Abstract

In this article we discuss a scheme of teleportation of atomic states making use of three-level lambda atoms. The experimental realization proposed makes use of cavity QED involving the interaction of Rydberg atoms with a micromaser cavity prepared in a coherent state. We start presenting a scheme to prepare atomic EPR states involving two-level atoms via the interaction of these atoms with a cavity. In our scheme the cavity and some atoms play the role of auxiliary systems used to achieve the teleportation.

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1 INTRODUCTION

Both entanglement and non-locality, which are closely related, are important concepts in quantum mechanics with applications in information processing and quantum computing [1, 2]. One of the most dramatic among various consequences of entanglement and non-locality with applications in information science is teleportation, put forward by Bennett at al [3]. The essentials of the teleportation scheme is that, given an unknown quantum state to the sender, making use of quantum entanglement and non-locality, it is possible to reproduce this state far apart in the quantum system of the receiver where, in the process, both the sender and the receiver follow a certain prescription and communicate with each other through a classical channel. In the end of the process the receiver has a quantum state similar to the quantum state of the sender and the quantum state of the sender is destroyed since, according to the no-cloning theorem [4, 1] it is not possible to clone a quantum state. Quantum teleportation is an experimental reality and it holds tremendous potential for applications in the field of quantum communication and computing [2, 1]. For instance, it can be used to build quantum gates which are resistant to noise and is intimately connected with quantum error-correcting codes [1]. The most significant difficulty for quantum teleportation to become an
useful tool in quantum communication and computing is how to avoid decoherence effects [3, 6]. A scheme of teleportation of atomic states, using cavity QED, has been proposed in Ref. [7]. For several proposals of realization schemes of teleportation see [2].

In this article we present a scheme of teleportation close to the original scheme presented by Bennett et al [3]. We will assume that Alice and Bob meet and then create an EPR atomic state involving atoms A2 and A4. Then Alice and Bob separate. Alice takes with her half of the EPR pair, that is, atom A2 and Bob keeps with him the other half of the EPR pair, that is, atom A4. Later on Alice are going to be able to teleport to Bob’s atom A4 an unknown state of an atom A1 making use of her half of the EPR pair, that is, atom A2.

In the discussion which follows we are going to consider Rydberg atoms of relatively long radiative lifetimes [8]. We also assume a perfect microwave cavity and we neglect effects due to decoherence. Concerning this point, it is worth to mention that nowadays it is possible to build up niobium superconducting cavities with high quality factors $Q$. It is possible to construct cavities with quality factors as high as $Q \sim 10^{12}$ [10], which, for frequencies $\nu \sim 50$ GHz gives us a cavity field lifetime of the order of a few seconds.

2 EPR STATES

Let us first show how we can get an EPR state [11] making use of three-level lambda atoms interacting with a cavity field. Consider a three-level lambda atom (see Fig. 1) interacting with the electromagnetic field inside a cavity $C$. The states of the atom, $|a\rangle$, $|b\rangle$ and $|c\rangle$ are so that the $|a\rangle \leftrightarrow |c\rangle$ and $|a\rangle \leftrightarrow |b\rangle$ transitions are in the far off resonance interaction limit. The time evolution operator for the atom-field interaction $U(t)$ is given by [12]

$$U(\tau) = -e^{i\varphi a^\dagger a}|a\rangle\langle a| + \frac{1}{2}(e^{i\varphi a^\dagger a} + 1)|b\rangle\langle b| + \frac{1}{2}(e^{i\varphi a^\dagger a} - 1)|c\rangle\langle c| + \frac{1}{2}(e^{i\varphi a^\dagger a} + 1)|c\rangle\langle b| + \frac{1}{2}(e^{i\varphi a^\dagger a} + 1)|c\rangle\langle c|,$$

(2.1)

where $a$ ($a^\dagger$) is the annihilation (creation) operator for the field in cavity $C$, $\varphi = 2g^2\tau/\Delta$, $g$ is the coupling constant, $\Delta = \omega_a - \omega_b - \omega = \omega_a - \omega_c - \omega$ is the detuning where $\omega_a$, $\omega_b$ and $\omega_c$ are the frequency of the upper level and of the two degenerate lower levels respectively and $\omega$ is the cavity field frequency and $\tau$ is the atom-field interaction time. For $\varphi = \pi$, we get

$$U(\tau) = -\exp\left(i\pi a^\dagger a\right)|a\rangle\langle a| + \Pi_+|b\rangle\langle b| + \Pi_-|c\rangle\langle b| + \Pi_-|c\rangle\langle c| + \Pi_+|b\rangle\langle c| + \Pi_-|b\rangle\langle c|,$$

