Directional coupling of emitters into waveguides: Which property determines the directionality?

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Recent experiments have demonstrated strongly directional coupling of light into the guided modes of waveguides. The applications of this effect depend on which properties of the electromagnetic field determine the directionality. In this Letter, we consider the coupling of an emitter into a nearby waveguide and study the separate impact that the handedness and the angular momentum of the emission have on the coupling directionality. We show that the handedness of the emission has no influence on the directionality. We also show that the directionality is mostly determined by the eigenvalue of the transverse component of angular momentum (an integer \(m\)). The sign of \(m\) determines the preferential coupling direction, and the degree of directionality depends exponentially on \(|m|\). We trace this exponential dependence back to an inherent property of the evanescent angular spectrum of the emissions. Our results are based on concurring full-wave simulations and analytical derivations, including symmetry-based arguments. According to our results, the directionality effect could be exploited for routing light depending on its angular momentum, or for detecting high-order multipolar transitions of discrete emitters. Yet, it is not suited for applications that require handedness sensitivity, like discriminating between the two enantiomers of chiral molecules.

I. INTRODUCTION

Several recent experiments have demonstrated directional coupling of light into waveguide modes. For example, pronounced directionality has been shown in the collection of atomic emissions by nearby optical fiber, and quantum dot emissions by nearby waveguides. Similarly, experiments have shown pronounced directional coupling of focused light beams into waveguides, either directly or mediated by a scatterer. The directionality effect has potential uses for routing light and classifying emitters, like fluorescent molecules or quantum dots, according to the electromagnetic properties that determine the direction of coupling. Different theoretical approaches have been developed to understand the effect. In particular, the concepts of transverse spin and spin-momentum locking in evanescent waves have been put forward as the origin of the directionality. The photon spin has been connected to both the angular momentum content of the field and its polarization handedness. While angular momentum and handedness can coincide for some particular beams, like circularly polarized Gaussian beams or single plane-waves, this is not true in general. For example, the handedness of vortex beams can be chosen independently from their angular momentum.

In the context of directional coupling, it is important to know which properties of the emission control the directionality. For example, the sensing of the dominant enantiomer in a solution of chiral molecules may be possible if the directionality is determined by the handedness of the emission.

In this Letter, we investigate the separate impact on the directional coupling of two different properties of the emission: Helicity(handedness), which is related to the chiral properties of the fields, and angular momentum, which is related to the rotational properties of the fields. We numerically analyze the coupling directionality of an emitter near a rectangular waveguide. The angular momentum and helicity of the emissions are changed independently. Our results clearly show that the directional coupling effect is independent of the helicity of the emission. When all the other parameters are kept constant, changing the helicity of the emission has no effect: Both the preferred direction and the degree of directionality are identical for the two different helicities. In contrast, our results show that the property of the emission that mostly determines the directionality is the angular momentum along the axis perpendicular to both the waveguide axis and the normal from the waveguide to the emitter. The sign of such transverse angular momentum determines the preferred direction, and its modulus affects the degree of directionality in an exponential way. We provide symmetry arguments that support the results, and trace the origin of the exponential dependence back to a similar dependence that the energy flux carried by the evanescent angular spectrum of the emissions has on the transverse angular momentum.

In the rest of the Letter, we will introduce the electromagnetic fields that we use to model the emissions, describe the waveguide system, and present and discuss the results before the concluding remarks.

II. DESCRIPTION OF THE EMISSIONS

Angular momentum and helicity are distinct. They are represented by different operators with a different range of integer eigenvalues, and they are connected to two different symmetries. Angular momentum generates rotations and helicity generates electromagnetic duality. Nevertheless, their
A frequency window of 40 THz centered at \( f \) of the emission is quantized along the \( \hat{z} \) axis, transverse to \( x \) for a total of \( 30 = (3 + 5 + 7) \). Their angular momentum \( j \) in Ref. 23. The helical multipoles, or multipoles of well-defined helicity, can be considered single-mode across the entire waveguide. The central frequency corresponds to a vacuum wavelength \( \lambda = 200 \text{ nm} \). We consider emissions up to the octupolar order \( j \geq 3 \) for both helicities and all possible values of \( m \), for a total of \( 30 = (3 + 5 + 7) \times 2 \) cases. The angular momentum of the emission is quantized along the \( \hat{z} \) axis, transverse to the \( x \) axis. We perform the numerical simulations over a frequency window of 40 THz centered at \( f_0 \approx 193.4 \text{ THz} \). The central frequency corresponds to a vacuum wavelength of 1550 nm, and the frequency span to a wavelength range between 1404 nm and 1729 nm. For practical purposes, the waveguide can be considered single-mode across the entire frequency band. At the central frequency, the first waveguide mode has an effective index of about 2.26, whereas the effective index of its second mode is only about 1.07. This second very weakly guided mode will be practically irrelevant since any surface defect will efficiently scatter the guided energy out of the waveguide. The simulations are performed in the time domain using CST MWS. Each emission is modeled with the help of an imaginary auxiliary box surrounding the emitter (see Fig. 1). For a given helical multipole \(| k j m_z \lambda \rangle \), the exact radiated electromagnetic fields on the surface of the box are computed using Eqs. (A7, A14). Their tangential components are then imprinted on the surface of the box in terms of electric and magnetic source currents. According to the surface equivalence principle (Chap. 3.5 in Ref. 24), the electromagnetic field produced by these sources outside the box is identical to the electromagnetic field emanating from the multipolar emission that takes place at the center of the box. After the emission, a portion of the radiated energy couples into the waveguide. Finally, the energy coupled to the first waveguide mode travelling towards either the \( \hat{x} \) or the \( -\hat{x} \) direction is recorded by two dedicated ports. We refer to the power coupled towards the \( \pm \hat{x} \) direction as \( C_{\pm \hat{x}} \). Appendix B contains further details about the numerical simulations.

