Effective photon mass in nuclear matter and finite nuclei

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Abstract

Electromagnetic field in nuclear matter and nuclei are studied. In the nuclear matter, because the expectation value of the electric charge density operator is not zero, different in vacuum, the \(U(1)\) local gauge symmetry of electric charge is spontaneously broken, and consequently, the photon gains an effective mass through the Higgs mechanism. An alternative way to study the effective mass of photon is to calculate the self-energy of photon perturbatively. It shows that the effective mass of photon is about \(5.42\text{MeV}\) in the symmetric nuclear matter at the saturation density \(\rho_0 = 0.16fm^{-3}\) and about \(2.0\text{MeV}\) at the surface of \(^{238}\text{U}\). It seems that the two-body decay of a massive photon causes the sharp lines of electron-positron pairs in the low energy heavy ion collision experiments of \(^{238}\text{U} + ^{232}\text{Th}\).

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I. INTRODUCTION

The observation of electron-positron pairs in high-Z heavy ion collision experiments at lower energies has been paid close attention \[1, 2, 3\]. From the Monte Carlo simulations of the lepton kinetic-energy and time-of-flight distributions, it seems that there exists a prompt two-body decay of a light neutral particle of mass 1.8\,MeV at rest in the center-of-mass frame. Although it was claimed that the sharp sum-energy lines in electron-positron pairs emission from heavy-ion collisions was not repeated by the ATLAS Positron Experiment (APEX) in 1995 \[4\], the APEX results have been suspected \[5, 6, 7, 8\]. In this paper, by studying the properties of the electromagnetic field in the nuclear matter and finite nuclei, we would show that the predicted light neutral particle of mass 1.8\,MeV is just a massive photon. In nuclear matter, we suggested that the photon gains mass through the interaction with nucleons, and the same mechanism may also be applied to the electromagnetic field in finite nuclei.

II. PERTURBATION CALCULATION OF EFFECTIVE PHOTON MASS IN NUCLEAR MATTER

In nuclear physics, we always define the ground state of nuclear matter as ”vacuum”, where the Fermi sea is filled by nucleons, and no anti-nucleons and holes exist. Because the proton has one unit of positive charge, and the expectation value of electric density operator in the nuclear matter is not zero, namely \( \langle \hat{\rho}_Q \rangle \neq 0 \). This ”vacuum” is different from the real vacuum where nothing exists.

If the electromagnetic field and proton field interaction is considered as a perturbation, the perturbative hamiltonian in the interactive representation can be expressed as

\[
\mathcal{H}_I = e \bar{\psi}_p(x) \gamma^\mu \psi_p(x) A_\mu(x),
\]

with \( \psi_p \) being the fields of the proton, and \( A_\mu \) being the electromagnetic field.

The S-matrix can be written as

\[
\hat{S} = \hat{S}_0 + \hat{S}_1 + \hat{S}_2 + \ldots,
\]

where

\[
\hat{S}_n = \frac{(-i)^n}{n!} \int d^4x_1 \int d^4x_2 \ldots \int d^4x_n T \left[ \mathcal{H}_I(x_1) \mathcal{H}_I(x_2) \ldots \mathcal{H}_I(x_n) \right].
\]
To study the proton-proton interaction via the photon exchange,

\[ \hat{S}_2 = \frac{(-i)^2}{2!} \int d^4x_1 \int d^4x_2 T [\mathcal{H}_I(x_1)\mathcal{H}_I(x_2)]. \]  

(4)

should be calculated.

The proton field operator \( \psi_p(x) \) and its conjugate operator \( \bar{\psi}_p(x) \) can be expanded in terms of a complete set of solutions to the Dirac equation:

\[ \psi_p(x) = \sum_{\lambda=1,2} \int \frac{d^3p}{(2\pi)^3} \sqrt{\frac{m}{E(p)}} A_{p,\lambda} U(p, \lambda) \exp(-ip_\mu x^\mu), \]  

(5)

\[ \bar{\psi}_p(x) = \sum_{\lambda=1,2} \int \frac{d^3p}{(2\pi)^3} \sqrt{\frac{m}{E(p)}} A_{p,\lambda}^\dagger \bar{U}(p, \lambda) \exp[ip_\mu x^\mu], \]  

(6)

where \( E(p) = \sqrt{p^2 + m^2} \), and \( \lambda \) denotes the spin of the proton, \( A_{p,\lambda} \) and \( A_{p,\lambda}^\dagger \) are the annihilation and creation operators of the nucleon respectively. In these two expressions, we have assumed that there are no antinucleons in the nuclear matter or finite nuclei, thus only positive-energy components exist. The photon field operator \( A_\mu(k, x) \) can be expressed as

\[ A_\mu(k, x) = a(k, \delta) \varepsilon_\mu(k, \delta) \exp(-ik \cdot x) + a^\dagger(k, \delta) \varepsilon_\mu(k, \delta) \exp(ik \cdot x), \]  

(7)

with \( k \) being the momentum of the photon.

