Revisiting the statistical isotropy of GRB sky distribution

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Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT
The assumption of homogeneity and isotropy on large scales is one of the main hypotheses of the standard cosmology. In this paper, we test the assumption of isotropy from the two-point angular correlation function of 2440 gamma-ray bursts (GRB) of the FERMI GRB catalogue. We show that the uncertainties in the GRB positions may induce spurious anisotropic signals in their sky distribution. However, when such uncertainties are taken into account no significant evidence against the large-scale statistical isotropy is found. This result remains valid even for the sky distribution of short-lived GRB, contrarily to previous reports.

Key words: Large-scale structure of Universe – methods: data analysis – gamma-ray burst: general

1 INTRODUCTION
One of the foundations of modern cosmology is the so-called Cosmological Principle (CP), which consists in the assumption that the Universe looks homogeneous and isotropic on large scales. Statistical analyses using recent cosmological observations bring evidence that the CP holds true at such scales, as obtained from the Cosmic Microwave Background (CMB) temperature anisotropies (Ade et al. 2016), cosmic distances from type Ia Supernovae (Andrade et al. 2018b; Deng & Wei 2018; Sun & Wang 2018; Andrade et al. 2018a; Zhao et al. 2019), galaxy number counts (Gibelyou & Huterer 2012; Yoon et al. 2014; Bengaly et al. 2018a; Rameez et al. 2018), the sky distribution of galaxy clusters (Bengaly et al. 2017; Migkas & Reiprich 2018). There is also evidence for a homogeneity scale in the counts of quasars and galaxies (Scrimegeour et al. 2012; Ntelis et al. 2017; Gonçalves et al. 2018a,b). However, some controversial claims have appeared in the literature, such as large-angle features in the CMB (Schwarz et al. 2016) and a large dipole anisotropy in radio source counts (Singal 2011; Rubart & Schwarz 2013; Bengaly et al. 2018b) (see also Dolfi et al. 2019.).

Gamma-ray bursts (GRB) have also been used to test the CP. These events are extremely energetic explosions, whose range lies between $10^{48} - 10^{55}$ erg, which exceeds hundred times the total energy radiated by a supernova. Also if they are not properly standard candles, they may reveal themselves as possible formidable distance indicators. For a detailed discussion on the topic, we refer the reader to Dainotti et al. 2018. They are usually classified into short-lived ($T_{90} < 2s$, SGRBs) and long-lived ($T_{90} > 2s$, LGRBs), where $T_{90}$ denotes the duration in which 90% of the burst fluence is accumulated. In the last decades, several authors have shown that the GRBs sky distribution is consistent with statistical isotropy (Hartmann & Blumenthal 1989; Hartmann et al. 1991; Meegan et al. 1992; Briggs et al. 1996; Tegmark et al. 1996). However, subsequent works suggested otherwise for SGRBs (Balazs et al. 1998; Meszaros et al. 2000; Magliocchetti et al. 2003; Vavrek et al. 2008; Tarnopolski 2017). Moreover, there are also claims for the existence of GRB structures of $\sim 2000$ Mpc at $z \sim 2$ (Horvath et al. 2014,
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2015; Ruggeri & Capozziello 2016) – for a different conclusion, see also (Bernui et al. 2008; Gibelyou & Hutener 2012; Li & Lin 2015; Ukwatta & Woźniak 2016; Tarnopolski 2017; Ripa & Shaﬁeloo 2017, 2018)\(^1\).

Given the relevance of the topic, and the current controversies, we revisit in this paper the question of the statistical isotropy in the GRB sky distribution. The analysis performed uses the two-point angular correlation function (2pACF) of 2440 gamma-ray bursts of the Fermi Gamma-ray Burst Monitor Burst catalogue. We show that, after removing objects with large sky positional errors, there is no evidence of anisotropy signatures for the whole GRB sample, as well as for LGRBs and SGRBs sub-samples.

This paper is organised as follows. In Sec. II the observational data set used in the analysis is discussed. The methodology and data analysis performed are presented in Sec. III. Sec. IV presents our main results whereas Sec. V discusses such results and summarises our main conclusions.

