On self-synchronization of inertial vibration exciters in a vibroimpact three-mass system

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Abstract. The dynamics of a vibration machine’s model in the form of a vibroimpact three-mass system with one shock pair, which oscillations are excited by two non-ideal inertial vibration exciters, is considered. The collision of the shock pair elements is modeled on the basis of the classical impact theory. A numerical simulation of the model dynamics at varying the excitation frequency, as well as the parameters of the initial gap and the coefficient of restitution, characterizing the conditions of contact interaction of the machine’s working bodies with the processed material, has been carried out. The influence of these parameters values on the frequency range of stable synchronization of vibration exciters and stable periodic vibroimpact modes of the system motion has been established. It is shown that at the same excitation frequency, a change in these parameters can lead to a change in the periodicity of the vibroimpact mode and the working bodies’ oscillations mode from synchronous-antiphase to practically synchronous-inphase.

1. Introduction

To excite oscillations of vibrating machines and mechanisms for various technological purposes, systems of several self-synchronizing inertial (unbalance) vibration exciters are widely used [1-3]. One of the main problems of their effective application is to ensure synchronous rotation of vibration exciters with given ratio of speeds and mutual phase, which is required for efficient implementation of the technological process, when interacting with the processed material.

The general theory of synchronization of dynamic systems and its application to the solution of some problems of self-synchronization of inertial vibration exciters in linear mechanical systems and in the systems with weak nonlinearity are considered in [4]. In [5-8], the issues of self-synchronization of vibration exciters in mechanical systems with nonlinear characteristics of elastic elements that simulate interaction with the processed material are considered for various types of nonlinearity. In this works the vibration exciters are attached to a single carrying body. In [1, 9-11], the dynamics of vibration machines with self-synchronizing vibration exciters located on various carrying bodies is analyzed. In these cases, the interaction with the processed material is taken into account on the basis of simplified representations, for example, in the form of an additionally attached mass, or a linear visco-elastic element. The interaction with the material in the form of a non-instantaneous impact of the supporting bodies is taken into account in [12, 13]. It is shown that taking into account the vibroimpact nature of the interaction has a significant effect on the frequency range of vibration exciter’s self-synchronization and the modes of system’s oscillation. At the same time, practical
application of such a model is complicated by the need to conduct relatively complex experiments to determine the dynamic characteristics of contact interaction.

In this paper, the dynamics of a model of a vibrating machine with two self-synchronizing vibration exciters located on different carrying bodies, which are its working bodies, is consider. Interaction with the processed material is taken into account in the form of an instant impact of the working bodies against each other on the basis of the classical theory of impact [14].

2. Mathematical model

The design scheme of the vibrating machine model is shown in figure 1. The model consists of a solid body of mass \( m_1 \), which simulates the frame of the machine, attached to a fixed foundation by means of a linear spring with the coefficients of stiffness and viscosity \( c_1 \) and \( b_1 \), respectively; and two rigid bodies with the same mass \( m_2 \), simulating the working bodies of the machine, symmetrically fixed to the frame using identical linear springs with the stiffness and viscosity coefficients \( c \) and \( b \), respectively. Motions of each of the bodies are described by the displacements \( x_1, x_2, x_3 \) of their centers of mass along the horizontal axis \( Ox \), measured relative to their equilibrium positions.

![Figure 1. The design scheme.](image)

