Abstract—Prior investigations on the Aloha network have primarily focused on its system throughput. Good system throughput, however, does not automatically translate to good delay performance for the end users. Neither is fairness guaranteed: Some users may starve, while others hog the system. This paper establishes the conditions for bounded mean queuing delay and nonstarved operation of the slotted Aloha network. We focus on the performance when collisions of packets are resolved using an exponential backoff protocol. For a nonsaturated network, we find that bounded mean delay and nonstarved operation can be guaranteed only if the offered load is limited to below a quantity called “safe bounded mean-delay (SBMD) throughput.” The SBMD throughput can be much lower than the saturation system throughput if the backoff factor \( r \) in the exponential backoff algorithm is not properly set. For example, it is well known that the maximum throughput of the Aloha network is \( \epsilon^{-1} = 0.3679 \). However, for \( r = 2 \), a value assumed in many prior investigations, the SBMD throughput is only 0.2158, a drastic penalty of 41% relative to 0.3679. Fortunately, using \( r = 1.3757 \) allows us to obtain an SBMD throughput of 0.3545, less than 4% away from 0.3679. A general conclusion is that the system parameters can significantly affect the delay and fairness performance of the Aloha network. This paper provides the analytical framework and expressions for tuning \( r \) and other system parameters to achieve good delay and nonstarved operation.

Index Terms—Access protocols, network performance, wireless LAN.

I. INTRODUCTION

The Aloha network has been studied extensively since the pioneer work by Abramson [1]. Prior work on the Aloha network has primarily focused on its overall system throughput. To achieve good system throughput, the transmission probabilities of the nodes must be adjusted dynamically according to the contention intensity in the network. An exponential backoff protocol can serve this purpose rather effectively [2].

Good system throughput, however, does not automatically translate to acceptable performance from the end user perspective. For example, if a real-time application such as a voice call running on top the Aloha network, delay performance is important. Even if the end application is not a real-time application, there is also the fairness issue, wherein some nodes in the Aloha network are starved while others enjoy good service. For example, for a TCP application, starvation could cause premature termination of its TCP connection due to time-out.

This paper is devoted to the study of how to ensure good delay and nonstarved performance in a slotted Aloha network operated with an exponential backoff protocol. In particular, we are interested in the setting of system parameters to attain not just good overall system throughput but also good delay and fairness performance. Within this context, this paper has two major contributions.

1) We establish an analytical framework for the study of queuing delay and starvation in the Aloha network.
2) Based on the analytical framework, we derive the dependency of delay and nonstarvation on the system parameters.

With respect to contribution 1), we unite the concepts of bounded mean-delay performance and nonstarvation, arguing that the conditions giving rise to them are one in the same in a nonsaturated Aloha network; namely, the service time at the heads of queues must be bounded. We find that the “saturation throughput,” a performance metric of focus in many prior studies, is not a sound measure of performance if we care about delay and nonstarved operation. In particular, to achieve good delay and nonstarved operation, the offered load must be below another quantity called “safe bounded mean-delay (SBMD) throughput,” which can be substantially lower than the saturation throughput. In establishing our analytical framework, we find that the delay analysis is much trickier than the saturation-throughput analysis in prior work. To better bring out the subtleties involved, we decompose our analysis into three steps: 1) a global analysis that captures the interaction among nodes; 2) a local analysis that captures the dynamic within a node; and 3) a coupling analysis that integrates 1) and 2) into a coherent whole. Steps 2) and 3) in the delay analysis, in particular, are a lot more involved than steps 2) and 3) in the saturation analysis.

With respect to contribution 2), we show that delay and nonstarvation can be very sensitive to the system parameters, much more so than the saturation throughput is. For example, it is well known that the maximum saturation throughput of a large slotted Aloha network with many nodes is \( \epsilon^{-1} = 0.3679 \). An exponential backoff factor of 2 (see Section II-A for the definition of the backoff factor) was commonly assumed in many prior studies [3], [4]. It can achieve a saturation throughput of 0.3466 [2]. Thus, a backoff factor of 2 is quite satisfactory when it comes to saturation throughput performance. However, if we desire bounded mean-delay and nonstarved performance, we...
must limit the system-offered load to below 0.2158, a drastic penalty of 41% with respect to the maximum throughput. Fortunately, using a backoff factor of 1.3757 instead of 2, the sustainable offered load can reach 0.3545—very close to the maximum throughput. This paper will present many other intricate relationships between system parameters and system operation and performance.

A. Related Work

Most prior investigations on the Aloha network (e.g., [2], [3], and [5]) consider the access delay (i.e., service time incurred by a packet at the head-of-line (HOL) of its queue). Less attention is paid to the overall queuing delay (i.e., waiting time plus service time). In [2], the saturation throughput (reciprocal of mean access delay) as a function of the backoff factor \( r \) was derived. A fundamental expression obtained in [2] is the dependency of saturation throughput on \( r \) for a large network:

\[
S_r = \frac{r - 1}{r} \ln \left( \frac{r - 1}{r - 1 + \ln r} \right)
\]

Higher moments of the access delay, however, were not considered. The work in [3] focused on the case of \( r = 2 \) only and investigated both the mean and variance of access delay. It was shown that the throughput must be below \((3(\ln 4 - \ln 3))/4 = 0.2158\) if variance of access delay is to be finite.

In contrast to these prior investigations, a focus of our work here is on the queuing delay, rather than the access delay, and for general \( r \). For the \( r = 2 \) case, bounded mean delay requires only the access-delay variance to be bounded. Hence, the sustainable offered load for bounded mean queuing delay is the same as that derived in [3] for bounded access-delay variance. For \( r \) smaller than 1.3757, however, we argue in this paper that an offered load that ensures bounded access-delay variance cannot safely guarantee bounded mean queuing delay and that the offered load must also be below the saturation throughput.

As in this paper, [6] also considered the nonsaturated scenario, but for 802.11 networks. Furthermore, the focus is on throughput rather than delay. It argued that the notion of saturation throughput is a pessimistic one in that the system throughput could be above the saturation throughput if the queues are forced to be emptied from time to time. We find that as far as the Aloha network is concerned, with an appropriate setting of \( r \), one could achieve throughput that is only less than 4% away from the maximum throughput of \( \sigma^{-1} \). This is achieved without forced emptying of queues and with delay performance taken into consideration.

In this paper, we consider a slightly different exponential backoff protocol than the prior work. Our model captures the main essence and principle of exponential backoff and has the advantage of being more amenable to analysis. Many of the saturation throughput results in [2] can be obtained within the space of less than one page with our model, as will be shown in Section II-A.

We are primarily interested in networks in which the number of nodes \( N \) is large. Our large-but-fixed \( N \) results are not to be confused with the results of the infinite-population model [7], in which nodes—each with one and only one packet to transmit—are created on the fly. In the former, the number of contending packets is bounded by \( N \), whereas in the latter, the number of contending packets can grow indefinitely. As a matter of fact, the saturation throughput of binary exponential backoff is 0.3466 in the limit of \( N \to \infty \) in the former, but zero in the latter [8].

The remainder of this paper is organized as follows. Section II presents our system model. We illustrate the use of the model in saturation analysis. Many expressions useful for queuing-delay and starvation analyses later are derived. Section III presents our queuing-delay analysis. We derive expressions that relate delay performance to system parameters. The materials presented in Section III show that queuing-delay analysis is much more subtle than the saturation-throughput analysis in Section II and in prior work. Section IV investigates in detail the effects of the backoff factor on the sustainable offered load for bounded mean-delay operation. Section V is devoted to the study of the starvation phenomenon. We derive the dependency of starvation on system parameters. Section VI concludes this paper.

II. SYSTEM MODEL AND SATURATION ANALYSIS

In this section, we first describe the system model under study, then perform a saturation analysis.

A. System Model

Real System: We consider a slotted Aloha network with \( N \) nodes. Each node has a queue to hold its backlog packets. When a fresh packet enters the HOL of its queue, it transmits with probability \((1/r_0)\) in each time slot, where \( r_0 \geq 1 \). When more than one node transmits a packet in a time slot, a collision occurs and the packets are corrupted. A collided packet will be retransmitted in a future time slot. Each time a HOL packet suffers a collision, the transmission probability in the future is divided by the backoff factor \( r > 1 \). Thus, a HOL packet that has suffered \( i \) prior collisions will be transmitted in a future time slot with probability \((1/(r_0^{p^i}))\). We refer to \( i \) as the backoff stage of a node. A HOL packet will be transmitted and retransmitted until it is successfully cleared without a collision, at which point the next-in-line packet, if any, will proceed to the HOL.

