Averaged transverse momentum correlations of hadrons in relativistic heavy-ion collisions

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We compile experimental data for the averaged transverse momentum $\langle p_T \rangle$ of proton, $\Lambda$, $\Xi^-$, $\Omega^-$ and $\phi$ at mid-rapidity in Au+Au collisions at $\sqrt{s_{NN}} = 200, 39, 27, 19.6, 11.5, 7.7$ GeV and in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, and find that experimental data of these hadrons exhibit systematic correlations. We apply a quark combination model with equal-velocity combination approximation to derive analytic formulas of hadronic $\langle p_T \rangle$ in the case of exponential form of quark $p_T$ spectra at hadronization. We use them to successfully explain the systematic correlations exhibited in $\langle p_T \rangle$ data of $p\Lambda$, $\Lambda\Xi^-$, $\Xi^-\Omega^-$ and $\Xi^-\phi$ pairs. We also use them to successfully explain the regularity observed in $\langle p_T \rangle$ of these hadrons as the function of $(dN_A/dy)/(N_{part}/2)$ at mid-rapidity in central heavy-ion collisions at both RHIC and LHC energies. Our results suggest that the constituent quark degrees of freedom and the equal-velocity combination of these constituent quarks at hadronization play important role in understanding the production of baryons and $\phi$ meson at these RHIC and LHC energies.

I. INTRODUCTION

In relativistic heavy-ion collisions, the hot nuclear matter is created at early collision stage by intensively inelastic collisions of colliding nucleons [1–6]. Subsequently, the matter expands, cools and finally decomposes into hadrons scattering out. The evolution of hot nuclear matter is a complex process governed by non-perturbative QCD and is mainly modeled by hydrodynamic models [7] and transport models [8–10] at present. Hadrons produced from hot nuclear matter always have certain transverse momentum $p_T$, the component of momentum which is perpendicular to the beam direction. The $p_T$ distribution of hadrons carries lots of information on hot nuclear matter such as thermalization, transverse collective flow generated by system expansion in both partonic and hadronic stage, and is an important physical observable in experiments of relativistic heavy-ion collisions.

Rich experimental data for $p_T$ spectra of identified hadrons at mid-rapidity are successively reported in heavy-ion collisions at RHIC and LHC over the past decade [11–22]. Based on these experimental data, lots of studies on properties of hadronic $p_T$ distribution are carried out, which greatly improve people’s understandings on property of the created hot nuclear matter and the mechanism of hadron production in relativistic heavy-ion collisions [23–34]. The average transverse momentum $\langle p_T \rangle$ of hadrons is obtained by integrating over $p_T$ spectra of hadrons. It is dominated by property of hadronic $p_T$ spectra in the low $p_T$ range and therefore it reflects the property of soft hadrons and correspondingly that of hot nuclear matters.

In this paper, we study the property of $\langle p_T \rangle$ of identified hadrons produced in relativistic heavy-ion collisions.

We compile experimental data for $\langle p_T \rangle$ of $\phi$, proton, $\Lambda$, $\Xi^-$ and $\Omega^-$ at mid-rapidity in Au+Au collisions at $\sqrt{s_{NN}} = 200, 39, 27, 19.6, 11.5, 7.7$ GeV and in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. We search the regularity in $\langle p_T \rangle$ data of these hadrons and, in particular, their dependence on hadron species and collision energy. We discuss what the underlying physics is responsible to the observed regularity. In particular, we study the effect of hadronization by an equal-velocity combination (EVC) mechanism of quarks and antiquarks [35, 36] in explaining the experimental data of $\langle p_T \rangle$. We derive analytic expression for $\langle p_T \rangle$ of identified hadrons in EVC mechanism so as to give clear quark flavor dependence of hadronic $\langle p_T \rangle$ and provide intuitive explanations of experimental data.

The paper is organized as follows. In Sec. II, we briefly introduce our EVC model and derive $\langle p_T \rangle$ of identified hadrons for the simplified quark distributions at hadronization. In Sec. III, we show our finding for the systematic correlation among experimental data for $\langle p_T \rangle$ of hadrons in relativistic heavy-ion collisions and give an intuitive explanation using our EVC model. In Sec. IV, we show the regularity on $\langle p_T \rangle$ of hadrons in central heavy-ion collisions as the function of $(dN_A/dy)/(0.5N_{part})$. In Sec. V, we discuss the influence of resonance decays on $\langle p_T \rangle$ correlations of hadrons. Finally, the summary and discussion are given in Sec. VI.

