Proton Stability In Supersymmetric $SU(5)$

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Within supersymmetric $SU(5)$ GUT we suggest mechanisms for suppression of baryon number violating dimension five and six operators. The mechanism is based on the idea of split multiplets (i.e. quarks and leptons are not coming from a single GUT state) which is realized by an extension with additional vector-like matter. The construction naturally avoids wrong asymptotic relation $M_D = M_E$. Thus, the long standing problems of the minimal SUSY $SU(5)$ GUT can be resolved.

In a particular example of flavor structure and with additional $U(1) \times Z_{3N}$ symmetry we demonstrate how the split multiplet mechanism works out. Namely, the considered model is compatible with successful gauge coupling unification and realistic fermion mass pattern. The nucleon decay rates are relatively suppressed and can be well compatible with current experimental bounds.

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I. INTRODUCTION

Baryon number violation is one of the predictions of Grand Unified Theories (GUT). In SUSY GUTs, usually dimension five ($d = 5$) operator induced proton decay dominates [1]. The sources for the latter are heavy color triplets’ couplings with ordinary matter supermultiplets. These couplings usually originate from the operators responsible for quark and lepton masses. Therefore, the observed Yukawa couplings and the baryon number violating operators may be closely related and this is the reason that it is not easy to satisfy the present experimental bound $\tau^{\exp}(p \to K\nu) \gtrsim 6.7 \cdot 10^{32}$ years [2] on proton lifetime [3]. On the other hand, it is not trivial to build realistic fermion mass pattern within GUTs. Therefore, the task is two fold: 1) within considered scenario care must be exercised to get realistic fermion masses and mixings, and 2) within the same framework the baryon number violating processes must be suppressed up to the required level. These are two main problems and for their resolution numerous mechanisms and specific examples have been suggested [4, 5, 6, 7, 8, 9, 10, 11]. It is a curious fact that the split multiplet mechanism, for suppressing the baryon number violation, more or less has been ignored (see however [2, 7]). Let us note that this mechanism is naturally realized within extra dimensional constructions [10, 11]. This is, most likely, the reason that within four dimensional constructions there were not many attempts to realize and apply this possibility. However, once the multiplet splitting is achieved (i.e quarks and leptons come from different GUT states), the baryon number can be conserved up to the needed level [11]. In this paper we suggest mechanisms for natural quark-lepton splitting within four dimensional SUSY $SU(5)$ [12]. We show that apart from suppressing the baryon number violation this splitting enables one to build realistic fermion mass pattern. The discussion of the mechanism and it’s needed ingredients are presented in the next section. In section [III] we show how $d = 6$ nucleon decay can be suppressed. In section [IV] for demonstrative purposes, we present particular example in which split multiplet mechanism is realized. It utilizes an additional $U(1) \times Z_{3N}$ symmetry which plays crucial role for adequate suppression of all unwanted baryon number violating couplings including Planck scale suppressed operators. An assumption on a particular flavor structure and simple minded SUSY spectrum near~1 TeV is made. These give perturbative gauge coupling unification and realistic fermion masses and mixings. At the same time, nucleon’s decay rate is compatible with current experimental bounds.

II. SUPPRESSION OF $d = 5$ BARYON NUMBER VIOLATION

In the minimal SUSY $SU(5)$ (MSSU5) GUT the matter sector consists of the $(10+5)$-plets per generation with the following decomposition under $SU(3)_C \times SU(2)_L \times U(1)_Y$:

$$10 = q(3, 2)_{-\frac{2}{3}} + u^c(3, 1)_1 + e^c(1, 1)_{-\frac{1}{3}} ,$$

$$\bar{5} = d^c(\bar{3}, 1)_{-\frac{1}{3}} + l(1, 2)_3 ,$$

where subscripts stand for the hypercharges in 1/$\sqrt{60}$ units [Y = $\frac{1}{\sqrt{60}}$ Diag (2, 2, 2, -3, -3)]. The pair of scalar superfields $H(5) + \bar{H}(\bar{5})$ has the following composition:

$$H(5) = h_u(1, 2)_{-3} + T(3, 1)_2 ,$$

$$\bar{H}(\bar{5}) = h_d(1, 2)_3 + \bar{T}(\bar{3}, 1)_{-2} ,$$

where $h_u, h_d$ denote the MSSM higgs doublet superfields, and $T, \bar{T}$ are their colored GUT partners. The renormalizable operators $10 \cdot 10H$ and $10 \cdot 5\bar{H}$ (the family indices

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are suppressed), together with ordinary Yukawa superpotential couplings, generate matter $T, T$ interactions:

$$\lambda_{10} \cdot 10H = \lambda (qu^c h_u + qqT + e^c w^c T) ,$$

$$\lambda'_{10} \cdot 5\tilde{H} = \lambda' (qd^c h_d + e^c l h_d + q\tilde{T} + w^c d^c \tilde{T}) .$$

Integration of $T, \tilde{T}$ states (with mass $M_T$) generates $d = 5$ operators

$$\frac{\lambda'}{M_T} (q ql)_F , \quad \frac{\lambda'}{M_T} (u^c u^c d^c e^c)_F ,$$

which induce the nucleon decay. Current experimental bound on nucleon lifetime requires $\lambda' \lesssim 10^{-9}$ (for $M_T \sim 10^{16}$ GeV and all soft SUSY breaking terms $\sim$ TeV). On the other hand, in MSSM $\lambda'$ is directly related to the quark and lepton Yukawa couplings and is typically $\sim 10^{-6}/\sin 2\beta$. This would lead to unacceptably fast proton decay. Note that couplings in (3) also lead to the wrong asymptotic mass relations $m_u = m_s$, $m_e/m_\mu = m_d/m_s$ at GUT scale. Thus, some modification should be done anyway in order to improve this situation. It is desirable to have a mechanism which simultaneously solve both - fermion mass problem and baryon number violation. Note that modification of either Yukawa sector or sparticle spectrum can improve the situation with baryon number violation. However, besides colored higgsino mediated 5 operators, there exist Planck scale suppressed baryon number violating couplings which in SUSY SU(5) have forms

$$\frac{\lambda_{10}}{M_{Pl}} (10 \cdot 10 \cdot 10 \cdot 5)_F \rightarrow \frac{\lambda_{10}}{M_{Pl}} (q ql + u^c u^c d^c e^c)_F .$$

This also mediates proton decay and in order to satisfy experimental bound one should arrange for appropriate couplings $\lambda_{10} \lesssim 10^{-7}$. Couplings $\lambda_{10}$ are completely independent from the Yukawa sector and therefore their smallness needs separate explanation, because at SUSY SU(5) level there is no symmetry argument for their suppression.

Below we present mechanism, different from existing ones, which within SUSY SU(5) GUT suppress (eliminate) baryon number violation and can solve the fermion mass problem.

