Nucleon electric dipole moment from polarized deep inelastic scattering

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In a previous paper [1], we have pointed out the connection between the CP-odd three-gluon (Weinberg) operator and certain twist-four corrections in polarized deep inelastic scattering. Based on this observation, we give a numerical estimate of the electric dipole moment of the proton and neutron induced by the Weinberg operator. Our result is smaller than the previous estimates based on QCD sum rules by a factor of about 3 or more.

I. INTRODUCTION

Permanent electric dipole moments (EDM) of elementary particles, the nucleons, nuclei and atoms are an unambiguous signal of CP-violation in Nature that has profound implications for the evolution and composition of the universe. While being definite predictions of the Standard Model (SM) and models beyond the SM (BSM), EDMs in fundamental particles have never been observed so far despite decades of searches by many experimental collaborations (see reviews [2, 3] and references therein). Currently, only upper bounds for various systems have been determined, and the race is on worldwide to lower these bounds by several orders of magnitude in the next decades to come. The sensitivity of measurement in the present experiments has already reached the region predicted by many BSM scenarios, thereby providing stringent constraints on model parameters.

Typically, some BSM physics at the TeV scale or above induces, at lower energies, high-dimensional CP-violating operators into the QCD Lagrangian via loop effects. From that point on, it becomes entirely a QCD problem to determine the EDM of hadrons and nuclei generated by these operators. Of particular interest is the purely gluonic, dimension-six operator called the Weinberg operator [4]

\[ \mathcal{O}_W = g f^{abc} \tilde{F}_\mu^a F_\mu^b F_\nu^c. \]

Unlike the other CP-violating operators which involve the quark fields, \( \mathcal{O}_W \) does not receive suppression due to the current quark masses. Moreover, unlike the topological operator \( \tilde{F}_{\mu\nu} F^{\mu\nu} \), \( \mathcal{O}_W \) is free of the strong CP problem and does not affect the vacuum structure of QCD. These considerations make \( \mathcal{O}_W \) a relatively 'clean' source of hadronic EDMs. However, purely-gluonic high-dimensional operators such as \( \mathcal{O}_W \) are quite difficult to deal with theoretically. The most promising approach, the lattice QCD simulation, is not yet at the practical level despite much progress in recent years [5, 6]. More phenomenological approaches [7–10] are subject to large theoretical uncertainties, but at least they can provide concrete numbers that are reasonable in terms of magnitude and can be used as a reference.

In this paper, we aim to improve on the phenomenological approach by suggesting a new method to evaluate an important part of the nucleon EDM, namely, the so-called 'one-nucleon reducible' diagrams in the classification of [7]. Our approach is based on the observation [1] that \( \mathcal{O}_W \) mixes with a certain twist-four, quark-gluon operator known in the context of higher-twist corrections in polarized deep inelastic scattering (DIS). We show that the matrix element of the latter operator directly affects the nucleon EDM through the one-nucleon reducible diagrams. Moreover, it can be extracted from the data of the existing and future polarized DIS experiments such as the Electron-Ion Collider (EIC) [11] in the US. This opens up a novel, unexpected connection between traditional QCD spin physics and the physics of the nucleon EDM. As a demonstration of our method, we employ a phenomenological value of the matrix element and make a simple estimate of the proton and neutron EDMs.

II. NUCLEON EDM FROM THE WEINBERG OPERATOR

Throughout this paper, we assume that the Weinberg operator is induced in QCD as a low-energy effective action due to some beyond the standard model (BSM) physics [4]

\[ \Delta \mathcal{L} = w \int d^4x \, g f^{abc} \tilde{F}_\mu^a F_\mu^b F_\nu^c \equiv w \int d^4x \, \mathcal{O}_W(x), \]

where \( \tilde{F}_\mu^a = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta} \) and \( \epsilon^{0123} = +1 \). The coefficient \( w \) has dimension \(-2\). Different normalizations of the operator are used in the literature. Here we include one factor of the QCD coupling \( g \) in \( \mathcal{O}_W \), and our convention is such that \( g \) enters the covariant derivative with a positive sign \( D^\mu = \partial^\mu + ig A^\mu \). This choice is convenient when...
establishing connections to higher twist operators in the QCD literature. It is also a preferred definition since the sign of the coupling $g$ is indefinite in QCD due to the symmetry $g, A_\mu, F_{\mu\nu} \rightarrow -g, -A_\mu, -F_{\mu\nu}$ of the Lagrangian. With our normalization, $w$ must be even under $g \rightarrow -g$.

