Ballpark prediction for the hadronic light-by-light contribution to the muon $\mu (g-2)\mu$

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Using the momentum dependence of the dressed quark mass and the well-known formulae for the mass dependent quark loop contribution to the light-by-light scattering insertions, we compute the hadronic light-by-light contribution to the the muon anomalous magnetic moment. We ascribe for the first time a systematic error on the calculation.

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I. INTRODUCTION

The anomalous magnetic moment of the muon is one of the most accurately measured quantities in particle physics. Any deviation from its prediction in the Standard Model of particle physics is a very promising signal of new physics.

The present world average experimental value of its deviation from the Dirac value, i.e., $a_\mu = (g_\mu - 2)/2$, is given by $a_{\mu}^{EXP} = 11659208.9(6.3) \times 10^{-10}$ [1,2]. This impressive result is still limited by statistical errors, and a proposal to measure the muon $(g-2)_\mu$, to a precision of $1.6 \times 10^{-10}$ has recently been submitted to FNAL [3].

At the level of the experimental accuracy, the QED contributions to $a_\mu$ from photons and leptons alone are very well known. Recently the calculation has been completed up to the fifth order $O(\alpha^5_{em})$ in the fine-structure constant $\alpha_{em}$, giving as result for the QED contribution $11658471.885(4) \times 10^{-10}$ [4].

The main uncertainties at present in the Standard Model calculation for $(g-2)_\mu$ originate from the hadronic vacuum polarization (HVP) as well as hadronic light-by-light scattering (HLBL) corrections. We show the present estimates and their uncertainties for the QED, HVP, HLBL, and the electroweak (EW) corrections in Table I.

| Contribution       | Result in $10^{-10}$ units | Ref. |
|--------------------|----------------------------|------|
| QED(leptons)       | 11658471.885 ± 0.004        | [4]  |
| HVP(leading order) | 692.3 ± 4.2                | [5]  |
| HVP(higher order)  | −9.84 ± 0.07               | [6]  |
| HLBL               | 11.6 ± 4.0                 | [7]  |
| EW                 | 15.4 ± 0.2                 | [8]  |
| Total              | 11659181.3 ± 5.8           |      |

The existing discrepancy between the experimental value for $(g-2)_\mu$ and its Standard Model prediction stands at about $3\sigma$.

In order to interpret the upcoming new experiment at FNAL, with an anticipated precision of $1.6 \times 10^{-10}$, there is an urgent need to improve on both the HVP as well as the HLBL contributions. The accuracy of the HVP contribution depends on the statistical error of the experimental data for the $e^+e^-$ annihilation cross-section into hadrons. With future experiments, in particular at BES-III [9], one foresees this error to quantitatively decrease. The HLBL cannot be directly related to any measurable cross section however, and requires the knowledge of Quantum Chromodynamics (QCD) contributions at all energy scales. Since this is not known yet, one needs to rely on hadronic models to compute it. Such models introduce some systematic errors which are difficult to quantify.

The main motivation of this work is to provide a ballpark prediction with a judicious error estimate for the HLBL scattering based on a duality argument between the hadronic degrees of freedom and the well-known quark loop contribution.

Such a duality estimate can be obtained by invoking a particular regime of QCD where one knows how to perform the quark loop integral responsible for the $a_\mu^{HLBL}$ (Fig. 1). This is the large-$N_c$ of QCD [10,11] where a quark-hadron duality is accounted for considering that hadronic amplitudes are described by an infinite set of non-interacting and non-decaying resonances. As shown in Ref. [12,13], the large-$N_c$ limit provides a very useful framework to approach this problem.

Using the large-$N_c$ counting and also the chiral counting, it was proposed in [12] to split the diagram of Fig. 1 into a set of different contributions where the numerically dominant contribution arises from the pseudo-scalar exchange diagram shown in Fig. 2.

The large-$N_c$ approach however has two shortcomings: firstly, the assumption of pion-exchange dominance implies that the remaining pieces are small enough to justify their omission. Although this seems reasonable [13], it might lead to an underestimation of the error. Secondly, calculations carried out in the large-$N_c$ limit demand an infinite set of resonances. As such sum is not known in practice, one ends up truncating the spectral function in a resonance saturation scheme, the so-called Minimal Hadronic Approximation [14]. The resonance masses used in each calculation are then taken as the physical ones from PDG [15] instead of the correspond-

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II. QUARK-HADRON DUALITY ESTIMATE

To perform a quark-hadron duality estimate for the HLBL contribution to $a_{\mu}$, we now discuss the direct contribution from the quark-loop diagram. The diagram with a light-quark running, with a mass of a few MeV, is only valid in the regime where quarks are not confined, so in the perturbative QCD regime. This is not the region dominating the loop integral since we know that the momentum circulating the loop covers the low- and high-energy ranges. From quark models, one can use a constituent quark mass model with mass around 200 MeV as a rough estimate, with such a value obtained after comparing the quark model with the experimental value of the HVP [20-23]. This, however, fails to reproduce the perturbative QCD result at high energies. One, then, desires a running momentum dependent mass to perform such integration.

