NUCLEOSYNTHESIS IN GAMMA-RAY BURST ACCRETION DISKS

JASON PRUET, 1 S. E. WOOSLEY, 2 and R. D. HOFFMAN 1

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ABSTRACT

We follow the nuclear reactions that occur in the accretion disks of stellar-mass black holes that are accreting at a very high rate, 0.01–1 M\(_\odot\) s\(^{-1}\), as is realized in many current models for gamma-ray bursts (GRBs). The degree of neutralization in the disk is a sensitive function of the accretion rate, black hole mass, Kerr parameter, and disk viscosity. For high accretion rates and low viscosity, material arriving at the black hole will consist predominantly of neutrons. This degree of neutralization will have important implications for the dynamics of the GRB-producing jet and perhaps for the synthesis of the r-process. For lower accretion rates and high viscosity, as might be appropriate for the outer disk in the collapsar model, neutron-proton equality persists, allowing the possible synthesis of \(^{56}\)Ni in the disk wind. \(^{56}\)Ni must be present to make any optically bright Type I supernova and, in particular, those associated with GRBs.

Subject headings: accretion, accretion disks — gamma rays: bursts — nuclear reactions, nucleosynthesis, abundances

1. INTRODUCTION

Growing evidence connects gamma-ray bursts (GRBs) to the birth of hyperaccreting black holes (Fryer, Woosley, & Hartmann 1999a), that is, stellar-mass black holes accreting matter from a disk at rates from \(-0.01\) to \(10\) M\(_\odot\) s\(^{-1}\). Such models include the collapsar (Woosley 1993; MacFadyen & Woosley 1999), merging neutron stars and black holes (Eichler et al. 1989; Janka et al. 1999; Ruffert & Janka 2001), supranovae (Vietri & Stella 1998, 1999), merging helium cores and black holes (Zhang & Fryer 2001), and merging neutron stars and black holes (Zhang & Fryer 2001), and merging neutron stars and black holes (Zhang & Fryer 2001), and merging neutron stars and black holes (Zhang & Fryer 2001). For such high accretion rates the disk is optically thick, except to neutrinos, and very hot, consisting in its inner regions of a (viscous) mixture of neutrons and protons. At its inner boundary, the disk connects to the black hole. Whatever processes accelerate the putative GRB-producing jet might therefore be expected to act on some mixture of disk material and other background medium (e.g., the collapsing star in the collapsar model). Farther out, material will be lost from the disk in a vigorous wind (MacFadyen & Woosley 1999; Narayan, Piran, & Kumar 2001). In fact, Narayan et al. (2001) suggest that most of the disk will be lost to a wind except for those models and in those regions where neutrino losses dominate the energy budget.

It is thus of some consequence to know the composition of such disks. In the least case, the nucleosynthesis may be novel and could account for rare species in nature, such as the r-process. At most, the presence of free neutrons may affect the dynamics of the GRB jet (Derishev, Kocharovsky, & Kocharovskiy 1999; Fuller, Pruet, & Abazajian 2000), the GRB neutrino signature (Bahcall & Mészáros 2000), light curve (Puet & Dalal 2002), and afterglow (e.g., via a “pre-acceleration” mechanism similar to the one discussed by Beloborodov 2002). It is also of some consequence to know whether the disk wind consists of radioactive \(^{56}\)Ni, as is necessary if a visible supernova is to accompany the GRB. If the electron mole number, \(Y_e = \Sigma Z_i (X_i / A_i)\), is less than 0.485, the iron group will be dominated by \(^{56}\)Fe and other more neutron-rich species (Hartmann, Woosley, & El Eid 1985), which will be incapable of illuminating the supernova. Since the jet itself is inefficient at heating sufficient matter to temperatures required for nuclear statistical equilibrium \((T \gtrsim 5 \times 10^9\) K), supernovae seen in conjunction with GRBs (Galama et al. 1998; Bloom et al. 2002, for example) would be difficult to understand.

We have thus undertaken a survey of the nucleosynthesis that happens in the disks of rapidly accreting black holes. To orient the reader, we show in Figure 1 an illustration of some of the processes discussed in the subsequent sections. The work is greatly facilitated by the existence of numerical and semianalytic solutions that yield the temperature-density structure and drift velocity (Popham, Woosley, & Fryer 1999). These solutions have been verified in the case of the collapsar model by direct numerical simulation (MacFadyen & Woosley 1999).

2. COMPUTATIONAL APPROACH

For a disk that is optically thin to neutrinos, the neutron-to-proton ratio, \(n/p\), is determined by the competition between electron and positron capture on nuclei and free nucleons. Since the baryon number per comoving volume is conserved, it is convenient to work with the electron fraction \(Y_e\) defined by

\[
Y_e = \Sigma Z_i (X_i / A_i),
\]

where \(Z_i\), \(X_i\), and \(A_i\) are the proton number, mass fraction, and atomic mass number (integer) of the species \(i\). We shall refer to compositions with \(Y_e < 0.5\) as “neutron-rich.”

