Research Article
Equilibrium Further Studied for Combined System of Cournot and Bertrand: A Differential Approach

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Abstract
In general, quantity competition and price competition exist simultaneously in a dynamic economy system. Whether it is quantity competition or price competition, when there are more than three companies in one market, the equilibrium points will become chaotic and are very difficult to be derived. This paper considers generally dynamic equilibrium points of combination of the Bertrand model and Cournot model. We analyze general equilibrium points of the Bertrand model and Cournot model, respectively. A general equilibrium point of the combination of the Cournot model and Bertrand model is further investigated in two cases. The theory of spatial agglomeration and intermediate value theorem are introduced. In addition, the stability of equilibrium points is further illustrated on celestial bodies motion. The results show that at least a general equilibrium point exists in combination of Cournot and Bertrand. Numerical simulations are given to support the research results.

1. Introduction
Quantity competition and price competition of oligopoly was debated for many years in an imperfectly competitive market. Some theorists [1–7] pointed out that the competitive equilibrium points, respectively, exist in the Cournot (quantity) model or Bertrand (price) model and compared the equilibrium with each other. Such as Singh and Vives first compared Cournot competition and Bertrand competition. For quantity equilibrium, the quantity in Cournot competition is larger than that in Bertrand competition in a vertically related duopoly [8]. On the condition that positive network externalities, Pal [9] pointed out that Cournot competition will yield higher profits [10] and prices and lower quantities, welfare [11], and consumers surplus than Bertrand competition. Haraguchi and Matsumura [12] revealed that if the number of companies is five or more, quantity competition may yield a larger profit for each company. Some scholars [1, 13–15] thought that Cournot competition is less efficient than Bertrand competition. Häckner [16] developed Singh and Vives, for more than two companies, and if commodities are complementary, prices may be lower under quantity competition than under price competition. That is called small profit but quick sale in marketing strategy. However, if commodities are substituted, it is beneficial to high-quality companies that they may earn higher profits under price competition than under quantity competition.

For price equilibrium, Vives [17] further pointed out, if the Bertrand equilibrium is unique, then it results are in lower prices than any Cournot equilibrium. Okuguchi [18] compared the equilibrium prices of the Bertrand competition and Cournot oligopolies with product differentiation, and under certain conditions, the Bertrand equilibrium prices are lower than the Cournot ones. Hamilton et al. [19] pointed out, for spatial discrimination of Bertrand competition and Cournot competition, equilibrium prices are lower under Bertrand competition, for low transport costs, profits are higher under Cournot competition, but for larger values of costs, profits are higher under the Bertrand model. The result is contrary to research findings [1].

Motivated from the above, it is not obvious that which kind of competition is more efficient. In reality, quantity competition and price competition exist simultaneously. Both quantity and price treated simultaneously as independent variables are very difficult to analyze. The proof of its uniqueness of the equilibrium point of combination of...
quantity and price becomes more difficult [20]. The equilibrium may be duplicated [21]. As the number of competing companies increases, the equilibrium of quantity and price will present unstable and chaotic phenomenon. A four-oligopoly mixed game of Cournot and Bertrand was detected, which is a complexity and chaotic system and showed the equilibrium point existed [22], and the stability region of the Cournot–Bertrand mixed model is bigger than that of the Cournot model or Bertrand model [23]. When the dynamic system of Bertrand and Cournot becomes stable, only one Nash equilibrium point exists [24]. How to find the general equilibrium points for combination of the Cournot model and Bertrand model is the question we address here.

For equilibrium of competition, some scholars showed the equilibrium results from spatial agglomeration. Such as Hotelling [25]’s linear city model, it concluded that the duopoly companies agglomerate at the center of the line model [26]; however, at center, there exists only a unique location equilibrium. With linear demand in every market of the Cournot game with spatial location choice, the number of companies of the Cournot model is more than two, and there exists a Nash location equilibrium [27]. A unique equilibrium point exists for spatial oligopoly [28]. According to Mayer [29], it appeared that central agglomeration equilibrium only arises in the particular case. d’Aspremont and Thisie [30] modified slightly for the version of Hotelling’s example, a price equilibrium solution exists everywhere. Pal and Sarkar [31] pointed out that spatial Cournot competition agglomerates at discrete equilibrium points. Gupta et al. [32] derived the results that an equilibrium exists for agglomeration of companies; however, the only equilibrium exists for agglomeration of duopolists, and a dispersed equilibrium exhibited for duopolists. Matsushima [33] revisited Cournot competition and spatial agglomeration, in a circular city, companies agglomerate at two points. Whether it is price competition or quality competition, many equilibrium points also exist [34], the equilibrium location exhibits spatial agglomeration by pairs [35]. However, the number of equilibrium points is not detected. In equilibrium points, the firms can charge a relative high price, and the prices are randomized [36]. So, it is noteworthy that there are many equilibrium points in oligopoly competition.

