A Redundancy-Aware Error Model for Kinematic Calibration of Redundantly Actuated Overconstrained Parallel Robots

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Abstract—For redundantly actuated overconstrained parallel robots (ROPRs), the existing general error models for kinematic calibration often omit their redundantly actuated and overconstrained characteristics, affecting the precision of the calibration. To address this issue, a general redundancy-aware error model (RAEM) is proposed. First, the configuration space of the ROPRs is implicitly constructed to expose the constraints on both the active joints and the geometric errors resulting from these two characteristics. To mitigate the consequent effects, the restrained deviations of redundant active joints are portrayed both in the RAEM and in the forward kinematics during each calibration iteration. A feasible error space is, thus, formulated to impose geometric error constraints on the identified errors. The introduced method can theoretically prevent the violation of configuration constraints. Finally, the superiority of the proposed RAEM is validated by kinematic calibration simulations and experiments. The results show that the proposed RAEM achieves better calibration precision than existing methods.

Index Terms—Error identification, error modeling, kinematic calibration, redundantly actuated parallel robot.

 NOMENCLATURE

e Error evaluation index.
f Degrees of freedom.
F Feasible error space matrix.
J Calibration Jacobian matrix.
k Minimal error vector.
K Coefficient matrix for k.
L Coefficient matrix for joint motion errors.
n Number describing the robot.
r Position vector.
R Rotation matrix.
T Homogeneous transformation matrix.
x Error vector in the feasible error space.
y Pose error vector of the end-effector.
ξ Twist vector.
θ Joint motion vector.
i Variable associated with limb i.
j Variable about the jth joint of the ith limb.
/ Nominal/Actual value of a variable.
i Variable at the ith measured point.

I. INTRODUCTION

REDUNDANTLY actuated parallel robots (PRs) have attracted considerable attention due to their ability to avoid singularities and reduce joint backlash effects [1], [2], offering advantages over classical nonredundant PRs. Typically, redundant PRs can be divided into two categories: 1) redundant nonoverconstrained PRs [3], [4], [5] and 2) redundant overconstrained PRs (ROPRs) [6], [7], [8]. Compared to the former, ROPRs are considered a promising solution for applications with high stiffness and large load capacity requirements, benefiting from the presence of common or redundant constraints [9].

Owing to manufacturing and assembly tolerances, an ROPR will inevitably suffer from geometric errors, resulting in reduced positioning precision. To tackle this issue, an economical yet efficient way of kinematic calibration is required [10], [11], which involves the procedures of error modeling, measurement, identification, and compensation. Among these procedures, error modeling is the most important theoretical basis and plays a decisive role in the precision of the kinematic calibration [12].

Due to the redundantly actuated characteristics of ROPRs, i.e., the number of active joints exceeding the ROPRs’ degrees of freedom (DoF), constraints exist among the active joint positions. Under geometric errors, a discrepancy arises between the actual and nominal kinematic models, hindering the nominal active joint positions from satisfying the actual active joint constraints. Meanwhile, the ROPR exhibits an overconstrained characteristic, where the constraints imposed on dependent joint positions are overdetermined based on geometric parameters. Hence, the constraint equations for the dependent joint positions will only be satisfied when the geometric errors meet specific conditions. Worse still, these two characteristics are coupled for both consequent effects, significantly complicating the error modeling of ROPRs. Since these situations would not occur
in the nonredundant and nonoverconstrained modes, their error modeling methods [12], [13], [14] cannot be directly applied to ROPRs, and error modeling for ROPRs with consideration of the two coupled characteristics is required.

To address these issues, there are two types of error modeling approaches for ROPRs developed in recent years. The first establishes the error model by differentiating the closed-loop vector equations [15], [16], [17], [18], [19], [20], [21], [22], which are the most widely utilized method due to the complex topologies of ROPRs. To simplify kinematic analysis and error modeling, some structures or links have to be assumed perfectly manufactured and assembled, leading to the removal of overdetermined equations. However, these assumptions can result in insufficient parameters in the error model, potentially degrading calibration precision. Regarding the redundantly actuated characteristic, it is either ignored [15], [16], [17], [18] or addressed on a case-by-case basis [19], [20], [21], [22], limiting its applicability to other ROPRs. For instance, in [18], to circumvent the effects of the two mentioned characteristics, it is assumed that no constraints exist between active joints and the geometric errors are limited in the nominal plane of the limbs. In [22], although the restrained deviations of active joints are formulated to meet actual active joint constraints, the error model still relies on certain assumptions, which might deteriorate the calibration precision and cannot be directly applied to other ROPRs.

With the intention of ensuring the generality of the error modeling, the second approach is further proposed [23], which establishes the error model through limbs. One typical example is the screw-based method [23], which establishes the error model of a spatial ROPR through limbs and can be easily applied to other ROPRs. Regretfully, this method overlooks the redundantly actuated and overconstrained characteristics, which assumes the absence of constraints between active joints and geometric errors. Based on these assumptions, the configuration constraints might be violated under geometric errors, affecting the precision of the calibration.

Considering these facts, a general error model for ROPRs that accounts for both redundantly actuated and overconstrained characteristics is still missing. On the one hand, the first approach, being case-specific and based on certain error assumptions, cannot be directly applied to other ROPRs and may impair calibration precision. On the other hand, as the second approach ignores the two inherent characteristics of ROPRs, it is difficult to satisfy the configuration constraints under geometric errors, affecting the calibration precision.

To remedy this issue, this work proposes a general redundancy-aware error model (RAEM) for ROPRs. First, an implicit representation of the ROPRs configuration space is constructed, thereby revealing the constraints on active joints and geometric errors caused by these two characteristics. To mitigate the consequent effects, the restrained deviations of redundant active joints are illustrated both in the RAEM and in the forward kinematics during each calibration iteration. A feasible error space is accordingly established to impose geometric error constraints on the identified errors. The introduced methods can theoretically prevent violations of configuration constraints.

Moreover, unlike the approach of eliminating redundant parameters through a set of principles in [23], our method establishes the error model for each limb using the adjoint error model [24]. This method effectively removes redundant error parameters of joints, meeting the minimal requirement. The kinematic calibration algorithm is thus built based on the proposed RAEM and the extended Kalman filter (EKF) algorithm [25]. Finally, the effectiveness of the proposed RAEM is validated by simulations and experiments.

The main contributions of this work are as follows.

1) A general error model for ROPRs, named RAEM, is proposed, accounting for the effects of redundantly actuated and overconstrained characteristics. Specifically, the restrained deviations of redundant active joints and the feasible error space are portrayed in the error model, providing excellent profits for calibration precision.

2) The RAEM is established based on the adjoint error model, which satisfies the minimal requirement, ensuring the stability of the calibration.

3) A kinematic calibration algorithm suitable for ROPRs is developed based on the RAEM. Extensive simulations and experiments are carried out to demonstrate its excellent precision and effectiveness.

The rest of this article is organized as follows. Section II outlines the article’s preliminaries. Error modeling and calibration methods for an ROPR are detailed in Sections III and IV, respectively. Section V presents the simulations and Section VI details the experiments. Finally, Section VII concludes this article.

