Simple Calculation Model of Combined Cycle Unit
Under Off-Design Conditions

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Abstract. The modeling of the combined cycle unit is often limited by the lack of characteristic curve and operating data. This paper provided a simple off-design model of combined cycle unit, which was based on theoretical derivation and experimental data. The simulation results showed that the model could reflect the characteristics of gas cycle components and realize the calculation of off-design conditions. The calculation error of heat transfer of HRSG and output of steam turbine under pure coagulation the condition was less than 0.5%.

Introduction

Due to the need of power grid peaking and heating supply, combined cycle units are often operating in off-design conditions. In order to better analyze the performance of the unit, an off-design model of combined cycle should be established.

Compared with the traditional thermal power unit, the difficulty in modeling off-design model of combined cycle unit is mainly concentrated on the components in the gas cycle. Gas cycle, as the top cycle working in high temperature zone, will determine the overall performance of the entire combined cycle [1]. The components of the gas cycle mainly include a compressor, a combustion chamber, a gas turbine, and a heat recovery boiler (HRSG).

The common used method of compressor modeling is based on component characteristics [2, 3, 4], which could be obtained by experiment or theoretical derivation. However, the accuracy of this type of modeling method is limited by characteristics performance curves. The primitives cascade method has received extensive attention, due to the capability of calculating performance of the compressor with IGV [5]. But a large amount of compressor structural data was required for primitives cascade method. Some data fitting methods have been used for compressor modeling [6, 7, 8, 9, 10], but those methods are difficult to promote to different types of compressors.

The modeling of gas turbines is mainly based on theoretical equations such as the elliptic equation derived from the isentropic flow equation of the nozzle and the Friguel formula [1,11,12], which are also difficult to perform well in both simplicity and accuracy.

Modeling of HRSG mainly focuses on the heat calculation. Different methods deal with heat transfer equations in different ways, such as taking heat transfer coefficient [14, 15], or as functions of the parameters of flue gas and working medium [13]. However, too many parameters are needed for those modeling methods.

In summary, the off-design model of gas cycle is mainly limited by two factors. One is the lack of performance characteristics data of compressors and gas turbines. Second, the mechanism model requires too much unit data, as well as large amount of calculation, and often difficult to apply to different kinds units. In view of the above problems, this paper synthesizes the modeling method of mechanism and data fitting, and a simple variable working condition model of combined cycle will be established.

Model building

The gas cycle process can be expressed as follows: the compressor draws air from the outside to form high-pressure gas, and then high-pressure gas is sent into the combustion chamber to be mixed...
with the fuel to generate high-temperature gas which produces power in a turbine by expansion. And the gas discharged from the gas turbine enters the heat recovery boiler and heats the working fluid. Each component in the process is separately modeled.

In order to minimize the need for structural parameters and performance data of various components and to establish an off-design model with strong applicability, a method combining theoretical derivation and experimental data will be applied in this paper.

**Compressor Modeling**

The model of the compressor can be derived by combining the statistical law and the data fitting method. Firstly, according to the statistical induction and theoretical analysis, the general form of the characteristic equation is obtained, and then the specific parameters of the equation are obtained by fitting the actual data, thereby establishing the compressor model [16].

The efficiency at the surge margin of compressor can be expressed as the reference point pressure ratio:

\[
\bar{\eta}_{su,i} = a_c \bar{n}^{-2} + b_c \bar{n} + c_c
\]  
(1)

In the formula, the subscript _su_ represents surge margin, _i_ represents an arbitrary working point, \(\bar{\eta}_{su,i}\) represents ratio operating point efficiency reference point seek efficiency, \(\bar{n}\) represents the ratio of the operating point speed and the reference point, \(a_c\), \(b_c\) and \(c_c\) are functions of the pressure ratio.

The optimal operating line of the compressor refers to a line formed by these points which have highest efficiency at each speed. Based on the derivation of [16], the relative values of efficiency, pressure ratio and flow rate on the optimal line can be expressed as functions of relative speed:

\[
\bar{\eta}_{opt} = a + b\bar{n}
\]  
(2)

\[
\bar{G}_{opt} = c + d\bar{n} + e\bar{n}^{-2}
\]  
(3)

\[
\bar{\pi}_{opt} = f + g\bar{n} + h\bar{n}^{-2}
\]  
(4)

Where, \(\bar{n}\) represents the relative flow, \(\bar{\eta}_{opt}\) represents relative efficiency of optimum operation curve, \(\bar{G}_{opt}\) represents relative flow of optimum operation curve, \(\bar{\pi}_{opt}\) represents relative ratio of optimum operation curve. The undetermined coefficients in the formula can be fitted by experimental data.