III. DESCRIPTION OF THE NUMERICAL SIMULATIONS

Figure 1 shows the geometry of the system. We consider an emitter at the origin of the coordinate system, and a nearby silicon waveguide of rectangular cross-section placed so that the waveguide is invariant under a reflection across the \( xOy \) plane, and its optical axis is parallel to the \( x \)-axis. The distance between the emitter and the axis of the waveguide is 590 nm. The width of the waveguide is 500 nm and its height is 200 nm. We consider emissions up to the octupolar order \( j = 3 \) for both helicities and all possible values of \( m \), for a total of \( 30 = (3 + 5 + 7) \times 2 \) cases. The angular momentum of the emission is quantized along the \( \hat{z} \) axis, transverse to the \( xOy \) plane. We perform the numerical simulations over a frequency window of 40 THz centered at \( f_0 \approx 193.4 \text{ THz} \). The central frequency corresponds to a vacuum wavelength of 1550 nm, and the frequency span to a wavelength range between 1404 nm and 1729 nm. For practical purposes, the waveguide can be considered single-mode across the entire frequency band. At the central frequency, the first waveguide mode has an effective index of about 2.26, whereas the effective index of its second mode is only about 1.07. This second very weakly guided mode will be practically irrelevant since any surface defect will efficiently scatter the guided energy out of the waveguide. The simulations are performed in the time domain using CST MWS. Each emission is modeled with the help of an imaginary auxiliary box surrounding the emitter (see Fig. 1). For a given helical multipole \(| k j m_z \lambda \rangle \), the exact radiated electromagnetic fields on the surface of the box are computed using Eqs. (A7, A14). Their tangential components are then imprinted on the surface of the box in terms of electric and magnetic source currents. According to the surface equivalence principle (Chap. 3.5 in Ref. 24), the electromagnetic field produced by these sources outside the box is identical to the electromagnetic field emanating from the multipolar emission that takes place at the center of the box. After the emission, a portion of the radiated energy couples into the waveguide. Finally, the energy coupled to the first waveguide mode travelling towards either the \( \hat{x} \) or the \( -\hat{x} \) direction is recorded by two dedicated ports. We refer to the power coupled towards the \( \pm \hat{x} \) direction as \( C_{\pm \hat{x}} \). Appendix B contains further details about the numerical simulations.

IV. RESULTS AND DISCUSSION

Figure 2 shows the logarithmic directionality of the in-coupled power \( D = \log_{10} [C_{+\hat{x}} / C_{-\hat{x}}] \) for varying angular momentum \( (m_z, j) \) and helicity \( \lambda \). A positive(negative) \( D \) indicates preferential coupling towards the \( +\hat{x}(-\hat{x}) \) direction, and \(|D|\) measures the degree of directionality in a logarithmic scale. For each \((m_z,j)\), the data in blue(red) corresponds to the positive(negative) helicity \(|j m_z + \rangle (|j m_z - \rangle)\). The color intensity encodes the frequency distribution of \( D \) as indicated by the insets. On the one hand, Fig. 2 clearly shows that the helicity does not influence the value of \( D \): Emissions with the
same multipolar content \( (m_z, j) \) but opposite helicity produce exactly the same values of \( D \). We will later show analytically that this follows from the symmetries of the system. The small discrepancies for large \( |m_z| \) are due to the low signal to noise ratio of the measurements in the non-preferred direction. On the other hand, Fig. 2 shows a clear dependence of \( D \) on the transverse angular momentum \( m_z \), which approximately follows the green dashed line corresponding to \( 2m_z \).

\[
D = \log_{10} \left| \frac{C_{+\lambda}}{C_{-\lambda}} \right| \approx 2m_z. \tag{2}
\]

The sign of \( m_z \) fixes the preferential coupling direction and, in a linear scale, the degree of directionality grows exponentially approximately as \( 2|m_z| \). We also observe that two emissions a and b, with \( (m_z, j)_a \) and \( -(m_z, j)_b \) will have \( D_a = -D_b \), and that for \( m_z = 0 \), \( D = 0 \). These observations will be also shown to follow from the symmetries of the system.

