Abstract

We quantize a generalized electromagnetism in 2 + 1 dimensions which contains a higher-order derivative term by using Dirac's method. By introducing auxiliary fields we transform the original theory in a lower-order derivative one which can be treated in a usual way.
I. INTRODUCTION

In the last years there has been an increasing interest in 2 + 1 dimensions gauge field theories [1] in part due to the fact that some of these theories exhibit features which could be associated to the high $T_c$ superconductors and the fractional quantum Hall effect [2]. In such cases the most studied theories have been those which includes a topological mass term to the gauge field (Chern-Simons term) that couples to the matter current.

Besides, the gauge field theories in lower dimensions are a good laboratory to study phenomena which are not well understood in four dimensions.

Particularly, the quantization of Chern-Simons theories has received much attention. Dirac’s quantization method [3] was used not only in the Abelian massive gauge theory [4] but also in the non-Abelian case and in the pure Chern Simons theory [5]. The Abelian massive gauge theory was also quantized in a manifestly covariant way [6].

On the other hand, theories with higher-order derivatives have also attracted considerable interest. Those terms were already introduced in the past century by Ostrogradskii. In the 1940’s, aiming to avoid divergences in Maxwell’s theory, Podolsky suggested the inclusion of higher-order derivative terms [7]. More recently, quantization aspects of this theory has been analysed by using Dirac’s method [8] as well as the Batalin-Fradkin-Vilkovisky formalism [9]. This interest rely on the possibility of soften the ultraviolet divergences, leading to a possible attenuation of the problem of renormalizability for theories like quantum gravity.

Most of the models with higher derivative terms present undesired properties such as non-renormalizability and tachyonic massive modes. Despite this, higher-order derivative terms have been used in gravitation in order to improve the ultraviolet behavior of the Einstein-Hilbert action [10]. Even in supersymmetry and string theory, higher-order terms play a certain role. In string theory, for instance, a term proportional to the extrinsic curvature of the world sheet was proposed and it has a great influence in the phase structure of the theory [11]. The idea was applied to the relativistic particles as well [12].
supersymmetry, higher-order derivative terms are a useful regularization, that preserves supersymmetry.

In this paper we intend to quantize the Maxwell-Chern-Simons-Podolsky theory by using Dirac’s method. It was shown that a Chern-Simons term can be generated in Quantum Electrodynamics in 2 + 1 dimension whenever one integrates the fermionic fields out [13-15]. As a matter of fact several other terms can also be generated, including those with higher-order derivatives of the gauge field. This can be seen, for example, in the derivative expansion method [16], like it was done in reference [14]. In this way, the study of properties of the present theory can be helpful to the better understanding of gauge effective actions in 2 + 1 dimensions.

This paper is organized as follows. In section II we define the model and derive the propagator. In section III we map the theory into another one which does not present higher-order derivative terms and quantize it; in section IV we discuss the results and make some remarks.

II. MAXWELL-CHERN-SIMONS-PODOLSKY THEORY

We start with the following Lagrangian density

\[ \mathcal{L} = -\frac{a}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\theta}{2} \varepsilon^{\mu\rho\nu} A_\mu \partial_\rho A_\nu - \frac{b^2}{2} \partial^\mu F_{\mu\nu} \partial_\lambda F^{\lambda\nu}, \]  

where \( a, \theta \) and \( b \) are free parameters which permit us taking the appropriate limits. The sign of the Podolsky term has been considered in conformity with the original work [7].

