THE HUBBLE CONSTANT INFERRED FROM 18 TIME-DELAY LENSES

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ABSTRACT

We present a simultaneous analysis of 18 galaxy lenses with time-delay measurements. For each lens, we derive mass maps using pixelated simultaneous modeling with shared Hubble constant. We estimate the Hubble constant to be $66^{+6}_{-4} \text{ km s}^{-1} \text{ Mpc}^{-1}$ (for a flat universe with $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$). We have also selected a subsample of five relatively isolated early-type galaxies, and by simultaneous modeling with an additional constraint on isothermality of their mass profiles, we get $H_0 = 76^{+3}_{-3} \text{ km s}^{-1} \text{ Mpc}^{-1}$.

Key words: cosmological parameters – gravitational lensing: strong

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1. INTRODUCTION

The Hubble constant is one of the most important parameters in cosmology. It determines the age of universe and the physical distances to objects, constrains the dark energy equation of state, and furthermore, it is used as a prior in many cosmological analysis. Hence, it is essential in cosmology to know the precise value of $H_0$ (Riess et al. 2009). The newest measurements give a quite well-defined $H_0$ with estimated errors at the 5%–10% level but unfortunately $H_0$ differs among different cosmological methods, and there is only marginal consistency between their 1σ errors (see Table 1).

Moreover, those methods are based on different physical principles and more importantly, they measure different consequences of $H_0$. Both supernovae (SNe) and Cepheids measure luminosity distance but at different scales (distant and local universe, respectively). On the one hand, Cepheids can provide a luminosity distance via the period–luminosity relation but only in the local universe; on the other hand, the significantly brighter SNe with their characteristic peak luminosity can measure cosmological distances but need to be calibrated with Cepheids. The Sunyaev–Zel’dovich (SZ) effect, which is based on high-energy electrons in a galaxy cluster distorting the cosmic microwave background (CMB) through inverse Compton scattering, is proportional to the gas density of the galaxy cluster and that combined with the cluster’s X-ray flux gives an estimate of the angular diameter distance. Finally, the angular power spectrum, especially strong degeneracies are between cosmological parameters which have correlated effects on the angular power spectrum of the CMB gives information about many clusters of galaxies. It is therefore important to explore complementary methods for measuring $H_0$. Gravitationally lensed quasars (QSOs) offer such an attractive alternative.

As shown byRefsdal (1964), the Hubble constant can be measured based on the time delay $\Delta t$ between multiply lensed images of QSOs because $H_0 \propto 1/\Delta t$, provided that the mass distribution of the lens is known. Time delays measure $\frac{D_0 D_{LS}}{D_{OL} D_{OS}}$, where $D_{OL}$, $D_{OS}$, and $D_{LS}$ are the angular diameter distances between observer and lens, observer and source, and lens and source, respectively. Gravitational lensing has its degeneracies but it is based on well-understood physics, and unlike distance ladder methods there are no calibration issues (Branch et al. 1996; Sandage et al. 2006).

Gravitational lensing has, up to now, not been seen as reliable as other leading cosmological methods. Determination of the Hubble constant using lensing is problematic, because the mass distribution of a lens strongly influences the result of $H_0$ and unfortunately we never have a complete knowledge of that. Hence, a choice of a lens model is needed.

Recently, however, time-delay lenses have successfully been used for $H_0$ estimation. In particular, Oguri (2007) used a Monte Carlo method to combine lenses and derived $H_0 = 70 \pm 6 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Saha et al. (2006) obtained $H_0 = 72^{+8}_{-11} \text{ km s}^{-1} \text{ Mpc}^{-1}$ using a combination of 10 lenses, Coles (2008) obtained $H_0 = 71^{+5}_{-3} \text{ km s}^{-1} \text{ Mpc}^{-1}$ using a combination of 11 lenses, and Suyu et al. (2009) found $H_0 = 69.7^{+4.9}_{-5.0} \text{ km s}^{-1} \text{ Mpc}^{-1}$ by detailed analysis of one gravitational lens, B1608+656.

This paper extends the work of Saha et al. (2006) and Coles (2008), where results on $H_0$ were presented using combined modeling of 10 and 11 lenses, respectively. We have used those systems with refined properties of lenses as a part of our sample and have added new systems that have been discovered and monitored during the past four years. We therefore now possess an almost doubled sample of systems with measured time delay, which demonstrates that gravitational lensing is a valuable method for $H_0$ estimation.

