On Coordinating Collaborative Objects

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A collaborative object represents a data type (such as a text document) designed to be shared by a group of dispersed users. The Operational Transformation (OT) is a coordination approach used for supporting optimistic replication for these objects. It allows the users to concurrently update the shared data and exchange their updates in any order since the convergence of all replicas, i.e. the fact that all users view the same data, is ensured in all cases. However, designing algorithms for achieving convergence with the OT approach is a critical and challenging issue. In this paper, we propose a formal compositional method for specifying complex collaborative objects. The most important feature of our method is that designing an OT algorithm for the composed collaborative object can be done by reusing the OT algorithms of component collaborative objects. By using our method, we can start from correct small collaborative objects which are relatively easy to handle and incrementally combine them to build more complex collaborative objects.

Key words: Collaborative Editors, Operational Transformation, Component-based design, Algebraic Specifications.

1 Introduction

Motivation. Collaborative editors constitute a class of distributed systems where dispersed users interact by manipulating simultaneously some shared objects like texts, images, graphics, etc. To improve data availability, the shared data is replicated so that the users update their local data replicas and exchange their updates between them. So, the updates are applied in different orders at different replicas of the object. This potentially leads to divergent (or different) replicas – an undesirable situation for collaborative editors. Operational Transformation (OT) is an optimistic technique which has been proposed to overcome the divergence problem [2]. This technique consists of an algorithm which transforms an update (previously executed by some other user) according to local concurrent ones in order to achieve convergence. It is used in many collaborative editors including CoWord [9] and CoPowerPoint [9] (a collaborative version of MicroSoft Word and PowerPoint respectively), and the Google Wave (a new Google platform\[1].

It should be noted that the data consistency relies crucially on the correctness of an OT algorithm. According to [7], the consistency is ensured iff the OT algorithm satisfies two convergence properties \(TP1\) and \(TP2\) (that will be detailed in Section 2). Finding such an algorithm and proving that it satisfies \(TP1\) and \(TP2\) is not an easy task because it requires analyzing a large number of situations. Moreover, when we consider a complex object (such as a filesystem or an XML document that are composite of several primitive objects) the formal design of its OT algorithm becomes very tedious because of the large number of updates and coordination situations to be considered if we start from scratch.

Related Work. Research efforts have been focused on automatically verifying the correctness of OT algorithms by using either a theorem prover [6] or a model-checker tool [1]. To the best of our knowl-
edge, [5] is the first work that addresses the formal compositional design of OT algorithms. In this work, two static constructions (where the number of objects to combine is fixed) have been proposed for composing collaborative objects: (i) the first construction has as a basic semantic property to combine components without allowing these components to interact; (ii) as for the second one it enables components to communicate by means of a shared part.

**Contributions.** As continuation of [5], we propose in this paper how to combine an arbitrary number of collaborative objects by using a dynamic composition in such a way the objects are created and deleted dynamically. The most important feature of our method is that designing an OT algorithm for the composed collaborative object can be done by reusing the OT algorithms of component collaborative objects. By using our method, we can start from correct small collaborative objects (i.e., they satisfy convergence properties) which are relatively easy to handle and incrementally combine them to build more complex collaborative objects that are also correct.

**Roadmap.** This paper is organized as follows: in Section 2 we give the basic concepts of the OT approach. The ingredients of our formalization for specifying the collaborative object and OT algorithm are given in Section 3. In Section 4 we present how to specify the dynamic composition of collaborative objects in algebraic framework. Section 5 gives the correctness of our dynamic composition approach. Finally, we give conclusions and present future work.

## 2 Operational Transformation Approach

Due to high communication latencies in wide-area and mobile wireless networks the replication of collaborative objects is commonly used in distributed collaborative systems. But this choice is not without problem as we will see in next sub-section.

### 2.1 Convergence Problems

One of the significant issues when building collaborative editors with a replicated architecture and an arbitrary communication of messages between users is the *consistency maintenance* (or *convergence*) of all replicas. To illustrate this problem, consider the following example:

**Example 2.1** Consider the following group text editor scenario (see Figure 7): there are two users (sites) working on a shared document represented by a sequence of characters. These characters are addressed from 0 to the end of the document. Initially, both copies hold the string “efecte”. User 1 executes operation \( \text{op}_1 = \text{Ins}(1, “f”) \) to insert the character “f” at position 1. Concurrently, user 2 performs \( \text{op}_2 = \text{Del}(5) \) to delete the character “e” at position 5. When \( \text{op}_1 \) is received and executed on site 2, it produces the expected string “effect”. But, when \( \text{op}_2 \) is received on site 1, it does not take into account that \( \text{op}_1 \) has been executed before it and it produces the string “effece”. The result at site 1 is different from the result of site 2 and it apparently violates the intention of \( \text{op}_2 \) since the last character “e”, which was intended to be deleted, is still present in the final string.

To maintain convergence, an OT approach has been proposed in [2]. It consists of application-dependent transformation algorithm such that for every possible pair of concurrent updates, the application programmer has to specify how to merge these updates regardless of reception order. We denote this algorithm by a function \( IT \), called *inclusion transformation* [8].
Example 2.2 In Figure 2 we illustrate the effect of IT on the previous example. When \( op_2 \) is received on site 1, \( op_2 \) needs to be transformed in order to include the effects of \( op_1 \): \( IT((Del(5),Ins(1,"f"))) = Del(6) \). The deletion position of \( op_2 \) is incremented because \( op_1 \) has inserted a character at position 1, which is before the character deleted by \( op_2 \). Next, \( op_2' \) is executed on site 1. In the same way, when \( op_1 \) is received on site 2, it is transformed as follows: \( IT(Ins(1,"f"),Del(5)) = Ins(1,"f") \): \( op_1 \) remains the same because "f" is inserted before the deletion position of \( op_2 \). Intuitively we can write the transformation \( IT \) as follows:

\[
\text{IT}(\text{Ins}(p_1,c_1),\text{Ins}(p_2,c_2)) = \begin{cases} 
\text{Ins}(p_1,c_1) & \text{if } (p_1 < p_2) \\
\text{Ins}(p_1+1,c_1) & \text{else}
\end{cases}
\]

2.2 Transformation Properties

Notation \([op_1; op_2; \ldots; op_n]\) represents an operation sequence. We denote \( Do(X,st) = st' \) when an operation (or an operation sequence) \( X \) is executed on a replica state \( st \) and produces a replica state \( st' \).

