Hyperon resonances coupled to pseudoscalar- and vector-baryon channels

K. P. Khemchandani\textsuperscript{1a}, A. Martínez Torres\textsuperscript{2b} and J. A. Oller\textsuperscript{3c}

\textsuperscript{1} Departamento de Ciências Exatas e da Terra, Universidade Federal de São Paulo, Campus Diadema, Rua Prof. Artur Riedel, 275, Jd. Eldorado, 09972-270, Diadema, SP, Brazil.

\textsuperscript{2} Instituto de Física, Universidade de São Paulo, C.P 66318, 05314-970 São Paulo, SP, Brazil.

\textsuperscript{3} Departamento de Física. Universidad de Murcia. E-30071 Murcia. Spain.

Abstract

We study hyperon resonances by solving coupled channel scattering equations. The coupled systems include pseudoscalar- and vector-baryon channels. The parameters of the model are restricted by making a $\chi^2$-fit to the cross section data on processes: $K^- p \rightarrow K^- p$, $K^- p \rightarrow K^0 n$, $K^- p \rightarrow \eta \Lambda$, $K^- p \rightarrow \pi^0 \Lambda$, $K^- p \rightarrow \pi^0 \Sigma^0$, $K^- p \rightarrow \pi^\pm \Sigma^\mp$. Data on the energy level shift and width of the 1s state of the kaonic hydrogen, as well as some cross section ratios near the threshold are also considered in the fit. Two type of fits are found as a result. In both cases, the properties of $\Lambda(1405)$ are well reproduced. In addition to this, a $\Sigma$ state is also found with mass around 1400 MeV. Cross sections, obtained with one of the two fits, are found to stay close to the data at energies away from the thresholds too, and as a result resonances with higher masses have also been studied.

\textsuperscript{a} kanchan.khemchandani@unifesp.br
\textsuperscript{b} amartine@if.usp.br
\textsuperscript{c} oller@um.es
I. INTRODUCTION

Investigating low-energy meson-baryon interaction, with nonzero strange quantum number, is of great importance to several interrelated topics in nuclear and hadron physics, such as the determination of the nature of the low-lying hyperons \[1\]–\[9\], the existence of kaonic-nuclear bound states, which has motivated several experiments \[10\]–\[12\], studies of kaon producing reactions which are, in turn, useful to understand the interactions of kaons in a dense medium \[13\], etc. The key motivational idea behind several related works is that the strangeness \(-1\) meson-baryon interaction is attractive in nature, and it is especially interesting in the \(s\)-wave since, as now widely accepted, it generates the isoscalar resonance \(\Lambda(1405)\) (the list of references on this topic is extensive, but for some of the recent works, we refer the reader to Refs. \[14\]–\[22\]).

There exist evidences for the presence of an isovector resonance too in nature, with its origin lying in the meson-baryon dynamics, with a mass similar to \(\Lambda(1405)\) \[3\], \[23\]–\[29\]. However, the case is less studied, as compared to \(\Lambda(1405)\), and the properties of the low-lying \(1/2^- \Sigma(s)\) obtained from different works are different. In Ref. \[3\], a coupled channel study of pseudoscalar-baryon systems was made using a kernel arising from \(s\), \(u\)-channel and a contact interaction obtained from the lowest order chiral Lagrangian, and the subtraction constant required to calculate the loop function were constrained by fitting relevant data available (\(K^-p \rightarrow \bar{K}N, \pi\Sigma, \pi\Lambda\) cross sections and different cross-section ratios among these processes at the \(K^-p\) threshold, as well as the \(\pi^+\Sigma^-\) mass distribution). As a result, in the case of isospin 1, two \(\Sigma\) states were found near the \(\bar{K}N\) threshold: \(1440 - i70\) MeV and \(1420 - i42\) MeV. The work was further extended by considering next-to-leading-order contributions from the chiral Lagrangian \[23\] and including data on the energy shift and width of the 1s state in kaonic hydrogen, cross sections on \(K^-p \rightarrow \eta\Lambda, \pi^0\pi^0\Sigma\), etc. In this latter work, the preferred Fit II gives rise to two poles with isospin 1 around the \(\bar{K}N\) threshold with pole positions: \(1376 - i33\) MeV and \(1414 - i12\) MeV. There is another fit to data in Ref. \[23\], called Fit I,
with no isospin 1 poles but it is disfavored by the photoproduction data of CLAS [6], because the two poles associated with \( \Lambda(1405) \) are both clearly above 1.4 GeV. Independent studies of Refs. [24–28] accumulate evidences for a \( J^\pi = 1/2^- \) \( \Sigma \) with a pentaquark nature, with mass and width 1380 MeV and 60 MeV, respectively, by studying processes different to those considered in Refs. [3, 23], like:

\[
K^–p \rightarrow \Lambda\pi^+\pi^−, \quad \gamma N \rightarrow K^+\pi\Lambda, \quad \Lambda p \rightarrow \Lambda p\pi^0, \quad \Lambda^+_c \rightarrow \eta\pi^+\Lambda.
\]

In addition to these works, the best fit to the data on \( \gamma + p \rightarrow K^+ + \Sigma^\pm, \Sigma^0 + \pi^\pm \) [30] re-quired inclusion of two \( 1/2^- \) states in the isospin one: \((1413 \pm 10) – i(26 \pm 5) \text{ MeV}\) and \((1394 \pm 20) – i(75 \pm 20) \text{ MeV}\). On the other hand, a recent partial-wave analysis (s- and p-wave) of \( S = -1 \) low-energy data, including differential cross sections (although it only considers pseudoscalar-baryon contact interactions), does not report finding of any \( 1/2^- \) \( \Sigma \) around 1400 MeV [31]. In the present scenario, it is not clear if an isospin one partner of \( \Lambda(1405) \) exists, and if it does, it is not clear if it corresponds to one or two close lying poles in the complex plane. A different analysis of the photoproduction data, consistent with chiral dynamics and unitarity in coupled channels, is conducted in Ref. [32] and a \( \Sigma^* \) state appears as a strong cusp around the \( \bar{K}N \) threshold, very similar to the \( a_0(980) \) shape around the \( K\bar{K} \) threshold.

Interestingly, in the previous study of \( S = -1 \) systems [29], two isospin 1 poles were found, though they lied deep in the complex plane, arising from coupled channel meson baryon dynamics (at \( 1427 – i145 \) MeV, \( 1438 – i198 \) MeV). However, the motivation of the work [29], done by two of the present authors, was to build the formalism to couple pseudoscalar- and vector-baryon systems, and it was beyond the scope of Ref. [29] to test if the resulting amplitudes reproduced different relevant data. Nonetheless, the poles of the well studied \( \Lambda(1405) \) were reproduced in agreement with other works. Besides, the kernels for the pseudoscalar-baryon (PB) systems in Ref. [29] were obtained from the contact interaction (the Weinberg-Tomozawa term) coming from the lowest order chiral Lagrangian and the vector-baryon (VB) interactions were calculated by evaluating s-, t- and u-channel diagrams and a contact interaction. The purpose of our present work is to improve the model used in
Ref. [29] by including the $s$-, and $u$-channel baryon-exchange diagrams to the kernels of the pseudoscalar-baryon, which have been found to play an important role in the generation of $\Sigma$ poles around 1400 MeV in Ref. [3]. The importance of these diagrams has been pointed out in other works too, like in Ref. [9], near the $K\Xi$ threshold. With the improved PB kernels, we constrain the parameters of the formalism (mainly the subtraction constants required to calculate the loop functions), in order to reproduce different available experimental data and test if the low lying $\Sigma$s found in Ref. [29] move closer to the real axis, and could correspond to the $\Sigma$s found in Refs. [3, 23–28]. The generation of the states like $\Lambda(1405)$ or $\Sigma$ with a similar mass is not expected to get important contributions from VB dynamics, but the inclusion of VB dynamics in the model can be very relevant in determining useful informations. For example, with our model we can obtain the R-VB couplings (where R is a resonance, like $\Lambda(1405)$, $\Lambda(1670)$, etc.), which are required in the calculations of $t$-channel diagrams, with a vector exchange, for processes like the photoproduction/electroproduction of $\Lambda(1405)$. Additionally, with the improved PB kernels and constrained PB amplitudes, we can obtain more reliable information on the properties of the hyperon resonances arising from the vector-baryon dynamics as well.

The manuscript is organized as follows. In section II we discuss the Lagrangians from which the meson-baryon interactions are obtained and used as kernels to study nonperturbative scattering in the systems. Towards the end of the same section, we discuss the idea of carrying out a $\chi^2$-fit, the parameters of the fit and the data to be considered in the fit. In section III we discuss the details on the results of the fits obtained. The properties of the resonances found in our study are also given in section III by categorizing them in different subsections on the basis of their spins and isospins. Finally, we present a summary of the work.
II. FORMALISM

The problem of hadron scattering gets typically more and more complex as the energy region to be scanned involves opening of more and more thresholds to possible coupled channels. To study hyperon resonances arising from hadron dynamics, with mass up to about 2 GeV, we implement a non-perturbative unitarization method by treating crossed-channel dynamics perturbatively as developed in Refs. [3, 33, 34]. There is a connection with this method and solving the Bethe-Salpeter equation for contact interactions [2, 35]. We take into account pseudoscalar- and vector-baryon channels, motivated by the fact that the thresholds of these channels are spread over the energy ranging from 1.25-2.2 GeV, and some of them lie close enough to couple to each other, for example $K\Xi, \bar{K}\gamma N$. The pseudoscalar meson-baryon interaction diagrams are deduced from the lowest order, $O(p)$, Lagrangian [1–3, 36–39]

$$\mathcal{L}_{PB} = \langle \bar{B}i\gamma^\mu \partial_\mu B + \bar{B}i\gamma^\mu [\Gamma_\mu, B] \rangle - M_B \langle \bar{B}B \rangle + \frac{1}{2} D' \langle \bar{B} \gamma^\mu \gamma_5 \{u_\mu, B\} \rangle + \frac{1}{2} F' \langle \bar{B} \gamma^\mu \gamma_5 [u_\mu, B] \rangle, \quad (1)$$

where $u_\mu = i u^\dagger \partial_\mu U u^\dagger$, and

$$\Gamma_\mu = \frac{1}{2} (u^\dagger \partial_\mu u + u u^\dagger \partial_\mu), \quad U = u^2 = \exp \left( \frac{i P}{f_P} \right), \quad (2)$$

with $f_P$ representing the pseudoscalar decay constant, and $P \ (B)$ denoting the matrices of the octet meson (baryon) fields:

$$P = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}} \eta & \sqrt{2} \pi^+ & \sqrt{2} K^+ \\ \sqrt{2} \pi^- & -\pi^0 + \frac{1}{\sqrt{3}} \eta & \sqrt{2} K^0 \\ \sqrt{2} K^- & \sqrt{2} \bar{K}^0 & \frac{2}{\sqrt{3}} \eta \end{pmatrix}, \quad B = \begin{pmatrix} \frac{1}{\sqrt{8}} \Lambda + \frac{1}{\sqrt{2}} \Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}} \Lambda - \frac{1}{\sqrt{2}} \Sigma^0 & n \\ \Xi^- & \Xi^0 & -\frac{1}{\sqrt{3}} \Lambda \end{pmatrix}.$$ 

The constants $F' = 0.46$ and $D' = 0.8$, in Eq. (1), reproduce the axial coupling constant of the nucleon: $F' + D' \approx g_A = 1.26$. 

