Effects of partial measurements on teleportation of quantum resources and quantum Fisher information in a relativistic scenario

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ABSTRACT
Quantum resources (QRs) and quantum Fisher information (QFI) of an input state, usually degrade during the teleportation under the Unruh effect. In this paper, we address the teleportation of a single and a two-qubit quantum state through the Unruh effect experienced by a mode of a free Dirac field, and study the effects of the partial measurement (PM) and partial measurement reversal (PMR) on the QRs and QFI of the teleported states. We investigated how the teleported QRs and QFI can be improved with the combined effect of PM and PMR for both single-qubit and two-qubit teleportation. We also consider how we can control the behavior of the QFI, quantum coherence (QC) as well as fidelity, fixing the acceleration, in single-qubit state teleportation. Our results show that QFI with respect to weight parameter ($F_{P^{M}_{\text{out}}}(\theta)$) enhances with the increase in measurements strength. In addition, we discuss in detail the optimal behavior of the QFI associated with the phase parameter ($F_{P^{M}_{\text{out}}}(\varphi)$), QC as well as fidelity with respect to the PM and PMR strength and examine the Unruh effect on optimal estimation. In particular, it is found that in the single-qubit scenario, the PM strength (PMR strength), with which the optimal estimation of phase parameter occurs, is the same strength with which the teleportation quality and the QC of the output single-qubit state reaches to its maximum value. On the other hand, generalizing the results for two-qubit teleportation and comparing the teleported QFIs in both single and two-qubit scenarios, we find that the encoded information in the weight parameter is better protected against the Unruh effect in the process of two-qubit teleportation. However, extraction of information encoded in the phase parameter is more efficient in single-qubit teleportation than the two-qubit one.

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1. Introduction

Quantum teleportation (1) is undoubtedly one of the most striking implications predicted by quantum mechanics and it is an important ingredient for quantum communication and quantum information processing (QIP) (2, 3). In the last decades
theoretical and experimental consideration of quantum teleportation has attracted many researchers’ attention (4–13). Quantum teleportation is described as a process by which an arbitrary unknown quantum state can be transmitted faithfully from one object to another, without physical traveling of the object itself. The system is isolated from the external forces in the original form of the teleportation (1), and a maximally entangled pair is used as the resource. However, decoherence (14, 15) is an inevitable phenomenon in open quantum systems which takes place due to the interaction between the system and environment. This leads to the degradation of quantum correlations, a fundamental resource for QIP, and therefore influences the fidelity in quantum state teleportation (16–18).

Relativistic quantum information (RQI) (19, 20) aims to realize the relationship between relativity as well as quantum information, and combine relativistic effects to amend quantum information tasks, e.g., quantum teleportation. Moreover, in RQI we try to understand how these protocols may be realized in curved space time. Unruh effect (21, 22), a significant prediction in quantum field theory, proposes that a uniformly accelerated observer in Minkowski spacetime (Rindler observer) associates a thermal bath of Rindler particles to the no particle state of inertial observer (called Minkowski vacuum). The decoherence effect, produced by the Unruh effect, suppresses the quantum resources (QR) such as quantum coherence (23), quantum discord (24, 25) and entanglement (25) in the case of bosonic or Dirac field modes. The degradation of QRs unavoidably decreases the confidence of some quantum information tasks like quantum teleportation. In this context it is really important to preserve QRs from decoherence during the teleportation process. Here we investigate the teleportation of a single and a two-qubit quantum state through the Unruh effect experienced by a mode of a free Dirac field, considered as a noise channel which we name it Unruh channel.

In addition to the teleportation of the whole quantum state, we also investigate the teleportation of the information encoded into a particular parameter. In contrast to quantum state teleportation where the quality of teleportation is characterized by fidelity, the credibility of teleportation of specific information is usually determined by quantum Fisher information (QFI) (26–29). QFI, representing the sensitivity of the state with respect to changes in a parameter, plays an important role in parameter estimation theory and is extensively employed in QIP. In particular, QFI has many applications in quantum information tasks such as entanglement detection (30, 31), specifying the non-Markovianity (32–34), and consideration of uncertainty relations (35–37). Hence it is of interest to study QFI in relativistic framework. Nevertheless, it is shown that the QFI is fragile and can be broken easily because of unavoidable decoherence effects (38–42). This is the most restricting factor in QFI applications for quantum teleportation. Therefore, protecting the QFI from decoherence is a fundamental subject.

In weak or partial measurement (PM), associated with a positive-operator valued measure (POVM), the system state does not completely collapse such that the initial state could be reversed with some operations. Recently, PMs together with partial measurement reversals (PMR) have been exploited as a practical method to protect quantum correlations of two-qubits as well as two-qutrits and the fidelity of a single-qubit, from amplitude damping (AD) decoherence (43–47). In ref. (48) the effect of partial measurements on the teleportation of QFI for a single-qubit state under the amplitude damping noise has been studied and it has been illustrated that the combination of PM and PMR could totally eliminate the influence of decoherence. The effects of PM and PMR on the enhancement of quantum coherence and QFI,
transmitted under a quantum spin-chain channel, have been considered in ref. (49). Moreover it has been shown that PM and PMR are able to improve the fidelity of teleportation when one or both qubits of the maximally entangled state shared between Alice and Bob suffer from the AD decoherence (50). It was also shown in ref (50) that this protocol works for the Werner states. However limited attention has been paid to protect the QRs and QFI against Unruh decoherence during the procedure of teleportation. Motivated by this, we study the enhancement effect of PM and PMR on teleportation of QRs and QFI through the Unruh noise channel for both single and two-qubit input quantum states.

In this paper, we have investigated the following scenario: the system consists of an inertial observer Alice and a uniformly accelerated observer Rob. Two PMs are performed before and after Robs acceleration, which are called PM and PMR, respectively. Then we use the above mentioned system as a resource in order to teleport a single and a two-qubit state, and consider how the degradation effect of the Unruh channel on the teleportation of QRs and QFI as well as teleportation fidelity can be improved by PM or PMR. According to our results, the combined effect of PM and PMR with the same strengths \( (p = q) \) may improve the teleportation of QRs and QFI with respect to phase parameter \( \varphi \), and also teleportation fidelity in both single-qubit and two-qubit scenarios.