Consider the electromagnetic form factor for of the nucleon in the presence of $\Delta L$

$$\langle P'S' | J_{em}^\mu (0) | PS \rangle = \bar{u}(P') S' \left[ \gamma^\mu F_1 (\Delta^2) + i g_{\mu\nu} \Delta^\nu (F_2 (\Delta^2) - i g_{\gamma 5} F_3 (\Delta^2)) \right] u(P)$$

where $\Delta^\mu = P'^\mu - P^\mu$ and $m$ is the nucleon mass. $J_{em}^\mu = \sum_f e_f | \bar{e}_f \gamma^\mu e_f \rangle$ is the electromagnetic current ($e_f = 2/3$ for the u-quark, etc). Our conventions are $\gamma_5 = -i \gamma^0 \gamma^1 \gamma^2 \gamma^3$ and $g_{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$. The spin 4-vector $S^\mu$ is defined by

$$2 S^\mu = \bar{u}(P) \gamma^5 \gamma^\mu u(P)$$

with the properties $S \cdot P = 0$ and $S^2 = -m^2$. The CP-violating $F_3$ form factor is nonzero due to the effective action [2]. Its value at vanishing momentum transfer is related to the EDM of the proton ($p$) and neutron ($n$)

$$\frac{F_3^{p,n}(0)}{2m} = d_{p,n}.$$ (5)

Finding the relation between $w$ and $F_3$ is a complicated nonperturbative problem of QCD. The usual argument, put forward in [7], is that diagrammatically there are three types of contributions as depicted in Fig. 1. The first two diagrams (a,b) are ‘one-nucleon reducible’ and feature the nucleon intermediate state. The last diagram (c) represents the sum of ‘one-nucleon irreducible’ contributions that are sensitive to physics at shorter distances. It has been argued in [7] that the reducible diagrams dominate, or at least provide the correct order of magnitude estimate, see also [8, 9]. The irreducible contribution has been recently calculated in the quark model in [10], and the result is indeed smaller than the contribution from the reducible diagrams calculated in [8, 9].

In the remainder of this section, we shall explicitly evaluate the reducible diagrams and confirm the result [7]

$$d_n \approx \frac{\mu_n}{\bar{u}_N J(0)} \langle N | \chi_{W | N} \rangle,$$ (6)

where $\mu_n$ is the magnetic moment of the neutron. While such an exercise may seem mundane (the derivation was omitted in [7]), it still has the merit of clarifying subtleties that are not always articulated in the literature. In particular, both the numerator and denominator of [6] vanish in the forward limit. This may seem problematic because $O_W$ is inserted at zero momentum transfer $\int d^4x O_W (x)$, so naively only the forward matrix elements are relevant. In [6], this issue is obscured by the notation $\langle N | \chi_{W | N} \rangle$ ($N$ for ‘nucleon’) which does not specify the momentum and spin states. We shall instead use the more transparent notation $\langle P'S' | \chi_{W | P'S'} \rangle$ throughout. We would like to also clarify whether $\mu$ in [6] is the total or anomalous magnetic moment, a question relevant when applying [6] to the proton case.

