The influence of installation angle of GGIs on full-tensor gravity gradient measurement

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Abstract. Gravity gradient plays an important role in many disciplines as a fundamental signal to reflect the information of the earth. Full-tensor gravity gradient measurement (FGGM) is an effective way to obtain the gravity gradient signal. In this paper, the installation mode of GGIs in FGGM is studied. It is expected that the accuracy of FGGM will be improved by optimizing the installation mode of GGIs. In addition, we analysed the relationship between GGIs’ installation angle and FGGM by establishing the measurement model of FGGM. Then the following conclusions was proved that there was no relationship between GGIs’ installation angle and the measurement result. This conclusion showed that there was no optimal angle for the GGIs’ installation in FGGM, and the installation angle only need to satisfy the relationship shown in the conclusion section of this paper. Finally, this conclusion was demonstrated by computer simulations.

1. Introduction
The gravitational gradient signal, as a high-precision signal for the change of the particle density in geological body, plays an important role in many disciplines [1-4]. Many measuring instruments have been developed for the measurement of gravity gradients [1, 5]. The earliest gravity gradiometer was torsion balance, developed by the Hungarian physicist Eötvös. This measurement device has poor precision and low efficiency, so it is quickly replaced by more efficient and high-precision measurement equipment.

One of the high-precision gravity gradient measurement equipment is rotating-accelerometer-based gravity gradiometer, which is widely used in mineral exploration. The core component of the gradiometer is the gravity gradient instrument (GGI) developed by Lockheed Martin. An independent component and a combined component of the gravitational gradient tensor can be measured by one GGI[6]. In order to achieve all the measurements of gravity gradient components, three GGIs are required. With these GGIs, three independent components and three combined components can be obtained.

As a kind of conservative field, the second derivative of gravitational potential satisfies Poisson equation. Therefore, there are only five independent components in the gravity gradient tensor. This means that there is a redundancy in the full-tensor gravity gradient measurement (FGGM). However, in the actual FGGM using Lockheed Martin’s full tensor gravity gradiometer (FTG) [7], there was no redundancy. The installation mode of GGIs in FTG is shown in figure 1: The rotational axes of the three GGIs, shown in figure 1, are orthogonal to each other. The angle between the projections of the axis of the axes on the x-y plane of the platform coordinate system is 120°, and the angle between the axes and the x-y plane is 35°.
2. The principle of GGI

The structure of GGI is shown in figure 2. There are four accelerometers symmetry mounted on a disk. The direction of the accelerometer sensitive axis is parallel to the tangent of the disk. The sensitivity of the two accelerometers is opposite. The radius of the disk is \( r \), and the disk is rotating at constant angular velocity \( \omega \).

The output value of the accelerometers on the turntable are denoted by \( f_i \), where \( i \) is the number of the accelerometers.

The output signals of the four accelerometers are combined as follows:

\[
U = f_i + f_3 - f_2 - f_4
\]  
(1)

Considering the direction of the accelerometer sensitive axis for accelerometer 1 and 3, 2, and 4 are opposite. The acceleration signal generated by the carrier motion in the combined signal \( U \) will be cancelled, thus the common-mode rejection for carrier motion acceleration was achieved. Finally, the relationship between the combined signal and the gravity gradient components is obtained as follows:

\[
U = 4r \left( \Gamma_{xx} + N_1 \right) \cos(2\omega t) - 2r \left( \Gamma_{yy} - \Gamma_{xx} + N_2 \right) \sin(2\omega t)
\]  
(2)

where \( \Gamma_{xx}, \Gamma_{yy} \), and \( \Gamma_{yy} \) are the gravitational gradient components; \( N_1 \) and \( N_2 \) respectively represent the disturbance signals that are coupled together with the measurement signal \( \Gamma_{yy} \) and \( \Gamma_{xx} - \Gamma_{yy} \), which can be calculated by the observation of the movement of the carrier.
Equation (2) shows that the gravity gradient signal is modulated to twice the rotation speed. The independent component $\Gamma_{nn}$ and the combined component $\Gamma_{nn} - \Gamma_{nn}$ can be achieved by demodulating the combined signal $U$ at the double frequency.

### 3. Relationship between GGIs installation angle and FGGM

In order to determine the relationship between GGIs installation angle and FGGM, the following coordinate frames as shown in figure 3 are used:

- **The Platform frame** ($p$-frame) with its origin at the location of the stable platform, and axes aligned with the directions of north, east, and the local vertical (down).
- **The GGI frame** ($gi$-frame, $i \in \{1, 2, 3\}$) with its origin at the center of the $i$-th GGI; the $x$-axis points to the starting position of the disc, and the $z$-axis points to the rotation axis of the disc, and the $y$-axis is in the plane of the disc.

**Figure 3.** The diagram of the platform frame ($p$-frame) and the GGI frame ($gi$-frame).

In figure 3, the GGIs are placed on a stable platform, which tracks the local geographic frame.

Furthermore, it is necessary to clarify the transformation relation of gravity gradient tensor in different coordinate systems. According to the study of Zhao L, et al. [8], the transformation relationship between the gravity gradient tensor in different coordinate systems is as follows:

$$\Gamma_{gi} = C_{gi}^p \Gamma_p C_{gi}^p$$  \hspace{1cm} (3)

where $\Gamma_{gi}$ and $\Gamma_p$ are the representation of gravity gradient tensor in the $gi$-frame and the $p$-frame, respectively; $C_{gi}^p$ is the direction cosine matrix from $p$-frame to $gi$-frame, and $C_{gi}^p$ is the transpose of $C_{gi}^p$.

