GLUON SATURATION AT SMALL X

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At very high energies, the relevant component of the hadron wavefunction can be described as a Color Glass Condensate, i.e., a state of high density gluonic matter whose distribution is random, but frozen over the relevant time scales. The weight function for this distribution obeys a renormalization group equation in the form of a functional Fokker-Planck equation. Its solution leads to an effective theory which predicts gluon saturation at sufficiently high energy, or small Bjorken’s $x$.

1 Introduction

Hadronic scattering at high energy, or small Bjorken’s $x$, uncovers a novel regime of QCD where the coupling is small ($\alpha_s \ll 1$) but the parton densities are so large that conventional perturbation theory breaks down, via strong non-linear effects (see [4, 5] for recent reviews and more references). Remarkable progress in recent years has resulted from the observation that this high density regime can be studied via semiclassical methods [3]. In what follows, I shall describe a classical effective theory [3, 6, 7, 8] which is well suited to study the non-linear phenomena at small $x$ and has a transparent physical interpretation: It portrays the gluon component of the hadron wavefunction (the relevant component at small $x$) as a Color Glass Condensate (CGC). This is a multiparticle quantum state with high occupation numbers, but to the accuracy of interest it can be represented as a stochastic color field with a probability law determined by a functional Fokker-Planck equation.

The latter is a renormalization group equation (RGE) which shows how to construct the effective theory by integrating out quantum fluctuations in the background of a strong color field (the CGC). The non-linear effects included in the RGE via this background field describe interactions among the gluons produced in the quantum evolution towards small $x$. This leads to a non-linear generalization of the BFKL equation which, remarkably, predicts the saturation of the gluon distribution at small $x$: Unlike the BFKL result for the gluon distribution, which increases exponentially with the rapidity $\tau \equiv \ln(1/x) \propto \ln s$ and thus violates the Froissart unitarity bound $\sigma \leq \ln^2 s$, our corresponding prediction grows only linearly with $\tau$, thus being consistent with the unitarity bounds.
2 The effective theory for the CGC

The effective theory applies to gluon correlations in the hadron wavefunction as measured in deep inelastic scattering at small Bjorken’s $x$. It is formulated in the hadron infinite momentum frame, where small $x$ corresponds to soft longitudinal momenta $k^+ = xP^+$, with $P^+$ the hadron momentum, and $x \ll 1$. The main observation is that the “fast” partons (i.e., the excitations with $p^+ \gg k^+$) can be replaced, as far as their effects on the soft correlation functions are concerned, by a classical random color source $\rho^a(x)$, whose gross properties are determined by the soft–fast separation of scales:

The fast partons appear to the soft gluons as a color charge distribution $\rho^a(x)$ which is static (i.e., independent of $x^+$), localized near the light-cone (within a small distance $\Delta x^- \sim 1/p^+ \ll 1/k^+$), and random (since this is the instantaneous color charge in the hadron “seen” by the soft gluons at the arbitrary time of their emission), with gauge-invariant probability density $W_\tau[\rho]$. (We use the rapidity $\tau \equiv \ln(P^+/k^+) = \ln(1/x)$ to indicate the dependence of the weight function upon the soft scale $k^+$.)

Gluon correlations at the scale $k^+ = xP^+$ are obtained as $(\vec{x} \equiv (x^-, x_\perp))$:

$$\langle A^{i^a}(x^+, \vec{x})A^{j^b}(x^+, \vec{y})\cdots \rangle_\tau = \int D\rho \ W_\tau[\rho] A^{i^a}(\vec{x})A^{j^b}(\vec{y})\cdots, \quad (1)$$

where $A^{i^a} = A^{i^a}[\rho]$ is the solution to the classical Yang-Mills equations

$$\left(D_\mu F^{\mu\nu}\right)_{a}(x) = \delta^{\nu+} \rho_a(x) \quad (2)$$

in the light-cone gauge $A^+ = 0$, which is the gauge which allows for the most direct contact with the gauge-invariant physical quantities. For instance, the gluon distribution function is obtained as

$$xG(x, Q^2) = \frac{1}{\pi} \int \frac{d^2k_\perp}{(2\pi)^2} \Theta(Q^2 - k^2) \left\langle |F^{+i^a}(\vec{k})|^2\right\rangle_\tau. \quad (3)$$

where $F^{+i^a} = \partial^+ A^{i^a}$ is the electric field associated to the classical solution $A^{i^a}[\rho]$, and $\vec{k} \equiv (k^+, k_\perp)$ with $k^+ = xP^+ = P^+ e^{-\tau}$.