Another closely related protocol often considered is a countdown-window protocol [2], [9], in which a countdown process is used to determine when a HOL packet is transmitted. The parameter \( r_0 \) in our protocol serves the same purpose as the initial window size \( W_0 \) of that model in determining the expected number of time slots until the first transmission of a HOL packet. The common backoff factor \( r \) serves the same purpose in both models—i.e., for dynamic adjustment of the transmission probabilities of nodes according to contention intensity. For a given \( r \), the two protocols have roughly the same behavior if \( r_0 \approx (W_0/2) \). Our model, however, is simpler to analyze. In Section II-A, for example, we show that many saturation results similar to those in [2] can be obtained in a few simple steps within the space of less than a page.

With our model, the “local state” of a queue can be described by a duple \((Q, B)\), where \( Q \) is the number of backlog packets in the queue, including the HOL packet, and \( B \) is the backoff stage of the HOL packet. The “global state” of the overall system consists of the aggregate local states of all \( N \) queues. One can, in principle, construct a multidimensional Markov chain to analyze the system. However, the analysis for even modest-size \( N \) is prohibitively complex, and not much insight can be gained from this brute-force analysis. Detailed and exact results, for example, are only available for the 2-node case [4].
Proxy System: For large $N$, an approximation technique that has often been used in saturation analysis is to replace the actual system model with a “proxy model” (e.g., used in [2] as well as [9] and many of its follow-up papers). This paper adopts the same approximations for saturation as well as nonsaturation analyses.

The proxy system makes two approximations: 1) the probability of collision $p_c$ experienced by a node is independent of its local state, and 2) as far as a local node is concerned, each of the other nodes transmits with a probability $p_k$ in a given time slot. Certainly these approximations are only valid under large $N$ when each local node only has a small effect on the overall system. Simulations of the actual system, referred to as the “real system” in this paper, can be used to check against the accuracy of the proxy-system analysis. This paper will show such verification results.

In this paper, for better exposition and understanding of the intricacies involved, we decompose the analysis of the proxy system into three steps. The first step is a “global analysis” linking $p_c$ and $p_k$: viz

$$p_c = 1 - (1 - p_k)^{N-1}$$

as $N \to \infty$.

$$p_k = \frac{1}{r} \left( 1 - \frac{r_0 S_k}{N} \right)$$

$$\rightarrow \frac{1}{r} \text{ as } N \to \infty.$$  \hspace{1cm} (3a)

Then $N \to \infty$, $S_k = G_s(1 - p_c)$, $S_k = G_s (1 - G_s/N)^{N-1}$ in (2) and $p_c = (1 - r_0 S_k/N)/r$ in (3), we can get

$$1 - p_c = \left( 1 - \frac{1 - p_c r}{r_0 (1 - p_c)} \right)^{N-1}.$$  \hspace{1cm} (4)

These equations in (2) and (3) also give

$$r - 1 = \frac{G_s N}{N - 1} \left( 1 + \frac{r_0 S_k}{N} \right)^N = \frac{(r - 1)}{r} \left( 1 + \frac{r_0 - 1}{r - 1} \frac{S_k}{N} \right)^{N-1}.$$  \hspace{1cm} (5)

For $N \to \infty$, $S_k = G_s(1 - p_c)$, $S_k = G_se^{-G_s}$ in (2) and $p_c = 1/r$ in (3) yield [below can also be obtained by taking limit in (5)]

$$G_s = \ln \left( \frac{r}{r - 1} \right)$$

$$S_k = \left( \frac{r - 1}{r} \right) \ln \left( \frac{r}{r - 1} \right).$$  \hspace{1cm} (6)

Note that while the solution for $S_k$ is in closed form in the asymptotic case, $S_k$ must be found numerically from (5) in the finite-$N$ case. Also, $S_k$ depends on $r_0$, $r$, $N$ in the finite-$N$ case but only on $r$ in the asymptotic case. The practical significance of (5) and (6) is that they allow us to study the dependency of the saturation throughput $S_k$ on system parameters $r$, $r_0$, $N$.

III. DELAY ANALYSIS

We now consider the nonsaturation analysis in which the queues of the nodes are not saturated. Unless otherwise stated, henceforth by “delay” we mean “queuing delay” rather than the “HOL access delay.” We assume the arrival process to each queue is Poisson with rate $\lambda_0 = S_k/N$, where $S_k$ is the offered load to the overall system, and $S_k/N$ is thus the offered load to a single queue.

For a nonsaturated system under equilibrium, the output rate (i.e., throughput) is equal to the input rate (i.e., offered load). Given a system with system parameters $r$, $r_0$, $N$, we could load it with different offered load $S_0$ and, therefore, obtain different throughput $S_0$. This is in contrast to a saturated system, in which the saturation throughput $S_k$ is a “fixed” quantity given $r$, $r_0$, $N$. 

Global Analysis: Consider the overall system consisting of the $N$ homogeneous nodes. Recall that $p_k$ is the probability of transmission of an arbitrarily chosen node in the proxy system, and $p_c$ is the collision probability of a transmitting node. By the homogeneity assumption of the proxy system, we have

$$p_c = 1 - (1 - p_k)^{N-1}$$

$$\rightarrow 1 - e^{-Np_k} \text{ as } N \to \infty.$$  \hspace{1cm} (1)

Define $G_s = Np_k$ as the global transmission attempt rate and $S_k$ as the saturation throughput of the overall system. Then, by definition

$$S_k = G_s(1 - p_c) = G_s \left( 1 - \frac{G_s}{N} \right)^{N-1}$$

as $N \to \infty$.  \hspace{1cm} (2)

The expression $S_k = G_se^{-G_s}$ for the asymptotic case, of course, is the well-known slotted Aloha throughput equation. The relationships in (2) govern the global dynamic of the system.

Local Analysis: Consider one particular node. Let $X$ be the HOL access delay of a packet. Then, by considering the successive additional expected access delays incurred conditioned on the number of collisions, we have $E[X] = r_0 + r_0 \tau_k + r_0 \tau_k^2 + \cdots = r_0/(1 - \tau_k)$. At saturation, the HOL is always occupied. Hence, by Little’s Law, we have $E[X] S_k/N = 1$, where $S_k/N$ is the saturation throughput of the local node. These two equations give

$$p_c = \frac{1}{r} \left( 1 - \frac{r_0 S_k}{N} \right)$$

$$\rightarrow \frac{1}{r} \text{ as } N \to \infty.$$  \hspace{1cm} (3b)

Coupling Analysis: We now couple the results from the global and local analyses. Overall, we can express any of the variables $S_k$, $G_s$, $p_c$, or $p_k$ in terms of the system parameters $r$, $r_0$, $N$. In the following, we only list the expressions that will be used later.

The dependency of $p_c$ on system parameters $r$, $r_0$, $N$ will be useful for our starvation analysis later. From $S_k = G_s(1 - p_c)$, $S_k = G_s (1 - G_s/N)^{N-1}$ in (2) and $p_c = (1 - r_0 S_k/N)/r$ in (3), we can get

$$1 - p_c = \left( 1 - \frac{1 - p_c r}{r_0 (1 - p_c)} \right)^{N-1}.$$  \hspace{1cm} (4)

For $N \to \infty$, $S_k = G_s(1 - p_c)$, $S_k = G_se^{-G_s}$ in (2) and $p_c = 1/r$ in (3) yield [below can also be obtained by taking limit in (5)]

$$G_s = \ln \left( \frac{r}{r - 1} \right)$$

$$S_k = \left( \frac{r - 1}{r} \right) \ln \left( \frac{r}{r - 1} \right).$$  \hspace{1cm} (6)
Different $S_0$, however, will give rise to different delay performances, and it is important not to overload the system. An issue of particular interest to us, which will be addressed by the end of this section, is the limit on $S_0$ that can ensure equilibrium and bounded-delay operation. We call this limit “safe bounded mean-delay throughput.” As will be shown, SBMD throughput depends on $r$, $r_0$, $N$ and may be lower than $S_0$.

As with the saturation analysis in Section II-B, we break down the delay analysis into global, local, and coupling analyses. It turns out that the local and coupling analyses are much more involved here.

