Orbital angular momentum modes do not increase the channel capacity in communication links

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Abstract
The orbital momentum of optical or radio waves can be used as a degree of freedom to transmit information. However, mainly for technical reasons, this degree of freedom has not been widely used in communication channels. The question is if this degree of freedom opens up a new, hitherto unused ‘communication window’ supporting ‘an infinite number of channels in a given, fixed bandwidth’ in free space communication as has been claimed? We answer this question in the negative by showing that on the fundamental level, the mode density, and thus room for mode multiplexing, is the same for this degree of freedom as for sets of modes lacking angular momentum. In addition we show that modes with angular momentum are unsuitable for broadcasting applications due to excessive crosstalk or a poor signal-to-noise ratio.

1. Introduction
Recently, the interest in using the angular momentum degree of freedom in both classical and quantum optics has increased. The attraction for both communities is the possibility to utilize angular momentum as a new degree of freedom. For example, orbital angular momentum (OAM) modes have found use in the rotation of optically trapped particles [1], in the detection of rotations through the rotational Doppler shift [2, 3], for the generation of angular momentum high-NOON states [4], and for generating unusual reflection characteristics [5]. Precise measurements of the OAM of light has been done to the single photon level [6]. In communication, OAM has been used for high-speed optical communication in free space [7–9] and in the radio domain [10, 11].

As for communication, claims have been made that using the OAM degree of freedom will increase the communication capacity per unit frequency and unit volume in radio links [10]. This claim has generated considerable excitement in the physics, electromagnetics, and microwave engineering communities [12, 13] and therefore it merits a critical examination. In [9] it is, e.g., stated that ‘...the light beam can in principle support an infinite number of OAM states at the same time (subject to the limitation imposed by the signal-to-noise ratio (SNR)). Therefore, OAM has the potential to significantly improve the spectral efficiency or photon efficiency of free-space and fiber optical communications’. This statement is correct, but it risks being misinterpreted as if only OAM modes have this potential. We claim that any orthogonal mode set may reach the same spectral efficiency as soon as the number of modes involved is significantly larger than one, because then the density of states is the same for any complete set of orthogonal modes.

To prove this claim, in section 2 we briefly discuss Weyl’s law and its implications. Applied to the wave equation, the law states that in the limit of a large volume containing the electromagnetic radiation, the mode density is independent of the shape of the volume. Thus, in the large volume that ‘free space’ implies, all sets of eigenmodes to the wave equation have a mode density dependent only of the frequency and the volume, so the
claim of an communication capacity increase due to the use of OAM is unfounded. Moreover, in section 3 we analyze the signal intensity and in section 4 we look at the crosstalk in a broadcast scenario relying on OAM mode multiplexing, which is a special case of mode multiplexing. This situation, where one transmitter sends the same signal to multiple receivers is the case that Tamburini et al are discussing in [10]. Our analysis show that an on-axis receiver achieves good mode discrimination (small crosstalk) but will receive very little of the transmitted power (poor SNR) in the modes with angular momentum. Placing the receiver off-axis may increase the received power for modes carrying OAM (resulting in a higher SNR), but it will increase the crosstalk to unacceptable levels. Thus, for broadcasting, where the receiver is in the far field of a transmitted beam (that is, the transmitted power is relatively directional such as in a satellite TV transmitter) and each receiver only have access to a fraction of the transmitted beam, we conclude that OAM modes are unsuitable. A very different case is a communication link where one transmitter sends information to one receiver, and the receiver intercepts essentially the whole transmitted mode (e.g. in an optical fiber). In this case OAM modes performs nominally a pair with any other set of optical modes. No better but no worse.

