On the thermal broadening of a quantum critical phase transition

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The temperature dependence of an integer Quantum Hall effect transition is studied in a sample where the disorder is dominated by short-ranged potential scattering. At low temperatures the results are consistent with a \((T/T_0)^{\alpha}\) scaling law where \(\Delta \nu_0\) from the critical value \((\nu_0 - \nu)^{\kappa}\), characteristic of a quantum critical phase transition. It is shown that the linear behaviour results from thermal broadening produced by the Fermi-Dirac distribution function and that the temperature dependence over the whole range depends only on the scaling parameter \(T_0\).

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In a recent paper \cite{1} (referred to hereafter as I) it was demonstrated, for a sample in which the disorder is dominated by short-ranged potential scattering, that the 2-1 integer quantum Hall transition was well described by a scattering parameter

\[
\sigma = \exp[-\Delta \nu/\nu_0(T)], \tag{1}
\]

where \(\Delta \nu = \nu - \nu_0\) is the deviation of the filling factor from the critical value \((\nu_c)\) and \(\nu_0(T)\), at least in the low temperature limit, obeys a scaling law \((T/T_0)^{\kappa}\) with \(\kappa \approx 3/7\). It was argued that this description, which is characteristic of a quantum critical phase transition, is also appropriate for transitions into the Hall insulator state and for the \(B = 0\) metal-insulator transition.

By contrast, Shahar et al \cite{2} (hereafter referred to as II) studying the Hall insulator transition in a variety of GaAs and InGaAs based samples, find the same exponential dependence on filling factor but a logarithmic slope \(\nu_0(T)\) that appears to vary as \(\alpha T + \beta\) rather than exhibiting \(T^{\kappa}\) scaling behaviour. The ratio \(\beta/\alpha\), defines a temperature that is found to be characteristic of the material system.

New results are presented here for the same p-SiGe sample studied in I but measured in more detail and over a wider temperature range. The same approximately linear dependence of \(\nu_0(T)\) seen in II is observed. It is attributed, not to a new transport regime, but rather to the situation where the behaviour is dominated by thermal broadening associated with the Fermi-Dirac distribution function.

Figure 1 shows resistivity components, \(\rho_{xx}\) and \(\rho_{xy}\), around the 2-1 quantum Hall transition, measured for a range of temperatures between 60 and 600mK. The sample and experimental procedures have been described previously. Although a fixed point can be seen in the Hall data the corresponding point in the longitudinal resistivity is progressively smeared out as the temperature is raised. This behaviour can be understood more clearly when the data is inverted to obtain conductivity components (Figure 2). The Hall conductivity shows a well defined fixed point, at \(s_{xy} = 1.5\) (in units of \(e^2/h\)) but the peak value of \(\sigma_{xx}\), which should correspond to a temperature independent critical point, deviates progressively from the expected value of 1/2 as the temperature increases. It should be noted, however, that there is a high degree of reflection symmetry, ie \(s_{xx}(\Delta \nu) = \sigma_{xx}(-\Delta \nu)\).

Following the approach outlined in I the scattering parameter \(s\) can be extracted from this data according to

\[
\sigma_{xx} = 2\sigma_{pk}(T)s/(1 + s^2), \quad \sigma_{xy} = 2 - s^2/(1 + s^2). \tag{2}
\]

A prefactor \(2\sigma_{pk}\), with a weak temperature dependence, has been introduced for \(\sigma_{xx}\) to account for the deviations from exact critical behaviour. As discussed in I these are attributed to the finite range of the impurity scattering potential. Although this is small, corresponding to predominantly large angle scattering, it is nevertheless finite so the momentum weighting factor, \((1-\cos \theta)\), plays a role in the transport processes.

Values of \(s\) deduced from \(\sigma_{xy}\) are shown in figure 3: very similar results are obtained from \(\sigma_{xx}\). In agreement with earlier work \cite{2} the field dependence near the critical point is well described by eqn. 1 and values...
FIG. 2. Conductivity data, plotted as a function of filling factor $\nu$, obtained by inverting the resistivities shown in Fig. 1.

