SUSY QCD corrections to the polarization and spin correlations of top quarks produced in $e^+e^-$ collisions

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Abstract:
We compute the supersymmetric QCD corrections to the polarization and the spin correlations of top quarks produced above threshold in $e^+e^-$ collisions, taking into account arbitrary longitudinal polarization of the initial beams.

Keywords: top quarks, supersymmetry, polarization, radiative corrections

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1 Introduction

A future linear $e^+e^-$ collider will be an excellent tool to search for and investigate extensions of the Standard Model (SM) of particle physics \[1\]. One particularly attractive extension of the SM is Supersymmetry (SUSY) \[2\], which solves several conceptual problems of the SM. Apart from their direct production, also virtual effects of SUSY particles may lead to observable deviations from the SM expectations. In particular, top quark pair production at a linear collider may be a sensitive probe of such effects. Very high energy scales are involved in the production and decay of top quarks. Moreover, since they decay very quickly, the spin of top quarks is not affected by hadronization effects and becomes an additional observable to probe top quark interactions. At a future linear $e^+e^-$ collider, the electron (and possibly also the positron) beam may have a substantial longitudinal polarization, which will be an asset to study top quark spin phenomena. We therefore study in this paper the impact of virtual effects of SUSY particles on spin properties of $t\bar{t}$ pairs in $e^+e^-$ collisions. We restrict ourselves here to the SUSY QCD sector of the Minimal Supersymmetric Standard Model (MSSM). SUSY QCD corrections to the (spin-summed) differential cross section for $e^+e^-\rightarrow t\bar{t}$ have already been studied quite some time ago \[3\], and we extend these results by keeping the full information on the $t\bar{t}$ spin state. The full MSSM corrections to the spin-summed differential cross section have been calculated in \[4\].

In section 2 we define the spin observables that we calculate in this paper and also discuss how they can be measured. Section 3 gives analytic results for these observables, and section 4 contains numerical results for specific choices of the SUSY QCD parameters. In section 5 we present our conclusions.

2 Spin observables

We consider the reaction

$$e^+(p_+,\lambda_+) + e^-(p_-,\lambda_-) \rightarrow (\gamma',Z^\ast) \rightarrow t(k_t) + \bar{t}(k_{\bar{t}}) + X,$$  \hspace{1cm} (1)

where $\lambda_-$ ($\lambda_+$) denotes the longitudinal polarization of the electron (positron) beam\(^1\). Within the Standard Model, spin effects of top quarks in reaction (1) have been analysed first in ref. \[5\]. QCD corrections to the production of top quark pairs, including the full information about their spins, can be found in refs. \[6, 7\]. Fully analytic results for the top quark polarization \[8\] and a specific spin correlation \[9\] to order $\alpha_s$ are also available.

The top quark polarization is defined as two times the expectation value of the top quark spin operator $S_t$. The operator $S_t$ acts on the tensor product of the $t$ and $\bar{t}$ spin spaces and is given by $S_t = \frac{\sigma_2}{2} \otimes 1$\(_L\), where the first (second) factor in the tensor product refers to the $t$ ($\bar{t}$) spin space. (The spin operator of the top antiquark is defined by $S_{\bar{t}} = 1 \otimes \frac{\sigma_2}{2}$. )

\(^1\)For a right-handed electron (positron), $\lambda_+ = +1$.\n
taken with respect to the spin degrees of freedom of the $t\bar{t}$ sample described by a spin density matrix $R$, i.e.

$$P_t = 2 \langle S_t \rangle = 2 \frac{\text{Tr}[RS_t]}{\text{Tr}R}. \tag{2}$$

For details on the definition and computation of $R$, see e.g. [6]. The polarization of the top antiquark $P_{\bar{t}}$ is defined by replacing $S_t$ by $S_{\bar{t}}$ in (2). For top quark pairs produced by CP invariant interactions, we have $P_t = P_{\bar{t}}$. The spin correlations between $t$ and $\bar{t}$ can be calculated by using the matrix

$$C_{ij} = 4 \langle S_{t,i}S_{\bar{t},j} \rangle = 4 \frac{\text{Tr}[RS_{t,i}S_{\bar{t},j}]}{\text{Tr}R}. \tag{3}$$

Using arbitrary spin quantization axes $\hat{a}$ and $\hat{b}$ for the $t$ and $\bar{t}$ spins, the spin correlation with respect to these axes is given by

$$c(\hat{a}, \hat{b}) = \frac{\hat{a}_i C_{ij} \hat{b}_j - (P_t \cdot \hat{a})(P_{\bar{t}} \cdot \hat{b})}{\sqrt{1 - (P_t \cdot \hat{a})^2} \sqrt{1 - (P_{\bar{t}} \cdot \hat{b})^2}}. \tag{4}$$

