Mass formulae and natural hierarchy in string effective supergravities

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Abstract

We study some conditions for the hierarchy $m_{3/2} \ll M_P$ to occur naturally in a generic effective supergravity theory. Absence of fine-tuning and perturbative calculability require that the effective potential has a sliding gravitino mass and vanishing cosmological constant, up to $O(m_{3/2}^4)$ corrections. In particular, cancellation of quadratically divergent contributions to the one-loop effective potential should take place, including the ‘hidden sector’ of the theory. We show that these conditions can be met in the effective supergravities derived from four-dimensional superstrings, with supersymmetry broken either at the string tree-level via compactification, or by non-perturbative effects such as gaugino condensation. A crucial role is played by some approximate scaling symmetries, which are remnants of discrete target-space dualities in the large moduli limit. We derive explicit formulae for the soft breaking terms arising from this class of ‘large hierarchy compatible’ (LHC) supergravities.
1 Introduction

If one tries to extend the validity of an effective field theory to energy scales much higher than its characteristic mass scale, and quantum corrections appear carrying positive powers of the cut-off scale $\Lambda$, one is faced with a scale hierarchy problem. The typical example is the gauge hierarchy problem \cite{1} of the Standard Model (SM) of strong and electroweak interactions, seen as a low-energy effective field theory. When the SM is extrapolated to cut-off scales $\Lambda \gg 1$ TeV, there is no symmetry protecting the mass of the elementary Higgs field, and therefore the masses of the weak gauge bosons, from large quantum corrections proportional to $\Lambda$. The most popular solution to the gauge hierarchy problem of the SM is \cite{2} to extend the latter to a model with global $N = 1$ supersymmetry, effectively broken at a scale $M_{SUSY} \lesssim 1$ TeV. These extensions of the SM, for instance \cite{3} the Minimal Supersymmetric Standard Model (MSSM), can be safely extrapolated up to cut-off scales much higher than the electroweak scale, such as the supersymmetric unification scale $M_U \sim 10^{16}$ GeV, the string scale $M_S \sim 10^{17}$ GeV, or the Planck scale $M_P \equiv G_N^{-1/2}/\sqrt{8\pi} \simeq 2.4 \times 10^{18}$ GeV.

In view of the following discussion, we would like to take a closer look at the properties that guarantee the stability of the gauge hierarchy against quantum corrections in the MSSM and its variants. Using a momentum cut-off $\Lambda$, the one-loop effective potential for a generic theory reads \cite{4}

\[ V_1 = V_0 + \frac{1}{64\pi^2} \text{Str} \mathcal{M}^0 \cdot \Lambda^4 \log \frac{\Lambda^2}{\mu^2} + \frac{1}{32\pi^2} \text{Str} \mathcal{M}^2 \cdot \Lambda^2 + \frac{1}{64\pi^2} \text{Str} \mathcal{M}^4 \log \frac{\mathcal{M}^2}{\Lambda^2} + \ldots, \]  

(1.1)

where the dots stand for $\Lambda$-independent contributions, $\mu$ is the scale parameter, and

\[ \text{Str} \mathcal{M}^n \equiv \sum_i (-1)^{2J_i}(2J_i + 1)m_i^n \]  

(1.2)

is a sum over the $n$-th power of the various field-dependent mass eigenvalues $m_i$, with weights accounting for the number of degrees of freedom and the statistics of particles of different spin $J_i$. In eq. (1.1), $V_0$ is the classical potential, which in the case of the SM (and of the MSSM) should contain mass terms at most of the order of the electroweak scale. The quantum correction to the vacuum energy with the highest degree of ultraviolet divergence is the $\Lambda^4$ term, whose coefficient $\text{Str} \mathcal{M}^0$ is always field-independent, and equal to the number of bosonic minus fermionic degrees of freedom. Being field-independent, this term can affect the discussion of the cosmological constant problem (when the theory is coupled to gravity), but does not affect the discussion of the gauge hierarchy problem. Anyway, this term is always absent in supersymmetric theories, which possess equal numbers of bosonic and fermionic degrees of freedom. The second most divergent term in eq. (1.1) is the quadratically divergent contribution, proportional to $\text{Str} \mathcal{M}^2$. In the SM, $\text{Str} \mathcal{M}^2$ depends on the Higgs field, and induces a quadratically divergent contribution to the Higgs squared mass, the well-known source of the gauge hierarchy problem. An early attempt to get rid of the quadratically divergent one-loop contributions to the SM Higgs
squared mass consisted in imposing the mass relation $[(\partial^2/\partial\varphi^2)\text{Str} \mathcal{M}^2(\varphi)]_{\varphi=v} = 0$; neglecting the light fermion masses, this amounts to requiring $3m_H^2 + 6m_W^2 + 3m_Z^2 - 12m_t^2 = 0$. It is clear (for recent discussions, see e.g. [6] and references therein) that such a requirement is modified at higher orders in perturbation theory, since it amounts to a relation among the dimensionless couplings of the SM that is not stable under the renormalization group. A more satisfactory solution of the problem is provided by $N = 1$ global supersymmetry. For unbroken $N = 1$ global supersymmetry, $\text{Str} \mathcal{M}_n$ is identically vanishing for any $n$, due to the fermion-boson degeneracy within supersymmetric multiplets [7]. The vanishing of $\text{Str} \mathcal{M}_2$ persists, as a field identity, if global supersymmetry is spontaneously broken in the absence of anomalous $U(1)$ factors [8]. Indeed, to keep the gauge hierarchy stable it is sufficient that supersymmetry breaking does not reintroduce field-dependent quadratically divergent contributions to the vacuum energy. This still allows for a harmless, field-independent quadratically divergent contribution to the effective potential, and is actually used to classify the so-called soft supersymmetry-breaking terms [9]. In the case of softly broken supersymmetry, the $A^2$ term of eq. (1.1) only contributes to the cosmological constant. With a typical mass splitting $M_{\text{SUSY}}$ within the MSSM supermultiplets, the logarithmic term in eq. (1.1) induces corrections to the Higgs mass terms (before minimization), which are at most $O(M_{\text{SUSY}}^2)$: the hierarchy is then stable if $M_{\text{SUSY}} \lesssim 1$ TeV.

To go beyond the MSSM, one must move to a more fundamental theory with spontaneous supersymmetry breaking. The only possible candidate for such a theory is $N = 1$ supergravity coupled to gauge and matter fields [10], where (in contrast with the case of global supersymmetry) the spontaneous breaking of local supersymmetry is not incompatible with vanishing vacuum energy. In $N = 1$ supergravity, the spin 2 graviton has for superpartner the spin 3/2 gravitino, and the only consistent way of breaking supersymmetry is *spontaneously*, via the super-Higgs mechanism. One is then bound to interpret the MSSM as an effective low-energy theory derived from a spontaneously broken supergravity [11]. The scale of soft supersymmetry breaking in the MSSM, $M_{\text{SUSY}}$, is related (in a model-dependent way) to the gravitino mass $m_{3/2}$, which sets the scale of the spontaneous breaking of local supersymmetry. One might naively think that, whatever mechanism breaks local supersymmetry and generates the hierarchy $m_{3/2} \ll M_P$, the condition $M_{\text{SUSY}} \sim m_{3/2} \lesssim 1$ TeV $\ll M_P$ remains sufficient to guarantee the stability of such a hierarchy against quantum corrections. To explain why this expectation is generically incorrect, and to motivate the present work, we need first to review some general facts about spontaneously broken $N = 1$ supergravity.

Even barring higher-derivative terms, the general structure of $N = 1$ supergravity still allows for a large amount of arbitrariness. First, one is free to choose the field content. Besides the gravitational supermultiplet, containing as physical degrees of freedom the graviton and the gravitino, one has a number of vector supermultiplets, whose physical degrees of freedom are the spin 1 gauge bosons $A^a_\mu$ and the spin 1/2 Majorana gauginos $\lambda^a$, transforming in the adjoint representation of the chosen gauge group. One is also free to choose the number of chiral supermultiplets, whose physical degrees of freedom are spin
1/2 Weyl fermions $\chi^I$ and complex spin 0 scalars $z^I$, and their transformation properties under the gauge group. Furthermore, one has the freedom to choose a real gauge-invariant Kähler function

$$g(z, \overline{z}) = K(z, \overline{z}) + \log |w(z)|^2, \quad (1.3)$$

where $K$ is the Kähler potential whose second derivatives determine the kinetic terms for the fields in the chiral supermultiplets, and $w$ is the (analytic) superpotential. One can also choose a second (analytic) function $f_{ab}(z)$, transforming as a symmetric product of adjoint representations of the gauge group, which determines the kinetic terms for the fields in the vector supermultiplets, and in particular the gauge coupling constants and axionic couplings,

$$g_{ab}^{-2} = \text{Re} f_{ab}, \quad \theta_{ab} = \text{Im} f_{ab}. \quad (1.4)$$

Once the functions $G$ and $f$ are given, the full supergravity Lagrangian is specified. In particular (using here and in the following the standard supergravity mass units in which $M_P = 1$), the classical scalar potential reads

$$V = V_F + V_D = e^G \left( G' G_I - 3 \right) + \frac{[(\text{Re} f)^{-1}]_{ab}}{2} \left( G_I T_a \overline{T}^j \right) \left( z^L T_a \overline{T}^j \right), \quad (1.5)$$

In our notation, repeated indices are summed, unless otherwise stated; we use Hermitian generators, $[(T_a)^I J]_J = T_a^J \overline{T}^J$; derivatives of the Kähler function are denoted by $\partial G/\partial z^I \equiv \partial_I G \equiv G_I$, and $\partial G/\partial \overline{z}^I \equiv \partial_I G \equiv G_I$, and the Kähler metric is $G_{IJ} = G_I J = K_{IJ} = K_{IJ}$. The inverse Kähler metric $G^{IJ}$, such that $G^{IJ} G_{JK} = \delta^I_K$, can be used to define

$$G^I \equiv G^{I\overline{J}} G_{\overline{J} \overline{I}}, \quad G^I \equiv G_J G^{J I}. \quad (1.6)$$

Notice that the $D$-term part of the scalar potential is always positive semi-definite, $V_D \geq 0$, as in global supersymmetry. However, in contrast with global supersymmetry, the $F$-term part of the scalar potential is not positive semi-definite in general. On the one hand, this allows for spontaneous supersymmetry breaking with vanishing classical vacuum energy, as required by consistency with a flat background. On the other hand, the requirement of vanishing vacuum energy imposes a non-trivial constraint on the structure of the theory,

$$\langle G' G_I \rangle = 3 \quad \text{if} \quad \langle V_D \rangle = 0. \quad (1.7)$$

The order parameter of local supersymmetry breaking in flat space is the gravitino mass,

$$m_{3/2}^2(z, \overline{z}) = e^G(z, \overline{z}) = |w(z)|^2 e^K(z, \overline{z}), \quad (1.8)$$

which depends on the vacuum expectation values of the scalar fields of the theory, determined in turn by the condition of minimum vacuum energy. The goldstino $\tilde{\eta}$ is given by

$$\tilde{\eta} = e^{G_I} G_I \chi^I + \frac{1}{2} G_I T_a^J \overline{T}^j \chi^a. \quad (1.9)$$
On the right-hand side of eq. (1.9), the two contributions are associated with $F$- and $D$-term breaking, respectively. In the following, we shall assume that the $D$ breaking is absent at tree level, as is the case in all interesting situations. For convenience, we shall also classify the fields as $z^I \equiv (z^\alpha, z^i)$, where the fields with Greek indices have non-vanishing projections along the goldstino direction, $\langle G^\alpha G_\alpha \rangle \neq 0$ (not summed), whereas the fields with small Latin indices have vanishing goldstino projection, $\langle G^i G_i \rangle = 0$. With these conventions, eq. (1.7) can be split as

$$\langle G^\alpha G_\alpha \rangle = 3, \quad \langle G^i G_i \rangle = 0,$$

and the goldstino just reads

$$\tilde{\eta} = e^{2\overline{G} I} \chi^I = e^{2\overline{G}_\alpha} \chi^\alpha.$$  (1.11)

If $N = 1$ local supersymmetry is spontaneously broken on a flat background, the coefficient of the one-loop quadratically divergent contributions to the vacuum energy is given by

$$\text{Str} \mathcal{M}^2(z, \overline{z}) = 2Q(z, \overline{z}) m^2_{3/2}(z, \overline{z}),$$  (1.12)

where

$$Q(z, \overline{z}) = N_{\text{TOT}} - G^i(z, \overline{z}) H_{i\overline{J}}(z, \overline{z}) \overline{G}^\overline{J}(z, \overline{z}),$$  (1.13)

$$H_{i\overline{J}}(z, \overline{z}) = R_{i\overline{J}}(z, \overline{z}) + F_{i\overline{J}}(z, \overline{z}),$$  (1.14)

$$R_{i\overline{J}}(z, \overline{z}) \equiv \partial_i \partial_{\overline{J}} \log \det G_{MN}(z, \overline{z}),$$  (1.15)

$$F_{i\overline{J}}(z, \overline{z}) \equiv -\partial_i \partial_{\overline{J}} \log \det \text{Re} [f_{ab}(z)].$$  (1.16)

Clearly, the only non-vanishing contributions to $\text{Str} \mathcal{M}^2$ come from the field directions $z^\alpha$ for which $\langle G^\alpha G_\alpha \rangle \neq 0$ (not summed). In eq. (1.13), $R_{i\overline{J}}$ is the Ricci tensor of the Kähler manifold for the chiral multiplets, whose total number is denoted by $N_{\text{TOT}}$. In eq. (1.14), $F_{i\overline{J}}$ has also a geometrical interpretation, since the way it is constructed from the gauge field metric is very similar to the way $R_{i\overline{J}}$ is constructed from the Kähler metric. It is important to observe that both $R_{i\overline{J}}$ and $F_{i\overline{J}}$ do not depend at all on the superpotential of the theory, but only depend on the metrics for the chiral and gauge superfields. This very fact allows for the possibility that, for special geometrical properties of these two metrics, the dimensionless quantity $Q(z, \overline{z})$ may turn out to be field-independent and hopefully vanishing.

