Density and Boundary Effects on Pion Distributions in Relativistic Heavy–Ion Collisions

Alejandro Ayala and Augusto Smerzi

1) Department of Physics, University of Illinois at Urbana-Champaign, 1110 W.Green St. Urbana, IL 61801-3080, USA
2) INFN, Istituto Nazionale di Fisica Nucleare, Italy

We compute the pion inclusive momentum distribution in a heavy–ion collision, assuming thermal equilibrium and accounting for boundary effects at the time of decoupling. We calculate the chemical potential corresponding to an average pion multiplicity in central collisions and explore the consequences of having the pion system produced close to the critical temperature for Bose–Einstein condensation.

PACS numbers: 03.65.Pm, 05.30.Jp, 25.75.-q

In recent years, much experimental effort has been devoted to the production of matter at high densities and temperatures in relativistic nuclear collisions. A main goal of such experiments is to detect the transition from hadronic matter to the quark gluon plasma (QGP). The expectation is that by colliding heavy systems, as opposed to single hadron–hadron collisions, the chances of producing a highly compressed state of matter, such as the QGP, are increased. Therefore, any experimental signal for the production of QGP is better distinguished if we compare similar signatures to those obtained from the collisions of smaller systems such as nucleon–nucleon (n–n) collisions.

In this spirit, recent experiments have concentrated their efforts on measuring one of those signatures, namely, the inclusive single particle momentum distributions for some of the most abundantly produced particles after a heavy ion reaction such as pions.

The most striking feature reported [1]–[5] is an enhancement at low as well as at high transverse momentum on the inclusive single pion distributions as compared to n–n collisions. This observation has sparked a great deal of theoretical effort trying to explain the origin of such peculiar behavior [6]–[14]. Li and Bauer [7] have supported the idea that a superposition of two Maxwell–Bolzmann (M–B) distributions with different temperatures might account for the observed behavior. Atwater et al [8] and Lee and Heinz [9] have suggested the possibility of transverse flow. Kataja and Ruuskanen [10] have fitted a non–zero chemical potential to the Bose–Einstein (B–E) distribution. Mostafa and Wong [11] stressed the importance of properly accounting for boundary effects at freezeout. Sollfrank et al [12] and G.E. Brown et al [13] (see also ref. [5]) included the influence of resonant decays into the picture. The effects of Coulomb final state interactions in describing the momentum distribution of charged particles has also been pointed out [14].

While the expansion, boundary and Coulomb effects have been amply discussed in the literature, one of the main features of systems described by B–E statistics has not been given enough consideration, namely, the fact that close to some temperature \( T_c \), bosons occupy predominantly the lowest energy state [15], [16]. This tendency of bosons to bunch together leads to B–E condensation at \( T_c \) and could be responsible for the enhancement of the distribution at low \( p_t \). It is remarkable, as we will latter show, that the pion multiplicities reached in some heavy ion experiments, together with the size of the pion source, as measured for example by Hanbury–Brown Twiss (HBT) interferometry, yield a value for \( T_c \) close to the temperatures which are consistent with typical abundances [17].

On the other hand, when the system of pions can be treated as being confined just before freezeout and the wave functions for the states satisfy a given condition at the confining boundary [1], the states form a discrete set. In this case, the density of states contributing to the momentum distribution is larger at high \( p_t \) compared to a simple B–E distribution. This effect, together with an initial transverse flow at freezeout, could account for the enhancement of the distribution at high \( p_t \).

In this paper, we compute the momentum distribution for pions produced in a relativistic heavy ion collision, assuming that at freezeout, the pions are in thermal equilibrium and incorporating boundary effects. We reserve the discussion of possible expansion effects for a following up work.

Recall that a system which consists of a fixed number \( N \) of weakly interacting, spin zero bosons, in thermal equilibrium at a temperature \( T \) is described by a grand–canonical ensemble obeying B–E statistics. If \( E_i \) represents the energy of a single particle state, labeled by \( i \), then the number of particles, chemical potential \( \mu \) and temperature are related by

*Current address: SISSA, Via Beirut 4, 34014, Trieste Italy
We start by estimating the critical temperature for the onset of B–E condensation. Fig.1 shows plots of $N$ versus $T$ for different values of $R$. Recall that the total multiplicity in a heavy ion reaction is a function of the invariant energy $\sqrt{s}$ in the collision. The multiplicity increases logarithmically with $\sqrt{s}$. At AGS energies ($\sqrt{s} \sim 5$ A GeV) for example, the average pion multiplicity per event produced in central collisions is on the order of 400–500 and thus the number of pions of a particular kind is roughly a third of the above. From Fig.1, we notice that the value of $T_c$ for a number of pions of a particular species between 100–200 is fairly high and decreases when the volume increases. From the discussion above, the contribution from the discrete energy states has to be accounted for. Then it is clear that when $T \rightarrow T_o$, one needs to return to the discrete picture and account for the contribution of the individual energy states to quantities such as the momentum distribution. For this purpose, we are required to make general assumptions about the evolution of the system.

