Quantized detector network POVMs and the Franson-Bell experiment

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We present a generalized POVM formalism for the calculation of quantum optics networks of arbitrary complexity and apply it in a detailed analysis of the Franson-Bell experiment. Our analysis suggests that when it comes to observations of quantum processes, laboratory apparatus cannot always be used according to classical principles.

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I. INTRODUCTION

The mathematical rules of standard quantum mechanics (SQM) are now well understood, but for over a hundred years it has been a serious challenge to understand what quantum mechanics (QM) means on an intuitive level. Part of the problem has been a natural tendency to regard QM as a direct replacement for classical mechanics (CM). This led to the belief that SQM is the most complete and correct description possible of systems under observation (SUOs) such as electrons, photons and atoms. This belief was challenged in the EPR paper [1] but SQM won the argument because to date all of its predictions have been supported empirically, including those for Bell inequality experiments. As a result, many theorists readily use phrases such as electron wave-function, photon polarization state and atomic orbital, as if these had some sort of physical existence independent of any context.

This view is understandable given the success of the Schrödinger equation when applied to important SUOs such as atoms and molecules. Some of the consequences of taking the Schrödinger equation literally were the development of Everett’s relative state formulation [2] and decoherence theory [3].

Whilst these developments contain much of interest, particularly in the case of decoherence theory, the finger of empirical evidence points firmly in another direction, towards the fundamental importance of apparatus in the interpretation of QM. Heisenberg and Bohr understood from the beginning that SUOs and apparatus cannot be separated in QM in the way that they are in CM. Indeed, Bohr’s counterattack to the EPR paper [4] relied on the contextuality of the processes of observation.

In recent times this point has been strongly reinforced by theorists such as Ludwig [5] and Kraus [6]. The work of these and others such as Peres [7] has supported the view that QM is much more about information exchange between SUOs and apparatus rather than being just an improved description of SUOs. Given this, it is natural to ask who or what is acquiring quantum information and how it happens, questions which focus attention on observers and apparatus. This was the motivation for this paper.

Whilst the rules of SUOs are those given by SQM and are therefore well understood mathematically, the quantum rules as they relate to apparatus remain to be fully understood and developed. There are surprises, as we shall show in our description of the Franson-Bell experiment, discussed in detail below.

In an effort to understand the processes of observation and measurement in QM, we have in recent years concentrated our efforts in developing quantized detector networks (QDN). This is a time-dependent description of apparatus based on quantum registers [8]. In this paper we combine QDN with the SQM rules of SUOs and the Ludwig-Kraus POVM formalism to provide a general framework for a wide class of modularized quantum processes.

In recent years, quantum optics has provided an ideal arena for the theoretical and experimental investigation of quantum principles, principally because photonics is a clean technology amenable to a modular approach. By this we mean that experimental apparatus of great complexity can be built up from basic components such as beam-splitters, Wollaston prisms, mirrors, polarizers and suchlike. Experiments based on such modules are the targets of our work described here.

In this paper we have two principal objectives. The first is to outline the generalized POVM formalism we have developed to describe modular quantum optical networks of arbitrary complexity. This formalism combines the two complementary aspects fundamental to all quantum experiments, viz., SUOs and apparatus. We have found this formalism well suited to algebraic computerization. All our reported POVM elements and coincidence rates have been calculated using a single computer algebra program, details of which can be supplied on request. This approach appears capable of dealing with modular networks of arbitrary complexity.

The second objective is to apply our formalism to a particular experiment first discussed theoretically by Franson [9] and referred to here as the Franson-Bell experiment. This is of considerable interest empirically because it appears to involve quantum non-locality and Bell-type violations of classical expectations. Our network approach to the Franson-Bell experiment shows that laboratory apparatus cannot under all circumstances...
be treated classically. Specifically, our analysis shows that how a photon detector’s dynamical output is determined depends on the observer’s knowledge of the photon states concerned. Another way of saying this is that the interpretation of what real equipment in a laboratory means is contextual and depends on the input states involved.

II. NOTATION, TERMINOLOGY AND RULES

A quantized detector network is a time-dependent collection of quantum nodes and quantum paths. Quantum nodes are those places in a quantized network at which an elementary yes/no observation is possible in principle. They are equivalent to effects in the Ludwig-Kraus formalism [6] and to elementary signal detectors (ESDs) in QDN [3]. They can serve as sources and detectors of quantum signals. Quantum path are those parts of a quantized network connecting two nodes. The quantum optics modules we discuss are always placed on quantum paths, i.e., between the quantum nodes where the detectors sit.

Time is treated with particular care in our formalism. Our notion of time is closer to proper time in relativity than to coordinate time and is a measure of information acquisition by observers, rather than a parameter dictating changes in SUOs. It always runs forwards and is irreversible. We assume from the outset that quantum experiments are dynamical processes in which not only states of SUOs evolve but apparatus itself can change.

In our approach, an observer extracts information during any given run of an experiment in a sequence of stages [11]. Usually a stage $\Omega_n$ can be identified as some instant of simultaneity of the observer’s laboratory time, but this is not always the case. A stage can involve parts of an experiment “on hold”, i.e., isolated from all other parts in such a way that to all intents and purposes no dynamical interaction or information flow can take place between them. A stage is a collection of detectors which are all effectively relatively spacelike, even though their laboratory times may indicate that some of them are relatively timelike. We have used this “multi-fingered” concept of time in a discussion of particle decay experiments and the quantum Zeno effect [10]. The Franson-Bell experiment provides an example where an intermediate stage in an experiment is not synonymous with a single instant of the observer’s laboratory time.

