Role of Tensor operators in $R_K$ and $R_{K^*}$

Debjyoti Bardhan,1,∗ Pritibhajan Byakta,2,† and Diptimoy Ghosh3,‡

1Department of Theoretical Physics, Tata Institute of Fundamental Research, 1 Homi Bhabha Road, Mumbai 400005, India
2Department of Theoretical Physics, Indian Association for the Cultivation of Science, 2A & 2B, Raja S.C. Mullick Road, Jadavpur, Kolkata 700 032, India
3Department of Particle Physics and Astrophysics, Weizmann Institute of Science, Rehovot 76100, Israel

The recent LHCb measurement of $R_{K^*}$ in two $q^2$ bins, when combined with the earlier measurement of $R_K$, strongly suggests lepton flavour non-universal new physics in semi-leptonic $B$ meson decays. Motivated by these intriguing hints of new physics, several authors have considered vector, axial vector, scalar and pseudo scalar operators as possible explanations of these measurements. However, tensor operators have widely been neglected in this context. In this paper, we consider the effect of tensor operators in $R_K$ and $R_{K^*}$. We find that, unlike other local operators, tensor operators can comfortably produce both of $R_{K^*}^{low}$ and $R_{K^*}^{central}$ close to their experimental central values. However, a simultaneous explanation of $R_K$ is not possible with only Tensor operators, and other vector or axial vector operators are needed. In fact, we find that combination of vector and tensor operators can provide simultaneous explanations of all the anomalies comfortably at the 1σ level, a scenario which is hard to achieve with only vector or axial vector operators. We also comment on the compatibility of the various new physics solutions with the measurements of the inclusive decay $B_d \to X_s \ell^+ \ell^-$.

1. Introduction

The LHCb collaboration has recently announced measurements of $R_{K^*} \equiv B(B_d \to K^+ \mu^+ \mu^-)/B(B_d \to K^+ e^+ e^-)$ in two $q^2 = (p_\mu^+ + p_{\ell^-})^2$ bins, [0.045, 1.1] and [1.1, 6] GeV$^2$ (referred to as low and central bins respectively) [1]. In both the bins, they observe deviation from the Standard Model (SM), at the $2.1-2.3 \sigma$ level in the low bin and at the $2.4-2.5 \sigma$ level in the central bin [1]. Interestingly, in the summer of 2014, a similar LHCb measurement of the ratio $R_K \equiv B(B^+ \to K^+ \mu^+ \mu^-)/B(B^+ \to K^+ e^+ e^-)$ for $q^2 \in [1, 6]$ GeV$^2$ also showed a 2.6σ deviation from the SM [2]. The experimental measurements as well as the latest SM predictions for these ratios are summarised in the first 3 rows of Table-I.

As the theoretical predictions of $R_K$ and $R_{K^*}$ in the SM are rather reliable [3, 4], these measurements highly suggest for lepton non-universal new physics (NP). This has spurred a lot of activities in the recent past, both in the language of model independent higher dimensional operators and specific models beyond the SM [3, 5–66]. In the context of dimension-6 NP operators, it has been pointed out that short distance NP operators of certain types can provide an overall good fit to the data. However, a discussion of the tensor operators was missing. In this paper, we fill this gap with a detailed analysis of the role of tensor operators in $R_K$ and $R_{K^*}$.

Note that, it is not possible to generate tensor operators at the dimension-6 level if the Standard Model gauge symmetry is imposed [19]. However, tensor operators can be generated at the dimension-8 level, see the end of section 4 for more details.

| Observable | SM prediction | Measurement |
|------------|---------------|-------------|
| $R_{K^*}^{cen}$ | 1.00 ± 0.01 [4, 60] | 0.66, 0.84 [2] |
| $R_{K^*}^{low}$ | 0.92 ± 0.02 [47] | 0.58, 0.77 [1] |
| $R_{K^*}^{cen}$ | 1.00 ± 0.01 [4, 60] | 0.60, 0.81 [1] |
| $B_{\mu\mu} \times 10^6$ | 3.57 ± 0.16 [70, 71] | 2.5, 3.5 [71–73] |
| $B_{ee} \times 10^4$ | 8.35 ± 0.39 [70, 71] | < 2.8 × 10$^4$ [74] |
| $B_{X_{\mu\mu}} \times 10^6$ | 1.59 ± 0.11 [75] | 0 [71] |
| $B_{X_{ee}} \times 10^4$ | 0.24 ± 0.07 [75] | 0.31, 0.91 [76] |
| $B_{X_{ee}} \times 10^6$ | 1.64 ± 0.11 [75] | 1.42, 2.47 [76] |
| $B_{X_{ee}} \times 10^6$ | 0.21 ± 0.07 [75] | 0.38, 0.75 [76] |

TABLE I. Observables, their SM predictions and experimental 1σ ranges. For $R_{K^*}^{cen}$, a more conservative SM prediction, 0.906 ± 0.028, has been recently reported in [4].