2 THE OBSERVATIONAL DATA SET

In our analysis we use the Fermi Gamma-ray Burst Monitor Burst catalogue, termed FERMI GRBST (Gruber et al. 2014; von Kienlin et al. 2014; Bhat et al. 2016). This catalogue is one of the most complete GRB catalogues currently available, comprising 2440 objects detected from July 14th 2008 until November 12th 2018\(^2\). Specifically, we make use of the following quantities:

(i) RA: The Right Ascension of the burst, given in J2000 decimal degree.

(ii) DEC: The Declination of the burst, given in J2000 decimal degree.

(iii) ErrorRadius: The uncertainty of the object position, in degrees. We term it as \(σ_r\).

(iv) \(T_{90}\): The duration, in seconds, during which 90% of the burst fluence was accumulated.

The sky distribution of the selected GRBs is displayed in the Fig. 1. Note that we removed one object because it does not contain information about to the \(T_{90}\) parameter. We test the statistical isotropy for three cases, namely (allGRB), the Long GRB sub-sample (LGRB), and the sub-set of short-lived GRBs (SGRB).

3 DATA ANALYSIS

3.1 Two-point angular correlation function

In order to summarise the distribution of data points in the sky, one can report the mean number of points at a given scale. However, in the presence of clustering, the mean may be an insufficient descriptor, as this measure is insensitive to it. The 2pACF is a statistic capable of characterising the clustering of objects in the sky, which we denote by \(ω(θ)\).

\(ω(θ) = \frac{⟨DD(θ)⟩−2⟨DR(θ)⟩+(RR(θ))}{⟨RR(θ)⟩}\), \(1\)

where the brackets denote the normalised number of all GRB pairs in the real data (\(DD(θ)\)), in the auxiliary random isotropic catalogue (\(RR(θ)\)), and between the data and the random catalogue (\(DR(θ)\)). The counts of pairs for each angular scale is carried out through the range \((θ−dθ/2, θ+dθ/2)\), where \(dθ\) is the bin width. We choose evenly spaced samples in our analysis, so that \(ω(θ)\) is calculated over the interval \((0°, 180°)\) with \(dθ = 1.8°\). The random isotropic catalogues are generated following a uniform distribution on a sphere, so that

\[
\begin{align*}
RA & = 0° + (360° − 0°) \ U[0,1), \\
DEC & = \arcsin (−90° + (180°) \ U[0,1)) .
\end{align*}
\]

We assess the error in the estimates of the 2pACF via bootstrap method (Tarnopolski 2017). This is done through 100 re-samplings between the data and the random catalogue. Finally, we achieve a bootstrap sampling distribution of the 2pACF whose uncertainty is the standard deviation of this sampling distribution.

We also compute the 2pACF for isotropic random sample through Monte Carlo (MC) method, and perform the bootstrap analysis on it as well. We denote the mean and the standard deviation of this distribution as the benchmark (Tarnopolski 2017), since it provides the 2pACF limits that a finite isotropic sample must satisfy. Any significant deviation from this benchmark would hint at departures from statistical isotropy.

3.2 Absolute sum test

In order to further investigate the readability of the anisotropic signal, we devised what we hereafter call “Ab-

\(^1\) In addition to test the CP, GRB have also been used to probe fundamental physics (Petitjean et al. 2016).

\(^2\) https://heasarc.gsfc.nasa.gov/W3Browse/fermi/fermigbrst.html.
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Figure 2. Upper: Comparison between the 2pACF of the MC-shuffle realisations and the benchmark. Blue dots represent the former, and the shaded regions represent the latter within the standard deviation bounds. Bottom: The absolute sum test using 20° size bin. The blue diamond markers represent the MC-shuffle, whereas orange pentagon markers represent the benchmark. Left panels show results obtained for GRBs with $\sigma_r \leq 20°$, and the right panels display results for $\sigma_r \leq 6°$.

