Evaluation of Localization by Extended Kalman Filter, Unscented Kalman Filter, and Particle Filter-Based Techniques

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Mobile robot localization has attracted substantial consideration from the scientists during the last two decades. Mobile robot localization is the basics of successful navigation in a mobile network. Localization plays a key role to attain a high accuracy in mobile robot localization and robustness in vehicular localization. For this purpose, a mobile robot localization technique is evaluated to accomplish a high accuracy. This paper provides the performance evaluation of three localization techniques named Extended Kalman Filter (EKF)-based localization, Unscented Kalman Filter (UKF)-based localization, and Particle Filter (PF)-based localization.

In the early period, Kalman Filter (KF) is used in an iterative manner that considers the prior information of the noise features to compensate for and to filter out the noise. But still, the issues arise during localization when attempting to model the noise that is only an approximation and does not specify the noise real distribution [5–7]. KF is only applicable for the linear stochastic procedures; however, for the nonlinear procedures, the EKF can be applied. The supposition of these two methods (KF and EKF) is that noise and process measurements are self-governing and with a normal probability distribution [8].

The authors analyzed the pertinency of the KF to the mobile robot self-positioning in [9–11]. These algorithms are only appropriate for linear systems. On the other hand, for robot self-positioning, EKF provides an alternative to the Bayesian filter. Therefore, in [12], the author proposed an EKF approach for the localization of four-wheel encoders and laser range-finder nodes. EKF is basically used for the nonlinear functions, which apply the Taylor series expansion to linearize the measurement models. Thus, the first-order nonlinear functions of the Taylor series is used. In the predictable statistics of the subsequent distributions, this linearization often encourages higher error. This is particularly obvious when the systems are vastly nonlinear, and it can lead to the deviation in the filter. Besides, the UKF does not estimate the nonlinear method, and the actual nonlinear
model is applied to the observation models. UKF basically estimate the distribution of random variables [13] by applying the scaled unscented alteration method. For the autonomous robot localization or Autonomous Underwater Vehicles (AUV) localization, the EKF and UKF performance is equated in [14]. The author concludes that the UKF performance is better concerning numerous characteristics such as average localization accuracy relative to the EKF. Besides this, UKF does not involve applying the Jacobian matrix; therefore, the UKF precision is higher.

The PFs have taken further consideration by the investigators because of their advantages against the EKF and the UKF [15]. PF can solve the issue of a measurement system that is affected by the non-Gaussian noise and accomplish the localization globally in the case in which there is no prior knowledge about the target [16]. The PFs are easier in implementation as compared to the EKF, but also exist some disadvantages such as ledton due to sample penury and substantial computational load. The ledton due to the sample penury occur more commonly and resultantly causes the failure of the PF algorithm. To address this issue, various PFs have been investigated such as Markov Chain Monte Carlo (MCMC) move step and Regularized PF (RPF) [17]. The proposed approaches cannot prevent it completely but can mitigate the sample penury. If the conventional PF flops because of the sample penury, it is unable to return to the normal conditions. Furthermore, a Hybrid Particle/Finite Impulse Response (FIR) Filter (HPFF) is investigated to solve this issue [18–21]. FIR filters are more stable as compared to other filters and robust, but FIR filters characteristically drop inaccuracy to the PF in the case of nonlinear systems [22–25]. Therefore, PF is applied as an elementary filter, which is susceptible to failure and deviation. The FIR filter is applied to recuperate the PF by restarting and reorganizing.

1.1. Contributions of This Work. This work aims to propose an effective vehicular localization technique for the mobile robot. For this purpose, the authors introduced three localization techniques. The proposed techniques’ performance is better in simulation concerning the abovementioned localization techniques in the literature. The contributions of this paper are divided into the following three phases.

1. EKF-based localization
2. UKF-based localization
3. PF-based localization

The performance of the abovementioned techniques is evaluated and analyzed in different scenarios. While considering numerous aspects of localization, the accuracy of these techniques is higher. A variety of vehicular localization techniques are investigated in the literature to analyze the presentation of the proposed techniques. These techniques are evaluated while considering numerous aspects of localization such as accuracy, time consumption, vehicle velocity, coverage area, and localization coverage. Furthermore, the presentation of the proposed localization techniques is compared with each other’s and also with other localization techniques.

