Heat Transfer of Thin Film Flow Over an Unsteady Stretching Sheet with Dynamic Viscosity

Ali Rehman¹, Zabidin Salleh¹*, Taza Gul²

¹ Department of Mathematics, Faculty of Ocean Engineering Technology and Informatics, Universiti Malaysia Terengganu, 21030 Kuala Nerus, Terengganu, Malaysia
² Department of Mathematics, City University of Science and Information Technology, Peshawar P/C, 25000, Pakistan

ARTICLE INFO

ABSTRACT

This research paper explains the impact of dynamics viscosity of water base GO-EG/GO-W nanofluid over a stretching sheet. The impact of different parameter for velocity and temperature are displayed and discussed. The similarity transformation is used to convert the partial differential equation to nonlinear ordinary differential equation. The solution of the problem is obtained by using the optimal homotopy analysis method (OHAM). The BVPh 2.0 package function of Mathematica is used to obtain the numerical results. The result of important parameter such as magnetic parameter, Prandtl number, Eckert number, dynamic viscosity, nanoparticles volume fraction and unsteady parameter for both velocity and temperature profiles are plotted and discussed. The BVPh 2.0 package is used to obtain the convergences of the problem up to 25 iteration. The skin friction coefficient and Nusselt number is explained in table form.

Keywords:
Water based GO-EG/GO-W nanofluids; Optimal homotopy analysis method; Stretching sheet; Dynamic viscosity

1. Introduction

The flow in which one dimensional is lesser than the other/s is usually named the flow of the thin film. This implies and observed practically inside many industries. In recent years the requirement for emerging higher thermal management and a more effective heat transporting system has frequently increase. The progressing of strong energy storage technologies is the core issue of the engineer and scientist. The aforesaid difficulties produce due to high rising demands of heating/cooling and other needs in manufacturing processes. It is the focal challenge for a wide research community to boost the heat exchange performance of common pure fluids such as water, various oils, and ethylene glycol, etc., which has weak thermal performances. It is a well-known fact that nano liquids can reduce the difficulties associated with the low thermal performance of traditional fluids and also fulfill the growing demand for heating/cooling and other needs in engineering processes. The development of heat transport via nanofluid has involve several scientists
due to a lot of uses in various sectors such as distillation and separation of bio-molecules, biosensors, atomic system cooling, and manufacture of glass fiber.

The homogeneous mixture of nanosized particle and base fluid is called nanofluid. The ordinary base fluids are water, oil and ethylene. It is known that the heat transfer rate of these nanofluids is very poor. To increase the thermal conductivity and heat transfer rate of the ordinary base fluid, we combine nanoparticles in the ordinary base fluid. Due to this combination of the ordinary base fluid and nanoparticles, the thermal conductivity and heat transfer of the ordinary base fluid increasing, but this method is very expensive and time vast. Remember that, because of these factors, many methods are introduced to increase the thermal conductivity of the ordinary fluid. One of these methods is the suspension of nanoparticle with diameter less than 50nm in the ordinary fluid.

The word of nanofluid was defined for the first time by Choi. Many researchers presented measurement for the thermal conductivity with different nanoparticle volume fraction material and dimension, and from all of these results, they observe the thermal conductivity of the nanofluid is more than that of the ordinary fluid. Haq et al., [1] used finite element method, they investigate the temperature will be enhance if we heated the domain partially. Prasannakumara et al., [2] do the comparison of linear and nonlinear stretching sheet and show that the Nusselt and Sherwood number is greater for the nonlinear. Rudraswamy et al., [3] investigated three dimensional nanofluid by using moving plate in the presence of magnetic field. Soomro et al., [4] used moving sheet to discuss the movement of nanoparticle, they use the passive method. Usman et al., [5] do the compression of skin fraction for shrinking channels and stretching channel, and show that the skin fraction for shrinking channel is greater than that of stretching for this compression by using analytical method. Takhar et al., [6] used stretching surface to discuss transient magnetohydrodynamic flow. Kumar et al., [7] study Marangoni convection effect of nanofluid treatment in the existence of heat source sink. Sheikholeslami and Rokni [8] discussed the effect of thermal radiation on nanofluid by using porous channel. Sheikholeslami and Shehzad [9] discussed the convective heat transfer of nanofluid in the presence of magnetic field. They investigate the electric field influence on nanofluid by using numerical method.

Laminar flow is very important area of research in the field of viscous fluid. There are mainly two types of laminar flows problems such as Blasius and Wedge flow problems. When the fluid velocity is constant and the free stream appear parallel to the surface, then the situation is called Blasius problem. Similarly, the situation in which surface form an angle with the free stream and velocity of the fluid is not constant, then such situation is known as Wedge problem. In current period, most of the researcher take interest in Wedge flow problem because due to its wide use in engineering and industries. Falkner and Skan [10] are the first to work in this area. They investigate wedge problem by using the 2D flow of incompressible and viscous fluid. Yih [11] investigated the problem of boundary film flow of forced convection through a wedge by using the condition of suction/injection. Watanabe [12] and Rajagopal et al., [13] examined different flows of wedge problems by using second grade fluid. Na [14] introduced a group of suitable transformation to transform the famous Falkner equation into two initial value problem and they solved the transformed problem by applying forward integration scheme. Also, Asaithambi [15] applied the finite difference scheme to explain the solution of Falkner and Skan equation.

