Selective Transport and Mobility Edges
in Quasi-1D Systems with a Stratified Correlated Disorder

F. M. Izrailev
Instituto de Física, Universidad Autónoma de Puebla,
Apartado Postal J-48, Puebla, Pue., 72570, México

N. M. Makarov
Instituto de Ciencias, Universidad Autónoma de Puebla,
Priv. 17 Norte No 3417, Col. San Miguel Hueyotlipan, Puebla, Pue., 72050, México

We present analytical results on transport properties of many-mode waveguides with randomly stratified disorder having long-range correlations. To describe such systems, the theory of 1D transport recently developed for a correlated disorder is generalized. The propagation of waves through such waveguides may reveal a quite unexpected phenomena of a complete transparency for a subset of propagating modes. We found that with a proper choice of long-range correlations one can arrange a perfect transparency of waveguides inside a given frequency window of incoming waves. Thus, mobility edges are shown to be possible in quasi-1D geometry with correlated disorder. The results may be important for experimental realizations of a selective transport in application to both waveguides and electron/optic nanodevices.

PACS numbers 72.10.-d; 72.15.Rn; 73.20.Fz; 73.20.Jc; 73.23.-b

During last few years there is a burst of interest to the problem of localization-delocalization transition in systems with correlated disorder (see, e.g., [1,2] and references therein). This fact is due to the possibility to observe an anomalous transport in 1D models with random potentials. In particular, it was shown [2] that specific long-range correlations give rise to a complete transparency of electron waves for given energy intervals. Experimental realization of such potentials for single-mode waveguides with delta-like scatters [3] has confirmed theoretical predictions.

The subject of wave propagation through disordered many-mode waveguides is important both from the theoretical viewpoint and for experimental applications such as optic fibers, remote sensing, radio wave propagation, shallow water waves, etc (see, for example, [4]). It has also a direct relation to the problem of electronic transport in mesoscopic conducting channels. So far, main results in this field have been obtained for random potentials of white-noise type. However, there are many physical situations where this assumption is not correct. Therefore, the understanding of generic properties of transport in the models with correlated disorder is important for the modern theory of electron/wave propagation. It should be also stressed that existing experimental technics allow for the construction of systems with sophisticated scattering potentials resulting in anomalous transport properties [5].

In this paper we study transport properties of quasi-1D waveguides with a stratified bulk disorder that has long-range correlations. Our main interest is in exploring the possibility of constructing such random potentials that result in frequency windows of a perfect propagation of waves.

In what follows we consider a plane waveguide (or conducting wire) of width \( d \), stretched along the \( x \)-axis. The \( z \)-axis is directed perpendicularly to the waveguide so that its lower edge is \( z = 0 \) and the upper one is \( z = d \). The waveguide is assumed to have a stratified (along the \( x \)-axis) region of the length \( L (|x| < L/2) \). As a physically plausible model for such a disorder we consider random potential \( V(x) \) with the zero average, \( \langle V(x) \rangle = 0 \). The angular brackets here and below stand for the statistical average over realizations of the random function \( V(x) \). Its correlator \( \langle V(x)V(x') \rangle = W(|x - x'|) \) is assumed to have a maximal value at \( x = x' \) and to decrease with an increase of \( |x - x'| \). For statistical treatment of the problem it is also important to assume that the characteristic scale of the decrease of \( W(|x - x'|) \) is much less than the length \( L \) of the scattering region.

We shall characterize transport properties of the waveguide by its transmittance \( T(L) \) that within the linear response theory is defined by the Kubo’s formula [6],

\[
T(L) = -\frac{4}{L^2} \int d\vec{r}d\vec{r}' \frac{\partial G(\vec{r},\vec{r}')}{\partial x} \frac{\partial G^*(\vec{r},\vec{r}')}{\partial x'}. \tag{1}
\]

Here the integration with respect to \( \vec{r} = (x, z) \) runs over the disordered region. Since the potential \( V(x) \) does not depend on \( z \), the retarded Green function has the form,

\[
G(\vec{r},\vec{r}') = \frac{2}{d} \sum_{n=1}^{N_d} \sin \left( \frac{\pi n z}{d} \right) \sin \left( \frac{\pi n z'}{d} \right) G_n(x,x'). \tag{2}
\]

Here \( N_d = [kd/\pi] \) is the total number of propagating waveguide modes (conducting channels) determined by
the integer part [...] of the ratio \( kd/\pi \). The wave number \( k \) is equal to \( \omega/c \) for a classical scalar wave of frequency \( \omega \), and to the Fermi wave number for electrons.

