Chirped pulse excitation of two-atom Rydberg states

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Abstract

We analyze the excitation of two ground state atoms to a double Rydberg state by a two-photon chirped optical pulse in the regime of adiabatic rapid passage (ARP). For intermediate Rydberg–Rydberg interaction strengths relevant for atoms separated by ~ten μm, adiabatic excitation can be achieved at experimentally feasible Rabi frequencies and chirp rates of the pulses, resulting in high transfer efficiencies. We also study the adiabatic transfer between ground and Rydberg states as a means to realize a controlled phase gate between atomic qubits.

Keywords: Rydberg atoms, adiabatic rapid passage, controlled phase gate

(Some figures may appear in colour only in the online journal)

1. Introduction

Neutral atoms in ground hyperfine states represent a clean, well-controlled system for quantum information processing providing long-term qubit storage, easy initialization, readout, manipulation with optical and magnetic fields, and scalability [1]. Two-qubit gates with atoms can be realized by transferring them into high-lying Rydberg states, in which the atoms can interact via strong and long-distance van der Waals and dipole–dipole interactions [2]. In the seminal proposal [3] of a controlled phase gate for atomic qubits based on interaction in Rydberg states, two realizations have been discussed. The first one works for close atoms in a regime when the interaction action strength $V_{\text{int}}$ exceeds the Rabi frequency $\Omega$ of excitation pulses coupling qubit states to the Rydberg ones: $V_{\text{int}} > \Omega$. In this regime of a Rydberg blockade the strong interaction prevents excitation of a second atom if the first atom has been excited. The blockade has been experimentally demonstrated for two atoms in separate dipole traps [4], followed by the realization of a CNOT gate [5] and entanglement [6]. A second approach to the controlled phase gate, discussed in [3], applies to smaller interaction strengths $V_{\text{int}} \lesssim \Omega$, for which the blockade cannot work. The gate then can be implemented by conditionally exciting both atoms to the Rydberg state, letting them interact to accumulate a π phase shift, and deexciting back to their original qubit states. This allows us to realize the gate between atoms separated by several sites in an optical lattice architecture and between atoms in distant individual microtraps, making the systems scalable. The second approach has not yet been realized experimentally and is currently a subject of active theoretical investigation [7–9]. Interactions in Rydberg states can also find applications in quantum simulation [10, 11], quantum repeaters [12], and in the realization of efficient and non-local nonlinearities [13], down to a single photon level [14].

Coherent control techniques such as STIRAP [15] and adiabatic rapid passage (ARP) [16] can provide robust excitation to the two-atom Rydberg state. The transfer of two atoms to the double Rydberg state by STIRAP was studied in [7], where it was shown that in a system of three-level atoms, typically used in experiments, having ground $|g\rangle$, intermediate $|i\rangle$ and Rydberg states $|r\rangle$ (shown in figure 1(a)), the only dark state does not connect the two-atom ground $|gg\rangle$ and Rydberg $|rr\rangle$ states. Application of pump and Stokes pulses, resonant with the $|g\rangle \rightarrow |i\rangle$ and $|i\rangle \rightarrow |r\rangle$ transitions, respectively, transfers the system from $|gg\rangle$ to an entangled $|ii\rangle - |gg\rangle \sqrt{2}$ state, not containing $|rr\rangle$. In a later work [9] it was found that a non-zero one-photon detuning $\Delta_p \sim V_{\text{int}}$ in the STIRAP scheme produces a dressed state directly connecting $|gg\rangle$ to $|rr\rangle$, but the transfer efficiency to that state was not optimal because of population loss from a fast decaying intermediate state. STIRAP-like excitation to the $|rr\rangle$ state with larger detunings $\Delta_p \sim$ hundreds MHz can also be realized using optimal control techniques by shaping the pulses such that the population of the intermediate state is minimized [17].

ARP with chirped optical pulses is well-known for providing efficient population transfer between quantum states,
and it will be studied in this work as a means to achieve robust excitation to double Rydberg states. In the two-photon ARP excitation scheme of figure 1(a) a large one-photon detuning from the fast-decaying intermediate state can be used, allowing us to minimize its population and obtain high transfer efficiencies. We will also analyze a controlled phase gate between two atomic qubits based on ARP excitation to the double Rydberg state along one of the dressed states, directly connecting the ground state qubits to the |rr⟩ state and back such that the fidelity of the controlled phase shift, and conclude in Section V.

