Joint estimation of direction of departure and direction of arrival for multiple-input multiple-output radar based on improved joint ESPRIT method

Daegun Oh¹, Ying-chun Li², Jasurbek Khodjaev¹, Jong-Wha Chong², Jong-Hun Lee¹

¹Robotics Systems, DGIST, 50-1, Sang-ri, Hyeonpung-Myeon, Dalseong-gun, Daegu, Korea
²Department of EE, Hanyang University, 222 Wangsimni-ro Seongdong-gu, Seoul, Korea
E-mail: jhlee@dgist.ac.kr

Abstract: This study presents an improved joint estimation method of signal parameters via rotational invariance technique (ESPRIT) algorithm for low-complexity simultaneous estimation of direction of departure (DOD) and direction of arrival (DOA) in a multiple-input–multiple-output radar system. The proposed algorithm is based on a data matrix and estimates DOD and DOA without a pairing operation. The computational complexity of the proposed joint ESPRIT algorithm is derived to be less than that of conventional two dimension multiple signal classification (2D-MUSIC) and reduced dimension multiple signal classification. The authors’ simulation results demonstrate that the proposed algorithm achieves performance very close to that of 2D-MUSIC and better performance than that of the ESPRIT algorithm.

1 Introduction

Multiple-input multiple-output (MIMO) radar has recently become the subject of increasing research interest because of its potential advantages, which can be obtained by utilising multiple antennas to simultaneously transmit diverse waveforms and receive reflected signals for high-resolution radar processing. Generally, any radar can be regarded as a MIMO radar if it detects targets by transmitting multiple signals separated in the temporal, spectral and/or spatial domains and similarly receiving such signals [1]. Recent work on MIMO radar can be grouped into two classes according to their antenna configurations. One class is configured with its antennas separated far away from each other [2], whereas the other is configured with its antennas co-located for coherent transmission and detection, which may be referred to as coherent MIMO radar [3–6].

Furthermore, based on the placement geometry of the transmitting and receiving antennas, coherent MIMO radar can be distributed into two classes, namely, mono-static MIMO radar [3, 4] and bi-static MIMO radar [5, 6]. In this paper, we focus on bistatic MIMO radar.

In bistatic MIMO radar, the transmitting antennas are positioned at quite different locations from the receiving antenna locations, but they are collocated to each other. Thus, the directions of a target towards all transmitting antennas, which are called direction of departure (DOD) are almost the same in the far-field scenario. Likewise, the directions of the target towards all receiving antennas, which are called direction of arrival (DOA), are almost the same in the far-field scenario.

DOD and DOA estimation algorithms for MIMO radar have been recently investigated in [7–12]. The estimation method of signal parameters via rotational invariance technique (ESPRIT) algorithm, which exploits the shift-invariant structure, was used for angle estimation in MIMO radar systems [7, 8]. Multiple signal classification (MUSIC), which exploits noise subspace for parameter estimation, has been also used in DOA estimation with MIMO radar systems as in [9, 10]. For joint estimation of DOD and DOA, it has been proved that the two dimension MUSIC (2D-MUSIC) algorithm can be used with MIMO radar systems. However, the requirement of two dimension (2D) pseudo-spectrum searching leads to much higher computational complexity as in [11, 12]. At this time, multi-dimensional pseudo-spectrum searching makes it almost impossible for the 2D MUSIC algorithm to track targets in real-time because of heavy computational complexity. Thus, for fast tracking without loss of estimation precision, a computationally efficient joint DOD and DOA estimation algorithm is required for bi-static radar. For this reason, a computationally efficient joint DOD and DOA estimation algorithm without pairing operations was proposed in [13] as an alternative to 2D algorithms. Then, reduced dimension (RD)-MUSIC was proposed in [14] for low-complexity joint DOD and DOA estimation in MIMO radar systems. Although the RD-MUSIC algorithm incurs less computational burden than 2D MUSIC, the opportunity to reduce complexity exists since MUSIC-type algorithms require iterative searching operations to detect spectral peaks.

In this paper, we propose an improved joint ESPRIT algorithm for joint estimation of DOD and DOA with
auto-pairing and compare the computational complexity of the proposed algorithm with those of 2D-MUSIC, RD-MUSIC and the conventional ESPRIT.

