Induced cosmology on a regularized brane in six-dimensional flux compactification

Eleftherios Papantonopoulos\textsuperscript{a,∗}, Antonios Papazoglou \textsuperscript{b,c,**} and Vassilios Zamarias\textsuperscript{a,***}

\textsuperscript{a} Department of Physics, National Technical University of Athens, Zografou Campus GR 157 73, Athens, Greece. 
\textsuperscript{b} APC\textsuperscript{1}, 10 rue Alice Domon et Léonie Duquet, 75205 Paris Cedex 13, France. 
\textsuperscript{c} GReCO/IAP\textsuperscript{2}, 98 bis Boulevard Arago, 75014 Paris, France.

Abstract

We consider a six-dimensional Einstein-Maxwell system compactified in an axisymmetric two-dimensional space with one capped regularized conical brane of codimension one. We study the cosmological evolution which is induced on the regularized brane as it moves in between known static bulk and cap solutions. Looking at the resulting Friedmann equation, we see that the brane cosmology at high energies is dominated by a five-dimensional $\rho^2$ energy density term. At low energies, we obtain a Friedmann equation with a term linear to the energy density with, however, negative coefficient in the small four-brane radius limit (i.e., with negative effective Newton’s constant). We discuss ways out of this problem.

∗ e-mail address: lpapa@central.ntua.gr  
** e-mail address: papazogl@iap.fr  
*** e-mail address: zamarias@central.ntua.gr

\textsuperscript{1}UMR 7164(CNRS, Université Paris 7, CEA, Observatoire de Paris) 
\textsuperscript{2}UMR 7095(CNRS, Université Paris 6)
1 Introduction

Six-dimensional brane world models are rather interesting to be studied for a number of reasons. Firstly, it was in the framework of brane theories with two extra transverse dimensions that the large (sub-millimeter) extra dimensions proposal for the resolution of the electroweak hierarchy problem was provided [1]. This scenario makes the study of extra dimensional theories, and string theory in particular, relevant to low energy phenomenology (colliders as well as astrophysical and cosmological observations) and is testable in the very near future. The different possibilities of realizing a brane world model in six dimensions must therefore be studied to allow for a comparison with experiment.

Quite a different motivation has been the proposal to ameliorate the cosmological constant problem, using codimension-2 branes (for a recent review on the subject see [2]). These branes have the interesting property that their vacuum energy instead of curving their world-volume, just introduces a deficit angle in the local geometry [3]. Models with this property which exhibit no fine-tuning between the brane and bulk quantities have been known as self-tuning (for early attempts to find similar models in five dimensions see [4]). Such self-tuning models with flux compactification [5, 6] have been extensively looked, but the flux quantization condition always introduces a fine tuning [7], unless one allows for singularities more severe than conical [8]. Alternative sigma-model compactifications have been shown to satisfy the self-tuning requirements [9]. However, the successful resolution of the cosmological constant problem would also require that there are no fine-tuning between bulk parameters themselves. No such self-tuning model has been found yet with all these properties.

A further motivation in studying such models with codimension-2 branes is that gravity on them is purely understood. The introduction of matter (i.e., anything different from vacuum energy) on them, immediately introduces malicious non-conical singularities [10]. A way out of this problem is to complicate the gravity dynamics by adding a Gauss-Bonnet term in the bulk or an induced curvature term on the brane, in which case the singularity structure of the theory is altered and non-trivial matter is allowed [11]. However, the components of the energy-momentum tensor of the brane and the bulk are tuned artificially and the brane matter is rather restricted [12]. Alternatively, one can regularize the codimension-2 branes by introducing some thickness and then consider matter on them [13]. For example, one can mimic the brane by a six-dimensional vortex (as e.g., in [14]), a procedure which becomes a rather difficult task if matter is added on it.

Another way of regularization was proposed recently, which consists merely of the reduction of codimensionality of the brane. In this approach, the bulk around the codimension-2 brane is cut close to the conical tip and it is replaced by a codimension-1 brane which is capped by appropriate bulk sections [15] (see [16] for a similar regularization of cosmic strings in flat spacetime). This regularization has been applied to flux compactification systems in six dimensions for unwarped “rugby-ball”-like solutions in [15], for warped solutions with conical branes (with or without supersymmetry) in [17] and for even more general warped solutions allowing non-conical branes in [18]. Specific brane energy-momentum tensor is required to build static solutions, involving a brane field with Goldstone-like dy-
namics. It is interesting to note that in the non-supersymmetric case, only quantized warpings are allowed [17].

In the present paper we will try to go a step further and consider an isotropic cosmological fluid on the above-mentioned regularized branes. To have a cosmological evolution on the regularized branes, the brane world-volume should be expanding and in general the bulk space should also evolve in time. Instead of tackling this problem in its full generality, which seems a formidable task, in the present work we will consider the motion of the regularized codimension-1 brane in the space between the bulk and the brane-cap which remains static (see e.g., [19]). In this way, a cosmological evolution will be induced on the brane in a similar way as in the mirage cosmology [20], but with the inclusion of the back-reaction of the brane energy density (i.e., the brane is not considered merely a probe one). Since in the mirage cosmology, the four-dimensional scale factor descends from the warp factor in the four-dimensional part of the bulk metric, we will discuss the regularized brane in the case of warped bulk [17], rather than unwarped bulk [15]. It is worth noting that the above procedure provided in five dimensions the most general isotropic brane cosmological solutions [21].

Solving the Israel junction conditions, which play the rôle of the equations of motion of the codimension-1 brane, we find the Friedmann equation on the brane which at early times is dominated by an energy density term proportional to $\rho^2$, like in the Rundall-Sundrum model in five dimensions. To recover four-dimensional cosmology at late times, we split as usual the energy-momentum tensor to a part which is the contribution of the static vacuum brane and to a part of additional matter. We find a regime which has a four-dimensional dependence on the energy density. However, in the interesting case where the brane moves close to its equilibrium point, which in turn is close to the would-be conical singularity, the coefficient of the linear to the energy density term is negative (i.e., we obtain negative effective Newton’s constant). Thus, we cannot recover the standard cosmology at late times. This seems to be the consequence of considering the bulk sections static. It is possible, that this behaviour is due to a ghost mode appearing among the perturbations of the system, after imposing the staticity of the bulk sections. Furthermore, the above result points out that there is a difference between the six-dimensional brane cosmology in comparison to the five-dimensional one. The study of brane cosmology in Einstein gravity in five dimensions, can be made either in a gauge where the bulk is time-dependent and the brane lies at a fixed position, or in a gauge where the bulk is static and the brane movement into the bulk induces a cosmological evolution on it [22]. This, however, does not seem to hold in six-dimensions anymore.

