Cherenkov friction on a neutral particle moving parallel to a dielectric

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Abstract
We describe a simple mechanism of quantum friction for a particle moving parallel to a dielectric, based on a fully relativistic framework and the assumption of local equilibrium. The Cherenkov effect explains how the bare ground state becomes globally unstable and how fluctuations of the electromagnetic field and the particle’s dipole are converted into pairs of excitations. Modeling the particle as a silver nano-sphere, we investigate the spectrum of the force and its velocity dependence. We find that the damping of the plasmon resonance in the silver particle has a relatively strong impact near the Cherenkov threshold velocity. We also present an expansion of the friction force near the threshold velocity for both damped and undamped particles.

Keywords: quantum friction, quantum electrodynamics, Cherenkov radiation, transition radiation, fluctuation forces

1. Introduction
The conversion of mechanical energy into heat is referred to as friction in most cases. Numerous mechanisms can be identified that cause friction, but it is still a challenge to infer macroscopic observations from microscopic phenomena. So far only very simple scenarios permit a detailed analysis of fundamental aspects. A prominent example is the theory of the quantized electromagnetic field applied to the case of two parallel moving plates separated by a small vacuum gap [1–5]; see [6–9] for reviews. Friction arises due to the spontaneous creation of particle pairs that propagate away into the plates or are dissipated there. A similar treatment can be applied to a body moving above a flat surface at constant speed [10–14]. Taking advantage of Lorentz invariance, one achieves treatments consistent with special relativity [15, 16], as it is also mandatory for the archetypal situation of high-energy charges being stopped in a medium. In a recent paper, we described such a formalism for a neutral, polarizable particle moving parallel to a flat interface [17]. At a typical distance of at least a few nanometers (larger than the atomic scale), the interaction depends on a few macroscopic parameters (refractive index, conductivity, surface impedance …). In the present paper we discuss a special configuration of this setting with a dielectric medium below the surface, and both particle and medium at zero temperature. This is what makes the friction a pure quantum-mechanical drag and closely relates it to the realm of Casimir phenomena [18]. Because of the growing interest in this field and some controversy surrounding it (see [13] for a review), the simple situation studied here might provide another testbed to compare current results and ideas in detail.

A nonzero friction force is found when the speed of the particle relative to the surface exceeds the velocity of light in the medium (c/n); this drag can thus be attributed to the Cherenkov effect. The situation is somewhat unusual because neither the surface nor the particle have to be dissipative. All that is required are spectral mode densities for the medium field and the particle.
In particular, we analyze in detail a spectral representation of the friction force that must be applied to move a small particle parallel to a flat dielectric surface. While this setup has obvious applications for micro- and nano-machines, our focus is on illustrating the underlying mechanisms. The basic physics is very similar to the seminal explanation of the Cherenkov effect [22] by Tamm and Frank [23]; for a certain sector of field modes, the Doppler shift flips the sign of the mode frequency (anomalous Doppler effect). This leads to scattering relations (S-matrix) in the form of a Bogoliubov transformation [7, 9]; incident waves are amplified, and pairs of elementary excitations (phonon-polaritons) can be created out of the bare ground state of the field and the particle’s internal dynamical variables. The frictional force arises from the power carried away by these excitations as they are absorbed or as they propagate into the bulk of the body. The recent paper by Barton [13] provides a particularly transparent calculation of these processes in a simplified setting (only surface plasmon modes are considered). A similar analysis has been given by Silveirinha [14]. The starting point we use here is based on the fluctuation electrodynamics developed by Rytov and co-workers [21]: the basic assumption is that the solid surface and the moving particle are in local thermodynamic equilibrium. This is a good approximation for a mesoscopic particle made from thousands of atoms, at least over time scales where its temperature can be considered constant (large heat capacity). The approximation is much more questionable for microscopic particles like atoms or molecules because they may settle into a non-thermal state due to spontaneous excitation.

