Arbitrary Phase Modulation of General Transmittance Function of First-Order Optical Comb Filter with Ordered Sets of Quarter- and Half-Wave Plates

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Abstract: Here we theoretically and experimentally demonstrated the arbitrary phase modulation of a general transmittance function (GTF) of the first-order optical comb filter based on a polarization-diversity loop structure, which employed two ordered waveplate sets (OWS’s) of a quarter-wave plate (QWP) and a half-wave plate (HWP). The proposed comb filter is composed of a polarization beam splitter (PBS), two equal-length polarization-maintaining fiber (PMF) segments, and two OWS’s of a QWP and an HWP with each set located before each PMF segment. The second PMF segment is butt-coupled to one port of the PBS so that its principal axis should be 22.5° away from the horizontal axis of the PBS. First, we explained a scheme to find four waveplate orientation angles (WOA’s) allowing the phase of a GTF to be arbitrarily modulated, using the way each component of the filter, such as a waveplate or PMF segment, affects its input or output polarization. Then, with the WOA finding method, we derived WOA sets of the four waveplates, which could give arbitrary phase retardations $\phi$’s from 0° to 360° to a GTF chosen here arbitrarily. Finally, we showed phase-modulated GTF’s calculated at eight selected WOA sets allowing $\phi$’s to be 0°, 45°, 90°, 135°, 180°, 225°, 270°, and 315°, and then the predicted results were verified by experimentally measured results. It is concluded from the theoretical and experimental demonstrations that the GTF of our filter based on the OWS of a QWP and an HWP can be arbitrarily phase-modulated by properly controlling the WOA’s of the four waveplates.

Keywords: optical filter; comb filter; polarization interference; general transmittance function; continuous phase modulation

1. Introduction

To date, optical comb filters have provided versatile spectrum controllability in the fields of interrogation of optical fiber sensors, wavelength routing in optical communications, and photonic processing of microwave signals [1–5]. In these comb filters, the capability of continuous wavelength tuning is a crucial function to pass desired spectral components or reject unwanted ones in optical signal processing. By utilizing a Mach-Zehnder interferometer structure [6,7], Sagnac birefringence loop structure [8,9], Lyot-type birefringence loop structure [10,11], and polarization-diversified loop structure (PDLS) [12–16], much effort has been exerted in implementing this continuous tunability of the absolute wavelength in comb filters. Among these filter configurations, optical comb filters based on a PDLS have superior flexibility in the wavelength switching or tuning of their output spectra [12–20]. Since 2017, successive works have been done to realize the continuous wavelength tuning of periodic transmission spectra in PDLS-based fiber comb filters [21–25]. A continuously
wavelength-tunable PDLS-based zeroth-order comb filter containing one polarization-maintaining fiber (PMF) segment as a birefringent element (BE) was reported by employing three ordered waveplate sets (OWS’s): an OWS of a half-wave plate (HWP) and a quarter-wave plate (QWP), another OWS of a QWP and an HWP, and two QWPs [21]. Apart from the zeroth-order comb filters, there have been several works reporting the implementation of the continuous wavelength tuning of flat-top [22,23], narrow [23,24], and arbitrary-shaped [25] passband transmission spectra, which could be obtained in a PDLS-based first-order comb filter having two PMF segments. In these previous studies on the first-order comb filters [22–25], only the OWS of an HWP and a QWP was adopted and placed before each PMF segment to continuously modulate the extra phase delay \( \phi \) in the filter transmittance function. Another OWS of a QWP and an HWP may also be a candidate to achieve the \( \phi \) modulation. To the best of our knowledge, the flexible wavelength tuning of arbitrary-shaped passband transmission spectra, that is, general transmittance functions (GTF’s) in addition to some special transmittance functions such as transmittance functions with flattened and narrowed passbands, was not achieved by using an OWS of a QWP and an HWP. Arbitrary phase modulation of the GTF of the first-order comb filter leading to its flexible wavelength tuning should not be underestimated but rather rigorously explored, because application-specific spectral shapes are demanded in some spectral flattening filters and label erasers and their wavelength tuning capability can provide great efficiency to related optical systems. In particular, the new OWS will bring completely different sets of waveplate orientation angles (WOA’s) where the phase \( \phi \) of the filter transmittance function can be arbitrarily modulated. On top of that, the conventional scheme utilized to obtain WOA’s theoretically in order to enable continuously tuning the filter wavelength [22–24], based on direct comparison between the filter transmittance function and the known analytic transmittance effortlessly found in literatures, is not suitable for finding theoretical WOA’s for continuous frequency tuning of the GTF, because the exact mathematical expression of a desired GTF needed for specific applications is often difficult to seek. Here, we theoretically and experimentally demonstrated the arbitrary phase modulation of a GTF of the PDLS-based first-order fiber comb filter employing two OWS’s of a QWP and an HWP. The proposed comb filter consists of a polarization beam splitter (PBS), two polarization-maintaining fiber (PMF) segments of equal length, and two OWS’s of a QWP and an HWP with each OWS positioned before each PMF segment. The second PMF segment is butt-coupled to one port of the PBS so that its principal axis should be 22.5° away from the transverse-magnetic (TM) polarization axis of the PBS. Basically, polarization conditions, which should be satisfied to modulate the desired phase of a GTF, are explained by taking advantage of the spectral evolution of the input and output states of polarization (SOP’s) of the second PMF in the narrowband transmittance function, on the basis of the continuous wavelength tuning mechanism of previously reported PDLS-based first-order narrowband comb filters [23,24]. Then, we explain a scheme to find four WOA’s for the arbitrary phase modulation of a GTF, using how each component of the filter, such as a QWP, an HWP, or a PMF segment, affects its input SOP (SOP\(_{in}\)) or output SOP (SOP\(_{out}\)). By making good use of this WOA finding method, we draw out WOA sets of the four waveplates, which can provide arbitrary phase retardations \( \phi \)'s from 0° to 360° to a GTF chosen here arbitrarily. Finally, we show phase-modulated GTF’s calculated at eight selected WOA sets allowing \( \phi \)'s to be 0°, 45°, 90°, 135°, 180°, 225°, 270°, and 315°, and then the theoretically predicted results are verified by experimentally measured ones. It is concluded from the theoretical and experimental demonstrations that the GTF of our filter based on the OWS of a QWP and an HWP can be arbitrarily phase-modulated, i.e., wavelength-tuned, by properly adjusting the WOA’s of the four waveplates.

