Testing Relativity with Orbiting Oscillators

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Abstract
Clock-comparison experiments using a satellite platform can give Planck-scale sensitivity to many parameters for Lorentz and CPT violation that are difficult to measure on Earth. A discussion of the theoretical framework for such tests is given, with emphasis on comparisons of output frequencies of atomic clocks and of electromagnetic cavity oscillators.

Introduction
Special relativity is an important underlying foundation for physics. For almost a century, experiments with ever-increasing precision have confirmed the validity of Lorentz symmetry. As precisions improve, there remains the possibility of detecting deviations from special relativity in experiments. High-precision atomic clocks and cavity oscillators planned for flight on the International Space Station (ISS) may be in a unique position to investigate this frontier in the coming years.

The Standard-Model Extension (SME) is a comprehensive framework detailing all possible coefficients that quantify Lorentz violation [1] and the associated CPT violation [2]. The breadth of the SME follows because all observable signals of Lorentz violation can be described by effective field theory [3]. The extensive literature on the SME in Minkowski spacetime has been complemented by recent work to include also gravitational effects [4]. A wide variety of experiments and theoretical investigations have been conducted to place bounds on the coefficients of the SME. Areas of physics affected include neutral mesons and baryogenesis [5, 6, 7], muon properties [8], Penning traps [9], comparisons of hydrogen with antihydrogen [10], and spin-polarized torsion pendula [11]. Recent work on neutrinos [12, 13] hints at the possibility of discovering Lorentz violation in that sector.

In many of these experiments, the sensitivity has attained the Planck-suppressed levels at which quantum-gravity effects may be expected on dimensional grounds.

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and is therefore of considerable interest. Despite dozens of experimental investigations, many of the numerous independent coefficients that quantify Lorentz violation remain unexplored. A large number of experiments will be needed to fully explore the SME parameter space.

The opportunity to place high-precision clock technology of various types on the ISS has the potential to open up several areas of the parameter space for Lorentz violation. Clock-comparison experiments will be possible by measuring the beat frequency between oscillators of various types, including hydrogen masers, rubidium and cesium atomic clocks, and microwave-cavity oscillators. Clock comparisons provide excellent tests of Lorentz symmetry [14]. A number of such tests have been conducted in Earth-based laboratories [15], and a detailed analysis of prospects for space tests exists [16].

Cavity oscillators provide access to the photon sector of the SME, so play a complementary role to the atomic clocks. There has been much interest in Lorentz violation in electromagnetism [17, 18]. The general effects of the SME in electromagnetism are known, and have yielded an exquisite test of the symmetry, bounding ten coefficients at the level of parts in $10^{32}$ [19]. Various earlier experimental results exist in this sector [20]. Cavity oscillators in the microwave and optical regimes have produced recent bounds on SME coefficients [21, 22], and there are plans for experiments of this genre to orbit on the ISS [23]. This proceedings summarizes aspects of the SME relevant to the experiments planned on the ISS. More details can be found in the references.

**Standard-Model Extension (SME)**

In the framework of the SME, the lagrangian describing a spin-$\frac{1}{2}$ Dirac fermion $\psi$ of mass $m$ in the presence of Lorentz violation is [3]:

$$\mathcal{L} = \frac{1}{2} \bar{\psi} \Gamma_\nu \partial^\nu \psi - \bar{\psi} M \psi ,$$

where

$$M := m + a_\mu \gamma^\mu + b_\mu \gamma_5 \gamma^\mu + \frac{1}{2} H_{\mu\nu} \sigma^{\mu\nu} \quad \text{and}$$

$$\Gamma_\nu := \gamma_\nu + c_{\mu\nu} \gamma^\mu + d_{\mu\nu} \gamma_5 \gamma^\mu + e_\nu + i f_\nu \gamma_5 + \frac{1}{2} g_{\lambda\mu\nu} \sigma^{\lambda\mu} .$$

