AN ADAPTIVE DYNAMIC PROGRAMMING METHOD FOR TORQUE RIPPLE MINIMIZATION OF PMSM

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Abstract. The imperfect sinusoidal flux distribution, cogging torque, and current measurement errors can cause periodic torque ripple in the permanent magnet synchronous motor (PMSM). These ripples are reflected in the periodic oscillation of the motor speed and torque, causing vibration at low speeds and noise at high speeds. As a high-precision tracking application, ripple degrades the application performance of PMSM. In this paper, an adaptive dynamic programming (ADP) scheme is proposed to reduce the periodic torque ripples. An optimal controller is designed by iterative control algorithm using robust adaptive dynamic programming theory and strategic iterative technique. ADP is combined with the existing Proportional-Integral (PI) current controller and generates compensated reference current iteratively from cycle to cycle so as to minimize the mean square torque error. As a result, an optimization problem is constructed and an optimal controller is obtained. The simulation results show that the robust adaptive dynamic programming achieves lower torque ripple and shorter dynamic adjustment time during steady-state operation, thus meeting the requirements of steady speed state and the dynamic performance of the regulation system.

1. Introduction. With obvious advantages over other motors, such as power density enhancement, efficiency improvement, and increased torque, PMSM has been widely developed in the electric drive system. However, the output of electromagnetic torque ripple greatly limits the application of PMSM in high control accuracy field\textsuperscript{22,26}. In recent years, the problem of speed fluctuations in PMSM has become a hot topic and researchers have proposed a variety of methods to suppress the torque ripple of PMSM. The researchers’ work is divided into two categories: the improvement in the main body structure design of motors and the optimization of software control algorithms\textsuperscript{13}.

The methods for improving the internal structure of the motor have focused on the stator slot \textsuperscript{11}, the fractional slot \textsuperscript{5} and the toothless stator structure \textsuperscript{12}. Although these methods weaken the torque ripple to a certain extent, they are difficult to implement and the design cost is high, which is not often considered in
practice. Since the control algorithm is an essential part of PMSM and no additional hardware equipment is required, the control algorithm is more suitable for ripple minimization. In terms of control algorithms, in the 20th century, the open-loop compensation method has been used to store the pre-programmed current in the look-up table to eliminate the torque fluctuation at a specific position. However, this method must obtain the close relationship between the torque ripple and the excitation current firstly. Due to the open loop control, some small changes may exacerbate torque ripple[7]. However, the popular method is based on the torque estimation of the motor system variable, and the torque is estimated online by the measured torque ripple component[3,14,24]. This method uses a torque sensor, which undoubtedly increases the error in current-to-voltage conversion and is not observed in many mechanical parts, such as cogging torque and oscillating load. Y. A. I. Mohamed and E. F. El-Saadany have used a current control scheme for torque ripple minimization and robust current regulation as well as an adaptive internal model of the PMSM vector drive[13]. Chu et al.[4] have proposed a speed loop control strategy, which used a PI regulator in parallel with the repeat controller. This configuration improves the steady-state characteristics of the system and suppresses the periodic velocity fluctuations without affecting the dynamic performance. However, it does not give rigorous mathematical proof to ensure convergence. The iterative learning control is a control algorithm for online learning compensation error at fixed time intervals[8,10]. Torque pulsation suppression can be regarded as a kind of periodic interference suppression. Iterative control can compensate for traditional PI control to achieve torque ripple suppression[17,23,25]. However, iterative learning control is especially sensitive to system parameter uncertainty and has a weak anti-interference ability.

