Kondo effect in the helical edge liquid of the quantum spin Hall state

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Following the recent observation of the quantum spin Hall (QSH) effect in HgTe quantum wells, an important issue is to understand the effect of impurities on transport in the QSH regime. Using linear response and renormalization group methods, we calculate the edge conductance of a QSH insulator as a function of temperature in the presence of a magnetic impurity. At high temperatures, Kondo and/or two-particle scattering give rise to a logarithmic temperature dependence. At low temperatures, for weak Coulomb interactions in the edge liquid the conductance is restored to unitarity with unusual power-laws characteristic of a ‘local helical liquid’, while for strong interactions transport proceeds by weak tunneling through the impurity where only half an electron charge is transferred in each tunneling event.

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The quantum spin Hall (QSH) insulator is a topologically non-trivial state of matter\textsuperscript{3} that has recently been observed in transport experiments carried out in HgTe quantum wells (QW)\textsuperscript{2} following its theoretical prediction\textsuperscript{3}. The two-dimensional QSH insulator has a charge excitation gap in the bulk but supports one-dimensional propagating states with opposite spin polarization. The QSH insulator is robust against weak single-particle perturbations which preserve time-reversal symmetry such as weak potential scattering\textsuperscript{4, 5, 6}.

This theoretical picture is consistent with experimental observations: the longitudinal conductance $G$ in a Hall bar measurement is approximately quantized to $G_0 = 2e^2/h$, independent of temperature, for samples of about a micron length\textsuperscript{2, 7}. However, larger samples exhibit $G < G_0$ and $G$ decreases with decreasing temperature\textsuperscript{7}. Deviations from the expected quantized value have been attributed to the presence of local doped regions due to potential inhomogeneities within the sample arising from impurities or roughness of the well/barrier interface\textsuperscript{7}. Although pure potential scattering cannot backscatter the edge states, the role of these potential inhomogeneities is to trap bulk electrons which may then interact with the edge electrons. These localized regions act as dephasing centers for the edge channels due to interaction effects and may cause backscattering.

In this work, we study theoretically the temperature dependence of the edge conductance of a QSH insulator. We consider the case where a local doped region in the vicinity of the edge contains an odd number of electrons and acts as a magnetic impurity coupled to the helical edge liquid. Our main results are as follows (Fig. 1):

1. At high temperatures, the conductance is logarithmic, $-\Delta G \equiv -(G - G_0) = \eta + \gamma \ln(D/T)$ where $\eta, \gamma$ are interaction-dependent parameters and $D$ is an energy scale of order the bulk gap.

2. For weak Coulomb interactions $K > 1/4$ where $K$ is the Luttinger parameter of the edge liquid, the conductance is restored to unitarity at $T = 0$ due to the formation of a Kondo singlet. This is in stark contrast with the Kondo problem in a usual spinful 1D liquid where the conductance vanishes at $T = 0$ for all $K_\rho < 1$ where $K_\rho$ is the Luttinger parameter in the charge sector\textsuperscript{3}. At low but finite $T$ the conductance decreases as an unusual power-law $\Delta G \propto -T^{2(4K-1)}$ due to correlated two-particle (2P) backscattering. The edge liquid being helical, the decrease in conductance is a direct measure of the spin-flip rate\textsuperscript{3}.

3. For strong Coulomb interactions $K < 1/4$, 2P
backscattering processes are relevant and the system becomes insulating at $T = 0$. At low but finite $T$, the conductance is restored by tunneling of excitations with fractional charge $e/2$ and we obtain $G(T) \propto T^{2(1/4K - 1)}$.

**Model.**—We model the impurity by a $S = \frac{1}{2}$ local spin coupled by exchange interaction to the 1D HL with Coulomb interactions. The HL having the same number of degrees of freedom as a spinless fermion, a single nonchiral boson $\phi$ is sufficient for its description in the bosonized language [3]. The system is described by the Hamiltonian $H = H_0 + H_x + H_2$ where $H_0$ is the usual Tomonaga-Luttinger Hamiltonian $H_0 = \frac{\pi}{2} \int dx \left[ K \Pi^2 + \frac{1}{4} \left( \partial_x \phi \right)^2 \right]$, with $K$ the Luttinger parameter and $v$ the edge state velocity. The Kondo Hamiltonian $H_K$ has the form

