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Thermally assisted domain walls quantum depinning at the high temperature range

A theoretical and numerical investigations of the quantum tunneling of the domain walls in ferromagnets and weak ferromagnets was performed taking into account the interaction between walls and thermal excitations of a crystal. The numerical method for calculations of the probability of a thermally stimulated quantum depinning as the function of temperature at the wide temperature range has been evolved.

Macroscopic quantum effects in magnetism are currently of great interest. Such phenomena are important in tests of quantum mechanics. In particular, magnetic domain walls tunneling seems to be one of most appropriate subject for investigations in this field. Together with detailed theoretical and experimental researches of domain walls in ferromagnets\[5,6,7]\, recently the similar phenomena were described in a weak ferromagnets\[6,7].

For the description of the domain walls dynamics it is convenient to apply the model, in which the wall is considered as a quasiparticle with some effective mass $m$. Such quasiparticle transferred via the crystal the change of a magnetic moments orientation. In the movement through a crystal the quasiparticle can be trapped by magnetic pinning center - as provided for example, by an impurity raising the anisotropy energy locally. The domain wall then can overcome this energy barrier in a following ways: either due to absorption of an external fields energy or due to thermal activation, and at last via tunneling.

Tunneling and thermal activation are usually considered as competing processes; thereat one think tunneling can be observable only at extremely low temperatures about 0.001 K - 1 K, whereas at higher temperatures tunneling is suppressed by thermal activation. However, it is not always true. The purpose of this paper is the theoretical and numerical investigations of the situations in which to some extent both discussed phenomena can cooperate. How that one comes to think of it, due to interaction with thermal excitations of a crystal and absorption of their energy, the wall "raised" in front of barrier. In this case effective height of barrier will decrease and, accordingly tunneling rate will increase. Further we spend detailed discussion of this mechanism for the Bloch walls and walls in weak ferromagnets.

1. Bloch walls thermally activated tunneling

A. Model and equations of motion

Let us consider in the beginning 180-degrees Bloch wall in an uniaxial crystal. The form of the rest wall has given by well-known Landau-Lifshitz\[2,3,4,5,7,8]\, exact solution

$$\sin \theta = \tanh\left(x/\sqrt{A/K_1}\right),$$

(1)

where $x$ is directed along easy axis, $A$ is constant of exchange and $K_1$ is constant of uniaxial anisotropy. For a travelling wall an additional "kinetic"energy $K$ arises. It may be represented in quadratic on velocity $v$ form $K = v^2m_d/2$, where $m_d = E_g/8\pi A\gamma^2 \sin^2 \theta$ is so-called Döring mass\[8] or, rather, surface density of the mass, and $E_g = \sqrt{AK_1}$ is surface energy density of a wall. In this terms equation of motion took on Newton’s form

$$m_d \frac{d^2 X}{dt^2} = 2I_s H,$$

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where \( X \) is coordinate of center of a wall. Let us consider interaction between wall and defect introducing the potential energy \( U(X) \); then last equation takes the form

\[
m_d \frac{d^2X}{dt^2} = 2I_aH - \frac{dU(X)}{dX}. \tag{2}
\]

In this paper we consider point-like repulsive impurity. In this case the potential energy of the wall will be change in manner \( U(\theta) = U_0 \cos^2 \theta \), where \( U_0 \) is maximal value of potential energy attainable when impurity lies in a center of the wall. If one take into consideration eq. (1) it is easy to obtain explicit function \( U(X) \):

\[
U(X) = U_0(1 - \tanh^2(X/\sqrt{A/K_1})) = U_0 \cosh^{-2}(X/a), \tag{3}
\]

where \( a = \sqrt{A/K_1} \) - characteristic width of both domain wall and potential \( U(X) \).

When eq. (2) was written, it was supposed that structure of moving wall does not vary and definition of \( m \) contains energy of the rest wall. However, Walker has shown that this assumption is justified only for slowly travelling walls. As it follows from exact solutions of the equation of motion for the Bloch wall, structure of a wall (in particular, its width and mass) strongly depends on velocity of movement. When walls velocity tends to some critical value \( c \), derivative of walls energy reduce to infinity, its mass tends to infinity and width tends to zero. Mostly, in usual ferromagnets the Walker limiting velocity \( c \) has value about several kilometers per a second, and in this case \( mc^2 \gg U_0 \). In this reason one may consider that condition \( v^2/c^2 \ll 1 \) in ferromagnets usually is satisfied, therefore in current section we shall be limited by a case of small velocities.