Ideally, lattice QCD calculations could be able to calculate such momentum dependent quark masses. As at present lattice QCD calculations are not yet fully feasible for physical pion masses, our proposal is to use the dressed-quark mass function $M(Q)$ in the chiral limit computed within the Dyson-Schwinger equation (DSE) framework. As shown in Fig. 3, the solution of the gap equation for a renormalized dressed-quark propagator using DSE provides us with the desired momentum dependent quark-mass function [24, 25]. We add a 10% relative error as an estimate for the extrapolation from the chiral limit to the physical light quark mass regime [26].

To determine the average momentum of the quark running in the loop on Fig. 1, we employ for our duality argument a hadronic model. Such model yields the average momenta entering the momentum dependent dressed quark mass $M(Q)$. In this averaging procedure, we employ both $Q = (Q_1 + Q_2)/2$ and $Q = \sqrt{Q_1 Q_2}$, and will ascribe the difference between both calculations to the theoretical uncertainty. With this mass, one can use...
the well-known formulae for the \( a_{\mu}^{H\!L\!B\!L} \) for spin-1/2

\[
a_{\mu}^{H\!L\!B\!L}(M(Q)) = \left( \frac{\alpha}{\pi} \right)^3 N_c \left( \sum_{q=u,d,s} Q_q^4 \right) \left[ \frac{3}{2}\zeta(3) - \frac{19}{16} \right] \frac{m_{\mu}^2}{M(Q)^2} + O \left( \frac{m_{\mu}^4}{M(Q)^4} \frac{\log^2 m_{\mu}^2}{M(Q)^2} \right) . \tag{1}
\]

In Eq. (1) we display the result up to first order in \( (m_{\mu}^2/M(Q)^2) \) but we include all terms up to the fourth order in our numerical calculations.

For our evaluation we will project the hadronic content of the quark loop of Fig. 1 onto the dominant hadronic piece of the HLBL. This is the pion-exchange contribution considered in [18]. We will use, instead of a hadronic model for the transition Form Factors (FF), a sequence of Padé Approximants [16] in two variables (called Chriisholm Approximants [31]) build up from the low-energy expansion of the pion FF obtained in [17] after a fit to the experimental data [32–35], to minimize a model dependence. The FF is considered to be off-shell (see Refs. [7, 36–40] where this point is addressed). To match the large momentum behavior with short-distance constraints from QCD, calculable using the OPE, we consider the relations obtained in Ref. [41].

In practice this amounts to use for the FFs (blobs in Fig. 2) the expression:

\[
F_{\pi^+ \gamma^+ \gamma^-}(p_{\pi}^2, q_1^2, q_2^2) = \frac{a - b q_1^2 - b q_2^2}{q_1^2 - q_2^2} (1 + c p_{\pi}^2) , \tag{2}
\]

where \( p_{\pi} = q_1 + q_2 \) and the free parameters are matched at low energies with the results in [17]: \( a \) is fixed by \( \Gamma_{\pi\rightarrow\gamma\gamma} = 7.63(16) \) eV from PDG [15] which already incorporates the recent PrimEx Collaboration result [12]; and \( b \) by a matching to the slope \( a_{\pi} = 0.0324(22) \) in [17]. The parameter \( c \) characterizes the off-shells of the pion and is obtained by imposing, along the lines of the Padé method, that

\[
\lim_{q_{1,2} \rightarrow \infty} F_{\pi^+ \gamma^+ \gamma^-}^{P_{01}}(q_{1,2}^2, q_{1,2}^2, 0) = f_\pi \chi/3 , \tag{3}
\]

where \( \chi = (-3.3 \pm 1.1) \) GeV\(^{-2} \), with an error of 30% as proposed in Refs. [7, 39, 41]. The results for the average momenta running in the quark loop in Fig. 1 using the FFs of Eq. (2) are shown in Table III where in parenthesis we quote the symmetrized errors from the input uncertainties. We remark that using \( Q_1 \) and \( Q_2 \) would yield similar results well within the range shown in Table III.

