Deep Electroproduction of Photons and Mesons on the Deuteron

F. Cano and B. Pire

1 DAPNIA/SPhN, CEA–Saclay, F-91191 Gif-sur-Yvette Cedex, France.
2 CPhT, École Polytechnique, F-91128 Palaiseau Cedex, France; UMR C7644 of CNRS.

July 17, 2003

Abstract. We study deeply virtual Compton scattering and deep exclusive meson electroproduction on a deuteron target. We model the Generalized Quark Distributions in the deuteron by using the impulse approximation for the lowest Fock-space state on the light-cone. We study the properties of the resulting GPDs, and verify that sum rules violations are quite small in the impulse approximation. Numerical predictions are given for the unpolarized cross sections and polarization asymmetries for the kinematical regimes relevant for JLab experiments and for HERMES at HERA. We conclude that the signal of coherent scattering on the deuteron is comparable to the one on the proton at least for low momentum transfer, providing support to the feasibility of the experiments. The short distance structure of the deuteron may thus be scrutinized in the near future.

PACS. 2 4.85.+p, 12.38.Bx, 25.30.-e

1 Introduction

The study of hard exclusive processes, such as deeply Virtual Compton Scattering (DVCS)
\[ eA \rightarrow e'\gamma A' \]  
(1)
and deep exclusive meson electroproduction (DEMP)
\[ eA \rightarrow e'MA' \]  
(2)
where A is a hadron (usually a nucleon, here a deuteron), and M a meson (usually a \( \rho \) or a \( \pi \)) or a pair of mesons (of relatively small invariant mass) in the kinematical domain of a large momentum transfer \( Q^2 \) between the leptons but a small momentum transfer (t) between the hadrons, has been recently demonstrated to open the possibility of obtaining a quite complete picture of the hadronic structure. The information which can be accessed through these experiments is encoded by the Generalized Parton Distributions (GPDs) [1,2] (for recent reviews see [3]), which give in particular information on the transverse location of quarks in the hadrons [4]. Recent measurements of the azimuthal dependence of the beam spin asymmetry in DVCS [5,6] have provided experimental evidence to support the validity of the formalism of GPDs and the underlying QCD factorization of short-distance and long-distance dominated subprocesses.

The theoretical arguments used in deriving factorization theorems in QCD for the nucleon [7] target case can be applied to the deuteron case as well, and therefore one can develop the formalism of GPDs for the deuteron [8]. From the theoretical viewpoint, it is the simplest and best known nuclear system and represents the most appropriate starting point to investigate hard exclusive processes off nuclei [9]. On the other hand, these processes could offer a new source of information about the partonic degrees of freedom in nuclei, complementary to the existing one from deep inelastic scattering. Experimentally, deuteron targets are quite common and as a matter of fact, DVCS experiments are being planned or carried out at facilities like CEBAF at JLab and HERMES at HERA, where some data have already been released [10]. One should of course distinguish the case where the deuteron serves merely as a source of slightly bound protons and neutrons from the case where the deuteron acts as a single hadron. In the former case, the scattering is incoherent and the deuteron will break up during the reaction. In the latter case, to which we devote our study, the deuteron stays intact after the scattering. The fact that this occurs in a non negligible fraction of events is not evident to everybody, since it is usual, but incorrect, to mix the concepts of hard and destructive reactions. Indeed, as estimates given below will show, the very nature of deep electroproduction in the forward region is that the target is not violently shattered by the hard probe. The fragile nature of the deuteron thus does not prevent it from staying intact. This picture should of course be experimentally tested through the comparison of rates for coherent and incoherent electroproduction. The need for a deuteron recoil detector is primordial in this respect.

The paper is organized as follows: In section 2 we remind the reader of the formalism of Generalized Parton Distributions for spin-1 targets in general and the
deuteron in particular. In section 3 we explain in detail the construction of the impulse approximation to evaluate the helicity amplitudes and in section 4 we derive the deuteron GPDs from the helicity amplitudes and study the properties and implications with a numerical model. In section 5 we give the useful formulae for calculating the DVCS cross section. In section 6 we show our numerical estimates for the usual observables in the DVCS case and comment on the feasibility of experiments. In section 7 we examine the electroproduction of mesons. Throughout the paper we will limit ourselves to the quark sector of the GPDs, which is a good approximation provided the Bjorken variable $x_{Bj}$ is not too small. This variable is defined as usual as $x_{Bj} = \frac{Q^2}{2\not{P} \cdot \not{Q}}$, i.e., it is given in lab frame by $x_{Bj} = \frac{Q^2}{2M}$, where $M$ is the deuteron mass and $\nu$ the virtual photon energy. Gluon effects will be needed for understanding higher energies experiments. Previous short reports on our results have been presented at recent conferences [11].

2 GPDs in the deuteron: definitions and basic properties

A parametrization of the non-perturbative matrix elements which determine the amplitudes in DVCS and DEMP on a spin-one target were given in terms of nine GPDs for the quark sector [8].

$$V_{\lambda'\lambda} = \int \frac{dx}{2\pi} e^{ix\cdot P' \cdot n} \langle P' | \lambda' | -\kappa n \rangle \gamma_n \psi(\kappa n) | P, \lambda \rangle$$

$$= \sum_{i=1,5} e^{ix_{Bj} V^{(i)}_{\beta\alpha}} e^{\alpha} H_i(x, \xi, t), \quad (3)$$

$$A_{\lambda'\lambda} = \int \frac{dx}{2\pi} e^{ix\cdot P' \cdot n} \langle P' | \lambda' | -\kappa n \rangle \gamma_n \gamma_5 \psi(\kappa n) | P, \lambda \rangle$$

$$= \sum_{i=1,4} e^{ix_{Bj} A^{(i)}_{\beta\alpha}} e^{\alpha} \tilde{H}_i(x, \xi, t), \quad (4)$$

where $|P, \lambda \rangle$ represents a deuteron state of momentum $P$ and polarization $\lambda$, $P = (P + P')/2$, and $n^\mu$ is a lightlike vector with $n^+ = 0$, $n^- = 0$. Due to the spin-one character of the target, there are more GPD’s than in the nucleon case, but at the same time the set of polarization observables which in principle could be measured is also richer. Not much is known about these non-perturbative objects which encode the way quarks are confined in deuterons, except a limited set of sum rules and some limiting case values. Sum rules [8] relate these GPDs to the usual deuteron form factors:

$$\int_{-1}^{1} dx H_i(x, \xi, t) = G_i(t) \quad (i = 1, 2, 3),$$

$$\int_{-1}^{1} dx \tilde{H}_i(x, \xi, t) = \tilde{G}_i(t) \quad (i = 1, 2), \quad (5)$$

or lead to a null average:

$$\int_{-1}^{1} dx H_4(x, \xi, t) = \int_{-1}^{1} dx \tilde{H}_3(x, \xi, t) = 0,$$

for $x > 0$. The corresponding relations for $x < 0$ involve the antiquark distributions at $-x$, with an overall minus sign in the expressions for $H_1$ and $H_3$.

In all this paper we will restrict to the quark contribution. Gluon contributions are expected to be small at medium energies but should be included in a more complete description of the process. Therefore we will limit ourselves to values of $x_{Bj}$ not smaller than 0.1.

3 Helicity Amplitudes in the Impulse Approximation

The impulse approximation is the zeroth order to explain the photon-nucleus interaction, but it is the first model one has to analyze since the bulk of the physics is already contained in it.

In the previous section we mentioned that the relevant quantities are the deuteron GPDs. For the sake of simplicity we will model the matrix elements $V_{\lambda'\lambda}$ [8] and $A_{\lambda'\lambda}$ [4] and recover the GPDs just by using the relations given in the appendix. Furthermore, since we are going to limit ourselves to the quark content of the deuteron and it is an isoscalar target, we will denote:

$$V_{\lambda'\lambda}^n = V_{\lambda'\lambda}^d \equiv V_{\lambda'\lambda}^q \quad (8)$$

and a similar relation holds for $A_{\lambda'\lambda}$. 

