The Role of Self-Assessment in Foundation of Mathematics Learning

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Abstract. This research is motivated by the low performance of students who took Foundations of Mathematics course. This study was aimed to describe (1) the learning outcomes of students who learned Mathematics Foundation after learning axiomatic applying self-assessment; (2) the difficulty of students and the alternative solutions; and (3) the response of students toward Foundation of Mathematics learning taught by applying self-assessment. This research was a descriptive research. The subjects were 25 mathematics students who studied Foundation of Mathematics in odd semester of the 2015/2016 academic year. Data collection was done using questionnaires, and testing methods. Based on the results of data analysis, it can be concluded that the learning outcomes of students were categorized as "good."; Student responses were positive; the difficulties lied in the sub material: Classification of Axiom Systems and the requirements, Theorem and how the formation, and finite geometry. The alternatives deal with these difficulties are to give emphasis and explanation as needed on these materials, as well as provide some more exercises to reinforce their understanding.

1. Introduction
This research was motivated by low performance of students who took Foundations of Mathematics course. Foundations of Mathematics course is basis of all Mathematics courses at the Mathematics Department. The foundation of Mathematics course consists of three topics, namely: (1) Logic, (2) Axiomatic, and (3) Set. The logic and set materials had been studied by students in senior high school, while the axiomatic material is a relatively new for them, and they often have difficulty to learn it. It can be seen from the learning outcomes of the midterm test of axiomatic material is always much lower than the other two topics. As an illustration, in the sub summative test ranged from 0 to 100, the three topics in last year three classes can be presented in the Table 1.

| Classes | Average of test score |
|---------|-----------------------|
|         | Logic | Axiomatic | Set       |
| C       | 70.3  | 44.8      | 68.2      |
| A       | 73.4  | 46.9      | 71.6      |
| U       | 71.1  | 48.7      | 70.0      |

The low scores in axiomatic material test showed that this material was difficult to understand by students. The cause of difficulties experienced by students has not yet known by the lecturers, so the lecturers could not respond optimally. For that, the researcher needs a certain technique to discover more about the causes of the difficulties of students, so that lecturers can determine solution alternatives. Meanwhile, self-assessment is an assessment technique that asks students to assess themselves with regard to the status, process and achievement levels of competency studies. Self-assessment techniques can be used to measure the competence of cognitive, affective and psychomotor [1]. Application of self-assessment can also help lecturers detect students’ difficulties, so they can plan an appropriate technique to overcome the students’ difficulties. Thus they can design a learning process that is expected to
overcome the difficulties of students. In the end, the application of self-assessment in learning process is expected to improve learning outcomes’ students.

Self-assessment is arguably the most powerful means for a tertiary education organisation to understand and improve its educational performance [2]. Besides he also explained that self-assessment is arguably the most powerful means for a tertiary education organization to understand and improve its educational performance. Sitzmann et al also found that many researchers continue to interpret self-assessments of knowledge as a valid indicator of learning [3]. The improvements in the mathematics results indicate that the use by young children of self-assessment techniques appears to be effective in fostering improvement in one area of academic achievement [4]. Moreover, self-assessment is an important skill to develop for lifelong learning [5]. Many researchers are turning to self-assessment as a means of harnessing this self-reflection phase to improve student learning. Researchers generally agree that self-assessment, or self-evaluation, can be defined as students judging their own work, based on evidence and explicit criteria, for the purpose of improving future performance[6,7].

The use of self-assessment as a type of formative assessment for learning is effective [8]. Researchers claim that self-assessment enhances motivation of beginners in a new area of study [9], focuses student attention on learning objectives and the assessments used to ensure them [7], and promotes a mastery goal orientation in which the focus is on improving knowledge, understanding, and skill [6]. Several studies on self-assessment and rubric use document positive effects on student self-efficacy [10, 11]. Challenges to the effective implementation of self-assessment include changing students’ views of their work to that of a set of goals [12], ensuring students have a clear understanding of the learning objectives [13], and allowing enough time for students to adequately develop self-assessment abilities [12]. In order to effectively implement self-assessment in their classrooms, teachers must develop self-assessment procedures that provide students with explicit criteria for evaluating their work, that involve students in the decision-making process, that encourage students to think about the quality of their performance, that manifest realistic goal setting, and that are integrated with the instruction [7].

