Obtaining Statistics of Turbulent Velocity from Astrophysical Spectral Line Data

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Abstract. Turbulence is a crucial component of dynamics of astrophysical fluids dynamics, including those of ISM, clusters of galaxies and circumstellar regions. Doppler shifted spectral lines provide a unique source of information on turbulent velocities. We discuss Velocity-Channel Analysis (VCA) and its offspring Velocity Coordinate Spectrum (VCS) that are based on the analytical description of the spectral line statistics. Those techniques are well suited for studies of supersonic turbulence. We stress that a great advantage of VCS is that it does not necessary require good spatial resolution. Addressing the studies of mildly supersonic and subsonic turbulence we discuss the criterion that allows to determine whether a traditional tool for such a research, namely, Velocity Centroids are dominated by density or velocity. We briefly discuss the use of higher order correlations as the means to study intermittency of turbulence. We discuss observational data available and prospects of the field.

Keywords: turbulence, molecular clouds, MHD

1. What can be learned from fluctuations of velocity?

As a rule astrophysical fluids are turbulent and the turbulence is magnetized. This ubiquitous turbulence determines the transport properties of interstellar medium (see Elmegreen & Falgarone 1996, Stutzki 2001, Cho et al. 2003) and intracluster medium (Inogamov & Sunyaev 2003, Sunyaev, Norman & Bryan 2003, see review by Lazarian & Cho 2004a), many properties of Solar and stellar winds, accretion disks etc. One may say that to understand heat conduction, transport and acceleration of cosmic rays, propagation of electromagnetic radiation in different astrophysical environments it is absolutely essential to understand the properties of underlying turbulence. The mysterious processes of star formation (see McKee & Tan 2002, Elmegreen 2002, Pudritz 2001) and interstellar medium (see Falgarone 1999 and references therein), shattering and coagulation of dust (see Lazarian & Yan 2003 and references therein) are also intimately related to properties of magnetized compressible turbulence (see reviews by Elmegreen & Scalo 2004).

From the point of view of fluid mechanics astrophysical turbulence is characterized by huge Reynolds numbers, $Re$, which is the inverse ratio of the eddy turnover time of a parcel of gas to the time required for
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viscous forces to slow it appreciably. For \( Re \gg 100 \) we expect gas to be turbulent and this is exactly what we observe in HI (for HI \( Re \sim 10^8 \)).

Statistical description is a nearly indispensable strategy when dealing with turbulence. The big advantage of statistical techniques is that they extract underlying regularities of the flow and reject incidental details. Kolmogorov description of unmagnetized incompressible turbulence is a statistical one. For instance it predicts that the difference in velocities at different points in turbulent fluid increases on average with the separation between points as a cubic root of the separation, i.e. \(|\delta v| \sim l^{1/3}\). In terms of direction-averaged energy spectrum this gives the famous Kolmogorov scaling \( E(k) \sim 4\pi k^2 P(k) \sim k^{5/3} \), where \( P(k) \) is a 3D energy spectrum defined as the Fourier transform of the correlation function of velocity fluctuations \( \xi(r) = \langle \delta v(x) \delta v(x + r) \rangle \).

Note that in this paper we use \( \langle ... \rangle \) to denote averaging procedure.

The example above shows the advantages of the statistical approach to turbulence. For instance, the energy spectrum \( E(k)dk \) characterizes how much energy resides at the interval of scales \( k, k + dk \). At large scales \( l \) which correspond to small wavenumbers \( k \) ( i.e. \( l \sim 1/k \) ) one expects to observe features reflecting energy injection. At small scales one should see the scales corresponding to sinks of energy. In general, the shape of the spectrum is determined by a complex process of non-linear energy transfer and dissipation. For Kolmogorov turbulence the spectrum over the inertial range, i.e. the range where neither energy injection nor energy dissipation are important, is characterized by a single power law and therefore self-similar. Other types of turbulence, i.e. the turbulence of non-linear waves or the turbulence of shocks, are characterized by different power laws and therefore can be distinguished from the Kolmogorov turbulence of incompressible eddies.

