Seismic isolation of buildings on two dimensional phononic crystal foundation

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Abstract. In order to realize the seismic isolation of buildings, we establish the two dimensional phononic crystal (PC) foundation which has the cell with the size close to the regular concrete test specimens, and is composed of the concrete base, rubber coating and lead cylindrical core. We study the in-plane band gap (BG) characteristics in it, through the analysis of the frequency dispersion relation and frequency response result. To lower the start BG frequency to the seismic frequency range, we also study the influences of material parameters (the elastic modulus of coating and density of cylindrical core) and geometry parameters (the thickness of coating, radius of cylindrical core and lattice constant) on BG ranges. The study could help to design the PC foundation for seismic isolation of building.

1. Introduction
Buildings usually undergo several kinds of vibrations from surrounding environment, such as earthquake, blasting, machine, traffic and pile driving, etc. Above all, vibrations caused by earthquake often cause a great loss on lives and properties, basically because of the damage of buildings. Thus, ensuring the safe operation of buildings under earthquake is a significant problem. Many seismic isolation methods have been developed and applied in civil engineering so far. Overall, these methods can be divided into three types, including passive, semi-active and active control methods [1]. Because the earthquake source cannot be controlled, we could just consider the buildings themselves and the wave propagation path between sources and buildings. The traditional seismic isolation methods generally rely on various dampers, wave barriers, as well as the most commonly used laminated rubber bearings [2]. Their strategy is energy dissipation by using damping, on the basis of changing the natural frequencies of systems. However, the traditional seismic isolation methods still have some shortages. For example, these methods basically need some additional equipment, which increases the complexity and costs of construction. Moreover, the additional equipment may affect the static behaviour of buildings, such as the poor vertical bearing capacity and large horizontal displacements when using laminated rubber bearings [3, 4]. Additionally, these methods usually emphasize the avoidance of resonance, thus the active control methods are developed to change the frequency of action [5]. While the frequency range of action of the traditional seismic isolation methods is still narrow. To cover the shortage, dampers are used to transfer and consume vibration energy. While dampers cannot just focus the frequency range of seismic wave, so the traditional seismic isolation methods still have relatively low efficiency. So could we develop a seismic isolation method which just covers the seismic frequency range without active control, has high vibration isolation efficiency, do not rely on additional equipment, and do not affect the static behaviour of buildings? However, at
least, the question is hard to answer, based on the traditional ideas on seismic isolation of buildings. The seismic isolation needs some new ideas.

Recently, the concept of phononic crystal (PC) \cite{6, 7} supplies a chance to answer the question. PCs are composed of specific periodical structures or materials. Because of the interaction of the periodicity and elastic waves, the elastic waves fall into some frequency ranges, which are called band gaps (BGs), cannot propagate. So PC has the ability of reducing and controlling elastic waves and vibrations. Due to the BG characteristics and other features, such as negative refraction, PCs could have many applications such as high-precision mechanical systems \cite{6}, noise control \cite{8}, vibration control \cite{9, 10}, vibration reduction \cite{11}, transducers and wireless communications \cite{12} and even acoustic cloak \cite{13}. On seismic isolation, some methods and prototypes have been proposed. The foundation is usually focused to introduce PC, such as the one dimensional layered foundation composed of concrete base and rubber \cite{14}, two dimensional PC foundation composed of concrete base, rubber coating and iron or steel core \cite{15, 16}, two dimensional PC foundations compose of soil base and concrete piles \cite{17, 18}, as well as the three dimensional PC foundations composed of concrete base, rubber coating and steel core \cite{19}. In order to meet the requirement of seismic frequency range, locally resonant PCs \cite{20} are generally designed to give very low frequency BGs. Although the seismic cloak \cite{13} could theoretically avoid the seismic waves, but the seismic waves would still release the destructive power behind the cloak. Thus in our opinion, for seismic waves, isolation and elimination are better than transfer. So here we still focus on the seismic isolation by using PC. However, the current studies are usually based on the quite large cells with lattice constants in meters. It is even hard to test the static behaviours of a cell, by using the regular testing machines. So we want to study the case with smaller cells which are close to the regular concrete test specimens. Considering the heavier lead core, concrete base and rubber coating, we will study the BG characteristics of the two dimensional PC foundation, in order to find whether it is feasible for seismic isolation of buildings.