$$H_K = \frac{J_0 a}{2\pi \xi} \left( S_- : e^{-i2\sqrt{\pi}\phi(0)} : + \text{h.c.} \right) - \frac{J_0 a}{\sqrt{\pi}} S_z \Pi(0), \quad (1)$$

where $S_{\pm} = S_x \pm is_y$ and $S_z$ are the spin operators for the impurity localized at $x = 0$. $a$ is the lattice constant of the underlying 2D lattice and corresponds to the size of the impurity (we assume that the impurity occupies a single lattice site). $\xi$, the penetration length of the helical edge states into the bulk, acts as a short-distance cutoff for the 1D continuum theory in the same way that the magnetic length, the penetration length of the chiral edge states, acts as a short-distance cutoff for the chiral Luttinger liquid theory of the quantum Hall edge [10].

In addition to Kondo scattering, 2P backscattering is allowed by time-reversal symmetry [3, 4]. In HgTe QW the wavevector at which the edge dispersion enters the bulk is usually much smaller than $\pi/2a$ such that the uniform 2P backscattering (umklapp) term requiring $4k_F = 2\pi/a$ can be ignored. The impurity potential can however provide a $4k_F$ momentum transfer and we must generally also consider a local impurity-induced 2P backscattering term $H_2 = \frac{\lambda_2}{2\pi \xi^2} \cos 4\sqrt{\pi}\phi(0) \equiv 2\pi \xi^2$: where $\lambda_2$ is the 2P backscattering amplitude.

**Weak coupling regime.**—We first consider the weak coupling regime where $J_1$, $J_2$, and $\lambda_2$ are small parameters. The calculation of the conductance proceeds in two steps. We first perform an explicit perturbative calculation of the conductance to quadratic order in the bare couplings $J_1$ and $\lambda_2$ using the Kubo formula. This result is then extended to include all leading logarithmic terms in the perturbation expansion by means of a weak coupling renormalization group (RG) analysis of the scale-dependent couplings $J_{H} (T)$ and $\lambda_2 (T)$ where the scale is set by the temperature $T$.

The forward scattering term $J_z$ can be removed from the Hamiltonian by a unitary transformation [12] of the form $U = e^{i\lambda S_z \phi(0)}$ with $\lambda = -J_z a / \sqrt{\pi} K \sqrt{\pi}$, at the expense of modifying the scaling dimension of the vertex operator: $e^{i2\sqrt{\pi} \phi}$. The transformed Hamiltonian reads $UHU^\dagger = H_0 + H_2 + \frac{J_1 a}{2\pi \xi} \left( S_- : e^{-i2\sqrt{\pi}\phi(0)} : + \text{h.c.} \right)$, where $\chi = 1 - \nu J_z / 2K$ with $\nu = a / \pi v$ the density of states of the HL. The scaling dimension of $e^{i2\sqrt{\pi} \phi}$ is $K \equiv K^2$.

Using the transformed Hamiltonian we obtain the correction to the conductance $\Delta G(T) = \Delta G_K (T) + \Delta G_2 (T)$ to quadratic order in the couplings $J_1$ and $\lambda_2$ using the Kubo formula, where $\Delta G_K$ is the correction due to spin-flip Kondo scattering [13],

$$\frac{\Delta G_K}{e^2 / h} = \frac{\Gamma(\frac{1}{2})F(\tilde{K})}{\Gamma(\frac{1}{2} + K)} \frac{\pi^2}{2} S(S + 1)(\nu J_z (T))^2, \quad (2)$$

for $\nu J_z (T) = \nu J_z (T / D)^{K-1}$ to $O(J_1)$ and $D = \hbar v / \pi \xi$ is a high-energy cutoff of order the bulk gap. $\Delta G_2$ is the correction due to 2P backscattering,

$$\frac{\Delta G_2}{e^2 / h} = \frac{\Gamma(\frac{1}{2}) \Gamma(4K)}{\Gamma(\frac{1}{2} + 4K)} \frac{a^4}{2\pi^2 K^2} \left( \frac{\lambda_2 (T)}{D} \right)^2, \quad (3)$$

where $\lambda_2 (T) = \lambda_2 (T / D)^{4K-1}$. There are no crossed terms of the form $O(J_1 \lambda_2)$ or $O(J_2 \lambda_2)$ for a $S = \frac{1}{2}$ impurity since the 2P backscattering operator flips two spins but a $S = \frac{1}{2}$ spin can be flipped only once. We however expect that such terms would be generated for impurities with higher spin.