The directions $\hat{a}, \hat{b}$ can be chosen arbitrarily. Different choices will yield different values for the spin correlation $c(\hat{a}, \hat{b})$. The spin properties of the top quarks and antiquarks can be measured by analysing the angular distributions of the $t$ and $\bar{t}$ decay products. For example, if both $t$ and $\bar{t}$ decay semileptonically, $t \rightarrow b\ell^+\nu_\ell, \bar{t} \rightarrow b\ell'^-\bar{\nu}_{\ell'}$, the following double differential lepton angular distribution is sensitive to the $t\bar{t}$ spin state:

$$\frac{1}{\sigma} \frac{d^2\sigma}{d\cos\theta^+ d\cos\theta^-} = \frac{1}{4} (1 + B_1 \cos\theta^+ + B_2 \cos\theta^- - C \cos\theta^+ \cos\theta^-), \tag{5}$$

with $\sigma$ being the cross section for the channel under consideration. In Eq. (5), $\theta^+ (\theta^-)$ denotes the angle between the direction of flight of the lepton $\ell^+ (\ell'^-)$ in the $t (\bar{t})$ rest frame and the chosen spin quantization axis $\hat{a}$ ($\hat{b}$). The coefficients $B_{1,2}$ and $C$ are related to the mean (averaged over the scattering angle) $t (\bar{t})$ polarization and spin correlation projected onto the directions $\hat{a}$ and $\hat{b}$. Using the double pole approximation [10] for the $t$ and $\bar{t}$ propagators, one obtains for the so-called factorizable contributions [11] [12]

$$B_1 = \kappa_+ \overline{P_t \cdot \hat{a}},$$

$$B_2 = -\kappa_- \overline{P_{\bar{t}} \cdot \hat{b}},$$

$$C = \kappa_+ \kappa_- \overline{\hat{a}_i C_{ij} \hat{b}_j}, \tag{6}$$

where the overline indicates the average over the scattering angle, e.g.

$$\overline{P_t \cdot \hat{a}} = \frac{\int_{-1}^{1} dy \text{Tr}[RS_t \cdot \hat{a}]}{\int_{-1}^{1} dy \text{Tr}R}, \tag{7}$$

etc., where $y$ is the cosine of the top quark scattering angle. In (6), $\kappa_\pm$ is the spin analysing power of the charged lepton $\ell^\pm$. At leading order, $\kappa_\pm = \pm 1$. QCD corrections to this result are at the per mill level [13]. SUSY QCD corrections to the spin analysing power $\kappa_\pm$ are exactly zero [14].
3 Analytic results

We now turn towards the calculation of the SUSY QCD corrections to the polarization and spin correlations of top quark pairs produced in $e^+e^-$ collisions. These corrections directly determine the SUSY QCD corrections to the double lepton distribution \( \kappa_{\pm} \) within the double pole approximation, since the corrections to the LO result \( \kappa_{\pm} = +1 \) are exactly zero and the non-factorizable contributions due to SUSY particles also vanish within that approximation.

The amplitude for reaction (1) including SUSY QCD corrections may be written as follows:

\[
i T_{fi} = i \frac{4 \pi \alpha}{s} \left\{ \chi(s) \bar{v}(p_+) \left( g_V \gamma_\mu - g_A \gamma_\mu s_5 \right) u(p_-) H_{Z\gamma}^\mu - \bar{v}(p_+) \gamma_\mu u(p_-) H_\gamma^\mu \right\},
\]

(8)

where \( g_V^e = -\frac{1}{2} + 2 \sin^2 \theta_W \) and \( g_A^e = -\frac{1}{2} \), with \( \theta_W \) denoting the weak mixing angle. The function \( \chi \) is given by

\[
\chi(s) = \frac{1}{4 \sin^2 \theta_W \cos^2 \theta_W} \frac{s}{s - m_Z^2},
\]

(9)

where \( m_Z \) stands for the mass of the Z boson. We neglect the Z width, since we work at lowest order in the electroweak coupling and the c.m. energy is far above \( m_Z \). The hadronic currents have a formfactor decomposition as follows:

\[
H_{Z\gamma}^\mu = \bar{u}(k_t) \left[ V_{Z\gamma}^\mu - A_{Z\gamma}^\mu \gamma_5 + S_{Z\gamma} \frac{(k_t - k_s)^\mu}{2m_t} \right] \gamma_\mu v(k_t)
\]

