The $\gamma p \to na^+_2(1320) \to n\rho^0\pi^+$ reactions within an effective Lagrangian approach

Yin Huang,1,2,3* Jun-Jun Xie,1,2,4† Xu-Rong Chen,1,2 Jun He,1,2,4 and Hong-Fei Zhang2,3

1Research Center for Hadron and CSR Physics, Institute of Modern Physics of CAS and Lanzhou University, Lanzhou 730000, China
2Institute of modern physics, Chinese Academy of Sciences, Lanzhou 730000, China
3School of Nuclear Science and Technology, Lanzhou University, Lanzhou 730000, China
4State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences

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We investigate the $a_2(1320)$ meson photoproduction in the $\gamma p \to na^+_2(1320)$ and $\gamma p \to n\rho^0\pi^+$ reactions within the effective Lagrangian method. For $\gamma p \to na^+_2(1320)$ reaction, by considering the contributions only from the $t$-channel $\pi^+$ exchange, we get a fairly good description of the current experimental total cross section data. We also studied the $\gamma p \to n\rho^0\pi^+$ reaction, which mainly contribute to the $\gamma p \to n\pi^+\pi^+\pi^-$ reaction. The latter reaction has measured by the CLAS Collaboration. The total cross sections, invariant mass distribution, and the Dalitz Plot of $\gamma p \to n\rho^0\pi^+$ reaction are shown, which can be tested by future experiments.

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I. INTRODUCTION

In the traditional constituent quark models (CQM), mesons are described as quark-anti-quark ($q\bar{q}$) states. This picture could explain successfully the ground states of the flavor SU(3) vector meson nonet. However, there are many meson (or meson-like) states could not be explained as $q\bar{q}$ states. For example, in the low energy scalar sector, the $\sigma(500)$, $a_0(980)$ and $f_0(980)$[3], which are proposed as the meson-meson dynamically generated states[4,5]. Besides, those meson states, with spin-parity-charge-parity $J^{PC}=0^{--}, 0^{+-}, 1^{--},$ and $2^{+-}$, etc, would be also existence, but they cannot be obtained by $q\bar{q}$ pairs within the CQM. These states are known as "exotics" and observation of them has been of great interest as it would be clear evidence for mesons beyond the classical CQM picture.

Indeed, on the experimental side, searching for the scalar-isoscalar mesons have been carried out in the $p\bar{p}$ annihilation[6], $pp$ interaction at high energies[7,8], $\pi N$ reactions[9,10], $J/\psi$ radiative decays[11,12], and photon production processes[13]. The results show that in the 1–2 GeV mass range there are several meson states that do not agree with the predictions of the CQM, and some of these states may have a significant non-$q\bar{q}$ component. As in Ref. [8], an exotic meson, $J^{PC}=1^{-+}$, $I^G=1^-$, with mass $1406\pm20$ MeV and width $180\pm30$ MeV, has been observed in the study of the exclusive reaction $\pi^-p\to\pi^0\eta n$ at 100 GeV. Ten years later, the E852 Collaboration reported another exotic resonant signal in the $\pi^{-}p\to\pi^{-}\eta p$ reaction at 18 GeV[13], the corresponding mass and width are $1370\pm16_{-30}^{+50}$ MeV and $385\pm40_{-105}^{+65}$ MeV, respectively. Furthermore, lattice and model calculation indicate that the lightest exotic meson should have $J^{PC}=1^{-+}$ and mass below 2 GeV[13,16].

Recently, the CLAS Collaboration at Jefferson Lab has contributed in looking for the exotic mesons in the $\pi^+\pi^-\pi^-$ system photoproduced by the charge exchange reaction $\gamma p \to \pi^+\pi^-\pi^-n$[17]. However, the partial wave analysis shows that the most contributions to the $\pi^+\pi^-\pi^-$ production are from the tensor mesons $a_2(1320)$ ($I^G(J^{PC})=1^-(2^{+-})$) and $\pi_2(1670)$. There is no evidence for the production of the exotic states $a_1(1260)$ and $\pi_1(1600)$ at expected levels.

