Multiplicity Fluctuation at Second-order Phase Transition on the Base of Squeezed States

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Abstract

We use the generalized squeezed state description in the framework of the Ginzburg-Landau theory. Multiplicity distributions of the squeezed states are studied at second-order phase transition at different squeeze factors. It is shown that the normalized factorial moments exhibit a specific behaviour as functions of the resolution scale. We obtain the values of the scaling exponent which coincides with experimental data at small squeeze factor.

Key words: Ginzburg-Landau theory, phase transition, squeezed states, factorial moment, multiplicity distribution, intermittency, scaling exponent.

PACS numbers: 12.38.Mh, 68.35.Rh, 73.43.Nq, 42.50.Dv

1 Introduction

With increasing of the collision energy in $e^+e^-, p\bar{p}, ep$, heavy-ion experiments the role of the multiparticle production and the collective effects of particle interaction becomes more significant. Large progress has been achieved in study of the considered processes within perturbative Quantum Chromodynamics (QCD) \cite{1,2,3,4}. Perturbative theory is not able to reproduce all consequences inherent to corresponding Lagrangian of the interaction, in particular, the collective aspects of the behaviour of considered systems as a whole can have fundamental significance for description of the features of the confinement and hadronization. Here methods of the statistical physics are fruitful since a mathematical apparatus used at investigation of the multiplicity distribution and particle correlations is common both for the statistical physics and for the processes of the multiparticle production \cite{5}. Investigations of the collective excited modes of the hadron (quark-gluon) medium began to carry out not long ago and have a phenomenological character.

For statistical systems the fluctuations are large near critical points. Therefore the multiplicity fluctuations of hadrons produced in high-energy heavy-ion collisions can be used as a measure of whether a quark-gluon system has undergone a phase transition \cite{6}. Until today the question concerning order of the parton-hadron PT in high-energy collisions is opened. Lattice gauge calculations indicate that for two flavors the PT is most likely of the second order \cite{7}. When strange quarks are included, it may become a weak first-order PT \cite{9}.
At present accelerator energies the average number of produced particles is large enough and therefore the scattering operator and other operators are inconvenient to consider in terms of number states. At the same time in such systems as lasers where the average number of photons is large it is conveniently to use the coherent state representation (P-representation) [10, 11]. There are a number of efforts to apply P-representation for investigation of the multiplicity fluctuations as a phenomenological manifestation of quark-hadron PT in the framework of the Ginzburg-Landau (GL) formalism both for the second-order [12, 13] and first-order [14, 15, 16] PT. In both cases it was supposed that multiplicity distribution of the hadrons without PT is a Poissonian and scaling behaviour of the factorial moments has been found. Moreover the scaling exponent $\nu$ is equal $1.305$ for second-order PT [13] and $1.32 < \nu < 1.33$ for the generalized GL model with first-order PT [16]. There was the disagreement between experiment and theory. Indeed, NA22 data on particle production in hadronic collisions give $\nu = 1.45 \pm 0.04$ [17], heavy-ion experiments $\nu = 1.55 \pm 0.12$ [12] and $\nu = 1.459 \pm 0.021$ [18].

At the same time the study of multiplicity fluctuations in a similar type of phase transition have carried out in nonlinear optics [19]. It was shown that second-order PT is related to the symmetry-changing instability of stationary non-equilibrium states. This fact may be additional evidence for the use of optical methods at investigation problems of the PT in the multiparticle production processes.

Indeed, the idea of applying stochastic methods developed for studying photon-counting statistics of light beams to particle production processes was used for explanation of the experimentally observed properties hadrons, in particular, the multiplicity distributions, factorial and cumulant moments. It was shown that the most general distribution that characterizes $e^+e^-$, $pp$, neutrino-induced collisions is a $k$-mode squeezed state distribution [20, 21]. Squeezed states (SS) involve coherent states as the specific case and posses uncommon properties: they display a specific behaviour of the factorial and cumulant moments [22] and can have both sub-Poissonian and super-Poissonian statistics corresponding to antibunching and bunching of photons [10], [23]-[25]. Although the SS are constructed in quantum optics (QO) their relevance to hadron production in high-energy collisions was recognized long ago [26, 27]. In particular, the multiplicity distribution of the pions has been explained by formalism of the squeezed isospin states [28]. In addition, study of the evolution of gluon states at the non-perturbative stage of jet development has obtained the new squeezed gluon states [29]-[32] which could be necessary element of hadronization and, in particular, QGP→hadrons. Then using the Local parton hadron duality it is easy to show that in this case behaviour of hadron multiplicity distribution in jet events is differentiated from the negative binomial one. Such specific behaviour of the multiplicity distribution is confirmed by experiments for $pp$, $p\bar{p}$-collisions [33]-[35].

