Estimation of the isothermal compressibility from event-by-event multiplicity fluctuation studies

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Abstract. The first estimation of the isothermal compressibility (kT) of matter is presented for a wide range of collision energies from √sNN = 7.7 GeV to 2.76 TeV. kT is estimated with the help of event-by-event charged particle multiplicity fluctuations from experiment. Dynamical fluctuations are extracted by removing the statistical fluctuations obtained from the participant model. kT is also estimated from event generators AMPT, UrQMD, EPOS and a hadron resonance gas model. The values of isothermal compressibility are estimated for the Large Hadron Collider (LHC) energies with the help of the event generators.

1 Introduction
To understand the behaviour of the system formed at high temperature and energy density, it is important to understand the thermodynamic state of matter formed in high-energy heavy-ion collisions. Thermodynamic properties of the system, i.e., a set of response functions, like specific heat, compressibility, and different susceptibilities are directly related to event-by-event fluctuation observables, which are experimentally measurable.

2 Methodology
Isothermal compressibility (kT) is the measure of relative change in volume with respect to change in pressure, at constant temperature.

In the Grand Canonical Ensemble (GCE), variance of particle multiplicity distribution is related to kT by [1, 2],

$$\sigma^2 = \frac{k_B T \langle N \rangle^2}{V} k_T$$

(1)

where k_B is Boltzmann constant. Charged particle multiplicity fluctuations can be defined by scaled variance (ωch), which is the variance scaled over the mean charged particle multiplicity (μ ≡ ⟨N⟩). Thus, we get a connection between the multiplicity fluctuations and kT through the following equation as,

$$\omega_{ch} = \frac{\sigma^2}{\mu} = \frac{k_B T \mu}{V} k_T$$

(2)

Thus, applying GCE properties to experimental measurements at mid-rapidity and with the help of Nch, T and V of the system, kT may be estimated at the chemical freeze-out, where the particle ratios get fixed and no new particle is generated later.

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3 Analysis

3.1 Extraction of dynamical multiplicity fluctuations

It is very important to extract dynamical multiplicity fluctuations in order to calculate $k_T$. A detailed study of the charged particle multiplicity fluctuations including centrality as well as beam-energy dependence, with proper acceptance corrections have been presented in [3]. The measured $\omega_{ch}$ have contributions from statistical as well as dynamical source; the dynamical part is related to thermodynamics. Here, the statistical fluctuations are estimated with the help of the participant model as described in [4], where $\omega_{ch}$ is given by,

$$\omega_{ch} = \omega_n + <n>\omega_{Npart}$$  \hspace{1cm} (3)

where $n$ is the number of charged per participant, $\omega_n$ and $\omega_{Npart}$ denote fluctuations in $n$ and number of participants, respectively. From proton-proton collision data, and the formulation given in [5], scaled variance from the participant model is estimated and by subtracting this statistical contribution from the experimentally found multiplicity fluctuations, the dynamical multiplicity fluctuations ($\omega_{ch,dyn}$) are extracted [6]. The results are presented in Fig. 1.

3.2 Multiplicity fluctuations from event generators

To validate the results from the experimental data, the results from three event generators, i.e, UrQMD, EPOS and AMPT (both default (def) and String Melting (SM) mode) are studied in details, for Au+Au collisions at $\sqrt{s_{NN}} = 7.7$ to 200 GeV. The centrality is selected using minimum bias distributions within $0.5 < |\eta| < 1$, and $0.2 < p_T < 2$ GeV/c. Narrow centrality bins are chosen to minimise geometrical fluctuations.

3.3 Estimation of $k_T$ from Hadron Resonance Gas (HRG) model

The calculations of $k_T$ in the HRG model are performed in terms of the species (denoted by subscript ‘i’) dependence of hadrons instead of the total number of charged particles. Considering the pressure $P$ as a function of $T$ and $\mu_i$, additionally, with fixed $N_i$, $k_T$ within HRG can be written as,
Following Eq. (4), \( k_T \) is estimated within the HRG model in Au+Au collisions as a function of collision energy. Results are compiled with the results from experiment, in Fig. 2.

4 Results and discussions

For the evaluation of \( k_T \) from experiment, the values of dynamical-scaled variances are used. The volume and chemical freeze-out temperatures \( T_{\text{ch}} \) are taken from Ref. [7]. The beam-energy dependence of \( k_T \) is shown in Fig. 2. From experimental data, we observe that \( k_T \) remains almost constant at all energies. Event generators show a decreasing trend from lower to higher energies. For LHC energies, \( k_T \) values are presented from AMPT and EPOS.

![Graph](image)

**Fig. 2.** Isothermal compressibility \( (k_T) \) as a function of collision energy for available experimental data for central (0-5%) Au+Au (Pb+Pb) collisions. Estimations of \( k_T \) from event generators and HRG model are presented, too.

The errors give the extent of the estimated values. For HRG, \( k_T \) is rapidly decreasing from 2 to 20 GeV, and remains constant at higher energies, both for full phase space and for \(|\eta| < 0.5\). This observation suggests that collision system is less compressible at higher energies. The estimation of isothermal compressibility \( (k_T) \), specific heat \( (c_v) \) [8], etc., help to obtain the Equation of State (EOS) of matter, hence, the measurements of these quantities are of immense importance.

This work used resources of the LHC grid computing centres at Variable Energy Cyclotron Centre and Bose Institute, Kolkata.

**References**

1. S. Mrowczynski, Phys. Lett. B 430 (1998) 9.
2. A. Adare et al. (PHENIX Collaboration), Phys. Rev. C 78 (2008) 044902.
3. M. Mukherjee et al., J. Phys. G: Nucl. Part. Phys. 43 (2015) 085102.
4. H. Heiselberg, Phys. Rept. 351 (2001) 161.
5. M. M. Aggarwal et al. (WA98 Collaboration), Phys. Rev. C 65 (2002) 054912.
6. M. Mukherjee et al., arxiv : 1708.08692 [nucl-ex].
7. S. Chatterjee et. al., Adv. High Energy Phys. (2015) 349013.
8. S. Basu et. al., Phys. Rev. C 94 (2016) 044901.
