A Switched Systems Approach to Path Following with Intermittent State Feedback

Hsi-Yuan Chen, Zachary I. Bell, Patryk Deptula, Warren E. Dixon

Abstract—Autonomous agents are often tasked with operating in an area where feedback is unavailable. Inspired by such applications, this paper develops a novel switched systems-based control method for uncertain nonlinear systems with temporary loss of state feedback. To compensate for intermittent feedback, an observer is used while state feedback is available to reduce the estimation error, and a predictor is utilized to propagate the estimates while state feedback is unavailable. Based on the resulting subsystems, maximum and minimum dwell time conditions are developed via a Lyapunov-based switched systems analysis to relax the constraint of maintaining constant feedback. Using the dwell time conditions, a switching trajectory is developed to enter and exit the feedback denied region in a manner that ensures the overall switched system remains stable. A scheme for designing a switching trajectory with a smooth transition function is provided. Simulation and experimental results are presented to demonstrate the performance of control design.

Index Terms—Intermittent state feedback, observer, predictor, switched systems theory, dwell time conditions

I. INTRODUCTION

Acquiring state feedback is at the core of ensuring stability in control designs. However, factors such as the task definition, operating environment, or sensor modality can result in temporary loss of feedback. For example, agents may be required to limit communication during predefined time frames or when traversing through certain regions. Motivated by such factors, various path planning and control methods have been developed that seek to ensure uninterrupted feedback (cf., [1]–[12]). Such results inherently constrain the trajectory or behavior of a system. For instance, visual servoing applications for nonholonomic systems can result in limited, sharp-angled or non-smooth trajectories to keep a target in the camera field-of-view (FOV) as illustrated in results such as [13]–[15]. Rather than trying to constrain the system to ensure continuous feedback is available, the approach in this paper leverages switched systems methods to achieve an objective despite intermittent feedback.

Solutions to relaxing the constant feedback constraint have been investigated. For example, methods to relax the requirement of keeping landmarks in the FOV have been developed in results such as [16] and [17]. In [16], multiple landmarks are linked together by a daisy-chaining approach where new landmarks are mapped onto the initial world frame and are used to provide state feedback when initial landmarks leave the FOV. Similar concepts were adopted in [17], where a wheeled mobile robot (WMR) is allowed to navigate around a landmark without constantly keeping it in the FOV by relating feature points in the background to the landmark and thus provide state feedback. Although the objective to eliminate the requirement of constant visual on the landmark is achieved, state feedback is assumed to be available during periods when the landmark is outside the FOV. Such daisy-chaining approaches provide state feedback in an ideal scenario, but the accuracy of the feedback may degrade or even diverge in the presence of measurement noise and disturbances in the dynamics.

Conventional approaches to the simultaneous localization and mapping (SLAM) problem, such as the works in [18]–[20], use relationships between features or landmarks to estimate the pose (i.e., position and orientation) of the sensor, usually a monocular camera, and simultaneously determine the position of landmarks with respect to the world frame. Typically, a feature rich environment with sufficient measurements are required for SLAM methods to provide state estimation. However, a well-known drawback with SLAM algorithms is that without proper loop closures the estimates will drift over time due to the accumulation of measurement noise (cf., [21], [22]). In this paper, a state estimate dynamic model propagates the state estimate when feedback is not available, and no additional feedback information is required. Sufficient conditions may be derived via a Lyapunov-based analysis to ensure the loop closures are achieved before the state estimates degrade beyond a desired threshold.

Stability of systems that experience random state feedback has been analyzed in previous literature. Typically, the intermittent loss of measurement is modeled as a random Bernoulli process with a known probability. Resulting trajectories are then analyzed in a probabilistic sense, where the expected value of the estimation error is shown to converge asymptotically, compared to the result in this paper which examines the behavior of the actual tracking and estimation errors.

The networked control systems (NCS) community has also examined systems with temporarily unavailable measurements. Results such as [23]–[29] rely on a decision maker that is independent of the estimator or controller to determine when to broadcast sensor information. The objective in these results is to minimize the cost of network bandwidth by reducing the frequency of data transmission. In [27]–[29], data loss is modeled as random missing outputs and noisy measurements. In each case, state estimates are propagated by a model of the controlled system during the periods when transmission is missing. On the contrary, the availability of sensor information

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in this paper is not controlled by a decision maker but instead
determined by the region in which the actual states are located.
Therefore, sensor information is only available when the states
are inside a feedback region.

It is well known that slow switching between stable subsystems
may result in instability as explained in [30]. For slow switching between stable subsystems, the underlying strategy
for proving stability involves developing switching conditions
to ensure the overall system is stable. If a common Lyapunov
function exists for all subsystems such that the time derivative
of the Lyapunov function is upper bounded by a common
negative definite function, the overall system is proven to be
stable in [30]. For cases where a common Lyapunov function
cannot be determined, multiple subsystem-specific Lyapunov
functions are used. In general, the overall Lyapunov function
is discontinuous and jumps may occur at switching interfaces.
Therefore, the stability of such a system is achieved by placing
switching conditions on the subsystems to enforce a decrease
in the subsystem-specific Lyapunov functions between each
successive activation of the respective subsystems. Typically,
these requirements manifest as (average) dwell time conditions
which specifies the duration for which each subsystem must
remain active, as described in [30].

