Operations on vague soft matrices and its applications in decision making

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Abstract. In this paper, we introduce the notion of vague soft matrix to represent vague soft sets in matrix form. By using this representation it is easy to store and manipulate vague soft sets in a computer. We then define basic operations like union, intersection, complement and product of vague soft matrices and their properties. Using vague soft product, we define the score matrix of two vague soft sets which will be useful in decision making problems. At the end, we provide a decision making algorithm using vague soft matrices and applied it for two real life problems.

Keywords : soft sets, vague soft sets, vague soft matrices, max-min product on VS-matrices, min-max product on VS-matrices, max-avg product on VS-matrices, min-avg product on VS-matrices, score matrix.

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1. Introduction
The algebraic structure of soft set theories and its applications have been studied in recent years which is a general mathematical tool for dealing with uncertainty which was initiated by D. Molodtsov [9]. Soft matrix theory was introduced by N.Cagman et al.[2]. They constructed a soft decision making model on the soft set theory. Later on, Maji et al.[7] have studied the theory of fuzzy soft sets by embedding the ideas of fuzzy sets. Y. Yong et al.[13] interpolated a matrix representation of fuzzy soft sets. After that, N.Cagman et al.[3] extended the study of the fuzzy soft matrices and several algebraic operations which are more functional to make theoretical studies in the fuzzy soft set theory. In later years, Intuitionistic fuzzy soft sets were initiated as an extension of soft sets by Maji et al.[8]. In 2012, Chetia et al.[4] proposed the concepts of intuitionistic fuzzy soft matrices which are a representation of intuitionistic fuzzy soft sets. By knowing the definition of such matrices, many authors like [6, 10, 11] extend their field of research to fuzzy soft matrices, intuitionistic fuzzy soft matrices by introducing a suitable method to solve problems in daily life. And the methods used in the problems are more practical and can be successfully applied to many problems that contain uncertainties. In 1993, W.L. Gua el al.[5] proposed the idea of a vague set which has a grade of membership value in the continuous subinterval of \([0,1]\). As a parameterization of the vague set, Xu et al.[12] introduced the vague soft set theory by combining the notions of the vague sets and the soft sets.

In this paper, we introduce the matrix form of vague soft sets and define some basic operations
on VS-matrices. At the end, we propose a new algorithm which helps to take some decisions in real life problems.

2. Preliminaries

**Definition 2.1.** [9] Let U be an initial universe, E be the set of all parameters, P(U) be the power set of U. A pair (F,E) is called a soft set over U, where F is a mapping given by $F : E \rightarrow P(U)$. In other words, the soft set is a parameterized family of subsets of the set U. Every set $F(e), e \in E$ from this family is called a e-approximate element of the soft set (F,E).

**Example 2.2.** Let $U=\{h_1, h_2, h_3\}$ be the set of three houses and $E=\{e_1(\text{beautiful}), e_2(\text{cheap}), e_3(\text{wooden})\}$ be set of parameters. Suppose that $F(e_1)=\{h_1, h_3\}$, $F(e_2)=\{h_2, h_3\}$ and $F(e_3)=\{h_3\}$. Then the soft set $(F,E)$ is given by $(F,E)=\{(\text{beautiful},\{h_1, h_3\}), (\text{cheap},\{h_2, h_3\}), (\text{wooden},\{h_3\})\}$.

**Definition 2.3.** [5] A vague set A in the universe $X=\{x_1, x_2, \ldots, x_n\}$ can be expressed by the following notion: $A=\{(x_i, [a_{i1}, 1-a_{i1}]) \mid x_i \in X\}$; that is, $A(x_i)=[a_{i1}, 1-a_{i1}]$, and the condition $0 \leq a_{i1} + (1-a_{i1}) \leq 1$ should hold for any $x_i \in X$, where $a_{i1}$ and $(1-a_{i1})$ are the membership degree and non-membership degree of the element $x_i$ to the vague set A.

**Definition 2.4.** [12] Let X be an initial universe set, V(X) the set of all vague sets on X, E a set of parameters, and $A \subseteq E$. A pair $(F,A)$ is called a vague soft set over X, where F is a mapping given by $F : A \rightarrow V(X)$. The set of all vague soft sets over X is denoted by $V\tilde{S}(X,E)$.

