Equivalence of Chern-Simons gauge theory and WZNW model using a BRST symmetry

Jens Fjelstad

and

Stephen Hwang

Department of Physics,
Karlstad University, S-651 88 Karlstad, Sweden

Abstract

The equivalence between the Chern-Simons gauge theory on a three-dimensional manifold with boundary and the WZNW model on the boundary is established in a simple and general way using the BRST symmetry. Our approach is based on restoring gauge invariance of the Chern-Simons theory in the presence of a boundary. This gives a correspondence to the WZNW model that does not require solving any constraints, fixing the gauge or specifying boundary conditions.

Submitted for publication to Physics Letters B

1email: jens.fjelstad@kau.se
2email: stephen.hwang@kau.se
In a seminal paper [1] on the 2+1 dimensional Yang-Mills theory with an action consisting purely of a Chern-Simons term ("Chern-Simons gauge theory"), Witten showed that the Chern-Simons theory on a compact surface was intimately connected with two-dimensional conformal field theory and more precisely to WZNW models with compact groups. The connection was stated in the somewhat abstract form that the physical Hilbert spaces obtained by quantization in 2+1 dimensions can be interpreted as the spaces of conformal blocks in 1+1 dimensions. A more concrete connection was also suggested by considering the Chern-Simons theory on a manifold with boundary. This connection was further elaborated in [3] and [2], generalizing the connection to gauged WZNW models with compact groups.

Although the treatment in [1]-[3] for a manifold with boundary is explicit it is not entirely satisfactory for several reasons. Firstly, the action on a manifold with boundary is not gauge invariant. A gauge transformation yields boundary terms. This implies that upon quantization there is no symmetry that restricts possible quantum corrections. However, the fact that the gauge invariance is broken at the boundary is precisely what is used in [3], [2] to show that Chern-Simons action reduces to a chiral WZNW action in a particular partial gauge. Thus, it seems that connection between the Chern-Simons theory and the WZNW model can only be made if gauge invariance is broken at the boundary and for a particular partial gauge. Related to this is the fact that in making the connection to the WZNW model one does not encounter the standard chiral WZNW Hamiltonian, but in fact one that is zero [2]. This is a consequence of the way the connection is made by solving the constraints. It really means that we still have an action not suitable for quantization since it is still not gauge fixed.

Secondly, as there is no gauge invariance on the boundary, one may even classically modify the Chern-Simons action with boundary terms. Different boundary terms will yield different boundary actions and different actions for the conformal field theory. An argument due to Regge and Teitelboim [4] may be invoked as a guiding principle for possible boundary terms. For our particular example this is done in [5]. Notice however that the situation in [4] is quite different from ours and we will see that in treating our model we use a completely different principle.

These objections to the explicit correspondence should be contrasted to the general connection involving the Hilbert space of the 2+1 dimensional theory and conformal blocks mentioned above. Although formulated for the case of a compact manifold without boundary and for a compact group, it does not refer to any particular gauge or any particular choice of boundary condition. It is natural to ask...
whether or not a more explicit formulation exists, which still is general with no reference to a particular gauge or choice of boundary condition. Some progress towards such a formulation have been achieved see e.g. [3], [4], but as far as we know there is no formulation which gives the connection between the CS and WZNW theories in a completely general and gauge invariant way and at the level of actions, and which does not depend on any particular boundary conditions. The objective of the present work is to provide such a formulation.

We will see that the correspondence may be formulated in terms of a BRST symmetry. Invariance with respect to the BRST transformations will imply that the two theories - Chern-Simons and WZNW - are equivalent. Particular forms of the action may then be found by different choices of the BRST invariant gauge fixing terms. The crucial step in our approach is to restore gauge invariance of the CS theory in the presence of a boundary. This will be achieved by adding new dynamical degrees of freedom at the boundary, namely the WZNW degrees of freedom. In spirit, our approach is similar to that of Dirac [5], where he advocated the introduction of new degrees of freedom describing the initial quantization surface and corresponding constraints to make the theory reparametrization invariant (see also [6]). An even closer resemblance is to the work on surface terms for Yang-Mills theories [7],[8] in connection with non-Abelian monopole solutions.

