Investigation on the fluctuation of hydraulic exciting force on a pump-turbine runner during the load rejection process

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Abstract. The flow-induced vibration is one of the important reasons causing the instability of pump-turbines during the load rejection process. However, the flow mechanism is not clear. In this study, the load rejection process of a pump-turbine was investigated, considering the clearance between the runner and stationary parts, through three-dimensional turbulent flow simulation with the dynamic mesh technology. Unsteady pressure boundary conditions were established at the inlet of the spiral-casing and outlet of the draft tube. The rotational speed of runner was calculated through coupling the rigid body motion with the water flow. The simulated rotational speed of runner shows a good consistency with the corresponding experimental data. Based on the numerical results, the fluctuation of the hydraulic thrust and hydraulic torque on the runner and their formation mechanism were analysed. The results show that there exists complex flow in the draft tube and in the vaneless space during the load rejection process. The complex flow leads to the severe fluctuation of hydraulic thrust and hydraulic torque on the runner with a low frequency, which is about 0.06–0.4 times rated rotational frequency of the runner. Consequently, the hydraulic exciting source may induce intense vibration of the runner potentially.

1. Introduction

Nowadays, with the development of clean energy such as wind power and solar power, the pumped storage power generation technology as a regulating tool in the electric grid has developed rapidly [1-4]. In the process of regulating power grid, pump-turbines undergo frequent start-stop and conversion processes between different operating regimes [5]. The operating characteristics and internal flow of pump-turbines are quite complex during the transition processes [6, 7]. There often accompany intense vibrations of pumped storage units [8]. The load rejection process is such a complicated transition process, which the pumped storage units always experience during the operation process [9]. It is closely related to safe and stable operations of pumped storage units. Therefore, the stability and vibration of pump-turbines during the load rejection process should be considered in the design stage of pump-turbines
turbines. In recent years, owing to the need of actual engineering, scholars have conducted a lot of studies on the load rejection process of pump-turbines.

The numerical calculation method on the transition processes of pump-turbines underwent a developing process which is from one-dimensional (1D) calculation method to three-dimensional (3D) calculation method, and then to the method of coupling 1D method with 3D method (1-3D). In early stage, 1D calculation method on the transition process of a pump-turbine is mainly the method of characteristic (MOC) [10]. The MOC is too simplified to offer fully detailed flow field distribution and non-linear characteristic of pressure pulsation which are used to optimize the pump-turbine and ensure the safe and stable operation of pump-turbines. Then, the fully 3D method was developed gradually [11, 12]. The fully 3D method could satisfy most requirements in the engineering in a certain sense. But this method still is not able to be widely applied in engineering under the limitation on the current computing capacity of computers. Comprehensively considering the advantages and disadvantages of 1D method and 3D method, a numerical calculation method of coupling 1D method with 3D method was proposed and developed. In this numerical method, the 1D equation which describes the transient flow in water conveyance pipelines and the Reynold-averaged Naiver Stokes equations which describe the incompressible turbulent flow in pump-turbines are solved simultaneously [13-18]. In order to further solve the problem that the MOC is inaccurate in the 1-3D method, Huang et al. [19] proposed an improved coupled method in which the results of the 1D are corrected with the 3D method. In contrast, Li et al. [20, 21] presented another numerical simulation method based on the experimental data of a dynamic transition process.

Coupling the aforementioned numerical simulation methods with experimental test, some research progresses on the dynamic instability of performances and pressure pulsation in the transition processes have been made. The relevant experimental tests show that there exists intense pressure pulsation in turbines during the transition processes [22-24]; the intense pressure pulsation is mainly caused by the rotor-stator interaction and the complex flow in turbines [25]. The hill chart of pump-turbines shows significant looping dynamic characteristic in the runaway process [26]. Zhang et al. [27, 28] analyzed the formation mechanism on the looping dynamic characteristic of pump-turbines in the runaway process with the numerical simulation method. Yin et al. [29] simulated the load rejection process and pointed out that the variation of the rotational speed is the direct factor which causes the dynamic instability of a pump-turbine. Liu et al. [30] analyzed the influence of inertia on the fluctuation of internal and performance characteristics in the runaway process. Fan et al. [31] pointed out that the fluctuation of hydraulic torque on the guide vane within a slight opening is caused by the repeating reversal of fluid in the guide vane region after load rejection. Nennemann et al. [32] investigated the effect of the stochastic dynamic load on the fatigue life of the runner. Xia et al. [33] investigated the correlation among the evolution of flow pattern in pump-turbines, dynamic pressure pulsation and hydraulic thrust on the runner, and found that the variation of discharge is the main reason for intense fluctuation of the hydraulic thrust on the runner in the runaway process.

