In this article we present a simple theoretical framework where the origin of the $\mu$-term and the matter-parity violating interactions of the minimal supersymmetric standard model can be understood from the spontaneous breaking of new Abelian gauge symmetries. In this context the masses of the $Z'$ gauge bosons, the $\mathcal{M}$-parity violating scale and the $\mu$-term are determined by the supersymmetry breaking scale. The full spectrum of the theory is discussed in detail. We investigate the predictions for the Higgs masses in detail showing that it is possible to satisfy the LEP2 bounds even with sub-TeV squark masses. The model predicts the existence of light colored fields, lepton and baryon number violation, and new neutral gauge bosons at the Large Hadron Collider.
discrete $Z_3$ symmetry is typically imposed to forbid the bilinear $\mu$-term, which is replaced by singlet field, whose vacuum expectation value (VEV) generates the $\mu$-term after symmetry breaking. This model is referred to as the next to minimal supersymmetric standard model (NMSSM) and it and its deviations are reviewed in Ref. [2]. Such a scenario expands the Higgs sector thereby potentially changing expectation for collider physics and causing cosmological concerns related to domain walls.

Our approach in this paper is to understand the possible origin of the discrete symmetries mentioned above from the spontaneous breaking of local symmetries. While it is maybe true that this simply amounts to replacing one symmetry by another, we think that the corresponding $Z'$ gauge bosons associated with local symmetries allow for a better handle on testing such ideas. Therefore, we propose a simple model where the origin of the $\mu$-term and the matter-parity violating interactions of the MSSM can be understood from the spontaneous breaking of two new Abelian gauge symmetries: $U(1)_{B-L}$ and $U(1)_{S}$ where only the third generation carries $U(1)_{S}$ charge. In order to define an anomaly free theory new colored triplets exotics are needed. $B - L$ is broken by the VEV of the “right-handed” sneutrino giving rise to lepton number violating $M$-parity violation and $U(1)_{S}$ is broken by the VEV of a SM singlet, $S$, which generates the $\mu$-term. The new $Z'$ associated with $U(1)_{S}$ give rise to flavor violation without experimental conflict. Symmetry breaking also allows for a consistent scenario for fermion masses, predicting a very small mixings between the third generation and the others. The numerical predictions for the lightest Higgs boson are investigated up to one-loop level showing the possibility to satisfy the experimental bounds from the LEP2 experiment with squark masses below 1 TeV. Finally, we make a brief discussion of how one could observe lepton and baryon number violation at the LHC in agreement with the experimental bounds on proton decay.

The remainder of this article is organized as follows: In Section II we expand on the issues of $M$-parity and the $\mu$-term and past attempts to address them. In Section III we propose our new theoretical framework where both issues can be solved and discuss the necessary symmetry breaking in Section IV. The properties of the full spectrum are presented in Section V, while in Section VI the main phenomenological aspects are presented. Our findings are summarized in Section VII.

II. THE $\mu$-PROBLEM AND $M$-PARITY

As mentioned above, different approaches to the $\mu$-problem and $M$-parity have significantly different consequences and it’s especially the presence or absence of the latter that answers one of the most important questions of the MSSM: the stability of the LSP. A brief review is therefore in order.

A. $M$-Parity Violating Interactions

The fate of $M$-parity in the MSSM has important cosmological and phenomenological implications. $M$-parity is defined as $M = (-1)^{3(B - L)}$, where $B$ and $L$ stand for total baryon number and lepton number, respectively. In general, the MSSM contains lepton and baryon number violating interactions in the superpotential:

$$W_{MV} = cLH_u + \lambda_\ell \ell\hat{L}e^c + \lambda' Q\hat{L}d^c + \lambda''\hat{u}d\hat{d}e^c$$  

In most phenomenological studies it is assumed that $M$-parity is conserved by hand, i.e. the above interactions are absent, or that only some of them are present: explicit $M$-parity breaking. Since these terms affect the most significant features of the MSSM, the origin of $M$-parity conservation or violation must be understood dynamically. It has long been realized that the simplest forum for this is $B - L$ symmetric theories [3]. Since $M$-parity is a subgroup of $B - L$, at the $B - L$ scale all the above interactions are absent. Local $B - L$ further requires the existence of right-handed neutrinos for anomaly cancelation which also provide the most minimal way of breaking $B - L$ [4]: the VEV of the right-handed sneutrino $\nu$. Therefore, in the simplest theory of $M$-parity, it is spontaneously broken and the $B$ and $L$ and the $M$-parity violating scales are determinate by the soft SUSY breaking scale. As has been emphasized in Ref. [4] after symmetry breaking only bilinear lepton number violating interactions are present and there are no dimension four contributions to proton decay. For a review on proton decay see Ref. [2].

B. The $\mu$-Problem and New Symmetries

The $\mu$ parameter is part of the $M$-parity conserving MSSM superpotential:

$$W_{MC} = Y_u\hat{Q}H_u\hat{u}e^c + Y_d\hat{Q}H_d\hat{d}e^c + Y_e\hat{L}\hat{H}d\hat{e} + \mu\hat{H}_u\hat{H}_d,$$  

and defines the mass of the Higgsinos and plays a very important role in electroweak symmetry breaking. This relates the $Z$ boson mass (which we can use to define the weak scale), the $\mu$ term and the soft terms in the Higgs sector:

$$\frac{1}{2}M_\mu^2 = -|\mu|^2 - \left(\frac{m_{H_u}^2\tan^2\beta - m_{H_d}^2}{\tan^2\beta - 1}\right),$$  

where $m_{H_u}$ and $m_{H_d}$ are the soft terms for the MSSM Higgses and $\tan\beta = v_u/v_d$. Notice that in order to satisfy the above equation the second term on the right-hand side must be negative and its magnitude must be larger than the $\mu$-term,

---

1 A scenario further motivated by string theory [5] [6].
for large $\tan \beta$, this translates into the condition that $\mu$ must be smaller in magnitude than the soft terms. At the same time $\mu$ is a mass dimensionful parameter in the superpotential and in principle it could be very large. This is the so-called $\mu$-problem. From chargino searches the $\mu$ lower bound is approximately $\mu \gtrsim 100$ GeV.

Many scenarios have been proposed to explain the origin of a SUSY-scale $\mu$-term \cite{8,14}. In the NMSSM one introduces a new singlet, $S$, and replaces the $\mu$ term in the superpotential by the term $\lambda S H_u H_d$. Then, the $\mu$-parameter is defined by the VEV of $S$ which is around the SUSY scale. In order to achieve this scenario a new discrete symmetry, a $Z_3$ symmetry, is introduced which forbids the mass term in the superpotential. See Ref. \cite{2} for a review of the NMSSM. However, the question of a dynamical origin for the $\mu$-term remains.

As in the $M$-parity case it is possible to find a gauge origin to the $\mu$-term by introducing a new abelian symmetry which is spontaneously broken at the TeV scale. However, unlike $B - L$ for $M$-parity, it is hard to pinpoint the simplest model. Various possibilities have been investigated by many groups \cite{10,12}. Since the $Z_3$ symmetry is replaced by a gauge symmetry, the cosmological problems associated with the spontaneous breaking of the discrete symmetry is avoided. We see such an approach as appealing because it connects the $\mu$ term to the existence of a new gauge boson which could experimentally relate to the mechanism for the dynamical generation of the $\mu$-term.

Combining the two possible solutions, discrete and local symmetries, to these two issues of $M$-parity and the $\mu$-problem affords four different frameworks for approaching these issues. Typically, most of the phenomenological studies have been performed in a model where a $Z_2$ (matter parity) and $Z_3$ is assumed. A second possibility is a simple extension of the model in Ref. \cite{4}, where $M$-parity is spontaneously broken along with $B - L$ and a $Z_2$ symmetry is assumed to explain the $\mu$-term. A third scenario was quoted as an example in Ref. \cite{10} where the generation of the $\mu$-term is defined by the scale where a new $U(1)'$ symmetry is broken and a $Z_2$ symmetry is assumed to avoid fast proton decay. Finally, one can consider a more complete framework with two Abelian symmetries for understanding dynamically the generation of the $M$-parity violating terms and the $\mu$-term.

The difficulty in flagging a simplest gauge solution to the $\mu$-problem is due to three issues that usually arise. Since the Higgs fields will now have a new charge, it is not a priori clear that Yukawa couplings generating fermion masses will be gauge invariant thus making fermion mass generation non-trivial. Anomaly cancellation usually requires the existence of new exotic color states. These will either couple to matter and induce rapid proton decay or form a separate sector with no couplings to matter resulting in the lightest exotic being stable. The latter scenario would lead to relic bound states, which could disagree with current cosmological data. We have found that several papers in the literature contain such traits, with a noteworthy example of the last one being Ref. \cite{11}, which solves both issues in a nice way but contains stable colored particles.

