The Construction Parameter Optimization of High-Speed Pantograph

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Abstract. The composition structure of high-speed pantograph is analyzed, which is transformed into a four-rod mechanism, and the relative motion relation and its position coordinate equations are built by constructing a kinematics model. Based on the equations of motion of pantograph and the simulation of motion pattern between elevation height and transverse deviation of bow head, angle of balance arm is obtained by programming and running program in Matlab. The main parameters which are affecting high-speed pantograph lateral offset and balance arm rotation angle are analyzed. The relevant parameters are optimized by using the sequential quadratic programming (SQP) algorithm in Matlab. We compare the results after optimization with the results before optimization. Finally, according to the optimized results, the pantograph is modeled in Solidworks. The pantograph model is imported into the finite element analysis software to load, and the displacement cloud diagram of the pantograph bow head is obtained, and compared with the theoretical value to verify the optimization results.

Keywords. High-speed pantograph; motion trajectory; parameter optimization; finite element analysis.

1. Introduction
With the rapid development of rail transit, the running speed of high-speed train is increasing, the off-line of pantograph and catenary becomes more and more frequent. The bad contact between pantograph and catenary directly affects the quality of receiving current and the performance of traction power supply. If the structure of pantograph is damaged at high speed during train operation, the overhead contact system will be damaged [1].

In recent years, the scholars have carried out a lot of research on the design of pantograph. Based on the study of the pantograph trajectory, the construction parameters of the high-speed pantograph are optimized to aim at the transverse displacement of the bow head and the deflection angle of the balance bar. Based on the results of static strength analysis and experimental design of pantograph, the influence of load and material properties on the reliability of pantograph is analyzed [2-4]. The contact pressure between catenary and pantograph is dynamic because of the lateral and longitudinal impact. The pantograph, as one of the key equipments for the vehicles to receive current, generally needs to control the contact pressure between pantograph and catenary at about 120N, and the positive and negative deviations cannot exceed 10N, so it is necessary to control the contact pressure between pantograph and catenary [5-8]. After analysis, to maintain a certain contact pressure makes the bow head nearly flat, at the same time to reduce the bow head lateral offset and bow head swing, the bow head balance arm rotation angle should be as small as possible [9-10].
2. The Structural Mode of Pantograph

To solve the problem of optimal design, the actual mechanism model is firstly abstracted into a mathematical model. By analyzing the movement of the pantograph and combining its main moving parts, the pantograph can be regarded as a combination of two four-bar mechanisms, which is shown in figure 1.

As is shown in figure 1, point A is the connection point between the lower arm and the bottom frame, point B is the connection point between the lower arm and the bottom frame, point C is the connection point between the upper arm rod and the upper frame, point D is the connection point between the lower arm rod and the upper frame, point E is the position where the upper arm rod is connected with the balance rod, point F is the position where the balance rod is in the balance arm rod of the bow head, and point Q is the position where the bow head is. Because the degree of freedom of the pantograph mechanism is 1, point A is taken as the reference point, the coordinate system is established as shown in figure 1. The lower arm rod AD is the active part, through controlling lift bow torque of the lower arm rod to determine the movement of the pantograph. The implications of the parameters in figure 1 are shown in table 1.

![Figure 1. Mathematical model of pantograph.](image)

**Table 1.** Design parameters and their implications.

| Design parameter | Implications |
|------------------|--------------|
| $L_1$            | Length of lower arm rod AD     |
| $L_2$            | Length of upper frame upper DQ |
| $L_3$            | Length of upper frame lower CD |
| $L_4$            | Length of tie rod BC            |
| $L_5$            | Length of AE                    |
| $L_6$            | Length of equalizing rod EF     |
| $L_7$            | Length of the bow head swing rod QF |
| $h$              | Longitudinal distance between the two hinged supports fixed on the under frame |
| $b$              | Transverse distance between the two hinged supports fixed by the under frame |
| $c$              | Angle between the upper part and the lower part of the balancing rod |
| $g$              | Angle between DA and EA         |
| $e$              | Angle between balance rod EF and horizontal plane |
| $\alpha$         | Ascending bow angle of lower arm rod AD |
| $\beta$          | Angle between tie rod BC and horizontal plane |
| $\Phi$           | Angle between CD and horizontal plane in lower part of upper frame |
| $\gamma$         | Angle between DQ and horizontal plane in lower part of upper frame |
| $\theta$         | Angle between QF and vertical direction of bow head swing rod |
According to the built coordinate system and the determined design parameters, the trajectory equations of the horizontal and vertical coordinates of point Q can be obtained as shown in equations (1)-(2).