5
Using this Lagrangian, we compute the following amplitudes for the contact interaction and $s$- and $u$-channel diagrams, which are in agreement with other works [3, 9, 23, 40],

$$V_{\text{cont}}(i \rightarrow j) = -\frac{1}{4f^2}\sqrt{\frac{M_i + E_i}{2M_i}} \sqrt{\frac{M_j + E_j}{2M_j}} A_{ij} \left[ (2\sqrt{s} - M_i - M_j) + \left(2\sqrt{s} + M_i + M_j \right) \right] \times \left( \frac{\vec{p}_i \cdot \vec{p}_j + i \chi_j^\dagger (\vec{p}_j \times \vec{p}_i) \cdot \vec{\sigma} \chi_i}{(M_i + E_i)(M_j + E_j)} \right),$$

(3)

$$V_s(i \rightarrow j) = \frac{1}{2f^2}\sqrt{\frac{M_i + E_i}{2M_i}} \sqrt{\frac{M_j + E_j}{2M_j}} \sum_k \frac{B_{ij}^k}{s - M_k^2} \left[ (\sqrt{s} - M_i) (\sqrt{s} - M_j) (\sqrt{s} - M_k) \right] \times \left( \frac{\vec{p}_i \cdot \vec{p}_j + i \chi_j^\dagger (\vec{p}_j \times \vec{p}_i) \cdot \vec{\sigma} \chi_i}{(M_i + E_i)(M_j + E_j)} \right) (\sqrt{s} + M_i) (\sqrt{s} + M_j) (\sqrt{s} + M_k),$$

(4)

$$V_u(i \rightarrow j) = -\frac{1}{2f^2}\sqrt{\frac{M_i + E_i}{2M_i}} \sqrt{\frac{M_j + E_j}{2M_j}} \sum_k \frac{C_{ij}^k}{s - M_k^2} \left[ \left( \sqrt{s} + M_k \right) + \sqrt{s} \left( M_j [M_i + M_k] \right) \right] \times \left( \frac{\vec{p}_i \cdot \vec{p}_j + i \chi_j^\dagger (\vec{p}_j \times \vec{p}_i) \cdot \vec{\sigma} \chi_i}{(M_i + E_i)(M_j + E_j)} \right) \left( u (\sqrt{s} - M_k) + \sqrt{s} (M_j [M_i + M_k] + M_j M_k) + M_j (M_i + M_j) (M_i + M_k) + M_i^2 M_k \right).$$

(5)

The summation in Eqs. (4), (5) corresponds to summing the diagrams with different allowed octet baryons exchanged in the $s$-, $u$-channel, respectively, for a given process $i \rightarrow j$, with $i$ ($j$) (here, and in Eqs. (3), (4), (5)) representing the initial (final) state. In these equations, $M_i$ ($E_i$) denotes the mass (energy) of the baryon in the initial/final/intermediate state, represented by a subindex $l = i/j/k$, respectively, $\vec{p}_l$ represents the center of mass momentum in the $l$th channel and $A_{ij}$, $B_{ij}$, $C_{ij}$ are isospin coefficients for different processes. The coefficients $B_{ij}$, $C_{ij}$, for isospin 0 and 1, are listed in Tables. A1, A2, A3, A4 in the appendix. We refer the reader to Ref. [2] for the constants, $A_{ij}$, related to the contact interactions.
These amplitudes can be projected on s-wave to obtain

\[ V^{L=0}_{\text{cont}}(i \to j) = -\frac{1}{4f^2_D} \sqrt{\frac{M_i + E_i}{2M_i}} \sqrt{\frac{M_j + E_j}{2M_j}} A_{ij} \left[ (2\sqrt{s} - M_i - M_j) \right], \]  

(6)

\[ V^{L=0}_s(i \to j) = \frac{1}{2f^2_D} \sqrt{\frac{M_i + E_i}{2M_i}} \sqrt{\frac{M_j + E_j}{2M_j}} (\sqrt{s} - M_i) (\sqrt{s} - M_j) \sum_k \frac{B^k_{ij}}{\sqrt{\sqrt{s} + M_k}}, \]  

(7)

\[ V^{L=0}_u(i \to j) = -\frac{1}{2f^2_D} \sqrt{\frac{M_i + E_i}{2M_i}} \sqrt{\frac{M_j + E_j}{2M_j}} \sum_k C_{ij} \left[ \sqrt{s + M_k} \right. \right.

\left. \left. - \frac{(M_i + M_k)(M_j + M_k)(\sqrt{s} + M_i + M_j - M_k)}{2(M_i + E_i)(M_j + E_j)} + \left( \frac{(M_i + M_k)(M_j + M_k)}{4|\vec{p}_i||\vec{p}_j|} \right) \right. \right.

\left. \times \left( (\sqrt{s} - M_i - M_j + M_k) - \frac{s + M_k^2 - m_i^2 - m_j^2 - 2E_iE_j}{2(M_i + E_i)(M_j + E_j)} (\sqrt{s} + M_i + M_j - M_k) \right) \right. \right.

\left. \times \ln \left( \frac{s + M_k^2 - m_i^2 - m_j^2 - 2E_iE_j - 2|\vec{p}_i||\vec{p}_j|}{s + M_k^2 - m_i^2 - m_j^2 - 2E_iE_j + 2|\vec{p}_i||\vec{p}_j|} \right) \right], \]  

(8)

where \( m_i \) \((m_j)\) represents the meson mass in the initial (final) state.

For the vector-baryon amplitudes, we follow the previous work \[41\], where the problem was studied in detail, using a Lagrangian based on hidden local symmetry, and it was found that \( s-, t-, u\)-channel diagrams and a contact interaction arising from two vector field terms give comparable contributions, and must all be considered. We take the following Lagrangian from Ref. \[41\]

\[ \mathcal{L}_{VB} = -g \left\{ \langle B\gamma_{\mu} [V_8^{\mu}, B] \rangle + \langle B\gamma_{\mu} B \rangle \langle V_8^{\mu} \rangle + \frac{1}{4M} \left( F \langle B\sigma_{\mu\nu} [V_8^{\mu\nu}, B] \rangle + D \langle B\sigma_{\mu\nu} \{ V_8^{\mu\nu}, B \} \rangle \right) \right\} \]  

(9)

+ \langle B\gamma_{\mu} B \rangle \langle V_0^{\mu} \rangle + \frac{C_0}{4M} \langle B\sigma_{\mu\nu} V_0^{\mu\nu} B \rangle \right\}, \]

In the case of \( |\vec{p}_i| = 0 \) or \( |\vec{p}_j| = 0 \), \( V^{L=0}_u \) can be obtained from Eq. 5 directly.
where the subscript \( 8 \) \((0)\) denotes the octet (singlet) part of the wave function of the vector meson (relevant in the case of \( \omega \) and \( \phi \)), \( V^{\mu \nu} \) represents the tensor field of the vector mesons,

\[
V^{\mu \nu} = \partial^{\mu} V^{\nu} - \partial^{\nu} V^{\mu} + ig [V^{\mu}, V^{\nu}],
\]

and \( V^{\mu} \) is the SU(3) matrix for the (physical) vector mesons

\[
V^{\mu} = \frac{1}{2} \begin{pmatrix}
\rho^0 + \omega & \sqrt{2} \rho^+ & \sqrt{2} K^+ \\
\sqrt{2} \rho^- & -\rho^0 + \omega & \sqrt{2} K^0 \\
\sqrt{2} K^- & \sqrt{2} K^0 & \sqrt{2} \phi
\end{pmatrix}^\mu.
\]

In Eq. (9), the coupling \( g \) is related to the vector meson decay constant, \( f_v \) through the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation

\[
\tag{12}
g = \frac{m_v}{\sqrt{2} f_v},
\]

with \( m_v \) being the mass of a given vector meson in the vertex and the constants \( D = 2.4, F = 0.82 \) and \( C_0 = 3F - D \) correctly reproduce the anomalous magnetic couplings of the \( \rho NN, \omega NN \) and \( \phi NN \) vertices \([42,44]\). Together with Eq. (9), and the kinetic term

\[
\tag{13}
\mathcal{L}_{3V} \equiv -\frac{1}{2} \langle V^{\mu \nu} V_{\mu \nu} \rangle,
\]

it is possible to calculate the \( s-, t-, u- \) channel amplitudes as well as the contact interaction by using \( [V^{\mu}, V^{\nu}] \) for \( V^{\mu \nu} \) in Eq. (9). It was found in Ref. \([41]\) that this contact interaction, apart from giving contributions comparable to other amplitudes, is important to guarantee the invariance of the Lagrangian under a gauge transformation.

Finally, the amplitudes for the transition between the pseudoscalar-baryon and the vector-baryon channels are deduced from the Lagrangian \([45]\)

\[
\tag{14}
\mathcal{L}_{PBVB} = \frac{-ig_{PBVB}}{2f_v} \left( F' \langle B \gamma_\mu \gamma_5 [P, V^\mu] , B \rangle + D' \langle B \gamma_\mu \gamma_5 \{[P, V^\mu] , B \} \rangle \right),
\]

which has been obtained by introducing the vector meson field as a gauge boson of the hidden local symmetry in the nonlinear sigma model. The procedure is, thus, like extending
the Kroll-Ruderman term for the photoproduction of a pion, replacing, inspired by the vector meson dominance, the photon by the vector meson \([45]\). The constants, \(F'\) and \(D'\) are the same as those defined for Eq. (1).

The formalism has been applied to study meson-baryon systems with various quantum numbers in Refs. [29, 46, 47] and, in fact, different vector-baryon amplitudes as well as those for the transition between pseudoscalar- and vector-baryon channels are taken from Ref. [29] for the present work. Though, it must be mentioned that the formalism in the present work is more elaborate, as compared to our previous works, since we include \(s\)- and \(u\)-channel diagrams for pseudoscalar-baryon interactions here. The contributions from these diagrams have been found to play an important role in the formation of isospin one resonances near 1400 MeV [3, 23] and it is the purpose of the present work to constrain our amplitudes to reproduce the experimental data in the low-energy region and investigate the formation of isospin 1 states around 1400 MeV.