2. Dressed states for two three-level atoms interacting with a two-photon chirped field

We consider two three-level atoms with internal states |g⟩, |i⟩ and |r⟩, shown in figure 1(a). Each atom interacts with two chirped laser pulses, pump and Stokes, on the transitions |g⟩ − |i⟩ and |i⟩ − |r⟩, respectively. In the Rydberg states |r⟩ they additionally interact with each other via dipole–dipole or van der Waals interactions. The system can be described by the Schrodinger equation assuming that all the interactions are much faster compared to decays of intermediate and Rydberg states. For simplicity of the analysis one can use ‘molecular’ states: |gg⟩, | ±⟩ = (|ig⟩ ± |gi⟩)/√2, |ii⟩, | ±⟩ = (|ir⟩ ± |ri⟩)/√2, |rr⟩. Expanded in these states the two-atom wave function has a form |Ψ⟩ = c_ig |gg⟩ + ∑_gi=± c_ig |i⟩ + ∑_ri=± c_ri |r⟩, where the amplitudes of the ‘molecular’ states are expressed via amplitudes of pure two-atom states as follows: c±_ig = (c_ig ± c_ig)/√2, c±_ri = (c_ri ± c_ri)/√2.

The Hamiltonian of the system in the rotating wave approximation is given by:

\[ H/\hbar = \sum_{j=1,2} (\Delta_p(t) |j⟩⟨j| + \delta(t) |r⟩⟨r|) + V_{\text{int}} |r⟩⟨r| - \sum_{j=1,2} (\mu_ig \vec{E}_p(t)/2\hbar |g⟩⟨l|) + \mu_i \vec{E}_S(t)/2\hbar |r⟩⟨l| + H. \text{ c.}, \]

(1)

where \( \Delta_p(t) = \omega_{ig} - \omega_p(t), \delta(t) = \omega_{ir} - \omega_p(t) - \omega_S(t) \) are the one and two-photon detunings of the pump and Stokes field frequencies \( \omega_p \) and \( \omega_S \) from the atomic transition frequencies \( \omega_{ig} \) and \( \omega_{ir} \), \( \mu_i \) and \( \mu_r \) are the dipole moments of the corresponding transitions and \( \vec{E}_p, \vec{E}_S \) are the amplitudes of the pump and Stokes fields. The Schrodinger equations for the ‘molecular’ state amplitudes are then given by:

\[ i \frac{dc_{ig}}{dt} = -\sqrt{2} \Omega \mu_ig, \]

\[ i \frac{dc_{+ig}}{dt} = \Delta_p c_{+ig} + \sqrt{2} \Omega \mu_ig c_{+ig} - \sqrt{2} \Omega \mu_i, \]

\[ i \frac{dc_{-ig}}{dt} = \Delta_p c_{-ig} - \sqrt{2} \Omega \mu_ig c_{-ig} - \sqrt{2} \Omega \mu_i, \]

\[ i \frac{dc_{ir}}{dt} = \Delta_p c_{ir} + \sqrt{2} \Omega \mu_r, \]

\[ i \frac{dc_{+ir}}{dt} = \Delta_p c_{+ir} - \sqrt{2} \Omega \mu_r c_{+ir} - \sqrt{2} \Omega \mu_i, \]

\[ i \frac{dc_{-ir}}{dt} = \Delta_p c_{-ir} - \sqrt{2} \Omega \mu_r c_{-ir} - \sqrt{2} \Omega \mu_i, \]
\[
\begin{align*}
\frac{dc_{+ig}}{dt} &= \Delta_p c_{+ig} - \Omega_S c_{+rg}, \\
\frac{dc_{i}}{dt} &= 2\Delta_p c_{i} - \sqrt{2}\Omega_p c_{+ig} - \sqrt{2}\Omega_S c_{+ri}, \\
\frac{dc_{+rg}}{dt} &= \delta c_{+rg} - \Omega_S c_{+ri}, \\
\frac{dc_{-rg}}{dt} &= \delta c_{-rg}, \\
\frac{dc_{+ri}}{dt} &= (\delta + \Delta_p) c_{+ri} - \sqrt{2}\Omega_S c_{i}, \\
\frac{dc_{-ri}}{dt} &= (\delta + \Delta_p) c_{-ri} - \Omega_p c_{-ri}, \\
\frac{dc_{rr}}{dt} &= (2\delta + V_{int}) c_{rr} - \sqrt{2}\Omega_S c_{+ri},
\end{align*}
\]

