Analysis and Prediction of Temperature Time Series Using Chaotic Approach

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Abstract. Analysis and prediction of temperature time series is important because temperature changes can affect human’s health. The objectives of this study are to analyse and predict the temperature series in Jerantut, Pahang, Malaysia using chaotic approach. Modelling through chaotic approach divided into two stages; reconstruction of phase space and prediction processes. Through the reconstruction of phase space, a single scalar time series is rebuilt into a multi-dimensional phase space. This multi-dimensional phase space is used to detect the presence of chaotic dynamics through phase space plot and Cao method. The results show that the observed time series is chaotic in dynamic. Therefore, one hour ahead prediction through local mean approximation method is done. The correlation coefficient value obtained is 0.9789. The value which is approaching one reflected that the predicted time series and observed time series are close to each other. Thus, the modelling through chaotic approach is considered succeed. It is hoped that the model can help Malaysian Meteorological Department and Department of Environment Malaysia in order to improve their weather services.

1. Introduction
Climate change give impacts on human life and also affecting the national economy [1] and human health [2]. Therefore, climatic changes need to be taken into account. According to Thinh et al. [3] and Pau et al. [4] stated that temperature changes had been afflicting and negatively impacting the agricultural sector. Moreover, studies conducted by Wichman [5] and Cheng et al. [6] have reported that elevated temperatures have also resulted in respiratory and cardiovascular-related diseases that can lead to death. Therefore, one temperature prediction model is needed to help the authorities to have better preparation in temperature changes.

This study focuses on the chaotic application approach to the temperature time series. The term first used by Li and Yorke in 1975 [7], while Lorenz was the first person to introduce a chaotic dynamic in 1963 [8]. Then, Takens introduced the Takens’ Embedding Theorem involving the use of phase space vectors to detect the presence of a chaotic dynamic in a time series [9]. This embedding theorem is a time-series transformation of one-dimensional scalar into a multi-dimensional phase space vector. Multi-dimensional phase space is used to detect the presence of chaotic dynamic. If the dynamic are present, a prediction model based on chaotic approach will be developed.

Studies by Indira et al. [10] and Echi, Tikyaa and Isikwue [11] find that the temperature time series was chaos using the Lyapunov exponential method and Poincare’ Map, while the studies by Adenan et al. [12] can classified the dynamic of the time series was also chaos by using the Cao method. Until today, the dynamical analysis of temperature time series has never been conducted in Malaysia. Therefore, this study will use the Cao method and the phase space plot reconstruction to detect the presence of a chaotic dynamic of temperature time series.

Chaotic approach had been widely used in various fields. Outside Malaysia, the chaotic approach successfully predicted rainfall time series by Fu et al. [13] in China, the river stream time series by Wilson et al. [14] in China and the sea level time series by Domenico et al. [15] in Australia. In Malaysia, a study on the river stream time series by Adenan et al. [12], time ozone series by Hamid, Adenan, and Nooraini [16] and Awang et al. [17] and PM10 by Hamid and Noorani [18] had also been successfully carried out using chaotic approach. However, the local approximation method never been
applied to temperature time series in Malaysia. Hence, this study will contribute to the escalation of chaotic application approach usage to modelling time series in Malaysia.

The objectives of this study are to identify the presence of a chaotic dynamic in the temperature time series and prediction temperature in Jerantut, Pahang using chaos model. In this study, modelling through a chaotic approach is divided into two stages: i) reconstruction of phase space and ii) prediction process. The first stages is to detect chaotic behaviour of temperature time series and the second is to predict the future temperature time series.

2. Data

Data used in this study is temperature time series. The observed temperature time series is recorded in the unit °C. This temperature data was observed at Batu Embun, Jerantut weather station located in Pahang state and is secondary data. Pahang is located in the east of Peninsular Malaysia (Figure 1) while Jerantut is located in the middle of all the districts in the state of Pahang. Jerantut district is bordered by Kelantan and Terengganu states in the north, Temerloh and Maran regions to the south, Lipis and Raub areas in the west and Kuantan and Terengganu states in the east [19]. It is the largest area in the state of Pahang (Figure 2). The Batu Embun weather station, Jerantut was selected because according to the Department of Environment, this station is a climate base station for all states in Malaysia. Moreover, the Jerantut area is the main entrance to Taman Negara and is one of the areas that supplies freshwater fish to all over the country in Malaysia. The temperature time series data we observed was collected based by hours for six months, from 1 January to 30 June 2014. The first five months of the data were the set of training (3624 data) used in the reconstruction of the phase space and the rest of 720 data was the test set used to test prediction performance. Of the total 4344 data, there was 0.48% of the lost data and was replaced by a time series of the same hour in the previous day. For example, the lost time series dated 01 February 2014 at 6 am, so the lost time series was replaced by a time series at 6am, January 31, 2014.

