Memory effect and BMS Symmetries for extreme black holes

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Abstract

We study horizon shells and soldering freedom for extreme black holes and show BMS like symmetries appear as soldering transformations. We determine the memory effect by considering the interaction of impulsive gravitational waves contained in horizon shells of extreme Reissner-Nordström black hole with test detectors. It is shown, test geodesics encode a memory of BMS symmetries into the components of the deviation vector and optical scalars for timelike and null test detectors respectively. We also discuss how supperrotation like symmetry can be recovered as soldering symmetry when one zooms in to the near-horizon region of extreme black holes.

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I. INTRODUCTION

Gravitational memory effect has been an active area of research and it has attracted researchers across the disciplines from classical gravity, gravitational wave astronomy [1–5, 12] to researchers in the area of quantum gravity [6, 7, 14, 27–29]. The feature that characterizes memory effect is a permanent displacement of test particles after the passage of a burst of gravitational waves. Memory effect may prove to be very promising field where many predictions of classical General Relativity (GR) can be tested in the coming advanced detectors like advanced LIGO or LISA [10, 11]. On the other hand its connection with Soft theorems and asymptotic symmetries of spacetimes have given an intriguing chance to dig into quantum structure of gravity in the low energy or infrared limit [6, 7, 14, 15].

Extreme black holes are important to the GR, Astrophysics and String community. Primarily because of its relevance in the calculation of black hole entropy, it got significant attention to the String theorists [9]. Extreme black holes allow conformal symmetry when we zoom in to its near horizon geometry, and this played a crucial role not only in the study of quantum black holes but also finding new kind of hairs in extreme black holes [8]. As it has been studied that almost 70% of the astrophysical black holes are near extremal and many super-massive black holes are also near extremal, more attention is needed to pay to explore the properties of these black holes.

In earlier studies [21, 22], it was shown that BMS like [13] asymptotic symmetries can be recovered at the horizon shell of black holes when one demands the induced metric on the horizon remains invariant under arbitrary coordinate transformations owing to satisfy the junction condition. These shells usually become history of an Impulsive Gravitational Wave (IGW) supported at the horizon. The corresponding memory effect has been discussed in [23]. Similar kind of studies in the context of plane-gravitational waves can be found in [28, 29, 31]. In this note, we explore the horizon shell in extreme black holes and the memory effect encoded in the geodesics crossing the shell. We again find the BMS like asymptotic symmetries are recovered at the horizon of Extreme Reissner-Nordst"orm (ERN) and Extreme-BTZ black holes. We also find the shell-intrinsic quantities in different situations. The recent extended version of BMS symmetry also contains superrotation symmetries which is defined as arbitrary conformal transformations on celestial sphere [16]. As near horizon ERN is described by an $AdS_2 \times S_2$ metric, it is shown how superrotation kind of symmetries in the form of conformal transformations are recovered near the horizon of ERN. We also touch upon the extreme BTZ case and discuss the soldering symmetries.

Next, we have studied interaction of impulsive gravitational waves (IGW)[32]. We show how BMS-like symmetries are encoded in the deviation vectors for a timelike congruence crossing the shells. We also show the jump in optical scalars for null geodesic congruence crossing the shell for an ERN black hole. The memory in all these cases is parametrized by BMS transformation parameters. In the end we briefly discuss the implication of Penrose limits in detection of memory effect.

Let us briefly summarize how the draft has been organized. In the next section we briefly touch upon the formulation based on which we would calculate the shell intrinsic properties. In the next section we study the horizon shell in ERN spacetime and calculated the intrinsic properties of the shell. The connection of ERN shell with conformal symmetries is elucidated via studying shells in the near horizon $AdS_2 \times S_2$ form of the metric. Next, horizon shells of
extreme BTZ black holes are discussed. The computational details are displayed in appendix. In the next two sections memory effect for timelike and null detectors are discussed for ERN spacetimes. Finally, we conclude the report with a discussion on the results and indicated some future outlooks.

II. HORIZON SHELLS, IGW AND BMS LIKE SOLDERING FREEDOM

We briefly discuss how IGW arise in the context of gluing two manifolds $\mathcal{M}_+\text{ and }\mathcal{M}_-$ across a common null hypersurface $\Sigma$. Let us setup the coordinates for $\mathcal{M}_\pm$ as $x_\mu^\pm$ with metrics $g_{\mu\nu}^\pm(x_\mu^\pm)$ and a common coordinate system $x_\mu$ across the null hypersurface $\Sigma$. We shall use Latin letters to denote hypersurface index and greek letters for spacetime indices. We denote $y^a$ as coordinates on the hypersurface with tangent vectors $e_\mu^a = \frac{\partial x_\mu}{\partial y^a}$. The hypersurface is defined as $\Phi(x) = 0$. Define the normal vector $n_\mu$ to the hypersurface as $n_\mu = g_{\mu\nu} \partial_\nu \Phi(x)$. We also define an auxiliary normal vector $N_\mu^\pm$, with normalization conditions $N_\mu^\pm N_\mu^\pm = 0$, $n_\mu N_\mu^\pm = -1$, and $e_\mu^A N_\mu^\pm = 0$, to study the extrinsic properties of the shell which completes the basis as well. In terms of common coordinate system, we have $[n_\mu] = [e_\mu^a] = [N_\mu^\pm] = 0$. The metric takes the following form in in this common coordinate system

$$g_{\mu\nu} = g_{\mu\nu}^+ \mathcal{H}(\Phi) + g_{\mu\nu}^- \mathcal{H}(-\Phi)$$

(II.1)

Where $\mathcal{H}(\Phi)$ is Heaviside step function. And the junction condition is,

$$[g_{ab}] = g_{ab}^+ - g_{ab}^- = 0$$

(II.2)

Using junction condition, we have,

$$R_{\nu\rho\sigma} = R_{\nu\rho\sigma}^+ \mathcal{H}(\Phi) + R_{\nu\rho\sigma}^- \mathcal{H}(-\Phi) + Q_{\nu\rho\sigma}^\mu \delta(\Phi)$$

(II.3)