One of the consequences of our approach is that in our formalism, quantum dynamics is always described in terms of mappings from one Hilbert space to another. There is no concept here of a state vector evolving in a fixed Hilbert space. Therefore, we are not dealing with the Schrödinger picture as we would normally in SQM. Also, because a jump from one Hilbert space to another may involve a change of Hilbert space dimension, we are not dealing with the Heisenberg picture either. In our diagrams, a symbol such as $A^i_n$ denotes the $i^{th}$ detector associated with stage $\Omega_n$. Note that $A^i_n$ need bear no causal relation to $A^j_m$, whatsoever, if $n \neq m$.

In all the discussions here, we shall concentrate exclusively on pure state physics. This is not only because mixed states are not expected to provide any great difficulties for the formalism, but also because the lessons to be learnt from the Franson-Bell experiment come from the fundamental level of pure state physics. We shall ignore all effects of quantum inefficiency and probability leakage, etc., as these bear the same relationship to our theory as friction does in CM, i.e., they are important aspects but do not contribute to the discussion.

In a typical experiment, we shall deal with tensor product states of the form

$$\Psi_n \equiv \sum_{i=1}^{d_n} \sum_{A=0}^{D_n-1} \Psi^A_n A^i_n A^j_n, \quad (1)$$

where $s$ refers to the SUO part, $a$ refers to the apparatus and the coefficients $\Psi^A_n$ are complex numbers. From the point of view of decoherence theory, $a$ would normally be replaced by $c$, to denote environment. However, there are crucial differences between that theory and our approach. Specifically, we do not use the Schrödinger equation to describe apparatus. Moreover, we do not exclude the state reduction concept, as this is associated with information exchange between observer and apparatus and is not interpreted by us as the collapse of any wave-like object.

In an expression such as [1] above, $s^i_n$ denotes a basis ket state $|s^i, n\rangle$ in some separable Hilbert space $H_n$ of dimension $d_n$, with dual space $H^*_n$. Generally, we use small Latin letters to label SUO basis states and such an index runs from 1 to $d_n$. The dual $\langle s^i, n\rangle$ of $s^i_n$ is denoted by $s^*_n$. The $s^i_n$ are orthonormalized, so we have the relation $s^i_n s^j_n = \delta_{ij}$. The ordering of symbols matters here, because $s^i_n s^j_n$ denotes the operator $|s^i, n\rangle \langle s^j, n|$. In our computer algebra program, ordering can be ignored, provided inner products are always immediately evaluated.

On the other hand, the apparatus basis states (referred to as labstates) $A^i_n$ are elements of a quantum register $R_n$ which is the tensor product of a finite number $r_n$ of qubits. $R_n$ is a Hilbert space with dimension $D_n = 2^{r_n}$. Generally we shall use capital Latin letters to index basis labstates and such labels will run from zero to $D_n - 1$. Full details are given in a recent review [8]. Typically, a preferred basis element $A^i_n$ for $R_n$ involves a number $k$ of detectors combined at stage $\Omega_n$ in the form

$$A^m_n \equiv A^+_{m_1,n} A^+_{m_2,n} \ldots A^+_{m_k,n} \langle 0, n \rangle, \quad (2)$$

where $M = 2^{m_1-1} + 2^{m_2-1} + \ldots + 2^{m_k-1}$, $A^+_{m,n}$ is the signal operator for the $m^{th}$ detector at stage $\Omega_n$ and $\langle 0, n \rangle$ is the void state for stage $\Omega_n$. Here the integers $m_1, m_2, \ldots, m_k$ are all necessarily different. We shall also use the notation $a^m_n \equiv A^+_{m_1,n} A^+_{m_2,n} \ldots A^+_{m_k,n}$.

The reason for the relative simplicity of our computer algebra approach is that whilst the physics of a network experiment can involve multiple detectors, such as in coincidence experiments, rule [2] associates a unique label

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III. DYNAMICAL EVOLUTION

In the following, all Hilbert spaces involved are complex and finite dimensional. Our quantum dynamics will be described in terms of probability conserving linear maps from one total Hilbert space $H_n \otimes R_n$ to the next, $H_{n+1} \otimes R_{n+1}$. To understand the details of the calculations, we need a few basic concepts.

We define a Born map to be a norm-preserving map from one Hilbert space $H$ into another Hilbert space $H'$. If $B : H \to H'$ is a Born map, then for any element $\Psi$ in $H$, the corresponding element $\Psi' \equiv B(\Psi)$ in $H'$ is such that $(\Psi, \Psi') = (\Psi', \Psi')$, where the inner product on the LHS is taken in $H$ whilst that on the RHS is taken in $H'$. From the basic definition of a Hilbert space, only the zero vector in $H$ can be mapped into the zero vector in $H'$ by a Born map.

Born maps can be non-linear. A semi-unitary map is just a linear Born map. Because linearity is fundamental to many quantum processes (but not all), we shall focus on such maps exclusively throughout this paper. The following theorem is fundamental to our work and is easy to prove:

**Theorem:** A semi-unitary map from Hilbert space $H$ into Hilbert space $H'$ exists if and only if $\dim H' \geq \dim H$.

If $U$ is a semi-unitary operator from $H$ to $H'$ then $U^+$ is a map from $H'$ to $H$ such that $UU^+ = I$, the identity operator for $H$. This means that a semi-unitary map not only preserves norms but also inner products, which turns out useful in the construction of the dynamics.

If $\dim H' = \dim H$, then $U^+$ is a semi-unitary map from $H'$ to $H$ such that $UU^+ = I'$, the identity operator for $H'$. In this special case, $U$ is said to be unitary. If however $\dim H' > \dim H$, then necessarily $UU^+ \neq I'$.

