MULTIRESOLUTION SIGNAL PROCESSING OF FINANCIAL MARKET OBJECTS

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ABSTRACT

Financial markets are among the most complex entities in our environment, yet mainstream quantitative models operate at predetermined scale, rely on linear correlation measures, and struggle to recognize non-linear or causal structures. In this paper, we combine neural networks, known to capture non-linear associations, with a multiscale decomposition to facilitate a better understanding of financial market data structures. Quantization keeps decompositions calibrated to market. We illustrate our approach via seven use cases.

Index Terms— Finance, multiresolution, VQ-VAE

1. INTRODUCTION

Financial markets are prototypical examples of systems exhibiting multiple scales of behavior. Signal processing (SP) techniques have been adapted from engineering to finance in search of excess returns. Their main challenge lies in dealing with the high dimensionality of the data. Machine learning (ML) applied to finance uses pattern learning paradigms closely connected to traditional statistical and numerical approaches [1]. ML has its own challenges: financial data is highly non-stationary, noisy, and in most cases, insufficient given the high dimensionality of the space to which it belongs.

One way to reduce dimensionality is to consider the intrinsic structure present in the data. For instance, bond yields, swap rates, inflation, and foreign exchange (FX) rates can be thought of as having 1D term-structures (e.g., zero-coupon, spot, forward, or basis curves). Similarly, volatilities implied by option prices are organized as (hyper-)surfaces in 2D or higher. These structures carry information that can help reduce complexity. A typical spot swap curve could have as many as fifty tenors. Studying the corresponding time series means working in 50D space. A 10 × 10 volatility grid maps to 100D space. Knowing that these data lie, in fact, on n-manifolds where n is relatively small, greatly reduces the burden of the learning task. We refer to these market data structures as market objects and focus on learning market behaviors from the shape and dynamics of these representations.

We propose a multiresolution decomposition of market objects generated with a novel architecture (FinQ-VAE) consisting of a pipeline of variational autoencoders (VAE) with latent space quantizations guided by financially meaningful constraints, e.g., market liquidity or trading views. Figure 1 shows a learned multiresolution decomposition of the US swap curve of March 21, 2022.

Fig. 1. Multiresolution decomposition of the swap curve of March 21, 2022. The input curve (a) is decomposed from coarse to fine (b)-(f), and calibrated to anchors on each layer.

2. RELATED WORK

Financial data, i.e., time series of prices, are typically non-stationary [2]. Hence, most SP and ML approaches operate on returns, i.e., changes in price from one time stamp to the
next. A time series of returns is typically more “stable” and likely to pass stationarity tests [3]. However, financial data also suffers from low signal-to-noise ratios. Hence, any signal found is wiped out by differencing. Multiresolution methods can be traced all the way back to Fourier transforms and wavelets [4, 5, 6]. They balance signal preservation with stationarity: a base shape acts as a noise smoother, retaining an average signal level, and a hierarchy of residuals adds refinements to the base shape while exhibiting desirable statistical properties like stationarity.

Autoencoders with their variational and conditional flavors [7, 8, 9, 10] have been adopted in finance mostly for single-resolution latent learning and its applications [11, 12, 13, 14, 15, 16]. We extend these ideas in two significant ways: (a) we define a new architecture of cascading VAEs that learns hierarchical decompositions of market objects for a variety of applications and (b) we introduce a quantization step [17] that takes into account financial constraints ensuring calibration to market at every scale.

3. FINQ-VAE

3.1. Modeling Background

VAEs facilitate the learning of probabilistic generative models from a set of observations in $x \in \mathbb{R}^d$ presumed to lie on a manifold of dimension $r \leq d$. The loss function to be optimized during training is according to [18]:

$$
\mathcal{L}(\theta, \phi, \beta; x, z) = 
\mathbb{E}_{q_\phi(z|x)}[\log p_\theta(x|z)] - \beta D_{KL}(q_\phi(z|x)||p(z))
$$

With its roots in signal processing theory, quantization helps compress continuous sets of values to a countable or even finite number of options beyond which the variability in values makes little or no contribution to modeling. For example, images can be viewed as containing discrete objects along with a discrete set of qualifiers such as color, shape, texture. Vector-quantized VAEs (VQ-VAE) have been developed to support discrete learning [17, 19]. Market objects also contain redundant information. Moreover, like higher-level objects in images, they are, by definition, discrete collections of sub-objects with qualifiers such as steepness, curvature, skews, smiles, liquid and illiquid regions. Therefore, it makes sense to consider the benefits of quantizing their latent spaces. By default, however, VAE outputs are not faithful to specific data. In finance, it is important to calibrate models to market-observed data or to enforce desired constraints. Instead of using a full-fledged VQ-VAE, we employ a simpler, non-learned quantization process that snaps encoder outputs to optimal latent locations via optimization with constraints. These dynamically quantized points are passed to the decoder to produce calibrated reconstructions, which are used to compute residuals for the next layer. Consequently, our multiresolution decompositions are calibrated to market at each scale.