Rehman et al., [16] used stretching cylinder to discuss the Marangoni convection of water-based CNT nanofluid. Rehman et al., [17] used stretching sheet to discuss viscous dissipation of thin film unsteady nanofluid. Due to the high oxidize structure, graphene is used as graphene oxide (GO). Graphene is a single layer and graphene oxide is two-dimension materials. Sir second Baronent was the first one introduced graphene oxide in 1859. Now graphene oxide is produced by modified hummeros method. The thermal conductivity of the metallic and nonmetallic nanofluid is very higher
than the base fluid. Due to these properties, ethylene glycol (EG) can be used as a cooling fluid and anti-freezing agent. The rotating disk model of nanofluid is the active research area of GO. Balandin et al., [18] discussed the improvement of thermal efficiency of the base fluid. For this, they used single layer graphene as a solvent. Wei et al., [19] discussed the stable diffusion of graphene oxide in ethylene glycol for the improvement of heat transfer. Gul and Firdous [20] used rotating disk model to discuss stable dispersion of graphene nanoparticle and graphene oxide water GO-W nanofluid. Recently, Gul et al., [21] discussed Marangoni convection by using water and ethylene glycol-based graphene oxide as a nanofluid.

Maleque [24] discussed the effects of binary on MHD Boundary layer heat and mass transfer flow with viscous dissipation and heat generation. Gireesha et al., [25] studied the exploration of activation energy and binary chemical reaction effects on nano Casson fluid flow with thermal and exponential space-based heat source. Prasnnakumara et al., [26] discussed the effects of chemical reaction and nonlinear thermal radiation on Williamson nanofluid slip flow over a stretching sheet embedded in a porous medium. Khodabandeh et al., [27] studied the energy saving with using of elliptic pillows in turbulent flow of two-phase water-silver nanofluid in a spiral heat exchanger. Rehman et al., [28] discussed the effect of dynamic viscosity of GO-W and GO-EG nanofluid by using stretching cylinder. Rehman et al., [29] discussed the effect of dynamic viscosity of CNTs nanofluid by using stretching sheet. Rehman et al., [30] used stretching sheet to discuss the effects of Marangoni convection, viscous dissipation and magnetic field of thin film unsteady nanofluid. Rehman et al., [31] used stretching sheet to discuss the effects of viscous dissipation and magnetic field of thin film unsteady nanofluid. Rehman et al., [32] used stretching sheet and discuss analytical study of unsteady squeezed flow of water base CNTs nanofluid with magnetic field and variable thermal conductivity. Raza [33] discussed thermal radiation and slip effects on magnetohydrodynamic (MHD) stagnation point flow of Casson fluid over a convective stretching sheet. Raza et al., [34] discussed heat transfer study of convective fin with temperature-dependent internal heat generation by hybrid block method. Alkasassbeh et al., [35] discussed the effects of Stefan blowing and slip conditions on unsteady MHD Casson nanofluid flow over an unsteady shrinking sheet. Lund et al., [36] discussed the magnetohydrodynamic flow of Cu-Fe3O4/H2O hybrid nanofluid with effect of viscous dissipation. Lund et al., [37] discussed the triple solutions of micropolar nanofluid in the presence of radiation over an exponentially preamble shrinking surface.

Inspired by the above analysis, the objective of this research paper is to provide an analytical method to explain the problem of water base GO-W and GO-EG nanofluid over an unsteady stretching sheet with dynamic viscosity. The model system of PDEs is converted to nonlinear ODEs by using similarity transformation. The analytical method OHAM is used to find analytical result of the model of nonlinear problem. Liao [23] used this method for the first time to solve nonlinear differential equation. The BVPh 2.0 package is used to obtain the convergences of the problem up to 25 iterations. The impact of different physical factors on the flow and heat transport features are discussed and presented in the graphical form. For the first time, this model is solved for dynamics viscosity analytically.

2. Mathematical Formulation

The unsteady flow of graphene oxide water and ethylene glycol (GO-W/GO-EG) based nanofluids with dynamic viscosity over a flexible sheet are considered. The unsteady flexible of the sheet in $x$ direction is $U_{\infty} = \frac{bx}{1-\gamma t}$ where $t$ is time and $b, \gamma$ is constant. The magnetic field is defined as $B(t) =$
\[ B_0 (1 - \gamma t)^{-\frac{1}{2}} \] where \( T_{\infty}(x, t) = T_0 - T_r \left( \frac{bx^2}{2v_f} \right) (1 - \gamma t)^{\frac{3}{2}} \) is the surface temperature vary with the distance \( x \) from the slit. The geometry of the proposed problem is sketched as follows (Figure 1).

The continuity, momentum and thermal boundary layer equations are stated, see [22]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_{nf} B^2(t)}{\rho_{nf}} u \quad (2)
\]

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} + \frac{\mu_{nf}}{(\rho C_p)_{nf}} \left( \frac{\partial u}{\partial y} \right)^2 \quad (3)
\]

where \( u \) and \( v \) are the velocity component along \( x \) and \( y \) direction, respectively. The thermal diffusivity of the base fluid is \( \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}} \).