**Example 2.5.** Suppose that $X=\{h_1, h_2, h_3\}$ is a set of three houses and $E=\{e_1(\text{expansive}), e_2(\text{beautiful}), e_3(\text{wooden}), e_4(\text{cheap})\}$ is a parameter set and $A=\{e_1, e_2, e_3\} \subseteq E$. The vague soft set $(F,A)$ describes the “attractiveness of the house” under the consideration of a decision maker to purchase which is given below $(F,A)=\{<e_1, [0.1, 0.6]_{h_1}, [0.2, 0.4]_{h_2}, [0.3, 0.7]_{h_3}>, <e_2, [0.4, 0.9]_{h_1}, [0.5, 0.6]_{h_2}, [0.2, 0.5]_{h_3}> , <e_3, [0.3, 0.4]_{h_1}, [0.2, 0.5]_{h_2}, [0.5, 0.9]_{h_3}> \}$.

3. Operations on Vague Soft Matrices

In this section we represent vague soft sets in terms of matrix called vague soft matrix. We define operations like union, intersection, complement and product of VS- matrices. We also define the score matrix of two V$\tilde{S}$- matrices using product of V$\tilde{S}$- matrices.

**Definition 3.1.** Let $X=\{x_1, x_2, x_3, \ldots, x_m\}$ be an initial universal set and E be the set of parameters given by $E=\{e_1, e_2, e_3, \ldots, e_n\}$ and $A \subseteq E$. Then the VS set $(F,A)$ can be written in matrix form as $[a_{ij}]_{m \times n} = F_A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$, where, $a_{ij} = \begin{cases} [I_{F(e_j)}(x_i), 1-I_{F(e_j)}(x_i)] & \text{if } e_j \in A, \\ [0, 0] & \text{if } e_j \notin A. \end{cases}$

for all $0 \leq i \leq m$, $0 \leq j \leq n$ and $I_{F(e_j)}(x_i), I_{F(e_j)}(x_i)$ are the truth degree matrix and false membership degree matrix of the element $x_i$, associated with the parameter $e_j$ in the vague set $F(e_j)$ such that $0 \leq I_{F(e_j)}(x_i) + I_{F(e_j)}(x_i) \leq 1$. According to the definition, a VS set $(F,A)$ is uniquely characterized by the VS- matrix $[a_{ij}]$. It means that a V$\tilde{S}$ set $(F,A)$ is formally equal to its VS$\tilde{S}$- matrix $[a_{ij}]_{m \times n}$. Therefore, we shall identify any VS set with its V$\tilde{S}$- matrix and these two concepts are interchangeable. The set of all V$\tilde{S}$- matrices of order $m \times n$ is denoted by $V\tilde{S}M_{m \times n}$.

**Example 3.2.** Consider the vague soft set $(F,A)$ as in Example 2.5. Then its V$\tilde{S}$- matrix is given by, $[a_{ij}]_{3 \times 4} = \begin{bmatrix} h_1 [0.1, 0.6] & [0.3, 0.9] & [0.3] & [0.0] \\ h_2 [0.2, 0.4] & [0.5, 0.6] & [0.2, 0.5] & [0.0] \\ h_3 [0.3, 0.7] & [0.2, 0.5] & [0.5, 0.9] & [0.0] \end{bmatrix}$

**Definition 3.3.** Let $[a_{ij}] \in V\tilde{S}M_{m \times n}$, $[b_{jk}] \in V\tilde{S}M_{n \times k}$ be two V$\tilde{S}$- matrices. Then
(i) the union of \([a_{ij}] \text{ and } [b_{ij}]\) is defined by \([a_{ij}] \cup [b_{ij}] = \max\{a_{ij}, b_{ij}\} = \max\{t_{F(e_j)}(x_i), t_{G(e_j)}(x_i)\},\]
\[\max\{1 - f_{F(e_j)}(x_i), 1 - f_{G(e_j)}(x_i)\}\] for all \(i, j\).

(ii) the intersection of \([a_{ij}] \text{ and } [b_{ij}]\) is defined by \([a_{ij}] \cap [b_{ij}] = \min\{a_{ij}, b_{ij}\} = \min\{t_{F(e_j)}(x_i), t_{G(e_j)}(x_i)\},\]
\[\min\{1 - f_{F(e_j)}(x_i), 1 - f_{G(e_j)}(x_i)\}\] for all \(i, j\).

(iii) the complement of \([a_{ij}]\) is defined by \([a_{ij}]^c = [f_{F(e_j)}(x_i), 1 - t_{F(e_j)}(x_i)]\) for all \(i, j\).

