OBSERVATIONAL CONSTRAINTS ON THE THEORY OF THE IMF

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Abstract

Observational constraints on the theory of the IMF are reviewed. These observations include the new result that star formation is very rapid, usually going from start to finish in only 1 or 2 dynamical times on a wide range of scales. This result, combined with the observation that the IMF is independent of cluster density over a factor of several hundred, implies that protostar coagulation during orbital motions in a cloud are not important for the IMF: there is not enough time. The observation that the IMF in individual clusters is about the same as the IMF in whole galaxies implies that stars of all masses form randomly in self-gravitating clouds of all masses. There cannot be a sequence of stellar masses in a cluster, based on stirring or heating for example, where the largest star that forms keeps increasing until the cloud is destroyed. The uniformity of the IMF over time and space argues for a process that is independent of specific properties of molecular clouds. This uniformity, along with the common observation of young stars in hierarchical clusters that resemble the structures of interstellar gas, suggests that the IMF, at least in the power law range, is a by-product of turbulence. Detailed physical processes may affect the turnover at low mass, but even this may have a universal character arising from a combination of turbulence-induced variations in the local cloud properties, and fragmentation in unstable cloud pieces.

1. Introduction

The stellar initial mass function (IMF) is a property of star formation that has recently garnered so much high-quality data that the general framework for its understanding may soon be at hand. Several relevant observations are reviewed here. Not all of these observations are directly related to the IMF; some are more about star formation in general than about the relative distribution of stellar masses in each particular region of star formation. Still, when taken as a whole, there is little alternative but the view that stars virtually freeze out of a gas that is structured by compressible turbulence, taking with them a universal signature of the mass distribution enforced by these motions. We are led to the conclusion that this process happens quickly on a dynamical time, and over a very wide range of scales, with little or no feedback and little sensitivity to the gaseous and galactic properties around it.

There have been two difficult aspects of the IMF problem: understanding what all of the star formation processes have in common, and sorting through the selection effects and assumptions that are implicit in each particular IMF observation.

For the first problem, the overall shape of the IMF may not depend much on the detailed processes of star formation: the IMF is an average over many of these processes and it always looks about the same. There should be a universal process at work, such as turbulence, that gives the basic IMF shape, while the detailed processes specific to each region may only modify the IMF by small amounts. If this is the case, then we should be able to understand most of the IMF using only the properties of turbulence.

The second of these problems is related to the observation that slight variations in the IMF are still present from region to region, even though the overall IMF is somewhat uniform (Scalo 1998). Many of these variations may be the result of three things: (1) pervasive selection effects, such as differential aging of high and low mass stars, which leads to the loss of some fraction at the high mass end when stars form more or less continuously in a large region for longer than several million years, or differential drift into the field resulting from the longer age or higher speeds of low mass stars; (2) mass segregation in clusters resulting in part from gas and small-star drag on the massive protostars, and (3) possible shifts in the basic mass scale for star formation in extreme environments. These systematic variations, along with stochastic variations from small number statistics in limited surveys, are always present to some degree in the observations. We discuss below another possible variation with the density of the region of star formation.

2. Star formation in a crossing time

After we thought for a long time that star formation must be slow in molecular clouds, perhaps to avoid the galactic...
catastrophe discussed by Zuckerman and Evans (1974), and after many attempts to explain this slowness by star-formation feedback (Norman & Silk 1980; Franco & Cox 1983; McKee 1989), magnetic support (Mouschovias 1976; McKee 1989), and turbulence (Bonazzola et al. 1987; Leorat, Passot, & Pouquet 1990; Vazquez-Semadeni & Gazol 1995), the observations now suggest just the opposite. Star formation seems to be fast on every scale in which it occurs, from sub parsecs to kiloparsecs (Elmegreen 2000a). Fast means that the star formation process begins and ends in only a few dynamical timescales in a cloud. Thus small scales form stars in a short time, measured in years, and, large scales form stars over a longer time, but both times are comparable to \((G\rho)^{-1/2}\) for average local density \(\rho\). Because of the way turbulence structures the gas, i.e., in fractal or multifractal patterns, bigger scales have smaller average densities, i.e., more and more of the volume is occupied by low density gas as the scale increases.