Since the $\rho\pi$ channel is the main decay channel of the $a_2(1320)$ ($\equiv a_2$) meson, the $\gamma p \to na^+_2(1320)$ reaction has been studied from the $\gamma p \to n(\rho\pi)^+$ reaction by several experiments[17–21]. On the theoretical side, a phenomenological analysis, for studying the production mechanism of the exotic states, has been done in Ref. [14] with the vector meson dominance (VMD) model. They found that in the photo-production the exotic $\pi_1(1600)$ and $a_2(1320)$ meson production should be comparable, which is disagreement with the recent precise experimental measurements[17]. Thus, more theoretical studies on this issue are needed.

In the present work, with the new experimental results from CLAS Collaboration[17], we study the $\gamma p \to na^+_2(1320) \to n\rho^0\pi^+$ reactions by using the effective Lagrangian approach and the isobar model. We consider the contributions from the $t$-channel $\pi^+$ exchange for the $\gamma p \to na^+_2(1320)$ reaction. For the low energy of the $\gamma p \to n\rho^0\pi^+$ reaction, we pay especially attention on the role of the $a_2(1320)$ meson.

This paper is organized as follows. In Sect. III we present the formalism and ingredients necessary for our calculations. The numerical results are also shown. A short summary is given in the last section.
II. FORMALISM AND NUMERICAL RESULTS

The basic tree level Feynman diagrams, for the $\gamma p \rightarrow na_2^+$ ([Fig. 1] (a)) and $\gamma p \rightarrow n\rho^0\pi^+$ ([Fig. 1] (b)) reactions, are shown in Fig. 1 where we consider only the $t$–channel $\pi^+$ exchange process, while the $s$–channel and $u$–channel processes are neglected since the information of those processes is scarce and we expect these contributions to be small.

FIG. 1: Feynman diagrams for $\gamma p \rightarrow na_2^+$ and $\gamma p \rightarrow n\pi^+\rho^0$ reactions.

A. $\gamma p \rightarrow na_2^+$ reaction

Firstly, we pay attention to the reaction $\gamma p \rightarrow na_2^+$. To compute the contributions from the $t$–channel $\pi^+$ exchange, as shown in Fig. 1 (a), we use the effective interaction Lagrangian densities as used in Refs. [23, 24],

\[
L_{a_2\gamma} = \frac{g_{a_2\gamma}}{m^2_\pi} \epsilon_{\mu\nu\rho\sigma} \partial^\rho a_2^{\lambda \gamma} \partial^\sigma A^\mu \partial_\nu \pi, \
\]

\[
L_{\pi N N} = -ig_{\pi \pi N} \bar{\pi} \gamma_5 \pi \cdot \pi N, \
\]

where $g_{\pi \pi N}^2/4\pi = 14.4$, while the value of the coupling constant $g_{a_2\gamma}$ can be determined from the partial decay width of $a_2 \rightarrow \pi\gamma$, which can be easily obtained with Eq. (1),

\[
\Gamma_{a_2 \rightarrow \pi\gamma} = \frac{g_{a_2\gamma}^2}{40\pi m_\pi^2} p_\gamma^5, \
\]

with

\[
p_\gamma = \frac{M^2 - m_\pi^2}{2M}, \
\]

where $M$ is mass of $a_2(1320)$ meson.

With mass ($M = 1318.3$ MeV, $m_\pi = 139.57$ MeV), total decay width ($\Gamma_{a_2} = 107$ MeV), and decay branching ratio of $a_2 \rightarrow \pi\gamma$ [Br($a_2 \rightarrow \pi\gamma$) = 2.68 x 10^{-3}], from Eq. (3), we obtain $g_{a_2\gamma} = 1.08 \times 10^{-2}$.

As we are not dealing with point-like particles, we ought to introduce the compositeness of the hadrons. This is usually achieved by including form factors in the finite interaction vertexes. In the present work, we adopt the following form factor as used in the Bonn model [27] for the exchanged $\pi^+$ meson in the $t$-channel,

\[
F_\pi(t) = \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - t}, \
\]

where $t = q^2$ with $q$ the four momentum of the exchanged $\pi$ meson, and $\Lambda_\pi$ is the cutoff parameter.

