The measurement method of tower grounding resistance with shorter measuring wire based on green’s theorem

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Abstract
The Tower grounding grids are important power facilities for transmission line grounding protection. The accurate measurement of grounding resistance provides data support for the safe operation of the grounding grids. The existing measurement methods for grounding resistance are mainly based on the 0.618 method, which has two disadvantages: (1) The distance between the voltage electrode and the measuring point should not be less than 2.5× the length of the scattered grounding electrode; (2) the long measuring wires ensure the measurement accuracy of the 0.618 method, but the long-distance between the electrodes and measuring point makes it difficult to carry out in the complex environment. To accurately solve the problems, this paper proposes a novel measurement method of grounding resistance with shorter measuring wire based on Green’s theorem. Combined with the physical model of the actual tower grounding grid, this paper establishes the potential equations in local space around the grounding grid and analyses the distribution of the equipotential surface. The results show that the proposed novel method of measuring tower grounding resistance can conveniently measure the grounding resistance, which greatly shortens the length of the measuring wire and has better adaptability for towers in a complex environment.

1 | INTRODUCTION

The distributed electrodes measurement is one of the most accurate methods for grounding resistance, but the disadvantage of this method is that it needs a long measuring wire. The terrain where the transmission line towers widely distribute is complex and changeable, especially in mountainous areas, corridors and urban densely populated areas, so the short-distance electrodes distribution measurement method has become the focus research of the grounding resistance [1, 2].

The research of measuring the grounding resistance is an important direction research in the grounding field. Dawalibi analysed the variation of the inductive coupling with the separation distance between the current and potential leads, operating frequency, and grounding grid size in uniform and multi-layer soils, which can be used as a reference to estimate possible errors in ground impedance measurements [3]. Parise described a methodology of using multiple current electrodes at short distances, modifying the classic fall-of-potential practice so that the measurements of touch voltage and step voltage are always conservative [4]. The method could help to monitor the evolution of the grounding system in time. The traditional measurement methods mentioned above have the problems of long measuring wires and poor adaptability to the complex terrain around the tower. Short-distance measuring wires methods for the grounding grid of power transmission becomes the research focus on the measurement of grounding resistance. Alcantara proposed an approximated procedure to determine the location of the auxiliary electrodes in the fall-of-potential method, and considered the influence of the size and location of the current electrodes in order to get the correct measuring point of the potential probe [5]. Li discussed the measuring methods of grounding resistance and proposed a new measuring method, which uses a higher-order spectrum [6]. Bai ignored the influence of the
injection current in the grounding grid on the ground potential near the current electrode, proposed the curve fitting method to measure grounding resistance [7]. Salam measured the soil resistivity near the testing site and the fill-of-potential method is used to measure the grounding resistance, proposed an empirical relationship between the grounding resistance and distance between grid and current probe [8]. Wu developed a simple short distance measurement software and compared the calculation results of the software with the measurement results of the traditional 0.618 method to verify the effectiveness of the short distance measurement method [9]. Analysed the influences of the reflective coefficient of the double-layer soil, the top-layer soil thickness and the length of vertical grounding rods on the resistance-reducing ratio by the CDEGS software package, and provided theoretical support for the reasonable selection of the length of vertical grounding rods [10]. Based on the electromagnetic field theory, Gouvalas derived the expression of Green's function in the horizontal double-layer soil structure using the complex image method, and compiled the grounding calculation program considering the resistance and mutual inductance of grounding conductor [11]. Zhou established the transient finite element analysis model of lightning impulse of grounding grid, and explained the calculation process of the model through theoretical modelling and software operation [12]. Yang studied the calculation method of the grounding grid parameters, solved various parameters of the grounding grid by the moment method, and calculated the soil layered structure by solving the Green's function in the multi-layer soil by the complex image method [13]. KOSTIC proposed an algorithm for estimating the total grounding resistance of complex grounding systems with the contact resistance included, and derive the simple formulas by which the total grounding resistance of the analysed grounding system can easily be calculated [14].

The above researches about traditional grounding resistance measurement methods, such as the 0.618 method, the 30 angle method and the multi-electrodes method, are based on the assumption that the grounding grid is equivalent to a hemispherical model, which means the soil structure of the hemispherical model is single and homogeneous [15, 16]. In AC grounding system, in the grounding system of AC transmission line tower, most of the current overflows and diffuses in the shallow layer of soil, which can hardly reach the deeper soil. In the study of the grounding performance of transmission line tower grounding grid, the uniform soil model can meet the engineering requirements without considering the multi-layer soil model.

The measurement method all have the problems of long measuring wires and poor adaptability in the complex environment, which can not meet the actual needs of the engineering project. The above researches about shorten the length of measuring wires mainly based on the theory of compensation method, which is practical but needs a long distance to make the equivalent hypothesis hold. To solve the above disadvantages, the measurement method of tower grounding resistance with shorter measuring wire based on Green's Theorem is proposed. Based on the physical model of tower grounding grid, the potential equation of tower grounding grid based on Green's theorem is established, the distribution of equipotential surface on earth affected by the test current in grounding grid is analysed, the calculation expression of tower grounding resistance is established to form the novel measurement method of tower grounding resistance with shorter measuring wire. Finally, the cases of tower grounding grids with different structure sizes in different soil resistivity are analysed, which verifies the applicability of the proposed measurement method. Therefore, the proposed measurement method has advantages of wide adaptability, high accuracy and shorter measuring wires, which can provide a new idea for the measurement of tower grounding resistance.

