Constraining R violation from Anomalous Abelian Family symmetry

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Abstract

The patterns of $R$ violation resulting from imposition of a gauged $U(1)$ horizontal symmetry, on the minimal supersymmetric standard model are systematically analyzed. We concentrate on a class of models with integer $U(1)$ charges chosen to reproduce the quark masses and mixings as well as charged lepton masses exactly or approximately. The $U(1)$ charges are further restricted from the requirement that very large bilinear lepton number violating terms should not be allowed in the super-potential. It is shown that all the trilinear $\lambda'_{ijk}$ and all but at most two trilinear $\lambda_{ijk}$ couplings vanish or are enormously suppressed.

1 Introduction

Supersymmetric Standard Models without R-parity contain large number (48) of free R-violating parameters and hence lack predictive power. However, models without R-parity are interesting in their own right in many ways. Models without lepton number can naturally accommodate neutrino masses as required by the present solar and atmospheric neutrino anomalies [1]. In such cases, the structure of lepton number violating couplings plays an important role as it determines the pattern of the mixing between the neutrino states.

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Since these trilinear L violating couplings are similar to the standard Yukawa couplings, it is generally argued that they are also hierarchical in nature. Given that the hierarchical nature of the standard Yukawa couplings can be attributed to an unbroken symmetry at a high scale, it would be interesting to study the structure of R-violating couplings under the influence of such a symmetry. In this talk, we present a systematic study of R-violating interactions in the framework of a U(1) family symmetry and their allowed patterns from low energy phenomenology.

2 U(1) SYMMETRY AND THE ε PROBLEM

The ratio of masses of fermions belonging to different generations (in quark and lepton sector both) when expressed in terms of Cabibo angle $\lambda = 0.22$, show a geometrical hierarchy. A mechanism that sets this orders of magnitude could be a broken U(1) family symmetry originally suggested by Froggatt and Nielsen (FN) [2]. We exploit the FN mechanism to shed light on the structures of R violating couplings [3].

Let us consider the MSSM augmented with a gauged horizontal U(1) symmetry. The standard super-fields ($L_i, Q_i, D_i, U_i, E_i, H_1, H_2$) are assumed to carry the charges ($l_i, q_i, d_i, u_i, e_i, h_1, h_2$) respectively with $i$ running from 1 to 3. The U(1) symmetry is assumed to be broken at a high scale by the vacuum expectation value (vev) of one gauge singlet super-field $\theta$ with the U(1) charge normalized to -1 or with two such fields $\theta, \overline{\theta}$ with charges -1 and 1 respectively. It is normally assumed that only the third generation of fermions have renormalizable couplings invariant under U(1). The rest of the couplings arise in the effective theory from higher dimensional terms [2]:

$$\Psi_i \Psi_j H \left( \frac{\theta}{M} \right)^{n_{ij}}$$

where $\Psi_i$ is a super-field, $H$ is the Higgs doublet and $M$ is some higher mass scale which could be Planck scale $M_p$ and $n_{ij} = \psi_i + \psi_j$ are positive numbers representing the charges of $\Psi_i, \Psi_j$ under U(1) respectively. This gives rise to an $ij^{th}$ entry of order $\lambda^{n_{ij}}$ in the mass matrix for the field $\Psi$. Identification $\lambda \sim 0.22$ and proper choice of the U(1) charges leads to successful quark mass matrices [4, 5] this way.

