AdS$_2$ and quantum stability in the CGHS model

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Abstract

The two-dimensional anti-de Sitter space(AdS$_2$) is constructed in terms of the CGHS model. The geometric solutions are composed of the AdS vacuum and the AdS black hole which are locally equivalent but distinguishable by their mass. The infalling classical fields do not play any role but the quantum back reaction is crucial in the formation of the AdS vacuum and AdS black hole. In the presence of the AdS black hole, there does not exist any radiation, which is consistent with the constraint equations. Therefore the transition from the AdS black hole to the AdS vacuum is impossible, and they are quantum mechanically stable. We discuss the reason why the vanishing Hawking radiation appears in the AdS$_2$ black hole in contrast to asymptotically flat black holes.

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I. INTRODUCTION

Recently, there has been much attention to the anti-de Sitter (AdS) spacetime in connection with a calculation of the statistical entropy of the Bañados, Teitelboim, and Zanelli (BTZ) black hole [1,2] which is related to the higher-dimensional black holes [3], and the supergravities on the boundary of AdS spacetime and conformal field theory correspondence [4]. It is now natural to study AdS spacetime which may be essential to resolve the quantum gravity puzzles.

On the other hand, black holes as geometric solutions of gravity theory are expected to have Hawking temperatures [5] which means that one can detect some radiation from the black hole through quantum fluctuations. The calculation of Hawking radiation can be done on the generic black hole backgrounds. At first sight, this Hawking radiation seems to be applied to both asymptotically flat and nonflat geometries. In the former case, the Tolman temperature as a local temperature is coincident with the Hawking temperature in the asymptotic infinity, however, they are not compatible with each other in the latter case, especially in AdS black holes in two dimensions since the local temperature $T_{\text{local}} = \frac{T_H}{\sqrt{g_{00}}} = \frac{\sqrt{M}}{2\pi \sqrt{r^2 - M_l^2}}$ vanishes in the asymptotic infinity while the Hawking temperature is finite. Therefore, it would be interesting to study whether Hawking radiation appears or not in this black hole.

Some years ago, black hole evaporation and the back reaction of the geometry has been studied by Callan-Giddings-Harvey-Strominger (CGHS) [6] and subsequently by Russo-Susskind-Thorlacius (RST) [7] and in the many literatures [8]. In two-dimensions, the quantum back reaction of the geometry is more tractable compared to the other higher dimensional cases and may solve various quantum gravity problems [9].

In this paper, we study the quantum-mechanical generation of constant curvature spacetime of AdS$_2$ in terms of quantum back reaction by using the CGHS (RST) model and obtain the AdS vacuum defined as a lowest energy state of geometry and AdS black hole which is regarded as a massive state in Sect. II. Similarly to the CGHS model, we take the large
$N$ limit where $N$ is a number of conformal matter fields in order to maintain the validity of semiclassical approximations. The crucial difference from the CGHS solution is that we shall assume the constant dilaton background instead of the linear dilaton or spacetime-dependent dilaton background. In Sect. III, we shall calculate the Hawking radiation in this AdS$_2$ black hole and infer the Hawking temperature from it without resort to the conventional definition of Hawking temperature. We find that the transition from the AdS black hole to the AdS vacuum is impossible, and they are quantum mechanically stable. Finally some remarks and discussion will be given in Sect. IV.

II. QUANTUM MECHANICAL GENERATION OF ADS$_2$

In this section, we obtain the AdS black hole solution from the CGHS model. Let us now consider the two-dimensional low-energy string theory given by

$$S_{DG} = \frac{1}{2\pi} \int d^2 x \sqrt{-g} e^{-2\phi} \left[ R + 4(\nabla \phi)^2 + \frac{2}{l^2} \right],$$  \hspace{1cm} (1)

where $\phi$ is a dilaton field and the cosmological constant is negative as $\Lambda = -\frac{1}{l^2}$. The action for the classical and quantum matter are written in the form of [6,7]

