Dynamic propagation of space tether system motion

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Abstract. This paper aims at solving motion equations for a Spatial Tether System, composed by a principal satellite and a sub-satellite, through an initial know condition. The translational motion of the sub-satellite around the principal satellite is described in spherical coordinates, described by distance and the angles that position the vector between both satellites. The rotational motion of a sub-satellite \( S_2 \) is described by Euler equations and the cinematic equations for 3-2-3 Euler angles. The results of dynamic propagation show that the sub-satellite moves around the principal satellite in a precession anticlockwise motion, and that also vertically oscillates throughout this motion, with an amplitude of approximately 10\(^{\circ}\), for the adopted conditions. The numeric propagator of sub-satellite trajectory throughout the principal satellite and rotational motion of sub-satellite can be adapted for other types of Space Tether Systems.

1. Introduction

The facility of how information is spread through the several means of communication enables the most recent discoveries made by space agencies, such as NASA or ESA, to reach the common public. A further exploration possibility of Mars, the incredible possibilities of space tourism and reutilization of rockets brought by the newest private companies of the sector are some of the factors that are bringing up the conversation of spatial [1], [2].

Before all these scenarios, it is very pertinent to study types of satellites than can be used to expand men’s knowledge frontiers about space. One kind of satellite systems, that has several interesting applications, are the Space Tether Systems – STS [2], [5]. These systems are constituted of 2 or more rigid bodies in different orbits, connected by one or more flexible and compressed cables, commonly called tethers. These tethers are composed of a fibrous material high resistant to normal traction and the length can vary from some hundreds of meters to tenths of kilometers, some of them can reach 50km or more, depending on the mission’s goal.

Generally, the satellites that compose STS are of different sizes, being that in case of two satellites, the biggest is called main satellite \( S_1 \) and the smallest is called sub-satellite \( S_2 \). The \( S_1 \) satellite normally stores both the tether and the Satellite \( S_2 \), until the moment of release. Not only the tether but also the Satellite \( S_2 \) can be released through a series of mechanisms, such as springs, centripetal force of the system or gravitational gradient effects. Figure 1 shows an example of the STS configuration with two satellites. The STS are not limited to only satellites: an astronaut making external repairs at the International Space Station – ISS, is connected to it by a tether. This astronaut/tether/ISS system is considered a STS.

There are several applications for STS [1], [2], [5] that vary since aerodynamic and electrodynamic studies, transport, among others. To exemplify, some practical applications of these systems can be mentioned, emphasizing its importance. A first STS application is found in the measurement of spatial geophysical gradients. A series of sub-satellites (\( S_2, S_3, S_4 \ldots \)) are distributed throughout a sole tether fixed to the space shuttle or at ISS, where the data collection would be made regarding different simultaneous altitudes. A very interesting application of STS is in aerodynamics. Using the space shuttle as a tug, it is possible to hold an experimental aircraft to a tether and study the aerodynamic effects over the real gas condition, without the issues found in common wind tunnels. One of the most important applications of STS is found in the electrodynamic field. The conducting
and isolated tether movement within the geomagnetic field would be capable of generating a DC tension in the wire, which would feed the electric instrumentation on board the main satellite, at cost of losing orbital energy. Plasma contactors would be fixed on both the tether edges to allow the generation of electric power. This last application of STS was put into practice in TSS missions Tether Satellite System, that has as the goal to launch a satellite, connected to a space shuttle through a tether, to make the data collection that help in the studies of space and plasmatic electrodynamics physics. TTS-1 was the first space mission of the program, with its launching happening in July 1992, onboard the Atlantis Space Shuttle. The mission was successful and could create several information about the STS dynamic, showing that the tether can be released, controlled and recovered. Sufficient tensions were also generated through the tether and big ionosphere currents were extracted.

Thus, the goal of this paper is to study the dynamic of STS movement, focusing on the sub-satellite motion around the main satellite and the rotational motion of sub-satellite, comprehending the parameters that govern its space behavior and making numerical integrations of the STS state equations using MATLAB software, as of these specific initial conditions available in [6].

2. Presenting the Problem
Consider the STS in Figure 2 [2]. The main satellite $S_1$, mass $m_1$, orbits the Earth in a circular trajectory of $R$ radius. It is connected to a smaller sub-satellite $S_2$, mass $m_2$, through a rigid and inflexible tether, length $l$ and mass $m_t$. This tether can be expanded or retracted, making a $T$ tension arise in the wire.

