Dark Vector-Gauge-Boson Model

Subhaditya Bhattacharya\textsuperscript{1}, J. Lorenzo Diaz-Cruz\textsuperscript{2},
Ernest Ma\textsuperscript{1}, and Daniel Wegman\textsuperscript{1}

\textsuperscript{1} Department of Physics and Astronomy, University of California,
Riverside, California 92521, USA
\textsuperscript{2} Facultad de Ciencias Fisico-Matematicas,
Benemerita Universidad Autonoma de Puebla, Puebla, Mexico

Abstract

A model based on $SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(2)_Y$ has recently been proposed,
where the $SU(2)_Y$ vector gauge bosons are neutral, so that a vector dark-matter candidate is possible and constrained by data to be less than about 1 TeV. We explore further implications of this model, including a detailed study of its Higgs sector. We improve on its dark-matter phenomenology, as well as its discovery reach at the LHC (Large Hadron Collider).
1 Introduction

The nature of dark matter [1] is under intense study. Whereas most assume that it is either a fermion or a scalar or a combination of both [2], the notion that it could be a vector boson just as well has also been proposed. In a theory of universal compact extra dimensions, the first Kaluza-Klein excitation of the standard-model $U(1)$ gauge boson $B$ is such a candidate [3]. The $T$–odd counterpart of $B$ in little Higgs models is another candidate [4]. Non-Abelian vector bosons from a hidden sector may also be considered [5]. All of the above involve “exotic” physics.

Recently, it was realized [6] that an existing conventional model [7] based on superstring-inspired $E_6$ has exactly the ingredients which allow it to become a model of vector-boson dark matter, where the vector boson itself ($X$) comes from an $SU(2)_N$ gauge extension of the Standard Model. In Sec. 2 we list all the necessary particles of this (nonsupersymmetric) model. In Sec. 3 we discuss in detail the complete Higgs potential and its minimization. In Sec. 4 we obtain the masses of all the gauge and Higgs bosons. In Sec. 5 we compute the annihilation cross section of the dark-matter vector boson $X$. In Sec. 6 we study the constraints from dark-matter direct-search experiments. In Sec. 7 we consider some possible signals at the Large Hadron Collider (LHC). In Sec. 8 there are some concluding remarks.

2 Particle content

Under $SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(2)_N \times S$, where $Q = T_{3L} + Y$ is the electric charge and $L = S + T_{3N}$ is the generalized lepton number, the fermions of this nonsupersymmetric model are given by [6]

$$\begin{pmatrix} u \\ d \end{pmatrix} \sim (3, 2, 1/6, 1; 0), \quad u^c \sim (3^*, 1, -2/3, 1; 0),$$

$$\begin{pmatrix} h^c, d^c \end{pmatrix} \sim (3^*, 1, 1/3, 2; -1/2), \quad h \sim (3, 1, -1/3, 1; 1),$$

(1) (2)
\[
\begin{pmatrix}
N & \nu \\
E & e
\end{pmatrix} \sim (1, 2, -1/2, 2; 1/2), \quad \begin{pmatrix}
E^c \\
N^c
\end{pmatrix} \sim (1, 2, 1/2, 1; 0), \quad (3)
\]
\[
e^c \sim (1, 1, 1; -1), \quad (\nu^c, n^c) \sim (1, 1, 0, 2; -1/2), \quad (4)
\]

where all fields are left-handed. The \(SU(2)_L\) doublet assignments are vertical with \(T_{3L} = \pm 1/2\) for the upper (lower) entries. The \(SU(2)_N\) doublet assignments are horizontal with \(T_{3N} = \pm 1/2\) for the right (left) entries. There are three copies of the above to accommodate the known three generations of quarks and leptons, together with their exotic counterparts.

It is easy to check that all gauge anomalies are canceled. The extra global \(U(1)\) symmetry \(S\) is imposed so that \((-1)^L\), where \(L = S + T_{3N}\), is conserved, even though \(SU(2)_N\) is completely broken.

The Higgs sector consists of one bidoublet, two doublets, and one triplet:

\[
\begin{pmatrix}
\phi_0^1 \\
\phi_0^2 \\
\phi_0^3
\end{pmatrix} \sim (1, 2, -1/2, 2; 1/2), \quad \begin{pmatrix}
\phi_1^1 \\
\phi_1^2
\end{pmatrix} \sim (1, 2, 1/2, 1; 0),
\]

\[(\chi_1^0, \chi_2^0) \sim (1, 1, 0, 2; -1/2), \quad \begin{pmatrix}
\Delta_0^0/\sqrt{2} \\
-\Delta_2^0/\sqrt{2}
\end{pmatrix} \sim (1, 1, 0, 3; 1).
\]