where \(\Omega_p = \mu_{p}/\hbar\), \(\Omega_S = \mu_{S}/\hbar\) are the Rabi frequencies of the pump and Stokes fields. The two-level atom scheme in terms of 'molecular' states along with pump and Stokes fields induced transitions are shown in figure 1(b). First, one can notice from equations (2) that the \(|+\rangle\) and \((-\rangle\) states decouple, i.e. laser fields connect states only within these subsystems. Secondly, the \(|gg\rangle\) state is laser coupled only to \(|+\rangle\) states such that initially the atoms are in the \(|gg\rangle\) state, the \(|-\rangle\) subsystem is never populated, and, therefore, can be discarded in the present model which neglects decays. Next, we assume that the one-photon detuning is large \(|\Delta| \gg \delta\), \(\Omega_p\), \(\Omega_S\) such that the amplitudes \(c_{+ig}\), \(c_{r}\) and \(c_{+ri}\) can be replaced by their steady-state solutions as follows:

\[
\begin{align*}
c_{+ig} &\approx \frac{\sqrt{2}\Omega_p c_{gg} + \Omega_S c_{+rg} + \sqrt{2}\Omega_p c_{i}}{\Delta_p}, \\
c_{-ri} &\approx \frac{\sqrt{2}\Omega_p c_{+i} + \Omega_S c_{+ri} + \sqrt{2}\Omega_p c_{rr}}{\Delta_p}, \\
c_{i} &\approx \frac{\sqrt{2}\Omega_p c_{+i} + \sqrt{2}\Omega_S c_{+ri}}{\Delta_p}.
\end{align*}
\]

Expressing further the \(c_{+ig}\) and \(c_{+ri}\) in terms of \(c_{gg}\), \(c_{+rg}\) and \(c_{rr}\), we eliminate the intermediate state \(|i\rangle\) and reduce the system equations (2) to three equations for states \(|gg\rangle\), \(|+\rangle_g\) and \(|-\rangle_r\):

\[
\begin{align*}
\frac{dc_{gg}}{dt} &= -\frac{2\Omega_p^2}{\Delta_p} c_{gg} - \sqrt{2}\Omega c_{+rg} - \frac{2\Omega_p^2}{\Delta_p} c_{rr}, \\
\frac{dc_{+rg}}{dt} &= \left(\delta - \frac{\Omega_p^2 + \Omega_S^2}{\Delta_p}\right) c_{+rg} - \sqrt{2}\Omega c_{gg} - \sqrt{2}\Omega c_{rr}, \\
\frac{dc_{rr}}{dt} &= \left(\delta + V_{int} - \frac{2\Omega_p^2}{\Delta_p}\right) c_{rr} - \sqrt{2}\Omega c_{+rg} - \frac{2\Omega_p^2}{\Delta_p} c_{gg},
\end{align*}
\]

where \(\Omega = \Omega_p \Omega_S / \Delta_p\) is the two-photon Rabi frequency. One can now obtain the dressed states of the two-atom system and their energies \(e\) from the energy equation:

\[
-\left(\delta - \frac{2\Omega_p^2}{\Delta_p}\right) \left(\delta - \frac{\Omega_p^2 + \Omega_S^2}{\Delta_p}\right) e - \frac{4\Omega_p^2}{\Delta_p} \delta - V_{int} = 0.
\]

Neglecting the \(\Omega^2/\Delta_p\) term and setting \(V_{int} = 0\) allows one to obtain analytical expressions for the dressed states and their energies in the absence of the Rydberg–Rydberg interaction:

\[
\begin{align*}
e_1 &= \delta - \frac{\Omega_p^2 + \Omega_S^2}{\Delta_p}, \\
e_{2,3} &= \delta - \frac{\Omega_p^2 + \Omega_S^2}{\Delta_p} \pm \sqrt{\left(\delta - \frac{\Omega_p^2 + \Omega_S^2}{\Delta_p}\right)^2 + 4\Omega_p^2 \Omega_S^2}.
\end{align*}
\]

where \(e_{2,3}\) is related to the \(\pm\) signs, respectively.