The iso-speed line model of the compressor describe the relationship between ratio, flow rate, and efficiency under the same speed, which can be expressed as the following formulas:

\[
\bar{\pi}_i = a_i \bar{G}_i^{-2} + b_i \bar{G}_i + c_i
\]  
(5)
In the formula, \( \overline{G_i} \) represents the relative flow rate on the i-th iso-speed line, \( \overline{\pi_i} \) represents the relative pressure ratio on the i-th iso-speed line. After calculating the relative flow rate and the relative pressure ratio, the efficiency on the equal speed line can be expressed as:

\[
x = \overline{G_i} \cdot \overline{\pi_i}^{\frac{1}{3}}
\]  

(6)

\[\eta_i = f(x)\]  

(7)

Where, \( \overline{\eta_i} \) represents the relative efficiency of the i-th iso-speed line. And to improve the accuracy of Eq. 5 and Eq. 7, different power rational polynomial could be introduced.

According to such a series of relations, the characteristic model of the compressor can be formed, which not only conforms to certain physical laws, but also facilitates popularization, at the same time, the actual data can be used for coefficient fitting to improve the accuracy of the model.

**Combustion Chamber Modeling**

The combustion chamber is mainly used for mixing high-pressure gas and natural gas to generate high-temperature and high-pressure gas. And gas parameters of outlet can be calculated according to the material balance and energy balance equation:

\[G_3 = G_2 + G_i\]  

(8)

\[H_\bullet G_3 = H_\bullet G_2 + G_i \bullet (H_i + \eta_c \bullet q_i)\]  

(9)

\[P_3 = \sigma_{cb} \bullet P_2\]  

(10)

Where, \( \eta_{cb} \) represents combustion efficiency, \( \sigma_{cb} \) represents total combustion chamber recovery coefficient, which can be taken as 0.97 ~ 0.99, \( G_2 \) represents combustion chamber inlet air flow, kg/s, \( G_3 \) represents combustion chamber outlet gas flow, kg/s, \( G_f \) represents fuel flow, kg/s, \( H \) represents enthalpy value, kJ/kg, \( P \) represents pressure, Pa.

**Gas Turbine Modeling**

Considering the generalization and precision of the gas turbine model, the modeling method is also based on a combination of theoretical analysis and experimental data. The characteristics of the existing turbine models are mostly described by the Freugaer formula, which is subject to the elliptical law and has poor precision. Thus, quasi-elliptic equation is used to describe the characteristics of the gas turbine. The quasi-elliptic equation can be expressed as

\[
\frac{G \sqrt{T_i / P_i}}{G_0 \sqrt{T_{so} / P_{so}}} = D_n \sqrt{\frac{a_z + b_z \delta - \delta^2}{a_z + b_z \delta_0 - \delta_0^2}}
\]

(11)

\[
D_n = \sqrt{1.78 - c_n \left(\frac{n}{n_0}\right) + b_n \left(\frac{n}{n_0}\right)^2}
\]

(12)

Where, the subscript 0 represents the design point parameter, \( \delta \) represents the turbo expansion ratio, \( a_z \) and \( b_z \) are undetermined coefficients, which are related to the turbine type, the average reaction degree, and the working adiabatic coefficient. \( b_n \) and \( c_n \) can be fitted by operating data, \( n \) represents turbine speed, rpm.

A turbine flow characteristic model can be obtained by the formula Eq. 11:

\[
\frac{G \sqrt{T_i / P_i}}{G_0 \sqrt{T_{so} / P_{so}}} = D_n \sqrt{\frac{1-(\frac{\delta - a_z y}{1-a_z})^p}{1-(\frac{\delta_0 - a_z y}{1-a_z})^p}}
\]

(13)
Where, $a_s$ and $b_s$ are undetermined coefficients, and $a_s = 0 \sim 0.6$, $b_s = 1.0 \sim 5$.