An important outcome of our treatment is that the action is completely fixed up to BRST exact terms. It is no longer possible to add boundary terms to the action without spoiling the BRST symmetry (with the exception of non-trivial BRST invariant terms, if they exist). Thus the apparent arbitrariness of adding boundary terms in the original formulation of the CS theory (which however can be partially reduced by appropriate boundary conditions) has been completely eliminated.

The use of the BRST symmetry in connection with the CS theory first appeared in [11]. Their treatment is, however, quite different from ours as gauge invariance is not restored (it uses a particular gauge and no new dynamical degrees of freedom are introduced at the boundary) and certain boundary conditions are assumed. Also the connection to the CS theory is not achieved on the level of actions.

Recently, the interest in the Chern-Simons theory has grown substantially due to the work of Maldacena [12] on the correspondence between string theory on anti-de Sitter spaces and certain conformally invariant theories associated with the boundary. This correspondence was formulated as the so-called holographic principle [13],[14] (ideas originally presented in [15],[16]). As our work here deals with a special (and simple) case of the holographic principle it may be of some interest that there exists
a simple and elegant way to implement this principle. However, due to the simplicity of the model with no propagating degrees of freedom in the bulk, it is not clear that this may be of use for a more non-trivial example.

The motivation of our work does not primarily come from AdS/CFT correspondence in itself, but rather from our desire to understand the calculations of the entropy of the 2+1 dimensional black hole presented in [17] (see also [18]). In particular, we wanted to understand the problem of choice of boundary terms in connection with the BTZ [19] black hole. Our results in connection with this problem will be presented elsewhere [20].

We start by briefly recalling the correspondence between Chern-Simons theory on a manifold with boundary and WZNW models as formulated in [1]-[3]. The Chern-Simons action is

\[ I_{CS} = -\frac{k}{2} \int_{\mathbb{R} \times \Sigma} Tr(A \wedge dA + \frac{2}{3} A \wedge A \wedge A). \] (1)

The action is invariant under the gauge transformation \( \delta A = d\epsilon + [A, \epsilon] \) if \( \Sigma \) has no boundary. If \( \Sigma \) has a boundary the variation yields a boundary term

\[ \delta I_{CS} = -\frac{k}{2} \int_{\mathbb{R} \times \partial \Sigma} Tr(\epsilon dA). \] (2)

The invariance of the action may be achieved by requiring \( \epsilon = 0 \) on the boundary. This is equivalent to saying that the full gauge group \( G \) is reduced to all gauge transformations \( G_1 \), that are one on the boundary. The variation of \( A_0 \) yields a constraint \((i, j = 1, 2)\)

\[ \epsilon^{ij} F_{aij} \approx 0, \ a = 1, \ldots n. \] (3)

This constraint implies that the vector field is pure gauge i.e. \( A_i = -\partial_i U U^{-1}, \) for some map \( U : \Sigma \rightarrow G \). The gauge invariance implies that we have an equivalence relation \( U \sim VU \) for any \( V \) that is one on the boundary. This implies that only the restriction of \( U \) to the boundary is relevant. If we use \( A_i = -\partial_i U U^{-1}, \ i = 1, 2, \) and fix the gauge partially by \( A^0 = 0, \) the Chern-Simons action reduces to

\[ I_{CS} = -\frac{k}{2} \left( \int_{\mathbb{R} \times \partial \Sigma} Tr(U^{-1} \partial_a U U^{-1} \partial_t U + \frac{1}{3} \int_{\mathbb{R} \times \Sigma} Tr(U^{-1} dU U^{-1} dU U^{-1} dU) \right). \] (4)

Here we have chosen \( \Sigma = D, \) a disc of radius \( R \) with a radial coordinate \( x^1 = r \) and an angular coordinate \( x^2 = \phi. \) We will for convenience make this choice for the remaining part of the paper. As remarked above, the Hamiltonian corresponding to this action is easily checked to be zero. Furthermore, the action corresponds
to a chiral WZNW action, where the current $J(z)$ in the conventional form of the WZNW theory here is identified with $A_{\phi} = -\partial_\phi UU^{-1}$, as can be seen from the form of the action (4). The Dirac brackets of these currents give that the current satisfies an affine Lie algebra of level $k$ \[[21]\]. If we add to the action boundary terms e.g. a term $-\frac{k}{2} \int_{R \times \partial \Sigma} A_\mu A_\nu C^{\mu\nu}$, for some matrix $C^{\mu\nu}$ ($\mu, \nu = 0, 1, 2$), then inserting again $A_i = -\partial_i UU^{-1}$ leads to modifications of (4) by boundary terms, which in turn implies that the resulting action is not the conventional chiral WZNW action. A boundary term of this form (for a particular $C^{\mu\nu}$) was e.g. suggested in \[[17]\] for the BTZ black hole.

