Collective scattering of an incident monochromatic circularly polarized wave in an unmagnetized non-equilibrium plasma

S Matsukiyo¹, Y Kuramitsu², K Tomita³

¹ Department of Earth System Science and Technology, Kyushu University, 6-1 Kasuga Koen, Kasuga, Fukuoka, 816-8580, Japan
² Department of Physics, National Central University, No. 300, Jhongda Rd., Jhongli City, Taoyuan 320, Taiwan
³ Department of Applied Science for Electronics and Materials, Kyushu University, 6-1 Kasuga Koen, Kasuga, Fukuoka, 816-8580, Japan
E-mail: matsukiy@esst.kyushu-u.ac.jp

Abstract. Collective scattering of an incident monochromatic circularly polarized electromagnetic wave in an unmagnetized non-equilibrium plasma is investigated. The wave equation of the scattered waves are numerically solved by giving wave spectra of the incident electromagnetic wave and a variety of longitudinal fluctuations of a plasma. In addition to the well known symmetric ion and electron features of frequency spectrum in an equilibrium plasma, some asymmetries are found in a frequency spectrum when anisotropic longitudinal wave propagation or the effect of the presence of a beam mode are taken into account.

1. Introduction
Thomson scattering has been widely used as a diagnostic technique to measure local parameters of a laboratory plasma [1]. A spectral density function of scattered waves provides collective features as well as non-collective ones of a plasma. The collective effects become remarkable if the wavelengths of incident and scattered waves are longer than the Debye length. An observed spectrum of scattered waves is fitted conventionally with the modeled spectral density function based on the assumption that the plasma is more or less quiet, i.e., in equilibrium [2].

Recent laboratory astrophysics, on the other hand, covers highly nonlinear and non-equilibrium phenomena like collisionless shocks, magnetic reconnection, etc. In such a plasma the scattering process of incident probe laser should be very complicated and scattered wave spectrum may be quite different from that in an equilibrium plasma. For instance, a beam component of a plasma is often seen upstream of a high Mach number collisionless shock [3]. Some incident ions are reflected at a high Mach number shock due to strong electrostatic potential so that the reflected ions behave as a beam. Such a beam may drive a variety of instabilities. The resultant local plasma should be highly turbulent and anisotropic. However, the theoretical framework of Thomson scattering in nonlinear and non-equilibrium plasmas has not been well established. In this study collective scattering of an incident monochromatic circularly polarized electromagnetic wave in some non-equilibrium plasmas is investigated by using numerical simulations.
2. Formulations and Basic Assumptions

We consider an unmagnetized electron-proton plasma, governed by the following equations.

\[ n_j m_j \left( \frac{\partial}{\partial t} + V_j \cdot \nabla \right) V_j = n_j q_j \left( E + \frac{V_j \times B}{c} \right) - T_j \nabla n_j \]  

(1)

\[ \frac{\partial n_j}{\partial t} = -\nabla \cdot (n_j V_j), \quad \nabla \cdot E = 4\pi \rho. \]  

(2)

\[ \nabla \cdot B = 0, \quad \frac{\partial B}{\partial t} = -c \nabla \times E. \]  

(3)

\[ \nabla \times (\nabla \times E) + \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = -\frac{4\pi}{c^2} \frac{\partial J}{\partial t}. \]  

(4)

Here, \( V_j = (u_j, v_j) \) is the velocity of \( j \)-th particle species in which \( u \) and \( v \) denote longitudinal and transverse components of the velocity, \( n_j \) the density, \( q_j \) the particle charge, \( m_j \) the particle mass, \( T_j \) the isotropic temperature which is assumed to be constant, \( c \) the speed of light, and \( E \) and \( B \) are the electric and the magnetic fields, respectively. The current and the charge densities are given by \( J = \sum_j q_j n_j V_j \) and \( \rho = \sum_j q_j n_j. \)

It is assumed that a circularly polarized electromagnetic incident wave propagating parallel to the \( x \)-direction is written as \( E_I = (\hat{E}_I / \sqrt{2}) e^{i\phi_I} \hat{\epsilon} + \text{c.c.} \) \( (\phi_I = k_I x - \omega_I t, \hat{\epsilon} = (\hat{y} - i\hat{z})/\sqrt{2}) \). One can easily confirm that the corresponding amplitudes in magnetic field and velocity are given by \( B_I = i(k_I c/\omega_I) \hat{E}_I \) and \( \dot{v}_{te} = -ie \hat{E}_I / m_e \omega_I, \) respectively. This incident wave may be scattered by the plasma containing longitudinal fluctuations given by \( \delta X = \sum_k (\delta \hat{X}(k)/2)e^{ik \cdot \mathbf{r}} + \text{c.c.} \) \( (\phi(k) = k \cdot \mathbf{r} - \omega t) \), where \( X \) can be either density, \( n_j \), longitudinal velocity, \( u_j \), or electrostatic field, \( E_s \). The scattered waves are expressed as \( \delta E_S = \sum_k (\delta \hat{E}_S(k_s)/\sqrt{2}) e^{i\phi_S(k_s)} \hat{\epsilon} + \text{c.c.} \) \( (\phi_S(k_s) = k_s \cdot \mathbf{r} - \omega s t) \).

