New perspectives in testing the general relativistic Lense–Thirring effect

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Summary. Testing the effects predicted by the General Theory of Relativity, in its linearized weak field and slow motion approximation, in the Solar System is difficult because they are very small. Among them the post-Newtonian gravitomagnetic Lense-Thirring effect, or dragging of the inertial frames, on the orbital motion of a test particle is very interesting and, up to now, there is not yet an undisputable experimental direct test of it. Here we illustrate how it could be possible to measure it with an accuracy of the order of 1%, together with other tests of Special Relativity and post-Newtonian gravity, with a joint space based OPTIS/LARES mission in the gravitational field of Earth. Up to now, the data analysis of the orbits of the existing geodetic LAGEOS and LAGEOS II satellites has yielded a test of the Lense-Thirring effect with a claimed accuracy of 20%-30%.

1 Introduction

The linearized weak–field and slow–motion approximation of the General Theory of Relativity (GTR) \cite{1} is characterized by the condition $g_{\mu\nu} \sim \eta_{\mu\nu} + h_{\mu\nu}$ where $g_{\mu\nu}$ is the curved spacetime metric tensor, $\eta_{\mu\nu}$ is the Minkowski metric tensor of the flat spacetime of Special Relativity and the $h_{\mu\nu}$ are small corrections such that $|h_{\mu\nu}| \ll 1$. Until now, many of its predictions, for the motion of light rays and test masses have been tested, in the Solar System, with a variety of techniques to an accuracy level of the order of 0.1% \cite{2,3}. It is not so for the gravitomagnetic Lense–Thirring effect due to its extreme smallness. It can be thought of as a consequence of a gravitational spin–spin coupling.

If we consider the motion of a spinning particle in the gravitational field of a central body of mass $M$ and proper angular momentum $J$, it turns out

\footnote{In the weak field and slow motion approximation of GTR the equations of motion of a test particle freely falling in the gravitational field of a central spinning body are formally analogous to those governing the motion of an electrically charged particle in an electromagnetic field under the action of the velocity–dependent Lorentz force. In the gravitational case the role of the magnetic field is played by the so called gravitomagnetic field which is generated by the off–diagonal terms of the metric $g_{0i}$ and whose source is the proper angular momentum $J$ of the central body.}
that the spin $s$ of the orbiting particle undergoes a tiny precessional motion [4]. The most famous experiment devoted to the measurement, among other things, of such gravitomagnetic effect in the gravitational field of Earth is the Stanford University GP–B mission [5] which should fly at the end of 2003, in spite of recent problems [6, 7].

If we consider the whole orbit of a test particle in its geodesic motion around $M$ as a sort of giant gyroscope, its orbital angular momentum $\ell$ undergoes the Lense–Thirring precession, so that the longitude of the ascending node $\Omega$ and the argument of pericentre $\omega$ of the orbit$^2$ of the test particle $\text{S}$ are affected by tiny secular precessions $\dot{\Omega}_{\text{LT}}, \dot{\omega}_{\text{LT}}$ [9, 10].

1.1 The LAGEOS-LAGEOS II Lense-Thirring experiment

Up to now, the only attempts to detect the Lense–Thirring effect on the orbit of test particles in the gravitational field of Earth are due to Ciufolini and coworkers [11] who analysed the laser data of the existing geodetic passive SLR (Satellite Laser Ranging) satellites LAGEOS and LAGEOS II over time spans of some years. The observable is a suitable combination of the orbital residuals of the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II according to an idea exposed in [12]. The relativistic signal is a linear trend with a slope of almost 60.2 milliarcseconds per year (mas yr$^{-1}$ in the following). The standard, statistical error is evaluated as 2%. The claimed total accuracy, including various sources of systematical errors, is of the order of 20% − 30%.

The main sources of systematical errors in this experiment are

– the unavoidable aliasing effect due to the mismodelling in the classical secular precessions induced on $\Omega$ and $\omega$ by the even zonal coefficients $J_l$ of the multipolar expansion$^3$ of the terrestrial gravitational field [13]
– the non–gravitational perturbations affecting especially the perigee of LAGEOS II [14, 15]. Their impact on the proposed measurement is difficult to be reliably assessed.