This change in thinking is based on direct and indirect observations. Direct observations are the age ranges for embedded and young clusters. The age range for Trapezium stars in Orion is 1 My or less (Prosser et al. 1994; Palla & Stahler 1999). In L1641 it is about the same (Hodapp & Deane 1993). NGC 1333 has a large number of short-lived jets and Herbig-Haro objects (Bally et al. 1996), so things are happening quickly there too. In NGC 6531, the age spread is immeasurably small (Forbes 1996).

These short time scales are all comparable to a few crossing times in the cloud cores. The average stellar density in the Trapezium cluster is \(\sim 10^3 \, M_\odot \, pc^{-3}\), corresponding to several thousand stars pc\(^{-3}\) (Prosser et al. 1994; McCaughrean & Stauffer 1994). The densities are about the same, sometimes a little less, in other young clusters too (see figure 5 in Testi et al. 1999). Considering that the efficiency to make a bound or nearly-bound cluster is around 50% (as measured in IC 348, for example – Lada & Lada 1995), this stellar density corresponds to an initial \(H_2\) density of around \(6 \times 10^4 \, H_2 \, cm^{-3}\), as is sometimes measured directly (Lada 1992). The corresponding dynamical time scale is \((G\rho)^{-1/2} \sim 0.3 \, My\).

Palla & Stahler (1999) suggest that as the Trapezium cloud contracted, the star formation rate increased, which means that it stayed at a rate roughly proportional to the instantaneous dynamical time during a factor of perhaps 10 in density enhancement.

Sometimes star formation occurs in several quick bursts, as in 30 Dor (Selman et al. 1999), but even then each burst seems to be fast, spanning a total time of about 2 My per burst in this case. Observations of longer durations are therefore suspect: the Pleiades cluster has been claimed to have a prolonged star formation period, perhaps 30 My (Siess et al. 1997; Belikov et al. 1998), but this could result from a mixture of multiple events (Bhatt 1989), or even uncertain stellar evolution times.

The age spread in a whole OB subgroup is generally larger than it is in any one compact core that might form in the subgroup. Age ranges of \(\sim 2 \, My\) seem typical (Blauu 1964; Massey et al. 1995a). The age range in a whole OB association, with several subgroups, is larger still, perhaps 10 My. These larger scales correspond to smaller densities, however, and to proportionally larger dynamical times. Even larger scales include star complexes (Efremov 1995), such as Gould’s Belt, which typically take 30–50 My to finish. Star complexes were originally discovered by Efremov (1978), and are defined by concentrations of supergiants and Cepheid variables, which are sensitive to this longer age range. Each type of star that is used for an observation is associated with a particular scale for clumping: the longer lived stars highlight larger regions.

This correlation between age range and size is not the result of expansion from a common center, which would make the age range increase linearly with size. It is not the result of stochastic propagation of star formation either; then the age range would increase with the square of the size. In fact, the age range increases with the square root of the size of the region, as determined from the distribution of Cepheid variables (Elmegreen & Efremov 1996) and clusters (Efremov & Elmegreen 1998) in the LMC. The square root dependence identifies turbulence as a controlling factor. Moreover, the constant of proportionality in the duration-versus-size correlation is about the same as in the analogous correlation between crossing time and size for molecular clouds and their clumps (Elmegreen 2000a). Thus star formation on a wide range of scales, from 20 pc to 1 kpc in the LMC, operates on about 1.5 turbulent crossing times for the associated gas. The result is a hierarchy of clusters in both age and position: small regions of star formation come and go, presumably in recycled gas, in the time it takes the larger region surrounding them to finish (see review in Elmegreen et al. 1999).

The hierarchical structure of young stars in star-forming regions provides indirect evidence for relatively rapid star formation: if we still see the stars inside a cluster with strong subclustering, reminiscent of the hierarchical structure in molecular gas, then the stars cannot have moved very far from their origins. They probably do not even have time to cross from one side of the cloud core to the other; if they did, they would mix up and not be hierarchical anymore (Elmegreen 2000a). Hierarchical structure in embedded young clusters is commonly seen; e.g., in IC 348 (Lada & Lada 1995), NGC 3603 (Eisenhauer et al. 1998), W33 (Beck et al. 1998), NGC 2264 (Piche 1993), and G 35.20-1.74 (Persi et al. 1997). Elson (1991) found spatial substructure in 18 LMC clusters, and Strobel (1992) found age substructure in 14 young clusters.

These observations, both direct and indirect, suggest that star formation is relatively rapid over a wide range of scales. If this is the case, then there is not enough time for a protostar to move around in a young cluster and either accrete the ambient gas as it moves or coalesce with other
protostars. This rules out a large class of models for the IMF.