Due to these possible complications, we take this opportunity to state our goals in addressing the $\mu$-term:

- No dimensionful parameters in the superpotential which would affect the electroweak symmetry breaking (EWSB) condition, Eq. (3). This includes the $\mu$-term as well as the $\epsilon$-term.
- Explain the long lifetime of the proton.
- No stable colored fields.
- Generation of all fermion masses and mixings.

Now, we are ready to discuss the simplest theoretical framework where these issues are addressed.

### III. THEORETICAL FRAMEWORK

In order to investigate how $M$-parity violating terms and the $\mu$-term are generating dynamically we introduce two extra Abelian symmetries, $U(1)_{B - L}$ and $U(1)_S$. The first is needed to understand the origin of $M$-parity while the second symmetry governs how the $\mu$-term is generated. Therefore, the model will be based on the local gauge symmetry

\[
SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{B - L} \otimes U(1)_S
\]

Inspired by the \textit{27} of $E_6$, we introduce four new fields: three generations of right-handed neutrinos necessary to gauge $B - L$, $S$ whose VEV generates the $\mu$-term and $T$ and $\bar{T}$ needed to cancel the $U(1)_S$ anomalies.

We assume a non-zero $z$ charge (the charge under $U(1)_S$) only for the third generation so that only one set of the latter three fields need be introduced (as opposed to one per generation). This further restricts the coupling of the exotics to third generation fermions only, significantly suppressing their contribution to proton decay. The anomaly cancellation conditions can be satisfied by the charges in Table \text{I} where $z_0$ is the charge of $\tilde{\nu}_3$, $z_T$ is the charge of $T$. These are the most general charges given the additional assumption that the top mass term is gauge invariant.

The most general superpotential that can be written with these charges is:

\[
W_1 = Y_L \tilde{Q}_3 \tilde{H}_u \tilde{\nu}^c_3 + Y_b \tilde{Q}_3 \tilde{H}_d \tilde{\nu}^c_3 + Y_T \tilde{L}_3 \tilde{H}_d \tilde{e}^c_3 + \lambda S \tilde{H}_u \tilde{H}_d + \lambda_1 \tilde{S} \tilde{T} \tilde{T}^c + \lambda_2 \tilde{Q}_3 \tilde{L}_3 \tilde{T} + \lambda_3 \tilde{\nu}_3 \tilde{d}^c_3 \tilde{T} + \lambda_4 \tilde{\nu}_3 \tilde{\nu}^c_3 \tilde{T}^c + \lambda_5 \tilde{d}^c_3 \tilde{e}^c_3 \tilde{T} + \lambda_6 \tilde{Q}_3 \tilde{Q}_3 \tilde{T}.
\]

The first and second rows allow for mass terms for the third generation only, the third for trilinear terms that, once $S$ acquires a VEV, generate mass terms for the MSSM Higgsinos and the colored exotics. The fourth and fifth rows sport third
TABLE I: Field Content ($\alpha = 1..2$).

| Field | $SU(3)$ | $SU(2)$ | $U(1)_{Y}$ | $U(1)_{B-L}$ | $U(1)_{S}$ |
|-------|---------|---------|------------|--------------|------------|
| $H_u$ | 1 2 3   | 1 2 3   | 1/6        | 1/3          | 0          |
| $H_d$ | 1 2 3   | 1 2 3   | -2/3       | -1/3         | 0          |
| $\hat{L}_a$ | 1 2 3   | 1 2 3   | 1/6        | 1/3          | 0          |
| $\tilde{c}_a$ | 1 1 1   | 1 1 1   | -2/3       | -1/3         | 0          |
| $\tilde{\nu}_a$ | 1 1 1   | 1 1 1   | -2/3       | -1/3         | 0          |
| $\tilde{\nu}_a$ | 1 1 1   | 1 1 1   | 1 1 1      | 1 1 1        | 0          |
| $\hat{L}_3$ | 1 2 3   | 1 2 3   | -1/2       | -1           | 0          |
| $\hat{c}_3$ | 1 1 1   | 1 1 1   | 1 1 1      | 1 1 1        | 0          |
| $\hat{\nu}_3$ | 1 1 1   | 1 1 1   | 1 1 1      | 1 1 1        | 0          |

This still leaves the first and second generation masses to be desired. However, as can be appreciated from Table I, there are still two degrees of freedom left: $z_T$ and $z_u$. This allows a choice between tree-level down- or up-type quark masses. We opt for tree-level up-type quark masses, which require $z_{H_u} = 0$ and yields the relationship $z_T = -1 - 2z_u$ and new contributions to the superpotential of the form:

$$W_2 = Y^{ab}_{u} \hat{Q}_a \hat{H}_u \hat{u}_b + Y^{ab}_{d} \hat{L}_a \hat{H}_u \hat{\nu}_b + \lambda^{ab}_{c} \frac{\tilde{S}}{\Lambda} \hat{Q}_a \hat{H}_d \hat{d}_b + \lambda^{ab}_{c} \frac{\tilde{S}}{\Lambda} \hat{L}_a \hat{H}_d \hat{e}_b,$$

where $a, b = 1..2$ only. In addition to the tree-level up-type masses, we can also generate down-type masses at the non-renormalizable level for the first and second generation. At this point, the only aspect of the fermionic sector missing is the mixings between the third generation and the others two. Fortunately, we have yet another charge degree of freedom.

There are three possible scenarios that give CKM-like mixings: 
$z_T = -3, z_u = 1; z_T = 1, z_u = -1;$ and $z_T = -1, z_u = 0$. Unfortunately, the latter two solutions introduce couplings between the colored exotic fields and the first two generations making proton decay unsafe. This leaves the first solution as the unique realistic case with charges given by

$$\tilde{Q}_3 \sim -1, \tilde{u}_3 \sim 1, \tilde{d}_3 \sim 2, \tilde{L}_3 \sim 4, \tilde{e}_3 \sim -3,$$

$$\tilde{\nu}_3 \sim -4, \tilde{H}_d \sim -1, \tilde{H}_u \sim 0, \tilde{S} \sim 1, \tilde{T} \sim -3, \text{and } \tilde{T} \sim 2.$$

Then, the additional superpotential terms allowed are:

$$W_3 = \lambda^a_{\tilde{S}} \hat{S} \hat{Q}_a \hat{H}_d \hat{d}_a + \lambda^a_{\tilde{S}} \frac{\tilde{S}^4}{\Lambda} \hat{L}_a \hat{H}_d \hat{e}_a$$

$$+ \lambda^a_{\tilde{S}} \frac{\tilde{S}^2}{\Lambda} \hat{Q}_a \hat{H}_d \hat{d}_a + \lambda^a_{\tilde{S}} \frac{\tilde{S}^4}{\Lambda} \hat{L}_a \hat{H}_d \hat{e}_a,$$

where the first and third terms allow for a realistic CKM matrix while the second and fourth terms are relevant for the mixing matrix in the leptonic sector, the PMNS matrix, given the appropriate scale, $\Lambda$. In order to generate the right value for the mass of strange quark ($m_s(M_Z) \approx 56 \text{ MeV}$, see for example Ref. [14]) we need a ratio, $\langle S \rangle / \Lambda \approx 10^{-4} - 10^{-3}$. This means one needs new degrees of freedom not very far from the TeV scale to understand the origin of these higher-dimensional operators. For example, one could integrate out some new fermions and generate the mass terms listed above.

In this paper we will ignore the origin of these terms and consider an effective theory where we can understand the origin of the $\mu$-term and $M$-parity violating interactions.

**IV. SYMMETRY BREAKING**

Symmetry breaking proceeds through the following VEVs:

$$\langle H_u^0 \rangle \equiv v_u / \sqrt{2} \text{ and } \langle H_d^0 \rangle \equiv v_d / \sqrt{2},$$

responsible for EWSB; $\langle \tilde{\nu}_a \rangle \equiv v_R / \sqrt{2}$ (we will assume only one generation of right-
handed sneutrinos acquires a VEV, breaking $B - L$. The VEV $(S) \equiv v_S / \sqrt{2}$ breaks $U(1)_S$, and $(\tilde{\nu}) \equiv v_L / \sqrt{2}$ is also generated. Due to the non-universality of the $U(1)_S$ charges, the minimization conditions and Higgs spectrum depend on which generation of right-handed sneutrino acquires a VEV. We will proceed in the most general way, designating the charges of the right-handed and left-handed sneutrino as $z_{\nu e}$ and $z_L$, respectively. This is of course zero for the first two generations and $\pm 4$ for the third. We also elucidate the relevant soft parameters:

$$-L_{\text{Soft}} = \left( a_v \tilde{L} H_u \tilde{\nu}^c + a_S S H_u H_d + \text{h.c.} \right) + m^2_S |S|^2 + m^2_{H_u} |H_u|^2 + m^2_{H_d} |H_d|^2 + m^2_L |\tilde{L}|^2 + m^2_{\nu e} |\nu e|^2.$$  \tag{8}

