Resonant absorption and amplification of circularly-polarized waves in inhomogeneous chiral media

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Abstract: It has been found that in the media where the dielectric permittivity $\varepsilon$ or the magnetic permeability $\mu$ is near zero and in transition metamaterials where $\varepsilon$ or $\mu$ changes from positive to negative values, there occur a strong absorption or amplification of the electromagnetic wave energy in the presence of an infinitesimally small damping or gain and a strong enhancement of the electromagnetic fields. We attribute these phenomena to the mode conversion of transverse electromagnetic waves into longitudinal plasma oscillations and its inverse process. In this paper, we study analogous phenomena occurring in chiral media theoretically using the invariant imbedding method. In uniform isotropic chiral media, right-circularly-polarized and left-circularly-polarized waves are the eigenmodes of propagation with different effective refractive indices $n_+$ and $n_-$, whereas in the chiral media with a nonuniform impedance variation, they are no longer the eigenmodes and are coupled to each other. We find that both in uniform chiral slabs where either $n_+$ or $n_-$ is near zero and in chiral transition metamaterials where $n_+$ or $n_-$ changes from positive to negative values, a strong absorption or amplification of circularly-polarized waves occurs in the presence of an infinitesimally small damping or gain. We present detailed calculations of the mode conversion coefficient, which measures the fraction of the electromagnetic wave energy absorbed into the medium, for various configurations of $\varepsilon$ and $\mu$ with an emphasis on the influence of a nonuniform impedance. We propose possible applications of these phenomena to linear and nonlinear optical devices that react selectively to the helicity of the circular polarization.

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References and links
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1. Introduction

Because of the possibility of many exotic photonic phenomena, there has been much recent interest in metamaterials with almost zero dielectric permittivity $\varepsilon$ and/or magnetic permeability $\mu$ [1–14]. These metamaterials have been termed epsilon-near-zero (ENZ), mu-near-zero (MNZ), or epsilon-and-mu-near-zero (EMNZ) metamaterials. Another area which has drawn much research interest is that of transition metamaterials, in which either $\varepsilon$ or $\mu$ or both change continuously from positive to negative values [15–21]. The common feature in both cases is the existence of the region where $\varepsilon$, $\mu$, or both vanish and the strong enhancement of the electromagnetic (EM) fields there. In the presence of an infinitesimally small amount of damping or gain in the resonance region, a large absorption or amplification of the EM wave energy occurs. The strong enhancement of the EM fields can also lead to enhanced nonlinear optical properties such as second harmonic generation and optical bistability [4, 6, 12, 20].

In Ref. 15, it has been pointed out that the strong absorption of the EM wave energy in transition metamaterials with vanishingly small damping is due to the mode conversion of transverse EM waves into longitudinal plasma oscillations. The derivation of the fact that EM waves in ordinary isotropic media are transverse requires that $\varepsilon$ and $\mu$ do not vanish anywhere. Therefore, in inhomogeneous media where $\varepsilon$ or $\mu$ vanishes in some region of space, longitudinal EM waves can be excited. A representative example is the mode conversion of EM waves into longitudinal plasma waves at the resonance points where $\varepsilon$ vanishes in cold unmagnetized plasmas [22–26]. The transfer of EM wave energy to the resonance region associated with more general kinds of mode conversion occurring in magnetized plasmas plays a central role in a wide range of phenomena in plasma physics [27–32].

In this paper, we are interested in a novel kind of mode conversion occurring in isotropic chiral media. These media are characterized by a coupling between the electric and magnetic fields, the strength of which is denoted by a parameter $\gamma$ termed chiral index [33, 34]. In the uniform case, right-circularly-polarized (RCP) and left-circularly-polarized (LCP) waves are $n_+ (= n + \gamma)$ and $n_- (= n - \gamma)$ respectively, where $n$ is the ordinary refractive in-
The difference between them gives rise to circular birefringence phenomena. In isotropic chiral media with a non-uniform impedance distribution, RCP and LCP waves are no longer eigenmodes and are coupled to each other. In naturally occurring chiral media, the value of $\gamma$ is typically very small. Recent rapid developments in metamaterials have made it possible to fabricate artificial chiral metamaterials, which show a greatly enhanced effect of chirality. Many interesting phenomena, including giant optical activity, strong circular dichroism and chirality-induced negative refractive index, can occur in these metamaterials [35–44].

