Magnetoroton modes at sub-Zeeman energies in a filling factor range around 1/3

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The nature of the excitations causing the modes at sub-Zeeman energies in a filling factor range around 1/3 first observed more than a decade ago by Dujovne et al. [Phys. Rev. Lett. 90, 036803 (2003)] still remains unresolved. We make a strong case that these modes are due to the “magnetorotons” in spinless neutral excitations. By calculating the dispersion of Girvin-MacDonald-Platzman mode using the recently proposed “unconventional” ground states for the fractional quantum Hall effect observed at the filling factors $\nu = 4/11, 4/13, 5/13, 5/17, 3/8, \text{and} 6/17$, we show a generic formation of the primary magnetorotons for all these $\nu$ at very low momenta with energies lower than a typical Zeeman energy in the semiconductor systems. We further propose an experimental criterion to test our theory.

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Cooperative behavior of the two-dimensional electrons in the presence of strong magnetic field leading to fractional quantum Hall effect (FQHE) [1] reflects in the collective modes [2] of excitations. The nature of these modes describes the underlying properties of the ground state arising from the collective quantum correlation. The observed collective modes [3-7] in FQHE states at filling factors $\nu = n/(2n \pm 1)$ confirm the prediction [8] of the presence of Landau-like levels emerged due to the creation of composite fermions (CFs), which are quasiparticles formed by electrons attached to an even number $(2p)$ of quantized vortices and are denoted as $2p$-CFs. The primary Jain sequence [8], $\nu = n/(2np \pm 1)$, of prominent states of FQHE in the partially filled lowest Landau level (LLL) can be envisaged through the model [8] of non-interacting CFs. Observation [9,11] of certain weaker FQHE states between two prominent states at filling factors in the range $2/5 > \nu > 1/3$ and $1/3 > \nu > 2/7$ indicates the importance of interaction between $2p$-CFs and $4p$-CFs respectively in the partially filled effective Landau-like levels of CFs, called $\Lambda$ levels (ALs), in producing the necessary energy gaps for such states. Hitherto these two types of states are referred to as Type-I and Type-II states respectively.

Neutral collective excitations in Type-I states correspond to the formation of excitons [12,14] of CFs when a CF is excited from one of the fully filled AL to an empty AL. Most of the low-lying modes among these excitations are observed [3,6,15] in various experiments. One of the intriguing features that the spin excitations are possible at below Zeeman energy, $E_Z$, due to the excitation of a CF by lowering [16,17] its AL and flipping its spin has been revealed by observing [7] modes at sub-$E_Z$ in resonant inelastic light scattering (RILS) experiments. Modes at sub-$E_Z$ for Type-II states have also been reported [18,19] and their presence further indicate the importance of interaction between the CFs in the partially filled AL. Indeed, interaction between the CFs in the partially filled AL leads to certain “unconventional” incompressible [20-24] FQHE states. However, determination of the nature of the excitations is necessary for a deeper understanding of the characteristics of these unconventional Type-II states.