We structure our analysis in the following way: some results of previous work are summoned to provide the basis for the Cherenkov effect. The quantum (Cherenkov) friction is then calculated and its physical properties are discussed. We link the friction force to an absorbed power that has to be provided to move the particle at constant speed. A relativistic argument put forward by Polevoi [3] attributes this power to carrying away the dissipated power (figure 1(left)).

\[ F_i = \frac{\hbar}{2\gamma} \int \frac{d\omega}{2\pi} \frac{d^2k}{(2\pi)^2} \left[ \text{sign}(\omega) - \text{sign}(\omega - \nu k_x) \right] \times k_x \text{Im} \alpha(\omega - \nu k_x) \sum_{m=\pm, \parallel} \phi_m(\omega, k_\parallel) \text{Im} \left( \frac{\varepsilon e^{-2\gamma}}{\kappa} \right) \]

The frequency \( \omega \) and parallel wave numbers \( k_\parallel = (k_x, k_y) \) are measured in the rest frame of the medium; the integral boundaries are \( \pm \infty \). The difference of sign functions arises from the thermal factors \( \text{coth}(\hbar \omega/(2 k_B T)) \) in the zero-temperature limit, evaluated in the respective rest frames of medium and particle (\( \omega' = \gamma(\omega - \nu k_x) \)). The particle polarizability is \( \alpha(\omega) \), \( \gamma \) is the Lorentz factor, and for the weight functions \( \phi_m \) we have (setting \( c = 1 \))

\[ \phi_p(\omega, k_\parallel) = \omega^2 - 2\gamma^2(v \times k_\parallel)^2 \left( 1 - \frac{\alpha^2}{k_\parallel^2} \right), \]

\[ \phi_s(\omega, k_\parallel) = \omega^2 - 2\gamma^2(k_\parallel^2 - (v \cdot k)^2) \left( 1 - \frac{\alpha^2}{k_\parallel^2} \right). \]

The reflection coefficients for \( p \)- and \( s \)-polarized light are

\[ k_r = \frac{i k - \kappa_r}{i k + \kappa_r}, \quad k_p = \frac{\nu k_x - \kappa_p}{\nu k_x + \kappa_r}, \]

where \( \kappa = \sqrt{k_\parallel^2 - \omega^2} \) and \( \kappa_p = \sqrt{\nu^2 \omega^2 - k_\parallel^2} \). The roots are chosen on the Riemann sheet where \( \text{Re} k_\parallel > 0, \text{Im} \alpha > 0, \text{Im} k_p > 0, \text{sign}(\omega) \text{ Re} k_p > 0 \), ensuring fields that propagate away from the vacuum-medium interface or decay with distance from it. Here, \( n \) is the refractive index of the medium.

Using symmetries and other properties we can further simplify the integral in equation (1). The integrand is even under the transformations \( (\omega, k_\parallel) \rightarrow (-\omega, -k_\parallel) \) and \( k_\parallel \rightarrow -k_\parallel \), so that it is sufficient to integrate over the domain \( \omega > 0, k_\parallel > 0 \). The difference of the sign-functions reduces to a factor of two for \( 0 < \omega < \nu v_\parallel \). This wedge-shaped domain in the \( k_\parallel, \omega \)-plane is below the projected light cone \( \omega = k_\parallel \), so that only fields that are evanescent at the particle’s location contribute to the force (figure 1(right)). We conclude that the factor \( e^{-2\gamma / k} \) is real-valued.

Another crucial insight is contained in the reflection coefficients (4): their imaginary part is nonzero only in the

2. The formalism

2.1. Friction force

In an earlier paper [17] we presented a covariant approach to the force on a particle that moves with arbitrary speed parallel to a flat surface that responds linearly to electromagnetic waves. We recovered the results of [15, 26]. The formalism allows for different temperatures of particle and surface, assuming local equilibrium states. The relative motion leads to Doppler shifts that are handled by Lorentz transforming an incident field into the frame co-moving with the particle or the surface. The Doppler-shifted frequencies of the equilibrium distributions are responsible for a non-equilibrium force that persists even when both temperatures \( T \rightarrow 0 \).