A good example is provided by the recent IR continuum observations of young protostars in Ophiuchus and Serpens (Motte, André, & Neri 1998; Testi & Sargent 1998). These protostars are very small compared to their angular separations and are not likely to collide with each other in even a few crossing times. Also, their distribution is clumpy like the gas; they should be more scattered if they are randomly orbiting from clump to clump inside the cloud core.

This result can be quantified by considering the cross section that would be necessary for an object to collide with another in one crossing time. If the density of protostars is \( n_3 \times 1000 \, \text{pc}^{-3} \), and the radius of the cluster is a typical \( R_{cl,0.2} \times 0.2 \, \text{pc} \) (Testi et al. 1999, Fig. 1), then the protostar cross section must be

\[
\pi R_{\text{protostar}}^2 = \sim \frac{(10^4 \, \text{AU})^2}{n_3 R_{cl,0.2}}.
\] (1)

This is such a large cross section that at typical cluster densities, only binary stars and disks should be affected by protostar interactions. Older models that assumed protostars move around for many crossing times in a cloud core got coalescence with smaller cross sections, but they also had to assume much higher cluster densities (Silk & Takahashi 1979; Bastien 1981; Larson 1990; Zinnecker et al. 1993; Price & Podsadamlofski 1995; Bonnell, Bate, & Zinnecker 1998).

Before we leave this topic, it is worth clarifying why rapid star formation of the type discussed here, i.e., hierarchical in position and time, does not lead to a catastrophic starburst in the whole galaxy, as envisioned by Zuckerman & Evans (1974). The reason has two observational sides: On a large scale, the star formation rate in a whole galaxy is slow because it follows the local dynamical time scale, just like individual clouds (Elmegreen 1997; Kennicutt 1998). This time scale is very large, comparable to the orbit time. On a small scale, most of the CO-emitting gas is not able to form stars: it is either too low in density as part of an interclump medium inside generally molecular clouds, or too high in density and transient because of intermittent turbulent effects. Only a small fraction, like 1% (McLaughlin 1999), of the mass in any molecular cloud actively participates in the star formation process. In this active gas, the efficiency of conversion of gas into stars is usually high, like 10% or more. To put it in another way, even though star formation always operates locally on a dynamical time scale, the total CO mass in the Galaxy is not turning into stars on the average dynamical time scale because the gas is fractal, i.e., mostly hollow, and the average or excitation density that is used to determine this average timescale rarely occurs in any local region.

3. IS THE IMF INDEPENDENT OF CLUSTER DENSITY?

The inability of stars or protostars to collide directly in even the densest environments suggests at first that the IMF should be independent of cluster density. The binary star fraction and relative disk fraction or disk mass should not be independent of cluster environment, but the star masses should. This conclusion is right in some sense, but a generalization of it to all stars might be a bit too hasty. Cluster density also affects the rate of accretion of ambient gas onto a protostar, and different densities might lead to different IMFs because of different accretion rates.

The geometry for the accretion of gas onto a protostar is not really known. In some models (e.g., Bonnell et al. 1998), a protostar is assumed to move around in a gaseous medium of uniform, or at least smooth, density, and to accrete this gas as it moves. Interstellar clouds are not uniform, however. They seem to be fractal with most of the mass occupying a small fraction of the volume. If this is the case, then the model of moving accretion would not build up much stellar mass: most of the time the moving stars would be in regions with very low densities.

On the other hand, if the star accretes virtually all of its mass from a clump, and consequently comoves with that clump because of momentum conservation, then the stellar mass is more a reflection of the clump mass, rather than the accretion rate multiplied by a time. We might as well assume, then, that the star mass is following the clump mass, and not worry too much about how the accretion actually occurs. This is the basic assumption in my recent IMF models (Elmegreen 1997, 1999a, 2000b,c).

There is a third possibility, however. It could also be the case that as the time by a dense core forms inside a molecular cloud, the gas is no longer highly fragmented and fractal. This could be because the Mach number of the turbulence is relatively low in this case; i.e., the line width may be nearly thermal (and the temperature may be elevated). If there is enough mass to form more than one star, then the model of moving accretion could apply. And in a dense cluster, there could also be enough time for such thermal cores to coalesce and build up in mass (because at the Jeans size, these cores would be fairly big). Then we are faced with the interesting possibility that dense clusters might end up with a different IMF than low density star-forming regions. That is, the high mass stars may accrete faster than the low mass stars in such a uniform environment, and so have a different contribution to the net IMF than would a low density region. This is an old idea (Larson 1978; Larson 1982; Zinnecker 1982). What do the observations say about it now?

