Compact binary evolutions with the Z4c formulation

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Outline

1. Background

2. Single compact objects

3. Compact binary evolutions

4. Conclusions
Current status of formulations and BCs in NR

Popular formulations are GHG and BSSNOK with puncture gauge:

- Excision for BHs.
- GHG is symmetric hyperbolic
- Constraint damping scheme.
- Wave-like constraints.
- Known to have a WP IBVP with CP.
- Radiation controlling CPBCs in frequent use.

- moving puncture BHs.
- BSSNOK strongly hyperbolic.
- No damping scheme.
- 0-speed mode.
- WP CP IBVP with frozen coefficients. Untested.
- Sommerfeld conditions used. IBVP overdetermined. Not CP.

Is there a formulation with the strengths of both?
Conformal decomposition of Z4 I

A natural choice seems to be a conformal decomposition (Bernuzzi & DH, ’09) of the Z4 formulation (Bona et. al. 04).

\[
\partial_t \chi = \frac{2}{3} \chi [\alpha (\hat{K} + 2\Theta) - D_i \beta^i],
\]

\[
\partial_t \gamma_{ij} = -2\alpha \ddot{A}_{ij} + \beta^k \gamma_{ij,k} + 2 \gamma_{k(l} \beta_{,j)}^k - \frac{2}{3} \gamma_{ij} \beta^k, k,
\]

\[
\partial_t \hat{K} = -D^i D_i \alpha + \alpha [\ddot{A}_{ij} \dddot{A}^j + \frac{1}{3} (\hat{K} + 2\Theta)^2]
+ 4\pi \alpha[S + \rho_{ADM}] + \alpha \kappa_1 (1 - \kappa_2) \Theta + \beta^i \ddot{K}, i
\]

\[
\partial_t \Theta = \alpha [\frac{1}{2} R - \frac{1}{2} \dot{\ddot{A}}_{ij} \dddot{A}^j + \frac{1}{3} (\hat{K} + 2\Theta)^2]
- 8\pi \rho_{ADM} - \kappa_1 (2 + \kappa_2) \Theta] + \mathcal{L}_\beta \Theta.
\]

- Z4c is strongly hyperbolic when coupled to the puncture gauge,
- Has a trivial wave-like constraint subsystem,
- Admits constraint damping scheme.

\[
\partial_t \ddot{A}_{ij} = \chi [-D_i D_j \alpha + \alpha (R_{ij} - 8\pi S_{ij})]^{tf}
+ \alpha [(\hat{K} + 2\Theta) \ddot{A}_{ij} - 2 \dddot{A}_k^i \dddot{A}_{kj}]
+ \beta^k \ddot{A}_{ij,k} + \dddot{A}_{ik} \beta^k,j - \frac{2}{3} \dddot{A}_{ij} \beta^k, k
\]

\[
\partial_t \dddot{\gamma}^i = -2 \dddot{\gamma}^j \alpha_j + 2 \alpha [\dddot{\gamma}^j \dddot{\gamma}^k - \frac{3}{2} \dddot{A}^j \ln(\chi), j
+ \frac{1}{3} \dddot{\gamma}^j (2\hat{K} + \Theta, j - 8\pi \dddot{\gamma}^j S_j) + \dddot{\gamma}^j \beta^j, jk
+ \frac{1}{3} \dddot{\gamma}^j \beta_{,kj} + \beta^i \dddot{\gamma}^i,j - \dddot{\gamma}^i \beta^i, j + \frac{2}{3} \dddot{\gamma}^i \beta^i, j
- 2\alpha \kappa_1 (\dddot{\gamma}^i - \dddot{\gamma}^i).]
\]
Conformal decomposition of Z4 II

First applications in spherical symmetry:

- The zero speed mode in the BSSNOK constraint subsystem (Beyer & Sarbach '02, Gundlach & M Garcia '04) causes Hamiltonian constraint violation. Z4c does not suffer from the problem.
The trivial structure of the constraint subsystem was used to construct constraint absorbing preserving boundary conditions (Ruiz et. al. ’10). Spherical numerics:

- A detailed examination of the constraint damping scheme was presented in spherical symmetry (Weyhausen et. al. ‘11).
- Evolutions of binary black holes were presented using CCZ4 (Alic et. al. ‘11).
Is it really worth making better boundary conditions?