1.2. Organization. The rest of the paper is sorted as follows. The related work is detailed in Section 2, and Section 3 presents the proposed techniques used for localization. The subsections of Section 3 present the proposed EKF, UKF, and PF localization techniques. Section 4 presents the discussion on the simulation results and comparison while Section 5 concludes the work.

2. Related Work

In order to expand the performance of mobile robot localization, several approaches have been introduced [26–28] to address vehicular localization [29] issues and errors of the NLoS environment. The method in [30] uses two receivers, and the Frequency Difference of Arrival (FDoA) signals to approximate the moving emitter velocity. There is a nonlinear measurement error which is occurred by the RF system noise in the geolocation environment with the Time Difference of Arrival (TDoA) [31, 32] and FDoA. For this problem, the authors proposed the iterated dual-EKF approach to compensate for the nonlinear estimation error. Using the iterated dual-EKF approach, the parameter estimation (PE) filter updates model uncertainties caused by exterior noises and has a higher convergence rate of the system parameters by the iteration method.

EKF is a traditional technique for positioning estimation [33, 34]. EKF extends models of nonlinear functions in the Taylor series close to the estimation state and shortens them to consider the linearization of the model in the first order. In the Line-of-Sight (LoS) environment, EKF based on the linear models achieves higher accuracy. But, if the channel is in the NLoS environment, the EKF shows a high error in localization because of measurement data deviation [35]. For mobile node localization, a variety of NLoS approaches have been presented [36–40]. The method of unscented transformation is used for the standard KF to produce the UKF, which attains a better estimation than the other methods. Another method is presented for the positioning of targets named Adaptive Iterated Unscented KF (AIUKF) which combines the adaptive factor and iterative approach to improve the localization performance. A PF localization approach based on the Monte Carlo technique is used for the positioning which exploits random sample group information for the approximation of the state Probability Density Function (PDF). Therefore, the performance of the PF-based localization in non-Gaussian is much better than demand. Perhaps, EKF is a well-known procedure for estimating a noisy measurement of the nonlinear system state. EKF is based on the nonlinear maps of the system around the estimated route. It is also based on the idea that the noises of measurement, input noise, and initial state are Gaussian. It
is acknowledged that the EKF is likely to deviate, mostly because of bad primary estimates and higher noises, but our experience does not know of any testable convergence conditions. The estimation divergence is extra probable when measurements are lost due to communication faults [41, 42], which is a communal disorder in mobile robotics.

In recent years, UKF was developed to tackle some problems such as the need for Gaussian noises and the properties of poor approximation [43]. The basic idea of UKF is to find a transformation that allows a random vector having a length of $n$ to approximate the covariance and mean when it is changed by the nonlinear map. It can be achieved by calculating a set of $2n + 1$ points, known as $\sigma$-points, based on the innovative vector mean and variance, transforming those points through the nonlinear map and then resembling the transformed vector mean and variance from the transformed $\sigma$-points. The authors investigated that the EKF estimate is precise to the first order, while in the case of Gaussian noises, the UKF is precise to the third order. Similarly, for the EKF, the covariance of the estimate is precise to the first order and second order for the UKF.

A procedure of tracking through PFs is presented in [44] to produce a probability distribution over the targeted area. The procedure applies the recursive Bayesian filters which are based on the sequential MC technique of sampling to estimate the location of the target posterior distribution by applying another distribution that can be prior arbitrary. PF begins with a regularly primed set of particles. Then, according to a movement model, all particle positions are modified. The authors considered the measure in the $(x, y)$ space that follows an arbitrary walk model for the representation of the human movement. The $x(t + \tau) = x(t) + \varphi(\tau)$ shows a random path of the user, where $\varphi$ denotes the random parameter which characterizes the probability of following a known direction in the tracking procedure next phase. In the measures achieved from the tags deployed in RFID, the model also takes into consideration the error model.