2 | POTENTIAL THEORY BASED ON GREEN'S THEOREM

2.1 | Cigrid potential

The potential equation proposed in this paper is based on Green’s theorem. The Green’s theorem is widely used in the field theory research because of its simple and flexible formula form [17–19].

The point to measure the tower grounding resistance is to measure the tower grounding potential. The potential distribution in the closed field in Figure 1 is studied. Based on this, the expression of the grounding potential is analysed.

Take any point in the field $V'$ surrounded by the closed surface $S'$ as the origin of the coordinate, and make the point $\vec{r}'$ under the coordinate as the potential reference point, $\vec{r}$ as any fixed point different from the point $\vec{r}'$ in the volume $V'$. The volume charge density distribution at any point is $\rho(\vec{r}')$, and $\varphi(\vec{r})$ represents the potential generated by all charges at the point $\vec{r}$. Bring Equations (2) and (3) into Green's second identity (1) [20].

$$\int_{V'} (\psi \nabla^2 \varphi - \varphi \nabla^2 \psi) \, dV' = \oint_{S'} (\psi \nabla \varphi - \varphi \nabla \psi) \cdot dS$$ (1)

$$\nabla^2 \varphi (\vec{r}) = \frac{\rho(\vec{r})}{\varepsilon(\vec{r})}$$ (2)

FIGURE 1 The potential distribution in the closed area
The effect of the Green boundary on the grounding grid potential

It is necessary to inject the test current into the tower grounding grid when measuring the tower grounding resistance. Because the grounding grid is generally composed of materials with good conductivity, the injection current will discharge into the earth through the grounding grid. If the burial depth of the grounding grid is ignored, the grounding grid can be approximately regarded as the composition of countless point current sources on the earth’s surface. Furthermore, it is also necessary to arrange several current-electrodes on the earth’s surface to reflux the injection current for forming a current circuit. The current-electrodes can be studied as the point current sources on the earth’s surface.

Boundary conditions are analysed when a single point current source discharges into uniform and flat earth. As shown in Figure 3, the outer surface of the enclosed area \( V' \) is consist of the flat upper surface \( S_1 \) and curved surface \( S_2 \). If the current source is injected at any point on the \( S_1 \) (which can be inside or outside \( S_1 \)), the potential at any point can be expressed as Equation (5):

\[
\phi(\vec{r}) = \frac{1}{4\pi\varepsilon} \int_{V'} \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} dV' + \frac{1}{4\pi} \oint_{\partial V} \frac{\partial \phi(\vec{r}'))}{\partial n} dS' - \frac{1}{4\pi} \int_{S_1 + S_2} \phi(\vec{r}')) \frac{\partial}{\partial n} \left( \frac{1}{|\vec{r} - \vec{r}'|} \right) dS'
\]  

(5)

In Figure 3, the Cartesian coordinate grid which regards the \( xoy \) plane with point current source origin is established. Under the action of the point current source, the potentials of any point \((x, y, z)\) can be expressed as Equation (6):

\[
\phi(\vec{r}) = \frac{\rho_r I}{2\pi \sqrt{(x^2 + y^2 + z^2)}}
\]

(6)

Where \( \rho_r \) is the soil resistivity, \( I \) is the injection current on the earth’s surface. Therefore, any point on \( S_1 \) satisfies (7):

\[
\frac{\partial \phi(\vec{r}')}{\partial n} \bigg|_{S_1} = -\frac{\rho_r I}{2\pi} \left( x^2 + y^2 + z^2 \right)^{-\frac{3}{2}} Z \bigg|_{S_1} = 0
\]  

(7)
Equation (7) shows that \( \frac{\partial \varphi'(r')}{\partial n} = 0 \) is valid for any point on \( S_1 \). Next, the term \( \frac{\partial \varphi'}{\partial n} \left( \frac{1}{| \vec{r} - \vec{r}' |} \right) \) for any point on \( S_1 \) is studied below.

\[ M(H_M, y_M, z_M) \]

is the corresponding point of \( \vec{7} \) in the enclosed area and \( N(x'_M, y'_M) \) is the corresponding point of \( \vec{r}' \) on the surface \( S_1 \). Obviously, \( z'_M = 0 \) is valid and Equation (8) can be obtained:

\[
|\vec{7} - \vec{r}'| = \sqrt{(x_M - x'_M)^2 + (y_M - y'_M)^2 + (z_M - z'_M)^2} \tag{8}
\]

It is not difficult to see that the positive direction of the \( Z \)-axis is the normal direction of \( S_1 \) from Figure 3, so Equation (9) can be obtained:

\[
\frac{\varphi'}{\partial n} \left( \frac{1}{| \vec{r} - \vec{r}' |} \right) = \left( (x_M - x'_M)^2 + (y_M - y'_M)^2 + (z_M - z'_M)^2 \right)^{-\frac{3}{2}} \left( z_M - z'_M \right) \tag{9}
\]

\( z_M = z'_M = 0 \) is valid when the buried depth of the grounding grid is ignored, thus \( \frac{\varphi'}{\partial n} \left( \frac{1}{| \vec{r} - \vec{r}' |} \right) = 0 \) for any point on \( S_1 \) can be obtained.

