**The Turbulent Star Formation Model. Outline and Tests**

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**Abstract.** We summarize the current status of the turbulent model of star formation in turbulent molecular clouds. In this model, clouds, clumps and cores form a hierarchy of nested density fluctuations caused by the turbulence, and either collapse or re-expand. Cores that collapse can be either internally sub- or super-sonic. The former cannot further fragment, and can possibly be associated with the formation of a single or a few stars. The latter, instead, can undergo turbulent fragmentation during their collapse, and probably give rise to a cluster of bound objects. The star formation efficiency is low because only a small fraction of the density fluctuations proceed to collapse. Those that do not may constitute a class of “failed” cores that can be associated with the observed starless cores. “Synthetic” observations of cores in numerical simulations of non-magnetic turbulence show that a large fraction of them have sub-sonic internal velocity dispersions, can be fitted by Bonnor-Ebert column density profiles, and exhibit “coherence” (an apparent independence of linewidth with column density near the projected core centers), in agreement with observed properties of molecular cloud cores.

**1. Introduction**

Two key questions related to star formation are a) What is the origin and nature of star-forming (and non-star-forming, if they exist) cores in turbulent molecular clouds? b) The star formation efficiency (SFE), measured as the fraction of a molecular cloud’s mass that ends up in stars during its lifetime, is low, of order 5-10% (see, e.g., the review by Evans 1999). Why? Different answers to these questions are given in the main two competing models of star formation. In the so-called “standard” model (Mouschovias 1976; Shu 1977; see the review by Shu, Adams & Lizano 1987), low-mass star-forming cores begin their lives as larger, magnetostatic, magnetically subcritical, partially-ionized clumps of low density contrast relative to their parent cloud. These slowly lose magnetic support and contract as the neutrals “slip” through the ions in a process commonly referred to as “ambipolar diffusion”, with characteristic time scale $\tau_{\text{ad}}$, until they finally become magnetically supercritical and proceed to gravitational collapse. The low SFE is explained in this case because $\tau_{\text{ad}}$ is much larger than the free-fall time scale $\tau_{\text{ff}}$, and thus constitutes the effective time scale for star formation, rather than $\tau_{\text{ff}}$. High-mass star formation is assumed to occur through the collapse of clumps that agglomerate enough mass to become supercritical and
thus collapse on a time scale $\tau_{ff}$, although clumps this massive are assumed to be rare. However, this model suffers from a number of shortcomings (see, e.g., Mac Low & Klessen 2003; Hartmann, this conference). In particular, it does not address the formation of the clumps, which are simply taken as initial conditions, nor how can such magnetostatic structures survive in the turbulent environment of their parent clouds.

In the competing, more recent turbulent scenario (e.g., Elmegreen 1993; Padoan 1995; Ballesteros-Paredes, Vázquez-Semadeni & Scalo 1999a, Ballesteros-Paredes, Hartmann & Vázquez-Semadeni 1999b; Klessen, Heitsch & Mac Low 2000; Heitsch, Mac Low & Klessen 2001; Hartmann, Ballesteros-Paredes & Bergin 2001; Padoan & Nordlund 2002; Vázquez-Semadeni, Ballesteros-Paredes & Klessen 2003; see also the reviews by Vázquez-Semadeni et al. 2000; Mac Low & Klessen 2003) the answer to question (a) above is that the cores are turbulent density fluctuations in the molecular clouds (which have typical rms Mach numbers $M_s \sim 10$) and that their statistical properties are therefore determined by the turbulent parameters in the parent cloud. The clouds are globally supported by turbulence, and in fact may be transient with short lifetimes (Ballesteros-Paredes et al. 1999b; Hartmann et al. 2001), but locally the density fluctuations induced by the turbulence may or may not collapse depending on their particular energy balance (Ballesteros-Paredes & Vázquez-Semadeni 1997; Vázquez-Semadeni, Shadmehri & Ballesteros-Paredes 2002). Then, the answer to question (b) above in the turbulent model is that the SFE is low because the cores contain only a fraction of the total cloud mass, only a fraction of them undergoes collapse, and, as is also the case in the standard model, only a fraction of the cores’ mass is involved in the collapse. That is, only a small fraction of the globally-turbulence-supported cloud is able to undergo collapse, through a kind of “turbulent colander”. Note that, while magnetic support is a key concept in the standard model of star formation, in the turbulent model it is not indispensable, as delay of the collapse is not necessary.