It turns out that the mismodelled classical precessions due to the first two even zonal harmonics of the geopotential $J_2$ and $J_4$ are the most insidious source of error for the Lense–Thirring measurement with LAGEOS and LAGEOS II. The combination of [12] is insensitive just to $J_2$ and $J_4$. According

$^2$ The longitude of the ascending node $\Omega$ is an angle in the reference $(x, y)$ plane, which usually coincides with the equatorial plane of the central body, counted from the reference $x$ axis to the line of the nodes. The line of the nodes is given by the intersection between the orbital plane and the reference plane. The argument of the pericentre $\omega$ is an angle in the orbital plane counted from the line of the nodes to the pericentre of the orbit which is the point of closest approach of the orbiting particle to the central body. In the original paper by Lense and Thirring the longitude of the pericentre $\varpi = \Omega + \omega$ is used instead of $\omega$.

$^3$ It accounts for the oblateness of Earth generated by its diurnal rotation.
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to the full covariance matrix of the EGM96 gravity model \[16\], the error due to the remaining uncanceled even zonal harmonics amounts to almost 13\% \[17\]. However, if the correlations among the even zonal harmonic coefficients are neglected and the variance matrix is used\[4\], the error due to the even zonal harmonics of the geopotential amounts to 46.6\% \[17\]. With this estimate the total error of the LAGEOS–LAGEOS II Lense–Thirring experiment would be of the order of 50%.

1.2 The LARES project

The originally proposed LAGEOS III–LARES mission \[19\] consists of the launch of a LAGEOS–type satellite–the LARES–with the same orbit of LAGEOS except for the inclination\[5\] \(i\) of its orbit, which should be supplementary to that of LAGEOS, and the eccentricity \(e\), which should be one order of magnitude larger in order to perform other tests of post–Newtonian gravity \[20, 21\]. In Table 1 the orbital parameters of the existing and proposed LAGEOS–type satellites are quoted.

| Orbital parameter          | LAGEOS | LAGEOS II | LARES  |
|----------------------------|--------|-----------|--------|
| \(a\) semi major axis (km) | 12270  | 12163     | 12270  |
| \(e\) eccentricity         | 0.0045 | 0.014     | 0.04   |
| \(i\) inclination (deg)    | 110    | 52.65     | 70     |
| \(\dot{\Omega}_{LT}\) (mas yr\(^{-1}\)) | 31     | 31.5      | 31     |
| \(\dot{\omega}_{LT}\) (mas yr\(^{-1}\)) | 31.6   | -57       | -31.6  |

The choice of the particular value of the inclination for LARES is motivated by the fact that in this way, by using as observable the sum of the

\[ Such approach is considered more realistic by some authors \[18\] because nothing assures that the correlations among the even zonal harmonics of the covariance matrix of the EGM96 model, which has been obtained during a multidecadal time span, would be the same during an arbitrary past or future time span of a few years as that used in the LAGEOS–LAGEOS II Lense–Thirring experiment or in the proposed LAGEOS–LARES mission.

\[ The inclination \(i\) is the angle between the orbital plane and the reference plane. It is counted from the reference plane so that equatorial orbits have \(i = 0\) deg. The semi major axis \(a\) and the eccentricity \(e\) fix the size and the shape, respectively, of the orbit of the test particle. For closed orbits \(0 < e < 1\). Circular orbits have \(e = 0\).
nodes of LAGEOS and LARES, it should be possible to cancel out to a very high level all the contributions of the even zonal harmonics of the geopotential, which depends on $\cos i$, and add up the Lense–Thirring precessions which, instead, are independent of $i$. The use of the nodes would allow to reduce greatly the impact of the non–gravitational perturbations to which such Keplerian orbital elements are rather insensitive [14, 15].

Of course, it would not be possible to obtain practically two orbital planes exactly 180 deg apart due to the unavoidable orbital injection errors which can be considered of the order of 0.5–1 deg. In Figure 1, page 4314 of [22] and Figure 1, page 1267 of [23] the impact of such source of error on the originally proposed LAGEOS–LARES mission has been shown. It could amount up to 4% for an injection error of 1 deg in the inclination of LARES.

In [22] an alternative observable based on the combination of the residuals of the nodes of LAGEOS, LAGEOS II and LARES and the perigee of LAGEOS II and LARES has been proposed. It would allow to cancel out the first four even zonal harmonics so that the error due to the remaining even zonal harmonics of the geopotential would be rather insensitive both to the orbital injection errors in the LARES inclination and to the correlations among the even zonal harmonic coefficients. It would amount to 0.02%–0.1% only [22, 23].

In regard to the present status of the LARES project, unfortunately, up to now, although its very low cost with respect to other much more complex and expensive space–based missions, it has not yet been approved by any national space agency or scientific institution.