\[
\begin{align*}
X_Q &= L_2 \cos \gamma - L_1 \cos \alpha \\
Y_Q &= L_1 \sin \alpha + L_2 \sin \gamma
\end{align*}
\]

(1)

According to equations (1)-(2), we know that the trajectory of point Q is related to \(L_1, L_2, \alpha \) and \(\gamma\), where the mathematical relationship of the intermediate parameter can be derived from equation (3).

\[
\gamma = \pi - [c + \alpha + \cos(\frac{L_3^2 + L_5^2 - L_1^2}{2L_1L_3})]
\]

(3)

According to the cosine theorem and the angle relationship, we can get equations (4)-(5).

\[
\begin{align*}
L_{AB} &= (a^2 + b^2)^{1/2} \\
L_{AB} &= L_4^2 + L_5^2 - 2L_4L_{AB}\cos[\pi - (\beta + \tan^{-1} \frac{h}{b})]
\end{align*}
\]

(4)

(5)

In summary, the locus of Q point is a function, in the same way, we can deduce the equations of the rotation angle and the motion of the balance arm, the concrete process is as follows.

Point E coordinates are shown in equations (6)-(7).

\[
\begin{align*}
X_E &= -L_5 \cos(\alpha - g) \\
Y_E &= L_5 \sin(\alpha - g)
\end{align*}
\]

(6)

(7)

Point F coordinates are shown in equations (8)-(9).

\[
\begin{align*}
X_F &= L_6 \cos(e) - L_5 \cos(\alpha - g) \\
Y_F &= L_6 \sin(e) + L_5 \sin(\alpha - g)
\end{align*}
\]

(8)

(9)

Among them, \(e\) is shown in equation (10).

\[
e = \tan^{-1} \frac{Y_F - Y_E}{X_F - X_E} - \cos^{-1} \frac{L_2^2 + L_5^2 - L_1^2}{2L_1L_5}
\]

(10)

\[
L_{EQ} = \left( (X_E - X_Q)^2 + (Y_E - Y_Q)^2 \right)^{1/2}
\]

(11)

\(\theta\) is shown in equation (12).

\[
\theta = \tan^{-1} \frac{X_F - X_Q}{Y_Q - Y_F}
\]

(12)

In equation (12), the relation of motion locus of synthetic point Q is a function of \(L_5, L_6, L_7\) and \(g\), which affect the bow head trajectory and the balance arm rotation angle \(\theta\). The optimization results can be achieved by setting reasonable corresponding parameter values. The optimal design is translated into a suitable parameter scheme to minimize the angle of the balance arm and the X-axis deviation of the bow head.

3. Optimization of Pantograph Objective Function

This paper is to minimize the rotation angle \(\theta\) of the balance arm.

\[
\text{Min}(\theta) = \text{Min}(\tan^{-1} \frac{X_F - X_Q}{Y_Q - Y_F})
\]

(13)

Constraint condition [11-13]:

1. When lowering the bow, the minimum retracting height shall be less than 300 mm, which is shown in inequality (14).
Min\( (Y_Q) < 300 \text{mm} \) (14)

(2) When ascending the arch, the maximum height of the ascending arch should be greater than 2400 mm, which is shown in inequality (15).