2. Weyl’s law

In the early 20th century Weyl and others studied the eigenvalue spectrum of the Laplace–Beltrami operator under certain boundary conditions [14] and derived what is called Weyl’s law. (A rich historical exposé of Weyl’s work on this problem can be found in [15].) A special case covered by Weyl’s law is the mode density of the wave equation enclosed in a perfectly reflecting cavity. The well known result is that in the asymptotic limit, where \( \sqrt{k} \gg 1 \), where \( V \) is the cavity volume and \( k \) is the wavevector length, the mode density can be written

\[
\frac{dN}{d\omega} = \frac{8\pi V c^2}{\epsilon^3} \quad \text{or} \quad \frac{dN}{d\omega} = \frac{V \omega^2}{\pi^2 c^3},
\]

where \( \omega \) denotes frequency, \( \omega \) denotes angular frequency, and \( c \) is the speed of light in vacuum. Since the result is independent of the shape of the volume, and therefore is valid for any eigenmode set, both those carrying OAM and those void of it, the use of OAM will not increase the information capacity in free space (nor in waveguides).

To exemplify this, we can look at the intensity-normalized Laguerre–Gauss (LG) mode amplitude function \( \Psi_{lm}(\phi, \rho) \) at a coordinate \( z \) and as a function of the distance \( \rho \) from the momentum (and symmetry) axis in the \( xy \) plane, where \( \phi \) is the angular coordinate. The amplitude is then given by [16]:

\[
\Psi_{lm}(\phi, \rho) = \frac{2m!}{\sqrt{\pi (|l| + m)!}} \frac{e^{-i\phi}}{w} \left( \frac{\rho \sqrt{2}}{w} \right)^{|l|} L_m^{|l|} \left( 2\rho^2/w^2 \right) e^{-\rho^2/2w^2 + i\mu},
\]

where \( w \) can be taken as a measure of the mode radius, \( l = 0, \pm 1, \pm 2, \ldots \) is the angular momentum quantal index, and \( m = 0, 1, 2, \ldots \) is the radial mode index. Here, for simplicity, we have neglected the phase-front curvature as the mode propagates (often expressed as a mode-index dependent Gouy phase) but approximated the phase-front to have a screw but no curvature as a function of \( \rho \).

If one pair-wise superimposes such modes with equal amplitude and radial mode index but opposite chirality (\( l \) and \( -l \)), then these new modes carry no OAM. The in-phase superposition will furthermore be orthogonal to the out-of-phase superposition so we will get an OAM-free, complete set of modes with pairwise correspondence to the OAM carrying LG-modes (except for the \( l = 0 \) mode that has no OAM anyhow). The mode densities of the two sets are of course identical. A more explicit derivation of the mode density for OAM modes can be found in [17].

The fact the use of OAM does not increase the channel capacity is also supported by Kish and Nevels [18] who used thermodynamic arguments to arrive at this conclusion. Should the mode density of OAM modes differ from any other set of modes, then thermodynamical results derived from mode density arguments, such as the black body radiation law (also studied by Weyl), would not be universal.

The conclusion is that the use of modes with angular momentum will not ‘open’ any previously unexploited degree of freedom. LG modes or other modes with angular momentum are simply orthonormal mode sets among others. The use of modes carrying OAM will not increase the channel capacity, neither in waveguided nor in free-space communication links. The OAM modes performs no worse, but also no better than any other set of orthogonal modes. In certain applications OAM mode multiplexing is already starting to be exploited [7–9], although broadcasting is neither a typical nor a suitable one for reasons that will become evident below.

3. On-axis, relative SNR in angular momentum mode broadcasting

Modes with OAM all have an axis defined by the field’s phase singularity around which the Poynting vector spirals. Suppose one uses several such modes in a beam in an attempt to increase the communication capacity of
the beam. Suppose further that one is considering free-space broadcasting, naturally defined as the case where no receiving antenna can intercept but a small fraction of the emitted beam (or lobe). That is, we consider a one-way, one-to-many communication link such as satellite TV broadcasting or a terrestrial microwave link. In figure 1 such a link is depicted, where for simplicity we have only drawn one of the receiving antennas.

