Creation of highly-charged calcium ions for the $g$-factor determination of the bound electron

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Abstract. High-precision measurements of the magnetic moment of the electron bound in hydrogen and lithium-like ions can be used to test bound-state quantum electrodynamical calculations. In the past measurements with relative experimental uncertainties as low as $2 \times 10^{-9}$ were performed on hydrogen-like carbon and oxygen ions. In the current experiment we plan to measure the $g$-factor of hydrogen-like and lithium-like calcium ions. A relative uncertainty $\delta g/g$ in the order of $10^{-9}$ is aspired. Here, we will give the motivation for the experiment, present the experimental techniques and first results.

1. Introduction

The $g$-factor of an electron is a proportionality constant between its angular momentum $\vec{J}$ and its magnetic moment $\vec{\mu}$

$$\vec{\mu} = -g\frac{e}{2m_e} \vec{J}.$$  (1)

For a Dirac-point-like free electron the $g$-factor is predicted to be exactly 2 [1]. Because of quantum electrodynamic [2] and relativistic [3] contributions the $g$-factor is shifted from the predicted value. In case the electron is bound to an ion, the electron is affected by the strong electromagnetic fields of the nucleons at its position. The strength of the e.m. fields ranges from $10^9$ V/cm for helium ($Z=2$) to $10^{15}$ V/cm for uranium ($Z=92$). Measurements on hydrogen-like carbon and oxygen ions were performed with relative uncertainties in the level of $10^{-9}$ [4, 5] to test highly accurate calculations [6] for the $g$-factor of a single electron bound to a spinless nucleus. In combination both led to the most precise determinations of the electron mass in atomic units [7, 8].

The effects of the bound-state quantum electrodynamics (BS-QED) increase with the nuclear charge $Z$, therefore heavier elements like calcium in the current experiment [9] are particularly interesting. The element calcium has two spinless isotopes (masses 40 u and 48 u), where the total spin of the system is the electron spin. The binding energy of the second last electron in the 1s-state of hydrogen-like calcium Ca$^{19+}$ is $\sim 5$ keV, which is within reach of the experimental setup. Not only the $g$-factor of hydrogen-like calcium Ca$^{19+}$ will be measured, it is also planned to investigate lithium-like calcium Ca$^{17+}$. Here the calculations take into account the electron-electron interactions and claim an accuracy of $9 \times 10^{-8}$ [10]. For studying nuclear structure effect one can compare $g$-factor measurements of the doubly-magic isotopes $^{40}$Ca and $^{48}$Ca, which is also planned.

1 The results which are presented here are part of the PhD thesis of Birgit Schabinger and Sven Sturm
2. g-factor measurement

The $g$-factor determination of a single stored ion is based on the “continuous Stern-Gerlach-effect” [11]. The Larmor frequency

$$\omega_L = g \frac{e}{2m_e} B$$  \hspace{1cm} (2)

is the spin precession frequency in a magnetic field $B$ and can be determined by measuring the spin flip probability for different excitation frequencies. Therefore the trapped ion is irradiated with microwaves at a frequency close to the Larmor frequency and the spin state before and after irradiation is observed. At the Larmor frequency the spin-flip probability [12] reaches a maximum for ions with negligible temperature. The value of the magnetic field at the ion’s position can be determined from its cyclotron frequency, given by

$$\omega_c = \frac{q}{m_{ion}} B,$$  \hspace{1cm} (3)

$q$ being the charge of the ion and $m_{ion}$ its mass. The $g$-factor is thus given by

$$g = 2 \frac{\omega_L q}{\omega_c e m_{ion}},$$  \hspace{1cm} (4)

where the masses of the ion and the electron are known from other experiments [8, 13].

2.1. Triple-Penning trap setup

The experimental setup is based on a set of three cylindrical Penning traps [14, 15], creation, analysis and precision trap, for more details see [16, 17]. The axial confinement of the ions is achieved with an electrostatic potential, superimposed to a constant magnetic field in the direction of the trap axis for radial confinement. The ion motion in a Penning trap is a composition of three independent oscillations. One is in the axial direction with the frequency

$$\omega_z = \left(qU_0/(m_{ion}d^2)\right)^{1/2},$$  \hspace{1cm} (5)

which depends on the trap potential $U_0$ and the characteristic trap size parameter $d$. In the radial direction the motion can be decomposed into two circular motions, with the reduced cyclotron frequency $\omega_+$ and the magnetron drift frequency $\omega_-$, given by

$$\omega_{\pm} = \omega_c/2 \pm \sqrt{\left(\frac{\omega_c}{2}\right)^2 - \omega_z^2/2}.$$  \hspace{1cm} (6)