A. Suppressing $qqT$ and eliminating $e^c u^c T$ operators

In order to demonstrate how the split multiplet mechanism works, we start considerations with one family only. The generalization to three families is straightforward and will be discussed later on. We extend the matter sector with vector like states in 15 and 15 representations of SU(5). In terms of $SU(3)_c \times SU(2)_L \times U(1)_Y$ they decompose as

$$15 = q(3,2)_{-1} + S(6,1)_{+} + \Delta(1,3)_{-6} ,$$

and conjugate transformations for the fragments of $\bar{15} = (\bar{q}, \bar{S}, \bar{\Delta})$. The state $q(\equiv q_{15})$ from 15-plet has transformation properties of the left handed quark doublet. The remaining $S$ and $\Delta$ states have 'exotic' quantum numbers. This feature of 15-plet can be used for the suppression of proton decay \[\bar{q}.\] With suitable couplings we can arrange that the light left handed quark doublet mainly comes from 15-plet. Consider the superpotential couplings

$$10\Sigma 15 + M_{15} 15 \cdot 15 ,$$

where $\Sigma$ is an adjoint 24-plet scalar superfield used for the breaking $SU(5) \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$. Substituting in (7) the GUT VEV $\langle \Sigma \rangle \equiv M_G$ with $M_{15} \ll \langle \Sigma \rangle$, we see that $q_{10}$ decouples by forming the massive state with $\bar{q}_{15}$. Namely, for the light $q$ and heavy $q_h$ states we have

$$q \simeq q_{15} , \quad q_h \simeq q_{10} + \frac{M_{15}}{M_G} q_{15} \Rightarrow$$

$$15 \supset q , \quad 10 \supset eq \quad \text{with} \quad \epsilon \equiv \frac{M_{15}}{M_G} .$$

The states $u^c$ and $e^c$ (from 10-plet) and fragments $(S, \Delta), (\bar{S}, \bar{\Delta})$ (from 15, \bar{15}) are not affected with this procedure. Therefore,

$$u^c, e^c \subset 10 ,$$

and masses of the decoupled states are given by

$$M(q_{10}, \bar{q}_{15}) \simeq M_G , \quad M_S = M_\Delta = M_{15} .$$

Now it is clear that the up quark mass will be generated through the Yukawa coupling of 15-plet with 10. Since 15-plet is the two index symmetric representation of SU(5), $\Sigma$ should participate in this coupling. Namely,

$$\frac{Y \Sigma}{M_s} 15 \cdot 10H \rightarrow Y_U (qu^c h_u + eq q T) ,$$

with $Y_U = \frac{\langle \Sigma \rangle}{M_s} Y$.

We see that the term $qqT$ is suppressed by factor $\epsilon$ in comparison to the up type quark Yukawa coupling. This occurred thanks to the splitting of the $q$-states living in 15 and 10plet superfields respectively. Note that no $e^c u^c T$ coupling arises from (1). The coupling $10 \cdot 10H$ is not needed at all and can be suppressed or completely eliminated in concrete scenario (discussed in sect. IV).

The scale $M_s$ in (1) is a cut off and one expects that it is much larger than the GUT scale $M_s \gg \langle \Sigma \rangle$ (in most conservative approach $M_s \sim M_{Pl} \approx 2.4 \cdot 10^{18}$ GeV - the reduced Planck mass). Thus, we can use this type of coupling for first two light families (i.e. for generation of up and charm quark masses). For the top quark mass we
need to have unsuppressed Yukawa coupling. If we do not apply this mechanism of \(qqT\) coupling suppression for the third generation, the top Yukawa can be due to the coupling \(10_310_2H\). However, the same coupling also generates unsuppressed \(q_3q_3T\) term. This would give sizable contribution to the nucleon decay \([3,13]\) through the mixings with light families. Thus, for suppressing \(q_3q_3T\) and generating the top Yukawa coupling at renormalizable level we suggest slight modification by introducing additional \(10^f + 10^f\) states and the couplings

\[
10\Sigma\bar{15} + 15\Sigma\bar{10}^f + M_{1515} \cdot \bar{15} + M_{10}10^f \cdot \bar{10}^f \tag{12}
\]

From these terms we can write down the mass matrices for appropriate fragments

\[
\begin{align*}
q_{10} & \quad \left( \begin{array}{cc} q_{10} & q_{15} \\ M_{10} & M_{15} \end{array} \right), \\
q_{15} & \quad \left( \begin{array}{c} q_{15} \\ M_{10} \end{array} \right),
\end{align*}
\]

The masses of the fragments \(S\) and \(\Delta\) are still given by \([10]\). With \(M_{15} < M_G, M_{10} \lesssim M_G\) the masses of remaining decoupled states are

\[
M(q_{10}, q_{15}) \sim M(q_{15}, q_{10}) \simeq M_G,
\]

\[
M(u_{10}', u_{15}') \sim M(e_{10}', e_{15}') \simeq M_{10},
\]

and distribution of light \(q, u^e, e^c\) fragments will be as follows

\[
10' \ni q, \quad 10' \ni u^e, e^c, e'q, \quad \text{with } e' \equiv \frac{M_{10}M_{15}}{M_G^2}.
\]

We will identify the \(q\) state from \(10'\) and \(u^e\) from \(10\) with the third generation matter. Therefore, the up quark mass is generated through the coupling \(10^f \cdot 10H\), while the \(qqT\) coupling will be suppressed. In more detail, taking into account \([15]\) we will have

\[
Y_U 10' \cdot 10H \rightarrow Y_U (qu^e h_u + e'qqT). \tag{16}
\]

Note that \(e^cu^cT\) coupling is still not generated from \([16]\).

With these simple mechanisms we will be able to suppress \(d = 5\) proton decay up to the needed level. If for \(i\)-th generation \((i = 1, 2, 3)\) the suppression factor of the corresponding \(qqT\) operator is \(\epsilon_i\) [see Eqs. \([15], [17]\) for definition of these factors], and the up quark Yukawa matrix (involved in the coupling \(qY_Uu^eh_u\)) in a family space has the form

\[
Y_U = \begin{pmatrix} q_1 & q_2 & q_3 \\ a_1 & a_{12} & a_{13} \\ a_{21} & a_2 & a_{23} \\ a_{31} & a_{32} & a_3 \end{pmatrix},
\]

then the coupling \(Y_{qq}\) (involved in \(qY_{qq}qT\)) will be

\[
Y_{qq} \simeq q_1 \begin{pmatrix} \epsilon_1 a_1 & \epsilon_2 a_{12} & \epsilon_3 a_{13} \\ \epsilon_{12} a_{12} & \epsilon_{22} a_2 & \epsilon_{23} a_{23} \\ \epsilon_{13} a_{13} & \epsilon_{23} a_{23} & \epsilon_3 a_3 \end{pmatrix},
\]

with \(\epsilon_{12} a_{12} = \frac{1}{2}(a_{12} \epsilon_2 + a_{21} \epsilon_1)\).

\[
\epsilon_{13} a_{13} = \frac{1}{2}(a_{13} \epsilon_3 + a_{31} \epsilon_1), \quad \epsilon_{23} a_{23} = \frac{1}{2}(a_{23} \epsilon_3 + a_{32} \epsilon_2).
\]

Note that since \(q\) and \(e^e\) states come from different \(SU(5)\) states, we can also avoid the asymptotic relation \(M_D = M_E\) common for minimal \(SU(5)\) GUT. This will be discussed in more detail later on.

