Cotunneling mechanism of single-electron shuttling

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The problem of electron transport by means of a dumbbell shaped shuttle in strong Coulomb blockade regime is solved. The electrons may be shuttled only in the cotunneling regime during the time spans when both shoulders of the shuttle approach the metallic banks. The conventional Anderson-like tunneling model is generalized for this case and the tunneling conductance is calculated in the adiabatic regime of slow motion of the shuttle. Non-adiabatic corrections are briefly discussed.

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I. INTRODUCTION

The idea of inducing charge transport in nanodevices by means of special type of time-dependent manipulations with the parameters of a device was formulated theoretically as adiabatic pumping and nanoelectromechanical shuttling mechanisms. Both phenomena are related to charge transfer through a nanoobject under perturbation $V(x,t)$ periodic both in time $t$ and a coordinate $x$. In case of pumping, $x$ is not necessarily a spatial coordinate. It may be, e.g., the point on the Coulomb diamond diagram for the tunnel conductance $G(v_{g1},v_{g2})$ in double quantum dots, which circumscribes closed trajectories as a function of the gate voltages $v_{g1}$ and $v_{g2}$ applied to two dots. Not only charge but also spin density may be pumped within this paradigm. Electron-electron interaction plays essential part in the adiabatic pumping. In particular, the resonance Kondo-like tunneling is essential for the spin-charge separation of the pumping component of tunnel current (see, e.g., Ref. 7 for discussion of this problem and references therein to other theoretical investigations of adiabatic pumping in quantum dots).

In pumping devices the nanoobject is immovable, and single-electron transport is induced by time-dependent control parameters. In the nanoelectromechanical shuttling (NEMS) the charge and spin transport occurs due to an interplay of electronic and mechanical degrees of freedom: a nanoobject (or its parts) move periodically in real space between the electrodes and transport electrons from one electrode to another in the course of this motion. In the ”classical” NEMS the shuttling instability develops because the charge accumulation in the shuttle induced by electron tunneling results in an electromechanical instability of the shuttle at a sufficiently large bias voltage and periodic mechanical motion of the shuttle arises in the system. Mechanical motion of the shuttle may be also induced in the ”dot” geometry, where a nanoisland is attached to a flexible pillar or cantilever. As a rule, shuttling in these devices is substantially nonadiabatic.

Usage of flexible nanowires in the shuttling circuits opens new exciting possibilities. The flexural vibration of a suspended nanowire in combination with a scanning tunnel microscope working as one of the two electrodes may be used for realization of a charge transport in the shuttling regime. On the other hand the nanoisland may be attached to such a string. Then mechanical motion of a shuttle will be provided by excitation of vibrational eigenmodes of this string. If the Coulomb blockade in the island is strong enough, the single electron shuttling regime may be realized under certain experimental conditions. Although the shuttle motion induced by the string flexure is slow in comparison with characteristic tunneling rate, the adiabaticity is usually violated when the shuttle approaches the electrodes and moves away.

In this paper we consider the single electron transport in suspended NEMS geometry. We study the shuttle shaped like a dumbbell (double quantum dot) rocking periodically around a fixed axis. Such turnstile transports electrons by means of cotunneling mechanism. First, we formulate the problem of cyclic tunneling in general terms. Second, we calculate the periodic time-dependent conductance in the adiabatic regime. Finally, the role of non-adiabaticity will be discussed qualitatively.

II. MODEL

As is known, cotunneling mechanism is a cornerstone of the Kondo regime of tunnel conductance through quantum dots under strong Coulomb blockade. The cotunneling amplitude $J_{tr}$, which arises in the second order perturbation theory in tunneling integrals $V_{l,r}$ between the dot and the left and right metallic leads is exponentially weak, $J_{tr} \sim V_l V_r / \Delta$ ($\Delta$ is the ionization energy of quantum dot). Due to formation of many-particle Abrikosov-Suhl resonances at the Fermi level, this amplitude approaches unity at low temperatures ($T \to 0$) and the zero bias anomaly (ZBA) in quantum conductance shows up in the Coulomb diamond window.

Shuttling provides another enhancement mechanism for single electron tunneling, which exists even in the absence of Kondo screening. This mechanism is an analog of the Debye-Waller effect in neutron scattering intensity: the average distance between the dot and the leads...
effectively reduces due to periodic motion of the shuttle between the source and drain electrodes. The exponential enhancement of effective tunneling transparency is controlled by the ratio $\langle x^2 \rangle/x_0^2$, where the numerator is the average mean square displacement of the shuttle position relative to its static distance $x_0$ from the leads (the device is supposed to be symmetric). Recently we have described one more enhancement mechanism unrelated to Kondo effect, which may be realized in half-metals with the energy gap for minority spin carriers. This mechanism may be evinced as a finite bias anomaly in conductance, which arises due to opening of a resonance tunneling channel for the minority spin carriers.