It is well-known that traditional control methods require the perfect knowledge of system dynamics suffers from the curse of dimensionality. To avoid these difficulties, Werbos first pointed out in [20] that adaptive approximation to the Hamilton-Jacobi-Bellman (HJB) equation can be achieved by designing appropriate reinforcement learning systems. On the basis of Webos and inspired by biological learning behavior, ADP is a non-model based on online information control to solve the optimal method[18,19,21]. The traditional dynamic programming requires a complete system from which a series of required parameters can be obtained. To avoid these difficulties, some researchers have proposed dynamic programming based on Linear Quadratic Regulator (LQR) theory[6], which can consider the impact of optimal policy to approximate the optimal cost or its gradient. Lewis[9] has proposed that ADP could obtain the optimal control strategy by numerical calculation under inaccurate conditions, solving HJB equation and algebraic curve (ARE) with online information, and proving the stability of ADP. ADP is developed to address the presence of dynamic uncertainty in linear and nonlinear dynamical systems. See Fig. 1 for a model for ADP.

As mentioned above, the main causes of torque ripples include cogging torque, flux harmonics and current measurement errors, which are uncertain and dynamic. The purpose of this paper is to apply the ADP control method to find online optimal stabilizing controllers for the torque ripples of PMSM. ADP can be considered as a special kind of feed-forward control that has a context memories as an additional layer, and the context memories as the extra memories that memorize the previous states and feed to processing module after one-step time delay. Therefore, the ADP control method does not need to consider uncertainty relative to other control
methods, but it needs some prior knowledge to ensure its precise system dynamics. In this paper, a new strategy utilizing iterative method which using context memories as feed-forward controllers to compensate for feedback corrections is proposed for finding the online adaptive optimal controllers for continuous-time linear systems with completely unknown system dynamics. The proposed method uses an approximate adaptive dynamic programming technique to iteratively solve the algebraic Riccati equation by using state and input of online information without the knowledge of the system matrix. In addition, all iterations can be performed by repeatedly using the same state and input information over some fixed time interval. This paper proves the convergence condition of the ADP controller through Lebesgue-p norm and Young-Inequality. Through experimental verification, the ADP control method can be considered to be used for torque ripple minimization in PMSM systems with prior knowledge. The remainder of this paper is organized as follows. The section 2 briefly describes the main causes of torque ripples. In Section 3, the mathematical model of the PMSM and the relationship between velocity ripple and torque ripple is given. The section 4 analyzes the adaptive dynamic programming algorithm and the section 5 proves convergence. The section 6 presents the experimental results. Finally, concluding remarks are made in the section 7.

2. Analysis of torque ripples. The main causes of torque ripples include cogging torque, flux harmonics, and current measurement errors, which are briefly analyzed as follows. Cogging torque is caused by the interaction between the magnetic flux and stator slots. It is manifested by the tendency of the rotor to align with a number of stable positions, even when the machine is unexcited. It arises from the tangential force between the permanent magnet and the armature tooth so that the rotor of the permanent magnet motor has a tendency to align with the stator in a certain direction, trying to position the rotor at certain positions. As a result, an oscillating torque is produced. The cogging torque causes the motor torque to fluctuate, generating vibration and noise, and the speed fluctuation occurs. The cogging torque prevents the motor from running smoothly and affects the performance of the motor.
Flux harmonics is another main source of torque ripples. Due to the non-sinusoidal flux density distribution in the air gap, the resultant flux linkage between the permanent and stator currents contains harmonics, which appear as 3rd, 5th, 7th, 11th... in the a-b-c frame. The triple harmonics are absent in the Y-connected stator windings.

\[
\begin{bmatrix}
\psi_{md} \\
\psi_{mq}
\end{bmatrix} = 
\begin{bmatrix}
\psi_1 + (\psi_5 + \psi_7) \cos(6\theta) + (\psi_{11} + \psi_{13}) \cos(12\theta) + \cdots \\
(-\psi_5 + \psi_7) \sin(6\theta) + (-\psi_{11} + \psi_{13}) \sin(12\theta) + \cdots \\
\psi_{d0} + \psi_{dq} \cos(6\theta) + \psi_{d12} \cos(12\theta) + \cdots \\
\psi_{q0} \sin(6\theta) + \psi_{q12} \sin(12\theta) + \cdots
\end{bmatrix}
\]

From the equation (1), when \(i_d=0\), the corresponding harmonics appear with the sixth and the multiples of the sixth-order harmonics in the d-q frame, which can be expressed as:

\[
T_m = T_0 + T_6 \cos(6\theta) + T_{12} \cos(12\theta) + \cdots
\]  

