THE CUNTZ SEMIGROUP, A RIESZ TYPE INTERPOLATION PROPERTY, COMPARISON AND THE IDEAL PROPERTY

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Abstract: We define a Riesz type interpolation property for the Cuntz semigroup of a $C^*$-algebra and prove it is satisfied by the Cuntz semigroup of every $C^*$-algebra with the ideal property. Related to this, we obtain two characterizations of the ideal property in terms of the Cuntz semigroup of the $C^*$-algebra. Some additional characterizations are proved in the special case of the stable, purely infinite $C^*$-algebras, and two of them are expressed in language of the Cuntz semigroup. We introduce a notion of comparison of positive elements for every unital $C^*$-algebra that has (normalized) quasitraces. We prove that large classes of $C^*$-algebras (including large classes of $AH$ algebras) with the ideal property have this comparison property.

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Key words: $C^*$-algebra, the Cuntz semigroup, a Riesz type interpolation property, ideal property, comparison of positive elements, $AH$ algebra.