Under off-design conditions, efficiency of a gas turbine can be expressed as

$$\eta_s = \frac{a}{\eta_{s,max}} \left( \frac{T_i}{T_{th}} \right)^{1-\delta_s} \left( a_s - (a_s - 1) \frac{a}{\eta_{s,max}} \right) \left( \frac{T_i}{T_{th}} \right)^{1-\delta_s}$$

(14)

$$\gamma = \frac{k - 1}{k}$$

(15)

Where, $\eta_s$ represents the gas turbine efficiency, subscript max indicates maximum efficiency of the turbine, $a_s$ represents undetermined coefficient, $k$-refrigerant adiabatic index.

When the expansion ratio and efficiency at off-design condition are obtained, the pressure and temperature of turbine exhaust gas can be calculated:

$$P_g = P_t \delta$$

(16)

$$T_g = T_t \left[ 1 - (1 - \delta^{-\gamma}) \eta_t \right]$$

(17)

Eq. 11-17 constitute an off-design model of gas turbines. The model can calculate the exit parameters based on the inlet parameters of the gas turbine, and can effectively use the turbine's operating data to improve the accuracy of the model.

**HRSG Modeling**

Too much data will be need in traditional methods of HRSG modeling. This paper uses the empirical formula method to calculate heat. The heat transfer amount and efficiency of HRSG can be expressed as

$$Q = Q_c \left( \frac{T_{ge}}{T_{ge,d}} \right)^2 \left( \frac{m_g}{m_{g,d}} \right)$$

(18)

$$\eta_H = \eta_{H,d} \left( \frac{T_{ge}}{T_{ge,d}} \right)^{0.35} \left( \frac{m_g}{m_{g,d}} \right)^{0.02}$$

(19)

Where, subscript $g$ denotes gas, subscript $d$ denotes design conditions, $Q$ represents heat exchange capacity of HRSG, kW, $\eta_H$ represents efficiency of HRSG, $T_{ge}$ represents exhaust gas temperature of gas turbine, K, $m_g$ represents gas mass flow, kg/s.

The index in the Eq. 18 and 19 can be fitted according to experimental data to expand the range of application of the empirical formulas. Compared with the traditional methods based on the heat transfer equations, the calculation of HRSG model based on the empirical formula is more convenient. At 50% load and above, the error of Eq. 18 and 19 is less than 1.5% [17].

**Combined Cycle Output**

Gas turbine output can be calculated based on thermodynamic parameters of gas. The output gas turbine is

$$\Delta H_g = m_a h_a - m_s h_s + m_{ca} h_{ca}$$

(20)

$$h = f(P, T)$$

(21)

$$W_g = \eta \Delta H_g$$

(22)

Where, the subscript $ca$ indicates cooling air, $W_g$ represents power out of turbine.
The output of the steam turbine is calculated by the heat transfer amount of HRSG. For steam turbines without reheat, the output can be written as

\[ W_{st} = W_{st,d} \eta_{c} \left( a_{st} + b_{st} \left( \frac{Q}{Q_{st}} \right) + c_{st} \left( \frac{Q}{Q_{st}} \right)^2 \right) \]  

(23)

\[ \eta_{c} = 1 - \frac{t_{0}}{T_{ge}} \]  

(24)

Where, the subscript \( d \) denotes design conditions; \( W_{st} \) represents steam turbine output power, kW, \( a_{st}, c_{st} \) and \( b_{st} \) are undetermined coefficients, \( \eta_{c} \) represents the efficiency of the Carnot cycle.

### Steam Turbine Output Correction

In actual operation, the efficiency of the steam turbine is not only related to the Carnot cycle, but also to the operating conditions of the turbine. In the off-design condition calculation, the isentropic efficiency of the steam turbine can be regarded as a function of the steam mass flow, and the steam turbine efficiency can be directly calculated according to the change of the steam mass flow [19].

\[ \frac{\eta_{st}}{\eta_{st,d}} = -1.0176 \left( \frac{m_{s}}{m_{s,d}} \right)^{0.1} + 2.4443 \left( \frac{m_{s}}{m_{s,d}} \right)^{0.3} - 2.1812 \left( \frac{m_{s}}{m_{s,d}} \right)^2 + 1.0535 \left( \frac{m_{s}}{m_{s,d}} \right) + 0.701 \]  