II. $\langle p_T \rangle$ OF HADRONS IN EVC MODEL

In this section, we apply a particular quark combination model [35, 37] to describe the production of hadrons at hadronization and derive $\langle p_T \rangle$ of different hadrons. The model assumes the constituent quarks and antiquarks as the effective degrees of freedom for the final parton system created in collisions at hadronization stage. Based on constituent quark model of internal
structure of hadrons at low energy scale, the model assumes that the equal velocity combination of these constituent quarks and antiquarks is main feature of hadron formation at hadronization. The quark masses are taken as the constituent masses so that the equal velocity combination of these constituent quarks and antiquarks can correctly construct the on-shell hadron. At mid-rapidity (i.e., taking \( y = 0 \)), \( p_T \) distribution of a hadron (\( dN/dp_T \)) is the product of those of (anti-)quarks

\[
\begin{align*}
 f_B, (p_T) &= \kappa_B, f_\bar{q}, (x_{1p_T}) f_{q_2} (x_{2p_T}) f_{q_3} (x_{3p_T}), \\
 f_M, (p_T) &= \kappa_M, f_\bar{q}, (x_{1p_T}) f_{q_2}, (x_{2p_T}).
\end{align*}
\]

Here, moment fraction \( x_i = m_i/(m_1 + m_2 + m_3) \) (\( i = 1, 2, 3 \)) in baryon formula satisfies \( x_1 + x_2 + x_3 = 1 \) and \( x_i = m_i/(m_1 + m_2) \) (\( i = 1, 2 \)) in meson formula satisfies \( x_1 + x_2 = 1 \). \( m_i \) is constituent mass of quark \( q_i \) and we take \( m_u = m_d = 0.3 \text{ GeV} \) and \( m_s = 0.5 \text{ GeV} \). Coefficients \( \kappa_B \) and \( \kappa_M \) are independent of \( p_T \), see Refs. [36] for their detailed expressions, and therefore are not involved in derivation of averaged transverse momentum \( \langle p_T \rangle \) in the following text.

The value of \( \langle p_T \rangle \) is dominated by the \( p_T \) spectrum of particles in the low \( p_T \) range. Therefore, here we focus on \( p_T \) spectra of quarks and antiquarks with low \( p_T \). Unfortunately, quarks of low \( p_T \) is governed by non-perturbative QCD dynamics and their distributions are difficult to calculate from first principle. Considering that experimental data for \( p_T \) spectra of hadrons at mid-rapidity in the low \( p_T \) range in relativistic heavy-ion collisions are generally well fitted by exponential function and/or Boltzmann distribution [14, 15, 17, 18, 21, 22], in this paper we take the following parameterization for quark \( p_T \) spectra at mid-rapidity

\[
f_q, (p_T) = \mathcal{N} p_T^\alpha \exp \left( -\frac{\sqrt{p_T^2 + m_q^2}}{T_i} \right),
\]

which is convenient to derive analytic results of hadronic \( \langle p_T \rangle \). Here, \( \mathcal{N} \) is coefficient to quantity the number of \( q_i \), which is irrelevant to \( \langle p_T \rangle \) calculations. \( T_i \) is the slope parameter to quantify the exponential decrease of the spectrum. Exponent \( k \) tunes the behavior of the spectrum at small \( p_T \). In the case of two-dimensional Boltzmann distribution in the rest frame we have \( k = 1 \), and in the one-dimensional case we have \( k = 0 \). If we directly apply Eq. (3) to fit the experimental data of \( p_T \) spectra of hadrons by Eq. 1, we should take \( k \approx 1/3 \) to properly describe baryon and \( k \approx 1/2 \) to properly describe meson (mainly \( \phi \)). In addition, the effect of strong collective radial flow should be included in the quark spectrum in the laboratory frame, which is dependent on collision energies in relativistic heavy-ion collisions. With these considerations, we take \( k \) as an relatively-free parameter in the range \([0, 1]\) in this study of hadronic \( \langle p_T \rangle \) in relativistic heavy-ion collisions at RHIC and LHC energies.