Since $w$ is small, it is enough to keep only the linear terms in $w$. The CP-violating part of the form factor then reads

$$\langle P'S' | J_{em}^\mu (0) | PS \rangle \sim iw \int d^4x \langle P'S' | T\{ J_{em}^\mu (x) O_W (x) \} | PS \rangle$$

$$= iw \int d^4x \langle P'S' | \theta(x^0) O_W (x) J_{em}^\mu (0) + \theta(-x^0) J_{em}^\mu (0) O_W (x) | PS \rangle.$$ (7)
One can isolate the reducible diagrams by inserting the complete set

\[
1 = \sum_{S''} \int \frac{d^3k}{(2\pi)^32E_k} |kS''⟩⟨kS''| + \cdots
\]

(8)

with \(k^0 = E_k = \sqrt{m^2 + \vec{k}^2}\) and keeping only the single nucleon state. The contributions from resonances \(N^*\) and multi-particle states \(\pi N, \pi\pi N, \cdots\) should be regarded as part of the irreducible diagram Fig.\[1\]c). This gives

\[
\begin{aligned}
& \int d^3x(P'[θ(x^0)O_W(x)P_{em}^{μ}(0)|P) \\
\approx & \sum_{S''} \int_0^∞ dx^0 \int d^3x \frac{d^3k}{(2\pi)^32E_k} e^{i(xP'-P)k} \langle P'|O_W(0)|kS''⟩⟨kS''|J^μ_{em}(0)|P) \\
= & \sum_{S''} \frac{i}{2E_k(E_{P'} - E_k + i\epsilon)} \langle P'|O_W(0)|kS''⟩⟨kS''|J^μ_{em}(0)|PS⟩ \bigg|_{k = \bar{P'}}
\end{aligned}
\]

(9)

The electromagnetic form factor in \([9]\) should that of CP-conserving theory. The matrix element of the Weinberg operator can be parameterized as \([1]\]

\[
\langle P'S'|O_W(0)|kS''⟩ = 4m^3E((P' - k^2))u(P'S')iγ_5u(kS'')
\]

\[
= 2m^2E((P' - k^2))(P' - k)u(P'S')γ^μiγ_5u(kS'')
\]

(10)

In \([9]\), there is a pole at \(E_{P'} = E_k\) and in \([10]\) there is a zero at \(P'^μ = k^μ\). Thus, the limit \(k \rightarrow P'\) has to be taken carefully. Adding the contribution from the second term in \([7]\) and using

\[
\begin{aligned}
\sum_{S''} u(kS'')\bar{u}(kS'') & = \bar{k} + m \\
\bar{u}(P'S')γ_5(\bar{k} + m) & = -(P' - k)\bar{u}(P'S')γ_5γ^μ \\
(\bar{k} + m)γ_5u(PS) & = (k - P')γ^μγ_5u(PS),
\end{aligned}
\]

(11)

one finds the relation

\[
\begin{aligned}
2iwm^3E(0) & \left( \lim_{k \rightarrow P'} \frac{P' - k}{E_{P'}(E_{P'} - E_k)} \bar{u}(P'S')γ_5[γ^μF_1 + \frac{iσ^{μν}\Delta^ν}{2m}F_2] \right) u(PS) \\
+ & \lim_{k \rightarrow P} \frac{E_{E_k} - E_{P'}}{E_{E_k} - E_{P}} \bar{u}(P'S') \left[ γ^μF_1 + \frac{iσ^{μν}\Delta^ν}{2m}F_2 \right] γ^νγ_5u(PS)
\end{aligned}
\]

\[
= \bar{u}(P'S') \frac{σ^{μν}\Delta^ν}{2m}γ_5F_3(\Delta^2)u(PS).
\]

(12)

Since the EDM is proportional to \(F_3\) at \(\Delta = 0\), one can write \(\bar{P} = \frac{P' + P}{2}\) and keep only linear terms in \(\Delta = P' - P\). Further, one can set \(S = S'\), and take the nonrelativistic limit in which \(\bar{P}, \bar{P'}, ̲\Delta\) are all small. The right hand side of \([12]\) then reduces to

\[
\begin{aligned}
\bar{u}(P'S)σ^{μν}γ_5σ_{μν}u(PS) & = \frac{-2i}{m} (P'μS'ν - P'νS'μ)Δ_μ \approx 2iS·̲\Delta g^{μ0}
\end{aligned}
\]

(13)

where we used \(\bar{P}·Δ = 0\) and the fact that \(S^0 = \frac{\bar{S}·P}{E_{P'}}\) is small in the nonrelativistic regime.

