Two-path interference in the resonance-enhanced few-photon ionization of atoms

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(Dated: April 19, 2022)

We investigate the resonance-enhanced few-photon ionization of an atomic Li target at a photon energy near the resonance between the 2s ground and 2p excited states. For this system, the ground-state ionization resembles an atomic “double slit”, because it can proceed through the 2p resonances with the magnetic quantum number $m_\ell$ being either $-1$ or $+1$. In our experiment, the target can be prepared in one of the polarized excited 2p states before subjecting it to the ionizing radiation, thereby effectively closing one of the two slits. This makes it possible to extract the interference term between the two pathways and obtain complex phase information on the final state. The analysis of our experimental results is supported by an ab initio model based on the numerical solution of the time-dependent Schrödinger equation.

INTRODUCTION

Two-path interference is one of the most intriguing and intensely studied phenomena in physics. It was first demonstrated in 1801 for optical light by Thomas Young in his well-known double-slit experiment. The historical importance of this experiment for the development of quantum theory is hard to overstate, because it reveals the wave nature of massive particles such as electrons, atoms, and even large molecules, thereby supporting de Broglie’s hypothesis of wave-particle duality. Until today, this phenomenon did not lose its appeal, and it has been observed in numerous systems. On one hand, it allows to extract phase information on wave functions, which is commonly not directly observable. On the other hand, it is exploited in many quantum-control schemes, because the manipulation of the relative amplitudes of the two pathways makes it possible to control the final state with high sensitivity. In atomic and molecular scattering processes, examples include well-known effects like Feshbach, shape, and Fano resonances, or atomic-scale double-slits formed by diatomic molecules exhibiting interferences in differential ionization cross sections due to ion, electron, or photon impact.

Multiphoton ionization processes of single atoms expose two- and multi-path interferences in a particularly clean way, because of the well-defined energy and limited angular momentum transfer in photon absorption reactions. A prominent example is RABBITT (Reconstruction of Attosecond Beating by interference of Two-Photon Transitions) spectroscopy, which has become the standard tool to characterize extreme-ultraviolet (XUV) attosecond pulse trains and allows the study of attosecond atomic dynamics in the time domain. Two-color ionization schemes using (lower) harmonic radiation enable the coherent control of the reactions’ final state via two-path interferences. Recently, other schemes have been considered, where double-slit structures in so-called Kramers-Henneberg states emerge through the distortion of a bound state by an external field, once again resulting in interference patterns. Two-path interference has not only been observed in laser pulses but also using two mutually incoherent (i.e., without mutual phase lock) continuous-wave (cw) lasers in the two-photon ionization of rubidium atoms, where the photon energies are tuned to two different resonances.

In the present study, two-path interference occurs in the ground-state ionization of lithium exposed to single-color femtosecond laser pulses, which are linearly polarized in the $y$ direction. The laser spectrum has its center wavelength at 660 nm and partially overlaps with the 2s−2p resonance at 671 nm. For the quantization axis chosen as the $z$ direction, the absorption of a single photon results in the excitation to the 2p state coherently populating the two magnetic sublevels with $m_\ell = +1$ and $-1$, respectively. These two eigenstates resemble the two “slits” in analogy to Young’s double-slit scheme (see Fig. 1). From these two excited levels, the atom is ionized without further resonance-enhancement by the absorption of two more photons from the same laser pulse. The final result is a superposition of electronic $p$ and $f$ continuum waves.

It is important to note that the distinction of these two pathways relies on the choice of the quantization direction. However, this choice is motivated by the experimental capability of preparing the atoms selectively in one of the two excited and polarized magnetic sublevels of the 2p state before exposing them to the femtosecond laser pulse. This enables us to measure not only the final intensity of the two interfering pathways, which corresponds to the differential cross sections for the ionization of the 2s state, but also the intensity of each pathway individually, which are the cross sections of 2p ionization.
The factor $A$ accounts for the fact that the first excitation step from the initial 2$s$ state to the intermediate 2$p$ levels alters the overall ionization probability. It should be noted that the picture developed here represents an approximation, because the time-dependent population dynamics is not fully accounted for. However, it will be shown below that the experimental observations and model calculations presented here are in very good agreement with the two-path picture outlined above. Equation (1) provides a direct and intuitive way to extract the complex phase difference of the continuum waves $\psi_+$ and $\psi_-$ as a function of the electron emission angle, thereby revealing the effect of the orientation of the initial electron orbital angular momentum on the final state’s phase.