![Fig. 1. Estimating the helicity amplitudes for the $\gamma^* D \to \gamma D$ in the impulse approximation. The final result is a convolution model between the deuteron wave function and the GPDs for the nucleon (upper blob).](image)
Let us denote by $P^\mu$ ($P'^\mu$) the momentum of the incoming (outgoing) deuteron and $\lambda$ ($\lambda'$) its polarization state, that sometimes we will denote by 0, + or −. To perform this analysis we will choose a symmetric frame where the average momentum $P^\mu = (P^\mu + P'^\mu)/2$ has no transverse components. We will also need a light-like vector $n^\mu$ to define, together with $P^\mu$, the light-cone and satisfying $P \cdot n = 1$. To be more concrete, it is convenient to choose a frame where $P^\mu$ moves fast to the right.

The momentum transfer $\Delta^\mu = P'^\mu - P^\mu$ has a longitudinal and a transverse component. The skewness controls the fraction of momentum transferred in the ’+’ direction:

$$\xi = - \frac{\Delta \cdot n}{(P + P') \cdot n} = - \frac{\Delta^+}{2P^+}$$

With these considerations, the four-vectors corresponding to each deuteron are:[1]

$$P^\mu = ((1 + \xi)P^+, \frac{M^2 + \Delta^2_{\parallel}/2}{2P^+(1 + \xi)}, - \frac{\Delta^\perp}{2}) \quad (10)$$

$$P'^\mu = ((1 - \xi)P^+, \frac{M^2 + \Delta'^2_{\parallel}/2}{2P^+(1 - \xi)}, - \frac{\Delta'^\perp}{2}) \quad (11)$$

The invariant momentum transfer is written as:

$$t = \Delta^2 = - \frac{4\xi^2 M^2 + \Delta^2_{\parallel}}{1 - \xi^2}$$

The positivity of $\Delta^2_{\parallel}$ implies that there is a minimal momentum transfer $t_0$ for a fixed $\xi$:

$$t_0 = - \frac{4\xi^2 M^2}{1 - \xi^2}$$

and at the same time, for a given $t$, there is an upper bound on the allowed values for $\xi$:

$$\xi^2 \leq \frac{-t}{4Mt^2 - t} \quad (14)$$

The GPDs depend in addition of one more variable $x$ which is defined as the fraction of average momentum carried by the partons in the ’+’ direction:

$$x = \frac{\bar{k} \cdot n}{P \cdot n}$$

with $\bar{k}^\mu = (k^\mu + k'^\mu)/2$. Therefore, the longitudinal momentum of the initial parton is $(x + \xi)\bar{P}^+$, whereas the final one has $(x - \xi)\bar{P}^+$, delivering a longitudinal transfer $\Delta^+ = -2\xi\bar{P}^+$ to the deuteron.

Now let us turn our attention to the kinematics at the nucleon level and let us define the fraction of longitudinal momentum carried by each nucleon in the deuteron as:

$$\alpha_i = \frac{p_i^+}{P^+}; \quad \alpha'_i = \frac{p'_i^+}{P'^+}$$

Therefore, we have that the relevant kinematical quantities for the nucleons that make up the initial deuteron are ($\alpha \equiv \alpha_1$):

$$p_1^+ = \alpha(1 + \xi)\bar{P}^+ \quad (17)$$

$$p_2^+ = (1 - \alpha)(1 + \xi)\bar{P}^+$$

and for the final deuteron we have($\alpha' \equiv \alpha'_1$):

$$p'_1^+ = \alpha'(1 - \xi)\bar{P}^+ \quad (18)$$

$$p'_2^+ = (1 - \alpha')(1 - \xi)\bar{P}^+$$

We can now use the decomposition of the deuteron states in terms of nucleon states and the wave function defined in Appendix A1, Eq. (71), to get:

$$V^\lambda_{\chi\chi'} = \frac{2}{(16\pi^3)} \int d\alpha d\alpha' dp_{1\perp} dp'_{1\perp} \sqrt{\frac{1 + \xi}{1 - \xi}} \frac{1}{\sqrt{a\alpha'}}$$

$$\cdot \delta^2(p_{1\perp} - p_{1\perp} - \Delta_{\perp})\delta(\alpha' - \alpha(1 + \xi) - 2\xi)$$

$$\cdot \Theta(\alpha(1 + \xi) - |x| - \xi) \Theta(\alpha(1 + \xi) - 2\xi) \quad (19)$$

$$\cdot \sum_{\chi_1, \chi_{1'}, \lambda_1, \lambda_2} \chi^*_\chi(\alpha', \kappa_1', \lambda_1', \lambda_2) \chi(\alpha, \kappa, \lambda_1, \lambda_2)$$

$$\cdot \frac{1}{2} \int \frac{dk}{2\pi} e^{i\kappa \cdot x} p_{1\perp}|\psi_{\alpha}(\frac{\kappa}{2} \gamma_\perp \cdot n\psi_{\alpha}(\frac{\kappa}{2} n)|p_1, \lambda_1)$$

In the equation above, the variables of the spectator nucleon have been eliminated just by using the normalization properties of the one-particle states. The arguments of the deuteron wave function (see appendix A1 for details), $\kappa_1'$ and $\kappa_{1\perp}$, are the transverse momentum of the active nucleon in a frame where $P_{1\perp} = 0$ and $P_{1\perp} = 0$ respectively. Their relationship with the transverse momentum in the symmetric frame is:

$$\kappa \equiv \kappa_{1\perp} = p_{1\perp} + \alpha\frac{\Delta_{\perp}}{2} \quad (20)$$

$$\kappa' \equiv \kappa'_{1\perp} = p'_{1\perp} - \alpha'\frac{\Delta_{\perp}}{2} \quad (21)$$

The Heaviside functions in Eq. (20) ensure the positivity of the 'plus' momentum carried by the nucleons and put a lower bound on the integration over $\alpha$. The first one stands for processes where $|x| > \xi$ whereas the second one acts when we are on the ERBL region where $|x| < \xi$.

Therefore, we end up with matrix elements of a non-local quark operator between one-nucleon states, which is parameterized in terms of nucleon GPDs. To take advantage of usual parameterizations found in the literature, it is convenient to keep on working on a symmetric frame.
However, since nucleons carry some transverse momentum within the deuteron, the symmetric frame for the deuteron is not the symmetric frame for the active nucleon.

By performing a transverse boost it is possible to evaluate the matrix element in (20) in a symmetric frame, which has the property:

$$\tilde{p}_+ + \tilde{p}_- = 0$$

where we have marked with a $\tilde{}$ the quantities in this boosted frame. Since it is a transverse boost, it does not change the $'+'$ components of the vectors. The light-like vector $n^\mu$ is not changed either, since it is a light-like vector with no transverse components, i.e. $\vec{n}^\mu = n^\mu$.

In that frame, the parameterization of the nucleon GPDs is made with variables that refer to the initial and final nucleon, i.e., we define:

$$x_N = \frac{\tilde{k} \cdot \tilde{n}}{\tilde{p}_+ \cdot \tilde{n}} = \frac{x}{\alpha(1 + \xi) - \xi}$$

$$\xi_N = -\frac{\tilde{\Delta} \cdot \tilde{n}}{2\tilde{p}_+ \cdot \tilde{n}} = \frac{\xi}{\alpha(1 + \xi) - \xi}$$

Due to the lower bound on the values of $\alpha$, we have $\xi_N \geq \xi$. Moreover, it can be checked that $\tilde{k} = \frac{\tilde{x}}{\xi}\tilde{n}$, which is consistent with the fact that we are probing $q\bar{q}$ distribution amplitudes in the deuteron only through the nucleon. In other words, when we enter the ERBL region in the deuteron (i.e. when $|x| = \xi$), we do so at the nucleon level (i.e. when $|x_N| = \xi_N$).