The paper is organized as follows. In Sec. 2 we review the static regularized brane solution in a bulk of general warping. In Sec. 3 we derive the equations of motion of the codimension-1 brane and in Sec. 4 we study the induced cosmological evolution on the moving brane. Finally in Sec. 5 we draw our conclusions.
2 Setup and static brane solutions

Before discussing the time-dependent scenario, let us remind ourselves of the static solution which we will use in the following for the brane motion. The bulk theory that we will use is a six-dimensional Einstein-Maxwell system which in the presence of a positive cosmological constant and a gauge flux, spontaneously compactifies the internal space [23]. The known axisymmetric solutions have in general two codimension-2 singularities at the poles of a deformed sphere [24]. We will study the case where only one (e.g., the upper) codimension-2 brane is regularized by the introduction of a ring-like brane at $r = r_c$ with an appropriate cap. The dynamics of the system is given by the following action

$$S = \int d^6x \sqrt{-g} \left( \frac{M^4}{2} R - \Lambda_i - \frac{1}{4} F^2 \right) - \int d^5x \sqrt{-\gamma_c} \left( \lambda + \frac{v^2}{2} (\tilde{D}_\mu \sigma)^2 \right) - \int d^4x \sqrt{-\gamma} - T ,$$

where $M$ is the six-dimensional fundamental Planck mass, $\Lambda_i$ are the bulk ($i = 0$) and cap ($i = c$) cosmological constants, $F_{MN}$ the gauge field strength, $T$ the tension of the lower codimension-2 brane, $\lambda$ the 4-brane tension, $\sigma$ the 4-brane Goldstone scalar field necessary for the regularization and $v$ the vev of the Higgs field from which the Goldstone field originates. For the coupling between the Goldstone field and the bulk gauge field we use the notation $\tilde{D}_\mu \sigma = \partial_\mu \sigma - e a_\mu$, with $a_\mu = A_M \partial_\mu X^M$ the pullback of the gauge field on the ring-like brane and $e$ its coupling to the scalar field. In the above action we omitted the Gibbons-Hawking term. The configuration is shown in more detail in Fig.1.

The solution for the bulk and cap regions depends on a parameter $\alpha$ which is a measure of the warping of the space (for $\alpha = 1$ we obtain the unwarped case) and is given by [17]

$$ds_6^2 = z^2 \eta_{\mu\nu} dx^\mu dx^\nu + R_i^2 \left[ \frac{dr^2}{f} + c_i^2 f \, d\varphi^2 \right] ,$$

Figure 1: The internal space where the upper codimension-2 singularity has been regularized with the introduction of a ring-like codimension-1 brane. The parameters of the action and the solution are denoted in the appropriate part of the internal space.
\[ F_{r\varphi} = -c_i R_i M^2 S \cdot \frac{1}{z^4}, \]  
(3)

with \( R_i^2 = M^4/(2\Lambda_i) \) and the following bulk functions

\[ z(r) = \frac{1}{2} [(1 - \alpha) r + (1 + \alpha)], \]  
(4)

\[ f(r) = \frac{1}{5(1 - \alpha)^2} \left[ -z^2 + \frac{1 - \alpha^8}{1 - \alpha^3} \cdot \frac{1}{z^3} - \alpha^3 \frac{1 - \alpha^5}{1 - \alpha^3} \cdot \frac{1}{z^6} \right], \]  
(5)

with \( S(\alpha) = \sqrt{\frac{3}{5} \alpha^3 \frac{1 - \alpha^5}{1 - \alpha^3}} \). The range of the internal space coordinates is \(-1 \leq r \leq 1\) and \(0 \leq \varphi < 2\pi\). Taking into account that in the limit \( r \to \pm 1\), it is \( f \to 2(1 \mp r)X_\pm \) with the constants \( X_\pm \) given by

\[ X_+ = \frac{5 + 3\alpha^8 - 8\alpha^3}{20(1 - \alpha)(1 - \alpha^3)}, \quad X_- = \frac{3 + 5\alpha^8 - 8\alpha^5}{20\alpha^4(1 - \alpha)(1 - \alpha^3)}, \]  
(6)

the cap is smooth at \( r = +1 \) as long as is \( c_c = 1/X_+ \). Furthermore, the metric is continuous if \( c_0 R_0 = c_c R_c \), which gives \( R_c = \beta_+ R_0 \) with \( \beta_+ = X_+ c_0 \). The conical singularity at \( r = -1 \) is supported by a codimension-2 brane with tension

\[ T = 2\pi M^4 (1 - c_0 X_-) \]  
(7)

while the parameters of the 4-brane \( \lambda, v \) are fixed by the radii \( R_0, R_c \) and the brane position \( r_c \) [17]. Furthermore, the gauge field is quantized as

\[ 2c_0 R_0 M^2 e^Y = N, \quad N \in \mathbb{Z}, \]  
(8)

with \( Y = \frac{(1 - \alpha^3)}{3\alpha^3(1 - \alpha)} S \) and the brane scalar field has solution \( \sigma = n\varphi \) with \( n \in \mathbb{Z} \). The two quantum numbers \( n, N \) are related through the junction conditions as

\[ n = \frac{N}{2} \frac{2}{(1 - \alpha^3)} \left[ \frac{5(1 - \alpha^8)}{8(1 - \alpha^5)} - \alpha^3 \right]. \]  
(9)

Since the quantities \( n, N \) are integers, the above relation imposes a restriction to the values of the admissible warpings \( \alpha \), which implies that static solutions are consistent only for discrete values of the warping \( \alpha \).

