Modeling of nano piezoelectric actuator based on block matching algorithm with optimal block size

WEI YangJie* & WU ChengDong

College of Information Science & Engineering, Northeastern University, Shenyang 110004, China

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In order to model the hysteresis behavior of a nano piezoelectric actuator (PA) on nano scale in a real time system, a new hysteresis modeling method based on an improved sub-pixel blocking matching algorithm with an optimal block size is proposed in this paper. First, Preisach model is introduced to model the hysteresis behavior of a piezoelectric actuator. Then, a real time block matching algorithm is researched and its block size is optimized with a standard object. Finally, experiments are performed with respect to a nanometer movement platform system, and the results show the feasibility and validity of the sub-pixel estimation based block matching algorithm and its application in modeling the hysteresis behavior of PA.

piezoelectric actuator, hysteresis modeling, optimal block size, nano

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1 Introduction

Lead zirconate titanate (PZT) is a ceramic perovskite material that shows a linear electromechanical interaction between mechanical and electrical states in crystalline materials with no inversion symmetry. Due to this effect, PZT with an applied voltage is often used as a displacement actuator. Compared to other actuators, it presents some advantages, such as small volume, no mechanical friction, high revolution and no noise, so it has been widely used in navigation, biology, and robotics [1–4].

However, due to the inherited nonlinearities, i.e. hysteresis, of PZT, the relationship between the applied voltage and the actuated displacement of a PZT actuator is complicated, it is necessary to model the hysteresis characteristic of a PZT actuator before using it in real applications. In the previous work, there are two basic methods to model it. The first method is based on the underlying physics of hysteresis with respect to some certain materials, as well as some empirical factors about the observed characteristics [5, 6]. Since most of the hysteresis characteristics are not completely understood, these physical models are applicable to a particular type of systems, and require special controller design techniques. The second method is based on the phenomenological nature, and it mathematically describes the observed phenomenon without physical insight into the problems [7–9]. Although this method can only approach the hysteresis to some certain degree, it is often used in real applications because of its simplification. Among these schemes, Preisach model is often referred to and has been used in many different systems [10–12].

In order to model the hysteresis of a PZT actuator, the first task is to measure its movement with high precision. In most applications, linear potentiometers and capacitance displacement sensors are used to measure movement of PZT actuators [13]. Although the precision and resolution of them are both high, they are greatly influenced by environment temperature and operation time. In some applications,
laser heterodyne interferometers are considered. However, they are expensive and have a large computational burden. Sometimes, Atomic Force Microscopy (AFM) system can measure a nano scale movement. But the measurement result is voltage, rather than displacement. Besides, it is time consuming [14].

In recent years, optical motion measurement methods based on images becomes attractive due to their rapid development [15–17]. Among them, image block matching is a typical technique [18–20] since it is not sensitive to random errors. Images, however, are typically processed assuming a uniform grid of pixels. While straightforward, the uniform grid representation does not scale well in a multi-scale setting for it requires an excessive amount of refinement to capture small details in the image, including low pixel count, sub-pixel resolution. The motion to be estimated is, for most applications, small and not integer. Therefore, in order to measure a nano scale movement, it is necessary to optimize the existing image block matching algorithms.

In this paper, we analyzed the basic performance of the image block matching algorithm and attained an optimal block size with a standard object image after a series of simulations. Then we proved its precision and used it to model a nano PZT actuator. The remainder of this paper is organized as follows. Firstly, in Section 2 the basic Preisach model principle is introduced; secondly, an idea of the sub-pixel block matching estimation is given, and a block size optimum is proposed to obtain a high precision PZT movement measurement method; subsequently, in Section 4, experimental results and detailed analysis are conducted. The conclusion is given in Section 5.