With the effective Lagrangian densities given above, we can easily construct the invariant scattering amplitude for the $\gamma p \rightarrow na_2^+$ reaction,

\[
M = \bar{u}(s_2, p_2) A_{\sigma, \nu, \delta} \varepsilon^\sigma(k_1, \lambda_1) T^\delta\nu(s_2, \lambda_2) u(s_1, p_1), \
\]

where $s_2, p_2$ and $s_1, p_1$ denote the spin polarization variables and the four-momenta of the outgoing neutron and the initial proton, respectively, while $k_1, \lambda_1$ and $k_2, \lambda_2$ are the four-momenta and spin polarization variables of the photon and $a_2(1320)$ meson, respectively. The $\bar{u}(s_2, p_2)$ and $u(s_1, p_1)$ are the Dirac spinors for the neutron and proton, respectively, while the $\varepsilon_\mu(k_1, \lambda_1)$ and $T^\nu\rho_\mu(k_2, \lambda_2)$ are the polarization vector and the polarization tensor of the photon and $a_2(1320)$ meson, respectively. The reduced $A_{\sigma, \nu, \delta}$ reads,

\[
A_{\sigma, \nu, \delta} = \frac{i g_{\pi NN} g_{a_2\gamma}}{m^2_\pi} F_\pi^2(t) \frac{\varepsilon_\mu p_2^{\nu} k_2^{\rho} k_1^{\sigma}}{t - m^2_\pi}, \
\]

Then, the differential cross section for $\gamma p \rightarrow na_2^+$ at center of mass (c.m.) frame can be expressed as

\[
\frac{d\sigma}{d\cos \theta} = \frac{m_p m_n}{8\pi s} \left| k_2^{c.m.} \right|^2 \left( \frac{1}{4} \sum_{s_1, s_2, \lambda_1, \lambda_2} |M|^2 \right), \
\]

where $\theta$ denotes the angle of the outgoing $a_2^+$ meson relative to beam direction in the c.m. frame, $k_2^{c.m.}$ and $k_1^{c.m.}$ are the 3-momentum of the initial photon and the final $a_2^+$ meson in the c.m. frame, respectively.

The sum over polarizations, in Eq. (5), can be easily done thanks to

\[
\sum_{\lambda} \varepsilon^n(k_1, \lambda) \varepsilon^{n*}(k_1, \lambda) = -g_{\mu\nu}, \
\]

for the photon, and

\[
\sum_{\lambda} T_{\mu\nu}(k_2, \lambda) T^{\mu*\nu*}(k_2, \lambda) = P_{\mu\nu\rho\sigma}, \
\]

\[
= \frac{1}{2} \left( \bar{g}_{\mu\nu'} \bar{g}_{\rho\sigma'} + \bar{g}_{\mu'\nu'} \bar{g}_{\rho\sigma} \right) - \frac{1}{3} \bar{g}_{\mu\nu} \bar{g}_{\rho\sigma}, \
\]

for the tensor $a_2(1320)$ meson, where $\bar{g}_{\mu\nu} = -g_{\mu\nu} + \frac{k_{\mu} k_{\nu}}{M^2}$.

With the ingredients shown above, we can easily calculate the total cross sections for $\gamma p \rightarrow na_2^+$ reaction, which is shown in Fig. 2 where the solid, dashed, and dotted lines are obtained with cut off parameter $A_\pi = 0.9, 0.7$, and 0.5 GeV, respectively. The experimental data from Refs. [17, 19, 21, 23] are also shown for comparing. We see that our theoretical results, which is obtained including only the contributions from the $t$–channel $\pi^+$ exchange, can give a reasonable description of the current experimental data.
where the value of the coupling constant $g_{a_2\rho\pi}$ can be determined from the $a_2 \to \rho \pi$ reaction. The decay processes are shown in Fig. 3.

![Feynman diagrams for $a_2 \to 3\pi$ reactions.](image)

FIG. 3: Feynman diagrams for $a_2 \to 3\pi$ reactions.

Next, we pay attention to the $\gamma p \to \eta\rho^0\pi^+$ reaction, which mainly contribute to the $\gamma p \to n\rho^0\pi^+$ reaction. In the present case, we need also the interaction Lagrangian density for the $a_2\eta\rho$ vertex,

$$
\mathcal{L}_{a_2\eta\rho} = \frac{g_{a_2\eta\rho}}{m_\pi^2} \epsilon_{\mu\nu\rho\sigma} \partial^\mu a_2^{\nu\sigma} \rho^\rho \partial_\delta \phi, \quad (11)
$$

where the value of the coupling constant $g_{a_2\eta\rho}$ can be determined from the $a_2 \to \eta\rho\eta$ decay. The decay processes are shown in Fig. 3.

