Coordination of Active and Reactive Power in Active Distribution Networks Based on Successive Linear Approximation

Jinhui Wang, Yue Yuan and Han Wu

College of Energy and Electrical Engineering, Hohai University

*wangjinhui@hhu.edu.cn

Abstract. Active distribution network (ADN) is an important part of the future power market, which undertakes the mission of accommodating high penetration distributed generation. However, traditional dispatch mode of the distribution system cannot accommodate high-penetrated renewable energy. In this paper, a novel optimal scheduling model is developed to optimize the accommodation of distributed renewable energy by coordinating active and reactive power. The proposed model takes capacitor bank (CB), energy storage systems (ESS) and static var generator (SVG) as management devices, optimize the day-ahead dispatchable resources of active distribution system in a centralized manner. Different from second-order cone programming method that cannot cooperate with the maximum DG penetration objective, we apply the successive linear approximation method to transform the original non-convex and nonlinear optimal dispatch model with AC balance equation into a mixed-integer linear programming (MILP) model, so that the model can be solved efficiently and accurately by commercial optimization software. A real distribution system in Nantong is employed to verify the proposed model.

1. Introduction

Against the background of growing worldwide energy shortage and environment problem, the high-penetrated renewable energy has connected to the distribution network [1]. Growing distributed generation (DG) brings problems such as voltage fluctuation, power flow reversal and difficulty in accommodating renewable energy[2]. After the concept of active distribution network is put forward, distribution system operation can comprehensively manage DG, ESS and controllable load in distribution network, and control the energy resources flexibly to attain the goal of energy-saving and increase the consumption rate of distributed power. Because the resistance and reactance of distribution network are similar, it is pivotal to pay attention to the active management of power regulation measures in the optimization process to achieve global optimization[3].

Since the models of OLTC, CB and ESS contain discrete variables, the distribution network scheduling is a complex mixed-integer nonlinear optimization program (MINOP). At present, most researches use convex relaxation technology to relax the nonlinear optimal power flow problem in power grid scheduling into a convex optimization problem and then use commercial optimization software package to solve it [4]. In [5], the semi-definite relaxation is introduced to transform the optimal power flow problem into the semidefinite program (SDP) solution. In [6], the MINLP problem is transformed into a second-order cone programming (SOCP) problem by adapting the second-order cone relaxation technical. Based on SOCP, reference [7] studies the optimization problem of minimum power loss and voltage deviation in distribution
network. However, the application scope of the above convex relaxation method is limited. For example, SDP is inefficient and difficult to be applied in practical operation [8]. When solving the programming problem, the second-order cone relaxation needs to satisfy constraints, such as the increasing function of line current and unlimited load power, etc. [9]. In some specific application scenarios, for example, the tightness of second-order cone relaxation is difficult to be satisfied because of the possibility of reverse power flow near the voltage limit and larger calculation error when the maximum absorption rate of renewable energy is used to study the scheduling of ADN, so we need to find new solutions.

2. A coordinating active and reactive power dispatch model for ADNs

In this section, the objection function, the operation limits of DG, ESS, CB and SVG are proposed, and dispatch model is formulated.

2.1. Objective function

In this paper, the aim of the ADN dispatch model is to maximize renewable energy consumption.

$$\text{Max} F_i = \sum_{i=1}^{T} \left( P_{DG,i} \right) \over \sum_{i=1}^{T} P_{DG,i}$$

(1)

Where: $P_{DG,i}$ and $P_{DG,i}$ denote the predicted and actual output of distributed renewable energy generator.

2.2. Constraints

a) Power output limits of DG: power output limits of DG can be expressed as follows,

$$\begin{align*}
0 \leq P_{DG,i} & \leq P_{DG,i} \\
-P_{DG,i} \tan \phi & \leq Q_{DG,i} \leq P_{DG,i} \tan \phi, \forall i \in B^{DG} \\
(P_{DG,i})^2 + (Q_{DG,i})^2 & \leq S_{DG,i}^2
\end{align*}$$

(2)

Where: $P_{DG,i}$ and $Q_{DG,i}$ are the actual active and reactive power output of node i connected to DG in interval t; $P_{DG,i}$ is the DG active power limit in interval t; $\phi$ and $\phi'$ are the upper and lower power factor angle.

