Steep Decay Phase Shaped by the Curvature Effect. II. Spectral Evolution

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Abstract
We derive a simple analytical formula to describe the evolution of spectral index \( \beta \) in the steep decay phase shaped by the curvature effect with the assumption that the spectral parameters and Lorentz factor of the jet shell are the same for different latitudes. Here, the value of \( \beta \) is estimated in the 0.3–10 keV energy band. For a spherical thin shell with a cutoff power-law (CPL) intrinsic radiation spectrum, the spectral evolution can be read as a linear function of observer time. For the situation with the Band function intrinsic radiation spectrum, the spectral evolution may be complex. If the observed break energy of the radiation spectrum is larger than 10 keV, the spectral evolution is the same as that shaped by jet shells with a CPL spectrum. If the observed break energy is less than 0.3 keV, the value of \( \beta \) would be a constant. For others, the spectral evolution can be approximated as a logarithmical function of the observer time in general.

Key words: gamma-ray burst: general

1. Introduction
Gamma-ray bursts (GRBs) are the most powerful electro-magnetic explosions in the universe. The so-called \( \gamma \)-ray prompt emission phase, which always triggers the observation of the Burst Alert Telescope (Barthelmy et al. 2005a), exhibits highly variable and diverse morphologies. The high variabilities in this phase may originate from the central engine activities (e.g., Ouyed et al. 2003; Proga et al. 2003; Lei et al. 2007; Liu et al. 2010; Lin et al. 2016; Zhang et al. 2016), the processes during jet propagation (Aloy et al. 2002; Morsony et al. 2007; Morsony et al. 2010), and the relativistic motion (e.g., mini-jets/turbulence) in the emission region (Lyutikov & Blandford 2003; Yamazaki et al. 2004; Kumar & Narayan 2009; Lazar et al. 2009; Narayan & Kumar 2009; Lin et al. 2013; Zhang & Zhang 2014). Following the prompt emission is a smooth steep decay phase, which is always observed at \( \sim 10^{2}–10^{3} \) s after the burst trigger and in the X-ray band (Vaughan et al. 2006; Cusumano et al. 2006; O’Brien et al. 2006). By extrapolating the prompt \( \gamma \)-ray light curve to the X-ray band, the smooth steep decay phase can connect to this extrapolated X-ray light curve smoothly. Thus, it is believed that the steep decay phase may be the “tail” of the prompt emission (Barthelmy et al. 2005b; Liang et al. 2006; O’Brien et al. 2006). The steep decay phase is also observed in the decay phase of flares (e.g., Jia et al. 2016; Mu et al. 2016; Uhm & Zhang 2016).

For the steep decay phase, the temporal decay index \( \alpha \) of the observed flux \( (F_{\mathrm{obs}} \propto (t_{\mathrm{obs}}^{-\alpha})^{-\alpha}) \) is found to be correlated with the spectral index \( \beta \), where \( t_{\mathrm{obs}} \) is the observer time after the trigger. This led to the development of the “curvature effect” model (Zhang et al. 2006; Yamazaki et al. 2006; Liang et al. 2006; Wu et al. 2006). When emission in a spherical relativistic jet shell ceases abruptly, the observed flux is controlled by high latitude emission of the jet shell. In this situation, the photons from higher latitude would be observed later and have a lower Doppler factor. Then, the observed flux would progressively decrease. For a power-law radiation spectrum \( (F \propto E^{-\beta}) \) with \( \beta = \) constant in the jet shell comoving frame, the relation between \( \alpha \) and \( \beta \) can be found, i.e., \( \alpha = 2 + \beta \) (see Uhm & Zhang 2015 for details; Kumar & Panaitescu 2000; Dermer 2004; Dyks et al. 2005). As shown in Nousek et al. (2006), the above relation is in rough agreement with the data on the steep decay phase of some Swift bursts. Adopting a time-averaged \( \beta \) in the steep decay phases, Liang et al. (2006) finds that \( \alpha = 2 + \beta \) is generally valid. Lin et al. (2017) points out that \( \alpha = 2 + \beta \) is only valid for a power-law radiation spectrum.

The strong evolution of spectral index \( \beta \) is always found in the steep decay phase (Butler & Kocevski 2007; Zhang et al. 2007, 2009; Starling et al. 2008; Mu et al. 2016). Several effects have been done to explain the strong \( \beta \) evolution in the scenario of the curvature effect. Since photons observed later may be from the higher latitude angle (\( \theta \)) of the jet shell, the softening spectrum may be due to a \( \theta \)-dependent spectral shape in the comoving frame of the jet shell (Zhang et al. 2007). In this scenario, the emitting region with the hardest spectrum in the jet shell should always be in the direction of \( \theta = 0 \) for most steep decay phases with strong spectral evolution (e.g., Mu et al. 2016). It seems to be contrived. Besides a \( \theta \)-dependent spectral shape, the softening spectrum may be due to a non-power-law intrinsic radiation spectrum with \( \theta \)-independent spectral parameters (Zhang et al. 2009). Since the radiation from different \( \theta \) is observed at different times and with different Doppler factors, the observed X-ray emission would be from different segments of the non-power-law intrinsic spectrum. Then, one would find a strong spectral evolution in the steep decay phases. This scenario has been studied in Zhang et al. (2009) for the steep decay phase of the prompt emission in GRB 050814. However, the evolution
pattern of the spectral index in the steep decay phase is not discussed in detail. Then, we try to derive an analytical formula to describe the $\beta$ evolution in the steep decay phase with a non-power-law intrinsic radiation spectrum and $\theta$-independent spectral parameters.

The paper is organized as follows. Since we try to test our analytical formula of $\beta$ evolution based on the numerical simulations, the procedure of our numerical simulations is presented in Section 2. The functional form of spectral evolution in the steep decay phase is derived and tested in Sections 3 and 4, respectively. Conclusions and a discussion are presented in Section 5.