Using an OT algorithm requires to satisfy two properties \([7]\), called transformation properties. Given three operations \( op, op_1 \) and \( op_2 \), with \( op_2' = IT(op_2,op_1) \) and \( op_1' = IT(op_1,op_2) \), the conditions are as follows:

- **Property TP1**: \( Do([op_1; op_2'], st) = Do([op_2; op_1'], st) \), for every state \( st \).
- **Property TP2**: \( IT(IT(op,op_1),op_2') = IT(IT(op,op_2),op_1') \).

TP1 defines a state identity and ensures that if \( op_1 \) and \( op_2 \) are concurrent, the effect of executing \( op_1 \) before \( op_2 \) is the same as executing \( op_2 \) before \( op_1 \). This condition is necessary but not sufficient when the number of concurrent operations is greater than two. As for TP2, it ensures that transforming \( op \) along equivalent and different operation sequences will give the same result. Properties TP1 and TP2 are sufficient to ensure the convergence property for any number of concurrent operations which can be executed in arbitrary order \([7]\).

3 Primitive Collaborative Objects

3.1 Basic Notions

In this sub-section we present terminology and notation that are used in the following sections. We assume that the reader is familiar with algebraic specifications. For more background on this topic see \([10, 4]\).
A many-sorted signature $\Sigma$ is a pair $(S,F)$ where $S$ is a set of sorts and $F$ is a $S^* \times S$-sorted set (of function symbols). Here, $S^*$ is the set of finite (including empty) sequences of elements of $S$. Saying that $f : s_1 \times \ldots \times s_n \rightarrow s$ is in $\Sigma = (S,F)$ means that $s_1 \ldots s_n \in S^*$, $s \in S$, and $f \in F_{s_1 \ldots s_n}$. A $\Sigma$-algebra $A$ interprets sorts as sets and operations as appropriately typed functions. A signature morphism $\Phi : \Sigma \rightarrow \Sigma'$ is a pair $(\mathsf{f}, \mathsf{g})$, such that $f : S \rightarrow S'$ and $g : \Sigma \rightarrow \Sigma'_{\mathsf{f}, \mathsf{g}}$, an $(S' \times S)$-sorted function. Usually, we ignore the distinction between $f$ and $g$ and drop all subscripts, writing $\Phi(s)$ for $f(s)$ and $\Phi(\sigma)$ for $g(\sigma)$ such that $\sigma \in F_{s_1 \ldots s_n}$. We denote the sort of booleans as $\mathsf{Bool}$.

Let $X$ be a family of sorted variables and let $T_\Sigma(X)$ be the algebra of $\Sigma$-terms. An equation is a formula of the form $l = r$ where $l, r \in T_\Sigma(X)$, for some sort $s \in S$. A conditional equation is a formula of the following form: $\bigwedge_{i=1}^n a_i = b_i \implies l = r$, where $a_i, b_i \in T_\Sigma(X)_{s_i}$. An algebraic specification is a pair $(\Sigma, E)$ where $\Sigma$ is a many-sorted signature and $E$ is a set of (conditional) $\Sigma$-equations, called axioms of $(\Sigma, E)$. A $(\Sigma, E)$-model is a $\Sigma$-algebra $A$ that satisfies all the axioms in $E$. We write $A \models E$ to indicate that $A$ is a $(\Sigma, E)$-model. Given a signature morphism $\Phi : \Sigma \rightarrow \Sigma'$ and $\Sigma$-algebra $A'$, the reduct of $A'$ to $\Sigma$, denoted $\Phi(A')$, represents carriers $A'_{\Phi(s)}$ for $s \in S$ and operations $\sigma_{\Phi(s)}$ for $\sigma \in S_{s_1 \ldots s_n}$. Given a $\Sigma$-equation $e$ of the form $l = r$. Then $\Phi(e)$ is $\Phi(l) = \Phi(r)$, where $\Phi : T_\Sigma(X) \rightarrow T_{\Sigma'}(X')$ and $X' = \Phi(X)$. An important property of these translations on algebras and equations under signature morphisms is called satisfaction condition, which expresses the invariance of satisfaction under change of notation:

**Theorem 3.1 (Satisfaction Condition [3])**. Given a signature morphism $\Phi : \Sigma \rightarrow \Sigma'$, a $\Sigma'$-algebra $A'$ and a $\Sigma$-equation $e$, $\Phi(A') \models e$ iff $A' \models E$.

An observational signature is a many-sorted signature $\Sigma = (S, S_{\text{obs}}, F)$ where $S_{\text{obs}} \subseteq S$ is the set of observable sorts. An Observational Specification is a pair $(\Sigma, E)$ where $\Sigma$ is an observational signature and $E$ is a set of axioms. We assume that axioms are conditional equations with observable conditions. A context is a term with exactly one occurrence of a distinguished variable, say $z$. Observable contexts are contexts of observable sort. Let $C_\Sigma(s, s')$ be the set of contexts of sort $s'$ that contain a distinguished variable of sort $s$. We write $c[t]$ for the replacement of distinguished variable $z$ by the term $t$. A $\Sigma$-algebra $A$ behaviorally satisfies an equation $l = r$, denoted $A \models^\Sigma_{\text{obs}} l = r$, iff $A \models^\Sigma c[t] = c[r]$ for every observable context $c$. A model of an observational specification $SP = (\Sigma, E)$ is a $\Sigma$-algebra $A$ that behaviorally satisfies every axioms in $E$. We write $A \models^\Sigma_{\text{obs}} SP$ or $A \models^\Sigma_{\text{obs}} E$. Also we write $E \models^\Sigma_{\text{obs}} e$ iff $A \models^\Sigma_{\text{obs}} E$ implies $A \models^\Sigma e$ where $e$ is a (conditional)-equation.

### 3.2 Component Specifications

Using Observational semantics we consider a Collaborative Object (CO) as a black box with a hidden (or non-observable) state [4]. We only specify the interactions between a user and an object. In the following, we give our formalization:

**Definition 3.2 (CO Signature)**. Given $S$ the set of all sorts, $S_b = \{\text{State}, \text{Meth}\}$ is the set of basic sorts and $S_d = S \setminus S_b$ is the set of data sorts. A CO signature $\Sigma = (S, S_{\text{obs}}, F)$ is an observational signature where the sort State is the unique non-observable sort. The set of function symbols $F$ is defined as follows:

1. $F_{\text{MethStateState}} = \{\text{Do}\}$, $F_{\text{MethMethMeth}} = \{\text{IT}\}$, $F_{\text{MethStateBool}} = \{\text{Poss}\}$, and $F_{0, s} = \emptyset$ for all other cases where $s \in S_b$ and $s \in S_b$.

2. A function symbol $f : s_1 \times s_2 \times \ldots \times s_n \rightarrow \text{Meth}$ is called a method if $s_1 \cdot s_2 \cdot \ldots \cdot s_n \in S_d$.

