Abstract

Darmstadt ν oscillations in decay of radioactive ion can only come from initial state wave function. Causality forbids any influence on transition probability by detection of ν or final state interference after decay. Energy-time uncertainty allows two initial state components with different energies to decay into combination of two orthogonal states with same energy, different momenta and different ν masses. Final amplitudes completely separated at long times have broadened energy spectra overlapping at short times. Their interference produces oscillations between Dicke superradiant and subradiant states having different transition probabilities. Repeated monitoring by interactions with laboratory environment at regular time intervals and same space point in laboratory collapses wave function and destroys entanglement. First-order time dependent perturbation theory gives probability for initial state decay during small interval between two monitoring events. Experiment measures momentum difference between two contributing coherent initial states and obtains information about ν masses without detecting ν. Simple model relates observed oscillation to squared ν mass difference and gives value differing by less than factor of three from values calculated from KAMLAND experiment. Monitoring simply expressed in laboratory frame not easily transformed to other frames and missed in Lorentz-covariant descriptions based on relativistic quantum field theory.
I. INTRODUCTION

A. Some basic questions

In the Darmstadt experiment [1] an initial radioactive “mother” ion enters a storage ring and eventually decays into a “daughter” ion and an electron neutrino which is a coherent mixture of at least two mass eigenstates with different masses. It moves with a relativistic velocity reversed by an accelerating magnetic field each time it has moved half way around the ring. Monitoring at regular intervals confirms that it has not yet decayed, thereby collapsing the wave function or destroying entanglement.

The following questions arise in any serious analysis of the decay rate of this mother ion.

1. How is a final state which has a coherent mixture of different $\nu$ mass eigenstates produced in a momentum-conserving weak interaction?

2. What is the initial state of an ion which decays by a momentum-conserving weak interaction into two states with different momenta?

3. The initial state must be a wave packet containing components with different momenta. How do these behave in going through complicated orbits in a storage ring?
   - Do these components have different time dilatations, as in the twin paradox?
   - What is the proper time and the time dilatation observed in the laboratory for an ion going through these accelerations and reversals of relativistic velocities?
   - How do the relative phases of different components of the initial wave function change in the complicated motion around the storage ring?
   - How is the repeated collapse of the wave function taken into account?

These questions are unfortunately completely ignored in most treatments of Darmstadt oscillations which also generally violate causality [2]. Any measurements or interference effects in the final state after decay cannot affect the transition probability before decay.
In this paper we attempt to understand and answer these questions and show that they are crucial for a reliable description of possible oscillations in the decay rate.

B. What is missed in most theoretical analyses of Darmstadt experiment

1. Theoretical papers saying this is impossible do not understand what is really measured.

2. They do not note that causality forbids any influence on the decay rate before decay by final state interference effects or the possible detection of the final $\nu$ after decay.

3. A crucial interaction with the environment is missed in nearly all papers
   - Experiment monitors initial state at same point and different times in laboratory
   - In all other frames these are different space points and different times
   - This interaction with environment is missed in all covariant treatments
   - They miss the collapse of the wave function at each monitoring

4. They do not consider the relativistic dynamics of motion in the storage ring
   - Initial ion undergoes relativistic time dilatation as in famous twin paradox.
   - Components of an ion wave packet having slightly different momenta in the laboratory have slightly different lifetimes in the laboratory system
   - The initial state is accelerated between relativistic velocities in opposite directions
   - Do we need general relativity to get proper time dilatation and decay probability?

C. Some attempts to answer the basic questions

The probability $P_i(t)$ that the initial “Mother” ion is still in its initial state at time $t$ and not yet decayed satisfies an easily solved differential equation,

$$\frac{d}{dt} P_i(t) = -W(t) P_i(t); \quad \frac{d}{dt} \log(P_i) = -W(t); \quad P_i(t) = e^{-\int W(t)dt} \quad (1.1)$$
where $W(t)$ denotes the transition probability per unit time at time $t$. If $W(t)$ is independent of time eq. (1.1) gives an exponential decay.