We follow the formalism as in Ref. [30] for the case of $N^*(1535)N\rho$ coupling. The $a_2 \to \pi\pi\pi$ partial decay width is related to the decay amplitude through

$$
d\Gamma_{a_2\to\rho^0\pi^+\to\pi^+\pi^+\pi^-} = \frac{1}{2M_5} \sum_{\mu\nu} |M_{a_2^+\to\rho^0\pi^+\to\pi^+\pi^+\pi^-}|^2 \times
$$

where $p_1, p_2, p_3$ and $E_1, E_2, E_3$ are the momenta and energies of the final mesons. The decay amplitude

$$
M_{a_2^+\to\rho^0\pi^+\to\pi^+\pi^+\pi^-} = \frac{g_{a_2\eta\rho}}{m_\pi^2} \epsilon_{\mu\nu\rho\sigma} \partial_\mu a_2^{\nu\sigma} \rho^\rho \partial_\sigma \phi, \quad (13)
$$

where $p_{a_2} = p_1 + p_2 + p_3$ and $p_\rho = p_2 + p_3$ are the four momenta of the $a_2$ meson and $\rho$ meson, respectively. The $F_\rho(p_\rho^2)$ and $G_{\sigma\beta}(p_\rho)$ are the form factor and the propagator for the $\rho$-meson, with the forms as in Ref. [30].

$$
F(p_\rho^2) = \frac{\Lambda_\rho^2}{\Lambda_\rho^2 + |p_\rho^2 - m_\rho^2|}, \quad (14)
$$

$$
G^{\sigma\beta}(p_\rho) = -i \frac{g^{\sigma\beta} - p_\rho^\sigma p_\rho^\beta}{p_\rho^2 - m_\rho^2 + im_p\Gamma_\rho}, \quad (15)
$$

with a cut-off parameter $\Lambda_\rho$ and the total decay width of $\rho$ meson, $\Gamma_\rho = 150$ MeV.

With the values of $g_{a_2\rho\pi}^2/4\pi = 2.91$ and the partial decay width $\Gamma_{a_2\to\rho^0\pi^+\to\pi^+\pi^+\pi^-} = 75$ MeV [1], we can get the coupling constant $g_{a_2\rho\pi}^2/4\pi$ as a function of the cut-off parameter $\Lambda_\rho$ as shown in Fig. 4. In the present calculation, we will take $\Lambda_\rho = 1.0$ GeV which leads to $g_{a_2\rho\pi}^2/4\pi = 1.9 \times 10^{-4}$.

![Coupling constant $g_{a_2\rho\pi}^2/4\pi$ versus the cut-off parameter $\Lambda_\rho$.](image)

FIG. 4: Coupling constant $g_{a_2\rho\pi}^2/4\pi$ versus the cut-off parameter $\Lambda_\rho$.

Turning now to the $\gamma p \to \eta\rho^0\pi^+$ reaction, by using the formalism and ingredients given above, the calculations of the differential and total cross sections for this reaction are straightforward,

$$
d\sigma(\gamma p \to \eta\rho^0\pi^+) = \frac{1}{8E_\gamma} \sum_{\alpha_1,\alpha_2} \sum_{\alpha_3,\alpha_4} |M|^2 \times
$$

$$
\frac{m_\eta d^3p_1 d^3p_2 d^3p_3}{E_1 E_2 E_3} \delta^4(M-p_1-p_2-p_3), \quad (16)
$$

where $p_i (i = 1, 2, 3, 4, 5)$ is the four momentum for photon, proton, neutron, $\rho^0$ meson, and $\pi^+$ meson, and $s_i$
(i = 1, 2, 3, 4) is the spin polarization variables of photon, proton, neutron and ρ0 meson, while $E_{\gamma}$ is the photon energy at the Lab frame. The scattering amplitude $\mathcal{M}$ is,