To evaluate the left hand side of \([12]\), some kind of regularization is necessary. We temporarilly treat the intermediate state as a ‘resonance’ and modify its mass as \(m \rightarrow m = m + \epsilon\). It then follows that \(\bar{P'} = \bar{k}\) but \(E_{P'} \neq \bar{E}_k\), so that only the \(ν = 0\) component remains. After cancelling \(E_{P'} - \bar{E}_k\) in the numerator and denominator, we take the limit \(\epsilon \rightarrow 0\). In this way, the brackets in \([12]\) reduces to

\[
\begin{aligned}
\bar{u}(P'S) & \left( \frac{1}{E_{P'}} γ_5γ^0 \left[ γ^μF_1 + \frac{iσ^{νμ}Δ_ν}{2m}F_2 \right] + \frac{1}{E_{P}} \left[ γ^μF_1 + \frac{iσ^{νμ}Δ_ν}{2m}F_2 \right] γ^0γ_5 \right) u(PS) \\
& \approx \frac{1}{m} \bar{u}(P'S') \left( 2g^{μ0}F_1γ_5 + \frac{iF_2}{2m}γ_52i(g^{0μ}γ^ν - g^{0ν}γ^μ)Δ_ν \right) u(PS)
\end{aligned}
\]

\[
\approx \frac{2}{m^2}(F_1 + F_2)S·̲\Delta g^{μ0}
\]

(14)
We thus arrive at
\[ d = \frac{F_3(0)}{2m} = 4wE(0)m^2 \frac{F_1(0) + F_2(0)}{2m} = 4wE(0)m^2 \mu \]  
(15)

In the present normalization, \( F_1(0) = |e| \) for proton and \( F_1(0) = 0 \) for neutron. Eq. (15) essentially agrees with [6], but now all the subtleties have been exposed. The present derivation also makes it clear that in the proton case the EDM is proportional to the total magnetic moment \( F_1 + F_2 \), not the anomalous magnetic moment \( F_2 \).

### III. OPERATOR MIXING

Now that we have the relation (15), the remaining task is to estimate the QCD matrix element \( E(\Delta^2 = 0) \). The central observation of [1] is that \( E \) is related to the matrix element of the following twist-four operator
\[
\langle PS|\bar{\psi}gF^{\mu\nu}\gamma_\mu\psi|PS \rangle = -2f_0m^2S^\mu
\]
(16)

(We have changed the normalization of \( f_0 \) by a factor of 2 with respect to [1].) More precisely, \( E \) and \( f_0 \) mix under renormalization group (RG) as follows\(^1\)
\[
E(\mu_0) - \frac{9N_c^2}{2(3N_c^2 + 4)}f_0(\mu_0) = \left( \frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \right)^\frac{7w}{30} \left( E(\mu) - \frac{9N_c^2}{2(3N_c^2 + 4)}f_0(\mu) \right)
\]
(17)

\[
f_0(\mu_0) = \left( \frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \right)^\frac{7w}{30} f_0(\mu), \quad \gamma_4 = \frac{8C_F}{3} + \frac{2n_f}{3}
\]
(18)

\[
w(\mu_0) = \left( \frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \right)^{-\frac{7w}{30}} w(\mu)
\]
(19)

where \( \beta_0 = \frac{11N_c}{3} - \frac{2n_f}{3} \) and
\[
\gamma_W = \frac{N_c}{2} + n_f + \frac{\beta_0}{2} = \frac{7N_c}{3} + \frac{2n_f}{3}
\]
(20)

is the anomalous dimension of the Weinberg operator [12]. We immediately see that the product \( wE \) that appears in [15] is not RG-invariant, contrary to what one naively expects from the fact that the effective action \( w \int d^4xO_W(x) \) is RG-invariant. This is because we are dealing with nonforward matrix elements. We take \( \mu \) to be the hadronic scale \( \sim 1 \) GeV, and \( \mu_0 \gtrsim 100 \) GeV is the high energy scale where the Weinberg operator is induced. Numerically, for \( n_f = 5 \),
\[
\gamma_W \approx 10.3, \quad \gamma_4 \approx 6.89, \quad \frac{9N_c^2}{2(3N_c^2 + 4)} \approx 1.31.
\]
(21)