The corresponding dressed states are as follows:

\[
\begin{align*}
|\Psi_1\rangle &= \frac{1}{\left(\delta + \frac{\Omega_p^2 + \Omega_S^2}{\Delta_p}\right)} \left(\sqrt{2}\Omega |gg\rangle - \frac{\Omega_p^2 - \Omega_S^2}{\Delta_p} |+\rangle_g - \sqrt{2}\Omega |rr\rangle\right), \\
|\Psi_{2,3}\rangle &= \frac{1}{\left(\delta + \frac{\Omega_p^2 - \Omega_S^2}{\Delta_p}\right)} \left(\frac{\Omega_p^2 + \Omega_S^2}{\Delta_p} |gg\rangle - \sqrt{2}\Omega |+\rangle_g - \frac{\Omega_p^2}{\Delta_p} |rr\rangle\right).
\end{align*}
\]

We can analyze the dressed states in the limits of a large two-photon detuning \(\delta\): \(|\delta| \gg \Omega_p^2/\Delta_p\), \(\Omega_S^2/\Delta_p\):

1) \(\delta > 0\):

\[
|\Psi_1\rangle \approx |+\rangle_g, \quad |\Psi_2\rangle \approx |rr\rangle, \quad |\Psi_3\rangle \approx |gg\rangle.
\]

2) \(\delta < 0\):

\[
|\Psi_1\rangle \approx |+\rangle_g, \quad |\Psi_2\rangle \approx |rr\rangle, \quad |\Psi_3\rangle \approx |gg\rangle.
\]

which shows that \(|\Psi_3\rangle \approx |gg\rangle\) for large \(\delta > 0\) and \(|\Psi_3\rangle \approx |rr\rangle\) for large \(\delta < 0\). As a result, one can transfer the two atoms from the \(|gg\rangle\) state to the \(|rr\rangle\) state using a negative chirp \(d\delta/dt < 0\) and back from the \(|rr\rangle\) to the \(|gg\rangle\) state using a
positive chirp $d\delta/dt > 0$. The same can be done using the $|Y_2\rangle$ state and a positive chirp to realize the $|gg\rangle \rightarrow |rr\rangle$ transfer, and a negative chirp to bring the system back into $|gg\rangle$.

Above it was assumed that the interaction in the Rydberg states is zero, which allowed us to obtain analytical expressions for dressed states and their energies. When $V_{\text{int}} \neq 0$ the energies and dressed states can be calculated numerically. We are interested in the case $V_{\text{int}} \approx \Omega$, when the dipole blockade is not working, and investigate how two atoms can be transferred to the double Rydberg state in this regime. The energies and the amplitudes of the $|gg\rangle$, $|+\rangle$, $|rg\rangle$ and $|rr\rangle$ components of the $|Y_2\rangle$ and $|Y_3\rangle$ states in the case $V_{\text{int}} = 0.5\Omega$ are shown in figures 2(a)–(c), respectively. One can see that for a large positive $\delta$ $|Y_3\rangle \approx |gg\rangle$ and for a large negative $\delta$ $|Y_2\rangle \approx |rr\rangle$ (for a large negative $\delta$ $|Y_3\rangle \approx |gg\rangle$ and for a large positive $\delta$ $|Y_2\rangle \approx |rr\rangle$), as expected from equation (6).

Figure 2 shows, therefore, that for intermediate interaction strengths the system can still be transferred from $|gg\rangle$ to $|rr\rangle$ if it adiabatically follows either $|Y_3\rangle$ for a negative chirp rate or $|Y_2\rangle$ for a positive one.

### 3. Efficiency of excitation to a double Rydberg state by ARP

In this section we numerically analyze the efficiency of two-photon excitation from $|gg\rangle$ to a double Rydberg state $|rr\rangle$ using ARP. We consider two three-level atoms interacting with pump and Stokes optical pulses and with each other via van der Waals or dipole–dipole interaction according to the Hamiltonian (1). We also take into account radiative decays from the intermediate $|i\rangle$ and Rydberg $|r\rangle$ states and describe the system using a density matrix equation:

$$\frac{d\rho}{dt} = i[H, \rho] + \mathcal{L}\rho,$$

where the Lindblad term, incorporating the decays, is as follows:

$$\mathcal{L}\rho = \sum_{j=1,2} \frac{\Gamma_j}{2} \left( 2\sigma_{rg}^+ \rho \sigma_{rg}^- - \sigma_{rg}^- \sigma_{rg}^+ \rho - \rho \sigma_{rg}^+ \sigma_{rg}^- \right)$$

$$+ \sum_{j=1,2} \frac{\Gamma_j}{2} \left( 2\sigma_{r}^+ \rho \sigma_{r}^- - \sigma_{r}^- \sigma_{r}^+ \rho - \rho \sigma_{r}^+ \sigma_{r}^- \right),$$