(2)

Where, \(T_0\), \(T_6\) and \(T_{12}\) are the dc component of the 6th and 12th harmonic torque magnitudes, respectively. Equation (2) indicates that the 6th and 12th torque harmonics are produced mainly due to the non-sinusoidal flux distributions’ periodic nature. In addition to the torque ripple discussed above, due to the physical limitations of the AD converter and its accuracy of the information transfer, the mathematical signal cannot fully represent the corresponding analog signal. The current offset and error will cause the torque to oscillate at a specific frequency.

In addition, the stator current is measured by a current sensor and converted to a voltage. However, any voltage fluctuation can affect measurement error. This analysis indicates that the electromagnetic torque consists of a fundamental component together with the 1st, 2nd, 6th, 12th harmonic components, etc. The control objective is to suppress these periodic torque ripples.

3. Mathematical model.

3.1. Mathematical model of A PMSM. With the assumptions that the PMSM iron core is unsaturated and the eddy currents and hysteresis losses are negligible, the voltage equations of d and q axis in the synchronously rotating reference frame can be expressed as follows:

\[
\begin{aligned}
\frac{du_d}{dt} &= -\frac{R_d}{L_d} i_d + p_n w i_q + \frac{1}{L_d} u_d \\
\frac{du_q}{dt} &= -\frac{R_q}{L_q} i_q - p_n w + \frac{1}{L_q} u_q - \frac{p_n \omega_f}{L_q} \\
\frac{dw}{dt} &= -\frac{T_L}{J} + B w - \frac{E_m}{J} \\
\end{aligned}
\]

(3)

Where, \(u_d, u_q, i_d\) and \(i_q\) are the d, q-axes stator voltages and stator currents respectively; \(R, L_d\) and \(L_q\) are the stator resistance and the d, q-axes equivalent inductances respectively; \(w\) is the rotor angular velocity; \(\phi_f\) is the permanent magnet flux; \(p_n\) is the number of pole pairs; \(J\) is the moment of inertia; \(B\) is the linear viscous friction coefficient; \(T_l\) is the load torque; \(T_m\) is the motor torque. For the above electromechanical model, the true states of \(\theta, w, i_d, i_q, i.e.\) are set. The mathematic model of the electromagnetic torque can be expressed as:

\[
T_m = 3 \frac{2}{p} (\psi_d i_q - \psi_q i_d).
\]

(4)

Setting \(i_d=0\), the maximum torque can be guaranteed. Equation (4) is converted to:

\[
T_m = 3 \frac{2}{p} \psi_d i_q = k_{t_q} i_q.
\]

(5)
3.2. **Speed ripples induced by torque ripples.** The transfer function between the motor mechanical angle speed $\omega_m$ and the electromagnetic torque $T_m$ is expressed as:

$$\omega(s) = \frac{T_m(s) - T_l(s)}{Js + B},$$

where, $J$ is the moment of inertia; $B$ is the linear viscous friction coefficient; $T_m(s) = f(\psi_d, i_q, \omega_m)$. It can be concluded that the speed will oscillate at the same harmonic frequencies as those of $T_m$, especially at low operating speeds. In order to reduce the torque ripple, the speed ripple needs to be reduced. Fig. 2 is the PMSM control block diagram based on ADP controller. From this figure, ADP can be considered as a special kind of feed-forward control that has a context memories as an additional layer, and the context memories as the extra memories that memorize the previous states and feed to processing module after one-step time delay.

![Figure 2. The controller block diagram of PMSM](image)

4. **Adaptive dynamic programming algorithm.** Adaptive Dynamic Programming is an algorithm for the design of robust optimal controller under dynamic uncertainties. For a qualitative description of the ADP operation, the following dynamic system is considered:

$$\dot{x} = Ax + Bu,$$

where, $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control input, $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$ are constant matrices, that are stabilizable. It is assumed that there is a known constant matrix $K_0$ such that $A - BK_0$ is a Hurwitz matrix. The known constant matrix $K_0$ can be obtained by calculating the value range from the simulation software under the condition that $A - BK_0$ is a Hurwitz matrix. The control objective is to find, if possible, an online optimal control policy $u^* = -K^*x$.

