On a numerical method for solving the hydrodynamic problem of underground leaching

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Abstract. Hydrodynamic problem of the process of underground leaching in the planned statement in a horizontal section is considered in the paper. An algorithm for solving the problem based on the longitudinal-transverse scheme and the method of flow sweep is given. The reliability of the developed algorithm is verified by example.

1. Introduction
As is known, an expansion of application boundaries of the method of underground leaching (UL) and a search for ways and means of its application for deposits containing mineral resources remains one of the main tasks in this sphere.

Observations for the groundwater level in UL regions are considered to be the important tasks of hydrogeological survey. They make it possible to estimate the ways and rates of filtration of technological solutions, possible losses of process solutions, hydraulic connection of the productive water-bearing horizon with non-ore horizons, the degree of stability of the hydrodynamic regime of the site under consideration, etc.

In [1, 2] it is shown that in UL process, filtration inhomogeneity of ores and ore-bearing rocks is of great importance.

Complete mathematical model of UL problem has been carried out in detail in [3]; on the basis of a computational experiment the reliability of the proposed numerical algorithms for solving complete mathematical model of UL has been proved. Numerical results are compared with test data physically corresponding to the real parameters of specific objects.

Inhomogeneity is the dependence of any parameter of the coordinate system within the limits of this system. The principal equations of UL of minerals are formulated in [4–5].

In this paper, a two-dimensional unsteady hydrodynamic problem of UL process is considered, the problem is accompanied by changes in the parameters of the solution and the productive water-bearing horizon along the (x, z) axes.

2. Statement of the problem
The problem is described by the system of equations

\[
\frac{\partial}{\partial x} \left( K(x,z) \frac{\partial H(x,z,t)}{\partial x} \right) + \frac{\partial}{\partial z} \left( K(x,z) \frac{\partial H(x,z,t)}{\partial z} \right) - \gamma = \theta(x,z,t)M(x,z,t) \frac{\partial H(x,z,t)}{\partial t} + F(x,z,t),
\]

\( (x,z) \in \Omega \{ 0 < x < a; \ 0 < z < c \}, \ t > 0 \)
with initial conditions
\[ H(x,z,0) = H_0(x,z) \] (2)

and with boundary conditions
\[ H(x,z,t)|_{\Gamma} = H_1(x,z,t), \quad (x,z) \in \Gamma, \] (3)

where \( H(x,z,t) \) is the variable head at the current point with coordinates \((x,z)\) at an arbitrary point in time \(t\); \( K(x,z) = k(x,z) \) is the filtration coefficient; \( H_0(x,z) \) is the initial head value; \( H_1(x,z,t) \) is the head value at the boundary of the ore-bearing horizon; \( \gamma \) is the specific weight of leached reagent; \( \theta = \frac{c}{\theta} \) is the ratio of the pure reagent in a unit of the pore volume of soil; \( c \) is the concentration of minerals; \( D \) is the area of ore-bearing horizon; \( \Gamma \) is the boundary of the area; \( M(x,z) = m\beta \); \( m \) is the porosity; \( \beta \) is the coefficient of elastic capacity; \( F(x,z,t) = \sum_{i=1}^{N} q_i(t) \delta(x-x_i,z-z_j) \) where \((x_i,z_j)\), \( q_i \), \( N \) are the coordinates, the flow rate of the solution and the number of \( i \)-th technological well, respectively; \( \delta \) is the delta function.

3. Method of solution
To solve the problem (1) - (3), proceed to the dimensionless variables by entering some characteristic values for \( L_x, L_z, H_x, k_x, \gamma_x, c_x \)

\[ \bar{x} = \frac{x}{L_x}, \quad \bar{z} = \frac{z}{L_z}, \quad \bar{H} = \frac{H}{H_x}, \quad \bar{k} = \frac{k}{k_x}, \quad \bar{c} = \frac{c}{c_x}, \quad \bar{\gamma} = \frac{\gamma}{\gamma_x}. \]

Having made some calculations and, for convenience omitting the dashes above the variables, the problem takes a form similar to (1) - (3).

To build a numerical algorithm of finite difference schemes, introduce a uniform grid

\[ \bar{\omega}_{x,z,t} = \left\{ (x_i = i h_x, z_j = j h_z, t_l = l \tau), h_x = \frac{1}{N_x}, h_z = \frac{1}{N_z}, i = 1, ..., N_x, j = 1, ..., N_z \right\}. \]

The obtained difference problem is solved by the methods of longitudinal-transverse scheme and the flow variant of the sweep method [6, 7].