Although the observed modes at sub-$E_Z$ in RILS experiments for Type-II states have been ascribed [18,19] as spin-flip (SF) excitations, the true nature of these excitations has not been resolved yet. We propose here that these modes are “magnetorotons” in the neutral spin-conservating excitations, not the SF excitations for the following reasons: (i) excitation below $E_Z$ is neither theoretically expected nor experimentally observed at $\nu = 1/3$; the observed sub-$E_Z$ mode at $\nu = 2/5$ which has been interpreted as “spin-roton” mode [7] in SF excitations, has energy only about $0.002 e^2/\epsilon l$ below $E_Z$, where $l = (\hbar c/eB)^1/2$ is the magnetic length and $\epsilon$ is the dielectric constant of the background material; deviations [13,19] of energies of the sub-$E_Z$ modes from $E_Z$ for Type-II states are about 4-5 times greater than that for $\nu = 2/5$; thus the continuity will not be restored if the observed sub-$E_Z$ modes for Type-II states are interpreted in terms of the SF excitations, (ii) the exact and the CF-diagonalization studies [25] for finite systems at $\nu = 4/11, 5/13$, and $3/8$ fillings suggest that the minimum energy required for neutral spinless excitation is within the range $0.002-0.01 e^2/\epsilon l$ which is substantially lower than a typical $E_Z$, $(\sim 0.02 e^2/\epsilon l)$ [20-24]. This is not surprising as the neutral excitations for these states involve intra-AL excitations. This minimum energy, however, does not ensure that the observed sub-$E_Z$ mode is due to the spinless neutral excitations because the modes detectable in RILS experiments correspond to the excitations with high density of states such as magnetorotons. In this letter, we show the formation of primary magnetoroton at a very low momentum in the dispersion of Girvin-MacDonald-Platzman (GMP) mode [2] whose energy is extremely low (sub-$E_Z$) for some of the filling...
The partially filled second Landau level with filling factors \( \nu \) of 2CFs (4CFs) is related with electronic filling factor as \( \nu = \nu^\ast/(2\nu^\ast + 1) \), where \( 0 < \nu < 1 \). We assume all the electrons are spin-polarized through out the paper except mentioning otherwise. The 2CFs with \( \bar{\nu} = \nu = 1/3, 2/3, 1/5, 4/5 \), and 1/2 respectively constitute \( \nu = 4/11, 5/13, 6/17, 9/23, 3/8 \). The partially filled second Landau level with filling factors \( \bar{\nu} = 1/3, 1/5, 4/5 \), and their respective particle-hole conjugates 2/3 and 4/5 are characterized by Wójc, Yi, and Quinn (WYQ) state \( ^2\Phi \) which is the groundstate of Haldane pseudopotential \( V_\theta \) that minimizes occupation with relative angular momentum 3 between any two particles. The even-denominator 3/8 state is characterized by the anti-Pfaffian pairing correlation \( ^2\Phi \) in the half-filled second Landau level that makes it a nonabelian FQHE state. The states of 4CFs at \( \nu = 4/13, 5/17, \) and 3/10 have identical character \( ^4\Phi \) with the states of 2CFs at \( \nu = 4/11, 5/13, \) and 3/8 respectively.

We employ the single mode approximation (SMA) \( ^2\Phi \) to obtain the dispersion of the GMP collective modes at the second generation filling factors. In the SMA, the spin-conserving excited state may be obtained by operating LLL-projected number-density operator on the ground state at a given filling factor. Thus the expression for the dispersion \( \Delta(k) \) of the excitation energies is given by

\[
\Delta(k) = 2[\bar{S}(k)]^{-1} \int \frac{dq}{(2\pi)^2} \sin^2 \left( \frac{k \times q}{2} \right) e^{-k^2l^2/2} \times \left[ v(|q - k|) e^{-k^2l^2/2} - v(q) \right] \bar{S}(q)
\]

with momenta \( k \) and \( q \), Fourier transform of the Coulomb potential \( v(q) = 4\pi\varepsilon^2/(\epsilon q) \). Here \( \bar{S}(k) = S(k) - 1 + e^{-k^2l^2/2} \) is the LLL projection of the static structure factor \( S(k) \). We thus determine \( S(k) \) below for calculating the GMP mode \( ^1\Phi \).

We begin with the calculation of pair-correlation functions \( g(r) \) between electrons using the ground state wave function for a certain filling factor in the spherical geometry \( ^2\Phi \), in which \( N \) electrons move on the surface of a sphere with radius \( R = \sqrt{Q} \), exposed to a magnetic quantum flux \( 2Q \) in the unit of a flux quantum \( \phi_0 = \hbar c/e \) produced by a magnetic monopole placed at the center of the sphere. The arc distance between two particles has been considered as \( r \). The relation between the total (integer) flux, \( 2Q \), and the total number of electrons, \( N \), for all the spin-polarized FQHE states in the range \( 2/5 > \nu > 2/7 \) as follows:

\[
2Q = \nu^{-1}(N - 1) - (3 - \nu^{-1})(\lambda + 2)
\]