In this paper, we further develop Matsushima [33] and Levitan and Shubik [20], explore the general equilibrium points for combination of Bertrand and Cournot from two stages on the basis of spatial agglomeration, and compare with monopoly. The results show that at least a general equilibrium point of combination of Cournot and Bertrand exists. Numerical simulations are given to support the research results. The paper is organized as follows. Section 2 analyzes the complex dynamical behaviors and general equilibrium point of Cournot competition and Bertrand competition separately and compares the general equilibrium aggregate profits, the general equilibrium price, and the general equilibrium quantity with monopoly. Section 3 takes into account combination of the Cournot model and Bertrand model. The final section concludes the paper.

Notations of selected symbols used in this paper are summarized in Table 1.

### Table 1: Symbol notation.

| Symbol | Meaning |
|--------|---------|
| $C_i(q_i)$ | Cost function of $i$ company |
| $m_i$ | Share of the market |
| $p$ | Price |
| $q$ | Quantity |
| $a$, $b$ | Parameters of inverse demand function model |
| $a$, $\beta$ | Parameters of demand function model |
| $q_{im}$ | General equilibrium quantity |
| $p_m$ | General equilibrium price |
| $\pi_m$ | General equilibrium profit |
| $\pi_{monopoly}$ | Profit of monopoly |
| $q_{qc}$ | General equilibrium quantity of combination |
| $p_{qc}$ | Price of general equilibrium quantity of combination |
| $\pi_{qc}$ | Profit of general equilibrium quantity of combination |
| $\pi_{pc}$ | Profit of general equilibrium price of combination |

### 2. Model and Analysis

2.1. Cournot Model. Based on the inverse demand function $p_i = a - bq$ and $Q = q_1 + q_2$, $i = 1, 2$, parameter $a > 0$ and $b > 0$, the dynamic Cournot model can be established. $p_i$ is the selling price. $q_i(t)$ denotes the quantity of $i$th company during the period $t = 0, 1, 2, \ldots$. Suppose that the products are identical in a market. The cost function is defined as $C_i(q_i) = c_i q_i^3$, $i = 1, 2$. $c_i$ is the industry marginal cost.

Then, the profit of the two companies can be obtained:

\[
\begin{align*}
\Pi_1(q_1, q_2) &= (a - b(q_1 + q_2))q_1 - c_1 q_1^3, \\
\Pi_2(q_1, q_2) &= (a - b(q_1 + q_2))q_2 - c_2 q_2^3.
\end{align*}
\]

The 1st company maximizes the profit with respect to $q_1$ and the profit of 2nd company with respect to $q_2$. The dynamic Cournot model is established:

\[
\begin{align*}
q_1(t + 1) &= a - 2bq_1(t) - bq_2(t) - 3c_1 q_1^2(t), \\
q_2(t + 1) &= a - 2bq_2(t) - bq_1(t) - 3c_2 q_2^2(t).
\end{align*}
\]

The equilibrium points of system (2) is complex. Bifurcation diagrams, attractors, and global maximal Lyapunov exponent [37] are used to show complex dynamical behaviors. The dynamic characteristics of map (2) can be showed by a numerical simulation. It is performed by fixing the parameters of model as follows: $b = 0.3$, $c_1 = 0.1$, $c_2 = 0.2$, and $0.1 < a < 2.3$. Criteria for selected parameters is referenced in the literature [38–40]. Figure 1 exhibits the bifurcation diagrams of $q_1$ and $q_2$ separately with the parameter changing. Figure 2 presents the bifurcation diagram of $q_1$ and $q_2$. Figure 3 displays the global maximal Lyapunov exponents. It shows that system (2) presents complex behaviors and becomes chaos. The attractor of the system further is explored. Figure 4 shows that the attractor of the system exists. It suggests the existence of deterministic chaotic.