2 Signal model of a multiple-input–multiple-output radar system

Consider a MIMO radar system with two uniform linear arrays for transmitting and receiving, as shown in Fig. 1. We assume that $M$ and $N$ elements are arranged with half-wavelength spacing between adjacent antennas for transmitting and receiving arrays, respectively. When there are $K$ targets, the output of the matched filters at the receiver is derived in [14], as

$$y(t) = [a(\phi_0) \otimes b(\theta_0), a(\phi_1) \otimes b(\theta_1), \ldots, \times a(\phi_{K-1}) \otimes b(\theta_{K-1})]f(t) + n(t)$$

(1)

where the steering vectors $a(\phi_k)=[1, \exp(-j\pi\sin(\phi_k)), \ldots, \exp(-j\pi(M-1)\sin(\phi_k))], b(\theta_k)=[1, \exp(-j\pi\sin(\theta_k)), \ldots, \exp(-j\pi(N-1)\sin(\theta_k))]$ contain the DOD $\phi_k$ and DOA $\theta_k$ for the $k$th target with respect to the transmitting array normal and the receiving array normal, respectively. In (1), $f(t) = [f_0(t), f_1(t), \ldots, f_{K-1}(t)]^T$ is the column vector containing $K$ sinusoids because of Doppler frequency $f_d(t) = \alpha_k \exp(-j2\pi f_d t)$, where $\alpha_k$ and $\beta_k$ denote the amplitude and frequency, respectively. The $MN \times 1$ noise vector $n(t)$ is assumed to be the independent, zero-mean complex Gaussian distribution with variance $\sigma_n^2$. The Kronecker product is denoted by $\otimes$. Assuming $L$ snapshots, the data matrix can be formed by stacking the sample vectors according to

$$D = [y(0), y(T_s), \ldots, y((L-1)T_s)]$$

(2)

In (2), when the data matrix is formed, the number of snapshots $L$ must be greater than or equal to $K$. Since $K$ targets are assumed, the signal subspace also should have $K$ independent column vectors to span the signal subspace. However, if $L < K$, it is impossible to derive $K$ dimensional signal subspace from $D$. In (1), the Doppler frequency is involved in the output of the matched filter. The Doppler frequency causes the initial phase in each column of $D$ to be changed; on the other hand, the phase relations between DOD and DOA within the columns are not affected. Since the proposed joint ESPRIT algorithm is based on the column space, the effect of the Doppler frequency is ignored as in [14].

3 Data model

The data matrix $D$ is defined to have $L$ column vectors of $MN$ elements as in (2), and the column vector $y(t)$ is defined as the sum of the Kronecker products of the DOD-induced steering vector and DOA-induced steering vector for $K$ targets in (1). Thus, considering no additive white Gaussian noise (AWGN), the matrix $D$ can be factorised in terms of the matrices associated with the DOD, DOA and Doppler frequency, such that

$$D = PRQ$$

(3)

where (see (4))

$$e_k = \exp(-j\pi \sin(\phi_k)), \quad \kappa_k = \exp(-j\pi \sin(\theta_k))$$

(5)

The factorisation, like (3), with an assumption of no AWGN has been derived based on the factorisation model for matrix pencil in [15]. In (4), the DOD-induced, DOA-induced and Doppler-induced phase shifts are defined by

$$v_0 = \exp(-j2\pi \beta_0 T_s)$$

respectively. As revealed in (3)–(4), the DOD-induced and DOA-induced phase shifts are combined in the columns of $P$. Thus, based on the factorisation of (3)–(4), the column space of $D$ is used for joint DOD and DOA estimation, respectively.

In the preceding section, we mentioned that the Doppler frequency is ignored in DOD and DOA estimation of the proposed method. In (3)–(4), it is shown that the Doppler frequency for each of $K$ targets is involved in $Q$ as $K$ row vectors. Since the proposed method exploits the DOD-induced and DOA-induced phase shifts within $P$, the effect of Doppler frequencies can be ignored for the basis of the ESPRIT.
proposed method. Now, in the presence of AWGN, applying singular value decomposition (SVD) with the matrix \( D \), \( D \) is rewritten in terms of signal and noise subspaces, such that

\[
D = U\Sigma V^T \quad \text{where} \quad U = [U_s, U_n],
\]

\[
\Sigma = \begin{bmatrix}
\Sigma_s & 0 \\
0 & \Sigma_n
\end{bmatrix} \quad \text{and} \quad V = [V_s, V_n]
\]