(2.2)

where

$$\Pi_+ = \frac{1}{2}(e^{i\pi a^\dagger a} + 1),$$

$$\Pi_- = \frac{1}{2}(e^{i\pi a^\dagger a} - 1).$$

(2.3)

Considering the non-normalized even and odd coherent states [13]

$$|\pm\rangle = |\alpha\rangle \pm | - \alpha\rangle,$$

(2.4)

in the following calculations we shall use the relations

$$\Pi_+|+\rangle = |+\rangle,$$

$$\Pi_+|-\rangle = 0,$$

$$\Pi_-|-\rangle = -|-\rangle,$$

$$\Pi_-|+\rangle = 0,$$

(2.5)
which are easily obtained from Eqs. (2.3) and (2.4), using \( e^{z^a} |\alpha\rangle = |e^z\alpha\rangle \).

Let us prepare the cavity \( C \) in the coherent state \(|\alpha\rangle\) and consider the atom \( A1 \) in the following state

\[ |\psi\rangle_{A1} = |b_1\rangle. \]

The initial state of the atom-cavity system is given by

\[
|\psi(0)\rangle_{A1-C} = |\psi\rangle_{A1}|\alpha\rangle = |b_1\rangle|\alpha\rangle = |b_1\rangle \frac{1}{2}(|+\rangle + |\rangle). \tag{2.6}
\]

We now let atom \( A1 \) fly through the cavity \( C \). The state of the system evolves according to the time evolution operator Eq. (2.2) yielding

\[
|\psi(\tau)\rangle_{A1-C} = U(\tau)|\psi(0)\rangle_{A0-C} = \frac{1}{2}\{ |b_1\rangle|+\rangle - |c_1\rangle|\rangle \}. \tag{2.7}
\]

Consider now another three-level lambda atom \( A2 \) prepared initially in the state \(|b_2\rangle\), which are going to pass through the cavity. Now, as initial state of the system, we have

\[
|\psi(0)\rangle_{A1-A2-C} = \frac{1}{2}\{ |b_1\rangle|+\rangle - |c_1\rangle|\rangle \}|b_2\rangle. \tag{2.8}
\]

After this second atom has passed through the cavity, the system evolves to

\[
|\psi(\tau)\rangle_{A1-A2-C} = U(\tau)|\psi(0)\rangle_{A1-C-A2} = \frac{1}{2}\{ |b_1\rangle|b_2\rangle|+\rangle + |c_1\rangle|c_2\rangle|\rangle \}. \tag{2.9}
\]

Now, we inject a coherent state \(|\alpha\rangle\) in the cavity, that is, we make use of \( D(\beta)|\alpha\rangle = |\alpha + \beta\rangle\), and we get

\[
|\psi\rangle_{A1-A2-C} = \frac{1}{2}\{ |b_1\rangle|b_2\rangle(|2\alpha| + |0\rangle) + |c_1\rangle|c_2\rangle|2\alpha| - |0\rangle) \}
= \frac{1}{2}\{ (|b_1\rangle|b_2\rangle + |c_1\rangle|c_2\rangle)|2\alpha\rangle + (|b_1\rangle|b_2\rangle - |c_1\rangle|c_2\rangle)|0\rangle \}. \tag{2.10}
\]

In order to disentangle the atomic states of the cavity field state we now send a two-level atom \( A3 \), resonant with the cavity, with \(|f_3\rangle\) and \(|e_3\rangle\) being the lower and upper levels respectively, through \( C \). If \( A3 \) is sent in the lower state \(|f_3\rangle\), under the Jaynes-Cummings dynamics [4] we know that the state \(|f_3\rangle|0\rangle\) does not evolve, however, the state \(|f_3\rangle|2\alpha\rangle\) evolves to \(|e_3\rangle|\chi_e\rangle + |f_3\rangle|\chi_f\rangle\), where \(|\chi_f\rangle = \sum_n C_n \cos(n t \sqrt{n + 1})|n\rangle\) and \(|\chi_e\rangle = -i \sum_n C_n+1 \sin(n t \sqrt{n + 1})|n\rangle\) and \( C_n = e^{-\frac{i}{2}|2\alpha|^2}(-2\alpha)^n/\sqrt{n!} \).

