Comprehensive Error Modeling of Four Axis Gear Machine Tool

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Abstract. The error modeling theory is studied. The configuration of the four axis gear machine tool is analyzed. Analysis and Describe of the motion of the four axis gear machine tool are investigated. The homogeneous transformation matrix of the relative static between the adjacent typical bodies is obtained and the homogeneous transformation matrix of the relative motion between the adjacent typical bodies is clarified. The comprehensive error model of four axis gear machine tool is established. The method and the integrated error model provide a theoretical basis for the machining and error analysis of complex surfaces.

Keywords: Machine tool, gear, comprehensive error, model.

1. Introduction
The main factors affecting the accuracy of machine tools include static geometric errors, errors caused by dynamic loads and machine vibration, thermal mechanical coupling errors. In addition, they are also affected by working environment, control strategy and control system [1], where the static geometric errors account for about 70% [2]. Aiming at geometric error which has great influence, the basic research includes error modeling and error measurement.

The classical modeling methods of geometric error are proposed in Ref [3-5]. A vertical three axis CNC milling machine time-varying positioning using dynamic fuzzy neural network is established in Ref [6]. A comprehensive error model of three-axis CNC is proposed in Ref [7]. On the basis of previous research, this paper studies the structural characteristics of four axis gear machine tool, and establishes the comprehensive error model of four axis gear machine tool.

2. Error modeling theory
The position and attitude of $B_i$ can be obtained by the motion transformation of its adjacent low order body $B_j$. If the Cartesian right-handed Cartesian coordinate systems $O_i-x_iy_iz_i$ and $O_j-x_jy_jz_j$ are established on the typical body $B_i$ and $B_j$ respectively and are fixed with the body, then the pose of the typical body $B_j$ is equivalent to that of the coordinate system $O_j-x_jy_jz_j$ and $O_i-x_iy_iz_i$, which can be described by homogeneous transformation matrix.

The two adjacent typical bodies in relative motion have two states: relative rest at the beginning of motion and relative motion after that. Therefore, when analyzing the pose of a typical body, the
relative static pose error caused by the relative static error, the ideal motion pose and the motion pose error caused by the motion error are considered

3. Establishment of configuration and coordinate system of four axis gear machine tool
Taking the four axis gear machine tool as the object, the unit component precision, the whole machine precision and the relationship between them are studied. The configuration of four axis gear machine tool is X, Z, A, B axis, which is shown in Fig. 1.

![Figure 1. The configuration of four axis gear machine tool.](image)

The analysis of error motion and ideal motion is shown in Fig. 2. The coordinate system O₀ is located in the bed. In the workpiece chain, there are mainly coordinate systems O₁, O₂ and O₃. In the tool chain, there are coordinates O₄, O₅ and O₆. To simplify model, it is assumed that the directions of Oₖ (k=1, 2, 3, 4, 5, 6, 7) in each coordinate system coincide with the directions of O₀.

![Figure 2. The analysis of error motion and ideal motion.](image)

4. Motion analysis and description of four axis gear machine tool
Based on the rigid body hypothesis, the errors caused by workpiece clamping, tool installation and wear are ignored, and the motion pair errors which have great influence are mainly considered.

4.1. Homogeneous transformation matrix of relative rest between adjacent typical bodies
The homogeneous transformation matrix (The following abbreviation is HTM) of each adjacent typical body of the machine tool is relatively static:
The initial state error only considers the perpendicularity error of $X$ and $Z$. Therefore, error homogeneous transformation matrix (The following abbreviation is EHTM) between workpiece and $B$, between $A$ and $X$, between tool and $A$ and between $B$ and $Z$ can be obtained:

$$T_{01p} = \begin{pmatrix} 1 & 0 & 0 & 100 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$ (1)

$$T_{04p} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 200 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$ (2)

$$T_{45p} = \begin{pmatrix} 1 & 0 & 0 & 100 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$ (3)

$$T_{12p} = T_{23p} = T_{56p} = I_{4x4}$$ (4)