First of all, the IMF is in fact steeper in low density regions than in clusters. A compilation of the observations in Elmegreen (1997, 1999a) demonstrated this. For example, the slope \( x \) in the power law part, written as \( M^{-1-\alpha} d\log M \), is in the range from 1.5 to 2 for the local field stars (Garmany, Conti, & Chiosi 1982; Scalo 1986;...
Humphreys & McElroy 1984; Blaha & Humphreys 1989; Basu & Rana 1992; Kroupa, Tout, & Gilmore 1993; Tsujimoto et al. 1997), whereas \( x = 1.35 \) for the Salpeter function is more appropriate for clusters. The same steep slope applies to the unclustered young stars in Orion, although the tightly clustered stars have \( x \) in the range from 1 to 1.5 (Ali & DePoy 1995). In the whole Orion field, the IMF is steep as well (Brown 1998). J.K. Hill et al. (1994), and R.S. Hill et al. (1995) also found that LMC clusters have significantly steeper slopes in regions of low young star density than high young star density. An even more extreme case is considered by Massey et al. (1995b), who find that the remote field in the LMC has \( x \sim 4 \).

The problem with these observations is that they all might contain selection effects. One example is differential drift, where the low mass stars, with their longer lives and possibly higher velocity dispersions, drift further into the field, or into the low density regions, than high mass stars. Segregation of high mass stars to the potential wells of young clusters could produce the steepening effect at low density too (and a relatively shallow IMF in the cluster cores). Such segregation would have to be rapid, however (Hillenbrand & Hartmann 1998; Bonnell & Davies 1998). Also important might be the failure to correct for the loss of evolved massive stars in a region that has been forming stars for a long time. The color magnitude diagram may show only the youngest high-mass stars, and the oldest could be gone by now. This might be the case for the IMF in a whole OB association, which could have been forming stars for a period of 10 My or more, much longer than the lifetime of the most massive stars. In fact, because the duration of star formation increases with the size of the region, as shown above, the IMF might be systematically steeper in the larger, lower density regions simply because of a lack of corrections for this subtle aging effect.

On the other hand, the IMFs in dense clusters are surprisingly invariant for the intermediate-to-high mass range, spanning a factor of several hundred in cluster star density (Massey & Hunter 1998; Luhman & Rieke 1998). Thus, whatever is happening in one cluster must be happening in all clusters. Moreover, this IMF is very close to the Salpeter IMF, namely with a slope in the range from -1 to -1.5 on a logarithmic scale at intermediate to high mass. The same slope applies to galaxies in general (sect. 4).

What complicates this bimodal approach is that the galaxy-wide IMF is not particularly sensitive to the slope of the IMF at the very highest masses, except for the requirement that there not be too many high mass stars, e.g. in the 50-100 M\(_\odot\) range, to avoid an unusually high abundance of oxygen compared to iron (Wang & Silk 1993). If the galaxy-wide IMF began to drop at around 50 M\(_\odot\), and had fewer 100 M\(_\odot\) stars than the number expected from an extrapolation of the Salpeter function, then we probably would not know this. However, dense clusters like 30 Dor have many O3 stars with masses of around 100 M\(_\odot\) and a Salpeter slope out to at least this value (Massey & Hunter 1998). Thus it is conceivable that dense clusters form proportionally more \( > 50 \) M\(_\odot\) stars than lower density regions. It may also be true in this case that the majority of stars cannot form in dense clusters, because then the galactic abundance of \( > 50 \) M\(_\odot\) stars, and perhaps of oxygen, would end up too high. Thus there remains a possibility that cluster density affects the IMF at the very highest masses in ways that are difficult to observe right now. A discussion of various constraints on the high mass IMF, including stochastic IMF models that extend into this range, is in Elmegreen (2000c).