As direct analogies to ENZ, MNZ, EMNZ and transition metamaterials, we consider uniform chiral slabs with near-zero $n_+$ or $n_-$ and chiral transition metamaterials, in which either $n_+$ or $n_-$ or both change from positive to negative values. We expect that the mode conversion of RCP or LCP waves or both into longitudinal modes and its inverse process occurs in these systems. In the presence of a vanishingly small amount of damping or gain in the resonance region, a large absorption or amplification of circularly-polarized waves will occur. When the impedance is not uniform throughout the space, there occurs a complication due to the coupling between RCP and LCP waves. This interesting mode conversion phenomenon has never been studied before. We aim to explore this phenomenon by calculating the mode conversion coefficient, which measures the wave absorption or amplification, in a numerically exact manner using the invariant imbedding method [23, 31, 45–48].

In Sec. 2, we describe the invariant imbedding method used in this paper. In Sec. 3, we present the results of numerical calculations for uniform chiral slabs and chiral transition metamaterials. Finally, in Sec. 4 we give a summary of the paper.

2. Method

In this section, we give a brief summary of the theoretical method used in the present study. More details can be found in [47]. We consider isotropic chiral media, the constitutive relations of which are given by

$$\mathbf{D} = \varepsilon \mathbf{E} + i \gamma \mathbf{H}, \quad \mathbf{B} = \mu \mathbf{H} - i \gamma \mathbf{E}. \quad (1)$$

The scalar quantities $\varepsilon$, $\mu$ and $\gamma$ are the dielectric permittivity, the magnetic permeability and the chiral index respectively [34]. We assume that the medium is stratified along the $z$ direction and $\varepsilon$, $\mu$ and $\gamma$ are functions of $z$ only. The wave is assumed to propagate in the $xz$ plane. Then, using Eq. (1) and the Maxwell’s equations, we derive the coupled wave equations satisfied by the $y$ components of the electric and magnetic fields, $E_y = E_y(z)$ and $H_y = H_y(z)$:

$$\frac{d^2 \psi}{dz^2} - \frac{d \mathcal{E}}{dz} \mathcal{E}^{-1}(z) \frac{d \psi}{dz} + \left[ k_0^2 \mathcal{E}(z) - q^2 I \right] \psi = 0, \quad (2)$$

where $\psi = (E_y, H_y)^T$, $I$ is a $2 \times 2$ unit matrix, $k_0 (= \omega/c)$ is the vacuum wave number, $q$ is the $x$ component of the wave vector, and the $2 \times 2$ matrix functions $\mathcal{E}$ and $\mathcal{M}$ are given by

$$\mathcal{E} = \begin{pmatrix} \mu & i \gamma \\ -i \gamma & \varepsilon \end{pmatrix}, \quad \mathcal{M} = \begin{pmatrix} \varepsilon & i \gamma \\ -i \gamma & \mu \end{pmatrix}. \quad (3)$$

We assume that an inhomogeneous chiral medium of thickness $L$ lies in the region $0 \leq z \leq L$. The waves are incident from the vacuum region where $z > L$ and transmitted to another vacuum region where $z < 0$. Our main interest is in calculating the $2 \times 2$ matrix reflection and transmission coefficients $r = r(L)$ and $t = t(L)$, which we consider as functions of $L$. The exact differential equations satisfied by $r$ and $t$ have been derived from Eq. (2) previously, using the
invariant imbedding method \[47\]:

\[
\frac{1}{i k_0 \cos \theta} \frac{d \rho}{d l} = r \rho + \rho + \frac{1}{2} \left( r + i \right) \left[ \mathcal{M} - \mathcal{E} + \tan^2 \theta \left( \mathcal{M} - \mathcal{E}^{-1} \right) \right] \left( r + i \right), \tag{4}
\]