Then we get

\[
|\psi\rangle_{A1-A2-A3-C} = \frac{1}{2}\{ (|b_1\rangle|b_2\rangle + |c_1\rangle|c_2\rangle)(|e_3\rangle|\chi_e\rangle + |f_3\rangle|\chi_f\rangle) + (|b_1\rangle|b_2\rangle - |c_1\rangle|c_2\rangle)|f_3\rangle|0\rangle \}. \tag{2.11}
\]

and if we detect atom \( A3 \) in state \(|e_3\rangle\), then we finally get the EPR state involving the atoms \( A1 \) and \( A2 \)

\[
|\Psi^+\rangle_{A1-A2} = \frac{1}{\sqrt{2}}(|b_1\rangle|b_2\rangle + |c_1\rangle|c_2\rangle), \tag{2.12}
\]

which is an entangled state of atoms \( A1 \) and \( A2 \).
In the above disentanglement process we can choose a coherent field with a photon-number distribution with a sharp peak at average photon number \( \langle n \rangle = |\alpha|^2 \) so that, to a good approximation, 
\[ |\chi_f\rangle \cong C_\pi \cos(\sqrt{\pi g \tau}|\pi), \quad |\chi_e\rangle \cong C_\pi \sin(\sqrt{\pi g \tau}|\pi), \]
where \( \pi \) is the integer nearest \( \langle n \rangle \), and we could choose, for instance \( \sqrt{\pi g \tau} = \frac{\pi}{2} \), so that we would have 
\[ |\chi_e\rangle \cong C_\pi |\pi\rangle \quad \text{and} \quad |\chi_f\rangle \cong 0. \]
In this case we atom \( A3 \) would be detected in state \( |e_3\rangle \) with almost 100% of probability. Therefore, proceeding this way, we can guarantee that the atomic and field states will be disentangled successfully as we would like.

Now, if we start from (2.9) and we inject a coherent state \( |-\alpha\rangle \) in the cavity and let a two-level atom \( A3 \) resonant with the cavity to fly through the cavity prepared initially in the state lower \( |f_3\rangle \) and detect the upper state \( |e_3\rangle \), we get
\[ |\Psi^-\rangle_{A1-A2} = \frac{1}{\sqrt{2}}(|b_1\rangle|b_2\rangle - |c_1\rangle|c_2\rangle), \]
which is also an EPR state.

Now, if we apply the rotation
\[ R = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \]
that is,
\[ R = |c_2\rangle\langle b_2| - |b_2\rangle\langle c_2|, \]
to the state (2.12) we get
\[ |\Phi^-\rangle_{A1-A2} = \frac{1}{\sqrt{2}}(|b_1\rangle|c_2\rangle - |c_1\rangle|b_2\rangle), \]
and applying (2.15) to the state (2.13) we get
\[ |\Phi^+\rangle_{A1-A2} = \frac{1}{\sqrt{2}}(|b_1\rangle|c_2\rangle + |c_1\rangle|b_2\rangle). \]
The rotation of the atomic states (2.14) can be performed as shown in [15]. The states (2.12), (2.13), (2.16) and (2.17) form a Bell basis [16].

3 TELEPORTATION

In this section we are going to discuss a teleportation scheme that is closely similar to the original scheme suggested by Bennett et al [3]. Let us assume that Alice and Bob meet and then they build up an EPR state involving two-level atoms \( A2 \) and \( A4 \) as described in section 2 (we use the notation \( A3 \) for the two-level atom used to disentangle the atomic states from the cavity state as in the previous sections). That is, as in section 2 they make use of a cavity prepared initially in a coherent state \( |\alpha\rangle \) and by sending \( A2 \) and \( A4 \) through this cavity where the atoms interact dispersively with the cavity, and following the recipe presented in that section they get
\[ |\Psi^+\rangle_{A2-A4} = \frac{1}{\sqrt{2}}(|b_2\rangle|b_4\rangle + |c_2\rangle|c_4\rangle), \]
Now, let us assume that Alice keeps with her the half of this EPR state consisting of atom \( A2 \) and Bob keeps with him the other half of this EPR state, that is, atom \( A4 \). Then they separate and let
Now, all Alice has to do is to inject or and Bob has got the right state and has to do nothing else.

If she detects ( or ), and again he has got the right state.

