The estimation and forecast of solar energy yield with Weibull distribution

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ABSTRACT

In the first and second chapter the article provides an overview of the currently used energy sources in Hungary and the most popular renewable energies. In addition, the Weibull estimation is presented, too. The subsequent chapter looks at some of the research results about the solar energy optimization with Weibull distribution. The study presented is a mathematical solution of the solar energy optimization with distribution. The final chapter contains a brief explanation of the results. This publication briefly summarizes a prototype solution for an estimation and forecast of solar energy and yield with Weibull distribution.

KEYWORDS

solar energy, Weibull distribution, renewable energy, energy sources in Hungary, mathematical modelling

1. INTRODUCTION

The increased use of renewable energy in the European Union is required by laws, conventions and several directives that are in line with EU legislation. Each Member State has set out commitments in its national energy policies reflecting the statement above, which Member States intend to achieve through a variety of methods. Renewable energy and related energy management are quite popular in several engineering fields, such as transportation or building service engineering - building energetics. Despite, there is resistance to new energy sources in several areas, one of the most popular examples of which might be the automotive industry. In construction, energetic renovation of buildings can be extremely costly, which each country seeks to help with subsidies. Nonetheless, so-called 'climate sceptics' must be considered, who do not see climate change as a real problem [1, 11, 12].

Various 'green movements and green organizations', such as Greenpeace and energy politicians in Brussels, have a strong view to halting climate change in Europe, for which an action plan has been drawn up for 2050. It is important to note that actions set up in this way may not be able to address climate change globally, as in the rest of the world, emission standards are not or not properly regulated in either energy or transportation. Currently (2021) the European continent accounts for 11% of the world’s energy consumption, of which Hungary consumes 0.2% [1, 11, 12].

The social, engineering, and scientific majority of Europe and Hungary are committed to the problem of climate change. Based on current facts and economic opportunities, it is possible to use renewable energy sources, but only in a moderate way. Excessive withdrawal from reality and possible naivety can generate a lot of damage in practice. Numerous research and engineering studies have highlighted that only partial decarbonization is possible by 2050. Nowadays the solar energy is very popular at buildings but in industrial field it is not stable enough [1, 11, 12, 17].
This means that the use of renewable energy sources may pose a problem not only in the short but also in the medium term; Károly Gerse, László Molnár and Géza Újhelyi have already delved into this matter on a high level in Hungary. Their findings and suggestions are quite remarkable [1, 13, 14, 15].

The goal of this study series is to provide an ideal mathematical formula to calculate solar energy demand in the future in Hungary.

2. RENEWABLE ENERGY RESOURCES

Main renewable energy sources [2]:

- wind energy: onshore, offshore (shallow and deep water),
- solar energy: solar collector, solar cell, solar energy production,
- hydropower: hydropower, tidal power plants,
- geothermal, thermal water, rock heat,
- biomass.

The availability of renewable energy sources is uncertain and stochastic. They generally require reserve. This problem is particularly significant for solar and wind energy. The integration of solar and wind farms poses control problems. The efficiency and success of the control are greatly facilitated by the forecasting and mathematical-probability description of the daily course of solar energy yield in the case of solar power plants. In this study, the authors aimed at describing the stochastic modelling of solar energy yield and the formalization of probability theory as an objective. This method offers hope for the efficient deployment of solar power plants and the reduction of load planning risks. With the modelling, it is possible to simulate forecasts and load runs. The processed data derive from the meteorological station of Debrecen International Airport which were provided for research by the National Meteorological Service.

3. THE SITUATION AND POSSIBILITIES OF THE USE OF RENEWABLE ENERGY SOURCES IN HUNGARY

With respect to wind energy utilization, opportunities in Hungary are less favourable than in Great Britain or a country whose geographical location makes it easier to use wind energy. It is important to mention that more favourable conditions are provided to produce energy with solar panels in Hungary due to the economic factors and actual government subsidies [1].