### B. Suppressing \(q1\bar{T}\) and \(u^e d^c \bar{T}\) Operators

Now we will present the mechanism for suppressing \(q1\bar{T}\) couplings. Recall that in \(SU(5)\) this type of terms originate from the couplings responsible for generation of down quark and charged lepton masses [see Eq. \([3]\)]. The suppression of \(q1\bar{T}\) can occur if the light \(l\) and \(d^c\) are coming from different \(SU(5)\) states. To realize such a splitting in a natural way we introduce additional vector like \(SU(5)\) matter \(5' + 5', \Psi(50) + \bar{\Psi}(50)\) and the following interaction terms

\[
M_5 \tilde{5} \cdot 5' + \rho \frac{\Sigma^2}{M_5} \tilde{5}' \bar{\Psi} + \bar{\rho} \frac{\Sigma^2}{M_5} \tilde{5}' \bar{\Psi} + M_\Psi \bar{\Psi} \bar{\Psi},
\]

\((\rho, \bar{\rho} \) are dimensionless couplings). The 50-plet does not contain the state with the quantum number of the lepton doublet \([14]\), however it includes the state with quantum numbers of \(d^c\). Therefore, after substituting appropriate VEVs in \([19]\), for the mass couplings of the corresponding fragments we will have

\[
\begin{pmatrix} d_5^c \\ d_5^e \end{pmatrix} = \begin{pmatrix} \tilde{d}_5^c & \tilde{d}_5^e \\ 0 & M_5 \end{pmatrix} \begin{pmatrix} \bar{a}_{12} M_5 e_G \\ \bar{a}_{23} M_\Psi \end{pmatrix},
\]

\[
\begin{pmatrix} l_5^c \\ l_5^e \end{pmatrix} = \begin{pmatrix} 0 \\ \rho M_5 e_G \end{pmatrix},
\]

where \(e_G = M_G/M_a\). As we see, \(l_5^c\) forms massive state with \(l_5\), and therefore the light lepton doublet emerges from \(5'\). However, the situation is different for \(d^c\). After integrating out \(d_5^c, d_5^e\) states, the \((2,1)\) element in the first matrix of \([20]\) receives the correction \(\tilde{M} = M_G^2 e_G^2/M_\Psi\). Assuming that \(\tilde{M} \gg M_5\), the light \(d^c\) state mostly remains in \(5\), while light lepton doublet \(l^c\) purely in \(5'\). Therefore, we have

\[
5 \ni d^c, \quad 5' \ni l, \epsilon'' d^c,
\]
\[
e' = \frac{M_5}{M} \ll 1, \quad \hat{M} \sim \rho^2 \frac{M_5^2}{M_\Psi} \sigma_5.
\]

The masses of the decoupled states are
\[
M(d^c_5, \overline{d}^c_5) = \hat{M}, \quad M(l_5, \overline{l}^c_5) = M_5,
\]
and all states from \(\Psi, \overline{\Psi}\) have mass \(M_\Psi\). From (21) we see that the light lepton doublet and \(SU(2)_L\) singlet down quark are coming from different \(SU(5)\) multiplets. This splitting will be crucial for suppression of \(qqT\) coupling. To see this, we should discuss the mass generation of the down quarks and charged leptons. Thus, it is important to know where the light left handed quark doublet \(q\) comes from. If the light \(q\) state emerges from 15-plet and \(\epsilon e\) state from 10 [the mechanism ensuring suppression of \(qqT\) coupling for 1st or/and 2nd family; see Eq. (11)], then the operators responsible for down quark and charged lepton masses are \(15 \cdot 5\bar{H} \) and \(10 \cdot 5'\bar{H}\) respectively. Namely, taking into account (8), (9) and (21) we have
\[
Y_D 15 \cdot 5\bar{H} \rightarrow Y_D dq \epsilon^\dagger h_d,
\]
\[
Y_E 10 \cdot 5'\bar{H} \rightarrow Y_E (\epsilon^\dagger lh_d + \epsilon q\bar{q}T + \epsilon'^\dagger u\bar{c}d\bar{T}) .
\]
As we see \(q\bar{q}T\) term emerges from coupling responsible for the charged lepton mass and is suppressed by factor \(\epsilon\). At the same time, the \(u\bar{c}d\bar{T}\) coupling is also suppressed. Since the \(\epsilon^\dagger u\bar{c}d\bar{T}\) coupling can be absent (see the discussion in the previous subsection) the corresponding right handed \(d = 5\) operator \(u\bar{c}d\bar{T}\) would not emerge at all. As far as the left handed operator is concerned, taking into account (11) and (4), it will have the form
\[
\epsilon^2 Y_L Y_E \frac{\epsilon'^\dagger}{M_T} eqq\bar{q} \bar{l}.
\]
Note that together with suppression of \(qqT\), also the relation \(M_D = M_E\) is avoided. The reason is simple: the Yukawa matrices \(Y_D\) and \(Y_E\) arise from completely independent interaction terms of (20) and (21), respectively.

Now let us show how the suppression of \(qqT\) coupling works for the case corresponding to Eq. (15) (suppression of \(qqT\) operator involving third family). In this case the terms \(10' \cdot 5\bar{H}\) and \(10 \cdot 5'\bar{H}\) are responsible for down type quark and charged lepton masses respectively. In particular, taking into account (15) and (21) we have
\[
Y_D 10' \cdot 5\bar{H} \rightarrow Y_D dq \epsilon^\dagger h_d,
\]
\[
Y_E 10 \cdot 5'\bar{H} \rightarrow Y_E (\epsilon^\dagger lh_d + \epsilon q\bar{q}T + \epsilon'^\dagger u\bar{c}d\bar{T}) .
\]
Therefore, the corresponding \(d = 5\) operator emerging from (15) and (20)
\[
(\epsilon')^2 \frac{Y_L Y_E}{M_T} eqq\bar{q} \bar{l} ,
\]
is suppressed by factor \((\epsilon')^2\), while \(u^\dagger u^\dagger d\bar{c}e\)-type operator is still absent.

As we see, in both cases [corresponding to (25) and (27)] the \(q\bar{q}T\) term emerges from the Yukawa couplings responsible for charged lepton masses. Thus, if \(Y_E\) in a family space has the structure
\[
Y_E = \left(\begin{array}{ccc}
\epsilon_1 & \epsilon_2 & \epsilon_3 \\
1 & 2 & 3 \\
a & b & c
\end{array}\right),
\]
then the matrix \(Y_q\) (involved in \(qY_ql\bar{T}\) coupling) will be
\[
Y_q \approx \left(\begin{array}{ccc}
q_1 & q_2 & q_3 \\
l_1 & l_2 & l_3 \\
1 & 2 & 3
\end{array}\right).
\]
Here, the factors \(\epsilon_i\) are the same as appeared in (18).

As we see, the split multiplet mechanisms we have discussed give good chance for the suppression of nucleon decay. Of course, one should make sure that all couplings which may lead to fast proton decay are absent. For example, the term \(u\bar{c}d\bar{T}\) can originate from the operator \(10 \cdot 10\bar{H}\). Therefore, some care should be taken to suppress such a coupling. Also, the Planck (cut off) scale suppressed \(d = 5\) baryon number violating operators must be adequately suppressed. In a concrete model, presented in sect. IV, we will show that all this can be achieved and justified by symmetry arguments.

### III. NATURALLY SUPPRESSED \(d = 6\) PROTON DECAY

In SUSY \(SU(5)\) the exchange of super-heavy \(V_X, V_l\) gauge superfields induce dimension six baryon number violating operators. The corresponding \(D\)-terms are \((qqu^\dagger l^\dagger)_{D}\) and \((glu^\dagger l^\dagger)_{D}\). Dimension six operators also emerge in non SUSY GUTs and may be more problematic if the GUT scale is lower than one in SUSY GUT (~\(10^{16}\) GeV with MSSM spectrum below \(M_G\) scale).

Thanks to the mechanism discussed in the previous section, these kind of operators can be also suppressed. Crucial role is played by splitting of appropriate matter. We will discuss the \(d = 6\) operator suppression on example of SUSY \(SU(5)\). Let us start consideration with 5-plet superfields which include states with the quantum numbers of \(d^c\) and \(l\). The \(D\)-terms including 5-plets are
\[
\left(5^1 \epsilon \overline{9V^5_5} + 5^1 \epsilon \overline{9V^5_5} \right)_{D},
\]
where \(V\) and \(g\) are \(SU(5)\) gauge superfield and the gauge coupling at scale \(M_G\) respectively. According to (21), the 5 states do not include light lepton doublets \(l\) at all.

\[
\]
and therefore the first term in (30) is irrelevant for us. However, from the second term of (30) we get
\[
\left(5^i e^{-gV 5'} \right)_D \to e'' g (l^l V_X d^c + d^c V_Y l)_D , \quad (31)
\]
As we see, the couplings of the heavy \( V_X, Y \) gauge superfields with the matter are suppressed by factor \( e'' \).