2. PIXELATED MODELING

Two different approaches for modeling lenses are commonly used. The first one, the non-parametric (pixelated) method, generates a large number of models which perfectly fit the data, each of them giving a different result which can then be averaged. For the second method, model fitting, one assumes parameterized models of the mass distribution of the lens.

Pixelated modeling has the advantage of allowing the lens shape and profile to vary freely. It does not presume any parameters and can provide models that would not be possible
to reproduce with parametric modeling. Using this approach to a combined analysis of a large sample of lenses is a powerful solution to the modeling problem in gravitational lensing.

In this paper, we use the non-analytical method created by Saha & Williams (2004)—PixeLens. PixeLens generates an ensemble of lens models that fit the input data. Each model consists of a set of discrete mass points, the position of the source, and optionally, if the time delays are known, \( H_0 \). The time delay \( \Delta t \) is the combined effect of the difference in length of the optical path between two images and the gravitational time dilation of two light rays passing through different parts of the lens potential well,

\[
\Delta t = \frac{1 + z_{ls}}{c} \frac{D_{os} D_{ol}}{D_{ls}} \left( \frac{1}{2} \kappa - \hat{\beta} \right)^2 - \Psi(\hat{\beta}).
\]

Here, \( \hat{\beta} \) and \( \bar{\hat{\beta}} \) are the positions of the images and the source, respectively, \( z_{ls} \) is the lens redshift, and \( \Psi \) is the effective gravitational potential of the lens.

The arrival time at position \( \hat{\beta} \) is defined in PixeLens, as a modeled surface,

\[
\tau(\hat{\beta}) = \frac{1}{2} \left| \hat{\beta} \right|^2 - \hat{\beta} \cdot \bar{\hat{\beta}} - \int \ln |\hat{\beta} - \hat{\beta}'| \kappa(\hat{\beta}') d^2\hat{\beta}'.
\]

where \( \kappa \) is surface mass density.

The errors of the positions of observed images and redshifts of source and lens are of the order of a few percent, thus can be ignored. The main source of errors in the data comes from time delays between images.

We have used PixeLens to generate a set of 100 models for a sample of lensing systems (see Section 3). PixeLens produces an ensemble of models with varying \( H_0 \), each consisting of sets of mass pixels, which exactly reproduce the input data. Moreover, it also models several lenses simultaneously, enforcing shared \( H_0 \) for all lenses. The image positions, the source and lens redshifts, and the time delay are assumed to be accurate enough for their errors to be ignored (Saha et al. 2006). PixeLens also imposes secondary constraints on mass maps: non-negative density; smoothness, where the density of a pixel must be no more than twice the average density of its neighbors; the mass profile is required to have 180° rotation symmetry (except if it appears very asymmetric); the shear is allowed within 45° of the chosen direction; circularly averaged mass profile should neither be shallower nor steeper than \( r^{-\alpha_{\text{min}}} \) and \( r^{-\alpha_{\text{max}}} \), respectively, where \( \alpha_{\text{min}} \) and \( \alpha_{\text{max}} \) are defined in PixeLens as the minimum and maximum steepness—while those values can be chosen by the user, the default PixeLens constraint is minimal steepness \( \alpha_{\text{min}} = 0.5 \); and finally, additional lenses as point masses can be constrained. PixeLens does not use flux ratios as constraints because of the possible influence of reddening by dust (Elías-Díaz et al. 2006), microlensing (Paraficz et al. 2006), or small-scale structure in the lens potential (Dalal & Kochanek 2002).

### 3. DATA SET

To date, there are 19 gravitational lens systems with published time delays. Table 2 summarizes the information about these 19 systems. We have made an attempt to use all the conclusions previously drawn about their shape, external shear, profile, etc. We have used the newest/best measurement of positions and redshifts of images and lens. Apart from the main lensing galaxies, we have also included all the galaxies that might contribute to the lensing. We added them whenever they are visible in the field. These systems are RX J0911+055, HE 1104–181, SBS 1520+530, B1600+434, and B1608+656. All the mass maps of the doubly imaged quasars are required to have 180° rotation symmetry and in the case of the quadruply imaged systems we allow the lens to be asymmetric if it has been reported asymmetric, which is the case for HE 0435–1223, SDSS J1004+4112, RX J1131–1231, and B1608+656. A constant external shear is allowed, for the lenses where the morphology shows evidence of external shear or the existence of external shear has been reported, which is the case for HE 0435–1223, RX J0911+055, FBQ J0957+561, SDSS J1004+4112, PG 1115+080, RX J1131–1231, SDSS J1206+4332, B1600+434, SDSS J1650+4251, and WFI J2033–4723.