In order to obtain the propagator one has to add the gauge-fixing Lagrangian

\[ \mathcal{L}_{gf} = \frac{1}{2\alpha} (\partial_\mu A^\mu)^2 \]  

to the original one, since (1) is not invertible; where \( \alpha \) is the gauge parameter. In such case the propagator, in momentum space, is given by
\[ G_{\mu\nu} = \frac{1}{[(a + b^2 k^2)^2 - \theta^2]} \left\{ (a + b^2 k^2)g_{\mu\nu} + \right. \\
\left. + \alpha \left[ (a + b^2 k^2)^2 - \theta^2 \right] k_\mu k_\nu + i \theta \varepsilon_{\mu\nu\rho} k_\rho \right\}, \] (3)

which agrees with reference [9] when the parameter \( \theta \) goes to zero. It is easy to see that the poles of the propagator are defined by the equation

\[ y^3 + a_1 y^2 + a_2 y + a_3 = 0 \quad ; \quad (y = k^2), \] (4)

where

\[ a_1 \equiv \frac{2a}{b^2} \quad ; \quad a_2 \equiv \frac{a_1^2}{4} \quad ; \quad a_3 \equiv -\frac{\theta^2}{b^4}. \] (5)

From the above equations we can see that the number of massive poles depends on the choice of the parameters.

By studying the solutions of above equation one sees that, in order to have only real roots, it is necessary to impose that the discriminant \( D \equiv Q^3 + R^2 \), where

\[ Q \equiv -\frac{a_1^2}{36}, \quad R \equiv \frac{1}{54} \left( \frac{a_1^3}{4} - 27a_3 \right), \] (6)

be lesser than or equal to zero. So, one have two possibilities:

i) \( D = 0 \): Here we have three real roots, where two are equal (leading to the appearing of ghosts). So we have that

\[ m_1^2 = 2R \frac{a_1}{3}, \quad m_2^2 = m_3^2 = -R \frac{a_1}{3}. \] (7)
However, the imposition $D = 0$, imply that one must have: $a_3 = 0$, which is a trivial solution with $\theta = 0$; or $a_3 = \frac{a_1^3}{54}$, but in this case we get a negative squared mass, revealing the existence of tachyonic excitations.

\textit{ii) $D < 0$:} Now we are faced with three different real roots of Eq.(4). The solutions can be written as:

\[
\begin{align*}
m_0^2 &= 2 \rho^3 \cos \left( \frac{\alpha}{3} \right) - \frac{a_1}{3}, \\
m_{\pm}^2 &= -\rho^3 \left[ \cos \left( \frac{\alpha}{3} \right) \pm \sqrt{3} \sin \left( \frac{\alpha}{3} \right) \right] - \frac{a_1}{3},
\end{align*}
\]

(8)

where

\[
\rho^3 = \left[ R^2 + \frac{1}{4} |a_3^2 - a_3 a_1^3/54| \right]^{\frac{1}{2}},
\]

(9a)

and

\[
\alpha = \arctan \left[ \frac{108 \sqrt{|a_3^2 - a_3 a_1^3/54|}}{a_1^3 - 108a_3} \right].
\]

(9b)

Besides, the imposition $D < 0$, leaves us with the possibilities:

1) $a_1 > 0:$ \hspace{1cm} $0 < a_3 < \frac{a_1^3}{54}$

2) $a_1 < 0:$ \hspace{1cm} $\frac{a_1^3}{54} < a_3 < 0$.

In this case however, it is hard to verify if the above values of $a_3$ and $a_1$ correspond to positive square masses, at least analytically. Nevertheless, doing a numerical analysis, it appears that the following rules hold:

a) For $a_1 > 0$, there is no possible way of keeping the three square masses simultaneously positive.

b) For $a_1 < 0$, one observes that along a reasonable range of variation for the parameter $a_1$ ($-0.01 \leftrightarrow -100$), that the region where the three masses are positive, it is always greater than that where $D < 0$. So one can see that the region defined above apparently leads to well behaved masses (no tachyons, no ghosts).
Unfortunately, the usual case where \( a = 1 \), and Podolsky’s parameter is negative, implies that \( a_1 \) be positive. Consequently the taquyons stays present in the theory.

III. DIRAC’S QUANTIZATION

In this section we perform the Dirac quantization through two different ways of gauge fixing the theory.