Where any non-zero component of the last term $Q_{\nu\rho\sigma}^\mu$ implicates the existence of IGWs on the null hypersurface. The full stress-energy tensor is given by,

$$T_{\mu\nu} = T_{\mu\nu}^+ \mathcal{H}(\Phi) + T_{\mu\nu}^- \mathcal{H}(-\Phi) + S_{\mu\nu} \delta(\Phi)$$

(II.4)

Where the stress-energy tensor $S_{\mu\nu}$ can be written as

$$S_{\mu\nu} = \mu n_\mu n_\nu + J^A (n_\mu e^\nu_A + n_\nu e^\mu_A) + p \sigma^{AB} e^A_\mu e^\nu_B$$

(II.5)

The $S_{\mu\nu}$ projected on $\Sigma$ is $S_{ab} = e^A_a e^B_b S_{\mu\nu}$. Where $\sigma^{AB}$ is non-degenerate metric for the spatial slice of the surface of the null shell; $A$ and $B$ denote the spatial indices. The physical intrinsic quantities of the shell are surface energy density, surface current and pressure dented as $\mu$, $J^A$ and $p$ respectively, have the following form[33–36],

$$\mu = -\frac{1}{8\pi} \sigma^{AB} [\mathcal{K}_{AB}] \quad J^A = \frac{1}{8\pi} \sigma^{AB} [\mathcal{K}_{VB}] \quad p = \frac{1}{8\pi} [\mathcal{K}_{VV}]$$

(II.6)

The properties of the shell are stored in $[\partial_\alpha g_{\mu\nu}] = \gamma_{\mu\nu} n_\alpha$ which says that the jump in the partial derivative of the metric is proportional to $\gamma_{\mu\nu}$. Furthermore, extrinsic curvature is related to induced metric $\gamma_{ab}$ on the shell in following way,

$$\gamma_{ab} = N_\mu [\partial_\mu g_{ab}] = 2[\mathcal{K}_{ab}]$$

(II.7)
Together with,
\[ \mathcal{K}_{ab} = e^\mu_a e^\nu_b \nabla_\mu N_\nu \] (II.8)
\[ \gamma_{ab} \] are related to transverse traceless components of \( \hat{\gamma}_{ab} \) by following relation,
\[ \hat{\gamma}_{ab} = \gamma_{ab} - \frac{1}{2} \gamma^* g_{ab} + 2 \gamma(a N_b) + \left( N_a N_b - \frac{1}{2} N.N g_{ab} \right) \gamma^\dagger \] (II.9)
Where, \( \gamma^\dagger = \gamma_{ab} n^a n^b \) and \( \gamma^* = g^{AB} \gamma_{AB} \)

Now let us touch upon how BMS-like transformations occur when we glue two spacetimes across \( \Sigma \). The details of the same can be seen in [21–23]. In general, BMS symmetries arise when we patch two spacetimes across the null hypersurface as the freedom of allowing the null direction to change by arbitrary diffeomorphisms which preserve the induced metric. These possible transformations thus can be calculated by solving the Killing equation on the hypersurface [21]. So we require,
\[ \mathcal{L}_Z g_{ab} = 0. \] (II.10)
Expanding this equation,
\[ Z^V \partial_V g_{AB} + Z^C \partial_C g_{AB} + \partial_A Z^C g_{BC} + \partial_B Z^C g_{AC} = 0, \] (II.11)
Considering that the metric \( g_{AB} \) to be independent of \( V \), \( Z^V \) remains unconstrained and can be of the form \( Z^V = F(V, \theta, \phi) \). Further, if we set \( \mathcal{L}_Z n^a = 0 \), one gets a restriction on \( Z^V \).
\[ \partial_V Z^V = 0 \Rightarrow Z^V = F(x^A) \] (II.12)
It generates supertranslation-like transformation which can be written as,
\[ V \to V + F(x^A) \] (II.13)
In [22], it was also if one relaxes the condition that \( g_{AB} \) is independent of \( V \), one can get superrotation like symmetries as conformal transformation. Next, we shall study the intrinsic properties of ERN and BTZ black hole.

**III. INTRINSIC PROPERTIES OF EXTREME RN BLACK HOLE**

In this section, we follow the recipe indicated in [22, 23]. We have a seed metric which is non-transformed in a given manifold \( \mathcal{M}_- \) and consider a metric for \( \mathcal{M}_+ \) manifold on which we do a supertranslation type coordinate transformation. The ERN metric in Eddington-Finkelstein (EF) coordinates for \( \mathcal{M}_- \) manifold is given by,
\[ ds^2 = -\left(1 - \frac{M}{r}\right)^2 dv^2 + 2dvdr + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \] (III.1)
We perform the supertranslation type transformation on \( v_+ \) coordinate and keep other coordinates unaltered,
\[ v_+ = v + F(\theta, \phi) \;; \; r_+ = r \;; \; \theta_+ = \theta \;; \; \phi_+ = \phi \] (III.2)
Under these transformations, the metric takes the following form,

\[ ds_+^2 = -\left(1 - \frac{M}{r}\right)^2 (dv + F_\theta(\theta, \phi)d\theta + F_\phi(\theta, \phi)d\phi)^2 + 2(dv + F_\theta(\theta, \phi)d\theta + F_\phi(\theta, \phi)d\phi)dr + r^2d\Omega_2^2 \]  

(III.3)

Where the horizon is situated at \( r = M \). For our choice of the auxiliary normal vector \( N_v = -1 \), satisfying normalization conditions, we use (II.8) to compute the extrinsic curvature \( K_{\theta\theta}, K_{\phi\phi} \) and \( K_{\theta\phi} \) on the horizon. Finally using (II.7), we get expressions for \( \gamma_{ab} \),

- \( \gamma_{\theta\theta} = 2F_\theta \)
- \( \gamma_{\phi\phi} = 2(F_{\phi\phi} + F_\theta \sin \theta \cos \theta) \)
- \( \gamma_{\theta\phi} = \gamma_{\phi\theta} = 2(F_{\theta\phi} - F_{\phi} \cot \theta) \)

Now using (II.9), one can also compute transverse traceless part of \( \gamma_{ab} \),

- \( \hat{\gamma}_{\theta\theta} = F_{\theta\theta} - \frac{F_{\phi\phi}}{\sin^2 \theta} - F_{\theta} \cot \theta \)
- \( \hat{\gamma}_{\phi\phi} = F_{\phi\phi} + F_{\theta} \sin \theta \cos \theta - F_{\theta\theta} \sin^2 \theta \)
- \( \hat{\gamma}_{\theta\phi} = \hat{\gamma}_{\phi\theta} = 2(F_{\theta\phi} - F_{\phi} \cot \theta) \)

Here, we observe that if one makes a constant shift in \( V_+ \) with a constant \( C \) i.e. \( V_+ = V + C \), \( \hat{\gamma}_{ab} \) vanishes. This is also called as Dray-'t Hooft shell [24]. We will come back to this in detail later.