So far, the existing 3D calculation methods on the transition process of hydraulic turbines have been relatively more mature compared with the previous calculation methods. However, the formation mechanisms of the dynamic instability of the performance characteristics and pressure pulsation are still not very clear. Especially, the formation mechanisms on the fluctuation of the hydraulic exciting force on the runner are more unclear during the load rejection process. Hence, the research gap was selected to investigate in this study.

In this study, firstly, the reasonable boundary conditions were determined according to the existing experimental data in the load rejection process. Next, the 3D turbulent flow in a pump-turbine during the load rejection process was simulated under considering the clearance between runner and stator. The simulated rotational speed of the runner was compared with the corresponding experimental data, the feasibility of predicting the actual load rejection process of a pump-turbine with the numerical simulation results was validated. Finally, the formation mechanisms on the fluctuation of the hydraulic exciting force on the runner during the load rejection process was analyzed according to the numerical
results with the method of coupling the time-frequency joint analysis and coherence analysis combining with the flow field.

2. Numerical Model and Computational Method

2.1. Computational domain

In this study, a pump-turbine in a pumped storage power plant was investigated. The pump-turbine includes a runner with 9 blades, 20 guide vanes, 20 stay vanes including a special one, a spiral casing and a draft tube. Nominal diameters of runner at inlet and outlet are 4850 mm and 2535 mm, respectively. The height of the guide vanes of the prototype pump-turbine is 425 mm. The size of the prototype is scaled to the size of model (1:9.25). The nominal diameter of the model runner outlet is 274 mm.

The entire computational domain consists of the spiral-casing region, the stay vane region, the runner region, the draft tube region and the clearance region between the runner and the stationary parts including the cover and the bottom ring as shown in figure 1 and 2.

![Figure 1. Computational domain.](image1)

![Figure 2. Clearance between the runner and the stationary parts and pressure monitoring points.](image2)

2.2. Mesh generation and mesh independency verification.

The structural grids were generated in all regions except the guide vane region. In order to simulate the dynamic closing process of the guide vane with the technology of dynamic mesh, the hybrid grids were generated in the guide vane region as shown in figure 3.

![Figure 3. Grids of guide vane region and schematic diagram of closing guide vane.](image3)

![Figure 4. Mesh independency verification (Discharge).](image4)

In this study, seven sets of grids were generated in the computational domain with a 25 mm guide vane opening. The number of nodes is increasing from five million to eight million. The relationship between discharge and the number of nodes is shown in figure 4. Comprehensively considering the accuracy and calculation time of the numerical simulation, the mesh with 7.27 million nodes was selected to simulate the transient flow in a pump-turbine during the load rejection process.

2.3. Boundary conditions.
The unsteady total pressure and static pressure boundary conditions were respectively determined at the spiral-casing inlet and the draft tube outlet as shown in figure 5 and 6. The outflow boundary condition was set at the outlet of the clearance between the hub and the cover.

Figure 5. Unsteady total pressure at the spiral casing inlet.

Figure 6. Unsteady static pressure at the draft tube outlet.

During the load rejection, the guide vane was closing gradually as shown in figure 3. The closing law of the guide vanes during the load rejection process is shown in figure 7. The dynamic closing process of the guide vane was simulated with the technology of dynamic mesh in FLUENT software.

During the load rejection process, the rotational speed of runner was calculated according to the equations (1) - (2) using the method of coupling of the rigid body motion with the water flow through a user defined function (UDF) in the FLUENT software.

\[ M = J \frac{d\omega}{dt} \]  

(1)

\[ \omega_{i+1} = \omega_i + \frac{M_i}{J} (t_{i+1} - t_i) \]  

(2)

where \( M \) denotes the resultant torque on the rotor; \( J \) denotes the inertia of the rotor; \( \omega \) denotes the angular speed of the rotor, \( t \) denotes the time. Subscripts ‘\( i \)’ and ‘\( i+1 \)’ respectively denote \( t_{i\text{th}} \) and \( t_{(i+1)\text{th}} \) time step.

2.4. Numerical scheme and solve control
In this study, the RNG \( k-\varepsilon \) turbulent model was selected to close the Reynolds-averaged governing equations. The turbulent flow in a pump-turbine was simulated with the commercial calculation software FLUENT. In the numerical simulation, the second-order upwind scheme was selected to discretize the convective term. The first-order upwind scheme was adopted to discretize turbulent kinetic energy, turbulent dissipation term. The discrete algebraic equations were solved with the SIMPLEC algorithm. The residual of physical quantities in the numerical calculation is \( 10^{-5} \). The time step of the unsteady computation is 0.0012 s. The maximum iterations are 25 in each time step. In this study, the transient flow in a pump-turbine during the load rejection process was initialized with the steady numerical result of the critical condition point where the pump-turbine just enters into the load rejection process.

