Two Novel $\text{C}_3\text{N}_4$ Phases: Structural, Mechanical and Electronic Properties

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Abstract: We systematically studied the physical properties of a novel superhard ($t$-$\text{C}_3\text{N}_4$) and a novel hard ($m$-$\text{C}_3\text{N}_4$) $\text{C}_3\text{N}_4$ allotrope. Detailed theoretical studies of the structural properties, elastic properties, density of states, and mechanical properties of these two $\text{C}_3\text{N}_4$ phases were carried out using first-principles calculations. The calculated elastic constants and the hardness revealed that $t$-$\text{C}_3\text{N}_4$ is ultra-incompressible and superhard, with a high bulk modulus of 375 GPa and a high hardness of 80 GPa. $m$-$\text{C}_3\text{N}_4$ and $t$-$\text{C}_3\text{N}_4$ both exhibit large anisotropy with respect to Poisson’s ratio, shear modulus, and Young’s modulus. Moreover, $m$-$\text{C}_3\text{N}_4$ is a quasi-direct-bandgap semiconductor, with a band gap of 4.522 eV, and $t$-$\text{C}_3\text{N}_4$ is also a quasi-direct-band-gap semiconductor, with a band gap of 4.210 eV, with the HSE06 functional.

Keywords: $\text{C}_3\text{N}_4$ allotropes; mechanical properties; electronic properties; superhard materials

1. Introduction

Studies on light element-based materials trace back to the middle of the last century. Since Lavoisier found that diamond was isostuctural to carbon and much denser than graphite, many studies have been devoted to its synthesis under high pressure [1–4]. More and more researchers have begun to investigate the carbon allotropes [5–15]. The second light element-based material to be evidenced was boron nitride. It includes three different structures: blende-, wurtzite- and graphitic-type structures. Cubic boron nitride (c-BN) was first elaborated upon in 1957 by Wentorf, who performed direct conversion using graphitic boron nitride (at 7 GPa and 1500 °C) [16]. Many boron nitride allotropes have been investigated by researchers, such as O-BN, Pbca-BN, Z-BN, W-BN, h-BN, bc-BN, P-BN, and cT8-BN. Interest in carbon nitrides has been initiated by studying materials that exhibit mechanical properties comparable with those of diamond. A fullerene is a molecule of carbon in the form of a hollow sphere, ellipsoid, tube, and many other shapes. Gueorguiev et al. [17,18] studied the formation mechanisms and structural features of fullerene-like carbon nitride (FL CNx), utilizing first-principles calculations.

Liu et al. first predicted $\beta$-$\text{C}_3\text{N}_4$ [19]; its structure originated in $\beta$-$\text{Si}_3\text{N}_4$, with carbon substituting for silicon. In the same way, $\alpha$-$\text{C}_3\text{N}_4$ has been deduced from $\alpha$-$\text{Si}_3\text{N}_4$, replacing silicon with carbon. The bulk of $\alpha$-$\text{C}_3\text{N}_4$ and $\beta$-$\text{C}_3\text{N}_4$ is 387 and 427 GPa, respectively, which are slightly smaller than that of diamond (431 GPa [20]). Therefore, there are sp$^2$ and sp$^3$ hybridizations on carbon and nitrogen in $\alpha$-$\text{C}_3\text{N}_4$ and $\beta$-$\text{C}_3\text{N}_4$, respectively. The pseudocubic-$\text{C}_3\text{N}_4$ structure is isostructural to $\alpha$-$\text{CdIn}_2\text{Se}_4$ [21] and was first proposed by Liu and Wentzcovitch [22]. The network structure of pseudocubic-$\text{C}_3\text{N}_4$ consists of corners-linked CN$_4$ tetrahedra in which the C-N-C angle is close to 109°, which ensures sp$^3$...
hybridization for nitrogen. The bulk modulus of pseudocubic-C$_3$N$_4$ is 448 GPa, which is slightly larger than that of diamond.

Cubic-C$_3$N$_4$ is another C$_3$N$_4$ phase and is isostructural to the high-pressure structure of Zn$_2$SiO$_4$, which was proposed by Teter and Hemley [23]. The structure of cubic-C$_3$N$_4$ is similar to that of pseudocubic-C$_3$N$_4$, including the hybridization. Mo et al. [24] and later Kroll [25] proposed a γ-C$_3$N$_4$ polymorph derived from a γ-Si$_3$N$_4$ spinel high-pressure structure. The largest difference between these structures involves the hybridization of nitrogen and carbon. In pseudocubic-C$_3$N$_4$ or cubic-C$_3$N$_4$, both carbon and nitrogen adopt $sp^3$ hybridization. Graphite C$_3$N$_4$ (γ-C$_3$N$_4$) consists of the stacking along the c-axis of graphitic planes. Teter and Hemley first described these graphitic planes as a hexagonal organization of C$_3$N$_3$ triazine cycles. Because of its graphitic structure, the bulk modulus is only 51 GPa [22,26].

We propose $m$-C$_3$N$_4$ ($m$ denotes Monoclinic symmetry, space group: Cm) and $t$-C$_3$N$_4$ ($t$ denotes Tetragonal symmetry, space group: $I$-42$m$), whose structures are based on $m$-Si$_3$N$_4$ and $t$-Si$_3$N$_4$ [27], respectively, with C substituting for Si. The mechanical and electronic properties of $m$-C$_3$N$_4$ and $t$-C$_3$N$_4$ are presented in this work.

