WOPPER –
A Monte Carlo Event Generator for
W Off-shell Pair Production
including Higher Order
Electromagnetic Radiative Corrections∗

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Abstract
We present the Monte Carlo event generator WOPPER for pair production of W’s and their decays at high energy e+e− colliders. WOPPER includes the effects from finite W width and focusses on the calculation of higher order electromagnetic corrections in the leading log approximation including soft photon exponentiation and explicit generation of exclusive hard photons.

1 Introduction

The precision experiments being performed at the e+e− colliders LEP at CERN and SLC at SLAC have confirmed the predictions of the Standard Model (SM) for the interactions between the gauge bosons and the fermions even at the level of electroweak radiative corrections. However, the non-Abelian structure of the gauge sector of the SM with its couplings between the electroweak gauge bosons has not been tested directly. In addition, the origin of electroweak symmetry breaking giving longitudinal components to the electroweak gauge bosons, is still obscure.

Anomalous couplings will disturb the extensive gauge cancellations taking place in the SM, and possible new physics will show up in particular in the cross section for the

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production of longitudinally polarized $W$ bosons. On the other hand, possible new physics is already severely constrained by LEP100 data, and the effects to be expected at LEP200 or at a 500 GeV linear collider are small.

In order to extract these small effects, one has to have a precise knowledge of the radiative corrections within the SM. The one-loop electroweak radiative corrections to the production of on-shell $W$’s are by now well established [1]. The influence of the finite $W$ width has been investigated in [2]. Also, the higher order QED corrections have been calculated in the leading log approximation (LLA) [3].

However, the experimental reconstruction of the $W$’s is complicated by the fact that they may decay either into leptons with an escaping neutrino, or into hadrons, where the jet energies may be poorly known due to undetected particles. In addition, the radiative corrections due to emission of photons produce a systematic shift of the effective center of mass energy towards smaller values. Such effects may best be studied with the help of a Monte Carlo event generator.

In this note, we report on the status of the new Monte Carlo event generator $WOPPER$ for the process $e^+e^- \rightarrow (W^+W^-)^* \rightarrow 4$ fermions. $WOPPER$ includes higher order electromagnetic radiative corrections in the LLA, with explicit generation of exclusive hard photons and the effects from finite width of the $W$’s. We present results from first simulations obtained with this generator, which will eventually become a full four fermion generator for high energy $e^+e^-$ colliders.

2 The Monte Carlo generator WOPPER

The Monte Carlo event generator $WOPPER$ is capable of a full simulation of the cross section for $e^+e^- \rightarrow 4$ fermions + $n\gamma$ via the resonant channel containing two $W$ bosons. The finite width of the $W$ bosons is included as well as QED radiative corrections in all orders of the leading logarithmic approximation. At very high energies these corrections are indeed the ones which are numerically most important, since

$$\frac{\alpha}{\pi} \log \left( \frac{s}{m_e^2} \right) \approx 6\% \quad \text{(at LEP200 and EE500 energies)} \quad (1)$$

The LLA is conveniently incorporated using the so-called structure function formalism [4]. In this formalism, the expression for the radiatively corrected cross section reads

$$\sigma(s) = \int_0^1 dx_+dx_- \, D(x_+, Q^2)D(x_-, Q^2) \, \hat{\sigma}(x_+x_-s), \quad (2)$$

where $\hat{\sigma}$ is the Born level cross section of the hard process, $D(x, Q^2)$ are the structure functions for initial state radiation, and $Q^2 \sim s$ is the factorization scale. The structure function satisfies the evolution equation

$$Q^2 \frac{\partial}{\partial Q^2} D(x, Q^2) = \frac{\alpha}{2\pi} \int_x^1 \frac{dz}{z} \left[ P_{ee}(z) \right]_+ D \left( \frac{x}{z}, Q^2 \right) \quad \text{with} \quad P_{ee}(z) = \frac{1 + z^2}{1 - z} \quad (3)$$
with initial condition $D(x, m^2) = \delta(1-x)$. The solution to eq. (3) automatically includes
the exponentiation of the soft photon contributions as well as a resummation of the large
logarithms of the form $\log(s/m^2)$ from multiple hard photon emission.

The radiatively corrected cross section (2) is implemented in a Monte Carlo event
generator by solving (3) by iteration. This procedure is well known from the corresponding
QCD applications \cite{5} and, as a by-product of the algorithm, the four-momenta of the
radiated photons may be generated explicitly. For more details we refer the reader to \cite{6}.

**WOPPER** also includes the effects from finite $W$ width. To introduce finite width for
the intermediate $W$ bosons one has various possibilities \cite{7}. The one chosen in the Monte
Carlo is to start from an off-shell cross section $\sigma^*$ for the process $e^+e^- \rightarrow W^+W^-$ with
arbitrary (timelike) four-momenta $k_\pm$ of the $W$ bosons.

$$\sigma^* = \sigma^*(s; k_+^2, k_-^2)$$ (4)

The cross section for the process $e^+e^- \rightarrow 4$ fermions is then obtained by convoluting $\sigma^*$
with propagators for the $W$ bosons multiplied by the decay probability for the subsequent
$W$ decay

$$\sigma = \int \frac{ds_+}{\pi} \frac{ds_-}{\pi} \frac{\sqrt{s_+ \Gamma_W(s_+)} \Gamma_W(s_-)}{(s_+ - M_W^2)^2 + s_+ \Gamma_W^2(s_+)} \frac{\sqrt{s_- \Gamma_W(s_-)} \Gamma_W(s_+)}{(s_- - M_W^2)^2 + s_- \Gamma_W^2(s_-)} \sigma^*(s; s_+, s_-)$$ (5)

where $\Gamma_W(s) \approx \sqrt{s} \Gamma_W/M_W$ is the effective off-shell decay width of the $W$’s. This means
that we neglect the contributions of the so-called background diagrams, which are sup-
pressed by a factor $\Gamma_W/M_W \sim 2.5\%$ for each non-resonant propagator. The contribution
of the background diagrams may be reduced further by appropriate cuts on the invariant
masses of the final state.

