Structures in Galaxy Clusters

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Abstract

The analysis of the presence of substructures in 16 well-sampled clusters of galaxies suggests a stimulating hypothesis: Clusters could be classified as unimodal or bimodal, on the basis of the sub-clump distribution in the 3-D space of positions and velocities. The dynamic study of these clusters shows that their fundamental characteristics, in particular the virial masses, are not severely biased by the presence of subclustering if the system considered is bound.

Subject headings: galaxies: clustering
1 Introduction

The presence or absence of substructures in galaxy clusters is very important in cluster evolution, as well as in the cosmological distribution of galaxy systems. In fact, subclustering may give constraints on the epoch of cluster formation, their present evolutionary status, the existence and distribution of dark matter, the morphology of galaxy members, etc. (see, e.g., Cavaliere et al. 1986; West, Oemler, & Dekel 1988; Fitchett 1988a). Moreover, the frequency of substructures in clusters offers the possibility of constraining the density parameter $\Omega$; in fact, structures (and substructures) should collapse earlier in low-density universes than in high-density universes (see, e.g., Richstone, Loeb, & Turner 1992; Lacey & Cole 1993; Kauffmann & White 1993; Mamon 1993).

Both Abell and Zwicky (see, e.g., Abell 1964; Abell, Neymann, & Scott 1969; Zwicky 1957; Zwicky et al. 1961-1968) considered the hypothesis that clusters contain substructures. Subsequently, contour maps in optical and X-ray domains (see, e.g., Baier 1979, 1980; Jones et al. 1979; Geller & Beers 1982; Ulmer, Crudace, & Kowalski 1985; Forman & Jones 1990) showed evidence of cluster substructures. West, Oemler, & Dekel (1988) contributed to the study of the problem of subclustering with alternative methods, which consider different kinds of spatial substructures. Other methods are the Lee-Method, which is based on maximum-likelihood (see Lee 1979; Fitchett & Webster 1987; Fitchett 1988); the Delta Statistics developed by Dressler & Shectman (1988), and the method devised by Gurzadyan, Harutyunyan, & Kocharyan (1991). These three methods are able to use both galaxy positions and velocity field, and they provide the statistical significance of the substructures detected in the cluster.

The recent literature contains numerous papers with observational evidence of cluster subclustering both in the optical and in the X-ray domains (see, e.g., Bahcall 1973; Quintana 1979; Forman et al. 1981; Henry et al. 1981; Forman & Jones 1982; Geller & Beers 1982; Ulmer & Crudace 1882; Bothun et al. 1983; Huchra, Davis, & Latham 1983; Forman & Jones 1984; Lucey, Currie, & Dickens 1986; Bingelli et al. 1987; Fitchett & Webster 1987; Colles & Hewett 1988; Mellier et al. 1988; Mushotzky 1988; Forman & Jones 1990; West & Bothun 1990; Beers et al. 1991; Briel, Henry, & Boehringer 1991; Rhee, van Harleem, & Katgert 1991; Jones & Forman 1992; Escalera et al. 1993a; White, Briel, & Henry 1993).

The observational situation is combined with a rich theoretical background. N-body codes have been widely used to describe cluster evolution, as well as the growing and the dumping of the subclustering (see, e.g., White 1976, Cavaliere et al. 1986; Fitchett 1988a; West, Oemler, & Dekel 1988; Cavaliere & Colafrancesco 1990; Cavaliere, Colafrancesco & Menci 1991; Schindler & Boehringer 1992). The possible presence of bimodal structures has been considered by Cavaliere et al. (1986).
The results of theoretical simulations and observational tests have been often debated and at present do not provide a final, coherent picture of subclustering in galaxy clusters.

In the present paper we use *Weighted Wavelet Analysis* to look for substructures in 16 well-sampled galaxy clusters. We plan to look for evidence of subclustering from a dynamic point of view, inasmuch as we combine the spatial distributions of the galaxies with their velocities.