3. A function symbol $f : s_1 \times s_2 \times \ldots \times s_n \rightarrow s$ is called an attribute if: (i) $s_1 \cdot s_2 \cdot \ldots \cdot s_n$ contains only one State sort; and (ii) $s \in S_d$. 

We use $\Sigma, \Sigma', \Sigma_1, \Sigma_2, \ldots$, as variables ranging over CO signatures.

The states of a collaborative object are accessible using the function $Do$ which given a method and a state gives the resulting state provided that the execution of this method is possible. For this we use a boolean function $Poss$ that indicates the conditions under which a method is enabled. The OT algorithm is denoted by the function symbol $IT$ which takes two methods as arguments and produces another method.

**Definition 3.3 ($\Sigma$-Morphism).** Given CO signatures $\Sigma$ and $\Sigma'$, then a $\Sigma$-morphism $\Phi : \Sigma \rightarrow \Sigma'$ is a signature morphism such that: (i) $\Phi(s) = s$ for all $s \in S_d$; (ii) $\Phi(f) = f$ for all $f \in \Sigma_{\omega, s}$ where $\omega \in S_d^*$ and $s \in S_d$; (iii) $\Phi(S_b) = S'_b$ (where $S'_b = \{\text{State}', \text{Meth}'\}$, $\Phi(\text{State}) = \text{State}'$ and $\Phi(\text{Meth}) = \text{Meth}'$).

The three conditions stipulate that $\Sigma$-morphisms preserve State sort, observable sorts and functions.

**Definition 3.4 (Collaborative Component Specification).** A collaborative component specification is a tuple $C = \langle \Sigma, M, A, T, E \rangle$ where: (i) $\Sigma$ is a CO signature; (ii) $M$ is a set of method symbols, i.e. $M = \{m \mid m \in \Sigma_{\omega, \text{Meth}}$ and $\omega \in S_d^*\}$; (iii) $A$ is a set of attribute symbols, i.e. $A = \{a \mid a \in \Sigma_{\omega, s}$ where $\omega$ contains exactly one State sort and $s \in S_d^*\}$; (iv) $T$ is the set of axioms corresponding to the transformation function; (v) $E$ is the set of all axioms. We let $C, C', C_1, C_2, \ldots$, denote collaborative component specifications.

In the following, we assume that all used (conditional) equations are universally quantified.

**Example 3.5** The following component specification $CCHAR$ models a memory cell (or a buffer) which stores a character value:

```latex
spec CCHAR =
  sort: Char Meth State
  opns: Do : Meth State -> State
        putchar : Char State -> Meth
        getchar : State -> Char
        IT : Meth Meth -> Meth
        maxchar : Char Char -> Char
  axioms:
(1) getchar(Do(putchar(c),st)) = c;
(2) IT(putchar(c1),putchar(c2)) = putchar(maxchar(c1,c2));
```

$CCHAR$ has one method putchar and one attribute getchar. Axiom (2) gives how to transform two concurrent putchar in order to achieve the data convergence. For that, we use function maxchar that computes the maximum of two character values. Note we could have used another way to enforce convergence.

As the previous specification $CNAT$ and $CCOLOR$ model a memory cell which stores respectively a natural number value and a color value:

```latex
spec CNAT =
  sort: Nat Meth State
  opns: Do : Meth State -> State
        putnat : Nat State -> Meth
        getnat : State -> Nat
        IT : Meth Meth -> Meth
        minnat : Nat Nat -> Nat
  axioms:
(1) getnat(Do(putnat(n),st)) = n;
(2) IT(putnat(n1),putnat(n2)) = putnat(minnat(n1,n2));
```
spec CCOLOR =
sort:
Color Meth State
ops:
Do : Meth State -> State
putcolor : Color -> Meth
getcolor : State -> Color
IT : Meth Meth -> Meth
axioms:
(1) getcolor(Do(putcolor(cl1),st)) = cl1;
(2) IT(putcolor(cl1),putcolor(cl11)) = putcolor(mincolor(cl1,c12));

To get data convergence we have used in CNAT (resp. CCOLOR) another function minnat (resp. mincolor) that computes the minimum value. The sorts Char, Nat and Color are built-in. □

For a concise presentation and without loss of generality, we shall omit the observable-sorted arguments from methods and attributes. We could suppose we have one function for each of its possible arguments. For instance, method putchar(c) may be replaced by putchar(c) for every c ∈ CHAR.

Definition 3.6 ((MA)-Complete). Given a component specification ‘C = (Σ,M,A,T,E). The set E is (MA)-complete iff all equations involving M have the form C ⇒ a(Do(m,x)) = t, where x is a variable of sort State, a ∈ M, m ∈ M, t ∈ TΣ,M(\{x\}) and C is a finite set of visible pairs t1 = t1′, t2 = t2′, ..., tn = t′n where t1, t1′ ∈ TΣ(X)s1, t2, t2′ ∈ TΣ(X)s2, ..., tn, t′n ∈ TΣ(X)sn.

In Example 3.5, component specification CCHAR is (MA)-complete as the only axiom involving methods (i.e., axiom (1)) has the required form. CNAT and CCOLOR are also (MA)-complete. In the remaining of this paper, we restrict our intention to component specification which are (MA)-complete.

As a component specification has an observational signature with one non-observable sort, State, then the observable contexts have the following form: a(Do(m1, ..., Do(m1,s)) where m1, ..., mn are methods and a is an attribute.

Definition 3.7 (Specification morphisms). Given two collaborative component specifications ‘C = (Σ,M,A,T,E) and ‘C′ = (Σ′,M′,A′,T′,E′), a specification morphism Φ : ‘C → ‘C′ is a signature morphism Φ : Σ → Σ′ such that: (i) Φ(M) ⊆ M′; (ii) Φ(A) ⊆ A′; (iii) E′ =Σobs Φ(e) for each e ∈ E.

Definition 3.7 provides a support for reusing component specification through the notion of specification morphism. Moreover, it exploits the fact that the source component specification is (MA)-complete by only requiring the satisfaction of finite number of equations (see condition (iii)). Note that Definitions 3.3 and 3.7 have been used for defining the static composition that enables us to build up a composite object from a fixed number of other collaborative objects [5]. For instance, SIZEDCHAR is the composition of CCHAR and CNAT denoted by SIZEDCHAR = CCHAR ⊕ CNAT. This composition may be associated to an object with a character value and an attribute for modifying the font size. Due to limited space, the reader is referred to [5] for more details.