A statistical method named Kullback-Leibler Distance (KLD) sampling is presented to increase the efficiency of the PFs by adapting the sample sets size on-the-fly [45]. The approach is used to link the error of approximation which is introduced by the PF sample-based illustration. The method selects a smaller number of samples if the density is observant to a small part of the state space, and rising samples if the uncertainty of the state is high. The mobile robot localization problem is used to test and demonstrate the approach to the adaptive PF. The localization of robots is the problem of estimating the position of a robot relative to the map of its operation area. This issue has been identified as one of the most important mobile robotics challenges which come in diverse essences. The simplest problem with localization is position tracking, where the primary location of the robot is determined, and location pursues to exact small, gradual errors in the robot’s odometry. The global localization is another stimulating problem for mobile robots, where a robot does not have prior knowledge about his position, but it has to be decided from scratch, instead.

Following the above discussion, it is reflected that to present efficient and accurate localization techniques, the issue of vehicular localization with limited features is previously important. Most of the previous localization techniques focused only on the vehicle localization accuracy while ignoring several aspects such the vehicle velocity, time consumption, and coverage. Besides this, we must take into account the positioning of the mobile robot in a precise manner. The author, therefore, considers three localization techniques named EKF, UKF, and PF in this paper. The proposed techniques are evaluated on the basis of their performances. The authors consider several aspects for localization such the vehicle velocity, time consumption, accuracy, and localization coverage.

3. Proposed Techniques of Localization

This section discusses the proposed localization techniques based on EKF, UKF, and PF. In the next pages, the proposed localization techniques are examined. Table I shows the notations used in this work.

3.1. Extended Kalman Filter-Based Localization. EKF is typically implemented by substitution of the KF for nonlinear systems and noise models. The models of observation and state transformation are nonlinear functions, but these can be differentiable functions. The observation and state transition models [46] are delineated as

$$ x_k = f(x_{k-1}, u_k) + w_k, $$

$$ z_k = h(x_k) + v_k, $$

In the above equations, $w_k$ and $v_k$ denote the process and observation noises and zero-mean Gaussian with the covariance $Q_k$ and $R_k$, where $x_k$ and $x_{k-1}$ are the current and previous robot state, while $u_k$ represents the input vector.

The state covariance is calculated as

$$ P_k = \begin{bmatrix} P_{x_k} & P_{x_k,1} \\ P_{x_k,1}^T & P_{11} \end{bmatrix} = \begin{bmatrix} P_{x_k} & P_{x_k,1} & \cdots & P_{x_k,1} \\ P_{x_k,1}^T & P_{11} & \cdots & P_{11,1} \\ \vdots & \vdots & \ddots & \vdots \\ P_{11,1}^T & \vdots & \cdots & P_{11} \end{bmatrix}, $$

where $P_k$ is the covariance of the estimated $n \times n$ matrix. Prediction and update states are the two stages of the EKF algorithm. To calculate the prediction state

$$ \tilde{x}_{k|k-1} = F \times \tilde{x}_{k-1|k-1} + B_k \times u_k, $$

where $F$ denotes the state transition and $B_k$ denotes the input matrix. Covariance from equation (3) will become

$$ P_{k|k-1} = F \times P_{k-1|k-1} \times F^T + Q, $$

where $P_{k|k-1}$ and $P_{k-1|k-1}$ are the current and previous states. $Q$ represents a covariance matrix for the noise process. The
Table 1: Index of notation.

| Notation | Description |
|----------|-------------|
| T        | Time interval |
| φ        | Random parameter |
| X_k      | Current state |
| X_{k-1}  | Previous state |
| Z_k      | Observation model |
| f        | Prediction state function |
| h        | Prediction measurement function |
| P_{k/k-1}| Current state covariance |
| P_{k-1/k-1}| Previous state covariance |
| Q        | Covariance matrix of the process noise |
| R        | Covariance matrix of the measurement noise |
| h(\tilde{X}_k) | Predictable estimation |
| S_k      | Covariance matrix of \hat{y}_k |
| H_k      | Predictable estimation Jacobian matrix |
| K_k      | Kalman optimal gain |
| P_{k/k}  | Covariance matrix subsequent state |
| F & B    | State transition matrices |
| \delta^2_x | Standard deviation |
| x, y     | Denote locations |
| P_e      | State information vector |
| H        | Output measurement matrix |
| v_{k-1}  | Process noise |
| n_k      | Measurement noise |
| w        | Weights |
| s, \gamma | Sigma point |
| x_i      | Sigma point where i = 1, 2, \ldots, 2n |
| x_{est}  | Estimation state |
| x_{true} | True state |
| n        | Dimension of x |
| s_k      | Set of particles |
| p(2k|x_k) | Importance factor |
| f_k(x_k) | Positive state function |
| T_s      | Robot sampling time |
| p(x_k|x_{true} : k) | Successive approximation |
| \delta(\cdot) | Dirac function |
| N        | Denote the samples |