Let us assume that the amplitude of the incident wave dominates those of the scattered waves. Then, we obtain an wave equation for the incident wave as \( c^2 \nabla \times (\nabla \times E_I) + \partial^2 E_I / \partial t^2 = -\omega_p^2 E_I \), where \( \omega_p^2 = \omega_{pe}^2 + \omega_{pi}^2 \) and \( \partial \mathbf{v}_{Ij} / \partial t = (q_j/m_j) \mathbf{E}_I \) have been used. The scattered waves, on the other hand, satisfy

\[ \nabla \times (\nabla \times \delta E_S) + \frac{1}{c^2} \frac{\partial^2 \delta E_S}{\partial t^2} = -\frac{4\pi}{c^2} \frac{\partial \delta J}{\partial t}, \]  

(5)

where \( \delta J = e n_0 (\delta \mathbf{v}_i - \delta \mathbf{v}_e) + e (\delta n_i \mathbf{v}_{Ii} - \delta n_e \mathbf{v}_{Ie}) \). Note that \( \delta \mathbf{v}_i \approx \mathbf{v}_{Ii} \approx 0 \) is a good approximation, since the transverse fluctuations are quite high frequency electromagnetic modes. Hence, if we assume \( \nabla = \hat{x} \partial / \partial x \), the wave equation for the scattered waves is written as

\[ \left( -\frac{\partial^2}{\partial x^2} + \frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \frac{\omega_p^2}{c^2} \right) \delta E_S = \frac{4\pi e}{c^2} \frac{\partial}{\partial t} (\mathbf{v}_{Ie} \delta n_e). \]  

(6)

If the density fluctuation is based on relatively high frequency Langmuir waves, the corresponding \( \delta E_x \) satisfies \( (\partial^2 / \partial t^2 - \omega_{te}^2 / \partial x^2 + \omega_{pe}^2) \delta E_x = -\omega_{pe}^2 / c (\mathbf{v}_{le} \times \delta \mathbf{B}_S + \delta \mathbf{v}_e \times \mathbf{B}_I) \), where \( \omega_{te}^2 = T_e / m_e. \) When the right hand side (RHS) is non-negligible, this represents the equation for stimulated Raman scattering. But if the incident wave is too weak to disturb the plasma so that the RHS is negligible, \( \delta E_x \) may be merely expressed as the superposition of longitudinal waves satisfying the dispersion relation \( \omega^2(k) = \omega_{pe}^2 + k^2 \omega_{te}^2. \) Hence, \( \phi(k) = k x + t \sqrt{\omega_{pe}^2 + k^2 \omega_{te}^2}. \) Eq.(6), then, represents the evolution of the waves scattered by collective electron motions.

If the density fluctuation is based on relatively low frequency ion acoustic (IA) waves, the corresponding \( \delta u_i \) satisfies \( (\partial^2 / \partial t^2 - C_s^2 \partial^2 / \partial x^2) \delta u_i = \left[ \partial (\delta \mathbf{j}_S \times \mathbf{B}_I + \mathbf{J}_I \times \delta \mathbf{B}_S) \right] / \partial t / (n_0 m_i c), \) where \( C_s^2 = (T_e + T_i) / m_i. \) When the RHS is non-negligible, this represents the equation for
stimulated Brillouin scattering. But if the incident wave is too weak to disturb the plasma so that the RHS is negligible, \( \delta u \) may be merely expressed as the superposition of longitudinal waves satisfying the dispersion relation \( \omega^2(k) = k^2 C_s^2 \). Note that in such a low frequency regime \( \delta n_i \approx \delta n_e \) (\( \delta u \approx \delta u_e \)), since the electron inertial is negligible. Eq.(6), then, represents the evolution of the waves scattered by collective ion motions.

Without losing generality, one can write as \( E_I = E_I(\cos \phi_I, \sin \phi_I), \quad v_I = (e \dot{E}_I/m_e \omega_I)(\sin \phi_I - \cos \phi_I), \) and \( \delta n_e = \sum_k (k n_0/\omega) \delta u \cos(\phi + \phi_{0k}), \) where \( E_I \) and \( \delta u \) are real. Here, \( \phi_{0k} \) denotes the initial phase of each longitudinal mode. Substituting the above into eq.(6) leads to

\[
\left(-\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial t^2} + 1\right) \delta E_S = -\frac{1}{\omega_I} \sum_k k \delta u (\cos(\phi + \phi_{0k} + \phi_I), \sin(\phi + \phi_{0k} - \phi_I)). \tag{7}
\]

Here, space and time have been normalized to \( c/\omega_{pe} \) and \( \omega_{pe}^{-1} \), the scattered electric field to \( \dot{E}_I \), respectively. The above equation is numerically solved for a number of different plasma conditions in the following.