We now proceed to find an alternative way of realizing the connection between the two theories. The Hamiltonian corresponding to the original action (1) is

$$H_{CS} = 2 \int_D d^2x \psi_\lambda A_0^\lambda - \frac{k}{2} \int_{\partial D} d\phi A_0^a A_{a0}.$$  \hspace{1cm} (5)

Let us neglect the surface term for the moment. Then the primary and secondary constraints are

$$P_a \approx 0 \text{ and } \psi_\lambda \equiv \frac{k}{4} \epsilon^{ij} F_{aij} \approx 0.$$ \hspace{1cm} (6)

Here $P_a$ is the momentum conjugate to $A_0^a$. Using the Poisson bracket (PB) 

$$\{A_0^a(x), A_0^b(x')\} = \frac{2}{k} \delta^a_b \delta^2(x - x')$$

one finds

$$\{\psi_\lambda(x), \psi_{\lambda'}(x')\} = f^{\lambda \lambda'}_{ab} \delta^2(x - x') - k (f^{\lambda \lambda'}_{ab} A_{c2} \delta^2(x - x') - \eta_{ab} \partial_\phi \delta^2(x - x') \delta(x^1 - R)).$$ \hspace{1cm} (7)

As the appearance of the boundary terms involving the delta function $\delta(x^1 - R)$ may seem surprising and is important in what follows, we will explain the calculation in a little more detail. In computing the PB one needs to use certain delta-function identities. These identities may be derived using test functions, that are usually assumed to be zero on the boundary. Then the constraint algebra may be shown to close. The extra terms on the right hand side of eq.(7) appear if one generalizes the delta-function identities to hold on test functions that are arbitrary on the boundary.\[3\]

The breakdown of a first class algebra due to a boundary was noted previously in \[[8]\] for Yang-Mills theory, in \[[22]\] for gravity in the Ashtekar formalism, and in \[[11]\] for the boundary was noted previously in \[[8]\] for Yang-Mills theory, in \[[22]\] for gravity in the Ashtekar formalism, and in \[[11]\] for gravity in the Ashtekar formalism.

\[3\] The practical way of making all computations here and below is in fact to use $\psi_\lambda = \int_D d^2x \psi_\lambda(x) \lambda^\lambda(x)$ for some $L_1$ functions on $D$ and compute $\{\psi_\lambda, \psi_{\lambda'}\}$. An alternative way of computing the PB is to introduce a complete set of functions on $D$ and then by using the PB of the modes with respect to this set, the PB can be computed yielding the same result.
for the present case. In [23] and [24] boundary corrections for Poisson brackets in general are discussed.

The appearance of the extra terms in eq.(7) implies that the constraints are not first class on the boundary i.e. gauge invariance is broken by boundary terms. This is of course connected to the fact that the action is not gauge invariant due to boundary contributions. It implies that a gauge may not be fixed on all of $D$. The breaking of gauge invariance occurs only at the boundary. If $x,x'$ are interior points, then the extra terms in eq.(7) are zero and the constraints are first class. Further implications of the breakdown of gauge invariance is that the time evolution of the secondary constraints yields additional constraints.

In the careful computation of eq.(7) lies also the hint on how to proceed. It is well-known that second class constraints may be converted into first class constraints by adding new degrees of freedom, whereby restoring the gauge symmetry of the theory. This will be the strategy in the following. Firstly one notices that $\tilde{\psi}_a(x) \equiv \psi_a(x) - \frac{k}{2} A_{a2}(x) \delta(x^1 - R)$ satisfies $\{\tilde{\psi}_a(x), \tilde{\psi}_b(x')\} = -f_{ab}^c \psi_c(x) \delta^2(x - x') - k \eta_{ab} \partial_\phi \delta^2(x - x') \delta(x^1 - R)$, so that the modification due to boundary terms are field independent. If we define a level $k$ WZNW current $J_a(t,\phi)$ (we will suppress the $t$-dependence henceforth) satisfying