The associated boundary condition is

\[
u = 0, T = T_\infty \text{ at } y = 0 \quad (4)
\]

\[
\frac{\partial u}{\partial y} = 0, v = \frac{\partial h}{\partial t} \text{ at } y = h(t), \quad (5)
\]

where \( h(t) \) is the film thickness. The similarity transformation is defined as

\[
\eta = \left( \frac{b}{\nu(1-\gamma t)} \right)^{\frac{1}{2}} y, \psi(x, y, t) = \left( \frac{\nu h}{(1-\gamma t)} \right)^{\frac{1}{2}} xf(\eta), \quad (6)
\]

\[
T(x, y, t) = T_0 - T_r \left( \frac{bx^2}{2v_f} \right) (1 - \gamma t)^{\frac{3}{2}} \beta(\eta).
\]

The stream function \( \psi(x, y) \) is defined as \( u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \) where \( \beta \) is the dimensionless film thickness and is defined as
\[ \beta = \left( \frac{b}{v(1-\gamma t)} \right)^{1/2} h(t). \]  

From Eq. (7),

\[ \frac{dh}{dt} = -\frac{\gamma \beta}{2} \left( \frac{v}{b} \right)^{1/2} (1 - \gamma t)^{-1/2}. \]  

Putting Eq. (8) into Eq. (2)-(5), we obtain the following coupled system of ordinary differential equation.

\[ \frac{d^3 f}{d\eta^3} + \frac{1}{(1+a\theta)(1-\phi)^2} \left( 1 - \phi + \phi \frac{\rho c_p}{\rho_f} \right) \left[ f \frac{d^2 f}{d\eta^2} - \left( \frac{df}{d\eta} \right)^2 - \frac{s}{2} \left( \frac{df}{d\eta} + \frac{\eta d^2 f}{d\eta^2} \right) \right] - M(1 - \phi)^{2.5} \frac{df}{d\eta} = 0 \]  

\[ \frac{k_{nf}}{k_f} \frac{(1 + \alpha \theta)}{d\eta^2} + Pr \left[ (1 + \alpha \theta) (1 - \phi) + \phi \frac{(\rho c_p)}{\rho_f} \right] \left[ f \frac{d\theta}{d\eta} - 2 \theta \frac{df}{d\eta} - \frac{s}{2} \left( 3 \theta + \eta \frac{d\theta}{d\eta} \right) \right] + Ec \left( \frac{d^2 f}{d\eta^2} \right)^2 = 0. \]  

The dimensionless parameters, \( S, M, Pr, \) and \( Ec, \) are defined as

\[ S = \frac{\gamma}{b}, M = \frac{\sigma_f B_0^2}{\rho B}, Pr = \frac{\nu_{nf}}{\alpha_f}, Ec = \frac{U_0}{C_p(T_{\infty} - T_0)} \]  

The transform boundary condition is

\[ f(0) = 0, \frac{df}{d\eta}(0) = 1, f(\beta) = \frac{S \beta}{2}, \frac{d^2 f}{d\eta^2}(\beta) = 0, \theta(0) = 1, \frac{d\theta}{d\eta}(\beta) = 0. \]  

The physical quantities of interest are the skin friction coefficient \( C_{nf}, \) the local Nusselt number \( Nu_x, \) and are defined as

\[ C_{nf} = \frac{\tau_{aw}}{2\rho U_{aw}^2}, Nu = \frac{q_{aw}}{k(T_{aw} - T_0)} x. \]  

Using the initial guessed values and auxiliary linear operators from Eq. (8) and Eq. (9), we obtain

\[ f_0(\eta) = \frac{3(2-S)}{2\beta^2} \left[ \frac{x^3}{6} - \frac{\beta x^2}{2} \right] + x, \quad \theta_0(\eta) = 1, \]  

\[ L_f = \frac{d^3 f}{d\eta^3}, \quad L_\theta = \frac{d^2 \theta}{d\eta^2}. \]  

### 3. Method of Solution

The analytical method namely OHAM is used to find the approximate analytical solution of the nonlinear ordinary differential equation. The BVPh 2.0 package function of Mathematica is used to obtain the numerical results of important parameters. The given Eq. (9) and Eq. (10) are solved analytically by analytical method, i.e., optimal homotopy asymptotic method (OHAM) which is described below.
\[ L(u(x)) + N(u(x)) + g(x) = 0, B(u(x)) \]  
\hspace{1cm} (16)

where \( L \) is linear operator, \( x \) is independent variable, \( g(x) \) is the unknown function, \( N \) is the nonlinear operator and \( B(u) \) is a boundary operator. By using this method, we first find a family of equations

\[ H(\phi(x), p) = (1 - p)[L(\phi(x, p)) + g(x)] - H(p)[L(\phi(x, p)) + g(x) + N(\phi(x, p))] = 0 \]

\[ B(\phi(x, p)) = 0 \]  
\hspace{1cm} (17)

where \( p \) is an embedding parameter and lies in \([0, 1]\), \( H(p) \) is nonzero auxiliary function for \( p \neq 0 \) and \( H(0) = 0 \) and \( \phi(x, p) \) is an unknown function.