With the expansion of Paks project, we expect an installed capacity of 2,400 MW, the availability of which is at least 8,000 hours/year. If we intend to replace the electricity to be produced by Paks II with renewable energy, for example using wind energy, installed wind power capacity of 19,000 MW would be necessary, which means that the availability of wind energy in Hungary is on average 1,000–1,200 hours/year. The area required for the installation of such a wind farm is approximately 3,000–4,000 km². Biomass-based replacement of Paks II would require the establishment of an energy plantation of 9,600 km². Another alternative: the total annual logging of Hungarian forest farms is ~ 9–10 million tons. That is all it would take for Paks II to replace wood-fired power plants [1, 11, 12].

The amount of electricity that can be obtained using solar panels is therefore extremely important and will be even more so in the future. During the analysis and measurements, we examined solar cells with a capacity of 275 Wp, which include both Asian and North American-made panels. The parameters of the selected modules are remarkably similar, their size is uniform, their efficiency is practically the same, this is illustrated in Fig. 1 [1].

The average annual electricity demand of an average Hungarian household is approximately 3,500 kWh. Figure 2 shows the amount of electricity that can be produced by a

![Fig. 1. Electric energy generation of one piece of solar panel per month [1]](image-url)
solar cell with a power of 275 Wp, an area of 1.63 m² and an efficiency of 16.8%, broken down by month [1].

During the winter months, generation by solar cells is extremely low. Overgeneration of the summer months, with careful sizing, compensates for this difference. Unfortunately, such long-term storage is currently unresolved [1].

The average daily electricity generation in winter months is 15% of the generation in the summer. The average annual electricity consumption can be achieved with 10 solar panels, each of 1.63 m², with a total of 16 m². In the absence of storability, cooperation with the electricity system is required and conventional, complementary capacities must be installed [1].

Let us talk about the storage options currently known and used. The minimum battery weight required to store the daily electricity needs of Hungary with state-of-the-art batteries is about 2 million tons. To put it into perspective, such a battery mass is or would be placed in about 4–5 million electric cars [1].

In general, we import large amounts of electricity because it is cheaper than producing it with domestic production. From an engineering point of view, however, the situation is much more complex. Over time, capacities will decrease, the only major power plant investment Paks II is predicted to be completed by 2030. Experts suggest the establishment of a solar panel capacity of 3,000–4,000 MW. It provides a good overview of the future development of power capacities [3].

The Hungarian power plant system relies primarily on fossil and nuclear energy. The existing energy generation system faces a major regulatory challenge regarding the integration of renewables, as their production is highly volatile. This large change in production yield over time is difficult to be tracked by the base power system, reducing the overall efficiency of the production system.

For all these reasons, it is crucial to provide a forecast of the current expected yields of renewable producers so that the energy regulatory system can handle it.

4. GENERAL OVERVIEW ABOUT THE MEASUREMENT

The measurements derive from Debrecen International Airport, and data were provided by the National Meteorological Centre (Fig. 3).

Depending on the geographical location, there is a different irradiation angle and daylight relationship, which affects the amount of radiation reaching the horizontal surface in different ways. Our measurement data and the conclusions drawn can be considered valid for the environment of the measurement point.

Since not only does the sun’s path affect the energy flow to the horizontal surface, but also meteorological disturbances such as semi-heat, we may face an area with the same sun’s path, distorting climate changes according to the climatic conditions can appear, which must be considered in calculations (Fig. 4). This provides the basis for the meteorological and climatic conditions of the region if we examine the possibilities of solar energy production (Fig. 5).

The monthly average of the radiant energy yield per horizontal surface at the measured point is summarized in the diagram below. The higher yields of the summer months are well-observed compared to the winter months.

Based on the visualization of the measurement results of 5 years, we conclude that it is not a stochastic process, but a near-cycle process.

The cyclicity seen above is partly explained by the cyclical alternation of the sun’s path and the angle of incidence, which was supplemented by the alternation of meteorological effects.

The figure below shows that the rate of radiation reaching the average daily horizontal surface is the lowest in the winter months and the highest in the summer months. This effect is reinforced by the fact that the average cloud cover in the winter months is significantly higher than in the summer, therefore the rate of direct radiation continues to decrease (Figs 6 and 7).
The degree of cloud cover was determined with meteorological data using the ‘n’ factor, the unit of which is octa, which shows how many eighths of the sky are covered by clouds.