$$\{J_a(\phi), J_b(\phi')\} = -f_{ab}^c J_c(\phi) \delta(\phi - \phi') + k \eta_{ab} \partial_\phi \delta(\phi - \phi'), \tag{8}$$

then

$$\psi'_a(x) \equiv \psi_a(x) - \frac{k}{2} A_{a2}(x) \delta(x^1 - R) - J_a(\phi) \delta(x^1 - R) \tag{9}$$

satisfies the closed algebra $\{\psi'_a(x), \psi'_b(x')\} = -f_{ab}^c \psi'_c \delta^2(x - x')$. Having first class constraints we now define a Hamiltonian

$$H' = 2 \int_D d^2 x A_0^a \psi'_a = H_{CS} + \int_{\partial D} d\phi (-\frac{k}{2} A_0^a A_{a2} + 2 J_a A_0^a). \tag{10}$$

This Hamiltonian is automatically gauge invariant even at the boundary. Notice that gauge invariance forces a modification of the Chern-Simons action by a unique boundary term in the vector fields. Let us pause and comment on the constraints of the new theory. These are given by $\psi'_a \approx 0$. Examining eq.(8) we see that these constraints consist of two pieces, a bulk and a boundary part. The boundary part enters much like a source term. For any point in the interior of $D$ we simply have the original constraints $\epsilon^{ij} F_{aij} \approx 0$. If we consider field configurations $A_i^a$ that are continuous and have continuos spatial derivatives at the boundary, then they will also satisfy $\epsilon^{ij} F_{aij} \approx 0$ on $\partial D$. In this case the constraint implies that $A_{a2}(\phi) \approx -\frac{k}{2} J_a(\phi)$.
on $\partial D$. Thus, we recover the connection between WZNW currents and $A^a_2$ mentioned above in the original formulation. For field configurations that are not smooth at the boundary this relation does not hold. It is not clear to us what the implications of this generalization mean.

A simple counting of the degrees of freedom shows that the additional degrees of freedom associated with the currents $J_a$, which are $n$ phase space degrees of freedom at every space point on the boundary, compensate exactly that the $n$ constraints are transformed from second class to first class constraints on the boundary. Let us now show in more detail that the theory defined by this Hamiltonian is equivalent to the Chern-Simons theory defined by the action (1) and a specific boundary term added to it.

First we impose the partial gauge $A^a_0 \approx 0$. Then $H_{CS} \approx H' \approx 0$ so that the resulting actions are identical. It remains to show that we can break the gauge invariance by imposing gauge fixing constraints on the boundary that eliminate the new degrees of freedom. We consider a gauge constraint $J_a(\phi)\delta(x^1 - R) \approx 0$. To see that this fixes the gauge at the boundary consider its gauge variation with parameter $\lambda_a$

$$\left\{ \int_D d^2x \psi'_b(x)\lambda^b(x), J_a(\phi')\delta(x^1 - R) \right\} \approx -\frac{k}{2} \partial_{\phi} \lambda_a(R, \phi').$$

Equating this to zero, we find $\partial_{\phi} \lambda_a(R, \phi') = 0$, from which we conclude that the gauge is fixed at the boundary apart from the zero mode part of $\psi'_a$. Neglecting this detail for the moment, the gauge fixing constraint eliminates the WZNW current degrees of freedom reducing the constraints $\psi'_a \approx 0$ to $\tilde{\psi}_a \approx 0$, which are second class on the boundary. Thus we are back to the original Chern-Simons theory (with a specific boundary term).

Let us now comment on the zero mode part. The gauge fixing of the zero mode part, $\int d\phi J_a(\phi) \approx 0$, is not a valid gauge choice. We may notice that in fact the $\phi$-independent part of the constraint generators $\psi'_a$ satisfy a closed algebra. Thus, we have really introduced slightly more degrees of freedom than necessary. This may be changed by subtracting the zero mode of $J_a$ from $\psi'_a$. Then the $\phi$-independent part of $A^a_2$ will play the rôle of the zero mode of the current. In the following we will for simplicity not concern ourself with this subtlety and leave $\psi'_a$ unaltered.