Particle decay experiments are examples of situations where the quantum evolution is semi-unitary but not unitary [10].

The dynamics underlying a given experiment is treated stage by stage, with semi-unitary evolution from an initial state $\Psi_0 \equiv \sum_{i=1}^{d_0} \sum_{A=0}^{D_0-1} \Psi_0^A s_0^A$ to some final state $\Psi_N \equiv \sum_{i=1}^{d_N} \sum_{A=0}^{D_N-1} \Psi_N^A s_N^A$, for $N > 0$. The dimension $d_n$ of the SUO Hilbert space $H_n$ can change from one stage to another, as happens in the case of parametric down conversion production of photon pairs from a single photon.

Consider evolution from stage $\Omega_n$ to $\Omega_{n+1}$. In our experiments, the observer is assumed to know which initial basis states need to be considered, because such contextual information comes with the construction of the apparatus. Typically there will be very many potential quantum register basis labstates, but most of these will be irrelevant in a given experiment. This occurs for example in experiments where photons come in from multiple correlated sources. Therefore, we need concentrate only on those basis states which are needed. These form a basis $B_n^{\text{eff}}$ for some effective subspace $H_n^{\text{eff}}$ of the total space $H_n \otimes R_n$. Provided we take all of the basis elements of $H_n^{\text{eff}}$ into account, we can restrict ourselves to that subspace in our calculations. Typically, for such an element $s_n^A$ in $B_n^{\text{eff}}$, we write

$$U_{n+1,n} s_n^A \overline{a}_n^A = \sum_{j=1}^{d_{n+1}} \sum_{B=0}^{D_{n+1}-1} U_{n+1,n}^{B,iA,j} s_n^A \overline{a}_{n+1}^B,$$

where the complex coefficients $U_{n+1,n}^{B,iA,j}$ satisfy the semi-unitarity conditions

$$\sum_{j=1}^{d_{n+1}} \sum_{B=0}^{D_{n+1}-1} [U_{n+1,n}^{B,iA,j}]^* U_{n+1,n}^{B,kC} = \delta_{ik} \delta_{AC},$$

and are determined from an empirical knowledge of the modules involved in the network (analogous to specifying a Hamiltonian in SQM). Provided we restrict the action of $U_{n+1,n}$ to $H_n^{\text{eff}}$, we may use “effective completeness”, i.e., resolution of the identity $I_n^{\text{eff}}$ in $H_n^{\text{eff}}$, to write

$$U_{n+1,n} \simeq \sum_{j=1}^{d_{n+1}} \sum_{B=0}^{D_{n+1}-1} \sum_{i} \sum_{A} s_n^{i,A} U_{n+1,n}^{B,iA,j} s_{n+1}^{B,jA} \overline{a}_n^A,$$

where we use the symbol $\simeq$ to denote effective representation and the summations for $\Omega_n$ are over the required effective ranges. Then semi-unitarity is equivalent to

$$U_{n+1,n}^+ U_{n+1,n} \simeq I_n^{\text{eff}}.$$

The complete effective evolution operator $U_{N,0}$ from $\Omega_0$ to $\Omega_N$ is given by $U_{N,0} = U_{N,N-1} U_{N-1,N-2} \cdots U_{2,1} U_{1,0}$ and is of the form

$$U_{N,0} \simeq \sum_{i=1}^{d_N} \sum_{A=0}^{D_N-1} \sum_{j} \sum_{B} s_n^{i,A} \overline{a}_N^{i,A,j} U_{N,0}^{B,iA,j} \overline{a}_0^B,$$

where the $U_{N,0}^{B,iA,j}$ are given by a product of all the interstage transition matrices involved. $U_{N,0}$ is semi-unitary provided each of the interstage evolution operators $U_{n+1,n}$ is semi-unitary.
Once the complete evolution operator has been determined, the generalized POVM operators are obtained in two steps. First we calculate the $D_N$ generalized Kraus operators $M^A_{N,0}$ by the rule

$$M^A_{N,0} = \bar{a}^A_0 U_{N,0} \equiv \sum_{i,j,k,B,C} \sum_{l,m,n} \psi_{i,j,k,B,C} U_{N,0}^i U_{N,0}^j s^B_0 \bar{a}^B_0,$$

(8)

For most networks, particularly those with large rank, most of these operators turn out to be zero. The elements $E_{N,0}$ of the POVM are then calculated by the rule

$$E^A_{N,0} \equiv M^A_{N,0} M^A_{N,0} \equiv \sum_{i,j,k,B,C} \psi_{i,j,k,B,C} U_{N,0}^i U_{N,0}^j s^B_0 \bar{a}^B_0. \quad (9)$$

They are positive operators over the initial total space $\mathcal{H}^0$ and satisfy the rule

$$\sum_{A=0}^{D_N-1} E^A_{N,0} = I_0^0.$$  

(10)

In this respect they differ from the standard POVM formalism, the POVMs of which are operators on the SUO Hilbert space $\mathcal{H}_0$.

Given a properly normalized initial state $\Psi_0 \equiv \sum_i \psi_i \phi_i$, then the outcome probability rates at stage $N$ are given by

$$\text{Pr}(a^A_N|\Psi_0) = \bar{\psi}_0 E^A_{N,0} \Psi_0,$$

(11)

where $\bar{\psi}_0 \equiv \sum_i \psi_i \phi_i$.

Care has to be taken in interpreting these quantities because QDN networks are time dependent and also because incoming states have finite wave-trains (or coherence times). In situations where wavetrains coming from different parts of a network fail to overlap in time at a detector, no interference can be expected to occur normally. This happens in the Franson-Bell experiment Scenario ii) discussed below. Note however that the Franson-Bell experiment Scenario iii) provides a spectacular exception to this rule.