(10)

with

\[
V_{Z\gamma} = V_{Z\gamma}^0 + V_{Z\gamma}^1,
\]

\[
A_{Z\gamma} = A_{Z\gamma}^0 + A_{Z\gamma}^1.
\]

(11)

In (11), \( V_{Z\gamma}^0 = Q_t \), where \( Q_t \) denotes the electric charge of the top quark in units of \( e = \sqrt{4 \pi \alpha} \), \( A_{Z\gamma}^0 = 0 \), and \( V_{Z\gamma}^1 = g_V^e = \frac{1}{2} - \frac{1}{4} \sin^2 \theta_W \), \( A_{Z\gamma}^1 = g_A^e = \frac{1}{2} \) are the tree level vector- and the axial-vector couplings of the top quark to the Z boson.

The one-loop SUSY QCD contributions to the different form factors are denoted by \( V_{Z\gamma}^1, A_{Z\gamma}^1 \) and \( S_{Z\gamma} \). Scalar and pseudoscalar couplings proportional to \( (k_t + k_s)^\mu \) and \( (k_t + k_s)^\mu \gamma_5 \) have been neglected in (8), since they induce contributions proportional to the electron mass. In addition CP violating formfactors proportional to \( (k_t - k_s)^\mu \gamma_5 \) are possible in SUSY QCD through a complex phase in the squark mass matrices. In [15] it has been shown that the dependence of the cross section on these phases is weak and that CP odd asymmetries are typically of the order of \( 10^{-3} \). We therefore set these phases to zero in the following. To make this paper self-contained we list the form factors \( V_{Z\gamma}^1, A_{Z\gamma}^1 \) and \( S_{Z\gamma} \) in the appendix. We have performed an analytic comparison to the corresponding results in [3] and found complete agreement.

We define

\[
f_{LL(LR)} = -Q_t + \chi(g_V^e + g_A^e)(g_V^e \pm g_A^e),
\]

\[
f_{RR(RL)} = -Q_t + \chi(g_V^e - g_A^e)(g_V^e + g_A^e),
\]

(12)
and

\[ P_\pm = 1 - \lambda_- \lambda_+ \pm (\lambda_- - \lambda_+). \] (13)

The electroweak couplings that enter the Born results are then given by

\[ g_{VV}^\pm = \frac{1}{8} [P_+(f_{RR} + f_{RL})^2 \pm P_-(f_{LL} + f_{LR})^2], \]

\[ g_{AA}^\pm = \frac{1}{8} [P_+(f_{RR} - f_{RL})^2 \pm P_-(f_{LL} - f_{LR})^2], \]

\[ g_{VA}^\pm = \frac{1}{8} [P_+(f_{RR} - f_{RL})^2 \pm P_-(f_{LL} - f_{LR})^2]. \] (14)

Likewise, defining

\[ g_{LL(LR)} = \chi (g_{VV}^\mp + g_{AA}^\mp) (V_1^L \pm A_1^L) - (V_1^L \pm A_1^L), \]

\[ g_{RR(RL)} = \chi (g_{VV}^\mp - g_{AA}^\mp) (V_1^L \pm A_1^L) - (V_1^L \pm A_1^L), \]

\[ s_{L(R)} = \chi (g_{VV}^\mp \pm g_{AA}^\mp) s_{SS} - s_{\gamma}. \] (15)

the SUSY QCD contributions may be written in terms of the following quantities:

\[ h_{VV}^\pm = \frac{1}{8} [P_+(f_{RR} + f_{RL})(g_{RR} + g_{RL}) \pm P_-(f_{LL} + f_{LR})(g_{LL} + g_{LR})], \]

\[ h_{AA}^\pm = \frac{1}{8} [P_+(f_{RR} - f_{RL})(g_{RR} - g_{RL}) \pm P_-(f_{LL} - f_{LR})(g_{LL} - g_{LR})], \]

\[ \text{Re} \ h_{VA}^\pm = \frac{1}{8} \text{Re} [P_+(f_{RR} g_{RR} - f_{RL} g_{RL}) \pm P_-(f_{LL} g_{LL} - f_{LR} g_{LR})], \]

\[ \text{Im} \ h_{VA}^\pm = \frac{1}{8} \text{Im} [P_+(f_{LR} g_{RR} - f_{RL} g_{RL}) \pm P_-(f_{LL} g_{LL} - f_{LR} g_{LR})], \]

\[ s_V^\pm = \frac{1}{4} [P_+(f_{RR} + f_{RL}) s_R \pm P_-(f_{LL} + f_{LR}) s_L], \]

\[ s_A^\pm = -\frac{1}{4} [P_+(f_{RR} - f_{RL}) s_R \pm P_-(f_{LL} - f_{LR}) s_L]. \] (16)

It is convenient to write the results in terms of the electron and top quark directions \( \hat{p} \) and \( \hat{k} \) defined in the c.m. system, the cosine of the scattering angle \( y = \hat{p} \cdot \hat{k} \), the scaled top quark mass \( r = 2m_t/\sqrt{s} \), and the top quark velocity \( \beta = \sqrt{1 - r^2} \).