When a subset of the subsystems is unstable, a layer of
complication is introduced to the analysis. A stability analysis
is provided in [31] for switched systems with stable and
unstable linear time invariant (LTI) subsystems, where an
average dwell time condition is developed. Similarly, the
authors in [32] developed dwell time conditions for nonlinear
switched systems with exponentially stable and unstable
subsystems. However, dwell time conditions typically require
the stable subsystems to be activated longer than the unstable
subsystems, as indicated in [31]. In [33], the authors developed
an observer to estimate the depths of feature points in a image
from a monocular camera and use a predictor to propagate the
state estimates when the features are occluded or outside the
FOV. Based on the error system formulation, the subsystem
for the observer is stable, while the subsystem for the predictor
is unstable. An average dwell time condition is developed to
ensure the stability of the switched system. However, the focus
of [33] is the estimation of feature depths and therefore have
not focused on achieving a control objective when feedback
is unavailable.

The development in this paper aims to achieve a path
following objective despite intermittent loss of feedback. The
novelty of this result is guaranteeing the stability of following
a path which lies outside a region with feedback while maxi-
mizing the amount of time the agent spends in the feedback-
denied environment. Switched systems methods are used to
develop a state estimator and predictor when state feedback is
available or not, respectively. Since switching occurs between
a stable subsystem when feedback is available and an unstationary
subsystem when feedback is not available, dwell time con-
ditions are developed that determine the minimum time that the
agent must be in the feedback region versus the maximum
time the agent can be in the feedback denied region. Using
these dwell time conditions, a switching trajectory is designed
based on the dwell time conditions that leads the agent in
and out of the feedback denied region so that the overall
system remains stable. The most similar result to this paper
is in [34], which includes state prediction and control for a
nonholonomic system moving around an obstacle. The goal
in [34] is to regulate a nonholonomic vehicle to a set-point in
the presence of intermittent feedback. However, the difficulty
of path following in the current paper arises when the system
is outside the feedback region.

The paper is organized as follows. In Section II the system
model is introduced. In Section III the tracking and estimation
objective is given and the respective error systems are defined.
Based on the error dynamics, a Lyapunov-based stability
analysis for the resulting switched system is performed in
Section IV to develop the dwell time conditions and to show
stability of the overall system. In Section V a strategy for
designing a switching trajectory is presented. A simulation
is provided in Section VI and an experiment is provided in
Section VII to demonstrate the performance of the approach.

II. SYSTEM MODEL

Consider a dynamic system subject to an exogenous distur-
ance as

\[ \dot{x}(t) = f(x(t), t) + v(t) + d(t), \] (1)

where \( x(t), \dot{x}(t) \in \mathbb{R}^n \) denote a generalized state and its
time derivative, \( f : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n \) denotes the locally Lipschitz drift
dynamics, \( v(t) \in \mathbb{R}^n \) is the control input, and \( d(t) \in \mathbb{R}^n \) is the
exogenous disturbance where the Euclidean norm is bounded
as \( \|d(t)\| \leq \tilde{d} \in \mathbb{R}_{\geq 0} \) with \( n \in \mathbb{N} \) and \( t \in \mathbb{R}_{\geq 0} \).

III. STATE ESTIMATE AND CONTROL OBJECTIVE

In this paper, the overall objective is to achieve path
following under intermittent loss of feedback. Specifically,
a known feedback region is denoted as a closed set \( \mathcal{F} \subset \mathbb{R}^n \),
where the complement region where feedback is unavailable is
denoted by \( \mathcal{F}^c \). That is, feedback is available when \( x(t) \in \mathcal{F} \)
and unavailable when \( x(t) \in \mathcal{F}^c \).

A desired path is denoted as \( x_d \subset \mathcal{F}^c \). It is clear that
state feedback is unavailable while attempting to follow \( x_d \),
and hence the system must return to the feedback region
\( \mathcal{F} \) intermittently to maintain stability. Therefore, a switching
trajectory, denoted by \( \bar{x}_d(t) \in \mathbb{R}^n \), is designed to overlay \( x_d \) as
much as possible while adhering to the subsequently developed
dwell time constraints. To quantify the ability of the controller
to track the switching trajectory, the tracking error \( e(t) \in \mathbb{R}^n \) is
defined as

\[ e(t) \triangleq e_1(t) + e_2(t), \] (2)

where the estimate tracking error \( e_1(t) \in \mathbb{R}^n \) is defined as

\[ e_1(t) \triangleq \hat{x}(t) - \bar{x}_d(t), \] (3)
and the state estimation error $e_2(t) \in \mathbb{R}^n$ is defined as

$$e_2(t) \triangleq x(t) - \hat{x}(t),$$

(4)

where $\hat{x}(t) \in \mathbb{R}^n$ is the state estimate.