From an energy point of view, the utilization of irradiation in clear or light, moderately cloudy periods is the most optimal. At our measurement point, 56% of the studied period was clear or slightly cloudy, during which time 77% of the total horizontal irradiation fell, which shows the energy relevance of the days with the above meteorological conditions.

In meteorological forecasts, the ‘n’ factor of cloud cover is given, which can be used to estimate the amount of actual radiation expected in each period, from which the amount of energy to be converted can be predicted.

Our goal is to find an estimating function that describes the irradiation model of clear and lightly cloudy, moderately cloudy days (0–6 octa) well.

5. PROBABILITY DENSITY FUNCTIONS AND WEIBULL DISTRIBUTION

To describe the functional relationship of stochastic data, it is worth using probability functions. In our research, we began to review the Weibull distribution. Since its special function forms include the distribution close to normal distribution, exponential distribution, the Rayleigh distribution, thus, in estimating the data, the applicability of the other distribution functions can be well examined by fitting the Weibull distribution function to the data set [4–7, 19].

In case of technical systems, it is extremely important to examine probability distributions, which several textbooks, articles, and PhD theses delve into. These days (2021) several mathematical methods are integrated into the engineering field including different soft computing methods and artificial intelligence tools. With these methods, it is possible to connect machines and equipment and extract and analyse data from...
the measured results in real time. These analyses and real-time data collections are an integral part of Industry 4.0 [4–7, 19].

Analysing and optimizing the entire energy mix with artificial intelligence [Fuzzy logic or Support Vector Machine (SVM) methods] is the result of possible results of long-term tests [4–7, 19].

The Weibull distribution has proven its applicability in several engineering fields. Its most important areas involve materials science and maintenance [15–17, 19].

In meteorology, it is also used to estimate the magnitude of potential wind energy in the study area [15–17, 19].

In the theory of probability calculation and in the field of statistics, the Weibull distribution is a continuous probability distribution. This distribution was named after Waloddi Weibull, who described it in detail in 1951. The distribution was discovered by Maurice Fréchet (1927) and was first used in 1933 to distribute granular particles (granules). This distribution was named after Waloddi Weibull, who described it in detail in 1951 [8–10, 19].

The probability density function of the Weibull ‘x’ probability variable can be described as follows [8–10, 19]:

where \( k > 0 \) is the shape parameter and \( \lambda > 0 \) is the scale parameter. The cumulative distribution function of the complement is the extended exponential function. The Weibull distribution is related to several other probability distributions, especially the exponential distribution \((k = 1)\) and the Rayleigh distribution \((k = 2)\). It can be considered as an interpolation of the Weibull distribution between the latter two.
In describing the time frequency of mechanical maintenance, the application of the Weibull distribution is as follows. If \( x \) is the value that indicates the time to failure, then the Weibull distribution indicates the failure frequency proportional to time. The interpretation of the shape parameter \( k \) is as follows [8–10]:

- \( k < 1 \) means that the failure rate decreases over time. This occurs when the initial failure is significant and therefore the failure decreases over time because potentially defective components have already been out of the system.
- In case of \( k = 1 \), the failure rate is constant over time. This means that errors are caused by random external events.

\[ \text{Fig. 7. Rate - Cloud n factor} \]

| Cloud n-factor | 1 | 2 | 3 | 4 | 5 |
|----------------|---|---|---|---|---|
| Time rate      | 0.18 | 0.18 | 0.20 | 0.13 | 0.31 |
| Energy rate    | 0.28 | 0.25 | 0.24 | 0.12 | 0.12 |

\[ \text{Fig. 8. a, Weibull distribution curve with different function images [18, 19]; b, Non-cumulative and cumulative curve form at time} \]
The irradiance values of the selected days were examined with the Weibull distribution described in the previous section. The time distribution Kwh/m² of solar cells is estimated with the Weibull density function. The figures were created with the so-called ‘k’ iterative approximation using an Excel Solver program.

Function generation: the radiant energy yield per hour on a horizontal surface for a given day. The mean value of the radiation yield was taken as the expected value of the Weibull distribution function.