2. Computational Method

Density functional theory (DFT) [28,29] calculations within Vanderbilt ultrasoft pseudopotentials [30] were performed using the Cambridge Serial Total Energy Package (CASTEP) code [31]. For the exchange and correlation functional, we used the Perdew–Burke–Ernzerhof (PBE) version of the generalized gradient approximation (GGA) [32]. For $\alpha$-C$_3$N$_4$, $\beta$-C$_3$N$_4$, d-ZB-C$_3$N$_4$, Pseudocubic-C$_3$N$_4$, Cubic-C$_3$N$_4$, $g$-C$_3$N$_4$, $m$-C$_3$N$_4$ and $t$-C$_3$N$_4$, an energy cutoff of 520 eV was used for the wave functions expansion. High dense $k$-point [33] sampling, with a grid spacing of less than $2\pi \times 0.025$ Å$^{-1}$ ($7 \times 17 \times 9$ for $m$-C$_3$N$_4$, $11 \times 11 \times 6$ for $t$-C$_3$N$_4$, $12 \times 12 \times 12$ for d-ZB-C$_3$N$_4$, $12 \times 12 \times 12$ for pseudocubic-C$_3$N$_4$, $8 \times 8 \times 8$ for cubic-C$_3$N$_4$, $10 \times 10 \times 6$ for $g$-C$_3$N$_4$, $7 \times 7 \times 8$ for $\alpha$-C$_3$N$_4$, and $7 \times 7 \times 18$ for $\beta$-C$_3$N$_4$) in the Brillouin zone, was used. The equilibrium crystal structures were achieved by utilizing geometry optimization in the Broyden–Fletcher–Goldfarb–Shanno (BFGS) [34] minimization scheme. The self-consistent convergence of the total energy was $5 \times 10^{-6}$ eV/atom; the maximum force on the atom was 0.01 eV/Å, the maximum ionic displacement was within $5 \times 10^{-4}$ Å, and the maximum stress was within 0.02 GPa. The electronic properties of $t$-C$_3$N$_4$ and $m$-C$_3$N$_4$ were calculated using the Heyd–Scuseria–Ernzerhof (HSE06) [35,36] hybrid functional.

3. Results and Discussion

3.1. Structural Properties

The crystal structures of $m$-C$_3$N$_4$ and $t$-C$_3$N$_4$ are shown in Figure 1. There are 14 (six carbon atoms and eight nitrogen atoms) atoms in a conventional cell of $m$-C$_3$N$_4$ and $t$-C$_3$N$_4$. Within this structure of $m$-C$_3$N$_4$, three inequivalent carbon atoms occupy the (0.8392, 0.0, 0.5479), (0.2917, 0.0, 0.8418) and (0.9864, 0.0, 0.3048) positions, and four inequivalent nitrogen atoms occupy the (0.2050, 0.0, 0.3856), (0.8843, 0.0, 0.8186), (0.5600, 0.0, 0.1253) and (0.5344, 0.5, 0.5771) positions, while for $t$-C$_3$N$_4$, two inequivalent carbon atoms occupy the (0.5, 0.5, 0.0) and (0.5, 0.0, 0.75) positions, and nitrogen atoms occupy the (0.7330, 0.2670, 0.8705) position, respectively. The basic building block of $m$-C$_3$N$_4$ is the six-membered zigzag carbon-nitrogen rings, which can be clearly observed in Figure 1a; the twelve-membered zigzag carbon-nitrogen rings in the [010] direction in the structure of $m$-C$_3$N$_4$ are shown in Figure 1b, while six-membered zigzag carbon-nitrogen rings and eight-membered gauche carbon-nitrogen rings exist in $t$-C$_3$N$_4$. The equilibrium lattice parameters of $m$-C$_3$N$_4$, $t$-C$_3$N$_4$, d-ZB-C$_3$N$_4$ (space group: $P$-43$m$), cubic-C$_3$N$_4$ (space group: $I$-43$d$), pseudocubic-C$_3$N$_4$ (space group: $P$-42$m$), $g$-C$_3$N$_4$ (space group: $P$-6$m$2), $\alpha$-C$_3$N$_4$ (space group: $P$3$\overline{1}$c) and $\beta$-C$_3$N$_4$ (space group: $P63/m$) at ambient pressure are listed in Table 1. The calculated parameters of $\alpha$-C$_3$N$_4$ and $\beta$-C$_3$N$_4$ are in excellent agreement with previous theoretical results (see Table 1).
Figure 1. The crystal structures of: \(m\text{-C}_3\text{N}_4\) (a,b); and \(t\text{-C}_3\text{N}_4\) (c) (black spheres denote carbon atoms, blue spheres denote nitrogen atoms).

Table 1. Calculated lattice parameters (Å) of \(m\text{-C}_3\text{N}_4\), \(t\text{-C}_3\text{N}_4\), \(d\text{-ZB-C}_3\text{N}_4\), Cubic-C\(_3\)N\(_4\), Pseudocubic-C\(_3\)N\(_4\) and \(g\text{-C}_3\text{N}_4\).

| Materials         | This Work | Other Works |
|-------------------|-----------|-------------|
| \(\alpha\text{-C}_3\text{N}_4\) | 6.512     | \(a = 6.489, c = 4.729^a\); \(a = 6.425, c = 4.715^b\); \(a = 6.453, c = 4.699^d\); \(a = 6.47, c = 4.71^e\); |
| \(\beta\text{-C}_3\text{N}_4\)   | 6.449     | \(a = 6.426, c = 2.418^a\); \(a = 6.419, c = 2.425^b\); \(a = 6.402, c = 2.404^c\); \(a = 6.394, c = 2.397^d\); \(a = 6.40, c = 2.40^e\); |
| \(d\text{-ZB-C}_3\text{N}_4\)   | 3.456     | \(a = 3.455\); \(a = 3.52\); \(a = 3.43\) |
| cubic-C\(_3\)N\(_4\)        | 5.398     | \(a = 5.395\); \(a = 5.40\) |
| pseudocubic-C\(_3\)N\(_4\)   | 3.456     | \(a = 3.41\); \(a = 3.44\) |
| \(g\text{-C}_3\text{N}_4\)   | 4.791     | \(a = 4.74\); \(a = 6.72\) |
| \(m\text{-C}_3\text{N}_4\)   | 8.032     | \(a = 6.246\) |
| \(t\text{-C}_3\text{N}_4\)   | 3.483     | \(a = 6.933\) |

\(a\) Reference [37]; \(b\) Reference [38]; \(c\) Reference [23]; \(d\) Reference [39]; \(e\) Reference [40]; \(f\) Reference [41]; \(g\) Reference [22]; \(h\) Reference [42].
The calculated pressure–volume relationships of \( m \)-\( C_3N_4 \) and \( t \)-\( C_3N_4 \), together with diamond, \( c \)-\( BN \), and other \( C_3N_4 \) allotropes, are shown in Figure 2. The highest incompressibility along the \( c \)-axis is due to \( m \)-\( C_3N_4 \) in the \( C_3N_4 \) allotropes, while along the \( c \)-axis, \( m \)-\( C_3N_4 \) yields the lowest incompressibility at pressures from 0 to 87 GPa; along the \( b \)-axis, \( m \)-\( C_3N_4 \) yields the lowest incompressibility at pressures from 87 to 100 GPa. For the crystal structure, \textit{pseudocubic}–\( C_3N_4 \) has the greatest incompressibility in the \( C_3N_4 \) allotropes discussed above, while \( m \)-\( C_3N_4 \) has the weakest incompressibility. However, the incompressibility of \( t \)-\( C_3N_4 \) is greater than that of \( c \)-\( BN \) and the incompressibility of \( m \)-\( C_3N_4 \) is weaker than that of \( c \)-\( BN \).