This paper is organized as follows: In Sec. II we give a brief description about teleportation, PM, PMR, QRs and QFI. The physical model is presented in Sec. III. We study the single-qubit teleportation as well as two-qubit teleportation under the Unruh noise channel in Sec. IV and Sec. V, respectively. Finally, Sec. VI is devoted to conclusion.

2. PRELIMINARIES

2.1. Teleportation

The main idea of quantum teleportation is transferring quantum information about an unknown quantum state to another location where it is spatially separated. An important factor in quantum teleportation is the channel connecting sender and receiver. In standard teleportation protocol \( T_0 \), local quantum operations, used to teleport the input state, includes Bell measurements and Pauli rotations. According to Bowen and Bose results, the standard teleportation protocol \( T_0 \) with mixed states as resource is tantamount to a generalized depolarizing channel (51).

2.1.1. Single-qubit teleportation

As mentioned above, teleportation protocol using a two-qubit mixed state as a resource, acts as a generalized depolarizing channel, therefore the output state for a teleported single-qubit state is obtained as follows (51)

\[
\rho_{\text{out}} = \Lambda(\rho_{\text{ch}}) \rho_{\text{in}},
\]

\[
= \sum_{i=0}^{3} \text{Tr}(B_i \rho_{\text{ch}}) \sigma_i \rho_{\text{in}} \sigma_i
\]  

(1)
where $B_i$ are the Bell states associated with the Pauli matrices $\sigma_i$,

$$B_i = (\sigma_0 \otimes \sigma_i) B_0 (\sigma_0 \otimes \sigma_i), \ i = 1, 2, 3$$

(2)

in which $\sigma_0 = I$, $\sigma_1 = \sigma_x$, $\sigma_2 = \sigma_y$ and $\sigma_3 = \sigma_z$. Moreover, we have $B_0 = \frac{1}{2} (|00\rangle + |11\rangle) (\langle 00 | + \langle 11 |)$, without loss of the generality.

2.1.2. Two-qubit teleportation

Teleportation of an unknown entangled state via two independent, equally entangled quantum channels has been studied by Lee and Kim (52). Actually, their protocol may be carried out by doubling the standard teleportation protocol $T_0$. Figure 1 displays the schematic drawing of entanglement teleportation. An unknown entangled state $\rho_{in}$ is generated by source $S$, and its particles are dispensed into $A_1$ and $A_2$. Besides, two independent entangled pairs (one of them numbered 3 and 5, the other pair numbered 4 and 6) are produced from source $E$. These pairs, each characterized by density matrix $\rho_{ch}$, play the role of the quantum channel. The measurement result at $A_i \ (i = 1, 2)$ is transmitted through the classical channel $C_i$ to $B_i$. Based on the information received by the classical communication, the unitary transformations are done on the particles 5 and 6 received at $B_i \ (i = 1, 2)$ to complete the teleportation.

![Figure 1. Schematic drawing of entanglement teleportation.](image)

Generalizing equation (1), the output state of the entanglement teleportation is found as follows

$$\rho_{out} = \sum_{ij} p_{ij} (\sigma_i \otimes \sigma_j) \rho_{in} (\sigma_i \otimes \sigma_j), \ i, j = 0, x, y, z.$$  (3)

where $p_{ij} = \text{Tr}(E^i \rho_{ch}) \text{Tr}(E^j \rho_{ch})$ and $\sum p_{ij} = 1$. Here $E^0 = |\psi^-\rangle \langle \psi^-|$, $E^1 = |\phi^-\rangle \langle \phi^-|$, $E^2 = |\phi^+\rangle \langle \phi^+|$, $E^3 = |\psi^+\rangle \langle \psi^+|$ and $|\psi^\pm\rangle = [01\pm00]/2$ as well as $|\phi^\pm\rangle = [00\pm11]/2$ are Bell states.

2.2. Partial measurement (PM) and partial measurement reversal (PMR)

We first give a brief introduction about the PM and PMR. In contrast with the standard von Neumann projective measurement, which completely collapses the measured system, PM, as a generalization of standard von Neumann projective measurement,
does not totally collapse the initial state into the eigenstates, and hence it is reversible. For a single-qubit, the PM is described by the following pair of measurement operators:

\[ M_0 = \sqrt{1-p} |0\rangle \langle 0| + |1\rangle \langle 1|, \]  
\[ M_1 = \sqrt{p} |0\rangle \langle 0|, \]

where \( p \) \((0 \leq p \leq 1)\) is the strength of PM and \( M_0^\dagger M_0 + M_1^\dagger M_1 = I \). \( M_1 \) is identical to von Neumann projective measurement and is irreversible, while \( M_0 \) is a PM that we are interested in this study. In order to reverse the effect of the PM, i.e., in order to recover the primary state, we need to use the inverse of \( M_0 \),

\[ M_0^{-1} = \frac{1}{\sqrt{1-q}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \sqrt{1-q} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{1}{\sqrt{1-q}} XM_0 X, \]

where \( X = |0\rangle \langle 1| + |1\rangle \langle 0| \) is the bit-flip operation. Last term of Eq.(6) implies that the reverse procedure \( M_0 \), can be implemented physically by the sequence of a bit-flip operation, another PM with measurement strength \( q \), and a second bit-flip operation.