3. Numerical results

We first assume that the incident wave frequency is \( \omega_I = 8.9 \) and IA and electron thermal velocities in the homogeneous thermal plasma are \( C_s = 0.01 \) and \( v_{te} = 0.1 \), respectively. The amplitudes of the IA waves are assumed to be one order larger than those of the Langmuir waves, while the different wave modes on the same dispersion branch have equal amplitudes. The initial phase, \( \phi_{0k} \), is random. In Fig.1(a) the frequency spectrum of the waves propagating in negative \( x \)-direction (backward scattered waves) is shown. The dominant two peaks near the incident wave frequency, \( \omega_I \), are due to the scattering by collective ion motions of the plasma. In other words, they are the result of the coupling between the incident wave and the IA waves. The subdominant two peaks are due to the scattering by collective electron motions of the plasma, which are the result of the coupling between the incident wave and the Langmuir waves. In a collective Thomson scattering the former peaks are often called as ion feature and the latter as electron feature. In the following only the ion feature is focused.

A plasma is assumed to be turbulent and the amplitude spectrum of the velocity fluctuations is given by a power law, \( \delta u(k) = Ak^{-2} \), where \( A \) denotes a constant. The integrated total amplitude \( W = \int_{k_{min}}^{k_{max}} |\delta u| dk \) is a parameter. Here, \( k_{min} \) and \( k_{max} \) are the minimum and the maximum wavenumbers of the longitudinal fluctuations. They are fixed throughout the following analysis as \( k_{min} = 2\pi/51.2 \) and \( k_{max} = 20\pi \). The incident wave frequency is \( \omega_I = 11.8 \).

Fig.1(b) represents the case that the IA waves are isotropic, i.e., the same spectra are assumed for all the four quadrant in \( \omega - k \) space with \( W = 0.1 \). In this case the symmetric two peaks of ion feature are clearly seen. Fig.1(c) shows the case that the IA waves propagating in negative \( x \)-direction, i.e., the second and the fourth quadrant in \( \omega - k \) space, are weakened to have smaller amplitudes by one order of magnitude. Then, the higher frequency peak (\( \omega_S > \omega_I \)) is reduced and the ion feature becomes asymmetric.

The influence of the presence of a drifting beam is shown in Fig.1(d) and (e). Here, the longitudinal fluctuations whose dispersion relation is \( \omega = ku_d \) (\( u_d = 0.2 \)) is added. Fig.1(d) shows the case that a beam instability is well developed and the amplitude spectrum of the beam mode is the same type of power law as the IA waves. The beam mode amplitudes are one order of magnitude larger than those of the isotropic IA waves. Apparently, in addition to the usual two ion feature peaks, one strong peak at \( \omega_S \approx 8 \) is seen. When a beam instability has not been well developed, it is often the case that only the limited region in \( k \) has wave power. Such a case is shown in Fig.1(e), in which the amplitude spectrum of the longitudinal
Figure 1. Frequency spectra of scattered waves in a variety of plasma conditions obtained by solving eq.(7). The longitudinal fluctuations in a plasma are given by (a) random noise on Langmuir and IA modes, (b) power law fluctuations of isotropic IA waves, (c) same as (b) but the waves propagating in negative x-direction have one order of magnitude smaller amplitudes, (d) power law fluctuations of a beam mode and isotropic IA waves, (e) same as (d) but the beam mode has a Gaussian spectrum centered at \( k = k_I/2 \).

fluctuations is given by a Gaussian centered at \( k = k_I/2 \). In contrast to Fig.1(d), the third peak in the frequency spectrum is shifted toward higher frequency.

4. Summary
The scattering of a monochromatic circularly polarized electromagnetic wave in an unmagnetized plasma is discussed. The wave equation of the scattered waves is solved numerically for a number of cases with different plasma conditions. The process considered here is essentially same as that of the collective Thomson scattering of an incident laser light in a laboratory plasma. Unlike an equilibrium plasma, the frequency spectrum of the scattered waves show asymmetric nature in non-equilibrium plasmas. As well known, when the IA waves have anisotropy in their propagation direction, the so-called ion feature of the scattered wave spectrum becomes asymmetric. If a plasma contains a beam, an additional peak appears in the frequency spectrum of the scattered waves.

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