The origination vector and covariance matrix for the EKF update state is

\[ \tilde{Y}_k = Z_k - h(\tilde{X}_k), \]
\[ S_k = H_k \times P_{k/k-1} \times H_k^T + R_k, \]  

where \( h(\tilde{X}_k) \) is the predictable estimations, \( S_k \) denotes the covariance matrix of \( \tilde{Y}_k \), and \( H_k \) is the predictable estimations Jacobian matrix. To calculate the subsequent state vector:

\[ \tilde{x}_{k|k} = \tilde{X}_{k|k-1} + K_k \times \tilde{Y}_k, \]
\[ K_k = P_{k/k-1} \times H_k^T \times S_k^{-1}, \]
\[ P_{k|k} = (I_n - K_k \times H_k) \times P_{k/k-1}, \]  

where \( K_k \) is the Kalman optimal gain and \( P_{k|k} \) denotes the covariance matrix subsequent state. To apply the prediction, motion, and observation models, the state vector is computed as

\[ X_k = [x \, y \, \gamma \, v]^T. \]  

Therefore, the state transition matrices \( F \) and \( B \) are calculated as

\[ F = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \]
\[ B = \begin{bmatrix} dt \cos(x(3)) & 0 \\ dt \sin(x(3)) & 0 \\ 0 & dt \\ 1 & 0 \end{bmatrix}, \]  

In the same way, the measurement noise \( R \) is calculated as

\[ R = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}. \]  

The Jacobian of the measuring system’s motion model and covariance can be computed as

\[ J_F = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -dt \sin(x(3)) & dt \, u(1) \, dt \cos(x(3)) & 1 & 0 \\ dt \cos(x(3)) & dt \sin(x(3)) & 0 & 1 \end{bmatrix}, \]
\[ P_0 = \begin{bmatrix} \sigma_x^2 & 0 & 0 & 0 \\ 0 & \sigma_y^2 & 0 & 0 \\ 0 & 0 & \sigma_{vx}^2 & 0 \\ 0 & 0 & 0 & \sigma_{vy}^2 \end{bmatrix}, \]  

where \( \sigma^{2}_x \) denotes the standard deviations of the estimated \( x, y \) locations and \( P_0 \) is the state information vector. The value of
the standard deviation, in this case, depends on certain variables such as the accuracy of GPS and the distance as shown in Figure 1. Thus, the measuring system Jacobian can be written as

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$  \hspace{1cm} (12)$$

where $H$ is the output measurement matrix and the Jacobian matrices of the state equation. The localization results of the EKF algorithm is shown in Figure 1.

### 3.2. Unscented Kalman Filter-Based Localization

UKF is a nonderivative filtering approach that immediately propagates the mean measurement covariance and the state estimates. Therefore, it precisely represents the allied distributions by any nonlinear transformations that include the measurement and state dynamics. In the process model, because of the nonlinearity in the state functions of the velocity, unscented approximation to the finest filtering explanation can be derived by implementing two steps of measurement and time update. In the implementation of the two phases, an unscented transform is approved for the sigma points formation. Furthermore, for the derivation of UKF, the Jacobian matrix calculation is not required as required for the EKF technique. The technique can proceed in the following way. Provided a nonlinear system with a discrete time-system model [47, 48]:

$$x_k = f(x_{k-1}, v_k, u_{k-1}),$$
$$\hat{y}_k = h(x_k, n_k, u_k),$$  \hspace{1cm} (13)$$