### 2.3. Quantum Fisher information

QFI is an important concept in parameter estimation theory. QFI of an unknown parameter \( \lambda \) encoded in quantum state \( \rho(\lambda) \) is defined as \((26, 57)\)

\[ F_Q(\lambda) = \text{Tr} \left[ \rho(\lambda) L^2 \right] = \text{Tr} \left[ (\partial_\lambda \rho(\lambda)) L \right], \]

where \( L \), the symmetric logarithmic derivative (SLD), is given by \( \partial_\lambda \rho(\lambda) = \frac{1}{2} (L \rho(\lambda) + \rho(\lambda) L) \), with \( \partial_\lambda = \partial / \partial \lambda \). Using the spectrum decomposition of \( \rho(\lambda) \), \( \rho(\lambda) = \sum_i p_i |\phi_i\rangle \langle \phi_i| \), where \( |\phi_i\rangle \) and \( p_i \) are eigenvectors and eigenvalues of the matrix \( \rho(x) \), respectively; one can rewrite the QFI as follows \((58)\)

\[ F_Q(\lambda) = \sum_{i,j} \frac{2}{p_i + p_j} |\langle \phi_i| \partial_\lambda \rho(\lambda) |\phi_j\rangle|^2 \]
\[ = \sum_i \frac{(\partial_\lambda p_i)^2}{p_i} + 2 \sum_{i \neq j} \left( \frac{p_i - p_j}{p_i + p_j} \right)^2 |\langle \phi_i| \partial_\lambda \phi_j \rangle|^2; \]

A simple and explicit expression can be acquired for the single-qubit state. Any qubit state can be expressed in the Bloch sphere representation as

\[ \rho = \frac{1}{2} (I + \omega \cdot \sigma) \]

where \( \omega = (\omega_x, \omega_y, \omega_z)^T \) is the Bloch vector and \( \sigma = (\sigma_x, \sigma_y, \sigma_z) \) indicates the Pauli matrices. Hence the QFI of the single-qubit state can be formulated as follows \((59)\)
\[ F_Q(\lambda) = \begin{cases} 
|\partial_\lambda \omega|^2 + \frac{(\omega \cdot \partial_\lambda \omega)^2}{1 - |\omega|^2}, & |\omega| < 1, \\
|\partial_\lambda \omega|^2 & |\omega| = 1.
\end{cases} \] (10)

where \(|\omega| < 1\) is used for a mixed state while \(|\omega| = 1\) is applicable for a pure state.

2.4. Quantum resources

Quantum coherence. Quantum coherence (QC) arising from the superposition principle is an important resource in quantum information and quantum computation processing. It plays a fundamental role in quantum mechanics. Various measures are expressed to quantify the coherence such as, trace norm distance coherence \((53)\), \(l_1\) norm, and relative entropy of coherence \((54)\). For a quantum state with the density matrix \(\rho\), the \(l_1\) norm measure of quantum coherence \((54)\) quantifying the coherence through the off diagonal elements of the density matrix in the reference basis, is given by

\[ C_{l_1}(\rho) = \sum_{i,j \neq i} |\rho_{ij}| \] (11)

Entanglement. Entanglement is recognized as a resource in quantum information processing (QIP) and is accountable to the advantage of many quantum computation and communication tasks. Actually, entanglement indicates correlations regarding non separability of the state of a composite quantum system. Entanglement of a bipartite system is quantified conveniently by concurrence \((55)\) which can be computed analytically for a X state as follows
\[ C(\rho) = 2\max \{0, C_1(\rho), C_2(\rho)\}, \]  \hspace{1cm} (12)

where \( C_1(\rho) = |\rho_{14}| - \sqrt{\rho_{22}\rho_{33}}, \) \( C_2(\rho) = |\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}, \) and \( \rho_{ij} \)’s are the elements of density matrix. Concurrence equals unity for maximally entangled states and vanishes for separable states.

**Quantum discord.** Quantum discord representing quantumness of the state of quantum system is a resource for certain quantum technologies. It can be preserved for a long time even when entanglement shows a sudden death. QD for any bipartite system is defined as difference between total correlations (i.e., quantum mutual information) and classical correlations. Computation of QD for general states is not usually a convenient task since it involves the optimization of the classical correlations. However, for a two-qubit X state system, the analytical expression of QD can be obtained as (56)

\[ \text{QD}(\rho_{AB}) = \min (Q_1, Q_2), \]  \hspace{1cm} (13)

where

\[ Q_j = H(\rho_{11} + \rho_{33}) + \sum_{i=1}^{4} \lambda_i \log_2 \lambda_i + D_j, \quad (j = 1, 2), \]

\[ D_1 = H \left( \frac{1 + \sqrt{[1 - 2(\rho_{33} + \rho_{44})]^2 + 4(|\rho_{14}| + |\rho_{23}|)^2}}{2} \right), \]  \hspace{1cm} (14)

\[ D_2 = -\sum_i \rho_{ii} \log_2 \rho_{ii} - H(\rho_{11} + \rho_{33}), \]

\[ H(x) = -x \log_2 x - (1 - x) \log_2 (1 - x), \]

and \( \lambda_i \)’s denote the eigenvalues of density matrix \( \rho_{AB} \).

3. Physical model

First we consider a free Minkowski Dirac field \( \psi \) in 3+1 dimensions

\[ i\gamma^\mu \partial_\mu \psi - m\psi = 0, \]  \hspace{1cm} (15)

where \( \gamma^\mu \) are the Dirac gamma matrices, \( m \) is the particle mass and \( \psi \) is a spinor wave function. The field can be represented from the perspective of inertial and uniformly accelerated observers (see details in (60)). We investigate a system including an inertial observer Alice (A) and a uniformly accelerated observer Rob (R). For the inertial observer, Minkowski coordinates \( (t, z) \) are the most proper coordinates to describe the field. The field can be expanded in terms of positive and negative frequency Minkowski modes \( \psi_k^+ \) and \( \psi_k^- \), which they form a complete orthonormal set,

\[ \psi = \int dk \left( a_k \psi_k^+ + b_k^\dagger \psi_k^- \right) \]  \hspace{1cm} (16)
where the wave vector $k$ represents the modes of massive Dirac fields. Moreover, $a_k^+, b_k^+$ and $a_k, b_k$ denote, respectively, the creation and annihilation operators for positive and negative frequency modes of momentum $k$.