4. THE CLUSTER IMF EQUALS THE GALAXY IMF

A surprisingly strong constraint on the theory of the IMF comes from the simple observation, mentioned above, that the cluster IMF slope is about the same as the galaxy-integrated IMF slope. The latter comes from observations of emission-line equivalent widths (Kennicutt, Tamblyn & Congdon 1994; Bresolin & Kennicutt 1997), in which the emission line measures the massive star flux and the continuum measures the low mass star flux. It also comes from color magnitude diagrams in the LMC and local dwarf galaxies (Greggio et al. 1993; Marconi et al. 1995; Holtzman et al. 1997; Grillmair et al. 1998), as well as from the relative abundances of Fe and O. The latter appear constant in a wide variety of systems, including QSO damped Ly\(\alpha\) lines (Lu et al. 1996) and Ly\(\alpha\) forest lines (Wyse 1998), the intracluster medium (Renzini et al. 1993; Wyse 1997, 1998), elliptical galaxies (Wyse 1998), and normal spirals, including the Milky Way.

This apparent equivalence between the cluster and integrated IMFs was not previously recognized as a constraint on the IMF models because for a long time it was not known that high mass stars always form along with low mass stars. Twenty years ago, bimodel star formation models proposed that high and low mass stars may form in different regions. These models are no longer popular, however (see review sections in Elmegreen 1997, 1999a). For the more realistic case in which high mass clouds make both high mass and low mass stars, giving a normal IMF, and for the common observation that low mass clouds make primarily low mass stars, we would have the unusual circumstance that far more low mass stars should be forming than high mass stars compared to the normal IMF were it not required that stars of all masses form randomly in clouds of all masses. That is, if high mass clouds make both high mass and low mass stars, but low mass clouds make only low mass stars, then the large number of low mass clouds compared to high mass clouds would give a summed IMF that is steeper than the IMF in each region.

Instead, it must be true that even low mass clouds occasionally form intermediate or high mass stars, although we rarely see this because of sampling effects; i.e. it takes...
a lot of stars to get an IMF sufficiently populated to form a high mass star. To put it differently, the observations seem to require that ten low mass clouds have the same probability of forming a high mass star as a single cloud with ten times the mass. In retrospect, considering the fractal structure of clouds, this is neither surprising nor unobserved. When viewed from a distance, small clouds are seen to be parts of large clouds, so when we see a high-mass star forming in an extended region of star formation with a large total cloud mass, closer examination should show that this star is really forming in a smaller subcloud, and that other smaller subclouds may have not have such massive stars at all. Only when we lose the multi-scale perspective, by studying only the local regions of star formation for example, do we have difficulties understanding how small clouds can make large stars. This statement is independent of the peculiarities of high mass star formation, which may involve hot cores or dense clusters, unlike low mass stars; it is only about sampling where such peculiarities are likely to occur among all of the clouds in a region.

We can also see how this constraint on unrestricted starbirth locations follows from the observations by considering a simple theoretical model (Elmegreen 1999a). Suppose that the cloud or cluster mass spectrum is

$$N(M_{cl})dM_{cl} \propto M_{cl}^{-\gamma}dM_{cl}$$

(2)

and that the largest star mass that forms in a cluster of mass $M_{cl}$ is $M_L \propto M_{cl}^\alpha$. Suppose also that the IMF in each region is

$$n(M)dM = n_0 M^{-1-x}dM$$

(3)

to the largest star mass, $M_L$. Setting $N(M_L)dM_L = N(M_{cl})dM_{cl}$, we can convert the individual IMFs into a summed IMF with the equation:

$$n_{gal}(M) = \int_M^\infty n(M|M_{cl})P(M_{cl})dM_{cl} \propto M^{-1-x_{eff}}$$

(4)

where $n(M|M_{cl})$ is the conditional probability of forming a star of mass $M$ in a region with a maximum mass $M_{cl}$. The integration gives $x_{eff} = (\gamma - 1)/\alpha$, which is remarkably independent of the local IMF slope, $x$.

Now, $\gamma \sim 2$ for clusters and probably for clouds also, considering the hierarchical structure (Fleck 1996; Elmegreen & Falgarone 1996; Elmegreen & Efremov 1997). Then the constraint that the local IMF equals the global IMF means that $x_{eff} = x$, which becomes $x = 1/\alpha$. That is, the largest star mass must increase with the cluster mass as $M_L \propto M_{cl}^{\gamma}$. This is in fact observed (Elmegreen 1983), but it also follows theoretically from purely random sampling with all stars equally likely to form in all clouds. The reason is that the largest mass star comes from the IMF through the equation

$$\int_{M_{cl}}^\infty n(M)dM = 1,$$

(5)

which implies that $n_0 = xM_{cl}$. The total cluster mass in the power law range is, similarly,

$$M_{cl} = \int_{M_{\text{smallest}}}^\infty Mn(M)dM = \frac{x}{x-1}M_L^{x-1}M_{\text{smallest}}$$

(6)

for smallest mass star $M_{\text{smallest}}$ in the power law range. These two equations give $M_L \propto M_{cl}^\alpha$.