Example 3.4. Let us consider the two \(2 \times 3\) VŠ-matrices \([a_{ij}] \in \text{VŠM}_{2 \times 3}, [b_{jk}] \in \text{VŠM}_{2 \times 3}\) as follows
\[
[a_{ij}] = \begin{bmatrix}
0.4 & 0.5 \\
0.1 & 0.2
\end{bmatrix},
[b_{jk}] = \begin{bmatrix}
0.2 & 1.1 \\
0.3 & 0.9
\end{bmatrix}.
\]

Then \([a_{ij}] \cup [b_{ij}] = \begin{bmatrix}
0.4 & 0.8 \\
0.1 & 0.3
\end{bmatrix}, [a_{ij}] \cap [b_{ij}] = \begin{bmatrix}
0.2 & 0.5 \\
0.2 & 0.6
\end{bmatrix}\] and \([a_{ij}]^c = \begin{bmatrix}
0.5 & 0.6 \\
0.8 & 0.9
\end{bmatrix}\).

Definition 3.5. Let \([a_{ij}] \in \text{VŠM}_{m \times n}, [b_{jk}] \in \text{VŠM}_{n \times k}\) be two VŠ-matrices. Then the max-min product of \([a_{ij}]\) and \([b_{jk}]\) is defined as follows
\[\oplus: \text{VŠM}_{m \times n} \times \text{VŠM}_{n \times k} \rightarrow \text{VŠM}_{m \times k}, [c_{ik}] = [a_{ij}] \oplus [b_{jk}] \text{ where, } c_{ik} = \max\{\min\{a_{ij}, b_{jk}\}\}, \text{ for all } i, k,\]

Definition 3.6. Let \([a_{ij}] \in \text{VŠM}_{m \times n}, [b_{jk}] \in \text{VŠM}_{n \times k}\) be two VŠ-matrices. Then the min-max product of \([a_{ij}]\) and \([b_{jk}]\) is defined as follows
\[\odot: \text{VŠM}_{m \times n} \times \text{VŠM}_{n \times k} \rightarrow \text{VŠM}_{m \times k}, [c_{ik}] = [a_{ij}] \odot [b_{jk}] \text{ where, } c_{ik} = \min\{\max\{a_{ij}, b_{jk}\}\}, \text{ for all } i, k,\]

Definition 3.7. Let \([a_{ij}] \in \text{VŠM}_{m \times n}, [b_{jk}] \in \text{VŠM}_{n \times k}\) be two VŠ-matrices. Then the min-avg product of \([a_{ij}]\) and \([b_{jk}]\) is defined as follows
\[\ominus: \text{VŠM}_{m \times n} \times \text{VŠM}_{n \times k} \rightarrow \text{VŠM}_{m \times k}, [c_{ik}] = [a_{ij}] \ominus [b_{jk}] \text{ where, } c_{ik} = \max\{\frac{a_{ij} + b_{jk}}{2}\}, \text{ for all } i, k,\]

Definition 3.8. Let \([a_{ij}] \in \text{VŠM}_{m \times n}, [b_{jk}] \in \text{VŠM}_{n \times k}\) be two VŠ-matrices. Then the min-avg product of \([a_{ij}]\) and \([b_{jk}]\) is defined as follows
\[\otimes: \text{VŠM}_{m \times n} \times \text{VŠM}_{n \times k} \rightarrow \text{VŠM}_{m \times k}, [c_{ik}] = [a_{ij}] \otimes [b_{jk}] \text{ where, } c_{ik} = \min\{\frac{a_{ij} + b_{jk}}{2}\}, \text{ for all } i, k,\]

Example 3.9. Let \(A = \begin{bmatrix}
0.2 & 0.4 \\
0.5 & 0.6
\end{bmatrix}\) and \(B = \begin{bmatrix}
0.1 & 0.5 \\
0.2 & 0.4
\end{bmatrix}\) be two VŠ-matrices in \(\text{VŠM}_{2 \times 3}\), then \(A \oplus B = \begin{bmatrix}
0.2 & 0.4 \\
0.5 & 0.7
\end{bmatrix}, A \odot B = \begin{bmatrix}
0.2 & 0.5 \\
0.2 & 0.4
\end{bmatrix}, A \ominus B = \begin{bmatrix}
0.35 & 0.45 \\
0.35 & 0.45
\end{bmatrix}\) and \(A \otimes B = \begin{bmatrix}
0.15 & 0.25 \\
0.15 & 0.25
\end{bmatrix}\).