In a general spontaneously broken $N = 1$ supergravity, the non-vanishing of $Q(z, \overline{z})$ induces, at the one-loop level, a contribution to the vacuum energy quadratic in the cut-off $\Lambda$. This leads to a very uncomfortable situation, not only in relation with the cosmological constant problem (a vacuum energy of order $m^2_{3/2} \Lambda^2$ cannot be cancelled by any physics

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\(^1\)We recall that, on non-flat backgrounds, stable vacuum states do not necessarily correspond to minima of the vacuum energy \(^2\). For the calculation of quadratic divergences on arbitrary backgrounds, see ref. \(^3\).
at lower energy scales) but also in relation with the gauge hierarchy problem, which asks for a gravitino mass not much larger than the electroweak scale. Since \( m_{3/2}(z, \bar{z}) \) is a field-dependent object, and its expectation value must arise from minimizing the vacuum energy, quadratically divergent loop corrections to the latter may generically destabilize [13] the desired hierarchy \( m_{3/2} \ll \Lambda \), attracting the gravitino mass either to \( m_{3/2} = 0 \) (unbroken supersymmetry) or to \( m_{3/2} \sim \Lambda \) (no hierarchy). This destabilization problem cannot be solved just by moving from the cut-off-regulated supergravity to the quantum supergravity defined by four-dimensional superstrings [16], since the only practical difference will be to replace the cut-off scale \( \Lambda \) by an effective scale of order \( M_S \). In a generic supergravity theory, we still have the freedom to evade this problem, by postulating the existence of an extra sector of the theory, which gives an opposite contribution to \( Q \), so that \( Q + \Delta Q = 0 \). Such a request, however, is very unnatural, and implies a severe fine-tuning among the parameters of the old theory and of the extra sector. In string-derived supergravities, the possibility of such a cheap way out is lost, since all the degrees of freedom of the theory are known and the total contribution to \( Q \) is well defined. We no longer have the freedom to compensate a non-zero \( Q \) by modifying the theory!

From the previous discussion, it is clear that a satisfactory solution of the hierarchy problem (\( m_{3/2} \ll M_P \)), and the perturbative stability of the flat background, at least up to \( \mathcal{O}(m_{3/2}^2) \) corrections, require the vanishing of \( Q(z, \bar{z}) \). It is also clear that, if such a solution exists, this will put strong constraints on the scalar and gauge metrics, see eqs. (1.12)–(1.16). In order to appreciate the geometrical meaning of the vanishing of \( Q(z, \bar{z}) \), we present here a simple working example (another, string-motivated example was previously given in [17]). Consider a model containing \( N_{TOT} \equiv N_c + 3 \) chiral superfields, three gauge singlets \((T, U, S)\) and \( N_c \) charged fields \( C_i \) \((i = 1, \ldots, N_c)\), with a gauge kinetic function given by \( f_{ab} = \delta_{ab} S \), a Kähler function parametrizing a \( SU(1, N_c + 1)/[U(1) \times SU(N_c + 1)] \times SU(1,1)/U(1) \times SU(1,1)/U(1) \) manifold,

\[
\mathcal{G} = -3 \log(T + \overline{T} - C_i \overline{C}_i) - k \log(U + \overline{U}) - \log(S + \overline{S}) + \log |w(C, U, S)|^2 , \tag{1.17}
\]

and a superpotential \( w(C, U, S) \), which depends non-trivially on all fields apart from the singlet field \( T \). One can easily prove that, thanks to the field identity \( \mathcal{G}^T \mathcal{G}_T \equiv K^T K_T \equiv 3 \), the scalar potential of such a model is automatically positive semi-definite, with a flat direction along the \( T \)-field, as in the ‘no-scale’ models of refs. [18,19]. At the minima that preserve the gauge symmetry, \( \mathcal{G}_S = \mathcal{G}_U = \mathcal{G}_C = 0 \), whereas \( \mathcal{G}_T \neq 0 \). The gauge coupling constant at the minimum is fixed to the value \( g^2 = (\text{Re} S)^{-1} \), and the VEV of the \( U \) field is also fixed by the minimization condition, whereas the gravitino mass \( m_{3/2}^2 = |w|^2 / [(S + \overline{S})(T + \overline{T})^3(U + \overline{U})] \) is classically undetermined, sliding along the \( T \) flat direction. To compute \( Q(z, \bar{z}) \) in this model, it is sufficient to realize that the Ricci

\footnote{We shall comment later on the possible contributions coming from string modes that remain massive in the limit of unbroken supersymmetry, and therefore do not appear in the effective theory below the string scale.}
tensors for the three factor manifolds have the simple expressions ($I = 0, 1, \ldots, N_c$):

$$R_{IJ} = \frac{N_c + 2}{3} G_{IJ}, \quad R_{S\bar{S}} = 2 G_{S\bar{S}}, \quad R_{U\bar{U}} = \frac{2}{k} G_{U\bar{U}}. \quad (1.18)$$

By just applying eqs. (1.13)–(1.16) to the present case, we find

$$Q = (N_c + 3) - 1 - G^T R T G^T = N_c + 2 - \frac{N_c + 2}{3} G^T G_T = 0. \quad (1.19)$$

In this simple example, we can clearly see that the vanishing of $Q(z, \bar{z})$ occurs at all minima of the potential along the flat direction $T$, and is completely independent of the details of the superpotential.

In the present work, we discuss how the previously mentioned conditions for the hierarchies $m_{3/2} \ll M_P$ and $\langle V \rangle \lesssim \mathcal{O}(m_{3/2}^4)$ can be realized in a generic effective supergravity theory. In particular, we go beyond the example of ref. [17], and identify a wide class of ‘large hierarchy compatible’ (LHC) models where, modulo $\mathcal{O}(m_{3/2}^2/M_P^2)$ corrections,

$$G^\alpha G_\alpha = 3 \quad \text{and} \quad Q = 0. \quad (1.20)$$

In section 2, we discuss the problem at the pure supergravity level, showing that the conditions of eq. (1.20) can be naturally achieved, as field identities, whenever the metrics for the gauge and chiral superfields are compatible with some (approximate) scaling properties. We also review some general mass formulae of supergravity, and explore the specific form they take in the case of the LHC models. In particular, we give explicit formulae for the resulting mass parameters at the level of the MSSM, which are subject to important restrictions and exhibit remarkable universality properties.

In section 3, we first review how the desired scaling properties naturally appear in the effective supergravity theories, which are extracted from four-dimensional superstring constructions in the low-energy limit, under the assumption that the fields $z^\alpha$, which trigger supersymmetry breaking, have large VEVs compared to the string scale. This is probably necessary in order to obtain a gravitino mass much smaller than the string scale, and is equivalent to neglecting the effects of winding modes in the low-energy effective Lagrangian. In the same limit, one recovers an approximate Peccei-Quinn symmetry for the moduli dependence of the overall scalar field metric $G_{IJ}$ of any chiral multiplet. This is consistent with the fact that this symmetry is restored when non-trivial topological effects on the world-sheet (exponentially suppressed in the large-volume limit for the internal space) can be neglected. We then apply the formulae of section 2 to a number of examples, corresponding to the effective supergravity theories of different four-dimensional superstring models (Calabi-Yau, orbifolds, fermionic constructions, \ldots), and to different mechanisms for supersymmetry breaking (coordinate-dependent compactifications, gaugino condensation, \ldots). In the final section, we critically rediscuss the interpretation of our results, in particular the role of string massive modes and higher-loop contributions to the effective potential, and describe some prospects for further work.

\textsuperscript{3} Results partially overlapping with ours were obtained in [20–22].
2 Supergravity mass formulae

2.1 General case

We assume here, as announced in the Introduction, spontaneous breaking of local $N = 1$ supersymmetry with vanishing vacuum energy and unbroken gauge symmetries (singlet goldstino). Before specializing to the case of LHC models, we would like to recall some general supergravity formulae for the boson and fermion mass matrices.

The expression for the gravitino mass, $m_{3/2}$, has already been given in eq. (1.8). The mass matrices for the spin 1/2 fermions are

\[
(M_{1/2})_{ab} = \frac{G^K f_{ab,K}}{2} m_{3/2} = \frac{G^\alpha f_{ab,\alpha}}{2} m_{3/2} \quad (2.1)
\]

for the gauginos, and, after projecting out the goldstino-gravitino mixing term, associated with the super-Higgs mechanism,

\[
(M_{1/2})_{IJ} = \left( g_{IJ} - \frac{1}{3} g_{IK} g_{LJ} + \frac{1}{3} g_{IL} g_{JK} \right) m_{3/2} \equiv \left( D_I g_J + \frac{1}{3} g_I g_J \right) m_{3/2} \quad (2.2)
\]

for the fermions in the chiral supermultiplets, where the covariant derivative $D_J$ is defined with respect to the Kähler connection $g^M_{JK} \equiv g^M g_{JK}$. The spin 0 mass matrices are

\[
(M_0^2)_{IJ} \equiv D_I D_J V = V_I V_J \quad (2.3)
\]

and

\[
(M_0^2)_{IJ} = D_I D_J V = V_I V_J - g^K_{IJ} V_K. \quad (2.4)
\]

In eqs. (2.1)–(2.4), we have written the different mass matrices as they appear in the supergravity Lagrangian: to compute the physical mass eigenvalues, one should not forget the presence of non-canonical kinetic terms, and rescale these expressions by the appropriate powers of the gauge and Kähler metrics, $[(\text{Re } f)^{-1/2}]^{cd}$ and $(G^{-1/2})^{K\overline{L}}$ for each gauge and chiral index, respectively. In the following, it will be useful to consider the fermion squared mass matrices,

\[
\left( M_{1/2} M_{1/2}^\dagger \right)_{ab} \equiv (M_{1/2})_{ac} [(\text{Re } f)^{-1}]^{cd} (M_{1/2}^\dagger)_{db}, \quad (2.5)
\]

\[
\left( M_{1/2} M_{1/2}^\dagger \right)_{IJ} \equiv (M_{1/2})_{IK} g^{K\overline{L}} (M_{1/2}^\dagger)_{\overline{L}J}. \quad (2.6)
\]

Remembering the general expression of the scalar potential, eq. (1.8), and the condition for a minimum with vanishing vacuum energy,

\[
V = V_I = 0 \quad (2.7)
\]

we can reexpress the spin 0 mass matrices as

\[
(M_0^2)_{IJ} = \left( M_{1/2} M_{1/2}^\dagger \right)_{IJ} + \left( g_{IJ} - g_K R^K_{IJL} g^L + \frac{1}{3} g_I g_J \right) m_{3/2}^2, \quad (2.8)
\]
and
\[(M^2_0)_{IJ} = V_{IJ} = [(\mathcal{G}^K D_K + 2)(\tilde{M}_{1/2})_{IJ}] m_{3/2}, \quad (2.9)\]
where \(R^K_{IL} = \partial_I G^K_{IL}\) and \((\tilde{M}_{1/2})_{IJ} = (M_{1/2})_{IJ}/m_{3/2}\). Incidentally, we can notice that eq. (2.7) implies \(G_I (M_{1/2})_{IJ} m_{3/2} = V_L - G_L V = 0, \quad (2.10)\)
consistently with the fact that the fermion mass matrix must have a vanishing eigenvalue with eigenvector in the goldstino direction.