The current-electrodes which are used to retrieve the injection current need to be distributed around the grounding grid when measuring tower grounding resistance. The grounding grid and the current-electrode can be regarded as a plurality of point current sources function on the earth surface, and the potential on \( S_1 \) is the superposition of all point current sources function. In particular, if the \( S_2 \) is an equipotential surface, Equation (10) can be obtained:

\[
\varphi(\vec{r}) = \frac{1}{4\pi \varepsilon} \int_{\vec{r}'} \frac{\varphi'(\vec{r}')}{| \vec{7} - \vec{r}' |} dV' + \left( \frac{1}{4\pi} \int_{S_2} \frac{\partial \varphi(\vec{r}')}{\partial n} dS' + \frac{\varphi(\vec{r}')}{4\pi \Omega} \right) \tag{10}
\]

Where \( \Omega = \int_{S_2} \frac{\vec{7} - \vec{r}}{| \vec{7} - \vec{r} |} dS' \) is the solid angle of the \( S_2 \) to the point \( \vec{7} \). The point \( \vec{7} \) is a point on the earth surface when the buried depth of the grounding grid is ignored, then \( \Omega = \int_{S_2} \frac{\vec{7} - \vec{r}}{| \vec{7} - \vec{r} |} dS' = 2\pi \) can be derived. Therefore, the grounding grid potential can be expressed as Equation (11):

\[
\varphi(\vec{r}) = \frac{1}{4\pi \varepsilon} \int_{\vec{r}'} \frac{\varphi'(\vec{r}')}{| \vec{7} - \vec{r}' |} dV' + \left( \frac{1}{4\pi} \int_{S_2} \frac{1}{| \vec{7} - \vec{r}' |} \frac{\partial \varphi(\vec{r}')}{\partial n} dS' + \frac{\varphi(\vec{r}')}{2} \right) \tag{11}
\]

The Equation (11) shows that the influence of the curved surface potential on the grounding grid potential can be directly expressed by \( \frac{\varphi(\vec{r}')}{2} \) if \( S_2 \) is an equipotential surface in Green’s boundary condition.

2.3 | The effect of space charge distribution on the grounding grid potential

The grounding grid potential is not only affected by the boundary potential condition but also related to the distribution of bulk charge density in the closed field of the enclosed ground grid. In the local space around the grounding grid, both electrical conductivity \( \gamma \) and dielectric constant \( \varepsilon \) are scalar functions in the local space. Equations (12) and (13) can be obtained from Gauss flux theorem and current continuity equation:

\[
\nabla \cdot \vec{E} = \nabla \varepsilon \cdot \vec{E} + \varepsilon \nabla \cdot \vec{E} = \rho \tag{12}
\]

\[
\nabla \cdot \vec{j} = \nabla \gamma \cdot \vec{E} + \gamma \nabla \cdot \vec{E} = 0 \tag{13}
\]

Equation (14) can be obtained from Equation (13):

\[
\nabla \cdot \vec{E} = -\frac{\nabla \gamma \cdot \vec{E}}{\gamma} \tag{14}
\]

Subtract Equation (13) from Equation (12) and substitute Equation (14) into it to obtain:

\[
\rho = \gamma E \cdot \frac{\nabla \varepsilon \gamma - \varepsilon \nabla \gamma}{\gamma^2} = \vec{j} \cdot \nabla \left( \frac{\varepsilon}{\gamma} \right) \tag{15}
\]

The Equation (15) shows that there is no volume charge in the soil if the soil is regarded as a homogeneous medium in the closed region which is composed of grounding grids and soil. At the interface of soil and grounding grid, \( \nabla \cdot \vec{E} \) is not zero because of the difference between the dielectric constant \( \varepsilon \) and the conductivity \( \gamma \) of the grounding grid and the soil. Therefore, there must be a volume charge accumulation at the interface between the soil and the grounding grid when the component of the overflow current perpendicular to the surface of the grounding grid flows through the interface between the grounding grid and the soil.

3 | SPACE POTENTIAL ANALYSIS OF GROUNDING GRID

The space where the grounding grid is located includes the space where the grounding grid itself is located and the surrounding soil space. Obtaining the grounding grid potential is necessary for measuring the tower grounding resistance. It can be known from the Equation (11) that grounding grid potential is related to soil potential distribution. In the analysis, space rectangular coordinate grid is established as shown in Figure 2.
3.1 Space potential distribution affected by the grounding grid current

The tower grounding grid with an injection current is simulated in the analysis. Low carbon steel is used for both horizontal and vertical grounding grid and the soil is equivalent to a hemispherical model. In the simulation model, the length of tower edge is 10 m, the length of the scattering grounding electrode is 20 m, the diameter of the grounding grid is 12 mm and the buried depth is 0.6 m. The conductivity of low carbon steel is 8.41 MS/m and the relative permittivity is 1. In the soil hemispherical model, set the model radius to 500 m and the relative permittivity of soil to 16. In the simulation analysis, the boundary of the soil hemispherical model is set as a physical boundary condition and the direct current of 1 A is injected into a ground lead. To study the universality of soil potential, different soil conductivities are used in soil parameters, inject 1 A DC test current at one of the ground leads. The parameter size of the grounding electrodes is much smaller than that of the hemispherical grounding model. First, the grounding grid is segmented, and the maximum and minimum unit sizes of the free triangle mesh are both 0.012 m. The simulation mainly studies the distribution of the current field, more intensive sectioning is set up at electrodes’ location, the maximum unit size of the free tetrahedron is 5 m, and the minimum unit size is 0.012 m. From the grounding electrodes to the boundary of the soil model, the coarser mesh is adopted, with the maximum unit size of 100 m and the minimum unit size of 0.5 m [21, 22]. The space potential distribution around the grounding grid is shown in Figure 4.