The initial “Mother” ion wave packet denoted by $|i(t)\rangle$ contains components with different momenta. Two components of the initial state denoted by $|i_1(t)\rangle$ and $|i_2(t)\rangle$ with slightly different unperturbed energies $E$ and $E + \delta E$ can both decay into the same final state. Their time development before the decay is written

$$|i_1(t)\rangle + |i_2(t)\rangle = e^{iH_0 t} [|i_1(0)\rangle + |i_2(0)\rangle] = e^{iEt} [|i_1(0)\rangle + e^{i\delta Et} |i_2(0)\rangle] \quad (1.2)$$

where the time $t = 0$ is defined as the time of entry into the apparatus and $H_0$ denotes the unperturbed Hamiltonian describing the motion of this wave packet in the electromagnetic fields constraining its motion in a storage ring.

The time between successive monitoring of the state of the initial ion is so short that the Fermi Golden Rule gives its decay probability during this interval. The transition from the initial state (1.2) to a final state denoted by $|f\rangle$ is

$$W(t) = \frac{2\pi}{\hbar} |\langle f | \text{T} e^{iEt} [i_1(0)\rangle + e^{i\delta Et} |i_2(0)\rangle] |^2 \rho(E_f) \approx \frac{4\pi}{\hbar} |\langle f | \text{T} |i(0)\rangle|^2 [1 + \cos(\delta E)t] \rho(E_f) \quad (1.3)$$

where $\text{T}$ is the transition operator, $\rho(E_f)$ is the density of final states and we have set $

\langle f | \text{T} | i_1(0) \rangle \approx \langle f | \text{T} | i_2(0) \rangle \equiv \langle f | \text{T} | i(0) \rangle$.

Equation (1.3) shows that the decay probability will oscillate in time with a frequency $\delta E$. This explains the occurrence of Darmstadt oscillations. But the evaluation of the oscillation frequency $\delta E$ depends upon the energy levels of the unperturbed Hamiltonian $H_0$ which includes the fields that create the orbit in the storage ring.

$W(t)$ depends upon the unperturbed propagation of the initial state before the time $t$ where its motion in the storage ring is described by classical electrodynamics. Any departure from exponential decay must come from the evolution in time of the initial unperturbed state. This can change the wave function at the time of the decay and therefore the value of the transition matrix element. What happens after the decay cannot change the wave function.
before the decay. Whether or not and how the final neutrino is detected cannot change the decay rate.

Here the transition probability depends upon propagation of the initial state during time $t$ between the entry of the ion into the apparatus and the time of the decay.

Although time-dependent perturbation theory might suggest the presence of a decay amplitude before the observed decay, the continued observation of the initial ion before the decay collapses the wave function and rules out any influence of any final state amplitude on the decay process. The time dependence of the decay depends only on propagation of the initial state and is independent of the final state amplitude created only at the decay point. Thus there is no violation of causality.

In the remainder of this paper we discuss the background physics leading to eq.(1.3) and show that a crude toy model for the oscillation gives a value for the squared neutrino mass difference which differs by less than a factor of three from the experimental value obtained from neutrino oscillations.

II. THE BASIC QUANTUM MECHANICS

A. The need for violation of energy conservation in producing an oscillating $\nu$

The electron-neutrino $\nu_e$ produced immediately after the weak interaction is a a mixed coherent state of $\nu$’s with different masses. This mixed coherent state cannot be created in a weak decay if energy and momentum are conserved. Neutrino oscillations can occur only if energy conservation is violated in the weak decay producing the $\nu$.

B. Darmstadt GSI experiment observes radioactive ion circulating storage ring

Oscillations in decay rate give information about $\nu$ masses without detecting $\nu$.

1. Standard weak interaction theory tells us that even if the $\nu$ is not detected
• The $\nu$ created when an electron absorbs a W boson is a weak flavor eigenstate $\nu_e$
• An electron disappears. Theory requires it to turn into an electron neutrino $\nu_e$

2. The weak flavor eigenstate $\nu_e$ is a coherent linear combination of mass eigenstates.
  • A single weak eigenstate $\nu_e$ is split by the mass difference into mass eigenstates.
  • Measurement of the $\nu_e$ component at short times shows oscillations.