A. Global Analysis

The global analysis of throughput is largely the same as that of the saturated system given the two approximations of the proxy system described in Section II-A. That is, (1) and (2) remain valid with the replacements of $S_0$ by $S_E$ and $G_0$ by $G_0$, where $G_0 = N p_k$ is the transmission attempt rate of the overall system when the offered load (throughput) is $S_0$. Parallel to (2), we have

$$S_0 = G_0 \left( 1 - \frac{G_0}{N} \right)^{N-1} = G_0 e^{-C_0 N} \text{ as } N \to \infty$$ (7)

where $G_0 = N p_k$ is the transmission attempt rate of the overall system when the offered load is $S_0$.

B. Local Analysis

The local analysis is more complicated than that in the saturated case since we need to consider the queuing dynamic at a node, not just the HOL contention dynamic. For Poisson arrival, a packet of a local queue generally arrives between the boundaries of two adjacent time slots. If it arrives to an empty queue, it must wait until the beginning of the next time slot before it can contend for transmission. Conceptually, it does not enter the HOL until the next time slot. It turns out that this local queue specification fits under the M/G/1 multiple-vacation queue model [10], as elaborated in the next paragraph. The intricate part of our analysis is in deriving the service-time distribution and the vacation-time distribution of the Aloha system to substitute into the equations of the M/G/1 vacation queue.

In the multiple-vacation queue model [10], the server leaves for a vacation when the queue becomes empty. The vacation length is a random variable $V$. Upon returning from a vacation, if the queue remains empty, the server immediately departs for another vacation. When a packet arrives to an empty queue in the Aloha network, the time until the beginning of the next time slot is part of the vacation time taken by the server. For slotted Aloha, the vacation time is fixed and equal to one slot time. The access delay incurred by a packet at the HOL corresponds to the service time of the M/G/1 vacation queue model.

For notation purposes, in the following, $F(z) = \sum_{i=0}^{\infty} \Pr [F = i] z^i$ denotes the $z$-transform of a discrete nonnegative random variable $F$, and $G*(s) = \int_0^\infty f_0(x) e^{-sx} dx$ denotes the Laplace transform of a continuous nonnegative random variable $G$. The M/G/1 vacation queue has the following solution:

$$Q(z) = \frac{(1 - \lambda_0 \bar{X})}{\lambda V} \frac{X* (\lambda_0 (1 - z)) [V^* (\lambda_0 (1 - z)) - 1]}{z - X* (\lambda_0 (1 - z))}$$

$$D*(s) = Q (1 - s / \lambda_0) = \frac{(1 - \lambda_0 \bar{X})}{V} \frac{X* (s) [V* (s) - 1]}{\lambda - s - \lambda_0 X* (s)}$$ (8)

where $Q$ is number of packets in the queue including the HOL packet; $D$ is queuing delay including the service time; $X$ is service time of a packet; $V$ is vacation time taken by the server when the queue is empty.

The expressions in (8) are generic expressions relating $Q$ and $D$ to $X$ and $V$. To use (8), however, we need to derive the distributions of $X$ and $V$ specific to our system. For slotted Aloha, each vacation lasts exactly one time slot, so that

$$V* (s) = e^{-s}$$ (9)

Recall that an approximation in the proxy system $p_k$ is that $p_k$ is constant and independent of the local state. We now derive $X$ in terms of $p_k$. Mathematically, the Laplace transform $X* (s)$ in (8) is related to the $z$-transform $X (z)$ by

$$X* (s) \equiv X (e^{-s})$$ (10)

To derive $X (z)$, let $C$ be the number of collisions experienced by a HOL packet before it is successfully transmitted. By conditional-probability argument, we have (11), shown at the bottom of the page. Thus,

$$X (z) = \sum_{k=0}^{\infty} (1 - p_k) p_k^k \prod_{j=0}^{k} \frac{z}{r_0 r_j - (r_0 r_j - 1) z}$$ (12)
Equations (8), (9), (10), and (12) allow us to derive moments of $D$ in terms of $r_0$, $r$, $p_c$. For the first moment $E[D]$, after some equation crunching, we can get (13), shown at the bottom of the page. We note that [11] independently obtained $X'(1)$ and $X''(1)$ for the $r_0 = 1$ case. Let us next consider the implications of (13).

**Bounded Mean-Delay Conditions:** We focus on the conditions to ensure bounded mean delay in the following. As mentioned above, higher moments of delay, such as delay variance, can also be obtained from (8)–(10) and (12) in principle. If desired, an argument similar to that below can also yield the conditions for bounded delay variance.

From (13), convergence of $E[D]$ requires $p_c r < 1$, $p_c r + \lambda_0 r_0 < 1$ and $p_c r^2 < 1$, but the first inequality is satisfied if the second is and can be eliminated. Thus, we have the following conditions for bounded $E[D]$:

$$p_c r + \lambda_0 r_0 = p_c r + \frac{\mu_0 S_0}{N} < 1 \quad \text{and} \quad p_c r^2 < 1.$$  \hspace{1cm} (14)

Note that at equilibrium, the mean service time is $X'(1) = r_0/(1 - p_c r)$. Applying Little’s law and the physical requirement that the average HOL occupancy is less than 1 when the system is nonsaturated, we have $\lambda_0 r_0/(1 - p_c r) < 1$, which is the same as the first inequality in (14). Thus, the first inequality is also the condition for nonsaturation.

The analysis thus far assumes steady-state equilibrium can be achieved. For a queuing system, steady state can be achieved if and only if $\Pr(Q = 0) > 0$ (see [12]). Since $\Pr(Q = 0) > 0$ means the queue is not saturated, the first inequality of (14) is also the necessary and sufficient condition for steady-state operation. In other words, nonsaturated operation is the same as steady-state operation.

The second inequality in (14) arises from the requirement to bound $V_{q(t)}(X) = X'(1) + X''(1)$ in (13). We note that unbounded $E[D]$ does not automatically imply that the system is saturated, although the converse is true. To see this, consider a hypothetical distribution of $Q$ that does not decay fast enough: $\Pr(Q = 0) = 1/2$, $\Pr(Q = i) = 3/(\pi i)^2$ for $i \geq 1$. It is easy to see that $\Pr(Q = i) = 1/2$, $\Pr(Q = 0) = 3/(\pi i)^2$ for $i \geq 1$ (hence, $E[D]$ also) is unbounded, but the system is not saturated because $\Pr(Q = 0) > 0$.

In short, bounded $E[D]$ requires both the system to be nonsaturated [first inequality in (14)] and the variance of the service time to be bounded [second inequality in (14)].

**C. Coupling Analysis**

The coupling analysis also involves many subtopics not present in the saturation case. The local analysis leaves us with (13), where mean delay is expressed in terms of $r$, $r_0$ and $p_c$. We need to use the result from the global analysis to remove the dependency on $p_c$. Numerically, $E[D]$ can be obtained as follows. For a given $S_0$, we compute $G_0$ from $S_0 = G_0(1 - G_0/N)^{N-1}$. We then substitute $p_c = 1 - S_0/G_0$ and $\lambda_0 = S_0/N$ into (13) to find $E[D]$. As elaborated below, a subtlety is that there are two possible solutions for $G_0$. We thus find ourselves in the quandary of having two possible $p_c$, which in turn gives rise to two possible $E[D]$ according to (13). We explore this subtlety below and argue that only one of the two possible $p_c$ is valid upon closer examination.

1) **Quantum Jump of Equilibrium Operating Point:** For exposition purposes, we consider the asymptotic $N \to \infty$ case here. A similar argument applies to the finite-$N$ case. First, we note that for saturated operation, for each fixed $S_0$, there are two possible $G_s$ according to the global-analytical result $S_0 = G_s e^{-G_s}$ from (2). These two $G_s$ correspond to two different backoff factors $r$ according to the coupling-analytical result (6). That is, two different $r$ can be used to achieve the same $S_0$ and they have different $G_s$. Fig. 1 is a pictorial illustration. The two $G_s$ are $G_s$ on the left and $G_s$ on the right, and the corresponding two $r$ are $r_l$ and $r_r$, respectively. From (6), we know that $G_s$ is a decreasing function of $r$, and therefore $r_l \geq r_r$.

Right after (14), we argued that the system must not be saturated in order that equilibrium can be achieved. Suppose that we load the system with $S_0 < S_s$ to ensure nonsaturated operation. Consider the two systems with $r_l$ and $r_r$, respectively. The global $S_0$-versus-$G_0$ and $S_s$-versus-$G_s$ curves have the same
form: \( S = G_o e^{-L_o} \). So, we can overlay the saturation and non-saturation operating points on the same graph, as in Fig. 1. As shown in Fig. 1, for the given \( S_0 < S_\infty \), we could draw a horizontal line below \( S_\infty \) to identify the corresponding \( G_o \). We find that for the given \( S_0 \), we have two possible \( G_o \): \( G_o \) and \( G_o \) with \( G_o < G_o \). Which of them is the “correct” operating point?