2. Phase Modulation Principle Based on Spectral Evolution of State of Polarization (SOP)

Figure 1a shows a schematic diagram of the proposed comb filter comprised of a PBS, two PMF segments whose lengths are equal (denoted by PMF 1 and PMF 2), two QWPs (denoted by QWP 1 and QWP 2), and two HWPs (denoted by HWP 1 and HWP 2). As shown in Figure 1a, one OWS of QWP 1 and HWP 1 is placed ahead of PMF 1, and the other OWS of QWP 2 and HWP 2 is located
between PMF 1 and PMF 2. Every OWS can effectively change the phase delay difference between two principal modes of the individual PMF segment, and the combination of QWP 2 and HWP 2, or the second OWS, has an additional function to effectively modify the relative angular difference between the principal axes of PMF 1 and PMF 2. The slow axis of PMF 2 butt-coupled to port 3 of the PBS is 22.5° away from the TM polarization axis (denoted by x axis) of the PBS. An input beam introduced into port IN of the filter is decomposed into two orthogonal linear polarization components, such as linear horizontal polarization (LHP) and linear vertical polarization (LVP), which circulate through the polarization-diversified loop of the filter in the clockwise (CW) and counterclockwise (CCW) directions, respectively. Figure 1b shows the two propagation paths of light travelling through the filter, designated as CW and CCW paths. When the SOP_in of the filter is LHP, input light propagates along the CW path and sequentially passes through a linear horizontal polarizer (x axis), QWP 1 (with its slow axis oriented at θQ1 for the x axis), HWP 1 (oriented at θH1), PMF 1 (oriented at θP1), QWP 2 (oriented at θQ2), HWP 2 (oriented at θH2), PMF 2 (oriented at θP2 = 22.5°), and a linear horizontal analyzer (x axis). In the case of the SOP_in of LVP, input light propagates through a linear vertical polarizer (y axis), PMF 2 (−θP2 oriented), HWP 2 (−θH2 oriented), QWP 2 (−θQ2 oriented), PMF 1 (−θP1 oriented), HWP 1 (−θH1 oriented), QWP 1 (−θQ1 oriented), and a linear vertical analyzer (y axis) in turn, circulating along the CCW path. F and S displayed on the waveplates and the PMFs indicate their fast and slow axes, respectively.

**Figure 1.** (a) Schematic diagram of the PDLS-based first-order fiber comb filter and (b) propagation path of light travelling through the filter.

Polarization interference is created by optical birefringence in an optical structure composed of two linear polarizers and BEs inserted between them. In the case of one BE like a PMF segment, the transmittance function of this optical structure becomes a sinusoidally varying function of Γ, the phase delay difference between two principal modes of the BE, which is represented by 2πBL/λ, where B, L, and λ are the birefringence of the BE, the length of the BE, and the free space wavelength, respectively. It is possible to tune the wavelength of this polarization interference spectrum by modifying Γ, and the modification of Γ can be done by adding a supplementary phase delay difference φ to Γ and modulating this φ [21]. In the same way, to tune the wavelength position of the first-order comb spectrum generated with two BEs, or two PMF segments [26], φ, which is added to Γ for individual PMF, should be modulated equally for both PMF 1 and PMF 2. This simultaneous φ modulation can be accomplished by changing the SOP_in’s of PMF 1 and PMF 2 with two OWS’s. When we increase φ from 0° to 360° by appropriately adjusting the SOP_in of each PMF, the comb spectrum makes a redshift by an interference period, i.e., a free spectral range (FSR). The filter transmittance function \( t_{\text{filter}} \) can be derived from the Jones transfer matrix [27] \( T \) given by (1) and is represented as (2). In this derivation,
no insertion losses are assumed to exist in any optical components of the filter, and all waveplates are regarded as wavelength-independent.

\[
T = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} T_{PMF2}(\theta_2) T_{HWP2}(\theta_2) T_{QWP2}(\theta_2) T_{PMF1}(\theta_1) T_{HWP1}(\theta_1) T_{QWP1}(\theta_1) \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} + \begin{bmatrix}
0 & 0 \\
0 & 1
\end{bmatrix} T_{QWP1}(\theta_1) T_{HWP1}(\theta_1) T_{PMF1}(\theta_1) \times T_{QWP2}(\theta_2) T_{HWP2}(\theta_2) T_{PMF2}(\theta_2) \begin{bmatrix}
0 & 0 \\
0 & 1
\end{bmatrix}
\]

(1)

\[
t_{\text{filter}} = \frac{1}{4} (P_0 + P_1 \cos \Gamma + P_2 \sin \Gamma)^2 + \frac{1}{4} (Q_0 + Q_1 \cos \Gamma + Q_2 \sin \Gamma)^2
\]

(2)

Here, \(T_{QWP1}, T_{HWP1}, T_{PMF1}, T_{QWP2}, T_{HWP2}, \) and \(T_{PMF2}\) mean the transfer matrices of QWP 1, HWP 1, PMF 1, QWP 2, HWP 2, and PMF 2, whose slow-axis orientation angles are \(\theta_{Q1}, \theta_{H1}, \theta_{P1}, \theta_{Q2}, \theta_{H2}, \) and \(\theta_{P2}\) (given by 22.5° here) for the x axis, respectively. \(P_0 = -\sin(\lambda_1 - \pi/4)\sin(\delta - 2\theta_1), \)
\(Q_0 = \cos(\lambda_1 - \pi/4)\cos(\delta - 2\theta_1), \)
\(P_1 = -\sin(\lambda_1 - \pi/4)\sin(\delta - 2\theta_1), \)
\(Q_1 = \cos(\lambda_1 - \pi/4)\cos(\delta - 2\theta_1), \)
\(P_2 = \cos(\lambda_1 - \pi/4)\cos(\delta - 2\theta_1), \)
\(Q_2 = \sin(\lambda_1 - \pi/4)\sin(\delta - 2\theta_1), \)
where \(\alpha = 2\theta_1 - (\theta_{Q1} + \theta_{Q2}), \beta = 2\theta_{H1} - (\theta_{Q1} + \theta_{Q2}), \gamma = 2\theta_{H2} + (\theta_{Q1} - \theta_{Q2}), \)
and \(\delta = 2\theta_{H1} - (\theta_{Q1} - \theta_{Q2}). \)