The following integral-quadratic cost is minimized:

$$J = \int_0^\infty (x^T Q x + u^T R u) d\tau,$$

where, $Q \geq 0$ and $R > 0$ real symmetric matrices with $A$ and $Q^{1/2}$ as an observable pair. To begin with, a stabilizing initial gain matrix $K_0$ is selected, which ensures that $A - BK_0$ is Hurwitz. Then, $u_0 = -K_0 x + e$ is applied as the control input with
Theorem 4.1. (Kleinman, 1968). Under the PE condition on ARE equation (10) in LQR theory, the condition for parameter convergence in adaptive control. The exploration noise error is persistently exciting, which is a standard condition for optimal solutions of the following equation:

\[ P_{t+1} = x^T P_k x = x^T P_k (A - BK) + (A - BK)^T P_k x + 2x^T P_k B(u_0 + K_k x) \]

\[ = -x^T (Q + K_k R K) x + 2(u_0 + K_k x)^T R K_{k+1} x. \] \hspace{1cm} (9)

Consequently, for \( k = 0, 1, 2, \ldots \), the following iterative equation can be derive:

\[ x^T P_k x |_{t_i}^{t_{i+1}} \int_{t_i}^{t_{i+1}} [-x^T (Q + K_k R K) x + 2(u_0 + K_k x)^T R K_{k+1} x] d\tau. \] \hspace{1cm} (10)

The existence and uniqueness of solution \((P_k, K_{k+1})\) in the above Eq. (10) rely on the fact that the exploration noise error is persistently exciting, which is a standard condition for parameter convergence in adaptive control.

The following result shows that these sequences \(\{P_k\}\) and \(\{K_k\}\) converge to their optimal solutions \(P^*\) and \(K^*\), respectively. \(P^*\) corresponds to the solution to the ARE equation (10) in LQR theory.

**Theorem 5.1.** (Kleinman, 1968). Under the PE condition on \(e(t)\), it is assumed that \(P_k = P_k^T\) and \(K_{k+1}\) can be uniquely solved from (4), for all \( k = 0, 1, \ldots, l - 1 \). Then, \( \lim_{k \to \infty} P_k = P^* \), \( \lim_{k \to \infty} K_k = K^* \) where \( K^* = R^{-1} B^T P^* \), \( P^* > 0 \) is the symmetric solution of the following equation:

\[ A^T P + PA + Q - PBR^{-1}B^T P = 0. \] \hspace{1cm} (11)

An online policy iteration algorithm is summarized as follows:

1. Let 0 → \( k \)
2. Solve \( P_k \) and \( K_{k+1} \) from equation (10).
3. Let \( k \leftarrow k + 1 \), and repeat until \(|P_k - P_{k-1}| \leq \varepsilon\) for \( k \geq 1 \), where the constant \( \varepsilon > 0 \) can be any predefined small threshold.
4. Use \( u = -K_k x \) as the approximated optimal control policy.

5. **Proof of convergence.** As known by formula (3), the PMSM mathematical model can be written as:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + f(x(t)) + Bu(t) \\
y(t) &= Cx(t)
\end{align*}
\] \hspace{1cm} (12)

where \( x(t) \in \mathbb{R}^3 \), \( y(t) \in \mathbb{R} \) and \( f(x(t)) \) is nonlinear function and

\[
A = \begin{bmatrix}
\frac{R}{L} & 0 & -p_n\phi_f \\
0 & \frac{R}{L} & 0 \\
p_n\psi_d & 0 & \frac{R}{L}
\end{bmatrix},
B = \begin{bmatrix}
0 & 0 \\
\frac{1}{L} & 0 \\
0 & \frac{1}{L}
\end{bmatrix},
C = \begin{bmatrix}
1 & 0 & 0 \\
\frac{L}{p} & -p_n\omega_i_d & p_n\omega_i_q
\end{bmatrix}.
\]

\[ f(x(t)) = \left[ -\frac{L}{p} -p_n\omega_i_d \right] p_n\omega_i_q \].