Our formalism is comprehensive, in that the computer algebra programme automatically calculates single detector outcome rates $\text{Pr}(A^A_N|\Psi_0)$, two photon coincidence rates $\text{Pr}(A^A_N A^A_0|\Psi_0)$, and all possible higher order multiple detector incidence rates, up to the saturation coincidence rate $\text{Pr}(A^A_2 A^A_1 A^A_0|\Psi_0)$. Most higher order incidence detector rates will be zero in typical experiments involving a single photon or a single photon pair process.

In the next three sections we show how to apply this formalism to three test cases: the Wollaston prism, the non-polarizing beam-splitter, and a more complicated network discussed by Brandt. Then we discuss the Franson-Bell experiment in detail.

### IV. THE WOLLASTON PRISM

A Wollaston prism is a quantum optics module which splits up a beam of light into two orthogonally polarized beams. Figure 1 is a schematic diagram of such a device. Here, symbols such as $A^A_n$ represent the $i$th detector in stage $\Omega_n$.

Consider an initial state $\Psi_0 = (\alpha s^1_0 + \beta s^2_0) a_0^1 \equiv \{|\alpha|s^1,0\} \otimes A^1_{\Omega_0}(0,0)$, where $s^i_0 \equiv |s^i,0\rangle$ is a polarization state vector in two-dimensional photon polarization Hilbert space $\mathcal{H}_0$, with $i = 1$ or 2 representing orthogonal polarizations such as horizontal and vertical. Here the coefficients $\alpha$ and $\beta$ are complex and we assume normalization to unity, viz., $|\alpha|^2 + |\beta|^2 = 1$.

**FIG. 1: The Wollaston prism.**

The evolution operator $U_{10}$ from stage $\Omega_0$ to stage $\Omega_1$ is assumed semi-unitary, mapping from a two-dimensional effective subspace of the four-dimensional initial Hilbert space $\mathcal{H}_0 \otimes \mathcal{R}_0$ to the final eight dimensional one $\mathcal{H}_1 \otimes \mathcal{R}_1$. The rules are

$$U_{10} s^1_1 a_0^1 = s^1_1 a_1^1, \quad U_{10} s^2_0 a_0^1 = s^2_1 a_1^1,$$

(12)

where $s^i_j a^j_k \equiv |s^i,1\rangle \otimes A^j_{\Omega_1}(0,1)$. Using effective completeness we may write

$$U_{10} \equiv s^1_1 s^1_0 a^1_0 a_0^1 + s^2_1 s^2_0 a^2_0 a_0^1,$$

(13)

which satisfies the semi-unitary relation

$$U_{10}^U U_{10} \equiv (s^1_1 s^1_0 a^1_0 a_0^1 + s^2_1 s^2_0 a^2_0 a_0^1) = I_0^0.$$  

(14)

There are two non-zero generalized Kraus matrices associated with stage $\Omega_1$, given by

$$M^1_{10} \equiv \frac{\partial}{\partial a^1_0} U_{10} = s^1_1 s^1_0 a_0^1,$$

$$M^2_{10} \equiv \frac{\partial}{\partial a^2_0} U_{10} = s^2_1 s^2_0 a_0^1.$$  

(15)
From these, the generalized POVM operators associated with $\Omega_1$ are given by
\begin{align}
E_{10}^1 &= M_{10}^1 + M_{10}^1 = s_0^1 s_0^1 a_0^1 a_0^1, \\
E_{10}^2 &= M_{10}^2 + M_{10}^2 = s_0^2 s_0^2 a_0^1 a_0^1, \\
E_{10}^i &= M_{10}^i + M_{10}^i = \delta_{ij} E_{10}^j \quad \text{(no sum over } i) \quad \text{and} \\
E_{10}^1 + E_{10}^2 &= I_0^{\text{eff}}. \quad (16)
\end{align}

In this particular case, these POVMs satisfy the relations $E_{10}^i E_{10}^j = \delta_{ij} E_{10}^j$ and
\[E_{10}^1 + E_{10}^2 = I_0^{\text{eff}}.\]

From these we find the conditional outcome rates
\[\Pr(A_1^i|\Psi_0) = |\alpha|^2, \quad \Pr(A_2^i|\Psi_0) = |\beta|^2, \quad (18)\]
assuming complete efficiency.

V. THE NON-POLARIZING BEAMSLIPPER

In this example, two beams of light enter the device through the two in-ports and are passed on to two outcome detectors, as shown in Figure 2.

![Beam-splitter diagram](image)

**FIG. 2:** The Beam-splitter.

Typically, a single photon scenario is involved, which we discuss here. For a single photon scenario, the initial state is given by
\[\Psi_0 = \alpha|s^1, 0\rangle \otimes A_{1,0}^+|0, 0\rangle + \beta|s^1, 0\rangle \otimes A_{2,0}^+|0, 0\rangle = \alpha s_0^1 a_0^1 + \beta s_0^1 a_0^2, \quad (19)\]
where the same photon polarization state $s^1$ is assumed for both in-ports. In the situation that different in-port states have different polarizations, the dynamics is easily modified. For the case concerned, the dynamics is given by the rules
\begin{align}
U_{10}s_0^1 a_0^1 &= t s_0^1 a_0^1 + i r s_0^1 a_0^2 , \\
U_{10}s_0^1 a_0^2 &= i r s_0^1 a_0^1 + t s_0^1 a_0^2, \quad (20)
\end{align}
where $t$ and $r$ are real and satisfy $t^2 + r^2 = 1$ and a relative phase change associated with reflection has been taken into account. Using effective completeness, we have
\[U_{10} \equiv s_0^1 s_0^1 \{t a_0^1 + i r a_0^2\} a_0^1 + s_0^1 s_0^2 \{i r a_0^1 + t a_0^2\} a_0^2, \quad (21)\]
which gives the non-zero POVM operators
\begin{align}
E_{10}^1 &= s_0^1 s_0^1 \{r^2 t a_0^1 a_0^1 - i r t a_0^1 a_0^2 + i r t a_0^2 a_0^2 + t^2 a_0^2 a_0^2\} \\
E_{10}^2 &= s_0^1 s_0^1 \{r^2 t a_0^1 a_0^1 + i r t a_0^1 a_0^2 - i r t a_0^2 a_0^2 + t^2 a_0^2 a_0^2\}. \quad (22)
\end{align}