Oscillations of the system are excited by two identical unbalance vibration exciters with an imbalance mass \( m_e \) and an eccentricity \( r \), rigidly fixed to each working body. The angular positions of the imbalances are described by the angles \( \phi_i \) (\( i = 1, 2 \) is the vibration exciter number) measured from the negative direction of the \( Ox \) axis. Each vibration exciter is driven by an asynchronous electric motor with the moment of inertia \( J \). The driving moment \( L_i \) of the \( i \)-th motor is taken into account by its static characteristic \( L_i = 2\sigma_i \Gamma M_{ci}/(s_{cl}/s_i - s_i/s_{cl}) \) [4], where \( M_{ci} \) and \( s_{ci} \) are critical torque and critical slip of the motor respectively, \( s_{cl} = (\omega_0 - \sigma_i \phi_i)/\omega_{cl}, \omega_{cl} = \omega_0/p_1 - \) synchronous speed of the motor, \( p_1 \) – quantity of the motor’s pole pair, \( \omega_0 \) – power supply frequency, \( \sigma_i = \pm 1 \) – coefficient taking into account the direction of the motor torque action. Friction in the bearings supporting the exciter’s shaft is considered as the moment of Coulomb friction forces \( L_{Ri} = 0.5m_e r d_s f_j \phi_i^2 \text{sign}(\phi_i) \), \( f_j \) – coefficient of Coulomb friction, \( d_s \) – diameter of vibration exciter’s shaft.

When the machine is operating, the processed material is fed into the space between the working bodies (working chamber). The material’s mass is significantly less than the mass of the working bodies and, therefore, is not taken into account. The filling of the working chamber with the material is characterized by the value \( \Delta \) (hereinafter "the initial gap"), which is the difference between the distance \( D \) between the working bodies at undeformed elastic elements of the system and the characteristic transverse dimension \( d \) of the material’s volume in the working chamber. A decrease in the value of \( \Delta \) corresponds to an increase in the fullness of the working chamber’s volume. The interaction of the working bodies with the material occurs when \( x_2 - x_3 = \Delta \), and is modeled by the instant impact of the working bodies against each other. Energy losses spent on material processing at impact are characterized by the coefficient of restitution \( R \). Velocities \( \dot{x}_{2+}, \dot{x}_{3+}, \dot{\phi}_{1+}, \dot{\phi}_{2+} \) of the shock pair elements after impact are defined by the expressions:
\[ \dot{x}_{2+} = \frac{((1 + R)A_2x_{3-} + (A_1 - RA_2)x_{2-})}{(A_1 + A_2)}, \]
\[ \dot{x}_{3+} = \frac{((1 + R)A_1x_{2-} + (A_2 - RA_1)x_{3-})}{(A_1 + A_2)}, \]
\[ \phi_{1+} = \phi_{1-} - m_e \sin(\phi(t)) \left( x_{2+} - x_{2-} \right)/J, \]
\[ \phi_{2+} = \phi_{2-} - m_e \sin(\phi(t)) \left( x_{3+} - x_{3-} \right)/J, \]

where \( A_1 = m_2 + m_e - (m_e \sin(\phi_1))^2/J, A_2 = m_2 + m_e - (m_e \sin(\phi_2))^2/J, J = J_0 + m_e r^2 \), \( x_{2-}, x_{3-}, \phi_{1-}, \phi_{2-} \) - velocities of the shock pair elements before impact.

Motion of the system between impacts is described by a system of equations (presented in dimensionless form):

\[
\begin{bmatrix}
\ddot{q}_1 + 2\lambda \mu_1 (\beta_1 \dot{q}_1 + \beta_2 \dot{q}_3) + n_1^2 q_1 = \mu_1 (\sum_{i=1}^2 (\dot{\phi}_i \sin(\phi_i) + \dot{\phi}_i^2 \cos(\phi_i)) \\
\ddot{q}_2 + 2\lambda \mu_2 (\beta_2 \dot{q}_1 + \beta_3 \dot{q}_3) + n_2^2 q_2 = \mu_2 (\sum_{i=1}^2 (\dot{\phi}_i \sin(\phi_i) + \dot{\phi}_i^2 \cos(\phi_i)) \\
\ddot{\phi}_1 = N_1(\dot{q}_1) + J_1(\ddot{q}_1 - \ddot{q}_2 + \ddot{q}_3) \sin(\phi_1) \\
\ddot{\phi}_2 = N_2(\dot{q}_2) + J_2(\ddot{q}_1 + \ddot{q}_2 + \ddot{q}_3) \sin(\phi_2)
\end{bmatrix},
\]

