NONFACTORIZABLE EFFECTS IN CHARMLESS B DECAYS AND B MESON LIFETIMES

HAI-YANG CHENG

Institute of Physics, Academia Sinica, Taipei, Taiwan 115, Republic of China
E-mail: phcheng@ccvax.sinica.edu.tw

Status of nonfactorizable effects in hadronic charmless B decays is reviewed. Implications of new CLEO measurements on \( B^0 \to \pi^+\pi^- \) and \( B \to \eta'K \) are discussed. Nonfactorizable effects due to color octet 4-quark operators are calculated using renormalization group improved QCD sum rules. The resultant \( B \)-meson lifetime ratio \( \tau(B^+) / \tau(B_d) \) agrees with experiment.

1 Generalized Factorization

The nonleptonic two-body decays of mesons are conventionally evaluated under the factorization hypothesis. In the factorization approach, the decay amplitude is expressed in terms of factorizable hadronic matrix elements multiplied by some combinations of Wilson coefficient functions. To be more specific, the factorization hypothesis assumes that the 3-body hadronic matrix element \( \langle h_1 h_2 | O | M \rangle \) for the decay \( M \to h_1 h_2 \) is approximated as the product of two matrix elements \( \langle h_1 | J \mu | 0 \rangle \) and \( \langle h_2 | J_\mu' | M \rangle \). However, it is known that this approach of naive factorization fails to describe the decays proceeding through the (class-I) color-suppressed internal \( W \)-emission diagrams, though it is at work for decay modes dominated by (class-II) external \( W \)-emission diagrams. This implies that it is necessary to take into account nonfactorizable contributions to the decay amplitude in order to render the color suppression of internal \( W \)-emission ineffective.

Because there is only one single form factor involved in the class-I or class-II decay amplitude of \( B \to PP \), \( PV \) decays, the effects of nonfactorization can be lumped into the effective parameters \( a_1 \) and \( a_2 \):

\[
a_{i,2}^{\text{eff}} = c_{i,2}^{\text{eff}} + c_{i,1}^{\text{eff}} \left( \frac{1}{N_c} + \chi_{1,2} \right),
\]

where \( \chi_i \) are nonfactorizable terms and receive main contributions from color-octet current operators. Since \( |c_1/c_2| \gg 1 \), it is evident from Eq. (1) that even a small amount of nonfactorizable contributions will have a significant effect on the color-suppressed class-II amplitude. If \( \chi_{1,2} \) are universal (i.e. process independent) in charm or bottom decays, then we have a generalized factorization scheme in which the decay amplitude is expressed in terms of factorizable contributions multiplied by the universal effective parameters \( a_{i,2}^{\text{eff}} \). For \( B \to VV \) decays, this new factorization implies that nonfactorizable terms contribute in equal weight to all partial wave amplitudes so that \( a_{i,2}^{\text{eff}} \) can be defined. It should be stressed that, contrary to the naive one, the improved factorization does incorporate nonfactorizable effects in a process independent form. Phenomenological analyses of two-body decay data of \( D \) and \( B \) mesons indicate that while the generalized factorization hypothesis in general works reasonably well, the effective parameters \( a_{i,2}^{\text{eff}} \) do show some variation from channel to channel, especially for the weak decays of charmed mesons. An eminent feature emerged from the data analysis is that \( a_{2}^{\text{eff}} \) is negative in charm decay, whereas it becomes positive in the two-body decays of the \( B \) meson.

\[
a_{2}^{\text{eff}}(B \to D \pi) \sim 0.20 - 0.28,
\]

\[
\chi_{2}(B \to D \pi) \sim 0.12 - 0.19.
\]

Phenomenologically, it is often to treat the number of colors \( N_c \) as a free parameter to model the nonfactorizable contribution to hadronic matrix elements and its value can be extracted from the data of two-body nonleptonic decays. Theoretically, this amounts to defining an effective number of colors \( N_{c}^{\text{eff}} \), called 1/\( \xi \) in [6], by \( 1/N_{c}^{\text{eff}} \equiv (1/N_c) + \chi \). It is clear from (2) that

\[
N_{c}^{\text{eff}}(B \to D \pi) = 1.8 - 2.2 \approx 2.
\]