This gives the non-zero outcome transition relative probability rates
\[\Pr(A_1^i|\Psi_0) = r^2|\alpha|^2 + i r t (\alpha^* \beta - \alpha \beta^*) + t^2|\beta|^2, \quad \Pr(A_2^i|\Psi_0) = r^2|\alpha|^2 - i r t (\alpha^* \beta - \alpha \beta^*) + t^2|\beta|^2, \quad (23)\]
assuming complete efficiency and wave-train overlap. The computer algebra program confirms that $\Pr(a_1^1|\Psi_0) = 0$ and $\Pr(a_1^1 A_2^1|\Psi_0) = \Pr(a_1^1|\Psi_0) = 0.$

VI. BRANDT’S NETWORK

The next example is a quantum optics network discussed by Brandt [12] in terms of conventional POVMs and is shown in Figure 3. Here and elsewhere $M$ refers to a mirror.

![Brandt’s network diagram](image)

**FIG. 3:** Brandt’s network.

Brandt’s analysis was in terms of non-orthogonal SUO state vectors. Our analysis avoids non-orthogonality issues directly. The initial state is given by
\[\Psi_0 = (\alpha s_0^1 + \beta s_0^2)a_0^1, \quad (24)\]
where $s_0^1, s_0^2$ denote orthogonal photon polarizations and $\alpha$ and $\beta$ satisfy $|\alpha|^2 + |\beta|^2 = 1.$ Evolution from $\Omega_0$ to $\Omega_1$ is given by the Wollaston prism dynamics given by [13]. The transition from $\Omega_1$ to $\Omega_2$ is determined by the effective transition rules
\begin{align}
U_{21}s_0^1 a_0^1 &= t s_0^1 a_0^1 + i r s_0^1 a_0^2, \\
U_{21}s_0^1 a_0^2 &= -s_0^1 a_0^1, \quad (25)
\end{align}
where $a_0^1 = A_{3,2}^+|0, 2\rangle = |23^{-1}, 2\rangle = |4, 2\rangle$ and $t^2 + r^2 = 1.$

The second equation in (25) represents a $\frac{1}{2}\pi$ rotation
of the photon polarization vector as it passes through the module labelled $R$ in Figure 3. It is assumed that the transmission and reflection parameters $t_1$, $r_1$ at $BS_1$ can be arranged to have specific values, so these are left undeclared at this point in the calculation. Hence

$$U_{21} = s_2^2 s_1^4 (t_1 a_2^1 + i r_1 a_2^2) a_2^1 - s_2^4 s_1^2 a_2^1 a_2^1. \quad (26)$$

In principle, detection can take place at $A_2^1$ during $\Omega_2$. However, this can be placed “on hold” until the final stage $\Omega_3$, which is more convenient. This is represented by the rule

$$U_{32} s_2^2 a_2^1 = s_3^1 a_3^1.$$  

The other parts of the process from $\Omega_2$ to $\Omega_3$ involve beam-splitter $BS_2$ and are given by

$$U_{32} s_2^2 a_2^2 = t_2 s_2^4 a_3^4 + i r_2 s_2^4 a_3^2,$$

$$U_{32} s_2^2 a_2^3 = i r_2 s_2^4 a_3^3 + t_2 s_2^4 a_3^2,$$

where $t_2^2 + r_2^2 = 1$, giving

$$U_{32} \simeq s_3^4 s_2^2 a_3^2 + s_3^8 (t_2 a_3^4 + i r_2 a_3^2) a_2^2$$

$$+ s_3^4 s_2^2 (i r_2 a_3^4 + t_2 a_3^2) a_2^2. \quad (28)$$

The complete effective evolution operator $U_{31} = U_{32} U_{21} U_{10}$ gives three non-zero POVMs:

$$E_{30} = t_1^2 s_1^4 a_3^1 a_3^1,$$

$$E_{32} = \{ t_2^2 s_2^2 s_3^2 a_3^2 + i r_2 s_2^2 s_3^2 a_3^2, t_2^2 s_2^2 s_3^2 a_3^2 + i r_2 s_2^2 s_3^2 a_3^2 \} a_3^1 a_3^1,$$

$$E_{31} = \{ i r_2 s_2^2 s_3^2 a_3^2 + t_2^2 s_2^2 s_3^2 a_3^2 \} a_3^1 a_3^1.$$  

which lead to the three non-zero outcome rates

$$\Pr(A_2^1 | \Psi_0) = t_1^2 |a|^2,$$

$$\Pr(A_3^2 | \Psi_0) = r_2^2 |a|^2 + r_2 t_2 (a^* \beta + a^* \beta^*) + t_2^2 |\beta|^2,$$

$$\Pr(A_3^3 | \Psi_0) = r_1^2 t_2 |a|^2 - r_2 r_1 t_2 (a^* \beta + a^* \beta^*) + r_2^2 |\beta|^2.$$

assuming perfect efficiency and wave-train overlap. When the reflection and transmission coefficients are chosen as by Brandt [12], these rates agree with his precisely.

