Mass Spectrum of Three-Pion System in Kaluza-Klein Picture

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Abstract

In this note we present additional arguments in favour of Kaluza and Klein picture of the world. In fact, we show that formula (1) provided by Kaluza-Klein approach with the fundamental scale early calculated [1] gives an excellent description for the mass spectrum of three-pion system.

In our previous papers [1, 2] we have presented the arguments in favour of that the Kaluza-Klein picture of the world has been been observed in the experiments at very low energies where the nucleon-nucleon dynamics has been studied. In particular we have found that geniusly simple formula for KK excitations provided by Kaluza-Klein approach gives an excellent description for the mass spectrum of two-nucleon system. In article [3] we have presented additional arguments in favour of Kaluza and Klein picture of the world. In fact, we have shown that simple formula provided by Kaluza-Klein approach with the fundamental scale early calculated [1] gives an excellent description for the mass spectrum of two-pion system. Surely, this was quite an event and, certainly, this very nice fact encouraged us to continue the study of the other hadronic systems in this respect.

Taking this line we have performed an analysis of experimental data on mass spectrum of the resonance states of three-pion system and compared them with calculated values provided by Kaluza-Klein scenario. In this note we present the results of this analysis.

As in the previous cases let us build the Kaluza-Klein tower of KK excitations for three-pion system by the formula

\[ M_{\pi^1\pi^2\pi^3} = \sqrt{m_{\pi^1}^2 + \frac{n_1^2}{R^2}} + \sqrt{m_{\pi^2}^2 + \frac{n_2^2}{R^2}} + \sqrt{m_{\pi^3}^2 + \frac{n_3^2}{R^2}}, \quad (n = 1, 2, 3, \ldots), \]  

(1)

where \( \pi^i (i = 0, +, -) = \pi^0, \pi^+, \pi^- \) and \( R \) is the same fundamental scale calculated early from the analysis of nucleon-nucleon dynamics at low energies [1, 2]

\[ \frac{1}{R} = 41.481 \text{ MeV} \quad \text{or} \quad R = 24.1 \text{ GeV}^{-1} = 4.75 \times 10^{-13} \text{ cm}. \]  

(2)

Kaluza-Klein tower such built is shown in Table 1 where the comparison with experimentally observed mass spectrum of three-pion system is also presented.
Table 1: Kaluza-Klein tower of KK excitations of three-pion system and experimental data.

| n | $M^{3\pi''}_n$ MeV | $M^{\pi\pm2\pi''}_n$ MeV | $M^{\pi\pm2\pi}_n$ MeV | $M^{3\pi\pm}_n$ MeV | $M^{3\pi}_{exp}$ MeV |
|---|---|---|---|---|---|
| 1 | 423.62 | 428.02 | 432.42 | 436.81 | \(\eta(0^{-+})[547]\) |
| 2 | 475.30 | 479.23 | 483.17 | 487.10 | |
| 3 | 550.77 | 554.17 | 557.57 | 560.98 | \(\eta'(0^{-+})[958]\) |
| 4 | 641.68 | 644.60 | 647.53 | 650.46 | |
| 5 | 742.38 | 744.91 | 747.44 | 749.98 | |
| 6 | 849.40 | 851.61 | 853.83 | 856.05 | |
| 7 | 960.62 | 962.58 | 964.55 | 966.51 | |
| 8 | 1074.75 | 1076.51 | 1078.26 | 1080.02 | |
| 9 | 1190.95 | 1192.53 | 1194.12 | 1195.70 | 1194 ± 14 |
| 10 | 1308.66 | 1310.10 | 1311.55 | 1312.99 | 1311.3±1.6 |
| 11 | 1427.51 | 1428.84 | 1430.16 | 1431.49 | 1419 ± 31 |
| 12 | 1547.25 | 1548.47 | 1549.69 | 1550.91 | |
| 13 | 1667.68 | 1668.81 | 1669.94 | 1671.08 | 1667 ± 4 |
| 14 | 1788.65 | 1789.71 | 1790.76 | 1791.82 | 1801 ± 13 |
| 15 | 1910.07 | 1911.06 | 1912.05 | 1913.04 | |
| 16 | 2031.86 | 2032.79 | 2033.72 | 2034.65 | 2030 ± 50 |
| 17 | 2153.95 | 2154.83 | 2155.70 | 2156.58 | 2090 ± 30 |
| 18 | 2276.29 | 2277.12 | 2277.95 | 2278.78 | |
| 19 | 2398.85 | 2399.64 | 2400.43 | 2401.22 | |
| 20 | 2521.69 | 2522.35 | 2523.10 | 2523.85 | |
| 21 | 2644.50 | 2645.22 | 2645.93 | 2646.65 | |
| 22 | 2767.54 | 2768.23 | 2768.91 | 2769.59 | |
| 23 | 2890.71 | 2891.36 | 2892.02 | 2892.67 | |
| 24 | 3013.97 | 3014.60 | 3015.23 | 3015.86 | |
| 25 | 3137.33 | 3137.94 | 3138.54 | 3139.14 | |
| 26 | 3260.78 | 3261.36 | 3261.94 | 3262.52 | |
| 27 | 3384.29 | 3384.85 | 3385.41 | 3385.97 | |
| 28 | 3507.87 | 3508.41 | 3508.95 | 3509.49 | |
| 29 | 3631.51 | 3632.03 | 3632.55 | 3633.08 | |
| 30 | 3755.21 | 3755.71 | 3756.21 | 3756.72 | |
Table 2: $M_9(1191-1196)$–Storey.