It is tempting to jump to the conclusion that in the system with \( r \), \( G_o \) is the operating point, and in the system with \( r \), \( G_o \) is the operating point. After all, this gives a smooth and continuous transition from the two operating points at saturation, \( G_i \) and \( G_i \), as \( S_0 \) is decreased slowly from \( S_\infty \). It turns out that this is not the case. As argued below, when the system is not saturated, the operating point is \( G_o \) for both \( r \) and \( r \); the operating point \( G_o \) is not tenable for either \( r \) or \( r \).

For the systems with \( r \) and \( r \), by definition their \( p \) at saturation are \( p = 1 - S_0 \) at the “potential” operating points \((G_o, S_0)\) and \((G_o, S_0)\) are \( p = 1 - S_0 \) at saturation. From the second line of (3), \( p \) at saturation. Thus, we have \( (1 - S_0) = 1 \) for \( r \) and \( r \), respectively.

At offered load \( S_0 \), the \( p \) at the “potential” operating points \((G_o, S_0)\) and \((G_o, S_0)\) are \( p = 1 - S_0 \) and \( p = 1 - S_0 \), respectively. If \((G_o, S_0)\) were the operating point under \( r \) and \( r \), we would respectively have the following:

\[
\begin{align*}
\text{under } r: & \quad (1 - S_0) r = 1 - S_0 G_i r = 1 \\
\text{under } r: & \quad (1 - S_0) r = 1 - S_0 G_i r = 1.
\end{align*}
\]

The inequalities in (15) can be seen as follows. Since \( G_o \) and \( G_i \), we have \( S_0 G_i = e^{-G_o} \) and \( S_0 G_i = e^{-G_o} \). The inequalities in (15) imply that \((G_o, S_0)\) cannot be the operating point under \( r \) or \( r \), because \( p \) \( r \) and \( p \) \( r \) violate the condition for nonsaturated and equilibrium operation (see (13) and the argument leading to (14) and thereafter).

By contrast, the operating point at \((G_o, S_0)\) satisfies \( p \) \( r \) for both \( r \) and \( r \), as seen from

\[
\begin{align*}
\text{under } r: & \quad (1 - S_0) r = 1 - S_0 G_i r = 1 \\
\text{under } r: & \quad (1 - S_0) r = 1 - S_0 G_i r = 1.
\end{align*}
\]

We therefore conclude that the correct \( G_o \) is the smaller of the two possible solutions to \( S_0 = G_o e^{-G_o} \). Note in particular that it does not matter what \( r \) is. The value of \( r \) only determines the saturation throughput \( S_\infty \). As long as we load the system with an offered load \( S_0 \) smaller than \( S_\infty \), \( G_o \) is independent of \( r \).

In Fig. 1, note also that for \( r \), as we decrease \( S_0 \) from \( S_\infty \) to \( S_\infty \) (i.e., moving from saturation operation to non-saturation operation), there is a quantum jump in the transmission attempt rate from \( G_i \) to \( G_o \) (hence, from \( p = 1 - S_0 / G_i \) to \( p = 1 - S_0 / G_o \)), as illustrated by the arrow in Fig. 1. We have performed simulations on the “real system” to verify this analytical conclusion. Fig. 2 shows the simulation results in which \((N, r) = (20, 10)\) and \( r = 1.2, 1.6, 2.0, \ldots, 1.2 \) which corresponds to \( r \). The right curve is the saturation throughput \( S_\infty \) versus-\( G_i \) curve. The left curve is the \( S_\infty \)-versus-\( G_i \) curve when we load the network with \( S_0 = 0.9S_\infty \) for each of the \( r \). The quantum jumps predicted analytically by the proxy system are obvious from the simulation results of the real system. We summarize our finding in Observation 1.

### Observation 1

For a given set of system parameters \( r, N, G \), if the resulting \((G, S_\infty)\) lies to the left of the peak of the \( S-G \) curve in (2), then the feasible nonsaturated operating region is all points \((G, S)\) to the left of \((G, S_\infty)\) on the \( S-G \) curve. On the other hand, if \((G, S_\infty)\) lies to the right of the peak of the \( S-G \) curve in (2), then the feasible nonsaturated operating region is all points to the left of \((G, S_\infty)\), where \((G, S)\) is the point to the left of the peak with the same saturation throughput \( S_\infty \).

2) Safe-Bounded-Mean-Delay Throughput \( 0_{SBM} \): It turns out that there are further subtleties in the coupling analysis. Although having \( S_0 < S_\infty \) will ensure nonsaturated operation, depending on the system parameters, this may or may not be sufficient for ensuring bounded-delay operation. That is, the feasible region established in Observation 1 pertains to nonsaturated operation only. To bound mean delay, we explain in the following that additionally \( S_0 \) cannot exceed another value, \( 0_{SBM} \), which we refer to as the boundary bounded-mean-delay throughput. Specifically, \( S_0 \) must be smaller than the minimum of \( S_\infty \) and \( 0_{SBM} \). We refer to \( 0_{SBM} = \min[S_\infty, S_0] \) as the safe bounded-mean-delay throughput.

The practical significance is as follows. If we load the system with \( S_0 > 0_{SBM} \), then the system will have unbounded mean delay. When that happens, one or both of the following may occur: 1) Queues may become saturated; and/or 2) different queues may experience widely different performance even though they all operate the same protocol and have the same homogeneous offered load \( S_0 \). Issue 2 will be discussed in more detail under the context of “starvation” in Section V. We first expound on the concept of \( 0_{SBM} \).
on the

there is the danger of the system running into

Again, with

The mechanic of the argument is

Indeed, what we observed was the

and cannot recover. On the

case. The mechanic of the argument is

To see the validity of Observation 2, note that

Fig. 3. Relative positions of \((G_{\text{BMD}}, S_{\text{BMD}})\) and \((G_{\ell}, S_{\ell})\) on the \(S = G e^{-G}\) curve and the associated feasible regions for bounded mean-delay, non-saturated operation (darkened lines) for (a) \(r = 2\), (b) \(r = 1.425\), (c) \(r = 1.3\), and (d) \(r = 1.125\).

We define the SBMD throughput as follows to correspond to

Recall that (14) is obtained from local analysis, and therefore

and (17) and (18) are outcomes of local analysis. The local analytical results (17) and (18) dictate which of the operating points on the global-analytical curve \(S_0 = G_0 e^{-G_0}\) are feasible and which are not for bounded mean-delay operation.

Observation 2: For a given \(r\), bounded mean delay requires the operating point \((G_{\ell}, S_{\ell})\) to lie to the left of \((G_{\text{BMD}}, S_{\text{BMD}})\) on the \(S_0 = G_0 e^{-G_0}\) curve.

To see the validity of Observation 2, note that \(p_1 = 1 - e^{-G_0}\), and therefore \(p_1\) increases with \(G_0\). Thus, for a given \(r\), in order for an operating point \((G_{\ell}, S_{\ell})\) to have \(p_1 r^2 < 1\), it must lie to the left of \((G_{\text{BMD}}, S_{\text{BMD}})\), where \(p_1 r^2 = 1\).

Observation 3: For a given \(r\), \((G_{\ell}, S_{\ell})\) is always to the right of \((G_{\text{BMD}}, S_{\text{BMD}})\) on the \(S = G e^{-G}\) curve.

To see the validity of Observation 3, note from (6) and (18) that \(G_{\ell} = \ln(r/r - 1) > \ln(r^2/(r - 1)(r + 1)) = G_{\text{BMD}}\).

To identify the feasible region for bounded mean delay and nonsaturated operation, in Fig. 3(a)–(d), we trace the movement of \((G_{\text{BMD}}, S_{\text{BMD}})\) according to (17) and (18) and the movement of \((G_{\ell}, S_{\ell})\) according to (6), as \(r\) decreases. Both points move to the right as \(r\) decreases. The darkened lines in Fig. 3(a)–(d) correspond to the feasible operating regions. We explain each of the four cases here.