The conventional Lorentz-preserving case is recovered from just the first term in each of $M$ and $\Gamma_\nu$ above. The additional terms in equations $M$ and $\Gamma_\nu$ contain conventional Dirac matrices $\{1, \gamma_5, \gamma^\mu, \gamma_5 \gamma^\mu, \sigma^{\mu\nu}\}$. They also contain parameters $a_\mu, b_\mu, c_{\mu\nu}, d_{\mu\nu}, e_\mu, f_\mu, g_{\lambda\mu\nu}$, and $H_{\mu\nu}$, that imply Lorentz violation in equation (1). Various mechanisms giving rise to these parameters are possible. They could for example arise as expectation values of Lorentz tensors in a fundamental theory with spontaneous Lorentz breaking [24]. The parameters in $M$ have dimensions of mass, and those in $\Gamma_\nu$ are dimensionless; $c_{\mu\nu}$ and $d_{\mu\nu}$ are traceless, while $H_{\mu\nu}$ is antisymmetric and $g_{\lambda\mu\nu}$ is antisymmetric in its first two indices.
The Lorentz-violation parameters in the lagrangian can be thought of as fixed geometrical background objects in spacetime. An experiment that rotates in space could in principle detect time-dependent projections of these geometric quantities. Similarly, two identical experiments with differing relative velocities could discern boost-dependent effects. Thus Lorentz violation is seen through comparisons of identical experiments with differing rotations and boosts, or through time dependence in a single experiment with nonzero acceleration. Lorentz symmetry is violated under these ‘particle transformations’ of entire experimental configurations. In contrast, experimenters observing one experimental system from different boosted or rotated inertial reference frames will find that the components of the parameters $a_\mu$, $b_\mu$, $c_{\mu\nu}$, $d_{\mu\nu}$, $e_\mu$, $f_\mu$, $g_{\lambda\mu\nu}$, and $H_{\mu\nu}$ transform like conventional tensors under Lorentz transformations of the coordinates. Thus, the SME preserves every aspect of conventional observer Lorentz symmetry.

The Lorentz-violation parameters are known to be minuscule, and so this formalism is well suited to treatment within perturbation theory. The theory has been studied in various contexts [25, 26, 27]. It is possible to pass to a Hamiltonian formalism and, with appropriate assumptions for the effective fermion $\psi$, to find the energy-level corrections for atoms within an atomic clock.

**General Clock-comparison Experiments**

An atomic clock operates by producing a stable output angular frequency based on an atomic energy-level transition. In many cases this frequency depends on a magnetic field that forms the quantization axis. Thus, if the third coordinate is defined to lie along this quantization axis, then the output frequency is $f(B_3)$. Stability of the clock is increased by operating the clock near a field-independent point and keeping the field $B_3$ as constant as possible.

In the presence of Lorentz violation, the SME provides a general framework that shifts the clock frequency, giving

$$\omega = f(B_3) + \delta \omega .$$

(4)

The quantity $\delta \omega$ contains all the contributions from Lorentz-violating terms in the SME lagrangian.

This small correction can contain terms that are orientation dependent, such as for example the dot product of $\vec{B}$ and the spatial part of $b^\mu$. It can also depend on the boost velocity of the clock relative to the inertial reference frame in which the four-vector $b^\mu$ is expressed. Even though the clock is usually stationary in the laboratory, the laboratory itself is a moving reference frame relative to the inertial reference frame. The exact form of $\delta \omega$ can be complicated and involves corrections for all the Lorentz-violating added terms in the SME lagrangian and requires a detailed knowledge of the motion of the clock laboratory.

In many realistic cases, the function $f$ can be inverted for values of $B_3$ ranging over those encountered experimentally. We therefore consider cases where $f^{-1}$
exists, and note that cases where the clock operates at a field-independent point can be handled by alternative methods. If the form of $f$ is known within the framework of conventional Lorentz-preserving physics, then for a clock running with frequency $\omega$ in the presence of Lorentz violation the inverse $f^{-1}(\omega)$ gives an effective magnetic field differing slightly from the actual magnetic field. If there is no Lorentz violation, this effective magnetic field is the actual $B_3$.

To search for evidence of Lorentz violation in practical terms means that the clock frequency has to be compared to a standard, which is essentially another atomic clock. Thus the comparison is made between two clocks, with frequencies $\omega_A$ and $\omega_B$, both of which are sensitive to Lorentz violation. One way to seek violations would be to monitor the frequency difference $\omega_A - \omega_B$. Any time dependence would indicate Lorentz violation. In conventional physics, this difference is non-zero and equals $f_A(B_3) - f_B(B_3)$. If the functions $f_A$ and $f_B$ have matching slopes then the difference would be constant even with variations in $B_3$. This matching of slopes is not in general possible for a given pair of atomic clocks.

A preferable measure of Lorentz violation that circumvents the difficulties with magnetic-field dependence is the modified frequency difference

$$\omega^\sharp := \omega_A - f_A \circ f_B^{-1}(\omega_B).$$

(5)

Using simple differentiation assumptions on the functions $f_A$ and $f_B$, it follows [16] that $\omega^\sharp$ is independent of $B_3$, equalling

$$\omega^\sharp = \delta \omega_A - v \delta \omega_B,$$

(6)

where the quantity $v$ is the dimensionless constant ratio of the two frequency gradients evaluated at zero field:

$$v = \left. \left( \frac{df_A}{dB_3} / \frac{df_B}{dB_3} \right) \right|_{B_3=0}.$$

(7)

Note also that in the absence of Lorentz violation, $\omega^\sharp$ vanishes.

