Diffuse TeV Emission at the Galactic Centre

Elizabeth Wommer,1⋆ Fulvio Melia,2† and Marco Fatuzzo3‡

1 Department of Physics, The University of Arizona, Tucson, Arizona 85721, USA
2 Department of Physics and Steward Observatory, The University of Arizona, Tucson, Arizona 85721, USA
3 Physics Department, Xavier University, Cincinnati, OH 45207

Submitted to MNRAS 2007 December 27

ABSTRACT
The High-Energy Stereoscopic System (HESS) has detected intense diffuse TeV emission correlated with the distribution of molecular gas along the galactic ridge at the centre of our Galaxy. Earlier HESS observations of this region had already revealed the presence of several point sources at these energies, one of them (HESS J1745-290) coincident with the supermassive black hole Sagittarius A*. It is still not entirely clear what the origin of the TeV emission is, nor even whether it is due to hadronic or leptonic interactions. It is reasonable to suppose, however, that at least for the diffuse emission, the tight correlation of the intensity distribution with the molecular gas indicates a pionic-decay process involving relativistic protons. In this paper, we explore the possible source(s) of energetic hadrons at the galactic centre, and their propagation through a turbulent medium. We conclude that though Sagittarius A* itself may be the source of cosmic rays producing the emission in HESS J1745-290, it cannot be responsible for the diffuse emission farther out. A distribution of point sources, such as pulsar wind nebulae dispersed along the galactic plane, similarly do not produce a TeV emission profile consistent with the HESS map. We conclude that only a relativistic proton distribution accelerated throughout the inter-cloud medium can account for the TeV emission profile measured with HESS.

Key words: Black Hole Physics, Cosmic Rays, Magnetic Fields, Turbulence, Galaxy: Centre

⋆ E-mail: ewommer@physics.arizona.edu
† Sir Thomas Lyle Fellow and Miegunyah Fellow E-mail: melia@as.arizona.edu
‡ E-mail: fatuzzo@xavier.edu

© 0000 RAS
Elizabeth Wommer, Fulvio Melia, and Marco Fatuzzo

Figure 1. Cloud distribution with the first assignment of $z$-coordinates. The left panel shows the line-of-sight view of the molecular gas. The right panel represents the same view, though from a vantage point slightly above the galactic plane.

1 INTRODUCTION

Observations of the galactic-centre ridge with the High-Energy Stereoscopic System (HESS) have revealed a surprisingly intense diffuse TeV emission, strongly correlated with the distribution of interstellar gas (Aharonian et al. 2006). This feature, along with the energy range accessible to HESS ($> 200$ GeV), suggests that the dominant component of this diffuse emission is almost certainly due to the decay of neutral pions produced in hadronic cascades initiated via the scattering of relativistic cosmic rays with protons in the ambient medium (see, e.g., Crocker et al. 2005; Ballantyne et al. 2007).

Earlier HESS observations of the galactic centre had already revealed the presence of several TeV point sources, one of these (HESS J1745–290) coincident with the supermassive black hole Sagittarius A* (Aharonian et al. 2004). A second nearby source (about $1^\circ$—or roughly 144 pc at that distance—toward positive longitude $l$ of the galactic centre) was identified with the supernova remnant/pulsar wind nebula G0.9+0.1. With HESS’s unprecedented sensitivity, it has been possible to subtract these (and other) point sources from the overall map of this region, to search for the fainter, diffuse emission. The latter extends along the galactic plane for over $2^\circ$, and is also spread out roughly $0.2^\circ$ in galactic latitude $b$.

At the distance to the galactic centre, an extension in latitude of $\sim 0.2^\circ$ corresponds to a scale height of about 30 pc, similar to that of giant molecular cloud (GMC) material in this region, as traced by its CO and CS line emission (see, e.g., Tsuboi et al. 1999). These data suggest that the Galaxy’s central region ($|l| < 1.5^\circ$ and $|b| < 0.25^\circ$) contains up to $\sim 10^8$ $M_\odot$ of molecular gas, providing a rich target of overlapping clouds for the incoming cosmic rays (see figure 1; the construction of this diagram is discussed in § 2 below).

However, a simple cosmic-ray interpretation for the diffuse TeV emission is problematic for several reasons. Chief among these is the observed gamma-ray spectrum, which does not appear to be consistent with the cosmic-ray distribution in the solar neighbourhood. The reconstructed
gamma-ray spectrum for the region $|l| < 0.8^\circ$ and $|b| < 0.3^\circ$ (with point-source emission subtracted) is a power law with photon index $\Gamma = 2.29 \pm 0.27$. Since for a power-law energy distribution the spectral index of the gamma rays tracks the spectral index of the cosmic rays themselves, the implied cosmic ray index ($\sim 2.3$) is much harder than that ($\sim 2.75$) measured at Earth. Of course, we must be aware of the fact that cosmic rays escape from the Galaxy as they diffuse outwards from the centre, on an energy-dependent time scale $t_{\text{esc}} \propto E^{-\delta}$, with $\delta \sim 0.4–0.6$ (Bhattacharjee 2000). Thus, if most of the cosmic rays detected at Earth indeed originate at the galactic centre, then the injected spectrum ought to be flatter (by a change in index of $\sim \delta$) than that observed here, and the two distributions may still be consistent with each other. This very interesting possibility deserves further analysis, which may be facilitated by the results presented in this paper.

