Robust linear control of nonconvex battery operation in transmission systems

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Abstract—We describe a robust multiperiod transmission planning model including renewables and batteries, where battery output is used to partly offset renewable output deviations from forecast. A central element is a nonconvex battery operation model which is used with a robust model of forecast errors and a linear control scheme. Even though the problem is nonconvex we provide an efficient and theoretically valid algorithm that effectively solves cases on large transmission systems.

Index Terms. Batteries, control, robust optimization.

I. INTRODUCTION

A great deal of recent work has focused on the integration of storage into transmission systems, often in conjunction with the use of renewables. A partial list includes [2], [3], [4], [5], [6], [7], [8], [9], [10], [11]. The main goal of this work is to incorporate nonlinear and nonconvex battery operation models within a control framework that accounts for uncertainty in grid operation, specifically with regards to renewables. The control framework we study in this paper works as follows (with assumptions discussed below).

1) We consider a time horizon comprised by \( T \) periods, each of length \( \Delta \) (on the order of half an hour to one hour). The output of each generator as well as a linear control scheme governing each battery are computed at time zero, using a forecast for renewable outputs.

2) At the start of each time period the average levels of renewable outputs for that period are estimated from real-time readings. These estimations are used to set the output for each battery, through the control policy described above, and so as to offset the deviations of renewable outputs from the forecast. The output of each battery will be held constant during the current period, with additional real-time changes in renewable output handled through a standard scheme such as frequency control.

This model simplifies many details, for brevity. For example, it assumes that loads are not subject to uncertainty. Furthermore, we propose a linear control, rather than affine. An affine control governing battery operation would result on batteries supporting loads (and/or charged by standard generators). This extension is straightforward. We omit these extensions for brevity.

This broad line of control modeling is similar to recent work; see e.g. [1], [12], [13]. Our main contributions are as follows:

(a) We optimize over an explicitly nonconvex model for battery operation. Batteries exhibit numerous complex nonlinear behaviors that can be very difficult to incorporate into an optimization framework. A standard model used in the literature relies on a constant charging and a constant discharging efficiency. If a (possibly negative) amount \( E \) of energy is input into a battery, then the energy state of the battery changes by the amount \( \eta_c E^+ - \eta_d E^- \) where \( 0 < \eta_c \leq 1 \) and (resp. \( 0 < \eta_d \leq 1 \)) is the charging (discharging) efficiency and \( E^+ \) and \( E^- \) are the positive and negative parts of \( E \). Such a model is inherently nonconvex and its incorporation in optimization requires the complementarity condition \( E^+ E^- = 0 \). This condition is sometimes modeled through the use of binary variables [14] and is not a guaranteed outcome from a convex formulation. We generalize this standard model by allowing state-dependent charging and discharging efficiencies. Additionally, we use a nonconvex charging speed model.

(b) We use a nonsymmetric and nonconvex robust model for renewable output deviations from forecast. Here we note that popular stochastic distributions for wind power are nonsymmetric, e.g. Weibull distributions. The use of nonsymmetries allows for specific risk stances with respect to renewable shortfall or excess.

(c) Despite the above nonconvexities, we describe a theoretically valid and computationally practicable optimization scheme that reduces our overall problem into a sequence of convex, linearly constrained optimization problems. Our algorithm is tested on realistic large transmission systems.

The detailed battery model is given in Section II and the forecast errors model is described in Section IV. Section V presents our optimization model, our algorithm is given in Section VI and experiments in Section VII.
II. NOMENCLATURE

- $B$ bus susceptance matrix
- $P_{k,t}^g$ standard generation at bus $k$, period $t$
- $P_{k,t}^l$ load at bus $k$, period $t$
- $\delta_k^t$ phase angle at bus $k$, period $t$
- $w_{k,t} + w_k,t$ renewable output at bus $k$, period $t$
- $\mathcal{W}$ set of deviations $w_{k,t}$
- $L_{km}$ line limit for line $km$
- $\lambda_{i,j}^t$ battery control at buses $i, j$, period $t$

III. BATTERY MODEL

The accurate modeling of battery behavior and operating constraints is a nontrivial task, made difficult by the wide variety of technologies and the complex nature of the underlying chemical processes. See e.g. [15], [16]. Our model seeks to incorporate relevant battery chemistry details while resulting in a computationally accessible formulation. We use charge-dependent piecewise-linear (or -constant) models for charge/discharge efficiency and charge/discharge speed. See e.g. [17] (page 19).