In Fig. 3(a), both \((G_{\text{BMD}}, S_{\text{BMD}})\) and \((G_{\ell}, S_{\ell})\) are to the left of the peak of the \(S-G\) curve, with \(S_{\text{BMD}} < S_{\ell}\). According to Observation 2, the feasible region for bounded mean-delay operation is to the left of \((G_{\text{BMD}}, S_{\text{BMD}})\), as shown in the figure. According to Observation 1, this region is also within the nonsaturated operating region. Overall, bounded mean-delay and nonsaturated operation can be ensured by limiting the offered load \(S_0 < S_{\text{BMD}}\). For \(S_0\) between \(S_{\text{BMD}}\) and \(S_\ell\), the system is nonsaturated but the mean delay is unbounded.

As \(r\) decreases, we have the situation in Fig. 3(b), where \((G_{\text{BMD}}, S_{\text{BMD}})\) is to the left and \((G_{\ell}, S_{\ell})\) is to the right of the peak of the \(S-G\) curve, with \(S_{\text{BMD}} > S_{\ell}\). Again, with the same argument as for Fig. 3(a), nonsaturation and bounded mean delay can be ensured by limiting \(S_0 < S_{\text{BMD}}\). Also, for \(S_0\) between \(S_{\text{BMD}}\) and \(S_\ell\), the system is nonsaturated, but the mean delay is unbounded.

As \(r\) decreases further, we have the situation in Fig. 3(c), where \((G_{\text{BMD}}, S_{\text{BMD}})\) is to the left and \((G_{\ell}, S_{\ell})\) is to the right of the peak, but \(S_{\text{BMD}} > S_{\ell}\). Decreasing \(r\) even further leads us to Fig. 3(d), where both \((G_{\text{BMD}}, S_{\text{BMD}})\) and \((G_{\ell}, S_{\ell})\) are to the right of the peak, with \(S_{\text{BMD}} > S_{\ell}\). For both of these cases, limiting \(S_0 < S_{\ell}\) will ensure nonsaturation and bounded mean delay.

For the two cases in Fig. 3(c) and (d), it may appear at first glance that we could load the system with \(S_0 > S_{\ell}\) and even \(S_0 > S_{\text{BMD}}\) while ensuring bounded mean-delay operation. To see this argument, suppose that we have an \(S_0\) as shown in Fig. 3(d). According to the argument immediately below Observation 2, at this \((G_{\ell}, S_0)\), \(p_1 r^2 < 1\), satisfying the bounded mean-delay condition. In the following, we argue that it is in fact not “safe” to load the system with \(S_0 > S_{\ell}\).

When \(S_0 > S_{\ell}\), there is the danger of the system running into saturation, at which point \(E[D]\) will go to infinity because the saturation throughput \(S_\ell\) cannot keep up with the input rate \(S_0\). That is, the equilibrium of the system as assumed in our local analysis in Section III-C does not apply anymore. In a simulation experiment, for a situation such as that depicted in Fig. 3(d), we intentionally caused the system to go into saturation with a sudden increase in the offered load and then decreased the offered load back to the \(S_0\) shown in the figure. The simulation results show that \(E[D]\) becomes unbounded thereafter. In other words, such a \(S_0\), which is larger than \(S_{\ell}\), is not a “safe” offered load, and it is obtained with an \textit{a priori} assumption of equilibrium and nonsaturation. If the system is already in saturation, \(E[D]\) is unbounded for such a \(S_0\) and cannot recover. On the other hand, in the simulation experiment, if we decreased the offered load further to below \(S_{\ell}\), then the system did clear up and \(E[D]\) became bounded. Indeed, what we observed was the “quantum-jump” phenomenon discussed in Section III-C1 as \(S_0\) crosses \(S_{\ell}\). Thus, \(S_0 < S_{\ell}\) is safe.

Combining the descriptions of all four cases above, we arrive at Observation 4.

Observation 4: The feasible region for \((G_{\ell}, S_0)\) in terms of bounded mean-delay and nonsaturated operation is the intersection of the two feasible regions in Observations 1 and 2.

We define the SBMD throughput as follows to correspond to Observation 4:

\[ S_{\text{BMD}}(r) = \min \left\{ S_{\text{BMD}}^2, S_{\ell}(r) \right\} \]

Finite-\(N\) Case \((S_{\text{BMD}})\) as Function of \(r\), \(r_0\), \(N\): We now consider the finite-\(N\) case. The mechanic of the argument is
similar to the $N \to \infty$ case. It can be shown that Observations 1, 2, and 4 remain intact on the finite-$N$ $S$-$G$ curve, $S = G(1 - G/N)^{N-1}$. However, Observation 3 may not be valid, as explained in the next paragraph. As a result, we cannot simply say $S_{\text{BM}}(r, r_0, N) = \min\{S_{\text{BM}}(r, r_0, N), S_0(r, r_0, N)\}$. Nevertheless, Observation 4 can still be used to identify the feasible region for $S_S$.

For finite $N$, the three equations $S_0 = G_0(1 - G_0/N)^{N-1}$, $p_c r^2 = 1$, and $p_o = 1 - S_0/G_0$ yield $S_{\text{BM}} = N(1 - 1/r^2) 1 - (1 - 1/r^2)^{1/(N-1)}$. Meanwhile, $S_{\text{BM}}(r, r_0, N)$ has no closed form but can be found numerically from (5). Numerically, we find that as $r$ decreases, it is possible for $(G_{\text{BM}}, S_{\text{BM}})$ to overtake $(G_S, S_S)$ so that it moves to the right of $(G_S, S_S)$. With respect to the situation in Fig. 3(d), if $(G_{\text{BM}}, S_{\text{BM}})$ is to the right of $(G_S, S_S)$, it is also below $(G_S, S_S)$. As a result, the intersected feasible region mentioned in Observation 4 includes the region where $S_{\text{BM}} \leq S_0 < S_S$ in addition to the region where $S_0 < S_{\text{BM}}$. In this case, $S_{\text{BM}}(r, r_0, N) = S_0(r, r_0, N)$, rather than $S_{\text{BM}}(r, r_0, N) = \min\{S_{\text{BM}}(r, r_0, N), S_0(r, r_0, N)\}$.

IV. EFFECTS OF BACKOFF FACTOR $r$

The analysis in the preceding section hinted that the backoff factor $r$ may have a significant impact on the system performance. This section is devoted to a detailed study of the effect of $r$.

A. Maximum SBM Throughput $r$

Let us now examine how $S_{\text{BM}}$ varies as $r$ is varied. We focus on the asymptotic $N \to \infty$ case here. Similar argument applies to the nonasymptotic case although the equations are more complicated. Fig. 4 plots $S_{\text{BM}}(r, r_0, N)$, $S_0(r, r_0, N)$, and $S_0(r)$ versus $r$ according to (17), (18), and (19).

For $r > 1.3757$, $S_{\text{BM}}(r, r_0, N) < S_0(r)$, and for $r \leq 1.3757$, $S_{\text{BM}}(r, r_0, N) \geq S_0(r)$. Specifically, the $r$ that maximizes $S_{\text{BM}}(r)$ is $r_{\text{opt}}^{\text{BM}} = 1.3757$, which is obtained by setting $S_{\text{BM}}(r) = S_0(r)$.

\[
\frac{r_{\text{opt}}^{\text{BM}}}{r_{\text{opt}}^{\text{BM}} - 1} \ln \left( \frac{r_{\text{opt}}^{\text{BM}}}{r_{\text{opt}}^{\text{BM}} - 1} \right) = \frac{r_{\text{opt}}^{\text{BM}}}{r_{\text{opt}}^{\text{BM}} - 1} \ln \left( \frac{r_{\text{opt}}^{\text{BM}}}{r_{\text{opt}}^{\text{BM}} - 1} \right) \cdot (20)
\]

Note that $r_{\text{opt}}^{\text{BM}} \neq r_{\text{opt}}^{\text{BM}} = e/(e-1)$, where the $r_{\text{opt}}^{\text{BM}}$ is the value of $r$ that maximizes the saturation throughput $S_0(r)$. The maximum saturation throughput is $S_0 = S_0(r_{\text{opt}}^{\text{BM}}) = e^{-1} = 0.3679$. However, $S_{\text{BM}}(r_{\text{opt}}) = 0.3636$, which is 17% below $S_0$. That is, if we set $r = r_{\text{opt}}^{\text{BM}}$, $S_0$ must be at least 17% below $S_0$ to ensure bounded-delay operation.