![Figure 2](image-url)  
Figure 2. The lattice constants \( a/a_0 \), \( b/b_0 \), \( c/c_0 \) and \( V/V_0 \) of compression as functions of pressure and temperature for: \( m \)-\( C_3N_4 \) (a); and \( t \)-\( C_3N_4 \) (b).

### 3.2. Elastic Properties and Hardness

In an effort to assess the thermodynamic stability of two novel \( C_3N_4 \) allotropes, enthalpy change curves with pressure for various structures were calculated, as presented in Figure 3. The dashed line represents the enthalpy of the summary of diamond and \( \alpha \)-\( N_2 \). It can be clearly seen that \( g \)-\( C_3N_4 \) has the lowest minimum value of enthalpy, which is in good agreement with previous reports and supports the reliability of our calculations [43]. \textit{Pseudocubic}–\( C_3N_4 \) has the greatest minimum value of enthalpy. The minimum value of total energy per formula unit of \( t \)-\( C_3N_4 \) is slightly larger than that of \( g \)-\( C_3N_4 \), \( \alpha \)-\( C_3N_4 \), \( m \)-\( C_3N_4 \), and \( \beta \)-\( C_3N_4 \) but much smaller than those of \textit{pseudocubic}–\( C_3N_4 \) and \textit{cubic}–\( C_3N_4 \), indicating that \( t \)-\( C_3N_4 \) and \( m \)-\( C_3N_4 \) should be thermodynamically metastable phases.

![Figure 3](image-url)  
Figure 3. Calculated enthalpies of different \( C_3N_4 \) structures relative to diamond and \( \alpha \)-\( N_2 \) as a function of pressure.
For monoclinic symmetry and tetragonal symmetry, there are different independent elastic constants. Monoclinic symmetry has thirty independent elastic constants \((C_{11}, C_{22}, C_{33}, C_{44}, C_{55}, C_{66}, C_{12}, C_{13}, C_{23}, C_{15}, C_{25}, C_{35}\) and \(C_{46}\)), while tetragonal symmetry has six independent elastic constants \((C_{11}, C_{33}, C_{44}, C_{66}, C_{12}\) and \(C_{13}\)). The mechanical stability criteria of monoclinic symmetry are given by [44,45]:

\[
C_{ii} > 0, i = 1 \sim 6.
\]

\[
[C_{11} + C_{22} + C_{33} + 2(C_{12} + C_{13} + C_{23})] > 0
\]

\[
(C_{33}C_{55} - C_{35}^2) > 0
\]

\[
(C_{44}C_{66} - C_{46}^2) > 0
\]

\[
(C_{22} + C_{33} - 2C_{23}) > 0
\]

\[
[C_{22}(C_{33}C_{55} - C_{35}^2) + 2C_{23}C_{55} - C_{23}^2C_{35} - C_{25}^2C_{33}] > 0
\]

\[
\Omega = 2[C_{15}C_{25}(C_{33}C_{12} - C_{13}C_{23}) + C_{15}C_{35}(C_{22}C_{13} - C_{12}C_{35}) + C_{25}C_{35}(C_{11}C_{23} - C_{13}C_{21})] - [C_{45}^2(C_{22}C_{33} - C_{23}^2) + C_{25}^2(C_{11}C_{33} - C_{13}^2)] + C_{55}g > 0
\]

\[
g = C_{11}C_{22}C_{33} - C_{11}C_{23}^2 - C_{22}C_{13}^2 - C_{33}C_{12}^2 + 2C_{12}C_{13}C_{23}
\]

The criteria for the mechanical stability of tetragonal symmetry are given by [44]:

\[
C_{ii} > 0, i = 1, 3, 4, 6.
\]

\[
(C_{11} - C_{12}) > 0
\]

\[
(C_{11} + C_{33} - 2C_{13}) > 0
\]

\[
[2(C_{11} + C_{12}) + C_{33} + 4C_{13}] > 0
\]

The calculated elastic constants of \(\alpha\)-C\(_3\)N\(_4\), \(\beta\)-C\(_3\)N\(_4\), \(t\)-C\(_3\)N\(_4\), \(d\)-ZB-C\(_3\)N\(_4\), pseudocubic-C\(_3\)N\(_4\), cubic-C\(_3\)N\(_4\) and \(m\)-C\(_3\)N\(_4\) are listed in Table 2. Elastic constants under high pressure were also studied. The elastic constants under ambient pressure and high pressure of \(t\)-C\(_3\)N\(_4\) and \(m\)-C\(_3\)N\(_4\) satisfied the mechanical stability criteria of monoclinic symmetry and tetragonal symmetry. Namely, \(t\)-C\(_3\)N\(_4\) and \(m\)-C\(_3\)N\(_4\) are mechanically stable. To confirm the stability of \(t\)-C\(_3\)N\(_4\) and \(m\)-C\(_3\)N\(_4\), their dynamical stability should also be studied under ambient pressure and high pressures. Thus, we calculated the phonon spectra for \(m\)-C\(_3\)N\(_4\) and \(t\)-C\(_3\)N\(_4\) at 0 and 100 GPa, as shown in Figure 4. No imaginary frequencies are observed throughout the whole Brillouin zone, signaling dynamically the stabilities of \(m\)-C\(_3\)N\(_4\) and \(t\)-C\(_3\)N\(_4\). The calculated elastic modulus of \(\alpha\)-C\(_3\)N\(_4\), \(\beta\)-C\(_3\)N\(_4\), \(d\)-ZB-C\(_3\)N\(_4\), pseudocubic-C\(_3\)N\(_4\), cubic-C\(_3\)N\(_4\), \(t\)-C\(_3\)N\(_4\) and \(m\)-C\(_3\)N\(_4\) are listed in Table 3. Bulk modulus \(B\) and shear modulus \(G\) were calculated by using the Voigt–Reuss–Hill approximation [46–48]. The Voigt and Reuss approximation of monoclinic symmetry is calculated using the following equations [44]:

\[
B_V = \frac{1}{9}[C_{11} + C_{22} + C_{33} + 2(C_{12} + C_{13} + C_{23})]
\]

\[
B_R = \Omega (a + b + c + d + e + f)^{-1}
\]

\[
a = (C_{33}C_{55} - C_{35}^2)(C_{11} + C_{22} - 2C_{12})
\]

\[
b = (C_{23}C_{35} - C_{25}C_{33})(2C_{12} - 2C_{11} - C_{23})
\]

\[
c = (C_{13}C_{35} - C_{15}C_{33})(C_{15} - 2C_{25})
\]

\[
d = (C_{13}C_{35} - C_{15}C_{33})(2C_{12} + 2C_{23} - C_{13} - 2C_{22})
\]
\[
e = (C_{13}C_{25} - C_{15}C_{23})(C_{25} - C_{15})
\]
\[
f = C_{11}(C_{22}C_{35} - C_{23}^2) - C_{12}(C_{12}C_{35} - C_{15}C_{23}) + C_{15}(C_{12}C_{25} - C_{15}C_{22}) + C_{25}(C_{23}C_{35} - C_{25}C_{33})
\]
\[
G_V = \frac{1}{15}[C_{11} + C_{22} + C_{33} + 3(C_{44} + C_{35} + C_{66}) - (C_{12} + C_{13} + C_{23})]
\]
\[
G_R = 15\left\{\frac{4(f + h + i + j + k + l)}{\Omega} + \frac{3\left(\frac{g}{\Omega} + \frac{C_{44} + C_{66}}{C_{44}C_{66} - C_{46}^2}\right)}{\Omega}\right\}^{-1}
\]
\[
h = (C_{35}C_{55} - C_{35}^2)(C_{11} + C_{22} + C_{12})
\]
\[
i = (C_{23}C_{55} - C_{25}C_{35})(C_{11} - C_{12} - C_{23})
\]
\[
j = (C_{13}C_{35} - C_{15}C_{33})(C_{15} + C_{25})
\]
\[
k = (C_{13}C_{55} - C_{15}C_{35})(C_{22} - C_{23} - C_{12} - C_{13})
\]
\[
l = (C_{13}C_{25} - C_{15}C_{23})(C_{15} - C_{25}).
\]

The Voigt and Reuss approximation of tetragonal symmetry is calculated using the following equations [44]:

\[
B_V = \frac{1}{9}[4C_{13} + C_{33} + 2(C_{11} + C_{12})]
\]
and pressures are shown in Figure 5a,b. Most of them increase with pressure, whereas materials and can be expressed by the universal anisotropic index (\(m\)) approximations of shear modulus for \(-\text{C}_3\text{N}_4\) and \(360.75\) GPa (534.66 GPa), respectively. The Reuss approximation values of the shear modulus for \(-\text{C}_3\text{N}_4\) and \(427\) of 18

\[
B_R = \frac{(C_{11} + C_{12})C_{33} - 2C_{13}^2}{C_{11} + C_{12} + 2C_{33} - 4C_{13}} \quad (29)
\]

\[
G_V = \frac{1}{30}[C_{11} + C_{12} + 2C_{33} - 4C_{13} + 3C_{11} + 12C_{44} + 6C_{66} - 3C_{12}] \quad (30)
\]

\[
G_R = 15\left\{\frac{18B_V}{(C_{11} + C_{12})C_{33} - 2C_{13}^2} + \frac{6}{C_{11} - C_{12}} + \frac{6}{C_{44}^2} + \frac{3}{C_{66}}\right\}^{-1} \quad (31)
\]

The Hill approximation of monoclinic symmetry and tetragonal symmetry is calculated using the following equation:

\[
B_H = \frac{B_V + B_R}{2}, \quad G_H = \frac{G_V + G_R}{2} \quad (32)
\]

Young’s modulus and Poisson’s ratio can be calculated using the following formulas, respectively: 

\[
E = 9B_HG_H/(3B_H + G_H), \quad \nu = (3B_H - 2G_H)/(6B_H + 2G_H) \quad [49].
\]

The relationships between elastic constants and pressures are shown in Figure 5a,b. Most of them increase with pressure, whereas \(C_{66}\) and \(C_{15}\) of \(-\text{C}_3\text{N}_4\) decrease with pressure. The dependence of the elastic constants on pressure of \(C_{22}\) of \(-\text{C}_3\text{N}_4\), i.e., \(dC_{22}/dP = 6.97\), means that \(C_{22}\) of \(-\text{C}_3\text{N}_4\) increases fastest among all elastic constants.

**Figure 5.** Elastic constants of \(-\text{C}_3\text{N}_4\) (a) and \(t\)-\text{C}_3\text{N}_4 (b) as a function of pressure, and the ratio \(B/G\) (c); Poisson’s ratio \(\nu\) (d); and \(A^{U}\) (e) of \(-\text{C}_3\text{N}_4\) and \(t\)-\text{C}_3\text{N}_4\) as a function of pressure.
Table 2. The calculated elastic constants (GPa) of $m$-$C_3N_4$, $t$-$C_3N_4$, $d$-$ZB$-$C_3N_4$, cubic-$C_3N_4$, pseudocubic-$C_3N_4$ and cubic-BN.