Before we begin to address the principle of the phase modulation of a GTF, let us consider the previous conclusions on the relationship between SOP changes and the continuous wavelength tuning of a narrowband transmittance \(t_n\) given by (3) in the comb filter composed of two PMF segments, two OWS’s of an HWP and a QWP, and the third HWP. \[23,24\].

\[
t_n = \left[3 - 4 \cos(\Gamma + \phi) + \cos 2(\Gamma + \phi)\right]/8
\]

(3)

The additional phase delay difference \(\phi\) in (3) defines the absolute wavelength position of the narrowband transmission spectrum. In the previous studies \[23,24\], eight WOA sets (from Set I\(_n\) to Set VIII\(_n\)) of four waveplates (except for the third HWP) were utilized to display eight wavelength-tuned spectra calculated from \(t_n\) with eight \(\phi\) values from 0° to 315° (increment: 45°). When the wavelength of input light increases from \(\lambda_0\) to \(\lambda_0 + \Delta \lambda\), where \(\Delta \lambda\) implies the FSR, the SOP\(_{in}\) of PMF 2 at Set I\(_n\) \((\phi = 0°)\) spectrally evolves on the Poincare sphere, as shown in Figure 2a. This spectral evolution indicated by pink circles is denoted by \(SE_{in}\). With increasing wavelength, the SOP\(_{in}\) moves along the \(SE_{in}\) trace rotating CW around the normal vector \(p_1\), when \(p_1\) points the observer. The rotation direction of the SOP\(_{in}\) can also be thought using a left-hand rule. If the direction of \(p_1\) is the same (as that of the left thumb), the direction of the rest of the fingers is the rotation direction of the SOP\(_{in}\) with the increase of the wavelength. In general, the SOP\(_{out}\) of the birefringent medium rotates around its slow axis on the Poincare sphere according to the wavelength \(\lambda\) of light passing through the medium. The vector \(AB\) connecting \(A (2\varepsilon = 0°, 2\psi = -135°)\) and \(B (2\varepsilon = 0°, 2\psi = 45°)\), which lie on the Poincare sphere, points the direction of the slow axis of PMF 2 \((\theta_{P2} = 22.5°)\) and is denoted by the vector \(p_2\). Here, \(2\varepsilon (-90° \leq 2\varepsilon \leq 90°)\) and \(2\psi (-180° \leq 2\psi \leq 180°)\) are the latitude and longitude of the Poincare sphere, respectively. For convenience, the plane whose normal vector is \(p_1\), that is, the plane of the \(SE_{in}\) trace, is designated as the SOP\(_{in}\) plane, and another plane with its normal vector of \(p_2\) is designated as the SOP\(_{out}\) plane. Actually, \(p_2\) is an axis of rotation, around which the \(SE_{in}\) trace moves as \(\phi\) varies. Hence, \(p_1\) and the SOP\(_{out}\) plane are parallel, and \(p_2\) and the SOP\(_{out}\) plane are parallel, ending up with \(p_1\) and \(p_2\) being perpendicular. At Set II\(_n\) \((\phi = 45°)\), the \(SE_{in}\) trace shown in Figure 2a rotates CCW by 45° around both \(p_1\) and \(p_2\), and thus \(p_1\) shown in Figure 2a rotates 45° CCW about \(p_2\) as well. Analogously, for the remaining six sets (III\(_n\)-VIII\(_n\)), the \(SE_{in}\) trace at Set I\(_n\) rotates CCW by 90°–315° (with an increment of 45°) around both \(p_1\) and \(p_2\).
Likewise, the SOP out’s of PMF 2 at Sets I n (φ = 0°) and II n (φ = 45°) spectrally evolve on the Poincare sphere over the same wavelength range of λ 0 to λ 0 + Δλ, as shown in Figure 2b. This spectral evolution also indicated by pink circles is designated as SE out. This SE out trace is embodied by making a CW rotation of the wavelength-dependent SOP in of PMF 2 about p 2 by an angle φ(λ) given by 360°(λ − λ 0)/Δλ. When PMF 2 is passed through by light at λ 0, the SOP in of PMF 2 at λ 0, C (2ε = 0°, 2ψ = 0°) shown in Figure 2a, rotates CW around p 2 by φ(λ 0) = 0°, and the SOP out of PMF 2 at λ 0 becomes C′ (2ε = 0°, 2ψ = 0°) shown in Figure 2b without a change in the position on the Poincare sphere. Similarly, the SOP in of PMF 2 at λ 0 + Δλ/4 rotates CW around p 2 by φ(λ 0 + Δλ/4) = 90° during its passage through PMF 2. If we put the SOP out of PMF 2 at a certain wavelength λ as O(λ), O(λ 0) at Set I n, marked by O 1(λ 0) in the figure, becomes C′. Owing to the wavelength-dependent transformation of the SOP in of PMF 2, O(λ) forms a non-circular trajectory SE out shaped like a droplet as λ increases from λ 0 to λ 0 + Δλ. During the wavelength increase of Δλ starting from λ 0, O(λ) goes round this entire SE out trace starting from C′ in the CW direction around the S 2 axis. If we assume the wavelength at which t n is maximized as λ peak, λ peak can be regarded as λ 0 at Set I n because t n reaches the maximum when O(λ) is nearest to LHP (2ε = 0°, 2ψ = 0°) on the Poincare sphere. At Set II n, O II (λ 0), i.e., the SOP out of PMF 2 at λ 0 becomes C′′ distant from O 1(λ 0) by an angular displacement that corresponds to Δλ/8 and also goes round the entire SE out trace starting from C′′ about the S 2 axis in the CW direction, as the wavelength increases from λ 0 to λ 0 + Δλ. Here, O 1(λ 0) or O II (λ 0) is the initial point of the spectral evolution of O(λ). The position difference between O 1(λ 0) and O II (λ 0) originates from the fact that, while the WOA set goes from Set I n to Set II n, the SE in trace at Set I n is rotated CCW by 45° not only about p 1 on the SOP in plane but also about p 2 on the SOP out plane. This means that simultaneously rotating the SE in trace CCW about both p 1 and p 2 can change the position of O(λ 0) along the SE out trace. On the SE out trace at Set II n, therefore, O II(λ 0 + Δλ/8) is located at C′′, and λ 0 + Δλ/8 becomes λ peak because C′′ is nearest to LHP. This implies that the narrowband transmission spectrum moves towards a longer wavelength region by Δλ/8, that is, φ is modulated by 45° making t n be redshifted by (45°/360°)Δλ = Δλ/8. Similarly, for remaining Sets III n–VIII n, φ is modulated by 90° to 315° with an increment of 45° making t n be redshifted by Δλ/4 to 7Δλ/8 with a step of Δλ/8, and λ peak changes from λ 0 + Δλ/4 to λ 0 + 7Δλ/8 with an increment of Δλ/8.
In the case of a GTF, or an arbitrary transmittance function, obtained in the proposed filter structure, the SOP\textsubscript{out} trace of PMF 2, designated as SE\textsubscript{out,GTF} trace, is presumed to be a completely different trajectory in comparison with the SE\textsubscript{out} trace shown in Figure 2b. However, it can be deduced from the aforementioned conclusions of the previous works that a location change of O(λ\textsubscript{0}) along the SE\textsubscript{out,GTF} trace, accompanied by a shift in λ\textsubscript{peak}, is responsible for a phase modulation in the GTF, resulting in a wavelength shift in the transmission spectrum. For a given SE\textsubscript{out,GTF} trace, the corresponding SOP\textsubscript{in} trace of PMF 2, designated as SE\textsubscript{in,GTF} trace, should be a circle on the Poincare sphere as well, since a wavelength-dependent BE through which light has passed is just one, PMF 1. However, in terms of the radius and the normal vector (p\textsubscript{1}) of the circular trace, this SE\textsubscript{in,GTF} trace will be quite different from the SE\textsubscript{in} trace shown in Figure 2a. Because it is assumed that all waveplates used here are independent of wavelength, they do not distort the shape of the SOP\textsubscript{in} or the SOP\textsubscript{out} trace. Thus, the SOP\textsubscript{out} trace of PMF 1 also has a circular shape identical to that of the SE\textsubscript{in,GTF} trace. Like the relationship between the SE\textsubscript{in} and SE\textsubscript{out} traces, the increase in λ\textsubscript{peak} results from the CCW revolution of the SE\textsubscript{in,GTF} trace about both p\textsubscript{1} on the SOP\textsubscript{in} plane and p\textsubscript{2} on the SOP\textsubscript{out} plane. If the SE\textsubscript{in,GTF} trace is revolved CCW about both p\textsubscript{1} and p\textsubscript{2} by an amount of ξ, the transmission spectrum makes a redshift of (ξ/360°)Δλ, resulting in an increase of (ξ/360°)λ\textsubscript{peak} in λ\textsubscript{peak}. That is to say, ξ can be regarded as φ or the additional phase delay difference. Thus, as ξ varies from 0° to 360°, the phase of the GTF also changes from 0° to 360°, which means that the GTF is able to be phase-modulated continuously by varying ξ. Briefly, one simultaneous CCW rotation of the SE\textsubscript{in,GTF} trace about both p\textsubscript{1} and p\textsubscript{2}, implemented by controlling four waveplates contained within the proposed filter, enables its transmission spectrum to be redshifted by an FSR (Δλ), in other words, its phase to be modulated by 360°.