By substituting Eq. (2) into Eq. (1), we come to the Landauer’s formula [7],

\[
T(L) = \frac{N_d}{n=1} T_n(L),
\]

(3)

with \( T_n(L) \) being obtained from Eq. (1) in which the integration is performed over \( x, x' \) only, and \( G \) is replaced by \( G_n \). The mode Green function \( G_n(x, x') \) is entirely determined by the 1D equation with the random potential \( V(x) \) and lengthwise wave number \( k_n = \sqrt{k^2 - (\pi n/d)^2} \),

\[
\left[ \frac{d^2}{dx^2} + k_n^2 - V(x) \right] G_n(x, x') = \delta(x - x').
\]

(4)

From Eq. (3) one can see that the total transmittance \( T(L) \) of a quasi-1D stratified structure is expressed as a sum of partial transmittances \( T_n \). Every transmittance \( T_n \) describes the transparency of the corresponding \( n \)th propagating mode. In such a way we reduce the transport problem for the quasi-1D disordered system to the 1D random model (4) of wave scattering in every \( n \)th channel. Eq. (4) can be solved by one of the methods developed in the transport theory of one-dimensional disordered systems, such as the perturbative diagrammatic method of Berezinskii [8], invariant imbedding method [9], or the two-scale approach [10].

The main result is that the average partial transmittance \( \langle T_n \rangle \) as well as all its moments, are described by the universal function that depends on one parameter \( \Lambda_n \equiv L/L_{loc}(k_n) \) only (one-parameter scaling). The quantity \( L_{loc}(k_n)/4 \) is, in fact, the backscattering length for \( n \)-th propagating mode. The inverse value \( L_{loc}^{-1}(k_n) \) is equal to the corresponding Lyapunov exponent that can be found in the transfer matrix approach, so that the quantity \( L_{loc}(k_n) \) is the localization length [11] associated with a given 1D channel.

Since general expression for the mode transmittance has quite complicated form (see, e.g., Ref. [10]), here we refer to limit cases only. Specifically, for large localization length, \( \Lambda_n = L/L_{loc}(k_n) \ll 1 \), the transmittance \( \langle T_n \rangle \) exhibits the ballistic behavior and the corresponding \( n \)th normal mode is practically transparent,

\[
\langle T_n \rangle \approx 1 - 4\Lambda_n.
\]

(5)

On the contrary, the transmittance is exponentially small when the localization length is much less than the length of the waveguide, \( \Lambda_n \gg 1 \),

\[
\langle T_n \rangle \approx \frac{\pi^{5/2}}{16} \Lambda_n^{-3/2} \exp(-\Lambda_n).
\]

(6)

This implies strong wave localization in a given \( n \)th channel. Since in this case the transmittance \( T_n \) is not a self-averaged quantity, it is more reasonable to deal with its average logarithm, \( \langle \ln T_n \rangle = -4\Lambda_n \).

Thus, in order to describe transport properties of a quasi-1D waveguide with 1D bulk disorder, the value of localization lengths \( L_{loc}(k_n) \) should be known. From the solution of the equation (4) one can obtain [8–10],

\[
L_{loc}^{-1}(k_n) = \frac{W(2k_n)}{16k_n^2},
\]

(7)

where \( W(k) \) is the Fourier transform of the binary correlator \( W(|x - x'|) \) for the scattering potential,

\[
W(|x - x'|) = \int_{-\infty}^{\infty} \frac{dk_x}{2\pi} \exp[ik_x(x - x')] W(k_x).
\]

(8)

The main feature of the expression (7) for \( L_{loc}(k_n) \) is its dependence on the mode index \( n \). One can see that the larger \( n \) is, the smaller is \( L_{loc}(k_n) \) and, consequently, the stronger is the coherent scattering within this mode. This dependence is quite strong due to the squared wave number \( k_n \) in the denominator of Eq. (7). Evidently, with an increase of the index \( n \) the value of \( k_n \) decreases.

An additional dependence appears because of the power spectrum \( W(2k_n) \). Since the correlator \( W(|x - x'|) \) is a decreasing function of \( |x - x'| \), the numerator \( W(2k_n) \) increases with \( n \) (it is a constant in the case of the delta-correlated potential only). Therefore, both the numerator and denominator contribute in the same way for the dependence of \( L_{loc}(k_n) \) on \( n \). As a result, we arrive at the concept of hierarchy of mode localization lengths,

\[
L_{loc}(k_N) < L_{loc}(k_{N-1}) < \ldots < L_{loc}(k_2) < L_{loc}(k_1).
\]

Thus, a remarkable phenomenon arises. On the one hand, the concept of one-parameter scaling holds for any of \( N_d \) channels whose partial transport is characterized solely by the value of \( \Lambda_n \). On the other hand, this concept turns out to be broken for the total waveguide transport. Indeed, due to the revealed hierarchy of \( L_{loc}(k_n) \), the total transmittance (3) depends on the whole set of scaling parameters \( \Lambda_n \). This fact is in contrast with quasi-1D bulk-disordered models, for which all transport properties are described by one parameter only. In this sense, our model is similar to quasi-1D waveguides with rough surfaces, for which the similar hierarchy of attenuation lengths was recently found [12,13].