Considering multiplicity distribution in different collisions without PT as squeezed one we generalize the coherent state representation taking into account the squeezed states within GL-approach for investigation of the multiplicity fluctuations at phase transition in QGP. Since the multiplicity fluctuations exhibit intermittency behaviour which is observed in a large number of experiments, we investigate conditions of appearance of this effect in depending on the parameters of GL model.
2 Multiplicity distribution of the squeezed states at second-order phase transition

It is convenient to start description of the squeezed-state formalism in GL theory with definition of the photon SS. Two basic kinds of single-mode ideal SS are used in QO: coherent squeezed state (CSS) and scaling SS (SSS) defined as

\[
|\psi,\eta\rangle = D(\psi)S(\eta)|0\rangle \quad (CSS),
\]

\[
|\psi,\eta\rangle = S(\eta)D(\psi)|0\rangle \quad (SSS),
\]

where \(D(\psi) = \exp\{\psi a^+ - \psi^* a\}\) is a displacement operator, \(S(\eta) = \exp\{\eta^* a^2 - \frac{\eta}{2}(a^+)^2\}\) is a squeeze operator, \(\psi = |\psi|e^{i\gamma}\) is an eigenvalue of non-Hermitian annihilation operator \(a\), \(|\psi|\) and \(\gamma\) are an amplitude and a phase of the coherent state correspondingly, \(\eta = re^{i\theta}\) is an arbitrary complex number, \(r\) is a squeeze factor, phase \(\theta\) defines the direction of squeezing maximum. Using general formula for two-photon coherent state distribution we can write the corresponding expression for CSS and SSS distributions in the form

\[
P_n = \frac{1}{\cosh(r)n!} \left(\frac{\tanh(r)}{2}\right)^n |H_n(\xi_1)|^2 e^{\xi_2},
\]

where \(H_n(\xi_1)\) is a Hermite polynomials, \(\xi_1\) and \(\xi_2\) are equal in case of CSS

\[
\xi_1 = \sqrt{\langle n \rangle - \sinh^2(r)} \left[ \cosh(r)e^{i(\gamma - \theta/2)} + \sinh(r)e^{-i(\gamma - \theta/2)} \right],
\]

\[
\xi_2 = \left[ \langle n \rangle - \sinh^2(r) \right] \left\{ \cosh(2r)[\tanh(r)\cos(2\gamma - \theta) - 1] + \sinh(2r)[\tanh(r) - \cos(2\gamma - \theta)] \right\}
\]

and for SSS

\[
\xi_1 = \sqrt{\langle n \rangle - \sinh^2(r)} e^{i(\gamma - \theta/2)} \left[ \cosh(2r) - \sinh(2r)\cos(2\gamma - \theta) \right]^{-\frac{1}{2}},
\]

\[
\xi_2 = \frac{\left[ \langle n \rangle - \sinh^2(r) \right] \left[ \tanh(r)\cos(2\gamma - \theta) - 1 \right]}{\cosh(2r) - \sinh(2r)\cos(2\gamma - \theta)}.
\]

Here \(\langle n \rangle\) is an average multiplicity. In particular case at \(\gamma = \theta = 0\) expression coincides with analogous expressions used for description of the multiplicity distribution in \(e^+e^-, p\bar{p}\), neutrino-induced collisions.