The allowed Yukawa couplings are thus

\[(d\phi_1^0 - u\phi_1^-)d^c - (d\phi_3^0 - u\phi_3^-)h^c, \quad (u\phi_2^0 - d\phi_2^+)u^c, \quad (h^c\chi_2^0 - d^c\chi_1^0)h, \quad (7)
\]
\[(N\phi_3^- - \nu\phi_1^- - E\phi_3^0 + e\phi_1^0)e^c, \quad (E\phi_2^+ - N\phi_2^0)n^c - (e\phi_2^+ - \nu\phi_2^0)\nu^c, \quad (8)
\]
\[(EE^c - NN^c)\chi_2^0 - (eE^c - \nu N^c)\chi_1^0, \quad n^c n^c \Delta_1^0 + (n^c \nu^c + \nu^c n^c)\Delta_0^2/\sqrt{2} - \nu^c \nu^c \Delta_3^0. \quad (9)
\]

There are five nonzero vacuum expectation values: \(\langle \phi_1^0 \rangle = v_1, \langle \phi_2^0 \rangle = v_2, \langle \Delta_1^0 \rangle = u_1, \) and \(\langle \chi_2^0 \rangle = u_2, \) corresponding to scalar fields with \(L = 0\), as well as \(\langle \Delta_3^0 \rangle = u_3\), which breaks \(L\) to \((-1)^L\). Thus \(m_d, m_e\) come from \(v_1\), and \(m_u, m_{\nu \nu} (= -m_{N\nu})\) come from \(v_2\), whereas \(m_h, m_E (= -m_{NN})\) come from \(u_2\), and \(n^c, \nu^c\) obtain Majorana masses from \(u_1\) and \(u_3\). The scalar fields \(\phi_3^{0,-}\) and \(\Delta_2^0\) have \(L = 1\), whereas \(\chi_1^0\) has \(L = -1\) and \(\Delta_3^0\) has \(L = 2\).

There are five neutral fermions per family. Two have odd \(L\) parity, i.e. \(\nu\) and \(\nu^c\). Their
The Higgs potential of this model is given by

\[ V = \mu_1^2 Tr(\phi_{13}^\dagger \phi_{13}) + \mu_2^2 \phi_1^\dagger \phi_2 + \mu_3^2 \chi \chi^\dagger + \mu_4^2 Tr(\Delta^\dagger \Delta) \]

where \( m_D \) comes from \( v_2 \) and \( M_3 \) from \( u_3 \). The other three have even \( R \) parity, i.e. \( N, N^c, \) and \( n^c \). Their \( 3 \times 3 \) mass matrix is given by

\[ \mathcal{M}_N = \begin{pmatrix} 0 & -m_E & -m_D \\ -m_E & 0 & 0 \\ -m_D & 0 & M_1 \end{pmatrix}, \]

where \( m_E \) comes from \( u_2 \) and \( M_1 \) from \( u_1 \). Note that without \( M_1 \), there would be a massless fermion in this sector. Since \((-1)^L\) is exactly conserved, \( \nu, \nu^c \) do not mix with \( N, N^c, n^c \).

Even though this model is nonsupersymmetric, \( R \) parity as defined in the usual way for supersymmetry, i.e. \( R \equiv (-)^{3B+L+2j} \), still holds, so that the usual quarks and leptons have even \( R \), whereas \( h, h^c, (N, E), (E^c, N^c), \) and \( n^c \) have odd \( R \). As for the scalars, \( (\phi_1^0, \phi_1^-), (\phi_2^+, \phi_2^0), \chi_2^0, \Delta_1^0, \) and \( \Delta_2^0 \) have even \( R \), whereas \( (\phi_3^0, \phi_3^-), \chi_1^0, \) and \( \Delta_3^0 \) have odd \( R \).

### 3 Higgs potential

The Higgs potential of this model is given by

\[ V = \mu_1^2 Tr(\phi_{13}^\dagger \phi_{13}) + \mu_2^2 \phi_1^\dagger \phi_2 + \mu_3^2 \chi \chi^\dagger + \mu_4^2 Tr(\Delta^\dagger \Delta) \]

\[ + \left( \mu_{12} \bar{\chi} \phi_1^\dagger \bar{\phi}_2 + \mu_{13} \bar{\chi} \phi_{13}^\dagger \phi_{13} + \mu_{23} \bar{\chi} \phi_{13}^\dagger \chi^\dagger + H.c. \right) + \frac{1}{2} \lambda_1 [Tr(\phi_{13}^\dagger \phi_{13})]^2 + \frac{1}{2} \lambda_2 (\phi_2^\dagger \phi_2)^2 \]