Charge efficiency. Generalizing the standard model described above, we assume piecewise-constant charging/discharging efficiencies. Charging is described by a battery-specific mono-tonically increasing, piecewise linear battery charging function $C(x)$, where $x$ represents electrical energy injection. Suppose that the battery holds (chemical energy) charge $y$, and that we input electrical energy ($= power \times time$) $\Sigma \geq 0$. Then the charge of the battery will increase to $y + C(C^{(-1)}(y) + \Sigma)$, where $C^{(-1)}$ is the inverse function of $C$. See Figure 1. The derivative of $C(x)$ (when it exists) is the charging efficiency at that point of the curve. We will assume a similar discharge function.

As in prior work (see e.g. [14]) we are given an operating range $[E^{\min}, E^{\max}]$ for battery charge. Our detailed model works as follows. Let $[c_0, c_1, \ldots, c_K]$ where $c_0 = C^{(-1)}(E^{\min})$ and $c_K = C^{(-1)}(E^{\max})$ be the breakpoints of the charging function. We impose conditions (a) and (b):

(a) We require that the battery charge always remain in the range $[E^{\min}, E^{\max}]$.

(b) For each $t > 1$, suppose that at the start of period $t$ battery charge lies in the range $[C(c_s), C(c_{s+1})]$ for some $s$. Let $t' > t$, and suppose our control mechanism never discharges the battery in periods $t, t+1, \ldots, t'$. Then we require that the maximum electrical energy input into the battery, in those periods, is at most $C^{(-1)}(E^{\max}) - c_s$. A similar statement is made in case we never charge the battery in periods $t, t+1, \ldots, t'$.

Constraint (b) amounts to a more detailed approximation of the charge and discharge functions than provided by (a) alone.

Charge speed. Here we want to constrain the maximum increase in battery charge in a period of length $\Delta$ as a function of the battery charge at the start of the period. It will be computationally more convenient to state it in terms of input electrical energy. We impose:

(c) Suppose that at the start of some period $t$ the charge of the battery, is in the range $[C(c_s), C(c_{s+1})]$ (for some $s$). Then during period $t$ we can input into the battery electrical energy at most $v_s$. Here, $v_0, \ldots, v_K$ are given limits and for convenience $c_0, \ldots, c_K$ are the breakpoints at the charging function. The definition is necessarily ambiguous at the breakpoints, and it implies state-dependent maximum (instantaneous) power injection limits.

IV. DATA ROBUSTNESS MODEL

Our linear control relies on estimations on the quantities $w_{k,t}$, formally defined as follows.

Definition. $w_{k,t}$ is the average deviation of renewable output at bus $k$ during period $t$.

We must therefore account for intrinsic stochastic variability on renewable output and also on measurement errors and noise. We rely on a robust optimization model, which we term a concentration model, which is given by nonnegative matrices $K^+$ and $K^-$, and a vector $b$, and corresponds to the set $\mathcal{W}$ of all $w$ satisfying

$$K^+ w^+ + K^- w^- \leq b$$

(1)

Here, $w^+$ is the vector with entries $w^+_{k,t} = \max\{w_{k,t}, 0\}$ and likewise with $w^-$. We note that the description (1) is nonconvex. It can be used to model bounds on the individual quantities $w^+_{k,t}$ and $w^-_{k,t}$, and allows for asymmetries and correlation both across time and buses. We assume that the set of $w$ satisfying (1) is full dimensional.
A special (though symmetric) case of this model is given by “uncertainty budgets” models (see [13, 19]), given for example by the conditions

$$|w_{i,k,t}| \leq \gamma_{t,k}, \text{ all } t \text{ and } k \tag{2}$$

$$\sum_{k} (\gamma_{t,k})^{-1} |w_{i,k,t}| \leq \Gamma^t \text{ all } t \tag{3}$$

Here, the $\gamma_{t,k}$ and $\Gamma^t$ are parameters used to model risk aversion, which can be chosen using data-driven techniques. See [20]. We can easily adapt these constraints so as to include cross-time correlation.

