On the thermalization of a Luttinger liquid after a sequence of sudden interaction quenches

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Abstract – We present a comprehensive analysis of the relaxation dynamics of a Luttinger liquid subject to a sequence of sudden interaction quenches. The critical exponent $\beta$ governing the decay of the steady-state propagator is expressed as an explicit functional of the switching protocol. At long distances $\beta$ depends only on the initial state while at short distances it is also history dependent. Continuous protocols of arbitrary complexity can be realized with infinitely long sequences. For quenches of finite duration we prove that there exists no protocol to bring the initial non-interacting system in the ground state of the Luttinger liquid, albeit thermalization occurs at short distances. The adiabatic theorem is then investigated with ramp switchings of increasing duration and several analytic results for both the propagator and the excitation energy are derived.

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Introduction. – The on-going experimental activities on ultracold atoms are continuously challenging our understanding of many-body quantum systems [1]. Laser- or evaporative-cooled below the $\mu$K, atoms crystallize in artificial lattices [2] thus providing nearly ideal realizations of bosonic [3–6] and fermionic [7–9] model Hamiltonians. The possibility of tuning the model parameters in real time brought the attention back to a fundamental issue in quantum-statistical physics: does a non-interacting bulk system relax toward the correlated ground state upon the switch-on of the interaction?

“Sudden quench” is the nomenclature coined for the sudden change of a parameter like, e.g., the convexity of a parabolic trap or the interaction strength, in an equilibrium system [10,11]. During the last five years experimental and theoretical investigations on the relaxation properties of quenched ultracold atoms enlightened the intriguing phenomenon of the thermalization breakdown [12]: either the system does not reach a steady state or, if it does, the steady state is not the ground state of the quenched Hamiltonian. A sufficient criterion for the occurrence of a steady state has been found, so far, only for integrable models [13–16] and it has been argued that steady-state values are calculable by averaging over a generalized, initial-state–dependent Gibbs ensemble [17]. The thermalization breakdown poses questions which are certainly conceptual in nature but may also be relevant to the growing field of optimal control theory [18–20]: what is the steady-state dependence on the initial state? and on the switching protocol? how the adiabatic limit is recovered? In this paper we provide a comprehensive analysis of the behavior of a Luttinger Liquid (LL) subject to arbitrary interaction quenches. We extend the study of Cazalilla for a sudden quench [21] to a sequence of $N$ sudden quenches using a recently proposed recursive method [22]. Continuous quenches of duration $T$ are then obtained in the limit $N \rightarrow \infty$ and allow us to address the adiabatic limit by making $T$ larger and larger. We calculate the equal-time one-particle propagator $G^{[N]}(x,t)$ as well as the excitation energy. At long and short distances the steady state $G^{[N]}(x,t) \sim x^{-\beta}$ and in both cases we are able to write the critical exponent $\beta$ as an explicit functional of the switching protocol. In the limit of continuous quenches ($N \rightarrow \infty$) the propagator at short distances thermalizes whereas at long distances it does not. An analytic formula for ramp-like switchings of duration $T$ valid for all $x$ is derived and is shown that $G^{[\infty]}$ and the ground-state propagator are the same up to

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a critical distance that diverges for $T \to \infty$. The recovery of the adiabatic limit is further illustrated from energy balance considerations. The calculation of the excitation energy $\Delta E(T)$ is reduced to the solution of a simple differential equation that can be used to find the optimal switching protocol of duration $T$ that minimizes $\Delta E(T)$. We prove that $\Delta E(T) > 0$ for all switching protocols of finite duration and provide an analytic expression for ramp-like switchings.

**Sequential quench in a Luttinger Liquid.** – The sudden interaction quench in a LL has been addressed in a series of papers [21,23–28]. At the distance $x$ the propagator $\mathcal{G}^{(1)}(x,t)$ exhibits the “light-cone” effect, i.e., a crossover between Fermi liquid behavior for times $t \ll x/2v$ (v being the quasiparticle velocity) and non-thermal LL behavior in the long-time limit. Sequential quenches yield an even richer phenomenology since additional time (and hence length) scales appear in the problem. We will show that different steady-state regimes emerge by probing the system at distances shorter or longer than the quenching time (in units of $v$) and that their nature depends on the switching protocol.

