We investigate the absorption properties of graphene-based anisotropic metamaterial structures where the metamaterial layer possesses an electromagnetic response corresponding to a near-zero permittivity. We find that through analytical and numerical studies, near perfect absorption arises over an unusually broad range of beam incidence angles. Due to the presence of graphene, the absorption is tunable via a gate voltage, providing dynamic control of the energy transmission. We show that this strongly enhanced absorption arises due to a coupling between light and a fast wave-mode propagating along the graphene/metamaterial hybrid.

With the refinement and advancement of fabrication techniques and theoretical methods, it is now possible to successfully create anisotropic metamaterial structures that have at least one vanishing principal component of the permeability $\mu$ or permittivity $\varepsilon$ tensors. These artificial hybrid structures have been shown to allow propagation of electromagnetic (EM) waves through narrow or mismatched channels with little reflection, and offer alternatives in wavefront shaping, beam steering, and EM energy control previously unseen in conventional systems. Typical experimental platforms involve various geometrical arrays of metallic rods or combinations of metallic and dielectric multilayers that have tunable system parameters to achieve the desired near-zero effective EM response. In addition to these platforms, other metamaterial architectures have been designed that manipulate the EM response to significantly reduce loss. These include, layered metal/dielectric structures, transparent conducting materials, superconducting nanoparticles, and plasmonic nanoshells with quantum dot cores. While there are a number of mechanisms to dynamically tune metamaterials, it would be preferable to have more robust, real-time tunability of their effective electric ($E$) or magnetic ($H$) responses.

These developments have led to a significant number of recent works devoted to studying graphene as a component in hybrid metamaterial designs. Enhanced absorbance in a hybrid graphene-metamaterial structure on a reflective substrate was proposed in ref. 26. In addition to its intrinsic 2D geometrical properties, graphene affords extensive control of its conductivity $\sigma$ due to its gate-tunable Fermi energy $E_F$. When EM waves interact with a graphene sheet, the gate voltage can tune the amount of energy that gets absorbed, depending on the relevant scattering processes, temperature, relaxation time, frequency of the incident beam and its polarization.

By placing graphene in contact with an anisotropic metamaterial that has vanishing principal components of the permittivity and permeability tensors over the frequencies of interest, one could expect the interplay between the two samples to present additional possibilities regarding the transmission of the light. More specifically, we pose the question if it is possible to utilize the dynamic and tunable conductivity of graphene to yield greater control over the way energy of EM waves is absorbed or reflected compared to when the materials are in isolation. Motivated by recent experimental and theoretical developments, we seek to establish if one may obtain tunable wide angle absorption in hybrid graphene/metamaterial layers where the tensor $\varepsilon$ describing the anisotropic response has components that are near zero.

To this end, we consider here the absorption characteristics of a layered system, comprised of monolayer graphene overlying a metamaterial slab that is connected to a perfectly conducting metal. In accordance with the discussion above, we will consider metamaterial structures that can possess an $\varepsilon$ near-zero (ENZ) response, where the metamaterial has a vanishingly small value of the real part of $\varepsilon$. In contrast to numerous past works in the RF and visible light regime, we focus here on operational wavelengths that correspond to the less exploited THz regime, where many new technologies are emerging. To accurately determine the absorption as a function of the geometrical and material parameters that describe graphene and the metamaterial, Maxwell’s equations are
solved using a transfer matrix approach that accounts for the presence of graphene through boundary conditions imposed on the magnetic field. We discuss the behavior of the absorption in terms of the energy flow throughout the entire structure.

Our main result is that by tuning $E_F$, we are able to determine experimentally relevant regimes where near perfect absorption can occur over an extremely wide range of incident angles for an ENZ metamaterial layer that has a small component of the permittivity tensor. We show that the enhanced absorption occurs due to coherent perfect absorption which couples light to a fast wave propagating along the graphene/ENZ hybrid.