Let us fix coordinates such that the \( x \)-axis points along the motion of the particle (velocity \( \nu \)), while the half-space \( z \leq 0 \) coincides with the medium (figure 1(left)). According to \([15–17, 26]\), the force component \( F_x \) acting on the particle (at distance \( z \) from the surface) is
annulus $\omega < k_t < n \omega$ (see figure 2(right)). With the condition derived from the sign functions we get $\omega < v n \cos \phi < v n \omega \cos \phi$, so that the condition for Cherenkov radiation follows

$$1 < v n \cos \phi$$

where $\phi$ is the angle between $v$ and $k_t$. The expression (1) for the force thus becomes:

$$F_s = \frac{4\hbar}{\gamma(2\pi)^3} \int_0^\infty d\omega \int_{a/v}^{nu/v} dk_t \int_0^{\sqrt{\omega^2 n^2 - k_t^2}} dk_x$$

$$k_x \Im \alpha(\omega - v k_t) \sum_{\alpha=1,2} \phi_\alpha(\omega, k_t) \Im (r_\alpha) e^{-2\pi \omega \kappa}.$$  

2.2. Photon emission and anomalous Doppler shift

The manipulations performed so far have a clear physical meaning within the theory of the Cherenkov effect [19, 22, 23] which is well understood. A kinematic explanation of the friction above the Cherenkov threshold can be given following the spirit of [27]. We start with the conservation of 4-momentum

$$p_\mu^{(1)} = \hbar k_x + p_x.$$  

The momenta $p_\mu^{(1)}$ describe the particle before and after the emission of a photon with momentum $\hbar k_x$, where $a = 1, 2$ labels the internal states (energy levels $\epsilon_{1,2}$). Although equation (7) and [27] deal with a particle moving through a medium, the physics is the same for the motion parallel to the...
dielectric medium. We have for the particle and the photon (recall that \(c = 1\))
\[
p^\mu_a = (E_a, \gamma m_a v), \quad m_a = M + \epsilon_a.
\] (8)
\[
E_a = \sqrt{m_a^2 + \gamma^2 m_a^2 v^2} = \gamma m_a v,
\] (9)
\[
k^\mu = (\omega, \mathbf{k}), \quad k = n\omega.
\] (10)

The greek indices run from 0 to 3, and toggling between co- and contravariant indices is done with the metric \(g_{\mu\nu} = \text{diag}(1, -1, -1, -1)\). It is understood that \(k = \sqrt{\mathbf{k}^2}\).

The masses \(m_a\) are associated with the particle’s energy levels. The photon is supposed to be emitted into the medium, hence the dispersion relation in equation (10). Because the particle is pushed by an external agent, the velocity \(v\) does not change during the emission. This is equivalent to neglecting the recoil [27] of the particle. Squaring equation (7) leads to
\[
(\epsilon_1 - \epsilon_2)(2M + \epsilon_1 + \epsilon_2) = 2E_0\hbar \omega(1 - v n \cos \phi),
\] (11)
with the same notation as in equation (5) above. We can reasonably make the approximation \(\epsilon_1, \epsilon_2 \ll M\) so that we recover
\[
\hbar \omega = - \frac{\epsilon_2 - \epsilon_1}{\gamma(1 - v n \cos \phi)}.
\] (12)

Now if the particle is faster than the speed of light inside the medium, \(c/\gamma n\), the denominator is negative (Cherenkov condition (3)). This is an illustration of the so-called anomalous Doppler effect where the photon frequency, as seen from the moving particle, \(\omega' = (\omega - v k z)\), is negative. The authors of [27] point out that this allows for the excitation of the particle to a higher energy level, \(\epsilon_2 > \epsilon_1\), while emitting a photon into the medium, the Cherenkov cone (see figure 2(left)). The power lost into the emission must be supplied by the force that keeps the particle on its track. In other words, considering quantum electrodynamics at a dielectric interface coupled to a polarizable particle moving faster than the Cherenkov threshold, it turns out that this is an example of an unstable field theory [7, 28], similar to electron-positron production in strong electric fields and Hawking radiation in a strong gravitational field.