In this paper we first summarize the current status of the turbulent model of core and star formation, together with key work that has led to its formulation (§2), and then we discuss evidence that indeed turbulence can produce structures which reproduce some important observational properties of “quiescent” molecular cloud cores (§3), thus implying a high plausibility for the turbulent model. In this paper, we assume that the turbulence in molecular clouds is continuously driven, as otherwise it would decay in roughly one crossing time (Mac Low et al. 1998; Stone et al. 1998; Padoan & Nordlund 1999). This is in fact expected if molecular clouds are an intermediate-scale part of the turbulent cascade in the ISM, since energy is then continuously cascading through them from the large energy injection scales to the small dissipative ones.

2. The turbulent model of core and star formation

It was already forseen by von Weizsäcker (1951) and Sasao (1973) that compressible turbulence in gas clouds must have a dual role: turbulent modes of size scale $\lambda$ provide “pressure” towards structures with sizes $> \lambda$, while simultaneously they induce the formation of large-amplitude density fluctuations of sizes $< \lambda$. The latter process is generally called “turbulent fragmentation”. Note
that, in order to be highly compressible, the turbulence must be supersonic, as is observed (see, e.g., Zuckerman & Palmer 1974), and therefore it overwhelms the thermal support at all but the smallest scales. This implies that turbulence-supported clouds may have masses much larger than their thermal Jeans mass.

Sasao (1973) already pointed out that the compressed regions may be driven to collapse, and Hunter & Fleck (1982) showed that within them the local Jeans length is strongly reduced. This implies that local collapsing cores generally may have masses smaller than even the thermal Jeans mass at the mean cloud density, and so their masses are in general much smaller than their parent cloud’s mass. Note additionally that the free fall time in the cores is given by $\tau_{\text{ff}} \sim (G\rho_{\text{core}})^{-1/2}$, and is therefore shorter than that of their parent cloud by a factor $\sqrt{\rho_{\text{cloud}}/\rho_{\text{core}}}$. This is the opposite to the case of linear (small-amplitude) perturbations, in which the fastest collapsing scale is the largest one (Larson 1985).

Moreover, not all the density fluctuations necessarily proceed to collapse; whether they do or do not depends on whether they acquire enough mass that their gravitational energy overwhelms all other forms of energy that provide support. If it does not, the fluctuations re-expand, as first shown numerically by Vázquez-Semadeni, Passot & Pouquet (1996) using non-isothermal, non-magnetic simulations subject to turbulent driving. In these, many generations of density fluctuations (cores) were seen to appear and disappear, until finally a (rather improbable) strong enough compressive event managed to produce a gravitationally unstable core, which then proceeded to collapse in roughly a local free-fall time. Moreover, the global density maximum in the simulations was seen to fluctuate chaotically in time (showing also the non-formation of any hydrostatic cores), until it suddenly began to increase at an accelerated pace, corresponding to the local, almost instantaneous collapse of a “core”.

Padoan (1995; see also Padoan & Nordlund 2002; Vázquez-Semadeni et al. 2003) proposed that the only fragments that collapse are those which are far enough down the turbulent cascade that their internal velocity dispersion is subsonic, and that in addition are gravitationally unstable. This scenario has the implication that, if one considers a succession of turbulent regimes in which subsonic velocity dispersions occur at progressively smaller scales, the fraction of the mass available for collapse, and therefore the SFE, should also be progressively smaller, because smaller regions contain smaller fractions of the total mass.