3. Finding Algorithm of Waveplate Orientation Angles (WOA’s) for Phase Modulation of General Transmittance Function (GTF)

Considering the aforementioned polarization conditions required for arbitrarily modulating the phase of a GTF or continuously tuning its wavelength, the WOA’s (θ\textsubscript{Q1}, θ\textsubscript{H1}, θ\textsubscript{Q2}, θ\textsubscript{H2}) for this arbitrary phase modulation (i.e., continuous wavelength tuning) are investigated with our unique WOA-finding algorithm. This algorithm is suggested as it is quite sophisticated to draw out an exact mathematical expression of a GTF having an application-specific unique spectrum. Moreover, although the mathematical expression t\textsubscript{GTF} of a GTF is given, it is cumbersome to solve (θ\textsubscript{Q1}, θ\textsubscript{H1}, θ\textsubscript{Q2}, θ\textsubscript{H2}) from trigonometric simultaneous equations obtained from direct comparison of t\textsubscript{GTF} with t\textsubscript{filter} in (2). To explain our angle-finding approach, let us choose a specific GTF I\textsubscript{GTF1}, which is procured at a certain selected WOA set (θ\textsubscript{Q1}, θ\textsubscript{H1}, θ\textsubscript{Q2}, θ\textsubscript{H2}) = (118.23°, 149.56°, 179.42°, 123.46°), and find (θ\textsubscript{Q1}, θ\textsubscript{H1}, θ\textsubscript{Q2}, θ\textsubscript{H2}) where I\textsubscript{GTF1} is wavelength-tuned. At the above selected WOA set indicated by Set I\textsubscript{GTF1} and fixed orientation angles of PMF 1 and PMF 2 (θ\textsubscript{P1} = 0° and θ\textsubscript{P2} = 22.5°), the SOP\textsubscript{out} trace of PMF 2, or SE\textsubscript{out,GTF1}, is obtained over a wavelength range from λ\textsubscript{0} (=1548.0 nm) to λ\textsubscript{0} + Δλ (=0.8 nm), as shown in Figure 3a. At Set I\textsubscript{GTF1}, O(λ\textsubscript{0}) indicated in Figure 3a is located nearest to LHP on the Poincare sphere, which makes the filter transmittance be maximized at λ\textsubscript{0}. If we put the wavelength at which I\textsubscript{GTF1} is maximized as λ\textsubscript{peak}, O(λ\textsubscript{0}) becomes O(λ\textsubscript{peak}), that is, λ\textsubscript{peak} = λ\textsubscript{0} = 1548.0 nm at Set I\textsubscript{GTF1}. The SOP\textsubscript{in} trace of PMF 2, or SE\textsubscript{in,GTF1} in Figure 3b, can be derived by utilizing the inverse matrix of T\textsubscript{PMF2} (T\textsubscript{PMF2}\textsuperscript{-1}) and the SE\textsubscript{out,GTF1} trace in Figure 3a. Based on the foregoing description on the phase modulation of the GTF using the SE\textsubscript{in,GTF} and SE\textsubscript{out,GTF} traces, another SE\textsubscript{in,GTF1} trace shown in Figure 3c can be obtained by rotating the SE\textsubscript{in,GTF1} trace at Set I\textsubscript{GTF1} CCW by 45° around both p\textsubscript{1} and p\textsubscript{2}, for the phase modulation of 45° (i.e., the wavelength tuning of Δλ/8 = 0.1 nm) in I\textsubscript{GTF1}. It is easily expected that this new SE\textsubscript{in,GTF1} trace will be obtained at another WOA set (θ\textsubscript{Q1}, θ\textsubscript{H1}, θ\textsubscript{Q2}, θ\textsubscript{H2}) designated later as Set II\textsubscript{GTF1}.  