Besides $R_K$ and $R_{K^*}$, we also consider the branching ratios of $B_s \to \ell^+ \ell^- (\ell = \mu, e)$ as they are reliably predicted in the SM. Furthermore, we also show the compatibility with measurements of the branching ratios of the inclusive decay $B_d \to X_s \ell^+ \ell^-$. The experimental measurements of these observables are summarised in Table I. In the table and the subsequent text, we use the following short-hand notations ($q^2$ is given in GeV$^2$).

$B_{X_{ee}}^{cen} \equiv B(B^+ \to K^+ \ell^+ \ell^-)$, $q^2 \in [1, 6]$

$B_{X_{ee}}^{low(cen)} \equiv B(B_d \to K^+ \ell^+ \ell^-)$, $q^2 \in [0.045, 1.1]$ ([1, 6])

$B_{X_{ee}} \equiv B(B_s \to \ell^+ \ell^-)$

$B_{X_{ee}}^{high} \equiv B(B_d \to X_s \ell^+ \ell^-)$, $q^2 \in [1, 6]$ ([14.2, 25])
2. Effective operators

The SU(3) × U(1) invariant effective Lagrangian at the dimension-6 level for $b \rightarrow s$ transition is given by

$$L_{\text{eff}} = - \frac{4G_F}{\sqrt{2}} \frac{\alpha_{\text{em}}}{4\pi} V_{tb} V_{ts}^* H_{\text{eff}}^{(i)} \pm \text{h.c.}$$

where, $H_{\text{eff}}^{(i)} = C_i O_i^i + C_2 O_2 + \sum_{i=3}^{6} C_i O_i + \sum_{i=7}^{10} C_i O_i$

In models beyond the SM, new operators can be generated. The complete basis of dimension-6 operators includes new operators given by

$$H_{\text{eff}}^{(i)} \text{, New} = \sum_{i=7,9,10} C_i O_i^i + \sum_{i=S,P,S',P',T,T5} C_i O_i$$

where, the various operators above are defined by

$$O_{7(1')} = \frac{1}{c} m_b [\sigma_{\mu\nu} P_{R(L)} b] F_{\mu\nu}$$
$$O_{9(10')} = [\gamma^\mu P_{R(L)} b] [\gamma^\nu], \quad O_{10(10')} = [\gamma^\mu P_{R(L)} b] [\gamma^\nu \gamma_5 l]$$
$$O_{S(S')} = [\gamma^\mu P_{R(L)} b] [\bar{l}], \quad O_{P(P')} = [\sigma_{\mu\nu} P_{R(L)} b] [\bar{l}]$$
$$O_T = [\sigma_{\mu\nu} b] [\bar{l}], \quad O_{T5} = [\sigma_{\mu\nu} \gamma_5 b]$$

Note that, the Wilson coefficients for the photonic dipole operators $O_7$ and $O_T$ are lepton universal by definition, and lead to lepton flavour non-universality only through lepton mass effects, which is not enough to provide explanation of the $R_{K^*}$ anomalies once bound from $B_d \rightarrow X_s \gamma$ is taken into account [9]. So we neglect NP effect in these operators. For all the other operators, we write their Wilson coefficients as $C_i = C_i^{\text{SM}} + \Delta C_i$ where $\Delta C_i$ corresponds to the shift in the Wilson coefficient from its SM value due to short distance NP.

3. Tensor operators

In this section, we study the effect of the two tensor operators, $O_T$, $O_{T5}$, on $R_K$ and $R_{K^*}$. In Eq. 3 - 5 below we show numerical formulae for the various branching ratios (normalised to their SM predictions) as functions of $\Delta C_{T5}^{\text{eff}}$ and $\Delta C_{T5}^{\text{cen}}$.

$$\frac{B_{K^\ell \ell/SM}}{B_{K^\ell \ell/SM}} \approx 1 + 0.02 |\Delta C_{T5}^{\text{eff}}|^2$$
$$\frac{B_{K^{*}e\mu(\mu)e/SM}}{B_{K^{*}e\mu(\mu)e/SM}} \approx 1 - 0.00(0.24) |\Delta C_{T5}^{\text{cen}}| + 0.30 |\Delta C_{T5}^{\text{cen}}|^2$$
$$\frac{B_{K^{*}e\mu(\mu)e/SM}}{B_{K^{*}e\mu(\mu)e/SM}} \approx 1 + 0.00(0.06) |\Delta C_{T5}^{\text{cen}}| + 0.53 |\Delta C_{T5}^{\text{cen}}|^2$$

The full set of numerical formulae valid in the presence of all the operators are presented in Appendix A. These formulae can be used to perform very quick analysis of models as the only required inputs in these formulae are the short distance Wilson coefficients.