In view of the above it is not surprising that attempts to obtain spectra of interstellar turbulence have been numerous. In fact they date as far back as the 1950s (see von Horner 1951, Munch 1958, Wilson et al. 1959). However, various directions of research achieved various degree of success (see Kaplan & Pickelner 1970, a review by Armstrong, Rickett & Spangler 1995). For instance, studies of turbulence statistics of ionized media were more successful (see Spangler & Gwinn 1990) and provided the information of the statistics of plasma density at scales \( 10^8 - 10^{15} \) cm. This research profited a lot from clear understanding of processes of scintillations and scattering achieved by theorists (see Narayan & Goodman 1989). At the same time the intrinsic limitations of the scincillations technique are due to the limited number of sampling directions, and relevance only to ionized gas at extremely small scales. Moreover, these sort of measurements provide only the density statistics, which is an indirect measure of turbulence.
Velocity statistics is much more coveted turbulence measure. Although, it is clear that Doppler broadened lines are affected by turbulence, recovering of velocity statistics was extremely challenging without an adequate theoretical insight. Indeed, both velocity and density contribute to fluctuations of the intensity in the Position-Position-Velocity (PPV) space.

2. What do velocity centroids tell us?

Let us consider “unnormalized” velocity centroids:

\[ S(X) = \int v_z \rho_s(X, v_z) \, dv_z, \]

where \( \rho_s \) is the density of emitters in the PPV space\(^1\).

In the case of emissivity proportional to the first power of density and provided that the turbulent region is thin for its radiation, analytical expressions for structure functions\(^2\) of centroids, i.e. \( \langle [S(X_1) - S(X_2)]^2 \rangle \) were derived in Lazarian & Esquivel (2003, henceforth LE03). In that paper the following criterion for centroids to reflect the statistics of velocity\(^3\) was established:

\[ \langle [S(X_1) - S(X_2)]^2 \rangle \gg \langle V^2 \rangle \langle [I(X_1) - I(X_2)]^2 \rangle, \]

where \( I(X) \) is the intensity \( I(X) \equiv \int \rho_s dV \) and the velocity dispersion \( \langle V^2 \rangle \) can be obtained using the second moment of the spectral lines:

\[ \langle V^2 \rangle \equiv \langle \int_X V^2 \rho_s dV \rangle / \langle \int_X \rho_s dV \rangle. \]

LE03 proposed to subtract the right hand sight of the expression (2) from the left hand sight of (2) to obtain modified velocity centroids that may still reflect velocity statistics even when ordinary centroids are dominated by density contribution. A numerical study in Esquivel & Lazarian (2004) confirmed that the criterion given by eq (2) is correct and revealed that for MHD turbulence simulations it holds for turbulence with Mach number less than 2. This was consistent with an earlier

\(^1\) Traditionally used centroids include normalization by the integral of \( \rho_s \). This, however does not substantially improve the statistics, but makes the analytical treatment very involved (Lazarian & Esquivel 2003).

\(^2\) Expressions for the correlation functions are straightforwardly related to those of structure functions (Monin & Yaglom 1975). The statistics of centroids using correlation functions was used in a later paper by Levier (2004).

\(^3\) LE03 showed that the solenoidal component of velocity spectral tensor can be obtained from observations using velocity centroids in this regime.
Table I. A summary of analytical results for channel map statistics derived in LP00.

| Slice thickness       | Shallow 3-D density | Steep 3-D density |
|-----------------------|---------------------|-------------------|
|                       | $P_n \propto k^n$, $n > -3$ | $P_n \propto k^n$, $n < -3$ |

2-D intensity spectrum for thin slice $\propto K^{n+m/2}$ $\propto K^{-3+m/2}$
2-D intensity spectrum for thick slice $\propto K^n$ $\propto K^{-3-m/2}$
2-D intensity spectrum for very thick slice $\propto K^n$ $\propto K^n$

*thin* means that the channel width < velocity dispersion at the scale under study.
*thick* means that the channel width > velocity dispersion at the scale under study.
*very thick* means that a substantial part of the velocity profile is integrated over.

$m$ is the power-law index of velocity structure function, i.e. $\langle (v(x+r)-v(x))^2 \rangle \sim r^m$.
$K$ is a 2D wavevector in the plane of the slice, $k$ a 3D wavevector.

Analysis of a different set of MHD turbulence data by Brunt & Mac Low who observed that velocity centroids poorly present velocity statistics for high Mach number turbulence. Esquivel & Lazarian (2004) studied modified velocity centroids and the statistical errors arising from subtracting the corresponding large numbers in order to find the residual fluctuations. They conclude that at the moment velocity centroids can be identified as a technique to study subsonic turbulence as well as very mildly supersonic turbulence.

### 3. How can supersonic turbulence be studied?

**What do channel maps tell us?**

Power spectra of fluctuations measured within narrow ranges of velocities have been observed by different authors at various times (see Green 1993). Those power spectra were guessed to be related to underlying turbulence, but what exactly those observed spectra mean was completely unclear.