**VII. THE FRANSON-BELL EXPERIMENT**

We now discuss the Franson-Bell experiment in detail. In the following, we refer to “photons” as if they were actual particles, because this is convenient and provides an intuitive picture. However, the quantum dynamics shows that such a picture can be misleading. A better interpretation of a photon is simply as a click in a detector.

The basic experiment consists of a coherent pair of photons sent in opposite directions towards a pair of separated Mach-Zehnder interferometers, as shown in Figure 4. Each photon passes through its own interferometer and depending on path taken, can suffer a change in phase $\phi$ and a time delay $\Delta T$. In the proposed experiment, this time delay is assumed the same for each interferometer, but the phase changes $\phi_1, \phi_2$ associated with the different interferometers can be altered at will.

![FIG. 4: The Franson-Bell experiment for $\Delta T < t_2 \ll t_1$.](image)

The experiment hinges on the relationship between three characteristic times, as discussed by Franson [3]. The first of these is the coherence time of each photon. In real experiments, a “monochromatic” single photon would be associated with a finite wave train of length $L$ moving at the speed of light $c$, which therefore takes a time $L/c$ to pass a given point. In this experiment, the correct coherence timescale $t_1$ is that associated with the production of the photon pair at $A_3^1$. This is a characteristic of the photon pair source and of the collimation procedures applied subsequently. Whilst $t_1$ cannot be altered, it can be determined empirically. We assume $t_1$ is the same for each photon.

The second characteristic time is $t_2$, the effective time interval within which both photons in a pair can be said to have been emitted. This can be measured by coincidence measurements with detectors $A_1^3$ and $A_2^3$ with all beam-splitters removed. It is assumed $t_2$ can be determined empirically and that $t_2 \ll t_1$. This last inequality is crucial to the Franson-Bell experiment because when this inequality holds, the observer has no way of knowing when a photon pair was created during the relatively long time interval $t_1$. It is this lack of knowledge which leads to quantum interference in scenario iii) discussed below. The spectacular aspect of the Franson Bell experiment is that unlike the double -slit experiment, where the observer cannot know from which point in space a photon came, here the observer does not know at which point in time the photon pair was produced.

The third characteristic time is $\Delta T$, the time difference between a photon travelling along the short arm of its interferometer and along its long arm. This time is adjustable and is assumed the same for each interferometer.

There are three scenarios we shall discuss: i) $\Delta T \ll t_2$, ii) $t_1 \ll \Delta T$ and iii) $t_2 \ll \Delta T \ll t_1$. In Franson’s original analysis [3], photon spin did not play a role. Therefore, in all scenarios considered here, photon spin is as-
and from this four non-zero POVMs were found. The total effective transition operator $U$ which gives the relative coincidence rates summed fixed once a given photon pair has been created.

VIII. SCENARIO i) $\Delta T \ll t_2$

The relevant figure for this scenario is Figure 4. The initial state is

$$\Psi_0 = s_0 A_0^1,$$

where $s_0$ represents the initial source spin state. The creation of a photon pair is represented by the evolution operator

$$U_{1,0} s_0 A_0^1 = s_1 A_1^1 A_1^1,$$

where $s_1$ represent the combined spin state of the photon pair at stage $\Omega$. Hence

$$U_{1,0} \simeq s_1 s_0 a_{1a_0}^1.$$

Next,

$$U_{2,1} s_1 A_1^1 A_1^1 = s_2 \{t_1 A_1^1 + i r_1 e^{i \phi_1} A_1^1\} \{t_2 A_1^1 + i r_2 e^{i \phi_2} A_1^1\},$$

where $\phi_1$ and $\phi_2$ are total phase change factors due to the increased path length of the long arms of the interferometers and phase-shift plates introduced in those long arms by the observer. This gives

$$U_{2,1} \simeq s_2 s_1 \{t_1 t_2 a_3^2 + i r_1 t_2 e^{i \phi_1} a_0^6 + i r_2 r_1 e^{i \phi_2} a_0^9 - r_1 r_2 e^{i \phi_1 + \phi_2} a_0^{12}\} a_1^3.$$

There are four terms to consider in the transition from $\Omega_2$ to $\Omega_3$ :

$$U_{3,2} s_3 A_2^2 A_2^1 A_2^1 = s_3 \{t_3 A_3^1 + i r_3 A_3^1\} \{t_4 A_3^1 + i r_4 A_3^1\},$$

$$U_{3,3} s_3 A_3^3 A_3^3 A_3^3 = s_3 \{t_3 A_3^3 + i r_3 A_3^3\} \{t_4 A_3^3 + i r_4 A_3^3\},$$

$$U_{3,4} s_3 A_4^4 A_4^4 A_4^4 = s_3 \{t_3 A_4^4 + i r_3 A_4^4\} \{t_4 A_4^4 + i r_4 A_4^4\},$$

which gives

$$U_{3,2} \simeq s_3 s_2 \{t_3 t_4 a_3^6 + i r_3 t_4 a_3^6 + i t_3 r_4 a_3^6 - r_3 r_4 a_3^{12}\} a_2^3$$

$$+ \{t_3 t_4 a_3^6 + i r_3 t_4 a_3^6 + i t_3 r_4 a_3^6 - r_3 r_4 a_3^{12}\} a_2^6$$

$$+ \{t_3 t_4 a_3^6 + i r_3 t_4 a_3^6 + i t_3 r_4 a_3^6 - r_3 r_4 a_3^{12}\} a_2^9$$

$$+ \{t_3 t_4 a_3^6 + i r_3 t_4 a_3^6 + i t_3 r_4 a_3^6 - r_3 r_4 a_3^{12}\} a_2^{12}.$$