2.3. Heating and frictional power

This simple kinematic analysis corresponds neatly to the integration domain in equations (1 and 6). Note in particular that the particle’s response function is evaluated at the Doppler-shifted frequency and yields \(\text{Im } a(\gamma(\omega - v k z)) < 0\) in the domain. This is a clear indicator that the anomalous Doppler effect in combination with the photon emission of photons into the Cherenkov cone indeed slows down the particle. Another quantity of interest is the rate of mass change in the particle’s co-moving frame. This is given by \(\dot{m} = u^a F_a\) where \(u_a\) is the particle’s 4-velocity. The full 4-vector of force \(F_a\) can be found in [17], and for our particle moving in the \(x\)-direction, we find
\[
\left(\frac{\dot{F}_0}{\dot{F}_x}\right) = \int_0^\infty d\omega \int_{\omega v}^{\omega v} d\omega' \int_0^{\omega a v - k z} dk \left\{ \frac{\hbar \omega}{\hbar k_z} \gamma(\omega, k_z) \right\}.
\] (13)

where the positive quantity
\[
\Gamma(\omega, k_z) = \frac{4}{\gamma(2\pi)^{d-2}} \text{Im } a(\gamma(\omega - v k_z)) \sum_{a=p} \phi_a(\omega, k_z) \text{Im } (\epsilon_a) e^{-2\pi k_z}.
\] (15)
can be identified as a spectrally resolved photon emission rate. (We have used the fact that \(\text{Im } a(\omega')\) is an odd function.) Note that the proper mass increases, \(m > 0\), because per emission event, a positive energy \(-\hbar \omega' = \hbar \gamma v(k_z - \omega)\) is dumped into the particle’s internal mass-energy, as discussed in the previous section. Indeed, we shall see with a simple oscillator model for the polarizability that the frequency \(\omega'\) in the co-moving frame is essentially fixed by the particle’s resonance.

To summarize this section, let us re-write the power balance as a sum of two positive terms:
\[
-v F_x = -F_0 + \frac{d m}{d \tau}
\] (16)

On the left-hand side, we see the frictional power spent to maintain the constant speed of the particle. The first term on the right-hand side gives the power of photon emission (energy \(\hbar \omega\) at rate \(\Gamma(\omega, k_z)\), see equation (14)), while the second gives the power absorbed in the particle. (The factor \(1/y\) gives the relativistic time dilation between the particle’s proper time \(\tau\) and the laboratory time \(t\).

3. Case study: relativistic nanoparticle

3.1. Numerical investigations

To illustrate further the physical features of the Cherenkov friction force, we provide some numerical estimates for a metallic nanoparticle. We chose a silver nano-sphere with radius \(a = 3\) nm that moves at a distance \(z = 10\) nm above a dielectric medium with refractive index \(n = 2\). For simplicity, frequency dispersion is neglected in the medium [25]. For the particle, we adopt a Drude model with the plasma frequency for silver \(\hbar \omega_p = 9.01\) eV and damping rate \(1/\tau\) (not to be confused with the proper time coordinate \(\tau\) above). For such a small particle, the first term of the Mie series will suffice so that its response is given by the electric dipole polarizability (for SI units, multiply with the Coulomb constant \(e_0\))
\[
\alpha(\omega) = 4\pi a^3 \epsilon(\omega) - \frac{1}{\epsilon(\omega)} = 4\pi a^3 \frac{\Omega^2}{\Omega^2 - \omega^2 - i \omega l \tau},
\] (17)
where \(\epsilon(\omega)\) is the metal permittivity. The resonance at \(\Omega = a \omega_p /\sqrt{3}\) corresponds to a plasmon mode localized on the particle. The calculations simplify considerably in the no-damping limit \(\tau \to \infty\) which gives
\[
\lim_{\tau \to \infty} \text{Im } a(\omega) = 2\pi^2 d\omega [\delta(\omega - \Omega) + \delta(\omega + \Omega)].
\] (18)