Let us now analyze a few particular cases in more detail. Figures 3(a,b) show \( D \) as a function of the frequency for the \( |m_z| = 0 \) and \( |m_z| = 1 \) cases, respectively. We observe in Fig. 3(a) that there is essentially no preferential coupling direction when \( |m_z| = 0 \) (note the vertical scale). Half the uncoupled energy travels towards each direction. This is also expected from symmetry considerations, as shown later on. The small fluctuations around \( D = 0 \) can be attributed to numerical noise. Figure 3(b) shows that the directionality of all multipolar emissions with \( m_z = +1 \) is positive along the whole spectrum, whereas the directionality of all multipolar emissions with \( m_z = -1 \) is negative along the whole spectrum. In Fig. 3(b) we clearly observe that the previously discussed behavior of \( D \) upon sign changes of the helicity \( \lambda \) and angular momentum \( m_z \) holds across the entire frequency range: There is a perfect spectral overlap of the directionality of multipolar emissions with equal \( (m_z, j) \) but opposite helicities, and, the directionality of a particular \( (m_z, j) \) emission is opposite to the directionality of the \( (m_z, j) \) emission. Finally, we note the sharp spectral features in the case of a dipolar emission at 205.3 THz, which is due to a pronounced dip in the frequency-dependent coupling into the guided mode towards one direction, leading to a spectral peak of the directionality towards the opposite direction.

We will now relate the results to the symmetries of the system. The waveguide system is invariant with respect to mirror reflections across the xOy and yOz planes. We can exploit these symmetries using the corresponding transformation properties of helical multipoles (see Appendix A):

\[
M_{\alpha} |k \ j \ m_z \ \lambda \rangle = (-1)^{m_z+1} |k \ j \ m_z - \lambda \rangle, \tag{3}
\]

\[
M_{\alpha} |k \ j \ m_z \ \lambda \rangle = (-1)^{j+m_z+1} |k \ j \ m_z - \lambda \rangle. \tag{4}
\]

where \( M_{\alpha} \) stands for the mirror transformation \( \alpha \to -\alpha \). Appendix C contains formal symmetry arguments. They show that the symmetry of the waveguide upon reflection across the xOy plane \( (M_x) \), together with Eq. (4) and that \( M_x (\pm \hat{x}) \to (\mp \hat{x}) \), imply that \( D \) must be identical for multipolar emissions with equal \( (m_z, j) \) and opposite helicity \( \lambda \). Hence, the coupling directionality cannot depend on helicity. Appendix C also shows that, given the previous symmetry condition (\( M_x \)), the symmetry of the waveguide upon reflection across the yOz plane \( (M_y) \), together with Eq. (3) and that \( M_y (\pm \hat{z}) \to (\pm \hat{z}) \), imply that multipolar emissions with opposite angular momentum \( m_z \) are going to have exactly the opposite directional coupling to the waveguide, and that for \( m_z = 0 \) the power coupled towards each direction must be identical. These regularities are clearly seen in our results. The same regularities will occur in any other geometry with the same symmetries. For
The exponential dependence of the directionality on the transverse angular momentum is remarkable. Its origin can be traced back to an intrinsic property of the evanescent plane-waves of the angular spectrum of the multipolar emissions. This is shown in Appendix D whose key points are the following ones: 1) Only the evanescent plane-waves in the angular spectrum of |k j m, λ⟩ can couple power into the waveguide because the propagation constant of the guided modes β is larger than k, the wavenumber outside the object; 2) The energy flux (real part of the Poynting vector) carried by those evanescent plane-wave components towards the ±z directions is proportional to a term that has a ±m_z exponential dependence, and; 3) Arguably, an approximate ≈ 2m_z exponential dependence of the directionality can then be expected. The origin of the exponential directionality is hence an intrinsic property of the emission. Such general origin, largely independent of the details of the system, is consistent with the wide variety of experimental setups where the directional coupling has been observed. The exponential directionality is also consistent with Ref. [11] where the transverse angular momentum content of evanescent plane-waves was shown to also depend exponentially on m_z (m_l in that work due to a different axis orientation). The fact that the result in Ref. [11] can be used to explain the way atomic transitions are excited by the evanescent tails of guided mode[26] strongly suggests that the directionality of the coupling of an emitter into guided modes should also depend exponentially on the transverse angular momentum: The excitation of a guided mode by an emitting object can be seen as the reciprocal situation from the one where the object is excited by the guided mode.

Regarding applications of the directional coupling to waveguides, on the one hand, the exponential dependence of the linear directionality with m_z may be exploited for detecting and classifying high order transitions of discrete nano-emitters, like atomic and molecular systems. On the other hand, it is clear that measurements of the directionality D do not allow to distinguish the helicity, chirality or handedness of the emission. For example, emissions from chiral molecules are typically composed of linear combinations of the two helicities like α, |k j m, λ = +1⟩ + α_− |k j m, λ = −1⟩, with imbalanced amplitudes |α_+| ≠ |α_-|. Whether |α_+| > |α_-| or |α_+| < |α_-| depends on the molecular enantiomer. According to our results, the coupling directionality will be the same for the two different enantiomers.