3. Like electron polarized in x direction with $\sigma_x = 1$ entering magnetic field in z direction
  • Stern-Gerlach experiment splits electron wave into two components with $\sigma_z = \pm 1$.
  • Short time after entering magnetic field spin precesses about z-axis
  • Measurement of $\sigma_x$ at short time shows oscillations,

4. Implications of causality
  • What happens to $\nu$ after it is produced cannot affect Darmstadt experiment.
  • Any detection of $\nu$ or $\nu$ oscillations following decay cannot influence decay rate.
  • Subsequent measurements can separate the neutrino into its mass eigenstates,
  • Measurements on final state cannot affect decay process that creates $\nu$.
  • A full analysis of Darmstadt oscillations can only be based on the properties of the ion wave function before the decay.

C. A “Which-Path” experiment with Dicke superradiance”

The initial single-particle wave packet has components with definite energies and momenta. For simplicity take two $\nu$ mass eigenstates, denoted by $\nu_1$ and $\nu_2$. There are two initial states of the radioactive ion with different energies and momenta which can produce this $\nu_e$, one via the neutrino $\nu_1$ and one via the neutrino $\nu_2$. We now note that:
1. At short times the final Breit-Wigner energy spectra are broadened.

2. Two broadened energy spectra with different centers can overlap and interfere.

3. In weak decay each of two components of initial state can create final state with $\nu_e$.
   
   - Each of these two components creates a different mass eigenstate, $\nu_1$ or $\nu_2$.
   
   - Two initial state components with different momenta and energies can decay into coherent state with two components having same momentum difference but same energy.
   
   - No measurement of the final state can determine which of the two components of the initial state created the final state.
   
   - Therefore this is a “which-path” or “two-slit” experiment in momentum space.
   
   - The Darmstadt oscillations are the “interference fringes” of this experiment.

Dicke has shown that whenever several initial states can decay into the same final state one linear combination called “superradiant” has maximum constructive interference in the decay, and an enhanced or “speeded up” decay rate. When there are only two states, the state orthogonal to the superradiant state is called subradiant and has a suppressed transition matrix element.

The ion created in the Darmstadt experiment is a wave packet containing two states that can decay into the same final state. Since these states have different energies, their relative phase changes with time and they oscillate between the speeded-up superradiant and slowed-down subradiant states. These oscillations in the decay probability are seen in the Darmstadt experiment.
III. MEASURING NEUTRINO MASSES WITHOUT DETECTING NEUTRINO

A. A missing mass experiment

Obtaining the values of the $\nu$ masses is possible without detecting the $\nu$ in a “missing mass” experiment. If the initial mother ion has a sharp energy spectrum and the daughter is detected in a high resolution spectrometer, energy and momentum conservation determine the $\nu$ mass. The daughter energy spectrum will have peaks corresponding to each $\nu$ mass.

But if the $\nu$ mass is determined, there can be no oscillations between final states with different masses. The experiments that observe $\nu$ oscillations cannot be missing mass experiments. Something must prevent the use of conservation laws from determining the $\nu$ masses.

The Darmstadt experiment does not have a sharp initial energy spectrum nor a high resolution detector. The experiment shows oscillations in time of the transition probability interpreted as coming from neutrino mass differences.

This leads to two questions:

1. What prevents the use of conservation laws from determining the $\nu$ masses in conventional $\nu$ oscillation experiments?

2. What information about $\nu$ masses is available in the Darmstadt experiment without detecting the $\nu$?

B. A more realistic experiment

To produce oscillations the final state of $\nu$’s emitted from the weak decay must be a linear combination of states with neutrinos having different masses and therefore different momenta and/or energies. If oscillations are detected in a macroscopic quantum-detector, all coherence between states of different energies is destroyed. The oscillating neutrino must be produced by interference between states with different momenta and the same energy
But in an initial one-particle wave packet all components with different momenta have different energies. Thus energy conservation must be violated in any experiment producing neutrino oscillations from a momentum-conserving weak decay.