To proceed further, we solve the Bethe-Salpeter equation in its on-shell factorized form [2, 3, 33, 35], as also used in several other works, like, in Refs. [3, 5, 7, 16, 21, 23, 48], with the lowest order amplitudes discussed above and make a \(\chi^2\)-fit to the data. The parameters of the fit are:

1. The subtraction constants required to calculate the loop integrals with the dimensional-regularization method

\[
G(\sqrt{s}) = \frac{i2M}{2\pi^2} \int \frac{d^4q}{(P - q)^2 - M^2 + i\epsilon} \frac{1}{q^2 - m^2 + i\epsilon} \frac{1}{\sqrt{s}}
\]

\[
= \frac{2M}{16\pi^2} \left\{ a(\mu) + \ln \frac{M^2}{\mu^2} + \frac{m^2 - M^2 + s}{2s} \ln \frac{m^2}{M^2} \right. \\
+ \frac{\bar{q}}{\sqrt{s}} \left[ \ln \left( s - (M^2 - m^2) + 2\bar{q}\sqrt{s} \right) + \ln \left( s + (M^2 - m^2) + 2\bar{q}\sqrt{s} \right) - s + (M^2 - m^2) + 2\bar{q}\sqrt{s} \right] \\
- \left. \ln \left( -s + (M^2 - m^2) + 2\bar{q}\sqrt{s} \right) - \ln \left( s - (M^2 - m^2) + 2\bar{q}\sqrt{s} \right) \right\},
\]
where $\tilde{P}$ is the total four momentum, $M (m)$ is the mass of the propagating baryon (meson), $\tilde{q} = \sqrt{s}(s, M^2, m^2)/2\sqrt{s}$, $a(\mu)$ is the subtraction constant at a regularization scale $\mu = 630$ MeV. Since a fit is made to both isospin 0 and 1 amplitudes, we have 14 subtraction constants as parameters, corresponding to the channels: $\bar{K}N$, $K\Xi$, $\pi\Sigma$, $\eta\Lambda$, $\pi\Lambda$, $\eta\Sigma$, $\bar{K}^*N$, $K^*\Xi$, $\rho\Sigma$, $\omega\Lambda$, $\phi\Lambda$, $\rho\Sigma$, $\omega\Sigma$, $\phi\Sigma$.

2. The decay constants of the mesons. In general, different mesons have different decay constants. We use an average value for the decay constant of the pseudoscalars, $f_P$, to be used in Eqs. (6), (7), (8), and another one for the vectors, $f_V$, to be used in vector-baryon amplitudes. These constants account for two additional parameters in the fit.

3. Finally, the coupling at the pseudoscalar-baryon–vector-baryon vertex, $g_{PBVB}$ in Eq. (14), is treated as a parameter to be fitted, whose value can be approximately estimated using Eq. (12). One gets $g_{PBVB} \sim 3.5$ by taking an average value for $m_v \sim 850$ MeV, $f_v \sim 170$ MeV. However, this value could be smaller if hadronic structure is taken into account by using a form factor. Note that if the pion decay constant $\sim 93$ MeV is used, instead of the vector decay constant, in Eq. (12), then $g_{PBVB} \sim 6$ (as in Refs. [29, 45, 46]). We, thus, allow $g_{PBVB}$ to vary between 1 and 6 in the fitting procedure.

The experimental data considered for the fit are:

1. The total cross sections of the processes: $K^- p \rightarrow K^- p$, $\bar{K}^0 n$, $\eta\Lambda$, $\pi^0\Lambda$, $\pi^0\Sigma^0$, $\pi^\pm\Sigma^\mp$, from the respective thresholds to about 30-50 MeV above the threshold. We consider the same data as in Ref. [23].

2. The energy level shift and width of the 1s state of the kaonic hydrogen measured by the SIDDHARTA collaboration [49]: $\Delta E = 283 \pm 36 \pm 6$ eV and $\Gamma = 549 \pm 89 \pm 22$ eV.
We use the relation between the energy shift and width of the 1s state of the kaonic hydrogen and the $K^-p$ scattering length, as obtained in Ref. [50]

$$\Delta E - i \frac{\Gamma}{2} = -2\alpha^3 \mu^2 a_{K^-p} [1 + 2\alpha \mu (1 - \ln \alpha) a_{K^-p}],$$

(16)

where

$$a_{K^-p} = - \frac{t_{K^-p}}{4\pi \sqrt{s_{th}}} M_p,$$

(17)

with $M_p$ being the proton mass and $\sqrt{s_{th}}$ denoting the $K^-p$ threshold energy.

3. The following ratios of the cross section at the threshold, taken from Ref. [23],

$$\gamma = \frac{\sigma(K^-p \rightarrow \pi^+\Sigma^-)}{\sigma(K^-p \rightarrow \pi^-\Sigma^+)} = 2.36 \pm 0.12,$$

$$R_c = \frac{\sigma(K^-p \rightarrow \text{charged particles})}{\sigma(K^-p \rightarrow \text{all})} = 0.664 \pm 0.033,$$

(18)

$$R_n = \frac{\sigma(K^-p \rightarrow \pi^0\Lambda)}{\sigma(K^-p \rightarrow \text{all neutral states})} = 0.189 \pm 0.015.$$

III. RESULTS AND DISCUSSIONS

In order to fit the data, the $\chi^2$ per degree of freedom, $\chi^2_{d.o.f}$, is calculated as [23, 40, 51–53],

$$\chi^2_{d.o.f} = \frac{\sum_{k=1}^{N} n_k}{N(\sum_{k=1}^{N} n_k - n_p)} \sum_{k=1}^{N} \frac{\chi^2_k}{n_k},$$

(19)

where $N$ is the number of different data sets, $n_k$ represents the number of data points in the $k$th data set, $n_p$ is the number of free parameters, and the $\chi^2$ for the $k$th data set is obtained as

$$\chi^2_k = \sum_{i=1}^{n_k} \frac{(y_{th;i} - y_{exp;i})^2}{\sigma^2_{k;i}},$$

(20)
with \( y_{k;i}^{\text{exp}} \) (\( y_{k;i}^{\text{th}} \)) representing the \( i \)th experimental (theoretical) point of the \( k \)th data set and \( \sigma_{k;i}^2 \) the standard deviation associated with it.

In the fitting procedure, we find that two types of solutions exist, which correspond to \( \chi^2_{\text{d.o.f}} \sim 1 \). The parameter sets related to the two solutions, which we label as Fit I and II, are given in Table I, together with the associated error bars. The central value and the associated error correspond to the mean value and the standard deviation, respectively, obtained for each parameter. The errors are estimated by admitting solutions satisfying the condition

\[
\chi^2 \leq \chi_0^2 + \sqrt{2\chi_0^2},
\]

(21)

where \( \chi_0^2 \) is the minimum \( \chi^2 \) value obtained, as in Refs. [54, 55].

**Table I.** Values of the parameters obtained by constraining the model amplitudes to reproduce experimental data (mentioned in section II). Here, \( a_i \) represents the subtraction constant for the channel \( i \) in the isospin base, \( f_P \) \( (f_v) \) is an average value for the decay constants of the pseudoscalar (vector) mesons, and \( g_{PBVB} \) is the coupling appearing in the \( PB \leftrightarrow VB \) vertices (see Eq. (14)).

| Parameters | Fit I | Fit II | Parameters | Fit I | Fit II | Parameters | Fit I | Fit II |
|------------|-------|--------|------------|-------|--------|------------|-------|--------|
| \( a_{KN} \) | \(-2.00 \pm 0.06\) | \(-2.12 \pm 0.10\) | \( a_{K^*N} \) | \(-4.34 \pm 0.08\) | \(-4.39 \pm 0.09\) | \( a_{\Xi} \) | \(-3.55 \pm 1.58\) | \(-3.65 \pm 1.34\) |
| \( a_{K \Xi} \) | \(-2.43 \pm 0.04\) | \(-2.43 \pm 0.06\) | \( a_{K^* \Xi} \) | \(-3.86 \pm 0.03\) | \(-3.33 \pm 0.06\) | \( a_{\phi \Xi} \) | \(-4.67 \pm 0.29\) | \(-2.51 \pm 0.39\) |
| \( a_{\eta \Sigma} \) | \(-1.09 \pm 0.07\) | \(-1.18 \pm 0.12\) | \( a_{\phi \Sigma} \) | \(1.17 \pm 1.29\) | \(-2.36 \pm 0.07\) | \( f_P \) (MeV) | \(94.62 \pm 1.46\) | \(97.24 \pm 1.56\) |
| \( a_{\eta \Lambda} \) | \(-1.25 \pm 0.03\) | \(-1.27 \pm 0.09\) | \( a_{\phi \Lambda} \) | \(-6.50 \pm 0.70\) | \(-3.86 \pm 2.09\) | \( f_v \) (MeV) | \(138.12 \pm 1.54\) | \(113.46 \pm 5.21\) |
| \( a_{\eta \Xi} \) | \(-0.84 \pm 0.26\) | \(-1.69 \pm 0.31\) | \( a_{\phi \Xi} \) | \(-6.83 \pm 0.60\) | \(-5.22 \pm 1.13\) | \( g_{PBVB} \) | \(2.19 \pm 0.09\) | \(1.81 \pm 0.07\) |
| \( a_{\eta \Sigma} \) | \(-3.62 \pm 0.44\) | \(-1.97 \pm 0.12\) | \( a_{\phi \Sigma} \) | \(-0.77 \pm 0.20\) | \(-0.49 \pm 0.47\) |