In Fig. 1 we show how $R_K^{\text{cen}}$, $R_{K^*}^{\text{low}}$ and $R_{K^*}^{\text{cen}}$ vary with $\Delta C_{T5}^{\text{cen}}$.

![FIG. 1. Variation of $R_K^{\text{cen}}$, $R_{K^*}^{\text{low}}$ and $R_{K^*}^{\text{cen}}$ with $\Delta C_{T5}^{\text{cen}}$. The horizontal bands correspond to the experimental 1σ upper and lower limits shown in Table-I.](image)

It can be seen from the left panel of Fig. 1 that $\Delta C_{T5}^{\text{cen}} \sim \pm 1$ not only explains $R_K^{\text{cen}}$ and $R_{K^*}^{\text{low}}$, simultaneously but also brings them close to the experimental central values. As pointed out by one of the authors in [55], this is not possible naturally by any other local operator at the dimension-6 level, and in this sense, the tensor operators are unique. However, as can be seen from the right panel of Fig. 1, $\Delta C_{T5}^{\text{cen}} \sim \pm 1$ cannot reduce $R_{K^*}^{\text{cen}}$ much from its SM value of unity, and hence a simultaneous explanation of $R_{K^*}^{\text{cen}}$, $R_{K^*}^{\text{cen}}$ and $R_{K^*}^{\text{low}}$ is not possible. All statements made here for $\Delta C_{T5}^{\text{cen}}$ applies equally for the other tensor Wilson coefficient $\Delta C_{T5}^{\text{eff}}$.

Note that, any non-zero value for $\Delta C_{T5}^{\text{eff}}$ and $\Delta C_{T5}^{\text{cen}}$ leads to values for $R_K$ and $R_{K^*}$ greater than their SM values.

---

Note however that, large deviations from the SM expectations in two $q^2$-bins of $P_L^{\mu\nu}$ have been claimed in the literature [14]. Interestingly, the Belle collaboration has provided the first measurement of $P_L^{\mu\nu}$ in the electron mode [85], and indeed, the central value for $P_L^{\mu\nu}$ deviates more than that of $P_E^{\mu\nu}$. However, at this point the statistics is low, and the jury is still out on this.
and thus, tensor operators in the muon sector are ruled out as possible explanation of these anomalies.

In the following section, we will investigate whether a simultaneous solution is possible when other additional operators are also considered. While we consider only unprimed operators in the main text, the effect of the primed operators in conjunction with the tensor operators can be found in Appendix B.

4. Combination of Vector and Tensor operators

In Fig. 2, we show the regions in $\Delta C_9^\mu - \Delta C_{T5}^e$ plane allowed by the experimental measurements of the various observables listed in Table-1. In the left panel, the blue, red and yellow shaded regions correspond to the $1\sigma$ experimental ranges of $R_{K^*}^{low}$, $R_{K^*}^{cen}$ and $R_{K^*}^{cen}$ respectively. The black shaded regions are the overlap of the three. It should be noticed that the black shaded region is outside the $\Delta C_{T5}^e = 0$ line, and hence no simultaneous solutions are possible with only $\Delta C_9^\mu$. In the right panel, we also show the regions allowed by $B_{X_{\mu\mu}}^{low}$ (in blue) and $B_{X_{ee}}^{high}$ (in red). The black shaded region from the left panel is also superimposed there. It can be seen that there is a small overlap of the black, blue and red regions in the right panel where all the constraints including those from the inclusive decay are satisfied.

![Fig. 2. Allowed regions in $\Delta C_9^\mu - \Delta C_{T5}^e$ plane. See text for more details.](image)

In Fig. 3, we show the allowed regions in the $\Delta C_9^\mu - \Delta C_{T5}^e$ plane. The various shaded regions in the left panel have the same meaning as in Fig. 2. The grey vertical (horizontal) band corresponds to the experimental $1\sigma$ allowed region of $B_{X_{\mu\mu}}^{low}$ ($B_{X_{ee}}^{high}$). Similar to the previous case, here also a simultaneous solution is not possible with only $\Delta C_9^\mu$, and non-zero tensor contribution is required. However, as can be seen from the right panel of Fig. 3, this scenario is in tension with the measurements of $B_{X_{\mu\ell}}^{high}$.