In (6), the sub-matrices \( U_s, \Sigma_s \) and \( V_s \) are associated with the signal subspace, and the sub-matrices \( U_n, \Sigma_n \) and \( V_n \) are associated with the noise subspace. The signal subspace and noise subspace are defined from [16] by

\[
D = U_s\Sigma_s V_s^T + U_n\Sigma_n V_n^T
\]

Practically, the separation between the sub-matrices for signal subspace \( (U_s, \Sigma_s \) and \( V_s) \) and the sub-matrices for noise subspace \( (U_n, \Sigma_n \) and \( V_n) \) cannot be done automatically with SVD. For the classification, like (7), between signal subspace and noise subspace, the derived singular values in \( \Sigma \) are used in a specific manner using the criterion of the minimum descriptive length of [17]. The subspace classification is illustrated in Fig. 2.

As proved in [16, 18], the steering matrix \( P \) of (4) and signal subspace in column space \( U_s \) can be related to each other with an assumption that the signal subspace is estimated correctly, such that

\[
U_s = PT^{-1}
\]

where \( T \) is a non-singular \( K \) by \( K \) matrix. Thus, based on the transformation relationship of (8), the phase shift-invariant structure in \( P \) for each of the DOD and DOA parameters can be used with \( U_s \). In \( P \), there are \( K \) columns of \( MN \) length arranged in conjunction with DOD-oriented and DOA-oriented phase shifts, which can be rewritten by

\[
P = \begin{bmatrix}
a_{0,0}b(\theta_0) & a_{1,0}b(\theta_0) & \cdots & a_{K-1,0}b(\theta_{K-1}) \\
a_{0,1}b(\theta_0) & a_{1,1}b(\theta_0) & \cdots & a_{K-1,1}b(\theta_{K-1}) \\
\vdots & \vdots & \ddots & \vdots \\
a_{0,M-1}b(\theta_0) & a_{1,M-1}b(\theta_0) & \cdots & a_{K-1,M-1}b(\theta_{K-1})
\end{bmatrix}
\]

(9)

where \( a_{m,n} \) denotes the \( m \)th element of \( a(\theta_0) \) and \( b(\theta_0) = [1, \exp(-j\pi\sin(\theta_0)), \ldots, \exp(-j\pi(N-1)\sin(\theta_0))]^T \). Now, two pairs of selection matrices \( \{J_0, J_1\} \) and \( \{J_2, J_3\} \) are defined for the generation of sub-matrices from \( U_s \) as

\[
J_0 := [I_M \otimes (I_N - 1)] \otimes I_N, \quad J_1 := [0_M \otimes (I_{M-1} - 1)] \otimes I_N
\]

\[
J_2 := I_M \otimes I_N, \quad J_3 := I_M \otimes I_N
\]

(10)

The selection matrices are multiplied by \( U_n \), yielding four sub-matrices \( U_0, U_1, U_2 \) and \( U_3 \), such that

\[
U_0 = J_0 U_s, \quad U_1 = J_1 U_s, \quad U_2 = J_2 U_s, \quad U_3 = J_3 U_s
\]

(11)

Now, to exploit the shift-invariant structure of \( P \), the four sub-matrices are also defined using the selection matrices, such that

\[
P_0 = J_0 P, \quad P_1 = J_1 P, \quad P_2 = J_2 P, \quad P_3 = J_3 P
\]

(12)

Then, it can be established from the definition of \( P \) in (4), with \( P_0 \) and \( P_1 \) for \( K \) DOD parameters, that

\[
P_1 = P_0 \Phi \quad \text{where} \quad \Phi = \text{diag}\left(\left[\exp(-j\pi\sin(\theta_0))\right]_k^{k=K-1}\right)
\]

(13)

In the same way, it can be established from the definition of \( P \) in (4), with \( P_2 \) and \( P_3 \) for \( K \) DOA parameters, that

\[
P_3 = P_2 \Theta \quad \text{where} \quad \Theta = \text{diag}\left(\left[\exp(-j\pi\sin(\theta_0))\right]_k^{k=K-1}\right)
\]

(14)