Figure 4(a–d) are soil potential distributions when the soil conductivity is 0.002, 0.008, 0.014 and 0.020 S/m respectively. It can be seen from Figure 4 that the soil potential distribution has similar properties in different soil conductivity. The potential of the tower grounding grid is the highest. With the increase of the distance from the grounding grid, the soil potential gradually decreases and tends to zero, and the distance between adjacent equipotential surfaces gradually decreases. According to the distribution of soil potential equipotential surface and Gauss flux theorem, the existence of equipotential surface around the tower grounding grid indicates that there must be accumulated charge at the interface between the grounding grid and the soil, which is the inevitable result of the difference between the dielectric constant $\varepsilon$ and conductivity $\gamma$ of the grounding grid and the soil, and is consistent with the above theoretical analysis. Besides, the simulation results show that there is a certain distance between the area with the hemispherical surface of the equipotential surface and the grounding grid, which is the root cause of the long measuring wires between the electrodes of the traditional measurement method.

3.1.1 Space equipotential surface distribution affected by the current-electrode current

To retrieve the test current injected into the grounding grid, several current electrodes need to be arranged in the process of measuring the grounding resistance, and then a measuring current circuit is formed. The above analysis is only to the space potential distribution affected by the grounding current alone, so it is necessary to consider the influence of current-electrode current on the potential distribution of soil and grounding grids. Because of the symmetry of the grounding grid, four
FIGURE 5 Soil potential distribution of soil affected by the current-electrode current in different soil conductivity (a) the soil conductivity is 0.002 S/m, (b) the soil conductivity is 0.008 S/m, (c) the soil conductivity is 0.014 S/m, (d) the soil conductivity is 0.020 S/m.

identical current-electrodes are placed at points (25,0,0), (0,25,0), (−25,0,0), (0,−25,0). The distance between the four current-electrodes and the tower centre equals the distance between the end of the scattering grounding electrode and the tower centre. In the simulation, the current excitation of each current electrode is −0.25 A. Four current electrodes are symmetrically distributed, and one current electrode is randomly selected for the amplification study. The potential distribution affected by the current-electrodes current is simulated and analysed under the condition that other parameters remain unchanged, and the equipotential surface distribution is shown in Figure 5.

Figure 5(a–d) are soil potential distributions when the soil conductivity is 0.002, 0.008, 0.014 and 0.020 S/m respectively. The results show that the equipotential surface is an approximately concentric hemispherical surface centred on the current electrode. There is no obvious equipotential surface surrounding the grounding grid in the area space near the grounding grid, which shows that the effect of charge accumulation on the interface between the grounding grid and the soil is not obvious affected by the current-electrode current, and the accumulated charge is close to zero. On the contrary, the dense equipotential surface distribution around the current electrodes indicates that there are more accumulated charges on the current electrodes, which is the inevitable result of high current flowing through the current-electrode and small current flowing through the grounding grid.

The characteristics of soil potential distribution affected by the current-electrode current show that when the current electrode is used to retrieve the overflow current in the soil, the accumulated charge on the interface between the current electrode and the soil is more, while the accumulated charge on the interface between the grounding body and the soil is less, which makes it possible to ignore the minimal accumulated charge on the current electrode when studying the space of the tower grounding grid affected by the current-electrode current. To further analyse the equipotential surface distribution in the soil affected by current-electrode current, the equipotential line distribution on the earth’s surface is plotted in Figure 6 based on the results in Figure 5.

The solid line in Figure 6 is an equipotential line on the earth’s surface only affected by current-electrode current. The results show that it is difficult to find a closed equipotential surface inside the four current-electrodes, and the dotted line in Figure 6 is a closed curve selected between equipotential line I and equipotential line II. Because the potential difference between adjacent equipotential surfaces is small, the red closed curve can be approximated as the first equipotential line, and an approximate equipotential surface can be similarly selected between the equipotential line I and equipotential line II. The approximate equipotential surface can be used as a Green’s
3.1.2 Grounding grid potential affected by the current-electrode current

The actual grounding resistance is the ratio of the grounding grid potential affected by the current-electrode current to the injection current on grounding grid. However, the grounding potential will be affected by the current-electrode current when measuring the grounding resistance. So it is necessary to analyse the grounding grid potential affected by current-electrode current.

For the convenience of explanation, “ι” represents the grounding grid, “ψ” represents the Green’s Boundary, “1” means the influence of the current in grounding grid, “2” means the influence of current-electrode current and “12” means that the current of grounding grid and current electrode act at the same time in the subscripts of the equation variables.

Since the grounding grid occupies a very small size in the soil space, the current density flowing through the grounding grid is inevitably small affected by the current in current-electrode. The current flowing into the grounding grid and the current flowing out of the grounding grid is equal, which produces potential of the point on the approximate equipotential plane as the potential of the equipotential Green’s boundary surface. Figure 7 is a distribution diagram of the soil equipotential surface affected by the current in current-electrode. The dotted line in Figure 6 is a hemispherical surface that surrounds the grounding grid. Except for the points on both extreme directions of the x-axis, the distance from the midpoint (5,0,0) to most points on $S_2$ is not much different. The dotted line in Figure 6 is a hemispherical surface with the point (5,0,0) as the centre, and the hemispherical surface is high coincident with the surface $S_2$. Therefore, the distance $|\vec{r} - \vec{r}'|$ from the point (5,0,0) to all points on the surface $S_2$ can be approximated as a constant value $r$. Considering that the electric field intensity in the normal direction of the upper surface $S_1$ is zero and the term $\frac{\partial \varphi_2(\vec{r}')}{\partial n}$ is small with little error due to the sparse equipotential surface distribution around the grounding grid, Equation (17) can be approximately expressed as Equation (18):

