The Supersymmetric Singlet Majoron

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Abstract

We study the supersymmetrized version of the singlet majoron model and, performing an analysis of the renormalization group equation improved potential, we find that a spontaneous breaking of $R$-parity can be achieved for a wide range of the parameters. Studying the finite temperature effective potential, we show that the phase transition leading to $R$-parity breaking can be of the first order and can occur at temperatures below the weak scale, thus avoiding any constraint coming from the requirement of the preservation of the baryon asymmetry in the early Universe.

♣ On leave of absence from INFN, Padova, Italy.

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September 1992
1. Introduction

The possibility that lepton number ($L$) is not an exact symmetry is of great interest, since it leads to the prediction of non-vanishing neutrino masses with important implications for particle physics, astrophysics, and cosmology. Theoretically one can envisage two schemes for $L$ violation: either $L$ is an approximate symmetry of the standard model explicitly broken, as in Grand Unified Theories (GUT’s)-inspired see-saw mechanisms [1], or it is an exact global symmetry spontaneously broken, leading to the existence of a Goldstone boson, the Majoron [2]–[3].

In supersymmetry, non-conservation of $L$ presents novel and characteristic features. First of all, because of the connection between $L$ and $R$-parity, the discrete symmetry that prevents the lightest supersymmetric particle from decaying, the breaking of $L$ has also important consequences for supersymmetric phenomenology. Furthermore, unlike the standard model, $L$ non-conserving renormalizable interactions are allowed by gauge invariance in supersymmetric models, even with a minimal choice of fields, providing a new source of explicit $L$ violation [4]. Spontaneous $L$-breaking is also possible in supersymmetry without introducing additional fields, since the scalar partner of the neutrino may acquire a non-vanishing vacuum expectation value [5]. Then the Majoron is mainly the supersymmetric partner of the neutrino and should be detected in $Z^0$ decays. Recent LEP measurements [6] of the $Z^0$ width have ruled out this possibility. Nevertheless, if the spontaneous $L$-breaking is triggered by the vacuum expectation value of the gauge-singlet right-handed neutrino, the existence of the Majoron is not in contradiction with the $Z^0$ width measurements. The feasibility of such a scheme was first shown in the model of ref. [7] by introducing seven new gauge singlet superfields, besides the usual particle content of the minimal supersymmetric standard model (MSSM). We extend the MSSM by simply including right-handed neutrino chiral superfields ($\hat{N}_i$) and an additional gauge singlet superfield $\hat{\Phi}$, carrying two units of $L$. The most general superpotential invariant under gauge symmetry and $L$ is

$$f = h_{ij}^d \hat{Q}_i \hat{d}_j^c \hat{H}_1 + h_{ij}^u \hat{Q}_i \hat{u}_j^c \hat{H}_2 + h_{ij}^e \hat{L}_i \hat{e}_j^c \hat{H}_1 + h_{ij}^\nu \hat{L}_i \hat{N}_j \hat{H}_2 - \mu \hat{H}_1 \hat{H}_2 + \lambda_{ij} \hat{N}_i \hat{N}_j \hat{\Phi},$$

(1)

where $i, j$ are the generation indices, $\hat{L}$, $\hat{Q}$ are the left-handed lepton and quark doublets respectively; $\hat{e}^c$, $\hat{u}^c$ and $\hat{d}^c$ are the (charge conjugate of) left-handed lepton and charge $-2/3$ and $1/3$ quark singlet superfields, respectively; $\hat{H}_1$ and $\hat{H}_2$ are the two Higgs doublets necessary to give masses to leptons and quarks through the Yukawa couplings $h_{ij}^e$, $h_{ij}^u$ and $h_{ij}^d$. The superpotential $f$ contains the usual terms of the MSSM augmented by Yukawa interactions for the right-handed neutrinos (with couplings $h_{ij}^\nu$) and an
interaction term for the new gauge singlet \( \hat{\Phi} \) (with couplings \( \lambda_{ij} \)). Our purpose in the present paper is to show that the supersymmetric version of the singlet Majoron model \([3]\) given in eq. (1) is a viable, and most economical, model for spontaneously broken \( L \)-symmetry and \( R \)-parity. In particular, we show that for a wide range of parameters a radiative breaking of \( R \)-parity can be obtained, as derived from the analysis of the renormalization group equations (RGE’s), which we solve in two limiting cases. After a brief discussion of the phenomenological consequences of the \( L \)-number spontaneous symmetry breaking, giving rise to constraints on the \( h_{ij}^{\nu} \) couplings coming from Majoron interactions and neutrino masses, we include the one-loop finite temperature corrections to the effective potential. This allows us to discuss the problem of baryogenesis in this class of models with spontaneous \( R \)-breaking. It is well known that instanton effects in the Standard Model violate \( B \) and \( L \) while conserving the combination \((B - L)\). There is a widespread consensus that they become important at temperatures above \( M_W \) \([8]\). If, in addition to these \( B \)- and \( L \)-violating effects, other interactions exist which violate \( B \), \( L \), and also the combination \((B - L)\), and they are in equilibrium at temperatures above \( M_W \), then no cosmological baryon asymmetry \( \Delta B \) can survive \([3]\). If \( R \)-parity is broken explicitly \([4]\), the \( L \)-violating interactions may wash out any preexisting baryon asymmetry, unless either very strong bounds are imposed on the \( R \)-breaking couplings \([4]\), or new mechanisms to generate or preserve \( \Delta B \) are invoked, see refs. \([10, 11, 12]\). Studying the phase transition leading to the spontaneous breaking of \( L \), we show that it may take place after the completion of the electroweak phase transition, when sphaleron interactions are frozen out and no wiping out of a net baryon asymmetry can occur any longer.

The paper is organized as follows. In Sect. 2 we present the model and discuss the minimization of the RGE’s improved scalar potential. In Sect. 3 a brief discussion of the phenomenology of the model is presented. We proceed to the analysis of the one-loop finite temperature corrected potential in Sect. 4. Finally, in Sect. 5 we discuss the results and present our conclusions.