In this paper we consider tunneling through a dumbbell shaped shuttle. A schematic sketch of the device is presented in Fig. 1 (left panel). We suppose that this shuttle is suspended on an elastic string oriented perpendicularly to the tunneling plane, e.g., by means of the method developed in Ref. 13. Another option is to suspend two islands on a string where the standing wave polarized in the tunneling plane is excited (Fig. 1, right panel).

Let us turn to the twisting turnstile as a more simple in practical realization object. We consider the idealized case where the equilibrium position with zero strain corresponds to the symmetric orientation of the dumbbell relative to the edges. Then the torque vibration mode may be excited in the string, which induces the in-plane rotation of the dumbbell. When the rotation angle approaches $\pi/2$, the distances between the dots and both leads become small and the tunneling amplitude increases simultaneously for the source and the drain lead (see Fig. 2). If the Coulomb blockade is strong enough, the number of electrons in the turnstile is fixed and an electron from the lead can tunnel from the source into a turnstile only provided another electron leaves the dot simultaneously. Thus effective elastic cotunneling arises as a ZBA in the tunnel conductance with strongly enhanced amplitude.

One may say that electrons tunnel between two banks in a shuttling regime, where a carrier may be injected from a bank to the moving island during the limited time span, when one of the two islands is passing this bank. In our idealized geometry this condition is satisfied by assuming that in the equilibrium position at rotation angle $\varphi = 0$ tunneling is exponentially weak and electron can jump onto the shuttle or leave it only at $\varphi \to \pm \pi/2$. Second, the characteristic frequencies of string vibrations are of the order of several MHz under the realistic experimental conditions. This means that the mechanically driven shuttle moves slowly in comparison with the characteristic times of electron motion, and the time dependent processes are adiabatic for the most part of the shuttling period. Adiabaticity may be violated only when the shuttle approaches the metallic banks and tunneling amplitudes grow exponentially with time reaching its maximum for $\varphi = \pm \pi/2$ and then again decreasing to nearly zero value. Thus the torque vibrations of a dumbbell-shape shuttle are transformed into periodically pulsing tunnel amplitudes with completely synchronized source-shuttle and shuttle-drain tunneling processes. Due to non-adiabatic character of switching, this tunneling is accompanied by formation of quasienergy levels, which contribute to the single-electron shuttling.

To separate the single electron shuttling from Kondo shuttling, we consider here spinless carriers. This approximation is valid provided the leads are fabricated from half-metals, i.e. materials in which the Fermi surface is formed by majority spin electrons, whereas the spectrum of minority spin electrons is gapped. The turnstile shuttle (TS) may be described within a frame-
work of time-dependent Anderson model with the Hamiltonian
\[ H = H_{\text{sh}} + H_L + H_R + H_{\text{tun}} \]  
(2.1)

Here the first term is Hamiltonian of the moving TS
\[ H_{\text{sh}} = \sum_{i=1,2} E_i n_i + U_{12} n_1 n_2; \]  
(2.2)

\( n_i = d_i^\dagger d_i \) is the occupation operator for the electrons in the wells of TS labeled as \( i = 1, 2 \); \( E_i \) are the corresponding discrete energy levels. Usually the time dependence is explicitly included in the shuttle coordinate, because the Pauli principle forbids double occupation of any well. We assume that the coupling potential because the Pauli principle forbids double occupation of any well. We suppose that the turnstile shuttle moves together with the acoustically excited string, and the time variable appears only in the last term in the Hamiltonian (2.1) (see below). The second term in Eq. (2.2) is the electrostatic interaction between electrons located in two potential wells. There is no need to introduce the single site Coulomb blockade potential because the Pauli principle forbids double occupation of any well. We assume that the coupling \( U_{12} \) is strong enough and the TS may be occupied only by one electron located either in the level \( E_1 \) or in the level \( E_2 \). The electrons in the left (L) and right (R) banks are represented by the Hamiltonian
\[ H_L + H_R = \sum_{\beta=L,R} \sum_\kappa \varepsilon_{\kappa} a_{\beta\kappa}^\dagger a_{\beta\kappa} \]  
(2.3)

with quasicontinuous spectra \( \varepsilon_{1\kappa} \). The last term in Eq. (2.2) is the tunneling Hamiltonian
\[ H_{\text{tun}} = \sum_{\beta\kappa} \sum_i \left[ W_{\beta,i\kappa}(t) a_{\beta\kappa}^\dagger d_i + \text{H.c.} \right]. \]  
(2.4)

Due to periodic shutting both wells of the TS enter tunneling contact with both banks. In the turnstile regime the time-dependent tunneling potential has the form of periodic pulses, \( W(t) = \sum_p w(t+pT_2) \). In a rough approximation these pulses have a rectangular shape of duration \( T_1 \) coming with the period \( T_2 \),
\[ w(t) = w_0 \left[ \theta(t + T_1/2) - \theta(t - T_1/2) \right]. \]  
(2.5)

This idealized picture of sudden approximation may be improved: if the real time-dependent tunnel integral for rotating TS is calculated, then the rectangular pulses transform into Gaussians,
\[ w(t) = w_0 \exp \left[ -\frac{t^2}{2(T_1/2)^2} \right]. \]  
(2.6)

(see Fig. 3).