The average radiant energy of the expected value was used, and the lambda factor influencing the distribution was defined as a function of the k factor and the expected value. Figure 8 illustrates different function images.

\[ E(X) = \lambda \Gamma \left( 1 + \frac{1}{k} \right) \]  

(1) shows the mean of a Weibull probability variable. Thus, the expected value depends on the lambda and k-factor values. The expected value was determined as the daily average radiant energy yield and thus the parameter to be determined is only the k-factor. The k-factor was approximated by the iterative method with Solver extension in such a way that our fitted daily curve must have an area under the same curve, therefore its expected value must be the same for the above reasons.

### Table 1. Data with respect to 7 June 2018

| kWh/m² | %    | Weibull | Weibull Power |
|--------|------|---------|---------------|
| 0      | 0    | 0       | 0             |
| 0      | 0    | 0       | 0             |
| 0      | 0    | 0       | 0             |
| 0      | 0    | 0       | 0             |
| 0.009167 | 0.001594 | 0.000321095 | 0.001846294 |
| 0.086389 | 0.015019 | 0.007604432 | 0.043725487 |
| 0.168889 | 0.029362 | 0.019559777 | 0.112467566 |
| 0.34    | 0.05911 | 0.052176293 | 0.30003686 |
| 0.434722 | 0.075578 | 0.073359841 | 0.42181086 |
| 0.64    | 0.111267 | 0.12401355 | 0.71308254 |
| 0.679167 | 0.118076 | 0.134149848 | 0.77136125 |
| 0.3175  | 0.055199 | 0.047425303 | 0.272695491 |
| 0.265   | 0.064071 | 0.036831776 | 0.211782711 |
| 0.553056 | 0.096151 | 0.10195191 | 0.586223483 |
| 0.764167 | 0.132954 | 0.156397956 | 0.899288248 |
| 0.691389 | 0.120201 | 0.137331106 | 0.789653859 |
| 0.356667 | 0.062008 | 0.05577028 | 0.320679109 |
| 0.210556 | 0.036606 | 0.026672014 | 0.15336408 |
| 0.195833 | 0.034046 | 0.024081855 | 0.138506893 |
| 0.036944 | 0.006423 | 0.002294646 | 0.01319317 |
| 0.0025  | 0.000435 | 5.13464E-05 | 0.00029614 |
| 0      | 0    | 0       | 0             |
| 0      | 0    | 0       | 0             |
| 0      | 0    | 0       | 0             |
| 0.007194 | 1    | 0.999999752 | 5.749998572 |

### 6. MEASUREMENT RESULTS AND CONCLUSIONS

The study was compiled with daily irradiation data. This is due to the fact that, regarding the energy control system, it is the instantaneous production yield what is necessary for the correct operation of the control system and while proper estimations are available for seasonal monthly irradiation data, an appropriate descriptive methodology has not been developed for the daily trace that considers meteorological characteristics such as cloud cover.

We received irradiation data for 5 years from the National Meteorological Service, on a daily basis. For an arbitrarily selected year, we were given an hourly breakdown of the days with monthly maximum and minimum irradiance values for any year. These datasets were represented by 24 functions. We observed that three curves characterize the annual irradiations as a function of cloud cover, therefore 3 summer days were selected for the study from the available database:

- 7 June 2018
- 3 July 2018
- 4 August 2018

An important consideration in selecting the days was to examine the summer months and to try to select nearly identical days. As a result, three summer months and their first days were selected. It is assumed that due to the examined curves and functions, the estimation method can be extended to other days of the year.

With respect to the science of materials, the shape parameter ‘k’ is known as the Weibull modulus. The density function of the Weibull distribution varies drastically depending on the ‘k’ value [8–10].

- In the range of 0 < k < 1, the density function moves toward ∞ if x approaches zero.
- In case of k = 1, the density function moves toward 1/λ when x approaches zero.
- In case of k > 1, the density function approaches zero, if x approaches zero, increases monotonically to the maximum, and then begins to decrease. It is crucial to note that the density function has a negative slope at x = 0 if 0 < k < 1; monotonically positive slope at x = 0 if 1 < k < 2 and flat at x = 0 if k > 2.
- In case of k = 2, the density has a monotonically positive slope at x = 0.

