Mass Function of Binary Massive Black Holes in Active Galactic Nuclei

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Abstract

If the activity of active galactic nuclei (AGNs) is predominantly induced by major galaxy mergers, then a significant fraction of AGNs should harbor binary massive black holes in their centers. We have studied the mass function of binary massive black holes in nearby AGNs based on the observed AGN black-hole mass function and the theory of evolution of binary massive black holes interacting with a massive circumbinary disk within the framework of the coevolution of massive black holes and their host galaxies. The circumbinary disk is assumed to be steady, axisymmetric, geometrically thin, self-regulated, self-gravitating but nonfragmenting with a fixed fraction of the Eddington accretion rate, which is typically one tenth of the Eddington value. The timescale of orbital decay is then estimated to be \( \sim 10^8 \) yr for equal mass black holes, being independent of the black-hole mass, semimajor axis, and viscosity parameter, but dependent on the black-hole mass ratio, Eddington ratio, and mass-to-energy conversion efficiency. This makes it possible for any binary massive black holes to merge within the Hubble time under the binary–disk interaction. We find that (1.5% ± 0.6%) of the total number of nearby AGNs for the equal-mass ratio and (1.3% ± 0.5%) for the one-to-ten mass ratio have close binary massive black holes with an orbital period of less than 10 yr in their centers, detectable with ongoing highly sensitive X-ray monitors, such as Monitor of All-sky X-ray Image and/or Swift/Burst Alert Telescope. Assuming that all binary massive black holes have the equal-mass ratio, about 10% of AGNs with black-hole masses of \( 10^6 - 10^7 M_\odot \) have close binaries, and thus provide the best chance to detect them.

Key words: accretion, accretion disks — binaries: general — black hole physics — galaxies: nuclei

1. Introduction

Most galaxies are thought to have massive black holes at their centers (Kormendy & Richstone 1995). Massive black holes play an important role not only in the activities of active galactic nuclei (AGNs) and quasars, but also in the formation and evolution of galaxies (Magorrian et al. 1998; Ferrarese & Merritt 2000; Gebhardt et al. 2000). Galaxy merger leads to mass inflow to the central region by tidal interactions, and then the nucleus of the merged galaxy is activated, and the black hole grows by gas accretion (Yu & Tremaine 2002). At some stage, the outflow from the central black hole sweeps away the surrounding gas and quenches the star formation and black hole growth. This also produces an observed correlation between the black hole mass and the velocity dispersion of individual galaxies (Di Matteo et al. 2005).

During a sequence of processes, binary massive black holes with a subparsec-scale separation are inevitably formed before two black holes merge by emitting gravitational radiation. Recent hydrodynamic simulations have shown rapid binary black hole formation on the parsec scale within several Gyr by the interaction between the black holes and the surrounding stars and gas in gas-rich galaxy mergers (Dotti et al. 2007; Mayer et al. 2007). Even if there are transiently triple massive black holes in a galactic nucleus, the system finally settles down to the formation of binary massive black holes by the merging of two black holes, or by ejecting one black hole from the system via a gravitational slingshot (Iwasawa et al. 2006).

In the coalescing process of two massive black holes, there has been the “final parsec problem”; it is still unknown how binary massive black holes evolve after their semimajor axis has reached the subparsec scale, where dynamical friction with the neighboring stars is no longer effective. Many authors have tackled the final parsec problem in the context of the interaction between the black holes and the stars, but there has still been extensive discussions (Begelman et al. 1980; Makino 1997; Quinlan & Hernquist 1997; Milosavljević & Merritt 2003; Sesana et al. 2007; Matsubayashi et al. 2007; Matsu & Habe 2009). There is another possible way to extract the energy and the angular momentum from binary massive black holes by the interaction between the black holes and the gas surrounding them. This kind of the binary–disk interaction could also be a candidate for resolving the final parsec problem (Ivanov et al. 1999; Gould & Rix 2000; Armitage & Natarajan 2002, 2005; Escala et al. 2005; Hayasaki 2009; Cuadra et al. 2009; Haiman et al. 2009) in spite of a claim (Lodato et al. 2009). Some authors have shown that there exist close binary massive black holes with a short orbital period of less than
10 yr and significant orbital eccentricity (Armitage & Natarajan 2005; Hayasaki 2009; Cuadra et al. 2009).

Hayasaki, Mineshige, and Sudou (2007) studied the accretion flow from a circumbinary disk onto binary massive black holes, using a smoothed particle hydrodynamics (SPH) code. They found that mass transfer occurs from the circumbinary disk to each black hole. The mass accretion rate significantly depends on the binary orbital phase in eccentric binaries, whereas it shows little variation with the orbital phase in circular binaries. Periodic behaviors of the mass accretion rate in binary systems with different system geometries or system parameters were also discussed by some other authors (Bogdanović et al. 2008; MacFadyen & Milosavljević 2008; Cuadra et al. 2009). Recently, Bogdanović, Eracleous, and Sigurdsson (2009) and Dotti et al. (2009) proposed the hypothesis that SDSS J092712.65+294344.0 consists of two massive black holes in binary, by interpreting the observed emission-line features as those arising from a mass-transfer stream from the circumbinary disk.