b) ESS operation model: The operation model of ESS is formulated as follows,

$$\begin{align*}
\left\{ \begin{array}{l}
u_{i,\text{char}}^{\text{ES}} + \nu_{i,\text{dischar}}^{\text{ES}} \leq 1 \\
\nu_{i,\text{char}}^{\text{ES}} \frac{P_{i,\text{char}}^{\text{ES}}}{P_{i,\text{dischar}}^{\text{ES}}} \leq \frac{P_{i,\text{char}}^{\text{ES}}}{P_{i,\text{dischar}}^{\text{ES}}}, \forall i \in B^{ESS}, \forall t \\
\nu_{i,\text{char}}^{\text{ES}} \frac{P_{i,\text{dischar}}^{\text{ES}}}{P_{i,\text{char}}^{\text{ES}}} \leq \frac{P_{i,\text{dischar}}^{\text{ES}}}{P_{i,\text{char}}^{\text{ES}}}
\end{array} \right.
\end{align*}$$

(3)

$$\begin{align*}
SOC_{i,t} = SOC_{i,t-1} + \frac{P_{i,\text{char}}^{\text{ES}}}{\eta_{i,\text{char}}^{\text{ES}}} \frac{\Delta t - P_{i,\text{dischar}}^{\text{ES}}}{\eta_{i,\text{dischar}}^{\text{ES}} \Delta t} / E_{\text{nominal}}^{\text{ES}} \\
SOC_{i,t} \leq SOC_{i,t} \leq SOC_{i,\text{end}} = SOC_{i,\text{set}} \\
SOC_{i,\text{end}} \leq SOC_{i,\text{set}}
\end{align*}$$

(4)

Where: $\nu_{i,\text{char}}^{\text{ES}}$ and $\nu_{i,\text{dischar}}^{\text{ES}}$ represent the charging condition and discharging condition of ESS i; $P_{i,\text{char}}^{\text{ES}}$, $P_{i,\text{dischar}}^{\text{ES}}$, $P_{i,\text{char}}^{\text{ES}}$, $P_{i,\text{dischar}}^{\text{ES}}$, $P_{i,\text{char}}^{\text{ES}}$ and $P_{i,\text{dischar}}^{\text{ES}}$ are severally the charging power and discharging power in interval t, the maximum and minimum charge-discharge power limits of the ESS i; $SOC_{i,t}$ is the remaining charge of ESS i in interval t; $E_{\text{nominal}}^{\text{ES}}$, $SOC_{i,\text{set}}$ and $SOC_{i,\text{end}}$ are the rated capacity, maximum and minimum of remaining charge of the ESS i; $SOC_{i,0}$ and $SOC_{i,\text{end}}$ are the SOC value at the initial and end of the scheduling cycle; $SOC_{i,\text{set}}$ is the set initial SOC value.
In order to ensure the same performance of ES in the next scheduling cycle, it is assumed that the initial value of SOC in this cycle is equal to that in the next cycle.

c) Operational limits of CB: the constraints of operational limits of CB can be expressed linearly as follows,

$$
\begin{align}
\gamma_{i,t} Q_{i,t}^{CB, opt} & = Q_{i,t}^{CB} \\
0 & \leq \gamma_{i,t}^{CB} \leq \gamma_{i,t}^{CB, max} \\
\forall t, \forall i \in B^{CB}
\end{align}
$$

(5)

Where: $B^{CB}$ is the node-set with CB; $Q_{i,t}^{CB}$ is the operation capacity of the capacitor at node $i$ in interval $t$; $Q_{i,t}^{CB, opt}$ is the compensation capacity of each gear; $\gamma_{i,t}^{CB}$ is an integer variable, which represents the number of operating banks of the capacitor in interval $t$; $\gamma_{i,t}^{CB, max}$ is the upper limit of the number of CB that can be operated. The limit of CB operation times can be expressed as follows, where: $N_{i,t}^{CB, max}$ is the upper limit of CB operation times in each scheduling cycle; $\Delta S_{i,t}$ is the change of CB compensation capacity at the adjacent time.

$$
\sum_{t=1}^{N_{i,t}^{CB, max}} \delta^{CB}_{i,t} \leq N_{i,t}^{CB, max} \\
-\delta^{CB, max}_{i,t} \leq \gamma_{i,t}^{CB} \leq \delta^{CB, max}_{i,t} \\
\forall t, \forall i \in B^{CB}
$$

(6)

d) Operational limits of SVG: SVG is a common continuous reactive power compensation device in distribution network. The constraint of the operational limits of SVG can be expressed as below,

$$
Q_{i,t}^{SVG, min} \leq Q_{i,t}^{SVG} \leq Q_{i,t}^{SVG, max} \\
\forall t, \forall i \in B^{SVG}
$$