2 The proposal of a joint OPTIS/LARES mission

2.1 The originally proposed OPTIS mission

OPTIS [24] is a recently proposed satellite–based mission\(^6\) which would allow for much improved tests of

– the isotropy of the velocity of light
– the independence of the velocity of light from the velocity of the laboratory
– the universality of the gravitational redshift.

This mission is based on the use of a spinning drag–free satellite in an eccentric, high–altitude orbit which should allow to perform a three orders of magnitude improved Michelson–Morley test and a two order of magnitude improved Kennedy–Thorndike test. Moreover, it should also be possible to improve by two orders of magnitude the tests of the universality of the gravitational redshift by comparison of an atomic clock with an optical clock. The proposed experiments are based on ultrastable optical cavities, lasers,

\(^6\) See also on the WEB [http://www.exphy.uni-duesseldorf.de/OPTIS/optis.html]
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an atomic clock and a frequency comb generator. Since it is not particularly
important for the present version of the mission, the final orbital configura-
tion of OPTIS has not yet been fixed; in \[24\] a perigee height\(^7\) of 10000 km
and apogee height of 36000 km are provisionally proposed assuming a launch
with a Ariane 5 rocket.

The requirements posed by the drag–free technology to be used, based on
the field emission electrical propulsion (FEEP) concept, yield orbital altitudes
not less than 1000 km. On the other hand, the eccentricity should not be too
high in order to prevent passage in the Van Allen belts which could affect
the on–board capacitive reference sensor. Moreover, the orbital period \(P_{\text{OPT}}\)
should be shorter than the Earth’s daily rotation of 24 hours. The orbital
configuration proposed in \[24\] would imply a semi major axis \(a_{\text{OPT}} = 29300\)
km and an eccentricity \(e_{\text{OPT}} = 0.478\). With such values the difference of the
gravitational potential \(U\), which is relevant for the gravitational redshift test,
would amount to

\[
\frac{\Delta U}{c^2} = \frac{G M_\oplus}{c^2 a} \left[ \frac{1}{(1 - e)} - \frac{1}{(1 + e)} \right] \sim 1.8 \times 10^{-10},
\]

where \(G\) is the Newtonian gravitational constant, \(M_\oplus\) the mass of Earth and
\(c\) the speed of light in vacuum. The result of \(1\) is about three orders of
magnitude better than that obtainable in an Earth–based experiment.

An essential feature of OPTIS is the drag–free control of the orbit. Drag–
free motion is required for the special and general relativistic tests which are
carried through using optical resonators. Even very small residual accelerations
of \(10^{-7} g\) may distort the resonators leading to error signals. As a by–
product, this drag–free control also guarantees a very high quality geodesic
motion which may be used, when being tracked, as probe of orbital relativistic
gravitational effects.

For a drag–free motion of the satellite a sensor measuring the actual accel-
eration and thrusters counteracting any acceleration to the required precision
are needed. The sensor, which is based on a capacitive determination of the
position of a test mass, has a sensitivity of up to \(10^{-12} \text{cm s}^{-2} \sqrt{\text{Hz}}\ \[24\].
These systems have a lifetime of many years.

2.2 The compatibility of OPTIS with the gravitomagnetic tests

In this contribution we wish to show that the rather free choice of the orbital
parameters of OPTIS and the use of a new drag–free technology open up the
possibility to extend its scientific significance with new important general
relativistic gravitomagnetic tests as well. Indeed, it would be of great impact
and scientific significance to concentrate as many relativistic tests as possible
in a single mission, including also measurements in geodesy, geodynamics and

\(^7\) With respect to Earth’s surface.
Earth monitoring. Another important point is that OPTIS is currently under serious examination by a national space agency—the German DLR. Then, even if it turns out that OPTIS would yield little or no advantages for the measurement of the Lense–Thirring effect with respect to the originally proposed LARES, if it will be finally approved and launched it will nevertheless be a great chance for detecting, among other things, the Lense-Thirring effect. The main characteristics of such a mission are the already mentioned drag–free technique for OPTIS and the SLR technique for tracking. Today it is possible to track satellites to an accuracy as low as a few mm. This may be further improved in the next years.

It seems that an orbital configuration of OPTIS identical to that of LARES of Table 1 would not be in dramatic contrast with the requirements for the other originally planned tests. For example, with the LARES orbital configuration the difference in the gravitational potential $\Delta U$ would be of the order of $3 \times 10^{-11}$, which is only one order of magnitude smaller than the one that could be obtained with the originally proposed OPTIS configuration.