We assume that the system of pions (of a particular species) produced after a heavy ion collision is in thermal equilibrium at its time of formation. This corresponds to assuming that the total collision rate before the time of decoupling is high compared, for instance, to the expansion rate.

For a given average multiplicity, we can then use a grand–canonical ensemble to describe the statistical properties of the pion system. But statistics alone is not enough to describe the system’s evolution, we also have to account for the fact that at the time of decoupling, the pion system has a finite size and is confined within the boundaries of a given volume. The shape of this volume is certainly an important issue. Bjorken dynamics is, for instance, implies an overall longitudinal expansion and thus that cylindrical geometry is better suited. Moreover, the time of decoupling is not necessarily the same over the entire volume. However, in order to gain physical insight into the problem, we will study the case in which the confining volume has spherical shape and the decoupling time is unique in the c.m. frame. We thus consider the following scenario: At the time of decoupling, when strong interactions have ceased, the system of pions (of a given species) is in thermal equilibrium and is confined within a sphere of radius $R$ (fireball) as viewed from the center of mass of the colliding nuclei.

We start by estimating the critical temperature for the onset of B–E condensation. Fig.1 shows plots of $N$ versus $T$ for different values of $R$. Recall that the total multiplicity in a heavy ion reaction is a function of the invariant energy $\sqrt{s}$ in the collision. The multiplicity increases logarithmically with $\sqrt{s}$. At AGS energies ($\sqrt{s} \sim 5$ A GeV) for example, the average pion multiplicity per event produced in central collisions is on the order of 400–500 and thus the number of pions of a particular kind is roughly a third of the above. From Fig.1, we notice that the value of $T_c$ for a number of pions of a particular species between 100–200 is fairly high and decreases when the volume increases. From the discussion above, the contribution from the discrete energy states has to be properly accounted for. Let us proceed to compute this contribution. We first solve for the relativistic wave function corresponding to the stationary states of a free particle inside a sphere of radius $R$

\[
\left(\frac{\partial^2}{\partial r^2} - \nabla^2 + m^2\right) \psi(\vec{r},t) = 0.
\]

We impose the boundary conditions corresponding to a rigid sphere, namely

\[
\psi(R,t) = 0,
\]

to describe the initial particle confinement at freezeout. The normalized solutions to eq. (3), together with the boundary condition, eq. (4), are [20]

\[
\psi_{k\ell m}(\vec{r},t) = \frac{1}{R J_{l+3/2}(kR)} \left(\frac{1}{r E_{k\ell}}\right)^{1/2} Y_{l+1/2} (\hat{r}) J_{l+1/2}(k r) e^{-i E_{k\ell} t},
\]

(5)

where $J_n$ is a Bessel function of the first kind and $Y_{lm}(\hat{r})$ are the spherical harmonics. The quantum number $k$ is given by the solution to

\[
N = \sum_{j=1}^{\infty} \frac{1}{e^{(E_j - \mu)/T} - 1}.
\]

Since $n(E_i)$ cannot be negative, we have the condition that $\mu < E_o$, where $E_o$ is the lowest single particle energy state. For a fixed number of particles, $\mu$ is a function of $T$. The peculiar characteristic of eq. (1) is that when $T$ approaches a certain critical value $T_c$, $\mu$ approaches the value $E_o$ and thus, this energy state becomes more populated as $T$ gets closer to $T_c$.