where $v_{k-1}$ and $n_k$ show the process noise and measurement noise. Further, to compute the sigma points, time, and measurement update, the $x_k$ estimates can be calculated. A set of test points and its related weights $w$ at time $k - 1$ is utilized for sigma points measurement such as $x_0, k - 1 = x_{k-1}$, where $x_0$ denotes the sigma point at state:

$$x_{\text{est}} = [0 \hspace{1cm} 0 \hspace{1cm} 0 \hspace{1cm} 0]^T,$$  \hspace{1cm} (14)$$

where $x_{\text{est}}$ is the estimated state. The state will be true if $x_{\text{true}} = x_{\text{est}}$. The corresponding weights for the sigma points can be calculated as

$$w^m = \frac{\lambda}{n + \lambda},$$
$$w^f = \left( \frac{\lambda}{n + \lambda} \right) + (1 - \alpha^2 + \beta),$$  \hspace{1cm} (16)$$

Similarly, at $t = 0$, the weights $w^m$ and $w^f$ can be written as

$$w_{0}^m = \frac{\lambda}{n + \lambda},$$
$$w_{0}^f = \frac{\lambda}{(n + \lambda) + (1 - \alpha^2 + \beta)},$$
$$w_{i}^m = w_{i}^f = \frac{1}{2(n + \lambda)},$$  \hspace{1cm} (17)$$

where the dimension of $x$ represented by $n$ and $\lambda = \alpha^2(n + k) - n$ is the parameter of scaling. Moreover, $\alpha$ is applied to find the sigma points spreading around the state variable $x$ mean, and $k$ is the second scaling parameter. To compute the prediction of sigma points with observation and motion model, the covariance and mean will become

$$x_{i,k|k-1} = f(x_{i,k-1}),$$
$$\hat{x}_{k|k-1} = \sum_{i=0}^{2n} w_i^f x_{i,k|k-1},$$
$$P_k = \sum_{i=0}^{2n} w_i^f [x_{i,k|k-1} - \hat{x}_{k|k-1}]^T [x_{i,k|k-1} - \hat{x}_{k|k-1}] + Q_k,$$
$$\bar{x}_{i,k|k-1} = \begin{bmatrix} x_{0.2,0,k|k-1} & x_{0.0,k|k-1} + \sqrt{Q_k} & x_{0.0,k|k-1} - \sqrt{Q_k} \end{bmatrix},$$  \hspace{1cm} (18)$$

With supplementary points gained from the process noise covariance matrix square root, the sigma points were increased.

$$\bar{y}_{i,k|k-1} = h(\bar{x}_{i,k|k-1}),$$
$$\bar{y}_{k|k-1} = \sum_{i=0}^{2n} w_i^f \bar{y}_{i,k|k-1},$$
$$P_{\bar{y},k} = \sum_{i=0}^{2n} w_i^f [\bar{y}_{i,k|k-1} - \bar{y}_{k}]^T [\bar{y}_{i,k|k-1} - \bar{y}_{k}] + R_k,$$
$$P_{\bar{y},k} = \sum_{i=0}^{2n} w_i^f [\bar{x}_{i,k|k-1} - x_{k}]^T [\bar{x}_{i,k|k-1} - x_{k}] + R_k,$$  \hspace{1cm} (19)$$
Figure 1: Localization comparison of EKF. In this phase, for all four iterations, the velocity is $v = 1$ m/s and time is $t = 60$ sec. In the legends, the red dashed line represents the EKF localization, where the green is the dead reckoning (DR), the pink solid line is the ground truth (GT), the blue color asterisks denote the GPS signals, and the blue line denotes the error ellipse (EE) during localization.
where $R_k$ denote the covariance matrix. The Kalman gain can be computed to update the state and the covariance.

