Thermodynamics of the universe bounded by the cosmological event horizon and dominated by the tachyon fluid

Fei-Quan Tu, Yi-Xin Chen*

Zhejiang Institute of Modern Physics, Zhejiang University, Hangzhou, 310027, P. R. China
Email: fqtuzju@foxmail.com, yxchen@zimp.zju.edu.cn

Abstract

Our aim is to investigate the thermodynamic properties of the universe bounded by the cosmological event horizon and dominated by the tachyon fluid. We give two different laws of evolution of our universe. Further, we show the first law and the generalized second law of thermodynamics (GSLT) are both satisfied in two cases, but their properties of the thermodynamic equilibrium are totally different. Besides, under our solutions, we find the validity of the laws of thermodynamics is irrelevant with the parameters of the tachyon fluid. Finally, we conclude that the universe bounded by the cosmological event horizon and dominated by the tachyon fluid has a good thermodynamic description. In turn, the thermodynamic description can provide a good physical interpretation for the dynamic evolution of our universe due to the equivalence between the first law of thermodynamics and the Friedmann equation to some extent.

1. Introduction

Numerous astro-observations tell us that our universe is in accelerating expansion at present[1,2]. The most common way to describe the effects of the current accelerating expansion is to introduce a cosmological constant in the Einstein’s equation. This model, called Λ—model, well explains the astro-observations. According to this model, about 73% of the total energy in our universe is the dark energy. However, the origin of the cosmological constant (or called the dark energy) is unknown. In 2002, Sen[3,4]showed that the classical decay of unstable D-branes in bosonic and superstring theories produces pressureless gas with non-zero energy density. Further he gave a description of this phenomenon

*Corresponding author
in an effective field theory. Afterwards, the idea was applied to cosmology by Gibbons, Padmanabhan, Einstein, etc. The effective action of the tachyon field associated with the decay of unstable D-branes was used as the fundamental action to investigate the inflation (see, for example, [7, 8, 9, 10]) or the current accelerating expansion (see, for instance, [6, 11, 12, 13, 14]). This way to study cosmology is attractive because either the dark energy which causes the current accelerating expansion or the inflation potential originates from a fundamental theory. Therefore, the tachyon field is a good candidate of the dark energy.

On the other hand, the feature which the field equation doesn’t have any direct physical interpretation and the lack of an elegant principle which can lead to the dynamics of gravity are unsatisfied as have been pointed out by Padmanabhan. He proposed that an interpretation should be based on the thermodynamics of horizons. Fortunately, since the black hole thermodynamics was discovered in the 1970’s, the relationship between thermodynamics and the horizon of black hole has been generally accepted. Furthermore, the Einstein equation can be derived from the proportionality of entropy and horizon area together with the relation $\delta Q = TdS$ connecting heat, entropy and temperature. The thermodynamic interpretation for the field equations in any diffeomorphism invariant theory of the gravity was also provided. The Friedmann equation with spatial curvature can also be derived from the first law of thermodynamics on the apparent horizon of the FRW universe. Hence, there exists a deep connection between the cosmological horizon and thermodynamics, and it is important and significant to investigate the thermodynamic properties of cosmological horizon. Besides, the thermal properties of dark energy have also been generally discussed, for example, see Ref. [20, 21, 22, 23].

In a cosmological model with a positive cosmological constant, the cosmological event horizon is the boundary of the past of the observer’s world line. This horizon is similar to the black-hole event horizon which can be described by thermodynamics. There exists an event horizon for our universe which is in accelerating expansion, so it is important and interesting to investigate the first law of thermodynamics and the GSLT of the universe bounded by the cosmological event horizon and dominated by the tachyon fluid (in this paper we should use terms like the tachyon field and the tachyon fluid interchangeably).