Since \( \gamma_W > \gamma_4 \), asymptotically \( \mu_0 \to \infty \) one has the relation
\[
E(\mu_0) \approx 1.31f_0(\mu_0),
\]
(22)

in the sense that the difference \( |E(\mu_0) - 1.31f_0(\mu_0)| \) is much smaller than \( |f_0(\mu_0)| \) and \( |E(\mu_0)| \). Eq. (22) does not say anything definite about the relation between \( E(\mu) \) and \( f_0(\mu) \) at low energy, but it does suggest that \( E(\mu) \) is in the ballpark of \( 1.3f_0(\mu) \) up to corrections of order unity. Another insight into the relative coefficient may come from the exact operator identity [1]
\[
O_W = \partial_\mu(\bar{\psi}gF^{\mu\nu}\gamma_\nu\psi) - \frac{1}{2} \tilde{F}_{\mu\nu}D^2F^{\mu\nu} \equiv O_4 + O_D,
\]
(23)

\(^1\) From now on, the argument of \( E \) is the RG scale, not the momentum transfer \( \Delta^2 \). The latter has been set equal to zero.
at high energy. We thus assume that

\[ E \approx \frac{f_0}{2} + \frac{1}{8\ln^2 \mu} \lim_{\Delta \to 0} \frac{\langle P'|F_{\mu\nu}D^2F^{\mu\nu}|P \rangle}{\Delta \cdot S} \]  

valid at any scale \( \mu \). While we do not know the value of the matrix element \( \langle F|D^2F \rangle \), its scale dependence can be inferred from the evolution of \( \mathcal{O}_D \) \([1]\):

\[ \frac{\partial \mathcal{O}_D}{\partial \ln \mu^2} = -\frac{\alpha_s}{4\pi} \left[ \gamma_W \mathcal{O}_D - \left( \frac{2N_c}{3} + \frac{8C_F}{3} \right) \mathcal{O}_4 \right] \]  

We see that, in addition to the strong suppression \( \sim \gamma_W \) with increasing \( \mu \), there is a compensating contribution proportional to \( \mathcal{O}_4 \sim f_0 \). This suggests the following scenario. At low energy, \( E(\mu) \sim 0.5f_0(\mu) \) simply from the first term in (24). As \( \mu \) increases, the \( \mathcal{O}_4 \) component of \( \mathcal{O}_D \) grows due to mixing, and this eventually leads to \( E \sim 1.3f_0 \) at high energy. We thus assume that \( E(\mu) \) sits somewhere between the two reference values

\[ 0.5f_0(\mu) < E(\mu) < 1.3f_0(\mu). \]  

with a slight preference towards the lower bound \( E \sim 0.5f_0 \). We shall use this estimate to calculate nucleon EDMs in the next section.

IV. POLARIZED DIS AT TWIST-FOUR AND NUCLEON EDM

Remarkably, the parameter \( f_0 \) is observable in experiments, as a higher twist correction in polarized deep inelastic scattering (DIS). Up to twist-four accuracy, the first moment of the \( g_1 \) structure function for proton \((p)\) and neutron \((n)\) measurable in polarized DIS can be written as \([15,16]\)

\[ \int_0^1 dx g_{1p,n}^p(x) = \left( \pm \frac{g_A}{12} + \frac{a_8}{36} \right) \left( 1 - \frac{\alpha_s}{\pi} + O(\alpha_s^2) \right) + \frac{\Delta \Sigma}{9} \left( 1 - \frac{33 - 8n_f \alpha_s}{33 - 2n_f \alpha_s} + O(\alpha_s^2) \right) \]

\[ + \frac{m^2}{9Q^2} (a_{2p}^p + 4d_{2p}^p + 4f_2^p), \]  