### 3 Moving brane junction conditions

To study the cosmological evolution on the 4-brane we introduce an energy momentum tensor of a perfect fluid on the brane. Then the total energy momentum tensor \( t^{(\text{br})}_{\mu\nu} = -(2/\sqrt{-\gamma_+})\delta S_{\text{br}}/\delta \gamma_+^{\mu\nu} \) (where \( S_{\text{br}} \) is the brane action) will be given by

\[ t^{(\text{br})}_{\mu\nu} = \text{diag}(\rho, P, P, P, \hat{P}) \]  
(10)

\^{3}In this brief presentation of the background, we have taken \( \xi = 1 \) in comparison with [17]. The physical quantities, however, are \( \xi \)-independent and depend only on \( \beta_+ \).
and a possible coupling of the brane matter to the bulk gauge field (consistent with the cosmological symmetries) by
\[ \delta S_{br}/\delta a^k = (l, L, L, \hat{L}) \] (11)

Splitting the above quantities to one part responsible for the static solution and another expressing the presence of matter on the brane, we have
\[ \rho = \lambda + \frac{v^2(n - e A^+_\varphi)^2}{2c_0^2 R_0^2 f(r_c)} + \rho_m \equiv \rho_0 + \rho_m \] (12)
\[ P = -\lambda - \frac{v^2(n - e A^+_\varphi)^2}{2c_0^2 R_0^2 f(r_c)} + P_m \equiv -\rho_0 + P_m \] (13)
\[ \hat{P} = -\lambda + \frac{v^2(n - e A^+_\varphi)^2}{2c_0^2 R_0^2 f(r_c)} + \hat{P}_m \] (14)
\[ l = l_m \] (15)
\[ L = L_m \] (16)
\[ \hat{L} = ev^2(n - e A^+_\varphi) + \hat{L}_m \] (17)

with all the other quantities vanishing and \( \rho_m, P_m, \hat{P}_m, l_m, L_m, \hat{L}_m \) the matter contributions.

In general, the inclusion of the above matter contributions will have the effect of giving some time dependence both to the brane as well as the bulk solutions. Since the study of the full time dependent problem is rather difficult, we will make in the present paper the approximation that the bulk remains static and that the brane matter merely makes the brane to move between the two static bulk sections, away from its equilibrium point \( r = r_c \).
This is to be regarded as a first step towards understanding the generic brane cosmological evolution.

To embed the brane in the static bulk, let us take the brane coordinates be \( \sigma^\mu = (\sigma, x^i, \varphi) \). [The brane-time coordinate \( \sigma \) is not to be confused with the Goldstone field \( \sigma \) which will not appear in our subsequent analysis.] Then the brane embedding \( X^M \) in the bulk is taken for both sections to be
\[ X^i = x^i \quad , \quad X^r = \mathcal{R}(\sigma) \quad \text{and} \quad X^\varphi = \varphi \] , (18)
while for the time coordinate embedding we choose for the outer bulk section
\[ X^0_{(\text{out})} = \sigma \] , (19)
and for the inner cap section
\[ X^0_{(\text{in})} = T(\sigma) \] . (20)

The continuity of the induced metric \( \gamma_{\hat{\mu}\hat{\nu}} = g_{MN} \partial_{\mu} X^M \partial_{\nu} X^N \), apart from the relation \( c_0 R_0 = c_c R_c \) as in the static case, gives a relation of the time coordinate \( T \) in the upper
cap region with the brane time coordinate $\sigma$ (dots are with respect to $\sigma$)

$$\dot{T}^2 \left( 1 - \beta_+ \frac{\ddot{R}}{T^2 \frac{R_0^2}{f z^2}} \right) = \left( 1 - \dddot{R} \frac{R_0^2}{f z^2} \right) .$$

(21)

Then the induced metric $\gamma_{\hat{\mu} \hat{\nu}}$ on the brane reads

$$d\sigma^2_{(5)} = -z^2 \left( 1 - \dot{R} \frac{R_0^2}{f z^2} \right) d\sigma^2 + z^2 d\vec{x}^2 + c_0^2 R_0^2 f d\varphi^2 .$$

(22)

The continuity of the gauge field, on the other hand, is guaranteed by the fact that its only non-vanishing component is $A_\varphi$ and $X^\varphi$ is $\sigma$-independent.

Apart from the continuity conditions we have to take into account the junction conditions for the derivatives of the metric and the gauge field, which read

$$\{ \hat{K}_{\hat{\mu} \hat{\nu}} \} = -\frac{1}{M^4} t^{(br)}_{\hat{\mu} \hat{\nu}} ,$$

(23)

$$\{ n_M \mathcal{F}_N^M \partial_k X^N \} = -\frac{\delta S_{br}}{\delta a^k} .$$

(24)

We denote $\{ H \} = H^m + H^{out}$ the sum of the quantity $H$ from each side of each brane. The extrinsic curvatures are constructed using the normal to the brane $n_M$ which points inwards to the corresponding part of the bulk each time (we use the conventions of [25]).

The left hand sides of the above equations are computed in detail in the Appendix A. Then the junction conditions for the metric are the following: The $(\sigma \sigma)$ component is

$$M^4 \left( 3 \frac{z'}{z} + \frac{f'}{2f} \right) \frac{\sqrt{f}}{R_0 \sqrt{1 - \ddot{R} \frac{R_0^2}{f z^2}}} \left( 1 - \frac{1}{\beta_+} |\dot{T}| \right) = \rho .$$

(25)

The $(ij)$ component is

$$\frac{R_0 M^4}{z^2 \sqrt{f} \left( 1 - \ddot{R} \frac{R_0^2}{f z^2} \right)^{3/2}} \left[ \dddot{R} - \beta_+ \ddot{R}^2 \left( \frac{\ddot{R}}{|\dot{T}|} \right) - 2 \ddot{R}^2 \left( 2 \frac{z'}{z} + \frac{f'}{2f} \right) (1 - \beta_+ |\dot{T}|) + \frac{f z^2}{R_0^2} \left( 3 \frac{z'}{z} + \frac{f'}{2f} \right) \left( 1 - \frac{1}{\beta_+} |\dot{T}|^3 \right) \right] = -P .$$

(26)

The $(\varphi \varphi)$ component is

$$\frac{R_0 M^4}{z^2 \sqrt{f} \left( 1 - \ddot{R} \frac{R_0^2}{f z^2} \right)^{3/2}} \left[ \dddot{R} - \beta_+ \ddot{R}^2 \left( \frac{\ddot{R}}{|\dot{T}|} \right) - \dddot{R}^2 \left( 5 \frac{z'}{z} + \frac{f'}{2f} \right) (1 - \beta_+ |\dot{T}|) + \frac{4 f z^2}{R_0^2} \left( 1 - \frac{1}{\beta_+} |\dot{T}|^3 \right) \right] = -\dot{P} .$$