Alternatively we could rely on chance-constrained models (or, better, on distributionally robust chance-constrained models). A technical hazard arising from the modeling of battery behavior can be outlined as follows. Suppose that $x(t)$, $t = 1, \ldots, T$ is a discrete-time stochastic process. Even if we understand this process well enough so that e.g. we can compute tail probabilities for each individual $x(t)$, the partial sums $S(k) = \sum_{t=1}^{k} x(t)$ will in general be much more complex random variables, except in special cases (i.e. gaussian distributions). This difficulty, combined with the goal of safe battery modeling, and the fact that we need to account for observation errors, leads us to rely on a robust model in this work.

V. OPTIMIZATION MODEL

Our optimization model will compute outputs $P_{k,t}^q$ for standard generation for each bus $k$ and period $t$ (fixed at zero if there is no generator at that bus) and control parameters $\lambda_{i,j}^t$ for each period $t$ and pair of buses $i, j$. This computation is assumed to take place at time zero based on forecast data as discussed above. We next describe the generic control scheme as well as a practical special case.

In the most general case, our control works as follows: at the start of period $t$ we compute estimates $w_{j,t}$ for all the values $w_{j,t}$, and during period $t$ the electrical power output of the battery at bus $i$ will be set to

$$- \sum_{j} \lambda_{i,j}^t w_{j,t}, \tag{4}$$

where $\lambda_{i,j}^t \geq 0$. A simplified scheme relies, for each bus $i$ holding a battery, on a set $R(i)$, the set of renewable buses that the battery at $i$ responds to. In this scheme the output at battery at $i$ in period $t$ will be of the form $-\lambda_{i,j}^t \sum_{j \in R(i)} w_{j,t}$. This simplified scheme only needs an estimate of the (random) aggregate quantities $\sum_{j \in R(i)} w_{j,t}$. Even more specialized cases are those where $R(i)$ = all buses, and where $R(i)$ = $i$ (a battery at each renewable bus). Regardless of the special case, let us denote by $\Lambda^t$ the matrix with entries $\lambda_{i,j}^t$, and by $w^t$ the vector with entries $w_{j,t}$. In what follows, $\Lambda$ is a vector that includes all parameters $\lambda_{i,j}^t$.

Our optimization problem is a robust multi-period DC-OPF-like problem, which we term BATTOPF:

$$\min_{P^g, \Lambda} \sum_{t} \sum_{k} c_{k,t}(P_{k,t}^q) \tag{5a}$$

s.t. $0 \leq P_{k,t}^q \leq P_{k,t}^{q \text{ max}}$ all period $t$ and buses $k$, (5b)

$0 \leq \lambda_{i,j}^t \text{ all period } t \text{ and buses } i,j, \tag{5c}$

and (5d)-(5f) feasible for all period $t$, and all $w \in \mathcal{W}$:

- $B \theta^t = P_{t}^{d} + \bar{w}_{t} + w_{t} - \Lambda^t w_{t} - P_{t}^{d}$ \tag{5d}
- line limits in period $t$, using phase angles $\theta^t$ \tag{5e}
- battery operation constraints in period $t$ \tag{5f}

In this formulation the $c_{k,t}$ are standard OPF generation cost functions (which we assume convex), $P_{k,t}^{q \text{ max}}$ is the maximum generation at bus $k$ in period $t$, $\Lambda^t$ is the matrix with entries $\lambda_{i,j}^t$, and $P_{t}^{d}, P_{t}^{d}$ and $w_{t}$ are (respectively) the vectors with entries $P_{i,j}^{d}$, $P_{i,j}^{d}$ and $w_{i,t}$. The formulation seeks a conventional generation plan plus a control algorithm so as to minimize generation costs subject to remaining feasible under all data scenarios. Constraint (5d) is the standard DC flow balance system.

