Modeling GDP Using Autoregressive Integrated Moving Average (ARIMA) Model: A Systematic Review

Benard Muma, Austin Karoki

Department of Management Science and Technology, Technical University of Kenya, Nairobi, Kenya
Email: mumabenard@gmail.com, gkaroki@gmail.com

Abstract
This paper examines the application of the Autoregressive Integrated Moving Average (ARIMA) Model in modeling GDP. The paper critically reviews the empirical literature on the application of ARIMA models in modeling GDP in various economies with the objective of establishing the appropriateness and popularity of ARIMA model in studying GDP. The paper concludes that ARIMA (2, 2, 2) and ARIMA (3, 1, 1) are significantly applied in studies involving 182-days treasury bills and ARIMA (1, 1, 0) 91-days treasury bills respectively. This paper proposes the application of Q-statistic for autocorrelation, Jarque-Bera (JB) test for normality and Box-Jenkins analysis for model identification and estimation, diagnosis as well as forecasting. The paper further presents that the correlogram of real GDP rate series can be used for GDP modeling, specifically recommending the application of ARCH and GARCH family models in studies involving the analysis of treasury bills. Lastly, this paper recommends the adoption of the expenditure approach to measure GDP and proposes a model that can be adopted in GDP modeling.

Subject Areas
Business Statistics and Analytics

Keywords
Autoregressive Integrated Moving Average Model, Modeling GDP

1. Introduction
Sustainable economic performance is a dream of most nations [1]. This is because the wellness of a nation as well as its people largely depends on its eco-
nomic performance [2]. According to United Nations (2015) [3], it is in the best interest of every nation to work towards sustainable economic growth. Sustainable economic performance is associated with enhanced economic and social outcomes [4]. In fact, literally, all nations are on the outlook for strategies that would enhance their economic suitability and stability [5]. Just like the developed nations, the report by United Nations (2015) [3] indicates that developing nations are on the run, towards the adoption of programs and initiatives that would enhance their economic performance. GDP is one of the most robust indicators of economic performance. In fact, United Nations Conference on Trade and Development (2020) indicates that GDP is applied globally as a measure of economic performance.

Literally, all nations are adopting more productive capabilities towards the development of better economies, including developing partnerships with private organizations and with governments, the adoption of structural transformations towards enhanced productivity, transition to the digital economy and more advanced technological capabilities among other initiatives [6]. While nations have been able to somehow predict economic performance through the national treasuries, the literature indicates that the economic environment is so volatile that in some cases, the economic forecasts just remain forecasts [7]. The outcome of the interaction between political, natural, social and economic factors in the economy is so unpredictable that no single indicator can be used to predict economic performance across nations [8].

Statistical models are arguably the most reliable tools for predicting future outcomes based on past data. One of the common statistical models in economic analysis is the ARIMA model. While literature exists on the application of the ARIMA model to forecast GDP in several nations [9] [10], empirical findings indicate that every nation has unique conditions that make it necessary to develop a unique ARIMA model. Findings indicate that there is a need to develop an economy specific model for reliable prediction of GDP. Through a systematic review, this systematic review intends to enhance understanding of the application of ARIMA models in GDP analysis and recommend appropriate models for such analyses [11].

2. Literature Review

2.1. Gross Domestic Product

Economic growth is a very popular topic among scholars, professionals and governments [6]. This is due to the fact that economic development determines the general wellbeing of a nation and its citizens [10]. According to the World Bank, Kenya is one of the fastest growing economies not only in Sub-Saharan Africa, but in the whole world [12]. Kenyan economy is characterized with stability in macroeconomic environment and resilience in service sector [13]. Just like any other Nation, Kenyan economy is affected by domestic and international market factors such as local fiscal and monetary policies, international trade,
global economic patterns and foreign investment [3]. In the local economic environment, rate of treasury bills and bonds can be used to assess the economic wellbeing.

The mostly used measure for economic growth is Gross Domestic Product (GDP) [14]. GDP is the market value of all final products produced within an economy within a specified time period. GDP is the most popularly used indicator for economic performance [1]. This is because; it measures both economic strength and national wealth [15]. In their study, Wabomba et al. (2016) [16] argue that GDP can be used as a measure of economic performance as to assess soundness of an economy.

Analysis of past studies indicates that scholars globally have adopted three approaches to determine GDP; income approach, production approach, and expenditure approach [17]. In a separate study, Yang et al. (2016) [15] argues that income and expenditure methods are more robust in measuring GDP. Income approach uses income earned by households and businesses in a year to measure GDP, production approach uses net product of output produced in a year while expenditure approach uses the values of expenditures made by households and businesses in an economy annually. On the other hand, Agrawal (2018) [9] presents that GDP can be measured in terms of factor cost using market price, indirect taxes and subsidies.