V. CONCLUSION

In this Letter, we have studied the directionality of the coupling of multipolar emissions with well-defined helicity and transverse angular momentum into a nearby single-mode waveguide. We have shown that there is no dependence of the directionality on the helicity(handedness). We have also shown that the directionality is mostly determined by the angular momentum, whose sign determines the preferential coupling direction, and whose absolute value affects the degree of directionality in an exponential way. The dependence(independence) of the directionality on angular momentum(helicity) has been shown by concurring full-wave simulations and formal symmetry arguments. We have also traced the origin of the exponential directionality to an inherent property of the evanescent angular spectrum of the emissions. While the numerical study has considered a single-mode waveguide, we expect the same conclusions to hold for multimode waveguides because the fundamental symmetry arguments would apply to each of the waveguide modes, and the exponential angular momentum dependence is inherent to the emissions.

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Appendix A: Multipoles of well-defined helicity

In this Appendix we present the basic properties of the multipolar emissions of well-defined helicity that we use in this work.

We assume a harmonic time dependence $e^{-i\omega t}$. Each monochromatic electromagnetic field component radiated by an emitter is an eigenstate of the Laplace operator with eigenvalue $-k^2$, i.e., they are radiating solutions of the monochromatic homogeneous vectorial Helmholtz equation. Morse and Feshbach showed that a complete basis of solenoidal solutions of the vectorial Helmholtz equation is given by the following two families of solutions $\nabla \times \tilde{\psi}(kr)$ and $\tilde{\Lambda} \nabla \times \tilde{\psi}(kr)$, where $\psi(kr)$ is a function that solves the scalar Helmholtz equation and $\Lambda$ is the helicity operator, which is defined as $\Lambda = \mathbf{J} \cdot \mathbf{P} / |\mathbf{P}| \equiv k^{-1} \nabla \times \times$, where $\mathbf{J}$ is the total angular momentum operator, $\mathbf{P}$ is the linear momentum operator, and the equivalence is true for monochromatic fields. Those two families of solutions were designed to be the TE and TM spherical solutions, i.e., spherical solutions that would describe electric and magnetic fields without a radial component, respectively. In literature they are known as magnetic and electric multipoles, respectively. Moreover, they are invariant under parity transformations because they are eigenstates of the parity operator $\Pi = M_x M_y M_z : (x, y, z) \rightarrow (-x, -y, -z)$, where $M_\alpha$ stands for the mirror transformation $\alpha \rightarrow -\alpha$. Hence, we may also call them multipoles of well-defined parity.

By solving the scalar Helmholtz equation in the spherical coordinate system, we obtain solutions like $\psi_{m, j}(kr) = \gamma_{m, j}^{(1)}(kr) \tilde{\psi}_{m, j}(kr) P^m_j(\cos \theta) e^{im\phi}$, where $\gamma_{m, j}^{(1)} = (2j+1)(j-m)!/\sqrt{4\pi(j+1)(j+m)!}$ is a normalization factor, $\tilde{\psi}_{m, j}(kr)$ is either the spherical Bessel function of the first kind $j^{(1)}_j$ or the spherical Hankel function of the first kind $\tilde{h}^{(1)}_j$ that corresponds to standing wave solutions, or the spherical Hankel function of the first kind $h^{(1)}_j$ that corresponds to radiating wave solutions. $P^m_j(\cos \theta)$ is the associated Legendre function of the first kind and $(j, m)$ are some integer indices that take the values $j = 1, 2, 3, ..., m = -j, -j+1, ..., 0, ..., j-1, j$, respectively. The index $j$ refers to the multipolar order of the emission, i.e., for $j = 1, 2, 3$ we have dipolar, quadrupolar, and octupolar emissions, respectively and $(\theta, \phi)$ are the polar and azimuthal angles of the spherical coordinate system, respectively. Both the scalar and vectorial solutions are simultaneous eigenstates of the total angular momentum squared operator $\mathbf{J}^2$ with eigenvalue $j(j+1)$, and of the angular momentum along the $z$-axis operator, $\mathbf{J}_z$, with eigenvalue $m$. We can refer to the radiating multipoles with well-defined parity $\tau = \pm 1$ with the following kets $|k \ j \ m_z \ \tau \rangle$:

\[
|k \ j \ m_z \ \tau = -1 \rangle = \nabla \times \left[ \tilde{\gamma}_{m, j}^{(1)} h^{(1)}_j (kr) P^m_j(\cos \theta) e^{im\phi} \right] \ \\
|k \ j \ m_z \ \tau = +1 \rangle = \tilde{\Lambda} |k \ j \ m_z \ \tau = -1 \rangle. \tag{A1}
\]

As we have mentioned, these fields are eigenstates of the parity operator $\Pi$:

\[
\Pi |k \ j \ m_z \ \tau \rangle = \tau(-1)^j |k \ j \ m_z \ \tau \rangle, \tag{A3}
\]

with corresponding eigenvalues $\tau(-1)^j$. The multipoles of magnetic type have a parity of $\tau = -1$ and the multipoles of electric type have a parity of $\tau = +1$.