**Theory**

The principal coordinate axes are chosen so that both the permittivity and permeability tensor in the anisotropic medium are both diagonal. A diagram of our setup is shown in Fig. 1. For concreteness, we focus on $p$-polarized incident light ($E_x, E_y, B_z \neq 0$). In the anisotropic medium (region 2), the nonzero components to the tensors are $(\varepsilon_{x,2}, \varepsilon_{z,2}, \mu_{y,2})$. It is backed by a perfect electric conductor (PEC) in region 1, where no electric or magnetic fields exist. The finite surface conductivity $\sigma$ of graphene will be effectively taken into account in the boundary condition for the magnetic field $H$ at the interface between air (region 3) and the anisotropic medium, where

$$\sigma = 2ie^2/k_BT((\pi\hbar^2)(\omega + i/t_{\text{rel}}))^{-1} \ln[2 \cosh(E_F/(2k_BT))].$$

When $E_T \gg k_BT$, $\hbar\omega$, we have,

$$\sigma = ie^2/[\pi\hbar^2(\omega + i/t_{\text{rel}})].$$

Thus, at a temperature $T$ of 300 K, and THz frequencies, we have $k_BT = 25$ meV, and $\hbar\omega \sim 1$ meV, meaning doping levels should exceed $E_F = 100$ meV when using Eq. (2). Here, $E_F$ is the Fermi energy, $\omega$ is the frequency of the EM wave, and $t_{\text{rel}}$ is the relaxation time. For frequencies in the THz region and lower, Eq. (2) reflects that the interband part to the conductivity is negligible compared its the intraband part, and consequently to the Drude-like behavior for $\sigma$.

We now consider Maxwell’s equation (in SI units) in the incident air region and in the anisotropic medium, both having no free charges or currents. For $p$-polarized incident light and diagonal tensors, we obtain the following set of equations determining the components of the fields when the time-dependence is harmonic ($e^{-i\omega t}$):

$$\varepsilon_x \partial_x E_x + \varepsilon_z \partial_z E_z = 0, \quad ic\omega B_y = -\partial_x E_z + \partial_z E_x, \quad -i\omega D_x = -\partial_y H_y, \quad -i\omega D_z = \partial_y H_y.$$ (3)

These equations are solved with a plane-wave ansatz for the field components where $E \propto e^{ik_zx}$ and $k_z$ is conserved due to translational invariance. The Methods section contains additional details of the numerical and analytical solutions of the above equations.

Once the equations have been solved, one may compute the absorbance $A$ from the scattering coefficients: $A_2$ and $B_2$ are associated with the transmitted and reflected electric field wave in region 2 whereas $B_1 \equiv r$ is the coefficient for the reflected wave in region 3. Since there is no transmission into the PEC layer, the absorbance $A$ can be computed from the incident wave after subtracting the reflected part: $A = 1 - |r|^2$. The reflection coefficient $r$ is obtained as

$$r = -e^{-2ik_z\tau}R_+ / R_-,$$ (4)

where we have defined

$$R_+ = \pm 2ie^{-ik_z\tau}\varepsilon_{x,2}k_{z,2}\varepsilon_{x,3} + 2cos(k_{z,2}\tau)\varepsilon_{x,2}k_{z,2}\omega - 2i\sin(k_{z,2}\tau)\sigma k_z k_{z,3}.$$ (5)

The coefficients for the other waves are given as: $A_2 = 2\varepsilon_{x,3}e^{-ik_z\tau}\omega k_{z,2}/R_-$, and $B_2 = -A_2$. We are now interested to see how the presence of the graphene layer modifies the absorption properties of the metamaterial, and
in particular determine the influence of the gate-controlled Fermi level which is tunable in situ. The analytical expression for the reflection coefficient \( r \) allows us to estimate which order of magnitude \( E_F \) should have in order to have any influence on the results. If the terms proportional to the conductivity \( \sigma \) are to be comparable in magnitude with the other terms, we see that e.g. \( \varepsilon_{33} \omega \sim k \sigma \) needs to be satisfied. Since we are considering an EM wave incident from air, \( \varepsilon_{33} = \varepsilon_{33} = \varepsilon_0 \). Inserting the expression for \( \sigma \), one finds that the above equation corresponds to a Fermi level in graphene of

\[
E_F \sim \varepsilon_0 c \sqrt{\frac{\hbar^2}{2m} + \frac{e^2}{\varepsilon_0} \cos \theta}.
\]  

(6)

For grazing incidence \( \theta \sim \pi/2 \), the graphene layer thus has no influence for any realistic values of \( E_F \). We also note that when \( \sigma \rightarrow \infty, r \rightarrow 1 \) such that the absorption vanishes, \( A \rightarrow 0 \). In order to gain information about the spatial distribution of the power flow in the structure, we also compute the Poynting vector \( S = E \times H \). We write \( E \sim \varepsilon \omega \) and \( H \sim H \omega \). For future use, we define the normalization constant for the power flow as the time-averaged incident power at \( r \rightarrow \infty \).