If ‘k’ goes to infinity, the Weibull distribution converges to the Dirac delta distribution with mean x = λ [8–10].
Under these boundary conditions, $k$-factor is iterated. The advantage of this method is that it uses the currently measured and recorded data (generally measured data), i.e., the old data sets can be used, the application of the method is extremely cost-effective and requires low computation.

This method can approximate different function images. Like the 8-month example, where it can also track rhapsodic function images from afternoon lamb clouds. In general, the approximation is not perfect for days with high scattered radiation, a good example is the period with a continuous cloud base, but since the total amount of energy is the least for such days and periods, therefore, the error of the process from an energy point of view is small even in the case of a large percentage deviation.

Thus, the function images for the selected three days are as follows (Tables 1–6):

7 June 2018

The value shown in blue in the figure is the kWh/m² characteristics of the solar cell, for which the Weibull function is plotted in yellow. It can be seen in Fig. 9 that a 'plunge' occurred in the middle of the day, presumably due to clouds.

7. SUMMARY

Renewable energy sources are essential today. This is backed by a number of studies and the strive of different countries

Table 2. Excel Solver variables

| mean          | 0.302734 kWh/m² |
|---------------|-----------------|
| $k$ factor    | 2.410887        |
| Expected value| 0.302734        |
| lambda        | 2.6             |
| $r^2$         | 0.984586        |

Table 3. Data with respect to 3 July 2018

| kWh/m² | % | Weibull srf | Weibull power |
|--------|---|-------------|---------------|
| 0      | 0 | 0           | 0             |
| 0      | 0 | 0           | 0             |
| 0      | 0 | 0           | 0             |
| 0      | 0 | 0           | 0             |
| 0      | 0 | 0           | 0             |
| 0.003611 | 0.000427 | 0.002826 | 0.001405232 |
| 0.064444 | 0.007623 | 0.088144 | 0.043834591 |
| 0.201944 | 0.023887 | 0.343495 | 0.17082153  |
| 0.371944 | 0.043995 | 0.704691 | 0.3504574   |
| 0.545   | 0.064465 | 1.091233 | 0.542674614 |
| 0.700556 | 0.082865 | 1.437241 | 0.714745762 |
| 0.835278 | 0.098801 | 1.725716 | 0.858205285 |
| 0.914444 | 0.108165 | 1.877332 | 0.938577481 |
| 0.958611 | 0.113389 | 1.974333 | 0.981843584 |
| 0.94    | 0.111888 | 1.937969 | 0.96375612  |
| 0.846111 | 0.100082 | 1.748226 | 0.869399713 |
| 0.735833 | 0.087038 | 1.514124 | 0.752979966 |
| 0.600556 | 0.070137 | 1.215714 | 0.60457464  |
| 0.436667 | 0.051651 | 0.848307 | 0.421866278 |
| 0.207222 | 0.024511 | 0.354163 | 0.176126403 |
| 0.081111 | 0.009594 | 0.115977 | 0.0576568   |
| 0.010833 | 0.001281 | 0.010489 | 0.005216437 |
| 8.454167 | 1       | 16.99998 | 8.454157372 |

Table 4. Excel Solver variables

| Mean          | 0.497304 kWh/m² |
|---------------|-----------------|
| $k$ factor    | 2.193886        |
| Expected value| 0.497304        |
| lambda        | 0.304603        |
| $r^2$         | 0.99911         |

