A Theory Perspective on Neutrino Oscillations

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Abstract

I summarize the status of neutrino oscillations that follow from current data, including the status of the small parameters $\alpha \equiv \Delta m_{\text{sol}}^2/\Delta m_{\text{ATM}}^2$ and $\sin^2 \theta_{13}$ characterizing the strength of CP violation in neutrino oscillations. I briefly discuss the impact of oscillation data on the prospects for probing the absolute scale of neutrino mass in neutrinoless double beta decay. I also comment on the theoretical origin of neutrino mass, mentioning recent attempts to explain current data from first principles.

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I. INTRODUCTION

Nowadays neutrino physics lies at the center of attention of the particle, nuclear and astrophysics communities. Here I summarize the determination of neutrino mass and mixing parameters from current neutrino experiments, given in detail in Ref. [1]. (Waiting for the verdictum of MiniBoone, we neglect the LSND data). The key concept in these neutrino oscillation studies is that of neutrino mixing, a characteristic feature of gauge theories of massive neutrinos. The most complete study of the structure of the neutrino mixing matrix was given in [2]. For the analysis of current neutrino oscillation data one assumes its simplest unitary, 3-by-3, and CP conserving form, as there is currently no sensitivity to CP violation. The interpretation of the data requires good calculations of solar and atmospheric neutrino fluxes [3, 4], neutrino cross sections and response functions, as well the inclusion of matter effects [5, 6] in the Sun and the Earth. The reader is referred to Ref. [1] for technical details and further references. For the discovery prospects of future neutrino oscillation studies see the Neutrino Oscillation Industry Web-Page [7]. Testing for the effect of leptonic CP phases and the absolute scale of neutrino mass constitutes the main upcoming challenge. Dirac CP phases will be probed in future oscillations studies, while Majorana phases will be tested in future searches for $\beta\beta_0\nu$ (neutrinoless double beta decay). I also briefly discuss the robustness of the oscillation hypothesis itself vis a vis the presence of non-standard physics.

Last, but no least, where does the neutrino mass come from? I briefly comment on two alternative views on the theoretical origin of neutrino mass, mentioning some of their possible signatures.

II. SOLAR AND REACTOR DATA

The solar neutrino data includes the rates of the chlorine experiment (2.56 ± 0.16 ± 0.16 SNU), the gallium results of SAGE (66.9 $^{+3.9}_{-3.8}$ $^{+3.6}_{-3.2}$ SNU) and GALLEX/GNO (69.3 ± 4.1 ± 3.6 SNU), as well as the 1496–day Super-K data (44 bins: 8 energy bins, 6 of which are further divided into 7 zenith angle bins). The SNO sample includes the data from the salt phase in the form of the neutral current (NC), charged current (CC) and elastic scattering (ES) fluxes, the 2002 spectral day/night data (17 energy bins for each day and night period) and the 391–day data. The analysis includes both statistical errors, and systematic uncertainties such as those of the eight solar neutrino fluxes.
Reactor anti-neutrinos from KamLAND are detected at the Kamiokande site by the process $\bar{\nu}_e + p \rightarrow e^+ + n$, where the delayed coincidence of the prompt energy from the positron and a characteristic gamma from the neutron capture allows an efficient reduction of backgrounds. Most of the incident $\bar{\nu}_e$'s come from nuclear plants at distances of $80 - 350$ km from the detector, far enough to probe large mixing angle (LMA) oscillations. To avoid large uncertainties associated with geo-neutrinos an energy cut at 2.6 MeV prompt energy is applied for the oscillation analysis.

The first KamLAND data corresponding to a 162 ton-year exposure gave 54 anti-neutrino events in the final sample, after cuts, whereas $86.8 \pm 5.6$ events are predicted for no oscillations with $0.95 \pm 0.99$ background events. This is consistent with the no–disappearance hypothesis at less than 0.05% probability, giving the first evidence for the disappearance of reactor neutrinos before reaching the detector, and thus the first terrestrial confirmation of oscillations with $\Delta m^2_{\text{sol}}$.