If momentum is conserved in the interaction, the initial state must also have coherent components with the same momentum difference as the final state. This momentum difference can provide information in the initial state on masses of neutrinos even when the neutrino is not detected, as in a missing mass experiment.

IV. WHAT IS ACTUALLY MEASURED IN THE EXPERIMENT?

The basic physics of the Darmstadt experiment is not simple. The initial state is a radioactive ion moving in a storage ring. The time of its decay is not measured directly. What is actually measured is not generally appreciated.

1. The observation of the decay

   • The ion is monitored at regular intervals during passage around the storage ring.
   • Each monitoring collapses the wave function (or destroys entanglement phase).
   • Time in the laboratory frame is measured at each wave function collapse.
   • The the decay of the initial state is observed by the disappearance of the ion between successive monitoring.

2. This cannot be a missing mass experiment. A conservation law must be violated

   • If energy and momentum are conserved this is a missing mass experiment
   • There are no \( \nu \) oscillations if the \( \nu \) mass is determined by conservation laws

3. A state with an oscillating \( \nu \) has two components with different \( \nu \) masses

   • The \( \nu \) state has two components with different momenta but the same energy
   • The momentum difference is determined by the \( \nu \) masses
• Measuring the momentum difference gives information about $\nu$ masses

• Momentum is conserved in the transition; energy is not

• The initial wave packet has two components with the same momentum difference

• Two components of the initial one-particle wave function with different momenta must have different energies

• Measuring the energy difference gives information about $\nu$ masses

4. Energy conservation is violated by energy-time uncertainty

• Short time between successive monitoring gives energy uncertainty

• At short times components of the initial wave function with different energies can decay into the same final state with the same energy

• Breit-Wigner amplitudes for transition have broadened widths at short times

• Broadened amplitudes with two different central energies can overlap and interfere at short times

5. The role of Dicke superradiance [4]

• Two components of the initial state with different energies can decay into the same $\nu_e$ final state

• The decay amplitude is the coherent sum of the amplitudes for the transitions from different components of the initial state to the same final state

• Dicke has shown that superradiance can arise when two different initial states can decay to the same final state

• The state for which the two amplitudes have maximum constructive interference is called the superradiant state

• Since these two amplitudes have different energies their relative phase changes linearly with time
• The relative phase change with time produces an oscillation between the superradiant state and the orthogonal “subradiant” state

6. The decay probability in the short time between successive monitorings is given by the Fermi “Golden Rule”

• The transition amplitude depends upon the initial state wave function

• The relative phases between transition amplitudes for each momentum component of the ion wave function oscillate between superradiance and subradiance in propagation through the storage ring

• These phase changes produce oscillations in decay probability

• The oscillations can give information about $\nu$ masses without detecting the $\nu$

V. THE BASIC PARADOX OF NEUTRINO OSCILLATIONS

A. Why do neutrinos oscillate? Textbooks don’t tell you

A $\nu$ at rest with definite flavor is a coherent mixture of energy eigenstates. Interference between these states produces oscillations in time between different flavors. Textbooks tell us $\nu$’s oscillate as coherent mixtures of states with different masses. They don’t tell us how such a mixed coherent state can be created and detected.