The approximation of difference equations in one time step consists in dividing this time step into two stages. In the first expression, the derivatives of the first variables are implicit, and the derivative of the second variable is explicit for the first half of the time step. For the second half of the time step, the derivative of the first variable is expressed explicitly, and the derivatives of the second variable are implicit. Then the task is reduced to the sequential solution of two one-dimensional problems, the typical form of which is:
The problem is solved by the method of flow sweep, the solution of which is sought in the form

\[ P_i = \frac{\tau}{h} \alpha_i W_{i-1/2} + \beta_i \]  

(5)

the final form of the algorithm is as follows

a) the values of the sweep coefficients \( \alpha_i, \beta_i \) are determined as:

\[
\alpha_N = -\left(\frac{h}{\tau} \lambda_2 \right)\left(0.5 \frac{h}{\tau} \theta_2 M_0 \lambda_2 + \lambda_2 \Phi_N \right);
\]

\[
\beta_N = \left(0.5 \frac{h}{\tau} \theta_2 \lambda_2 M_N + \lambda_2 \right)\;
\]

\[
\alpha_i = -\left(\frac{h^2}{\tau} - K_{i+1/2} \alpha_{i+1}\right)\;
\]

\[
K_{i+1/2} + \theta_2 M_i \left(\frac{h^2}{\tau} - K_{i+1/2} \alpha_{i+1}\right)
\]

(6)

\[
\beta_i = \left(\frac{K_{i+1/2} (\beta_{i+1} - h\gamma) + \Phi_i \left(\frac{h}{\tau} - K_{i+1/2} \alpha_{i+1}\right)}{K_{i+1/2} + \theta_2 M_i \left(\frac{h^2}{\tau} - K_{i+1/2} \alpha_{i+1}\right)}\right), \quad i = N-1, \ldots, 1.
\]

b) the sought for function \( H_i \) is determined by the following relations

\[
H_0 = -\lambda_4 \left(\frac{h}{\tau} - K_{1/2} (\beta_1 - h\gamma) - (f_1 + 0.5 \lambda_4 \theta_1 (\Phi_0)\left(\frac{h^2}{\tau} - K_{1/2} \alpha_1\right))\right)\left(0.5 \lambda_4 \theta_1 M_0 - \lambda_4 \left(\frac{h^2}{\tau} - K_{1/2} \alpha_1\right) + \frac{h}{\tau} \lambda_4 K_{1/2}\right);
\]

(7)

\[
H_{i+1} = \left(\frac{(H_i + h\gamma)K_{i+1/2} \alpha_{i+1} + \frac{h^2}{\tau} \beta_{i+1}}{\frac{h^2}{\tau} - K_{i+1/2} \alpha_{i+1}}\right), \quad i = 0, \ldots, N - 1.
\]
4. Results

According to the above algorithm, a module program in the DELPHI algorithmic language was compiled and implemented, and the hypothetical hydrodynamic problem of UL process was solved in the domain shown in Fig. 1 at the following initial data (in dimensional form).

The width and thickness of the reservoir - 50 m and 25 m, respectively; the filtration coefficient - 5.8 m/day; the specific weight of fluid is 0.91 g/cm$^3$; coordinates and flow rates of injection and production wells (1 - 20 m, 7.5 m, 50 m$^3$/day; 2 - 25 m, 15 m., 100 m$^3$/day; 3 – 30 m, 7.5 m, 50 m$^3$/day).

Figure 1. The layout of the filter in injection ($\times$) and in production (o) wells.

To solve the problem, a grid of 100 by 50 is chosen. The resultant field of pressure in dimensionless form on the 30th day is presented in the form of an isoline shown in Fig.2.

Figure 2. Pressure isolines on the 30th day of well operation.

The symmetry of the obtained results confirms the fact that the developed algorithm can be used to solve problems similar to the set one. At the initial point in time, the pressure field is taken with account of specific weight of the fluid, i.e. pressure values increase in the direction of the (OZ) axis in the form of equal lines. Over time, the pressure values near the injection wells increase, and in the pumping well decrease. Such layout of wells certifies the loss of the extracted solution, i.e. it does not pollute the environment.
5. Conclusion
The numerical solution of the hydrodynamic problem of the UL process in the horizontal formulation makes it possible to determine the pressure distribution in the vicinity of the wells and to control the uniform distribution of the UL process.

References

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