Looking at the original Lagrangian (1), it is possible to verify that it is equivalently described by the Lagrangian density

\[
\mathcal{L} = -\frac{a}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\theta}{2} \varepsilon^{\mu\rho\nu\sigma} A_\mu \partial_\rho A_\nu + \frac{1}{2} Z_\mu Z^\mu + \frac{b}{2} F_{\mu\nu} Z^{\mu\nu},
\]

where \( Z_\mu \) is an auxiliary field and \( Z_{\mu\nu} \), defined by

\[
Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu,
\]

is the associated field strength. For sake of curiosity, it is interesting to observe that we could also perform a suitable change of variables, capable of disentangle the term coupling the field strengths. Thus we would obtain that

\[
\overline{\mathcal{L}} = -\frac{a}{4} F_{\mu\nu} F^{\mu\nu} + -\frac{1}{2} \left| \frac{b^2}{a} \right| (2 - \text{sign}(-a)) Z_{\mu\nu} Z^{\mu\nu} + \frac{1}{2} Z_\mu Z^\mu +
\]

\[
+ \frac{\theta}{2} \varepsilon^{\mu\rho\nu\sigma} \overline{A}_\mu \partial_\nu \overline{A}_\rho - \frac{\theta}{4} \frac{b^2}{|a|} \varepsilon^{\mu\rho\nu\sigma} Z_\mu \partial_\nu Z_\rho - \frac{|b|}{\sqrt{a}} \varepsilon^{\mu\rho\nu\sigma} \overline{A}_\mu \partial_\nu \overline{Z}_\rho.
\]

In this case it can be seen easily that we would get a Chern-Simons-Maxwell field interacting with a Chern-Simons-Proca one through crossed Chern-Simons terms.

However, we will work with the Lagrangian density (10) instead of that one of equation (1). Here we do not have to define the momentum associated with the field time derivative. In any case, the introduction of the auxiliary fields duplicates the number of variables we are dealing with.
The equations of motion are

\[ a \partial_\mu F^{\mu \nu} + \theta \varepsilon^{\nu \mu \alpha} F_{\mu \alpha} = b \partial_\mu Z^{\mu \nu} \quad , \tag{12a} \]
\[ b \partial_\mu F^{\mu \nu} = Z^{\nu} \quad . \tag{12b} \]

Taking the divergence in equation (12b) and using the antisymmetry property of \( F^{\mu \nu} \) we get

\[ \partial_\nu Z^{\nu} = 0 \quad , \tag{13} \]

which is a Lagrangian constraint.

The equation of motion of the potential \( A_\mu \) has the form

\[ (a - b^2 \Box) A^\nu + 2 \theta \varepsilon^{\nu \beta \mu} \partial_\beta A_\mu - (a - b^2 \Box) \partial^\nu \partial^\mu A_\mu = 0 \quad . \tag{14} \]

From the Lagrangian density (10) we have the following primary constraints

\[ \Omega_1 = \pi_{0A} \approx 0 \quad , \tag{15a} \]
\[ \Omega_2 = \pi_{0Z} \approx 0 \quad , \tag{15b} \]

which are, respectively, the momenta associated to the components \( A_0 \) and \( Z_0 \).

The space-components of the momenta are

\[ \pi_i^A = a F_{i0} - \frac{\theta}{2} \varepsilon_{ij} A_j - b Z_{i0} \quad , \tag{16a} \]
\[ \pi_i^Z = -b F_{i0} \quad . \tag{16b} \]

We can now construct the primary Hamiltonian

\[ H_p = \int d^2 \vec{x} \left[ A_0 \partial_t \pi_{i}^A + Z_0 \partial_t \pi_{i}^Z - \frac{\pi_{i}^A \pi_{i}^Z}{b} - a \frac{\pi_{i}^Z}{2b^2} \left( \pi_{i}^Z \right)^2 + \right. \]
\[ + \frac{\theta}{2b} \varepsilon_{ij} \pi_i^Z A_j - \frac{\theta}{2} \varepsilon_{ij} A_0 \partial_i A_j - \frac{Z_0^2}{2} + \frac{Z_j^2}{2} + \frac{a}{4} F_{ij}^2 \]
\[ - \frac{b}{2} Z_{ij} F_{ij} + \lambda_1 \Omega_1 + \lambda_2 \Omega_2 \right] \quad , \tag{17} \]
where $\lambda_i$ are the Lagrange multipliers.