Before concluding this section, we would like to give more explicit formulae for the mass terms in the sector of the theory corresponding to the fields \(z^i\), for which \(\langle \mathcal{G}^i \rangle = \langle \mathcal{G}_i \rangle = 0\). In realistic supergravity models satisfying our assumptions, such a sector should include the chiral superfields of the MSSM. One can easily find
\[(M_{1/2})_{ij} = \left( G_{ij} - G_{ij} \tilde{G}^{T} \right) m_{3/2}, \quad (2.11)\]
\[(M^2_0)_{ij} = \left( M_{1/2} M^{\dagger}_{1/2} \right)_{ij} + \left( G_{ij} - G_{ij} R_{i\bar{j}} \tilde{G}^{\bar{j}} \right) m_{3/2}, \quad (2.12)\]
\[(M^2_0)_{ij} = V_{ij} = \left[ (G^a D_a + 2)(\tilde{M}_{1/2})_{ij} \right] m_{3/2}^2, \quad (2.13)\]
In the sector under consideration, one can obtain a particularly simple expression also for
\[D_i D_j D_k V = V_{ijk} = \left[ (G^a D_a + 3) \hat{V} \right] m_{3/2}^2, \quad (2.14)\]
where \(\hat{V} \equiv D_i D_j D_k (G^L G_L) = w_{ijk}. \quad (2.15)\)

### 2.2 LHC models

We begin this section by considering a restricted theory with only the \(n\) fields \(z^\alpha\), and a Kähler function of the form
\[G(r^\alpha) = -\log Y(r^\alpha) + \log |w|^2, \quad (2.16)\]
where \(Y\) is a homogeneous function of degree \(p\), depending only on the combinations
\[r^\alpha \equiv z^\alpha + \overline{z}^\alpha = 2 \text{Re } z^\alpha, \quad (2.17)\]
and \(w\) is assumed not to depend on \(z^\alpha\), \(\partial w/\partial z^\alpha = 0\). In other words, we shall assume from now on that
\[r^\alpha Y_\alpha = p Y, \quad (2.18)\]
where it is unambiguous to define \(Y_\alpha \equiv \partial Y/\partial r^\alpha \equiv \partial Y/\partial z^\alpha \equiv \partial Y/\partial \overline{z}^\alpha\). From eq. (2.18), it immediately follows that the Kähler metric for the fields \(z^\alpha\) is a homogeneous function of degree \((-2)\),
\[r^\alpha G_{\alpha\beta\gamma} = -2 G_{\alpha\beta}, \quad (2.19)\]
and that
\begin{align}
G^\alpha &= -r^\alpha, \\
G^\alpha G_\alpha &= p, \\
G^\alpha R_{\alpha\beta} G^\beta &= 2n.
\end{align}

For \( p = 3 \), one obtains the structure of the ‘no-scale’ models: for a non-vanishing \( w \), supersymmetry is broken with vanishing tree-level vacuum energy, and the gravitino mass, \( m_{3/2}^2 = |w|^2/Y \), is sliding along the flat \( z^\alpha \) directions \[18\].

We now move to the full theory, containing the \( N_{TOT} \) fields \( z^I \equiv (z^\alpha, z^i) \). To obtain a simple expression for the full Ricci tensor, and inspired by the effective theories of four-dimensional superstrings, to be discussed in the following section, we assume some generic scaling (homogeneity) properties for the Kähler potential associated with the \( z^i \) fields. In a suitable parametrization, such that one can expand for small field fluctuations around \( \langle z^i \rangle = 0 \), we write the full Kähler function as
\begin{equation}
G = -\log Y(r^\alpha) + \sum_A K_{iAJ}^A (r^\alpha) z^i A \bar{z}^A + \frac{1}{2} \sum_{A,B} [P_{iAJB} (r^\alpha) z^i A z^j B + \text{h.c.}] + \log |w(z^i)|^2 + \ldots,
\end{equation}
where \( K_{iAJ}^A \) is an \( n_A \times n_A \) matrix and a homogeneous function of degree \( \lambda_A \), i.e.
\begin{equation}
r^\alpha K_{iAJ}^A \alpha = \lambda_A K_{iAJ}^A, \quad \sum_A n_A = N - n,
\end{equation}
and the dots stand for cubic or higher-order terms in the fields \( z^i \). To compute the coefficient \( Q \) of the one-loop quadratic divergences, we do not need to make any particular assumption about the form of the functions \( P_{iAJB} \). However, we shall see in the following section that, in the effective theories of four-dimensional superstrings, also the functions \( P_{iAJB} \) have scaling properties analogous to eq. \( (2.24) \),
\begin{equation}
r^\alpha P_{iAJB} \alpha = \rho_{iAJB} P_{iAJB}.
\end{equation}

The Kähler metric associated with the full Kähler function of eq. \( (2.23) \) has the form
\begin{equation}
G_{i\bar{j}} = \begin{pmatrix}
G_{\gamma\delta} \\
& \cdots \\
& K^A_{iAJ} \\
& & \cdots
\end{pmatrix},
\end{equation}
and from eqs. \( (2.24) \) and \( (2.26) \) it immediately follows that \( G^\alpha G_\alpha \equiv 3 \),
\begin{equation}
G^\alpha (\partial_\alpha \partial_\beta \log \det G_{\gamma\delta}) G^\beta = 2n, \quad G^\alpha (\partial_\alpha \partial_\beta \log \det K_{iAJ}^A ) G^\beta = -\lambda_A n_A,
\end{equation}
so that
\begin{equation}
G^I R_{\bar{i}j} G^\bar{j} = G^\alpha R_{\alpha\beta} G^\beta = 2n - \sum_A \lambda_A n_A.
\end{equation}
To include the possibility of $F_{I\bar{J}} \neq 0$, we also assume that the gauge field metric, $\text{Re} \ f_{ab}$, is a homogeneous function of the variables $r^\alpha$ of degree $\lambda_f$, i.e.

$$r^\alpha (\text{Re} \ f_{ab})_\alpha = \lambda_f \text{Re} \ f_{ab}.$$  \hfill (2.29)

Observe that, because of the analyticity of $f$, the only possible solutions to eq. (2.29) correspond to $\lambda_f = 0, 1$, and in the latter case $f_{ab}$ must be a linear function of the fields $z^\alpha$. Denoting by $d_f$ the dimension of the gauge group, we then get

$$G^I F_{I\bar{J}} G^\bar{J} = G^\alpha F_{\alpha \beta} G^\beta = \lambda_f d_f.$$  \hfill (2.30)

This allows us to rewrite the general expression for $Q$, eq. (1.13), in the final form

$$Q = \sum_A (1 + \lambda_A) n_A - n - \lambda_f d_f - 1.$$  \hfill (2.31)

From eq. (2.31) we can immediately read the contributions to $Q$ from the chiral and gauge multiplets, once their scaling weights $\lambda_A$ and $\lambda_f$ are given. For example, chiral multiplets do not contribute to $Q$ if $\lambda_A = -1$ and give a positive contribution if $\lambda_A = 0$, whereas the $z^\alpha$ multiplets ($\lambda = -2$) and the massive gauginos always give a negative contribution. We shall comment later on the possibility of chiral multiplets with $\lambda_A < -1$, which would provide additional negative contributions to $Q$. For the moment, it is important to stress again that, within our class of supergravity models with approximate scaling properties, requiring that $Q = 0$ amounts to a field-independent but highly non-trivial constraint, which couples the hidden and the observable sectors.

For the LHC models under consideration, the general supergravity mass formulae of the previous paragraph undergo dramatic simplifications, especially if one also assumes the scaling properties (2.25) for the functions $P_{iAJ}$. The (non-normalized) squared mass matrix for gauginos can be written as

$$\left( M_{1/2}^1 M_{1/2}^\dagger \right)_{ab} = \lambda_f^2 (\text{Re} \ f)_{ab} m_{3/2}^2.$$  \hfill (2.32)

Moving to the (non-normalized) mass terms for the component fields of the chiral supermultiplets, and distinguishing between the indices $\alpha$ and $i$, after some simple algebra we find

$$(M_{1/2})_{\alpha \beta} = \left( -G_{\alpha \beta} + \frac{1}{3} G_{\alpha} G_{\beta} \right) m_{3/2},$$  \hfill (2.33)

$$(M_0^2)_{\alpha \beta} = 0,$$  \hfill (2.34)

and

$$(M_{1/2})_{iAJB} = \left[ \frac{w_{iAB}}{w} + P_{iAJB} (1 + \rho_{iAJB}) \right] m_{3/2}$$  \hfill (2.35)

$$= w_{iAJB} e^{K/2} + P_{iAJB} (1 + \rho_{iAJB}) m_{3/2}.$$  \hfill (2.36)
\[ (M_0^2)_{i\lambda jB} = \left[ (2 + \lambda_A + \lambda_B) \frac{w_{i\lambda jB}}{w} + (2 + \lambda_A + \lambda_B - \rho_{i\lambda jB})(1 + \rho_{i\lambda jB})P_{i\lambda jB} \right] m_{3/2}^2 \]

\[ = (2 + \lambda_A + \lambda_B) w_{i\lambda jB} e^{K/2} m_{3/2}^2 + (2 + \lambda_A + \lambda_B - \rho_{i\lambda jB})(1 + \rho_{i\lambda jB})P_{i\lambda jB} m_{3/2}^2 \cdot \]

\[ V_{\lambda jB}^D = (3 + \lambda_A + \lambda_B + \lambda_D) \frac{w_{i\lambda jB}^D}{w} m_{3/2}^2 \]

\[ = (3 + \lambda_A + \lambda_B + \lambda_D) w_{i\lambda jB}^D e^{K/2} m_{3/2}^2 . \]

A number of important consequences can be derived from eqs. (2.32)–(2.38) already at this level. Even more stringent ones will be derived in the following section, by using additional constraints on the functions \( w, K_{i\lambda jA} \) and \( P_{i\lambda jB} \) coming from generalized target-space duality symmetries.

First, notice that the spin 0 fields \( z^a \) in the supersymmetry breaking sector have always masses \( O(m_{3/2}/M_P) \), i.e. in the \( 10^{-3} - 10^{-4} \) eV range if the gravitino mass is at the electroweak scale, with interesting astrophysical \([34]\) and cosmological \([33]\) implications, including a number of potential phenomenological problems. After subtracting the goldstino, eq. (1.11), their spin 1/2 partners \( \chi^a \) have all masses equal to the gravitino mass \( m_{3/2} \), as can be easily verified by noticing that the canonically normalized mass matrix \( M_\chi \) is real and symmetric, and satisfies \( M_\chi^2 = m_{3/2} M_\chi \), tr \( M_\chi^2 = (n - 1)m_{3/2}^2 \).

Furthermore, by remembering that the chiral superfields \( z^i \) should contain the quark, lepton and Higgs superfields of the MSSM, one can derive some predictions for the explicit mass parameters of the MSSM. Similar predictions were derived, for special goldstino directions and under slightly different assumptions, in ref. \([22]\), and we agree with these results when applicable. For the gaugino masses one finds that, if there is unification of the gauge couplings, \( (Re f)_{ab} = \delta_{ab}/g_i^2 \), then

\[ m_{1/2}^2 = \lambda_f^2 m_{3/2}^2 , \quad (\lambda_f = 0, 1) . \]

As for the spin 1/2 fermions \( \chi^i \), we should distinguish two main possibilities. Those in chiral representations of the gauge group, such as the quarks and the leptons, cannot have gauge-invariant mass terms. Those in real representations of the gauge group, such as the Higgsino fields \( \tilde{H}_1 \) and \( \tilde{H}_2 \) of the MSSM, can have both a ‘superpotential mass’, proportional to \( w_{i\lambda jB} \), and a ‘gravitational’ mass, proportional to \( P_{i\lambda jB} \), but the distinction between the two terms is not invariant under analytic field redefinitions. Both these terms can in principle contribute to the superpotential ‘\( \mu \)-term’ of the MSSM, and to the associated off-diagonal (analytic-analytic) scalar mass term \( m_\mu^2 \): we do not give here their explicit expressions, since much simpler ones will be obtained in the following section, within the effective theories of four-dimensional superstrings. We anticipate here that in these examples either the superpotential or the gravitational contribution to \( \mu \) will be present, not both. Writing then \( (M_0^2)_{i\lambda jB} = (B)_{i\lambda jB} (M_{1/2})_{i\lambda jB} \), in analogy with the MSSM notation, we shall find

\[ B_{H_1 H_2} = (2 + \lambda_{H_1} + \lambda_{H_2}) m_{3/2} , \]

\[ (2.40) \]
or

\[ B_{H_1H_2} = (2 + \lambda_{H_1} + \lambda_{H_2} - \rho_{H_1H_2})m_{3/2}, \]

respectively. Moving further to the spin 0 bosons \( z^i \) in chiral representations (squarks, sleptons, \ldots), they can only have diagonal (analytic-antianalytic) mass terms, of the form

\[ (m_0^2)_A = (1 + \lambda_A)m_{3/2}^2. \]

From the previous formula, we can see, as already observed in [22], that the scaling weights of the quark and lepton superfields must respect the inequality

\[ \lambda_A \geq -1, \]

since otherwise one would develop charge- and colour-breaking minima. Weights smaller than \((-1)\) are allowed, instead, for the Higgs fields, since in that case a negative contribution to \( m_0^2 \) can be compensated by an extra positive contribution coming from the \( \mu \)-term. Similarly, a general formula can be obtained for the coefficients of the cubic scalar couplings of the MSSM potential,

\[ (A)_{iAjbkd} = (3 + \lambda_A + \lambda_B + \lambda_D)m_{3/2}. \]

It is remarkable that the only field dependence of the soft breaking terms in eqs. (2.39)–(2.43) is via the gravitino mass \( m_{3/2} \): this fact is welcome both to fulfil the stringent constraints on soft mass terms that come from flavour-changing neutral currents [36] and to generate dynamically the hierarchy \( m_{3/2} \ll M_P \) via MSSM quantum corrections [19,37].

### 3 String examples

In this section, we apply the mass formulae obtained for the LHC supergravity models to some concrete examples, corresponding to the effective theories of different four-dimensional superstring models, and to different possible mechanisms for spontaneous supersymmetry breaking. Our purpose is twofold: we want not only to illustrate the previous results on a number of representative cases, but also to justify our assumptions, which at the pure supergravity level might appear plausible but not really compulsory.