Secondly, the diffuse spectrum is remarkably similar to that of the central point source HESS J1745–290 itself ($2.29 \pm 0.27$ versus $2.25 \pm 0.10$). Though other sources (such as pulsar wind nebulae) along the galactic ridge also have similar spectra, the central TeV source is by far the brightest and may therefore be the dominant hadron accelerator in this region. It is indeed of considerable interest to know whether a single source (say, the supermassive black hole) could be responsible for producing most of the relativistic particles in the Galaxy’s central region, regardless of whether or not these should rightly be regarded as members of the overall cosmic-ray population.

In an earlier paper (Ballantyne et al. 2007), we examined the possible role of Sagittarius A* in producing the central point source HESS J1745–290, and concluded that stochastic acceleration within the inner 20–30 Schwarzschild radii of the black hole’s event horizon could produce both the relativistic electrons responsible for Sagittarius A*’s mm-spectrum, as well as an outflowing flux of relativistic protons that diffuse outwards to fill the inner 2–3 pc region, where they scatter predominantly with molecular gas in the circumnuclear disk to produce the TeV signal. Only $\sim 1/3$ of these protons encounter the disk, however, so up to this point, it is still not known whether the remainder of these particles can diffuse outwards to much larger radii to produce the diffuse emission along the galactic ridge.

In addition, it is not yet clear whether Sagittarius A* itself is the main contributor of relativistic hadrons to the source HESS J1745–290. At least two alternative models have been proposed, including a plerion scenario with Sagittarius A* as the wind source (Atoyan and Dermer 2004), and a pulsar wind nebula discovered recently within only a light-year of Sagittarius A* (Wang, Lu, and Gotthelf 2005). But all leptonic models for the TeV emission in HESS J1745–290 (and possibly elsewhere along the ridge) are subject to the extremely rapid cooling rates resulting from...
Elizabeth Wommer, Fulvio Melia, and Marco Fatuzzo

the intense photon fields and relatively strong magnetic fields in and around the molecular clouds in this part of the Galaxy.

One of the principal goals of this paper is therefore to examine whether relativistic protons accelerated by Sagittarius A* can in fact fill the ∼ 2° region surrounding the black hole to account for the diffuse TeV emission as well. We will conclude that this scenario is very unlikely. However, adhering to the idea that hadronic cascades, rather than inverse-Compton-scattering lepton distributions, are more likely to produce the diffuse TeV signal, we will also consider other source configurations that produce a TeV emissivity consistent with the HESS data.

In subsequent sections, we will first assemble the data pertaining to the molecular gas distribution at the galactic centre, and then describe our approach in setting up a turbulent magnetic field through which the hadrons must diffuse. As they wind their way outwards from their respective source(s), the protons lose energy steadily before finally scattering with other protons, and we will examine the processes that dominate this energy loss rate. We will describe our technique for calculating the proton propagation through this medium, which does not rely on a “standard” diffusion approach, but is instead more robust and less dependent on unknown factors, such as the diffusion coefficients. Finally, we will summarize our method for calculating the particle cascade (once a collision has occurred), and describe our simulations and results.

2 THE MOLECULAR CLOUD DISTRIBUTION

Observations of the inner few hundred parsecs of the Galaxy reveal a large concentration (up to ∼ 10⁸ M⊙) of dense molecular gas (Güsten and Philipp 2004). Much of this material is concentrated within giant molecular clouds, with a size ∼ 50–70 pc. Compared with their counterparts in the galactic disk, the galactic centre (GC) molecular clouds are denser and warmer. Emission maps of density-sensitive molecular species, e.g., CS (Bally et al. 1985), H₂CO (Güsten and Henkel 1983), and HC₃N (Walmsley et al. 1986), indicate that the GC clouds are “clumpy”, with high-density (∼ 10⁵ cm⁻³) regions embedded within a less dense (∼ 10³.⁷ cm⁻³) intra-cloud medium. The average cloud density is then roughly 10⁴ cm⁻³, which is quite large compared with the value ∼ 10².⁵ cm⁻³ for a typical disk cloud, but necessary if the GC clouds are to survive the strong tidal forces in the galactic-centre potential.

The temperature of GC clouds is also relatively high (∼ 30–60 K) and fairly uniform (Morris et al. 1983). Cloud temperatures were first estimated using measurements of metastable transitions of ammonia (NH₃) and later confirmed by AST/RO (Antarctic Submillimeter Telescope and
Remote Observatory) observations of CO line emission (see, e.g., Kim et al. 2002). Diffuse X-ray emission detected from the galactic centre by Chandra indicates the presence of “soft” (0.8 keV) and “hard” (8 keV) plasma components coexisting with the giant molecular clouds (Muno et al. 2004). While supernova shock waves can provide an explanation for the cooler plasma, the origin of the hard component is still uncertain. Clearly, though, the galactic centre is a warm environment and many possible heating mechanisms have been proposed to explain the observed cloud temperatures. Direct heating by energy dissipation via collisions with dust has mostly been discounted due to the comparatively low temperature of the dust particles (21 ± 2 K; Pierce-Price et al. 2000). Other possibilities include magnetic viscous heating, small scale dissipation of supersonic turbulence, large scale J- and C-shocks, and UV-heating in exposed photo-dissociation layers. More than likely, all of these mechanisms contribute on some level, but the dissipation of supersonic turbulence appears to be the most promising means by which the majority of the molecular clouds are warmed as the heating rate of this process is comparable to the cooling rate of the gas.