In quantum field theory (multi-mode case) the average number of particles in observed state is defined as

\[
\langle n \rangle = \left\langle \int_V dza^+(z)a(z) \right\rangle
\]
and then an average multiplicity for CSS and SSS is equal correspondingly to

\[ \langle n \rangle = \int_V |\psi(z)|^2 dz + \sinh^2(r), \]

\[ \langle n \rangle = \left( \int_V |\psi(z)|^2 dz \right) \left[ \cosh(2r) - \sinh(2r) \cos(2\gamma - \theta) \right] + \sinh^2(r). \]  \hspace{1cm} (6)

Here for simplicity we regard that phase of the coherent state and of squeezing effect are the same for whole space that is quantities \( \gamma, r, \theta \) are parameters. The probability density of finding \( n \) particles in SS is

\[ |\langle n |\psi(z), \eta \rangle|^2 = P_{0n}. \]

Then using the expressions (6) for average multiplicity we can write \( P_{0n} \) in the form

\[ P_{0n} = \frac{1}{\cosh(r)n!} \left( \frac{\tanh(r)}{2} \right)^n \left[ \int_V |\psi(z)|^2 dz \right]^{\frac{1}{2}} \left( \int_V |\psi(z)|^2 dz \right)^{\frac{1}{2}} \times \exp \left\{ \int_V |\psi(z)|^2 dz F_2(r, \gamma, \theta) \right\}, \]

\[ \times \exp \left\{ \int_V |\psi(z)|^2 dz F_1(r, \gamma, \theta) \right\}, \]

\[ \text{where } F_1(r, \gamma, \theta), F_2(r, \gamma, \theta) \text{ are functions of the parameters } r, \gamma, \theta \text{ and in case of CSS are equal to} \]

\[ F_1(r, \gamma, \theta) = \frac{\cosh(r)e^{i(\gamma-\theta/2)} + \sinh(r)e^{-i(\gamma-\theta/2)}}{\sqrt{\sinh(2r)}}, \]

\[ F_2(r, \gamma, \theta) = \cosh(2r)[\tanh(r) \cos(2\gamma - \theta) - 1] + \sinh(2r)[\tanh(r) - \cos(2\gamma - \theta)]. \] \hspace{1cm} (9)

and for SSS

\[ F_1(r, \gamma, \theta) = \frac{e^{i(\gamma-\theta/2)}}{\sqrt{\sinh(2r)}}, \quad F_2(r, \gamma, \theta) = \tanh(r) \cos(2\gamma - \theta) - 1. \] \hspace{1cm} (10)

From Fig.1 it is obviously that at \( \theta = 0 \) we have a sub-Poissonian distribution and at \( \theta = \pi \) — super-Poissonian one. If the squeeze factor is more than one we have oscillations of given distribution (Fig.1: CSS distribution). Obviously that at \( r \to 0 \) (the squeezing effect is absent) the probability density of finding \( n \) particles is Poissonian

\[ P_{0n}^0 = \frac{1}{n!} \exp \left\{ - \int_V |\psi(z)|^2 dz \right\} \left( \int_V |\psi(z)|^2 dz \right)^n. \] \hspace{1cm} (11)
Within standard GL model the free energy of the system is

\[ F[\psi] = \int dz \{ a|\psi(z)|^2 + b|\psi(z)|^4 + c|\partial\psi/\partial z|^2 \}, \tag{12} \]

where \( \psi(z) \) is introduced to serve as a complex order parameter. Then the hadron multiplicity distribution can be given by the functional integral of the type \[ P_n = Z^{-1} \int D\psi P_0^n e^{-F[\psi]}, \tag{13} \]

here \( Z = \int D\psi e^{-F[\psi]} \). Thus the probability of having a large \( n \) in volume \( V \) is controlled by deviation of \( \psi \) from \( \psi_0 \) (minimum of the GL potential) as specified by the thermodynamical factor \( e^{-F[\psi]} \).