Free-space modes are typically, effectively orthogonal only if the receiver antenna detects the vast majority of the emitted power. However, modes with angular momentum have the particularity that different modes are effectively orthogonal even if only a small portion of the mode is intercepted, provided that the receiver antenna is placed symmetrically around the mode central axis. However, on this axis the field of the modes carrying OAM essentially vanishes. Therefore, there is a significant power penalty for using modes with OAM if one wants to discriminate between them. If the receiving antenna is moved off-axis, the received power may increase, but at the expense of significant crosstalk as we will show in the next section. To analyze the on-axis situation we shall employ the LG modes defined in (1). One could criticize our omission of the curvature of the phase-front, but as long as the receiver antenna radius is much smaller than the beam width, that is $r \ll w$, this is a valid approximation for the estimation of the received power. In figure 2 the amplitude and phase of the fundamental mode with $l = m = 0$ and in figure 3 the mode with $l = 1, m = 0$ are plotted. Note the vanishing amplitude at the center of the latter.

The assumed receiver antenna area is $A = \pi r^2$, and we will assume that the detector is optimally capable of distinguishing orthogonal angular momentum modes. The incident power $p_0$ on an on-axis receiver antenna for the $l = 0, m = 0$ mode when $r/w \ll 1$ is

$$p_0 \propto \left( 1 - e^{-2\pi^2 w^2 / r^2} \right) \approx \frac{2r^2}{w^2}.$$  \hspace{1cm} (2)

Similarly, the incident power $p_1$ for the $l = \pm 1, m = 0$ angular momentum mode becomes

$$p_1 \propto \left( 1 - e^{-2\pi^2 w^2 / r^2} \left( 1 + \frac{2r^2}{w^2} \right) \right) \approx \frac{2r^4}{w^4}.$$  \hspace{1cm} (3)

Thus the relative power in the two broadcasted modes are

$$\frac{p_1}{p_0} = \left( \frac{r}{w} \right)^2.$$  \hspace{1cm} (4)
If one assumes that the noise, e.g., due to quantum or thermal noise is mode independent, which is reasonable, the relative SNR between the two modes is also given by the right hand side of (4), showing that there is a significant penalty to be paid for using the \( l = \pm 1 \) mode on axis. For higher order \( l \)-modes the power penalty scales with progressively higher power in \( w/r \). Equation (1) in fact implies that the incident power \( p \) on an on-axis receiver antenna for the \( l \) mode scales as

\[
p_l / p_0 = (w/r)^{2l}.
\]

Thus, communication channel multiplexing using OAM modes with on-axis detection comes at either a significant power cost, or at a significant reduction of the SNR. This was already pointed out in [19, 20] by using a two-antenna multiple-input multiple-output (MIMO) model and in [21] using many modes. Assuming that the received noise was mode-independent and that the initial SNR was 30 dB, the authors of [21] found that for transmitter to receiver distances above the Rayleigh distance \( R_c^2 / 2 \), where \( R \) is the transmitting antenna radius, the capacity increase gained by OAM mode multiplexing vanishes quickly with distance e.g., at distances 300 times the Rayleigh distance the use of multiple OAM modes offers essentially no capacity gain compared to the use of the fundamental, \( l = 0 \) mode only. The on-axis power, and hence the SNR, of the \( l \neq 0 \) modes is simply too low to give any appreciable contribution to the channel capacity. A subsequent analysis by Zhang et al [22] where the receiving antenna was placed on the beam central axis but tilted with the respect of the axis found that the channel capacity decreases rapidly with increasing tilt. The receiver antenna in this case consisted of 4–16 omnidirectional small sub-antennas placed on a circle and having optimized phase-delays to maximize the channel capacity. The study confirmed Edfors et al result that the channel capacity also decreased rapidly with distance. Observe that Zhang et al simulations did not consider antenna displacements but antenna tilt, but in some respects a tilt and a displacement induces very similar degradation for small tilts and small displacements, respectively.

As a further example, consider broadcasting with modes described by (1), one with \( l = 0 \) (the fundamental mode, lacking OAM) and one with \( l = 1 \). Assume furthermore that the same total power is transmitted into the two modes. If a receiving antenna is centered on the beam central axis and it has \( r/w \approx 2.25 \text{ m}/67 \text{ m} = 0.034 \) as in the experimental vortex mode communication paper by Tamburini et al [10], then the \( l = \pm 1 \) mode will have an almost 30 dB lower SNR than the \( l = 0 \) mode. If instead one had dimensioned the transmission link in terms of a fixed, desired receiver SNR, then the \( l = \pm 1 \) modes would have had to be fed with 800 times higher power than the \( l = 0 \) mode.