If we neglect lepton capture on bound nuclei and approximate the disk material as consisting of a mixture of free nucleons and \(\alpha\) particles, the evolution of \(Y_e\) with the radius is given by (Puet, Fuller, & Cardall 2001).

\[
u \cdot \nabla Y_e = \left( \frac{1 - 2GM/rc^2}{1 - |\nabla V(r)|/c^2} \right)^{1/2} \frac{dY_e}{dr} = -\lambda_e \left( \frac{Y_e - 1 - \frac{X_{\text{free}}}{2}}{2} \right) + \lambda_e \left( 1 - Y_e - \frac{1 - \frac{X_{\text{free}}}{2}}{2} \right).
\]

1 N-Division, Lawrence Livermore National Laboratory, Livermore, CA 94550.
2 Department of Astronomy and Astrophysics, University of California, Santa Cruz, CA 95064.
Here $X_{\text{free}}$ is the mass fraction of free nucleons, $u$ is the 4-velocity of the flow, $M$ represents the black hole mass, and $V(r)$ is the radial drift velocity as measured in an inertial frame corotating with the disk. In the middle term in equation (2), we have assumed steady state conditions as well as cylindrical symmetry, and we have adopted the relatively simple Schwarzschild metric relevant for most of our calculations. Generally, relativistic effects do not make a big difference in our calculations of the electron fraction, although they are important for determining the structure of the disk.

In equation (2) $\lambda_{e^-p}$ and $\lambda_{e^+n}$ are the rates for the processes,

$$e^- + p \rightarrow n + \nu_e ,$$

$$e^+ + n \rightarrow p + \bar{\nu}_e .$$

These rates are given by (Fuller, Fowler, & Newman 1980)

$$\lambda_{e^-p} = K \int_{m/m_e}^{\infty} w^2 \left( \frac{w - \delta m}{m_e} \right)^2 G_-(1,w)S_-(T, U_F) dw ,$$

$$\lambda_{e^+n} = K \int_{1}^{\infty} w^2 \left( \frac{w + \delta m}{m_e} \right)^2 G_+(1,w)S_+(T, U_F) dw .$$

Here $K \approx 6.414 \times 10^{-4} \, \text{s}^{-1}$ determines the free neutron lifetime, $\delta m \approx 1.293 \, \text{MeV}$ is the neutron-proton mass difference, and $m_e$ is the electron mass. The functions $S_-$ and $S_+$ are the electron and positron distribution functions, while the $G_\pm$ are the Coulomb wave correction factors discussed in Fuller, Fowler, & Newman (1980).

The electron Fermi energy $U_F$ is found by inverting the expression for the net electron number density,

$$n_e^+ - n_e^- = \rho Y_e N_A = \frac{1}{\pi^2} \left( \frac{m_e c}{h} \right)^3 \int_0^{\infty} \rho^2 |S_-(T, U_F) - S_+(T, U_F)| d\rho ,$$

with $\rho = (w^2 - 1)^{1/2}$.

We solve for the evolution of $Y_e$ by integrating equation (2) for the results presented in Popham et al. (1999). Those authors assumed $Y_e = \frac{1}{3}$. Because we will show that in some cases $Y_e \leq \frac{1}{2}$ and, consequently, that the electron Fermi energy and degeneracy pressure are substantially smaller than for $Y_e = \frac{1}{3}$, our results are not entirely self-consistent but should suffice, given the very approximate nature of a one-dimensional calculation.

3. RESULTS

3.1. Scaling with Disk Viscosity and Accretion Rate

We present results for several different disk models. The parameters describing these models, as well as the electron fraction near the event horizon at $r \approx 10^6 \, \text{cm}$, are given in Table 1. In all cases we assume $Y_e = \frac{1}{3}$ at large radii. For simplicity, we have adopted a constant black hole mass of 3 $M_\odot$, although the scaling relations for mass are obvious in Popham et al. (1999). All else being equal, a larger mass will give less electron capture.

Table 1 shows a clear trend with disk viscosity and accretion rate; electron capture in the inner disk becomes more pronounced with decreasing viscosity and increasing mass accretion rate. This arises because low-viscosity flows inefficiently advect angular momentum outward and are, for a given mass accretion rate, denser than high-viscosity flows. Dense flows are electron degenerate, so that pair $e^\pm$ creation is suppressed and electron capture dominates over positron capture. Similarly, a larger accretion rate also implies a denser disk—for a given value of $a$—and more capture.