The VEVs of the potential in the phenomenologically appropriate limit of very small $v_L$ and $Y_L$ and in the one family approximation are

$$\langle V_F \rangle = -\frac{1}{2} Y_{\nu} \lambda v_L v_d v_R v_S + \frac{1}{4} \lambda^2 (v_u^2 + v_d^2) v_S^2 + \frac{1}{4} \lambda^2 v_u^2 v_d^2,$$  \tag{9}

$$\langle V_S \rangle = \frac{1}{2} m^2_{H_u} v_u^2 + \frac{1}{2} m^2_{H_d} v_d^2 + \frac{1}{2} v_u^2 m^2_L + \frac{1}{2} v_d^2 m^2_{\nu e} + \frac{1}{2} m^2_S v_S^2 + \frac{1}{\sqrt{2}} a_v v_L v_u v_R - \frac{1}{\sqrt{2}} a_S v_d v_u v_S,$$  \tag{10}

$$\langle V_D \rangle = \frac{1}{32} (g^2_1 + g^2_2) (v_u^2 - v_d^2 - v_L^2)^2 + \frac{1}{32} g^2_2 (v_S^2 - v_d^2 + z_{\nu e} v^2_R - z_{\nu e} v_L^2)^2 + \frac{1}{32} g^2_2 (\tilde{v}_R - \tilde{v}_L)^2.$$  \tag{11}

Focusing now on the scenario where the first or second generation sneutrinos acquire a VEV and assuming that $v_S, v_R \gg v_u, v_d$, so that the two sectors decouple, yields the following familiar MSSM-like results:

$$\frac{2b}{\sin 2\beta} = M^2_{H_u} + M^2_{H_d} + 2|\mu|^2,$$  \tag{12}

$$\frac{1}{2} M^2_Z = -|\mu|^2 - \left( \frac{M^2_{H_u} \tan^2 \beta - M^2_{H_d}}{\tan^2 \beta - 1} \right),$$  \tag{13}

where the difference from the MSSM is in the definition of $M_{H_u}$ and $M_{H_d}$.

$$M^2_{H_d} = m^2_{H_d} - \frac{1}{8} g^2_2 (v^2_S - v^2 \cos^2 \beta) + \frac{1}{2} \lambda^2 v^2 \sin^2 \beta,$$  \tag{14}

$$M^2_{H_u} = m^2_{H_u} + \frac{1}{2} \lambda^2 v^2 \cos^2 \beta.$$  \tag{15}

Here, $v^2 = v_u^2 + v_d^2$. The non-MSSM VEVs can be approximated as

$$v^2_R = -\frac{8 m^2_{\nu e}}{g^2_{BL}},$$  \tag{16}

$$v^2_S = \left( \frac{8 m^2_2 + 4 \lambda^2 v^2 - g^2_2 v^2 \cos^2 \beta}{g^2_S} \right),$$  \tag{17}

$$v_L = \frac{v_R (\lambda Y_\nu v_d v_S - \sqrt{2} a_v v_u)}{2 \left( m^2_L - \frac{1}{8} g^2_{BL} v^2_R + \frac{1}{8} (g^2_1 + g^2_2) v^2 \cos (2\beta) \right)}.$$  \tag{18}

Notice that using Eqs. (17) and (19) one can understand that the $\mu$ term generated after symmetry breaking is determined by the soft mass $m_S$. Then, in this way one can say that SUSY breaking scale sets the size of this mass term in the MSSM superpotential.

The first two VEVs require the numerator to be positive meaning in general that $m^2_S, m^2_{\nu e} < 0$, i.e. tachyonic right-handed sneutrino and singlet masses. A tachyonic $S$ can easily be generated through a radiative mechanism if its coupling to the exotic triplets is large enough, while — for non-universal right-handed sneutrino masses — a tachyonic right-handed sneutrino can be generated via the mechanism discussed in Ref. [6]. Alternatively, its possible that $\lambda_S$ is of order one, which will drive the right-handed sneutrino negative in the traditional way, however this would require much smaller values for the exotic triplet couplings to quarks to compensate for proton decay. Regardless of how the tachyonic masses are generated, the VEVs and therefore the symmetry breaking scales are defined by the SUSY breaking mass scale. This is very appealing since it tethers the corresponding $Z'$ masses to this scale as well, giving hope that the underlying mechanism for the $\mu$-term and $M$-parity violation can be tested at the LHC.

V. SPECTRUM

In this section we will outline the spectrum in the different sectors of this theory.

1. Charged Fermion Masses

It is crucial to show that a consistent scenario for fermion masses is possible in this context. A detailed analysis is beyond the scope of this article but a brief discussion is presented. The charged fermion masses are generated after the
symmetry breaking and are given by

\[ M_u = \begin{pmatrix} A_u & 0 \\ B_u & C_u \end{pmatrix} \]

(21)

where \( A_u = Y_u v_u / \sqrt{2} \) is a 2 by 2 matrix, \( B_u = \lambda_i v_S v_u / 2 \Lambda \) is a 2 by 1 matrix, and \( C_u = Y_t v_u / \sqrt{2} \). In the case of the down sector we find

\[ M_d = \begin{pmatrix} A_d & 0 \\ B_d & C_d \end{pmatrix} \]

(22)

where \( A_d = \lambda_d v_d / 2 \Lambda \) is a 2 by 2 matrix, \( B_d = \lambda_0 v_S^2 v_d / 2 \sqrt{2} \Lambda^2 \), and \( C_d = Y_d v_d / \sqrt{2} \). The mass matrix for charged leptons reads as

\[ M_e = \begin{pmatrix} A_e & 0 \\ B_e & C_e \end{pmatrix} \]

(23)

with \( A_e = \lambda_e v_e v_d / 2 \Lambda \) is a 2 by 2 matrix, \( B_e = \lambda_0 v_S^4 v_e / 4 \sqrt{2} \Lambda^3 \), and \( C_e = Y_e v_d / \sqrt{2} \).

There are two interesting results we should discuss: The first is that the new gauge symmetry \( U(1)_s \) is basically a flavor symmetry since after symmetry breaking one obtains specific textures for all fermion mass matrices. Second, the fact that \( B_{d,e} \ll A_{d,e} \ll C_{d,e} \) implies that the mass matrices for down-quark and charged leptons can be diagonalized approximately by a matrix containing a submatrix in the 2 by 2 sector. As we will explain carefully later, this ensures that the new physical couplings of the gauge boson associated to \( U(1)_s \) will never induce large flavor violation is the down sector. This is important for avoiding the strong bounds from flavor changing neutral currents and proton decay. Notice that the same argument holds also for the up-quark sector, but with a less strong hierarchy.

2. Neutral Gauge Bosons

We have two new neutral gauge bosons associated with the two new \( U(1) \) groups. We proceed by assuming a sneutrino VEV in the first or second generation only and no kinetic mixing terms. The mass matrix for the four neutral gauge bosons in the basis \((B_\mu^Y, W_\mu^3, B_\mu^S, B_\mu^{BL})\) is then

\[
\begin{pmatrix}
\frac{1}{2} g_1^2 v^2 & -\frac{1}{4} g_1 g_2 v^2 & \frac{1}{4} g_1 g_S v_d^2 & 0 \\
-\frac{1}{4} g_1 g_2 v^2 & \frac{1}{2} g_2^2 v^2 & -\frac{1}{4} g_2 g_S v_d^2 & 0 \\
\frac{1}{4} g_1 g_S v_d^2 & -\frac{1}{4} g_2 g_S v_d^2 & \frac{1}{2} g_S^2 (v_d^2 + v_S^2) & 0 \\
0 & 0 & 0 & \frac{1}{2} g_{BL} v_R^2
\end{pmatrix},
\]

(24)

where \( v^2 \equiv v_d^2 + v_S^2 \approx (246 \text{ GeV})^2 \). Thus we can immediately see that the \( B_{BL}^\mu \) gauge boson does not mix with the other neutral gauge bosons and decouples. We define the mass eigenstate as \( Z_{BL} \) with mass \( \sqrt{\frac{1}{2} g_{BL} v_R^2} \). Rotating by the weak angle \( \theta_W \) projects out the photon zero-mode which decouples and leaves the two-by-two mass matrix in the basis \((Z_\mu^0, B_\mu^S)\)

\[ M_{Z\mu}^2 = \begin{pmatrix} M_{Z\mu}^2 & \Delta \\ \Delta & M_{ZS}^2 \end{pmatrix}, \]

(25)

where

\[ M_{Z\mu}^2 = \frac{1}{4} v^2 \left( g_1^2 + g_2^2 \right), \]

(26)

\[ M_{ZS}^2 = \frac{1}{4} g_S^2 \left( v_d^2 + v_S^2 \right), \]

(27)

\[ \Delta = -\frac{1}{16} v^2 g_S \sqrt{g_1^2 + g_2^2} \]

\[ = -\frac{g_S}{\sqrt{g_1^2 + g_2^2}} M_{Z\mu}^2 \cos^2 \beta. \]

(28)