The differential cross section including the SUSY QCD corrections reads:

\[
\frac{d\sigma}{dy} = \frac{d\sigma^0}{dy} + \frac{d\sigma^\pm}{dy} = \sigma_{pt} \frac{3N_C\beta}{8} \left\{ 2 - \beta^2(1 - y^2) \right\} \left( g_{VV}^\pm + 2\text{Re} \ h_{VV}^\pm \right) \\
+ \beta^2(1 + y^2) \left( g_{AA}^\pm + 2\text{Re} \ h_{AA}^\pm \right) + 4\beta y \left( g_{VA}^\pm + 2\text{Re} \ h_{VA}^\pm \right) - 2\beta^2(1 - y^2)\text{Re} \ s_V^\pm \right\}, \] (17)
where

\[ \sigma_{pt} = \frac{4\pi\alpha^2}{3s}, \]  

(18)

and \( d\sigma^0/dy \) is obtained by setting \( h_{VV}^+ = h_{AA}^+ = h_{VA}^+ = s_{\nu}^+ = 0 \). We further introduce a vector perpendicular to \( \mathbf{k} \) in the production plane \( \mathbf{k}^\perp = \mathbf{\hat{p}} \times \mathbf{\hat{k}} \) and a vector normal to this plane, \( \mathbf{n} = \mathbf{\hat{p}} \times \mathbf{\hat{k}} \). The top quark polarization including the SUSY QCD corrections is equal to the top antiquark polarization and reads:

\[
\mathbf{P}_t = \mathbf{P}_t^0 + \mathbf{P}_t^1
\]

\[
= \sigma_{pt} \frac{3N_c\beta}{4} \left\{ \beta (1+y^2) (g_{VA} - 2\text{Re} h_{VA}) + y (g_{VV} + 2\text{Re} h_{VV}) + \beta^2 y (g_{AA} + 2\text{Re} h_{AA}) \right\} \mathbf{\hat{k}}
\]

\[
+ r \left[ \beta y (g_{VA} + 2\text{Re} h_{VA}) + g_{VV} + 2\text{Re} h_{VV} - \frac{\beta^2}{r^2} \left( \text{Re} s_{\nu} - \beta \text{Re} s_A \right) \right] \mathbf{k}^\perp
\]

\[
+ \left[ 2\beta y \text{Im} h_{VA} + \frac{\beta^2}{r} (y \text{Im} s_{\nu} - \beta \text{Im} s_A) \right] \mathbf{n} \left\{ \frac{d\sigma^0}{dy} \right\}^{-1} - \mathbf{P}_t^0 \frac{d\sigma^1}{dy} \left\{ \frac{d\sigma^0}{dy} \right\}^{-1}.
\]

(19)

For the matrix \( C_{ij} \) defined in (4) we find

\[
C_{ij} = C_{ij}^0 + C_{ij}^1 = \frac{1}{3} \delta_{ij} \left[ 1 + \frac{d\sigma^1}{dy} \left( \frac{d\sigma^0}{dy} \right)^{-1} \right]
\]

\[
+ \sigma_{pt} \frac{3N_c\beta}{4} \left\{ \frac{d\sigma^0}{dy} \right\}^{-1} \left( \left[ g_{VV}^+ + 2\text{Re} h_{VV}^+ - \beta^2 (g_{AA}^+ + 2\text{Re} h_{AA}^+) \right] \left[ k_i^+ k_j^+ - \frac{1}{3} \delta_{ij} (1-y^2) \right]
\]

\[
+ \left[ (y + \beta^2(1-y^2)) (g_{VV}^+ + 2\text{Re} h_{VV}^+) + \beta^2 y (g_{AA}^+ + 2\text{Re} h_{AA}^+) + 2\beta y (g_{VA}^+ + 2\text{Re} h_{VA}^+) \left[ k_i^+ k_j^+ - \frac{1}{3} \delta_{ij} \right]
\]

\[
+ 2\beta^2(1-y^2) \text{Re} s_{\nu} \left[ k_i^+ k_j^+ - \frac{1}{3} \delta_{ij} \right]
\]

