A Multi-Objective Bus Rapid Transit Energy Saving Dispatching Optimization Considering Multiple Types of Vehicles

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ABSTRACT Reducing energy consumption and promoting sustainable mobility solutions, including public transport (PT), are increasingly becoming key objectives for policymakers worldwide. Energy saving dispatching optimization for bus rapid transit (BRT) is one of the most efficient strategies for reducing traffic congestion and energy conservation. The purpose of this paper is to address the BRT dispatching problem while taking into account the association between the vehicle type, the waiting time of passengers and the energy consumption of vehicles. This paper presents a mechanical model to describe the level of energy used in different vehicles based on engine universal characteristics considering the characteristics of the vehicle, engine, road, and driving type. The load factor and the passenger average waiting time are used to estimate the quality of service. Furthermore, in order to determine the vehicle scheduling scheme, a multi-objective energy saving dispatching optimization model of BRT is developed aiming to minimize the waiting time of passengers and energy consumption of vehicles. Moreover, a two-phase algorithm is employed in order to solve this multi-objective model. The results show that the designed algorithm is valid for solving the dispatching optimization model of BRT, and the energy consumption and passenger waiting time can be reduced by using an appropriate dispatching scheme.

INDEX TERMS BRT dispatching, energy consumption, multiple types of vehicles, multi-objective, niched genetic algorithm.

I. INTRODUCTION

Bus rapid transit (BRT) systems have grown in popularity in recent years, spurred by government initiatives, the increasing cost of rail transit and market realities. However, the absence of a control system in a BRT system tends to result in vehicle bunching due to the stochastic nature of traffic flows and passenger demand at the stations. It also leads to an evident increase in bus headway variance and a consequent worsening of both the magnitude and variability of the average waiting time. This in turn impacts heavily the level of service [1]. In addition, surface transportation has a significant impact on the environment. The transportation sector uses a large amount of energy and is accountable for a significant amount of carbon dioxide (CO₂) emissions.

The transport sector in the United States accounts for 70% of U.S. petroleum consumption [2]. For an average European passenger car in 2015, reported to emit slightly more than 120 g CO₂/km, this gap translates into an extra 36-48 g CO₂/km or an increase of fuel consumption of approximately 1.5 to 2 l/100 km (petrol equivalents) [3]. China’s urban passenger transport associated motorized travel, energy consumption and life-cycle GHG emissions reached 2,815 billion passenger kilometers (pkm), 77 million tons of oil equivalent (toe) and 335 million tons of CO₂ equivalent in 2010. On the national level, GHG emissions by transit buses accounted for 10.5% of the total [4]. Although public transport is responsible for only a fraction of total emissions, improvements in efficiency and a reduction in energy usage are desirable. Therefore, reducing energy consumption and promoting sustainable mobility solutions, including public transport, are increasingly becoming key objectives for policymakers worldwide. Energy use in public transport is mainly...
affected by the numbers of scheduled trips in timetables and the quality of the vehicle schedule. However, one cannot simply reduce the number of trips to minimize the emissions. This would affect the level of service and attractiveness of public transport and might in turn lead to people switching to less efficient transport modes [5]. Therefore, public transport operators will face considerable challenges, having to adapt their current dispatching schemes to save energy. A good scheduling scheme should comprehensively consider the bus operation cost and quality of service from the perspective of operators and passengers. Thus, it is important to optimize the BRT dispatching strategy to improve the service level and reduce energy consumption.

Much of the focus of the literature on the bus dispatching problem is, therefore, focused on the issues of departure frequency and multiple types of vehicles.

A. OPTIMIZATION OF BUS DEPARTURE FREQUENCY

Furth and Wilson [6] proposed a constrained resource-allocation model to maximize net social benefit that consists of the ridership benefit and wait-time savings by setting frequencies. Yao et al. [7] developed an optimal model for bus frequency of a bus line, which aimed to minimize the total cost of passengers and bus operators and gave attention to the benefits of both the bus operators and the passengers. Ruiz et al. [8] proposed a bus frequency optimization methodology to improve harmonization between service level and social equity in public transport. Ma [9] applied a bi-level approach for the optimal line frequencies in a transit network, meaning the frequencies that minimize passengers travel time plus the operating cost. The lower level problem (route choice) was solved by using a Cross Entropy Learning algorithm, which was able to find the user equilibrium in transport networks. The upper level problem (optimizing line frequencies) used the Hooke-Jeeves algorithm to find improvements in the current solution. Considering the same problem of finding the optimal frequency for a bus network, Yu et al. [10] applied a bi-level programming model with the objective to reduce passengers’ total travel time. The upper level determined the bus frequencies by a genetic algorithm while the lower level assigned transit trips to the bus route network by use of a label-marking method. The two levels were solved sequentially until convergence. Furthermore, Huang and Ren [11], Parbo et al. [12] and Yu et al. [13] also used a bi-level optimization model to solve the bus scheduling problem by optimizing bus frequencies. Huang et al. [14] presented a novel bus dispatching model to minimize the waiting time of passengers based on dynamic arrival times and passenger flow predictions.

B. MULTI-VEHICLE DISPATCH

Ceder [15] addressed the vehicle scheduling problem, while taking into account the association between the characteristics of each trip (urban, peripheral, inter-city, etc.) and the vehicle type required for the particular trip. Hassold and Ceder [16] demonstrated how to make public-transit services more attractive by using two simultaneous objectives: minimizing the average estimated passenger waiting time and minimizing the discrepancy from a desired occupancy level on the vehicles. The first objective will improve the service and attract more users, and the second objective will assure economical operation. A network-based procedure was used to create timetables with multiple vehicle types to solve this bi-objective problem. Furthermore, a new methodology based on a minimum-cost network flow model utilizing sets of Pareto-optimal timetables for the multiple vehicle types vehicle scheduling problem (MVT-VSP) has been proposed by them [17]. Yang et al. [18] established a multi-objective departure frequency model of BRT considering multiple types of buses, capacity of bus and BRT and platform to optimize the average loading rate, operating costs and queuing time of buses. Furthermore, the problem was divided into two sub-problems and an analytical method was put forward to find this multi-objective model’s Pareto solution set. Peña et al. [19] studied the problem of vehicle scheduling in urban public transport systems taking into account the vehicle-type and size as a multi-objective combinatorial optimization problem and proposed a heuristic algorithm based on Multi-Objective Cellular evolutionary algorithm. Luo et al. [20] proposed an optimization model for dynamic bus dispatching to minimize the overall waiting time of passengers in a transit system considering multiple types of real-time information such as dynamic passenger flows and road traffic status. In this model, multiple bus routes and waiting times at the transfer stations were taken into consideration. Accordingly, a genetic algorithm with memory-based initialization was developed to solve the model.