This problem was addressed in Lazarian & Pogosyan (2000, henceforth LP00) who found the relation between the spectrum of intensity fluctuations in channel maps and underlying spectra of velocity and density. They found that the power index of the intensity fluctuations depends on the thickness of the velocity channel (see Table 1).

It is easy to see that both steep and shallow underlying density the power law index *steepens* with the increase of velocity slice thickness. In the thickest velocity slices the velocity information is averaged out and it is natural that we get the density spectral index $n$. The velocity fluctuations dominate in thin slices, and the index $m$ that characterizes
the velocity fluctuations, i.e. $|\Delta v| \sim l^{m/2}$, can be obtained using thin velocity slices (see Table 1). Note, that the notion of thin and thick slices depends on a turbulence scale under study and the same slice can be thick for small scale turbulent fluctuations and thin for large scale ones. The formal criterion for the slice to be thick is that the dispersion of turbulent velocities on the scale studied should be less than the velocity slice thickness. Otherwise the slice is thin.

One may notice that the spectrum of intensity in a thin slice gets shallower as the underlying velocity get steeper. To understand this effect let us consider turbulence in incompressible optically thin media. The intensity of emission in a slice of data is proportional to the number of atoms per the velocity interval given by the thickness of the data slice. Thin slice means that the velocity dispersion at the scale of study is larger than the thickness of a slice. The increase of the velocity dispersion at a particular scales means that less and less energy is being emitted within the velocity interval that defines the slice. As the result the image of the eddy gets fainter. In other words, the larger is the dispersion at the scale of the study the less intensity is registered at this scale within the thin slice of spectral data. This means that steep velocity spectra that correspond to the flow with more energy at large scales should produce intensity distribution within thin slice for which the more brightness will be at small scales. This is exactly what our formulae predict for thin slices (see also LP00).

The result above gets obvious when one recalls that the largest intensities within thin slices are expected from the regions that are the least perturbed by velocities. If density variations are also present they modify the result. When the amplitude of density perturbation becomes larger than the mean density, both the statistics of density and velocity are imprinted in thin slices. For small scale asymptotics of thin slices this happens, however, only when the density spectrum is shallow, i.e. dominated by fluctuations at small scales.

Is spatial resolution necessary to study statistics of velocity?

Spatial resolution is essential for centroids and channel maps. However, the velocity fluctuations are also imprinted on the fluctuations of intensity along the velocity coordinate direction. The corresponding 3D PPV spectra were derived in LP00. Table 3 in LP00 states that two terms, one depending only on velocity and the other depending both on velocity and density, contribute to the spectrum measured along velocity coordinate. for steep density the intermediate scaling therefore is $k^{2n/m}$ ($n$ is negative), while the small scale asymptotics scales as $k^{-6/m}$. If the density is shallow the situation is referred, namely, at larger scales $k^{-6/m}$ asymptotics dominates, while $k^{2n/m}$ asymptotics
is present at smaller scales. The transition from one asymptotics to another depends on the amplitude of density fluctuations. Therefore both density and velocity statistics can be restored from the observations this way. If the measurements are done with an instrument of poor spatial resolution these are the expected scalings. A further study of these interesting regime is done in Chepurnov & Lazarian (2004). The only requirement for the VCS to operate is for the turbulence to be supersonic and for the instrument to have an adequate spectral resolution.

4. What is the effect of absorption?

The issues of absorption were worrisome for the researchers from the very start of the research in the field (see Munch 1958). The erroneous statements about the effects of absorption on the observed turbulence statistics are widely spread in the literature. For instance, a fallacy that absorption allows to observe density fluctuations localized in the thin surface layers of clouds, i.e. 2D turbulence, exists (see discussion in LP04).

Using transitions that are less affected by absorption, e.g. HI, allows frequently to avoid the problem. However, it looks foolish to disregards the wealth of spectroscopic data only because absorption is present. A study of absorption effects is given in LP04. There it was found that for sufficiently thin slices the scalings obtained in the absence of absorption still hold provided that the absorption on the scales under study is negligible. A similar criterion is valid for the VCS. From the practical point of view, absorption imposes an upper limit on the scales for which the statistics can be recovered.