\[ + \frac{1}{2} \lambda_3 Tr(\phi_{13}^\dagger \phi_{13} \phi_{13}^\dagger \phi_{13}) + \frac{1}{2} \lambda_4 (\chi \chi^\dagger)^2 + \frac{1}{2} \lambda_5 [Tr(\Delta^\dagger \Delta)]^2 + \frac{1}{4} \lambda_6 Tr(\Delta^\dagger \Delta - \Delta \Delta^\dagger)^2 \]

\[ + f_1 \chi \phi_{13}^\dagger \phi_{13} \chi^\dagger + f_2 \chi \bar{\phi}_{13}^\dagger \bar{\phi}_{13} \chi^\dagger + f_3 \phi_2^\dagger \phi_{13} \phi_{13}^\dagger \phi_2 + f_4 \bar{\phi}_{13}^\dagger \bar{\phi}_{13} \phi_2^\dagger \phi_2 + f_5 (\phi_2^\dagger \phi_2)(\chi \chi^\dagger) \]

\[ + f_6 (\chi \chi^\dagger) Tr(\Delta^\dagger \Delta) + f_7 \chi (\Delta^\dagger \Delta - \Delta \Delta^\dagger) \chi^\dagger + f_8 (\phi_2^\dagger \phi_2) Tr(\Delta^\dagger \Delta) \]

\[ + f_9 Tr(\phi_{13}^\dagger \phi_{13}) Tr(\Delta^\dagger \Delta) + f_{10} Tr(\phi_{13} (\Delta^\dagger \Delta - \Delta \Delta^\dagger)) \phi_{13}^\dagger, \]

where

\[ \tilde{\phi}_2 = \left( \begin{array}{c} \phi_2^0 \\ -\phi_2^- \end{array} \right), \quad \tilde{\phi}_{13} = \left( \begin{array}{c} \phi_{13}^0 \\ -\phi_{13}^- \end{array} \right), \quad \tilde{\chi} = (\chi_2^0, -\chi_1^0), \]

\[ \tilde{\phi}_2 = \left( \begin{array}{c} \phi_2^0 \\ -\phi_2^- \end{array} \right), \quad \tilde{\phi}_{13} = \left( \begin{array}{c} \phi_{13}^0 \\ -\phi_{13}^- \end{array} \right), \quad \tilde{\chi} = (\chi_2^0, -\chi_1^0), \]
and the $\mu_{23}$ term breaks $L$ softly to $(-1)^L$.

The minimum of $V$ is determined by

$$V_0 = \mu_1^2 v_1^2 + \mu_2^2 v_2^2 + \mu_3^2 u_3^2 + \mu_\Delta (u_1^2 + u_3^2) + 2\mu_{22} v_1 v_2 u_2 + 2\mu_{12} u_1 u_2^2 + 2\mu_{23} u_3 u_2^2$$
$$+ \frac{1}{2}\lambda_1 v_1^4 + \frac{1}{2}\lambda_2 v_2^4 + \frac{1}{2}\lambda_3 v_3^4 + \frac{1}{2}\lambda_4 u_2^4 + \frac{1}{2}\lambda_5 (u_1^2 + u_3^2)^2 + \frac{1}{2}\lambda_6 (u_1^2 - u_3^2)^2$$
$$+ f_2 v_1^2 u_2^2 + f_4 v_1^2 v_2^2 + f_5 v_2^2 u_2^2 + f_6 u_3^2 (u_1^2 + u_3^2) + f_7 u_2^2 (u_1^2 - u_3^2)$$
$$+ f_8 v_2^2 (u_1^2 + u_3^2) + f_9 v_1^2 (u_1^2 + u_3^2) + f_{10} v_1^2 (u_1^2 - u_3^2),$$

where

$$0 = \mu_1^2 + (f_9 + f_{10}) u_1^2 + 2u_2^2 + (f_9 - f_{10}) u_3^2 + (\lambda_1 + \lambda_3) v_1^2 + 4v_2^2 + \frac{\mu_{22} v_2 u_2}{v_1},$$

$$0 = \mu_2^2 + f_8 u_1^2 + f_5 u_2^2 + f_8 u_3^2 + f_4 v_1^2 + \lambda_2 v_2^2 + \frac{\mu_{22} v_1 v_2}{v_2},$$

$$0 = \mu_\Delta + (f_6 - f_7) u_1^2 + \lambda_4 v_1^2 + (f_6 + f_7) u_3^2 + 2v_1^2 + f_5 v_2^2 + \frac{\mu_{22} v_1 v_2}{u_2}$$
$$+ 2\mu_{12} u_1 + 2\mu_{23} u_3,$$

$$0 = \mu_\Delta + (\lambda_5 + \lambda_6) u_1^2 + (f_6 - f_7) u_2^2 + (\lambda_5 - \lambda_6) u_3^2 + (f_9 + f_{10}) v_1^2 + f_8 v_2^2 + \frac{\mu_{12} u_2}{u_1},$$

$$0 = \mu_\Delta + (\lambda_5 - \lambda_6) u_1^2 + (f_6 + f_7) u_2^2 + (\lambda_5 + \lambda_6) u_3^2 + (f_9 - f_{10}) v_1^2 + f_8 v_2^2 + \frac{\mu_{23} u_2^2}{u_3}.$$  