2. Procedure for Simulating Jet Emission

The emission of a spherical thin jet shell with jet opening angle $\theta_{\text{em}}$ radiating from $r_0$ to $r$, is our focus, where $r_0$ and $r$ are the location of jet shell estimated with respect to the jet base. We assume that (1) the central axis of jet shell coincides with the observer’s line of sight and (2) the jet shell has no $\theta$-dependent spectral parameters and Lorentz factor. The procedure for simulating jet emission is detailed in Lin et al. (2017). In this section, we present a brief description about this model.

We assume the jet shell is located at radius $r$ for time $t$, where $r$ is measured with respect to the jet base. For performing the simulations about the jet radiation, the jet shell is modeled with a number of emitters randomly distributed among the jet shell. The observed time of photons from an emitter located at $(r, \theta)$ is

$$t_{\text{obs}} = \left\{ \int_{r_{\text{obs}}}^{r} \left[ 1 - \beta_{\text{jet}}(l) \right] \frac{dl}{c \beta_{\text{jet}}(l)} + \frac{r(1 - \cos \theta)}{c} \right\}(1 + z),$$

(1)

where $c \beta_{\text{jet}}(l) = c d r / d l$ is the velocity of jet shell at radius $r = l$, $c$ is the light velocity, $\theta$ is the polar angle of the emitter with respect to the line of sight in spherical coordinates (the origin of coordinate is at the jet base), and $z$ is the redshift of the explosion producing the jet shell.

The radiation mechanism of an emitter is always discussed in the synchrotron process or the inverse Compton process. In our work, the shape of the radiation spectrum is more important than the detailed radiation processes. Following the work of Uhm & Zhang (2015), the radiation spectrum of an electron with $\gamma_{\text{e}}' (\gg 1)$ is assumed to be

$$P'(E') = P'_0 \frac{H'(E'/\tilde{E}_0')}{\tilde{E}_0'},$$

(2)

where $P'_0$ describes the spectral power observed in the jet shell comoving frame and $\tilde{E}_0'$ is the characteristic photon energy of electron emission. Thus, the total radiation power of an emitter in the comoving frame is $n'_e P'_0 \frac{H'(E'/\tilde{E}_0')}{\tilde{E}_0'}$, where $n'_e$ is the total number of electrons with $\gamma_{\text{e}}'$ in an emitter. For the functional form of $H'(x)$, we study following two cases.

Case (I): $H'(x)$

$$= \begin{cases} \chi^{\bar{\alpha} + 1} \exp(-x), & x \leq (\bar{\alpha} - \bar{\beta}), \\ (\bar{\alpha} - \bar{\beta})^{\bar{\alpha} - \bar{\beta}} \chi^{\bar{\alpha} - \bar{\beta}} \exp(\bar{\beta} - \bar{\alpha}) x^{\bar{\alpha} + 1}, & x \geq (\bar{\alpha} - \bar{\beta}), \end{cases}$$

(3)

where $\bar{\alpha}$ and $\bar{\beta}$ are constants. The spectral shape in Case (I) is the so-called “Band function” spectrum (Band et al. 1993).

In some bursts, the observed prompt (or X-ray flare) emission can be fitted with a cutoff power-law (CPL) spectrum, i.e., Case (II). Then, the spectral evolution for a jet shell with Case (II) is also studied. A photon in the comoving frame with energy $E'$ is boosted to $E = DE'/(1 + z)$ in the observer’s frame, where $D$ is the Doppler factor described as

$$D = \left[ \Gamma (1 - \beta_{\text{jet}} \cos \theta) \right]^{-1},$$

(4)

and $\Gamma$ is the Lorentz factor of the jet shell. During the shell’s expansion for $dt (\sim 0)$, the observed spectral energy $dU$ from an emitter into a solid angle $d\Omega$ in the direction of the observer is given as (Uhm & Zhang 2015)

$$\delta U_E(t_{\text{obs}}) = (D^2 \delta \Omega) \left( \frac{\delta t}{\Gamma} \right) \frac{1}{4 \pi} n'_0 P'_0 \frac{H'(E(1 + z))}{D \tilde{E}_0'} = \delta U_t(t_{\text{obs}}),$$

(5)

where the emission of electrons is assumed isotropically in the jet shell comoving frame (see Beniamini & Granot 2016; Geng et al. 2017).

The procedures for obtaining the observed flux is shown as follows. First, an expanding jet is modeled with a series of jet shells at radius $r_0$, $r_1 = r_0 + \beta_{\text{jet}}(r) c d t$, $r_2 = r_1 + \beta_{\text{jet}}(r_1) c d t$, ..., $r_n = r_{n-1} + \beta_{\text{jet}}(r_{n-1}) c d t$, ..., appearing at the time $t = 0$, $\delta t$, $2 \delta t$, ..., $n \delta t$, ... with velocity $c \beta_{\text{jet}}(r)$, $c \beta_{\text{jet}}(r_1)$, $c \beta_{\text{jet}}(r_2)$, ..., $c \beta_{\text{jet}}(r_n)$, ..., respectively. During the shell’s expansion for $\delta t$, the shell move from $r_{n-1}$ to $r_n$ with the same radiation behavior for emitters. Second, we produce $N$ emitters centered at $(r_n, \theta, \phi)$ in spherical coordinates, where the value of $\cos \theta$ and $\phi$ are randomly picked up from linear space of $[\cos \theta_{\text{jet}}$, 1] and [0, $2\pi$], respectively. The observed spectral energy from an emitter during the shell’s expansion from $r_{n-1}$ to $r_n$ is calculated with Equation (5). By discretizing the observer time $t_{\text{obs}}$ into a series of time intervals, i.e., $[0, \delta t_{\text{obs}}], [\delta t_{\text{obs}}, 2\delta t_{\text{obs}}], ..., [(k - 1)\delta t_{\text{obs}}, k\delta t_{\text{obs}}], ...,$ we can find the total observed spectral energy

$$U_E(k - 1)\delta t_{\text{obs}}, k\delta t_{\text{obs}}) = \sum_{(k - 1)\delta t_{\text{obs}}, \leq \delta t_{\text{obs}}, \leq k\delta t_{\text{obs}}} \delta U_E(t_{\text{obs}}),$$