$$S_{Cl} = \frac{1}{2\pi} \int d^2 x \sqrt{-g} \left[ -\frac{1}{2} \sum_{i=1}^{N} (\nabla f_i)^2 \right],$$  \hspace{1cm} (2)

$$S_{Qt} = \frac{\kappa}{2\pi} \int d^2 x \sqrt{-g} \left[ -\frac{1}{4} R \frac{1}{\Box} R - \frac{\gamma}{2} \phi R \right],$$  \hspace{1cm} (3)

where the anomaly coefficient is given by $\kappa = \frac{N-24}{12}$ and for a good semiclassical approximation, we take the large number of conformal matter fields. The parameter $\gamma$ is chosen associated with the models such as $\gamma = 0$ for the CGHS model and $\gamma = 1$ for the RST model. The constant $\gamma$ will be in fact restricted in later. The nonlocal Polyakov action [12] in Eq. (3) is written as, by introducing an auxiliary field $\psi$ for later convenience,

$$S_{Qt} = \frac{\kappa}{2\pi} \int d^2 x \sqrt{-g} \left[ \frac{1}{4} R \psi - \frac{1}{16} (\nabla \psi)^2 - \frac{\gamma}{2} \phi R \right].$$  \hspace{1cm} (4)

Then the effective total action is
\[ S_T = S_{DG} + S_M, \] 

(5)

where the matter part of the action is composed of two pieces of \( S_M = S_{Cl} + S_{Qt} \).

The equations of motion and the constraint equations with respect to metric for the action (5) are

\[ G_{\mu\nu} = T^M_{\mu\nu} \] 

(6)

where

\[
G_{\mu\nu} = \frac{2\pi}{\sqrt{-g}} \frac{\delta S_{DG}}{\delta g^{\mu\nu}} \\
e^{-2\phi} \left[ 2\nabla_\mu \nabla_\nu \phi + 2g_{\mu\nu} \left( (\nabla \phi)^2 - \Box \phi - \frac{1}{2l^2} \right) \right], \tag{7}
\]

\[
T^M_{\mu\nu} = -\frac{2\pi}{\sqrt{-g}} \frac{\delta S_M}{\delta g^{\mu\nu}} \\
= \frac{1}{2} \sum_{i=1}^{N} \left[ \nabla_\mu f_i \nabla_\nu f_i - \frac{1}{2} (\nabla f_i)^2 \right] + \frac{\kappa}{4} \left[ \nabla_\mu \nabla_\nu \psi + \frac{1}{4} \nabla_\mu \psi \nabla_\nu \psi - g_{\mu\nu} \left( \Box \psi + \frac{1}{8} (\nabla \psi)^2 \right) \right] \\
- \frac{\gamma \kappa}{2} \left[ \nabla_\mu \nabla_\nu \phi - g_{\mu\nu} \Box \phi \right]. \tag{8}
\]

The remaining equations of motion with respect to dilaton, conformal matter fields, and auxiliary field are given by respectively

\[ e^{-2\phi} \left[ R + 4\Box \phi - 4(\nabla \phi)^2 + \frac{2}{l^2} \right] = -\frac{\gamma \kappa}{4} R, \] 

(9)

\[ \Box f_i = 0, \] 

(10)

\[ \Box \psi = -2R. \] 

(11)

The trace of Eq. (6) yields

\[ e^{-2\phi} \left[ -2\Box \phi + 4(\nabla \phi)^2 - \frac{2}{l^2} \right] = \frac{\kappa}{2} R + \frac{\gamma \kappa}{2} \Box \phi \] 

(12)

where the right hand side of Eq. (12) are conformal anomaly and local counter terms. In the CGHS and RST models, the vacuum is a linear dilaton vacuum with a flat metric. The dilaton charge \( Q \) is nonzero, which is explicitly fixed through the condition of the asymptotic flatness of the black geometry,
\[ \phi(r) = -\frac{1}{2} Qr = -\frac{1}{\sqrt{2l}} r \]  

(13)

and it also receives quantum corrections \cite{8,7}.