Figure 3 is the diagram of temperatures time series in Pahang and a statistical explanation can be referenced in Table 1. Table 1 show that the average temperature in Pahang is 26.6°C. Whereas, the minimum recorded temperature is 18.5°C and the maximum temperature is 35.7°C.

Figure 1. Pahang’s Location in Malaysia (Source: https://www.google.com.my)
3. Method
The chaotic approach involves two stages which are the reconstruction of the phase space and the prediction process. Through the phase space reconstruction, single dimensional scalar data is reconstructed into multi-dimensional phase space. Using this multi-dimensional phase space, the phase space plot method and the Cao method will detect the chaotic dynamics of temperature time series in Pahang. If the chaotic dynamics are present in time series, then the local mean approximation method (LMAM) will be used for the prediction process.
3.1. Reconstructing phase space
For chaotic approach, reconstruction of phase space is important for starting the temperature time series process. This technique is known as the reconstruction of phase space because the observed time series by scalar (single dimension) is reconstructed into a multi-dimensional phase space vector. Time series \( X \) is recorded by scalar which is:

\[
X = \{x_1, x_2, x_3, \ldots, x_N\}
\]

with \( N \) is the total number of time series. Time series \( X \) is divided into two parts. The first part is a training time series, while the second part is a time series test. The training time series is used to detect the presence of chaotic dynamics and calculating parameters. Time series test is stored and will be used for comparison purposes with prediction time series. The time series will be reconstruction into \( m \)-dimension phase space:

\[
Y^m = \left(x_{i}, x_{i+\tau}, x_{i+2\tau}, x_{i+3\tau}, \ldots, x_{i+(m-1)\tau}\right)
\]

with \( \tau \) parameter is delay time and \( m \) parameter is embedded dimension. Both parameters need to be sought first to reconstruct the complete \( m \)-dimension phase space.

This \( \tau \) parameter needs to be carefully selected so that the attractor structure in the phase space can be fully described [20]. According to Zhan et al. [21] and Islam and Sivakumar [22], if the value \( \tau \) is too small, the coordinates of the phase space cannot reflect the dynamics of a time series. If the value \( \tau \) is too large, different coordinates cannot be correlated and will result in loss of original time series information. Previous studies by Adenan and Noorani [23] have use \( \tau = 1 \) and the prediction results for the study were satisfactory. Therefore, the fixed \( \tau = 1 \) is used in this study.

According to Takens [9] and Regonda et al. [24], \( m \) embedding dimension is the minimum number of variables required to represent the dynamic time series. In this study, the Cao method is selected to find the parameter \( m \) value because this method does not depend on the number of available data and can distinguish between chaos or random property.

3.2. Cao’s method
Cao method is used to find \( m \) parameter values [25]. The Cao method selected is because this method does not contain any subjective parameters except the \( \tau \) delay time and does not rely on much of the data available. \( m \) from the Cao method is calculated through:

\[
E_1(m) = \frac{E(m+1)}{E(m)}
\]

which is

\[
E(m) = \frac{1}{N-m\tau} \sum_{n=1}^{N-m\tau} \|Y^m_{n+1} - Y^m_{ij}\| \|Y^m_n - Y^m_{ij}\|
\]

and \( \|\cdot\| \) is the maximum norm. \( Y^m_{ij} \) is the nearest neighbour to \( Y^m_{n} \). The \( m \) graph against \( E_1(m) \) is plotted. If \( E_1(m) \) stops changing when the \( m \) value is larger than \( m_0 \), then \( m_0 + 1 \) is the minimum embedded dimension size to look for. Cao’s method can also be used to determine whether this time series is random or chaos. If the \( E_1(m) \) value continues to saturate with the \( m \), increasing, then, the time series is chaos. If no saturation occurs, the time series is random. [25] also introduced \( E_2(m) \) calculation which is:

\[
E_2(m) = \frac{E(m+1)}{E(m)}
\]
which,

\[ E(m) = \frac{1}{N-m} \sum_{n=1}^{N-m} |x_{n+mc}^m - x_{n+mc}^m| \]  

(6)

If the chaotic dynamic is present in a time series, there are some \( m \) or at least one \( m \) which \( E2(m) \neq 1 \), therefore, if \( E2(m) \neq 1 \), exists, then the chaotic dynamic are present in the observed time series.

3.3. Phase space plot method

The phase space condition can represent the complete condition of a system at a time. Phase space dynamics can be observed through point-to-point evolution through a trajectory in the phase space. For actual time series, phase space plots are important to observe to determine the dynamics of the time series. For the \( \tau \) value obtained, the plot of the phase space will be constructed in the \( \{x(t), x(t+\tau)\} \) plane. The existence of attractor region shows the presence of a chaotic dynamic in a time series [20].