To utilize equation (6), several options are possible. If the functions $f_A$ and $f_B$ are known in detail, then one way to proceed is to record the values of $\omega_A$ and $\omega_B$ at each instant, and then to combine them using this equation. This gives an experimental value of $\omega^\sharp$ that can be compared with the theoretical calculation. This method requires a detailed knowledge of the functions $f_A$ and $f_B$, which may not be possible in practise.

An alternative method to experimentally determine $\omega^\sharp$ is to use feedback control to keep the frequency $\omega_B$ of clock B fixed relative to itself. Then in equation (5), $f_A[f_B^{-1}(\omega_B)]$ is constant, making $\omega^\sharp = \omega_A + \text{constant}$. The constant value that arises even in the absence of Lorentz violation is irrelevant, since the experimental procedure requires monitoring only the variations in $\omega^\sharp$. This method could be useful in situations where a detailed knowledge of $f_A$ and $f_B$ is not known but
the two clocks are in the same magnetic field $B_3$. This may offer advantages for potential clock-comparison experiments on the ISS, where the magnetic field is likely to fluctuate. The signal to be monitored would thus be

$$\omega_A = \omega^* - \text{constant} = \delta \omega_A - v \delta \omega_B - \text{constant} . \quad (8)$$

### Applying the SME in the Laboratory Frame

The atoms comprising an atomic clock are a complex system of protons, neutrons, and electrons. To calculate the effect of the SME on such systems, a variety of simplifying assumptions are made to model the system with a single wave function $\psi$. The Hamiltonian for this system can be split into two portions: a conventional part describing the atom within the chosen model, and a perturbative Lorentz-violating part $h'$ arising from the SME. It can be expressed as a sum of perturbative Hamiltonians for each proton, electron, and neutron (indexed by $w$) in the atom:

$$h' = \sum_w \sum_{N=1}^{N_w} \delta h_{w,N} . \quad (9)$$

In this expression, the atom or ion $W$ has $N_w$ particles of type $w$, and $\delta h_{w,N}$ is the Lorentz-violating correction for the $N$th particle of type $w$. Since each of the three particle species in the atom has a set of Lorentz-violation parameters, a superscript $w$ must be placed on each of the parameters $a_{\mu}$, $b_{\mu}$, $c_{\mu\nu}$, $d_{\mu\nu}$, $e_{\mu}$, $f_{\mu}$, $g_{\lambda\mu\nu}$, and $H_{\mu\nu}$.

The symmetry-breaking energy-level shifts are calculated by finding the expectation value of the perturbative Hamiltonian $h'$ in the desired unperturbed state of the atoms. In most cases, the quantization axis is defined by a magnetic field, and the total angular momentum $\vec{F}$ of the atom or ion and its projection along the quantization axis are conserved to a good approximation. Thus quantum states for the atomic-clock atoms can be labelled by the corresponding quantum numbers $|F,m_F\rangle$. We define the third coordinate of the laboratory reference frame to be this quantization axis.

In the laboratory frame, the energy-level shift for state $|F,m_F\rangle$ is

$$\delta E(F,m_F) = \langle F,m_F|h'|F,m_F\rangle$$

$$= \tilde{m}_F \sum_w (\beta_w \tilde{b}_3^w + \delta_w \tilde{d}_3^w + \kappa_w \tilde{g}_d^w) + \tilde{m}_F \sum_w (\gamma_w \tilde{c}_q^w + \lambda_w \tilde{g}_q^w) . \quad (10)$$

In this expression, $\tilde{m}_F$ and $\tilde{m}_F$ are particular ratios of Clebsch-Gordan coefficients. The quantities $\beta_w$, $\delta_w$, $\kappa_w$, $\gamma_w$, and $\lambda_w$ are expectation values of combinations of spin and momentum operators in the extremal states $|F,m_F = F\rangle$. They can not in general be calculated exactly since a detailed description of the nuclear forces is not known. For further details of these quantities, see reference [14].
In equation (10) the quantities with tildes are specific combinations of Lorentz-violation parameters, and importantly, are the only possible parameter combinations to which clock-comparison experiments are sensitive. For the case of $\tilde{b}_3^w$, the combination is
\[
\tilde{b}_3^w := b_3^w - m_w d_{30}^w + m_w g_{120}^w - H_{12}^w .
\] (11)
The other tilde quantities can be found in reference [14].