When \( p = 0 \), Eq. (17) become as

\[ L(\phi(x, p)) + g(x) = 0. \]  
\hspace{1cm} (18)

Let the solution of Eq. (18) be

\[ \nu_0(x). \]  
\hspace{1cm} (19)

When \( p = 1 \), Eq. (17) become as

\[ L(\phi(x, p)) + g(x) + N(\phi(x, p)) = 0, \]  
\hspace{1cm} (20)

which have a solution, let the solution is

\[ \nu(x). \]  
\hspace{1cm} (21)

Thus, by increasing \( p \) from 0 to 1, the solution change from \( \nu_0(x) \) to \( \nu(x) \). Choose the supplementary function as

\[ H(P) = PC_1 + P^2C_2 + P^3C_3 + \ldots \]  
\hspace{1cm} (22)

where \( C_1, C_2, C_3, \ldots \) are constants to be determined. Expanding \( \phi(x, p) \) in a series with respect to \( p \), we have

\[ \phi(x, p, C_i) = \nu_0(x) + \sum_{k=1}^{\infty} \nu_k(x, C_i)p^k \text{ for } i = 1, 2, 3, \ldots \]  
\hspace{1cm} (23)

Put Eq. (23) into Eq. (22) and comparing the coefficient of different power of \( p \), we get differential equations with boundary condition. By solving this equation, we find a series solution in the form of functions \( \nu_0(x), \nu_1(x, C_1), \nu_2(x, C_2) \) and the solution of Eq. (22) can be written as

\[ \nu(x, C_i) = \nu_0(x) + \sum_{k=1}^{\infty} \nu_k(x, C_i). \]  
\hspace{1cm} (24)
The residual is as follow

\[ R(x, C_i) = L(\nu(x, C_i)) + N(\nu(x, C_i)) + g(x). \]  

(25)

In Eq. (25), if \( R(x, C_i) = 0 \), then Eq. (24) have exact solution but for nonlinear differential equation, it is not possible. So, we try to minimize the \( R(x, C_i) \) to get best approximate analytical solution of Eq. (17).

4. Result and Discussion

The main objective of this section is to study the effect of various model factors like \( \phi, M, \alpha, Pr, Ec, S, \beta \) (nanoparticle volume fraction, magnetic field, dynamic viscosity, Prandtl number, Eckert number, unsteady parameter and dimensionless film thickness parameter on velocity and temperature distribution. In Table 1 and Table 2, the numeric results illustrate the impacts of different model factors on the skin friction coefficient and Nusselt number of both GO-W and GO-EG. The effect of different parameters on local skin friction coefficient is shown in Table 1. Table 1 depicts that skin friction coefficient enhances in the case of GO-W and reduce in the case of GO-EG for the increasing values of both unsteady parameter \( S \) and magnetic field \( M \). Whereas, Table 3 shows the Nusselt number coefficient decline in the case of both GO-W and GO-EG for rising magnitude of Prandtl number and Eckert number. The convergence for the both GO-W and GO-EG nanofluids has been obtained up to the 25th iteration in Table 4 and Table 5 one by one. Table 4 and Table 5 show that by increasing the number of iterations, reduces the order of residual error and strong convergence attained. Figure 1 is the physical geometry of the problem. Figure 2 shows the effect of the dynamic viscosity against the velocity field. The relation between \( \frac{df}{d\eta} (\eta) \) velocity field and dynamic viscosity \( \alpha \) is inverse relation. The increasing values of the dynamic viscosity \( \alpha \) decreases the \( \frac{df}{d\eta} (\eta) \) velocity field as displayed in Figure 2. Physically, by increasing dynamic viscosity, it will produce resistant forces due to which fluid particles does not move easily, so velocity field is decreasing by increasing the dynamic viscosity \( \alpha \). Relatively, this effect is strong in the GO-W. Figure 3 shows the impact of the nanoparticles volume fraction versus the velocity field. The relation between \( \frac{df}{d\eta} (\eta) \) and \( \phi \) is inverse relation. For the growing magnitude of \( \phi \), decreases the \( \frac{df}{d\eta} (\eta) \) as displayed in Figure 3, because the large value of \( \phi \) produce viscous forces due to which velocity field is decreasing. Relatively this effect is strong in the GO-W as equated to GO-EG.

| Table 1 |
| --- |
| The thermo physical properties |
| Physical properties | Thermal conduct \( k \) (\( W/\text{mk} \)) | Specific heat \( c_p \) (\( J\text{kg}^{-1}K^{-1} \)) | Density \( \rho \) (\( \text{kgm}^{-3} \)) |
| Base fluid | 0.492 | 3594 | 1053 |
| Solid particles | GO-W | 2400 | 1600 | 3600 |
| | GO-EG | 3000 | 2600 | 1500 |
### Table 2
Effect of unsteady parameter $S$, magnetic field $M$ on skin friction coefficient