**Theorem 5.1.** Assuming a vector function \( f : [0, T_0] \to \mathbb{R}^m \), \( f(t)[f^1(t), \ldots, f^m(t)]^T \).

\[
\|f(\cdot)\|_p = \left( \int_0^{T_0} \left( \max_{1 \leq i \leq m} |f^i(t)| \right)^p dt \right)^{1/p}, 1 \leq p \leq \infty.
\] \hspace{1cm} (13)
As known from the reference [1], \( \lim_{p \to \infty} \| f(\cdot) \| = \| f(\cdot) \|_\infty = \| f(\cdot) \|_{\sup} \), the upper bound norm \( \| f(\cdot) \|_{\sup} = \sup (\max \left| f(\cdot) \right|) \) is a special case of the Lebesgue-p norm.

**Lemma 5.2.** (Young inequality). Assuming that the scalar function \( g \in R \) and \( h \in R \) are integrable, and \( 1 \leq p, q, r \leq \infty, 1/r = 1/p + 1/q - 1, \) then \( g \cdot h \in R \) and \( \| g \cdot h(\cdot) \|_r \leq \| g(\cdot) \|_p \| h(\cdot) \|_q \), when \( r = p, q = 1 \), the Young inequality can be equal \( \| g \cdot h(\cdot) \|_1 = \| g(\cdot) \|_p \| h(\cdot) \|_q \).

**Theorem 5.3.** Let \( \{ e_k(\cdot) \} = \{ e_k(t) \} e_k(t) \in R, k = 1, 2, ..., t \in [0, T_0] \) be a set of convergence functions whose error limit is \( e_k^*(\cdot) \), when \( k \to \infty \), there is \( \Delta e_k(t) = e_k(t) - e_k^*(t) \), \( \lim_{k \to \infty} \| e_k(\cdot) - e_k^*(\cdot) \|_p = 0 \) and it is defined as:

\[
q_p = \{(e_k(\cdot), e_k(\cdot)) = \begin{cases} 
0, & \text{if } \Delta e_k(t) \equiv 0, t \in [0, T_0] \\
\lim_{k \to \infty} \sup_{t \in [0, T_0]} \frac{\| \Delta e_k(\cdot) \|_p}{\| e_k(\cdot) \|_p}, & \Delta e_k(t) \neq 0, t \in [0, T_0]
\end{cases}
\]

Setting \( \omega^*(t) \) as the expected speed of the motor, the adaptive dynamic programming control strategy is constructing as:

\[
u_{k+1}(t) = u_k(t) + \Phi_p e_k(t) + \Phi_d \dot{e}_k(t), t \in [0, T_0], k = 1, 2...
\]

Where, \( u_k(t) \) is q-axis reference compensation current \( \Delta i_q^* \); \( e_k(t) \) is speed error \( \omega^*(t) - \omega(t) \); \( \Phi_p \) and \( \Phi_d \) are proportional and differential gains, respectively; \( k \) is the number of iterations. Combining (13) and (14), system tracking error can be obtained:

\[
e_{k+1}(t) = \omega^*(t) - \omega_{k+1}(t)
\]

\[
= \left[ \omega^*(t) - \omega_k(t) \right] - \left[ \omega_{k+1}(t) - \omega_k(t) \right]
\]

\[
= e_k(t) - \left[ \omega_{k+1}(t) - \omega_k(t) \right]
\]

\[
= e_k(t) - C \int_0^t \exp(A(t - \tau)) B \Phi_p e_k(\tau) d\tau - C \int_0^t \exp(A(t - \tau)) B \Phi_d \dot{e}_k(\tau)
\]

\[
= e_k(t) - C \int_0^t \exp(A(t - \tau))(B \Phi_p + A B \Phi_d) e_k(\tau) d\tau - C \exp(A(t - \tau)) B \Phi_d e_k(\tau)|_{\tau=0}^{\tau=t}
\]

