Modal analysis of monorail rocket sled and its cabin connections

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Abstract. Structural stiffness is an important dynamic parameter for the rocket sled. In this paper, both modal test and finite element modal analysis are carried out on the certain monorail sled and its two different cabin connection kinds which are bolt and thread. By comparing the natural frequencies and modal shapes obtained by test and simulation methods respectively, it is found that the stiffness of bolted cabins is consistent with the simulation results, while the threaded cabins performs a severe stiffness degradation due to the thread clearance, which also causes the natural frequencies of the completed sled to be reduced and the modal shapes to be inconsistent with the simulation results. The finite element model of threaded connections is modified by giving anisotropic material properties to the connected elements, and the simulation deviation is reduced. According to the analysis results of two different kinds of cabin connections, threaded cabins on the final assembled rocket sled are strengthened by welding. The modal test results of the reinforced threaded cabin connection indicate that negative influence of the threaded connections is eliminated and the natural frequency and vibration mode are very close to the simulation results.

1. Introduction
Rocket sled test is a high-speed ground-based test system, in which the sled is powered by rocket and coasted along a specially built track with high-precision. Typical rocket sled test items include impact and penetration test, interceptor tracking test, material erosion and ablative test, flight crew escape system test, bullet separation test and inertial navigation system test, etc [1]. The test speed of rocket sled test ranges from subsonic to hypersonic velocity regime, and the latest world land speed record of Mach 8.5 or 6,416 mph was established at the Holloman High-Speed Test Track (HHSTT) on September 11, 2019.

Figure 1. Typical monorail rocket sled of HHSTT.
Figure 1 shows the typical monorail rocket sled of HHSTT [2]. Sleds, or the vehicles that propel and carry payloads down the track, are restrained to the rail by “slippers”. Due to rail manufacturing imperfections and rail alignment tolerances, the inner cavities of the slippers are oversized, and there shall be slipper gap between the slippers and the track. When the sleds are coasting down the track at high-speed, the slippers impact the rail repeatedly. The rail irregularities together with the slippers and their gaps generate a severe vibration environment. As the result, the vibration levels experienced on-board the sleds are much more severe than those experienced in a flight environment [3-4].

In general, the vibration degree of the sleds is strongly affected by the structural dynamic parameters. Among which, the structural stiffness is an important dynamic parameter [5]. The sled should be rigid enough to withstand the vibration environment and perform its assigned tasks, but not too rigid to result in higher impact loads. At the same time, once the natural frequency of the sleds is close to the excitation frequency of the track, the sled resonance will be induced and the track will be damaged [6]. As a result, it is necessary to analyze the mode of the sled to ensure the safety of the structure, and provide design parameters for vibration isolation and vibration absorption measures. In design of the rocket sled, modal analysis mainly includes the analysis of system’s natural frequencies and the corresponding mode shapes [7].

In a certain sled test, the monorail sled was designed, the finite element modal analysis was carried out, the result indicated that the sled stiffness could meet the design requirement. Then the sled was processed and the modal test was carried out, but the test results are quite different from the simulation. In order to find out the reason, the simulation and modal test of both bolt and thread connections were compared, and the problem was located in the stiffness degradation of the threaded connection. Finally, the screw joint is reinforced and the results of the new modal test are consistent with the simulation values.

2. Modal analysis of the initial sled

The Initial monorail sled was modeled with UG NX10 software [8]. Figure 1 shows the structure diagram of the sled. The sled adopts an integrated design, which is mainly composed of missile simulator, front and rear slipper and instrumentation system. The missile simulator is composed of 4 cabins with a thickness of 6mm, and the cabins is shown in Figure 2. Cabin 1 and cabin 2 are connected by flange bolts, and cabin 2 is joined to cabin 4 with screw type by the thread at both ends of cabin 3. The lower part of cabin 2 and cabin 4 is bolted with one slipper respectively. The material of the missile simulator and the slippers is structural steel, the elastic modulus is 210GPa, the density is 7850kg/m³, and the Poisson's ratio is 0.3. The material of instrumentation system is aluminum alloy, the elastic modulus is 710GPa, the density is 2770kg/m³, the Poisson's ratio is 0.33. After completion of the initial model design, two methods of finite element modal analysis and modal test are used to obtain its modal parameters.