The binary backoff factor of $r = 2$ is assumed in the majority of prior work and in many practical multiple-access networks such as the Ethernet and WiFi. For slotted Aloha, the corresponding saturation throughput $S_0(2) = 0.3666$ is reasonably close to $S_0 = 0.3679$, and one could hardly raise objection to adopting $r = 2$ on the basis of saturation throughput. However, if bounded mean delay is desired, we have $S_{\text{BM}}(2) = 0.2158$. That is, there is a drastic 41% penalty with respect to $S_0$. Therefore, $r = 2$ is a bad choice from the delay consideration.

Fortunately, the maximum SBM throughput, obtained by setting $r = r_{\text{opt}}^{\text{BM}} = 1.3757$, is rather close to $S_0$. Specifically, $S_{\text{BM}} = S_{\text{BM}}(r_{\text{opt}}^{\text{BM}}) = 0.3545$. The penalty with respect to $S_0$ is only less than 4%. Overall, we conclude that using the proper $r$ is important to ensuring a good throughput under the bounded-delay requirement, perhaps more so than when saturation throughput is the only concern. This can be seen from Fig. 4, which shows that $S_{\text{BM}}(r)$ rises and falls much more sharply with $r$ than $S_0(r)$ does.

B. Mean Delay Versus Offered Load

Fig. 5 plots $E[D]$ versus $S_0$ for the case of $N \geq 30$. Numerically, $E[D]$ is obtained as follows. For a given $S_0$, we compute $G_0$ from $S_0 = G_0(1 - G_0/N)^{N-1}$. Recall from the discussion in Section III-C1 that this will yield two solutions, $G_{\alpha_1}$ and $G_{\alpha_2}$, but that the smaller $G_{\alpha_1}$ is the correct operating point. We substitute $p_c = (G_{\alpha_1} - S_0)/G_{\alpha_1}$ and $\lambda_0 = S_0/N$ into (13) to find $E[D]$.

In Fig. 5(a), $(r_0, r, N) = (10, 1.5, 82, 30)$. For this case, $S_{\text{BM}} = 0.3140 < 0.3675 = S_0$. This case corresponds to the situation in Fig. 3(a). $S_{\text{BM}}$ is limited by $S_{\text{BM}}$ rather than the saturation throughput $S_0$. The solid line in Fig. 5(a) is the result from analysis. The cross points are simulation results of the proxy system (see definition in Section II-A) in which the dynamic of a single node is simulated with fixed $p_c = (G_{\alpha_1} - S_0)/G_{\alpha_1}$ computed numerically. The dotted points are simulation results of the real system with 30 queues. The results are consistent in that, for offered load $S_0$ near $S_{\text{BM}}$, $E[D]$ begins to build up quickly.

An interesting observation is that near $S_{\text{BM}}$, the simulated $E[D]$ does not converge in either the proxy or the real system. The fact that the simulation results of the proxy system can fluctuate below and above the numerical results of the proxy system is noteworthy, although the simulation experiment simulates exactly the same proxy system as that in the analysis. In other words, this nonconvergence is not due to the proxy system not being able to approximate the real system well. On the contrary, the proxy system suggests that similar nonconvergence may happen in the real system, which is borne out by our simulation results. In fact, for the same $S_0$ near $S_{\text{BM}}$, different simulation runs will produce rather different $E[D]$ even if we let each run last a long time. The underlying cause of such nonconvergence will be further discussed in Section V-B.
to jolt the system into saturation, and then decreased $S_0$ back to the original value $S_0 > S_{BBMD}$. The system did not get out of saturation, $E[D]$ became unbounded, and (14) is not satisfied thereafter. That is, the bounded $E[D]$ as in Fig. 5(b) would elude us once the system is saturated. Even if we did not jolt the system into saturation as aforementioned, the system may eventually evolve to the saturation state with a constant $S_0 > S_{BBMD}$. How soon it does depends on how close $S_0$ is to the peak of the $S$-$G$ curve. The intricate dynamic on how long the system can remain stable at an offered load above $S_{BBMD}$ is an interesting subject for further research.

V. Starvation

Starvation occurs when some nodes do not get to transmit their packets for an exceedingly long time. This may happen, for example, when the nodes back off exponentially to a large backoff stage. Other nodes with a smaller backoff stage will then hog the channel. As far as we know, the “qualitative” observation of the starvation phenomenon was first made in [2] (although under a different backoff protocol). The authors attributed the discrepancy between their simulation and analytical results to starvation. Left open are three major outstanding issues.

1) What is the appropriate “quantitative” definition of starvation? To study starvation systematically, we need a starvation metric that is measurable, much like delay is measurable.

2) Why does starvation lead to a discrepancy between simulation and analytical results? What is the root cause of this phenomenon?

3) How are system parameters $r$, $r_0$, $N$, $S_0$ in our system model related to starvation quantitatively?

Sections V-A, V-B, and V-C address issues 1), 2), and 3), respectively.

A. Definition of Starvation

Fundamentally, starvation is related to HOL service. There is a vague notion that when a HOL packet does not receive service for a long time, the associated queue is then starved. Thus, an attempt to define starvation quantitatively could focus on the property of the HOL service time $X$.

Consider all the busy times of all nodes. Suppose that we randomly choose a node and a point within its busy time to observe the service time of the HOL packet into which the random point falls. Then, the random variable that we observe is not $X$. It is another random variable $Y$, whose probability distribution $Pr(Y = y)$ is related to the probability distribution of $X$, $Pr_X(x) = Pr[X = x]$ by $Pr_Y(y) = yPr_X(y)/E[X]$. The weight $y$ is due to the fact that the random point we sample is proportionately more likely to fall within a long service time than a short service time. The denominator $E[X]$ is a normalization factor so that $Pr_Y(y)$ sums up to one. This “node-centric” sampling makes sense as far as starvation is concerned since we are interested in whether a busy node is suffering from a long service time at a randomly chosen time.

A number of definitions of starvation around $Y$ are possible. For example, we could say that there is no starvation if and only if $Pr[Y > y_{target}] < \varepsilon$ for some $y_{target} > 0$ and $\varepsilon > 0$. Another possibility is $E[Y] < \sum target$ for some target mean $\sum target > 0$.  

We have also simulated the setting $(r_0, r, N) = (10, 1.200, 30)$. For this case, $S_{BBMD} = 0.3762 > 0.3561 = S_0$, and $S_{BBMD}$ is limited by $S_0$ rather than $S_{BBMD}$. This is a rather subtle case corresponding to Fig. 3(d), in which both $S_{BBMD}$ and $S_0$ are to the right of the peak of the $S$-$G$ curve. It is possible to load the system with $S_0$ above $S_{BBMD}$ and yet satisfy the convergence condition as dictated by (14). The analytical and simulation results in Fig. 5(b) confirm that. Such an $S_0 > S_{BBMD}$ that has a finite $E[D]$ in Fig. 5(b), however, may be “unstable” in another sense. In a simulation experiment, we used an even larger $S_0$.

Fig. 5. $E[D]$ versus $S_0$, for (a) $(r_0, r, N) = (10, 1.582, 30)$ and (b) $(r_0, r, N) = (10, 1.200, 30)$.
For the rest of this paper, we adopt the simple definition that requires \(E[X^2]\) to be finite.

**Definition of Nonstarvation:** A system is nonstarved if and only if \(E[Y]\) (hence, \(E[X^2]\)) is finite.

That \(E[Y]\) is finite does not mean it is small. The implicit understanding behind this definition is that for whatever condition we come up with that can meet the finite \(E[Y]\) requirement, we need to use a condition that is somewhat tighter in actual implementation. This is analogous to the definition of \(S_{\text{SBMMD}}\) where we need to make \(S_0\) smaller than \(S_{\text{SBMMD}}\) by a sufficient margin if we want to meet certain targeted mean delay (i.e., we cannot simply set \(S_0 = S_{\text{SBMMD}}\)).

With this definition, we can now relate the condition for nonstarvation to the condition for bounded mean delay in a non-saturated system. Mathematically, it can be easily shown from \(P_\gamma(y) = yP_\gamma(y)/E[X]\) that \(E[Y]\) is bounded if and only if \(E[X^2]\) is bounded. According to (13), if \(E[D]\) is not bounded, then \(E[X^2]\) is also not bounded. The practical significance and interpretation is as follows. When \(E[X^2]\) is large, not only will the delay performance be bad, but the performance among different nodes may also vary widely because some are starved while others are not.

Our definition of starvation allows us to unite the notions of nonstarved operation and bounded mean-delay operation since a root cause giving rise to both of them is the same: large \(E[X^2]\).