2. Minimization of the effective potential.

The tree level potential for the neutral scalar fields \( H_1^0, H_2^0, \nu \) (respectively belonging to the weak doublet superfields \( \hat{H}_1, \hat{H}_2, \hat{L}_i \)) and \( \Phi, \Phi, \Phi \) (belonging to the gauge singlet superfields) can be decomposed as

\[
V_{\text{tree}} = V_{H}^{\text{tree}}(H_1^0, H_2^0) + V_{N\Phi}(N_i, \Phi) + V_{\nu}(\nu_i, \Phi, H_1^0, H_2^0).
\]  (2)
$V_H^{\text{tree}}$ is the usual MSSM Higgs potential. Assuming for simplicity all $\lambda_{ij}$ real, we can work in a basis for the $N_i$ fields where the couplings $\lambda_{ij}$ are diagonal ($\lambda_{ij} = \lambda_i \delta_{ij}$) and write

$$V_{N\Phi}^{\text{tree}}(N_i, \Phi) = 4 \sum_i |\lambda_i N_i \Phi|^2 + \sum_i \lambda_i N_i^2 |\Phi|^2 - \left( \sum_i A_i \lambda_i N_i^2 \Phi + \text{h.c.} \right),$$

where the last three terms contain the soft supersymmetry breaking masses ($m_{N_i}, m_\Phi$) and trilinear couplings ($A_i$).

It is plausible to expect that the coupling constants $h_\nu$ are of the same order of magnitude of $h_e$, the Yukawa couplings for the charged leptons, and thus numerically small ($h_\nu \simeq 10^{-2} - 10^{-6}$). Assuming then that $|h_\nu| \ll 1$ and keeping only the leading terms of an expansion in $h_\nu$, we obtain

$$V_\nu^{\text{tree}}(\nu_i, N_i, \Phi, H_0^1, H_0^2) = \frac{g^2 + g'^2}{8} \left[ \left( \sum_i |\nu_i|^2 \right)^2 + 2 \sum_i |\nu_i|^2 \left( |H_0^1|^2 - |H_0^2|^2 \right) \right]$$

$$+ \sum_i m_{\nu_i}^2 |\nu_i|^2 + \left[ \sum_{ij} h_{\nu ij} \nu_i \left( 2 \lambda_j N_j^* \Phi^* H_2 - \mu N_j H_0^j - A_{ij}^{(h)} N_j H_0^j \right) + \text{h.c.} \right] + O(h_\nu^2),$$

where we have again introduced the appropriate soft supersymmetry breaking mass parameters ($m_{\nu_i}$) and trilinear couplings ($A_{ij}^{(h)}$), and $g$ and $g'$ are the $SU(2)_L$ and $U(1)_Y$ gauge couplings, respectively.

As is usually done for the MSSM, we will assume that the supersymmetry breaking terms have a common origin and are therefore related at some GUT scale ($M_{\text{GUT}} \simeq 10^{16}$ GeV), such as all mass parameters and trilinear couplings are equal to a universal mass $\tilde{m}$ and a universal coupling $A$. The values of the supersymmetry breaking parameters appearing in the scalar potential, eqs.(2-4), are then derived by solving the relevant RGE’s with boundary conditions at $M_{\text{GUT}}$. It is well known that, for an appropriate choice of initial conditions at $M_{\text{GUT}}$, the effect of running the parameters in $V_H^{\text{tree}}(H_0^1, H_0^2)$ from $M_{\text{GUT}}$ to low energy is to drive electroweak symmetry breaking with non-vanishing vacuum expectation values for the Higgs fields, $\langle H_0^1 \rangle = \frac{v}{\sqrt{2}} \cos \beta$, $\langle H_0^2 \rangle = \frac{v}{\sqrt{2}} \sin \beta$, with $v = 246$ GeV.

We will minimize the renormalization group improved potential (3) and safely neglect the logarithmic term $\text{Str} \mathcal{M}^4 \ln(\mathcal{M}^2/Q^2)$ by choosing the renormalization scale $Q$ at the order of the weak scale. Moreover, assuming $CP$-conservation, we may take all the parameters in eq. (3) to be real.
By assuming that the $\langle \nu_i \rangle$'s are much smaller than the typical weak scale $v$, we can neglect the quartic terms in eq. (4). We obtain the approximate solution

$$v_{\nu_i} = v \frac{\sum_j h_{ij}^j v_{N_i}}{\sqrt{2}} \left[ m_{v_{\nu_i}} + \cos 2\beta \right] + O(h^2),$$

where $v_\Phi \equiv \langle \Phi \rangle$, $v_{N_i} \equiv \langle N_i \rangle$. Notice that, for consistency, we must require $v_\Phi \simeq v_{N_i} = O(1)$ TeV and $h^2 \ll 1$.

The values of $v_\Phi$ and $v_{N_i}$ are determined by the minimum of $V_{tree}^{N\Phi}$, eq. (3)

$$v_\Phi = \frac{x}{4\lambda_3}, \quad v_{N_3} = \frac{m_\Phi^2}{4\lambda_3^2 (A_3 - x)}, \quad v_{N_2} = v_{N_1} = 0,$$

where $x$ is a solution of the cubic equation:

$$x^3 - 3A_3 x^2 + 2 \left( 2m_{N_3}^2 - m_\Phi^2 + A_3^2 \right) x - 4A_3 m_{N_3}^2 = 0,$$

under the condition

$$A_3 > 1.$$  

We have chosen $\lambda_3$ to be the smallest of the non-zero values of $\lambda_i$. In the case $\lambda_1 = \lambda_2 = \lambda_3$, a minimum with two non-vanishing $v_{N_i}$ is possible. However, such a minimum requires $m_\Phi^2 > m_{N_i}^2$, in contradiction with the solution of the RGE’s, see eqs. (21-22) and the Appendix.