II. PRELIMINARIES

A. Robot Description

Fig. 1 depicts an f-DoF ROPR with \( n_0 \) limbs. It is equipped with \( n_a \) active joints. Joints having more than 1-DoF are decomposed into several equivalent 1-DoF joints. Hence, each limb comprises \( f_i \) 1-DoF joints, resulting in a total of \( n_0 = \sum_{i=1}^{n_l} f_i \) joints.

Definition 1: The redundantly actuated characteristics of an ROPR mean that the number of active joints exceeds its DoF, i.e., \( n_a > f \). Redundant actuation is categorized into two types: 1) nonlimb-redundant, involving the activation of passive joints
without adding a new limb, and 2) limb-redundant, characterized by the introduction of additional active limbs [26]. In this article, we first focus on the limb-redundant ROPR. Without loss of generality, we assume that the \((f+1)\)th to \(n_l\)th limb is redundant, and the first joint in limb \(i\) is actuated, as shown in Fig. 1. Then, it can be easily generalized to the other type.

**Definition 2:** The overconstrained characteristics of an ROPR indicate that the number of its closed-loop equations is greater than the number of dependent joints, i.e., \(6(n_l-1) > n_\theta - f\) [27].

### B. Adjoint Error Model of Each Limb

To establish the overall error model of an ROPR, the adjoint error model [24] for each limb is first established in the following, which theoretically eliminates redundant error parameters of each joint, meeting the minimal requirement.

1) **Geometric Background:** The special Euclidean group \(SE(3)\) are composed of \(4 \times 4\) matrices of the form \(T = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}\), where \(R \in SO(3)\) and \(t \in \mathbb{R}^3\). Given \(T = (R, t) \in SE(3)\), its adjoint transformation is expressed as \(Ad(T) = \begin{bmatrix} R & 0 \\ tR & R \end{bmatrix}\), where the operator \(\wedge\) maps \(t\) from \(\mathbb{R}^3\) into \(so(3)\). This operator \(\wedge\) can also transform a given twist \(\xi = [\omega^T, \nu^T]^T\) from \(\mathbb{R}^6\) to \(se(3)\), and \(\vee\) denotes the reverse operation.

2) **Error Modeling:** The forward kinematics for limb \(i\) is described by the product of exponentials (POE) formula

\[
T_i(\theta_i) = \prod_{j=1}^{f_i} \xi_{i,j}^{\theta_{i,j}} T_{i,0} \tag{1}
\]

where \(T_{i,0} \in SE(3)\) denote the current and initial end-effector pose of limb \(i\), respectively; the joint motion vector of limb \(i\) is written as \(\theta_i = [\theta_{i,1}, \ldots, \theta_{i,f_i}]^T\); \(\xi_{i,j} \in se(3)\) denotes a twist associated with the \(j\)th joint of limb \(i\) at the home configuration, which is expressed in \(\{S\}\). According to the adjoint error model [24], the relationship between the actual twist \(\xi_{i,j}^a\) and the nominal twist \(\xi_{i,j}^n\) is given by

\[
\xi_{i,j}^a = Ad(e^{\tilde{\delta}n_{i,j}})\xi_{i,j}^n \tag{2}
\]

where \(\delta n_{i,j} = B_{i,j}k_{i,j}\) is the adjoint error of \(j\)th joint in limb \(i\); \(k_{i,j} \in \mathbb{R}^3\) or \(\mathbb{R}^2\) denotes the minimal errors of a rotation (R) or a parasitic (P) joint. The matrix \(B_{i,j}\) streamlines the error model of each limb, removing redundant parameters of joints and satisfying minimal requirements. It belongs to \(\mathbb{R}^{6 \times 4}\) for an R joint and \(\mathbb{R}^{6 \times 2}\) for a P joint. See Appendix C (Supplementary material) for details.

Define \(Ad_{i,j} = \prod_{k=1}^{f_i} Ad(e^{\tilde{\delta}n_{i,k}})\), \(Ad_{i,0} = I\). By linearizing (1) at each configuration using (2), we obtain the error model for limb \(i\) as [24]

\[
(\delta T_i T_i^{-1})^\vee = \sum_{j=1}^{f_i} (Ad_{i,j-1} - Ad_{i,j}) B_{i,j}k_{i,j} + \sum_{j=1}^{f_i} Ad_{i,j-1} \delta \theta_{i,j} + Ad_{i,f_i} \delta n_{i,sl} \tag{3}
\]

where \((\delta T_i T_i^{-1})^\vee \in \mathbb{R}^6\) and \(\delta n_{i,sl} = (\delta T_i T_i^{-1})^\vee \in \mathbb{R}^6\) denote the current and initial end-effector error in limb \(i\), respectively \(\delta \theta_{i,j}\) is the joint motion error of \(\theta_{i,j}\). The matrix form of (3) is written as

\[
y_i = K_i k_i + K_{sl,i} k_{sl,i} + L_i \delta \theta_i \tag{4}
\]

where \(y_i = (\delta T_i T_i^{-1})^\vee \in \mathbb{R}^6\); \(k_i = [k_{i,1}^T, \ldots, k_{i,f_i}^T]^T \in \mathbb{R}^{n_l}\), contains the geometric errors for each joint twists and \(k_{sl,i} = \delta n_{i,sl} \in \mathbb{R}^6\) is the initial end-effector error, in limb \(i\); \(\theta_i = [\theta_{i,1}, \ldots, \theta_{i,f_i}]^T \in \mathbb{R}^{f_i}\) is the joint motion errors in limb \(i\).

The coefficient matrices can be given as

\[
K_i = [D_{i,1}B_{i,1}, \ldots, D_{i,f_i}B_{i,f_i}], D_{i,j} = Ad_{i,j-1} - Ad_{i,j}
\]

\[
K_{sl,i} = Ad_{i,f_i}, L_i = [\xi_{i,1}, Ad_{i,1}\xi_{i,2}, \ldots, Ad_{i,f_i}\xi_{i,f_i}].
\]

### C. Error Modeling Ignoring the Two Characteristics

One alternative for establishing the overall error model is to directly integrate the limb error models without considering the redundantly actuated and overconstrained characteristics, as done in the screw-based method [23].

Given that the end-effector is shared by all limbs, we define \(y \triangleq y_1 = \ldots = y_n, k_{sl} \triangleq k_{sl,1} = \ldots = k_{sl,n_l}\). Integrating the error models of each limb in (4) yields

\[
\Omega y = Kk + L \delta \theta \tag{5}
\]

where \(k = [k_{1,1}^T, \ldots, k_{n_l,1}^T, k_{n_l,1}^T]^T \in \mathbb{R}^{n_l}\) and \(\delta \theta = [\delta \theta_{1,1}^T, \ldots, \delta \theta_{n_l,1}^T]^T \in \mathbb{R}^{n_l}\) represent the geometric errors and the joint motion errors of the whole ROPR, respectively. The coefficient matrices can be given as

\[
\Omega = [I_0, \ldots, I_0]^T \in \mathbb{R}^{6n_l \times 6}, L = \text{blockdiag}(L_1, \ldots, L_{n_l})
\]

\[
K = [\text{blockdiag}(K_1, \ldots, K_{n_l}), [K_{sl,1}^T, \ldots, K_{sl,n_l}^T]]^T.
\]

Neglecting an ROPR’s inherent characteristics leads to two assumptions: 1) zero motion errors in all active joints and 2) unconstrained geometric errors \(\delta\) during calibration. Accordingly, the ROPR’s overall error model, excluding the terms related to the motion errors of the active joints, is derived from (5) as

\[
\Omega y = Kk + L_p \delta \theta_p \tag{6}
\]

where \(\delta \theta_p \in \mathbb{R}^{n_l-n_a}\) denotes the motion errors of all passive joints; \(L_p\) is derived from \(L\) by removing columns corresponding to the motion errors of the active joints.