where $\sigma_{rg}^+ = \hat{d}^\dagger \hat{g}$, $\sigma_{rg}^- = \hat{r} \hat{i}$ and $\sigma_{r}^+ = (\sigma_{rg}^+)^\dagger$, $\sigma_{r}^- = (\sigma_{rg}^-)^\dagger$ are the raising and lowering operators for the $j$th atom, $I_r$ and $I_i$ are radiative decay rates of the intermediate and Rydberg states. The pump and Stokes pulses Rabi frequencies have a Gaussian form $\Omega_{p,S}(t) = \Omega_{0p,0S} \exp(-\mu(t-t_0)^2/2\tau_{p,S}^2)$ and detunings are $\Delta_p(t) = \omega_{p} - \omega_{p}(t) = \Delta_{0p} + \alpha (t-t_0)$, $\Delta_i(t) = \omega_{i} - \omega_{i}(t) = \Delta_{0i} + \alpha (t-t_0)$, where $\alpha$ is the linear chirp rate of the pulses, assumed equal for both.

The population $\rho_{rr}$ of the double Rydberg state, i.e. the excitation efficiency, is shown in figure 3(a) for a range of two-photon Rabi frequencies and chirp rates. In calculation parameters of $^{87}$Rb atoms were used with $|i\rangle = 5P_{3/2}$ with decay rate $\Gamma_i = 6$ MHz and $|r\rangle = 80$ S with $\Gamma_r = 485$ MHz. The Rydberg state decay has contributions from spontaneous emission with the rate $\Gamma_{i,p}^\text{BBR} = 300$ Hz and from the atom’s interaction with black-body radiation (BBR) at 300 K, which includes BBR induced decay with the rate $\Gamma_{i,p}^\text{BBR,â} = 0.42\Gamma_{i,p}^\text{BBR} \approx 126$ Hz, excitation to higher Rydberg states with the rate $\Gamma_{i,p}^\text{BBR} = 0.15\Gamma_{i,p}^\text{BBR} \approx 45$ Hz, and ionization with the rate $\Gamma_{i,p}^\text{BBR,ion} = 0.045\Gamma_{i,p}^\text{BBR} \approx 14$ Hz, resulting in the total decay rate $\Gamma_i = \Gamma_{i,p}^\text{BBR} + \Gamma_{i,p}^\text{BBR,â} + \Gamma_{i,p}^\text{BBR} + \Gamma_{i,p}^\text{BBR,ion} \approx 485$ Hz [18]. One can see that the efficiency reaches ~97% for sufficiently high two-photon Rabi frequencies and chirp rates, providing adiabatic interaction between atoms and laser pulses. Adiabaticity requires that $|d\delta/dt| \ll \Omega^2$ and $|d\delta/dt| \tau_{p,S}^2 \gg 1$ [19], where $d\delta/dt = 2\alpha$ is the two-photon chirp rate, as well as equal Rabi frequencies $\Omega_{p} = \Omega_{S}$, pulse durations $\tau_p = \tau_S$ and chirp rates of the pump and Stokes.
The interaction in Rydberg states can be used to realize a controlled phase gate $C_Z$, which acts on two-qubit states $|nm\rangle$ ($n, m = 0, 1$) as $|nm\rangle \rightarrow e^{i\phi_{nm}} |nm\rangle$. The gate can be implemented using ARP excitation in the following way. In the first step, four-level atoms (shown in figure 1(a)) interact with a chirped two-photon pulse such that the system initially in $|gg\rangle = |11\rangle$ evolves along the $|\Psi_1\rangle$ dressed state. At the end of the pulse the atoms will be transferred into $|rr\rangle$ and acquire a phase factor $\int e^\iota(t)dt$. At the same time the states $|gg\rangle = |10\rangle$ and $|gg\rangle = |01\rangle$ will evolve into $|r0\rangle$ and $|0r\rangle$, respectively, and acquire a phase factor $\int e^{-\iota(t)}dt$, where $\iota = \delta/2 - \frac{\Delta^2}{2p^2} \Omega^2 + \sqrt{\delta^2 + \Omega^2}$ are the dressed state energies for a single atom interacting with the two-photon chirped pulse [22]. One can see from equation (4) that if $V_{\text{int}} = 0$ and $\Omega_p = \Omega_S$, $\epsilon_{23} = 2\epsilon_s$ as expected. When $V_{\text{int}} \neq 0$ we can estimate the eigenenergies using equation (3) in the limit of small interaction strengths assuming $V_{\text{int}} \ll |\delta|$, $\Omega_p^2/|\Delta_p|$, $\Omega_S^2/|\Delta_p|$:}