The transverse momentum of the nucleon that interacts with the photon is, after the boost:

$$\tilde{p}_+ = -\frac{\tilde{\Delta}_+}{2}$$

$$\tilde{p}_- = \frac{\tilde{\Delta}_-}{2}$$

$$\tilde{\Delta} = (1 + \xi_N)\tilde{\Delta}_+ + 2\xi_N\tilde{p}_-$$

Now we can use the parameterization of the nucleon matrix element given in the appendix, and after some algebra and changes in the integration variables we reach the final result for $V_{N\lambda}^q$:

$$V_{N\lambda}^q = \frac{2}{(16\pi^3)} \int_{\alpha_{min}}\, d\alpha\, d\alpha'\, d\tilde{k}_\perp\, d\tilde{k}_\perp' \sqrt{\frac{1 + \xi}{1 - \xi} - \frac{1}{\sqrt{\alpha\alpha'}}}$$

$$\cdot \delta^2(\tilde{k}_\perp - \tilde{k}_\perp') - \frac{1 - \alpha}{1 - \xi} \tilde{\Delta}_+ \delta(\alpha') - \frac{\alpha(1 + \xi) - 2\xi}{1 - \xi} \tilde{\Delta}_-$$

$$\sum_{\lambda_1,\lambda_2} \chi_{\lambda_1}^{\alpha}(\alpha', \tilde{k}_\perp', \lambda_1, \lambda_2)\chi_{\lambda}(\alpha, \tilde{k}_\perp, \lambda_1, \lambda_2)$$

$$\times \left[ (1 - \xi_N^2) H^{IS}(x_N, \xi_N, t) + \frac{\xi_N^2}{\sqrt{1 - \xi_N^2}} E^{IS}(x_N, \xi_N, t) \right]$$

$$\delta(\lambda_1 - \lambda) + \frac{\sqrt{t_0 - t}}{2m} \eta_{\lambda} E^{IS}(x_N, \xi_N, t) \delta(\lambda_1 - \lambda)$$

and a similar expression for $A_{N\lambda}^q$.

4 Deuterons GPDs

Once we have obtained the helicity amplitudes it is straightforward to get from them the deuteron GPDs, just from the definitions given in Eq. (34). To do so in a simple way, one can use the light-cone polarization vectors given in [8]. The analytical expressions are summarized in the appendix.

At this point one may argue that definitions of deuteron GPDs were not actually necessary to reach the results of the preceding section and that one can derive the cross sections and observables, directly from Eqs. (25) and (29). But it should be emphasized that the genuine objects that parametrize the hadronic structure are the GPDs, the rest being just kinematics. The GPDs have well defined properties in some limits and their analysis could help us in testing the soundness of a model, as a complementary check to the comparison with experimental data.
momentum distribution of the nucleon in the deuteron as:
large skewness.
be strongly suppressed in the impulse approximation for
guess that the formation of the coherent final state will
dimensional momentum between the nucleons and one may thus
one cannot have a very asymmetrical sharing of longitu-
deuteron. Since the deuteron is a loosely bound system,
final state of this active nucleon still has to fit into the final
To be more quantitative let us define the longitudinal
momentum distribution of the nucleon in the deuteron as:
\[ n_\lambda(\alpha) = \sum_{\lambda_1, \lambda_2} \int \frac{d k_\perp}{(16\pi)^3} \frac{d \beta}{2} \chi_\lambda(\beta, k_\perp, \lambda_1, \lambda_2)^2 \delta(\alpha - \beta) \]
and in the generalized Bjorken limit this is entirely fixed by the
kinematics of the virtual photon (\( \xi \approx x_{Bj}/2 \)). In
the impulse approximation, this momentum transfer has
to be provided by the active nucleon, and after that, the
final state of this active nucleon still has to fit into the final
deuteron. Since the deuteron is a loosely bound system,
one cannot have a very asymmetrical sharing of longitudi-
dimensional momentum between the nucleons and one may thus
guess that the formation of the coherent final state will
be strongly suppressed in the impulse approximation for
large skewness.

4.2 Modelling nucleon GPDs

Let us first consider the helicity conserving nucleon GPDs. Following [13] we have taken a factorized ansatz for the t
dependence of the nucleon GPDs:
\[ H^u(x_N, \xi_N, t) = h^u(x_N, \xi_N) \frac{1}{2} F_1^u(t) \]
\[ H^d(x_N, \xi_N, t) = h^d(x_N, \xi_N) F_1^d(t) \]
\[ H^q(x_N, \xi_N, t) = h^q(x_N, \xi_N) \tilde{F}^q(t) \]
and neglected the strange quark contributions \( H^s \). The
flavour decomposition of the proton and neutron Dirac
form factor, for which we have taken the usual dipole parameterizations [14], gives:

\[
F^{0}(t) = 2F_{0}^{g} + F_{0}^{h} \tag{40}
\]

\[
F^{1}(t) = 2F_{1}^{g} + F_{1}^{h} \tag{41}
\]

For the axial form factor we have taken \( \tilde{F}^{q}(t) = (1 - t/M_{A}^{2})^{-2} \) with \( M_{A} = 1.06 \text{ GeV} \) [15]. For \( \tilde{h}^{g} \) and \( \tilde{h}^{h} \) we follow the ansatz based on double distributions:

\[
\tilde{h}^{g}(x_{N}, \xi_{N}) = \int_{0}^{1} dx' \int_{-1 + x'}^{1 - x'} dy' \left[ \delta(x_{N} - x' - \xi_{N}y') \tilde{q}(x') \right] \pi(x', y'), \tag{42}
\]

\[
\tilde{h}^{h}(x_{N}, \xi_{N}) = \int_{0}^{1} dx' \int_{-1 + x'}^{1 - x'} dy' \left[ \delta(x_{N} - x' - \xi_{N}y') \tilde{q}(x') \right] \pi(x', y'), \tag{43}
\]

\[
\pi(x', y') = \frac{3}{4} \frac{(1 - x')^{2} - y'^{2}}{(1 - x')^{3}}. \tag{44}
\]

where we have only considered the polarization of the valence quarks. To avoid numerical problems with the integrals in the low \( x \) region we have followed the procedure explained in [15]. Throughout this work we have taken the parameterization MRST 2001 NLO [16] for the unpolarized parton distributions and the parameterization LSS01 [17] for the polarized ones.

Concerning the helicity flip nucleon GPDs, \( E^{q} \) and \( \tilde{E}^{q} \), we can safely neglect the latter since we deal with an isoscalar target. The former is suppressed in the \( V_{\lambda \lambda} \) amplitudes, Eq. [29] by kinematical factors. However, one might think that there could be physical situations which could be sensitive to this GPD: in the amplitudes where \( \lambda \neq \lambda \) the transition with \( H^{1S} \) is done at the cost of using the D-wave of the deuteron, i.e., by making use of angular momentum. The term with \( E^{1S} \) could flip the helicity at the nucleon level and therefore, the (rather small) D-wave admixture in the deuteron is not necessary.

Unfortunately, most of the observables are dominated by amplitudes with \( \lambda = \lambda \). We have checked that effects due to \( E^{1S} \) are negligible and we have only shown how tiny they are for the sum rules, just for illustrative purposes. Furthermore, when modelling \( E^{q} \) following the steps explained in [13], one realizes that the isoscalar combination is suppressed. Recall that \( E^{q} \) is normalized to the Pauli form factor, that in the forward limit gives just the anomalous magnetic moment, very small for the isoscalar case.

Let us stress that the available models of GPDs are fraught with uncertainties, in particular in the ERBL region. There, GPDs describe the emission of a \( q\bar{q} \) pair from the target, and an ansatz only using the information from usual parton densities should be used with care. Notice also that, while for \( x > \xi \) GPDs are bounded from above [15], no analogous constraints are known in the ERBL region. A particular type of contribution in the ERBL region is the Polyakov-Weiss D-term [19], which we will not include in our analysis.

4.3 Results

With the ingredients mentioned before we plot in figures [4] and [5] the corresponding generalized quark distributions, which is flavorblind for the deuteron case. The support of these functions is \(-1 < x < -1\) but we have plotted only the central region. In addition, due to the assumption made when modelling \( \tilde{H} \) for the nucleon (the non-contribution of the polarized sea) we have that \( \tilde{H}_{i}(x \leq \xi, \xi, t) \) vanishes.