Additional KamLAND data with a larger fiducial volume of the detector were presented at Neutrino 2004, corresponding to a 766.3 ton-year exposure. In total 258 events have been observed, versus $356.2 \pm 23.7$ reactor neutrino events expected in the case of no disappearance and $7.5 \pm 1.3$ background events. This leads to a confidence level of 99.995% for $\bar{\nu}_e$ disappearance. Moreover they obtain evidence for spectral distortion consistent with oscillations.

A very convenient way to bin the latest KamLAND data is in terms of $1/E_{\text{pr}}$, rather than $E_{\text{pr}}$. Various systematic errors associated to the neutrino fluxes, backgrounds, reactor fuel composition and individual reactor powers, small matter effects, and improved $\bar{\nu}_e$ flux parameterization are included [1]. Assuming CPT invariance one can directly compare the information obtained from solar neutrino experiments with the KamLAND reactor results.

The KamLAND data single out the LMA solution from the previous “zoo” of alternatives [8]. The stronger evidence for spectral distortion in these data also leads to improved $\Delta m^2_{\text{sol}}$ determination, substantially reducing the allowed region of oscillation parameters. More than just cornering the oscillation parameters, however, KamLAND has eliminated all previously viable non-oscillation solutions. From this point of view KamLAND has played a key role in the resolution of the solar neutrino problem.
III. ATMOSPHERIC AND ACCELERATOR DATA

The first evidence for neutrino conversions was the zenith angle dependence of the $\mu$-like atmospheric neutrino data from the Super-K experiment in 1998, an effect also seen in other atmospheric neutrino experiments. At the time there were equally good non-oscillation solutions, involving non-standard interactions [9]. Thanks to the accumulation of upgoing muon data, and the observation of the dip in the $L/E$ distribution of the atmospheric $\nu_\mu$ survival probability, the signature for atmospheric neutrino oscillations has now become convincing. The data include Super-K charged-current atmospheric neutrino events, with the $e$-like and $\mu$-like data samples of sub- and multi-GeV contained events grouped into 10 zenith-angle bins, with 5 angular bins of stopping muons and 10 through-going bins of up-going muons. We do not use $\nu_\tau$ appearance, multi-ring $\mu$ and neutral-current events, since an efficient Monte-Carlo simulation of these data would require further details of the Super-K experiment, in particular of the way the neutral-current signal is extracted from the data. As far as atmospheric neutrino fluxes are concerned, we employ the latest three-dimensional calculations given in [4].

The disappearance of $\nu_\mu$'s over a long-baseline probing the same $\Delta m^2$ region relevant for atmospheric neutrinos is now available from the KEK to Kamioka (K2K) neutrino oscillation experiment. Neutrinos produced by a 12 GeV proton beam from the KEK proton synchrotron consist of 98% muon neutrinos with a mean energy of 1.3 GeV. The beam is controlled by a near detector 300 m away from the proton target. Comparing these near detector data with the $\nu_\mu$ content of the beam observed by the Super-K detector at a distance of 250 km gives information on neutrino oscillations.

The first phase (K2K-I data sample, corresponding to $4.8 \times 10^{19}$ protons on target) gave 56 events in Super-K, whereas $80.1^{+6.2}_{-5.4}$ were expected for no oscillations. The second phase (K2K-II data, corresponding to $4.1 \times 10^{19}$ protons on target) gave 108 events in Super-K, to be compared with $150.9^{+11.6}_{-10.0}$ expected for no oscillations. Out of the 108 events 56 are so-called single-ring muon events. This data sample contains mainly muon events from the quasi-elastic scattering $\nu_\mu + p \rightarrow \mu + n$, and the reconstructed energy is closely related to the true neutrino energy. The K2K collaboration also finds that the observed spectrum is consistent with the one expected for no oscillation only at a probability of 0.11%, whereas the best fit oscillation hypothesis spectrum has a probability of 52%.