From Figures 1 and 2, there are many Nash equilibrium points in system (2). It is very difficult to derive the number of equilibrium points.
2.2. General Equilibrium Point of Quantity Competition.

In duopoly Cournot competition, the dispersion equilibria exist [41], and we can get the inner optional equilibrium output. However, when the number of companies is increasing, the equilibrium points cannot be derived. Gao and Du [42] further investigated general equilibrium points for the Cournot model. The aggregate production of companies is assumed to be

$$Q = q_1 + q_2 + \cdots + q_n. $$

Their products are identical or similar. If product differentiation is higher, it may destabilize the Nash equilibrium of Cournot [43]. On the identical or similar products, the market price is defined as follows: $P(Q) = P(q_1 + q_2 + \cdots + q_n)$. The cost function of any company assumes $C_i(q_i)$, $C_i(q_i) = c_i q_i^j$, $i = 1, 2, \ldots, n$, $j = 1$. $c_i$ is the marginal cost.

The model is expressed as

$$P(q_j) = a - b \sum_{j=1}^n q_j, \quad j = 1, 2, \ldots, n. \quad (3)$$

Figure 1: Bifurcation diagrams.

Figure 2: $q_1$ and $q_2$ bifurcation diagram.

Figure 3: The global maximum Lyapunov exponent.

Figure 4: $q_1$ and $q_2$ attractors.
The profit of company $i$ can be denoted:
\[
\Pi_i(q_1, q_2, \ldots, q_n) = P(q_1 + q_2 + \cdots + q_n)q_i - C_i(q_i).
\] (4)

When $i = j = 1$, the optimal profit condition of the $i$th company is satisfied as
\[
f'(Q) = f'(q) = -2bq + (a - c) = 0,
\] (5)
with parameters
\[
a > 0, \\
b > 0, \\
a > c.
\] (6)

If there exists only a company in one market, the market structure is monopoly. From (5), its production is
\[
q = \frac{a - c}{2b}.
\] (7)

Its price is denoted:
\[
p = \frac{a + c}{2}.
\] (8)

Its profit can be obtained:
\[
\pi_{\text{monopoly}} = \frac{(a - c)^2}{4b}.
\] (9)

When $j = n$, the market becomes a perfect competitive market. The quantity can be derived as follows:
\[
q = \frac{a - c}{b}.
\] (10)

Based on differential Lagrange's intermediate value theorem, from (7) and (10), the general equilibrium quantity of Cournot $q_m$ can be derived:

\[
\begin{cases}
\Pi_1 = \alpha p_1 - \beta p_1^2 - \beta p_1 p_2 - c_1 \alpha^3 + c_1 \beta^3 (p_1 + p_2)^3 + 3 c_1 \alpha^2 \beta (p_1 + p_2) - 3 c_1 a \beta^2 (p_1 + p_2)^2, \\
\Pi_2 = \alpha p_2 - \beta p_2^2 - \beta p_1 p_2 - c_2 \alpha^3 + c_2 \beta^3 (p_1 + p_2)^3 + 3 c_2 \alpha^2 \beta (p_1 + p_2) - 3 c_2 a \beta^2 (p_1 + p_2)^2.
\end{cases}
\] (14)

The 1st company maximizes the profit with respect to $p_1$ and the profit of 2nd company with respect to $p_2$. The dynamic Bertrand model is established:

\[
\begin{cases}
p_1(t + 1) = a - 2 \beta p_1(t) - \beta p_2(t) + 3 c_1 \alpha^2 \beta (p_1(t) + p_2(t))^2 + 3 c_1 a \beta^2 - 6 c_1 a \beta^2 (p_1(t) + p_2(t)), \\
p_2(t + 1) = a - 2 \beta p_2(t) - \beta p_1(t) + 3 c_2 \alpha^2 \beta (p_1(t) + p_2(t))^2 + 3 c_2 a \beta^2 - 6 c_2 a \beta^2 (p_1(t) + p_2(t)).
\end{cases}
\] (15)

The equilibrium points of system (15) is complex. Bifurcation diagrams, attractors, and global maximal Lyapunov exponent [37] are used to show complex dynamical behaviors. We can see the dynamic characteristics of map (15) by a numerical simulation. It is performed by fixing the parameters of the model as follows: $b = 0.328$, $c_1 = 0.001$, $c_2 = 0.002$, and $1.1 < a < 300$. Criteria for selected parameters is referenced in the Cournot model. Figure 5 exhibits the bifurcation diagrams of $p_1$ and $p_2$ separately with the parameter changing. Figure 6 displays the global maximal Lyapunov exponents. It shows that system (15) presents complex behaviors and becomes chaos. The attractor of the system further is explored. Figure 7 shows that the attractor of the system exists. It suggests the existence of deterministic chaotic.
From Figure 5, there are many Nash equilibrium points in system (15). However, we cannot derive the equilibrium points so far.