Problem BATTOPF presents challenges because of the non-convexities in the modeling of (5f). Further, the concentration model $\mathcal{W}$ is provided through a nonconvex description, and, as a result, the standard linear programming duality approach in robust optimization cannot be applied. In Section VI we will provide an efficient algorithm for solving this problem. We remark that (5d), which must hold for all $w \in \mathcal{W}$, requires that for all $t$,

$$0 = \sum_{i} \left( P_{i,t}^{d} + \bar{w}_{i,t} + w_{i,t} - \sum_{j} \lambda_{i,j}^t w_{j,t} - P_{i,t}^{d} \right). \tag{5d}$$

Since $0 \in \mathcal{W}$ in particular we have that $0 = \sum_{i} (P_{i,t}^{d} + \bar{w}_{i,t} - P_{i,t}^{d})$, and as a result for all $w \in \mathcal{W}$

$$0 = \sum_{i} w_{j,t} \left(1 - \sum_{i} \lambda_{i,j}^t \right). \tag{5d}$$

Since the set $\mathcal{W}$ is assumed full-dimensional, and (5d) describes a hyperplane in $w$-space, we conclude that (5d) is feasible (for a given $P^d$ and $\Lambda$) if and only if

$$1 = \sum_{i} \lambda_{i,j}^t \quad \forall \ t, j. \tag{5d}$$

VI. ALGORITHM

As described above it appears difficult to produce an explicit, practicable convex formulation for BATTOPF. Here instead we provide an efficient cutting-plane procedure. Even though the outline of the algorithm below is standard [21], Steps 2 and 3 are novel and critical.

The algorithm relies on a linearly constrained relaxation for BATTOPF termed the master formulation, with objective (5f). At the start of the procedure the master formulation includes constraints (5b), (5c) and (7). Each iteration produces
a candidate solution \((\hat{p}^g, \hat{\lambda})\) for BATTOPF. If this candidate is infeasible for BATTOPF, that is to say there is a realization of renewable deviations under which the candidate fails to satisfy \(5c-5f\), then the algorithm identifies a linear inequality that is valid for all feasible solutions for BATTOPF, yet violated by the candidate. This inequality is then added to the master formulation. Thus, at each iteration the master formulation is a relaxation for BATTOPF, and hence if at some iteration the current candidate is feasible for BATTOPF then it is optimal. Formally, the algorithm is as follows:

1. Solve the master formulation, with solution \((\hat{p}^g, \hat{\lambda})\).
2. Check whether \((\hat{p}^g, \hat{\lambda})\) is feasible for BATTOPF, that is to say, \((\hat{p}^g, \hat{\lambda})\) satisfies \(5c-5f\) for every \(w \in W\). If so \((\hat{p}^g, \hat{\lambda})\) is optimal for BATTOPF. STOP.
3. Otherwise, there is \(\hat{w} \in W\) such that \((\hat{p}^g, \hat{\lambda})\) does not satisfy \(5c-5f\), when the renewable deviations are given by \(\hat{w}\). Compute an inequality

\[
\alpha^T P^g + \beta^T \Lambda \geq \alpha_0
\]

which is satisfied by all solutions to BATTOPF, but violated by \((\hat{p}^g, \hat{\lambda})\), add it to the master formulation, and go to 1.

Steps 2 and 3 are both nontrivial because of the nonconvexity of the battery model and the nonconvex description of the uncertainty set \(W\). See Section VII for details.

VII. NUMERICAL EXPERIMENTS

The algorithm was implemented using Gurobi [22] as the LP solver. The first set of numerical tests study the scalability of the algorithm as the number of time periods increases. For these tests we used the winter peak Polish grid from MATPOWER [23], with 2746 buses, 3514 branches, 388 generators, base load (approx.) 24.8 GW, 32 wind farms, with forecast output 4.5 GW and 32 batteries, with total initial charge approx. 3.2 GWh after unit conversion. We used the uncertainty budget robustness model with an implied forecast output 4.5 GW, amounts to almost 50% of total load (315 MW). The minimum (DC-OPF) generation cost for Case9 is approximately 5216, and due to the large renewable penetration, in our example the cost in the nominal case (no forecast errors) is much lower: 2384.75. However, this solution is certainly not robust.

Next we describe, in greater detail, a one-period example derived from the “Case9” dataset in Matpower. This case was modified by adding renewables and batteries.