Substituting Eq. (3) into Eqs. (1) and (2), we obtain

\[
\begin{align*}
\langle p_T \rangle_B &= \frac{\int f_B (p_T) p_T dp_T}{\int f_B (p_T) dp_T} = \frac{\int p_T^{3k+1} \exp \left[ -\left( \frac{p_T^2}{T_1^2} + \frac{p_T^2}{T_2^2} + \frac{p_T^2}{T_3^2} \right) \right] dp_T}{\int p_T^{3k} \exp \left[ -\left( \frac{p_T^2}{T_1^2} + \frac{p_T^2}{T_2^2} + \frac{p_T^2}{T_3^2} \right) \right] dp_T},
\end{align*}
\]

\[
\begin{align*}
\langle p_T \rangle_M &= \frac{\int f_M (p_T) p_T dp_T}{\int f_M (p_T) dp_T} = \frac{\int p_T^{3k+1} \exp \left[ -\left( \frac{p_T^2}{T_1^2} + \frac{p_T^2}{T_2^2} \right) \right] dp_T}{\int p_T^{3k} \exp \left[ -\left( \frac{p_T^2}{T_1^2} + \frac{p_T^2}{T_2^2} \right) \right] dp_T},
\end{align*}
\]

where \( m_B = m_{q_1} + m_{q_2} + m_{q_3} \) and \( m_M = m_{q_1} + m_{\bar{q}_2} \). We use the integral formula

\[
\int_0^\infty p_T^n \exp \left[ -a \sqrt{p_T^2 + m^2} \right] dp_T = m^{n+1/2} \frac{\Gamma(n+1/2) \Gamma(n+1)}{\sqrt{\pi} a^{n+1/2}}
\]

where \( a = m, \Gamma(z) \) is Gamma function and \( K_n(z) \) is the modified Bessel function of the second kind. We obtain

\[
\begin{align*}
\langle p_T \rangle_B &= (m_{q_1} + m_{q_2} + m_{q_3}) \sqrt{\frac{2}{\alpha_B} \frac{\Gamma(3k/2 + 1)}{\Gamma(3k/2 + 1/2)} \frac{K_{3k/2 + 3/2}(\alpha_B)}{K_{3k/2 + 1}(\alpha_B)}},
\end{align*}
\]

\[
\begin{align*}
\langle p_T \rangle_M &= (m_{q_1} + m_{\bar{q}_2}) \sqrt{\frac{2}{\alpha_M} \frac{\Gamma(k + 1)}{\Gamma(k + 1/2)} \frac{K_{k+3/2}(\alpha_M)}{K_{k+1}(\alpha_M)}}
\end{align*}
\]

with

\[
\alpha_B = \frac{m_{q_2}}{T_1} + \frac{m_{q_3}}{T_2} + \frac{m_{q_2}}{T_3} = \alpha_{q_1} + \alpha_{q_2} + \alpha_{q_3},
\]

and

\[
\alpha_M = \frac{m_{q_1}}{T_1} + \frac{m_{\bar{q}_2}}{T_2} = \alpha_{q_1} + \alpha_{\bar{q}_2}.
\]
As shown by these expressions, \( \langle p_T \rangle \) of different hadrons is correlated by the simple combination of slope parameter \( \alpha_q \) of quarks at hadronization.

Bessel function \( K_{\nu}(\alpha) \) and Gamma function \( \Gamma(z) \) usually have complex expressions. Here, we present the numerical approximations for \( \langle p_T \rangle_B \) and \( \langle p_T \rangle_M \)

\[
\langle p_T \rangle_B \approx (m_{q_1} + m_{q_2} + m_{q_3}) \left( 0.26 + 0.024k + \frac{0.96 + 2.99k}{\alpha_B} \right),
\]

\[
\langle p_T \rangle_M \approx (m_{q_1} + m_{q_2}) \left( 0.25 + 0.03k + \frac{0.97 + 1.99k}{\alpha_M} \right)
\]

in order to see their dependence on \( \alpha \) and \( k \) in a numerically intuitive way. The relative errors of two approximations are less than about 3% for the physical range of \( \langle p_T \rangle_B \) and \( \langle p_T \rangle_M \) in heavy-ion collisions at RHIC and LHC energies studied in this paper.