Based on (3) and (4), the control objective is to ensure that $e_1(t)$ and $e_2(t)$ converge, and therefore $e(t)$ will converge. To facilitate the subsequent development, let the composite error vector be defined as $z(t) \triangleq [e_1^T(t) \ e_2^T(t)]^T$.

IV. CONTROLLER AND UPDATE LAW DESIGN

To facilitate the subsequent analysis, two subsystems are defined to indicate when the states are inside or outside the feedback region. When $x(t) \in \mathcal{F}$, an exponentially stable observer can be designed using various approaches (e.g., observers such as [33], [35], [36] could be used). The subsequent development is based on an observer update law designed as

$$\dot{\hat{x}}(t) = f(\hat{x}(t), t) + v(t) + v_r(t),$$

(5)

where $v_r(t) \in \mathbb{R}^n$ is a high-frequency sliding-mode term designed as

$$v_r(t) = k_2 e_2(t) + \tilde{d} \text{sgn}(e_2(t)),$$

(6)

where $k_2 \in \mathbb{R}_{++}^{n \times n}$ is a constant, positive definite gain matrix. When $x(t) \in \mathcal{F}^c$, the state estimate is updated by a predictor designed as

$$\dot{\hat{x}}(t) = f(\hat{x}(t), t) + v(t).$$

(7)

Since the state is required to transition between $\mathcal{F}$ and $\mathcal{F}^c$, a switched systems analysis is used to investigate the stability of the overall switched system. To facilitate this analysis, the error systems for $e_1(t)$ and $e_2(t)$ are expressed as

$$\dot{e}_1(t) = f_{1p}(\bar{x}_d(t), \hat{x}(t), t),$$

(8)

$$\dot{e}_2(t) = f_{2p}(x(t), \hat{x}(t), t),$$

(9)

where $f_{1p}, f_{2p} : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_{++} \rightarrow \mathbb{R}^n$, $p \in \{a, u\}$, $a$ is an index for subsystems with available feedback, and $u$ is an index for subsystems when feedback is unavailable. Based on (8) and the subsequent stability analysis, the controller is designed as

$$v(t) = \begin{cases} \dot{x}_d(t) - f(\hat{x}(t), t) - k_1 e_1(t) - v_r(t), & p = a, \\ \dot{x}_d(t) - f(\hat{x}(t), t) - k_1 e_1(t), & p = u. \end{cases}$$

(10)

1Once $x(t) \in \mathcal{F}$, a simple reset scheme (i.e. setting $\hat{x}(t) = x(t)$) could be used. The reset scheme would eliminate the subsequently developed minimum dwell time for which $x(t)$ is required to remain in the feedback region $\mathcal{F}$. However, the subsequent development is based on the continued use of the observer to illustrate a more general stability condition for systems that require an observer or $\hat{x}(t) \in \mathcal{L}_{ac}$. 

2In cases where a piecewise-continuous controller is required, the robustifying term in (5) may be designed as $v_r(t) = k_2 e_2 + \frac{d^2}{T} e_2$, where $\epsilon \in \mathbb{R}_{++}$ is a design parameter.

where $\dot{x}_d(t) \in \mathbb{R}^n$, and $k_1 \in \mathbb{R}^{n \times n}$ is a constant, positive definite gain matrix. By taking the time derivative of (3) and substituting (5) into the resulting expression, (5) can be expressed as

$$\dot{e}_1(t) = -k_1 e_1(t), \quad \forall p.$$ 

(11)

After taking the time derivative of (3) and substituting (10) into the resulting expression, the family of systems in (9) can be expressed as

$$\dot{e}_2(t) = \begin{cases} f(x(t), t) - f(\hat{x}(t), t) + d(t) \\ -\tilde{d} \text{sgn}(e_2(t)) - k_2 e_2(t), & p = a, \\ f(x(t), t) - f(\hat{x}(t), t) + d(t), & p = u. \end{cases}$$

(12)

V. SWITCHED SYSTEM ANALYSIS

To further facilitate the analysis for the switched system, let $t_i^n \in \mathbb{R}_{++}$ denote the time of the $i^{th}$ instance when $x(t)$ transitions from $\mathcal{F}^c$ to $\mathcal{F}$, and $t_i^u \in \mathbb{R}_{++}$ denote the time of the $i^{th}$ instance when $x(t)$ transitions from $\mathcal{F}$ to $\mathcal{F}^c$, for $i \in \mathbb{N}$. The dwell time in the $i^{th}$ activation of the subsystems $a$ and $u$ is defined as $\Delta t_i^a := t_i^a - t_i^n \in \mathbb{R}_{++}$ and $\Delta t_i^u := t_{i+1}^u - t_i^u \in \mathbb{R}_{++}$, respectively. By Assumption [1] subsystem $a$ is activated when $t = 0$, and consequently $t_i^u > t_i^a, \forall i \in \mathbb{N}$.