The rapid falloff of the GPDs with \( x \) reflect the fact that the impulse approximation, i.e., the single nucleon contribution cannot account for very large longitudinal momentum.

Notice also the huge differences in the scales of the various GPDs: for the vector sector, \( H_{3} \) dominates over the others, whereas \( H_{2} \) or \( H_{5} \) are very small. The respective sizes may be related to the values of the different deuteron form factors for the GPDs that have a sum-rule connection to them (see Eq. [5]).

The form factors that we have used in the current \( (G_{1}, G_{2}, G_{3}) \) are related with the usual charge monopole, \( G_{C} \), magnetic dipole, \( G_{M} \) and charge quadrupole, \( G_{Q} \) in the following way:

\[
G_{1}(t) = G_{C}(t) = \frac{2}{3} \eta G_{Q}(t)
\]

\[
G_{2}(t) = G_{M}(t)
\]

\[
(1 + \eta)G_{3}(t) = G_{M}(t) - G_{C}(t) + \left( 1 + \frac{2}{3} \eta \right) G_{Q}(t)
\]

whith \( \eta = \frac{3t}{MV_{D}} \). With the flavour decomposition of the form factors for the deuteron, we have: \( G_{1i}^{q} = G_{1i}^{d} \equiv G_{1i}^{\eta} = 3G_{i} \). The dominant form factor is \( G_{3i}^{d} \) due mainly to the size of \( G_{Q}(t) \) (see [21]), and if we consider the form factors as a normalization condition for the GPDs, it is natural that \( H_{3} \) dominates over the other GPDs.

Notice, however, that the fact that a GPD is large does not mean necessarily that it plays a major role in the observables; it has to be multiplied by the corresponding kinematical coefficients.

It is worthwhile mentioning that we have plotted the GPDs at a particular value of \(-t\), i.e., we cannot set \( t = 0 \) to study the \( \xi \) and \( x \) behaviour. The reason is that, even if we assume a factorized form for the \( t \) dependence in the nucleon, in the deuteron we cannot isolate this \( t \) dependence. In fact, there are two sources of \( t \) dependence in \( H_{i} \) and \( \tilde{H}_{i} \): first the explicit \( t \) dependence in the nucleon GPDs and, the most important one, the transverse momentum in the deuteron wave function. Then, we cannot circumvent the kinematical relationship between \( \xi \) and \( t \), Eq. [12] and for a non-vanishing \( \xi \) we have inevitably a non-vanishing \( t \).
4.4 Sum Rules: tests and discussions

In the impulse approximation we have retained only the lowest Fock-space state of the deuteron (Fig. 2 a and b). As we see, the $q\bar{q}$ components which are tested in the region $|x| < \xi$, are considered only within the nucleon itself (Fig. 2 b).

We have also neglected the $N\bar{N}$ components in the deuteron wave function, which could give rise to diagrams like the one in Fig. 2 c. When one evaluates the elastic deuteron form factor [21,22] this is an exact approximation since one can always choose a frame where the momentum transfer is purely transverse and in that case, $\Delta^+ = 0$ and no pairs can be created from or annihilated into a photon.

In the DVCS and DEMP cases there is a non-vanishing momentum transfer in the longitudinal direction, controled by the skewness parameter $\xi$. Therefore, there are necessarily diagrams where the final photon goes out from the anhilation of, for example, a $N\bar{N}$ pair or a $q\bar{q}$ (see figure 2 c). One has to include these higher Fock-space components to recover Lorentz invariance.

Lorentz invariance is the physical reason why the sum rules (5), obtained by integrating the GPDs over $x$, become $\xi$-independent. When we perform that integration we have matrix elements of local operators (form factors) that cannot depend on the artifacts of the kinematics, i.e. of the separation between transverse and longitudinal momentum transfer (see [23] for a more detailed discussion on this point).

We can make use of this relationship between the $\xi$-dependence of the sum rules and the contribution of higher Fock-space states in the deuteron to check how accurate the impulse approximation is. We have plotted in figure 6 the quantities:

$$ I_i(\xi) = \int_{-1}^{1} dx H^i(x, \xi, t) $$

For a fixed $t$, the functions $I_i(\xi)$ would be constant if the impulse approximation was exact: straight lines in the figure show the 'theoretical' values of the sum rule according to the experimental parameterization of the form factors. Any residual $\xi$ dependence of $I_i(\xi)$ is a measure of the importance of the higher Fock-space states that we have not included in our description. Looking at this figure and to the corresponding one for the axial case, we can see that this dependence is fairly mild, which indicates that, in the kinematical regime that we are interested in, the deuteron is essentially a two-nucleon state. As $\xi$ increases the impulse approximation would become a too rough approximation with respect to Lorentz invariance.

One should distinguish between the variation in $\xi$ of the quantities $I_i(\xi)$ and the particular values they take, which are sensitive also to the details of the employed 2 We checked that the inclusion of the nucleon GPD $E^{IS}$ does not introduce any improvement at all: its contribution vanishes exactly at $\xi = 0$, and is always small at other values of $\xi$. 

---

Fig. 4. Generalized Quark Distributions for the deuteron at $Q^2 = 2$ GeV$^2$, $\xi = 0.1$ and $t = -0.25$ GeV$^2$.

Fig. 5. Generalized polarized Quark Distributions for the deuteron at $Q^2 = 2$ GeV$^2$, $\xi = 0.1$ and $t = -0.25$ GeV$^2$. $H_1$ (upper-left), $H_2$ (upper-right), $H_3$ (lower-left), $H_4$ (lower-right).
model. In figure 6 we see that the points obtained with our calculations are quite close to the experimental parameterization. Obviously, the models works better at smaller $\xi$, for the reasons exposed above. In fact, in figure 7 we show the $t$-dependence of the sum rules at $\xi = 0$, where it is clearly seen that results agrees quite well with the experimental parameterization, when available, or with the values imposed by time reversal or Lorentz invariance. This comes as no surprise since it is well known that the light-cone deuteron wave function is able to give the deuteron form factors at the momentum transfer we are working (see [24] for a recent review). In the context of our discussion, at $\xi = 0$ the pair creation or annihilation with the photon vanishes in the light-cone formalism.

One final remark concerns also the subtleties of the light-cone formalism: nucleons are on-shell, i.e., they verify, with the notation employed in the previous section, that:

\[ p_i^- = \frac{m^2 + P_i^2}{2p_i^+} \]  

But, they are off light-cone energy shell, and as a consequence, if $P^+ = p_1^+ + p_2^+$ and $P_\perp = p_1\perp + p_2\perp$ (as it is the case), one has that $P^- \neq p_1^- + p_2^-$. Therefore, one has that the momentum transfer $t$ defined from the deuteron variables does not coincide with the one defined from the variables of the active nucleon. Moreover, the upper limit over $\xi_N^2$ is not $\frac{m^2}{2t}$. If nevertheless one enforces $\xi_N^2$ to have this upper limit, this leads only to a tiny shift in the value of $\alpha_{\text{min}}$ in the integrals. From the practical point of view, these differences are too tiny to be seen in the numerical calculation, unless one goes to very large values of $-t$. This just reflects the fact that the off-shellness effects in the light-cone energy are of the order of the binding energy over the longitudinal momentum [25]:

\[ P^- - (p_1^- + p_2^-) \propto \frac{V}{P^+} \]  

and in our case this is of the order of the binding energy of the deuteron over the center of mass energy, i.e., very small.