2 Nonfactorizable Effects in Hadronic Charmless B Decays

What are the nonfactorizable effects in hadronic charmless B decays? We note that the effective Wilson coefficients appear in the factorizable decay amplitudes in the combinations \( a_{2i} = c_{2i}^{\text{eff}} + \frac{1}{N_{e}^{\text{eff}}} c_{2i-1}^{\text{eff}} \) and \( a_{2i-1} = c_{2i-1}^{\text{eff}} + \frac{1}{N_{e}^{\text{eff}}} c_{2i}^{\text{eff}} (i = 1, \cdots, 5) \). As discussed in Sec. 1, nonfactorizable effects in the decay amplitudes of \( B \to PP \), \( VP \) can be absorbed into the parameters \( a_{i}^{\text{eff}} \). This amounts to replacing \( N_c \) in \( a_{i}^{\text{eff}} \) by \( (N_{c}^{\text{eff}})_{i} \). Explicitly,

\[
a_{2i}^{\text{eff}} = c_{2i}^{\text{eff}} + \frac{1}{(N_{c}^{\text{eff}})_{2i}} c_{2i-1}^{\text{eff}},
\]

\[
a_{2i-1}^{\text{eff}} = c_{2i-1}^{\text{eff}} + \frac{1}{(N_{c}^{\text{eff}})_{2i-1}} c_{2i}. \tag{4}
\]

It is customary to assume in the literature that \( (N_{c}^{\text{eff}})_{1} \approx (N_{c}^{\text{eff}})_{2} \cdots \approx (N_{c}^{\text{eff}})_{10} \); that is, the nonfactorizable term is usually assumed to behave in the same way in tree and
penguin decay amplitudes. A closer investigation shows that this is not the case. We have argued [3] that nonfactorizable effects in the matrix elements of \((V - A)(V + A)\) operators are different from that of \((V - A)(V - A)\) operators. One reason is that the Fierz transformation of the \((V - A)(V + A)\) operators \(O_{5,6,7,8}\) is quite different from that of \((V - A)(V - A)\) operators \(O_{1,2,3,4}\) and \(O_{9,10}\). Hence, we will advocate that

\[
N_c^{\text{eff}}(LL) = (N_c^{\text{eff}})_{1,2,3,4,9,10},
N_c^{\text{eff}}(LR) = (N_c^{\text{eff}})_{5,6,7,8},
\]

and that \(N_c^{\text{eff}}(LR) \neq N_c^{\text{eff}}(LL)\). In principle, \(N_c^{\text{eff}}\) can vary from channel to channel, as in the case of charm decay. However, in the energetic two-body \(B\) decays, \(N_c^{\text{eff}}\) is expected to be process insensitive as supported by data.

2.1 Nonfactorizable effects in spectator amplitudes

To study \(N_c^{\text{eff}}(LL)\) in spectator amplitudes, we focus on the class-III decay modes sensitive to the interference between external and internal \(W\)-emission amplitudes. Good examples are the class-III modes: \(B^+ \to \omega \pi^\pm, \pi^0 \pi^\pm, \eta \pi^\pm, \eta' \pi^\pm, \cdots\). Considering \(B^+ \to \omega \pi^\pm\), we find that the branching ratio is sensitive to \(1/N_c^{\text{eff}}\) and has the lowest value of order \(2 \times 10^{-6}\) at \(N_c^{\text{eff}} = \infty\) and then increases with \(1/N_c^{\text{eff}}\). The 1997 CLEO measurement yields

\[
B(B^+ \to \omega \pi^\pm) = (1.1\pm 0.6_{-0.5}^{+0.2}) \times 10^{-5}.
\]