In this paper, firstly, we define the model of the two dimensional PC foundation. Then, we calculate the frequency dispersion relation and analyse the BG characteristics. Next, from the frequency response analysis of the corresponding finite specimen, we verify the BG results. Finally, we study the influences of the materials parameters and geometry parameters on BGs, in order to get lower frequency BGs for seismic isolation of buildings.

2. Model

Figure 1 shows a periodical arrangement of two dimensional PC foundation, which contains the base and coated cylindrical cores. Buildings are constructed on it. There is translational invariance in the direction z parallel to the cylinder cores, and the foundation has two dimensional periodicity in the xy plane. When earthquake happens far away from the PC foundation, the seismic wave would propagate approximately as plane wave in the foundation. Here, we just consider the plane wave component in the xy plane, and then the polarization modes of the elastic waves can be decoupled into the in-plane (xy) mode and the anti-plane (z) mode. The z mode governs the motion of each particle in the PC foundation along the direction z, which drives the vertical motion of buildings. While the xy mode governs the motion of each particle in the PC foundation in the xy plane, which drives the horizontal motion of buildings. Considering the horizontal shake is the main reason of the building collapse, we would focus the plane waves in xy mode below.

In the xy plane, a minimum periodical cell is enough to illustrate the information of the two dimensional PC foundation. The PC foundation has the square lattice in the xy plane. Figure 2(a) shows that the cell is composed of the concrete base, rubber coating and lead core. The lattice constant is a, the diameter of cylindrical core is d_c, and the outer diameter of coating is d_s. So the thickness of coating is (d_c-d_s)/2. To determine the BG ranges, the frequency dispersion relation should be given firstly. We use the finite element method to calculate the frequency dispersion relation according to the wave vectors in the irreducible Brillouin zone shown in figure 2(b). The details could be seen in \cite{21}.
3. Elastic BGs

Based on the model mentioned above, we construct the two dimensional PC foundation with actual materials. The concrete base has the elastic modulus and Poisson's ratio of $E_{\text{concrete}}=3.0\times10^{10}$ Pa and $\nu_{\text{concrete}}=0.2$. The lead core has the two parameters of $E_{\text{lead}}=4.08\times10^{10}$ Pa and $\nu_{\text{lead}}=0.369$. And the rubber coating has the two parameters of $E_{\text{rubber}}=1.175\times10^{5}$ Pa and $\nu_{\text{rubber}}=0.469$. Considering the sizes of actual foundations and specimens for test, we firstly choose the lattice constant as 0.3 m. The diameter of lead cores is 0.2 m. And the thickness of rubber coating is 0.02 m. Then we calculate the frequency dispersion relation, which is shown in figure 3. The shadow zone illustrates the frequency range without corresponding wave vectors, and gives the BG range of 29.3~58.5 Hz. Clearly, several flat straight curves shown in the frequency dispersion relation, which agrees with the feature of the locally resonant PCs. Moreover, based on the similar lattice constant and concrete base, the Bragg scattering mechanism controlled PC would give the BGs with the frequency of several thousand Hz. We use the locally resonant PC to reduce the BG range close to the seismic frequency range.