| Materials          | $C_{11}$ | $C_{12}$ | $C_{13}$ | $C_{33}$ | $C_{34}$ | $C_{35}$ | $C_{56}$ | $C_{66}$ |
|--------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| $a$-$C_3N_4$       | 848       | –         | 906       | 319       | –         | 335       | 179       | 131       |
| Reference [37]     | 851       | –         | 906       | 326       | –         | 334       | 183       | 129       |
| $\beta$-$C_3N_4$   | 852       | –         | 1150      | 286       | –         | 312       | 228       | 111       |
| Reference [37]     | 833       | –         | 1049      | 289       | –         | 287       | 259       | 110       |
| Reference [22]     | 834       | –         | 1120      | 305       | –         | 279       | 138       | –         |
| $d$-$ZB$-$C_3N_4$  | 791       | –         | –         | 443       | –         | 184       | –         | –         |
| Reference [41]     | 794       | –         | –         | 431       | –         | 184       | –         | –         |
| Cubic-$C_3N_4$     | 889       | –         | –         | 518       | –         | 309       | –         | –         |
| Reference [26]     | 861       | –         | –         | 469       | –         | 300       | –         | –         |
| Pseudocubic-$C_3N_4$ | 790    | –         | 792       | 445       | –         | 444       | 188       | 187       |
| Reference [37]     | 804       | –         | 805       | 439       | –         | 439       | 183       | 183       |
| $m$-$C_3N_4$       | 564       | 1002      | 859       | 209       | 331       | 213       | 195       | 36        |
| $t$-$C_3N_4$       | 702       | –         | 767       | 428       | –         | 424       | 212       | 195       |
| Cubic-BN           | 823       | –         | –         | 479       | –         | 185       | –         | –         |
| Reference [50]-Exp.| 820       | –         | –         | 480       | –         | 190       | –         | –         |

Table 3. The calculated elastic modulus (GPa), $B/G$, hardness $Hv$ (GPa) and the universal anisotropic index of $m$-$C_3N_4$, $t$-$C_3N_4$, $d$-$ZB$-$C_3N_4$, cubic-$C_3N_4$, pseudocubic-$C_3N_4$ and cubic-BN.

| Materials          | $B$ | $G$ | $B/G$ | $E$ | $v$ | $Hv$ | $A^U$ |
|--------------------|-----|-----|-------|-----|-----|------|-------|
| $a$-$C_3N_4$       | 386.83 | 386.82 | 386.65 | 335.03 | 387 | 337 | 1.15 | 784 | 0.16 | 78 | 0.053 |
| Reference [48]     | 387.60 | 387.60 | 341.85 | 338.96 | 388 | 340 | 1.14 | 790 | 0.16 | 82 | 0.043 |
| $\beta$-$C_3N_4$   | 406.11 | 405.57 | 330.77 | 320.96 | 406 | 326 | 1.25 | 772 | 0.18 | 78 | 0.154 |
| Reference [48]     | 408.21 | 407.84 | 322.09 | 311.12 | 408 | 317 | 1.29 | 755 | 0.19 | 63 | 0.176 |
| $d$-$ZB$-$C_3N_4$  | 386.59 | 386.59 | 387.23 | 374.14 | 387 | 381 | 1.02 | 861 | 0.13 | 81 | 0.175 |
| Reference [51]     | –     | –     | –      | –      | 387 | 379 | 1.03 | 850 | 0.13 | 63 | –     |
| Cubic-$C_3N_4$     | 502.48 | 502.48 | 426.82 | 394.18 | 502 | 411 | 1.22 | 969 | 0.18 | 88 | 0.414 |
| Reference [48]     | –     | –     | –      | –      | 496 | 401 | 1.24 | 948 | 0.18 | 90 | 0.87 b |
| Pseudocubic-$C_3N_4$ | 388.21 | 388.21 | 387.66 | 373.98 | 388 | 381 | 1.02 | 861 | 0.13 | 81 | 0.183 |
| Reference [48]     | 390.02 | 390.02 | 384.75 | 376.86 | 390 | 382 | 1.02 | 864 | 0.13 | 70 | 0.04 t |
| $m$-$C_3N_4$       | 332.22 | 320.80 | 293.26 | 254.44 | 327 | 274 | 1.19 | 643 | 0.17 | 37 | 0.798 |
| $t$-$C_3N_4$       | 374.81 | 374.47 | 360.75 | 340.05 | 375 | 350 | 1.07 | 801 | 0.14 | 80 | 0.305 |
| c-BN               | 397.43 | 397.43 | 414.90 | 398.88 | 397 | 407 | 0.98 | 910 | 0.12 | 63 | 0.201 |
| –                  | –     | –     | –      | –      | –   | –   | –    | –   | –   | –   | –     |

a Reference [52]; b Reference [51]; c Reference [53]; d Reference [50]; e Reference [54]; f Reference [55].

The dependence of bulk modulus, shear modulus and Young’s modulus on pressure of $m$-$C_3N_4$ and $t$-$C_3N_4$ is 3.49, 0.59, and 2.15 and 3.41, 1.59, and 4.35, respectively. Young’s modulus of $t$-$C_3N_4$ increases faster than other elastic modulus, while the increase in the shear modulus of $m$-$C_3N_4$ is the slowest. At ambient pressure, the bulk modulus of $a$-$C_3N_4$, $\beta$-$C_3N_4$, $m$-$C_3N_4$ and $t$-$C_3N_4$ are 387 GPa, 406 GPa, 327 GPa and 375 GPa, respectively. The calculated hardness of $a$-$C_3N_4$, $\beta$-$C_3N_4$, $m$-$C_3N_4$, $t$-$C_3N_4$, Cubic-$C_3N_4$, $d$-$ZB$-$C_3N_4$, Pseudocubic-$C_3N_4$ and c-BN are shown in Table 3. The bulk modulus of $t$-$C_3N_4$ is 375 GPa, which is slightly larger than that of c-BN, while the bulk modulus of $m$-$C_3N_4$ is slightly smaller than that of c-BN. The hardness of $m$-$C_3N_4$ is only 37 GPa, which is approximately half of that of $a$-$C_3N_4$, $\beta$-$C_3N_4$, $d$-$ZB$-$C_3N_4$, pseudocubic-$C_3N_4$ and $t$-$C_3N_4$. The reason for this phenomenon is that the mechanical properties of $m$-$C_3N_4$ are not excellent compared with the other $C_3N_4$ allotropes and the bulk modulus, shear modulus and Young’s modulus are all smaller than those of other $C_3N_4$ allotropes.