Consequently, \(1/N_c^{\text{eff}} > 0.35\) is preferred by the data. Because this decay is dominated by tree amplitudes, this in turn implies that \(N_c^{\text{eff}}(LL) < 2.9\). If the value of \(N_c^{\text{eff}}(LL)\) is fixed to be 2, the branching ratio for positive \(\rho\), which is preferred by the current analysis, will be of order \((0.9 - 1.0) \times 10^{-5}\), which is very close to the central value of the measured one. Unfortunately, the significance of \(B^+ \to \omega \pi^\pm\) is reduced in the recent CLEO analysis and only an upper limit is quoted [11]. \(B(B^+ \to \pi^\pm \omega) < 2.3 \times 10^{-5}\). Nevertheless, the central value of \(B(B^+ \to \pi^\pm \omega)\) remains about the same as (6). The fact that \(N_c^{\text{eff}}(LL) \sim 2\) is preferred in charmless two-body decays of the \(B\) meson is consistent with the nonfactorizable term extracted from \(B \to (D, D^*)\pi, \rho\) decays: \(N_c^{\text{eff}}(B \to D \pi) \approx 2\). Since the energy release in the energetic two-body decays \(B \to \omega \pi, B \to D \pi\) is of the same order of magnitude, it is thus expected that \(N_c^{\text{eff}}(LL)\) is roughly \(2\).

In analogy to the decays \(B \to D^{(*)}\pi, \rho\), the interference effect of spectator amplitudes in class-III charmless \(B\) decay can be tested by measuring the ratios:

\[R_1 \equiv 2 \frac{B(B^- \to \pi^- \pi^0)}{B(B^- \to \pi^- \pi^+)},\]

\[R_2 \equiv \frac{B(B^- \to \rho^- \pi^0)}{B(B^- \to \rho^- \pi^+)},\]

\[R_3 \equiv \frac{B(B^- \to \pi^- \rho^0)}{B(B^- \to \pi^- \rho^+)}.\]

The ratios \(R_i\) are greater (less) than unity when the interference is constructive (destructive). Hence, a measurement of \(R_i\) (in particular \(R_3\)), which has the advantage of being independent of the Wolfenstein parameters \(\rho\) and \(\eta\), will constitute a very useful test on the effective number of colors \(N_c^{\text{eff}}(LL)\).

During this conference, CLEO has reported the updated limits on \(B^0 \to \pi^+\pi^-\) and \(B^- \to \pi^-\pi^0\) [12].

\[B(B^0 \to \pi^+\pi^-) < 0.84 \times 10^{-5},\]

\[B(B^- \to \pi^-\pi^0) < 1.6 \times 10^{-5}.\]

In particular, the limit on \(B^0 \to \pi^+\pi^-\) is improved by a factor of 2. It appears that this decay provides a stringent constraint on the form factor \(F_0^{B^{\pm}}\). Irrespective of the values of \(N_c^{\text{eff}}\), the predicted branching ratio for \(B^0 \to \pi^+\pi^-\) will easily exceed the current limit if \(F_0^{B^{\pm}}(0) \gtrsim 0.30\). Note that the decay rate of \(B^0 \to \pi^+\pi^-\) increases slightly with \(N_c^{\text{eff}}(LL)\) as it is dominated by the tree coefficient \(a_1\). For \(F_0^{B^{\pm}}(0) = 0.30\), we find \(N_c^{\text{eff}}(LL) \lesssim 0.20\).

2.2 Nonfactorizable effects in penguin amplitudes

The penguin amplitude of the class-VI mode \(B \to \phi K\) is proportional to the QCD penguin coefficients \((a_3 + a_4 + a_5)\) and hence sensitive to the variation of \(N_c^{\text{eff}}(LR)\) since \(a_4\) is \(N_c^{\text{eff}}\)-stable, but \(a_3\) and \(a_5\) are \(N_c^{\text{eff}}\)-sensitive. Neglecting \(W\)-annihilation and space-like penguin diagrams, we find that \(N_c^{\text{eff}}(LR) = 2\) is evidently excluded from the present CLEO upper limit [13].