2  Preisach model

Normally, the hysteresis behavior of PZT is shown as Figure 1. In the Preisach model, the hysteresis characteristic is modeled by the weighted sum of relays termed as Preisach elemental operators, as shown in Figure 2, where $\alpha$ and $\beta$ are called upper and lower switching values of input, respectively. The hysteresis behavior is described as the following equation:

$$f(t) = \int_{\alpha>\beta} \mu(\alpha, \beta) \gamma_{\alpha, \beta}[u(t)] d\alpha d\beta,$$

(1)

where $u(t)$ and $f(t)$ are the input and the output of the system at time instant $t$, respectively; $\gamma_{\alpha, \beta}[u(t)]$ denotes the elementary hysteresis operator and it can be defined as

$$\gamma_{\alpha, \beta}[u(t)] = \begin{cases} 1, & u(t) > \alpha, \\ -1, & u(t) < \beta, \\ \text{maintain}, & \beta < u(t) < \alpha, \end{cases}$$

where $\mu(\alpha, \beta)$ is the density function of the Preisach function corresponding to $\alpha$ and $\beta$ which should be determined with some experimental data.

Suppose the input of a hysteresis system is as Figure 3(a), Figure 3(b) explains how to ensure the integration domain of the Preisach model eq. (1) in $\alpha$-$\beta$ plane. The following steps describe the procedure.

Step 1. During $[0, t_1]$, the system input increases from initial $0$ to $\alpha_1$ (the first local maximum); thus the integration domain at time $t$ is a closed region surrounded by the following three straight lines: $\alpha$ axis, $\alpha = \beta$ and $\alpha = u(t_1)$.

Step 2. During $[t_1, t_2]$, the system input decreases from $\alpha_1$ to $\beta_1$ (the first local minimum), and the integration domain decreases while the right border moves from right to left, and finally obtains a trapezoidal region surrounded by four lines at time $t_2$: $\alpha$ axis, $\alpha = \beta$, $\alpha = \alpha_1$ and $\beta = \beta_1$.

Step 3. During $[t_2, t_3]$, the system input increases again from $\beta_1$ to $\alpha_2$ (the second local maximum). At the same time, the integration domain of the Preisach model is composed of two parts: one is the trapezoidal region obtained at time instant $t_2$, and the other is a triangle region on its right side,
surrounded by three straight lines: $\beta = \beta_1$, $\alpha = \alpha_2$ and $\alpha = \beta$.

**Step 4.** During $[t_3, t_4]$, the system input decreases from $\alpha_2$ to $\beta_1$ (the second local minimum), and the integration domain of the Preisach model decreases too. The process is similar to step 2, and the final result at time $t_4$ is composed of two parts: one is the trapezoidal region obtained at step 2, the other is the trapezoidal region surrounded by four lines: $\alpha = \beta$, $\beta = \beta_2$, $\alpha = \alpha_2$ and $\beta = \beta_1$.

The preceding four steps explain the local memory property of PZT, i.e., the output of PZT relates not only the current input, but also the preceding local minimal and maximal inputs.

In many references, the Preisach model is identified through estimating the density function $\mu(\alpha, \beta)$. However, these strategies are time consuming, because they either deal with the double integral problem or solve partial differential equations, especially when the hysteresis model needs to be updated in a real time control system. While in ref. [10], the authors introduced an interesting substitute to avoid these drawbacks, since they identified the following integral instead of the density function itself:

$$F(x, y) = \int \int_{\Omega} \mu(x, y) d\alpha d\beta,$$

where $\Omega$ is a domain of integration surrounded by three straight lines: $\alpha = \beta$, $\beta = y$, $\alpha = x$.

For the identified function $F(x, y)$, one can easily describe the output of the Preisach model in the preceding four steps as following equations:

$$
\begin{align*}
  y(t_1) &= F(\alpha_1, 0), \\
  y(t_2) &= F(\alpha_2, 0) - F(\alpha_1, \beta_1), \\
  y(t_3) &= F(\alpha_1, 0) - F(\alpha_1, \beta_1) + F(\alpha_2, \beta_2), \\
  y(t_4) &= F(\alpha_1, 0) - F(\alpha_1, \beta_1) + F(\alpha_2, \beta_2) - F(\alpha_1, \beta_1). \\
\end{align*}
$$

(4)

Thus, with some mathematical tools to approach $F(x, y)$, it is easy to describe the relationship between the input and output of the hysteresis characteristic using algebraic equations. In this paper, the neural network method is utilized to model the characteristics of PZT.

3 Relative blurring

3.1 Block matching method

The foundational principle of the block matching algorithm (BMA) is to find a matching block from an image $X$ in some other image $Y$ through measuring difference, such as distance or similarity, between two images.