The optimization of bus departure frequency and dispatch problems for multiple types of vehicles have been widely studied in the context of public transit systems. A commonly used technique in the literature for vehicle scheduling problems is multi-objective optimization, in which the passenger waiting time, the operating cost, the average loading rate, the multiple vehicle types, etc. were always taken into account. However, almost all of these studies did not take the energy consumption into account.

C. ENERGY CONSUMPTION ANALYSIS OF BUSES

For buses, unlike the private car, the passenger load should not be ignored for bus energy consumption estimation because the load changes during the trip. Substantial research has been conducted to demonstrate the potential of private cars to improve energy efficiency and to reduce emissions. There are only a few relevant research papers that present an energy consumption analysis of buses, not to mention energy-saving dispatch.

Frey et al. [21] found that the passenger load had a significant effect on fuel consumption, particularly in the middle and high-speed ranges. The increased passenger load could increase the modal average emission and fuel consumption rates. In another study, eight buses were tested with 1.0 and 2.5 t load mass respectively for comparison.
The average fuel consumption was increased by 4.6 ± 3.6% with 2.5 t load mass compared to 1.0 t load mass [22]. Wang, et al. [23] compared on-road emission and fuel consumption levels for Euro III and IV buses fueled on diesel and compressed natural gas, and analyzed emission and fuel consumption characteristics of buses in Beijing using a portable emissions measurement system. Ma et al. [24] proposed a method for summarizing driving style characteristic parameters on the basis of vehicle-engine combined model to study the influence of driving style on fuel consumption. Hassold and Ceder [5] examined the benefits that can be derived by using multiple vehicle types for even-headway timetables. Results showed that timetables with multiple vehicle types could increase passenger occupancy of the vehicles and reduce total energy consumption. Yu et al. [25] quantified the influence of passenger load on diesel bus emissions and fuel consumption based on the real-world on-road emission data measured by the Portable Emission Measurement System (PEMS) on urban diesel buses. The results show that the influence of passenger load on emission and fuel consumption rates were related to vehicle’s speed and acceleration. Wang and Rakha [26], [27] enhanced bus fuel consumption modeling by circumventing the bang-bang control problem using the Virginia Tech Comprehensive Power-based Fuel consumption Model (VT-CPFM) framework, and the model is calibrated for a series of diesel-powered buses using in-field second-by-second data. Xia et al. [28], [29] constructed a double-objective mathematical model and the corresponding adaptive tabu search (TS) algorithm for solving Low-Carbon Logistics Vehicle Routing Problem.

Few works have so far addressed the BRT dispatching problem with explicit consideration of the energy consumption. In view of the above, this study proposes a multi-objective energy saving dispatching optimization model of BRT to minimize both the waiting time of passengers and energy consumption of vehicles.

The remainder of the paper is organized as follows: First, some mathematical symbols are defined in Section 2.1, a mechanical model to describe the level of energy used in different vehicles based on engine universal characteristics is provided in Section 2.2, and the operation of BRT and the related mathematical model are introduced in Sections 2.3 and 2.4, respectively. Second, a genetic algorithm (GA) based on a niche selection operator as the first phase of the two-phase algorithm is proposed in Section 3.1, and the second phase, a proposed methodology for integrating objective weighting factors, is implemented in Section 3.2. Additionally, the proposed model and algorithm are tested via a numerical experiment in Section 4. Finally, the conclusions are presented in Section 5.

II. THE ENERGY-SAVING DISPATCHING MODEL

A. BASIC NOTATIONS

Let \( S = \{ s | s = 1, 2, \cdots, N \} \) be the BRT station set, where \( N \) is the total station number of a BRT line. Let \( B = \{ b | b = 1, 2, \cdots, M \} \) be the vehicle set, where \( M \) is the total number of vehicle departures. \( V_{C_b} \) denotes the capacity of vehicle \( b \in B, \ U^b_s, D^b_s \) denote average passenger arrival rate and passenger alighting rate of vehicle \( b \) at station \( s \), which are fixed constants computed by historical data for the stop, respectively. Therefore, the numbers of passengers boarding and passengers alighting can be denoted by \( U^b_s, D^b_s \) respectively. Let \( p^b_s \) be the number of passengers remaining on vehicle \( b \) at station \( s \), and \( q^b_s \) be the number of passengers on vehicle \( b \) when it arrives at station \( s \). \( t^1_s, t^2_s \) denote the average running time and the distance between stations \( s - 1 \) and \( s \), and \( t_{bs} \) denotes the dwell time of vehicle \( b \) at station \( s \). \( C^{b-1}_s \) denotes the energy consumption of vehicle \( b \) from station \( s - 1 \) to \( s \). \( f^b_s \) denotes the arrival time of vehicle \( b \) at station \( s \). Let \( h_1, h_2 \) be the minimum and maximum vehicle headway, respectively, and \( W_1, W_2 \) be the dispatch time window.

