Dynamic analysis of slender launching system connected by clamp band joint using harmonic balance method

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Abstract. Clamp band joints are widely used to fasten spacecrafts onto launching systems. Due to the unilateral constraints and the frictional slippage at the joint interface, clamp band joints may bring nonlinearity into launching systems during launching process. In this paper, the dynamics of a slender launching system with clamp band joint is investigated using harmonic balance method. Firstly, the formulas for the joint stiffness of the clamp band joint are proposed. Then, the finite element model for the launch vehicle and the spacecraft connected by the clamp band joint is developed, where the clamp band joint is represented by a massless beam element. Finally, harmonic balance method is applied to calculate the steady state response of the launching system.

1. Introduction
Spacecrafts are commonly fastened onto launch vehicles by the clamp band joints in modern aerospace applications. The continuity of the interface between the launch vehicle and the spacecraft is guaranteed by the preload imposed onto the clamp band joint. Though the magnitude of the preload is strictly controlled to prevent joint separation, there may exist frictional slippage at the joint interface under severe loading conditions, which results in variations of the local stiffness and in turn leads to nonlinear dynamic behavior of the launching system.

Dynamics of launch vehicles have been extensively investigated. For different research purposes, various dynamic models for launch vehicles incorporating the propulsion [1], aerodynamics [2], guidance and control [3], and structural flexibility [4] were developed. When modeling launching systems, the approximation of rigid connection between launch vehicles and spacecrafts is usually adopted without accommodating the influence of clamp band joint.

The authors have developed the finite element model for a launching system, where the clamp band joint is represented by a massless beam element [5]. The modal characteristics of the launching system were discussed in detail considering the variations of the mass and length of the launching system due to the fuel combustion and stage jettisons during the ascent flight; whereas, the steady state response was simply calculated for certain excitation frequencies by numerical integration.

Compared with numerical integration, harmonic balance method (HBM) is a more efficient approach and has been widely used for solving strongly nonlinear problems [6-9].

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In this paper, the launching system model accommodating the nonlinear influence of the clamp band joint proposed in [5] is adopted. HBM is applied to computing the steady state response of the launching system and evaluating the influence of the clamp band joint on the system response.

2. Modelling of Launching System with Clamp Band Joint

2.1. Clamp Band Joint Model

The configuration of the clamp band joint is shown in Figure 1. Under preload, the joint generates axial and bending constraints and fastens the interface ring and the payload adapter together, which connect to the spacecraft and launch vehicle respectively. The transverse deformation at the joint interface is resisted by auxiliaries such as lips, pins or splines. Hence, the joint is assumed to be rigid in the lateral direction.

![Figure 1](image.png)

**Figure 1.** Sketch of the joint interface between the launch vehicle and the spacecraft.

The relative axial deformation at the joint interface under the axial load $T$ is given by [5]

$$Y = \frac{r_o T}{\pi R_o} - 2(R_i - R_{f_h}) f_e + \left( t_r - \frac{t_{f_m}}{2} \right) S_{n} \frac{K_o}{R_o}$$

(1)

where, $r_o = R_i - R_0 - \frac{\nu R_0 A (\lambda t_0 + 1)}{2 \lambda^2 R_0^2 A + 2 \lambda R_0 t_0}$, $f_e = \frac{E A I_t}{2 \lambda^2 R_0^2 A + 2 \lambda R_0 t_0}, \quad k_o = \frac{E A I_t}{4 \lambda^3 R_0^2 A + 4 \lambda^2 R_0 t_0}$,

$$t_r = \frac{R_i}{2 \lambda^2 R_0^2 A + 2 \lambda R_0 t_0}$$

The relative deflection angle at the joint interface under the bending moment $M$ is [5]

$$\Psi = \frac{R_i - R_0}{R_3} \left( \psi^0 - \psi^* \right)$$

(2)

where,

$$\psi^0 = \frac{\pi R_0}{k_r + \frac{E I_t}{R}}$$

\[ \frac{t_r}{2} \frac{(\sin^2 \phi - \mu \cos^2 \phi) (R_i - R_{f_h})}{2 (\sin \phi + \mu \cos \phi)} \left( \frac{t_{f_m}}{4} \right) S R_o + \frac{r_o}{\pi R_0} \frac{(R_i - R_{f_h}) \mu S}{(\sin^2 \phi - \mu \cos^2 \phi) R_0 M_m} \]