If the usual form of Hawking temperature $T = 1/(2\pi R)$ where $R$ is the radius of the horizon is used to investigate the cosmological event horizon, then the conclusion that the cosmological event horizon is unphysical from the point of view of the laws of thermodynamics is obtained. However, if the modified Hawking temperature is taken, the correct laws of thermodynamics on the cosmological event horizon can be obtained. We will study the laws of thermodynamics of the universe bounded by the cosmological event horizon and dominated by the tachyon field through using the modified temperature in this paper. First, the tachyon field which only changes with time is taken as the form $\phi = At^\beta$ where $A$ and $\beta$ are positive constants in the flat FRW universe. By substituting this form into the Friedmann equation and the continuity equation (this is also the equation of motion of the tachyon field $\phi$), we obtain the laws
of growth of the scale factor of the universe. Then we restrict the constant $\beta$ to make it satisfy $\frac{1}{2} < \beta \leq 1$ by comparing the results of the theoretical calculation with that of the astronomical observations. In the case of $\beta = 1$, the tachyon potential is proportional to inverse square of the tachyon field and we get the restriction $0 < A < \sqrt{\frac{\pi}{3}}$ because our universe is in accelerating expansion. In the case of $\frac{1}{2} < \beta < 1$, $\beta = \frac{2}{3}$ is taken due to the similarity of the properties of evolution of the universe and $A = 1$ is chosen in order to see the properties of evolution of the universe more clearly. By investigating the above two cases, we show that they both satisfy the first law of thermodynamics and the GSLT, but their properties of the thermodynamic equilibrium are totally different. The universe dominated by the field $\phi = At$ can’t reach the thermodynamic equilibrium while the universe dominated by the field $\phi = At^\beta$ where $\frac{1}{2} < \beta < 1$ can reach the thermodynamic equilibrium. Besides, under our solutions, we find the validity of the laws of thermodynamics is irrelevant with the parameters of the tachyon fluid. In the paper the units are chosen with $c = \hbar = k_B = 1$ and the signature of the spacetime metric is taken as $(+,-,-,-)$.

2. Properties of evolution of the universe dominated by the tachyon fluid

The effective Lagrangian density of the tachyon field in string theory is taken as

$$L = -V(\phi)\sqrt{1-g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi},$$

(1)

where $\phi$ is the tachyon field and $V(\phi)$ is the tachyon potential.

Thus, the action of coupling this tachyon field with gravity is

$$S = \int d^4x \sqrt{-g} \left( \frac{R}{16\pi G} + L \right),$$

(2)

where $R$ is the scalar curvature. Varying the action (2) with respect to the metric $g^{\mu\nu}$, we obtain the Einstein equation

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu},$$

(3)

where the stress tensor is defined as

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\partial}{\partial g^{\mu\nu}} \left( \frac{\partial_\mu\phi\partial_\nu\phi}{\sqrt{1-\partial_\lambda\phi\partial^\lambda\phi}} + g_{\mu\nu}\sqrt{1-\partial_\lambda\phi\partial^\lambda\phi} \right) V(\phi),$$

(4)

for the tachyon field. For the perfect fluid, its form is

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}. $$

(5)

If the tachyon field is considered as an effective fluid, then we can obtain the corresponding energy density, pressure and velocity

$$\rho = \frac{V(\phi)}{\sqrt{1-\partial_\lambda\phi\partial^\lambda\phi}}, \quad p = -V(\phi)\sqrt{1-\partial_\lambda\phi\partial^\lambda\phi}, \quad u_\mu = \frac{\partial_\mu\phi}{\sqrt{1-\partial_\lambda\phi\partial^\lambda\phi}},$$

(6)
In the flat FRW universe, the metric is
\[ ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \] (7)
where \( a(t) \) is the scale factor. From the metric, we know the tachyon field is irrelevant with the space coordinates. The Friedmann equation is
\[ H^2 = \frac{8\pi G}{3} \rho = \frac{8\pi G}{3} \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}, \] (8)
where a dot refers to a derivative with respect to the cosmic time and \( H = \frac{\dot{a}(t)}{a(t)} \) is the Hubble constant.

Varying the action (2) with respect to the tachyon field \( \phi \), we obtain the following equation of motion
\[ \ddot{\phi} + 3H\dot{\phi} + \frac{V'}{V}\dot{\phi} = 0. \] (9)
This equation is equivalent to the continuity equation which is
\[ \dot{\rho} + 3H(\rho + p) = 0. \] (10)
Combining Eq.(8) with Eq.(10), we obtain
\[ \dot{H} = -\frac{3}{2} \dot{\phi}^2 H^2. \] (11)

Because the specific potential form for the tachyon field isn’t fixed by the string theory, we can investigate the different form to see the properties of evolution of the universe. In this paper, the tachyon field \( \phi \) is taken as the following form
\[ \phi = At^\beta, \] (12)
where \( A \) and \( \beta \) are positive constants.