Eqs. $\dagger$ are those for a glass (here, a color glass): There is a stochastic color charge, that is averaged over. This is frozen over short time scales, of the order of the lifetime $\Delta x^+ \sim 1/k^- \propto k^+$ of the soft gluons, but it changes randomly over the larger time scale $1/p^- \gg 1/k^-$, which is the natural time scale for the dynamics of the fast partons. This is like a glass which is disordered and is a liquid on long time scales but seems to be a solid

\[a\] I use light-cone vector notations, e.g., $k^\mu = (k^+, k^-, k_\perp)$, with $k^+ \equiv (k^0 + k^3)/\sqrt{2}$, $k^- \equiv (k^0 - k^3)/\sqrt{2}$, $k_\perp \equiv (k^1, k^2)$, and $k \cdot x = k^- x^+ + k^+ x^- - k_\perp \cdot x_\perp$. 

on short time scales. We shall see later that, at saturation, the gluons are very densely packed, with a phase-space density $\sim 1/\alpha_s$, which is the maximum density allowed by their mutual interactions. They form a Bose condensate.

In the effective theory, this Color Glass Condensate is described by strong classical color fields $A_i^a \sim 1/g$. The classical dynamics is then fully non-linear, so one needs the exact solution to eq. (2), which reads

$$A_i^a(x) = \frac{i}{g} U(x) \partial^i U^\dagger(x),$$

$$U^\dagger(x^-, x_{\perp}) = P \exp \left\{ ig \int_{-\infty}^{x^-} dz^- \alpha(z^-, x_{\perp}) \right\},$$

$$-\nabla_{\perp}^2 \alpha(x) = \rho(x),$$

Since $\rho(x)$, and therefore $\alpha(x)$, are localized near $x^- = 0$, so is the electric field, which appears effectively as a $\delta$-function to any probe with low longitudinal resolution (i.e., with momenta $q^+ < k^+$):

$$F^+(x) \approx \delta(x^-) \frac{i}{g} V(\partial^i V^\dagger), \quad V^\dagger(x_{\perp}) \equiv P \exp \left\{ ig \int dx^- \alpha^a(x^-, x_{\perp}) T^a \right\}. \quad (7)$$

3 The Renormalization Group Equation

The weight function $W_\tau[\rho]$ for the effective theory at the scale $k^+ = x P^+$ is obtained by integrating out the quantum fluctuations with $p^+ > k^+$ in layers of $p^+$, to “leading logarithmic accuracy” — i.e., by retaining only the terms enhanced by the large logarithm $\ln(1/x)$ —, but to all orders in the background fields $A_i^a$ generated at the previous steps $6, 7$. The background fields are an essential ingredient: their interactions with the quantum gluons correspond, in the more conventional picture of the “parton cascades”, to rescatterings among the soft gluons radiated in the quantum evolution towards small $x$ (e.g., recombinations of gluons from different cascades).

The quantum evolution can be formulated as a renormalization group equation (RGE) for the flow of $W_\tau[\rho]$ with $\tau = \ln(1/x)$. To motivate the structure of this equation, I succinctly describe one step in this RG procedure. Assume that we know the effective theory at some initial scale $\Lambda^+$ — as specified by $W_\Lambda[\rho] \equiv W_\tau[\rho]$, with $\tau = \ln(P^+/\Lambda^+)$ —, and we are interested in correlations at the softer scale $k^+ \sim b \Lambda^+$ with $b \ll 1$ and $\alpha_s \ln(1/b) < 1$. Our purpose is to construct the new weight function $W_{b\Lambda}[\rho] \equiv W_{\tau+\Delta\tau}[\rho]$ ($\Delta\tau \equiv \ln(1/b)$), which would determine the gluon correlations at this softer scale. As compared to $W_\Lambda[\rho]$, the new weight function must include also the
quantum effects induced by the “semi-fast” gluons with \( p^+ \) in the strip

\[ b\Lambda^+ < |p^+| < \Lambda^+. \]  