Thus large mass clouds are more likely to form high mass stars, but only because they form more stars overall. This is the implication of the observation that the cluster IMF equals the galaxy-wide IMF.

As a result of this implication, we can also state that star mass does not increase monotonically with time in a cluster as more and more stars form, because of some gaseous heating or heightened turbulence for example. This rules out a large class of sequential IMF models. If a model has this character, it may explain the IMF in any one cloud, but it cannot explain the IMF in the sum of all clouds.

There may be an interesting exception to this equality between the cluster IMF and the galaxy IMF in the observation by Massey et al. (1995b) that the slopes of the IMFs in the extreme field regions of the LMC and Milky Way are much steeper than the slopes in clusters. This goes in the same direction as the density dependence discussed in Section 3, so the result may have something to do with protostellar accretion, but there is also a simpler explanation: In the extreme field, the pressure and cloud density are likely to be low and the overall star formation rate low. Then OB stars may more readily disrupt their clouds and halt further star formation once they appear. If we consider the disruptive power of Lyman Continuum radiation and how it scales with cloud and cluster mass, then the value of $\alpha$ used above in the expression equals about 0.25 (Elmegreen 1999a). This gives $x_{eff} = 4$, which is what Massey et al. (1995b) observe. Elsewhere, $\alpha \sim 1/x$ and $x_{eff} = x \sim 1.35$, so massive stars should not readily halt star formation in their clouds. Presumably this is because star formation is usually too fast.

5. Young star fields are fractal, like the gas

The final clue to the origin of the IMF mentioned in the introduction is the observation that young star fields are hierarchically clumped, or fractal, just like the gas. Reviews of this property are in Elmegreen & Efremov (2000) and Elmegreen et al. (1999). Sometimes the hierarchy can be traced for 5 levels, from the scale of a giant patch of star formation in a spiral arm to subclumping inside a compact cluster (e.g., the cases of W3 and Orion are described in detail in Elmegreen et al. 1999). Various observations of subclustering of stars inside clusters were already mentioned above.

The gas has a similar structure, as is well known from fractal cloud studies (e.g., Scalo 1985; Falgarone et al. 1991; Stutzki et al. 1998). The mass spectrum of clouds probably comes from this structure too (Elmegreen & Falgarone 1996). The origin of all this fractal structure is presumably turbulence. Computer models reproduce it as well as can be expected at the present time (MacLow et al. 1997; Elmegreen 1999b).
The implication is that the stars freeze out of turbulent gas rather quickly, without moving much from their formation sites. Moreover, if they form in turbulence-generated clumps, then they probably have masses that are proportional to the turbulence-generated masses, i.e., to the local clump masses in a turbulent, self-gravitating medium. Models for this process are in Elmegreen (1993) and Klessen et al. (2000). In that case the origin of the IMF power law, or, if not an exact power law, then something close to a power law, such as a log-normal distribution, is a natural consequence of turbulence far removed from any boundaries. Turbulence produces power law velocities and scale-free structures, like fractals or multi-fractals (Sreenivasan 1991). The question is, how exactly does turbulence make the IMF? We do not know yet, but we can come close with a simple model that has all the essential physics.

6. A MODEL FOR THE IMF IN TURBULENT CLOUDS

An interstellar cloud is somewhat self-similar over a wide range of scales. The thermal Jeans mass hardly shows up in the correlations between total linewidth and size, or density and size. For this reason, the formation processes of stars are probably self-similar too for a wide range of masses, at least in the power-law part of the IMF. This means that in a hierarchical cloud, the mass that goes into a star can come from any level in the hierarchy, provided the corresponding clump is sufficiently self-gravitating at some time in its life to make a star.