![Fig. 3. Allowed regions in $\Delta C_9^\mu - \Delta C_{T5}^e$ plane. See text for more details.](image)

We now consider the two cases $\Delta C_9^\mu = -\Delta C_{T5}^e$ vs. $\Delta C_{T5}^e$ and $\Delta C_9^\mu = -\Delta C_{T5}^e$ vs. $\Delta C_{T5}^e$. In Fig. 4 we show our results. It can be seen from the upper panel that $\Delta C_9^\mu = -\Delta C_{T5}^e$ alone (i.e., with $\Delta C_{T5}^e = 0$) can not explain $R_{K^*}^{low}$, $R_{K^*}^{cen}$, and $R_{K^*}^{cen}$ simultaneously within their experimental $1\sigma$ regions. However, a simultaneous solutions is possible if a non-zero $\Delta C_{T5}^e \sim \pm 0.6$ is considered. Note that, the Wilson coefficient $C_{10}^\mu$ also modifies $B_{\mu\mu}$ which gives a bound $0 \lesssim \Delta C_{10}^\mu \lesssim 0.7$ at the $1\sigma$ level [55]. Hence, the black overlap region in the upper left panel is allowed by $B_{\mu\mu}$. However, as in Fig. 3, this scenario also is in tension with the measurements of $B_{X_{\mu\ell}}^{high}$.

![Fig. 4. Allowed regions in $\Delta C_9^\mu = -\Delta C_{T5}^e$ vs. $\Delta C_{T5}^e$ plane (upper panel) and $\Delta C_9^\mu = -\Delta C_{10}^\mu$ vs. $\Delta C_{T5}^e$ plane (lower panel). See text for more details.](image)

The situation is better for $\Delta C_9^\mu = -\Delta C_{T5}^e$ vs. $\Delta C_{T5}^e$ as shown in the bottom panel of Fig. 4. Here, a simultaneous solutions to not only $R_{K^*}^{low}$, $R_{K^*}^{cen}$ and $R_{K^*}^{cen}$, but...
also the inclusive decay $B_d \to X_s e^+ e^-$ is possible. This corresponds to the small overlap of the black, red and blue shaded regions in the lower right panel of Fig. 4.

Before closing this section, we would like to mention that the tensor operators do not get generated at the dimension-6 level if SU(2) × U(1) gauge invariance is imposed, which was also pointed out in [19]. However, it can be generated at the dimension-8 level. For example, one can write down the operator $(1/\Lambda^4)(\overline{s}_T P_L l) (\overline{e}_T P_L b)$ which, after electroweak symmetry breaking, generates the operator $(v^2/2\Lambda^4)(\overline{s}_T P_L l) (\overline{e}_T P_L b)$ and tensor operator $(v^2/8\Lambda^4)(\overline{s}_T P_L P_L l) (\overline{e}_T P_L P_L b)$. For more details, see Appendix C.

5. Summary

Motivated by the recent measurements of $R_{K^*}$ in two $q^2$ bins by the LHCb collaboration, we have performed a detailed analysis of the role of tensor operators in $R_K$ and $R_{K^*}$, for the first time in the literature. We show that, unlike the vector, axial vector, scalar or pseudo scalar operators, tensor operators can comfortably explain $R_{K^*}^{cen}$ and $R_{K^*}^{low}$ simultaneously. Hence, if the experimental measurement of $R_{K^*}$ in the low $q^2$ bin stays in the future, either a very light vector boson (as shown by one of the authors in [20]) or the existence of tensor operators would be unavoidable. However, we find that a simultaneous explanation of $R_{K^*}$ also would require the existence of other Wilson coefficients (of vector and/or axial vector operators, for example) in conjunction with the tensor operators. We study the interplay of the vector and axial vector operators with the tensor structures, and obtain the regions allowed by the $1\sigma$ experimental values of $R_K$ and $R_{K^*}$. We further show that the measured branching ratios for the inclusive $B_d \to X_s e^+ e^-$ decay provide very important constraints on the various solutions. We also present completely general numerical formulae which can be used to effortlessly compute $R_{K^*}^{cen}$, $R_{K^*}^{cen}$, $R_{K^*}^{low}$, and the inclusive branching fractions just knowing the short distance Wilson coefficients at the $m_b$ scale.

Appendix A: Complete expressions for the branching ratios

\[
\frac{B_{K^{*e}}^{cen}}{B_{K^{*e}}^{cen, SM}} = 1 + 0.2429[\Delta C_5^0] + 0.0274[\Delta C_5^0]^2 + 0.2427[\Delta C_5^0 \Delta C_5^0] + 0.0549[\Delta C_5^0][\Delta C_5^0] + 0.0274[\Delta C_5^0]^2 - 0.225[\Delta C_5^0] + 0.0274[\Delta C_5^0]^2 - 0.225[\Delta C_5^0] + 0.0274[\Delta C_5^0]^2 + 0.0092[\Delta C_5^0]^2 + 0.0184[\Delta C_5^0][\Delta C_5^0] + 0.0092[\Delta C_5^0]^2 + 0.0184[\Delta C_5^0][\Delta C_5^0] + 0.0092[\Delta C_5^0]^2 + 0.0184[\Delta C_5^0][\Delta C_5^0] + 0.0092[\Delta C_5^0]^2 + 0.0184[\Delta C_5^0][\Delta C_5^0] + 0.0092[\Delta C_5^0]^2 (A1)
\]