If \( \beta = 1 \), we get the solution of Eq.(11)
\[ a(t) = \left(\frac{1}{5A^2}\right)^{\frac{1}{3}}, \] (13)
where we have assumed \( H(0) = \infty \). From this form, we know the universe is in accelerating expansion when \( A^2 < \frac{2}{3} \). So the parameter \( A \) has the restriction \( 0 < A < \frac{\sqrt{6}}{\sqrt{3}} \) when the field is \( \phi = At \). Then the energy density, the potential and the pressure are respectively read as
\[ \rho = \frac{1}{6\pi GA^4t^2}, \] (14)
\[ V(t) = \frac{\sqrt{1 - A^2}}{6\pi GA^4t^2} \] (15)
and
\[ p = \frac{A^2 - 1}{6\pi GA^4 t^2}. \] (16)

Further we can obtain the cosmological event horizon
\[ R_E = a(t) \int_t^\infty \frac{dt'}{a(t')} = \frac{3A^2}{2 - 3A^2} t. \] (17)

If \( \beta \neq 1 \), we get the solution of Eq.(11)
\[ a(t) = \exp \left( \frac{(2\beta - 1)t^2 - 2\beta}{3A^2\beta^2(1 - \beta)} \right), \] (18)
where we have taken the initial values \( a(0) = 1 \) and \( H(0) = \infty \). We obtain the Hubble constant
\[ H = \frac{2(2\beta - 1)}{3A^2\beta^2} t^{1-2\beta}. \] (19)

From this form, we know the universe is in accelerating expansion when \( \frac{1}{2} < \beta < 1 \) and \( \beta > 1 \).

The properties of evolution of the universe when \( \beta = \frac{2}{3} \) are similar to that of \( \frac{1}{2} < \beta < 1 \) while the properties of evolution of the universe when \( \beta = 2 \) are similar to that of \( \beta > 1 \), so we take the specific values \( A = 1, \beta = \frac{2}{3} \) and \( \beta = 2 \) in order to see the properties of evolution more clearly.

When \( A = 1 \) and \( \beta = \frac{2}{3} \), we have the scale factor
\[ a(t) = \exp \left( \frac{3}{4} t^{2/3} \right). \] (20)

Then we obtain the Hubble constant
\[ H = \frac{1}{2t^{1/3}}, \] (21)
and the energy density
\[ \rho = \frac{3}{32\pi Gt^{2/3}}. \] (22)

The potential and the pressure are respectively
\[ V(t) = \sqrt{1 - \frac{4}{9t^{2/3}} \frac{3}{32\pi Gt^{2/3}}} \] (23)
and
\[ p = \left( \frac{4}{9t^{2/3}} - 1 \right) \frac{3}{32\pi Gt^{2/3}}. \] (24)

Moreover, the cosmological event horizon is
\[ R_E = a(t) \int_t^\infty \frac{dt'}{a(t')} = \exp \left( \frac{3}{4} t^{2/3} \right) \frac{2}{3} \left[ 3 \exp \left( -\frac{3}{4} t^{2/3} \right) t^{1/3} + \sqrt{3\pi} erf \left( \frac{\sqrt{3}}{2} t^{1/3} \right) \right]. \] (25)
where \( \text{erf } c(x) \) is the error function, its definition is \( \text{erf } c(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-t^2} dt \).

When \( A = 1 \) and \( \beta = 2 \), we have the scale factor
\[
a(t) = \exp\left(-\frac{1}{4t^2}\right). \tag{26}
\]

Then we obtain the potential
\[
V(t) = \sqrt{1 - 4t^2} \frac{3}{32\pi G t^6}. \tag{27}
\]

From the above potential, we know the time \( t \) should satisfy \( 0 < t < \frac{1}{2} \), so the cosmological horizon doesn’t exist. Besides, the rate of the expansion is so slow that it isn’t consistent with the result of the astronomical observation. Hence we should discard such solution.

In conclusion, we obtain the solution \( \frac{1}{2} < \beta \leq 1 \) when we take the form of the field \( \phi(t) = At^\beta \). Regardless of \( \beta = 1 \) or \( \frac{1}{2} < \beta < 1 \), the law of evolution of the universe can explain the inflation in the early universe or the current accelerating expansion of the universe when the appropriate positive constants are taken. Here, we would like to point out that the author in Ref[7] assumed the form of scale factor of the universe to study the potential form of the tachyon field. However, we take the solutions of the equation of motion of the tachyon field to investigate the evolution of the universe. Our way to the results are more natural. Moreover, we can assume the tachyon field has the other form and analyze the relevant solutions, but the process is analogous.