Then, first Alice prepares a cavity \( C \) in a coherent state \( | \alpha \rangle \) and sends \( A1 \) and \( A2 \) through \( C \). Taking into account (2.2) with \( \varphi = \pi \), after atoms \( A1 \) and \( A2 \) fly through the cavity she gets

\[
| \psi \rangle_{A1-A2-A4-C} = \frac{1}{\sqrt{2}} \{ | b_1 \rangle | b_2 \rangle \{ | \xi \rangle + | \xi \rangle \} | b_4 \rangle + | c_1 \rangle | c_2 \rangle \{ | \xi \rangle + | \xi \rangle \} | b_4 \rangle + | b_1 \rangle | c_2 \rangle \{ | \xi \rangle + | \xi \rangle \} | b_4 \rangle + | b_1 \rangle | c_2 \rangle \{ | \xi \rangle + | \xi \rangle \} | b_4 \rangle + \}
\]

Now, all Alice has to do is to inject \( | \alpha \rangle \) in the cavity, send a two-level atom \( A3 \) resonant with the cavity in the lower state \( | f_3 \rangle \) through the cavity and detect the upper state \( | e_3 \rangle \) and detect ( ( \( b_1 \) | \( b_2 \) ), ( \( c_1 \) | \( c_2 \) ) ), ( \( b_1 \) | \( c_2 \) ) or ( \( c_1 \) | \( b_2 \) ) ) and call Bob informing him the result of her atomic detection. If she detects ( ( \( b_1 \) | \( b_2 \) ) )

\[
| \psi \rangle_{A4} = \zeta | b_4 \rangle + | c_4 \rangle,
\]

and Bob has got the right state and has to do nothing else.

If Alice detects ( ( \( c_1 \) | \( c_2 \) ) ) Bob gets

\[
| \psi \rangle_{A4} = \zeta | b_4 \rangle + \xi | c_4 \rangle,
\]

and again he has got the right state.

If Alice detects ( ( \( b_1 \) | \( c_2 \) ) ) Bob gets

\[
| \psi \rangle_{A4} = \xi | b_4 \rangle + \zeta | c_4 \rangle,
\]

and he needs to perform the rotation

\[
R_4 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},
\]
on the states of $A_4$ and he gets the right state. The rotation $\xi$ can be performed with an intense field for which $a \rightarrow \sqrt{n}e^{i\theta}$, $a^\dagger \rightarrow \sqrt{n}e^{-i\theta}$, and $\varphi a^\dagger a \rightarrow \varphi n$ with $\varphi n = \pi$ in (2.1).

Finally if Alice detects $(|c_1\rangle \langle b_2|)$ Bob gets

$$|\psi\rangle_{A_4} = \xi |b_4\rangle + \zeta |c_4\rangle,$$

and again he needs to perform the rotation $\xi$ on the states of $A_4$ and he gets the right state. In Fig. 2 we show the set-up of the teleportation scheme discussed here.

We can also present a representation in terms of qbits of the teleportation we have discussed. Making use of

$$|b_k\rangle = |0_k\rangle_{A},$$

$$|c_k\rangle = |1_k\rangle_{A},$$

$$|+\rangle = |0\rangle_{C},$$

$$|-\rangle = |1\rangle_{C},$$

we have

$$|\alpha\rangle = \frac{1}{2}(|+\rangle + |\rangle) = \frac{1}{2}(|0\rangle_{C} + |1\rangle_{C}),$$

$$|-\alpha\rangle = \frac{1}{2}(|+\rangle - |\rangle) = \frac{1}{2}(|0\rangle_{C} - |1\rangle_{C}),$$

we can write for the initial state of our system taking into account (3.20) and the fact that the cavity is initially in a coherent state $|\alpha\rangle$ as

$$|\psi\rangle_{A_1-A_2-A_4-C} = \frac{1}{2}\{\xi(|0_1\rangle \langle 0_2| \langle 0_4| + |0_1\rangle \langle 1_2| \langle 1_4|) +$$

$$\xi(|1_1\rangle \langle 0_2| \langle 0_4| + |1_1\rangle \langle 1_2| \langle 1_4|)(|0\rangle_{C} + |1\rangle_{C}).$$

(3.30)

The atom-field interaction in terms of qbits is represented as

$$|A\rangle \langle B| \langle C| \rightarrow |A \oplus C\rangle \langle B \oplus C| \langle C|,$$

(3.31)

where the symbol $\oplus$ means addition modulo 2 or it refers to the logical exclusive-OR (XOR) operation, which is a three qbit quantum gate and $\xi$ can be written as

$$\frac{1}{\sqrt{2}}\{0_1\rangle_{A} \langle 0_2|_{A}[\xi |0\rangle_{C} \langle 0_4|_{A} + \xi |1\rangle_{C} \langle 1_4|_{A}] +$$

$$|1_1\rangle_{A} \langle 1_2|_{A}[\xi |0\rangle_{C} \langle 1_4|_{A} + \xi |1\rangle_{C} \langle 0_4|_{A}] +$$

$$|0_1\rangle_{A} \langle 1_2|_{A}[\xi |0\rangle_{C} \langle 1_4|_{A} + \xi |1\rangle_{C} \langle 0_4|_{A}] +$$

$$|1_1\rangle_{A} \langle 0_2|_{A}[\xi |0\rangle_{C} \langle 0_4|_{A} + \xi |1\rangle_{C} \langle 1_4|_{A}].$$

