Multiple Attractors in the Response of a Flexible Rotor in Active Magnetic Bearings with Geometric Coupling

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Abstract. Numerical results on the response of a flexible rotor supported by nonlinear active magnetic bearings are presented. Nonlinearity arising from the magnetic actuator forces that are nonlinear functions of the coil current and the air gap between the rotor and the stator, and from the geometric coupling of the magnetic actuators is incorporated into the mathematical model of the flexible rotor - active magnetic bearing system. For relatively large values of the geometric coupling parameter, the response of the rotor with the variation of the speed parameter within the range $0.5 \leq \Omega \leq 5.0$ displayed a rich variety of nonlinear dynamical phenomena including sub-synchronous vibrations of periods-2, -3, -6, -9, and -17, quasi-periodicity and chaos. Numerical results also reveal the occurrence of bi-stable operation within certain ranges of the speed parameter where multiple attractors may co-exist at the same speed parameter value depending on the operating speed of the rotor.

1. Introduction

Active magnetic bearings are increasingly being favored over the conventional fluid-film and rolling-element bearing types in rotating machinery applications. This is mainly due to their higher mechanical efficiency since the absence of contact between the rotor and the stator during operation of the machine reduces the losses due to friction. The magnetic bearings are, however, highly nonlinear and their interaction with the rotor that they support can lead to various nonlinear phenomena in the rotor’s response. The main source of nonlinearity in active magnetic bearings is the relationship between the forces generated in the electromagnetic actuator and the coil current and the air gap between the rotor and the stator. The force is proportional to the current squared and inversely proportional to the gap squared. Cross-coupling between the electromagnetic forces acting in two orthogonal directions is also a source of nonlinearity in magnetic bearing systems. One of the main causes of the cross-coupling effect is attributed to the geometry of the actuators. The air gap at a point on a magnetic pole is actually not constant over the entire pole area due to the geometrical curvature of the pole. This results in a normal force, which is perpendicular to the principal force, which in turn causes geometric coupling between these forces. Other causes of cross-coupling are attributed to gyroscopic and eddy-current effects.

The effect of nonlinearity arising from cross-coupling due to gyroscopic motion on the response of a rigid rotor in magnetic bearings examined in [1] showed the occurrence of Hopf bifurcation at

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certain values of operating speed. Multiple co-existing solutions were found at primary resonance of a rigid rotor response in magnetic bearings incorporating nonlinearity due to geometric coupling of the actuators [2]. The effects of geometric coupling on the response of a rigid rotor in magnetic bearings investigated in [3] and [4] revealed the existence of quasi-periodic and period-2 vibrations, as well as jump phenomena. Numerical integration and numerical continuation methods were used to investigate the unbalance response of a rigid rotor in magnetic bearings [5]. This work showed the occurrence of symmetry-breaking and period-doubling bifurcations. The response of a flexible rotor supported by magnetic and auxiliary bearings investigated numerically in [6] revealed the occurrences of sub-synchronous vibrations of periods-2, -4 and -8, and quasi-periodic and chaotic vibrations. The stability and bifurcations of a flexible rotor supported by radial and thrust magnetic bearings were examined using the Floquet theory in [7]. This work showed the importance of incorporating thrust magnetic bearings into the mathematical model of the rotor-bearing system, as they significantly influence the nonlinear dynamics of the system.

The effect of geometric coupling parameter on the response of a flexible rotor in radial active magnetic bearings is numerically investigated in this work. Nonlinearity arising from cross-coupling due to the actuators’ geometry, as well as from the magnetic actuator forces that are nonlinear functions of the coil current and the air gap between the rotor and the stator is incorporated into the mathematical model of the rotor-bearing system.

2. Governing Equations

The governing equations of a flexible rotor in active magnetic bearings are derived with the following assumptions being valid: (i) rotor is symmetric with part of its mass lumped at the rotor mid-span and the remainder at the bearing stations, (ii) rotor speed is constant, (iii) rotor and support stiffness are radially symmetric, (iv) damping force acting on the disc at rotor mid-span due to air dynamics is viscous, (v) rotor imbalance is defined in a single plane on the disc at the rotor mid-span, (vi) rotor motion in the axial direction is neglected, (vii) gyroscopic effect is neglected, (viii) flux leakage is neglected, i.e., the flux runs entirely through the iron except in the air gap, (ix) fringing effect, i.e., the spreading of flux in the air gap, is neglected, (x) magnetic iron operates below saturation level and well within the linear range of the iron magnetization curve, which implies constant permeability of the iron, and (xi) flux is homogeneous both in the iron and in the air gap and runs entirely within the magnetic loop, and the cross-section of the iron that is assumed constant along the entire loop is equal to that of the air gap. Accounting for the external forces acting on the rotor mid-span and bearing journal that include the rotor imbalance force, shaft elastic force, viscous damping force, magnetic bearing forces, and gravity, the governing equations can be expressed in non-dimensional form by equation (1).