1. No experiment has ever seen a $\nu$ at rest

2. Detectors in experiments observing $\nu$ oscillations do not measure time

3. Detectors destroy all interference between states with different energies

B. Coherence in $\pi - \mu$ decay

The neutrinos emitted in $\pi - \mu$ decay must be linear combinations of $\nu$ mass eigenstates with a definite relative magnitude and phase to produce only muons and no electrons in
detectors a short distance from the source. If the initial pion in $\pi - \mu$ decay has a sharp momentum, the $\nu$'s emitted with different masses have different momenta and there is no coherence or interference between amplitudes for two $\nu$ mass eigenstates and no cancellation of the electron transition in the detector. The initial pion wave packet must have pairs of components with just the right momentum difference to produce the two $\nu$ mass eigenstates coherently. The strength of the transition depends on the relative magnitudes and phases of these components. The existence of $\nu$ oscillations shows that the final state produced in a weak interaction contains pairs of coherent states with a momentum difference related to the $\nu$ mass difference. Momentum is conserved in the weak interaction; therefore the initial state must also contain pairs of coherent states with the same momentum difference. Since the amplitudes produced by these pairs contribute coherently to the transition, the transition amplitude depends upon their relative phase. These initial pairs have different energies; their relative phase changes with time and oscillates between constructive and destructive interference with a period that depends upon the momentum difference. Measuring this oscillation can give the mass squared difference between $\nu$ mass eigenstates even when the $\nu$ is not detected.

C. The problem

1. The original Brookhaven experiment [5] detecting neutrinos showed a $\nu$ emitted in a $\pi \to \mu \nu$ decay entering a detector and producing only muons and no electrons.

2. The $\nu$ enters detector as coherent mixture of mass eigenstates with right relative magnitudes and phases to cancel the amplitude for producing an electron at the detector.

3. A macroscopic detector destroys all coherence between different energies

4. $\nu$ wave function must have states with different masses and momenta; same energy

5. In initial one-particle state components with different momenta have different energies.
6. Brookhaven experiment [5] can’t exist if energy and momentum are conserved

D. The Solution

1. If momentum is conserved in the interaction, violation of energy conservation needed.

2. Energy-time uncertainty in the laboratory frame allows components of initial wave packet with different energies to produce same final $\nu_e$ with the same single energy.

3. Transition probability depends on relative phase between two components

E. Darmstadt application

Radioactive ion circulates in storage ring before decay [1]

1. Relative phase and transition probability change in propagation through storage ring.

2. Phase changes produce oscillations in decay probability

3. Oscillations can give information about $\nu$ masses without detecting the $\nu$

F. A simple example of resolution of the paradox

We now show how two initial states with energies $E_f - \delta$ and $E_f + \delta$ can decay into the same final state with energy $E_f$. Time dependent perturbation theory shows violation of energy conservation by energy-time uncertainty in sufficiently short times [6].

The time dependent amplitude $\beta_f(E_i)$ for the decay from a single initial state with energy $E_i$ into a final state with a slightly different energy $E_f$ is

$$\frac{\beta_f(E_i)}{g} \cdot (E_i - E_f) = \left[e^{-i(E_i - E_f)t} - 1\right] \cdot e^{-2iE_f t} \quad (5.1)$$

where we have set $\hbar = 1$ and $g$ is the interaction coupling constant.
We now generalize this result to the case of an initial wave packet with two components having energies $E_i \pm \delta$ and define $x \equiv E_i - E_f$

$$e^{2iE_ft} \frac{g}{2} \cdot [\beta_f(E_f + x - \delta) + \beta_f(E_f + x + \delta)] = \left[ e^{-i(x-\delta)t} - 1 \right] + \left[ e^{-i(x+\delta)t} - 1 \right]$$ (5.2)

The square of the transition amplitude denoted by $T$ is then given by

$$\frac{|T|^2}{g^2} \equiv \left[ \beta_f(E_f + x - \delta) + \beta_f(E_f + x + \delta) \right]^2$$ (5.3)

$$\frac{|T|^2}{g^2} = 4 \cdot \left[ \frac{\sin^2[(x-\delta)t/2]}{(x-\delta)^2} + \frac{\sin^2[(x+\delta)t/2]}{(x+\delta)^2} + \frac{2\sin^2[\delta t/2]}{x^2} + 2\sin^2[xt/2]\cos[\delta t] - \sin^2(\delta t) \right]$$ (5.4)