where we suppress the scale \( (Q^2) \) dependence of the low-energy constants for simplicity. \( a_8^p = a_8^n = g_A \) is the isovector axial charge and we wrote \( a_8^p = a_8^n \equiv a_8 \). \( \Delta \Sigma \) is the quarks’ helicity contribution to the nucleon spin. Usually, \( \Delta \Sigma \) is the main object of interest as it is the crucial building block of the nucleon spin sum rule. Here, however, we focus on the higher twist corrections in the second line of (27) which are suppressed by the characteristic factor \( m^2/Q^2 \). The terms proportional to \( a_2 \) and \( d_2 \) are the so-called target mass corrections (TMCs) important in the low-\( Q^2 \) region. They originate from the finiteness of the nucleon mass and can be expressed by the third moments of the \( g_1, g_2 \) structure functions

\[ a_{2p}^p = 2 \int_0^1 dx x^2 g_{1p}^p(x), \quad d_{2p}^p = \int_0^1 dx x^2 (2g_{1p}^p(x) + 3g_{2p}^p(x)). \]  

Eq. (28) shows that the TMCs are related to twist-two and twist-three matrix elements, although in practice they appear as twist-four effects \( \sim 1/Q^2 \) in the cross section. The ‘genuine’ twist-four contribution can be written as

\[ f_{2,3,8}^p = 2 \left( \pm \frac{f_3}{12} + \frac{f_8}{36} + \frac{f_0}{9} \right) \]  

where \( f_{0,3,8} \) are defined through the following matrix elements

\[ \langle PS|\bar{\psi}gF_{\mu\nu}\gamma_\nu T^a\psi|PS \rangle = -2f_{2,3,8}^p m^2 S^a \]  

with \( T^{a=0,3,8} \) being \( 3 \times 3 \) flavor matrices

\[ T^0 = 1, \quad T^3 = \text{diag}(1, -1, 0), \quad T^8 = \text{diag}(1, 1, -2). \]  

\(^2\) In DIS, it is common to choose \( \mu^2 = Q^2 \), the virtuality of the photon emitted from the incoming lepton.
The minus sign in \([30]\), which is absent in most literature, is due to our sign convention of the coupling \(g\), namely \(D^\mu = \partial^\mu + igA^\mu\). In \([29]\) we wrote \(f_{0,8}^p = f_{0,8}^n = f_0\) and \(f_3^p = -f_3^n = f_3\). Note that \(f_0\) is the same as in the previous definition \([16]\).

There have already been several attempts to extract the twist-four contributions from the world data of polarized DIS \([17,20]\). The original result in \([17]\) is

\[
f_2^p = 0.10 \pm 0.05, \quad f_2^n = 0.07 \pm 0.08 \quad (Q^2 = 1 \text{ GeV}^2)
\]

(The sign convention of \(f_2\) in \([17]\) is opposite to ours, so we flipped the sign.) However, the precision of the early data used in this reference was not very good. The more recent results for the neutron based on experiments at the Jefferson Lab are\(^3\)

\[
\begin{align*}
\text{Ref.}[19] & \quad f_2^n = 0.034 \pm 0.043 \quad (Q^2 = 1 \text{ GeV}^2) \\
\text{Ref.}[21] & \quad f_2^p = 0.053 \pm 0.026 \quad (Q^2 = 3.21 \text{ GeV}^2)
\end{align*}
\]

Once \(f_2^n\) are known, one can extract the linear combination

\[
f_3(Q^2) = \frac{f_0(Q^2)}{9} + \frac{f_0(Q^2)}{36} \frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)} \gamma_4^{N_S/\beta_0} + \frac{f_0(Q^2)}{9} \frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)} \gamma_4^{N_S/\beta_0}
\]

where \(\gamma_4^{N_S} = \frac{2g^2}{3}\) and \(\gamma_4\) is the same as in \([18]\). This in principle allows one to separately determine \(f_0\) and \(f_3\).