In the Monte Carlo generator **WOPPER** the four fermion final states are generated accord-
ing to the distribution \cite{8}. In the partial decay widths of the $W$’s, the QCD corrections
to the hadronic decays have been taken into account up to first order in $\alpha_s$. The decay
angular correlations of the fermions are calculated by using the polarization density ma-
trix of the intermediate $W$’s, which is obtained from the off-shell helicity amplitudes. For
more details see ref. \cite{6}.

## 3 Results

Since **WOPPER** is a full Monte Carlo event generator, one can in principle study the correc-
tions due to finite width and electromagnetic radiation for any exclusive quantity. How-
ever, for the sake of comparison with other work we will consider here mainly corrections
to the total cross section and several simple distributions.

In figure 1 we plot the total cross section in the energy range from the $W$ pair produc-
tion threshold to 1 TeV. The dotted, dashed and full lines show the cross section obtained
from **WOPPER** at Born level in the narrow width approximation, including finite $W$ width, and
with all corrections turned on, respectively. Figure 2 gives the corrections relative to
the lowest order cross section due to finite $W$ width and QED corrections in the region
from 200 GeV to 1 TeV. As it was pointed out earlier \cite{8}, the effects from the finite $W$
width do not vanish at high energies but rather enhance the cross section by about 6% at 1 TeV.

As has been mentioned above, the longitudinal modes of the electroweak gauge bosons play a specific rôle in investigating the origin of electroweak symmetry breaking and in extracting the effects of physics beyond the SM. Hence a determination of the $W$ helicities bosons from the decay products is mandatory. Analyses of this type have been performed for all $W$ decay channels but without QED corrections in [4].

The polarizations of the $W$'s may be determined from their decay angular distribution in their rest frame, which is obtained from the energy spectrum of the decay fermions. E.g. for the leptonic decays, one defines the decay angle $\cos \theta^*$ by:

$$\cos \theta^* = \frac{1}{\beta} \left( \frac{2E_{\text{Lepton}}}{E_{\text{Beam}}} - 1 \right) \quad \text{where} \quad \beta = \sqrt{1 - \left( \frac{M_W}{E_{\text{Beam}}} \right)^2}$$

(6)

Here $E_{\text{Lepton}}$ is the energy of the charged lepton in the lab system, and $\beta$ is the $W$ velocity. Figure 3 shows the corresponding distribution reconstructed from the decay leptons for a total of $10^5$ events generated at 500 GeV. One can easily see that the QED corrections heavily distort this angular distribution and may therefore make the determination of the $W$ helicities difficult. In particular, the longitudinal components of the $W$ bosons seem to be strongly enhanced; this is due to the large boost between lab and c.m. system when hard photons are radiated from the initial state. A simple reconstruction scheme, which accounts for this boost effect for the special case of semi-leptonic events but assuming the narrow width approximation, has been presented in [10].

Finally, since WOPPER is a true multiphoton Monte Carlo event generator, we show in figure 4 the multiplicity distribution of events with multiple registered hard photons at a c.m.s. energy of 500 GeV. In this plot, a photon is counted if its energy lies above a given cut $E_{\gamma, \text{min}}$ and if its polar angle lies in the range $5^\circ < \theta_\gamma < 175^\circ$ with respect to the beam line. For realistic energy and angular resolutions of the detector, one expects a cross section of the order of $5 \cdot 10^{-2} \text{pb}$ for events with at least two detected photons, which corresponds to about 500 events for a typical year of running at a luminosity of $10^{33} \text{cm}^{-2}\text{s}^{-1}$.

The present version of WOPPER does not contain weak corrections. In a forthcoming version of the Monte Carlo generator, weak corrections will be included in the framework of effective Born cross sections [11]. Also, an interface to hadronization Monte Carlos [12] for a realistic description of the hadronic decays of the $W$'s will be added. Anomalous couplings for the $\gamma WW$ and $ZW W$ vertices may also be included in a future version of the generator.

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**Figure Captions**

**Figure 1:** Total cross section from WOPPER (full line: fully corrected, dashed line: finite width only, dotted line: Born formula).

**Figure 2:** Corrections relative to the Born cross section in the energy range from 200 GeV to 1 TeV (stars: fully corrected, open symbols: finite width only).

**Figure 3:** Decay angle of $W$’s into charged leptons, as given by eq.(6), for $10^5$ events at 500 GeV.

**Figure 4:** Cross section for multiphoton events at 500 GeV. Photons are counted above $E_{\gamma,\text{min}}$ and in the angular range $5^\circ < \theta < 175^\circ$. 
Figure 1

Figure 2

Figure 3

Figure 4