$$|A \cdot \psi_{2s}|^2 = |\psi_+ + \psi_-|^2 = |\psi_+|^2 + |\psi_-|^2 + 2|\psi_+||\psi_-| \cos \Delta \phi,$$

where $\psi_{2s}$, $\psi_+$, and $\psi_-$ represent the electronic continuum wave functions for the ionization of the initial 2$s$ ($m_\ell = 0$), 2$p$ ($m_\ell = +1$), and 2$p$ ($m_\ell = -1$), respectively.

### RESULTS AND DISCUSSION

In the present study, lithium atoms in the 2$s$ ground state and 2$p$ excited state are ionized in a laser field with a central wavelength of 660 nm at intensities well below $10^{11}$ W/cm$^2$. This situation corresponds to Keldysh parameters larger than 20, and hence the system is expected to be well described in a multi-photon picture. The two initial states are ionized by the absorption of (at least) three or two photons, respectively, resulting in a final electron energy of about 200 meV. The measured and calculated electron momentum spectra shown in Fig. 2 are in excellent agreement with each other. Before proceeding to the analysis of the two-path interference introduced above, two important features of the data should be mentioned, even though they were already reported previously in several recent studies.

First, while the photoelectron momentum distributions (PMDs) for 2$s$ ionization exhibit reflection symmetry with respect to the laser electric field direction (the vertical direction in the momentum spectra shown in Fig. 2), this symmetry is broken for ionization of the polarized 2$p$ state. Consequently, the main electron emission direction appears to be shifted. The dependence of this phenomenon, known as magnetic dichroism, on the laser wavelength and intensity was recently investigated by Acharya et al. [31]. In this earlier study, these asymmetries were explained in a partial-wave picture. They were traced back to a nonvanishing mean orientation of the final electron orbital angular momentum $\langle m_\ell \rangle \neq 0$. This “remnant” of the initial target polarization is par-

![Few-photon ionization scheme in lowest-order perturbation theory. The ionization pathways from the 2$p$ state with $m_\ell = +1$ and $m_\ell = -1$ are shown as red dashed and blue dotted arrows, respectively. The 2$s$ ionization corresponds to the superposition of both.](image)

This fact can approximately be expressed as

$$|A \cdot \psi_{2s}|^2 = |\psi_+ + \psi_-|^2 = |\psi_+|^2 + |\psi_-|^2 + 2|\psi_+||\psi_-| \cos \Delta \phi,$$

where $\psi_{2s}$, $\psi_+$, and $\psi_-$ represent the electronic continuum wave functions for the ionization of the initial 2$s$ ($m_\ell = 0$), 2$p$ ($m_\ell = +1$), and 2$p$ ($m_\ell = -1$), respectively. The factor $A$ accounts for the fact that the first excitation step from the initial 2$s$ state to the intermediate 2$p$ levels alters the overall ionization probability.

It should be noted that the picture developed here represents an approximation, because the time-dependent population dynamics is not fully accounted for. However, it will be shown below that the experimental observations and model calculations presented here are in very good agreement with the two-path picture outlined above. Equation (1) provides a direct and intuitive way to extract the complex phase difference of the continuum waves $\psi_+$ and $\psi_-$ as a function of the electron emission angle, thereby revealing the effect of the orientation of the initial electron orbital angular momentum on the final state’s phase.

### METHODS

The experimental technique and the theoretical method are identical to those reported in previous studies on very similar systems [29, 31]. Therefore, only some key features are repeated here and parameters specific to the present study are mentioned.