In Fig. 1 we show the cross sections of the different processes, as obtained by the parameter set labelled as Fit I. The shaded bands in the panels correspond to the results obtained by using the criteria given in Eq. (21). The data considered in the fit are shown as (red) filled circles in Fig. 1. These data are same as those considered in Ref. [23] and are taken from the same sources as in this reference. We have included more data points from Ref. [56] and which are shown as (blue) filled squares in Fig. 1 going to about 100-200 MeV above
FIG. 1. Cross sections of different processes studied in our work. The shaded region represents the results found with the parameters listed under the label Fit I, in Table [I]. Data shown as (red) filled circles were used in the $\chi^2$ fitting procedure explained in the text.
the threshold for these reactions. It can be seen that the results stay close to the data points at higher energies too, even though the data at these energies were not used in the fit. At energies farther from the reaction threshold, the cross sections are expected to get contributions from interactions in higher partial waves, and, thus, the s-wave amplitudes, which are the ones we calculate, are not expected to be sufficient to describe data at such energies. For a better description of the data we need to include some well known resonances in the formalism, such as Λ(1520)(3/2−), Λ(1600)(1/2+), Σ(1620)(1/2+), which are related to p-, d-wave pseudoscalar-baryon interactions. Such states can be taken into account by including channels, like, meson–decuplet-baryon \[57\], two meson-one baryon \[58\], etc. Such extensions of our work can be done in future. Still it is reassuring to see that the cross sections obtained at higher energies do not differ much from the experimental data. It is worth mentioning that the coupling to vector-baryon channels is useful in improving this agreement, at energies away from the threshold. Although, the presence of the vector-baryon coupling is more significant in the case where the reaction threshold is higher (closer to the VB thresholds). Such is the case of the process \(K^-p \rightarrow \eta \Lambda\), whose threshold is about 140 MeV away from the \(\bar{K}^*N\), keeping in mind the finite width of \(K^*\). The finite widths of the vector mesons are taken into account in the formalism by folding the relevant loop function over the variable mass range of the vector mesons as \[41\] \[59\]

\[\tilde{G}_j(\sqrt{s}) = \frac{1}{N_j} \int \frac{d\tilde{m}^2}{(m_j - 2\Gamma_j)^2} \left(-\frac{1}{\pi}\right) G_j(\sqrt{s}) \text{Im} \left\{ \frac{1}{\tilde{m}^2 - m_j^2 + im_j \Gamma(\tilde{m})} \right\}, \tag{22}\]

where the subscript \(j\) refers to the \(j\)th meson-baryon channel in the loop, \(m_j\) (\(\Gamma_j\)) is the central mass (width) of the meson in the loop, \(G_j(\sqrt{s})\) is calculated using Eq. (15) and

\[N_j = \int \frac{d\tilde{m}^2}{(m_j - 2\Gamma_j)^2} \left(-\frac{1}{\pi}\right) \text{Im} \left\{ \frac{1}{\tilde{m}^2 - m_j^2 + im_j \Gamma(\tilde{m})} \right\}. \tag{23}\]

The variable width in Eqs. (22), (23) for the \(j\)th meson decaying to mesons \(a\) and \(b\) is
calculated as
\[ \Gamma(\tilde{m}) = \Gamma_j \left( \frac{m_j^2}{\tilde{m}^2} \right) \left( \frac{\lambda^{1/2}(\tilde{m}^2, m_a^2, m_b^2)/2\tilde{m}}{\lambda^{1/2}(m_j^2, m_a^2, m_b^2)/2m_j} \right)^3 \theta(\tilde{m} - m_a - m_b). \]

In Fig. 2, we show the cross sections of the processes $K^-p \rightarrow \eta\Lambda$ and $K^-p \rightarrow \bar{K}^0n$ obtained by decoupling PB and VB channels. As can be seen, the coupling to the VB channels plays a more important role in the case of the process with a higher threshold.

Before discussing the results found, within the Fit I, for the energy shift and width of the $1s$ state of the kaonic hydrogen and cross section ratios mentioned in Eqs. (16), (18), as well as the poles found in the complex plane, we show the cross sections found with the parameter set labelled as Fit II. It can be seen that the cross sections are in good agreement with the data in the energy region corresponding to the filled circles (which are used to minimize the $\chi^2$), as expected, and the results stay near the data points at higher energies except for the case of $K^-p \rightarrow \eta\Lambda$. This finding may indicate, when comparing the two fits, that the results related to the poles found in the complex plane may be more reliable in the case of Fit I, at energies beyond $\sim 1.68$ GeV (which corresponds to the laboratory momentum of about 0.77 GeV shown in Figs. 1, 3). At lower energies, though, the two fits are of similar
FIG. 3. Cross sections obtained with the parameter set Fit II given Table I. The data are taken from the same source as in Fig 1.
quality, implying that the poles obtained in amplitudes for both fits, in the complex plane, should be reliable at energy below $\sim 1.68$ GeV. Besides this finding, the cross section ratios, as well as the energy shift of the 1s state of the kaonic hydrogen found within the two fits, as given in Table II, are in good agreement with the experimental data (see the values given in section II). We, thus, find it useful to discuss the remaining results for both fits. In Tables III-VI, we list the poles found in the complex plane, with the amplitudes obtained within both fits. In the following subsections we also compare the properties of the states found in our analysis with those available from other theoretical/experimental works. Before proceeding, though, we would like to discuss the procedure to calculate the $T$-matrix in the complex energy plane, which is needed to look for resonances/bound states formed in the systems under investigation. For this, we calculate the loop function for the $i$th channel in the first (I) and second (II) Riemann sheet as [4, 35]:

$$G_i(\sqrt{s}) = \begin{cases} 
G_i^{(I)}(\sqrt{s}), & \text{for } \text{Re}\{\sqrt{s}\} < (m_i + M_i) \\
G_i^{(II)}(\sqrt{s}), & \text{for } \text{Re}\{\sqrt{s}\} \geq (m_i + M_i)
\end{cases}$$

|          | $\Delta E$(eV) | $\Gamma$(eV) | $\gamma$ | $R_c$ | $R_n$ |
|----------|----------------|--------------|----------|-------|-------|
| Fit I    | 300 $\pm$ 3   | 448 $\pm$ 6  | 2.357 $\pm$ 0.005 | 0.663 $\pm$ 0.003 | 0.191 $\pm$ 0.002 |
| Fit II   | 301 $\pm$ 6   | 474 $\pm$ 17 | 2.364 $\pm$ 0.008 | 0.668 $\pm$ 0.003 | 0.193 $\pm$ 0.002 |
| Data (from Refs. [23, 49]) | 283 $\pm$ 36 $\pm$ 6 | 549 $\pm$ 89 $\pm$ 22 | 2.36 $\pm$ 0.12 | 0.664 $\pm$ 0.033 | 0.189 $\pm$ 0.015 |

TABLE II. Results found for the energy shift and width of the 1s state of the kaonic hydrogen and the cross section ratios defined in Eq. (18). The central value and errors correspond to the mean value and the standard deviation, respectively, determined from the solutions satisfying Eq. (21).
where

\[ G_i^{(I)}(\sqrt{s}) = G_i(\sqrt{s}) \]  \hspace{1cm} (24) \\
\[ G_i^{(II)}(\sqrt{s}) = G_i^{(I)}(\sqrt{s}) - 2i \text{Im}\{G_i^{(I)}\} \]
\[ = G_i^{(I)}(\sqrt{s}) + \frac{i M_i q_i^{(I)}}{2\pi \sqrt{s}}, \]  \hspace{1cm} (25)

with \( m_i, M_i \) being the masses of the \( i \)th-meson and baryon, and \( q_i^{(I)} \) the center of mass momentum of the same channel on its first Riemann sheet (with a positive imaginary part). If a pole appears in the complex plane, it can be seen in the complex amplitude for all the channels. Depending on the threshold of a given channel, the pole can appear below or above the threshold (i.e, on the corresponding first or second Riemann sheet of that channel).

A. Isospin = 0, spin = 1/2

In the case of \( I(J^P) = 0 \, (1/2^-) \), in both types of fits, a double pole structure is found in the energy region around 1400 MeV (see Table III), which can be related to \( \Lambda(1405) \). The double pole nature of \( \Lambda(1405) \) is widely discussed in the literature \cite{4, 16, 19, 21, 40, 51, 53}. Our results are compatible with the pole values obtained in these former works, as well as with those determined by the CLAS collaboration \cite{6} from the data on the electroproduction of \( \Lambda(1405) \), with the lower mass pole being near 1368 MeV and the higher mass pole near 1423 MeV.

We give the couplings of these poles to the different meson-baryon channels considered in the present work in Table III. The coupled channel treatment of pseudoscalar-baryon and vector-baryon systems is a particular feature of our formalism and it allows us to obtain the information on the coupling of the low lying resonances, like, \( \Lambda(1405) \), to both type of channels. The information on the coupling of the states to vector-baryon channels is useful in studies of processes like the photoproduction/electroproduction of \( \Lambda(1405) \) through a \( t \)-channel vector exchange (as done in Refs. \cite{6, 60, 63}).
TABLE III. Pole positions and couplings of the $I(J^P) = 0(1/2^-)$ states found. The central values and errors were obtained as explained in the caption of Table I (for the sake of space, the errors are represented as superscripts). Masses and widths are given in MeV. The coupling of the state to a given channel are written as rows in the Table for Fit I and II (the first (second) row is related to the results for Fit I (Fit II)).