We now present new geometric solution called AdS$_2$ for the constant dilaton background. Of course, the AdS$_2$ does not appear at the classical level, however if we consider the quantum back reaction of the geometry, then the nontrivial geometry appears due to the conformal anomaly. The dilaton field is now assumed to be a constant,

\[ \phi = \phi_0. \]  

(14)

Note that the constant dilaton solution is inconsistent at the classical level of $\kappa = 0$ as easily seen from Eq. (12) and it is only possible in the quantized theory. This means that in quantum gravity there may appear new kinds of geometries depending on the dilaton backgrounds. By using Eqs. (11) and (12), the $\phi_0$ is chosen as

\[ \phi_0 = -\frac{1}{2} \ln \left[ \frac{\kappa(2 - \gamma)}{4} \right]. \]  

(15)

From Eqs. (12) and (15) on the constant dilaton background, the effective curvature scalar is obtained as

\[ R = -\frac{2}{l_{\text{eff}}^2} \]  

(16)

where $l_{\text{eff}}^2 = \frac{l^2}{2 - \gamma}$. The parameter $\gamma$ is restricted to $\gamma < 2$ to obtain the negative curvature scalar. Then from Eq. (14), we can see the anomaly coefficient should be $\kappa > 0$ ($N > 24$), which is automatically valid in the large $N$ limit. Especially, for $\gamma = 1$ corresponding to the RST model \cite{1}, by adding ghost decoupling term \cite{10}, $\kappa$ can be shifted to $\frac{N}{12}$.

It is interesting to note that the constant curvature appears due to the quantum back reaction of the metric. This interesting feature is essentially on the basis of constant dilaton field. Most of cases, the linear dilaton vacuum and its time-dependence has been assumed in contrast to the present case.

\footnote{\textsuperscript{1}The nonflat solution for the RST model has been obtained in Ref. \cite{23}.}
In the conformal gauge,

\[ ds^2 = -e^{2\rho(\sigma^+, \sigma^-)} d\sigma^+ d\sigma^-, \]  

(17)

the equations of motion and constraints \( \{3\} \) are given by

\[ \partial_+ \partial_- \rho + \frac{1}{4l_{\text{eff}}^2} e^{2\rho} = 0, \]  

(18)

\[ \partial_+ \partial_- f = 0, \]  

(19)

\[ T_{\pm \pm}^{M} = T_{\pm \pm}^{\text{cl}} + T_{\pm \pm}^{\text{Qt}} = 0, \]  

(20)

where \( T_{\pm \pm}^{\text{cl}} = \frac{1}{2} \sum_{i=1}^{N} (\partial_\pm f_i)^2 \), \( T_{\pm \pm}^{\text{Qt}} = -\kappa [ (\partial_\pm \rho)^2 - \partial_\pm^2 \rho] - \kappa t_\pm \), and \( t_\pm \) reflects the nonlocality of the conformal anomaly \( \{3\} \). Solving the equations of motion in the conformal gauge yields \( \{13-16\} \)

\[ e^{2\rho} = \frac{M}{\sinh^2 \left[ \sqrt{M} (\sigma^+ - \sigma^-) / 2l_{\text{eff}} \right]}, \]  

(21)

\[ f_i = f_i^{(+)}(\sigma^+) + f_i^{(-)}(\sigma^-), \]  

(22)

where \( M \) is an integration constant. The AdS\(_2\) vacuum is now defined by

\[ e^{2\rho} = \frac{4l_{\text{eff}}^2}{(y^+ - y^-)^2}, \]  

(23)

and we assume that \( M \geq 0 \). For \( M \to 0 \), the solution \( \{21\} \) exactly comes down to the AdS\(_2\) vacuum and the local geometries are equivalent in that the curvature scalar is independent of the parameter \( M \). The parameter \( M \) describes just only the existence of the horizon of the geometry. Furthermore, the solution should be satisfied with the following constraint equations,

\[ \frac{1}{2} \sum_{i=1}^{N} (\partial_\pm f_i^{(\pm)})^2 (\sigma^{\pm}) - \frac{\kappa M}{4l_{\text{eff}}^2} - \kappa t_\pm (\sigma^{\pm}) = 0. \]  