$$k_k = P_{yy,k} \times P_{yy,k}^{-1},$$

$$\bar{x}_k = \bar{x}_k' + k_k (y_k' - \bar{y}_k'),$$

$$p_k = P_k - k_k \times P_{yy,k} \times k_k^T.$$

By considering the primary estimation state, i.e., $x_0' = E[x_0]$ and $p_0 = E[(x_0 - x_0') \times (x_0 - x_0')^T]$}

$$R = \begin{bmatrix} \cos (\theta) & \sin (\theta) \\ -\sin (\theta) & \cos (\theta) \end{bmatrix},$$

$$x_{k+1} = Fx_k + Bv_k,$$

where $R$ denotes the covariance matrix for measuring noise and $F$ and $B$ can be calculated as

$$F = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$B = \begin{bmatrix} dt. \cos (x(3)) & 0 \\ dt. \sin (x(3)) & 0 \\ 0 & dt \\ 1 & 0 \end{bmatrix},$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

In the above equations, $F$ and $B$ denote the state matrices of transition, while $H$ represents the observation model output measurement matrix. The localization results of the UKF technique is presented in Figure 2.

### 3.3. Particle Filter-Based Localization

The authors analyzed the performance of the PF-based localization technique in this part. As similar to the tracking method, a PF is implemented to construct a probability distribution over the field of the targeted area of operation. This technique uses a recursive Bayesian filter based on the sampling of MC to calculate the target location subsequent distribution by applying another distribution which can be random a priori [40]. The PF approach is less intensive computationally in comparison to the EKF and UKF techniques. Besides this, in contrast to the KF, PF can avoid any supposition related to the intrinsic prominent attribute of the process and uncertainty related to the node information. The PF required an optimum interval of average but does not include the statistics on noise. Moreover, when the robot is traveling during the process of localization, the sensor particles in the surrounding of the robot can communicate with the robot and transfer information to the robot [45]. The mobile robot can receive a range of data from RFID that its position is known. The localization of the robot can communicate with the robot and transfer information of the robot can be calculated as

$$s_k = [(x_i', u_i') | i = 1, 2, 3, \ldots, n],$$

$$w_k = \frac{p(x_i' | z^k_i, u^k)}{p(x_{i-1} | z^k_i, u^k)},$$

$$w_k = \eta p(z_k | x_i'),$$

where $s_k$ is the set of particles, the numerator is the target distribution, and the denominator is the distribution proposal. Moreover, $\eta$ is the constant and $p(z_k | x_i')$ denotes the importance factor. For the state function, $f_k(x_i)$ is supposed to be a positive function where the PF algorithm produces the samples from $f_k(x_i)p(x_i | z^k, u^k)$ where at the initial stage, the samples are at position $f_0(x_i).$

To calculate the innovative samples, a random particle $x_{i-1}$ is computed from $X_{i-1}$ and being distributed for $n$ relation to $f_k(x_{i-1})p(x_{i-1} | z^k_i, u^k_i).$ At state $x_i \sim p(x_i | u_i, x_i), the importance weights $w$ can be written with the $f_k(x_i)$ function.

$$w_k = \frac{f_k(x_i)p(x_i | z^k_i, u^k)}{f_k(x_{i-1})p(x_{i-1} | z^k_i, u^k_i)},$$

$$w_k = \frac{f_k(x_i)p(z_k | x_i', u_i)}{f_k(x_{i-1})p(x_{i-1} | z^k_i, u^k_i)}.$$

To replace the constant of proportionality, the above equation can be written as

$$w_k \propto \frac{f_k(x_i)}{f_k(x_{i-1})}.$$
The state $\tilde{x}_{k|k-1}$ estimate objectives can also be delineated by $\tilde{x}_{k|k-1} = f (\tilde{x}_{k-1|k-1})$.

$$p_{k|k-1} = \sum_{i=1}^{N} w_i^k \times \left[ \tilde{x}_i^k - \tilde{x}_{k|k-1} \right] \times \left[ \tilde{x}_i^k - \tilde{x}_{k|k-1} \right]^T + Q.$$  \hfill (27)