A. Shell-Intrinsic properties

With the help of extrinsic curvature expressions, we directly compute the surface energy density of the shell as

\[ \mu = -\frac{1}{8M^2\pi} \triangle^{(2)} F(\theta, \phi) \]  

(III.4)

And the surface current \( J^A \) and pressure of the shell is given by,

\[ J^A = 0 \quad ; \quad p = 0 \]  

(III.5)

Interestingly, the current turns out to be zero in EF coordinates unlike the Schwarzschild or non-extreme case [21]. It is easy to see from (III.4), the energy density is conserved along the null direction of the shell i.e. \( \partial_\mu \mu = 0 \). However, there is no finite charge corresponding to this supertranslation like BMS translation, as it vanishes when evaluated on the spherical surface.

Now let us try to see if we can get a shell without matter supporting only gravitational waves. For this one needs to see if there is any regular solution of the equation obtained by setting \( \mu \) equal to zero.

\[ \triangle^{(2)} F(\theta, \phi) = 0 \]  

(III.6)
This is a Laplace’s equation on a sphere. We know this Laplacian has spherical harmonics $Y^m_l(\theta, \phi)$ as eigenfunctions with $-l(l+1)$ as eigenvalues. However, here we have only $l=0$ as a feasible solution and that corresponds to a constant only. Therefore, the allowed shift in the $v$ direction is of the form-

$$v \to v + c.$$  

(III.7)

Where $c$ is a constant. As we have already mentioned, this correspond to Dray-’t Hooft shell [24]. Direct substitution of this into the components of $\hat{\gamma}_{ab}$ yields zero. So there can’t be a shell supporting pure gravitational waves. A similar situation also obtained for Schwarzschild case [21]. Next, we would like to see if conformal symmetries can be recovered as soldering freedom in ERN spacetime. To see this one must remember that shells in constant curvature spacetimes can produce conformal isometries as soldering freedom when Penrose’s cut-and-paste approach is employed [32, 33]. The basic technique is to glue two metrics across a null surface after performing a conformal transformation and a subsequent shift in the null direction on one side and then attaching the other side. Penrose’s plane-wave and spherical wave generation out of flat Minkowski space are the prominent examples. It is to be remembered that in those cases, one also generates shells with purely impulsive gravitational wave without any matter [32]. In the following subsection we try to find conformal symmetries in a near horizon version of ERN metric.

B. ERN black hole in $AdS_2 \times S^2$ form

In this section, we consider a near horizon limit of extreme RN which has $AdS_2 \times S^2$ form. This means, the near horizon geometry is conformally flat. As $AdS_2$ contains conformal symmetries as its isometries, this study may be more illuminating than the earlier. We shall start with the non-extreme metric written as following,

$$ds^2 = \left( \frac{(r - r_+)(r - r_-)}{r^2} \right) dt^2 + \left( \frac{(r - r_+)(r - r_-)}{r^2} \right)^{-1} dr^2 + r^2 d\Omega^2_2$$

(III.8)

Now consider the near extremal limit of the metric as $\lambda = \frac{r-r_0}{r_0} \ll 1$. Where $r_0 = M$ is the horizon of the extreme RN. In this limit, the metric takes the following form [18, 19],

$$ds^2 \simeq -\lambda^2 dt^2 + \frac{r_0^2}{\lambda^2} d\lambda^2 + r_0^2 d\Omega^2$$

(III.9)

It is to remember, this static coordinates are not global coordinates and they only cover a portion of entire AdS space. In this form $r_0$ is playing the role of radius of curvature for both the $AdS_2$ or $S^2$ part. If one now Further, transforms to ingoing Eddington-Finkelstein coordinates by

$$v = t - \frac{1}{\lambda},$$

(III.10)

and sets $M = 1$,the metric for $M_-$ manifold becomes ,

$$ds^2 = -\lambda^2 dv^2 + 2dv d\lambda + (d\theta^2 + \sin^2 \theta d\phi^2).$$

(III.11)
For this metric, the auxiliary normal is $N_v = -1$ and we define the similar coordinate transformation for supertranslation mentioned in (III.2). The supertranslated metric has the following form,

$$ ds^2 = -\lambda^2 (dv + F_\theta(\theta, \phi) d\theta + F_\phi(\theta, \phi) d\phi)^2 + 2(dv + F_\theta(\theta, \phi) d\theta + F_\phi(\theta, \phi) d\phi) d\lambda + d\Omega_2^2 $$  

(III.12)

Employing the same method as described in the last subsection we derive the surface energy density,

$$ \mu = -\frac{1}{8\pi} \Delta^{(2)} F(\theta, \phi) \quad (\text{III.13}) $$

We show that at the horizon shell placed on the Poincare Killing horizon $\lambda = 0$, the surface energy density takes the same form as it appears for ERN in the previous section. Again the shell doesn’t allow pure IGW. But here we should have a direct connection with the conformal transformation as the underlying spacetime obeys the conformal symmetry. To see that, we may parametrize the compact part of the metric in terms of complex coordinates \(^1\) The metrics at the two sides with two different radii $r_{\pm}$ should match on the hypersurface means, in terms of stereographic coordinates for 2 sphere,

$$ r_{\pm}^2 \gamma_{zz} dz_+ d\bar{z}_+ = r_{\pm}^2 \gamma_{zz} dz_+ d\bar{z}_+. \quad (\text{III.14}) $$

Here $r_{\pm}/M_{\pm}$ are the radial coordinates corresponding to the two metrics across the shell and $\gamma_{zz}$ is the round metric on unit sphere. Now, one can make a $z_+ \to w(z_+)$ transformation on the + side of sphere and compensate that by a corresponding shift (angle dependent) to $M_{\pm}$ by expressing it as a function of $M_{-}, w(z_-), \bar{w}(\bar{z}_-)$. In this process one introduces superrotation (local conformal transformations) like symmetries on the shell. It will be interesting to study memory in this kind of frameworks [41].