(3.32)

Then the teleportation process is accomplished detecting $(|0_1\rangle_{A} \langle 0_2|_{A}),(|1_1\rangle_{A} \langle 1_2|_{A}),(|0_1\rangle_{A} \langle 1_2|_{A})$ or $(|1_1\rangle_{A} \langle 0_2|_{A}$ followed by a rotation of the atomic states if necessary as described above.

**Figure Captions**
Fig. 1- Energy level scheme of the three-level lambda atom where \( |a\rangle \) is the upper state with atomic frequency \( \omega_a \), \( |b\rangle \) and \( |c\rangle \) are the lower states with atomic frequency \( \omega_b \) and \( \omega_c \), \( \omega \) is the cavity field frequency and \( \Delta = \omega_a - \omega_b - \omega = \omega_a - \omega_c - \omega \) is the detuning.

Fig. 2- Alice and Bob meet and generate an EPR state involving atoms A2 and A4. Then they separate and Alice keeps atom A2 with her and Bob keeps atom A4 with him. Later on Alice decides to teleport an unknown state prepared in atom A1 to Bob. She sends atoms A1 and A2 through a cavity C prepared initially in a coherent state \( |\alpha\rangle \). After atoms A1 and A2 have flown through C Alice injects \( |\alpha\rangle \), sends a two-level atom A3 prepared initially in the lower state \( |f_3\rangle \) and resonant with the cavity through C and detects the upper state \( |e_3\rangle \). Then she detects the states of atoms A1 and A2 and calls Bob and inform him the result of her atomic detections. Depending on the results of Alice’s atomic detections Bob has or not to perform an extra rotation \( R_4 \) on the states of his atom A4.

References

[1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*, (Cambridge Univ. Press, Cambridge, 2000).

[2] G. Chen and R. K. Brylinski Eds., *Mathematics of Quantum Computation*, (Chapman & Hall/CRC, London, 2002).

[3] C. H. Bennett, C. Brassard, C. Crépeau, R. Jozsa, A. Peres and W. K. Wootters, Phys. Rev. Lett. 70, 1895 (1993).

[4] W. K. Wootters and W. H. Zurek, Nature 299, 802 (1982).

[5] M. Orszag, *Quantum Optics*, (Springer-Verlag, Berlin, 2000).

[6] W. H. Zurek, Phys. Today, October 44, 36 (1991); Phys. Rev. D 24, 1516 (1981); Phys. Rev. D 26, 1862 (1982).

[7] L. Davidovich, N. Zagury, M. Brune, J. M. Raimond and S. Haroche, Phys. Rev A 50 (1994), R895.

[8] A. A. Radzig and B. M. Smirnov, *Reference Data on Atoms, Molecules, and Ions* (Springer-Verlag, Berlin, 1985); T. F. Gallagher, *Rydberg Atoms* (Cambridge Univ. Press, Cambridge, 1984).

[9] M. Brune, F. Schmidt-Kaler, A. Mauli, J. Dreyer, E. Hagley, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. 76, 1800 (1996).

[10] G. Rempe, F. Schmidt-Kaler and H. Walther, Phys. Rev. Lett. 64, 2783 (1990).

[11] A. Einstein, B. Podolsky and N. Rosen, Phys. Rev. 47, 777 (1935).

[12] P. L. Knight, Phys. Scr. T12, 51 (1986); S. J. D. Phoenix and P. L. Knight, J. Opt. Soc. Am. B 7, 116 (1990).

[13] E. S. Guerra, B. M. Garraway and P. L. Knight, Phys. Rev. A 55, 3482 (1997).
[14] W. H. Louisell, *Quantum Statistical Properties of Radiation*, (Wiley, New York, 1973).

[15] E. S. Guerra and J. C. Retamal, J. Mod. Optics 46, 295 (1999).

[16] S. L. Braustein, A. Mann and M. Revzen, Phys. Rev. Lett. 68, 3259 (1992).
Fig. 1 - E. S. Guerra

\[ \begin{align*} &\Delta \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad |a\rangle \\
&\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad |b\rangle \\
&\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad |c\rangle \end{align*} \]
Fig. 2 - E. S. Guerra