Hayasaki, Mineshige, and Ho (2008) have, furthermore, performed a new set of simulations at higher resolution with an energy equation based on the blackbody assumption, adopting the same set of binary orbital parameters that we previously used (Hayasaki et al. 2007) ($a = 0.01$ pc, eccentricity $e = 0.5$, and mass ratio $q = 1.0$). By this two-stage simulation, they found that while the Optical/NIR light curve exhibits little variation, the X-ray/UV light curve shows significant orbital modulation in the triple-disk system, which consists of an accretion disk around each black hole and a circumbinary disk around them. X-ray/UV periodic light variations originate from a phase-dependent mass transfer from a circumbinary disk (cf. Hayasaki & Okazaki 2005). The one-armed spiral wave on the accretion disk induced by the phase-dependent mass transfer causes the mass to accrete onto each black hole within one orbital period. This is repeated every binary orbit. These unique light curves are, therefore, expected to be one of the observational signatures of binary massive black holes.

Highly sensitive X-ray monitors over a wide area provide a unique opportunity to discover close binary massive black holes in an unbiased manner, based on detecting the orbital flux modulation. Monitor of the All-sky X-ray Image (MAXI: Matsuoka et al. 2009), a Japanese experimental module attached to the International Space Station, is now successfully in operation since its launch in 2009 July. MAXI, covering the energy band of 0.5–30 keV, achieves a significantly improved sensitivity compared to Swift/BAT, based on the observed black-hole mass function of nearby AGNs and the probability of finding binary massive black holes, based on the evolutionary scenario, as described in section 2. Brief discussions and conclusions are summarized in section 4.

2. Final-Parsec Evolution of Binary Massive Black Holes

We first describe the evolution of binary massive black holes, while focusing on interactions with surrounding gaseous disks within the framework of the coevolution of massive black holes and their host galaxies.

Does the black-hole mass correlate with the velocity dispersion of the bulge in individual galaxies despite that there is a single or binary in their center? This is one of fundamental problems within the framework of the coevolution of massive black holes and their host galaxies. In some elliptical galaxies, there is a core with the outer steep brightness and inner shallow brightness. Binary massive black holes are considered to be closely associated with such a core structure with the mass of the stellar light deficit (Ebisuzaki et al. 1991; Milosavljević & Merritt 2001). Recently, Kormendy and Bender (2009) showed tight correlations among black-hole masses, velocity dispersions of host galaxies, and the masses of stellar light deficits, using the observational data of 11 elliptical galaxies with cores. This suggests that these correlations still hold even if there are not only a single massive black hole, but also binary massive black holes in the cores of elliptical galaxies.

Binary massive black holes are considered mainly to evolve via three stages (Begelman et al. 1980). Firstly, each black hole sinks independently toward the center of the common gravitational potential due to dynamical friction with neighboring stars. If the binary can be regarded as being a single black hole, its gravitational influence radius to the field stars is defined as

$$ r_{inf} = \frac{G M_{bh}}{\sigma_s^2} \sim 3.4 \text{[pc]} \left( \frac{M_{bh}}{10^7 M_{\odot}} \right)^{1-(2/\beta_2)} , $$

where the tight correlation between the black-hole mass and the one-dimensional velocity dispersion, $\sigma_s$, of the stars, the “$M_{bh}-\sigma_s$ relation”; $M_{bh}/10^7 M_{\odot} = \beta_1 (\sigma_s/200 \text{km s}^{-1})^{\beta_2}$ is made use of. Unless otherwise noted, $\beta_1 = 16.6$ and $\beta_2 = 4.86$ (Merritt & Milosavljević 2005) are adopted in what follows.

When the separation of two black holes becomes less than 1 pc or so, angular momentum loss by dynamical friction slows down due to the loss-cone effect, and a massive hard binary is formed. This is the second stage. The binary hardens at the radius where the kinetic energy per unit mass of the star with $\sigma_s$ equals the binding energy per unit mass of the binary (Quinlan 1996). Its hardening radius is defined as

$$ a_h = \frac{1}{4} \left( 1 + q \right)^2 r_{inf} \sim 8.5 \times 10^{-1} \text{[pc]} \frac{q}{(1 + q)^2} \left( \frac{M_{bh}}{10^7 M_{\odot}} \right)^{1-(2/\beta_2)} , $$

where $q$ is the mass ratio and $r_{inf}$ is the influence radius.
where \( q \) is the black-hole mass ratio.

Finally, the semimajor axis of the binary decreases the radius at which the gravitational radiation dominates, and then a pair of black holes merge into a single supermassive black hole. The detailed timescale in each evolutionary phase will be described in the following three subsections.