Due to the redundantly actuated and overconstrained characteristics, under geometric errors, there exist constraints among all actual active joint positions, and the geometric errors also need to meet specific constraint conditions. Assuming zero motion errors in active joints, along with unconstrained geometric errors, will lead to the violation of configuration constraints and subsequently compromise the calibration precision.

### III. Redundancy-Aware Error Model

To tackle this issue, we propose the RAEM. First, the effects of the two characteristics on the overall error model are analyzed based on the configuration space of the ROPR. To mitigate these effects, the restrained deviations of the redundant active
joints are portrayed in the RAEM and a feasible error space is accordingly established to impose geometric error constraints on the identified errors. The introduced methods can theoretically prevent the violation of configuration constraints, providing benefits for calibration precision.

A. Effect of the Two Characteristics

To demonstrate the effects of these two characteristics, we begin by introducing the configuration space (C-space) of an ROPR. In an ROPR, the end-effector is shared by all limbs, thus \( T_1 = \ldots = T_{n_l} \). Denote the joint motion vector and kinematic parameters of the whole ROPR as \( \theta \) and \( \xi \), respectively. In such a case, the C-space of an ROPR can be implicitly constructed based on the configuration constraints as

\[
C(\theta, \xi) = \{ \xi \in \mathbb{R}^{n_a} : C(\theta, \xi) = 0 \} \tag{7}
\]

where \( C(\theta, \xi) = [(\log(T_1^{-1}T_1))\ldots, (\log(T_{n_l}^{-1}T_{n_l}))]^{T} \in \mathbb{R}^{(n_l-1)} \) is the pose deviations between adjacent limbs.

1) Effect of Redundantly Actuated Characteristics: For an \( f\)-DoF ROPR with \( n_a \) actuations, the active joints can be divided into two categories: 1) \( f \) independent active joints and 2) \( n_a - f \) redundant active joints. There is no intrinsic difference between them, so the choice of redundant active joints is arbitrary. Since the DoF of the ROPR is \( f \), the dimension of this C-space is also \( f \). This indicates that given the motions of \( f \) independent active joints, the remaining joint motion, including redundant active joint motion \( \theta_{ra} \in \mathbb{R}^{n_a-f} \) and passive joints motion \( \theta_{p} \in \mathbb{R}^{n_a-n_a} \), can be strictly determined by (7).

With geometric errors, it can be assumed that given the motion of an independent active joint, deviation of the dependent joint motion \( \delta \theta_{d} = [\delta \theta_{p}^{T}, \delta \theta_{ra}^{T}]^{T} \in \mathbb{R}^{n_a-f} \) should satisfy the actual configuration constraints. The deviations of the redundant active joints \( \delta \theta_{ra} \) are termed restrained deviations. This restrained deviation is comprised of encoder indexing errors and joint deformation, which cannot be directly measured by the joint encoder [20], [21], [22].

For the independent active joints, it can be considered that only constant zero offsets exist, which can be equated to geometric errors. This concept, previously demonstrated in serial robots, is now extended to the ROPR. Detailed evidence supporting this extension is presented in Appendix A (see Supplementary material).

2) Effect of Overconstrained Characteristics: Due to the overconstrained characteristics of an ROPR, the geometric error \( k \) should satisfy specific constraints to avoid violating the configuration constraints. The subsequent analysis will detail these constraints. To investigate this effect, we first perform an additional transformation of (7) using the properties of the logarithm function, we can express each component in (7) as

\[
(\log(T_iT_{i+1}^{-1}))^{\vee} = (\log(T_i^{-1}T_{i+1})^{-1})^{\vee} = (-log(T_{i}^{T}T_{i}^{-1}) + log(T_{i+1}^{T}T_{i+1}^{-1}))^{\vee} \tag{8}
\]

where \( T_{i}^{w} \) denote the actual end-effector pose of the ROPR; \( i = 1, \ldots, n_l - 1 \). Considering the property \( (\log(T_{i}^{w}T_{i}^{-1}))^{\vee} \approx y_{i} \) [28], [29] and combining it with (7)–(8), the configuration constraints can be approximated as

\[
C(\theta, \xi) = [y_{1}^{T} \ldots y_{n_l}^{T} \ldots y_{n_l-1}^{T} - y_{n_l}^{T}]^{T} = 0. \tag{9}
\]

Under geometric errors, by substituting (4) into (9) and omitting terms related to independent active joints, we establish the relationship between geometric error \( k \) and motion errors of dependent joints \( \delta \theta_{d} \)

\[
Wk + Y \delta \theta_{d} = 0 \tag{10}
\]

where the coefficient matrices are

\[
W = \begin{bmatrix}
-K_1 & K_2 & 0 & 0 & K_{st,2} - K_{st,1} \\
0 & \cdots & \cdots & 0 & \vdots \\
0 & 0 & -K_{n_l-1} & K_{n_l} & K_{st,n_l} - K_{st,n_l-1}
\end{bmatrix},
\]

\[
Y = \begin{bmatrix}
-L_{d,1} & L_{d,2} & 0 & 0 & \vdots \\
0 & \cdots & \cdots & 0 & \vdots \\
0 & 0 & -L_{d,n_l-1} & L_{d,n_l}
\end{bmatrix} \in \mathbb{R}^{6(n_l-1) \times (n_a-f)}
\]

and the dependent joint motion errors are explicitly represented as \( \delta \theta_{d} = [\delta \theta_{d,1}^{T}, \ldots, \delta \theta_{d,n_l}^{T}]^{T} \in \mathbb{R}^{n_a-f} \), including passive and redundant active ones. For a nonredundant limb \( i(\leq f) \), \( L_{d,i} = L_{i}([0_{(f_i-1)\times1}, I_{f_i-1}])^{T} \in \mathbb{R}^{6 \times (f_i-1)} \), \( \delta \theta_{d,i} = [0_{(f_i-1)\times1}, I_{f_i-1}][\delta \theta_{i}^{T}] \in \mathbb{R}^{f_i-1} \); for a redundant limb \( i(f \geq f) \), \( L_{d,i} = L_{i} \), and \( \delta \theta_{d,i} = \theta_{i} \).