The kinetic \( D \)-term of 15-plet is irrelevant for the baryon number violation because from light states 15-plet includes only \( q \). For the case corresponding to Eqs. (29), (9) only 10-plet's \( D \)-term is relevant:
\[
(10^1 e^{gV} 10)_D \to \epsilon g \left( V_X (q^1 e^c + qu^c) + V_Y (qe^c + q^1 u^c) \right)_D , \quad (32)
\]
producing couplings with the suppression factor \( \epsilon \).

Upon integration of the \( V_X, Y \) states with mass \( \approx M_G \) from (31) and (32) we get the following baryon number violating \( d = 6 \) operators
\[
g^2 \left[ \epsilon^2 e^{6u^c} e^{6c} + \epsilon'' q u^c d^c + h.c. \right]_D . \quad (33)
\]
As we see, two \( d = 6 \) operators in (33) are naturally suppressed by factors \( \epsilon^2 \) and \( \epsilon'' \) respectively. Note that if we are dealing with case corresponding to Eq. (15), then the factor \( \epsilon \) in (33) must be replaced by \( \epsilon' \). Once more, this mechanism for the suppression of \( d = 6 \) nucleon decay also can be applied within non SUSY \( SU(5) \).

**IV. EXAMPLE OF REALISTIC SUSY \( SU(5) \)**

The possibilities for suppressing the proton decay in SUSY \( SU(5) \) GUT discussed above can be successfully applied for the realistic model building. By proper selection of the appropriate mass scales we can get suppression \( [\epsilon, \epsilon'] \) in Eqs. (8), (15) as strong as we wish. This requires the scales \( M_{15} \) and \( M_5 \) to be below \( M_G \). However, this introduces an additional states below the GUT scale, and the running of gauge couplings will be altered. In order to maintain successful gauge coupling unification, an additional constraint on these scales should be imposed. Suppression of \( q \bar{q} T \) coupling brings the states \( (S + \bar{S})_{15} \) and \( (\Delta + \bar{\Delta})_{15} \) below \( M_G \). With their masses \( M_S = M_\Delta = M_{15} \), one can see that the states \( S, S \) contribute stronger to the running of \( \alpha_3 \) in comparison of \( \Delta, \bar{\Delta} \)'s contribution into the \( \alpha_2 \) running. To compensate this dis-balance an additional \( SU(2)_L \) states are required. This occurs naturally if the mechanism for \( q \bar{q} T \) suppression is invoked. In this case below \( M_G \) we also have an additional \( SU(2)_L \) doublets (see Eq. (22)). This offers possibility for successful gauge coupling unification.

Now, we present an example of SUSY \( SU(5) \) realizing ideas discussed above. Considering three families of quarks and leptons, the appropriate couplings (such as of Eqs. (17), (11), (19), (24)) should be promoted to the matrices in a family space.

**TABLE I: \( U(1) \times Z_{3N} \) charges \( Q, q \) of the scalar superfields.**

| \( X \) | \( Z \) | \( \Sigma(24) \) | \( H(5) \) | \( \bar{H}(5) \) |
|---|---|---|---|---|
| 1 | -1 | 0 | -1/3 | -2/3 |
| 0 | 3 | 0 | -1 | 1 |

Thus, we introduce three pairs of 15-plets: \( (15 + \bar{15})_i \) \( (i = 1, 2, 3) \) and the pair \( 10' + \bar{10}' \) (needed for renormalizable top Yukawa coupling), and also \( (5' + \bar{5}')_i, (\Psi + \bar{\Psi})_i \).

In addition, we introduce \( U(1) \times Z_{3N} \) symmetry, where as will turn out \( U(1) \) is an anomalous and \( Z_{3N} \) is discrete symmetry. Importance of these symmetries will become obvious soon. The anomalous \( U(1) \) factors can appear in effective field theories from strings and cancellation of its anomalies occurs through the Green-Schwarz mechanism [17]. Due to the anomaly, the Fayet-Iliopoulos term \(-\xi \int d^4V_A \) is always generated [18] and the corresponding \( D_A \)-term has the form [19]
\[
g^2 \frac{2}{8} D_A^2 = g^2 \frac{2}{8} \left( -\xi + \sum Q_i |\phi_i|^2 \right)^2 , \quad \xi = g^2 M_D^2 \frac{2}{192\pi^2} \text{Tr} Q , \quad (34)
\]
where \( Q_i \) is the \( U(1) \) charge of superfield \( \phi_i \). The transformations under \( U(1) \) and \( Z_{3N} \) are respectively
\[
U(1) : \quad \phi_i \to e^{iQ_i} \phi_i ,
\]
\[
Z_{3N} : \quad \phi_i \to e^{i\omega} \phi_i , \quad \text{with} \quad \omega = \frac{2\pi}{3N} . \quad (35)
\]
The anomalous \( U(1) \) can be very useful for building models with realistic phenomenology [20], and we also take advantage of it here for avoiding unwanted couplings. The symmetry \( Z_{3N} \) also will play a crucial role. We introduce two \( SU(5) \) singlet superfields \( X \) and \( Z \) which will be used for \( U(1) \times Z_{3N} \) breaking. The \( Q_i \) and \( q_i \) charges of scalar superfields are given in Table I. Let us first discuss the VEV generation for scalar components of \( X \) and \( Z \) superfields. The lowest superpotential coupling for these superfields, allowed by \( U(1) \times Z_{3N} \) symmetry, is
\[
W(X, Z) = \sigma M_3^3 \left( \frac{X Z}{M_3^2} \right)^N , \quad (36)
\]
where \( \sigma \) is dimensionless coupling. With \( \xi > 0 \), in unbroken SUSY limit the conditions \( D_A = 0, F_X = F_Z = 0 \) give \( \langle X \rangle = \sqrt{\xi} \) and \( \langle Z \rangle = 0 \). However, the non zero VEV for \( Z \) can be generated after including the soft SUSY breaking potential terms
\[
V_{SB} = m_{3/2}^2 (|X|^2 + |Z|^2) - A m_{3/2} (W + W^\dagger) \quad (37)
\]
Thus, we should minimize the whole potential
\[
V = g^2 \frac{2}{8} D_A^2 + |F_X|^2 + |F_Z|^2 + V_{SB} , \quad (38)
\]
where
\[ D_A = -\xi |X|^2 - |Z|^2, \quad F_X = \frac{\partial W}{\partial X}, \quad F_Z = \frac{\partial W}{\partial Z}. \] (39)

Considering soft breaking contribution as a perturbation to the potential’s leading part, it is natural that by proper selection of \( N \) we will get \( \langle Z \rangle \ll \langle X \rangle \). Therefore we parameterize the vacuum as
\[ \langle |X|^2 \rangle = \xi (1 - \kappa^2) + \alpha m_{3/2}^2, \quad \langle |Y|^2 \rangle = \kappa^2 \xi, \] (40)

where
\[ \alpha \sim 1, \quad \kappa \ll 1, \] (41)

should be found from the minimization. Note that with this parameterization \( D_A \)-term is shifted by the SUSY scale\( \sim m_{3/2}^2 \). Minimizing the potential (38) with real \( A \) and conditions in (38), we find analytically
\[ \alpha = -\frac{4}{g_A^2} + O(\kappa^2), \quad \kappa = \alpha \left( \frac{m_{3/2}}{M_*} \right)^{\frac{-1}{2}} \left( \frac{\sqrt{\xi}}{M_*} \right)^{-2\frac{N-1}{N}}, \]
with \( \alpha = \left( \frac{A \pm \sqrt{A^2 - 8\langle N - 1 \rangle}}{2\sigma N \langle N - 1 \rangle} \right)^{\frac{1}{2}} \). (42)

With \( m_{3/2} \sim 1 \text{ TeV}, M_* \sim 10^{17} \text{ GeV} \) (we will comment on this value of the cut off scale below), \( \sqrt{\xi} \sim 10^{16} \text{ GeV} \) and \( N = 6 \) we have \( \kappa \sim 0.1 \) and therefore initial assumption (38) is justified. Thus, finally we have
\[ \frac{\langle X \rangle}{M_*} \sim \frac{\sqrt{\xi}}{M_*} = 0.1, \quad \frac{\langle Z \rangle}{M_*} \sim \kappa \frac{\sqrt{\xi}}{M_*} \sim 10^{-2}. \] (43)

Below we will use these values obtained by \( U(1) \times Z_{3N} \) (\( N = 6 \)) symmetry.