\[B(B \to \phi K^+) < 0.5 \times 10^{-5},\]

and that \(1/N_c^{\text{eff}}(LR) < 0.23\) or \(N_c^{\text{eff}}(LR) > 4.3\). A similar observation was also made in [14]. The branching ratio of \(B \to \phi K^*\), the average of \(\phi K^{*+}\) and \(\phi K^{*0}\) modes, is also measured recently by CLEO with the result [15].

\[B(B \to \phi K^*) = (1.1_{-0.5}^{+0.6} \pm 0.2) \times 10^{-5}.\]

We find that the allowed region for \(N_c^{\text{eff}}(LR)\) is \(4 \gtrsim N_c^{\text{eff}}(LR) \gtrsim 1.4\). This is in contradiction to the constraint \(N_c^{\text{eff}}(LR) > 4.3\) derived from \(B^\pm \to \phi K^\pm\). In fact, the factorization approach predicts that \(\Gamma(B \to \phi K^+) \approx \Gamma(B \to \phi K^-)\) when the \(W\)-annihilation type of contributions is neglected. The current CLEO measurements [11] and [15] are obviously not consistent with the prediction based on factorization. One possibility is that generalized factorization is not applicable to \(B \to VV\).
Therefore, the discrepancy between $B(B \to \phi K)$ and $B(B \to \phi K^*)$ will measure the degree of deviation from the generalized factorization that has been applied to $B \to \phi K^*$. It is also possible that the absence of $B \to \phi K$ events is a downward statistical fluctuation. At any rate, in order to clarify this issue and to pin down the effective number of colors $N_{\text{eff}}(LR)$, we urgently need measurements of $B \to \phi K$ and $B \to \phi K^*$, especially the neutral modes, with sufficient accuracy.

2.3 $B \to \eta' K$ decays

The published CLEO results on the decay $B \to \eta' K$

$$B(B^\pm \to \eta' K^\pm) = (6.5^{+1.5}_{-1.1} \pm 0.9) \times 10^{-5},$$

$$B(B^0 \to \eta' K^0) = (4.7^{+2.7}_{-2.0} \pm 0.9) \times 10^{-5},$$  

(11)

are several times larger than previous theoretical predictions [14,15,18] in the range of $(1-2) \times 10^{-5}$. It was pointed out last year by several authors [13,14,15] that the decay rate of $B \to \eta' K$ will get enhanced because of the small running strange quark mass at the scale $m_b$ and sizable $SU(3)$ breaking in the decay constants $f_\eta$ and $f_0$. Ironically, it was also realized last year that the above-mentioned enhancement is partially washed out by the anomaly effect in the matrix element of pseudoscalar densities, an effect overlooked before. Specifically,

$$\langle \eta' | \bar{s} \gamma_5 s | 0 \rangle = -\frac{m_s^2}{2m_s} (f_\eta^u - f_\eta^d),$$  

(12)

where the QCD anomaly effect is manifested by the decay constant $f_\eta^u$. Since $f_\eta^u \approx \frac{1}{2} f_\eta^d$, it is obvious that the decay rate of $B \to \eta' K$ induced by the $(S-P)(S+P)$ penguin interaction is suppressed by the anomaly term in $\langle \eta' | \bar{s} \gamma_5 s | 0 \rangle$. As a consequence, the net enhancement is not large. If we treat $N_{\text{eff}}(LL)$ to be the same as $N_{\text{eff}}(LR)$, as assumed in previous studies, we would obtain typically $B(B^\pm \to \eta' K^\pm) = (2-3) \times 10^{-5}$ (see the solid curve in Fig. 1).