These results can be complemented by a RG analysis. The RG equation for $\lambda_2$ follows by dimensional analysis $d\lambda_2 / d\ell = -(1 - 4K) \lambda_2$ with $\ell = -\ln (D / T)$, so that $\lambda_2$ is relevant for $K < 1/4$ and irrelevant for $K > 1/4$. The renormalized coupling is $\lambda_2 (T) = \lambda_2 (T / D)^{4K-1}$, and second order renormalized perturbation theory $\Delta G_2 (T) \propto -\lambda_2 (T)^2$ simply reproduces the Kubo formula result [3]. Perturbation theory fails for $T \lesssim T_2^* = \sqrt{\alpha / \lambda_2^2}$ where $T_2^* \propto (\lambda_2^2)^{1/(1-4K)}$ is a scale for the crossover from weak to strong 2P backscattering with $\lambda_2^2$ the bare 2P backscattering amplitude.

The one-loop RG equations [3, 14] for the Kondo couplings $J_{H} (T)$, $J_z$ read

$$\frac{dJ_{H}}{d\ell} = (1 - K)J_{H} + \nu J_{H} J_z, \quad \frac{dJ_z}{d\ell} = \nu J_z^2. \quad (4)$$

The family of RG trajectories is indexed by a single scaling invariant $c = (\nu J_z)^2 - (\nu J_z)^2$ where $\nu J_z \equiv \nu J_z + 1 - K$, which is fixed by the couplings at energy scale $D$. In contrast to the spinful case [3], the absence of spin-flip forward scattering in the HL preserves the stability of the ferromagnetic fixed line, as is the case in the usual Kondo problem. The renormalized spin-flip amplitude $J_{H} (T)$ is given in terms of the bare parameters $J_{H}^0, J_z^0$ by

$$\nu J_{H} (T) = \frac{\alpha \nu J_z^0}{\sinh \left( \alpha \nu J_z^0 \ln (T / T_{K}^*) \right)}, \quad (5)$$

such that $\Delta G_K$ is obtained to all orders in perturbation theory in the leading-log approximation by substituting Eq. [5] in Eq. [2]. In Eq. [5], $\alpha = [(J_z^0 / J_{H}^0)^2 - 1]^{1/2}$.
is an anisotropy parameter \( \lambda_2 \) and \( T^*_K \) is the Kondo temperature,

\[
T^*_K = D \exp \left( -\frac{1}{\nu J_0} \frac{\arcsinh \alpha}{\alpha} \right).
\]

In the isotropic case \( \alpha = 0 \), one recovers the usual form \( T^*_K = D e^{-1/\nu J_0} \). In the limit \( 1 - K \gg \nu J_0, \nu J_0^0 \), Coulomb interactions dominate over Kondo physics and we obtain

\[
T^*_K \approx D \left( \frac{\nu J_0^0}{1-K} \right)^{1/(1-K)},
\]

a power-law dependence similar to results previously obtained \( \text{[10]} \) for Kondo impurities in spinful Luttinger liquids. From the scaling exponent we see that \( T^*_K \) corresponds to the scale of the mass gap opened in a spinless Luttinger liquid by a point potential scatterer of strength \( \nu J_0^0 \) and the corresponding crossover is that of weak to strong single-particle backscattering.

In the high temperature limit \( \max\{T^*_Z, T^*_K\} \ll T \lesssim D \), both the Kondo and 2P scattering processes contribute logarithmically to the suppression of the conductance,

\[
-\Delta G(T) \sim \eta + \gamma \ln(D/T)\quad \text{where } \eta, \gamma \text{ are functions of the bare couplings } K, J_0^0, J_0^0, \text{ and } \lambda_2^0.
\]

**Strong coupling regime.**—We now investigate the low temperature regime below the crossover temperatures \( T \ll \min\{T^*_Z, T^*_K\} \). The topological nature of the QSH edge state as a ‘holographic liquid’ living on the boundary of a 2D system \( \text{[9]} \) results in a drastic change of the low-energy effective theory in the vicinity of the strong coupling fixed point as compared to that of a usual 1D quantum wire. As suggested by the perturbative RG analysis, the nature of the \( T = 0 \) fixed point depends on whether \( K \) is greater or lesser than \( 1/4 \).