One finds that the neutrino mass-squared difference inferred from the $\nu_\mu$ disappearance in
K2K agrees with atmospheric neutrino results, providing the first terrestrial confirmation of oscillations with $\Delta m^2_{\text{atm}}$ with accelerator neutrinos. Due to low statistics the current K2K data sample gives a rather weak constraint on the mixing angle. However, although the determination of $\sin^2 \theta_{\text{atm}}$ is completely dominated by atmospheric data, K2K data already start constraining the allowed $\Delta m^2_{\text{atm}}$ values [1]. For example, there is a constraint on $\Delta m^2_{\text{atm}}$ from below, which is important for future long-baseline experiments, since their sensitivities are drastically affected if $\Delta m^2_{\text{atm}}$ lies in the lower part of the $3\sigma$ range indicated by current atmospheric data.

IV. THREE-NEUTRINO OSCILLATION PARAMETERS

Lepton mixing is a characteristic feature of gauge theories of massive neutrinos. The first systematic study of the effective form of the lepton mixing matrix was given in [2]. For 3-neutrinos the simplest form of this matrix can be taken as

$$K = \omega_{23} \omega_{13} \omega_{12}$$

(1)

where each factor in the product of the $\omega$’s is effectively $2 \times 2$, characterized by an angle and a CP phase. Two of the three angles are involved in solar and atmospheric oscillations, so we set $\theta_{12} \equiv \theta_{\text{sol}}$ and $\theta_{23} \equiv \theta_{\text{atm}}$. The last angle in the three–neutrino leptonic mixing matrix is $\theta_{13}$,

$$\omega_{13} = \begin{pmatrix} c_{13} & 0 & e^{i\phi_{13}} s_{13} \\ 0 & 1 & 0 \\ -e^{-i\phi_{13}} s_{13} & 0 & c_{13} \end{pmatrix},$$

for which only an upper bound currently exists. All three phases are physical [10]. Two of the phases are fundamental, and arise at the two-generation level, being associated to the Majorana nature of neutrinos. They show up only in lepton-number violating processes, like neutrinoless double beta decay, not in conventional neutrino oscillations [10, 11]. The other phase corresponds to the phase present in the quark sector (Dirac-phase) and exists only with three (or more) neutrinos. This phase affects neutrino oscillations.

Such unitary form for the lepton mixing matrix holds in models where neutrino masses arise in the absence of right-handed neutrinos. To a good approximation, it also holds if neutrino masses are induced by a high-energy-scale seesaw mechanism (see below).

In our analysis we follow the simplest unitary form in Eq. 1. Since current neutrino oscillation experiments are insensitive to CP violation, we also neglect all phases in
the analysis. In this approximation oscillations depend on the three mixing parameters $\sin^2 \theta_{12}, \sin^2 \theta_{23}, \sin^2 \theta_{13}$ and on the two mass-squared splittings $\Delta m^2_{\text{sol}} \equiv \Delta m^2_{21} \equiv m^2_2 - m^2_1$ and $\Delta m^2_{\text{atm}} \equiv \Delta m^2_{31} \equiv m^2_3 - m^2_1$ characterizing solar and atmospheric neutrinos. The hierarchy $\Delta m^2_{\text{sol}} \ll \Delta m^2_{\text{atm}}$ implies that one can set $\Delta m^2_{\text{sol}} = 0$, to a good approximation, in the analysis of atmospheric and K2K data. Similarly, one can set $\Delta m^2_{\text{atm}}$ to infinity in the analysis of solar and KamLAND data. Apart from the data already mentioned, the analysis also includes the constraints from "negative" reactor experiments, CHOOZ and Palo Verde.