2.4. General Equilibrium Point of Price Competition. The Nash equilibrium of Bertrand becomes unstable as the speed of parameters adjustment increases [44]. In duopoly pricing, it can also produce persistent motion, periodic or chaotic [45]. With the increasing number of companies, the equilibrium points cannot be derived. We expand and develop the previous work of Puu [45]. Now, suppose that there are \( n \) companies in a same or similar market. All companies have an identical cost function. We replace the cost function with the average cost of the industry. In this paper, we focus on oligopoly competition. Without loss of generality, we assume the market demand function:

\[
Q_i = \alpha - \beta p_i, \quad i = 1, 2, \ldots, n, \tag{16}
\]

where \( Q_i \) is the quantity of company \( i \), \( \alpha \) and \( \beta \) are parameters, and \( p \) is the price of product.

All companies produce an identical or similar product. In a certain industry, the total cost function is \( C_i(q_i), i = 1, 2, \ldots, n \). As usual in the literature, we assume that all companies have the same cost function, and that the cost function is linear. Therefore, \( C(Q) = cQ \). It means that all companies have the same marginal cost and the same average cost and are equal to \( c \).

In the monopoly structure, the profit function of a company can be denoted:

\[
R(p) = (p - c)Q = -\beta p^2 + (\beta c + \alpha)p - ac. \tag{17}
\]

The condition of the inner optimal profit for the monopoly company \( i \) is that formula (12) is taken as a partial derivative:
\[ R'(p) = -2\beta p + \beta c + \alpha. \]  
(18)

From formula (17), we may obtain the optimal price:
\[ p = \frac{\beta c + \alpha}{2\beta}, \]  
(19)

with parameters
\[ \alpha > 0, \]
\[ \beta > 0, \]  
(20)
\[ \alpha > \beta c. \]

In a perfectly competitive structure market, the price is equal to the average cost (marginal cost):
\[ p = c. \]  
(21)

With the development of economy, the congeneric market will become more and more stable, and the equilibrium of competition will result in aggregation. A spatial aggregation concept model is built to illuminate imperfectly competitive equilibrium. That is to say, Bertrand competition is an imperfect competition. Now, we define \( p_m \), as a general equilibrium price that we will seek for. The region between \( u \) and \( v \) is the chaotic district of the Bertrand model when there are \( n \) companies in the congeneric market, such as Figure 8.

The \( u \) endpoint represents the monopoly market, and the \( v \) endpoint is the perfectly competitive market.

Suppose 1. Based on the differential Lagrange’s intermediate value theorem, there exists some \( z \) points on the open interval \([u, v]\) (where \( u < v \)). Let function \( f: \[u, v]\rightarrow \mathbb{R} \) be a continuous function on the closed interval \([u, v]\) and differentiable on the open interval \((u, v)\). The derivative of function \( f \) at \( z \) equals the difference quotient \( \Delta f(u, v) \). That is to say, there exists a point \( z \) in \((u, v)\), such that
\[ \Delta f(u, v) = f'(z) = \frac{f(v) - f(u)}{v - u}. \]  
(22)

\( p_m \) is one of the salient points of the Bertrand model. We can derive \( p_m \) on the intermediate value theorem.

Suppose 2. The endpoint \( u = (\alpha + \beta c)/2\beta \), and the other endpoint \( v = c \).

On the closed interval \([ (\alpha + \beta c)/2\beta , c]\),
\[ f'(z) = f'(p_m) = \frac{f(v) - f(u)}{v - u} = \frac{\alpha - \beta c}{2}. \]  
(23)

Substituting (23) back into (18) yields finally, we can get at least a general equilibrium point of price in the Bertrand model:
\[ p_m = \frac{3\beta c + \alpha}{4\beta}. \]  
(24)

By a numerical simulation, in order to test the stability of results, without loss of generality, fix the parameters \( c = 2, 2 < \alpha < 3, \) and \( 1 < \beta < 2 \). From Figure 9, the distribution state of \( p_m \) is a complex surface. The surface is composed of a set of solutions, and it is a lower convex surface. The convex surface is also called an attractor. In general, the attractor of the value of \( p_m \) is stable. Simulation result demonstrates the validity and stability of the new algorithm. That is to say, a convex point exists in the Bertrand model. At least, an equilibrium point \( p_m \) is derived between the monopoly market structure and perfect market structure. However, with the increasing number of companies, it will not become unstable.