(24)

We shall assume there does not exist the classical flux, which is in fact of no relevance to the formation and evaporation of black hole as seen in Eq. \( \{18\} \). In fact, this classical flux of infalling matter fields cannot be connected with \( M \) unless \( t_\pm = 0 \). If we fix \( t_\pm = 0 \)
in the black hole background, the Virasoro anomaly appears in the constraints under the coordinate transformation and the theory becomes inconsistent. On the other hand, if the classical matter fields exist, then the boundary condition is just changed according to Eq. (20). Either way, the classical matter field is not crucial in AdS$_2$ since the constant curvature is independent of infalling matter energy density. Therefore we simply set $T_{\pm \pm}^{cl} = 0$.

III. HAWKING RADIATION OF ADS$_2$ BLACK HOLE

Let us now study the Hawking radiation of this AdS$_2$ black hole. The Hawking radiation in two dimensions is usually given by the anomalous transformation of the energy-momentum tensor. This fact comes from the requirement of the Virasoro anomaly free condition of the energy-momentum tensors. Therefore $T^{Qt}_{-\sigma^{-}}$ should be a tensor without any anomaly under the coordinate transformation. The Hawking radiation seems to be a global effect and it is determined by the boundary effect given by the integration constant $t_-$. Then it is given by

$$-\kappa t_-(\sigma^-) = -\frac{\kappa}{2}\{y^-, \sigma^-\} = \frac{\kappa M}{4l_{\text{eff}}^2}$$  \hspace{1cm} (25)

where $y^\pm = \frac{2l_{\text{eff}}}{\sqrt{M}}\tanh\frac{\sqrt{M}\sigma^\pm}{2l_{\text{eff}}}$ and $\{y^-, \sigma^-\}$ is a Schwartzian derivative. At first sight, the Hawking radiation seems to be a constant and it is compatible with the Hawking temperature given by $T_H = \frac{\sqrt{M}}{2\pi l_{\text{eff}}}$ since $-\kappa t_- = \kappa\pi^2T_H^2$. However, this is not the case. The quantum-mechanical energy-momentum tensor is defined as

$$h(\sigma^+, \sigma^-) = T^{Qt}_{-\sigma^-}(\sigma^+, \sigma^-) = T^\text{Bulk}_{-\sigma^-} + T^\text{boundary}_{-\sigma^-} = 0$$  \hspace{1cm} (26)

$^2$See Ref. [11] for extensive reviews
since $T_{-}^{Q_t}$ is composed of both bulk and boundary contribution

$$T_{-}^{\text{Bulk}} (\sigma^+, \sigma^-) = -\kappa \left[ (\partial_- \rho)^2 - \partial_-^2 \rho \right]$$

$$= -\frac{\kappa M}{4l_{\text{eff}}^2}, \quad (27)$$

$$T_{-}^{\text{boundary}} (\sigma^+, \sigma^-) = -\kappa t_-$$

$$= \frac{\kappa M}{4l_{\text{eff}}^2}$$

respectively, and they are exactly canceled out, which is consistent with Eq. (20). The negative contribution of the bulk part Eq. (27) is calculated by the use of Eq. (21). It is interesting to note that this part is constant, which is in contrast with the asymptotically flat case, for instance, the CGHS black hole. For the asymptotically spatial infinity, this null relation is valid, and this means that there does not exist Hawking radiation on the AdS black hole background. Therefore, any quantum transition is impossible from the black hole state to the AdS$_2$ vacuum through the Hawking radiation.