4. Crosstalk in angular momentum mode broadcasting

To increase the received power for modes with \( l \neq 0 \) one can put the receiving antenna off-axis by a distance \( o \), in the annular region where these modes have their intensity maxima, see figure 3. The problem then becomes crosstalk, because off-axis, the parts of the LG modes detected by the receiver antenna are no longer orthogonal. This can easily be seen in figure 3, from which it is clear that away from the central axis, the phase-front of the \( l = 1 \) mode becomes relatively flat and therefore resemble the flat phase-front of the fundamental \( l = 0 \) mode shown in figure 2. In addition, the amplitude for the two modes has similar fall-off for \( o/w > 1 \). Therefore, even an optimally designed, but small receiver antenna will not be able to discriminate the two modes particularly well, but experience significant crosstalk.

The crosstalk can be estimated by computing the mode overlap between the nominally orthogonal modes over a square with side \( 2r \), horizontally offset from the mode axis by a distance \( o \). (Here we assume that the

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{The amplitude (left) and phase (right) of an intensity normalized LG-mode with \( l = 1 \) and \( m = 0 \).}
\end{figure}
The complex crosstalk amplitude \( c(o, r) \) between a \( \text{LG}_{0,0} \) and a \( \pm \text{LG}_{1,0} \) mode can be computed from the local mode overlap as

\[
c(o, r) = \frac{2\sqrt{2P}}{\pi w^2} \int_{-r+w}^{r+w} \int_{-r}^{r} \sqrt{x^2 + y^2} e^{-2(x^2+y^2)/w^2} e^{2\pi i\tan(y/x)} \, dy \, dx,
\]

where \( P \) is the total broadcasted power. The amplitude in a \( \text{LG}_{0,0} \) mode collected by a \( \text{LG}_{0,0} \) antenna can be expressed

\[
a_{0,0}(o, r) = \frac{2\sqrt{P}}{\pi w^2} \int_{-r+w}^{r+w} \int_{-r}^{r} e^{-2(x^2+y^2)/w^2} \, dy \, dx.
\]

The shape of the curves is easy to understand. A receiving antenna designed for the \( \text{LG}_{0,0} \) fundamental mode will not experience any crosstalk when it is placed on an \( \text{LG}_{l=0} \) beam axis. However, if it is displaced radially to the maximum intensity of the \( \pm \text{LG}_{1,0} \) mode or beyond, where the phase front of this mode is relatively flat, it will be a good receiving antenna also for this mode. Far from the beam axis the \( \text{LG}_{0,0} \) mode intensity will have an exponential decrease. The \( \pm \text{LG}_{1,0} \) mode will have the same fall-off but will also have the multiplicative factor \( x^2 + y^2 \), see (6). Therefore the ratio between the intensities will be proportional to \( o^2 \) which is reflected by the slope of 2 of the dashed, red curve in figure 4. A similar argument shows why a receiving antenna designed for the \( \pm \text{LG}_{1,0} \) will have a decreasing crosstalk, scaling like \( o^{-2} \) when \( o \gg w \), see (6) and (8). Since the crosstalk is zero on axis, the blue curves must have a maxima somewhere around \( o = w \), depending a bit on the relative receiving antenna size \( r/w \). The crosstalk into the \( \text{LG}_{0,0} \) mode receiver is almost independent of \( r \) for a given offset ratio \( o/w \) as long as \( r/w \leq 0.01 \). Therefore only one line is displayed for this case.

From this analysis one sees that it is possible to obtain a good single channel communication link by transmitting the \( \text{LG}_{0,0} \) fundamental mode and placing the receiving antenna on-axis. However, if one uses the orbital angular degree of freedom for multiplexing in broadcasting, then crosstalk quickly becomes an issue for a receiving antenna located off-axis. Antennas must be located very close to the mode axis to achieve good crosstalk suppression. However, as seen from equation (4) and figure 3, and discussed above, on-axis antennas

![Figure 4](image-url)
optimized for LG-modes with \( l \neq 0 \) will not receive much power unless they are so large \( (r/w \approx 1) \) that they intercept most of the mode. Therefore, such modes are not a viable option in a broadcasting scenario.