## 2 Data

For the detection of substructures, only well-sampled, complete (or quasi-complete) clusters of galaxies are suitable for study. The 16 systems we studied are nearby clusters with a limited range of redshifts (chosen to minimize possible evolutionary effects). For some clusters, several samples were available, at different levels of completeness in velocity — 85%, 95%, and 100%. The completeness in velocity was measured by the ratio of the number of cluster members with redshift over the total number of cluster galaxies present in the field, up to a given magnitude. We then removed those samples containing fewer than some 30 galaxies, since for poorly populated structures it is not possible to obtain reliable results on their dynamics.

In Table 1, Column (1) lists the names of clusters; Column (2) the richness classes $R$, mainly taken from Abell, Corwin, & Olowin (1989); Columns (3) and (4) give the relevant references for velocities and magnitudes, respectively; Columns (5) and (6) give the coordinates $\alpha_0$ and $\delta_0$, respectively, of the origin of the map (bottom-right corner); Columns (7) and (8) show the extension of the map along the X and Y axes, respectively, given in arcmin.

The criteria with which the cluster members have been selected are the same as in Girardi et al. (1993). A maximum aperture of $3 \, h^{-1} \, Mpc$ picks out our clusters (we use $H_0 = 100 \, h \, km \, s^{-1} \, Mpc^{-1}$ throughout). Our virial masses have been obtained by means of robust estimates of the velocity dispersions (see, e.g., Girardi et al. 1993)

## 3 Wavelet Analysis

The Wavelet Transform is the convolution between the signal (i.e., the angular positions of galaxies) and an analysing wavelet function. As a result of such a convolution, the signal is transformed into a set of ”coefficients”, the *wavelet coefficients*, which contain all the information we need for the analysis of cluster structures.

The particularities of a wavelet versus other more ”classical” functions are the following:
A wavelet is naturally bound, and invariant with translation: so it is particularly suitable for a local analysis (Time-Frequency, or, for the present work, Position-Size);

(ii) It is also invariant with dilatation, allowing to a multi-scale analysis.

So the main analysis consists in performing the Wavelet Transform at each point of the signal, using a full set of scales: thus no structure in the signal will escape detection, whatever its location and size.

A wavelet is a function that obeys some concrete mathematical conditions (continuity, differentiability, ... etc.; see Escalera et al. 1993a for details). It always has a null mean value. Thus a constant signal will lead to null coefficients. In this work we use the 2D radial wavelet, called Mexican Hat, which comes from a second derivation of a gaussian: as a consequence, a constant gradient will also produce null coefficients. This is particularly useful for detecting small structures inserted in the gradient of a big one.

If now the signal presents some discontinuities, the wavelet coefficients will react accordingly, producing a local maximum. The stronger the discontinuity correlated to the analysing wavelet (size/scale), the stronger the value of the corresponding local maximum.

The data form a map of the analysed cluster with galaxies plotted as points. So our signal is a 2D distribution of δ-functions, and the structures are then defined as a local excess of density. Considering the above working of the transform, we understand that areas of uniform distribution will give null coefficients, while substructures of a given size will produce high local values of the wavelet coefficients (hereafter called local maxima). In all cases the edge effects are taken into account, since wavelets are very sensitive to density contrasts (see also Escalera & Mazure 1992). In this work the densities reach zero value before the edge of the field, so the edge effects can be easily removed by analysing areas greater than the cluster, with null areas outside the limits of the true field.

Therefore, as mentioned above, the main analysis consists in performing the wavelet transform with a set of different scales.

For a given scale $s$, the Mexican Hat explores, at each pixel, a circular area within a radius roughly equal to $4s$. The largest scale that we consider is $s = 0.25R_f$ (where $R_f$ is the radius of the analysed field), so the main cluster should be detected as a single structure. On the other hand, the lower limit of the wavelet scales is estimated from the mean inter-particle distance: the corresponding explored area contains only one galaxy. Thus, the lowest value of the wavelet scale $s$ is given by the relation $(4s)^2 = R_f^2/N_g$ (where $N_g$ is the number of galaxies considered). The usual values of $N_g$ generally mean that the smallest scale is equal to $0.05R_f$.