The potential $V_{tree}^{N\Phi}$ at the minimum is

$$V_{tree}^{N\Phi}(v_{N_i}, v_\Phi) = \left( \frac{m_\Phi x}{4\lambda_3} \right)^2 \left[ 1 - \left( \frac{m_\Phi}{A_3 - x} \right)^2 \right].$$

The condition for spontaneous $L$-breaking is that $V_{tree}^{N\Phi}(v_{N_i}, v_\Phi)$ is smaller than the value of the potential at the origin:

$$A_3 - m_\Phi < x < A_3 + m_\Phi.$$  

Moreover the requirement that the potential $V_{tree}^{N\Phi}$ is bounded from below implies the condition:

$$m_\Phi^2 > 0.$$  

As stated above, the parameters in the potential $V_{tree}^{N\Phi}$, eq. (3), are related by an initial condition at $M_{GUT}$. Their dependence upon the energy scale $Q$ is

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1Here, in order to avoid non-vanishing $\langle \nu_i \rangle$'s for $h^2 = 0$, we are assuming $m_{\nu_i}^2 + (M_Z^2/2) \cos 2\beta > 0$. This is however not very restrictive, since it holds for almost all of the supersymmetric parameters which lead to a correct electroweak symmetry breaking.
determined by the following RGE’s \((t \equiv \log Q/M_{GUT})\)

\[
\frac{d}{dt} \lambda_i = \frac{5}{8\pi^2} \lambda_i^3, \\
\frac{d}{dt} m_{N_i}^2 = \frac{1}{2\pi^2} \left(2m_{N_i}^2 + m_{\Phi}^2 + |A_i|^2\right) \lambda_i^2, \\
\frac{d}{dt} m_{\Phi}^2 = \frac{1}{4\pi^2} \sum_i \left(2m_{N_i}^2 + m_{\Phi}^2 + |A_i|^2\right) \lambda_i^2, \\
\frac{d}{dt} A_i = \frac{1}{4\pi^2} \left(4A_i \lambda_i^2 + \sum_j A_j \lambda_j^2\right),
\]

with boundary conditions at \(Q = M_{GUT}\):

\[
m_{N_i}(0) = m_{\Phi}(0) = \tilde{m}^2, \quad A_i(0) = A, \quad \lambda_i(0) = \lambda_i^{(0)}.
\]

Eqs. (12-15) can be easily solved either in the limit of vanishing \(\lambda_1\) and \(\lambda_2\):

\[
\lambda_{1,2}(Q) = 0, \quad \lambda_3(Q) = \lambda_3^{(0)} K, \\
A_{1,2}(Q) = AK^{2/5}, \quad A_3(Q) = AK^2,
\]

\[
m_{\Phi}^2(Q) = \tilde{m}^2 \left[2 + (3 - A^2)K^2 + A^2 K^4\right], \\
m_{N_{1,2}}^2(Q) = \tilde{m}^2, \quad m_{N_3}^2(Q) = 2m_{\Phi}^2(Q) - \tilde{m}^2, \\
K \equiv \left(1 + \frac{5}{4\pi^2} \lambda_3^{(0)} \log \frac{M_{GUT}}{Q}\right)^{-1/2} = \left(1 - \frac{5}{4\pi^2} \lambda_3^2(Q) \log \frac{M_{GUT}}{Q}\right)^{1/2},
\]

or in the limit \(\lambda_1 = \lambda_2 = \lambda_3\):

\[
\lambda_i(Q) = \lambda_i^{(0)} K, \quad A_i(Q) = AK^{14/5}, \\
m_{\Phi}^2(Q) = \tilde{m}^2 \left[-2 + 3(3 - A^2)K^{14/5} + 3A^2 K^{28/5}\right], \\
m_{N_i}^2(Q) = \frac{2}{3} m_{\Phi}^2(Q) + \frac{1}{3} \tilde{m}^2.
\]

In summary, \(L\)-symmetry and \(R\)-parity are spontaneously broken if eq. (7) allows a solution which satisfies the conditions (8), (10), and (11), with the supersymmetry breaking parameters fixed at low energy by the RGE’s (12-15). In figs. 1-2 we show the region of parameters \(\lambda_3(Q^2 = M_W^2)\) and \(A_3(Q^2 = M_W^2)\) where \(L\) is spontaneously broken in the case \(a)\ \lambda_1 = \lambda_2 = 0, \lambda_3 \neq 0\) (where eqs. (16-19) can be applied) and \(b)\ \lambda_1 = \lambda_2 = \lambda_3\) (where eqs. (20-22) can be applied). An upper bound on \(|A|\) follows from the constraint of avoiding undesirable color-breaking minima [13]

\[
|A|^2 < 3(3\tilde{m}^2 + \mu^2),
\]

\(^{2}\)In such a case only one gauge singlet superfield \(\hat{N}\) is responsible for all the light neutrino masses.
at the GUT scale. Since the value of \( \mu \), the Higgs mixing parameter, see eq. (1), does not explicitly enter in our analysis, we have chosen to plot in figs. 1-2 the constraint \(|A| < 5\bar{m}\), as representative of eq. (23). For small values of \( \lambda_i \), eq. (7) can be solved analytically and the solution which satisfies conditions (8), (10), and (11), at the weak scale, is

\[
x = \frac{A_3}{2} \left(1 + \sqrt{1 - \frac{8\bar{m}^2}{A_3^2}}\right) \quad \text{for} \quad |A_3| > 3\bar{m}.
\]  

(24)

In the case \( a \), for larger values of \( \lambda_3 \) (\( \lambda_3 > 0.1 \)), solutions for smaller values of \(|A_3|\) are possible, and they correspond mainly to parameters such that \( m_{N_i}^2 < 0 \), see fig. 1. In the case \( b \) such solutions are not present, since the RGE’s imply now \( m_{N_i}^2 < m_{N_i}^2 \) and the condition (11) cannot be satisfied if \( m_{N_i}^2 < 0 \). In the case \( b \) there is also an approximate (broken by Yukawa terms) \( O(3) \) generation symmetry of the right-handed neutrinos, which is spontaneously broken to \( O(2) \), giving rise to two pseudo-Goldstone bosons (see Appendix). This is not necessarily a problem, since these light familons would be very weakly coupled to ordinary particles.