Max\( (Y_Q) > 2400 \text{mm} \) (15)

(3) In order to ensure the stability of the current received by the pantograph, it is stipulated that the transverse offset \( x \) of the pantograph head should be as small as possible when the pantograph is lifted or lowered. The deviation is less than 30 mm, which is shown in inequality (16).

\[
\text{Min}(X_{Q_{\text{Max}}} - X_{Q_{\text{Min}}}) < 30 \text{ mm}
\]

\( X_{Q_{\text{Max}}} \) is the maximum value of the bow head in the horizontal direction respectively; \( X_{Q_{\text{Min}}} \) is the minimum values of the bow head in the horizontal direction respectively.

\[
\begin{align*}
X_{Q_{\text{Max}}} &= \text{Max}(L_2 \cos \gamma - L_1 \cos \alpha) \\
X_{Q_{\text{Min}}} &= \text{Min}(L_2 \cos \gamma - L_1 \cos \alpha)
\end{align*}
\] (17)

(4) The design principle of the four-bar linkage is combined, and the constraint conditions are shown in inequalities (18), when the constraint conditions are considered, the kinematic and technological requirements of the planar linkage are satisfied.

\[
\begin{align*}
1400 \text{ mm} &\leq L_1 \leq 1750 \text{ mm} \\
1800 \text{ mm} &\leq L_2 \leq 1900 \text{ mm} \\
150 \text{ mm} &\leq L_3 \leq 400 \text{ mm} \\
900 \text{ mm} &\leq L_4 \leq 1250 \text{ mm} \\
130 \text{ mm} &\leq h \leq 180 \text{ mm} \\
760 \text{ mm} &\leq b \leq 770 \text{ mm} \\
8^\circ &\leq c \leq 20^\circ
\end{align*}
\] (18)

3.1. Solution Method

The sequential quadratic programming (SQP) algorithm is used by fmincon function to solve the middle-scale optimization problem in Matlab [14-17]. The optimization in this paper belongs to the middle-scale optimization problem, and the sequential quadratic programming method is used to solve it. The flow diagram of the SQP algorithm is shown in figure 2.

3.2. Structure Optimization and Result Analysis

This article focuses on optimization design by using the optimization toolbox, which can solve many problems [18-20]. The constraints of design variables are linear inequality constraints, so this optimization design selection, function (multi-variable nonlinear function with constraints) to optimize the solution to the function of the maximum problem. Its mathematical model is Min\( f(x) \) s.t. \( C(x) \leq 0 \), the meaning of the mathematical expression is to find a set of \( x \), and make the objective function value minimum, and meet the constraints \( C(x) \leq 0 \), s.t. meaning subject to, that needs to meet after the constraints.

After understanding the call format of the function, to program in Matlab. By means of the objective function and constraints, a set of optimal solutions satisfying the design requirements can be obtained by using the SQP algorithm. Through comparing the changes of the parameters of the design variables and the changes of the related graphs before and after the optimization, which are showed in table 2.
The correlation analysis of the optimization design is as follows: The transverse displacement of the bow head before and after optimization was obtained by rewriting the optimized parameters into the program in figure 3. It can be seen that the transverse displacement of the bow head after optimization is 14 mm, which is much smaller than that of 260 mm before optimization.

In figure 4, we can see that the angle of the balance arm after optimization is less than 1°, which is much smaller than 5.5° before optimization. It can meet the requirements of optimization design.

### Table 2. Design variables before and after optimization.

| Design variables | Before optimization | After optimization |
|------------------|---------------------|--------------------|
| $L_1$ (mm)       | 1590                | 1545               |
| $L_2$ (mm)       | 1889                | 1900               |
| $L_3$ (mm)       | 311                 | 351                |
| $L_4$ (mm)       | 1127                | 1135               |
| $L_5$ (mm)       | 1547                | 1525               |
| $L_6$ (mm)       | 1852                | 1879               |
| $L_7$ (mm)       | 95                  | 94                 |
| h (mm)           | 160                 | 170                |
| b (mm)           | 770                 | 765                |
| c (rad)          | 0.1806              | 0.1403             |
| g (rad)          | 0.0252              | 0.0264             |
From the comparison chart before and after optimization in figure 5, we can draw a conclusion that there is no obvious change of ascending angle after optimization. The change of the pantograph bow angle of with different design variables does not affect the change of the bow angle, which is controlled artificially at the initial time and is not affected by the bow movement.