One could argue that the crosstalk between modes will decrease by using different radial modes (with different \( m \)) for multiplexing. This is true to some extent, but since the mode index does not change the OAM of the mode, such mode multiplexing has little bearing on using the OAM degree of freedom in broadcast multiplexing. We will therefore leave this issue without consideration.

The example above was calculated for modes with \( l = 0 \) and \( l = \pm 1 \). However, the situation only becomes marginally better for modes with larger angular momentum (difference). The fundamental reason is the same, namely that the phase front of any LG-mode becomes essentially flat as the distance from the central axis increases. Thus, the crosstalk will be substantial between any two modes if they are detected by an off-axis antenna. The conclusion is thus that the OAM degree of freedom is not suitable for communication multiplexing in a broadcasting system. This obstacle was already hinted at in [11], but that experiment was performed with the receiver in the near-field, placed on-axis. Thus, the demonstration was more akin to a near-field link than to a broadcasting scenario. In fact, one of the few degrees of freedom in broadcasting that gives good discrimination between modes relatively independently of the location of the receiving antenna (but within the transmitted beam) is the polarization. Not surprisingly, this degree of freedom is already exploited in even relatively simple systems such as satellite TV-broadcasting.

It should be stressed that the cross-talk conclusions above pertain to OAM-modes in a broadcasting scenario. If OAM modes are used in a near-field [7, 8] or in a wave-guided communication system [23, 24], then OAM-modes are no worse (but \( a \ priori \) no better) than other sets of modes for mode multiplexing. In, e.g., [9] it is stated that ‘In addition, the inherent orthogonality of the various OAM states may reduce crosstalk, … compared to cases employing standard MIMO methods’. We would agree to the first half of the statement with the provision that the entire OAM mode needs to be detected to have mode orthogonality. One should also have in mind that OAM modes are no more orthogonal than any other eigen-mode set given by Maxwell’s equations and appropriate boundary conditions. As for the second half of the statement we disagree. MIMO is a generic technique of generating different modes, and in principle MIMO can generate or detect any mode, including OAM modes. Hence, one can prefer one way of generation or detection over another for various reasons, but the mode orthogonality will not be any different just because MIMO was used.

5. Conclusions

On the basis of Weyl’s law it is clear that the mode density of modes carrying OAM has the same scaling with frequency, angular frequency or wave vector as, for example, a plane wave set of modes has. From this we conclude that the angular momentum carried by electro-magnetic waves does not increase the channel capacity compared to a mode set void of OAM. (Such a mode set can be constructed from equally weighted sums and differences of angular momentum modes with opposite chiralities). Using one set of eigenmodes one codes the information in another degree of freedom than with the other set, but the number of power-orthogonal modes (or ‘channels’) per unit frequency bandwidth remains the same. This argument is also supported by thermodynamical considerations.

We also found that apart from this fact, OAM modes are not suitable for broadcasting. When only a small portion of the mode is detected by the receiver antenna, as is the case in broadcasting per definition, the different detected modes have a large overlap, resulting in a very significant crosstalk between the modes. Only on the central axis of the beam are the modes orthogonal, but here the intensity of any mode carrying optical angular momentum vanishes. Hence, only the fundamental mode, having no OAM (but linear or circular polarization), is really viable for on-axis (and off-axis) broadcast. In some applications, such as quantum cryptography, the fact that a significant portion of the mode has to be detected to gain any appreciable information could possibly be exploited. An evesdropper with access to only a small part of the beam can then not distinguish between different states as efficiently as the legitimate recipient who has access to a larger portion of the beam. However, this scenario is different from the broadcasting scenario discussed in [10] and in this paper.

Thus, while OAM may have advantages over other types of modes in certain applications, and while they certainly can carry information that is difficult or impossible to detect with antennas insensitive to the waves’ angular momentum [25], they do not extend the channel capacity of free space as has been claimed.

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