5 DVCS Amplitudes and Cross Sections

There are two processes that contribute to the deeply virtual Compton scattering amplitude of Eq. 1 under consideration. The first one is the Bethe-Heitler process where the outgoing photon is produced from the lepton line. Its amplitude (for either electrons or positrons) is given by

\[ T_{\text{BH}} = -\frac{e^3}{L} \epsilon_\mu(q', \lambda') j_\nu(0) L^{\mu\nu} \]  

where $e$ is the proton charge. The deuteron current is given by:

\[ j_\mu = -G_1(t)(\epsilon^\mu \cdot \epsilon)2\hat{P}_\mu + G_2(t) \left[ (\epsilon^\mu \cdot 2\hat{P})\epsilon_\mu + (\epsilon \cdot 2\hat{P})\epsilon_\mu \right] - G_3(t)(\epsilon^\mu \cdot 2\hat{P})(\epsilon \cdot 2\hat{P}) \frac{\hat{P}_\mu}{M^2} \]  

where the three form factors have been measured in the low and medium momentum transfer ranges [26]. The leptonic tensor $L^{\mu\nu}$ is given by

\[ L^{\mu\nu} = \bar{u}(k', h') \left[ \gamma^{\mu} \frac{1}{(k' + \gamma')^2} \gamma^{\nu} + \frac{1}{(k - \gamma')^2} \gamma^{\mu} \right] u(k, h) \]  

The Bethe-Heitler process is thus completely known in terms of already measured form factors.

The second process where the photon is emitted from the hadronic part is more interesting in terms of the study of the hadronic structure. It is called virtual Compton scattering since it can be decomposed in a $\gamma^* A \rightarrow \gamma B$ process. Its amplitude $T_{\text{VCS}}$ is written as

\[ T_{\text{VCS}} = \frac{-e^3}{Q^2} \sum_{\lambda} \Omega(h, \lambda) M_{H^\lambda H^\lambda} \]  

where the upper sign is for electrons and the lower one for positrons and the function $\Omega$ comes from the decompo-
Fig. 7. Sum rules for the vector GPDs at $Q^2 = 2 \text{ GeV}^2$ and fixed $\xi = 0$ as a function of $t$. Solid lines represent the expected theoretical values whereas points are the results of our evaluation.

The polarization vector of the virtual photon

$$\bar{u}(k', h)\gamma^\nu u(k, h) = \sum_\lambda \frac{Q}{\sqrt{1-\epsilon}} \hat{\Omega}(h, \lambda) \epsilon^\nu(q, \lambda)$$

(53)

$$\hat{\Omega}(h, \lambda) = \left[ \delta_{\lambda 0} \sqrt{2} - \frac{\lambda}{\sqrt{2}} (\sqrt{1+\epsilon} + 2h \lambda \sqrt{1-\epsilon}) e^{-i\lambda \phi} \right]$$

Sometimes we will also use:

$$\Omega(h, \lambda) = \frac{Q}{\sqrt{1-\epsilon}} \hat{\Omega}(h, \lambda)$$

(54)

The photon-deuteron helicity amplitudes are defined as:

$$M_{H^{\nu'}, H\lambda} = \epsilon^*_\nu(q', \lambda') \epsilon_\nu(q, \lambda) H^{\mu\nu}$$

and

$$H^{\mu\nu} = (g^{\mu\nu} - \bar{\nu}' \hat{\nu}' - \bar{\nu} \hat{\nu}')$$

(55)

Fig. 8. Sum rules for the axial-vector GPDs at $Q^2 = 2 \text{ GeV}^2$ and $t = -0.5 \text{ GeV}^2$.

$$\int_{-1}^{1} dx \left( \frac{1}{x - \xi - i\eta} + \frac{1}{x + \xi - i\eta} \right) \sum_q \epsilon^2 q^{V\lambda\lambda}$$

$$+ i\epsilon^{\mu\nu\alpha\beta} \tilde{p}_\alpha \tilde{n}_\beta$$

$$\int_{-1}^{1} dx \left( \frac{1}{x - \xi} - i\eta \right) \frac{1}{x + \xi - i\eta} \sum_q \epsilon^2 q^{A\lambda\lambda}$$

(56)

with the convention $\epsilon_{123} = +1$.

For completeness, let us first write down the formula for the cross section of the Bethe Heitler process on the deuteron

$$\sum [T_{BH}]^2 = \frac{(4\pi\alpha_{em})^3}{t^2} |\mathcal{K}_A(t) + \mathcal{K}_B(t)|$$

(57)

where $A, B$ are the elastic structure functions of the deuteron, which are well known in the low momentum region, and the kinematical coefficients are:

$$\mathcal{K}_B = -\frac{2M^2}{(k \cdot q')(k' \cdot q')} [(2(k \cdot q') + t)^2 + (2(k \cdot q') + Q^2)^2]$$

$$\mathcal{K}_A = -\mathcal{K}_B + \frac{4t}{(k' \cdot q')} (M^2 + s + Q^2 - 2s_{kp})$$

(58)

$$+ \frac{t}{2(k \cdot q')(k' \cdot q')} \left\{ (2M^2 + Q^2 - 2s_{kp})^2 - (Q^2 - 2t)^2 + 4(Q^2 + s - s_{kp})^2 + 4t(t + s - s_{kp}) \right\}$$
with $s_{kp} = (k + p)^2$.

The VCS amplitude gives a contribution to the cross section which may be decomposed in terms of its azimuthal dependence as

$$\sum |T_{VCS}|^2 = \frac{1}{3} \frac{(4\pi\alpha_{em})^3}{Q^2(1 - \epsilon)} \sum_{H,H'} (2|M_{H',H1}|^2 + 2|M_{H',H-1}|^2 + 4\epsilon|M_{H',H0}|^2$$

$$\sum_{H,H',\lambda,h} 2h \text{Re}\{\epsilon_\mu^{\ast}(q^\prime, \lambda^\prime = +1) j_\mu L^{\mu\nu} \tilde{f}(h, \lambda)\}$$

$$\sum_{H,H',\lambda,h} \text{Re}\{M_{H',H\lambda}\}$$

where the upper sign stands for electrons and the lower one for positrons.

The study of the initial electron helicity dependence may be expressed through the following weighted contributions to the cross section

$$\sum 2h |T_{BH}|^2 = 0$$

$$\sum 2h |T_{VCS}|^2 = \frac{4}{3} \frac{(4\pi\alpha_{em})^3}{Q^2} \sqrt{\frac{\epsilon}{1 - \epsilon}}$$

$$\sum_{H,H'} \sin \phi \text{Im}\{M_{H',H0}M_{H',H1} - M_{H',H0}M_{H',H-1}\}$$

$$\sum_{H,H',\lambda,h} 2h (T_{VCS} T_{BH}^\ast + T_{VCS} T_{BH}^\ast) = \frac{2}{3} \frac{(4\pi\alpha_{em})^3}{Q^2} \sqrt{\frac{\epsilon}{1 - \epsilon}}$$

$$\sum_{H,H',\lambda} \text{Im}\{\epsilon_\mu^{\ast}(q^\prime, \lambda^\prime = +1) j_\mu L^{\mu\nu} \tilde{f}(h, \lambda)\}$$

where the upper sign stands for electrons and the lower one for positrons.

### 6 Numerical Results for DVCS

Our model enables us now to estimate the cross section of coherent deeply virtual Compton scattering on the deuteron. We are particularly interested by the forthcoming experiments at JLab and Hermes at DESY, and we thus shall present results for the kinematics of these experimental set ups.

We present in Fig. 9 the unpolarized cross section for low (left panel) and medium (right panel) energy reactions. The Bethe-Heitler and VCS contributions are shown as well as their interference. The relative importance of these contributions depend much on the production angle of the final photon, as can be read from the figure. To discuss the feasibility of the experiment, a comparison to the proton target case is welcome. This is shown on Fig 10 for medium energy reactions.

Coherent deep VCS is certainly not a negligible effect at small values of $t$ and we can expect that this process should soon become observable so that some knowledge of the deuteron GPDs will become accessible. More than testing the validity of the impulse approximation, the goal of such an experiment is to observe some definite deviation from the impulse approximation predictions, thereby indicating some non-trivial short distance content of the deuteron. To scrutinize such effects, it is interesting to turn to some more specific observables, such as spin and charge asymmetries.