Lithium atoms are cooled and confined in a volume of about 1 mm diameter in a near-resonant all-optical atom trap (AOT) [32] with a fraction of about 25% being in the polarized excited 2$p$ ($m_\ell = +1$) state and about 75% in the 2$s$ ground state. The atoms are ionized in the field of a femtosecond laser based on a Ti:Sa oscillator with two noncollinear optical parametric amplifier (NOPA) stages. For the present study, the laser wavelength was chosen to center at 660 nm with pulse durations (FWHM of intensity) of about 65 fs and a peak intensity of about $3 \times 10^{10}$ W/cm$^2$. The three-dimensional electron momentum vectors are measured with a resolution of about 0.01 a.u. [33] in a reaction microscope (e.g., [34, 35]). It is important to note that this experimental setup enables us to obtain differential cross-normalized data for the ionization of the 2$s$ and the 2$p$ initial states simultaneously.

In our theoretical model, the lithium atoms are approximated as a single active electron moving in the field of a $1s^2$ ionic core. The latter is described by a static Hartree potential [36, 37], which is supplemented by phenomenological terms to account for the core polarizability as well as exchange between the valence electron and those in the core [29]. The (complex) final-state wave function is obtained after propagating the initial state in time by numerically solving the time-dependent Schrödinger equation (TDSE).
tion. This evident violation of LOPT close to the 2s resonance was reported and discussed in our previous study as well: It is explained by the coupling between the atomic initial orbital angular momentum (i.e., perpendicular to the drawing plane). Typically preserved throughout the ionization process. The angular shifts and observed asymmetries are then a result of the interference between (phase-shifted) partial waves with different \( m_\ell \).

Second, the azimuthal photoelectron angular distributions (PADs) for 2p ionization feature six peaks. As discussed below, this indicates beyond lowest-order contributions to the ionization cross section. Generally, the dependence of the differential cross sections on the azimuthal angle \( \varphi \) is given by \[ (1) \]

\[
\frac{d\sigma}{d\Omega} = \sum_{m_\ell} c_{m_\ell} e^{im_\ell \varphi} \left| \psi_{m_\ell} \right|^2 ,
\]

where the factors \( c_{m_\ell} \) relate to the complex amplitudes of the partial waves. In lowest-order perturbation theory (LOPT), the absorption of only the minimum number of photons is considered. For the present initial 2p \((m_\ell = +1)\) state, this corresponds to two-photon absorption. In the electric dipole approximation, this results in partial waves with \( m_\ell = -1, +1, \) and \(+3\) contributing to the final state (cf. Fig. 1). For this set of dipole-allowed \( m_\ell \) values, therefore, the above expression results in a photoelectron angular distribution with no more than four peaks, in contrast to the six peaks observed in both the experiment and the \textit{ab initio} calculation. This evident violation of LOPT close to the 2s – 2p resonance was reported and discussed in our previous study as well: It is explained by the coupling between the 2s and 2p states in the external field giving rise to Rabi oscillations between these two states and resulting in a contribution of \( m_\ell = -3 \) to the final state. Accounting for this additional pathway, the expression in Eq. (2) allows for angular distributions with up to six peaks, which is consistent with experiment and calculation.

The validity of the two-path interference expression given in Eq. (1) can be tested by using our theoretical description. In a first step, the final-state wave functions \( \psi_- \) and \( \psi_+ \) (i.e., their magnitudes and phases) for the ionization of the 2p initial states with \( m_\ell = -1 \) and +1, respectively, are calculated. Their absolute squares, corresponding to the differential ionization cross sections, are shown as a function of the photoelectron momentum in Fig. 2 (left) considering only electron emission in the \( xy \) plane (i.e., for a polar angle \( \vartheta = 0 \)). Due to symmetry considerations, the systems with opposite initial orbital angular momentum \( m_\ell = +1 \) and \(-1\) are mirror images of one another with the mirror plane spanned by the laser polarization direction (the \( y \) axis) and the direction of the initial atomic polarization (the \( z \) axis). Specifically, \[ (2) \]

\[
\psi_+(\pi/2 + \varphi) = \psi_-(\pi/2 - \varphi).
\]

In the second step, Eq. (1) is tested by comparing the calculated momentum distribution for 2s ionization (corresponding to \( |\psi_{2s}|^2 \)) with the intensity of the superposition of the two wave functions for 2p ionization \( |\psi_+ + \psi_-| \)^2. The momentum distributions obtained by these two different methods are shown in Fig. 3 (right and center, respectively). They are in overall very good agreement indeed, although the \( |\psi_{2s}|^2 \) distribution has a slightly larger diameter. This small discrepancy is a result of slightly different (by approximately 10 %) photoelectron energies for 2s and 2p ionization, because the wavelength of the ionizing field is off the 2s – 2p resonance by about 10 nm.