To investigate obtained expression we identify \( V = \delta^d \) and regard that \( |\psi(z)| \) is constant in every bin width \( \delta \) (\( d \) is a dimension). Then multiplicity distribution after phase transition is

\[ P_n = \frac{1}{2\pi \cosh(r)} \int D\psi \frac{\tanh^n(r)}{2^n n!} \exp \left\{ -F[\psi] + \int_V |\psi(z)|^2 dz F_2(r, \gamma, \theta) \right\} \]

\[ \times \left| H_n \left( \left[ \int_V |\psi(z)|^2 dz \right]^{\frac{1}{2}} F_1(r, \gamma, \theta) \right)^2 \right|. \tag{14} \]

To investigate obtained expression we identify \( V = \delta^d \) and regard that \( |\psi(z)| \) is constant in every bin width \( \delta \) (\( d \) is a dimension). Then multiplicity distribution after phase transition is

\[ P_n = \frac{1}{2\pi \cosh(r)} D_{-1}^{-1} \left( -|a| \sqrt{\frac{\delta^d}{2b}} \right) \]

\[ \int_0^{2\pi} d\gamma \exp \left\{ \frac{\delta^d F_2(r, \gamma, \theta)(F_2(r, \gamma, \theta) + 2|a|)}{8b} \right\} \]

\[ \times \sum_{k=0}^{n/2} \sum_{l=0}^{n/2} (-1)^{k+l} \frac{(2k - 1)!! (2l - 1)!! n! (n - k - l)!}{(2k)!(2l)!(n - 2k)!(n - 2l)!} \left( \frac{2\delta^d}{b} \right)^{\frac{1}{2}(a-k-l)} F_1^{n-2k}(r, \gamma, \theta) \]

\[ \times (F_1^*)^{n-2l}(r, \gamma, \theta) D_{-(n-k-l+1)} \left( -|a| + F_2(r, \gamma, \theta) \sqrt{\frac{\delta^d}{2b}} \right), \tag{15} \]
where $D_f(w)$ is a function of the parabolic cylinder. Obviously, this expression for $P_n$ is not depended on the phase which defines the direction of squeezing maximum since integrand is a harmonic function of this squeeze parameter. Influence of the phase transition on behaviour of the multiplicity distributions is shown on Fig. 2.

Figure 2: Multiplicity distributions of the squeezed states with taking into account of the phase transition.

This expressions for $P_n$ (14), (15) will be essential at analysis phenomenon of intermittency.

3 Intermittency

One of the effective way to manifest the nature of the multiplicity fluctuations in high-energy collisions is to examine the dependence of the normalized factorial moments $F_q$ [17, 38]

$$F_q = \frac{\langle n(n-1) \cdots (n-q+1) \rangle}{\langle n \rangle^q} = \frac{f_q}{f_1^q}$$

(16)
on the bin width $\delta$ in rapidity. Here $f_q = \langle n(n-1) \cdots (n-q+1) \rangle$, $n$ is the number of hadrons detected in $\delta$ in an event, and the average are taken over all events. The multiplicity fluctuations can exhibit intermittency behaviour which is manifested by power-law behaviour of $F_q$ on $\delta$ [17]

$$F_q \propto \delta^{-\varphi_q},$$

(17)

where $\varphi_q$ is referred to as the intermittency index. Indeed, apart from collision energy and nuclear size we can vary only the size of a cell $\delta$ in phase space that is just the central theme of intermittency. This effect has been observed in a large number of experiments: $e^+e^-, \mu p, pp, pA$ and AA collisions [17].

Therefore the intermittency analysis is used to explore universal characteristics of quark-hadron PT in the GL model. In this section we examine whether (17) is valid under taking into account PT. Since

$$f_q = \sum_{n=q}^{\infty} \frac{n!}{(n-q)!} P_n,$$

(18)

$^{1}$Values of the parameters $a, b, r$ correspond to case then we have intermittency and the scaling exponent value is equal to 1.459.
using (14) and (12) we obtain the next explicit form of \( f \)

\[
f_q = \frac{1}{2Z \cosh(r)} \int D\psi \ e^{-F[\psi]} \exp \left\{ \int_V |\psi(z)|^2 dz \ F_2(r, \gamma, \theta) \right\}
\]

\[
\times \sum_{n=q}^{\infty} \frac{1}{(n-q)!} \left( \frac{\tanh(r)}{2} \right)^n \left[ H_n \left( \left[ \int_V |\psi(z)|^2 dz \right]^{1/2} F_1(r, \gamma, \theta) \right) \right]^2,
\]

where \( V \) is the volume of the cell in which the factorial moment is measured. Taking into account the formula \textsuperscript{39}

\[
\sum_{k=0}^{\infty} t^k H_{k+m}(x) H_{k+n}(y) = (1 - 4t^2)^{-(m+n+1)/2} \exp \left[ \frac{4xyt - 4t^2(x^2 + y^2)}{1 - 4t^2} \right]
\]