Now, using those multipolar states of well-defined parity as a basis we construct multipolar emissions that are eigenstates of the helicity operator $\Lambda$. This transition to helical multipoles is quite straightforward once we notice that $\Lambda$:

\[
\Lambda |k \ j \ m_z \ \tau \rangle = |k \ j \ m_z \ \tau - \tau \rangle. \tag{A4}
\]

By making use of this property we can define multipoles of well-defined helicity $\lambda = \pm 1$:

\[
|k \ j \ m_z \ \lambda \rangle = |k \ j \ m_z \ \tau = +1 \rangle \sqrt{2} / |1 - \lambda(1 + \lambda)|, \tag{A5}
\]

for $\lambda = \pm 1$, which will have the following property:

\[
\Lambda |k \ j \ m_z \ \lambda \rangle = \lambda |k \ j \ m_z \ \lambda \rangle, \tag{A6}
\]

i.e., they will be eigenstates of the helicity operator with eigenvalues $\lambda = \pm 1$. From the definition of Eq. (A5), together with the analytic expressions of Eqs. (A1,A2), and after some algebra, we can obtain the expanded analytical formula for the radiating multipoles of well-defined helicity:

\[
|k \ j \ m_z \ \lambda \rangle = \frac{\lambda}{\sqrt{2}} |k \ j \ m_z \ \tau = -1 + \lambda |k \ j \ m_z \ \tau = +1 \rangle, \tag{A7}
\]

\[
\frac{1}{2} \left[ \frac{d}{d kr} \right] (kr h^{(1)}_j) + i h^{(1)}_j \right] A_{\lambda, m, j} (\tilde{\epsilon} \tilde{\lambda} (\tilde{\epsilon}) \tilde{\epsilon}_\lambda (\tilde{\epsilon}), \tag{A8}
\]

\[
\tilde{\epsilon}_\lambda (\tilde{\epsilon}) = \frac{-\lambda \theta(\tilde{\epsilon}) - i \phi(\tilde{\epsilon})}{\sqrt{2}}, \tag{A9}
\]

and we suppressed for brevity the arguments of $P^m_j(\cos \theta)$, $h^{(1)}_j (kr)$, and $(\tilde{\epsilon}, \tilde{\theta}, \tilde{\phi})$, and where $\tilde{\epsilon} = \tilde{\phi} \times \tilde{x}, \tilde{\phi} = \tilde{x} \times \tilde{y} / |\tilde{x} \times \tilde{y}|$ are the polar and azimuthal unit vectors that correspond to $\tilde{\epsilon}$.

From Eq. (A7), we can obtain the following spatial symmetry properties of the multipoles of well-defined helicity:

\[
M_\lambda |k \ j \ m_z \ \lambda \rangle = (-1)^{m_z+1} |k \ j \ m_z - \lambda \rangle, \tag{A10}
\]

\[
M_\lambda |k \ j \ m_z \ \lambda \rangle = -|k \ j \ m_z - \lambda \rangle, \tag{A11}
\]

\[
M_\lambda |k \ j \ m_z \ \lambda \rangle = (-1)^{j+m_z+1} |k \ j \ m_z - \lambda \rangle, \tag{A12}
\]

\[
\Pi |k \ j \ m_z \ \lambda \rangle = (-1)^{j+1} |k \ j \ m_z - \lambda \rangle. \tag{A13}
\]
Finally, by making use of Eq. (A6) together with the Maxwell–Faraday equation, \( \mathbf{H} = \Lambda \mathbf{E}/iZ \) where \( Z \) is the wave impedance of the hosting medium, we end up with the formulas below that give the electromagnetic field that is radiated by some particular multipolar emission of well-defined helicity \( \lambda \) and angular momentum \( m_z \):

\[
\mathbf{E}_{\lambda,m_z,j}(kr) = |k\ j\ m_z\ \lambda\rangle, \quad \mathbf{H}_{\lambda,m_z,j}(kr) = \frac{\lambda}{iZ} \mathbf{E}_{\lambda,m_z,j}(kr). \tag{A14}
\]

Appendix B: Details about the numerical simulation with CST MWS.

We pick the size of the auxiliary box to be 190 nm and assign a mesh step of 3.8 nm across its surface. Picking a fine mesh here is crucial because of the need to accurately model the evanescent fields that will be generated by the multipolar emission, as they are actually responsible for the near-field coupling to the waveguide. For this reason, we picked a mesh size of 0.02\(\lambda_0\) that allows us to correctly model fast varying evanescent fields with a spatial periodicity of even about 0.1\(\lambda_0\).

The waveguide ports that collect the energy guided from the emitter to each side of the waveguide are placed at a large distance of 6\(\mu\)m from the yOz plane. This needs in order to sufficiently reduce the background noise inside our simulation domain, since the radiated fields that do not overlap with the guided mode profiles are going to be reflected from the waveguide ports back into the simulation domain, which is of course undesired.