The convergence of the PA sequence to a meromorphic function is guaranteed by Pomerenke’s theorem [43]. The problem is to know how fast this convergence is and also how to ascribe a systematic error on each element of that sequence. For the particular case of a meromorphic function (such as a Green’s function in large-\( N_c \) QCD), the simplest way of evaluating a systematic error is by comparing the difference between two consecutive elements on the PA sequence [18].

In our approach to the FF, we evaluate the systematic error by computing a second element on the PA sequence [18] and compare it with the result using Eq. (2). The second element is:

\[
F_{\pi^+ \gamma^+ \gamma^-}^{P_{02}}(p_{\pi}^2, q_1^2, q_2^2) = \frac{a + b q_1^2 - b q_2^2}{q_1^2 - q_2^2} (1 + c p_{\pi}^2) , \tag{4}
\]

with five coefficients to be matched with \( \Gamma_{\pi\rightarrow\gamma\gamma} \), the slope \( a_{\pi} \), the curvature of the pion FF \( b_{\pi} = 1.06(26) \times 10^{-3} \) [18], with \( \chi \) in Eq. (3) and the first vector meson resonance \( M_{\rho} = 0.776(77) \) GeV (where the error in parenthesis is obtained with the half-width rule method which accounts effectively for 1/\( N_c \) corrections on meson masses when using PDG values in large-\( N_c \), calculations [44, 46]). The results for the average momenta running in the quark loop in Fig. 1 using the approximant of Eq. (1) are shown in Table III where again the errors are due to the input ones. We assert that the quark masses produced using Eq. (1) from our average momenta are in excellent agreement with the constituent quark masses obtained in previous ballpark determinations of the HLBL [20, 22] based on the experimental value of the HVP.

TABLE II. Collected results for the average momenta running in the quark loop in Fig. 1 its corresponding mass \( M(Q) \) in Fig. 2 and the \( a_{\mu}^{H\!L\!B\!L} \) result in accordance to Eq. (1), Fig. 4 for both \( P_0 \) and \( P_1 \) parameterizations for \( F_{\pi^+ \gamma^+ \gamma^-} \).

| \( P_0 \) | (\( Q_1 + Q_2 \))/2 | 0.49(6) | 0.167(17) | 15.0(2.4) |
| \( P_1 \) | (\( Q_1 + Q_2 \))/2 | 0.48(9) | 0.169(25) | 14.7\( ^{+4.3}_{-3.2} \) |
| | \( \sqrt{Q_1, Q_2} \) | 0.44(5) | 0.182(18) | 12.9(1.9) |
| | \( \sqrt{Q_1, Q_2} \) | 0.44(8) | 0.182(23) | 13.0\( ^{+3.2}_{-2.5} \) |

The similarity of the results obtained within both approximants indicates that the low-energy region (up to 1–2 GeV) dominates the contribution to \( a_{\mu}^{H\!L\!B\!L} \) [17, 48]. To evaluate the error on our approximation, we look for the maximum of the difference in the region up to 1 GeV between the \( P_0 \) and \( P_1 \) parameterizations for \( F_{\pi^+ \gamma^+ \gamma^-} \), as explained in Ref. [18]. Of course, this difference depends on the energy, and grows as the energy increases. At 1 GeV, the relative difference is about 5% [19], and we take this error as a conservative estimate of the error on the
whole low-energy region. We should add this error to the $a_{\mu}^{HLBL}$ results shown in Table IV.

For comparison with other approaches [13, 23, 30, 36, 47, 50, 51], we quote what would be the pion-exchange piece to the light-by-light scattering contribution if one would use the models in Eqs. (2) and (4) with such purpose. Eq. (2) yields $a_{\mu}^{HLBL,\pi^0} = 6.49(56) \times 10^{-10}$ and Eq. (4) $a_{\mu}^{HLBL,\pi^0} = 6.51(71) \times 10^{-10}$, in good agreement with the result $a_{\mu}^{HLBL,\pi^0} = (7.2 \pm 1.2) \times 10^{-10}$ obtained in Ref. [4]. For the pion-pole contribution (i.e., when the offshellness of the pion is swich-off, $c = 0$ in Eqs. (2) and (4), we would obtain $a_{\mu}^{HLBL,\pi^0} = 5.52(27) \times 10^{-10}$ and $5.55(34) \times 10^{-10}$, respectively, in accordance to the result $a_{\mu}^{HLBL,\pi^0} = (5.8 \pm 1.0) \times 10^{-10}$ reported in Ref. [30]. These results reassure the stability of the PA sequence.