3. Phenomenology of the model.

The spontaneous breaking of \( L \)-symmetry will give rise to a Goldstone boson, the Majoron. Since the \( \langle \nu_i \rangle \)'s carry both \( L \) and hypercharge quantum numbers, the physical Goldstone boson is given by:

\[
J \simeq \text{Im} \left[ \left( \sum_i \frac{v_{\nu_i}^2}{v_{L}} \right) \sqrt{2} \sin \beta H_2^0 - \cos \beta H_1^0 \right] \\
+ \sum_i \left( \frac{v_{\nu_i} v_{\nu_i}}{v_{L}} - \frac{v_{N_i} v_{N_i}}{v_{L}} \right) + 2 \frac{v_{\Phi} v_{\Phi}}{v_{L}},
\]  

(25)

\[
v_{L}^2 \equiv 4v_{\Phi}^2 + \sum_i v_{N_i}^2,
\]  

(26)

where we have used the fact that the \( v_{\nu_i} \)'s are much smaller than the other vacuum expectation values.

The strongest bounds on the Majoron couplings come from the observed duration of helium burning in red-giants. Ref. [14] gives

\[
g_e \lesssim 3 \times 10^{-13} \quad \text{and} \quad g_\gamma \lesssim 10^{-10} \text{ GeV}^{-1},
\]  

(27)
where $g_e$ is the coupling constant of the pseudoscalar interaction of the Majoron with electrons and $g_\gamma$ is the coupling constant of the Majoron-photon interaction term

$$-\frac{g_\gamma}{4} F^{\mu\nu} \tilde{F}_{\mu\nu} J. \quad (28)$$

Since the $L$ current is anomalous, a coupling of the form (28) is generated at the one loop level by a triangular graph with charginos and charged leptons running inside the loop. Although the exact result for $g_\gamma$ depends on the particular choice of the supersymmetric parameters, we can give an estimate of it. We first assume that the Yukawa couplings $h_{\nu ij}$ have a hierarchical structure in generation indices (like $h_{e ij}$), with $h_{\nu 31} < \sim h_{\nu 32} < \sim h_{\nu 33} < 1$. Taking now the supersymmetric parameters of the same order of magnitude as the weak scale and all coupling constants, except Yukawas, of order unity, the one-loop induced Majoron-photon coupling is:

$$g_\gamma \simeq \frac{\alpha}{2\pi v_L} h_{\nu ij}^2. \quad (29)$$

and from eq. (27) we obtain the bound $h_{\nu 33}^\nu \lesssim 10^{-2} \sqrt{v_L/\text{TeV}}$.

A more stringent bound comes from the Majoron-electron coupling. Under the same assumptions as before, we can estimate $g_e \simeq h_{\nu 33}^\nu h_{\nu 11}^e$ from the Higgs component of the Majoron, and $g_e \simeq h_{\nu 13}^\nu$ from the scalar neutrino component of the Majoron. Barring the possibility of accidental cancellations, eq. (27) implies $h_{\nu 33}^\nu \lesssim 0.4 \times 10^{-3}$ and $h_{\nu 13}^\nu \lesssim 0.5 \times 10^{-6}$. Of course this is to be understood as merely an estimate of the bound, which actually depends on the various input parameters of the MSSM. However it is interesting to observe that $h_{\nu 33}^\nu \approx 10^{-3}$ and $h_{\nu 13}^\nu \approx 10^{-6}$ are the values suggested by an analogy with the charged lepton Yukawa couplings.

After spontaneous symmetry breaking, neutrinos acquire masses. The three families of left-handed neutrinos mix with the right-handed neutrinos and with the neutralinos in a $11 \times 11$ mass matrix. The neutrino masses are therefore complicated functions of the supersymmetric parameters and coupling constants. However, in the above-mentioned approximation of keeping a single energy scale for the supersymmetric parameters and keeping all the coupling constants, except the Yukawas, equal to one, we find

$$m_{\nu_r} \simeq h_{\nu 33}^\nu \tilde{m}. \quad (30)$$

For $h_{\nu 33}^\nu \approx 10^{-3}$ and $\tilde{m} \approx 1 \text{ TeV}$, the tau neutrino mass is $m_{\nu_r} \approx 1 \text{ MeV}$, an interesting value for phenomenology. The other two neutrinos $\nu_e$ and $\nu_\mu$ are expected to be almost massless, since the mass matrix has two approximate zero eigenvalues (even for $h_{\nu 13}^\nu, h_{\nu 23}^\nu \neq 0$). The massive $\nu_r$ does not cause cosmological problems since it is short lived

$$\tau(\nu_r \to \nu_i J) \approx \frac{16\pi}{m_{\nu_r}} h_{\nu 33}^\nu h_{\nu 13}^\nu \simeq 3 \times 10^{-6} \left( \frac{h_{\nu 33}^\nu}{10^{-3}} \right)^{-4} \left( \frac{h_{\nu 13}^\nu}{10^{-4}} \right)^{-2} \left( \frac{\tilde{m}}{\text{TeV}} \right)^{-1} \text{s.} \quad (31)$$
Notice that the Majoron interacts with ordinary particles with strength always proportional to $h^\nu$ and therefore decouples in the limit $m_\nu \to 0$.

Even if the Majoron is coupled with ordinary matter weakly enough to escape bounds from stellar cooling ($h^\nu_{33} \lesssim 10^{-3}, h^\nu_{13} \lesssim 10^{-6}$), it still could be detected in rare $L$-violating tau and muon decays. Processes like $\tau \to \mu J$, $\tau \to eJ$, and $\mu \to eJ$ occur with rates observable at future dedicated experiments, as has been exhaustively discussed in ref. [13]. On the other hand, for $h^\nu \lesssim 10^{-3}$, the Majoron cannot be detected in LEP experiments looking for direct Majoron production in $Z^0$ decays accompanied by either $\gamma$, $Z^0^*$, or a light scalar boson.