3.2. Problem Statement

Given a market object $O_{mkt}$, our goal is to learn a multiresolution decomposition such that:

1. It consists of a base shape object $O_{base}$ and a number $n+1$ of residual layers, $L_0, L_1, ..., L_n$ and objects $\delta_0, \delta_1, ..., \delta_n$ such that $O_{mkt} = O_{base} + \delta_0 + \delta_1 + ... + \delta_n$.

2. Anchor points are selected on each of the base and residual layers $L_0, L_1, ..., L_{n-1}$: $a_i^j$, where $i$ is the index of the anchor and $j$ is the layer index. The selection criteria reflect financial considerations: e.g., some points are more liquid or tradeable.

3. Each intermediate reconstruction with $j < n$ layers $O_j = O_{base} + \delta_0 + \delta_1 + ... + \delta_{j-1}$ is respectively calibrated to anchor points on $L_0, L_1, ..., L_{j-1}$. The residuals in the last layer $L_n$ are computed to recover the input market object exactly: $\delta_n = O_{mkt} - O_{n-1}$.

An example of user-defined anchors distributed according to market liquidity in the case of a swap curve might be: $\{2Y, 5Y, 10Y, 30Y\}$ as $L_0$ anchors, $\{7Y, 20Y\}$ as $L_1$, $\{3Y, 25Y\}$ as $L_2$, etc. This applies to market objects of varied dimensions with scattered constraints that need not be regularly spaced.

3.3. The FINQ-VAE Architecture

Our FINQ-VAE architecture is shown in Figure 2. The training inputs are market objects $O_{mkt}$ and anchors $A$ (Figure 1 (a)). Each layer is a VAE neural net, consisting of an Encoder, a Decoder, and a latent space that entails a quantization that maps encoded latent vectors into constraint-optimized vectors to be passed to the Decoder for reconstruction.

The base layer learns a coarse general shape (Figure 1 (b)). In this example, the $Encoder_{base}$ takes as input the full swap curve specified by a set of $m = 18$ swap tenors and a set of base anchor points, e.g., $A = \{2Y, 5Y, 10Y, 30Y\}$. The output of $Encoder_{base}$ is a latent vector $z_{base}$ in a latent space of fixed dimension. In this example we used a 3D latent space. Without quantization, $Decoder_{base}$ maps the latent vector $z_{base}$ into a base output curve $O_{base}$, i.e., a reconstruction representing the “learned” global shape.

This representation can be likened to a PCA reconstruction using the same number of principal components. Unlike a PCA output (also shown in Figure 1 (b)), the VAE-generated curves are better fitted to anchors by design. We modify the VAE loss function to include an anchor calibration term:

$$
\mathcal{L}_{recon}(x, Decoder(z)) = 
||x - Decoder(z)||_2^2 + \alpha ||x|_A - Decoder(z)|_A||_2^2
$$

where $\alpha \geq 0$ and the anchors $A$ serve as constraints.

While the reconstructed base curve fits the overall shape, we would like the anchor points to be fitted even better. We quantize the latent vector $z_{base}$ to $z_{base}^{\delta}$ via optimization:
\[ z^q_j = \arg\min_{z} \| x_j - \text{Decoder}_j(z) \|_2 \]  

where \( j \) is the layer index. The base curve \( O^q_{\text{base}} \) reconstructed from the quantized vector is shown in Figure 1 (b). Figure 3 shows curves with the same base shape before and after quantization.

The first residual layer takes as input the difference object \( \delta_0 = O_{\text{mkt}} - O^q_{\text{base}} \) and another VAE is trained to learn the \( \delta_0 \) residuals. The resulting quantized residual \( \delta^q_0 \) is applied to the base object \( O^q_{\text{base}} \) to produce our \( L_0 \) market object reconstruction: \( O_0 = O^q_{\text{base}} + \delta^q_0 \), see Figure 1 (c).

The second residual layer takes as input the difference object \( \delta_1 = O_{\text{mkt}} - O_0 \) and the process is repeated. Figures 1 (d)-(e) show the reconstructions on layers 1 and 2. The final residuals \( \delta_3 = O_{\text{mkt}} - O_2 \) are computed to recover the input \( O_{\text{mkt}} \) exactly (Figure 1 (f)).