The GC molecular clouds are also threaded by a pervasive magnetic field, whose strength is revealed, e.g., by the presence of non-thermal filaments (NTFs) in the diffuse interstellar medium (ISM). The NTFs appear in radio images as long thin strands (tens of pc in length while only fractions of a pc wide) more or less perpendicular to the plane of the galaxy (Morris 2007). The strongly polarized synchrotron emission from the NTFs indicates that the magnetic field points along the filaments, whose apparent rigidity when they interact with molecular clouds and the turbulent interstellar medium suggests field strengths on the order of a few milligauss (see, e.g., Yusef-Zadeh and Morris 1987). Whether the NTFs are manifestations of a large-scale poloidal magnetic field or are localized structures has not been firmly established.

Mid- and far-infrared thermal dust emission from magnetically aligned dust grains provides additional information about the direction of the magnetic field within the warm, dense molecular clouds. In direct contrast to the ISM field, the magnetic field within the clouds is, for the most part, parallel to the galactic plane (Werner et al. 1988). Clues regarding how the galactic centre can have both poloidal and toroidal magnetic field configurations are provided by submillimeter polarization measurements of the central molecular zone by Chuss et al. (2003), which show that the field orientation is linked to the density of the region. Differential motion between dense material and the surrounding medium shears the poloidal magnetic field making it more toroidal, whereas less dense regions (like the cloud envelope) do not have sufficient mass to distort the initial field.

The cause of the stability of the GC giant molecular clouds has also been the subject of much discussion. It had been shown that, in general, the CO luminosity of a molecular cloud scales...
with its virial mass (Young and Scoville 1991). Using this result, the molecular mass of a region could be determined from its measured CO luminosity by using a standard conversion factor $X \equiv N(H_2)/I_{CO} = 3.0 \times 10^{20}$ (cm$^{-2}$ [K km s$^{-1}$]$^{-1}$). However, measurements by Oka et al. (1998) show that GC molecular clouds do not follow the $L_{CO}-M_{VT}$ trend of galactic disk clouds. Simply changing the $X$-factor used for GC clouds is not an option since its value is set by observations of $\gamma$-ray (Blitz et al. 1985) and far-infrared (Cox and Laureijs 1989) emission. Based on these findings, Oka et al. (1998) conclude that GC clouds larger than $\sim 30$ pc are not bound by their self-gravity but are instead in equilibrium with the external pressure of the galactic centre environment. However, the observed pressure due to a hot plasma $P_{\text{plasma}} \sim 10^{-9.2}$ erg cm$^{-3}$ is an order of magnitude smaller than that required since the turbulent pressure within the clouds is $P_{\text{turb}} \sim 10^{-8}$ erg cm$^{-3}$ (Güsten and Philipp 2004). Clouds may instead be bound by their own magnetic fields. Equating the turbulent and magnetic ($B^2/8\pi$) energy densities gives field strengths of $\sim 0.5$ mG within the clouds. Then, using the pressure-bound assumption, Oka et al. (1998) infer the value $2 \times 10^7 M_\odot$ for the lower limit to the mass of molecular gas within the inner 400 pc of the galaxy.

In spite of this abundant material, however, it is somewhat surprising that the star formation rate (SFR) in the galactic centre is currently not especially high ($\sim 0.3$–$0.6$ $M_\odot$ yr$^{-1}$ in the central 500 pc as opposed to $\sim 5.5$ $M_\odot$ yr$^{-1}$ in the disk; Güsten 2004, and references cited therein). Detection of 6.7 keV line-emission due to a helium-like iron K-shell transition by Yamauchi et al. (1990), though, suggests that either $\sim$1,000 supernova explosions, or one $10^{54}$-erg explosion, occurred in this region within the past $10^5$ years. Their results lend independent support to the earlier work of von Ballmoos, Diehl, and Schönhfelder (1987), who investigated the 1.8 MeV $\gamma$-ray emission from $^{26}$Al, produced in nuclear reactions during energetic events like supernova explosions. These observations indicate that the galactic centre may have been a more active star-forming region in the recent past.

Other than the physical conditions within these clouds, the remaining ingredient required to
assemble a reasonable representation of the molecular gas distribution along the galactic ridge is the three-dimensional spatial positioning of the cloud centroids. The positions in the plane of the sky of the 14 dominant giant molecular clouds (see Table 1) in the galactic centre region ($|l| \leq 2^\circ$, $|b| \leq 1^\circ$) were taken from Oka et al. (1998). For convenience, we divided the longitudinal coordinate into 10 bins, each with a width $\Delta l = 0.4^\circ$, and tabulated the number of clouds whose centre occurs in the range between $l_i$ and $l_i + \Delta l$. The cloud spatial distribution along the line-of-sight (i.e., the $z$-direction) is not known, but we used a randomization procedure to attribute a value of this coordinate to each cloud, constrained by the requirement that the distribution in $z$ matches the distribution in $l$. For each cloud in bin $i$ in the longitudinal direction, we assigned a randomly chosen cloud to the corresponding bin along the line-of-sight direction. Then, each cloud in bin $i$ was given a line-of-sight position $z = z_i + \chi \Delta z$, where $z_i$ is the lower bound on bin $i$, $\chi$ is a randomly chosen number between 0 and 1, and $\Delta z = \Delta l = 0.4^\circ$ is the bin size. Clouds were re-binned and/or re-positioned if overlap between clouds occurred. Using this method, we produced two different molecular cloud maps, shown in figures 1 and 2.