In a context of the discussed problem, the following physical situation will be interested us. Let the domain wall with kinetic energy \( K \) arrived at a potential barrier with the height \( U_0 \), which simulates its interaction with a defect of the crystal. If \( K < U_0 \) the segment of wall in immediate proximity to barrier will be trapped in a metastable minimum. It is acceptably to consider such segment as an isolated quasiparticle with the effective mass determined as an integral over the area of defect

As stated above, there are three ways for the wall to overcoming barrier: with the help of an external field, due to thermal activation and via tunneling. Let us consider very weak field \( H \ll \frac{1}{k_B} |\frac{dU}{dX}| \). Such field can’t to disengage the wall, but it will create asymmetry for a displacement of a wall in front of barrier and behind of barrier. For the invariability of the walls structure, the condition \( U_0 \ll mc^2 \) must be satisfied. On the other hand, height of barrier should not be too small inasmuch as thermal activation will become the prevale and tunneling will be suppressed. Together, both these restrictions give a rather narrow corridor for reasonable parameters. Nevertheless, values which we shall accept are usual for ferromagnets, therefore it is possible to use them for calculations. Let us suppose height of barrier \( U_0 = 10^{-14} \) erg, quasiparticles mass \( m = 10^{-26} \) gm , width of potential \( a = 10^{-6} \) cm and area of defect \( S = 10^{-13} \text{cm}^2 \).

B. Calculations and results

By virtue of the preset assumption on weakness of an external field, it is possible to believe that metastable minimums width is lot greater than barrier’s width, whereas its depth is considerably smaller than the barrier height. In such case the quasiparticles in front of barrier has a quasi-continuous spectrum and using of Maxwellian distribution for the analysis of a problem is acceptable.

Let us suppose a quasiparticle in a thermal equilibrium with a crystal. We consider an ensemble containing \( N \) such particles. We shall perform computations in according to the next computational scheme. Interval of energy from \( 0 \) up to \( U_{\text{max}} \) was divided into equal subintervals \( \delta w \). The number of particles for each subinterval one can found from the expression

\[
N_w = \int_{w-\delta w/2}^{w+\delta w/2} \frac{2N}{\sqrt{\pi}} (k_B T)^{-1.5} \sqrt{w} \exp(-\frac{w}{k_B T})dw. \tag{4}
\]

\(^1U_{\text{max}} \) was adjusted so as there will be neglected number of particles outside interval for each given value of a temperature.
In the next step we assign to all particles within the limits of given subinterval an identical average value $w$. The validity of this device was checked numerically, i.e. number of dissections $N$ adjusted so as to guarantee the stability of computing scheme as a whole.

Further, the barrier penetrability $D$ was calculated for each subinterval by well-known formulae:

$$D = \frac{\sinh^2(\pi k a)}{\sinh^2(\pi k a) + \text{cosh}^2\left(\frac{\pi k a}{\sqrt{2}}\right)} \sqrt{\frac{2\pi \hbar}{m a^2}}$$

when $\frac{32\pi^2 m U_0 a^2}{\hbar^2} < 1$ and

$$D = \frac{\sinh^2(\pi k a) - \text{cosh}^2\left(\frac{\pi k a}{\sqrt{2}}\right)}{\sinh^2(\pi k a) + \text{cosh}^2\left(\frac{\pi k a}{\sqrt{2}}\right)} \sqrt{\frac{2\pi \hbar}{m a^2}}$$

when $\frac{32\pi^2 m U_0 a^2}{\hbar^2} > 1$ (5)

where $k = \frac{\sqrt{2\pi m w}}{k_B T}$. We emphasize that eq. (5) is the exact solution of Shr"odinger equation for the potential (3).

Product $D \times N_w$ gives $N_{w0}$ or a number of particles from given subinterval which overcomes the barrier. The total sum of all $N_{w0}$ gives $N_0$ - number of all particles in ensemble transmitted the barrier. Then $F = \frac{N_0}{N}$ will be effective barrier penetrability. Let us note that magnitude of $F$ is determined not only by tunneling but by over-barrier reflection too. Really, even in case when energy of particle large then $U_0$, $D$ may be less then 1.

The probability of thermal activation $G$ one can easily found if to calculate fraction of particles with energy above $U_0$

$$G = \int_{U_0}^{\infty} \sqrt{\pi} (k_B T)^{-1.5} \sqrt{w} \exp\left(-\frac{w}{k_B T}\right) dw.$$ (6)

In numerical calculations on upper bound we of course use substitution $\infty \rightarrow U_{\text{max}}$ with precautions described above.

The dependencies both of effective barrier penetrability $F$ and probability of thermal activation $G$ on the temperature are plotted in fig. 1 and 2. Fig. 1 represented influence of discussed mechanism in wide temperature range from 0 to 300 K. In fig. 2 for most clearness plotted same results in narrow region - from 0 to 20 K, where exist greatest difference between $F$ and $G$.

We hope that these results demonstrated severity of the tunneling exposure to the depinning processes.