At this stage, it seems to be appropriate to compare Hawking radiation in the CGHS model with the present AdS$_2$ black hole. For an asymptotically flat black hole of the CGHS model, the Hawking radiation is just given by [3]

$$h(\sigma^-) = T_{-}^{Q_t} (\sigma^+, \sigma^-) |_{\sigma^+ \to \infty}$$

$$= T_{-}^{\text{Bulk}} |_{\sigma^+ \to \infty} + T_{-}^{\text{boundary}} |_{\sigma^+ \to \infty}$$

$$\approx -\kappa t_-(\sigma^-) \quad (29)$$

where

$$T_{-}^{\text{Bulk}} (\sigma^+, \sigma^-) |_{\sigma^+ \to \infty} = -\kappa \left[ (\partial_- \rho)^2 - \partial_-^2 \rho \right] |_{\sigma^+ \to \infty}$$

$$= -\frac{1}{96l^2} \left[ 1 - \frac{1}{\left( 1 + \sqrt{2a\epsilon\sqrt{\kappa_0}(\sigma^- - \sigma^+ + \sigma_0) \right)}^2 \left( \sigma^+ \to \infty \right) \right]$$

$$= 0$$

$$T_{-}^{\text{boundary}} (\sigma^+, \sigma^-) |_{\sigma^+ \to \infty} = -\kappa t_-$$
\[
= \frac{1}{96l^2} \left[ 1 - \frac{1}{\left( 1 + \sqrt{2}a e^{\frac{1}{2\sqrt{l}}(\sigma^-)} \right)^2} \right] \quad (\sigma^+ \to \infty)
\]

and \(a\) is a proportional to the infalling flux \[3\]. In the asymptotic null infinity \((\sigma^+ \to +\infty)\), Hawking radiation is only due to the boundary term \(t_\pm\) since the bulk contribution vanishes at the null infinity, while for the AdS\(_2\) case both bulk and boundary effects are simultaneously considered in the Hawking radiation process since they are all constants.

One may reconsider whether the infalling matter field affects the formation and evaporation of AdS black hole or not. This problem can be studied by using the Jackiw-Teitelboim model \[18\],

\[
S_{JT} = \int d^2x \sqrt{-g} \Phi \left[ R + \frac{2}{l^2} \right]
\]

where \(\Phi\) is an auxiliary field. From the beginning, we assume the AdS vacuum or AdS black hole background and consider the infalling conformal matter field as in the CGHS model. Then the dynamical equation of motion with respect to \(\rho\) in the conformal gauge does not contain any information of matter fields similarly to our model.

We now exhibit some of equations different from the CGHS model \((\gamma = 0)\) when we consider the Polyakov induced gravity action \(3\) with the JT model,

\[
\partial_+ \partial_- \Phi - \frac{1}{4l^2} e^{2\rho} \left( \frac{\kappa}{2} - \Phi \right) = 0,
\]

\[
\partial^2 \Phi - 2\partial_\pm \rho \partial_\pm \Phi = T^M_{\pm\pm}.
\]

In this case, the solutions are given by

\[
\Phi^{-1} = -\frac{1}{M} \tanh \left[ \frac{\sqrt{M}(\sigma^+ - \sigma^-)}{2l} \right],
\]

\[
e^{2\rho} = \frac{M}{\sinh^2 \left[ \frac{\sqrt{M}(\sigma^+ - \sigma^-)}{2l} \right]},
\]

where we simply assume \(T^c_{\pm\pm} = 0\). Note that the constraint equations \(3\) should be Virasoro anomaly free such that \(T^c_{\pm\pm}\) can be transformed as the primary operator in conformal field theory,
In this case also, the bulk and boundary effects contribute to the Hawking radiation and they are exactly canceled out as

\[ T_{\pm \pm}^\text{Bulk}(\sigma^+, \sigma^-) = -\kappa \left[ (\partial_- \rho)^2 - \partial_-^2 \rho \right] = -\frac{\kappa M}{4l^2}, \]

(36)

\[ T_{\pm \pm}^\text{boundary}(\sigma^+, \sigma^-) = -\kappa t_- = \frac{\kappa M}{4l^2}. \]

(37)

