ON SPACETIME DIFFERENTIAL ELEMENTS AND THE DISTRIBUTION OF BIO-HAMILTONIAN COMPONENTS

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Abstract. Various Hamiltonian models have been derived for chemical structures belonging to living organisms while the Hamiltonian concept was not applied to life as a whole. However, Hamiltonian components were recently defined for living organisms on the condition to take in consideration their evolutionary implications (Bounias, 2001: CASYS’01). This paper identifies differential elements of Spacetime, from which it delimits a probabilistic fuzzy-like invariance standing for conservativity of biological Hamiltonians. The distributions of potential and kinetic components in an individual bio-Hamiltonian, and the distribution of such individual Hamiltonians of living organisms interacting in more complex systems are shown to behave as a non-linear generalized convolution of functions.

Key words. Biological Hamiltonian; Convolution of functions; Spacetime differential; fuzzy-invariance

PACS: 03.65.B2. Foundations, theory of measurements, miscellaneous theories.

Introduction

While living organisms do not behave independently from the properties of matter (Bounias, 1990), for long, no Hamiltonian, nor wave function nor Schrödinger equation was considered for living systems (Rosen, 1989). The concept of a Hamiltonian of a system was originally defined for physical systems in classical and quantum mechanics, them for simple chemical systems. In the recent past years, Hamiltonian treatment has been tried for components of living organisms. Structures were addressed in proteins, for solitons in Raman scattering (Xiao-Feng, 1998) and beta sheet to alpha
helix conformations (Ito, 1999), in DNA helix-coil transition (Morozov et al., 2000), in plant light-harvesting chromoprotein complexes (Tretiak et al., 2000). Functions were considered in electron transfer tunneling (Balabin et al., 1998), and energy storage for cellular motion (Nakagawa et al., 2000). All such works have been dealing with Hamiltonian treatment of structures involved in living organisms, that is concerning chemical molecules rather than the living phenomenon in its whole.

However, while Hamiltonian and wave equations are used in Physics to try to predict the evolution of a system, up to the evolution of universe, if similar parameters were to be identified for living organisms, they would contribute to predict the behavior of ecosystems in connection with the status of their embedding medium, namely Planet Earth. The main components of the Hamiltonian of life have recently been shown to include: (i) kinetic components as the manifold $WK = \{ WK_m, WK_M, WK_E \}$ of microstructural and metabolic interactions, macroscopic activity and anticipatory behavior leading to homeostatic and evolutionary adaptation; (ii) potential components $WP = \{ WP_m, WP_M, WP_E \} \cup (WP_g)$, the latter including the selection of expressed characters from DNA existing structures, and the construction of new genomic components by evolutionary processes (Bounias, 2001).

Since living organisms are interacting in more complex systems and ecosystems, where they are embedded it was necessary to examine by which kind of relations their respective Hamiltonians, which may be only partly conservative, could themselves be connected within more conservative supersystems. This study will address first the distribution of kinetic and potential components of an individual Hamiltonian throughout the time-related sequence of configurations, and then the distribution of interacting Hamiltonians inside a more complex system.

1 On differential elements of spacetime

Former works have demonstrated that our observable spacetime can be formally identified with a ordered sequence $\{ S_1 \}$ of 3-D Poincaré sections embedded in a 4-D topological space (Bonaly and Bounias, 1995). Mappings of one into the next section wear the form of a momentum and stand for infinitesimal increments of time and space (Bounias, 1997). The embedding topological 4-space is provided with a natural metrics as the set distance,
i.e. the symmetric differences between sets (Bounias and Bonaly, 1996; Bounias, 1997), which is compatible with the definition of a topology on a space. Each section is mapped to the next one by a moment of junction (MJ) which connects either the distances or the objects, i.e. their complementaries or "instans" (Bounias, 1997). In short, space is subdivided into sets intersections, standing for objects (or "instans") denoted by \(m < A, B, ...>\) and their complementaries, i.e. the set distances denoted by \(\Delta(A, B, ...).\)