**Extreme BTZ:**

In the case of extreme BTZ black hole, we also observed there can’t be a non-trivial shell without matter supporting pure IGW. However unlike ERN, for zero surface energy density, one does not get harmonic solutions. For a non-rotating extreme BTZ black hole, the solution is logarithmic and the periodicity of $\phi$ will spoil the solution to be meaningful see Appendix A. It seems in 3 dimension one can’t make any construction by which something similar like superrotations can be induced. Even one uses the near horizon metric which will be of the $AdS_2 \times S^1$ form or similar, due to the periodic identification of angular coordinate, the construction will not produce a feasible solution.

**IV. MEMORY EFFECT: EXTREME RN BLACK HOLE**

In this section, we shall study the memory effect for ERN black hole on timelike geodesics. We will adopt a Kruskal extension that unambiguously places the shell at $U = 0$, and also better suited for memory effect analysis. One can obviously try the same thing with EF

\(^1\) This is discussed for constant curvature spaces in [33], and recently in [22], in the context of black holes.
coordinates. We would consider a Kruskal metric that is regular on the horizon\cite{25}. The following transformations for the metric (III.1) are to be made,

\[ u = -\psi(-U) \quad ; \quad v = \psi(V) \quad (IV.1) \]

Where we consider \( \psi(V) \) to be of the form,

\[ \psi(V) = 4M\left(lnV - \frac{M}{2V}\right) \quad (IV.2) \]

We also observe that the tortoise coordinate has a pole term which makes the metric singular on the horizon in traditional Kruskal transformation.

\[ r_* = r + 2M\left(ln(r - M) - \frac{M}{2(r - M)}\right) + \text{constant} \quad (IV.3) \]

Near the horizon,

\[ r_* \sim \frac{1}{2}\psi(r - M) \quad (IV.4) \]

Under this assumption, metric is given by,

\[ ds^2 = -\frac{(r - M)^2}{r^2}\psi(-U)\psi(V)'dUdV + r^2d\Omega_2^2 \quad (IV.5) \]

Where prime denotes the derivative with respect to \( V \) and \( d\Omega_2^2 \) is 2-sphere metric. The transformations are not well defined if the metric is degenerate on the horizon. However, we construct the asymptotic form of the metric which assumes that asymptotically, one can have \( t \sim r_* \) then \( u \sim -2r_* \sim -\psi(r - M) \). Therefore, the inverse transformation is,

\[ U = -\psi^{-1}(-u) \sim -\psi^{-1}(\psi(r - M)) = -(r - M) \quad (IV.6) \]

Together with,

\[ \psi(-U)' \sim \frac{4M}{r - M} + \frac{2M^2}{(r - M)^2} \quad (IV.7) \]

And \( \psi(V)' \) is regular as it is finite and non-zero everywhere. Thus we have the asymptotic form of the metric for which Kruskal coordinates are well defined on the horizon. The metric is written as \cite{25}

\[ ds^2 = -\frac{2M^2}{r^2}\psi(V)'dUdV + r^2(U)d\Omega_2^2 \quad (IV.8) \]

We would consider this metric and study the off-shell extension of the transformations and memory effect. The construction here follows the one considered in \cite{23}
A. Off-shell extension of the soldering transformation

We now extend the soldering transformation off the shell in $\mathcal{M}_+$ manifold for extreme RN black hole to the linear order of $U$ such that metric components remain continuous across the shell. Here $U = -(r - M)$ and we see that on the horizon $r = M$, $U = 0$. Here we follow the method adopted in [21, 23]. The off-shell soldering transformations to the linear order in $U$ is given by,

$$U_+ = UC(V, x^A) \ ; \ V_+ = F(V, x^A) + UA(V, x^A) \ ; \ x^i_+ = x^i + UB^i(V, x^A) \quad (IV.9)$$

We first need to determine functions $A(V, x^A)$, $C(V, x^A)$ and $B(V, x^A)$. For this, we take transformed metric and compare $g_{\alpha\beta}$ components with the non-transformed metric. We determine,

$$C = \frac{\partial_V \psi(V)}{\partial_V \psi(F)} \quad (IV.10)$$
$$A = \frac{M^2}{2} \frac{F_V}{\partial_V \psi(V)} \sigma_{AB} B^A B^B \quad (IV.11)$$
$$B^A = \partial_V \psi(V) \frac{1}{M^2 F_V} \sigma^{AB} F_B \quad (IV.12)$$

Here we specialize our calculation for BMS case $V \rightarrow V + F(\theta, \phi)$. Now we can explicitly write the metric component wise.

$$Ug^{(1)+}_{ab} dx^a dx^b = 2M^2 U \left[ \left( - \frac{\psi(V)''}{M^2} + \frac{\psi(V)'}{M^2 \psi(F)} \psi(F)'' \right) dV^2 + \frac{\psi(V)'}{M^2} \left( \frac{\partial_A \psi(F)}{\psi(F)} \right)' + F_A \frac{\psi(F)''}{\psi(F)} \right] dV dx^A$$
$$+ \frac{\psi(V)'}{M^2} \left( F_{AB} + F_B \frac{\partial_A \psi(F)'}{\psi(F)} - \frac{M \sigma_{AB}}{\psi(F)} \right) dx^A dx^B$$
$$- \frac{2}{M^2} \frac{\psi(V)'}{\psi(F)} F_\phi \cot \theta d\theta d\phi + B^a \sin \theta \cos \theta d\phi^2 \right] \quad (IV.13)$$