![Chart](image.png)

FIG. 4. $a_{\mu}^{HLBL}$ results using the expansion in Eq. (1) at order $(m_{\mu}^2/M(Q)^2)^0$ (dashed blue) and at order $(m_{\mu}^2/M(Q)^2)^8$ (solid black) in terms of the running quark mass $M(Q)$. The red point indicates the exact result for the point $M(Q) = m_{\mu}$. The horizontal band shows the ballpark result from Eq. (5). In units of $10^{-10}$ for $(Q_1 + Q_2)/2$ and $\sqrt{Q_1Q_2}$ respectively. We are using the DSE to obtain the momentum dependent dressed quark masses. Once the lattice calculations will reach the physical quark mass values, this 10% error will disappear.

III. RESULTS AND CONCLUSIONS

To quote a final number for $a_{\mu}^{HLBL}$ implies to consider several sources of error:

- Firstly, we have the error coming from the experimental inputs used to build up our approximants, which arises mainly from the fit to the experimental data on the FF, and the offshellness of the pion. With the new forthcoming experimental data on that FF at BES-III [9] we intuit lower input errors on our results.

- Secondly, we have a 10% error due to the departure from the chiral limit shown in Fig. 3. This error is absolutely asymmetric and moves the $M(Q)$ values in Table IV upwards. That implies a decrease of about 15% on $a_{\mu}^{HLBL}$ (down to 12.8 and 11.0

- Furthermore, we also have a systematic error from the PA sequence used. We estimate this to be around 5% (see [49] for details on how to obtain such estimation). That implies an error on $a_{\mu}^{HLBL}$ about $\pm 0.8$ for $(Q_1 + Q_2)/2$ and $\pm 0.6$ for $\sqrt{Q_1Q_2}$ in units $10^{-10}$. Since both $P_1^\mu$ and $P_2^\mu$ parameterizations for $F_{\pi\gamma\gamma}$ give almost the same results for $a_{\mu}^{HLBL}$, no extra error due to the difference between them should be included on $a_{\mu}^{HLBL}$. We should remark that the FFs employed here, although constrained by the experimental data, do not have the correct behavior when both photon virtualities $(q_1$ and $q_2$) are very large where a behavior $\sim 1/(q_1^2 + q_2^2)$ is predicted [52–54]. This fact does not affect our calculation since $a_{\mu}^{HLBL}$ is very largely dominated by the low-energy region [47, 48].

- The last source of error comes from the evaluation of $a_{\mu}^{HLBL}$ using the order $(m_{\mu}^2/M(Q)^2)^8$ in Eq. (1) instead of the full result. The difference is so smooth, Fig. 4 that no extra error should be considered so far.

Our ballpark estimate lies then in the range:

$$a_{\mu}^{HLBL} = [10.5(2.0) \div 15.0(2.5)] \times 10^{-10},$$

where the error in parenthesis is the combined systematic and input errors as commented above and the two numbers represent the range due to the two momenta considered in our computations after the chiral extrapolation. Although different in nature, our ballpark nicely agrees with previous HLBL estimates [20–22]. Similar result is found if instead of our FF we use the one in Ref. [30]. With this ballpark estimate and the numbers collected in Table IV, we draw a new window for the existing discrepancy between the experimental value for the $(g - 2)_{\mu}$ and its Standard Model prediction from 3.7σ to 3.0σ considering the lower and upper extremes of your ballpark, respectively.

For comparison with approaches that considered a pseudoscalar-pole contribution (i.e, $c = 0$ in Eqs. (2) and (4)) instead of a pseudoscalar-exchange, we also report what would be our ballpark estimation for such scenario:

$$a_{\mu}^{HLBL} = [8.2(0.4) \div 12.5(0.7)] \times 10^{-10}. (6)$$

In summary, we presented a ballpark estimate of the hadronic light-by-light scattering contribution to the $(g - 2)_{\mu}$ based on a duality argument, and estimated the average momentum flowing in the quark loop diagram of Fig. 1 from a hadronic model. This average momentum then allowed us to calculate the momentum dependent
quark mass in the quark-loop result from $a_{\mu}^{\text{HLBL}}$. Most of the recent phenomenological calculations of the $a_{\mu}^{\text{HLBL}}$ fall into the range obtained in this work (\[13, 20, 36, 50, 51\]), including previous ballpark estimates (\[20, 22\]). The shift of our estimate due to the offshellness of the pseudoscalar (compare Eq. (5) with Eq. (6)) suggest further investigations for such kind of effects. Beyond this, we notice that contributions of higher pseudoscalar resonances as well as higher spin resonances not included so far in the HLBL may explain our slightly higher results compared to other determinations. Measurements of two-photon decay widths of $\eta(1295)$, $\pi(1300)$, $\eta(1405)$, $\eta(1475)$, as well as scalar, axial-vector, and tensor states would help along these lines.

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