(27)
Finally, the ($\varphi$) component of the gauge field junction condition is

\[
- \frac{c_0 M^2 S \sqrt{f}}{z^4 \sqrt{1 - \dot{R}^2 R_0^2}} \left( 1 - \frac{1}{\beta_+^2} |\dot{T}| \right) = \hat{L} ,
\]

while its other components dictate that the couplings of the brane matter to the other bulk gauge components vanish

\[
l = L = 0 .
\]

Equation (25) gives a Friedmann equation for the brane coordinate $\mathcal{R}$, while equation (26) gives an acceleration equation. On the other hand, (27) and (28) are constraint equations for the matter on the brane. We will for the moment assume that the brane matter is such that $\hat{P}$ and $\hat{L}$ are given by the latter equations. In a concrete model of matter on the brane the latter two equations will introduce some additional constraint for its evolution. It is easy to see from (27) and (28), that in the static limit $\hat{P}$ and $\hat{L}$ tend to zero.

### 4 The cosmological dynamics of the 4-brane

For discussing the dynamics of the moving brane, we must study the Friedmann (25) and the acceleration (26) equations. Firstly, in order to bring the Friedmann equation to a more standard form, we rewrite the metric in the form

\[
ds_{(5)}^2 = -d\tau^2 + a^2(\tau)d\vec{x}^2 + b^2(\tau)d\varphi^2 ,
\]

with $a = z(\mathcal{R}(\tau))$ and $b = c_0 R_0 \sqrt{f(\mathcal{R}(\tau))}$. The brane proper time is given by

\[
\dot{\tau}^2 = z^2 \left( 1 - \dot{\mathcal{R}}^2 R_0^2 \frac{R^2}{f z^2} \right) .
\]

From now on we will assume without loss of generality that $\dot{\tau} > 0$. It is evident from the above, that cosmological evolution from mere motion of the brane in the static bulk is possible only when there is warping in the bulk. In the unwarped version of this model [15], mirage cosmology is impossible and some bulk time-dependence is compulsory.

The Hubble parameters for the two scale factors are given by

\[
H_a \equiv \frac{1}{a} \frac{da}{d\tau} = \frac{z'}{z^2} \frac{\dot{\mathcal{R}}}{\sqrt{1 - \dot{\mathcal{R}}^2 R_0^2 \frac{R^2}{f z^2}}} ,
\]

and

\[
H_b \equiv \frac{1}{b} \frac{db}{d\tau} = \frac{f'}{2f z} \frac{\dot{\mathcal{R}}}{\sqrt{1 - \dot{\mathcal{R}}^2 R_0^2 \frac{R^2}{f z^2}}} .
\]
Then the ratio of $H_a$ and $H_b$ gives a precise relation between them for our particular model. It is given by

$$H_b = \frac{zf'}{2fz'}H_a,$$

(34)

and we notice that since in the model we study it is always $f' < 0$, in the neighborhood of $r = 1$, the two Hubble parameters have opposite sign. This means that if the four dimensional space expands, the internal space shrinks.

From (32) and (21), we can express the brane velocity $\dot{R}$ and the time embedding $\dot{T}$ as a function of the Hubble parameter $H_a$ as

$$\dot{R} = \frac{z^2}{z'} \frac{H_a}{\sqrt{1 + \mathcal{A} H_a^2 R_0^2}}, \quad |\dot{T}| = \sqrt{\frac{1 + \beta^2 \mathcal{A} H_a^2 R_0^2}{1 + \mathcal{A} H_a^2 R_0^2}},$$

(35)

with $\mathcal{A} = \frac{z^2}{f z'}$ evaluated on the brane.

Then substituting $\dot{R}$ and $|\dot{T}|$ from (35) back to (25), we obtain the Friedmann equation as a function of the Hubble parameter

$$\frac{M^4}{R_0} \mathcal{C} \left( \sqrt{1 + \mathcal{A} H_a^2 R_0^2} - \frac{1}{\beta_+} \sqrt{1 + \beta_+^2 \mathcal{A} H_a^2 R_0^2} \right) = \rho,$$

(36)

where $\mathcal{C} = \sqrt{f \left( 3z' + \frac{f'}{z} \right)}$ is evaluated on the brane. We can solve the above equation for $H_a$ and obtain a more familiar form

$$H_a^2 = \frac{1}{4 M^8 C^2 A} \rho^2 + \frac{M^8 C^2 (1 - \beta_+^2)^2}{4 R_0^6 \beta_+^4 A} \cdot \frac{1}{\rho^2} - \frac{1}{2 \beta_+^2 R_0^2 A}.$$

(37)

The above Friedmann has unconventional dependence on the energy density. In particular the inverse square dependence is known to occur for motions in backgrounds of asymmetrical warping [26].

The equilibrium point $r_c$ of the system is found if we set $H_a = 0$, which gives the brane energy density without matter

$$\rho_0 = -\frac{M^4 C_c}{R_0 \beta_+} (1 - \beta_+),$$

(38)

where $C_c$ is the value of $\mathcal{C}$ at $r_c$ and is the same appearing in (12). Let us note that there exists a second root of (37) which is not compatible with (36). [This is because we squared twice (36) to take (37).] The behaviour of the function $\mathcal{C}$ is that, as $r \to -1$ it limits to $\mathcal{C} \to \infty$ and monotonically decreases and limits to $\mathcal{C} \to -\infty$ as $r \to 1$. On the other hand $\mathcal{A}$, is always positive with $\mathcal{A} \to \infty$ as $r \to \pm 1$, and $O(1)$ in the intermediate region. In the unwarped limit $\alpha \to 1$ the Friedmann equation becomes as expected trivial, i.e., $H_a = 0$.