### 3 THE TURBULENT MAGNETIC FIELD

After leaving the acceleration zone, protons random-walk their way outwards scattering off of the relatively strong turbulent magnetic field. Our treatment here differs from the usual diffusion approach, which is often limited by poorly known factors, such as the diffusion coefficients. Instead, we follow the motion of individual particles by solving the Lorentz force equation with a magnetic field whose spatial profile is consistent with Kolmogorov turbulence. To create this magnetic field, we adopt the prescription of Giacalone and Jokipii (1994). While their original aim was to use the simulated field to calculate the Fokker-Planck coefficients based on the motion of individual
particles and then compare those values to analytic theory, we will simply use their algorithm to track the particle trajectories themselves.

For a particle of mass $m$ and charge $q$ moving in a magnetic field $B(r)$, we define $\Omega(r) = qB(r)/mc$, where $c$ is the speed of light. The total field $\Omega(r)$ is then written as the sum of two terms: $\Omega_0$, corresponding to the background field and $\delta\Omega$, which is the fluctuation about the mean and is not necessarily small. A three-dimensional field is then obtained by summing over a number of randomly polarized transverse waves. A form for $\delta\Omega$ that satisfies Gauss’s law $\nabla \cdot B = 0$ is

$$\delta\Omega(r) = \sum_k \Omega(k)[\cos \alpha(k)\hat{y}' \pm i \sin \alpha(k)\hat{z}'] \exp[ikx' + i\beta(k)],$$

(1)

where $\alpha(k)$ and $\beta(k)$ are random numbers between 0 and $2\pi$. The primed and unprimed coordinates are related by the rotation matrix

$$r' = \begin{pmatrix} \cos \theta & \cos \phi & \cos \theta \sin \phi & \sin \theta \\ -\sin \phi & \cos \phi & 0 \\ -\sin \theta \cos \phi & -\sin \theta \sin \phi \cos \phi & \sin \theta \end{pmatrix} r.$$  

(2)

The angles $\theta$ and $\phi$ are functions of $k$, such that $0 \leq \theta(k) \leq \pi$ and $0 \leq \phi(k) \leq 2\pi$. Thus, to create a three-dimensional field, five random numbers ($\alpha, \beta, \theta, \phi, \pm$) are needed for each value of $k$.

Assuming the irregular field is generated by Kolmogorov turbulence, we set

$$\Omega(k) = \Omega(k_{\text{min}})(k/k_{\text{min}})^{\Gamma/2},$$

(3)

where $k_{\text{min}}$ is the wavenumber corresponding to the longest wavelength and $\Gamma = 5/3$ is the power of the Kolmogorov spectrum. Although we have chosen this specific value of $\Gamma$, we do not expect that a particular choice of turbulence significantly affects the results of the simulation.

The value of $\Omega(k_{\text{min}})$ can be found using the total energy density

$$S = \sum_k \frac{B^2(k)}{8\pi} = \frac{m^2c^2}{8\pi q^2} \Omega^2(k_{\text{min}}) \sum_k (k/k_{\text{min}})^{-\Gamma/2},$$

(4)

taking $S = B_0^2/8\pi$, so that there is as much energy density contained in the fluctuations as there is in the background field. For our simulations, we use 200 values of $k$ evenly spaced on a logarithmic scale with wavelengths between 0.1 and $10r_{\text{gyr}}$ where $r_{\text{gyr}}$ is the gyroradius of a proton in the background magnetic field $B_0$.

There are two approaches to implementing a turbulent magnetic field generated by this method. The first approach (and the one used by Giacalone and Jokipii 1994) is to calculate the magnetic field at every time step for each particle position. For a particle moving with relativistic velocity $u \equiv \gamma v$ (in terms of the velocity vector $v$), its position is found by solving the equation of motion

$$\frac{du}{dt} = \frac{1}{\gamma} u \times \Omega(r).$$

(5)
For each time step, the same set of random numbers is used for each particle so that only $r$ changes. In the second approach, the magnetic field is generated for a given volume at the beginning of the simulation. Giacalone and Jokipii (1994) estimate that the required volume is $V \sim 100 \lambda_{\text{max}}^3$, where $\lambda_{\text{max}}$ is the longest wavelength of the fluctuations. Taking the interpolation distance between volume lattice points to be $0.25 \lambda_{\text{min}}$ ($\lambda_{\text{min}}$ being the shortest wavelength) results in $6.4 \times 10^9$ computations. Since we have $\lambda_{\text{max}} = 100 \lambda_{\text{min}}$, $6.4 \times 10^9$ computations would have to be performed before any simulations of particle motion can begin. This is not only time-consuming, but also very memory-intensive. So, like Giacalone and Jokipii (1994), we adopt the former approach. In this way, the magnetic field is generated only where needed, the overwhelming amount of computer memory required by the second approach is eliminated, and the particles are not confined to any pre-assigned volume.

4 ENERGY LOSS RATES

The protons lose energy through various interactions with the environment as they diffuse. The dominant energy loss mechanisms are as follows:

(i) $pp$ scattering

Defining the energy loss rate as

$$R \equiv -\frac{1}{E} \left(\frac{dE}{dt}\right),$$

we have for relativistic protons cooling due to their inelastic collisions with ambient protons of density $n_p$

$$R_{pp} = n_p c \sigma_{pp} K_{pp}. \quad (7)$$

The cross section $\sigma_{pp}$ depends only weakly on proton energy, increasing from $\simeq 30$ mbarn at $E_{\text{proton}} \sim$ few GeV to $40$ mbarn at $10^3$–$10^4$ GeV (Karol 1988), so for simplicity we approximate it with a constant value $\sigma_{pp} = 40$ mbarn. Likewise, although the inelasticity parameter $K_{pp}$ depends on the centre of momentum energy $\sqrt{s}$ (i.e., $K_{pp} = 1.35 s^{-0.12}$ for $\sqrt{s} \geq 62$ GeV, and $K_{pp} = 0.5$ for $\sqrt{s} \leq 62$ GeV; see Markoff et al. 1997, 1999), we use only the low-energy $K_{pp} = 0.5$ in our calculations.