|        | $\Lambda(1405)$ | $\Lambda(1670)$ | $\Lambda(1800)$ |
|--------|-----------------|-----------------|-----------------|
| Fit I  | $1373^{+5}_{-5} - i \, 114^{+9}_{-9}$ | $1426^{+1}_{-1} - i \, 16^{+2}_{-2}$ | $1681^{+1}_{-1} - i \, 16^{+2}_{-2}$ |
| Fit II | $1385^{+5}_{-5} - i \, 124^{+10}_{-10}$ | $1426^{+1}_{-1} - i \, 15^{+2}_{-2}$ | $1681^{+2}_{-2} - i \, 7^{+1}_{-1}$ |
| $K^0 N$ | $0.84^{\pm 0.14}_{-0.13} - i \, 1.91^{\pm 0.06}_{-0.05}$ | $2.44^{\pm 0.05}_{-0.05} + i \, 0.69^{\pm 0.08}_{-0.08}$ | $0.33^{\pm 0.02}_{-0.02} - i \, 0.38^{\pm 0.03}_{-0.03}$ | $0.14^{\pm 0.05}_{-0.05} - i \, 0.12^{\pm 0.07}_{-0.07}$ | $-0.36^{\pm 0.06}_{-0.06}$ |
| $K^0 \Xi$ | $0.66^{\pm 0.35}_{-0.33} - i \, 1.93^{\pm 0.12}_{-0.11}$ | $2.43^{\pm 0.16}_{-0.16} + i \, 0.63^{\pm 0.23}_{-0.23}$ | $0.15^{\pm 0.06}_{-0.06} - i \, 0.19^{\pm 0.13}_{-0.13}$ | $0.39^{\pm 0.02}_{-0.02} - i \, 0.45^{\pm 0.03}_{-0.03}$ | $0.04^{\pm 0.02}_{-0.02} - i \, 0.36^{\pm 0.06}_{-0.06}$ |
| $\pi^0 \Sigma$ | $-0.51^{\pm 0.05}_{-0.05} + i \, 0.49^{\pm 0.06}_{-0.06}$ | $0.59^{\pm 0.09}_{-0.09} - i \, 0.19^{\pm 0.04}_{-0.04}$ | $2.74^{\pm 0.26}_{-0.26} + i \, 0.25^{\pm 0.22}_{-0.22}$ | $1.26^{\pm 0.06}_{-0.06} - i \, 0.39^{\pm 0.28}_{-0.28}$ | $-0.36^{\pm 0.15}_{-0.15} + i \, 0.12^{\pm 0.08}_{-0.08}$ |
| $\eta \Lambda$ | $-0.55^{\pm 0.13}_{-0.13} + i \, 0.27^{\pm 0.06}_{-0.06}$ | $0.72^{\pm 0.14}_{-0.14} - i \, 0.14^{\pm 0.08}_{-0.08}$ | $0.33^{\pm 0.04}_{-0.04} + i \, 0.28^{\pm 0.34}_{-0.34}$ | $3.30^{\pm 0.11}_{-0.11} - i \, 0.14^{\pm 0.29}_{-0.29}$ | $-0.36^{\pm 0.15}_{-0.15} + i \, 0.12^{\pm 0.08}_{-0.08}$ |
| $K^+ N$ | $-0.24^{\pm 0.07}_{-0.07} + i \, 0.29^{\pm 0.08}_{-0.08}$ | $-0.87^{\pm 0.06}_{-0.06} - i \, 0.05^{\pm 0.09}_{-0.09}$ | $0.27^{\pm 0.02}_{-0.02} + i \, 0.42^{\pm 0.06}_{-0.06}$ | $0.09^{\pm 0.05}_{-0.05} - i \, 0.14^{\pm 0.07}_{-0.07}$ | $-0.13^{\pm 0.02}_{-0.02} - i \, 0.05^{\pm 0.07}_{-0.07}$ |
| $K^+ \Xi$ | $-0.23^{\pm 0.08}_{-0.08} - i \, 0.18^{\pm 0.14}_{-0.14}$ | $0.04^{\pm 0.36}_{-0.36} + i \, 0.23^{\pm 0.19}_{-0.19}$ | $0.50^{\pm 0.92}_{-0.92} + i \, 0.01^{\pm 0.10}_{-0.10}$ | $0.21^{\pm 0.09}_{-0.09} + i \, 0.03^{\pm 0.19}_{-0.19}$ | $0.95^{\pm 0.16}_{-0.16} + i \, 0.09^{\pm 0.10}_{-0.10}$ |
| $\rho \Sigma$ | $1.23^{\pm 0.11}_{-0.11} - i \, 0.08^{\pm 0.09}_{-0.09}$ | $-0.36^{\pm 0.12}_{-0.12} + i \, 0.42^{\pm 0.05}_{-0.05}$ | $-2.05^{\pm 0.25}_{-0.25} + i \, 0.22^{\pm 0.13}_{-0.13}$ | $1.01^{\pm 0.47}_{-0.47} + i \, 0.22^{\pm 0.18}_{-0.18}$ | $-0.18^{\pm 0.72}_{-0.72} + i \, 0.20^{\pm 0.53}_{-0.53}$ | $-1.45^{\pm 0.57}_{-0.57} - i \, 0.07^{\pm 0.20}_{-0.20}$ |
| $\omega \Lambda$ | $0.16^{\pm 0.11}_{-0.11} + i \, 0.29^{\pm 0.07}_{-0.07}$ | $-0.24^{\pm 0.09}_{-0.09} - i \, 0.01^{\pm 0.02}_{-0.02}$ | $0.23^{\pm 0.16}_{-0.16} - i \, 0.09^{\pm 0.08}_{-0.08}$ | $-0.28^{\pm 0.28}_{-0.28} - i \, 0.04^{\pm 0.03}_{-0.03}$ | $-0.22^{\pm 0.02}_{-0.02} - i \, 0.02^{\pm 0.02}_{-0.02}$ |
| $\phi \Lambda$ | $0.57^{\pm 0.24}_{-0.24} + i \, 0.41^{\pm 0.19}_{-0.19}$ | $-0.47^{\pm 0.42}_{-0.42} + i \, 0.03^{\pm 0.18}_{-0.18}$ | $-1.76^{\pm 2.58}_{-2.58} + i \, 0.10^{\pm 0.37}_{-0.37}$ | $-1.26^{\pm 0.32}_{-0.32} + i \, 0.06^{\pm 0.42}_{-0.42}$ | $-2.26^{\pm 0.52}_{-0.52} - i \, 0.02^{\pm 0.21}_{-0.21}$ |

Table III also shows a pole around 1680 MeV, which is related to $\Lambda(1670)$. The mass and width of this state range, according to the particle data group (PDG) [66], between 1670-1680 MeV and 25-50 MeV, respectively. The pole position found with Fit I: $(1681 \pm 1) - (i(16 \pm 2)$ MeV is in better agreement with the properties of $\Lambda(1670)$ from the PDG [66]. We have determined the branching ratios of this state for channels $\bar{K}N$, $\pi^0 \Sigma$ and $\eta \Lambda$ and find them,
respectively, to be 28%, 34% and 25% with the central values of the parameters in Fit I and 19%, 61% and 7% with the central values in Fit II (given in Table I). The former values are more in agreement with the values: 20-30%, 25-55% and 10-25% given in Ref. [66]. This finding is in line with the earlier discussions on the reliability of the results obtained within Fit II beyond $\sim 1680$ MeV, due to the disagreement of the $K^−p → \eta\Lambda$ cross sections at energies $\gtrsim 1680$ MeV.

In view of the results found in our work, and as widely accepted, both $\Lambda(1405)$ and $\Lambda(1670)$ can be interpreted as states arising from pseudoscalar-baryon dynamics. We find a pole with $I(J^P) = 0 (1/2^-)$, which gets contribution from vector-baryon dynamics as well, with mass around 1730 MeV in Fit I. In the case of Fit II though, two poles are found in the energy region 1700-1800 MeV. Only one $1/2^-$ $\Lambda$ state is listed in this energy region by the PDG [66], $\Lambda(1800)$, which encompasses $I(J^P) = 0 (1/2^-)$ states with masses ranging from 1720-1850 MeV and widths ranging over 100-600 MeV. It is then quite possible that more than one state get classified under the same label $\Lambda(1800)$. From our study and in light of the information available from the PDG [66], it can be said that a state is found around 1730 MeV with a width around 40 MeV. However, missing channels not considered in the present work could have an impact on the width of this state and make it larger. A more detailed study involving such channels and considering data on reactions producing $VB$ channels should be done in future to investigate further properties of this state.

A comment regarding the widths of the states found in our work is here in order. The half widths of the states with mass around or above 1800 MeV have been determined from the real axis (by reading the full width at the half maximum of the related peaks appearing in the squared amplitudes, on the real axis, a quite common procedure in this kind of problem [64]). This is done because the widths of the vector mesons, here and throughout the work, are not taken into account when calculating the amplitudes in the complex plane, since such a consideration would imply a variable mass of the vector meson and, hence, a not well defined branch cut in the complex plane. However, as explained earlier, the amplitudes on the real
axis have been obtained by taking the finite widths of the vector mesons into account. Thus, a better estimation of the widths of the resonances is obtained from the real axis.

B. Isospin = 1, spin = 1/2

In the case of $1^{-/2} \pi$ isovector scattering amplitudes studied in the complex plane, two poles appear around 1400 MeV with the parameter set Fit I (see Table IV), while only one pole is obtained with Fit II. It can be seen from Figs. [1] [3] and the results in Table [1] that the quality of both fits is similar in the energy region near the threshold. Thus, from our work, it is difficult to distinguish the possibility of the existence of one or two isovector poles around 1400 MeV. But even if two poles exist in nature, they may be related to the same state due to the proximity of the masses and the widths. Thus, it can be affirmed that a $\Sigma$ state does seem to appear in this energy region. As mentioned in the introduction of this article, the information on the light $\Sigma$'s is less abundant when compared to light $\Lambda$’s. Still, we can compare our results with other works [3, 23, 28, 30], where almost all agree on the existence of one $\Sigma$ state around 1380 MeV with the width of about 60 MeV. An evidence for two poles around 1400 MeV, in isospin 1 amplitudes, has been discussed in Refs. [3, 23, 30], out of which Ref. [23] finds one of the poles to be narrower, as is the case of the results for Fit I listed in Table IV. Suggestions have been done to find this state in the $\chi_{c0}$ decay into $\bar{\Sigma}\Sigma\pi$ [65] and in the $\Lambda^+$ decay into $\eta\pi^+\Lambda$ [28].

In the energy region where vector-baryon thresholds are open, we find no isovector poles with Fit II, though two poles with Fit I are found in the energy region 1600-1900 MeV, which can be related to $\Sigma(1620)$ and $\Sigma(1900)$, respectively, listed by the PDG [66]. Actually, we have studied the possibility of relating the state at $1630\pm 33 - i(104\pm 13)$ MeV to $\Sigma(1670)$ as well as $\Sigma(1620)$. Little is known about both these $\Sigma$s and the PDG [66] indicates that each of them may be related to two states, of which the spin-parity of only one (in each case) is known. The spin-parity of one of the $\Sigma(1620)$s is given as $1/2^-$ by the PDG and for one of
TABLE IV. Pole positions and couplings of the $I(J^P) = 1(1/2^-)$ states found in our work. The central values and errors were obtained as explained in the caption of Table I (for the sake of space, the errors are represented as superscripts).