If the time is sufficiently short so that the degree of energy violation denoted by $x$ is much larger than the energy difference $\delta$ between the two initial states, $x \gg \delta$ and

$$x \gg \delta; \quad |T|^2 \approx 8g^2 \cdot \left[ \frac{\sin^2[xt/2]}{x^2} \right] \cdot [1 + \cos \delta t]$$ (5.5)

The transition probability is given by the Fermi Golden Rule. We integrate the the square of the transition amplitude over $E_i$ or $x$, introduce the density of final states $\rho(E_f)$ and assume that $\delta$ is negligibly small in the integrals.

$$\int_{-\infty}^{+\infty} |T|^2 \rho(E_f) dx \approx \int_{-\infty}^{+\infty} 8g^2 \cdot \left[ \frac{\sin^2[xt/2]}{x^2} \right] \cdot [1 + \cos \delta t] \rho(E_f) dx$$ (5.6)

The transition probability per unit time $W$ is then

$$W \approx 4g^2 \cdot \int_{-\infty}^{+\infty} du \left[ \frac{\sin^2 u}{u^2} \right] \cdot \rho(E_f) [1 + \cos(\delta t)] \cdot t = 4\pi g^2 \rho(E_f)$$ (5.7)

The interference term between the two initial states is seen to be comparable to the direct terms when $\cos(\delta t) \approx 1$; i.e. when the time $t$ is so short that the energy uncertainty is larger than the energy difference between the two initial states.

This example shows in principle how two initial states with a given momentum difference can produce a coherent final state containing two neutrinos with the same energy and the
given momentum difference. A measurement of the momentum difference between the two initial states can provide information on neutrino masses without detecting the neutrino.

In this simple example the amplitudes and the coupling constant $g$ are assumed to be real. In a more realistic case there is an additional extra relative phase between the two terms in eq.(5.2) which depends upon the initial state wave function. In the GSI experiment [1] this phase varies linearly with the time of motion of the initial ion through the storage ring. This phase variation can produce the observed oscillations.

VI. A SIMPLIFIED MODEL FOR DARMSTADT OSCILLATIONS

A. The initial and final states for the transition matrix

The initial radioactive “Mother” ion is in a one-particle state with a definite mass moving in a storage ring. There is no entanglement [7] since no other particles are present. To obtain the required information about this initial state we need to know the evolution of the wave packet during passage around the storage ring. This is not easily calculated. It requires knowing the path through straight sections, bending sections and focusing electric and magnetic fields. We neglect these complications in the present calculation and assume an approximation in which the relative phase $\delta \phi(E)$ between amplitudes having energies $E$ and $E + \delta E$ changes with time with time $t$ as

$$\delta \phi(E) \approx \delta E \cdot t$$

(6.1)

The final state is a “Daughter” ion and a $\nu_e$ neutrino, a linear combination of several $\nu$ mass eigenstates. This $\nu_e$ is a complicated wave packet containing different masses, energies and momenta. The observed oscillations arise only from $\nu$ components with different masses and different momenta and/or energies.
B. Kinematics for a simplified two-component initial state.

We first consider the transition [8,9] for an initial state having momentum $\vec{P}$ and energy $E$. The final state has a recoil ion with momentum denoted by $\vec{P}_R$ and energy $E_R$ and a neutrino with energy $E_\nu$ and momentum $\vec{p}_\nu$. If both energy and momenta are conserved,

$$E_R = E - E_\nu; \quad \vec{P}_R = \vec{P} - \vec{p}_\nu; \quad M^2 + m^2 - M_R^2 = 2EE_\nu - 2\vec{P} \cdot \vec{p}_\nu$$  \hspace{1cm} (6.2)

where $M$, $M_R$ and $m$ denote respectively the masses of the mother and daughter ions and the neutrino. We now consider a simplified two-component initial state for the “mother” ion having two components denoted by $|\vec{P}\rangle$ and $|\vec{P} + \delta \vec{P}\rangle$ having momenta $\vec{P}$ and $\vec{P} + \delta \vec{P}$ with energies $E$ and $E + \delta E$. The final state denoted by $|f\rangle$ has two components having neutrino momenta $p_\nu$ and $p_\nu + \delta p_\nu$ with energies $E_\nu$ and $E_\nu + \delta E_\nu$ together with a recoil ion having the same momentum and energy for both components. The changes in these variables produced by a small change $\Delta(m^2)$ in the squared neutrino mass are seen from eq. (6.2) to satisfy the relation