Together with $L$-symmetry, $R$-parity is also spontaneously broken. As a consequence, the lightest supersymmetric particle can decay, modifying the standard experimental signals of supersymmetry. Assuming that the neutralino ($\chi^0$) is the lightest supersymmetric particle, $\chi^0 \to J\nu_\tau$, $\chi^0 \to Z^0\nu_\tau$, $\chi^0 \to W^\pm\tau^\mp$ are possible decay modes. If $\chi^0$ is heavier than $Z^0$ and $W^\pm$ gauge bosons, the three decay modes are expected to occur at similar rates (with details depending upon the choice of supersymmetric parameters), and the $\chi^0$ lifetime is:

$$ \tau(\chi^0) \simeq \frac{16\pi}{h^\nu_{33}^2 m_{\chi^0}} \simeq \left( \frac{10^{-3}}{h^\nu_{33}} \right)^2 \left( \frac{300 \text{ GeV}}{m_{\chi^0}} \right) 10^{-19} \text{s}. $$

(32)

The two-body decay is fast, so that a $\chi^0$ produced in collider experiments decays within the detector, unless $h^\nu_{33}$ is extremely small. If $\chi^0$ is lighter and the gauge bosons produced in the decay are virtual, $\chi^0 \to J\nu_\tau$ becomes the dominant mode, leaving no visible trace of the neutralino decay. On the other hand, the chargino could instead be the lightest supersymmetric particle, with a decay mode $\chi^{\pm} \to J\tau^{\mp}$.

4. Baryogenesis and spontaneous $R$-breaking.

As pointed out in the Introduction, one of the major reasons for interest in breaking $R$-parity spontaneously instead of explicitly is the possibility of avoiding the tough constraint imposed by the survival of the cosmic baryon asymmetry $\Delta B$ [9]. To make this point clear, let us first review why baryogenesis so severely constrains explicit $R$-breaking.

The absence of $R$-symmetry in the initial superpotential implies that either $L$- or $B$- number is explicitly violated (in principle, both these two numbers could be violated, but, in that case, an unbearably fast proton decay would occur). If the $L$- or $B$-violating interactions, which derive from these $R$-violating terms in the superpotential, are in thermal equilibrium at temperatures for which the
\((B + L)\)-violating anomalous electroweak interactions are operative, any possible preexisting \(\Delta B\) will be erased. Requiring the \(R\)-violating interactions to be constantly out of equilibrium implies very stringent upper bounds on their couplings in the superpotential of the order of \(10^{-7} - 10^{-8}\). Clearly, if this is the case, \(R\)-violating effects are completely invisible in accelerator physics experiments.

Two possible loopholes have been proposed in the literature: \(i\) \(R\)-breaking in the leptonic sector leaves some partial lepton number unbroken \(\left( L_{\text{unb}} \right)\), so that \(\left( \frac{1}{3}B-L_{\text{unb}} \right)\) replaces the usual \(B-L\) number \([12]\); \(ii\) some new mechanism for a late \(\Delta B\) production at the Fermi scale is operative. In this latter case, one can try to make use of an interesting interplay between \(R\)-violating interactions in the leptonic sector, giving rise to a net \(\Delta L\) at the weak scale, and the still operative anomalous interactions, converting this \(\Delta L\) into a net \(\Delta B\) \([11]\).

If \(R\)-parity is an exact symmetry to start with, and only subsequently broken in a spontaneous way, then we can envisage what we consider the most attractive resolution of the conflict between \(R\)-breaking and baryogenesis. Namely, \(R\) could remain a good symmetry throughout the whole interval between the Planck and Fermi scales, exhibiting a spontaneous breaking at temperatures so low that sphalerons are no longer operative. If \(R\) is broken in such a way in the leptonic sector, there is obviously no harm at all for any preexisting \(\Delta B\). The crucial question to answer is then: is it possible to achieve a spontaneous breaking of \(L\) at a temperature below that at which the electroweak phase transition occurs? We will show that this is possible at least for a range of the \(\lambda\) parameters of our model.

First we include the one-loop finite temperature corrections into the potential of eq. (3) \([16]\). In both cases \(a\) and \(b\), see Sect. 2, only one out of the three fields \(N_i\), henceforth \(N\), acquires a VEV, so that we reduce our analysis along the \(N\) and \(\Phi\) directions.

In the limit \(h^\nu \to 0\) and in terms of \(N_R = \text{Re} N/\sqrt{2}, \phi_R = \text{Re} \Phi/\sqrt{2}\), the finite temperature contribution to the effective potential is

\[
V_T^{N\Phi} (N_R, \phi_R) = \frac{1}{2} \mu_N^2(T) N_R^2 + \frac{1}{2} \mu_\Phi^2(T) \phi_R^2 - \frac{T}{12 \pi} \sum_{iB} m_i^3 \\
+ \sum_{iF} N_i \frac{m_i^4 (N_R, \phi_R)}{64 \pi^2} \ln \left[ \frac{m_i^2 (N_R, \phi_R)}{A_f T^2} \right] \\
- \sum_{iB} N_i \frac{m_i^4 (N_R, \phi_R)}{64 \pi^2} \ln \left[ \frac{m_i^2 (N_R, \phi_R)}{A_b T^2} \right],
\]

where

\[
\mu_N^2(T) = m_N^2 + \lambda^2 T^2, \\
\mu_\Phi^2(T) = m_\Phi^2 + \frac{1}{2} \lambda^2 T^2,
\]

are the \(N\) and \(\Phi\) plasma masses; \(m_i\)'s are the mass eigenvalues of any particle with \(N_i\) degrees of freedom, \(A_f = \pi^2 \exp(3/2 - 2\gamma_E)\), \(A_b = 16 A_f\), and \(\gamma_E \simeq 0.57\).
Note that we are assuming $\mathcal{CP}$-conservation so that $\text{Im}\langle \Phi \rangle = \text{Im}\langle N \rangle = 0$ at all the temperatures.