### 2.1. Star Driven Phase

#### 2.1.1. The dynamical friction

Each black hole sinks into the common center of mass due to dynamical friction with ambient field stars. The merger rate of two black holes (Binney & Tremaine 1987) is given by

\[
\dot{a}(t) = -\frac{0.428}{\sqrt{2}} \ln \Lambda GM_{bh} \left. \frac{1}{a(t)} \right|_{\sigma_s} a^2(t), \tag{3}
\]

where \( a(t) \) is the separation between two black holes and \( \ln \Lambda \approx 10 \) is the Coulomb logarithm. The decaying timescale of black-hole orbits is then written as

\[
t_{\text{df}} = \frac{a(t)}{\dot{a}(t)} \sim 8.4 \times 10^6 \text{yr} \left( \frac{a(t)}{a_0} \right)^2 \left( \frac{M_{bh}}{10^7 M_\odot} \right)^{(1/\beta_2)-1}, \tag{4}
\]

where \( a_0 = 100 \text{ pc} \) is the typical core radius of the host galaxy. Integration of the merger rate gives

\[
a(t) = \left( 1 - \frac{t}{t_{\text{df}}} \right)^{1/2}, \tag{5}
\]

where

\[
t_{\text{df}} \sim 4.2 \times 10^6 \text{yr} \left( \frac{a_0}{100 \text{ pc}} \right)^2 \left( \frac{M_{bh}}{10^7 M_\odot} \right)^{(1/\beta_2)-1}. \tag{6}
\]

Recall that \( \beta_2 \sim 5 \), and hence the index of mass dependence is \( -4/5 \) for both \( t_{\text{df}} \) and \( t_c \).

#### 2.1.2. Stellar scattering

Even after hardening of the binary, the orbital decay continues to decay by the loss cone refilling of stars due to two-body relaxation. In addition, repeated gravitational slingshot interactions with stars make the orbital decay significantly rapid for a black hole with mass less than a few \( 10^6 M_\odot \). For a system with a singular isothermal sphere, the timescale is given as (Milosavljević & Merrit 2003)

\[
t_{\text{ss}} = \frac{a(t)}{\dot{a}(t)} \sim 3.0 \times 10^3 \text{yr} \left( \frac{a_b}{a(t)} \right) \left( \frac{M_{bh}}{10^7 M_\odot} \right). \tag{7}
\]

For a black hole with mass greater than \( 10^6.5 M_\odot \), this mechanism contributes inefficiently to the orbital decay of the binary on the subparsec scale. Instead, the binary–disk interaction is likely to be a dominant mechanism of the orbital decay. Quite recent \( N \)-body simulations show that stellar dynamics alone can also resolve the final parsec problem (e.g., Berentzen et al. 2009).

### 2.2. Gaseous-Disk Driven Phase

The circumbinary disk would be formed inside the gravitational influence radius after hardening of the binary. The inner edge of the circumbinary disk is then defined as

\[
r_{\text{in}} = \left( \frac{m + 1}{l} \right)^{2/3} a(t) \sim 1.8 [\text{pc}] \left( \frac{q}{1 + q} \right)^2 \left( \frac{M_{bh}}{10^7 M_\odot} \right)^{1-(2/\beta_2)} \left[ \frac{a(t)}{a_h} \right]. \tag{8}
\]

where \( a(t) \) is the semimajor axis of the binary, and \( m = 2 \) and \( l = 1 \) are adopted unless otherwise noted (Artemowicz & Lubow 1994).

For the sake of simplicity, the circumbinary disk is assumed to be steady, axisymmetric, and geometrically thin with a differential rotation and fraction of the Eddington accretion rate:

\[
M_{\text{acc}} = \eta \dot{M}_{\text{Edd}} \sim 2.2 \times 10^{-2} \left( \frac{M_\odot}{\text{yr}} \right) \left( \frac{\eta}{0.1} \right) \left( \frac{0.1}{c} \right) \left( \frac{M_{bh}}{10^7 M_\odot} \right). \tag{9}
\]

where \( \eta, \epsilon, \) and \( \dot{M}_{\text{Edd}} \) are the Eddington ratio, mass-to-energy conversion efficiency, and Eddington accretion rate, defined by

\[
\dot{M}_{\text{Edd}} = (1/\epsilon)4\pi GM_{bh}M_{\odot}/c\sigma_T, \tag{40}
\]

where \( m_p, c, \) and \( \sigma_T \) show the proton mass, light velocity, and Thomson-scattering cross section, respectively.

The surface density of the circumbinary disk can then be written as

\[
\Sigma = \frac{M_{\text{acc}}}{2\pi v} \frac{d \ln r}{d \ln \Omega}, \tag{10}
\]

where the eddy viscosity is defined as \( \nu = \alpha c_{\text{ev}} H \) with the Shakura–Sunyaev viscosity parameter (Shakura & Sunyaev 1973), \( \alpha \), the characteristic velocity, \( c_{\text{ev}} \), and the disk scale-height, \( H \).