(7)

Where: $B^{SVG}$ is the set of nodes containing SVG; $Q_{i,t}^{SVG, min}$ and $Q_{i,t}^{SVG, max}$ are the upper and lower limits of SVG compensation power; $Q_{i,t}^{SVG}$ is the compensation capacity of SVG in interval $t$.

e) Power flow constraints: The constraints of power balance at each node are expressed as follows,

$$
\begin{align}
P_{i,t} = & \sum_{(i,j)} P_{ij}^q + \sum_{j=1}^{N} G_{ij} v_{ij}^2 + P_{ij}^{DG} + P_{ij}^{Essdis} - P_{ij}^{Esschar} - P_{ij}^{Load} \\
Q_{i,t} = & \sum_{(i,j)} Q_{ij}^q + \sum_{j=1}^{N} -B_{ij} v_{ij}^2 = Q_{ij}^{DG} + Q_{ij}^{Ess} + Q_{ij}^{CB} + Q_{ij}^{SVG} - Q_{ij}^{load}
\end{align}
$$

(8)

$$
P_{i,t}^2 + Q_{i,t}^2 \leq S_{i,max}^2
$$

(9)

Where, $P_{i,t}, Q_{i,t}$, and $\theta_{i,t}$ are active power, reactive power, and phase angle difference of line $ij$ in interval $t$; $S_{i,max}$, $g_{ij}$ and $b_{ij}$ are severally the maximum apparent power, conductance, and susceptance of the line $ij$; $G_{ij}$ and $B_{ij}$ are severally the conductance and susceptance of the node to the ground; $v_{ij}$ is the voltage in interval $t$; $P_{ij}^q$, $P_{ij}^{DG}$, $P_{ij}^{Essdis}$, $P_{ij}^{Esschar}$, $P_{ij}^{Ess}$, $P_{ij}^{Ct}$ and $P_{ij}^{Load}$ are the injection power, the active power of distributed renewable power supply, the charging and discharging power of ESS, and active power consumed by load; $Q_{ij}^q$, $Q_{ij}^{DG}$, $Q_{ij}^{Ess}$, $Q_{ij}^{CB}$, $Q_{ij}^{SVG}$ and $Q_{ij}^{load}$ are the injected reactive power of node $i$ in interval $t$, the power of PV inverter and the energy storage inverter, and the reactive power consumed by load.
3. Linear approximation of constraints

Considering the nonlinear constraints in the model, the above active and reactive power coordination problem is a nonlinear problem, so that it is inefficient to be solved by common commercial optimization software. In this section, the nonlinear terms in the model are linearized to improve the efficiency of the solution.

3.1 Successive linearization of power flow equations

Expression of line capacity limit (10) is a nonlinear constraint in the form of an ellipse, which can be approximated linearly by multiple rectangular constraints in the way.

\[
egin{align*}
-S_{\text{g, max}} &\leq P_{ij,t} \leq S_{\text{g, max}} \\
-S_{\text{q, max}} &\leq Q_{ij,t} \leq S_{\text{q, max}} \\
-\sqrt{2}S_{\text{g, max}} &\leq P_{ij,t} + Q_{ij,t} \leq \sqrt{2}S_{\text{g, max}} \\
-\sqrt{2}S_{\text{g, max}} &\leq P_{ij,t} - Q_{ij,t} \leq \sqrt{2}S_{\text{g, max}}
\end{align*}
\]

(11)

This equation (8) can be a linear approximate equation by making a first-order Taylor series expansion.

First, the \(\sin \theta_{ij,t}\) and \(\cos \theta_{ij,t}\) terms in the equation (8) are expanded by the first-order Taylor series near the given initial value \(\left(v_{ij,t}, \theta_{ij,t}\right)\).

\[
\begin{align*}
\sin \theta_{ij,t} &\approx \cos \theta_{ij,t} \theta_{ij,t} + \sin \theta_{ij,t} \theta_{ij,t} \\
\cos \theta_{ij,t} &\approx -\sin \theta_{ij,t} \theta_{ij,t} + \cos \theta_{ij,t} \theta_{ij,t}
\end{align*}
\]

(12)

By substituting equations (12) into equation (8), the branch current equation can be obtained as below,

\[
\begin{align*}
P_{ij,t} &= v_{ij,t}^2 g_{ij,t} - v_{ij,t} v_{i,t} \left(g_y c_{ij,t} + b_y s_{ij,t}\right) - v_{ij,t} v_{j,t} \theta_{ij,t} \\
Q_{ij,t} &= -v_{ij,t}^2 b_{ij,t} + v_{i,t} v_{j,t} \left(-g_y s_{ij,t} + b_y c_{ij,t}\right) - v_{ij,t} v_{j,t} \theta_{ij,t}
\end{align*}
\]