One lens system has been excluded from our analysis, B1422+231. Raychaudhury et al. (2003) indicated that the time-delay measurements made by Patnaik & Narasimha (2001) are possibly inaccurate. Patnaik & Narasimha (2001) reported \( \Delta t_{12} = 7.6 \pm 2.5 \) days, whereas Raychaudhury et al. (2003) lens modeling predicts \( \Delta t_{12} = 0.4 \) h⁻¹ days. According to Raychaudhury et al. (2003) this value would not be expected to show up in the Patnaik & Narasimha (2001) data, which sampled every four days. We also follow the Raychaudhury et al. (2003) prediction because in our analysis the system gives an unreasonably low Hubble constant \( H_0 = 12 \pm 3 \) km s⁻¹ Mpc⁻¹, and we exclude the system in what follows.

Table 1 shows a mosaic of the average mass distributions for the remaining 18 lenses.

### 4. FULL SET RESULTS

Our resulting \( H_0 \) distribution is shown in Figure 2. We have performed the calculation of 18 systems for a flat universe with \( \Omega = 0.7 \), \( \Omega_m = 0.3 \), and we obtain \( H_0 = 66^{+6}_{-5} \) km s⁻¹ Mpc⁻¹ at 68% confidence and \( H_0 = 66^{+8}_{-7} \) km s⁻¹ Mpc⁻¹ at 90% confidence. We also note that for \( \langle z_s \rangle = 0.6 \), \( \langle z_l \rangle = 1.8 \), which are the average lens and source redshifts of our sample, the inferred \( H_0 \) should increase by 2% for an open universe with \( \Omega = 0.0 \), \( \Omega_m = 0.3 \) and decrease by 7% for an Einstein-de Sitter universe \( \Omega = 0.0 \). \( \Omega_m = 1.0 \).

Figure 3 presents the comparison between estimation from our sample of lenses and samples from previous lensing studies using 15 lenses¹ (Oguri 2007) and 10 lenses (Saha et al. 2006). The \( H_0 \) distribution of the samples of 10 and 15 lenses are slightly different than the original results of Oguri (2007) and Saha et al. (2006).² Oguri (2007) used 16 lensed quasar systems (40 image pairs) to constrain the Hubble constant. For each image pair, he computed the likelihood as a function of the Hubble constant. He then computed the effective \( \chi^2 \) by summing up the logarithm of the likelihoods. The first summation runs over lens systems, whereas the second summation runs over

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¹ Excluding B1422+231.
² Oguri (2007) obtained \( H_0 = 70 \pm 6 \) km s⁻¹ Mpc⁻¹, and Saha et al. (2006) obtained \( H_0 = 72^{+7}_{-6} \) km s⁻¹ Mpc⁻¹.
### Table 2
Properties of Time-delay Lenses