This matrix describes the \( Z_\mu^0, B_\mu^S \) mixing. The non-diagonal element is proportional to \( v_d \) and thus the mixing will be suppressed for large values of \( \tan \beta \). We label the physical states \( Z \) and \( Z' \) whose masses are

\[ M_{Z,Z'}^2 = \frac{1}{2} \left[ M_{Z\mu}^2 + M_{ZS}^2 + \sqrt{(M_{Z\mu}^2 - M_{ZS}^2)^2 + 4\Delta^2} \right], \]

(29)

which in the limit \( M_{Z\mu}^2 \ll M_{ZS}^2 \) simplifies to

\[ M_{Z}^2 \approx M_{Z\mu}^2 + \frac{g_S^2}{g_1^2 + g_2^2} M_{Z\mu}^2 \cos^4 \beta, \]

(30)

\[ M_{Z'}^2 \approx M_{ZS}^2 - \frac{g_S^2}{g_1^2 + g_2^2} M_{Z\mu}^2 \cos^4 \beta. \]

(31)

The mixing angle, defined such that

\[ Z_\mu^0 = Z_\mu \cos \theta_{ZZ'} - Z_S^0 \sin \theta_{ZZ'}, \]

(32)

\[ Z_S^0 = Z_\mu \sin \theta_{ZZ'} + Z_\mu^0 \cos \theta_{ZZ'}, \]

(33)

\[ \theta_{ZZ'} = \frac{1}{2} \arctan \left( \frac{2 \Delta}{M_{Z\mu}^2 - M_{ZS}^2} \right) \]

\[ \approx \frac{g_S}{\sqrt{g_1^2 + g_2^2}} \cos^2 \beta \epsilon + O(\epsilon^2). \]

(34)

Here \( \epsilon \equiv \frac{M_{Z\mu}^2}{M_{ZS}^2} \). Notice that the \( O(\epsilon) \)-terms in \( M_{Z,Z'}^2 \) and \( \theta_{ZZ'} \) have an additional suppression for large values of \( \tan \beta \). For a recent discussion of the constraints on \( \theta_{ZZ'} \), see Ref. [15].

In order to illustrate the possible numerical values for the mixing angle \( \theta_{ZZ'} \) in Fig. 1 we show the values when \( v_S = 2 \text{ TeV} \) and for different values of \( g_S \) and \( \tan \beta \). Notice that in the whole parameter space the mixing angle is very small, i.e. \( \theta_{ZZ'} < 10^{-3} \). Before going to the next subsection, let us make some comments about the case where the third generation right-handed sneutrino acquires a nonzero VEV. In that
case we have the following Z-mass matrix
\[
\begin{pmatrix}
\frac{1}{4} v^2 (g_1^2 + g_2^2) & -\frac{1}{4} v^2 g s \sqrt{g_2^2 + g_1^2} & 0 \\
-\frac{1}{4} v^2 g s \sqrt{g_2^2 + g_1^2} & \frac{1}{2} g_2^2 (v_2^2 + v_3^2 + 16v_R^2) & -gbLgs v_R^2 \\
0 & -gbLgs v_R^2 & \frac{1}{2} g_{BL} v_R^2
\end{pmatrix}
\]

This is a more complicated case since the two new Z bosons do mix, and they also mix with the SM Z-boson. If one of the new Z-bosons is much heavier than the other, then it decouples and we are in the usual Z-Z' scenario, whereas if both have similar masses then one has a Z-Z-Z'' situation where the expressions are more involved (see Ref. [17] for an analysis of the kinetic and mass mixing of three neutral gauge bosons). In any case, the mixing of the heavy states with the SM Z-boson and the contribution to the SM Z-mass are still dominated by the quantity \( \frac{v_3}{v^2} \cos \beta \) in such a way that, as in the previous case, they are suppressed for large values of tan \( \beta \) and \( v_{S,R} \).

The phenomenology of a \( B-L \) gauge boson has been studied extensively in the literature and relevant bounds can be found in Ref. [16]. The reach at the LHC for a \( B-L \) Z' is studied in Ref. [18] and the effects of SUSY decays are shown in Ref. [19]. While a Z' that couples only to the third family does not have as much coverage it has been studied in Ref. [20].

3. Z' Couplings to Fermions

Here we study the case where only the \( U(1)_S \) Z' and the SM Z boson mix. The neutral current interactions of the fermions are described by the Lagrangian
\[
\mathcal{L}_{\text{Z'}} = g_1 J^\mu_{Z'} B_\mu + g_2 J^\mu_{Z'} W^3_\mu + g_s J^\mu_{Z'} B^S_\mu + g_{BL} J^\mu_{BL} B^{BL}_\mu \\
= e J^\mu_{em} A_\mu + g_0 J^\mu_{Z'} Z'_\mu + g_s J^\mu_{Z'} B^S_\mu + g_{BL} J^\mu_{BL} B^{BL}_\mu \\
= e J^\mu_{em} A_\mu + g_2 J^\mu_{Z'} Z'_\mu + g_s J^\mu_{Z'} Z'_\mu + g_{BL} J^\mu_{BL} Z^{BL}_\mu,
\]

where \( J^\mu_{Z'}, J^\mu_{em}, J^\mu_{BL} \) are the well known SM currents. The electromagnetic and \( B-L \) currents are not modified whereas \( J^\mu_{Z'} \) and \( J^\mu_{BL} \) are

\[
J^\mu_{Z'} = \bar{u} \gamma^\mu (C^u_{dL} P_L + C^u_{dR} P_R) u \\
+ \bar{d} \gamma^\mu (C^d_{dL} P_L + C^d_{dR} P_R) d,
\]

\[
J^\mu_{BL} = \bar{u} \gamma^\mu (C^u_{dL} P_L + C^u_{dR} P_R) u \\
+ \bar{d} \gamma^\mu (C^d_{dL} P_L + C^d_{dR} P_R) d,
\]

where \( u^T = (u, c, t) \), \( d^T = (d, s, b) \) and the \( C^u \) matrices are three-by-three charge matrices in flavor space. The currents \( J^\mu_{Z'} \) and \( J^\mu_{BL} \) have the same structure, and the relation between the \( C \)-matrices in the different bases is the following

\[
g_Z C_x = g_0 C^0_x \cos \theta_{ZZ'} + g_s C^S_x \sin \theta_{ZZ'},
\]

\[
g_Z C'_x = -g_0 C^0_x \sin \theta_{ZZ'} + g_s C^S_x \cos \theta_{ZZ'},
\]

where \( x = uL, uR, dL, dR \). The \( C^0_x \) matrices are those of the SM and are proportional to the identity (flavor universal interaction), whereas the \( C^S_x \) matrices are non-universal because only the third generation feels the \( U(1)_S \) interaction. So far
we have only taken into account the effects of the EWSB in the
gauge sector, with the associated mixing among Z bosons, but
mixing in the fermion sector must also be taken into account.
Starting with the Yukawa matrices that have been introduced
in the previous sections and performing the usual rotation to
mass-eigenstates:

\[ u_{L,R} \to U_{L,R} u_{L,R}, \quad (40) \]
\[ d_{L,R} \to D_{L,R} d_{L,R}, \quad (41) \]

we end up with a Lagrangian with the usual CKM matrix in
the charged current sector \( V_{CKM} = U_{L}^{\dagger} D_{L} \) (we neglect pos-
sible extra phases). In the neutral current sector we have the
same structures \([36]\) and \([37]\), but making the substitutions

\[ C_{uL} \to \tilde{C}_{uL} \equiv U_{L}^{\dagger} C_{uL} U_{L}, \quad (42) \]

and the same transformation holds for \( C_{dL}, C_{dR} \) and \( C_{dR} \),
and for the Z’-current. As it is well-known, the SM \( C_{x}^{S} \) mat-
rices remain unchanged by this rotation because they are pro-
tional to the identity, but things are different for the \( C_{x}^{S} \) mat-
rices, where we will have

\[ \tilde{C}_{dL}^{S} = D_{L}^{\dagger} C_{dL}^{S} D_{L} \approx C_{dL}^{S}, \quad (43) \]
\[ \tilde{C}_{dR}^{S} = D_{R}^{\dagger} C_{dR}^{S} D_{R} \approx C_{dR}^{S}, \quad (44) \]
\[ \tilde{C}_{uL}^{S} = U_{L}^{\dagger} C_{uL}^{S} U_{L} \approx V_{CKM} C_{uL}^{S} V_{CKM}^{\dagger}, \quad (45) \]
\[ \tilde{C}_{uR}^{S} = U_{R}^{\dagger} C_{uR}^{S} U_{R}. \quad (46) \]

where we have neglected the non-diagonal elements in the
Yukawa couplings in the down sector involving the third fam-
ily and that the \( C_{x}^{S} \) matrices are zero except for the (3,3) ele-
ment. Thus one can see that, apart from the very small mixing,
the only new effect in the coupling of the Z-boson to the down
quarks is in the diagonal Z\( b \bar{b} \) coupling, which will be slightly
modified. We have only discussed things in the quark sector,
but the leptonic sector is identical.