From (16) and (24), we can derive general equilibrium quantity:
\[ q_m = \frac{3(\alpha - \beta c)}{4}. \]  
(25)

From (17), (24), and (25), the general equilibrium profit of Bertrand is derived:
\[ \pi_m = \frac{3(\alpha - \beta c)^2}{16\beta}. \]  
(26)

When there is only a company in the market, the company is a monopoly. The price is expressed as
\[ p = \frac{\beta c + \alpha}{2\beta}. \]  
(27)

Its production is denoted:
\[ q = \frac{\alpha - \beta c}{2}. \]  
(28)

The profit of the monopoly is denoted:
\[ \pi_{\text{monopoly}} = \frac{(\alpha - \beta c)^2}{4\beta}. \]  
(29)

From above, we can conclude that the value of (23) is larger than that of (24), and the value of (29) is also larger.
than that of (26). It shows that the competition of the Bertrand can cause lower general equilibrium price than monopoly competition. The result is not the same as that of [46], which showed the competition of Bertrand’s model can yield higher prices than monopoly. The profit of Bertrand’s model is also lower than monopoly. It is clear that the general equilibrium profit of Bertrand’s model is not better than monopoly. However, at least a general equilibrium point of the Bertrand model is derived. In the imperfectly competitive market, the company can obtain optimal profit by the general equilibrium price.

3. General Equilibrium of Combination of Quantity and Price

In practice, both price competition and quantity competition exists simultaneously in competitive markets. According to Vernon [47], there are three stages in a product life cycle: introduction, maturity, and standardized stage. Monopoly is in the stage of location of new products. The competition of Cournot and that of Bertrand locate in two other stages. Such as in Figure 10, with the constant dynamic competition of companies, quantity competition and price competition will stabilize. This situation is located in the maturity stage and standardized stage. Obviously, quantity competition and price competition exist and are applied simultaneously for company competition in the maturity stage and standardized stage. This paper built a dynamic aggregation game model of two stage to detect the general equilibrium point of combination of quantity competition and price competition based on the general equilibrium point of quantity competition and general equilibrium point of price competition, respectively. In the first stage, quantity competition is the dominant competitive strategy with price competition. It is debated that whether or not an equilibrium point of quantity competition exists in combination of quantity competition and price competition. In the second stage, price competition is the dominant competitive strategy with quantity competition, and a general competitive equilibrium point of price will also be explored.

3.1. General Competitive Equilibrium of Dominant Quantity

Define 1. \( q_{qc} \) is the general equilibrium point of dominant quantity competition with price competition.

In this stage, many companies compete mainly by quantity. The more products the companies produce, the more competitive it becomes. It must result in low price. However, the profit is sometimes not better. If the company produces less, unless the product is priced high, the profit is also less. Other companies compete with each other by price. In this economic system with quantity competition and price competition, it becomes more significant to find general equilibrium quantity for combination of quantity and price. It can help companies to obtain better profit.

From Section 2, based on the differential Lagrange’s intermediate value theorem, we define the endpoint \( u = 3(a - c)/4b \) and other endpoint \( v = 3(a - \beta c)/4 \):

\[
f'(q_{qc}) = \frac{3b^2 (a - \beta c)^2 - 4b (a - c)(a - \beta c) - 8(a - c)^2}{4(a - c) - 4b (a - \beta c)}
\]  

(30)

On the open interval \( (3(a - c)/4b, 3(a - \beta c)/4) \), from (3) and (25), the general equilibrium quantity \( q_{qc} \) of combination of quantity and price can be derived:

\[
q_{qc} = \frac{12(a - c)^2 - 3b^2 (a - \beta c)^2}{8b(a - c) - 8b^2 (a - \beta c)}
\]  

(31)

Figures 11 and 12 show that the distribution of \( q_{qc} \) is a complex surface. The surface is called the attractor in the chaotic system. We change the values of parameter \( a, \beta, a, \) and \( b \), the results have no effect on the distribution of \( q_{qc} \) value. It shows that the value of \( q_{qc} \) is stable. Based on the three-body problem in the space, the oligarch companies will gather with a dynamic economic system. The solutions of the system are dynamic and stable. Therefore, the value of \( q_{qc} \) is fixed and stable. That is to say, a convex equilibrium point of quantity exists in the model of combination of quantity and price. It is clear that at least a general equilibrium point \( q_{qc} \) can be derived for combination of Bertrand and Cournot.