The beam spin asymmetry is defined as

$$A_{LU}(\phi) = \frac{d\sigma^\uparrow(\phi) - d\sigma^\downarrow(\phi)}{d\sigma^\uparrow(\phi) + d\sigma^\downarrow(\phi)}$$

where $\phi$ is the angle between the lepton and hadron scattering planes. The numerator is proportional to the interference between the Bethe-Heitler and the VCS amplitudes.

Our predictions calculated with our modelized deuteron GPD’s are shown on Fig. 11 for JLab and Hermes energies. The sign of the asymmetry is reversed for a positron beam. Such a sizable asymmetry should be quite easily measured. It will constitute a crucial test of the validity of any model.

It has been shown that, asymptotically, the beam spin asymmetry exhibits a $\sin(\phi)$ azimuthal dependence. We have performed a Fourier decomposition of the asymmetry $A_{LU}$ obtained for the deuteron and, indeed, we have checked the dominance of the $\sin(\phi)$ component, even
at relatively low values of $Q^2$. But we also have a sizeable $\sin(2\phi)$ component (see figure (6)), which is less suppressed than in the nucleon case. This is likely to come from the following fact: the Bethe-Heitler propagators exhibit when $t \neq t_0$ an azimuthal dependence in $\cos(\phi)$ which goes with terms of the order of $t/Q^2$, but also with terms of the order $m^2/Q^2$. In the nucleon case this $\sin(2\phi) = 2\sin(\phi)\cos(\phi)$ component was already seen under some kinematical conditions, and the larger deuteron mass enhances it. In that respect, we expect that the scaling regime signed by the dominance of the $\sin(\phi)$ component is likely to be reached later in the deuteron case than in the proton case.

The beam charge asymmetry

$$A_C(\phi) = \frac{d\sigma^{e+} - d\sigma^{e-}}{d\sigma^{e+} + d\sigma^{e-}}$$  \hspace{1cm} (65)

is also proportional to the interference of the Bethe-Heitler and the VCS processes. Its characteristic azimuthal dependence is shown on Fig[13] Its size is large enough for a feasible experimental evaluation.

7 Deep exclusive meson electroproduction

Deep exclusive meson electroproduction\cite{26} may be discussed along the same lines as the DVCS reaction. The same QCD factorization property exists which allows to separate a short distance subprocess from long-distance matrix elements, provided the initial virtual photon is longitudinally polarized. The same GPDs appear in principle in the deuteron sector, but selection rules select some of the GPDs. The meson production is described within the well established collinear approximation\cite{27} through the introduction of a distribution amplitude $\Phi(z,Q^2)$ which is generally parametrized through its asymptotic (in the sense of its $Q^2$ evolution) expression.

$$\Phi^\rho(z,Q^2) = 6f^\rho \rho z(1-z)$$  \hspace{1cm} (66)

for the $\rho$ meson, and

$$\Phi^\pi(z,Q^2) = 6\sqrt{2}f^\pi z(1-z)$$  \hspace{1cm} (67)

for the $\pi$ meson, with $f^\rho = 216$ MeV and $f^\pi = 92$ MeV. The resulting amplitudes read ($\lambda,\lambda'$ denoting deuteron polarizations as above):

$$M^{\rho}_{\lambda',\lambda} = -ie\frac{16\pi\alpha_S}{9} \int_0^{t_0} dz \frac{\Phi^\rho(z)}{z}$$
Fig. 13. Azimuthal dependence of the beam charge asymmetry $A_C$ for the kinematics of HERMES shown in previous figures.

$$
\int_0^1 dx (\frac{1}{x - \xi + i \epsilon} + \frac{1}{x + \xi - i \epsilon}) \frac{1}{\sqrt{2}} V_{\lambda, \lambda}^{\rho}(68)
$$
for the vector meson case, and

$$
M_{\lambda, \lambda}^{\pi} = -ie \frac{16\pi \alpha_s}{9}  \int_0^1 dz \frac{\phi_r(z)}{z}
\int_0^1 dx (\frac{1}{x - \xi + i \epsilon} + \frac{1}{x + \xi - i \epsilon}) \frac{1}{\sqrt{2}} A_{\lambda, \lambda}^{\pi}(69)
$$
for the pseudoscalar meson case, where the isosinglet nature of the deuteron has been used to simplify the results. The factor $\frac{1}{2\epsilon}$ in front of the matrix elements $V_{\lambda, \lambda}$ and $A_{\lambda, \lambda}$ come from the flavour decomposition of the $\rho^0$ and the $\pi^0$, i.e., $\frac{1}{\sqrt{2}}(\langle \bar{u}u \rangle - \langle \bar{d}d \rangle)$.

An interesting feature of meson electroproduction is the absence of a competing subprocess such as the Bethe-Heitler process in DVCS. Rates are also higher than in the DVCS case by a factor of $H^2$ and for $Q^2 = 4$ GeV$^2$, $x_{Bj} = 0.1$ and for $Q^2 = 3$ GeV$^2$, $x_{Bj} = 0.2$. As in the proton case, the vector meson production is enhanced with respect to the pseudoscalar meson production. The pseudoscalar production is quite small since the isosinglet nature of the deuteron forbids any enhanced $E$ contribution due to $\pi^0$ exchange in the t-channel.

![Fig. 14. $\rho^0$ (left) and $\pi^0$ (right) virtual photoproduction cross sections at $x_{Bj} = 0.1$, $Q^2 = 4$ GeV$^2$ (solid lines) and $x_{Bj} = 0.2$, $Q^2 = 3$ GeV$^2$ (dashed lines).](image)

8 Conclusion

We have demonstrated that deeply virtual Compton scattering and deep exclusive meson electroproduction on the deuteron is a feasible and promising field of study of the deuteron structure. More theoretical work is obviously needed before one can draw definite conclusions from forthcoming data. In particular, some of the so-called higher twist terms are needed and should be estimated, at least those coming from target mass effects that may expect to be non-negligible. A first estimate of this effect has been given in the second reference of Ref. 9. The inverse process, photoproduction of a lepton pair, sometimes called timelike Compton scattering is also feasible, and should give complementary information as was shown in Ref. 31 for the nucleon case. Double deeply virtual Compton scattering may also be studied along the same lines. We expect that a deepened understanding of the short distance structure of the deuteron will emerge from these studies.

We acknowledge useful discussions with M. Diehl, M. Garçon and F. Sabatie. Special thanks to P.A.M. Guichon and M. Vanderhaeghen for their help with technical calculations. This work was supported by the EC–IHP Network ESOP, Contract HPRN-CT-2000-00130.

A Appendix

A.1 Light-cone Deuteron Wave Function

We have chosen a covariant normalization for the one-particle states on the light cone:

$$
\langle p^{+}, p_{\perp}^{+}, \lambda^{+}\mid p^{+}, p_{\perp}, \lambda \rangle = (2\pi)^3 2p^+ \delta(p^{+} - p^+) \cdot \delta^{(2)}(p_{\perp}^{+} - p_{\perp}) \delta_{\lambda N} \delta_{\lambda' N'} \quad (70)
$$
where $p^+$ is defined in terms of the ordinary vector components as $p^+ = \frac{1}{\sqrt{2}}(p^0 + p^3)$ and $p_{\perp}$ corresponds to the
components in the transverse direction. We have to evaluate matrix elements of a quark operator between deuteron states but eventually we have to deal with two quasi free nucleon states. The definition of the state of a deuteron with momentum $P$ and polarization $\lambda$ in terms of two-nucleon state is:

$$|P^+, P_\perp, \lambda\rangle = \frac{1}{(16\pi^3)^3} \sum_{\lambda_1, \lambda_2} \int \frac{d\xi_1 d\xi_2}{\sqrt{\xi_1 \sqrt{\xi_2}} \delta(1 - \xi_1 - \xi_2)} \cdot dp_{1\perp} dp_{2\perp} \cdot \delta^{(2)}(P_\perp - p_{1\perp} - p_{2\perp}) \chi_\lambda(\xi_1, k_{1\perp}, \lambda_1; \xi_2, k_{2\perp}, \lambda_2) \cdot |p_{1\perp}^+, p_{1\perp}, \lambda_1; p_{2\perp}^+, p_{2\perp}, \lambda_2\rangle$$