One may wonder whether with charge assignments given is Table II desirable GUT symmetry breaking can occur or not. Also, the color triplets from \( H, \bar{H} \) should be super-heavy, while doublets should remain massless (doublet-triplet (DT) splitting). Since the adjoint \( \Sigma \) is not transformed under \( U(1) \times Z_{3N} \) symmetry, the renormalizable superpotential includes couplings
\[ W(\Sigma) = M_2 \bar{T}_3 \Sigma^2 + \lambda_2 \bar{T}_3 \Sigma^3, \] (44)

and the non zero VEV \( \langle \Sigma \rangle = V_Z \cdot \text{Diag} (2, 2, 2, -3, -3) \) with \( V_Z = \frac{\sqrt{2}}{\alpha} \) is obtained. This insures the breaking \( SU(5) \to SU(3)_c \times SU(2)_L \times U(1)_Y \). As far as the DT splitting is concerned, without invoking some particular mechanism it can be achieved by fine tuning if couplings \( (M_H + \lambda_H \Sigma) \bar{H} \). However, these couplings are forbidden by \( U(1) \times Z_{3N} \) symmetry. Instead, the operators \( \lambda XHH + X \Sigma \bar{H} / H^\prime \) are allowed. They lead to the DT splitting with \( M' \sim \langle X \rangle \) and \( \lambda \sim V_Z / \langle X \rangle \). The operator \( \lambda XHH / H^\prime \) can be generated by decoupling of additional states with mass \( M' \). For example, introducing \( H'(5), \bar{H}'(5) \) states with \( (Q, q) \) charges \(-1/3, -1\) respectively, the relevant couplings are
\[ \lambda' XHH + \lambda \Sigma \bar{H} / H^\prime. \] (45)

After integrating out the heavy \( H', \bar{H}' \) states, we remain with effective superpotential couplings
\[ \lambda' XHH - \lambda \Sigma \bar{H} / M' \Sigma HH. \] (46)

With a selection \( \lambda' = -3 \sigma_1 \sigma_2 V_Z / M' \) the MSSM Higgs doublets remain massless, while the color triplets occur mass \( M_T = 5 \lambda' \langle X \rangle / 3 ~ M_Z \) with \( \lambda' \sim V_Z / \langle X \rangle \). Therefore, we see that within SUSY \( SU(5) \) GUT, augmented with \( U(1) \times Z_{3N} \) symmetry, it is possible to build self consistent scalar sector.

Now we are ready to discuss the fermion sector. \( U(1) \times Z_{3N} \) charge assignments for matter states are displayed in Table II. Note that with this prescription all matter parity violating operators are forbidden. Therefore, thanks to the \( U(1) \times Z_{3N} \) symmetry the \( R \)-parity is automatic. The reason for this is the fact that by VEVs \( \langle X \rangle, \langle Z \rangle \) the \( U(1) \times Z_{3N} \) is not completely broken. Namely, the subgroup \( Z_3^A \times Z_3 \) remains unbroken. The transformations under \( Z_3^A \) and \( Z_3 \) are respectively
\[ Z_3^A : \quad \phi_i \to e^{i2\pi q_i} \phi_i, \]
\[ Z_3 : \quad \phi_i \to e^{i\pi/3} \phi_i, \quad \text{with} \quad \pi = \frac{2\pi}{3}. \] (47)

The superfields \( X, Z \) are neutral under \( Z_3^A \) and \( Z_3 \).

In understanding of observed hierarchies between charged fermion masses and mixings crucial role plays the flavor structure of the Yukawa sector. Same is true in connection of the color higgsino mediated and Planck scale suppressed \( d = 5 \) operator induced nucleon decay. Their structures determine the signature of the nucleon decay. Definite structures as well as predictions can be obtained by flavor symmetries. Indeed, symmetry principle is very powerful for a predictive power. We will not introduce here generation symmetries, and instead consider one particular example demonstrating realization of the split multiplet mechanism.

Thus we promote the couplings of \( \rho, \omega \) in the flavor space as
\[ \lambda_{10,15\Sigma} + \lambda_{15,15\Sigma} + \lambda_{15,15\Sigma} + M_{10,10'10}, \] (48)
where for simplicity we have assumed that the matrices $\lambda$, $\lambda^2$ are diagonal and only 15s couples with $10'$. Moreover, we take

$$\lambda_i \sim \lambda \sim 1, \quad M_{10} \sim M_G,$$

$$M_{15_1}, M_{15_2} = M_{15_3} \equiv M_{15} \ll M_G,$$

where $M_{15_3} = \lambda^2 i (Z)$. Thus, with

$$\epsilon_1 = \frac{M_{15_1}}{\lambda_1 M_G}, \quad \epsilon_2 = \frac{M_{15_2}}{\lambda_2 M_G},$$

$$\epsilon_3 = \frac{M_{10} M_{15_3}}{\lambda_3 M_G^2}, \quad (\epsilon_2 \sim \epsilon_3 \equiv \epsilon),$$

and carrying out analysis analogous done in sect. [11A] we will have

$$q_{1,2} \subset 15_{1,2}, \quad q_3 \subset 10', \quad (u^c, e^c)_{1,2,3} \subset 10_{1,2,3},$$

$$10_1 \supset \epsilon_1 q_1, \quad 10_2 \supset \epsilon q_2, \quad 10_3 \supset \epsilon q_3.$$

The couplings $\frac{1}{\lambda} 10' \Sigma_i 15_i$ because of the suppression factor $\sim \langle Z \rangle / M_* \sim 10^{-2}$ do not change these relations. They cause $15_3 \supset 10^{-2} q_3, 10_i \supset 10^{-2} q_3$ which will not have any impact on our studies. The mass spectrum of the decoupled states is

$$M(q_{10}, \bar{q}_{10}) \sim M(q_{15_3}, \bar{q}_{10}) \sim M(u^c_{10}, \bar{u}^c_{10}) \simeq \epsilon M_G,$$

$$M(e^c_{10}, \bar{e}^c_{10}) \simeq M_G, \quad M_{S_1} = M_{\Delta_1} = M_{15_3} \simeq \epsilon_1 M_G,$$

$$M_{S_{2,3}} = M_{\Delta_{2,3}} = M_{15} \simeq \epsilon M_G.$$

Moreover, the couplings in [19] will be replaced by $U(1) \times Z_{3N}$ invariant terms

$$\tilde{5}_l \left( \begin{array}{ccc} 5_i' & \Psi_i & 0 \\ M_5 & 0 & \rho \Sigma_2 / M_* \end{array} \right),$$

$$\tilde{5}'_i \left( \begin{array}{ccc} M_5' Z / M_* & \rho \Sigma_2 / M_* \end{array} \right).$$

