Vegetation patterns in the arid and the semi-arid climatic zones are an interesting example of spontaneous symmetry breaking in complex systems. Competition of shrubs for a limited supply of water is the relevant process that dictates the spatial organization. The struggle for water induces an indirect interaction among shrubs, as the flora extinct if its water supply is insufficient.

The situation becomes more complicated if the resource admits spatial dynamics. Recent theoretical and experimental work reveals the dynamics of competing populations in water, where light, the limiting resource, is consumed gradually by the upper layers of aquatic phytoplankton. This model may be extended to include spatial dynamics of the fauna, but it does not support time independent patterning.

Vegetation patterns are an example of one species (shrubs) and one resource (water) system, where field studies revealed wide variety of stable, or almost stable, spontaneous segregation modes. Understanding the underlying mechanism for generation of such patterns and their observed resilience is considered as an important step toward a comprehension of the desertification process, where environmental effects like climate changing and grazing destroys the natural balance toward stable aridity.

Technically, the water-biomass system has been considered as a spatially extended nonlinear system, that, in some parameter range, may yield stripes, spots, labyrinth and other ordered arrangements attributed to a positive feedback mechanism, i.e., to the inhibition of water runoff and evaporation by the flora. However, the typical perennial vegetation patterns in the semi-arid zone are disordered, as one can easily see in Fig. (1). The generic spatial organization of perennial flora varied along the precipitation gradient: from scattered "green spots" in the arid zone through clusters of shrubs in the semi-arid zone, to an almost full coverage of the soil by biomass in the humid/subhumid climate. Analysis of the transverse correlations in the three panels of Fig. (1) shows that the correlation length in the semi-arid zone is larger (by factor of 2-3) than in the other regions, and seems to indicate weak long-range oscillations in the Mediterranean site, perhaps a precursor of Turing instability.

In this letter we present a general and simple model of the water-shrubs reaction that is able to yield all these features. Our model takes into account the intrinsic "noise", i.e., the amplification of initial fluctuations due to the minimal size needed for the survival of perennial flora. The resulting pattern is disordered but robust, thus it may be considered as the reactive equivalent of a glass. Although the concept of free energy, deep local minima and thermal equilibrium is absent in reactive systems, it still presents an example of a spontaneous breakdown of symmetry toward a disordered, long-living meta-stable state.

To present the model, let us begin from its zero dimensional ("flower pot") deterministic and continuous dynamics. With water supplied to the system at some rate \( R \) and continuum vegetation growth, the time evolution of the water-shrub system is described by the following, nondimensionalized rate equations:

\[
\frac{\partial B}{\partial t} = wB - \mu B \tag{1}
\]

\[
\frac{\partial w}{\partial t} = R - w - \lambda wB
\]

Where \( w \) stands for the available water density, \( B \) is the density of shrubs biomass, the term \( wB \) represent shrubs growth as they consume water while \( -\lambda wB \) is the...
a reaction-diffusion equations of the form, 

\[ \frac{\partial B}{\partial t} = wB - \mu B \]

\[ \frac{\partial w}{\partial t} = D\nabla^2 w + \mathbf{v} \cdot \nabla w + R - w - \lambda wB, \]

where \( \mathbf{v} \) pointed down the hillslope. Simple linear analysis implies that in the absence of cross-diffusion effects (like those considered recently by [7]), no Turing-like instability exists in that system; the steady state is a uniform covering of all the plane by the same amount of flora which corresponds to the stable fixed point and fluctuations of wavenumber \( k \) decay like \( e^{-\kappa^2 t} \).

In the desert area considered here there are two seasons, dry summer and humid winter. Eqs. (4,3) presents the winter, with “smeared” rain events. While annual flora wilt in the summer, perennial shrubs have to survive, so they must reach some threshold size before the dry season. If the winter is not long enough to allow for a full development of the plant to its stable fixed point, the survival of a shrub depends on its size at the end of the rainy season, which, in turn, depends on small fluctuations in its initial size and the consumption of water by its neighbors.