One can estimate the value of $T_c$ by considering the continuum limit of eq. (1). The result is that for a system of weakly interacting relativistic bosons, $T_c$ is given implicitly by

\[
N = \frac{V m^3}{2 \pi^2} \sum_{j=1}^{\infty} \left(\frac{T_c}{m_j}\right) e^{m_j/T_c} K_2 \left(\frac{m_j}{T_c}\right)
\]

where $V$ is the volume, $m$ is the boson’s mass and $K_2$ is the modified Bessel function of the second kind and order 2. However, the above limit assigns a weight zero to the lowest energy state. Then it is clear that when $T \rightarrow T_o$, one needs to return to the discrete picture and account for the contribution of the individual energy states to quantities such as the momentum distribution. For this purpose, we are required to make general assumptions about the evolution of the system.
The energy eigenvalues are related to $k$ by
\begin{equation}
E_{kl} = \sqrt{k^2 + m^2}.
\end{equation}

The normalized contribution to the momentum distribution from the energy state with quantum numbers $k, l, m'$ is proportional to the absolute value squared of the space Fourier transform of eq. (2)
\begin{equation}
\phi_{klm'}(\vec{p}) = (2E_{kl})|\psi_{klm'}(\vec{p})|^2,
\end{equation}
where
\begin{equation}
\psi_{klm'}(\vec{p}) = \frac{1}{(2\pi)^{3/2}} \int d^3r e^{-i\vec{p} \cdot \vec{r}} \psi_{klm'}(\vec{r}).
\end{equation}

Due to the azimuthal symmetry of the problem, the wave function in momentum space does not depend on the quantum number $m'$ and is a function only of the momentum magnitude
\begin{equation}
\psi_{klm'}(\vec{p}) = \psi_{kl}(p) \delta_{m'0}.
\end{equation}

Accordingly, the thermal momentum distribution is given by
\begin{equation}
\frac{d^3N}{d^3p} = \sum_{k,l} \phi_{kl}(p) e^{(E_{kl} - \mu)/T - 1}.
\end{equation}
where $\phi_{kl}(p)$ is given explicitly by
\begin{equation}
\phi_{kl}(p) = \frac{(2l + 1)}{2\pi p} \left[ \frac{k J_{l+1/2}(pR)}{(k^2 - p^2)} \right]^2
\end{equation}
and $\mu$ is computed from
\begin{equation}
N = \sum_{k,l} \frac{4(2l + 1)}{e^{(E_{kl} - \mu)/T - 1}}
\end{equation}
for fixed $N$. In general, we can expect an enhancement at high transverse momentum, relative to a simple B–E distribution, due to the finite size of the system at freezeout, which results in a higher density of states contributing at high $p$ than at low $p$. The shape of the distribution at low $p$ should also be affected by a finite chemical potential.

Fig. 2 shows the transverse momentum distribution $md^2N/m^2dmdy$ at fixed $y (= 1.4)$, as a function of $p_t$ calculated from eqs. (11) and (12) for several values of $R$, $T$ and $N$ and for $\mu$ computed from eq. (13). For the chosen set of parameters, the main effect is the bending upwards of the distribution at high $p_t$ relative to a simple exponential with the same temperature. We also find, in agreement with reference [11], that the distribution starts deviating from a simple exponential fall off at smaller intermediate values of $p_t$ as $R$ decreases.

Fig. 3 shows the distribution for $R = 8$ fm, $T = 150$ MeV, $N = 150$, compared to a simple B–E distribution with the same parameters and a chemical potential corresponding to the same number of particles. Notice how the distribution in terms of discrete states deviates from the simple B–E distribution at high momentum and that, since the parameters are far from the critical region for B–E condensation, both distributions coincide low $p_t$. The situation changes at low $p$ when the parameters are close to the critical region, this is depicted in fig. 4 where we show the distributions for $R = 6$ fm, $T = 120$ MeV, $N = 200$. The difference stems from the way the condensate contribution is included in the discrete states picture compared to the continuous case. In the former, the condensate contribution is spread over different states, although it is mainly concentrated in the lowest ones; in the latter, the condensate contribution is restricted to the lowest energy state, $p = 0$.

In summary, we propose to compute the pion inclusive distribution accounting for density effects at decoupling by calculating the chemical potential corresponding to a given multiplicity. Possible boundary effects can be included by a description in terms of a set of discrete states. If the pion system is produced close to the critical region for B–E condensation, a description in terms of a discrete set of states lends itself for the inclusion of the contribution from the lowest energy states. We also emphasize that the condensation is a high density phenomenon and thus depends on both, the pion multiplicity and the freezeout volume. The presence or absence of a condensate could be used to indicate for instance the degree of transparency in a central collision and have consequences for correlation experiments.
A.A. thanks G. Baym, J. Kapusta, G. Bertsch and B. Vanderheyden for insightful discussions and suggestions, the Institute for Nuclear Theory at the University of Washington for its hospitality and the Department of Energy for partial support during the completion of this work. A.S. thanks V.R. Pandharipande for his kind hospitality at the physics department of the UIUC. We are indebted to J. Stachel from the E877 collaboration, for discussing and making available their data and for very useful comments and suggestions. Support for this work has been received in part by the US National Science Foundation under grant NSF PHY94–21309.