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coincides with the $S_1$ axis on the Poincare sphere as the orientation angle $\theta_{P1}$ of PMF 1 is 0°, as shown in Figure 3e. Then, $\theta_{H2}$ and $\theta_{Q2}$ can be determined by finding the SOPout trace of PMF 1 to satisfy this constraint for $p_1$.

Figure 3. Wavelength-dependent variation of SOP’s in $t_{GTF1}$ obtained at Set I $GTF_1$: (a) SOP_out and (b) SOP_in traces of PMF 2 at Set I $GTF_1$. Spectral evolution of SOP’s in the phase-modulated version of $t_{GTF1}$, obtained at Set II $GTF_1$: (c) SOP_in trace of PMF 2, (d) SOP_out trace of QWP 2, (e) SOP_out trace of PMF 1, (f) SOP_out of HWP 1, and (g) SOP_out trace of PMF 2. (h) Two calculated transmission spectra obtained at Sets I $GTF_1$ and II $GTF_1$, indicated as blue and red solid lines, respectively.
What we should focus on is a way to obtain Set II_{GTF1}, another WOA set described above. Basically, the SOP out of one component of the filter is equivalent to the SOP in of the following component in any propagation path shown in Figure 1b. As our discussion is confined to the CW path, as an example, the SOP out of PMF 1 is equivalent to the SOP in of QWP 2. The same thing can be said for SOP traces. For example, the SOP in of HWP 1 is the same as the SOP out of QWP 1. A QWP and an HWP rotate input polarization CCW by 90° and 180° about their slow axes on the Poincaré sphere, respectively. If an SOP trace, composed of multiple different SOPs distributed on the Poincaré sphere, enters a waveplate, therefore, not its shape but its position is altered by the waveplate. For a given SE_{in,GTF1} trace at Set II_{GTF1} (in Figure 3c), a certain SOP out of QWP 2 can be obtained by using the SE_{in,GTF1} trace (i.e., the SOP out of HWP 2) and the inverse matrix of T_{HWP2} or T_{HWP2^{-1}}, if θH2 is fixed as one value (unknown yet). With this SOP out of QWP 2, shown in Figure 3d, a corresponding SOP out of PMF 1, which is transformed into the SOP out of QWP 2 after passing through QWP 2, can also be derived similarly by using the predetermined SOP out of QWP 2 and T_{QWP2^{-1}}, if θQ2 is fixed as another value (unknown either), as shown in Figure 3e. Because the SOP rotation due to a QWP or an HWP applies equally to all points comprising an SOP trace, the shape of the SOP trace created by PMF 1 remains the same after passage through HWP 2 or QWP 2, and thus the trajectory shape of the SOP out of QWP 2 or PMF 1 is equal to that of the SE_{in,GTF1} trace. In terms of the SOP out of PMF 1, however, pT, the normal vector of the trace, coincides with the S1 axis on the Poincaré sphere as the orientation angle θp1 of PMF 1 is 0°, as shown in Figure 3e. Then, θH2 and θQ2 can be determined by finding the SOP out trace of PMF 1 to satisfy this constraint for pT.

After both θH2 and θQ2 are elucidated, the SOP out of HWP 1, which is displayed as an SOP location on the Poincaré sphere, can simply be determined using the SOP out trace of PMF 1 and T_{PMF1^{-1}}, as shown in Figure 3f. In particular, the validity of θH2 and θQ2 determined above can be verified by checking whether this SOP in of PMF 1 is a single SOP point or not. That is to say, if θH2 and θQ2 predetermined abiding by the above constraint for pT are suitable for the WOA set at Set II_{GTF1}, the SOP in of PMF 1 should be unchanged even with variations in the wavelength from λ0 to λ0 + Δλ. As θH2 and θQ2 are found from the SE_{in,GTF1} trace at Set II_{GTF1}, this SOP in of PMF 1 (i.e., the SOP out of HWP 1) is the starting point to unveil θH1 and θQ1. By using the SOP out of HWP 1 (in Figure 3f) and T_{HWP1^{-1}}, the SOP out of QWP 1 can be obtained, if we set θH1 as a fixed value (unknown yet). Moreover, for a fixed value (unknown either) of θQ1, a corresponding SOP in of QWP 1 can be drawn out from the above SOP out of QWP 1 and T_{QWP1^{-1}}. Then, to find θH1 and θQ1, we can harness the fundamental SOP constraint that the SOP in of QWP 1 should be LHP because light emerging from port 2 of the PBS is linear horizontally polarized. Through this reverse tracing algorithm described above, (θQ1, θH1, θQ2, θH2) at Set II_{GTF1} could be found to be (108.49°, 132.67°, 201.92°, 134.71°), and it was also confirmed from additional calculations, although not suggested here, that several degenerate WOA sets could also exist for l_{GTF1} at Set II_{GTF1}. It is figured out from the SOP out trace of PMF 2 at Set II_{GTF1}, the SE_{out,GTF1} trace in Figure 3g, that O(λ0) at Set II_{GTF1} differs from O(λ0) at Set I_{GTF1}, although the shape of the SE_{out,GTF1} trace at Set II_{GTF1} remains unchanged in comparison with the shape of the SE_{out,GTF1} trace at Set I_{GTF1}. From O(λ0) at Set I_{GTF1}, O(λ0) at Set II_{GTF1} is distant by an angular displacement that corresponds to Δλ/8 along this SE_{out,GTF1} trace towards decreasing wavelength. This implies that O(λ0 + Δλ/8) at Set II_{GTF1} is the same as O(λ0) at Set I_{GTF1} and also as O(λ_{peak}) at Set II_{GTF1}, and λ_{peak} becomes λ0 + Δλ/8 at Set II_{GTF1}, resulting in a wavelength shift of Δλ/8 in l_{GTF1} (i.e., a phase modulation of 45°). Figure 3h shows two transmission spectra of I_{GTF1}, which are calculated at Sets I_{GTF1} and II_{GTF1} and displayed by blue and red solid lines, respectively. It is apparent that the transmission spectrum at Set II_{GTF1} is redshifted by Δλ/8 (= 0.1 nm) compared with that at Set I_{GTF1}. In short, for a phase modulation of η, or a wavelength tuning of (η/360°)Δλ, in GTF1, one should find a new SE_{in,GTF1} trace first, obtained by revolving the SE_{in,GTF1} trace at Set I_{GTF1} CCW by η in angle about p1 and p2. Then, one can find (θQ1, θH1, θQ2, θH2) for this new SE_{in,GTF1} trace through the WOA-searching procedures addressed above. If we pick η within 0° to 360°, I_{GTF1} can be arbitrarily phase-modulated. This arbitrary phase modulation can be utilized directly for the continuous wavelength tuning of I_{GTF1}.
4. WOA’s for Phase Modulation of GTF and Its Phase-Modulated Spectra