**B. Starvation and Nonconvergence of Simulations**

This section explores why nonconvergence of simulation results happens to occur whenever the system is starved, a phenomenon observed in [2] as well as in our simulation experiments. Underlying this phenomenon is a fundamental cause: the immeasurability of performance when starvation occurs, as explained in the following.

**Saturated Case:** Starvation can occur in a saturated or non-saturated system. We first focus on the saturated case. Suppose we want to measure the average service time \(E[X]\) at saturation (note: \(S_0 = 1/E[X]\) by Little’s law). In the following, we argue that, for a starved system, \(E[X]\) cannot be estimated accurately. For our measurement, imagine that we perform \(m\) experiments, \(m \gg 1\). Each experiment \(j \in \{1, 2, \ldots, m\}\) is conducted over a long time so that we could gather the HOL service times of \(n_p \gg 1\) packets of a particular queue. For each trace \(j\), we can compute the average service time as

\[
\bar{X}_j = \frac{\sum_{i=1}^{n_p} X_{j,i}}{n_p}, \quad j = 1, \ldots, m
\]

(21)

where \(X_{j,i}\) is sample \(i\) of trace \(j\). From the large set of \(m\) experiments, we have \(m\) samples of \(\bar{X}_j\) from (21). From the samples, we can then construct the probability density of \(\bar{X}_j\). Let us make \(n_p\) very large for each of the experiments. We wish that the Law of Large Numbers would then apply, and the spread of this density would then become very narrow. If so, we could estimate \(E[X]\) accurately by defining \(E[X] = \bar{X}_j\) for any \(j\) since \(\bar{X}_j\) for different \(j\) converges. If not, we really do not know which \(\bar{X}_j\) is to be believed, and a definitive measure of \(E[X]\) would elude us. Note the caveat that if \(\bar{X}_j\) does not converge as \(n_p\) increases, \(E[X]\) alternatively defined as \(E[X] = (\sum_j \bar{X}_j)/m\) does not converge either since this is equivalent to increasing the sample size \(n_p\), which does not help.

We show in the following that if the system is starved and \(E[X^2]\) is unbounded, then \(E[\bar{X}_j^2]\) is unbounded. Hence, \(f_{\bar{X}_j} (\bar{x})\) does not “narrow” with large \(n_p\). The expectation in (22) is the ensemble average over a large number of experiments.

\[
E[\bar{X}_j^2] = \frac{1}{n_p^2} E \left[ \left( \sum_{i=1}^{n_p} X_{j,i} \right)^2 \right] 
\geq \frac{1}{n_p} E \left[ \sum_{i=1}^{n_p} X_{j,i}^2 \right] = \frac{E[X^2]}{n_p},
\]

(22)

Thus, \(E[\bar{X}_j^2]\) is unbounded if \(E[X^2]\) is unbounded. Of course, in experiments, our measurement is time-limited by the duration of our experiment, and we will not observe \(\bar{X}_j^2\) to be infinite. Nevertheless, the above points out that it is likely that \(\bar{X}_j\) will not converge in experiments.

Fig. 6 presents our experimental results. We set \((\tau_0, \tau, N) = (10, 1.5, 82, 15)\), a starved case where \(E[X^2]\) is unbounded. From Fig. 6(a) and (b), we observe the throughputs of different queues are quite different, and \(\bar{X}_j\) converges to a common value as \(n_p\) increases. For the same reason that \(\tau_1\) and \(\tau_2\) do not converge, neither does the average throughput of measured performance results are intricately tied, and we observe the throughputs of different queues are quite different even if we average the throughputs over a long stretch of time. Unfairness tends to persist.

In summary, the phenomena of starvation and nonconvergence of measured performance results are intricately tied, and they have the same root cause: unbounded \(E[X^2]\).

**Nonstarved Case:** The above has focused on the saturated case. Nonconvergence also occurs in the nonstarved case. In the nonstarved case, the offered load \(S_0\) is a factor as to whether starvation occurs.

Besides the nonconvergence of measured \(E[X]\), which occurs when \(E[X^2]\) is unbounded, the measured \(E[D]\) may not converge either. For the same reason that \(E[X^2]\) is \(\infty\) does not allow converged measurement of \(E[X]\), \(E[D^2]\) is \(\infty\) does not allow converged measurement of \(E[D]\) either. It can be shown from (8) that \(E[D^2]\) goes to infinity before \(E[X^2]\) does (omitted here to conserve space). This is borne out by Fig. 5(a), in which the measured \(E[D]\) begins to diverge before \(S_0\) reaches \(S_{\text{SBMMD}}\). Again, the nonconvergence of the measured \(E[D]\) has nothing to do with the inequality of the proxy system with respect to the real system. Even for the proxy-system simulation,
as shown in Fig. 5(a), there is a spread in the measured $E[D]$ due to the fundamental reason of immeasurability.

C. Impact of System Parameters on Starvation

We now investigate how system parameters affect starvation.

Saturated Case: For the study of the saturated case, we note that the expression of $X(z)$ in (12) for the nonsaturated case is also valid for the saturated case because it is parameterized on $p_c$. Following (12), (14) indicates that bounded $E[X^2]$ requires $p_c r^2 < 1$. We just need to be careful to substitute the $p_c$ obtained from the global analysis of the saturated case rather than that from the nonsaturated analysis.

For fixed $r$, $r_0$, it turns out that starvation sets in when the number of nodes $N$ is beyond a certain value. Here, we are interested in this critical value of $N$. It can be shown from (4) that $N$ is an increasing function of $p_c$ for $r > 1$. Rearranging (4), we have

$$N = 1 + \frac{\ln(1 - p_c)}{\ln(1 - \frac{1}{r_0(1 - p_c)})}.$$  \hfill (23)

Substituting $p_c < 1/r^2$ (condition for bounded $E[X^2]$) gives

$$N < 1 + \frac{\ln(1 - 1/r^2)}{\ln(1 - \frac{1}{r_0(1 - 1/r^2)})}$$

$$= \frac{\ln \left( \frac{r}{r_0} \right) - \ln \left( 1 + \frac{1}{r_0} - \frac{1}{r} \right)}{\ln \left( \frac{r+1}{r} \right) - \ln \left( 1 + \frac{1}{r} - \frac{1}{r_0} \right)} \triangleq N^*_s.$$  \hfill (24)

where $N^*_s$ is the critical value we seek. Note that $N^*_s$ increases with $r_0$ but decreases with $r$.

To illustrate the phenomenon of starvation, we present in Fig. 7 a simulation trace of a real system with $(r_0, r, N) = (10, 1.2, 30)$. According to (24), this parameter setting will result in starvation. We simulated a total of 20 million time slots, and examined one particular node. Specifically, we looked at the number of cleared packets of the node within each time window of $7500 \approx 100N/S_s$ slots. Thus, the expected number of cleared packets per time window is 100. Fig. 7 plots the number of cleared packets for successive time windows. Note that besides the large spread in the number of cleared packets, there are two occasions during which the node receives no service at all for a very long time. The first occasion lasts for 1.1 million slots, and the second occasion lasts for 0.33 million slots.

Before concluding the discussion here, we would like to point out that the study of the saturated case is particularly relevant to the scenario in which each node is a TCP source. TCP is a greedy transport-layer protocol. For long-lasting TCP applications, such as FTP or P2P file sharing, a TCP connection will attempt to keep the queue at the MAC layer occupied at all
times, thus causing the system to operate in saturation. Relationship (24) allows us to determine the maximum number of active nodes in an Aloha network before starvation sets in and how this number depends on $r$ and $r_0$. When the number of active nodes is too large, some of them will be starved, leading to unfairness. Generally, smaller $r$ is more robust against starvation (see Fig. 8). However, bear in mind that the overall saturation throughput will also go down if $r$ is too small [according to (5)]. Thus, there is a tradeoff between system throughput and fairness. Relationships (5) and (23) allow us to engineer the right balance by tuning $r$ and $r_0$.

**Nonsaturated Case:** For the nonsaturated case, the offered load $S_0$ is a design parameter in addition to $r$, $r_0$, $N$. In general, there is a feasible region for nonstarved operation within the space $(r, r_0, N, S_0)$. Unlike in the saturated case, the additional degree of freedom in $S_0$ in the nonsaturated case allows us to support large $N$. For any $N$, we could make $S_0$ small enough to avoid starvation.

