The Role of Cerenkov Radiation in the Pressure Balance of Cool Core Clusters of Galaxies

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Abstract

Despite the substantial progress made recently in understanding the role of AGN feedback and associated non-thermal effects, the precise mechanism that prevents the core of some clusters of galaxies from collapsing catastrophically by radiative cooling remains unidentified. In this Letter, we demonstrate that the evolution of a cluster’s cooling core, in terms of its density, temperature, and magnetic field strength, inevitably enables the plasma electrons there to quickly become Cerenkov loss dominated, with emission at the radio frequency of \( \lesssim 350 \text{ Hz} \), and with a rate considerably exceeding free–free continuum and line emission. However, the same does not apply to the plasmas at the cluster’s outskirts, which lacks such radiation. Owing to its low frequency, the radiation cannot escape, but because over the relevant scale size of a Cerenkov wavelength the energy of an electron in the gas cannot follow the Boltzmann distribution to the requisite precision to ensure reabsorption always occurs faster than stimulated emission, the emitting gas cools before it reheats. This leaves behind the radiation itself, trapped by the overlying reflective plasma, yet providing enough pressure to maintain quasi-hydrostatic equilibrium. The mass condensation then happens by Rayleigh–Taylor instability, at a rate determined by the outermost radius where Cerenkov radiation can occur. In this way, it is possible to estimate the rate at \( \approx 2 M_\odot \text{ year}^{-1} \), consistent with observational inference. Thus, the process appears to provide a natural solution to the longstanding problem of “cooling flow” in clusters; at least it offers another line of defense against cooling and collapse should gas heating by AGN feedback be inadequate in some clusters.

Key words: galaxies: clusters: general

1. Introduction: Statement of the Problem

It has been known for some time (Lea et al. 1973; Cowie & Binney 1977; Fabian & Nulsen 1977) that the plasma conditions in the center of many clusters of galaxies enable the region to cool by free–free emission on timescales well within one Hubble time and, consequently, one expects copious star formation and central mass deposition triggered by this very large cooling rate. Specifically, the cooling time of free–free continuum emission is

\[
t_{\text{ff}} = 4.24 \times 10^8 \left( \frac{T}{5 \times 10^5 \text{ K}} \right)^{1/2} \left( \frac{n_e}{10^{-2} \text{ cm}^{-3}} \right)^{-1} \text{ years},
\]

and the inclusion of line cooling would shorten this time by \( \approx 40 \) times for gas at temperatures of \( \approx 5 \times 10^5 \text{ K} \) and an abundance of 0.5 solar; see Figure 1 of Schure et al. (2009; note the figure assumes full solar abundance). Such losses are responsible for a theoretical mass deposition rate of

\[
M = \frac{2 L_X \mu m_p}{5 kT} = 100 \left( \frac{L_X}{10^{44} \text{ erg s}^{-1}} \right) \left( \frac{kT}{3 \text{ keV}} \right)^{-1} \times \left( \frac{\mu}{0.5} \right) M_\odot \text{ year}^{-1},
\]

where \( L_X \) is the X-ray luminosity of the cool core and \( kT \) is the temperature. The actual \( M \) inferred from observations at various wavelengths is inconsistent with such a large value, and the ensuing “cooling flow” problem has lasted for several decades despite numerous attempts in finding a solution; see, e.g., the rather detailed analysis of Hudson et al. (2010).

Although there has recently been a lot of progress made in ascertaining the role of the central AGN in terms of its feedback effects (e.g., Gaspari et al. 2013; Dasadia et al. 2016), the exact physical mechanism that provides the necessary pressure to hold off the inflow and further cooling of such large quantities of matter remains a mystery.