4. APPLICATIONS AND RESULTS

In this section we present seven use cases. Objects reconstructed with FinQ-VAE are not only plausible as in [20], but also calibrated to market. The choice of anchors at different scales is fully configurable. We trained two FinQ models: one on daily USD spot swap curves and one on bond curves [21] between Jan 2001 and the end of 2019. The test data period is Jan 2020 to Jul 2022. Using the magnitude of the \( L_0 \) residuals as a measure of the quality of fit between the learned base curves and the actual market data around the most liquid points, we note that the model appears robust over the test data, starting in 2020. This includes the COVID-19 pandemic, the inflationary period that follows, and the start of the rate hike cycle by the US Fed, as evident in Figure 4 (top left).

Hierarchical Factor Analysis

In contrast to PCA, which typically operates on returns and ignores market levels, our multiresolution decomposition learns a global shape level on its base layer and residuals at various scales. The variational feature of the model ensures smooth navigation through latent space. The quantization feature ensures that outputs are calibrated to user-specified important points. The latter is a powerful property, as traditionally, VAEs have only been used in finance for generating plausible, uncalibrated data which could be very different from market-observed values. The hierarchy of latent spaces induces a hierarchy of factors (Figure 6) enabling more granular analyses of market object dynamics.

Scenario Generation

Standard practice in financial risk management is to shock market objects either in absolute or percentage terms. There are two main considerations: the shape of a scenario and its overall size. PCA-based techniques are appealing because of their simplicity, however proper calibration of scenario
size is challenging and global dependence on non-intuitively weighted linear combinations of points is difficult to interpret.

Our multiresolution framework splits responsibilities: base shapes account for market levels and are under the control of a few key drivers that are easier to intuit by human experts. Their views define anchor points and desired stresses. Dependencies that are more difficult to synthesize through human experience are generated algorithmically (Figure 5).

Fig. 5. Full curve scenarios conditional on user moves.

**Synthetic Data Generation**

Synthetic market objects are composable in hierarchical fashion. This can be done artificially by sampling the latent spaces of the VAEs from coarse to fine, or by using historical or trader-specified moves of anchor points for the lower layers in the hierarchy and randomly sampling the higher ones. Figure 6 (a)-(c) illustrate synthetic samples on several layers.

**Nowcasting**

Nowcasting is the “forecasting” of the present or the near future in the absence of complete information about the current state of the market. It has two main components: an understanding of what is already priced in the market and a view on future conditions. We allow for such views to be incorporated. For example, option portfolios require full implied volatility surfaces to price, yet option prices may not be available for all \((\text{expiry}, \text{underlier})\) pairs. The most liquid points can be used as anchors, while missing values can be sampled from the latent distributions. In the examples of Figure 5, scenario curves are reconstructed from the known current move of a single point (driver) and previous residuals. Note that our approach does not require training with conditional labels.

**Residuals as Signals**

Residual time series could be used as signals for systematic strategies. \(A_0\) anchor points should have residuals that are close to zero on \(L_0, L_1, \cdots\). If this is not the case, their deviation from zero may be used as a signal that the shapes being reconstructed are difficult to fit to the constraints. Such difficulties are harbingers of unusual market conditions.

**Relative Value Analysis**

We applied FinQ to learning US Treasury bond curves. While bonds and interest rate swaps capture similar macroeconomic developments, swap curves tend to be smoother. Hence, swap curve reconstructions could be used to identify viable swap spread trades. Figure 4 (c) indicates that buying 30Y bonds and selling the same maturity swaps might be a good strategy.

**Outlier Detection**

The variational aspect of autoencoders ensures relatively compact clusters of latent encodings. Samples that stand out from their clusters are likely to be outliers. Figure 6 (d) singles out such a point corresponding to May 6, 2010; upon closer inspection, we notice that on this date the 20Y value seems stale, causing a non-smooth curve compared the learned shapes (the “flash crash” occurred intraday, which may have contributed to noise in the end-of-day data).

**5. CONCLUSIONS**

FinQ-VAE is a novel architecture for multiresolution signal processing of market objects. Market-calibrated representations are learned using a layered approach. User-specified constraints can be incorporated at different scales to generate quantized embeddings that lead to calibrated reconstructions of market objects. To our knowledge, this is the first time multiresolution analysis is combined with quantized VAEs and applied to financial modeling in a way that accommodates constraints such as trading views and liquidity.
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