|                | $\Sigma$'s around 1400 MeV | $\Sigma(1620)$ or $\Sigma(1670)$ | $\Sigma(1900)$ |
|----------------|-----------------------------|---------------------------------|---------------|
| Fit I          |                             |                                 |               |
| $K\pi$         | $0.18^{+0.03} + i 0.14^{+0.05}$ | $0.08^{+0.04} + i 0.52^{+0.73}$ | $1.47^{+0.08}$ | $i 0.017^{+0.07}$ | $-0.86^{+0.03}$ | $+ i 0.79^{+0.02}$ | $-$ | $-$ |
| $K\Xi$         | $1.06^{+0.22} + i 1.45^{+0.12}$ | $0.62^{+0.47} + i 0.42^{+1.00}$ | $2.89^{+0.26}$ | $- i 0.65^{+0.24}$ | $0.84^{+0.03}$ | $- i 0.39^{+0.05}$ | $-$ | $-$ |
| $\pi\Sigma$    | $-0.17^{+0.09} - i 0.020^{+0.03}$ | $0.77^{+0.06} - i 0.67^{+1.22}$ | $0.71^{+0.33}$ | $- i 1.63^{+0.19}$ | $-0.02^{+0.04}$ | $+ i 0.32^{+0.08}$ | $-$ | $-$ |
| $\pi\Lambda$   | $0.03^{+0.10} + i 0.07^{+0.06}$ | $-0.91^{+1.32} + i 0.39^{+0.81}$ | $-0.26^{+0.34}$ | $- i 0.23^{+0.18}$ | $0.36^{+0.2}$ | $+ i 1.54^{+0.04}$ | $-$ | $-$ |
| $\eta\Sigma$   | $-0.43^{+0.03} - i 0.23^{+0.09}$ | $0.31^{+0.31} - i 0.59^{+1.12}$ | $-2.14^{+0.24}$ | $- i 0.13^{+0.11}$ | $0.07^{+0.03}$ | $- i 0.43^{+0.02}$ | $-$ | $-$ |
| $K^*\pi$       | $0.04^{+0.10} + i 0.15^{+0.07}$ | $-1.69^{+1.19} + i 0.31^{+0.68}$ | $-0.31^{+0.09}$ | $- i 0.11^{+0.16}$ | $0.71^{+0.05}$ | $- i 0.05^{+0.02}$ | $-$ | $-$ |
| $K^*\Xi$       | $-0.50^{+0.22} - i 0.38^{+0.08}$ | $1.40^{+2.11}$ | $- i 1.10^{+2.38}$ | $-1.89^{+0.47}$ | $- i 0.37^{+0.14}$ | $-0.98^{+0.14}$ | $- i 0.72^{+0.06}$ | $-$ | $-$ |
| $\rho\Sigma$   | $-0.15^{+0.07} - i 0.14^{+0.04}$ | $0.76^{+1.02}$ | $- i 0.58^{+0.85}$ | $-0.76^{+0.18}$ | $- i 0.53^{+0.49}$ | $-1.10^{+0.04}$ | $- i 0.34^{+0.03}$ | $-$ | $-$ |
| $\rho\Lambda$  | $0.36^{+0.18} + i 0.29^{+0.07}$ | $-0.95^{+1.15} + i 0.93^{+1.84}$ | $2.44^{+0.50}$ | $+ i 0.94^{+0.27}$ | $1.51^{+0.29}$ | $+ i 0.82^{+0.09}$ | $-$ | $-$ |
| $\omega\Sigma$ | $-0.15^{+0.11} - i 0.14^{+0.05}$ | $1.03^{+1.35}$ | $- i 0.55^{+1.10}$ | $-0.14^{+0.23}$ | $- i 0.44^{+0.14}$ | $-0.64^{+0.10}$ | $- i 0.23^{+0.04}$ | $-$ | $-$ |
| $\phi\Sigma$   | $0.27^{+0.17} + i 0.24^{+0.08}$ | $-1.73^{+2.27}$ | $+ i 0.90^{+1.82}$ | $0.42^{+0.38}$ | $+ i 0.53^{+0.24}$ | $1.04^{+0.20}$ | $+ i 0.39^{+0.07}$ | $-$ | $-$ |

the $\Sigma(1670)$s as $3/2^-$. For a better analysis, we study the following decay ratios known
for $\Sigma(1670)$ (with unknown spin-parity) \[66\],
\[
\frac{\Gamma(\Sigma(1670) \to \bar{K}N)}{\Gamma(\Sigma(1670) \to \pi\Sigma)} < 0.75,
\]
\[
0.05 \lesssim \frac{\Gamma(\Sigma(1670) \to \pi\Lambda)}{\Gamma(\Sigma(1670) \to \pi\Sigma)} \lesssim 0.85,
\]
and from the state in our Fit I, the former ratio is obtained to be $\sim 0.5$ and the latter one is found $\sim 0.06$. In case of $\Sigma(1620) \ (1/2^-)$, the following partial widths are known from different partial-wave analyses \[66\]
\[
0.08 < \frac{\Gamma(\Sigma(1620) \to \bar{K}N)\Gamma(\Sigma(1620) \to \pi\Sigma))^{1/2}}{\Gamma_{\text{total}}} < 0.35,
\]
\[
0.1 < \frac{\Gamma(\Sigma(1620) \to \bar{K}N)\Gamma(\Sigma(1620) \to \pi\Lambda))^{1/2}}{\Gamma_{\text{total}}} < 0.15,
\]
\[
0.08 < \frac{\Gamma(\Sigma(1620) \to \bar{K}N)}{\Gamma_{\text{total}}} < 0.35,
\]
and we obtain them to be 0.37, 0.10, and 0.26, respectively. This analysis shows that our state can be associated to $\Sigma(1620)(1/2^-)$ as well as to $\Sigma(1670)$ with unknown spin-parity, which, in turn, may imply that both these states are not different. It may be useful to give the branching ratios of our state $1630 \pm 33 - i(104 \pm 13) \text{ MeV}$ here. We find that decay ratios to $\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$, $\eta\Sigma$, and $K\Xi$ are 26.3\%, 52.2\%, 3.5\%, 7.9\% and 7.6\%, respectively. Not much is known about $\Sigma(1900)$ either, it has been found in the partial-wave analysis of Ref. \[67\]. The mass and width in Ref. \[66\] of $\Sigma(1900)$ are in agreement with those in Table IV.

C. Isospin = 0, spin = 3/2

The vector-baryon systems can have a total spin 1/2 or 3/2 in s-wave interactions. Thus, we can study states with spin-parity $(J^P) = (3/2^-)$ too. Such states arise purely from vector-baryon dynamics. In the case of the $I(J^P) = 0(3/2^-)$ configuration, in both type of fits, we find a state in the energy region 1700-1800 MeV, while in fit II, one more state appears
TABLE V. Pole positions and couplings of the $I(J^P) = 0 \ (3/2^-)$ states found. The central values and errors were obtained as explained in the caption of Table I. Since $PB$ systems in $s$-wave can only have $J^P = 1/2^-$, there is no coupling between the states listed in this table and the $PB$ channels in our model. Masses and widths are in MeV. The width gets contribution from the widths of the vector mesons (see text for more details).

|       | $\Lambda$ (1690)                               | $\Lambda$ (2050)                               |
|-------|-----------------------------------------------|-----------------------------------------------|
| Fit I | $1802^{+7}_{-8} - i 1.3^{+0.8}_{-0.3}$         | $1901^{+18}_{-15} - i 15.9^{+5.6}_{-7.3}$     |
| Fit II| $1744^{+29}_{-28} - i 1.8^{+1.0}_{-0.6}$       |                                               |

around 1900 MeV (see Table V). Though we may rely more on the results obtained with Fit I, we look for known $3/2^- \Lambda$ states listed by the PDG [66], and find that there are two such $\Lambda$s in 1690-2050 MeV: $\Lambda$(1690), with mass and width of $1697 \pm 6$ MeV and $65 \pm 14$ MeV, respectively, and $\Lambda$(2050), with mass and width listed as $2056 \pm 22$ and $493 \pm 60$ MeV, respectively, out of which the latter one has been catalogue in Ref. [66], so far, only motivated by the partial-wave analysis of $K N$ multichannel reactions done in Ref. [67]. A full comparison is difficult in this case, since in our formalism, the $J^P = 3/2^-$ VB channels do not couple to $J^P = 1/2^- PB$ channels in $s$-wave. The small widths of the states given in Table V are due to the finite widths of the vector mesons involved in the dynamics. For a more reliable determination of the widths, PB and VB channels should be coupled in this
sector too, including other mechanisms, like those in Ref. [68] and including decuplet baryons in our formalism. In addition to this, reactions involving VB final states might be included in the set of data fitted in the analysis. Such extensions of our work should be done in future.

D. Isospin = 1, spin = 3/2

Some states, with $I(J^P) = 1\ (3/2^-)$, are also found in our work, as shown in Table VI. With Fit I, a pole is found around 1617 MeV which can be associated with the $3/2^-$

D. Isospin = 1, spin = 3/2

Some states, with $I(J^P) = 1\ (3/2^-)$, are also found in our work, as shown in Table VI. With Fit I, a pole is found around 1617 MeV which can be associated with the $3/2^-$

D. Isospin = 1, spin = 3/2

Some states, with $I(J^P) = 1\ (3/2^-)$, are also found in our work, as shown in Table VI. With Fit I, a pole is found around 1617 MeV which can be associated with the $3/2^-$

D. Isospin = 1, spin = 3/2

Some states, with $I(J^P) = 1\ (3/2^-)$, are also found in our work, as shown in Table VI. With Fit I, a pole is found around 1617 MeV which can be associated with the $3/2^-$

D. Isospin = 1, spin = 3/2

Some states, with $I(J^P) = 1\ (3/2^-)$, are also found in our work, as shown in Table VI. With Fit I, a pole is found around 1617 MeV which can be associated with the $3/2^-$.TABLE VI. Pole positions and couplings of the $I(J^P) = 1(3/2^-)$ states found in our work. The central values and errors were obtained as explained in the caption of Table I. Since $PB$ systems in s-wave can only have $J^P = 1/2^-$, there is no coupling between the states listed in this table and the $PB$ channels (in our model).

|                | $\Sigma(1670)$          | $\Sigma(1940)$       |
|----------------|-------------------------|----------------------|
| Fit I          | $1617^{+37}_{-i\ 2^{+1}}$ | $-$                  |
| Fit II         | $1818^{+34}_{-i\ 9.5^{+6.8}}$ | $2030^{+41}_{-i\ 150^{+45}}$ |
| $K^*N$         | $0.41^{+0.13}_{-0.015}$ + $i\ 0.003^{+0.015}_{-0.002}$ | $-$                  |
| $K^*\Xi$       | $3.84^{+1.48}_{-0.19}$ + $i\ 0.14^{+0.19}_{-0.18}$ | $-$                  |
| $\rho\Sigma$   | $0.44^{+0.22}_{-0.07}$ + $i\ 0.03^{+0.07}_{-0.09}$ | $-$                  |
| $\rho\Lambda$  | $-1.02^{+0.43}_{-0.03}$ + $i\ 0.04^{+0.06}_{-0.03}$ | $-$                  |
| $\omega\Sigma$ | $-1.25^{+0.52}_{-0.07}$ + $i\ 0.05^{+0.07}_{-0.05}$ | $-$                  |
| $\Phi\Sigma$   | $2.59^{+1.01}_{-0.13}$ + $i\ 0.10^{+0.13}_{-0.10}$ | $-$                  |

$\Sigma(1640)$ [66], whose mass and width range in the interval $1669\pm7$ MeV and $64^{+10}_{-14}$, respectively. As mentioned earlier, in our model there is no coupling between the PB and VB channels
in the spin 3/2 configuration, and, thus, the states get small widths owing to the instability of the vector mesons, which is taken into account by calculating the loop functions as in Eq. (22). For a better estimation of the widths, it may be important to consider transitions from vector-baryon to pseudoscalar-baryon channels in spin 3/2 too, but it is beyond the scope of the present work.

In the case of Fit II, we find two poles which may, both, be associated to the $3/2^- \Sigma(1940)$, since the larger width of the pole $(2030 \pm 41) - i(150 \pm 45)$ would appear like a background in the presence of the pole $(1818 \pm 34) - i(9.5 \pm 6.8)$ in the experimental data. However, Fit II may not be considered good at energies $\gtrsim 1680$ MeV.

