Measuring $\theta_{12}$ Despite an Uncertain Reactor Neutrino Spectrum

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Abstract

The recently discovered 5 MeV bump highlights that the uncertainty in the reactor neutrino spectrum is far greater than some theoretical estimates. Medium baseline reactor neutrino experiments will deliver by far the most precise ever measurements of $\theta_{12}$. However, as a result of the bump, such a determination of $\theta_{12}$ using the theoretical spectrum would yield a value of $\sin^2(2\theta_{12})$ which is more than 1% higher than the true value. We show that by using recent measurements of the reactor neutrino spectrum the precision of a measurement of $\theta_{12}$ at a medium baseline reactor neutrino experiment can be improved appreciably. We estimate this precision as a function of the $^9$Li spallation background veto efficiency and dead time.
In about 5 years the largest liquid scintillator detectors ever built will be used to detect reactor neutrinos at the experiments JUNO [1] and RENO 50 [2]. The often-stated goal of these experiments is the determination of the neutrino mass hierarchy, following the strategy of Petcov and Piai [3]. Obtaining the required precision for a determination of the hierarchy will be challenging [4, 5, 6]. On the other hand, whether or not this precision can be achieved, there is no doubt that such experiments can provide by far the most precise measurement yet of $\theta_{12}$.

Imperfect knowledge of the reactor neutrino spectrum is a leading source of uncertainty in the measurement of $\theta_{12}$. Studies of this measurement use the latest reactor neutrino flux model from Ref. [7]. They also use the uncertainties quoted in that paper. Nonetheless, as the author himself stated in Ref. [8], the uncertainty quoted in Ref. [7] reflects only a subset of the sources of uncertainty in the analysis and so in fact yields only a lower bound on the true uncertainty. Indeed, a widely accepted explanation for the reactor anomaly of Ref. [9] is that the uncertainty of reactor neutrino fluxes is systematically underestimated in the literature.

Recently a 5 MeV bump in the ratio of the measured reactor neutrino spectrum to the theoretical spectrum of [7] has been discovered by RENO [10, 11], confirmed by Double Chooz [12] and quantified by Daya Bay [13]. The amplitude of this bump is more than 10%, corresponding to $4\sigma$ in terms of the theoretical reactor flux uncertainties of Ref. [7]. Therefore it is clear that the difference between the true reactor spectrum and that of Ref. [7] is appreciably larger than the subset of the uncertainties which were quantified in that work.

What effect does this have on a determination of $\theta_{12}$? Let us fix the neutrino mass splittings to be

$$\Delta M_{31}^2 = 2.4 \times 10^{-3} \text{eV}^2, \quad \Delta M_{21}^2 = 7.5 \times 10^{-5} \text{eV}^2$$

with the normal mass hierarchy and the relevant neutrino mixing angles to be

$$\sin^2(2\theta_{13}) = 0.089, \quad \sin^2(2\theta_{12}) = 0.857.$$  

We normalize the $\nu_e$ flux at JUNO by setting the number of IBD events to be $10^5$ for a 6 year run at a baseline of 58 km, but we adapt the correct baselines from Refs. [6, 1].

For the calculation of $\chi^2$, in addition to $\theta_{12}$, we minimize three pull parameters corresponding to the normalization of the reactor spectrum and background, with uncertainties of 5% and 1% respectively, and also the value $\theta_{13}$ with an uncertainty of 0.01. Variations of the later two uncertainties have little effect on our results. Then, assuming a perfectly
understood nonlinear energy response for the detector, we find that if the true reactor spectrum is that observed by Daya Bay in Ref. [13] but it is fit to the theoretical spectrum of Ref. [7] then the lowest $\chi^2$ fit would arise with a value of $\sin^2(2\theta_{12})$ which is, depending on the background rejection efficiency, between 0.010 and 0.011 too high. This is because it leads to less events at the solar oscillation maximum, around 3 MeV, and more events at higher energies, away from the maximum.

By comparison, studies in the literature on the precision of a measurement of $\sin^2(2\theta_{12})$ using the uncertainty reported in Ref. [7] estimate a precision of, for example, 0.3% including the uncertainty caused by a model of the detector’s nonlinear energy response [14]. Thus, were $\theta_{12}$ determined using the theoretical model [7] of the reactor spectra then the value obtained would differ from the true value by four times the expected uncertainty.