The total effective transition operator $U_{3,0} = U_{3,2} U_{2,1} U_{1,0}$ was evaluated via computer algebra and from this four non-zero POVMs were found. Combining these with the initial state and setting $t_s = r_1 = 1/\sqrt{2}$, $i = 1, 2, 3, 4$, as assumed by Franson [1], gives the relative coincidence rates

$$\text{Pr}(A_1^1 A_2^1 | \Psi_0) = \sin^2(\frac{1}{2} \phi_0) \sin^2(\frac{1}{2} \phi_2),$$

$$\text{Pr}(A_1^1 A_2^2 | \Psi_0) = \cos^2(\frac{1}{2} \phi_0) \sin^2(\frac{1}{2} \phi_2),$$

$$\text{Pr}(A_1^1 A_3^3 | \Psi_0) = \sin^2(\frac{1}{2} \phi_0) \cos^2(\frac{1}{2} \phi_2),$$

$$\text{Pr}(A_1^3 A_2^4 | \Psi_0) = \cos^2(\frac{1}{2} \phi_0) \cos^2(\frac{1}{2} \phi_2).$$

FIG. 5: The Franson-Bell experiment for $t_2 \ll t_1 \ll \Delta T$.

Each of these rates shows angular dependence due to independent “photon self-interference” within each separate interferometer. This form of interference will be referred to as local. There are no global interference effects involving both interferometers and no post selection of data is required.

IX. SCENARIO ii) $t_1 \ll \Delta T$

In this situation, the photon wavetrains $A_2^1$, $A_2^1$ reflected at $BS_1$ and $BS_2$ respectively travel along the long arms of their respective interferometers at the speed of light or less, depending on the medium through which they move. Therefore, these wavetrains arrive at $BS_3$ and $BS_4$ long after the transmitted wavetrains $A_2^1$ and $A_2^1$ have impinged on $BS_3$ and $BS_4$. In consequence, no local or global interference can take place. In fact, the observer can now obtain total information concerning the timing of each coincidence outcome in every run of the experiment and know precisely what path was taken by each photon.

Under this circumstance, the four original detectors $A_1^1$, $A_1^1$, $A_2^2$, and $A_2^2$ assumed in the previous section have to be regarded as eight separate detectors, $A_i^1$, $i = 1, 2, \ldots, 8$, as shown in Figure 5. The first four of these register photon clicks from short path photons whilst the last four signal clicks from those that have traveled the long paths. This information is specific to each photon and does not involve any photon pairs. Note that the final stage quantum register involved in this scenario and the next is 256 dimensional. However, our computer algebra programme has no difficulty dealing with this because it works only within effective Hilbert spaces of greatly reduced dimensions.

This demonstrates a fundamental point about apparatus. In the conventional usage of apparatus, experimentalists tend to regard their equipment as having some sort
of “trans-temporal” identity, or persistence. In Figure 5, for example, beam-splitters BS3 and BS4 would most likely persist in the laboratory during the long interval $\Delta T$ between their interaction with wave-trains $A_2^1$ and $A_2^2$ and with the delayed wave-trains $A_2^3$ and $A_2^4$. Even classically, however, this need not be the case. It is conceivable that $\Delta T$ could be so long, such as several years, so that the beam-splitters could be destroyed and rebuilt at leisure between the observation of any short-arm photons and any long-arm photons.

Whatever the actuality in the laboratory, from a quantum point of view, the beam splitters receiving short and long-arm photons should be considered as completely separate pieces of equipment in this scenario (but not in the next). In other words, apparatus and how it is used is time dependent. The analysis in the next section shows that the rules for doing this can be quite non-classical and appear to violate the ordinary rules of causality.

For the current scenario, $t_1 \ll \Delta T$, the dynamics follows the same rules as in the previous section up to the transition from $\Omega_2 \rightarrow \Omega_3$. At this point, the transformation rules have to take into account the possibility that the observer could know the timings of all events. The rules for this transition are now

$$U_{3,2} s_2 A_2^2 A_2^3 = s_3 \{t_3 A_3^3 + i r_3 A_3^2\} \{t_4 A_4^3 + i r_4 A_4^2\},$$

$$U_{3,2} s_2 A_2^4 A_2^5 = s_3 \{t_3 A_3^5 + i r_3 A_3^4\} \{t_4 A_4^5 + i r_4 A_4^4\},$$

$$U_{3,2} s_2 A_2^6 A_2^7 = s_3 \{t_3 A_3^7 + i r_3 A_3^6\} \{t_4 A_4^7 + i r_4 A_4^6\},$$

$$U_{3,2} s_2 A_2^8 A_2^9 = s_3 \{t_3 A_3^9 + i r_3 A_3^8\} \{t_4 A_4^9 + i r_4 A_4^8\},$$

which should be compared with (36). This gives

$$U_{3,2} \cong s_3 s_2 \{t_3 t_4 a_3^3 + i r_3 t_4 a_3^2 + i t_3 r_4 a_9^3 - r_3 t_4 a_{12}^2\} a_2^3 + \{t_3 t_4 a_{33}^3 + i r_3 t_4 a_{32}^2 + i t_3 r_4 a_{39}^3 - r_3 t_4 a_{312}^2\} a_2^3 + \{t_3 t_4 a_{33}^3 + i r_3 t_4 a_{32}^2 + i t_3 r_4 a_{39}^3 - r_3 t_4 a_{312}^2\} a_2^3 \quad (40)$$

which should be compared with (37).