The three–neutrino oscillation parameters that follow from the global oscillation analysis in Ref. [1] are summarized in Fig. I and in Table I. In the upper panels of the figure the $\Delta \chi^2$ is shown as a function of the parameters $\sin^2 \theta_{12}, \sin^2 \theta_{23}, \sin^2 \theta_{13}, \Delta m^2_{21}, \Delta m^2_{31}$, minimized with respect to the undisplayed parameters. The lower panels show two-dimensional projections of the allowed regions in the five-dimensional parameter space. The best fit values and the allowed $3\sigma$ ranges of the oscillation parameters from the global data are summarized in Table I. This table gives the current status of neutrino oscillation parameters.
Table I: Neutrino oscillation parameters [1].

| parameter | best fit | 3σ range |
|-----------|----------|----------|
| $\Delta m^2_{21} \ [10^{-5} \text{eV}^2]$ | 7.9 | 7.1–8.9 |
| $\Delta m^2_{31} \ [10^{-3} \text{eV}^2]$ | 2.2 | 1.4–3.3 |
| $\sin^2 \theta_{12}$ | 0.30 | 0.24–0.40 |
| $\sin^2 \theta_{23}$ | 0.50 | 0.34–0.68 |
| $\sin^2 \theta_{13}$ | 0.000 | $\leq 0.043$ |

Figure 2: Determination of $\alpha \equiv \Delta m^2_{\text{sol}}/\Delta m^2_{\text{atm}}$ and bound on $\sin^2 \theta_{13}$ from current data, from [1].

Note that in a three–neutrino scheme CP violation disappears when two neutrinos become degenerate [2] or when one angle vanishes [12]. As a result its effects involve both the small mass hierarchy parameter $\alpha \equiv \Delta m^2_{\text{sol}}/\Delta m^2_{\text{atm}}$ as well as the small mixing angle $\theta_{13}$. The left panel in Fig. 2 gives the parameter $\alpha$, namely the ratio of solar over atmospheric splittings, as determined from the global $\chi^2$ analysis. The right panel in Fig. 2 gives $\Delta \chi^2$ as a function of $\sin^2 \theta_{13}$ for different data samples. One finds that the KamLAND-2004 data have a surprisingly strong impact on this bound. Before KamLAND-2004 the overall bound on $\sin^2 \theta_{13}$ was dominated by the CHOOZ reactor experiment, together with the determination of $\Delta m^2_{31}$ from atmospheric data.

In Fig. 3 we show the upper bound on $\sin^2 \theta_{13}$ as a function of $\Delta m^2_{\text{atm}}$ from CHOOZ data alone compared to the bound from an analysis including solar and reactor neutrino data. One sees that, although for larger $\Delta m^2_{\text{atm}}$ values the bound on $\sin^2 \theta_{13}$ is dominated by CHOOZ, this bound deteriorates quickly as $\Delta m^2_{\text{atm}}$ decreases (see Fig. 3), so that for $\Delta m^2_{\text{atm}} \lesssim 2 \times 10^{-3}\text{eV}^2$ the solar and KamLAND data become relevant.
In summary the overall improvement is especially important for lower $\Delta m^2_{\text{atm}}$ values. The new overall global bound on $\sin^2 \theta_{13}$ is 0.043 at 3$\sigma$ for 1 d.o.f. Such an improved $\sin^2 \theta_{13}$ bound follows mainly from the strong spectral distortion found in the 2004 KamLAND sample.

Future long baseline reactor and accelerator neutrino oscillation searches [13], as well as studies of the day/night effect in large water Cerenkov solar neutrino experiments such as UNO or Hyper-K [14] could bring more information on $\sin^2 \theta_{13}$ [15]. With neutrino physics entering the precision age it is necessary to scrutinize also the validity of the unitary approximation of the lepton mixing matrix in future experiments, given its theoretical fragility [2]. Indeed, any model where neutrino masses follow "a-la-seesaw" gives corrections to this approximation, which may be sizeable in some cases.

V. ABSOLUTE NEUTRINO MASS SCALE

Neutrino oscillation data are insensitive to the absolute scale of neutrino masses and also to the fundamental issue of whether neutrinos are Dirac or Majorana particles [10, 11]. On general grounds neutrino masses are expected to be Majorana [2], a fact that may explain their relative smallness with respect to other fermion masses. The significance of
the neutrinoless double beta decay stems from the fact that, in a gauge theory, irrespective of the mechanism that induces $\beta\beta_{0\nu}$, it necessarily implies a Majorana neutrino mass \[16\], as illustrated in Fig. 4. Hence the importance of searching for neutrinoless double beta decay \[17\]. Quantitative implications of the “black-box” argument are model-dependent, but the theorem itself holds in any “natural” gauge theory.