2. Cerenkov Radiation in the Magnetized Intracluster Medium and Its Competition Against Free–Free Emission

In this Letter, we suggest that another, hitherto ignored, emission process at play in the plasma under the relevant conditions may hold the key to the cooling flow problem. It is known that the plasma of the intracluster medium (ICM) has a frozen-in magnetic field, which for the cool cluster cores can be as high as 30–50 \( \mu \)G (Taylor & Perley 1993; Taylor et al. 2006; Fabian et al. 2008) or as low as a few \( \mu \)G (Govoni et al. 2006; McNamara & Nulsen 2012). As conditions of the core, we further assume that the central temperature can cool to \( kT = 0.05 \text{ keV} \) and the density reaches \( n_e = 0.01 \text{ cm}^{-3} \) so that the cooling time is

\[
t_{\text{cool}} \approx 10^7 \text{ years}
\]

as obtained from (1) after taking into account the factor of 40 decrement due to line emission, as mentioned in the text after (1). Note that in this parameter regime of choice the magnetic pressure equals the gas pressure, while in the outskirts of the cluster where \( B \approx 1 \mu \text{G} \) and \( n_e \approx 10^{-3} \text{ cm}^{-3} \) the gas pressure dominates. Indeed, the importance of magnetic and cosmic-ray pressure in cluster cool cores was addressed (see, e.g., Laganá et al. 2010 and references therein).
Turning to the focus of this Letter, we wish to point out that in addition to free–free continuum radiation and resonance line emission, electrons in the magnetized plasma of cool cluster cores can also lose energy by Cerenkov radiation (emission, electrons in the magnetized plasma of cool cluster cores can also lose energy by Cerenkov radiation (McKenzie 1963, 1967; Ginzburg1979; Melrose & McPhedran 1991) provided the criterion

$$\cos^2 \theta = \frac{\omega^2}{\mu^2(\theta) \nu^2}$$

is fulfilled. In (4), \(\theta\) is the angle w.r.t. the local magnetic field \(B\) at which the radiation is emitted, \(\nu\) is the speed of the electron parallel (or anti-parallel) to \(B\), and \(\mu = \mu(\theta)\) is the refractive index of the magnetoplasmonic medium that depends on the frequency of the emitted wave as well as \(\theta\) in a complicated way. Yet, as noted on p600 of the seminal paper of McKenzie (1963), provided the magnetic field is strong enough and \(\nu\) is small, such that the cyclotron frequency \(\Omega = eB/m_e\) satisfies the inequality

$$\frac{\nu^2 \omega^2_p}{c^2 \Omega^2} < 1,$$

where \(\omega_p = \sqrt{ne^2/(\varepsilon_0 m_e)}\) is the electron plasma frequency, there exists a principal mode of emission extending from \(\omega = 0\) to a maximum frequency of

$$\omega_m = \Omega \left(1 - \frac{\nu^2 \omega^2_p}{c^2 \Omega^2}\right),$$

the relevance of which we shall demonstrate. The radiation is fairly isotropic if \(\nu \ll c\). Note also in passing that there is a secondary mode of emission (both modes are consequences of the Alfvén–Whistler approximation limit), by which the electron loses energy at a relatively negligible rate, and will not be discussed here.

Before continuing further, one should be aware of the caveats. The underlying assumptions of the last three equations are threefold. First, the wavelength \(\lambda\) of the emitted radiation far exceeds the size of any homogeneous subregion of the ICM within which \(B\) and \(\omega_p\) do not vary spatially. Since the typical scale size of field smoothness is of the order of the gyroradius of a proton in the same field \(B\), this size is indeed \(\ll \lambda\). Second, the frequency of collisions of plasma particles is small w.r.t. \(\Omega\) and \(\omega_p\) (the approximation of a collisionless plasma); as we shall show below in (14), this too is always the case for the ICM. Third, the results apply to the regime of a cold plasma, i.e., if thermal motions are included there will be relatively small corrections, which we neglected.

Another interesting feature about (5) is that although the criterion holds for the cooling core parameters, it does not for the outskirts of a cluster where \(\nu \approx 8 \times 10^8\) cm s\(^{-1}\) \((kT \approx 5\) keV\), \(n_e \approx 10^{-3}\) cm\(^{-3}\), and \(B \approx 1\) \(\mu\)G. The plasma is essentially unmagnetized and cannot support Cerenkov radiation.