Electron cotunneling through moving turnstile may be mimicked by means of time dependent tunneling through an immovable two-channel double dot shown in Fig. 4. The appropriate time-dependence of the parameters \( W_{1i}(t) \) in this construction is provided by the time-dependent gate voltages regulating the magnitude of these parameters in such a way that in the first half-period the channels \( 1L \) and \( 2R \) are open and the channels \( 1R \) and \( 2L \) are blocked. As a result we get periodic in time tunnel integrals, containing two sequences of pulses shifted by half a period (see below).

III. SINGLE ELECTRON SHUTTLING IN PERIODIC REGIME

We formulate the periodic time-dependent problem in the following way (see Fig. 2). Let the infinitesimally small bias voltage be directed from the left bank to the right bank. At some moment, say \( t = 0 \), the \emph{singly occupied} shuttle with an electron captured in the site 2 is in the equilibrium position with the rotation angle \( \varphi = 0 \). Then within a quarter period \( t = T_2/4 \) shuttle rotates to the angle \( \varphi = -\pi/2 \), the electron leaves well 2 for the right bank, thus releasing the Coulomb blockade \( U_{12} \), so that the next electron from the left bank is allowed to enter the well 1. At \( t = 3T_2/4 \) the rotation angle is \( \varphi = \pi/2 \), and cotunneling process is allowed again. At \( t = T_2 \) the shuttle loaded with one electron returns into
initial position with $\varphi = 0$ and an electron in the site 2 and the next cycle begins. Two electrons are shuttled from the left lead to the right one during one cycle.

This picture implies that only two charge states of TS are involved in cotunneling cycle, namely the one-electron states $|\Lambda\rangle = |10\rangle$ and $|\Lambda\rangle = |01\rangle$ where an electron occupies the site 1 or 2, respectively. Two other states, namely the state with empty wells $|\Lambda\rangle = |00\rangle$, and the doubly occupied state $|11\rangle$ appear only as virtual intermediate states. For the sake of simplicity we assume that the state $|11\rangle$ is completely suppressed by the Coulomb blockade potential $U_{12}$. It is worthwhile to rewrite the Hamiltonian (2.1) in terms of the operators $X^{A\Lambda'} = |\Lambda\rangle\langle\Lambda'|$. Then the first term acquires the form

$$H_{sh} = \sum_{\Lambda=0,1,2} E_\Lambda X^{A\Lambda}$$  \hspace{1cm} (3.1)

Here notation $|0\rangle = |00\rangle$, $|1\rangle = |10\rangle$, $|2\rangle = |01\rangle$ is used.

We consider a symmetric shuttle, where $E_1 = E_2$ is the energy of a singly occupied shuttle and $E_0 = 0$ is the energy of the empty TS. The tunneling Hamiltonian reads

$$H_{tun} = \sum_{\beta_{\kappa}} \sum_{i=1,2} \left[ W_{\beta_{\kappa}}(t) a_{\beta_{\kappa} i}^\dagger X^{0i} + \text{H.c.} \right].$$  \hspace{1cm} (3.2)

Four tunneling parameters, which are assumed to be $\kappa$-independent are grouped in two periodic series shifted by half period,

$$W_{L1}(t) = W_{R2}(t) = W_a(t)$$
$$W_{L2}(t) = W_{R1}(t) = W_b(t) = W_a(t - T_2/2),$$  \hspace{1cm} (3.3)

so that

$$W_a(t) = \sum_{p=0}^{+\infty} w(t + pT_2),$$
$$W_b(t) = \sum_{p=0}^{+\infty} w[t + (2p - 1)T_2/2]$$  \hspace{1cm} (3.4)

with $w(t)$ defined in Eq. (2.5) or (2.6) ($p$ is integer).