III. CORRELATIONS AMONG \( \langle p_T \rangle \) OF DIFFERENT HADRONS

In this section, we study the correlation among \( \langle p_T \rangle \) of different hadrons. In Fig. 1 (a), we present \( \langle p_T \rangle \) of proton as horizontal axis and \( \langle p_T \rangle \) of \( \phi \) at correspondingly collision energy and centrality as the vertical axis to study the correlation between them. As we know, the mass of proton is close to that of \( \phi \) but the quark flavor composition of proton (\( uud \)) is completely different from that of \( \phi \) (\( s\bar{s} \)). Except a few datum points in central Au+Au collisions at \( \sqrt{s_{NN}} = 200 \text{ GeV} \), we see an relatively stable correlation between \( \langle p_T \rangle \) of proton and that of \( \phi \). In Fig. 1 (b) and (c), we show correlations among proton, \( \Lambda \) and \( \Xi^{-} \) in the manner of successive strangeness. In Fig. 1 (d), we show the correlation between \( \phi \) and \( \Xi^{-} \) which both have two strange (anti-)quarks. We see the systematic correlations among these hadrons. In Fig. 1 (e-h), we show the correlation among \( \langle p_T \rangle \) data of \( \phi \) and anti-baryons and also find systematic correlations among them.

As we know, the hot nuclear matters created in heavy-ion collisions at different collision energies and centralities have different size and geometry, different evolution times in partonic phase and in the subsequent hadronic re-scattering stage, etc. The correlations shown in Fig. 1 seem to assign these difference into a systematic manner and therefore may indicate some underlying physics which is universal in heavy-ion collision at both RHIC and LHC energies. We think that the universal hadronization mechanism may be a possible physical reason.

Therefore, we apply the EVC model in Sec. II to understand the above correlations in experimental data of hadronic \( \langle p_T \rangle \). Hadrons in Fig. 1 are all made up of up, down, strange quarks and their antiquarks. In our model, exponent parameter \( k \) and slope parameters \( \alpha_u, \alpha_d, \alpha_s, \alpha_{\bar{u}}, \alpha_{\bar{d}}, \alpha_{\bar{s}} \) and \( \alpha_\Xi \) are needed to fix in order to calculate \( \langle p_T \rangle \) of hadrons according to Eqs. (7-10). Here, we can assume the isospin symmetry between up and down quarks \( \alpha_u = \alpha_d \) at mid-rapidity in relativistic heavy-ion collisions. We also assume the charge conjugation symmetry for strange quarks and antiquarks \( \alpha_s = \alpha_{\bar{s}} \), which is found to be a good approximation at mid-rapidity in relativistic heavy-ion collisions at RHIC and LHC ener-
gories [36]. Finally, only three slope parameters $\alpha_u$, $\alpha_\bar{u}$ and $\alpha_s$ are left in addition to the exponent parameter $k$. If we know the correlation between $\alpha_u$ and $\alpha_s$, we can calculate correlations among $\langle p_T \rangle$ of baryons and $\phi$, and therefore use them to explain experimental data in Fig. 1(a). Applying Eqs. (7) and (8) to proton and $\phi$, we have

$$\langle p_T \rangle_p = 3m_u \sqrt{\frac{2}{\alpha_p} \frac{\Gamma(3k/2 + 1) K_{3k/2 + 1}(\alpha_p)}{\Gamma(3k/2 + 1/2) K_{3k/2 + 1}(\alpha_p)}}, \quad (13)$$

$$\langle p_T \rangle_\phi = 2m_s \sqrt{\frac{2}{\alpha_\phi} \frac{\Gamma(k + 1) K_{k + 1/2}(\alpha_\phi)}{\Gamma(k + 1/2) K_{k + 1}(\alpha_\phi)}}, \quad (14)$$