Where $\psi(F)'$ denotes derivative with respect to function $F$ itself and $\psi(V)'$ is for derivative with respect to $V$. Whereas for $\mathcal{M}_-$ manifold, the metric is

$$Ug^{(1)-}_{ab} = -2MU \sigma_{AB} dx^A dx^B \quad (IV.15)$$

Since the metric is written in $(U, V, \theta, \phi)$ and at $r = M \Rightarrow U = 0$. Keeping in mind the normalization conditions $n^\mu n_\mu = 0$ and $n^\mu N_\mu = -1$, we have auxiliary normal $N^U = 1$ to the hypersurface. Now we can extract all components of $\gamma$-tensor using,

$$\gamma_{ab} = N^\alpha [\partial_\alpha g_{ab}] \quad (IV.16)$$
The $\gamma_{ab}$ components are given by,

\[ \gamma_{VV} = 2 \left( -\psi(V)'' + \frac{\psi(V)'^2}{\psi(F)'} \psi(F)'' \right) \]  
\[ (IV.17) \]

\[ \gamma_{VA} = 2 \frac{\psi(V)'}{\psi(F)'} F_A \]  
\[ (IV.18) \]

\[ \gamma_{\theta\theta} = 2 \psi(V)' \left( F_{\theta\theta} + \frac{F_{\theta}^2 \psi(F)''}{\psi(F)'} - \frac{M \sin^2 \theta}{\psi(F)'} + F_{\theta} \sin \theta \cos \theta - \frac{M \sin^2 \theta}{\psi(V)' \psi(F)'} \right) \]  
\[ (IV.19) \]

\[ \gamma_{\phi\phi} = 2 \psi(V)' \left( F_{\phi\phi} + \frac{F_{\phi}^2 \psi(F)''}{\psi(F)'} - \frac{M}{\psi(F)'} F_{\phi} \sin \theta \cos \theta - \frac{M \sin^2 \theta}{\psi(V)' \psi(F)'} \right) \]  
\[ (IV.20) \]

\[ \gamma_{\theta\phi} = 2 \psi(V)' \left( \frac{F_{\theta} F_{\phi}}{\psi(F)'} \psi(F)'' + F_{\theta\phi} - F_{\phi} \cot \theta \right) \]  
\[ (IV.21) \]

\[ \gamma_{\phi\theta} = 2 \psi(V)' \left( \frac{F_{\phi} F_{\theta}}{\psi(F)'} \psi(F)'' + F_{\phi\theta} - F_{\theta} \cot \theta \right) \]  
\[ (IV.22) \]

Now we can directly compute the transverse traceless components of $\hat{\gamma}_{ab}$, for spherical part,

\[ \hat{\gamma}_{\theta\phi} = \hat{\gamma}_{\phi\theta} = \gamma_{\theta\phi} \]  
\[ (IV.23) \]

\[ \hat{\gamma}_{\theta\theta} = \frac{1}{2} \left( \gamma_{\theta\theta} - \frac{\gamma_{\phi\phi}}{\sin^2 \theta} \right) \]  
\[ (IV.24) \]

\[ \hat{\gamma}_{\phi\phi} = \frac{1}{2} \left( \gamma_{\phi\phi} - \gamma_{\theta\theta} \sin^2 \theta \right) \]  
\[ (IV.25) \]

Other components can also be calculated in the same way. We can estimate surface energy density in the following manner,

\[ \mu = - \frac{1}{16 \pi M^2} \left( \gamma_{\theta\theta} + \frac{1}{\sin^2 \theta} \gamma_{\phi\phi} \right) \]  
\[ (IV.26) \]

Therefore,

\[ \mu = - \frac{\psi(V)'}{8 \pi M^2} \left( \triangle^{(2)} F(\theta, \phi) - 2M \left( \frac{1}{\psi(F)'} + \frac{1}{\psi(V)'} \right) + \left( \frac{F_{\theta}^2}{\sin^2 \theta} + \frac{F_{\phi}^2}{\psi(F)'} \right) \psi(F)'' \right) \]  
\[ (IV.27) \]

The surface current and surface pressure has the following form,

\[ J^A = \frac{1}{8 \pi M^2} \delta^{AB} \left( F_B \psi(F)'' \right) \]  
\[ (IV.28) \]

\[ p = - \frac{1}{8 \pi (\psi(V))' \psi(V)''} \left( -\psi(V)'' + \frac{\psi(V)'}{\psi(F)'} \psi(F)'' \right) \]  
\[ (IV.29) \]

Where $\delta^{AB}$ is inverse of the unit sphere metric. Here unlike the EF shell we get a finite current and pressure.

**B. Memory effect for timelike geodesics**

Let us consider two timelike geodesics crossing the null hypersurface supporting IGWs. Here we briefly review the interaction of IGW and timelike geodesics [34]. We study the
measurable effects between two nearby geodesics before and after the passage of the IGWs. We observe the change in the deviation vector between the two adjacent geodesics. This amounts to solve the geodesic deviation equation together with junction conditions. Here we work in a local coordinate system in which the metric is continuous but its first derivative is discontinuous across the null surface. Further details can be found in \[23, 33, 34, 38–40\]. Let us consider \( T^\mu \) be a unit timelike vector field with,

\[ g_{\mu\nu}T^\mu T^\nu = -1 \]  

(IV.30)

Integral curve of \( T^\mu \) passes through the null shell situated at \( \Sigma \). Consider the deviation vector between two nearby timelike geodesics is \( X^\mu \) with \( g_{\mu\nu}T^\mu X^\nu = 0 \). The geodesic equation for \( X^\mu \) is

\[ \ddot{X}^\mu = -R^\mu_{\rho\sigma\delta}T^\rho X^\sigma T^\delta \]  

(IV.31)

We decompose \( \gamma_{ab} \) of the hypersurface \( U = 0 \) into transverse and traceless part in order to see the effect of wave and shell separately

\[ \gamma_{ab} = \tilde{\gamma}_{ab} + \bar{\gamma}_{ab} \]  

(IV.32)

where,

\[ \bar{\gamma}_{ab} = 16\pi \left( g_{ac}S^{cd}N_dN_b + g_{bc}S^{cd}N_dN_a - \frac{1}{2}g_{cd}S^{cd}N_aN_b - \frac{1}{2}g_{ab}S^{cd}N_cN_d \right) \]  

(IV.33)

From here, we obtain,

\[ \tilde{\gamma}_{VB} = 16\pi g_{BC}S^{VC}; \quad \bar{\gamma}_{AB} = -8\pi S^{VV}g_{AB} \]  

(IV.34)