In this note we would like to observe that the precise measurements of the reactor spectrum by the Daya Bay [13] and at the RENO near detector [11] in fact allow for a precise determination of $\theta_{12}$. Of course the uncertainty in $\theta_{12}$ is highly dependent upon the shape of the difference between the true spectrum and measured spectrum. This shape is not known. However from Fig. 5 of Ref. [13] or Fig. 6 of [11] one can estimate the magnitude of the uncertainty and also, due to the binning, one can estimate the variation of the spectrum on energy scales of order the bin size or larger.

As the entire spectrum, as measured at JUNO or RENO 50, corresponds to only half of a $1 - 2$ flavor oscillation, only such broad features of the spectrum will be important for measuring $\theta_{12}$. Therefore even if the underlying reactor spectrum has a rich structure at scales of order 200 keV or smaller, this will have no effect on the determination of $\theta_{12}$. On the other hand the determination of the hierarchy depends on $1 - 3$ oscillations which have a much shorter wavelength and so may be affected by such a substructure in the reactor spectrum [15], an effect which may even be amplified by the self-calibration of Ref. [1].

To estimate the effect of the unknown spectrum on the determination of $\theta_{12}$, we need to choose a model for the spectrum which is consistent with the uncertainties reported in Refs. [11, 13]. For simplicity we choose two models of the deformation of the spectrum. The first is linear model and the second is a Gaussian model peaked near the $1 - 2$ oscillation maximum. The corresponding $1\sigma$ deformations $\delta N(E)$ in the spectrum $N(E)$ at energy $E$ are defined to be

$$\frac{\delta N(E)}{N(E)} = \epsilon \left( \frac{E}{\text{MeV}} - 4.6 \right) \quad \text{and} \quad \frac{\delta N(E)}{N(E)} = -\epsilon \exp \left[ -\left( \frac{E}{\text{MeV}} - 3 \right)^2 \right]$$

respectively. The zero-point, 4.6 MeV, is chosen for convenience so as to fix the total number of events. In Fig. 1 we plot these deformations with the parameter $\epsilon$ chosen to roughly reflect
the uncertainty reported in Refs. [11, 13]. This estimation is necessarily imprecise for several reasons. First, we do not have the covariance matrix for the uncertainties in the various bins. Second, the reactor spectrum depends on the isotope ratios which depend on the reactor and are also time-dependent. While current experiments may use the time dependence to obtain the spectrum for each isotope, the uncertainty in this extrapolation will necessarily be larger than the total uncertainty in the reactor spectrum used here. There are also subdominant reactor-dependent contributions arising from spent fuel at the reactor site.

We use the cosmogenic $^9$Li background of Ref. [16] and calculate the shift $\delta(\sin^2(2\theta_{12}))$ of the best fit $\sin^2(2\theta_{12})$ resulting from the shift in the spectrum (3) as a function of the background rejection efficiency and dead time for a 6 year JUNO run. We have checked that the shift in each model is, to a very good approximation, proportional to $\epsilon$. Note that $\delta(\sin^2(2\theta_{12}))$ can be affected by a shift in the best fit pull parameters. This effect depends entirely upon the shape of the unknown spectrum deformation and for various models amplifies or reduces $\delta(\sin^2(2\theta_{12}))$. In Fig. 2 we demonstrate this effect by comparing the shift with and without fitting the pull parameters. Here, a lower background rejection allows the degeneracy between the background normalization, reactor flux normalization and $\theta_{12}$ to generate a spurious shift in $\theta_{12}$. To avoid this spurious dependence on our choice of model, below we fix the pull-parameters when determining $\delta(\sin^2(2\theta_{12}))$. As a consequence of this choice we will systematically underestimate the uncertainty in $\theta_{12}$, although we believe that this bias is smaller than many other uncertainties in our analysis.

Assuming a true value of $\sin^2(2\theta_{12}) = 0.8570$, we find, in the case of the $\epsilon = 0.01$
Figure 2: The magnitude of the shift in the best fit value of $\sin^2(2\theta_{12})$ corresponding to the linear model with deformation $\epsilon = 0.01$ (dashed) the Gaussian model with $\epsilon = 0.025$ (dot-dashed) and a 0.004 shift of $\sin^2(2\theta_{12})$ (solid) are shown in black. The shifts with these deformations divided by two are shown in blue, showing that to a good approximation the shift in $\theta_{12}$ is proportional to $\epsilon$. On the left the pull parameters are fit, on the right they are fixed to their default values.