Finally, we note that all our formulas have an implicit \( e^{-i\omega t} \) time dependency, whereas CST MWS adopts the opposite time convention. We therefore need to take special care so that we give as an input the correct real fields. Specifically we need to modify the relevant formulas in Eqs. (A7/A14) by changing the sign of the magnetic field, the sign of \( m_z \), and also shifting from Hankel functions of the first kind to Hankel functions of the second kind in order to obtain radiating fields.

Appendix C: Demonstration of symmetry arguments

In this Appendix we will demonstrate that the regularities observed in the numerical results follow from symmetry arguments, namely that:

1. The directionality must be identical for multipolar emissions with equal \( (k, j, m_z) \) and opposite helicity \( \lambda \).
2. the directionality must be opposite for multipolar emissions with equal \( (k, j, \lambda) \) and opposite angular momentum \( m_z \), and that
3. the directionality must be zero for \( m_z = 0 \).

We will show that: i) The first statement follows from the mirror symmetry of the waveguide across the xOy plane, \( M_z \);

ii) The second statement follows from the first, plus the mirror symmetry of the waveguide across the yOz plane, \( M_x \); and

iii) The third statement follows from the second one when \( m_z = 0 \).

We will model the coupling between the emission of a multipole \( |k\ j\ m_z\ \lambda\rangle \) and the power guided along the \( \pm \hat{x} \) direction of the waveguide as:

\[
C_{\pm \hat{x}}|_{(k,j,m_z,\lambda)} = |\sigma_{\pm \hat{x}}\rangle |S\ |k\ j\ m_z\ \lambda\rangle|^2 \tag{C1}
\]

where \( \sigma_{\pm \hat{x}} \) is a guided mode of the waveguide in the \( \pm \hat{x} \) direction, and \( S \) is the S-matrix of the system that represents the coupling mechanism. The directionality \( D \) is defined as:

\[
D|_{(k,j,m_z,\lambda)} = \log_{10} \left[ \frac{C_{\pm \hat{x}}|_{(k,j,m_z,\lambda)}}{C_{\mp \hat{x}}|_{(k,j,m_z,\lambda)}} \right]. \tag{C2}
\]

We will now use Eqs. (A12/C1/C2), as well as the invariance of \( S \) under \( M_z \): \( M_z^\dagger SM_z = S \). The guided modes in the \( \pm \hat{x} \) direction must be invariant under \( M_z \), therefore we can write \( M_z| \sigma_{\pm \hat{x}}\rangle = e^{i\varphi_{\pm \hat{x}}}| \sigma_{\pm \hat{x}}\rangle \), with \( e^{i\varphi_{\pm \hat{x}}} \) being a phase term. Thus,

\[
C_{\pm \hat{x}}|_{(k,j,m_z,\lambda)} = |\langle \sigma_{\pm \hat{x}}| S |k\ j\ m_z\ \lambda\rangle|^2 = |\langle \sigma_{\pm \hat{x}}| M_z^\dagger SM_z |k\ j\ m_z\ \lambda\rangle|^2 = |e^{-i(\varphi_{\pm \hat{x}} + \pi(j + m_z + 1))}|\sigma_{\pm \hat{x}}\rangle S |k\ j\ m_z - \lambda\rangle|^2
\]

\[
= |\langle \sigma_{\mp \hat{x}}| S |k\ j\ m_z - \lambda\rangle|^2 = C_{\mp \hat{x}}|_{(k,j,m_z,-\lambda)}, \tag{C3}
\]

As a result, we obtain

\[
D|_{(k,j,m_z,\lambda)} = D|_{(k,j,m_z,-\lambda)}, \tag{C4}
\]

which proves statement 1 above.

We now use Eqs. (A10/C1/C2), as well as the invariance of \( S \) under \( M_x \): \( M_x^\dagger SM_x = S \). Due to the fact that the energy in the waveguide travels from one end to the other, the guided modes of the waveguide are not eigenstates of \( M_x \). Instead, they are transformed into each other as \( M_x| \sigma_{\pm \lambda}\rangle = e^{i\varphi_{\pm \lambda}}| \sigma_{\mp \lambda}\rangle \), with \( e^{i\varphi_{\pm \lambda}} \) being a phase term. Therefore, we have that:

\[
C_{\pm \lambda}|_{(k,j,m_z,\lambda)} = |\langle \sigma_{\pm \lambda}| S |k\ j\ m_z\ \lambda\rangle|^2 = |\langle \sigma_{\pm \lambda}| M_x^\dagger SM_x |k\ j\ m_z\ \lambda\rangle|^2 = |e^{-i(\varphi_{\pm \lambda} + j m_z + 1)}|\sigma_{\pm \lambda}\rangle S |k\ j - m_z - \lambda\rangle|^2
\]

\[
= |\langle \sigma_{\mp \lambda}| S |k\ j - m_z - \lambda\rangle|^2 = C_{\mp \lambda}|_{(k,j,m_z,-\lambda)}, \tag{C5}
\]

which implies:

\[
D|_{(k,j,m_z,\lambda)} = -D|_{(k,j,-m_z,-\lambda)}, \tag{C6}
\]