**Definition 1.** The Moments of Junction are defined as follows for \(G = (m \text{ or } \Delta)\) and \(X = \{A, B, ...\}:\)

\[
\text{MJ}_G(i, j) = G_i[X] \perp f_{i,j}(X) \quad (1)
\]

where function \(f\) takes values \(0 \leq f_{i,j}(X) \leq 1\) (Bounias, 1997), depending on the indicatrix functions \(l(x)\) of each point \(x\) of a section \((S_i)\) mapped into the topologies of the next \((S_{i+1})\), or generally to any further one \((S_j)\). For any closed and open subparts \(P_i(X)\) in \((S_i)\), one has for any \(x:\)

\[
1_i(x) = \begin{cases} 
1 & \text{iff } x \in (P_i), \\
0 & \text{iff } x \notin (P_i), 
\end{cases}
\]

then:

\[
f_{i,j}(x) = \begin{cases} 
1 & \text{iff } l_i(x) = l_j(x), \\
0 & \text{iff } l_i(x) \neq l_j(x). 
\end{cases}
\]

**Theorem 1.** The Moment of Junction provides a differential element of spacetime.

Proof. Let a space increments from \((S_i)\) to \((S_{i+1})\) be as small as a difference in one point. Thus, for the mapping of \((S_i)\) into \((S_{i+1})\) the Moment of Junction \(\text{MJ}_{(i,i+1)}\) differs by a distance defined by \(d(x'_i, x_{i+1})\) where \(x'_i\) is the projection of \(x_i\) on \((S_{i+1})\). Two such points can be adjacent though nonequal, that is the distance \(d(x'_i, x_{i+1})\) can be as small as needed, while \(\text{MJ}_{(i,i+1)}\) remains the same mathematical object. Therefore, \(\text{MJ}_{(i,i+1)}\) stands for a differential element of space.

Then, as far as there exists at least one point \(x_i\) such that \(d(x'_i, x_{i+1})\), then \((S) \cap (S) \neq \emptyset\) and the Moment of Junction is positive. Hence, \(\text{MJ}_{(i,i+1)}\) represents in this case the smallest interval separating two states of the considered space. This interval exists, it is non-null, though it has no measurable duration. This denotes a differential element of time.
Gathered together, these two statements define a differential of both space and time, that is of spacetime, which completes the proof.

2 Interaction mappings of bio-Hamiltonian components

2.1 Distribution of components of individual Hamiltonians

Lemma 2.1.1. The moment of junction of the Hamiltonian of a conservative system is distributive for its components.

Proof. Let $W$ denote the kinetic component and $V$ the potential one in $H=(W+V)$. A variation $(W-dW)$ is accompanied by a correlated $(V+dV)$. In the spacetime sequence, $f_{(i)}(X_i - dX_i) \mapsto f_{(j)}(X_j - dX_i) = f_{(j)}(X_j) + f_{(j)}(dX_i)$ for objects composing the set $X$ in which $W$ and $V$ can ultimately be measured. Then:

$$MJ(W \cup V) = MJ[(W \setminus dW) \cup (V \cup dV)]$$

$$= MJ(W) \cup MJ(V) \cup (dV \setminus dW)$$

with $(dV \setminus dW) = \emptyset$ iff $dV = dW$.

(Note that the denotation $A \setminus B$ above signifies the complementary of $B$ in $A$.)

Thus:

$$MJ(W \cup V) = MJ(W) \cup MJ(V)$$

iff the system is conservative.

Lemma 2.1.2. The Hamiltonian of a individual organism is affected a boundary of invariance.