In the up-quark sector things are different and FCNC are in
principle possible

\[ \left[ \tilde{C}_{uL}^{S} \right]_{ij} = (V_{CKM})_{i3} \left[ C_{uL}^{S} \right]_{33} (V_{CKM})_{j3}^{*} \]
\[ \quad = - (V_{CKM})_{i3} (V_{CKM})_{j3}^{*}, \quad (47) \]
\[ \left[ \tilde{C}_{uR}^{S} \right]_{ij} = [U_{R}]^{*}_{i3} [C_{uR}^{S}]_{33} [U_{R}]_{j3} = - [U_{R}]^{*}_{i3} [U_{R}]_{j3}. \quad (49) \]

However, the argument given for the down-quark sector can
be applied also here in the limit where we neglect the higher
dimensional operators. Therefore the FCNC in the up-quark
sector are suppressed by the smallness of the elements \([M_{u}]_{i3}\)
and \([M_{u}]_{3i}\) (\( i = 1, 2 \)) in our model, although in the left-
handed sector this is related to the CKM matrix. Thus we see
that FCNC in the down-quark and charged-lepton sector,
where the strongest constraints appear \((K^{0} - \bar{K}^{0} \) mixing, \( \mu - e \) conversion, ... \) \([12]\) are suppressed in our model. In the up-
quark sector we have found that the FCNC are suppressed by
the tiny \([M_{u}]_{i3,3i}\) elements and also either by the small mixing
\( \theta_{ZZ'}, \) or by the mass of the Z’ boson. One can actually check
that the Yukawa suppression is so strong that even for a Z’ bo-
son lighter than the SM Z boson one satisfies the constraints
coming from \( D^{0} - \bar{D}^{0} \) mixing \([22]\). A detailed analysis of all
the constraints coming from flavor violation will be published
in a future publication.

4. Higgs Sector

The Higgs sector is composed of the MSSM Higgs dou-
blets, \( H_{u} \) and \( H_{d} \), and the singlet \( S \). After symmetry break-
ing lepton number is broken and the Higgses will mix with the
sleptinos from the Higgs bosons (although these effects can be
important in the decays of the LSP). Keeping this in mind, the
physical Higgs sector contains one CP-odd scalar \( A \), similar
to the MSSM but now with some small admixture of \( S \).
It also contains four CP-even scalars: \( h \), the SM-like Higgs;
and \( H_{1}, H_{2} \) and \( H_{3} \). The latter three are some combination of
the MSSM Higgs bosons, \( S \) and the right-handed sneutrino.
These are labeled from lightest to heaviest. Of course, there
is also the charged Higgs of the MSSM, \( H^{\pm} \), whose composi-
tion is purely MSSM Higgs bosons.

The mass of the CP-odd Higgs is given by

\[ m_{A}^{2} = \frac{2b}{\sin 2\beta} + \frac{b v_{S}^{2} \sin 2\beta}{2 v_{S}^{2}}. \quad (50) \]

There are two limits in which the \( Z - Z' \) mixing is phe-
nomenologically viable: \( M_{2}^{2}/M_{Z}^{2} \ll 1 \) (Eq. \([34]\)) which
implies \( v_{S}^{2} \ll 1 \) and when \( \tan \beta \) is quite large. Both
cases imply the second term in the \( m_{A}^{2} \) expression is negli-
gible therefore yielding \( m_{A} \sim 2b/\sin 2\beta \) as in the MSSM.
This value is always positive. The goldstone boson associated
with the \( U(1)_{S} \) and \( U(1)_{B-L} \) will predominantly be composed of
a linear combination of the CP-odd part of \( S \) and \( \bar{\nu}^{c} \) depend-
going the kinetic mixing between those two sectors and which
generation of right-handed sneutrino acquires a VEV.

The most general mass matrix for the CP-even scalars, \( M_{2}^{2} \),
in the basis \( \sqrt{2} \text{Re} (H_{d}, H_{u}, S, \bar{\nu}^{c}) \), has the following elements:

\[ M_{S_{11}}^{2} = \frac{1}{4} (g_{1}^{2} + g_{2}^{2} + g_{3}^{2}) v_{S}^{2} \cos^{2} \beta + b \tan \beta, \quad (51) \]
\[ M_{S_{12}}^{2} = -b + \frac{1}{8} (4x^{2} - g_{1}^{2} - g_{2}^{2}) v_{S}^{2} \sin 2\beta, \quad (52) \]
\[ M_{S_{13}}^{2} = \frac{1}{4} (4x^{2} - g_{3}^{2}) v_{S} \cos \beta - b \frac{v}{v_{S}} \sin \beta, \quad (53) \]
\[ M^2_{S_{31}} = -\frac{1}{4} g_S (\xi g_{BL} + z_{\nu^c} g_S) v_R v \cos \beta, \]  
(54)
\[ M^2_{S_{22}} = \frac{1}{4} \left( g_1^2 + g_2^2 \right) v^2 \sin^2 \beta + b \cot \beta, \]  
(55)
\[ M^2_{S_{23}} = -b v \frac{v}{v_S} \cos \beta + \lambda^2 v_S v \sin \beta, \]  
(56)
\[ M^2_{S_{24}} = 0, \]  
(57)
\[ M^2_{S_{33}} = \frac{1}{4} g_S^2 v_S^2 + \frac{1}{2} b v_S^2 \sin 2\beta, \]  
(58)
\[ M^2_{S_{34}} = \frac{1}{4} g_S (\xi g_{BL} + z_{\nu^c} g_S) v_R v_S, \]  
(59)
\[ M^2_{S_{44}} = \frac{1}{4} \left( z_{\nu^c} g_S^2 + 2 \xi z_{\nu^c} g_{BL} + g_{BL} \right) v_R^2, \]  
(60)

where \( \xi \) is the kinetic mixing between \( U(1)_{BL} \) and \( U(1)_S \) and \( z_{\nu^c} \) is the \( U(1)_S \) charge of \( \nu^c \): zero for the first two generations and negative four for the third. In the case where these two parameters are zero, the right-handed sneutrino has a mass equal to the \( Z_{BL} \) mass: \( g_{BL} v_R v/2 \). For completeness, we also present the important one-loop corrections, \[23\], to the up-type three-by-three matrix from top/stop loops presented in Ref. \[2\] and repeated here only for the sake of consistent notation. Some of these are implemented by the redefinition in the tree-level mass matrix:
\[ b \rightarrow b + \frac{3}{16\sqrt{2} \pi^2} \lambda v^2 A_t F_t v_S \]  
(61)
while the rest are given by
\[ \Delta M^2_{S_{11}} = -\frac{3Y^2}{32\pi^2} \mu^2 G_t, \]  
(62)
\[ \Delta M^2_{S_{22}} = \frac{3Y^2}{32\pi^2} \times \left( -A_t^2 G_t + 4A_t E_t + 4m_l^2 \ln \left( \frac{M^2_{l_R}}{m^2_{l_L}} \right) \right), \]  
(63)
\[ \Delta M^2_{S_{33}} = -\frac{3Y^2}{64\pi^2} \lambda^2 v^2 \cos^2 \beta G_t, \]  
(64)
\[ \Delta M^2_{S_{44}} = \frac{3Y^2}{32\pi^2} \mu (A_t G_t - 2 E_t), \]  
(65)
\[ \Delta M^2_{S_{12}} = -\frac{3Y^2}{32\pi^2} \lambda \mu v \cos \beta (4E_t - G_t), \]  
(66)
\[ \Delta M^2_{S_{23}} = -\frac{3Y^2}{32\sqrt{2} \pi^2} \lambda \mu v \cos \beta (A_t G_t - 2E_t), \]  
(67)

where \( m_t \) is the top mass, \( M_{t_L} \) and \( M_{l_R} \) are the lighter and heavier stop masses respectively and \( A_t \) is the trilinear \( \mu \) term for the up-type Higgs and stops: \( V_{soft} \supset Y_t A_t Q H_u t^c \). Finally, the loop functions are given by
\[ F_t = \frac{1}{M^2_{l_R} - M^2_{l_L}} \left( M^2_{l_R} \ln \left( \frac{M^2_{l_R}}{M^2_{l_L}} \right) - M^2_{l_L} \ln \left( \frac{M^2_{l_L}}{M^2_{l_R}} \right) \right) - 1, \]  
(68)
\[ G_t = \sin^2 2\theta_t \left( \frac{M^2_{l_R} + M^2_{l_L}}{M^2_{l_R} - M^2_{l_L}} \ln \left( \frac{M^2_{l_R}}{M^2_{l_L}} \right) - 2 \right), \]  
(69)
\[ E_t = -m_t \sin 2\theta_t \ln \left( \frac{M^2_{l_R}}{M^2_{l_L}} \right), \]  
(70)

where \( \theta_t \) is the mixing angle in the stop sector and \( M_{SUSY} \) is typically taken to be \( \sqrt{M_{t_L} M_{l_R}} \). The physical stop masses are derived by diagonalizing the stop mass matrix:
\[ M^2_t = \left( \begin{array}{cc} m^2_{\tilde{Q}} + m^2_t + D_L & m_t X_t \\ m_t X_t & m^2_{\tilde{t}_L} + m^2_t + D_R \end{array} \right), \]  
(71)

where
\[ D_L = \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) M_Z^2 \cos 2\beta, \]  
(72)
\[ D_R = \frac{2}{3} \sin^2 \theta_W M_Z^2 \cos 2\beta, \]  
(73)
\[ X_t = A_t - \mu \cot \beta. \]  
(74)

The radiative correction to the Higgs mass is maximized for maximal mixing, defined as \( X_t = \sqrt{6} M_S \), where \( M_S^2 = \frac{1}{2} \left( M^2_{t_L} + M^2_{l_R} \right) \) and we use notation similar to Ref. \[24\].