\[
+ r \left[ y (g_{VV}^+ + 2\text{Re} h_{VV}^+) + \beta (g_{VA}^+ + 2\text{Re} h_{VA}^+) - \frac{\beta^2}{r^2} (y \text{Re} s_{\nu} - \beta \text{Re} s_A^+) \right] \left[ k_i^+ k_j^+ - \frac{1}{3} \delta_{ij} \right]
\]

\[
+ 2\beta \text{Im} h_{VA} \left[ k_i^+ n_j + k_j^+ n_i \right] + \beta \left[ 2y \text{Im} h_{VA} + \frac{\beta}{r} (\text{Im} s_{\nu} - \beta \text{Im} s_A^+) \right] \left[ k_i n_j + k_j n_i \right]
\]

\[
- C_{ij} \frac{d\sigma^1}{dy} \left( \frac{d\sigma^0}{dy} \right)^{-1} \right\}.
\]

(20)

The Born results \( \mathbf{P}_t^0 \) and \( C_{ij}^0 \) are obtained from (19) and (20) by setting \( h_{VA}^+ = h_{VV}^- = h_{AA}^- = s_{\nu}^- = s_A^+ = d\sigma^1/dy = 0 \).

For fully polarized electrons (or positrons) a so-called ‘optimal spin basis’ can be constructed. This is an axis \( \mathbf{\hat{d}} \) with respect to which the \( t \) and \( \bar{t} \) spins are 100% correlated at the tree level in the Standard Model for any velocity and scattering angle [16]. This axis \( \mathbf{\hat{d}} \) is the solution of the equation

\[ \mathbf{\hat{d}} \mathbf{C}_{ij}^0 \mathbf{\hat{d}} = 1. \]

(21)
One gets
\[ \hat{d} = x \hat{k} + \sqrt{1 - x^2} \hat{k}^\perp, \]  
(22)

with \( x \in [-1, 1] \) only if either \( P_+ = 0 \) or \( P_- = 0 \). For \( P_+ = 0 \), which can be realized with left-handed electrons (\( \lambda_- = -1 \)), one finds
\[ x = -\frac{f_{LL}(\beta + y) + f_{LR}(y - \beta)}{\left[(1 + y\beta)^2 f_{LL}^2 + (1 - y\beta)^2 f_{LR}^2 + 2(y^2\beta^2 + 1 - 2\beta^2)f_{LL}f_{LR}\right]^{1/2}}. \]
(23)

For right-handed electrons, the optimal basis is obtained by replacing \( f_{LL} \rightarrow f_{RR}, f_{LR} \rightarrow f_{RL} \) in Eq. (23). Note that at threshold \( \hat{d} \rightarrow 0 \), i.e. the optimal basis at threshold is defined by the direction of the beam, while in the high-energy limit \( \hat{d} \rightarrow \hat{k} \), i.e. the optimal basis coincides with the helicity basis. By analytically evaluating \( \hat{d}_i C^i_{ij} \hat{d}_j \) we find that the virtual SUSY QCD corrections to the \( t\bar{t} \) spin correlations in the optimal basis are exactly zero.

## 4 Numerical results

In this section we present numerical results for the SUSY QCD corrections to the top quark polarization and \( t\bar{t} \) spin correlations. We also include a discussion of the corrections to the differential cross section and compare our results to the literature.

We take into account the effects of mixing of the chiral components of the top squark. The stop mass matrix can be expressed in terms of MSSM parameters as follows:
\[ M_{\tilde{t}}^2 = \begin{pmatrix} M_{\tilde{Q}}^2 + m_t^2 + m_Z^2 (\frac{1}{2} - Q_s s_w^2) \cos 2\beta & m_t (A_t - \mu \cot \beta) \\ m_t (A_t - \mu \cot \beta) & M_{\tilde{U}}^2 + m_t^2 + m_Z^2 Q_s s_w^2 \cos 2\beta \end{pmatrix}, \]
(24)

where \( M_{\tilde{Q}}, M_{\tilde{U}} \) are the soft SUSY-breaking parameters for the squark doublet \( \tilde{q}_L (q = t, b) \) and the top squark singlet \( \tilde{t}_R \), respectively. Further, \( A_t \) is the stop soft SUSY-breaking trilinear coupling, and \( \mu \) is the SUSY-preserving bilinear Higgs coupling. The ratio of the two Higgs vacuum expectation values is given by \( \tan \beta \), and we use the abbreviation \( s_w = \sin \theta_w \). The squared physical masses of the stops are the eigenvalues of the above matrix. In order to simplify the discussion, we set \( \tan \beta = 1 \) for all following results. Further, we assume that the sbottom mass matrix is diagonal with degenerate mass eigenvalues, \( M_{\tilde{b}}^2 = \text{diag}(m_{\tilde{b}}^2, m_{\tilde{b}}^2) \). Neglecting \( m_{\tilde{b}} \) in the sbottom mass matrix this leads to \( M_{\tilde{Q}} = m_{\tilde{b}} \), and the stop mass matrix simplifies under the above assumptions to
\[ M_{\tilde{t}}^2 = \begin{pmatrix} m_t^2 + m_t^2 & m_t M_{LR} \\ m_t M_{LR} & M_{\tilde{U}}^2 + m_t^2 \end{pmatrix}, \]
(25)