(6)

in the time interval $[(k - 1)\delta t_{\text{obs}}, k\delta t_{\text{obs}}]$ based on Equations (1) and (5). Then, the observed flux at the time $(k/2 - 1)\delta t_{\text{obs}}$ is

$$F_E = \frac{U_E(k - 1)\delta t_{\text{obs}}, k\delta t_{\text{obs}})}{D_L \delta t_{\text{obs}} \delta \Omega},$$

(7)

where $D_L$ is the luminosity distance of the jet shell with respect to the observer.

In our numerical simulations, the jet shell is assumed to begin radiation at radius $r = 10^{14}$ cm with a Lorentz factor $\Gamma(r) = \Gamma_0 = 300$ and $\tilde{E}_0' = \tilde{E}_0$. The evolution of Lorentz factor is given by $\Gamma(r) = \Gamma_0$. If assumption (2) does not hold, the emitting region with the hardest intrinsic spectrum in the jet shell may always be in the direction of our sight. It seems to be contrived. Then, the $\theta$-dependence of the spectral parameters and Lorentz factor may be weak.
factor $\Gamma$ and $E'_0$ are assumed as

$$\Gamma(r) = \Gamma_0 \left( \frac{r}{r'} \right)^{n}.$$  

The value of $n$, $P'_0$ is assumed to increase with time $t'$ in the jet comoving frame, i.e., $n'_P = n'_0 P'_0 t'$, and $t' = 0$ is set at the radius $r$, where $n'_0$ and $P'_0$ are constants. The value of $N \gg 1$, $\delta t < t_{c,r}$, $\theta_{jet} \gg 1/\tau_0$, and $\delta t_{obs} = 0.005 t_{c,r}$ are adopted and remain as constants in a numerical simulation, where $t_{c,r} = (r(1+z))^{1/2}$. By changing the observed photon energy $E$ and running the numerical simulation again, we can find the observed flux $F_E$ at different $E$. Since the observational energy band of the Swift X-ray Telescope (XRT) used to estimate the spectral index is $[0.3 \text{ keV}, 10 \text{ keV}]$, we obtain the observed flux $F_E$ at photon energy $E = 0.3 \text{ keV}$, $0.3 \times 1.12 \text{ keV}$, $0.3 \times 1.12^2 \text{ keV}$, ..., $10 \text{ keV}$. With $F_E$ observed at different $E$, we fit the spectrum with a power-law function $(E/1 \text{ keV})^{-\beta}$ to find the value of $\beta$. The total duration of our light curves is set as $50 t_{c,r}$. Then, the obtained data would be significantly large. To reduce the file size of our figures, we only plot the data in the time interval with $k$, satisfying $(k-1) \delta t_{obs} < 1.1^{m} \times 0.01 t_{c,r} + t_0 < k \delta t_{obs}$, where $m \geq 0$ is any integer and $t_0$ is the observer time set for $t_{obs} = 0$ (see Section 3). The spectral evolution pattern in these figures are the same as those plotted based on all of data from our numerical simulations.

3. Analytical Formula of Spectral Evolution

We first analyze the spectral evolution for radiation from an extremely fast cooling thin shell (EFCs). For this situation, we assume the radiation behavior of the jet shell to be unchanged during the shell’s expansion time $\delta t (\sim 0)$. Then, we have $r = r_0 + \beta_{jet} c \delta t \sim r_0$ and Equation (1) can be reduced to

$$t_{obs} = (r/c)(1 - \cos \theta)(1 + z),$$  

which describes the delay time of photons from ($r$, $\theta$) with respect to those from ($r$, $\theta = 0$). It should be noted that the beginning of the phase shaped by the curvature effect in this situation ($\delta t \sim 0$) is at around $t_{obs} = 0$. With $\Gamma \gg 1$, $D$ can be reduced to

$$D \approx \left\{ \Gamma - \Gamma \left( 1 - \frac{1}{2 \Gamma^2} \right) \left[ 1 - (1 - \cos \theta) \right] \right\}^{-1} \approx \left[ \frac{1}{2 \Gamma} + \Gamma (1 - \cos \theta) \right]^{-1},$$  

or

$$D \approx \frac{2 \Gamma}{1 + t_{obs}/t_{c,r}},$$  

where $t_{c,r}$ is the characteristic timescale of the shell curvature effect at radius $r$;

$$t_{c,r} = \frac{r (1 + z)}{2 \Gamma c}.$$  

The difference between $D$ and $2 \Gamma/(1 + t_{obs}/t_{c,r})$ can be neglected for a significantly large value of $\Gamma$. Then, we use $D = 2 \Gamma/(1 + t_{obs}/t_{c,r})$ in our analysis.

For the observer time interval $\delta t_{obs}$, the observed total number of the emitter is $N \delta (\cos \theta)/\left( 1 - \cos \theta_{jet} \right)$ with $|\delta (\cos \theta)| = c \delta t_{obs}/r(1 + z)$ derived based on Equation (9), where $\theta_{jet}$ is the jet opening angle. Then, the observed flux at the time $t_{obs}$ is

$$F_E = \frac{\delta U_{E_{0}}(t_{obs}) N \delta (\cos \theta)}{D^{2} \delta t_{obs} d \Omega},$$  

or,

$$F_E \propto D^{2} H’(E(1 + z))/D^{2} E_{0}/G.$$  

The method used to derive Equation (14) is from Uhm & Zhang (2015); see their paper for details.