Here we still assumed that the appropriate entries are diagonal and universal. We assume that $M_5 \sim M_5'$ (the smallness of both these scales, with respect to $M_G$ or $M_*$, may have same origin. However, this is not explained here). Therefore, carrying out similar analysis presented in previous section, with

$$\epsilon'' \equiv \frac{M_5}{M} \ll 1, \quad \tilde{M}_0 \sim \rho \frac{M_G^2 (Z)}{M_5 M_*} \epsilon'^2,$$

we have

$$\tilde{5}_l \supset d^c_i, 10^{-2} l_i, \quad \tilde{5}'_i \supset l_i, \quad \epsilon'' d^c_i.$$

The decoupled states will have the masses

$$M(l_5, l_{5'}) = M_5 ,$$

$$M(d^c_i, d_{5'}) = \tilde{M}, \quad M_{\Psi_i} = M_\Psi.$$

Note that in $(1,2)$ entry of [53] the operator $\Sigma^3 (XZ)^5 / M_*^{11}$ is allowed. However, it would induce strongly suppressed correction $\sim 10^{-17} M_G$ and is not relevant. Now we can discuss the gauge coupling unification. The latter suggests the particular selection for appropriate mass scales.

A. Gauge coupling unification

We assume that the masses of the matter $50_3$-plets are close to the cut off scale $M_\Psi \simeq M_* - \text{much higher than the GUT scale}$. Thus, they do not affect the gauge coupling running. Moreover, with $M_{S_5} \sim M_G$ and $\lambda_5 \sim 1$ in [14] for colored octet and $SU(2)_L$ triplet (from adjoint $\Sigma$) masses we get $m_{S_8} = m_{S_3} \sim M_G$ and with $M_G \ll M_*$, higher order operators will not affect this relation. Also, with color triplets’ mass (from $H, \bar{H}$) near $M_G$, these states will not contribute to the gauge coupling running and at the leading order will not play role in determination of the GUT scale (unlike the proposals of [14]). However, for the masses of the three vector like pairs we have $M(d^c_{15}, d_{15'}) \equiv \tilde{M}$ (see Eq. [20]). Apart these states, below $M_G$ we have $3 \times (l_5 + l_{5'})$ and $3 \times (S + \bar{S} + \Delta + \bar{\Delta})_{15}$ states with masses given in [52] and [56]. Thus, for the strong gauge coupling constant at $M_Z$ scale in 1-loop approximation we get:

$$\alpha_3^{-1} = \left( \alpha_3^0 \right)^{-1} - \frac{3}{2\pi} \ln (\epsilon'^2) - \frac{27}{14\pi} \ln \frac{\tilde{M}}{M_5},$$

where $\alpha_3^0$ is the value of the strong coupling constant within MSSM and is $\alpha_3^0 (M_Z) \simeq 0.126$ [22]. The additional in [57] allow to obtain the value compatible with experiments $\alpha_3^{\text{exp}} (M_Z) \simeq 0.1176$ [3]. This can be achieved with $\frac{M_5}{M} \simeq 14^{15} (\epsilon'^2) / 7^{9}$. In order to have more accurate estimate we have performed calculations in two loop approximation. The picture of gauge coupling unification is given in Fig. 1.

For simplicity we have taken all squark, slepton, higgsino and gaugino masses all near the TeV scale. In particular

$$m_{\tilde{q}} = m_{\tilde{t}} = m_{\tilde{b}} = m_{\tilde{\mu}_{susy}} = 10^{2.9} \text{ GeV},$$

$$M_{\tilde{W}} = m_{\tilde{\mu}} \frac{\alpha_2}{\alpha_3} \bigg|_{\mu = m_{\tilde{\mu}_{susy}}} \simeq 287 \text{ GeV}.$$

Also, we have taken

$$\epsilon_1 = 1/3, \quad \epsilon = 0.1, \quad \frac{M_5}{M} \simeq 2.2 \cdot 10^{-2},$$
The appropriate factor \( A_L \) is the long range factor which is mainly due to QCD running.

Let us start with calculation of the short range factor corresponding to \( d = 5 \) operators. Note that in our model the \( qfT \) coupling is related to the charged lepton Yukawa matrix. Therefore, generalizing expression given in \([23]\), we will have

\[
A^S_{d=5} = A^S_{d=5, uc} = A(\lambda_t) \prod_{i,a>b} \frac{\alpha_i(\mu_a)}{\alpha_i(\mu_b)} \frac{\epsilon_i}{(10 a-b)} ,
\]

where \( b_i(\mu_{a-b}) \) denotes gauge coupling 1-loop \( b \)-factors in the mass interval \( \mu_b - \mu_a \) and \( A(\lambda_t) \) includes the renormalization effect due to the top Yukawa coupling. We have evaluated \( A^S_{d=5} \) for our scenario (more details of the Yukawa sector is given in section \([IVB]\) in 2-loop approximation for \( \lambda_t(M_Z) \approx 1 \) and obtained

\[
A^S_{d=5} \approx 2.03 ,
\]

(to be compared with the factor obtained in MSSU5 \( A^S_{d=5} \approx \) 2.06). As far as the \( d = 6 \) operators are concerned, as it will turn out, the first type operator of Eq. (63) will be relevant. It’s 1-loop short range renormalization factor is given by

\[
A^S_{d=6} = \prod_{i,a>b} \frac{\alpha_i(\mu_a)}{\alpha_i(\mu_b)} \frac{\epsilon_i}{(10 a-b)} , \quad \bar{c}_i = \left( \frac{23}{30} , \frac{3}{2} , \frac{4}{3} \right) .
\]

In our model numerically we get \( A^S_{d=6} \approx 2.23 \). Also long range renormalization factor \( A_L \) should be taken into account. The latter is \( A_L \approx 1.34 \) \([24]\), and finally for \( d = 6 \) operator renormalization factor we have

\[
A^R_{d=6} = A_L A^S_{d=6} \approx 2.99 .
\]

### B. Proton life time

The nucleon decay via \( d = 5 \) operators crucially depends on Yukawa sector. Therefore, first we briefly discuss how desirable fermion pattern can be obtained. Since we have arranged the multiplet splitting, displayed in Eqs. (61), (65), will not be difficult to get realistic fermion masses. Once more we stress that we consider one particular example with simple flavor structure. Starting with up type quarks, we will write appropriate couplings in such a way that the up quark mass matrix will be diagonal. Relevant terms consistent with \( U(1) \times Z_{3N} \) symmetry are

\[
\sum_{M} \frac{1}{M^\alpha} (\gamma_1 15 10_1 + \gamma_2 15 2 10_2) H + \gamma_3 10^3 10_3 H ,
\]

**FIG. 1: Gauge coupling unification.** \( \alpha_3(M_Z) \approx 0.1176, M_G \approx 2.2 \cdot 10^{15} \) GeV.

\[
\text{with } \tilde{M} \approx 2.1 \cdot 10^{11} \text{ GeV} .
\] (For stronger suppression of nucleon decay smaller values of \( \epsilon_1, \epsilon \) are required. However, this would make new states lighter and the constraint from the coupling unification does not give much flexibility.) All this and input values \( \alpha_1^{-1}(M_Z) = 59.0, \alpha_2^{-1}(M_Z) = 29.6 \) provides the successful unification with

\[
\alpha_3(M_Z) = 0.1176 , \quad M_G \approx 2.2 \cdot 10^{15} \text{ GeV} ,
\]

\[
\alpha_G^{-1}(M_G) \approx 14.91 .
\] As we see, due to the new states the unification scale \( M_G \) is reduced, while the unified gauge coupling \( \alpha_G \) is enhanced:

\[
\frac{M_G}{M_G^0} \approx \frac{1}{9.1} , \quad \frac{\alpha_G}{\alpha_G^0} \approx 1.6 ,
\] [superscript ‘0’ indicates the values obtained within minimal SUSY \( SU(5) \)]. These values will be useful for estimating of the proton decay rates in this model.