We first consider the case where \( K > 1/4 \). In this case, 2P backscattering is irrelevant and \( \Delta G_2 \) flows to zero. On the other hand, for antiferromagnetic \( J_z \) the Kondo strong coupling fixed point \( J_{1/2}, J_z \to +\infty \) is reached at \( T = 0 \), with formation of a local Kramers singlet and complete screening of the impurity spin by the edge electrons. As a result, the formation of the Kondo singlet effectively removes the impurity site from the underlying 2D lattice (Fig. 2). In a strictly 1D spinful liquid, this has the effect of cutting the system into two disconnected semi-infinite 1D liquids (Fig. 2b) and transport is blocked at \( T = 0 \) for all \( K > 1/8 \). In contrast, due to its topological nature, the QSH edge state simply follows the new shape of the edge and we expect the unitarity limit \( G = 2e^2/h \) to be restored at \( T = 0 \). For finite \( T \ll T^*_K \), the effective low-energy Hamiltonian contains the leading irrelevant operators in the vicinity of the fixed point. In the case of spinful conduction electrons, the lowest-dimensional operator causing a reduction of the conductance is single-particle backscattering. However, the helical property of the QSH edge states forbids such a term and it is natural to conjecture that the leading irrelevant operator must be the 2P backscattering operator with scaling dimension 4\( K \). We thus expect a correction to the conductance at low temperatures \( T \ll T^*_K \) for \( K > 1/4 \) of the form \( \Delta G \sim -(T/T^*_K)^{2(4K-1)} \).

In particular, in the noninteracting case \( K = 1 \) we predict a \( T^4 \) dependence in marked contrast to both the usual Fermi liquid \( \text{[17]} \) and spinful 1D liquid \( \text{[8]} \) behaviors. This dependence characteristic of a ‘local helical Fermi liquid’ can be understood from a simple phase space argument. The Pauli principle requires the 2P backscattering operator to be defined through a point-splitting procedure \( \text{[6]} \) with the short-distance cutoff \( \xi \), which translates into a derivative coupling in the limit of small \( \xi \),

\[
\psi_R^\dagger(0)\psi_R^\dagger(\xi)\psi_L(0)\psi_L(\xi) \to \xi^2\psi_R^\dagger \partial_x \psi_R^\dagger \psi_L \partial_x \psi_L.
\]

In the absence of derivatives, the four fermion term contributes \( T^2 \) to the inverse lifetime \( \tau_k^{-1} \). The derivatives correspond to four powers of momenta close to the Fermi points in the scattering rate \( \Gamma_{k,k'} \to p' \propto (k-k')^2 (p-p')^2 \), which translates into an additional factor of \( T^4 \). Furthermore, since at temperatures \( T \ll T^*_K \) suppression of the conductance is entirely due to two-electron scattering, we expect the effective charge \( e^* \equiv S/(2|\langle I_R \rangle|) \) obtained from a measurement of the shot noise \( S \) in the backscattering current \( \langle I_R \rangle \) to be \( e^* = 2e \). \( \text{[11, 18]} \)

For \( K < 1/4 \), the \( |\lambda_2| \to \infty \) fixed point is reached at \( T = 0 \) and the system becomes insulating. The field \( \phi(x = 0, \tau) \) is pinned at the minima of the cosine potential \( H_2 \) located at \( \pm (2n+1)\sqrt{\pi}/4 \) for \( \lambda_2 > 0 \) and \( \pm 2n\sqrt{\pi}/4 \) for \( \lambda_2 < 0 \), with \( n \in \mathbb{Z} \). The conductance is restored at finite temperatures by instanton processes corresponding to the tunneling between nearby minima separated by \( \Delta \phi = \sqrt{\pi}/2 \). From the relation \( j_\phi = e\partial_\phi \phi/\sqrt{\pi} \) between electric current \( j_e \) and \( \phi \) field, the