Owing to the overconstrained characteristics of ROPRs, i.e., \( 6(n_l - 1) > n_{0} - f \), the row number of \( Y \) exceeds its column number. Consequently, in (10), a solution for \( \delta \theta_{d} \) may not exist, leading to potential violations of configuration constraints if each component of \( k \) is independent and constant across different configurations. In other words, for \( \delta \theta_{d} \) to be solvable, \( Wk \) must lie within the column space of \( Y \), imposing constraints on the geometric errors \( k \).

B. Redundancy-Aware Error Model

1) Tackle Restrained Deviations: As detailed in Section III-A1, to mitigate the effects of redundantly actuated characteristics, it is essential to portray the restrained deviations of redundant active joints in the overall error model (6). Adding these deviations and the corresponding elements to (6) yields

\[
\Omega y = Kk + L_{d}\delta \theta_{d} \tag{11}
\]

where \( L_{d} = \text{blockdiag}(L_{d,1}, \ldots, L_{d,n_l}) \). Given that \( \delta \theta_{d} \) varies with the configuration, these errors should be eliminated from the overall error model. Define the left nullspace of \( L_{d} \) as \( N = \text{null}(L_{d}^{T}) \). By left-multiplying (11) with \( N^{T} \) and applying the Moore–Penrose pseudoinverse [30], we can eliminate \( \delta \theta_{d} \). This process yields a reduced error model as

\[
y = (N^{T}\Omega)^{\dagger}N^{T}Kk. \tag{12}
\]

2) Feasible Error Space: As discussed in Section III-A2, addressing the overconstrained characteristics involves determining constraints on geometric errors to prevent violations of (10). Our goal is to find the space that meets these constraints, which we represent through a matrix \( F \), denoted as the feasible error space.

**Proposition 1:** To satisfy the overdetermined equation in (10) across \( m \) configurations, the error vector \( k \) needs to be
characterized as

\[ k = Fx \]  

(13)

where \( F \) is the feasible error space, which can be derived from (10) across \( m \) configurations. \( x \in \mathbb{R}^{n_x} \) is the error component in \( F \). The specific expression of \( F \) is

\[
F = P \begin{bmatrix} -R^{-1} & 0 \end{bmatrix} \begin{bmatrix} 0_{(n_x-r) \times r} & I \end{bmatrix} P^T \in \mathbb{R}^{n_x \times n_x}
\]  

(14)

where \( P = \begin{bmatrix} -\bar{R} & 0 \end{bmatrix} \) is the pivoted QR decomposition of \( \bar{A} \) [31]; \( \bar{A} \in \mathbb{R}^{18m \times n_x} \) represents the nullspace of \( k \) in \( m \) configurations, and its rank is \( r \).

**Proof:** Given in Appendix B (see Supplementary material). \( \blacksquare \)

3) **Error Model of an ROPR:** Substituting (13) into (12), the REAM of an ROPR considering redundantly actuated and overconstrained characteristics can be derived as

\[
y = (N^T \Omega)^{-1} N^T K F x := J x.
\]  

(15)

The error modeling procedure of the proposed RAEM is depicted in Fig. 2.

**C. Generalization of the RAEM**

The proposed RAEM is also applicable to nonlimb-redundant ROPRs and redundant nonoverconstrained PRs.

**Proposition 2:** For nonlimb-redundant ROPRs \((n_a > n_l = f)\), the main difference lies in reformulating the Jacobian matrix in (11) as \( L_{d,i} = L_i \begin{bmatrix} 0_{(f_l-1) \times 1} & I_{f_l-1} \end{bmatrix} \in \mathbb{R}^{6 \times (f_l-1)} \), and the joint motion errors in limb \( i \) should be modified as \( \delta \theta_{d,i} = \begin{bmatrix} \delta \theta_{d,i} \end{bmatrix} \in \mathbb{R}^{f_l-1} \).

**Proposition 3:** For redundant nonoverconstrained PRs, without overconstrained characteristics, the matrix \( Y \) related to joint motion errors in (10) is not overdetermined, i.e., \( Y \in \mathbb{R}^{n_a \times n_x} \), and its geometric errors are not subject to constraints. In such a case, its error model can be simplified to (12), with \( F = I \).

**IV. CALIBRATION USING RAEM**

This section introduces a calibration procedure that involves iterative error identification and parameter updates until the iteration is nonimproving.

**A. Error Identification in Each Calibration Iteration**

After measuring \( m \) configurations and based on the error model in (15), the error identification in each iteration can be formulated as

\[
\min_x \| \tilde{y} - J x \|_2^2
\]  

(16)

where \( \tilde{y} = [y^{(1)}T, \ldots, y^{(m)}T]^T \) denotes the overall pose error; \( \bullet^{(i)} \) denotes the value of a variable at the \( i \)th measured configuration; \( y \) can be measured by \( y = (\log(T^T \hat{J}^n(T)^{-1})) \); \( \hat{\theta} \) and \( \theta \) denote the nominal and actual value of a variable, respectively; \( \hat{J} = [J^{(1)}T, \ldots, J^{(m)}T]^T \) is the calibration Jacobian matrix; \( \| \bullet \|_2 \) denotes the 2-norm.

Although the least square method can efficiently solve (16), it is noise-sensitive [32]. In contrast, the EKF method has been proven to be robust in the kinematic calibration for nonlinear systems with measurement noise [25], [32], [33]. Thus, for stable calibration results, the EKF algorithm is employed to solve (16). In the EKF algorithm, the geometric errors \( x \) and the pose errors \( y \) are considered as state variables and observer variables of the EKF, respectively. It consists of two steps: 1) prediction and 2) updating. During the prediction step for \( i \)th configuration, the geometric error \( x \) and the estimated error covariance \( P \in \mathbb{R}^{n_x \times n_x} \) are given by

\[
x^{(i)(i-1)} = x^{(i-1)(i-1)}
\]

\[
P^{(i)(i-1)} = P^{(i-1)(i-1)} + Q_a
\]  

(17)

where \( \bullet^{(i)(i-1)} \) represents a priori estimate of a variable at the measured configuration \( i \), and \( \bullet^{(i)(i)} \) represents a posteriori estimate; \( Q_a \in \mathbb{R}^{n_x \times n_x} \) is the process noise covariance. The updating step of the \( i \)th configuration, the Kalman gain \( G \in \mathbb{R}^{n_x \times 6} \), the estimation of \( x \), and the estimated error covariance \( P \) are determined as follows:

\[
G^{(i)} = P^{(i)(i-1)} J^{(i)(i)} \left( J^{(i)(i)} P^{(i)(i-1)} J^{(i)(i)}^T + Q_a \right)^{-1}
\]

\[
x^{(i)(i)} = x^{(i)(i-1)} + G^{(i)} \left( y^{(i)} - J^{(i)} x^{(i)(i-1)} \right)
\]

\[
P^{(i)(i)} = \left( I - G^{(i)} J^{(i)} \right) P^{(i)(i-1)}
\]  

(18)

where \( Q_m \in \mathbb{R}^{6 \times 6} \) is the measurement noise covariance.