where the so-called “flux-shift” \( \lambda \) for the CFs in the partially filled Landau level is defined in terms of the magnitude of the effective flux as \( 2Q^\ast = \bar{\nu}N - (\lambda + 2) \) with \( \bar{N} = N - (2Q^\ast + 1) \) being the number of CFs in the partially filled Landau level, and \( 2Q^\ast + 1 \) is the degeneracy in the lowest Landau level. Recall \( ^4\Phi \), that the choice of \( \lambda \) characterizes the nature of correlation in the partially filled Landau level: \( \lambda = 7 \) for \( \nu = 4/11 \) and 4/13; \( \lambda = -2 \) for \( \nu = 5/13 \) and 5/17; \( \lambda = 9 \) for 6/17; and \( \lambda = -1 \) for 9/23, and 3/8. The composite-fermion-diagonalization (CFD) method \( ^4\Phi \) which is almost exact \( ^2\Phi \) for finite number of particles has been used to show that the ground states at these filling factors with these particular \( \lambda \) are incompressible. Excepting 3/8 state for which we consider the CFD ground state, we have considered the proposed \( ^2\Phi \) CFD-WYQ wavefunctions, \( ^4\Phi \), respectively for the states corresponding to 2CFs and 4CFs, for calculating \( g(r) \) as they have significantly high overlap with the CFD ground state, which allows us calculate for higher number of particles. (We have checked that the pair-correlation calculated using the CFD ground states for smaller number of particles agrees with that calculated using wavefunctions up to maximum distance accessible using the CFD ground states.) Here spherical spinor variables \( u = \cos(\theta/2)e^{-i\phi/2} \) and \( v = \sin(\theta/2)e^{i\phi/2} \), \( P_{LLL} \) denotes projection into the LLL, and \( \Phi_{WYQ} \) denotes wavefunction at filling factor 1 + \( \bar{\nu} \). When the partially filled second Landau level would have WYQ correlation \( ^4\Phi \), with \( \pm \) referring to the sign of effective magnetic flux, \( 2Q^\ast \).

We employ the Monte Carlo method with Metropolis algorithm to calculate \( g(r) \) for \( \nu = 4/11, 5/13, 6/17, \) and 9/23 using the state \( ^2\Phi \) for \( N = 32, 36, 26, \) and 27 respectively, \( \nu = 4/13 \) and 5/17 using the state \( ^4\Phi \) for \( N = 24 \) and 26 respectively, and \( \nu = 3/8 \) using the CFD ground state for \( N = 24 \). We are restricted to consider more number of particles because of determining huge number (equal to the number of basis states in a system with particle \( \bar{N} \) and flux \( 2(Q^\ast + 2) \) of \( \bar{N} \times N \) determinants in each step of the Monte Carlo and also the projection into the LLL of the states involving 4CFs with negative effective flux. We find that \( g(r) \) oscillates around its long range value 1.0 with smaller decay rate in comparison to the neighboring Jain states, but the amplitude of the oscillation in the calculated data for finite systems does not die out completely to obtain required \( g(r) = 1 \) at large distances. We therefore extrapolate the
numerical data up to large distances using the damped-oscillatory form $g(r) = 1 + A(r/l)^{-\alpha} \sin(\beta r/l - \gamma)$ used earlier [31], where numerical constants $A$, $\alpha$, $\beta$, and $\gamma$ which are different for different filling factors, are determined by fitting the available numerical data. We thus obtain more oscillations in $g(r)$ before it converges to its large distance limit. For example, in the case of $\nu = 2/7$, there are three oscillations [21] obtained using the CFD ground state for $N = 28$ and four oscillations using the wavefunction $\Psi_1$ up to $N = 32$ for the finite systems, but our thermodynamically extrapolated $g(r)$ acquires three more oscillations before it converges to unity. Figure 1 shows $g(r)$ for the thermodynamic systems at different filling factors in the range $2/5 \geq \nu \geq 2/7$. While $g(r)$ for $\nu = 1/3$ has one maximum, $\nu = 2/5$ and $2/7$ have two maxima each, and for any state in Jain sequence [8] with $\nu = n/(2n \pm 1)$ has $n$ maxima [32], $g(r)$ for Type-II states have several maxima and the number of maxima does not match with the numerator of the filling factor. This has direct consequence on the static structure factor and hence on the energy dispersion of the collective modes. Recall [32] that the energy of the primary roton at $\nu = 5/11$ is much less than the same at $\nu = 2/5$ because the number of oscillations in $g(r)$ for the former is more than the latter.

The structure factors may then be calculated using the relation $S(k) = 1 + n_0 \int dr e^{ikr}[g(r) - 1]$, where mean electron density $n_0 = \nu/(2\pi l^2)$. As suggested by GMP [2], an appropriate form of pair-correlation function of a quantum liquid in a FQHE state at the LLL is given by

$$g(r) = 1 - e^{-r^2/4l^2} + \sum_m \frac{2}{m!} \frac{r^2}{4l^2}^m c_m e^{-r^2/4l^2}$$

where prime indicates summation over odd $m$ only. The numerically calculated $g(r)$ and its thermodynamic extrapolation data are fitted with the functional form [5] and the upper cut-off value of $m$ is taken to be very large for picking up oscillations in $g(r)$. The coefficients $c_m$’s are constrained with the charge neutrality, perfect screening, and the compressibility sum rules [2] [33] as an analogy with the two-dimensional one component plasma is invoked, can be expressed in terms of the respective moments of the pair-correlation functions: $M_0 = -1$, $M_1 = -1$, and $M_2 = 2(\nu^{-1} - 2)$ with $M_n = n! n_0 \int dr (r^2/2)^n g(r) - 1$. These sum rules ensure that the projected structure factor into the LLL behaves as $S(k) \rightarrow (kl)^4(1 - \nu)/8\nu$ as $k \rightarrow 0$.