3. Thermodynamics of the universe dominated by the tachyon fluid

In this section, our main aim is to investigate the first and generalized second law of thermodynamics for the universe bounded by the cosmological event horizon and dominated by the tachyon fluid \( \phi \) whose forms are taken as that of the solutions in the above section.

3.1. First law of thermodynamics of the universe dominated by the tachyon fluid

In the homogenous and isotropic universe, the metric can be expressed as
\[
ds^2 = h_{ij}dx^i dx^j + R^2d\Omega_2^2, \tag{28}
\]
where \( i, j \) can take values 0 and 1, \( R = a(t)r \) and the 2-dimensional metric \( h_{ab} = \text{diag}(-1, a^2/(1 - \kappa r^2)) \) in which \( \kappa \) is the spatial curvature constant. A scalar quantity is defined as
\[
\chi = h^{ij}\partial_i R \partial_j R. \tag{29}
\]
If the scalar quantity $\chi = 0$, it gives $R_A = \frac{1}{\sqrt{H^2 + \frac{\kappa}{\pi^2}}}$ which is called apparent horizon. The surface gravity on the apparent horizon is defined as

$$\kappa_A = -\frac{1}{2} \frac{\partial \chi}{\partial R} \bigg|_{r=R_A} = \frac{1}{R_A}$$

and the corresponding Hawking temperature is defined as

$$T_A = \frac{\kappa_A}{2\pi} = \frac{1}{2\pi R_A}.$$  \hfill (30)

Based on Ref. [26], the Hawking temperature on the cosmological event horizon $R_E$ is defined as

$$T_E = \frac{\kappa_E}{2\pi} = \frac{1}{2\pi R_E}.$$  \hfill (31)

In this paper, we take the flat FRW metric (7), so the apparent horizon $R_A = \frac{1}{H}$. For the tachyon field which is $\phi = At$, the scale factor grows with time as Eq.(13) and the cosmological event horizon is Eq.(17). The amount of energy flux across the horizon during the time interval $dt$ is

$$-dE_H = 4\pi R_H^3 T_\mu^\nu k^\mu k^\nu dt$$

with $k^\mu = \frac{1}{\sqrt{v}}(v, -v, 0, 0)$ a null vector. Hence, for the cosmological horizon, we obtain

$$-dE = 4\pi R_E^3 H(\rho + p) dt = \frac{12A^2 dt}{(2 - 3A^2)^3 G},$$

where we have used the relation $v = HR_E$. According to the area-entropy relation $S_E = \frac{\pi R_E^2}{G}$, we get

$$T_E dS_E = \frac{12A^2}{(2 - 3A^2)^3 G} dt.$$  \hfill (35)

Thus we see the first law of thermodynamics $-dE = T dS$ is kept on the cosmological event horizon when the tachyon field is taken as $\phi = At$.

For the tachyon field $\phi = t^2$, the scale factor grows with time as Eq.(20) and the cosmological event horizon is Eq.(25). Compared to the term $3 \exp \left(\frac{-3}{4} t^{2/3}\right) t^{1/3}$, the error function term $\sqrt{3\pi} \operatorname{erf} \left(\frac{x}{2}\right)$ is very small when $t$ is relatively large (for example $t > 10$), so we can discard it. Thus the cosmological event horizon is

$$R_E \approx 2t^{1/3}.$$  \hfill (36)

The amount of energy flux across the cosmological event horizon is

$$-dE = 4\pi R_E^3 H(\rho + p) dt = \frac{2dt}{3Gt^{2/3}}.$$  \hfill (37)

On the other hand, we can obtain
Hence the first law of thermodynamics \(-dE = TdS\) is approximately satisfied on the cosmological event horizon when the tachyon field is taken as \(\phi = t^4\).

Due to the similarity of the properties, the first law of thermodynamics should be correct on the cosmological event horizon when the tachyon field is \(\phi = At^\beta\) where the parameter \(\beta\) satisfies \(\frac{1}{2} < \beta < 1\).