4 **Gauge and Higgs boson masses**

After the spontaneous breaking of $SU(2)_N \times SU(2)_L \times U(1)_Y$, the gauge bosons $X_{1,2,3}$ and $W, Z$ acquire masses as follows:

$$m_{W}^2 = \frac{1}{2}g_2^2 (v_1^2 + v_2^2), \quad m_{X_{1,2}}^2 = \frac{1}{2}g_N^2 [u_2^2 + v_1^2 + 2(u_1 \mp u_3)^2],$$

$$m_{Z,X_3}^2 = \frac{1}{2} \begin{pmatrix} (g_1^2 + g_2^2)(v_1^2 + v_2^2) & -g_N \sqrt{g_1^2 + g_2^2} v_1^2 \\ -g_N \sqrt{g_1^2 + g_2^2} v_1^2 & g_N^2 [u_2^2 + v_1^2 + 4(u_1 \mp u_3)^2] \end{pmatrix},$$

Whereas the usual gauge bosons have even $R$, two of the $SU(2)_N$ gauge bosons $X_{1,2}$ have odd $R$ and $X_3 (= Z')$ has even $R$. Assuming that $X_1$ is lighter than $X_2$, the former becomes
a good candidate for dark matter. There is also $Z - Z'$ mixing in this model, given by $- (\sqrt{g_1^2 + g_2^2/g_N}) [v_1^2/(u_2^2 + 4u_1^2 + 4u_3^2)]$. This is constrained by precision electroweak data to be less than a few times $10^{-4}$. If $m_{Z'} \sim 1$ TeV, then $v_1$ should be less than about 10 GeV. Now $m_b$ comes from $v_1$, so this model implies that $\tan \beta = v_2/v_1$ is large and the Yukawa coupling of $bb'\phi_1^0$ is enhanced. This will have interesting phenomenological consequences \[8\].

There are 22 scalar degrees of freedom, 6 of which become massless Goldstone bosons, leaving 16 physical particles. Their masses are given below:

$$
m_2(\phi_3^\pm) = (f_1 - f_2)u_2^2 + 2f_{10}(u_3^2 - u_1^2) - \lambda_3 v_1^2 + (f_3 - f_4)v_2^2 - \mu_{22}v_2u_2/v_1,
$$

$$
m_2(\sin \beta \phi_1^+ + \cos \beta \phi_2^+) = [f_3 - f_4 - \mu_{22}u_2/v_1 v_2]\sqrt{v_1^2 + v_2^2},
$$

where $\tan \beta = v_2/v_1$ and the orthogonal combination $\cos \beta \phi_1^+ - \sin \beta \phi_2^+$ is massless, corresponding to the longitudinal component of $W^\pm$. The $5 \times 5$ mass-squared matrix spanning $(\phi_{1I}, \phi_{2I}, \chi_{2I}, \Delta_{1I}, \Delta_{3I})$ is given by

$$
\begin{pmatrix}
-\mu_{22}v_2u_2/v_1 & -\mu_{22}u_2 & -\mu_{22}v_2 & 0 & 0 \\
-\mu_{22}u_2 & -\mu_{22}v_1u_2/v_2 & -\mu_{22}v_1 & 0 & 0 \\
-\mu_{22}v_2 & -\mu_{22}v_1 & -\mu_{22}v_1v_2/u_2 - 4\mu_{12}u_1 - 4\mu_{23}u_3 & -2\mu_{12}u_2 & 2\mu_{23}u_2 \\
0 & 0 & -2\mu_{12}u_2 & -2\mu_{23}u_2/\mu_1 & 0 \\
0 & 0 & 2\mu_{23}u_2 & 0 & -\mu_{23}u_2/\mu_3
\end{pmatrix}
$$