What is the role played by the intrinsic charm content of the $\eta'$ to $B \to \eta' K$? It has been advocated that the new internal W-emission contribution coming from the Cabibbo-allowed process $b \to c\bar{c}s$ followed by a conversion of the $c\bar{c}$ pair into the $\eta'$ via two gluon exchanges is potentially important since its mixing angle $V_{cb}V_{cs}^*$ is as large as that of the penguin amplitude and yet its Wilson coefficient $a_2$ is larger than that of penguin operators. The decay constant $f_\eta^e$, defined by $\langle 0 | \bar{\eta}' | \eta \rangle = i f_\eta^e q_\eta$, has been calculated theoretically and extracted phenomenologically from the data of $J/\psi \to \eta \gamma, J/\psi \to \eta' \gamma$ and of the $\eta\gamma$ and $\eta'\gamma$ transition form factors [18] and it lies in the range $-2.3 \text{ MeV} \leq f_\eta^e \leq -18.4 \text{ MeV}$. The sign of $f_\eta^e$ is crucial for the $\eta'$ charm content contribution. For a negative $f_\eta^e$, its contribution to $B \to \eta' K$ is constructive for $\alpha_2 > 0$. Since $\alpha_2$ depends strongly on $N_{\text{eff}}(LL)$, we see that the $c\bar{c} \to \eta'$ mechanism contributes constructively at $1/N_{\text{eff}}(LL) > 0.28$ whereas $\alpha_2 > 0$, whereas it contributes destructively at $1/N_{\text{eff}}(LL) < 0.28$ where $\alpha_2$ becomes negative. In order to explain the abnormally large branching ratio of $B \to \eta' K$, an enhancement from the $c\bar{c} \to \eta'$ mechanism is certainly welcome in order to improve the discrepancy between theory and experiment. This provides another strong support for $N_{\text{eff}}(LL) \approx 2$.

If $N_{\text{eff}}(LL) = N_{\text{eff}}(LR)$, then $B(B \to \eta' K)$ will be suppressed at $1/N_{\text{eff}} \leq 0.28$ and enhanced at $1/N_{\text{eff}} > 0.28$ (see the dashed curve in Fig. 1 for $f_\eta^e = -15 \text{ MeV}$). If the preference for $N_{\text{eff}}$ is $1/N_{\text{eff}} \lesssim 0.2$ (see e.g. [15]), then it is quite clear that contribution from the $\eta'$ charm content will make the theoretical prediction even worse at small $1/N_{\text{eff}}$! On the contrary, if $N_{\text{eff}}(LL) \approx 2$, the $c\bar{c}$ admixture in the $\eta'$ will always lead to constructive interference irrespective of the value of $N_{\text{eff}}(LR)$ (see the solid curve in Fig. 1).

Figure 1: The branching ratio of $B^\pm \to \eta' K^\pm$ as a function of $1/N_{\text{eff}}(LR)$ with $N_{\text{eff}}(LL)$ being fixed at the value of 2 and $\eta = 0.34$, $\rho = 0.16$. The charm content of the $\eta'$ with $f_\eta^e = -15 \text{ MeV}$ contributes to the solid curve but not to the dotted curve. The anomaly contribution to $\langle \eta' | \bar{s} \gamma_5 s | 0 \rangle$ is included. For comparison, predictions for $N_{\text{eff}}(LL) = N_{\text{eff}}(LR)$ as depicted by the dot-dashed curve with $f_\eta^e = 0$ and dashed curve with $f_\eta^e = -15 \text{ MeV}$ is also shown. The solid thick lines are the preliminary updated CLEO measurements (13) with one sigma errors.

At this conference we learned that a recent CLEO reanalysis of $B \to \eta' K$ using a data sample 80% larger in previous studies yields the preliminary results

$$B(B^\pm \to \eta' K^\pm) = (7.4^{+0.8}_{-1.0} \pm 1.0) \times 10^{-5},$$

$$B(B_d \to \eta' K^0) = (5.9^{+1.8}_{-1.6} \pm 0.9) \times 10^{-5},$$  

(13)

suggesting that the original measurements (11) were not an upward statistical fluctuation. This favors a slightly larger $f_\eta^e$ in magnitude. For $N_{\text{eff}}(LL) = 2$ and $f_\eta^e = -15 \text{ MeV}$, which is consistent with all the known theoretical and phenomenological constraints, we show in Fig. 1 that
\( \mathcal{B}(B^\pm \to \eta'K^\pm) \) at 1/\( N_c^{\text{eff}}(LR) \leq 0.2 \) is enhanced considerably from (2.5–3) \( \times 10^{-5} \) to (4.6–5.9) \( \times 10^{-5} \). In addition to the \( \eta' \) charm content contribution, \( N_c^{\text{eff}}(LL) \approx 2 \) leads to constructive interference in the spectator amplitudes of \( B \to \eta'K \) and an enhancement in the term proportional to \( 2(a_3 - a_5)X_{\text{LL}}^{BK,\gamma'} + (a_3 + a_4 - a_5)X_{\text{LL}}^{BK,\eta'} \).