The stability criterion for self-gravitation of the disk is defined by the Toomre \( Q \)-value:

\[
Q = \frac{\Omega c_{\text{ev}}}{\pi G \Sigma}. \tag{11}
\]

If the disk structure obeys a standard disk with \( M_{\text{acc}} \) with \( \eta = 0.1 \), \( Q \) is much less than 1. This means that the disk is massive enough to be gravitationally unstable. Therefore, we introduce a self-regulated, self-gravitating disk model (Mineshige & Umemura 1996; Bertin 1997). The condition of a self-regulated disk is given by

\[
Q \approx 1. \tag{12}
\]

From equations (10) and (11), the effective sound velocity of a self-gravitating disk is written as

\[
c_{\text{sg}} = \left( \frac{G M_{\text{acc}}}{2 \alpha_{\text{sg}} \ln r} \right)^{1/3}, \tag{12}
\]

where we adopt \( \alpha = \alpha_{\text{sg}} \lesssim 0.06 \) and \( c_{\text{ev}} = c_{\text{sg}} \). If the radiative cooling in the disk is so efficient, the disk would fragment. The criterion whether the disk fragments or not is given by

\[
\alpha_{\text{sg}} = 0.06 \tag{13}
\]

if \( \alpha_{\text{sg}} > 0.06 \), fragmentation occurs in the disk and causes subsequent star formation. Such a situation is beyond the scope of our disk model. Assuming that the disk face radiates as a blackbody, the sound velocity can be written as

\[
c_s = \left( \frac{R_s}{\mu} \right)^{1/2} \left( \frac{3GM_{bh}M_{\text{acc}}}{8\pi r^3\sigma} \right)^{1/8}, \tag{13}
\]

where \( R_s, \mu, \) and \( \sigma \) are the gas constant, molecular weight, and Stefan–Boltzmann constant, respectively.

The self-gravity of the disk is stronger than the gravity of the central black hole at the self-gravitating radius where the total disk mass equals the black-hole mass. The rotation velocity of the disk then becomes flat outside the self-gravitating radius:
where we put $c_{\text{cv}} = c_{\text{sg}}$. Inside $r_{\text{sg}}$, the angular frequency of the disk corresponds to the Keplerian one, where the gravity of the central black hole dominates the dynamics of the disk. When $c_{\text{cv}} = c_{\text{sg}}$, the disk transits from a self-regulated, self-gravitating disk to the standard disk. Its radius is given as

$$
r_{\text{stsg}} \approx \left( \frac{R_{\odot}}{\mu} \right)^{4/3} \left( \frac{GM_{\text{bh}}}{2a_{\text{sg}}} \right)^{-8/9} \left( \frac{3GM_{\text{bh}}M_{\text{acc}}}{8\pi\sigma} \right)^{1/3}
$$

$$
\sim 6.4 \times 10^{-4} \text{[pc]} \left[ \frac{\alpha_{\text{sg}}}{0.06} \right]^{8/9} \left( \frac{0.1}{\epsilon} \right) \left[ \frac{M_{\text{bh}}}{10^{7}M_{\odot}} \right]^{-2/9}.
$$

As the binary evolves, the disk structure comes to depend on the black hole mass. Since $r_{\text{sg}}(a_{\text{bh}})$ is less than $r_{\text{sg}}$ and more than $r_{\text{stsg}}$ in the all black-hole mass range, the circumbinary disk is initially modeled as a self-regulated, self-gravitating disk with the Keplerian rotation. When the semimajor axis decays into the decoupling radius, defined by equations (25) and (26), $r_{\text{in}}(a_{\text{d}})$ is less than $r_{\text{stsg}}$ for $10^{3}M_{\odot} \lesssim M_{\text{bh}} \lesssim 3 \times 10^{6}M_{\odot}$. The disk structure for $10^{2} \lesssim M_{\text{bh}} \lesssim 3 \times 10^{6}M_{\odot}$ can then be described by the standard disk theory, whereas the disk still remains to be a self-regulated, self-gravitating disk with Keplerian rotation for other black-hole mass ranges (see the dotted line of figure 2).

The circumbinary disk and binary exchange the energy and angular momentum through the tidal/resonant interaction. For a moderate orbital-eccentricity range, the torque of the binary potential dominantly acts on the circumbinary disk at the 1:3 outer Lindblad resonance radius where the binary torque balances with the viscous torque (Artymowicz & Lubow 1994).

On the other hand, the circumbinary disk is deformed to be elliptical by the tidal/resonant interaction. The density of gas is locally enhanced by the gravitational potential at the closest two points from each black hole on the inner edge of the circumbinary disk (Hayasaki et al. 2007). The angular momentum of gas is removed by the locally enhanced viscosity, and thus the gas overflows from the two points to the central binary. An accretion disk is then formed around each black hole by the transferred gas (Hayasaki et al. 2008). The mass transfer therefore adds its angular momentum to the binary via two accretion disks (Hayasaki 2009).