Assumption 1. The system is initialized in a feedback region (i.e. $x(0) \in \mathcal{F}$).

To analyze the switched system, a common Lyapunov-like function is designed as

$$V_r(z(t)) = V_1(e_1(t)) + V_2(e_2(t)),$$

(13)

where the candidate Lyapunov functions for the tracking error and the estimation error are selected respectively as

$$V_1(e_1(t)) = \frac{1}{2} e_1^T(t) e_1(t),$$

(14)

$$V_2(e_2(t)) = \frac{1}{2} e_2^T(t) e_2(t).$$

(15)

The common Lyapunov-like function $V_r(z(t))$ globally exponentially converges while $x(t) \in \mathcal{F}$ and exhibits an exponential growth when $x(t) \in \mathcal{F}^c$. Hence, a desired maximum bound $V_M$ and a minimum threshold $V_T$ on $V_r(z(t))$ may be imposed such that $V_r(z(t)) \leq V_M$ and $V_r(z(t')) \leq V_T$. A representative illustration for the evolution of $V_r(z(t))$ is shown in Figure [II]. A lower threshold, $V_T$, enforces the convergence of $\|z(t)\|$ to an arbitrary small value. When implementing a high-frequency controller, $V_T$ may be selected arbitrarily close to zero. However, the closer $V_T$ is selected to zero, the longer $x(t)$ is required to remain in $\mathcal{F}$, and therefore the selection of $V_T$ is dependent of the individual application tolerance. When a high-gain controller (e.g., $v_r(t) = k_2 e_2(t) + \frac{d}{T} e_2(t)$) is implemented, $V_T$ should be selected such that $V_T \geq \epsilon$, where $\epsilon$ is a design parameter.
Theorem 1. The composite error system trajectories of the switched system generated by the family of subsystems described by (11), (12), and a piecewise constant, right continuous switching signal \( \sigma : [0, \infty) \rightarrow p \in \{ a, u \} \) are globally uniformly ultimately bounded provided \( u \) the switching signal satisfies the minimum feedback availability dwell time condition

\[
\Delta t^n_u \geq -\frac{1}{\lambda_u} \ln \left( \min \left( \frac{V_T}{V_\sigma(z(t^n_u))}, 1 \right) \right) \\
\text{(16)}
\]

and the maximum loss of feedback dwell time condition

\[
\Delta t^n_u \leq \frac{1}{\lambda_a} \ln \left( \frac{V_M + \frac{d^2}{2\lambda_a}}{V_\sigma(z(t^n_u)) + \frac{d^2}{2\lambda_a}} \right), \\
\text{(17)}
\]

where \( \lambda_a \) and \( \lambda_u \) are subsequently defined known positive constants.

Proof: By taking the time derivative of (14) and substituting for (11) yields

\[
\dot{V}_1(e_1(t)) \leq -2k_1V_1(e_1(t)), \forall t, \tag{18}
\]

where \( k_1 \) is the minimum eigenvalue of \( k_1 \). By using (12), the time derivative of (15) can be expressed as

\[
\dot{V}_2(e_2(t)) = \begin{cases} 
-(2(k_2 - c)V_2(e_2(t)), & t \in [t^n_u, t_{u+1}^a), \\
\lambda_aV_2(e_2(t)) + \frac{1}{2}d^2, & t \in [t^n_u, t_{u+1}^a). 
\end{cases} \tag{19}
\]

where \( c \in \mathbb{R}_{>0} \) is a Lipschitz constant, \( k_2 > c \in \mathbb{R} \) is the minimum eigenvalue of \( k_2 \), and \( \lambda_u \triangleq c + 1 \in \mathbb{R}_{>0} \).

From (18) and (19), the time derivative of the common Lyapunov-like function can be expressed as

\[
\dot{V}_\sigma(z(t)) \leq \begin{cases} 
-\lambda_\sigmaV_\sigma(z(t)), & t \in [t^n_u, t_{u+1}^a), \\
\lambda_\sigmaV_\sigma(z(t)) + \frac{1}{2}d^2, & t \in [t^n_u, t_{u+1}^a), \forall i \in \mathbb{N}, \tag{20}
\end{cases}
\]

where \( \lambda_\sigma = 2\min(k_1, k_2 - c) \in \mathbb{R}_{>0} \). The solutions to (20) for the two subsystems are

\[
\begin{align*}
V_\sigma(z(t)) &\leq V_\sigma(z(t^n_u))e^{-\lambda_\sigma(t-t^n_u)}, \quad t \in [t^n_u, t_{u+1}^a), \\
V_\sigma(z(t)) &\leq V_\sigma(z(t^n_u))e^{\lambda_u(t-t^n_u)} - \frac{d^2}{2\lambda_u} \left( 1 - e^{\lambda_u(t-t^n_u)} \right), \quad t \in [t^n_u, t_{u+1}^a). \tag{21}
\end{align*}
\]