(13)

The square term \(v_{ij,t}^2\) of node voltage can be regarded as a variable, and the nonlinear term in the power flow equation is transformed into \(v_{ij,t} v_{i,t} \theta_{ij,t}\) and \(v_{ij,t} v_{j,t} \theta_{ij,t}\). In order to decompose these two terms, we take \(v_{ij,t} v_{j,t}\) as a whole,

\[
v_{ij,t} v_{j,t} \theta_{ij,t} \approx v_{ij,t} v_{j,t} v_{i,t} \theta_{ij,t} + (v_{ij,t} v_{i,t} - v_{j,t} v_{j,t}) \theta_{ij,t}
\]

(14)

By substituting formula (14) into formula (13), the following branch power flow equation can be obtained,

\[
\begin{align*}
P_{ij,t} &= g_y v_{ij,t}^2 \left[-\left(g_y c_{ij,t} + b_y s_{ij,t}\right) + \left(g_y c_{ij,t} + b_y s_{ij,t}\right) \theta_{ij,t}\right] v_{i,t} v_{j,t} - \left(g_y c_{ij,t} + b_y s_{ij,t}\right) v_{i,t} v_{j,t} v_{j,t} v_{j,t} \left(\theta_{ij,t} - \theta_{ij,t}\right) \\
Q_{ij,t} &= -b_{ij,t} v_{ij,t}^2 \left[-\left(-g_y s_{ij,t} + b_y c_{ij,t}\right) + \left(-g_y s_{ij,t} + b_y c_{ij,t}\right) \theta_{ij,t}\right] v_{i,t} v_{j,t} + \left(-g_y s_{ij,t} + b_y c_{ij,t}\right) v_{i,t} v_{j,t} v_{j,t} v_{j,t} \left(\theta_{ij,t} - \theta_{ij,t}\right)
\end{align*}
\]

(15)

In which \(v_{ij,t} v_{j,t}\) can be determined as below,

\[
v_{ij,t} v_{j,t} = \frac{1}{2} [v_{ij,t}^2 + v_{j,t}^2 - (v_{ij,t} - v_{j,t})^2] = \frac{v_{ij,t}^2 + v_{j,t}^2}{2} \Rightarrow \frac{v_{ij,t}^2}{v_{ij,t}^2 + v_{j,t}^2} = \frac{U_{ij,t} + U_{j,t} - v_{j,t}}{2}
\]

(16)

The square term \(v_{ij,t}^2\) of voltage can be represented by the variable \(U_{ij,t}\). The Taylor expansion \(v_{ij,t}^2\) can be further approximated as below,

\[
v_{ij,t}^2 \approx 2 v_{ij,t} v_{j,t} \theta_{ij,t} - v_{ij,t}^2 \approx 2 v_{ij,t} v_{j,t} v_{ij,t} + v_{j,t} v_{ij,t} - v_{ij,t}^2 \approx 2 \frac{v_{ij,t}^2 - v_{ij,t} v_{j,t} (U_{ij,t} - U_{j,t})}{v_{ij,t}^2 + v_{j,t}^2} = v_{ij,t}^2
\]

(17)
The approximate linear term $v_{ij,t}^2$ is defined as $v_{ij,t,L}^\prime$.

$$
\begin{align*}
\begin{cases}
P_{ij,t}^e &= g_y U_{ij,t} + g_{ij,p} U_{ij,t} + b_{ij,p} (\theta_{ij,t} - \theta_{ij,t_{in}}) + g_{ij,p} v_{ij,t,L}^\prime \\
Q_{ij,t} &= -b_y U_{ij,t} + b_{ij,q} U_{ij,t} - g_{ij,q} (\theta_{ij,t} - \theta_{ij,t_{in}}) - b_{ij,q} v_{ij,t,L}^\prime 
\end{cases}
\end{align*}
$$

In reference [10], [11], many numerical examples show that the average approximate error of the successive linearized power flow method is about 1E-5, which has a very high calculation accuracy.