The SM-like Higgs mass will depend on the various parameters and the one-loop effects. In Fig. \[2\] we plot curves of constant \( m_h \) in the (a) \( \tan \beta - \lambda \) plane for \( \mu = 400 \text{ GeV} \), (b) \( \mu - \lambda \) plane for \( \tan \beta = 10 \) and (c) \( \mu - \tan \beta \) plane for \( \lambda = 0.1 \); the red curves correspond to the LEP2 bound of 114 GeV. We furthermore use \( a_\lambda = 100 \text{ GeV} \), \( g_S = 0.4 \) and a top mass of 173 GeV. Dashed purple curves of constant \( \theta_{ZZ'}/ = 1 \times 10^{-3} \) are also included as a conservative upper bound. This calculation is done in the maximal mixing scenario (\( X_t = \sqrt{6} M_S \)), for \( m_{\tilde{Q}} = m_{\tilde{t}_L} = 1000 \text{ GeV} \). This corresponds to \( m_{\tilde{t}_{1,2}} \sim 800, 1180 \text{ GeV} \). We further assume no mixing between the \( B - L \) and \( U(1)_S \) sectors, i.e. no kinetic mixing and no VEV for the third generation sneutrino. Varying \( a_\lambda \) also has an effect the contours, namely elongating the corners of the curves in (a) in (b) towards the right and in (c) towards the left but does not influence the maximum Higgs mass value.

Fig. \[2\] indicates that the Higgs mass is maximized for small \( \lambda \) and large \( \tan \beta \). Small \( \lambda \) is one of the necessary limits to recover the MSSM, while increased Higgs mass with increased \( \tan \beta \) is a behavior shared with the MSSM. In fact, in both cases, the maximum is at around \( m_h \sim 130 \text{ GeV} \) for this value of the stop masses and stop mixing. The reason for the strong resemblance to the MSSM is that the NMSSM-like parame-
ter space that allows for a Higgs mass surpassing the MSSM value—large \( \lambda \), relatively small \( \mu \) and small \( \tan \beta \)—is ruled out here due to \( \theta_{ZZ'} \), see Fig. 2. However it might be possible to relax this bound on \( \theta_{ZZ'} \) since \( Z_S \) couples only to the third generation. While a more detailed study of this is required, this part of parameter space could open up new NMSSM-like possibilities such as the lightest Higgs being mostly singlet thereby pushing up the mass of the SM-like Higgs. Since the mostly singlet Higgs and \( Z_S \) have correlated masses, this would further mean a light \( Z_S \) which could alleviate a tension that usually exists in models with gauge origins for the \( Z \) thereby possibly inducing new lepton and baryon or baryon number violating decays for the LSP.

Finally, the mass of the charged Higgs is

\[
m_{H^\pm}^2 = \frac{2b}{\sin 2\beta} + M_W^2 - \frac{1}{2} \lambda^2 v^2,
\]

(75)

where \( M_W \) is the mass of the \( W \) boson of the SM and where in general the above expression could be negative but will typically be dominated by the positive contribution from the \( b \)-term.

5. \( M \)-Parity Violation, Neutralinos and Neutrinos

Above the SUSY scale, the \( B - L \) symmetry guarantees \( M \)-parity conservation. Once the right-handed sneutrino acquires a VEV bilinear \( M \)-parity violating terms (which break lepton number) are generated. Schematically, these include \( Y_{\nu} v_R (L H_u) \), the effective \( \epsilon \) term and the only significant contribution from the superpotential, and gaugino-lepton mixing, e.g.

\[
g_{BL} v_R \left( \nu^c B_{BL} \right), g_2 v_L \left( \nu \tilde{W}^0 \right), g_2 v_L \left( \nu \tilde{W}^+ \right).
\]

(76)

In addition to mediating the decay of the LSP, these terms mix SUSY and SM particles contributing to the neutralino mass matrix. In the basis \( \left( \nu, \nu^c, B_{BL}, \tilde{S}, \tilde{B}_S, B, \tilde{W}, H_d, H_u \right) \) the neutralino mass matrix is given by

\[
M_{\chi^0} = \left( \begin{array}{cc} M_{BL} & \Gamma \\ \Gamma^T & M_{\chi^0} \end{array} \right),
\]

(77)

where

\[
M_{BL} = \left( \begin{array}{ccc} 0 & \frac{Y_{\nu} v_R}{\sqrt{2}} & -\frac{g_{BL} v_R}{\sqrt{2}} \\ \frac{g_{BL} v_R}{\sqrt{2}} & 0 & M_{BL} \\ -\frac{g_{BL} v_R}{\sqrt{2}} & M_{BL} & 0 \end{array} \right),
\]

(78)

\[
\Gamma = \left( \begin{array}{ccc} 0 & \frac{g_{ST} v_R}{2} & \frac{g_{ST} v_R}{2} & 0 \\ \frac{g_2 v_R}{2} & 0 & 0 & 0 \\ 0 & \frac{g_2 v_R}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right),
\]

(79)

and

\[
M_{\chi^0} = \left( \begin{array}{cccc} 0 & g_{ST} v_R & 0 & -\frac{\lambda v}{\sqrt{2}} \\ g_{ST} v_R & M_S & 0 & 0 \\ 0 & 0 & M_1 & \frac{g_{2} v_R}{2} \\ -\frac{\lambda v}{\sqrt{2}} & \frac{g_{2} v_R}{2} & \frac{g_{2} v_R}{2} & 0 \\ 0 & \frac{g_{2} v_R}{2} & \frac{g_{2} v_R}{2} & -\frac{\lambda v}{\sqrt{2}} \end{array} \right),
\]

(80)

where the lower four-by-four block of \( M_{\chi^0} \) is the MSSM mass matrix and \( M_{BL} \) is the \( B - L \) part and decouples from the rest if the third generation sneutrino is not VEVed and there is no significant \( (B - L) \sim S \) mixing. If only one sneutrino acquires a VEV, there will be one heavy right-handed neutrino and two with active neutrino masses.

In the case of a third generation right-handed sneutrino VEV, an additional \( M \)-parity violating term is generated, which mixes the right-handed bottom quark (squark) with the triplino (triplet) via the \( \lambda_5 \) couplings in Eq. 5. Unlike the \( M \)-parity violating terms which mix the neutrinos with the neutralinos, this mixing term does not generate neutrino masses and can therefore be large in comparison (although proton decay would then dictate smaller baryon number violating interactions for the exotic triplets). Such a large coupling would make this the most important source of \( M \)-parity violation thereby possibly inducing new lepton and baryon or baryon number violating decays for the LSP.

6. Colored Triplet

As discussed earlier, a pair of colored triplets, \( \tilde{T} \sim (3, 1, 1/3, 2/3, z_T) \) and \( \tilde{T} \sim (3, 1, -1/3, -2/3, -6 - z_T) \), are necessary for \( U(1)_S \) anomaly cancellation. The triplinos acquire mass as do the Higgsinos, from the VEV of \( S \):

\[
M_T = M_{\tilde{T}} = \frac{\lambda v_S}{\sqrt{2}}.
\]

(81)

The triplets themselves also accrue mass from the soft terms:

\[
\mathcal{L}_{soft} \supset -m_{T}^2 |T|^2 - m_{T}^2 |T|^2 + (B_T T \tilde{T} + \text{h.c.}),
\]

(82)

where \( B_T \) is the product of a trilinear \( a \)-term and the VEV of \( S \), and the \( D \)-terms. Their physical masses are

\[
M_{T_{1,2}} = \frac{1}{2} \left( M_T^2 + M_{\tilde{T}}^2 \right) \mp \sqrt{(M_T^2 - M_{\tilde{T}}^2)^2 + 4 |B_T|^2}
\]