Hence, Cerenkov losses are significant only in the core. The spectral emissivity of this mode is

$$\frac{dl}{d\omega} = \frac{e^2 \mu_0 e^2}{4\pi \nu \omega^2_p \Omega^2} (\Omega^2 - \omega^2)$$

(see Equations (6.2) of McKenzie 1963 and (33) of McKenzie 1967). The total power emitted into the frequency range \(0 < \omega < \Omega\) is

$$I = 1.16 \times 10^{-12} \left( \frac{\nu}{2.37 \times 10^8\ \text{cm s}^{-1}} \right)^{-1} \left( \frac{B}{20\ \mu\text{G}} \right)^4 \left( \frac{n_e}{10^{-2}\ \text{cm}^{-3}} \right)^{-1} \text{eV s}^{-1},$$

(8)

where

$$\Omega = \frac{e B}{m_e} = 351.2 \left( \frac{B}{20\ \mu\text{G}} \right) \text{Hz},$$

(9)

\(\nu\) is the projected average speed along the \(+B\) or \(-B\) direction of the particles in an isotropic thermal gas, namely,

$$\nu = \frac{2kT}{\pi m_e} \approx 2.37 \times 10^{8} \left( \frac{kT}{0.05\ \text{keV}} \right)^{1/2}\ \text{cm s}^{-1}.$$  

(10)

In enlisting only the parallel component of \(\nu\) for the sake of simplicity, one may underestimate \(dl/d\omega\) somewhat. The power spectrum associated with the full helical motion of the electron may be found in McKenzie (1967).

### 3. Comparison to Conventional Cooling Rate: Reabsorption

In (8), the reader is alerted to the particle density and speed dependence of the Cerenkov intensity; they are counterintuitive because one is used to thinking about a conventional emissivity like free–free and resonant transitions, which increase with \(n_e\) and \(\nu\). In fact, from (8) one can readily estimate the lifetime of a \(kT = 0.05\) keV emitting electron, as

$$t_{\text{Cerenkov}} \approx 1.44 \times 10^6 \left( \frac{kT}{0.05\ \text{keV}} \right)^{3/2} \left( \frac{B}{20\ \mu\text{G}} \right)^{-4} \times \left( \frac{n_e}{10^{-2}\ \text{cm}^{-3}} \right) \text{years},$$

(11)

which is considerably shorter than the lifetime against free–free and line cooling (3). It is also shorter than the lifetime against
cooling by cyclotron radiation

\[ t_{\text{cyclotron}} \approx 4.1 \times 10^{11} \left( \frac{kT}{0.05 \text{ keV}} \right)^{-1} \left( \frac{B}{20 \mu \text{G}} \right)^{-2} \text{years} \]

(12)

by six orders of magnitude.

Thus, it is clear that cool cluster cores possess certain unique characteristics not commonly shared by other astrophysical environments, namely, they are venues where Cerenkov radiation can take place on large, 10–100 kpc, scales. In Figure 1, we plot the critical temperature and density values for which \( t_{\text{Cerenkov}} = t_{\text{cool}} \) at two values of \( B \), where \( t_{\text{cool}} \) is the X-ray cooling time, assumed to be shorter than (1) by 40 times because of line emission. One can see that even for fields as weak as a few \( \mu \text{G} \) the gas temperature cannot cool below 0.5 keV at a density of \( n_e = 0.01 \text{ cm}^{-3} \). It should be emphasized, however, that Cerenkov radiation can prevent cooling flow only when the parameters of the entire cluster core place it below the corresponding critical curve. Nevertheless, the fact that for every parcel of cooling gas in the ICM this must happen at some stage can also be understood from general physical arguments. If the ICM cools and condenses, \( B \propto 1/r^2 \) by flux conservation and \( n_e \propto 1/r^3 \) by mass conservation. The quantity on the left side of (5) that is proportional to \( n_e/B^2 \propto r \) would then decrease with \( r \) to quickly evolve the gas to satisfy this inequality of the Cerenkov condition, and the lifetime against Cerenkov radiation (11) would decrease even faster, as \( n_e/B^2 \propto r^2 \) to become less than the X-ray cooling time. Indeed, Soker & Sarazin (1990) provided a formula for the radius at which the magnetic pressure of a cooling flow cluster will inevitably dominate the gas pressure. Thus, the purpose of this Letter is to point out that even in the absence of heating by AGN feedback, there exists another mechanism that may offer a second line of defense against cooling flow.

