Gravitational Waves from a Population of Galactic Neutron Stars

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Abstract. The existence of a large number of asymmetric, rotating neutron stars, each individually emitting periodic or quasi-periodic gravitational waves in the frequency band around 100 Hz, raises the possibility of detecting their combined signals, by exploiting the amplitude modulation of the received waves as the antenna changes its orientation with respect to fixed stars. This modulation is directly related to the amount of anisotropy present in the source distribution, and, if detected, could give valuable information about the spatial distribution of neutron stars in our Galaxy.

1 Introduction

It is possible that a fraction of the gravitational wave (GW) stochastic background is generated astrophysically within our Galaxy, e.g., by a large population of galactic sources emitting long-duration individual signals which overlap in frequency. This is the case, for example, in the low-frequency region, where the expected signal from unresolved close binary systems represents an effective sensitivity limit for planned spaceborn interferometers, at frequencies below $\sim 3 \text{ mHz}$ [3, 7].

A similar situation may arise for ground-based interferometers, when considering waves emitted by a large population of non-axisymmetric, rotating neutron stars (NS). This contribution to the background signal, in any case, would not have the same relevance as the galactic binaries background has for space interferometers, for the following reasons: first, we do not know how many rotating NS are present in our Galaxy, nor how efficiently they emit GW, and therefore we do not know whether their contribution to the background signal is relevant, when compared to other stochastic sources. Also, one should consider that the stochastic nature of this signal, in the sense that individual sources cannot be resolved, results from the finite duration of the observation time (i.e. from the finite size of the resolution bin in the frequency spectrum), which in the terrestrial case does not have any a priori upper limit. Finally, even if one assumes that the number of sources is large enough for the emitted waves to be unresolved in a given integration time, one can always rely on a network of detectors, which would allow disentangling this signal from other stochastic sources. One can thus conclude that the overall signal from galactic

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NS should not interfere with the search of other sources, unless the NS birthrate is considerably higher than today's estimates (see Eq. (5) below).

It has been suggested, however, that detecting and measuring this signal with a single detector will provide direct and valuable information about the spatial distribution of NS in the Galaxy [5], and in some cases could even be easier than detecting individual sources [6]. This detection strategy is based on the fact that the antenna response is not isotropic, and therefore the change of orientation of the interferometer with respect to fixed stars generates an amplitude modulation of the stochastic signal, given that the sources are not distributed isotropically with respect to us. The important point is that this modulation is totally deterministic, and for a terrestrial interferometer it can be easily expressed as the sum of four sinusoidal oscillations, with frequencies $k/T_d (k = 1, \ldots, 4)$, where $T_d$ is the duration of the sidereal day [4]. The amplitude of these four periodic oscillations is related to the amount of anisotropy in the spatial distribution of sources, being identically zero for a perfectly isotropic population (as it should be, since in this case the detector's motion is totally irrelevant). Thus, by measuring the modulations of the variance of the stochastic signal one should be able to obtain valuable information about how the NS are distributed in our Galaxy [5].

This contribution summarizes some recent results on this subject. Full details of the calculations can be found in [5].

2 The strength of the stochastic signal

In general, deformations produced by internal forces in a slowly-rotating NS are responsible for creating a slightly non-axisymmetric configuration (described by the equatorial oblateness $\epsilon_e$), and for making the star’s angular velocity precess around the near-symmetry axis (we call 'wobble angle' $\theta_W$ the constant angle between the rotation and symmetry axes). Both effects result in the emission of sinusoidal gravitational radiation, respectively at twice the rotation frequency $f_1 = \omega_{\text{rot}}/\pi$, and at its precessional sideband $f_2 = (\omega_{\text{rot}} + \omega_{\text{prec}})/(2\pi) \simeq f_1/2$.

The amplitudes of the two components are

\begin{align}
    h_{\text{rot}} &= \sqrt{\frac{32}{5} \pi^2 \epsilon_e I f_1^2 / r}, \\
    h_{\text{prec}} &= \sqrt{\frac{32}{5} \pi^2 \epsilon_p \theta_W I (2f_2)^2 / r},
\end{align}

where $I$ is the star's moment of inertia, $\epsilon_p$ is the poloidal oblateness, mainly due to centrifugal forces, and $r$ is the distance to the earth.