FIG. 2: Strong coupling regime: the Kondo singlet effectively removes one site from the system. a) Luttinger liquid with \( K_\rho < 1 \): a punctured 1D lattice is disconnected, b) QSH edge liquid with \( K > 1/4 \): the edge liquid follows the boundary of the deformed 2D lattice. c) Half-charge tunneling for \( K < 1/4 \) by flips of the Ising order parameter \( m \).
charge pumped by a single instanton (Fig. 2) is obtained as $\Delta Q_{\text{inst}} = \frac{\pi}{\nu} \Delta \phi = \frac{\pi}{2}$. This fractionalized tunneling current can be understood as the Goldstone-Wilczek current \cite{10, 21, 22} for 1D Dirac fermions with a mass term $\delta \mathcal{L} = g \Psi (m_1 + i \gamma^3 m_2) \Psi$ where $g \sim \lambda_2$, and the mass order parameters $m_1 = \cos 2\sqrt{\pi} \phi$, $m_2 = \sin 2\sqrt{\pi} \phi$ change sign during an instanton process with $\Delta \phi = \sqrt{\pi}/2$. The order is Ising-like because the $2P$ backscattering term explicitly breaks the spin $U(1)$ symmetry of the helical liquid $H_0 = \frac{-2i}{\nu} \int dx (K_2 (m_1 + m_2))$ down to $\mathbb{Z}_2$, where $\sigma_z = \rho_+ - \rho_-$ and $\rho = \rho_+ + \rho_-$ are the spin and charge densities, and $\rho_{\pm}$ are the chiral densities for the two members of the Kramers pair.

Fractionalization of the tunneling current is confirmed by a saddle-point evaluation of the path integral for large $\lambda_2$ in the dilute instanton gas approximation, which yields a Coulomb gas representation of the partition function that can be mapped exactly to the boundary sine-Gordon theory

$$S[\theta] = \frac{K}{\beta} \sum_{n=1}^{\infty} [\omega_n ]|\theta(\omega_n)|^2 + \int_0^\beta d\tau \cos \sqrt{\pi} \theta(\tau),$$

where $\theta$ is the instanton fugacity. The RG equation for $t$ follows as $\frac{dt}{dt} = (1 - \frac{1}{\nu^2}) t$ and the conductance $G(T) \propto t(t^2)$ is a power-law $G \propto (T/T_0^* )^{2(\nu/4K-1)}$ for $T \ll T_0^*$, $K < 1/4$. In contrast to the strong coupling regime in a usual Luttinger liquid where $t$ corresponds to a single-particle hopping amplitude \cite{22}, the unusual scaling dimension of the tunneling operator in the present case corresponds to half-charge tunneling. In particular, we calculate the shot noise in the strong coupling regime using the Keldysh approach \cite{22} and find $S = 2e^*|\langle I \rangle|$ where $\langle I \rangle$ is the tunneling current and $e^* = e/2$.

Experimental realization.—We find that the experimental results of Ref. \cite{7} are consistent with our theoretical expressions for the weak coupling regime with a weak Luttinger parameter $K \simeq 1$, but the small number of available data points does not allow for a reliable determination of the model parameters. The temperature dependence of the conductance being exponentially sensitive to $K$, our predictions can be best verified in QW with stronger interaction effects. Due to reduced screening of the Coulomb interaction \cite{22}, we expect to see a steeper decrease of conductance with decreasing temperature in HgTe samples with only a backgate.

Because of lower Fermi velocities $v_F$, we expect even stronger interaction effects to occur in InAs/GaSb/AlSb type-II QW \cite{22} which have been recently predicted to achieve the QSH effect \cite{21}. For QW widths $w_{\text{InAs}} = w_{\text{GaSb}} = 10 \text{ nm}$ in the inverted regime \cite{21}, and considering only screening from the front gate closest to the QW layer, from a $\mathbf{k} \cdot \mathbf{p}$ calculation of material parameters we obtain $K \simeq 0.2 < 1/4$, making the insulating phase observable at low temperatures. Although the backgate will cause additional screening, $v_F$ can be further decreased by adding a thin AlSb barrier layer between the InAs and GaSb QW layers. The Fermi velocity is controlled by the overlap between electron and hole subband wavefunctions \cite{3} which are localized in different QW layers in the type-II configuration \cite{22}, and an additional barrier layer will decrease this overlap. A lower $v_F$ also translates into higher Kondo temperatures since $\nu J \propto 1/v_F^2$, where one power of $v_F$ comes from the matrix element of the localized impurity potential between edge states, and one power comes from the density of states $\nu$. Since $T_K^*$ depends on $\nu J$ exponentially, we expect experimentally accessible Kondo temperatures in type-II QW.