In order to describe the shuttling mechanism shown in Fig. 2 in mathematical terms, we represent the Hamiltonian $H_{tun}$ in the form of discrete Fourier series. For this sake we expand the tunneling parameters (3.3):

$$W_a(t) = \sum_{n=0}^{\infty} W_n \cos(\Omega_n t)$$
$$W_b(t) = \sum_{n=0}^{\infty} (-1)^n W_n \cos(\Omega_n t)$$  \hspace{1cm} (3.5)

where $(\Omega_n = n\Omega = 2\pi n/T_2)$. The Fourier coefficients for two types of tunneling pulses (2.5), (2.6) are

$$W_0 = w_0 \tau, \hspace{0.5cm} W_n = \frac{2w_0 \sin \pi n \tau}{\pi n}$$  \hspace{1cm} (3.6)

for the rectangular pulses and

$$W_0 = \sqrt{\frac{\pi}{2}} w_0 \tau, \hspace{0.5cm} W_n = \sqrt{2\pi} w_0 \tau \exp \left[ -\frac{(\pi n \tau)^2}{2} \right]$$  \hspace{1cm} (3.7)

for the Gaussian pulses. Here $\tau = T_1/T_2$.

Then inserting (3.5) in (3.2) we see that the tunneling Hamiltonian

$$H_{tun} = H_{tun}^{(e)} + H_{tun}^{(o)},$$  \hspace{1cm} (3.8)

is separated into two parts

$$H_{tun}^{(e)} = \sum_{n} W^{(e)}(t) a_{\kappa e}^\dagger X_e + \text{H.c.},$$
$$H_{tun}^{(o)} = \sum_{n} W^{(o)}(t) a_{\kappa o}^\dagger X_o + \text{H.c.}$$  \hspace{1cm} (3.9)

containing the contributions of the even and odd harmonics,

$$W^{(e)}(t) = \sum_{m=0}^{\infty} 2W_{2m} \cos(\Omega_{2m} t),$$
$$W^{(o)}(t) = \sum_{m=0}^{\infty} 2W_{2m+1} \cos(\Omega_{2m+1} t).$$  \hspace{1cm} (3.10)

Even and odd creation operators for the lead and dot electrons are defined as

$$a_{\kappa e}^\dagger = \frac{1}{\sqrt{2}} (a_{\kappa L}^\dagger + a_{\kappa R}^\dagger), \hspace{0.5cm} a_{\kappa o}^\dagger = \frac{1}{\sqrt{2}} (a_{\kappa L}^\dagger - a_{\kappa R}^\dagger),$$
$$X_e^\dagger = \frac{1}{\sqrt{2}} (X^{10} + X^{20}), \hspace{0.5cm} X_o^\dagger = \frac{1}{\sqrt{2}} (X^{11} - X^{21})$$  \hspace{1cm} (3.11)

which implies

$$X_e^\dagger X_e = X^{ec} = \frac{1}{2} (X^{11} + X^{22} + X^{12} + X^{21}),$$
$$X_o^\dagger X_o = X^{oo} = \frac{1}{2} (X^{11} + X^{22} - X^{12} - X^{21}),$$
$$X_e^\dagger X_o = X^{eo} = \frac{1}{2} (X^{11} - X^{22} - X^{12} + X^{21}),$$
$$X_o^\dagger X_e = X^{oe} = \frac{1}{2} (X^{11} - X^{22} + X^{12} - X^{21})$$  \hspace{1cm} (3.12)

The Hamiltonian of symmetric shuttle may be also rewritten in these variables as

$$H_{sh} = E_d (X^{ec} + X^{oo})$$  \hspace{1cm} (3.13)

with $E_d = E_1 = E_2$. A similar transformation for the symmetric leads ($\varepsilon_{\kappa L} = \varepsilon_{\kappa R} = \varepsilon_{\kappa}$) transforms the Hamiltonian (2.3) into

$$H_{lead} = \sum_{\kappa} \varepsilon_{\kappa} (a_{\kappa e}^\dagger a_{\kappa e} + a_{\kappa o}^\dagger a_{\kappa o}).$$  \hspace{1cm} (3.14)
As a result the Fourier transformed Hamiltonian of periodic shuttling is reduced to the system (3.8), (3.13), (3.14). Any Fourier harmonic \( W_n \) is a result of averaging the sequence of periodic pulses \( w(t) \) with the corresponding exponent. In the case of sudden rectangular pulses the convergence of Fourier harmonics (3.10) is very poor, so that the whole series should be taken into account, and even in this case the problem of convergence known as the Gibbs defect remains. In the case of more realistic Gaussian pulses the Fourier harmonics (3.7) fall exponentially, so that the whole series should be taken into account, where the convergence of the series in the Hamiltonian (3.8) is much better,

\[
W_{n+2}/W_n = \exp[-\pi^2\tau^2(2n + 2)]
\]

The parameter \( \tau \) may be estimated in a simplified model, where the torque mode is imitated by rotation of a sphere with the radius \( d \) along the orbit with the radius \( R \), so that the distance between the leads equals \( 2(R + d) \). In this approximation \( \tau = d/\pi \sqrt{R(d + R)} \), so that the dwelling time \( T_1 \) of the shuttle near the bank is controlled only by the geometrical factor \( d/R \). In the torque regime the interval \( T_1 \) is longer because the turnstile slows down around the turning point.