Since the primary constraints must be maintained in the time, their consistency conditions generates two other constraints (secondary constraints).

$$\Omega_3 = \dot{\Omega}_1 = \partial_i \pi_i^A - \frac{\theta}{2} \varepsilon_{ij} \partial_i A_j \approx 0$$  \hspace{1cm} (18a)$$

and

$$\Omega_4 = \dot{\Omega}_2 = \partial_i \pi_i^Z - Z_0 \approx 0 .$$ \hspace{1cm} (18b)$$

The definition of $\pi_i^Z$ in equation (16b) and equation (18b) is nothing more than one of the Lagrangian constraints of equation (12b). Furthermore, equation (18b) can be seen as a “Gauss law with sources”.

$$\partial_i E_i \approx -\frac{Z_0}{b} ,$$ \hspace{1cm} (19)$$

where $E_i = F_{i0}$.

It is possible to verify that the consistency of $\Omega_3$ is identically fulfilled,

$$\dot{\Omega}_3 \equiv 0 .$$ \hspace{1cm} (20)$$

On the other hand, consistency of $\Omega_4$ gives a condition to $\lambda_2$

$$\dot{\Omega}_4 = -\partial_i Z_i + \lambda_2 \approx 0$$

$$\lambda_2 \approx \partial_i Z_i .$$ \hspace{1cm} (21)$$

Up to now we have four constraints. $\Omega_2$ and $\Omega_4$ are second class constraints and $\Omega_1$ and $\Omega_3$ are first class. In order to transform those last into second class constraints we have to do a gauge choice. This can be done through two different ways. One can introduce additional second class constraints, or break the gauge invariance directly at Lagrangian level by using the Faddeev-Popov’s trick. Here we will apply the two approaches respectively.

Equation (14) suggests the following gauge choices

$$\text{(a)} - \text{b}^2 \partial_i A_i = 0 ,$$ \hspace{1cm} (22)$$
\[ \partial_i A_i = 0 \quad . \] (23)

We choose the last one, which is going to be our fifth constraint

\[ \Omega_5 = \partial_i A_i \approx 0 \quad , \] (24)

whose consistency generates a sixth constraint

\[ \Omega_6 = \dot{\Omega}_5 = \nabla^2 A_0 + \frac{\partial_i \pi_i}{b} \approx 0 \quad , \] (25)

that, in its turn, gives a condition to \( \lambda_1 \)

\[ \dot{\Omega}_6 = \nabla^2 \lambda_1 \frac{\partial_i Z_i}{b} \approx 0 \quad , \] (26)

and no more constraints are generated at all.

It is worth mentioning that the gauge choice (23) can be satisfied if we choose the gauge function as [5]

\[ \Lambda = -\frac{1}{\nabla^2} \partial_i A_i \quad , \] (27)

for, starting from the gauge transformation

\[ A'_j = A_j + \partial_j \Lambda \] (28)

we get

\[ \partial_j A'_j = \partial_j A_j + \nabla^2 \Lambda = 0 \quad , \] (29)

and, since

\[ A'_0 = A_0 + \partial_0 \Lambda \quad , \] (30)

we have

\[ \nabla^2 A'_0 = \nabla^2 A_0 - \partial_0 \partial_i A_i = \partial_i F_{0i} = -\frac{\partial_i \pi_i}{b} \quad , \] (31)

where we have used equation (16b). This way we recover the constraint \( \Omega_6 \), when this is strongly imposed,
Now we have only second class constraints and we can invert the constraint matrix. The commutation relations among the constraints are

\[
\{ \Omega_1(\vec{x}), \Omega_6(\vec{y}) \} = -\nabla^2 \delta(\vec{x} - \vec{y}) ,
\]
\[
\{ \Omega_2(\vec{x}), \Omega_4(\vec{y}) \} = \delta(\vec{x} - \vec{y}) ,
\]
\[
\{ \Omega_3(\vec{x}), \Omega_5(\vec{y}) \} = -\nabla^2 \delta(\vec{x} - \vec{y})
\]