Meanwhile, the axioms can play many roles in mathematics [14]. In addition, axiomatic is indispensable knowledge in mathematics, it is needed to understand all branches of mathematics, because mathematics is a knowledge using axiomatic deductive systems. Therefore, an understanding of axiomatic is indispensable for students. Thus all students should understand the axiomatic material well. If students have difficulty in understanding the axiomatic material, then most likely they will have difficulty in understanding almost all of mathematics materials.

The objectives of this research are to describe: (a) the learning outcomes’ students at the course of foundations of mathematics after learning axiomatic applying self-assessment; (b) the difficulties’ students and the alternative solutions in the mathematics foundation course after learning axiomatic applying self-assessment; and (c) responses of students who studied foundation of mathematics taught by applying self-assessment.

2. Self Assessment
Self-assessment is an assessment technique that asks students to assess themselves with regard to the status, process and achievement levels of competency studies [1]. Self-assessment techniques can be used to measure the competence of cognitive, affective and psychomotor: (a) Assessment of cognitive competence in the classroom, for example: students were asked to assess the mastery of knowledge and skills of thinking; (b) Assessment of affective competency assessment, for example, students can be asked to create text that contains the outpouring of feelings towards a particular object. Furthermore, students are asked to make an assessment based on criteria or reference were prepared; (c) In connection with psychomotor competency assessment, students can be asked to assess the skills that they have learned based on criteria or reference were prepared.

The use of self-assessment can provide a positive impact on the personality development of students. The use of self-assessment also has some advantages, among others: (a) can foster confidence students, because they were trusted to assess themselves; (b) students realize their strengths and weaknesses, because when they were given the confidence, they should introspect on their strengths and weaknesses;
(c) can encourage, familiarize and train students to be honest, because they are required to be honest and objective in making judgments.

Self-assessment conducted based on clear and objective criteria. Therefore, self-assessment in the classroom should be done through the following steps: (a) Determine the competence or the capability to be assessed, (b) Determine the assessment criteria that will be used, (c) Formulate the assessment format, can be a scoring guidelines, a list of the check, or the scale, (d) Ask students to assess themselves, (e) Examines random sample assessment results, to encourage the students in order to continually assess ourselves accurately and objectively, (f) Give feedback to the students based on results of the study on the samples of assessment results taken randomly.

3. Foundation of Mathematics Material
Foundation of Mathematics is a basic course in the Department of Mathematics, which consists of three main topics, namely: (1) Logic, (2) axiomatic, and (3) Set. In this study, it is limited to the axiomatic material, comprising: (a) Facts, concepts, relations, operations and principles, (b) Inductive and Deductive Mindsets, (c) Undefined term and Initial Statement, (d) Axiom system and the requirements and classification, (e) The types and elements of definition, the intension and extension, (f) Theorem, the elements and formation, and (g) Finite geometry.

An axiomatic system is a list of undefined terms together with a list of statements (called “axioms”) that are presupposed to be “true.” A theorem is any statement that can be proven using logical deduction from the axiom [15]. In order for a set of axioms can constitute a system, the necessary requirements are important. They are: (1) Consistent, (2) Independent, (3) Complete, and (4) Economical.

