Universal Democracy Instead of Anarchy

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Abstract

We propose for the flavour structure of both the quark and lepton sectors, the principle of Universal Democracy (UD), which reflects the presence of a $Z_3$ symmetry. In the quark sector, we emphasize the importance of UD for obtaining small mixing and flavour alignment, while in the lepton sector large mixing, including the recently measured value of $U_{e3}$, is obtained in the UD framework through the seesaw mechanism. An interesting correlation between the values of $U_{e3}$ and $\sin^2(\theta_{23})$ is pointed out, with the prediction of $\sin^2(\theta_{23}) \approx 0.42$ in the region where $U_{e3}$ is in agreement with the DAYA-BAY experiment.

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1 Introduction

The recent discovery of a relatively large $U_{e3}$, together with an indication that atmospheric neutrino mixing is not maximal, has had a significant impact on attempts at understanding the principle, if any, behind the observed pattern of fermion masses and mixing. Until recently, leptonic mixing was in agreement with the Ansatz of tri-bi-maximal mixing (TBM) in the leptonic sector, which in turn triggered the suggestion of various family symmetries like $A_4$ which can lead to TBM. Since in this ansatz $U_{e3}$ vanishes, the recent discovery of a relatively large $U_{e3}$, rules out the TBM scheme. This has led to an intense activity in the construction of models which can accommodate a non-vanishing $U_{e3}$, within a variety of frameworks. It has also been suggested that the recent data on leptonic mixing enforces the idea that in fact there is no symmetry principle behind the observed leptonic mixing and instead anarchy prevails with the mixing arising from a random distribution of unitary $3 \times 3$ matrices.

In this paper instead of anarchy, we advocate the principle of universal democracy (UD) for fermion masses and mixing, with UD prevailing both in the quark and lepton sectors. The UD principle reflects the presence of a $Z_3$ family symmetry, which leads in leading order to fermion mass matrices proportional to the so-called democratic flavour structure, where all matrix elements have equal value. Previously, the democratic flavour structure has been applied to the quark sector; here we point out that starting with the same democratic flavour structure for all leptonic mass matrices, namely charged lepton, Dirac neutrino and right-handed Majorana mass matrices, one can reproduce the observed pattern of leptonic masses and mixing, accommodating in particular the recently measured value of $U_{e3}$.

This paper is organized as follows. In the next section we present the UD framework. In section 3, we show how to obtain natural democracy through a $Z_3$ family symmetry and discuss both the quark and lepton mixing. Our numerical results are combined in section 4 and in section 5 we present our conclusions.

2 The Universal Democracy Framework

We suggest that all fermion mass matrices are in leading order proportional to the so-called democratic matrix, denoted by $\Delta$, with all elements equal to the unit:

$$\Delta = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(1)

For simplicity, we assume that the neutrino masses are generated in the framework of an extension of the Standard Model (SM), consisting of the addition of three right-handed neutrinos
to the spectrum of the SM:

\[ -\mathcal{L} = Y_{ij}^{ij} \bar{L}_i \phi l_{jR} + Y_{ij}^{ij} \bar{L}_i \tilde{\phi} v_{jR} + \frac{1}{2} v_{jR}^T C (M_R)^{ij} v_{jR} + h.c., \]

where \( L_i, \phi \) denote the left handed lepton and Higgs doublets, and \( l_{jR}, v_{jR} \) the right handed charged lepton and neutrino singlets. This automatically leads, through the seesaw mechanism \[7\] to an effective neutrino mass matrix at low energies, with naturally small neutrino masses.

In the UD framework, the fermion mass matrices have the form:

**Quark Sector:**

\[ M_d = c_d [\Delta + \varepsilon_d P_d] \]
\[ M_u = c_u [\Delta + \varepsilon_u P_u] \]

**Lepton Sector:**

\[ M_l = c_l [\Delta + \varepsilon_l P_l] \]
\[ M_D = c_D [\Delta + \varepsilon_D P_D] \]
\[ M_R = c_R [\Delta + \varepsilon_R P_R] \]

where the notation is self-explanatory. In particular, \( c_l, c_D, c_R \) are over-all constants, \( M_l, M_D, M_R \) denote the charged lepton mass matrix, and neutrino Dirac and right-handed neutrino mass matrices, respectively. Finally, the \( \varepsilon_iP_i \) denote small perturbations to universal democracy.