![Figure 2. The structure diagram of the initial monorail rocket sled.](image)

2.1. Finite element modal analysis

2.1.1. Theory and model. For the structural system with multiple degrees of freedom, any motion can be synthesized by its free vibration mode. The essence of the modal analysis is to solve the modal vector of the motion equation with finite degrees of freedom in the state of undamped and unloaded
(since the damping of the structure has little influence on its modal frequency and mode shape, it can be ignored). For the undamped system and for free vibrations, we have a special case of equation:

$$[M][\mu] + [K][\mu] = 0$$  \hspace{1cm} (1)

both $[M]$ and $[K]$ in Equation (1) are real symmetric matrices for the linear structure system, and the equation has the following simple harmonic motion solutions:

$$\{\mu(x,y,z,t)\} = \{\phi(x,y,z)\} \exp(j\omega t)$$  \hspace{1cm} (2)

Where $\phi(x,y,z)$ is the amplitude of the displacement vector, which defines the spatial distribution of the displacement vector, $\omega_n$ is the angular frequency of simple harmonic motion. After substituting Equation (2) into Equation (1), the following relations can be obtained:

$$[K - \omega_n^2 M] \{\phi\} \exp(j\omega t) = \{0\}$$  \hspace{1cm} (3)

Equation (3) is true at any time, so remove the term containing $t$:

$$[K - \omega_n^2 M] \{\phi\} = \{0\}$$  \hspace{1cm} (4)

Equation (4) is a typical real eigenvalue problem, and the condition of $\{\phi\}$ having a non-zero solution is that the value of the determinant of its coefficient is zero:

$$[K - \lambda M] = 0$$  \hspace{1cm} (5)

Where $\lambda = \omega_n^2$. A set of discrete roots $\lambda_i = (i = 1,2,3,\ldots,n)$ can be obtained by solving Equation (5), and the corresponding vector $\{\phi_i = (i = 1,2,3,\ldots,n)\}$ can be obtained by substituting Equation (5) back to Equation (4), where $\lambda_i$ is the square of the ith order natural frequency of the structural system and the $\phi_i$ is called the ith modal shape. Finite element modal analysis is the process of establishing a finite element model and performing numerical analysis.

The sled model is reasonably simplified to improve the efficiency of calculation while maintaining precision of analysis. After removing the sensor and geometrically details like fillets, chamfers, bolts and holes that has little effect on the mode, the model is meshed mainly using hexahedral 20-node grid, and a few tetrahedral and triangular prismatic grids are used in the transition. The result of meshing is shown in Figure 3. The grid size is 0.2cm to 1cm, and the total number of grids is 155,838.

![Figure 3. The finite element model of the initial sled.](image)

2.1.2 Analysis result. The modal analysis of the rocket sled in free state was carried out in ABAQUS, and Lanczos algorithm was used, which extracts eigenvalues and eigenvectors with high precision and fast operation speed [9]. Lanczos is an algorithm that transforms symmetric matrix into symmetric tridiagonal matrix through orthogonal similarity transformation. The rigid body modes are removed after calculation to ensure the convergence of the results. The first 5 order mode shapes are shown in Figures 4-8, and the corresponding natural frequencies are shown in Table 1.
The 1st order natural frequency of the rocket sled is about 150Hz, which conforms to the sled design criteria. Then the finite element model was used to analyze and optimize the dynamic response of the rocket sled. After which, the sled was manufactured, and the modal test was conducted.

2.2. Modal test analysis

2.2.1. Principles and methods. Based on the linear superposition principle, complex structural systems can be decomposed into the superposition of many modes. Such a decomposition process is called modal analysis. If the decomposition process is completed by experimental methods, it is called modal test. As we know, the linear multi-degree-of-freedom system can be decomposed into N single-degree-of-freedom subsystems. The frequency response function of the system is the weighted sum of the frequency response functions of each single-degree-of-freedom subsystem, namely:

$$H(\omega) = \sum_{i=1}^{N} \frac{\{\phi_i\} \{\phi_i\}^T}{K_i - \omega^2 M_i + j\omega C_i}$$

where $M_i, K_i, C_i$ are system mass matrix, stiffness matrix and damping matrix. Every row or column of the frequency response function matrix of the structure system contains all the modal parameters of the structure system. The basic principle of the modal test is to obtain the modal parameters of the structure by obtaining the frequency response.