**B. Parameter Update in Each Calibration Iteration**

Using the identified \( x \), the nominal parameters are updated as

\[
\xi_{i,j}^n = \text{Ad} (\hat{\theta}^{m_{i,j}}) \xi_{i,j}^n, \ T_{i,0}^n = e^{\hat{K}_{i,0}} T_{i,0}^n
\]  

(19)

with \( \delta m_{i,j} = B_{i,j} k_{i,j}, k = F x \). Next, kinematics is updated. In this step, the motions of the independent joints \( \theta_{d,i} \), including redundant active and passive joints, are required to be updated via forward kinematics, which is inherently different from the forward kinematics in nonredundant PRs [14]. With the optimization objective being the sum of the pose deviations between adjacent limbs, the forward kinematics is numerically calculated using the Newton–Raphson method, yielding the following iterative scheme:

\[
\delta \theta_d = (Y^T Y + \lambda I)^{-1} Y^T \delta e_f
\]  

(20)

where \( \delta e_f = C(\theta, \xi) \) denotes the vector combined by pose deviations between the adjacent limbs, which has been defined in (7); \( Y \) is the transformation matrix between \( \theta_d \) and \( \delta e_f \), referring to (10); the Levenberg–Marquardt (LM) algorithm is utilized...
here to prevent the denominator term from ill-condition, and $\lambda$ is a damping parameter. After updating the motions of dependent joints $\theta_{ij}$, the next iteration will be started until $\|\delta e_f\|$ is less than a given threshold, or the iteration is nonimproving. The end effector pose $T^n$ is also updated based on the forward kinematics.

V. NUMERICAL EVALUATION

This section conducts kinematic calibration simulation on two robots, including a redundant nonoverconstrained PR and an ROPR. The calibration of the redundant nonoverconstrained PR is for verifying the proposed RAEMs effectiveness in handling redundant actuation characteristics. The calibration of the ROPR aims to confirm the RAEM’s capability in addressing redundantly actuated and overconstrained characteristics.

In addition, two error models are compared, which are as follows.

1) RAEM: The proposed redundancy-aware error model.

For a redundant nonoverconstrained PR, the RAEM only considers the redundantly actuated characteristics and can be simplified as described in Proposition 3.

2) oRAM: It serves as a baseline representative of the existing general error model for ROPRs, which ignores the redundantly actuated and overconstrained characteristics. In oRAM, deviations of all active joints are assumed to be zero in both the error model (15) and the forward kinematics (20) while constraints on geometric errors are neglected, i.e., $F = I$. Given that the adjoint error model in oRAM can be equivalent to other suitable general error models [24], [34], such as the screw-based method, with the main distinction being the approach to eliminating redundant error parameters, oRAM can, indeed, be regarded as a representative of existing general error models.

To quantitatively evaluate the calibration results, the position and orientation errors, $e_p = \|t^a - t^n\|_2$ and $e_o = \arccos(\omega_o \cdot \omega^n)$, are defined, where $t$ and $\omega$ are the position and orientation vectors of the end-effector pose $T$, respectively.

In subsequent evaluations, the logic for selecting the covariance in EKF is as follows. Considering that the geometric error $x$ remains almost unchanged in an experiment, we set the process noise covariance to $Q_x = 1 \times 10^{-7}I_{n_x \times n_x}$. The initial value is set to $x = 0_{n_x \times 1}$, so the initial estimation error covariance for the different component of $x$ can be considered to be the same, i.e., $P^{(00)} = pI_{n_x \times n_x}$. Due to the complexity of noise in the output equation, including sensor measurement noise, nongeometric errors, etc., it is relatively difficult to specify an initial guess for matrix $Q_m$ in advance. Thus, we assume that $Q_m$ is an identical diagonal matrix, i.e., $Q_m = q_m I_{6 \times 6}$. On this basis, the covariances $Q_m$, $P^{(00)}$ are determined through discrete sampling within a given range to minimize $\text{Ave}(e_p) + \text{Ave}(e_o)$. In practice, for a prototype calibrated in meters and with a sensor measurement noise of 5 $\mu$m, it is recommended to take $q_m = 1, p = 1$ as the initial parameters and search for discretization around it. In cases of ROPR with minor geometric errors, it is advisable to suitably decrease the initial values of $q_m$ and $p$.

A. Simulation I: Calibration of a Redundant Nonoverconstrained PR

1) Structure Description: The 3PRS-PUS PR, shown in Fig. 3, contains a fixed base, an end-effector, a PUS limb, and three identical PRS limbs. Each limb connects to the fixed base and the end-effector at points $B_i$ and $C_i$, respectively. The spatial frame $\{S\}$ is attached to the fixed base located at point $O$, in which the $z$-axis is perpendicular to the plane of $A_1, A_2, A_3, A_4$, and the $x$-axis points from $A_1$ to $A_3$ with the $y$-axis satisfying the right-hand rule. The tool frame $\{T\}$ is attached to the end-effector located at the center of the end-effector surface $E_2$, in which the $x$-axis is parallel to $C_1C_3$, and the $y$-axis points from $C_2$ to $C_4$ with the $w$-axis satisfying the right-hand rule.

The DoF of this PR is three, including two rotations about the $u$ and $y$ axes, and one translation along the $z$-axis [35]. The end-effector’s motion is described using three generalized coordinates $[z, \alpha, \beta]$, where $z$ denotes the $z$-coordinate of $E_1$ concerning $\{S\}$; $\alpha$ and $\beta$ denote the rotation angles about the $u$ and $y$ axes, respectively. With $z, \alpha, \beta$, the pose of $\{T\}$ in $\{S\}$ can be written as

$$ T = \begin{bmatrix} R & 0r_{E_2}^E_1 + r_{E_1} \\ 0 & 1 \end{bmatrix}, $$

where $R = \exp(\hat{u}_y \beta) \exp(\hat{u}_x \alpha)$ is the rotation matrix of $\{T\}$ concerning $\{S\}$, in which $u_x = [1, 1, 0]^T$, $u_y = [0, 1, 0]^T$, $u_z = [0, 0, 1]^T$ are the unit vectors in $\mathbb{R}^3$, $r_{E_2}^E_1 \in \mathbb{R}^3$ is the position of point $E_2$ regarding the frame $E_1$-uvw, which is a constant vector; $r_{E_1}$ is the position of $E_1$ regarding $\{S\}$, which can be expressed by the generalized coordinates [35].