Table 5. Data with respect to 4 August 2018

| kWh/m² | % | Weibull srf | Weibull power |
|--------|---|-------------|---------------|
| 0      | 0 | 0           | 0             |
| 0      | 0 | 0           | 0             |
| 0      | 0 | 0           | 0             |
| 0      | 0 | 0           | 0             |
| 0      | 0 | 0           | 0             |
| 0.020833 | 0.00286 | 0.014112 | 0.006425    |
| 0.124722 | 0.017122 | 0.163054 | 0.074232    |
| 0.286667 | 0.039355 | 0.505403 | 0.23009     |
| 0.454444 | 0.062388 | 0.933516 | 0.424993    |
| 0.605556 | 0.083133 | 1.349031 | 0.614161    |
| 0.728333 | 0.099989 | 1.689883 | 0.769337    |
| 0.826667 | 0.113488 | 1.956123 | 0.890545    |
| 0.932778 | 0.128056 | 2.229425 | 1.014969    |
| 0.85    | 0.121496 | 2.108592 | 0.959958    |
| 0.856389 | 0.117569 | 2.034427 | 0.926194    |
| 0.521389 | 0.071578 | 1.115726 | 0.507946    |
| 0.404722 | 0.055562 | 0.801499 | 0.364891    |
| 0.377778 | 0.051863 | 0.731514 | 0.330329    |
| 0.199722 | 0.027419 | 0.309764 | 0.141023    |
| 0.058333 | 0.008008 | 0.05771  | 0.026273    |
| 0.000833 | 0.000114 | 0.00173  | 7.86E-05    |
for the future. New energy sources also play a prominent role in Hungary. As it turned out in the introduction, Hungary has several opportunities and limitations regarding different energies; for example, no major installations will be carried out in the future regarding wind farms, while the construction of solar systems is becoming increasingly popular. These solar systems are not only used in public or industrial environments but are also becoming more popular among the Hungarian population due to increasingly favourable prices and various state subsidies, which were created precisely to make renewable energy sources as popular and widespread as possible.

To write the article and carry out the research, data measured at the meteorological station of Debrecen International Airport were used, which were provided by the National Meteorological Service. After arranging the data into a single table, power functions were determined, and the irradiation values of the selected days were examined with the Weibull distribution. The time distribution of solar cells Kwh/m² was obtained with the Weibull density function, which are included in the tables described in this article. The figures were calculated with the so-called ‘k’ iterative approximation using an Excel Solver program. During the analysis of 3 summer days, it can be noticed that the Weibull density function gives an extremely good approximation. 3 days studied represented near-ideal conditions during the study. For further analysis, more data are needed that can be examined.

In this article, a stochastic description of the solar energy trace has been implemented, with the aim of making a preliminary estimate and developing a new device. This tool is of paramount importance in engineering, as the stochastic behaviour and stochastic nature that appears to be implemented in technical systems also appears in the engineering sciences. The probability description is the only tool to study the prediction and course of solar energy that can be extended with Weibull distribution. Based on previous studies, Weibull distribution may be more successful than other normal distributions.

As a further research task, it would be expedient to process data from several years, with the help of which an even more accurate forecast may be reached. From a technical point of view, the calculation requirement of the Weibull distribution is not high, therefore it can be excellently implemented in a real-time data processing unit, which can further strengthen the practical application of the

| Table 6. Excel Solver variables |
|--------------------------------|
| Mean                         | 0.45526 |
| k factor                     | 2.368087 |
| Predicted value              | 0.45526 |
| lambda                       | 0.295121 |
| r²                           | 0.993042 |

Fig. 9. Solar energy curve – 7 June 2018; a-non-cumulative curve; b-cumulative curve

Fig. 10. Daily change – 3 July 2018
method. In the next development, the Weibull distribution and the function images obtained above will be applied to study the flexibility of power plants.

REFERENCES

[1] G. László, H. Rita, and K. Zoltán, “Fenntartható energia, mellékelészés nélkül, Az európai Energia unió terv,” Fizikai szemle, vol. 69, nos 7–8, pp. 237–42, 2019, 6p. Available: http://real-j.mtak.hu/12758/37/FizSzem-2019_07-08.pdf#page=23 (Downloaded: 2021.03.16. 13:27).

[2] J. C. David, Mackay: Fenntartható energia mellékeszés nélkül. Budapest: Vertis Zrt. és Typotex Kiadó, 2011, ISBN 978-963-279-575-1.

[3] L. Sándor, “Gondolatok a 2033-as erőművi kapacitásokról,” Available: https://magyarenergetika.hu/wp-content/uploads/2019/07/Lang-ME_kefe.pdf, (Downloaded: 2021.03.16. 14:26).