Having found the shifts of the energy levels $|F,m_F\rangle$, the effect of the SME on the frequency corresponding to the transition $(F,m_F) \to (F',m'_F)$ is found from the difference
\[
\delta\omega = \delta E(F,m_F) - \delta E(F',m'_F) .
\] (12)
This is the $\delta\omega$ appearing in equation (4).

The Standard Inertial Reference Frame

The SME indicates that Lorentz violation can occur in nature through the existence of a variety of background observer Lorentz tensors. The objective of experimental tests of Lorentz symmetry is to measure these tensor components, or to place bounds on them if experiments are not able to resolve them. The inertial reference frame in which the components of these tensors are measured needs to be standardized to allow different experiments to compare independent measurements of the same quantities.

By convention, the inertial reference frame used to present results for bounds on SME quantities has origin at the center of the Sun. The axes are labelled $X$, $Y$, and $Z$, with $Z$ axis parallel to the axis of the Earth in the northerly orientation. The $X$ axis points towards the vernal equinox on the celestial sphere, and the $Y$ axis completes the right-handed system. This frame is close to inertial over periods of thousands of years, as opposed to any Earth-based frame in which the inertial approximation breaks down after a week or two.

The time variable in this frame is denoted by $T$, and is measured by a clock considered to be at the center of the Sun with $T = 0$ taken to be at the vernal equinox in the year 2000.

We define the laboratory coordinate system to have third coordinate along the quantization axis. The laboratory frame $(x_1,x_2,x_3)$ is not in general inertial, and quantities measured in the laboratory frame must be transformed into the standard inertial reference frame. The specifics of the transformation to the inertial reference frame will depend on the experiment. Clock-comparison experiments have been done in Earth-based laboratories, and others are planned for satellites encircling the Earth. We focus here on the latter.

For our purposes, the motion of a satellite-based experiment can be considered to be a superposition of two circular motions, where one is the motion of the Earth around the Sun, and the other is the motion of the satellite around the Earth. The Earth moves on a circle in a plane tilted at angle $\eta \approx 23^\circ$ to the equatorial plane,
passing through the positive $X$ axis at the vernal equinox. The orbit of the satellite can be specified by giving the inclination angle $\zeta$ to the inertial $Z$ axis, and the right ascension $\alpha$ of the ascending node of the orbit, where the satellite cuts the equatorial plane in the northward direction.

There are various possibilities for the orientation of the laboratory reference frame within the satellite. For definiteness, we define the quantization axis $x_3$ to be directed along the velocity vector of the satellite relative to the Earth, and the $x_1$ axis to be directed radially towards the center of the Earth.

With these definitions, the laboratory-frame observable quantities in clock-comparison experiments can be expressed in terms of the corresponding inertial-frame quantities. Since the motion is a composition of two circular motions, the expressions are long, even with simplifying assumptions such as zero eccentricity for the orbits. The general form for the expression $\tilde{b}_3$ is:

$$
\tilde{b}_3 = \cos \omega_s T_s \left\{ \left[ -\tilde{b}_X \sin \alpha \cos \zeta + \tilde{b}_Y \cos \alpha \cos \zeta + \tilde{b}_Z \sin \zeta \right] + \beta_3 \text{[seasonal terms...]} \right\} \\
+ \sin \omega_s T_s \left\{ \left[ -\tilde{b}_X \cos \alpha - \tilde{b}_Y \sin \alpha \right] + \beta_3 \text{[seasonal terms...]} \right\} \\
+ \cos 2\omega_s T_s \left\{ \beta_s \text{[constant terms...]} \right\} + \sin 2\omega_s T_s \left\{ \beta_s \text{[constant terms...]} \right\} \\
+ \beta_s \text{[constant terms...]} .
$$

(13)

Oscillations involving the satellite orbital frequency appear with single and double frequencies $\omega_s$, and $2\omega_s$. The much slower orbital frequency $\Omega_\oplus$ of the Earth as it moves around the Sun appears in the seasonal terms as indicated. The suppressions due to the speed $\beta_\oplus \approx 10^{-4}$ of the Earth and $\beta_s \approx 10^{-5}$ for the ISS are also shown explicitly. For further details of $\tilde{b}_3$, see [16], and see [14] for details concerning the nonrelativistic limit.