In materials science, ductility is a solid material’s ability to deform under tensile stress. If a material is brittle, when subjected to stress, it will break without significant deformation (strain). Additionally, these material properties are dependent on pressure. Pugh [56] proposed a simple relationship to judge the plastic properties of materials based on their elastic modulus, i.e., $B/G$. If the ratio $B/G$ is
larger than 1.75, a material exhibits the ductile property; otherwise, the material is brittle. Moreover, Poisson’s ratio $v$ is another criterion for judging the plastic properties of materials [57]. A larger $v$ value ($v > 0.26$) for a material indicates ductility, while a smaller $v$ value ($v < 0.26$) usually denotes brittleness. At ambient pressure, the ratio $B/G$ and $v$ of $\alpha$-C$_3$N$_4$, $\beta$-C$_3$N$_4$, $d$-ZB-C$_3$N$_4$, Cubic-C$_3$N$_4$, Pseudocubic-C$_3$N$_4$, $m$-C$_3$N$_4$ and $t$-C$_3$N$_4$ are as listed in Table 3. The ratio $B/G$ and $v$ of four $C_3N_4$ allotropes are all less than 1.75 and 0.26, respectively, which indicates that the four $C_3N_4$ allotropes are all brittle. $t$-C$_3$N$_4$ has the most brittleness, while $\beta$-C$_3$N$_4$ has the least brittleness. The pressure dependence of $B/G$ and Poisson’s ratio $v$ are shown in Figure 5c,d, respectively. In Figure 5c,d, the $B/G$ and $v$ of $m$-C$_3$N$_4$ and $t$-C$_3$N$_4$ increase with increasing pressure. $m$-C$_3$N$_4$ is found to change from brittle to ductile at 71 GPa, while $t$-C$_3$N$_4$ does not change from brittle to ductile in this pressure range.

The elastic anisotropy of a solid is closely related to the possibility of inducing microcracks in materials and can be expressed by the universal anisotropic index ($A^U$) [58]. The universal anisotropic index is defined as $A^U = 5G_N/G_R + B_N/B_R - 6$. The calculated results of universal anisotropic index are also shown in Table 3. The universal anisotropic index of $\alpha$-C$_3$N$_4$ is only 0.073, which is approximately one-third that of $\beta$-C$_3$N$_4$, approximately one-sixth that of $t$-C$_3$N$_4$, and approximately one-sixteenth that of $m$-C$_3$N$_4$. Namely, $\alpha$-C$_3$N$_4$ and $m$-C$_3$N$_4$ exhibit the smallest and largest elastic anisotropy in $A^U$, respectively. The pressure dependence of the universal anisotropic index is shown in Figure 5e. The universal anisotropic index of $m$-C$_3$N$_4$ increases faster than that of $t$-C$_3$N$_4$. The reason for this phenomenon is that the difference between the value of Voigt and Reuss approximations of shear modulus for $m$-C$_3$N$_4$ are greater than that of $t$-C$_3$N$_4$. At 0 GPa (100 GPa), the Voigt approximation values of the shear modulus for $m$-C$_3$N$_4$ and $t$-C$_3$N$_4$ are 293.26 GPa (403.14 GPa) and 360.75 GPa (534.66 GPa), respectively. The Reuss approximation values of the shear modulus for $m$-C$_3$N$_4$ and $t$-C$_3$N$_4$ are 254.44 GPa (262.45 GPa) and 340.05 GPa (484.94 GPa) at 0 GPa (100 GPa), respectively. The difference between the values of the Voigt and Reuss approximations of the shear modulus for $m$-C$_3$N$_4$ ranges from 38.8 to 140.7 GPa at 0 GPa and 100 GPa, respectively. Nevertheless, the difference between the value of the Voigt and Reuss approximations of the shear modulus for $t$-C$_3$N$_4$ only ranges from 20.7 to 49.7 GPa at 0 GPa and 100 GPa, respectively. Thus, the universal anisotropic index $m$-C$_3$N$_4$ increases faster than that of $t$-C$_3$N$_4$.

