A ROBUST ARTIFICIAL NOISE AIDED TRANSMIT DESIGN FOR MISO SECRECY

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ABSTRACT

This paper considers an artificial noise (AN) aided secrecy rate maximization (SRM) problem for a multi-input single-output (MISO) channel overheard by multiple single-antenna eavesdroppers. We assume that the transmitter has perfect knowledge about the channel to the desired user but imperfect knowledge about the channels to the eavesdroppers. Therefore, the resultant SRM problem is formulated in the way that we maximize the worst-case secrecy rate by jointly designing the signal covariance $W$ and the AN covariance $\Sigma$. However, such a worst-case SRM problem turns out to be hard to optimize, since it is nonconvex in $W$ and $\Sigma$ jointly. Moreover, it falls into the class of semi-infinite optimization problems. Through a careful reformulation, we show that the worst-case SRM problem can be handled by performing a one-dimensional line search in which a sequence of semidefinite programs (SDPs) are involved. Moreover, we also show that the optimal $W$ admits a rank-one structure, implying that transmit beamforming is secrecy rate optimal under the considered scenario. Simulation results are provided to demonstrate the robustness and effectiveness of the proposed design compared to a non-robust AN design.

Index Terms— secrecy capacity, convex optimization, semidefinite program (SDP), artificial noise.

1. INTRODUCTION

In the last decade, multi-antenna techniques have been extensively investigated from the perspective of providing high throughput. Recently, there has been much interest in using multiple antennas to achieve secure communication, which is known as physical-layer secrecy. In a traditional single-input single-output scenario, the idea of physical-layer secrecy is to add some structured redundancy in the transmitted signal such that the desired user can correctly decode the confidential information, but for the eavesdropper he/she cannot retrieve anything from the observation [1]. To make physical-layer secrecy viable, a prerequisite is that the desired user’s channel has to be better than the eavesdropper’s. However, this may not be satisfied if the transmitter has only a single antenna; e.g., when the eavesdropper is closer to the transmitter with lower reception noise power than the desired user. To alleviate the dependence of the channels, recent studies are mainly focused on employing multiple antennas to transmit, since multiple transmit antennas provide additional spatial degree of freedom to degrade the reception of the eavesdropper. A possible way to do this is transmit beamforming, which premultipies the signal by a weight vector such that the power radiation is concentrated over the direction of the desired user. In addition to concentrating the signal power, a more active way is to use part of the power to artificially generate some noise to interfere the eavesdropper. This artificial noise (AN) approach was first proposed in [2], and has been shown to be effective in improving secrecy rates [2–7].

Current studies on AN aided transmit design are mainly based on a premise that AN lies in the orthogonal complement subspace of the desired user’s channel in an isotropic fashion, e.g., [2–6]. This isotropic AN design has an advantage that no eavesdropper’s channel state information (CSI) is needed at the transmitter, thereby making it very suitable for the passive eavesdropper scenario. On the other hand, as demonstrated in [4, 8], when the eavesdropper’s CSI is perfectly known at the transmitter, e.g., when the eavesdropper is also a user of the system, we can block the eavesdropper more effectively by aligning AN to the eavesdropper’s direction through judicious optimization, rather than keeping the AN isotropically. However this perfect CSI assumption may be too stringent. Thus, in this paper we consider a scenario where the transmitter has only imperfect CSIs of the eavesdroppers, and we attempt to maximize the worst-case secrecy rate by jointly optimizing the signal and the AN covariances. Through a careful reformulation, we show that the AN aided worst-case secrecy rate maximization (SRM) problem can be handled by performing a one-dimensional line search in which a sequence of semidefinite programs (SDPs) are involved. Moreover, we prove that the optimal transmit covariance for the signal part admits a rank-one structure, and thus, transmit beamforming is an optimal transmit strategy for the scenario considered.

There are some related works worth mentioning. In [4, 8], AN aided transmit beamforming designs are considered from a QoS perspective. Specifically, they focus on the signal-to-interference-and-noise-ratio (SINR) at each receiver (including the eavesdroppers), instead of the secrecy rate considered here. Moreover, this work considers imperfect CSI in a worst-case sense. In [7], the authors consider the AN design for secrecy rate maximization under the stochastic CSI uncertainty model, as opposed to the worst-case deterministic model considered here.

