Wealth distribution in modern societies: collected data and a master equation approach

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Abstract

A mean-field like stochastic evolution equation with growth and reset terms (LGGR model) is used to model wealth distribution in modern societies. The stationary solution of the model leads to an analytical form for the density function that is successful in describing the observed data for all wealth categories. In the limit of high wealth values the proposed density function has the accepted Tsallis-Pareto shape. Our results are in agreement with the predictions of an earlier approach based on a mean-field like wealth exchange process.

Keywords: growth and reset process, master equation, stationary distributions, transient dynamics

1. Introduction

Since the last decade of the 19\textsuperscript{th} Century, when the pioneering studies on inequalities in socio-economic systems were performed by Vilfredo Pareto \cite{1}, the distribution of wealth and income have been intensively studied \cite{2}. According to Pareto’s well-known result the richest end of the cumulative distribution for wealth and income in a given society follows a power-law function characterized by the so-called Pareto exponent. Probably this was the first encounter of the scientific community with non-Gaussian, scale-free distributions in complex socio-economic systems. It was lately recognized that the low and middle range of the wealth and income distributions, follow a different trend, which was many times approximated by a Boltzmann-Gibbs type exponential distribution \cite{3–7}.

Nowadays, many electronic databases containing a large amount of data on income and wealth in different countries, are accessible for researchers \cite{8–10}. Such data can be a goldmine for those, who wish to explore social inequalities, universalities or dynamics in the distribution of socio-economic proxies. The data for both quantities (wealth and income) can be studied on the level of individuals or groups (families, companies, settlements, …) \cite{11–13}, with an immediate influence on the shape of the obtained distribution. Although, there might be correlation between the income and wealth of the individuals \cite{14, 15}, there is no direct connections between these two economical measures. A universal and striking similarity between wealth and income is that for both of them Pareto’s law apply: the tail is power-law like. Nevertheless in the limit of small and middle wealth/income values the distribution function for these two quantities can have a different shape \cite{3, 14}. Due to the fact that the income of individuals in a society is directly derivable from tax data, there are excellent exhaustive databases for this socio-economic measure making the investigations more easy \cite{16}. Sampling for wealth distribution is more peculiar and less accurate than it is in case of the income. The data that is available for wealth is mostly based on using some indirect measures (proxies), estimations and annual surveys. A relevant difference relative to income is also the fact that wealth can be negative, meaning debts. In such a view the distribution function of wealth should be more complex, and should not be limited to the $[0, \infty]$ interval.

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In the last few decades many theoretical work were done both by economists [17–19] and physicists [3–7, 12, 20–27] to understand the measured income and wealth distributions. Based on how these models target the dynamics of the relevant economic quantity (wealth or income) the majority of them can be grouped in a few relevant classes [3]. The first type models from the literature are based on simple analogies with thermodynamic systems. The relevant distributions are usually derived either from maximization of the Shannon-Gibbs entropy under different conditions or by the generalization of the concept of entropy [4, 7, 28]. The second type models describe the relevant economic phenomenon as stochastic exchange processes based on predefined dynamical rules. Such an approach can be either an agent-based computation [29] or a mean-field type analytical model. As an example for such a process, that one should mention here, is given by the multiplicative growth and exchange model, elaborated by J.P. Bouchaud and M. Mezard [20], that will be discussed in more detail later. Approaching the dynamics of income and wealth based on master equations with average growth, decrease or reset rates [5, 25] is also a common modeling paradigm. Apart of these two main categories there are also a large variety of models stepping over the mean-field type approximations and considering exchange processes on lattice [4, 6] or random networks [12, 23, 24].

Recently, we proposed a simple model based on mean-field like stochastic growth and reset processes for describing income distribution in modern societies [25]. The used master-equation contains growth and reset rates derived from real world data. The model offered an excellent description for the income distribution in all income categories [25]. Here we aim to show that a similar approach is successful in modeling the collected data for wealth as well. After our knowledge a model that is successful in describing analytically the distribution of wealth for all wealth categories is still missing. Therefore, as a step forward relative to the presently available wealth distribution models, we plan to give a unified and compact analytical description for the density function of wealth distribution on the entire wealth interval. Our manuscript is organized in the following manner: (1) first we present the used modeling framework and apply it to the wealth dynamics in social systems, (2) we than derive from statistical data the relevant density functions for wealth distribution in modern societies and compare them with the analytical prediction of our model, (3) finally we discuss the agreement between these data and our model predictions and comment on the appropriateness of the applied stochastic growth and reset rates.