We have checked that at this distance and for velocities above the Cherenkov threshold, both polarizations contribute roughly the same amount to the force. This is at variance with the more familiar regime of short (non-retarded) distances and slow (non-relativistic) atoms where the p-polarization dominates and an electrostatic calculation suffices.
Figure 3. Impact of particle velocity and plasmon damping on quantum friction. (left) Frequency spectrum $F_{\omega}(\omega)$ of the friction force for a silver nanoparticle at different velocities above the Cherenkov threshold $c/n = 0.5$, obtained by integrating $F_{\omega}(\omega, k_{||})$ over $k_{||}$. The arrows give the apex of the hyperbola (equation (20)) shown in figure 2(left). We used the quite arbitrary normalization factor $4\hbar(4\pi a^3)(2\pi)^{-3} 10^{-4} (\omega_{pl}/c)^3 = 3.4 \text{aN/m}$ for the force spectrum. We took a damping time fixed by $\Omega = 32.5$, shorter than in bulk, which can be attributed to electron scattering at the nanoparticle surface [30–33]. (right) Comparison of the lossless case $1/\tau = 0$ and a particle resonance with a finite width (same parameters as in figure 2). The arrow indicates the frequency $\omega_0$ (equation (20)) where the particle resonance $\omega' = -\Omega$ intersects the light cone in the medium (the apex of the hyperbola in figure 2(left)). Same normalization as in figure 3(left). Dashed lines: equation (23) for $1/\tau = 0$ and equation (25) for $1/\tau > 0$.

Figure 1(right) illustrates the simple appearance of the integration volume, which determines most of the features of the force spectrum: it lies between the zero-frequency plane $\omega' = 0$ and the medium light cone $\omega = k_\parallel /n$. The opening angle $\phi_{\text{max}}$ of the intersection (measured in the $k_\parallel$-plane, relative to the direction of the velocity $v$) is given by the Cherenkov formula

$$\cos \phi_{\text{max}} = \frac{\omega/v}{n\omega} = \frac{1}{nv}$$

(19)

where $v$ is the particle velocity (scaled to $c$).

Figure 2(left) shows the impact of the particle’s plasmon resonance: the plane $\omega' = -\Omega$ and the medium light cone $\omega = k_\parallel /n$ intersect in a hyperbola whose opening angle (projected onto the $k_\parallel$-plane) is again given by the Cherenkov formula (19)—the higher the speed of the particle, the more inclined is this plane. The integrand roughly peaks near the apex of the hyperbola whose position is easily calculated to be

$$\omega_a = \frac{\Omega}{\gamma (nv - 1)}, \quad k_{\text{sa}} = \frac{n\Omega/c}{\gamma (nv - 1)}, \quad k_{\text{sa}} = 0.$$  

(20)

In figure 2(right), we plot a slice at constant frequency through the spectral density $F_{\omega}(\omega, k_{\parallel})$ of the friction force given by

$$F_{\omega}(\omega, k_{\parallel}) = \frac{4\hbar k_{\parallel}}{(2\pi)^3} \text{Im} \left( a(\omega - v) \right) \sum_{\alpha = s, p} \frac{2\kappa_{\alpha} a_{\beta}(\omega, k_{\parallel})}{q_{\alpha}^2 k_{\parallel}^2 + k_{\beta}^2} e^{-2v t} ,$$

(21)

where the imaginary part of the reflection amplitudes $r_{\alpha}$ was worked out from equations (4), and $q_{\alpha} = 1, q_p = n^2$. The density plot reveals how the resonances of the polarizability $a(\omega')$ select narrow stripes in the $k_{\parallel}$ plane. Only the resonance $\omega' = -\Omega$ (blue) lies in the integration domain relevant for quantum friction.