(8)

To characterize these effects, consider the change in the 2-point function \( \langle A^i_a(x)A^j_b(y) \rangle_A \) with decreasing \( \Lambda^+ \). At the original scale \( \Lambda^+ \), we can use the effective theory to write \( \langle A^i(x)A^i(y) \rangle_A = \langle A^i(x)A^i(y) \rangle_{W_\Lambda} \), with \( A^i[\rho] \) the classical solution in eq. (\ref{eq:1}), and the average over \( \rho \) computed as in eq. (\ref{eq:2}). At the new scale \( b\Lambda^+ \), \( A^i_a = A^i_a + \delta A^i_a \), with \( \delta A^i_a(x) \) representing the quantum fluctuations with momenta \( p^+ < \Lambda^+ \). Thus,

\[
\langle A^i(x)A^j(y) \rangle_{b\Lambda} = \left\langle \left\langle (A^i + \delta A^i)(x)(A^j + \delta A^j)(y) \right\rangle_{\rho} \right\rangle_{W_\Lambda} = \langle A^i(x)A^j(y) \rangle_{W_{\Lambda}A},
\]

(9)

where the internal brackets \( \langle \cdots \rangle_{\rho} \) in the first line stand for the quantum average over semi-fast fluctuations (cf. eq. (\ref{eq:3})) at fixed \( \rho \), while the external brackets \( \langle \cdots \rangle_{W_\Lambda} \) denote the classical average over \( \rho \) with weight function \( W_\Lambda[\rho] \). By definition, the result of this double averaging (i.e., of the classical plus quantum calculation) in the original effective theory at the scale \( \Lambda^+ \) must be the same as the result of a purely classical calculation in the new effective theory at the scale \( b\Lambda^+ \). This is the content of the second line in eq. (\ref{eq:2}).

The quantum corrections in the first line of eq. (\ref{eq:2}) must be computed to lowest order in \( \alpha_s \ln(1/b) \), but to all orders in the background fields \( A^i \). This is essentially an one-loop calculation, but with the exact background field propagator of the semi-fast gluons. By matching its result with the classical calculation in the second line in eq. (\ref{eq:2}), one can deduce the functional change \( \Delta W = W_{\tau + \Delta \tau} - W_\tau \) necessary to absorb these new correlations. Since \( \Delta W \propto \Delta \tau \), the evolution is formulated as a (functional) RGE, which is most conveniently written for \( W_\tau[\alpha] \equiv W_\tau[\rho = -\nabla^2 \alpha] \), cf. eq. (\ref{eq:5}). It reads:

\[
\frac{\partial W_\tau[\alpha]}{\partial \tau} = \int_{x_\perp, y_\perp} \frac{1}{2} \delta \eta_{ab}(x_\perp, y_\perp) \frac{\delta}{\delta \alpha_b^a(x_\perp)} W_\tau[\alpha] = -HW_\tau[\alpha],
\]

(10)

where \( \eta_{ab}(x_\perp, y_\perp) \) it itself a non-linear functional of the color field \( \alpha_a(x) \), via the Wilson lines \( V \) and \( V^\dagger \) defined in eq. (\ref{eq:6}):

\[
\eta_{ab}(x_\perp, y_\perp) = \frac{1}{\pi} \int \frac{d^2 z_\perp}{2\pi} \frac{2(x_\perp^2 - z_\perp^2)(y_\perp^2 - z_\perp^2)}{2(y_\perp^2 - z_\perp^2)^2} \left\{ 1 + V_x^a V_y^b - V_x^b V_y^a - V_z^a V_y^b \right\}.
\]

(11)

\[b\] In Ref. \cite{1}, Weigert has shown that Balitsky’s equations \cite{2} can be summarized into a functional equation equivalent to (\ref{eq:2}). More recently, a simplified derivation of eq. (\ref{eq:1}) has been given by Mueller, from the dipole point of view.\cite{4}
The functional derivatives in eq. (10) are taken with respect to $\alpha^a(x_\perp) \equiv \alpha^a(x^- = x^-_\perp, x_\perp)$, where $x^-_\perp \equiv 1/\Lambda^+ \sim e^\tau$. This is so since the quantum corrections are located in the strip $1/\Lambda^+ \lesssim x^- \lesssim 1/b\Lambda^+$, i.e., on top of the original field which has support at $0 \leq x^- \lesssim 1/\Lambda^+$. Thus, by integrating out the quantum modes in layers of $p^+$, one constructs the classical field (or source) in layers of $x^-$, with a one-to-one correspondence between $p^+$ and $x^-$ which reflects the uncertainty principle $\Delta x^- \Delta p^+ \sim 1$.