and the others vanish. The inverse of the constraint matrix

\[
C(\vec{x} - \vec{y})^{-1} = \begin{pmatrix}
0 & 0 & \nabla^{-2} \\
0 & -1 & 0 \\
0 & 0 & \nabla^{-2}
\end{pmatrix} \delta(\vec{x} - \vec{y}) ,
\]

and the only Dirac brackets among the dynamical variables which do not vanish are

\[
\{ A_i(\vec{x}), \pi_j^A(\vec{y}) \}_{\text{D.B.}} = -\left( \delta_{ij} - \frac{1}{\nabla^2} \partial_i \partial_j \right) \delta(\vec{x} - \vec{y}) ,
\]
\[
\{ Z_i(\vec{x}), \pi_j^Z(\vec{y}) \}_{\text{D.B.}} = -\delta_{ij} \delta(\vec{x} - \vec{y}) ,
\]
\[
\{ Z_i(\vec{x}), A_0(\vec{y}) \}_{\text{D.B.}} = -\frac{1}{b} \frac{1}{\nabla^2} \partial_i \delta(\vec{x} - \vec{y}) ,
\]
\[
\{ Z_i(\vec{x}), Z_0(\vec{y}) \}_{\text{D.B.}} = \partial_i \delta(\vec{x} - \vec{y})
\]

and

\[
\{ \pi_i^A(\vec{x}), \pi_j^A(\vec{y}) \}_{\text{D.B.}} = \frac{\theta}{2} \left( \varepsilon_{jk} \frac{1}{\nabla^2} \partial_k \partial_i - \varepsilon_{ik} \frac{1}{\nabla^2} \partial_k \partial_j \right) \delta(\vec{x} - \vec{y})
\]

From equation (16b) and the Hamilton equation for the momentum \( \pi_i^Z \),

\[
\dot{\pi}_i^Z = Z_i + b \partial_k F_{ki}
\]
we get the “Ampere law in the presence of a source”

\[ \dot{E}_i + \varepsilon_{ki} \partial_k B = -\frac{Z_i}{b}, \tag{37} \]

where \( B = \varepsilon_{ij} \partial_i A_j \) is the magnetic field.

Now we perform the Dirac quantization by following the second approach quoted before. For this we add the following gauge fixing term in the Lagrangian density (10),

\[ \mathcal{L}_{GF} = \frac{\alpha}{2} \sigma^2 + \sigma (\partial_\mu A^\mu), \tag{38} \]

where \( \sigma \) is an auxiliar field, used to permit that the Lorentz condition be introduced as a linear constraint. Its introduction produces the following set of primary second class constraints:

\[ \Omega_1 = \pi^A_0 - \sigma; \Omega_2 = \pi^Z_0; \Omega_3 = \pi_\sigma, \tag{39} \]

whose preservation in time generates the secondary constraints:

\[ \Omega_4 = \alpha \sigma - \vec{\nabla} \cdot \vec{A}; \Omega_5 = \vec{\nabla} \cdot \vec{\pi}^Z - Z_0; \]
\[ \Omega_6 = \alpha \vec{\nabla} \cdot \vec{A} \pi^A + \frac{\alpha}{2} \varepsilon^{ij} \partial_i A_j - \nabla^2 A_0 + \frac{1}{|b|} \vec{\nabla} \cdot \vec{\pi}^Z, \tag{40} \]

and also eliminate the Lagrange multipliers,

\[ \nabla^2 \lambda_1 = -\frac{1}{|b|} \vec{\nabla} \cdot \vec{Z}; \lambda_2 = \vec{\nabla} \cdot \vec{Z}; \lambda_3 = \vec{\nabla} \cdot \vec{\pi}^A + \frac{\theta}{2} \varepsilon^{ij} \partial_i A_j. \tag{41} \]