In order for universal democracy to satisfy ’tHooft naturalness principle \[8\], there should be a symmetry of the Lagrangian which leads to exact UD. In the quark sector, it is straightforward to find a symmetry which leads to UD. In particular, since all quarks enter on equal footing in the gauge sector, it is natural to assume that in leading order the Yukawa couplings obey a \( S^Q_{3L} \times S^u_{3R} \times S^d_{3R} \) family permutation symmetry, acting on the left-handed quark doublets, the right-handed up quarks and right-handed down quarks, respectively. It is clear that this symmetry leads to UD in the quark sector.

However, in the lepton sector, and taking into account the observed large leptonic mixing, the UD extension is not straightforward. For definiteness, let us consider that there is a lepton number violation mechanism at high energies leading at low energies to the effective Majorana mass term for the light neutrinos. If one trivially extends to the lepton sector the above symmetry by considering a \( S^L_{3L} \times S^l_{3R} \) acting on the lepton doublets and charged leptons, one obtains a charged lepton mass matrix proportional to \( \Delta \). However, the effective neutrino Majorana mass matrix is then proportional to \( (c \Delta + c' I) \), where one expects \( c, c' \) to be of the same order, since both terms are allowed by the family symmetry. It can be shown \[9\] that no large leptonic mixing can be obtained in this case.

In the next section, we explore the possibility of obtaining the observed quark and leptonic mixing, including the results of the DAYA-BAY experiment, through a small perturbation of a \( Z_3 \) symmetry imposed on the quark and lepton sectors. This \( Z_3 \) symmetry leads to UD in leading order in all fermion sectors.
3 The \( Z_3 \) Symmetry

3.1 Natural Democracy

We impose a \( Z_3 \) family symmetry on the Lagrangean, realized in the following way.

**Quark Sector:**

\[
\begin{align*}
Q_{Li} & \rightarrow P_{ij}^\dagger Q_{Li} \\
u_{Ri} & \rightarrow P_{ij} \nu_{Ri} \\
d_{Ri} & \rightarrow P_{ij} d_{Ri}
\end{align*}
\]

where \( \omega = e^{\frac{2\pi i}{3}} \).

It can be readily verified that this is indeed a \( Z_3 \) symmetry since \( P^2 = P^\dagger, P^3 = I \). Then, the Lagrangean, and in particular the quark mass terms \( \overline{Q}_{Li} M_{ij}^u u_{Rj} \) and \( \overline{Q}_{Li} M_{ij}^d d_{Rj} \), are invariant, if the quark mass matrices \( M^u,d \) obey the following relation

\[
P \cdot M \cdot P = M
\]

Notice that we do not have \( P^\dagger \cdot M \cdot P = M \). It is crucial for our results that Eq. (6) holds and it immediately follows that \( \text{det}(M) = 0 \), since \( \text{det}(P) \) is not real. Thus, \( M \) must have one or more zero eigenvalues, and in [9] it was indeed shown that \( M \) is proportional to the democratic matrix \( \Delta \).

**Lepton Sector:**

In the lepton sector, we impose the \( Z_3 \) symmetry in exactly the same way:

\[
\begin{align*}
L_i & \rightarrow P_{ij}^\dagger L_j \\
\nu_{iR} & \rightarrow P_{ij} \nu_{jR} \\
\nu_{iR} & \rightarrow P_{ij} \nu_{jR}
\end{align*}
\]

It is clear that for a \( Z_3 \) symmetry realized in the way indicated in Eqs. (5, 6, 7), all fermion mass matrices, \( M_d, M_u, M_l, M_D \) and \( M_R \), are proportional to \( \Delta \). In particular, in the exact \( Z_3 \) limit, \( M_R \) will not contain a term \( a I \), since this term is not allowed by the \( Z_3 \) symmetry.