$$
\frac{1}{4\pi \varepsilon} \int_{S_2} \frac{1}{|\vec{r} - \vec{r}'|} \varphi_2(\vec{r}') dS' \approx \frac{1}{4\pi \varepsilon} \int_{S_2} \frac{1}{|\vec{r} - \vec{r}'|} \overrightarrow{E}_2 dS' \quad (17)
$$

$$
\frac{1}{4\pi \varepsilon} \int_{S_2} \frac{1}{|\vec{r} - \vec{r}'|} \varphi_2(\vec{r}') dS' \approx \frac{1}{4\pi \varepsilon r} \oint_{S_1 + S_2} \overrightarrow{D}_2 dS' \quad (18)
$$

Where $r$ is the approximate constant of $|\vec{r} - \vec{r}'|$, $\overrightarrow{D}_2$ is the electric displacement vector only affected by the current-electrode current, and $\oint_{S_1 + S_2} \overrightarrow{D}_2 dS'$ is the total net charge enclosed by $S_1$ and $S_2$, also known as the accumulated net charge on the grounding grid. Equation (19) can be obtained if the accumulated net charge is ignored:

$$
\frac{1}{4\pi} \int_{S_2} \frac{1}{|\vec{r} - \vec{r}'|} \varphi_2(\vec{r}') dS' = 0 \quad (19)
$$
The \( Q_s \) in Equation (19) is the accumulated net charge at the interface between soil and grounding grid. Then the Equation (20) can be derived through Equations (19), (16) and (19):

\[
\varphi_{b_2}(\vec{r}) \approx -\varphi_{b_2}(\vec{r}') / 2
\]

(20)

When measuring the tower grounding resistance, it is necessary to calculate the influence of the current-electrode current on the grounding grid potential. There are two ways to calculate the influence of current-electrode current on the grounding grid potential: one way is to treat the soil around the grounding grid as a homogeneous medium, and then the grounding grid potential can be obtained by the superposition principle. The other way is to obtain the grounding grid potential indirectly by the boundary potential \( \varphi_{b_2} \) according to Equation (20). However, as discussed in Section 3.2, the space where the grounding grid locates becomes an equipotential body and is no longer a pure soil medium owing to the presence of the grounding grid. Therefore, whether the soil around the grounding grid can be treated as a homogeneous medium when calculating the grounding grid potential according to the superposition principle is worthy of further analysis.

To study whether the space medium of the grounding body can be treated as a completely homogeneous soil medium, the potential distribution with buried grounding grid in soil and the potential distribution without buried grounding grid in the soil are compared. If the calculated space potential distribution in the two different models is quite different, it means that the local space of the grounding grid cannot be regarded as the approximate calculation of the complete homogeneous soil. If space potential distribution in the two models is the same in a certain area, it means to some extent that part of the space between the grounding grid can be treated as the approximate calculation of the complete homogeneous soil, the potential of any point on the surface can be calculated by the superposition principle of multi-point current sources:

\[
V' = \sum_{i=1}^{4} \frac{\rho \times I}{8\pi r_i}
\]

(21)

Where \( V' \) is the potential of the point different from the four current electrodes on \( S_1 \), \( r_i \) is the distance from the calculated point to the \( i \)th current electrode, \( \rho \) is the soil resistivity, and \( I \) is the total current injected on the four current electrodes with the value of \(-1A\). To facilitate the analysis, the potential distribution without a buried grounding grid is also calculated by simulation, which is the same as the calculation result through the superposition principle.

The earth potential distribution along the extending direction of the scattering grounding electrode (as shown in Figure 2) is shown in Figure 8. Figure 8(a) shows the potential distribution with buried grounding grid in soil, Figure 8(b) shows the potential distribution without buried grounding grid in soil. Figure 8 shows the horizontal axis between 7m and 19m is the area where the scattering grounding electrode locates. It can be seen that the potential distribution with buried grounding grids and the potential distribution without buried grounding grids are very different. In Figure 8(a), because the grounding grid is equipotential, the axial potential distribution between 7 and 19 m is a straight curve, while in Figure 8(b), the axial potential in this area does not have such a straight line, but a smooth curve. Due to the weakening of the effect of the potential body, the local effect of the grounding grid can be ignored in the area far away from the grounding grid, the physical model with the grounding body embedded, so the simulation results of the two are relatively close, with little difference.

The simulation results show that the soil close to the grounding grid cannot be regarded as the approximated homogeneous medium when using the superposition principle to calculate the grounding potential affected by current-electrode current. When using the superposition principle to calculate the potential of a point far away from the grounding grid including \( \varphi_{b_2} \), the soil can be approximated as a homogeneous medium, which will generate little error. Therefore, the influence of current-electrode current on the grounding grid potential can be obtained by calculating the boundary potential \( \varphi_{b_2} \) which is far away from the grounding grid.
3.2 Spatial potential distribution affected by the grounding grid current and current-electrode current

In the process of potential measurement, it is necessary to find the supposed zero potential point, and then obtain the potential of the tested point by measuring the voltage between the tested point and the supposed zero potential point.