\[ k_r + \frac{E I_t}{R} \]
\[ \psi^* = \begin{Bmatrix} \frac{t_{10}}{2} \left( \cos \varphi - \mu \sin \varphi \right) \left( R_i - R_{10} \right) \times \frac{S}{R_0} k_r + \frac{EI_m}{R} \\ \frac{t_{10}}{2} \left( \sin \varphi + \mu \cos \varphi \right) \left( R_i - R_{10} \right) \times \frac{M}{R_0} k_r + \frac{EI_m}{R} \end{Bmatrix}, \quad \psi^* > 0, \]
\[ \begin{Bmatrix} 0, \quad \psi^* \leq 0 \end{Bmatrix} \]

Correspondingly, the axial stiffness \( K_a \) and bending stiffness \( K_m \) of the clamp band joint can be expressed in the incremental form
\[ K_a = \frac{\partial T}{\partial \psi}, \]  
\[ K_m = \frac{R_m \delta M}{\left( R_i - R_{10} \right) \left( \delta \psi^0 - \delta \psi^* \right)}, \]  
(3)  
(4)

The relationships between the external loads and relative deformations at the joint interface calculated based on Eqs. (3) and (4) are shown in Figure 2, where the structural parameters of the 1194 adapter of LM-3A and the preload of 25 kN are adopted. It is revealed in Fig. 2 that the both the relation curves are piecewise linear.

(a) Axial force vs. axial deformation  
(b) Bending moment vs. deflection angle

**Figure 2.** Relationship between external loads and relative deformation at the joint interface.

### 2.2. Launching System Model

The launching system is divided into five components, that is, the 1st, 2nd and 3rd stages of the launch vehicle, the payload fairing and the spacecraft. Those components are modeled separately using the beam element shown in Fig. 3. By assuming that the launch system is axisymmetric and neglecting its torsional vibration, the motion of launching system is in the \( xy \) plane. Correspondingly, there are three degrees of freedom (dofs) in each node, and the vector of generalized displacements of the beam element is
\[ \mathbf{\delta}^* = \begin{bmatrix} u_1 & v_1 & \theta_1 & u_2 & v_2 & \theta_2 \end{bmatrix}^T \]  
(5)

The mass and stiffness matrices of the beam element are developed based upon Timoshenko beam theory. The propellant in the launch vehicle is modeled as additional mass without rotary inertia and imposed to the relevant nodes of the beam models to represent the influence of the liquid on the structural dynamics. The element mass matrix of the liquid propellant can be written as
\[ \mathbf{M}_L = \text{diag} \begin{bmatrix} \frac{m'}{2n} & \frac{m'}{2n} & 0 & \frac{m'}{2n} & \frac{m'}{2n} & 0 \end{bmatrix} \]  
(6)

Then the element mass matrix for each stage of the launch vehicle is
\[ \mathbf{M}_e = \mathbf{M}_L + \mathbf{M}_L \]  
(7)
By assembling the component models together, the linear model for the launching system without considering the influence of the clamp band joint can be obtained as

$$M\ddot{\delta} + C\dot{\delta} + K\delta = F(t)$$

(8)

The clamp band joint connecting the launch vehicle and the spacecraft is treated as a massless beam element as shown in Fig. 4, where the joint stiffness matrix $K_{cb}$ involves the axial and bending stiffnesses as bellow

$$K_{cb} = \begin{bmatrix} K_a & 0 \\ 0 & K_m \end{bmatrix}$$

(9)

Neglecting the joint mass, the general form of equation for the launching system accommodating the clamp band joint is

$$M\ddot{\delta} + C\dot{\delta} + K\delta = F(t) - KJ\delta$$

(10)

where, $K_J$ is the joint stiffness matrix extended from $K_{cb}$, whose expression is

$$K_J = \begin{bmatrix} K_{cb} & -K_{cb} \\ -K_{cb} & K_{cb} \end{bmatrix}$$

(11)

The damping matrix of the launching system model is built according to Rayleigh damping law

$$C = \alpha M + \beta K$$

(12)

where the constants $\alpha$ and $\beta$ are such as the modal damping for the first and six modes of the system are equal to 0.4%.