Here, we explore WOA sets for continuous phase modulation of another GTF $l_{GTF2}$, which is defined at another different $(\theta_{Q1}, \theta_{H1}, \theta_{Q2}, \theta_{H2}) = (105^\circ, 154^\circ, 17^\circ, 2^\circ)$ with $\theta_{P1} = 0^\circ$ and $\theta_{P2} = 22.5^\circ$, by using our reverse tracing method. Specifically, 360 sets of $(\theta_{Q1}, \theta_{H1}, \theta_{Q2}, \theta_{H2})$, where an additional phase shift of $1^\circ$–$360^\circ$ (increment: $1^\circ$) can be induced in $l_{GTF2}$, were found with our method. Figure 4a shows the 360 sets of $(\theta_{Q1}, \theta_{H1}, \theta_{Q2}, \theta_{H2})$ as functions of $\phi$ ranging from $0^\circ$ to $360^\circ$ (step: $1^\circ$), displayed by blue circles ($\theta_{Q1}$), green squares ($\theta_{H1}$), red diamonds ($\theta_{Q2}$), and violet triangles ($\theta_{H2}$), respectively, which are found at fixed values of $\theta_{P1} = 0^\circ$ and $\theta_{P2} = 22.5^\circ$. In contrast with WOA sets for continuous wavelength tuning of an analytic transmittance $I_a$ [23,24], all four WOA’s ($\theta_{Q1}, \theta_{H1}, \theta_{Q2}, \theta_{H2}$) do not take the form of simple mathematical functions of $\phi$ and have similar patterns either. Whereas $\theta_{Q1}(\phi)$ and $\theta_{H1}(\phi)$ have a period of $180^\circ$, the period of $\theta_{Q2}(\phi)$ or $\theta_{H2}(\phi)$ is $360^\circ$. $\theta_{Q1}, \theta_{H1}, \theta_{Q2},$ and $\theta_{H2}$ are bounded in the WOA ranges of $63.5^\circ$–$116.6^\circ$, $115.4^\circ$–$154.6^\circ$, $-37.2^\circ$–$37.2^\circ$, and $-18.1^\circ$–$18.1^\circ$, respectively. Because the period of $\theta_{H2}(\phi)$ is two times longer than that of $\theta_{H1}(\phi)$ or $\theta_{Q1}(\phi)$, both loci have two-fold symmetry. It is readily seen that the density of the point $(\theta_{H2}, \theta_{H1})$ or $(\theta_{H2}, \theta_{Q1})$, the interval between two locus points, is different along the locus. In terms of the locus of $(\theta_{H2}, \theta_{H1})$, the lower inner part of the locus is sparser than its other parts. This implies that the $\phi$ variation is smaller at the lower inner part of the locus for the same amounts of changes in $\theta_{H2}$ and $\theta_{H1}$. Similarly, in terms of the locus of $(\theta_{H2}, \theta_{Q1})$, the inner part of the locus is denser than its lower part, which means that the $\phi$ variation is larger at the upper part of the locus for the same amounts of changes in $\theta_{H2}$ and $\theta_{Q1}$. Next, the loci of $(\theta_{Q1}, \theta_{H1})$ and $(\theta_{Q1}, \theta_{Q2})$, displayed by blueish squares and reddish circles, are plotted with $\theta_{Q1}(\phi)$, $\theta_{H1}(\phi)$, and $\theta_{Q2}(\phi)$ in Figure 4a at $\phi$ ranges of $0^\circ$–$180^\circ$ and $0^\circ$–$360^\circ$, respectively, as shown in Figure 4c. In terms of the locus of $(\theta_{Q1}, \theta_{H1})$, the point of $(\theta_{Q1}, \theta_{H1})$ makes a round along this elliptical locus from $(105^\circ, 154^\circ)$ in the CW direction, while $\phi$ increases from $0^\circ$ to $180^\circ$. It can be seen that the right side of this elliptical locus is much denser than its left side, which suggests that an equal displacement of the point $(\theta_{Q1}, \theta_{H1})$ on the locus results in a relatively greater change in $\phi$ at the right side. Even if the $(\theta_{Q1}, \theta_{H1})$ locus is not represented using analytic mathematical expressions as suggested in the previous work [23], the desirable phase $\phi$, i.e., the desirable transmission dip (or peak) wavelength, can be set and be continuously changed by selecting and shifting $(\theta_{Q1}, \theta_{H1})$ on this locus, respectively, while letting $\theta_{Q2}$ and $\theta_{H2}$ follow the WOA curves of $\theta_{Q2}(\phi)$ and $\theta_{H2}(\phi)$ in Figure 4a, respectively. In terms of the locus of $(\theta_{Q1}, \theta_{Q2})$, the point of $(\theta_{Q1}, \theta_{Q2})$ makes a closed parabolic trace for $\phi$ increasing from $0^\circ$ to $360^\circ$. Like the $(\theta_{H2}, \theta_{H1})$ and $(\theta_{H2}, \theta_{Q1})$ loci in Figure 4b, this $(\theta_{Q1}, \theta_{Q2})$ locus also has a two-fold symmetry with respect to the $\theta_{Q1}$ axis because $\theta_{Q2}(\phi)$ has a period two times longer than $\theta_{Q1}(\phi)$. Moreover, the density difference on the locus is analogous to that of the $(\theta_{Q1}, \theta_{H1})$ locus. Figure 4d shows the loci of $(\theta_{Q2}, \theta_{H1})$ and $(\theta_{Q2}, \theta_{H2})$, displayed by blueish squares and reddish circles, respectively. $\theta_{Q2}(\phi)$, $\theta_{H1}(\phi)$, and $\theta_{H2}(\phi)$ in Figure 4a are utilized to plot these loci at a $\phi$ range of $0^\circ$–$360^\circ$. $(\theta_{Q2}, \theta_{H1})$ forms a locus shaped like inverted Spiderman’s eyes with increasing $\phi$ (from $0^\circ$ to $360^\circ$), which also has a two-fold symmetry with respect to the $\theta_{H1}$ axis owing to the period of $\theta_{Q2}(\phi)$ two times longer than that of $\theta_{H1}(\phi)$. It can also be found that the lower outer part of the $(\theta_{Q2}, \theta_{H1})$ locus is sparser than its other parts. The locus of $(\theta_{Q2}, \theta_{H2})$ forms a parallellogram-shaped trace, which makes one CW revolution along the locus, starting at $(17^\circ, 2^\circ)$, while $\phi$ varies from $0^\circ$ to $360^\circ$. The left and right sides of the $(\theta_{Q2}, \theta_{H2})$ locus are slightly sparser than its other parts. This indicates that an identical displacement of the point $(\theta_{Q2}, \theta_{H2})$ along the locus causes a relatively smaller modification of $\phi$ at these parts of the locus. In sum, it is confirmed from Figure 4 that one can always find a WOA set $(\theta_{Q1}, \theta_{H1}, \theta_{Q2}, \theta_{H2})$ with respect to any $\phi$ (increasing from $0^\circ$ to $360^\circ$ by $1^\circ$). This means
that the phase of \( t_{GTF2} \) can be continuously modulated, also indicating that \( t_{GTF2} \) can be continuously frequency-tuned. As long as an initial WOA set where any GTF is obtained is given, one can always find a WOA set inducing an arbitrary phase in this GTF.

