We study current fluctuations in a Y-shaped conductor connected to external leads with finite impedances. We show that, due to voltage fluctuations in the circuit, the moments of the transferred charges cannot be obtained from simple rescaling of the bare values already in the second moments. The cross-correlation between the output terminals can change from negative to positive under certain parameter regimes.

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Feedback effects on the current correlations in Y-shaped conductors

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Current fluctuations in mesoscopic systems are of intense interest recently. Besides the fact that they inevitable exist and become more important in electric circuits when these are miniaturized, it is now well-appreciated that they contain interesting and fundamental physics (see, e.g., [1, 2, 3]). For example, Fermi statistics of the electrons are responsible for the reduction of the shot noise as compared with a corresponding system of classical or Bosonic particles. Furthermore, the same Fermi statistics of the charge carriers have important implications in multi-terminal setups. The quantity of interest in this case is the cross-correlation between different terminals [4, 5, 6]. Experiment of this type is the solid-state analogue of the Hanbury Brown-Twiss experiment in quantum optics [7]. It has been shown that generally Fermi statistics implies that the cross-correlations between the currents in two different output arms are negative [4, 5, 6]. This theoretical result assumes non-interacting electrons and is supported by experiments available so far [5].

The sign of this cross-correlation has triggered many investigations into the question as to under what circumstances it would be reversed. Several mechanisms have been proposed. A few invoke possible electronic ground states that are not normal-metallic (Landau-Fermi liquid) states, such as superconducting (e.g. [8]), quantum Hall, or Luttinger liquids states (e.g. [9]). Still another mechanism has been proposed relying on finite frequency and capacitive couplings [10], and one in ferromagnetic systems based on “bunching” of transferred electrons due to spin blockade [11].

Here we show that a feedback mechanism in the presence of external impedances can also lead to this sign change. We consider the system as shown in Fig. 1. The “sample” A is a Y-shaped conductor in which electrons propagate coherently. It can consist of tunnel barriers or diffusive conductors, and is connected to external resistors \(Z_a\), \(Z_b\) and \(Z_c\). Our considerations are also of relevance for practical measurements (c.f. [13]). In measurements of cross-correlations, one injects an incident beam of charge carriers (here from reservoir a) and then splits the beam into two parts using a “beam splitter”, such as the Y-shaped conductor here. One would like to measure the current correlation between two output terminals, here b and c. In most current measurements however, one needs to couple the sample to external measuring circuits. For example, here we are considering the case where the current measurements are actually made by voltage measurements across the impedances \(Z_b\) and \(Z_c\). If the external measuring circuit can be idealized as having zero impedance, then the voltage across the sample would be non-fluctuating and the current fluctuations are entirely due to intrinsic properties of the sample and the carriers. With finite external impedances, the voltage across the sample then becomes fluctuating and the current correlations will be modified.

Previously, feedback due to the presence of external impedances has been considered for two-terminal conductors [1], and more recently, in the context of third moment of the shot noise [12, 13, 14, 15, 16]. The results based on the Langevin formalism [11] concluded that the second moments of current fluctuations can be obtained from the corresponding zero-impedance (intrinsic or “bare”) values by a simple scaling. However, it was shown, using both a Keldysh technique [14, 15] and the Langevin formalism [12, 16], that this rescaling breaks down at the third moment. In this work we show that, for our three-terminal setup, even the second moment cannot be obtained from a rescaling of the corresponding bare value. For instance, the cross-correlation acquires contributions from auto-correlators. Since the bare auto-correlators are always positive, it is then possible to have positive cross-correlations in appropriate parameter regimes. The effect of external impedances on current fluctuations of multi-terminal circuits has also been considered by Büttiker and his collaborators [11] using the Langevin formalism. However, they considered a multiprobe measurement of a two-terminal conductor and thus not directly our geometry here.