The equilibrium price \( p_{qc} \) also is derived on inverse demand function:

\[
p_{qc} = a - \frac{12(a - c)^2 - 3b^2 (a - \beta c)^2}{8(a - c) - 8b(a - \beta c)}
\]  

(32)

The equilibrium profit \( \pi_{qc} \) is denoted:

\[
\pi_{qc} = \frac{3b^2 (a - \beta c)^2 - 4(a - c)^2 - 8b(a - c)(a - \beta c)}{b(8(a - c) - 8b(a - \beta c))^2}.
\]  

(33)

By comparing the value of (7) and that of (31) and the value of (9) and that of (33), it is obviously that the profit of the monopoly is the biggest. However, the quantity of monopoly is smaller than that of combination of quantity and price.

3.2. General Competitive Equilibrium of Dominant Price

Now, we will further explore the general equilibrium price in combination of Bertrand and Cournot for \( n \) companies. In this stage, many companies compete mainly by price. If the
If the product is priced lower, companies will make less profit. If the product is priced higher, the sales will fall. To be corresponding to this, other companies compete by quantity. Whether it is price competition or quantity competition, maximizing profit is the goal of any company. From Section 2, with the intermediate value theorem, the general equilibrium price of combination of quantity and price maybe detected. It can maximize the companies’ profit.

\[ f'(p_{pc}) = \frac{3\alpha^2 + 4\alpha^2\beta^2 - 4\alpha^2\beta - \alpha\beta^3 c - 3\alpha\beta c}{4\alpha(1 - \beta)} \]  \hspace{1cm} (35)

From (13) and (30), \( p_{pc} \) can be derived:

\[ p_{pc} = \frac{\alpha + 7\beta c - 3\beta^2 c - 4\alpha\beta^2}{8\beta(1 - \beta)} \]  \hspace{1cm} (36)

From Figure 13, the state of \( p_{pc} \) is a complex convex surface by a number simulation. Based on the solution of the three-body problem, the surface exists attractor, the value of \( p_{pc} \) is also stable. The solution set forms a lower convex surface. It is obvious that a convex equilibrium point of price exists in the model of combination of quantity and price. We find the equilibrium point \( p_{pc} \) in the combination of Bertrand and Cournot. If a company will enter into a relative market by price strategy, the result can offer a method to estimate the equilibrium price of the market.

Here, assume \( r = \alpha + 7\beta c - 3\beta^2 c - 4\alpha\beta^2 \), and formula (31) is also expressed as (32):
complexity, the distributing of $p_m$, $q_{qc}$, and $p_{pc}$ is a complex surface, respectively. The surface is called the attractor and it is stable. It means that these general equilibrium points are stable. Numerical simulations are given to support the research results.

The results show that no matter which market structure it is, the profit of monopoly is the biggest. However, some previous studies compared the Bertrand equilibrium prices with the Cournot equilibrium price under the same market structure. Such as [18, 19], the results show that the Bertrand equilibrium prices are lower than the Cournot ones. Other [10] results show that Cournot competition can yield lower quantities than Bertrand competition. However, in this paper, we compare the general Cournot’s equilibrium quantity with that of monopoly and the general Bertrand’s equilibrium price with that of monopoly. It is significant that the general equilibrium price of combination of quantity and price can generate higher price than monopoly. By contrast, the general equilibrium quantity of combination of quantity and price can also generate higher quantity than monopoly.

This paper provides a theoretical foundation on how to make the company itself to obtain the optimal quantity strategy or the optimal price strategy. In a dynamic economy system, when a company is about to enter into a relative market, according to the industry marginal cost (the average cost), the demand function, inverse demand function of a competitive company, the general equilibrium quantity of combination, and general equilibrium price of combination can be separately calculated.

Data Availability

The authors declare that no data were used to support this study.

Conflicts of Interest

No conflicts of interest for all authors.

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