For convenience of description, the system model given in Eq. (8) is referred to as the fixed model; whereas, that given by Eq. (10) is referred to as the clamped model.

3. Harmonic Balance Method with Condensation

3.1. Harmonic Balance Formulation
HBM is employed to calculate the steady state response of the launching system. Considering the response to be periodic, the displacements $\delta(t)$ can be represented as a truncated Fourier series with $N$ harmonics

$$\delta(t) = \sum_{n=0}^{N} \left( \hat{a}_n \cos n\Omega t + \hat{b}_n \sin n\Omega t \right)$$ (13)

Denote $R_{cb} = K_{cb} \delta$, which represents the nonlinear force caused by the clamp band joint. Correspondingly, the nonlinear force $R_{cb}(t, \delta)$ can also be written as a truncated Fourier series

$$R_{cb}(t, \delta) = \sum_{n=-N}^{N} \left( \hat{c}_n \cos n\Omega t + \hat{d}_n \sin n\Omega t \right)$$ (14)

Substituting the expressions of $\delta(t)$ and $R_{cb}(t, \delta)$ from Eqs. (13) and (14) into Eq. (10) and balancing the harmonic terms yields

$$\Lambda \mathbf{Z} + \mathbf{d} - \mathbf{F} = 0$$ (15)

where

$$\mathbf{Z} = [\hat{a}_0^T, \hat{a}_1^T, \hat{b}_1^T, \ldots, \hat{a}_N^T, \hat{b}_N^T]^T$$

and $\mathbf{d}$ is the vector of Fourier coefficients of the displacement and nonlinear force and excitation force, $\mathbf{F}$ is the vector of Fourier coefficients of the excitation force, and $\Lambda$ is a block-diagonal matrix of $(4m+2)(2N+1)\times(4m+2)(2N+1)$ dimension, whose expression is

$$\Lambda = \begin{bmatrix} K_{nn} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & K_{nn} \end{bmatrix}$$ (16)

where

$$\Lambda_n = \begin{bmatrix} K - (k\Omega)^2 M & k\Omega (C + \Omega G) \\ -k\Omega (C + \Omega G) & K - (k\Omega)^2 M \end{bmatrix}$$ (17)

The Fourier coefficients $\mathbf{d}$ are nonlinear functions of $\mathbf{Z}$ in an implicit form in Eq. (15), where the relationship between $\mathbf{d}$ and $\mathbf{Z}$ can be obtained by using the alternate frequency/time domain (AFT) method [10] sketched as follow:

$$\mathbf{Z} \xrightarrow{\text{IDFT}} \delta(t) \xrightarrow{\text{DFT}} \mathbf{R}_{cb}(t, \delta) \xrightarrow{\text{IDFT}} \mathbf{d}$$

Then only $(4m+2)(2N+1)$ unknowns of $\mathbf{Z}$ elements left in Eq. (15). And those simultaneous algebraic equations can be solved by using the Netwon-Raphson method iteratively, given a proper guess of the initial values.

3.2. Condensation

The finite element model for the launching system can contain quite a large number of dofs, which makes the solving procedure time consuming. On the contrary, the nonlinear influence of the clamp band joint is only related to the axial and deflection dofs at the joint interface. Therefore, condensation is carried out to reducing the problem order.

The equations in Eq. (15) are reorganized into $p$ linear dofs and $q$ nonlinear dofs for each harmonic as follow

$$\begin{bmatrix} A_{qq} & A_{qp} \\ A_{pq} & A_{pp} \end{bmatrix} \begin{bmatrix} \mathbf{Z}_q \\ \mathbf{Z}_p \end{bmatrix} = \begin{bmatrix} \mathbf{F}_q \\ \mathbf{F}_p \end{bmatrix} - \begin{bmatrix} \mathbf{d}_q \\ \mathbf{d}_p \end{bmatrix}$$ (18)

From Eq. (18), the Fourier coefficients $\mathbf{Z}_q$ of the displacement of the nonlinear dofs can be expressed as
\[ \dot{A}_{pq} Z_q = \ddot{F}_q \cdot d_q \]  