We have performed the calculations using both the Langevin and Keldysh formalisms [17]. The results are identical. To illustrate the physics more clearly, let us first consider a simplified case where only \(Z_b \neq 0\) using the Langevin formalism. Let us first introduce some
short-hand notations. We denote the conductances of the three arms of our sample as \( G_a, G_b, G_c \). We shall define \( G \equiv G_a + G_b + G_c \) and also the dimensionless parameters \( \eta_a \equiv G_a / G \) etc (thus \( \eta_a + \eta_b + \eta_c = 1 \)). In the present situation, the potentials \( \phi_1 \) and \( \phi_3 \) at points 1 and 3 are given by the external potentials \( V_a \) and \( V_c \) respectively and do not fluctuate. However, the quantity \( \phi_2 \) is a fluctuating quantity. The current in an arm, say, is a linear combination of two contributions, one being linear in the bias potentials \( \phi \)'s and another due to the Langevin noise. Thus we have

\[
I_b = G \eta_b [\eta_a (V - \phi_2) - \eta_c \phi_2] + \delta I_2 .
\]

(1)
The first term follows easily from circuit theory. \( \delta I_2 \) is the Langevin noise whose expectation value is zero. We shall specify its variance later. Similarly for arm \( c \), we have

\[
I_c = G \eta_c [\eta_a V + \eta_b \phi_2] + \delta I_3 .
\]

(2)
The fluctuating potential \( \phi_2 \) is related to \( I_b \) by

\[
\phi_2 = I_b Z_b .
\]

(3)
(We are interested in the zero frequency limit so the current along arm \( A_b \) is equal to that through the resistor \( Z_b \).) By taking the expectation values of \( (1)-(3) \), we can obtain \( T_b, T_c \) and \( \bar{\phi}_2 \). In particular, we have

\[
\bar{\phi}_2 = \frac{1}{z_t} Z_b G \eta_a \eta_b V .
\]

(4)
Here \( z_t \equiv 1 + Z_b G \eta_b (\eta_a + \eta_c) \) is a dimensionless quantity. Subtracting the expectation values of Eqs. (1)-(3) from these equations themselves, we find, by eliminating \( \phi_2 - \bar{\phi}_2 \) in favor of \( I_b - T_b \),

\[
\Delta I_b \equiv I_b - T_b = \frac{1}{z_t} \delta I_2 ,
\]

(5)
\[
\Delta I_c \equiv I_c - T_c = \frac{1}{z_t} Z_b G \eta_b \eta_c \bar{\delta} I_2 + \delta I_3 .
\]

(6)
From these, we can readily obtain the fluctuations \( \langle \Delta I_b \Delta I_c \rangle \) etc. (We leave out the frequency variables for simplicity here. See below for more accurate notations). In particular,

\[
\langle \Delta I_b \Delta I_c \rangle = \frac{1}{z_t^2} Z_b G \eta_b \eta_c \langle \delta I_2 \delta I_2 \rangle + \frac{1}{z_t} \langle \delta I_2 \delta I_3 \rangle .
\]

(7)
This shows immediately that the cross-correlation has several contributions. Besides one which is a rescaling of the “bare” correlator \( \langle \delta I_2 \delta I_3 \rangle \), there is another contribution being proportional to \( \langle \delta I_2 \delta I_2 \rangle \). The origin of this latter term is obvious also from the above derivation, that is, the sample is “driven” by the potential \( \phi_2 \) which is itself fluctuating. To complete the calculation we need the expressions for \( \langle \delta I_2 \delta I_2 \rangle \) and \( \langle \delta I_2 \delta I_3 \rangle \). For this, we have to notice that the sample is now biased at voltages \( V \) at point 1, \( \phi_2 \) at point 2, and 0 at point 3. Let us define the bare (superscript \( 0 \)) correlators \( C_{bc}^{(0)} \) by the expression \( \langle \delta I_2 (\omega) \delta I_1 (\omega') \rangle = 2 \pi \delta (\omega + \omega') C_{bc}^{(0)} \) etc. In the shot noise (temperature \( T \to 0 \) regime, we expect these correlators to be linear combinations of contributions that are proportional to the average potential differences, i.e.,

\[
C_{bb}^{(0)} / e = s_{bb}^{(b)} (V - \bar{\phi}_2) + s_{bb}^{(c)} (V - 0) ,
\]

(8)
\[
C_{bc}^{(0)} / e = s_{bc}^{(b)} (V - \bar{\phi}_2) + s_{bc}^{(c)} (V - 0) .
\]

(9)
The values of the coefficients \( s_{bb}^{(b)} \) etc will be given below. Here the superscripts are denoted according to the potentials relative to \( a \) and the subscripts, the currents. Writing \( \langle \Delta I_b (\omega) \Delta I_c (\omega') \rangle = 2 \pi \delta (\omega + \omega') C_{bc} \), we can now obtain the “renormalized” correlator \( C_{bc} \) (which is also proportional to the correlators for transferred charges) from Eq. (7), using (11), (8) and (9):

\[
C_{bb} = \frac{e V}{z_t^2} \left\{ (P + S) \left[ P^2 s_{bb}^{(b)} + Q^2 s_{bc}^{(b)} + 2 PQ s_{bb}^{(b)} \right] + (Q + R) \left[ P^2 s_{bb}^{(c)} + Q^2 s_{cc}^{(c)} + 2 PQ s_{bb}^{(c)} \right] \right\} ,
\]