For this study three reference system are needed, which are represented in Figure 2. One system of coordinates $F_0$ with origin in O in the centre of the mass (CM) of the main satellite $S_1$, and is related to its orbital movement, being that the axis $X_0$ points in the direction of the orbital speed vector that connects the $S_1$’s CM to the Earth’s CM. One system $F$ with origin in CM of the sub-satellite $S_2$, whose directions of the axis are associated to the characteristics of $S_2$, this system is considered as fixed, i.e., does not follow the satellite rotation and as the sub-satellite spins, the directions of the system $F$ do not alter. A coordinate system, $F’$, is fixed in the centre of mass of the sub-satellite $S_2$, with its axis $x’$, $y’$ and $z’$ coinciding with the principal inertia axis of $S_2$.

To determine the sub-satellite position regarding the principal satellite, the spherical coordinates $(l, \alpha, \beta)$ are used: $\alpha$ represents the angle on the plan between the $0Y_0$ and the projection of the $S_2$ position in the plan $0Y_0X_0$, $\beta$ represents the angle out of the plan between $l$ and the projection of the $S_2$ position in the plan $0Y_0X_0$. The tension in the tether $T$ is represented in the coordinates system $F_0$. 

![Figure 1. Configuration of a STS with 2 satellites [3].](image-url)
2.1. Motion equations for the $S_2$ motion around $S_1$

The translational motion equations of the sub-satellite $S_2$ around the $S_1$ are written as follows [3]:

\[
\ddot{l} = M_{21} l \dot{\beta}^2 + M_{21} l (\dot{\alpha} + \dot{u})^2 \cos(\beta)^2 + M_{21} \dot{u}^2 l (3 \cos(\alpha)^2 \cos(\beta)^2 - 1) - \frac{T}{m_2} \tag{1}
\]

\[
\ddot{\alpha} = - M_{23} \left( \ddot{l} \right) (\dot{\alpha} + \dot{u}) - 2 \dot{\beta} (\dot{\alpha} + \dot{u}) \tan(\beta) - 3 \dot{u}^2 \sin(\alpha) \cos(\alpha) \tag{2}
\]

\[
\ddot{\beta} = - M_{23} \left( \ddot{l} \right) \dot{\beta} - (\dot{\alpha} + \dot{u})^2 \sin(\beta) \cos(\beta) - 3 \dot{u}^2 \cos(\alpha)^2 \sin(\beta) \cos(\beta) \tag{3}
\]

Where: $u$ – latitude argument defined by the sum of perigee argument and the true anomaly, associated to the orbital movement of the main satellite, $\mu$ – Earth’s gravitational parameter and

\[
M_{21} = \left[ \frac{m_2 t_3}{m_2 + m_t} \right], \quad M_{23} = \left[ \frac{m_2 + \left( \frac{m_t}{2} \right)}{m_2 + \frac{m_t}{3}} \right], \quad \dot{u} = \sqrt{\frac{\mu}{R^3}} \tag{4}
\]

On [6] a law is introduced for the distension of the tether length in the time and its variation rate, which does not depend on other state variables ($\alpha$, $\beta$, $\dot{\alpha}$, $\dot{\beta}$). Thus, it no longer considers $l$ and $\dot{l}$ as state variables, reducing the system’s degree of freedom. This same control law is presumed in this paper and is given by [6]:

\[
l = (l_{\text{initial}} - l_{\text{final}}) e^{-ct} \tag{5}
\]

Where $l_{\text{initial}}$ and $l_{\text{final}}$ are the initial and final length of the tether, $c = 0.005s^{-1}$ [6] is a constant and $t$ is the time.

Assuming the control law established by the Eq. (5) for the tether length, the motion equations are summarized in the equations (2) and (3).

In this paper the initial data available in [6] will be considered. With the goal of reducing the complexity of a system to be estimated, on [6] it is assumed if the principal satellite mass is much
bigger than the sub-satellite mass and the tether mass, and that the tether length is much smaller that the orbit radius, i.e., \( m_1 >>> m_2 \), \( m_1 >> l \), \( l << R \), which entails \( M_2 \leftarrow l \) \( c \) \( M_3 = l \). The equations presented in [6] divert from the Eq. (2) regarding the equation of second order in the plan angle \( \alpha \), related to the first term of Eq. 2. Thus for comparison purposes with the result presented in [6], the Eq. (2) was replaced in one of simulations by:

\[
\ddot{a}_{\text{modified}} = -M_{23} \left( \frac{1}{l} \right) (\ddot{u}) - 2\beta (\dot{\alpha} + \dot{u}) \tan(\beta) - 3\vec{u}^2 \sin(\alpha) \cos(\alpha)
\] (6)

