Simulation of the trajectories described by a space vehicle around the asteroid 243 Ida and its natural satellite Dactyl

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Abstract. The asteroid 243 Ida located in the asteroid belt, between Mars and Jupiter, is the fourth largest asteroid of the Koronis asteroid family, with an average diameter of 31.3 km and a mass around $4.2 \times 10^{16}$ kg, and a small moon, Dactyl. In order to study the dynamics of this system, orbital trajectories are simulated around Ida considering, besides the gravitational attraction of Dactyl, the non-central gravitational field of the asteroid, defined by a polyhedral model that defines the shape and the non-uniform mass distribution of the body. In this way, the magnitude and the behaviour of such forces, and also their influence on the orbital elements that define the trajectory of the space vehicle, are evaluated and analysed.

1. Introduction
The asteroid 243 Ida located in the asteroid belt, between Mars and Jupiter, belongs to the family of asteroids Koronis whose origin is supposed to be the result of the collision of two major bodies [1]. The asteroid Ida is the fourth largest of the Koronis family having an average diameter of 31.3 km and a mass around $4.2 \times 10^{16}$ kg. This asteroid has another interesting feature: it has a small moon, called Dactyl, discovered by the Galileo spacecraft during its approach to the asteroid [2-4]. But the orbit of this small moon was not precisely determined since the images sent by Galileo were insufficient to accurately determine Dactyl’s orbit around Ida, as illustrated in Figures 1 and 2.

![Figure 1. Ida and Dactyl](Source: NASA/JPL [5])

![Figure 2. Possible orbits of Dactyl around Ida](Source: NASA/JPL - Dactyl potential orbits [5])

It was considered in this work the orbital motion of a spacecraft around Ida, perturbed by the gravitational attraction of Dactyl, taking into account the Dactyl orbit almost circular with radius of 90 km [6], mass of 2380 kg, and the non-central gravitational field of Ida. The gravitational field was
defined by a polyhedral model of the asteroid volume, provided by NASA [7], associated to a model of mass concentrations allocated in the barycenter of the polyhedra that compose the volume of the asteroid [8-12]. Using this approach and the Spacecraft Trajectory Simulator (STRS) [13-16], it is possible to map the gravitational field around the asteroid and determine the trajectory that would be described by a spacecraft around Ida / Dactyl, which characterizes the main objective of this work.

2. Gravitational Perturbation

The gravitational field of an asteroid such as 243 Ida cannot be considered as a central force field since its shape does not resemble that of a sphere. Due to the irregular shape of the asteroid, the trajectory of a spacecraft in its orbit is perturbed by the gravitational potential generated by the asteroid. In this way the modeling of the gravitational field and the orbital perturbation imposed on the vehicle is fundamental for the simulation of the movement dynamics.

In this work we use the polyhedral model provided by NASA to model the asteroid mass distribution [7 and 17]. It was considered that in the barycenter of each polyhedron is allocated a mass concentration equivalent to the mass of the respective polyhedron. In this way, the gravitational force applied to the vehicle due to each of the mass concentrations can be calculated. By effecting the integral of all forces we obtain the resultant gravitational force. The comparison between this resultant with the gravitational force, that would be generated if the force field were central, provides the disturbing force that is applied to the vehicle at each step of the simulation [10-11 and 18], as shown in Figure 3.

\[ U = -G \int_V \frac{\rho \, dV}{||r||} \]  

Equation (1)

The gravitational force applied to the spacecraft relative to the secondary centers of attraction with mass \( M_i \) of each volume \( V_i \) in the barycenter of each tetrahedron, called secondary centers of attraction, as modeled in STRS and used in [8-12] and [18].

\[ F_{G_i} = \frac{-GmM_i \hat{i}}{||r_i||^2} + \frac{-GmM_i \hat{j}}{||r_i||^2} + \frac{GmM_i \hat{k}}{||r_i||^2}, \quad i = 1, 2, 3 ..., n \]  

Equation (2)

Therefore, the resultant of the perturbing gravitational force \( \overrightarrow{F_{GP}} \) applied to the spacecraft can be modeled as the difference between the force referring to the central field and the sum of the gravitational forces due to the concentrations masses, given by equation (3), where \( M \) is the total mass.

Figure 3. Simulation flowchart

The gravitational potential \( U \) of the asteroid is given by equation (1). \( \rho \) is density and \( dV \) is a small differential volume relative to the asteroid. Due to the complexity of the calculation of equation (1) to model the gravitational potential considering the irregular shape of the asteroid, the total volume \( V \) can be divide into \( n \) tetrahedrons, concentrating the mass \( M_i \) of each volume \( V_i \) in the barycenter of each tetrahedron, called secondary centers of attraction, as modeled in STRS and used in [8-12] and [18].
of the asteroid.

\[
\vec{F}_{GP} = -\frac{GmM}{\|\vec{r}\|^2} \hat{r} + \sum_{i=1}^{n}\left[\frac{GmM_i}{\|\vec{r}_{Gi}\|^2} \hat{r} + \frac{GmM_i}{\|\vec{r}_{Gi}\|^2} \hat{j} + \frac{GmM_i}{\|\vec{r}_{Gi}\|^2} \hat{k}\right]
\]