4 General properties and consequences

Eq. (10) has the structure of a diffusion equation: It is a second-order (functional) differential equation whose r.h.s. is a total derivative, as necessary to conserve the total probability. It describes quantum evolution as the diffusion (with “time” $\tau$) of the probability density $W_\tau[\alpha]$ in the functional space spanned by $\alpha_a(x^-, x_\perp)$.

For comparison, consider the usual Fokker-Planck equation describing Brownian motion in flat space ($P(x, t) \equiv$ the probability density to find the particle at point $x$ at time $t$ knowing that it was at $x = 0$ at time 0):

$$\frac{\partial P(x, t)}{\partial t} = D \frac{\partial^2}{\partial x^i \partial x^i} P(x, t) - \frac{\partial}{\partial x^i} \left( F^i(x) P(x, t) \right),$$

where $D$ is the diffusion constant and $F^i = -\partial V / \partial x^i$ is an external force. If $F^i = 0$, the corresponding solution:

$$P(x, t) = \frac{1}{(4\pi D t)^{3/2}} \exp \left\{ -\frac{x^2}{4Dt} \right\}$$

(13)

goes smoothly to zero at any $x$ when $t \to \infty$ (runaway solution). But for $V \neq 0$, there exists a non-trivial stationary solution $P_0(x) \sim \exp[-V(x)/D]$, i.e., an equilibrium distribution that is asymptotically reached by the system.

There is manifestly no force term in the RGE (10). This property relies on subtle compensations between real and virtual corrections and entails a purely diffusive behaviour, as in eq. (13). Thus, there is no fixed point, or stationary distribution, for the flow described by eq. (10).

From the functional RGE (10), one can derive ordinary evolution equations for all the observables which can be computed as an average over $\alpha$. Particularly interesting quantities are the gauge-invariant products of Wilson lines $\langle \text{tr}(V^i_1 V^j_2) \text{tr}(V^i_3 V^j_4) \cdots \rangle_\tau$, for which eq. (10) predicts the same evolution equations as obtained by Balitsky via operator product expansion. In par-
ticular, the 2-point function $\langle \text{tr}(V_x^\dagger V_y) \rangle_\tau$ satisfies

$$\frac{\partial}{\partial \tau} \langle \text{tr}(V_x^\dagger V_y) \rangle_\tau = -\frac{\alpha_s}{2\pi^2} \int d^2z_\perp \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2(y_\perp - z_\perp)^2} \times \langle N_c \text{tr}(V_x^\dagger V_y) - \text{tr}(V_x^\dagger V_z)\text{tr}(V_z^\dagger V_y) \rangle_\tau$$

which in the large $N_c$ limit becomes a closed equation, due to Kovchegov, that can be recognized as a non-linear generalization of BFKL. Its solution determines the cross section $\sigma_{\text{dipole}}(\tau, r_\perp)$ for the scattering of a “color dipole” of size $r_\perp = x_\perp - y_\perp$ off the hadron. Recent progress with this equation exhibits color transparency ($\sigma_{\text{dipole}} \propto r_\perp^2 Q_s^2(\tau)$) at small distances $r_\perp \ll 1/Q_s(\tau)$, and saturation ($\sigma_{\text{dipole}} \approx \text{const}$) at large distances $r_\perp \gg 1/Q_s(\tau)$, in qualitative agreement with a phenomenological model by Golec-Biernat and Wüsthoff which successfully describes the HERA data. In the above estimates, $Q_s(\tau)$ is the saturation scale to be discussed in the next section.

5 Gluon Saturation and Unitarity

Except in the large $N_c$ limit, Balitsky’s equations do not close individually, but rather form an infinite hierarchy of coupled equations. It is then more convenient to study directly the functional RGE (10). As shown in Ref. approximate solutions to this equation can be obtained in limiting kinematical regimes, by combining mean field approximations (MFA) and kinematical simplifications. Moreover, an exact solution in the form of a path-integral in 2+1 dimensions, can be also written down, which may be used in lattice calculations. (See also the second Ref. for a different path-integral formulation.) Here, I shall briefly describe the analytic solutions, with emphasis on the phenomenon of gluon saturation.