The inequality in (21) indicates that \( \| z(t) \| \leq \| z(t^n_u) \| e^{-\frac{1}{2}\lambda_\sigma(t-t^n_u)}, \quad t \in [t^n_u, t_{u+1}^a) \). The minimum threshold \( V_T \) is selected to enforce the convergence of \( \| z(t) \| \) to desired threshold before allowing \( x(t) \) to transition into \( \mathcal{F}^c \). This condition can be expressed as \( V_\sigma(z(t^n_u))e^{-\lambda_\sigma(t-t^n_u)} \leq V_T \), and therefore the condition in (16) is obtained after algebraic manipulation. If \( \frac{V_T}{V_\sigma(z(t^n_u))} > 1 \), the value of \( V_\sigma(t^n_u) \) is already below the threshold and thus no minimum dwell time is required for the subsystem.

When \( t \in [t^n_u, t_{u+1}^a) \), the inequality in (21) indicates that \( \| z(t) \| \leq \sqrt{\| z(t^n_u) \|^2 e^{\lambda_\sigma(t-t^n_u)} - \frac{d^2}{2\lambda_u} \left( 1 - e^{\lambda_u(t-t^n_u)} \right)} \), and hence, the maximum bound \( V_M \) is selected to limit the growth of errors, where \( V_M > V_T \). The maximum dwell time condition for each of the \( n \) unstable periods is expressed as \( V_\sigma(z(t^n_u))e^{\lambda_\sigma\Delta t^n_u} - \frac{d^2}{2\lambda_u} \left( 1 - e^{\lambda_\sigma\Delta t^n_u} \right) \leq V_M \), and therefore the condition in (17) can be obtained.

Therefore, the composite error system trajectories generated by (11) and (12) are globally uniformly ultimately bounded as depicted in Figure 1.

VI. SWITCHING TRAJECTORY DESIGN

Since \( x_d \) lies outside the feedback region, i.e. \( x_d \in \mathcal{F}^c \), \( \forall t \), and cannot be followed for all time, the switching trajectory \( \tilde{x}_d(t) \) is designed to enable \( x(t) \) to follow \( x_d \) to the extent possible given the dwell time conditions in (16) and (17). A design challenge for \( \tilde{x}_d(t) \) is to ensure \( x(t) \) re-enters \( \mathcal{F} \) to satisfy the sufficient condition in (17). While \( x(t) \) transitions through \( \mathcal{F}^c \), \( e(t) \) may grow as indicated by (12), and this growth must be accounted for when designing \( \tilde{x}_d(t) \). To facilitate the development of the switching trajectory \( \tilde{x}_d(t) \), \( x_d(t) \in \mathbb{R}^n \) is defined as the closest orthogonal projection of \( \tilde{x}_d(t) \) on the boundary of \( \mathcal{F} \).

When the maximum dwell time condition is reached, \( \| e(t) \| \leq 2\sqrt{V_M} \). This bound implies there exist a set \( B = \{ y \in \mathbb{R}^n \mid \| y - \tilde{x}_d(t) \| \leq 2\sqrt{V_M} \} \) such that \( x(t) \in B, \forall t \). Therefore, the switching trajectory must penetrate a sufficient distance into \( \mathcal{F} \) to compensate for the error accumulation. The distance to compensate for error growth motivates the design of a cushion that ensures the re-entry of the actual states when the maximum dwell time is reached. Based on \( x_b(t) \), the cushion \( x_c(t) \in \mathbb{R}^n \) is selected as

\[
x_c(t) = x_b(t) + \Phi, \tag{23}
\]

where \( \Phi \in \mathbb{R}^n \), such that \( \| \Phi \| \geq 2\sqrt{V_M} \) and \( B \subseteq \mathcal{F} \). The general design rule is that the switching trajectory must
where\( H \parallel \text{composite error} \) and \( t \parallel \text{by respectively. Based on the desired error bound and threshold,} \) and to ensure a smooth and continuous

\[ S(\rho) = 6\rho^5 - 15\rho^4 + 10\rho^3 \] (24)

where \( \rho \in [0, 1] \) is the input parameter. Given the transition function in (24), the switching trajectory is designed as

\[
\bar{x}_d(t) = \begin{cases} 
H \left( S(\rho^0_i), x_b(t), x_e(t) \right), & t_i^q \leq t < t_i^k, \\
H \left( S(\rho^{i+1}_i), g(x_d, t), x_b(t) \right), & t_i^k \leq t < t_{i+1}^q, \\
H \left( S(\rho^{i+2}_i), g(x_d, t), g(x_d, t) \right), & t_{i+1}^q \leq t < t_{i+1}^k, \\
H \left( S(\rho^{i+3}_i), x_e(t), x_d(t) \right), & t_{i+1}^k \leq t < t_i^{q+1}, 
\end{cases}
\]