However, in order to do so, very accurate data over a very wide range in \(Q^2\) are needed. Since such data are not available at the moment, we resort to model calculations of \(f_3\) from QCD sum rules \([14,22]\) and instantons \([23,24]\).

Here we quote the result of \([24]\) since the predicted value \(f_2^p\) agrees well with \([33]\):

\[
\begin{align*}
f_0 &= 0.01, \quad f_3 = -0.25, \quad f_8 = -0.11 \\
f_2^n &= -0.046, \quad f_2^p = 0.038
\end{align*}
\]

at the defining scale of the model \(Q^2 = 0.4 \text{ GeV}^2\). The isoscalar form factor \(f_0\) is small partly because it is subleading in the large-\(N_c\) counting \(f_0 \sim 1/N_c\).

We are now ready to present numerical results. Varying \(E\) in the range \([20]\) and using \(f_0 = 0.01\) and the known constants\(^4\)

\[
\mu_p = 2.79\mu_N, \quad \mu_n = -1.91\mu_N, \quad \mu_N = 0.105\text{ e fm}, \quad m^2\mu_N \approx 470\text{ e MeV},
\]

we find\(^5\)

\[
26w\text{ e MeV} < d_p < 69w\text{ e MeV}, \quad -47w\text{ e MeV} < d_n < -18w\text{ e MeV},
\]

Let us compare \([37]\) with the previous results based on QCD sum rules \([3,9]\). The results in these two references are consistent with each other, so we only quote the result of \([9]\). Our normalization of \(O_W\) differs from \([9]\) by a factor of \(g\), and our sign convention of \(g\) is opposite. Since Ref. \([9]\) assumed \(g[9] = 2.13\) is positive, we have to divide our result by \(g = -g[9] = 2.13\) to get \((w' \equiv gw)\)

\[
-32w'\text{ e MeV} < d_p < -12w'\text{ e MeV} \quad 8.4w'\text{ e MeV} < d_n < 22w'\text{ e MeV}.
\]

This should be compared to \([9]\)

\[
d_p = -109\text{ e MeV}, \quad d_n = 74\text{ e MeV}.
\]

\((w'\) has been set to unity in \([9]\).) The signs agree, but our results \([38]\) are smaller by a factor of about 3 even in the maximal case \(E = 1.3f_0\) and this factor becomes \(8 \sim 9\) in the minimal case \(E = 0.5f_0\). The suppression is mainly attributed to the small twist-four matrix element \(f_0 \sim 0.01 \,[24]\), but we have not included systematic uncertainties in it.

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\(^3\) Ref. \([20]\) also studied the proton case, but the authors only extracted the sum \(d_3^p + f_2^p\).

\(^4\) Note that we use the total magnetic moment of the proton as explained earlier.

\(^5\) We do not know the sign of \(w\). Here we assume it is positive, but if it turns out to be negative, the inequality symbols should be modified accordingly.
V. CONCLUSIONS

In this paper, we have presented a novel prediction for the nucleon EDMs originating from the Weinberg operator. This has been made possible owing to the recent observation \[1\] that the matrix element of the Weinberg operator is related to the parameter \(f_0\) that can be extracted from polarized DIS experiments. The main result is shown in Eq. (37). Admittedly, our estimate is rather crude and ignores unknown systematic errors in the particular model prediction for \(f_0\) \[35\], not to mention uncertainties associated with (26). However, at the moment we do not expect that one can significantly improve on this point. Ideally, \(f_0\) should be determined from polarized DIS experiments, and after all, this is our main message. In order to reliably extract \(f_0\), we need very precise data over a wide range in \(Q^2\) (see (34)), while the existing data are rather limited in \(Q^2\). The future polarized DIS experiments at the EIC in the US. and the EICc (EIC in China) may help in this regard. Hopefully our work triggers discussions on the feasibility of such studies in these experiments.

Concerning the result, the obtained values \(d_{p,n}\) are much smaller (by a factor of 3 \(\sim\) 9) than the previous results based on QCD sum rules \[8, 9\]. It is very difficult to identify the origin of the discrepancy, since the method used in \[8, 9\] is totally different from ours. To better understand this point, it may be helpful to revisit the sum rule calculations of \(f_0\) \[14, 22\].