solute sum test”. We defined it as the summation of the absolute 2pACF values within a specific angular scale, i.e.,

$$\text{Absolute sum} = \sum_i |\omega(\theta)| \forall i \in \Delta\theta,$$

where $\Delta\theta$ is the angular range at which we split the $\omega(\theta)$, so that we perform this summation in a tomographic fashion within this range, i.e., $i_1 = (0°, 20°), i_2 = (20°, 40°), \ldots, i_9 = (160°, 180°)$. We choose $\Delta\theta = 20°$ in order to have at least 10 values of $\omega(\theta)$ within this interval (since $d\theta = 1.8°$) and thus a robust statistics for each $i$-bin. Hence we can compare the 2pACF at different angular scales for two distributions, e.g. the real data versus the benchmark. Again, any deviation beside the error bars (as obtained from standard uncertainty propagation) suggests potential deviations from statistical isotropy, after all, the 2pACF values should agree at different angular if this assumption holds.

3.3 Statistical significance estimate

We assess the statistical significance of our analysis by means of nonparametric tests between two different samples as follows:

(i) Kolmogorov-Smirnov (KS): It consists of a distribution-free test which compares the empirical cumulative distribution function (ECDF) of two samples (Ivezić et al. 2014). This test relies on a metric which measures the maximum distance of the two ECDF $F_m(x)$ and $G_n(x)$,

$$D = \max |F_m(x) - G_n(x)|.$$  \hspace{1cm} (5)

(ii) Anderson-Darling (AD): This test is based on the following statistics (Scholz & Stephens 1987):

$$A^2_{mn} = \frac{mn}{N} \int_{-\infty}^{\infty} \frac{(F_m(x) - G_n(x))^2}{H_N(x)\{1 - H_N(x)\}} dH_N(x).$$  \hspace{1cm} (6)

$F_m$ and $G_n$ are ECDF for two independent samples that may have different number of points, namely $n$ and $m$, respectively. $H_N(x) = (mF_m + nG_n) / N$ is the ECDF of the pooled sample, where $N = m + n$.

Our null hypothesis is that the two sample are drawn from the same distribution. KS test is sensitive to the location, the scale and the shape of the distribution, while AD test is only sensitive to the shape. Moreover, AD test is more sensitive to the tails differences than KS test, which in turn is more sensitive to the differences near the centre of the distribution. In this sense, these two non-parametric tests are complementary.

We choose $\alpha = 0.05$ as the significance level in which we reject the null hypothesis. Hence, a p-value lower than $\alpha$ when we compare the real data with the benchmark, for example, would denote that the samples are not drawn from the same distribution - and thus the data is not statistically isotropic. We used the routines KS_2SAMP and ANDER-
3.4 GRB positional uncertainties

Previous works showed that the GRB positional uncertainties might affect the measurement of the 2pACF and angular power spectrum of their celestial distribution (Hartmann et al. 1991; Tegmark et al. 1996). Here, in order to investigate the impact of such uncertainties on our analysis, we produce 1000 Monte Carlo (MC) realisations with the following prescription:

\[
R_{\text{new}} = N \left( R, \sigma^2 \right); \quad (7)
\]

\[
D_{\text{Cnew}} = N \left( D, \sigma^2 \right). \quad (8)
\]

Due to the shuffling of GRB celestial positions, we henceforth call this test MC-shuffle. We compute the 2pACF for each of these realisations, then we take their mean and standard deviation, and compare them with the benchmark. We repeat this procedure for different positional uncertainty cutoffs, which we denote by \( \sigma \). If we find significant discrepancy between these MC-shuffle and benchmark realisations for less restrictive cutoffs, this hints at spurious anisotropies arising due to such uncertainties.

4 RESULTS

We depict the impact of GRB positional errors on the 2pACFs in Fig. 2. By comparing the MC-shuffle and the benchmark with an upper limit of \( \sigma = 20^\circ \) and \( \sigma = 6^\circ \), we can clearly see that the former case agrees less with the benchmark, especially on smaller angular scales. This result shows that large positional errors indeed introduce spurious anisotropic signatures, as previously reported in Hartmann et al. (1991); Tegmark et al. (1996), and therefore we should impose an upper cut in our working sample. The GRB sub-sample with \( \sigma \leq 6^\circ \) will be hereafter taken as our real data sample, since it alleviates this feature, and we do not lose a large fractional of GRB, as we still retain 1634 GRBs in total, i.e., 1476 LGRBs and 158 SGRBs.