The MFA consists in replacing $\eta_{ab}(x_\perp, y_\perp)$ in the kernel of the RGE by its expectation value $\langle \eta_{ab}(x_\perp, y_\perp) \rangle_\tau = \delta^{ab} \gamma_\tau(x_\perp, y_\perp)$. Then, the equation can be solved exactly, and the solution $W_\tau$ — a Gaussian in $\alpha$ (or $\rho$) — can then be used to self-consistently determine the approximate kernel $\gamma_\tau(x_\perp, y_\perp)$.

The kinematical approximations rely on the fact that there is an intrinsic transverse scale in the problem, the saturation scale $Q_s(\tau)$, which is the inverse correlation length for Wilson lines:

$$\langle V_x^\dagger V_y \rangle_\tau \approx \begin{cases} 1, & \text{for } |x_\perp - y_\perp| \ll 1/Q_s(\tau) \\ 0, & \text{for } |x_\perp - y_\perp| \gg 1/Q_s(\tau) \end{cases}$$

To get an estimate for $Q_s(\tau)$, one starts the analysis at sufficiently short distances $|x_\perp - y_\perp| \ll 1/Q_s(\tau)$, where the fields are weak and their dynamics
is linear \(V_x^1 \approx 1 + ig\alpha(x_\perp)\), and study the onset of non-linearities with increasing \(|x_\perp - y_\perp|\). In this regime, one obtains (cf. eq. (8)):

\[
\langle V_x^1 V_{y} \rangle_{\tau} \approx \exp\left\{ -\frac{\bar{\alpha}_s \tau^2}{2 R^2} xG(x, 1/r_\perp^2) \right\},
\]

(\(\bar{\alpha}_s = \alpha_s N/\pi\), \(r_\perp = x_\perp - y_\perp\), and \(R\) the hadron radius) with \(xG(x, Q^2)\) satisfying the standard evolution equation in the double log approximation:

\[
\frac{\partial^2}{\partial \tau \partial \log Q^2} xG(x, Q^2) = \bar{\alpha}_s xG(x, Q^2).
\]

(Here, this equation arises via the self-consistency requirement.) Non-linear effects, which are negligible at sufficiently small \(r_\perp\), become important for \(r_\perp \sim 1/Q_s(\tau)\) where the exponent in eq. (16) becomes of order one. Thus,

\[
R^2 Q_s^2(\tau) \sim \bar{\alpha}_s xG(x, Q_s^2(\tau)),
\]

which together with eq. (17) implies that \(Q_s(\tau)\) increases exponentially:

\[
Q_s^2(\tau) = Q_s^2(\tau_0) e^{c\bar{\alpha}_s (\tau - \tau_0)} \quad (c = 4 \text{ in DLA}).
\]

The weight function at high-momenta \(k_\perp \gg Q_s(\tau)\) is the Gaussian:

\[
W_\tau^{\text{high}-k_\perp}[\rho] \approx \exp\left\{ -\frac{1}{2} \int_{Q_s(\tau)} xG(x, k_\perp^2) \rho^a(k_\perp) \rho^a(-k_\perp) \right\},
\]

where \(\mu_\tau(k_\perp) \propto (\partial/\partial \log k_\perp^2) xG(x, k_\perp^2)\) is only slowly varying with \(k_\perp\). Eq. (17) is the McLerran-Venugopalan model of independent colour charges, amended by the standard, linear, quantum evolution, cf. eq. (17).