- Renewables are located at buses 4 and 8, with forecast output 50 and 100 MW, respectively. We used a concentration model given by the constraints \(-50 \leq w_{4,1} \leq 0, -100 \leq w_{5,1} \leq 0, 2|w_{4,1} + w_{5,1}| \leq 100\). In particular the system may experience a loss of up to 100 MW in renewable power.
- Identical batteries are located at buses 4 and 9, with charging efficiency 1.0 and discharging efficiency 0.8 (note that since we only consider renewable decreases, only discharges will take place). Both batteries start with 80 units of charge, and the instantaneous maximum (power) discharge rate is set at 100 MW. Each of the batteries to the aggregation of renewable errors, i.e. the output of each battery \(i\) is of the form \(-\lambda_i (w_{4,1} + w_{5,1})\).
- The limits of lines \(4-5, 5-6, 6-7, 7-8, 8-9\) and \(9-4\) were reduced to 50, 75, 50, 90, 100 and 70 MW (resp.)

The forecast renewable generation, 150 MW, amounts to almost 50% of total load (315 MW). The minimum (DC-OPF) generation cost for Case9 is approximately 5216, and due to the large renewable penetration, in our example the cost in the nominal case (no forecast errors) is much lower: 2384.75. However, this solution is certainly not robust.

The solution to the robust problem has cost 2188.05 (less than one percent increase over the non-robust solution) and is given by \(\lambda_4 = 0.36, \lambda_9 = 0.64, \hat{p}^g\). We can briefly examine the feasibility of this solution as follows. Consider, first, the battery charge constraints, and suppose renewable output drops by 100 MW (the maximum allowed by the model), then the charge of the battery at bus 9 will decrease by 0.64 \times 100 / 8 = 80 units. Since the battery starts with 80 units it will therefore drain completely (but not go negative). Likewise, the charge of the battery at bus 4 will likewise drop by 0.36 \times 100 / 8 = 45 units from its original 80 units.

VIII. TECHNICAL DETAILS

Here we outline efficient procedures for 2 and 3. The battery constraints are the most delicate. For brevity we focus on battery speed, and the next result is key.

Lemma 2. Suppose that \((\hat{p}^g, \hat{\lambda})\) is an infeasible candidate solution for BATTOPF. Let \(\hat{w} \in W\) be a realization of renewable deviations under which \((\hat{p}^g, \hat{\lambda})\) fails to satisfy the battery operation constraint at some period \(t\) and bus \(k\). Then, without loss of generality, \(\hat{w}\) is sign consistent for all periods \(h < t\), that is to say, either \(\hat{w}_{i,h} \geq 0\) for all buses \(i\) and periods \(h < t\), or \(\hat{w}_{i,h} \leq 0\) for all buses \(i\) and periods \(h < t\).

Lemma 2 (proved later) has an important corollary. Namely, Step 2 gives rise, for each time period, to two linear programs for each battery speed constraint. We discuss these points next.

Consider, first, the battery speed constraint for a battery at bus \(i\) and period \(t\). Given \(\hat{\lambda}\) this constraint will fail to hold if
there is \( \dot{w} \in \mathcal{W} \) such that in period \( t \) the battery charge is in some interval \( (C(e_s), C(e_{s+1})) \) and yet the electrical power input into the battery, as per \( \Lambda \), exceeds \( v_s \). Since we can assume that \( \dot{w} \) is sign-consistent consider the case where all \( w_{i,t} \geq 0 \) for \( i < t \). If at time zero the battery has charge \( E_0 \), then

\[
\begin{align*}
\lambda_s & \leq E_0 + \sum_{h=1}^{t-1} \sum_j \lambda_{i,h} \dot{w}_{j,t} \leq e_{s+1} \quad \text{and} \quad (9a) \\
\sum_j \lambda_{i,j} \dot{w}_{j,t} & > v_s. \quad (9b)
\end{align*}
\]

Thus, the existence of such a vector \( \dot{w} \) can be tested by solving the linear program where we maximize \( \sum_j \lambda_{i,j} \dot{w}_{j,t} \) subject to \( (9a) \), \( K^+ \dot{w} \leq b \) (the positive part of the concentration model), and \( \dot{w} > 0 \). The case \( \dot{w} \leq 0 \) is similar. If a vector \( \dot{w} \) satisfying \( (9) \) is found, then it proves that any control \( \Lambda \) feasible for \textsc{battopf} must violate at least one of the three inequalities in \( (9) \), giving rise to a “disjunctive cut” \( [24] \).