The energy shift expressed in equation (10) also depends on the specific atoms in the clock and the transition being used for the clock. Species planned for flight on the ISS include rubidium 87, cesium 133, and hydrogen. Further details are available in [16].

The Photon Sector

Another highly-stable clock is the cavity oscillator, in which a resonant frequency of an electromagnetic oscillation is excited. There is interest in such oscillators for use on the ISS [23]. The SME provides a complete and unified framework detailing all possible Lorentz violations in the photon sector[19]. The photon-sector lagrangian can be expressed as

$$
\mathcal{L} = \frac{1}{2} \left[ (1 + \tilde{\kappa}_{tr}) \tilde{E}^2 - (1 - \tilde{\kappa}_{tr}) \tilde{B}^2 \right] \\
+ \frac{1}{2} \tilde{E} \cdot (\tilde{\kappa}_{e+} + \tilde{\kappa}_{e-}) \cdot \tilde{E} - \frac{1}{2} \tilde{B} \cdot (\tilde{\kappa}_{e+} - \tilde{\kappa}_{e-}) \cdot \tilde{B} + \tilde{E} \cdot (\tilde{\kappa}_{o+} + \tilde{\kappa}_{o-}) \cdot \tilde{B}.
$$

Here, $\tilde{\kappa}_{tr}$ is a single number, $\tilde{\kappa}_{e+}$, $\tilde{\kappa}_{e-}$, $\tilde{\kappa}_{o-}$ are traceless symmetric $3 \times 3$ matrices, and $\tilde{\kappa}_{o+}$ is an antisymmetric $3 \times 3$ matrix, giving a total of 19 independent
coefficients for Lorentz violation in the photon sector. The \( \tilde{\kappa} \) quantities are tensor-like geometric quantities that can in principle be detected with electromagnetic experiments.

This lagrangian leads to modified Maxwell equations and hence to a modified wave equation for light. Among the effects is a rotation of the polarization vector that is frequency dependent. To observe such a rotation, the propagation distance should be maximized. Using polarization data for light from 16 distant cosmological sources, a bound [19] has been placed on 10 linear combinations of the \( \tilde{\kappa} \) coefficients, denoted by \( k^a \) for \( a = 1, \ldots, 10 \). The result is

\[
|k^a| < 2 \times 10^{-32}.
\]  

These bounds are among the tightest bounds in the photon sector, but apply only to 10 linear combinations of coefficients. Of the 9 remaining independent coefficients, seven have been bounded in experiments with electromagnetic cavity oscillators at the level of parts in \( 10^{15} \) [21].

For resonant cavities, the SME lagrangian (14) leads to a fractional resonant-frequency shift \( \delta \nu / \nu \) that depends on the geometrical relationship between the axis of the cylindrical cavity and the SME background tensor-like fields. For optical frequencies, the general form of the relative frequency shift is

\[
\frac{\delta \nu}{\nu} = \frac{1}{2|E_0|^2} [\vec{E}_0^* \cdot (\kappa_{DE})_{\text{lab}} \cdot \vec{E}_0 / \varepsilon - (\hat{N} \times \vec{E}_0^*) \cdot ((\kappa_{HB})_{\text{lab}} \cdot (\hat{N} \times \vec{E}_0)],
\]

where \( \hat{N} \) is the axis of the cylindrical cavity, \( \vec{E}_0 \) is the electric field vector, and \( \kappa_{DE}, \kappa_{HB} \) are specific linear combinations of the \( \tilde{\kappa} \) matrices in the laboratory reference frame.

Since the laboratory is noninertial, being either on the surface of the Earth or on a satellite orbiting the Earth, equation (16) must be expressed in terms of the standard inertial reference frame as discussed for atomic clocks above. If the laboratory is Earth-based, the oscillator output contains sinusoidal variations with frequencies \( \omega_\oplus \) and \( 2\omega_\oplus \) as well as slower seasonal variations due to the tilt of the Earth relative to the Sun. Similar single- and double-frequency effects can be expected for oscillators on a satellite. Several bounds on these coefficients have been obtained in Earth-based experiments using optical and microwave frequencies [21, 22].

**Discussion**

No evidence exists for Lorentz violation at present, although an interesting possibility relating to the apparent mass of neutrinos may change this [12, 13]. The SME provides the full parameter space for testing Lorentz symmetry. While parts of this space have been investigated with experiments from all corners of the physics globe, there are still some regions that are inaccessible. Experiments on the ISS
will open up a path to several outlying regions of this world. Atomic clocks [16]
and microwave cavities [19, 21, 22] aboard the ISS may give a new perspective on
whether nature is Lorentz symmetric.

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