Therefore, as far as the energy-momentum tensor of vacuum state vanishes \( T_{\pm \pm}^\text{Qt}(y^\pm) = 0 \), the radiation is impossible. Note that in Refs. [25, 21], the Hawking radiation is proportional to the black hole mass due to Eq. (37). The crucial difference between them comes from the contribution of so-called bulk part of energy-momentum tensors. However, in Refs. [25, 20, 21], the models are in fact different in that the dilaton field is not constant, and the back reaction of the geometry may depend on the dilaton field with the metric, so the quantum-mechanical energy-momentum tensor may be different from that of our CGHS model in Sect. III.

IV. DISCUSSION

In this work, we have shown that the AdS black hole solution is possible in the CGHS model. This is in fact realized in the quantized theory by assuming the constant dilaton background. Therefore, in the quantum level, there exist two kinds of black hole solutions, the CGHS and AdS solutions depending on the dilaton charge.

However, one might think that the present result on null Hawking radiation of the two-dimensional AdS black hole is doubtful. And it may be concluded that the result may be dependent on details of some boundary conditions of AdS geometry. So we now reconsider another way to clarify whether the Hawking radiation comes out in this black hole or not.
A massless scalar field as a test field is considered on the AdS$_2$ black hole background, then the greybody factor of this black hole can be evaluated through the wave function of the test field. In this case, if we allow boundary condition compatible with the equation of motion of scalar field, then the wave function at the horizon is decomposed into ingoing and outgoing modes in the asymptotic infinity. Remarkably, the amplitude of outgoing wave corresponding to the Hawking radiation is zero for the massless scalar field, and the absorption coefficient is 1. This peculiar phenomena of vanishing Hawking radiation will be discussed in detail in elsewhere [24] by comparing with other models.

On the other hand, there may be another reason why the above trivial result comes out. It seems to be that the two-dimensional AdS black hole is in fact locally equivalent to the AdS vacuum in that the curvature scalar is constant which is independent of the parameter $M$. This parameter may be a coordinate artifact since it can be removed by using the coordinate transformation as $y^\pm = \frac{2l_{\text{eff}}}{\sqrt{M}} \tanh \sqrt{\frac{M}{2l_{\text{eff}}}} \sigma^\pm$ where $y^\pm$ describes the vacuum geometry while $\sigma^\pm$ does the AdS black hole. Therefore, one can think that the two-dimensional AdS black hole and vacuum in some sense belong to the equivalent class. The only difference between them comes from the fact that the parameter $M$ just globally describes the location of horizon in the geometry. Furthermore, if the parameter $M$ turns out to be a coordinate artifact, then it is meaningless to interpret it as a conserved quantity as a black hole mass. The explicit ADM mass calculation on the background metric of AdS vacuum [25], gives interestingly vanishing ADM mass as far as we consider the constant dilaton background in two dimensions, which is in contrast with the case of the three-dimensional BTZ black hole. This supports that the parameter $M$ is not a conserved mass but just a gauge artifact. At this stage, one might again think that even though $M$ is trivial, the horizon exists for the nonvanishing $M$, so there may be thermal radiation similar to the Rindler space. In the Rindler space, the accelerated observer detects the thermal radiation which is related to the coordinate change, corresponding to the Schwartzian discussed in Eq. (25) and (28) in our case. As pointed out in Sec. III, in the AdS case, there exists another contribution, viz, the bulk radiation Eq. (27). Therefore, the net radiation can be zero.
The final point to be mentioned is that intuitively how come the AdS$_2$ black hole does not radiate. The similar phenomena can be found in near horizon geometry of extremal charged black holes of two-dimensional Maxwell-dilaton gravity \cite{22} or spherically symmetric reduced Reissner-Nordström solution or most of D-brane solutions. Since the AdS geometry comes from the extremal cases from the string theory point of view, so that Hawking radiation does not occur. Therefore the present AdS$_2$ black hole might be an effective theory of extremal black holes whose Hawking temperature is zero.

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