\[
\frac{B_{K^{*\mu\mu}}^{cen}}{B_{K^{*\mu\mu}}^{cen, SM}} = 1 + 0.2427[\Delta C_5^0] + 0.0274[\Delta C_5^0]^2 + 0.2427[\Delta C_5^0 \Delta C_5^0] + 0.0548[\Delta C_5^0][\Delta C_5^0] + 0.0274[\Delta C_5^0]^2 - 0.2253[\Delta C_5^0] + 0.0275[\Delta C_5^0]^2 - 0.225[\Delta C_5^0] + 0.0275[\Delta C_5^0]^2 + 0.0092[\Delta C_5^0]^2 + 0.018[\Delta C_5^0][\Delta C_5^0] + 0.0092[\Delta C_5^0]^2 - 0.187[\Delta C_5^0][\Delta C_5^0] + 0.0046[\Delta C_5^0][\Delta C_5^0] + 0.0091[\Delta C_5^0]^2 + 0.0182[\Delta C_5^0]^2 + 0.0091[\Delta C_5^0]^2 + 0.0168[\Delta C_5^0]^2 + 0.0185[\Delta C_5^0]^2 (A2)
\]

\[
\frac{B_{K^{*e}}^{low}}{B_{K^{*e}}^{low, SM}} = 1 + 0.0764[\Delta C_5^0] + 0.0136[\Delta C_5^0]^2 - 0.1048[\Delta C_5^0][\Delta C_5^0] - 0.0257[\Delta C_5^0][\Delta C_5^0] + 0.0136[\Delta C_5^0]^2 - 0.1118[\Delta C_5^0] + 0.0136[\Delta C_5^0]^2 + 0.1054[\Delta C_5^0][\Delta C_5^0] - 0.0257[\Delta C_5^0][\Delta C_5^0] + 0.0136[\Delta C_5^0]^2 + 0.0066[\Delta C_5^0][\Delta C_5^0] + 0.0136[\Delta C_5^0]^2 + 0.1054[\Delta C_5^0][\Delta C_5^0] - 0.0257[\Delta C_5^0][\Delta C_5^0] + 0.0136[\Delta C_5^0]^2 + 0.0066[\Delta C_5^0][\Delta C_5^0] + 0.0066[\Delta C_5^0][\Delta C_5^0] - 0.0015[\Delta C_5^0][\Delta C_5^0] + 0.2901[\Delta C_5^0][\Delta C_5^0] - 0.0013[\Delta C_5^0][\Delta C_5^0] + 0.2901[\Delta C_5^0][\Delta C_5^0] (A3)
\]

\[
\frac{B_{K^{*\mu\mu}}^{low}}{B_{K^{*\mu\mu}}^{low, SM}} = 1 + 0.0806[\Delta C_5^0] + 0.0144[\Delta C_5^0]^2 - 0.1103[\Delta C_5^0][\Delta C_5^0] - 0.027[\Delta C_5^0][\Delta C_5^0] + 0.0144[\Delta C_5^0]^2 - 0.1167[\Delta C_5^0] + 0.0142[\Delta C_5^0]^2 - 0.27[\Delta C_5^0][\Delta C_5^0] + 0.0142[\Delta C_5^0]^2 + 0.0006[\Delta C_5^0]^2 - 0.012[\Delta C_5^0][\Delta C_5^0] + 0.006[\Delta C_5^0]^2 - 0.0078[\Delta C_5^0][\Delta C_5^0] + 0.0019[\Delta C_5^0][\Delta C_5^0] - 0.019[\Delta C_5^0][\Delta C_5^0] + 0.0006[\Delta C_5^0][\Delta C_5^0] + 0.0078[\Delta C_5^0][\Delta C_5^0] - 0.019[\Delta C_5^0][\Delta C_5^0] + 0.0019[\Delta C_5^0][\Delta C_5^0] - 0.013[\Delta C_5^0][\Delta C_5^0] + 0.0006[\Delta C_5^0]^2 - 0.015[\Delta C_5^0][\Delta C_5^0] + 0.0165[\Delta C_5^0][\Delta C_5^0] - 0.0165[\Delta C_5^0][\Delta C_5^0] + 0.3057[\Delta C_5^0]^2 - 0.2676[\Delta C_5^0][\Delta C_5^0] + 0.0088[\Delta C_5^0][\Delta C_5^0] + 0.0088[\Delta C_5^0][\Delta C_5^0] + 0.305[\Delta C_5^0]^2 (A4)
\]
\[
\frac{B_{K^{\mu}\mu}^{\text{cen}}}{B_{K^{\mu}\mu}^{\text{cen,SM}}} = 1 + 0.2187[\Delta C_{9'}^\mu] + 0.032[\Delta C_{9'}^\mu]^2 - 0.1998[\Delta C_{9'}^\mu] - 0.0474[\Delta C_{9'}^\mu][\Delta C_{9'}^\mu] + 0.032[\Delta C_{9'}^\mu]^2 - 0.2695[\Delta C_{10'}^\mu] + 0.032[\Delta C_{10'}^\mu] + 0.1945[\Delta C_{10'}^\mu] - 0.0474[\Delta C_{10'}^\mu][\Delta C_{10'}^\mu] + 0.032[\Delta C_{10'}^\mu]^2 + 0.0067[\Delta C_{S}^\mu]^2 - 0.0134[\Delta C_{S}^\mu][\Delta C_{S}^\mu] + 0.0052[\Delta C_{S}^\mu][\Delta C_{S}^\mu] - 0.0076[\Delta C_{S}^\mu][\Delta C_{S}^\mu] + 0.0549[\Delta C_{T}^{2\mu}] - 0.0003[\Delta C_{T}^{5\mu}] + 0.0001[\Delta C_{0\mu}][\Delta C_{T}^{5\mu}] - 0.0001[\Delta C_{0\mu}][\Delta C_{T}^{5\mu}] + 0.5349[\Delta C_{T}^{5\mu}]^2 (A5)
\]