In the above equation, $T_s$ is the robot sampling time, $Q$ represents the covariance matrix, whereas the weight is represented by $w$. The successive approximation $p(x_k | z_{1:k})$ and the state $\tilde{x}_{k|k}$ estimation can be calculated by:

$$p(x_k | z_{1:k}) = \sum_{i=1}^{N} w_i^k \times \delta (x_k - \tilde{x}_i^k),$$  \hfill (28)

$$\tilde{x}_{k|k} = E[ x_k | z_{1:k} ],$$  \hfill (29)
\[ \bar{x}_{kk} = \int x_k \times p(x_k | z_{1:k}) dx_k = \sum_{i=1}^{N_i} w_i^k \times x_i^k, \]  
\[ P_{kk} = \sum_{i=1}^{N_i} w_i^k \times [\bar{x}_i^k - \bar{x}_{kk}] \times [\bar{x}_i^k - \bar{x}_{kk}]^T, \]

where \( \delta(\cdot) \) is the function of Dirac delta. In the above equation, the subsequent approximation is set as a subsequent function which is approximated by \( N \) samples set [49].

\[
F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (32)
\]

\[
B = \begin{bmatrix} dt \cos(x(3)) & 0 \\ dt \sin(x(3)) & 0 \\ & 0 & dt \end{bmatrix}. \quad (33)
\]

As mentioned above in equations (32) and (33), \( F \) and \( B \) denote the motion and measurement model state transition matrices.

### 4. Simulation Discussion and Comparison

In the above sections, the authors investigated the localization performance of the proposed EKF, UKF, and PF techniques. Each technique of localization performs well in its context; however, their efficiency differs from one another in some factors. A separate simulation is conducted for each localization technique, but some parameters are consistent through different iterations in these approaches. These parameters include such as those of the first case; the velocity is chosen as \( v = 1 \) m/s and the time is \( T = 10 \) sec as presented in Figures 1, 2, and 3, respectively. Table 2 shows the parameters used in the simulations. Furthermore, the initial time is chosen to be \( t = 0 \) and \( t = 60 \) sec which represent the end time, while the global time is chosen to be \( dt = 0.1 \) sec. For navigation, Dead Reckoning (DR) is the method of measuring one’s current location by using a previously defined location, using speed and course estimates over time. On the other hand, Ground Truth (GT) is required for real-time localization. The first case state vector is \( x_{est} = [0 \ 0 \ 0]^T \) where the true state can be \( x_{true} = x_{est} \). At the initial stage, the observation vector is calculated in such way \( z = [0 \ 0 \ 0 \ 0]^T \); also, the matrix of covariance \( Q \) for motion and covariance matrix for observation \( R \) are calculated. While calculating \( Q \) and \( R \), the sigma points are calculated. For the PF technique, the function \( f(x_k) \) is used to calculate the importance of weights \( w_i \). The subsequent approximation \( p(x_k | z_{1:k}) \) and the state estimation \( \bar{x}_{kk} \) are calculated in equations (28) to (31). The Jacobian matrices \( F \) of the state equation calculated in equation (32) and Jacobian of motion model \( JF \) is calculated in equation (33). The coverage area in all three methods is almost similar, while the localization accuracy is varying. Besides this, the time consumption for all three techniques is dissimilar as can be seen in Table 3 where the velocity and time are reserved constant during four iterations.

In the next step, the velocity \( v \) is varied to \( v = 2, v = 3, v = 4, v = 5, v = 6, \ldots, v = 10 \) m/s. By the variable speed, the coverage range is enlarged as shown in Figures 4 and 5. The velocity variation is considered for the EKF and UKF techniques similarly, but the effect is different in each technique. The coverage of localization in the UKF-based technique is higher (see Figure 5) than the EKF algorithm (see Figure 4). In this phase, the localization coverage is higher, but unfortunately, the localization accuracy is also affected. When the robot is traveling with a higher speed during the process of localization, the mobile robot has a lower communication with the sensors in the surrounding, but if the robot is traveling with a lower speed, the process of communication is more reliable and convenient as compared to the case of high speed. Therefore, the author concludes that the proposed techniques are more accurate and reliable for lower velocities.