(24)

with two zero mass eigenvalues, spanned by the states $v_1\phi_{1I} - v_2\phi_{2I}$ and $-(v_1/2)\phi_{1I} - (v_2/2)\phi_{2I} + u_2\chi_{2I} - 2u_1\Delta_{1I} + 2u_3\Delta_{3I}$, corresponding to the longitudinal components of $Z$ and $Z'$. In the $(\chi_{1I}, \Delta_{2I}, \phi_{3I})$ sector, the mass-squared matrix is given by

$$
[(f_1 - f_2)v_1^2 + 2f_{10}(u_1^2 - u_3^2) - \mu_{22}v_1v_2/u_2 - 2(\mu_{12} - \mu_{23})(u_1 - u_3)]\chi_{1I}^2
$$

$$
+ 2\sqrt{2}u_2[\mu_{23} - \mu_{12} + f_1(u_1 + u_3)]\chi_{1I}\Delta_{2I} + 2[\mu_{22}v_2 - (f_1 - f_2)v_1u_2]\chi_{1I}\phi_{3I}
$$

$$
+ [\lambda_0(u_1 + u_3)^2 - \mu_{12}u_2^2/2u_1 - \mu_{23}u_2^2/2u_3]\Delta_{2I}^2 - 2\sqrt{2}f_{10}v_1(u_3 + u_1)\Delta_{2I}\phi_{3I}
$$

$$
+ [(f_1 - f_2)u_2^2 + 2f_{10}(u_3^2 - u_1^2) - \mu_{22}v_2u_2/v_1]\phi_{3I}^2,
$$

(25)
with one zero mass eigenvalue, corresponding to the longitudinal component of \(X_1\). The mass-squared matrix of the \((\chi_{1R}, \Delta_{2R}, \phi_{3R})\) sector is analogously given by

\[
[(f_1 - f_2)v_1^2 + 2f_7(u_1^2 - u_3^2) - \mu_{22}v_1 v_2 / u_2 - 2(\mu_{12} + \mu_{23})(u_1 + u_3)] \chi_{1R}^2 \\
+ 2\sqrt{2}u_2[\mu_{23} + \mu_{12} + f_7(u_3 - u_1)]\chi_{1R}\Delta_{2R} - 2[\mu_{22}v_2 - (f_1 - f_2)v_1 u_2] \chi_{1R}\phi_{3R} \\
+ [\lambda_6(u_1 - u_3)^2 - \mu_{12}u_3^2/2u_1 - \mu_{23}u_3^2/2u_3]\Delta_{2R}^2 + 2\sqrt{2}f_{10}v_1(u_3 - u_1)\Delta_{2R}\phi_{3R} \\
+ [(f_1 - f_2)u_2^2 + 2f_{10}(u_3^2 - u_1^2) - \mu_{22}v_2 v_2 / v_1] \phi_{3R}^2.
\]

with one zero mass eigenvalue, corresponding to the longitudinal component of \(X_2\). The remaining 5 scalar fields \((\phi_{1R}, \phi_{2R}, \chi_{2R}, \Delta_{1R}, \Delta_{3R})\) form a mass-squared matrix

\[
\begin{pmatrix}
2(\lambda_1 + \lambda_3)v_1^2 & 2f_1v_1 v_2 & 2f_2v_1 u_2 & 2(f_9 + f_{10})v_1 u_1 & 2(f_9 + f_{10})v_1 u_3 \\
2f_1v_1 v_2 & 2\lambda_2 v_2^2 & 2f_5v_2 u_2 & 2f_8v_2 u_1 & 2f_8v_2 u_3 \\
2f_2v_1 u_2 & 2f_5v_2 u_2 & 2\lambda_4 u_2^2 & 2(f_6 - f_7)u_1 u_2 & 2(f_6 + f_7)u_2 u_3 \\
2(f_9 + f_{10})v_1 u_1 & 2f_8v_2 u_1 & 2(f_6 - f_7)u_1 u_2 & 2(\lambda_5 + \lambda_6)u_1^2 & 2(\lambda_5 - \lambda_6)u_1 u_3 \\
2(f_9 - f_{10})v_1 u_3 & 2f_8v_2 u_3 & 2(f_6 + f_7)u_2 u_3 & 2(\lambda_5 - \lambda_6)u_1 u_3 & 2(\lambda_5 + \lambda_6)u_3^2
\end{pmatrix}
\]

\[
+ \begin{pmatrix}
-\mu_{22}v_2 v_2 / v_1 & \mu_{22}v_2 & \mu_{22}v_2 & 0 & 0 \\
\mu_{22}v_2 & -\mu_{22}v_2 / v_2 & \mu_{22}v_1 & 0 & 0 \\
\mu_{22}v_2 & \mu_{22}v_1 & -\mu_{22}v_2 / u_2 & 2\mu_{12} u_2 & 2\mu_{23} u_2 \\
0 & 0 & 2\mu_{12} u_2 & -\mu_{12} u_2 / u_1 & 0 \\
0 & 0 & 2\mu_{23} u_2 & 0 & -\mu_{23} u_2 / u_3
\end{pmatrix}.
\]