One can see that near \( 10^{17} \) GeV scale the unified gauge coupling becomes strong \( \frac{\alpha_G}{\alpha} \approx 0.26 \). Thus, for cut off scale one should take \( M_* \approx 10^{17} \) GeV. With this, the perturbative regime is kept in a quite wide range above the GUT scale. As we will see shortly, the values of \( \epsilon_1 \) and \( \epsilon \) selected here provide an adequate suppression of the proton decay.

Finally, we calculate short range renormalization factors which will be used in the next subsection. The appropriate baryon number violating \( d = 5 \) and \( d = 6 \) operators, generated at GUT scale, should be defined at scale \( \mu = 1 \) GeV. Thus two ranges are relevant for renormalization. Due to running from \( M_G \) down to \( M_Z \) (or SUSY scale) the appropriate factor \( A^S \) is called short range renormalization factor. From scale \( M_Z \) down to
where $\gamma_{1,2,3}$ are dimensionless constants. Taking into account \([51]\) we will have

$$Y_U = \text{Diag} (\lambda_u, \lambda_c, \lambda_t) , \quad \lambda_{u,c} \sim \gamma_{1,2} \frac{(\Sigma)}{M^4} , \quad \lambda_t = \gamma_3.$$  \(\text{(67)}\)

Thus, the CKM mixings ($V_{ij}$) should come from the down quark sector. The relevant couplings for the latter are

$$(15,1,15,2,10') Y_D \begin{pmatrix} 5_1 & 5_2 & 5_3 \end{pmatrix} \tilde{H} ,$$

\(\text{(68)}\)

where the non-diagonal Yukawa matrix $Y_D$ is responsible for CKM mixings.

Thanks to the mechanism discussed in sect. \(\text{III}\) the charged lepton Yukawa matrix elements are independent from $Y_D$. For simplicity we take diagonal couplings

$$Y_{\ell} 10 \bar{5}' i \tilde{H} ,$$

\(\text{(69)}\)

which with \([51], [55]\), give

$$Y_E = \text{Diag} (\epsilon_1 \lambda_u , \epsilon_1 \lambda_c , \epsilon_1 \lambda_t) , \quad \lambda_{u,c,\mu,\tau} = Y_{1,2,3} .$$

\(\text{(70)}\)

Note, that the $U(1) \times Z_{3N}$ symmetry provides the suppression of the coupling $15 \cdot 5 \tilde{H}$ by factor $\sim \langle Z \rangle / M_\ast \sim 10^{-2}$ in comparison of \([68], [69]\) operators, and therefore can be ignored. As far as the $10 \cdot 5 \tilde{H}$ type couplings are concerned, they are strongly suppressed($\sim \langle X(Z) \rangle^2 \sim 10^{-16}$). From all this and Eqs. \([18], [29]\) $Y_{qq}$ and $Y_{ql}$ matrices will be

$$Y_{qq} = \text{Diag} (\epsilon_1 \lambda_u , \epsilon_1 \lambda_c , \epsilon_1 \lambda_t) ,$$

\(\text{(71)}\)

These couplings induce $qqql$ type $d = 5$ left handed operators.

Before estimating the proton life time, let us note that the couplings $10,10,10 \tilde{H}$ are forbidden by $U(1) \times Z_{3N}$ symmetry. Only the higher order operators $X^a X^b X^c M_\ast^{20,10,10 \tilde{H}}$ are allowed. For the VEVs given in \([63]\), induced operator $u^a e^b T$ is suppressed by factor $\sim 10^{-16}$. This makes color triplet induced $d = 5$ right handed baryon number violating operators completely irrelevant. As far as the cut of scale suppressed $d = 5$ baryon number violating operators are concerned, one can easily see that they also involve high powers of $\langle Z \rangle / M_\ast$ and $\langle X \rangle / M_\ast$. Namely, the allowed couplings are $X(Z X)^4 10,10,10 \tilde{H}$, $(X Z)^{4} 10,10,10 \tilde{H}$, etc. Thus the suppression by factors $\sim 10^{-15}$ is guaranteed. Therefore, we conclude that in our model only sources for the proton decay are the couplings given in \([71]\) and $X, Y$ boson induced decay which we discuss afterwards.

The appropriate $d = 5$ left handed operator is converted to four fermion operators through the Wino dressings. Those, relevant for nucleon decay, have form

$$- \frac{1}{M_T} \mathcal{F} \alpha_{ij}(u^d)(u^e) ,$$

\(\text{(72)}\)

where, together with other family independent factors, $\mathcal{F}$ includes the loop integral, and for simplicity we have assumed that the squarks and sleptons of all families have universal mass. The flavor dependent couplings $\alpha_{ijk}$ and $\alpha_{ij'}$ are given by \([22], [71]\)

$$\alpha_{ijk} = (L_d^i Y_{qq} L_e)_{jk} (V^T L_u^i Y_{qq} L_d^j)^{11} + (L_d^i Y_{qq} L_u^j V_{ij})_{kl} (V^T L_u^i Y_{qq} L_d^j)_{kl} + (L_d^i Y_{qq} L_u^j V_{ij})_{kl} (L_u^j Y_{qq} L_e)_{kl} ,$$

\(\text{(73)}\)

$$\alpha_{ij'} = -(L_u^i Y_{qq} L_u^j)^{11} (V^T L_u^i Y_{qq} L_e)_{ij} + (L_u^i Y_{qq} L_u^j V_{ij})_{kl} (L_u^j Y_{qq} L_e)_{kl} + (L_u^i Y_{qq} L_u^j V_{ij})_{kl} (L_u^j Y_{qq} L_e)_{kl} ,$$

\(\text{(74)}\)

$L_{u,d,c}$ are unitary matrices transforming the left handed fermion states in order to diagonalize corresponding mass matrices.

For the considered case here we have $L_u = L_c = 1$, $L_d = V^*$. Therefore, the only non-diagonal matrix is the CKM matrix. In particular, using \([71]\) we have

$$\alpha_{ijk} = 2 \delta_{1k} \lambda_\epsilon \epsilon_1 (V^T Y_{qq} V_{ij}) .$$

These factors are responsible for the decays with neutrino emission. The dominant decay mode is $p \to K^+ \nu_e$ and the corresponding amplitude is proportional to $\frac{1}{M_T} 2 \lambda_\epsilon \lambda_\theta \epsilon_1 \epsilon_1 (\theta_e = 0.22$ is a Cabibbo angle). Note that in MSSU5 the amplitude of the dominant decay mode $p \to K^+ \nu_e$ is $\frac{1}{M_T} 2 \lambda_\epsilon \lambda_\theta \epsilon_1 \epsilon_1$.