Each NS is slowly spinning down due to the loss of energy and angular momentum in the form of electromagnetic and gravitational waves. The exact evolution of the star’s rotation frequency, and consequently of the GW frequency and amplitude, depends on which of the two emission mechanisms is dominant. This also affects in a significant way the spectral properties of the cumulative
signal from the whole population of NS. For example, for a given birthrate \( \nu \) and a given observing time \( T_{\text{obs}} \), there exists a threshold frequency \( f_c \) below which the signal from the NS population behaves like a stochastic signal, in the sense that one expects, on average, more than one source per frequency resolution bin. It turns out that the relatively fast evolution due to electromagnetic emission makes this continuum frequency \( f_c \) being located below the frequency band of interest \( (f_c < f_{\text{min}} \simeq 20 \text{ Hz}) \), unless

\[
\nu \gtrsim 2 \times 10^3 \text{yr}^{-1},
\]

(3) a birthrate considerably higher than the present estimate \( \nu_{\text{est}} \simeq 10^{-2} \text{yr}^{-1} \).

If, however, a significant fraction of sources are born with a small magnetic field, so that GW radiation reaction dominates the (much slower) evolution, then the limit (3) drops to

\[
\nu \gtrsim 5.5 \times 10^{-3} \text{yr}^{-1}.
\]

(4)

In deriving eqs. (3) and (4) it has been assumed a magnetic field strength \( |\vec{B}| \simeq 10^{12} \text{ Gauss} \), an ellipticity \( \epsilon \simeq 10^{-6} \), a moment of inertia \( I \simeq 10^{45} \text{ g cm}^2 \), and \( T_{\text{obs}} = 1 \text{ yr} \); a more general expression for the minimum birthrate \( \nu \) can be found in [5].

In view of the fact that unresolved NS are likely to be spinning down by gravitational radiation reaction, we will consider only this mechanism from now on. Also, for the sake of brevity, the precessional component \( h_{\text{pre}} \) of the radiation will be neglected, by imposing \( \theta_W = 0 \). The rotational component \( h_{\text{rot}} \) gives, in the confusion region \( f \leq f_c \), a stochastic signal of spectral amplitude

\[
S_h(f) = \frac{I \nu}{5 f} \int \frac{\rho(r)}{r^2} d\vec{r} \simeq 1.5 \times 10^{-51} \text{ Hz}^{-1} \left( \frac{f}{100 \text{ Hz}} \right)^{-1}
\]

\[
\times \left( \frac{\nu}{10^{-2} \text{yr}^{-1}} \right) \left( \frac{I}{10^{45} \text{ g cm}^2} \right) \left[ \frac{\int \rho(r) d\vec{r}}{(10 \text{kpc})^{-2}} \right].
\]

(5)

Note that the spectral level (5) does not depend on the oblateness of the sources, a direct consequence of the fact that gravitational radiation reaction is assumed to be the dominant spindown mechanism.

As one can infer from Eq. (5), a birthrate of one pulsar each 100 years gives a signal well below the expected sensitivity of planned terrestrial detectors. Unfortunately, the actual rate is rather unknown, since a few hundred pulsars out of perhaps \( 10^8 \) rotating NS in our Galaxy have been detected, and presumably the strongest sources have not yet been detected electromagnetically. However, as shown in the next section, this stochastic signal can be detected even if its strength is much less than that of the instrumental noise, by exploiting the peculiar time dependence of the signal’s amplitude.
3 Amplitude modulation of the stochastic signal

As already mentioned in the Introduction, the stochastic signal generated by any anisotropic distribution of sources, as seen by an interferometer anchored on the surface of the earth, is non-stationary. It is now possible to quantify this effect, and relate it to the actual distribution of sources via a simple integral.

In particular, we will focus our attention to the signal’s expected variance, which can be expressed as

$$\sigma^2(t) = \int df \, S(f, t), \quad (6)$$

where $S(f, t)$ is the generalization of Eq. (5) which takes into account the actual response of the interferometer, through its antenna pattern $P(\theta, \phi, t)$, i.e.

$$S(f, t) = \frac{I\nu}{2f} \int \frac{P(\theta, \phi, t)\rho(\vec{r})}{r^2} \, dr. \quad (7)$$