where

$$\alpha_p = 2\alpha_u + \alpha_d = 3\alpha_u, \quad (15)$$

$$\alpha_\phi = \alpha_s + \alpha_\bar{s} = 2\alpha_s \quad (16)$$

according to Eqs. (9) and (10). By fitting experimental data of $\langle p_T \rangle$ of proton and $\phi$ shown in Fig. 1(a), we can reversely extract the correlation between $\alpha_u$ and $\alpha_s$, which can be parameterized as

$$\alpha_u(\alpha_s) = c_0 + c_1 \alpha_s \quad (17)$$

where $c_0$ and $c_1$ are two coefficients. Because the extraction is also dependent on exponent parameter $k$, we list values of $c_0$ and $c_1$ at several different $k$ in Table. I. The resulting fitted correlations between proton and $\phi$ with these different extractions are shown as lines with different types in Fig. 2(a). In order to reduce to the bias in choice of $k$, we let these different extractions to represent the same correlation between $\langle p_T \rangle$ of proton and $\phi$, i.e., these lines are coincident with each other. By fitting experimental data of $\langle p_T \rangle$ of anti-proton and $\phi$, we also obtain the correlation between $\alpha_\bar{u}$ and $\alpha_s$ with the parameterized form

$$\alpha_\bar{u}(\alpha_s) = d_0 + d_1 \alpha_s. \quad (18)$$

The values of coefficients $d_0$ and $d_1$ at several $k$ are shown in Table. I and the corresponding fit is shown in Fig. 2(b). At low RHIC energies where $\alpha_s$ is large, $\alpha_\bar{u}$ is different from $\alpha_u$ to a certain extent, which is because the finite baryon density at low collision energies will cause the asymmetry between up/down quarks and their anti-quarks.

Table I. Coefficients in Eqs. (17) and (18) at different $k$, which are extracted from experimental data of $\langle p_T \rangle$ of (anti-)proton and $\phi$ at mid-rapidity in heavy-ion collisions at RHIC and LHC energies.

| $k$  | $c_0$ | $c_1$ | $d_0$ | $d_1$ |
|------|-------|-------|-------|-------|
| 0.0  | -0.05 | 0.69  | -0.05 | 0.71  |
| 0.1  | -0.07 | 0.76  | -0.07 | 0.78  |
| 0.3  | -0.11 | 0.84  | -0.11 | 0.86  |
| 0.5  | -0.15 | 0.89  | -0.15 | 0.91  |
| 1.0  | -0.23 | 0.94  | -0.25 | 0.97  |

With these two relationship, we can calculate correlations among $\langle p_T \rangle$ of various hadrons by Eqs. (7) and (8) with Eqs. (15), (16) and

$$\alpha_\Lambda = 2\alpha_u + \alpha_s, \quad (19)$$

$$\alpha_\Xi = 2\alpha_s + \alpha_u, \quad (20)$$

$$\alpha_\Omega = 3\alpha_s \quad (21)$$

and the corresponding anti-baryons according to Eqs. (9) and (10). In Fig. 3, we present theoretical results for $\langle p_T \rangle$ correlations of $pA$, $\Lambda\Xi^-$, $\Xi^-$, $\Omega^- \phi$ pairs, and these of anti-baryons, and we compare them with experimental data. Theoretical results at different $k$ with corresponding coefficients in Table. I are shown as lines with different types. These lines are different to a certain extent, which shows the theoretical uncertainties due to the selection of exponent parameter $k$. Overall, we see that the systematic feature of the correlations exhibited by $\langle p_T \rangle$ data of these hadrons can be described by our theoretical model.

This result is quite interesting. Here we only consider the effect of hadronization by EVC mechanism without
any considerations on other dynamical ingredients such as system size and geometry, evolution time, hadronic re-scattering, etc. We run the event generators URQMD 3.4 [8] and AMPT 2.26 (1.26) [10] which practically include those dynamical processes and we do not find better description on systematic correlations in Fig. 1 when we use default parameter values of event generators. Therefore, our results in Fig. 3 indicate the important role of hadronization by EVC mechanism in describing $\langle p_T \rangle$ of those hadrons in relativistic heavy-ion collisions at both RHIC and LHC energies.