In this scenario, we find sixteen non-zero coincidence rates, each of the form $\Pr(A_i^j A_{i3}^m|\Phi_0)$, where $i = 1, 3, 5, 7$ and $j = 2, 4, 6, 8$. All of them are constant, i.e., independent of $\phi_1$ and of $\phi_2$. For example, $\Pr(A_1^2 A_{13}^5|\Phi_0) = t_1^1 t_2^2 t_3^3 t_4^4$. And so on. The case of symmetrical beam-splitters, where $t_1 = r_1 = 1/\sqrt{2}$, all sixteen rates are equal.

For this scenario, the detectors behave in a manner consistent with the notion that photons are classical-like particles propagating along definite paths.

**X. SCENARIO iii) $t_2 \ll \Delta T \ll t_1$**

This is the scenario discussed by Franson [1]. In the following, $S$ stands for “short path” and $L$ for “long path.” The fundamental change induced by the observer’s setting of $\Delta T$ such that $t_2 \ll \Delta T \ll t_1$ is that unlike the previous scenario, the observer cannot now use individual times of detector clicks to establish which of the coincidences $S - S$ or $L - L$ has occurred in a given run of the experiment. The relevant diagram is Figure 6, which is identical to Figure 5 except now $A_3^3$ is replaced by $A_1^{3\rightarrow 5}$, $A_3^5$ replaced by $A_1^{1\rightarrow 7}$, $A_3^5$ is replaced by $A_1^{4\rightarrow 6}$ and $A_3^5$ replaced by $A_1^{4\rightarrow 8}$, where for example $3\rightarrow 5$ means “3 or 5”. Which alternative is taken depends on the contextual information available in principle to the observer.

The dynamics for this scenario is identical to that for the previous one, except for the last equation in (39), which is replaced by

$$U_{3,2} s_2 A_2^2 A_2^3 = s_3 \{t_3 A_3^3 + i r_3 A_3^2\} \{t_4 A_4^3 + i r_4 A_4^2\}. \quad (41)$$

This gives

$$U_{3,2} \cong s_3 s_2 \{t_3 t_4 a_3^{13} + i r_3 t_4 a_3^{12} + i t_3 r_4 a_9^{13} - r_3 t_4 a_{12}^{12}\} a_2^{13} + \{t_3 t_4 a_{33}^{13} + i r_3 t_4 a_{32}^{12} + i t_3 r_4 a_{39}^{13} - r_3 t_4 a_{312}^{12}\} a_2^{13} + \{t_3 t_4 a_{33}^{13} + i r_3 t_4 a_{32}^{12} + i t_3 r_4 a_{39}^{13} - r_3 t_4 a_{312}^{12}\} a_2^{13} \quad (42)$$

instead of (40).

Assuming a total production rate normalized to unity and symmetrical beam-splitters, we find the coincidence rates

$$\Pr(A_i^j A_{i3}^m|\Phi_0) = \Pr(A_i^j A_{i3}^m|\Phi_0) = \frac{1}{4} \cos^2(\phi_1 + \phi_2),$$

$$\Pr(A_i^j A_{i3}^m|\Phi_0) = \Pr(A_i^j A_{i3}^m|\Phi_0) = \frac{1}{4} \sin^2(\phi_1 + \phi_2), \quad (43)$$

which demonstrate non-locality. The other non-zero coincident rates involve $A_1^3 A_3^3$, $A_1^3 A_3^5$, $A_1^3 A_3^2$, $A_1^3 A_3^7$, $A_1^3 A_3^6$, $A_1^3 A_3^8$, $A_1^3 A_3^5$ and $A_1^3 A_3^7$ and are all $\frac{1}{4}$. Note that in actual Scenario iii experiments, the observer would have to measure the times at which coincidence clicks were obtained and then post-select, i.e., filter out, those coincidences corresponding to the $\{S - S, L - L\}$ processes and those corresponding to the $\{S - L, L - S\}$.
XI. COMMENTARY AND CONCLUSIONS

Since Franson’s original paper, there has been great interest in empirical confirmation of the Scenario iii) predictions. Whilst there still appears some room for debate concerning the interpretation of the experiment, the results of Kwiat et al. \[13\] vindicate Franson’s prediction, which corresponds to $\Pr(A_3^z A_2^x | \Psi_0) = \frac{1}{2} \cos^2(\frac{\phi_1 + \phi_2}{2})$ in our approach.

Assuming the quantum theoretical interpretation of this experiment is correct, then there is an extraordinary lesson to be learnt, not about SUOs in particular, but about the rules concerning the use of apparatus and how these can differ spectacularly to those expected classically. The interference of the $S – S$ and $L – L$ amplitudes in Scenario iii) cannot be envisaged in a classical way to occur locally in time. Any attempt to think about it in terms of photons as actual particles seems to lead to bizarre concepts which would never be acceptable conventionally. It has to be recognized that a two-photon state is not equivalent under all circumstances to a state with two separate photons. A more recent quantum optics experiment with similar conclusions has been reported by Kim \[14\].

The weight of evidence points to the conclusion that quantum outcome amplitudes are dynamically affected by contextual information held in principle by the observer. When some information is absent, then quantum interference can occur. This supports the position of Heisenberg and Bohr concerning the fundamental principles and interpretation of quantum physics. Quantum optics experiments such as the Franson-Bell experiment are providing more and more evidence that quantum mechanics is not just a theory of SUOs, but also a fundamental perspective on the laws of observation in physics. It is our view that the surface of those laws has only been scratched to date. The formalism we have developed and presented here, coupled with modern computer algebra technology, appears to give us some potential to dig deeper into those laws.

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