Where \( \tilde{\gamma}_{VB} \) is symmetric in lower indices. We impose a condition on the test particles that initially they reside on 2-dimensional surface. Thus we set \( X_{(0)V} = V_{(0)V} = 0 \), where \( X_{(0)\mu} \) are components of deviation vector before the passage of the IGW at the horizon, and \( V_{(0)a} = \frac{dX_{(0)a}}{dU} \bigg|_{U=0} \). Finally using (IV.34),

\[ X_V = \frac{1}{2}U\tilde{\gamma}_{VB}X_{(0)}^B = 8\pi U g_{BC}S^{VC}X_{(0)}^B \]  

(IV.35)

\[ X_A = X_{A(0)} + \frac{U}{2}\gamma_{AB}X_{(0)}^B + UV_{(0)}^A \]  

(IV.36)

\( \gamma_{AB} \) term represents the distortion effect of the wave on the test particle. Note that, in the \( X_V \) part of the equation, if the surface current is non-zero i.e. \( S^{VC} \neq 0 \) then \( X_V \neq 0 \). Which means the test particle will no longer reside on 2-dimensional surface. It gets displaced off the surface. We also observe that in the case under study we have a non-zero current or non-zero \( S^{VC} \). Hence the effect of passage of impulsive wave is to deflect the particles off the 2-d surface.

The expression for the \( X_\theta \) component of the deviation vector.

\[ X_\theta = X_{\theta(0)} + \frac{U}{2}\left( \gamma_{\theta\theta}X_{(0)}^\theta + \gamma_{\theta\phi}X_{(0)}^\phi \right) + UV_{(0)}^\theta \]
The $X_\phi$ component can also be recovered in the same way. The expressions of $\gamma_{AB}$ are to be replaced from above. Thus the deviation between two timelike geodesics is determined in terms of supertranslation parameter $F(\theta, \phi)$ contained in $\gamma_{AB}$. We can integrate these respective deviation equations and obtain the shift with respect to the parameter of the geodesics, gives us displacement memory effect. For physical shells having $S^{VV} > 0$, we can also see there is a diminishing effect for the transverse components of the deviation vectors carrying the BMS like memories.

V. B-TENSOR AND NULL GEODESICS

In this section, we shall study the effect of IGWs on null geodesics passing orthogonally through the hypersurface. We would estimate the jump in optical parameters such as shear, expansion. Our analysis is described in detail in [23, 31]. Let us consider a null congruence whose tangent vector is denoted by $N$ and it is orthogonally crossing the hypersurface $\Sigma$ supporting IGWs. The normalization condition will remain same as $n.N = -1$, $n.n = 0$ and $N.e = 0$. At first, we compute $N_+$ to the $'+'$ side of the null shell in common coordinate system and similarly $N_-$ for the $'-'$ side of the null shell.

The major object of interest is the failure tensor or the $B$-tensor with respect to the vector $N_0$, expressed in the continuous coordinate at the shell.

$$\tilde{B}_{AB}(x^\mu) = \frac{\partial x^M_0}{\partial x^A} \frac{\partial x^N_0}{\partial x^B} e^\alpha_A e^\beta_B \nabla^\beta N_0(\mu) \tag{V.1}$$

Here the coordinates $x_0$ are designated for on the hypersurface and $x^\mu = x_0 + U N_0^\mu(x^\mu)$. We first compute $B$-tensor off the shell and pull it back to an infinitesimal away point from $\Sigma$. We expect a non-vanishing change in optical parameters which depicts the memory of IGWs.

A. Memory effect for null geodesics

We consider extreme RN black hole in Kruskal coordinates in asymptotic form of the metric given by eqn.(IV.8). The initial tangent vector to the null congruence is $N_+ = \lambda \partial_U$. Components of congruence $N_0$ on hypersurface in continuous coordinates are given by the inverse Jacobian transformation (see Appendix B),

$$N^\alpha_0 = \left(\frac{\partial x^\beta_+}{\partial x^\alpha} \right)^{-1} N^\beta_+ \big|_{\Sigma} \tag{V.2}$$

We first find the tangent vector at the hypersurface,

$$N_0 = \lambda \left( \frac{F_\phi}{\psi(V)^\prime} \right) \partial_U - \frac{(\psi(V)^\prime - 2)}{2M^2F_V} \left( F_\theta^2 + \frac{F_\phi^2}{\sin^2 \theta} \right) \psi(V)^\prime \partial_V - \frac{F_\theta \psi(F)^\prime}{M^2} \partial_\theta - \frac{F_\phi \psi(F)^\prime}{M^2 \sin^2 \theta} \partial_\phi \bigg|_{\Sigma} \tag{V.3}$$

From $U$-component of $N_0$, we determine,

$$\lambda = \frac{\psi(V)^\prime}{\psi(F)^\prime F_V} \tag{V.4}$$
Using eqn. (V.2) and $N_0$ expression, we get,

$$B_{AB} = 2\frac{\psi(V)'}{F_V^2} F_A F_B V - \frac{\psi(V)'}{F_V^3} F_A F_B F_V^2 - \frac{\psi(V)'}{F_V} F_{AB} + \frac{F_A F_B}{F_V^2} \psi(V)'' - \Gamma^\delta_{AB} N_\delta \quad (V.5)$$

If we specialize our case for BMS supertranslation, the non-vanishing change in the optical parameter.

$$[\Theta] = \frac{1}{M^2} \left( - \psi(V)' \Delta^{(2)} F(\theta, \phi) + \psi(V)'' \sigma^{AB} F_A F_B - 2M + \frac{2}{M} \right) \quad (V.6)$$

$$[\Sigma_{\theta\theta}] = \frac{1}{2} \left( - F_{\theta\theta} + \frac{F_{\phi\phi}}{\sin^2 \theta} + F_\theta \cot \theta \right) + \frac{1}{2} \left( \psi(V)'' - \frac{F_\theta^2}{\sin^2 \theta} \right) - \frac{1}{M} \quad (V.7)$$

$$[\Sigma_{\phi\phi}] = \frac{1}{2} \left( - F_{\phi\phi} + F_{\theta\theta} \sin^2 \theta - \frac{F_\theta \sin 2\theta}{2} \right) + \frac{1}{2} \left( \psi (V)'' - \frac{F_\theta^2 - F_\phi^2 \sin^2 \theta}{\sin^2 \theta} \right) - \frac{\sin^2 \theta}{M} \quad (V.8)$$

$$[\Sigma_{\theta\phi}] = - \psi(V)' F_{\theta\phi} + F_{\theta} F_{\phi} \psi(V)'' + \psi(V)' F_\phi \cot \theta \quad (V.9)$$

Here, we have non-vanishing jumps for expansion and shear comprising BMS parameters $F(\theta, \phi)$. The jump in the expansion is emerging from shell stress-energy tensor while in shear it is due to existence of IGW. These jumps in shear and expansion arising from $B$-tensor concludes memory effect on null geodesics.