This suggestion was tested numerically by Vázquez-Semadeni et al. (2003). For a series of SPH simulations of turbulent clouds, they empirically measured the “sonic scale” $\lambda_s$, i.e., the scale at which the turbulent velocity dispersion equals the sound speed. Then they also operationally defined and measured the SFE as the fraction of mass in collapsed sink particles after two turbulent crossing times. With these data, they showed that the SFE correlates well with $\lambda_s$, and in fact scales as $\text{SFE} \approx e^{(-\lambda_0/\lambda_s)}$, where $\lambda_0$ is a suitable reference scale. This suggests that indeed the sonic scale is one fundamental parameter in determining the SFE. Additionally, Vázquez-Semadeni et al. (2003) also showed that the SFE does not correlate well with the driving scale, and that previous

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1Note that the existence of a subsonic range is simply the consequence of a turbulent cascade, and does not necessarily imply that subsonic cores are at the scale of turbulence dissipation.
results suggesting so (Klessen et al. 2000) were only an artifact of maintaining the same total kinetic energy in runs with different driving scales, as this caused higher energies per unit wavenumber interval in runs forced at smaller scales, decreasing the sonic scale.

However, it is also clear that not only subsonic cores can collapse (if they are super-Jeans), but also supersonic cores, if gravity anyway overwhelms all other forces. In this case, not contemplated by either Padoan (1995), Padoan & Nordlund (2002) nor Vázquez-Semadeni et al. (2003), turbulence will be able to still produce fragments, but not support the cloud as a whole. Given the shorter free-fall time of the smaller, nonlinear scales, these will collapse first. As soon as they do, they will collectively become a non-dissipative system, and thus will not collapse to a singularity, but instead form a bound cluster of collapsed objects (see Bate, this volume). Thus, one can tentatively associate the global collapse of an unsupported supersonic region with the “clustered” mode of star formation, while the individual collapse of subsonic regions may be associated with the “isolated” mode, in which each core may produce one or a few stars.

Finally, there is the issue of the cores that do not collapse, but instead rebound after the compression is over, and merge back with their surrounding medium. Vázquez-Semadeni et al. (2002) have performed a simple calculation to estimate the re-expansion time $\tau_{\text{exp}}$, defined as the time it takes to double the initial radius. This is shown in fig. 1 as a function of the core radius at maximum compression, with $\tau_{\text{exp}}$ given in units of the free-fall time, and the radius in units of the equilibrium radius at which internal pressure balances self-gravity. We see that the minimum re-expansion time is of the order of a few free-fall times, or $\sim 1$ Myr. This picture is consistent with observational estimates of core lifetimes, and, in particular, the proposed existence of failed cores is consistent with the known existence of a large number of starless cores (at least half of the total; e.g., Lee & Myers 1999). These cores have traditionally been considered to be in a pre-protostellar evolutionary stage, but in the turbulent scenario they may well never form stars (Taylor, Morata & Williams 1996; Vázquez-Semadeni et al. 2002).

We conclude this section by noting that it has all been based on the concept and numerical simulations of the non-magnetic case. Thus, within the context of the turbulent star formation scenario, the magnetic field is not an indispensable ingredient. The isolated and clustered modes of star formation, as well as the low SFE, can be well understood in terms of purely hydrodynamic, self-gravitating turbulence. We expect the presence of a magnetic field to introduce only quantitative modifications to this scenario.

3. Comparisons with observations

In order to assess the feasibility of the turbulent model of star formation, it is necessary to compare its results with the observations. A question that is frequently asked in relation to the turbulent model is whether it can explain the relatively quiescent nature of observed cores, where by “quiescent” it is normally meant subsonic, or transonic at most. However, as discussed in §2, the existence of subsonic velocity dispersions at small enough scales is a natural consequence
Figure 1. Re-expansion time of a core, in units of the free-fall time, defined as the time necessary to double the initial core’s radius, as a function of the initial radius $r_1$, normalized to the equilibrium radius.

of the turbulent cascade, and in fact, the “sonic scale” appears to play a key role in the control of the SFE, as described there.

Additionally, Barranco & Goodman (1998) have observationally found that the linewidth in a (small) sample of cores exhibits the interesting property of becoming independent of the offset from the core center at small offsets. This asymptotic value is slightly larger than the thermal value, and was interpreted by those authors as an indication that the non-thermal velocity dispersion becomes scale-independent in the cores, saturating at a slightly subsonic level. They referred to this phenomenon as “coherence” of the velocity field in the innermost regions of the cores, within scales $\lesssim 0.1$ pc.