(71)

where $\xi_i = p_i^2$ and the transverse momenta $k_{i\perp}$ are the transverse momenta of the nucleons in a frame where $P_\perp = 0$ (hadron frame). We have also kept the notation $p_{i\perp}$ to make explicit the fact that the states are defined in an arbitrary frame but the wave function always refers to the hadron frame. The relationship between both sets of coordinates is given by

$$k_{i\perp} = p_{i\perp} - \xi_i P_\perp$$

(72)

Let us define the measures that take part in the integrals as:

$$[d\xi] = d\xi_1 d\xi_2 \delta(1 - \xi_1 - \xi_2)$$

(73)

$$[dp_{\perp}] = dp_{1\perp} dp_{2\perp} \cdot \delta^{(2)}(P_\perp - p_{1\perp} - p_{2\perp})$$

(74)

We can also perform the integrals over the transverse momenta in the hadron frame with the measure:

$$[dk_{\perp}] = dk_{1\perp} dk_{2\perp} \cdot \delta^{(2)}(k_{1\perp} + k_{2\perp})$$

(75)

The wave function of the deuteron with polarization $\lambda$ is $\chi_\lambda(\xi_1, k_{1\perp}, \lambda_1; \xi_2, k_{2\perp}, \lambda_2)$ with $\lambda_i$ being the polarization of the nucleons. It is normalized according to

$$\sum_{\lambda_1, \lambda_2} \int [d\xi] [dk_{\perp}] |\chi_\lambda(\xi_1, k_{1\perp}, \lambda_1; \xi_2, k_{2\perp}, \lambda_2)|^2 = 1$$

(76)

By taking advantage of the properties $\xi_1 \equiv \xi = 1 - \xi_2$ and $k_{1\perp} \equiv k_\perp = -k_{2\perp}$ we can further simplify the notation in the wave function:

$$\chi_\lambda(\xi_1, k_{1\perp}, \lambda_1; \xi_2, k_{2\perp}, \lambda_2) \equiv \chi_\lambda(\xi, k_\perp; \lambda_1, \lambda_2)$$

(77)

A last remark concerns the connection between the light-cone wave function of the deuteron and the ordinary (instant-form) relativistic wave function obtained with different phenomenological potentials. Whereas the first one is expressed in terms of light-cone coordinates, the latter one is a function of the ordinary three-vectors $k_i$ and fulfills, in general, a Schrödinger type equation. It can be shown that, if we define the longitudinal momentum from the light-cone coordinates as:

$$k_z \equiv M_0(\xi - 1/2)$$

(78)

where $M_0$ is the free mass mass:

$$M_0^2 = m^2 + k_\perp^2$$

(79)

then the eigenvalue equation fulfilled by $\chi_\lambda(\xi, k_\perp; \lambda_1, \lambda_2)$ can be interpreted as a Schrödinger equation and therefore, it can be related to the wave function obtained from phenomenological potentials in the instant-form formalism. More explicitly:

$$\chi_\lambda(\xi, k_\perp; \lambda_1, \lambda_2) = \sum_{\mu_1, \mu_2} \left[ \frac{M_0}{4\xi(1 - \xi)} \right]^{1/2} \langle \lambda_1 | R_M^\dagger(\xi, k_\perp) | \mu_1 \rangle \cdot \langle \lambda_2 | R_M(1 - \xi, -k_\perp) | \mu_2 \rangle \chi_\lambda(c; \mu_1, \mu_2)$$

(80)

The global factor in the r.h.s. of the equation above is just the Jacobian of the transformation from the variables $\{\xi, k_\perp\}$ to $\{k\}$. We also have the matrix elements of the Melosh rotation, which relate the spin in the light-front with the spin in the instant-form of the dynamics. Finally, the (canonical) deuteron wave function $\chi_\lambda(k; \mu_1, \mu_2)$ is written as [33]:

$$\chi_\lambda(k; \mu_1, \mu_2) = (16\pi^3)^{1/2} \sum_{L, m_L, m_s} \frac{1}{\sqrt{2}} \left\langle \mu_1 | \mu_2 m_s \right\rangle (L 1 1 | m_L m_s \lambda) Y_{L, m_L}(k) u_L(k)$$

(81)

A.2 Helicity amplitudes and GPDs

We give here the kinematical coefficients that relate the helicity amplitudes evaluated with the light-cone helicity vectors and the generalized parton distributions. To be consistent with the choice of a right-handed set of polarization vectors made in [3], these expressions should be used only if $\Delta_x < 0$ in the explicit evaluation of $V_{\lambda', \lambda}$ or $A_{\lambda', \lambda}$.

By applying parity properties we can reduce the number of independent helicity amplitudes $V_{\lambda', \lambda}$ down to five (that we have chosen to be $V_{++}$, $V_{00}$, $V_{-+}$, $V_{0+}$, $V_{+0}$) and to four in the pseudovector case $A_{\lambda', \lambda}$ (we keep $A_{++}$, $A_{+-}$, $A_{0+}$, $A_{+0}$). We recall these parity and time reversal properties:

$$V_{\lambda', \lambda} = (-1)^{\lambda'-\lambda} V_{-\lambda', -\lambda}$$

(82)

$$A_{\lambda', \lambda} = (-1)^{\lambda'-\lambda} A_{-\lambda', -\lambda}$$

(83)

$$V_{\lambda', \lambda}(\xi) = (-1)^{\lambda'-\lambda} V_{\lambda', -\lambda}(\xi)$$

(84)

$$A_{\lambda', \lambda}(\xi) = (-1)^{\lambda'-\lambda} A_{\lambda', -\lambda}(\xi)$$

(85)

(86)

We denote:
where the sum covers only the helicity amplitudes that we have chosen as independent ones and the non-vanishing coefficients are:

\[
\begin{align*}
H_i &= \sum_{\lambda', \lambda} c_{i}^{\lambda', \lambda} V_{\lambda', \lambda} \quad (87) \\
\tilde{H}_i &= \sum_{\lambda', \lambda} c_{i}^{\lambda', \lambda} A_{\lambda', \lambda} \quad (88)
\end{align*}
\]

\[
\begin{align*}
c_{i}^{++} &= \frac{1}{3(1 - \xi^2)^2} (3\xi^4 - 7\xi^2 - 2D(1 - \xi^2) + 2) \\
c_{i}^{00} &= \frac{1}{3(1 - \xi^2)} \\
c_{i}^{+} &= \frac{1}{D(1 - \xi^2)} \\
c_{i}^{0} &= -\frac{1}{1 - \xi} \sqrt{\frac{1 + \xi}{2D(1 - \xi)}} \\
c_{i}^{+0} &= -c_{i}^{0+} (\xi \rightarrow -\xi) (89) \\
c_{i}^{2+} &= \frac{2}{1 - \xi^2} \\
c_{i}^{20} &= \frac{2\xi^2}{D(1 - \xi^2)} \\
c_{i}^{02} &= \frac{1}{1 - \xi} \sqrt{\frac{1 + \xi}{2D(1 - \xi)}} \\
c_{i}^{00} &= -c_{i}^{0+} (\xi \rightarrow -\xi) (90) \\
c_{i}^{3+} &= \frac{1}{D(1 - \xi^2)^2} \\
c_{i}^{30} &= \frac{2\xi}{2(1 - \xi^2)^2} \\
c_{i}^{03} &= \frac{1}{1 - \xi} \sqrt{\frac{1 + \xi}{2D(1 - \xi)}} \\
c_{i}^{00} &= c_{i}^{0+} (\xi \rightarrow -\xi) (91) \\
c_{i}^{4+} &= \frac{1}{(1 - \xi^2)^2} (\xi^2 + 2D(1 - \xi^2) + 1) \\
c_{i}^{40} &= \frac{1}{(1 - \xi^2)} \\
c_{i}^{04} &= \frac{1}{D(1 - \xi^2)^3} (2\xi^2 + D(1 - \xi^4)) \\
c_{i}^{00} &= \frac{1}{2(1 - \xi^2)^2} \sqrt{\frac{1 + \xi}{2D(1 - \xi)}} (D(1 - \xi^2) + \xi) \\
c_{i}^{00} &= -c_{i}^{0+} (\xi \rightarrow -\xi) (92)
\end{align*}
\]

In the forward limit \((\xi \rightarrow 0, t \rightarrow 0)\) we get the simplified expressions:

\[
\begin{align*}
H_1(x, 0, 0) &= \frac{1}{3}(2V_{++} + V_{00}) \quad (97) \\
H_5(x, 0, 0) &= -(V_{++} + V_{00}) \quad (98) \\
\tilde{H}_1(x, 0, 0) &= A_{++} \quad (99)
\end{align*}
\]

The integral over \(H_1(x, 0, 0)\) is the parton number, which imposes a serious check on the contruction of the helicity amplitudes in the forward limit.