The principle is shown in Figure 4. Generally, $X(i, j)$ and $Y(i, j)$ are the model image and the target image, respectively. $F(i, j)$ is the continuous image function, $\varepsilon_{x,y}$ represents additive noise, $s=(s_x, s_y)$ is the shift between the model image and the target image.

$$
\begin{align*}
  X_{i,j} &= F(i, j) + \varepsilon_x, \\
  Y_{i,j} &= F(i - s_x, j - s_y) + \varepsilon_y.
\end{align*}
$$

(5) (6)

The BMA based techniques usually can be divided into two classes according to the measurement criterion: minimal difference and maximal similarity. The widely used object functions based on difference measurements include Sum-of-Squared-Differences (SSD), Sum-of-Absolute-Diff-

![Figure 4](image-url)
The shift between the model image and the target image can be obtained from

\[ X_{i,j} = x_{i,j} - \frac{1}{n \times m} \sum_{i=0}^{n-1} \sum_{j=0}^{m-1} X_{i,j} = x_{i,j} - X_{i,j}, \]  

\[ Y_{i\alpha,j\nu} = y_{i\alpha,j\nu} - \frac{1}{n \times m} \sum_{i=0}^{n-1} \sum_{j=0}^{m-1} Y_{i\alpha,j\nu} = y_{i\alpha,j\nu} - Y_{i\alpha,j\nu}, \]  

where \( x_{i,j}, y_{i,j} \) are the original gray value of the model and target image respectively; \( X_{i,j}, Y_{i\alpha,j\nu} \) are the mean of each image inside their respective “block”; \( u, v \) are the coordinates of the model image block when the original point is in the top left corner, and \( R(u, v) \) is the value of object function between model block and target block.

The shift between the model image and the target image can be denoted as

\[ s = s_x + s_y, \]  

where \( s_x = (n_x, n_y) \) are the integer shifts and \( s_y = (\varepsilon_x, \varepsilon_y) \) are the sub-pixel shifts. If the evaluating step is one pixel, \( s_x \) can be obtained from

\[ R(n_x, n_y) = \max \{ R(u, v) \}. \]  

However, the measurement precision and the resolution of an image based algorithm are limited by the pixel size, i.e., the position of the peak can only be solved with pixel-level accuracy, which is not enough when high precision is needed in nano positioning and nano manipulation, so the sub-pixel idea is used to further improve the measurement precision.

The basic idea for the sub-pixel movement estimation is to use an interpolation strategy, and a quadratic curve fitting around the peak \( s_x \) is usually used to estimate the sub-pixel shift \( s_y \) as follows:

\[ s_x = \frac{R'(-1,0) - R'(1,0)}{2[R'(1,0) + R'(0,0) - 2R'(0,0)]}, \]  

\[ s_y = \frac{R(0,-1) - R'(0,1)}{2[R'(0,1) + R'(0,0) - 2R'(0,0)]}, \]  

where \( R'(u-n_x, v-n_y) = R(u, v) \).

### 3.2 Optimal block size

In BMA, the parameters of a block can influence the speed and the measurement bias. For a target object with “orthogonal” edges, i.e., edges that have derivatives along the vertical and horizontal directions of the pixel grid, choosing the block only containing these orthogonal edges will minimize the summation containing cross derivatives, as well as the coupling bias [19]. However, the difficulty in selecting proper block size is a well-known disadvantage of the BMA algorithm, because there is a trade-off between having a block that is as large as possible to average out noise, a block as small as possible to guarantee that there is no deformation within it. Therefore, in this section, we researched estimation error with different block sizes on a synthetic standard object and attained an optimal block size respect to both the integral and sub-pixel motion.

First, we made a synthetic object image as shown in Figure 5(a), and chose a block, shown in Figure 5(b), with orthogonal edges. Then we shifted the original image with a step of sub-pixel, and the step size is from 0.2 pixel to 1.0 pixel. Through changing the block size, we could get different motion results with NCC and sub-pixel estimation method in Section 2. The motion measurement results are shown in Tables 1 and 2, where the block size is from 20x20 pixels to 80x80 pixels. Since the true motion of each step is known, it is convenient to analyze the estimation error between the true motion and the estimated motion with different block sizes. The error analysis result along x axis and y axis is shown in Figures 6 and 7, respectively.