2.2. Application of ARIMA Models in GDP Modeling

Application of Autoregressive Integrated Moving Average (ARIMA) model in modeling practical economic and societal issues is evident in various parts of the world. There are various ARIMA models that have been put forward for modeling of possible practical problems. The general model takes the form, ARIMA (p, d, q) which is general used as a modeling equation. In the model, p denotes the number of lag observations in the model; q denotes the degree of differencing while d is the number of differences that makes the time series stationary. From the general modeling equation, specific models are derivable, such as, the first order autoregressive model, ARIMA (1, 0, 0), the random walk model, ARIMA (0, 1, 0), the differenced first order autoregressive model, ARIMA (1, 1, 0), simple exponential smoothing model, ARIMA (0, 1, 1), linear exponential smoothing model, ARIMA (0, 1, 1) or (0, 2, 2) and damped trend linear exponential smoothing model.

In their study, Logubayom, Nasiru & Luguterah (2013) [14] argue that Exponential Smoothing Method (ESM) and ARIMA are the mostly used approaches to forecast GDP. In a study by Agrawal (2018) [9], ARIMA model was used in modeling of Indian GDP. The study used time series data on real GDP for a period of 20 years. The study established that ARIMA (1, 1, 0) and ARIMA (1, 1, 1) are not significantly different in terms of application in economic modeling. Yang et al. (2016) [15] argues that adoption of appropriate ARIMA model can produce more objective forecast of GDP. Specifically, they recommend ARIMA (2, 2,
2) for prediction of GDP. ARIMA models are more effective and accurate in modeling GDP [18]. In their study based in Kenya, Wabomba et al. (2016) [16] too propose ARIMA (2, 2, 2) and present that ARIMA models are more accurate and robust in short term forecasts as compared to other models such as vector error correction models, and Autoregressive Conditional Heteroskedasticity (ARCH) based models. A similar study was conducted by Yang et al. (2016) [15] in China. The study applied ARIMA model to predict Chinese GDP using data between 1978 and 2014. The study recommends that ARIMA (2, 2, 2) can be applied in short term forecasting of GDP.

In a study, Hisham & Amin (2020) [6] reviewed ARIMA model to study GDP in Sudan. The study settled on ARIMA (1, 1, 1) as an effective and efficient model in modeling annual GDP. The study also applied Autoregressive (AR) model, Moving-Average (MA) model and Autoregressive Moving-Average (ARMA) model. Similar study was conducted by Abonazel and Abd-Elftah (2019) [19] in Egypt between 1965 and 2016, the study recommended ARIMA (1, 2, 1) as the most suitable model for forecasting of GDP. In an examination, Sehgal, Bijoy & Deisting (2012) [12], on the other hand, applied ARIMA model predicting Sudanese GDP and established that ARIMA (0, 1, 0) is the most suitable model for Sudanese GDP.

In Nigeria, Omekara, Okereke & Ehighibe (2016) [20] applied ARIMA model to conduct a time series analysis of economic performance as influenced by interest rate. The study established that owing to volatility of interest rates, ARMA-GARCH is the most suited model in forecasting interest rates. The study proposed application of ARIMA (2, 1, 3) in analyses involving modeling of interest rates. A related study in Rwanda by Nyoni & Bonga (2019) [21], that used data between 1960 and 2017 established that ARIMA (3, 1, 1) is best suited the Rwandan Economy. The model was established to be stable for economic modeling. ARIMA models have equally been applied in forecasting treasury bills rate. However, Logubayom, Nasiru & Luguterah (2013) [14] recommended that ARIMA (3, 1, 1) and ARIMA (1, 1, 0) are suitable for 91 treasury bills and 182 treasury bills rates, especially in the short run.

In their analysis, Abonazel & Abd-Elftah (2019) [19] applied ARIMA models to forecast Kenyan GDP. The study recommends ARIMA (2, 2, 2) as the most appropriate model for the Kenyan economy. On the other hand, Ondieki (2014) [22] applied ARCH and GARCH family models to analyze Treasury bill rates. The study used GARCH model (1, 1) and incorporated 91 and 182 days’ treasury bills. The study modeled 91 and 182 separately. The study proposed ARIMA (3, 1, 1) for 91 days treasury bills and ARIMA (1, 1, 0) for 182 treasury bills rate in the Kenyan context.

3. Methodology
This paper critically analyses empirical literature on application of ARIMA models in studies involving modeling of GDP. 10 papers are randomly selected and
analyzed with the objective of identifying the models used. The paper also consolidates the procedures used in the analyses in the studies in order to propose a step by step approach for future studies. Lastly, the paper proposes a model that can be considered by researchers in future studies involving GDP, specifically using the expenditure approach.

4. Summary and Conclusions

From the literature reviewed, it was evident that various ARIMA models have been proposed for the modeling of GDP. Table 1 presents the application of various ARIMA models in selected studies.