2. Thermostimulated domain walls tunneling in a weak ferromagnets. Accounting of the quasi-relativism.

As stated above the thermal stimulation of the tunneling for Bloch walls can be realized only under rigid restrictions, especially for parameter $U_0$. For high barriers the bordering $U_0 < mc^2$ may be broken and structure of the wall will be varied; the quasi-relativistic phenomena arise from this reason. The accounting of such peculiarities will be demonstrated by example of walls in a weak ferromagnet. Theory, which successfully described high-energy dynamics in such materials, was evolved in Ref. [3]. The weak ferromagnets has very suitable properties for comparison theory with experimental data. Walls in weak ferromagnets, as a rule, has a mass by one or two order of magnitude smaller then in ferromagnets, and its width essentially smaller too. Both this factors leads to increasing of the tunneling rate. Let us also denote, that very pure samples with low defects concentration are available now, this factor gives good reproducibility of measurements results.

A. Model

Let us consider, for example, a weak ferromagnet of a therbium orthoferrits type within the two-lattice approximation using the ferro- and antiferromagnetics vectors $\mathbf{m}$ and $\mathbf{l}$, respectively. Its thermodynamical potential will be [3]

$$\Phi_0(l, m) = J m^2 + A(|\mathbf{m}|)^2 - m \mathbf{H} + d_1 m_x l_z - d_3 m_z l_x + K_m l_z^2 + K_{ab} l_x^2,$$

where $J$ and $A$ are, respectively, constants of uniform and non-uniform exchange, $K_m$ and $K_{ab}$ - constants of anisotropy, $\mathbf{H}$ - total external field acting on the wall, $d_1$ and $d_3$ are Dzyaloshinsky exchanges constants. After minimization on $m$ one can obtain [4]

$$\Phi_0 = A(|\mathbf{m}|)^2 - \frac{\lambda_j}{2}(H^2 - (\mathbf{H L})^2) - M_0 H_z l_x - M_0 H_x l_z + K_m l_z^2 - K_{ab} l_x^2,$$

$$\begin{cases}
\Phi_0 l_z = J m_z^2 + A m^2 + \frac{\lambda_j}{2} (H^2 - (\mathbf{H L})^2) - \frac{M_0^2}{2} + K_m l_z^2 - K_{ab} l_x^2,
\Phi_0 l_x = J m_x^2 + A m^2 + \frac{\lambda_j}{2} (H^2 - (\mathbf{H L})^2) - \frac{M_0^2}{2} + K_m l_x^2 - K_{ab} l_z^2,
\Phi_0 l_y = J m_y^2 + A m^2 + \frac{\lambda_j}{2} (H^2 - (\mathbf{H L})^2) - \frac{M_0^2}{2} + K_m l_y^2 - K_{ab} l_z^2,
\Phi_0 l_z = J m_z^2 + A m^2 + \frac{\lambda_j}{2} (H^2 - (\mathbf{H L})^2) - \frac{M_0^2}{2} + K_m l_z^2 - K_{ab} l_x^2,
\end{cases}$$

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\Phi_0 l_y = J m_y^2 + A m^2 + \frac{\lambda_j}{2} (H^2 - (\mathbf{H L})^2) - \frac{M_0^2}{2} + K_m l_y^2 - K_{ab} l_z^2,
\Phi_0 l_z = J m_z^2 + A m^2 + \frac{\lambda_j}{2} (H^2 - (\mathbf{H L})^2) - \frac{M_0^2}{2} + K_m l_z^2 - K_{ab} l_x^2,
\end{cases}$$

3
where $\chi_\perp$ is transverse susceptibility, $M_0^0$ and $M^0_\parallel$ are the values of magnetization in phases $\Gamma_4(1||x)$ and $\Gamma_2(1||z)$ respectively. Without loss of generality, we can consider $ac$-type walls only. In spherical coordinates corresponding Lagrange density will be
\[
\mathcal{L} = \frac{\gamma z}{2c^2} (\frac{\partial z}{\partial t})^2 - \frac{\gamma}{2} \mathbf{H}[1, \frac{\partial}{\partial r}] - \Phi_0
\]

The appropriate Lagrange function per unit area of the domain wall has essentially quasi-relativistic form
\[
L = -m^*c^2\sqrt{1 - v^2/c^2} - U(x)
\]  

(7)

where $m^*c^2 = 4\sqrt{AK}$ and $c^* = \gamma_\perp A/\chi_\perp$ is spin-wave velocity; $U(x)$ - potential of wall-defect interaction. Associated with eq. (7) Hamiltonian then will be
\[
\mathcal{H}_p(p, x) = c^*\sqrt{p^2 + m^*c^2c^2} + U(x),
\]  

(8)

where $p = \frac{mv}{\sqrt{1 - v^2/c^2}}$ - canonical impulse. The kinetic energy then will be $K = pc^*$.