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\begin{align*}
    x''_D &= -\frac{2\zeta}{f} x'_D - \frac{1}{f^2} (x_D - x_J) + U\Omega^2 \cos \Omega \tau \\
    y''_D &= -\frac{2\zeta}{f} y'_D - \frac{1}{f^2} (y_D - y_J) - \frac{W}{f^2} + U\Omega^2 \sin \Omega \tau \\
    x'_J &= F_{X_+} - F_{X_-} + \alpha_x (F_{Y_+} + F_{Y_-}) - \frac{1}{f^2 \gamma} (x_J - x_D) \\
    y'_J &= F_{Y_+} - F_{Y_-} + \alpha_y (F_{X_+} + F_{X_-}) - \frac{1}{f^2 \gamma} (y_J - y_D) - \frac{W}{f^2}
\end{align*}
\]

(1)

where
The motion of the system can be described by the non-dimensional displacements $x_D$ and $y_D$ of the geometric center of the rotor mid-span, and the displacements $x_J$ and $y_J$ of the geometric center of the journal. $\zeta$ is half the viscous damping ratio on the disc at the rotor mid-span. $f$, the frequency ratio, is the ratio of the linear natural frequency of the magnetic bearing system, $\omega_n$, to the pin-pin natural frequency of the flexible rotor, $\omega$. The unbalance parameter, which is a measure of the rotor imbalance, is defined as the ratio of the eccentricity of the rotor center of mass from its geometric center of rotation, to the nominal air gap of the magnetic bearing. $\Omega$, the speed parameter, is the ratio of the rotor operating speed, $\omega$, to the linear natural frequency of the magnetic bearing system, $\omega_n$. $\tau$ is the non-dimensional time. $W$, the gravity parameter, represents the unidirectional static force acting on the disc at the rotor mid-span, and at the bearing stations. $\alpha$ is the geometric coupling parameter, which is the ratio of the attractive, on-axis force between each magnet and the bearing journal to the normal, off-axis force. $\gamma$, the mass ratio, is the ratio of the journal mass, $m_J$, to half-mass of the disc at the rotor mid-span, $m_D$. $P$ and $D$ are respectively the non-dimensional proportional and derivative feedback gains of the controller. $F_{X+}$, $F_{X-}$, $F_{Y+}$, and $F_{Y-}$ are the magnetic bearing forces, and their derivation can be found in [8].

3. Numerical Results and Discussion

In the numerical simulation performed, the values of $U$, $W$, $\zeta$, $P$, $D$, $\gamma$, $f$, and $\alpha$ were respectively fixed at 0.1, 0.0, 0.001, 1.1, 0.03, 0.2, 1.5 and 0.28. $\Omega$ was varied from 0.05 to 5.0 at intervals of 0.01 in order to investigate the effect of the speed parameter on the response of the magnetically supported flexible rotor. Equation (1) was numerically integrated using the MATLAB software package, which utilizes a variable-step continuous solver based on an explicit Runge-Kutta (4,5) formula, the Dormand-Prince pair. The results of the numerical simulation are illustrated using bifurcation diagrams, rotor whirl orbits, Poincaré maps and power spectrum plots. The rotor whirl orbit, which is the plot of $x_J$ versus $y_J$, represents the instantaneous position of the journal center in the magnetic bearing clearance circle. Poincaré map is obtained by sampling the trajectory of the rotor whirl orbit at constant interval of the forcing period of $T = 2\pi / \Omega$ and projecting the outcome on the $x_J(nT)$ versus $y_J(nT)$ plane. The variation of $x_J(nT)$ in the Poincaré map with $\Omega$ is then plotted to form the bifurcation diagram. The power spectrum, which exhibits the frequency contents of the rotor response at the bearing station, is determined from the Fourier transformation of the time series of the journal response in the $X$-direction.