(19)

where, \( \dot{A}_{pq} = A_{pq} - \lambda_{pq} \lambda_{pq}^{-1} A_{pq} \), \( \ddot{F}_q = F_q - \lambda_{pq} \lambda_{pq}^{-1} F_p \). The Netwon-Raphson method can be applied to solving Eq. (19) to obtain \( Z_q \) with a huge saving in computational cost. Consequently, the Fourier coefficients \( Z_p \) of the linear dofs can be obtained using the relation

\[ Z_p = \lambda_{pq}^{-1} (F_p - \lambda_{pq} Z_q) \]  

(20)

4. Computations and Results

4.1. Model Parameters

The three-stage expendable LM-3A is studied here, of which the configuration is shown in Fig. 5. The LM-3A is a liquid fueled launch vehicle, of which the mass values of the three stages and payload fairing are listed in Table 1 [11]. The spacecraft mounted on the launch vehicle is assumed to be a beam with equivalent length of 3.5 m and diameter of 0.4 m. When performing response analyses, both lateral and longitudinal excitations are imposed at the bottom of the launching system model.

![Figure 5. Schematic configuration of LM-3A launch vehicle (Units: m).](image)

**Table 1. Parameter of LM-3A launch vehicle (Units: kg).**

| Component          | Structural mass | Propellant mass |
|--------------------|-----------------|-----------------|
| 1st stage          | 11.23×10³       | 172.05×10³      |
| 2nd stage          | 3.561×10³       | 30.555×10³      |
| 3rd stage          | 2.742×10³       | 18.193×10³      |
| Payload fairing    | 500             | —               |

4.2. Validation of Harmonic Balance Method

Prior to investigating the steady state response of the launching system using HBM, numerical integration is undertaken to validate the HBM. A harmonic excitation with the amplitude of 1g and the frequency of 15 Hz is applied to the launching system model in the longitudinal direction. The displacement response at the top of the spacecraft and the relative displacement at the joint interface are computed by numerical integration and HBM are illustrated in Fig. 6, where harmonic terms of 3, 7 and 9 are considered for the HBM.

It can be seen from Fig. 6(a) that the spacecraft response calculated from HBM with different retained harmonic terms shows good correlation with that of the numerical integration. However,
obvious divergence from the numerical integration results can be observed for the case of \( n = 3 \) when examining the relative displacement at the joint interface shown in Fig. 6(b). Seven harmonic terms provide sufficient accuracy with sound agreement between the simulation results of the HBM and those of the numerical integration. Hence, \( n = 7 \) is adopted in the following analyses.

4.3. Steady State Response of Launching System
The frequency response curve of the launching system subjected to the lateral excitation with the amplitude of 50 kN and frequencies ranging from 1-50 Hz is shown in Fig. 7. It is revealed from Fig. 7 that for relatively lower frequencies the results calculated from the clamped model and the fixed one show little difference. As the excitation frequency increases, the response amplitude of the clamped model becomes slightly larger than that of the fixed model; whereas, along with the further increment of the excitation frequency, the response amplitude of clamped model gets smaller compared with that of the fixed model.

The response amplitudes at the top of the spacecraft under the longitudinal excitation of 1g in the frequency band of 1-50 Hz is illustrated in Fig. 8, where no significant difference between the simulation results of clamped and fixed models is observed. Though there exists relative displacement between the launch vehicle and the spacecraft in the clamped model, the relative displacement is so small compared with the response that it has little effect on the launching system response.
5. Conclusions

HBM was employed to calculate the steady state response of the launching system, where the clamp band joint is modelled as a massless beam element with nonlinear stiffnesses in both the lateral and longitudinal directions. It was revealed that the nonlinear joint stiffnesses had little influence on the longitudinal response of the launching system and affected the lateral response for relatively high frequency band.

Actually, there do exist relative displacements at the joint interface due to the nonlinear joint characteristics. Since the relative displacement is too small compared with the launching system response, its influence on the system response is insignificant. However, the nonlinearity brought by the clamp band joint may affect the stability of the launching system, which requires further investigation.

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