Apart from the existence of this radiation, there is the question of its fate after emission. Unlike the X-ray, EUV, and visible photons, which can usually stream out of the optically thin core, Cerenkov radiation consists of very low frequency radio waves (9), which can only propagate in the magnetionic medium of the core at fixed angles to the local magnetic field (McKenzie 1963). Since the field lines are unlikely to be smooth and radially directed on 10–100 kpc scales, the radiation will not escape the core region; even if they do, the outer ICM region of relatively unmagnetized plasmas has a plasma frequency far in excess of (9), i.e., it will reflect the radiation back to the center.

Can the Cerenkov photons be re-absorbed by the core and reheat the gas there? Since they are unable to escape, the answer is: yes, eventually. In fact, even for an isotropic unmagnetized plasma, provided the temperature is finite, longitudinal plasma waves can propagate in lieu of electromagnetic waves, and the reabsorption of such waves is the same process as Landau damping (see Ginzburg 1979, p. 142 and 260). Yet, there is another related point that must also be taken into account. Absorption is effective only if its rate is higher than stimulated emission. For a perfectly Maxwellian thermal gas, the rate of absorption is indeed higher than stimulated emission, fractionally by the amount \( 1 - \exp \left( -\frac{\omega}{\lambda \omega} \right) \approx \frac{\omega}{\lambda \omega} \) (see, e.g., Bekefi 1966).

Since the maximum emitted frequency \( \omega \) corresponds to \( \omega \approx 4.4 \times 10^{-15} \), the Maxwell–Boltzmann energy distribution \( f(\epsilon) \propto \exp \left( -\epsilon / (kT) \right) \) has to be extremely stable to ensure that absorption is always higher, everywhere in the gas. In fact, it is reasonable to assert that if the emitted Cerenkov radiation is re-absorbed and cannot as a result exert any formidable pressure on the gas, the mean energy fluctuation \( \delta \epsilon / \epsilon \) of a particle in any volume of gas of size \( \approx 1 \) radiation wavelength \( \lambda \) has to be \( \ll \hbar \omega / (kT) \). Given that \( \lambda^3 \approx 1.6 \times 10^{-26} \text{ cm}^3 \), for \( n_e \approx 0.01 \text{ cm}^{-3} \), one has \( N \approx 10^{24} \) particles, so that \( \delta \epsilon / \epsilon \approx 1/\sqrt{N} \approx 10^{-12} \gg \hbar \omega / (kT) \); the gas is not stable enough to ensure absorption prevails. One must therefore assume, due to the much larger random fluctuations in the particle energy, there is population inversion among half of the electrons, i.e., at least a significant fraction of the emitted Cerenkov radiation can survive absorption. Of course, the absorbing fraction of the electron population cannot reheat itself to the extent of taking back from the radiation the energy of the remaining (unabsorbed) fraction. The net result is continuous cooling of the electrons, as they keep emitting more net radiation than they can absorb, until all their energy is converted to Cerenkov radiation.

Can the gas be reheated by electron scattering with the Cerenkov photons? Since the photon energy \( \hbar \omega \) is smaller than the electron, there can be no energy transfer from the former to the latter. The only possibility is the nonlinear Compton process, but this requires the incident wave to have a sufficiently large amplitude, satisfying the condition \( \Gamma = eE/(m_e c \omega) \gg 1 \) with \( \Gamma = E^2 / (3 m_e c^2 kT/2) \) (see, e.g., Sokolov & Ternov 1986, p. 266). The reader can readily verify that this \( \Gamma \) parameter of the Cerenkov radiation here is not large enough; in fact,

\[ \Gamma = 0.123 \left( \frac{\eta}{0.30} \right)^{1/2} \left( \frac{kT}{0.05 \text{ keV}} \right)^{1/2} \left( \frac{\omega}{350 \text{ Hz}} \right)^{-1}, \]

(13)

where \( \eta \) is the fraction of gas energy converted to Cerenkov radiation. In any case, the mean free path of electron scattering is much larger than the size of the cluster core for there to be meaningful acceleration of gas electrons by this mechanism.