In this paper, each analysed field corresponds to a map of $256 \times 256$ pixels. The wavelet scales used, given in pixel units, are $s = 64, 32, 24,$ and $16$; for all the analysed samples, the largest and smallest scales used are, respectively, 64 and 16 pixels. The field analysed with the largest scale will produce a
wavelet image showing generally a single central structure, hereafter called A-structure, which corresponds to the detection of the whole cluster. If the scale is decreased, many other events can be observed. The central A-structure may remain regular at every scale, or may split into substructures. Minor substructures may also appear in any region of the cluster field.

The results of such an analysis, as extracted from the local maxima, are the following:

(a) a given structure is located in the field by the position of the local maximum of the wavelet coefficients;

(b) its size is estimated from the scale which best detects the structure, i.e. the scale producing the greatest value of the local maximum of the wavelet coefficients; in the following, such an optimal wavelet scale is called W-scale;

(c) the statistical significance is derived directly from this local maximum, and gives the individual probability of existence of the concerned structure (via random simulations, see below).

Thus a given structure, even if detected at many scales, is only defined in terms of position-dimension-probability by its corresponding W-scale. In this way we detect all the structures lying on the map, whatever their location and size may be.

As a fundamental point of our analysis, we note that the above results are individually obtained for each structure present in the signal. We may also recall that in this paper we perform the Weighted Wavelet Analysis. This method gives weights to the galaxies by considering the velocity data, in a way that takes into account the correlations that exist in physical substructures. As a weight we used the δ parameter introduced by Dressler & Shectman (1988); see also Escalera & Mazure (1992) and Escalera (1992) for further details on the weighting procedure. Thus the detected structures contain 3D information. All these individual 3D results make possible an interpretation of the meaning of the detected structures from a dynamic point of view.

The wavelet method quantifies the statistical validity of the detection of substructures by using sets of Monte Carlo simulations of the analysed clusters. Such simulations are performed in order to remove the real correlations (in positions and velocities) that may exist in the analysed cluster. Thus, a given local maximum of the wavelet coefficients, which corresponds to a true structure, should not be reproduced in the simulations. In fact, we check how many times a structure similar to that detected in the real cluster appears in these random simulations. The natural parameter which makes this comparison possible is the value of the local maximum of the wavelet coefficients. This procedure gives us, for each structure, the probability of its occurring by chance (see Escalera et al., 1993a, for details). As an additional check, we coupled the analysis with the bootstrap resampling technique (see, e.g., Efron & Tibshirani 1986), which can also provide every substructure with a significance level. In the present paper, almost all the sub-clumps
detected have significance levels $\geq 99\%$. Probabilities from both procedures are associated with the results.

4 Different Morphologies

When one considers the largest wavelet scales, two cases seem sufficient to describe the morphology of our set of clusters:

- Case (U) : *Unimodal Clusters*. These clusters show a single dominant central structure, which is located at the dynamic center of the cluster when a detailed analysis is carried out. Case (U) concerns also clusters with a structure that is single but considerably elongated, without any well-defined features.

- Case (B) : *Bimodal Clusters*. These clusters show two dominant structures, roughly equivalent in size and richness, with two well-separated centers.

In this paper, due to the limited number of samples, we chose to include all ”elongated” cases in the (U) class, although these structures may mask true bimodal clusters, e.g., ones closely aligned with the line of sight. Obviously the study of a richer set of clusters might allow us to remove this ambiguity.

When we now explore smaller and smaller cluster-areas with decreasing scales, two kinds of events enrich this morphological scenario:

- Event (sc) : *structures in the core*. The central structures (dominant at the largest scale) can remain single with decreasing scales or can split into smaller substructures of group-sizes. This concerns also the bimodal cases, when the splitting occurs in at least one of the two main components.

- Event (sf) : *structures in the field*. Small but statistically relevant substructures appear in the whole cluster area when small scales are used.