By appropriately tuning \( E_F \), graphene-dielectric stacks can exhibit a wavevector dispersion that traces out a hyperbolic contour at a given frequency. Such hyperbolic metamaterial (HMM) systems are described by a permittivity tensor whose principle components are opposite in sign, and allow for the propagation of evanescent modes in subwavelength imaging, and can be designed to exhibit substantial near-field absorption. Such structures are often nonresonant and possess low loss. There have been a variety of HMM systems that have been fabricated, including semiconductor hybrids, metallic layers, and silver nanowires. Gate tunable HMM systems have also been made that allow control of their optical response. Here, we will determine how the absorption properties of HMM structures are modified by the presence of a graphene layer over a broad range of angles \( \theta \) and Fermi energies \( E_F \).

We first consider the standard Drude-like frequency response in the metamaterial, such that

\[
\varepsilon_{33}/\varepsilon_0 = 1 - \frac{\sigma^2}{\varepsilon_0} \left[ 1 + (\alpha f)^2 \right] + i \frac{\sigma f}{\varepsilon_0} \left[ 1 + (\alpha f)^2 \right].
\]  

(7)

Here, \( \alpha = \frac{\lambda}{\lambda_c} \) and \( f = 0.02 \), where \( \lambda_c \) is the characteristic wavelength, with \( \lambda_c = 1.6 \) mm. Designing a metamaterial within effective medium theory to possess the prescribed dielectric response in the THz regime can be challenging with current fabrication and materials capabilities. It may be possible however to design a multilayer dielectric structure or rod lattice with polaritonic components to achieve the desired HMM response since the permittivity of polaritonic materials mimic metals in the optical regime. Therefore construction of an anisotropic HMM that has a longitudinal ENZ response should be possible at THz frequencies.

It will also be of interest to see how the absorption properties depend on the sign of \( \varepsilon_{33} \), determined by the frequency of the incident wave, and also its magnitude. We set \( \lambda \sim \lambda_c \), so that \( \varepsilon_{33} \sim 0 \). If we take the limit \( \varepsilon_{33} \rightarrow 0 \), the reflection coefficient simplifies to

\[
r = \frac{\varepsilon_0 c \cos \theta}{\varepsilon_0 c + \sigma \cos \theta}.
\]  

(8)

In the absence of graphene (\( \sigma = 0 \)), we have zero absorption since \( r^2 = 1 \). However, when graphene is present (\( \sigma \neq 0 \)) an interesting opportunity arises. It is seen from the above equation that \( r = 0 \), meaning perfect absorption \( A = 1 \) when the following condition is satisfied:

\[
\varepsilon_0 c = \sigma \cos \theta.
\]  

(9)

In the low THz regime and below (\( \omega \ll 10 \) THz), we have \( \omega \ll \tau \) and full absorption takes place for angles \( \theta \), satisfying

\[
\cos \theta = \frac{223.67}{E_F (\text{meV})}.
\]  

(10)

Although this equation represents the criterion for 100% absorption, we show in the next section that the absorption in fact is close to 100% for an extremely wide range of incident angles.

**Results and Discussion**

To establish how perfect absorption can be achieved, we discuss how the incident beam can couple to the intrinsic EM modes of the system. Our approach involves finding the poles of the reflection coefficient, Eq. (8), by setting the denominator equal to zero. To find the complete set of poles including the bound and leaky wave contributions, it becomes necessary to extend \( k_z = \beta + i \omega \) to the complex plane and solve the pole dispersion equation (see the Methods section) for \( \beta \) and \( \alpha \) (at a given \( \omega \)). This dispersion equation admits four types of solutions for the complex wavevector. Of these solutions, only those which correspond to perfect absorption modes are retained, since not all solutions correspond to coupling of the incident beam with the structure. The perfect absorption modes of interest are those which are activated via phase-matching of the incident plane wave to solutions corresponding to \( k_z/k_0 < 1 \), and \( \Im(k_z) = 0 \). These modes represent a coherent superposition of waves that propagate without loss along the surface. Indeed, only when the incident wave is incident at the perfect absorption angle, \( \theta_A \), does the direction of energy flow in the air region, \( \theta_A = \arctan(S/S_E) \), precisely coincide with the incident angle \( \theta_0 \).
We set the characteristic wavelength of the incident EM wave with the permittivity components perpendicular and parallel to the interfaces are of opposite sign. The corresponding HMM dispersion relations are then identified as either type I if \( \varepsilon_{x} > 0 \) and \( \varepsilon_{z} < 0 \), or type II if \( \varepsilon_{x} < 0 \) and \( \varepsilon_{z} > 0 \). The operating wavelength of the incident beam is tuned around \( \lambda_{0} \) to control the sign of \( \varepsilon_{z} \). The component of \( \varepsilon \) parallel to the interface always has the same magnitude, but can differ in sign according to \( \varepsilon_{z} = (\pm 4 + 0.1)i\varepsilon_{0} \).