(ii) $p\gamma$ scattering

An inelastic scattering between a proton and photon may lead to pair production, $p\gamma \rightarrow pe^+e^-$, and photo-pion production, $p\gamma \rightarrow p\pi^0$ and $p\gamma \rightarrow np^+$. The cross-section and inelasticity for these processes depend on the photon energy (see, e.g., Begelman, Rudak, and Sikora 1990). In the
proton rest frame, the threshold photon energy for pair production is $E_{th}^{(e)} = 2m_e \sim 1$ MeV and for pion production is $E_{th}^{(\pi)} = m_\pi \left(1 + m_\pi/2m_p\right) \sim 145$ MeV.

For the conditions of interest to us here, it can be shown easily by way of estimate that the energy-loss rate due to $p\gamma$ interactions is insignificant compared to that from $pp$ scatterings, so we may safely ignore this process here.

(iii) Synchrotron Processes

The synchrotron cooling rate is

$$R_{\text{synch}} = \frac{4}{3} \left(\frac{m_e}{m_p}\right)^3 \frac{c\sigma_T u_B}{m_e c^2 \gamma_p},$$

where $u_B = B^2/2\mu_0$ is the (total) magnetic field energy density and $\sigma_T = 0.665 \times 10^{-28} \text{ m}^2$ is the Thomson cross-section.

(iv) Compton scattering

Finally, the cooling rate due to Compton scattering is

$$R_C = \frac{u_{\text{rad}}[x < \frac{m_e}{m_p} \gamma_p^{-1}]}{u_B} R_{\text{synch}},$$

where $u_{\text{rad}}$ is the radiation energy density.

The various cooling rates are plotted in Figure 3 as functions of the proton Lorentz factor $\gamma_p$, both inside the molecular clouds and in the region between the clouds. Evidently, for proton energies $E_p \leq 10^{18}$ eV, the energy loss rate is dominated by $pp$ scatterings. For simplicity, we will therefore also ignore the energy-loss rates due to synchrotron radiation, and Compton scattering in calculating the particle trajectories.
5 PROTON PROPAGATION

There are two basic approaches to simulating the propagation of protons through the interstellar medium. These may be summarized as follows:

(i) solving the Lorentz force equation, \( \mathbf{F} = q \mathbf{v} \times \mathbf{B} \), for each individual particle, or

(ii) solving the diffusion equation,

\[
\frac{\partial W}{\partial t} + \sum_{i} \left( \frac{\partial}{\partial x_i} (W v_i) - \frac{\partial}{\partial x_i} \left( \kappa_{ij} \frac{\partial W}{\partial x_j} \right) \right),
\]

for the distribution function \( W(x_i, t) \), in terms of the diffusion coefficients \( \kappa_{ij} \).

Calculating the positions of a large number of protons using the Lorentz force equation is very time-consuming. On the other hand, we only have rough estimates of the diffusion coefficients needed to solve the diffusion equation. For this study, we have devised a hybrid approach in which we calculate these coefficients using the motion of individual particles traced with the Lorentz equation. Our method is a two-step approach in which we first follow the trajectories of a select number of individual protons, and then use the statistics from this short simulation to model their motion over a much longer period of time. In this way, we make no assumption about the protons’ diffusion and the time needed to run the simulation is significantly reduced.

5.1 Individual Proton Trajectories

We solve the Lorentz force equation for 1,000 protons with kinetic energy \( T_p \) moving in a uniform medium with conditions representative of a giant molecular cloud interior, and then of the region between the clouds. The protons diffuse away from their point of origin by random-walking through the turbulent magnetic field, generated at each point along their path using the method described in section 3. Unlike the application of Giacalone and Jokipii (1994), however, we use 200 values of the wavenumber \( k \) evenly spaced on a logarithmic scale between \( k_{\text{min}} = 2\pi/10r_{\text{gyr}} \) and \( k_{\text{max}} = 2\pi/0.1r_{\text{gyr}} \), in terms of the gyroradius \( r_{\text{gyr}} = \gamma p m_p v/eB \) of a proton with Lorentz factor \( \gamma p \).

For each set of pre-selected environmental conditions, the total time \( \tau \) we follow each proton is chosen small enough that it loses no more than 1% of its energy (\( \Delta E/E < 0.01 \)), yet large enough that it will gyrate many times. This time interval is further sub-divided into much smaller time steps \( dt = \chi 2\pi \gamma p m_p v/eB \), where \( \chi \) is a random number such that \( 0 \leq \chi \leq 1 \).

To calculate the proton’s position after each \( dt \), we first rotate to a primed set of coordinates in which the magnetic field points along the \( z' \)-direction (\( B' = B_0 \hat{k}' \)), solve the Lorentz force
equation, and then rotate back to the original, unprimed coordinate system. This process is repeated at each step of the random-walk until the total time limit has been reached.

5.2 Proton-Propagation Statistics

The data for the individual protons whose trajectories have been calculated in the manner described above are plotted in Figure 5, showing their occupation as a function of distance from their point of origin. The distribution is strongly dependent on the proton energy, so this process must be repeated for a wide range of proton kinetic energies $T_p$. The distribution can be modeled adequately with a Gaussian function

$$N(r) = N_0 e^{-(r-\bar{r})^2/2\sigma^2},$$

where the average distance $\bar{r}$ and standard deviation $\sigma$ are to be determined using a $\chi^2$ fitting of the data with the Levenberg-Marquardt method.