Note that Eq. (6) becomes identical to Eq. (5) if the spatial distribution is isotropic with respect to us, since $\int d\Omega P(\theta, \phi, t) \equiv 8\pi/5$. For a terrestrial interferometer in the equatorial reference frame one finds

$$P(\theta, \phi, t) = \sum_{k=0}^{4} \{a_k(\theta, \phi) \cos(k\omega d t) + b_k(\theta, \phi) \sin(k\omega d t)\}, \quad (8)$$

where the Fourier coefficients $a_k, b_k$ are explicitly given in Eq. (5), and $\omega_d$ is the earth sidereal frequency $\omega_d \simeq 7 \times 10^{-5}$ rad s$^{-1}$. Inserting Eq. (8) in Eq. (7) one trivially obtains, upon integration over frequency

$$\sigma^2(t) = \sum_{k=0}^{4} \{\alpha_k \cos(k\omega d t) + \beta_k \sin(k\omega d t)\}, \quad (9)$$

where

$$\alpha_k \propto \int d\Omega a_k(\theta, \phi) \int dr \, \rho(r, \theta, \phi), \quad \beta_k \propto \int d\Omega b_k(\theta, \phi) \int dr \, \rho(r, \theta, \phi),$$

and analogous for $\beta_k$, with $a_k$ replaced by $b_k$. Thus, the variance of the stochastic signal is a deterministic function of time; its Fourier decomposition reveals four discrete frequencies, harmonics of the earth sidereal frequency $\omega_d$. The amplitudes of these four spectral lines depend on the particular form of $\rho$, i.e. on the spatial distribution of the sources. By measuring them, one can therefore obtain information about the global distribution of sources in the Galaxy.

The detection strategy could be as follows: the data record, of duration $T_{\text{obs}} \gg T_d$, will be divided into several batches of duration $\tau \ll T_d$. For each batch $J$ we obtain an estimate $s_J^2$ of the variance $\sigma^2(t_J)$. According to Eq. (3),
the variables \( \{ s_J^2 \} \) are expected to show four sinusoidal modulations, of known frequencies \( \omega_d \) and its first three harmonics) and phases (depending on the initial orientation of the detector with respect to the Galaxy). The amplitude of these oscillations can be estimated with a simple Fourier analysis, in the hypothesis that the instrumental noise is stationary. Note that, in order for this search to be successful, it is not required that \( S_h > S_{\text{noise}} \), since we are not directly comparing the GW stochastic signal against the instrumental noise; instead, we are trying to detect a deterministic and periodic signal \( \sigma^2(t) \) buried in a stationary random noise of amplitude \( \sigma_{\text{noise}}^2 \). The main problem in this measurement comes from those noise sources that, far from being stationary, present the same diurnal variations as the gravitational signal, due for example to temperature changes in the mirrors. This issue has been addressed in great detail by Giazotto et al. \[6\], where a careful monitoring of the temperature fluctuations in the mirrors has been advocated.

In order to estimate the magnitude of the effect, and in particular the amplitude of the periodic terms of \( \sigma^2(t) \) with respect to the constant one, we have considered different galactic populations, like the bulge, the disk, the halo, etc. Table 1 shows some results in this direction: for each of the three interferometers presently under construction – the two LIGO detectors in the United States \[1\], and the VIRGO detector in Italy \[2\] –, we have estimated the amplitudes \( \gamma_k \equiv \sqrt{\alpha_k^2 + \beta_k^2} \) \((k = 0, \ldots, 4)\) generated by, respectively, (i) a spherical halo, (ii) an exponential disk, and (iii) a cluster of sources located in the galactic center.

| k | Halo | Disk | Bulge |
|---|------|------|-------|
| | L-1 | L-2 | V     | L-1 | L-2 | V     | L-1 | L-2 | V     |
| 0 | -3.96 | -3.96 | -3.96 | -0.53 | -0.55 | -0.54 | -2.27 | -2.25 | -2.28 |
| 1 | -6.87 | -6.93 | -6.87 | -1.07 | -1.15 | -1.08 | -2.32 | -2.34 | -2.29 |
| 2 | -7.24 | -7.04 | -7.20 | -1.42 | -1.25 | -1.41 | -2.70 | -2.45 | -2.59 |
| 3 |     |     |     | -1.97 | -2.12 | -2.02 | -2.96 | -3.09 | -3.02 |
| 4 |     |     |     | -2.21 | -2.18 | -2.12 | -3.22 | -3.20 | -3.13 |

Table 1: Summary of the Fourier coefficients \( \gamma_k \equiv \sqrt{\alpha_k^2 + \beta_k^2} \), for various galactic components. For each population, the columns marked L-1, L-2, and V are obtained taking into account the orientation on the earth surface of, respectively, the two LIGO detectors and the VIRGO detector. For further details, see \[5\].

From Table 1, one can conclude that the largest contribution to the periodic terms of \( \sigma^2 \) comes from the galactic disk. Also important is the signal coming from a localized cluster of sources, like the galactic bulge or even a rich globular cluster. The periodic contributions from the spherical halo are expected to be
negligible.

References

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