Consider the simplifying case of \(f_7 = f_{10} = 0\) and \(\mu_{12} = \mu_{23}\), then from Eqs. (18) and (19), we find \(u_1 = u_3\). The massless states of Eqs. (25) and (26) are then easily identified: \(u_2\chi_{1I} + v_1\phi_{3I}\) and \(u_2\chi_{1R} + 2\sqrt{2}u_1\Delta_{2R} - v_1\phi_{3R}\) for the longitudinal components of \(X_1\) and \(X_2\) respectively. Three exact mass eigenstates are:

\[
(\Delta_{1I} + \Delta_{3I}) / \sqrt{2} : \quad m^2 = -\mu_{12} u_2^2 / u_1.
\]

\[
\Delta_{2I}, \quad (\Delta_{1R} - \Delta_{3R}) / \sqrt{2} : \quad m^2 = 4\lambda_6 u_1^2 - \mu_{12} u_2^2 / u_1.
\]

Using the approximation \(v_{1,2} << u_{1,2}\), we also have

\[
\phi_{3R}, \phi_{3I} : \quad m^2 = (f_1 - f_2)u_2^2 - \mu_{22} v_2 v_2 / v_1,
\]
\( \frac{(v_2 \phi_1 + v_1 \phi_2)}{\sqrt{v_1^2 + v_2^2}}: \quad m^2 = -\mu_{22}u_2(v_1^2 + v_2^2)/v_1v_2, \) (31)

\( \frac{(2\sqrt{2}u_1 \chi_{1R} - u_2 \Delta_{2R})}{\sqrt{8u_1^2 + u_2^2}}: \quad m^2 = -\mu_{12}(8u_1 + u_2^2/u_1), \) (32)

\( \frac{(4u_1 \chi_{2I} + u_2 \Delta_{1I} - u_2 \Delta_{3I})}{\sqrt{16u_1^2 + 2u_2^2}}: \quad m^2 = -\mu_{12}(8u_1 + u_2^2/u_1). \) (33)

This pattern shows that \((\phi_1^0, \phi_1^-)\) and \((\phi_2^+, \phi_2^0)\) behave as the conventional two Higgs doublets with the former coupling to \(d\) quarks and the latter to \(u\) quarks. The new feature here is that \((\phi_1^0, \phi_1^-)\) also interact with the \(SU(2)_N\) gauge bosons. An interesting possibility for example is \(Z' \rightarrow \phi_1^0 \bar{\phi}_1^0 \rightarrow (b\bar{b})(b\bar{b})\).

5 \(X_1X_1\) annihilation

We assume that \(X_1\) is the lightest particle having odd \(R\). It is thus stable and a possible candidate for dark matter. In the early Universe, \(X_1X_1\) will annihilate to particles of even \(R\), i.e. \(d\bar{d}\) through \(h\) exchange, \(e^-e^+\) through \(E\) exchange, \(\nu\bar{\nu}\) through \(N\) exchange, and \(\phi_1 \bar{\phi}_1\) through \(\phi_3\) exchange (and direct interaction). There is also the direct-channel process, such as \(X_1X_1 \rightarrow \phi_1R \rightarrow d\bar{d}\), which is suppressed by \(m_d\) so it is negligible here. However, the corresponding process for dark-matter direct search, i.e. \(X_1d \rightarrow X_1d\) through \(\phi_1R\) exchange, may be important as discussed in the next section. Note that there is no tree-level contribution from \(Z'\) because the only allowed triple-vector-boson coupling is \(X_1X_2Z'\) and \(X_2\) is too heavy to be involved.

In Fig. 1 we show the various annihilation diagrams, resulting in the nonrelativistic cross section \(\times\) relative velocity given by

\[
\sigma v_{rel} = \frac{g_4^4m_X^2}{72\pi} \left[ \sum_h \frac{3}{(m_h^2 + m_X^2)^2} + \sum_E \frac{2}{(m_E^2 + m_X^2)^2} \right]
+ \frac{2}{(m_{\phi_3}^2 + m_X^2)^2} + \frac{1}{m_X^2(m_{\phi_3}^2 + m_X^2)} + \frac{3}{8m_X^4},
\]

where the sum over \(h, E\) is for 3 families. The factor of 3 for \(h\) is the number of colors, and
Figure 1: Annihilation of $X_1X_1$ to standard-model particles.

the factor of 2 for $E$ is to include $N$ which has the same mass of $E$. For the scalar final states $\phi_1\bar{\phi}_1$, in addition to the exchange of $\phi_3$, there is also the direct $X_1X_1\phi_1\bar{\phi}_1$ interaction. Since $v_1 \ll v_2$, both $\phi_1^0$ and $\phi_1^{-}$ are physical particles to a very good approximation. Assuming as we do that $m_X$ is the smallest mass in Eq. (34), we must have

$$\sigma v_{rel} < \frac{41g_N^4}{576\pi m_X^2}. \quad (35)$$

This puts an upper bound on $m_X$ for a given value of $\sigma v_{rel}$. Assuming $\sigma v_{rel} > 0.86$ pb from the requirement of relic abundance, and $g_N^2 (\approx g_2^2) = 0.4$, we then obtain

$$m_X < 1.28 \text{ TeV}. \quad (36)$$

In other words, whereas the scale of $SU(2)_N$ breaking is a priori unknown, the assumption of $X$ dark matter constrains it to be of order 1 TeV and be accessible to observation at the LHC.