Remark 3.10. For any VS-matrices \(A \in \text{VŠM}_{m \times m}, B \in \text{VŠM}_{m \times m}\), we have \(A \oplus B \neq B \oplus A, A \odot B \neq B \odot A, A \ominus B \neq B \ominus A\) and \(A \otimes B \neq B \otimes A\).

Proposition 3.11. Let \(A \in \text{VŠM}_{m \times n}, B \in \text{VŠM}_{n \times k}\) be two VS-matrices. If \(A^c, B^c\) are the complements of these vague soft matrices, then

(i) \(A^c \oplus B^c = (A \odot B)^c\) and \(A^c \odot B^c = (A \oplus B)^c\).

(ii) \(A^c \ominus B^c = (A \otimes B)^c\) and \(A^c \otimes B^c = (A \ominus B)^c\).

Proof: It is follows from the Definitions 3.3 (iii), 3.5, 3.6, 3.7 and 3.8.
Example 3.12. Consider the VS-matrices as in Example 3.9, then

\[
A^c \oplus B^c = \begin{bmatrix}
0.5 & 0.8 & 0.9 & 0.9 & 0.8 & 1 \\
0.6 & 0.8 & 0.5 & 0.9 & 0.6 & 0.8 \\
0.6 & 0.8 & 0.4 & 0.9 & 0.6 & 0.7
\end{bmatrix},
\]

\[
A^c \otimes B^c = \begin{bmatrix}
0.6 & 0.8 & 0.4 & 0.9 & 0.6 & 0.7 \\
0.5 & 0.8 & 0.6 & 0.9 & 0.5 & 0.9 \\
0.5 & 0.6 & 0.8 & 0.9 & 0.5 & 0.9
\end{bmatrix},
\]

\[
A^c \odot B^c = \begin{bmatrix}
0.75 & 0.85 & 0.95 & 0.95 & 0.9 & 1 \\
0.6 & 0.85 & 0.7 & 0.9 & 0.65 & 0.9 \\
0.7 & 0.9 & 0.85 & 0.95 & 0.7 & 0.95
\end{bmatrix}
\text{ and } A^c \odot B^c = \begin{bmatrix}
0.55 & 0.65 & 0.35 & 0.65 & 0.35 & 0.6 \\
0.5 & 0.7 & 0.5 & 0.75 & 0.45 & 0.7 \\
0.25 & 0.45 & 0.55 & 0.65 & 0.35 & 0.6
\end{bmatrix}.
\]

Now compare these matrices with the matrices in Example 3.9 we get, \(A^c \oplus B^c = (A \oplus B)^c\), \(A^c \odot B^c = (A \odot B)^c\), \(A^c \odot B^c = (A \odot B)^c\) and \(A^c \odot B^c = (A \odot B)^c\).

Definition 3.13. Let \(A \in V \tilde{S}M_{m \times n}\), \(B \in V \tilde{S}M_{n \times k}\) be two VS-matrices. If \(A^c, B^c\) are the complements of these vague soft matrices then the score matrix of A and B is defined as \(S(A,B) = \frac{A^c \oplus B^c}{A^c \odot B^c}\), where \(V = [t_{ABB} + (1 - f_{A^cB^c})]\) and \(W = [t_{A^cB^c} + (1 - f_{ABB})]\), here \(\emptyset\) denotes any one of the above four products of the VS-matrices.

4. Decision making algorithm using vague soft matrices

Let \(X = \{x_1, x_2, \ldots, x_n\}\) be an initial universal set and consider the parameter sets E and \(E'\).

Algorithm:

Step 1: Input the \(V \tilde{S}\) sets (F,E) and \((E, E')\) for the given set of parameters.

Step 2: Construct the VS-matrices A and B corresponding to (F, E) and \((E, E')\), \(A^c\) and \(B^c\) corresponding to (F, E)\(^c\) and \((E, E')\)^c\) respectively.

Step 3: Find the convenient composite product of the VS matrices among \(\oplus, \odot, \ominus\) and \(\otimes\), say \(\oplus\). And Compute \(A \oplus B\) and \(A^c \oplus B^c\).

Step 4: Find the score matrix \(S(A, B)\) using the Definition 3.13.