Proof. Let $H(W,V)$ be the Hamiltonian of an organism $A \in (X)$ and $\varphi$ a function such that: $MJ(W \cup V) = \varphi(MJ\{A\})$. Then, $H(W \cup V) = \varphi(H\{A\})$. Assuming that the system $A$ is measured by continuous variables, the moment $MJ$ of $\varphi(W,V)$ can be written using the joined probability density of $W$ and $V$, i.e. $f(W,V)$ (Ruegg, 1988):

$$MJ[\varphi(W,V)] = \int \int \varphi(W,V)f(W,V)dWdV$$

(3)
Assume the particular case where $\varphi(W,V) = W \cup V$. Then:

$$MJ(W \cup V) = \int \int (W \cup V)f(W,V)dWdV \quad (4)$$

The repartition function of $H = W \cup V$ is $F(h)$, for $H = \{h_1, ..., h_n\}$ is:

$$F(h) = \int \int_{W \cup V \subseteq h} f(W)f(V) dWdV \quad (5)$$

where $h$ appears as a boundary delimiting the range of invariance of $H$.

**Remarks.**

(i) The distribution function $f_{(i,j)}(A)$ is valued in $[0,1]$ and such is valued the distribution of components giving the measure of $W$ and $V$. Therefore, the invariance boundary introduces the notion of a fuzzy invariance for the Hamiltonian of a biological organism whose components are provided a apparent stability by flows of matter and energy from exchanges with the surrounding milieu.

(ii) Function $f_{(i,j)}(A)$ defines the balance of system $(A)$ between $W$ and $V$ forms:

At extrema of global values, $f_{(i,j)}(A) = 1$ denotes a absolutely motionless state ($W=0$) while $f_{(i,j)}(A) = 0$ depicts a state of absolute motion ($V=0$).

### 2.2 Distributions of Hamiltonian functions for two interacting organisms

**Definitions 2.2.1.** Denote by $X=\{A,B,Q\}$ the set of species, habitat and resources, respectively. The global ecosystem is a space of magmas $[4]$ $E=\{(X), (\Phi)\}$, where $(\phi)$ is a functional. Call $(O)$ and $(\perp)$ two kinds of mappings connecting Hamiltonians $H(x_i)$ and $H(x_j)$ for any two members of $(X)$ and $(T^\perp)$ the family of mappings from $(\perp)$ to some $(O)$. Call $(\varphi)$ the specific kind of relationship which maps two components $H(x_i)$ and $H(x_j)$ contained in $H\{(x_i), (x_j)\}$. Let $H[(x_i) \cup (x_j)]\mapsto \varphi[H(x_i), H(x_j)]$ be a function (approximated as $H(x_i) \cup H(x_j)$ in section 3.1). Note that $dH(x_i) \neq 0$, $dH(x_j) \neq 0$ during interaction, with $dH(x_i, x_j \approx 0$ for $\varphi(x_i, x_j) \subseteq \{x_i, x_j\} \subseteq (X) \subseteq (E) \subseteq (etc.)$.

Repartition functions still are denoted by $F$ and distribution functions by $f$. 
Theorem 2.2.2. Hamiltonians of individual components of an invariant pair in a system with higher order of complexity are mapped by non-linear convolution-like functions.

Proof. For continued variables, let $H(z) = \varphi(H(x_i), H(x_j))$. Then:

$$F(H(z) = \int_{\varphi(H(x_i), H(x_j)) \subseteq H\{x_i, x_j\}} f(H(x_i), H(x_j)) \, dH(x_i) \cdot dH(x_j) \quad (6)$$

where $H\{x_i, x_j\}$ stands for the former fuzzy invariant boundary $h$ of relation (4).

For discrete variables one would have the following distribution of probabilities:

$$P(\varphi(H(x_i), H(x_j))) = \bigcup_{k \in (X)} P\{(\varphi(H(x_i) = k) \cap (\varphi(H(x_j)) = c_{\varphi(H(x_j))}) \quad (7)$$

where $c_A(B)$ denotes the complementary of $B$ in $A$, also denoted by $A\setminus B$.