To examine if the WOA sets found by our angle-finding method can embody the predicted continuous phase modulation of \( t_{GTF2} \), the transmission spectra of \( t_{GTF2} \) were calculated at eight WOA sets, which were chosen from the WOA sets found above for the phase modulation of \( t_{GTF2} \), with \( \theta_{P1} \) and \( \theta_{P2} \) set at 0° and 22.5°, respectively. The chosen WOA sets, denoted by Sets I_{GTF2}–VIII_{GTF2}, enable \( \phi \)'s to be chosen as 0°, 45°, 90°, 135°, 180°, 225°, 270°, and 315°, respectively. The transmission spectra of the GTF \( t_{GTF2} \), shown in Figure 5, were calculated at the eight WOA sets (Sets I_{GTF2}–VIII_{GTF2}) in a wavelength range of 1548–1552 nm. For the calculation of the transmission spectrum, the length \( L \) and birefringence \( B \) of PMF (PMF 1 or PMF 2) were set as 7.2 m and 4.166 \( \times 10^{-4} \), respectively, so that \( \Delta \lambda \) became ~0.8 nm at 1550 nm. It can be seen from the figure that the first-order comb spectrum of deformed narrow passbands makes a redshift as \( \phi \) is modulated as 0° to 315° (increment: 45°). If the peak wavelength of one passband is designated as \( \lambda_{\text{peak},\text{GTF}} \) at Set I_{GTF2} (\( \phi = 0° \)), one step transition in the WOA set (e.g., from Set II_{GTF2} to Set III_{GTF2}) leads to an increase of 45° in \( \phi \) and of 0.1 nm in \( \lambda_{\text{peak},\text{GTF}} \). Linear movement of \( \lambda_{\text{peak},\text{GTF}} \) with respect to \( \phi \), displayed as red-dotted arrows, directly tells us that the continuous or the desired arbitrary phase modulation of \( t_{GTF2} \) can be realized. These calculation results clearly manifest that \( t_{GTF2} \) can be continuously wavelength-shifted within \( \Delta \lambda \) using the WOA sets shown in Figure 4, implying the arbitrary phase modulation capability of \( t_{GTF2} \) within 360°.

Figure 4. (a) Four WOA’s \( \theta_{Q1} \) (blue circles), \( \theta_{H1} \) (green squares), \( \theta_{Q2} \) (red diamonds), and \( \theta_{H2} \) (violet triangles) as a function of extra phase difference \( \phi \) (from 0° to 360° with a step of 1°) for phase modulation of another GTF \( t_{GTF2} \) at \( \theta_{P1} = 0° \) and \( \theta_{P2} = 22.5° \). (b) Loci of \((\theta_{Q2}, \theta_{H1})\) and \((\theta_{H2}, \theta_{Q1})\), displayed by blueish squares and reddish circles, respectively. (c) Loci of \((\theta_{Q1}, \theta_{H1})\) and \((\theta_{Q2}, \theta_{H2})\), displayed by blueish squares and reddish circles, respectively. (d) Loci of \((\theta_{Q2}, \theta_{H1})\) and \((\theta_{Q2}, \theta_{H2})\), displayed by blueish squares and reddish circles, respectively.
within $\Delta \lambda$ using the WOA sets shown in Figure 4, implying the arbitrary phase modulation capability of $t_{\text{GTF}2}$ through experiments, our filter was manufactured with a fiber-pigtailed PBS (OZ Optics) with four ports, two fiber-pigtailed QWPs (OZ Optics), two fiber-pigtailed HWPs (OZ Optics), and two ~7.12 m-long segments of bow-tie PMF (Fibercore) with a birefringence of $-4.166 \times 10^{-4}$, as shown in Figure 1a. Figure 6 shows an actual experimental setup for measuring the transmission spectrum of the constructed filter. As shown in Figure 6, we employed a broadband light source (BLS, Fiberlabs FL7701) covering S, C, and L bands and an optical spectrum analyzer (OSA, Yokogawa AQ6370C) to monitor the transmission spectra of the constructed filter. The input and output ports (ports 1 and 4) of the filter were connected to the BLS and OSA, respectively, with optical fiber patchcords (FC/PC type). For acquisition of high resolution and contrast optical spectra, we set the sensitivity and resolution bandwidth of the OSA as HIGH1 and 0.05 nm, respectively. To avoid unwanted position changes of all the optical components comprising the filter, we taped them up on the optical table so that they were immobilized during the spectrum measurement.