2. The growth and reset process

A simple local growth and global reset master equation (LGGR model) proved to be successful in modeling relevant statistics in several complex phenomena [30]. The evolution equation contains both local and long distance transitions: uni-directional local growth and reset to a given new state [30]. In order to present this model let us consider an ensemble of identical elements that can have different numbers of quanta of a relevant quantity. An immediate example in the line of the problem considered here, are the individuals in a society, owning different amount of wealth. Let us denote here by \( P_n(t) \) the probability that a person has exactly \( n \) quanta of wealth at time \( t \). Normalization requires \( \sum_n P_n(t) = 1 \). In case the reset is only to the state with \( n \) quanta the dynamics of the growth and reset process in the space of the wealth quanta \( n \) is sketched in Figure 1a. For this case the evolution equation for the \( P_n(t) \) probabilities writes as:

\[
\frac{dP_n(t)}{dt} = \mu_{n-1} P_{n-1}(t) - \mu_n P_n(t) - \gamma_n P_n(t) + \delta_{n,0} \langle \gamma \rangle (t). \tag{1}
\]

Here we denoted the growth-rate from state \( n \) to \( n + 1 \) by \( \mu_n \) and the reset rate from the state with \( n \) quanta to state \( n = 0 \) as \( \gamma_n \). The last term on the right side is re-feeding at state \( n = 0 \), ensuring the preservation of normalization for \( P_n(t) \):

\[
\langle \gamma \rangle (t) = \sum_j \gamma_j P_j(t). \tag{2}
\]

The dynamical process from above can be generalized to continuous quanta, \( n \rightarrow x \in \mathbb{R} \). Instead of the discrete \( P_n(t) \) probabilities we shift to a continuous \( \rho(x, t) \) probability density, with the normalization condition \( \int_{\mathbb{R}} \rho(x, t) dx = 1 \). As it is detailed in [30] the continuous limit of the master equation (1) becomes:

\[
\frac{\partial \rho(x, t)}{\partial t} = \frac{\partial}{\partial x} \left[ \mu(x) \rho(x, t) \right] - \gamma(x) \rho(x, t) + \langle \gamma(x) \rangle (t) \delta(x). \tag{3}
\]
The re-feeding at $x = 0$ and conservation of the normalization is ensured by the term with the $\delta(x)$ Dirac functional and by considering:

$$\langle \gamma(x) \rangle(t) = \int_{-\infty}^{\infty} \gamma(x) \rho(x, t) dx$$  \hspace{1cm} (4)

In previous studies it was proven, that the above dynamical evolution equation converges to a steady-state with the $\rho_s(x)$ stationary probability density $[30, 31]$

$$\rho_s(x) = \frac{\mu_0 p_0(0)}{\mu(x)} e^{-\int_{-\infty}^{x} \frac{\mu(u)}{\mu_0} du},$$  \hspace{1cm} (5)

with:

$$\langle \gamma(x) \rangle = \int_{-\infty}^{\infty} \gamma(x) \rho_s(x) dx$$  \hspace{1cm} (6)

As we discussed in several recent publications, many important distributions that are frequently encountered in complex systems can be explained in the framework of the growth and reset model by properly selecting the state-dependent local growth rate $\mu(x)$ and reset rate $\gamma(x)$. Among these studies our recent one of income distribution $[25]$ by the LGGR model encourages us to attempt a similar approach to wealth.