We have already stressed that the structure of a generic \( N = 1 \) supergravity has a large amount of arbitrariness. Such arbitrariness is significantly reduced if one considers the particular class of theories that are obtained, in the low-energy limit, from some underlying four-dimensional superstring model. Even if there are infinitely many four-dimensional superstring vacua with unbroken \( N = 1 \) supersymmetry, the form of their low-energy effective theories is subject to important restrictions. For each of these vacua, the gauge group and the multiplet content are uniquely specified, and so are the Kähler and the gauge kinetic functions, which, as we shall describe in the following, do indeed exhibit the remarkable geometrical properties assumed in the previous section. Moreover,
as an effect of the string unification of all interactions, these theories do not contain any explicit mass parameter besides the string mass scale $M_S$, in the sense that all couplings and masses of the low-energy effective theory are associated with the VEVs of some moduli fields.

There is a vast literature concerning the effective supergravities corresponding to four-dimensional superstring models with unbroken $N = 1$ local supersymmetry, both at the classical [23, 24] and at the quantum [38] level. The typical structure that emerges is the following. The vector multiplets are fixed by the four-dimensional gauge group characterizing a given class of string solutions. As for the chiral multiplets, there is always a universal ‘dilaton-axion’ multiplet, $S$, singlet under the gauge group, which at the classical level entirely determines the gauge kinetic function,

$$f_{ab} = \delta_{ab} S.$$ (3.1)

Notice that, in the notation of eq. (2.29), $\lambda_f = 1$ if $S \in \{z^\alpha\}$, $\lambda_f = 0$ otherwise. In addition to $S$, there are in general other singlet chiral superfields, called ‘moduli’, which do not appear in the superpotential and thus correspond to classically flat directions of the scalar potential. They parametrize the size and the shape of the internal compactification manifold, and will be denoted here by the generic symbols $T$ and $U$. Finally, there are other chiral superfields, which are in general charged under the gauge group, or at least have a potential induced by some superpotential coupling: for the moment, we shall denote them with the generic symbol $C$, understanding that in realistic models this class of fields should contain the matter and Higgs fields of the MSSM.

The remarkable fact is that in the known four-dimensional string models, in the limit where the $T$ and/or $U$ moduli are large with respect to the string scale $M_S$, the Kähler manifold for the chiral superfields obeys the properties assumed in section 2, with well-defined scaling weights of the Kähler metric with respect to the real combinations of moduli fields $s \equiv (S + \bar{S})$, $t_i \equiv (T_i + \bar{T}_i)$ and $u_i \equiv (U_i + \bar{U}_i)$. As we are going to explain, these scaling weights are remnants of the target-space duality symmetries [39], which survive in the limit of large $T$ and/or $U$ moduli. More precisely, the Kähler potential can be written as

$$K = -\log Y(s, t, u) + K^{(C)}(C, \bar{C}; T, \bar{T}; U, \bar{U}).$$ (3.2)

The function $Y$ factorizes into three terms,

$$Y = Y^{(S)}(s) \cdot Y^{(T)}(t) \cdot Y^{(U)}(u),$$ (3.3)

where

$$Y^{(S)} = s,$$ (3.4)

so that

$$s Y_s = Y.$$ (3.5)

Another general feature involves the moduli $T_i$, corresponding to harmonic (1, 1) forms, associated with deformations of the Kähler class of the internal compactified space. Even
if their number is model-dependent, the fact that three of them are related to the three complex coordinates of the internal compactification manifold implies, in the limit of large $T$ moduli,

$$t_i Y_i = 3 Y.$$  \hspace{1cm} (3.6)

The moduli $U_i$ are associated with harmonic $(1,2)$ forms, correspond to deformations of the complex structure of the internal compactified space, and their existence, number and properties are more model-dependent. In general, in the limit of large $U$ moduli one can write a relation of the form

$$u_i Y_{ui} = p_U Y,$$ \hspace{1cm} (3.7)

where $p_U = 0, 1, 2, 3$ depends on the superstring model under consideration. Finally, keeping only quadratic fluctuations of the $C$ fields (sufficient to evaluate the Kähler metric and the mass terms around $C = \bar{C} = 0$), one can in general write

$$K^{(C)} = \sum_A K^A_{iAJ} (t, u) C'^A C'^{\bar{A}} + \frac{1}{2} \sum_{A,B} \left[ P_{iAJB} (t, u) C'^A C'^{JB} + \text{h.c.} \right] + \ldots,$$ \hspace{1cm} (3.8)

with generic scaling properties of the form

$$t_i \left( K^A_{iAJ} \right) = \lambda^A t_i K^A_{iAJ},$$ \hspace{1cm} (3.9)

$$u_i \left( K^A_{iAJ} \right) = \lambda_u u_i K^A_{iAJ},$$ \hspace{1cm} (3.10)

$$t_i (P_{iAJB}) = \rho_t^{iA} P_{iAJB},$$ \hspace{1cm} (3.11)

$$u_i (P_{iAJB}) = \rho_u^{iA} P_{iAJB},$$ \hspace{1cm} (3.12)

where the scaling weights of $K^A_{iAJ}$ and of $P_{iAJB}$ are now correlated

$$\rho_t^{iA} = \frac{\lambda^A + \lambda^B}{2}, \quad \rho_u^{iA} = \frac{\lambda_u^A + \lambda_u^B}{2}.$$  \hspace{1cm} (3.13)

As we shall see in the following examples, the fact of having definite values for the scaling weights $\lambda_t, \lambda_u$ will amount to significant restrictions on the possible values of the tree-level MSSM mass parameters.

The remarkable scaling properties (3.9) (3.12) follow from the discrete target-space dualities, which are symmetries of the full Kähler function $G$. Under a generic duality transformation, of the form

$$z^\alpha \rightarrow f(z^\alpha),$$ \hspace{1cm} (3.14)

the Kähler potential transforms as

$$K \rightarrow K + \phi + \bar{\phi},$$ \hspace{1cm} (3.15)

where $\phi$ is an analytic function of the moduli fields $z^\alpha$, and in particular it must be that

$$Y \rightarrow Y e^{\phi + \bar{\phi}}.$$ \hspace{1cm} (3.16)
Also, it is not restrictive for our purposes to consider the case in which the fields $C_A$ transform with a specific modular weight $\lambda_A$,

$$C_A \rightarrow e^{-\lambda_A \phi} C_A. \quad (3.17)$$

The fact that target-space duality is a symmetry then implies a definite transformation property for the superpotential,

$$w \rightarrow e^{-\phi} w, \quad (3.18)$$

which in turn puts very strong restrictions on the superpotential couplings, for example the cubic Yukawa couplings of the form $h_{iAjk} C^i A C^j B C^k D$. If $h_{iAjk} D$ is such that, in the large moduli limit, it goes to a non-vanishing constant (or, more generally, to a modular form of weight zero), then we must have

$$\lambda_A + \lambda_B + \lambda_D = 1. \quad (3.19)$$

For example, in $Z_2 \times Z_2$ orbifolds the $h_{iAjk}$ are constants, whereas in Calabi-Yau manifolds they are modular forms of weight zero, which approach a constant in the large volume limit for the associated moduli.

In the case of unbroken supersymmetry, and in the large moduli limit, the classical superpotential $w$ is independent of the $(S, T, U)$ moduli fields, and at least quadratic in the $C$ fields. From the previous scaling properties, it also follows that around $C = 0$ one can write

$$K_s K_s = 1, \quad K^h K_i = 3, \quad K^u K_u = p_U. \quad (3.20)$$

Armed with this result, we are ready to discuss spontaneous supersymmetry breaking in the superstring effective supergravities. As already explained, to have broken supersymmetry and vanishing vacuum energy one needs $w \neq 0$ and $G^I G_I = 3$ at the minima of the tree-level potential. If one takes the effective supergravities derived from the four-dimensional superstring models with unbroken supersymmetry, one consistently obtains a positive semi-definite scalar potential, admitting $C = 0$ minima with unbroken supersymmetry and vanishing vacuum energy, and flat directions along the $S$, $T$ and $U$ moduli fields. To obtain supersymmetry-breaking minima with unbroken gauge symmetries, i.e. the situation discussed in the previous section, one must then introduce a superpotential modification, which generates minima with $C = 0$, $w \neq 0$, $G^I G_I = 3$ when the summation index $I$ runs over the $(S, T, U)$ moduli, $G^I G_I = 0$ when $I$ runs over the $C$ fields. This means, however, that the superpotential modification must depend on at least some of the $(S, T, U)$ moduli, since otherwise we would get, when summing over the moduli indices, $G^I G_I = 4 + p_U$, which would make the scalar potential strictly positive-definite and thus not allow for the desired minima. As for the origin of possible superpotential modifications, we must refer to the two types of mechanisms for supersymmetry breaking considered so far in the framework of four-dimensional string models. The first one corresponds to exact tree-level string solutions, in which supersymmetry is broken via orbifold compactification. The second one is based on the assumption that supersymmetry breaking is induced by
non-perturbative phenomena, such as gaugino condensation or something else, at the level of the string effective field theory. These will be the two possibilities considered in the following examples. Before moving to the examples, we would like to present some general results for the superpotential modification, which can be obtained as a consequence of target-space duality.

In the case of non-perturbative supersymmetry breaking, in the absence of a second-quantized string formalism one can assume that, at the level of the effective supergravity, the superHiggs mechanism is induced by a superpotential modification which preserves target-space duality [40]. The relevant transformations are those acting non-trivially on the moduli fields $z^\alpha$ associated with supersymmetry breaking. If, for example, the modified superpotential has the form

$$w = w_{\text{SUSY}} + A(z^\alpha) + B_{iAJB}(z^\alpha)C^{iA}C^{jB} + \ldots,$$

(3.21)

target-space duality requires then the following transformation properties:

$$A(z^\alpha) \rightarrow A(z^\alpha)e^{-\phi}, \quad B_{iAJB}(z^\alpha) \rightarrow B_{iAJB}(z^\alpha)e^{-(\lambda_A+\lambda_B)\phi}.$$

(3.22)

Unfortunately, the form of the function $A(z^\alpha)$ cannot be uniquely fixed by the requirement that it is a modular form of weight ($-1$). However, another important constraint comes from the physical requirement that the potential must break supersymmetry and generate a vacuum energy at most $O(m_3^4/2)$ in the large moduli limit. This implies that $A(z^\alpha) \rightarrow \text{constant} \neq 0$ for $z^\alpha \rightarrow \infty$. This is not the case for the models of supersymmetry breaking with minima of the effective potential at small values of $T$, which make use of the Dedekind function $\eta(T)$ in the superpotential modification [33]: either they do not break supersymmetry or they do so with a large cosmological constant, in contradiction with the assumption of a constant flat background. In the case of the function $B_{iAJB}(z^\alpha)$, it is sufficient to assume that, in the large moduli limit, $B_{iAJB}(z^\alpha) \rightarrow \text{constant}$. For the moduli fields that are not involved in the breaking of supersymmetry, these asymptotic conditions are not necessary and can be relaxed. The requirement that $A(z^\alpha) \rightarrow \text{constant} \neq 0$ for $z^\alpha \rightarrow \infty$ defines an approximate no-scale model, with minima of the effective potential corresponding to field configurations that are far away from possible $z^\alpha \simeq O(1)$ self-dual minima with unbroken supersymmetry ($G_\alpha = 0$) and negative vacuum energy $O(M_P^2)$. Between these two classes of extrema, there may exist other extrema of the effective potential with $G_\alpha \neq 0$, but those are generically unstable and/or have non-vanishing vacuum energy [33]. As for the VEVs of the moduli fields that do not contribute to supersymmetry breaking (those with $G^iG_i = 0$), they are generically fixed to some extended symmetry points (e.g. the self-dual points).

In the string models with tree-level supersymmetry-breaking, the superpotential modifications in the large-moduli limit are fully under control, since in that case the explicit form of the one-loop string partition function is known, and one can derive the low-energy effective theory without making any assumption. As we shall see later, one obtains automatically the desired scaling properties of the kinetic terms, which in some cases can
produce a LHC model. In this class of models, the large-moduli limit is a necessity, since for small values of the moduli (close to their self-dual points) there exist Hagedorn-type instabilities, induced by some winding modes that become tachyonic in flat space-time \[^{28,30}\]. At the self-dual point there is a new stable minimum with unbroken supersymmetry and negative cosmological constant, as expected. We should stress here that the prescription for a consistent effective field theory in the region of small moduli requires the addition of extra degrees freedom, corresponding to the winding modes which can become massless or tachyonic for some values of the \(T\) and/or \(U\) moduli close to the self-dual points. In the large-moduli limit, however, we can disregard the effects of these extra states and not include them in the effective field theory. In this limit, as we shall see, the superpotential modification associated with supersymmetry breaking seems to violate target-space duality. On the other hand, \(w_{SUSY}\) and the Kähler potential maintain the same expressions as in the case of exact supersymmetry, with the desired scaling properties that can produce a LHC supergravity model.