The LL Hamiltonian describes interacting spinless electrons confined in a 1D wire of length $L$ and reads

$$H = \frac{1}{2} \sum_{\alpha = \text{L,R}} \int_{-\frac{L}{2}}^{\frac{L}{2}} dx \left[ -2i e \alpha v_F \psi_{\alpha}^\dagger (x) \partial_x \psi_{\alpha} (x) 

+ g_\alpha \psi_{\alpha}^2 (x) + 2 \rho_\alpha (x) \rho_\alpha (x), \right] $$

(1)

where $\alpha$ denotes the chirality of the electrons with Fermi velocity $e \alpha v_F$ ($\alpha = \pm 1$) and density $\rho_\alpha \equiv \psi_{\alpha}^\dagger \psi_{\alpha}$; “,” being the normal ordering. The coupling constants $g_{2(4)}$ refer to forward scattering processes between electrons of opposite (identical) chirality. We consider the system non-interacting ($g \equiv g = 0$) and in the ground state before a series of $N$ interaction quenches at times $0 \equiv t_0 < t_1, \ldots, < t_{N-1} \equiv T$ takes place. Let $g_{2n}$ be the value of the $n$th coupling $g_{2n}$ between $t_{n-1}$ and $t_n$. The corresponding LL Hamiltonian. Each $H_n$ can be bosonized [29] in terms of the scalar fields $\phi$, defined from $\psi_{\alpha} (x) = \frac{\sqrt{2\pi} e^{i\phi(x)}}{\sqrt{2\pi}}$, with $\alpha$ the anticommuting Klein factors and $a$ a short-distance cutoff. The result is a simple quadratic form

$$H_n = \frac{v_n}{2} \int_{-\infty}^{\infty} dx [K_{\phi}^{-1}(\partial_x \phi(x))^2 + K_{\theta}(\partial_x \theta(x))^2],$$

(2)

where $\phi = \phi_R + i \phi_L$ and $\theta = \phi_R - i \phi_L$ are conjugated fields, $v_n = \sqrt{2\pi e \alpha v_F + g_{4n}}^2 - g_{2n}^2}^{1/2}$ is the renormalized velocity and the parameter $K_n = \sqrt{2\pi e \alpha v_F + g_{4n}^2}/\sqrt{2\pi e \alpha v_F + g_{2n}^2}$ measures the interaction strength. Note that $0 < K_n \leq 1$ for repulsive interactions; $K_n = 1$ corresponds to non-interacting systems while small values of $K_n$ indicate a strongly correlated regime. In our case $K_0 = 1$ but no complications arise from arbitrary values of $K_0$, which is therefore left unspecified. As we shall see, such freedom permits to address general initial-state dependences.

**Excitation energy.** The excitation energy $\Delta E(t)$ is defined as the difference between the energy of the LL and the ground-state energy of the LL Hamiltonian at the same time $t$. Since we are interested in $\Delta E(T)$ after the interaction quench is completed we consider $t > t_{N-1}$ and write

$$\Delta E(t) = \langle \Psi_0 (t) | H_N | \Psi_0 (t) \rangle - \langle \Psi_N | H_N | \Psi_N \rangle, \quad (3)$$

where $|\Psi_0 (t)\rangle = e^{-iH_L(t-t_{N-1})}|\Psi_0 \rangle$ and here and in the following $|\Psi_n \rangle$ is the ground state of $H_n$. To calculate $\Delta E(T)$ we expand the scalar fields in $H_n$ as

$$\phi (x) = \sum_{q > 0} \frac{\sqrt{2\pi} e^{i\phi_q}}{\sqrt{2\pi}} [b_q e^{-iqx - \epsilon_q} + b_q^* e^{iqx + \epsilon_{q}^*}] + \sqrt{\pi} N_\alpha, \quad n_\alpha = \# \text{number of electrons with chirality } \alpha.$$