The above expression is valid for all the temperatures $T$ larger than any $m_i$, provided that $N_R$ and $\phi_R$ are in thermal equilibrium. The interactions coming from the terms $h^\nu L H \tilde{N}$ and $\lambda N \tilde{N} \Phi$ in eq. (1) keep these fields in equilibrium as long as their rate $\Gamma$ is larger than the expansion rate of the Universe $H \simeq g_*(T)T^2/M_P$, where $g_*(T)$ is the number of the relativistic degrees of freedom present at temperature $T$ and $M_P$ is the Planck mass. This requirement is fulfilled for $T \lesssim h^2 M_P$ and $T \lesssim \lambda^2 M_P$, which, for reasonable values of $h$ and $\lambda$, are certainly satisfied for all the temperature range of interest.

At very high temperature, $V_{N\Phi}^T(N_R, \phi_R)$ is approximated by

$$V_{N\Phi}^T(N_R, \phi_R) \simeq \frac{1}{2} \mu_N^2(T)N_R^2 + \frac{1}{2} \mu_{\phi}^2(T)\phi_R^2,$$

so that the minimum is at $\langle N_R \rangle = \langle \phi_R \rangle = 0$, and $L$ is preserved.

Let us now consider the range $\lambda = \mathcal{O}(1)$. As we have previously seen, in the case a) of Sect. 2, we found numerical solutions for $\lambda_3 \gtrsim 0.1$ with $m_N^2$ driven to negative values at the weak scale, as one can immediately deduce from the RGE’s, see eqs. (16-19). In such a case, the critical temperature $T_C$ may be defined as the temperature at which the effective potential develops a flat direction at the origin. From eq. (34) one can infer that

$$T_C^2 \simeq -\frac{m_N^2}{\lambda^2},$$

and we conclude that in this case the phase transition related to $L$-breaking can occur after the electroweak phase transition, since $m_N^2$ can be driven to sufficiently small values at the weak scale. Indeed, the $L$-violation can occur at temperatures low enough for the sphaleron transition to be completely out of equilibrium. Moreover, if $T_C$ is small enough, the minimum of the effective potential for $T \approx T_C$ may be already formed near the vacuum at $T = 0$. In fact, if $T_C \ll m_i$, the finite temperature corrections to the effective potential become negligible around the vacuum at $T = 0$ due to an exponential Boltzmann suppression [17].

In this situation the transition is expected to be of first order. However, one might still wonder whether the reheating temperature at the end of the phase transition is high enough to reestablish the dangerous equilibrium coexistence between anomalous electroweak interactions and $L$-violating phenomena. We proceed to estimate the reheating temperature $T_{RH}$. In the limit of vanishing $m_N^2$, eq. (7) can be exactly solved and the energy at the minimum is given by

$$V_{N\Phi}^{\min} = -\frac{1}{64} \frac{A^4}{\lambda^2} f(k),$$

where

$$f(k) = \frac{1}{2} - 10k - 4k^2 + \frac{1}{2} (1 + 8k)^{3/2},$$

and

$$T_{RH} \simeq \frac{m_i}{A^2}.$$
and $k \equiv (m_\Phi^2/A^2)$. Eqs. (8) and (10) require $0 \leq k \leq 1$, and therefore $f(k)$ varies between 1 and 0.

Comparing eq. (37) with

$$|V_{N\Phi}^{\text{min}}| = \frac{\pi^2}{30} g_*(T) T_{RH}^4,$$

we find

$$T_{RH} \simeq 0.4 g_*^{-1/4}(T) \frac{A}{\sqrt{\lambda}} f(k)^{1/4}.$$  

From the above expression we estimate that the reheating temperature can be lower than the critical temperature of the electroweak phase transition in the whole range for $\lambda$ under consideration, i.e. $\lambda \gtrsim 0.1$, and $k$ close to 1. For instance, for $A \simeq 1 \text{ TeV}$, $T_{RH} \lesssim 100 \text{ GeV}$ for $\lambda \gtrsim 0.1$ and $k \gtrsim 0.8$. Hence, for $\lambda$ large enough, we can avoid a sizeable reheating and the preexisting $\Delta B$ safely survives.

We are left with the other region in the parameter space where $\lambda \ll 1$. To discuss this case it is useful to parametrize the potential in eq. (33) in such a way which makes evidence of the dependence of the terms on $\lambda$. This is readily achieved by defining the following dimensionless quantities

$$\tilde{\phi}_R \equiv \lambda \frac{\phi_R}{m}, \quad \tilde{N}_R \equiv \lambda \frac{N_R}{m}, \quad \tilde{T} \equiv \lambda \frac{T}{m},$$

and

$$\tilde{m}_N^2 \equiv \lambda^2 \frac{m_N^2}{m^2}, \quad \tilde{m}_\Phi^2 \equiv \lambda^2 \frac{m_\Phi^2}{m^2}, \quad \tilde{m}_i^2 \equiv \lambda^2 \frac{m_i^2}{m^2}.$$  

Now the effective potential becomes

$$V_{N\Phi}\left(\tilde{N}_R, \tilde{\phi}_R\right) = \tilde{m}_i^4 \left[ F\left(\tilde{N}_R, \tilde{\phi}_R, \tilde{m}_N^2, \tilde{m}_\Phi^2, \tilde{T}\right) - \frac{\lambda}{12\pi} \tilde{T} \sum_i \tilde{m}_i^3 + O\left(\lambda^2\right) \right],$$

where $F$ can be easily deduced from eq. (33).