with \( M_{LR} = A_t - \mu \). The stop mass eigenstates are obtained from the chiral states by a rotation:
\[ \begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_t & \sin \theta_t \\ -\sin \theta_t & \cos \theta_t \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}, \]
(26)
Figure 1: Relative correction to the total cross section as a function of the mixing parameter $M_{LR}$ for a fixed mixing angle $\theta_{\tilde{q}} = \pi/4$. Shown is the correction for two different gluino masses $m_{\tilde{g}} = 150$ GeV and $m_{\tilde{g}} = 250$ GeV.

Maximal mixing ($\theta_{\tilde{t}} = \pi/4$ and $M_{LR} \neq 0$) corresponds to $M_{U}^2 = m_{\tilde{b}}^2$. The latter relation will also be assumed for $M_{LR} = 0$, leading to the following stop mass eigenvalues:

$$m_{1,2} = \sqrt{m_{\tilde{b}}^2 + m_{\tilde{t}}^2 \pm m_t M_{LR}}.$$  \(27\)

Note that we use here the same set of assumptions on the squark mass matrices as we did in our study of the SUSY QCD corrections in the decay of polarized top quarks \[14\]. Further we use $\sin^2 \theta_W = 0.2236$, $\alpha_s = 0.11$, and we set the top mass to $m_t = 174$ GeV and the sbottom mass that enters Eq. (27) to $m_{\tilde{b}} = 100$ GeV.

Fig. 1 shows the relative SUSY QCD correction $\sigma^1/\sigma^0$ to the total cross section for $e^+e^- \rightarrow t\bar{t}$ with unpolarized beams at $\sqrt{s} = 500$ GeV as a function of the mixing parameter $M_{LR}$, where $\sigma^0$ and $\sigma^1$ are obtained from Eq. (17) by integrating over $y$. Shown are the relative corrections for two different gluino masses, namely $m_{\tilde{g}} = 150$ GeV and $m_{\tilde{g}} = 250$ GeV. For a large mixing parameter $M_{LR}$ and a small gluino mass of $m_{\tilde{g}} = 150$ GeV we find a large negative correction. The correction decreases as the gluino mass increases. A mixing parameter of $M_{LR} = 200$ GeV corresponds to a light stop mass of $m_{\tilde{t}_2} = 74$ GeV, which is above the current experimental lower limit [17]. With our choice of the masses, we are far away from the threshold singularity at $m_t = m_{\tilde{g}} + m_{\tilde{t}}$, where a more sophisticated calculation is necessary.

Fig. 2 shows the differential cross section $d\sigma/dy$, again for two different gluino masses $m_{\tilde{g}} = 150$ GeV and $m_{\tilde{g}} = 250$ GeV, and for the cases of 'no mixing' ($M_{LR} = 0$) and 'mixing' ($M_{LR} = 200$ GeV and $\theta_{\tilde{q}} = \pi/4$), again at $\sqrt{s} = 500$ GeV. We have compared our results for $\sigma$ and $d\sigma/dy$ with [3] and found agreement with their Fig. 3 (no mixing case), while we disagree with the results depicted in Fig. 4 ($\sigma$ and forward-backward asymmetry with stop mixing). We have also compared our results including the mixing with [4, 18] and find complete agreement.

\[2\] Note that by fixing $\theta_{\tilde{t}} = \pi/4$ the light stop can be either $\tilde{t}_1$ or $\tilde{t}_2$ depending on the sign of $M_{LR}$. 7
We now turn towards the discussion of the SUSY QCD corrections to the $t\bar{t}$ spin properties.