Since Rob is the uniformly accelerated observer, in order to describe what he sees, Rindler coordinates $(\tau, \xi)$ should be used. As it can be seen in Fig. 2, Rindler spacetime manifests two regions I and II, causally disconnected. Because of the eternal acceleration, Rob travels on a hyperbola compelled in the region I. Compared with Eq. (16), Dirac field can be expanded in terms of positive and negative frequency Rindler modes

$$\psi = \int dk \left( c_k^+ \psi_k^{I+} + d_k^+ \psi_k^I + c_k^+ \psi_k^{II+} + d_k^+ \psi_k^{II} \right)$$

where $(c_k, c_k^\dagger)$ represent the annihilation and creation operators for Rindler particle and $(d_k, d_k^\dagger)$ denote those of the antiparticle, in the region $n$ with $n = I, II$. The Minkowski and Rindler creation and annihilation operators are connected via the Bogoliubov transformation

$$a_k = \cos r c_k - \sin r d_k^\dagger, \quad b_k^\dagger = \sin r c_k + \cos r d_k^\dagger,$$

where $r = \arccos \sqrt{1 + e^{-2\omega a}}$, $\omega$ is the Dirac particle frequency and $a$ is the acceleration. Since $0 < a < \infty$, therefore $r \in [0, \pi/4]$.

Let Alice and Rob initially share the following entangled state at a point in Minkowski spacetime

$$|\Psi(0)\rangle = \sin \frac{\theta}{2} |0\rangle_A |0\rangle_R + \cos \frac{\theta}{2} |1\rangle_A |1\rangle_R.$$  

As it can be seen in Fig. 2, Rindler spacetime manifests two regions I and II, which are causally disconnected. Because of the eternal acceleration, Rob travels on a hyperbola compelled in the region I.

With the single-mode approximation, the Minkowski vacuum state can be expressed in terms of the Rindler regions I and II states (60):

$$|0\rangle_M = \cos r |0\rangle_I |0\rangle_{II} + \sin r |1\rangle_I |1\rangle_{II},$$

and the excited state is given by

$$|1\rangle_M = |1\rangle_I |0\rangle_{II}$$

Note that the observers in regions I and II are causally disconnected. Since the mode corresponding to II in not observable, it should be traced out.

We assume that Rob first performs a PM of the form (4) on his particle, and then uniformly accelerates. In the next step, a PMR is carried out by Rob in region I. Since Rob is restricted to region I due to the causality condition, we trace the state over region II. Provided that the PM and PMR are successfully accomplished, the following mixed state between Alice and Rob is obtained (61)
\[
\rho_{A,I} = \frac{1}{N_2} \begin{pmatrix}
\sin^2 \frac{\vartheta}{2} p \cos^2 r & 0 & 0 & \sin \frac{\vartheta}{2} \cos \frac{\vartheta}{2} \sqrt{pq} \cos r \\
0 & \sin^2 \frac{\vartheta}{2} \frac{pq}{2} \sin^2 r & 0 & 0 \\
0 & 0 & 0 & 0 \\
\sin \frac{\vartheta}{2} \cos \frac{\vartheta}{2} \sqrt{pq} \cos r & 0 & 0 & \cos^2 \frac{\vartheta}{2} q
\end{pmatrix}.
\]

(22)

where \( N_2 = \sin^2 \frac{\vartheta}{2} p \cos^2 r + \sin^2 \frac{\vartheta}{2} \frac{pq}{2} \sin^2 r + \cos^2 \frac{\vartheta}{2} q \) is the normalization factor, \( p = 1 - \bar{p} \) and \( q = 1 - \bar{q} \), in which \( p \) and \( q \) represent the first and the second PM strengths.

4. Preparation probability

The probability of preparing the system in state (22) via the above prescription is obtained as

\[
P = P_1 P_2,
\]

\[
= \bar{p} \sin^2 \frac{\vartheta}{2} \cos^2 r + \bar{q} \left( \bar{p} \sin^2 \frac{\vartheta}{2} \sin^2 r + \cos^2 \frac{\vartheta}{2} \right).
\]

(23)

where \( P_1 = \text{tr} \left( M_0^\dagger M_0 \rho (0) \right) \) in which \( \rho (0) = |\Psi (0)\rangle \langle \Psi (0)| \) and \( P_2 = \text{tr} \left( M_0^\dagger M_0 \rho (2) \right) \) where \( \rho (2) = \text{tr}_\Pi (|\Psi \rangle \langle \Psi |) \), with \( |\Psi \rangle = \frac{1}{\sqrt{N_1}} \left[ \sin \frac{\vartheta}{2} \sqrt{\bar{p}} (\cos r |0_A1|0_I1) + \sin r |0_A0|1|1_I1) + \cos \frac{\vartheta}{2} |1_A0|0|1_I1) \right] \) and \( N_1 = \sin^2 \frac{\vartheta}{2} \bar{p} + \cos^2 \frac{\vartheta}{2} \). In Fig. 3, the probability is plotted versus acceleration parameter \( r \), for \( \vartheta = \pi//2 \). We see that the probability of preparing the state of the system decreases with increase in \( p \) or \( q \). Moreover, we see that \( P(p,q) \) is larger than \( P(q,p) \) provided that \( p > q \), i.e., when the strength of the first measurement is larger than the second measurement the probability becomes greater. The same results are obtain for \( 0 < \vartheta < \pi//2 \).