$$\mathcal{M} = i \sqrt{g_{\gamma a_2 \pi^0} g_{\gamma a_2 \pi^0} g_{\pi^0 N}} A_{\pi^0},$$

with

$$A_{\pi^0} = \bar{u}_n (p_3) \gamma_5 u_p (p_2) \frac{F_2^2 (t)}{t - m_\pi^2} \epsilon_{\mu \nu \rho \sigma} q_{a_2}^\rho p_{1}^\sigma (p_1) (p_2 - p_3) \delta$$

$$\times \frac{F_{a_2} (q_{a_2}^2) P^{\rho \delta} \omega}{q_{a_2}^2 - M^2 + i M \Gamma_{a_2}} \epsilon_{\alpha \beta \gamma \lambda} q_{a_2}^\alpha \epsilon^{\lambda^*} (p_4) p_5 \omega,$$

where $F_{a_2} (q_{a_2}^2)$ is the form factor for the off shell $a_2(1320)$ meson, which has the form,

$$F_{a_2} (q_{a_2}^2) = \frac{\Lambda_{a_2}^4}{\Lambda_{a_2}^2 + (q_{a_2}^2 - M^2)^2},$$

where $q_{a_2}$ is the four momentum of the $a_2(1320)$ meson and the cut off parameter $\Lambda_{a_2} = 1.0$ GeV. On the other hand, the $P_3^\rho \omega \delta$, in Eq. [18], has been defined in Eq. [10].

Then, the total cross section versus the beam energy $E_{\gamma}$ of the photon for the $\gamma p \rightarrow n \rho^0 \pi^+$ reaction is calculated by using a Monte Carlo multi-particle phase space integration program. Our predictions, with $\Lambda_{a_2} = 0.5$, 0.7 and 0.9 GeV, for the beam energies $E_{\gamma}$ from just above the production threshold 1.36 GeV to 6.0 GeV are shown in Fig. [5] by dotted, dashed and solid curves, respectively.

Furthermore, the corresponding $\rho^0 \pi^+$ invariant mass spectrum, and the Dalitz Plot for the $\gamma p \rightarrow n \rho^0 \pi^+$ reaction at beam energy $E_{\gamma} = 5.1$ GeV, which is reachable for the CLAS experiment [17], are calculated and shown in Fig. [6]. The dashed lines are pure phase space distributions, while, the solid lines are full results from our model. From Fig. [6], we can see that there is a clear peak in the $\rho^0 \pi^+$ invariant mass distribution, which is produced by including the contribution from the $a_2(1320)$ meson. Those theoretical predictions can be checked by the future experiments.

### III. SUMMARY

In this work, with the new experimental results from CLAS Collaboration [17], we perform a calculation of the $a_2(1320)$ meson photon-production in the $\gamma p \rightarrow na_2^+ (1320)$ and $\gamma p \rightarrow n \rho^0 \pi^+$ reactions within the effective Lagrangian method and the isobar model. For the $\gamma p \rightarrow na_2^+(1320)$ reaction, by considering the contributions from only the $t$-channel $\pi^+$ exchange, we get a fairly good description of the current experimental total cross section data.

The recent results of the $\gamma p \rightarrow n \rho^0 \pi^+$ reaction from CLAS Collaboration [17] show that the most contributions to the $\pi^+ \pi^- \pi^+$ production are from the tensor mesons $a_2(1320)$ and $a_2(1670)$. So, basing on our results of $\gamma p \rightarrow na_2^+(1320)$ reaction, we have studied the $\gamma p \rightarrow n \rho^0 \pi^+$ reaction, which mainly contribute to the $\gamma p \rightarrow n \pi^+ \pi^- \pi^+$ reaction. In this case, we pay especially attention on the role of the $a_2(1320)$ meson. We have calculated the total cross sections, invariant mass distribution, and the Dalitz Plot for the $\gamma p \rightarrow n \rho^0 \pi^+$ reaction. Those theoretical predictions can be tested by the future experiments.

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FIG. 6: The $\rho^0\pi^+$ invariant mass spectrum (upper panel), and the Dalitz Plot (lower panel) for the $\gamma p \rightarrow np^0\pi^+$ reaction at beam energy $E_\gamma = 5.1$ GeV. The dashed lines are pure phase space distributions, while, the solid lines are full results from our model.

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