In the next winter, the existence of a shrub causes a depletion of the available soil moisture in its immediate neighborhood (roughly speaking, in an area of typical linear size \( \sqrt{1/\mu} \)), and the chance for another shrub that pops out in the depletion region to reach the threshold is lowered. The whole area is then segregated into a mosaic of water accepting and water contributing patches. This is a non-Turing mechanism that has nothing with the effect of shrubs on the overland flow. While Turing instability is characterized by some typical wavelength that sets the linear size of vegetation and bare soil patches to be equal, our model allows for clusters of arbitrary size, as indicated in Fig. (1).

The optimal segregation of the hillslope, that gives maximal biomass per unit area, is an ordered array of shrubs, each located in the lower end (or, on a flat plane, in the middle) of its contributing area. A regular or distorted “lattice” of flora is then formed, similar to the structure of atoms in a two dimensional crystal. However, this optimal scenario is rarely accomplished in nature, due to the stochastic character of the growth process itself. For simplicity, let us assume that the seed bank in the soil ensures the development of a perennial shrub if some water exist at the site. As the first shrub pops out in an empty region, the soil moisture in its surroundings (primarily downhill) is depleted, and the next shrub will not grow in this “shadowed” area. Nevertheless, as the next shrub also occurs at random, its position will be uncorrelated with the first, except that it can not pop out at the shadowed region of the first one, and so on. The process continues until all the slope is shaded (“jammed”). This stochastic growth process yields a ran-
than winters. The average amount of flora is much smaller and then persists, with negligible fluctuations, up to 500 dynamics will increase the efficiency of biomass growth), (in 1-2 rainy seasons) to a robust configuration (slower present. In the simulation, the system freezes rapidly of a threshold-noise model on a "flat" (no slope) land are perhaps in the form of grazing or climate changes. very rare, unless some intervene come from the outside, essential changes, such as the death of a plant and a for- there is not enough source area for the next shrub. Es-

Fig. 2: Numerical results of forward Euler integration of the reaction-diffusion equations (1) on 100X100 sites grid with periodic boundary conditions. A grey spot is plotted around the location of each shrub, with the size of the spot proportional to its biomass. The dotted lines present soil moisture contours around the shrubs. The simulation parameters are \( v = 0 \) (no slope) and \( D = 10, \mu = 0.2, R = 0.5, \lambda = 1.2 \) (this implies \( B_1 = 1.25 \) and \( w_1 = 0.2 \)). Initial conditions are no water and a seed of biomass taken from a square distribution between [0,0.01]. Effect of the summer is modelled at the end of each "winter" (21 time cycles) by setting all water to zero while the flora at a site is dropped to the seed level if the biomass is smaller than a threshold, with \( B_{th} = 1.2 \). The average (per site) values of water (about 0.32) and flora (0.57) reflect the inefficient use of water attributed to the glass-like structure.

Random covering of the slope by shrubs, with typical distance between nearest neighbors, but no long-range structure. The random arrangement is, however, extremely robust; although the slope is covered inefficiently by the shrubs, there is not enough source area for the next shrub. Essential changes, such as the death of a plant and a formation of another one, are discontinuous, and though are very rare, unless some intervene come from the outside, perhaps in the form of grazing or climate changes.

In Figure (3), typical results of a numerical simulation of a threshold-noise model on a “flat” (no slope) land are presented. In the simulation, the system freezes rapidly (in 1-2 rainy seasons) to a robust configuration (slower dynamics will increase the efficiency of biomass growth), and then persists, with negligible fluctuations, up to 500 winters. The average amount of flora is much smaller than \( B_1 \), and the average amount of water is larger than \( w_1 \), i.e., there is an inefficient use of the water due to the stochastic arrangement of the shrubs. To guide the eye, soil moisture contours are also plotted (dotted lines, with the water level inside smaller than 0.46) and reveal the depletion zone around each shrub. No empty site in the region maintains enough water to allow for new shrub to develop, and the whole region is “shaded” by the existing flora. Figure (4) present the results of a simulation with the same growth and diffusion parameters, but with nonzero downhill slope, and the effect on the moisture depletion zones is evident.