The observed spectral index $\beta$ is always estimated with XRT observations. The observational energy band of XRT is $[0.3 \text{ keV}, 10 \text{ keV}]$. Then, $\beta$ can be approximately described as

$$\beta \approx \beta_{es} = - \frac{\log (F_{10 \text{ keV}}/F_{0.3 \text{ keV}})}{\log (10 \text{ keV}/0.3 \text{ keV})}.$$  

For Case (II), we have

$$\beta_{es} = - \alpha - 1 + \frac{10 \text{ keV} - 0.3 \text{ keV}}{E_{0,r} [\ln (10 \text{ keV}/0.3 \text{ keV})]},$$  

where $E_{0,r}$ is read as

$$E_{0,r}(t_{obs}) = \frac{E_{0,0}(D)}{1 + z} = \frac{E_{0,0}(D)}{(1 + t_{obs}/t_{c,r})(1 + z)}.$$  

For Case (I), however, the relation of $\beta_{es}$ and $E_{0,r}$ may be different for different values of $E_{0,0}$ ($t_{obs}$).

For Case (I) and $10 \text{ keV} \leq (\hat{\alpha} - \hat{\beta}) E_{0,r}$, we have

$$\beta_{es} = - \hat{\alpha} - 1 + \frac{10 \text{ keV} - 0.3 \text{ keV}}{E_{0,r} [\ln (10 \text{ keV}/0.3 \text{ keV})]},$$  

which is the same as Equation (16). For Case (I) and $0.3 \text{ keV} \leq (\hat{\alpha} - \hat{\beta}) E_{0,r}$, one can find

$$\beta_{es} = - \hat{\beta} - 1.$$  

For Case (I) and $0.3 \text{ keV} \leq (\hat{\alpha} - \hat{\beta}) E_{0,r} < 10 \text{ keV}$, we have

$$\beta_{es} = \frac{\log (F_{10 \text{ keV}}/F_{E_{0,r}}) + \log (F_{E_{0,r}}/F_{0.3 \text{ keV}})}{\log (10 \text{ keV}/0.3 \text{ keV})},$$  

or,

$$\beta_{es} = - \frac{(\hat{\beta} + 1) [\ln (10 \text{ keV}/E_{0,r})]}{\ln (10/0.3)} - (\hat{\alpha} + 1) \times \frac{\ln (E_{0,r}/0.3 \text{ keV})}{\ln (10/0.3)} + E_{0,r} - 0.3 \text{ keV}.$$  

For Case (I), we compare the value of $\beta$ and $\beta_{es}$ in the upper-left panel of Figure 1 by changing the value of $E_{0,r}$, where the value of $\hat{\alpha} = -1$ and $\hat{\beta} = -2.3$ are adopted. The value of $\beta$ is obtained by fitting $F_{0.3 \text{ keV}}, F_{0.3 \times 1.12 \text{ keV}}, \ldots, F_{10 \text{ keV}}$ with $(E/1 \text{ keV})^{-\hat{\beta}}$. In this panel, the value of $\beta$ and $\beta_{es}$ are shown.
with black “+” and blue dashed line, respectively. The value of $e_{sb}$ presents a good estimation of the spectral index $\beta$. However, the deviation of $e_{sb}$ with respect to $\beta$ can be easily found for $E_{0}=0.3$ keV $\ll E_{0,r} \ll 20$ keV.

Then, we would like to present a better estimation ($e_{sb}^{\prime}$) about the value of $\beta$. For Case (II) or Case (I) with $E_{0,r} \geq 10$ keV/$(\hat{\alpha} - \hat{\beta})$, the deviation of $e_{sb}$ relative to $\beta$ is due to the fact that we use a power-law function to fit a CPL spectrum. In Figure 1, one can easily find the behavior of $e_{sb}^{\prime}$. Then, we adopt

$$e_{sb}^{\prime} = -\alpha - 1 + 0.857 \frac{10 \text{ keV} - 0.3 \text{ keV}}{E_{0,r} \ln(10 \text{ keV}/0.3 \text{ keV})}$$

(22)

to estimate the spectral index $\beta$.

For Case (I) and $E_{0,r} \leq 0.3$ keV/$(\hat{\alpha} - \hat{\beta})$, we adopt

$$e_{sb}^{\prime} = -\beta - 1.$$  

(23)

For Case (I) and $0.3$ keV $< (\hat{\alpha} - \hat{\beta})E_{0,r} < 10$ keV, we have

$$e_{sb} = (\hat{\beta} - \hat{\alpha}) \ln(E_{0,r}/1 \text{ keV}) - \frac{0.3 \text{ keV}}{E_{0,r} \ln(10/0.3)}.$$  

(24)

Then, the following form

$$e_{sb}^{\prime} \propto x \frac{(\hat{\beta} - \hat{\alpha}) \ln(E_{0,r}/1 \text{ keV})}{\ln(10/0.3)} - y \frac{0.3 \text{ keV}}{E_{0,r} \ln(10/0.3)}$$

(25)

is used to estimate the spectral index $\beta$. Here, the value of $x$ and $y$ are obtained by requiring the continuity of $e_{sb}^{\prime}$ at $E_{0,r} = 0.3$ keV/$(\hat{\alpha} - \hat{\beta})$ and $10$ keV/$(\hat{\alpha} - \hat{\beta})$, i.e.,

$$0.2371 + 0.2766 \frac{x}{y} - 1 = 0.$$  

(26)

Then, we adopt $x = 1.22$ and $y = 1.65$ in this work, i.e.,

$$e_{sb}^{\prime} = A + 1.22 \frac{(\hat{\beta} - \hat{\alpha}) \ln(E_{0,r}/1 \text{ keV})}{\ln(10/0.3)} - 1.65 \frac{0.3 \text{ keV}}{E_{0,r} \ln(10/0.3)}.$$  