The interplay between the hierarchy of \( L_{loc}(k_n) \) and the one-parameter scaling for \( \langle T_n \rangle \) leads to that, in general, quasi-1D stratified structures exhibit three transport regimes.

(I) If the largest of the localization lengths, \( L_{loc}(k_1) \), is much less than the waveguide length \( L \),

\[
L_{loc}(k_1) \ll L,
\]

(9)

all the propagating modes are localized and the waveguide is non-transparent.
(II) On the contrary, when the smallest localization length \( L_{loc}(k_{N_d}) \) is much larger than \( L \),

\[
L_{loc}(k_{N_d}) \gg L, \tag{10}
\]

all the propagating modes are open (\( \langle T_n \rangle \approx 1 \)) and the waveguide has almost perfect transparency. The total ballistic transmittance in this case is equal to the total number of the propagating modes, \( \langle T(L) \rangle = N_d \).

(III) The intermediate situation arises when \( L_{loc}(k_1) \) of the first mode is larger, while \( L_{loc}(k_{N_d}) \) of the \( \text{"last"} \) \( N_d \)-th mode is smaller than the waveguide length \( L \),

\[
L_{loc}(k_{N_d}) \ll L \ll L_{loc}(k_1). \tag{11}
\]

In this case a very interesting phenomenon of the coexistence of ballistic and localized transport occurs. Namely, while lowest modes are in the ballistic regime, highest modes are strongly localized.

As a demonstration, let us consider waveguides with a large number of conducting channels, \( N_d = |kd/\pi| \approx kd/\pi \gg 1 \), and with potentials having the widely used Gaussian correlator,

\[
W(|x|) = W_0 k_0 \pi^{-1/2} \exp(-k_0^2 x^2), \quad W(k_x) = W_0 \exp(-k_x^2/4k_0^2). \tag{12}
\]

It is convenient to introduce two parameters

\[
\alpha = \frac{L}{L_{loc}(k_1)} = \frac{W_0 L}{16k_0^2}, \quad \delta = \frac{L_{loc}(k_{N_d})}{L_{loc}(k_1)} = \frac{2|kd/\pi|}{(kd/\pi)} \ll 1, \tag{13}
\]

where \( L_{loc}(k_1) \) and \( L_{loc}(k_{N_d}) \) refer, respectively, to the largest and the smallest localization lengths in the limit case of the white-noise potential (\( k_0 \rightarrow \infty \)), and \( \{kd/\pi\} \) is the fractional part of the parameter \( kd/\pi \).

By applying the above discussed approach, one can find that for Gaussian correlations (12) all the propagating modes are localized when \( \alpha \gg \exp(k^2/k_0^2) \). This inequality is stronger than that valid for the white-noise case, \( \alpha \gg 1 \). The intermediate situation occurs when

\[
\exp(k^2/k_0^2) \gg \alpha \gg \delta \exp(\delta k^2/k_0^2). \tag{14}
\]

One can see that the longer range \( k_0^{-1} \) of the correlated disorder is, the simpler are the conditions (14) of the coexistence of ballistic and localized transport.

Finally, the waveguide is almost perfectly transparent in the case when \( \alpha \ll \delta \exp(\delta k^2/k_0^2) \). In particular, at any given value of \( \alpha \gg 1 \) (fixed values of the waveguide length \( L \), wave number \( k \) and the disorder strength \( W_0 \)) the ballistic transport of all the conducting channels can be realized by a proper choice of the stratified correlated disorder. From Eq. (14) one can also understand under what conditions the coexistence of ballistic and localized modes can be achieved for the standard white-noise case.