B. EQUATION FOR ESTIMATING ENERGY CONSUMPTION

For transit diesel buses, unlike the private car, the passenger load should not be ignored for bus energy consumption estimation because the load changes during the trip [25]. As stated, it is not feasible to characterize the real-world usage of all vehicles. Thus, we used a mechanical model in this paper to describe the level of energy used in different vehicles based on engine universal characteristics. This model has been popularly used in the estimation of vehicular engine consumption [30], [31]. The rates of energy consumption in this model depend on the energy demand in the road transport sector. These rates also depend on various factors including occupancy, speed and length of the trip. This mechanical model is based on universal characteristics of the vehicle, engine, road, and operating mode [31] and can obtain the microscale energy consumption of vehicles under different road conditions and passenger loads. The framework for estimating energy consumption of this mechanical model is depicted in Fig. 1.

Fig.2 shows the forces acting on a BRT vehicle during movement according to the mechanical model.

The main forces acting in the longitudinal direction on the vehicle are the traction force as well as the driving resistance forces. A vehicle’s total driving resistance \( F_t \) (given in (1)) is equal to the sum of the air resistance \( F_w \), the total rolling resistance \( F_r \), of all wheels, the climbing resistance \( F_i \) and the acceleration resistance \( F_j \) [31], [32].

\[
F_t = F_r + F_w + F_i + F_j
\]

(1)

The rolling resistance force \( F_r \) depends on the force that presses the vehicle onto the surface and is a function of the road slope angle \( \alpha \). The parameters are the total vehicle mass \( m \) (kg), the gravitational constant \( g \) and the rolling resistance coefficient of the tires \( f \) [32].

\[
F_r = mg \cos \alpha
\]

(2)

On a sloped roadway, the sine component of the total vehicle weight force acts according to \( mg \) in the vehicle’s
The vehicle related to the shape of the projected front surface area of
be made with an error of less than 5% [33]:

For roadways with slope less than 30%, which equals a
slope angle less than $\alpha = 17^\circ$, the following substitution can
be made with an error of less than 5% [33]:

$$\cos \alpha \approx 1, \quad i = \tan \alpha \approx \sin \alpha.$$ (4)

Thus, the rolling resistance force $F_r$ and the climbing resistance $F_i$ can be simplified as follows:

$$F_r + F_i = mgf + mgi.$$ (5)

The air resistance $F_w$ results from the friction and the displacement of the surrounding air. It is a function of the square of the driving speed $v$ (km/h). The coefficients are related to the shape of the projected front surface area of the vehicle $A$ (m$^2$), air drag coefficient $C_D$ and the air density $[32], [33]$.

$$F_w = \frac{C_DA v^2}{21.15}$$ (6)

When the vehicle moves upon acceleration, mass inertia forces arise in the opposite direction of the acceleration. The inertia force in the overall road resistance is called acceleration resistance. The acceleration resistance $F_j$ can be written as:

$$F_j = \delta m \frac{dv}{dt},$$ (7)

where $\delta$ is the correction coefficient of rotating mass ($\delta > 1$), which is mainly related to the moment of inertia of flywheel and wheel, and the transmission ratio $\frac{dv}{dr}$ is the acceleration of the vehicle, (m/s$^2$).

The power requirement for a given vehicle can be calculated from the road load equation, which includes rolling resistance, air resistance, climbing resistance and inertial power required to accelerate the vehicle. Equation 8 shows the road load equation, where $P_e$ (kW) represents the propulsion power demanded by the vehicle at the drive wheels, and $\eta_i$ represents the mechanical efficiency.

$$P_e = \frac{v}{3600 \eta_i} \left( mgf + mgi + \frac{C_D A v^2}{21.15} + \delta m \frac{dv}{dt} \right)$$
$$= \frac{1}{\eta_i} \left( mgfv + mgv + \frac{C_D A v^4}{76140} + \delta mv \frac{dv}{dt} \right)$$ (8)

The consumption rate (g/sec) can thus be calculated by

$$Q_s = \frac{P_e g_e}{3600}.$$ (9)

where $g_e$ denotes the specific fuel consumption (g/(kW.h)).

The specific fuel consumption $g_e$ can be determined according to rotating speed $n_e (r \cdot \text{min}^{-1})$ and torque $T_e (N \cdot m)$ from the universal characteristics of the engine. The rotating speed $n_e$ can be calculated by the relationship with auto speed and engine rotating speed [34].

$$n_e = \frac{v_l i_0}{0.377 r},$$ (10)

where $i_g$ and $i_0$ are the speed ratio of gearbox and driving axle, respectively, and $r$ denotes the speed ratio of gearbox and driving axle.

The energy consumption of vehicle $b$ from station $s-1$ to $s$, $C_{b}^{s-1,s}$, can thus be calculated.

C. OPERATION ANALYSIS OF BRT

1) THE DWELL TIME

Dwell time of a vehicle at the station comprises the time of passenger boarding and alighting and the time required to open and close the doors. Equation 12 shows the dwell time calculation method.

$$t_{b_{s}} = t_{1}(D_{b} + U_{b})/n + t_{2},$$ (12)

where $t_{b_{s}}$ denotes the dwell time of vehicle $b$ at station $s$ (s), $t_{1}$ is the boarding or alighting passenger service time (s/p), $n$ is the number of doors, and $t_{2}$ indicates door opening and closing time (s). BRT requires no on-board fare payment, and the vehicle has multiple available doors. The boarding or alighting passenger service time would be 0.72 (s/p) [35].
Usually, the time required to open and close the doors is approximately 10 seconds. However, the time for long-sized vehicles to enter and leave the station is longer. Thus, the door opening and closing time can be estimated from (13):

\[ t_2 = 10 + l_b/6, \quad (13) \]

where \( l_b \) is the length of vehicle \( b \).