| System     | \(z_f\) | \(z_s\) | P.A. | Point Massa | \(x^\circ\) | \(y^\circ\) | \(\Delta^\circ\) (days) | Reference |
|------------|---------|---------|------|-------------|-------------|-------------|-------------------------|----------|
| PG 1115+080 | 0.311   | 1.722   | −45°| 0           | 0.381       | 1.344       | 13.3±0.9                | 1, 2, 3, 4 |
| HE 2149−275 | 0.495   | 2.030   |     | 0           | 0.714       | −1.150      | 103 ± 12                | 5, 6, 7   |
| SDSS J1206+4332 | 0.748 | 1.789   | 50°  | 0           | 0.663       | −1.749      | 116±4                  | 8, 9      |
| HE 0435−1223 | 0.454   | 1.689   | −65°| 0           | −1.165      | 0.573       | 2.1±0.7                 | 10, 11    |
| SDSS J1650+4251 | 0.577 | 1.547   | 80°  | 0           | 0.017       | 0.872       | 422.6±0.6               | 12, 13    |
| QSO J0957+561 | 0.356   | 1.410   | −30°| 0           | 1.408       | 5.034       | 0                       | 14, 15, 16 |
| RX J1131−1231 | 0.295   | 0.658   | −35°| 0           | −1.984      | 0.578       | 1.5±0.2                 | 17, 3     |
| RX J0911+055 | 0.769   | 2.800   | 90°  | 1           | 2.226       | 0.278       | 416 ± 8                 | 18, 19, 20, 21 |
| SDSS J1004+4112 | 0.680 | 1.734   | 90°  | 0           | 3.943       | −8.868      | 821.6±2.1               | 5, 22, 23, 24 |
| WFI J2033−4723 | 0.661   | 1.660   | 0°   | 0           | −1.439      | −0.311      | 35.5±1.4                | 25, 26, 27 |
| PKS 1830−211 | 0.885   | 2.507   |     | 0           | −0.498      | 0.456       | 26±4.5                  | 28, 29, 30, 31 |
| B1600+434  | 0.410   | 1.590   | 90°  | 1           | 0.610       | 0.814       | 51±4                    | 5, 32, 33, 34 |
| B0218+357  | 0.685   | 0.944   |     | 0           | 0.250       | −0.119      | 10.1±1.1                | 35, 36, 37, 38 |
| B1608+656  | 0.630   | 1.394   |     | 1           | −1.155      | −0.896      | 31.5±1.5                | 39, 40, 41, 42 |
| HE 1104−181 | 0.729   | 2.319   |     | 2           | −1.936      | −0.832      | 152.2±2.8               | 43, 44, 45 |
| SBS 1520+530 | 0.761   | 1.855   |     | 1           | 1.130       | 0.387       | 46.47±0.49              | 46, 47, 48, 49 |
| FBQ J0951+263 | 0.24    | 1.246   |     | 0           | 0.750       | 0.459       | 16±2                    | 50, 51    |
| SBS 0909+532 | 0.830   | 1.376   |     | 0           | 0.572       | 0.494       | 45±3                    | 5, 52, 53 |
| B1422+231  | 0.339   | 3.62    | 10°  | 0           | 1.014       | −0.168      | 7.6±2.5                 | 54, 55, 3  |
image pairs for each lens system. He derived the best-fit value and its error of Hubble constant in the standard way using a goodness-of-fit parameter.

The difference with Oguri (2007) is mainly due to the use of a different modeling method and a different statistical and modeling approach to obtain $H_0$. In the case of Saha et al. (2006) the method is identical and the difference comes from the use of other/newer data, and the use of different rules for constraining shear, adding secondary lenses, etc. Using the Saha et al. (2006) sample of 10 lenses we have obtained $H_0 = 63^{+6}_{-5}$ $\text{km} \text{s}^{-1} \text{Mpc}^{-1}$ and using the Oguri (2007) sample of 16 lenses (minus B1422+231), we obtained $H_0 = 66^{+7}_{-5}$ $\text{km} \text{s}^{-1} \text{Mpc}^{-1}$.

5. “ELLIPTICAL” SAMPLE

The strongest degeneracy in lens modeling is the so-called mass-sheet degeneracy between time delays and the steepness of the mass profile. Without changes in the position of the images, we can change the steepness of the mass profile, and hence, the resulting $H_0$. Thus, if the steepness and $\Delta r$ are known, $H_0$ is well constrained. Although the time delays are also influenced by other more complicated degeneracies involving details of the shape of the lens, these effects are secondary (Saha & Williams 2006).

In our sample of 18 systems, we have galaxies with a variety of steepness. Without detailed observations the profile steepness of each lens is not known, except possibly for elliptical galaxies.

Several studies have shown that elliptical galaxies may be considered as approximately isothermal $\rho \propto r^{-2}$ (Koopmans et al. 2006, 2009; Oguri 2007; Gerhard et al. 2001). Hence, by selecting from our 18 system only elliptical galaxies we expect to get a uniform sample with known slopes of mass profiles.

We have selected five systems that are relatively isolated, elliptical galaxies: PG 1115+080 (Impey et al. 1998), HE 2149−275 (Lopez et al. 1998), SDSS J1206+4332 (Paraficz et al. 2009), HE 0435−1223 (Kochanek et al. 2006), and SDSS J1650+4251 (Vuissoz et al. 2007; Morgan et al. 2003).

Five other lenses were also found to be elliptical but the cluster or group to which they belong has strong influence on the lensing system, and thus it is difficult to model them: QSO 0957+561 (Oscoz et al. 1997; Bernstein & Fischer 1999); RX J1131−1231 (Claeskens et al. 2006); RX J0911+055 (Kneib et al. 2000), SDSS J1004+4112 (Sharon et al. 2005), and WFI J2033−4723 (Eigenbrod et al. 2006).

Three of the lenses are most probably spiral galaxies: PKS 1830−211 is a face-on spiral galaxy (Winn et al. 2002), B1600+434 is a spiral galaxy with a companion (Jaunsen & Hjorth 1997), and B2018+357 is an isolated spiral galaxy (Koopmans & THE CLASS Collaboration 2001).