The bifurcation diagram for the rotor response with increasing $\Omega$ is shown in figure 1. For the range $0.05 \leq \Omega \leq 0.23$, the response of the rotor was synchronous, i.e., period-1. Chaotic motion of the rotor was observed in the range $0.24 \leq \Omega \leq 0.49$, except for $\Omega = 0.26$ and 0.29, where sub-synchronous response of period-9 was seen, and for $\Omega = 0.27$ where the response was synchronous. The rotor whirl orbit, Poincaré map and power spectrum plot of the period-9 response of the rotor for $\Omega = 0.26$ are shown in figure 2. With further increase of $\Omega$, sub-synchronous response of period-3 and chaotic vibrations of the rotor were observed alternately. Period-3 response was observed to exist in the ranges $0.50 \leq \Omega \leq 0.61$, $0.66 \leq \Omega \leq 0.70$ and $0.82 \leq \Omega \leq 0.94$. Chaotic response of the rotor was seen in the ranges $0.62 \leq \Omega \leq 0.65$ and $0.71 \leq \Omega \leq 0.80$. Sub-synchronous rotor response of period-6 was found to exist for $\Omega = 0.81$. The response of the rotor was found to be synchronous for

$$F_{X+} = \frac{1}{4(P-1)} \left[ \frac{(1-Px_J-Dx_J)^2}{(1-x_J)^2} \right]$$

$$F_{Y+} = \frac{1}{4(P-1)} \left[ \frac{(1-Py_J-Dy_J)^2}{(1-y_J)^2} \right]$$

$$F_{X-} = \frac{1}{4(P-1)} \left[ \frac{(1+Px_J+Dx_J)^2}{(1+x_J)^2} \right]$$

$$F_{Y-} = \frac{1}{4(P-1)} \left[ \frac{(1+Py_J+Dy_J)^2}{(1+y_J)^2} \right]$$
0.95 \leq \Omega \leq 1.64$, except for a small range $1.48 \leq \Omega \leq 1.50$ where quasi-periodic vibrations were seen. For $1.65 \leq \Omega \leq 2.29$, the response of the rotor was quasi-periodic except for values of $\Omega$ between 2.10 and 2.19, where chaos was observed. For the range $2.30 \leq \Omega \leq 5.0$, the response of the rotor was synchronous.

Figure 1. (a) Bifurcation diagram of the rotor response with increasing $\Omega$. Enlargement of figure 1(a) for: (b) $0.05 \leq \Omega \leq 0.5$, (c) $0.5 \leq \Omega \leq 1.0$, (d) $2.0 \leq \Omega \leq 2.5$.

Figure 2. Rotor response for $\Omega = 0.26$: (a) rotor whirl orbit, (b) Poincaré map, (c) power spectrum.
The response of the rotor with decreasing $\Omega$ is shown in the bifurcation diagram of figure 3. For $5.0 \geq \Omega \geq 2.29$, the response of the rotor was synchronous. Quasi-periodic vibration was seen in the rotor’s response for the ranges $2.28 \geq \Omega \geq 2.20$ and $2.08 \geq \Omega \geq 1.65$. For the range $2.19 \geq \Omega \geq 2.10$, the rotor response was chaotic and for $\Omega = 2.09$, a period-17 attractor was observed. With further decrease in $\Omega$, synchronous vibration response which was seen to exist in the range $1.64 \geq \Omega \geq 0.77$ eventually underwent a period-doubling bifurcation resulting in periodic response of period-2 for the range $0.76 \geq \Omega \geq 0.73$. Chaotic vibration was largely seen to dominate the rotor’s response for the range $0.72 \geq \Omega \geq 0.24$, except at specific frequencies where periodic vibrations were observed; period-1 at $\Omega = 0.27$, period-3 at $\Omega = 0.5$ and period-9 at $\Omega = 0.26$ and 0.29. For the range $0.23 \geq \Omega \geq 0.05$, the response of the rotor was synchronous.