In the \( B_s \) system, we find that \( B_s \to \eta'\eta' \), the analogue of \( B^0 \to \eta'K^0 \), has also a large branching ratio of order \( 2 \times 10^{-5} \).

### 2.4 Summary

Tree-dominated rare \( B \) decays, \( B^\pm \to \omega \pi^\pm \) and \( B^0 \to \pi^+\pi^- \), favor a small \( N_r^{\text{eff}}(LL) \), namely \( N_r^{\text{eff}}(LL) \approx 2 \). The constraints on \( N_r^{\text{eff}}(LR) \) derived from the penguin-dominated decays \( B^\pm \to \phi K^\pm \) and \( B \to \phi K^* \), which tend to be larger than \( N_r^{\text{eff}}(LL) \), are not consistent with each other. Our analysis of \( B \to \eta'K \) clearly indicates that \( N_r^{\text{eff}}(LL) \approx 2 \) is favored and \( N_r^{\text{eff}}(LR) \) is preferred to be larger. The preliminary updated CLEO measurements of \( B \to \eta'K \) seem to imply that the contribution from the \( \eta' \) charm content is important and serious.

### 3 Final-state interactions and \( B \to \omega K \)

The CLEO observation of a large branching ratio for \( B^\pm \to \omega K^\pm \)

\[
\mathcal{B}(B^\pm \to \omega K^\pm) = (1.5^{+0.7}_{-0.6} \pm 0.2) \times 10^{-5}, \tag{14}
\]

is difficult to explain at first sight. Its factorizable amplitude is of the form

\[
A(B^- \to \omega K^-) \propto (a_4 + Ra_6)X^{(B\omega,K)} + (2a_3 + 2a_5 + 1/2a_9)X^{(B\omega,K)} + \cdots,
\]

with \( R = -2m_K^2/(m_b m_s) \), where ellipses represent for contributions from W-annihilation and space-like penguin diagrams. It is instructive to compare this decay mode closely with \( B^- \to \rho K^- \)

\[
A(B^- \to \rho^0 K^-) \propto (a_4 + Ra_6)X^{(B\rho,K)} + \cdots. \tag{16}
\]

Due to the destructive interference between \( a_3 \) and \( a_6 \) penguin terms, the branching ratio of \( B^\pm \to \rho^0 K^\pm \) is estimated to be of order \( 5 \times 10^{-7} \). The question is then why is the observed rate of the \( \omega K^- \) mode much larger than the \( \rho K^- \) mode? By comparing (15) with (16), it is natural to contemplate that the penguin contribution proportional to \( (2a_3 + 2a_5 + 1/2a_9) \) accounts for the large enhancement of \( B^\pm \to \omega K^\pm \). However, this is not the case: The coefficients \( a_3 \) and \( a_5 \), whose magnitudes are smaller than \( a_4 \) and \( a_6 \), are not large enough to accommodate the data unless \( N_r^{\text{eff}}(LR) < 1.1 \) or \( N_r^{\text{eff}}(LR) > 20 \) (see Fig. 9 of[2]).