We have already seen that our theory reduces to the Chern-Simons theory for a particular choice of partial gauge. We will now show that for another choice of gauge we may eliminate the vector field degrees of freedom leaving us with only the WZNW degrees of freedom, thus establishing classically the equivalence of the two
theories. Imposing the gauge constraints $A_0^a \approx A_1^a \approx 0$, it is easily checked that this fixes the gauge both in the interior of $D$ and at its boundary. This eliminates all vector field degrees of freedom (as $A_2^a$ is conjugate to $A_1^a$). The Dirac bracket is, therefore, only non-zero for brackets containing the WZNW degrees of freedom and, therefore, we have found the promised reduction. Possible forms for the Hamiltonian will be discussed below.

We have established the connection between the Chern-Simons theory and the chiral WZNW model by imposing different gauges. A more fundamental way of manifesting the equivalence is to introduce a BRST charge. As the constraints are first class this is straightforward using the BFV formalism \[25\], \[26\]. We have

$$
\Omega \equiv \int_D d^2x \left[ \frac{1}{4} \epsilon^{ijkl} F_{aij} c^a + \frac{1}{2} f_{ab} c^a c^b + P_a \bar{b}^a \right] - \int_{\partial D} d\phi \left[ \left( \frac{1}{2} A_{a2} + J_a \right) c^a \right].
$$

(12)

We note that the BRST charge consists of two distinct parts. In the first line we have a bulk part and in the second a boundary part. The ghosts $c^a, \bar{b}^a$ and their momenta $b_a, \bar{c}_a$ satisfy the usual PB. It is easily checked that $\{\Omega, \Omega\} = 0$. A BRST invariant Hamiltonian may now be constructed in the standard way

$$
H_{\text{tot}} = \{\Omega, \chi\}
$$

(13)

for some gauge fermion $\chi$. We will now discuss some possible choices of $\chi$.

(i) $\chi = \int_D d^2x \left( b_a A_0^a + \bar{c}_a \left( \hat{A}_0^a + \chi'^a [A] \right) \right) + \int_{\partial D} d\phi \bar{c}_a J^a,$

where $\chi'^a$ is some gauge fixing functional of the vector fields. Inserting this into (13) yields a Hamiltonian

$$
H_{\text{tot}} = \int_D d^2x \left[ P_a \hat{A}_0^a + \bar{b}_a c^a + \psi_{\text{tot}}^a A_0^a + P_a \chi'^a + c^a \left\{ \psi'_{\text{tot}}^a, \chi'^b \right\} \bar{c}_b \right] + \int_{\partial D} d\phi \left[ f_{ab} J^b c^a + P_a J^a \right].
$$

(14)

Here $\psi_{\text{tot}}^a \equiv \{\Omega, b_a\}$. $P_a$ acts as Lagrange multipliers for the gauge fixing constraints. The eq. of motion for $A_0^a$ give $\chi'_a + J_a \delta(x^1 - R) = 0$, which for smooth vector fields at the boundary yields $\chi'_a = 0$ on $D$ and $J_a = 0$ on $\partial D$. This Hamiltonian, therefore, corresponds to eliminating the WZNW degrees of freedom.

(ii) $\chi = \int_D d^2x \left( b_a A_0^a + \bar{c}_a \left( \hat{A}_0^a + A_1^a \right) \right) + \int_{\partial D} d\phi \bar{c}_a \left( J^a - \frac{q_a}{2} P^a \right).

This choice gives a Hamiltonian

$$
H_{\text{tot}} = \int_D d^2x \left[ P_a \hat{A}_0^a + \bar{b}_a c^a + \psi_{\text{tot}}^a A_0^a + P_a A_1^a + c^a \left\{ \psi'_{\text{tot}}^a, A_1^b \right\} \bar{c}_b \right] + \int_{\partial D} d\phi \left[ f_{ab} J^b c^a + P_a (J^a - \frac{q_a}{2} P^a) \right].
$$

(15)
Here the eq. of motion for $P_a$ imply $A_1^a + (J^a - \alpha P^a)\delta(x^1 - R) = 0$, which for smooth fields give $A_1^a = 0$ in $D$ and $J^a - \alpha P^a = 0$ on $\partial D$. This choice corresponds to eliminating the vector field at the boundary. Inserting the second equation into the Hamiltonian gives

$$H_{tot} = \int_{\partial D} d\phi \left[ \frac{1}{2\alpha} J^a J_a + f_{ab} J_c \bar{c}^a c^b \right] + H_{bulk}. \quad (16)$$

The first term is precisely the Sugawara Hamiltonian (for an appropriate choice of $\alpha$). The second term is a ghost correction at the boundary. The third term is a bulk Hamiltonian. Since the degrees of freedom are completely eliminated in the bulk, this term will not contribute to any Dirac brackets. In fact, we may set this term to zero and still have a BRST invariant Hamiltonian.