IV. $\langle p_T \rangle$ OF HADRONs AS THE FUNCTION OF $\langle dN_{ch}/dy \rangle / (0.5N_{part})$

Experimental data for hadronic $\langle p_T \rangle$ shown in the form of Fig. 1 reveal correlations among $\langle p_T \rangle$ of different hadrons, which are mainly relevant to hadronization mechanism according to our studies in the previous section. In this section, we study another aspect of $\langle p_T \rangle$ of hadrons, i.e., their absolute values, and search some regularity underlying these experimental data in relativistic heavy-ion collisions at both RHIC and LHC energies.

There are many physical ingredients that influence $\langle p_T \rangle$ of hadrons. Generally speaking, there are two main sources of generating the transverse momentum of hadrons. The first source is the intensive parton interactions at early collision stage which form the thermal bulk nuclear matter and generate primordial thermal or stochastic momentum of particles. Another source is the expansion of hot nuclear matter in both partonic phase (if exists) and hadronic phase, which generates the collective radial flow and therefore strengthens the $\langle p_T \rangle$ of hadrons. The effects of two sources are both influenced by collision parameters such as collision energy, collision centrality and collision system. These collision parameters influence the size and geometry of the bulk nuclear matter, the intensity of soft parton/particle interactions, the time of system expansion and correspondingly the magnitude of collective radial flow. In view of these complex ingredients, it seems to be difficult to find a simple and perfect regularity for $\langle p_T \rangle$ of hadrons by directly analyzing the experimental data of $\langle p_T \rangle$ of hadrons in relativistic heavy-ion collisions.

Here, we try to take $(dN_{ch}/dy) / (0.5N_{part})$ to quantify the excitation of hadronic $\langle p_T \rangle$. $dN_{ch}/dy$ is the rapidity density of charged particles at mid-rapidity. It can characterize the size of the created hot nuclear matter. In general case, i.e., as other conditions (such as collision energy and collision system) are not changed, the larger system means more intensive particle excitation (i.e., higher stochastic momentum or higher temperature) and more expansion (i.e., more radial flow). Experimental observation have shown that $\langle p_T \rangle$ of hadrons generally positively responds to the $dN_{ch}/dy$ at given collision energy and collision system [14, 15, 17, 18, 21, 22, 38–40]. Therefore, we take $dN_{ch}/dy$ as the main relevant ingredients parameterizing $\langle p_T \rangle$ of hadrons. $N_{part}$ is the number of participant nucleons calculated in Glauber model [41], which depends on collision energy and collision system and impact parameter. It can characterize the total amount of energy deposited in the collision region and therefore characterize the initial size and energy density of the created nuclear matter. The ratio $(dN_{ch}/dy) / (0.5N_{part})$ quantifies the average number of charged particles produced by a pair of participant nucleons. It can roughly characterize the average number of particles produced by a pair of participant nucleons.
charged particles produced by an “unit” effective energy deposited by the collision of a pair of nucleons. In general, the higher \( (dN_{ch}/dy)/(0.5N_{part}) \) means more intensive particle excitation which needs more intensive parton interactions and also means more momentum generation. Therefore, we expect \( (dN_{ch}/dy)/(0.5N_{part}) \) should positively correlate with \( \langle p_T \rangle \) of hadrons.

The geometry property of hot nuclear matter, mainly controlled by impact parameter, also influence \( \langle p_T \rangle \) of hadrons. In particular, in peripheral collisions where impact parameter is large, various-order anisotropic flows are generated and will influence the \( \langle p_T \rangle \) to a certain extent by, for example, asymmetric distribution of \( p_x \) and \( p_y \). Therefore, in order to remove the freedom of impact parameter which is quite complex to parameterize, we only use experimental data of hadronic \( \langle p_T \rangle \) in central collisions to search their possible regularity with respect to \( (dN_{ch}/dy)/(0.5N_{part}) \).

In Fig. 4, we compile experimental data for \( \langle p_T \rangle \) of \( \phi \) and (anti-)baryons at mid-rapidity in central heavy-ion collisions at different collision energies. We see that these data of hadronic \( \langle p_T \rangle \) exhibit a clear regularity when we plot them as the function \( (dN_{ch}/dy)/(0.5N_{part}) \). Here data of \( dN_{ch}/dy \) at mid-rapidity and \( N_{part} \) are taken from Refs. [15–17, 22]. In calculation of \( (dN_{ch}/dy)/(0.5N_{part}) \), only experimental uncertainties of \( dN_{ch}/dy \) are included.