3.2 Linearization of DG output constraint equation

The quadratic form of the capacity constraint (2) of the distributed power inverter can also use the rectangular constraint as for external approximation to achieve its linear expression, as follows,

$$
\begin{align*}
\begin{cases}
\Delta_{ij} &\leq P_{ij}^{DG} \leq S_{ij}^{DG} \\
\Delta_{ij} &\leq Q_{ij}^{DG} \leq S_{ij}^{DG} \\
\sqrt{2} \Delta_{ij} &\leq P_{ij}^{DG} + Q_{ij}^{DG} \leq \sqrt{2} S_{ij}^{DG} \\
\sqrt{2} \Delta_{ij} &\leq P_{ij}^{DG} - Q_{ij}^{DG} \leq \sqrt{2} S_{ij}^{DG}
\end{cases}
\forall i, \forall i \in B^{DG}
\end{align*}
$$

Thus, the model is transformed into a MILP model.

4. Solution method

It is different for commercial software to directly solve the MILP with an explicit iterative process. In this paper, the specific calculation flow is shown in Figure 1.

1) Internal circulation: In the inner loop, the fixed discrete variable value is adapted to solve the original scheduling model in the way of successive linearization to provide a high-quality initial value of continuous variables for the outer loop. The convergence of the results is judged by the power flow linearization error $\Delta_{ij}$.

$$
\Delta_{ij} = \max_{(i,j) \in \chi} \left( \sqrt{\left( P_{ij} \right)^2 + \left( Q_{ij} \right)^2} - \sqrt{\left( P_{ij}^e \right)^2 + \left( Q_{ij}^e \right)^2} \right),
\forall (i,j) \in \chi
$$

Where: When $\Delta_{ij} < \Delta_{nol}$, the result converges. In this paper, $\Delta_{nol}$ is taken as 1E-2.

2) External circulation: In the outer loop, the optimal solution of an integer variable is obtained based on the initial value of the continuous variable calculated in the inner loop. The integer variables in the model mainly affect the voltage amplitude and reactive power distribution but have little influence on the active power distribution and phase angle.
The successive linearization algorithm has higher calculation efficiency because of the higher accuracy of power flow linearization, fewer iterations in the solution process, and the MILP model of active and reactive power coordinated scheduling, which has higher calculation efficiency.

5. Case study

5.1. Introduction of the case
A real distribution network in Nantong, China is employed to verify the proposed model and algorithm. The network topology is given in Figure 2. A 2.5431MWp PV system is linked with bus 15, and a 0.5MWh ESS is connected to bus 13. The ESS operates in the mode of maximum income, with no more than two times of charging and discharging per day, rated power of 0.5MW, initial SOC of 0.5, charging efficiency of 88%, discharge efficiency of 90% and DOD of 90%. The scheduling interval is set as 1 h.
Figure 2. Chaoyang line topology

A typical summer sunny day is used to simulate the practical operation of ADN. The predicted load and PV output curve in the day is shown in Figure 3.

Figure 3. The output curve of Typical PV and load

Figure 3 shows that active demand of the distribution network is distributed between 0.9MW-2.3MW. Overall, the load demand in the daytime is large, while the load demand at night is small. At 4:45 a.m., the minimum load demand is 0.997MW; at 10:15 a.m., the maximum load is reached, and the peak load is 2.227MW. In addition, it can find that the load is greater than the PV output under the existing PV output so that the distribution network can absorb more PV under the regulation of active
management measures such as ESS, SVG and CB. Table 1 shows the load rate of each load point in the calculation example.

Table 1. Load rate of each load point

| Load                      | Node number | Capacity (kVA) | Peak load (kVA) | Load rate (%) |
|---------------------------|-------------|----------------|-----------------|---------------|
| China Mobile              | 6           | 30             | 12.994          | 43.313        |
| Alphay Biological Technology Co | 7           | 2400           | 1172.866        | 48.869        |
| Property Development Co   | 8           | 1030           | 212.866         | 20.665        |
| Maritime Logistics Co      | 10          | 160            | 127.445         | 79.665        |
| Garment Co                | 11          | 250            | 110.166         | 44.067        |
| Engineering Headquarters  | 12          | 100            | 37.006          | 37.006        |
| China ZTT                 | 13          | 1000           | 551.495         | 55.149        |
| MITRE                     | 14          | 630            | 454.949         | 72.214        |

5.2. Result analysis
In this paper, we take the maximum PV absorption rate as the objective function and the maximum PV day in summer as the typical day. After the GAMS optimization calculation, the PV absorption rate reaches 100% in typical days, and the total network loss is 87.49kwh, which is significantly higher than the original grid. At this time, the PV output curve and the output curve of large power grid are shown in Figure 4.