Before studying various limits of the above equation, let us define the effective four dimensional matter energy density $\rho_m^{(4)}$ by averaging over the azimuthal direction (we assume that $\rho_m$ is independent of $\varphi$)

$$\rho_m^{(4)} = \int d\varphi \sqrt{g_{\varphi\varphi}} \rho_m = \frac{2\pi \beta_+}{X_+} R_0 \sqrt{f} \rho_m,$$

(39)
Figure 2: The generic form of the effective Newton’s constant $G_{\text{eff}}$ as a function of the brane position $\mathcal{R}(\sigma)$. As the brane approaches the equilibrium point $r_c$, we always have $G_{\text{eff}} < 0$. Additionally, $G_{\text{eff}}$ diverges as $r \to \pm 1$ and at one point $r_d$ in between.

with similar definitions for the other 4-brane quantities.

Let us suppose now that initially $-1 \ll \mathcal{R}(\sigma) < r_c < 1$, with $1 - r_c \ll 1$. The goal is to find how $\mathcal{R}(\sigma)$ behaves. To recover a four-dimensional Friedmann equation at late times we can assume that the brane energy density is small in comparison with the static case energy density, i.e., $\rho_m^{(4)} \ll \rho_0$, so we can expand (37) in powers of $\rho_m^{(4)}$ and obtain the following four dimensional form of the Friedmann equation

$$H^2 = \frac{8\pi}{3} G_{\text{eff}} \rho_m^{(4)} + \Delta(a) + \mathcal{O}(\rho_m^{(4)2}),$$

(40)

where the effective Newton’s constant is

$$G_{\text{eff}} = \frac{3X_c c_c (1 - \beta_+)}{32\pi^2 R_0^2 \beta_+^2 M^4 \mathcal{A} c^2 \sqrt{f}} \left[ \frac{C_c^4}{c_c^2} \left( \frac{1 + \beta_+}{1 - \beta_+} \right)^2 - 1 \right].$$

(41)

The quantity $\Delta(a)$ depends on the parameters of the bulk and plays the rôôle of the the mirage matter induced on the brane from the bulk and it is given by

$$\Delta(a) = \frac{(1 - \beta_+)^2}{4 R_0^2 \beta_+^2 \mathcal{A}} \left[ \frac{C_c^2}{c_c^2} + \frac{C_c^2}{c_c^2} \left( \frac{1 + \beta_+}{1 - \beta_+} \right)^2 - 2 \frac{1 + \beta_+^2}{(1 - \beta_+)^2} \right].$$

(42)

The behaviour of $G_{\text{eff}}$ as a function of $\mathcal{R}(\sigma)$ is given in Fig. 2 and has the following important features: At the points where the geometry becomes conical ($r \to \pm 1$) the effective Newton’s constant is negative and diverging. In between, there is a point $r_d$, with

$$z_d = \left( \frac{3(1 - \alpha^8)}{8(1 - \alpha^3)} \right)^{1/5},$$

(43)

which is a root of $\mathcal{C}$ and $G_{\text{eff}}$ diverges to $+\infty$. At this point the matter energy density is bound to vanish. It is important to note that even in the region where $G_{\text{eff}}$ is positive,
there is always a strong time variation of $G_{\text{eff}}$, which for
\[
\frac{1}{G_{\text{eff}}} \frac{dG_{\text{eff}}}{d\tau} = H_a \delta ,
\] (44)
has $\delta > O(10)$, in contradiction with observations [27] which dictate that $\delta < 0.1$. But even more important is the fact that close to the static equilibrium point, where the cosmology is supposed to mimic best the one of a codimension-2 brane, we get negative Newton’s constant
\[
G_{\text{eff}}(R = r_c) = \frac{3X_+}{8\pi^2 R_0^2 \beta_+(1 - \beta_+) M^4 A C_c \sqrt{f}} < 0 ,
\] (45)
since we have that for any value of the parameters and for $r_c$ in the neighborhood of $r = +1$, it is $C_c < 0$.

Let us also look at the mirage matter contribution $\Delta$, which is depicted in Fig. 3. As expected, it vanishes for the static equilibrium point $r_c$, it is finite at the boundaries $r = \pm 1$ and diverges at the root $r_d$ of $C$. Again there is no region in the brane position interval where the contribution of $\Delta$ is constant enough to resemble a cosmological constant contribution to the four dimensional Friedmann equation.

On the opposite limit, that the matter energy density is much larger than the static case energy density, i.e., $\rho_m^{(4)} \gg \rho_0$, we get the expected asymptotics
\[
H_a^2 = \frac{1}{4M^8 C^2 A} \rho_m^2 ,
\] (46)
which is a five-dimensional Friedmann law (with time-varying five-dimensional Newton’s constant) at early times.

Taking under consideration the difficulties of the model to give a Friedmann equation with the correct sign of $G_{\text{eff}}$, we will not proceed with the presentation of the analysis of the acceleration equation (26). With this equation, one finds even more difficulties towards obtaining a realistic four-dimensional evolution. For example, in the low energy limit, one
gets a coefficient of the linear energy density term, which is not related to the $G_{\text{eff}}$, that we obtained from the Friedmann equation, in the way it does in standard four-dimensional General Relativity.

This pathological features of the low energy density limit, where the expansion does not appear to have an effective four-dimensional limit, can find a potential explanation when looking at the energy continuity equation on the brane. Taking the covariant divergence of the Israel junction condition (23), we can obtain the following equation

$$\nabla^{(4)}_{\hat{\mu}} t^{(br)}_{\hat{\nu}} = -M_4 \{ \nabla^{(4)}_{\hat{\mu}} \hat{K}^{\hat{\nu}} \} .$$  \hspace{1cm} (47)

Then using the Codazzi equation

$$\nabla^{(4)}_{\hat{\mu}} \hat{K}^{\hat{\nu}} = G_{K\Lambda} n^\Lambda h^K_N \partial_{\hat{b}} X^N ,$$  \hspace{1cm} (48)

and the bulk Einstein equation, we arrive at the simple expression [25]

$$\nabla^{(4)}_{\hat{\mu}} t^{(br)}_{\hat{\nu}} = -\{ T^{(B)}_{K\Lambda} n^\Lambda h^K_N \partial_{\hat{b}} X^N \} .$$  \hspace{1cm} (49)

Because of the jump of the bulk energy momentum tensor across the 4-brane, the energy-momentum tensor on the 4-brane is not conserved. This is a usual feature of moving brane cosmologies in asymmetrically warped backgrounds [28]. In more detail, the form of the above equation for the particular model is given by

$$\frac{d\rho}{d\tau} + 3(\rho + P)H_a + (\rho + \hat{P}) \frac{zf'}{2fz'} H_a = -\frac{S^2 \rho H_a}{z'z^7 C \sqrt{f}} ,$$  \hspace{1cm} (50)

where in the right hand side we have used the Friedmann equation (37). In a straightforward but lengthy calculation, one can see that the latter equation can be derived from (25), (26), (27). The problems in the four-dimensional limit, that we faced previously, can be traced to large energy dissipation off the brane as well as a large work done during the contraction of the ring-brane.