Figure 9(a, b) are distribution diagrams of equipotential surface affected by the current of grounding grid and current-electrode when the soil conductivity is 0.01 S/m. The yellow line mark in Figure 9 is the point (40,0, 0) on the equipotential surface, the distance is twice the length of the scattering tower grounding electrode. From the simulation results, it can be seen that the soil equipotential surface affected by the grounding grid current and the current-electrode current basically coincides outside the yellow line mark, that is, the electric field intensity outside the yellow line mark is the same affected by the two currents, and the direction is opposite. Because the potential affected by two kinds of current is zero potential point at infinite distance, it can be inferred from the superposition principle that when the test current is injected into the grounding grid and four current electrodes are used to retrieve the current, the potential outside the yellow line tends to zero value, and then the point in the soil that is more than twice the length of the scattered grounding electrode from the grounding grid is taken as the supposed zero potential point directly.

In order to verify the correctness of above inference, the earth potential affected by the grounding grid current and current-electrode current is simulated and analysed. In the simulation model, the current of 1 A is injected into the grounding grid and the current of −0.25 A is injected into all current-electrodes to retrieve the injection current. Figure 10 shows the results when the soil conductivity is 0.01 S/m.

The simulation results show that the potential at the grounding grid is the highest, the potential at the current electrodes is the lowest, and the grounding surface potential far away from the grounding grid tends to zero, which is consistent with the above inference results. The above results show the feasibility of the point in the soil area twice the length of the scattered grounding electrode from the grounding grid as supposed zero potential points.

4 THE MEASUREMENT METHOD OF TOWER GROUNDING RESISTANCE WITH SHORTER MEASURING WIRE BASED ON GREEN’S THEOREM

The potential of the tower grounding grid and its surrounding space potential have been analysed in detail in the previous paper. In this chapter, based on the simplified potential expression affected by the current-electrode current, the measurement method of tower grounding resistance with shorter measuring wire based on Green’s Theorem is proposed.

4.1 Space potential distribution affected by the grounding grid current

The calculation expression of tower grounding resistance can be obtained by the principle of potential superposition. The grounding grid potential affected by the current in grounding grid and current-electrode is \( \varphi_{g1} \), the grounding grid potential affected by the grounding current can be expressed as Equation (22):

\[
\varphi_{g2} = \varphi_{g1} - \frac{\varphi_{e2}}{2}
\]
The expression of tower grounding resistance can be obtained as Equation (23):

\[ R = \frac{\varphi_{g1} - \varphi_{g2}}{I} \]  

(23)

In Equation (23), \( \varphi_{g12} \) can be obtained by measuring the voltage between the grounding grid and the actual zero potential point. The analysis results in the previous chapters show that the ground potential can be selected as the supposed zero potential points at the twice length of scattering ground electrode away from the grounding grid along the direction of the scattering ground electrode.

The point of the novel measurement method is to arrange the electrodes appropriately, measure the grounding potential affected by the grounding grid current and current-electrode current, and compensate the influence of the current-electrode current on the grounding grid potential with the Green’s boundary potential, to obtain the grounding grid potential affected by the current-electrode current alone. The ratio of the potential to the grounding grid current is the tower grounding resistance value.

### 4.2 Distance of current electrode distribution and location of Green boundary reference point

The distribution distance of current electrodes and the proposed zero points are important factors affecting the measurement results of grounding resistance. It has been proposed that the midpoint of any sideline of the tower grounding grid can be used as the proposed zero points, and the equipotential surface can be approximately a hemispherical surface with the point as the spherical centre. A more accurate approximate equipotential surface, i.e. Green’s boundary surface, can be formed by determining the proper position of the current electrodes. Choosing a reasonable reference potential point can guarantee the correct choice of Green’s boundary, so the conclusion that the effect of current-electrode current on the grounding grid potential \( \varphi_{g2}(\mathbf{r}) \approx -\varphi_{g1}(\mathbf{r}) / 2 \) can be obtained. Therefore, the determination of the current electrodes distribution distance and the Green’s boundary reference point location becomes the focus of this paper. In the analysis, the Green’s boundary reference point is selected on the x-axis, and the earth’s surface which is twice the length of the scattering grounding electrode from the tower foundations centre is taken as the supposed zero potential points.

To study the distribution distance of current electrodes and the location of Green’s boundary reference point, different points on the x-axis are selected as Green’s boundary reference points to calculate the grounding resistance while different current pole distribution distances are used. The 1/3 segment point (12, 0, 0) between the current electrode and the tower edge which is more closed to the tower edge, the midpoint (15, 0, 0) between the current electrode and the tower edge and the 1/3 segment point (18, 0, 0) between the current electrode and the tower edge which is more closed to the current electrode are regarded as the Green’s boundary reference point to calculate the grounding resistance. Through the simulation analysis, the actual potential and the actual grounding resistance of the tower grounding grid are obtained when measuring the grounding resistance by electrodes distribution. The grounding resistance measured by the method proposed in this paper is calculated by the Equation (23). The error between the measured tower grounding resistance and the actual tower grounding resistance are listed in Table 1.

The data in Table 1 shows that the relative error of the measurement results will be greatly different with the change of the current-electrode distance and the location of the reference point, the relative error is smallest with the value of 0.7% when the 1/3 segment point (12, 0, 0) is selected as the Green’s boundary reference point. The above analysis shows that when the soil conductivity is 0.01 s/m, the potential reference point is (12, 0, 0), the calculation error of the grounding grid potential will be less than 0.045V, and the relative error of the grounding resistance will be less than \( \frac{0.045}{0.25} \times 100\% = 2.2\% \).