$$
\epsilon_{23} = \delta - \frac{2\Omega_p^2}{\Delta_p} + \sqrt{\left(\delta + \frac{V_{\text{int}}}{2}\right)^2 + 4\Omega^2} \approx \delta - \frac{2\Omega_p^2}{\Delta_p} + \sqrt{\delta^2 + 4\Omega^2} + \frac{\delta V_{\text{int}}}{2\sqrt{\delta^2 + 4\Omega^2}},
$$

where $\Omega_p = \Omega_S$ was set again. It shows that for $V_{\text{int}} \neq 0$, the phase accumulated by the state $|11\rangle$, when it is transferred to $|rr\rangle$, differs from twice the phase of the states $|01\rangle$ and $|10\rangle$ when they are transferred to $|0r\rangle$ and $|r0\rangle$, respectively. We also assume that the state $|00\rangle$ is not interacting with the pulse, which can be realized, e.g. by choosing a specific polarization of the pump field. As a result, the state $|00\rangle$ will acquire zero phase.

In the second step the system returns back into qubit subspace with useful phases. To realize it one can apply the following trick [23]: (i) the chirp in the first and the second steps has the same sign such that $\delta(T + t) = -\delta(T - t)$, where $T$ is the time boundary between the steps, as shown in figure 4(a). The system then returns from $|rr\rangle$ to $|gg\rangle$ along $|\Psi_2\rangle$, and from $|rr\rangle$ and $|0r\rangle$ to $|10\rangle$ and $|01\rangle$ along $\epsilon_s$; (ii) the one-photon detuning changes sign in the second step with respect to the first one $\Delta_p \rightarrow -\Delta_p$. Provided the pump and Stokes pulses are applied symmetrically in time $\Omega_p, \Omega_S (T + t) = \Omega_p, \Omega_S (T - t)$ (see figure 4(a)), the conditions (i) and (ii) allow us to cancel the overall phase accumulated by the $|10\rangle$ and $|01\rangle$ states: $\phi_{01} = \phi_{00} = \int_{\text{step \, I}} e^\iota(t)dt + \int_{\text{step \, II}} e^\iota(t)dt = 0$. At the same time, the phase accumulated by the $|11\rangle$ state $\phi_{11} = \int_{\text{step \, I}} e^\iota(t)dt + \int_{\text{step \, II}} e^\iota(t)dt \neq 0$. By adjusting the pulse parameters and interpulse time $\phi_{11} = \pi(2n + 1)$ can be realized, which will produce the gate. For small interaction strengths one can estimate the $\phi_{11}$ phase.

4. Controlled phase gate using ARP excitation to a double Rydberg state

The interaction in Rydberg states can be used to realize a controlled phase gate $C_Z$, which acts on two-qubit states $|nm\rangle$.
using equation (9):

\[
\phi_{11} = \int_{T-\tau_a}^{T} e_1(t) dt + \int_{T}^{T+\tau_a} e_2(t) dt
\]

\[
= \int_{T-\tau_a}^{T} \left( 2e_1(t) - \frac{\delta V_{\text{int}}}{2\sqrt{\delta^2 + 4\Omega^2}} \right) dt
\]

\[
+ \int_{T}^{T+\tau_a} \left( 2e_2(t) + \frac{\delta V_{\text{int}}}{2\sqrt{\delta^2 + 4\Omega^2}} \right) dt
\]

\[
= -\int_{T-\tau_a}^{T} \frac{\delta V_{\text{int}}}{2\sqrt{\delta^2 + 4\Omega^2}} dt
\]

\[
+ \int_{T}^{T+\tau_a} \frac{\delta V_{\text{int}}}{2\sqrt{\delta^2 + 4\Omega^2}} dt
\]

\[
= 2 \int_{T}^{T+\tau_a} \frac{\delta V_{\text{int}}}{2\sqrt{\delta^2 + 4\Omega^2}} dt,
\]

where \( \tau_\text{sp} \) is the time duration of each step, assumed equal, and \( \delta(T + t) = -\delta(T - t) \) was applied in the last line. Using linearly chirped pulses with \( \delta = 2\alpha(t - t_c) \) where \( T < t_c < T + \tau_a \) and setting \( \Omega = \text{const} \) for simplicity we have

\[
\phi_{11} = \frac{V_{\text{int}}}{\alpha} \sqrt{\delta^2 + 4\Omega^2} \left[ \int_{T-\tau_a}^{T} e_1(t) dt + \int_{T}^{T+\tau_a} e_2(t) dt \right]
\]

\[
= \frac{V_{\text{int}}}{\alpha} \left( \sqrt{4\alpha^2(T + \tau_a - t_c)^2 + 4\Omega^2} - \sqrt{4\alpha^2(T - t_c)^2 + 4\Omega^2} \right).
\]

