Evanescent waves in photonic crystals and image of Veselago lens

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It is shown that negative electric permittivity \( \epsilon \) and magnetic permeability \( \mu \) recently discovered in a photonic crystal in the vicinity of the \( \Gamma \)-point are properties of propagating modes only. The evanescent modes rather decay than increase in the bulk of the crystal though they may be amplified by surface waves. If surface support such waves, the evanescent waves may improve the image of a thin Veselago lens. It is shown that a “perfect lens” contradicts to the wave optics and a criterion of “superlensing” is formulated.

Left handed medium (LHM), defined by Veselago as a medium with simultaneously negative \( \mu \) and \( \epsilon \), recently attracted much attention mostly because of the negative refraction at the interface with a regular medium (RM). This effect allows creation of a unique device called the “Veselago lens”. This lens is a slab of LHM inside a RM with a condition that both media have the same isotropic refractive index and the same impedance. The interest in LHM was significantly amplified by the work of Pendry who argued that the Veselago lens is a “perfect lens” in the sense that it gives a perfect image of the point source. This statement is based upon the observation that the evanescent waves (EW’s) of a form \( \exp(ik_{xy} - \kappa x) \) that usually decay in the near field region are amplified by the LHM. Pendry claimed that the amplified EW’s may restore a perfect image. Following Veselago, Pendry considered a “hypothetical” LHM (HLHM) with negative and real \( \mu \) and \( \epsilon \). He did not present a solution in a coordinate space. This was done by Ziolkowski and Heyman and their solution reveals a fundamental problem with Pendry’s idea. The solution diverges exponentially at each point of a 3D domain near the \( \Gamma \)-point has been presented in Ref. [12].

In this paper we consider p-polarization for both EW’s and propagating modes in a uniaxial PC. Fig. (a) shows the elementary cell in the plane \( x - y \) and the spectrum of p-polarized propagating waves. The dashed line is our working frequency. All data below are obtained for this frequency.

From the same numerical solution of microscopic Maxwell’s equation that gives the spectrum one gets microscopic fields \( h_z(x,y), e_y(x,y) \) and \( e_z(x,y) \) corresponding to a given frequency of propagating modes. The microscopic fields are the Bloch functions wile the macroscopic fields have a form \( B_z = <h_z>, E = <e> \), where \( <\ldots> \) means averaging over the unit cell. Near the \( \Gamma \)-point the Bloch functions have a small \( k \), and the macroscopic fields of the propagating modes are plane waves that obey the macroscopic Maxwell equations.

As follows from Ref. [8], the values of \( h_{zz} = \mu \) and \( e_{xx} = \epsilon_{yy} = \epsilon \), that describe propagation of these waves, are negative because their group velocity is negative. Here \( z \) is the axis of the crystal. It has been shown previously that the values of \( \epsilon \) and \( \mu \) can be expressed through the integrals of Bloch’s function at \( k = 0 \). It is important that in a system with a spatial dispersion the separation of magnetization and displacement currents is impossible. Our parameters \( \epsilon(\omega) \) and \( \mu(\omega) \) are not exclusive properties of the medium at a given frequency, as in macroscopic electrodynamics without spatial dispersion. Rather they are properties of a particular mode of this medium, whose fields are used for the calculations. We argue below that they are not applicable to the EW’s. The theorem connecting signs of both \( \epsilon(\omega) \) and \( \mu(\omega) \) with the sign of group velocity is not applicable.
to the EW’s as well. Near the Γ-point \( \mu \sim k^2 \). Since the dispersion law \( \omega(k) \) is also isotropic one obtains \( \mu \) that are function of \( \omega \) only. This approach is good for an infinite PC, but may create problems with boundary conditions because in coordinate space \( k^2 \) should be considered as Laplacian operator. We are going to refer to this problem somewhere else. For the EW’s this approach fails completely because their dispersion law is anisotropic.

To demonstrate Veselago lens one should know \( \mu \) and \( \epsilon \) of the RM medium around the PC. In this paper we find the value of of refractive index \( n = \sqrt{\epsilon \mu} \) from the function \( \omega(k) \), shown in Fig. 1(a), using definition \( \omega = kc/n \). The second condition that provides matching of impedance is the absence of reflection of the incident plane wave in a wide range of incident angles.