3. Results and Discussion

3.1. The verification of the numerical results
In order to verify the correctness of the numerical results of the transient flow during the load rejection process, the unit rotational speed was compared with the experimental data as shown in figure 8. The relative rotational speed is defined as equation (3).
where \( n_r \) denotes the relative rotational speed, \( n \) denotes the rotational speed, \( n_0 \) denotes the rotational speed at the initial instant of the load rejection process. It can be seen from figure 8, the rotational speed obtained from simulation is basically consistent with the experimental data on the overall trend. In addition, the maximum relative runaway speed of simulation is 126.47\%, while the corresponding experimental value is 128.17\%. The difference is only 1.70\% between the simulation and experiments. It indicates that the numerical result of the transient flow is reasonable in a certain error range.

\[
 n_r = \frac{n}{n_0} \times 100\% 
\]

(3)

**Figure 8.** Comparison of rotational speed between simulation and experiments.

### 3.2. Analysis of hydraulic thrust on runner

In order to analyse the vibration of runner during the load rejection process, the hydraulic thrust on the runner including its radial component, axial component and torque are shown in figures 9~12.

**Figure 9.** Variation of radial hydraulic thrust including \( F_x \) and \( F_y \) on runner with time

**Figure 10.** Distributing trajectory of radial hydraulic thrust on runner

**Figure 11.** Variation of axial hydraulic thrust on the runner with time.

**Figure 12.** Variation of hydraulic torque on the runner with time.

Figures 9, 11 and 12 show that the hydraulic thrust and hydraulic torque on runner fluctuate intensely near the point of the first runaway condition (\( t= 4.8 \) s). Meanwhile the figures 9 and 10 show that the distributing trajectory of radial hydraulic thrust on runner seriously deviates from the center of the runner. It indicates that there exists intense flow-induced vibration of runner.
In order to determine the flow-induced vibrational frequency of runner, the analysis method of short time Fourier transform (STFT) was used to analyze the radial hydraulic thrust, axial hydraulic thrust and hydraulic torque on runner as shown in figures 13~16 ($f_0$ is the rated frequency of runner at initial instant of load rejection process).

It can be seen that the fluctuating intensity of various frequency components near the point of the first runaway condition (t= 4.8 s) is higher than the corresponding intensity at other instants. Additionally, the fluctuating intensity of the lower frequency components is higher than the fluctuating intensity of the high frequency components. There are similar distributing characteristics near the instants (t=11 s, 15 s, and 18 s) with near the instant (t= 4.8 s). Moreover, there also exist blade passing frequency and its corresponding harmonic frequencies in figures 13~16.

In order to distinguish the lower frequency components, the analysis method of continuous wavelet transform (CWT) was adopted to analyze the lower frequency components of the hydraulic thrust and hydraulic torque on runner as shown in Figures 17-20 ($f_0$ is the rated frequency of the runner at initial instant of load rejection process). It is clear that the fluctuating intensity of the low frequency components of the axial hydraulic thrust $F_z$ on runner is stronger than the corresponding fluctuating intensity of radial hydraulic thrust $F_x, F_y$ and hydraulic torque on the runner. The stronger low frequency fluctuating components are about 0.06~0.4$f_0$.

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**Figure 13.** STFT result of radial hydraulic thrust $F_x$ on the runner.

**Figure 14.** STFT result of radial hydraulic thrust $F_y$ on the runner.

**Figure 15.** STFT result of axial hydraulic thrust $F_z$ on the runner.

**Figure 16.** STFT result of hydraulic torque $T$ on the runner.

**Figure 17.** CWT result of radial hydraulic thrust $F_x$ on runner.

**Figure 18.** CWT result of radial hydraulic thrust $F_y$ on runner.
3.3. Analysis of hydraulic pressure near runner

The hydraulic pressure near runner is closely related to the fluctuation of hydraulic thrust on the runner. The monitor points of pressure are arranged near the runner as shown in figure 2. In the meridian plane through the center of the outlet of draft tube, VL is the monitor point in vaneless space, DT is the monitor point at the inlet of draft tube, CRC is the monitor point in the clearance between the runner and the cover, CRB is the monitor point in the clearance between the runner and the bottom ring. The results of STFT on the pressure signal are shown in Figures 21~24 ($f_0$ is the rated frequency of runner at initial instant of load rejection process).

