3D Numerical Modeling for Inductive Processes

A. Gagnoud¹, Y. Du Terrail-Couvrat¹, O. Budenkova²

¹ Univ. Grenoble Alpes, CNRS, Grenoble INP, SIMAP, 38000 Grenoble, France

Corresponding author : annie.gagnoud@simap.grenoble-inp.fr

Abstract
In order to analyze the performance and to optimize inductive processes we develop a dedicated software coupling integral method and finite element method. The integral method is dedicated to model electromagnetic phenomenon and the finite element method is used to model heat transfer and fluid mechanic. This software is developed for 2D and 3D configurations. In the article some results are given for various inductive processes.

Key words : induction processes, numerical modeling, coupled phenomena, integral method, finite element method

Introduction
Inductive processes are used in many industrial processes and applied to various materials such as metals, semiconductors and oxides within a large frequency domain from Hz to MHz. These applications involve coupled phenomena: electromagnetism, heat exchange and fluid mechanics. The phenomena manifest themselves and interact depending on the properties of the material, frequency and geometry of the process. Our aim was to develop dedicated software to model a large diversity of electromagnetic processes. Usually, electromagnetic phenomenon is modeled with the finite element method (FEM). Thus, various 3D potential formulations are available[1]. To take into account thin electromagnetic skin depth, boundary impedance formulation can be used [2]. The management of these formulations is complex. We choose to model electromagnetic phenomenon with Integral Method (IM) which is better adapted to inductive processes [3] [4] compared to the FEM. The main advantage of this method is that the calculating mesh is limited to conductive parts that allows one to perform calculations for significantly different scales. For example: some parts of installation can be very large whereas the space between them can be extremely small, or a thin electromagnetic skin depth can be easily modeled. For numerical modeling of heat exchange and fluid mechanic we choose Finite Element Method (FEM) with Galerkin’s projection method which allows for describing various formulations. In the software IM and FEM use a discretization with finite elements and Lagrange polynomial interpolation. IM and FEM are coupled using interpolation between the mesh of each method. Depending on the physical process under consideration, different algorithms can be adopted: for strong coupling phenomena one non-linear system for all physical variables is solved [5] and for weak coupling the IM and FEM are used sequentially.

Electromagnetic model
Usually, the different parts of the inductive processes are electrical conductors which can be metal, semi-conductors or melt oxides. In this case, the integral method is based on two equations : the local Ohm’s law (1) and the conservation of current equation (2). A sinusoidal alternating potential difference is imposed at the edges of the coil, the equations are written in their complex form:

\[ \mathbf{J} = -\sigma \nabla \mathbf{V} - i \omega \mathbf{A} \]  

(1)

\[ \nabla \cdot \mathbf{J} = 0 \]  

(2)

where \( \mathbf{J} \) is the complex current density, \( \sigma \) is the electrical conductivity of the material, \( \mathbf{V} \) is the electrical scalar potential, \( i \) is the imaginary number, \( \omega \) is the angular frequency and \( \mathbf{A} \) is the magnetic vector potential. The magnetic vector potential is expressed by Biot and Savart law (3):

\[ \mathbf{A} = \frac{\mu_0}{4\pi} \oint_{\partial \mathbf{V}} \frac{\mathbf{J}}{r} \, dv \]  

(3)

where \( \mu_0 \) is the magnetic permeability of the vacuum, \( \mathbf{V} \) is the volume of the conductors, \( r \) is the distance between a point of the conductor and the calculation point of the magnetic potential. By introducing the expression of the vector potential (3) in the relation (1) an integral formulation is obtained. A first order finite elements mesh is constructed for the electrical conductors of the system, i.e. for the inductor coil, crucible, load, etc. If the electromagnetic skin depth of a conductor is thin compared to the size of the object, a surface mesh is done, otherwise a volume mesh is done. Equation (1) is written on each node of the mesh and equation (2) is integrated on each element of the mesh. We impose zero normal current density at the surface of the conductors except at the boundary of the inductor where electrical potential is imposed. Finally, a linear system is obtained where unknowns are the complex...
current density and the complex electrical potential. The coefficients of the matrix are integrals which are calculated numerically by Gauss method, like in FEM.

**Heat exchange and fluid mechanic model**

To model the heat exchange and fluid mechanic a finite element method formulation is obtained by Galerkin's projection. For finite element method various algorithms are developed: linear, non linear, transient and non linear transient.

As an example, we consider static heat transfer equation without phase change (4).

\[
\rho C_p \frac{\partial T}{\partial t} + \rho \mathbf{U} \cdot \nabla T + \nabla \cdot (-k \nabla T) = Q_{th}
\]

where \( T \) is the temperature, \( \rho \) is the density, \( C_p \) is the calorific capacity, \( \mathbf{U} \) is the velocity, \( k \) is the thermal conductivity, \( t \) is the time and \( Q_{th} \) is the source term. For the temperature, we use second order elements, so the temperature is interpolated by second order Lagrange polynomial functions. For the projection of heat exchange equation second order Lagrange functions are used as test functions.