Since a perturbative single nucleon loop would contribute to the self energy of the photon a term of \( -e^2 k^2 C(k^2) \), which can be eliminated by the renormalization procedure[9], the diagrams in Fig.1 should be calculated.

The expectation value of \( \hat{S}_2 \) can be written as

\[ \langle k_2, \varepsilon_\mu(k_2, \delta_2) \mid \hat{S}_2 \mid k_1, \varepsilon_\nu(k_1, \delta_1) \rangle = -ie^2 (2\pi)^4 \delta^4(p_1 + k_1 - p_2 - k_2) \varepsilon_\mu(k, \delta) \varepsilon_\nu(k, \delta) \]

\[ \sum_{\lambda=1,2} \int \frac{d^3p}{(2\pi)^3} \frac{m}{E(p)} \theta(p_F - |\vec{p}|) \]

\[ \bar{U}(p, \lambda) \left( \gamma^\nu \frac{1}{\bar{p} - \vec{k} - m} \gamma^\mu + \gamma^\mu \frac{1}{\bar{p} + \vec{k} - m} \gamma^\nu \right) U(p, \lambda), \]  

(8)

where \( k_1 = k_2 = k \), and \( p_1 = p_2 = p \), and \( \theta(x) \) is the step function.

As the situation in Fig.1 is considered, we obtains the photon propagator \( G(k) \) in nuclear
matter as

\[
G(k) = \frac{1}{(2\pi)^4} \frac{-ig_{\mu\nu}}{k^2 + i\varepsilon} + \frac{1}{(2\pi)^4} \frac{-ig_{\mu\alpha}}{k^2 + i\varepsilon} \sum_{\lambda=1,2} (-ie^2)(2\pi)^4 \int \frac{d^3p}{(2\pi)^3 E(p)} \theta(p_F - |p|)
\]

\[
\bar{U}(p, \lambda) \left( \gamma^\beta \frac{1}{p - k - m} \gamma^\alpha + \gamma^\alpha \frac{1}{p + k - m} \gamma^\beta \right) U(p, \lambda) \frac{1}{(2\pi)^4} \frac{-ig_{\beta\nu}}{k^2 + i\varepsilon}
\]

\[
= \frac{-i}{(2\pi)^4} \frac{g_{\mu\nu}}{k^2 + i\varepsilon} + \frac{-i}{(2\pi)^4} \frac{-e^2}{k^2 + i\varepsilon} \sum_{\lambda=1,2} \int \frac{d^3p}{(2\pi)^3 E(p)} \frac{m}{\theta(p_F - |\vec{p}|)}
\]

\[
\bar{U}(p, \lambda) \left( \gamma^\nu \frac{1}{p - k - m} \gamma^\mu + \gamma^\mu \frac{1}{p + k - m} \gamma^\nu \right) U(p, \lambda) \frac{1}{k^2 + i\varepsilon}.
\]  \hfill (9)

According to the Dyson equation

\[
\frac{-ig_{\mu\nu}}{k^2 - \mu^2 + i\varepsilon} = \frac{-ig_{\mu\nu}}{k^2 + i\varepsilon} + \frac{-i}{k^2 + i\varepsilon} g_{\mu\nu} \mu^2 \frac{1}{k^2 + i\varepsilon},
\]  \hfill (10)

we obtain

\[
g_{\mu\nu} \mu^2 = -e^2 \sum_{\lambda=1,2} \int \frac{d^3p}{(2\pi)^3} \frac{m}{E(p)} \theta(p_F - |\vec{p}|)
\]

\[
\bar{U}(p, \lambda) \left( \gamma^\nu \frac{1}{p - k - m} \gamma^\mu + \gamma^\mu \frac{1}{p + k - m} \gamma^\nu \right) U(p, \lambda).
\]  \hfill (11)

