RELAXING NEAR THE CRITICAL POINT*

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I present an analysis of the relaxation rate for long-wavelength fluctuations of the order parameter in an \(O(N)\) scalar theory near the critical point. Our motivation is to model the non-equilibrium dynamics of critical fluctuations near the chiral phase transition in QCD. In the next-to-leading order in the large \(N\) expansion we find a critical slowing down regime, i.e., an increasing of the relaxation time of long wavelengths fluctuations. This result suggests, for near critical systems, relevant deviations from thermal equilibrium for the distribution functions of low-energy particles and could have important phenomenological consequences in Heavy Ions Collision and in the Early Universe Cosmology.

For QCD with only two flavors of massless quarks it has been argued that the chiral phase transition at finite temperature is of second order and described by the universality class of \(O(4)\) Heisenberg ferromagnets. Second order critical points are characterized by strong critical long-wavelength fluctuations and a diverging correlation length that could lead to important experimental signatures. These signatures would be akin to critical opalescence near the critical point in binary fluids and could be observed in an event-by-event analysis of the fluctuations of the charged particle transverse momentum distribution (mainly pions).

Critical slowing down of long-wavelength fluctuations near a second order critical point is the statement that the long-wavelength Fourier components of the order parameter relax very slowly towards equilibrium. In mean-field theory in classical critical phenomena, the relaxation time for homogeneous fluctuations diverges as \(\tau \propto \xi^z\) with \(\xi\) the correlation length, or, at critical point, as \(\tau(k) \propto k^z\) with \(z\) a dynamical critical exponent.

The phenomenological importance of critical slowing down for the QCD phase transition both in Relativistic Heavy Ion Collisions as well as in Early Universe Cosmology motivates us to study this phenomenon in a model Quantum Field Theory that bears on the low energy (chiral) phenomenology of QCD, the \(O(N)\) linear sigma model.

We point out that the analysis requires non-perturbative techniques since near the critical points perturbation theory in the quartic coupling \(\lambda\) breaks down. This

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can be directly understood from a two-loop computation of the relaxation rate. We compute both the critical damping rate $\Gamma(k, T_c)$ for inhomogenous fluctuations of wavevector $k$ and the near critical damping rate $\Gamma_0(m_T, T)$ for homogenous fluctuations, where $m_T \propto |T - T_c|^{1/2} \ll T_c$ is the effective thermal mass.

We find respectively $\Gamma(k, T_c) \propto \lambda^2 T_c^2 / k$ and $\Gamma_0(m_T, T) \propto \lambda^2 T^2 / m_T$. These results hold in the semisoft region $T \gg k \gg \lambda T$ and clearly reveal the breakdown of the perturbative expansion in the long wavelength limit $k \to 0$ at $T = T_c$ and for $T \to T_c$ and $k = 0$. In order to face this problem analytically, we implement a non-perturbative resummation of bubble-type diagrams via the large $N$ approximation and compute the damping rate in the next-to-leading order in the large $N$ limit (alternatively, the use of the nonperturbative thermal renormalization group approach has been advocated by various authors). This resummation is akin to that obtained via the renormalization group with the one loop beta function and reveals the softening of the scattering amplitude and the crossover to an effective three dimensional theory for momenta $q \ll \lambda T$.

A detailed analysis of the different contributions to the two-loops relaxation rate shows that the rate is dominated by very soft loop momentum $q \ll \lambda T$ which in the weak coupling limit $\lambda \ll 1$ are classical. The implementation of the non-perturbative large $N$ resummation effectively entails a screening of the contribution from these momenta. Consequently the most important contribution to the relaxation rate arises both from the semisoft classical region of loop momentum $T \gg q \gg \lambda T$ and also from the hard region $q \geq T$. A detailed analysis of the contribution from the loop momenta reveals a non-perturbative ultrasoft scale

$$k_{us} \simeq \frac{\lambda T}{4\pi} e^{-\frac{4}{\pi} \lambda^2},$$

which is exponentially small in the small coupling regime.

We find that for soft momenta $\lambda T \gg k \gg k_{us}$ the damping rate is dominated by classical semisoft loop momenta and given by

$$\Gamma(k, T_c) \simeq \frac{\lambda T}{2\pi N} \left[ 1 + O\left( \frac{1}{\ln \frac{k}{k_{us}}} \right) \right],$$

i.e. it is essentially $k$–independent and depends linearly on the coupling constant. For $k \ll k_{us}$ the classical approximation breaks down and the damping rate is given by

$$\Gamma(k, T_c) \simeq \frac{4\pi T_c}{3N \ln \frac{k}{k_{us}}} \left[ 1 + O\left( \frac{1}{\ln \frac{k}{k_{us}}} \right) \right].$$

For homogeneous fluctuations near the critical point ($k = 0$, $m_T \neq 0$) the damping rate is nearly constant for $m_T \gg k_{us}$

$$\Gamma_0(m_T, T) \simeq \frac{\lambda T}{2\pi N} \left[ 1 + O\left( \frac{1}{\ln \frac{k_{us}}{m_T}} \right) \right].$$
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whereas in the ultrasoft region is logarithmically vanishing

\[ \Gamma_0(m_T, T) \sim k_{us} \leq m_T \ll \frac{4\pi T}{3N\ln \frac{T}{m_T}} \left[ 1 + O\left( \frac{1}{\ln \frac{T}{m_T}} \right) \right]. \]  

(4)

Thus critical slowing down, i.e., the vanishing of the quasiparticle width \( \Gamma \) for long-wavelengths emerges in the ultrasoft limit \( k \ll k_{us} \) or very near the critical point \( m_T \ll k_{us} \). We notice that in such regimes the rate is independent of the coupling \( \lambda \).

The large \( N \) approximation is not limited to weak coupling and our results apply just as well to a strong coupling case \( \lambda > 1 \). In such a case we have \( k_{us} \sim \lambda T \gg k \), therefore the classical approximation is not valid and the relaxation rate for \( k \ll T \) or \( m_T \ll T \) is given by Eqs. (2) and (4) respectively. The weak coupling analysis instead clearly reveals that there emerges a hierarchy of widely separated scales for loop momenta: from hard \( q \geq T \) to semisoft \( T \gg q \gg \lambda T \), and soft \( \lambda T \gg q \) that lead to different contributions to the relaxation rate. Which is the relevant scale for the damping rate is determined by the wavevector of the fluctuation of the order parameter and the proximity to the critical temperature. For \( k \), \( m_T \gg k_{us} \) the classical approximation does apply and the damping rate is dominated by the soft and semisoft classical loop momenta [with the results (1) and (4)], whereas for \( k \), \( m_T \ll k_{us} \) the classical approximation breaks down and the damping rate is dominated by hard loop momenta \( q \geq T \) [with the results (2) and (3)].

Further studies are needed in order to understand the effects of the next-to-next-to-leading corrections in \( 1/N \), which can be be relevant for finite \( N \) in the critical limits \( k \to 0 \) at \( T = T_c \) and \( m_T \to 0 \) at \( k = 0 \).

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