We consider Eq. (34) as a function of $m_X$ and $\delta = m_h/m_X - 1$, with all three $h$’s having the same mass. We then consider the two extreme cases for the other contributions: one where all heavy masses are equal to $m_X$; and the other where all heavy masses (except $m_X$) are equal to the (arbitrary) value $2.5m_X$ to ensure that no Yukawa or quartic coupling
gets too large. In the $\delta - m_X$ plane, for a given value of $\sigma_{v_{rel}}$, the region between these two lines is then the allowed parameter space for $m_X$ and $m_h$. We show this in Fig. 2 for $\sigma_{v_{rel}} = 0.91 \pm 0.05$ pb [10].

![Figure 2: Allowed region in $\delta = m_h/m_X - 1$ versus $m_X$ (in TeV) from relic abundance and from CDMS direct search.](image)

6 Direct dark matter search

In Fig. 3 we show the tree-level diagrams for $X_1 d \to X_1 d$ through the direct-channel exchange of $h$ and the cross-channel exchange of $\phi_{1R}$. Taking into account twist-2 operators and gluonic contributions calculated recently [9] and assuming that $m(h_d) = m(h_s) = m(h_b) = m_h$, we find

$$\frac{f_p}{m_p} = 0.052 \left[ -\frac{g_N^2}{4m_\phi^2} - \frac{g_N^2}{16} \frac{m_h^2}{(m_h^2 - m_X^2)^2} \right] + \frac{3}{4}(0.222) \left[ -\frac{g_N^2}{4} \frac{m_X^2}{(m_h^2 - m_X^2)^2} \right]$$
\[
- (0.925) \left( (1.19) \frac{g_N^2}{54 m_\phi^2} + \frac{g_N^2}{36} \left[ (1.19) \frac{m_h^2}{6(m_h^2 - m_X^2)^2} + \frac{1}{3(m_h^2 - m_X^2)} \right] \right). \tag{37}
\]

To obtain \( f_n/m_n \), the numerical coefficients \((0.052, 0.222, 0.925)\) in the above are replaced by \((0.061, 0.330, 0.922)\). The spin-independent elastic cross section for \( X_1 \) scattering off a nucleus of \( Z \) protons and \( A - Z \) neutrons normalized to one nucleon is then given by

\[
\sigma_0 = \frac{1}{\pi} \left( \frac{m_X}{m_N} \right)^2 \left| \frac{Z f_p + (A - Z) f_n}{A} \right|^2. \tag{38}
\]

Here we will use \( ^{73}\text{Ge} \) with \( Z = 32 \) and \( A - Z = 41 \) to compare against the recent CDMS result [11]. In the range \( 0.3 < m_X < 1.0 \) TeV, the experimental upper bound is very well approximated by [12]

\[
\sigma_0 < 2.2 \times 10^{-7} \text{ pb} \ (m_X/1 \text{ TeV})^{0.86}. \tag{39}
\]

In Fig. 2 this appears as a solid line for \( m_\phi = 120 \) GeV, to the right (left) of which is allowed (forbidden) by the CDMS data. If \( m_\phi > 120 \) GeV, this line will move slightly to the left. It is seen that the relic-abundance constraint is indeed allowed, but direct search is still far away from testing this model.
Figure 4: Normalized signal and background distributions as functions of missing transverse energy.

7 Collider phenomenology

The dark-matter gauge boson $X_1$ may be produced at the Large Hadron Collider in association with the lightest exotic heavy quark $h$ through $d + $gluon $\rightarrow h + X_1$. Consider the following mass spectrum:

$$m_h > m_{X_2} > m_{E,N} > m_{X_1}. \quad (40)$$

In that case, $h$ may decay into $X_1d$ and $X_2d$, then $X_2$ will decay into $E^+l^-, E^-l^+, \bar{N}\nu, \bar{N}\bar{\nu}$, and $E^+ \rightarrow X_1l^+, E^- \rightarrow X_1l^-, \bar{N} \rightarrow X_1\bar{\nu}$, $N \rightarrow X_1\nu$. This means that about 1/4 of the time, $pp \rightarrow hX_1$ will end up with one quark jet + missing energy + $l_i^+l_j^-$ and $pp \rightarrow hh$ will end up with two quark jets + missing energy + $l_i^+l_j^-$. Some of these two-lepton final states could involve different flavors because of mixing of families in the $SU(2)_N$ sector. Note that $X_2 \rightarrow X_1 +$ virtual $X_3 \rightarrow X_1 + d\bar{d} (l^-l^+)$ is also possible, but very much suppressed if
\[ m_{E,N} < m_{X_2}. \]