Next, we can make use of the relationships in (13) and (14) for DOD and DOA parameters for the sub-matrices \( U_0, U_1 \), \( U_2 \) and \( U_3 \) based on the transformation of (8). Then, the DOD and DOA parameter estimations become equivalent to the eigenvalue decomposition (EVD) with \( \{U_0, U_1\} \) and \( \{U_2, U_3\} \), respectively, such that

\[
E_{\phi} = U_0^\dagger U_1 = T\Phi T^{-1} \quad \text{and} \quad E_{\theta} = U_2^\dagger U_3 = T\Theta T^{-1}
\]

(15)

where the superscript \( \dagger \) denotes the Moore–Penrose pseudo inverse. For joint estimation of \( \phi \) and \( \theta \) without a pairing process, 2D-based shift-invariant algorithms, such as 2D-MUSIC and 2D-ESPRIT, can be employed based on the relationship in (15). However, high computational
complexity is required for 2D algorithms because of the joint diagonalisation [19, 20] needed for auto-pairing of \( \phi_k \) and \( \theta_k \).

4 Improved joint estimation method of signal parameters via rotational invariance technique algorithm

In this paper, we are interested in an algebraic method by which \( \phi_k \) and \( \theta_k \) can be jointly estimated as a pair without high computational complexity. As shown in (15), two matrices \( \hat{E}_\phi \) and \( \hat{E}_\theta \) are jointly diagonalisable via the non-singular matrix \( \hat{T} \). Thus, iterative joint diagonalisation methods [19, 20] and multi-dimensional joint parameter estimation techniques, such as those in [18, 21], can be employed. However, for the proposed algorithm, the joint estimation of \( \phi_k \) and \( \theta_k \) can be achieved without the use of joint diagonalisation and multi-dimensional searching.

The key steps for the proposed joint ESPRIT algorithm are the following:

**Step 1:** Operate SVD of the data matrix \( D \) and derive \( \hat{U}_k \) involving \( K \) singular vectors spanning the signal subspace.

**Step 2:** Extract \( \hat{U}_0, \hat{U}_1, \hat{U}_2 \) and \( \hat{U}_3 \) from \( \hat{U}_k \) as in (11) using the selection matrices \( J_0, J_1, J_2 \) and \( J_3 \).

**Step 3:** Construct the matrices \( \hat{E}_\phi \) and \( \hat{E}_\theta \) as

\[
\hat{E}_\phi = \hat{U}_0^\dagger \hat{U}_1 \quad \text{and} \quad \hat{E}_\theta = \hat{U}_2^\dagger \hat{U}_3
\]

In (16), the matrices \( \hat{E}_\phi \) and \( \hat{E}_\theta \) are perturbed by AWGN. So, there cannot be the ideal transformation matrix \( \hat{T} \), which can jointly diagonalise \( \hat{E}_\phi \) and \( \hat{E}_\theta \), such that

\[
\hat{E}_\phi = \hat{T} \hat{\Phi} \hat{T}^{-1} \quad \text{and} \quad \hat{E}_\theta = \hat{T} \hat{\Theta} \hat{T}^{-1}
\]

Thus, instead of trying to estimate the ideal \( \hat{T} \) of (17), the proposed method makes use of \( \hat{E}_\phi \) and \( \hat{E}_\theta \) in separate ways for joint DOD and DOA estimation.

\[
\hat{T}_\phi = \hat{U}_0^\dagger \hat{U}_1 \quad \text{and} \quad \hat{T}_\theta = \hat{U}_2^\dagger \hat{U}_3
\]

Step 4: Based on (13) and (15), the EVD on \( \hat{E}_\phi \) gives

\[
\hat{E}_\phi = \hat{T}_\phi \hat{\Phi}_\phi \hat{T}_\phi^{-1}
\]

where \( \hat{\Phi}_\phi = \text{diag} \left[ \lambda_{\phi,k} \right]_{k=0}^{K-1} \) and

\[
\lambda_{\phi,k} = \exp \left(-j \pi \sin(\phi_k) \right) + \xi_{\phi,k}
\]

In (18), \( \xi_{\phi,k} \) denotes the noise contribution. Through EVD of \( \hat{E}_\phi \), the \( K \) eigenvalues containing DOD parameters and the corresponding eigenvectors \( \hat{T}_\phi \) are obtained as a pair. If there is no perturbation because of AWGN in \( \hat{E}_\phi \), the estimated eigenvectors \( \hat{T}_\phi \) from \( \hat{E}_\phi \) can diagonalise not only \( \hat{E}_\phi \) but also \( \hat{E}_\theta \) correctly without distortion based on the relationship of (15). Using this relationship, we can achieve paired DOA estimation for \( \hat{E}_\theta \) using the pre-estimated eigenvectors in \( \hat{T}_\phi \), such that (see (19)).