**Figure 3.** The transverse displacement of bow before and after optimization. **Figure 4.** A comparison of the angle of the head balance arm.

**Figure 5.** Contrast diagram of ascending bow angle before and after optimization. **Figure 6.** The bow angle and the angle of the balance arm before and after optimization.

Comparing the angle of the bow and the angle of the balance arm before and after optimization in figure 6, and combining the angle of the balance arm, we can get the reason that the angle of the balance arm changes before and after optimization. Figure 6 is a more intuitive representation of the dynamic relationship between the bow angle and the angle of balance arm. The rotation angle of the balance arm is greatly reduced, and the optimization goal is realized after optimization.

4. The Finite Element Analysis of Pantograph
The use of Ansys Workbench for product analysis is divided into the following steps [21]: Step 1: Start the Ansys Workbench and set up the units; Step 2: Select the type of analysis and generate the analysis flow chart; Step 3: Edit and create material properties; Step 4: Analyse importing geometry, choosing analysis type, setting boundary condition, dividing grid, loading, solving and displaying results; Step 5: Save the results. Material Properties of pantograph components are shown in table 3.
Table 3. Material properties of pantograph components.

| Components     | Material       | Poisson’s ratio | Elastic modulus (GPa) | Density (kg/m³) |
|----------------|----------------|-----------------|-----------------------|-----------------|
| Under frame    | Carbon steel   | 0.29            | 207                   | 7800            |
| Lower arm rod  | Carbon steel   | 0.29            | 207                   | 7800            |
| Draw rod       | Stainless steel| 0.305           | 190                   | 7750            |
| Upper frame    | Aluminium alloy| 0.33            | 71.7                  | 2740            |
| Balancing rod  | Aluminium alloy| 0.33            | 71.7                  | 2740            |
| Bow head       | Aluminium alloy| 0.33            | 71.7                  | 2740            |

Displacement nephogram of the loading model on the right side of the bow head before optimization is shown in figure 7. Displacement nephogram of the loading model on the left side of the bow head before optimization is shown in figure 8. Model is carried out by using the optimized parameters, the model entity is introduced into the finite element analysis software for static analysis [22-23]. The optimized bow head displacement nephogram was obtained by applying 300N on the left and right sides of the bow head respectively. The maximum load displacement of the bow head right side is 16.947 mm, and the maximum load displacement of the bow head left side is 23.006 mm. After optimization, the displacements of the bow head left and right sides are less than 30 mm, which meet the requirements of optimization design, and further verifies the correctness of optimization. Loading displacement nephogram on the bow head right side after optimization is shown in figure 9. Loading displacement nephogram on the left side of bow head after optimization is shown in figure 10.
According to the displacement nephogram obtained from the results of static analysis, the bow head displacement before and after optimization is calculated. Displacements of before and after optimization in bow head loaded is shown in Table 4.

| Displacement direction | Before optimization | After optimization |
|------------------------|---------------------|--------------------|
| Right (mm)             | 127.1               | 16.9               |
| Left (mm)              | 172.5               | 23.0               |

5. Conclusion
The movement process of pantograph is studied to ensure the quality of current received by high-speed train, and the main factors affecting the stability of current received are obtained by analyzing the movement track of Pantograph. Based on the movement model of Pantograph, the process of pantograph climbing is simulated and analyzed. The geometric parameters of the pantograph are optimized by the sequential quadratic programming method and optimization function according to the performance requirements of pantograph. The optimum parameters of the pantograph structure are obtained by optimization and comparison, which can be used as a reference for relevant design. Secondly, the optimized parameters are used for modeling, and the correctness of the optimized design is further verified through the finite element static analysis, the transverse offset of the bow head is controlled below 30 mm, and the angle between the balance arm and the bow head is less than 3°. The optimization goal is well achieved.

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