The transmission spectra of the GTF $t_{\text{GTF}2}$, shown in Figure 7, were measured at the eight WOA sets (Sets I$_{\text{GTF}2}$–VIII$_{\text{GTF}2}$) in a wavelength range of 1548–1552 nm. From the measured spectra, the FSR of the constructed filter was evaluated as ~0.8 nm around 1550 nm. This FSR is determined by $L$ (~7.12 m) and $B$ (~4.166 $\times 10^{-4}$) of PMF used here and increases with wavelength. As confirmed from the theoretical spectra in Figure 5, while the WOA set is switched from Set I$_{\text{GTF}2}$ ($\phi = 0^\circ$) to Set VIII$_{\text{GTF}2}$ ($\phi = 315^\circ$), the comb spectrum of deformed narrow passbands, which seems to be analogous to the comb spectrum shown in Figure 5, shifts towards a longer wavelength region step by step by ~0.1 nm, resulting in a total wavelength displacement of ~0.7 nm. The inset shows the variation of $\lambda_{\text{peak,GTF}}$ (indicated as a red-dotted arrow) according to eight values of $\phi$, i.e., $0^\circ$, $45^\circ$, $90^\circ$, $135^\circ$, $180^\circ$, $225^\circ$, $270^\circ$, and $315^\circ$, which are obtained at Sets I$_{\text{GTF}2}$, II$_{\text{GTF}2}$, III$_{\text{GTF}2}$, IV$_{\text{GTF}2}$, V$_{\text{GTF}2}$, VI$_{\text{GTF}2}$, VII$_{\text{GTF}2}$, and VIII$_{\text{GTF}2}$, respectively. Blue squares designated as I$_{\text{GTF}2}$–VIII$_{\text{GTF}2}$ indicate the $\lambda_{\text{peak,GTF}}$ positions of transmission spectra obtained at eight Sets I$_{\text{GTF}2}$–VIII$_{\text{GTF}2}$. It can be figured out from the inset that

![Figure 5](image-url)
\( \lambda_{\text{peak,GTF}} \) very linearly increases with \( \phi \) showing an adjusted \( R^2 \) value of \( \sim 0.99946 \) and the tuning of the spectrum wavelength can be mediated by the phase modulation. In particular, through additional experiments, continuous frequency tuning capability was confirmed with respect to numerous \( \phi \) values arbitrarily selected from 0° to 360° except integer multiples of 45° as well. Thus, it is experimentally verified that the GTF \( t_{\text{GTF2}} \) can be arbitrarily phase-modulated by properly controlling \((\theta_{Q1}, \theta_{H1}, \theta_{Q2}, \theta_{H2})\), specifically, appropriately selecting \((\theta_{Q1}, \theta_{H1}, \theta_{Q2}, \theta_{H2})\) that satisfies the WOA sets suggested in Figure 4. Ultimately, we conclude that any GTF generated in the proposed comb filter can be phase-modulated, or wavelength-tuned, continuously by taking advantage of WOA sets that can always be drawn out with our reverse tracing algorithm.

5. Experimental Verification of Phase-Modulated Spectra

In order to verify the predicted phase modulation capability of \( t_{\text{GTF2}} \) through experiments, our filter was manufactured with a fiber-pigtailed PBS (OZ Optics) with four ports, two fiber-pigtailed QWPs (OZ Optics), two fiber-pigtailed HWPs (OZ Optics), and two ~7.12 m-long segments of bow-tie PMF (Fibercore) with a birefringence of \( \sim 4.166 \times 10^{-4} \), as shown in Figure 1a. Figure 6 shows an actual experimental setup for measuring the transmission spectrum of the constructed filter. As shown in Figure 6, we employed a broadband light source (BLS, Fiberlabs FL7701) covering S, C, and L bands and an optical spectrum analyzer (OSA, Yokogawa AQ6370C) to monitor the transmission spectra of the constructed filter. The input and output ports (ports 1 and 4) of the filter were connected to the BLS and OSA, respectively, with optical fiber patchcords (FC/PC type). For acquisition of high resolution and contrast optical spectra, we set the sensitivity and resolution bandwidth of the OSA as HIGH1 and 0.05 nm, respectively. To avoid unwanted position changes of all the optical components comprising the filter, we taped them up on the optical table so that they were immobilized during the spectrum measurement.

![Figure 6. Actual experimental setup for measurement of phase-modulated transmission spectra of GTF (\( t_{\text{GTF2}} \)).](image-url)
We demonstrated the arbitrary phase modulation of a GTF, which could be obtained in the first-order fiber comb filter based on the PDLS. The proposed filter was composed of a PBS, two OWS’s of an HWP and a QWP, and two PMF segments. Each OWS was located before each PMF segment, and the second PMF segment was butt-coupled to one port of the PBS so that its principal axis should be 22.5° away from the TM polarization axis of the PBS. Basically, polarization conditions, which should be satisfied to modify the phase of a GTF as a desired value, were explained using the spectral evolution of the SOP\textsubscript{in} and SOP\textsubscript{out} of the second PMF in the narrowband transmittance function (t\textsubscript{GTF}), on the basis of the continuous wavelength tuning mechanism of previously reported PDLS-based first-order narrowband comb filters. Then, we explained a systematic scheme to find four WOA’s for the arbitrary phase modulation of a GTF using the effect of each component of the filter, such as a waveplate or PMF segment, on its SOP\textsubscript{in} or SOP\textsubscript{out}. By exploiting this WOA finding method, we derived WOA sets of the four waveplates, which could give arbitrary phase delays ϕ’s from 0° to 360° to a GTF chosen here arbitrarily. Finally, we showed phase-modulated GTF’s calculated at eight selected WOA sets allowing ϕ’s to be 0°, 45°, 90°, 135°, 180°, 225°, 270°, and 315°, and then the theoretically predicted results were experimentally verified. It is concluded from the theoretical and experimental demonstrations that the GTF of our filter based on the OWS of a QWP and an HWP can be arbitrarily phase-modulated by appropriately controlling the WOA’s of the four waveplates. Our demonstration is expected to be beneficial to fiber-optic applications, which demand frequency-tunable comb filters with specific transmittances, such as microwave photonic signal processing, optical sensor interrogation, and multiwavelength lasing.

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**Figure 7.** Experimental phase-modulated transmission spectra of GTF (t\textsubscript{GTF}), measured at eight WOA sets (Sets IGTF\textsubscript{2}–VIIIGTF\textsubscript{2}).

6. Conclusions

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