In a steady state, the mass-transfer rate equals the accretion rate, $M_{\text{acc}}$. Since it is much smaller than the central transfer rate, defined by equation (41) of Hayasaki (2009), the effect of the mass-transfer upon the torque can be neglected. The orbital-decay rate is then be approximately written by using equation (22) of Hayasaki (2009) as

$$\frac{\dot{a}(t)}{a(t)} \approx \frac{\dot{J}_{\text{chd}}}{J_{b}} \sqrt{1 - \epsilon^{2}},
$$

where $\epsilon$ is the orbital eccentricity of the binary and the net torque, $\dot{J}_{\text{chd}}$, from the binary to circumbinary disk can be approximately written as

$$\dot{J}_{\text{chd}} \approx \frac{3^{4/3}(1 + q)^{2}}{q} \frac{1}{\tau_{\text{vis, in}}} \frac{M_{\text{bh}}}{M_{\text{d}}} \frac{J_{b}}{\sqrt{1 - \epsilon^{2}}},
$$

where $\tau_{\text{vis, in}} = \frac{r_{\text{in}}^{2}}{2GM_{\text{bh}}}$ is the viscous timescale measured at the inner edge of the disk and $M_{\text{d}}$ is the local disk mass, defined as $M_{\text{d}} = \pi r_{\text{in}}^{2} \Sigma_{\text{in}}$. From equation (10), the product of the viscous timescale and ratio of the black-hole mass to the local disk mass is given by

$$\tau_{\text{vis, in}} M_{\text{bh}} \frac{M_{\text{d}}}{M_{\text{bh}}} \approx 2 \frac{M_{\text{bh}}}{M_{\text{acc}}} \frac{d \ln \Omega}{d \ln \eta}.
$$

Substituting equation (17) with equation (18) into equation (16), the orbital-decay rate can be expressed by

$$\frac{\dot{a}(t)}{a(t)} = -\frac{1}{t_{\text{chd}}}.
$$

where $t_{\text{chd}}$ is the characteristic timescale of orbital decay due to the binary–disk interaction:

$$t_{\text{chd}} \approx \frac{3^{4/3}(1 + q)^{2}}{q} \frac{1}{\tau_{\text{vis, in}}} \frac{M_{\text{bh}}}{M_{\text{d}}} \frac{J_{b}}{\sqrt{1 - \epsilon^{2}}},
$$

where we adopt $\Omega = \Omega_{\text{K}}$. Note that $t_{\text{chd}}$ is independent of the black-hole mass, semimajor axis, and viscosity parameter, but dependent on the black-hole mass ratio, Eddington ratio, and mass-to-energy conversion efficiency. These arise from the assumptions that the disk is axisymmetric with a fraction of the Eddington accretion rate, and that its angular momentum is outwardly transferred by the viscosity of Shakura–Sunyaev type.

Integrating equation (19), we obtain

$$\frac{a(t)}{a_{\text{bh}}} = \exp \left( -\frac{t}{t_{\text{chd}}} \right).
$$

Following equations (29) of Hayasaki (2009), the orbital eccentricity increases with time in the present disk model. The orbital eccentricity is, however, expected to saturate during the disk-driven phase, because the angular momentum of the binary is mainly transferred to the circumbinary disk when the binary is at the apastron. The saturation value of the orbital eccentricity becomes $\epsilon = 0.57$. This value is estimated by equating the angular frequency at the inner edge of the circumbinary disk with the orbital frequency at the apastron.

2.3. Gravitational-Wave Driven Phase

The merging rate by the emission of gravitational wave can be written (Peters 1964) as

$$\frac{\dot{a}(t)}{a(t)} = -\frac{64G^{3}M_{\text{bh}}^{3}}{5c^{5}a^{4}(1 + q)^{2}(1 - \epsilon^{2})^{5/2}},
$$

where $f(\epsilon) = 1 + 73e^{2}/24 + 37e^{4}/96$. The coalescent timescale is then given as

$$t_{\text{gw}} = \frac{a(t)}{\dot{a}(t)} = \frac{5}{8} \left[ \frac{a(t)}{r_{\text{S}}} \right]^{4} \frac{r_{\text{S}}(1 + q)^{2} (1 - \epsilon^{2})^{7/2}}{c q f(\epsilon)},
$$

where $c_{\text{S}}$ is the speed of light.
be written as 

\[ r = \frac{8 c t_{\text{gas}}}{5 r_{S}} \frac{q}{(1 + q)^2 (1 - e^2)^{7/2}} \cdot f(e)^{1/4} \]  

(24)

When the timescale of orbital decay by the emission of gravitational radiation is shorter than the viscous timescale measured at the inner edge, the circumbinary disk is decoupled with the binary. The decoupling radius is then defined by 

\[ \frac{a_d}{r_{S}} = \left[ \frac{8 c t_{\text{vis,S}}}{5 r_{S}} \frac{q}{(1 + q)^2 (1 - e^2)^{7/2}} \right]^{4/11} \]  

(25)

for the standard disk, and 

\[ \frac{a_d}{r_{S}} = \left[ \frac{8 c t_{\text{vis,S}}}{5 r_{S}} \frac{q}{(1 + q)^2 (1 - e^2)^{7/2}} \right]^{2/7} \]  

(26)

for the self-regulated, self-gravitating disk. Here, \( t_{\text{vis,S}} \) is the viscous timescale measured at the inner edge of the circumbinary disk when \( a(t) = r_{S} \). Depending on the disk model, it can be written as 

\[ t_{\text{vis,S}} \sim 5.9 \times 10^2 [\text{yr}] \left( \frac{a_{\text{SS}}}{0.1} \right)^{1/4} \left( \frac{\eta}{0.1} \right)^{1/4} \left( \frac{M_{\text{bh}}}{10^7 M_{\odot}} \right)^{5/4} \]  

(27)

for the standard disk, and 

\[ t_{\text{vis,S}} \sim 5.6 \times 10^6 [\text{yr}] \left( \frac{a_{\text{SG}}}{0.06} \right)^{1/3} \left( \frac{\eta}{0.1} \right)^{2/3} \right]^{1/3} \]  

(28)

for the self-regulated, self-gravitating disk.