The total mass range for clumpy structure in clouds is \(\sim 10^{10}\), whereas the mass range for stars in the power law part of the IMF is only \(\sim 100\). Because of this large difference in mass range, stars have to come from only the part of the cloud hierarchy that is fairly close to the thermal Jeans mass at the total cloud pressure. Below that, the gas cannot collapse easily. The break in the IMF from the power law part at intermediate to high mass and the relatively flat part at low mass occurs at this thermal Jeans mass, which is \(\sim 0.3\ M_\odot\) for typical conditions (Larson 1992; Elmegreen 1997). If there is no preference for scale above the thermal Jeans mass, then stars are essentially coming from a more-or-less random sample of clumps in the hierarchical gas distribution. Random samples from a hierarchy produce an \(M^{-2}dM\) mass distribution (Fleck 1996), which is already close to the Salpeter function of \(M^{-2.35}dM\). Random sampling from Fourier k-space produces a \(k^2dk = M^{-2}dM\) spectrum too (Elmegreen 1993). This is a good start to look for a theory of the IMF.

The next step is to recognize that the sampling process cannot be completely uniform on all scales. As we have seen, dynamical events work faster at higher densities. In a fractal cloud, the smallest fragments have the highest densities, so we should really be considering a random sample with a rate proportional to the dynamical rate, which scales as the square root of the local density. In this case, smaller mass clumps form stars more often than higher mass clumps, and that steepens the IMF from a slope of 2 to about 2.15 (Elmegreen 1997). The same steepening occurs if we think of the turbulent rate as a function of wavenumber \(k\) in phase space too, because the turbulent rate varies approximately as \(k^{1/2} \propto M^{-1/3}\), which steepens the IMF for purely turbulent sampling to \(k^{5/3}dk \propto M^{-13/6}\) (Elmegreen 1993).

Now there are two additional effects, one that takes care of itself automatically in the above picture, and another that is likely to happen anyway, and which requires a bit more physics. The first effect is a competition for gas: when a dense, low-mass clump turns into a star, the gas that made it is no longer available to make another star. This effect steepens the IMF to a slope equal to about the Salpeter value, 2.3 (or 1.3 for intervals of log \(M\)). The reason is that each mass structure surrounding the first star in the hierarchy of structures has a little less gas to make its second star.

The other effect is that structures that are very large will evolve so slowly that the gas inside of them should turbulently remix and make new dense cores before the larger scale itself can make a star. This differs from the competition for mass above, which works even if the turbulent structures are static. With the second effect, the gas that is left over after the formation of a low mass star in an original low mass core can get recycled by turbulent motions into forming more low mass cores, and this may form more low mass stars without ever getting into a large mass star on the larger scale. This constraint involves timing. Any scale that has a star formation time much larger than the turbulent mixing time of the smallest star-forming scale inside of it is not likely to form a single large star, but will form numerous smaller stars instead.

This second effect has been simulated in a computer model (Elmegreen 2000c) by rejecting any previously chosen clump of mass \(M\) with a probability of \(1 - e^{-t(M)/t(M_J)}\) for crossing time \(t(M)\) on scale \(M\) and for minimum mass \(M_J\), which is the Jeans mass or some other minimum. If \(M \approx M_J\) and \(t(M) \approx t(M_J)\), then stars freeze out at scales equal to or less than \(M\) without much turbulent remixing. If \(t(M) > t(M_J)\), then even after all the initial low mass clumps turn into stars, more low mass clumps will still have time to form from the residual gas and turn into more low mass stars. Then nothing is left over for a high mass star of mass \(M\). This ratio \(t(M)/t(M_J)\) is the average number of turbulent crossings for scale \(M_J\) that occur during a turbulent crossing time at scale \(M\). It is the mean waiting time for significant mixing on scale \(M\). Thus \(e^{-t(M)/t(M_J)}\) is the Poisson probability that no significant remixing occurs. The clump mass is related to the crossing time by \(M/M_J \propto [t(M)/t(M_J)]^{5}\). The observed IMF requires that \(t(M)/t(M_J)\) be no more than order unity, perhaps at most 2, limiting \(M/M_J\) to less than several hundred before the fall off in the high mass IMF becomes
steep. This limit in $t(M)/t(M_J)$ is consistent with the requirement of rapid star formation, discussed in Section 2.

Model IMFs with this timing constraint are shown in Elmegreen (2000c). The slope at intermediate mass steepens a little, from 2.3 to 2.35, which is in fact the Salpeter value, and it steepens a lot above $\sim 100 \, M_\odot$, making the formation of extremely massive stars as unlikely as the observations require.