\[
\hat{E}_\theta = \hat{T}_\theta \hat{\Theta}_\theta \hat{T}_\theta^{-1}
\]

where \( \hat{\Theta}_\theta = \text{diag} \left[ \lambda_{\theta,k} \right]_{k=0}^{K-1} \) and

\[
\lambda_{\theta,k} = \exp \left(-j \pi \sin(\theta_k) \right) + \xi_{\theta,k}
\]

Step 5: In step 4, we apply EVD with \( \hat{E}_\phi \). In this step, we apply EVD with \( \hat{E}_\theta \) as in step 4. The procedures in this step are the same as the step 4 with the replacement of \( \hat{E}_\phi \) with \( \hat{E}_\theta \).

Applying the EVD with \( \hat{E}_\theta \)

\[
\hat{E}_\theta = \hat{T}_\theta \hat{\Theta}_\theta \hat{T}_\theta^{-1}
\]

where \( \hat{\Theta}_\theta = \text{diag} \left[ \lambda_{\theta,k} \right]_{k=0}^{K-1} \) and

\[
\lambda_{\theta,k} = \exp \left(-j \pi \sin(\theta_k) \right) + \xi_{\theta,k}
\]

Step 6: Through steps 4 and 5, we obtained two pairs of DOD-induced and DOA-induced phase shifts, denoted by \( \{ \hat{\phi}_k, \lambda_{\phi,k} \} \) and \( \{ \hat{\theta}_k, \lambda_{\theta,k} \} \), respectively, for \( k = 0, \ldots, K-1 \). In step 4, the sub-matrices \( \hat{U}_0 \) and \( \hat{U}_1 \) are used with
EVD. In step 5, \( \hat{U}_2 \) and \( \hat{U}_3 \) are used with EVD in the same way. Thus, we can obtain diversity gain through averaging the paired estimates \{\( \lambda_{\phi, k}, \hat{k}_i \)\} and \{\( \hat{k}_i, \lambda_{\theta, k} \)\}, for \( k = 0, \ldots, K - 1 \). Before averaging, the estimated DOD-induced and DOA-induced phase shifts in steps 4 and 5 are transformed into the DOD and DOA estimates by

\[
\hat{\phi}_{\text{step4,}k} = \sin^{-1}(-\text{angle}(\lambda_{\phi, k})) \quad \text{and} \quad \hat{\theta}_{\text{step4,}k} = \sin^{-1}(-\text{angle}(\hat{k}_i)) \tag{22}
\]

and

\[
\hat{\phi}_{\text{step5,}k} = \sin^{-1}(-\text{angle}(\hat{\epsilon}_k)) \quad \text{and} \quad \hat{\theta}_{\text{step5,}k} = \sin^{-1}(-\text{angle}(\lambda_{\theta, k})) \tag{23}
\]

where \( \text{angle}(\cdot) \) is a function that maps the complex value onto a real-valued phase angle. Then, the final DOD and DOA estimates are given by

\[
\hat{\phi}_k = \frac{\hat{\phi}_{\text{step4,}k} + \hat{\phi}_{\text{step5,}k}}{2} \quad \text{and} \quad \hat{\theta}_k = \frac{\hat{\theta}_{\text{step4,}k} + \hat{\theta}_{\text{step5,}k}}{2} \quad \text{for} \quad k = 0, \ldots, K - 1 \tag{24}
\]

5 Cramer–Rao lower bound analysis

To derive a lower bound for any kind of the unbiased estimators has proven to be useful for performance verification. The most famous one of such lower bounds is Cramer–Rao lower bound (CRLB). CRLB provides a benchmark against the proposed method as well as the conventional methods. Moreover, it tells us the impossibility of finding an unbiased estimator whose variance is less than the bound. In this section, we derive the CRLB for the data model for MIMO radar, denoted by \( y(t) \) in (1).