Extremely wide-angle absorption for type-I and type-II anisotropic structures is demonstrated clearly in Fig. 2, where we provide results for these two cases for different thicknesses of the metamaterial and values of the Fermi level. The difference between Type I and Type II for a fixed thickness primarily occurs around normal incidence, \( \theta = 0 \), where the absorption is much higher in the Type I case. It is seen that when the thickness is close to the wavelength \( \tau \lambda = 0.85 \) in panels (c) and (f), the presence of graphene provides very high absorption for a remarkably wide range of angles. This should be contrasted to the case without graphene (dashed line), where the absorption peaks for an angle \( \theta < 45^\circ \) and then rapidly declines. For sufficiently small metamaterial thicknesses, and smaller angles, the metamaterial layer becomes insignificant and the EM wave interacts with mainly the graphene sheet and metal backing. Tuning \( E_{F} \) is seen to improve the absorption dramatically not only in magnitude, but also in terms of which angles of incidence that become absorbed. Remarkably, the absorption coefficient can be tuned to be nearly perfect, for an unusually broad range of angles. This is distinct from a scenario involving conventional dielectrics where isolated absorption peaks can appear at specific angles of incidence.

To pinpoint the mechanism of the observed perfect absorption, the mode characteristics of the system should be identified. By correlating the longitudinal wavevector component \( k_{x} \) of the incident EM wave with the permitted EM modes of the graphene-based ENZ structure, the critical coupling responsible for perfect absorption can be identified. Since \( k_{x}/k_{0} < 1 \), and \( \Im(k_{x}) = 0 \), we search for the allowed fast-wave EM modes (whereby the phase velocities exceed \( c \) that the structure supports\(^{51} \)). By solving the transcendental equation \( R_{\imath} = 0 \) [Eq. (5)], we find the permitted incident wavevector components that yield \( A_{\imath} = 0 \). The obtained \( k_{x} \), are converted to a critical coupling angle via \( \theta = \arcsin(k_{x}/k_{0}) \), giving the necessary incident beam angles for fast-wave mode coupling to occur. To illustrate this, in Fig. 3 the perfect absorption curves (solutions to \( R_{\imath} = 0 \)) are shown as a function of \( E_{F} \) and \( \theta \). The wavelength considered \( \lambda = 1.01\lambda_{0} \) corresponds to a negative real part of \( \varepsilon_{z} \) that is also near zero. In (a) \( \varepsilon_{z} = (4 + 0.1)i\varepsilon_{0} \) corresponding to a type I HMM, while in (b) both permittivity components are of the same sign, since now \( \varepsilon_{z} = (-4 + 0.1)i\varepsilon_{0} \). The dispersion curves are shown together with density plots of the absorption \( A \), which gives a global view of the high absorption regions spanned by \( E_{F} \) and \( \theta \). Clearly the fast-wave dispersion curves overlap with the regions of perfect absorption. A broader range of incident angles leading to perfect absorption is found in the graphene/HMM structure (a) compared to (b) where both \( \Re[\varepsilon_{z}] \) and \( \Im[\varepsilon_{z}] \) are of
the same sign. It is evident that incident angles corresponding to perfect absorption occur when a fast-wave mode is excited in the structure, due to the coherent superposition of waves with $k_j$ below the light line. These modes are distinct from conventional guided wave modes that have $k_j/k_0 > 1$, and consequently the EM fields would be evanescent in the air region near the graphene layer. They also differ from leaky wave modes that dissipate when propagating along $x$ since in that case, $\Im[k_j] \neq 0$. Therefore, by coupling the incident beam to fast wave modes of the structure, the enhanced absorption features provided by the HMM in the ENZ regime, complemented with the tunable energy flow afforded by graphene, generates a broad range of angles for perfect absorption to arise.