By integrating equation (22), the coalescent time from the transition radius is approximately written as 

\[ t_{\text{gw}} \sim 3.8 \times 10^{11} [\text{yr}] \left[ \frac{a(t)}{a_{t}} \right]^{4} \left( \frac{M_{\text{bh}}}{10^7 M_{\odot}} \right)^{-3} \left( \frac{1 + q}{q} \right)^2 \]  

(29)

where \( e_0 = 0.57 \) is the initial orbital eccentricity at the transition radius.

2.4. Observable Period Range for Binary Black Holes

The resonant/tidal interaction causes mass transfer from the circumbinary disk to the accretion disk around each black hole (Hayasaki et al. 2007). The ram pressure by mass transfer acts on the outer edge of accretion disk, and gives a one-armed oscillation on the disk (cf. Hayasaki & Okazaki 2005). The one-armed wave propagates from the outer edge to the black
Fig. 3. (a) Orbital-decay timescale, \( |a/\dot{a}| \), of evolution of binary massive black holes with a semimajor axis from 100 pc to \( 10^{-2} \) pc. The total black hole mass is \( M_{\text{bh}} = 10^{7.5} M_\odot \) with equal mass ratio, \( q = 1.0 \). (b) Corresponding elapsed time of evolution of binary massive black holes. In both panels, the dashed line shows the first evolutionary phase in which two black holes get close each other toward the hardening radius, \( a_h \), by their angular momentum loss due to the dynamical friction with surrounding field stars. The solid line shows the second evolutionary phase from \( a_h \) to \( a_t \) in which the angular momentum of the binary is removed by binary–disk interaction. The binary evolves from \( a_t \) to \( a_t \), where the stage changes from the disk-driven phase to the gravitational-wave driven phase in which binary massive black holes finally coalesce by the emission of the gravitational radiation. There is the decoupling radius, \( a_d \), in the third evolutionary phase shown by the dotted line. In panel (a), the dash-dotted line shows the timescale of orbital decay by the stellar scattering.

Figure 4. Same format as in panel (a) of figure 3, but for \( M_{\text{bh}} = 10^6 M_\odot \).

hole, which allows gas to accrete onto the black hole within the orbital period. This is repeated by every binary orbit. This mechanism therefore causes periodic light variations synchronized with the orbital period (Hayasaki et al. 2008).

For observational purposes, \( a_{10} \) is defined as the semimajor axis corresponding to the feasible orbital period, 10 yr, detectable with MAXI and/or Swift/BAT by

\[
a_{10} \sim 4.9 \times 10^{-3}[\text{pc}] \left( \frac{M_{\text{bh}}}{10^7 M_\odot} \right)^{1/3}.
\]

(30)

Figure 1 shows the mass dependence of each orbital period evaluated at \( a_t \), \( a_d \), and \( a_{10} \) for equal-mass binary massive black holes. The dashed line, dotted line, and horizontal dash-dotted line show the orbital periods at \( a_t \), \( a_d \), and \( a_{10} \), respectively. The area filled in with the solid line shows the existential region of binary black-hole candidates with periodic light-curve signatures detectable with MAXI and/or Swift/BAT.

Figure 2 shows the mass dependence on each characteristic semimajor axis, \( a_h \), \( a_t \), \( a_d \), and \( a_{10} \), for the equal-mass binary. All of the semimajor axes become longer since the black-hole mass is more massive. The decoupling radius is described by equation (25) when \( M_{\text{bh}} \lesssim 3 \times 10^6 M_\odot \), whereas it is described by equation (26) when \( M_{\text{bh}} \gtrsim 3 \times 10^6 M_\odot \). Note that \( a_{10} \) is on the track where the binary evolves by the binary–disk interaction when \( M_{\text{bh}} \lesssim 5 \times 10^7 M_\odot \), whereas \( a_{10} \) is on the track where the binary evolves by the emission of gravitational wave when \( M_{\text{bh}} \gtrsim 5 \times 10^7 M_\odot \).

Figure 3 shows the orbital-decay timescale, \( |a/\dot{a}| \), of binary massive black holes with \( M_{\text{bh}} = 10^{7.5} M_\odot \) in panel (a) and the corresponding elapsed time in panel (b). The dashed line shows the timescale in the first evolutionary phase where the orbit decays by dynamical friction (hereafter, dynamical-friction driven phase). The solid line shows the timescale in the second evolutionary phase where the orbit decays by the binary–disk interaction (hereafter, disk-driven phase). The dotted line
shows the timescale in the final phase where the orbit decays by the dissipation due to the emission of the gravitational wave (hereafter, gravitational-wave driven phase). The dash-dotted line shows the orbital-decay timescale of stellar scattering (hereafter, stellar-scattering driven phase). We note that the orbital-decay timescale of stellar-scattering driven phase is too long for stellar scattering to be an efficient mechanism in binary evolution. The binary therefore evolves toward coalescence via the first dynamical-friction driven phase, second disk-driven phase, and final gravitational-wave driven phase. The orbital-decay timescale of the disk-driven phase is longer than the other two.