In the limit \( \alpha |T - t_c| \gg \Omega \) one obtains \( \phi_{11} \approx 2V_{\text{int}} \tau_a \), showing that the duration of each step is \( \tau_a = \pi/2V_{\text{int}} \), of the same order as the time between two STIRAP pulse sequences required to accumulate a \( \pi \) phase shift in [9]. For small interaction strengths the gate fidelity is expected to be low, because gate duration can become comparable to the Rydberg state decay time, which will result in decay induced error. For intermediate strengths \( V_{\text{int}} \sim \Omega \) the fidelity is expected to be high, and for large interaction strengths \( V_{\text{int}} \gg \Omega \) the fidelity will decrease again due to the appearance of a blockade. We numerically calculated the fidelity for a range of interaction strengths \( 0 \leq V_{\text{int}}/\Omega \leq 1 \) and the results are shown in figure 4(b), where the fidelity is defined as

\[
F = \langle \Psi_{\text{ideal}} | \rho(T + \tau_a) | \Psi_{\text{ideal}} \rangle.
\]

The expected state of the system after the gate in the absence of errors is

\[
|\Psi_{\text{ideal}}\rangle = \frac{1}{2} \left( |00\rangle + |01\rangle + |10\rangle - |11\rangle \right)
\]

and \( \rho(T + \tau_a) \) is the density matrix at the end of the second step taking into account population and coherence decays. An initial state of the system \( |\Psi\rangle = |g\rangle = |1\rangle \), i.e. there was no decay into the \( |g'\rangle = |0\rangle \) state. It

\[
\int_{T-\tau_a}^{T+\tau_a} e(t) dt
\]

\[
\int_{T-\tau_a}^{T+\tau_a} e(t) dt
\]
was also assumed that the $|0\rangle$ state was not interacting with the chirped pulse. The same parameters of $^{87}$Rb were used: the radiative decay rates $\Gamma_0 = 6$ MHz and $\Gamma = 485$ Hz, corresponding to the $5P_{1/2}$ and $80$S states [18], respectively, and the energy splitting between the qubit states of 6.835 GHz, corresponding to the hyperfine splitting between ground state $F = 2$ and $F = 1$ sublevels. During calculations the conditions of the first and second steps were applied: the Rabi frequencies were symmetric, and chirp rate and one-photon detuning were antisymmetric with respect to the step time boundary. In order to accumulate the $\psi$ phase shift in the $|11\rangle$ state the time delay between two pulse sequencies $(t_c - T)$ was adjusted every time $V_{int}$ was changed. Figure 4(c) shows the time dependence of the dressed state energy during both steps for $V_{int} = 0.5 \Omega$, with the phase $\phi_{11}$ given by the integral $\int_{T-t}^{T+t} e(t)dt$. Fidelities $\sim 94\%$ were obtained for $V_{int}/\Omega \sim 0.5$, limited by an incomplete conversion of $|gg\rangle$ into $|rr\rangle$ during the first step, i.e. less than 100% transfer efficiency $\eta_{rr}$, which resulted in a small admixture of the $|ry\rangle$ dressed state to the $|yy\rangle$ state during the second step, when the system returned from $|rr\rangle$ to $|gg\rangle$. The obtained fidelities are comparable to the ones expected in the STIRAP-based excitation scheme [9]. We checked the effect of the intermediate state decay on the fidelity, which was an important source of error in [9], and found that complete cancellation of the decay gives the fidelity increase $\sim 1\%$. The intermediate state decay is less important in our scheme due to a large one-photon detuning. Decay from the Rydberg state also does not significantly affect the gate in our case due to its short duration $\sim 2(t_c - T) \approx 300$ ns and small decay rate $\Gamma = 485$ Hz.

Another possible way to implement the controlled phase gate is to follow the same dressed state, e.g. the $|y1\rangle$, in both steps, which can be done if the chirp rate changes sign in the second step such that $\phi(T + t) = -\phi(T - t)$. In this case at the end of the second step the state $|11\rangle$ will acquire the phase shift $\phi_{11} = \int_{T-t}^{T+t} e_1(t)dt$, and the $|01\rangle$ and $|10\rangle$ states will acquire the shift $\phi_{01} = \phi_{10} = \int_{T-t}^{T+t} e_2(t)dt$. The simplest way to realize this is to adjust pulse parameters in such a way that $\phi_{01,10} = \pi$ and $\phi_{11} = 3\pi$ (for small $V_{int}$ $\phi_{11} \approx 2\phi_{01,10}$), for larger $V_{int}$ $\phi_{11} > 2\phi_{01,10}$. However, this scheme might be more challenging than the one discussed above, because two phases have to be simultaneously tuned to specific values.