where \( q = \begin{bmatrix} q_1, q_2, q_3 \end{bmatrix}^T, x = \begin{bmatrix} x_1, x_2, x_3 \end{bmatrix}^T, U = \begin{bmatrix} u_{11} & 0 & u_{31} \\
1 & -1 & 1 \\
1 & 1 & 1 \end{bmatrix}, \)

\[ u_{11} = -\frac{2 + \frac{c_1 - m_1}{m_1} \sqrt{\delta}}{2m_1}, \quad u_{31} = -\frac{2 + \frac{c_1 - m_1}{m_1} + \sqrt{\delta}}{2m_1}, \quad D = 4(1 + \frac{c_1}{c} + \frac{m_1}{m}) + \frac{c_1}{m_1} \]

\[ n_1^2 = \frac{2 + \frac{c_1}{c} + \frac{m_1}{m}}{2m_1}, \quad n_2^2 = \frac{2 + \frac{c_1}{c}}{2m_1}, \quad n_2^2 = 1, \]

\[ T_1 = \sqrt{m/c}, \quad \mu_1 = m_{22}/m_{11}, \quad \mu_2 = m_{22}/m_{33}, \quad \beta_1 = b_{11}/b_{22}, \quad \beta_2 = b_{33}/b_{22}, \]

\[ \lambda = b_{32}^2 T_2/2 m_{22}, \quad \Delta = \Delta/X, \quad X = m_e r/m_{22}, \]

where \( m_{ij}, b_{ij} \) - corresponding elements of inertia matrix \( M^* \) and damping matrix \( B^* \), determined as follows: \( M^* = U^T MU, B^* = U^T BU, M = \begin{bmatrix} m_1 & 0 & 0 \\
0 & m_0 & 0 \\
0 & 0 & m \end{bmatrix}, B = \begin{bmatrix} 2b + b_1 & -b & -b \\
-b & b & 0 \\
-b & 0 & b \end{bmatrix}, m = m_2 + m_e, \)

dots denote differentiation with respect to dimensionless time \( \tau = t/T_1 \).

3. Simulation results and their analysis

The simulation of the model dynamics was carried out numerically in Matlab. Integration of the system of differential equations (2) was performed by standard integration functions by the Runge-Kutta method of the 4th order with an additional option to accurately determine the moments of working bodies’ collision. At the moments of collision, the integration was interrupted, and the velocities were recalculated according to (1), after which the integration continued.

To study the dynamic characteristics of the system, the dimensionless frequency of the supply voltage \( \tilde{\omega}_e = \omega_e/n_1 \) was set, which gradually changed in a stepwise manner in the range of values \( 0.1 \leq \tilde{\omega}_e \leq 3.6 \) with a frequency step of \( \Delta \tilde{\omega}_e = 0.1 \). The maximum torque of the motors was considered constant in the frequency range. The calculations were performed both with increasing and decreasing \( \tilde{\omega}_e \). At each value of the frequency, the calculations were carried out during a period of time sufficient for the transient processes decay and the steady oscillations establishment. After the steady oscillations were achieved, the averaged frequency \( \bar{\omega}_e \) and the mutual phase \( \Delta \phi \) of the exciters
rotation, the peak-to-peak values and period of the working bodies’ oscillations, as well as the speed of their collisions (impact rate), were determined. To establish the influence of $\Delta^*$ and $R$ on the dynamic characteristics of the system, the calculations were repeated at their different values in the ranges $-5 \leq \Delta^* \leq 15$ and $0.1 \leq R \leq 0.9$. The simulation was carried out for the following values of the system parameters: $n_1 = 0.166$, $n_3 = 1.524$, $\mu_1 = 0.576$, $\mu_2 = 0.423$, $\beta_1 = 0.473$, $\beta_2 = -0.579$, $\beta_3 = 6.279$, $\lambda = 0.03$, $f_r = 0.013$, $M_{c1}/L_1 = M_{c2}/L_2 = 0.248$, $s_{c1} = s_{c2} = 0.15$, $0.5m_ervd_f/L_1 = 1 \cdot 10^{-4}$.