So far we have neglected three effects in the consideration of \( B^\pm \to \omega K^\pm \): W-annihilation, space-like penguin diagrams and final-state interactions (FSI). It turns out that FSI may play a dominant role for \( B^\pm \to \omega K^\pm \). The weak decays \( B^- \to K^-\pi^0 \) via the penguin process \( b \to s u \bar{u} \) and \( B^- \to K^0\pi^- \) via \( b \to s d \bar{d} \) followed by the quark rescattering reactions \( \{K^*-\pi^0, K^{*-0}\pi^-\} \to \omega K^- \) contribute constructively to \( B^- \to \omega K^- \) (see Fig. 2), but destructively to \( B^- \to \rho K^- \). Since the branching ratios for \( B^- \to K^-\pi^0 \) and \( K^{*-0}\pi^- \) are, of order \( (0.5–0.8) \times 10^{-5} \), it is conceivable that a large branching ratio for \( B^\pm \to \omega K^\pm \) can be achieved from FSI via inelastic scattering. Moreover, if FSI dominate, it is expected that \( B(B^\pm \to \omega K^\pm) \approx (1 + \sqrt{2/5})B(B^0 \to \omega K^0) \).

![Figure 2: Contributions to \( B^- \to K^-\omega \) from final-state interactions via the weak decays \( B^- \to K^-\pi^0 \) and \( B^- \to K^{*0}\pi^- \) followed by quark rescattering.](image)

### 4 Non spectator Effects and \( B \) Meson Lifetimes

In the heavy quark limit, all bottom hadrons have the same lifetimes, a well-known result in the parton picture. With the advent of heavy quark effective theory and the OPE approach for the analysis of inclusive weak decays, it is realized that the first nonperturbative correction to bottom hadron lifetimes starts at order \( 1/m_b^2 \) and it is model independent. However, the \( 1/m_b^2 \) corrections are small and essentially canceled out in the lifetime ratios. The non spectator effects such as W-exchange and Pauli interference due to four-quark interactions are of order \( 1/m_Q^2 \), but their contributions can be potentially significant due to a phase-space enhancement by a factor of \( 16\pi^2 \). As a result, the lifetime differences of heavy hadrons come mainly from the above-mentioned non spectator effects.

The four-quark operators relevant to inclusive non-leptonic \( B \) decays are

\[
O_{V-A} = \bar{b}_L \gamma_\mu q_L \bar{q}_L \gamma^\mu b_L,
\]
From which one can follow to define four hadronic parameters $B_1, B_2, \varepsilon_1, \varepsilon_2$ relevant to our purposes:

\[
\frac{1}{2m_B} \langle \mathcal{B}(O_{V-A(S-P)}) | B \rangle = f_B \frac{m_B}{8} B_{1(2)},
\]

\[
\frac{1}{2m_B} \langle \mathcal{B}(T_{V-A(S-P)}) | B \rangle = f_B \frac{m_B}{8} \varepsilon_{1(2)}. \tag{18}
\]

Under the factorization approximation, $B_1 = 1$ and $\varepsilon_i = 0$. To the order of $1/m_b^2$, the $B$-hadron lifetime ratios are given by

\[
\frac{\tau(B^-)}{\tau(B_d^0)} = 1 + \left( 0.043 B_1 + 0.0006 B_2 - 0.61 \varepsilon_1 + 0.17 \varepsilon_2 \right),
\]

\[
\frac{\tau(B_s^0)}{\tau(B_d^0)} = 1 + \left( -1.7 \times 10^{-5} B_1 + 1.9 \times 10^{-5} B_2 - 0.0044 \varepsilon_1 + 0.0050 \varepsilon_2 \right). \tag{19}
\]

It is clear that even a small deviation from the factorization approximation $\varepsilon_i = 0$ can have a sizable impact on the lifetime ratios.

We have derived in heavy quark effective theory the renormalization-group improved QCD sum rules [28] for the hadronic parameters $B_1, B_2, \varepsilon_1, \varepsilon_2$. The results are

\[
B_1(m_b) = 0.96 \pm 0.04, \quad B_2(m_b) = 0.95 \pm 0.02,
\]

\[
\varepsilon_1(m_b) = -0.14 \pm 0.01, \quad \varepsilon_2(m_b) = -0.08 \pm 0.01, \tag{20}
\]

to the zeroth order in $1/m_b$. The resultant $B$-meson lifetime ratios are $\tau(B^-)/\tau(B_d) = 1.11 \pm 0.02$ and $\tau(B_s)/\tau(B_d) \approx 1$, to be compared with the world averages [29] $\tau(B^-)/\tau(B_d) = 1.07 \pm 0.03$ and $\tau(B_s)/\tau(B_d) = 0.94 \pm 0.04$. Therefore, our prediction for $\tau(B^-)/\tau(B_d)$ agrees with experiment.