Under the on shell condition of protons

\[
p^2 - m^2 \approx 0,
\]  \hfill (12)

the self-energy of the real, on-shell photon can be easily derived as

\[
\mu^2 = \frac{e^2}{m} \int \frac{d^3p}{(2\pi)^3} \frac{m}{E(p)} \theta(p_F - |\vec{p}|)
\]

\[
= \frac{e^2 \rho_p}{2m}.
\]  \hfill (13)

In this expression, \( e^2 = 4\pi\alpha \), with \( \alpha = \frac{1}{137} \) being the fine structure constant, \( m \) is the mass of proton, and \( \rho_p \) denotes the scalar density of protons

\[
\rho_p = 2 \int \frac{d^3p}{(2\pi)^3} \frac{m}{(p^2 + m^2)^{1/2}}.
\]  \hfill (14)

Thus, the photon gains the effective mass

\[
\mu = \sqrt{\frac{e^2 \rho_p}{2m}}
\]  \hfill (15)

in nuclear matter.
It should be mentioned that the effective mass of photon is only related to the scalar density of protons in the nuclear matter, but not the momentum of the photon. In a symmetric nuclear matter, where the densities of proton and neutron are the same, if the nucleon density is \(0.16 \text{fm}^{-3}\), the effective mass of photon is about 5.42 MeV.

In a finite nucleus, a two-parameter Fermi model of nuclear charge-density-distribution can be written as [10]:

\[
\rho_p(r) = \frac{\rho_c}{1 + \exp\left(\frac{r - c}{z}\right)},
\]

where \(c = 6.874 \text{fm}, z = 0.556 \text{fm}\) for \(^{238}\text{U}\), and \(\rho_c = 0.0635 \text{fm}^{-3}\) is determined by the equation

\[
4\pi \int \rho_p(r)r^2 dr = Z.
\] (17)

By assuming that in a finite nucleus, the proton number density is equal to the nuclear charge-density, the proton densities and corresponding effective photon masses at different radii around the surface region of \(^{238}\text{U}\) can be calculated and the corresponding results are listed in Table 1. It seems that at the surface of \(^{238}\text{U}\), the photon has effective mass of about 2.0 MeV, which is consistent with the predicted neutral particle mass in the low energy \(^{238}\text{U} + ^{232}\text{Th}\) heavy-ion collisions near Coulomb barrier [1, 2, 3], so the massive photon at the surface of \(^{238}\text{U}\) or \(^{232}\text{Th}\) might decay into an electron-positron pair in the disturbances of the nuclear Coulomb field. It might be the reason of the discovery of sharp line 800 keV \(e^+e^-\) pairs in the \(^{238}\text{U} + ^{232}\text{Th}\) heavy-ion collisions.

The electromagnetic interaction between protons is realized by exchanging virtual photons in nuclear matter. In the relativistic Hartree approximation shown in Fig. 2, the momentum of virtual proton \(k = 0\), consequently \(k^2 = 0\). Then the argument for the real photon mentioned above is suitable for the virtual photon exchanged between protons in nuclear matter. The contribution of the photon mass term should be included in the calculation of relativistic Hartree approximation or relativistic mean-field approximation in the finite nuclei.

### III. HIGGS MECHANISM

Again in the nuclear matter, because \(\langle \rho_p \rangle \neq 0\), the \(U(1)\) local gauge symmetry of electric charge is spontaneously broken, and the Higgs field should be included.
The Lagrangian density of Higgs field $\phi$ is\[11\]

$$L = (D^\mu \phi)^*(D_\mu \phi) - V(\phi^* \phi),$$

(18)

with

$$D^\mu = \partial^\mu + ieA^\mu,$$

(19)

$$V(\phi^* \phi) = \lambda (\phi^* \phi - \phi_0^2)^2 (\phi_0 \neq 0).$$

(20)

The Lagrangian is invariant under the local gauge transformation

$$A^\mu(x) \longrightarrow A^\mu(x) + \partial^\mu \omega(x),$$

$$\phi(x) \longrightarrow e^{-ie\omega(x)} \phi(x),$$

$$\phi^*(x) \longrightarrow e^{ie\omega(x)} \phi^*(x),$$

(21)

where $\omega(x)$ is an arbitrary real function. The local gauge symmetry is spontaneously broken when $\phi_0 \neq 0$.