![Figure 3. The probability of preparing the state of the channel, for \( \vartheta = \pi//2 \).](image)

Next we discuss how PM and PMR affect the degradation of QRs and QFI teleportation through the Unruh noise channel (Eq. (22)).
5. Single-qubit teleportation under the Unruh noise channel

In this section, we investigate the teleportation of QFIs and QC related to single-qubit state, through the Unruh noise channel. We consider \( |\psi_{in}\rangle = \cos \theta/2 |0\rangle + e^{i\varphi} \sin \theta/2 |1\rangle \), \( 0 \leq \theta \leq \pi \), \( 0 \leq \varphi \leq 2\pi \) as the input state in the process of teleportation, where \( \theta \) and \( \varphi \) are the weight and phase parameters, respectively. We use the shared state between Alice and Rob, Eq. (22), as the resource \( \rho_{AI} = \rho_{ch} \) to teleport the single-qubit input state. Using Eq. (1), the output state can be obtained as follows

\[
\rho_{out}^{PM} = \frac{1}{N^2} \begin{pmatrix}
A \cos^2 \frac{\theta}{2} + D \sin^2 \frac{\theta}{2} & \mathcal{F} e^{-i\varphi} \sin \theta \\
\mathcal{F} e^{i\varphi} \sin \theta & A \sin^2 \frac{\theta}{2} + D \cos^2 \frac{\theta}{2}
\end{pmatrix},
\]

where

\[
A = \sin^2 \frac{\theta}{2} \sqrt{pq} \cos^2 r + \cos^2 \frac{\theta}{2} \sqrt{pq},
\]

\[
D = \sin^2 \frac{\theta}{2} \sqrt{pq} \sin^2 r,
\]

\[
\mathcal{F} = \sin \frac{\varphi}{2} \cos \frac{\theta}{2} \sqrt{pq} \cos r.
\]

For input state \( |\psi_{in}\rangle = \cos \theta/2 |0\rangle + e^{i\varphi} \sin \theta/2 |1\rangle \), the QFIs with respect to parameters \( \theta \) and \( \varphi \) are easily found to be \( F_{in}^{\theta} = 1 \) and \( F_{in}^{\varphi} = \sin^2 \theta \), respectively. It is seen that \( F_{in}^{\varphi} \) is dependent on \( \theta \) and is maximized for \( \theta = \frac{\pi}{2} \) while \( F_{in}^{\theta} \) is independent of weight parameter \( \theta \) and has a constant value. Therefore, the balance-weighted input state is preferable. Using Eqs. (10) and (24), QFI with respect to weight and phase parameters are found, respectively, as follows

\[
F_{out}^{PM}^{\theta} = \frac{1}{N^2} \left[ (A - D)^2 \sin^2 \theta + 4 \mathcal{F}^2 \cos^2 \theta + \frac{1}{N^2} \left( (A - D)^2 - 4 \mathcal{F}^2 \right)^2 \sin^2 2\theta \right],
\]

\[
F_{out}^{PM}^{\varphi} = 4 \left| \frac{\mathcal{F} \sin \theta}{N^2} \right|^2.
\]

In Fig. 4, QFI with respect to weight parameter, \( F_{out}^{PM}^{\theta} \), for single-qubit state teleportation through the pure Unruh decoherence and for the case that the combination of PM and PMR have been applied, is plotted as a function of acceleration parameter \( r \). It can be seen that after teleportation under pure Unruh channel (i.e., \( p = q = 0 \)), when the acceleration increases, QFI decays monotonously for all values of the initial parameter \( \theta \). Studying the behavior of \( F_{out}^{PM}^{\theta} \), when the PM and PMR are applied on the channel, we observe that applying either PM (i.e., \( q = 0 \)) or PMR (i.e., \( p = 0 \)) may improve \( F_{out}^{PM}^{\theta} \) for all initial states of the channel (see Figs. 4(a) and 4(b) ). For sufficiently strong measurement strength (\( p \rightarrow 1 \) or \( q \rightarrow 1 \)), the precision of estimating weight parameter can be enhanced remarkably and it is almost robust against the Unruh decoherence.
Figure 4. Single-qubit teleported QFI with respect to weight parameter, $\theta$, as functions of acceleration parameter $r$ by fixing $\theta = \frac{\pi}{2}$ and for $0 < \vartheta < \pi$ for (a) $q = 0$, (b) $p = 0$.

Figure 5. Single-qubit teleported QFI with respect to weight parameter, $\theta$, as functions of PM strength, $p$, for $r = 0.6$, and different values of PMR strength.

The important question that comes up is that if the acceleration is constant, how one can control the QFI by applying PM and PMR. In Fig. 5 we consider the $F_{\text{out}}^{\text{PM}}(\theta)$ behavior versus $p$. It is observed that in the absence of PMR ($q = 0$), $F_{\text{out}}^{\text{PM}}(\theta)$ enhances with increase in $p$ (space dashed purple line) for all values of the channel parameter $\vartheta$. It is also seen that with the combined effect of PM and PMR, estimation precision of weight parameter is also improved. We obtain the same results investigating the behavior of $F_{\text{out}}^{\text{PM}}(\theta)$ versus $p$.

In Fig. 6, $F_{\text{out}}^{\text{PM}}(\varphi)$ for single-qubit state teleportation, is plotted as functions of PM as well as PMR strength for fixed value of acceleration parameter $r = 0.6$ and the maximally entangled input state ($\theta = \pi/2, \varphi = 0$). It is seen from Fig. 6(a) that for $\frac{\pi}{2} \leq \vartheta < \pi$, with increase in PM strength $F_{\text{out}}^{\text{PM}}(\varphi)$ increases to reach a maximum value and then it decreases with more increase of $p$. Moreover, comparing the behavior of $F_{\text{out}}^{\text{PM}}(\varphi)$ for different values of PMR strength, we see that with increase in $q$, optimal estimation of the phase parameter occurs for larger values of $p$. Nevertheless, increase of the PMR strength interestingly raises the optimal value of the QFI, leading to enhancement of the phase parameter estimation. We also see, in that range of $\theta$, while for small values of $p$, the QFI may fall with an increase in $q$, it can enhance as $q$ increases for larger values of $p$. We obtain the same results investigating the behavior of $F_{\text{out}}^{\text{PM}}(\varphi)$ versus $q$ for $0 < \vartheta \leq \frac{\pi}{2}$. In particular, in this range, the QFI may decrease with an increase in $p$ for small values of $q$, while it can exhibit increasing behavior as
Figure 6. Single-qubit teleported QFI with respect to phase parameter, $\varphi$, as functions of (a) PM strength, $p$, fixing $\vartheta = \frac{3\pi}{4}$ and (b) PMR strength, $q$, fixing $\vartheta = \frac{\pi}{4}$; where we have chosen the acceleration parameter $r = 0.6$.

