Corrigendum

Frames and semi-frames
Jean-Pierre Antoine and Peter Balazs 2011 J. Phys. A: Math. Theor. 44 205201

Received 3 October 2011
Published 3 November 2011

Introduction

The paper [1] contains a few minor errors, which could be confusing for the reader. Hence this corrigendum, which concerns three different sections.

2.3.1. An alternative. The line after equation (2.16) should be read as follows:

In this relation, \( J_{\Theta} := C \Psi S = C(R_S) \), with norm \( \| G^{-3/2} f \|_{L^2} \).

2.6. An example of a non-regular upper semi-frame

(1) The first line on page 12 should be read as follows:

where \( H_n := L^2(\mathbb{R}^+, r^{n-1} \, dr) \), \( n = \text{integer} \geq 1 \ldots \)

(i.e. with \( r^{n+1} \) instead of \( r^{n+1} \)).

This error, which goes back to [2], propagates in the following expressions.

(2) Equation (2.23) should read

\[
\frac{1}{(2\pi)^2} \int_{\mathbb{R}^+} \frac{r^{1-n}}{|\hat{\psi}(r)|^4} |f(r)|^2 \, dr < \infty. \tag{2.23}
\]

(3) Equation (2.24) should read

\[
(C \Psi f)(x) = \langle \psi_x, f \rangle = \int_{\mathbb{R}^+} e^{iur} \frac{\hat{\psi}(r)}{|\psi(r)|^2} f(r) r^{n-1} \, dr. \tag{2.24}
\]

(4) The expressions for \( G^{-1} \) and \( G \) should read, respectively,

\[
G^{-1} F(x) = C \Psi S^{-1} C^{-1} \Psi^{-1} F(x) = \frac{1}{2\pi} \int_{\mathbb{R}^+} e^{iur} \frac{\hat{F}(r)}{|\hat{\psi}(r)|^2} |\psi(r)|^2 r^{n-1} \, dr,
\]

\[
G F(x) = C \Psi S C^{-1} \Psi^{-1} F(x) = 2\pi \int_{\mathbb{R}^+} e^{iur} \hat{F}(r) |\psi(r)|^2 r^{n-1} \, dr.
\]

(5) The norms on the top of page 13 should read, respectively,

- For \( J_\Psi \) : \( \| F \|_{\Psi} = (F, G^{-1} F)_{L^2} = \int_{\mathbb{R}^+} \frac{\hat{F}(r)^2}{|\psi(r)|^2} r^{n-1} \, dr \);
- For \( J_0 \) : \( \| F \|_0^2 = \| F \|_{L^2}^2 \);
- For \( J_\Psi \) : \( \| F \|_{\Psi} = (F, GF)_{L^2} = 4\pi^2 \int_{\mathbb{R}^+} \frac{\hat{F}(r)^2}{|\psi(r)|^2} r^{n-1} \, dr \).

(6) Page 13, line 7: the map \( C \Psi \) should read

\[
(C f)(x) = \langle \psi_x, f \rangle = \int_{\mathbb{R}^+} e^{iux} \frac{\hat{\psi}(r)}{|\psi(r)|^2} f(r) r^{n-1} \, dr.
\]
3.6. Generalization of discrete frames

The symbols for the analysis and synthesis operators for fusion frames have been mixed up. Here is the correct version.

(i) The *analysis operator* $C_{W,v} : \mathcal{H} \to \mathcal{H}^\oplus$ defined by

$$
C_{W,v} f = \{v_j \pi_{\mathcal{H}_j} f\}_{j \in J}.
$$

(ii) The *synthesis operator* $D_{W,v} = C_{W,v}^* : \mathcal{H}^\oplus \to \mathcal{H}$, which is given by

$$
D_{W,v} f = \sum_{j \in J} v_j f_j \quad \text{for all } f = \{f_j\} \in \mathcal{H}^\oplus.
$$

Note that the series on the rhs converges unconditionally.

References

[1] Antoine J-P and Balasz P 2011 Frames and semi-frames *J. Phys. A: Math. Theor.* 44 205201
[2] Ali S T, Antoine J-P and Gazeau J-P 1993 Continuous frames in Hilbert space *Ann. Phys.* 222 1–37