Comparing the STFT results of the pressure signal with the STFT results of the hydraulic thrust on the runner in figures 13~16, it can be seen that almost all the time-frequency characteristics of the pressure signals at each monitor points are generally similar with the time-frequency characteristics of the hydraulic thrusts on runner. It indicates that the vibration of runner are related to the fluctuation of pressure near the runner during the load rejection process. However, there are no quite significant difference among the pressure signals at monitor points VL, DT, CRC and CRB except the fluctuating intensity of pressure signal at VL, which is obviously stronger than others. Thus, it is not clear which pressure signal concretely causes the fluctuation of various hydraulic thrusts on the runner.
3.4. **Coherence analysis among hydraulic thrusts on runner and pressure**

In order to determine which pressure signal concretely causes the fluctuation of various hydraulic thrusts on runner, the method of coherence analysis is applied to discuss the relationship among the hydraulic thrusts and pressure signals as shown in figures 25–28 ($f_0$ is the rated frequency of runner at initial instant of load rejection process). Figure 25 and Figure 26 show the radial hydraulic thrust $F_x$ and $F_y$ are mainly related to the pressure at VL, CRC and CRB. Figures 27 and 28 show that the axial hydraulic thrust $F_z$ and hydraulic torque $T$ on the runner are mainly related to the pressure at DT.

![Figure 25. Coherence analysis among the radial hydraulic thrust $F_x$ and the pressure signals at VL, DT, CRC and CRB](image)

![Figure 26. Coherence analysis among the radial hydraulic thrust $F_y$ and the pressure signals at VL, DT, CRC and CRB](image)

![Figure 27. Coherence analysis among the axial hydraulic thrust $F_z$ and the pressure signals at VL, DT, CRC and CRB](image)

![Figure 28. Coherence analysis among the hydraulic torque $T$ and the pressure signals at VL, DT, CRC and CRB](image)

3.5. **Flow mechanism analysis on the fluctuation of hydraulic exciting force**

In order to further analyse the flow mechanism which causes the radial hydraulic thrust on runner and the pressure in the vaneless space fluctuating intensely, the surface streamline in the vaneless space was extracted as shown in figure 29.

![Figure 29. Surface streamlines on three different surfaces in the vaneless space](image)
At the instant \((t= 3 \text{ s})\), the surface streamline in the vaneless space is very smooth. However, at instants \((t= 4.8 \text{ s} \text{ and } 5.76 \text{ s})\), there appear obvious vortex flow in the vaneless space. Hence, the radial hydraulic thrust on runner and pressure in the vaneless space fluctuate intensely near the point of the first runaway condition \((t= 4.8 \text{ s})\).

Adopting the similar analysis method with above mentioned method in figure 29, the surface streamline in draft tube was extracted as shown in figure 30. At all instants \((t= 3.0 \text{ s}, 4.8 \text{ s}, 5.76 \text{ s} \text{ and } 6.72 \text{ s})\) during the load rejection process, there exist significant back-flow in the draft tube. The back-flow in the draft tube directly impacts the surface of runner. So the axial hydraulic thrust and hydraulic torque on runner are closely related to the flow in draft tube. In addition, at the instants \((t= 3.0 \text{ s}, 4.8 \text{ s} \text{ and } 5.76 \text{ s})\), there are obvious vortex flow in the draft tube. Therefore, the axial hydraulic thrust and hydraulic torque on runner fluctuate intensely near the point of the first runaway condition \((t= 4.8 \text{ s})\).

![Surface streamline on six different surfaces in the draft tube](image)

**Figure 30.** Surface streamline on six different surfaces in the draft tube

### 4. Conclusions

In this study, the 3-D transient load rejection process of a pump-turbine was simulated. The simulated rotational speed of runner shows a good consistency with the experimental data. According to simulated results, the fluctuation of hydraulic thrust, hydraulic torque on the runner and underlying flow mechanism were analysed. Some conclusions are obtained as following.

During the load rejection process, the hydraulic thrust and hydraulic torque on the runner fluctuate intensely with low frequency (The low frequencies are about 0.06~0.4 times rated rotational frequency of the runner), which may cause the serious vibration of the runner and even threaten the safe and stability of the pump-turbine potentially.

The fluctuation of axial hydraulic thrust and hydraulic torque on the runner is mainly related to the pressure pulsation at the inlet of draft tube; While the fluctuation of radial hydraulic thrust is mainly related to the pressure pulsation in the vaneless space and in the clearance between the runner and stationary parts.

The fluctuation of axial hydraulic thrust and hydraulic torque on runner is caused by the back-flow and vortex flow in the draft tube. The fluctuation of radial hydraulic thrust on the runner is caused by the vortex flow in the vaneless space. Therefore, the flow-induced vibration of the runner is mainly caused by the complex flow in the draft tube and in the vaneless space during the load rejection process. It means that it is possible to decrease the flow-induced vibration of runner through controlling the flow in the draft tube and in the vaneless space during the load rejection process.

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