Before discussing in some detail the evolution of $Y_e$, we note that the influence of black hole spin on the composition is also important. This is because the angular momentum imparted to the black hole via the accreting matter will typically drive the Kerr parameter high. For example, a Kerr parameter of $a \approx 0.9$ is typical for collapsars. A larger Kerr parameter allows the disk to move to smaller radii before entering the event horizon, so that more electron capture and a smaller $Y_e$ will result. This is demonstrated by model G, which corresponds to a black hole with $a = 0.95$. In this model $n/p$ near the event horizon is about 10 times larger than $n/p$ for the same model except with $a = 0$ (model B). The influence of the black hole spin on $Y_e$ is limited to regions quite near the event horizon. For $r > 10^{6.5} \, \text{cm}$, $Y_e$ in model B is essentially the same as in model G. These considerations imply that as far as the

| Model   | $M^a$ | $a^b$ | $Y_e$ |
|---------|-------|------|-------|
| A........ | 0.01  | 0.1  | 0.510 |
| B........ | 0.1   | 0.1  | 0.435 |
| C........ | 0.1   | 0.03 | 0.119 |
| D........ | 0.1   | 0.01 | 0.045 |
| E........ | 1.0   | 0.1  | 0.105 |
| F........ | 0.03  | 0.1  | 0.527 |
| G........ | 0.1   | 0.01 | 0.077 |

| a | Accretion rate. |
|---|-----------------|
| b | Disk viscosity.  |

* Disk properties inferred from the scaling relations in Popham et al. 1999 for this model.
* Kerr parameter $a = 0.95$ for this model.
In composition is concerned, the main difference between flows around zero angular momentum holes and those around large angular momentum holes will be that the jet in the large angular momentum case will likely be more neutron-rich. The composition in the bulk of the wind coming from the disk, however, will likely be similar in the two cases.

In Figure 2 we show the evolution of $Y_e$ with radius for an accretion disk with $\alpha = 0.1$ and a relatively low mass accretion rate (at least for standard collapsar models), $M = 0.01 \, M_\odot \, s^{-1}$. This flow is not dense enough to make the electrons degenerate. Consequently, $Y_e$ is governed by thermal $e^\pm$ capture. In this case the only asymmetry is the neutron-proton mass difference, and $Y_e$ actually increases slightly owing to the threshold for the rate in equation (3), becoming greater than 0.5. The free nucleon mass fraction appearing in Figure 2 is calculated from the simple estimate provided in Qian & Woosley (1996). Lepton capture on heavy nuclei is negligible for this disk.

Results for a hotter and denser flow, $M = 0.1 \, M_\odot \, s^{-1}$, $\alpha = 0.1$, are shown in Figure 3. The material at radii $r \lesssim 10^{7.5}$ cm is mildly degenerate, and the electron fraction is driven to $Y_e \approx 0.44$. The quantity $\lambda_{e^-p}(r/V)$ is the product of the electron capture rate and a rough measure of the dynamic timescale, or time left before the infalling material crosses the event horizon. Because $Y_e$ cannot come to equilibrium unless $\lambda_{e^-p}(r/V) \approx 1$, $Y_e$ is not in equilibrium in this flow. Also, because $\lambda_{e^-p}(r/V) \ll 1$ when bound nuclei are present, weak processes on heavy nuclei are not important even in the extreme limit, where the bound nucleons behave as free nucleons with respect to electron capture.

Figures 4 and 5 show the influence of viscosity on the composition of the flow. For $\alpha = 0.03$, electron capture becomes important at $r \approx 10^{7.6}$ cm and $Y_e$ is in close equilibrium with $e^\pm$ capture until $r \approx 10^{6.5}$ cm. There is at most $\sim M_\odot$ of an electron capture per bound nucleus in the disk. For $\alpha = 0.01$, the inflowing material becomes degenerate and neutron-rich early on. Most of the free protons are locked into $\alpha$ particles until the $\alpha$ particles dissociate. Positron capture on the excess free neutrons results in a brief increase in $Y_e$, as seen in the bump at $r \approx 10^{7.3}$ cm in Figure 5. The final $n/p$ in this disk is very large, $\sim 20$.

The influence of the mass accretion rate on the composition is seen in Figure 6, where we plot results for $M = 1 \, M_\odot \, s^{-1}$ and $\alpha = 0.1$. This disk is quite similar to the...
\(M = 0.1 M_\odot \text{ s}^{-1}, \alpha = 0.3\) case. Indeed, this is just what is expected from the scaling relations in Popham et al. (1999). For \(M \approx 1 M_\odot \text{ s}^{-1}\) and larger, the assumption that the disk is optically thin to neutrinos begins to break down (MacFadyen & Woosley 1999; Di Matteo, Perna, & Narayan 2002). This influences both the structure of the disk and, because neutrino capture becomes important, the evolution of \(Y_e\). Because neutrino trapping likely influences the structure of the disk at the same level as the assumption that \(Y_e = \frac{1}{2}\), we do not calculate \(Y_e\) for disks calculated under the assumption of partial neutrino trapping (and \(Y_e = \frac{1}{2}\)).