Figures 2-4 show stable periodic vibroimpact modes of the model motion and areas of synchronous rotation of vibration exciters with different values of the mutual phase $\Delta\phi$ depending on the frequency of the exciters rotation $\tilde{\omega}$ and the value of $\Delta^*$, calculated for $R = \{0.1, 0.3, 0.5\}$ at an increase in the frequency $\tilde{\omega}_e$ (figures 2-4 (a)) and with its decrease (figures 2-4 (b)). Single markers denote vibroimpact modes of motion with different periods relative to the period of the exciters rotation.
$T = 1/\tilde{\omega}$ and one collision during the period found: markers "O" denote modes with a period equal to the main excitation period $T$, markers "□" denote modes with period $2T$, markers "◊" denote modes with a period greater than $2T$. Vibroimpact modes of the corresponding periodicity with several impacts found during the period (multi-impact modes) are marked with similar, but double markers.

To display changes in the steady-state mutual phase $\Delta \varphi$, the entire range of possible values was divided into intervals $\{(-90^{\circ}; -15^{\circ}), [-15^{\circ}; 15^{\circ}], (15^{\circ}; 165^{\circ}), [165^{\circ}; 195^{\circ}], (195^{\circ}; 270^{\circ})\}$. Regions of the varied parameters values at which $\Delta \varphi$ is set within the corresponding interval are shown in different colors. Note that the values of $\Delta \varphi \in [-15^{\circ}; 15^{\circ}]$ correspond to the rotation of vibration excitors close to synchronous-in-phase, and $\Delta \varphi \in [165^{\circ}; 195^{\circ}]$ - close to synchronous-antiphase, required for normal operation of the machine.

**Figure 3.** Motion modes for $R=0.3$. 

(a) 

(b)
The presented results show that in the frequency range \( n_1 < \tilde{\omega} < 1 \) (pre-resonant frequency range), \( T \)-periodic vibroimpact modes are usually excited, characterized by a synchronous-antiphase mode of the exciters rotation and working bodies oscillations, low impact rates, as well as weak sensitivity to changes of \( R \). The vibroimpact mode characteristic, determined by the number of collisions per period and their impact rates, significantly depends on the value of the initial gap \( \Delta^* \). A decrease in \( \Delta^* \) leads to excitation of multi-impact modes. The frequency range of vibroimpact modes excitation practically coincides with the frequency range of synchronous-antiphase rotation of vibration exciters, the boundaries of which differ significantly depending on the method of reaching a given excitation mode - with an increase or decrease in the excitation frequency.

![Figure 4](image-url)

**Figure 4.** Motion modes for \( R=0.1 \).
In the frequency range $1 < \tilde{\omega} < n_3$ (inter-resonant frequency range), it is possible to excite $T$-periodic vibroimpact modes, characterized by a synchronous-antiphase rotation of vibration exciters and oscillations of working bodies, as well as relatively high impact rates (see figures 2-4 (a)). The implementation of these modes turns out to be possible due to the change in the resonance characteristics of the system because of the impacts occurrence. However, since in the absence of impacts in this frequency range a synchronous-in-phase mode of vibration exciters rotation is realized (see figures 2-4 (b)), to excite these vibro-shock modes, certain conditions must be created, for example, due to initial excitation vibroimpact oscillations in the adjacent frequency ranges, followed by adjustment of the excitation frequency to the specified range. With a decrease in the values of both $\Delta^r$ and $R$, the frequency range of excitation of stable vibroimpact modes narrows, due to the shift to the left of its upper boundary.