Then, the Hamiltonian takes the diagonal form

$$H_n = \sum_{q > 0} \sum_\alpha v_n q b_{aqn}^\dagger b_{aqn} + E_n,$$

(4)

with $E_n$ the zero point energy and $b_{aqn}^\dagger$ the annihilation (creation) operators for the elementary excitations of $H_n$ with chirality $\alpha$ and momentum $q$. These operators are related to the non-interacting $b_{aqn}^\dagger$ of the $\phi_q$ expansion via the Bogoliubov transformations $b_{aqn}^\dagger = b_{aqn}^\dagger \cosh \varphi_q + i b_{aqn}^\dagger \sin \varphi_q$, with $\varphi_q = \frac{i}{2} \sum_n \tanh^{-1}[(1 - K_q^2)/(1 + K_q^2)]$. Following the bosonization the average over the ground state $|\Psi_n \rangle$ is converted into an average over the vacuum of the $b_{aqn}$-excitations and hence $\Delta E(T)$ takes the form

$$\Delta E(T) = \sum_{q > 0} v q n_q (t),$$

(5)

where $n_q (t)$ is the average of the number operator $\sum_\alpha b_{aqn}^\dagger b_{aqn}$ over $|\Psi_0 (t)\rangle$. The relation between two consecutive boson operators are easily found and read

$$b_{aqn+1}^\dagger = b_{aqn} \cosh \Delta \varphi_n + b_{aqn}^\dagger \sinh \Delta \varphi_n,$$

(6)

where $\Delta \varphi_n = \varphi_{n+1} - \varphi_n$. After a cascade of transformations (6) to express the $b_{aqn}$ in terms of $b_{aq0}$ we obtain $n_q (t) \equiv n_{q0}$, as the solution of the recursive system of equations

$$P_{q,n} = (c_q^2 + s_q^2) P_{q,n+1} + 2 c_q s_q R e [Q_{q,n+1} e^{-i\epsilon_q \Delta t_n}],$$

$$Q_{q,n} = 2 c_q R_s P_{q,n+1} + c_q^2 R_s Q_{q,n+1} e^{-i\epsilon_q \Delta t_n} + s_q^2 R_s Q_{q,n+1}^* e^{i\epsilon_q \Delta t_n},$$

(7)

$$\rho_{q,n} = \rho_{q,n+1} + 2 c_q^2 R_s P_{q,n+1} + 2 s_q R_s Q_{q,n+1} e^{-i\epsilon_q \Delta t_n},$$

where $\Delta t_n = t_{n+1} - t_n$, $t_N \equiv t$, $c_q = \cosh \Delta \varphi_n$, $s_q = \sin \Delta \varphi_n$, $\epsilon_q = 2 v q$ and boundary conditions $P_{q,n} = Q_{q,n} = R_s = N_0 = 0$. In eq. (7) we assumed that $v_n = v$ for all $n$; the general recursive scheme is simply obtained by replacing $v_n \to v_{n+1}$. We observe that the function $Q_{q,0}$ obtained from the above system is also useful to evaluate the squared energy fluctuations

$$\sigma^2 (t) = \langle \Psi_0 (t) | H_N^2 | \Psi_0 (t) \rangle - \langle \Psi_0 (t) | H_N | \Psi_0 (t) \rangle^2 = \sum_{q > 0} 4 (v q^2 | Q_{q,0} |^2).$$

(8)
such quantity violates the generalized Gibbs-ensemble hypothesis. For a single quench our result for $\sigma^2$ agrees with ref. [24]. However, for $N$ quenches the entanglement between modes of opposite chirality is more complex since $Q_{q,0}$ carries memory not only of the initial state, but also of the entire switching process.

In fig. 1 we plot $n_q(t)$ and $|Q_{q,0}|^2$ as a function of $q$ (for $t > t_{N-1} = T$ there is no dependence on $t$ since the switching protocol finishes at $T$) for a series of $N = 4, 6, 7$ and $\infty$ quenches at times $t_n = nT/(N-1)$ with $K_n = 1 - 0.35n/N$ and $T = 1$. Momentum $q$ is in units of $1/a$ and time in units of $a/v$.