The new physics, intrinsically non-linear, shows up at large distances \((r_\perp \gg 1/Q_s(\tau))\), or low momenta \((k_\perp \ll Q_s(\tau))\), where color fields are strong, \(\alpha^a \sim 1/g\), and Wilson lines are decorrelated, \(V_x^1 V_{y} \approx 0\). Then one can neglect the bilinears involving Wilson lines in eq. (17), so that the corresponding weight function is very simple:

\[
W_\tau^{\text{low}-k_\perp}[\rho] \approx \exp\left\{ -\frac{1}{2} \int_{Q_s(\tau)} xG(x, k_\perp^2) \rho^a(k_\perp) \rho^a(-k_\perp) \right\}.
\]

This describes a 2-dimensional Coulomb gas, with “dielectric constant”

\[
\varepsilon_\tau(k_\perp) \equiv \tau - \bar{\tau}(k_\perp) = \frac{1}{\bar{\alpha}_s} \ln \frac{Q_s^2(\tau)}{k_\perp^2},
\]

which is the rapidity window for quantum evolution in the saturation regime. Indeed, for given \(k_\perp \ll Q_s(\tau)\), only those partons are saturated whose rapidities \(y\) are large enough for \(Q_s(y) > k_\perp\). This requires \(\bar{\tau}(k_\perp) < y < \tau\), with \(\bar{\tau}(k_\perp)\) the rapidity where \(Q_s(\bar{\tau}(k_\perp)) = k_\perp\).
Saturation is an immediate consequence of eq. (21), which implies the following gluon density per unit transverse phase space at \( k_\perp < Q_s(\tau) \):

\[
\frac{d^2(xG)}{d^2k_\perp d^2b_\perp} = \frac{N^2 - 1}{4\pi^4} \left( \tau - \bar{\tau}(k_\perp) \right) = \frac{N^2 - 1}{4\pi^4 c} \frac{1}{\bar{\alpha}_s} \ln \frac{Q_s^2(\tau)}{k_\perp^2}.
\] (23)

At fixed \( \tau \), the gluon density is almost constant for \( k_\perp < Q_s(\tau) \), and of order \( O(1/\alpha_s) \). This is the Color Glass Condensate. When \( \tau \sim \ln s \) increases, the gluon density at \( k_\perp < Q_s(\tau) \) increases only linearly with \( \tau \) (i.e., logarithmically with the energy \( s \)), but the saturation scale \( Q_s(\tau) \) itself increases as a power of \( s \). That is, with increasing energy, the new partons are predominantly produced at large transverse momenta \( \gtrsim Q_s(\tau) \), so they cannot be discriminated by an external probe with resolution \( Q^2 \approx Q_s^2(\tau) \). Thus, although the total number of gluons keeps growing when the energy increases, there is no contradiction with unitarity since, beyond some energy, the partons produced by quantum evolution are too tiny to contribute to the cross section at fixed \( Q^2 \). Saturation is a natural mechanism to restore unitarity.

References

1. L. V. Gribov, E. M. Levin, M. G. Ryskin, Phys. Rept. 100 (1983), 1.
2. A. H. Mueller, Nucl. Phys. B335 (1990), 115; ibid. B558 (1999), 285.
3. L. McLerran and R. Venugopalan, Phys. Rev. D49 (1994), 2233; ibid. 49 (1994), 3352; ibid. 50 (1994), 2225.
4. L. McLerran, hep-ph/0104283, and references therein.
5. E. Levin, hep-ph/0105207, and references therein.
6. J. Jalilian-Marian, A. Kovner, A. Leonidov and H. Weigert, Nucl. Phys. B504 (1997), 415; Phys. Rev. D59 (1999), 014014.
7. E. Iancu, A. Leonidov and L. McLerran, Nucl. Phys. A692 (2001), 583; Phys. Lett. B510 (2001) 133; E. Ferreiro et al., hep-ph/0109115.
8. E. Iancu and L. McLerran, Phys. Lett. B510 (2001) 145.
9. H. Weigert, hep-ph/0004044.
10. I. Balitsky, Nucl. Phys. B463 (1996), 99; hep-ph/0101042.
11. A. H. Mueller, hep-ph/0110160.
12. Yu. V. Kovchegov, Phys. Rev. D60 (1999), 034008; D61 (2000), 074018.
13. K. Golec-Biernat and M. Wiśniewski, Phys. Rev. D59 (1999), 014017; ibid. D60 (1999), 114023; Eur. Phys. J. C20 (2001) 313.
14. J.-P. Blaizot, E. Iancu and H. Weigert, in preparation.

\footnote{Eq. (23) shows only a mild, logarithmic, dependence upon \( k_\perp \), to be contrasted with the \( 1/k_\perp^2 \) behaviour at large \( k_\perp \), due to bremsstrahlung.}