Other choices of gauge fermions are of course possible e.g. one which makes the Lagrange multiplier field $A_0^a$ dynamical. We will, however, not discuss this further. Instead let us end with some concluding remarks.

We have demonstrated the classical equivalence between the Chern-Simons gauge theory and the chiral WZNW model. It should be emphasized that the equivalence between the bulk and boundary theories follows without any specifications of gauge or boundary conditions. It is the gauge invariance or more generally the BRST invariance that makes the different choices of Hamiltonian physically equivalent. The quantum equivalence is ensured once the nilpotency of the BRST operator is established and non-trivial BRST invariant states are found. Thus, for this particular example one may state that the holographic principle is translated into a statement about BRST symmetry.

Our considerations have for simplicity been for the disc, but they may easily be generalized to other cases. Then the delta function $\delta(x^1 - R)$ is replaced by a generalized delta function $\delta(x \in \partial \Sigma)$, which is defined by $\int_{\Sigma} [f(x) \delta(x \in \partial \Sigma)] = \int_{\partial \Sigma} f(x)$. Note also that it is irrelevant where the boundary actually is i.e. we have the freedom to move it at will. This is a freedom that follows from our formulation as it does not rely on any particular boundary conditions. For the case of the black hole it will imply that the calculation of the entropy will be (almost) independent of where we actually put the boundary. We can choose the horizon, but just as well another surface. We will discuss this in more detail in [20].

Acknowledgement: We would like to thank Steve Carlip for enlightening discussions concerning his work. We are also indebted to Ioannis Bakas for pointing out the work in [3] and [8].
References

[1] E. Witten, Comm. Math. Phys. **121** (1989) 351

[2] G. Moore and N. Seiberg, Phys. Lett. **B220** (1989) 422

[3] S. Elitzur, G. Moore, A. Schwimmer and N. Seiberg, Nucl. Phys. **B326** (1989) 108

[4] T. Regge and C. Teitelboim, Ann. Phys. (N.Y.) **88** (1974) 286

[5] M. Bañados, Phys. Rev. **D52** (1995) 5816

[6] P.A.M. Dirac, Phys. Rev. **114** (1959) 924

[7] J.L. Gervais, B. Sakita and S. Wadia, Phys. Lett. **63B** (1976) 55

[8] S. Wadia, Phys. Rev. **D15** (1977) 3615

[9] A.P. Balachandran, G. Bimonte, K.S. Gupta and A. Stern, Int. J. Mod. Phys **A7** (1992) 4655

[10] M.-I. Park, Nucl. Phys. **B544** (1999) 377

[11] A. Blasi and R. Collina, Phys. Lett. **B243** (1990) 99

[12] J. Maldacena, Adv. Theor. Math. Phys **2** (1998) 231-252

[13] G.'tHooft, *Dimensional Reduction in Quantum Gravity*, [hep-th/9310026](http://arxiv.org/abs/hep-th/9310026)

[14] L. Susskind, J.Math.Phys. **36** (1995) 6377-6396

[15] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, Phys. Lett. **B428** (1998) 105

[16] E. Witten, Adv.Theor.Math.Phys. **2** (1998) 253-291

[17] S. Carlip, Phys. Rev. **D51** (1995) 632

[18] S. Carlip, Class. Quantum Gravity **15** (1998) 3609

[19] M. Bañados, C. Teitelboim and J. Zanelli, Phys. Rev. Lett **69**(1992) 1849

[20] J. Fjelstad and S. Hwang, in preparation.

[21] E. Witten, Comm. Math. Phys. **92** (1984) 455

[22] V.O. Soloviev, Phys. Lett. **B292** (1992) 30
[23] K. Bering, *Putting an edge to the Poisson bracket*, hep-th/9806042

[24] V.O. Soloviev, *Bering’s proposal for boundary contribution to the Poisson bracket*, hep-th/9901112

[25] E.S. Fradkin and G.A. Vilkovisky, Phys. Lett 55B (1975) 224

[26] I.A. Batalin and G.A. Vilkovisky, Phys. Lett 69B (1977) 309