### 3.2. Effects of Neutrino Capture

The effect of neutrino capture on the evolution of \(Y_e\) can be addressed in an approximate way. In particular, we are interested in the rate \(\nu_e n\) for the process

\[\nu_e + n \rightarrow p + e^- .\] (8)

The capture of electron antineutrinos can be neglected because of both the low proton number density and the paucity of positrons in the flow. The number of neutrinos captured per neutron is roughly \(r_{\nu_e, \text{cap}} n_0 / n_n \approx Y_e \tau_{\nu_e, \text{cap}} n_0 / n_p\), where \(\tau_{\nu_e, \text{cap}}\) is the optical depth for the process in equation (8), and \(n_{\nu_e} / n_0 (n_0 / n_p)\) is the number of electron neutrinos produced per neutron (proton) in the flow. The ratio \(R\) of the number of neutrino captures per neutron to the number of electron captures per proton is

\[R \approx \frac{\lambda_{\nu_e}}{\lambda_{e^- p}} = \tau_{\nu_e, \text{cap}} Y_e .\] (9)

This equation implies that an optical depth of \(1\) to \(\nu_e\) capture is roughly equivalent to a doubling of \(\lambda_{e^- n}\). We note here that while the \(\nu_e\)'s produced in electron-capture reactions are not thermal, their average capture section is only approximately a factor of 2 higher than the average capture cross section calculated under the assumption of a thermal distribution for the electron neutrinos (at the electron Fermi energy and temperature) for conditions of interest in accretion disks.

Equation (9) only includes the influence of electron neutrinos produced via electron capture on protons. This is justified because the contribution to \(\nu_e\) production from \(e^+ e^-\) annihilation is small when neutrinos are trapped in accretion disks. To see this, note that the ratio of the \(\nu_e\) luminosity arising from \(e^+ e^-\) annihilation to the \(\nu_e\) luminosity arising from electron capture on protons is \(L_{\text{annihilation}} / L_{e^- p} \approx (1/3)(T / 10^{11} \text{ K})^3 (10^{10} \text{ g cm}^{-3} / \rho Y_e)\) (e.g., Popham et al. 1999). This equation assumes nondegenerate conditions. Because electron degeneracy suppresses positron formation, the contribution to the neutrino luminosity from \(e^+ e^-\) annihilation will be somewhat smaller than this estimate. Di Matteo et al. (2002) give estimates of \(T^3 / \rho\) for flows in which neutrino trapping occurs. In all cases, \(L_{\text{annihilation}} / L_{e^- p} < 1\). For example, for the \(M = 1 M_\odot \text{ s}^{-1}, \alpha = 0.1\) disk, \(L_{\text{annihilation}} / L_{e^- p} \lesssim (1/134 Y_e)\). For flows with mass accretion rates much larger than \(M = 1 M_\odot \text{ s}^{-1}\), a weak equilibrium description of the disk is more appropriate than our description of the flow as being marginally optically thick to neutrinos. We do not study such flows in this paper.

It is difficult to quantitatively calculate the influence of neutrino capture on \(Y_e\) in a postprocessing step. This is because of the complexities of neutrino transport, as well as because of the feedback between \(Y_e\), the electron capture rate, and the disk dynamics. However, the following simple considerations argue that neutrino capture is unlikely to have a dramatic influence on \(Y_e\).

To discuss some specific cases, first consider the \(M = 0.1 M_\odot \text{ s}^{-1}, \alpha = 0.1\) disk. Di Matteo et al. (2002) argue that neutrinos will have an absorptive optical depth of unity at \(r \approx 3 \times 10^6\) cm for this disk. Because weak processes have essentially been frozen out by this radius [see the \(\lambda_{e^- p}(r/V)\) curve in Fig. 3], neutrino capture will have a small influence on \(Y_e\).

Denser flows trap neutrinos more efficiently but are still expected to remain neutron-rich. For example, for the \(M = 1 M_\odot \text{ s}^{-1}, \alpha = 0.1\) disk, Di Matteo et al. (2002) show that the neutrino absorptive optical depth will be greater than unity for \(r < 2 \times 10^7\) cm. As weak processes are rapid compared to the dynamic timescale in this disk, the result will be an increase in \(Y_e\). To estimate the magnitude of this increase, we adopt the simple procedure of artificially increasing the positron capture rate, \(\lambda_{e^- n} \rightarrow 5 \lambda_{e^- n}\), which is likely an overestimate of the protonization rate. The increase in \(\lambda_{e^- n}\) results in an electron fraction of \(Y_e \approx 0.166\) near the event horizon. This is approximately 50% higher than the electron fraction calculated by neglecting the influence of \(\nu_e\) capture, although still quite neutron-rich.