2) ARRIVAL TIME

The arrival time of vehicle \( b \) at station \( s \) is equal to the sum of the arrival time at the last station, the dwell time \( b \) at the last station, and the average running time between stations \( s - 1 \) and \( s \).

\[ f_b^s = f_b^{s-1} + t_{hs} - 1 + t_s^{s-1}. \quad (14) \]

3) NUMBERS OF PASSENGERS BOARDING AND ALIGHTING

The number of passengers alighting from the bus \( b \) at the stop \( s \) can be computed as follows:

\[ D_b^s = q_b^s d_b^s. \quad (15) \]

The boarding passengers include two groups: the passengers left by the bus \( b \) and the passengers arriving during the arrival interval time of two buses. Thus, the number of passengers who expect to board the bus can be formulated as follows:

\[ (f_b^s - f_b^{s-1}) u_b^s + p_b^{s-1}. \quad (16) \]

Thus, the number of boarding passengers \( U_b^s \) can be computed as follows:

\[ U_b^s = \min\{Q_b - (q_b^s - D_b^s), (f_b^s - f_b^{s-1}) u_b^s + p_b^{s-1}\}. \quad (17) \]

Then, the number of passengers left by the bus \( b \) can also be yielded as follows:

\[ p_b^s = (f_b^s - f_b^{s-1}) u_b^s + p_b^{s-1} - U_b^s. \quad (18) \]

In addition, the number of passengers on vehicle upon arriving at station \( s \) can be obtained:

\[ q_b^s = q_b^{s-1} - D_b^{s-1} + U_b^s. \quad (19) \]

D. THE MATHEMATICAL MODEL

For a good scheduling scheme, a critical trade-off should be made between the level of service and the operating costs. Long waiting times and crowded conditions represent a poor quality of service from the passenger perspective. Accordingly, the total energy consumption of vehicles can represent the operation cost.

1) TOTAL WAITING TIME OF PASSENGERS

As shown in Larsen and Sunde [36] the expected waiting time for randomly arriving passengers can be calculated as follows:

\[ \sum_{s=1}^{N} \sum_{b=1}^{M} \frac{(f_b^s - f_b^{s-1})^2 u_b^s}{2} + \sum_{b=2}^{M-1} (f_b^{s-1} - f_b^s) p_b^s. \quad (20) \]

2) TOTAL ENERGY CONSUMPTION OF VEHICLES

The total energy consumption of vehicles during the dispatch time window can be calculated by

\[ \sum_{s=2}^{N} \sum_{b=1}^{M} C_b^{1-s}. \quad (21) \]

3) PASSENGER LOADS

Transit is less attractive when transit vehicles are highly crowded. Crowded vehicles also slow down transit operation, as it takes more time for passengers to board and disembark. From the passenger’s perspective, passenger loads reflect the comfort level of the on-board vehicle portion of a transit trip. Passenger loads for buses usually use the measure of—area per passenger.

Accordingly, the load factors (area per passenger) given in Table 1 can be used to estimate quality of service (QOS) [37]. At level of service (LOS) E, a transit vehicle will be as full as passengers will normally tolerate.

| Table 1. Passenger load QOS. |
|-----------------------------|
| QOS | A | B | C | D | E | F |
|-----|---|---|---|---|---|---|
| m²/p | >1.0 | 0.5 - 1.0 | 0.4 - 0.49 | 0.3 - 0.39 | 0.2 - 0.29 | <0.2 |

Furthermore, Liu and [38] and Yang et al. [39] calculated the effective standing area acceptable to each standing passenger in a transit vehicle of 0.25 m². Therefore, the vehicle load factor refers to actual capacity usage as a percentage of the maximum passenger capacity. Generally, it is not advisable to plan to operate at a load factor of 100%. Here, \( \alpha, \beta \) are defined as the maximum and minimum capacity usage, respectively. During the simulation period, the average load factor degree of vehicles is thus controlled between \( \beta \) and \( \alpha \).

\[ \beta \leq \frac{\sum_{b=1}^{M} \max(q_b^s)}{\sum_{b=1}^{M} V C_b} \leq \alpha. \quad (22) \]

Then, the energy saving dispatching optimization model can be formulated as follows:

\[ \min z_1 = \sum_{s=1}^{N} \left[ \sum_{b=2}^{M} \frac{(f_b^s - f_b^{s-1})^2 u_b^s}{2} + \sum_{b=2}^{M-1} (f_b^{s-1} - f_b^s) p_b^s \right] + \sum_{b=2}^{M-1} (f_b^{s-1} - f_b^s) p_b^s \]

\[ \min z_2 = \frac{\sum_{s=2}^{N} \sum_{b=1}^{M} C_b^{1-s}}{\sum_{b=1}^{M} V C_b} \]

\[ s.t. \quad W_1 \leq f_b^1 \leq W_2, \quad b \in B \quad (23) \]

\[ h_1 \leq f_b^1 - f_b^{b-1} \leq h_2, \quad b \in B \quad (24) \]

\[ \beta \leq \frac{\sum_{b=1}^{M} \max(q_b^s)}{\sum_{b=1}^{M} V C_b} \leq \alpha, \quad s \in S \]
Constraint (23) ensures that the departure time is within the dispatch period, while constraint (24) indicates the arrival interval limits.

III. SOLUTION ALGORITHMS
In this section, a two-phase algorithm is employed in order to solve this problem. The first phase is to find a Pareto set. In this phase, we can obtain multiple Pareto solutions. The second phase is to obtain satisfactory schemes from the Pareto set based on an integrated weighting method.

A. GENERATING THE PARETO SET BASED ON THE NICHED GENETIC ALGORITHM
In the first phase, the genetic algorithm (GA) is used to find the Pareto set.

1) CHROMOSOME STRUCTURE
In this study, \( CH = (F, V) \) is designed as a chromosome, where gene
\[
F = (F_1, F_2, \ldots, F_M) = (f_1^1 - W_1, f_2^1 - f_1^1, \ldots, f_M^1 - f_{M-1}^1)
\]
represents the arrival interval time of vehicles at the first station and gene \( V = (VC_1, VC_2, \ldots, VC_M) \) denotes the capacity of vehicles.