Using these two conditions we get that at the working frequency the RM material should have \( \epsilon = 1.125, \mu = 0.08 \). The physical reasons for the appearance of \( \mu \) in a non-magnetic PC are discussed in Ref. [7]. In all computations below we use these values for homogeneous regions surrounding the PC slab.

Fig. 1(b) shows the result of our simulation for light propagation through the PC slab surrounded by a homogeneous medium. One can see the negative refraction of light coming in and going out of the PC with equal absolute values of incident and refracted angles and without any visible reflections from the interfaces. We checked it in a wide range of the incident angles (from 0 to 60 degrees). An animation shows that the wave fronts are moving to the right outside the PC slab and to the left inside the slab. All of these phenomena exactly correspond to Veselago’s theory.

Now we come to EW’s in the same PC slab imbedded into the same RM as in Fig 1(b). Our results are shown in Fig. 2. The boundary conditions implied for an incident wave at \( x = 0 \) are \( h_z = \exp(ik_yy) \) and \( dh_z/dx = -\kappa h_z \). Note that due to reflection the total field shown in Fig. 2 may not coincide at \( x = 0 \) with the incident one. To obey Maxwell’s equation we imply the dispersion law of EW’s in a form \( k_y^2 - \kappa^2 = \omega^2 n^2/c^2 \). The distance from the left boundary of the RM (point \( x = 0 \)) to the PC slab is 1/2 of the slab thickness. In Fig. 2(a) the PC slab is substituted by the HLHM that has negative \( \epsilon \) and \( \mu \) with the absolute values same as in the RM. One can see a strong amplification of the EW first discovered by Pendry. However the PC slab gives a very different picture. The result strongly depend on the way the surface is cut. Our surfaces are perpendicular to [10] direction, but they cut either across the holes (AH) or between the holes (BH). In the case of the BH surface (dashed lines) the EW decreases in the PC slab in the same way as in RM without any signature of amplification. However, there is an amplification at the left AH surface for a thick slab and at both AH surfaces for a thin slab. We think that this amplification are due to the surface waves (SW), excited by EW’s. Since the picture is not symmetric with respect to left and right interfaces this excitation is not resonant. These SW’s have nothing in common with left-handed properties and with polarons in the HLHM, considered by Ruppin because they depend on the properties of the surface. We think these SW’s appear just due to the termination of periodic structure. They have been studied before by many authors with and without connection to the focusing.

Thus, we have found that EW’s cannot be described in terms of the same negative \( \epsilon \) and \( \mu \) that are found...
FIG. 2: (Color online) Magnetic field of the EW in the HLHM (a) and in the PC slabs (b),(c) surrounded by a regular homogeneous medium at the following parameters of EW:

\[ k_y = \left( \sqrt{\frac{5}{2}} \right) k_0, \kappa = 0, \] 

\[ k_0 = \frac{\omega n}{c}, n = \sqrt{\mu} = 0.3. \]

The cross section \( y = 0 \) is shown. The thickness of the PC slab is 20d (b) and 10d (c). The full lines (black) are for the AH surfaces, the dashed lines (red) are for BH surfaces. The period of oscillation is \( d \).

for the propagating waves. The formal reason is that the dispersion law of EW is anisotropic even near the Γ-point. Therefore in this case \( \mu \) cannot be represented in \( k \)-independent form as in the case of propagating waves. The physical explanation reads that the EW in the direction of decay is the same for both regular and backward wave. Therefore in this direction there is no difference between LHM and RM. We have verified, however, that if an EW propagates (has a real \( k \)) in a direction not parallel to the surface of the PC slab, it shows a negative refraction, similar to that shown in Fig. 1(b).

Now we describe the image of the Veselago lens. The point source is the 2D Green function \( H_s = iH_0 H_{0}^{(1)}(\rho k_0) \exp^{-i\omega t} \), where \( k_0 = \omega n/c \) and the Hankel function \( H_{0}^{(1)} = J_0 + iN_0 \). It is located in the RM at a distance \( a \) to the left of the PC slab.