To summarize, \(Y_e\) in disks with high viscosity and modest accretion rates will not be affected by neutrino capture simply because neutrinos are not trapped or because weak processes are too slow. The electron fraction in disks with neutrino absorptive optical depths of a few will increase somewhat but will remain neutron-rich owing to the sharp rise in \(\lambda_{e^- p}\) with \(Y_e\).

### 3.3. Transport to the Surface of the Disk

In order for a low electron fraction to have observable implications, the electron fraction must not come to equilibrium at \(Y_e = \frac{1}{2}\) as the material travels out of the plane of the disk. To estimate the evolution of the electron fraction in convective blobs moving out of the disk, we parameterize the convective timescale by the turbulent convection speed (Narayan et al. 2001),

\[v_{\text{turb}} \approx \alpha \nu_K = 6.3 \times 10^8 \alpha_{-1} r_{-1}^{-1/2} \text{ cm s}^{-1} ,\] (10)
where $\alpha_1 = \alpha/0.1$ and $r_7 = r/10^7$ cm. This implies a time to go 1 pressure scale height,

$$\tau_{\text{conv}} \sim r/v_{\text{turb}} \sim 16\alpha_1^{-1} r_7^{1/2} \text{ ms} \ . \ (11)$$

We also assume that the convective blob expands adiabatically. An estimate for the entropy per baryon in the disk is given by Qian & Woosley (1996),

$$s/k_b \approx 0.052 \frac{T_{\text{MeV}}^3}{\rho_{10}} + 7.4 + \ln \left( \frac{T_{\text{MeV}}^{3/2}}{\rho_{10}} \right) \ . \ (12)$$

Here, $T_{\text{MeV}}$ is the temperature in MeV and $\rho_{10}$ is the density in units of $10^{10}$ g cm$^{-3}$. The first term on the right-hand side of equation (12) is the contribution to the entropy from relativistic light particles ($\gamma/e^\pm$), while the next two represent the contribution from free nucleons. Equation (12) is not appropriate when electrons are degenerate. For degenerate electrons, the contribution of thermal $e^\pm$ pairs is small and the coefficient 0.052 should be closer to 0.02, representing just the entropy of the photons. However, changing the coefficient to 0.02 makes little difference for our purposes.

When relativistic particles dominate the entropy, the adiabat satisfies $T^3 \propto \rho$. When free nucleons dominate the entropy, the adiabat satisfies $T^{3/2} \propto \rho$. The evolution of the electron fraction in a convective blob is crucially sensitive to the adiabat on which the blob travels.

To illustrate this, consider first a disk with $M = 0.01$, $\alpha = 0.1$. In Figure 7 we show the evolution of the electron fraction in matter originating from two different points in this disk and for different assumptions about $\tau_{\text{conv}}$. The top curves in Figure 7 correspond to material originating from $r = 10^6$ cm in the disk. At this point the entropy is relatively high, $s/k_b \sim 23$ by the estimate in equation (12), and consequently the adiabat is closer to $T^3 \propto \rho$ than to $T^{3/2} \propto \rho$. The temperature decreases quite slowly relative to the density, allowing for the formation of pairs that efficiently drive $n/p$ to equality. For large $\tau_{\text{conv}}$, this process occurs more efficiently than for small $\tau_{\text{conv}}$. The lower curves in Figure 7 correspond to material originating from $r = 10^7$ cm in the disk. At this point the entropy is $\approx 17$, so the adiabat is near $T^{3/2} \propto \rho$. For low entropies, then, the temperature and density decrease at a comparable rate and pair formation is somewhat suppressed, keeping $Y_e$ low.

For flows in which the electron Fermi energy is large enough to make the inner disk neutron-rich, the entropy is generally dominated by nonrelativistic particles, and the adiabat again preserves a low $Y_e$. This is illustrated in Figure 8, where we plot the evolution of electron fraction in an adiabatic convective bubble for two different disks characterized by a low $Y_e$. In this figure we show the evolution of a fluid element from only a single initial radius ($10^7$ cm) because the entropy in these disks is never dominated by relativistic particles when $Y_e$ is low, so that the behavior shown in Figure 8 is generic.

The above considerations outline the general features of how $Y_e$ evolves in material as it travels from the center to the surface of the disk. In flows where $Y_e$ becomes small, the degeneracy of the electrons implies that free nucleons dominate the entropy. A low $Y_e$, then, will remain low. By contrast, in flows where $Y_e$ is close to $\frac{1}{3}$, the adiabat can be close enough to $T^3 \propto \rho$ for $Y_e$ to be driven to $\frac{1}{3}$ and the neutron excess at the center of the disk is larger than the neutron excess in material finding its way out of the disk.