2.2. Ion creation

The charge breeding of the ions is performed inside the trap with an electron beam ion source (EBIS) in combination with a Penning trap located inside a sealed vacuum chamber [16, 17] to achieve extreme vacuum conditions ($p \leq 10^{-16}$ mbar) at liquid helium temperature. Electrons are emitted from a single field emission point (FEP) by applying a voltage difference between an acceleration electrode and the FEP. The electron beam is guided by the magnetic field through the creation trap and is reflected by the strong negative potential at the reflector electrode. The electrons oscillate several times between the reflector and the FEP until the Coulomb-repulsion causes the beam to expand radially and hit the target made out of graphite with a sputtered layer of mainly $^{40,44,48}$Ca and some contamination of silicon. The atoms are ablated and ionized by the electrons. The ions are trapped by the negative potential of the creation trap where charge breeding takes place by consecutive electron-impact ionization.
2.3. Mass spectrum in the Penning trap

The charge breeding process creates several charge states of different elements, for example, $^{12}\text{C}^{6+}$, $^{16}\text{O}^{7+}$, and $^{28}\text{Si}^{11+}$ (see Fig. 1 a). The axial frequency of an ion in a Penning trap is determined by measuring the frequency of the image current induced by the ion across an electronic resonance circuit attached to the trap electrodes. To identify the different ions one can take advantage of the dependence of the axial motion on the trapping potential $U_0$. By scanning the potential several species with different $q/m$ come into resonance with the axial tank circuit. To create a mass spectrum as shown in figure (Fig. 1 a) a special amplifier is used. The ion signal is already amplified by a cryogenic preamplifier and again amplified and filtered by a second amplifier at room temperature. After a narrow bandpass filter, the signal is rectified and integrated. With the knowledge of one $q/m$ all other signals can be related to the various ion species.

2.4. Measurement of the axial frequency of a single ion

If the isolated ion is in thermal equilibrium with the tank circuit a minimum in the Johnson noise density of the resonator appears exactly at the ion’s oscillating frequency (Fig. 1 b). Because the axial frequency depends on the trapping potential $U_0$ it is possible to bring ions with different charge-to-mass ratios $q/m$ into resonance with the tank circuit at 754 kHz in case of the precision Penning trap.

\[ \omega_c = \sqrt{\omega_+^2 + \omega_-^2 + \omega_z^2}. \] (7)

2.5. Determination of the free cyclotron frequency

The free cyclotron frequency is not a real eigenmotion of the ion in the Penning trap, therefore it cannot be measured directly. The relation between the three eigenmotions and the free cyclotron frequency is expressed in the invariance theorem [18]:

2.6. Sideband coupling

The reduced cyclotron and the magnetron frequency can be determined by resonant coupling of two ion motions [12]. Therefore a suited electric radio-frequency field (rf) is irradiated, causing an amplitude

\[ \omega_c = \sqrt{\omega_+^2 + \omega_-^2 + \omega_z^2}. \]
modulation of the corresponding motions. In this experiment the “double-dip method” is used, where a rf signal at the sideband frequency $\omega_+ - \omega_z$ causing the axial resonance to split into two. The components $\omega_{r,l}$ are given by

$$\omega_{r,l} = \omega_z - \delta \pm \sqrt{\delta^2 + |A|^2}$$

where $|A|$ is a proportion for the measure of the coupling in units of frequency and $\delta$ is a measure for the detuning of the coupling frequency to the sideband frequency $\omega_+ - \omega_z$. Finally, the reduced cyclotron frequency can be expressed as

$$\omega_+ = (\omega_+ - \omega_z + \delta) + \omega_z + 2 \left( \frac{\omega_+ + \omega_{r,l}}{2} - \omega_z \right)$$

where “$\omega_+ - \omega_z + \delta$” is the known irradiated coupling frequency and $\omega_z$ and $\omega_{r,l}$ respectively are observables. For the magnetron motion $\omega_-$ an analogous analysis can be used.

By using the “double-dip method” the reduced cyclotron frequency $\omega_+$ was determined for several highly charged ions, e.g. $^{28}\text{Si}^{12+}$, with an accuracy of $10^{-6}$. This allowed the determination of the field strength of the superconducting magnet to $B = 3.7642 \, \text{T}$.

3. Current status

Highly charged ions have been created in a cryogenic EBIS and transported to the precision trap where single species have been prepared. The axial and reduced cyclotron frequency was measured for several species. Presently, the charge state breeding process is being investigated in more as required to create lithium-like calcium ions. The next steps will be the detection of single highly-charged ion and finally the determination of the $g$-factor. In the future, we plan to extend our $g$-factor measurements to heavy highly-charged ions up to uranium $^{238}\text{U}^{91+}$ at the HITRAP facility [19, 20].

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