According to behavior of experimental data in Fig. 4 and the approximated formula of hadronic \( \langle p_T \rangle \) in Eqs. (11) and (12), we parameterize the \( (dN_{ch}/dy)/(0.5N_{part}) \) dependence of slope parameter \( \alpha_q \) of quarks as the following form

\[
\alpha_q = \left[ g_q + h_q \left( \frac{dN_{ch}/dy}{N_{part}/2} \right)^{2/3} \right]^{-1}.
\] (22)

Coefficients \( g \) and \( h \) of \( u, \bar{u} \) and \( s \) quarks can be fixed by using Eqs. (7) and (8) to fit experimental data of proton, anti-proton and \( \Omega^- + \bar{\Omega}^+ \). Values of \( g \) and \( h \) at different \( k \) are shown in Table. II. These fittings to experimental data of proton, anti-proton and \( \Omega^- + \bar{\Omega}^+ \) are shown in Fig. 5 (a), (e) and (h) as lines of different types. In order to avoid the bias in selection of exponent parameter \( k \), we let these different fitting groups generate the same \( (dN_{ch}/dy)/(0.5N_{part}) \) dependence for \( \langle p_T \rangle \) of proton, anti-proton and \( \Omega^- + \bar{\Omega}^+ \), that is, lines of different types are coincident with each other in Fig. 5 (a), (e) and (h). This treatment can enable us to study theoretical uncertainty in prediction of other hadrons. Results for \( \langle p_T \rangle \) of other hadrons as the function of \( (dN_{ch}/dy)/(0.5N_{part}) \) are shown in Fig. 5(b-d) and (f-g) and are compared with experimental data.
to study the effect of resonance decays. In these results at different 
which are in good agreement with experimental data. In this section, we study the influence of resonance decay on the correlations among \( \langle p_T \rangle \) of different hadrons. We apply the quark combination model developed in previous works [35, 37] to calculate the influence of resonance decay on \( \langle p_T \rangle \) of hadrons. Following experimental corrections, results of \( \Lambda \) and \( \Xi^- \) do not include weak decay contributions but results of proton and anti-proton include them. We adopt the following strategy to quantify the effect of resonance decays. First, we use the model to calculate the \( \langle p_T \rangle \) of final-state (anti-)proton and that of \( \phi \) with the parameterized quark \( p_T \) spectra in Eq. (3). We apply the model to fit the \( \langle p_T \rangle \) correlation between experimental data of (anti-)protons and those of \( \phi \) to obtain the correlation between \( \alpha_u \) and \( \alpha_s \) with the parameterization form Eq. (17) and that between \( \alpha_g \) and \( \alpha_s \) with Eq. (18). The newly obtained coefficients \( c_0, c_1, d_0, d_1 \) are slightly different from those in Table I due to the effect of resonance decays. In the fitting process, we keep the same \( p_\phi \) and \( \bar{p}_\phi \) correlations shown as lines in Fig 2. Second, we calculate \( \langle p_T \rangle \) correlations among other hadron pairs and compare them with results in Fig. 3 to study the effect of resonance decays.

In Fig. 6, we show \( \langle p_T \rangle \) correlations among different final-state hadrons at given slope parameter \( k = 0.3 \) and compare them, dashed lines, with results of directly-produced hadrons at same \( k \), the dot-dashed lines, which are borrowed from Fig. 3. Experimental data [14, 15, 17–22, 38] are also presented. We see that two sets of results are only slightly different, which indicates the weak influence of resonance decays on \( \langle p_T \rangle \) correlations of these hadrons. Results at other values of \( k \) are quite similar and therefore not presented.
In the similar way, we further study the effect of resonance decays on hadronic $\langle p_T \rangle$ as the function of $(dN_{ch}/dy)/(0.5N_{part})$ in central heavy-ion collisions. In Fig. 7, the dashed lines denotes results for $\langle p_T \rangle$ of hadrons with resonance decays and dot-dashed lines denote results without resonance decays. Symbols are experimental data [14–22, 38]. Experimental data of protons (a), anti-protons (c) and $\Omega^-$ (h) are used to determine parameters of (anti-)quarks at hadronization, and we keep the same extent in reproducing these experimental data, i.e., dashed lines are coincident with the dot-dashed lines in three panels. In Fig. 7 (b-d) and (f-g), we see the difference between dashed lines and dot-dashed lines is small, which indicate the weak influence of resonance decays on $\langle p_T \rangle$ correlations of these hadrons.