A priori the model has eighteen free parameters: Fifteen U(1) charges for quark and lepton super-fields, two U(1) charges for two Higgs super-fields and the parameter $x = q_3 + d_3 + h_1$, which determines the tan $\beta$ as, $\tan \beta \sim \lambda^x (m_t/m_b)$. However, the requirements of reproducing correct quark and lepton mass matrices along with the CKM matrix and the cancellations of the extra $U(1)$ anomalies (through the Green-Schwarz mechanism [6]) would reduce the number of parameters to four. The appropriate values for the rest of the parameters which are now written in terms of charge differences of form $q_3 = q_i - q_3$ etc, have been studied in the literature [5, 7] and we present them here in Table I.
Models

| Models | $l_{13} + e_{13}$ | $l_{23} + e_{23}$ | $d_{13}$ | $q_{23}$ | $u_{13}$ | $u_{23}$ | $d_{13}$ | $d_{23}$ |
|--------|-------------------|-------------------|-----------|----------|---------|---------|---------|---------|
| IA     | 4                 | 2                 | 3         | 2        | 5       | 2       | 1       | 0       |
| IIA    | 4                 | 2                 | 4         | 3        | 4       | 1       | 1       | -1      |
| IIIA   | 4                 | 2                 | 4         | 2        | 5       | 2       | -1      | -1      |
| IVA    | 4                 | 2                 | -2        | -3       | -10     | 7       | 6       | 5       |
| IB     | 5                 | 2                 | 4         | 3        | 4       | 1       | 1       | -1      |
| IIB    | 5                 | 2                 | 3         | 2        | 5       | 2       | 1       | 0       |
| IIIB   | 5                 | 2                 | 4         | 3        | 4       | 1       | 1       | -1      |
| IVB    | 5                 | 2                 | -2        | -3       | -10     | 7       | 6       | 5       |

Table 1: In the above, Models in the class A reproduce exactly the lepton mass matrix, whereas models of class B take into consideration $O(\lambda)$ variations as the $U(1)$ symmetry predictions are exact only upto $O(1)$. Inclusion of the R-violating parameters can lead to additional constraints. Let us consider, as a starting point only lepton number violating couplings which are given as:

$$W_{R_p} = \lambda'_{ijk} L_i Q_j D_k^c + \lambda_{ijk} L_i L_j E_k^c + \epsilon_i L_i H_2$$

where we have used the standard notation. We will comment on the baryon number violating couplings later on. Under the $U(1)$ symmetry the order of magnitudes of these couplings are also predicted, which are constrained by low energy phenomenology. The most stringent constraint comes from the possible choice of charges for the parameter $\epsilon_i$. Similar in nature to the $\mu$ parameter, the $\epsilon_i$ would have the following structure under the $U(1)$ symmetry (if $l_i + h_2 \geq 0$):

$$\epsilon_i \sim M \lambda^{l_i + h_2}$$

where $\lambda$ is the Cabibo angle. Unless the charges $l_i + h_2$ are appropriately chosen, the predicted value for the $\epsilon_i$ can grossly conflict with (a) The scale of $SU(2) \times U(1)$ breaking and (b) neutrino masses, as the neutrino mass generated in the presence of these couplings is directly proportional to $\epsilon_i$. In order to prevent very large $\epsilon_i$ one must ensure,

a) $l_i + h_2 \geq 24$ (which has been derived by choosing $\lambda = 0.22$ and $M = 10^{16} GeV$) or

b) $l_i + h_2 < 0$ so that the analyticity of $W$ will not allow such a term. This constraint has been so far neglected in the literature while considering the structure of R-violation under the influence of the $U(1)$ symmetry. As we will discuss below, this constraint plays an important role in deciding the allowed patterns of R-violation.

### 3 STRUCTURES OF TRILINEAR COUPLINGS

In this section, we will consider the effect of the $U(1)$ symmetry on the L-violating couplings, $\lambda'_{ijk}$, $\lambda_{ijk}$. The magnitudes and the structures of these
The trilinear couplings are determined by the following equations:

\[
\lambda'_{ijk} = \theta(n'_{ijk})\lambda''_{ijk} \quad (3)
\]

\[
\lambda_{ijk} = \theta(n_{ijk})\lambda''_{ijk} \quad (4)
\]