To model fluid mechanic for an incompressible fluid we consider the Navier-Stokes equation (5) and the continuity equation (6). The density is considered constant.

\[
\rho \frac{\partial \mathbf{U}}{\partial t} + \rho \nabla \cdot \mathbf{U} = \mathbf{g} + \mathbf{F} + \nabla \cdot \mathbf{\Sigma}
\]

where \( \mathbf{g} \) is the gravity vector, \( \mathbf{F} \) is the external force and \( \mathbf{\Sigma} \) is the stress tensor.

\[
\nabla \cdot \mathbf{U} = 0
\]

The velocity is interpolated by second order Lagrange polynomial functions and the pressure is interpolated by first order Lagrange polynomial functions. The Navier-Stokes equation is projected on second order Lagrange functions. The continuity equation is projected on first order Lagrange functions. To establish the fluid mechanic formulation we use P2P1 polynomial functions.

**Coupling algorithms**

In the case of inductive systems the source term of heat exchange is equal to time average Joule power density (7), and the external force of fluid mechanic is equal to time average electromagnetic force density (8). So we have:

\[
Q_{th} = \frac{\mathbf{J} \cdot \text{conj}(\mathbf{J})}{2}
\]

\[
\mathbf{F} = \frac{1}{2} \text{real}(\mathbf{J} \times \text{conj}(\mathbf{B}))
\]

where \( \mathbf{B} \) is the complex magnetic induction, and \( \text{conj}(\mathbf{B}) \) is conjugate of \( \mathbf{B} \). To model the interaction between phenomena three kinds of algorithm are developed:
- sequential algorithm with IM in the first step and FEM in the second step. In this case coupling terms are source terms of thermo-hydrodynamic equation: Joule power density and electromagnetic force density. These quantities are interpolated at the integration points of the elements of the finite element mesh on the IM mesh.
- iterative algorithm between the two methods IM and FEM. In this case coupling terms are the source terms of the thermo-hydrodynamic equation and the electrical conductivity in the electromagnetic formulation which is interpolated at the integration point of the elements of the integral method on the FEM mesh.
- strong coupling: one system of equation is constituted by the formulations of the two methods.

**Results**

Three cases of inductive cases are presented. We consider first an axisymmetrical configuration of the electromagnetic stirring at 100kHz. The inductor coil is constituted of two wires. The radius of the load is equal to 4cm and its eight is 6cm. A sequential algorithm is used. The thin skin depth model is applied, so the electromagnetic quantities are calculated on the boundary of the electrical conductors: load and wires. A laminar fluid flow is calculated with the FEM formulation. The electromagnetic force is presented in the Fig. 1, and the velocity in the Fig. 2. Secondly we model the electromagnetic levitation at 164kHz. The radius of the load is equal to 0.494cm. The configuration and the electromagnetic force density are presented in Fig. 3 and the fluid flow in the Fig. 4. In the last example a 3D case is presented: inductor coil and cylindrical load at 30kHz frequency. The radius of the load is 2.6cm and its eight is 4cm. The thin skin depth model is applied. A sequential algorithm is used to solve electromagnetism and thermal problem. In the Fig. 5 the configuration and the density power is presented. In the Fig. 6 the temperature at the surface of the load is presented.
Fig. 1: Vectors of the electromagnetic force density on the boundary of the melt load

Fig. 2: Fluid flow on the melt load, maximum velocity: $4.2 \times 10^{-3}$ m/s

Fig. 3: Electromagnetic levitation: geometry and electromagnetic force density on the melt load

Fig. 4: Fluid flow on the melt load, maximum velocity: $2.03 \times 10^{-2}$ m/s

**Conclusion**

A software dedicated to the inductive problem has been developed based on the coupling between the integral method and the finite element method. The integral method is well adapted to model inductive systems because the mesh is limited to active part of the installation. So even a small air gap between the objects of the installation can be modeled. Furthermore, a model adapted to the case of thin skin depth is developed. The use of finite element method permits one to implement various formulations for heat exchange and fluid mechanic. A developed approach allows use of different coupling algorithms.
Fig. 5: 3D configuration: geometry and Joule power density and real part of the current density vectors.

Fig. 6: Temperature on the surface of the load.

References
1. O. Biro, K. Preis, IEEE Trans. Magn. 25 (4) (1989), 3145-3159.
2. S. Yuferev, N. Ida, IEEE Trans. Magn. 35 (3) (1999), 1486-89.
3. R. Scapolan, A. Gagnoud, Y. Du Terrail, IEEE Trans. Magn. 50 (2) (2014), 7023504
4. A. Gagnoud, IEEE Trans. Magn. 40 (1) (2004), 29-36
5. P. Triwong, A. Gagnoud, Serbian Journal of Electrical Engineering 5 (1) (2008), 87-98