\[
\frac{B_{K^{\mu}\mu}^{\text{cen}}}{B_{K^{\mu}\mu}^{\text{cen,SM}}} = 1 + 0.2194[\Delta C_{9'}^\mu] + 0.0321[\Delta C_{9'}^\mu]^2 - 0.2004[\Delta C_{9'}^\mu] - 0.0476[\Delta C_{9'}^\mu][\Delta C_{9'}^\mu] + 0.0321[\Delta C_{9'}^\mu]^2 - 0.2622[\Delta C_{10'}^\mu] + 0.0324[\Delta C_{10'}^\mu] + 0.1949[\Delta C_{10'}^\mu] - 0.0475[\Delta C_{10'}^\mu][\Delta C_{10'}^\mu] + 0.0324[\Delta C_{10'}^\mu]^2 + 0.0066[\Delta C_{S}^\mu]^2 - 0.0132[\Delta C_{S}^\mu][\Delta C_{S}^\mu] + 0.0055[\Delta C_{S}^\mu][\Delta C_{S}^\mu] - 0.0038[\Delta C_{S}^\mu][\Delta C_{S}^\mu] + 0.0034[\Delta C_{S}^\mu][\Delta C_{S}^\mu] - 0.0034[\Delta C_{S}^\mu][\Delta C_{S}^\mu] + 0.0067[\Delta C_{T}^{2\mu}] - 0.0138[\Delta C_{T}^{2\mu}] + 0.0034[\Delta C_{T}^{2\mu}] - 0.0034[\Delta C_{T}^{2\mu}] + 0.0134[\Delta C_{T}^{2\mu}] + 0.0034[\Delta C_{T}^{2\mu}] - 0.0034[\Delta C_{T}^{2\mu}] + 0.0537[\Delta C_{T}^{5\mu}] + 0.0539[\Delta C_{T}^{5\mu}]^2 (A6)
\]

\[10^6 B_{X_{\ell\ell}}^{\text{low}} = 10^6 B_{X_{\ell\ell}}^{\text{low,SM}} + 0.4156[\Delta C_{9'}^\mu] + 0.0147([\Delta C_{9'}^\mu]^2 + [\Delta C_{9'}^\mu]^2 + [\Delta C_{10'}^\mu]^2) - 0.5308[\Delta C_{10'}^\mu]
+ 0.00184([\Delta C_{S}^\mu]^2 + [\Delta C_{S}^\mu]^2 + [\Delta C_{T}^{2\mu}]^2 + [\Delta C_{T}^{2\mu}]^2) + 0.8614([\Delta C_{T}^{5\mu}]^2 + [\Delta C_{T}^{5\mu}]^2) (A7)
\]

\[10^6 B_{X_{\ell\ell}}^{\text{high}} = 10^6 B_{X_{\ell\ell}}^{\text{high,SM}} + 0.1187[\Delta C_{9'}^\mu] + 0.0143([\Delta C_{9'}^\mu]^2 + [\Delta C_{9'}^\mu]^2 + [\Delta C_{10'}^\mu]^2) - 0.1171[\Delta C_{10'}^\mu] + 0.0143[\Delta C_{10'}^\mu]^2
+ 0.0063([\Delta C_{S}^\mu]^2 + [\Delta C_{S}^\mu]^2 + [\Delta C_{T}^{2\mu}]^2 + [\Delta C_{T}^{2\mu}]^2) + 0.1272([\Delta C_{T}^{5\mu}]^2 + [\Delta C_{T}^{5\mu}]^2) (A8)
\]