Figure 3: The magnitude of the shift in the best fit value of $\sin^2(2\theta_{12})$ corresponding to the $\epsilon = 0.01$ linear model (left) and the $\epsilon = 0.025$ Gaussian model (right). Each curve represents a different dead time, in ascending order from 0% to 50% in steps of 10%.
The uncertainty \( \sigma \) in the best fit value of \( \sin^2(2\theta_{12}) \), assuming a perfectly understood reactor spectrum, optimizing all pull parameters to minimize \( \chi^2 \) linear model, that the best fit value of \( \sin^2(2\theta_{12}) \) corresponding to Eq. (3) is between 0.8586 and 0.8591 for every value of the dead time and background rejection efficiency sampled, corresponding to a 0.2% overestimation of \( \sin^2(2\theta_{12}) \). The shift \( \delta(\sin^2(2\theta_{12})) \) as a function of the rejection efficiency and dead time is plotted in Fig. 3. While this result of course does depend on our choice of model (3) for the unknown part of the reactor spectrum, we expect that the shape of our final uncertainty will be roughly model-independent. In a companion paper we will use this shape to determine the optimal \(^9\text{Li} \) veto strategy and muon tracking requirements for a measurement of \( \theta_{12} \).

To determine other contributions to the precision of a measurement of \( \sin^2(2\theta_{12}) \), we fix the reactor flux to the model of Ref. [7] and use the Asimov data set to determine the value of \( \sin^2(2\theta_{12}) \) for which, when choosing the pull parameters to minimize \( \chi^2 \), one obtains \( \chi^2 = 1 \). The resulting 1\( \sigma \) uncertainties are summarized in Fig. 4. In Fig. 5 we add the result in quadrature to \( \delta(\sin^2(2\theta_{12})) \) to obtain the final uncertainty \( \sigma_{\text{tot}}(\sin^2(2\theta_{12})) \). As can be seen, using the recent measurements [11, 13] one can reduce the uncertainty in \( \sin^2(2\theta_{12}) \) to about 0.3%−0.5%, which is roughly in line with the stated goals of the experimental collaboration. A more precise measurement of the reactor spectrum in the future may reduce this [15], but not beyond the uncertainty displayed in Fig. 4.

To determine the precision of a measurement of \( \sin^2(2\theta_{12}) \) if the third and fourth Taishan reactors are not built is straightforward. These account for 26% of the total thermal power expected at the Taishan and Yangjiang reactor complexes. Therefore, if the dead time is \( \tau \), one can still read the resulting uncertainties off of Fig. 4. However, instead of the curve
Figure 5: The x-axis and curves are as in Fig. 3. The sum in quadrature $\sigma_{tot}$ of the uncertainty $\sigma$ and the shift $\delta$ for the $\epsilon = 0.01$ linear (left) and $\epsilon = 0.025$ Gaussian (right) models corresponding to $\tau$, one must use the curve corresponding to a dead time

$$\tau' = 0.76\tau + 0.24. \quad (4)$$

At first glance the fact that our final precision is of the same order as that obtained in previous studies might suggest that this analysis has been trivial. However we would like to point out that this coincidence is accidental, caused by the fact that the theoretical uncertainties of Ref. [7] are similar in magnitude to last year’s observational uncertainty [11, 13]. Had we used older data, or last year’s data from Double Chooz [12] then the new uncertainty would have been much larger. Indeed the two analyses are quite different. Traditional estimates of the precision of a measurement of $\theta_{12}$, such as that in Ref. [14], are quite precise as they use the uncertainty in [7] for which the full covariance matrix is given. However, for an analysis using the theoretical spectrum of Ref. [7], they are nonetheless inaccurate as that uncertainty was always intended as a lower bound and is now known to be smaller than the error by a factor of four. On the other hand, as the uncertainties in our analysis are observational, there is no such bias. Nonetheless, as we do not have a covariance matrix for these uncertainties, we were forced to adopt a model for the unknown deformation in the spectrum and our result is, at this time, somewhat model-dependent. This dependence on the model may be removed by a reanalysis once a covariance matrix for the uncertainty in the spectrum resulting from each of the four dominant fissile isotopes is released.

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