The behavior of the energy flow is exhibited in Fig. 4, where the Poynting vector components are largest near the interfaces, except for near normal angles of incidence, where the energy flows along the metal backing. The flow of $S_z$ in the metamaterial is opposite to the incident beam, demonstrating negative refraction that arises from the small negative permittivity component $\varepsilon_{zz}$. The perfect absorption features seen in Fig. 3 are also consistent with Fig. (d) for $\theta = 41^\circ$ where the energy flow is maximal in the vacuum region at this angle, indicating purely downward energy flow with no upward contributions from surface reflections.

**Methods**

We here provide some details for how to obtain the solution of the Maxwell equations in the presence of graphene. The wave-equations for $E_j$ and $H_j$ take the same form in region $j = 2, 3$:

$$\frac{\partial^2 E_j}{\partial z^2} + E_j(\omega^2 \varepsilon_{xx,j} H_{j,y} - \varepsilon_{zz,j} k_0^2 \varepsilon_{zz,j}) = 0, \quad k = x, z. \tag{12}$$

The dispersion relation then reads:

$$k_j^2 = \varepsilon_{zz,j} k_0^2 \varepsilon_{zz,j}, \quad k_j = k_{zz,j} \omega.$$

The fast-wave solutions to the wave equations for $E_j$ in region $j$ is then written:

$$E_j = E_{0j} e^{i k_{zz,j} z} + E_{3j} e^{i k_{zz,j} z}. \tag{13}$$

where we defined $k_{zz,j} = k_{zz,j} \omega$. Using this relation, we may now write down the solution of the electric field in region $j$ which takes into account both transmitted and reflected waves:

$$E_j = A_j (\hat{x} + \hat{z} k_j) e^{ik_{zz,j} z} + B_j (\hat{x} - \hat{z} k_j) e^{ik_{zz,j} z}.$$

Since the light is incident from air, we take $A_1 = 1$. The magnetic field is obtained as

$$H_{j,y} = \frac{\varepsilon_{yy,j}}{k_{zz,j}} (B_j e^{ik_{zz,j} z} - A_j e^{-ik_{zz,j} z}) e^{ik_{zz,j} z}. \tag{14}$$

There are three coefficients to be determined: $\{A_2, B_2, B_3\}$. To do this, we need to specify the boundary conditions at the air/anisotropic medium interface ($z = \tau$) and the anisotropic medium/PEC interface ($z = 0$). Since the tangential component of $E$ is always continuous, it follows that $E_z = E_{03} |_{z=\tau} = E_{13} |_{z=\tau}$, and $E_z = E_{33} |_{z=0} = 0$. The presence of the graphene layer now enters in the boundary condition for the tangential component of $H$ by writing the free current as $J = -\sigma E$: $H_{y,3} |_{z=\tau} = H_{y,3} |_{z=\tau} + \sigma(E_{3} \cdot \hat{x}) |_{z=\tau}$. Specifically, we find that the boundary conditions may be written in the form $Ma = b$ where

$$a = (A_2, B_2, B_3), \quad b = (0, e^{-ik_{zz,3} \tau}, -e_{xx,3} e^{-ik_{zz,3} \tau} k_{zz,3}),$$

$$M = \begin{bmatrix} 1 & 1 & 0 \\ e^{-ik_{zz,3} \tau} & e^{ik_{zz,3} \tau} & -e^{-ik_{zz,3} \tau} \\ -e^{-ik_{zz,3} \tau} & e^{ik_{zz,3} \tau} & -e^{-ik_{zz,3} \tau} e_{xx,3} k_{zz,3} \end{bmatrix}. \tag{15}$$
where we defined \( \varepsilon = \varepsilon_{x,2}^z \pm \sigma \). Thus, the coefficients are solved by \( a = M^{-1} b \), which in turn fully determines \( E \) and \( H \) throughout the entire structure.

We also provide an explicit expression for the time-averaged Poynting vector \( \langle S \rangle \) in the metamaterial region 2 is:

\[
\langle S \rangle_x = \frac{1}{2} \frac{k_z}{k_{x,2}} \Re \left\{ \varepsilon_{x,2} \left( |A|^2 e^{2 \Im(k_{x,2}) z} + |B|^2 e^{-2 \Im(k_{x,2}) z} - 2 \Re \{ A^* B e^{2 \Re(k_{x,2}) z} \} \right) \right\},
\]

\[
\langle S \rangle_z = \frac{1}{2} \Re \left\{ \frac{\varepsilon_{x,2}}{k_{x,2}} \left( |B|^2 e^{-2 \Im(k_{x,2}) z} - |A|^2 e^{2 \Im(k_{x,2}) z} - 2 \Im \{ A^* B e^{2 \Re(k_{x,2}) z} \} \right) \right\}.
\]