References

1. H.Y. Cheng, Phys. Lett. B 395, 345 (1994).
2. A.N. Kamal, A.B. Santra, T. Uppal, and R.C. Verma, Phys. Rev. D 53, 2506 (1996).
3. H.Y. Cheng, Z. Phys. C 69, 647 (1996).
4. H.Y. Cheng and B. Tseng, Phys. Rev. D 51, 6295 (1995).
5. M. Neubert and B. Stech, CERN-TH/97-99 hep-ph/9705294.
6. M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C 34, 103 (1987).
7. For a review of CLEO measurements on charmless $B$ decays, see K. Lingel, T. Skwarnicki, and J.G. Smith, hep-ex/9804013.
8. H.Y. Cheng and B. Tseng, hep-ph/9803457, to appear in Phys. Rev. D.
9. CLEO Collaboration, M.S. Alam et al., CLEO CONF 97-28 (1997).
10. F. Parodi, P. Roudeau, and A. Stocchi, hep-ph/9802289.
11. CLEO Collaboration, T. Bergfeld et al., Phys. Rev. Lett. 81, 272 (1998).
12. CLEO Collaboration, J. Roy, invited talk presented at the XXIX International Conference on High Energy Physics, Vancouver, July 23-28, 1998.
13. N.G. Deshpande, B. Dutta, and S. Oh, OITS-644 hep-ph/9712443.
14. CLEO Collaboration, B.H. Behrens et al., Phys. Rev. Lett. 80, 3710 (1998).
15. L.L. Chau, H.Y. Cheng, W.K. Sze, H. Yao, and B. Tseng, Phys. Rev. D 43, 2176 (1991); D 58, (E)019090 (1998).
16. G. Kramer, W.F. Palmer, and H. Simma, Z. Phys. C 66, 429 (1995); Nucl. Phys. B 428, 77 (1994).
17. D.S. Du and L. Guo, Z. Phys. C 75, 9 (1997); D.S. Du, M.Z. Yang, and D.Z. Zhang, Phys. Rev. D 53, 249 (1996).
18. A. Ali and C. Greub, Phys. Rev. D 57, 2996 (1998).
19. A.L. Kagan and A.A. Petrov, UCHEP-27 hep-ph/9707354.
20. N.G. Deshpande, B. Dutta, and S. Oh, Phys. Rev. D 57, 5723 (1998); A. Datta, X.G. He, and S. Pakvasa, Phys. Lett. B 419, 369 (1998).
21. A. Ali, J. Chay, C. Greub, and P. Ko, Phys. Lett. B 424, 161 (1998).
22. F. Araki, M. Musakhanov, and H. Toki, hep-ph/9803356 and hep-ph/9808290.
23. A.A. Petrov, Phys. Rev. D 58, 054003 (1998); T. Feldmann, P. Kroll, and B. Stech, hep-ph/9802409.
24. A.A. Petrov, hep-ph/9712313; J. Cao, F.G. Cao, T. Huang, and B.Q. Ma, hep-ph/9807508.
25. A. Ali, G. Kramer, and C.D. Lü, hep-ph/9804363.
26. CLEO Collaboration, B.H. Behrens et al., CLEO CONF 98-09 (1998).
27. Y.H. Chen, H.Y. Cheng, and B. Tseng, IP-ASTP-10-98.
28. M. Neubert and C.T. Sachrajda, Nucl. Phys. B 483, 339 (1997).
29. H.Y. Cheng and K.C. Yang, hep-ph/9805222.
30. S. Willloco, invited talk presented at the XXIX International Conference on High Energy Physics, Vancouver, July 23-28, 1998.