Figure 6. Correlations among $\langle p_T \rangle$ of hadrons at mid-rapidity in relativistic heavy-ion collisions at different collision energies and collision centralities. Symbols are experimental data [14, 15, 17–22, 38]. The dashed lines are theoretical results including resonance decays and the dot-dashed lines are these not including resonance decays.

Figure 7. $\langle p_T \rangle$ of hadrons the function of $(dN_{ch}/dy)/(0.5N_{part})$ at mid-rapidity in central heavy-ion collisions at different collision energies. Symbols are experimental data [14–22, 38]. The dashed lines are theoretical results including resonance decays and the dot-dashed lines are these not including resonance decays.
VI. SUMMARY AND DISCUSSION

In this paper, we have applied a quark combination model with equal-velocity combination approximation to study the averaged transverse momentum ($\langle p_T \rangle$) correlations of proton, $\Lambda$, $\Xi^-$, $\Omega^-$ and $\phi$ in relativistic heavy-ion collisions. We derived analytic formulas of hadronic $\langle p_T \rangle$ in the case of exponential form of quark $p_T$ spectra at hadronization, which can clarify correlations among $\langle p_T \rangle$ of identified hadrons based on the constituent quark structure of hadrons. We used these analytic formulas to explain the systematic correlations exhibited in $\langle p_T \rangle$ data of $p\Lambda$, $\Lambda\Xi^-$, $\Xi^-\Omega^-$ and $\Xi^-\phi$ pairs and those of anti-baryons. We discussed the regularity for $\langle p_T \rangle$ of these hadrons as the function of $\langle dN_{ch}/dy \rangle/\langle N_{part}/2 \rangle$ at mid-rapidity in central heavy-ion collisions at both RHIC and LHC energies, and used our model to self-consistently explain $\langle p_T \rangle$ of these hadrons as the function of $\langle dN_{ch}/dy \rangle/\langle N_{part}/2 \rangle$. In these studies, we use experimental data of (anti-)protons to fix the property of up/down (anti-)quarks and those of $\Omega$ or $\phi$ to fix that of strange quarks at hadronization. Then we predict correlations among $\langle p_T \rangle$ of other hadron pairs and compare with experimental data to test the theoretical consistency. Moreover, we studied the effects of resonance decays on $\langle p_T \rangle$ correlations of hadrons and find they are weak in comparison with hadronization.

Our studies shown that the $\langle p_T \rangle$ correlations among experimental data of proton, $\Lambda$, $\Xi^-$, $\Omega^-$ and $\phi$ in Au+Au collisions at RHIC energies and Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV can be self-consistently described by the equal-velocity combination mechanism of constituent quarks and antiquarks at hadronization. This indicates the important role of constituent quarks and antiquarks as the effective degrees of freedom of the hot nuclear matter at hadronization stage and their equal-velocity combination as the main feature of hadron formation in relativistic heavy-ion collisions. The current study is consistent with our previous works in studying the elliptic flow of these hadrons and the quark number scaling property of $p_T$ spectra of $\Omega^-$ and $\phi$ in relativistic heavy-ion collisions at RHIC and LHC energies using the same quark combination mechanism [42, 43].

In relativistic heavy-ion collisions, re-scatterings of hadrons after hadronization will influence momentum of hadrons to a certain extent. For example, the signal of $\phi$ may be lost by the scattering of their decay daughters with the surrounding hadrons and $\phi$ may be also generated by the coalescence of two kaon. We will study this hadronic rescattering effect in the future works. In addition, we will also carry out a systematic study on $p_T$ spectra of identified hadrons at mid-rapidity in different centralities in Au+Au collisions at STAR BES energies. $p_T$ spectra of identified hadrons contain more dynamical information than their $\langle p_T \rangle$, which can be used to further test our quark combination model at low RHIC energies.

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