In terms of Steps 2 and 3 of our algorithm we have to consider, lastly, line limits. To fix ideas consider the constraint that the power flow on line \( k,m \) does not exceed its limit \( L_{km} \) in period \( t \). Using shift factors (i.e., a pseudoinverse for \( B \)) we need to check that

\[
\nu_{km}^T \left( P_{k,m}^p - P_{k,m}^d + \dot{w}_{k,m} - \dot{A}_i w_{l,i} \right) \leq L_{km} \quad (10)
\]

for all \( w \in \mathcal{W} \), where \( \nu_{km} \) is an appropriate vector. It suffices to maximize the right-hand side of \( (10) \) over all \( w \in \mathcal{W} \). This can be done by solving the LP

\[
\begin{align*}
\max \nu_{km} \left( P_{k,m}^p - P_{k,m}^d + \dot{w}_{k,m} - \dot{A}_i w_{l,i} \right) \\
\text{s.t.} \quad K^+ \dot{w'} \leq b \\
\dot{w'} = \dot{w}(p) - \dot{w}(n), \\
\dot{w}(p) \geq 0, \quad \dot{w}(n) \geq 0. \quad (13)
\end{align*}
\]

Here \( \dot{w} \) represents \( w \), and \( \dot{w}(p) \) and \( \dot{w}(n) \) represent, respectively, \( w^+ \) and \( w^- \). Since \( K^+ \geq 0 \) and \( K^- \geq 0 \) there is an optimal solution to this LP satisfying the complementarity condition \( \dot{w}_{k,m}^{(p)} \dot{w}_{k,m}^{(n)} = 0 \) for all buses \( k \) and periods \( h \). Thus \( \dot{w} \in \mathcal{W} \) and \( (13) \) yields the concentration model \( (1) \).

If the value of the LP exceeds \( L_{km} \), then, where \( \dot{w} \in \mathcal{W} \) is an optimal solution satisfying complementarity, the inequality

\[
\nu_{km} \left( P_{k,m}^p - P_{k,m}^d + \dot{w}_{k,m} - \dot{A}_i w_{l,i} \right) \leq L_{km} \quad (14)
\]

is valid for \textsc{battopf} but violated by \( (P_{k,m}^{\dot{w}'}, \dot{A}) \).

**Proof of Lemma 2.** We sketch a proof of Lemma 2 in the case of the charge speed constraints. Suppose that \( \Lambda \) violates the charge speed constraint for some battery \( k \) during period \( t \), under deviations \( w \in \mathcal{W} \). Let \( E_0 \) be the battery charge at time zero (known), and the energy state at the start of period \( t \), under deviations \( w \), be \( E' \). Then \( E' \in (e_s, e_{s+1}) \) (for some \( s \)) and the energy input into the battery during period \( t \) exceeds the maximum \( v_s \). Assume that \( C^{-1}(E') \geq C^{-1}(E_0) \) (the reverse case is similar). Let \( \bar{w}_{i,h} = w_{i,h}^+ \) for all buses \( i \) and periods \( h < t \), and \( \bar{w}_{i,t} = w_{i,t}^+ \) otherwise. Then \( \bar{w} \in \mathcal{W} \). The deviations \( \bar{w} \) do not cause discharging before period \( t \), and so the at the start of period \( t \) the energy state will be at least \( E' \). Since the charging function is monotonically increasing and continuous, for some value \( 0 \leq \kappa \leq 1 \) the vector \( \bar{w} \) defined by \( \bar{w}_{i,h} = \kappa \bar{w}_{i,h} \) for \( h < t \), and \( \bar{w}_{i,t} = w_{i,t}^+ \) otherwise, is such that under deviations \( \bar{w} \) the energy state at the start of period \( t \) will be exactly \( E' \). Yet, clearly, \( \bar{w} \in \mathcal{W} \), \( \bar{w}_{i,h} \geq 0 \) for all \( i \) and \( h < t \).

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