2.2. Rotational Motion equation of sub-satellite \( S_2 \)

To analyze the rotational motion of a sub-satellite \( S_2 \) in this paper are used the Euler ‘equations, equations 7-9, and the cinematic equations, equations 10-12, for 3-2-3 Euler angles [7], [8]:

\[
\begin{align*}
\dot{\omega}_x &= \left( \frac{1}{I_{zz}} \right) \left[ (I_{yy} - I_{xx}) \omega_y \omega_z - d_y \vec{T}_y + d_x \vec{T}_x \right] \\
\dot{\omega}_y &= \left( \frac{1}{I_{yy}} \right) \left[ (I_{xx} - I_{zz}) \omega_z \omega_x - d_z \vec{T}_z + d_y \vec{T}_y \right] \\
\dot{\omega}_z &= \left( \frac{1}{I_{xx}} \right) \left[ (I_{yy} - I_{xx}) \omega_x \omega_y - d_x \vec{T}_x + d_z \vec{T}_z \right] \\
\dot{\phi} &= \left( \frac{1}{\sin(\theta)} \right) [ -\cos(\psi) \omega_x + \sin(\psi) \omega_y ] \\
\dot{\theta} &= \sin(\psi) \omega_x + \cos(\psi) \omega_y \\
\dot{\psi} &= \left( \frac{1}{\sin(\theta)} \right) [ \cos(\theta) \cos(\psi) \omega_x - \cos(\theta) \sin(\psi) \omega_y + \sin(\theta) \omega_z ]
\end{align*}
\] (7-12)

Where \( \omega_x, \omega_y, \omega_z \), \( \vec{T}_x, \vec{T}_y, \vec{T}_z \) and \( d_x, d_y, d_z \) are components of the \( S_2 \) rotational spin \( \vec{\omega} \), the tension \( \vec{T} \) on the cable and distance of the point application of \( \vec{T} \), respectively, in a reference system \( F'x'y'z' \), fixed in the sub-satellite \( S_2 \), (see the Figure 8), \( I_{xx}, I_{yy}, I_{zz} \) are principal moments of inertia of \( S_2 \), and the Euler angle \( \phi \) (precession angle), \( \theta \) (nutation angle) and \( \psi \) (spin angle). It is important to note that there is singularity in the cinematic equation for \( \theta = n\pi, n \in \mathbb{N} \). Then in the numerical application it is necessary a special treatment close to this point.

3. Results

In the same way as [6], the focus of this paper is related to the tether collection phase (\( l_{\text{initial}} > l_{\text{final}} \)). For the results comparison purposes, the STS data available in [6] were used, with:

\[
\begin{align*}
m_1 &= 14000 \text{ kg}, \ m_2 = 1000 \text{ kg}, \ R = 6787 \text{ km}, \ l_{\text{initial}} = 50 \text{ km}, \ l_{\text{final}} = 45 \text{ km}, \\
\mu &= 398778,48 \text{ km}^3/\text{s}^2, \ \alpha(0) = 0^\circ, \ \beta(0) = 10^\circ, \ \dot{\alpha}(0) = 0.13^\circ/\text{s}, \ \dot{\beta}(0) = 0^\circ/\text{s}.
\end{align*}
\]

The numerical integrations of the equations were made by the 4th Order Runge- Hutta method, in MATLAB software programming.

The results obtained [4] for the \( S_2 \) movement around \( S_1 \) are presented in Figure 3 and 4 and are associated to the following propagations: Greene – uses equations 2 and 3; Yong – uses equations 6 and 3.

Observing the curves of Figures 3a, 4a and 4b it is noticed a good convergence between the results. In Fig. 3b it is observed that the results show more differences and it is justified because the equations used for the second order equation in the plan angle \( \alpha \) in [3] and [6] are different. However, the result obtained by Greene, equations 2 and 3, is a result closer to the real one, since the equations are complete.

For a qualitative interpretation of the results for the plan angle \( \alpha \), it is possible to conclude that the sub-satellite motion in plan \( X_0Y_0 \) (orbital plan of \( S_1 \)) varies in a counterclockwise way, see Figure 5, and, in the instant \( t = 2555s \) it completes its first turn around the main satellite, \( \alpha = 360^\circ \). The Figure 6
shows the projection of sub-satellite $S_2$ on the orbital plane $OXY_0$ for 5000 sec and it is possible to note that the length of the tether retracted from 50 km to 45 km.