7. Conclusions

The processes of star formation are not yet well enough understood to trace in detail the sequence of events that differentiates high and low mass stars. There is so much uniformity in the IMFs from different regions, however, that many of these factors may not be important for the final distribution anyway. Somehow the averaging inherent in plotting a histogram of stellar masses erases the memory of the different physical processes that were involved in forming the stars. Triggered regions of star formation have about the same IMFs as quiescent regions; large region have about the same functions as small regions, aside from sampling statistics, and moderately old regions are about the same as the youngest. What this means is that stars with the same mass could have formed in very different environments, even with different processes, but we would not necessarily know this from a mere histogram of final star mass. The averaging has erased the details.

The previous sections have proposed that the approximately power-law part of the IMF is somehow tracing the power-law conditions in star-forming clouds that are continuously established by pervasive turbulence. If true, then we do not need a theory of star formation to explain the IMF, but rather a theory of turbulence. This is the reverse of what most studies have been after: previous theories of the IMF began with a model for how stars form, usually ignoring the turbulent properties of the gas that goes into these stars, and then sampled the parameter space or the competitive processes until the final stellar masses were obtained. Now we suspect that any reasonable theory of star formation can give the same final IMF, even many of the theories that have already been proposed on physical grounds, provided only that they operate in a fractal, hierarchically-structured and turbulent medium. This may be why random sampling from hierarchical clouds at a rate that scales with the square root of the local density gives the right result: all theories of star formation have this basic scaling for the rate at which things happen.

The flattening at low mass down to the brown dwarf state may be a different matter. Here there is a characteristic mass, the mass at the lower limit to the Salpeter power law, and there may be a different reason for the mass distribution function below this limit than above. At the most fundamental level, however, this low-mass distribution is not that different from the high mass distribution: both are power laws for a factor of $\sim 100$ in mass range; they just have different powers. Maybe some scale-free properties of cloud or collapse dynamics are involved at low mass too. Regardless, the boundary itself has to contain some other physics to get the mass scale. For this reason, the mass at the boundary between the high-mass power law and the low-mass flat part may be expected to vary over the extreme range of star-forming environments. Such variations have been suggested for starburst regions, but direct observations at the boundary mass are still lacking.

The previous sections also emphasized the importance for the IMF of the dynamically rapid timescale for star formation in most regions. This seems to rule out a class of models that depends strongly on protostellar orbits in the cloud or on protostellar interactions. The observation of a similar IMF in clusters and in whole galaxies suggests further that stars of all masses can form in clouds of all masses. The issue here is that clouds are basically fractal in structure, so clouds of all masses are contained within, or are parts of, clouds with all higher masses. The definition of a cloud mass is vague. One uncertainty is whether dense clusters really have different high-mass IMFs than the average for all stars. Dense clusters may, in fact, have a slight excess of high-mass stars compared to the average for all stars; it would be very difficult to know this at the present time. If true, then accretion and coalescence effects may be important in dense cluster cores, but the fraction of stars which form under these conditions would have to be low. The Salpeter function found in dense clusters out to $100 \, M_\odot$ or more cannot continue indefinitely. If it did, then a galaxy the size of ours would have a few $1000 \, M_\odot$ stars (at birth), just by sampling the IMF for a large total mass.

Progress in understanding the IMF should come from two fronts: observations of statistically significant IMFs in a variety of different environments, sampling extremes in density, temperature, and pressure, and computational modeling of mass segregation processes in self-gravitating, turbulent, magnetic gas. Further IMF modeling in whole galaxies and careful studies of the timescale for star formation would be useful too.

References

Ali, B., & Deyo, D.L. 1995, AJ, 109, 709
Bally, J., Devine, D., & Reipurth, B. 1996, ApJ, 473, L49
Bastien, P. 1981, A&A, 93, 160
Basu, S., & Rana, N.C. 1992, ApJ, 393, 373
Beck, S. C., Kelly, D. M., & Lacy, J. H. 1998, AJ, 115, 2504
Belikov, A. N., Hirte, S., Meusinger, H., Piskunov, A. E., & Schilbach, E. 1998, A&A, 332, 575
Bhatt, H. C. 1989, A&A, 213, 299
Blauw, A. 1964, ARAA, 2, 213
Blaaha, C., & Humphreys, R.M. 1989, AJ, 98, 1598
Bonazzola, S., Heyvaerts, J., Falgarone, E., Perault, M., & Puget, J. L. 1987, A&A, 172, 293
Bonnell, I.A., & Davies, M.B. 1998, MNRAS, 295, 691