\[
\frac{1}{i k_0 \cos \theta} \frac{d \rho}{d l} = i \rho + \frac{1}{2} \left[ \mathcal{M} - \mathcal{E} + \tan^2 \theta \left( \mathcal{M} - \mathcal{E}^{-1} \right) \right] \left( r + i \right), \tag{5}
\]

where \( \theta \) is the incident angle. We integrate the coupled differential equations (4) and (5) numerically from \( l = 0 \) to \( l = L \) using the initial conditions \( r(0) = 0 \) and \( t(0) = 1 \) and obtain \( r \) and \( t \) as functions of \( L \).

The matrix component \( r_{11} \) (or \( r_{21} \)) is the reflection coefficient when the incident wave is \( s \)-polarized and the transmitted wave is \( s \)-polarized (\( p \)-polarized). Similar definitions are applied to \( r_{22} \) and \( r_{12} \) and to the transmission coefficients. For circularly-polarized incident waves, we obtain a new set of the reflection and transmission coefficients \( r_{ij} \) and \( t_{ij} \), where \( i \) and \( j \) are either + or −, from

\[
\begin{align*}
    r_{++} & = \frac{1}{2} \left( r_{22} + r_{11} \right) + i \frac{1}{2} \left( r_{21} - r_{12} \right), & r_{--} & = \frac{1}{2} \left( r_{22} - r_{11} \right) + i \frac{1}{2} \left( r_{21} + r_{12} \right), \\
    r_{+-} & = \frac{1}{2} \left( r_{22} - r_{11} \right) - i \frac{1}{2} \left( r_{21} + r_{12} \right), & r_{-+} & = \frac{1}{2} \left( r_{22} + r_{11} \right) - i \frac{1}{2} \left( r_{21} - r_{12} \right), \\
    t_{++} & = \frac{1}{2} \left( t_{22} + t_{11} \right) + i \frac{1}{2} \left( t_{21} - t_{12} \right), & t_{--} & = \frac{1}{2} \left( t_{22} - t_{11} \right) + i \frac{1}{2} \left( t_{21} + t_{12} \right), \\
    t_{+-} & = \frac{1}{2} \left( t_{22} - t_{11} \right) - i \frac{1}{2} \left( t_{21} + t_{12} \right), & t_{-+} & = \frac{1}{2} \left( t_{22} + t_{11} \right) - i \frac{1}{2} \left( t_{21} - t_{12} \right). \tag{6}
\end{align*}
\]

\( r_{++} \) (or \( r_{--} \)) represents the reflection coefficient when the incident wave is RCP and the reflected wave is RCP (LCP), \( r_{+-} \) and \( t_{ij} \)’s are defined similarly. The absorptance \( A_1 \) (\( A_2 \)) is defined as the fraction of the incident wave energy absorbed into the medium when an \( s \) (\( p \)) wave is incident. Similarly, \( A_+ \) (\( A_- \)) is the fraction of the incident wave energy absorbed into the medium when an RCP (LCP) wave is incident. These quantities can be obtained from

\[
\begin{align*}
    A_1 & = 1 - |r_{11}|^2 - |r_{21}|^2 - |t_{11}|^2 - |t_{21}|^2, \\
    A_2 & = 1 - |r_{12}|^2 - |r_{22}|^2 - |t_{12}|^2 - |t_{22}|^2, \\
    A_+ & = 1 - |r_{++}|^2 - |r_{--}|^2 - |t_{++}|^2 - |t_{--}|^2, \\
    A_- & = 1 - |r_{+-}|^2 - |r_{-+}|^2 - |t_{+-}|^2 - |t_{-+}|^2. \tag{7}
\end{align*}
\]

If \( A_i \) (\( i = 1, 2, +, - \)) is negative, the wave is amplified inside the medium due to some gain mechanism. In that case, \(|A_i| = -A_i|\) is a natural measure of wave amplification. When mode conversion or its inverse process occurs, \( A_i \) is nonzero even in the presence of an infinitesimally small loss or gain. We define \(|A_i|\) to be the mode conversion coefficient in such cases.