3.2 Quark Sector: Small Mixing and Alignment

Significant features of the observed pattern of quark masses and mixing include hierarchical quark masses, small mixing and the observed alignment between the spectrum of the masses in the up and down quark sectors. This observed alignment is rarely mentioned in the literature and yet it is an important feature observed in the quark sector. So it is worth defining in a precise manner what alignment means. Let us consider a set of quark mass matrices \( M_d, M_u \)
where the quark masses are hierachical and the mixing is small. Small mixing means that there is a weak-basis where both \( M_d \) and \( M_u \) are close to a diagonal matrix. Even for this set of matrices, and taking into account that in the SM, the Yukawa couplings for the up and down quark sectors are entirely independent, it is as likely that in the basis where \( M_u \) is close to \( \text{diag}(m_u, m_c, m_t) \), \( M_d \) is close to \( \text{diag}(m_d, m_s, m_b) \), meaning alignment, as in contrast having \( M_d \) close to \( \text{diag}(m_b, m_d, m_s) \) meaning misalignment. Note that the ordering in one of the sectors is arbitrary, but the relative ordering is physically meaningful. For a set of random matrices, even if one assumes hierarchical masses and small mixing, the probability of having alignment is only 1/6. For a set of arbitrary masses, one can verify whether one has small mixing and alignment through the use of weak-basis invariants [10].

In order to see this, it is convenient to define the dimensionless matrices with unit trace:

\[
h_{u,d} = \frac{H_{u,d}}{\text{Tr}[H_{u,d}]}\]

(8)

where \( H_u \equiv M_u M_u^\dagger \) and similarly for \( H_d \). As we have previously mentioned, alignment means that in the weak-basis where \( H_u = \text{diag}(m_u^2, m_c^2, m_t^2) \), \( H_d \) is close to \( \text{diag}(m_d^2, m_s^2, m_b^2) \). Then defining \( A = h_d - h_u \) and taking into account that \( \text{Tr}(A) = 0 \) by construction, in turn implying that \( |\chi(A)| = \frac{1}{2} \text{Tr}[A^2] \), one can show that the condition for alignment is

\[
|\chi(A)| \ll 1
\]

(9)

where \( \chi(A) \) is the second invariant of \( A \), namely \( \chi(A) = a_1 a_2 + a_1 a_3 + a_2 a_3 \), with the \( a_i \) denoting the eigenvalues of \( A \). In order to see that the condition Eq. (9) leads to alignment, consider the limit where \( m_t, m_b \) go to infinity. Assuming alignment, one has \( H_{u,d} = \text{diag}(0,0,1) \) so that \( \chi(A) = 0 \). If one considers instead that there is small mixing but no alignment, then in the weak-basis where \( H_u = \text{diag}(m_u^2, m_c^2, m_t^2) \), one may have \( H_d \) close to \( \text{diag}(m_d^2, m_s^2, m_b^2) \). One can check that in this case \( \chi(A) \approx 1 \), indicating total misalignment. A very interesting feature of universal democracy is the fact that a small perturbation of UD as indicated in Eq. (3), automatically leads to alignment of the heaviest generation. Alignment of the two light generations depends, of course, on the specific breaking of UD through \( e_u P_d, e_u P_d \) in Eq. (3). It is indeed a salient feature of the universal democracy hypothesis for the quark sector [3] that it guarantees exactly these two important phenomenological properties: small mixing and alignment.