The above fidelity calculations assume that the atomic motion is frozen during the gate. This assumption can be violated due to mechanical forces acting on atoms in the double Rydberg state [24]. The forces can result in excitation of higher motional states for atoms trapped in microtraps and optical lattices, resulting in undesirable entanglement between the motional and qubit states. We estimate the probability of excitation from a ground to the first excited motional state for atoms in an optical lattice. The amplitude of the first motional state after the system is deexcited from the double Rydberg state is $e_{max} = F\sigma(1 - \exp(-2\omega_0(t_c - T)))/\omega_0$, where $F = \partial V_{int}/\partial r = 6C_0/r^7 = 6V_{int}/r$, is the force acting between two atoms interacting via the van der Waals $C_0/r^6$ interaction, $\omega_0$ is the trapping potential oscillation frequency and $\delta = \sqrt{\hbar/m\omega_0}$ is the ground motional state wavefunction width, $r$ is the distance between atoms, and $2(t_c - T)$ is the time the system spends in the double Rydberg state. Assuming $V_{int} = 5$ MHz, corresponding to $r = 9.8 \mu m$ for atoms in the 80S state [25], $\omega_0 = 100$ kHz, $\delta r = 35$ nm and $t_c - T \approx 150$ ns, the probability of the motional state excitation $|e_{max}|^2 \sim 0.02$, which gives the additional error in the fidelity.

Our analysis shows that the ARP and STIRAP-type excitation [9] to the double Rydberg state, which use simple analytic pulse sequences, predict similar high controlled phase gate fidelities $\sim 94\%$ and $\sim 97\%$ in the former and the latter cases, respectively. However, these values are not good enough to allow fault-tolerant quantum computation, which requires a gate fidelity $\geq 99.9\%$ [26]. One of the strategies to increase the fidelity is to apply more complex coherent control techniques such as optimal control [27] and genetic [28] algorithms to shape laser pulses. Optimal control of a STIRAP based blockaded controlled phase gate has been analyzed recently in [20], where it was shown that the gate error can be decreased by an order of magnitude (from $10^{-3}$ to $10^{-6}$) if one uses optimized pulse sequences instead of analytic. Optimization of chirped pulses, first proposed in [29], is successfully used to achieve efficient population transfer between molecular states [30] and might help to improve the fidelity of the ARP based gate, which will be the subject of a future work.

5. Conclusion
In conclusion, we analyzed the excitation of two ground state atoms to a double Rydberg state by a chirped two-photon pulse using ARP. During ARP-type excitation, dressed states of the coupled atoms-light system provide direct connection of the $|gg\rangle$ to the double Rydberg $|rr\rangle$ state contrary to the case of resonant STIRAP [7]. Numerical analysis taking into account population and coherence decays predicts robust transfer to the $|rr\rangle$ state that can reach a high efficiency $\sim 97\%$ in the case of $^{87}$Rb atoms for intermediate interaction strengths $V_{int} \sim \Omega$. The high transfer efficiency is possible due to a large one-photon detuning allowed in the ARP scheme, minimizing losses from the fast-decaying intermediate state. The large one-photon detuning has to be compensated by high Rabi frequencies $\sim 200$ MHz of the pump and Stokes pulses to achieve adiabaticity, but they are currently within experimental reach along with required chirp rates $\sim$ hundreds MHz $\mu s^{-1}$. We also considered a controlled phase gate for two atomic qubits based on ARP transfer to interacting Rydberg states. Applying antisymmetric one- and two-photon detunings and symmetric Rabi frequencies during excitation and deexcitation steps one can cancel the phases of the $|01\rangle$ and $|10\rangle$ qubit states and tune the phase of the $|11\rangle$ state to $\pi$, producing the gate. Gate fidelities $\sim 94\%$ were numerically...
predicted at $V_{\text{int}} \sim \Omega$ for $^{87}$Rb atoms, limited by incomplete switching of the dressed states between the excitation and deexcitation steps. Our analysis shows that ARP and STIRAP-type double Rydberg state excitations using simple analytic pulse sequences are expected to achieve comparable transfer efficiencies and controlled phase gate fidelities, which are high but still insufficient for fault-tolerant quantum computation. One of the ways to increase the transfer efficiency and therefore the gate fidelity is to use more complex optimized chirped pulses.

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