Obviously, the lowest-energy solution is

$$A^\mu(x) = 0,$$

(22)

$$\phi(x) = \phi_0 e^{i\alpha_0}.$$ 

(23)

To study the classical modes near such a solution, the most convenient way is staying in the "unitary gauge", in which $\phi(x)$ is real under a continuous local gauge transformation. Thus we can always write

$$\phi(x) = \rho(x) \quad (real).$$

(24)

Then the equations of motion become

$$\frac{\partial F_{\mu\nu}}{\partial x_\nu} = 2e^2 \rho^2 A_\mu - e\bar{\psi}_p \gamma_\mu \psi_p,$$

(25)

$$(\partial^\mu + ieA^\mu)(\partial_\mu + ieA_\mu)\rho = 2\lambda \rho(\phi_0^2 - \rho^2),$$

(26)
(iγ_μ D^μ - m) ψ_p = 0, \hspace{2cm} (27)

(iγ_μ ∂^μ - m) ψ_n = 0. \hspace{2cm} (28)

Since ∂_ν ∂_μ F^{μν} = 0, we must have

∂_μ A^μ(x) = 0, \hspace{2cm} (29)

wherever ρ(x) ≠ 0. Taking

ρ(x) = φ_0 + η(x) \hspace{2cm} (30)

and treating η(x) and A^μ(x) as small quantities, one can deduce Eqs. (25) and (26) into linearized forms

\frac{∂F_{μν}}{∂x_ν} = 2e^2 φ_0^2 A_μ - e \bar{ψ}_p γ_μ ψ_p, \hspace{2cm} (31)

and

(∂^μ ∂_μ - 4λφ_0^2) η = 0. \hspace{2cm} (32)

These equations show that the fields A^1, A^2, φ and φ* in a fixed gauge are now replaced by A^1, A^2, A^3 and η. The spin-1 particle has a mass of √2eφ_0, and the spin-0 particle has a mass of 2√λφ_0. In other word, the photon obtains mass in the nuclear matter.

From above discussion, we find that in the nuclear matter, the effective mass of photon can either be obtained by the Higgs mechanism or the perturbative calculation. This result also implies that the Higgs mechanism is not a unique method to solve problems when the local gauge symmetry is spontaneously broken.

IV. SUMMARY

In summary, the electromagnetic field in nuclear matter is studied. Because in the nuclear matter the expectation value of the electric charge density operator is not zero, the U(1) local gauge symmetry of electric charge is spontaneously broken, consequently the photon gains a effective mass. Alternatively, this mass can also be obtained from the perturbative calculation of the photon self-energy in the nuclear matter. It is shown that the effective mass is related to the scalar density of protons. The effective mass of photon is about 5.42 MeV in the symmetric nuclear matter at the saturation density of ρ_0 = 0.16 fm^{-3}, and about
2.0MeV in the surface of $^{238}U$. It just fits the mass of the neutral particle suggested in the high-Z heavy ion collision experiments at low energies near Comloub barrier, in which the sharp lines of electron-positron pair was observed. Obviously, the two-body decay of such a massive photon would produce a pair of electron and positron. The property of electromagnetic field in the nuclear matter or finite nuclei is different from that in the vacuum. Because the photon in the nuclear matter has an effective mass, this fact should also carefully be considered in the investigation of the nuclear many-body problems. In the future work, we will continue to study the relevant effects on this aspect.

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TABLE I: Effective photon mass $\mu$ and corresponding radii $r$ in the nucleus of $^{238}U$ for different proton number density $\rho_p(r)$

| $\rho_p(r)$ (fm$^{-3}$) | $r$ (fm) | $\mu$ (MeV) |
|-------------------------|----------|-------------|
| 0.005                   | 8.242    | 1.37        |
| 0.006                   | 8.131    | 1.50        |
| 0.007                   | 8.035    | 1.62        |
| 0.008                   | 7.951    | 1.73        |
| 0.009                   | 7.876    | 1.83        |
| 0.010                   | 7.807    | 1.93        |
| 0.011                   | 7.743    | 2.03        |
| 0.012                   | 7.684    | 2.12        |
FIG. 1: Feynman diagrams for the photon propagator in nuclear matter.
FIG. 2: Feynman diagrams for the electromagnetic interaction in the relativistic Hartree approximation in finite nuclei.