A. Time-dependent canonical transformation for shuttling Hamiltonian

Having in mind the approximate adiabaticity of shuttling transport, we use the method of time-dependent canonical transformation \( \tilde{L} \) for derivation of cotunneling Hamiltonian as well as for calculation of current operator (see also Ref. [24], where the time-independent canonical transformation diagonalizing the Anderson impurity Hamiltonian in the mean-field approximation was proposed).

As is shown in Ref. [19], the tunneling terms may be ousted from the time-dependent Schrödinger operator \( \mathcal{L} = -i\hbar \partial /\partial t + \tilde{H} \) with \( \tilde{H} = H_e + H_o \) in Eqs. (3.8) – (3.13) by means of the transformation matrix \( e^{S_s(t) + S_o(t)} \) applied to the field operator \( A^\dagger \) with components (3.11). To eliminate the tunneling terms from the Schrödinger equation defined by the operator

\[
\tilde{L} = -i\hbar \frac{\partial}{\partial t} + \tilde{H} ,
\]

one should take \( \tilde{H} \) in the following form

\[
\tilde{H} = e^{S_s + S_o} (H_e + H_o) e^{-S_s - S_o} + S_1 ,
\]

\[
S_1 = i\hbar \int_0^1 d\epsilon \lambda\epsilon (S_s + S_o) \left( S_e + S_o \right) e^{-\lambda (S_s + S_o)}
\]

with

\[
S_p = \sum_\kappa \left( u_{pc} X_p \right) a_{pc} - \text{H.c.}
\]
into $H_{\text{cotun}}(t) = H_p + H_\tau$ with

$$H_p(t) = H_{LL} + H_{RR} =$$

$$\frac{1}{2} \sum_{\kappa \kappa'} (J_{\kappa \kappa'}^{ee} X^{ee} + J_{\kappa \kappa'}^{oo} X^{oo}) (a^\dagger_{L\kappa} a_{L\kappa'} + a^\dagger_{R\kappa} a_{R\kappa'}) -$$

$$\frac{1}{2} \sum_{\kappa \kappa'} (J_{\kappa \kappa'}^{eo} X^{oe} + J_{\kappa \kappa'}^{oe} X^{eo}) (a^\dagger_{L\kappa} a_{R\kappa'} - a^\dagger_{R\kappa} a_{L\kappa'})$$

$$H_\tau(t) = H_{LR} + H_{RL} =$$

$$\frac{1}{2} \sum_{\kappa \kappa'} (J_{\kappa \kappa'}^{ee} X^{ee} - J_{\kappa \kappa'}^{oo} X^{oo}) (a^\dagger_{L\kappa} a_{R\kappa'} + a^\dagger_{R\kappa} a_{L\kappa'}) +$$

$$\frac{1}{2} \sum_{\kappa \kappa'} (J_{\kappa \kappa'}^{eo} X^{oe} - J_{\kappa \kappa'}^{oe} X^{eo}) (a^\dagger_{L\kappa} a_{R\kappa'} - a^\dagger_{R\kappa} a_{L\kappa'})$$

Since only the states near the Fermi level with $\varepsilon_\kappa \sim \varepsilon_F$ contribute to the current at small enough $\Omega$, we neglect the $\kappa \kappa'$ dependence in the exchange integrals.

By construction the difference $J^{ee} - J^{oo}$ stems from interlacing of the components $W_0(t)$ and $W_\delta(t)$ in the SW transformation. Namely, the 2$m$-th harmonic of this difference is

$$J^{ee} - J^{oo} = \frac{4}{\Delta} \sum_{m=0}^{\infty} W_0(t) W_\delta(t) \cos \Omega_{2m+1} \cos \Omega_{2m\ell},$$

where the bar denotes averaging over the shuttling period. One can check by inspection of Eqs. (3.4), (2.6) and Fig. 3 that these averages are exponentially small. Correspondingly, the sum $J^{ee} + J^{oo}$ contains only the averaged products $W_0^2(t) = W_\delta^2(t)$. These averages give the main contribution to the electron shuttling and we keep them in the tunneling current. Below we will be interested only in a few first harmonics of tunneling current, which stem from the cotunneling parameters

$$J^{ee} = \frac{8W_0^2 + 4W_\delta^2}{\Delta} \cos 2\Omega t,$$

$$J^{oo} = \frac{4W_\delta^2}{\Delta} (1 + \cos 2\Omega t),$$

$$J^{eo} = \frac{8W_0 W_\delta + 4W_\delta W_\delta}{\Delta} \cos \Omega t.$$  

In these expressions only zero to second harmonics in Eq. (3.10) are taken into account. The higher orders terms $\sim \cos n\Omega t$ may be obtained within the same procedure.