Then, substituting the direction cosine matrix $C_{gi}^p$ and gravity gradient tensor expressed in the $p$-frame $\Gamma_p$ into equation (3) yields the gravity gradient components $\Gamma_{xx} - \Gamma_{yy}$ and $\Gamma_{yy}$ which are measured by the $i$-th GGI:

$$\begin{align*}
\Gamma_{xx} - \Gamma_{yy} &= \left((c_{11}')^2 - (c_{21}')^2\right)\Gamma_{xx} + 2\left(c_{11}'c_{11}' - c_{21}'c_{21}'\right)\Gamma_{xy} + 2\left(c_{11}'c_{13}' - c_{21}'c_{23}'\right)\Gamma_{xz} + \left((c_{12}')^2 - (c_{22}')^2\right)\Gamma_{yy} \\
&\quad + 2\left(c_{11}'c_{13}' - c_{21}'c_{23}'\right)\Gamma_{xy} + \left((c_{13}')^2 - (c_{23}')^2\right)\Gamma_{xz} \\
\Gamma_{yy} &= c_{12}'c_{12}'\Gamma_{xx} + \left(c_{21}'c_{21}' + c_{22}'c_{22}'\right)\Gamma_{xy} + \left(c_{21}'c_{23}' + c_{23}'c_{21}'\right)\Gamma_{xz} + c_{22}'c_{22}'\Gamma_{yy} + \left(c_{23}'c_{23}' + c_{12}'c_{12}'\right)\Gamma_{xy} + c_{13}'c_{13}'\Gamma_{xz}
\end{align*}$$  \hspace{1cm} (4)

where:
For the three GGIs in figure 2, the relationship between the measured value and \( \Gamma \) is as follows:

\[
G = MP
\]

in which

\[
G = \left( \Gamma_{xx}^i - \Gamma_{yy}^i, \Gamma_{yy}^i, \Gamma_{zz}^i, \Gamma_{zz}^i, \Gamma_{xy}^i, \Gamma_{xz}^i, \Gamma_{yx}^i, \Gamma_{zx}^i, \Gamma_{yz}^i, \Gamma_{zy}^i \right)
\]

\[
M = \begin{bmatrix}
(c_{11}^i - c_{21}^i)^2 + (c_{12}^i - c_{22}^i)^2 + (c_{13}^i - c_{23}^i)^2 \\
2(c_{11}^i - c_{21}^i)(c_{12}^i - c_{22}^i)(c_{13}^i - c_{23}^i) \\
2(c_{11}^i - c_{21}^i)(c_{12}^i - c_{22}^i)(c_{13}^i - c_{23}^i) \\
2(c_{12}^i - c_{22}^i)^2 + (c_{13}^i - c_{23}^i)^2 + (c_{13}^i - c_{23}^i)^2 \\
2(c_{12}^i - c_{22}^i)(c_{13}^i - c_{23}^i)(c_{13}^i - c_{23}^i) \\
2(c_{12}^i - c_{22}^i)(c_{13}^i - c_{23}^i)(c_{13}^i - c_{23}^i) \\
2(c_{13}^i - c_{23}^i)^2 + (c_{13}^i - c_{23}^i)^2 + (c_{13}^i - c_{23}^i)^2 \\
2(c_{13}^i - c_{23}^i)(c_{13}^i - c_{23}^i)(c_{13}^i - c_{23}^i) \\
2(c_{13}^i - c_{23}^i)(c_{13}^i - c_{23}^i)(c_{13}^i - c_{23}^i)
\end{bmatrix}
\]

\[
P = \left( \Gamma_{xx}^p, \Gamma_{yy}^p, \Gamma_{zz}^p, \Gamma_{xy}^p, \Gamma_{xz}^p, \Gamma_{yz}^p, \Gamma_{zx}^p, \Gamma_{zy}^p \right)
\]

Substituting the installation angles of GGIs into equation (7), it can be seen that when there is no parallel between the three GGIs, the following conclusion is established:

\[
\text{Rank}(M) = 5 < 6
\]

Thus, there is no unique solution to the equation (6). The result also explains why there is no redundancy in the FGGM using the Lockheed Martin’s FTG.

4. Simulation

In order to verify the above conclusion, the simulation is carried out according to the following steps: first, setting up the installation angles of GGIs; second, calculate the direction cosine matrix \( C_p \) and \( M \); third, calculate the rank of \( M \), and check whether it is 6 and then we know that whether the redundant component is measurable.

The steps of the simulation are shown in figure 4.

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**Figure 4. Simulation steps.**
The installation angle of the three GGIs are selected as shown in table 1.

**Table 1.** The installation angle of the three GGIs.

|     | θ (rad) | ψ (rad) | φ (rad) |
|-----|---------|---------|---------|
| GGI-1 | π/10    | π/5     | 3π/10   |
| GGI-2 | π/4     | 17π/36  | 5π/36   |
| GGI-3 | π/12    | 5π/12   | π/5     |

Then, the rank of \( M \) in equation (7) is calculated as follows:

\[
\text{rank}(M) = 5
\]

Thus, equation (6) has no unique solution, and the measurement of redundancy can not be achieved.

5. Conclusion

In this paper, we have demonstrated the following conclusion: in the FGGM, redundant signal is non-measurable, the installation angles of GGIs have no influence on FGGM. That is, regardless of how the installation angle is selected, no redundant information will be obtained. This conclusion of the study shows that in the FGGM, there is no relationship between the measurement of the gravity gradient components and the installation angle of the three GGIs. Therefore, when installing GGIs, the only conditions that need to be satisfied is:

\[
\begin{vmatrix}
\theta_1 & \psi_1 & \phi_1 \\
\theta_2 & \psi_2 & \phi_2 \\
\theta_3 & \psi_3 & \phi_3 \\
\end{vmatrix} \neq 0
\]

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