Similar considerations apply to the evolution of $Y_e$ in fluid elements after they have left the disk—low-entropy flows remain neutron-rich, while high-entropy flows tend toward $Y_e = \frac{1}{3}$ (or greater). A complete description of the composition in the disk wind involves knowing, among other things, how much entropy is deposited by the mechanism driving outflow from the disk. For the sake of estimating an upper bound to the increase in $Y_e$, we make the assumption that the heating mechanism deposits enough entropy that the outflow becomes radiation-dominated. In this case, thermal $e^\pm$ pair capture determines $Y_e$. For temperatures greater than an MeV, the rate for pair $e^+ + e^-$ capture is $\lambda_{\text{cap}} \approx 0.4 (T/1 \text{ MeV})^3 \text{s}^{-1}$. Assuming that the temperature evolves as $T = T_0 \exp(-t/3\tau_{\text{dyn}})$, with $\tau_{\text{dyn}}$ the timescale characteristic of expansion outside of the disk, the total number of positron captures is roughly $0.03 (T_0/1 \text{ MeV})^3 (\tau_{\text{dyn}}/0.1 \text{ s})$. Here, $T_0$ is the peak temperature of the ejecta after it has left the disk. A reasonable approximation is that $T_0$ corresponds to the temperature after the
convective bubble has gone up 1 density scale height (this is true for neutrino-driven winds from nascent neutron stars, where the temperature outside of the neutrinosphere is a decreasing function of radius). The asymptotic (at infinity) electron fractions obtained under the above assumptions for the flows corresponding to $\tau_{\text{conv}} = 50 \text{ ms}$ in Figure 7 are $Y_e = 0.45$ for the $r = 10^7 \text{ cm}$ curve and $Y_e = 0.61$ for the $r = 10^6 \text{ cm}$ curve, while for the flows corresponding to $\tau_{\text{conv}} = 50 \text{ ms}$ in Figure 8 the asymptotic electron fractions are $Y_e = 0.29$ for the $M = 0.1 M_\odot \text{ s}^{-1}$ curve and $Y_e = 0.3$ for the $M = 1 M_\odot \text{ s}^{-1}$ curve. For all of these estimates, we have taken $\tau_{\text{dyn}} = 50 \text{ ms}$. The general trend in asymptotic $Y_e$ is similar to the trend of $Y_e$ within the disk. Very neutron-rich flows tend to remain neutron-rich, while flows with $Y_e$ near $\frac{1}{3}$ are driven to higher $Y_e$. Our estimates here are simple, and ultimately the details of the composition above the disk will depend on the hydrodynamics of the outflow.

4. IMPLICATIONS

4.1. Radioactivity in the Disk Wind

The most obvious consequence of our calculations is that $^{56}\text{Ni}$ will be absent from the winds of hyperaccreting black holes unless (1) the accretion rate is low ($\lesssim 0.1 \dot{M}_\odot \text{ s}^{-1}$) and (2) the disk viscosity is high, $\alpha \lesssim 0.1$. Interestingly, modern views regarding $\alpha$ disks and GRB models favor values close to these.

Typical accretion rates in the collapsar model are $0.05$–$0.1 \dot{M}_\odot \text{ s}^{-1}$ (MacFadyen & Woosley 1999; their Fig. 5 and Fig. 10). The neutron excess will be smaller in Type II collapsars powered by fallback rather than direct black hole formation (MacFadyen, Woosley, & Heger 2001). Considerable accretion into the hole and mass loss from the disk may continue, at a declining rate, even after the main GRB-producing event ($\sim 20 \text{ s}$) is over (Zhang & Woosley 2002). It thus seems likely that the collapsar model will be able to provide the $^{56}\text{Ni}$ necessary to make the supernovae that accompany GRBs (although only if $\alpha \gtrsim 0.1$). This is also true of the slower accreting models like helium-star black hole mergers and black hole–white dwarf mergers. However, any wind from merging compact objects, or similar models like the supranova, will be neutron-rich. Although perhaps of interest for nucleosynthesis, they will not produce $^{56}\text{Ni}$, at least during the black hole accretion epoch.

4.2. Neutron Excess in GRB Jets

For disks with high accretion rates, and certainly for merging neutron stars or black hole–neutron star mergers, the matter near the event horizon will be very neutron-rich. If this material pollutes the outgoing jet, the GRB jet will itself, at least initially, contain free neutrons.

The dynamics of accelerating neutron-rich jets can differ dramatically from the dynamics of pure proton jets. Fuller et al. (2000) showed that a high Lorentz factor fireball that is neutron-rich can lead to two very distinct kinematic components: a slow neutron outflow and a fast proton outflow. This arises because the uncharged neutrons are weakly coupled to the radiation-dominated plasma and are accelerated principally via strong neutron-proton scatterings (Derishev et al. 1999). Strong scatterings freeze out when they become slow compared, with the dynamic timescale, and at this point the neutrons coast. If this decoupling occurs while the jet is still accelerating, then the Coulomb-coupled protons go on to have a larger Lorentz factor than the neutrons.

