Quasi-Stationary Temperature Profile and Magnetic Flux Jumps in Hard Superconductors

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Abstract
In the present paper, the temperature distribution in the critical state of hard superconductors is investigated in the quasi-stationary approximation. It is shown that the temperature profile can be essentially inhomogeneous in the sample, which affects the conditions of initiation of a magnetic flux jumps.

Key words: Critical state, flux flow, flux jump, instability.

While dealing with instabilities of the critical state in hard superconductors, the character of the temperature distribution $T(x, t)$ and that of the electromagnetic field $\vec{E}(x, t)$ are of substantial practical interest [1]. This derives from the fact that thermal and magnetic destructions of the critical state caused by Joule self-heating are defined by the initial temperature and electromagnetic field distributions. Hence, the form of the temperature profile may noticeably influence the criteria of critical-state stability with respect to jumps in the magnetic flux in a superconductor. Earlier (cf., e.g., [2]), in dealing with this problem, it was usually assumed that the spatial distribution of temperature and field were either homogeneous or slightly inhomogeneous. However, in reality, physical parameters of superconductors may be inhomogeneous along the sample as well as in its cross-sectional plane. Such inhomogeneities can appear due to different physical reasons. First, the vortex structure pinning can be inhomogeneous due to the existence of weak bonds in the superconductor. Second, inhomogenety of the properties may be caused by their dependence on the magnetic field $H$. Indeed, the field $H$ influences many physical quantities, such as the critical current density $j_c$, the differential conductivity $\sigma_d$, and the heat conductivity $k$.

In the present paper, the temperature distribution in the critical state of hard superconductors is investigated in the quasi-stationary approximation. It is shown that the temperature profile can be essentially inhomogeneous, which affects the conditions of initiation of a magnetic flux jumps.

The evolution of thermal ($T$) and electromagnetic ($\vec{E}, \vec{H}$) perturbations in superconductors is described by a nonlinear heat conduction equation [3],

$$\nu \frac{dT}{dt} = \nabla[\kappa \nabla T] + \vec{j} \vec{E}, \tag{1}$$
a system of Maxwell’s equations,

$$\text{rot} \vec{E} = -\frac{1}{c} \frac{d\vec{H}}{dt},$$  \hspace{1cm} (2)$$

$$\text{rot} \vec{H} = \frac{4\pi}{c} \vec{J},$$ \hspace{1cm} (3)

and a critical-state equation

$$\vec{J} = \vec{J}_c(T, \vec{H}) + \vec{J}_r(\vec{E}).$$ \hspace{1cm} (4)

Here $\nu = \nu(T)$ is the specific heat, $\kappa = \kappa(T)$ is the thermal conductivity respectively; $\vec{J}_c$ is the critical current density and $\vec{J}_r$ is the active current density.

We use the Bean-London critical state model to describe the $j_c(T, H)$ dependence, according to which $j_c(T) = j_0[1 - a(T - T_0)]$ [4], where the parameter $a$ characterizes thermally activated weakening of Abrikosov vortex pinning on crystal lattice defects, $j_0$ is the equilibrium current density, and $T_0$ is the temperature of the superconductor.

The $j_r(E)$ dependence in the region of sufficiently strong electric fields ($E \geq E_f$; where $E_f$ is the limit of the linear region of the current-voltage characteristic of the sample [2]) can be approximated by a piecewise-linear function $j_r \approx \sigma_f E$, where $\sigma_f = \frac{\eta c^2}{H\Phi_0} \approx \frac{\sigma_n H_{c2}}{H}$ is the effective conductivity in the flux flow regime and $\eta$ is the viscous coefficient, $\Phi_0 = \frac{\pi h c}{2e}$ is the magnetic flux quantum, $\sigma_n$ is the conductivity in the normal state, $H_{c2}$ is the upper critical magnetic field. In the region of the weak fields ($E \leq E_f$), the function $j_r(E)$ is nonlinear. This nonlinearity is associated with thermally activated creep of the magnetic flux [5].