[1] H. Ströbele et al (NA35 Collaboration), Z. Phys. C 38, (1988), 89.
[2] R. Albrecht et al (WA80 Collaboration), Z. Phys. C 47, (1990), 367.
[3] F. Videbaek et al (E802/866 Collaboration), Proceedings Quark Matter 95, Nucl. Phys. A590, (1995), 249c.
[4] J. Barrette et al (E877 Collaboration), Proceedings Quark Matter 95, Nucl. Phys. A590, (1995), 259c.
[5] J. Barrette el al (E814 Collaboration), Phys. Lett. B 351, (1995), 93.
[6] H.W. Barz, G. Bertsch, D. Kusnezov and H. Schultz Phys. Lett. B 254, (1991), 332.
[7] B.–A. Li and W. Bauer, Phys. Lett. B 254, (1991), 335.
[8] T.W. Atwater, P.S. Freier and J.I. Kapusta, Phys. Lett. B 199, (1987), 30.
[9] K.S. Lee and U. Heinz, Z. Phys. C 43, (1989), 425.
[10] M. Kataja and P.V. Ruuskanen, Phys. Lett B 243, (1990), 181.
[11] M.G.–H. Mostafa and C.–Y. Wong, Phys. Rev. C 51, (1995), 2135.
[12] J. Sollfrank, P. Koch and U. Heinz, Phys. Lett. B 252, (1990), 256.
[13] G.E. Brown, J. Stachel and G.M. Welke, Phys. Lett. B 253, (1991), 19.
[14] M. Gyulassy and S.K. Kauffmann, Nucl. Phys. A362, (1981), 503.
[15] See however P. Gerber and H. Leutwyler, Phys. Lett. B 246, (1990), 513.
[16] S. Pratt, Phys. Lett. B 301, (1993), 159.
[17] P. Braun–Munzinger, J. Stachel, J.P. Wessels and N. Xu, Phys. Lett. B 344, (1995), 43.
[18] J.D. Bjorken, Phys. Rev. D 27, (1983), 140.
[19] D. Kusnezov and G. Bertsch, Phys. Rev. C 40, (1989), 2075.
[20] See for example L. D. Landau and E. M. Lifshitz Quantum Mechanics, Non–Relativistic theory, first English edition, Addison–Wesley Publishing Co. Inc., (1958).

Figure Captions

FIG. 1. Critical temperature vs. Number of particles for different values of the radius R at decoupling.

FIG. 2. Invariant momentum distribution $md^2N/m^2dmdy$ at $y = 1.4$, for several values of the parameters $R$, $T$, $N$ and with the computed values of $\mu$. Notice the bending upwards of the distribution at high $p_t$ and the shape at low $p_t$.

FIG. 3. Invariant momentum distribution $md^2N/m^2dmdy$ at $y = 1.4$, for $R = 8$fm, $T = 150$MeV and $N = 150$ corresponding to a value of $\mu = 94.3$MeV. Shown also is a simple B–E distribution with the same set of parameters corresponding to a value of $\mu = 67.4$MeV.

FIG. 4. Invariant momentum distribution $md^2N/m^2dmdy$ at $y = 1.4$, for $R = 6$fm, $T = 120$MeV and $N = 200$ corresponding to a value of $\mu = 169.1$MeV. Shown also is a simple B–E distribution with the same set of parameters corresponding to a value of $\mu = 83.5$MeV.
Fig. 1

The graph shows the variation of the number of particles with temperature for different radii: R=8 fm (solid line), R=7 fm (dotted line), and R=6 fm (dashed line). As the temperature (T) increases, the number of particles (Num. of Part.) also increases.

The x-axis represents the temperature (T) in MeV, ranging from 100 to 200. The y-axis represents the number of particles, ranging from 0 to 800.
Fig. 2

md^2 N/m_t^2 d^2 m_t dy (GeV)^2

R=8 fm
T=150 MeV
N=200

R=8 fm
T=130 MeV
N=200

R=8 fm
T=110 MeV
N=200

R=8 fm
R=7 fm
R=6 fm

T=150 MeV
N=200

R=8 fm

T=150 MeV
N=250

T=150 MeV
N=200

T=150 MeV
N=150
Fig. 3

Discrete States
Simple BE
Fig. 4

Discrete States

Simple BE