(83)

where

\[
M_T^2 = |\lambda_1|^2 \frac{v_S^2}{2} + m_T^2 + \frac{g_{BL}^2 v_T}{12} - \frac{3}{8} g_{2} v_S^2, \quad (84)
\]

\[
M_{\tilde{T}}^2 = |\lambda_1|^2 \frac{v_S^2}{2} + m_{\tilde{T}}^2 - \frac{g_{BL}^2 v_T}{12} + \frac{1}{4} g_{2} v_S^2. \quad (85)
\]
and we neglect electroweak $D$-term contributions. Using the couplings of the colored triplet fields with matter allows us to write their interactions with the physical fermions. For the triplet $T$:

\begin{align}
\lambda_2 & \ U_{ij} \ E_{3j} \ u_i \ e_j \ T, \\
\lambda_2 & \ D_{3i} \ N_{3j} \ d_i \ \nu_j \ T, \\
\lambda_3 & \ U_{3i}^c \ D_3^c \ u_i^c \ d_j^c \ T.
\end{align}

FIG. 2: Curves of constant $m_h$ in the (a) $\tan \beta - \lambda$ plane for $\mu = 400$ GeV, (b) $\mu - \lambda$ plane for $\tan \beta = 10$ and (c) $\mu - \tan \beta$ plane for $\lambda = 0.1$. We use $a_\lambda = 100$ GeV, $g_S = 0.4$ and a top mass of 173 GeV. Here $B - L$ is broken by the second generation sneutrino only and there is no mixing between the $B - L$ and $U(1)_S$ sectors. We further assume the maximal mixing scenario for the stop masses ($X_t = \sqrt{6} M_S$) with the soft mass parameters $m_{\tilde{Q}} = m_{\tilde{t}_c} = 1$ TeV. The red contour is the LEP2 bound on $m_h$ of 114.4 GeV and the dashed purple lines indicate constant $\theta_{ZZ'} = 10^{-3}$: a conservative upper bound on the $Z - Z'$ mixing.
Here we use the standard convention for the diagonalisation of the fermion mass matrices, $U^TU^c = Y^{diag}_d$, $D^TY^cD = Y^{diag}_d$, $E^TE = Y^{diag}_{e}$, $N^TY_N = Y^{diag}_{\nu}$. We also define $V = U^TD$ and $V_{PMNS} = E^TN$. In the case of the field $T$ we find the following interactions:

$$\lambda_4 U^c_{3i} E^{c}_{j3} u^c_i e_j \bar{T},$$  \hspace{1cm} (89)
$$\lambda_5 D^c_{j3} N^c_{j3} d^c_i \nu^c_j \bar{T},$$  \hspace{1cm} (90)
$$2\lambda_6 U^c_{3i} D_{3j} u_i d_j \bar{T}. $$ \hspace{1cm} (91)

We are now ready to study the proton decay aspect of this theory in the next section.

VI. PHENOMENOLOGICAL ASPECTS

A. Proton Stability

In the previous section we have discussed the main properties of the interactions of the colored fields, $T$ and $\bar{T}$. Integrating out the Higgs $T$ and using the above interactions we find that the amplitude for $p \rightarrow \pi^0 e^+_\alpha$ is given by

$$A_T(p \rightarrow \pi^0 e^+_\alpha) \sim \frac{\lambda_2 \lambda_3}{M_T^2} U^c_{3i} D^c_{3j} U_{3i} E_{3\alpha} \lesssim 10^{-30} \text{GeV}^{-2}. $$ \hspace{1cm} (92)

Assuming $M_T = 1 \text{ TeV}$, $U^c_{31} = U^c_{32} = D^c_{31} = E_{3\alpha} \approx V^{13}_{CKM}$, one gets the bound $\lambda_2 \lambda_3 < 10^{-12}$. One can do something similar using the bound on the decay $p \rightarrow K^+ \bar{\nu}$:

$$A_T(p \rightarrow K^+ \bar{\nu}) \sim \frac{\lambda_3 \lambda_6}{M_T^2} U^{c}_{31} \left( D^c_{31} D^c_{32} + D^c_{32} D^c_{32} \right) N_{3i} \lesssim 10^{-30} \text{GeV}^{-2}. $$ \hspace{1cm} (93)

The field $\bar{T}$ can mediate proton decay as well. For the channels $p \rightarrow \pi^0 e^+_\alpha$ the amplitude reads as

$$A_T(p \rightarrow \pi^0 e^+_\alpha) \sim \frac{2\lambda_4 \lambda_6}{M_T^2} U^c_{31} D_{31} U^c_{3i} E_{3\alpha} \lesssim 10^{-30} \text{GeV}^{-2}. $$ \hspace{1cm} (94)

Then, one gets the bound $\lambda_3 \lambda_6 < 5 \times 10^{-13}$ if $M_T = 1 \text{ TeV}$ and assuming $U^c_{31} = D_{31} = U^c_{31} = D_{32} \approx V^{13}_{CKM}$. The same happens to the amplitude

$$A_T(p \rightarrow K^+ \bar{\nu}) \sim \frac{2\lambda_6 \lambda_5}{M_T^2} U^c_{31} \left( D_{31} D^c_{32} + D_{32} D^c_{32} \right) N_{3i} \lesssim 10^{-30} \text{GeV}^{-2}. $$ \hspace{1cm} (95)

Notice that it is difficult to set the bounds on the couplings, $\lambda_2, \ldots, \lambda_6$ depend on the size of the elements of flavor matrices for all quarks and leptons. Since the mixing between the third generation and the others two is very small in the down quark and charged lepton sectors, $D^c_{3i} = D_{3i} = E_{3i} \approx \delta_{3i}$, and the bounds discussed above can be avoided. However, one has to investigate the bounds coming from proton decay at loop level. It is important to notice that $(V^{*}_{CKM})_{i3} \approx U_{3i}$ and $N_{3i} \approx (V_{PMNS})_{3i}$, and one has contributions to the channel $p \rightarrow \pi^+ \bar{\nu}$ at two loop level:

$$A(p \rightarrow \pi^+ \bar{\nu}) \sim \frac{2\lambda_2 \lambda_3}{(16\pi^2)^2 M_T^2} \left( V^{13}_{CKM} \right)^3 \lesssim 10^{-30} \text{GeV}^{-2}. $$ \hspace{1cm} (96)

In the case of $p \rightarrow K^+ \bar{\nu}$ one gets

$$A(p \rightarrow K^+ \bar{\nu}) \sim \frac{2\lambda_6 \lambda_2}{(16\pi^2)^2 M_T^2} \left( V^{13}_{CKM} \right)^2 \lesssim 10^{-30} \text{GeV}^{-2}. $$ \hspace{1cm} (97)

Now, using this equation one can set a bound to the product: $\lambda_2 \lambda_6 < 10^{-12}$. Unfortunately, the bounds on $\lambda_4, \lambda_5$ and $\lambda_6$ depend on unknown mixing matrices $U^c$ and $N^c$.

B. Baryon and Lepton Number Violation at the LHC

In this model $M$-parity is spontaneously broken after symmetry breaking and one expects the typical signals for bilinear R-parity violation. In this subsection we will focus mainly on the properties of the new exotic fields needed to define an anomaly free theory. The high energy analogue of the proton decay mediated by the new exotic colored triplets is potential exciting signals of baryon and lepton number violation at the LHC. While studies have shown that typically detecting lepton number violation at LHC is manageable, in this case it will be much more challenging since the exotic triplets couple only to the tau, which can decay hadronically, obscuring its lepton number. Detecting lepton number violation would then crucially depend on how well one can see the $\tau$ leptons. Furthermore observing baryon number violation is always tricky at the LHC due to lack of information on the initial and final states. Specifically, the baryon number of the initial state can have one of five values: 0 for two gluons or for $q\bar{q}$, $\pm 1/3$ for a gluon and a quark and $\pm 2/3$ for two quarks. Therefore, one must be able to observe a final state with a baryon number different than these: an insurmountable task when observing light jets. Fortunately, the fact that the exotics $T$ and $\bar{T}$ couple purely to the third generation of quarks and leptons in the flavor basis helps here since it is possible to tag tops and bottoms. While such issues require an in-depth study, we shall proceed by simply elucidating the processes that may be amiable to such a study.

Production of the colored triplets in the most efficient manner proceeds through pair production via gluon fusion. This is of course a strong process with large cross sections, equivalent to squark pair production from gluon fusion. Decay proceeds through the coupling to third generation matter; the possible final states violating baryon and lepton number are

$$gg \rightarrow T_i T^*_i \rightarrow t t b \tau \quad \text{and} \quad gg \rightarrow T_i T^*_i \rightarrow t t b \bar{\nu},$$

where baryon and lepton number are both violated by one unit, as expected from proton decay operators. The decay width of the colored triplets depend on the size of the relevant
Yukawa couplings discussed in the section about proton decay. Then, one can have different scenarios for given values of $\lambda_2, \lambda_3, \lambda_4, \lambda_5,$ and $\lambda_6$ couplings. For example, one can have a scenario where the main decays are into quarks and charged leptons if $\lambda_2$ and $\lambda_5$ are suppressed. As for detectability, in principle at least baryon number violation can be observed in this process since the final baryon number of $\pm 1$ is different than any of the initial state baryon number possibilities listed above. However, this is crucially dependent on correctly identifying that these are all like-sign quarks. It goes without saying that lepton number violation can only be measured in the $\tau$ channel. A detailed analysis of the signals is beyond the scope of this article.