Various aspects of this competition scenario are similar to the adsorption of large particles at an interface. In the model of random sequential adsorption, "hard" particles are added sequentially to a D dimensional volume at random positions with the condition that no trial particle can overlap previously inserted ones. The addition process is then repeated until the system reaches its "jamming limit", at which the density saturates. The adsorbed particles density at the jamming limit is lower than the close-packed form, and the configuration of adsorbed particles is "frozen" at some disordered pattern. A shrub above threshold, with its excluded volume of depleted moisture is similar to an adsorbed “disc”. The shrubs-water model, although non-local (excess water transferred downhill) and reversible (new shrub may remove an existing plant by depleting its water resources), yields a similar jamming disordered and inefficient covering of the slope.

The above considerations about glassy structure elucidates the existence and the robustness of the vegetation patterns in the arid zone, but fails to explain the aggregation of shrubs (larger correlation length) in the semi-arid zone, or the Turing patterns [6, 7]. To explain patchiness one should consider the positive-feedback mechanism, i.e., the inhibition of water dynamics induced by the shrubs themselves. In the absence of shrubs (and other meso topography factors), water flow downhill, with some typical lateral displacement per unit length. In the presence of shrubs the flow in their vicinity is slower than that in bare soil, as a result of higher infiltration rates. In addition the microclimate under shrub is characterized by less direct radiation and smaller evaporation rate. This means that close to the shrub there are favor- able soil water conditions and more flora may grow. The “repulsive” interaction among shrubs due to the struggle.
FIG. 4: Effect of the positive feedback mechanism. While at low precipitation \((R = 0.45, \text{left panel})\) the repulsive interaction wins and no clustering occurs, at higher humidity \((R = 0.47, \text{middle panel})\) clustering is more pronounced, and at \(R = 0.5\) (right) vegetation stripes may be recognized. All other parameters are the same as in Figure (3).

for water is then balanced by an “attraction”. Accordingly, the size of a typical cluster changes along the precipitation gradient, from a single shrub at the arid limit to large clusters in the semi-arid regime. The response of the system to external parameters (climate change) and grazing seems to depend on its phase. In particular, it seems that hysteresis loops (desertification transition) like those described in [3] are a characteristic of the clustered phase. A detailed discussion of these issues will be presented elsewhere.

In Fig. (4), the results of the same simulation program with positive feedback are presented. The only new ingredient added to the simulation is a suppression of the asymmetry in the downhill water flow, with the asymmetry term \(v\) multiplied by \(exp[-5B(r)/B_{th}]\) (this is a cross-convection effect). Diffusion and evaporation remain the same as in Fig. (3). As the downhill flow becomes smaller, water tend to accumulate at shrub’s neighborhood. The transition from the arid (left) zone, with no clustering and glassy structure, to more clustered patterns and even some linear order in the semi-arid (right) is evident.

In conclusion, the generic spatial patterns due to the struggle for water are disordered frozen patterns, and the threshold-noise process dictates the vegetation spatial organization in the arid zone. The instability that yields this glassy structure is not Turing-like, thus the inhomogeneity is not characterized by typical length scale. Shrub clusters and Turing patterns emerge as further instability of this glassy structure if the precipitation is large enough, where the positive feedback dominates. It then leads first to clustering of shrubs and then to a global order in the form of “tiger bush” patterns.

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[8] Actual field experiments usually finds the soil moisture under a shrub larger than the moisture few meters away from it. The reason for that has to do with the positive feedback mechanisms that reduce water losses in this region. However, up to some limit, this water resource serves the shrub during the summer, and is not available for the use of another planet. One may regard this fraction of soil moisture as part of the biomass, while the contours in Figs. 2, 3, 4 represent available water density.
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