$$\frac{\Delta(m^2)}{2} = E\delta E_\nu + E_\nu\delta E - P\delta p_\nu - p_\nu\delta P = -E\delta E \cdot \left[ 1 - \frac{\delta E_\nu}{\delta E} + \frac{p_\nu}{P} - \frac{E_\nu}{E} \right] \approx -E\delta E$$  \hspace{1cm} (6.3)

where we have neglected transverse momenta and noted that momentum conservation in the transition requires $P\delta p_\nu = P\delta P = E\delta E$, $E$ and $P$ are of the order of the mass $M$ of the ion and $p_\nu$ and $E_\nu$ are much less than $M$. To enable coherence the two final neutrino components must have the same energy, i.e. $\delta E_\nu = 0$. Then eq.(6.3) requires $\delta E \neq 0$ and we are violating energy conservation.

Equation (6.3) relates $\delta E$ to the difference between the squared masses of the two neutrino mass eigenstates. Thus the relative phase $\delta \phi(E)$ at a time $t$ between the two states is given in the approximation (6.1)

$$E \cdot \delta E = -\frac{\Delta(m^2)}{2}; \quad \delta \phi(E) \approx -\delta E \cdot t = -\frac{\Delta(m^2)}{2E} \cdot t = -\frac{\Delta(m^2)}{2\gamma M} \cdot t$$  \hspace{1cm} (6.4)

where $\gamma$ denotes the Lorentz factor $E/M$. 

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C. Dicke superradiance and subradiance in the experiment

Consider the transition in the simplified two-component model satisfying equation. (6.4)

The final state denoted by \( |f(E_\nu)\rangle \) has a “daughter” ion and an electron neutrino \( \nu_e \) which is a linear combination of two neutrino mass eigenstates denoted by \( \nu_1 \) and \( \nu_2 \) with masses \( m_1 \) and \( m_2 \). To be coherent and produce oscillations the two components of the final wave function must have the same energy \( E_\nu \) for the neutrino and the same momentum \( \vec{P}_R \) and energy \( E_R \) for the “daughter” ion.

\[
|f(E_\nu)\rangle \equiv |\vec{P}_R; \nu_e(E_\nu)\rangle = |\vec{P}_R; \nu_1(E_\nu)\rangle \langle \nu_1 | \nu_e \rangle + |\vec{P}_R; \nu_2(E_\nu)\rangle \langle \nu_2 | \nu_e \rangle \quad (6.5)
\]

where \( \langle \nu_1 | \nu_e \rangle \) and \( \langle \nu_2 | \nu_e \rangle \) are elements of the neutrino mass mixing matrix, commonly expressed in terms of a mixing angle denoted by \( \theta \).

\[
\cos \theta \equiv \langle \nu_1 | \nu_e \rangle; \quad \sin \theta \equiv \langle \nu_2 | \nu_e \rangle; \quad |f(E_\nu)\rangle = \cos \theta |\vec{P}_R; \nu_1(E_\nu)\rangle + \sin \theta |\vec{P}_R; \nu_2(E_\nu)\rangle \quad (6.6)
\]

Since the states \( \nu_1(E_\nu) \) and \( \nu_2(E_\nu) \) have the same energies and different masses, they have different momenta. After a very short time two components with different initial state energies can decay into a final state which has two components with the same energy and a neutrino state having two components with the same momentum difference \( \delta \vec{P} \) present in the initial state.