Step 5: According to our choice \(\oplus\) (resp. \(\odot, \ominus\) and \(\otimes\)) find the maximum (resp. maximum, minimum, minimum) score for each object of the initial universe in the score matrix. The corresponding parameters in the score matrix of these values are the Optimum choice for the decision makers.

Example 4.1. Let us consider the situation that the company conducts an interview for the following posts “Business Correspondent”, “Marketing officer”, “Manager” and “Programmer”. The company is going to select the suitable post for the candidates based on their skills. The following model helps to identify the selection.

Consider the set of four candidates \(X = \{x_1, x_2, x_3, x_4\}\) which may be characterized by the set of four parameters \(E = \{e_1, e_2, e_3, e_4\}\) for \(j = 1, 2, 3, 4\) the parameter \(e_j\)'s stands for Communication Skills, Marketing, Expert in coding, Team management respectively. We give a suitable decision using the product of VS-matrices. First, we construct the VS- set (F,E) which represents the skills of the candidates. Assume that \(E' = \{v_1, v_2, v_3, v_4\}\) for \(k = 1, 2, 3, 4\) the parameter \(v_k\)'s stands for the post Business Correspondent, Marketing officer, Manager, Programmer respectively. And we construct another VS- set \((E, E')\). It focuses the skills for the suitable posts.

Step 1: First, we Input the following two VS- sets

\[
(F, E) = \begin{bmatrix}
< e_1, & [0.5, 0.6, 0.2, 0.3, 0.3, 0.3, 0.3, 0.3] >, & < e_2, & [0.0, 0.1, 0.5, 0.6, 0.1, 0.5, 0.5, 0.9] > \\
< e_3, & [0.2, 0.3, 0.2, 0.5, 0.8, 0.9, 0.6, 0.7] >, & < e_4, & [0.3, 0.4, 0.2, 0.3, 0.1, 0.5, 0.7, 0.8] >
\end{bmatrix},
\]

\[
(E, E') = \begin{bmatrix}
< v_1, & [0.6, 0.7, 0.0, 0.2, 0.1, 0.2, 0.3, 0.5] >, & < v_2, & [0.2, 0.3, 0.7, 0.8, 0.1, 0.5, 0.0, 1] > \\
< v_3, & [0.1, 0.2, 0.3, 0.3, 0.0, 0.2, 0.8, 0.9] >, & < v_4, & [0.5, 0.6, 0.3, 0.5, 0.7, 0.8, 0.1, 0.2] >
\end{bmatrix}
\]

Step 2: And we construct the vague soft matrices A and B corresponding to (F,E) and \((E, E')\),
In order to minimize the cost of the products, they can select the best delivering company with minimum shipping charge for the products which vary from company to company like Shopclues, Flipkart, Amazon, Snapdeal. The shipping charge for the products which vary from company to company like Shopclues, Flipkart, Amazon, Snapdeal, vary from company to company like Shopclues, Flipkart, Amazon, Snapdeal.

\[ A = \begin{bmatrix} 0.5, 0.6 & 0.2, 0.3 & 0.3, 0.4 \\ 0.7, 0.8 & 0.2, 0.5 & 0.3, 0.7 \\ 0.3, 0.5 & 0.2, 0.6 & 0.3, 0.1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0.6, 0.7 & 0.2, 0.3 & 0.1, 0.2 & 0.5, 0.6 \\ 0.7, 0.8 & 0.3, 0.3 & 0.3, 0.5 \\ 0.1, 0.2 & 0.1, 0.5 & 0.2, 0.7 & 0.8, 0.9 \\ 0.3, 0.5 & 0.1, 0.1 & 0.8, 0.9 & 0.1, 0.2 \end{bmatrix}. \]

Then its vague soft complements are given by,

\[ A^c = \begin{bmatrix} 0.4, 0.5 & 0.9, 1 & 0.7, 0.8 & 0.6, 0.7 \\ 0.7, 0.8 & 0.4, 0.5 & 0.5, 0.8 & 0.7, 0.8 \\ 0.5, 0.7 & 0.5, 0.9 & 0.1, 0.2 & 0.5, 0.9 \\ 0.7, 0.7 & 0.1, 0.5 & 0.3, 0.4 & 0.2, 0.3 \end{bmatrix} \quad \text{and} \quad B^c = \begin{bmatrix} 0.3, 0.4 & 0.7, 0.8 & 0.8, 0.9 & 0.4, 0.5 \\ 0.8, 1 & 0.2, 0.3 & 0.7, 0.7 & 0.5, 0.7 \\ 0.8, 0.9 & 0.5, 0.9 & 0.8, 1 & 0.2, 0.3 \\ 0.5, 0.7 & 0.9, 1 & 0.1, 0.2 & 0.8, 0.9 \end{bmatrix}. \]