Figure 7. The optimal value of PM and PMR strengths as functions of acceleration parameter $r$ for (a) $q = 0.6$ and $\vartheta = \frac{3\pi}{4}$ and (b) $p = 0.6$ and $\vartheta = \frac{\pi}{4}$.

$p$ increases for large values of $q$ (see Fig.6(b)).

Considering the optimal behavior of single-qubit teleported QFI with respect to phase parameter as functions of $p$ and $q$, we obtain $p_{\text{opt}}$ and $q_{\text{opt}}$ as follows

$$ p_{\text{opt}} = \frac{q \cos^2 r \sin^2 \vartheta - \cos \vartheta (1 - q)}{\sin^2 \vartheta (1 - q \sin^2 r)} $$

$$ q_{\text{opt}} = \frac{\sin^2 \vartheta (p + 2 (1 - p) \cos^2 r) - 1}{\sin^2 \vartheta (p + (1 - p) \cos^2 r) - 1} $$

(28)

Figure 7. shows how optimal values of $p$ and $q$ vary in terms of acceleration parameter $r$. Decrease of the optimal value of $p_{\text{opt}}$ with increase in acceleration (see Fig. 7(a).) indicates when $r$ increases the optimal estimation of phase parameter can be realized by weaker PM. However, Fig. 7(b) shows that more strong PMR is required for attaining the optimal QFI when the accelerated observer moves with more larger acceleration.

Behavior of the QFI with respect to phase parameter, $F_{\text{out}}^{PM}(\varphi)$, as a function of ac-
Figure 8. Single-qubit teleported QFI with respect to phase parameter, $\varphi$, as functions of acceleration parameter $r$ by fixing $\theta = \pi/2$ for (a) $0 < \vartheta < \pi/2$, (b) $\vartheta = \pi/2$ and (c) $\pi/2 < \vartheta < \pi$.

Acceleration parameter, $r$, for different ranges of the channel parameter, $\vartheta$, is investigated in Fig. 8. It is seen that for teleportation under pure Unruh channel (i.e., $p = q = 0$), there is monotonous degradation in $F_{PM}^{out}(\varphi)$ with increase in $r$. However, we find that the combined effect of PM and PMR with the same strength, $(p = q)$, leads to partially improvement of the estimation precision of the phase parameter. Besides, when this common measurement strength increases $F_{PM}^{out}(\varphi)$ is protected much better for $\pi/2 \leq \vartheta < \pi$; it even increases surprisingly with increase in acceleration for $\pi/2 < \vartheta < \pi$, in the limit $p \rightarrow 1$ and $q \rightarrow 1$. In addition, our numerical calculation shows that in order to protect the QFI with respect to $\varphi$ and QR of the teleported state against the Unruh effect, we can use the following special choice for PMR strength $(61)$

$$q_s = 1 - (1 - p) \cos^2 r. \quad (29)$$

In fact, the Unruh noise may be approximately eliminated provided that the PM strength is sufficiently strong $(p \rightarrow 1)$ and the above choice for the PMR is applied (see blue dashed lines in Fig. 8).

If we intend to teleport only the information encoded into the phase parameter, we can manage the input state by choosing the weight parameter as $\theta = \pi/2$, to estimate the phase parameter with the best precision; i.e., the best estimation of phase parameter is obtained if the input state is maximally entangled (see Fig. 9).

In the following, the effect of PM or PMR on QC teleportation of single-qubit are studied. Using the $l_1$-norm measure (Eq. (11)), QC for the density matrix (24), can be obtained as follows

$$C_{l_1}(\rho_{out}^{PM}) = \frac{\sin \theta \sqrt{pq \cos \vartheta \sin \theta}}{N_2}. \quad (30)$$
Figure 9. Single-qubit teleported QFI with respect to phase parameter $\varphi$ as functions of $\theta$, fixing the acceleration parameter $r = 0.6$.

Figure 10. Quantum coherence of the teleported single-qubit state as functions of acceleration parameter $r$ by fixing $\theta = \frac{\pi}{2}$ for (a) $0 < \vartheta < \frac{\pi}{2}$, (b) $\vartheta = \frac{\pi}{2}$ and (c) $\frac{\pi}{2} < \vartheta < \pi$.

In the case of teleportation without application of PM or PMR on the Unruh channel, i.e., $p = q = 0$ and then $N_2 = 1$, we find

$$C_{l_1}(\rho_{out}) = |\sin\theta \cos r \sin \theta|$$  \hspace{1cm} (31)

which is the teleported quantum coherence under the pure Unruh decoherence.

Investigating QC of the teleported single-qubit state as functions of $r$ or studying its behavior versus PM and PMR strength for fixed value of the acceleration parameter, one can see that the results, qualitatively similar to $F_{out}^{PM}(\varphi)$, are observed (see Figs.
Figure 11. Quantum coherence of the teleported single-qubit state as functions of (a) PM strength, \( p \), fixing \( \vartheta = \frac{3\pi}{4} \) and (b) PMR strength, \( q \), fixing \( \vartheta = \frac{\pi}{4} \); for fixed value of the acceleration parameter \( r = 0.6 \).

\begin{align*}
\text{(a)} & \quad \text{Figure 12. Fidelity of the single-qubit teleportation as functions of acceleration parameter } r \\
\text{(b)} & \quad \text{by fixing } \theta = \frac{\pi}{2} \text{ and } \varphi = 0 \text{ for (a) } 0 < \vartheta < \frac{\pi}{2}, (b) \vartheta = \frac{\pi}{2} \text{ and (c) } \frac{\pi}{2} < \vartheta < \pi.}
\end{align*}

In order to determine the quality of teleportation, i.e., closeness of the teleported state to the input state, the fidelity (62) between \( \rho_{\text{in}} \) and \( \rho_{\text{out}} \) defined as
\[
\begin{align*}
f(\rho_{\text{in}}, \rho_{\text{out}}) &= \frac{\text{Tr}}{N^2} \left( \sqrt{(\rho_{\text{in}})^{1/2} \rho_{\text{out}} (\rho_{\text{in}})^{1/2}} \right)^2 \\
&= \langle \psi_{\text{in}} | \rho_{\text{out}} | \psi_{\text{in}} \rangle,
\end{align*}
\]
should be computed. Therefore, we obtain
\[
f = \frac{1}{N^2} \left[ \left( \frac{A-B}{2} + F \cos 2\varphi \right) \sin^2 \theta + D \right]. \tag{32}
\]

In Fig. 12, the teleportation fidelity versus acceleration parameter \( r \) has been plot-
Figure 13. Fidelity of the single-qubit teleportation as functions of (a) PM strength, \( p \), fixing \( \vartheta = \frac{3\pi}{4} \) and (b) PMR strength, \( q \), fixing \( \vartheta = \frac{\pi}{4} \); for fixed value of the acceleration parameter \( r = 0.6 \).