In \( y(t) \) of (1), DOD and DOA parameters are related with each other via Kronecker product, such that

\[
[a(\phi_0) \otimes b(\theta_0), a(\phi_1) \otimes b(\theta_1), \ldots, a(\phi_{K-1}) \otimes b(\theta_{K-1})]
\]

Solving this Kronecker product of (25)

\[
[a(\phi_0) \otimes b(\theta_0), a(\phi_1) \otimes b(\theta_1), \ldots, a(\phi_{K-1}) \otimes b(\theta_{K-1})] = \begin{bmatrix} B & B\Phi & \cdots & B\Phi^{K-1} \end{bmatrix}
\]

where \( \Phi \) denotes DOD-induced diagonal matrix, defined in (13), and \( B \) denotes DOA-induced matrix, such that, \( B = [b(\theta_0), b(\theta_1), \ldots, b(\theta_{K-1})]^T \) and \( b(\theta_i) = [1, \exp(-j\pi \sin(\theta_i)), \ldots, \exp(-j\pi(N-1)\sin(\theta_i))]^T \). Then, the vector \( y(t) \) can be rewritten by

\[
y(p_T) = \begin{bmatrix} B & B\Phi & \cdots & B\Phi^{K-1} \end{bmatrix} f(p_T) + n(p_T)
\]

Since we assume \( K \) unknown targets as in (1), the parameters to be estimated from \( y(p_T) \) of (27) can be defined as the 2 \( K \) vector \( \kappa \)

\[
\kappa = [\phi^T \Phi^T]^T
\]

where \( \Phi = [\theta_0, \theta_1, \ldots, \theta_{K-1}]^T \) and \( \Phi = [\phi_0, \phi_1, \ldots, \phi_{K-1}]^T \).

In [18], the CRLB derivation for multiple parameter estimation was performed in conjunction with joint angle and frequency estimation (JAFE). Since the data model of (27), which is represented in terms of \( \Phi \) and \( B \), has the same structure as the data model for JAFE in [18], we make use of the derived CRLB results in [18] to derive the CRLB of the data model of (27).

As we define

\[
G_\theta = \begin{bmatrix} \partial b(\theta_0) & \cdots & \partial b(\theta_{K-1}) \end{bmatrix}
\]


\[
I_\kappa(\kappa) \text{ can be defined by}
\]

\[
I_\kappa(\kappa) = \frac{\alpha_s^2}{\alpha_n^2} \text{Re} \begin{bmatrix} G_\rho(\Phi)G_\rho(\Phi)^H & G_\rho(\Phi)G_\rho(\Phi)^H \end{bmatrix}^H \tag{31}
\]

where \( \alpha_s^2 \) and \( \alpha_n^2 \) denote the signal power and noise power, respectively. The Fisher information matrix can be derived from [18] as

\[
I(\kappa) = \sum_{p=0}^{L-1} I_\rho(\kappa)
\]

\[
= \frac{\alpha_s^2}{\alpha_n^2} \text{Re} \begin{bmatrix} G_\rho(\Phi)G_\rho(\Phi)^H & G_\rho(\Phi)G_\rho(\Phi)^H \end{bmatrix} \tag{32}
\]

Then,

\[
\text{CRLB}(\kappa) = \left[I^{-1}(\kappa)\right]_{ii} \tag{33}
\]

The generalised CRLB derivation is equivalent to derive the
Fisher matrix as in [22]. Since the Fisher information matrix is derived as in (32), the CRLB for estimating the \( \eta \) parameter in \( \kappa \) is obtained from the inverse of the Fisher information matrix \( I_{\eta}(\kappa) \), as shown in (33).

### 6. Complexity analysis

The computational complexity costs for the ESPRIT and 2D MUSIC algorithms were derived to be \( O(M^3P^3) \) and \( O(M^2P^2 + M^3P^3 + n^2M^2P^2) \), where \( P \) denotes the oversampling rate in [23, 24], and \( n \) denotes the iteration number, respectively. RD-MUSIC requires \( O(M^2N^2 + M^3N + n(M^2N + M^2)(M + K^2)) \) in [14]. Costs of the individual operations are summarised in Table 1. In general, the iteration number should be much larger than \( L, M, P \), that is, \( n \gg L, M, P \), in order to achieve high-resolution estimation from the pseudo-spectrum as in [25]. This requirement is common to both 2D-MUSIC and RD-MUSIC. On the other hand, the proposed algorithm does not include an iterative operation. It consists of SVD of the data matrix, pseudo-inverses, multiplications between matrices, and EVD of \( \hat{E}_\phi \) and \( \hat{E}_\theta \). Thus, the cost of the proposed algorithm is derived from Table 1 as \( O(M^2P^2N + MPN^2 + N^3 + 2K^3(M - 1)P + K + 2(M - 1)^2P^3 + 2(M - 1)^3P^3) \approx O(MP(MPN + N^2 + K^2 + MP^2) + N^3 + K^3) \). We assumed that \( P = 1 \) for non-oversampling, and the cost of the proposed algorithm can be approximated as \( O((M, N))^3 \). When \( P = 1 \) and \( n \gg L, M, P \), the costs of 2D MUSIC, RD-MUSIC and ESPRIT are simply approximated to be \( O(n^3M^2) \), \( O(n^3M^3N) \) and \( O(M^2P^2 + M^3P^3 + K^3) \), respectively.