**Appendix B: Primed operators**

Earlier we considered only the unprimed vector and axial vector operators, namely, \(C_{9'}^{\mu,e}\) and \(C_{10'}^{\mu,e}\), and neglected their primed counterparts \(C_{9'}^{\mu,e}'\) and \(C_{10'}^{\mu,e}'\). It has been shown (see for example, [20]) that the primed operators alone are unable to produce the experimental measurements of \(R_K\) and \(R_K'\) simultaneously. In this section, we will investigate whether the situation can improve in the presence of tensor operators.

Fig. 5 shows the allowed regions in \(\Delta C_{9'}^{\mu} - \Delta C_{T}^{5\mu}\) plane.

Fig. 6. Allowed regions in \(\Delta C_{9'}^{\mu} - \Delta C_{T}^{5\mu}\) plane. See text for more details.

However, this solution is in tension with \(B_{X_{\ell\ell}}^{\text{low}}\) as can be seen from the grey region in the left panel of Fig. 5. Note that, in the right panel of Fig. 5 the blue region covers the whole plane, and hence this solution is consistent with \(B_{X_{\ell\ell}}^{\text{high}}\). Similar statements can be made also for \(\Delta C_{10'}^{\mu}\), as can be seen from Fig. 6.

Fig. 7 and Fig. 8 show the allowed regions in \(\Delta C_{9'}^{\mu}\) vs. \(\Delta C_{T}^{5\mu}\) and \(\Delta C_{9'}^{\mu}\) vs. \(\Delta C_{T}^{5\mu}\) planes respectively. In these cases also, the primed operators can be allowed if a large tensor contribution exists at the same time.

It can be seen that in order to satisfy \(R_{K}^{\text{cen}}, R_{K}^{\text{low}}\) and \(R_{K}^{\text{high}}\) simultaneously in the presence of \(\Delta C_{9'}^{\mu}\), large value of \(\Delta C_{T}^{5\mu} \approx \pm 1.3\) is also needed.
2. \( C_s L e Q \frac{[\sigma_{R} Q_{3} H]}{A^4} [\sigma_{R} L_{4} H] \)
\[ \rightarrow C_s L e Q \left( \frac{1}{2} \frac{[\sigma_{R} Q_{3} H]}{A^4} [\sigma_{R} L_{4} H] + \frac{1}{8} [\sigma_{R} \sigma_{\mu \nu} Q_{3} H] [\sigma_{R} \sigma_{\mu \nu} L_{4} H] \right) \]
\[ = \frac{1}{8} C_s L e Q \frac{v^2}{A^4} \left( O_{S'} - O_{P'} + \frac{1}{4} O_T - \frac{1}{4} O_{T5} \right) \] (C2)

It is hard to generate only the tensor operators in a complete field theory model. The second operator above is much easier to generate (it can be generated even at the tree level). In this case, however, both scalar and tensor operators are generated with the following relations among the Wilson coefficients,

\[ \Delta C_{S'} = -\Delta C_{P'} = 4\Delta C_{T5} = -4\Delta C_{T5} \]. (C3)

Note that, gauge invariance at the dimension 6 level always leads to the relation \( \Delta C_{S'} = +\Delta C_{P'} \) [19], which is now broken by the dimension 8 operators. In Fig. 9, we show the various allowed regions in the \( \Delta C_{S'} (= -\Delta C_{P'}) \) vs. \( \Delta C_{T5} (= -\Delta C_{T5}) \) plane. It is interesting that the black overlap regions in the left panel satisfy Eq. (C3) approximately. In fact, there is tiny region in the right panel which satisfies the inclusive measurements too.

Note that, the value of \( \Delta C_{S'} = -\Delta C_{P'} \approx 3 \) corresponds to a NP scale \( \Lambda \sim (C_s L e Q)^{1/4} 1.5 \text{ TeV} \). While the scale is rather low, it is still intriguing that one local operator in Eq. (C2) can explain all the anomalies (including \( R_{K^0}^{low} \)) simultaneously. Unfortunately, for such large value \( \Delta C_{S'} = -\Delta C_{P'} \approx 3 \), \( B_{ee} \) exceeds the experimental upper bound, and some cancellation, either from other dimension-8 operators or from dimension-6 operators would be necessary for this operator to be viable. More detailed exploration of such dynamics is left for future work.