3.3 Convergence Properties

Before stating the properties that a component specification ‘C = (Σ,M,A,T,E) has to satisfy for ensuring convergence, we introduce some notations. Let m1, m2, ..., mn and s be terms of sorts Meth and State respectively:

1. applying a method sequence on a state is denoted as:

(s)[m1;m2;...;mn] ≜ Do(mn,...,Do(m2,Do(m1,s))...)
where $m_1$, $m_2$, and $m_3$ are two disjoint sets of methods, we define convergence:

**Definition 4.1 (Composition Pattern).**

Let $M' \subseteq M$ be a set of methods, we denote $CP1|_{M_1,M_2}$ as the restriction of $CP1$ to $M'$. Let $M_1,M_2 \subseteq M$ be two disjoint sets of methods, we define $CP1|_{M_1,M_2}$ as:

$$CP1|_{M_1,M_2} \triangleq (\text{Legal}(seq_1,s) = \text{true} \land \text{Legal}(seq_2,s) = \text{true}) \implies (s)seq_1 = (s)seq_2$$

where $seq_1 = [m_1; IT(m_2,m_1)]$ and $seq_2 = [m_2; IT(m_1,m_2)]$.

Let $M' \subseteq M$ be a set of methods, we denote $CP2|_{M_1}$ as the restriction of $CP2$ to $M'$. Let $M_1,M_2 \subseteq M$ be two disjoint sets of methods, we define $CP2|_{M_1,M_2}$ as:

$$CP2|_{M_1,M_2} \triangleq IT^*(m_3,[m_1; IT(m_2,m_1)]) = IT^*(m_3,[m_2; IT(m_1,m_2)])$$

such that $m' \in M_i$, $m'' \in M_j$ and $m \in M_k$ for all $i,j,k \in \{1,2\}$ with $k \neq i$ or $k \neq j$.

The following definition gives the conditions under which a component specification ensures the data convergence:

**Definition 3.8 (Consistency).** $\mathcal{C}$ is said consistent iff $\forall \mathcal{C} \models_{\text{obs}} CP1 \land CP2$.

## 4 Dynamic Composition

In this section, we present a construction that enables us to combine an arbitrary number of the same collaborative object according to a given structure (we will call it composition pattern). In other words, such objects are created and deleted dynamically. Thus, the obtained object has no static structure.

### 4.1 Basic Definitions

**Definition 4.1 (Composition Pattern).** A composition pattern is a parametric specification $\overline{\mathcal{C}} = (PA, \mathcal{C})$ where:

- $PA = (\Sigma_{PA}, E_{PA})$, called formal parameter, is an algebraic specification;
- $\mathcal{C} = (\Sigma, M, A, T, E)$, called body, is a collaborative component specification (or a component);
such that the following conditions hold: (i) $S_{PA} = \{\text{Elem, Bool}\}$; (ii) $\Sigma_{PA} \subset \Sigma$; (iii) $E_{PA} \subset E$; (iv) there exists a method symbol $m \in M$ containing at least one argument of Elem sort; this method is called parametric method (v) there exists an attribute symbol $a \in A$ such that either its result is of Elem sort or one of its arguments is Elem sort; $a$ is called parametric attribute.

We let $\mathcal{C}, \mathcal{C}_1, \mathcal{C}_2, \ldots$ denote the composition patterns. □

Example 4.2 The composition pattern $PSET = (PA, \mathcal{C})$ describes finite sets with parametric element:

Formal parameter $PA$ gives the properties of parameter sort Elem:

```
spec PA =
  sorts:
  Elem Bool
  opns:
  eq : Elem Elem -> Bool
  axioms:
  (1) eq(x,y)=eq(y,x);
  (2) eq(x,y)=true, eq(y,z)=true => eq(x,z)=true;
```

Body $\mathcal{C}$ is collaborative object representing data set of Elem sort:

```
spec C =
  sorts:
  Set Elem Bool
  opns:
  empty : -> Set
  Do : Meth Set -> Set
  nop : -> Meth
  add : Elem -> Meth
  remove : Elem -> Meth
  Poss : Meth Set -> Bool
  iselem : Elem Set -> Bool
  IT : Meth Meth -> Meth
  axioms:
  (1) Poss(nop, st)=true;
  (2) Poss(add(x), st)=true;
  (3) iselem(x, st)=true => Poss(remove(x), st)=true;
  (4) iselem(x, st)=false => Poss(remove(x), st)=false;
  (5) eq(x,y)=true => iselem(x, Do(add(y), st))=true;
  (6) eq(x,y)=false => iselem(x, Do(add(y), st))=iselem(x, st);
  (7) eq(x,y)=true => iselem(x, Do(remove(y), st))=false;
  (8) eq(x,y)=false => iselem(x, Do(remove(y), st))=iselem(x, st);
  (9) eq(x,y)=true => IT(add(x), add(y))=nop;
  (10) eq(x,y)=false => IT(add(x), add(y))=add(x);
  (11) IT(add(x), remove(y))=add(x);
  (12) eq(x,y)=true => IT(remove(x), remove(y))=nop;
  (13) eq(x,y)=false => IT(remove(x), remove(y))=remove(x);
  (14) IT(remove(x), add(y))=remove(x);
```

In the following definition, we give under which conditions a collaborative component can substitute a formal parameter in a composition pattern.

Definition 4.3 (Admissibility). Given $\mathcal{C} = (PA, \mathcal{C})$ a composition and $\mathcal{C}_1 = (\Sigma_1, M_1, A_1, T_1, E_1)$ a component such that $(\Sigma \setminus \Sigma_{PA}) \cap \Sigma_1 = \emptyset$ (i.e. no similar names). Component $\mathcal{C}_1$ is said admissible for $\mathcal{C}$ if for all axioms $e \in E_{PA}, E_1 \models_{\text{obs}} \Phi(e)$, where $\Phi : \Sigma_{PA} \to \Sigma_1$ is a signature morphism with $\Phi(\text{Elem}) = \text{State}_{\mathcal{C}_1}$ and $\Phi(\text{Bool}) = \text{Bool}$. □
Consider the character component $\text{CCHAR} = (\Sigma_1, M_1, A_1, T_1, E_1)$ given in Example 3.5. This component is admissible for the pattern PSET (see Example 4.2) by using the following morphism: \( \Phi(\text{Elem}) = \text{State}_{\text{CCHAR}} \) and \( \Phi(\text{eq}) = (=_{\text{CCHAR}}) \). This enables us to build up a set of characters.