To analyze the anisotropy of $m$-C$_3$N$_4$ and $t$-C$_3$N$_4$ more systematically, we will investigate the anisotropy of $m$-C$_3$N$_4$ and $t$-C$_3$N$_4$ for Poisson’s ratio, the shear modulus and Young’s modulus by utilizing the ELAM codes [20,59]. The two-dimensional representations of Poisson’s ratio in the $xy$ plane, $xz$ plane and $yz$ plane for $m$-C$_3$N$_4$ and $t$-C$_3$N$_4$ are shown in Figure 6. The blue, red and cyan lines represent Poisson’s ratio at 0, 50 and 100 GPa, while the solid line and dash-dot line represent the minimum and maximum values of Poisson’s ratio in the $xy$ plane, $xz$ plane and $yz$ plane, respectively. From Figure 6, it is clear that the anisotropy of Poisson’s ratio for $m$-C$_3$N$_4$ and $t$-C$_3$N$_4$ increases with increasing pressure. The maximum value of Poisson’s ratio for $m$-C$_3$N$_4$ is 0.47, 0.61 and 0.76 at 0, 50 and 100 GPa, while the minimum value of Poisson’s ratio for $m$-C$_3$N$_4$ is 0.01; the maximum and minimum values of Poisson’s ratio are the same for $m$-C$_3$N$_4$ in the $xy$ plane, $xz$ plane and $yz$ plane. The maximum value of Poisson’s ratio for $t$-C$_3$N$_4$ is 0.30, 0.38 and 0.44 at 0, 50 and 100 GPa, respectively, while the minimum value of Poisson’s ratio for $t$-C$_3$N$_4$ is 0.0; the maximum and minimum values of Poisson’s ratio are the same for $t$-C$_3$N$_4$ in the $xy$ plane, $xz$ plane and $yz$ plane. The difference between the maximum and minimum values of Poisson’s ratio for $t$-C$_3$N$_4$ and $m$-C$_3$N$_4$ shows that $m$-C$_3$N$_4$ exhibits greater anisotropy with respect to Poisson’s ratio.
Figure 6. 2D representation of Poisson’s ratio in the $xy$ plane, $xz$ plane and $yz$ plane for: $m$-$C_3N_4$ (a–c); and $t$-$C_3N_4$ (d–f). The solid line represents the minimum, and the dashed line represents the maximum. The blue, red and cyan lines represent Poisson’s ratio at $p = 0$, 50 and 100 GPa, respectively.
The 2D representations of the shear modulus in the $xy$ plane, $xz$ plane and $yz$ plane for $m$-$C_3N_4$ and $t$-$C_3N_4$ are illustrated in Figure 7. For $m$-$C_3N_4$ in Figure 7a–c, the maximum value of the shear modulus occurs in the deviation from the $x$ axis or $y$ axis of approximately 15 degrees in the $xz$ plane. Moreover, the minimum value of the shear modulus occurs along the $y$ axis in the $xy$ plane and $yz$ plane, respectively. The maximum and the minimum values of $m$-$C_3N_4$ are 163 GPa and 455 GPa at ambient pressure, respectively, and the ratio $G_{\text{max}}/G_{\text{min}} = 2.79$. At the same time, the maximum and minimum values of $m$-$C_3N_4$ are 153 GPa (110 GPa) and 611 GPa (738 GPa), respectively, at 50 GPa (100 GPa); the ratio $G_{\text{max}}/G_{\text{min}} = 3.99$ at $p = 50$ GPa, and the ratio $G_{\text{max}}/G_{\text{min}} = 6.71$ at $p = 100$ GPa. The anisotropy of $m$-$C_3N_4$ increases with increasing pressure. The average shear modulus of $m$-$C_3N_4$ is 266 GPa, 312 GPa and 328 GPa, respectively. For $t$-$C_3N_4$ in Figure 7d–f, the maximum value of the shear modulus for $t$-$C_3N_4$ in the $xy$ plane, $xz$ plane and $yz$ plane appears along the coordinate axis, while the minimum value of the shear modulus for $t$-$C_3N_4$ in the $xy$ plane, $xz$ plane and $yz$ plane appears in the deviation from the coordinate axis of approximately 45 degrees. The maximum and minimum values of the shear modulus for $t$-$C_3N_4$ are 245 GPa, 291 GPa, 323 GPa and 428 GPa, 559 GPa, 669 GPa at 0 GPa, 50 GPa, 100 GPa, respectively. The ratio $G_{\text{max}}/G_{\text{min}} = 1.75$ at $p = 0$ GPa, the ratio $G_{\text{max}}/G_{\text{min}} = 1.92$ at $p = 50$ GPa, and the ratio $G_{\text{max}}/G_{\text{min}} = 2.07$ at $p = 100$ GPa. It is clear that the ratio $G_{\text{max}}/G_{\text{min}}$ for $t$-$C_3N_4$ is much smaller than that for $m$-$C_3N_4$. In other words, $m$-$C_3N_4$ exhibits greater anisotropy than $t$-$C_3N_4$. This agrees well with our previous prediction of anisotropy with respect to the universal anisotropic index and Poisson’s ratio.

As a valid method to describe the elastic anisotropic behavior of a crystal completely, the 3D surface constructions of the directional dependences of reciprocals of Young’s modulus are practically useful. The results are shown in Figure 8 for Young’s modulus. For an isotropic system, the 3D shape reflects the content of anisotropy [60]. In Figure 8a,c, the 3D shape of Young’s modulus shows that $m$-$C_3N_4$ exhibits greater anisotropy than $t$-$C_3N_4$. As the pressure increases, the anisotropy of Young’s modulus for $m$-$C_3N_4$ and $t$-$C_3N_4$ increases, but $m$-$C_3N_4$ still exhibits greater anisotropy than $t$-$C_3N_4$. To analyze the anisotropy of $m$-$C_3N_4$ and $t$-$C_3N_4$ in detail, the 2D representations of Young’s modulus in the $xy$ plane, $xz$ plane and $yz$ plane for $m$-$C_3N_4$ and $t$-$C_3N_4$ are depicted in Figure 9. From Figure 9, it is clear that $m$-$C_3N_4$ has a larger anisotropy and that the anisotropy will become larger with increasing pressure.

Young’s modulus of $t$-$C_3N_4$ has the same value in different planes, while that of $m$-$C_3N_4$ has different values in different planes. For example, at ambient pressure, the maximum and minimum values of Young’s modulus for $t$-$C_3N_4$ are 928 GPa and 612 GPa in the $xy$ plane, $xz$ plane and $yz$ plane, while at $p = 100$ GPa, they are 1510 GPa and 864 GPa, respectively. However, the maximum value of Young’s modulus is 996 GPa in the $xy$ plane and $yz$ plane for $m$-$C_3N_4$, but in the $xz$ plane, it is 995 GPa, and the minimum value is always 476 GPa. At 100 GPa, the difference reaches a larger degree; in the $xy$ plane and $yz$ plane, the maximum value of Young’s modulus for $m$-$C_3N_4$ is 1638 GPa, while the maximum value is 1634 GPa. This also proves that $m$-$C_3N_4$ has a larger anisotropy from the other side.
Figure 7. 2D representation of shear modulus in the \(xy\) plane, \(xz\) plane and \(yz\) plane for: \(m\)-C\(_3\)N\(_4\) (a–c); and \(t\)-C\(_3\)N\(_4\) (d–f).
3.3. Electronic Structures