Furthermore, the proposed localization techniques’ performance is evaluated in different scenarios. The performance is assessed by varying the time of \( T \) in all three techniques of localization. However, the velocity is kept constant in all three approaches as can be seen in Table 4. By changing the time \( T \), the localization performance is also varying as can be seen in Figures 6, 7, and 8. In the case of EKF-based localization, by changing the time \( T \), the localization coverage is decreasing as shown in Figure 6. Secondly, in the case of UKF-based localization, by varying the time \( T \), the localization coverage is decreasing as presented in Figure 7. Similarly, by changing the time of \( T \) in the PF-based localization, the coverage area is decreasing gradually. It shows that the time \( T \) is inverse proportional to the coverage area of localization in all three techniques of localization. However, the time consumption is different in all three techniques as can be seen in Table 4. Among all three localization approaches, PF is the lowest time consumer as compared to other approaches. To compare the time consumption of our proposed PF technique with other techniques, the authors in [50] presented a PF technique Wi-Fi target localization. The technique is used to solve the localization problem of radio source by applying the RSSI measurements. The technique exploits the behavior of wireless signals in free space which obtains the estimates of positions from the signal strength. The authors considered the time-varying strategy to evaluate the performance of localization. The time is enlarged to \( T = 1 \) sec, \( T = 20 \) sec, \( T = 50 \) sec, and \( T = 100 \) sec, but unfortunately, by increasing the time more and more, the performance is gradually decreasing. However, in our proposed localization techniques, by varying the time \( T \), the performance is still much better as compared to the other techniques.

As mentioned before, every method performs well in its domain, but concerning some aspects, their performance is varying. To compare the proposed methods, the authors evaluated their performance while considering several factors such as localization coverage, time consumption, and localization accuracy. In the PF algorithm,
the time consumption is lower than in the EKF and UKF algorithms as shown in Table 3. Moreover, UKF is less time consuming as compared to the EKF technique. However, the localization accuracy of EKF and UKF is higher than the PF localization technique. In comparison with other techniques, the performance of the proposed technique is better than the previous as investigated in the literature. A number of localization approaches are presented by the researchers [14, 43, 51, 52]. Each approach focused on the localization performance, but a limited number of aspects are considered such as most of them focused only on the accuracy of localization. However, our proposed methods consider several aspects at once such as the localization coverage, localization accuracy, and consumption of time. Therefore, to the best of the author’s knowledge, the proposed techniques are performing well in comparison with other techniques in this field.

5. Conclusion

To conclude, in this paper, the authors addressed three localization strategies based on EKF, UKF, and PF techniques. The authors evaluated the efficiency of the proposed techniques of localization by considering many factors such as the scope of localization, the accuracy of localization, and time of consumption. Basically, two steps are used to investigate the proposed localization techniques. Firstly, the authors kept the velocity of the robot constantly i.e., $v = 1 \text{ m/s}$, and the process is repeated for four iterations as shown in Figure 3: Localization comparison of PF technique. In this phase, for all four iterations, the velocity is $v = 1 \text{ m/s}$ and time is $t = 60 \text{ sec.}$
Figures 1, 2, and 3. In the second case, the velocity of the robot is varied to $v = 2 \text{ m/s}$, $v = 3 \text{ m/s}$, and so on. As a result, the localization scope often differs by changing the velocity of the robot, as shown in Figure 4 (EKF) and Figure 5 (UKF). Furthermore, the authors evaluated the time consumption of the proposed localization techniques. Among all these techniques, the PF’s time consumption is lower compared to the localization techniques of EKF and UKF as shown in Table 2. However, the localization accuracy of EKF and UKF is better than the PF-based localization. Finally, the proposed methods are compared with each other’s and also with other standard approaches. Therefore, the proposed localization methods performed better as compared to the other techniques as mentioned in the above section of comparison.
**Figure 6:** Localization performance of EKF with (a) $T = 30$ sec and (b) $T = 90$ sec.

**Figure 7:** Localization performance of UKF with (a) $T = 30$ sec and (b) $T = 90$ sec.
In the future, the authors will perform more experiments to study the effects and performance of these localization methods. In addition, our future study will also have a look on the performance of these localization techniques in combination with the simultaneous robotic localization and mapping.

**Data Availability**

Since the funding project is not closed and related patents have been evaluated, the simulation data used to support the findings of this study are currently under embargo while the research findings are commercialized. Requests for data, based on the approval of patents after project closure, will be considered by the corresponding author.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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