If integrated intensity of spectral lines is studied in the presence of absorption non-trivial effects emerge. Indeed, for optically thin medium the spectral line integration results in intensity reflecting the density statistics. LP04 showed that this may not be any more true for lines affected by absorption. Depending on the spectral index of velocity and density fluctuations the contributions from either from density or velocity dominate the integrated intensity fluctuations. When velocity is dominant a very interesting regime for which intensity fluctuations show universal behavior, i.e. the power spectrum $P(K) \sim K^{-3}$ emerges. If density is dominant, the spectral index of intensity fluctuations is the same as in the case an optically thin cloud. Conditions for these regime as well as for some more interesting intermediate asymptotic regimes are outlined in LP04.
5. How can intermittency be studied?

Velocity and density power spectra do not provide a complete description of turbulence. Intermittency of turbulence (its variations in time and space) and its topology in the presence of different phases are not described by the power spectrum. Recent numerical research that employed higher order correlation functions (Muller & Biskamp 2000, Cho, Lazarian & Vishniac 2002b, 2003) showed them to be a promising tool. For instance, the distinction between the old Iroshnikov-Kraichnan and the GS95 model is difficult to catch using power spectra with a limited inertial range, but is quite apparent for fourth order statistics. The difference in physical consequences of whether the turbulence dissipates in shocks or in intermittent vortices may be very substantial. Recent research (see review by Lazarian & Cho 2004) suggests that dissipation of interstellar motions via vortices is very important. Although the scaling of vortex intermittency with the Reynolds number \( R \) is not clear, it is suggestive that the intermittency is increasing with \( R \).

Higher order statistics obtained from observational data were reported for observed velocity\(^4\) in Falgarone et al. (1994) and for density in Padoan et al. 2003. According to Falgarone & Puget (1995) and Falgarone et al. (1995), the intermittency in vorticity distribution can result in the outbursts of localized dissipation that make tiny regions within cold diffuse clouds chemically active. This is an extremely important conclusion that stimulates more intensive studies of higher moment statistics from observations.

6. What are the niches for different techniques?

Astrophysical fluids demonstrate turbulent motions at very different Mach numbers. The “old and good” velocity centroids are shown (LE03, Esquivel & Lazarian 2004) to be reliable only at low Mach numbers. This makes some of the earlier results obtained, e.g. for hypersonic motions in HI using velocity centroids (see Meville-Deschene et al. 2003), as well as some results on molecular clouds somewhat questionable. Observational testing whether the criterion (2) is satisfied is essential for a confident use of centroids to study velocity statistics.

For supersonic turbulence VCA and VCS present the best bet at the moment. The analytical description of intensity fluctuations in PPV space obtained in LP00 and LP04 allows to reliably separate velocity

\(^4\) Whether in all cases the used centroids reflected the actual velocity statistics is not sure because the criterion by Lazarian & Esquivel (2003) has not been applied to the data.
and density contributions to the observed fluctuations. VCS looks to be the most promising tool as it requires only frequency resolution to be adequate. It opens an avenue for studies of turbulence in poorly resolved objects, e.g. for extragalactic research.

Another limitation on the use of the techniques arises from the density-velocity correlations in the data. LP00 analytically studied the effect of velocity-density correlations for the VCA and concluded that even for the maximal possible level of correlation, the scaling of intensity fluctuations in thin velocity channels is not affected. Further numerical studies in Lazarian et al. (2001) and Esquivel et al. (2003) confirmed that the velocity-density correlations present in compressible MHD flows do not change the statistics obtained with the VCA. Our ongoing research shows that velocity centroids seem to be more affected by the velocity-density correlations, but this is not a dominant effect for at least for the low Mach number flows for which the centroids are applicable. Currently we study the effect of correlations arising from gravity.

For studies of higher order statistics velocity centroids present the best bet for the moment. This places limitations on the data sets to be studied (e.g. the criterion (2) should be satisfied), and stimulates the search for an alternative techniques.

We also would like to stress that the different techniques discussed are complementary. For instance, it was noted in LE03 that the velocity centroids are sensitive only to solenoidal motions, while VCA is affected by both potential and solenoidal motions. Therefore combining the two techniques it should be possible to study the effects of compressibility in the astrophysical flows. In addition, different techniques are affected differently by gas temperature. This allows to get insight into the temperature distribution along the line of sight.

Obtaining the statistics of velocity turbulence may provide sometimes unexpected bonuses. For instance, the damping scale of turbulence can be determined if the velocity dispersion is known at the injection scale. This scale could provide a standard yardstick for finding the distances to clouds, e.g. to high velocity clouds.