We compare the 2pACF between the real data and the benchmark in the upper panels of Fig. 3, while the bottom ones display the results from the absolute sum test as described in 3.2. The left, middle and right panels represent the allGRBs, LGRBs and SGRBs samples results, respectively. Once more the shaded regions in the upper panels provide the allowed region which an intrinsic isotropic sample can vary due to randomness. From these plots, we can conclude that both the 2pACF and the absolute sum show a good agreement between the allGRBs, LGRBs and SGRBs and the benchmark, and therefore no evidence against statistical isotropy.

In addition, we show the results obtained from the non-parametric tests in Table 1. We obtain that we cannot reject the null hypothesis at significance level of \( \alpha = 0.05 \) for all data samples, including SGRBs, although they yield the lowest p-value among all. We thence confirm that the sky distribution of FERMIGBRST catalogue of GRBs is statistically isotropic, as expected from the CP.

|            | allGRBs | LGRBs | SGRBs |
|------------|---------|-------|-------|
| p-value    | 0.15    | 0.12  | 0.17  |
| AD         | 0.10    | 0.16  | 0.56  |
| p-value    | 0.31    | 0.29  | 0.19  |

5 DISCUSSION AND CONCLUSION

One of the major concepts of modern cosmology is the assumption of the statistical homogeneity and isotropy on large scales. Together with the Einstein’s field equations, they are at the basis of what we know as modern cosmology.

In this paper, we probed the statistical isotropic hypothesis of the CP by means of the 2pACF of the GRB sky distribution. To perform our analysis, we compared the 2pACF of the Fermi GRB catalogue with the 2pACF of the isotropic synthetic sample. We also investigated how the uncertainty in the GRBs position might affect ours conclusion by drawing 1000 MC simulations with new GRB positions inside the radius of the observational positional uncertainty.

We found that large positional uncertainties lead to spurious anisotropy detection, as shown in the left panel of Fig. 2. For this reason, we perform cuts on the position uncertainty, choosing \( \sigma = 6^\circ \) as an optimal upper cut in which we can avoid spurious anisotropy without losing too many sources. Then, we split the data in three samples: all GRBs, LGRBs and SGRBs, containing 1634, 1476 and 158 each, respectively, after this cutoff. Fig. 3 shows that all these data-sets are in good agreement with the statistical isotropy hypothesis, since the 2pACF agreed with the isotropy allowed region and the absolute sum is consistent between data and the benchmark sample. This result is confirmed by the KS and AD tests between the real data and the benchmark, whose p-values were displayed in Table 1. None of these p-values were smaller than 0.05, meaning we cannot reject the null hypothesis at this significance level. We remark that in the case of SGRB, despite a lower p-value, one still cannot reject the null hypothesis.

We conclude that there is no significant evidence for isotropy departure in the currently available GRB sky distribution, even including the SGRBs sub-sample. This result is in good agreement with Tarnopolski (2017); Ukwatta & Wozniak (2016); Ripa & Shafieloo (2017, 2018), in which we used an updated sample of GRB and a different estimator. Therefore, our result confirms the validity of statistical isotropy of the GRB distribution across the sky, which should be definitely underpinned in light of forthcoming Gamma-Ray surveys like e.g. THESEUS (Amati et al. 2018). Specifically, this space mission is aimed to exploit GRBs in view of investigating the early Universe and then providing a substantial advance in time-domain astrophysics. Besides,
considering the proposed large range of redshift investigation, it would be extremely interesting for multi-messenger astrophysics.

ACKNOWLEDGEMENTS

U.A. acknowledges financial support from CAPES. C.A.P.B. acknowledges financial support from the South African SKA Project. J.S.A. acknowledges support from CNPq (grant Nos. 310790/2014-0 and 400471/2014-0) and FAPERJ (grant No. E-26/203.024/2017). S.C. acknowledges support from INFN (Initiativa Specifica QGSKY) and CANTATA COST action CA15117.

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