Figure 14. Comparing teleported \( F^{PM}_{\text{out}}(\varphi) \), QC as well as teleportation fidelity for single-qubit state by fixing \( \theta = \frac{\pi}{2}, \varphi = 0 \), (a) versus \( p \) in the absence of PMR, \( q = 0 \), (b) versus \( p \) in the presence of PMR, \( q = 0.6 \), (c) versus \( q \) in the absence of PM, \( p = 0 \), and (d) versus \( q \) in the presence of PM, \( p = 0.6 \).

It is seen that the results obtained for fidelity is the same as the obtained results for QC and \( F^{PM}_{\text{out}}(\varphi) \), i.e., fidelity degrades with increase in \( r \) under pure Unruh effect. However, the combined effect of PM and PMR for channel parameter lying in the region \( \frac{\pi}{2} \leq \vartheta < \pi \), can improve the quality of teleportation and it may even enhance with increase in acceleration for \( \frac{\pi}{2} < \vartheta < \pi \) in the limit \( p, q \to 1 \). Moreover, Unruh decoherence is approximately eliminated for all values of \( \vartheta \), with \( q = q_{\text{opt}} \) and in the limit \( p \to 1 \), consequently the teleportation process may be implemented with better
quality.

Now we investigate the fidelity of the single-qubit teleportation as functions of PM as well as PMR strength. As it is seen in Fig. 13(a), similar to $F_{\text{out}}^{\text{PM}}(\varphi)$ and QC, with proper selection of the channel parameter $\vartheta$ the quality of teleportation may be enhanced with increase in $p$ or $q$ to reach a maximum value. Besides, in the range $0 < \vartheta \leq \frac{\pi}{2}$ ($\frac{\pi}{2} < \vartheta < \pi$), analyzing the QFI behavior as a function of $q$ ($p$), we see that the QFI may be decreased (improved) with an increase in $p$ ($q$) for small values of $q$ ($p$), while it can exhibit increasing behavior as $p$ ($q$) increases for large values of $q$ ($p$). In addition, optimal teleportation fidelity becomes greater with increase in $q$ or $q$, hence the teleportation process is done more successfully.

Finally, in Figs. 14(a) and 14(b) we compare and illustrate the harmonic behavior of $F_{\text{out}}^{\text{PM}}(\varphi)$, QC and teleportation fidelity as functions of PM strength for $\frac{\pi}{2} < \vartheta < \pi$, and PMR strength for $0 < \vartheta \leq \frac{\pi}{2}$, in the case of single-qubit teleportation. We can conclude that for both $q = 0$ and $q \neq 0$, the PM strength which optimizes the estimation precision of the phase parameter, is the strength at which the quality of teleportation is the best and the coherence of the output single-qubit state reaches to its maximum value. Investigating the behavior of the above mentioned quantities as functions of PMR strength, we achieve the same results (see Figs. 14(c) and 14(d)).

6. Two-qubit teleportation under the Unruh noise channel

In order to study QRs and QFI teleportation of a two-qubit state through the Unruh channel, $|\psi_{in}\rangle = \cos \theta/2|10\rangle + e^{i\varphi} \sin \theta/2|01\rangle$, $0 \leq \theta \leq \pi$, $0 \leq \varphi \leq 2\pi$ is considered as the input state in the teleportation process. We follow Kim and Lee's two-qubit teleportation protocol (52), and use two copies of the shared state between Alice and Rob as the quantum channel. Using Eq. (3), we obtain the output state as

$$
\rho_{\text{out}}^{\text{PM}} = \frac{1}{N^2} \begin{pmatrix}
AD & 0 & 0 & 0 \\
0 & A^2 \cos^2 \theta \frac{\pi}{2} + D^2 \sin^2 \theta \frac{\pi}{2} & 2F^2 e^{-i\varphi} \sin \theta & 0 \\
0 & 2F^2 e^{i\varphi} \sin \theta & A^2 \sin^2 \theta \frac{\pi}{2} + D^2 \cos^2 \theta \frac{\pi}{2} & 0 \\
0 & 0 & 0 & AD
\end{pmatrix},
$$

where $A$, $D$ and $F$ are determined by Eq. (25).

Now we apply PM or PMR on Rob's particle, before teleporting the two-qubit state. Then, we study the influence of PM or PMR on the degradation effect of the Unruh noise channel on QRs and QFI teleportation. Using density matrix (33), the corresponding quantum coherence is obtained as follows

$$
C_l\left(\rho_{\text{out}}^{\text{PM}}\right) = 4\left|\frac{F^2 \sin \theta}{N^2} \right|, 
$$

The results, obtained for teleportation of two-qubit QC under Unruh noise channel, are similar to single-qubit teleportation.

According to the Eqs. (12) and (33), the entanglement of the teleported two-qubit state is obtained as

$$
C\left(\rho_{\text{out}}^{\text{PM}}\right) = 2\text{Max}\left\{0, 2\left|\frac{F^2 \sin \theta}{N^2} \right| - \frac{AD}{N^2}\right\},
$$
Figure 15. Entanglement of the teleported two-qubit state as functions of acceleration parameter $r$ by fixing $\theta = \frac{\pi}{2}$ for (a) $0 < \vartheta \leq \frac{\pi}{2}$, and (b) $\frac{\pi}{2} < \vartheta < \pi$.