Let the perpendicularity errors between $X$ and $Z$ axes at the initial position $(x_P, z_P)$ be respectively $\eta_{zx}$ and $\eta_{xz}$. According to the small angle assumption, the error caused by $\eta_{zx}$ is $\Delta z_P = x_P \sin \eta_{zx} \approx x_P \eta_{zx}$, and the error caused by $\eta_{xz}$ is $\Delta x_P = z_P \sin \eta_{xz} \approx z_P \eta_{xz}$, so EHTM of $X$-axis to bed is:

$$\Delta T_{01p} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & x_P \eta_{zx} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$ (6)

EHTM between $Z$ and bed can be obtained:

$$\Delta T_{12p} = \begin{pmatrix} 1 & 0 & 0 & z_P \eta_{iz} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$ (7)

4.2. Homogeneous transformation matrix of relative motion between adjacent typical bodies

HTM and EHTM of Z axis relative to bed motion $z$ can be expressed as:
HTM and EHTM of \( B \) with respect to \( Z \) rotation \( \beta \) about \( y \) axis are

\[
T_{12s} = \begin{pmatrix}
cos \beta & 0 & sin \beta & 0 \\
0 & 1 & 0 & 0 \\
-sin \beta & 0 & cos \beta & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\]  \quad (10)

\[
\Delta T_{12s} = \begin{pmatrix}
1 & -\epsilon_{zB} & \epsilon_{yB} & \delta_{xB} \\
\epsilon_{zB} & 1 & -\epsilon_{xB} & \delta_{yB} \\
-\epsilon_{yB} & \epsilon_{xB} & 1 & \delta_{zB} \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\]  \quad (11)

There is no relative motion error between the workpiece and \( B \) fixed.

\[
T_{23s} = I_{4\times4}
\]  \quad (12)

\[
\Delta T_{23s} = I_{4\times4}
\]  \quad (13)

In the same way, HTM and EHTM of \( x \)-axis relative to bed motion \( X \) are \( T_{05s} \) and \( \Delta T_{05s} \). HTM and EHTM of a relative to \( x \) rotation about \( X \) axis are \( T_{45s} \) and \( \Delta T_{45s} \).

There is no relative motion error between the tool and \( a \), so \( T_{56s} = I_{4\times4} \) and \( \Delta T_{56s} = I_{4\times4} \).

In the above formulas, \( \delta_{ij} \) is linear displacement error. \( \epsilon_{ij} \) is angular displacement error. \( i \) is error direction, \( i = x, y, z \). And \( j \) is nominal direction of motion.

5. Comprehensive error modeling of four axis gear machine tool

The comprehensive error model of 4-axis gear machine tool in workpiece coordinate system is obtained by Section 4:
\[ E = \begin{pmatrix} e_x & e_y & e_z & 0 \end{pmatrix} \]

\[ = \left( \begin{array}{cccc} T_{04p} & \Delta T_{04p} & T_{04s} & \Delta T_{04s} & T_{45s} & \Delta T_{45s} & T_{45p} & \Delta T_{45p} & T_{56p} & \Delta T_{56p} & T_{56s} & \Delta T_{56s} \\ T_{01p} & \Delta T_{01p} & T_{01s} & \Delta T_{01s} & T_{12p} & \Delta T_{12p} & T_{12s} & \Delta T_{12s} & T_{23p} & \Delta T_{23p} & T_{23s} & \Delta T_{23s} \end{array} \right) \times P_i \]

\[ - \left( \begin{array}{cccc} T_{04p} & \Delta T_{04p} & T_{04s} & \Delta T_{04s} & T_{45s} & \Delta T_{45s} & T_{45p} & \Delta T_{45p} & T_{56p} & \Delta T_{56p} & T_{56s} & \Delta T_{56s} \end{array} \right) \times P_i \] (14)

6. Conclusions
This paper established the comprehensive error model of four axis gear machine tool. The error modeling theory is studied. The configuration of the four axis gear machine tool is analyzed. Analysis and Describe of the motion of the four axis gear machine tool are investigated. Homogeneous transformation matrix of the relative static between the adjacent typical bodies is obtained and the homogeneous transformation matrix of the relative motion between the adjacent typical bodies is clarified. The method and the integrated error model provide a theoretical basis for the machining and error analysis of complex surfaces.

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