Effect of shadowing and antishadowing corrections to the evolution of gluon density at small-$x$

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Abstract. We have obtained a semi-analytical solution of GLR-MQ-ZRS equation at small $x$ for gluon density using Regge like ansatz of $G(x, Q^2)$ in the leading twist approximation. It is observed that, our solution is valid in the vicinity of saturation scale where modification in the QCD evolution can not be ruled out due to salient gluon fusion processes. Our computed results of nonlinear gluon density are then compared with different global parameterizations and it is enticing to note that, the rapid growth of gluon densities towards small-$x$ is tamed due to the nonlinear corrections.

1. Introduction
The study of the gluon density at small-$x$ is particularly important because here gluons are expected to dominate the hadron structure function. However in the region of very small-$x$, the sharp growth of the gluon density is expected to slow down eventually in order to restore the Froissart bound on physical cross sections. A remarkable effect known as the gluon fusion, which induces nonlinear corrections to the linear QCD evolution, are considered to be responsible for this taming behaviour. The shadowing and antishadowing corrections are dynamically produced by gluon fusions and the coexistence of both the processes in QCD evolution of gluon densities restores the local momentum conservation. Gribov-Levin-Ryskin(GLR) and Mueller-Qiu (MQ) in their pioneering works [1] first studied the nonlinear effects at the twist-4 level by summing up the gluon recombination diagrams using the Abramovsky-Gribov-Kancheli (AGK) cutting rules. But the nonlinear term in the GLR-MQ equation seems to violate the momentum conservation. Later, Zhu and his collaborators suggested a new evolution equation, popularly known as the GLR-MQ-ZRS equation [2], considering gluon recombination in the leading logarithmic ($Q^2$) approximation using time ordered perturbation theory (TOPT) instead of the AGK cutting rules.

Here we have made an attempt to find a semi-analytical solution of GLR-MQ-ZRS equation by employing Regge like ansatz of $G(x, Q^2)$ to study the behavior of nonlinear gluon density at leading twist approximation in the photon virtuality $Q^2$ and compared our results with the results of various global PDF groups viz., HERAPDF[3] and CT10[4].

2. Formalism
In terms of gluon density, the GLR-MQ-ZRS can be mathematically expressed as [2]
conveniently be expressed as

\[ \frac{dG(x, Q^2)}{d \ln(Q^2)} = \frac{\alpha_s}{\pi} \int_x^1 \frac{dx_1}{x_1} G(x_1, Q^2) + \frac{9\alpha_s^2}{2\pi R^2 Q^2} \frac{N_c^2}{N_c^2 - 1} \int_{x/2}^1 \frac{dx_1}{x_1} G^2(x_1, Q^2) \]

\[ - \frac{9\alpha_s^2}{\pi R^2 Q^2} \frac{N_c^2}{N_c^2 - 1} \int_x^{1/2} \frac{dx_1}{x_1} G^2(x_1, Q^2) \]  

(1)

The first term on the R.H.S. of the above equation is the usual DGLAP equation at Double Logarithmic Approximation (DLA). The second and the third term represents the antishadowing and shadowing contributions from the correlative gluons inside the hadrons respectively. \( N_c \) is the number of color charges and \( R \) represents the correlative radius of the gluons inside the hadrons. Here, \( \alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2)} \) with \( \Lambda \) being the QCD cut off parameter and \( \beta_0 = 11 - \frac{2}{3}N_f \), where \( N_f \) is the number of active quark flavours.

To solve the GLR-MQ equation we have taken into account a simple form of Regge like behaviour of gluon distribution function as [5]

\[ G(x, Q^2) = J(Q^2)x^{-\lambda_g}, \]  

(2)

where \( J(Q^2) \) is a function of \( Q^2 \) and \( \lambda_g \) is the Regge intercept for gluon. Regge theory provides a highly ingenious parametrization of all total cross sections and is supposed to be applicable for very small-\( x (x \leq 0.01) \).

In terms of the new variable \( t = \ln(Q^2/\Lambda^2) \) and on further simplifications, Eq. (1) can conveniently be expressed as

\[ \frac{\partial G(x, t)}{\partial t} = \Upsilon_1(x) G(x, t) - \frac{\Upsilon_2(x)}{t^2} G^2(x, t). \]  

(3)

where, \( \Upsilon_1(x) \) and \( \Upsilon_2(x) \) are functions of the Bjorken variable \( x \). The analytical solution of Eq.(3) has the following form

\[ G(x, t) = \frac{\Gamma(\Upsilon_1(x) G(x, t_0))}{-t_0^{\Upsilon_1(x)} + \Upsilon_2(x) \left( \Gamma[-1 + \Upsilon_1(x), t_0] + \Upsilon_2(x) \Gamma[-1 + \Upsilon_1(x), t] \right) G(x, t_0)}. \]  

(4)

This expression allows us to investigate the effect of shadowing and anti-shadowing corrections on the \( Q^2 \) evolution of gluon distribution at some fixed values of small-\( x \).

3. Result and discussion

The effects of nonlinear corrections to the \( Q^2 \) evolution of gluon distribution function, \( G(x, Q^2) \), is examined at small-\( x \) by solving GLR-MQ-ZRS equation. We have performed our analysis in the kinematic region \( 2 \leq Q^2 \leq 40 \text{ GeV}^2 \) and \( 10^{-4} \leq x \leq 10^{-2} \) where the suggested solution is found to be legitimate. Fig. 1(a-d) show the \( Q^2 \) dependence of \( G(x, Q^2) \) with nonlinear corrections obtained from Eq.(4) for both \( R = 2 \text{ GeV}^{-1} \) and \( R = 5 \text{ GeV}^{-1} \) and compared with CT10 and HERAPDF parametrizations respectively.

To summaries, in this paper the nonlinear GLR-MQ-ZRS equation for \( G(x, Q^2) \) is solved in leading twist approximation and the effect of shadowing and antishadowing corrections has been investigated. The suggested solution is found to be valid only in the vicinity of gluon saturation and a taming behaviour of the rapid growth of the gluon densities is observed towards small \( x \). Moreover results show that the antishadowing contributions increases with the increase in correlation radius \( R \) among the interacting gluons as expected. It is observed that our results well reproduce the general trend of global PDFs, but lie slightly below the CT10 and HERAPDF.
Figure 1. $Q^2$ dependence of $G(x, Q^2)$ with combined effect of shadowing and antishadowing corrections for four fixed values of $x$.

results. This may be due to the fact that, in this work we have considered only the leading twist contribution of the gluon-gluon splitting terms to the evolution of $G(x, Q^2)$. The strong coupling constant $\alpha_s$ also controls the growth of gluon distribution towards small-$x$. However, inclusion of higher order terms in the gluon splitting as well as in the expansion of $\alpha_s$ may enhance the compatibility of our results with the global PDF data and it will be always interesting to study the effect of higher twist corrections in the evolution of $G(x, Q^2)$.

References
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