### 3.1 String tree-level breaking

At the level of explicit four-dimensional \(N = 1\) heterotic string constructions, the only known mechanism for spontaneous supersymmetry breaking is the tree-level one, based on generalized coordinate-dependent compactifications \[^{26,30}\], which is analogous to the one proposed by Scherk and Schwarz for extended supergravity theories \[^{41}\]. This mechanism was also considered by Fayet \[^{42}\] and by Rohm \[^{43}\] in the context of \(N = 2\) extended supergravity and of type-II superstrings, respectively. In the case of \(N = 1\) chiral theories, this mechanism is inconsistent at the field theory level, whereas it can be consistently formulated in the framework of orbifold string constructions, thanks to the existence of string ‘twisted states’, which contain non-trivial chiral sectors. The effective \(N = 1\) supergravities, corresponding to superstring models where supersymmetry is spontaneously broken by this mechanism, were derived in \[^{27}\] for fermionic constructions \((Z_2 \times Z_2\) orbifolds), and can be easily generalized to a large class of orbifold models. The main features of these effective theories are the following:

1. The Kähler potential and the gauge kinetic function of the effective theory are the same as those obtained in the limit of unbroken supersymmetry, so that, up to analytic field redefinitions, supersymmetry breaking is indeed induced only by a superpotential modification.

2. For \(C = 0\), the Kähler manifold for the \(T\) and \(U\) moduli can be decomposed into the product of two factor manifolds. The first one, described by a Kähler potential \(K'\), involves one \(T\) and one \(U\) field, to be called here \(T'\) and \(U'\)

\[
K' = -\log[(T' + \bar{T}')(U' + \bar{U}')], \quad (C = 0),
\]

and the second one, described by a Kähler potential \(K''\), involves all the remaining \(T\) and \(U\) moduli.
3. The superpotential modification associated with supersymmetry breaking does not involve the fields $S$, $T'$ and $U'$, so that $G_S = K_S$, $G_{T'} = K_{T'}$, $G_{U'} = K_{U'}$. The condition $G_\alpha G_\alpha = 3$, which must be satisfied at the minima, is identically saturated by the fact that for $C = 0$ it is $G_S G_S = G_{T'} G_{T'} = G_{U'} G_{U'} = 1$. The goldstino direction is then along some linear combination of the $(S, T', U')$ fields.

4. The superpotential modification associated with supersymmetry breaking involves the fields appearing in $K''$, so that, restricting the sum over $I$ to these fields, the condition $G_I G_I = 0$ can be satisfied at all minima.

As an illustrative example, we describe here in some detail the models based on $Z_2 \times Z_2$ orbifolds, i.e. fermionic constructions. In that case the Kähler manifold is known \cite{24}, and the Kähler potential reads

$$K = K_0 + K' + K'' + K_1 + K_2 + K_3,$$

(3.24)

where

$$K_0 = - \log Y^{(S)},$$

(3.25)

$$K' = - \log Y_1,$$

(3.26)

$$K'' = - \log Y_2 - \log Y_3,$$

(3.27)

$$K_1 = \frac{z^{a_1} \bar{z}^{\overline{a_1}}}{Y_2^{1/2} Y_3^{1/2}},$$

(3.28)

$$K_2 = \frac{z^{a_2} \bar{z}^{\overline{a_2}}}{Y_1^{1/2} Y_3^{1/2}},$$

(3.29)

$$K_3 = \frac{z^{a_3} \bar{z}^{\overline{a_3}}}{Y_1^{1/2} Y_2^{1/2}},$$

(3.30)

and

$$Y_1 = (T' + \overline{T'})(U' + \overline{U'}) - (y^{i_1} + \overline{y}^{\overline{i_1}})^2,$$

(3.31)

$$Y_2 = 1 - y^{i_2} \overline{y}^{\overline{i_2}} + \frac{1}{4} (y^{i_2} y^{i_2} ) (\overline{y}^{\overline{i_2}} \overline{y}^{\overline{i_2}}),$$

(3.32)

$$Y_3 = 1 - y^{i_3} \overline{y}^{\overline{i_3}} + \frac{1}{4} (y^{i_3} y^{i_3} ) (\overline{y}^{\overline{i_3}} \overline{y}^{\overline{i_3}}).$$

(3.33)

The expressions for $K_1, K_2, K_3$ are valid only for quadratic fluctuations around $z = 0$, whereas those for $K_0, K', K''$ are valid for arbitrary fluctuations of the associated fields. Indeed, $Y_1, Y_2$ and $Y_3$ all parametrize manifolds of the $SO(2, 2 + n)/[SO(2) \times SO(2 + n)]$ type, with $n = n_{y_1}, n_{y_2}, n_{y_3}$ respectively (here $i_{1,2,3} = 1, \ldots, n_{y_1}, n_{y_2}, n_{y_3} + 2$), and in principle they can all be written in the same functional form. However, we have used here the freedom of performing analytic field redefinitions to move to a field basis where the superpotential
assumes a particularly simple form, reducing to a constant $k$ for $y = z = 0$. Omitting explicit indices to avoid too heavy a notation, in the chosen basis the modified superpotential can be formally written as

$$w = k + \mu (y_2y_2 + y_3y_3) + y_1y_2y_3 + z_1z_1y_1 + z_2z_2y_2 + z_3z_3y_3,$$

(3.34)

where (in Planck units) the constants in the superpotential can be written as

$$k = \frac{e_1 + e_2}{2}, \quad \mu = \frac{e_1 - e_2}{2},$$

(3.35)

and $(e_1, e_2)$ are two quantized charges of order unity. One can easily show [21] that the superpotential (3.34) gives rise to a positive-semi-definite potential, with an infinity of supersymmetry-breaking minima at vanishing vacuum energy. Concentrating here on the minima with $y = z = 0$, the gravitino mass is

$$m_{3/2}^2 = \frac{k^2}{(S + \overline{S})(T' + \overline{T')}(U' + \overline{U'})} = \frac{(e_1 + e_2)^2 g_U^2}{2R^2},$$

(3.36)

where $R \equiv 2(T' + \overline{T')^{1/2}(U' + \overline{U'})^{1/2}$ can be interpreted as a radius in the internal space. The right-hand side of the above formula clearly displays the so-called decompactification problem [44]: since in string models $(e_1, e_2)$ are quantized and of order unity, the internal radius must be pushed to very large values, $R^{-1} \simeq 1$ TeV, in order to have $m_{3/2} \simeq 1$ TeV. One consequence of this fact is the existence, for all the states of the spectrum with $R$-dependent masses, of an infinite tower of Kaluza-Klein excitations, with masses that are integer multiples of the gravitino mass. This fact, however, is still compatible with the present experimental data [45]. The real problem resides in the fact that, in general, one expects large threshold corrections to the gauge and Yukawa couplings, due to the contributions of the massive excitations to the corresponding beta functions [46]. In the framework of field theory, this problem has no solution. As will be discussed later, however, in the framework of string theory this problem can be avoided [29], even if no realistic string model with the desired features has been constructed yet.

Having the explicit form of the effective supergravity theory, it is easy to determine the scaling weights of the different fields with respect to $z^a \equiv (S, T', U')$,

$$\lambda_f = 1, \quad \lambda_{z_1,y_2,y_3} = 0, \quad \lambda_{z_2,z_3} = -1, \quad \lambda_{S,T',U',y_1} = -2,$$

(3.37)

and to apply eq. (2.31) to compute the value of $Q$,

$$Q = -d_f - n_{y_1} + n_{y_2} + n_{y_3} + n_{z_1}.$$

(3.38)

The previous result is extremely interesting, since it shows that $Q$ can be zero if there is a relation among the number of fields belonging to the chiral multiplets and the dimension of the gauge group. From the pure supergravity point of view, one could always arrange for an ad hoc cancellation by using the arbitrariness in the choice of the gauge group and
of the chiral multiplet content, but such a solution would appear extremely unnatural. In string models of this kind, however, this freedom is not present: in each model, one has just to compute the resulting value of \( Q \) and check whether it is zero or not.

We now apply eqs. (2.32)–(2.38) to compute the mass spectrum and comment on its most relevant features. The gaugino masses are universal and equal to the gravitino mass: in the standard notation of the MSSM, \( m_{1/2} = m_{3/2} \). As for the fields belonging to the chiral supermultiplets, it is important to observe that they have mass terms coming both from the superpotential and from the Kähler potential. Furthermore, one can observe that, in the chosen parametrization and around the minima with \( z = y = 0 \), the only analytic bilinear terms in the \( y \) and \( z \) fields appearing in the Kähler potential are those proportional to \( y_1 y_1 \), with scaling weights

\[
\rho_{y_1 y_1} = -2.
\]  

(3.39)

It is then immediate to see that, among the \( z \) scalars containing the chiral families, whose fermionic partners have all vanishing masses around \( y = z = 0 \), \( (z_2, z_3) \) have vanishing scalar masses, whereas \( z_1 \) have a universal ‘gravitational’ mass equal to the gravitino mass. Moving now to the \( y \) fields, the \( \tilde{y}_2 \) and \( \tilde{y}_3 \) fermions have superpotential masses equal to \( \mu^2 e^K = (e_1 - e_2)^2 g_0^2 / (2R^2) \), whereas the \( \tilde{y}_1 \) fermion fields and the moduli fermions \( \tilde{S}, \tilde{T}', \tilde{U}' \) have ‘gravitational’ masses equal to the gravitino mass. The \( y_1 \) scalar fields, which for \( z = 0 \) belong to the same \( SO(2, n_1 + 2) / [SO(2) \times SO(n_1 + 2)] \) manifold as the \( T' \) and \( U' \) moduli, have vanishing diagonal (analytic-antianalytic) masses, as a result of a cancellation between a negative and a positive gravitational contribution. Moreover, also the off-diagonal (analytic-analytic) masses for the \( y_1 \) scalars are vanishing for a similar cancellation. The \( (y_2, y_3) \) scalars have contributions to the diagonal masses coming both from the superpotential, \( \mu^2 e^K \), and from the Kähler potential, \( k^2 e^K \), whereas the off-diagonal contributions come only from the superpotential and are given by \( 2\mu ke^K \). For each of the two sectors \( y_2 \) and \( y_3 \), then, one has scalar mass eigenvalues given by \( (\mu - k)^2 e^K \) and \( (\mu + k)^2 e^K \). The moduli scalars \( S, T' \) and \( U' \) have all vanishing masses, associated to the classically flat directions of the potential. Finally, one can observe that the \( A \) terms, associated to the terms of the scalar potential that are cubic in the charged fields, are universal and given by \( A = 1 \).

We would like to end this paragraph with some general considerations on string models with tree-level spontaneous supersymmetry breaking, going beyond the specific orbifold example discussed above. In these models, \( Q \) is also field-independent and given by expressions similar to (3.38). Although in field theories relations among the dimension of the gauge group and the number of degrees of freedom in the different scaling sectors for the chiral superfields look in general unnatural in the absence of symmetry reasons, in string theory such relations can be a consequence of the consistency of the underlying superconformal symmetry and of the requirement of modular invariance. Indeed, there exist many four-dimensional string solutions, based on orbifold and fermionic constructions, which exhibit spontaneously broken \( \tilde{N} = 1 \) supersymmetry and a vacuum energy scaling like \( m_{3/2}^4 \).
with no contributions of order $m_{3/2}^2 M_P^2$. This statement can be explicitly verified not only at the level of the effective theory, which includes only the states that are massless in the limit of unbroken supersymmetry, but also when including the contributions of the massive string and compactification modes. This ‘miraculous’ string cancellation can be seen as a consequence of some hidden symmetries of the string spectrum, which imply some level of fermion-boson mass degeneracy also in the phase with broken supersymmetry. As will be now discussed, this reorganization of the mass spectrum is related to the properties of spontaneously broken $N = 2$ and $N = 4$ supergravities.

On orbifolds, the string partition function can be written as a sum over different sectors, with different amounts of space-time supersymmetry: (i) one $N = 4$ sector; (ii) one, two or three $N = 2$ sectors; (iii) the $N = 1$ sectors, which in realistic models must contain the chiral families. In all known string models with spontaneous supersymmetry breaking at tree level, boson-fermion mass splittings are generated in the $N = 4$ sector and in some of the $N = 2$ sectors, but there are no tree-level mass splittings in the $N = 1$ sectors. This property is due to the fact that the $N = 1$ sectors are twisted ones, and thus their spectrum cannot carry any dependence on the $T'$ and $U'$ moduli fields, associated with the size of the internal compactification manifold. In a certain class of models, the vanishing of the $m_{3/2}^2 M_P^2$ contributions to the one-loop partition function follows from the absence of $N = 2$ sectors with non-zero fermion-boson mass splittings. In that case, the only states with non-zero mass splittings are those in the $N = 4$ sector, and the absence of $m_{3/2}^2 M_P^2$ contributions to the vacuum energy (including also the contributions from the string massive states) can be understood in terms of the old result that $\text{Str} \mathcal{M}^2 = 0$ in $N = 4$ extended supergravity [11,17]. In other words, even if the model has only $N = 1$ broken supersymmetry, the organization of the mass splittings, both in the light and in the heavy sectors, obeys the constraints of $N = 4$ extended supersymmetry. In principle, there is another possibility of avoiding a non-vanishing $\text{Str} \mathcal{M}^2$, which allows for the presence of $N = 2$ sectors with non-zero boson-fermion mass splittings, at the condition of having vanishing contributions to the $N = 2$ beta functions coming from these sectors. In this case, the vanishing of $\text{Str} \mathcal{M}^2$ is a consequence of the finiteness of the associated $N = 2$ theory, and is also necessary in order to avoid enormous threshold corrections to the string gauge coupling constant, which would spoil the string perturbative expansion.