### A.3 Parameterization of the nucleon matrix elements

In a symmetric frame, where \(P^\mu = (p^\mu + p'^\mu)/2\) has no transverse momentum we take the following parameterization:
where the integration path must be understood along the ‘-$\gamma$’ direction. By using light-cone helicity states, which is close to the usual helicity in frames where the particle moves fast to the right and allows to get compact and elegant expressions, we have to insert in the previous equations the following results:

\[
\begin{align*}
\bar{u}(p', \lambda') &\gamma^+ u(p, \lambda) = 2p^+ \sqrt{1 - \xi^2} \delta_{\lambda\lambda'} \\
\bar{u}(p', \lambda') &\gamma^+ \gamma_5 u(p, \lambda) = 2\lambda_2 p^+ \sqrt{1 - \xi^2} \delta_{\lambda\lambda'} \\
\bar{u}(p', \lambda') &= \frac{\pi^+ \Delta^+}{2m} u(p, \lambda) = 2p^+ \\
&\left( -\frac{\xi^2}{\sqrt{1 - \xi^2}} \delta_{\lambda\lambda'} + \frac{\lambda_0 - t}{2m} \eta \delta_{\lambda,-\lambda'} \right) \\
\bar{u}(p', \lambda') &\gamma^+ \gamma_5 u(p, \lambda) = 2\lambda_2 p^+ \\
&\left( -\frac{\xi^2}{\sqrt{1 - \xi^2}} \delta_{\lambda\lambda'} + \frac{\lambda_0 - t}{2m} \eta \delta_{\lambda,-\lambda'} \right)
\end{align*}
\]

where

\[
\eta_\lambda = \frac{2\lambda \Delta_x - i \Delta_y}{\Delta_z}
\]

and \(\Delta^\mu = p'^\mu - p^\mu\).

References

1. D. Müller, D. Robaschik, B. Geyer, F. M. Dittes and J. Hofeij, Fortsch. Phys. 42 (1994) 101; X. Ji, Phys. Rev. Lett. 78 (1997) 610 and J. Phys. G 24 (1998) 1181; A. V. Radyushkin, Phys. Rev. D56 (1997) 5524; J. Blümlein, B. Geyer and D. Robaschik, Nucl. Phys. B560, 283 (1999); A.V. Radyushkin, Phys. Rev. D 59 (1999) 014030.

2. M. Diehl et al, Phys. Lett. B411(1997) 193.

3. A. Blaizovic and M. Vanderhaeghen, Phys. Rev. D 56 (1997) 818; A. V. Radyushkin, Phys. Rev. D 59 (1999) 014030.

4. M. Burkard, Phys. Rev. D62 071503 (2000); J.P.Ralston and B. Pire, Phys. Rev. D 66, 115101 (2002); M. Diehl, Eur. Phys. J. C 25, 223 (2002).

5. A. Airapetian et al, Phys. Rev. Lett. 87 (2001) 182001.

6. S. Stepanyan et al. [CLAS Collaboration], Phys. Rev. Lett. 87 (2001) 182001.

7. J.C. Collins, L. Frankfurt and M. Strikman, Phys. Rev. D56, 2982 (1997); J. C. Collins and A. Freund, Phys. Rev. D59, 074009 (1999).

8. E.R. Berger, F. Cano, M. Diehl and B. Pire, Phys. Rev. Lett. 87 (2001) 142302.

9. A. Kirchner and D. Müller, hep-ph/0202279; A. Kirchner and D. Müller, hep-ph/0302007; V. Gusey and M. Strikman, hep-ph/0301210.

10. A. V. Efremov and A.V. Radyushkin, Phys. Lett. B 23, 73 (2002).

11. M. Lacombe et al., Phys. Lett. B 101 (1981) 139.

12. K. Goeke, M.V. Polyakov and M. Vanderhaeghen, Prog. Part. Nucl. Phys. 47 (2001) 401.

13. S. Boffi et al., Phys. Rep. 226 (1993) 1.

14. S. Stepanyan et al. [HERMES Collaboration], arXiv:hep-ex/0211008.

15. E. R. Berger, F. Cano, M. Diehl and B. Pire, Phys. Rev. D 5524 (1997) 5524; A. V. Efremov and A.V. Radyushkin, Phys. Lett. B 101 (1999) 101; X. Ji, Phys. Rev. D 59 (1999) 014030.

16. M. Vanderhaeghen, P. Guichon and M. Guidal, Phys. Rev. D 65, 077504 (2002) and Phys. Rev. D 65, 114015 (2002).

17. M. V. Polyakov and C. Weiss, Phys. Rev. D 60, 114017 (1999).

18. M. Garçon and J.W. Van Orden, Adv. Nucl. Phys. 26 (2002) 294.

19. L. L. Frankfurt and M.I. Strikman, Nucl. Phys. B 148 (1979) 107.

20. L.L. Frankfurt, T. Frederico and M. Strikman, Phys. Rev. C48 (1993) 2182.

21. S.J. Brodsky, M. Diehl and D.S. Hwang, Nucl. Phys. B596 (2001) 99.

22. J. R. Cooke and G.A. Miller, Phys. Rev. C 66, 034002 (2002); J. R. Cooke, arXiv:nucl-th/0112029.

23. S.J. Brodsky and G.P. Lepage, in Perturbative QCD, edited by A.H. Mueller (World Scientific, Singapore, 1989).

24. M. Vanderhaeghen, P. Guichon and M. Guidal, Phys. Rev. D 60 (1999) 094017.

25. G.P. Lepage and S.J. Brodsky, Phys. Lett. B87, 359 (1979); A.V. Efremov and A.V. Radyushkin, Phys. Lett. B94, 245 (1980).

26. M. Diehl et al., Phys. Rev. Lett. 81 (1998) 1782; M. Diehl et al., hep-ph/9901233; M. V. Polyakov, Nucl. Phys. B 555 (1999) 231; B. Lehmann-Dronke et al., Phys. Lett. B 475 (2000) 147; B. Lehmann-Dronke et al., Phys. Rev. D 63 (2001) 114001; P. Hagedorn et al., Phys. Lett. B 535, 117 (2002) [Erratum-ibid. B 540, 324 (2002)] and Eur. Phys. J. C 26, 261 (2002).

27. I. V. Anikin, B. Pire and O. V. Teryaev, Phys. Rev. D 62, 071501 (2000).

28. A.V. Efremov and A.V. Radyushkin, Phys. Lett. B94, 245 (1980).

29. M. Diehl et al., Phys. Rev. Lett. 81 (1998) 1782; M. Diehl et al., hep-ph/9901233; M. V. Polyakov, Nucl. Phys. B 555 (1999) 231; B. Lehmann-Dronke et al., Phys. Lett. B 475 (2000) 147; B. Lehmann-Dronke et al., Phys. Rev. D 63 (2001) 114001; P. Hagedorn et al., Phys. Lett. B 535, 117 (2002) [Erratum-ibid. B 540, 324 (2002)] and Eur. Phys. J. C 26, 261 (2002).

30. A.V. Efremov and A.V. Radyushkin, Phys. Lett. B94, 245 (1980).