From Tables 1, 2 and Figures 6, 7, the following conclusions can be attained:

(a) The estimation error of NCC is sensitive to the block size and it has the similar trends along x and y axes.

(b) When the block size is 20x20 pixels and 80x80 pix-
Table 1 Relationship between X motion and block size

| Motion (pixel) | 20×20 | 30×30 | 40×40 | 50×50 | 60×60 | 70×70 | 80×80 |
|---------------|-------|-------|-------|-------|-------|-------|-------|
| 0.2           | 0.328 | 0.167 | 0.122 | 0.099 | 0.087 | 0.078 | −0.125|
| 0.4           | 0.310 | 0.394 | 0.324 | 0.290 | 0.271 | 0.257 | −0.012|
| 0.6           | 0.609 | 0.626 | 0.622 | 0.619 | 0.618 | 0.616 | 0.223 |
| 0.8           | 0.953 | 0.883 | 0.861 | 0.851 | 0.844 | 0.839 | 0.640 |
| 1.0           | 1.128 | 1.029 | 1.002 | 0.989 | 0.981 | 0.976 | 0.816 |

Table 2 Relationship between Y motion and block size

| Motion (pixel) | 20×20 | 30×30 | 40×40 | 50×50 | 60×60 | 70×70 | 80×80 |
|---------------|-------|-------|-------|-------|-------|-------|-------|
| 0.2           | 0.328 | 0.167 | 0.121 | 0.099 | 0.086 | 0.078 | −0.125|
| 0.4           | 0.310 | 0.395 | 0.324 | 0.291 | 0.270 | 0.257 | −0.012|
| 0.6           | 0.609 | 0.628 | 0.623 | 0.620 | 0.617 | 0.617 | 0.224 |
| 0.8           | 0.953 | 0.883 | 0.861 | 0.851 | 0.844 | 0.840 | 0.640 |
| 1.0           | 1.128 | 1.029 | 1.002 | 0.989 | 0.981 | 0.976 | 0.816 |

Figure 6 Estimation error along x axis with different block sizes.

Figure 7 Estimation error along y axis with different block sizes.

els, the relative estimation error is more than 1/2; while when the block size is between 30×30 pixels and 70×70 pixels, the relative error is less than 1/3.

(c) Considering the estimation error of each step along x and y axes, the optimal block size respect to a standard object can be concluded: 30×30 pixels.

Therefore, in the next experiment, we used the NCC object function and the optimal block size to measure the movement of our actuator on nano scale.

4 Experiments

In order to verify the feasibility of the optimal block size in the last section and model the hysteresis behavior of our PZT actuator, experiments were performed with respect to a nanometer movement platform as shown in Figure 8 and the standard grid micrograph used as the target object is shown in Figure 9.

The experimental setup is composed of a piezoelectric actuator, a microscope, a 2D nanometer movement platform, a CCD and a computer. In the experiment, the controller of the piezoelectric actuator regulates the input voltage and drives the platform to move. The microscope captures the sequence images of the standard grid micrograph during the movement of the platform and outputs them into the computer. The computer gets the images and calculates the movement using the corresponding processing algorithm. In this experiment, the microscope is KH7700 designed by HiRox, and its magnification is 60×; the nanometer movement platform is designed by Physik Instrumente (PI) company.

4.1 Experiment I

First, in order to validate the precision of our algorithm, we used the PI nano platform to control the motion of the standard grid and calculated it with our parameters. Because the platform can output the nano motion, the shift of the grid is known. The KH-7700 microscope was used to capture the images of the grid, in which the size of every pixel is 57.47 nm×57.47 nm, and the each step shift of the PI
nano platform is 50 nm. Since the horizontal pixel is 57.47 nm, the true shift value of each step should be 0.89 pixel.

Figures 10 and 11 are the practical shift calculation results with our optimal block size. Figure 10 is the practical shift of the integral shift and Figure 11 is the practical shift of sub-pixel, where the vertical axis denotes the 2D motion, with unit of pixel, and the horizontal axis denotes the shift steps; the curve with stars is the true movement, the curve with “o” is the calculation movement when the block size is 60×60 pixels, and the curve with “Δ” is the calculation movement when the block size is the optimal value: 30×30 pixels. The start positions of these two blocks are the same, and both of them have “orthogonal” edges like Figure 5(b).