3. Wealth distribution in view of the LGGR model

In order to realistically model income a linearly increasing growth rate

$$\mu(x) = \sigma (x + b),$$  \hspace{1cm} (7)

and a smart-reset rate was used in $[25]$, allowing both the appearance and disappearance of individuals in different income categories. For low income values the reset rate was chosen to be negative, while for higher income values the reset rate became positive, saturating to a finite value. Such a flow is illustrated schematically for the discrete probability space in Figure 1. The negative reset rate at low $x$ values describes an appearance of new individuals in the system with that income, the positive reset rate on the other hand means disappearance of individuals from the income category at $x$. The investigation of an exhaustive ten year social security data in Cluj county (Romania) confirmed the linearly increasing growth rate with $b = 0$, and was supporting a kernel function for the reset rate in the form:

$$\gamma(x) = K - \frac{r}{x + q}, \hspace{0.5cm} (K, b, q \in \mathbb{R}_+).$$  \hspace{1cm} (8)

Using such an approach the growth and reset model yields a Beta Prime stationary distribution for income, in excellent agreement with recent statistical data $[25]$.

The main difference between the distributions of income and wealth is that the total wealth of the individuals can be negative, i.e. debts. In such a context the wealth distribution function is defined usually on an interval $[-b, \infty]$, while income distribution is defined on the $[0, \infty]$ interval. The value $b$ characterizes the maximum amount of debts that are accepted for the individuals by the financing system. This is the amount of debt that is considered to be reimbursable.

Similar to income, one can admit that the linearly increasing growth rate is a reasonable assumption also for wealth, in agreement with the Matthew’s principle: "The rich gets richer”. The average increase in wealth over a fixed time is usually not by given amount but rather by a given percentage of the already existing wealth. In this sense the linear growth rate expressed in Equation (7) should be a valid approximation for wealth, too. This growth rate is positive for the whole $[-b, \infty]$ interval. For $x > -b$ it allows a growth in wealth, therefore the reimbursement of the debts. For $x < -b$ the growth rate becomes negative, and therefore accumulated debts cannot be reimbursed.

For the reset process a smart-reset rate similar to the one used for income, eq. (8), is a reasonable approximation for the dynamics in wealth. The growth process starts mainly at negative or low wealth values, while individuals will leave the society with higher wealth values. Therefore, similar to the income, the reset-rate should be negative at negative wealth values and saturate at a positive value for high wealth values. Since the negative wealth means debt,
this reset rate indicates that the growth usually starts either from debts accumulated by loans or losses in transactions or from an initial low wealth value. Among other possibilities this includes new individuals appearing in the society or wealthy individuals resetting their wealth by unsuccessful transactions. Again, in order to prevent debts below the \( b \) value, one has to choose in eq. (8) \( q = b \).

According to the above arguments, for a simple growth and reset master equation approach to wealth dynamics one could use the following kernel functions for the growth and reset rates

\[
\mu(x) = \sigma (x + b) \\
\gamma(x) = \sigma \left( k - \frac{\alpha}{x + b} \right),
\]

with \( \alpha, k, b \in \mathbb{R}_+ \).

In this approach it is easy to show that the normalized \( \rho_s \) stationary distribution defined on the \([-b, \infty]\) interval becomes:

\[
\rho_s(x) = \frac{\alpha^k}{\Gamma(k)} e^{-\frac{x}{\alpha}} (b + x)^{-1-k}.
\]

The reset and growth rates defined by the equations (9) and the above stationary distribution (10) ensures the conservation of the total number of actors \( (N_{tot}) \) and the total amount of wealth \( (W_{tot}) \) in the system. One can derive this directly by using the master equation (3) in the stationary limit, or by inspecting the following integrals:

\[
\Delta N_{tot} \propto \langle \gamma(x) \rangle = \int_{-b}^{\infty} \gamma(x) \rho_s(x) dx = \int_{-b}^{\infty} \sigma \left( k - \frac{\alpha}{x + b} \right) \frac{\alpha^k}{\Gamma(k)} e^{-\frac{x}{\alpha}} (b + x)^{-1-k} dx = 0,
\]

\[
\Delta W_{tot} \propto \int_{-b}^{\infty} [\mu(x) - x \gamma(x)] \rho_s(x) dx = \\
= \int_{-b}^{\infty} \sigma \left( x + b \right) - k x + \frac{\alpha x}{x + b} \frac{\alpha^k}{\Gamma(k)} e^{-\frac{x}{\alpha}} (b + x)^{-1-k} dx = 0
\]

The stationary distribution function (10) leads to the average wealth value,

\[
\langle x \rangle_s = \int_{-b}^{\infty} x \rho_s(x) dx = \frac{\alpha}{k - 1} - b = a b
\]

with

\[
a = \frac{\alpha}{b(k - 1)} - 1.
\]
It is easy to show that the distribution function for the wealth normalized by the average wealth, \( w = x/\langle x \rangle \), writes as:

\[
\rho(w) = \frac{a (a + 1)^k (k - 1)^k}{\Gamma(k)} e^{-\frac{(a x - 1)}{a w}} (1 + a w)^{-1-k}.
\]  

Here \( w \in [-1/a, \infty] \), and the above distribution function is normalized on this interval.