IV. SUMMARY AND OUTLOOK

A simultaneous fit to several relevant data has been made to study hyperon resonances. Low-lying hyperon resonances have been studied earlier in several works, by solving pseudoscalar-baryon coupled-channel scattering equations. We have included both pseudoscalar- and vector-baryon dynamics and find that the properties of the widely known hyperons, like, $\Lambda(1405)$, are well reproduced. The formalism used in the previous work on this topic [29] has been extended by including $s$- and $u$-channel diagrams to study pseudoscalar-baryon interactions. We find that an isospin 1 state, around 1400 MeV, also exists, though it is not clear if it is related to one or two poles in the complex plane. The data fitted in the present work are related to the production of pseudoscalar-baryon channels. Still the cross sections at somewhat higher energies are found to follow the data, in one of the two fits obtained in the present work. Thus, hyperons resonances with higher masses have also been studied. The present work can further be improved by considering data on reactions with vector-baryon as final states and by including decuplet baryons in our formalism.
ACKNOWLEDGEMENTS

The authors sincerely thank Prof. Eulogio Oset for reading the manuscript and giving useful suggestions. K.P.K and A.M.T gratefully acknowledge the financial support received from FAPESP (under the grant number 2012/50984-4) and CNPq (under the grant numbers 310759/2016-1 and 311524/2016-8). J.A.O thanks partial financial support from the MINECO (Spain) and EU grant FPA2016-77313-P.
Appendix: Isospin coefficients of different pseudoscalar-baryon amplitudes

TABLE A1. Coefficients for the $s$-channel amplitudes in the isospin 0 base. We indicate in the first column the exchanged particles. For example, the only non zero contribution to a $s$-channel diagram for $\bar{K}N \rightarrow \bar{K}N$, in the isospin 0, comes from a $\Lambda$ exchange.

|     | $\bar{K}N$ | $K\Xi$ | $\pi\Sigma$ | $\eta\Lambda$ |
|-----|------------|--------|--------------|--------------|
| $\Sigma$ | 0          | 0      | 0            | 0            |
| $\Lambda$ | $\frac{(D+3F)^2}{3}$ | 3$F^2 - \frac{D^2}{3}$ | $\sqrt{\frac{2}{3}}D(D + 3F)$ | $\sqrt{\frac{2}{3}}D(D + 3F)$ |
| $N$ | 0          | 0      | 0            | 0            |
| $\Xi$ | 0          | 0      | 0            | 0            |

|     | $K\Xi$ | $\pi\Sigma$ | $\eta\Lambda$ |
|-----|--------|--------------|--------------|
| $\Sigma$ | 0      | 0            | 0            |
| $\Lambda$ | $3F^2 - \frac{D^2}{3}$ | $\frac{(D-3F)^2}{3}$ | $-\sqrt{\frac{2}{3}}D(D - 3F)$ |
| $N$ | 0      | 0            | 0            |
| $\Xi$ | 0      | 0            | 0            |

|     | $\pi\Sigma$ | $\eta\Lambda$ |
|-----|--------------|--------------|
| $\Sigma$ | $\sqrt{\frac{2}{3}}D(D + 3F)$ | $\frac{2D^2}{3}$ |
| $\Lambda$ | $-\sqrt{\frac{2}{3}}D(D - 3F)$ | $\frac{2D^2}{\sqrt{3}}$ |
| $N$ | 0          | 0            |
| $\Xi$ | 0          | 0            |
TABLE A2. Coefficients for the $s$-channel amplitudes in the isospin 1 base. We indicate in the first column the exchanged particles. For example, the only non zero contribution to a $s$-channel diagram for $\bar{K}N \rightarrow \bar{K}N$, in isospin 1, comes from a $\Sigma$ exchange.

|     | $\bar{K}N$ | $\bar{K}\Xi$ | $\pi\Sigma$ | $\pi\Lambda$ | $\eta\Sigma$ |
|-----|------------|-------------|-------------|--------------|--------------|
| $\bar{K}N$ | | | | | |
| $\Sigma$ | $(D - F)^2$ | $D^2 - F^2$ | $2F(D - F)$ | $-\sqrt{\frac{2}{3}}D(D - F)$ | $-\sqrt{\frac{2}{3}}D(D - F)$ |
| $\Lambda$ | 0 | 0 | 0 | 0 | 0 |
| $N$ | 0 | 0 | 0 | 0 | 0 |
| $\Xi$ | 0 | 0 | 0 | 0 | 0 |
| $\bar{K}\Xi$ | | | | | |
| $\Sigma$ | $D^2 - F^2$ | $(D + F)^2$ | $2F(D + F)$ | $-\sqrt{\frac{2}{3}}D(D + F)$ | $-\sqrt{\frac{2}{3}}D(D + F)$ |
| $\Lambda$ | 0 | 0 | 0 | 0 | 0 |
| $N$ | 0 | 0 | 0 | 0 | 0 |
| $\Xi$ | 0 | 0 | 0 | 0 | 0 |
| $\pi\Sigma$ | | | | | |
| $\Sigma$ | $2F(D - F)$ | $2F(D + F)$ | $4F^2$ | $-2\sqrt{\frac{2}{3}}DF$ | $-2\sqrt{\frac{2}{3}}DF$ |
| $\Lambda$ | 0 | 0 | 0 | 0 | 0 |
| $N$ | 0 | 0 | 0 | 0 | 0 |
| $\Xi$ | 0 | 0 | 0 | 0 | 0 |
| $\pi\Lambda$ | | | | | |
| $\Sigma$ | $-\sqrt{\frac{2}{3}}D(D - F)$ | $-\sqrt{\frac{2}{3}}D(D + F)$ | $-2\sqrt{\frac{2}{3}}DF$ | $\frac{2D^2}{3}$ | $\frac{2D^2}{3}$ |
| $\Lambda$ | 0 | 0 | 0 | 0 | 0 |
| $N$ | 0 | 0 | 0 | 0 | 0 |
| $\Xi$ | 0 | 0 | 0 | 0 | 0 |
| $\eta\Sigma$ | | | | | |
| $\Sigma$ | $-\sqrt{\frac{2}{3}}D(D - F)$ | $-\sqrt{\frac{2}{3}}D(D + F)$ | $-2\sqrt{\frac{2}{3}}DF$ | $\frac{2D^2}{3}$ | $\frac{2D^2}{3}$ |
| $\Lambda$ | 0 | 0 | 0 | 0 | 0 |
| $N$ | 0 | 0 | 0 | 0 | 0 |
| $\Xi$ | 0 | 0 | 0 | 0 | 0 |

29
TABLE A3. Coefficients for the \( u \)-channel amplitudes in the isospin 0 base. We indicate in the first column the exchanged particles. For example, a \( \Sigma \) and a \( \Lambda \) exchange in the \( u \)-channel give non zero contributions to the process \( \bar{K}N \rightarrow \bar{K}\Xi \), in isospin 0

|     | \( \bar{K}N \) | \( \bar{K}\Xi \) | \( \pi\Sigma \) | \( \eta\Lambda \) |
|-----|----------------|-----------------|----------------|----------------|
| \( \bar{K}N \) | \( \Sigma \) | 0 | \( -\frac{1}{3}(D^2 - F^2) \) | 0 | 0 |
|     | \( \Lambda \) | 0 | \( -\frac{1}{6}(D^2 - 9F^2) \) | 0 | 0 |
|     | \( N \) | 0 | 0 | \( -\sqrt{\frac{3}{2}}(D^2 - F^2) \) | \( \frac{D^2 - 9F^2}{3\sqrt{2}} \) |
|     | \( \Xi \) | 0 | 0 | 0 | 0 |
| \( \bar{K}\Xi \) | \( \Sigma \) | \( \frac{1}{2}(D^2 - F^2) \) | 0 | 0 | 0 |
|     | \( \Lambda \) | \( \frac{1}{6}(D^2 - 9F^2) \) | 0 | 0 | 0 |
|     | \( N \) | 0 | 0 | 0 | 0 |
|     | \( \Xi \) | 0 | 0 | \( \sqrt{\frac{3}{2}}(D^2 - F^2) \) | \( -\frac{D^2 - 9F^2}{3\sqrt{2}} \) |
| \( \pi\Sigma \) | \( \Sigma \) | 0 | 0 | \( 2F^2 \) | \( \sqrt{\frac{3}{2}} \) |
|     | \( \Lambda \) | 0 | 0 | \( \frac{3}{2}F^2 \) | 0 |
|     | \( N \) | \( -\sqrt{\frac{3}{2}}(D^2 - F^2) \) | 0 | 0 | 0 |
|     | \( \Xi \) | 0 | \( \sqrt{\frac{3}{2}}(D^2 - F^2) \) | 0 | 0 |
| \( \eta\Lambda \) | \( \Sigma \) | 0 | 0 | \( -\frac{2D^2}{\sqrt{3}} \) | 0 |
|     | \( \Lambda \) | 0 | 0 | 0 | \( \frac{2D^2}{3} \) |
|     | \( N \) | \( \frac{3}{3\sqrt{2}}(D^2 - 9F^2) \) | 0 | 0 | 0 |
|     | \( \Xi \) | 0 | \( \frac{D^2 - 9F^2}{3\sqrt{2}} \) | 0 | 0 |
TABLE A4. Coefficients for the $u$-channel amplitudes in the isospin 1 base. We indicate in the first column the exchanged particles. For example, a $\Sigma$ and a $\Lambda$ in the $u$-channel give non zero contributions to the process $\bar{K}N \rightarrow \bar{K}\Xi$, in isospin 1.