We then observe an analytic result for $N \to \infty$ and show that both $n_q$ and $Q_{q,0}$ vanish exponentially at large $q$. As $n_q$ and $Q_{q,0}$ are not identically zero the LL does not thermalize. Below we show how the thermalization breakdown is reflected on the equal-time one-particle propagator.

The equal-time propagator. The equal-time one-particle propagator is defined as

$$G^{[N]}(x,t) = \langle \Psi_0(t)|\psi_R(x)\psi_R^+(0)|\Psi_0(t)\rangle,$$

where the superscript $N$ specifies the number of sudden quenches of the switching protocol. To calculate $G^{[N]}$ we express the fermion fields in terms of the boson fields, expand the latter in elementary excitations and exploit the transformations (6) between two consecutive $b$ operators. We then obtain

$$G^{[N]}(x,t) = \frac{1}{2\pi a^2} \sum_{q>0} e^{-iqt} e^{\frac{1}{2} \left[ \sum_{n=1}^{N} (2\pi q x - |F_{n,q}|^2 - |F_{q,0}|^2) \right]},$$

where the functions $F_{n,q,0}(x,t)$ are solutions of the recursive relations

$$F_{n,q,n+1} = F_{q,n+1}e^{-i\varphi_{n+1}}c_n - F_{q,n+1}e^{i\varphi_{n+1}}c_n,$$

with boundary conditions $F_{R_{N,N}} = (e^{i\varphi} - 1)\cosh\varphi_N$, $F_{L_{4,N}} = (e^{-i\varphi} - 1)\sinh\varphi_N$. Like in eq. (7) the general recursive scheme is obtain by replacing $vt_n \rightarrow v_n t_n$.

The full analytic expression of $G^{[N]}$ grows in complexity with increasing $N$. To illustrate the typical features of $G^{[N]}$ in fig. 2 we report the contour plot of $\ln|G^{[N]}(x,t)|$ for $N = 3$ as a function of time and distance. The parameters are $K_0 = 1$, $K_1 = 0.1$, $K_2 = 0.5$, $K_3 = 0.9$ and $T_0 = 0$, $T_1 = 5$, $T_2 = 10$. Time is in units of $10a/v$ and distance is in units of $10a$.
an out-of-equilibrium LL subject to a sequence of interaction quenches [22]. Note that $\beta_{sd}$ only depends on the interaction parameters $K_n$ and not on the switching times $t_n$ (see footnote 1). Equation (12) returns the well-known exponent of a LL in the ground state for $K_0 = K_1 \ldots = K_N \equiv K$ since $\beta_{sd} = (K + 1/K)/2 - 1$, and the exponent of a LL after a single interaction quench for $K_0 = K_1 = \ldots = K_n = 1$ and $K_{n+1} = \ldots = K_N = K$ since $\beta_{sd} = (1 + K^2)^2/(4K^2) - 1$, in agreement with ref. [21].

The situation is radically different at long distances. In this limit the dependence on $x$ of the steady-state propagator is again a power law

$$\lim_{x \to \infty} \lim_{t \to \infty} G^{[N]}(x,t) \approx \frac{i}{2\pi(x + ia)} \left[ \frac{R[t_n, K_n]}{x} \right]^{\beta_{sd}[K_n]},$$

but $R[t_n, K_n]$ is a length depending on all intermediate switching times $t_n$ and interactions $K_n$. As for the exponent $\beta_{sd}$ a striking cancellation of the intermediate $K_n$ occurs and we find

$$\beta_{sd}[K_n] = \frac{(K_0^2 + K_N^2)(1 + K_N^2)}{4K_0K_N^2} - 1,$$

which depends only on the initial and final states.

To summarize, the steady-state propagator does not thermalize for discontinuous switchings, and the thermalization breakdown manifests in different ways at short and long distances. In the next section we address the evolution of this behavior when the sequential protocol approaches a continuous protocol.