4. Connection to the wealth distribution function proposed by Bouchaud and Mezard

Bouchaud and Mezard considered a Langevin type equation with stochastic multiplicative growth and exchange terms to model the wealth distribution in a closed society [20]. In their approach only positive wealth values were allowed and the time-evolution of the wealth \( W_i \) of the individual \( i \), is approximated as:

\[
\frac{dW_i(t)}{dt} = \eta_i(t) W_i(t) + \sum_{j \neq i} [J_{ij} W_j(t) - J_{ji} W_i(t)].
\]  

The first term on the right hand-side describes multiplicative growth governed by the noise term, \( \eta_i \), that is assumed to have a normal distribution with mean \( \langle \eta \rangle \) and variance \( 2\Theta \). The equation is invariant under a scale transformation \( W_i \rightarrow k W_i, (k \in \mathbb{R}_+) \). In the mean-field limit \( \langle J_{ij} = J/N \rangle \) and for wealth values normalized to the mean \( \langle w_i = W_i/\langle W \rangle \rangle \) the above evolution equation leads to an analytically solvable Fokker-Planck equation, that has the equilibrium solution [20]:

\[
\rho_{BM} = \frac{g^e}{\Gamma(g)} e^{-\frac{z}{w^{-(2+g)}}}.
\]

with \( g = J/\Theta \). The form of this distribution function is rather similar to the more general distribution function [14] derived in the previous section from the LGGR model. The obvious difference is that \( \rho_{BM} \) is defined only for \( \omega \in [0, \infty] \) and does not incorporate the possibility of having debts. While the distribution function proposed by Bouchaud and Mezard has only one free fitting parameter, the generalized version given by equation [14] has two adjustable parameters, allowing for more freedom in fitting the observed real world distributions.

At this point it is interesting to note that two very different mean-field type approaches for the wealth dynamics leads to a similar form of the stationary distribution function. The approach considered by Bouchaud and Mezard considered mean-field like exchange, and allowed a diffusion governed by a multiplicative noise. In contrast, our approach in the present study is based on a simple growth and reset master-equation, coarse-graining over the diffusion and exchange terms and incorporating these in the phenomenological growth and reset processes.

5. Comparison with data

Due to the nature of wealth, which may be considered as the total sum of valuable possessions of an individual or a household, quantifying its value is a complex task. Unlike income, there are no simple proxies that would offer the possibility of constructing an exhaustive dataset for the wealth distribution in a given geographic region. The methods used nowadays were already described in the last century [32], and reconsidered in the recent years [33]. Our wealth data are obtained from the World Inequality Database [34]. These percentile datasets were derived from the National Accounts, Survey data, Tax data and Rich Lists using complex methods, detailed in a working paper [33]. First, we extracted data for USA and Russia (two economies with very different history) for several consecutive years. The probability density function (PDF) of the normalized wealth was computed for each country in each year, i.e. wealth was always normalized to the average wealth, \( \langle W \rangle \), for the given year \( (w = W/\langle W \rangle) \). Using this method the wealth distributions for each country in different years collapsed on a master curve, as it is illustrated in Figures [2] and [3]. On Figure [2] we use log-log scales and plot the \( W > 0 \) part of the density function. On Figure [3] we consider log-normal scales and plot the part for negative (debts) and small wealth values. Taking into account the collapse visible in Figures [2] and [3] it becomes possible to derive the averaged density function for each country in part, plotted on the respective graphs as continuous black lines.

Interestingly, the trends both for USA and Russia are very similar and they can be compared in a better manner after plotting the average trends on the same curve. At this point one can attempt a fit with the density function [14].
Figure 2: Probability density function of the normalized (rescaled) wealth, \( w = W/\langle W \rangle \), for the \( W > 0 \) region using log-log scales. Results for USA and Russia for the years indicated indicated in the legends. The continuous black curve illustrates the average of the density functions for the considered years.