(25)

where \( H \left( S(\cdot), q(t), r(t) \right) \equiv S(\cdot)q(t) + [1 - S(\cdot)]r(t) \) for \( q(t), r(t) \in \mathbb{R}^n \), \( g : x_d \times \mathbb{R} \rightarrow \mathbb{R}^n \) gives the desired state on \( x_d \) at time \( t \), \( \rho^0_i, \rho^{i+1}_i, \rho^{i+2}_i \) and \( \rho^{i+3}_i \) are designed as \( \rho^0_i \equiv \frac{b - t_i^q}{\Delta t_i^q} \) and \( \rho^{i+1}_i \equiv \frac{t_{i+1}^{k+1} - t_i^q - \sum_{k=1}^{i+1} p_k \Delta t_k}{p_i + \Delta t_i} \), \( j \in \{0, 1, 2\} \), the weights used to partition the maximum dwell time are denoted by \( p_k \in [0, 1] \), and the corresponding partitions are denoted by \( t_{i+1}^{k+1} \). The final partition, \( t_{i+3}^q \), coincides with \( t_i^{q+1} \). To avoid singularity in \( \rho^0_i \) and to ensure a smooth and continuous switching trajectory, \( \Delta t_i^q \) must be arbitrarily lower bounded above zero (see Remark 2).

Remark 1. Other trajectories satisfying the dwell time conditions in Theorem 1 may also be implemented, such as the work in [34].

Remark 2. Lower bounding \( \Delta t_i^q \) by an arbitrary value, \( \alpha \in \mathbb{R}_{>0} \), does not violate Theorem 1 since the system is allowed to remain in the feedback region longer than the minimum dwell time, implying that \( \Delta t_i^q \leq \alpha \leq (t_i^q - t_i^r) \) holds. Other trajectory designs may not require \( \Delta t_i^q \) to be lower bounded.

VII. SIMULATION

A simulation is performed to illustrate the performance of the controller given intermittent loss of state feedback. Based on the system model given in [1], \( f(x(t), t) \) is selected as \( f(x(t), t) = Ax \) where \( A = 0.5I_3 \), and \( d(t) \) is drawn from a uniform distribution between \([0, 0.06]\) meters per second. The initial states and estimates are selected as \( x(0) = [0.1m \ 0.2m \ 0\ rads] \) and \( \hat{x}(0) = [0.2m \ 0.3m \ 0\ rads] \). The observer and the controller gains were selected as \( k_1 = 3I_3 \) and \( k_2 = 3I_3 \), respectively. The desired upper bound and lower threshold for the composite error \( ||z(t)|| \) are selected as 0.9 and 0.02 meters, respectively. Based on the desired error bound and threshold, the Lyapunov function bound and threshold are determined as \( V_M = 0.2025 \) and \( V_T = 1 \times 10^{-4} \).

The desired path \( x_d \) is selected as a circular trajectory with a radius of 2 meters centered at the origin. The boundary of the feedback region is selected as a circle with a 1-meter radius about the origin. The switching trajectory \( \bar{x}_d(t) \) were designed as described in Section VI and follows \( x_d \) at \( \frac{\pi}{T} \) radians per second, where the partition weights are selected as \( p_0 = 0, \ p_1 = 0.3, \ p_2 = 0.4, \ p_3 = 0.3 \). Figure 2 depicts the agent’s planar trajectory and shows that when the agent was inside the region with state feedback, both the estimation and tracking errors, \( ||e_1(t)|| \) and \( ||e_2(t)|| \), exo-
nentially converged. When the agent was outside the feedback region, the tracking error converged while the predictor error exhibited exponential divergence.

The average maximum and minimum dwell times between switches are 2.16 and 0.26 seconds, respectively. Based on the simulation result, the system is allowed to remain 8.23 times longer outside the feedback region than inside on average. Furthermore, 40% of the maximum dwell time is dedicated to following the desired path, which translates to 36% of the combined duration of the maximum and minimum dwell times per cycle.

In Figure 3, the composite error \( \| z(t) \| \) is shown. Figure 3 indicates that \( \| z(t) \| \) remained below 0.9 meters for all time and less than or equal to 0.02 (indicated by the black dashed line) by the end of each stable period, which demonstrates the robustness of the presented control design under the dwell time condition constraints and disturbances. Since an exact model of the system was used in this simulation, the resulting tracking error is bounded well below the maximum bound, and hence emphasizing the conservative nature of the Lyapunov analysis method.

