Localization of distributed radiation sources from a printed circuit board in the near field using wavelet transform

I V Skvortsov¹, V V Bochkarev¹ and R R Latypov¹
¹ Kazan Federal University, Institute of Physics, Kazan, 420008, Russia
E-mail: skvorcov_ilya@mail.ru

Abstract.
In this paper we describe algorithms for localization of distributed radiation sources from a printed circuit board in the near field using wavelet transform. The radiation sources are presented in the form of a set of simple oscillators - Hertz dipoles. Comparison of the two methods of localization for distributed radiation sources using wavelet transform is performed using Tikhonov regularization and LASSO regression. The model examples show that the application of LASSO regression using wavelet transform gives better results.

1. Introduction
One of the problems of electromagnetic compatibility is the localization of the most active sources of radiation and the determination of their parameters. The most well-known method for determining radiation sources located on a printed circuit board is to scan the amplitudes of the component fields at each point of space [1, 2]. This method is usually used to carry out measurements in the far field to improve the measurement accuracy and reduce the requirements for the experimental setup. However, it can also be used to perform measurements in the near field [2]. Values of the field amplitudes for each component at each point of space are used as initial data for solving the ill-posed problem. The printed circuit board where the radiation sources are located is usually represented in the form of a set of Hertz dipoles, the equations for which are well-known [3-6]. Tikhonov regularization is considered to be the classical method of solving the ill-posed problem [7,8]. This method allows us to determine both point and distributed sources of radiation [8]. Lately, LASSO (least absolute shrinkage and selection operator) regression has been increasingly used [9-12]. This method also allows solving inverse problems. The main advantage of this method is that it allows obtaining low-dimensional solutions (with a small number of non-zero coefficients). This feature of the algorithm does not allow us to directly determine the distributed radiation sources while solving the inverse problem. This feature of LASSO regression is described in this paper. Recently, many problems have been solved using wavelet transform. This method is widely used because wavelet transform allows us to represent a signal in the form of a series with a small number of nonzero coefficients. In this paper, we propose to use LASSO regression with wavelet transform to solve the problem of finding the most active regions of radiation from a printed circuit board.

2. Model of equivalent dipoles
Usually, an equivalent model of a printed circuit board is a combination of electric or magnetic dipoles located in the plane of an object. To build a dipole model of a printed circuit board, the results of modelling the tangential components of the electric or magnetic fields in the measurement plane are used [7, 8]. In our model, we use only magnetic dipoles.

Let us consider an elementary dipole with a dipole moment \( \vec{p} \), which is at the point \((x_0, y_0, z_0)\) in the plane of the object. The magnetic field generated by an elementary dipole at the observation point \((x, y, z)\) and located at a distance \(r\), is determined by the expression [6]:

\[
\vec{H}(\vec{r}) = j \omega \left( \frac{1}{r} + \frac{1}{r^3} \right) \cdot \left( \hat{p} \times \vec{I}_r \right) \cdot G(\vec{r}),
\]

Here \( G(\vec{r}) = \frac{e^{-jkr}}{4\pi r} \) is the Green's function in free space, \( \vec{p} = p_x \cdot \vec{I}_x + p_y \cdot \vec{I}_y + p_z \cdot \vec{I}_z \) is the magnetic dipole moment, \( \vec{I}_r = \frac{\vec{p}}{r} \) is the unit radial vector.
If the dipole is located in the XOY plane, the expression has the form:

\[
(\mathbf{p} \times \mathbf{r}) = p_y \cdot \mathbf{i}_x \cdot \frac{z - z_0}{r} - p_x \cdot \mathbf{i}_y \cdot \frac{z - z_0}{r} + p_x \cdot \mathbf{i}_z \cdot \frac{y - y_0}{r} - p_y \cdot \mathbf{i}_z \cdot \frac{x - x_0}{r}
\]  

(2)

The expressions for the components of the magnetic field have the form:

\[
\begin{align*}
H_x &= j \omega \frac{e^{-i kr}}{4 \pi r^2} \left( |k + \frac{1}{r} | (z - z_0) \cdot p_y = A_x(r) \cdot p_y, \\
H_y &= -j \omega \frac{e^{-i kr}}{4 \pi r^2} \left( |k + \frac{1}{r} | (y - y_0) \cdot p_x = A_y(r) \cdot p_x, \\
H_z &= j \omega \frac{e^{-i kr}}{4 \pi r^2} \left( |k + \frac{1}{r} | (x - x_0) \cdot p_y = A_{zy}(r) \cdot p_y - A_{xz}(r) \cdot p_x
\end{align*}
\]  