Figure 3: Probability density function of the normalized (rescaled) wealth, \( w = W/\langle W \rangle \), for the negative (debts) and small wealth region. Results for USA, and Russia for the years indicated in the legends. The continuous black curve illustrates the average of the density functions for the considered years.

obtained by our simple master equation approach with growth and reset terms. As it is illustrated in Figure 4 the averaged PDF for the renormalized wealth is rather similar for the USA and Russia. We also learn from Figure 4 that one can obtain a qualitatively fair fit for the whole wealth interval using the parameters \( k = 1.4, a = 6.5 \) in the PDF from equation (14).

According to this result one would assume thus an even a stronger universality for the PDF in the \( W/\langle W \rangle \) variable. Not only the PDF for different years collapse, but Figure 4 suggests that also the PDF for different countries might collapse on a universal trend. Performing an analysis on wealth distributions for other countries as well, we learn however that this is not the case. We can take for example the case of France, and extract the PDF of wealth distribution for several years from [24]. Data for different years collapse again as it is indicated in the left hand-side panel from Figure 5. (In case of France the data does not have information on negative wealth values, we used thus only the log-log plot for \( W > 0 \).) Plotting together the averaged PDF with the ones obtained for USA and Russia suggests that the fit parameters in this case should vary since the scaling in the limit of high wealth values is obviously different. In
Figure 4: Probability density function of the normalized (rescaled) wealth, $w = W/\langle W \rangle$. Qualitative comparison between data for USA and Russia and the PDF given by our model, equation (14). Disks with different colors (consult the legends of the figures) are the averaged results for the studied years while the continuous thick line is the fit with the PDF given by eq. (14), using the parameters: $k = 1.4, a = 6.5$. Please be aware of the log-log scale for the figure on the left and the log-normal scale for the figures on the right hand-side.

consequence, the collapse for the PDF in case of Russia and USA seems to be only a simple coincidence.

Figure 5: Probability density function of the normalized (rescaled) wealth, $W/\langle W \rangle$, for the $W > 0$ region using log-log scales. The Figure on the left shows results for France for the years indicated in the legends. The figure on the right hand-side compares the averaged PDF-s (average on all considered years) for USA, Russia and France.

6. Discussion

Comparison with collected data shows that the PDF given by equation (14), obtained from a simple mean-field type growth and reset master equation, offers a good description for wealth inequalities in modern societies. Our results are also in agreement with the form of the PDF suggested by the alternative approach of Bouchaud and Mezard, considering a mean-field type exchange [20]. The advantage of the present model is however, that it allows to consider negative wealths (debts), too. Seemingly the two free parameters in the PDF from (14), allows for an improved fitting even in the $W > 0$ limit. In comparison with the fit given for $W > 0$ in Figure 2, on the Figure 6 we illustrate the best fit ($g = 0.4$, leading to the same scaling law) that one obtains with the PDF from equation (16). Evidently our two-parameter LGGR model gives an improvement in the small wealth limit. This should not be a surprise, since the PDF from (16) has only one fit parameter. The same observation is true if one considers the data for France. As it is visible in the right hand-side panel of Figure 6, the best fit with equation (14), (fit parameters: $k = 1.68$ and $a = 7$) is
better for the small wealth limit when compared to the fit with equation (16), (fit parameter: \( g = 0.68 \)). The scaling exponents in these cases coincide: \( -2.68 \).

Figure 6: Best fit for the data on the PDF of wealth distribution for USA, Russia, and France in the \( W > 0 \) limit using the model of Bouchaud and Mezard (16). On left hand-side panel we illustrate the fit with equation (16) using \( g = 0.4 \). On the graph from the right hand-side we compare the best fits with the PDF's from equation (14) to (16). The corresponding parameters are: \( k = 1.68 \), \( a = 7 \) and \( g = 0.68 \).