When we deal with circularly-polarized waves propagating in weakly nonuniform chiral media, it is convenient to introduce the fields \( \mathbf{F}_+ \) and \( \mathbf{F}_- \) defined by

\[
\mathbf{F}_\pm = \mathbf{E} \pm i \eta \mathbf{H}, \quad \eta = \sqrt{\frac{\mu}{\varepsilon}}, \tag{8}
\]

where \( \mathbf{F}_+ \) and \( \mathbf{F}_- \) describe RCP and LCP waves respectively and \( \eta \) is the wave impedance. These fields satisfy

\[
\nabla \times \mathbf{F}_\pm = \pm \left[ k_\pm \mathbf{F}_\pm + \frac{1}{2 \eta} \nabla \eta \times (\mathbf{F}_+ - \mathbf{F}_-) \right], \quad k_\pm = n_\pm k_0, \tag{9}
\]
Fig. 1. Mode conversion coefficients $A_+$ and $A_-$ for RCP and LCP waves incident on a uniform chiral slab of thickness $L$ versus incident angle, when (a) $\epsilon_R = \mu_R = \gamma = \pm 2$ and (b) $\epsilon_R = \mu_R = -\gamma = \pm 2$. The other parameters used are $k_0L = 5\pi$ and $\varepsilon_I = \mu_I = 10^{-5}$. In (a), strong absorption for LCP waves occurs, while $A_+$ for RCP waves vanishes, because $n - \gamma = 0$. In (b), strong absorption for RCP waves occurs, while $A_-$ for LCP waves vanishes, because $n + \gamma = 0$. The $A_-$ curve in (a) is identical to the $A_+$ curve in (b).

where

$$n_{\pm} = n \pm \gamma.$$  \hspace{1cm} (10)

In positive index media, the refractive index $n$ is equal to $\sqrt{\varepsilon \mu}$, while in negative index media with simultaneously negative real parts of $\varepsilon$ and $\mu$, $n$ is equal to $-\sqrt{\varepsilon \mu}$. We notice that in uniform chiral media, RCP and LCP waves are the eigenmodes of the wave equation with the effective refractive indices $n_+$ and $n_-$ respectively. In inhomogeneous media with nonuniform impedance, these two modes are no longer eigenmodes and are coupled to each other.

3. Numerical results

We first consider the simplest situation in which circularly-polarized waves are incident on a single uniform slab made of an isotropic chiral medium. We have verified numerically that in the limit of very small absolute values of the imaginary parts of $\varepsilon$ and $\mu$, significant optical absorption or amplification occurs only when $n_+$ vanishes for RCP waves, or when $n_-$ vanishes for LCP waves. If the imaginary parts of $\varepsilon$ and $\mu$ are positive, resonant absorption occurs, whereas if they are negative, resonant amplification does.

In Fig. 1, we plot the absorptance, or the mode conversion coefficient, $A_{\pm}$ for RCP and LCP waves incident on a uniform slab of thickness $L$ with $\varepsilon = \epsilon_R + i\epsilon_I$ and $\mu = \mu_R + i\mu_I$ versus the incident angle $\theta$. We consider the four cases where $\epsilon_R = \mu_R = \gamma = \pm 2$ and $\epsilon_R = \mu_R = -\gamma = \pm 2$, which include both positive and negative index media. The values of $\epsilon_I$ and $\mu_I$ are $10^{-5}$ and the parameter $k_0L$ is equal to $5\pi$. Strong absorption for LCP waves occurs at very small incident angles when $n_- = n - \gamma = 0$, while $A_+$ for RCP waves vanishes. This corresponds to the cases where $\epsilon_R = \mu_R = \gamma = 2$ and $\epsilon_R = \mu_R = \gamma = -2$. In contrast, strong absorption for RCP waves
Fig. 2. Mode conversion coefficients in the absorbing case, $A_+$ and $A_-$, and those in the amplifying case, $|A_+|$ and $|A_-|$, for RCP and LCP waves incident on a uniform chiral slab of thickness $L$ versus incident angle, when (a), (c) $\varepsilon_R = \mu_R = \gamma = 2$ and (b), (d) $\varepsilon_R = 4$, $\mu_R = 1$, $\gamma = 2$. The parameter $k_0L$ is equal to $5\pi$. Strong absorption occurs in (a) and (b), where $\varepsilon_I = \mu_I = 10^{-5}$, whereas strong amplification occurs in (c) and (d), where $\varepsilon_I = \mu_I = -10^{-5}$. In (c), $A_+$ is identically zero and is not shown on the logarithmic plot.