These geometric considerations carry over when we integrate over $k_{\parallel}$ and $k_{\parallel}$ and consider the force spectrum. This is illustrated in figure 3. Photon emission resonant with the particle plasmon resonance becomes dominant at velocities well above the Cherenkov threshold (figure 3(left)). Close to threshold, contributions at lower frequency arise from photons that are off-resonant, more precisely quasi-static, in the frame co-moving with the particle. Similar to Cherenkov radiation, they are boosted into the visible range by the Doppler shift. These photons arise from the nonzero value of the polarizability at low frequencies

$$\omega' \ll \Omega: \quad \text{Im} \left( a(\omega') \right) \approx 4\pi a^3 \frac{\omega'}{\Omega^2}$$

(22)

Note that the only material parameter in this regime is the metal conductivity $\varepsilon_0 \Omega^2$, see also [4, 29]. Our interpretation is confirmed in figure 3(right) where the spectrum is also calculated in the lossless limit, using the approximate polarizability (18). Off-resonant photon emission is suppressed, and the frequency $\omega_0$ from equation (20) provides a sharp threshold.

Finally, the total friction force is plotted as a function of the particle velocity in figure 4. Note again the relatively large difference between finite damping and the lossless limit near the Cherenkov threshold (arrows).

3.2. Approximations near threshold

The integrals can be calculated approximately when the opening angle of the Cherenkov cone is very narrow ($v = 1/n$). The main features are captured by the reflection coefficient in $p$-polarization, expanded for small $k_n$ (see equation (4)). (See [34] for more details.) The formulas of this section are represented in dashed (gray) lines on figures 3 and 4; the agreement is quite satisfactory.

For a particle polarizability with a very narrow resonance, we find the approximate spectrum

$$\omega \gg \omega_a: \quad F_{\omega}(\omega) d\omega \approx \frac{4\hbar (4\pi a^3)}{(2\pi)^3} \frac{\pi^2 \Omega}{8nv^2} d\omega a_{\omega} a_{\omega} \text{Re} \left( \frac{\omega^2}{\omega^2 + \omega_0^2 + (nv - 1)(\omega + \omega_a)} \right)$$

(23)

$$k_{\text{max}}^2 = (nv - 1)^2 \frac{\omega - \omega_a}{\nu^2} [2\omega + (nv - 1)(\omega + \omega_a)]$$

(24)

where $\omega_a$ is given by equation (20), and $k_{\text{max}}$ parametrizes the width of the hyperbola in figure 2(left). This spectrum has a
sharp threshold (dashed gray lines in figure 3). If the polarizability includes damping, the contribution from quasi-static frequencies can be computed similarly, using the approximation (22). Assuming for simplicity that $\tau$ does not depend on frequency (non-radiative damping), the resulting spectrum is

$$\omega_0 - \omega : \quad F_\omega \left( \omega \right) \approx - \frac{4h(4\pi \alpha^3)}{(2\pi)^3} \frac{n^2}{\lambda^3} \frac{\omega_0}{\lambda} \frac{(v - 1)^2}{v^2 - 1} \frac{v^4}{15}$$

and peaks roughly at the inverse roundtrip time $1/(z\sqrt{n^2 - 1})$ (dashed lines in figure 3(right)). As illustrated in the Figure, this approximation becomes quite poor away from the threshold, as frequencies above the validity of the low-frequency approximation (22) for $\text{Im} \sigma(\omega')$ become relevant.