Notation: $A^H$, $\text{Tr}(A)$ and rank($A$) represent Hermitian transpose, trace and rank of a matrix $A$; $I$ is an identity matrix of appropriate size; $A \succeq 0$ ($A > 0$) means $A$ is Hermitian positive semidefinite (definite) matrix; $\mathbb{H}^N$ denotes the set of $N$-by-$N$ Hermitian matrices; $\mathbf{x} \sim \mathcal{CN}(\mathbf{\mu}, \Sigma)$ means that $\mathbf{x}$ is a random vector following a complex circular Gaussian distribution with mean $\mathbf{\mu}$ and covariance $\Sigma$; $E\{\cdot\}$ is the expectation operator.

2. SYSTEM MODEL AND PROBLEM STATEMENT

We consider a wireless network that consists of a single transmitter with $N_t$ antenna elements, and $K+1$ single antenna receivers. Among these $K+1$ receivers, only one receiver is legitimate while
where $s(t)$ is the confidential information intended for Bob and we define its covariance as $\mathbf{W} = \mathbb{E}\{s(t)s^H(t)\}$; $\mathbf{z}(t)$ is the noise vector artificially created by Alice to interfere Bob. In this work, we assume that $\mathbf{z}(t)$ and $s(t)$ are independent, and $\mathbf{z}(t) \sim \mathcal{CN}(0, \mathbf{Z})$. Generally speaking, we wish to design $\mathbf{W}$ and $\mathbf{\Sigma}$ so that a good information secrecy can be achieved.

To describe the imperfect CSI model for Eves, we assume that Alice knows only channel estimates of $\mathbf{g}_k$, i.e.,

$$\mathbf{g}_k = \bar{\mathbf{g}}_k + \Delta \mathbf{g}_k, \quad k = 1, \ldots, K,$$

where $\bar{\mathbf{g}}_k$ is the channel estimate at Alice; $\Delta \mathbf{g}_k$ represents the channel uncertainty. These uncertainties are assumed to be deterministic unknowns with bounds on their magnitudes:

$$\|\Delta \mathbf{g}_k\|_2 \leq \epsilon_k, \quad k = 1, \ldots, K$$

for some $\epsilon_1, \ldots, \epsilon_K > 0$.

Under the above described uncertainty model, the proposed robust SRM formulation is given by [9]

$$R'(P) = \max_{\mathbf{W}, \mathbf{\Sigma}} \left\{ \min_{k=1,\ldots,K} f_k(\mathbf{W}, \mathbf{\Sigma}) \right\},$$

subject to

$$\text{Tr} (\mathbf{W} + \mathbf{\Sigma}) \leq P, \quad \mathbf{W} \succeq 0, \quad \mathbf{\Sigma} \succeq 0,$$

where

$$f_k(\mathbf{W}, \mathbf{\Sigma}) = \log \left( 1 + \frac{\mathbf{h}_k^H \mathbf{W} \mathbf{h}_k}{1 + \mathbf{h}_k^H \mathbf{\Sigma} \mathbf{h}_k} \right) - \max_{\mathbf{g}_k \in B_k} \log \left( 1 + \frac{\mathbf{g}_k^H \mathbf{W} \mathbf{g}_k}{1 + \mathbf{g}_k^H \mathbf{\Sigma} \mathbf{g}_k} \right),$$