(11)
\[
C_{cc} = \frac{e V}{z_t^2} \left\{ (P + S) \left[ S^2 s_{bb}^{(b)} + R^2 s_{cc}^{(b)} + 2 SR s_{bb}^{(b)} \right] + (Q + R) \left[ S^2 s_{bb}^{(c)} + R^2 s_{cc}^{(c)} + 2 SR s_{bb}^{(c)} \right] \right\} ,
\]

(12)
\[
C_{bc} = \frac{e V}{z_t^2} \left\{ (P + S) \left[ PS s_{bb}^{(b)} + QR s_{cc}^{(b)} + (PR + QS) s_{bc}^{(b)} \right] + (Q + R) \left[ PS s_{bb}^{(c)} + QR s_{cc}^{(c)} + (PR + QS) s_{bc}^{(c)} \right] \right\} .
\]

(13)
In these equations

\[
\begin{align*}
z_t &\equiv \eta_a (1 + GZ_a \eta_b) (1 + GZ_b \eta_c) \\
&\quad + \eta_b (1 + GZ_c \eta_a) (1 + GZ_a \eta_c) \\
&\quad + \eta_c (1 + GZ_a \eta_b) (1 + GZ_b \eta_c)
\end{align*}
\]

(14)
stand for
\[
\begin{align*}
P &= 1 + Z_a G \eta_a \eta_c + Z_c G \eta_c (\eta_a + \eta_b), \\
Q &= -Z_a G \eta_a \eta_b + Z_c G \eta_c \eta_b, \\
R &= 1 + Z_a G \eta_a \eta_c + Z_b G \eta_b (\eta_a + \eta_c), \\
S &= -Z_c G \eta_a \eta_c + Z_b G \eta_b \eta_c.
\end{align*}
\]
(15)

The coefficients \(s_{bb}^{(b)}\) etc. in Eqs. (11)-(13) are the same as those entering Eqs. (5). Generalization of the intrinsic ("bare") correlation between arms \(\alpha\) and \(\beta\) in the shot noise regime is thus (c.f. Eqs. (5), (11))
\[
C_{\alpha\beta}^{(0)} = s_{\alpha\beta}^{(b)} (\phi_1 - \phi_2) + s_{\alpha\beta}^{(c)} (\phi_1 - \phi_3).
\]
(16)

These coefficients take different forms for tunnel junctions and for diffusive wires. For tunnel junctions, if \(\phi_1 \geq \phi_2 \geq \phi_3\), they are given by
\[
\begin{align*}
s_{bb}^{(b)} &= G \eta_b (\eta_a (1 - 2 \eta_a \eta_b) - \eta_c (1 - 2 \eta_a \eta_c)), \\
s_{bb}^{(c)} &= G \eta_b \eta_c (1 - 2 \eta_a \eta_c), \\
s_{cc}^{(b)} &= -G \eta_c (2 \eta_a \eta_c - 2 \eta_b \eta_c), \\
s_{cc}^{(c)} &= G \eta_c [\eta_a (1 - 2 \eta_a \eta_c) + \eta_b (1 - 2 \eta_a \eta_c - 2 \eta_b \eta_c)], \\
s_{bc}^{(b)} &= G \eta_b \eta_c (1 - 2 \eta_a \eta_c - 2 \eta_b \eta_c), \\
s_{bc}^{(c)} &= -G \eta_c \eta_b (1 - 2 \eta_a \eta_c - 2 \eta_b \eta_c).
\end{align*}
\]
(17)