Before leaving this paragraph, a final comment on supersymmetry breaking at the string tree level is in order. It is true that at present the only known mechanism for spontaneous supersymmetry breaking at the string tree level implies that the gravitino mass scales like the inverse of the radius of an internal dimension, see eq. (3.36). However, we cannot exclude the possibility of constructing string models where supersymmetry is also spontaneously broken at the tree level, but the gravitino mass has different scaling properties with respect to the radius $R$, for example

$$m_{3/2} = O(1) R^{-n},$$

(3.40)
with \( n \leq 3 \). To obtain a gravitino mass at the 1 TeV scale, for \( n = 1 \) one needs the scale associated with \( R^{-1} \) to be also 1 TeV, but such a scale can be pushed to \( 3 \times 10^{10} \) GeV for \( n = 2 \) and to \( 10^{13} \) GeV for \( n = 3 \). In the last two cases, the first Kaluza-Klein states would have masses very much above the electroweak scale: only in the case \( n = 1 \) they have a chance to be accessible at presently envisaged accelerators. An argument which supports the possibility of having models with \( n = 2, 3 \) is the following. It is known \([27,30]\) that the existing models with tree-level supersymmetry breaking are equivalent, from the point of view of the effective theory, to particular ‘gaugings’ of \( N = 4 \) supergravity \([48]\), based on the \( E_2 \in SO(6,6) \times SU(1,1)_S \) group. However, from the field theory point of view there are many other flat gaugings, corresponding to different subgroups \( H \) of \( SO(6,6) \), for example \( H = SU(2) \times SL(2, R) \), which give rise to cases in which \( n = 2 \) or \( n = 3 \). Some technical obstacles must be overcome in order to explicitly formulate string models corresponding to these more general gaugings, and we hope to return to this problem in the near future.

### 3.2 Non-perturbative breaking

We now discuss the other proposed mechanism for spontaneous supersymmetry breaking in string-derived supergravity models, i.e. the possibility of non-perturbative phenomena, which at the level of the effective supergravity theory can again be described by a modification of the superpotential. In order to obtain a consistent model, with broken supersymmetry and classically vanishing vacuum energy, the superpotential modification must be such that \( \mathcal{G}^I \mathcal{G}_I = 3 \) around \( C = 0 \). As already discussed at the beginning of this section, the superpotential modification must then contain some dependence on the \((S, T, U)\) moduli fields in order to avoid a strictly positive potential.

For example, the simplest choice \( w = k + O(C^2) \), with \( w \) independent of the \((S, T, U)\) moduli, would give \( \mathcal{G}^I \mathcal{G}_I = K^I K_I = 4 + p_V \), where the index \( \tilde{I} \) runs over the \((S, T, U)\) moduli, and therefore a potential around \( C = 0 \) of the form \( V = (1 + p_V)|k|^2 e^K = (1 + p_V)|k|^2/Y \). Since the quantity at the numerator is field-independent and strictly positive-definite, there is no stationary point for the potential, with the exception of the boundaries of moduli space, \( Y \to \infty \), for example the decompactification limit \( Y^{(T)} Y^{(U)} \to \infty \) or the zero-coupling limit \( Y^{(S)} \to \infty \). It is then clear that, to avoid this problem, the superpotential must depend on some of the moduli, in order to fix some of the VEVs associated with the moduli directions.

The simplest superpotential modification follows from the conjecture of gaugino condensation, and includes a non-trivial dependence on the \( S \) modulus. Such an assumption is made plausible by the fact that the gauge coupling constant of the theory is determined by the VEV of the \( S \) field. An \( S \)-dependent superpotential modification can allow for minima with \( \mathcal{G}_S = 0 \), and fix the VEV of the \( S \) modulus at the minima.

Irrespectively of the details of the \( S \) dependence of the superpotential, as long as there is a field configuration of \( S \) such that \( \mathcal{G}_S = 0 \), this is sufficient to create a well-
behaved positive-semi-definite potential in the absence of $U$-type moduli ($p_U = 0$). When such moduli are present, one must make the further assumption that the superpotential contains also a non-trivial $U$-dependence, so that minima with $G_U = 0$ can be allowed: otherwise, the scalar potential would still remain strictly positive-definite. Notice that the stabilization of the VEVs of the $U$-type moduli can be performed either at the string level, by moving to the points of extended symmetry associated with the $U$ moduli, or at the level of the effective theory, by extending the assumption made for the $S$ field.

A particularly interesting scenario \cite{49} for a non-perturbative $S$-dependent superpotential is the requirement of an $SL(2, \mathbb{Z})$ duality, as suggested by a `dual' description of strongly coupled strings in terms of (weakly coupled) toroidally compactified five-branes \cite{50}. In this regime, the $T$ and $S$ duality symmetries are interchanged, and it is consistent to treat the $T$ modulus classically, i.e. in the field-theory limit. Then the superpotential is

$$w(S) = \frac{F[J(S)]}{\eta^2(S)},$$

(3.41)

where $F$ defines a section of a flat holomorphic bundle over $SL(2, R)/[SO(2) \times SL(2, \mathbb{Z})]$, $\eta(S) \equiv e^{-\pi S/12} \prod_{n \geq 0}(1 - e^{-2\pi nS})$ is the Dedekind $\eta$-function for argument $(iS)$, and $J(S)$ is the generator of modular functions of weight $0$. Further restrictions on $F$ occur if we impose that $w(S) \to 0$ for $\text{Re } S \to \infty$ (weakly coupled string). The potential is of the no-scale type, $V \geq 0$, and its global minimum is at $G_S = 0$ [with $w(S) \neq 0$], which occurs for $\text{Re } S = 1, T$ arbitrary. The sliding gravitino mass is

$$m^2_{3/2}(T + \overline{T}) = \frac{k}{Y(T)}, \quad k = \left| \frac{w(S)^2}{Y(S)} \right|_{\text{Re } S = 1}. \quad (3.42)$$

Note that, without the $T$-flat directions, the dilaton potential would not be positive-semi-definite, and its minimization would give an unacceptable vacuum energy. The resolution of this puzzle is that the sliding $T$ singlet allows for supersymmetry breaking ($G_T \neq 0$) with vanishing vacuum energy. On the contrary, without $T$ field, the extremum with $G_S = 0$ would correspond to unbroken supergravity in anti-deSitter space.

In the following, we shall denote a superpotential modification with non-trivial $S$ and $U$ dependence with the name of ‘$T$-breaking’, since in that case the condition of vanishing vacuum energy, $G^I G_I = 3$, is saturated by the $T$ fields only. For the models in which $p_U = 3$, we may alternatively assume a superpotential modification, which depends only on $S$ and $T$, so that $G_S = G_{T_i} = 0$ at the minima, and the condition $G^0 G_I = 3$ is entirely saturated by the $U$ moduli: we shall call this scenario ‘$U$-breaking’. In the cases in which the Kähler manifolds for the $T$ and $U$ moduli are factorized, one may also consider intermediate scenarios of $S/T/U$-breaking, in which the superpotential modification is such that, at the minima of the potential with $C = 0$, it is identically $G^S G_S + G^{T_i} G_{T_i} + G^{U_i} G_{U_i} = 3$, with non-vanishing contributions from more than one sector. We should stress again that in all these scenarios the resulting value of $Q$, the coefficient appearing in $\text{Str } \mathcal{M}$, does not depend on the details of the superpotential modification, but only on the scaling properties.
of the different fields with respect to the moduli for which $G^a G_a \neq 0$ (not summed) at the minima.

After these general considerations, some illustrative examples are in order.

1. **Calabi-Yau manifolds**

We begin by recalling some generic features of the effective supergravity theories corresponding to Calabi-Yau compactifications. The gauge group is $E_6 \times E_8$, and the matter fields $C$ can be divided into two groups: we shall denote by $A^r a$ the matter fields in 27 representations of $E_6$, in one-to-one correspondence with the $T$ moduli, by $B^i j$ the matter fields in $\overline{27}$ representations of $E_6$, in one-to-one correspondence with the $U$ moduli.

The Kähler manifold for the $T$ and $U$ moduli fields factorizes into the direct product of two submanifolds with ‘special geometry’ \[51\]. The Kähler potential for the matter fields, taking into account only quadratic fluctuations, is

$$ K^{(A,B)}(A, T; B, \overline{T}; U, \overline{U}) = e^{K_T - K_{\overline{T}}}(K_1)_{ij} A^i A^j + e^{K_U - K_{\overline{U}}}(K_2)_{ij} B^i B^j + (P_{ij} A^i B^j + \text{h.c.}) \tag{3.43} $$

where $K_1 = -\log Y(T)$, $K_2 = -\log Y(U)$, and the function $P_{ij}$ is such that \[52\]

$$ \partial_{ij} P_{ij} = (K_1)_{ij} (K_2)_{ij} + \ldots $$

which integrated gives $P_{ij} = (K_1)_{ij} + \ldots$, where the dots stand for terms annihilated by $\partial_{ij}$. The situation of interest to us is the one where the moduli-dependence of the superpotential is such that

$$ G^s G_s = 3 r_s, \quad G^t G_t = 3 r_t, \quad G^u G_u = 3 r_u, \tag{3.45} $$

with

$$ r_s + r_t + r_u = 1, \tag{3.46} $$

so that $G^i G_i = 3$ as desired. In this case, one can easily rewrite eq. (2.31) as

$$ Q = (N_{TOT} - 1) - r_s d_f + \sum_i \left( \lambda^s_i r_t + \lambda^u_i r_u \right), \tag{3.47} $$

where now the index $i$ runs over all moduli and matter fields. The relevant scaling weights for Calabi-Yau manifolds are the following

- **$S$**: $\lambda_t = 0$, $\lambda_u = 0$;
- **$T$**: $\lambda_t = -2$, $\lambda_u = 0$;
- **$U$**: $\lambda_t = 0$, $\lambda_u = -2$;
- **$A, B$**: $\lambda_t = -1$, $\lambda_u = -1$.

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We neglect here the possible presence of extra singlets besides the $T$ and $U$ moduli. However, their presence does not affect the quadratic divergences if they have scaling weights $\lambda_t = \lambda_u = -1$. The previous formulae for the scaling weights can also be used to discuss the possibility of a gravitational contribution to the MSSM ‘$\mu$-term’. In Calabi-Yau manifolds, from the general properties of the $P_{ij,j'}$ coefficients and of the scaling weights for the matter fields (which should contain the MSSM Higgs doublets, with the ‘$\mu$-term’ originating from a $27 \cdot \overline{27}$ coupling), we deduce that a non–vanishing $\mu$ may occur when the goldstino mixes non-trivially the $(S, T, U)$ directions.

1a: Pure $T$- or pure $U$-breaking

In the notation of eq. (3.3), and in the limit of large $T$ moduli and small $U$ moduli, corresponding to the classical limit of Kaluza-Klein compactifications, we can write

$$Y^{(T)}(t) = d_{ij,j'} k_t t^{i} t^{j'} t^{k_t} + \tau ,$$

and

$$Y^{(U)}(u) = \sigma + \eta_{ij,j'} u^{i} u^{j'} + \ldots .$$

For $\tau = 0$, eq. (3.49) corresponds to a particular solution of eq. (3.4), where $d_{ijk}$ are the (topological) intersection matrix coefficients, and can be interpreted as a consequence of the ‘special geometry’ of Calabi-Yau moduli space. The constant $\tau$ in (3.49) is a perturbative correction coming from the $\alpha'$-expansion of the associated $\sigma$-model [53]. As one can easily verify for each of the supersymmetry-breaking mechanisms considered in the following, this correction gives rise to harmless $O(m^4)$ contributions to the scalar potential and to equally harmless $O(m^2/\overline{M}^2)$ contributions to $Q$. In the following, we shall set for simplicity $\tau = 0$.

The simplest possibility is to assume a superpotential modification that depends on the $S$ and $U$ moduli, but not on the $T$ moduli. Equivalently, one could directly write down the effective theory in the large-radius limit for the Kähler class moduli $T$ only, assuming that the VEVs of the complex structure moduli $U$ have already been fixed by physics at the Planck scale, and consider a superpotential modification that depends on the $S$ field only. One gets a positive-semi-definite potential, thanks to the field identity $G^{T_i} G_{T_i} \equiv 3$ and to the fact that there are values of the $S$ and $U$ moduli satisfying the conditions $G_S = K_S + (\log w)_S = 0$ and $G_{U_i} = K_{U_i} + (\log w)_{U_i} = 0$. Making the identification $z^\alpha \equiv T_i$, one obtains from eq. (3.47) that

$$Q^{(T)} = h_{2,1} - h_{1,1} = \frac{\chi}{2} ,$$

where $\chi$ is the Euler characteristic of the Calabi-Yau manifold, $h_{2,1}$ is the number of the $(2, 1)$ moduli $U_i$, and $h_{1,1}$ is the number of the $(1, 1)$ moduli $T_i$. In this case, the tree-level spectrum is particularly simple. The only massive states are the physical $T$ fermions and the $U$ and $S$ scalars, all with masses equal to the gravitino mass.
Because of mirror symmetry [54], we can also consider the limit specular to the previous one, i.e. the limit of large $U$ moduli and small $T$ moduli, in which case the roles of $K_1$ and $K_2$ are interchanged, with $d_{trjrk} \to d_{ijujuU}$, where $d_{ijujuU}$ corresponds to the classical limit of the mirror Calabi-Yau manifold. We can then assume that the superpotential modification depends non-trivially on the $S$ and $T$ moduli, but not on the $U$ moduli, so that $G^U_i G_{U_i} \equiv 3$, due to the geometry of the $(1, 2)$ moduli, and $G_S = G_{T_i} = 0$ at the minima of the potential. Making the identification $z^\alpha \equiv U_i$, one obtains from eq. (3.47) that

$$Q(U) = h_{1,1} - h_{2,1} = -\chi_2.$$ (3.52)

Specularly to the previous case, the only massive states are the physical $U$ fermions and the $T$ and $S$ scalars, all with masses equal to the gravitino mass.