An axiom system is said to be "consistent" when the statements in a collection of the axioms is not contradictory. Non-contradictory statement is not only in the sense only, but also in terms of terms and symbols used. It is said to be "independent" if each of these statements in a collection of axioms that are not mutually dependent. It means that one of axiom of the system must not be derived or obtained from the other axioms. It is said to be "complete" if each statement which is derived from the system can be proven whether it is true or false. (Of course, it is in the scope of the dichotomous logic). When the axioms in an axiomatic system is not complete, then we can not obtain the theorems. It is said to be "economical" when the symbols or terminology used is not redundant, and all of the statements in the system of axioms have no same meaning.

In every science, there is a way to classify the materials, and each classification has a certain basis. It was held not to be intended to complicate to someone who study it, instead it will be easier for those who study it. In mathematics, it is known some classification of axiom. There are two ways to classify it, namely: (1) axiom of "self evident truth" and "non-self-evident truth", (2) axiom of "material", "formal" and "formalized".

Consider this statement: "The highest judge in mathematics that can be used to determine whether a statement is true or false is the structure. The highest judge in the SCIENCE is Reality".

3.1. The concept and its formation
A concept is an abstract idea that can be used to perform classification of objects. It can be established through an abstraction. As a simple example in daily life we can say that the bicycle, train, car, rickshaw is a vehicle. But houses, trees, stones is not vehicle. It means "vehicle" is a concept. The concept of vehicle can be viewed as an abstraction of a particular vehicle. In mathematics, it is known many concepts. Eg: "triangle", "square" and so on. It is also known the concept of "metric space", "group", and there are many others.

How is the formation of a concept? It can be conducted through: (1) an abstraction, for example: the establishment of concept number though two times of abstraction. (2) an idealization, for example: "flatness" of a plane and "straightness" of a line. (3) abstraction and idealization, for example: the formation of "cube" or "cone" concepts, (4) an additional terms in the previous concept, for example: the formation of "rhombus" from the "parallelogram" concept.

3.2. Definition
Definition of a concept is "an expression that can be used to restrict the concept". There is a freedom to make a definition of a concept, but the most important is the consistency of the definition. There are many types of definitions, namely: (a) **Analytical Definition.** A definition is said to be analytical definition if it is mentioned proximum genus and specific differentiation, (b) **Ginetical Definition.** A definition is said to be ginetical definition if it shows or reveals the occurrence of a defined concept, (c) **Definition using the formula.** A definition is not always expressed in the form of regular expressions. It can also be expressed in mathematical sentences.

### 3.3 Theorem

A theorem is statement that must be proved the correctness. It can be stated as an implication (if . . . then . . .). The specific properties of the theorem is not always obtained by deductive reasoning, but also may be found through field experience or empirical data. However, finally the truth must be proven by deductive thought patterns in the structure.

Thus, a theorem or a specific property can be obtained through inductive steps, and then verified by way of deductive reasoning. The properties of a sequence can be "found" by trial and error, and then can be verified by using mathematical induction. Likewise, some properties or theorems in the theory of network or graph. It has been argued that the form of a theorem is an implication. But there is also in a bi-implication form. In contrast, the definition is the sentence must always be interpreted as a biimplication. It is included the theorem: the "lemma" and "corollary".

### 4. Method

This is a descriptive research. It was conducted at Mathematics Department of Surabaya State University. The subjects were 25 mathematics students who studied Mathematics Foundation in odd semester of the 2015/2016 academic year.

Data collection methods of this research were test and questionnaire. Test was used to obtain data on how was the students’ ability after the learning process of Foundation of Mathematics. Besides, the students’ work also can be used to know the students’ difficulties by seeing which material of the test has been answered incorrectly. There are two questionnaires, namely: (1) Self-assessment questionnaire (2) Responses questionnaire.

The test results were analyzed and determined average score, then it was categorized as follows.

\[
\begin{align*}
60 \leq \bar{x} \leq 100 & \quad \text{good} \\
40 \leq \bar{x} < 60 & \quad \text{good enough} \\
0 \leq \bar{x} < 40 & \quad \text{not good}
\end{align*}
\]

The results of questionnaire were analyzed by grouping the types of students’ difficulties, and then determined the percentage of each types of students’ difficulties. These difficulties were also cross checked with the students’ work in midterm test. The results of responses questionnaire of students were analyzed by calculating percentage of students who gave positive responses, then determined the category of students’ response. The students’ response was said to be positive if at least 70% of the students responded positively to at least 80% aspect in question.