The approach based on the growth and reset master equation is based on two hypotheses, formulated on the growth and reset rates, expressed mathematically by equations (9). It would be therefore in order to discuss here also the appropriateness of the rates used in our model. Concerning the chosen form for the growth rates we mentioned that it is in agreement with the generally accepted preferential growth hypothesis (Matthew’s principle). According to this, the wealth of individuals should grow with a speed that is proportional to their wealth values. This inevitably leads to an exponential increase, interrupted stochastically by the reset process.

In order to support this hypothesis we investigated the growth of the wealth for the richest people in the world. We extracted from the Forbes database the 15 leading persons who were constantly in the top-list between 2001 and 2019. For each of them we followed their wealth \( W_i(t) \) in each year, \( t \), relative to the wealth from 2001, \( \omega_i(t) = W_i(t)/W_i(2001) \), and then studied the average increase \( \omega(t) = \langle \omega_i(t) \rangle_i \) as a function of time. The result for \( \omega(t) \) is plotted on Figure 7 with log-normal scales. The apparently linear trend from Figure 7 is in agreement with an exponentially trend: \( \omega(t) \approx \exp[-0.075(t - 2001)] \).

Concerning the reset rate, unfortunately we do not have any direct information supporting the used kernel function. It is possible however to show what are the consequences of this reset rate in view of the fitted PDF for USA and Russia. Using the fit parameters from Figure 4 (\( k = 1.4 \) and \( a = 6.5 \)), we determine the variations per unit time of the
fraction of population in a unit wealth interval \( n(w) = N(w, w + dw)/dw \) due to the smart reset process:

\[
\frac{dn(w)}{dt} \propto -\gamma(w) \rho(w) = -\left( k - \frac{(a + 1)(k - 1)}{1 + a w} \right) a (a + 1)^k (k - 1)^k e^{-\frac{(a + 1)(k - 1)}{1 + a w}} (1 + a w)^{-1-k}
\]  

Results in such sense are plotted in Figure 8. From here we learn that the majority of people start their dynamics in accumulated wealth in the region of small and negative wealth values, well below the average wealth in the society \( w < 0.2 \) and leave the statistics with positive wealth. This is in nice agreement with our everyday-life experience.

![Figure 8: Variation per unit time](image)

Figure 8: Variation per unit time \( dn(w)/dt \propto -\gamma(w) \rho(w) \) of the fraction of population with wealth around \( w \) in a unit wealth interval \( n(w) = N(w, w + dw)/dw \) solely due to the smart reset process. The used model parameters are the ones given for the fit to the real world density functions from Figure 4 \( k = 1.4 \) and \( a = 6.5 \).

7. Conclusion

Following the modeling paradigm offered by the mean-field description of a stochastic growth and reset process we proposed a simple analytical model for wealth distribution in modern societies. The challenge we faced was to derive a compact mathematical formula for describing the density function of wealth distribution for all wealth categories, including the negative part (debts) as well. We used growth and reset rates similar with the ones used for modeling income distribution [25]. The preferential growth rate is supported by wealth dynamics data of the world’s leading billionaires. The smart reset rate is negative for the debtor and low wealth part of the society and becomes positive for middle and high wealth values. This is in agreement with our everyday observation that young people usually start their life with debts or low wealth values and leave the society at an older age in the higher wealth categories. Moreover, bad transactions in the wealthier part of the society could reset the wealth values to negative or low values. Following this view we consider that the rates used by us are appropriate for approaching in a mean-field manner the dynamics of wealth in a society. The minor modifications in the growth and reset rates relative to the ones used for targeting the income distributions, allowed the extension to negative wealth values. As a consequence of these modifications we find that the shape of the stationary distribution function becomes different from the one obtained for the income. In [25] we argued that for income a Beta Prime distribution function offers a good description of the collected data. Here instead we find that a slightly modified version of the distribution function proposed by Bouchaud and Mezard [20] works most properly. The formula proposed in equation (14) presents the Tsallis-Pareto type tail, and it gives a good fit for the data on the whole wealth interval. As a step forward relative to many earlier attempts for modeling wealth, this PDF describes in an acceptable manner the negative wealth limit, too. It is also important to note that we found striking similarities between the wealth distributions in the USA and Russia. It is surprising in the view of their very different economic history. Similar to the income distribution, a rescaling by the average, the PDF-s of wealth for different years coincide on a master curve.

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