**Definition 4.4 (Instantiation parameter).** Let \( \overline{C_1} = (PA, \xi_1) \) be a composition pattern. Given \( C_2 = (\Sigma_2, M_2, A_2, T_2, E_2) \) an admissible component for \( \overline{C_1} \) via a signature morphism \( \Phi : \Sigma_{PA} \rightarrow \Sigma_2 \). The instantiation of \( \overline{C_1} \) by \( C_2 \), denoted by \( \overline{C_1}[PA \leftarrow C_2]_\Phi \), is the specification \( (\Sigma, M, A, T, E) \) such that:

1. \( \Sigma = \Sigma_2 \cup \Phi(\Sigma_{PA}) \cup (\Sigma \setminus \Sigma_{PA}) \);
2. \( M = \Phi(M_1) \);
3. \( A = \Phi(A_1) \);
4. \( T = \Phi(T_1) \);
5. \( E = E_2 \cup \Phi(E_1) \).

\( \square \)

Although the above definition (see Definition 4.5) may seem rather complicated to understand, it is just a mathematical formulation of some simple ideas how to build a complex component – with dynamic structure – from a composition pattern \( \overline{C_1} \) and an admissible component \( C_2 \):

- The formal parameter of \( \overline{C_1} \) is replaced by an admissible component \( C_2 \) in order to build a new component \( C \).
- This new component \( C \) is extended by a new method \( \text{Update} \) whose role is to connect the \( \overline{C_1} \)'s state space with the \( C_2 \)'s state space. In other words, the use of \( \text{Update} \) means that changing the state of \( C_2 \) implies changing the state of \( \overline{C_1} \).
- Axioms given in (iv) show how to transform \( \text{Update} \). On the one hand, we have to add axioms to define how to transform \( \text{Update} \) against other methods of \( \overline{C_1} \). On the other hand, when modifying the same object of \( C_2 \) we use the transformation function related to \( C_2 \). But, the modification of two distinct objects of \( C_2 \) can be performed in any order (there is no interference).
- Axioms given in (v) state how attributes are altered by the method \( \text{Update} \).

**Definition 4.5 (Dynamic Composition).** Given a composition pattern \( \overline{C_1} = (PA, \xi_1) \), a component \( C_2 = (\Sigma_2, M_2, A_2, T_2, E_2) \) and a signature morphism \( \Phi : \Sigma_{PA} \rightarrow \Sigma_2 \). Let \( \text{Update} : s_1 \ldots s_n \cdot \text{State}_{C_2} \rightarrow \text{Meth} \) be a method symbol. The specification \( C = (\Sigma, M, A, T, E) \) is said a dynamic composition of \( C_2 \) with respect to \( \overline{C_1} \) (denoted \( \overline{C_1}[C_2] \)) iff \( C_2 \) is admissible for \( \overline{C_1} \) via \( \Phi \), and \( C = \overline{C_1}[PA \leftarrow C_2]_\Phi \cup \{ \text{Update} \} \) such that:

1. \( \Sigma = (S', F') \) with \( S' = S_2 \cup \Phi(S_1) \) and \( F' = \{ \text{Update} \} \).
2. \( M' = \{ \text{Update}(U, x, y) \mid x, y \) are variables of sort \( \text{State}_{C_2} \) and \( U \) is a variable of sort \( S^a \))
3. \( A' = \emptyset \);
4. Let \( u_1 = \text{Update}(U, x, Do_{C_2}(m_1, x)) \) and \( u_2 = \text{Update}(U', x', Do_{C_2}(m_2, x')) \) be two methods where \( m_1, m_2 \in C_2 \). For every method \( m \in \Phi(M_1) \), we have:

\[ T' = \text{Ax}(IT(u_1, m)) \cup \text{Ax}(IT(m, u_1)) \cup \text{Ax}(IT(u_1, u_2)) \]

such that \( \text{Ax}(IT(u_1, u_2)) \) contains the following axioms:

\[ U = U' \wedge x = x' \implies IT(u_1, u_2) = u'_1 \]
\[ x \neq x' \implies IT(u_1, u_2) = u_1 \]
\[ U \neq U' \implies IT(u_1, u_2) = u_1 \]

with \( u'_1 = \text{Update}(U, Do_{C_2}(m_2, x'), Do_{C_2}(IT_{C_2}(m_1, m_2), Do_{C_2}(m_2, x'))) \).
(v). For each attribute symbol \( a : s'_1 \ldots s'_m \to s' \), we have
\[
E' = \text{Ax}(\text{Poss}(\text{Update}(U,x,y),st)) \cup \text{Ax}(a(\text{Do}(\text{Update}(U,x,y),st)))
\]
where \( \text{Ax}(a(\text{Do}(\text{Update}(V,x,y),st))) \) is defined as follows:
(a) \( a \) is the instance of a parametric attribute whose one of its arguments is of sort \( \Phi(\text{Elem}) \):
\[
C[Z,x',U,x,y,\text{st}] \implies a(Z,x',\text{Do}(\text{Update}(U,x,y),\text{st})) = \text{cst}
\]
\[
\neg C[Z,x',U,x,y,\text{st}] \implies a(Z,x',\text{Do}(\text{Update}(U,x,y),\text{st})) = a(Z,x',\text{st})
\]
with \( \text{cst} \) is constant of sort \( s' \) and \( C[Z,x',V,x,y,\text{st}] \) is its negation is a formula (containing free variables) built up of conjunction of observable equations in such a way that \( C[Z,x',U,x,y,\text{st}] \land C[Z,x',U',x,y,\text{st}] \) is false whenever \( U \neq U' \).
(b) \( a \) is the instance of a parametric attribute with \( s' = \Phi(\text{Elem}) \):
\[
C'[Z,U,\text{st}] \implies a(Z,\text{Do}(\text{Update}(U,x,y),\text{st})) = y
\]
\[
\neg C'[Z,U,\text{st}] \implies a(Z,\text{Do}(\text{Update}(U,x,y),\text{st})) = a(Z,\text{st})
\]
where \( C'[Z,U,\text{st}] \) (and its negation) is a formula (containing free variables) built up of conjunction of observable equations in such a way that \( C'[Z,U,\text{st}] \land C'[Z,U',\text{st}] \) is false whenever \( U \neq U' \).
(c) \( a \) is not the instance of a parametric attribute: \( a(Z,\text{Do}(\text{Update}(U,x,y),\text{st})) = a(U,\text{st}) \).