Continuous quenches. – Continuous switching protocols are obtained as a limiting case of a sequential quench. Let us start by analyzing again the excitation energy $\Delta E(t)$ and squared energy fluctuations $\sigma^2(t)$. Taking the quenching times $t_n = nt/(N-1)$ equally spaced and letting $N \to \infty$ the variable $t_n$ becomes a continuous variable between 0 and $t$ and we can construct the differentiable functions $P_q(s)$, $Q_q(s)$ and $\varphi(s)$ according to $P_q(t_n) = P_{q,n}$, $Q_q(t_n) = Q_{q,n}$ and $\varphi(t_n) = \varphi_n$. Expanding eqs. (7) to first order in $\Delta t = t/(N-1)$ and $\Delta \varphi_n$ we find a coupled system of differential equations

$$\frac{dP_q}{ds} = -2Re[Q_q] \frac{d\varphi}{ds},$$

$$\frac{dQ_q}{ds} = -2P_q \frac{d\varphi}{ds} + 2ivqQ_q,$$

that should be solved with boundary conditions $P_q(t) = 1$ and $Q_q(t) = 0$. The average occupation number $n_q(t)$ is then simply given by $1 - P_q(0)$. The most popular continuous protocol is the ramp switching [31] $\varphi(s) = \varphi_0/t$ with $0 < s < t$. In this case the system (15) can be solved exactly and we find

$$n_q(t) = 1 - \cos(2\varphi_0\sqrt{q^2 - 1})/q^2 - 1,$$

with $\varphi = qvt/\varphi_0$. Similarly we obtain for the energy fluctuations $|Q_q(0)|^2 = 2\sin(\varphi_0\sqrt{q^2 - 1})(1 + \cos(2\varphi_0\sqrt{q^2 - 1}) - 2q^2(q^2 - 1)^{-1})$. The above quantities correctly approach zero for $t \to \infty$ (adiabatic limit), whereas for any finite $t$ are exponentially suppressed at large $q$, see the curve $N \to \infty$ in fig. 1. We also checked that inserting eq. (16) into eq. (5) and taking the long-time limit $t \to \infty$ and the weak-coupling limit $\varphi \ll qa/t$ the excitation energy vanishes as

$$\lim_{t \to \infty} \Delta E(t) \sim \frac{\ln t}{t^2}, \quad \varphi \ll qa/t,$$

in agreement with ref. [28]. It is interesting to observe that the dependence of $\Delta E \sim \delta^4/n$ in $\delta$ on the ramp rate $\delta = \varphi_0/t$ is not a simple power law $\delta^n$ and, therefore, does not belong to the non-analytic regimes contemplated in ref. [32]. On the contrary the asymptotic expansion of the energy fluctuations at weak coupling yields $\lim_{t \to \infty} \sigma^2(t) \sim L^2/t^2$.

The system of differential equations (15) is exact (non-perturbative) and we now exploit it to prove that there exist no switching protocol of finite duration capable to drive the initial non-interacting system in the interacting ground state of a LL. Since $\Delta E(t)$ is the sum of non-negative $n_q$’s it is sufficient to show that $n_0(t) = 1 - P_0(0)$ cannot be zero. The optimal switching protocol $\varphi(s)$ that reproduces a target $P_0(s)$ follows from eqs. (15) with $q = 0$; in this case $\varphi(s)$ depends only on the instantaneous value of $P_0(s)$, and reads

$$\varphi(s) = \varphi - \frac{1}{2} \cosh^{-1} P_0(s),$$

where $\varphi$ is the value of the correlation angle at the end of the quench. Thus, $P_0(0) = 1$ only provided that the initial and final strength of the interaction is the same, $\varphi(0) = \varphi$. In particular $n_0(t) > 0$ for all switching
protocols that connect an initial interaction strength \( K_0 \) to a final interaction strength \( K \neq K_0 \).