Thus, in our model for the corresponding partial life time we expect

$$\tau_{d=5}(p \to K^+ \nu_e) =$$

$$\left( \frac{\lambda_\epsilon \theta_e}{\lambda_\epsilon \epsilon_1} \right)^2 \left( \frac{M_G}{M_i^2} \right)^2 \left( \frac{(A_{d=5})^0}{A_{d=5}^0} \right)^2 \tau_0 (p \to K^+ \nu_e) ,$$

\(\text{(75)}\)

where $\tau_0$ is proton life time in MSSU5. Taking all SUSY breaking soft terms near TeV scale, we have

$$\tau_{d=5}(p \to K^+ \nu_e) \simeq$$

$$3.8 \cdot 10^{3} \cdot \left( \frac{0.1}{\epsilon} \right)^2 \left( \frac{1/3}{\epsilon_1} \right)^2 \left( \frac{M_G}{2.2 \cdot 10^{15} \text{GeV}} \right)^2 \tau_0 .$$

\(\text{(76)}\)

In \([71]\) we used $A_{d=5}^0 = 2.03$ calculated for our model [see Eq. \([63]\)]. The decays with emission of the charged leptons are due to $\alpha'$ factors. The dominant mode is
\[ p \rightarrow K^0 \mu^+ \] (with corresponding factor \( \alpha'_{22} \approx 2 \lambda_a \lambda_c \epsilon_1 \)) with the life time

\[ \tau = 5(p \rightarrow K^0 \mu^+) \approx \frac{5.4 \cdot 10^3 \cdot \left( \frac{0.1}{\epsilon} \right)^2 \left( \frac{1/3}{\epsilon_1} \right)^2 \frac{M_G}{2.2 \cdot 10^{15} \text{GeV}}}{\tau_0} \cdot (78) \]

As we see, both decay modes of Eqs. (77), (78) are suppressed in comparison to the dominant decay mode of MSSU5. In order to make an estimate of proton life time one should make selection of sparticle spectrum. With soft terms near TeV, given in (55), we have \( \tau_0 \approx 3.5 \cdot 10^{30} \) years. Thus, we will have

\[ \tau = 5(p \rightarrow K^0 \nu_e) \approx 0.7 \cdot \tau = 5(p \rightarrow K^0 \mu^+) \approx \frac{1.3 \cdot 10^{34}}{\sin 2\beta} \cdot \frac{2}{2} \cdot (79) \]

These are above current experimental bounds \( \tau_{\exp}(p \rightarrow K^+ \nu_e) \approx 6.7 \cdot 10^{32} \) years and \( \tau_{\exp}(p \rightarrow K^0 \mu^+) \approx 1.2 \cdot 10^{32} \) years. Ongoing and planned experiments give promise to probe partial lifetimes given in (79) (these life times decrease with increase of tan \( \beta \)).

Since in our model the GUT scale is reduced nearly by factor 10 and the unified gauge coupling is stronger, the \( d = 6 \) operators become relevant. However, due to multiplet splitting, the suppression still occurs [see Eq. (33)]. The dominant \( d = 6 \) operator is \( \epsilon_2 \frac{\alpha}{\alpha} q_1^{\dagger} u_1^{\dagger} e_1^{\dagger} D \), where subscripts label the flavor indices. This operator induces the process \( p \rightarrow \pi^0 e^+ \) with a decay width:

\[ \Gamma_{d=6}(p \rightarrow \pi^0 e^+) = \frac{m_p}{16 \pi f_{\pi}^2} \tilde{\alpha}^2 (1 + D + F) \left( \frac{g^2}{M_G^2} \left( A_{\pi}^d \right)^2 \right) \cdot (80) \]

With \( f_{\pi} = 0.13 \text{ GeV}, \tilde{\alpha} = 0.015 \text{ GeV}^3, D = 0.8, F = 0.47, A_{\pi}^d = 2.99 \) and \( \epsilon_1 = 1/3 \) we get

\[ \tau_{d=6}(p \rightarrow \pi^0 e^+) = \frac{1}{\Gamma_{d=6}(p \rightarrow \pi^0 e^+)} \approx 5 \cdot 10^{33} \text{ years} , \]

which is slightly above the experimental limit \( \tau_{\exp}(p \rightarrow \pi^0 e^+) \approx 1.6 \cdot 10^{33} \) yrs.] This (possibly) dominant decay mode is a characteristic signature of our model. Future experiments will probe such decays and test viability of the particular model presented here.

V. CONCLUSIONS

In this paper we have suggested the mechanism for suppressing the nucleon decay within SUSY SU(5) GUT. The mechanism is based on idea of split multiplets and also helps to build realistic fermion pattern. For transparent demonstration of the presented mechanism we have considered simple example consistent with gauge coupling unification, realistic fermion mass pattern and the proton life time compatible with experiments.

The suggested possibilities can be applied for building various realistic SU(5) scenarios with interesting phenomenological implications. In particular, it would be interesting, in this context, to address the problem of flavor and try to gain a natural understanding of observed hierarchies between fermion masses and mixings. Also, it is desirable to understand the origin of hierarchies between various mass scales appearing in the construction. For all this an additional symmetries, such as flavor symmetry, may play crucial role guaranteeing the robustness of predictions. In a concrete model, for realizing suggested mechanisms and for suppression of unwanted baryon number violation, we have applied \( U(1) \times U_3 \) symmetry (also providing an automatic R-parity). It will be interesting to use such a symmetry as a flavor symmetry.

Finally, here we have not attempted to have natural solution of the doublet-triplet splitting problem. For the latter GUTs such as SO(10) \( [27] \) and SU(6) \( [28] \), \( [7] \) are more motivated. One can attempt to realize the split multiplet mechanism within these constructions and also study other phenomenology. These and related issues will be discussed elsewhere.

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[1] N. Sakai and T. Yanagida, Nucl. Phys. B 197 (1982) 533; S. Weinberg, Phys. Rev. D 26 (1982) 287.
[2] This decay mode dominates in simple minded SUSY SU(5) and SO(10) GUTs.
[3] W. M. Yao et al. [Particle Data Group], J. Phys. G 33 (2006) 1.
[4] G. R. Dvali, Phys. Lett. B 287 (1992) 101; K. S. Babu and S. M. Barr, Phys. Rev. D 48 (1993) 5354; hep-ph/9306242; J. Hisano, T. Moroi, K. Tobe and T. Yanagida, Phys. Lett. B 342 (1995) 138 hep-ph/9406147; B. Berezhiani, hep-ph/9602325.
[5] I. Gogoladze and A. Kobakhidze, Phys. Atom. Nucl. 60 (1997) 126 hep-ph/9610389; Z. Chacko and R. N. Mohapatra, Phys. Rev. D 59 (1999) 011702 hep-ph/9808458.
[6] Z. Berezhiani, Z. Tavartkiladze and M. Vysotsky, hep-ph/9809301.
[7] Q. Shafi and Z. Tavartkiladze, Phys. Lett. B 451 (1999) 129 hep-ph/9901243; Phys. Lett. B 459 (1999) 563 hep-ph/9904249.
[8] Q. Shafi and Z. Tavartkiladze, Nucl. Phys. B 573 (2000) 40 hep-ph/9905202.
[9] B. Bajc, P. Fileviez Perez and G. Senjanovic, Phys. Rev.
Of course, it is desirable to have more appealing mecha-
nisms to explain observed baryon number violation.

This property of 50-plet can be used to realize natural
doublet-triplet splitting in the scalar sector by miss-
ning partner mechanism: H. Georgi, Phys. Lett. B 108
(1982) 283; B. Grinstein, Nucl. Phys. B 206 (1982)
387; A. Masiero, D. V. Nanopoulos, K. Tamvakis and
T. Yanagida, Phys. Lett. B 115 (1982) 380.

For a review and extended list of references see P. Nath
and R. Arnowitt, Phys. Rev. D 67 (2003) 075004.

Although we will discuss the mechanism in an example of
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in order to suppress GUT gauge boson mediated d = 6
baryon number violation.

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ning partner mechanism: H. Georgi, Phys. Lett. B 108
(1982) 283; B. Grinstein, Nucl. Phys. B 206 (1982)
387; A. Masiero, D. V. Nanopoulos, K. Tamvakis and
T. Yanagida, Phys. Lett. B 115 (1982) 380.

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