So we finish with six second class constraints, whose inverse matrix is given by

\[ C(\vec{x} - \vec{y})^{-1} = \frac{1}{2} \begin{pmatrix}
0 & 0 & 1 & 0 & 0 & -\nabla^{-2} \\
0 & 0 & 0 & 0 & -2 & 0 \\
-1 & 0 & 0 & \alpha^{-1} & 0 & 0 \\
0 & 0 & -\alpha^{-1} & 0 & 0 & -\alpha^{-1}\nabla^{-2} \\
0 & 2 & 0 & 0 & 0 & 0 \\
\nabla^{-2} & 0 & 0 & \alpha^{-1}\nabla^{-2} & 0 & 0
\end{pmatrix} \delta(\vec{x} - \vec{y}). \tag{42} \]
After using the constraints strongly, we eliminate the fields $A_0$, $\pi_0$, $\sigma$, $\pi_\sigma$, $B_0$, $\pi_0^B$. Obtaining for the remaining fields the following nonvanishing Dirac brackets:

\begin{align}
\{A_i, \pi_j^A\}_{\text{D.B.}} &= -\left[\delta_{ij} - \frac{\partial_i \partial_j}{\nabla^2}\right] \delta(\vec{x} - \vec{y}); \\
\{Z_i, \pi_j^A\}_{\text{D.B.}} &= \frac{1}{2\alpha|b|} \frac{\partial_i \partial_j}{\nabla^2} \delta(\vec{x} - \vec{y}); \\
\{Z_i, \pi_j^Z\}_{\text{D.B.}} &= -\delta_{ij} \delta(\vec{x} - \vec{y}); \\
\{\pi_i^A(\vec{x}), \pi_j^A(\vec{y})\}_{\text{D.B.}} &= \frac{\theta}{2} \left(\varepsilon_{jk} \frac{1}{\nabla^2} \partial_k \partial_i - \varepsilon_{ik} \frac{1}{\nabla^2} \partial_k \partial_j\right) \delta(\vec{x} - \vec{y}).
\end{align}

The reduced Hamiltonian is then written as

\begin{align}
H_r &= \frac{1}{2\alpha} (\vec{\nabla} \cdot \vec{A})^2 + \alpha (\vec{\pi}_A)^2 + \alpha \theta \varepsilon^{ij} \pi_i^A A_j + \frac{1}{2} (\vec{\nabla} \cdot \pi^B)^2 + \\
&\quad - \frac{\vec{Z}^2}{2} - \frac{a}{2b^2} (\vec{\pi}_Z)^2 + \frac{a}{2} \partial_i A_j F^{ij} + \frac{\alpha \theta}{2|b|} (\vec{A})^2 + \\
&\quad + \frac{\theta}{|b|} \varepsilon^{ij} \pi_i^Z A_j + |b| \partial_i Z_j F^{ij}.
\end{align}

It is be interesting to note that in this case, as might be expected, the Dirac brackets have some dependence in the gauge parameter. Furthermore some of the brackets are equal to that obtained using the former approach to fixing the gauge.

**IV. CONCLUSIONS**

We have performed Dirac’s quantization of a generalized electromagnetism in $2 + 1$ dimension which contains Maxwell, Chern-Simons and Podolsky’s terms. Our results are in complete agreement with the literature when the proper limits of the parameters are taken. Moreover the freedom to choose these parameters allow us to have distinct massive poles for the fields, including the existence of tachyonic modes which, unfortunately, can not be eliminated consistently.

We used the auxiliary field method to reduce the model to a lower-order derivative one and consequently pay the price of duplicating the number of dynamical variables. This
is an equivalent price one would have to pay if auxiliary field were not used, since one has to consider the time-derivative field as well as its conjugated momentum as independent variables.

As a consequence of introducing auxiliary fields we can see the Maxwell equations, equations (19) and (37), in a different point of view. The higher-derivative fields can be seen as sources to the fields themselves.

If one considers the effective action to the gauge field generated from the integration of fermion fields, the theory treated here should present interesting features. Since Chern-Simons and Podolsky’s terms can be generated dynamically it would be of interest to analyse the dependence with the temperature of Chern-Simons [14,15] and Podolsky’s parameters, consequently the spectrum of massive excitation.

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