Having $\lambda \ll 1$, we can neglect terms of higher order in $\lambda$ and in this case the phase transition related to the spontaneous breaking of $L$-number is likely to occur at a temperature higher than that characteristic of the electroweak phase transition. Indeed, from eq. (45), one can infer that the dimensionless critical temperature has to be $O(1)$, that is the critical temperature is expected to be $O(\tilde{m}/\lambda)$. Hence, there exists an interval during which $L$ and $(B + L)$ are simultaneously violated. We can envisage two strategies for a net $\Delta B$ to be present at the end of the electroweak transition:

i) the $L$-violating interactions which arise from the spontaneous breaking of $R$ are never in equilibrium throughout the entire time sphalerons are still active;

ii) the presence of $L$-violation allows the production of a net $\Delta B$ via sphaleron interactions.
Option i) leads to a situation similar to that described at the beginning of this Section for the case of explicit $R$-breaking. Obviously, also in this case it might happen that some partial lepton number remains unbroken and then $B$ minus this number remains a good symmetry of the theory. In the case ii), one can envisage the possibility of producing particles with $L$-violating decays via bubble collisions at the $L$-breaking phase transition. The distributions of particles originated in this way are automatically out of thermal equilibrium. As for $\mathcal{CP}$-violation, the other crucial ingredient to obtain a net $\Delta L$, the situation is more promising than in the Standard Model, since there are additional couplings (the $h\nu$’s) which can be complex and whose phases are not severely limited by the existing phenomenological constraints on $\mathcal{CP}$-violation. Provided that these $L$-violating decays are fast enough, we can invoke processes induced by sphalerons to convert $\Delta L$ into $\Delta B$. One can work out an explicit example making use of the production and subsequent decay of the lightest ”right-handed neutrino” $N$ to give rise to a net $\Delta L$ along the lines of an analogous situation studied in ref. 11. The main difficulty for these scenarios is not so much the creation of a sizeable $\Delta L$, but rather the threat of washing out such $\Delta L$ by those same $L$-violating interactions which gave rise to it. The point is that in the case $\lambda \ll 1$ the $L$-breaking phase transition takes place at a temperature of $\mathcal{O}(\tilde{m}/\lambda)$, high enough to produce all the usual particles of the Standard Model through bubble collisions. In particular, at this temperature processes like $\nu\nu \rightarrow hh$ occur. It is easy to see that, if some $h\nu$ couplings are large enough to give rise to a sizeable $\Delta L$ in the $N$ decays, then some neutrino annihilations into Higgs pairs (or the inverse process) are fast enough to be in equilibrium, hence possibly erasing the $\Delta L$. In some contrived situations it is possible to avoid (at least) a large washing out of $\Delta L$, but it is likely that in the case $\lambda \ll 1$ it is quite difficult to preserve the dynamical generation of $\Delta L$ which resulted from the $L$-breaking phase transition. For such a reason we believe that the case of $\lambda = \mathcal{O}(1)$ is much more appealing as far as the problem of preserving a net $\Delta B$ is concerned, since the limits on the coupling constants are either absent or much less severe than those present in models with explicit $R$-parity breaking, unless new mechanisms are called into play, see refs. 10, 11 and 12.
5. Conclusions.

The survival of the cosmic matter-antimatter asymmetry imposes severe constraints on the $B$- and $L$-violating terms which are otherwise allowed by the gauge symmetry and global $N = 1$ supersymmetry in low energy supersymmetric extensions of the Standard Model. Imposing ordinary $R$-parity seems to be the safest solution for this problem. However, in view of the large ignorance surrounding the original theory from which low energy supersymmetry arises, we think that it is worthy to study how alternatives with explicit or spontaneous violations of $R$-parity address the problem of the $\Delta B$ survival. For models with Baryon Parity [18], i.e. a discrete symmetry which preserves $B$ while allowing for $L$-violation in the superpotential, the best way to avoid the washing out of $\Delta B$ seems to be the conservation of a residual lepton-flavour number [12]. Alternatively, one can try to produce a new $B$ asymmetry at the electroweak phase transition through an articulated interplay of $L$- and $(B + L)$-violating interactions [11]. However this late dynamical production of $\Delta B$ depends on the bubble dynamics which is still a rather open subject.

In this paper we have focused on the survival of $\Delta B$ in models where the initial supergravity Lagrangian possesses $R$-symmetry, which is subsequently broken in a spontaneous way at a temperature of the order of the Fermi energy scale. We have related the spontaneous breaking of $R$-parity to that of the lepton number $L$. After LEP results on the $Z^0$ lineshape the only available possibility seems to be through the VEV of a scalar which is singlet under $SU(2)_L \otimes U(1)_Y$. Hence, the simplest extension of the MSSM which allow for such a breaking of $L$ includes some new singlet superfield carrying $L$. We have studied here the possibility of supersymmetrizing the original singlet majoron model of Chikashige, Mohapatra and Peccei [3]. We have showed that it is possible to achieve a radiative breaking of $R$ for a wide range of parameters. This class of models with radiative breaking of $L$ exhibits a crucial feature for the problem of the survival of the cosmic $\Delta B$: we can find initial conditions allowing for a very late radiative violation of $L$ at temperatures such that sphaleron induced processes are no longer operative. On the other hand, some choices of these initial conditions allow for effects of broken $R$-parity potentially accessible in accelerator experiments.

In conclusion, the claim that an observable $R$-parity violation implies a complete washing out of the cosmic $\Delta B$ finds an interesting way out: it is possible to construct classes of supersymmetric theories where $R$-parity remains a good symmetry throughout all the history of the early Universe down to temperatures of $O(M_W)$, being subsequently broken in a spontaneous way by radiative effects when the anomalous $(B+L)$-violating effects are out of equilibrium. The concrete, phenomenologically viable illustration that we provide here should stimulate further effort in the direction of a cosmologically acceptable $R$-violation with visible effects at collider physics.
After accomplishing our work, we received the paper "Phase Transition for $R$-parity Breaking" by M. Chaichian and R. Shabad in which the same point of a late spontaneous $R$ breaking is illustrated in a different supersymmetric realization.

Acknowledgements

We thank F. Feruglio for many useful discussions and for participating in some stages of this work. Two of us (M.P. and A.R.) would like to thank the Laboratori Nazionali del Gran Sasso, where the present work was completed, for the kind hospitality. K. Enqvist is also acknowledged for useful discussions.
Appendix.

In this Appendix we wish to discuss in more detail the minimization of the effective potential in the case \( \lambda_1 = \lambda_2 = \lambda_3 \equiv \lambda \), such that \( A_1 = A_2 = A_3 \equiv A \).