|       | $\bar{K}N$ | $K\Xi$ | $\pi\Sigma$ | $\pi\Lambda$ | $\eta\Sigma$ |
|-------|------------|--------|--------------|--------------|--------------|
| $\bar{K}N$ | $\Sigma$ | 0 | $-\frac{D^2-F^2}{2(D^2-9F^2)}$ | 0 | 0 | 0 |
|       | $\Lambda$ | 0 | 0 | 0 | 0 | 0 |
|       | $N$       | 0     | $D^2-F^2$ | $\frac{(D+F)(D+3F)}{\sqrt{6}}$ | $\frac{(D-F)(D-3F)}{\sqrt{6}}$ |
|       | $\Xi$     | 0     | 0 | 0 | 0 | 0 |
| $K\Xi$ | $\Sigma$ | 0 | $\frac{-D^2-F^2}{2(D^2-9F^2)}$ | 0 | 0 | 0 |
|       | $\Lambda$ | 0 | 0 | 0 | 0 | 0 |
|       | $N$       | 0     | 0 | 0 | 0 | 0 |
|       | $\Xi$     | 0     | $F^2-D^2$ | $\frac{(D-F)(D-3F)}{\sqrt{6}}$ | $\frac{(D+F)(D+3F)}{\sqrt{6}}$ |
| $\pi\Sigma$ | $\Sigma$ | 0 | 0 | $2F^2$ | $2\sqrt{\frac{2}{3}}DF$ | $-2\sqrt{\frac{2}{3}}DF$ |
|       | $\Lambda$ | 0 | 0 | $-\frac{2D^2}{3}$ | 0 | 0 |
|       | $N$       | 0     | $D^2-F^2$ | 0 | 0 | 0 |
|       | $\Xi$     | 0     | $F^2-D^2$ | 0 | 0 | 0 |
| $\pi\Lambda$ | $\Sigma$ | 0 | 0 | $\frac{2\sqrt{2}}{3}DF$ | $\frac{2D^2}{3}$ | 0 |
|       | $\Lambda$ | 0 | 0 | 0 | 0 | $\frac{-2D^2}{3}$ |
|       | $N$       | 0     | 0 | 0 | 0 | 0 |
|       | $\Xi$     | 0     | 0 | 0 | 0 | 0 |
| $\eta\Sigma$ | $\Sigma$ | 0 | 0 | $-2\sqrt{\frac{2}{3}}DF$ | 0 | $\frac{2D^2}{3}$ |
|       | $\Lambda$ | 0 | 0 | 0 | $\frac{-2D^2}{3}$ | 0 |
|       | $N$       | 0     | 0 | 0 | 0 | 0 |
|       | $\Xi$     | 0     | 0 | 0 | 0 | 0 |
[1] N. Kaiser, P. B. Siegel and W. Weise, Nucl. Phys. A 594, 325 (1995).
[2] E. Oset and A. Ramos, Nucl. Phys. A 635 (1998) 99.
[3] J. A. Oller and U. G. Meiße, Phys. Lett. B 500, 263 (2001).
[4] D. Jido, J. A. Oller, E. Oset, A. Ramos and U. G. Meiße, Nucl. Phys. A 725, 181 (2003).
[5] T. Hyodo and D. Jido, Prog. Part. Nucl. Phys. 67, 55 (2012).
[6] H. Y. Lu et al. [CLAS Collaboration], Phys. Rev. C 88, 045202 (2013).
[7] C. Garcia-Recio, C. Hidalgo-Duque, J. Nieves, L. L. Salcedo and L. Tolos, Phys. Rev. D 92, no. 3, 034011 (2015).
[8] H. Kamano and T.-S. H. Lee, Phys. Rev. C 94, no. 6, 065205 (2016).
[9] A. Ramos, A. Feijoo and V. K. Magas, Nucl. Phys. A 954, 58 (2016).
[10] M. Skurzok et al., Acta Phys. Polon. B 49, 705 (2018).
[11] A. Dot, T. Inoue and T. Myo, JPS Conf. Proc. 17, 082006 (2017).
[12] F. Sakuma [J-PARC E15 Collaboration], JPS Conf. Proc. 13, 010002 (2017).
[13] D. Cabrera, L. Tolos, J. Aichelin and E. bratkovskaya, J. Phys. Conf. Ser. 668, no. 1, 012048 (2016).
[14] M. Mai, Few Body Syst. 59, no. 4, 61 (2018).
[15] K. Miyahara, T. Hyodo and W. Weise, arXiv:1804.08269 [nucl-th].
[16] L. Roca, J. Nieves and E. Oset, JPS Conf. Proc. 17, 071002 (2017).
[17] C. Wang, L. l. Liu and X. H. Guo, Phys. Rev. D 96, no. 5, 056002 (2017).
[18] Z. W. Liu, J. M. M. Hall, D. B. Leinweber, A. W. Thomas and J. J. Wu, Phys. Rev. D 95, no. 1, 014506 (2017).
[19] Y. Kamiya, K. Miyahara, S. Ohnishi, Y. Ikeda, T. Hyodo, E. Oset and W. Weise, Nucl. Phys. A 954, 41 (2016).
[20] R. Molina and M. Dring, Phys. Rev. D 94, no. 5, 056010 (2016) Addendum: [Phys. Rev. D 94, no. 7, 079901 (2016)].
[21] E. Oset and L. Roca, EPJ Web Conf. 97, 00023 (2015).
[22] A. Martinez Torres, M. Bayar, D. Jido and E. Oset, Int. J. Mod. Phys. Conf. Ser. 26, 1460057 (2014).
[23] Zhi-Hui Guo and J. A. Oller, Phys. Rev. C. 87, 035202 (2013).
[24] J. J. Wu, S. Dulat and B. S. Zou, Phys. Rev. D 80, 017503 (2009).
[25] J. J. Wu, S. Dulat and B. S. Zou, Phys. Rev. C 81, 045210 (2010).
[26] P. Gao, J. J. Wu and B. S. Zou, Phys. Rev. C 81, 055203 (2010).
[27] J. J. Xie, J. J. Wu and B. S. Zou, Phys. Rev. C 90, no. 5, 055204 (2014).
[28] J. J. Xie and L. S. Geng, Phys. Rev. D 95, no. 7, 074024 (2017).
[29] K. P. Khemchandani, A. Martinez Torres, H. Nagahiro and A. Hosaka, Phys. Rev. D 85, 114020 (2012).
[30] K. Moriya et al. [CLAS Collaboration], Phys. Rev. C 87, no. 3, 035206 (2013).
[31] D. Sadasivan, M. Mai and M. Döring, arXiv:1805.04534 [nucl-th].
[32] L. Roca and E. Oset, Phys. Rev. C 88, 055206 (2013).
[33] J. A. Oller and E. Oset, Phys. Rev. D 60, 074023 (1999).
[34] J. A. Oller, Phys. Lett. B 477, 187 (2000)
[35] J. A. Oller and E. Oset, Nucl. Phys. A 620, 438 (1997) Erratum: [Nucl. Phys. A 652, 407 (1999)].
[36] U. G. Meißner, Rept. Prog. Phys. 56, 903 (1993).
[37] G. Ecker, Prog. Part. Nucl. Phys. 35 (1995) 1.
[38] A. Pich, Rep. Prog. Phys. 58 (1995) 563.
[39] J. A. Oller, M. Verbeni and J. Prades, JHEP 0609, 079 (2006).
[40] B. Borasoy, R. Nissler and W. Weise, Eur. Phys. J. A 25, 79 (2005).
[41] K. P. Khemchandani, H. Kaneko, H. Nagahiro and A. Hosaka, Phys. Rev. D 83 (2011) 114041.
[42] E. E. Jenkins, M. E. Luke, A. V. Manohar, M. J. Savage, Phys. Lett. B302, 482-490 (1993).
[43] U. -G. Meißner, S. Steininger, Nucl. Phys. B499, 349-367 (1997).
[44] D. Jido, A. Hosaka, J. C. Nacher, E. Oset, A. Ramos, Phys. Rev. C66, 025203 (2002).
[45] K. P. Khemchandani, A. Martinez Torres, H. Kaneko, H. Nagahiro and A. Hosaka, Phys. Rev. D 84 (2011) 094018.
[46] K. P. Khemchandani, A. Martinez Torres, H. Nagahiro and A. Hosaka, Phys. Rev. D 88 (2013) 114016; K. P. Khemchandani, A. Martinez Torres, H. Nagahiro and A. Hosaka, Int. J. Mod. Phys. Conf. Ser. 26 (2014) 1460060.
[47] K. P. Khemchandani, A. Martinez Torres, A. Hosaka, H. Nagahiro, F. S. Navarra and M. Nielsen, Phys. Rev. D 97, no. 3, 034005 (2018).
[48] A. Feijoo, V. K. Magas and A. Ramos, J. Phys. Conf. Ser. 1024, no. 1, 012020 (2018). doi:10.1088/1742-6596/1024/1/012020
[49] M. Bazzi et al. [SIDDHARTA Collaboration], Phys. Lett. B 704, 113 (2011).
[50] U. G. Meißner, U. Raha and A. Rusetksy, Eur. Phys. J. C 35, 349 (2004).
[51] C. Garcia-Recio, J. Nieves, E. Ruiz Arriola and M. J. Vicente Vacas, Phys. Rev. D 67, 076009 (2003)
[52] Y. Ikeda, T. Hyodo and W. Weise, Nucl. Phys. A 881, 98 (2012).
[53] M. Mai and U. G. Meißner, Nucl. Phys. A 900, 51 (2013).
[54] M. Albaladejo and J. A. Oller, Phys. Rev. Lett. 101, 252002 (2008).
[55] A. Etkin et al., Phys. Rev. D 25, 1786 (1982).
[56] Landolt and Börsntein, Numerical data and Functional Relationships in Science and Technology, Group I, Volume 12, Sub-volume a, Total-Cross Sections for reactions of high energy physics, A. Baldini, V. Flaminio, W. I. G. Moorhead, D. R. O. Morrison, Edited by H. Schopper.
[57] L. Roca, S. Sarkar, V. K. Magas and E. Oset, Phys. Rev. C 73, 045208 (2006).
[58] A. Martinez Torres, K. P. Khemchandani and E. Oset, Phys. Rev. C 77, 042203 (2008).
[59] E. Oset and A. Ramos, Eur. Phys. J. A 44 (2010) 445.

[60] S. i. Nam, J. H. Park, A. Hosaka and H. C. Kim, J. Korean Phys. Soc. 59, 2676 (2011).

[61] H. Kohri et al. [LEPS Collaboration], Phys. Rev. Lett. 104, 172001 (2010).

[62] S. H. Kim, S. i. Nam, A. Hosaka and H. C. Kim, Phys. Rev. D 88, no. 5, 054012 (2013).

[63] S. i. Nam, K. S. Choi, A. Hosaka and H. C. Kim, Phys. Rev. D 75, 014027 (2007).

[64] E. Oset, A. Ramos, E. J. Garzon, R. Molina, L. Tolos, C. W. Xiao, J. J. Wu and B. S. Zou, Int. J. Mod. Phys. E 21, 1230011 (2012).

[65] E. Wang, J. J. Xie and E. Oset, Phys. Lett. B 753, 526 (2016) doi:10.1016/j.physletb.2015.12.060 [arXiv:1509.03367 [hep-ph]].

[66] M. Tanabashi et al. [ParticleDataGroup], Phys. Rev. D 98, no. 3, 030001 (2018).

[67] H. Zhang, J. Tulpan, M. Shrestha and D. M. Manley, Phys. Rev. C 88, no. 3, 035205 (2013).

[68] E. J. Garzon and E. Oset, Eur. Phys. J. A 48, 5 (2012)