The notation \( \text{Ax}(f) \) means the set of axioms used for defining function \( f \). □

![Figure 3: Dynamic Composition.](image)

**Example 4.6** Figure 4.1 shows the dynamic composition of CCHAR (see Example 3.5) with respect to PSET (see Example 4.2), via the following morphism \( \Phi(\text{Elem}) = \text{State}_{\text{CCHAR}} \) and \( \Phi(\text{eq}) = (=_{\text{obs}}) \). Note that \( \Theta \) and \( \Theta' \) are only inclusion morphisms [10, 4]. The composition proceeds by the following steps:

1. The instantiation of PSET via \( \Phi \), i.e. \( \text{SETCHAR} = \Phi(\text{PSET}) \).
2. Add to SETCHAR a new method \( \text{Update} : \text{State}_{\text{CCHAR}} \text{State}_{\text{CCHAR}} \to \text{Meth} \) with the following axioms:
   (a) Transforming Update methods (see Definition 4.5.(iv)):
   \[
   \begin{align*}
   (16) & \; c1 = c2 \implies \text{IT}(\text{Update}(c1,c2),\text{Update}(c3,c4)) = \text{Update}(c4,c') \\
   (17) & \; c1 \not\sim c2 \implies \text{IT}(\text{Update}(c1,c2),\text{Update}(c3,c4)) = \text{Update}(c1,c2)
   \end{align*}
   \]
   where \( c' = \text{Do}_{\text{CCHAR}}(\text{IT}_{\text{CCHAR}}(m1,m2),c4) \), \( m1 \) and \( m2 \) are methods of CCHAR such that \( c2 = \text{Do}_{\text{CCHAR}}(m1,c1) \) and \( c4 = \text{Do}_{\text{CCHAR}}(m2,c3) \).
(b) Axioms for defining function Poss:

(18) iselem(c, st) = true => Poss(Update(c, c'), st) = true
(19) iselem(c, st) = false => Poss(Update(c, c'), st) = false

(c) axioms for all attributes observing the effects of Update (see Definition 4.5(v)):

(20) c = c2 => iselem(c, Do(Update(c1, c2), st)) = true
(21) c1 ≠ c2 => iselem(c, Do(Update(c1, c2), st)) = iselem(c, st)

4.2 Illustrative Example

In word processor softwares (such as MicroSoft Word), a document has a hierarchical structure. It contains not only text but also formatting objects (font, color, size, etc). Typically, a document is divided into pages, paragraphs, phrases, words and characters. A formatting object may be found in each of these levels. Several collaborative editors rely on this document structure, as CoWord [9] that is a collaborative version of MicroSoft Word. Now we will present how to model this document structure using a dynamic composition. Note that each level has a linear structure, except of characters. So, we use a composition pattern STRING that represents a sequence of elements. The formal parameter Elem of STRING can be substituted by any component. Moreover, this pattern has two methods: (i) Ins(p, e, n) to add element e at position p; (ii) Del(p, n) to remove the element at at position p. The argument n is the identity of the issuer (user or) site.

Suppose we want to equip the document with formatting objects such as the size and color. So, consider the components CCHAR (a character component), CNAT (a size component) and CCOLOR (a color component) (see Example 3.5). The basic element in our structure document is the formatted character (an object character with color and size attributes), FCHAR that is obtained by a static composition [5]:

\[ FCHAR = CCHAR \oplus CNAT \oplus CCOLOR. \]

A formatted word is a sequence of formatted characters that is built up by dynamic and static compositions:

\[ \text{WORD} = \text{STRING}[FCHAR] \]
\[ \text{FWORD} = \text{WORD} \oplus \text{CNAT} \oplus \text{CCOLOR}. \]

The remaining levels are built up in the same way:

\[ \text{SENTENCE} = \text{STRING}[\text{FSENTENCE}] \]
\[ \text{FSENTENCE} = \text{SENTENCE} \oplus \text{CNAT} \oplus \text{CCOLOR} \]
\[ \text{PARAGRAPH} = \text{STRING}[\text{FSENTENCE}] \]
\[ \text{FPARAGRAPH} = \text{PARAGRAPH} \oplus \text{CNAT} \oplus \text{CCOLOR} \]
\[ \text{PAGE} = \text{STRING}[\text{FPARAGRAPH}] \]
\[ \text{FPAGE} = \text{PAGE} \oplus \text{CNAT} \oplus \text{CCOLOR} \]

5 Correctness

In this section, we present the correctness of our dynamic composition by enumerating the following properties.

Lemma 5.1 Let \( a : s_1 \ldots s_n \text{ State} \rightarrow s \) be an attribute such that \( a \) is the instance of a parametric attribute. Given two methods \( u_1 = \text{Update}(U, x, x') \) and \( u_2 = \text{Update}(V, y, y') \). If \( U \neq V \) or \( x \neq y \) then:

\[ a(Z, (st)[u_1; u_2]) = a(Z, (st)[u_2; u_1]) \]

for all states \( st \). □

Proof. Two cases are considered:

First case: there is only one argument \( s_i = \Phi(Elem) = \text{State}_{C2} \) with \( i \in \{1, \ldots, n\} \) such that: \( a : s_1 \ldots s_{n-1} \text{ State}_{C2} \text{ State} \rightarrow s \). According to Definition 4.5 we have:

\[ a(Z, z, (st)[\text{Update}(U, x, x'); \text{Update}(V, y, y')]) = a(Z, z, (st)[\text{Update}(V, y, y'); \text{Update}(U, x, x')]) \]
To prove Equation (1) we have to consider two cases:

1. **Basis induction:**
   
   (a) if \( C[Z, z, U, x, x', st] \land C[Z, z, V, y, y', st] \) is true then \( cst = cst' \);
   
   (b) if \( C[Z, z, U, x, x', st] \land C[Z, z, V, y, y', st] \) is true then \( cst = cst' \);
   
   (c) if \( C[Z, z, U, x, x', st] \land C[Z, z, V, y, y', st] \) is true then \( cst = cst' \);
   
   (d) if \( C[Z, z, U, x, x', st] \land C[Z, z, V, y, y', st] \) is true then \( a(Z, z, st) = a(Z, z, st) \);

2. **Second case:** \( s = \Phi(Elem \subset State_{E_2}) \) such that: \( a : s_1 \ldots s_{n-1} \rightarrow State_{E_2} \).

   According to Definition [4.5], we have:

   \[ a(Z, (st)[\text{Update}(U, x, x'); \text{Update}(V, y, y')]) = a(Z, (st)[\text{Update}(V, y, y'); \text{Update}(U, x, x')]) \]

   So, we have the following cases:

   (a) if \( C'[Z, U, st] \land C'[Z, U', st] \) is true then \( x' = x' \);

   (b) if \( C'[Z, U, st] \land C'[Z, U, st] \) is true then \( x' = x' \);

   (c) if \( C'[Z, U, st] \land C'[Z, U, st] \) is true then \( a(Z, st) = a(Z, st) \); \( \square \)

If two \textit{Update} methods \( u_1 \) and \( u_2 \) modify two distinct objects respectively then both sequences \([u_1; u_2]\) and \([u_2; u_1]\) have the same effect.