| $M$ | $S$ | $\frac{d^2 f}{d\eta^2}(0)$ | $\varphi = 0.01$ | $\frac{d^2 f}{d\eta^2}(0)$ | $\varphi = 0.02$ | $\frac{d^2 f}{d\eta^2}(0)$ | $\varphi = 0.01$ | $\frac{d^2 f}{d\eta^2}(0)$ | $\varphi = 0.2$ |
|-----|-----|-----------------------------|------------------|-----------------------------|------------------|-----------------------------|------------------|-----------------------------|------------------|
|     | 0.1 | GO-W                        | 30.3162          | 30.3335                     | 28.7696          | 30.3535                     | GO-W             | 30.3412                     | 28.3040          |
|     | 0.2 | GO-W                        | 30.5140          | 30.3635                     | 28.2918          | 30.3293                     | GO-W             | 30.5861                     | 25.2830          |
|     | 0.3 | GO-W                        | 30.5991          | 30.3735                     | 27.2629          | 30.3129                     | GO-W             | 30.7777                     | 25.1609          |
|     | 0.1 | GO-EG                       | 30.6162          | 30.3735                     | 27.2629          | 30.3129                     | GO-EG            | 30.6883                     | 25.2830          |
|     | 0.2 | GO-EG                       | 30.6883          | 30.5861                     | 25.2830          | 30.264                      | GO-EG            | 30.7430                     | 25.1609          |
|     | 0.3 | GO-EG                       | 30.7430          | 30.7777                     | 25.1609          | 28.7778                     | GO-EG            | 30.7777                     | 25.1609          |

### Table 3
Effect of Prandtl number $Pr$ and Eckert number $Ec$ on Nusselt number coefficient

| $r$ | $Ec$ | $\frac{d^2 f}{d\eta^2}(0)$ | $\varphi = 0.01$ | $\frac{d^2 f}{d\eta^2}(0)$ | $\varphi = 0.02$ | $\frac{d^2 f}{d\eta^2}(0)$ | $\varphi = 0.01$ | $\frac{d^2 f}{d\eta^2}(0)$ | $\varphi = 0.2$ |
|-----|-----|-----------------------------|------------------|-----------------------------|------------------|-----------------------------|------------------|-----------------------------|------------------|
|     | 0.1 | GO-W                        | 1.1433           | 0.4409                      | 1.437            | 0.441                       | GO-W             | 1.1143                      | 0.4371           |
|     | 0.2 | GO-W                        | 1.1245           | 0.4371                      | 1.224            | 0.257                       | GO-W             | 1.1011                      | 0.4369           |
|     | 0.5 | GO-W                        | 1.1143           | 0.4371                      | 1.131            | 0.231                       | GO-W             | 1.0976                      | 0.4236           |
|     | 0.6 | GO-W                        | 1.1011           | 0.4369                      | 1.121            | 0.229                       | GO-W             | 0.5870                      | 0.4211           |
|     | 0.3 | GO-W                        | 1.0976           | 0.4236                      | 0.8700           | 0.2223                      | GO-W             | 0.5870                      | 0.4211           |
|     | 0.7 | GO-W                        | 1.0976           | 0.4236                      | 0.8700           | 0.2223                      | GO-W             | 0.5870                      | 0.4211           |

### Table 4
Individual averaged squared residual errors for GO-W when $Pr = 6.5, Ec = 0.5, M = 0.1, S = 0.4, = 0.1, \beta = 0.3, \phi = 0.1$

| $m$ | $\varepsilon_m^f$ GO-W | $\varepsilon_m^\theta$ GO-W |
|-----|-------------------------|-----------------------------|
| 5   | 2.1321 x 10^{-2}        | 1.9911 x 10^{-1}            |
| 10  | 2.5721 x 10^{-4}        | 1.2211 x 10^{-2}            |
| 15  | 3.15721 x 10^{-6}       | 2.2211 x 10^{-4}            |
| 20  | 3.22721 x 10^{-6}       | 3.2211 x 10^{-5}            |
| 25  | 3.9911 x 10^{-7}        | 4.2211 x 10^{-6}            |

### Table 5
Individual averaged squared residual errors for GO-EG when $Pr = 6.5, Ec = 0.5, M = 0.1, S = 0.4, = 0.1, \beta = 0.3, \phi = 0.1$