(27)

being used for the situations with Case (I) and $0.3$ keV $< (\hat{\alpha} - \hat{\beta})E_{0,r} < 10$ keV, where $A = -1 + 0.0573\hat{\alpha} - 1.0573\hat{\beta} + 0.3479(\hat{\beta} - \hat{\alpha}) \ln(\hat{\alpha} - \hat{\beta})$. The value of $e_{sb}^{\prime}$ for situations with Case (I) and different $E_{0,r}$ can be found in Figure 1 with red solid lines. One can find that the deviation of $e_{sb}^{\prime}$ relative to $\beta$ is very small for an EFCS with Case (I). Then, we use $e_{sb}^{\prime}$ to describe the spectral index for situations with Case (I). In addition, Equation (22) is adopted to describe the spectral index for situations with Case (II). In Figure 1, one can also find that the value of $e_{sb}^{\prime}$ with respect to $E_{0,r}$ is almost the same for different $\hat{\alpha}$ and $\hat{\beta}$. This reveals that the evolution pattern of the spectral index would be almost the same for different $\hat{\alpha}$ and $\hat{\beta}$. Then, we only discuss Case (I) with $\hat{\alpha} = -1$ and $\hat{\beta} = -2.3$.

By substituting Equation (17) into $e_{sb}^{\prime}$, the analytical formula of $t_{\text{obs}}$-dependent $\beta$ can be obtained, i.e.,
where $\kappa$, $a$, $b$, and $c$ are defined as follows.

$$\kappa = \frac{0.3 \text{ keV}}{E_{0,r}(t_{\text{obs}} = 0)},$$ (29)

$$a = -1 + 0.4762\hat{\alpha} - 1.4762\hat{\beta} + 0.3479(\hat{\beta} - \hat{\alpha})\ln(\hat{\alpha} - \hat{\beta}) + 0.3479(\hat{\alpha} - \hat{\beta})\ln \kappa + b,$$ (30)

$$b = -1.65 \frac{0.3 \text{ keV}}{E_{0,r}(t_{\text{obs}} = 0)\ln(10/0.3)} = -0.4706\kappa,$$ (31)

$$c = 1.22 \frac{- \hat{\alpha} - \hat{\beta}}{\ln(10/0.3)} = 0.3479(\hat{\alpha} - \hat{\beta}).$$ (32)

For situations with Case (I), $(\hat{\alpha} - \hat{\beta})E_{0,r}(t_{\text{obs}} = 0)$ is the break energy of Band function observed at $t_{\text{obs}} = 0$; for situations with Case (II), $E_{0,r}(t_{\text{obs}} = 0)$ is the cutoff energy of CPL spectrum observed at $t_{\text{obs}} = 0$. In practice, we may be interested on the steep decay phase with $t_{\text{obs}} \geq t_0$ (≥0). By defining $\tilde{t}_{\text{obs}} = t_{\text{obs}} - t_0$, Equation (11) is reduced to

$$D(\tilde{t}_{\text{obs}}) = \frac{2\Gamma}{1 + \tilde{t}_0/\tilde{t}_{c,r}} \frac{1}{1 + \tilde{t}_{\text{obs}}/(\tilde{t}_{c,r} + t_0)} = \frac{D_0}{1 + \tilde{t}_{\text{obs}}/\tilde{t}_{c,r}}.$$ (33)

where $D_0 = 2\Gamma/(1 + \tilde{t}_0/\tilde{t}_{c,r})$ is the Doppler factor of the emitter observed at $\tilde{t}_{\text{obs}} = 0$ (or $t_{\text{obs}} = t_0$) and $\tilde{t}_{c,r} = \tilde{t}_{c,r} + t_0$ is adopted. With Equation (33), we have

$$\beta(\tilde{t}_{\text{obs}}) = \left\{ \begin{array}{ll}
\frac{a_1 + 7.9\hat{\kappa}\tilde{t}_{\text{obs}}/\tilde{t}_{c,r}}{1 + \tilde{t}_{\text{obs}}/\tilde{t}_{c,r}}, & \text{Case (I):} \\
\frac{a_2 + b\tilde{t}_{\text{obs}}/\tilde{t}_{c,r} + c \ln(1 + \tilde{t}_{\text{obs}}/\tilde{t}_{c,r})}{1 + \tilde{t}_{\text{obs}}/\tilde{t}_{c,r}}, & \text{Case (II):}
\end{array} \right.$$ (34)

$$\beta(\tilde{t}_{\text{obs}}) = \left\{ \begin{array}{ll}
\frac{a_1 + 7.9\hat{\kappa}\tilde{t}_{\text{obs}}/\tilde{t}_{c,r}}{1 + \tilde{t}_{\text{obs}}/\tilde{t}_{c,r}}, & \text{Case (I):} \\
\frac{a_2 + b\tilde{t}_{\text{obs}}/\tilde{t}_{c,r} + c \ln(1 + \tilde{t}_{\text{obs}}/\tilde{t}_{c,r})}{1 + \tilde{t}_{\text{obs}}/\tilde{t}_{c,r}}, & \text{Case (II):}
\end{array} \right.$$ (35)

where

$$\tilde{t}_{c,r} = -\hat{\alpha} - 1 + 7.9\kappa(1 + t_{\text{obs}}/t_{c,r}),$$ (36)

$$\beta(\tilde{t}_{\text{obs}}) = \left\{ \begin{array}{ll}
\frac{a_1 + 7.9\hat{\kappa}\tilde{t}_{\text{obs}}/\tilde{t}_{c,r}}{1 + \tilde{t}_{\text{obs}}/\tilde{t}_{c,r}}, & \text{Case (I):} \\
\frac{a_2 + b\tilde{t}_{\text{obs}}/\tilde{t}_{c,r} + c \ln(1 + \tilde{t}_{\text{obs}}/\tilde{t}_{c,r})}{1 + \tilde{t}_{\text{obs}}/\tilde{t}_{c,r}}, & \text{Case (II):}
\end{array} \right.$$ (37)