By assuming for OPTIS the same orbital configuration of LARES the following combination yields high accuracy

$$\dot{\Omega}_{\text{LAGEOS}} + c_1 \dot{\Omega}_{\text{LAGEOS II}} + c_2 \dot{\Omega}_{\text{OPTIS}} + c_3 \omega_{\text{OPTIS}} = 61.8 \mu_{\text{LT}}, \quad (2)$$

with

$$c_1 \sim 3 \times 10^{-3}, \ c_2 \sim 9.9 \times 10^{-1}, \ c_3 \sim 1 \times 10^{-3}. \quad (3)$$

The parameter $\mu_{\text{LT}}$ is 0 in Galileo–Newton mechanics and 1 in GTR. The resulting relativistic signal would be a linear trend with a slope of 61.8 mas yr$^{-1}$. The combination of (2) cancels out the first three even zonal harmonics so that the systematic error due to the remaining harmonics of higher degree amounts to $(\delta \mu_{\text{LT}}/\mu_{\text{LT}})_{\text{even zonals}} = 3 \times 10^{-3}$. The variance matrix of EGM96 up to degree $l = 20$ has been used. It can be shown that this result is insensitive to the orbital injection errors in the inclination of OPTIS.

With regard to the non-gravitational perturbations on the LAGEOS satellites, only the contributions of the nodes of LAGEOS and LAGEOS II, weighted by the small coefficients of (3), have to be considered. This is quite relevant in the final error budget because, as already pointed out, the nodes of the LAGEOS satellites, contrary to the perigees of these laserranged satellites, are orbital elements much less sensitive to the action of the non–gravitational perturbations. With regard to the effect on the non–gravitational perturbations on the OPTIS satellite, it turns out that, by assuming a residual unbalanced acceleration of $10^{-12}$ cm s$^{-2}$, the impact of the node of OPTIS would be of the order of $10^{-3}$ and that of the perigee of OPTIS would be of the order of $10^{-4}$ in view of the coefficients of (3). So, according also to the evaluations of Table 2 and Table 3 of [22], over $T_{\text{obs}} = 7$ years $(\delta \mu_{\text{LT}}/\mu_{\text{LT}})_{\text{NGP}} \sim 3 \times 10^{-3}$. Last but not least, note that the impact of the perigee of LAGEOS II, difficult to be modelled at a high level of accuracy, is absent.
So, the total final systematic error budget in measuring the Lense–Thirring effect with (2) should be better than 1%.

Perhaps the major point of conflict between the original designs of the OPTIS and LARES missions is represented by the eccentricity $e$ of the orbit of the spacecraft. Indeed, while for the gravitational redshift test, given by (1), a relatively large value of $e$ is highly desirable, the originally proposed LARES orbit has a smaller eccentricity. The point is twofold: on one hand, it is easier and cheaper, in terms of requirements on the performances of the rocket launcher, to insert a satellite in a nearly circular orbit, and, on the other, the present status of the ground segment of SLR would assure a uniform tracking of good quality for such kind of orbits. In fact, the large eccentricity of the originally proposed OPTIS configuration, contrary to the other existing geodetic satellites of LAGEOS–type, would not allow for a uniform coverage of the laser–ranged data in the sense that certain portions of the orbit might remain poorly tracked.

However, the originally proposed OPTIS mission implies the use of a ARiane 5 rocket to insert the spacecraft into a GTO orbit and, then, the use of a kick motor. Moreover, it may be reasonable to assume that, when OPTIS/LARES will be launched, the network of SLR ground stations will have reached a status which will allow to overcome the problem of reconstructing rather eccentric orbits to a good level of accuracy.

Then, a reasonable compromise between the OPTIS and LARES requirements could be an eccentricity of, say, $e = 0.1$. In that case (1) yields a gravitational redshift of $\Delta U_c^2 = 7.3 \times 10^{-11}$. With regard to the Lense–Thirring effect, it turns out that the error due to the even zonal harmonics of geopotential would amount, from (2), to 1.5%, according to the variance matrix of EGM96 up to degree $l = 20$. Also in this case it would be insensitive to the orbital injection errors in the inclination. However, the forthcoming Earth gravitational models from CHAMP [26] and GRACE [27] should greatly improve such estimates. With a larger eccentricity the impact of the non–gravitational perturbations would be reduced and, on the other hand, the accuracy of the measurement of the various post–Newtonian gravitational effects on the OPTIS/LARES perigee would be increased.

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