The modal test of the initial sled is made by using hammer-hitting pulse-inspirit method and MIMO (multiple input multiple output) parameter identification method. According to the finite
element analysis results of the sled, 82 measuring points were arranged by cylindrical coordinate system, and 6 representative points were selected on the sled to set the one-way acceleration sensor. The force hammer was used to strike the measuring points, and 82 measurement points were struck in turn. The frequency response and mode shape are obtained by collecting the signal into LMS Test Lab system [10]. Figure 9 shows the scene of the modal test.

**Figure 9.** Scene of modal test of the initial sled.

### 2.2.2. Modal test result

The first 5 order mode of the rocket sled were analyzed. The natural frequencies comparison is shown in Table 1, and the mode shapes are shown in Figures 10-14.

| Mode order | The natural frequency in simulation (Hz) | The natural frequency in test (Hz) |
|------------|----------------------------------------|----------------------------------|
| The 1st order | 117 | 59 |
| The 2nd order | 338 | 118 |
| The 3rd order | 386 | 260 |
| The 4th order | 417 | 305 |
| The 5th order | 545 | 338 |

**Figure 10.** The 1st order modal shape.  
**Figure 11.** The 2nd order modal shape.  
**Figure 12.** The 3rd order modal shape.  
**Figure 13.** The 4th order modal shape.
The natural frequencies comparison is shown in Table 1. It can be seen that the simulation and the test results do not correspond well. From natural frequency perspective, the value of 1st order natural frequency of modal test is only half of the simulation value, only the 5th order frequency of 338Hz corresponds to the 2nd order frequency of 338Hz of the simulation results. From natural mode shape perspective, the results are completely different. Accordingly, the specific reason needs to be investigated.

3. Cause analysis

3.1. Model check
The simulation model is carefully reviewed. The mass of the finite element model is 156 kg, while the actual mass of the sled is about 162 kg. Consequently, there is a certain disparity between the simulation model and the physical model, but it’s not enough to introduce such a large simulation error. To test and verify the influence of different types of units on the calculation results, C3D8, C3D8R, C3D20, C3D2OR and the shell element are adopted respectively for the calculation, the results have certain difference, but not significantly. There is only one explanation of this error, that is the simulation model can’t represent the sled cabin connections well, because the finite element model adopts the method of node equivalence to connect the cabins of the whole sled, while the cabins are connected by thread or bolt method for actual physical model, which may lead to a large difference in stiffness at the joints.

3.2. Modal analysis of cabins
In order to verify the preliminary conclusion of the cause analysis and assess the stiffness reduction for different types of connections, the sled was disassembled and the modal test and simulation were carried out for two joint cabins, which are connected by thread and bolt respectively. Cabin 1 and cabin 2 were connected by flange bolts, while cabin 3 and cabin 4 were connected by thread. The free modal analysis of these two kinds cabin connection was conducted in ABAQUS. In the simulation model, the cabins are still connected with the method of node equivalent, namely the degree of freedom coupling at the cabin joint is the same. The modal tests were also carried out after reassembling the two kinds of connecting cabins. The first 3 order natural frequencies obtained are shown in Table 2.

|                  | Bolted cabins | Threaded cabins |
|------------------|---------------|-----------------|
|                  | Simulation    | Test            | Simulation | Test |
| The 1st order natural frequency (Hz) | 202           | 201             | 575        | 243  |
| The 2nd order natural frequency (Hz) | 319           | 327             | 625        | 455  |
| The 3rd order natural frequency (Hz) | 390           | 388             | 764        | 524  |

Results in Table 2 imply that the values of simulation and test results of the bolted connection of cabin 1 and cabin 2 are very close, which indicates that the stiffness degradation of this kind of connection is very small. The deviation of the threaded connection results of cabin 3 and cabin 4 is...
huge, the 1st natural frequency of the test results is only half of the value of the simulation, which indicates that the finite element model failed to simulate the threaded connection of the physical model. There must be an assembly space for the actual screw thread connection, even if the preload is applied. The connection between cabin 3 and cabin 4 is close to the flexible connection with degrees of bending and slipping, resulting in a serious stiffness degradation.