Making use of cartesian coordinates

\[
\begin{align*}
N_i &= (X_i + iY_i)/\sqrt{2}, \\
\Phi &= (F + iG)/\sqrt{2},
\end{align*}
\]

the effective potential, see eq. (3), reads

\[
V^\text{tree}_{N\Phi} = \lambda^2 (F^2 + G^2)(\vec{X}^2 + \vec{Y}^2) + \frac{\lambda^2}{4} \left[ (\vec{X}^2 - \vec{Y}^2)^2 + 4(\vec{X} \cdot \vec{Y})^2 \right] \\
&+ \frac{m_N^2}{2}(\vec{X}^2 + \vec{Y}^2) + \frac{m_\Phi^2}{2}(F^2 + G^2) \\
&- \frac{\lambda}{\sqrt{2}} A \left[ (\vec{X}^2 - \vec{Y}^2)F - 2G\vec{X} \cdot \vec{Y} \right].
\]

(A.1)

Since \( V^\text{tree}_{N\Phi} \) is now invariant under an \( O(3) \otimes U(1)_L \) symmetry, the possible vacua lay on \( O(3) \otimes U(1)_L \) orbits. In particular, by an appropriate \( O(3) \) rotation, it is not restrictive to consider the following vacuum

\[
F \neq 0, \quad G \neq 0,
\]

\[
\begin{pmatrix}
0 & 0 \\
X_2 & 0 \\
X_3 & Y_3
\end{pmatrix}.
\]

(A.2)

The stationarity conditions read

\[
\begin{align*}
\partial_{X_2} V^\text{tree}_{N\Phi} &= \left[ 2\lambda^2 (F^2 + G^2) + \lambda^2 (\vec{X}^2 - \vec{Y}^2) + m_N^2 - \sqrt{2}\lambda A F \right] X_2 = 0, \\
\partial_{X_3} V^\text{tree}_{N\Phi} &= \left[ 2\lambda^2 (F^2 + G^2) + \lambda^2 (\vec{X}^2 - \vec{Y}^2) + m_N^2 - \sqrt{2}\lambda A F \right] X_3 + 2\lambda^2 (\vec{X} \cdot \vec{Y})Y_3 \\
&+ \sqrt{2}\lambda A G Y_3 = 0, \\
\partial_{Y_3} V^\text{tree}_{N\Phi} &= \left[ 2\lambda^2 (F^2 + G^2) - \lambda^2 (\vec{X}^2 - \vec{Y}^2) + m_N^2 + \sqrt{2}\lambda A F \right] Y_3 + 2\lambda^2 (\vec{X} \cdot \vec{Y})X_3 \\
&+ \sqrt{2}\lambda A G Y_3 = 0, \\
\partial_F V^\text{tree}_{N\Phi} &= \left[ 2\lambda^2 (\vec{X}^2 + \vec{Y}^2) + m_\Phi^2 \right] F - \frac{\lambda A}{\sqrt{2}} (\vec{X}^2 - \vec{Y}^2) = 0, \\
\partial_G V^\text{tree}_{N\Phi} &= \left[ 2\lambda^2 (\vec{X}^2 + \vec{Y}^2) + m_\Phi^2 \right] G + \sqrt{2}\lambda A (\vec{X} \cdot \vec{Y}) = 0.
\end{align*}
\]
We discuss separately the two cases $X_2 \neq 0$ and $X_2 = 0$.

\(i\) $X_2 \neq 0$: we can set $G = 0$ by a $U(1)_L$ rotation and from the stationarity conditions we get

\[(\tilde{X} \cdot \tilde{Y}) Y_3 = 0.\]

(A.5)

If $Y_3 \neq 0$, then $X_3 = 0$ and from the eqs. (A.4) we get

\[F^2 = \left(\frac{-m_N^2}{2\lambda^2}\right),\]

(A.6)

from which it is seen that $m_N^2$ must be negative. Similarly, stationarity conditions give

\[\frac{A}{\lambda^2} \sqrt{-m_N^2} = \tilde{X}^2 - \tilde{Y}^2 \leq \tilde{X}^2 + \tilde{Y}^2 = \frac{A^2 - m_\Phi^2}{2\lambda^2}.\]

(A.7)

The above expressions implies the hierarchy

\[m_N^2 \leq m_\Phi^2 < A^2.\]

(A.8)

Correspondingly, the value of $V_{tree}^{N\Phi}$ at the minimum is

\[V_{tree}^{N\Phi} = \frac{1}{2\lambda^2} m_N^2 \frac{A^2 - m_\Phi^2}{2} < 0.\]

(A.9)

However, this minimum breaking $R$-parity is forbidden by RGE’s, see eq. (22), which give $m_\Phi^2 < m_N^2$.

On the other hand, if $X_2 \neq 0$ and $Y_3 = 0$, using the $O(3)$ symmetry, we can take all the fields, except $F$ and $X_3$, to be vanishing. In this case the original $O(3) \otimes U(1)_L$ breaks to $O(2)$, giving rise to one Majoron and two familons. The minimization of the effective potential is similar to that discussed in the text in sect. 2.

\(ii\) $X_2 = 0$: using a $U(1)_L$ rotation, one can choose $Y_3 = 0$ ($F, G \neq 0$). Stationarity conditions imply

\[\sqrt{2}\lambda A G X_3 = 0,\]
which gives $G = 0$ for $X_3 \neq 0$ (the opposite case $X_3 = 0$ and $G \neq 0$ does not give a minimum). The analysis of such a case reduces to the previously analyzed case i) with $Y_3 = 0$.

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**Figure Captions**
1) Allowed region in the \((\log_{10} \lambda, A)\) parameter space for the breaking of \(R\)-parity (denoted by dots), in the case \(\lambda_1 = \lambda_2 = 0, \lambda_3 = \lambda\). The region above the continuous line is forbidden by the \(|A| < 5\tilde{m}\) condition imposed at the GUT scale. The regions below the dotted and the dashed lines correspond to positive values for \(m_{\Phi}^2\) and \(m_{N^c}^2\), respectively. \(A\) is given in units of \(\tilde{m}\).

2) Same as in fig. 1), but for \(\lambda_1 = \lambda_2 = \lambda_3 = \lambda\).