We repeat this procedure over the energy range $10^{11}$ eV $\leq T_p < T_{p,\text{trans}}$, where $T_{p,\text{trans}}$ is the energy at which the protons transition from diffusive random-walk behavior to (effectively) straight-line motion. From a practical standpoint, this transition may be taken to occur when the total distance traveled by a proton becomes less than $10r_{\text{gyr}}$. The time elapsed during the simulation (see Table 2) is fixed by the requirement that protons lose a given fraction of their initial kinetic energy. In our calculations, these values are $\Delta T_p/T_p = 0.01$ for environmental conditions matching
Figure 5. This figure shows the distribution of 1000 1-TeV protons as a function of position after $3.33 \times 10^8$ seconds. The dots are data from the short simulation, the line is the Gaussian fit to the data with $\bar{r} = 4.05 \times 10^{14}$ meters and $\sigma = 1.91 \times 10^{14}$ meters.

Table 2. Fitting Coefficients as a Function of Magnetic Field Strength, Ambient Proton Density, and Elapsed Time

| $B$ (µG) | $n_p$ (cm$^{-3}$) | $t$ (s) | $a_r$ | $b_r$ | $a_\sigma$ | $b_\sigma$ |
|---------|------------------|--------|-------|-------|------------|------------|
| 10      | 100              | $1.67 \times 10^9$ | 9.98033 | 0.499622 | 9.68435 | 0.497083 |
| 100     | 100              | $1.67 \times 10^9$ | 9.50314 | 0.497989 | 9.16946 | 0.498592 |
| 1000    | $10^4$           | $1.67 \times 10^9$ | 8.48292 | 0.499238 | 8.13816 | 0.500453 |

those found within the molecular clouds, and $\Delta T_p / T_p = 0.001$ in the inter-cloud medium. We have found the following formulations of the average net distance traveled and standard deviation in terms of the proton kinetic energy to be useful representations of the data:

$$\ln(\bar{r}/1\,\text{m}) = a_r + b_r \ln(T_p/1\,\text{eV})$$  \hspace{1cm} (11a)\\

$$\ln(\sigma/1\,\text{m}) = a_\sigma + b_\sigma \log(T_p/1\,\text{eV})$$, \hspace{1cm} (11b)

where $a_r$, $b_r$, $a_\sigma$, and $b_\sigma$ are constants determined from $\chi^2$ optimization. A sample of these constants is given in Table 2 for three different magnetic field intensities.

5.3 The Proton Distribution

The use of equations (10), (11a), and (11b) permits us to propagate the protons with any given energy $T_p$ quickly and accurately. If $T_p \geq T_{p,\text{trans}}$, the proton moves along a straight line to its next interaction point (usually a $pp$ scattering event). If $T_p < T_{p,\text{trans}}$, the motion is diffusive and Monte-Carlo methods are employed with the use of equations (10), (11a), and (11b).
6 THE PARTICLE CASCADE

As the protons diffuse, a fraction of them undergo a $pp$ scattering event after traveling a total path length $dx$, according to the rate

$$\frac{dN}{dx} = -Nn_H \sigma_{pp}. \quad (12)$$

These $pp$ scatterings create pions that decay, ultimately producing gamma rays, electrons, positrons, and a host of neutrinos. A simulated image of the gamma ray emission may be obtained by calculating and mapping the spectra of the secondary particles produced in this cascade.

Inelastic scatterings between high-energy protons $n_p(E_p)$ and ambient protons $n_H$ result in a $\pi^0$ emissivity

$$Q_{\pi^0}^{pp} = c n_H \int_{E_{th}(E_{\pi^0})} dE_p n_p(E_p) \frac{d\sigma(E_{\pi^0}, E_p)}{dE_{\pi^0}}, \quad (13)$$

where $E_{th}(E_{\pi^0})$ is the minimum proton energy needed to produce a pion with energy $E_{\pi^0}$. The neutral pion decay $\pi^0 \to 2\gamma$ leads to a gamma-ray emissivity

$$Q_{\gamma}(E_{\gamma}) = 2 \int_{E_{\pi^0}^{min}(E_{\gamma})}^{E_{\pi^0}(E_{\gamma})} dE_{\pi^0} \frac{Q_{\pi^0}^{pp}}{(E_{\pi^0}^2 - m_{\pi^0}^2 c^4)^{1/2}}, \quad (14)$$

where $E_{\pi^0}^{min}(E_{\gamma}) = E_{\gamma} + m_{\pi^0}^2 c^4/(4E_{\gamma})$.

For protons with energies less than 3 GeV, the differential $\pi^0$ cross section may be determined using the isobar model of Stecker (1970) (see also Dermer 1986). The expressions needed to calculate the low-energy cross section are quite detailed and lengthy so they are not repeated here, but may be found in, e.g., the Appendix of Fatuzzo and Melia (2003). Above 7 GeV, we use the scaling approximation of Blasi and Colafrancesco (1999),

$$\frac{d\sigma(E_p, E_{\pi^0})}{dE_{\pi^0}} = \frac{\sigma_0}{E_{\pi^0}} f_{\pi^0}(x), \quad (15)$$

where $x = E_{\pi^0}/E_p$, $\sigma_0 = 32$ mbarn, and

$$f_{\pi^0}(x) = 0.67(1 - x)^{3.5} + 0.5e^{-18x} \quad (16)$$

takes into account the energy-dependent pion multiplicities that occur at high energies. In the intermediate (3-7 GeV) range, a linear combination of the low- and high-energy forms is used.