Assume that the number of vehicles \( M = 8 \), the dispatch period \( W_1 = 7 : 00 \), \( W_2 = 8 : 00 \), the minimum and maximum vehicle headways \( h_1 = 3, h_2 = 10 \), and the capacity of vehicles is 95 or 195 people. Fig.3 is a chromosome.

\[
\begin{align*}
F &= (8, 7, 7, 9, 8, 7, 6, 8) \\
V &= (95, 95, 95, 195, 195, 95, 95, 195)
\end{align*}
\]

FIGURE 3. A simple example of a chromosome.

2) INITIALIZATION
The initialization is an important process in GA, serving the role of randomly initializing the solution. Let \( \text{popsize} \) be the number of chromosomes. To describe the problem conveniently, we briefly take the index \( K \) to denote the type number of vehicle capacity and set the arrival interval to an integer multiple of 30 seconds. Details to initialize a chromosome are summarized in the following algorithm.

Step1. Let \( i = 1 \). For \( i = 1 \) to \( \text{popsize} \), repeat Step 2 to Step 4.

Step2. Let \( j = 1 \).

Step3. Generate a random integer number \( R \) from uniform distribution \( u(2h_1, 2h_2) \), and let \( f_j^1 = R/2 \). Generate a random number \( k \) from uniform distribution \( u(1, K) \), and let \( VC_j^k \) be the \( k \)th type capacity.

Step4. If \( \sum_{m=1}^{i} F_m^i \geq W_2 - W_1 \), let \( F_j^i = W_2 - W_1 - \sum_{m=1}^{i-1} F_m^i \) and \( M = j \), set \( i = i+1 \), and go to Step2. Otherwise, set \( j = j + 1 \) and go to Step3.

It is clear that the obtained chromosomes meet constraints (23) and (24).

3) CROSSOVER OPERATOR
Let \( P_c \in (0, 1) \) be the crossover probability. To determine the parents for crossover operation, we repeat the following process from \( i = 1 \) to \( \text{popsize} \): randomly generate a real number \( r_c \) from \( u(0, 1) \), where the chromosome \( CH_i \) is selected as the parent if \( r_c < P_c \).

Allow us to illustrate the crossover operator on the chromosome \( CH_i \). First, randomly generate two unequal integers \( n_1, n_2 \in [1, M] \), and then exchange the values of \( F_{n_1} \) and \( F_{n_2} \), \( VC_{n_1} \) and \( VC_{n_2} \). We can obtain a new chromosome \( CH'_{i} \). Assume that \( n_1 = 2, n_2 = 5 \). The crossover process is illustrated in Fig.4.

4) MUTATION OPERATOR
A parameter \( P_m \in (0, 1) \) is defined as the probability of mutation. In a similar manner, \( i = 1 \) to \( \text{popsize} \): generate a real number \( r_m \) from \( u(0, 1) \); if \( r_m < P_m \), then the chromosome \( CH_i \) is selected to be mutated. The mutation includes two independent operators. The first operator optimizes the capacity of vehicles. The second operator adjusts the arrival interval.

\( a: \) THE FIRST OPERATOR
This first operator is only utilizable for gene \( V \). Randomly select \( S \) elements from gene \( V \) and re-select these capacities. Assume that \( VC_1 \) and \( VC_2 \) are selected. The process of the first operator is shown in Fig.5.

\( b: \) THE SECOND OPERATOR
This second operator is just utilizable for gene \( F \). The purpose of this operation is to change the structure of chromosomes. These operators are summarized as follows:

Step1. Randomly generate a mutation position \( j \in [1, M] \), and regenerate this vehicle’s arrival interval \( F_j \).

Step2. For \( i = j \) to \( M \), repeat Step 3 to Step 6.

Step3. If \( \sum_{m=1}^{i} F_m \geq W_2 - W_1 \), let \( F_i = W_2 - W_1 - \sum_{m=1}^{i-1} F_m \), and when \( M = i \), stop. Otherwise, go to Step 4.

Step4. If \( W_2 - W_1 - \sum_{m=1}^{M-1} F_m \leq h_2 \), then \( F_M = W_2 - W_1 - \sum_{m=1}^{M-1} F_m \); stop. Otherwise, proceed to Step5.

Step5. Generate a random integer number \( R \) from uniform distribution \( u(2h_1, 2h_2) \), let \( F_M = R/2 \). Generate a random number \( k \) from uniform distribution \( u(1, K) \), and let \( VC_M \) be the \( k \)th type capacity.

Step6. Set \( M = M + 1 \), go to Step 4.

The second operator can be divided into the following three cases by the approach proposed above.

Case 1. \( \sum_{m=1}^{i} F_m \geq W_2 - W_1 \). In this case, the number of vehicles will decrease. Next, we use a simple chromosome
in Fig.6 as an example to illustrate the mutation process for gene $F$, where the number of vehicles $M$ is 9. Assume that $F_4$ is chosen to be mutated and regenerate $F_4 = 10$. We can calculate that $\sum_{m=1}^{8} F_m \geq W_2 - W_1$, and $F_8 = W_2 - W_1 - \sum_{m=1}^{7} F_m = 60 - 54 = 6$. In this situation, the number of vehicles is decreased to 8.

**Case 2.** $\sum_{m=1}^{M} F_m < W_2 - W_1$ and $W_2 - W_1 - \sum_{m=1}^{M-1} F_m \leq h_2$.

In this case, the number of vehicles will remain unchanged. Fig.7 is the mutation process of gene $F$ for this situation. Assume that $F_4$ is also chosen for mutation and regenerate $F_4 = 7$. Then, we can calculate that $\sum_{m=1}^{8} F_m < W_2 - W_1$, and $W_2 - W_1 - \sum_{m=1}^{7} F_m = 2 \leq h_2$. In this situation, according to Step 4, we can calculate that $F_8 = W_2 - W_1 - \sum_{m=1}^{M-1} F_m = 10$.