\section{Conclusions}

In the present paper we made a first step towards the study of the cosmology of a codimension-2 brane, which is regularized by the method of lowering its codimensionality (cutting the space close to the conical tip and replacing it by a ring brane with an appropriate cap). As a first approximation, we assumed that the bulk and the cap remain static as the brane moves between them. The motion of the brane then induces a cosmological evolution for the matter on the brane. The junction conditions provide the Friedmann and acceleration equations on the brane.

We can see already from the Friedmann equation that we cannot recover standard cosmological evolution of the brane at low energies. The effective Newton’s constant is negative in the interesting limit that the brane approaches its equilibrium point, close to
the would-be conical singularity. In other words, we obtain antigravity in this limit. Even away from this point, i.e., when the brane moves away from its equilibrium point, the Newton’s constant varies significantly in contradiction with standard cosmology. At one position of the internal space, the Newton’s constant even diverges and forces the matter energy density to vanish. Taking all the above into account, we did not present the further analysis of the system by considering the acceleration equation.

This result is not altered by supersymmetrizing the model [6]. The bulk and cap solutions are different from the non-supersymmetric case due to the presence of the dilaton. However, the only difference in the Friedmann is a redefinition of the quantities $A$ and $C$. In the supersymmetric case, these quantities read

$$A_{\text{susy}} = 4 \frac{z^2}{f z^2} \quad \text{and} \quad C_{\text{susy}} = \sqrt{f} \left( \frac{3z'}{2z} + \frac{f'}{2f} \right).$$

(51)

It is easy to see again that the effective Newton’s constant is negative for the motion near the would-be conical tip and that even away from that point, it is very strongly varying.

The reason for this unconventional cosmological evolution, is the use of the specific restricted ansatz for the solution of the system’s equation of motion. The staticity of the bulk was proved to be an oversimplification. We imagine that this restriction on the system may result to the appearance of some scalar mode in the perturbative analysis of [29], which for a certain region of the brane motion (close to the pole of the internal manifold) is ghost-like. This mode may then be responsible for the negative effective Newton’s constant. The unrestricted perturbative analysis in [29] resulted in a linearized four-dimensional Einstein equation for distances larger than the compactification scale. The same happened in [15] in the unwarped model perturbation analysis$^4$ (see [30] for a related analysis with a brane induced gravity term), even though it would not have mirage evolution, as we studied it here, because of the absence of a warp factor. The appearance of the standard four-dimensional linearized dynamics, shows that they are realized when the bulk is necessarily time-dependent.

Clearly, the next step should be the study of the system in a setup where the bulk is also time-dependent. In that respect, brane cosmology in six dimensions seems to be different for the one in five dimensions. In Einstein gravity in five dimensions, one can always work in a gauge where the bulk is static and the brane acquires a cosmological evolution by moving into the bulk. However, in six dimensions there are more degrees of freedom which make this gauge choice not general. In the present paper, we have frozen these degrees of freedom hoping to find consistent cosmological solutions, but it turned out that this does not give a viable cosmology. There are several known time-dependent backgrounds which can be used to look for realistic brane cosmologies [31]. We plan to address this issue in the near future.

$^4$The question of stability of the regularized models was not discussed in [15, 29] and it could be that these compactifications have modes with negative mass squared.
Addendum

At the last stages of this work, we became aware of the work of [32], where a cosmological evolution, similar in spirit with the present paper, is discussed. More precisely, the time evolution of the modulus of a complex scalar field on the brane is studied, while keeping the bulk static. Our results are in agreement with [32] and in many respects complementary. In particular, as in our case, four-dimensional late time cosmology is not recovered.

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Appendix A: Computation of extrinsic curvatures

In this Appendix we will calculate the extrinsic curvatures on the brane positions, which will be used in the main text to evaluate the junction conditions. Let the brane position in the bulk be $X^M(\sigma^\mu)$, from which we evaluate the induced metric on the brane as $\gamma_{\mu\nu} = g_{MN}\partial_\mu X^M \partial_\nu X^N$. Firstly, we should calculate the normal vector of the brane $n_M$, which is orthogonal to all the tangent vectors of the brane $\partial_\nu X^M$, that is $\partial_\nu X^M n_M = 0$, and is normalized as $n_M n_N g^{MN} = 1$.

Once the normal vector is computed, one can evaluate the projection tensor $h_{MN} = g_{MN} - n_M n_N$, which is related to the induced metric as $h^{MN} = \gamma_{\mu\nu} \partial_\mu X^M \partial_\nu X^N$ and satisfies also $\gamma_{\mu\nu} = h_{MN} \partial_\mu X^M \partial_\nu X^N$ due to the orthogonality of the normal to the tangent vectors. Afterwards, the extrinsic curvature is given by $K_{MN} = h^A \partial_A n_M$ (the covariant derivative computed with $g_{MN}$) and with trace $K = g^{MN} K_{MN}$. The pullback of the extrinsic curvature on the brane is given by $\hat{K}_{\mu\nu} = K_{MN} \partial_\mu X^M \partial_\nu X^N$ and has the property that $K = \gamma_{\mu\nu} K_{\mu\nu} = K$. The combination which appears in the junction conditions is $\hat{K}_{\mu\nu} = K_{\mu\nu} - K_{\mu\nu}$.

As is discussed in [17], the metric component $g_{rr}$ in the radial coordinate direction need not be continuous. This then introduces normal vectors $n_M$ for each side of the brane which are not opposite (since they are normalized with different $g_{rr}$). Nevertheless, the junction conditions make perfect sense if the discontinuous $g_{rr}$ and the non-opposite $n_M$’s are adequately used.