From Figures 10 and 11, we can see that when the block size is the optimal value 30×30 pixels, the integral pixel measurement is exactly equal to the true value, and the maximal sub-pixel measurement error is 0.18±0.13=0.05 pixel (the relative error is 5.6%); while when the block size is 60×60 pixels, the maximal error of the integral pixel measurement and the sub-pixel measurement is 0.01 pixel and 0.12 pixel, respectively (the maximal relative error is 13%). Therefore, compared to the result with a block size of 60×60 pixels, the measurement method with the optimal block size can reduce the relative error by 7.4%, and the maximal estimation error is 0.05×57.47=2.9 nm.

4.2 Experiment II

Secondly, we used the optical motion measurement method mentioned in Section 2 to measure the practical motion of our PZT actuator. In this experiment, in order to compare different motions with different driving voltages, we used two driving voltages. The measurement result is shown in Figure 12, where the vertical axis denotes the motion, with unit of nm, and the horizontal axis denotes the driving voltage, with unit of V. Figure 12(a) is the driving curve when the voltage increases to 200 V and then decreases to 0 V smoothly; Figure 12(b) is the driving curve when the variation routine of the voltage is 0 V-200 V-0 V, 0 V-150 V-0 V, 0 V-100 V-0 V, 0 V-50 V-0 V.

The measurement results of the piezoelectric actuator driving characteristic are consistent with the physics analysis of PZT in Figure 12. Furthermore, the proposed method, which is simple in manipulation and credible in measurement results, coincides with the requirement of the micro/nano measurement on nano scale.

4.3 Experiment III

In this experiment, we identified the Preisach model of the
PZT actuator using the proposed BMA with optimal block size based on Experiment I and Experiment II. First, a sequence of known voltages was applied to the actuator and its movement at each step was calculated with the proposed motion measurement algorithm. Then, according to the relationship between voltages and displacements, we constructed the Preisach model of the actuator. Finally, the model was verified with a new group of voltages.

The neural network used to approach the double integral $F(x,y)$ as eq. (3) is shown in Figure 13. It is a typical feedforward network with three hidden layers. Layer 1 and layer 2 have 10 and 5 neurons respectively and both have tanh active function. Layer 3 has only 1 neuron with pure linear active function.

In order to identify $F(x,y)$, the input voltage is denoted in Figure 14. By training the neural network, we could obtain the approached $F(x,y)$ as Figure 15.

Furthermore, Figures 16 and 17 show the comparison between the real outputs of the system and the output of the Preisach model using neural network. From them, we can see that the output of the Preisach model matches the real data perfectly. According to Figure 17, most approaching errors are less than 50 nm, while the absolute movement is as much as 10000 nm. So the relative error is only 0.5%. Finally, we used another group inputs to verify the feasibility of the new constructed Preisach model. The inputs are as following equation:

$$u(t) = \begin{cases} 
(10t)V, & 0s < t < 10s, \\
(200 - 10t)V, & 10s < t < 20s. 
\end{cases} \quad (14)$$

The experimental result is as Figure 18, from which the approaching output of the Preisach model also matches the real measurements very well. The average error computed by eq. (15) is 12.196 nm, compared to the maximal real output of 5200 nm.

$$E = \frac{1}{N} \sum_{i=1}^{N} (\text{error}(i))^2, \quad (15)$$
where $E$ is the defined average error; $N$ is the number of the measured data; $error(i)$ is the $i$th approaching error.

5 Conclusions

In this paper, we presented a new PZT hysteresis behavior modeling method on nano scale based on a sub-pixel optical motion measurement algorithm with an optimal block size. The main contributions of this paper mainly include the following three aspects:

1) An improved real time sub-pixel block matching algorithm was proposed and researched in detail, and an optimal block size was obtained.

2) The practical driving curve of an actuator was measured with the proposed method and the optimal parameter was validated.

3) The preceding measurement method was used to model the hysteresis behavior of the PZT actuator, combined with the Preisach model. The experiments were conducted and the results show the feasibility of both the measurement method and the modeling method.

Furthermore, the proposed optical motion measurement algorithm can also be used to obtain the real motion variables of a platform online since only a 30x30 pixels block is required, and then with the measurement result, the modeling error of the hysteresis behavior can be compensated and a more precise motion control can be attained.

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