We calculate $S(k)$ for all the Type-II states considered here and show in Fig. 2. $S(k)$ for $\nu = 1/3$, 2/5, and 2/7 has also been recalculated for comparison with $S(k)$ of Type-II states and shown as insets in Fig. 2. Owing to the many more oscillations in $g(r)$ for Type-II states, change in the sign of the slope of $S(k)$ occurs several times compared to the same for the neighboring Jain states. Also, the spectral weights at low through moderate $k$, $(k \lesssim 1.5l^{-1})$, for Type-II states are more than the Jain states.

We next calculate the energy dispersion [1] $\Delta(k)$ of the GMP modes for the unconventional FQHE states at

![Figure 1](image1.png)

![Figure 2](image2.png)
the filling factors $\nu = 4/11, 5/13, 3/8, 6/17, 9/23, 4/13,$ and $5/17$ and show in Fig. 3. We also show the dispersion of the GMP modes for neighboring Jain states, viz, $\nu = 1/3, 2/5,$ and $2/7$ as insets of Fig. 3. While $\nu = 1/3$ has one magnetoroton at $k \simeq 1.5l^{-1}$, $\nu = 2/5$ and $2/7$ have two magnetoroton modes, of which the primary roton minimum occurs at $k \simeq 0.4l^{-1}$ and the secondary roton minimum occurs at $k \simeq 1.6l^{-1}$ and $1.4l^{-1}$ respectively. The unconventional FQHE states in the ranges $2/5 > \nu > 1/3$ and $1/3 > \nu > 2/7$ have two prominent magnetoroton each, of which the position of the secondary one replicates that for $\nu = 2/5$ and $2/7$ in the respective ranges and the primary one forms at $k \simeq 0.2–0.3l^{-1}$. Several other weaker magnetorotons also form for these states due to the appearance of several changes in the sign of the slopes of $S(k)$ at low through moderate $k$, caused by the several oscillations in $g(r)$. The energy of the primary roton is generically very small and it lies in the range $0.004–0.011 e^2/\epsilon l$. As the decay rate of the amplitude of the oscillation is less in $g(r)$ for the unconventional Type-II FQHE states compared to the neighboring Jain states, and the corresponding $S(k)$ has higher spectral weight at moderate $k$, the energy of the primary roton becomes extremely small.

Apart from ignoring ubiquitous disorder and Landau level mixing, we have also not considered the contribution of finite thickness because our calculation of the dispersion is based on the SMA which is not the best for quantitative evaluations. Nonetheless, our study has merit in showing the formation of primary magnetorotons at very low momenta with sub-Zeeman energies as the SMA has proved itself a good approximation in the case of the primary sequence of states at $\nu = n/(2n + 1)$: It predicts correct numbers of magnetorotons, shows correct qualitative behavior at low momenta, and provides comparable (within 10–35% deviation) estimation of energy of the primary magnetoroton to the same predicted in a more robust excitonic theory of the CFs.

The sub-$E_Z$ modes have been observed in RILS experiments using depolarized geometry where the directions of polarizations of the incident and scattered light are perpendicular to each other. Not only the spin excitations but also the spinless excitations are selected in the depolarized spectra. On the other hand, polarized spectra in which the polarizations of the incident and scattered light are parallel selects only the spinless excitations. Therefore observing a sub-$E_Z$ mode for Type-II states in polarized spectra of the RILS experiments should be a confirmatory test of the theory predicted here.

Although we have considered here the spin-polarized FQHE states only for the sake of simplicity in determining the GMP modes [2], we believe that the neutral spinless modes [35,36] for other spin-polarizations [22,24,26] also will have very low energy magnetorotons because the energy for the relevant intra-ΛL excitations is expected to be small.

In summary, we show that the magnetorotons form at very low energies in the dispersions of the spinless neutral GMP modes at $\nu = 4/11, 14/3, 5/13, 5/17, 3/8, 6/17, 9/23$ described by the unconventional FQHE states. These energies are substantially smaller than a typical Zeeman energy for a realistic experimental situation. We thus explain the observed sub-Zeeman energy for the relevant intra-ΛL excitations is expected to be small.

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