Let us first compute the quantities we are interested in for the outer bulk section. The normal vector is

$$n_M^{\text{out}} = -\frac{R_0}{\sqrt{J}} \cdot \frac{1}{\sqrt{1 - \hat{R}^2 R_0^2 / J^2}} (-\hat{R}, 0, 1, 0)$$ (A.1)
[From now on we suppress the notation (\textit{out}) which is to be understood for all subsequent quantities.]

The projection tensor $h_{MN} = g_{MN} - n_M n_N$ has components

$$h^0_0 = \frac{1}{1 - \mathcal{R}^2 R_0^2 f z^2}, \quad h^0_0 = \mathcal{R} h^0_0, \quad h^0_r = -\mathcal{R} \frac{R_0^2}{f z^2} h^0_0, \quad h^r_r = -\mathcal{R}^2 \frac{R_0^2}{f z^2} h^0_0 \quad \text{(A.2)}$$

$$h^i_j = \delta^i_j, \quad h^\phi_\phi = 1 \quad \text{(A.3)}$$

The Christoffel symbols for the bulk metric that we need are

$$\Gamma^r_{\nu r} = \frac{z'}{z} \delta_\mu^\nu, \quad \Gamma^r_{\mu r} = -\frac{z' f}{R_0} \eta_{\mu \nu} \quad \text{(A.4)}$$

$$\Gamma^r_{r r} = -\frac{f'}{2 f}, \quad \Gamma^r_{\phi \phi} = -\frac{1}{2} c_0^2 f f' \quad \text{(A.5)}$$

[Also $\Gamma^r_{r \phi} = f'/(2 f)$ but we don’t need it. All other are zero.]

We can now compute the extrinsic curvatures $K_{MN} = h^K_M h^K_N \nabla_K n_A$. Note that since we have expressed the normal vectors in the brane coordinates, the bulk derivative is understood to be taken using the projection rule $h^K_M \partial_M = g^M_K \partial^\hat{\nu} X^M \partial_{\hat{\nu}}$. The calculation gives

$$K_{00} = (h^0_0)^2 n_r \left[ -\mathcal{R} - \Gamma^r_{00} + \mathcal{R}^2 (2 \Gamma^r_{0 r} - \Gamma^r_{r r}) \right] \quad \text{(A.6)}$$

$$K_{05} = -\mathcal{R} \frac{R_0^2}{f z^2} K_{00}, \quad K_{55} = \mathcal{R}^2 \frac{R_0^4}{f^2 z^4} K_{00} \quad \text{(A.7)}$$

$$K_{ij} = -\Gamma^r_{ij} n_r = \frac{z' f}{R_0} n_r \delta_{ij} \quad \text{(A.8)}$$

$$K_{\phi \phi} = -\Gamma^r_{\phi \phi} n_r = \frac{1}{2} c_0^2 f f' n_r \quad \text{(A.9)}$$

The pullbacks of the extrinsic curvatures on the brane $K_{\hat{\mu} \hat{\nu}} = K_{MN} \partial_\mu X^M \partial_\nu X^N$ are

$$K_{\sigma \sigma} = \left( 1 - \mathcal{R}^2 \frac{R_0^2}{f z^2} \right)^2 K_{00} = n_r \left[ -\mathcal{R} - \frac{z' f}{R_0^2} + \mathcal{R}^2 \left( \frac{2 z'}{z} + \frac{f'}{2 f} \right) \right] \quad \text{(A.10)}$$

$$K_{\iota j} = K_{ij} = \frac{z' f}{R_0^2} n_r \delta_{ij} \quad \text{(A.11)}$$

$$K_{\phi \phi} = K_{\phi \phi} = \frac{1}{2} c_0^2 f f' n_r \quad \text{(A.12)}$$

Now we can construct the hatted quantities $\hat{K}_{\hat{\mu} \hat{\nu}} = K_{\mu \nu} - K_{\gamma \mu \nu}$. They read

$$\hat{K}_{\sigma \sigma} = n_r \frac{f z^2}{R_0^2} \left( 1 - \mathcal{R}^2 \frac{R_0^2}{f z^2} \right) \left( \frac{3 z'}{z} + \frac{f'}{2 f} \right) \quad \text{(A.13)}$$
\[ K_{ij} = \frac{n_r \delta_{ij}}{(1 - R^2 \frac{R_0}{Tz^2})} \left[ -\ddot{R} + 2\dot{R}^2 \left( \frac{z'}{z} + \frac{f'}{2f} \right) \right] \] (A.15)
\[ \dot{K}_{\varphi\varphi} = \frac{n_r c_0^2 R_0 f}{z^2 (1 - R^2 \frac{R_0}{Tz^2})} \left[ -\ddot{R} - 4 \frac{z'f}{R_0^2} + \dot{R}^2 \left( \frac{5z'}{z} + \frac{f'}{2f} \right) \right] \] (A.16)

Let us now compute the quantities we are interested in for the *inner cap section*. The normal vector is
\[ n^i_M = \frac{\beta_+ R_0}{\sqrt{f}} \cdot \frac{1}{\sqrt{1 - \beta_+^2 \frac{R_0^2}{T^2 f z^2}}} \left( -\frac{\dot{R}}{T}, 0, 1, 0 \right) \] (A.17)

[From now on we suppress the notation \((in)\) which is to be understood for all subsequent quantities]

The projection tensor \( h_{MN} = g_{MN} - n_M n_N \) has components
\[ h_0^0 = \frac{1}{1 - \beta_+^2 \frac{R_0^2}{T^2 f z^2}} \quad , \quad h_0^r = \frac{\dot{R}}{T} h_0^0 \quad , \quad h_0^\varphi = -\beta_+^2 \frac{\dot{R}^2 R_0^2}{T^2 f z^2} h_0^0 \quad , \quad h_r^r = -\frac{1}{2} c_0^2 f f' \] (A.18)

The Christoffel symbols for the bulk metric that we need are
\[ \Gamma^\mu_{\nu\rho} = \frac{z'}{z} \delta^\mu_\nu \quad , \quad \Gamma^r_{\mu\nu} = -\frac{z'f}{\beta_+^2 R_0^2} \eta_{\mu\nu} \] (A.19)
\[ \Gamma^r_{rr} = -\frac{f'}{2f} \quad , \quad \Gamma^r_{\varphi\varphi} = -\frac{1}{2} c_0^2 f f' \] (A.20)