Both events sc and sf describe group-size features, each containing some 10% of the total cluster population.

The distinction between ”core” and ”field” areas is not induced only by the map inspections since we took into account the 3-D data, i.e. the positions and velocities relative to the parameters of the central A-structure. In this way, the sf and sc classifications are not subjective.

In Table 2 one can see the results of our analysis: 12 of our clusters are unimodal and 4 are bimodal. In particular, Column (1) lists the names of the clusters; Column (2) gives the sample used, with different velocity completeness (** = 85%, * = 90%, no symbol = 100%); Column (3) shows the number of galaxy members considered in the analysis; in Column (4) the morphological classification of substructures is coded with the symbols explained above, $U$ and $B$ referring to the classification at the largest wavelet scale, and sc and sf concerning the classification at smaller scales.

Table 3 describes, cluster by cluster, the main structure (or the binary structure) and the substructures detected. Substructures with fewer than 4 galaxies are not listed, since for such poorly populated structures it is not
possible to produce reliable conclusions from a dynamic analysis. Column (1) lists the cluster name; Column (2) lists the samples used, with different velocity completeness (the coding is the same as in Table 2); Column (3) identifies the structure \( M = \) main cluster, \( A = \) dominant central structure, \( A1, A2 = \) two components in bimodal cases, \( B, C,... = \) substructures; Column (4) gives the wavelet scales at which the structures were detected (the so-called W-scale). Values are given in pixel units for a map of \( 256 \times 256 \) pixels. Column (5) gives the number of galaxies belonging to the concerned structures; Column (6) lists the probability that each structure is due to random fluctuations (see §3); since our procedure for assigning this probability checks the significance of subclustering by comparing the observed distribution of galaxies to many random representations of a regular cluster (single central structure), it is obvious that no probability can be assigned to the A-structure; Column (7) gives the probabilities \( P_{Lee} \), according to the Lee-method; Column (8) gives the probabilities \( P_\Delta \), according to Dressler & Schectman’s (1988) method. In all cases (Columns (6),\( 7), \) and (8)) no substructure is considered as detected when the associated probability is \(< 95\%\).

There is a good agreement between our wavelet method and the others two methods.

In order to make a compact presentation, we display only two wavelet images per cluster although the multiplicity of structures in some particular cases would require three or more images to show the W-scales of all clumps.

For each cluster we show the most complete sample, as listed in Table 2. In each figure, right ascension grows from left to right. The coordinates of origin (bottom-right corner of the map) as well as a map scaling correspondence are given in Table 1. The wavelet images are superimposed over the cluster galaxy plots. Solid lines represent the isopletes of the wavelet coefficients. Labelled structures correspond to those discussed in Table 2. Non-labelled features are those not statistically relevant or containing fewer than 4 galaxies.

Figures 1A to 16A show the wavelet images of our 16 clusters obtained by using the large scales. In this way it is possible to identify unimodal \( (U) \) and bimodal \( (B) \) clusters. Figures 1B to 16B show the wavelet images of the same clusters obtained by using the small scales. Minor substructures appear in some clusters, following the above-mentioned scenarios – cases \( (sc) \) and \( (sf) \). Notice that in general the central A-structure is still seen at small scales, although the corresponding physical structure is only defined by the largest scales.

5 Substructure Dynamics

The dynamics of the clusters and their corresponding substructures and, in particular, the estimates of galaxy velocity dispersions and virial masses, help
to corroborate the morphological description of clusters given above.

Table 4 lists the most important physical parameters for the clusters and their substructures. In particular, Column (1) lists the name of the clusters; Column (2) lists the samples used, the degree of velocity completeness being coded as in Table 2; Column (3) identifies the structure: $M =$ main cluster, $A =$ dominant central structure, $A_1, A_2 =$ two components in bimodal cases, $B, C, \ldots =$ substructures, and $M - BC =$ environment, i.e. main cluster with substructures $B, C, \ldots$ removed. Each environment file well describes the cluster as a whole, in particular when the removed substructures are very far from the center, in terms of both position and velocity. Column (4) gives the number $N_g$ of galaxies of the concerned structure or substructure; Column (5) lists the mean velocity (biweight estimate), $V$, in $Km s^{-1}$; Column (6) lists the robust velocity dispersion, $\sigma$, in $Km s^{-1}$; Column (7) lists the virial mass, $M_v$, in units of $10^{14} M_\odot$; Column (8) gives the virial radius, $R_v$, in Mpc. Obviously we have to put more weight on the results obtained at a higher completeness level.