### 3.3 Generating large leptonic mixing through the breaking of \( Z_3 \)

A breaking of \( Z_3 \) generates leptonic mass matrices of the form of Eq. (4), where the \( \epsilon_i \ll 1 \) \((i = l, D, R)\) and the \( P_i \) are of order 1. We assume that the perturbation \( P_R \) of the right-handed heavy Majorana neutrinos is such that the inverse of \( \Delta + \epsilon_R P_R \) exists, and we find that generically \( [\Delta + \epsilon_R P_R]^{-1} \) is of the form:

\[
[\Delta + \epsilon_R P_R]^{-1} = \frac{c_p}{\epsilon_R} [L + \epsilon_R Q] \quad ; \quad c_p = \frac{1}{q + \epsilon_R p}
\]

(10)
where \( p \) and \( q = \sum Q_{ij} \) are cubic and quadratic polynomials in the elements \((P_R)_{ij}\) and \( L, Q \) are matrices with respectively linear and quadratic elements in \((P_R)_{ij}\). Obviously \( p, q, L \) and \( Q \) are in general of order 1. It is possible to have special cases where either \( p \) or \( q \) vanish, but not both, since we require that the inverse of \( \Delta + \varepsilon_R P_R \) exists. Furthermore, it is a general characteristic of this inverse that the linear matrix \( L \) and the quadratic matrix \( Q \) satisfy the relations:

\[
\Delta L = 0 \quad ; \quad \Delta Q \Delta = q \Delta
\]

(11)

Applying these algebraic relations to the effective neutrino mass matrix formula one obtains a transparent formula for \( M_{\text{eff}} \):

\[
M_{\text{eff}} = c \left( q \Delta + \varepsilon_D \Delta Q P_D + \varepsilon_R P_D Q P_T + \varepsilon_R^2 P_D L P_T \right)
\]

(12)

with \( c = -c_R^2 / c_R (q + \varepsilon_R p) \).

This expression obtained for the effective neutrino mass matrix tells us when to expect large mixing for the lepton sector in the case of an aligned hierarchical spectrum for the charged leptons, as well as for the Dirac and heavy Majorana neutrinos. In general, i.e., for a generic perturbation \( P_R \) in the right-handed Majorana sector, there will be more than one element \((P_R)_{ij}\) of order one, thus implying that the quadratic polynomial \( q = \sum Q_{ij} \) is also of order one. So, if the term in \( M_{\text{eff}} \) proportional to \( \varepsilon_R^2 / \varepsilon_D \) is small, it is clear that the effective neutrino mass matrix will be, just like the charged lepton mass matrix, to leading order, proportional to \( \Delta \), and, thus, there will be no large mixing. Therefore, if one wants to avoid small mixing, one must have the term proportional to \( \varepsilon_D^2 / \varepsilon_R \), in Eq. (12), to be of order one or larger.

4 Numerical Results

Next, we give a numerical analysis of our model. In particular, we give a specific point in parameter space which exhibits all features described here and is in excellent agreement with the experimental evidence and in particular with the recent DAYA-BAY result. In accordance with the universal democracy hypothesis, given in Eq. (4), we write the leptonic mass matrices as proportional to \( \Delta \) plus some matrix with elements of order of a power in \( \lambda \equiv 0.2 \).

For the charged lepton mass matrix, we have \( M_l = c_l \left[ \Delta + \lambda \ P_l \right] \), where,

\[
M_l = c_l \begin{pmatrix} 1 & 1 & 1 \\ 1 & e^{i \delta_1} \lambda^4 & 1 \\ 1 & 1 & e^{i \delta_2} \lambda \end{pmatrix} ; \quad c_l = 593.3 \text{ MeV} \quad ; \quad \delta_1 = -1.078 \quad ; \quad \delta_2 = -1.335
\]

(13)

For the neutrino Dirac and right-handed mass matrices, we have:

\[
M_D = c_D \left[ \Delta + \lambda \ P_D \right] ; \quad M_R = c_R \left[ \Delta + \lambda^3 \ P_R \right]
\]

(14)
where

\[
\lambda P = \begin{pmatrix}
0.408 \cdot \lambda & 0.490 \cdot \lambda & 0.817 \cdot \lambda \\
-1.021 \cdot \lambda^2 & -0.817 \cdot \lambda & 0.851 \cdot \lambda^3 \\
-1.443 \cdot e^{i\frac{\pi}{4}} \cdot \lambda^2 & 1.021 \cdot i \cdot \lambda^2 & 0.408 \cdot i \cdot \lambda
\end{pmatrix}
\]

and

\[
\lambda^3 P = \begin{pmatrix}
0 & 0 & 0 \\
0 & -0.319 \cdot i \cdot \lambda^3 & 0 \\
0 & 0 & -1.512 \cdot \lambda^5
\end{pmatrix}
\]

with

\[
c_D = \frac{m_{top}^3}{2} : \quad c_R = 3.4 \times 10^{15} \text{ GeV}
\]