VII. SUMMARY AND OUTLOOK

We have proposed a simple model where the origin of the $\mu$ term and the matter-parity violating interactions of the MSSM can be understood from the spontaneous breaking of two new Abelian gauge symmetries. We have found the following results:

- The new symmetries are $U(1)_{B-L}$ and $U(1)_S$, where the latter is relevant only to the third generation. In order to satisfy $U(1)_S$ anomalies new exotics, the colored triplets $T$ and $\tilde{T}$, are needed.

- The local $B-L$ gauge symmetry is broken by the VEV of the “right-handed” sneutrinos giving rise to lepton number violating $M$-parity violation and $U(1)_S$ is broken by the VEV of $S$, generating the $\mu$-term.

- The new $Z'$ associated with $U(1)_S$ gives rise to flavor violation without conflict with experiments.

- We have shown that it is possible to have a consistent scenario for fermion masses after symmetry breaking. In this case one has well-defined textures for charged fermion masses and the mixings between the third generation and the others is very small.

- The numerical predictions for the lightest Higgs boson have been investigated up to one-loop level showing the possibility to satisfy the experimental bounds from LEP2 experiment. We have found that the upper bound on the lightest Higgs mass is $m_h \sim 130$ GeV if the stop masses are below 1 TeV.

- We made a brief discussion of how one could observe lepton and baryon number violation at the LHC in agreement with the experimental bounds on proton decay.

In our opinion this framework opens up the possibility to test the origin of the MSSM interactions ($\mu$ term and lepton number violating interactions) at the LHC. The collider signals and the predictions for fermion masses will be investigated in a future publication.

Acknowledgment

P. F. P. is supported in part by the US DOE under contract No. DE-FG02-95ER40896, by the Wisconsin Alumni Research Foundation and by the James Arthur Fellowship, CCPP-New York University. S. S. is supported in part by the US DOE under contract No. DE-FG02-95ER40896 and by the Wisconsin Alumni Research Foundation. M. G.-A. is supported by the US DOE contract DE-FG02-08ER41531 and by the Wisconsin Alumni Research Foundation.

[1] M. Drees, R. Godbole and P. Roy, “Theory and Phenomenology of Sparticles,” (World Scientific, Singapore, 2004).
[2] U. Ellwanger, C. Hugonie and A. M. Teixeira, “The Next-to-Minimal Supersymmetric Standard Model,” Phys. Rept. 496 (2010) 1 [arXiv:0910.1785 [hep-ph]].
[3] C. S. Aulakh, R. N. Mohapatra, “Neutrino as the Super-symmetric Partner of the Majoron,” Phys. Lett. B119, 136 (1982); M. J. Hayashi and A. Murayama, “Radiative Breaking of $SU(2)_L \times U(1)_{B-L}$ gauge symmetry induced by broken N=1 Supergravity in a Left-Right symmetric model,” Phys. Lett. B 153, 251 (1985); R. N. Mohapatra, “Mechanism For Understanding Small Neutrino Mass In Superstring Theories,” Phys. Rev. Lett. 56, 561-563 (1986); L. M. Krauss and F. Wilczek, “Discrete Gauge Symmetry in Continuum Theories,” Phys. Rev. Lett. 62 (1989) 1221; A. Font, L. E. Ibanez and F. Quevedo, “Does Proton Stability Imply the Existence of an Extra Z0?”, Phys. Lett. B 228 (1989) 79; S. P. Martin, “Some simple criteria for gauged R-parity,” Phys. Rev. D 46 (1992) 2769 [arXiv:hep-ph/9207218].
[4] P. Fileviez Perez, S. Spinner, “Spontaneous R-Parity Breaking and Left-Right Symmetry,” Phys. Lett. B673 (2009) 251-254 [arXiv:0811.3424 [hep-ph]]; V. Barger, P. Fileviez Perez and S. Spinner, “Minimal gauged U(1)(B-L) model with spontaneous R-parity violation,” Phys. Rev. Lett. 102 (2009) 181802 [arXiv:0812.3661 [hep-ph]].
[5] V. Braun, Y.-H. He, B. A. Ovrut, T. Pantel, “The Exact MSSM spectrum from string theory,” JHEP 0605, 043 (2006). [hep-th/0512177].
[6] M. Ambroso, B. Ovrut, “The B-L/Electroweak Hierarchy in Heterotic String and M-Theory,” JHEP 0901, 011 (2009). [arXiv:0904.4509 [hep-th]]; M. Ambroso, B. A. Ovrut, “The B-L/Electroweak Hierarchy in Smooth Heterotic Compactifications,” Int. J. Mod. Phys. A25, 2631-2677 (2010). [arXiv:0901.1129 [hep-th]].
[7] P. Nath and P. Fileviez Perez, “Proton stability in grand unified theories, in strings and in branes,” Phys. Rept. 441 (2007) 191 [arXiv:hep-ph/0601023].
[8] G. F. Giudice and A. Masiero, “A Natural Solution to the mu Problem in Supergravity Theories,” Phys. Lett. B 206 (1988) 480.
[9] J. E. Kim and H. P. Nilles, “The mu Problem and the Strong CP Problem,” Phys. Lett. B 138, 150 (1984).
[10] M. Cvetic, D. A. Demir, J. R. Espinosa, L. L. Everett, P. Langacker, “Electroweak breaking and the mu problem in supergravity models with an additional U(1),” Phys. Rev. D56, 2861 (1997), [hep-ph/9703317].
[11] M. Aoki and N. Oshimo, “A Supersymmetric model with an extra U(1) gauge symmetry,” Phys. Rev. Lett. 84 (2000) 5269 [arXiv:hep-ph/9907481].
[12] H. S. Lee, K. T. Matchev and T. T. Wang, “A U(1) ′-prime solution to the μ − problem and the proton decay problem in supersymmetry without R-parity,” Phys. Rev. D 77 (2008) 015016 [arXiv:0709.0763 [hep-ph]].

[13] H. C. Cheng, B. A. Dobrescu and K. T. Matchev, “A Chiral supersymmetric standard model,” Phys. Lett. B 439 (1998) 301 [arXiv:hep-ph/9807246].

[14] I. Dorsner, P. Fileviez Perez and G. Rodrigo, “Fermion masses and the UV cutoff of the minimal realistic SU(5),” Phys. Rev. D 75 (2007) 125007 [arXiv:hep-ph/0607208].

[15] J. Erler, P. Langacker, S. Munir, E. R. Pena, “Improved Constraints on Z-prime Bosons from Electroweak Precision Data,” JHEP 0908 (2009) 017. [arXiv:0906.2435 [hep-ph]].

[16] M. S. Carena, A. Daleo, B. A. Dobrescu and T. M. P. Tait, “Z′-prime gauge bosons at the Tevatron,” Phys. Rev. D 70 (2004) 093009 [arXiv:hep-ph/0408098].

[17] J. Heeck and W. Rodejohann, “Kinetic and mass mixing with three abelian groups,” arXiv:1109.1508 [hep-ph].

[18] L. Basso, A. Belyaev, S. Moretti, G. M. Pruna and C. H. Shepherd-Themistocleous, “Z′ discovery potential at the LHC in the minimal B − L extension of the Standard Model," arXiv:1002.3586 [hep-ph].

[19] C.-F. Chang, K. Cheung, T.-C. Yuan, “Supersymmetric Decays of the Z′ Boson," [arXiv:1107.1133 [hep-ph]].

[20] A. A. Andrianov, P. Osland, A. A. Pankov, N. V. Romanenko, J. Sirkka, “On the phenomenology of a Z′ coupling only to third family fermions,” Phys. Rev. D58, 075001 (1998). [hep-ph/9804389].

[21] P. Langacker, M. Plumacher, “Flavor changing effects in theories with a heavy Z′ boson with family nonuniversal couplings,” Phys. Rev. D62 (2000) 013006. [hep-ph/0001204].

[22] E. Golowich, J. Hewett, S. Pakvasa, A. A. Petrov, “Implications of D0 - D0 Mixing for New Physics," Phys. Rev. D76 (2007) 095009. [arXiv:0705.3650 [hep-ph]].

[23] G. Degrassi, P. Slavich, “On the radiative corrections to the neutral Higgs boson masses in the NMSSM," Nucl. Phys. B825, 119-150 (2010). [arXiv:0907.4682 [hep-ph]].

[24] M. S. Carena, H. E. Haber, “Higgs boson theory and phenomenology," Prog. Part. Nucl. Phys. 50, 63-152 (2003). [hep-ph/0208209].

[25] D. K. Ghosh, G. Senjanovic, Y. Zhang, “Naturally Light Sterile Neutrinos from Theory of R-parity,” Phys. Lett. B698, 420-424 (2011). [arXiv:1010.3968 [hep-ph]]; V. Barger, P. Fileviez Perez, S. Spinner, “Three Layers of Neutrinos," Phys. Lett. B696, 509-512 (2011). [arXiv:1010.4023 [hep-ph]].