In this subsection, we show that the first law of thermodynamics is correct or approximately correct on the cosmological event horizon in the flat FRW universe dominated by the tachyon field whose rate of change is \(\phi = At^\beta\left(\frac{1}{2} < \beta \leq 1\right)\). Furthermore, the first law of the thermodynamics derives from the Friedmann equation, so they are equivalent to some extent.

### 3.2. GSLT and thermodynamic equilibrium of the universe dominated by the tachyon fluid

In this subsection, our main aim is to investigate whether the universe bounded by the cosmological event horizon and dominated by the tachyon fluid satisfies the GSLT and approaches a thermodynamic equilibrium or not. By means of the entropy functions, the GSLT and the thermodynamic equilibrium configuration can be expressed as \[\text{[29, 30]}\]

\[
\dot{S}_h + \dot{S}_{fh} \geq 0, \quad (1) \quad \ddot{S}_h + \ddot{S}_{fh} < 0 \quad (39)
\]

where \(S_h\) and \(S_{fh}\) represent the entropies of the horizon and the tachyon fluid respectively.

By using some thermodynamic relations and the assumption which the temperature of the fluid is same as that of the horizon, the relations

\[
\dot{S}_E + \dot{S}_{fE} = \frac{8\pi^2 R_E(\rho + p)}{H} \left( R_E - \frac{1}{H} \right) \quad (40)
\]

and

\[
\ddot{S}_E + \ddot{S}_{fE} = 8\pi^2(\rho + p) \left( R_E - \frac{1}{H} \right) \left[ - \left\{ \frac{R_E}{2} \left( 1 - 3\frac{p}{\rho} \right) + \frac{1}{H} + 3R_E \frac{p}{\rho} \right\} + R_E \left\{ 1 - \frac{3(1 + \frac{p}{\rho})}{2(2H - 1)} \right\} \right] \quad (41)
\]

were obtained in Ref. [30].

Now, we analyze the GSLT and the thermodynamic equilibrium for the universe dominated by the following different potential of the tachyon fluid:

1. The potential \(V(t) = \frac{\sqrt{1 - A^2 t^2}}{\sqrt{G}A^4 t^2}\). For this potential, our universe evolves as Eq.\((13)\). For entropy variations, Eq.\((40)\) and Eq.\((41)\) become

\[
\dot{S}_E + \dot{S}_{fE} = \frac{8\pi^2 R_E A^2 \rho}{H} \frac{9A^4 t}{2(2 - 3A^2)} \quad (42)
\]
Figure 1: In the picture, \( y \) represents \( F(t) - \frac{2}{3(HR_E - 1)t^{2/3}} \). We can know \( y < 0 \) and \( y \) tends to 0 when time \( t \) is large.

and

\[
\dot{S}_E + \dot{S}_{fE} = 8\pi^2 A^2 \rho \left( R_E - \frac{1}{H} \right)^2. \tag{43}
\]

From the Eq.(42), we know \( \dot{S}_E + \dot{S}_{fE} > 0 \) because of \( A^2 < \frac{2}{\pi} \), which indicates that the total entropy is increasing, so the GSLT is satisfied. However, \( \dot{S}_E + \dot{S}_{fE} > 0 \) which indicates our universe can’t reach the thermodynamic equilibrium from the Eq.(43).

2. The potential \( V(t) = \sqrt{1 - 4t^2 \frac{3}{32\pi G t^2}} \). For this potential, our universe evolves as Eq.(20). For entropy variations, Eq.(40) and Eq.(41) become

\[
\dot{S}_E + \dot{S}_{fE} = \frac{32\pi^2 R_E \rho}{9Ht^{2/3}} \left( R_E - \frac{1}{H} \right) \tag{44}
\]

and

\[
\ddot{S}_E + \ddot{S}_{fE} = \frac{32\pi \rho}{9Ht^{2/3}} R_E \left( R_E - \frac{1}{H} \right) \left[ F(t) - \frac{2}{3(HR_E - 1)t^{2/3}} \right] \tag{45}
\]

where \( F(t) = 1 - \left\{ \frac{2}{H} \left( 1 - 3\frac{\rho}{p} \right) + \frac{2}{HR_E} + 3\frac{p}{\rho} \right\} \). From the Eq.(44), we know \( \dot{S}_E + \dot{S}_{fE} \geq 0 \) because \( R_E - \frac{1}{H} \geq 0 \). This indicates that the total entropy is increasing, so the GSLT is satisfied. Further, we see \( \dot{S}_E + \dot{S}_{fE} = 0 \) when \( t \to \infty \). From the Figure 1, we know that \( \ddot{S}_E + \ddot{S}_{fE} \leq 0 \) which indicates that our universe is approaching a thermodynamic equilibrium configuration.