**Lemma 5.2** Let \( u_1 = \text{Update}(U, x, x') \) and \( u_2 = \text{Update}(V, y, y') \) be two methods. For all states \( st \), if \( U \neq V \) or \( x \neq y \) then \((st)[u_1; u_2] =_{\text{obs}} (st)[u_2; u_1] \). \( \square \)

**Proof.** Consider an arbitrary context \( C[st] = a \cdot m_1 \ldots \cdot m_n \) for \( n > 0 \) with \( a \in A \) and \( m_i \in M \) such that \( i \in \{1, \ldots, n\} \). Next we have: \( C[(st)[u_1; u_2]] = C[(st)[u_2; u_1]] \).

It is sufficient to prove by induction that:
\[ a(Z, (st)[u_1; u_2; m_1(X_1); \ldots; m_n(X_n))] = a(Z, (st)[u_2; u_1; m_1(X_1); \ldots; m_n(X_n)]) \]

**Basis induction:** For \( n = 0 \) and \( C[st] = a \) we have:
\[ a(Z, (st)[u_1; u_2]) = a(Z, (st)[u_2; u_1]) \] \hspace{1cm} (1)

To prove Equation (1) we have to consider two cases:

(i) \( a \) is the instance of a parametric attribute: Equation (1) is then true by using Lemma 5.1.

(ii) \( a \) is not the instance of a parametric attribute: According to Definition 4.5 we have:
\[ a(Z, (st)[u_1; u_2]) = a(Z, st) \] and \( a(Z, (st)[u_2; u_1]) = a(Z, st) \).
Induction hypothesis: For \( n > 0 \) \( a(Z, (st)[u_1; u_2; m_1(X_1); \ldots; m_n(X_n)]) = a(Z, (st)[u_2; u_1; m_1(X_1); \ldots; m_n(X_n)]) \)

Induction step: We show now if \( C'[st] = a \cdot m_1 \cdot \ldots \cdot m_n \cdot m_{n+1} \) then \( C[(st)[u_1; u_2]] = C[(st)[u_2; u_1]] \). Let \( st_1 = (st)[u_1; u_2; m_1(X_1); \ldots; m_n(X_n)] \) and \( st_2 = (st)[u_2; u_1; m_1(X_1); \ldots; m_n(X_n)] \). By induction hypothesis we deduce that \( st_1 = \text{obs} st_2 \). As \( = \text{obs} \) is a congruence then \( a(Z, (st_1)[m_{n+1}]) = a(Z, (st_2)[m_{n+1}]) \).

The dynamic composition of a consistent component with respect to a consistent composition pattern produces a new component that satisfies \( CP1 \) for all \( \text{Update} \) methods.

**Theorem 5.3** Given a composition pattern \( \mathcal{C}_1 = (PA, \mathcal{C}_1) \) and a component \( \mathcal{C}_2 = (\Sigma_2, M_2, A_2, T_2, E_2) \). Let \( \mathcal{C} = (\Sigma, M, A, T, E) \) be the dynamic composition of \( \mathcal{C}_2 \) with respect to \( \mathcal{C}_1 \). If \( \mathcal{C}_1 \) and \( \mathcal{C}_2 \) are consistent then \( E \vdash \text{obs} CP1 \mid M \) with \( M' \) is the set of \( \text{Update} \) methods.

**Proof.** \( CP1 \mid M' \) is defined as follows:

\[
(st)[\text{Update}(X, u, v); IT(\text{Update}(Y, u', v'), \text{Update}(X, u, v))] =
(st)[\text{Update}(Y, u', v'); IT(\text{Update}(X, u, v), \text{Update}(Y, u', v'))]
\]

where \( v = \text{Do}_{\mathcal{C}_2}(m_1(V), u) \) and \( v' = \text{Do}_{\mathcal{C}_2}(m_2(W), u') \) with \( m_1 \) and \( m_2 \) are methods in \( \mathcal{C}_2 \). According to Definition 4.5 we consider two cases:

**First case:** \( X = Y \) and \( u = u' \)

\( CP1 \mid M' \) is rewritten as follows:

\[
(st)[\text{Update}(X, u, v); \text{Update}(Y, v, Do_{\mathcal{C}_2}(IT_{\mathcal{C}_2}(m_2(W), m_1(V)), v))]
= (st)[\text{Update}(Y, u', v'); \text{Update}(X, v', Do_{\mathcal{C}_2}(IT_{\mathcal{C}_2}(m_1(V), m_2(W)), v'))]
\]

As \( v = \text{Do}_{\mathcal{C}_2}(m_1(V), u), v' = \text{Do}_{\mathcal{C}_2}(m_2(W), u) \) and \( \mathcal{C}_2 \) is consistent then

\[
\text{Do}_{\mathcal{C}_2}(IT_{\mathcal{C}_2}(m_2(W), m_1(V)), v) = \text{Do}_{\mathcal{C}_2}(IT_{\mathcal{C}_2}(m_1(V), m_2(W)), v') = u''
\]

Thus we get: \( (st)[\text{Update}(X, u, v); \text{Update}(Y, v, u'')] = (st)[\text{Update}(Y, u', v'); \text{Update}(X, v', u'')] \) that is true.

**Second case:** \( X \neq Y \) or \( u \neq u' \)

\( CP1 \mid M' \) is rewritten as follows:

\[
(st)[\text{Update}(X, u, v); \text{Update}(Y, u', v')] = (st)[\text{Update}(Y, u', v'); \text{Update}(X, u, v)]
\]

This equation is always true according to Lemma 5.2.

The dynamic composition of a consistent component with respect to a consistent composition pattern produces a new component that satisfies \( CP2 \) for all \( \text{Update} \) methods.

**Theorem 5.4** Given a composition pattern \( \mathcal{C}_1 = (PA, \mathcal{C}_1) \) and a component \( \mathcal{C}_2 = (\Sigma_2, M_2, A_2, T_2, E_2) \). Let \( \mathcal{C} = (\Sigma, M, A, T, E) \) be the dynamic composition of \( \mathcal{C}_2 \) with respect to \( \mathcal{C}_1 \). If \( \mathcal{C}_1 \) and \( \mathcal{C}_2 \) are consistent then \( E \vdash \text{obs} CP2 \mid M \) with \( M' \) is the set of \( \text{Update} \) methods.