Analytic results can be obtained for the one-particle propagator as well. Let us start by analyzing the exponents \( \beta \) of the long- and short-distance power-law behavior of \( G^{[N]} \). For a continuous switching we can construct the differentiable function \( K(s) \) according to \( K(t_n) = K_n \), with \( K(0) = K_0 \) and \( K(t) = K \). Approximating \( K(t_{n-1}) = K(t_n - \frac{2}{N}) \approx K(t_n) - \frac{2}{N} K'(t_n) \) and taking the logarithm of eq. (12) we find

\[
\ln \left[ \frac{2K_0(\beta_{sd} + 1)}{1 + K^2} \right] = \lim_{N \to \infty} \sum_{n=1}^{N} \log \left[ 1 - \frac{T}{N} K'(t_n) \right] = - \int_{0}^{t} ds \frac{K'(s)}{K(s)} = \ln \frac{K_0}{K},
\]

and hence

\[
\beta_{sd}[K(s)] = \frac{1}{2} \left( K + \frac{1}{K} \right) - 1,
\]

which coincides with the exponent of a ground-state LL. Therefore, at short distances the initial-state dependence as well as the history dependence are washed out by differentiable switchings and the propagator thermalizes. Discontinuous switchings\(^2\) do, instead, introduce a history dependence through the ratio between the values of \( K(s) \) across the discontinuity. To the contrary the long-distance exponent \( \beta_{sd} \) in eq. (14) is always history independent and the steady-state propagator does never thermalize. Our conclusions agree with recent perturbative results by Dóra et al. in ref. [28].

To study the crossover from short to long distances we must calculate the steady-state propagator at all \( \ell \). For continuous switching the recursive system of eq. (10) reduces to a system of differential equations

\[
\frac{d}{ds} F_{aq} = i v q F_{aq} + F_{aq} \frac{d\varphi}{ds},
\]

that should be solved with boundary conditions \( F_{Rd}(t) = (e^{iqx} - 1) \cos \varphi(t) \) and \( F_{Ld}(t) = (e^{-iqx} - 1) \sin \varphi(t) \). The equal-time propagator \( G^{[\infty]}(x,t) \) can then be calculated from eq. (9) with \( F_{aq,0} = F_{aq}(0) \). In the special case of a ramp protocol \( \varphi(s) = \bar{\varphi} s/T \) for \( s < T \) and \( \varphi(s) = \bar{\varphi} \) for \( T < s < t \) the system of equations (20) can be solved analytically and we get

\[
F_{Rd}(0) = \alpha_{q} (1 - e^{iqx}) \sinh \gamma_q \left[ e^{iq(T-t)} \bar{\varphi} \sinh \bar{\varphi} + e^{-iq(T-t)} (i v q T - \gamma_q e^{2\gamma_q} + 1) \cosh \bar{\varphi} \right],
\]

where \( \gamma_q = \sqrt{\bar{\varphi}^2 - (vqT)^2} \); the function \( F_{Ld}^* (0) \) is obtained from \( F_{Rd}(0) \) simply by exchanging \( \sinh \bar{\varphi} \leftrightarrow \cosh \bar{\varphi} \) and by replacing \( v \to -v \). In fig. 4 we plot the steady-state propagator \( G^{[\infty]} \) as a function of the distance \( x \) for two different ramp switching times and \( \bar{\varphi} = 0.46 \), which corresponds to the LL parameter \( K = 0.4 \). The results clearly agree with the scenario outlined above: at distances smaller than the characteristic length \( 2\bar{\varphi} \) the steady-state propagator behaves thermally while at large distances it behaves like the sudden-quench (non-thermal) propagator of ref. [21], albeit shifted upwards by a history-dependent constant (see also ref. [28]).