Analyzing the GSLT and the thermodynamic equilibrium configuration for the universe dominated by the above different potential, we show they both satisfy the GSLT, but their properties of the thermodynamic equilibrium are totally different. The universe dominated by the potential \( V(t) = \sqrt{1 - \frac{A^2}{6\pi G A^4 t^2}} \)
can’t reach the thermodynamic equilibrium while the universe dominated by the potential 
\[ V(t) = \sqrt{1 - 4t^2 \frac{3}{32\pi Gt}} \] can reach the thermodynamic equilibrium.

4. Conclusion and discussion

Thermodynamics of the universe bounded by the cosmological event horizon has been investigated in the perfect fluid dark model (that is, the equation of state of the dark energy is \( p = \omega \rho \) where \( \omega < -\frac{1}{3} \) and \( \omega \) is a constant) and in the holographic dark energy model\[26, 31\]. The thermodynamical properties related to the apparent horizon and the cosmological event horizon were studied in Ref.\[25\] where the dark energy was also taken as holographic fluid. However, all the cosmological constant, the perfect fluid dark energy and the holographic dark energy don’t have a basic physical interpretation. In this paper, the tachyon field is taken as the fluid of dark energy in our universe. As have been pointed out in introduction, the tachyon field originates from the decay of unstable D-branes in string theory, so it has a good physical origin. This is our main motivation to choose the tachyon fluid as the dark energy. Besides, a large number of authors have studied the field as the dark energy which causes the current accelerating expansion or the inflation field and their conclusions showed the tachyon field is a good candidate of the dark energy. As we have known, the cosmological event horizon is the boundary of the past of the observer’s world line. On the other hand, the horizon of the black hole is the boundary of observable area for the outside observers. Hence the properties of the cosmological event horizon is similar to that of the black hole. Now that the horizon of the black hole can be described by the laws of thermodynamics, then the cosmological event horizon should also be described by thermodynamics. Therefore, the cosmological event horizon has a better physical interpretation than the Hubble horizon or the apparent horizon under the description of thermodynamics of the cosmological horizon. This is the main reason that we choose the cosmological event horizon as the boundary of the universe in this paper.

First, the tachyon field which changes with time is taken as the form \( \phi = At^\beta \) in the flat FRW universe. Then, by combining the Friedmann equation with the continuity equation and analyzing the laws of growth of the scale factor, we find that \( \frac{1}{2} < \beta \leq 1 \) is a good interval to describe the effects of accelerating expansion. The law of growth of the scale factor is \( a(t) = t^{\frac{1}{1-\beta}} \) with \( 0 < A < \sqrt{\frac{\pi}{3}} \) when \( \beta = 1 \), whose form is similar to that of standard cosmological model. While, the scale factor is \( a(t) = e^{\frac{\left(2\beta - 1\right)t^{2-2\beta}}{3\beta^2(1-\beta)A^2}} \) when \( \frac{1}{2} < \beta < 1 \). However, regardless of \( \beta = 1 \) or \( \frac{1}{2} < \beta < 1 \), the law of evolution of the universe can explain the inflation in the early universe or the current accelerating expansion of the universe when the appropriate positive constants are taken. In the case of \( \beta = 1 \), the first law of thermodynamics and the GSLT are satisfied but the thermodynamic equilibrium can’t be reached. On the other hand, in the case of \( \frac{1}{2} < \beta < 1 \), the first law of thermodynamics is approximately correct ( because
the error function can be not considered when \( t \) is relatively large), the GSLT are satisfied and the thermodynamic equilibrium can be reached. Besides, under our solutions, we find the validity of the laws of thermodynamics is irrelevant with the parameters of the tachyon field. Finally, we can conclude the universe bounded by the cosmological event horizon and dominated by the tachyon fluid has a good thermodynamic description. In turn, the thermodynamic description can provide a good physical interpretation for the dynamic evolution of our universe due to the equivalence between the first law of thermodynamics and the Friedmann equation to some extent.

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