We now combine Eq. (C4) and Eq. (C6) to obtain that, for systems that are invariant under both \( M_x \) and \( M_z \):

\[
D|_{(k,j,m_z,\lambda)} = -D|_{(k,j,-m_z,-\lambda)}, \tag{C7}
\]

which proves statement 2 above. The last statement is readily shown by particularizing Eq. (C7) for \( m_z = 0 \):

\[
D|_{(k,j,0,\lambda)} = -D|_{(k,j,0,\lambda)} \Rightarrow D|_{(k,j,0,\lambda)} = 0. \tag{C8}
\]
Appendix D: Plane-wave spectrum of multipoles of well-defined helicity and the directionality of its evanescent part

In this Appendix we will examine the plane-wave decomposition of the multipolar emissions with well-defined helicity in order to investigate the origin of the observed exponential dependence of the directionality on the transverse component of the angular momentum. We will show that the exponential directionality comes from an intrinsic characteristic of the evanescent part of the angular spectrum of the emissions: The energy flux carried by evanescent plane-waves with opposite \(k_x\) is proportional to a term that has a \(\pm im\) exponential dependence on the transverse angular momentum.

1. Plane-wave spectrum

In Ref. [28] Eqs. (B1a, B1b), Devaney and Wolf expand the fields of multipolar emissions of well-defined parity in a series of plane-waves containing both propagating and evanescent components. By using our introduced conventions, normalizations, and from the definition of the multipoles of well-defined helicity [Eq. (A5)], we can reach the following formula that expands the helical multipoles as an integral series of plane-waves:

\[ |k_j m_\lambda\rangle \equiv \frac{1}{2\pi i^{j-1}} \int_0^{2\pi} \sin \theta_k d\theta_k A_{\lambda, m_j} (k) \hat{e}_\lambda (k) e^{ikr}, \quad \text{for } z \geq 0, \]  

\[(D1)\]

where

\[ \hat{k}(\theta_k, \phi_k) = \hat{x} \sin \theta_k \cos \phi_k + \hat{y} \sin \theta_k \sin \phi_k + \hat{z} \cos \theta_k \]  

\[(D2)\]

is the wavevector direction of each plane-wave component and the integration contour for the polar angles is \(C^\pm = [0, \pi \pm i\infty] \) for \(z \geq 0\), with the imaginary polar angles \(\theta_k\) accounting for the evanescent waves.

In our case we are interested in the plane-wave spectrum for the half-space \(y < 0\), because this is the half-space that hosts the waveguide (see Fig. 1 of the main text). We exploit the analytical continuation of Eq. (A3) in the complex space of the angles \((\theta_k, \phi_k)\) for obtaining the angular spectrum that expands the radiated fields of the emission at the half-space \(y < 0\). The plane-waves that expand the fields at the \(y < 0\) half-space correspond to \(k_x\) and \(k_y\) components that take real values varying from \(-\infty\) to \(+\infty\); and \(k_y = -\sqrt{k_x^2 - k_y^2 - k_z^2}\) takes values on the negative real(imaginary) axis for propagating(evanescent) plane-waves that propagate(decay) along the \(-\hat{y}\) direction. We are going to use the formulas:

\[ \theta_k = \arccos (k_x/k) = -i \ln \left[ k_x/k + i \sqrt{1 - (k_x/k)^2} \right], \]  

\[(D3)\]

\[ \phi_k = \arctan (k_x, k_y) = -i \ln \left[ \frac{k_x + ik_y}{\sqrt{k_x^2 + k_y^2}} \right], \]  

\[(D4)\]

in order to calculate the complex angles \((\theta_k, \phi_k)\) that are the input arguments for the plane-wave spectrum \(A_{\lambda, m_j} (k)\).

We note that, for the evanescent part of the spectrum, the norm of the polarization vector \(\hat{e}_\lambda (k)\) stops being unitary. Specifically, we have that:

\[ \hat{e}_\lambda (k) = \frac{1}{\sqrt{2}} \left( -\lambda \cos \theta_k \cos \phi_k + i \sin \theta_k \right) + \frac{\hat{y}}{\sqrt{2}} \left( -\lambda \cos \theta_k \sin \phi_k - i \cos \theta_k \right) + \frac{\hat{z}}{\sqrt{2}} \sin \theta_k, \]  

\[(D5)\]

\[ |\hat{e}_\lambda (k)\rangle = \cosh (\text{Im} \{\theta_k\}) \cosh (\text{Im} \{\phi_k\}) \lambda \cos (\text{Re} \{\theta_k\}) \sinh (\text{Im} \{\phi_k\}). \]  

\[(D6)\]

We will later use the following properties:

\[ |\hat{e}_\lambda (k_x, k_z)\rangle = |\hat{e}_\lambda (k_x, -k_z)\rangle, \quad \text{and}, \]  

\[(D7)\]

\[ |\hat{e}_\lambda (k_x, k_z)\rangle = |\hat{e}_\lambda (-k_x, -k_z)\rangle. \]  

\[(D8)\]