3.4. Prediction model construction

Prediction through a chaotic approach can be translated by the equation:

\[ Y_{j+1}^m = f\left(Y_j^m\right) \]  

(7)

with \( Y_j^m \) which is the final phase space and \( Y_{j+1}^m \) is an hour phase space in the future. Through a local approach, \( Y_{j+1}^m \) prediction is based on the value of the nearest neighbour \( Y_j^m \) to a neighbourhood. \( k \) which is the nearest neighbour to \( Y_j^m \) is chosen based on the \( Y_j^m - Y_j^m \) minimum value with \( j' < j \). If only \( (k = 1) \) is the nearest neighbour, then the approximation of \( Y_{j+1}^m \) is \( Y_{j+1}^m \). Because normally \( k > 1 \), prediction \( Y_{j+1}^m \) is usually taken as an average for the \( Y_{j+1}^m \) value which is

\[ Y_{j+1}^m = \frac{\sum_{q=1}^{k} Y_{j+q}^m}{k} \]  

(8)

\( k \) value is determined by trial and error while prediction performance is measured using correlation coefficients. The correlation coefficient values approach one indicates a predicted time series is near with the actual time series. So, the closer the coefficient of collation to one, the better the prediction model is built.

4. Results and discussion

4.1. Chaos presence identification

Cao’s method and phase space plots are two methods for identifying the presence of a chaotic over the temperature series. Figure 4 (a) is the result of the Cao’s method for a temperature time series in the state of Pahang. Observed in the figure, it is found that the \( E1(m) \) value starts to stop changing at the \( m_0 = 3 \) value. Then the sought embedded dimension value is four \((m_0 + 1 = 4)\). Whereas the \( E1(m) \) value continues to saturate with the increasing of \( m \) and there is some \( m \) or at least one \( m \) which is \( E2(m) \neq 1 \). So this supports that the chaotic dynamic is presents in the temperature time series at Pahang.

Figure 4 (b) is a phase space plot using \( \tau = 1 \), phase space plot is built into two dimensions plane \( \{t, t+1\} \). Based on the diagram, the coordinates of the phase space are seen accumulate in the middle. This convergence is also known as an attractor. With the existence of attractor in the temperature time series space of the observed temperature series in Pahang, then the chaotic dynamic is present in the
tested time series. In conclusion, both the results of the Cao Method and the phase space plot state that the chaos dynamics are present in the temperature series of Pahang.

![Figure 4](image_url)

**Figure 4.** (a) Cao’s Method and (b) Phase Space

4.2. **Factors influencing temperature time series**

Results from the Cao’s method find at least four variables affecting the temperature time series. According to a study conducted by Scott et al. [26], the factors that influence temperature changes are due to widespread deforestation. Meanwhile, studies conducted by Adiwijaya, Wisesty, and Nhita [27] find that temperature changes are influenced by meteorological factors such as volcanic radiation, solar radiation, humidity, air pressure, rainfall and wind speed. From the list in the above paragraph, there are more than four factors that affect the temperature. Therefore, the finding of the $m$ value of the Cao’s method is compatible with the number of factors listed.

4.3. **Prediction results**

By using $\tau = 1$ and $m = 4$, the phase space of equation (2) is constructed for the purposes of prediction. Furthermore, a training time series is predicted through LMAM using the phase space. Figure 5 shows the predicted time series are almost equal to the actual time series, so this proves that the temperature time series can be predicted well. Correlation coefficient value between actual data and prediction data is 0.9789. This shows that there is a high correlation between actual data and predicted data because the value is nearing one. Therefore, the LMAM model is excellent in prediction the temperature time series.
5. Conclusion

Phase space plot and Cao’s method shows the appearance of the chaos property in the temperature time series observed in Jerantut, Pahang. Six months of data predicted using LMAM shows good performance with a coefficient of correlation close to one. It is hoped that these findings will help to realizing the first, second, fourth and fifth Malaysian meteorological departments of strategic plans. The following are the strategic plans of the Malaysian Meteorological Department [28]:

1. Improve the effectiveness of weather services to reduce disaster risk.
2. Strengthen flight meteorology services to ensure flight safety and wellbeing.
3. Empower earthquake and tsunami services to reduce the risk of earthquake and tsunami.
4. Strengthening climate service for national prosperity.
5. Empowering human capital development.

Also, it is hoped that this study will help the Department of Environment to achieve the second and third strategic thrusts. The following are the strategic thrusts of the Department of Environment [29]:

Core 1: Management of water, geology and sustainable land.

Strategy i: Ensuring the effectiveness of water resources management.

Strategy ii: Strengthen the management of effective geological resources.

Strategy iii: Effective and efficient of land and geospatial management and administration management.

Core 2: Environmental management, biodiversity and sustainable forest.

Strategy i: Environmental quality management.

Strategy ii: Effectiveness biodiversity conservation and sustainable forest management.

Core 3: Effective and proficient management of organizational governance.

Strategy i: Cultural organization values.

Strategy ii: Reinforce the ministry's governance.

6. Appreciation

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7. References

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