It is easy to show that that the perfect image of this source is impossible. Indeed, if the image has the same form as the source, the magnetic field near the focus would obey inhomogeneous Helmholtz equation with a delta-function at the focus, while the focal point would not contain any source. Thus, this solution does not obey homogeneous Helmholtz equation near the focal point. Note that this argument does not specify the nature of a lens.

We simulated the Veselago lens with the same PC slab and the same parameters of the RM. Similar to geometry of Fig. 2 the distance \( a \) is 1/2 of the thickness of the PC slab. Fig. 3 shows the spatial distributions of magnetic energy near the focus in both lateral (y) and perpendicular (x) directions. The distribution in the lateral direction is symmetric for \( y = -y \) so it is shown for positive \( y \) only.

Our results shown in Fig. 2 give a possibility to calculate analytically the distribution of energy near the focus of a thick Veselago lens with both AH and BH surfaces or of a thin lens with BH surface. In these cases evanescent waves do not reach the focus. At positive \( x \) the Green function of the source can be represented in a
form $H = H_p + H_{ev}$, where

$$H_p = i(H_0/\pi) \int_{-k_0}^{k_0} \exp\left(iky + x\sqrt{k_0^2 - k^2} - \omega t\right) dk$$

contains only propagating modes while

$$H_{ev} = (H_0/\pi) \int_{|k| > k_0} \exp\left(iky - x\sqrt{k_0^2 - k^2} - i\omega t\right) dk$$

contains only EW's.

If the EW's decay inside the PC slab, only term $H_p$ contributes to the focus if the slab is thick enough. On the other hand our results show that for the propagating modes the PC slab works exactly like a HLHM. Using the Fresnel equation it is easy to show that the field near the focus is given by $H_p(x',y')$, where the arguments originate at the focal point. This function is not symmetric with respect to $x'$ and $y'$: the width of the distribution in a perpendicular to the slab direction is larger than in the lateral direction (See Fig. 3). This general result originates from the withdrawal of EW's. Note that the source field depends only on $\rho$. Another important feature of this imaging is that the image in the lateral direction is $H_p(0'y') = iH_0 \rho_0(y'k_0) \exp(-i\omega t)$. If we consider real fields, the part of the source field that is proportional to $\sin(\omega t)$, namely $H_0 \rho_0(\rho k_0) \sin(\omega t)$, has a perfect image in the lateral direction while the part of the source with $\cos(\omega t)$ has a zero image in this direction. This follows only from the plane (non-axial) geometry of the lens that is assumed to be infinite in the lateral direction. The perfect lateral imaging of the Bessel function $J_0$ does not contradict to general principles because this Bessel function obeys homogeneous Helmoltz equation.

In our simulation, however, the slab has a finite length $h$ in y-direction. In this case some propagating modes go outside the slab. To take this into account one should integrate in Eq. (1) from $-k_m$ to $k_m$, where $k_m = k_0(h/2a)/\sqrt{1+(h/2a)^2}$, $a$ is the distance from the source to the slab. In our case $h/2a = 4$ for thick and thin lenses respectively. Due to this factor the lateral distribution slightly differs from $J_0^2$. These calculations are shown in Fig. 3 for both thick (20d) and thin (10d) lenses. They are different because of different values of $a$. For the thick lens analytical calculation is in a very good agreement with the computational data.

A sharp first peak of a Bessel-like function in lateral direction is often considered as a result of "superlensing". We think, however, that any result that follows from Eq. (1) has nothing to do with "superlensing". Following Pendry we would define superlensing as a result of amplification of EW's, but Eq. (1) does not contain EW's at all. It gives just a diffraction pattern of the Veselago lens that was obtained in Ref. [3] for a 3D-lens. Then the energy distribution given by this equation should be called "regular lensing". Thus, our thick Veselago lens based upon PC does not provide any superlensing. The thin lens with AH sufrace gives substantially sharper image than it follows from Eq. (1) with a proper $h/2a$-correction (See Fig. 3). This is due to amplification of EW's shown in Fig. 2(c) and this effect can be considered as a superlensing.

In conclusion we have shown that the Veselago lens imaging has some peculiar features, such as a perfect imaging of the phase shifted part of the source. We show that a thin lens in a near field regime may provide a substantial superlensing depending on the way the slab is terminated. The physics of this superlensing is not connected with the left-handed properties of a medium.

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