The BG obtained from the frequency dispersion relation is based on the ideal PC foundation model, which has infinite periodicity in the xy plane. While for the actual finite structure, we still need to verify the BG results. We choose the specimen with 12×2 cells which is shown in figure 4. We just focus the wave propagation along the ΓX direction which corresponds to the frequency dispersion curves between ΓX, so in this direction we give 12 cells, while in another direction we just give 2 cells. We apply the harmonic displacement impulses which sweep over frequency range of 0~0.6 to one end of specimen, and then get the frequency responses at point A on the other end. We just consider the impulse which corresponds to the in-plane transverse vibration. Figure 5 shows the frequency responses at point A. The distinct attenuation area illustrates the actual BG range, which covers 28.7~57.9 Hz. The result matches the theoretical BG range very well. We also give the motions of the specimen under the impulses of 21 Hz (out of the BG) and 31 Hz (in the BG), which are shown in figure 6. Evidently, the 21 Hz impulse makes the specimen move at each part. While the 31 Hz impulse basically transfers the vibration energy to several coated cores close to the excitation source, and block the vibration in the concrete base. These also verifies the vibration isolation effectiveness of the two dimensional PC foundation.
Figure 3. Frequency dispersion relation of the two dimensional PC foundation with concrete base, lead core and rubber coating. The lattice constant is 0.3 m. The shadow zone is the BG range.

Figure 4. Finite specimen of the two dimensional PC foundation under the in-plane transverse vibration impulse.

Figure 5. The frequency response is obtained at point A marked in figure 4.

Figure 6. Motions of the specimen under the impulses with (a) 21 Hz and (b) 31 Hz.

Of course, one can see that the BG range still needs to be adjusted to enter the general seismic frequency range, in order to use the BG characteristic. So we furtherly study the influences of material parameters (the elastic modulus and density) and geometry parameters (the lattice constant, size of core and coating) on the BG range, which are shown in figure 7. The studies are all based on the two dimensional PC foundation sample mentioned above. When we focus on the influence of one parameter on BGs, we just change this parameter without changing any other parameters. For material parameters, we only consider the elastic modulus of coating and density of cylindrical core, because these two parameters directly determine the start frequency of locally resonant BG. Figure 7(a) and (b) show the influences of these two parameters on BG range. The softer coating and heavier cylindrical core could bring lower resonant frequency, and give lower BG. Additionally, the softer coating and heavier cylindrical core also brings wider BG. For geometry parameters, we consider the thickness of coating, radius of cylindrical core and lattice constant. Figure 7(c)–(d) show the influences of these...
three parameters on BG range. Increasing the thickness of coating and size of the cylindrical core could respectively reduce the stiffness of coating and increase the mass of core, which both helps to lower the resonant frequency and leads to the reduction of the start BG frequency. While the former decreases the BG range, the latter widens the BG range. Moreover, increasing the lattice constant could give lower but narrower BG. Considering the limitation of actual materials and geometry arrangement, to lower the start BG frequency to the seismic frequency range, the most effective measures are increasing the lattice constant and using the softer coating. In spite of this, the BG range of a kind of 2 dimensional PC foundation could still be hard to cover the seismic frequency range. But one can combine the BGs with different frequency ranges in several kinds of PC foundations to obtain super wide BGs, which could meet the requirement of seismic isolation of buildings.

**Figure 7.** Influences of (a) elastic modulus of coating, (b) density of cylindrical core, (c) thickness of coating, (d) radius of cylindrical core and (e) lattice constant on the BG range.
4. Conclusions
In summary, we mainly study the in-plane BG characteristics of the two dimensional PC foundation, in order to realize the seismic isolation of buildings. The two dimensional PC foundation we designed is composed of the concrete base, rubber coating and lead cylindrical core. Through the analysis of the frequency dispersion relation and frequency response result, we determine the existence of the low frequency locally resonant BGs. While, even using the heavy lead core, the cell with the size close to the regular concrete test specimens, is hard to bring the BGs in the general seismic frequency range. The study of the influences of material parameters and geometry parameters shows that, increasing the lattice constant and using the softer coating are the most effective measures to lower the start BG frequency to the seismic frequency range. However, one kind of PC foundation would never give a wide BG which covers the seismic frequency range, thus we still need other methods to make a wider BG which covers a bigger frequency range of seismic waves, such as combining different BG ranges by using several kinds of PC foundations.

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