In Fig. 3 we investigate the expectation value of the top spin operator as a function of the centre-of-mass energy. We have computed the average projected polarization defined in Eq. (7) for three choices of the quantization axis $\hat{a}$, namely for $\hat{a} = \hat{k}$ (flight direction of the top), for $\hat{a} = \hat{p}$ (electron beam direction), and for $\hat{a} = \hat{n}$ (normal to the event plane). These quantities are shown in three different plots, where thin curves correspond to the tree level results and the thick curves are the relative corrections in percent. The corrections are shown for the case of mixing ($\theta_{\tilde{q}} = \pi/4$ and $M_{LR} = 200$ GeV) and a gluino mass of $m_{\tilde{g}} = 150$ GeV. For the polarizations of the initial beams we choose $\lambda^+ = 0$ and consider the three cases $\lambda^- = -1, 0, +1$. The projection of the top quark polarization onto $\hat{n}$ vanishes at tree level, and thus we only show the contribution from SUSY QCD absorptive parts in percent. In all cases SUSY QCD effects change the tree level results by less than 1% and vanish at threshold.

In Fig. 4 we show the averaged spin correlations $\hat{a}_i C_{ij} \hat{b}_j$ for the choices $\hat{a} = \hat{b} = \hat{k}$ (helicity correlation), $\hat{a} = \hat{b} = \hat{p}$ (beamline correlation), and $\hat{a} = \hat{p}, \hat{b} = \hat{k}$, for the same choice of parameters as in Fig. 3. Again the SUSY QCD correction are tiny. Fig. 5 shows the correlations for the choices $\hat{a} = \hat{k}, \hat{b} = \hat{n}$ and $\hat{a} = \hat{p}, \hat{b} = \hat{n}$. The first of these two choices of spin quantization axes leads to SUSY QCD effects slightly larger than 1% around c.m. energies of 700 GeV and for a fully polarized electron beam.

5 Conclusions

In this paper we have derived analytic expressions for the SUSY QCD corrections to the polarization and spin correlations of $t\bar{t}$ pairs produced in $e^+e^-$ annihilation with longitudinally
Figure 3: Average projected top quark polarization $\overline{P_t \hat{a}}$ defined in (7) for the choices $\hat{a} = \hat{k}$ (top), $\hat{a} = \hat{p}$ (middle), $\hat{a} = \hat{n}$ (bottom) as a function of the centre-of-mass energy. In each plot we show the tree level results (thin lines) and the relative corrections in percent (thick lines) for unpolarized positrons and the three cases $\lambda_\perp = -1, 0, +1$. 
Figure 4: Same as Fig. 3, but for the quantities $\hat{k}_C i j k_j$ (top), $\hat{p}_C i j \hat{p}_j$ (middle), and $\hat{p}_i C_i j \hat{k}_j$ (bottom).
Figure 5: Same as Fig. 3, but for the quantities $\hat{k}_i C_{ij} \hat{n}_j$ (top) and $\hat{p}_i C_{ij} \hat{n}_j$ (bottom).
polarized beams. The results depend in particular on the gluino mass and the masses of the scalar partners of the top quark. The latter masses depend on the mixing in the stop sector. For maximal mixing, the SUSY QCD corrections to the cross section are negative and reach values of about $-1.3\%$ ($-5\%$) for a gluino mass of 250 GeV (150 GeV) and a light stop mass of 74 GeV. For the same choice of parameters, the $\tilde{t}\bar{t}$ spin observables typically receive SUSY QCD corrections well below 1%.

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**Appendix**

Here we list explicit results for the formfactors defined in Eqs. (10), (11). Apart from the Standard Model parameters $m_t$, $\alpha_s$, and electroweak couplings defined in section 3, the formfactors depend on the gluino mass $m_{\tilde{g}}$, the masses of the two physical top squarks $m_{\tilde{t}_1,2}$, and the mixing angle $\theta_{\tilde{t}}$ that determines how the top squark mass eigenstates are related to the weak eigenstates, cf. Eq. (26).

We find (with $C_F = (N_C^2 - 1)/2N_C = 4/3$):

\[
V_1^\gamma = \frac{\alpha_s}{2\pi} C_F Q_t \left[ C_{24}^{11} + C_{24}^{22} \right] + Q_t \frac{\delta Z_R + \delta Z_L}{2},
\]

(A.1)

\[
V_1^Z = \frac{\alpha_s}{\pi} C_F \left[ (g_A' \cos^2 \theta_t - Q_t \sin^2 \phi_W) C_{24}^{11} + (g_A' \sin^2 \theta_t - Q_t \sin^2 \phi_W) C_{24}^{22} \right] + g_V \frac{\delta Z_R + \delta Z_L}{2} - g_A' \frac{\delta Z_R - \delta Z_L}{2},
\]

(A.2)

\[
A_1^\gamma = \frac{\alpha_s}{2\pi} C_F Q_t \left[ C_{24}^{11} - C_{24}^{22} \right] \cos 2\theta_t - Q_t \frac{\delta Z_R - \delta Z_L}{2},
\]