Both for the $T$-breaking and for the $U$-breaking, the coefficient $Q$ of the one-loop quadratic divergences is non-zero in all physically relevant models, since in all Calabi-Yau vacua the number of chiral fermion families is also proportional to the Euler characteristic. We may interpret this result in the sense that, for Calabi-Yau string solutions, pure $T$-breaking and pure $U$-breaking are incompatible with the stability of the hierarchy $0 \neq m_{3/2} \ll M_P$. Notice, however, that the mirror symmetry maps $Q$ into $-Q$: this suggests an interesting possibility, to be described below.

1b: mixed $T/U$-breaking

From the string point of view, as an additional possibility for supersymmetry breaking we can consider the case in which both the $T$ and the $U$ moduli are large, and introduce a superpotential modification such that the potential admits minima with $G_{T_i} \neq 0, G_{U_i} \neq 0, G_S = 0$. The vanishing of the classical vacuum energy then implies

$$G^{T_i} G_{T_i} = 3 \cos^2 \theta \quad \text{and} \quad G^{U_i} G_{U_i} = 3 \sin^2 \theta.$$ (3.53)

In the notation of eq. (3.45), $r_s = 0, r_t = \cos^2 \theta, r_u = \sin^2 \theta$. Since, as we shall see, with this mixed $T/U$-breaking it is possible to have $Q^{(T/U)} = 0$ in Calabi-Yau models, it is appropriate to discuss this case in some detail. First, we shall show that there are superpotential modifications that break supersymmetry with a positive-semi-definite potential. Neglecting the contributions of the $C$ fields, using hatted indices for the fields $t^\hat{\alpha}$ and dotted indices for the fields $u_{\dot{\alpha}}$, and considering for the moment a generic superpotential $w$, we can write the scalar potential as

$$V = e^K \left[ |w|^2 G^s G_s + (K_{\hat{\alpha}} w + w_{\hat{\alpha}}) K^{\hat{\alpha}\hat{\beta}} \left(K_{\hat{\beta}} w + w_{\hat{\beta}}\right) + (K_{\dot{\alpha}} w + w_{\dot{\alpha}}) K^{\dot{\alpha}\dot{\beta}} \left(K_{\dot{\beta}} w + w_{\dot{\beta}}\right) - 3|w|^2 \right].$$ (3.54)

Using the scaling properties of the Kähler manifolds associated with the moduli fields $t^\hat{\alpha}$ and $u_{\dot{\alpha}}$, $K^{\hat{\alpha}} K_{\hat{\alpha}} = K^{\dot{\alpha}} K_{\dot{\alpha}} = 3$, after some simple algebra we can rewrite $V$ in the more
suggestive form

\[ V = e^K \left\{ |w|^2 \mathcal{G}_s \mathcal{G}_s + 3 \left| w + \frac{1}{3} K^{\dot{\alpha}} w_\dot{\alpha} + \frac{1}{3} K^{\dot{\alpha}} w_\dot{\alpha} \right|^2 + w_\dot{\alpha} \left( K^{\dot{\alpha} \dot{\beta}} - \frac{1}{3} K^{\dot{\alpha}} K^{\dot{\beta}} \right) \frac{1}{w_{\dot{\beta}}} \right\} \]

\[ + \left. w_\dot{\alpha} \left( K^{\dot{\alpha} \dot{\beta}} - \frac{1}{3} K^{\dot{\alpha}} K^{\dot{\beta}} \right) \frac{1}{w_{\dot{\beta}}} - \frac{1}{3} \left[ \left( K^{\dot{\alpha}} w_\dot{\alpha} \right) \left( K^{\dot{\alpha}} \frac{1}{w_{\dot{\beta}}} \right) + \text{h.c.} \right] \right\} . \tag{3.55} \]

For a generic superpotential \( w \), the above scalar potential is not manifestly positive-semidefinite. However, we shall now show that there exists a generic class of superpotential modifications that ensure this desired property. Consider first an \( S \)-dependence of \( w \) such that the equation \( \mathcal{G}_S = 0 \) can have a solution at some finite value of \( S \). Then assume that \( w \) depends only on a linear combination of the \( T^{\dot{\alpha}} \) and the \( U^{\dot{\alpha}} \) moduli,

\[ w = w \left( d_\dot{\alpha} T^{\dot{\alpha}} + d_\dot{\alpha} U^{\dot{\alpha}} \right) . \tag{3.56} \]

With the above choice of superpotential, the scalar potential becomes

\[ V = e^K \left\{ 3 \left| w + \frac{1}{3} K^{\dot{\alpha}} d_\dot{\alpha} w' + \frac{1}{3} K^{\dot{\alpha}} d_\dot{\alpha} w' \right|^2 + |w'|^2 d_\dot{\alpha} \left( K^{\dot{\alpha} \dot{\beta}} - \frac{1}{3} K^{\dot{\alpha}} K^{\dot{\beta}} \right) \frac{1}{d_{\dot{\beta}}} \right\} \]

\[ + \left. |w'|^2 d_\dot{\alpha} \left( K^{\dot{\alpha} \dot{\beta}} - \frac{1}{3} K^{\dot{\alpha}} K^{\dot{\beta}} \right) \frac{1}{d_{\dot{\beta}}} - \frac{1}{3} |w'|^2 \left[ \left( K^{\dot{\alpha}} d_\dot{\alpha} \right) \left( K^{\dot{\alpha}} \frac{1}{d_{\dot{\beta}}} \right) + \text{h.c.} \right] \right\} . \tag{3.57} \]

Since, as already noticed, \( K^{\dot{\alpha}} = -t^{\dot{\alpha}} \) and \( K^{\dot{\alpha}} = -u^{\dot{\alpha}} \), the last term of the previous equation, which is not manifestly positive-semi-definite, identically vanishes if the \( d_\dot{\alpha} \) are purely real and the \( d_\dot{\alpha} \) purely imaginary (or, more generally, whenever their relative phase is equal to \( e^{i\pi/2} \)). The minima with broken supersymmetry are those for which both \( w \) and \( w' \) are different from zero. On the other hand, having vanishing vacuum energy at the minima implies

\[ d_\dot{\alpha} \left( K^{\dot{\alpha} \dot{\beta}} - \frac{1}{3} K^{\dot{\alpha}} K^{\dot{\beta}} \right) \frac{1}{d_{\dot{\beta}}} = d_\dot{\alpha} \left( K^{\dot{\alpha} \dot{\beta}} - \frac{1}{3} K^{\dot{\alpha}} K^{\dot{\beta}} \right) \frac{1}{d_{\dot{\beta}}} = 0 . \tag{3.58} \]

It is clear that the required conditions can be simultaneously satisfied, since both the effective metrics \( K^{\dot{\alpha} \dot{\beta}} - (1/3) K^{\dot{\alpha}} K^{\dot{\beta}} \) and \( K^{\dot{\alpha} \dot{\beta}} - (1/3) K^{\dot{\alpha}} K^{\dot{\beta}} \) are positive semi-definite and have always one zero eigenvalue, as a consequence of the field identities \( K^{\dot{\alpha}} K_{\dot{\alpha}} = K^{\dot{\alpha}} K_{\dot{\alpha}} = 3 \). In the case in which there are only one \( T \) and one \( U \) moduli, the second and the third terms in the potential (3.57) are identically vanishing due to the above identities, and one is just left with the first positive-semi-definite term. In the case of many \( T \) and \( U \) moduli, the minima of the potential must correspond to configurations of the \( T \) and \( U \) moduli such that

\[ \left( K^{\dot{\alpha} \dot{\beta}} - \frac{1}{3} K^{\dot{\alpha}} K^{\dot{\beta}} \right) \frac{1}{d_{\dot{\beta}}} = \left( K^{\dot{\alpha} \dot{\beta}} - \frac{1}{3} K^{\dot{\alpha}} K^{\dot{\beta}} \right) \frac{1}{d_{\dot{\beta}}} = 0 . \tag{3.59} \]

Since \( K^{\dot{\alpha}} \) and \( K^{\dot{\alpha}} \) are linear in \( t^{\dot{\alpha}} \) and \( u^{\dot{\alpha}} \), respectively, it is convenient to perform the field redefinitions

\[ T' \equiv d_\dot{\alpha} T^{\dot{\alpha}} \quad \text{and} \quad U' \equiv -i d_\dot{\alpha} U^{\dot{\alpha}} . \tag{3.60} \]
In terms of the redefined fields $T'$ and $U'$, the potential \((3.57)\) becomes

\[
V = 3 \left| w - \frac{1}{3} t' w_T - \frac{i}{3} u' w_U \right|^2 ,
\]

which is manifestly positive-semi-definite, and vanishes whenever

\[
w = \frac{1}{3} t' w_T + \frac{i}{3} u' w_U ,
\]

with $w = w(T' + iU')$. Using the minimization condition \((3.62)\), and disregarding at first the matter fields, one finds

\[
G^{T_i} G_{T_i} \equiv G^\hat{a} G_{\hat{a}} = 3 \cos^2 \theta , \quad G^{U_i} G_{U_i} \equiv G^\hat{a} G_{\hat{a}} = 3 \sin^2 \theta ,
\]

where

\[
\cos^2 \theta \equiv \frac{u'^2}{t'^2 + u'^2} , \quad \sin^2 \theta \equiv \frac{t'^2}{t'^2 + u'^2} ,
\]

so that $G^{T_i} G_{T_i} + G^{U_i} G_{U_i} \equiv G^\hat{a} G_{\hat{a}} + G^\hat{a} G_{\hat{a}} \equiv 3$ for any value of $\theta$. The directions $t' \equiv (T' + T')$ and $u' \equiv (U' + U')$ are arbitrary, while $(T - T)$ and $(U - U)$ are fixed in terms of $t'$ and $u'$. Once the direction of the goldstino has been specified, the computation of the coefficient of quadratic divergences is a straightforward application of eq. \((3.47)\), and one gets

\[
Q^{(T/U)} = (h_{2,1} - h_{1,1}) \cos 2\theta = \frac{\cos 2\theta \chi}{2} .
\]

As a result, this type of breaking allows for $Q^{(T/U)} = 0$ in the direction associated with $\theta = \pi/4$, corresponding to the case in which the $T$ and $U$ moduli contribute to supersymmetry breaking with equal strength, $G^{T_i} G_{T_i} = G^{U_i} G_{U_i} = 3/2$. Despite the presence of $T/U$ mixing, the fact that the charged scalars have all weight (-1) implies that both their soft masses and the associated $\mu$ term are classically vanishing.

Even though we concentrated here on a specific example, eq. \((3.65)\) can be easily extended to all string-derived supergravity models in which, at the supersymmetry breaking minima of the potential, $G^S G_S = 0$, $G^{T_i} G_{T_i} + G^{U_i} G_{U_i} = 3$, and there is at least one field direction, of the form $a_{\hat{a}} t^\hat{a} + b_{\hat{a}} u^\hat{a}$, with $a$ and $b$ arbitrary vectors, in which the classical potential is flat.

As a final comment, we observe that the considerations made here for Calabi-Yau manifolds can be easily extended to arbitrary symmetric orbifolds, whose effective theories are known \[23\]. Without going into details, consider for example the case of full $T$-breaking, so that $z^\alpha = T_i$. In this case the scaling weights are known, and can be summarized as follows: $\lambda_T = -2$ for the $T_i$ moduli; $\lambda_F = -1$ for the untwisted families, in 27 representations of $E_6$; for the twisted moduli, $\lambda_{TM} = -3$ when there are no $N = 2$ sectors, $\lambda_{TM} = -2$ in the presence of $N = 2$ sectors; for the twisted families, $\lambda_{TF} = -2$ when there are no $N = 2$ sectors, $\lambda_{TF} = -1$ in the presence of $N = 2$ sectors. The fact that one can have twisted families with $\lambda_{TF} = -2$ implies that the generic symmetric orbifolds, with the only exception of the $Z_2 \times Z_2$ case, are not compatible with full $T$-breaking.
2. \( Z_2 \times Z_2 \) orbifolds

In these models, the Kähler potential is identical to the one discussed in the string example of the previous paragraph, but we prefer here to perform an analytic field redefinition, in order to write \( Y_1, Y_2 \) and \( Y_3 \) all in the same functional form

\[
Y_A = (T_A + \overline{T}_A)(U_A + \overline{U}_A) - (y^{iA} + \overline{y}^{iA})^2, \quad (A = 1, 2, 3). \tag{3.66}
\]

For \( z = 0 \), and for an arbitrary superpotential \( w \), the scalar potential of the models under consideration reads \([17]\)

\[
V = V_0 + \sum_{A=1}^{3} V_A + V_D, \tag{3.67}
\]

where

\[
V_0 = e^K \left| w - (S + \overline{S})w_S \right|^2, \tag{3.68}
\]

and

\[
V_A = e^K \left[ \left| w - w_{TA}(T_A + \overline{T}_A) - w_{UA}(U_A + \overline{U}_A) \right|^2 - w_{iA}(y_{iA} + \overline{y}_{iA}) \right]^2 + Y_A \left( \frac{|w_{iA}|^2}{2} - \overline{w}_{TA} w_{UA} - \overline{w}_{UA} w_{TA} \right). \tag{3.69}
\]

One can easily extend the calculation to include the dependence on the \( z \) fields \([17]\), but this is not necessary for the following considerations. Notice that the last term in \( V_1, V_2, V_3 \) is not manifestly positive semi-definite. In the following, however, we shall consider superpotential modifications such that the last term vanishes and supersymmetry breaking minima with vanishing vacuum energy are generated.

