter, Prandtl number, Biot number, and Lewis number on the velocity, energy, and concentration profile are plotted in Figures 3 to 16. Figures 3 and 4 explains \( A \) effect on velocity \( f'(\xi) \). These plots expose development in boundary layer thickness when \( A > 1 \) and diminishes momentum boundary layer thickness for \( A < 1 \). The role of \( M \) on \( f'(\xi) \) is evaluated through Figure 5. Physically, higher \( M \) values produce a
resistive force, namely the Lorentz force, which lowers fluid flow. The variations of Pr against \( \Theta(\zeta) \) are offered in Figure 6. This graph stated a shrinking in both \( \Theta(\zeta) \) and thermal boundary layer via larger Pr. As thermal diffusivity diminishes when Pr is increased, which corresponds to a decrease in energy transference and eventually it produces a decay in the thermal boundary layer. Figure 7 shows the outcome of \( Nt \) on \( \Theta(\zeta) \). The temperature gradients in the boundary layer induce a thermophoretic force on the nanoparticles and that leads to a fast flow away from the stretching surface. Hence, additional fluid is heated away from the surface, and consequently, as \( Nt \) upsurges, the temperature inside the boundary layer rises. Figure 8 displays the impact of convective heating parameter called \( Bi \) on \( \Theta(\zeta) \). Physically, \( Bi \) is the ratio of convection at the surface to conduction within the surface of a body. As the effect of \( Bi \) rises, temperature at the surface upsurges, due to an increase in the boundary layer thickness. Figure 9 displays the performance of \( Nb \) on \( \varphi(\zeta) \). From the graphical sketch, it is obvious that an augmentation in \( Nb \) provides an increase in \( \varphi(\zeta) \) within the region of the boundary layer. Figure 10 displays disparity of fluid concentration \( \varphi(\zeta) \) for varied \( Le \), where, \( Le \) is the fraction of the thermal diffusivity to mass diffusivity, and
when the values of $Le$ are enhanced the thermal diffusion rate goes past the rate of mass diffusion and ultimately downfall $\phi(\xi)$. This occurrence is detected in Figure 10. Figure 11 depict the variation in $\phi(\xi)$ for different values of Pr. It is observed that, the Lewis number decreases distribution of $\phi(\xi)$ due to a decline in mass diffusion. Like the effects of $Le$, the Pr decline the nanoparticle volume fraction due is to the lessening in the fluid temperature as Pr rises. Figures 12 and 13 emphasize the influence of velocity ratio parameter and magnetic parameter on drag force. Here, drag force diminishes when these parameters are augmented. Moreover, the attributes of local Nusselt number, which represents the heat transfer rate at the surface, increase for larger Prandtl number and Brownian movement while decreasing for thermophoretic parameters shown in Figures 14 to 16. Table 1 is made to present the variation in drag force for several values of velocity ratio parameter and magnetic parameter. It is noticed that the drag force diminishes for larger values
the of velocity ratio parameter, but the drag force boosts through larger magnetic parameter. Table 2 displays the numerical values of the Nusselt number for the different values of physical parameters. It is reported that the heat transfer rate at the surface is a diminishing function of Brownian and thermophoretic constraints, whereas an opposite trend can be found with the increasing Prandtl number.

**Closing remarks**

A theoretical problem of magneto-hydrodynamics thermal boundary layer stagnation point flow of nanofluid over a stretching surface with effect of heat transfer and convective boundary conditions have been examined numerically. A similarity solution is presented which effects, magnetic, Brownian motion, thermophoretic force, and velocity ratio parameter. The governing equations were transmuted to nonlinear system of ODEs through suitable transformation techniques. A numerically solutions to nonlinear flow equations were obtained by shooting method. The precise findings of this study are summarized below:
• As noticed that velocity profiles escalate with increment in velocity ratio parameter when \( A > 1 \).
• The velocity and boundary layer thickness diminishes subject to increase in the magnetic field strength.
• The thermal boundary layer curves develop with increment in thermophoretic parameter while reverse trends perceived for concentration profile.
• The fluid temperature and relative thickness dwindles when velocity ratio parameter and Prandtl number is incremented.
• An upsurge in magnetic parameter develops the surface drag force and diminishes with larger velocity ratio parameter.
• The heat transfer rate increases with both Prandtl number and Brownian parameter, whereas drop through larger thermophoretic parameter.

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**Appendix**

**Notation**

| Symbol | Description |
|--------|-------------|
| A      | Velocity ratio parameter |
| C      | Concentration at surface |
| C∞     | Ambient concentration |
| F      | Dimensionless stream function |
| M      | Magnetic parameter |
| Pr     | Prandtl number |
| T      | Temperature of the fluid inside the boundary layer |
| U∞     | Free stream velocity |
| B0     | Magnetic field strength |
| Cf     | Skin friction |
| DB     | Brownian diffusion coefficient |
| κ      | Thermal conductivity |
| Nb     | Brownian motion parameter |
| qw     | Wall heat flux |
| Tf     | Temperature of a hot fluid |
| (u, v) | Velocity component |
| Bi     | Biot number |
| Ntu    | Nusselt number |
| DT     | Thermophoresis diffusion coefficient |
| Le     | Lewis number |
| Nt     | Thermophoresis parameter |
| Re     | Local Reynolds number |
| Tw     | Ambient temperature |
| Γ      | Parameter defined by $\Gamma = \frac{\langle pc \rangle}{\langle pc_f \rangle}$ |

**Greeks’ symbols**

| Symbol | Description |
|--------|-------------|
| ζ      | Dimensionless similarity variable |
| ρf     | Density of the base fluid |
| ξ      | Stream function |
| Θ      | Dimensionless temperature |
| μ      | Dynamic viscosity of the fluid |
| pcf    | Heat capacity of the base fluid |
| α      | Thermal diffusivity |
| τw     | Wall shear stress |
| ν      | Kinematic viscosity of the fluid |
| ρcp    | Effective heat capacity of a nanoparticle |
| σ      | Electrical conductivity |
| φ      | Dimensionless concentration |