Now that oscillations are experimentally confirmed we know that $\beta\beta_{0\nu}$ must be induced by the exchange of light Majorana neutrinos, the so-called ”mass-mechanism”. The corresponding amplitude is sensitive both to the absolute scale of neutrino mass as well as the two Majorana CP phases that characterize the minimal 3-neutrino mixing matrix \[2\], none of which can be probed in oscillations.

Fig. 5 shows the estimated average mass parameter characterizing the neutrino exchange contribution to $\beta\beta_{0\nu}$ versus the lightest neutrino mass. The calculation takes into account the current neutrino oscillation parameters in \[1\] and the nuclear matrix elements of \[18\] and compares with experimental sensitivities. The upper (lower) panel corresponds to the cases of normal (inverted) neutrino mass spectra. In these plots the “diagonals” correspond to the case of quasi-degenerate neutrinos \[19, 20\], which give the largest $\beta\beta_{0\nu}$ amplitude. In the normal hierarchy case there is in general no lower bound on the $\beta\beta_{0\nu}$ rate since there can be a destructive interference amongst the neutrino amplitudes. In contrast, the inverted neutrino mass hierarchy implies a “lower” bound for the $\beta\beta_{0\nu}$ amplitude. A normal hierarchy model with no lower bound on $\beta\beta_{0\nu}$ is given in Ref. \[21\].

Future experiments like GERDA, SuperNEMO, CUORE, COBRA and others will extend the sensitivity and provide an independent check of the Heidelberg-Moscow claim \[22, 23\]. More information on the absolute scale of neutrino mass will also come from future beta decays searches (KATRIN) \[24\] as well as cosmology \[25\].

Figure 4: Neutrinoless double beta decay and Majorana mass are theoretically equivalent \[16\].
VI. THE ORIGIN OF NEUTRINO MASS

It is well-known that the effective dimension-five operator $\ell \ell \phi \phi$ where $\phi$ the $SU(2) \otimes U(1)$ Higgs doublet and $\ell$ is a lepton doublet induces neutrino masses once the electroweak symmetry breaks down through a nonzero vacuum expectation value $\langle \phi \rangle$. Nothing is known from first principles about the mechanism giving rise to this operator, its associated mass scale or flavour structure. The resulting Majorana neutrino masses are therefore unpredicted in general. However the very fact that Majorana neutrino masses violate lepton number may explain, irrespective of the underlying physics, why neutrinos are much lighter than the other fermions.

One possibility is that the dimension-five operator is suppressed by a large scale in the denominator (top-bottom scenario). Alternatively, it may be suppressed by a small scale in the numerator (bottom-up scenario). Both scenarios are viable and can be made natural, the first being closer to the idea of unification.
The most well-studied realization of the top-bottom scenario is the seesaw mechanism, which induces small neutrino masses from the exchange of heavy states that might come from unification. Small neutrino masses are induced either by heavy $SU(2) \otimes U(1)$ singlet “right-handed” neutrino exchange (type I) or heavy scalar bosons exchange (type II), in a nomenclature opposite from the one given in [2]. The effective triplet seesaw term has a flavor structure different from the type-I term, contributing to the lack of predictivity of general seesaw schemes, where both co-exist. Predictivity within the seesaw approach may be obtained by appealing to extra symmetries, as given, for example, in [20]. The model predicts maximal $\theta_{23} = \pi/4$, $\theta_{13} = 0$, and $\theta_{12} = \mathcal{O}(1)$, though unpredicted. Moreover, if CP is violated $\theta_{13}$ becomes arbitrary but the Dirac CP violation phase is maximal [27]. The model leads to a variety of phenomenological implications. For example, it gives a lower bound on the absolute neutrino mass $m_\nu \gtrsim 0.3$ eV. It also requires a light slepton below 200 GeV, and gives large rates for flavour violating processes. A survey of related models is given in Ref. [28].