Comparing \( O((M, N))^3 \) with \( O(n^3M^2) \) and \( O(n^3M^3N) \), we conclude that the cost of the proposed algorithm is much less than that of 2D MUSIC or RD-MUSIC based on the requirement of \( n \gg L, M, P \). Comparing \( O((M, N))^3 \) with \( O(M^2P^2 + M^3P^3 + K^3) \), it is straightforward that the highest order for both of them are 3. Thus, it can be said that the computational costs of the proposed method and ESPRIT are similar.

### 7. Simulation results

We carried out 10 000 Monte-Carlo simulations to compare the performance of the proposed algorithm with that of ESPRIT, RD-MUSIC and 2D-MUSIC. The derived CRLB is also compared. We define the RMSE as

\[
\text{RMSE} = \left( \frac{1}{K} \sum_{k=1}^{K} \left( \frac{1}{10000} \sum_{n=1}^{10000} (\hat{\theta}_{k,n} - \theta_n)^2 \right) \right)^{1/2}
\]

where \( \hat{\theta}_{k,n} \) and \( \theta_n \) are the estimate of DOD and DOA for the \( n \)th Monte-Carlo trial, respectively. In the subsequent simulations, we assume two non-coherent sources located at angles \( (\phi_0, \theta_0) = (17^\circ, 25^\circ) \) and \( (\phi_1, \theta_1) = (20^\circ, 55^\circ) \), with \( \beta_0 = 1e3, \beta_1 = 2e3, |\alpha_0| = 1 \) with uniformly distributed phase, \( |\alpha_1| = 1 \) with uniformly distributed phase and \( T_c = 1e-6 \). In Figs. 4 to 7, we compare our algorithm with ESPRIT [8], 2D-MUSIC [11], RD-MUSIC [14] and combined MUSIC with ESPRIT algorithm of [26] in \( 3 \times 3, 4 \times 4, 5 \times 5 \) transmitting and receiving arrays. In the case of the combination of MUSIC with ESPRIT, the DOD parameter is estimated by MUSIC and the DOA parameter is estimated by ESPRIT in a specific manner. Our proposed method is labelled as Joint ESPRIT in the following figures.

Since the numbers of transmitting antennas and receiving antennas are the same, the RMSEs of DOD and DOA estimates are seen to be almost identical to each other in the figures. In low SNR areas, the proposed algorithm shows...
almost the same performance as 2D MUSIC, RD-MUSIC and the combination of MUSIC with ESPRIT, and it shows better performance than the ESPRIT algorithm. Since conventional ESPRIT cannot provide pair-wise DOD and DOA estimates for the \(k\)th target, an additional pairing process between the DOD and DOA estimates is essential for the localisation of each target. This kind of separate DOD and DOA estimation of conventional ESPRIT leads to performance degradation in comparison with the proposed algorithm, 2D-MUSIC, RD-MUSIC and the combination of MUSIC with ESPRIT, which support the auto-pairing mechanism.

The resolution of the MUSIC-based parameter estimations is affected by the iteration number \(n\). When \(n\) is small, the required complexity decreases, whereas the RMSEs of the algorithms are more likely to be saturated because of inadequate resolution in the pseudo-spectrum. We can see the effect of the number of pseudo-spectrum points \(n\) in Fig. 3.