Step 3: Now we compute \( A \) the vague soft matrices \( A \oplus B \) and \( A^c \oplus B^c \) using max-avg product as follows,

\[ A \oplus B = \begin{bmatrix} 0.55, 0.65 & 0.35, 0.45 & 0.55, 0.65 & 0.5, 0.6 \\ 0.45, 0.65 & 0.45, 0.7 & 0.45, 0.7 & 0.75, 0.85 \\ 0.5, 0.65 & 0.6, 0.85 & 0.75, 0.85 & 0.65, 0.75 \end{bmatrix} \quad \text{and} \quad A^c \oplus B^c = \begin{bmatrix} 0.85, 1 & 0.75, 0.85 & 0.8, 0.9 & 0.7, 0.9 \\ 0.65, 0.85 & 0.8, 0.9 & 0.75, 0.85 & 0.75, 0.85 \\ 0.65, 0.95 & 0.7, 0.95 & 0.65, 0.85 & 0.65, 0.9 \\ 0.55, 0.65 & 0.7, 0.75 & 0.75, 0.8 & 0.5, 0.65 \end{bmatrix}. \]

Step 4: The score matrix of \( A \) and \( B \) is given by \( S(A, B) = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix} = \begin{bmatrix} 1.525 & 1.2 & 1.45 & 1.35 \\ 1.2 & 1.5 & 1.325 & 1.35 \\ 1.325 & 1.4 & 1.325 & 1.575 \\ 1.2 & 1.45 & 1.575 & 1.275 \end{bmatrix}. \)

Step 5: It is clear from the above matrix that the maximum values give the optimum choice. Therefore, the candidate \( x_1 \) suitable for the post “Business Correspondent”, candidate \( x_2 \) suitable for the post “Marketing officer”, candidate \( x_3 \) suitable for the post “Programmer” and candidate \( x_4 \) suitable for the post “Manager”.

**Example 4.2.** Let us consider the four different companies say X= \{c_1, c_2, c_3, c_4\} each of which manufactures the products like “Television”, “Air-conditioner”, “Refrigerator” and “Washing machine”. In order to minimize the cost of the products, they can select the best delivering company with minimum shipping charges. They hope that it increase the sales. The decision is taken by the help of the following model.

Assume that the products \( E= \{p_1, p_2, p_3, p_4\} \) for \( j=1,2,3,4 \) the parameter \( p_j \)'s stands for Television, Air-conditioner, Refrigerator and Washing machine respectively. Suppose that \( E' = \{d_1, d_2, d_3, d_4\} \) denotes companies like Shopches, Flipkart, Amazon, Snapdeal. The shipping charge for the products which vary according to the size of the products, travel distance, cost, delivery time. Now we consider the VS-set (F,E) denotes the production cost of products and the VS-set (E, E’) denotes the shipment cost of the products. Step 1: We input the two VS-sets by

\[
(F,E) = \begin{cases} < p_1, \begin{bmatrix} 0.3, 0.5 \\ 0.7, 0.8 \end{bmatrix}, \begin{bmatrix} 0.4, 0.8 \\ 0.3, 0.5 \end{bmatrix}, \begin{bmatrix} 0.1, 0.5 \\ 0.5, 0.8 \end{bmatrix}, \begin{bmatrix} 0.5, 0.6 \\ 0.3, 0.5 \end{bmatrix}, \begin{bmatrix} 0.4, 0.6 \\ 0.7, 0.8 \end{bmatrix}, \begin{bmatrix} 0.4, 0.7 \\ 0.6, 0.7 \end{bmatrix}, \begin{bmatrix} 0.5, 0.6 \\ 0.7, 0.8 \end{bmatrix}, \begin{bmatrix} 0.6, 0.7 \\ 0.8, 0.9 \end{bmatrix}, \begin{bmatrix} 0.7, 0.8 \\ 0.8, 0.9 \end{bmatrix}, \begin{bmatrix} 0.8, 0.9 \\ 0.9, 1 \end{bmatrix}, \begin{bmatrix} 0.9, 1 \\ 1, 0 \end{bmatrix}, \begin{bmatrix} 1, 0 \end{bmatrix}, \begin{bmatrix} 1, 0 \end{bmatrix}> \end{cases}
\]