$$B_k = \{ \mathbf{g}_k \in \mathbb{C}^{N_t} \mid \|\mathbf{g}_k - \bar{\mathbf{g}}_k\|_2 \leq \epsilon_k \}.$$
Now consider (8c). It can be interpreted as the following implication:
\[
\Delta g_k^H \Delta g_k - \xi^2 \leq 0 \Rightarrow \Delta g_k^H M \Delta g_k + 2\mathrm{Re}\{\Delta g_k^H M \Delta g_k\} + \Delta g_k^H M g_k - (\beta - 1)\xi \leq 0, \quad \forall k
\]
where \( M = Z - (\beta - 1)Q \). By Lemma 1, we can rewrite the above implication equivalently as the following matrix inequality:
\[
T_k(Z; Q, \beta, \mu_k, \xi) = \begin{bmatrix}
\mu_k I - M & -M g_k \\
-\xi^2 M & -\xi^2 \mu_k - \xi^2 H M g_k + (\beta - 1)\xi
\end{bmatrix} \succeq 0,
\]
with \( \mu_k \geq 0 \). Substituting (10) into (8), we have
\[
\max_{\bar{Z},\bar{Q},\beta,\mu_k,\xi} \phi(\beta) = \begin{cases}
\xi + h^H(\bar{Z} + \bar{Q})h & \text{s.t. } 1 \leq \beta \leq 1 + P||h||^2, \\
\xi + h^H(\bar{Z} + \bar{Q})h & \text{s.t. } 1 \leq \beta \leq 1 + P||h||^2,
\end{cases}
\]
where
\[
\phi(\beta) = \max_{\bar{Z} \succeq 0, \bar{Q} \succeq 0, \xi \geq 0, \mu_k \geq 0} \phi(\beta) = \begin{cases}
\phi(\beta) & \text{s.t. } \xi + h^H(\bar{Z} + \bar{Q})h = 1, \\
\phi(\beta) & \text{s.t. } \xi + h^H(\bar{Z} + \bar{Q})h = 1.
\end{cases}
\]

The proof of Theorem 1 is given in the Appendix. A key ingredient of proving Theorem 1 is to consider a secrecy rate related power minimization problem and investigate its Karush-Kuhn-Tucker (KKT) conditions. Also note that the proof is constructive and hence the rank-one \( W^* \) can be found in practice. Theorem 1 provides a useful physical-layer design guideline that transmit beamforming is an optimal transmit strategy for the artificial noise aided secure transmission under the scenario considered.

4. SIMULATION RESULTS AND CONCLUSIONS

We provide two simulation examples to test the performance of the robust AN design proposed in Section 3 and compare it with an equal-power-splitting isotropic AN design [6], which splits half of the power to transmit the confidential information over the direction of \( h \), while the remaining half, as artificial noise, is spread isotropically in the orthogonal complement subspace of \( h \). In the following simulations, we denote the normalized channel uncertainty \( \alpha_k = \epsilon_k/\sqrt{\mathbb{E}[||g_k||^2]} \). We set \( \alpha_1 = \ldots, \alpha_K = \alpha \), i.e., the same uncertainty level for all Eve’s channel links. The elements of \( h \) and \( g_k \) are i.i.d. complex Gaussian distributed with mean 0 and variance 1. All results were averaged over 1000 independent channel realizations. Fig. 1 evaluates the relationship between the worst-case secrecy rate (i.e., the objective value in (3)) and the transmit power level for different number of Eves. It can be seen from the figure that the proposed robust AN design outperforms the non-robust AN design over the whole power range tested. In particular, for \( P = 20\mathrm{dB}, K = 3 \) and \( \alpha_1 = 0.1 \), the worst-case secrecy rate gap between these two designs is about 1.5 bps/Hz. Fig. 2 shows the impact of channel uncertainty on the worst-case secrecy rate for different number of Eves. We see in Fig. 2 that the proposed robust design achieves a higher worst-case secrecy rate than the non-robust design over the whole uncertainty region tested.

This paper has proposed a joint optimization approach to the AN aided covariances design for the robust secrecy rate maximization problem. We have shown that the corresponding robust optimization problem can be handled by performing a one-dimensional line search, in which a sequence of SDPs are involved. The present work considers a worst-case achievable secrecy rate problem under deterministic channel uncertainties. As a future work, it would be interesting to study how this work may be extended to deal with stochastic channel uncertainties, e.g., through an outage-based formulation.

5. APPENDIX

Proof of Theorem 1: The proof consists of two steps: First, we consider a secrecy rate related power minimization (PM) problem and show that the optimal solution of the PM problem is also optimal to our main problem (3); second, we show that the optimal \( W \) of the PM problem has to be of rank one, and thus establish the existence of a rank-one optimal \( W \) for Problem (3).