Finally, several recent observational works have fitted, with various degrees of success, Bonnor-Ebert (BE; Ebert 1955; Bonnor 1956) column density profiles to observed cores. BE spheres are hydrostatic, non-magnetic configurations, bounded by a hot, tenuous medium that contributes external pressure, but not to the gravitational potential of the system. The presence of such a hot confining medium is indispensible for the stability of the system, since otherwise the configurations need to smoothly extend to infinity, and in this case are generally unstable, as is, for example, the case of the singular isothermal sphere. Vázquez-Semadeni et al. (2002) have pointed out that such configurations cannot be realized in single-temperature media (as molecular clouds appear to be), since in this case the requisite of the hot, tenuous confining medium cannot be fulfilled. However, it is then puzzling that the observations seem to reasonably fit the column density profiles of BE spheres reasonably well.

In a couple of recent papers, we have addressed these two questions. Ballesteros-Paredes et al. (2003) considered a sample of 378 projections of the cores in the numerical simulations onto a hypothetical plane of the sky (POS). Column density profiles, averaged over the position angle on the POS, were
produced, and the scatter around this average was recorded. Then, BE column density profiles were fitted to the average profiles, and accepted if the error between the fitted and average profile was smaller than the scatter observed in the profiles contributing to the average. Otherwise, the fits were rejected. This procedure yielded 65% of “acceptable” BE fits to the profiles, showing that the transient cores in the simulations have nevertheless angle-averaged profiles that often resemble BE ones. This can probably be attributed to the strong smoothing introduced by the angle-averaging.

The issue of a sub- or transonic nature of the cores has been addressed by Klessen et al. (2003), who investigated the velocity structure for the core sample studied by Ballesteros-Paredes et al. (2003). They found that, in simulations with gravity turned off, roughly half the cores have subsonic velocity dispersions. In simulations with gravity on, the number of subsonic cores decreases, but still the vast majority has velocity dispersions that do not exceed twice the sound speed, and roughly 10% have subsonic velocity dispersions. Moreover, the velocity dispersion was measured in this plot out to column densities of 1/5 times the maximum, which probably increases the measured velocity dispersion, in comparison to the standard procedure of measuring out to the FWHM only. Thus, the simulations are entirely consistent with the fact that molecular cloud cores are generally transonic and often even subsonic.

Klessen et al. (2003) also probed the cores in the simulations for “coherence”. Figure 2 shows the velocity dispersion vs. column density along all lines of sight through a few randomly selected cores, clearly showing a trend to acquire a constant value, often subsonic, towards high column densities. Thus, the cores in the simulations also exhibit “coherence” when projected on the POS. Since such independence of scale of the turbulent motions has no counterpart in normal turbulent flows, Klessen et al. (2003) suggested that the “coherence” may be an observational effect, probably due to the fact that near the projected core centers, the linewidth along a given line of sight is dominated by its length rather than by its typical scale on the POS (i.e., the contributing material lies within a cylinder that is longer than it is wide). If the column length does not vary much around the projected core maximum, neither will the linewidth, causing the apparent “coherence”.

4. Conclusions

In this paper we have given a global outline of the turbulent model of molecular clouds and star formation, and then summarized a number of tests of the simulations showing that the cores in the non-magnetic simulations studied exhibit similar density and velocity features to those observed in real cores, strongly supporting the validity of the turbulent model of molecular clouds. Provided that the turbulence in molecular clouds is continuously driven, the model explains in a natural way the origin and structure of the cores as transient turbulent density fluctuations in the clouds, and the low efficiency of star formation, since a) the cores contain a small fraction of the cloud’s mass, and b) not all cores are destined to collapse. Those that do not, rebound and merge back into their environment, and may generally correspond to the observed “starless” cores. This scenario does not necessarily rely on the magnetic field to operate, although
Figure 2. Velocity dispersion vs. column density along all lines of sight through a few randomly selected cores in one turbulent simulation driven at large scales. Compare to fig. 3 of Barranco & Goodman (1998).

the presence of such a field will most certainly introduce quantitative corrections into the model. Although further testing is still clearly necessary, and the role of the magnetic field needs to be quantified, a quantitative theory of star formation in turbulent molecular clouds appears to be under way.

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