The discussion above shows that the final momentum distribution for 2s ionization (which corresponds to

**FIG. 2.** Experimental (left) and theoretical (right) PMDs projected onto the \( xy \) plane for few-photon ionization of the 2s (top row) and 2p \((m_\ell = +1)\) (middle row) initial states by linearly polarized laser pulses of 65 fs duration with a center wavelength of 660 nm and a peak intensity of 3.1 \( \times 10^{10} \) W/cm\(^2\). The laser polarization direction is along the \( y \) axis (i.e., vertical), while the atomic initial orbital angular momentum is oriented in the \( z \) direction (i.e., perpendicular to the drawing plane).

**FIG. 3.** Absolute square of the calculated wave functions \( \psi_± \) and \( \psi_- \) (left), of their coherent sum \( \psi_± + \psi_- \) (center), and of \( \psi_{2s} \) (right) in the \( xy \) plane in momentum space. See text for details.
$|\psi_{2s}\rangle$ can to a good approximation be calculated from the (complex) final-state wave function $\psi_+$ for ionization of the polarized $2p (m_\ell = +1)$ initial state by exploiting Eq. (1) and the mirror symmetry between $\psi_+$ and $\psi_-$. Evidently, this is not possible with the experimental data, because only the absolute square of the final-state wave function $\psi_+$ is directly measured, but not its phase. However, the relative phase between $\psi_-$ and $\psi_+$ can be extracted by reversing the above procedure and solving Eq. (1) for the phase difference. This yields

$$\cos \Delta \phi = \frac{|A| \cdot |\psi_{2s}|^2 - |\psi_+|^2 - |\psi_-|^2}{2 |\psi_+| |\psi_-|}. \tag{3}$$

Generally, complex phases of continuum wave functions depend on both the photoelectron emission angle and energy. The reconstruction of the phase difference $\Delta \phi$ between the two wave functions $\psi_+$ and $\psi_-$ in three-dimensional momentum space from experimental data using Eq. (3) requires matching photoelectron energies for $2s$ and $2p$ ionization, which is not strictly fulfilled for the present laser wavelength. The question arises whether, in spite of the small photoelectron energy mismatch, Eq. (3) is still applicable, if only the dependence on the electron emission angle is considered (i.e., for the electron energy fixed at the peak energy). This can be the case, if the angular distributions do not significantly vary with small shifts of the photoelectron energy. This is tested by comparing the calculated angular distributions for $2s$ ionization (extracted from $|\psi_{2s}|^2$) with the distribution obtained for the interfering wave functions $|\psi_+ + \psi_-|^2$. The corresponding angular distributions are shown in Fig. 4. Indeed, the angular spectrum obtained from the interfering wave functions closely resembles the distribution calculated for $2s$ ionization. Their difference is barely noticeable. There is only a small deviation in the relative intensity of the main peak in the polarization direction and the side peaks. Therefore, we conclude that Eq. (3) makes it possible to extract the phase difference $\Delta \phi$ of $\psi_+$ and $\psi_-$ (in a good approximation) as a function of the azimuthal angle for the peak photoelectron energy.

Before Eq. (3) can be employed to calculate the phase difference $\Delta \phi$ from the experimental data, the factor $A$ has to be determined. This is straightforward, because in Young’s double-slit scenario the total flux is generally conserved, i.e., the final total intensity has to equal the sum of the intensities going through each slit individually. Therefore, the interference term does not change the total intensity, and hence the factor $A$ must fulfill the condition $\int d^3p \cdot |A\psi_{2s}|^2 = \int d^3p \cdot (|\psi_+|^2 + |\psi_-|^2)$.

The experimental PADs are shown in Fig. 5 (left). While the distributions for the ionization for the $2s$ and the $2p (m_\ell = +1)$ initial states are measured directly in our experiment, the data for $2p (m_\ell = -1)$ are obtained by reflecting the data for the opposite target polarization on the laser polarization axis. Using these angular distributions, the cosine of the phase difference is calculated with Eq. (3), plotted in Fig. 5 (right), and compared to the theoretical predictions.