The remaining five systems were not included into the “elliptical” sample due to various other issues: B1608+656 has two lensing galaxies inside the Einstein ring; HE 1104+180 has little starlight, suggesting a dark matter dominated lens (Poindexter et al. 2007; Vuissoz et al. 2008); SBS 1520+530 has a steeper than isothermal slope, probably due to mergers (Auger et al. 2008); FBQ 0951+263 is a complicated system which is hard to model (Peng et al. 2006); and SBS 0909+532 is probably early type (Lehár et al. 2000), but according to Motta et al. (2002) is not very typical due to lots of dust.

Using the sample of five elliptical galaxies and constraining the steepness of their mass profiles to be $\alpha_{\text{min}} = 1.8$ and $\alpha_{\text{max}} = 2.2$, we ran the PixeLens simultaneous modeling. The sample of five lensing systems gives us a Hubble constant estimation $H_0 = 79^{+5}_{-3}$ $\text{km} \text{s}^{-1} \text{Mpc}^{-1}$ at 68% confidence and $H_0 = 79^{+5}_{-3}$ $\text{km} \text{s}^{-1} \text{Mpc}^{-1}$ at 90% confidence. We have also combined the five constrained systems with the rest of the unconstrained sample to perform simultaneous modeling and obtained very well determined Hubble constant $H_0 = 76 \pm 3$ $\text{km} \text{s}^{-1} \text{Mpc}^{-1}$ at 68% confidence and $H_0 = 76 \pm 5$ $\text{km} \text{s}^{-1} \text{Mpc}^{-1}$ at 90% confidence. The results are presented in Figure 4.

6. CONCLUSIONS

Non-parametric modeling using PixeLens was applied to an ensemble of 18 lenses to determine a new value for the Hubble constant. We have obtained $H_0 = 66^{+2}_{-1}$ $\text{km} \text{s}^{-1} \text{Mpc}^{-1}$ for a flat universe with $\Omega_\Lambda = 0.7$, $\Omega_m = 0.3$. We have also compared our results with the two previous attempts to estimate $H_0$ from time delays (Saha et al. 2006; Oguri 2007; Figure 3).

Our additional result was based on studies of a selected sample of five lensing galaxies that have mass profiles close to $\rho \propto r^{-2}$. The Hubble constant recovered using the selected sample of elliptical galaxies combined with the rest of the systems is $H_0 = 76^{+3}_{-2}$ $\text{km} \text{s}^{-1} \text{Mpc}^{-1}$.

The gravitational lensing method that constrains $H_0$ has difficulties due to a couple of degeneracies between mass and time delay. The major degeneracy, the mass-sheet degeneracy, as we have shown, can be addressed by a careful choice of galaxies and the others partially by a combined, pixelated analysis of a large sample of lenses. Pixelated lens modeling provides insight into the structure of galaxies and the distribution of dark matter.
Figure 1. Ensemble of 18 average mass maps of the lenses. The larger tick marks in each panel correspond to 1''0. Red and cyan dots represent the positions of the images and the sources, respectively. The contours are in logarithmic steps, with critical density corresponding to the third contour from the outside. The systems are grouped according to their morphology described in Section 5.

(A color version of this figure is available in the online journal.)
which together with precise measurements of time delays gives a reliable cosmological method.
Lensing can already determine the Hubble constant approaching the accuracy level of other leading measurements. Nevertheless, more observations are still needed. Future data with precise time-delay measurements and better lens models will give even better constraints on $H_0$, perhaps turning lensing into a very competitive method.

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Figure 2. Histogram of the ensembles of $H_0$ values estimated from 18 lensing systems. We have assumed $\Omega_\Lambda = 0.7$ and $\Omega_m = 0.3$.

Figure 3. Comparison of the three histograms of the ensembles of $H_0$ values. Histogram plotted with the solid line represents our results for 18 systems; the dashed line represents the results of 10 lenses (Saha et al. 2006) and the dotted line is a result of 16 lenses (Oguri 2007). All calculations are done for a flat universe with $\Omega_\Lambda = 0.7$ and $\Omega_m = 0.3$.

Figure 4. Histogram of the distribution of $H_0$ values for the selected sample of five elliptical galaxies having constrained steepness of mass profiles in the range 1.8–2.2 (solid line) and the selected elliptical sample combined with the rest of the systems (dashed line). All calculations are done for a flat universe with $\Omega_\Lambda = 0.7$ and $\Omega_m = 0.3$.