**Case 3.** $\sum_{m=1}^{M} F_m < W_2 - W_1$ and $W_2 - W_1 - \sum_{m=1}^{M-1} F_m > h_2$.

In this case, the number of vehicles will increase. Fig.8 is the mutation process of gene $F$ for this situation. Assume that $F_4$ is also chosen for mutation and regenerate $F_4 = 4$. We can calculate that $\sum_{m=1}^{8} F_m < W_2 - W_1$, and
Step 1. Normalizing the decision matrix and obtaining normalized matrix \( R_{ij} = [r_{ij}]_{p \times 2} \). Retrieving the normalized matrix \( H_{ij} = [h_{ij}]_{p \times 2} \).

Step 2. Computing the information entropy of objective \( j \):

\[
\text{en}_j = -(\ln p) - 1 \sum_{i=1}^{p} h_{ij} \ln h_{ij}, \quad j = 1, 2
\]  

(If \( h_{ij} = 0 \), then \( h_{ij} \ln h_{ij} = 0 \).

Step 3. Computing the weight of objective \( j \):

\[
\omega_j' = \frac{1 - \text{en}_j}{\sum_{k=1}^{2} (1 - \text{en}_k)}, \quad j = 1, 2.
\]  

(38)

Step 4. Computing the weighted objective value of solution \( i \):

\[
z_i' = \sum_{j=1}^{2} \omega_j' r_{ij}, \quad i = 1, 2, \ldots, P.
\]  

(39)

2) SUBJECTIVE WEIGHTING MODULE

In the second module, subjective weighting using the multi-criteria analysis method is performed.

The long waiting time and crowded conditions represent a poor quality of service from the passenger perspective. Thus, when the waiting time is long, both passengers and operators regard waiting time as an important indicator. Accordingly, the average waiting time of BRT given in Table 2 can be used to estimate the quality of service. By assigning the subjective preferences to impact weighting factors of objectives, the subjective weight \( \omega_j''(j = 1, 2) \) can be acquired.

| TABLE 2. Passenger average waiting time QOS and subjective weight. |
|-------------------------------------------------------------|
| QOS          | A   | B   | C   | D   | E   | F   |
|----------------|-----|-----|-----|-----|-----|-----|
| Waiting time /min | 0-1.5 | 1.5-3 | 3-4.5 | 4.5-6 | 6-7.5 | >7.5 |
| weight        | Z_1 | 0.15 | 0.35 | 0.5  | 0.65 | 0.75 | 0.85 |
| weight        | Z_2 | 0.85 | 0.65 | 0.5  | 0.25 | 0.15 | 0.15 |

3) WEIGHTING FACTOR INTEGRATION MODULE

After obtaining the weight factors based on the above two methods, the synthesized weight can be determined by the integrated weighting method as follows:

\[
\omega_j = k_1 \omega_j' + k_2 \omega_j'' \quad (j = 1, 2)
\]  

(30)

where \( k_1, k_2 \) denote the coefficients of importance for two methods, which can be calculated by

\[
k_1 = \frac{(\sum_{i=1}^{P} \sum_{j=1}^{2} \omega_j' r_{ij})^2}{(\sum_{i=1}^{P} \sum_{j=1}^{2} \omega_j' r_{ij})^2 + (\sum_{i=1}^{P} \sum_{j=1}^{2} \omega_j'' r_{ij})^2}.
\]  

(31)

\[
k_2 = 1 - k_1.
\]  

(32)

B. COMPUTING SYNTHESIZED OBJECTIVE VALUES OF PARETO SOLUTIONS

To perform dispatching scheme analysis in the result interpretation phase, the proposed methodology for integrating objective weighting factors is implemented based on the use of the information entropy method and subjective weighting method, which consists of three modules.

1) INFORMATION ENTROPY WEIGHTING MODULE

Information entropy, based on the discrepancy driven mechanism, is one of the commonly used objective weighting methods. It accurately reflects the amount of information utility for each objective and avoids the human influence factors.

Suppose that the number of Pareto solutions is \( P \). Then, a decision matrix \( M_{AP}^{P \times 2} \) is obtained with \( P \) Pareto solutions and 2 objectives. Thus, the calculation process for the weighted objective value of each solution by the information entropy method is stated as follows:

\[
W_2 - W_1 - \sum_{m=1}^{2} F_m = 13 > h_2. \]

In this situation, according to Step 5 and Step 6, we can randomly generate \( F_8 \) and calculate \( F_9 \).

5) SELECTION PROCESS

When GA is applied to solve the single-objective optimization, the fitness that gives the ranking criteria of the chromosomes is assigned to each chromosome. However, the fitness assignment in terms of the Pareto solution for the multi-objective problem is more difficult than those of single-objective in GA [40].

In this paper, a niche selection operator is designed to implement the selection of the genetic algorithm. The process is stated as follows:

Step1. Generating a selection population \( E \) which is composed of \( \text{popsize} \) chromosomes of the current population and \( \text{size} \) chromosomes randomly chosen from the Pareto set \( P \).

Step2. Choosing two individuals \( CH_1, CH_2 \) from the selection population randomly, and comparing \( CH_1 \) and \( CH_2 \). If one dominates the other, it is selected: stop. Otherwise, go to Step 3.

Step3. Calculating the niche radius. Set \( d_1, d_2 \) to be the difference between the maximum and minimum values of the objective function \( z_1, z_2 \), and let the niche radius

\[
d = \frac{\theta \sqrt{(d_1)^2 + (d_2)^2}}{\text{popsize} + \text{size} - 1}, \quad (25)
\]

where \( \theta \in [2, 4] \).

Step4. Calculating the niche count \( k_1, k_2 \) of \( CH_1, CH_2 \).

Step4-1. For all chromosomes \( CH_i \in M \), if

\[
\sqrt{(z_1(CH_1) - z_1(CH_i))^2 + (z_2(CH_1) - z_2(CH_i))^2} \leq d,
\]  

then \( k_1 = k_1 + 1 \). Otherwise, \( k_2 = k_2 + 1 \).