7 RESULTS AND DISCUSSION

Our first attempt to match HESS’s map of the diffuse emission is based on the assumption that all of the cosmic-ray protons originate from Sagittarius A*, since only $\sim 1/3$ of the hadrons required to account for the central TeV source HESS J1745-290 actually interact with the circumnuclear disk surrounding the black hole. The remaining proton efflux could in principle fill the interstellar
medium out to $|l| \sim 1^\circ$, and account for the diffuse TeV glow associated with the molecular gas dispersed along the galactic ridge (see Figures 1 and 2).

The gamma-ray count map resulting from our simulation with Sagittarius A* as the sole source of high-energy protons, calculated from equations (13) and (14), is shown in Figure 6. The top panel is a plot of the full range of photon counts spanning eight orders of magnitude in intensity. However, the actual HESS diffuse TeV emission map (Aharonian et al. 2006) covers a much smaller dynamic range, dropping by only a factor of two from the highest intensity pixels down to the lowest. Thus, in order to make a direct comparison with the HESS data, we also show in the lower panel the corresponding calculated intensity map with the same factor-2 intensity range. In
Figure 7. Same as figure 6, except here for protons injected into the inter-cloud medium by 5 distinct sources. The diffuse emission is more pronounced than that in figure 6, though the emission tends to be concentrated on the point sources, and is only weakly correlated with the molecular gas distribution shown in figures 1 and 2.

either case, it is clear that the protons from Sagittarius A* acting as the lone source of cosmic rays cannot explain the observed diffuse gamma-ray emission. The very evident peak in the TeV emission associated with the black hole itself may still be a viable explanation for the central source HESS J1745-290, but subtracting this from the rest of the diffuse emission produces a TeV map centred on Sagittarius A*, extending out only a fraction of a degree, or at most only about 1° if we include the full 8 orders of magnitude in intensity.

Our conclusion is that as long as the magnetic field surrounding Sagittarius A* is at least partially turbulent (we here assumed that half of the magnetic energy is turbulent), then the hadronic diffusion away from the galactic centre is too slow to account for $pp$ scattering events with molecular gas more than a fraction of a degree (i.e., only tens of parsecs) out along the ridge.
We next considered a situation in which the cosmic-ray protons originate from several point sources distributed along the galactic plane. In addition to diffuse emission, HESS has discovered several points sources in this region, including HESS J1745-290, typically associated with supernova remnants or pulsar wind nebulae. The HESS map reveals five TeV “hot spots” in the region $|l| \leq 1.25^\circ$ and $|b| \leq 0.5^\circ$. In the second simulation, we therefore distributed the proton injection among 5 individual point sources, assigning them the observed latitudes and longitudes, and a random $z$ (line-of-sight) coordinate consistent with positions between the molecular clouds.

To further test the dependence of our proton diffusion (and the associated synthetic TeV map) on the magnetic field intensity, we simulated the hadronic propagation for two values of $B$, 10 $\mu$G (Figure 7) and 100 $\mu$G (Figure 8). As expected, increasing the strength of the magnetic field con-
fines the protons more strongly, and therefore restricts the TeV emission to more compact regions surrounding the sources. Here too, a dominant feature of the gamma-ray maps is the evident concentration of TeV emission near the injection sites, which might account very well for the point sources themselves. But the diffusion scale is still too small, for both values of magnetic field, to permit the protons to fill the medium between the clouds with a sufficiently dispersed cosmic-ray population to account for the HESS intensity map. Decreasing the magnetic field further, to a value $\sim 1 \mu G$ helps, but the synthetic map always contains intensity gradients revealing the location of the point sources, in contrast with the actual TeV map, which shows a strong correlation of the gamma-ray emissivity only with the measured concentration of molecular gas and none of the individual point sources.

It is quite evident from these two sets of simulations that if the TeV photons are to be produced in hadronic interactions involving the molecular gas, the relativistic protons themselves must be accelerated in situ, throughout the interstellar medium. The fact that their diffusion length is at most only a fraction of a degree, means that even a remote acceleration site, well away from the molecular clouds, cannot produce the required cosmic-ray distribution.

To examine whether protons accelerated throughout the inter-cloud medium can in fact produce the observed diffuse TeV emission, we therefore also simulated a situation in which the protons emerge uniformly via, e.g., second-order Fermi acceleration off the turbulent magnetic field. In this calculation, the starting point for each proton falls somewhere between the molecular clouds but is otherwise chosen randomly. Figure 9 shows the synthetic TeV intensity map resulting from this model, demonstrating a strong correlation with the location (in projection) of the molecular gas (see figures 1 and 2), in excellent agreement with the HESS observations.

8 CONCLUSION

In summary, then, we have found that the conditions at the galactic centre preclude a point-source origin for the cosmic rays responsible for producing the diffuse TeV emission correlated with the molecular gas distributed along the ridge. The supermassive black hole Sagittarius A* may be associated with the central object HESS J1745-290, but its hadronic efflux cannot extend out to $\sim 1^\circ$ since the protons lose their energy or scatter with the ambient medium on much smaller scales. Distributed point sources offer the possibility of extending the diffuse TeV emission to greater distances from the centre, but they too would produce a morphology with centrally peaked
Unlike the simulations displayed in figures 6, 7, and 8, the gamma-ray intensity map shown here is produced entirely by relativistic protons injected throughout the inter-cloud medium (e.g., by second-order Fermi acceleration). The magnetic field is assumed to have an average value of $10 \mu G$. The correlation between the gamma-ray emissivity and the molecular gas distribution (figures 1 and 2) is quite evident. However, the protons responsible for producing this diffuse TeV emission do not have the same distribution as the cosmic-ray population measured at Earth. These hadrons therefore apparently represent a distinct population at the galactic centre.