![Figure 4. The output curve of PV and large grid](image1)

As shown in Figure 4, the actual PV output and the theoretical PV output basically coincide. The accommodation of PV has been significantly improved through the coordination of active and reactive power. Because of the large light intensity, so it is unnecessary to purchase power from the superior grid at 11:00 a.m., and the local PV can complete the power supply of all loads.

As we can see from the Figure 5 that The SOC of ESS fluctuates throughout the day, but there is no case that the ESS cannot be adjusted due to excessive PV output or load. The ESS can absorb the PV power in the period of PV power generation to ensure the absorption of PV. According to the line loss and other indicators at this time, although the PV output is increased, it is still in a good operating condition with adjustable control and small loss and voltage offset.

6. Conclusion
In this paper, considering the management of SVG, CB and ESS, an optimal active and reactive power dispatching model of ADN is established to the accommodation of distributed renewable energy. The method of successive linear approximation adopted in this model can simplify the power flow, so that the original problem can be transformed to MILP, which is solved by using CPLEX solver on the GAMS. The following conclusions can be obtained from theoretical analysis and case calculation:
1) The accommodation of distributed renewable energy can be improved by active operation control of distribution network. Through the active control of reactive comprehensive device’s operation strategy and ESS, the distributed energy can be fully accommodated on the condition of ensuring voltage quality.

2) The complex MINLP model can be transformed into MILP model by means of successive first-order Taylor series expansion, piecewise linearization and polygon approximation, so that the model can have higher applicability and solution efficiency in general planning software.

**Acknowledge**

This work is supported by the National Key Research and Development Program of China (Grant 2016YFB0900100).

**References**

[1] CQ Kang, LZ Yao, “Key Scientific Issues and Theoretical Research Framework for Power Systems with High Proportion of Renewable Energy,” *Automation of Electric Power Systems*, vol. 41, no. 9, pp. 2-11, Jun. 2017.

[2] JY Liu, HJ Gao, Z Ma, YX Li, “Review and prospect of active distribution system planning,” *Journal of Modern Power Systems and Clean Energy*, vol. 3, no. 4, pp. 457-467, Nov. 2017.

[3] Georgilakis P S, Hatziargyriou N D, “A review of power distribution planning in the modern power systems era: Models, methods and future research,” *Electric Power Systems Research*, vol. 121, pp. 89-100, Apr. 2015.

[4] HJ Gao, XD Shen, R Xu, “Optimal power flow research in active distribution network and its application examples,” *Proceedings of the CSEE*, vol. 37, no. 6, pp. 1637-1644, Aug. 2017.

[5] Philipp F, Turhan D, “Linear/Quadratic Programming-Based Optimal Power Flow using Linear Power Flow and Absolute Loss Approximations,” *International Journal of Electrical Power & Energy Systems*, vol. 107, pp. 680-689, May. 2019.

[6] WY Zheng, WC Wu, BM Zhang, HB Sun, YB Liu, “A Fully Distributed Reactive Power Optimization and Control Method for Active Distribution Networks,” *IEEE Transactions on Smart Grid*, vol. 7, no. 2, pp. 1021-1033, Jul. 2014.

[7] QY Guo, JK Wu, MO Chao, HH Xu, “A Model for Multi-objective Coordination Optimization of Voltage and Reactive Power in Distribution Networks Based on Mixed Integer Second-order Cone Programming,” *Proceedings of the CSEE*, vol. 38, no. 5, pp. 1385-1396, May. 2018.

[8] TY He, ZN Wei, GQ Sun, YH Sun, Q Gao, “Quasi Direct Current Optimal Power Flow Based on Modified Semi-Definite Programming Algorithm,” *Power System Technology*, vol. 39, no. 09, pp. 2553-2558, Sep. 2015.

[9] B M. Alzalg, “Stochastic second-order cone programming: Applications models,” *Applied Mathematical Modelling*, vol. 36, no. 10, pp. 5122-5134, Oct. 2012.

[10] Z Yang, H Zhong, Q Xia, HW Zhong, CQ Kang, “Optimal power flow based on successive linear approximation of power flow equations,” *IET Generation, Transmission & Distribution*, vol. 10, no. 14, pp. 3654-3662, Jul. 2016.

[11] Z Yang, A Bose, HW Zhong, N Zhang, Q Xia, CQ Kang, “Optimal Reactive Power Dispatch With Accurately Modeled Discrete Control Devices: A Successive Linear Approximation Approach,” *IEEE Transactions on Power Systems*, vol. 32, no. 3, pp. 2435-2444, May. 2017.