(3-5)

Equations (3-5) can also be written briefly as

\[
\begin{bmatrix}
0 & A_x \\
A_y & 0
\end{bmatrix}
\begin{bmatrix}
p_x \\
p_y
\end{bmatrix} = \begin{bmatrix}
H_x \\
H_y
\end{bmatrix}
\]  

(6)

The components of the magnetic field and the transition matrix A are assumed to be known, but the dipole moments p are unknown. The system of equations (6) is expressed in matrix form as \( A \cdot p = H \), and its formal solution can be written as \( p = A^{-1} \cdot H \) (where \( A^{-1} \) is the inverse of A). In our case, the matrix A is normally ill-conditioned, and therefore a small change in the right-hand side in (6) will result in a large change in the solution. Thus, the inverse problem of determining the location of radiation sources by measuring amplitudes of the electromagnetic field is incorrectly posed. To solve incorrectly posed problems, regularization procedure is usually used.

3. **Ridge regression (Tikhonov regularization) and LASSO regression**

Regularization is used to find a stable solution of ill-posed problems by taking into account some a priori information. For example, using Tikhonov regularization, a solution with a minimal Euclidean norm is chosen from all possible solutions that ensure a small disparity of equations (6). To find such a solution, it is necessary to minimize the functional \( \Omega(y, \lambda) \):

\[
\Omega(p, \lambda) = \| A \cdot p - H \|^2_2 + \lambda \| p \|^2_2
\]  

(7)

where \( \lambda \) is the regularization parameter. This optimization problem has an explicit solution [13]:

\[
p_\lambda = (A^H A + \lambda^2 I)^{-1} A^H \cdot H
\]  

(8)

Among other possible methods of regularization, LASSO regression is of particular interest for this task. In this case, a solution with a minimum L1-norm is chosen from possible solutions. Such problem is equivalent to minimizing the functional

\[
\Omega(p, \lambda) = \| A \cdot p - H \|^2_1 + \lambda \| p \|_1
\]  

(9)

This regularization method allows us to obtain solutions of low dimensionality (as a rule, most of the coefficients in the solution vector are equal to zero).

There is no explicit solution to the problem of minimizing the functional (9), but there is an effective numerical method [14]. Thus, minimizing the LASSO \( \Omega(p, \lambda) \) functional and choosing the parameter \( \lambda \), we obtain vectors \( p \) for two tangential components of the electromagnetic radiation at each point.

4. **Wavelet transform**

Wavelet transform is similar to Fourier transform, in particular to a Fourier window transform. However, wavelet transform has a different evaluation function. The main difference is that Fourier transform decomposes a signal into components in the form of sines and cosines, that is, functions that
are localized in the Fourier space [15] Wavelet transform uses functions localized both in time and Fourier space. In the general case, wavelet transform can be expressed by the following equation:

$$F(a, b) = \int_{-\infty}^{+\infty} f(x)\psi^*_{(a,b)}(x) \, dx$$

(10)

where * is the symbol of complex conjugacy, f (x) is the initial data, F (a, b) is the wavelet coefficient vector, a is the stretching parameter, b is the position parameter and the function $\psi_{(a,b)}(x)$ is some function called a wavelet. A discrete wavelet transform decomposes the original data into a set of wavelets (functions) that are orthogonal to their parallel translation and scaling. The main advantage of using wavelet transform is that this method allow us to represent complex signals in the form of a set of wavelet coefficients, a few of which are nonzero.

5. Application of regularization using wavelet analysis for estimation of radiation sources

In real conditions, the sources of electromagnetic radiation on the printed circuit board are unevenly distributed throughout the entire plane of the object. Sources are usually located in certain areas. These are areas of the location of the printed tracks and various electronic components of the board, as the most active sources of radiation.

To test the algorithms under consideration, let us define a model with two distributed sources located close to each other (see Fig.1). The distributed sources are obtained by applying an inverse wavelet transformation to a predetermined vector of wavelet coefficients, few of which only are nonzero.

Fig. 1 Model with two distributed sources

The grid size is 4096 × 4096 nodes. The distance from the measurement plane to the object under study is 6 times larger than the distance between the grid nodes. The wavelength is set 19 times higher than the grid spacing. The distribution of the nodule of the magnetic field strength generated by the two sources is shown in Fig. 2.