When deriving this Hamiltonian, multiplication rules (3.12) have been used.

Before turning to calculation of the tunneling current, we simplify the effective Hamiltonian in order to retain only the terms responsible for the electron cotunneling though the turnstile. First, we omit the term $H_p(t)$ (3.22) containing creation of electron-hole pairs in the left and right leads. These terms are essential in the problems where the infrared orthogonality catastrophe modifies the current (Kondo or edge singularity mechanism). In the absence of such channels they yield only higher order corrections to the shuttling current. The remaining term $H_\tau(t)$ rewritten in the original variables reads

$$H_\tau(t) = \frac{1}{4} \sum_{\kappa \kappa'} [(J^{ee} - J^{oo})(X^{11} + X^{22}) + (J^{ee} + J^{oo})(X^{12} + X^{21})] (a^\dagger_{L\kappa} a_{R\kappa'} + a^\dagger_{R\kappa} a_{L\kappa'})$$

$$+ \frac{1}{4} \sum_{\kappa \kappa'} (J^{eo} + J^{oe})(X^{12} - X^{21}) (a^\dagger_{L\kappa} a_{R\kappa'} - a^\dagger_{R\kappa} a_{L\kappa'})$$

The three lowest harmonics of the tunneling potential $W(t)$ contribute also to the diagonal part of the transformed shuttle Hamiltonian

$$\tilde{H}_\text{sh} = \tilde{E}_d^{ee} X^{ee} + \tilde{E}_d^{oo} X^{oo}$$

with

$$\tilde{E}_d^{ei}(t) = E_d - D_\rho(t) \sum \frac{1}{\Delta_{\kappa}},$$

$$D_e = 4W_0^2 + 2W_\delta^2 + 8W_0 W_\delta \cos 2\Omega t$$

$$D_o = 2W_\delta^2 (1 + \cos 2\Omega t).$$

The periodic in time perturbation (3.28) results in a reconstruction of the energy spectrum of the shuttle, which may be described in terms of quasienergies, $E_m = E_d^p + m\Omega$ (see, e.g., Ref. [21]). These states are also involved in the tunneling current, but the corresponding tunneling channels open only in the higher than fourth order correction in $W_{\beta i}$.

**IV. ELECTRON TRANSPORT THROUGH A TURNSTILE SHUTTLE**

We start discussion of shuttling mechanism with general remarks. It should be emphasized that the TS can transport electrons preferably in one direction only provided its mirror symmetries $1 \leftrightarrow 2$ and/or $L \leftrightarrow R$ are slightly violated. Indeed, if the state $|2\rangle$ with the electron in the site 2 is chosen as the initial state, and the cycle shown in Fig. 2 is realized, then the electron charge
is transported from the left to the right. If the configuration \( |1\rangle \) is chosen as the initial state, then the same cycle results in the electron transport from the right to the left. Another way to change the current direction is to shift to \( \pi \) the phase of torque vibration. Certainly, two directions of single-electron cotunneling are equivalent if the mirror symmetry is perfect, so that the charge transport through the TS arises as a response to violation of this symmetry, whatever is the microscopic mechanism of this violation.

To find the tunnel current through the turnstile, one should calculate the time-dependent probabilities of the transitions from the initial state at \( t = t_0 \) to the states with \( 1, 2, \ldots 2M \) electron-hole pairs in the leads at \( t = MT \) in accordance with Fig. 3. In the problems of this type, one should distinguish between the weak coupling and strong coupling regimes. In the weak coupling regime the tunnel conductance is a superposition of elastic ZBA and the set of inelastic replica, in which the finite bias anomalies (FBA) arise because of excitations of higher harmonics of the periodic coupling strength (or higher Floquet states in terms of quasienergies)\(^{20,21}\). In the strong coupling regime the whole Floquet spectrum is involved in the time evolution of the system, determined by the evolution operator

\[
U(t_0, t) = |\Psi(t)\rangle\langle\Psi(t_0)|. \tag{4.1}
\]

This problem is quite complicated because the highly excited Floquet states belonging to continuum are involved in tunneling and the relaxation channels, which open due to Auger-type processes of multiple creation of electron-hole pairs with finite energy and accompany the electron cotunneling. Leaving this regime for future studies, we consider here only the perturbative regime.