- Consider central density in evolution of single TOV star.
- Sommerfeld gives perturbation. Does not converge away with resolution. BCs can effect physics.
- This example by construction maximises effect. Boundaries close.
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Compact binary evolutions with the Z4c formulation
Radiation controlling CPBCs

Boundary conditions for constraints:

\[
\left( l^a \partial_a \right)^L \Theta \approx 0, \quad \left( l^a \partial_a \right)^L \tilde{Z}^i \approx 0.
\]

With GW controlling condition:

\[
\left( l^a \partial_a \right)^{L-1} \psi_0 \approx \partial_t^2 h \psi_0.
\]
Recipe for BCs implementation

Implementation?

- Conformal decomposition.
- Populate ghostzones.
- Standard bulk FDs at Boundary.
- Normal EoMs for metric.

Do they work?

\[
\begin{align*}
\partial_t \Theta & \doteq - \alpha (\partial_s \Theta + \frac{1}{r} \Theta) + \beta^i \partial_i \Theta, \\
\partial_t \tilde{A}_s & \doteq -\alpha \chi \left\{ 2 \tilde{D}^i \tilde{A}_{is} - \frac{2}{3} \tilde{D}_s (2 \tilde{K} + \Theta) - \frac{2}{3} R_{ss} \\
& + \frac{2}{3} \chi \partial_s \left[ \tilde{r}^s - (\tilde{r}_d)^s \right] - \frac{1}{3} \chi \partial_A \left[ \tilde{r}^A - (\tilde{r}_d)^A \right] \\
& + \frac{1}{3} R_{qq} - 3 \tilde{D}^i (\ln \chi) \tilde{A}_{is} - \kappa_1 \left[ \tilde{r}_s - (\tilde{r}_d)_s \right] \right\} \\
& + \alpha \left[ \tilde{A}_{ss} (\tilde{K} + 2 \Theta) - 2 \tilde{A}_s^i \tilde{A}_{is} \right] - \frac{2}{3} \chi D_s D_s \alpha \\
& + \frac{1}{3} \chi D^A D_A \alpha + \mathcal{L}_A \tilde{A}_{ss}, \\
\partial_t \tilde{A}_{AB}^{TF} & \doteq - \alpha \left[ \tilde{D}_s \tilde{A}_{AB} - \tilde{D}_{(A} \tilde{A}_{B)s} + \frac{1}{2} \tilde{A}_{s(A} \tilde{D}_{B)} (\ln \chi) \\
& - \frac{1}{2} \tilde{A}_{AB} \tilde{D}_s (\ln \chi) + \tilde{A}_A^i \tilde{A}_{iB} - \frac{2}{3} \tilde{A}_{AB} (\tilde{K} + 2 \Theta) \right]^{TF} \\
& - \chi D_A D_B^{TF} \alpha + \mathcal{L}_\beta \tilde{A}_{AB}^{TF}.
\end{align*}
\]
Incoming violation as Sommerfeld boundary causally connected.

CPBCs reduce violation.

At late times BSSNOK has large violation sitting at boundary.
Incoming violation kicks star like in spherical symmetry.
Effect tiny compared to that in spherical symmetry.
Does not converge away.
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Compact binary evolutions with the Z4c formulation
Hamiltonian constraint violation in BNS reduced in Z4c data.

Growth in constraint after merger with either formulation (cf. Bernuzzi et al. 2012).
- ADM mass integral BSSNOK.
- Noise after outgoing signal reaches extraction sphere.
- ADM mass integral $Z_{4c}$.
- Mass integral plus emitted GW energy constant

\[
\frac{E_{\text{ADM}}(r)}{M_{\text{ADM}}(t=0)} + E_{\text{gw}}
\]
Angular momentum

Angular momentum type integral:

\[
\frac{J_{\text{ADM}}(r)}{J_{\text{ADM}}(t=0)}
\]

Jump in BSSNOK when extraction sphere causally connected to outer boundary!
Gravitational waves

Accuracy in GW Amplitude in BBH:

Roughly a factor of 2 improvement.
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Summary

- Implemented CPBCs. Improvement over BSSNOK with Sommerfeld in constraints and angular momentum conservation.

- In BNS evolutions Hamiltonian constraint violation is reduced by more than an order of magnitude. Significant improvement in mass conservation.

- Improvement in GW error a factor between roughly 2 and 4 in BBH. Similar in BNS until merger.