where $\kappa$, $a$, $b$, and $c$ are constants, $\tilde{t}_{c,r}$ is the decay timescale of the phase with $\tilde{t}_{\text{obs}} \geq 0$. $E_0(\tilde{t}_{\text{obs}}) = E_0(\tilde{t}_{\text{obs}})/\tilde{t}_{c,r}$ with $E_{0,0} = E_0(\tilde{t}_{\text{obs}} = 0)$ being the observed photon energy at

$$\hat{b} = -0.4706\hat{\kappa},$$

and $E_{0,r}(t_{\text{obs}} = 0)$ is the observed characteristic photon energy of the radiation spectrum at $t_{\text{obs}} = 0$. With $\tilde{t}_{c,r} = t_{c,r} + t_0 - t_{\text{obs},r}$. $D_0 = 2\Gamma/[1 + (t_0 - t_{\text{obs},r})/t_{c,r}]$, and $t_0 \geq t_{\text{obs},r}$. Equation (34) is applicable to describe the spectral evolution for the radiation from an EFCS located at any $r$, where

$$t_{\text{obs},r} = (1 + z) \int_{r_0}^{r} \left[ 1 - \beta_{\text{jet}}(l) \right] \frac{dl}{c\beta_{\text{jet}}(l)}$$ (38)

is the observed time for the first photon from a radiating jet shell located at $r$. If $t_0 = t_{\text{obs},r}$, we have $a_1 = -\hat{\alpha} - 1 + 7.9\hat{\kappa}$ and $a_2 = a$ with $\kappa$ being replaced by $\hat{\kappa}$ for an EFCS based on Equation (28).

In general, the shell may radiate from $r_0$ to $r_\infty$ with $r_\infty > r_0$. An expanding jet in our work is modeled with a series of jet shells located at radius $r_0$, $r_1 = r_0 + \beta_{\text{jet}}(r_1)c\hat{t}, r_2 = r_1 + \beta_{\text{jet}}(r_1)c\hat{t}, ..., r_n = r_{n-1} + \beta_{\text{jet}}(r_{n-1})c\hat{t}$, ... with appearing time $t = 0s$, $\hat{t}_l, 2\hat{t}_l, ..., n\hat{t}_l, ...$, respectively. The radiation behavior of the jet shell during the time interval $[(n-1)\hat{t}_l, n\hat{t}_l]$ does not change. This behavior is similar to that of an EFCS’s radiation discussed above. Then, the radiation of our jet can be regarded as the radiation from a series of EFCSs located at $r_0$, $r_1$, $r_2$, ..., $r_n$, ... with appearing time $t = 0s$, $\hat{t}_l, 2\hat{t}_l, ..., n\hat{t}_l, ...$. Thus, we would like to use the observed photon energy $E_0(\tilde{t}_{\text{obs}})$ of the radiation spectrum to replace $E_{0,r}(\tilde{t}_{\text{obs}})$, i.e.,

$$10 \text{ keV} \leq (\hat{\alpha} - \hat{\beta})E_{0,r}(\tilde{t}_{\text{obs}}),$$ (39)
\( \tilde{t}_{\text{obs}} = 0, \tilde{\kappa} = 0.3 \text{ keV}/(E_{0,0}), \) and \( \tilde{b} = -0.4706\tilde{\kappa}. \) For situations with Case (I), the value of \( (\tilde{\alpha} - \tilde{\beta})E_0 \) is the observed break energy of the Band function; for situations with Case (II), the value of \( E_0 \) is the observed cutoff energy of the CPL spectrum. The value of \( f \sim 1 \) is introduced by considering that the observed flux is from a series of EFCs located at different \( r \) with different \( t_{\text{obs}}, t_{\text{cr}}, t_{\text{tp}}. \) As discussed in Section 4, the exact value of \( f \) depends on the behavior of the jet’s dynamics and radiation, and thus is difficult to estimate in reality. It is interesting to note that the value of \( f \) is around unity for our studying cases (see Figure 3). Moreover, Equation (38) with \( f = 1 \) presents a good estimation of the spectral evolution (see Figure 3). Then, we suggest the use of \( f = 1 \) in practice.

Equations (38) and (39) are our obtained analytical formula of the spectral evolution in the steep decay phase. Since Equation (39) is involved in Equation (38), we only test Equation (38) with our numerical simulations. For Equation (38), if \( 10 \text{ keV} \lesssim (\tilde{\alpha} - \tilde{\beta})E_0,0 \) is satisfied, the value of \( a_1 \) would be the spectral index at \( \tilde{t}_{\text{obs}} = 0. \) In this situation, \( a_1 = \beta(\tilde{t}_{\text{obs}} = 0) \) is adopted in our testing process and the value of \( a_2 \) is appropriately taken in order to maintain the continuity of \( \beta(\tilde{t}_{\text{obs}}) \) at \( fE_0 = 10 \text{ keV} / (\tilde{\alpha} - \tilde{\beta}). \) If \( 0.3 \text{ keV} < (\tilde{\alpha} - \tilde{\beta})E_0,0 < 10 \text{ keV} \) is satisfied, the value of \( a_2 \) would be the spectral index at \( \tilde{t}_{\text{obs}} = 0. \) In this situation, \( a_2 = \beta(\tilde{t}_{\text{obs}} = 0) \) is adopted in our testing process. It is interesting to find that for a significantly large value of \( E_{0,0}, \) the value of \( \tilde{\kappa} \) would be low and thus \( \beta(\tilde{t}_{\text{obs}}) \approx a_2 + c \ln(1 + \tilde{t}_{\text{obs}}/\tilde{t}_0) \) can be found.