(A.3)

\[
A_1^Z = \frac{\alpha_s}{2\pi} C_F \left\{ 2 \left[ (g_A' \cos^2 \theta_t - Q_t \sin^2 \phi_W) C_{24}^{11} - (g_A' \sin^2 \theta_t - Q_t \sin^2 \phi_W) C_{24}^{22} \right] \cos 2\theta_t \right. \\
+ \left. g_V' \delta Z_R - \delta Z_L \right\} - g_V' \frac{\delta Z_R - \delta Z_L}{2} + g_A' \frac{\delta Z_R + \delta Z_L}{2},
\]

(A.4)
The quantities of the top quark field in the on-shell renormalization scheme. They are given explicitly by Passarino and Veltman [19],

\[ S_Y = \frac{\alpha_s}{2\pi} C_F Q_i m_t \left[ s_{11}^{11(22)} + s_{12}^{22} \right], \quad \text{(A.5)} \]

where

\[ s_{11(22)} = m_t \left( C_{11}^{(22)} + C_{21}^{(22)} \right) \pm m_\tilde{g} \sin 2\theta_t \left( C_{01}^{(22)} + C_{11}^{(22)} \right), \quad \text{(A.6)} \]

\[ S_Z = \frac{\alpha_s}{2\pi} C_F m_t \left\{ 2 \left( g_A^* \cos^2 \theta_t - Q_t \sin^2 \bar{\theta}_W \right) s_{11}^{11} + 2 \left( g_A^* \sin^2 \theta_t - Q_t \sin^2 \bar{\theta}_W \right) s_{12}^{22} - g_A^* \sin 2\theta_t \cos 2\theta_t m_\tilde{g} \left( C_{01}^{12} + C_{11}^{12} + C_{01}^{21} + C_{11}^{21} \right) \right\}. \quad \text{(A.7)} \]

In the above expressions, the one-loop integrals \( C_{ij}^{11} \ldots C_{ij}^{24} \) are defined by the decomposition of Passarino and Veltman [19],

\[ C_{ij}^{0 \mu \nu \nu} = \frac{(2\pi\mu)^{4-d}}{i\pi^2} \int d^d l \frac{1; l_\mu; l_\nu}{\left[ (l^2 - m_\tilde{g}^2) + i\epsilon \right] \left[ (l - k_\nu)^2 - m_{i_\nu}^2 + i\epsilon \right] \left[ (l + k_\nu)^2 - m_{i_\nu}^2 + i\epsilon \right]} \quad \text{(A.8)} \]

with \((k_\nu = k_\nu + k_\tilde{t})\):

\[ C_{ij}^{1j} = -k_{i_\mu} C_{ij}^{11} + k_{i_\nu} C_{ij}^{12}, \]

\[ C_{ij}^{2j} = k_{i_\mu} k_{i_\nu} C_{ij}^{21} + k_{i_\nu} k_{i_\mu} C_{ij}^{22} - (k_{i_\mu} k_{i_\nu} + k_{i_\nu} k_{i_\mu} + g_{i_\mu} C_{ij}^{21} + g_{i_\mu} C_{ij}^{24}). \quad \text{(A.9)} \]

The quantities \( \delta Z_{R,L} \) denote the one-loop renormalization constants for the chiral components of the top quark field in the on-shell renormalization scheme. They are given explicitly by

\[ \delta Z_{R,(L)} = \frac{\alpha_s C_F}{4\pi} \left\{ 2m_t^2 \left[ (B_1^i)' + (B_1^i)' \right] + 2m_\tilde{g} m_t \sin 2\theta_t \left[ (B_0^i)' - (B_0^i)' \right] \right\} + B_1^i + B_1^i \pm \cos 2\theta_t \left[ B_1^i - B_1^i \right], \quad \text{(A.10)} \]

where

\[ B_{0,1}^i = \frac{(2\pi\mu)^{4-d}}{i\pi^2} \int d^d l \frac{1; l_\mu}{\left[ (l^2 - m_\tilde{g}^2) + i\epsilon \right] \left[ (l - k_\tilde{t})^2 - m_{i_\nu}^2 + i\epsilon \right] \left[ (l + k_\tilde{t})^2 - m_{i_\nu}^2 + i\epsilon \right]} = B_{0,1}^i; (-k_{i_\mu}) B_1^i, \quad \text{(A.11)} \]

and

\[ (B_{0,1}^i)' = -\frac{dB_{0,1}^i}{dk_\tilde{t}} |_{k_\tilde{t}^2 = m^2}. \quad \text{(A.12)} \]

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