The current operator is defined as

\[
\hat{I} = e\frac{1}{2}(\hat{N}_R - \hat{N}_L) \tag{4.2}
\]

where \( \hat{N}_\beta = \frac{d}{dt}(\sum_x a_\beta^\dagger a_\beta) \). Calculation of these derivatives by means of equations of motion gives for \( \hat{I} \) following equation

\[
\hat{I} = \frac{ie\hbar}{4\omega} \sum_{\kappa, \kappa'} (J^{ee} + J^{oo}) (X^{12} + X^{21}) \left( a_{L\kappa}^\dagger a_{R\kappa} - a_{R\kappa}^\dagger a_{L\kappa} \right) + \frac{ie\hbar}{4\omega} \sum_{\kappa, \kappa'} (J^{co} + J^{oe}) (X^{12} - X^{21}) \left( a_{L\kappa}^\dagger a_{R\kappa} + a_{R\kappa}^\dagger a_{L\kappa} \right). \tag{4.3}
\]

Having in mind the exponential smallness of the terms \( \propto (J^{ee} - J^{oo}) \) discussed in the previous section we have kept in this equations only the terms corresponding to the processes shown in Fig. 2 where the electron transfer from the left to the right bank or vice versa is accompanied by repopulation of the two "seats" in the shuttle.

We calculate the tunneling conductance \( G(\omega) \) arising as a response to weak periodic in time bias \( eV(t) \) by means of the Kubo-Greenwood formula

\[
G(\omega, t) = \frac{-i}{\hbar \omega} \left[ D(t, \omega) - D(t, \omega = 0) \right] \tag{4.4}
\]

Here retarded Green function \( D(t, t') \) and its Fourier transform are defined as

\[
D(t, t') = -i \langle G | [\hat{I}(t), \hat{I}(t')] | G \rangle \Theta(t - t') \]
\[
D(t, \omega) = \int dt' \Theta(t' - t) e^{-i\omega(t' - t)} \tag{4.5}
\]

The initial "pure" state \( |G\rangle \) is defined as a state with one electron in one of the two sites (say, 2) and the filled Fermi spheres in the bank electron reservoirs. This state is doubly degenerate:

\[
|G\rangle_{e,o} = |\bar{b}\rangle (|1\rangle \pm |2\rangle) / \sqrt{2}
\]

Here \( |\bar{b}\rangle \) stands for the filled Fermi spheres in the leads. When choosing the Kubo-Greenwood equation in the form \( A \frac{d}{dt} \), we have taken into account the fact that the time-dependent terms are present both in the Hamiltonian of unperturbed itinerant electrons in the two banks \( E, \omega \) with applied bias

\[
H_b(t) = H_R + H_L - \mu(N_R + N_L) - eV(t) \left( \frac{N_R - N_L}{2} \right) \tag{4.6}
\]

and in the tunneling Hamiltonian \( H_{\text{tun}} \).

The leading time independent contribution to the current comes from the \( n = 0 \) harmonic of the shuttling current \( J^{ee(0)}/4 \) and is proportional given by the time-independent component \( \sim J^{ee(0)}/4 \) in the first term of Eq. \( 1.20 \). By means of Eq. \( 1.3 \) and \( 3.22 \) we obtain the conventional expression for the zero bias conductance

\[
G_0 = \frac{2\pi e^2}{h} \rho_F^2 |J^{ee(0)}/4|^2, \tag{4.7}
\]

where \( \rho_F \) is the electron density of states in the leads.

The prefactor 2 in this spinless model appeared due to the specific charge transfer mechanism in a turnstile shuttle:
unlike the standard shuttle with a single "seat", the two-seat shuttle transfers two electrons within a period, as is seen from Fig. 2.

Equation (4.7) determines the time-averaged background of a tunnel current through a turnstile. It is important that the coupling constant \( J_{\text{exc}(0)} \) contains the maximal values of tunneling matrix elements \( w_0 \) corresponding to the closest contact between the shuttle and metallic banks, although the coupling strength is essentially weakened due to the factor \( \tau \) which characterizes the dwelling time of the shuttle in the nearest vicinity of the banks [see Eq. (3.7)]. Taking into account corrections given by the first harmonic \( J_{\text{exc}(1)} \) we obtain the average conductance

\[
\overline{G} = \pi e^2 / 8h \rho_F^2 J_{\text{exc}(0)}^2 [1 + 4e^{-(\pi \tau)^2}].
\] (4.8)

To find full time-dependent conductance let us rewrite Eq. (4.3) for the current operator in the form

\[
\hat{I}(t) = i e \hbar \left[ I_0 (X^{12} + X^{21}) + I_2 (X^{12} + X^{21}) \cos 2\Omega t \right] \sum_{\kappa} \sum_{\kappa'} \left( a_{L\kappa}^\dagger a_{R\kappa} - a_{R\kappa}^\dagger a_{L\kappa} \right) \\
+ i e \hbar I_1 (X^{12} - X^{21}) \cos \Omega t \sum_{\kappa} \sum_{\kappa'} \left( a_{L\kappa}^\dagger a_{R\kappa} + a_{R\kappa}^\dagger a_{L\kappa} \right)
\] (4.9)

with

\[
I_0 = \frac{2W_0^2 + W_1^2 + W_2^2}{\Delta}, \\
I_1 = \frac{4W_0 W_1 + 2W_1 W_2}{\Delta}, \\
I_2 = \frac{W_1^2 + 4W_0 W_2}{\Delta}.
\] (4.10)