(25)

Wherein, \( \eta_{st} \) represents steam turbine efficiency, \( m_{s} \) represents steam mass flow rate, kg/s, the subscript \( d \) represents design condition. The corrected steam turbine output can be expressed as:

\[ W_{st,c} = W_{st,d} \eta_{st,c} \left( a_{st,c} + b_{st,c} \left( \frac{Q}{Q_{st}} \right) + c_{st,c} \left( \frac{Q}{Q_{st}} \right)^2 \right) \]  

(26)

The coefficients and power of the correction factor used in Eq. 26 can be adjusted according to different units to improve the accuracy and application range of the model.

For the turbines containing extraction steam heating, the power output of steam is affected by the extraction steam. Based on the straight condensing condition, the effect of the extraction steam on the steam turbine output can be fitted by the ratio of the extraction steam to the main steam:

\[ \frac{W_{st,e}}{W_{st}} = \frac{a_{st,e} \left( \frac{G_{e}}{G_{0}} \right)^{0.1} + b_{st,e} \left( \frac{G_{e}}{G_{0}} \right)^{0.3} + c_{st,e} \left( \frac{G_{e}}{G_{0}} \right)^2 + d_{st,e}}{W_{st}} \]  

(27)

Wherein, the subscript \( c \) represents the extraction conditions, \( W_{st} \) represents output of the steam turbine under straight condensing condition, kW, \( G_{e} \) represents mass flow rate of extraction steam, kg/s, \( G_{0} \) represents main steam mass flow, kg/s. the undetermined coefficients are coefficients to be fitted by the operating data.

### Simulation Analysis

#### Compressor Simulation

In this simulation, eight sets of compressor experimental data at different relative rotational speeds were collected, and four of them were selected for fitting while the other four were used for validation. The results are shown in Fig. 2. It shows that the parameters of the operating line can be well fitted, and the relative efficiency has a little deviation, which may be caused by that the selected reference working point is not the highest efficiency.

Fig. 3 and Fig. 4 show the simulation result of compressor relative pressure ratio and relative efficiency under different rotational speed. It can be seen that both curves fit the data well, which can reflect the variation characteristics of the pressure ratio and efficiency, but the accuracy of pressure ratio is poor in the high speed and high flow area, which is mainly due to the blockage in
the blade channel. As the negative angle of attack increases, increase in flow is limited, and the pressure ratio curve become steeper.

Figure 2. The fitting results of compressor optimal operating line.

Figure 3. The fitting results of compressor relative ratio under different speeds.

Figure 4. The fitting results of compressor relative efficiency under different speeds.

**HRSG Simulation and Calculation of Steam Turbine Output**

The model of the HRSG and steam turbine were also verified. The waste heat boiler model is mainly used to calculate the heat transfer during the off-design condition. HRSG heat transfer data of different loads under different work conditions were collected. Change of working condition leads to change of exhaust and environment parameters. The steam turbine model is for workout calculation. Fig.5 and Fig.6 show the calculation results which indicated that both heat of HRSG and the estimated output of the steam turbine under different working conditions is in good agreement with the actual operating data with a little deviation.

The steam turbine output with extraction steam heating can be obtained by Eq. 27. The results were shown in Fig.7, wherein the y axis coordinates is the ratio of the output of extraction conditions to the turbine output of condensing condition, the x axis coordinate is the ratio of the amount of extraction steam to the amount of main steam flow. The fitting error is within 3%, which meets the analytical requirements.
Conclusion

The paper established the off-design model of the gas cycle, which adopted the combination of theoretical derivation and data fitting and can effectively solve the calculation problem of the variable working condition of the combined cycle unit. The model is simple and easy to generalize. Based on the previous operational data, the model can analyze the output of the entire combined cycle, and can also be used for units with extraction heating. The accuracy of the model is verified by simulation, wherein the calculation error of the heat transfer of HRSG and steam turbine under pure condensing conditions were 0.5% or less, while the error of the steam turbine output under extraction heating is 3% or less.

Due to the lack of specific data, this paper is not able to validate the gas turbine model, also failed to verify the wider applicability of the off-design model. Therefore, the next step is to make a more comprehensive verification of the off-design model.
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