The momentum conserving transition matrix elements between the two initial momentum components to final states with the same energy and momentum difference \( \delta \vec{P} \) are

\[
\langle f(E_\nu) | T | \vec{P} \rangle = \cos \theta \langle \vec{P}_R; \nu_1(E_\nu) | T | \vec{P} \rangle; \quad \langle f(E_\nu) | T | \vec{P} + \delta \vec{P} \rangle = \sin \theta \langle \vec{P}_R; \nu_2(E_\nu) | T | \vec{P} + \delta \vec{P} \rangle \quad (6.7)
\]

The Dicke superradiance [4] analog here is seen by defining superradiant and subradiant linear combinations of these states

\[
|Sup(E_\nu)\rangle \equiv \cos \theta |P\rangle + \sin \theta |P + \delta P\rangle; \quad |Sub(E_\nu)\rangle \equiv \cos \theta |P + \delta P\rangle - \sin \theta |P\rangle \quad (6.8)
\]

The transition matrix elements for these two states are then
\[
\frac{\langle f(E_\nu) | T | Sup(E_\nu) \rangle}{\langle f | T | P \rangle} = [\cos \theta + \sin \theta]; \quad \frac{\langle f(E_\nu) | T | Sub(E_\nu) \rangle}{\langle f | T | P \rangle} = [\cos \theta - \sin \theta] \quad (6.9)
\]

where we have neglected the dependence of the transition operator \( T \) on the small change in the momentum \( P \). The squares of the transition matrix elements are

\[
| \langle f(E_\nu) | T | Sup(E_\nu) \rangle |^2 = [1 + \sin 2\theta] \quad | \langle f(E_\nu) | T | Sub(E_\nu) \rangle |^2 = [1 - \sin 2\theta] \quad (6.10)
\]

For maximum neutrino mass mixing, \( \sin 2\theta = 1 \) and

\[
| \langle f(E_\nu) | T | Sup(E_\nu) \rangle |^2 = 2 | \langle f | T | P \rangle |^2; \quad | \langle f(E_\nu) | T | Sub(E_\nu) \rangle |^2 = 0 \quad (6.11)
\]

This is the standard Dicke superradiance in which all the transition strength goes into the superradiant state and there is no transition from the subradiant state.

Thus from eq. (6.8) the initial state at time \( t \) varies periodically between the superradiant and subradiant states. The period of oscillation \( \delta t \) is obtained by setting \( \delta \phi(E) \approx -2\pi \),

\[
\delta t \approx \frac{4\pi \gamma M}{\Delta(m^2)}; \quad \Delta(m^2) = \frac{4\pi \gamma M}{\delta t} \approx 2.75\Delta(m^2)_{exp} \quad (6.12)
\]

where the values \( \delta t = 7 \) seconds and \( \Delta(m^2) = 2.22 \times 10^{-4} eV^2 = 2.75\Delta(m^2)_{exp} \) are obtained from the GSI experiment and neutrino oscillation experiments [10].

**VII. CONCLUSIONS**

Neutrino oscillations cannot occur if the momenta of all other particles participating in the reaction are known and momentum and energy are conserved. A complete description of the decay process must include the interaction with the environment and violation of energy conservation. A new oscillation phenomenon providing information about neutrino mixing is obtained by following the initial radioactive ion moving in a storage ring before the decay. The ion is monitored at regular intervals, showing that the “mother” ion has not decayed and collapsing the wave function. The decay between two successive monitorings is detected by the disappearance of the mother ion. The probability of the decay during this interval is given by the Fermi golden rule. The dependence of the decay probability changes
between successive monitorings and causes oscillations in time. Difficulties introduced in conventional $\nu$ experiments by tiny neutrino absorption cross sections and very long oscillation wave lengths are avoided. Measuring each decay enables every $\nu$ event to be observed and counted without the necessity of observing the $\nu$ via the tiny absorption cross section. The confinement of the initial ion in a storage ring enables long wave lengths to be measured within the laboratory.

The theoretical value (6.12) obtained with minimum assumptions and no fudge factors is in the same ball park as the experimental value obtained from completely different experiments. Better values obtained from better calculations can be very useful in determining the masses and mixing angles for neutrinos.

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