From both approximations for the spectra, the velocity-dependent friction force can be calculated, leading to:

- no damping: $F_i \approx - \frac{4h(4\pi \alpha^3)}{(2\pi)^3} \frac{n^2}{\lambda^3} \frac{\omega_0}{\lambda} (v - 1)^2 \frac{v^4}{15} e^{-2v\sqrt{\zeta_{pl} - 1}}$.
- with damping: $F_i \approx - \frac{4h(4\pi \alpha^3)}{(2\pi)^3} \frac{n^2}{\lambda^3} \frac{\omega_0}{\lambda} (v - 1)^2 \frac{v^4}{15} e^{-2v\sqrt{\zeta_{pl} - 1}}$.

In both cases, we have simplified the complicated polynomial in $v$ to the lowest order above $1/n$. The dependence on the threshold frequency $\omega_0 \sim (v - 1/n)^{-1}$ makes the no-damping result exponentially small at threshold, while damping leads to a cubic power law $\sim (v - 1/n)^3$. We also emphasize the different power laws with distance; the corrections in the second line of equation (26) are quite significant for our parameters, as we have the relatively large value $\omega_0 \zeta = 2.2$ at $v = 0.55$.

The numerical calculation for a particle with damping agrees quite well with formula (27) close to the threshold velocity. Around $v \sim 0.53$ the contribution from the resonance takes over and the dependence on the damping constant becomes negligible.

4. Conclusion

We investigated the force on a neutral particle moving in close proximity parallel to a dielectric. Studying the expression (1) that was derived from a relativistic extension of the fluctuation-dissipation theorem, we provided a connection to a fundamental and simple friction mechanism. If the particle moves faster than the speed of light inside the medium (Cherenkov condition), it can dissipate energy by creating pairs of excitations. Unlike as in [13], the pairs are formed by internal excitations of the particle and photon modes propagating in the medium. These photon modes change the sign of their frequency under the Doppler shift (anomalous Doppler effect [23, 35]). This leads to a S-matrix in the form of a Bogoliubov transformation that, when applied to the bare ground state, ‘spontaneously’ excites the particle and generates a photon emitted into the medium [7, 9, 27]. The mechanism we described is another example of an unstable vacuum state in a quantum field theory [28]. The main features of Cherenkov friction were recovered in geometrical terms by analyzing the frequency spectrum of the force. In order to provide a concrete example, we considered a metallic nanoparticle whose polarizability is dominated by a plasmon resonance. We found a remarkable agreement of the numerical data with an expansion of the force in $(v - c/n)$ near the threshold. The approximate expressions illustrate the roles played by low frequencies in the particle’s polarizability and frequencies around the plasmon resonance, respectively.

To make contact with the current discussion on quantum friction [5, 12, 13, 36], we note that in its simplest form, Cherenkov friction does not require damping neither in the particle nor in the surface. We studied the impact of dissipation in the particle (as described by a damped plasmon mode) and found that this significantly contributes to the friction force just above the Cherenkov velocity, while maintaining strictly zero friction below it.

If absorption is allowed for in the surface, the friction force also changes qualitatively because the Cherenkov threshold is lost. From equation (1), it is easy to show for a metallic surface described by the Drude dielectric function that $F_i(v)$ is nonzero, however small is $v$. A similar behavior is also found when the temperature is raised. More details will be reported elsewhere. Our general result for the radiative force is identical to that of [16]. The general setting for the field quantization (lossless and non-dispersive dielectric) is the same as in [25], however, a different particle is considered there (self-energy of a moving electron). The approach of [37] is limited to friction forces linear in the relative velocity of two systems which are both at the same temperature. The vanishing of linear friction at $T = 0$ is consistent with our analysis. The investigation of [13] uses a different model for the particle’s polarizability.
a microscopic two-level system with radiative damping only. In the description of the field modes near the surface, damping (absorption) is allowed for, and only electrostatic fields are considered (non-relativistic limit). We emphasize in particular that the excitations that lead to frictional losses are pairs of surface plasmons in [13]. A comprehensive picture where the weight of this excitation process can be compared directly to the spontaneous particle excitation studied here still needs to be developed. The simple setting put forward in this paper may provide a route towards such a picture.

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