Analytical determination of the stop band tuning of photonic crystals infiltrated with liquid crystals

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Abstract

We demonstrate that the tuning of the optical properties of a photonic crystal infiltrated with liquid crystal can be calculated using the Von-Laue diffraction condition. We present a simple formula to predict the shift of the stop band for all the diffraction orders using an effective index of the composite structure. We consider that our formula is useful to determine in a simple manner the shift of the optical properties of tunable photonic crystals. We compare the accuracy of our method with calculations obtained with the Plane Wave Method.

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I. INTRODUCTION

Photonic crystals (PC) are a new kind of materials which facilitate the control of the light. PC exhibits Photonic Band Gaps (PBG) that forbids the radiation propagation in a specific range of frequencies. In the past ten years has been developed an intensive effort to study and micro-fabricate PBG materials in one, two or three dimensions.

However, for many applications it is advantageous to obtain some degree of tuning of the PBG. This can be made by changing the refractive index of the constitutive materials by means of an external parameter, such the temperature or voltage. One of the most promising routes to achieve such tuning is the use of liquid crystals (LC). Recently it has been reported several works of PC infilled with LC with robust ranges of tuning in the optical regime. In most of the cases, the theoretical analysis is performed via the calculation of the band structure or light reflection (transmission) using plane wave expansion or Transfer Matrix Methods (TMM). In this work we illustrate the possibility of determining the degree of tuning in a simple manner by using the Von-Laue condition (VLC). The VLC takes account of the variation of the constitutive parameters of the unitary cell introducing an effective refractive index that averages the value of the constitutive parameters. This formula is useful to determine in an easy way the tuning of the recently reported experimental configurations. The performance and limits of our approximation is compared with exact calculations obtained with the Plane Wave Method (PWM).

![Unit cell of an one dimensional photonic crystal composed by liquid crystal and dielectric.](image)

The refractive indices are $n_{LC}$ and $n_{dïeï}$, respectively. The width of each slab is $d_{LC}$ and $d_{dïeï}$. The width of the unit cell is $d = d_{LC} + d_{dïeï}$.
Reduced frequency \( (d/\lambda) \)

Wave vector \( (\pi/d) \)

\[
\lambda = 60 \, \mu m \\
\lambda = 30 \, \mu m \\
\lambda = 20 \, \mu m \\
\lambda = 15 \, \mu m \\
\lambda = 12 \, \mu m \\
\lambda = 10 \, \mu m \\
\lambda = 8.5 \, \mu m \\
\lambda = 7.5 \, \mu m \\
\lambda = 6.6 \, \mu m \\
\lambda = 6.0 \, \mu m
\]

FIG. 2: Photonic band structure of a PC composed of a dielectric \( n_{\text{die}} = 3.42 \) and LC \( n_{\text{LC}} = 1.49 \). The filling fraction is \( f = 0.4 \).

II. THEORY

We start our analysis by considering the Von-Laue diffraction condition, \[15\]

\[
k^2 = (k + G)^2. \tag{1}
\]

\( k \) and \( G \) are the wave vector and reciprocal lattice vector, respectively. If the vectors \( k \) and \( G \) are parallels, it can be written

\[
2|k| = |G|, \tag{2}
\]

where the absolute value of the reciprocal lattice vector \(|G|\) is defined as
\[ |G| = \frac{2\pi}{d} m, \quad (3) \]

\( d \) and \( m \) are the lattice period and the diffraction order, respectively. The wave vector \( |k| \) is defined as

\[ |k| = n_{eff} \frac{\omega}{c}, \quad (4) \]

where \( n_{eff} \) is the effective index of the composite medium. Using eqs. (3), (4) and (2) the VLC is

\[ \frac{d}{\lambda_m} = \frac{m}{2n_{eff}}, \quad (5) \]

where we have introduced the definition of reduced frequency \( \omega d/2\pi c = d/\lambda_m \), with \( \lambda_m \) as the wavelength for the diffraction order \( m \). Fig. 1 shows a unit cell composed of liquid crystals and dielectric, with refractive indices \( n_{LC} \) and \( n_{dielectric} \), respectively. The width of each layer is \( d_{LC} \) and \( d_{dielectric} \). The width of the unit cell is \( d = d_{LC} + d_{dielectric} \). We define the filling fraction as the space filled by the dielectric material over the total space in the unit cell, \( f = d_{dielectric}/d \). The effective index \( n_{eff} \) is taken as a simple average in the unit cell in the form

\[ n_{eff} = fn_{LC} + (1 - f)n_{dielectric}, \quad (6) \]