In Fig. 3, it can be illustrated that the more points the pseudo-spectrum has, the more accurate estimation can be achieved. The shapes of the two curves are the same as each other. However, when they are sampled with different \(n\), the estimation error \(\eta_{11}\) for the first peak in the case of \(n = 11\) is larger than \(\eta_{22}\) in the case of \(n = 22\), even if the curves are the same. When \(n = 11\), the estimation error becomes \(\eta_{11} = 0.7\) for the first peak as shown in Fig. 3a. However, when the number of pseudo-spectrum points \(n\) increases to 22 for the same pseudo-spectrum, the estimation error becomes \(\eta_{22} = 0.3\), which is smaller than \(\eta_{11}\), as illustrated in Fig. 3b. This kind of estimation error occurs not because of the lack of SNR and the number of samples, but because of the iteration number \(n\). Thus, even if a high SNR and a large number of samples are given, there can be some saturation because of relatively small \(n\).

We can also observe the effect of \(n\) on RMSEs from Figs. 4–6. In Fig. 4, two coherent sources are assumed with three transmitting \((M = 3)\) and three receiving antennas \((N = 3)\), and six snapshots \((L = 6)\) are assumed. The 2D-MUSIC and RD-MUSIC algorithms show almost the same performance regardless of \(n\) with SNRs below 16 dB. However, saturation occurs only for MUSIC-type algorithms with \(n = 40\). Especially for the combination of MUSIC with ESPRIT, only the DOD parameter is estimated by MUSIC. Thus, saturation occurs in only the RMSE of the DOD parameter for the combination of MUSIC with ESPRIT when \(n = 40\), as shown in Fig. 4.

When the numbers of transmitting and receiving antennas are increased to \(M = 4\) and \(N = 4\) without any change in \(L\), the accuracy is also improved as shown in Fig. 5 because of the increased samples of \(y(t)\). Therefore the RMSEs become lower than those shown in Fig. 4. Thus, saturations appears in the both cases of \(n = 40\) and \(n = 80\) in Fig. 5 for the MUSIC-type algorithms.

When \(M = 5\) and \(N = 5\) with \(L = 6\), better DOD and DOA estimation results are obtained for the algorithms as shown in Fig. 6.

Comparing the RMSEs of the figures to each other, it can be seen that the SNR at which the saturation of the RMSEs begins differs according to \(n\) regardless of \(M\) and \(N\). From Figs. 4–6, saturation appears at RMSEs of about \(4 \times 10^{-4}\) and \(1.5 \times 10^{-4}\) for \(n = 40\) and 80, respectively, regardless of \(M\) and \(N\).

As shown in the above figures, we performed Monte-Carlo simulations using the AWGN model. Additionally, we
**Fig. 5** RMSE of DOD and DOA estimates with $M = 4$, $N = 4$ and $L = 6$

- **a** RMSEs for DOD
- **b** RMSEs for DOA

**Fig. 6** RMSE of DOD and DOA estimates with $M = 5$, $N = 5$ and $L = 6$

- **a** RMSEs for DOD
- **b** RMSEs for DOA
performed Monte-Carlo simulations using the clutter model of [27]. In our paper, the popular Weibull clutter model was used. The coherent Weibull random variable $\omega = u + jv$ was generated from a coherent Gaussian random variable via the zero-memory non-linear (ZMNL) transformations [27].

Define

$$u = x(x^2 + y^2)^{1/a-1/2} \quad \text{and} \quad v = y(x^2 + y^2)^{1/a-1/2}$$ (34)

where $x$ and $y$ are zero-mean joint Gaussian variables with variance $\sigma^2$. This transformation can be interpreted as the ZMNL transformation of the amplitude with the phase kept constant. The joint probability density function of $(u, v)$ is given by

$$p(u, v) = \frac{1}{2\pi\sigma^2} \frac{a}{(u^2 + v^2)^{(a/2-1)}} \exp\left[-\frac{1}{2\sigma^2} (u^2 + v^2)^{(a/2)}\right]$$ (35)

where $a$ is the skewness parameter of the Weibull variable.

Next, we tested the conventional methods and the proposed method using the Weibull clutter model. For $N=5$, $M=5$, $L=8$ and $n=80$, the RMSEs in the Weibull clutter model are shown in Fig. 7.

Consequently, it can be said that the proposed algorithm shows almost the same performance as the conventional RD-MUSIC and 2D-MUSIC algorithms with reduced complexity without saturation.

\section{Conclusion}

An improved joint ESPRIT algorithm was proposed for estimation of DOD and DOA with low computational complexity. The proposed algorithm reduces computational complexity and eliminates the pairing process based on the fact that the eigenvectors of DOD and DOA are shared. Furthermore, the estimation performance of the proposed algorithm is comparable with those of RD- and 2D-MUSIC.

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