Appendix C: SU(2) \times U(1)_Y gauge invariance

As mentioned in the main text, the tensor operators do not get generated at the dimension-6 level if SU(2) \times U(1)_Y gauge invariance is imposed\(^1\). However, they can be generated at the dimension-8 level. Here we show a few examples,

1. \( \frac{C_{Y_{1} Y_{1}}}{A^4} [\sigma_{R} \sigma_{\mu \nu} Q_{3} H] [\sigma_{R} \sigma_{\mu \nu} L_{4} H] \)
\[ \rightarrow \frac{1}{2} C_{Y_{2} Y_{1}} \frac{v^2}{A^4} [\sigma_{R} \sigma_{\mu \nu} b_{L}] [\sigma_{R} \sigma_{\mu \nu} c_{L}] \]
\[ = \frac{1}{4} C_{Y_{2} Y_{1}} \frac{v^2}{A^4} (O_{T} - O_{T5}) \] (C1)

\[ \Delta C_{S'} = -\Delta C_{P'} = 4\Delta C_{T5} = -4\Delta C_{T5} \] (C3)

2. \( C_s L e Q \frac{[\sigma_{R} L_{4} H]}{A^4} [\sigma_{R} Q_{3} H] \)
\[ \rightarrow C_s L e Q \left( \frac{1}{2} \frac{[\sigma_{R} Q_{3} H]}{A^4} [\sigma_{R} L_{4} H] + \frac{1}{8} [\sigma_{R} \sigma_{\mu \nu} Q_{3} H] [\sigma_{R} \sigma_{\mu \nu} L_{4} H] \right) \]
\[ = \frac{1}{8} C_s L e Q \frac{v^2}{A^4} \left( O_{S'} - O_{P'} + \frac{1}{4} O_T - \frac{1}{4} O_{T5} \right) \] (C2)

FIG. 7. Allowed regions in \( \Delta C_{S'} - \Delta C_{T5} \) plane. See text for more details.

FIG. 8. Allowed regions in \( \Delta C_{10'} - \Delta C_{T5} \) plane. See text for more details.

FIG. 9. Allowed regions in \( \Delta C_{S'} (= -\Delta C_{P'}) \) vs. \( \Delta C_{T5} (= -\Delta C_{T5}) \) plane. See text for more details.
Tensor operators have also been considered in the context of the charged current anomalies $R_D$ and $R_{D^*}$, see for example [89, 90]. In that case, however, tensor operator can be generated already at the dimension 6 level [90].
[72] CMS, S. Chatrchyan et al., Phys. Rev. Lett. 111 (2013) 101804, 1307.5025.
[73] LHCb, R. Aaij et al., (2017), 1703.05747.
[74] CDF, T. Aaltonen et al., Phys. Rev. Lett. 102 (2009) 201801, 0901.3803.
[75] T. Huber, T. Hurth and E. Lunghi, Nucl.Phys. B802 (2008) 40, 0712.3009.
[76] BaBar, J.P. Lees et al., Phys. Rev. Lett. 112 (2014) 211802, 1312.5364.
[77] P. Ball, G.W. Jones and R. Zwicky, Phys. Rev. D75 (2007) 054004, hep-ph/0612081.
[78] A. Khodjamirian et al., JHEP 1009 (2010) 089, 1006.4945.
[79] M. Dimou, J. Lyon and R. Zwicky, Phys. Rev. D87 (2013) 074008, 1212.2242.
[80] S. Jager and J. Martin Camalich, JHEP 05 (2013) 043, 1212.2263.
[81] J. Lyon and R. Zwicky, (2014), 1406.0566.
[82] M. Ciuchini et al., JHEP 06 (2016) 116, 1512.07157.
[83] S. Jager and J. Martin Camalich, Phys. Rev. D93 (2016) 014028, 1412.3183.
[84] B. Capdevila et al., JHEP 04 (2017) 016, 1701.08672.
[85] Belle, S. Wehle et al., Phys. Rev. Lett. 118 (2017) 111801, 1612.05014.
[86] HPQCD, C. Bouchard et al., Phys. Rev. D88 (2013) 054509, 1306.2384, [Erratum: Phys. Rev.D88,no.7,079901(2013)].
[87] A. Bharucha, D.M. Straub and R. Zwicky, JHEP 08 (2016) 098, 1503.05534.
[88] G. Hiller and M. Schmaltz, Phys. Rev. D90 (2014) 054014, 1408.1627.
[89] P. Biancofiore, P. Colangelo and F. De Fazio, Phys. Rev. D87 (2013) 074010, 1302.1042.
[90] D. Bardhan, P. Byakti and D. Ghosh, JHEP 01 (2017) 125, 1610.03038.