Reducing relations (6) and (7) to the particular case where one would have: $\varphi(H(x_i), H(z) = (H(x_i) + (x_j))$ would give for a discrete variable:

$$P(H(x_i) + H(x_j)) = \sum_{k=0}^{Hz} \{P(Hx_i = k) \cap (Hx_j = Hz - k) \quad (8)$$

and for a continuous variable the repartition function:

$$F(Hz_i) = \int_{-\infty}^{+\infty} f(Hx_i) \cdot F(Hz - Hx_i) \, dHx_i \quad (9)$$

that is also the distribution, with commutativity between $Hx_i$ and $Hx_j$:

$$f(Hz_i) = \int_{-\infty}^{+\infty} f(Hx_i) \cdot f(Hz - Hx_i) \, dHx_i \quad (10)$$

which denotes the convolution $f(Hx_i) \ast f(Hx_j)$.

This allows an extension of the general case of the functional $(\Phi)$. In effect: let $i$ and $j$ be indexed on Card($X$), $k$ be indexed on a spatial distribution within any of Poincarè sections $(S_\alpha)$ of the ordered sequence $\{S\}_\alpha$,
and $L$ be indexed on the sequence $(n \in L)$. Then, the mappings of $(\Phi)$ are involved in the following two expressions:

\[
\left( (Hx)_i \perp^L (Hx)_j \right)_{L+t} = T^L_L \left( (Hx)_i \mathcal{O}^k (Hx)_j \right)_L,
\]

(11a)

\[
\left( (Hx)_i \perp^k (Hx)_j \right)_{k+p} = T^k_k \left( (Hx)_i \mathcal{O}^k (Hx)_j \right)_k,
\]

(11b)

that is, by gathering (11a) and (11b) into one single form:

\[
\left( (Hx)_i \perp^{L\ast k} (Hx)_j \right)_{(L+t)\ast(k+p)} = T^{L\ast k}_{L\ast k} \left( (Hx)_i \mathcal{O}^{L\ast k} (Hx)_j \right)_{L\ast k},
\]

(12)

which denotes a nonlinear generalized convolution in the sense of Bolivar-Toledo et al. (1985).

\[\text{(QED)}\]

### 2.3 Boundaries of the system

Now, some preliminary consideration should be added about the area of validity of the above functionals.

**Definitions 2.3.1.** We will call "canonic functions" the conditions for the functionality of ecosystems which apply to all members as equivalence relations or in a commutative way (which includes the Abelian groups for all binary relations operating with relevant kinds of mappings). Examples are the founding conditions (Bonaly and Bounias, 2000) of continuity, complementarity and mutualism.

We will call "specific functions" those which connect interspecific relations as order relations. An example is the relation "feeding on" in predator-to-prey relations.

**Proposition 2.3.2.** The domain of the convolution of Hamiltonians [equation (13)] belongs to the set of canonical functions, and its range belongs to the complete system of canonical plus specific functions.

How specific functions are involved will be matter of further developments.

All these results provide a perspective for further exploration of relationships connecting Hamiltonian components of the Hamiltonian of a global system.
3 Discussion and Conclusion

3.1 Outside components in potential and kinetic energies

The bio-Hamiltonian has been shown to be under influence of external factors, though it represents an internal sum of energy. A potential energy \( W_P \) or is the product of a scalar \( \mu \) (characteristic of components of mass of an object) by a distance of functions \( d[\zeta(x_i), \zeta(x_j)] \) of its positions, where \( \zeta \) maps a causality factor applying on \( \mu \). It is noteworthy that \( E_{\text{Pot}} \) of a system involves the work that forces (i.e. causality components) acting on a system are able to perform, taking into account the parameters of position, shape, configuration, of this system. Thus, components outside the system are involved.