The long-to-short distance crossover is particularly transparent in the weak coupling limit \( \bar{\varphi} \ll 1 \). In this case we can expand \( F_{aq,0} \) to second order in \( \bar{\varphi} \), perform the sum over \( q \) and find the steady-state propagator \( G^{[\infty]} \) at all \( x \)

\[
\lim_{t \to \infty} G^{[\infty]}(x,t) \approx \frac{i}{2\pi(x + ia)} \frac{a^2}{x^2 - (2vT)^2} \frac{1}{x + 2vT} \left[ \frac{2vT}{x} \right]^{2\bar{\varphi}^2},
\]

with power-law exponents depending on \( x \). Equation (22) reproduces with remarkable accuracy the long-to-short distance crossover, which in the same approximation read

\[
\lim_{x \to 0} \lim_{t \to \infty} G^{[\infty]}(x,t) \approx \frac{i}{2\pi(x + ia)} \frac{a}{x} \left[ \frac{2\bar{\varphi}^2}{x} \right],
\]

\[
\lim_{x \to \infty} \lim_{t \to \infty} G^{[\infty]}(x,t) \approx \frac{i}{2\pi(x + ia)} \frac{R_0}{x} \left[ \frac{2\bar{\varphi}^2}{x} \right],
\]

\( R = \sqrt{2vT a} \) being a history-dependent length (cf. eq. (13)). The high accuracy stems from the fact that the correlation angle \( \varphi \) remains small also in the intermediate coupling regime characterized by \( g_2 \approx g_4 \approx v_F \) for which

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\( \varphi \approx 0.1 \). Finally we observe that from fig. 4 and eq. (22) the adiabatic limit is recovered when the switching time \( T \to \infty \). As \( T \) increases the steady-state propagator equals the equilibrium propagator up to larger and larger values of \( x \).

**Conclusions.** – We provided a comprehensive analysis of the relaxation dynamics of a LL after the quenching of the electron-electron interaction for different switching protocols. The bosonization method is combined with a recursive procedure to address arbitrary sequences of sudden quenches and hence, as a limiting case, continuous protocols. The approach allows us to evaluate the excitation energy \( \Delta E \) and the equal-time one-particle propagator \( G \). We found that for a sequence of sudden quenches \( \Delta E \) is always larger than zero. The thermalization breakdown has a dramatic impact on the propagator both at finite times and at the steady state. In particular the steady-state \( G \) exhibits a power-law behavior \( |x|^{-\beta} \) with different exponents at long and short distances. Remarkably we are able to express \( \beta \) as explicit functionals of the switching protocol. We found that at long distances \( \beta = \beta_{ad} \) carries informations on the initial state but not on the history of the switching protocol, and coincides with the exponent of a sudden quench \([21]\). At short distances \( \beta = \beta_{ad} \) is, instead, both initial-state and history dependent: it is only for continuous protocols that memory is washed out and \( \beta_{ad} \) equals the exponent of a LL in equilibrium.

The continuous limit of the recursive procedure leads to a simple system of differential equations that can be solved numerically for the excitation energy \( \Delta E \) we proved that there exist no switching protocols of finite duration capable to bring a system from a non-interacting ground state to the ground state of a LL. It is only in the adiabatic limit that thermalization occurs. These results may be relevant to design the optimal switching protocol that minimizes the total excitation energy or the energy \( a \) of a given \( q \)-excitation. For the propagator \( G \) we clarify how the adiabatic limit is attained by studying ramp switching protocols of increasing duration \( T \). At weak coupling we derive an explicit expression for \( G(x,t \to \infty) \) that is valid for all \( x \) and around the crossover distance \( x \approx 2\sqrt{t} \) is dominated by a power law \( |x - 2\sqrt{t}|^{-\beta(x)} \) with a \( x \)-dependent exponent. The formula further reproduces with high accuracy the different power laws at long and short distances. Our predictions could be experimentally confirmed by measurements in ultracold fermionic atoms loaded in optical lattices, recently proposed \([33,34]\) as candidates to realize highly controllable and tunable LLs.