**Proof.** Let \( up \triangleq \text{Update}(R, v, w), up_1 \triangleq \text{Update}(P, x, y) \) and \( up_2 \triangleq \text{Update}(Q, z, t) \) be three methods, where \( w = \text{Do}_{\mathcal{C}_2}(m(Z), v), y = \text{Do}_{\mathcal{C}_2}(m_1(V), x) \) and \( t = \text{Do}_{\mathcal{C}_2}(m_2(W), z) \) with \( m, m_1 \) and \( m_2 \) are methods in \( \mathcal{C}_2 \). Condition \( CP2 \mid M' \) is defined as follows:

\[
IT^*(up, [up_1; IT(up_2, up_1)]) = IT^*(up, [up_2; IT(up_1, up_2)])
\]
According to Definition 4.5 we consider two cases:

First case: $P = Q$ and $x = z$

CP2 $|_{M'}$ is rewritten as $IT^*(up, [up_1; up_2]) = IT^*(up, [up_2; up_1'])$ where:

$$up_1' \triangleq \text{Update}(P, Do\_\mathcal{E}_2(m_2(W), z), Do\_\mathcal{E}_2(\text{IT}_\mathcal{E}_2(m_1(V), m_2(W)), Do\_\mathcal{E}_2(m_2(W), z)))$$

and

$$up_2' \triangleq \text{Update}(Q, Do\_\mathcal{E}_2(m_1(V), x), Do\_\mathcal{E}_2(\text{IT}_\mathcal{E}_2(m_2(W), m_1(V)), Do\_\mathcal{E}_2(m_1(V), x)))$$

Two cases are possible:

1. $R = P$ and $v = x$. In this case we get:

$$\text{Update}(R, u_1, Do\_\mathcal{E}_2(m', u_1)) = \text{Update}(R, u_2, Do\_\mathcal{E}_2(m'', u_2))$$

where

$$u_1 \triangleq Do\_\mathcal{E}_2(\text{IT}_\mathcal{E}_2(m_2(W), m_1(V)), Do(m_1(V), x))$$

$$m' \triangleq \text{IT}_\mathcal{E}_2^*(m(Z), [m_1(V); \text{IT}_\mathcal{E}_2(m_2(W), m_1(V))])$$

$$u_2 \triangleq Do\_\mathcal{E}_2(\text{IT}_\mathcal{E}_2(m_1(V), m_2(W)), Do(m_2(W), z))$$

$$m'' \triangleq \text{IT}_\mathcal{E}_2^*(m(Z), [m_2(W); \text{IT}_\mathcal{E}_2(m_1(V), m_2(W))])$$

Since $\mathcal{E}_2$ is consistent, then $u_1 = u_2$ and $m' = m''$. Consequently, the above equation is true.

2. $R \neq P$ or $v \neq x$. We have $IT^*(up, [up_1; up_2']) = up$ and $IT^*(up, [up_2; up_1']) = up$.

Second case: $P = \bar{Q}$ or $x \neq z$ CP2 $|_{M'}$ is rewritten as follows:

$$IT^*(\text{Update}(R, v, w), [\text{Update}(P, x, y); \text{Update}(Q, z, t)]) =$$

$$IT^*(\text{Update}(R, v, w), [\text{Update}(Q, z, t); \text{Update}(P, x, y)])$$

Three cases are considered:

1. $R = P$ and $v = x$. We get:

$$\text{Update}(R, Do\_\mathcal{E}_2(m_1(V), x), Do\_\mathcal{E}_2(\text{IT}_\mathcal{E}_2(m(Z), m_1(V)), Do(m_1(V), x))) =$$

$$\text{Update}(R, Do\_\mathcal{E}_2(m_1(V), x), Do\_\mathcal{E}_2(\text{IT}_\mathcal{E}_2(m(Z), m_1(V)), Do(m_1(V), x)))$$

2. $R = Q$ and $v = z$. We get:

$$\text{Update}(R, Do\_\mathcal{E}_2(m_2(W), z), Do\_\mathcal{E}_2(\text{IT}_\mathcal{E}_2(m(Z), m_2(W)), Do(m_2(W), z))) =$$

$$\text{Update}(R, Do\_\mathcal{E}_2(m_2(W), z), Do\_\mathcal{E}_2(\text{IT}_\mathcal{E}_2(m(Z), m_2(W)), Do(m_2(W), z)))$$

3. $R \neq P$, $R \neq Q$, $v \neq x$ or $v \neq z$. We get $\text{Update}(R, v, w) = \text{Update}(R, v, w)$.

The following theorem is very important since it stipulates that the consistency property is preserved by dynamic composition.

**Theorem 5.5** Given a consistent composition pattern $\mathcal{C}_1 = (P, \mathcal{E}_1)$ and a consistent component $\mathcal{E}_2 = (\Sigma_2, M_2, A_2, T_2, E_2)$. Let $\mathcal{C} = (\Sigma, M, A, T, E)$ be the dynamic composition $\mathcal{E}_2$ with respect to $\mathcal{E}_1$ via the morphism $\Phi$. If $E \equiv_{\text{obs}} CP1 |_{M', \Phi(M_1)}$ and $E \equiv_{\text{obs}} CP2 |_{M', \Phi(M_1)}$ then $\mathcal{C}$ is consistent where $M'$ is the set of update methods. □
Proof. Assume that $E \models_{obs} CP_1 \mid_{M', \Phi(M_1)}$ and $E \models_{obs} CP_2 \mid_{M', \Phi(M_1)}$. By definition, $\mathcal{C}$ is consistent iff $E \models_{obs} CP_1 \land CP_2$.

1. Proof of $E \models_{obs} CP_1$. Condition $CP_1$ can be expressed as follows:

$$CP_1 \triangleq CP_1 \mid_{M'} \land \Phi(CP_1 \mid_{M_1}) \land CP_1 \mid_{M', \Phi_2(M_1)}$$

As $\mathcal{C}_1$ is consistent and according to Theorem 5.3, $CP_1$ is then satisfied.

2. Proof of $E \models_{obs} CP_2$. Condition $CP_2$ can be given as follows:

$$CP_2 \triangleq CP_2 \mid_{M'} \land \Phi(CP_2 \mid_{M_1}) \land CP_2 \mid_{M', \Phi_2(M_1)}$$

Since $\mathcal{C}_1$ is consistent then $CP_2$ is true (By Theorem 5.4). □

6 Conclusion

In this work, we have proposed a formal component-based design for composing collaborative objects. We have dealt with the composition of arbitrary number of collaborative objects by using a dynamic composition in such a way the objects are created and deleted dynamically. Moreover, we have provided sufficient conditions for preserving $TP_1$ and $TP_2$ by the dynamic composition.

As future work, we intend to study the semantic properties of static and dynamic compositions.

Finally, we want to implement these compositions on top of the verification techniques given in [6] [1].

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