**B. Memory effect in off-shell extension of null congruence**

Now we perform the off-shell extension of the soldering transformation to estimate the off-shell $B$-tensor. The Jacobian of transformation can be written as,

$$\frac{\partial x^M}{\partial x^A} = \frac{\delta^B_A}{(\delta^B_M - U \frac{\partial N_B}{\partial x^M})} = \frac{I}{W}.$$

Where $W = (\delta^B_M - U \frac{\partial N_B}{\partial x^M})$, and inverse of the $W$ matrix is then given by,

$$P^M_A \equiv \left( \delta^B_M - U \frac{\partial N_B}{\partial x^M} \right)^{-1} = \frac{1}{\text{det}(W)} \left( 1 + \frac{U \psi(V)'}{M^2} \frac{F_{\phi\phi}}{\sin^2 \theta} - \frac{2 \cos \theta F_\phi}{\sin^3 \theta} \right) \left( 1 + \frac{U \psi(V)'}{M^2} F_{\theta\theta} \right) \left( 1 + \frac{U \psi(V)'}{M^2} F_{\theta\phi} \right), \quad (V.10)$$

where determinant of $W$ is given by,

$$\text{det}(W) = \left( 1 + \frac{U \psi(V)'}{M^2} \frac{F_{\phi\phi}}{\sin^2 \theta} \right) \left( 1 + \frac{U \psi(V)'}{M^2} F_{\theta\theta} \right) - \frac{U^2 \psi(V)'^2}{M^2} F_{\phi\phi}^2 \left( \frac{F_{\theta\theta}}{\sin^2 \theta} - \frac{2 \cos \theta F_\phi}{\sin^3 \theta} \right).$$

The expression is quite hue to be written here. But in a compact way one can define inverse matrices as $P^M_A$ and $Q^N_B$ for transformed $B_{AB}$ tensor. Thus, we can write

$$B_{AB} = P^M_A Q^N_B B_{MN}. \quad (V.11)$$

However, if one sets $U = 0$, off-shell $B$-tensor reduces to on-shell $B$-tensor which we have already computed in above section. However, we also see that the transformed failure tensor depends linearly on $U$. It is clear that off-shell $B$-tensor also suffers jump across the null shell. Thus, memory effect is again maintained in the off-shell extension of the soldering transformation together with the linear dependence of null geodesics on $U$.  

13
1. Memory effect and Penrose limit of Extreme black holes

It is known that any spacetime metric can be reduced to a plane wave metric around a point of a null geodesic. This was shown by Penrose and the resulted metric is known as Penrose limit of a spacetime \([42]\). As Penrose limits produce plane wave spacetimes (pp-wave, plane symmetric, homogeneous waves etc.), it is comparatively much easy to study geodesic deviation vectors and optical tensors in those backgrounds \([43]\) and at the same time these studies can provide some useful theoretical setups that may prove useful in the future detection schemes of memory effect. In terms of Brinkmann coordinates the Penrose limit takes a generic form as:

\[
\begin{align*}
\text{ds}^2 &= 2du dv + A_{ab}(u)dx^a dx^b du^2 + \delta_{ab}dx^a dx^b \\
&= 2du dv + A_{11}(u)dx^1 dx^1 du^2 + A_{22}(u)dx^2 dx^2 du^2
\end{align*}
\]  

(V.12)

The Penrose limit actually captures many properties of the original spacetime. Since near horizon limit of ERN black hole has \(AdS_2 \times S_2\) geometry. Penrose limit of this structure should also capture the properties of \(AdS_2 \times S_2\) \([20]\). We know, for a \(AdS \times S\) space, \(A_{ab}\) becomes constant, this comes from a hereditary property of AdS space. Explicit calculation for the metric (III.9) shows the non-vanishing components of \(A_{ab}\) are:

\[
\begin{align*}
A_{11} &= -\frac{4L^2}{M^4} \\
A_{22} &= -\frac{2L^2}{M^4}.
\end{align*}
\]  

(V.13) (V.14)

Here \(L\) and \(M\) are the two constants of motion namely angular momentum and Mass of ERN spacetime. Clearly the components of \(A_{ab}(u)\) are constant. This is expected as our underlying space is AdS, whose Riemann tensor has covariant divergence producing a constant \(A_{ab}\) (profile function) producing symmetric plane waves. It will be interesting to study the memory in such symmetric plane waves.

VI. DISCUSSIONS

The motivation of this work is to find the BMS memory effect with the help of intrinsic properties of the null shell for extreme black holes. Let us conclude summarize the whole work,

1. We started with estimating the intrinsic properties of the null shell for ERN black hole and then for Extreme BTZ together with \(J = 0\). The calculation details for BTZ black hole is shown in the Appendix A. We observe that we do not get harmonic solutions for zero surface energy density in the case of ERN and Extreme BTZ. For ERN the shell reduces to Dray ‘t Hooft shell if the matter is not present. For extreme BTZ, there is no such feasible solution exists.

2. We also discussed how conformal symmetries may arise as soldering freedom when we zoom in very near to the black hole transforming the usual ERN metric in \(AdS \times S_2\) form. We have related this with the Penrose’s cut-paste construction. An exact solution has not been given here, which we hope to report in the future.
3. Next, we performed the off-shell extension of soldering transformation for ERN black hole. We used a Kruskal like double null metric that is regular at the horizon and obtained the off-shell extension, to the linear order of $U$. We then computed shell-intrinsic tensors. Thereafter, we obtained the expressions of deviation vectors in terms BMS like parameters. We show the test particles initially at rest get displaced from its initial 2-d plane. This corresponds to the memory effect for timelike geodesics crossing the null shell supporting IGWs. The memory is non-oscillatory in nature.