FIG. 3. Scattering parameter $s$, defined in the text, derived from the $\sigma_{xy}$ data shown in Fig. 2, plotted on a logarithmic scale.

of $\nu_0(T)$, derived from the slopes at $\Delta\nu = 0$, are shown in Fig. 4. Also shown are values obtained from the $\sigma_{xx}$ data, deduced from the width of the peak measured at half-maximum height. In agreement with the results reported in II the temperature dependence of $\nu_0(T)$, over this relatively large range, is not given by a $T^\kappa$ scaling behaviour but is better described by a linear variation of the form $\alpha T + \beta$. For the data shown the characteristic temperature given by $\beta/\alpha$ is $0.08K$, very similar to the values found in II for GaAs/GaAlAs samples. At the lowest temperatures, however, the data is consistent with an asymptotic approach to the expected scaling behaviour with $\kappa \approx 3/7$.

To support the identification of the low temperature behaviour with scaling it is important to establish experimentally that change in $s$ around $T = 0.1K$ seen in Fig. 4a is a genuine feature and not just the result of temperature saturation in the sample produced by spurious heating effects. Fortunately, there is available here an independent measure of the temperature of the holes. Figure 5 shows $\rho_{xx}$ data between filling factors $\nu = 3$ and 2 at dilution refrigerator temperatures of 120mK, 60mK and "base" temperature (about 30mK). In addition to the 3-2 integer quantum hall transition at 5 tesla there is extra structure, at higher fields, that is strongly dependent on both temperature and measuring current. It has been established by activation measurements that this corresponds to a paramagnetic/ferromagnetic phase transition induced by exchange enhancement of the spin splitting.

As shown in the inset the amplitude of this peak varies approximately linearly with temperature down to 50mK, the lowest temperature for which the Ge resistance thermometer, mounted in the mixing chamber, is calibrated. As the main graph shows, the peak height continues to decrease as the mixing chamber temperature is further lowered. This provides some confidence that at the lowest temperature shown in Fig. 4 (60mK) the deviation of temperature of the holes in the sample from that measured by the Ge thermometer is at most a few mK.

It is argued therefore that at the lowest temperatures scaling as $T^\kappa$ is observed but as the temperature is raised
the calculation therefore gives the correct behaviour for higher values of $T$. In light of the relatively crude model used to define the energy dependence of $s(E)$ the agreement with experiment is very good. Similar behaviour is also observed when $\sigma_{xx}$ is used as a basis for the calculation although there is some difficulty then in knowing how, precisely, to deal with the deviation of the peak value (cf eqn. 2) from $0.5e^2/h$. The thermal broadening not only gives the linear increase of $\nu_0(T)$ but also a temperature dependent reduction of the peak height, of the correct order of magnitude. A proper treatment of this effect, however, requires the inclusion of the momentum weighting term $(1-\cos \theta)$.

The good agreement between the calculated and experiment values of $\nu_0(T)$ therefore confirms that the linear dependence should be associated with thermal broadening and does not represent a new transport regime. The characteristic temperature $\beta/\alpha$ is not a new temperature scale but rather just depends on $T_0$. Indeed, it provides a means of determining this parameter even when the low temperature scaling regime happens to be experimentally inaccessible.

For the data presented here $T_0$ is 290K and the ratio of the two temperatures is about $2.7 \times 10^{-4}$. Similar values also pertain to the GaAs/GaAlAs data shown in II. For the InGaAs/InP data shown in II, where $\beta/\alpha$ is about 0.5K, scaling behaviour is not seen. However, for a similar sample $T_0$ is approximately 3400K with a corresponding ratio of about $1.5 \times 10^{-4}$, of the same order of magnitude. This suggests therefore, that thermal broadening provides a means whereby these two apparently conflicting experimental results, obtained in nominally identical samples, can be reconciled.

In summary, for a sample where the disorder is dominated by short-ranged potential scattering, so the transport data is an accurate reflection of the quantum critical processes, the temperature dependence of the 2-1 integer quantum Hall transition is characterised by two regimes. In a low temperature regime the behaviour is consistent with scaling having the expected exponent ($\kappa = 3/7$), while at higher temperatures, in agreement with earlier work, there is a linear dependence. It is suggested, and confirmed by calculation, that the linear dependence should be attributed to thermal broadening from the Fermi-Dirac distribution function.

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