7. What do observations tell us?

Application of the VCA to the Galactic data in LP00 and to Small Magellanic Cloud in Stanimirovic & Lazarian (2001) revealed spectra of 3D velocity fluctuations consistent with the Kolmogorov scaling. LP00 argued that the same scaling was expected for the magnetized turbulence appealing to the Goldreich-Shridhar (1994) model. Esquivel
et al. (2003) used simulations of MHD turbulent flows to show that in spite of the presence of anisotropy caused by magnetic field the expected scaling of fluctuations is Kolmogorov. Studies by Cho & Lazarian (2002, 2003) revealed that the Kolmogorov-type scaling is also expected in the compressible MHD flows. These studies support MHD turbulence model for SMC.

Studies of turbulence are more complicated for the inner parts of the Galaxy, where (a) two distinct regions at different distances from the observer contribute to the emissivity for a given velocity and (b) effects of the absorption are important. However, the analysis in Dickey et al. (2001) showed that some progress may be made even in those unfavorable circumstances. Dickey et al. (2001) found the steepening of the spectral index with the increase of the velocity slice thickness. They also observed the spectral index for strongly absorbing direction approached $-3$ in accordance with the conclusions in LP04.

21-cm absorption provides another way of probing turbulence on small scales. The absorption depends on the density to temperature ratio $\rho/T$, rather than to $\rho$ as in the case of emission. However, in terms of the VCA this change is not important and we still expect to see emissivity index steepening as velocity slice thickness increases, provided that velocity effects are present. In view of this, results of Deshpande et al. (2001), who did not see such steepening, can be interpreted as the evidence of the viscous suppression of turbulence on the scales less than 1 pc. The fluctuations in this case should be due to density and their shallow spectrum $\sim k^{-2.8}$ may be related to the damped magnetic structures below the viscous cutoff (Cho, Lazarian & Vishniac 2002b).

Studies of velocity statistics using velocity centroids are numerous (see O’Dell 1986, Miesch & Bally 1994, Miesch, Scalo & Bally 1999). The analysis of observational data in Miesch & Bally (1994) provides a range of power-law indexes. Their results obtained with structure functions if translated into spectra are consistent with $E(k) = k^\beta$, where $\beta = -1.86$ with the standard deviation of 0.3. The Kolmogorov index $-5/3$ falls into the range of the measured values. L1228 exhibits exactly the Kolmogorov index $-1.66$ as the mean value, while other low mass star forming regions L1551 and HH83 exhibit indexes close to those of shocks, i.e. $\sim -2$. The giant molecular cloud regions show shallow indexes in the range of $-1.9 < \beta < -1.3$ (see Miesch et al. 1999). It worth noting that Miesch & Bally (1994) obtained somewhat more shallow indexes that are closer to the Kolmogorov value using autocorrelation functions. Those may be closer to the truth as in the presence of absorption in the center of lines, minimizing the regular velocity used for individual centroids might make the results more
reliable. Whether the criterion given by (2) is satisfied for the above
data is not clear. If it is not satisfied, which is quite possible as the
Mach numbers are large for molecular clouds, then the spectra above
reflect the density rather than velocity. If the criterion happen to be
satisfied a more careful study taking absorption effects into account is
advantageous.

Apart from testing of the particular scaling laws, studies of tur-
bulence statistics should identify sources and injection scales of the
turbulence. Is turbulence in molecular clouds a part of a large scale
ISM cascade (see Armstrong et al. 1997)? How does the share of the en-
ergy within compressible versus incompressible motions vary within the
Galactic disk? There are examples of questions that can be answered
in future.