Figure 4 shows the orbital-decay timescale of the binary with the same format as in panel (a) of figure 3, but for $M_{\text{bh}} = 10^6 M_\odot$. Note that stellar scattering is efficient after hardening of the binary, as shown in the dash-dotted line of figure 4.

### 3. Mass Function of Binary Massive Black Holes

The observed black-hole mass function of nearby AGNs allows one to study mass functions of binary massive black holes based on the evolutionary scenario described in the previous section. Figure 5 shows the fractional mass function of hard X-ray selected AGNs in the local universe, where the black-hole mass is estimated from the $K$-band magnitudes of their host galaxies, as compiled by Winter et al. (2009). This mass function is based on the 99% complete uniform sample of local Seyfert galaxies, and hence there is no uncertainty as to the sample.

Assuming that two galaxies start randomly to merge during the interval $t_{\text{ls}}$, where $t_{\text{ls}}$ is the look-back time from present universe, the probability for finding binary massive black holes in the present universe can be expressed by $|a/\dot{a}|/t_{\text{ls}}$. The number of binary massive black holes, $N_{\text{bh}}$, in AGNs can be obtained from $N_{\text{bh}} = f_c((a/\dot{a})/t_{\text{ls}})N_{\text{AGN}}$, where $N_{\text{AGN}}$ is the number of AGNs and $f_c$ is a dimensionless fraction parameter. More than 25% of the host galaxies of Swift/BAT AGNs show evidence of on-going mergers (Koss et al. 2010), where the separation between central two cores in the merging galaxies is more than a kpc-scale. Therefore, we set $f_c = 0.75$ unless otherwise noted, since we discuss binary black holes with a separation of a much smaller scale.
Finding binary massive black holes. The mass functions evaluate the black-hole mass function of AGNs by the probability for finding binary massive black holes in AGNs. The mass function is defined by multiplying 

$$ q $\n
from equations (20), (21), and (29).

The probability evaluated at $ t_{AGN} $ keeps constant over all of mass ranges.

The integral probability estimated for $ t_{AGN} $ into $ t_{\text{acc}} $, where

$$ \Delta t = \begin{cases} t_{\text{gas}}^{\text{in}} (a_{10}/a_0) + t_{\text{sw}}^{\text{in}} (a_0) - t_{\text{sw}}^{\text{in}} (a_d) & a_{10} \geq a_0, \\ t_{\text{gas}}^{\text{in}} (a_{10}) - t_{\text{sw}}^{\text{in}} (a_d) & a_{10} \leq a_0, \end{cases} $$

from equations (20), (21), and (29).

The integrated probability estimated for $ a_d \leq a \leq a_{10} $ is a monotonically decreasing function of black-hole mass. Note that they rapidly decrease as the black-hole mass becomes greater than $ 6 \times 10^6 M_\odot $ for $ q = 1.0 $ and $ 2 \times 10^7 M_\odot $ for $ q = 0.1 $.

Figure 7 shows mass functions of binary massive black holes in AGNs. The mass function is defined by multiplying the black-hole mass function of AGNs by the probability for finding binary massive black holes. The mass functions evaluated at the hardening radius, $ a_h $, and the transition radius, $ a_t $, are exhibited in panels (a) and (b), respectively. In both panels, the solid line and the dashed line show the mass functions with $ q = 1.0 $ and $ q = 0.1 $, respectively.

There is less population of binaries with mass less than $ 10^{6.5} M_\odot $ in panel (a), because the binary more rapidly evolves by stellar scattering than by disk–binary interaction in the mass range, as shown in the dash-dotted line of figure 4. The total fractions of binary black holes over all mass ranges are $ 4.3\% \pm 2\% $ for $ q = 0.1 $ and $ 13\% \pm 4\% $ for $ q = 1.0 $ in both panels (the quoted errors reflect the statistical uncertainties in the Swift/BAT AGN mass function). It is noted that from both panels the binary black holes of $ M_{bh} = 10^{6.5-7} M_\odot $ are the most frequent in the nearby AGN population.

Figure 8 shows the mass functions of binary black holes with the constraint that the orbital period is less than $ 10 \text{yr} $ in both cases of $ q = 1.0 $ and $ q = 0.1 $. In the figure, binary black holes of $ M_{bh} = 10^{6.5-7} M_\odot $ for $ q = 1.0 $ and those of $ M_{bh} = 10^{7.5-8} M_\odot $ for $ q = 0.1 $ are the most frequent in the nearby AGN population. It is notable that, assuming that all the binaries have an equal black-hole mass ratio, $ 13\% $ of AGNs with black hole of $ 10^{6.5-7} M_\odot $ have binary black holes.