With these, we obtain the following physical observables: charged lepton, light neutrino masses

\[
m_e = 0.511 \text{ MeV} \quad m_{\nu_1} = 0.000496 \text{ eV} \quad M_1 = 8.15 \times 10^{11} \text{ GeV}
\]
\[
m_\mu = 105.66 \text{ MeV} \quad m_{\nu_2} = 0.00875 \text{ eV} \quad M_2 = 5.82 \times 10^{12} \text{ GeV}
\]
\[
m_\tau = 1776.8 \text{ MeV} \quad m_{\nu_3} = 0.0505 \text{ eV} \quad M_3 = 1.02 \times 10^{16} \text{ GeV}
\]

and mixing

\[
|U^{PMNS}| = \begin{pmatrix}
0.8101 & 0.5658 & 0.1536 \\
0.4456 & 0.6258 & 0.6402 \\
0.3810 & 0.5369 & 0.7527
\end{pmatrix}
\]

\[
|U_{e3}|^2 = 0.0236 \\
\sin^2 \theta_{12} = 0.328 \\
\sin^2 \theta_{23} = 0.420 \\
J = 0.0347 \\
|m_{ee}| = 0.00195 \text{ eV}
\]

Note that all observables are within the experimental bounds, in particular we have a sufficient large \( |U_{e3}| \), in agreement with the recent DAYA-BAY experimental results. We have also a large hierarchy for the heavy Majorana neutrinos due to the perturbation term in \( M_R \), which is proportional to higher power in \( \lambda \) in Eq. (14).

In addition to this point, we have explored a stable region in parameter space around this point which is in agreement with experiment. This was done using a numerical analysis with a Monte Carlo type algorithm. We generated random perturbations of at most 10% away from the initial parameters, by means of a uniform distribution. Each of the plots in Figs. 1, 2, 3 represents the normalized density of points belonging to the data set, as a function of the values of a pair of observables. Our parameter space point, given in Eqs. (13, 14), is also depicted in the figures as the intersection of the dashed horizontal line with the dashed vertical line.
A salient feature illustrated by this numerical analysis is an interesting correlation between the values of $U_{e3}$ and $\sin^2(\theta_{23})$. In the region where values of $U_{e3}$ are obtained in agreement with the DAYA-BAY experiment, values of $\theta_{23} \neq 45^\circ$ are favoured around $\sin^2(\theta_{23}) \approx 0.42$.

5 Conclusions

We have presented a unified view of the flavour structure of the quark and lepton sectors, based on the principle of Universal Democracy coming from a $Z_3$ symmetry, where in leading order all fermion mass matrices are proportional to the democratic matrix. In the quark sector, small mixing arises from the breaking of UD which also generates the masses of the light generations. In the lepton sector, the breaking of UD is also small, but in the presence of the seesaw mechanism, this small breaking of UD is able to generate large leptonic mixing, including a value of $U_{e3}$ in agreement with the DAYA-BAY result. The smallness of the breaking of $Z_3$ is natural in the 'tHooft sense, since in the limit of exact UD, the Lagrangean acquires the $Z_3$ symmetry.

In conclusion, our analysis shows that the observed pattern of fermion masses and mixing, both in the quark and lepton sectors, may reflect the principle of Universal Democracy, rendered natural by a $Z_3$ symmetry.

Acknowledgments

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Figure 1: Density of data points as a function of $\sin^2 \theta_{23}$ and $|U_{e3}|$

Figure 2: Density of data points as a function of $\sin^2 (\theta_{12})$ and $|U_{e3}|$
Figure 3: Density of data points as a function of $m_{\nu_1}$ and $|U_{e3}|$