(16)

For \( e_k(0) = \omega^*(0) - \omega_k(0) = 0 \), the above formula can be transformed as:

\[
e_{k+1}(t) = (1 - C B \Phi_d) e_k(t) - C \int_0^t \exp(A(t - \tau))(B \Phi_p + A B \Phi_d) e_k(\tau) d\tau
\]

(17)

Calculating the Lebesgue-p norm for the above formula, and using convolution Young inequality, it can be calculated as:

\[
\| e_{k+1}(\cdot) \|_p \leq |1 - C B \Phi_d| \| e_k(\cdot) \|_p + |C \exp(A(\cdot))(B \Phi_p + A B \Phi_d)|_1 \| e_k(\cdot) \|_p
\]

\[
= (|1 - C B \Phi_d| + |C \exp(A(\cdot))(B \Phi_p + A B \Phi_d)|_1) \| e_k(\cdot) \|_p
\]

(18)

Let \( \alpha = (|1 - C B \Phi_d| + |C \exp(A(\cdot))(B \Phi_p + A B \Phi_d)|_1) \), we can get:

\[
\| e_{k+1}(\cdot) \|_p \leq \alpha \| e_k(\cdot) \|_p, k = 1, 2, 3...
\]

(19)

Consequently, the sufficient condition for the controller to converge is: \( \lim_{k \to \infty} e_k(t) = 0 \) and \( \alpha < 1 \).
6. Implementation of drive system. Fig. 3 is the controller block diagram of minimum ripple system under the ADP controller. Firstly, the expected reference speed $\omega^*$ is given. Under the instantaneous state, the reference speed is expected to provide a reference compensation current $i^*_q$. In order to minimize the torque ripple, the ADP controller provides additional compensation current combined with $\Delta i_q$ to achieve a stable state. $i_d$ and $i_q$ generate voltage through d axis and q axis current controllers respectively. Then, FOC is used to control PMSM. Finally, speed and rotor information can be obtained from sensors and feedback the desired speed to achieve a complete closed loop.

The parameters of the surface-mounted PMSM (delta-connected stator windings) used are listed in Table 1.

| Characteristic             | Symbol | Value       |
|----------------------------|--------|-------------|
| Stator phase resistance    | R      | 2.875       |
| d and q-axes               | $L_d = L_q$ | 8.5mH     |
| Number of pole pairs       | $p_n$  | 4           |
| Viscous damping            | B      | 0.008 N.m.s |
| Torque constant            | $K_t$  | 1.05 N.m    |
| Rotational inertia         | J      | 0.003kg.m2  |

The effectiveness of ADP will be verified from two perspectives of low speed, high speed and low load, high load. The performance is compared with that of conventional PI torque control. In the simulation, the simulation step is $T_s=1e-6$s, the three-phase SVPWM inverter with a switching frequency of kHz.

The parameters of the current PI controllers in this method are q-axis controllers: $K_{ip} = L_q \times 1100$ and $K_{iq} = R \times 1100$; d-axis controllers: $K_{dp} = L_q \times 1100$ and $K_{dq} = R \times 1100$. The parameters of the ADP controllers $\Phi_p = 0.07$ and $\Phi_p = 0.9$. To verify the effectiveness of the proposed ADP schemes, simulations are carried out by using different expected speed as $\omega^* = 50$ and 500RPM different extra load 1.6Nm and 9.0Nm. Considering the cost, 20 iterations are more appropriate.