Amongst “bottom-up” models we mention those where neutrino masses are given as radiative corrections [29] and models where low energy supersymmetry is the origin of neutrino mass [30]. The latter are based on the idea that R parity spontaneously breaks [31], leading to a very simple effective bilinear R parity violation model [32]. In this case the neutrino mass spectrum typically follows a normal hierarchy, with the atmospheric scale generated at the tree level and the solar scale radiatively “calculable” [33]. In order to reproduce the masses indicated by current data, typically the lightest supersymmetric particle decays inside the detector. More strikingly, its decay properties correlate with neutrino mixing angles. For example, if the LSP is the lightest neutralino, it is expected to decay 50/50 to muons and taus, since the observed atmospheric angle is close to $\pi/4$ [30, 33]. This constitutes a characteristic feature of the proposal that supersymmetry is the origin of neutrino mass and opens the tantalizing possibility of testing neutrino mixing at high energy accelerators, like
the "Large Hadron Collider" (LHC) and the "International Linear Collider" (ILC).

VII. ROBUSTNESS OF THE OSCILLATION INTERPRETATION

The general effective model-independent description of the seesaw at low-energies is characterized by \((n, m)\), \(n\) being the number of \(SU(2) \otimes U(1)\) isodoublet and \(m\) the number of \(SU(2) \otimes U(1)\) isosinglet leptons [2]. This leads to a very rich and complex structure of the charged current lepton mixing matrix (non-unitary) and non–diagonal neutral current [2]. For example, the \((3,3)\) seesaw model has 12 mixing angles and 12 CP phases (both Dirac and Majorana-type) characterizing its full 3×6 lepton mixing matrix [2].

The nontrivial structure of charged and neutral current weak interactions is a general feature of seesaw models [2] and leads to dimension-6 terms non-standard neutrino interactions (NSI), as illustrated in Fig. 7. Such sub-weak strength \(\varepsilon G_F\) operators can be of two types: flavour-changing (FC) and non-universal (NU). In inverse seesaw-type models [34], the non-unitary piece of the lepton mixing matrix can be sizeable and hence the induced non-standard interactions may be phenomenologically important [35]. "Large" NSI strengths may also be induced by scalar boson exchanges in models with radiatively induced neutrino masses [29], and in supersymmetric unified models [36].

Non-standard physics may in principle affect neutrino propagation properties and detection cross sections [37]. In their presence, the Hamiltonian describing atmospheric neutrino propagation has, in addition to the standard oscillation part, another term \(H_{NSI}\)

\[
H_{NSI} = \pm \sqrt{2} G_F N_f \left( \begin{array}{c} 0 & \varepsilon \\ \varepsilon & \varepsilon' \end{array} \right).
\]

(2)

Here \(+(-)\) holds for neutrinos (anti-neutrinos) and \(\varepsilon\) and \(\varepsilon'\) parameterize the NSI: \(\sqrt{2} G_F N_f \varepsilon\) is the forward scattering amplitude for the FC process \(\nu_\mu + f \rightarrow \nu_\tau + f\) and \(\sqrt{2} G_F N_f \varepsilon'\)
represents the difference between $\nu_\mu + f$ and $\nu_\tau + f$ elastic forward scattering. Here $N_f$ is the number density of the fermion $f$ along the neutrino path. In the 2–neutrino approximation, the determination of atmospheric neutrino parameters $\Delta m^2_{\text{ATM}}$ and $\sin^2 \theta_{\text{ATM}}$ was shown to be practically unaffected by the presence of NSI on down-type quarks ($f = d$) [38]. Future neutrino factories will substantially improve this bound [39].

In contrast, the oscillation interpretation of solar neutrino data is more “fragile” with respect to the presence of non-standard interactions in the $e - \tau$ sector [40]. On the other hand, it has been shown [41] that, even a small residual non-standard interaction of neutrinos in the $e - \tau$ channel leads to a drastic loss in sensitivity in the $\theta_{13}$ determination at a neutrino factory. It is therefore important to improve the sensitivities on NSI, another window of opportunity for neutrino physics in the precision age.

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