The above results show that when the novel measurement method proposed in this paper is used to measure the grounding resistance, the arrangement of the current electrode at a position equivalent to the distance of the scattered grounding electrode can form an approximate equipotential hemispherical surface, and the 1/3 segment point close to the tower foundation is a more accurate Green’s boundary reference point.

The Green’s boundary reference point location plays an important role in the accuracy of grounding resistance measurement, some slight deviations on reference point location may result in large errors in the measurement results. The influence of different Green’s boundary reference points on the measurement accuracy of ground resistance is studied, the reference points selected in the analysis are on the x-axis, which near the 1/3 segment point close to the tower foundation.

It can be seen from Figure 11 that there is little difference in earth’s surface potential near the 1/3 segment point (12, 0, 0). When the soil conductivity is 0.002 S/m, the potentials at point (10, 0, 0) and point (14, 0, 0) are –3.28 V and –3.52 V, respectively. If the grounding resistance is measured with these two points.
potential distribution on the x-axis affected by the current-electrode current

**TABLE 2** Relative difference error table

| Soil conductivity [S/M] | Actual earth resistance [Ω] | (10,0,0) point potential [V] | (14,0,0) point potential [V] | Relative error [%] |
|-------------------------|-----------------------------|------------------------------|------------------------------|-------------------|
| 0.002                   | 10.100                      | −3.282                       | −3.525                       | 1.188             |
| 0.004                   | 5.053                       | −1.641                       | −1.762                       | 1.197             |
| 0.006                   | 3.371                       | −1.094                       | −1.175                       | 1.201             |
| 0.008                   | 2.529                       | −0.821                       | −0.881                       | 1.186             |
| 0.010                   | 2.025                       | −0.656                       | −0.705                       | 1.209             |
| 0.012                   | 1.688                       | −0.547                       | −0.588                       | 1.214             |
| 0.014                   | 1.448                       | −0.469                       | −0.504                       | 1.209             |
| 0.016                   | 1.268                       | −0.410                       | −0.441                       | 1.222             |
| 0.018                   | 1.127                       | −0.365                       | −0.392                       | 1.198             |
| 0.02                    | 1.015                       | −0.328                       | −0.352                       | 1.182             |

The relative difference is 1.188% for the actual grounding resistance at this soil conductivity is 10.1 Ω. Similarly, the relative difference in different soil conductivity can be obtained as shown in Table 2.

Table 2 shows that the measurement error is small when the points around point (12, 0, 0) is selected as the Green’s boundary reference point. There is a large selection range for a reference point that can control the measurement error of the grounding resistance within the allowable measurement error range of the engineering. The slight deviation of the Green’s boundary reference point will not bring a huge error in the measurement result of the grounding resistance, and the stability of the measurement result is good.

**TABLE 3** Grounding resistance of tower with different measurement methods

| Tower mark          | The 0.618 method [Ω] | The clamp method [Ω] | The short distance method [Ω] |
|---------------------|----------------------|----------------------|-------------------------------|
| Chenxue West line 12| 2.55                 | 2.58                 | 2.53                          |
| Pingxue West line 11| 3.35                 | 3.39                 | 3.45                          |
| Pingxue East line 12| 3.41                 | 3.57                 | 3.34                          |
| Pingxue East line 13| 2.62                 | 2.67                 | 2.56                          |
| Xueshan North line 5| 2.38                 | 2.52                 | 2.44                          |
| Xuezhong South line 6| 3.45                 | 3.58                 | 3.47                          |

5 | APPLICABILITY OF MEASUREMENT METHOD

The measurement method of tower grounding resistance with shorter measuring wire based on Green’s theorem is very applicable, and it has a great advantage with shorter measuring wire compared with the traditional method. The applicability and superiority of the method proposed are demonstrated below.

5.1 | Simulation cases analysis of the novel measurement method

To verify the applicability of the method, different grounding grid models are applied for simulation analysis. The distance from the four current electrodes to the tower foundation centre is the same as the distance from the end of the scattering grounding electrodes to the tower centre. The 1/3 segment point between the current electrode and the tower edge which is more closed to the tower edge is selected as the Green’s boundary reference point and the point twice the length of the scattering grounding electrode from the tower foundation centre is taken as the supposed zero potential points. Tables 3-6 show the simulation comparison analysis results in different grounding grid models.

The dates in tables indicate that the accurate measurement results in different grounding grids can be obtained by the proposed measurement method in this paper. In general, the measurement error can be controlled within 5% effectively which can meet the actual measurement accuracy requirements in engineering. These cases show that the proposed method has high measurement accuracy and great applicability.

5.2 | Field experiment analysis of the novel measurement method

To verify the accuracy of this method in the actual field, the grounding resistance of some towers in Jiangjin District of
TABLE 4  The simulation results of the grounding grid root 5m, the scattering grounding grid 15 m, and the current pole distance 20 m

| Soil conductivity [S/M] | Actual earth resistance [Ω] | Measured resistance value of ground electrode [Ω] | Relative error [%] |
|-------------------------|-----------------------------|--------------------------------------------------|-------------------|
| 0.002                   | 19.407                      | 19.409                                           | 0.0103            |
| 0.004                   | 9.705                       | 9.706                                            | 0.0103            |
| 0.006                   | 6.471                       | 6.472                                            | 0.0155            |
| 0.008                   | 4.854                       | 4.856                                            | 0.0412            |
| 0.01                    | 3.884                       | 3.885                                            | 0.0257            |
| 0.012                   | 3.237                       | 3.238                                            | 0.0309            |
| 0.014                   | 2.775                       | 2.777                                            | 0.0721            |
| 0.016                   | 2.429                       | 2.430                                            | 0.0412            |
| 0.02                    | 1.944                       | 1.945                                            | 0.0514            |