\[
(E,E') = \begin{cases} < d_1, \begin{bmatrix} 0.3, 0.5 \\ 0.7, 0.8 \end{bmatrix}, \begin{bmatrix} 0.4, 0.8 \\ 0.3, 0.5 \end{bmatrix}, \begin{bmatrix} 0.1, 0.2 \\ 0.2, 0.4 \end{bmatrix}, \begin{bmatrix} 0.2, 0.4 \\ 0.3, 0.5 \end{bmatrix}, \begin{bmatrix} 0.3, 0.5 \\ 0.4, 0.5 \end{bmatrix}, \begin{bmatrix} 0.4, 0.5 \\ 0.5, 0.6 \end{bmatrix}, \begin{bmatrix} 0.5, 0.6 \\ 0.6, 0.7 \end{bmatrix}, \begin{bmatrix} 0.6, 0.7 \\ 0.7, 0.8 \end{bmatrix}, \begin{bmatrix} 0.7, 0.8 \\ 0.8, 0.9 \end{bmatrix}, \begin{bmatrix} 0.8, 0.9 \\ 0.9, 1 \end{bmatrix}, \begin{bmatrix} 0.9, 1 \\ 1, 0 \end{bmatrix}, \begin{bmatrix} 1, 0 \end{bmatrix}, \begin{bmatrix} 1, 0 \end{bmatrix}> \end{cases}
\]

Step 2: The vague soft matrices A and B corresponding to (F,E) and (E,E’) are constructed as
In this paper, we have defined max-min product, min-max product, max-avg product and min-avg product.

Step 5: Finally, the minimum score value of \( S(A, B) \) gives the optimum choice, which means that the company selects Flipkart, the company selects Amazon, the company selects Snapdeal and the company selects Amazon respectively for their needs.

5. Conclusion
In this paper, we have defined max-min product, min-max product, max-avg product and min-avg product operators of vague soft matrices. A decision making algorithm is constructed using these operators and discussed with two real life problems. It can be successfully applied to many other problems that has uncertainties.

References
[1] Bora M, Bora B, Neog T J and Kumar D 2013 Intuitionistic fuzzy Soft matrix theory and its application in medical diagnosis Ann. Fuzzy Math. Inform. pp 1-11
[2] Cagman N and Enginoğlu S 2010 Soft matrix theory and its decision making Comput. Math. Appl. 59 pp 3308-3314
[3] Cagman N and Enginoğlu S 2012 Fuzzy Soft matrix theory and its application in decision making Iran. J. Fuzzy Syst. 9(1) pp 109-119
[4] Chetia B and Das P K 2012 Some results of intuitionistic fuzzy matrix theory Adv. Appl. Sci. Res. 3(1) pp 421-423
[5] Gau W L and Buehrer D J 1993 Vague sets IEEE Trans. Systems Man and Cybernet. 23(2) pp 610-614
[6] Inthumathi V, Chitra V and Jayasee S 2017 Fuzzy soft min-max Decision Making and its Applications Journal of Informatics and Mathematical Sciences 9(3) pp 827-834
[7] Maji P K, Biswas R and Roy A R 2001 Fuzzy Soft sets J. Fuzzy Math. 9(3) pp 589-602
[8] Maji P K, Roy A R and Biswas R 2004 On intuitionistic Fuzzy Soft sets J. Fuzzy Math. 12(3) pp 669-683
[9] Molodtsov D 1999 Soft set theory - first results Comput. Math. Appl. 37 (4-5) pp 19-31
[10] Rajarajewari P and Dhanalakshmi P 2013 Intuitionistic Fuzzy Soft Matrix Theory and its application in decision making International Journal of Engineering Research and Technology 2(4) pp 1100-1111
[11] Shanmugasundaram P, Seshaiyah C V and Rathi K 2014 Intuitionistic Fuzzy Soft Matrix Theory in Medical Diagnosis Using Max-Min Average Composition Method Journal of Theoretical and Applied Information Technology 67(1) pp 180-190
[12] Xu W, Ma J, Wang S and Hao G 2010 Vague soft set and their properties Comput. Math. Appl. 59(2) pp 787-794

[13] Yang Y and Ji C 1965 Fuzzy Soft Matrices and their Applications Artificial Intelligence and Computational Intelligence AICI 2011 pp 618-627