The total fraction of binary black holes over all mass ranges is $ 1.5\% \pm 0.6\% $ for $ q = 1.0 $ and $ 1.3\% \pm 0.5\% $ for $ q = 0.1 $. We can therefore observe 10–27 candidates for 1300 AGNs detectable with MAXI, assuming that the activities of all nearby AGNs last for $ t_{AGN} = M_{bh}/M_{\text{acc}} $. Note that MAXI covers the softer energy band (2–30 keV) than Swift/BAT (15–200 keV), and hence the ratio of type-1 (unabsorbed) AGNs to type-2 (absorbed) AGNs will be higher in the MAXI survey ($ \approx 8:5 $ based on the model by Ueda et al. 2003) than in the Swift/BAT survey ($ \approx 1:1 $; Tueller et al. 2010). Here, we have referred to the same AGN mass function; however, we do not find any significant statistical difference between the observed mass functions of type-1 and type-2 AGNs based on the Kolmogorov–Smirnov test.
4. Summary and Discussion

We have considered mass functions of binary massive black holes on the subparsec scale in AGNs based on the evolutionary scenario of binary massive black holes with surrounding gaseous disks within the framework of the coevolution of massive black holes and their host galaxies.

As very recent progress in observations of binary massive black holes with the Sloan Digital Sky Survey (SDSS), there is a claim that two broad emission-line quasars with multiple redshift systems are subparsec binary candidates (Borson & Lauer 2009). The temporal variations of such emission lines are attributed to the binary orbital motion (Shen & Loeb 2009; Loeb 2010). These can be used as complementary approaches to search for binary massive black holes with MAXI and/or Swift/BAT.

Recently, Volonteri, Miller, and Dotti (2009) predicted the fraction of binary quasars at \( z < 1 \) based on a theoretical scenario for the hierarchical assembly of supermassive black holes in a ΛCDM cosmology. They adopted the merging timescale of binary black holes with a circumbinary disk estimated by Haiman, Kocsis, and Menoce (2009), in order to explain the observed paucity of binary quasars in the SDSS sample (2 out of 10000: Bogdanović et al. 2009; Dotti et al. 2009; Borson & Lauer 2009). For the black hole mass range of \( \sim 10^8 M_\odot \), which these SDSS quasars likely have, our calculation gives a similar merging timescale (\( \sim 10^8 \) yr, independent of mass). Hence, our model will also be compatible with the SDSS results when applied to the same cosmological model. In a lower mass range, however, we predict a significantly longer merging timescale, by a factor of 10 at \( \sim 10^7 M_\odot \), than that estimated by Haiman, Kocsis, and Menoce (2009), which rapidly decreases with decreasing mass. Hence, much larger fractions of subparsec binary black holes are expected in our model than in Volonteri, Miller, and Dotti (2009) if low-mass black-holes are considered.

Kocsis and Sesana (2010) studied the nHz gravitational-wave background generated by close binary massive black holes with orbital periods of 0.1–10 yr, taking account of both the cosmological merger rate and the binary–disk interaction such as the planetary (type II) migration (Haiman et al. 2009). The orbital-decay timescale for low black-hole mass binaries (\( M_{\text{bh}} \leq 10^7 M_\odot \)) is much shorter than that of our model. This suggests that little stochastic gravitational wave background is attenuated by applying our model for their scenario, because the amplitude of gravitational wave background is proportional to the root of the ratio of the orbital-decay timescale of the disk-driven phase to that of the gravitational-wave driven phase.

Our main conclusions are summarized as follows:

1. Binary massive black holes on the subparsec scale can merge within the Hubble time by the interaction with a triple disk consisting of an accretion disk around each black hole and a circumbinary disk surrounding them. Assuming that the circumbinary disk is steady, axisymmetric, geometrically thin, self-regulated, self-gravitating but non-fragmenting with a fixed fraction of Eddington accretion rate, its orbital-decay timescale is given by \( \sim 3.1 \times 10^8 q(1 + q)^{-2}(0.1/\eta)(\epsilon/0.1)[\text{yr}] \), where \( q, \eta \), and \( \epsilon \) show the black-hole mass ratio, Eddington ratio, and mass-to-energy conversion efficiency, respectively.

2. Binary black holes of \( M_{\text{bh}} = 10^{8.5-9} M_\odot \) in the disk-driven phase are the most frequent in the AGN population. Assuming that the activities of all nearby AGNs last on the accretion timescale, \( M_{\text{bh}}/M_{\text{acc}} \), the total fractions of binaries with the semimajor axis evaluated at the hardening radius and with that at the transition are estimated to be 4.3% ± 2% and 13% ± 4%, respectively.

3. Assuming that all binary massive black holes have the equal mass ratio (\( q = 1.0 \)), \( \sim 13 \% \) of AGNs with \( M_{\text{bh}} = 10^{8.5-9} M_\odot \) harbor binary black holes with an orbital period of less than 10 yr in their centers. This black-hole mass range therefore provides the best chance to find such close binary black holes in AGNs.

4. The total fraction of close binary massive black holes with an orbital period of less than 10 yr, as it is detectable with MAXI and/or Swift/BAT, can be estimated to be 1.5% ± 0.6% for \( q = 1.0 \) and 1.3% ± 0.5% for \( q = 0.1 \).

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