4. Testing

In this section, we test Equation (38) based on the numerical simulations. Figure 2 shows the evolution of \( \beta \) for an EFC. Here, \( \tilde{t}_{\text{obs}} = 0 \) is the beginning of the steep decay phase dominated by the shell curvature effect. For each part of this figure, the upper half plots the integrated flux in the 0.3–10 keV energy band and the lower half shows the spectral index \( \beta. \) In the left panel, the violet “□,” red “●,” black “+,” and green “×” represent the data from the numerical simulations with \( 2E_{0,0}^{d} \Gamma _0/(1 + z) = 0.1 \text{ keV}, 1 \text{ keV}, 5 \text{ keV}, \) and 50 keV, respectively. The violet, red, black, and green solid lines represent the value of \( \beta \) estimated with Equation (38), \( \tilde{t}_c = t_{c,\text{tp}}, f = 1, \) and \( E_{0,0} = 0.1 \text{ keV}, 1 \text{ keV}, 5 \text{ keV}, \) and 50 keV, respectively. It has been found that Equation (38) can present a good estimation of the spectral evolution for an EFC. In the right panel of this figure, we plot the light curves and spectral evolution with \( t_0 = t_{c,\text{tp}}. \) The meaning of symbols are the same as those in the left panel. In the lower half of the right panel, the decay timescale is \( \tilde{t}_c = t_{c,\text{tp}} + t_0 = 2t_{c,\text{tp}} \) according to Equation (33). In addition, one can find \( E_{0,0} = E_{0,0}^{d}D_{t_0}/(1 + z) = E_{0,0}^{d} \Gamma_0/(1 + z) \) at \( \tilde{t}_{\text{obs}} = t_0. \) Then, we plot the value of \( \beta \) estimated with Equation (38), \( \tilde{t}_c = 2t_{c,\text{tp}}, f = 1, \) and \( E_{0,0} = 0.05 \text{ keV} \) (violet solid line), 0.5 keV (violet solid line), 2.5 keV (black solid line), and 25 keV (red solid line), respectively. It has been found that Equation (38) presents a good estimation of the spectral evolution in the steep decay phase for an EFC.

In Figure 3, we show the results for situations with a spherical thin shell radiating from \( r_0 \) to \( 2r_0, \) where the data from numerical simulations with different \( s \) and \( w \) are plotted in sub-figures (a)–(i), respectively. In each sub-figure, the upper-left panel shows the evolution of integrated flux in the 0.3–10 keV energy band, and the lower-left panel (right part) shows the spectral evolution for \( t_{\text{obs}} \geq 0 \) \( (t_{\text{obs}} \geq t_p) \). Here, \( t_p \) is the peak time of the integrated flux in the 0.3–10 keV energy band, and \( t_p = 2.32t_{c,\text{tp}}, 1.01t_{c,\text{tp}}, \) and 0.50\( t_{c,\text{tp}}, \) are found for situations with \( s = -1, 0, \) and 1, respectively. It should be noted that the phase with \( t_{\text{obs}} \geq t_p \) is dominated by the shell curvature effect in our simulations (Ulhm & Zhang 2015; Lin et al. 2017). The violet “■,” red “●,” black “+,” and green “×” in Figure 3 represent the data from the numerical simulations with \( 2E_{0,0}^{d} \Gamma_0/(1 + z) = 0.1 \text{ keV}, 1 \text{ keV}, 5 \text{ keV}, \) and 50 keV, respectively. For comparison, the \( \beta \) estimated with Equation (38) and \( E_{0,0} = E_{0,0}^{d}(2r_0)\Gamma(2r_0) \) is shown with solid lines in the right part of each sub-figure, where \( \tilde{t}_c = 6.41t_{c,\text{tp}}, 1.96t_{c,\text{tp}}, \) and 0.69\( t_{c,\text{tp}}, \) are adopted for situations with \( s = -1, 0, \) and 1 (Lin et al. 2017), respectively. In general, Equation (38) with \( f \sim 1 \) presents a good estimation of the spectral evolution
Figure 3. Left panel: evolution of flux and spectral index for a thin jet shell radiating from $r_0$ to $2r_0$; right panel: comparison of $\beta$ from simulations and that estimated with Equation (36) for the steep decay phase, where different values of $s$ and $w$ in the simulations are studied. For clarity, the flux is shifted by dividing 1.5 for adjacent light curves in the plot.
Figure 3. (Continued.)
Figure 3. (Continued.)
according to the results shown in Figure 3. Then, we can conclude that Equation (38) can describe the spectral evolution.