2) Geometric Error Modeling: The parameters related to the error model are described in the following. The PRs DoF is $f=3$, with $n_1 = 4$ limbs, $n_a = 4$ actuators, and the first joint in each limb being active. The DoF of each limb is $f_1 = f_3 = f_4 = 5$, $f_2 = 6$, and the total number of joints is $n_0 = 21$. These variables satisfy the equation that $6(n_1 - 1) = n_q - f$, indicating that this PR is non-overconstrained according to Definition 2. Limb 4 (PRS) is considered as the redundant limb. The home configuration is defined as $\alpha = 0$ rad, $\beta = 0$ rad, $z = 0.2$ m. At this configuration, the nominal coordinates of the joint points

Fig. 3. Structure of the 3PRS-PUS PR.
regarding \( \{S\} \) are (unit: m):
\[
\begin{align*}
\mathbf{r}_{B_1} &= \begin{bmatrix} -0.196, 0, -0.0568 \end{bmatrix}^T, & \mathbf{r}_{C_1} &= \begin{bmatrix} -0.116, 0, 0.2 \end{bmatrix}^T \\
\mathbf{r}_{B_2} &= \begin{bmatrix} 0, -0.196, -0.0568 \end{bmatrix}^T, & \mathbf{r}_{C_2} &= \begin{bmatrix} 0, -0.116, 0.2 \end{bmatrix}^T \\
\mathbf{r}_{B_3} &= \begin{bmatrix} 0.196, 0, -0.0568 \end{bmatrix}^T, & \mathbf{r}_{C_3} &= \begin{bmatrix} 0.116, 0, 0.2 \end{bmatrix}^T \\
\mathbf{r}_{B_4} &= \begin{bmatrix} 0.196, 0, -0.0568 \end{bmatrix}^T, & \mathbf{r}_{C_4} &= \begin{bmatrix} 0.116, 0, 0.2 \end{bmatrix}^T \\
\end{align*}
\]
(22)
where \( \mathbf{r}_{B_i}, \mathbf{r}_{C_i}, \mathbf{r}_{D_i} \in \mathbb{R}^3 \) denote the nominal coordinates of the joint points \( B_i, C_i \), and \( D_i \), respectively. On this basis, the twist of each limb can be calculated. For limbs 1 and 3, their joint twists are
\[
\begin{align*}
\xi_{i,1} &= \begin{bmatrix} 0_{1 \times 3}, \mathbf{u}_z \end{bmatrix}^T, \quad \xi_{i,2} = \begin{bmatrix} \mathbf{u}_y, (\mathbf{r}_{B_i} \times \mathbf{u}_y) \end{bmatrix}^T \\
\xi_{i,3} &= \begin{bmatrix} \mathbf{u}_y, (\mathbf{r}_{C_i} \times \mathbf{u}_y) \end{bmatrix}^T, \quad \xi_{i,4} = \begin{bmatrix} \mathbf{u}_z, (\mathbf{r}_{C_i} \times \mathbf{u}_z) \end{bmatrix}^T \\
\xi_{i,5} &= \begin{bmatrix} \mathbf{u}_z, (\mathbf{r}_{C_i} \times \mathbf{u}_z) \end{bmatrix}^T.
\end{align*}
\]
(23)
Similarly, for limbs 2 and 4, the twist of their joints can be obtained by
\[
\begin{align*}
\xi_{2,1} &= \xi_{4,1} = \begin{bmatrix} 0_{1 \times 3}, \mathbf{u}_z \end{bmatrix}^T, \quad \xi_{2,2} = \begin{bmatrix} \mathbf{u}_y, (\mathbf{r}_{B_2} \times \mathbf{u}_y) \end{bmatrix}^T \\
\xi_{2,3} &= \begin{bmatrix} \mathbf{u}_z, (\mathbf{r}_{B_2} \times \mathbf{u}_z) \end{bmatrix}^T, \quad \xi_{2,4} = \begin{bmatrix} \mathbf{u}_y, (\mathbf{r}_{C_2} \times \mathbf{u}_y) \end{bmatrix}^T \\
\xi_{2,5} &= \begin{bmatrix} \mathbf{u}_y, (\mathbf{r}_{C_2} \times \mathbf{u}_y) \end{bmatrix}^T, \quad \xi_{2,6} = \begin{bmatrix} \mathbf{u}_z, (\mathbf{r}_{C_2} \times \mathbf{u}_z) \end{bmatrix}^T \\
\xi_{4,2} &= \begin{bmatrix} \mathbf{u}_z, (\mathbf{r}_{B_2} \times \mathbf{u}_z) \end{bmatrix}^T, \quad \xi_{4,3} = \begin{bmatrix} \mathbf{u}_y, (\mathbf{r}_{C_4} \times \mathbf{u}_y) \end{bmatrix}^T \\
\xi_{4,4} &= \begin{bmatrix} \mathbf{u}_y, (\mathbf{r}_{C_4} \times \mathbf{u}_y) \end{bmatrix}^T, \quad \xi_{4,5} = \begin{bmatrix} \mathbf{u}_z, (\mathbf{r}_{C_4} \times \mathbf{u}_z) \end{bmatrix}^T.
\end{align*}
\]
(24)
Based on the previous analyses, the RAEM and oRAM of this redundant nonoverconstrained PR can be obtained. During calibration, the inverse kinematics problem under nominal parameters is also needed, which can be solved using Padé–Kahan subproblems [36] and is not focused on here.

3) Evaluation Setup: The simulation procedure for calibration is illustrated in Fig. 4. In the data generation, 97 measured configurations are generated by discretizing the generalized workspace as \( \alpha \in [-25, 25] \) deg \( \times \beta \in [-25, 25] \) deg \( \times z \in [300, 600] \) mm, for the calibration. In addition, 72 validated configurations are randomly generated in the workspace. Geometric errors \( \mathbf{x} \), following the normal distribution \( \mathcal{N}(1 \times 10^{-3}, 1 \times 10^{-6}) \), are randomly selected and then added to the nominal parameters, based on (19). The actual poses are then obtained by the forward kinematics. Considering that the measurement accuracy of the laser tracker is \( 5 \mu \) m, random errors following the normal distribution \( \mathcal{N}(5 \times 10^{-6}, 2.5 \times 10^{-11}) \) are added to the position of \( \{T\} \), while random errors following \( \mathcal{N}(5 \times 10^{-5}, 2.5 \times 10^{-9}) \) are injected into the orientation of \( \{T\} \).

In the parameter calibration, the geometric errors can be identified by EKF by the measured end pose error. Based on the identified errors, the nominal parameters and current kinematics can be updated. Repeat these steps until the residual pose error between adjacent iterations is less than \( 5 \times 10^{-5} \). By means of discrete optimization, the covariances of the EKF for the RAEM are selected as \( Q_m = 3 \times I_{6 \times 6} \) and \( P_{(0/0)} = 50 \times I_{82} \). The covariances of the EKF for the oRAM are chosen as \( Q_m = 70 \times I_{6 \times 6} \) and \( P_{(0/0)} = 50 \times I_{82} \).

4) Results and Analyses: The comparative results are illustrated in Fig. 5 and Table I. It can be seen that the proposed RAEM outperforms oRAM in the calibration precision. Specifically, compared with the oRAM, the proposed RAEM can effectively reduce the residual Ave(\( \varepsilon_\beta \)) and Ave(\( \varepsilon_\alpha \)) of the oRAM by 86.35% and 62.84%, respectively, at the measured configurations. At the validated configurations, the residual Ave(\( \varepsilon_\beta \)) and Ave(\( \varepsilon_\alpha \)) of the oRAM can be reduced by 90.11% and 75.84%, respectively. This improvement is attributed to RAEM accounting for the restrained deviations in both the error model and forward kinematics, ensuring configuration constraints are maintained throughout the calibration process, thereby enhancing precision. The previous results confirm the capability of the proposed RAEMs in handling redundantly actuated characteristics and provide a foundation for the subsequent kinematic calibration of an ROPR.