[4] I. Csáky and F. Kalmár, “Effects of solar radiation asymmetry on buildings’ cooling energy needs,” J. Buildin Phys., 2015, First Published August 11 https://doi.org/10.1177/1744259115597444.

[5] M. József, S. Robert, “Artificial intelligence applied for technical status diagnostics of the batteries of automated guided vehicles,” in 2019 International Conference on Military Technologies (ICMT), V. Krivánek (szerk.) Brno, Csehország: IEEE-INST ELECTRICAL ELECTRONICS ENGINEERS INC, 2019, pp. 1–8, 8p.

[6] P. László, Rendszerek és folyamatok modellezése. Debrecen: Campus Kiadó, 2008.

[7] M. József and K. Ferenc, “Investigation of thermal comfort responses with fuzzy logic,” Energies, vol. 12, no. 9, pp. 1–13, 2019, Paper no. 1792, 13 p.

[8] J. Obádovics, Gyula: Valószínűségszámítás és matematikai statisztika. Scolar Kft., 2009, ISBN 978-963-244-0675.

[9] F. Sándor, Valószínűségszámítás és matematikai statisztika 2. Available: https://www.uni-miskolc.hu/~matfs/MF_02.pdf (Downloaded: 2021.03.16. 15:03).

[10] Pannon Egyetem: 4. fejezet – Megbízhatóság, elérhetőség és biztonság, Available: http://moodle.autolab.uni-pannon.hu/Mecha_tananyag/biztonsagkritikus_rendszer/2019/04.pdf (Downloaded: 2021.03.16. 15:42).

[11] J. Tóth, F. Kalmár, G. Husi, A. Csai, I. Budai, I. Bartha, B. Kulcsár, I. Kocsis, and Z. Tiba, Napelem, a jövő energiaforrása. Debrecen: CEZE Kft., 2009, p. 172, ISBN 9789638861481.

[12] I. Bartha, J. Tóth, and G. Husi, “Effect of a four-way rotating device to the electric energy production with solar cell,” in Proceedings of the Annual Session of Scientific Papers: IMT Oradea - 2011: Extended Abstracts, vol. 31, T. Vesseleyi, C. Bungau, F. Sandu Blaga, M. Teodor, A. Rus, and R. Catalin Tarca, Eds., Oradea: Editura Universitatii din Oradea, 2011, ISBN 9786061005086.

[13] I. Gács, “és társai: az új magyar energiastratégia,” in Új Széchenyi Terv, Budapest, 2010, (made on behalf of the Government of Hungary).

[14] K. Gerse, Regionális piacon, importforrások, kapacitásigény. Magyar Energetika XXIII/6. Gerse K.: Mekkorak a valakió villa-mosenergia-árákat? Ki fizeti őket? Kiadatal kérnát, 2016, pp. 2–18.

[15] L. Molnár, “A klímaváltozás elleni politikák és következményeik az energetikára,” Energiaigazdálkodás, 59/1–2, p. 3, 2018, Molnár L.: Villamosenergia-ellátásbiztonság az EU-ban a 2020-as években. Energiaigazdálkodás 59/3–4 (2018) 62.

[16] I. Csáky, Épületek nyári hőtermelésének energetikai vizsgálata, PhD Dissertation, University of Debrecen, 2015, Available: https://doktori.hu/index.php?menuid=192&lang=HU&vid=14864 (Downloaded: 2021.05.16. 12:26).

[17] “Industry today: do solar panels work on commercial buildings?” Available: https://industrytoday.com/do-solar-panels-work-on-commercial-buildings/, (Downloaded: 2021.09.13., 08:59).

[18] A. Anand, R. Aggarwal, and O. Singh. “Using Weibull distribution for modeling bimodal diffusion curves: a naive framework to study product life cycle,” Int. J. Innovation Technol. Manag., vol. 16, no. 7, 2019, Paper no. 1950050, 17p, https://doi.org/10.1142/S021987701950050.

[19] Wikipedia: Weibull distribution, Available: https://en.wikipedia.org/wiki/Weibull_distribution, (Downloaded: 2021. 10. 11).