Step 1: Consider the following power minimization problem:
\[
\min_{W, \Sigma} \mathrm{Tr}(W + \Sigma) \tag{14a}
\]
\[
s.t. \sum_{k=1}^{K} \min_{f_k} f_k(W, \Sigma) \geq R^*, \quad W \succeq 0, \quad \Sigma \succeq 0 \tag{14b}
\]
where \( f_k(W, \Sigma) \) is denoted in (3); \( R^* \) is the optimal value of Problem (3). Here Problem (14) aims to minimize the total transmit power given a minimum secrecy rate specification \( R^* \). Let \( (W, \Sigma) \) be optimal solutions of Problems (3) and (14), respectively. Apparently, \( (W, \Sigma) \) is feasible to (14). Thus, we have that
\[
\mathrm{Tr}(W + \Sigma) \leq \mathrm{Tr}(W + \Sigma) \leq P, \tag{15}
\]
which further implies that $(\hat{W}, \hat{\Sigma})$ is feasible to Problem (3), i.e.,
\[
\min_{k=1,\ldots,K} f_k(\hat{W}, \hat{\Sigma}) \leq R^*.
\] (16)

On the other hand, as an optimal solution of (14), $(\hat{W}, \hat{\Sigma})$ must satisfy (14b). Therefore, combining (14b) and (16), we get
\[
\min_{k=1,\ldots,K} f_k(\hat{W}, \hat{\Sigma}) = R^*
\]
i.e., $(\hat{W}, \hat{\Sigma})$ is also optimal to Problem (3).

**Step 2:** To prove that the optimal $W$ of (14) has to be of rank one, we first re-express (14) as the following problem by using a similar approach presented in Section 3:
\[
\min_{W, \Sigma, \alpha, \lambda} \text{Tr}(W + \Sigma)
\]
\[
s.t. \ h^H(W + (1 - \alpha)\Sigma)h + 1 - \alpha \geq 0 \quad (17b)
\]
\[
A_k(W, \Sigma, \alpha, \lambda_k) \geq 0, \ k = 1, \ldots, K \quad (17c)
\]
\[
W \succeq 0, \ \Sigma \succeq 0, \ \lambda_k \geq 0, \ k = 1, \ldots, K \quad (17d)
\]

where
\[
A_k(W, \Sigma, \alpha, \lambda_k) = A_{k,1}(\lambda_k, \alpha) - G_k^H(W + (1 - 2^{-R^*} \alpha)\Sigma)G_k,
\]
\[
A_{k,1}(\lambda_k, \alpha) = \begin{bmatrix} \lambda_k I & 0 \\ 0 & -\lambda_k e_k^T + 2^{-R^*} \alpha - 1 \end{bmatrix}, \ G_k \equiv [I \ g_k].
\]

We list part of the KKT conditions of (17) below
\[
I - \eta hh^H + \sum_{k=1}^K G_kB_kG_k^H - Y = 0 \quad (18a)
\]
\[
WY = 0 \quad (18b)
\]
\[
W \succeq 0, \ Y \succeq 0, \ \eta \geq 0, \ B_k \succeq 0, \ \forall k \quad (18c)
\]

where $Y, B_k$ and $\eta$ are dual variables associated with $W, A_k$ and (17b), respectively.

Premultiplying (18a) by $W$ and making use of (18b)-(18c), we have
\[
W^H(I + \sum_{k=1}^K G_kB_kG_k^H) = \eta Whh^H.
\] (19)

Therefore, the following relation holds
\[
\text{rank}(W) = \text{rank}(W^H(I + \sum_{k=1}^K G_kB_kG_k^H)) \leq 1 \quad (20a)
\]
\[
= \text{rank}(\eta Whh^H) \leq 1 \quad (20b)
\]

where (20a) follows from $I + \sum_{k=1}^K G_kB_kG_k^H > 0$; (20b) follows from (19), and the fact that $hh^H$ is a rank-one matrix. Since $R^* > 0$, $W = 0$ is infeasible to Problem (14). Thus, $\text{rank}(W) = 1$ must hold true, which completes the proof.

6. REFERENCES

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Fig. 1: Worst-case secrecy rate versus transmit power. $N_t = 4, P = 20$ dB.

Fig. 2: Worst-case secrecy rate versus channel uncertainty. $N_t = 4, P = 20$ dB.