TABLE 5  The simulation results of the grounding device root 8m, the scattering grounding device 15 m, and the current pole distance 20 m

| Soil conductivity [S/M] | Actual earth resistance [Ω] | Measured resistance value of ground electrode [Ω] | Relative error [%] |
|-------------------------|-----------------------------|--------------------------------------------------|-------------------|
| 0.002                   | 11.745                      | 11.387                                           | −3.048            |
| 0.004                   | 5.874                       | 5.696                                            | −3.030            |
| 0.006                   | 3.918                       | 3.799                                            | −3.037            |
| 0.008                   | 2.939                       | 2.850                                            | −3.028            |
| 0.01                    | 2.352                       | 2.281                                            | −3.019            |
| 0.012                   | 1.961                       | 1.902                                            | −3.001            |
| 0.014                   | 1.682                       | 1.631                                            | −3.032            |
| 0.016                   | 1.472                       | 1.427                                            | −3.057            |
| 0.02                    | 1.179                       | 1.143                                            | −3.053            |

TABLE 6  The simulation results of the grounding grid root 10m, the scattering grounding grid 25 m, and the current pole distance 35 m

| Soil conductivity [S/M] | Actual earth resistance [Ω] | Measured resistance value of ground electrode [Ω] | Relative error [%] |
|-------------------------|-----------------------------|--------------------------------------------------|-------------------|
| 0.002                   | 9.155                       | 8.766                                            | −4.249            |
| 0.004                   | 4.580                       | 4.386                                            | −4.236            |
| 0.006                   | 3.055                       | 2.926                                            | −4.223            |
| 0.008                   | 2.293                       | 2.196                                            | −4.230            |
| 0.01                    | 1.836                       | 1.758                                            | −4.248            |
| 0.012                   | 1.531                       | 1.466                                            | −4.246            |
| 0.014                   | 1.313                       | 1.257                                            | −4.265            |
| 0.016                   | 1.149                       | 1.101                                            | −4.178            |
| 0.02                    | 1.022                       | 0.979                                            | −4.207            |

FIGURE 12  The relative error of grounding resistance

Chongqing is tested on site, and the error analysis of this method and the actual measurement value of grounding resistance is carried out. The tower grounding resistances with different methods are shown in Table 3.

The 0.618 method is recognized as a more accurate measurement method of grounding resistance. If the 0.618 method is taken as the reference value of the actual tower grounding resistance, the relative error of the clamp meter method and the short distance method proposed in this paper compared with the 0.618 method can be obtained. The relative error distribution is shown in Figure 12.

Figure 12 shows that the relative error of the grounding resistance measurement is controlled within 3% when using the short distance method proposed in this paper; while the relative error of the clamp method is within 6%. Using the novel method to measure the grounding resistance can produce less measurement error and meet the actual needs of the project.

5.3  Distance analysis of electrodes distribution

The measurement accuracy of the proposed method is generally <5%, which is similar to the high measurement accuracy of the 0.618 method. However, the 0.618 method has the disadvantage of long measuring wire, which makes the applicability of the 0.618 method poor. The existing grounding resistance measurement methods are based on the assumption that the grounding grid is equivalent to a hemispherical model, which requires the distribution of the voltage electrode must be in the area where the injection current overflows in the form of a hemispherical surface. In engineering, it is generally believed that the overflow current outside the scattering grounding electrode about 2.5× from the tower foundation centre is evenly overflowed in the form of a hemispherical surface. Therefore, it is required that
the position of the voltage electrode P is outside the scattering grounding electrode 2.5 times the tower foundation centre, and the position of the current electrode C is outside the scattering grounding electrode 4 times from the grounding grid. The principle of the conventional straight-line three-pole method is shown in Figure 13.

When using the measurement method proposed in this paper to measure the tower grounding resistance, the farthest point is the supposed zero potential point. The supposed zero potential point is in the area where the test current of the grounding grid coincides with the equipotential surface generated by the current-electrode current, while in the area where the grounding grid current just coincides with the current-electrode current, the overflow current of the grounding grid has not been evenly distributed in a hemispherical form. Therefore, the electrode distribution distance of zero potential point is shorter than that of 2.5 times the scattering grounding electrode. The calculation results in Section 5.2 also show that the earth potential tends to zero in the area beyond the two times of the length of the scattering grounding electrode from the grounding grid, which greatly shortens the measuring wire, and shows the superiority of the measurement method proposed in this paper.

6 | CONCLUSION

This paper proposes a measuring method of tower grounding resistance with shorter measuring wire based on Green's Theorem. According to the measurement theory of tower grounding resistance, combined with the potential calculation model of the tower grounding grid to get the novel short-distance measurement method. The results verify the proposed measurement method. The measurement error of the proposed short-distance method is smaller than that of the three electrodes method when the distribution distance of the zero potential point is the same as that of the compensation point, and the distance of electrodes is shorter than that of the three electrodes method when the supposed zero potential point is located at twice length of the scattering grounding electrode. In summary, the short-distance measurement method proposed in this paper can effectively shorten the distance of electrodes, improve the adaptability of the measurement method in complex environment and lay a solid engineering foundation for the accurate measurement of grounding resistance.

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