In the case \(\phi_1 \geq \phi_3 \geq \phi_2\), the corresponding coefficients can be obtained from the above expressions by exchanging the indices \(b\) and \(c\). For example, \(s_{cc}^{(b)}\) can be obtained from the above expression for \(s_{bb}^{(c)}\) with all indices of its right hand members making the exchange \(b \leftrightarrow c\). For diffusive wires, if \(\phi_1 \geq \phi_2 \geq \phi_3\),
\[
\begin{align*}
s_{bb}^{(b)} &= G \frac{3}{\eta_c} (\eta_a - \eta_c), \\
s_{bb}^{(c)} &= G \frac{3}{3} \eta_c \eta_c (1 + 2 \eta_a), \\
s_{cc}^{(b)} &= G \frac{3}{\eta_c} \eta_c (2 \eta_a - 1), \\
s_{cc}^{(c)} &= G \frac{3}{3} \eta_c (\eta_a + \eta_b), \\
s_{bc}^{(b)} &= G \frac{3}{3} \eta_c \eta_c (1 - 2 \eta_a), \\
s_{bc}^{(c)} &= -G \frac{3}{3} \eta_c \eta_c.
\end{align*}
\]
(18)

Again, for \(\phi_1 \geq \phi_3 \geq \phi_2\), one can get the coefficients by exchanging \(b \leftrightarrow c\) in the formulas above. These coefficients have been calculated using generalization of the methods proposed by Nazarov (14). Some of these coefficients can also be deduced from the literature (e.g. (6), (19), and (20)).

Equations (11)-(13) are the main analytic results of this paper. The auto-correlators \(C_{bb}\) and \(C_{cc}\) can be shown to be positive definite (17). We shall concentrate on the cross-correlation \(C_{bc}\) for the rest of the paper.

For \(C_{bc}\), we can show that (17) it is always negative if \(Z_a \eta_a\) is larger than \(Z_b \eta_b\) and \(Z_c \eta_c\). Hence, we shall focus on the rest of the parameter space. We show the results for two particular examples, \(Z_a = 0\), \(Z_b = Z_c = 1/G\) in Fig. 2 and \(Z_a = 0\), \(Z_b = Z_c = 10/G\) in Fig. 3. We see that, for sufficiently large \(Z_b\) and \(Z_c\), it is indeed possible to have positive \(C_{bc}\). The positive region starts near small \(\eta_a\), and grows with increasing \(Z_b\) and \(Z_c\). Indeed, for \(Z_b\) and \(Z_c\) both \(\rightarrow \infty\), one can show that the cross-correlation actually becomes positive for any \(\eta\)'s. (This sign change is not confined to \(Z_a = 0\), for more examples, see (17).)

We can understand this behavior physically as follows. (For more quantitative statements, see (17).) When there is a positive fluctuation of the current through, say the arm \(b\), there is a corresponding increase in the potential at point 2 in Fig. 1. This voltage fluctuation in turn will lead to an extra current through the arm \(c\), thus giving a positive contribution to the cross-correlation \(C_{bc}\). This contribution will in particular be large for small \(\eta_a\), since most of this fluctuating current will flow through \(c\). We have a net positive \(C_{bc}\) if this contributions overwhelm the "bare" negative correlation contribution (see Eq. (13)). In particular, since \(s_{bc}^{(b)} + s_{bc}^{(c)} < 0\) is proportional to \(\eta_2^2\) for tunnel junctions whereas it is proportional to \(\eta_a\) for diffusive wires for small \(\eta_a\), it is therefore easier to get positive \(C_{bc}\) for tunnel junctions than for diffusive wires.

Our mechanism for sign change is distinct from that due to bunching (c.f. (12)). We have calculated also the Fano factors and found no bunching in the injected current (17).

In conclusion, we have shown that, for a multi-terminal conductor connected to external leads with finite impedances, the moments of the transferred charges cannot be obtained from simple rescaling of the bare moments. The cross-correlation between the output terminals can even become positive under certain parameter regimes.

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FIG. 1: Schematic for the circuit considered in this paper. The arms \( A_a, A_b, A_c \) of the Y-shaped conductor \( A \) are connected to external leads biased, respectively, at voltages \( V_a = V, V_b, \) and \( V_c \) \((V_b = V_c = 0 \) in this paper). The leads are assumed to have impedances \( Z_a, Z_b, \) and \( Z_c \), which are schematized as external resistors connected to the sample arms. The nodes 1, 2, 3 between the sample arms and the resistors are where voltage fluctuations set in.

FIG. 2: Plots for the cross-correlations of Y-shaped conductors with (a) tunnel junctions and (b) diffusive wires in the arms. Here the external impedances are \( Z_a = 0 \) and \( Z_b = Z_c = 1/G \). \( C_{bc} \) is in units of \( eGV \) and the thick lines over the surfaces mark the contour \( C_{bc} = 0 \).

FIG. 3: Same as Fig. 2 except that impedances are \( Z_a = 0 \) and \( Z_b = Z_c = 10/G \).