The distribution features six crests and troughs whose positions agree very well between theory and experiment. However, some discrepancies in the magnitude persist. While the calculated curve reaches the maximum and minimum values of $+1$ and $-1$, respectively, the oscillation is weaker in the experimental data. Generally, a value of $+1$ for $\cos \Delta \phi$ corresponds to maximum constructive interference, which is expected at angles where the angular distribution for $2s$ ionization has a local maximum. Correspondingly, $\cos \Delta \phi = -1$ means complete destructive interference, which should occur at local minima in the differential $2s$ ionization data.

There are two effects that might blur these interferences in the experimental data: 1) There is a small, but nonnegligible experimental angular uncertainty. 2) The experimental data represent an average over a laser intensity range, as already discussed above.
CONCLUSIONS AND OUTLOOK

We studied the details of electron emission in few-photon ionization of lithium atoms initially either in the 2s ground state or in the polarized 2p \((m_\ell = +1)\) excited state by radiation close to the 2s \(- 2p\) resonance. We exploited the fact that the 2s state can be ionized through two possible pathways, specifically via the 2p resonance with either \(m_\ell = +1\) or \(-1\). These two pathways interfere in the final state and resemble a double-slit. Because our experiment allows us to obtain the differential cross sections for the 2s and the 2p initial states separately, we are able to measure the final wave with both "slits" open, or with one "slit" closed. Therefore, the data makes it possible to extract the interference term, thereby providing information on the phases in the wave functions. The experimentally obtained phase differences are in very good agreement with our theoretical predictions.

Moreover, several interesting features are observed in the present data, which were reported for similar systems in preceding studies: First, the photoelectron angular distributions after ionization of the polarized 2p state are not symmetric with respect to the laser polarization. Instead, the peaks are shifted. The wavelength and intensity dependence of this effect, known as magnetic dichroism, was systematically studied in \[31\]. Second, the peak structures in the present angle-differential spectra are in direct contradiction to the predictions of lowest-order perturbation theory. This is remarkable, since LOPT is generally expected to yield reasonable results at the rather moderate intensities used in our experiments. These discrepancies are explained by Rabi oscillations due to the coupling of the 2s and 2p states in the ionizing laser field. See \[31\] for further discussion.

It is worth noting that the present method is not the only way to access information regarding the final state's phase. The angular distributions can be fitted using model functions described by a superposition of partial waves as expressed in Eq. \[2\]. Under certain conditions, this make it possible to extract the relative phases between the complex amplitudes of partial waves contributing to the final state. For single-photon ionization, such complete studies were pioneered in the 1990s using polarized atomic targets \[35-40\]. In the multiphoton ionization regime, phase information was obtained by ionizing atoms with elliptically polarized light \[41, 42\]. In contrast, the present scheme, which exploits the resonance enhancement through two magnetic sublevels, provides direct, complete, and intuitive access to the interference term and the final-state phase.

Two- or multi-path interferences in few-photon ionization are well suited for quantum control schemes, if the relative phases and intensities of the different paths can be regulated (e.g., \[26\]). It is interesting to conceive such a scheme for the present system. In fact, controlling the relative (complex) amplitudes of the transient \(2p (m_\ell = -1)\) and \(2p (m_\ell = +1)\) populations is experimentally straightforward. The transitions from the 2s ground state to the two polarized excited 2p levels are driven by left- and right-handed circularly polarized laser radiation, respectively, propagating in the z-direction. The superposition of these two fields with equal intensity and fixed relative phase corresponds to the linearly polarized light used in the present experiment. Changing the rel-
ative phase corresponds to a rotation of the polarization direction in the $xy$ plane.

Furthermore, a change in the relative intensities can be achieved by introducing an ellipticity to the radiation. In the present scheme, quasi-monochromatic light is used and changes of the laser polarization would also affect the ionization steps after populating the resonant $2p$ levels. However, the effect on the excitation process and the ultimate ionization could be de-coupled by using bichromatic laser fields with a weak contribution close to the $2s - 2p$ resonance and a stronger contribution off resonance. Such an experiment would allow to prepare an atomic target in a coherent superposition of excited magnetic sublevels before ionizing it, thereby providing numerous possibilities to analyze and control the final state.