VIII. EXPERIMENTS

In Section VIII, an experiment is performed to verify the theoretical results where a single integrator dynamics is used instead of the exact system model. The overall goal of the experiment is to represent a scenario where an unmanned air vehicle is tasked with following a path where feedback is not available (e.g., inside an urban canyon). Specifically, the objective is to demonstrate the boundedness of the tracking error \( e(t) \) through multiple cycles of switching between the feedback-available and unavailable regions based on the dwell time constraints established in Section VI. A Parrot Bebop 2.0 quadcopter is used as the unmanned air vehicle. The quadcopter is equipped with a 3-axis gyroscope, a 3-axis accelerometer, an ultrasound sensor, and an optical-flow sensor. The on-board sensors provide an estimate of the linear and angular velocities of the quadcopter at 5Hz. To control the quadcopter, the `bebop_autonomy` package developed by [38] is utilized to send velocity commands generated from an on-board computer running Robotic Operating System (ROS) Kinetic in Ubuntu 16.04. The communication link between the computer and the quadcopter is established through a WiFi channel at 5GHz.

A NaturalPoint, Inc. OptiTrack motion capture system is used to simulate a feedback signal and record the ground truth pose of the quadcopter at a rate of 120Hz. While the quadcopter is inside the feedback region, pose information from the motion capture system is directly used as feedback in the controller and update laws designed in Section VI. When the quadcopter operated outside of the feedback region, the pose feedback is discarded. During these times, the onboard velocity measurements are used to feedforward the state estimate. Although the OptiTrack system continue to record the pose of the quadcopter, the pose information is only used as ground truth for illustration purposes.

Utilizing the motion capture system, a circular region of available feedback is centered at the origin of the Euclidean world frame with a radius of 1 meter. Since torque level control authority is not available, single integrator dynamics, \( \dot{q}(t) = u(t) + d(t) \), are assumed for the quadcopter where \( q(t) = [ x(t) \ y(t) \ z(t) \ \alpha(t) ]^T \) and \( x(t), y(t), z(t), \alpha(t) \in \mathbb{R} \) are the 3-D Euclidean coordinates and yaw rotation of the quadcopter with respect to the inertial frame. The disturbance is assumed to be upper bounded as \( \ddot{d} = 0.035 \). To compensate for the disturbance, a high-gain robust controller is implemented to ensure a continuous control command. The controller and update law gains are selected as \( k_1 = 0.4I_3, \ k_2 = 0.6I_3, \) and \( \epsilon = 0.1 \). To regulate and match the actual velocity output to the control command, a low level PID controller is implemented.

The desired upper bound and lower threshold on \( \| z(t) \| \) are selected as 0.9 and 0.14 meters, respectively. Since single integrator dynamics are assumed for the quadcopter dynamic, a less conservative minimum dwell time condition can be derived (details are given in the Appendix). The desired path is defined as a circular path centered at the origin with a radius of 1.5 meters. Following the design method outlined in Section VI, a switching trajectory is designed to follow \( x_d \) with an angular velocity of \( \pi \) radians per second. To prevent the quadcopter from drifting out of the feedback region prematurely, an intermediate trajectory is designed to be \( x_{\text{int}}(t) = 0.7x_3(t) \) to replace \( x_3(t) \) in (25) as a safety measure.

The partitions for the maximum dwell time are selected as \( p_0 = 0, \ p_1 = 0.4, \ p_2 = 0.2, \ p_3 = 0.4 \).

Initially, the quadcopter is launched inside \( F \) along with the switching trajectory, which transitions between \( F \) and \( x_d \) over the prescribed time span. The experimental results demonstrate that the quadcopter is capable of intermittently leaving \( F \) to follow \( x_d \) for some period of time and then return to \( F \) consistently. The supplementary video accompanying this paper, available for download at http://ieeexplore.ieee.org, gives a recording of the experiment with the motion of the quadcopter and the switching trajectory projected on the floor. The overall path following plot, including the desired path, switching trajectory and actual states, is shown in Figure 4 where a total of 8 cycles of leaving and re-entering \( F \) occurred. During the periods when the quadcopter is outside the feedback region, large odometry drifts are apparent and the actual tracking error diverges as the dynamic models in Section VI indicate. Table II indicates the maximum and minimum dwell times for each cycle. On average, the quadcopter was allowed to reside approximately 6 times longer in \( F^c \) than \( F \), and 20% of which is dedicated to following \( x_d \). Specifically, the quadcopter is allowed 19.85 seconds in \( F^c \) and is required to remain in \( F \) for 3.31 seconds on average. Based on the partition weights of the maximum dwell time, Table II describes the partitions and the duration for each partition. During partition 1, \( \bar{x}_d(t) \) transitions from the \( x_3(t) \) to \( x_d \) where the partition weight was set to 50%. The relatively large partition allot more time in transition to yield a slower...
velocity profile, which produces less overshoot in the tracking performance. The distance between $x_d$ and $\mathcal{F}$ is also a major factor in distributing partition weights in the sense that the closer $x_d$ is to $\mathcal{F}$, the less time is required for transition and more time can be allocated to follow $x_d$.