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[1] For a review, see M. König et al., J. Phys. Soc. Jpn 77, 031007 (2008).
[2] M. König et al., Science 318, 766 (2007).
[3] B. A. Bernevig, T. L. Hughes, and S. C. Zhang, Science 314, 1757 (2006).
[4] C. L. Kane and E. J. Mele, Phys. Rev. Lett. 95, 226801 (2005).
[5] C. Wu, B. A. Bernevig, and S. C. Zhang, Phys. Rev. Lett. 96, 106401 (2006).
[6] C. Xu and J. E. Moore, Phys. Rev. B 73, 045322 (2006).
[7] M. König, Ph.D. thesis, University of Würzburg (2007).
[8] A. Furusaki and N. Nagaosa, Phys. Rev. Lett. 72, 892 (1994).
[9] M. Garst, P. Wölfle, L. Borda, J. von Delft, and L. Glazman, Phys. Rev. B 72, 205125 (2005).
[10] X.-G. Wen, Phys. Rev. B 44, 5708 (1991).
[11] D. Meidan and Y. Oreg, Phys. Rev. B 72, 121312(R) (2005).
[12] V. J. Emery and S. Kivelson, Phys. Rev. B 46, 10812 (1992).
[13] This result is valid in the high-temperature regime $hv/L \ll T < D$ for Fermi liquid leads where $L$ is the length of the QSH region (see Ref. \cite{27}).
[14] A. Schiller and K. Ingersent, Phys. Rev. B 51, 4676 (1995).
One can show that the Kondo model derived from the Anderson model for a single level coupled to the HL is isotropic due to time-reversal symmetry with $J_0 = J_z = (|t|^2 + |u|^2) \left( \frac{1}{\epsilon_F + \epsilon_d} + \frac{1}{\epsilon_d + U - \epsilon_F} \right)$ where $\epsilon_F$ is the Fermi energy, $\epsilon_d$ is the impurity level with on-site Coulomb repulsion $U$, and $u$ and $t$ are the spin-flip and non-spin-flip hopping amplitudes, respectively. Coulomb interactions ($K \neq 1$) may however induce an effective anisotropy ($\alpha \neq 0$) even if the original Kondo model is isotropic.

D.-H. Lee and J. Toner, Phys. Rev. Lett. 69, 3378 (1992).

P. Nozières, J. Low. Temp. Phys. 17, 31 (1974).

In the weak coupling or high temperature regime $T \gg T^*_{K}$, both the Kondo ($\epsilon^* = \epsilon$) and 2P backscattering ($\epsilon^* = 2\epsilon$) contributions to the effective carrier charge are present, such that we expect a non-universal value for the Fano factor.

J. Goldstone and F. Wilczek, Phys. Rev. Lett. 47, 986 (1981).

X. L. Qi, T. L. Hughes, and S. C. Zhang, Nature Phys. 4, 273 (2008).

K can be estimated by $K = [1 + \alpha \ln(d/\ell)]^{-1/2}$ where $\alpha = \frac{2e^2}{\pi^2 \hbar v_F}$ and $\epsilon$ is the bulk dielectric constant. The distance $d$ from the QW layer to a nearby metallic gate acts as a screening length for the Coulomb potential, and $\ell$ is a microscopic length scale $\ell = \max\{\xi, w\}$ which acts as a short-distance cutoff for the Coulomb potential, where $w$ is the thickness of the QW layer.

L. J. Cooper, N. K. Patel, V. Drouot, E. H. Linfield, D. A. Ritchie, and M. Pepper, Phys. Rev. B 57, 11915 (1998).

C. Liu, T. L. Hughes, X. L. Qi, K. Wang, and S. C. Zhang, Phys. Rev. Lett. 100, 236601 (2008).

D. L. Maslov, Phys. Rev. B 52, R14368 (1995).