occurs when $n_+ = n + \gamma = 0$, while $A_-$ for LCP waves vanishes. This corresponds to the cases where $\varepsilon_R = \mu_R = -\gamma = 2$ and $\varepsilon_R = \mu_R = -\gamma = -2$. The $A_-$ curve in Fig. 1(a) is identical to the $A_+$ curve in Fig. 1(b). We note that the maximum value of the mode conversion coefficient occurring at $\theta \approx 0.065^\circ$ is very close to 0.5. We have checked numerically that this maximum value remains the same for smaller values of $\varepsilon_I$ and $\mu_I$ and larger values of $k_0L$.

In Fig. 2, we compare the results for the two slabs with $\varepsilon_R = \mu_R = \gamma = 2$ and $\varepsilon_R = 4$, $\mu_R = 1$, $\gamma = 2$. In both cases, $n$ is equal to $\gamma$ and strong mode conversion occurs for LCP waves. The main difference between the two is that the impedance $n$ in the former case is equal to 1, while that in the latter is equal to 0.5. Because the slabs are surrounded by a vacuum with $\eta = 1$, $n$ is uniform throughout the space when $\varepsilon_R = \mu_R = \gamma = 2$. In this case, RCP and LCP waves can never cause mode conversion. On the other hand, when $\varepsilon_R = 4$, $\mu_R = 1$, $\gamma = 2$, $\eta$ changes discontinuously at the interfaces of the slab. Incident RCP waves can generate LCP waves at the interfaces, which can subsequently be mode-converted. Therefore strong absorption or amplification occurs for both RCP and LCP waves in this case as shown in Figs. 2(b) and 2(d). Since the coupling between RCP and LCP waves occurs only at the interfaces and is not very strong, the absolute value of $A_+$ is substantially smaller than that of $A_-$. We notice that an extremely large amplification results when inverse mode conversion occurs, as shown in Figs. 2(c) and 2(d).

Next, we consider nonuniform slabs, which contain discrete resonance planes satisfying $n_\pm = 0$. In Fig. 3, we plot the mode conversion coefficients for RCP and LCP waves incident on a nonuniform slab of thickness $L$, where $\varepsilon_R = \mu_R = 2(z/L) - 1$ ($0 \leq z \leq L$) and $\gamma = 0.8$, versus incident angle. The parameter $k_0L$ is equal to $20\pi$. In Fig. 3(a), $\varepsilon_I$ and $\mu_I$ are equal to $10^{-8}$ and in Fig. 3(b), they are equal to $-10^{-8}$. The whole curves remain identical for smaller absolute values of $\varepsilon_I$ and $\mu_I$, which signifies that the absorption or the amplification is not due
Fig. 3. Mode conversion coefficients in the absorbing case, $A_+$ and $A_-$, and those in the amplifying case, $|A_+|$ and $|A_-|$, for RCP and LCP waves incident on a nonuniform slab, where $\varepsilon_R = \mu_R = 2(z/L) - 1 \ (0 \leq z \leq L)$ and $\gamma = 0.8$, versus incident angle. Circularly-polarized waves are assumed to be incident from the region where $z > L$. The parameter $k_0L$ is equal to $20\pi$. In (a), $\varepsilon_I = \mu_I = 10^{-8}$ and in (b), $\varepsilon_I = \mu_I = -10^{-8}$.