Roughly, dynamic neutron decoupling is only expected for fast jets. Here, the precise meaning of “fast” depends, among other things, on the fireball source size. For relativistic flows originating from compact objects, the dynamic timescale characterizing the acceleration of the flow is smaller than 1 ms, and the final Lorentz factor must be greater than $\sim 300$ in order for dynamic neutron decoupling to occur. For jets in the collapsar model, the timescale characterizing the acceleration is set by the surface of last interaction of the jet with the stellar envelope at $\sim 10^{11} \text{ cm}$. In this case, the entropy per baryon in the jet has to be greater than approximately $3 \times 10^{6}$ in order for dynamic neutron decoupling to occur. It is not clear whether or not such high entropies are reached in jets from collapsing stars.

There may be a detectable neutrino signature of dynamic neutron decoupling. Neutron-proton collisions occurring during decoupling will generate pions. In turn, these pions will decay and lead to the generation of neutrinos with energies of a few GeV in our reference frame. These neutrinos should be detectable at the rate of a few per year in next-generation neutrino telescopes (Bahcall & Mészáros 2000). In addition, there may also be a direct electromagnetic signature of neutron decoupling. For some bursts arising from external shocks, a slow and decoupled neutron shell will decay and shock with the outer proton shell as the outer shell plows into the interstellar medium. This leads to a characteristic two-peaked structure in the burst (Pruet & Dalal 2002).

Regardless of whether dynamic neutron decoupling occurs, neutrons in the jet may have observable implications for the GRB afterglow. This is particularly true when the shocking radius, i.e., the radius at which the shocks giving rise to the observed gamma-rays occur, is smaller than the length over which free neutrons decay. This condition is

$$r_{\text{shock}} \lesssim \gamma_2^2 \tau_n c \approx 10^{15} \gamma_2 \text{ cm}, \quad (13)$$

where $r_{\text{shock}}$ is the shocking radius, $\gamma_2$ is the Lorentz factor of the outflow in units of 100, and $\tau_n \sim 1000 \text{ s}$ is the free neutron lifetime. Equation (13) is generally satisfied for bursts from internal shocks and bursts from external shocks in a very dense medium. When equation (13) holds, the neutrons can stream ahead of the slowing proton shell (the complement of the dynamic neutron decoupling discussed above) and deposit energy as they decay. Implications for the resulting afterglow in such a case have been discussed by Beloborodov (2002).

In § 4.3 we discuss the $r$-process in winds and jets from accretion disks. However, we first note that GRB jets are characterized by interesting light-element synthesis. Pruet, Guiles, & Fuller (2002) and Lemoine (2002) calculated the thermal synthesis of light elements in GRB-like outflows. They find that the final deuterium mass fraction can be of order 1%, some 3 orders of magnitude larger than the primordial yield. For kinematically well-coupled flows, the nucleosynthesis depends sensitively on $Y_e$, with deuterium yield decreasing with increasing neutron excess. For flows in which dynamic neutron decoupling occurs, high-energy neutron-$\alpha$ collisions will spill deuterons and can result in final deuteron mass fractions as high as 10% (Pruet et al. 2002).
4.3. The r-Process

There are two possible sites for the r-process here in those cases where \( Y_e \) is low—in the jet and in the disk wind.

4.3.1. In the Jet

The jet has the merit of originating in the vicinity of the black hole where the neutron excess is likely to be greatest. If magnetic fields drive the heating and initial acceleration of the jet, then this neutron excess will likely be preserved. If neutrinos drive the outflow (MacFadyen & Woosley 1999), \( Y_e \) will be reset to some extent by the weak interactions. However, if the details of the outflow above the black hole are similar to spherically symmetric neutrino-driven ultrarelativistic winds, then only \( \nu\bar{\nu} \) annihilation will be important, and the flow will remain neutron-rich (Prue, Fuller, & Cardall 2001).

It is clear that the rapid expansion of the jet will be favorable to freezing out with a large abundance of free neutrons, and this is conducive to the r-process (Hoffman, Woosley, & Qian 1997). However, a pure nucleonic jet coming from the adiabatic expansion of a fireball with energy loading \( \eta = E_{\text{internal}}/\rho \omega^2 \gg 100 \) is too much of a good thing when it comes to entropy and rapid expansion. The entropy was given in equation (12) (Qian & Woosley 1996),

\[
\frac{s}{k} = \frac{11\pi^2}{45\left(\frac{kT}{\hbar c}\right)^3} m_n \rho. \tag{14}
\]

The internal energy (ergs g\(^{-1}\)) is

\[
\epsilon = \frac{11\pi^2}{60\left(\frac{kT}{\hbar c}\right)^4} \frac{1}{\rho}, \tag{15}
\]

and hence, for \( \eta = 200 \) and \( kT \approx 2 \text{ MeV} \),

\[
\frac{s}{k_B} = \frac{4}{3} \frac{\eta \epsilon^2 m_n}{kT} \approx 10^5. \tag{16}
\]

The timescale for the expansion, \( \tau/\epsilon \), is less than a millisecond. The high entropy implies a low density at the temperature when \( n \) and \( \rho \) can start to recombine. Coupled with the rapid expansion, one might conclude that the jet will remain pure nucleons and light nuclei and that an r-process is impossible.