In Fig. 15, we plot the concurrence of teleported two-qubit state as a function of acceleration parameter $r$ for different strengths of PM and PMR. It is clear that the entanglement absolutely decreases with increase in acceleration under the pure Unruh decoherence. However, it can be amplified with combined action of PM and PMR for all values of initial channel parameter $\vartheta$. In fact, when the strength of PM increases, the entanglement degradation decreases, especially in the limit $p = q \to 1$, entanglement is approximately protected against the Unruh decoherence. Surprisingly, as seen in Fig. 15, in that limit, the teleported entanglement may increase under the Unruh effect for initial channel parameter lying in the region $\frac{\pi}{2} < \vartheta < \pi$. In addition, it is seen that the entanglement is also improved by applying $q_s$ even without first PM (i.e., $p = 0$) for $0 < \vartheta \leq \frac{\pi}{2}$.

Considering the behavior of QD as a function of acceleration parameter $r$ for different PMs strength, we see that combined action of PM and PMR can raise QD for $\vartheta$ lying in the range $\frac{\pi}{2} \leq \vartheta < \pi$ (see Fig. 16). In particular, in the limit $p, q \to 1$, QD may increase with applying PMs for $\frac{\pi}{2} < \vartheta < \pi$. Moreover, if we choose $q = q_{opt}$, QD can increase even in the absence of first PM (i.e., $p = 0$) for $0 < \vartheta \leq \frac{\pi}{2}$.

Using Eqs. (8) and (33), we find the two-qubit teleported QFIs with respect to weight and phase parameters as follows

$$F_{\text{out}}^{\text{PM}}(\theta) = \frac{1}{N^2} \zeta + \frac{8A^2D^2\left(\zeta^2 - 16F^4\right)}{\zeta\left[(A^2 - D^2)^2 - 16F^4\right] \cos 2\theta - (\zeta^2 + 4(A^2D^2 - 4F^4))},$$

$$F_{\text{out}}^{\text{PM}}(\varphi) = \frac{16F^4\sin^2\theta}{\zeta N^2},$$

where $\zeta = A^2 + D^2$. Surprisingly, we obtain the results similar to single-qubit teleportation, investigating the teleportation of two-qubit QFI under Unruh channel.

In Figs. 17 and 18, we compare teleportation of QFI in both single and two-qubit cases (supposing that $\theta$ or $\varphi$ carries the same information in both cases). In Fig. 17, we see that the information encoded in the weight parameter $\theta$ is better protected against Unruh effect during teleportation of two-qubit state, comparing it with the single-
Figure 16. QD of the teleported two-qubit state as functions of acceleration parameter $r$ by fixing $\theta = \frac{\pi}{2}$ and $\varphi = 0$ for (a) $0 < \vartheta < \frac{\pi}{2}$, (b) $\vartheta = \frac{\pi}{2}$, and (c) $\frac{\pi}{2} < \vartheta < \pi$

Figure 17. Comparing teleported $F_{\text{out}} (\theta)$ for single and two-qubit states, fixing $\theta = \frac{\pi}{2}$ and $\vartheta = \frac{\pi}{2}$ (a) in the absence of measurements, (b) in the presence of measurements.

qubit scenario. Nevertheless, extraction of information encoded into phase parameter $\varphi$, is more efficient in single-qubit teleportation than the two-qubit one (see Fig. 18). Therefore, depending on what parameter we want to teleport, we use single or two-qubit state to encode the required information.

Finally, fidelity for the two-qubit teleportation under the Unruh channel, are found to be

$$f = \frac{1}{N^2} \left[ \left( \frac{A^2 - D^2}{4} + F^2 \cos 2\varphi \right) 2\sin^2 \theta + D^2 \right], \quad (38)$$

We get the results similar to single-qubit teleportation fidelity, investigating the
Comparing teleported $F_{\text{PM}}(\varphi)$ for single and two-qubit states by fixing $\theta = \frac{\pi}{2}$ and $\vartheta = \frac{\pi}{2}$ (a) in the absence of measurements, (b) in the presence of measurements.

Comparing the fidelity of single and two-qubit teleportation, fixing $\theta = \frac{\pi}{2}$, $\varphi = 0$ and $\vartheta = \frac{\pi}{2}$ for (a) in the absence of measurements, (b) in the presence of measurements.

Comparing the fidelity of single and two-qubit teleportation under the Unruh noise channel with and without applying the measurements.

Comparing the fidelity of single and two-qubit teleportation in Fig. 19, we observe that quality of teleportation is better in single-qubit case than the two-qubit one. It means that single-qubit teleportation is more robust against the Unruh decoherence.

7. Summary and conclusions

Teleportation of QRs and QFI of single and two-qubit states, under the Unruh effect experienced by a mode of a free Dirac field, was discussed in this paper. We investigated the conditions under which the degradation effect of the Unruh effect on QRs and QFI teleportation can be improved by PMs, and found that the value of initial parameter of the channel $\vartheta$ plays a key role in this scenario. Moreover, we examined how the partial measurements can be performed to eliminate the Unruh effect or how they may be designed such that the Unruh effect can be used to enhance the quantum communication. Besides, fixing the acceleration and considering the behavior of the
QFI, QC and teleportation fidelity as functions of PM strength (PMR strength), we found that $F_{\text{PM}}^\text{out} (\varphi)$, QC and teleportation fidelity harmonically increase to reach a maximum value and then decrease with more increase in $p$ ($q$). We also analytically analysed the optimal behavior of the QFI associated with the phase parameter. Finally, comparing the teleportation of QFI for single and two-qubit cases as functions of acceleration, we showed that the information encoded in the weight parameter $\theta$ is better protected against the Unruh effect in the case of two-qubit teleportation. However, in the case of single-qubit teleportation, encoding information in the phase parameter $\varphi$ is more efficient. Therefore, we encode the information into either the weight or phase parameter, depending on either the two or single-qubit scenario, respectively, is used for the teleportation.

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