Let us consider a superconducting sample placed into an external magnetic field $\vec{H} = (0, 0, H_e)$ increasing at a constant rate $\frac{d\vec{H}}{dt} = \dot{H} = \text{const}$. According to the Maxwell equation (2), a vortex electric field $\vec{E} = (0, E_e, 0)$ is present. Here $H_e$ is the magnitude of the external magnetic field and $E_e$ is the magnitude of the back-ground electric field. In accordance with the concept of the critical state, the current density and the electric field must be parallel: $\vec{E} \parallel \vec{j}_c$. The thermal and electromagnetic boundary conditions for the Eqs. (1)-(4) have the form

$$\kappa \frac{dT}{dx}\bigg|_{x=0} + w_0[T(0) - T_0] = 0,$$

$$T(L) = T_0,$$

$$\frac{dE}{dx}\bigg|_{x=0} = 0,$$

$$E(L) = 0.$$ \hspace{1cm} (5)

For the plane geometry (Fig.) and the boundary conditions $H(0) = H_e$, $H(L) = 0$, the magnetic field distribution is $H(x) = H_e(L - x)$, where $L = \frac{cH_e}{4\pi j_c}$ is the depth of magnetic flux penetration into the sample and $w_0$ is the coefficient of heat transfer to the cooler at the equilibrium temperature $T_0$. 

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The condition of applicability of Eqs. (1)-(4) to the description of the dynamics of evolution of thermomagnetic perturbations are discussed at length in [2].

In the quasi-stationary approximation, terms with time derivatives can be neglected in Eqs. (1)-(4). This means that the heat transfer from the sample surface compensates the energy dissipation arising in the viscous flow of magnetic flux in the medium with an effective conductivity \( \sigma_f \). In this approximation, the solution to Eq. (2) has the form

\[
E = \dot{H} \frac{L - x}{c}. \tag{6}
\]

Upon substituting this expression into Eq. (1) we get an inhomogeneous equation for the temperature distribution \( T(x, t) \),

\[
\frac{d^2 \Theta}{d\rho^2} - \rho \Theta = f(\rho). \tag{7}
\]

Here we introduced the following dimensionless variables

\[
f(\rho) = -[1 + r \omega \rho] \frac{j_0}{\alpha L_0}, \quad \Theta = \frac{T - T_0}{T_0}, \quad \rho = \frac{L - x}{r},
\]

and the dimensionless parameters \( \omega = \frac{\sigma_f \dot{H}}{c j_0} \), and, \( r = \left( \frac{c \kappa}{a \dot{H} L^2} \right)^{1/3} \), where \( r \) characterizes the spatial scale of the temperature profile inhomogeneity in the sample. Solutions to Eq. (7) are Airy functions, which can be expressed through Bessel functions of the order 1/3 [6]

\[
\Theta(\rho) = C_1 \rho^{1/2} K_{1/3} \left( \frac{2}{3} \rho^{3/2} \right) + C_2 \rho^{1/2} I_{1/3} \left( \frac{2}{3} \rho^{3/2} \right) + \Theta_0(\rho), \tag{8}
\]

\[
\Theta_0(\rho) = \rho^{1/2} K_{1/3} \left( \frac{2}{3} \rho^{3/2} \right) \int_0^\rho \left[ 1 + r \omega \rho_1 \right] \rho_1^{3/2} I_{1/3} \left( \frac{2}{3} \rho_1^{3/2} \right) d\rho_1 - \rho^{1/2} I_{1/3} \left( \frac{2}{3} \rho^{3/2} \right) \int_0^\rho \left[ 1 + r \omega \rho_1 \right] \rho_1^{3/2} K_{1/3} \left( \frac{2}{3} \rho_1^{3/2} \right) d\rho_1,
\]