ACKNOWLEDGMENTS

The experimental material presented here is based upon work supported by the National Science Foundation under Grant No. PHY-1554776. The theoretical part of this work was funded by the NSF under grants No. PHY-2012078 (N.D.) and PHY-1803844 (K.B.), and by the XSEDE supercomputer allocation No. PHY-090031.

[1] T. Young, Philosophical Transactions of the Royal Society of London 92, 12 (1802).
[2] C. J. Davison and L. H. Germer, Proceedings of the National Academy of Sciences 14, 317 (1928).
[3] C. Jonsson, Zeitschrift für Physik 161, 454 (1961).
[4] D. W. Keith, C. R. Ekstrom, Q. A. Turchette, and D. E. Pritchard, Physical Review Letters 66, 2693 (1991).
[5] M. Arndt, O. Nairz, J. Vos-Andreae, C. Keller, G. van der Zouw, and A. Zeilinger, Nature 401, 680 (1999).
[6] L. de Broglie, J. Physique (serie 6) VIII, 225 (1927).
[7] H. Feshbach, Annals of Physics 5, 357 (1958).
[8] U. Fano, Physical Review 124, 1866 (1961).
[9] G. J. Schulz, Reviews of Modern Physics 45, 378 (1973).
[10] C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Reviews of Modern Physics 82, 1225 (2010).
[11] N. Stolterfoht, B. Sulik, V. Hoffmann, B. Skogvall, J. Y. Chesnel, J. Randanga, F. Frémont, D. Henneceart, A. Cas simi, X. Husson, A. L. Landers, J. A. Tanis, M. E. Galassi, and R. D. Rivalo, Physical Review Letters 87, 023201 (2001).
[12] D. Misra, U. Kadhane, Y. P. Singh, L. C. Tribedi, P. D. Fainstein, and P. Richard, Physical Review Letters 92, 153201 (2004).
[13] K. N. Egódpáti, S. Sharma, A. Hasan, A. C. Laforge, D. H. Madison, R. Moshammer, and M. Schulz, Physical Review Letters 106, 153202 (2011).
[14] S. Zhang, D. Fischer, M. Schulz, A. Voitkiv, A. Sensleben, A. Dorn, J. Ullrich, X. Ma, and R. Moshammer, Physical Review Letters 112, 023201 (2014).
[15] D. S. Milne-Brownie, M. Foster, J. Gao, B. Lohmann, and D. H. Madison, Physical Review Letters 96, 233201 (2006).
[16] X. Li, X. Ren, K. Hossen, E. Wang, X. Chen, and A. Dorn, Physical Review A 97, 022706 (2018).
[17] H. D. Cohen and U. Fano, Physical Review 150, 30 (1966).
[18] M. Kunitski, N. Eicke, P. Huber, J. Köhler, S. Zeller, J. Voigtsberger, N. Schott, K. Henrichs, H. Sann, F. Trinter, L. P. H. Schmidt, A. Kalinin, M. S. Schöller, T. Jahnke, M. Lein, and R. Dörner, Nature Communications 10, 10.1038/s41467-018-07882-8 (2019).
[19] H. Muller, Applied Physics B 74, s17 (2002).
[20] K. Klünder, J. M. Dahlström, M. Gisselbrecht, T. Fordell, M. Swoboda, D. Guénôt, P. Johnson, J. Caillat, J. Mauritsson, A. Maquet, R. Taieb, and A. L’Huillier, Physical Review Letters 106, 143002 (2011).
[21] M. Isinger, R. J. Squibb, D. Busto, S. Zhong, A. Harth, D. Kroon, S. Nandi, C. L. Arnold, M. Miranda, J. M. Dahlström, E. Lindroth, R. Feifel, M. Gisselbrecht, and A. L’Huillier, Science 358, 893 (2017).
[22] D. Bharti, D. Atri-Schuller, G. Menning, K. R. Hamilton, R. Moshammer, T. Pfeifer, N. Douguet, K. Bartschat, and A. Harth, Physical Review A 103, 022834 (2021).
[23] Y.-Y. Yin, C. Chen, D. S. Elliott, and A. V. Smith, Physical Review Letters 69, 2355 (1992).
[24] P. Ehlotzky, Physics Reports 345, 175 (2001).
[25] C. Brif, R. Chakrabarti, and H. Rabitz, New Journal of Physics 12, 075008 (2010).
[26] L. Giannessi, E. Allaria, K. C. Prince, C. Callegari, G. Sansone, K. Ueda, T. Morishita, C. N. Liu, A. N. Grum-Grzhimailo, E. V. Gryzlova, N. Douguet, and K. Bartschat, Scientific Reports 8, 10.1038/s41598-018-25833-7 (2018).
[27] P.-L. He, Z.-H. Zhang, and F. He, Physical Review Letters 124, 163201 (2020).
[28] J. Pursehouse, A. J. Murray, J. Wältzel, and J. Berakdar, Physical Review Letters 122, 053204 (2019).
[29] A. D. Silva, D. Atri-Schuller, S. Dubey, B. Acharya, K. Romans, K. Foster, O. Russ, K. Compton, C. Rischbieter, N. Douguet, K. Bartschat, and D. Fischer, Physical Review Letters 126, 10.1103/physrevlett.126.023201 (2021).
[30] A. H. N. C. D. Silva, T. Moon, K. L. Romans, B. P. Acharya, S. Dubey, K. Foster, O. Russ, C. Rischbieter, N. Douguet, K. Bartschat, and D. Fischer, Physical Review A 103, 053125 (2021).
[31] B. P. Acharya, M. Dobson, S. Dubey, K. L. Romans, A. H. N. C. D. Silva, K. Foster, O. Russ, K. Bartschat, N. Douguet, and D. Fischer, Phys. Rev. A 104, 053103 (2021).
[32] S. Sharma, B. P. Acharya, A. H. N. C. D. Silva, N. W. Parris, B. J. Ramsey, K. L. Romans, A. Dorn, V. L. B. de Jesus, and D. Fischer, Physical Review A 97, 043427 (2018).
[33] F. Thini, K. L. Romans, B. P. Acharya, A. H. N. C. de Silva, K. Compton, K. Foster, C. Rischbieter, O. Russ, S. Sharma, S. Dubey, and D. Fischer, Journal of Physics B: Atomic, Molecular and Optical Physics 53, 095201 (2020).
[34] R. Hubele, M. Schuricke, J. Goullon, H. Lindenblatt, N. Ferreira, A. Laforge, E. Brühl, V. L. B. de Jesus, D. Globig, A. Kelkar, D. Misra, K. Schneider, M. Schulz,
M. Sell, Z. Song, X. Wang, S. Zhang, and D. Fischer, Review of Scientific Instruments 86, 033105 (2015).