| $m$ | $\varepsilon_m^f$ GO-EG | $\varepsilon_m^\theta$ GO-EG |
|-----|-------------------------|-----------------------------|
| 5   | 1.2211 x 10^{-1}        | 1.4711 x 10^{-1}            |
| 10  | 3.2211 x 10^{-3}        | 3.4711 x 10^{-3}            |
| 15  | 4.2211 x 10^{-5}        | 5.4711 x 10^{-5}            |
| 20  | 5.2211 x 10^{-6}        | 7.1111 x 10^{-6}            |
| 25  | 6.4711 x 10^{-7}        | 5.5232 x 10^{-7}            |
Figure 4 shows the impact of the magnetic field versus the velocity field. The relation between $M$ and $\frac{df}{d\eta}(\eta)$ is inverse relation. For the growing magnitude of $M$, decreases the $\frac{df}{d\eta}(\eta)$ as displayed in Figure 4. Physically, by increasing magnetic field $M$, resistance forces are produced so that decreases the velocity profile. Relatively this effect is strong in the GO-W as equated to GO-EG. Figure 5 shows the impact of the unsteady parameter $S$ versus the velocity field. The relation between $S$ and $\frac{df}{d\eta}(\eta)$ is inverse relation. For the growing magnitude of $S$, decreases the $\frac{df}{d\eta}(\eta)$ because surface is greater for higher estimation of $S$. Therefore, the higher value of $S$ augmenting the opposing force cause decline the fluid velocity as displayed in Figure 5. Relatively this effect is strong in the GO-W as equated to GO-EG.
Figure 6 shows the impact of the dimensionless film thickness parameter versus the velocity field. The dimensionless film thickness parameter shows dull effect on velocity profile, in the starting point velocity is decreasing near the sheet surface and after the point of convection the velocity profile is increasing as shown Figure 6. Relatively this effect is strong in the GO-W as equated to GO-EG. Similarly Figure 7 displays the effect of the $\phi$ against the $\theta(\eta)$ temperature field. From the figure, we observe that the relation between temperature field and $\phi$ is direct relation. So, by increasing values of the $\phi$ decreases the $\theta(\eta)$ temperature field as displayed in Figure 7. Relatively this effect is strong in the GO-W.
Figure 6 shows the effect of the $Pr$ against the $T(\eta)$ temperature field. The relation between temperature field and $Pr$ is inverse relation, so by enhance the $Pr$ decreasing the temperature profile as displayed in Figure 8. In fact, the thickness of the momentum boundary layer is to be larger than that of the thermal boundary layer, or that the viscous diffusion is larger than the thermal diffusion and therefore, the larger amount of the $Pr$ reduces the thermal boundary layer. Relatively this effect is strong in the GO-W.

Figure 7 shows the impact of the Eckert number $Ec$ versus the temperature field. From the figure, we notice that the relation between temperature field and $Ec$ is direct relation, that is by increase the $Ec$, increasing the temperature profile as displayed in Figure 9. In fact, the thickness of the momentum boundary layer is to be small than that of the thermal boundary layer, or that the viscous diffusion is less than the thermal diffusion and therefore, the larger amount of the $Ec$ increases the thermal boundary layer. Relatively this effect is strong in the GO-W.
5. Conclusion

Laminar (wedge flow problem) thin film flow of water-based GO-EG and GO-W nanofluids over the stretching sheet of dynamic viscosity has been examined in this research. The two sorts GO-EG/GO-W water base nanofluids have been used for the impact of dynamic viscosity. The solution of the problem is obtained by using the optimal homotopy analysis method (OHAM). The BVPh 2.0 package function of Mathematica is used to obtain the numerical results. The result of important parameters such as magnetic parameter, Prandtl number, Eckert number, dynamic viscosity, nanoparticles volume fraction and unsteady parameter for both velocity and temperature profiles are plotted and discussed. The BVPh 2.0 package is used to obtain the convergences of the problem up to 25 iteration. The skin friction coefficient and Nusselt number is explained in table form. The obtained outputs are deliberated as follows

- It is observed that the increasing nanoparticles volume fraction $\phi$ decreases the velocity field.
- It is observed that by increasing dynamics viscosity $\alpha$ reduces the velocity field.
- It is observed that by increasing the values of the thin film thickness parameter $\beta$ decreases the velocity in the starting and then increasing the velocity field.
- It is observed that by increase the Eckert number $Ec$ increasing the temperature profile.
• It is observed that by increasing the Prandtl number $Pr$, decreasing the temperature profile.

Acknowledgement
This research has been partially supported by Universiti Malaysia Terengganu under the Postgraduate Research Grant (PGRG) vote no. 55193/3.

References
[1] Haq, Rizwan Ul, S. Naveed Kazmi, and Toufik Mekkaoui. "Thermal management of water based SWCNTs enclosed in a partially heated trapezoidal cavity via FEM." International Journal of Heat and Mass Transfer 112 (2017): 972-982. https://doi.org/10.1016/j.ijheatmasstransfer.2017.05.041

[2] Prasannakumara, B. C., B. J. Gireesha, M. R. Krishnamurthy, and K. Ganesh Kumar. "MHD flow and nonlinear radiative heat transfer of Sisko nanofluid over a nonlinear stretching sheet." Informatics in Medicine Unlocked 9 (2017): 123-132. https://doi.org/10.1016/j.imu.2017.07.006

[3] Rudraswamy, N. G., S. A. Shehzad, K. Ganesh Kumar, and B. J. Gireesha. "Numerical analysis of MHD three-dimensional Carreau nanoliquid flow over bidirectionally moving surface." Journal of the Brazilian Society of Mechanical Sciences and Engineering 39, no. 12 (2017): 5037-5047. https://doi.org/10.1007/s40430-017-0897-3

[4] Soomro, Feroz Ahmed, Rizwan-ul Haq, Zafar Hayat Khan, and Qiang Zhang. "Passive control of nanoparticle due to convective heat transfer of Prandtl fluid model at the stretching surface." Chinese Journal of Physics 55, no. 4 (2017): 1561-1568. https://doi.org/10.1016/j.cjph.2017.05.007

[5] Usman, Muhammad, Rizwan Ul Haq, Muhammad Hamid, and Wei Wang. "Least square study of heat transfer of water-based Cu and Ag nanoparticles along a converging/diverging channel." Journal of Molecular Liquids 249 (2018): 856-867. https://doi.org/10.1016/j.molliq.2017.11.047