Substitution of (3.7) yields

\[
I_0 = \frac{\pi (w_0 \tau)^2}{\Delta} [1 + 2e^{-y} + 2e^{-4y}], \\
I_1 = \frac{4\pi (w_0 \tau)^2}{\Delta} e^{-y/2} [1 + e^{-2y}], \\
I_2 = \frac{2\pi (w_0 \tau)^2}{\Delta} e^{-y} [1 + 2e^{-y}].
\] (4.11)

with \( y = (\pi \tau)^2 \) (see (2.10)). The parameter \( \exp(-y) \) controls the smallness of the contribution of higher harmonics.

Operators \( X^{12} \) describing elastic processes with zero frequency do not contribute to the cotunneling dynamics and give the combinations \( X^{11} \pm X^{22} \) in the current correlation functions in (3.3). After averaging these combinations yield the factors 1 and 0, respectively in the resulting equations for the conductance. Inserting Eqs. (3.10), (4.10) into Eq. (4.3) and then into Eq. (4.4) we calculate the electron-hole loops for itinerant carriers in the standard way and get finally

\[
G(0, t) = \frac{e^2}{h} \rho_F^2 \left[ \left( I_0^2 - \frac{I_1^2}{2} \right) + 2 \left( I_0 I_2 - \frac{I_2^2}{4} \right) \cos 2\Omega t \right].
\] (4.12)

This equation describes adiabatic two-electron shuttling. We see that the oscillating motion of a turnstile with the frequency \( \Omega \) results in an appearance of periodic component with the frequency \( 2\Omega \) in the time-dependent conductance. Higher harmonics \( 2m\Omega \) in the odd modes and \( (2m + 1)\Omega \) in the coupling constants (3.20) may also be taken into account. These harmonics result in an appearance of the higher even harmonics \( \propto \cos 2n\Omega \) with smaller amplitudes in oscillating conductance. An example of oscillating conductance including 4 harmonics is shown in Fig. 5.

In principle, we may consider nonadiabatic effects as well. One may take into account weak non-adiabatic corrections, which stem from the terms \( \tilde{v}_{ee}(t) \) and \( \tilde{v}_{oh}(t) \) in Eq. (3.13). These corrections may be observed as phase shifts in the cosine functions. Truly non-adiabatic effects arise when the contact between the shuttle and the
banks results in reconstruction of the spectra of electrons in the shuttle due to a fast enough periodic passage of two islands forming a turnstile along the banks. Such reconstruction may be treated in terms of quasienergies, which are just a realization of the Floquet theorem for the time-periodic systems. In this case the energy conserves in the process of cotunneling under the perturbation with the period \( T_i = 2\pi/\Omega \) to within the “Umklapp” processes, \( E_f = E_i + m\Omega \), where \( E_i,f \) are the initial and final states of cotunneling act. As a result, the satellites \( \sim 2n\hbar\Omega \) should arise in the conductance as finite bias anomalies. These satellites cannot be incorporated in the above calculation scheme. Development of an appropriate non-perturbative description is beyond the framework of the present paper.

V. CONCLUSIONS

Our analysis of periodic shuttling within a time-dependent Anderson model with periodic tunneling perturbing odd and even modes of the shuttle and bath subsystems has shown that the shuttle works as a harmonic analyzer, which transforms the input signal \( E \) into the output adiabatic signal \( E(\pm\Omega) \). In the approximate equation (4.12) only two first harmonics \( \Omega \) and \( 2\Omega \) are taken into account. Numerical estimates presented in Fig. 5 show that even in the case of large enough parameter \( \exp(-y) \) the higher harmonics only weakly perturb the basic features of the effect, namely permanent background \( \propto I_0^2 \), where the magnitude of cotunneling strength is controlled by the parameter \( \tau^4 \) [see Eq. (4.11)], and the periodic temporal oscillations with the leading harmonic \( 2\Omega \).

We have found three realizations of this model, namely the turnstile suspended on an elastic string in two configurations (Fig. 1) and a double quantum dot with gate-controlled tunneling parameters \( W_{ij} \) (Fig. 1). In principle turnstile configurations may be realized also in molecular motors. In this paper we considered only the charge transport induced by electrical bias, but the ratchet-like turnstiles could also be proposed.

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