To determine the fast-wave modes of the system, we solve Maxwell’s equations for the EM fields in the absence of an incident field, thereby effectively treating the structure as a waveguide. This is equivalent to finding the poles of the reflection coefficient for a given sign of \( k_z \). The corresponding transcendental equation determines the allowed \( \omega \) and \( k_x \) pair that a source field can excite in the metamaterial system. From the many possible solutions, we consider only fast-waves corresponding to \( k_x < k_0 \), and \( \Im \{ k_x \} = 0 \). The dispersion equation that must be solved is,

\[
-\varepsilon + \varepsilon_{x,2}^z k_{x,2} \cot(k_{x,2} \tau) + \frac{\sigma k_{x,2}}{\omega} = 0,
\]

where \( k_{x,2} \) can be either positive or negative. When the system under consideration is isotropic, the transcendental equation for the modes in the ENZ limit (\( \varepsilon_{x,2} = \varepsilon_{z,2} \rightarrow 0 \)), reduces to,

\[
-\varepsilon_0 + \frac{\sigma k_{x,2}}{\omega} = 0.
\]

The \( k_x \) is converted into a coupling angle \( \theta \), that the incident beam must be directed at in order for fast-wave modes to be excited: \( \theta = \arcsin(k_x/k_0) \). What constitutes a zero numerically in the root finding algorithm that solves Eq. (17) is a tolerance that yields absorption with \( A > 99\% \).

For completeness, we also briefly show results for the spectral dependence of the absorbance, i.e. when moving away from the ENZ regime by varying the wavelength \( \lambda \) of the incident wave. This is shown in Fig. 5 for different thicknesses of the metamaterial layer. As seen, the broad and high absorbance only appears when \( \lambda/\lambda_z \) is relatively close to 1. For large deviations from \( \lambda/\lambda_z = 1 \), a conventional scenario with isolated absorption peaks arises.

Figure 4. Components of the normalized Poynting vector: (a) \( S_x/S_0 \) (along the interface) and (b) \( S_z/S_0 \) (normal to the interface) as functions of position \( z/\tau \) and angle \( \theta \). The Fermi energy is set at \( E_F = 200 \text{meV} \). The bottom panels reveal the spatial behavior of \( S_x \) and \( S_z \) for several \( \theta \). Vertical lines identify the vacuum/metamaterial interface at \( z/\tau = 1 \). The system parameters are the same as those used in Fig. 3(a).
We also examine the effects of loss on the absorption properties of the graphene-based metamaterial. In Fig. 6, we consider the same system that was studied in Fig. 2 except now substantial losses are included whereby the component of the permittivity orthogonal to the interface, $\varepsilon_{z,2}$, has a large imaginary component. Although the wide-angle absorption properties are seen to be reduced in the type-II case, remarkably for the type-I HMM, wide-angle absorption remains intact when considerable losses are present.

Figure 5. Absorbance vs. angle of incidence for different wavelengths of the incident wave, in order to show the behavior as one moves away from the ENZ regime. The thickness of the metamaterial is set to (a) $\tau/\lambda = 0.01$, (b) $\tau/\lambda = 0.6$, (c) $\tau/\lambda = 1.5$. We have fixed $E_F = 300$ meV, $\varepsilon_{z,2} = (4+0.1i)\varepsilon_0$, and $\lambda_z = 1.6$ mm.

Figure 6. Absorption vs. angle of incidence for the Type I case (top row) and Type II case (bottom row). Here we consider the high loss regime where $\text{Im}(\varepsilon_{z,2})/\varepsilon_0 = 1$, while all other parameters are the same as those used in Fig. 2.

We also examine the effects of loss on the absorption properties of the graphene-based metamaterial. In Fig. 6, we consider the same system that was studied in Fig. 2 except now substantial losses are included whereby the component of the permittivity orthogonal to the interface, $\varepsilon_{z,2}$, has a large imaginary component. Although the wide-angle absorption properties are seen to be reduced in the type-II case, the top panels (b) and (c) reveal that remarkably for the type-I HMM, wide-angle absorption remains intact when considerable losses are present.
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K.H. and J.L. contributed to the analytical and numerical calculations as well as the discussion of the results and the writing of the manuscript.

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