![Figure 3. (a) Bifurcation diagram of the rotor response with decreasing $\Omega$. Enlargement of figure 3(a) for: (b) $0.05 \leq \Omega \leq 0.5$, (c) $0.5 \leq \Omega \leq 1.0$, (d) $2.0 \leq \Omega \leq 2.5$.](image)

A comparison of the bifurcation diagram in figure 1 for increasing $\Omega$ with that in figure 3 for decreasing $\Omega$ revealed the existence of multiple attractors for certain speed parameter ranges. The speed parameter ranges where multiple attractors were seen in the rotor’s response are summarized in table 1. For the ranges $0.51 \leq \Omega \leq 0.61$ and $0.66 \leq \Omega \leq 0.70$, the response of the rotor may be periodic of period-2 or chaotic depending on whether its operating speed is increasing or decreasing. The period-3 and chaotic response of the rotor for $\Omega = 0.55$ is illustrated in figure 4 using the rotor whirl
orbit, Poincaré map and power spectrum plot. The possibility of chaotic motion to co-exist with periodic motion of period-1 and period-2 was respectively observed in the ranges $0.77 \leq \Omega \leq 0.80$ and $0.73 \leq \Omega \leq 0.76$. Attractors of period-1 and period-3 were seen to co-exist for the range $0.82 \leq \Omega \leq 0.94$. Period-1 attractors were also seen to co-exist with period-6 attractors for $\Omega = 0.81$. For the range $1.48 \leq \Omega \leq 1.50$ and for $\Omega = 2.29$, quasi-periodic attractors co-existed with period-1 attractors. A quasi-periodic attractor was seen to co-exist with a period-17 attractor for $\Omega = 2.09$, figure 5.

Table 1. Range of speed parameter ($\Omega$) where multiple attractors were found to exist.

| Speed Parameter ($\Omega$) Range | Increasing $\Omega$ | Decreasing $\Omega$ |
|----------------------------------|--------------------|---------------------|
| $0.51 \leq \Omega \leq 0.61$    | Period-3           | Chaos               |
| $0.66 \leq \Omega \leq 0.70$    | Period-3           | Chaos               |
| $0.73 \leq \Omega \leq 0.76$    | Chaos              | Period-2            |
| $0.77 \leq \Omega \leq 0.80$    | Chaos              | Period-1            |
| $\Omega = 0.81$                 | Period-6           | Period-1            |
| $0.82 \leq \Omega \leq 0.94$    | Period-3           | Period-1            |
| $1.48 \leq \Omega \leq 1.50$    | Quasi-Periodic     | Period-1            |
| $\Omega = 2.09$                 | Quasi-Periodic     | Period-17           |
| $\Omega = 2.29$                 | Quasi-Periodic     | Period-1            |

Figure 4. Rotor whirl orbit, Poincaré map and power spectrum of the rotor response for $\Omega = 0.55$ with initials conditions: (a) $(x_j, y_j, x_D, y_D, x_j', y_j', x_D', y_D') = (-0.2, -0.8, -0.2, -0.8, -0.1, -0.05, -0.1, -0.05)$, (b) $(x_j, y_j, x_D, y_D, x_j', y_j', x_D', y_D') = (-0.2, -0.8, -0.2, -0.8, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0)$. 
Figure 5. Rotor whirl orbit, Poincaré map and power spectrum of the rotor response for $\Omega = 2.09$ with initials conditions: (a) $(x_j, y_j, x_D, y_D, x_j', y_j', x_D', y_D') = (0.05, -0.05, 0.05, -0.05, 0.0, 0.0, 0.0, 0.0)$, (b) $(x_j, y_j, x_D, y_D, x_j', y_j', x_D', y_D') = (0.7, 0.3, 0.75, 0.75, 0.0, 0.0, 0.0, 0.0)$.

4. Concluding Remarks
A rich variety of nonlinear dynamical phenomena were observed in the response of a flexible rotor supported by active magnetic bearings for relatively large values of geometric coupling parameter $\alpha$. In particular, sub-synchronous vibrations of periods-2, -3, -6, -9 and -17, as well as quasi-periodic and chaotic vibrations were seen to exist in the rotor’s response within the speed parameter range $0.05 \leq \Omega \leq 5.0$. For certain speed parameter ranges, bi-stable operation was found to occur where multiple attractors may co-exist at the same speed parameter value depending on whether the operating speed of the rotor is increasing or decreasing. In practical rotating machinery supported by active magnetic bearings, one should not discard the possibility of synchronous rotor vibration to become non-synchronous or even chaotic when subjected to external excitations due to preloads or fluid forces that may cause the rotor’s initial conditions to move from one basin of attraction to another. Non-synchronous and chaotic vibrations should be avoided in the operation of rotating machinery as they cause fluctuating stresses, which in turn may rapidly induce fatigue failure of its main components.

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