where \( C_1 \) and \( C_2 \) are integration constants, which are determined by the boundary conditions to be

\[
C_1 = 0, \quad C_2 = \left. -w_0 L \Theta(0) + \kappa \frac{d\Theta}{d\rho} \right|_{\rho = \frac{L}{r}} \left[ w_0 \left( \frac{L}{r} \right)^{1/2} I_{1/3} \left( \frac{2}{3} \rho^{-3/2} \right) - 2 \frac{d}{d\rho} \left( \rho^{1/2} I_{1/3} \left( \frac{2}{3} \rho^{3/2} \right) \right) \right] \bigg|_{\rho = \frac{L}{r}}
\]

From the Maxwell equation (2), the temperature inhomogeneity parameter can be expressed in the form

\[
\alpha = \frac{r}{L} = \left[ \frac{4 \pi \nu j_0 H_e}{a H_e^2 \dot{H} \kappa} \right]^{1/3} \tag{9}
\]
It is evident that $\alpha \sim 1$ near the threshold for a flux jump, when $\frac{aH_e^2}{4\pi \nu j_0} \sim 1$, even under the quasi-stationary heating condition $\frac{\dot{H} t_\kappa}{H_e} << 1$; where $t_\kappa = \frac{\nu L^2}{\kappa}$ is the characteristic time of the heat conduction problem.

Let us estimate the maximum heating temperature $\Theta_m$ in the isothermal case $w = \frac{\kappa}{L} \geq 1$. The solution to Eq. (7) can be represented in the form

$$\Theta(x) = \Theta_m - \rho_0 \frac{(x-x_m)^2}{2}, \quad (10)$$

near the point at which the temperature is a maximum, $x = x_m$ (Fig.).

With solution (10) being approximated near the point $x_m = \frac{L}{2}$ with the help of the thermal boundary conditions, the coefficient $\rho_0$ can be easily determined to be $\left( \frac{8}{L^2} \right) \Theta_m$ and the temperature can be written as

$$\Theta(x) = \Theta_m \left[ 1 - \frac{4}{L^2} \left( x - \frac{L}{2} \right)^2 \right], \quad (11)$$

Substituting this solution into Eq.(7), the superconductor maximum heating temperature due to magnetic flux jumps can be estimated as

$$\Theta_m = \frac{\left[ j_0 + \frac{\sigma_f \dot{H}}{c}(L-x_m) \right] \frac{\dot{H}}{c \kappa T_0} (L-x_m)}{\frac{\gamma}{L^2} - \frac{a \dot{H}}{c \kappa} (L-x_m)} . \quad (12)$$

For a typical situation when $\frac{\gamma}{L^2} << \frac{a \dot{H}}{c \kappa} (L-x_m)$ the estimation for $\Theta_m$ is

$$\Theta_m \approx \left[ j_0 + \frac{\sigma_f \dot{H}}{c}(L-x_m) \right] \frac{\dot{H} L^2}{c \kappa T_0} (L-x_m). \quad (13)$$

Here, the parameter $\gamma \sim 1$ (for a parabolic temperature profile $\gamma \sim 8$). It is easy to verify that for typical values of $j_0 = 10^6 A/cm^2$, $\dot{H} = 10^4$ Gs/sek, and $L = 0.01$ cm the heating is sufficiently low: $\Theta_m << 1$. In the case of poor sample cooling, $w = 1 - 10 erg/(cm^2 sK)$, the $\Theta_m$ is

$$\Theta_m = \frac{\dot{H} j_0 L^2}{c w_0 T_0} \approx 0.5; \quad (14)$$

i.e., the heating temperature can be as high as $\delta T_m = T_0 \Theta_m \sim 2K$. One can see that in the case of poor sample cooling, the heating can be rather noticeable and influence the conditions of the thermomagnetic instability of the critical state in the superconductor.

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FIGURE

Fig. The distribution of the temperature profile $\Theta(x)$. 