Fig. 4. Mode conversion coefficients in the absorbing case, $A_+$ and $A_-$, and those in the amplifying case, $|A_+|$ and $|A_-|$, for RCP and LCP waves incident on a nonuniform slab, where $\varepsilon_R = \mu_R = z/L \ (0 \leq z \leq L)$ and $\gamma = 0.1, 0.9$, versus incident angle. Circularly-polarized waves are assumed to be incident from the region where $z > L$. The parameter $k_0L$ is equal to $20\pi$. In (a), $\varepsilon_I = \mu_I = 10^{-8}$ and in (b), $\varepsilon_I = \mu_I = -10^{-8}$. In (b), $A_+$ is identically zero and is not shown on the logarithmic plot.
Fig. 5. Mode conversion coefficients in the absorbing case, $A_+$ and $A_-$, and those in the amplifying case, $|A_+|$ and $|A_-|$, for RCP and LCP waves incident on a nonuniform slab, where $\varepsilon_R = z/L$, $\mu_R = (z/L)^2$, and $\gamma = 0.5$, versus incident angle. Circularly-polarized waves are assumed to be incident from the region where $z > L$. The parameter $k_0L$ is equal to $20\pi$. In (a), $\varepsilon_I = 10^{-8}$ and in (b), $\varepsilon_I = -10^{-8}$.

In Fig. 4, we consider a nonuniform slab with a different configuration such that $\varepsilon_R = \mu_R = z/L (0 \leq z \leq L)$. In this case, $n_+$ never vanishes, while $n_- = 0$ at $z/L = \gamma$. Because the impedance is uniform and equal to 1 everywhere, RCP waves are unable to cause mode conversion and $A_+$ is identically zero. We compare the results for the two cases where $\gamma = 0.1$ and $\gamma = 0.9$. Resonant absorption and amplification are substantially stronger for $\gamma = 0.9$, especially at larger incident angles, than for $\gamma = 0.1$, which is due to the fact that waves have to propagate for longer distances to reach the resonance plane experiencing further reflections in the case of $\gamma = 0.1$.

Finally, in Fig. 5, we consider the configuration where $\varepsilon_R = z/L$ and $\mu_R = (z/L)^2 (0 \leq z \leq L)$. Then the real parts of the refractive index and the impedance are given by $(z/L)^{3/2}$ and $(z/L)^{1/2}$ respectively. Because the impedance is nonuniform, RCP and LCP waves are coupled to each other. Similarly to the previous case, $n_+$ never vanishes and $n_- = 0$ at $z/L = \gamma^{2/3}$. The inhomogeneity of $\eta$ causes a strong coupling between RCP and LCP waves, which results in a substantial amount of $|A_+|$, as shown in Fig. 5.
4. Conclusion

In this paper, we have studied the mode conversion and the resonant absorption and amplification of circularly-polarized electromagnetic waves in stratified chiral media theoretically using a generalized version of the invariant imbedding method. In uniform isotropic chiral media, RCP and LCP waves are the eigenmodes with different effective refractive indices $n_+$ and $n_-$, whereas in the chiral media with a nonuniform impedance variation, they are not the eigenmodes and are coupled to each other. We have found that both in uniform chiral slabs where $n_+$ or $n_-$ is near zero and in chiral transition metamaterials where $n_+$, $n_-$ or both change from positive to negative values, a strong absorption or amplification of circularly-polarized waves occurs in the presence of a vanishingly small amount of damping or gain. We have presented numerical calculations of the mode conversion coefficient for various spatial configurations of $\varepsilon$ and $\mu$ with an emphasis on the influence of a nonuniform impedance. Based on the investigations reported here, it is possible to develop an efficient absorber or amplifier that reacts selectively to the helicity of the circular polarization. The strong enhancement of the electromagnetic fields can be exploited in developing novel nonlinear optical devices operating with circularly-polarized waves.

Acknowledgments

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