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References

Armstrong, J. W., Rickett, B. J., & Spangler, S. R. 1995, ApJ, 443, 209
Brunt, C.M., Mac Low, M.M. 2004, ApJ, 604, 196
Chepurnov, A. & Lazarian, A. 2004, ApJ, submitted
Cho, J., Lazarian, A. 2002, Phy. Rev. Lett., 88, 245001 (CL02)
Cho, J., Lazarian, A. 2003a, MNRAS, 345, 325
Cho, J., Lazarian, A., Honein, A., Knaepen, B., Kassinos, S., & Moin, P. 2003, ApJ, 589, L77
Cho, J., Lazarian, A., & Vishniac, E. 2002a, ApJ, 564, 291
Cho, J., Lazarian, A., & Vishniac, E. 2002b, ApJ, 566, L49
Cho, J., Lazarian, A., & Vishniac, E. 2003b, ApJ, 595, 812
Deshpande, A.A., Dwarakanath, K.S., Goss, W.M., 2000, ApJ, 543, 227
Elmegreen, B. 2002, ApJ, 577, 206
Elmegreen, B. & Falgarone, E. 1996, ApJ, 471, 816
Elmegreen, B. & Scalo, J. 2004, ARA&A, in press
Esquivel, A. & Lazarian, A. 2004, ApJ, submitted
Esquivel A., Lazarian A., Pogosyan D., & Cho J. 2003, MNRAS, 342, 325
Falgarone, E. 1999, in Interstellar Turbulence, ed. by J. Franco, A. Carraminana,
   CUP, (henceforth Interstellar Turbulence) p.132
Falgarone, E., Lis, D. C., Phillips, T. G., Pouquet, A., Porter, D. H., Woodward, P. R. 1994, ApJ, 436, 728
Falgarone, E., Panis, J.-F., Heithausen, A., Perault, M., Stutzki, J., Puget, J.-L.,
   Bensch, F. 1998, A& A, 331, 669
Falgarone, E. & Puget, J.-L. 1995, A& A, 293, 840
Falgarone, E., Pineau des Forets, G., & Roueff, E. 1995, A&A, 300, 870
Goldreich, P. & Sridhar, S. 1993, ApJ, 438, 763
Green, D.A. 1993 MNRAS, 262, 328
von Horner, S. 1951, Zs.F. Ap., 30, 17
Inogamov, N.A. & Sunyaev, R.A. 2003, Astronomy Letters, 29, 791.
Kolmogorov, A. 1941, Dokl. Akad. Nauk SSSR, 31, 538
Kaplan, S.A. & Pickelner, S.B. 1970
Lazarian, A. & Cho, J. 2004, ApJSS, in press
Lazarian, A. & Esquivel, E. 2003, ApJ, 592, L37 (LE03)
Lazarian, A. & Pogosyan, D. 2000, ApJ, 537, 720 (LP00)
Lazarian, A. & Pogosyan, D. 2004, ApJ, in press (LP04)
Lazarian, A. & Pogosyan, D., & Esquivel, A. 2002, in Seeing Through the Dust, ASP Conf. Proc. Vol 276, eds by A. R. Taylor et al. (San Francisco), p. 182
Lazarian, A. & Pogosyan, D., Vazquez-Semadeni, E., & Pichardo, B. 2001, ApJ, 555, 130
Lazarian, A. & Yan, H. 2003, in “Astrophysical Dust” eds. A. Witt & B. Draine, APS, in press
Levier, F. 2004, ApJ, in press
McKee, Christopher F.; Tan, Jonathan C. 2002, Nature, 416, 59
Miesch, M.S., & Bally, J. 1994, ApJ, 429, 645
Miesch, M. S., Scalo, J., & Bally, J. 1999, ApJ, 429, 645
Miville-Deschenes, M.A., Levrier, F., & Fulgarone, E. 2003, ApJ, in press, Miville-Deschenes, M.A., Joncas, G., Fulgarone, E., & F. Boulanger, 2003, A&A, in press, astro-ph/0306570
Monin, A.S., & Yaglom, A.M. 1975, Statistical Fluid Mechanics: Mechanics of Turbulence, vol. 2, The MIT Press
Müller, W.-C. & Biskamp, D. 2000, Phys. Rev. Lett., 84(3), 475
Narayan, R., & Goodman, J. 1989, MNRAS, 238, 963
Munch, C. 1958, Rev. Mod. Phys., 30, 1035
O’Dell, C.R. 1986, ApJ, 304, 767
Ossenkopf, V. & Mac Low, M.-M. 2002, A&A, 390, 307
Padoan, P., Boldyrev, S., Langer, W., & Nordlund, A. 2003, ApJ, 583, 308
Pudritz, R. E. 2001, From Darkness to Light: Origin and Evolution of Young Stellar Clusters, ASP , Vol. 243. Eds T. Montmerle and P. Andre. San Francisco, p.3
Stutzki, J. 2001, Astrophysics and Space Science Supplement, 277, 39
Stanimirovic, S. & Lazarian, A. 2001, ApJ, 551, L53
Spangler, S.R., & Gwinn, C.R. 1990, ApJ, 353, L29
Sunyaev, R.A., Norman, M.L., & Bryan, G.L. 2003, Astronomy Letters, 29, 783.
Wilson, O.C., Munch, G., Plather, E.M., & Coffeen, M.F. 1959, ApJS, 4, 199