where \( n'_{ijk} = c_i + n^d_{jk} \), \( n_{ijk} = c_i + n^l_{jk} \), \( c_i = l_i + x + h_2 - h \), \( n^d_{jk} = q_{j3} + d_{k3} \), \( n^l_{jk} = l_{j3} + e_{j3} \) with \( n^d_{jk} \) and \( n^l_{jk} \) being completely fixed for a given model displayed in Table I. The parameter \( h = h_1 + h_2 \). The analyticity requirement of the \( W \) would lead to some of the trilinear couplings being zero if the corresponding exponent is negative. As mentioned above, the low energy phenomenology has stringent constraints on the magnitudes of these couplings \[\Box\]. However, it turns out that the constraint from the \( K^0 - \bar{K}^0 \) mass difference alone is sufficient to rule out the presence of the trilinear couplings in most models. Here, we use a conservative estimate of the above limit given as:

\[
\lambda'_{i12}\lambda'_{l21} \leq \lambda^{12} \sim 1.3 \cdot 10^{-8} \quad (5)
\]

In addition to the above constraint one has to consider the constraints on the \( c_i \) parameters mentioned above, which leads to conditions (a) or (b). After taking these constraints into consideration, the allowed structures of the trilinear couplings can be studied in the models presented in Table I. Firstly, we observe from eq.\[\Box\] that imposing the constraint (a) \( l_i + h_2 \gtrsim 24 \) would lead to trilinear couplings having orders of magnitude \( \sim \lambda^{10} \sim 10^{-12} \). Such a small value of the trilinear couplings would not have any phenomenological consequence. We will now consider imposing the second choice (b) \( l_i + h_2 \leq 0 \). To present the analysis in this case, we choose the phenomenologically most preferred model, namely model IA. In this case, the trilinear \( \lambda'_{ijk} \) couplings are explicitly given as,

\[
\lambda'_{ijk} = \lambda^{l_i + h_2 + x} \left( \begin{array}{ccc}
\lambda^4 & \lambda^3 & \lambda^3 \\
\lambda^3 & \lambda^2 & \lambda^2 \\
\lambda & 1 & 1
\end{array} \right). \quad (6)
\]

where it is implicit that some element is zero if the corresponding exponent is negative. The above matrix is expressed in terms of the corresponding matrix for the down-quarks, \( \epsilon(M_d)_{jk} \). Hence, for negative \( l_i + h_2 \), it follows that the \( \lambda'_{ijk} \) is either larger than the matrix element \((M_d)_{jk}\) or is zero for every \( i \). In the former case, the phenomenological requirement of the constraint, eq.\[\Box\] is not easily met. Specifically, equation for the \( c_i \) gets translated to,

\[
c_i \equiv l_i + h_2 + x < -3 \quad \text{or} \quad \geq 3 \quad (7)
\]

This condition ensures that \( \lambda'_{i12}\lambda'_{l21} \) either satisfies constraints from eq.\[\Box\] (when \( c_i > 3 \)) or is identically zero when \( c_i < -3 \). But \( c_i \geq 3 \) is untenable since \( l_i + h_2 \leq 0 \) and \( \tan \beta \sim \lambda^{\nu}(m_t/m_b) \geq O(1) \) needs \( x \leq 2 \) leading to \( c_i \leq 2 \). As a result one must restrict \( c_i \) to less than \(-3\) for all \( i \). The choice \( c_i = -4 \) is also ruled out as mixing of fields in kinetic term would now produce the additional couplings which are constrained by eq.\[\Box\]. Thus one concludes that only viable possibility from phenomenology is to require vanishing \( \lambda'_{ijk} \) for all values of \( i,j,k \).
The above arguments also serve to restrict the trilinear couplings \( \lambda_{ijk} \). Defining \((\Lambda_k)_{ij} \equiv \lambda_{ijk}\), we have,

\[
(\Lambda_1)_{ij} = \lambda^4 \begin{pmatrix}
0 & \lambda^{c_2} & \lambda^{c_3} \\
-\lambda^{c_2} & 0 & \lambda^{c_3+l_2-l_1} \\
-\lambda^{c_3} & -\lambda^{c_3+l_2-l_1} & 0
\end{pmatrix},
\]