Step5. Selecting a chromosome. If \( k_1 < k_2 \), \( CH_1 \) is selected. Otherwise, \( CH_2 \) is selected.
\[ \mathbf{SZ}_i = \sum_{j=1}^{2} a_{ij} r_{ij}, \quad i = 1, 2, \ldots, P. \tag{33} \]

IV. NUMERICAL EXPERIMENTS

A. THE LINE AND VEHICLES OF LANZHOU BRT

A support vector machine (SVM) method based on affinity propagation (AP) was developed to forecast short-term passenger flow of the bus stations of the Lanzhou BRT [41]. In general, the passenger flow can be categorized according to three patterns, i.e., slack hour, normal hour, and rush hour. Furthermore, each pattern has several time segments with similar passenger flow. Based on the passenger flow analysis from [14], [41], [42], we can find that the time segment with similar passenger flow is approximately 60 min. In a segment, the same scheduling strategy can be used. In this paper, the dispatch approach for vehicles has been tested with data for the morning peak (07:00-08:00) of BRT in the city of Lanzhou, China. There is one line in operation. The total length of the system is 9.1 kilometers (Fig. 9) and there are in total 15 stations: the distances between stations are shown in Table 3. There are currently 50 special 12-meter and 20 18-meter BRT buses. During the simulation period, the average load factor degree of vehicles is controlled between 70% and 80%.

![FIGURE 9. BRT line in Lanzhou city of China.](image)

| Stations | Distance/m | Stations | Distance/m |
|----------|------------|----------|------------|
| 1,2      | 740        | 8,9      | 690        |
| 2,3      | 500        | 9,10     | 500        |
| 3,4      | 570        | 10,11    | 450        |
| 4,5      | 500        | 11,12    | 1000       |
| 5,6      | 670        | 12,13    | 600        |
| 6,7      | 650        | 13,14    | 770        |
| 7,8      | 760        | 14,15    | 700        |

The detailed information for two types of buses is listed in Table 4. The minimum and maximum vehicle headway, \( h_1 \) and \( h_2 \), were set to 1 and 5 min, respectively. Table 5 provides information about the average passenger arrival rate and passenger alighting rate of 15 stations (the direction from Liujiapu square station to Lanzhou west station is upward).

B. THE ESTIMATION OF ENERGY CONSUMPTION

Fig.10 shows the performance maps of universal characteristics for WP7.270 and WP10.375 engines. Fig.11 presents the speed-specific driving cycles of BRT vehicles used in the simulated operation.

![FIGURE 10. The performance map for engines.](image)

Based on the mechanical model in Section II and specific fuel consumption maps, the energy consumptions of 12-meter and 18-meter BRT vehicles can be obtained for situations with different passenger loads and running distances, which are shown in Table 6 (assumed per passenger weight of 65 kg).

The results presented in Table 6 show that the engine energy consumption \( C \) can be regarded as a function of the passenger load \( x \) and distance \( y \), then the energy consumption equations can be expressed as follows by curve fitting (the slope of the road section is ignored):

Energy consumption of 12-meter BRT vehicle:

\[ C = 0.05 + 0.0006q + 0.00019l - 1.47 \times 10^{-6}q^2 + 4.059 \times 10^{-7}lq, \]

Energy consumption of 18-meter BRT bus:

\[ C = 0.081 + 0.00045q + 0.00027l - 4.52 \times 10^{-7}q^2 + 4.965 \times 10^{-7}lq. \]
where $C$ is the energy consumption (L); $q$ is the number of passengers on vehicle (passenger load); $l$ is the distance between the stations (m).

### C. ANALYSIS AND DISCUSSION OF RESULTS

The designed chromosome length is 15. For testing the stability and reliability of the niched GA, we perform this algorithm 10 times under each different initial parameter, and record the obtained average Pareto solution number, runtime and iteration number in Table 7. It is shown that when $\text{popsize} = 50$, $P_c = 0.5$ and $P_m = 0.3$ are shown in Fig.12, and the distribution of solutions is shown in Fig.13.

After using the information entropy method and subjective weighting method, respectively, synthesized objective values are obtained according to the integrated weighting factors and the top 22 scheduling schemes, whose values exceed 0.7, are listed in Table 8. As seen from Table 8, there are 11 schemes utilizing 13 vehicles and 6 schemes utilizing 14 vehicles. This shows that the requirement can be fulfilled by using a smaller number of vehicles.

#### TABLE 4. Information for buses.

| Vehicle type | 12-meter | 18-meter |
|--------------|----------|----------|
| Engine model | WP7270   | WP10375  |
| Carb weight (Kg) | 11 200   | 17 850   |
| Gross vehicle weight (Kg) | 17 600   | 28 000   |
| Rated passenger capacity /p | 98       | 195      |
| Maximum speed (Km/h) | 80       | 70       |
| Front surface area $A/m^2$ | 7.78     | 8.16     |
| Wheel size | 275/70R22.5 | 275/70R22.5 |
| $i_s$  | 5.82,3.23,1.95,1.26,1.00 | 5.82,3.49,2.07,1.36,1.00 |
| $i_s$  | 5.571    | 6.14     |
| $\eta_s$ | 0.9      | 0.9      |
| Number of doors | 2        | 3        |

#### TABLE 5. The average arrival rate and alighting rate of stations.

| $s$ | $u_s^u$ (p/min) | $d_s^u$ (%) | $u_s^d$ (p/min) | $d_s^d$ (%) |
|-----|-----------------|-------------|-----------------|-------------|
|     | up  | down | up  | down | up  | down | up  | down |
| 1   | 7   | 0    | 0.0 | 100.0 | 9   | 13   | 8    | 40.4 | 18.2 |
| 2   | 2   | 0    | 0.5 | 42.7  | 10  | 10   | 8    | 34.1 | 20.8 |
| 3   | 8   | 0    | 6.7 | 33.6  | 11  | 10   | 8    | 31.7 | 18.2 |
| 4   | 7   | 1    | 6.4 | 29.9  | 12  | 2    | 2    | 7.5  | 2.4  |
| 5   | 2   | 1    | 4.7 | 9.5   | 13  | 8    | 8    | 29.0 | 5.7  |
| 6   | 7   | 1    | 13.5| 19.4  | 14  | 0    | 2    | 10.3 | 0.2  |
| 7   | 7   | 2    | 11.6| 17.1  | 15  | 0    | 22   | 100.0| 0.0  |
| 8   | 5   | 3    | 8.7 | 15.4  |     |      |      |      |      |