To illustrate the stability of the control scheme, the Euclidean norm of the estimate tracking error, $e_1(t)$, and the estimation error, $e_2(t)$, are displayed in Figure 5 and 6. The estimate tracking error exhibits exponential stability regardless of feedback availability.

![Figure 5. Estimate tracking error $\|e_1(t)\|$. As indicated by the analysis, the estimate tracking error exhibits exponential stability regardless of feedback availability.](image)

Table I

| Cycle | Max. D. T. (s) | Min. D. T. (s) |
|-------|---------------|---------------|
| 0     | -             | 3.50          |
| 1     | 19.12         | 4.55          |
| 2     | 19.38         | 4.09          |
| 3     | 19.25         | 3.20          |
| 4     | 19.72         | 3.34          |
| 5     | 20.21         | 1.67          |
| 6     | 19.08         | 2.55          |
| 7     | 19.65         | 3.73          |
| 8     | 22.35         | 3.16          |
| Avg   | 19.85         | 3.31          |

Table II

| Cycle | Maximum dwell times (s) |
|-------|-------------------------|
|       | Part. 1 (40%) | Part. 2 (20%) | Part. 3 (40%) |
| 1     | 7.65            | 3.82          | 7.65          |
| 2     | 7.75            | 3.88          | 7.75          |
| 3     | 7.70            | 3.85          | 7.70          |
| 4     | 7.89            | 3.94          | 7.89          |
| 5     | 8.08            | 4.04          | 8.08          |
| 6     | 7.63            | 3.82          | 7.63          |
| 7     | 7.86            | 3.93          | 7.86          |
| 8     | 8.94            | 4.47          | 8.94          |

A novel method that utilizes a switched systems approach to ensure path following stability under intermittent state feedback is presented. The presented method relieves the requirement of state feedback at all times. State estimates are used in the tracking control to compensate for the intermittence of state feedback. A Lyapunov-based, switched systems analysis is used to develop maximum and minimum dwell time conditions to guarantee stability of the overall system. The dwell time conditions allow the desired path to be completely outside of the feedback region, and a switching trajectory is designed to bring the states back into the feedback region before the error growth exceeds a defined threshold. The candidate

IX. Conclusion

A novel method that utilizes a switched systems approach to ensure path following stability under intermittent state feedback is presented. The presented method relieves the requirement of state feedback at all times. State estimates are used in the tracking control to compensate for the intermittence of state feedback. A Lyapunov-based, switched systems analysis is used to develop maximum and minimum dwell time conditions to guarantee stability of the overall system. The dwell time conditions allow the desired path to be completely outside of the feedback region, and a switching trajectory is designed to bring the states back into the feedback region before the error growth exceeds a defined threshold. The candidate
Figure 6. Estimation error $\|e_2(t)\|$. As indicated by the analysis, the estimation error converges when $x(t) \in F$ and diverges when $x(t) \in F^c$.

Figure 7. Evolution of $\|z(t)\|$. The dash-dot (vertical) lines indicate the switching interface of minimum and maximum dwell times, and the dashed (horizontal) lines indicate the prescribed upper bound and lower threshold.

Figure 8. Actual tracking error $\|e(t)\|$. The dash-dot lines indicate the switching interface of minimum and maximum dwell times.

switching trajectory switches between the desired path and the feedback region using smootherstep transition functions. A simulation and an experiment were performed to illustrate the robustness of the control and trajectory design. Future research will focus on development of an approximate optimal control approach using adaptive dynamic programming concepts to yield approximately optimal results. Further efforts will also examine cases where the feedback region is time-varying or unknown.

APPENDIX

When using single integrator dynamics, $\dot{x}(t) = u + d(t)$, the resulting estimation error dynamics for the unstable sub-system is $\|\dot{e}_2(t)\| \leq \bar{d}$, and the corresponding Lyapunov-like function derivative is $\dot{V}_\sigma(t) \leq d\|e_2(t)\|$. By solving the ordinary differential equation for $\dot{e}_2(t)$, the estimation error $e_2(t)$ exhibits a linear growth that can be bounded as $e_2(t) \leq e_2(t^u_n) + \bar{d}(t - t^u_n)$. After substituting in the linear bound on $e_2(t)$, it follows that $\dot{V}_\sigma(t) \leq \bar{d} \|e_2(t^u_n)\| + \bar{d}^2(t - t^u_n)$, and solving the ordinary differential equation yields $V_\sigma(t) \leq \frac{1}{2} \bar{d}^2(t - t^u_n)^2 + \bar{d} \|e_2(t^u_n)\| (t - t^u_n) + V_\sigma(z(t^u_n))$. After imposing $V_\sigma(t) \leq V_M$ as the upper bound constraint, the maximum dwell time can be derived by solving the quadratic equation and taking the positive root as

$$\Delta t^u_i \leq \frac{\sqrt{\|e_2(t^u_n)\|^2 - 2(V_\sigma(z(t^u_n)) - V_M) - \|e_2(t^u_n)\|^2}}{\bar{d}}.$$

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