The kinetic energy \( w_k \) or \( E_{\text{kin}} \) is a function of some expression of the mass \( M \) of a system (\( M=\cup m_i \)) and of the square of the velocities \( (v_i)^2 \) of its components, in a Newtonian, a relativistic and related forms. Importantly, the theorem of the kinetic energy states that the variation of kinetic energy of a system during a time lapse is the sum of all works of all forces (i.e. causality parameters) acting on the system during this interval, thus including internal, external and connection or interaction forces. Since \( v_i = \frac{dx_i}{dt} \), the position of objects is again involved.

3.2 Wave function for macroscopic objects

In classical quantum mechanics, the wave function \( \psi \) is determined by the frequency \( \nu \) and by the de Broglie wavelength (\( \lambda_{\text{deBroglie}} \)) of a particle (Krasnoholovets, 2001b). So far, no physical interpretation was possible for \( \psi \) as the root of a probability of localization. However, recently the wave function of a macroscopic object has alternatively been shown to be conceivable in terms of specific deformations of space, by Krasnoholovets (2001a,b). The period and amplitude of a system composed of a peculiar form of deformation of space (standing for a particle whose mass is proportional to the deformations) periodically communicated partly to the surrounding space (giving a "inerton cloud") and then back to the particle. During this cycle, the velocity of a moving particle oscillates between an initial value and zero, and its mass components oscillate between the particle and its inertons cloud (Krasnoholovets, 1997).
This approach provides a physical meaning to the de Broglie and Compton wavelengths as well as to the frequency of the system, and the corresponding formalism has been shown to reach a classical form. Let \( \{ \pi \} \) be a set of vector parameters describing all of the mass components of the corpuscular system and \( \hat{c}_\pi \) a limit in the velocity of transmission of space deformations; then, \( \ddot{\pi} - \hat{c}_\pi^2 \nabla \pi = 0 \) (Krasnoholovets, 2002). Wave function components of one particle can thus be extended to those of an entire organism and to all massive objects. Furthermore, the theory consistently allows gravity and relativity to be deduced from submicroscopic properties (Krasnoholovets, 1997, 2000, 2001a). Therefore, a deterministic macroscopic wave function \( \psi(X,t) \) becomes conceptually accessible and it can be associated with the Hamiltonian of living organisms. In a preliminary work (Bounias, 2001) it has been pointed out that the trace of the macroscopic wave function of a ecosystem in the sequence \( \{ S_i \} \) of Poincaré sections stands for the historical of the ecosystem, a non-linear causality factor identified by Landis (1996).

3.3 Specific conservativity status of the bio-Hamiltonian

Studying the Hamiltonian of a living organism rather than just biochemical components raises a property of fuzzy-like conservativity which contrasts with the status of physical objects. However, no physical structure is strictly conservative: the ceaseless motion does not exist, and all corpuscles have limited duration of life. In a molecule, atoms have different Hamiltonians, and the Hamiltonian of the molecule itself is subjected to the nature of interactions with its environment.

In a more complex system like a ecosystem, all components of individual Hamiltonians are interacting in a dynamical steady state. It has been demonstrated (Bounias and Bonaly, 2000) that the state of such an ecosystem is determined by the properties of the orbit of each component (which includes species, habitat and resources) by the manifold of functions. All combinations of these parameters are timely non-linear and the evolution of the system is logically determined by a non-linear convolution: this supports the result obtained here from a more fundamental approach involving the moments of junction as differential elements of spacetime.

The fuzzy-invariance component appearing in biological systems represents a term with topological meaning. In effect, the convolution of bio-
Hamiltonians correlates all their components in a compact space since it is finite and discrete. The Heine-Borel-Lebesgue theorem states that a finite subcover can exist from any finite subcover: the latter is necessarily finite and it involves all possible correlations, of which some actually are reflected in a finite section of spacetime. This lets a choice about which components are selected in a redundant system as Life, and therefore the presence of a fuzzy operator is justified. On the other hand, while the invariance of moments originates in empirical observations, and remains to be formally proved from a completely independent theory, conservativity has been shown to be fulfilled through a continuum of the geometry of physical objects in a 4-manifold, where only their traces in 3-D sections have a physical meaning.

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