#### FIGURE 11. The speed-specific driving cycles of BRT vehicles.

#### FIGURE 12. The distribution of obtained Pareto solutions.

#### FIGURE 13. The Pareto solution distribution of vehicle number.
Vehicle operation schedules for Schemes 1, 21 and 22 are shown in Fig.14. For Scheme1, which uses 13 vehicles (9 18-meter buses and 4 12-meter buses), the energy consumption of vehicles is 99.39 L and the total waiting time of passengers is 33 486 min, while the average waiting time for each passenger is 3.69 min.

For Scheme 22, which also uses 13 vehicles (13 18-meter buses), the energy consumption of vehicles is 109.83 L and the total waiting time of passengers is 43,956 min, while the average waiting time for each passenger is 4.85 min.

From the result, we can see that in the Pareto solution set, the proportion of 18-meter vehicles is larger in the schemes with fewer vehicles. Generally, when the number of vehicles increases, passenger waiting time decreases, the energy consumption increases and the energy consumption can be reduced through vehicle type combination.
TABLE 7. Performance comparison between diversification settings.

| Parameters | Solutions | Runtime (min) | Iteration number |
|------------|-----------|---------------|-----------------|
| Genetic operator | popsize | number |               |               |
| $P_r=0.5$ | 30 | 74 | 2.6 | 7,500 |
| $P_r=0.3$ | 40 | 89 | 4.5 | 9,800 |
| $P_r=0.1$ | 50 | 101 | 6.5 | 11,200 |
| $P_r=0.01$ | 60 | 112 | 9.3 | 15,600 |
| $P_r=0.5$ | 40 | 78 | 2.6 | 7,100 |
| $P_r=0.3$ | 50 | 87 | 4.2 | 8,250 |
| $P_r=0.1$ | 60 | 94 | 8.9 | 14,850 |
| $P_r=0.5$ | 40 | 73 | 1.9 | 6,800 |
| $P_r=0.3$ | 50 | 84 | 3.5 | 7,650 |
| $P_r=0.1$ | 60 | 92 | 8.2 | 11,900 |

TABLE 8. The Pareto solutions with higher synthesized objective values.

| Scheme | Waiting time (min) | Energy consumption (litre) | Vehicle number | Synthesized objective value |
|--------|-------------------|---------------------------|---------------|----------------------------|
| 1      | 33.486            | 99.39                     | 13            | 0.737                      |
| 2      | 29.217            | 102.23                    | 13            | 0.736                      |
| 3      | 31.593            | 101.55                    | 14            | 0.734                      |
| 4      | 31.286            | 101.81                    | 14            | 0.734                      |
| 5      | 25.768            | 105.08                    | 13            | 0.732                      |
| 6      | 26.463            | 104.84                    | 13            | 0.731                      |
| 7      | 26.807            | 104.63                    | 13            | 0.731                      |
| 8      | 31.042            | 102.06                    | 13            | 0.730                      |
| 9      | 28.265            | 104.53                    | 14            | 0.725                      |
| 10     | 36.229            | 99.21                     | 14            | 0.724                      |
| 11     | 24.632            | 107.51                    | 13            | 0.721                      |
| 12     | 40.392            | 96.24                     | 14            | 0.721                      |
| 13     | 37.616            | 98.65                     | 15            | 0.720                      |
| 14     | 25.244            | 107.43                    | 13            | 0.719                      |
| 15     | 38.499            | 98.40                     | 15            | 0.718                      |
| 16     | 41.841            | 96.12                     | 14            | 0.714                      |
| 17     | 39.803            | 98.21                     | 16            | 0.712                      |
| 18     | 42.627            | 95.93                     | 15            | 0.711                      |
| 19     | 23.590            | 109.86                    | 13            | 0.710                      |
| 20     | 24.034            | 109.84                    | 13            | 0.708                      |
| 21     | 43.356            | 95.37                     | 16            | 0.708                      |
| 22     | 24.594            | 109.83                    | 13            | 0.706                      |

V. CONCLUSION

For a good scheduling scheme, a critical trade-off is made between the level of service and the operating costs. This paper focused on the transit operations-planning activities: vehicle-scheduling with different vehicles types considering the level of service and the operating costs. In terms of the service level, mainly the passenger waiting time and the passenger loads are adopted, and the cost aspect mainly uses the energy consumption to carry out the manifestation.

This paper presents a mechanical model to describe the level of energy used in different vehicles based on engine universal characteristics, mainly considering the characteristics of the vehicle, engine, road, and driving type. Furthermore, in order to determine the vehicle scheduling scheme, a multi-objective energy saving dispatching optimization model of BRT is developed to minimize the waiting time of passengers and energy consumption of vehicles. Moreover, a two-phase algorithm is employed in order to solve this multi-objective model. In the first phase, a genetic algorithm based on a niche selection operator is used to find the Pareto set. In the second phase, a proposed methodology for integrating objective weighting factors is implemented to perform dispatching scheme analysis.

Data of BRT system in Lanzhou City, China, is collected to test the model and the algorithm. The results show that the designed algorithm is valid for solving the dispatching optimization model of BRT, and the energy consumption and passenger waiting time can be reduced by using an appropriate dispatching scheme. Moreover, the proposed decoding, mutation and selection ideas can be adopted in other genetic algorithms to design the related algorithm for the public transit dispatching problem. In the future, possible extensions of this study will be the regional bus scheduling problem.

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