(3)

3. Simulations and results

Simulations were carried out with a duration of 30 and 100 days. The initial semi-major axis was considered to be 60 km in all simulations. Several simulations for different orbital inclinations were performed, varying the inclination from zero to 90°, but just the results referring to the inclination of 35° will be shown here. Although the orbit used is close to a circular orbit, the altitude varies significantly due to the irregular asteroid shape. The gravitational perturbation due to the irregular shape of the asteroid is a function of the position of the vehicle and the rotation of the asteroid. These parameters depend on the orbital plane adopted. The effect of the radiation pressure was taken into account considering the solar irradiance constant for a distance of 2.7 AU of 187.38 W/m², and the optical properties of the spacecraft surface (absorption coefficient 0.3; specular coefficient 0.3; diffuse coefficient 0.4) for a projection area of 10 m² in all directions. However, the graphs related to the radiation pressure were omitted since the velocity increment due to this perturbation remains approximately constant, around 10⁶ m/s. It was considered a spacecraft with 1000 kg.

For both simulations the following initial conditions were adopted: \(a = 60 \text{ km}; \quad e = 0.000001; \quad i = 35°; \quad \Omega = 0; \quad \omega = 0\). The Simulation 1 considers a simulation time of 30 days and the Simulation 2 of 100 days. For the simulations 1 and 2, respectively, the figures 4 and 5 show the velocity increment applied to the vehicle considering all the perturbations (Cowell’s method) for each axis (green, magenta and blue) and also for the absolute value of the disturbance (red). The variation of the perturbation observed in the figure occurs mainly due to the non-central gravitational field of Ida. Figures 6 and 7 show the velocity increment due to the Dactyl gravitational perturbation for each axis (green, magenta and blue) and also for the absolute value (red). It can be seen in the figures that occur peaks in the moments of greatest approximation with the Dactyl along the trajectory of the spacecraft.

![Figure 4. Total perturbation (sim. 1)](image1)

![Figure 5. Total perturbation (sim. 2)](image2)

Figures 8 and 9 show the disturbing velocity increment due to the gravitational pull of the Sun, once again, for each of the axes and for the absolute value. Figures 10 and 11 show the sum of the disturbance velocity increment, due to all the combined perturbations, throughout the simulations. The sum of the disturbance velocity increment can be considered analogous to the integral of the disturbances acting on the vehicle along its trajectory, during the simulated time interval. This sum can be used as a parameter to be minimized during the analysis of a mission of a spacecraft that aims to orbit the asteroid, in order to find less disturbed trajectories that may require a lower fuel consumption to maintain the trajectory close to nominal trajectory and, in this way, enable the execution of the mission for which the vehicle was designed. Figures 12 to 19 show the variation of the orbital parameters of the spacecraft due to the action of the gravitational perturbation generated by the non-homogeneity of the mass distribution of Ida and the gravitational attractions of the Sun and Dactyl. It can be seen in figures 12 and 13 that the semi-major axis of the orbit presents short-period variations,
oscillating with maximum amplitude around 25 m. However, there is a long-term trend, which is most evident in Figure 13 for the second simulation, which causes the semi-major axis to deviate more and more over time. This variation of the semi-major axis is corroborated by the variation of the eccentricity of the orbit that can be observed in figures 14 and 15. It is clear that there is a possibility of the vehicle escaping from the region of influence of Ida if the deviations in the semi-major axis and eccentricity continue to increase over time. This fact illustrates the need to find minimally disturbed trajectories by evaluating the sum of the disturbance shown in Figures 10 and 11.

Figures 16 and 17 show that the perturbation of the non-central field of Ida is also capable of altering the orbital plane of the spacecraft trajectory. Although the figures show small values for inclination deviations, it is clear from Figure 17 that there is a tendency of increase the deviation over time. Finally, figures 18 and 19 show the altitudes of the vehicle with respect to the asteroid Ida. Even considering almost circular trajectories a significant variation of the altitude is observed, with maximum amplitude of variation around 16 km. This is due to the irregular shape of the asteroid. It is evident in the figures that the soft landing of a spacecraft in an irregularly shaped body such as the asteroid Ida presents a great challenge [12], since the relative velocity of the vehicle and the surface of the body may vary suddenly as a function of the altitude variation, as shown in figures 18 and 19.
4. Conclusions
The simulations showed that due to the irregular shape of the asteroid the spacecraft approaches the surface even when using an almost circular initial orbit. This is a characteristic of the orbits around asteroids, in which the gravitational perturbation can oscillate expressively as a function of the altitude. The main disturbance in the trajectory of the spacecraft was due to the gravitational potential of Ida. The disturbance caused by Dactyl is insignificant since the estimated mass of the natural satellite of Ida is very small, and besides that, the Dactyl orbit was not adequately determined and varies due to
the gravitational perturbation of Ida. An initial orbit with an orbital period of 1.53 terrestrial days and small eccentricity was adopted, but other possible orbits should be considered. Thus, this work represents a starting point for the study of trajectories around Ida and its moon Dactyl.

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