| $R(I^G J^P C)$ | $M_R$ MeV | $\Gamma_R$ MeV | Reaction | Collab. |
|----------------|-----------|----------------|----------|---------|
| $h_1(0^{-1+})$ | 1190 ± 60 | 320 ± 50       | $\pi p \to 3\pi n$ | SPEC 81 |
| $\pi(1^{-0}+)$ | 1190 ± 30 | 440 ± 80       | $\pi^+ Z \to Z3\pi$ | SPEC 84 |
| $a_1(1^{-1+})$ | 1194 ± 14 | 462 ± 56       | $\tau^+ \to \pi^+\pi^+\pi^-\nu$ | MRK 86 |
| $\omega(1^{-1+})$ | 1208 ± 15 | 430 ± 50       | $pp \to pp\pi^+\pi^-\pi^0$ | OMEG 90 |

Table 3: $M_{10}(1309-1313)$–Storey.

| $R(I^G J^P C)$ | $M_R$ MeV | $\Gamma_R$ MeV | Reaction | Collab. |
|----------------|-----------|----------------|----------|---------|
| $\pi(1^{-0}−)$ | 1342 ± 20 | 220 ± 70       | $\pi^- p \to p3\pi$ | OMEG 81 |
| $a_1(1^{-1+})$ | 1280 ± 30 | 300 ± 50       | $\pi^- p \to p3\pi$ | CNTR 81 |
| $\omega(1^{-1+})$ | 1285 – 1331 | 619–814 | $\tau^- \to \nu_\tau[3\pi]^-$ | CLEO 99 |
| $\omega_2(1^{-2+})$ | 1317 ± 3  | 120 ± 10       | $pp \to pp\pi^+\pi^-\pi^0$ | WA102 98 |
|                | 1311.3±1.6| 103.0±6.0      | $\pi^- p \to \pi^+\pi^-\pi^0 n$ | VES 96 |
|                | 1310 ± 5  | 120 ± 10       | $pp \to pp\pi^+\pi^-\pi^0$ | OMEG 90 |
|                | 1317 ± 2  | 96 ± 9         | $\pi^- p \to 3\pi p$ | SPEC 80 |
|                | 1318 ± 7  | 112 ± 18       | $\pi^+ n \to p(3\pi)^0$ | DBC 75 |
|                | 1305 ± 14 | 120 ± 40       | $\gamma p \to \eta\pi^+\pi^-\pi^-$ | SHF 93 |
|                | 1310 ± 2  | 97 ± 5         | $\pi^- p \to 3\pi p$ | OMEG 81 |
|                | 1306 ± 4  | 79 ± 12        | $\pi^+ p \to 3\pi p$ | HBC 70 |

We have used Review of Particle Physics [4] where the experimental data on mass spectrum of the resonance states of three-pion system have been extracted from. Again we see from Table 1 that there is a quite remarkable correspondence of the calculated KK excitations for three-pion system with the experimentally observed mass spectrum of three-pion resonance states, which we consider as an additional strong evidence of Kaluza-Klein picture of the world.

Some known experimental information concerning the experimentally observed mass spectrum of three-pion system is collected in separate tables: Table 2 – Table 8. Certainly, here we have a much more poor experimental data set compared to the case of two-pion system. Nevertheless we can learn from these tables a few remarkable facts as well.

First of all, as it was mentioned in our previous paper [3], many different three-pion resonances with the different quantum numbers may occupy one and the same storey in KK tower. This is a peculiarity of the systematics provided by Kaluza-Klein picture.

Table 4: $M_{11}(1428-1432)$–Storey.

| $R(I^G J^P C)$ | $M_R$ MeV | $\Gamma_R$ MeV | Reaction | Collab. |
|----------------|-----------|----------------|----------|---------|
| $\omega(0^{-1--})$ | 1400^{+100}_{-200} | 187 ± 15 | $e^+e^- \to \pi^+\pi^-\pi^0$ | RVUE 98 |
|                | 1419 ± 31 | 174 ± 59       | $e^+e^- \to \rho\pi$ | DM2 92 |
Table 5: \(M_{13}(1668 - 1671)\)-Storey.