(20)

and identifying \( V = \delta^d \), regarding that \( |\psi(z)| \) is constant in every bin width \( \delta \), we rewrite the expression \textsuperscript{19} taking into account the explicit form of \( F_1(r, \gamma, \theta), F_2(r, \gamma, \theta) \) for CSS \textsuperscript{1} and SSS \textsuperscript{11} correspondingly in the next form

(CSS)

\[
f_q = (2Z)^{-1} \sinh^2 q(r) \int_0^{2\pi} d\gamma \int_0^{\infty} d|\psi|^2 e^{-F[\psi]} \sum_{n=0}^{q} \frac{(q!)^2}{(q-n)!(2\tanh(r))^n} \frac{1}{(q-n)!} \left[ H_n \left( \left[ \frac{|\psi|^2 \delta^d}{\sinh(2r)} \right]^{1/2} e^{i(\gamma - \theta/2)} \right) \right]^2,
\]

(SSS)

\[
f_q = (2Z)^{-1} \sinh^2 q(r) \int_0^{2\pi} d\gamma \int_0^{\infty} d|\psi|^2 e^{-F[\psi]} \sum_{n=0}^{q} \frac{(q!)^2}{(q-n)!(2\tanh(r))^n} \frac{1}{(q-n)!} \left[ H_n \left( \left[ \frac{|\psi|^2 \delta^d}{\sinh(2r)} \right] \sinh(r) e^{i(\gamma - \theta/2)} - \cosh(r) e^{-i(\gamma - \theta/2)} \right) \right]^2
\]

(22)

Integrating obtained expressions we can represent their as

\[
f_q = \frac{J_q}{J_0},
\]

(23)

where in case CSS

\[
J_q = \frac{\pi}{\sqrt{2b\delta^d}} \exp \left\{ \frac{|a|^2 \delta^d}{8b} \right\} \sinh^2 q(r) \sum_{n=0}^{q} \frac{(q!)^2}{(q-n)!(2\tanh(r))^n} \left[ \sum_{k=0}^{n/2} \frac{(2k-1)!!}{(2k)!} \right]^2
\]

\[
\times \frac{\sinh(2r))^{2k-n}}{(n-2k)!} \left( \frac{2\delta^d}{b} \right)^{(n-2k)/2} D_{-(n-2k+1)} \left( -|a| \sqrt{\delta^d/2b} \right)
\]

(24)
and for SSS

\[ J_q = \frac{\pi}{\sqrt{2b\delta^d}} \exp \left\{ \frac{|a|^2\delta^d}{8b} \right\} \sinh^{2q}(r) \sum_{n=0}^{q} \frac{(q!)^2}{(q-n)!} \tanh^{-n}(r) \sum_{k=0}^{n/2} \sum_{l=0}^{n/2} \frac{\binom{2k-1}{2k}! \binom{2l-1}{2l}!}{(2k)! (2l)!} \]

\times (\sinh(2r))^{k+l-n} (n-k-l)! \left( \frac{2\delta^d}{b} \right)^{\frac{1}{2} \left( n-k-l \right)} D_{(n-k-l+1)} \left( -|a|\sqrt{\delta^d} \right) 

\times \sum_{j=0}^{n-2l} \frac{(\sinh(r))^{l-k+2j} (\cosh(r))^{2n-k-2l-2j}}{j! (l-k+j)! (n-k-l-j)! (n-2l-j)!}.

(25)

Then according to (16), (23) the normalized factorial moments \( F_q \) have the next form

\[ F_q = J_q J_{-q} J_{0}^{q-1}. \]

(26)

On Fig. 3 and Fig. 4 we represent the results of analysis of the dependences of \( \ln F_q \) on \( (-\ln \delta^d) \) and of \( \ln F_q \) on \( (\ln F_2) \) correspondingly for the squeeze factors \( r = 2.48 \) (CSS) and \( r = 0.3876 \) (SSS) and for the next values of the parameters of the GL model \( a = -10, b = 0.20055 \).