An important question is therefore whether the particle acceleration occurs within the interstellar medium, or whether the cosmic rays emerge from any known population of sources distributed throughout the galactic ridge. From our simulations, we infer that the gamma ray emissivity associated with any given object drops by a factor $\sim 2$ within a distance of roughly $0.1^{\circ}$. Since this is effectively the contour range of the HESS maps, individual sources would not be evident as long as their angular separation were less than this value. In a projected area $\sim 2^{\circ} \times 1^{\circ}$, this would require about 50 individual sources. But the total number of TeV sources detected by HESS (many emission regions not consistent with the HESS map. Only cosmic rays accelerated throughout the inter-cloud medium can produce a diffuse TeV glow consistent with the observations.
of them presumably pulsar wind nebulae) was far smaller than this. In addition, only \(~5\) low-mass X-ray binaries have been identified in this region (Bird et al. 2007), and no other class of object with a volume density greater than this is known to be a strong source of relativistic hadrons. It is therefore likely that the particles are not being injected into the interstellar medium by individual objects.

Since the relativistic protons pervading the inter-cloud medium at the galactic centre are accelerated throughout the plane, second-order Fermi acceleration may be dynamically important in this region. A reasonable extension of this work will therefore be the inclusion of stochastic particle acceleration by the same turbulent magnetic field responsible for the particle diffusion.

Future extensions of this model should include a self-consistent calculation of the longer wavelength emission produced by particles in the \(pp\)-induced cascade. At the very least, the synchrotron emission due to the secondary leptons spiraling around the assumed magnetic field must not exceed the diffuse radio glow along the ridge. In principle, the TeV and radio intensity maps, used together, might produce a unique measurement of the magnetic field under the assumption that a single hadronic process is responsible for both spectral components. The results of this calculation will be reported elsewhere.

ACKNOWLEDGMENTS

This research was partially supported by NSF grant 0402502 at the University of Arizona, and a Miegunyah Fellowship at the University of Melbourne.

REFERENCES

Aharonian, F. et al. 2004, AA, 425, L13
Aharonian, F. et al. 2006, Nature, 439, 695
Atoyan, A., and Dermer, C. D. 2004, ApJ Letters, 617, L123
Ballantyne, D., Melia, F., and Liu, S. 2007, ApJ Letters, 657, L13
Bally, J., Stark, A., Wilson, R., and Henkel, C. 1985, ApJ Supp., 65, 13
Begelman, M., Rudak, B., and Sikora, M. 1990, ApJ, 362, 38
Bhattacharjee, P. 2000, Physics Reports, 327, 109
Bird, A. J. et al. 2007, ApJ Supplements, 170, 175
Blasi, P., and Colafrancesco, S. 1999, Astropart. Phys., 12, 169
Blitz, L., Bloemen, J.B.G.M., Hermsen, W., Bania, T.M. 1985, AA, 143, 267
Chodorowski, M., Zdziarski, A., and Sikora, M. 1992, ApJ, 400, 181
Di
ffuse TeV Emission

Chuss, D. et al. 2003, ApJ, 599, 1116
Cox, P., and Laureijs, R. 1989, in IAU Symp. 136, The Center of the Galaxy, ed. M. Morris (Dordrecht: Kluwer), 121
Crocker, R. et al. 2005, ApJ, 622, 892
Dermer, C.D. 1986, AA, 157, 223
Fatuzzo, M., and Melia, F. 2003, ApJ, 596, 1035
Giacalone, J., and Jokipii, J. R. 1994, ApJ Letters, 430, L137
Güsten, R., and Henkel, C. 1983, AA, 125, 136
Güsten, R., and Philipp, S. 2004, arXiv:astro-ph/0402019
Karol, P.J. 1988, ApJ, 332, 615
Kim, S., Martin, C., Stark, A., and Lane, A. 2002, ApJ, 580, 896
Markoff, S., Melia, F., and Sarcevic, I. 1997, ApJ Letters, 489, L47
Markoff, S., Melia, F., and Sarcevic, I. 1999, ApJ, 522, 870
Maximon, L.C. 1968, J.Res. NBS, 72B, 79
Morris, M. 2007, arXiv:astro-ph/0701050
Morris, M., Polish, N., Zuckerman, B., and Kaifu, N. 1983, AJ, 88, 1228
Muno, M.P. et al. 2004, arXiv:astro-ph/0402087
Oka, T. et al. 1998, ApJ, 493, 730
Pierce-Price, D. et al. 2000, ApJ Letters, 545, L121
Stecker, F.W. 1970, Astrophys. Space Sci., 6, 377
Tsuboi, M., Toshihiro, H., and Ukita, N. 1999, ApJ Supp., 120, 1
von Ballmoos, P., Diehl, R., and Schönfelder, V. 1987, ApJ, 318, 654
Walmsley, C. et al. 1986, AA, 155, 129
Wang, Q. D., Lu, F. J., and Gotthelf, E. V. 2005, MNRAS, 367, 937
Werner, M.W. et al. 1988, ApJ, 333, 729
Yamauchi, S. et al. 1990, ApJ, 365, 532
Young, J.S., and Scoville, N.Z. 1991, ARAA, 29, 581
Yusef-Zadeh, F., and Morris, M. 1987, AJ, 94, 1178

© 0000 RAS, MNRAS 000, 000–000