| \(R(I^GJ^{PC})\) | \(M_R\) MeV | \(\Gamma_R\) MeV | Reaction | Collab. |
|------------------|-------------|---------------|-----------|---------|
| \(\omega(0^{-1}--)\) | 1670 ± 20 | 160 ± 20 | \(\gamma p \rightarrow 3\pi X\) | OMEG 83 |
|                  | 1679 ± 34 | 99 ± 49 | \(e^+e^- \rightarrow 3\pi\) | FRAM 80 |
|                  | 1652 ± 17 | 42 ± 17 | \(e^+e^- \rightarrow 3\pi\) | OSPK 79 |
| \(\omega_3(0^{-3}--)\) | 1665.3±5.2 | 149±19 | \(\pi^- p \rightarrow \pi^+\pi^-\pi^0n\) | VES 96 |
|                  | 1673 ± 12 | 173 ± 16 | \(\pi^+ p \rightarrow \Delta 3\pi\) | HBC 78 |
|                  | 1650 ± 12 | 253 ± 39 | \(\pi^- p \rightarrow N3\pi\) | OMEG 78 |
|                  | 1669 ± 11 | 173 ± 28 | \(\pi^+ p \rightarrow \Delta^{++}3\pi\) | HBC 75 |
|                  | 1678 ± 14 | 167 ± 40 | \(\pi^+ n \rightarrow p3\pi^0\) | DBC 74 |
|                  | 1679 ± 13 | 155 ± 40 | \(\pi^+ n \rightarrow p3\pi^0\) | DBC 71 |
|                  | 1670 ± 20 | 100 ± 40 | \(\pi^+ n \rightarrow p3\pi^0\) | DBC 69 |
| \(\pi_2(1^{-2}+)\) | 1667 ± 4 | 168 ± 10 | AVERAGE | PDG 00 |
|                  | 1676 ± 6 | 260 ± 20 | \(\pi^- p \rightarrow 3\pi p\) | OMEG 81 |
|                  | 1657 ± 14 | 219 ± 20 | \(\pi p \rightarrow 3\pi X\) | SPEC 80 |
|                  | 1662 ± 10 | 285 ± 60 | \(\pi^+ p \rightarrow p3\pi\) | HBC 77 |
|                  | 1672 ± 3.5 | 259 ± 11 | AVERAGE | PDG 00 |

Table 6: \(M_{14}(1789 - 1792)\)-Storey.

| \(R(I^GJ^{PC})\) | \(M_R\) MeV | \(\Gamma_R\) MeV | Reaction | Collab. |
|------------------|-------------|---------------|-----------|---------|
| \(a_2(1^{-2}++)\) | 1752±21±4 | 150±110±34 | \(\gamma\gamma \rightarrow \pi^+\pi^-\pi^0\) | L3 97 |
| \(X(1^{-?}++)\) | 1763±20 | 192±60 | \(\gamma p \rightarrow n\pi^+\pi^-\pi^-\) | SHF 91 |
|                  | 1787±18 | 118±60 | \(\gamma p \rightarrow (p\pi^+)(\pi^+\pi^-\pi^-)\) | SHF 91 |
| \(\pi(1^{-0}+)\) | 1776±13 | 155±40 | AVERAGE | PDG 00 |
|                  | 1775±7 | 190±15 | \(\pi^- A \rightarrow \pi^+\pi^-\pi^-A\) | VES 95 |
|                  | 1770 ± 30 | 310 ± 50 | \(\pi^- A \rightarrow 3\pi A\) | SPEC 82 |
|                  | 1801 ± 13 | 210 ± 15 | AVERAGE | PDG 00 |

It is especially pleased for us to emphasize that the experimental measurement of the \(a_2\) meson mass made by Protvino VES Collaboration 5 with the best world precision

\[ M(a_2) = 1311.3 \pm 1.6\text{(stat)} \pm 3.0\text{(syst)} \text{MeV} \]

is in excellent agreement with the theoretically calculated value

\[ M_{\pi^+\pi^-\pi^0} = 1311.55 \text{MeV}. \]

The same is true for the \(\omega_3\) meson where the theoretically calculated mass of KK excitation in \(3\pi^0\) system \(M_{13}^{\omega_3} = 1667.68 \text{MeV}\) is in a very good agreement with PDG AVERAGE value \(M(\omega_3) = 1667 \pm 4 \text{MeV}\). Moreover, it is very interesting to point out that theoretical calculation of KK excitations in \(\rho\pi\) system by the formula

\[ M_n^{\rho\pi} = \sqrt{m_\rho^2 + \frac{n^2}{R^2}} + \sqrt{m_\pi^2 + \frac{n^2}{R^2}}, \quad (n = 1, 2, 3, \ldots), \quad (3) \]
where we use $m_\rho = 769.3 \text{MeV}$ for the $\rho$ meson mass from [4], gives $M_{10}^{\rho \pi^0} = 1310.28 \text{MeV}$ and $M_{10}^{\rho \pi^\pm} = 1311.67 \text{MeV}$ which accurately agree with the experimental measurement of the $a_2$ meson mass provided by VES Collaboration. This means that $a_2$ meson may manifest itself as a configuration of $\rho \pi$ system in the main, and this is a very nontrivial fact. For example, that statement is not true for the $\omega_3$ meson.

Of course, it would be very desirable to state new experiments to search a further justification of the systematics provided by Kaluza and Klein picture of the world, e.g. to fill the empty cells in Table 1. We believe that this is a quite promising subject of the investigations in particle and nuclear physics.

References

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