Fig. 2 The module of the magnetic field strength from two closely spaced sources
The region with the highest radiation intensity is shown in the centre of Fig. 1. Axes x and y are oriented in the plane of measurement.

To solve the inverse problem and determine the position of the sources, we used Tikhonov regularization and LASSO regression with a wavelet transform.

The results of the operation of the two algorithms without the use of wavelet transformation and using it are shown in Fig. 3 and Fig. 4.

![Fig. 3 Distribution of power of the distributed sources using Tikhonov regularization A) without wavelet transform, B) with wavelet transform](image3)

![Fig. 4 Distribution of power of the distributed sources using LASSO regression A) without wavelet transform, B) with wavelet transform](image4)

As can be seen from Fig. 3, when we are solving an ill-posed problem using Tikhonov regularization, the results of the dipole distribution without wavelet transform and with wavelet transform are identical. However, as can be seen from Fig. 4B, when we are solving an ill-posed problem using LASSO regression, the results obtained using the wavelet transform are close to the initial distribution. The criterion for estimating the solution is the norm of the difference between the initial distribution and the reconstructed distribution divided by the norm of the initial distribution. The decision error when using wavelet transform is 66.8% for Tikhonov regularization and -1.7% for LASSO regression.

Table 1 shows solution errors obtained for Tikhonov regularization and LASSO regression using wavelet transform. The calculations were carried out at different signal-to-noise ratio (normal white noise was added to the model values of the field strength). As can be seen from Table 1, the application of LASSO regression using wavelet transform gives significantly better results.
Table 1. Solution errors for Tikhonov regularization and LASSO regression using wavelet transform for different signal-to-noise ratios

|                       | 120 dB | 106 dB | 100 dB |
|-----------------------|--------|--------|--------|
| Tikhonov regularization| 67.2%  | 77.3%  | 99.8%  |
| LASSO regression      | 1.7%   | 1.7%   | 1.7%   |

6. Conclusion
This article describes two algorithms of estimating the localization of electromagnetic radiation sources from a printed circuit board in the near field using wavelet transform. To restore the power distribution of sources from measurements of the field strength, Tikhonov regularization and LASSO regression were used. Comparison of the model examples showed that LASSO regression using wavelet transform gives a more accurate estimation of source localization than Tikhonov's regularization using wavelet transform. Estimates of source power distributions using LASSO regression with wavelet transform are more resistant to noise.

Acknowledgments
The work is performed according to the Russian Government Program of Competitive Growth of Kazan Federal University

References
[1] Gregson S McCormick J Parini C 2007 Principles of Planar Near-Field Antenna Measurements. (London: The Institution of Engineering and Technology)
[2] Weng H Beetner D G DuBroff R E Shi J 2007 IEEE Transactions on Electromagnetic Compatibility 49(3) 805–815
[3] Sarkar T K Taaghol A 1999 IEEE Transactions on Antennas and Propagation 47(3) 566–573
[4] Tong X Thomas D Nothofer A Sewell P Christopoulos C 2010 IEEE Trans. Electromagn. Compat 52(2) 462–470
[5] Nishina B Qiang C 2016 IEEE Transactions on Antennas and Propagation 64(4) 1334 - 1341
[6] Sophoeces J Orfanidis 2002 Electromagnetic Waves and Antennas (Piscataway: Rutgers University)
[7] Tong X 2010 Simplified equivalent modelling of electromagnetic emissions from printed circuit boards (Nottingham: University of Nottingham)
[8] Gorbunova A Baev A Konovalyuk M Kuznetsov Y 2015 IEEE International Symposium on Electromagnetic Compatibility (EMC) 450 – 455
[9] Do H N Choi J Lim C Maiti T 2015 IEEE Conf. of Advanced Intelligent Mechatronics 984-89
[10] Wu Y Chen Y Shi Y Lu C Song D 2016 IEEE Conf. on Digital Signal Processing 190-93
[11] Jones L K 2009 IEEE Transactions of Information theory 55(12) 5700-27
[12] Zhang B Geng J Lai L 2015 IEEE Transactions of Signal Processing 63(9) 2209-24
[13] Tikhonov A N 1963 151(4) Soviet Mathematics 1035–1038
[14] Tibshirani R 1996 Journal of the Royal Statistical Society. Series B 58 (1) 267–88
[15] Meyer, Yves (1992). Wavelets and Operators. Cambridge: Cambridge University Press