[35] D. Fischer, in Ion-Atom Collisions edited by M. Schulz (De Gruyter, 2019) pp. 103–156.

[36] B. J. Albright, K. Bartschat, and P. R. Fliche, Journal of Physics B: Atomic, Molecular and Optical Physics 26, 337 (1993).

[37] M. Schuricke, G. Zhu, J. Steinmann, K. Simeonidis, I. Ivanov, A. Kheifets, A. N. Grum-Grzhimailo, K. Bartschat, A. Dorn, and J. Ullrich, Physical Review A 83, 10.1103/physreva.83.023413 (2011).

[38] M. Pahler, C. Lorenz, E. v. Raven, J. Rüder, B. Sonntag, S. Baier, B. R. Müller, M. Schulze, H. Staiger, P. Zimmermann, and N. M. Kabachnik, Phys. Rev. Lett. 68, 2285 (1992).

[39] U. Becker, Journal of Electron Spectroscopy and Related Phenomena 96, 105 (1998).

[40] K. Godehusen, P. Zimmermann, A. Verweyen, A. von dem Borne, P. Wernet, and B. Sonntag, Phys. Rev. A 58, R3371 (1998).

[41] F. Dulieu, C. Blondel, and C. Delsart, Journal of Physics B: Atomic, Molecular and Optical Physics 28, 3845 (1995).

[42] Z.-M. Wang and D. S. Elliott, Physical Review Letters 84, 3795 (2000).