This conclusion would be erroneous on several counts. First, the jet may not always have such high-energy loading. There may be cases where no gamma-ray burst is produced and the entropy loading of the jet is much less. The entropy would then decrease and the expansion timescale would increase. Whether to call these “dirty fireballs” or simply an extension of the “disk wind” is simply a matter of taste (see below).

Of more relevance to GRB models, and particularly to those that involve massive stars, is that the jet will not escape the star without interaction. Along the walls of the jet, the Kelvin-Helmholtz instability will occur (Zhang & Woosley 2002). The jet will also encounter one or more shocks where it will be abruptly slowed. Most interesting is the “jet head,” which moves through the star subrelativistically with a speed that varies but is of order \( c/3 \). At this head the jet mixes with stellar material and the mixture is swept backward (in the moving frame), forming a cocoon. It typically takes 5–10 s for the jet head to penetrate the roughly 1 solar radius of overlying Wolf-Rayet star in the collapsar model.

During this time, neutrons in the jet are mixed with heavy nuclei—He, O, Si, Ne, Mg and the like—which can serve as seeds for an r-process. The overall dynamics is likely to be quite complex, and its study would require a multidimensional relativistic simulation well beyond the scope of this paper. However, Zhang & Woosley (2002) find typical densities and timescales in the jet prior to breakout are \( 10^{-4} \) to \( 10^{-1} \) g cm\(^{-3}\) and seconds. This is enough that, if not all of the neutrons in the jet would capture.

Still, the overall yield in gamma-ray bursts is too small to account for the solar r-process. Assume there is one GRB for each 100 supernovae. Each burst has \( 10^{51} \)–\( 10^{52} \) ergs of relativistic ejecta (both jets) and a Lorentz factor that is at least 200 (Lithwick & Sari 2001). The jet will be some mixture of entrained stellar material and jet, but assume that the initial jet is half or more of the relativistic ejecta. This gives an equivalent yield of about \( 10^{-7} M_\odot \) per supernova. Probably, the r-process will be at most 10% of this (the rest may be alpha particles and entrained nuclei). This then is several orders of magnitude less than required to make the solar r-process. However, GRB jets might still be important sources of the r-process in metal-deficient stars or of select rare nuclei in the Sun.

4.3.2. The r-Process in the Disk Wind

A more significant r-process could come from the nonrelativistic ejecta that comprise the disk wind, provided that the entropy is increased by magnetic processes to well above its value in the middle of the disk, as in, for example, Daigne & Mochkovitch (2002). Assume that the accretion is 50% efficient, that is, that half the material that enters the disk eventually enters the black hole and the other half is blown away (Narayan et al. 2002). A typical collapsar-powered GRB involves the accretion of from one to several solar masses, so \( \sim 1 \) \( M_\odot \) is ejected from the disk. Most of this will not be neutron-rich enough or expand fast enough to make the r-process, but consider the implications if only 1% of the mass that is lost comes from the inner disk at times when the accretion rate is over 0.1 \( M_\odot \) s\(^{-1}\) and \( Y_e < 0.4 \).

So close to the hole, significant entropy will be added by any acceleration process that produces a strong outflow. A crude estimate is \( s/k_B \sim GMm_n/(rT_{\text{max}}) \sim 400 r_6^{-1} \) (2 MeV/T\(_{\text{max}}\)) (Qian & Woosley 1996), where \( T_{\text{max}} \) is the temperature at which the energy deposition is the greatest. In fact, the conditions in the inner disk for accretion rates \( \sim 0.1 \) \( M_\odot \) s\(^{-1}\) are not so different—in terms of neutrino luminosity, neutron excess, temperature, and gravitational potential—from the neutrino-driven wind in an ordinary supernova, and one might expect a similar r-process (Woosley et al. 1994). The gravitational potential, however, can in principle be greater, and a larger entropy and shorter timescale might be realized. Assume that the ejecta will be very rich in alpha particles and contain of order 10% r-process by mass. Putting the numbers together then gives an r-process synthesis equivalent to \( 10^{-5} \) \( M_\odot \) per supernova, making this possibility well worth further investigation.

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