Equation (D7) follows because \(\theta_k\) does not depend on \(k_z\), and \(\text{Im} \{\phi_k(k_x, k_z)\} = -\text{Im} \{\phi_k(k_x, -k_z)\} = \ln \sqrt{k_x^2 + k_z^2} - \ln |k_x + i k_z|\). Equation (D8) follows because \(\theta_k\) is chosen for each direction by fixing \(k_x\) to either \(+\beta\) or \(-\beta\), a single plane-wave of the spectrum is chosen for each direction by fixing \(k_z\). We want to compute the ratio between the energy flux (real part of the Poynting vector) carried by the two chosen plane-waves. Such ratio can be shown to be equal to:

\[ R_{\lambda, m_j} (k_z) = \log_{10} \left[ \frac{|A_{\lambda, m_j} (k_x = +\beta, k_z) \hat{e}_\lambda (k_x = +\beta, k_z) |^2}{|A_{\lambda, m_j} (k_x = -\beta, k_z) \hat{e}_\lambda (k_x = -\beta, k_z) |^2} \right]. \]

\[(D9)\]

We now use Eqs. (A8,D3,D4,D6,D7) to decompose Eq. (D9) into two terms:

\[ R_{\lambda, m_j} (k_z) = 2 \log_{10} \left[ \frac{|e^{i \phi_k (k_x = +\beta, k_z)}|}{e^{i \phi_k (k_x = -\beta, k_z)}|} \frac{|\hat{e}_\lambda (k_x = +\beta, k_z)\rangle}{|\hat{e}_\lambda (k_x = -\beta, k_z)\rangle} \right] + 2 \lambda \log_{10} \left[ \frac{|\hat{e}_+ (k_x = +\beta, k_z)\rangle}{|\hat{e}_+ (k_x = -\beta, k_z)\rangle} \right] \]

\[ = 2 m_z f(k_z) + 2 \lambda g(k_z), \]

\[(D10)\]
where we have defined:

\[
f(k_z) = \log_{10} \left[ \frac{e^{i\phi(k_z=\pm\beta,k_z)}}{e^{i\phi(k_z=-\beta,k_z)}} \right]
\]

\[
f(k_z) = \log_{10} \left[ \frac{\beta - i \sqrt{1 - \left( \frac{\beta}{k} \right)^2 - \left( \frac{\kappa}{k} \right)^2}}{\beta + i \sqrt{1 - \left( \frac{\beta}{k} \right)^2 - \left( \frac{\kappa}{k} \right)^2}} \right],
\]

and:

\[
g(k_z) = \log_{10} \left[ \frac{\hat{e}_+ (k_z = \pm \beta, k_z)}{\hat{e}_+ (k_z = -\beta, k_z)} \right].
\]

We note that \( f(k_z) \) is always positive since \( \beta > k \) and also has an even symmetry: \( f(k_z) = f(-k_z) \). On the other hand, \( g(k_z) \) is a function with odd symmetry: \( g(k_z) = -g(-k_z) \). Both functions have singularities at \( |k_z| = k \) and approach zero in the limit of \( |k_z| \to \infty \). In Fig. 4 we plot the two functions for the case of \( \beta/k = 2.26 \).

It can be shown that the inequality \( f(k_z) \geq |g(k_z)| \) always holds true. This has as a consequence that the sign of \( R_{2, m, j}(k_z) \) solely depends on the sign of the transverse component of the angular momentum \( m_z \). The 2\( m_z \)-dependent term in Eq (D10) will be later argued to be the origin of the exponential dependence of the directionality \( D \) on \( m_z \). Additionally, \( R_{2, m, j}(k_z) \) does not depend on the multipolar order \( j \), and it has the symmetry property of \( R_{2, m, j}(k_z) = R_{-2, -m, j}(-k_z) \).

The overall directionality \( D \) on \( (k, j, m_z, \lambda) \) will be due to the coherent sum of all its evanescent components with \( k_z = \pm \beta \). The cross-section of the waveguide, the multipolar order, and the distance between the emitter and the waveguide will affect the way in which the different \( k_z \)-components will be combined. It is not possible to compute \( D \) from our results, but we can argue what the expected trends are using the last line of Eq (D10). First, as shown in Appendix C, the overall directionality \( D \) does not depend on the helicity \( \lambda \) when the system has the mirror symmetry. This means that the final result cannot have any \( \lambda \)-dependent term like the \( 2\lambda g(k_z) \) in Eq (D10). The other term in Eq (D10), with a \( 2m_z \)-dependence appears for each of the \( k_z \)-components, and we therefore expect that the final \( D \) should show a similar exponential dependence on \( m_z \). This expectation is confirmed by the numerical results.

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We note that there is nothing special about aligning the waveguide along the x-axis. All the results will be the same under rotations of the waveguide along the z-axis since the multipolar fields are eigenstates of $J_z$. Equation (D10) indicates that what is important when it comes to directionality is that the angular momentum is quantized along the axis perpendicular to the plane defined by the optical axis of the waveguide and the location of the emitter. This defines the transverse component of angular momentum.

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