B. Simulation II: Kinematic Calibration of an ROPR

This section further applies the proposed RAEM to perform kinematic calibration on the 2PPRRR-PSS-PRS ROPR [35], named M2, to verify the effectiveness of the proposed method for kinematic calibration on ROPR.
1) Structure Description: The CAD model and the schematic model of the M2 ROPR are shown in Fig. 6. Due to the space limitation, only the differences between M2 ROPR and the 3PRS-PUS PR are described in the following. On the one hand, limbs 1 and 3 of the M2 ROPR are PRRR while limbs 1 and 3 of the 3PRS-PUS PR are PRS. On the other hand, limb 2 of the M2 ROPR is PSS while that of the 3PRS-PUS PR is PUS.

2) Geometric Error Modeling: The local DoF existing in the S-S chain of limb 2 (PSS) is ignored in the error modeling, and the DoF of each limb is thus, \( f_1 = 4, f_2 = 6, f_3 = 4, f_4 = 5 \), respectively. The DoF of this ROPR is \( f = 3 \), the number of limbs is \( n_l = 4 \), and the total number of joints is \( n_q = 19 \). These variables clearly satisfy Definition 2, i.e., \( 6(n_l - 1) > n_q - f \), indicating the ROPR’s overconstrained characteristics. Similar to Simulation 1, given each joint twist, the error model of the M2 ROPR can be derived from Section IV. The specific error modeling procedure can be found in Appendix C (see Supplementary material).

3) Evaluation Setup: The selection of measured configurations, validated configurations, and the measurement noise of the 3PRS-PUS PR are described in the following. On the one hand, limbs 1 and 3 of the M2 ROPR are PRRR while limbs 1 and 3 of the 3PRS-PUS PR are PRS. On the other hand, limb 2 of the M2 ROPR is PSS while that of the 3PRS-PUS PR is PUS.

4) Results and Analyses: The mean position and orientation errors of the two compared models during iterative calibration are shown in Fig. 7. As shown in the figure, RAEMs calibrations precision and convergence speed outperforms those of oRAM. Specifically, after three iterations, the mean position errors of the proposed RAEM converge to the same level as the given measurement errors. Compared with oRAM, we can infer that without considering the two characteristics in RAEM, the mean position error \( \text{Ave}(e_p) \) and mean orientation error \( \text{Ave}(e_o) \) will increase to nearly 546% and 276% of the original values, respectively, at the measured configuration. This phenomenon can be explained as follows. First, to tackle the redundantly actuated characteristics, the RAEM proposed portrays the restrained deviations of redundant active joints in the error model. Second, considering the overconstrained characteristics, the RAEM projects all errors into the feasible error space to enforce the geometric error constraints. Therefore, the proposed RAEM can prevent the violation of configuration constraints, which is demonstrated in Fig. 8. Here, \( \| \delta e_f \|_2 \) is used to qualify the degree of violation of the configuration constraints, which are defined in (20). As shown in Fig. 8, the violation degree of the oRAM is nearly 544% and 213%, respectively. These results confirm that thanks to the consideration of the two characteristics, the proposed RAEM can achieve better calibration precision than oRAM. Furthermore, the residual errors at the validated configurations are qualified in Table II. At the validated configurations, compared to RAEM, \( \text{Ave}(e_p) \) and \( \text{Ave}(e_o) \) of oRAM are deteriorated by nearly 544% and 213%, respectively. These results validate the RAEMs effectiveness and highlight the advantages of addressing the redundantly actuated and overconstrained characteristics in ROPRs.

VI. EXPERIMENT VERIFICATION

To further showcase the RAEM’s effectiveness, calibration experiments are conducted on the prototype of the M2 ROPR. 

Fig. 6. Schematic model of the M2 ROPR. (a) CAD model. (b) Schematic model.

Fig. 7. Simulation II: Mean position and orientation errors during the calibration process of the M2 ROPR.

Fig. 8. Simulation II: Forward kinematics convergence errors of RAEM and oRAM at the measured configurations.

TABLE II

| Model   | Measured Configurations \( \text{Ave}(e_p) \) (mm) | Validated Configurations \( \text{Ave}(e_p) \) (mm) | \( \text{Ave}(e_o) \) (mrad) | \( \text{Ave}(e_o) \) (mrad) |
|---------|-----------------------------------------------|--------------------------|-------------------------|-------------------------|
| Before  | 4.273                                         | 7.702                    | 4.335                   | 6.682                   |
| RAEM    | 0.010                                         | 0.029                    | 0.009                   | 0.028                   |
| oRAM    | 0.556                                         | 0.830                    | 0.499                   | 0.623                   |

The bold entries emphasize the superiority of the proposed RAEM.
the whole generalized workspace, the measured configurations are generated to validate the calibration results. To guarantee the robot’s geometric errors, and additional 59 configurations are similarly with (25) by the SMRs 

\[ T^M_S = \begin{bmatrix} q_x & q_y & q_z & t^M_{S} \end{bmatrix} \]  

(25)

where \( t^M_{S} = \frac{1}{2}(r_{S1} + r_{S2}), q_x = \frac{r_{S2} - t^M_{S}}{\|r_{S2} - t^M_{S}\|}, q_z = \frac{(r_{S1} - r_{S3}) \times (r_{S2} - r_{S1})}{\| (r_{S1} - r_{S3}) \times (r_{S2} - r_{S1}) \|}, \) \( q_y = q_z \times q_x. \) Here, \( r_{Si}, i = 1, 2, 3 \) denotes the position of \( S_i \) measured by the laser tracker.

Due to the limited visibility when attaching the SMRs directly to the end-effector’s surface, a calibration board is designed. This board is rigidly attached to the end-effector and provides a convenient way to measure the pose of the end-effector. Three SMRs are attached to this calibration board, denoted by \( T_1, T_2, T_3. \) By measuring the pose of the tool frame \( \{T\} \) regarding the measurement coordinate system \( \{M\} \), the coordinate transformation describing the pose of the frame \( \{T\} \) regarding \( \{S\} \) can be obtained as

\[ T^a = T^M_S^{-1} T^M_T \]  

(26)

where \( T^a \) and \( T^M_T \) denote the pose of \( \{T\} \) with respect to \( \{S\} \) and \( \{M\} \), respectively. The pose of the tool frame is constructed similarly with (25) by the SMRs \( T_1, T_2, \) and \( T_3. \)

As shown in Fig. 10, 95 measured configurations are selected to identify the robot’s geometric errors, and additional 59 configurations are generated to validate the calibration results. To guarantee that the calibration results can be used as a reference for the whole generalized workspace, the measured configurations are evenly distributed in the workspace. In addition, considering the number of error parameters is \( n_e = 74 \), the number of measured configurations \( m = 95 \) aligns with the condition \( m \geq 5n_e/6 \) [37], providing a foundation for good calibration results. Specifically, the measured configurations are generated within the ROPR’s generalized workspace as \( \alpha \in [-20, 20] \) deg \( \times \beta \in [-20, 20] \) deg \( \times z \in [403.4, 433.34] \) mm, and some configurations are excluded because they are blocked by the prototype and cannot be measured. The validated configurations are generated by randomly selecting configurations in the \( z = 410.9 \) mm and \( z = 425.9 \) mm planes, to verify the nonrandomness and effectiveness of the calibration result. The calibration process is the same as shown in Fig. 4. The covariances of the EKF for the RAEM are selected as \( Q_m = 0.0011 \times I_{6 \times 6} \) and \( P^{(0)} = 31 \times I_{74} \). The covariances of the EKF for the oRAM are \( Q_m = 40 \times I_{6 \times 6} \) and \( P^{(0)} = 100 \times I_{74} \).