In the frequency range $\tilde{\omega} > n_3$ (above-resonant frequency range), near its lower boundary, there is a frequency range in which $T$-periodic vibroimpact modes are excited, characterized by a synchronous-antiphase mode of vibration exciters rotation and oscillations of the working bodies, as well as high impact rates. A change in the parameters $\Delta^r$ and $R$ leads to a change in the width of this frequency range due to a shift in its upper boundary. As $\Delta^r$ decreases, this frequency range expands. At positive values of $\Delta^r$, a decrease in $R$ leads to a narrowing of this frequency range, and at negative values of $\Delta^r$ to expansion. Regardless of the sign of $\Delta^r$ at the same excitation frequency, a decrease in $R$ leads to a decrease in the impact rates.

In addition, at $\Delta^r \geq 0$ and far from $n_3$, a frequency range in which it is possible to excite stable vibroimpact $2T$-periodic modes, characterized by a synchronous-antiphase mode of vibration exciters rotation and working bodies oscillations, as well as high impact rates, is observed. Note that for excitation of these modes, it is necessary that the amplitudes of the working bodies relative oscillations, which in the resonant frequency range in the absence of impacts decrease with an increase in the excitation frequency, exceed the value of $\Delta^r$. As $R$ decreases, this frequency range expands due to the leftward shift of its lower boundary. In this case, at the same excitation frequency, the impact rates practically do not change. In the remaining frequency range of the above-resonant range, predominantly chaotic vibroimpact modes are observed.

For $\Delta^r < 0$ at a distance from $n_3$, there is a frequency range in which $T$-periodic vibroimpact modes are excited, characterized by a synchronous mode of vibration exciters rotation with stable values of $\Delta \phi$ in the range $-90^\circ < \Delta \phi < 0^\circ$, tending to $0^\circ$ as the excitation frequency increases. In these modes, the oscillations of the working bodies occur almost in-phase, but with different amplitudes. As a result, their collisions occur at low impact rates. With a decrease in $R$, the frequency range of stable excitation of these modes expands due to the shift to the right of its upper boundary.

4. Conclusion
As a result of the model dynamics analysis, the influence of the parameters $\Delta^r$ and $R$, characterizing the conditions of the working bodies’ interaction with the processed material, on the frequency range of self-synchronization of vibration exciters and periodic vibroimpact modes of the system motion is established. In the pre-resonant and inter-resonant frequency ranges, periodic vibroimpact modes with period equal to the period of excitation ($T$-periodic modes) are predominantly excited. An increase in the excitation frequency leads to an increase in the impact rates of the working bodies in these modes, while a decrease in both $\Delta^r$ and $R$ leads to a narrowing of the frequency range of their excitation.

In the above-resonant frequency range at $\Delta^r \geq 0$, there are two frequency ranges with $T$-periodic vibroimpact modes, near the third natural frequency, and $2T$-periodic vibroimpact modes, at a distance from it, with synchronous antiphase rotation of vibration exciters and oscillations of working bodies and high impact rates. With a decrease in $R$, the frequency range of excitation of $T$-periodic modes narrows, and of $2T$-periodic modes expands. At the same time, the impact rate of the working bodies’ collision in $T$-periodic modes decreases, and in $2T$-periodic modes it practically does not change. When $\Delta^r < 0$, near the third natural frequency, there is a frequency range with the required $T$-periodic vibroimpact modes, which expands with a decrease in $R$. In the remaining part of this frequency range,
$T$-periodic vibroimpact modes arise when the vibration exciters rotate close to synchronous-in-phase mode. In this case, the oscillations of the working bodies occur almost in phase, but with different amplitudes, as a result of which their collisions occur at low impact rates. The upper boundary of the frequency range of stable excitation of these modes shifts significantly to the right with decreasing $R$.

It is shown that at the same excitation frequency, a change in the parameters of the machine’s working bodies interaction with the material can lead to a change in both the periodicity of the vibroimpact mode and the form of working bodies oscillations from synchronous-antiphase to practically synchronous-inphase.

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