The tuning of the diffraction condition can be written as

\[ \frac{d}{\lambda_m} = \frac{m}{2[f n_{LC} + (1 - f)n_{dielectric}].} \quad (7) \]

III. NUMERICAL RESULTS

To verify our analytical formula, we have performed a detailed calculation of a particular PC. We have chosen the experimental parameters recently reported by Tolmachev et al. in Ref. [8]. We consider the PC as a stack of alternating slabs of dielectric and LC with refractive index \( n_{dielectric} = 3.42 \) and \( n_{LC} = 1.49 \), respectively. The filling fraction is \( f = 0.4 \). In Fig. 1 we show the Photonic Band Structure (PBS) calculated with the PWM. [14] We observe the existence of four band gaps. We have plot the PBS in the usual way with wave vector \( (\pi/d) \) and reduced frequency \( (d/\lambda) \) in the abscissa and ordinate, respectively.
FIG. 3: Evolution of the Photonic Band Gap as a function of the variation of the liquid crystal refractive index, $n_{LC}$. In panels (a) - (d) we present the variation for the first four photonic band gaps. Additionally, in the right side of the ordinate axis, we have plot a wavelength scale. This has be done because most of the experimental results are usually presented in this form in LC experiments. In this case, we have illustrated the wavelength scale for a period of $d = 6\mu m$.

We consider a change in $n_{LC}$ as the result of the variation from the homeotropic ($n_{LC} = 1.49$) to the pseudoisotropic ($n_{LC} = \sqrt{(2n_o^2 + n_e^2)/3} = 1.56$) state under the influence of an external voltage. In Fig. 3 we present the shift of the PBG as the variation of $n_{LC}$. In panels (a) - (d) we present the evolution of the first four PBG, which are illustrated in the gray zones. Inside each gray zone, we have plotted the evolution of the VLC eq. (7). We observe that for the first PBG in panel (a), the VLC lies in the middle. For the rest of the cases, the VLC does not defines the center of the PBG. However, the interesting fact is that
the shift of the PBG’s are predicted. In order to define the stop band shift, we introduce the formula

\[ \Delta \lambda_m = \lambda_m(n_{LC} = 1.56) - \lambda_m(n_{LC} = 1.49) \]  

(8)

where \( \lambda_m(n_{LC} = 1.56) \) and \( \lambda_m(n_{LC} = 1.49) \) are the diffraction for the order \( m \). In table 1 we present the numerical values obtained with the eqs. (7) and (8).

| \( m \) | \( \lambda_m(n_{LC} = 1.49) \) | \( \lambda_m(n_{LC} = 1.56) \) | \( \Delta \lambda_m \) |
|-------|-----------------|-----------------|----------------|
| 1     | 27.144\( \mu m \) | 27.648\( \mu m \) | 0.50\( \mu m \) |
| 2     | 13.572\( \mu m \) | 13.824\( \mu m \) | 0.25\( \mu m \) |
| 3     | 9.048\( \mu m \) | 9.216\( \mu m \) | 0.16\( \mu m \) |
| 4     | 6.786\( \mu m \) | 6.912\( \mu m \) | 0.12\( \mu m \) |

We observe that the greatest shift is given for the diffraction order \( m = 1 \). For the following diffraction orders, the shift decreases. It is important to note that for the diffraction order \( m = 3 \), we obtain the experimental value obtained in ref. (9).

IV. CONCLUSION

In conclusion, we have numerically demonstrated the advantage to use the VLC to predict the tuning of the stop band in PC structures. Previously theoretical reports on tuning were based on calculation on PWE or TMM. We have shown that our formulation agree well with experimental data reported in Ref. (9) We consider that our formula can be useful to determine in an easy way the tuning of the stop band in tunable PC.

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