[6] Takhar, Harminder S., Ali J. Chamkha, and Girishwar Nath. "Unsteady three-dimensional MHD-boundary-layer flow due to the impulsive motion of a stretching surface." Acta Mechanica 146, no. 1 (2001): 59-71. https://doi.org/10.1007/BF01178795

[7] Kumar, K. Ganesh, B. J. Gireesha, M. R. Krishnamurthy, and B. C. Prasannakumara. "Impact of convective condition on Marangoni convection flow and heat transfer in Casson nanofluid with uniform heat source sink." Journal of Nanofluids 7, no. 1 (2018): 108-114. https://doi.org/10.1166/jn.2018.1439

[8] Sheikholeslami, M., and Houman B. Rokni. "Numerical simulation for impact of Coulomb force on nanofluid heat transfer in a porous enclosure in presence of thermal radiation." International Journal of Heat and Mass Transfer 118 (2018): 823-831. https://doi.org/10.1016/j.ijheatmasstransfer.2017.11.041

[9] Sheikholeslami, M., and S. A. Shehzad. "Simulation of water based nanofluid convective flow inside a porous enclosure via non-equilibrium model." International Journal of Heat and Mass Transfer 120 (2018): 1200-1212. https://doi.org/10.1016/j.ijheatmasstransfer.2017.12.132

[10] Falkneb, V. M., and Sylvia W. Skan. "LXXV. Solutions of the boundary-layer equations." The London, Edinburgh, and Dublin Philosophical and Journal of Science 12, no. 80 (1931): 865-896. https://doi.org/10.1080/1478643109461870

[11] Yih, K. A. "Uniform suction/blowing effect on forced convection about a wedge: uniform heat flux." Acta Mechanica 128, no. 3 (1998): 173-181. https://doi.org/10.1007/BF01251888

[12] Watanabe, T. "Thermal boundary layers over a wedge with uniform suction or injection in forced flow." Acta Mechanica 83, no. 3 (1990): 119-126. https://doi.org/10.1007/BF01172973

[13] Rajagopal, Kumbakonam R., A. S. Gupta, and Tsung-Yen Na. "A note on the Falkner-Skan flows of a non-Newtonian fluid." International Journal of Non-Linear Mechanics 18, no. 4 (1983): 313-320. https://doi.org/10.1016/0020-7462(83)90028-8

[14] Na, Tsung Yen. Computational methods in engineering boundary value problems. Academic press, New York, 1980.

[15] Asaithambi, Asai. "A finite-difference method for the Falkner-Skan equation." Applied Mathematics and Computation 92, no. 2-3 (1998): 135-141. https://doi.org/10.1006/amco.1997.2334

[16] Rehman, Ali, Taza Gul, Zabidin Salleh, Safyan Mukhtar, Fawad Hussain, Kottakkaran Sooppy Nisar, and Poom Kumam. "Effect of the Marangoni convection in the unsteady thin film spray of CNT nanofluids." Processes 7, no. 6 (2019): 392. https://doi.org/10.3390/pr7060392

[17] Rehman, Ali, Zabidin Salleh, Taza Gul, and Zafar Zaheer. "The impact of viscous dissipation on the thin film unsteady flow of GO-EG/GO-W nanofluids." Mathematics 7, no. 7 (2019): 653. https://doi.org/10.3390/math7070653

[18] Balandin, Alexander A., Suchismita Ghosh, Wenzhong Bao, Irene Calizo, Desalegne Teweldebrhan, Feng Miao, and Chun Ning Lau. "Superior thermal conductivity of single-layer graphene." Nano Letters, 8, no. 3 (2008): 902-907. https://doi.org/10.1021/nl0731872
[19] Yu, Wei, Huaqing Xie, and Dan Bao. "Enhanced thermal conductivities of nanofluids containing graphene oxide nanosheets." Nanotechnology 21, no. 5 (2009): 055705. https://doi.org/10.1088/0957-4484/21/5/055705

[20] Gul, Taza, and Kiran Firdous. "The experimental study to examine the stable dispersion of the graphene nanoparticles and to look at the GO-H2O nanofluid flow between two rotating disks." Applied Nanoscience 8, no. 7 (2018): 1711-1727. https://doi.org/10.1007/s13204-018-0851-4

[21] Gul, Taza, Waqas Noman, Muhammad Sohail, and Muhammad Altaf Khan. "Impact of the Marangoni and thermal radiation convection on the graphene-oxide-water-based and ethylene-glycol-based nanofluids." Advances in Mechanical Engineering 11, no. 6 (2019): 1687814019856773. https://doi.org/10.1177/1687814019856773

[22] Qasim, M., Z. H. Khan, R. J. Lopez, and W. A. Khan. "Heat and mass transfer in nanofluid thin film over an unsteady stretching sheet using Buongiorno's model." The European Physical Journal Plus 131, no. 1 (2016): 1-11. https://doi.org/10.1140/epjp/i2016-16016-8

[23] Liao, Shijun. "An optimal homotopy-analysis approach for strongly nonlinear differential equations." Communications in Nonlinear Science and Numerical Simulation 15, no. 8 (2010): 2003-2016. https://doi.org/10.1016/j.cnsns.2009.09.002

[24] Maleque, Kh. "Effects of binary chemical reaction and activation energy on MHD boundary layer heat and mass transfer flow with viscous dissipation and heat generation/absorption." International Scholarly Research Notices 2013 (2013). https://doi.org/10.1155/2013/284637

[25] Gireesha, B. J., M. Archana, B. Mahanthesh, and B. C. Prasannakumara. "Exploration of activation energy and binary chemical reaction effects on nano Casson fluid flow with thermal and exponential space-based heat source." Multidiscipline Modeling in Materials and Structures (2019).