Band theory is one of the most stringent tests of the physics of semiconductors. For example, silicon, calcite and copper all contain similar densities of electrons, but they have different physical properties, all inexplicable without quantum mechanics [61]. Thus, it is necessary to understand the band structure and density of states. The band structures and density of states of m-C₃N₄ and t-C₃N₄ at different pressures are shown in Figure 10. The band structure calculations show that m-C₃N₄ is a quasi-direct band gap semiconductor, with a band gap of 4.52 eV (see Figure 10a), and at 100 GPa, m-C₃N₄ remains a quasi-direct band gap semiconductor, with a band gap of 5.68 eV. t-C₃N₄ has a quasi-direct band gap of 4.21 eV at (0.322 0.322 0.0) along the M–Γ direction and Γ point, while the direct band gap is 4.22 eV at the Γ point. At 100 GPa, t-C₃N₄ has a quasi-direct band gap band gap of 4.79 eV at (0.322 0.322 0.0) along the M–Γ direction and Γ point, while the direct band gap is 4.81 eV at the Γ point. At 0 GPa, the valence band maximum (VBM) of m-C₃N₄ is located at the Z point, the energy of VBM near the Fermi level of m-C₃N₄ is 10.37 eV, and the energy of the Z point near the Fermi level is 10.39 eV; thus, m-C₃N₄ is a quasi-direct band gap semiconductor. At 100 GPa, the Fermi level of m-C₃N₄ increases to 14.04 eV, and the energy of the Z point near the Fermi level of m-C₃N₄ is 14.02 eV. The VBM of m-C₃N₄ is located at the point along the Z and Γ directions; its energy is 14.04 eV. The conduction band minimum (CBM) is at the Γ point for m-C₃N₄ at 0 and 100 GPa. The energy of CBM is 14.91 and 19.72 eV. At 0 GPa, the Fermi level of m-C₃N₄ is 10.39 eV, which is slightly smaller than that of t-C₃N₄ (10.52 eV). For t-C₃N₄, the CBM is at the Γ point for t-C₃N₄ at 0 and 100 GPa; the energy of CBM is 14.73 and 18.82 eV, respectively. The VBM of t-C₃N₄ is located at the point along the M and Γ directions; the energy is 10.52 and 14.03 eV, respectively. Moreover, the energy of the Γ point near the Fermi level of t-C₃N₄ is 10.51 eV and 14.01 eV at 0 and 100 GPa, respectively. Thus, t-C₃N₄ is a quasi-direct band gap semiconductor. Interestingly, the band gaps of m-C₃N₄ and t-C₃N₄ both increase with increasing pressure. At 100 GPa, m-C₃N₄ increases by 25.61%, and t-C₃N₄ increases by 13.66% compared with that at 0 GPa.
Figure 9. 2D representation of Young’s modulus in the $xy$ plane, $xz$ plane and $yz$ plane for: $m$-C$_3$N$_4$ (a–c); and $t$-C$_3$N$_4$ (d–f). The blue, red and cyan lines represent Poisson’s ratio at $p = 0\, 50$ and $100$ GPa, respectively.
3.3. Electronic Structures

Band theory is one of the most stringent tests of the physics of semiconductors. For example, silicon, calcite and copper all contain similar densities of electrons, but they have different physical properties, all inexplicable without quantum mechanics [61]. Thus, it is necessary to understand the electronic band structure and density of states. The band structures and density of states of the two novel C\textsubscript{3}N\textsubscript{4} allotropes are both brittle. The \( B/G \) and \( v \) of the two novel C\textsubscript{3}N\textsubscript{4} phases are both less than 1.75 and 0.26, respectively, which indicates that the two novel C\textsubscript{3}N\textsubscript{4} allotropes are both brittle. The \( B/G \) and \( v \) of \( m\text{-C}_3\text{N}_4 \) and \( t\text{-C}_3\text{N}_4 \) increase with increasing pressure. \( m\text{-C}_3\text{N}_4 \) is found to change from being brittle to ductile at 71 GPa, while \( t\text{-C}_3\text{N}_4 \) does not change from being brittle to ductile in this pressure range. The elastic anisotropy calculations show that \( m\text{-C}_3\text{N}_4 \) and \( t\text{-C}_3\text{N}_4 \) both exhibit large anisotropy with respect to Poisson’s ratio, the shear modulus and Young’s modulus and universal anisotropic index. The band structure calculations show that \( m\text{-C}_3\text{N}_4 \) and \( t\text{-C}_3\text{N}_4 \) are a quasi-direct-band-gap semiconductor and a quasi-direct-band-gap semiconductor, respectively. Moreover, the band gaps of \( m\text{-C}_3\text{N}_4 \) and \( t\text{-C}_3\text{N}_4 \) continue to be a quasi-direct band-gap and quasi-direct band gap at 100 GPa, respectively. The band gaps of \( m\text{-C}_3\text{N}_4 \) and \( t\text{-C}_3\text{N}_4 \) are 4.522 and 4.210 eV, respectively, and these materials are both wide-band-gap semiconductors. Due to their quasi-direct band gaps, they are attractive for luminescent device applications.

4. Conclusions

In conclusion, we have predicted two novel C\textsubscript{3}N\textsubscript{4} allotropes, \( i.e., m\text{-C}_3\text{N}_4 \) and \( t\text{-C}_3\text{N}_4 \), with space groups \( Cm \) and \( I-42m \), which are both mechanically and dynamically stable up to at least 100 GPa. The bulk modulus of \( t\text{-C}_3\text{N}_4 \) is 375 GPa, which is slightly larger than that of \( c\text{-BN} \), while the bulk modulus of \( m\text{-C}_3\text{N}_4 \) is slightly smaller than that of \( c\text{-BN} \). The hardness of \( t\text{-C}_3\text{N}_4 \) is larger than that of \( c\text{-BN} \), thereby making it a superhard material with potential technological and industrial applications. The ratio \( B/G \) and \( v \) of the two novel C\textsubscript{3}N\textsubscript{4} phases are both less than 1.75 and 0.26, respectively, which indicates that the two novel C\textsubscript{3}N\textsubscript{4} allotropes are both brittle. The \( B/G \) and \( v \) of \( m\text{-C}_3\text{N}_4 \) and \( t\text{-C}_3\text{N}_4 \) increase with increasing pressure. \( m\text{-C}_3\text{N}_4 \) is found to change from being brittle to ductile at 71 GPa, while \( t\text{-C}_3\text{N}_4 \) does not change from being brittle to ductile in this pressure range. The elastic anisotropy calculations show that \( m\text{-C}_3\text{N}_4 \) and \( t\text{-C}_3\text{N}_4 \) both exhibit large anisotropy with respect to Poisson’s ratio, the shear modulus and Young’s modulus and universal anisotropic index. The band structure calculations show that \( m\text{-C}_3\text{N}_4 \) and \( t\text{-C}_3\text{N}_4 \) are a quasi-direct-band-gap semiconductor and a quasi-direct-band-gap semiconductor, respectively. Moreover, the band gaps of \( m\text{-C}_3\text{N}_4 \) and \( t\text{-C}_3\text{N}_4 \) continue to be a quasi-direct band-gap and quasi-direct band gap at 100 GPa, respectively. The band gaps of \( m\text{-C}_3\text{N}_4 \) and \( t\text{-C}_3\text{N}_4 \) are 4.522 and 4.210 eV, respectively, and these materials are both wide-band-gap semiconductors. Due to their quasi-direct band gaps, they are attractive for luminescent device applications.

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