The fundamentally different situation arises when the stratified medium has specific long-range correlations. To show this, we would like to note that the localization length \( L_{loc}(k_n) \) of any \( n \)-th conducting channel are entirely determined by the dependence \( W(k_x) \), see Eq. (7). Therefore, if \( W(2k_n) \) abruptly vanishes within some interval of wave number \( k_n \), then \( L_{loc}(k_n) \) diverges and the corresponding propagating mode appears to be fully transparent for any length of the waveguide. Thus, it is naturally to ask how to construct such random profiles of stratified medium that result in a complete transparency of waveguides within any desired frequency region. The answer to this question can be found from a generalization of the methods developed in Refs. [2,14]. Specifically, having a desirable dependence for the randomness power spectrum \( W(k_x) \), one should obtain the function \( \beta(x) \) whose Fourier transform is \( W^{1/2}(k_x) \). Then, the random potential \( V(x) \) can be constructed as a convolution of white noise \( Z(x) \) with the function \( \beta(x) \),

\[
V(x) = \int_{-\infty}^{\infty} dx' Z(x-x') \beta(x'). \tag{15}
\]

We emphasize that the transition between localized and ballistic wave/electron transport can be arranged in an abrupt way at any given point inside the allowed interval for \( k_n \). In order to achieve such a transition, it is convenient to take \( W(k_x) \) with a discontinuity at the desired point. In other words, the random potential \( V(x) \) should be of a specific form with long-range correlations along the waveguide.

To show how to arrange a sharp transition, let us take a random potential \( V(x) \) with the following power spectrum \( W(k_x) \),

\[
W(k_x) = W_0 \Theta(|k_x| - 2k_0), \tag{16}
\]

where \( \Theta(x) \) is the unit-step function and the characteristic wave number \( k_0 > 0 \) is a correlation parameter to be specified. According to Eq. (15), the random potentials having such correlator can be constructed as

\[
V(x) = W_0^{1/2} \left[ Z(x) - \int_{-\infty}^{\infty} dx' Z(x-x') \frac{\sin(2k_0 x')}{\pi x'} \right].
\]

Now one can see that in the case under consideration all low modes with wave numbers \( k_n > k_0 \) have finite localization lengths while for high modes with \( k_n < k_0 \) the localization lengths diverge,

\[
L_{loc}^{-1}(k_n > k_0) = W_0/16k_n^2, \quad L_{loc}^{-1}(k_n < k_0) = 0. \tag{17}
\]

Remarkably, in contrast to the potentials with Gaussian correlations (see above), all propagating modes with

\[
n > N_{loc} = [(kd/\pi)(1 - k_0^2/k^2)^{1/2}] \Theta(k - k_0), \tag{18}
\]

exhibit the ballistic behavior. Since their mode transmittance \( T_n = 1 \), such modes form a coset of completely
transparent channels. As for other propagating modes with low indices $n \leq N_{loc}$, they remain to be localized for large enough length size $L$ when the condition (9) holds, i.e. for $W_0 L / 16 k_0^2 \gg 1$.

The expression (18) determines the total number $N_{loc}$ of localized modes and total number $N_{tr} = N_d - N_{loc}$ of completely transparent modes. Since localized modes do not contribute to the total transmittance (3), the latter is equal to the number $N_{tr}$ of completely transparent modes and do not depend on the waveguide length $L$,

$$\langle T \rangle = \left[ kd / \pi \right] - [kd / \pi] (1 - k_0^2 / k^2)^{1/2} \Theta (k - k_0). \quad (19)$$

We remind that square brackets stand for the integer part of the inner expression.

For $k_0 \ll k$ the number of localized modes $N_{loc} \approx \left[ (kd / \pi) (1 - k_0^2 / 2k^2) \right]$ is of the order of $N_d$. Consequently, the number of transparent modes $N_{tr}$ is small, or there are no such modes at all. Otherwise, if $k_0 \to k$, the integer $N_{loc} \approx \sqrt{2} (kd / \pi) (1 - k_0 / k)^{1/2}$ turns out to be much less than the total number of waveguide modes $N_d$, and the number of transparent modes $N_{tr}$ is large. When $k_0 > k_1$, the number $N_{loc}$ vanishes and all modes become fully transparent. In this case the correlated disorder results in a perfect transmission of waves. One can see that the value $k_0$ is, in essence, the total mobility edge that separates the region of complete transparency from that where lower modes are localized.

In conclusion, we have studied quasi-1D waveguides with a stratified disorder that has long-range correlations. We have shown that with a proper choice of correlations one can arrange a complete transparency of transverse channels in a given frequency window of incoming waves. It is worthwhile to emphasize a non-monotonic step-wise dependence of the total transmittance (19) on the wave number $k$. Specifically, within the region $k < k_0$ for fixed values of $k_0$ (when $W_0 L d^2 / 16 \pi^2 > (k_0 d / \pi)^2 \gg 1$), all propagating modes are transparent and the transmittance exhibits step-wise increase with an increase of $k$. Each step up arises for an integer value of $kd / \pi$ when new conducting channel emerges. This effect is similar to that known to occur in quasi-1D ballistic (non-disordered) structures (see, e.g., [15]).