(8)

\[
(\Lambda_2)_{ij} = \lambda^2 \begin{pmatrix}
0 & \lambda^{c_1} & \lambda^{c_3+l_1-l_2} \\
-\lambda^{c_1} & 0 & \lambda^{c_3} \\
-\lambda^{c_3+l_1-l_2} & -\lambda^{c_3} & 0
\end{pmatrix},
\]

(9)

\[
(\Lambda_3)_{ij} = \begin{pmatrix}
0 & \lambda^{c_2+l_1-l_3} & \lambda^{c_1} \\
-\lambda^{c_2+l_1-l_3} & 0 & \lambda^{c_2} \\
-\lambda^{c_1} & -\lambda^{c_2} & 0
\end{pmatrix},
\]

(10)

where \( c_i \) are the same coefficients defined in the context of the \( \lambda' \) and are required to be \(< -4\) as argued above. It then immediately follows from the charge assignments of the Model IA that all the \( \lambda_{ijk} \) except \( \lambda_{123}, \lambda_{231} \) and \( \lambda_{312} \) are forced to be zero. Moreover, \( \lambda_{312} \) and \( \lambda_{231} \) cannot simultaneously be zero.

Thus, we have demonstrated an important conclusion that Model IA can be consistent with phenomenology only if all \( \lambda'_{ijk} \) and all \( \lambda_{ijk} \) except at most two are zero. Similar exercise has been done with the other models leading to similar conclusion. Taking into consideration all the constraints on the models, we have numerically looked for integer solutions while restricting the absolute values of the charges \( q_3, u_3, d_3, l_1, l_2, l_3 \) to be \( \leq 10 \). This requirement is imposed for simplicity. As an example, we present here the allowed values for the Model IIB in the Table II.

From the above analyses we can derive the following conclusions:

(1) While all \( \lambda'_{ijk} \) are forced to be zero, some models do allow one or two non-zero \( \lambda_{ijk} \) which need not always be compatible with phenomenology.

(2) Although the term \( L_i H_2 \) is not directly allowed, it can be generated through the mechanism proposed by Giudice and Masiero \[9\]. The order of magnitudes of the \( \epsilon_i \) given in this case can still be of phenomenological relevance \[8\].

(3) Though we have not considered the baryon number violating couplings, we have also not imposed baryon parity. Solutions show that the operator \( U_i^c D_j^a D_k^b \) carries large -ve charge in all models. Thus Baryon number violation is automatically forbidden from super-potential. It can also be shown that the \( \lambda'' \) generated from effective U(1) violating D-term are extremely suppressed, \( O(10^{-15}) \) \[8\]. So Proton stability is automatically explained in all models.
Table II: Here we display the possible U(1) charge assignments for the Model IIB of Table I, consistent with the phenomenological constraints listed in the text. Absolute values of $q_3, u_3, d_3, l_1, l_2, l_3$ have been restricted to $\leq 10$ for simplicity.

4 SUMMARY

The supersymmetric standard model allows 39 lepton number violating parameters which are not constrained theoretically. We have shown in this talk that the U(1) symmetry, invoked to understand fermion masses can play an important role in constraining these parameters. We have shown that only phenomenologically consistent possibility, in this context is that all the trilinear $\lambda'$ and all but two $\lambda$ couplings to be zero or extremely small of $O(10^{-4})$. While the patterns of $R$ violation have been earlier discussed in the presence of U(1) symmetry, the systematic confrontation of these pattern with phenomenology leading to this important conclusion was not made to the best of our knowledge. This way, U(1) symmetry is shown to require that only four or five of the total 39 lepton number violating couplings could have magnitudes in the phenomenologically interesting range!

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