We can now compute the extrinsic curvatures as
\[ K_{00} = (h_0^0)^2 n_r \left[ -\frac{1}{T} \left( \frac{\dot{R}}{T} \right) - \Gamma^r_{00} + \frac{\dot{R}^2}{T^2} (2\Gamma^0_0 - \Gamma^r_{rr}) \right] \] (A.21)
\[ = (h_0^0)^2 n_r \left[ -\frac{1}{T} \left( \frac{\dot{R}}{T} \right) - \frac{z'f}{R_0^2} + \frac{\dot{R}^2}{T^2} \left( \frac{2z'}{z} + \frac{f'}{2f} \right) \right] \] (A.22)
\[ K_{05} = -\beta_+^2 \frac{\dot{R}}{f z^2} \right) K_{00} \quad , \quad K_{55} = \beta_+^4 \frac{\dot{R}^2 R_0^4}{T^2 f z^4} K_{00} \] (A.23)
\[ K_{ij} = -\Gamma^r_{ij} n_r = \frac{z'f}{\beta_+^2 R_0^2} n_r \delta_{ij} \] (A.24)
\[ K_{\varphi\varphi} = -\Gamma^r_{\varphi\varphi} n_r = \frac{1}{2} c_0^2 f f' n_r \] (A.25)

The pullbacks of the extrinsic curvatures on the brane \( K_{\mu\nu} = K_{MN} \partial_\mu X^M \partial_\nu X^N \) are
\[ K_{\sigma\sigma} = \dot{T}^2 \left( 1 - \beta_+^2 \frac{\dot{R}^2 R_0^2}{T^2 f z^2} \right)^2 K_{00} = \dot{T}^2 n_r \left[ -\frac{1}{T} \left( \frac{\dot{R}}{T} \right) - \frac{z'f}{\beta_+^2 R_0^2} + \frac{\dot{R}^2}{T^2} \left( \frac{2z'}{z} + \frac{f'}{2f} \right) \right] \] (A.26)
\[ K_{ij} = \mathcal{K}_{ij} = \frac{z' f}{\beta_+^2 R_0^2} n_r \delta_{ij} \quad (A.27) \]

\[ K_{\varphi \varphi} = \mathcal{K}_{\varphi \varphi} = \frac{1}{2} c_0^2 f f' n_r \quad (A.28) \]

Now we can construct the hatted quantities \( \hat{K}_{\mu \nu} = K_{\mu \nu} - K_{\gamma \mu \nu} \). They read

\[ \hat{K}_{\sigma \sigma} = n_r \frac{f z^2}{\beta_+^2 R_0^2} \frac{1}{f z^2} \left( 1 - \beta_+^2 \frac{\dot{R}}{T^2} \right) \left( 3 \frac{z'}{z} + \frac{f'}{2f} \right) \quad (A.29) \]

\[ \hat{K}_{ij} = \frac{n_r \delta_{ij}}{\left( 1 - \beta_+^2 \frac{\dot{R}}{T^2} \right) z^2} \left[ -\frac{1}{T} \left( \frac{\dot{R}}{T} \right) - 2 \frac{\dot{R}^2}{T^2} \left( \frac{2 z'}{z} + \frac{f'}{2f} \right) - \frac{f z^2}{\beta_+^2 R_0} \left( 3 \frac{z'}{z} + \frac{f'}{2f} \right) \right] \quad (A.30) \]

\[ \hat{K}_{\varphi \varphi} = \frac{n_r c_0^2 R_0^2 f}{z^2 \sqrt{f} \left( 1 - \beta_+^2 \frac{\dot{R}}{T^2} \right) z^2} \left[ -\frac{1}{T} \left( \frac{\dot{R}}{T} \right) - 4 \frac{z' f}{\beta_+^2 R_0} \left( \frac{5}{z} + \frac{f'}{2f} \right) \right] \quad (A.31) \]

We can now write down the jumps of the hatted extrinsic curvatures, which participate in the junction conditions

\[ \{\hat{K}_{\sigma \sigma}\} = -\left( 3 \frac{z'}{z} + \frac{f'}{2f} \right) \frac{z^2 \sqrt{f}}{R_0} \sqrt{1 - \frac{\dot{R}^2 R_0^2}{f z^2}} \left( 1 - \frac{1}{\beta_+^2 |\dot{T}|} \right) \quad (A.32) \]

\[ \{\hat{K}_{ij}\} = \frac{R_0 \delta_{ij}}{\sqrt{f} \left( 1 - \beta_+^2 \frac{\dot{R}}{T^2} \right)^{3/2}} \left[ \ddot{\mathcal{R}} - \beta_+^2 \frac{\dot{T}^2}{|\dot{T}|} \right] - 2 \frac{\dot{R}^2}{T^2} \left( \frac{2 z'}{z} + \frac{f'}{2f} \right) \left( 1 - \beta_+ |\dot{T}| \right) \]

\[ + \frac{f z^2}{R_0} \left( 3 \frac{z'}{z} + \frac{f'}{2f} \right) \left( 1 - \frac{1}{\beta_+^2 |\dot{T}|^3} \right) \quad (A.33) \]

\[ \{\hat{K}_{\varphi \varphi}\} = \frac{c_0^2 R_0^3 f}{z^2 \sqrt{f} \left( 1 - \beta_+^2 \frac{\dot{R}}{T^2} \right)^{3/2}} \left[ \ddot{\mathcal{R}} - \beta_+^2 \frac{\dot{T}^2}{|\dot{T}|} \right] - \frac{\dot{R}^2}{T^2} \left( \frac{5}{z} + \frac{f'}{2f} \right) \left( 1 - \beta_+ |\dot{T}| \right) \]

\[ + \frac{4 f z' z}{R_0^2} \left( 1 - \frac{1}{\beta_+ |\dot{T}|^3} \right) \quad (A.34) \]

Furthermore, the jump of the gauge field is given by

\[ \{n_r \mathcal{F}_{\varphi}\} = \frac{c_0 M^2 S \sqrt{f}}{z^4 \sqrt{1 - \beta_+^2 \frac{\dot{R}}{f z^2}}} \left( 1 - \frac{1}{\beta_+^2 |\dot{T}|} \right) \quad (A.35) \]
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