Fig. 4(a), 5(a) show the speed response and Fig. 4(b), 5(b) are the Fourier analysis of the corresponding speed of the ADP controller.
Figure 4. Speed response with ADP controller at 500r/min

Figure 5. Speed response Fourier analysis with ADP controller at 500r/min

Figure 6. Speed response with ADP controller at 50r/min

Figure 7. Speed response Fourier analysis with ADP controller at 50r/min
Fig. 4 shows the speed response with a reference speed of 500 r/min under the ADP controller, and the system reaches a stable speed after the setting time $t_s = 0.016s$. The stabilized speed range is 499.8802-500.1253 r/min, and the fluctuation range is 0.2451. Fig. 8 (a) shows the speed response with a reference speed of 500 r/min under the PI controller. After adjusting the time $t_s = 0.002s$, the system reaches a stable speed. The velocity range after stabilization was 495.6341-502.8779 r/min, and the fluctuation range was 7.2438. The comparison is shown in Table 2.

| Speed at 500r/min | Speed range(r/min) | Fluctuation error |
|-------------------|--------------------|-------------------|
| ADP controller    | 499.8802-500.1253  | 0.2451            |
| PI controller     | 495.6341-502.8779  | 7.2438            |

Table 2. response at different reference speed.

Fig. 5 and Fig. 8(b) are the corresponding Fourier spectrum analysis. In the low-frequency region, the PI controller has a higher distribution than ADP controller. Although ADP control lags behind PI controller in adjusting time, it is obviously superior in the range of speed ripple, and the optimization is about 3.3%. The
difference between high-speed and low-speed ripples is that these torque pulsations are naturally filtered out by the rotor or load inertia to some extent during high-speed operation, so the fluctuation of speed and phase current is smaller than that during low-speed operation.

Fig. 6 shows the speed response with a reference speed of 50r/min under the ADP controller, and the system reaches a stable speed after the setting time $t_s = 0.001s$. The stabilized speed range is 49.7712-50.2942r/min, and the fluctuation range is 0.5230. Fig. 7 is the Fourier analysis of the speed response. Fig. 9(a) shows the speed response with a reference speed of 50 r/min under the PI controller. After adjusting the time $t_s = 0.001s$, the system reaches a stable speed. The velocity range after stabilization was 46.3281-53.5526r/min, and the fluctuation range was 7.2279. Since the reference speed set is relatively low, it is easy to reach the preset value. The setting time of both controllers is very small, but the advantage of ADP controller in speed ripple cannot be ignored.

Table 3. response at different load torque.

| Load torque at 1.6Nm | Torque range(Nm)  | Fluctuation error |
|---------------------|-------------------|------------------|
| ADP controller      | 1.5603-1.6428     | 0.0825           |
| PI controller       | 0.3482-3.0570     | 2.7088           |
| Load torque at 9.0Nm| Torque range(Nm)  | Fluctuation error|
| ADP controller      | 8.9603-9.1562     | 0.1959           |
| PI controller       | 6.9523-12.1328    | 5.1805           |

Fig. 10 and Fig. 11 are the responses of different controllers under different load torques. The experimental results demonstrate that when the load is 1.6Nm and
the reference speed is 500r/min, the torque fluctuation is 1.5603-1.6428Nm under the ADP controller. However, under the control of the PI, the torque fluctuation is 0.3482-3.0570Nm. The torque fluctuation increases to 6.9523-12.1328Nm when the load torque increases to 9.0Nm. Under ADP control, this number is reduced to 8.9603-9.1562Nm. The error is controlled to be within 0.1Nm with the ADP method. It can be concluded that the ADP control has better performance in the minimization of torque ripples compared to the PI method.

According to the above mentioned experimental results, the effectiveness of the proposed ADP control scheme to suppress torque ripple is verified at different operating speeds and load torque.

7. Conclusion. The purpose of this paper is to adopt a new strategy of the iterative method using ADP to find an online adaptive optimal controller for continuous-time linear systems with unknown system dynamics for minimizing the torque ripple. After analyzing the formation of the torque ripple, the periodicity of the ripple is obtained, and the method of compensating the current by using adaptive dynamic programming is more suitable. In addition, as an additional control module, the solution is easy to be implemented and can be added to any existing controller without the need to accurately understand the motor parameters. This paper proves the convergence of the controller and verifies the feasibility of the algorithm through simulation experiments. In the future, we can design an adaptive ADP controller from the perspective of robustness, so that it can automatically adjust parameters to achieve better robustness.

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