On the other hand, for $k > k_0$ the transmittance reveals a step down decrease due to successive localization of low modes. In contrast with the steps up, every step down occurs at the local mobility edge of the corresponding channel, when the second term in Eq. (19) takes the integer value. In general, these values do not coincide with the integer values of $kd / \pi$, thus resulting in a new kind of step-wise dependence for the transmittance. Finally, for very large values $k^2 \gg W_0 L / 16$, the transmittance starts to increase again, due to a successive delocalization of the modes. Our results can be used for fabrications of electron or optic nanodevices with a selective transport.

This research was supported by CONACYT grant 34668-E, and by BUAP grant II-104G02 (México).

[1] P. Carpena, P. Bernaola-Galván, P. Ch. Ivanov, and H. E. Stanley, Nature, 418, 955 (2002); F. A. B. F. de Moura, M. D. Coutinho-Filho, E. P. Raposo, and M. L. Lyra, cond-mat/0209304; L. I. Deych, M. V. Ermentchouk, and A. A. Lisiansky, Phys. Rev. B 67, 024205 (2003).

[2] F. M. Izrailev and A.A. Krokhin, Phys. Rev. Lett. 82, 4062 (1999); A. A. Krokhin and F. M. Izrailev, Ann. Phys. (Leipzig) 8, 153 (1999); F. M. Izrailev, A. A. Krokhin, and S. E. Ulloa, Phys. Rev. B 63, 041102(R) (2001).

[3] U. Kuhl, F. M. Izrailev, A. A. Krokhin, and H.-J. Stöckmann, Appl. Phys. Lett. 77, 633 (2000).

[4] I. Ohlidal and K. Navratil, Progress in Optics, 34, 249 (1995); C. Amra, Applied Optics, 32, 5481 (1993); I.M. Fuks, in Proceedings of IGARSS (June 24-28, 2002, Toronto, Canada), Vol. 2, pp. 1251; A.A. Bulgakov, S.A. Bulgakov and M. Nieto-Vesperinas, Phys. Rev. B 52, 10788 (1995); S.A. Bulgakov and M. Nieto-Vesperinas, J. Opt. Soc. Amer. 13, 500 (1996).

[5] U. Kuhl and H.-J. Stöckmann, Phys. Rev. Lett. 80, 2323 (1998); V. Bellani, et al., Phys. Rev. Lett. 82, 2159 (1999).

[6] R. Kubo, J. Phys. Soc. Japan 12, 570 (1957).

[7] R. Landauer, Physica Scripta T42, 110 (1992).

[8] V. O. Berezins, Zh. Eksp. Teor. Fiz. 65, 1251 (1973) [Sov. Phys.-JETP 38, 620 (1974)]; A. A. Abrikosov and I. A. Ryzhin, Adv. Phys. 27, 147 (1978).

[9] R. Bellman and G. M. Wing, An Introduction to Invariant Imbedding (N.Y.: Wiley, 1975); V. I. Klyatskin, The Invariant Imbedding Method in a Theory of Wave Propagation (Moscow: Nauka, 1986) (in Russian).

[10] N. M. Makarov and Yu. V. Tarasov, J.Phys.: Condens. Matter 10, 1523 (1998); N. M. Makarov and Yu. V. Tarasov, Phys. Rev. B 64, 235306 (2001); N.M. Makarov, lectures on Spectral and Transport Properties of One-Dimensional Disordered Conductors, www.ifuap.buap.mx/virtual/page_vir.html.

[11] I. M. Lifshits, S. A. Gredevsul, and L. A. Pastur, Introduction to the Theory of Disordered Systems (Wiley; New York, 1988).

[12] J. A. Sánchez-Gil, V. Freilikher, I. V. Yurkevich, and A. A. Maradudin, Phys. Rev. Lett. 80, 948 (1998); J. A. Sánchez-Gil, V. Freilikher, A. A. Maradudin, and I. V. Yurkevich, Phys. Rev. B 59, 5915 (1999).

[13] A. García-Martín, J. A. Torres, J. J. Sáenz, and M. Nieto-Vesperinas, Phys. Rev. Lett. 80, 4165 (1998).

[14] F. M. Izrailev, N. M. Makarov, Opt. Lett. 26, 1604 (2001); Phys. Rev. B 67, 113402 (2003).

[15] B. J. van Wees, H. van Houten, C. W. J. Beenakker, et al., Phys. Rev. Lett. 60, 848 (1988).