[26] Prasannakumara, B. C., B. J. Gireesha, Rama SR Gorla, and M. R. Krishnamurthy. "Effects of chemical reaction and nonlinear thermal radiation on Williamson nanofluid slip flow over a stretching sheet embedded in a porous medium." Journal of Aerospace Engineering 29, no. 5 (2016): 04016019. https://doi.org/10.1061/(ASCE)AS.1943-5525.0000578

[27] Khodabandeh, Erfan, Davood Toghraie, A. Chamkha, Ramin Mashayekhi, Omidali Akbari, and Seyed Alireza Rozati. "Energy saving with using of elliptic pillows in turbulent flow of two-phase water-silver nanofluid in a spiral heat exchanger." International Journal of Numerical Methods for Heat & Fluid Flow 30, no. 4 (2019): 2025-2049. https://doi.org/10.1108/HFF-10-2018-0594

[28] Rehman, Ali, Zabidin Salleh, and Taza Gul. "Influence of Dynamics Viscosity on the Water Base Graphene Oxide-Ethylene Glycol/Graphene Oxide-Water Nanofluid Flow Over A Stretching Cylinder." Journal of Nanofluids 8, no. 8 (2019): 1661-1667. https://doi.org/10.11600/jon.2019.1722

[29] Rehman, Ali, Zabidin Salleh, and Taza Gul. "Influence of dynamics viscosity on the water base CNTs nanofluid flow over a stretching surface." Cogent Engineering 7, no. 1 (2020): 1772945. https://doi.org/10.1080/23311916.2020.1772945

[30] Rehman, Ali, Zabidin Salleh, and Taza Gul. "The impact of marangoni convection, magnetic field and viscous dissipation on the thin film unsteady flow of go-eg/go-w nanofluids over an extending sheet." JP Journal of Heat and Mass Transfer 18, no. 2 (2019): 477-496. https://doi.org/10.17654/HM018020477

[31] Rehman, Ali, Zabidin Salleh, and Taza Gul. "The impact of the magnetic field and viscous dissipation on the thin film unsteady flow of GO-EG/GO-W nanofluids." In Journal of Physics: Conference Series, vol. 1366, no. 1, p. 012031. IOP Publishing, 2019. https://doi.org/10.1088/1742-6596/1366/1/012031

[32] Rehman, Ali, Zabidin Salleh, and Taza Gul. "Analytical study of unsteady squeezed flow of water base cnts nanofluid with magnetic field and variable thermal conductivity over a stretching surface." Frontiers in Heat and Mass Transfer (FHMT) 14 (2020). https://doi.org/10.5098/hmt.14.20

[33] Raza, Jawad. "Thermal radiation and slip effects on magnetohydrodynamic (MHD) stagnation point flow of Casson fluid over a convective stretching sheet." Propulsion and Power Research 8, no. 2 (2019): 138-146. https://doi.org/10.1007/jpapr.2019.01.004

[34] Raza, J., Fateh Mebarez-Oudina, Paras Ram, and S. Sharma. "MHD flow of non-Newtonian molybdenum disulfide nanofluid in a converging/diverging channel with Rosseland radiation." In Defect and Diffusion Forum, vol. 401, pp. 92-106. Trans Tech Publications Ltd, 2020. https://doi.org/10.4028/www.scientific.net/DDF.401.92

[35] Alkasasbeh, Mohammad, Zurni Omar, Fateh Mebarez-Oudina, Jawad Raza, and Ali Chamkha. "Heat transfer study of convective fin with temperature-dependent internal heat generation by hybrid block method." Heat Transfer-Asian Research 48, no. 4 (2019): 1225-1244. https://doi.org/10.1002/htj.21428

[36] Lund, Liaquat Ali, Zurni Omar, Jawad Raza, and Ilyas Khan. "Magnetohydrodynamic flow of Cu-Fe304/H2O hybrid nanofluid with effect of viscous dissipation: dual similarity solutions." Journal of Thermal Analysis and Calorimetry 10, no. 1007 (2020): 1-13. https://doi.org/10.1007/s10973-020-09602-1
[37] Lund, Liaquat Ali, Zurni Omar, Jawad Raza, and Ilyas Khan. "Triple solutions of micropolar nanofluid in the presence of radiation over an exponentially preamble shrinking surface: Convective boundary condition." *Heat Transfer* 49, no. 5 (2020): 3075-3093. [https://doi.org/10.1002/htj.21763](https://doi.org/10.1002/htj.21763)