When reading Table 4, we may remark that, in some particular cases, a substructure detected at a small scale is totally included in the dominant structure detected at the largest scale (e.g., see $A$ and $B$ for the sample A2670). On the other hand, in some cases the central $A$-structure detected at the largest scale does not differ from the whole cluster (e.g., $A = M$ for the sample S0463).

The robust velocity dispersion of the whole cluster does not seem to be strongly biased by the presence of subclustering in most of our 16 clusters. Generally, the virial masses of the dominant structures (designated $A$, $A_1$ and $A_2$ in Tables) fairly closely approximate the total virial mass of the main cluster (designated $M$). In particular, the ratio of masses is rather well related to the ratio of populations (numbers of galaxies involved in the respective systems). This can be easily checked by comparing the masses and populations of structures $M$ and $A$ for unimodal clusters, and structures $M$ and $A_1 + A_2$ for bimodal clusters. The difference between the total mass and the subclump masses turns out to be huge only when the detected substructures do not belong to a single cluster, but are part of physically unbound systems. This only happened once in our set, in the case of A0151, where two probably unbound galaxy systems are seen in projection close to one another, looking like a binary cluster. The assumption that a system is bound may be based on the *Newton Gravitational Criterion* (see, e.g., Beers et al. 1982). So the virial mass estimates of our clusters do not seem to be severely biased by the presence of their substructures. Moreover, the masses of the small substructures detected are generally one order of magnitude smaller than the total mass of the cluster. This is consistent with the relative populations of these small systems, which mostly represent, as mentioned above, some 10% of the total numbers of galaxies.

However, the virial analysis we have performed is but a partial test, and
these results should be checked both by extending the analysis to a larger sample of clusters and by performing numerical simulations.

5.1 Conclusions

In our set of 16 clusters, subclustering seems to be able to classify cluster structures quite naturally as Unimodal and Bimodal.

We wish to consider this result as a stimulating hypothesis for cluster classification. In fact, the possibility of classifying galaxy clusters by means of their structure would allow us to combine their present dynamic statuses, and therefore their evolutionary histories, with their morphological characteristics. It may be interesting to note that our results are in agreement with the cluster simulations by Cavaliere et al. (1986); these Authors claim the existence of a clear bimodal configuration in about 30% of their runs, and we have 4 bimodal systems in our 16 clusters. Salvador-Solé, Sanromá, & González-Casado (1993) and Salvador-Solé, González-Casado, & Solanes (1993) find that at least 50% of apparently relaxed clusters contain significant substructures, and we have only 3 clusters without significant substructures. Moreover, both subclustering and binary structures have been detected also in cluster X-Ray-maps (see, e.g., Forman et al. 1981; Henry et al. 1981; Ulmer & Cruddace 1882; Forman & Jones 1984; Mushotzky 1988; Jones & Forman 1992; White, Briel, & Henry 1993), in particular Mushotzky (1988) suggests that a significant fraction of all clusters (∼ 25%) shows X-Ray double images, and Jones & Forman (1992) estimate that double and complex structures are about 20%.

The preliminary results concerning the dynamics of clusters and their substructures seem to stress the robustness of the virial mass estimate and the corresponding limited biases induced by subclustering. The virial masses of substructures are closely related, in our clusters, to the number of galaxies belonging to the substructures themselves. The typical masses of these substructures are often ∼ 10% of the parent-cluster total mass. This is in agreement with the merging-hierarchical-scenario suggested by Cavaliere, Colafrancesco, & Menci (1992) for galaxy clusters, where small galaxy systems survive the merging processes inside the clusters.