2a: \((T_1, T_2, T_3)\)-breaking

Consider a superpotential modification with a non-trivial dependence on \( S \) and on the moduli \((U_1, U_2, U_3)\), but no dependence on the moduli \((T_1, T_2, T_3)\). Then, if \( G_S = G_{UA} = 0 \) is allowed in configuration space, supersymmetry is broken with \( V = 0 \) and \( z^0 \equiv (T_1, T_2, T_3) \).

The scaling weights of the different fields are

\[
\lambda_f = \lambda_{S, U_A} = 0, \quad \lambda_{y_{A}, z_A} = -1, \quad \lambda_{T_A} = -2. \tag{3.70}
\]

The interesting result is that in all \( Z_2 \times Z_2 \) models of this type one finds \([17]\)

\[
Q = 0. \tag{3.71}
\]

As for the particle spectrum, after observing that \( \rho_{y_1 y_1} = \rho_{y_2 y_2} = \rho_{y_3 y_3} = -1 \), one finds that, besides possible superpotential masses for the \( S \) and \( U \) superfields, the only states with non-vanishing supersymmetry breaking masses, all identical to the gravitino mass \( m_{3/2} \), are the physical \( T \) fermions and the \( S \) and \( U \) bosons.

A completely symmetric result holds if one assumes full \( U \)-breaking instead of full \( T \)-breaking.
2b: \((T_1, U_1, T_2)\)-breaking

We assume here, for the sake of discussion, the existence of a superpotential modification that generates a positive-semi-definite potential, with supersymmetry-breaking minima such that \(z^\alpha \equiv (T_1, U_1, T_2)\). In such case, the gravitino mass would scale like \(R^{-2}\). The scaling weights of the different fields would be
\[
\lambda_f = \lambda_{S,T_3,U_2,U_3,y_3} = 0, \quad \lambda_{y_2} = \frac{3}{2}, \quad \lambda_{y_3} = -1, \quad \lambda_{z_1} = -\frac{1}{2}, \quad \lambda_{T_1,U_1,T_2,y_1} = -2, \quad (3.72)
\]
and the coefficient of the quadratic divergences would read
\[
Q = -n_{y_1} + n_{y_3} + \frac{1}{2} n_{z_1} - \frac{1}{2} n_{z_2}. \quad (3.73)
\]
Notice, however, that this type of breaking can only be consistent if \(n_{z_3} = 0\), since for the fields \(z_A\), which transform in spinorial representations of the unbroken orthogonal gauge group, and therefore cannot have gravitational mass terms, it is not permitted to have \(\lambda_{z_A} < -1\), which would generate a negative squared mass for the associated scalars. One can verify that models with the latter property can be formulated via fermionic string constructions. On the other hand, the vanishing of \(Q\) would not be automatic, but would require a non-trivial relation among the field representations.

2c: \((S/T_2, T_1, T_3)\)-breaking

Another interesting situation occurs when the \(S\) moduli and one of the compactification moduli are mixed, in a way similar to the way the \(T/U\) mixing was occurring in the previous Calabi-Yau example.

Imagine a superpotential of the form
\[
w = w_{SUSY} + w_0(T_2 + i S, U_A), \quad (3.74)
\]
with no explicit dependence on the moduli \(T_2 - i S, T_1, T_3\). One can explicitly verify that the scalar potential is positive semi-definite and, provided that the configurations with \(G_U = G_y = G_z = 0\) are allowed, it admits supersymmetry breaking minima with
\[
G^S G_S = \sin^2 \theta, \quad G^{T_2} G_{T_2} = \cos^2 \theta, \quad G^{T_1} G_{T_1} = G^{T_3} G_{T_3} = 1, \quad (3.75)
\]
where
\[
\sin^2 \theta = \frac{t_2^2}{t_2^2 + s_2^2} \quad (3.76)
\]
corresponds to a flat direction. In this case, one finds
\[
Q = \sin^2 \theta \left( n_{z_1} + n_{z_3} + n_{y_2} - d_f \right), \quad (3.77)
\]
and for \(\sin^2 \theta = 0\) one recovers the result of pure \(T\)-breaking. However, one can have \(Q = 0\) also when \(\sin^2 \theta \neq 0\) but its coefficient in eq. \([3.77]\) vanishes, which implies a non-trivial relation among the field representations. The latter case would allow for non-vanishing gaugino masses \(m_1^{2} = \sin^2 \theta m_3^{2}\).
Imagine finally a superpotential modification that generates a positive-semi-definite potential, with supersymmetry-breaking minima corresponding to

\[
G^S G_S = \sin^2 \theta, \quad G^{T_2} G_{T_2} = \cos^2 \theta, \quad G^{T_1} G_{T_1} = G^{U_1} G_{U_1} = 1 ,
\]

where

\[
\sin^2 \theta = \frac{t_2^2}{t_2^2 + s^2}
\]

(3.79)

corresponds to a flat direction. In this case, one would find

\[
Q = \sin^2 \theta \left( -d_f + n_{y_2} + \frac{n_{z_1} + n_{z_3}}{2} \right) + \left( n_{y_3} - n_{y_1} + \frac{n_{z_1} - n_{z_3}}{2} \right).
\]

(3.80)

For \( \sin^2 \theta = 1 \) we recover the result of the string tree-level breaking, for \( \sin^2 \theta = 0 \) the result of \((T_1, U_1, T_2)\) breaking.

### 4 Conclusions and outlook

In this paper we have argued that, in realistic models of spontaneously broken supergravity, the desired hierarchy \( m_Z, m_{3/2} \ll M_P \) can be stable, and eventually find a natural dynamical explanation, when quantum loop corrections to the effective potential do not contain terms quadratic in the cut-off scale, controlled at the one-loop level by \( Q \), as defined in eqs. (1.12)–(1.16). Requiring broken supersymmetry with vanishing vacuum energy and vanishing \( Q \) (modulo corrections suppressed by \( m_{3/2}^2 / M_P^2 \) or exponentially) defines a highly non-trivial constraint on the Kähler potential \( K \) and the gauge kinetic function \( f_{ab} \), including both the observable and the hidden sectors of the theory, as well as on the mechanism for spontaneous supersymmetry breaking. In the presence of some approximate scaling properties of the gauge and Kähler metrics, with respect to the fields with non-vanishing components in the direction of the goldstino, the contributions to \( Q \) of the different degrees of freedom that get mass via supersymmetry breaking depend only on their scaling weights \( \lambda_i \), and not on the VEVs of the sliding singlet fields in the hidden sector. We have derived a similar result for the individual mass matrices of the theory, and in particular for the explicit mass parameters of the MSSM, \( (m_{1/2}, m_0, A, \mu, m_Z^2) \), which take very simple expressions in terms of the assumed scaling weights. These expressions for \( Q \) and for the mass parameters of the MSSM find a deeper justification in the effective theories of four-dimensional superstrings, where supersymmetry breaking is described either at the string tree-level or, by assuming some non-perturbative phenomena, only in the effective field theory. In these theories, the full particle content and the approximate scaling weights are completely fixed. The origin of the approximate scaling properties of the superstring effective theories is due to target-space modular invariance, and the scaling weights \( \lambda_i \) are nothing but the target-space duality weights with respect to the
moduli fields, which participate in the supersymmetry-breaking mechanism. Indeed, in the limit of large moduli the discrete target-space duality symmetries are promoted to some accidental scaling symmetries of the gauge and matter kinetic terms in the effective supergravity theory. As an application, we gave explicit expressions for $Q$ and for the MSSM mass terms in the effective theories of some representative four-dimensional superstring models.

When one changes the direction of the goldstino in the space of the moduli fields, the value of $Q$ changes accordingly, in a very simple way. As an example, we discussed the case of mixed $T/U$-breaking in Calabi-Yau compactifications, and we showed that the case of diagonal breaking, $G^T G_T = G^U G_U = 3/2$, corresponds to vanishing $Q$. In the effective theories of fermionic constructions ($Z_2 \times Z_2$ asymmetric orbifolds), the case of full $T$-breaking, $G^{TA} G_{TA} = 3$, also corresponds to vanishing $Q$, but one can also conceive cases of mixed $(S, T, U)$ breaking in which $Q = 0$ may be realized. As for the explicit string constructions in which supersymmetry is spontaneously broken at the tree level, all the presently known solutions have $G^T G_T = G^U G_U = G^S G_S = 1$, and in some of them the constraint $Q = 0$ is satisfied, due to some accidental organization of the massive string spectrum, which is very similar to the one of spontaneously broken $N = 4$ extended supergravities. Thus we have identified in $Q = 0$ another criterion for a consistent choice of the supersymmetry breaking directions, $\{z^a\}$ such that $G^a G_a \neq 0$, in the string-induced effective supergravities.

Some comments on our results are in order, so that we can better identify their interpretation. The first one concerns the contribution to $Q$ of the massive string modes. In the case of string tree-level breaking, one can easily compute such a contribution directly at the string level, since one knows the explicit form of the one-loop string partition function. One can identify a class of physically relevant four-dimensional orbifold models where the result is the desired one,

$$V_{eff} = (\text{constant}) \times m_{3/2}^4 + \ldots ,$$

where the constant depends on the number of residual bosonic and fermionic massless string states after supersymmetry breaking, and the dots represent the contributions of the massive string states, which, in the large moduli limit (large compactification radius $R$ with respect to the string scale $\alpha'^{1/2}$), are exponentially suppressed, as $e^{-cR^2/\alpha'}$. The validity of the above behaviour follows from two facts: (i) the absence of the $N = 2$ sector associated with the $R$-dependent threshold corrections; (ii) the fact that the contributions to the vacuum energy come only from the $N = 4$ sector, where, including also the string massive states, one still has the well-known sum rules of $N = 4$ supergravity, $\text{Str} \, \mathcal{M}^n = 0$ for $n < 4$ and $\text{Str} \, \mathcal{M}^4 = (\text{constant}) \times m_{3/2}^4$. This result explicitly displays two remarkable properties. The first is the exponential suppression of the infinitely many massive string states in the large moduli limit: the contribution to $Q$, and also to the terms of the one-loop vacuum energy $O(m_{3/2}^4)$, is entirely given by the one calculated in the effective supergravity theory, in analogy to what happens when computing string threshold corrections to gauge
and Yukawa couplings. This fact suggests the possibility that, at least for string tree-level breaking, the coefficient $Q$ is a topological number, calculable from the effective field theory data only, without the need of knowing all the massive string modes. If this conjecture turns out to be true, then it is not inconceivable that $Q$, in analogy with anomalies, is a purely one-loop phenomenon: if so, the hierarchy problem could be solved in the class of models where $Q = 0$. This conjecture would be much more difficult to prove in the case of non-perturbative breaking at the field-theory level. Still, there are some indications that $Q$ could be related to an anomaly coefficient constructed from some composite currents. We hope to return to this point in a future publication.

Among the models we have considered, the more realistic and constrained ones appear to be those where $G_S \neq 0$, since they exhibit a simple tree-level generation of the scalar and gaugino mass terms, as well as of the $\mu$-term, for the MSSM. However, the cancellation of quadratic divergences requires in these models an interplay between real and chiral representations of the gauge group. Moreover, with $G_S \neq 0$ the flatness of the potential along some directions requires a specific mixing with some other moduli, to obtain vanishing tree-level cosmological constant with sliding gravitino mass. In this spirit, dilaton dominance [55] seems difficult to reconcile with a sliding gravitino mass.

Since in this paper we looked at tree-level mass formulae and at one-loop quadratic divergences, we did not consider string loop corrections to the effective supergravity theory, which usually modify the Kähler potential and the gauge couplings. In the framework of a consistent perturbative expansion in the string coupling constant, these corrections are in general negligible in first approximation. As observed in [22], however, they could be relevant in the cases in which some physical parameters are accidentally vanishing at the tree level.

One of the assumptions of the present work was the alignment of the goldstino along directions that are gauge singlets at the Planck scale. This implies, in particular, the existence of scalar particles with interactions of gravitational strength and masses of order $m^2_{3/2}/M_P$, with interesting astrophysical and cosmological implications, including a number of potential phenomenological problems. More general situations may however be possible, when some underlying symmetry controls the hidden sector physics [56]. It is still an open question whether in these models one can have a naturally vanishing cosmological constant, modulo $O(m^4_{3/2})$ corrections, and at the same time guarantee the stability of the scale hierarchies.

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