B. Results and Analyses

The comparative results between RAEM and oRAM are shown in Fig. 11. The figure shows that RAEM has better convergence speed and much less calibration errors than oRAM. Specifically, the iteration process of RAEM converges after two iterations while oRAM converges after four iterations. After calibration, at the measured configurations, the mean position errors \( \text{Ave}(e_p) \) of RAEM and oRAM decrease to \( 1.2 \times 10^{-4} \) m and \( 3.1 \times 10^{-4} \) m, respectively, and their mean orientation errors \( \text{Ave}(e_o) \) decrease to \( 4.9 \times 10^{-4} \) rad and \( 5.9 \times 10^{-4} \) rad, respectively. These results indicate that if the two inherent characteristics are not considered as in the oRAM, \( \text{Ave}(e_p) \) and \( \text{Ave}(e_o) \) will deteriorate by 158.3% and 20.41%, respectively. The results of the validation configurations are shown in Table III. In this case, \( \text{Ave}(e_p) \) of RAEM and oRAM reduce to \( 1.2 \times 10^{-4} \) m and \( 3.1 \times 10^{-4} \) m, respectively, and their \( \text{Ave}(e_o) \) decrease to \( 5.9 \times 10^{-4} \) rad and \( 6.7 \times 10^{-4} \) rad, respectively. In summary, these results validate that the proposed RAEM can effectively improve the kinematic calibration precision compared to the existing general error models, which do not consider the two intrinsic characteristics.
TABLE III
EXPERIMENT: COMPARED RESULTS BEFORE/AFTER CALIBRATION

| Model | Measured Configurations | Validated Configurations |
|-------|-------------------------|-------------------------|
|       | Ave($e_p$)(mm) | Ave($e_o$)(mrad) | Ave($e_p$)(mm) | Ave($e_o$)(mrad) |
| Before | 4.61 | 17.97 | 4.60 | 17.86 |
| RAEM   | 0.12 | 0.49 | 0.12 | 0.59 |
| oRAM   | 0.31 | 0.59 | 0.31 | 0.67 |
| sRAM   | 0.38 | 0.71 | 0.37 | 0.78 |
| CLVM   | 0.46 | 5.01 | 0.45 | 4.60 |

The bold entities emphasize the superiority of the proposed RAEM.

It is worth noting that there are still residual errors after kinematic calibration, and Ave($e_o$) is larger than Ave($e_p$). The reasons are as follows. First, many unmodeled errors, such as joint clearance, limit the calibration precision. Second, since the orientation of the end-effector is calculated based on the measured position of the SMRs, the magnitude of this orientation error is much larger than its position error, limiting the precision of the orientation in kinematic calibration.

To further verify the necessity of removing the zero offsets of active joints, as demonstrated in Section III-A, we compare the RAEM with an error model, called sRAM, which considers constant offsets in all active joints and ignores the two intrinsic characteristics. In sRAM, the active joint deviation in (5) is assumed to be nonzero and cannot be eliminated. In addition, the active joint motion needs to be updated in each calibration iteration. The rest of the processing in sRAM is the same as oRAM. In sRAM, the covariance matrices of the EKF are selected as $Q_m = 70 \times I_{6 \times 6}$, and $P^{(100)} = 10 \times I_{74}$.

The residual errors after calibration are shown in Fig. 12 and Table III. It is shown that the calibration precision of the proposed RAEM is the highest among the three error models. This indicates that under geometric errors, the configuration constraints are violated when considering all active joint deviations as zero, such as oRAM, or all active joint deviations as nonzero constants, such as sRAM, leading to a degradation in calibration precision. In addition, it is worth noting that the calibration precision of sRAM is lower than that of oRAM. The primary reason for this is that the constant zero offset in active joints are equivalent to geometric errors, making them unidentified, as shown in Appendix A (see Supplementary material). Since zero offsets of active joints and geometric errors are considered in sRAM simultaneously, the identifiability will be affected, leading to a deterioration of the calibration precision.

Moreover, comparisons with the closed-loop vector method are performed. In CLVM, the redundantly actuated characteristic is ignored and some error assumptions are made. The key assumption is that each limb is confined on a corresponding plane for the sake of a closed-form solution to the ROPR’s inverse kinematics [16], [38], [39]. The details of the error modeling process based on the CLVM are presented in Appendix D (see Supplementary material). As a result, a 36-parameter error model can be obtained. Meanwhile, the same measurements are used for kinematic calibration and precision evaluation using the CLVM. The compared results are listed in Table III.

Compared with the conventional CLVM, the proposed RAEM can effectively reduce the residual Ave($e_p$) and Ave($e_o$) of the CLVM by 73.91% and 90.22%, respectively, at the measured configurations. At the validated configurations, the residual Ave($e_p$) and Ave($e_o$) of the CLVM can be reduced by 73.33% and 87.17%, respectively. The reduced calibration precision of CLVM is primarily due to its error assumptions, which result in an incomplete representation of error components, particularly those causing orientation errors such as the axis deviations of R joints. Addressing these deviations would significantly increase the complexity of kinematic analysis and error modeling in CLVM, particularly due to the absence of a general modeling procedure for such a complex robot. Contrarily, the RAEM, with its complete error components and general framework, not only improves calibration precision but can also be easily extended to any other ROPRs.

It should be noted that since more error components have been utilized in the proposed calibration model, the computational cost will be higher than the CLVM. In addition, using the proposed kinematic calibration method, the control scheme of ROPRs will be more complex because an extra searching process is required to determine the actual inverse kinematics.

VII. CONCLUSION

This study proposes a general error model, called RAEM, considering the redundantly actuated and overconstrained characteristics proposed for the kinematic calibration of ROPRs. First, the configuration space is implicitly represented to expose the constraints on the active joints and the geometric errors resulting from the two characteristics. To mitigate these effects, the restrained deviations of redundant active joints are portrayed in both the RAEM and the forward kinematics in each calibration iteration. A feasible error space is, thus, formulated to impose geometric error constraints on the identified errors. In this way, the proposed RAEM effectively prevents the violation of configuration constraints. In addition, the RAEM is established based on the adjoint error model, ensuring the minimal requirement. Taking the M2 ROPR as an example, simulations and experiments are conducted to verify the performance of the proposed RAEM. The experiment shows that with the proposed RAEM, the mean position and orientation errors are decreased by at least 97.40% and 97.27%, respectively. Compared with the existing general error models that neglect the two inherent characteristics, the residual mean position error of the proposed RAEM is reduced by 61.29%. In the future, it is worth conducting further research on nongeometric calibration for ROPRs.
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