The presence of these substructures and their frequency may also contribute to the debated determination of the density parameter Ω. In fact, structures (and substructures) should collapse earlier in low-density universes than in high-density universes. Therefore, these subclumps should not often be detected today in clusters if Ω is small. However, this point is sensitive to a correct estimate of the survival time of substructures in a collapsed cluster (see, e.g., Richstone, Loeb, & Turner 1992; Lacey & Cole 1993; Kauffmann & White 1993; Mamon 1993).

Of course, the limited number of our clusters advise us to be wary and to look for further confirmation of all our results in other optical cluster
samples, which should be more extended in number, richness classes, and redshift, as soon as they become available.

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Captions to Figures

Figs. 1A to 16A: Wavelet images of our 16 clusters obtained by using the largest scales: 64 pixels for all clusters, except 32 pixels for A2151 and A3716 (all for a $256 \times 256$ map). See §3 for details. Unimodal ($U$) and bimodal ($B$) clusters are easily identified.

Figs. 1B to 16B: Wavelet images of the 16 samples obtained by using the smallest scales: 16 pixels for all clusters (for a $256 \times 256$ map). See §3 for details. Minor substructures (labelled B, C, ...) appear in some clusters, corresponding to the events $sc$ and $sf$ (see text).
Captions to Tables

Table 1. The Data-Sample. Column (1): Cluster name; Column (2): Richness class; Columns (3) and (4): References to velocities and magnitudes, respectively; [1] Proust et al. (1992); [2] Dressler (1980); [3] Kent & Sargent (1983); [4] Zwicky et al. (1961–1968); [5] Ostriker et al. (1988); [6] Nilsson (1973); [7] Dressler & Schectman (1988b); [8] Richter (1987); [9] Richter (1989); [10] Sharples, Ellis, & Gray (1988). Columns (5) and (6): Coordinates of the origin of the map (bottom-right corner), $\alpha_0$ and $\delta_0$ respectively; Columns (7) and (8): Extension of the map along the X and Y axes respectively, given in arcmin.

Table 2. Cluster classification. Column (1): Cluster name; Column (2): Sample used, with different velocity completeness (** = 85%, * = 90%, no symbol = 100%); Column (3): Number of galaxy members; Column (4): labels $U$ and $B$, respectively Unimodal and Bimodal, refer to the classification at the largest wavelet scales; labels $sc$ and $sf$ refer to events detected at smaller wavelet scales.

Table 3. Cluster Structures and Substructures. Column (1): Cluster name; Column (2): Sample used, with different velocity completeness (** = 85%, * = 90%, no symbol = 100%); Column (3): Structures ($M$ = main cluster, $A$ = dominant central structure, $A_1,A_2$ = two components in bimodal cases, $B,C,...$ = substructures); Column (4): Wavelet scales used for the detection of the concerned structures, given in pixel units for a $256 \times 256$ map. Column (5): Number of galaxies involved in the concerned structure or substructure; Column (6): Wavelet probability of subclustering; Column (7): Subclustering Lee probabilities; Column (8): Subclustering Dressler & Schectman probabilities.

Table 4. Dynamical Quantities of Clusters and Cluster Substructures. Column (1): Cluster name; Column (2): Sample used, with different velocity completeness (** = 85%, * = 90%, no symbol = 100%); Column (3): Structures ($M$ = main cluster, $A$ = dominant central structure, $A_1,A_2$ = two components in bimodal cases, $B,C,...$ = substructures, $M-BC,...$ = environment, i.e. main cluster with substructures $B,C...$ removed.); Column (4): Number of galaxies involved in the concerned structure; Column (5): Mean velocity (biweight estimate), in $Km \ s^{-1}$; Column (6): Robust velocity dispersion, in $Km \ s^{-1}$; Column (7): Virial mass, in units of $10^{14} \ M_\odot$; Column (8): Virial radius, in Mpc.