The values of $f$ adopted in Figure 3 are estimated based on the following discussion. For the phase with $t_{\text{obs}} \gtrsim t_p$, we have $t_0 = t_p$ and $t_{\text{obs}} = t_{\text{obs}} - t_p$. As discussed in Section 3, the observed flux in the phase with $t_{\text{obs}} \geq 0$ is from a series of EFCSs located at $r_0, r_1, r_2, \ldots, r_n, \ldots$ with appearing time $\tilde{t} = 0$, $\tilde{t}, 2\tilde{t}, \ldots, n\tilde{t}, \ldots$. Then, any EFCS can exert more or less influence on the spectral evolution in the steep decay phase. It should be noted that the observed time for the first photon from an EFCS located at $r$ is $t_{\text{obs}}$. Thus, the spectral evolution for the radiation from an EFCS located at $r$ can be described with Equation (34) by adopting $\tilde{t}_{c,r} = t_{c,r} + t_0 - t_{\text{obs}}$, $D_r = 2\Gamma/[1 + (t_0 - t_{\text{obs}})/t_{c,r}]$, and $t_0 \geq t_{\text{obs}}$. That is to say, the spectral evolution in the phase with $t_{\text{obs}} \geq 0$ for the radiation from an EFCS is controlled by two parameters: $\tilde{E}_{0,r}(\tilde{t}_{\text{obs}} = 0) = 2\Gamma(\tilde{r})E_0(\tilde{r})/[1 + t_p - t_{\text{obs}}]/t_{c,r}$ and $t_{c,r} = t_{c,r} + t - t_{\text{obs}}$. Since the radiation of the jet shell can be regarded as the radiation from a series of EFCSs located at different $r$ with different $t_{\text{obs}}$, a different pattern of $r$-dependent $\tilde{E}_{0,r}(\tilde{t}_{\text{obs}} = 0)$ and $t_{c,r}$ for EFCSs would require a different value of $f$ to describe the spectral evolution with Equation (38). Taking the situation with $s = 0$ and $w = 0$ as an example, it can be found that the decay timescale $\tilde{t}_{c,r}$ for radiation from any EFCS is the same, i.e., $\tilde{t}_{c,r} = \tilde{t}_{c,r}$. However, the value of $\tilde{E}_{0,r}(\tilde{t}_{\text{obs}} = 0) = 2\Gamma_0E_{0,0}r_{c,r}/\tilde{t}_{c,r}$ increases with the location of EFCSs. Then, the spectral evolution should be fitted with Equation (38) and $f \lesssim 1$ if $E_{0,0} = 2E_0(2r_0)\Gamma(2r_0)$ is satisfied, where $E_{0,0} = 2E_0(2r_0)\Gamma(2r_0)$ is found in our simulation. This behavior can be found in Figure 3(a), where Equation (38) with $f = 0.85$ presents a better estimation about the spectral evolution. For the situation with $s = 0$ and $w = -1$, $t_{c,r}$ and $\tilde{E}_{0,r}(\tilde{t}_{\text{obs}} = 0) = 2\Gamma_0E_{0,0}r_{c,r}/(\tilde{t}_{c,r}r)$ remain constants for different EFCSs. Then, the spectral evolution can be described with Equation (38), $E_{0,0} = 2E_0(2r_0)\Gamma(2r_0)$, and $f = 1.0$. This can be found in Figure 3(b), where Equation (38) with $f = 1$ presents a perfect estimation about the spectral evolution. Then, Equation (38) with $f = 1$ is not shown in this sub-figure, and the bottom-right panel of Figure 3(b) is empty. The value of $f$ adopted in other situations can be analyzed in the same way as that shown above. In reality, however, the exact value of $f$ is difficult to estimate. Figure 3 shows that Equation (38) with $f = 1$ can present a good estimation of the spectral evolution. Then, we conclude that Equation (38) with $f = 1$ can present a good estimation of the spectral evolution in the steep decay phase.

5. Conclusions and Discussion

This work focuses on the spectral evolution in the steep decay phase shaped by the curvature effect. We study the radiation from a spherical relativistic thin shell with a significantly large jet opening angle ($\theta_{\text{jet}} \gg 1/\Gamma$). In addition, we assume that (1) the central axis of jet shell coincides with the observer’s line of sight (2) and the jet shell has no $\theta$-dependent spectral parameters and Lorentz factor. For shells with a CPL intrinsic radiation spectrum, we find that the spectral evolution can be described as Equation (39), i.e., $\beta\tilde{E}_{\text{obs}} \approx 7.9\tilde{E}_{\text{obs}}/\tilde{t}_{c}$. This equation reveals that $\beta\tilde{E}_{\text{obs}}$ is a linear function of the observer time.

Our formula can be used to confront the curvature effect with observations and estimate the decay timescale of the steep decay phase. In observations, the steep decay phase has been observed in the decay phase of the prompt emission phase and flares in GRBs. Since the value of $E_{0,0} (\sim 0.3–1$ MeV) in the prompt emission phase is significantly larger than 10 keV, the $\beta$ in the steep decay phase of prompt emission would be a linear function of observer time based on Equation (38). This behavior has been observed in a number of bursts, such as GRBs 050814 (Zhang et al. 2009, please see the spectral evolution in the $\beta - t_{\text{obs}}$ space), 051001 etc. The linear relation between $\beta$ and $t_{\text{obs}}$ is also found in the steep decay of flares (e.g., Mu et al. 2016). For a linear function $\beta$ of $t_{\text{obs}}$, one can obtain the slope of the linear function, which is equal to $7.9\tilde{E}_{\text{obs}}/\tilde{t}_{c} = 2.37$ keV/[$E_{0,0}(t_{\text{obs}} = 0)\tilde{t}_{c}]$ based on Equations (38).
or (39). Then, the decay timescale $\tilde{\tau}_c$ of our studying phase can be estimated if the value of $E_0(\tilde{\tau}_{obs} = 0)$ is known. We fit the spectral evolution in the decay phase of a flare ($\sim 172$ s) in GRB 060904B (Mu et al. 2016) with a linear function. The fitting result, i.e., $\tilde{\beta} = 0.84 \pm 0.02 \tilde{\tau}_{obs} = \tilde{\tau}_{obs} - \tilde{\tau}_p$, is shown in the left panel of Figure 4 with a red solid line. Then, we have $E_0(\tilde{\tau}_{obs} = 0)\tilde{\tau}_c = 119$ keV s, which is around that found in Mu et al. (2016), i.e., $2.54$ keV $\times 65.39$ s. For a flare ($\sim 116$ s) in GRB 131030A, the spectrum at the beginning of the steep decay phase can be fitted with a Band function. With the fitting result found in Mu et al. (2016), i.e., $\tilde{\alpha} = -0.91$, $\tilde{\beta} = -3.19$, and $E_0(\tilde{\tau}_{obs} = 0) = 1.95$, we plot the evolution of $\tilde{\beta}$ based on Equation (38) in the right panel of Figure 4 with a red solid line. Here, we adopt $\tilde{\tau}_c = 54$ s (Mu et al. 2016), which can also be roughly estimated based on the flux evolution. It can be found that Equation (38) describes the spectral evolution approximately, which may reveal that a more appropriate value of parameters (e.g., $\tilde{\tau}_c$) may be required for this source. The agreement of our analytical formula and observational data shows that the assumption (2) given at the beginning of Section 2 (i.e., the jet shell has no $\theta$-dependent spectral parameters or Lorentz factor) is applicable in reality.

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