In the following, we choose \( m_{X_1} = 700 \text{ GeV}, m_{E,N} = 735 \text{ GeV}, m_{X_2} = 770 \text{ GeV}, \) and \( m_h = 980 \text{ GeV}. \) We find that at the LHC \( (E_{cm} = 14 \text{ TeV}), \) the cross section of \( dX_1X_1l^-l^+ \) production is 5.5 fb. We show in Fig. 4 the distribution of this signal versus the expected standard-model background (dominated by \( t\bar{t} \)) as a function of missing transverse energy, using the cut \( p_T > 20 \text{ GeV} \) for each lepton with \( |\eta| < 2.5, \) and \( p_T > 50 \text{ GeV} \) for the one hadronic jet. We use \texttt{CalcHEP} \cite{13} in combination with \texttt{Pythia} \cite{14} in this calculation. We show in Table 1 that a cut on missing transverse energy of 200 GeV would eliminate the standard-model background which is dominated by \( t\bar{t} \) events.

| Event rates for \( \ell^+\ell^- + 1 \text{jet} + E_T \) with \( p_T > 20 \text{, } p_T > 50 \) |
|-----------------|-----------------|-----------------|
|                | \( E_T > 100 \) | \( E_T > 200 \) | \( E_T > 300 \) |
| Signal          | 3.1             | 1.6             | 0.59             |
| Background      | 237             | 0               | 0                |

Table 1: Event rates (fb) for LHC with \( E_{cm} = 14 \text{ TeV}, \) using CTEQ6L parton distribution functions, and the average of final state particle masses as partonic \( E_{cm}. \)

8 Concluding remarks

The (nonsupersymmetric) dark vector-gauge-boson model \cite{6} is studied in some detail. Its complete particle content is delineated and analyzed, including the most general Higgs potential and its minimization. The identification of the \( X_1 \) boson as a dark-matter candidate (to account for the observed relic abundance) constrains the \( SU(2)_N \) breaking scale to be about 1 TeV. We have updated the theoretical cross section for \( X_1 \) to interact in underground direct-search experiments. The present CDMS bound is shown to be much below what is expected in this scenario. On the other hand, the prognosis for observing the consequences of this model at the LHC with \( E_{cm} = 14 \text{ TeV} \) and integrated luminosity of 10 fb\(^{-1}\) is good,
with an expected signal in our specific example of 16 events (dimuon + jet + missing energy) against negligible background for \( m_{X_1} = 700 \) GeV and \( m_h = 980 \) GeV.

**Acknowledgements**

This work is supported in part by the US Department of Energy under Grant No. DE-FG03-94ER40837, and by CONACYT-SNI (Mexico). SB would like to thank Dr. Ehsan Noruzifar for technical help and Dr. Asesh Krishna Datta for valuable comments on the numerical simulation.

**References**

[1] For a review, see for example G. Bertone, D. Hooper, and J. Silk, Phys. Rept. **405**, 279 (2005).

[2] Q.-H. Cao, E. Ma, J. Wudka, and C.-P. Yuan, arXiv:0711.3881 [hep-ph].

[3] G. Servant and T. M. P. Tait, Nucl. Phys. **B650**, 391 (2003).

[4] J. Hubisz and P. Meade, Phys. Rev. **D71**, 035016 (2005).

[5] T. Hambye, JHEP **0901**, 028 (2009).

[6] J. L. Diaz-Cruz and E. Ma, Phys. Lett. **B695**, 264 (2011).

[7] D. London and J. L. Rosner, Phys. Rev. **D34**, 1530 (1986).

[8] C. Balazs, J. L. Diaz-Cruz, H.-J. He, T. M. P. Tait, and C.-P. Yuan, Phys. Rev. **D59**, 055016 (1999).

[9] J. Hisano, K. Ishiwata, N. Nagata, and M. Yamanaka, arXiv.1012.5455 [hep-ph].
[10] Particle Data Group: K. Nakamura et al., J. Phys. G: Nucl. Part. Phys. 37, 075021 (2010).

[11] Z. Ahmed et al., Science 327, 1619 (2010).

[12] S. Khalil, H.-S. Lee, and E. Ma, Phys. Rev. D81, 051702(R) (2010).

[13] A. Pukhov, arXiv:hep-ph/0412191.

[14] T. Sjostrand, S. Mrenna and P. Skands, JHEP 0605, 026 (2006).