From the summary, it is evident that ARIMA (2, 2, 2) and ARIMA (3, 1, 1) have significantly been applied in studies involving 182-days treasury bills while ARIMA (1, 1, 0) has been used in studies involving 91-days treasury bills. However, various studies seem to generate a specific model for each economy, an indication of the diverse nature of the economies. This review concludes that time series analysis is preferable in studies involving short-run projections using large data sets such as modeling of GDP in agreement with [19]. The paper presents that the ARIMA model, through the application of univariate time series, involving autoregressive, differencing and moving averages, can regress stationary and non-stationary data, and is, therefore, the most preferred model for the analysis of GDP. The maximum likelihood estimator can be used to develop the models with

Table 1. Selected papers on application of ARIMA models in modeling of GDP.

| Author                        | Study Focus       | ARIMA Model Used               |
|-------------------------------|-------------------|--------------------------------|
| Agrawal (2018) [9]            | Modeling GDP in India | ARIMA (1, 1, 0); ARIMA (1, 1, 1) |
| Yang et al. (2016) [15]       | Modeling GDP in China  | ARIMA (2, 2, 2)               |
| Wabomba et al. (2016) [16]    | Modeling GDP in Kenya | ARIMA (2, 2, 2), ARIMA (1, 1, 0) |
| Hisham & Amin (2020) [6]      | Modeling GDP in Sudan | ARIMA (1, 1, 1)              |
| Abonazel and Abd-Elftah (2019) [19] | Modeling GDP in Egypt | ARIMA (1, 2, 1)            |
| Sehgal, Bijoy & Deisting (2012) [12] | Modeling GDP in Sudan | ARIMA (0, 1, 0)           |
| Omekara, Okereke & Ehigibe (2016) [20] | Modeling GDP in Nigeria | ARIMA (2, 1, 3)          |
| Nyoni & Bonga (2019) [21]     | Modeling GDP in Rwanda | ARIMA (3, 1, 1)          |
| Ondieki (2014) [22]           | Modeling GDP in Kenya | ARIMA (3, 1, 1); ARIMA (1, 1, 0) |
| Logubayom, Nasiru & Luguterah (2013) [14] | Modeling GDP in Ghana | ARIMA (3, 1, 1); ARIMA (1, 1, 0) |
the aid of STATA where the order of auto terms (p), order of integration (d) and
the number of moving average terms (e) are used as the parameters. For the model
to reliably forecast a situation, it must be accurate, must use adequate data and
the data must have been collected over an adequate time period [16]. Diagnostic
analysis can be conducted using Q-statistic for autocorrelation and Jarque-Bera
(JB) test for normality. Furthermore, Box-Jenkins analysis, involving data gener-
ation and collection, identification of stationary in time series, model identifica-
tion and estimation, diagnostic checking and forecasting [9] is recommended for
GDP modeling using ARIMA. The correlogram of the real GDP rate series can be
used in such analyses. Lastly, the paper recommends the application of ARCH and
GARCH family models in studies involving the analysis of treasury bills.

The review acknowledges that economic conditions vary from a nation to the
other. An individualized approach to the modeling of GDP is recommended, where
each economy is modeled taking into consideration the unique prevailing economic
conditions. This paper recommends the adoption of an expenditure approach to
measure GDP. This is due to the holistic nature of the expenditure approach. The
paper proposes the following model for studying the GDP:

\[
GDPExp = C_{GRS} + I_B + G_{FGRS} + Ex - Im
\]  

where:

- \( C_{GRS} \)—consumer spending on goods and services;
- \( I_B \)—investor spending on capital goods;
- \( G_{FGRS} \)—government spending on public goods and services;
- \( Ex \)—export;
- \( Im \)—import.

The paper proposes ARIMA modeling as presented. Autoregressive (AR) model is expressed as:

\[
X_t = c + \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \cdots + \alpha_p X_{t-p} + \epsilon_t, \quad t = 1, 2, \ldots, T
\]  

Moving averages (MA) model is expressed as:

\[
X_t = \epsilon_t - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2} - \cdots - \theta_q \epsilon_{t-q}
\]  

ARMA model, incorporating AR and MA models is expressed as:

\[
X_t = c + \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \cdots + \alpha_p X_{t-p} + \epsilon_t - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2} - \cdots - \theta_q \epsilon_{t-q}
\]  

Non-seasonal, ARIMA model is expressed as:

\[
\Delta X_t = c + \alpha_1 \Delta X_{t-1} + \alpha_2 \Delta X_{t-2} + \cdots + \alpha_p \Delta X_{t-p} + \epsilon_t - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2} - \cdots - \theta_q \epsilon_{t-q}
\]  

where:

\[
\Delta X_t = X_t - X_{t-1}
\]  

In Model (4), if \( p = q = 0 \), the model becomes a random walk model (ARIMA 0, 1, 0).

**Conflicts of Interest**

The authors declare no conflicts of interest.
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