### 5. RESULT AND DISCUSSION

The research was conducted during the course of learning Foundation of Mathematics for material axiomatic that apply self-assessment in Mathematics Education. Learning axiomatic materials was carried out within 3 (three) meetings, on Wednesday, 9, 16 and 23 September 2015. Having carried out three times of learning process, the researcher held sub summative test on September 30, 2015.

The results of the midterm test of Foundations of Mathematics course with the axiomatic material including the category of "Good" with an average score of 75.68. At the end of the second and third meetings of the Foundation of Mathematics learning course, the students were given self-assessment questionnaire of cognitive aspect, and at the end of the third meeting, they were given self-assessment questionnaire of affective aspects. The results of the self-assessment questionnaire of cognitive and affective aspects that are given to all students can presented as follows. All of students have understood the materials: Characteristics of Mathematics, Types of base objects of Mathematics, Inductive Mindset,
Undefined Term and Initial Statement. About 88% students have understood “Structure of Axiomatic Deductive”, 68% students have understood System of axioms and the requirements, and 75% have understood Classification of axioms.

The most difficult of the eight sub-topics are “Classification of Axiom”, “System of axioms and the requirement”, and “Theorems and the formation”. The reasons given were: (1) Because it is a new material, do not understand the material, (2) Because the material is difficult to understand, confusing material, (3) Because if the problem was changed, it often made be confused.

Almost all the students can understand the material being taught. Even for sub-topics 1 to 4 and sub-topic 6, all students (100%) can understand the material being taught. However, there are some students (72%) stated that among the eight topics covered, the material “Classification Axiom” is the most difficult material, (32%) stated that System of axioms and the requirement is the most difficult material, while as many as 20% of the students stated that "theorems and the formation" material is the most difficult material. Based on these results, the researcher tried to give emphasis and explanations as necessary on three materials, as well as provide some more exercises to strengthen their understanding. Furthermore, based on the results of the self-assessment questionnaire of affective, it can be stated that all of students do the tasks given, pay attention to the direction given faculty, try to follow well the learning process of Mathematics Foundation for axiomatic material. Most of them (80.95%) said that during the discussion groups, they listen and pay attention if their friends or lecturer speak.

It also be stated that the attitudes of students towards learning course Foundations of Mathematics for axiomatic materials are positive, because 100% aspects of their self-assessment items have been responded positively by the students. Based on data analysis presented in Table 6, it can be stated that the students’ response is "positive," because every item of questionnaire was responded positively at least 71.43% of students.

Based on all data analysis, it can be stated that the students learning outcomes at the Foundations of Mathematics course after learning that applied self-assessment was categorized as "good". Difficulties of students are in sub-material: Axiom, Classification of Axiom Systems and the requirements; theorem, the components and formation, as well as finite geometry. Many students confuse to differentiate between definition and theorem. It seemed when they were asked to provide an example of a theorem, they provide example of definition and vice versa. The alternative solutions are to give emphasis and explanations as necessary on both the materials, as well as provide some more exercises to strengthen their understanding. The response of students toward Mathematics Foundation for axiomatic material that is performed by applying self-assessment is positive.

6. Conclusion and Recommendation
Based on the research result, it can be concluded that self-assessment is effective used to improve students’ performance. Self-assessment can help lecturers detect students’ difficulties, so they can plan an appropriate technique to overcome the students’ difficulties.

Based on the conclusion, there are two recommendations to the lecturers: (1) to apply self-assessment in their lessons; (2) to overcome the difficulties experienced by the students as soon as possible.

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