In Figure 7 is observed a periodic variation for the angle outside the plan $\beta$, showing that the coordinate $Z_0$ of the sub-satellite oscillated as far as $\beta$ oscillates (approximately between $-13^\circ$ and $+13^\circ$). This figure shows two extreme positions of the sub-satellite in plan $Y_0Z_0$, in the $t_1$ e $t_2$ instants. Figure 8 shows the 3-dimensional representation of the $S_2$ motion around $S_1$ and can be observed motion outside the plane $X_0Y_0$ plane.

![Figure 7](image1.png)

![Figure 8](image2.png)

**Figure 3.** Temporal evolution of the plan angle $\alpha$ and its derivative.

For this work, the sub-satellite has the shape of a solid rectangular parallelepiped with uniform distribution of mass as it is shown in the Figure 9. It is assumed that the material of $S_2$ satellite is aluminum and their dimensions are:

- height ($z' \text{axis}$) = 0.50 m,
- length ($y' \text{axis}$) = 2 m,
- width ($x' \text{axis}$) = 0.37 m.
Figure 4. Temporal evolution of outside plan angle $\beta$ and its derivative.

Initial Conditions with rotation about the $x'$ axis are:
\[ \phi = 0^\circ, \theta = 0.5^\circ, \psi = 0^\circ, \omega_x = 0.0001 \text{ rad/s}, \omega_y = 0 \text{ rad/s}, \omega_z = 0 \text{ rad/s}. \]
Figure 5. Illustration of the $S_2$ motion around $S_1$ in the orbital plan of $S_1$.

Figure 6. Bi-dimensional representation of the $S_2$ motion around $S_1$ for 5000 sec.

Figure 7. Extreme positions of the sub-satellite in plan $Y_0$. 
The results associated with $S_2$ rotational motion [2] are shown in the Figure 10 and 11, after a special treatment around the singularity for the nutation angle equal $n\pi$.

4. Final Comments
The importance of studying STS, satellite systems connected by tethers, is due to the vast range of their applications in several aerospace knowledge areas [1], [5]. Thus it is necessary to understand the several parameters involved and in order to do this is important to manipulate and solve the motion equations of these systems.

This paper numerically integrated the motion equations associated to the sub-satellite movement around the principal satellite, using known initial condition to be able to predict the involved temporal parameters of this system. The integration used a $4^{th}$ Order Runge-Kutta method, with a MATLAB software. The results were compared to the results presented in [6] to validate the data obtained by the propagator.

The results have shown to be suitable, converting with the known data. It has been identified that sub-satellite $S_2$ has a precession movement in a counter clockwise direction in the orbital plane $X_0Y_0$ and that it oscillates vertically through this movement, with maximum and minimum amplitude approximated to $13^\circ$ for the initial adopted data.
To analyze the rotational motion of a sub-satellite $S_2$, it was assumed the rotation about the $x'$ axis. A special treatment was given for the cinematic equation along the singularity for 3-2-3 Euler angles. By the results, it is possible to note similar behavior for spin components $\omega_x$ and $\omega_z$, and it happened due to mass distribution around these axes. The spin components had high amplitude oscillation in the beginning and after they got stabilization. Temporal behavior analysis of spin velocity components allows inferring the instant in which the control system has to be actuated in order to prevent high frequency oscillations. Such phenomenon with high frequency may hinder the Tether System mission.

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References
[1] Cartmell M P and Mckenzie D J 2008 A review of space tether research. Progress in Aerospace sciences, 44, 1, pp 1-21.
[2] Cosmo M L and Lorenzini E C 1997 Tethers is Space Handbook (3. ed. Cambridge: Smithsonian Astrophysical Observatory).
[3] Greene M E and Denney T S 1991 IEEE Trans. on Aeros. and Electr. Syst. 27, 4, 689.
[4] Livio B 2017 Propagação dinâmica do movimento do Sistema de satélites tether (Trabalho de Graduação, UFABC, São Bernardo do Campo).
[5] Yi Chen, Rui Huang, Xianlin Ren and Ye He 2013 History of the Tether concept and tether missions: a review, ISRN Astronomy and Astrophysics, 2013, article ID 502973.
[6] Yong H; Bin L; Wenfu X; Cheng L 2009 ICIA 2009: Intern. Conf. on Inf. and Autom. Zhuhai/Macau, 148.
[7] Wertz J R 1978 Spacecraft Attitude Determination and Control (D. Reidel. Dordrecht. Holanda).
[8] Zanardi M C 2018 Dinâmica do Voo Espacial (Editora da UFABC, Santo André –SP).
Figure 10. Temporal variation for Euler angles
Figure 11. Temporal variation for the components of spin velocity.