**REFERENCES**

[1] Kinoshita T., Wenger T. and Weiss D. S., *Nature*, 440 (2006) 900.
[2] Bloch I., Dalibard J. and Zwerger W., *Rev. Mod. Phys.*, 80 (2008) 885.
[3] Greiner M., Mandel O., Esslinger T., Hänsch T. W. and Bloch I., *Nature*, 415 (2002) 39.
[4] Stoferle T., Moritz H., Schori C., Köhl M. and Esslinger T., *Phys. Rev. Lett.*, 92 (2004) 130403.
[5] Winkler K., Thalhammer G., Lany F., Grimm R., Hecker Denschlag J., Daley A. J., Kantian A., Büchler H. P. and Zoller P., *Nature*, 441 (2006) 853.
[6] Spielman I. B., Phillips W. D. and Porto J. V., *Phys. Rev. Lett.*, 98 (2007) 080404.
[7] Köhl M., Moritz H., Stoferle T., Günter K. and Esslinger T., *Phys. Rev. Lett.*, 94 (2005) 080403.
[8] Jördens R., Strohmaier N., Günter K., Moritz H. and Esslinger T., *Nature*, 455 (2008) 204.
[9] Schneider U., Hackermüller L., Will S., Best T., Bloch I., Costi T. A., Helmes R. W., Rasch D. and Rosch A., *Science*, 322 (2008) 1520.
[10] Kollath C., Läuchli A. M. and Altman E., *Phys. Rev. Lett.*, 98 (2007) 180601.
[11] Mannmana S. R., Wessel S., Noack R. M. and Muramatsu A., *Phys. Rev. Lett.*, 98 (2007) 210405.
[12] Rigol M., *Phys. Rev. Lett.*, 103 (2009) 100403.
[13] Barthele T. and Schollwöck U., *Phys. Rev. Lett.*, 100 (2008) 100601.
[14] Stefenucci G. and Almbladh C.-O., *Phys. Rev. B*, 69 (2004) 195318; *Europhys. Lett.*, 67 (2004) 14.
[15] Stefenucci G., *Phys. Rev. B*, 75 (2007) 195115.
[16] Conneau H. D. et al., *Ann. Inst. Henri Poincaré*, 10 (2009) 61.
[17] Rigol M., Dunjko V., Yurovsky V. and Olshanii M., *Phys. Rev. Lett.*, 98 (2007) 050405.
[18] Werschnik J. and Gross E. K. U., *J. Phys. B: At. Mol. Opt. Phys.*, 40 (2007) R175.
[19] Brif C., Chakrabarti R. and Raditz H., *New J. Phys.*, 12 (2010) 075008.
[20] Rahmani A. and Chamon C., arXiv:1011.3061v2.
[21] Cazalilla M. A., *Phys. Rev. Lett.*, 97 (2006) 156403.
[22] Perfetto E., Stefenucci G. and Cini M., *Phys. Rev. Lett.*, 105 (2010) 156802.
[23] Perfetto E., *Phys. Rev. B.*, 74 (2006) 205123.
[24] Iucci A. and Cazalilla M. A., *Phys. Rev. A.*, 80 (2009) 063619.
[25] Uhrig G. S., *Phys. Rev. A*, 80 (2009) 061602(R).
[26] Zhou Zong-Li and Lou Ping, *Commun. Theor. Phys.*, 51 (1139) 2009.
[27] Zhou Zong-Li, Zhang Guo-Shiun and Lou Ping, *Chin. Phys. Lett.*, 27 (2010) 067305.
[28] Dóra B., Haque M. and Zaránd G., *Phys. Rev. Lett.*, 106 (2011) 156406.
[29] Giamarchi T., *Quantum Physics in One Dimension* (Clarendon, Oxford) 2004.
[30] Calabrese P. and Cardy J., *Phys. Rev. Lett.*, 96 (2006) 136001.
[31] Eckstein M. and Kollar M., *New J. Phys.*, 12 (2010) 055012; Hickichi T., Suzuki S. and Sengupta K., *Phys. Rev. B*, 82 (2010) 174305; Venumadhav T., Haque M. and Moessner R., *Phys. Rev. B*, 81 (2010) 054305.
[32] Polkovnikov A. and Gritsev V., *Nat. Phys.*, 4 (2008) 1177.
[33] Günter K., Stoferle T., Moritz H., Köhl M. and Esslinger T., *Phys. Rev. Lett.*, 95 (2005) 230401.
[34] Pedri P., De Palo S., Orignac E., Citro R. and Chiofalo M. L., *Phys. Rev. A*, 77 (2008) 015601.