4. Pressure of Cerenkov Radiation, Quasi-hydrostatic Equilibrium, and Limit to the mass Deposition Rate

For reasons explained in the last section, the energy of the electrons in the thermal gas of a cluster’s cool core is emitted as Cerenkov radiation at a sufficiently early stage, before further cooling by X-ray and EUV emission can act. In this way, the entire gas will lose its energy because the ions will follow suit: the ion–electron collision timescale

\[ t_{\text{collision}} = 23.6 \left( \frac{kT}{0.05 \text{ keV}} \right)^{3/2} \left( \frac{n_e}{10^{-2} \text{ cm}^{-3}} \right)^{-1} \times \left( \frac{\ln \Lambda}{17} \right) \text{ years}, \]

(14)

where \( \Lambda = b_{\text{max}}/b_{\text{min}} \) and \( b \) is impact parameter, is instantaneously short. Thus, by the time all the gas in the core has cooled and condensed, the Cerenkov radiation holds essentially the same reservoir of energy density as the gas initially. And

\footnote{One can also arrive at the same conclusion by applying a \textit{reductio ad absurdum} argument. If the electron energy distribution strictly follows the Boltzmann factor \( \exp \left( -\epsilon/kT \right) \) even at the resolution of \( \delta \epsilon / \epsilon \ll 1/\sqrt{N} \), the mean energy per particle in the volume of \( \lambda^3 \) will inevitably have to satisfy the inequality \( \delta \epsilon / \epsilon \ll 1/\sqrt{N} \) in violation of a standard statistical mechanics result that may be found in textbooks like Mandl (1991).}
because the radiation is trapped by the overlying plasma, it cannot escape.

If the pre-cooled gas was in hydrostatic equilibrium in the cluster potential well, the radiation trapped in the core by the overlying layers of hot gas will, after some configurational adjustment of scale heights (because radiation having the same energy density as the gas has only half the pressure) to be worked out later in a more detailed paper, prevent further episodes of central condensation and accelerated cooling. The purpose of this Letter is to highlight the importance of a much faster electron loss mechanism than any of the ones hitherto known by enlisting a set of parameters representative of the cooling gas at some radius within the core of a cluster. Of course, we merely provided a working example, as the gas parameters depend on radius, but nevertheless they serve to demonstrate the existence of an entire central region of Cerenkov radiation pressure domination, defined by a boundary radius beyond which no emission can take place because the inequality (5) is no longer satisfied.

The exact value of such a boundary radius may vary from cluster to cluster and is given by the point in Figure 1 where the gas parameters first find themselves below the curve corresponding to the ICM core magnetic field (note the argument given after (12), which proves that as the gas cools the density and temperature must quickly place it below the curve). Thus, e.g., for an ICM with \( kT \approx 0.05 \text{ keV}, n_e \approx 2 \times 10^{-3} \text{ cm}^{-3} \) and \( B \approx 4 \text{ \mu G} \) (these parameters marginally satisfy (5), \( \dot{M} \approx 2 M_\odot \text{ year}^{-1} \), which is not far away from the observationally inferred values and limits). Owing to the central (Cerenkov) radiation pressure, the actual mass deposition can only occur sporadically by Rayleigh–Taylor instability, when the overlying matter piles up enough to break through. Of course, the above parameters are not the only possible combination, i.e., there can be a range of allowable \( \dot{M} \), though it is clear the rate cannot be as large as (2). At least one may conclude that Cerenkov radiation should not henceforth be completely omitted from consideration when modeling of cool-core clusters; it might even be a vital ingredient to be included.

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