Figure 3: Log-log plot of \( F_q \) vs \( 1/\delta^d \) in case of the squeezed states.

Figure 4: Log-log plot of \( F_q \) vs \( F_2 \) in case of the squeezed states.

If the local slope of \( \ln F_q \) vs \( \ln F_2 \) is approximately constant then we would have the scaling behaviour (Ochs-Wosiek scaling law)

\[ F_q \propto F_2^{\beta_q}, \]

(27)
which is valid for intermittent systems \cite{38}. The slopes $\beta_q$ are well fitted by the formula \cite{12}

$$\beta_q = (q-1)^\nu,$$  \hspace{1cm} (28)

where $\nu$ is a scaling exponent. Dependence of $\nu$ on squeeze factor $r$ is represented on the Fig.5 at the same values of the parameters of the GL model.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure5.png}
\caption{Dependence of the scaling exponent on squeeze factor $r$.}
\end{figure}

It is obvious from Fig.5 that we have intermittency in case of the CSS when $\nu = 1.066$ at $r = 2.48$ and $\nu = 1.459$ at $r = 0.3876$ for SSS. Thus scaling behaviours of the normalized factorial moments for the scaling squeezed states are characterized by obtained scaling exponent $\nu$ that agrees with experimental NA22 data on particle production in heavy-ion experiments \cite{18}. Parameters at which the scaling exponent values agree with various experimental data \cite{17, 12} are represented in the Tab.1.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
\multicolumn{2}{|c|}{a} & \multicolumn{2}{|c|}{\nu = 1.450} & \multicolumn{2}{|c|}{\nu = 1.459} & \multicolumn{2}{|c|}{\nu = 1.550} \\
\hline
 & b & r & b & r & b & r \\
\hline
-1 & 0.00663 & 0.32363 & 0.00592 & 0.32663 & 0.00220 & 0.34375 \\
-2 & 0.02761 & 0.31312 & 0.02455 & 0.31826 & 0.00992 & 0.32663 \\
-3 & 0.05217 & 0.32917 & 0.04988 & 0.32216 & 0.0275 & 0.31387 \\
-4 & 0.07313 & 0.35251 & 0.06956 & 0.34795 & 0.03760 & 0.31758 \\
-5 & 0.09535 & 0.36548 & 0.09052 & 0.36190 & 0.05582 & 0.31572 \\
-6 & 0.11815 & 0.37385 & 0.11213 & 0.37079 & 0.07140 & 0.32305 \\
-7 & 0.14138 & 0.37970 & 0.13398 & 0.37693 & 0.08570 & 0.33040 \\
-8 & 0.16472 & 0.38401 & 0.15609 & 0.38144 & 0.09871 & 0.33740 \\
-9 & 0.18819 & 0.38730 & 0.17834 & 0.38489 & 0.11190 & 0.34261 \\
-10 & 0.21187 & 0.38983 & 0.20055 & 0.38758 & 0.12518 & 0.34664 \\
\hline
\end{tabular}
\caption{Parameters at which the scaling exponent values agree with experimental data.}
\end{table}

In case of the CSS the scaling exponent values are not agree with various experimental data \cite{17, 12} at any values of the parameters $a, b, r$.

\section{Conclusion}

We study multiplicity fluctuations and intermittency in second order phase transition from QGP to hadrons within of the GL model. Generalizing P-representation to squeezed state one (in particular,
for two types: CSS, SSS) we obtain the explicit expressions for the probability of finding \( n \) particles and for the normalized factorial moments \( F_q \) which include additional parameters \( r, \theta \) inherent to the squeezing effect.

Changing new parameters we can more successfully apply GL model for description of the phase transitions. Indeed, at \( a = -10, b = 0.20055 \) and at \( r = 0.3876 \) in case of the scaling squeezed states of the hadrons we have intermittency when the value of the scaling exponent is equal to 1.459. Obtained value of the scaling exponent agrees with experimental data \([12, 17, 18]\).

We hope that squeezed state approach will be available for description of fluctuations in the phase transition from quark-gluon plasma to hadrons in processes where an energy density is very high, for example, in heavy ion collisions at high energy.

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