3.3. Model modification
The nodal equivalent method used in the finite element analysis cannot describe the thread connection state of the cabins accurately. The coupling relationship of the thread connection cabins is modified to MPC (Multi-point constrains), and anisotropic material properties are assigned to the connection unit. The connection unit possess an elastic modulus of 90GPa along the axial direction, and the other directions remain unchanged (210GPa). The modal analysis process of the new model is as described above, the first 5 natural frequencies are shown in Table 3.

| Mode order | The natural frequency in the 2nd simulation (Hz) | The natural frequency in initial test (Hz) |
|------------|-----------------------------------------------|------------------------------------------|
| The 1st order | 61 | 59 |
| The 2nd order | 286 | 118 |
| The 3rd order | 325 | 260 |
| The 4th order | 361 | 305 |
| The 5th order | 468 | 338 |

The mode shapes are shown in Figures 15-19. Natural frequencies of the modified model are significantly lower than that of the original model, and the simulation error is reduced. The 1st order natural frequency is very close, but there are still significant differences between the other-order frequencies and modal shapes. In other words, the modified model can accurately describe the stiffness of the connection under a certain vibration mode, but it is not enough to describe the influence of the threaded connection on the entire modal analysis.

Figure 15. The 1st order modal shape.  
Figure 16. The 2nd order modal shape.  
Figure 17. The 3rd order modal shape.  
Figure 18. The 4th order modal shape.
4. Modal analysis of the final sled

The modal parameters of the sled obtained from the test are inconsistent with the simulation values, indicating that the dynamic analysis based on the above simulation model is invalid. The sled structure cannot be used in rocked sled test unless it is reinforced. According to the analysis results, the lower natural frequency of the rocket sled is the main reason for the stiffness reduction of threaded cabin connections. After completion of the sled assembly, the threaded cabins are welded into one unit, and the new second modal test was carried out. The measuring point setting and excitation mode are as same as the previous test, and the first 5 orders of natural frequencies and mode shapes of the sled are obtained as shown in Table 4 and Figures 20-24.

Table 4. Comparison of simulation and test values of the reinforced sled.

| Mode order | The natural frequency in initial simulation (Hz) | The natural frequency in the 2nd test (Hz) |
|------------|-------------------------------------------------|------------------------------------------|
| 1st order  | 117                                             | 117                                      |
| The 2nd order | 338                                        | 330                                      |
| The 3rd order | 386                                         | 401                                      |
| The 4th order | 417                                         | 426                                      |
| The 5th order | 545                                         | 607                                      |

Figure 19. The 5th order modal shape.

Figure 20. The 1st order modal shape.

Figure 21. The 2nd order modal shape.

Figure 22. The 3rd order modal shape.

Figure 23. The 4th order modal shape.
Figure 24. The 5th order modal shape.

The modal test of the final sled is consistent with the 1st order natural frequency of the initial finite element modal analysis at 117 Hz, and the errors of the remaining frequencies are within 20%. The reason for the error may be that although the connection between the cabins has been improved by welding, there are still some non-linear characteristics, and the finite element model is not perfectly equivalent to the physical model. Among the first 5 modes obtained by the test, the 1st order mode is a half-sine wave, the 2nd and 3rd order are two half-sine waves, the 4th order is the breathing mode at the front of the sled, and the 5th order is the rear breathing. The overall results correspond well with the initial modal simulation results, indicating that the dynamic analysis based on the initial model is credible and the sled can be used for subsequent rocket sled test.

5. Conclusions

Based on the modal analysis theory, the two methods of finite element modal analysis and modal test are combined to obtain the modal of a certain monorail rocket sled in this paper. The following conclusions can be drawn from the analysis process:

(1) The simulation value of the 1st order natural frequency of the initial rocket sled is 167 Hz, which is double the result of the modal test, and the mode shapes of the two are also inconsistent. Therefore, it is not appropriate to use the node equivalent method or MPC method to deal with the connection of rocket sled cabins.

(2) The measured 1st order natural frequency of the threaded connection cabins is only half of the simulated value, the cause of the simulation deviation is located on the threaded connection. When the model is revised by giving anisotropic material properties to the connected elements, although the screw connection cabin cannot be perfectly simulated, the simulation deviation is decreased.

(3) After welding reinforce of threaded cabins of the final assembled rocket sled, the second modal test shows that the negative influence of the threaded connections is eliminated, and the natural frequency and mode shapes are very close to the simulated values.

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