Consider the asymptotic $N \rightarrow \infty$ case, and suppose that we load the system with $S_0 < S_*$ to ensure nonsaturated operation. The feasible region is then governed by $S_0 < S_{\text{SEMD}}(r)$ in (19), which is independent of $r_0$ as well as $N$. Essentially, the feasible region for nonstarved operation is the same as that for bounded mean-delay operation. This observation again ties together the notions of bounded mean delay and nonstarvation. The largest possible offered load for nonstarved operation is therefore $S_{\text{SEMD}}(r) = 0.3545$, obtained when $r = r_{\text{SEMD}} = 1.3757$ (see Section IV-A).

![Fig. 8. $N^*_g$ versus $r_0$ for the case of $r = 1.2, 1.3757,$ and $2.$](image)

VI. CONCLUSION

We have presented an analytical framework for the study of queuing delay and starvation in the slotted Aloha network operated with the exponential backoff protocol. Based on the framework, we have derived the dependency of queuing delay and nonstarved operation on the system parameters, including the backoff factor, the initial transmission probability, and the number of nodes in the network.

With respect to delay performance, we showed that the system-offered load $S_0$ must be below a “safe bounded mean-delay throughput,” $S_{\text{SEMD}}$, in order for the mean delay to be bounded. Specifically, for the case in which the number of nodes is large, the sustainable offered load must be limited as follows:

$$S_0 < S_{\text{SEMD}} \triangleq \min \left[ \frac{r^2}{r^2 - 1} \ln \left( \frac{r^2}{r^2 - 1} \right), \frac{r - 1}{r} \ln \left( \frac{r}{r - 1} \right) \right]$$

(25)

where $r$ is the backoff factor. The first term in the min function is due to the need to bound the service-time variance, and the second term is the saturation throughput. It is worth noting from (25) that $S_{\text{SEMD}}$ is smaller than or equal to the well-known saturation throughput $S_*$. This means that we cannot automatically assume we could load the system with offered load up to the saturation throughput when delay performance is a concern.

With respect to starvation, for a nonsaturated system, we argued that the conditions for bounded mean delay and nonstarvation are one in the same, thus unifying these two notions. For a large Aloha network, for example, limiting the offered load to below the $S_{\text{SEMD}}$ given in (25) can ensure bounded mean-delay and nonstarved operations.

Starvation is also a concern in a saturated system. Saturation can occur, for example, when the applications at the nodes run the TCP transport protocol on top of the MAC protocol. TCP connections, being greedy in nature, will keep the queues occupied at all times, thus saturating the system. Unlike in the nonsaturated case, the number of nodes $N$ rather than the offered load $S_0$ in the saturated case must be limited. The bound on $N$ is given by $N_g^*$

$$N < N_g^* \triangleq \frac{\ln \left( \frac{r_0}{r_0 - 1} \right) - \ln \left( 1 + \frac{1}{r} - \frac{1}{r_0} \right)}{\ln \left( \frac{r + 1}{r} \right) - \ln \left( 1 + \frac{1}{r} - \frac{1}{r_0} \right)}$$

(26)

where $1/r_0$ is the initial transmission probability.

A general conclusion is that delay and nonstarved performance can be very sensitive to the system parameters; indeed, much more so than the saturation throughput is. Careful tuning of the system parameters is important. For example, consider a large Aloha. It is well known that the maximum throughput is $c^{-1} = 0.3679$. The binary backoff factor $r = 2$ is assumed in many prior investigations, and the corresponding saturation throughput is $S_*(2) = 0.3466$, which is close to the maximum of 0.3679. However, if we want to bound mean delay and prevent starvation, according to (25), the offered load $S_0$ must be below $S_{\text{SEMD}}(2) = 0.2158$, a drastic 41% lower than 0.3679. Therefore, setting $r = 2$ is not desirable from the standpoint of good delay and nonstarvation performance, although it may achieve good saturation throughput. By tuning $r$ to $r_{\text{SEMD}} = 1.3757$, $S_{\text{SEMD}}$ can be maximized. The corresponding result is $S_{\text{SEMD}}(1.3757) = 0.3545$, which is less than 4% below 0.3679. Thus, $r = r_{\text{SEMD}}$ allows us to achieve good overall system throughput, good delay performance, and nonstarvation at the same time.

Last but not least, although a main focus of this paper is mean delay, the analytical framework is general enough that higher moments of delay can also be studied using similar procedures propounded in this paper. Specifically, the Laplace Transform of delay in (8) can be used to generate higher moments of delay, and the three-step global-local-coupling analysis in this paper...
can then be used to derive conditions needed to bound the higher moments.

Finally, two natural generalizations of the methods and results in this paper are for carrier-sense multiple-access (CSMA) networks and networks with multiple-packet-reception (MPR) capability [13]. A companion paper of ours [14] is an attempt in that direction.

REFERENCES

[1] N. Abramson, “The Aloha system—Another alternative for computer communication,” in Proc. Fall Joint Comput. Conf., AFIP Conf., 1970, vol. 44, pp. 281–285.
[2] B.-J. Kwak, N.-O. Song, and L. E. Miller, “Performance analysis of exponential backoff,” IEEE/ACM Trans. Netw., vol. 13, no. 2, pp. 343–353, Apr. 2005.
[3] Y. Yang and T.-S. P. Yum, “Delay distribution of slotted Aloha and CSMA,” IEEE Trans. Commun., vol. 51, no. 11, pp. 1846–1857, Nov. 2003.
[4] J. Goodman, A. G. Greenberg, N. Madras, and P. March, “Stability of binary exponential backoff,” JACM, vol. 35, no. , 3, pp. 579–602, Jul. 1988.
[5] F. A. Tobagi, “Distributions of packet delay and interdeparture time in slotted Aloha and Carrier Sense Multiple Access,” JACM, vol. 29, no. 4, pp. 907–927, Oct. 1982.
[6] T. Javidi, M. Liu, and R. Vijayakumar, “Saturation rate in 802.11 revisited.” [Online]. Available: http://www.eecs.umich.edu/techreports/sys-
tems/cspl/cspl-371.pdf
[7] D. P. Bertsekas and R. G. Gallager, Data Networks. New York: Wiley, 1992.
[8] D. Aldous, “Ultimate instability of exponential back-off protocol for acknowledgement-based transmission control of random access communication channels,” IEEE Trans. Inf. Theory, vol. IT-33, no. 2, pp. 219–223, Mar. 1987.
[9] G. Bianchi, “Performance analysis of the IEEE 802.11 distributed coordination function,” IEEE J. Sel. Areas Commun., vol. 18, no. 3, pp. 535–547, Mar. 2000.
[10] B. T. Doshi, “Queueing systems with vacations—a survey,” Queueing Syst., Theory Application, vol. 1, no. 1, 1986.
[11] T. T. Lee and L. Dai, “A statistical theory of wireless networks—Part I. Queueing analysis of spatial interferences,” Chinese Univ. of Hong Kong, 2008, Technical Report.
[12] L. Kleinrock, Queueing Systems. New York: Wiley, 1975, vol. 1, Theory.
[13] P. X. Zheng, Y. J. Zhang, and S. C. Liew, “Multipacket reception in wireless local area networks,” in Proc. IEEE ICC, Jun. 2006, pp. 3670–3675.
[14] Y. J. Zhang, S. C. Liew, and D. R. Chen, “Delay analysis for wireless local area networks with multipacket reception under finite load,” Chinese Univ. of Hong Kong [Online]. Available: http://arxiv.org

Ying Jun (Angela) Zhang (S’01–M’05) received the B.Eng. degree with honors in electronic engineering from Fudan University, Shanghai, China, in 2000 and the Ph.D. degree in electrical and electronic engineering from the Hong Kong University of Science and Technology in 2004.

Since 2005, she has been with the Department of Information Engineering at Hong Kong University of Science and Technology, where she is currently an Assistant Professor. Her research interests include wireless communications and mobile networks, adaptive resource allocation, cross-layer design and optimization, wireless LAN, and MIMO signal processing.

Dr. Zhang won the Hong Kong Young Scientist Award 2006 as the only winner in the category of engineering science. She has served as a TPC Co-Chair of the Communication Theory Symposium of IEEE ICC 2009, Track Chair of ICCCN 2007, and Publicity Chair of IEEE MASS 2007. She is on the editorial boards of IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS and Security and Communications Journal.

Da Rui Chen (S’07) received the B.Eng. and M.Phil. degrees in information engineering from Xi’an Jiaotong University, Xi’an, China, in 2005 and the Chinese University of Hong Kong, Hong Kong, China, in 2007, respectively.

Currently, he is a Research Assistant in the Department of Information Engineering at the Chinese University of Hong Kong. His research interests include mobile and ad hoc networks, cross-layer design, and wireless MAC.