As a matter of fact, numerically our result turns out to be comparable to the contribution from the one-nucleon irreducible diagram Fig. 1(c) recently calculated in \[10\].\footnote{To compare with the result of \[10\], the numbers in \[38\] have to be divided by 3.} This calls in question the original argument that the reducible diagrams dominate over the irreducible ones. However, as we already mentioned, our result contains unknown, possibly large systematic errors. Further efforts in both theory and experiment are certainly needed to draw definitive conclusions.

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[1] Y. Hatta, Phys. Rev. D 102, 094004 (2020), 2009.03657.
[2] T. Chupp, P. Fierlinger, M. Ramsey-Musolf, and J. Singh, Rev. Mod. Phys. 91, 015001 (2019), 1710.02504.
[3] N. Yamanaka, B. Sahoo, N. Yoshinaga, T. Sato, K. Asahi, and B. Das, Eur. Phys. J. A 53, 54 (2017), 1703.01570.
[4] S. Weinberg, Phys. Rev. Lett. 63, 2333 (1989).
[5] V. Cirigliano, E. Mereghetti, and P. Stoffer, JHEP 09, 094 (2020), 2004.03576.
[6] M. D. Rizik, C. J. Monahan, and A. Shindler (SymLat), Phys. Rev. D 102, 034509 (2020), 2005.04199.
[7] I. I. Bigi and N. Uraltsev, Nucl. Phys. B 353, 321 (1991).
[8] D. A. Demir, M. Pospelov, and A. Ritz, Phys. Rev. D 67, 015007 (2003), hep-ph/0208257.
[9] U. Haisch and A. Hala, JHEP 11, 154 (2019), 1909.08955.
[10] N. Yamanaka and E. Hiyama (2020), 2011.02531.
[11] A. Prokudin, Y. Hatta, Y. Kovchegov, and C. Marquet, eds., Proceedings, Probing Nucleons and Nuclei in High Energy Collisions: Dedicated to the Physics of the Electron Ion Collider: Seattle (WA), United States, October 1 - November 16, 2018 (WSP, 2020), 2002.12333.
[12] A. Morozov, Sov. J. Nucl. Phys. 40, 505 (1984).
[13] E. V. Shuryak and A. Vainshtein, Nucl. Phys. B 201, 141 (1982).
[14] I. Balitsky, V. M. Braun, and A. Kolesnichenko, Phys. Lett. B 242, 245 (1990), [Erratum: Phys.Lett.B 318, 648 (1993)], hep-ph/9310316.
[15] X.-D. Ji and P. Urnau, Phys. Lett. B 333, 228 (1994), hep-ph/9308263.
[16] H. Kawamura, T. Uematsu, J. Kodaira, and Y. Yasui, Mod. Phys. Lett. A 12, 135 (1997), hep-ph/9603338.
[17] X.-D. Ji and W. Melnitchouk, Phys. Rev. D 56, 1 (1997), hep-ph/9703363.
[18] E. Leader, A. V. Sidorov, and D. B. Stamenov, Phys. Rev. D 67, 074017 (2003), hep-ph/0212085.
[19] Z. Meziani et al., Phys. Lett. B 613, 148 (2005), hep-ph/0404066.
[20] E. Leader, A. V. Sidorov, and D. B. Stamenov, Phys. Rev. D 75, 074027 (2007), hep-ph/0612360.
[21] D. Flay et al. (Jefferson Lab Hall A), Phys. Rev. D 94, 052003 (2016), 1603.03612.
[22] E. Stein, P. Gornicki, L. Mankiewicz, and A. Schafer, Phys. Lett. B 353, 107 (1995), hep-ph/9502323.
[23] J. Balla, M. V. Polyakov, and C. Weiss, Nucl. Phys. B 510, 327 (1998), hep-ph/9707515.
[24] N.-Y. Lee, K. Goeke, and C. Weiss, Phys. Rev. D 65, 054008 (2002), hep-ph/0105173.