The quasi-relativistic type of Hamiltonian (8) carried to essential peculiarities in wall dynamics. It is important for us that energy distribution for walls (quasiparticles) will appreciably changed, and effective barrier penetrability $F$ and probability of thermal activated depinning $G$ may changed too. Therefore it is necessary to define energy distribution for a quasi-relativistic particles. For this purpose we shall use the Gibbs canonical distribution
\[
dz_p = a^0 \exp(-K(p)/k_B T)dp,
\]

where $a^0$ one can found by using normality condition $\int_{-\infty}^{\infty} dz_p = 1$. As since $K(p) = cp$ then
\[
 z(w) = \frac{1}{2(k_BT)^3}w^2 \exp\left(-\frac{w}{k_BT}\right).
\]

(9)

Obviously, maximum of this distribution in comparison with the same for classical distribution will shifted into high-energy region. Therefore fraction of "vigorous" particles will increase and correspondingly will changed $F$ and $G$.

B. Results of calculations

From results of numerical calculations submitted below it will be visible that the walls in weak ferromagnets are really more suitable subjects for the study of the tunneling in the high temperature range. Let us designate the main parameters. Effective mass of a wall let will be $m = 10^{-13}gm/cm^2$. We shall take both width of an energy barrier of the defect and width of a wall $\Delta$ of the same order $10^{-4}cm$. Both area of the defect and area of the tunneling segment of the wall will be $S = 10^{-13}cm^2$, hence quasiparticle mass will be $m^* = 10^{-26}gm$. The height of barrier one can find from the value of coercive force $D^0 = 10^{-13}erg$. On the face of it such values seems little suitable for the tunneling. After formal substitution this parameters in eq. (5) one can found negligible range for $D$ - about $10^{-80}$ or smaller. But account of the quasi-relativism essentially changed the physical situation.

Let us consider this situation more detailed. Now, as before, we shall carry out modeling of the barrier by function (3). The sorting particles of an ensemble by the energy we shall execute in according with algorithm presented in Sec. 1. But taking into account eq. (9) we find now for $N_w$
\[
N_w = \int_{w-\delta w/2}^{w+\delta w/2} \frac{N_w^2}{2(k_BT)^3}w^2 \exp(-w/k_BT)dw.
\]

(10)

Accordingly, the probability of thermal activated depinning will be
\[
G = \int_{U_0}^{\infty} \frac{w^2}{2(k_BT)^3} \exp(-w/k_BT)dw
\]

The calculations of $D$ for a wall in a weak ferromagnet were done numerically with using of the technique offered in Ref[4]. The final computational results for $F$ and $G$ are plotted in figure 3. It is visible that with accepted
parameters the difference between probability of quantum thermal-stimulated depinning $F$ and thermal activated depinning $G$ more distinct than in usual ferromagnets. In particular, at 50 K $F$ two times as large $G$, and even at room temperature difference between $F$ and $G$ amounts to 0.05. Thus, account of quantum effects in mechanism of the depinning is actually topical.

In Sec. 1 we have considered Bloch walls tunneling in usual ferromagnet. The results obtained there are valid for low-energy walls only. At high energy walls dynamics also becomes quasi-relativistic and for its description Walker technique is necessary. The structure of Walker solution corresponds formally to Hamiltonian (9), therefore one should think reasonable to propagate results of Sec. 2 on the Bloch walls. However, nature of maximal velocity in this case is other. In ferromagnet, when walls velocity tends to maximal (not limiting) value, the form of the wall can essentially changed. In this case it is necessary to take into account additional energy, connected with a leakage fields, because of propagation of the results of the given section into Bloch walls required an additional analysis.

3. Admissibility of the experimental testing

Domain walls depinning via tunneling was investigated usually at very low temperatures (see, for example, Review4). Unfortunately, we have no data on a tunnel depinning at high temperatures. The data concerning to the basic parameters of a problem, such as form, width and height of the energy barrier was obtained indirectly and at present time is not sufficiently reliable. This fact made difficult the comparison theory with experimental data. Therefore the special importance has experiments, where the depinning of solitary walls is investigated. Problems, connected with a statistical nature of a depinning, will be in this case eliminated. In this respect it is very interesting experimental technique used in ref.5, where a solitary wall tunneling in a superthin wire was investigated. In our opinion, expansion this technique on high temperature region may be very useful for testing physical mechanism described in present paper. Testing of the presented results may be carried out also with using the magnetic noise technique. At any rate, this is in urgent need of search for the departures from classical temperature dependence for any physical quantities associated with walls depinning.

Author is grateful to A K Zvezdin and V V Dobrovitski for helpful discussion.

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