4. Further, we computed the memory effect for null geodesics passing orthogonally through the null shell on ERN supporting IGWs. We observed that there is non-vanishing change in optical parameters such as expansion and shear which again shows the memory effect for null geodesics crossing orthogonally to the hypersurface. We also find a finite jump in the expansion and shear, for the congruence at off-shell points. The only thing which differs from on-shell $B$-tensor is that in transformed $B$-tensor, terms will linearly depend on $U$. But when we set $U = 0$, the off-shell $B$-tensor takes the initial form i.e. non-transformed $B$-tensor or on-shell $B$-tensor.

5. We briefly touch upon the possibility of studying memory effect in a plane wave background that will be produced by taking Penrose limit of extreme black holes. For the ERN case near the horizon, we get a symmetric plane wave metric. It will be interesting to study the geodesic congruences and memory for the symmetric plane waves.

One of the useful extension of this work towards studying different flux-balance laws as depicted in [44]. Studying quantum effects in such IGW spacetimes generated in extreme black holes could be another interesting study where semiclassical features of near horizon symmetries may show up [45].

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**Appendix A: Soldering freedom in Extreme BTZ black hole**

First we present the intrinsic quantities for extreme BTZ black hole. We take two rotating extreme BTZ metrics with horizon situated at $r_0 = r_{\pm} = l \sqrt{\frac{M}{2}}$, can be seen from (A.3). For $\mathcal{M}_{-}$ manifold, in EF coordinate system, the metric takes the following form,

$$ds^2 = -\frac{(r^2 - r_0^2)^2}{r^2 l^2} dv^2 + 2dvdr + r^2 (d\phi + N^\phi dv)^2$$  \hspace{1cm} (A.1)

Where,

$$N^\phi = -\frac{J}{2r^2} \quad ; \quad J = Ml$$  \hspace{1cm} (A.2)
In general,
\[ r_\pm^2 = \frac{ML^2}{2} \left\{ 1 \pm \left[ 1 - \left( \frac{J}{ML} \right)^2 \right]^{\frac{1}{2}} \right\} \tag{A.3} \]

Here, we see that for \( J = ML \), \( r_\pm \) becomes \( r_0 \). Now, considering the superteanslation type transformations as,
\[ v_+ = v + F(\phi) \; ; \; r_+ = r \; ; \; \phi_+ = \phi \tag{A.4} \]

we obtain intrinsic quantities of the shell,
\[ \mu = -\frac{1}{8\pi} \sigma^{AB}[\kappa_{AB}] = -\frac{1}{8\pi r_0^2} \left( \Gamma_{\phi\phi}^{v_+} + 2F_\phi \Gamma_{v_+\phi}^{v_+} + F_\phi^2 \Gamma_{v_+v_+}^{v_+} + r_0 \right) \tag{A.5} \]
\[ J^A = \frac{1}{8\pi} \sigma^{AB}[\kappa_{vB}] = \frac{1}{8\pi r_0^2} \left( \Gamma_{v_+\phi}^{v_+} + F_\phi \Gamma_{v_+v_+}^{v_+} \right) \tag{A.6} \]
\[ p = -\frac{1}{8\pi} [\kappa_{vv}] = -\frac{1}{8\pi} (\Gamma_{v_+v_+}^{v_+} - \Gamma_{vv}^{v_+}) \tag{A.7} \]

We find that there are no major changes in the physics of estimated quantities except the long expressions for Christoffel symbols. The expressions are not so illuminating here so we turn our attention to non-rotating BTZ case.

\[ a. \; Extreme \; BTZ \; black \; hole: \; J=0 \]

In this case, we take \( J = 0 \). In this approximation the metric, with horizon at \( r_0 = l\sqrt{M} \), takes the following form,
\[ ds^2 = -\left( -M + \frac{r^2}{l^2} \right) dv^2 + 2dvdr + r^2 d\phi^2 \tag{A.8} \]

We may add a charge also to this metric, the results will not alter qualitatively. Again we perform the supertranslation on \( v \) coordinate. The transformed metric is,
\[ ds^2 = -(-M + \frac{r^2}{l^2})(dv + \partial_\phi F(\phi)d\phi)^2 + 2(dv + \partial_\phi F(\phi)d\phi)dr + r^2 d\phi^2 \tag{A.9} \]

Now for calculating the surface density, current and pressure,
\[ \mu = -\frac{1}{8\pi} \sigma^{AB}[\kappa_{AB}] = -\frac{1}{8\pi r_0^2} \left( F_{\phi\phi} + 4r_0 F_\phi^2 \right) \tag{A.10} \]
\[ J^\phi = \frac{1}{8\pi} \sigma^{AB}[\kappa_{vB}] = \frac{F_\phi}{4\pi r_0 l^2} \tag{A.11} \]
\[ p = -\frac{1}{8\pi} [\kappa_{vv}] = 0 \tag{A.12} \]

In the extreme BTZ black hole, we again observe there can be a non-trivial shell without matter supporting pure IGW. However unlike ERN, for zero surface energy density, one does not get harmonic solutions, the solution is logarithmic and the periodicity of \( \phi \) will spoil the solution to be meaningful. Hence unlike we ERN case, we don’t get a shell with purely impulsive wave.
Appendix B: Inverse Jacobi transformation

In this section, we provide the inverse Jacobi transformation for extreme RN case which we use in the memory effect for null geodesics section. Components of congruence $N_0$ on hypersurface in continuous coordinates are given by the inverse Jacobian transformation,

$$
\left( \frac{\partial x^\alpha}{\partial x^\beta} \right)^{-1} = \begin{pmatrix}
\frac{\psi'(F)F_\nu}{\psi'(V)} & 0 & 0 & 0 \\
-(\psi'(V)-2\psi'(F)(\csc^2(\theta)F_\theta^2+F_\phi^2)) & 1 & F_\theta & F_\phi \\
\frac{2M^2F_\nu}{\psi'(F)F_\phi} & 0 & 1 & 0 \\
-\frac{\csc^2(\theta)\psi'(F)F_\theta}{M^2} & 0 & 0 & 1
\end{pmatrix} \tag{B.1}
$$

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