Dark matter annihilation near a black hole: plateau vs. weak cusp

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I. INTRODUCTION

One of the most promising methods of indirect dark matter search is the detection of annihilation signal from so-called dark matter “spikes” around a black hole (either a supermasive black hole in galactic center or an intermediate-mass black hole). The existence of spikes is predicted by adiabatic growth model [1]. If the initial density profile is cusped, \( \rho \propto r^{-\gamma} \), then the adiabatically grown spike will also have power-law profile \( \rho' \propto r^{-\gamma'} \), with \( \gamma' = (9 - 2\gamma)/(4 - \gamma) \). If the initial profile is cored, then the spike will have \( \rho' \propto r^{-3/2} \).

It is evident that densities can reach very high values for small radii (but greater than \( r_h = 2GM_{bh}/c^2 \)). The annihilation rate of dark matter particles is therefore high enough to reduce the density. The usual argument is the following: consider the dark matter distribution function \( f(r, v) \). The annihilation rate at a certain point is given by

\[
\frac{\partial f(r, v)}{\partial t} = -\frac{\rho(r)}{m_\chi} \langle \sigma v \rangle f .
\]

Here \( m_\chi \) is the particle mass, \( \langle \sigma v \rangle \) is the annihilation cross-section times relative velocity, which is independent of \( v \).

If particles have circular orbits and the density is spherically symmetric, then one may integrate \( f \) over \( dv \) and obtain

\[
\frac{\partial \rho(r)}{\partial t} = -\frac{\rho^2}{\rho_a t} , \quad \rho(r) = \frac{\rho_0(r) \rho_a}{\rho_0 + \rho_a} , \quad \rho_a = \frac{m_\chi}{\langle \sigma v \rangle t} .
\]

Here \( \rho_0 = \rho(r, t = 0) \) is the initial density, and \( \rho_a \) is called annihilation plateau density. One can see that indeed for small radii the density approaches \( \rho_a \), and the “height” of the plateau decreases in time.

In the general case of non-circular orbits (but still having spherical symmetry) it is easier to switch from \( \{ r, v \} \) variables to \( \{ E, L \} \) variables (energy and angular momentum per unit mass). Then Eq. (1) becomes

\[
\frac{\partial f(E, L)}{\partial t} = -\frac{\tilde{\rho}}{\rho_a t} f , \quad \tilde{\rho} = \frac{1}{T} \int \rho(r) \frac{dr}{v_r} .
\]

\( \tilde{\rho} \) is the orbit-averaged density. For convenience, we replace \( L \) for \( R = L^2/L_c^2 \), where \( L_c = GM_{bh}/\sqrt{-2E} \) is the angular momentum of a circular orbit (and hence \( 0 \leq R \leq 1 \)). The radius of a circular orbit is \( r_c = GM_{bh}/(-2E) \), and we substitute \( r = x r_c \). Then we have

\[
\tilde{\rho} = \rho(r_c) \int_{1-\sqrt{1-R}}^{1+\sqrt{1-R}} \frac{dx}{\rho(r_c) \pi \sqrt{2/x - 1 - R/x^2}} .
\]

Assume we have density profile \( \rho(r) \propto r^{-\gamma} \), and a velocity anisotropy parameter \( \beta = 1 - \sigma_t^2/2\sigma_r^2 \). The case \( \beta = -\infty \) corresponds to circular orbits, \( \beta = 0 \) is the isotropic case, \( 0 < \beta < 1 \) is the case of radial velocity anisotropy.

The caveat is that in the isotropic case the density profile cannot be shallower than \( r^{-0.5} \) in the center (or, more generally, \( \gamma \geq \beta + 1/2 \)). This means that we cannot get an annihilation plateau with constant density for initially isotropic velocity distribution. One might argue that the annihilation introduces tangential velocity anisotropy to the degree compatible with flat density distribution, but in fact \( \tilde{\rho}(R) \) is increasing function of \( R \) for \( 0 < \gamma < 1 \), and the circular orbits are indeed depopulated. Thus, a careful examination is needed in the general case, as noted in [5]. This is the aim of the present study.

II. A QUALITATIVE ARGUMENT

A simple argument can explain the impossibility to create density profile more shallow than \( r^{-(\beta + 1/2)} \).

Consider the distribution function of the following broken power-law form:

\[
f(E, R) = f_0 R^{-\beta} \left\{ \begin{array}{ll} (E/E_0)^{p_1} , & |E| > |E_0| \\ (E/E_0)^{p_2} , & |E_0| > |E| > 0 . \end{array} \right.
\]

\( p_1 \) and \( p_2 \) are broken power-law indices.
We assume that \( p_1 < p_2 \), i.e. the distribution function is convex in \( \log E - \log r \) coordinates. It is a simple exercise to show that the velocity anisotropy parameter equals exactly \( \beta \) in the above expression.

We now demonstrate that if \( p_1 > \beta - 1 \), then the density in the region \( r \ll r_0 = GM_{bh}/(-E_0) \) is determined by the distribution function at \( |E| > |E_0| \), and \( \rho \propto r^{-(p_1+3/2)} \) (steeper than \( r^{-(\beta+1/2)} \)). In the opposite case, however, the density in the region \( r < r_0 \) is determined by the distribution function outside that region (i.e. with \( |E| < |E_0| \)), provided that \( p_2 - \beta > -1 \).

The density is given by expression

\[
\rho(r) = \sqrt{2\pi} \left( \frac{GM_{bh}}{r} \right)^{3/2} \int_0^1 \frac{d\varepsilon}{\varepsilon} \int_0^{4\varepsilon(1-\varepsilon)} dR \frac{f(-\varepsilon GM_{bh}/r, R)}{\sqrt{1-\varepsilon - R/4\varepsilon}}.
\]

(6)

Here \( \varepsilon = -E/rGM_{bh} \) is dimensionless energy.

If we are interested in \( r \ll r_0 \), then the integral can be split in two constituents: \( \rho_1 = \int_{\varepsilon_0}^1 \cdots \) and \( \rho_2 = \int_0^{\varepsilon_0} \cdots \), representing the contribution of inner and outer areas, correspondingly. \( \varepsilon_0 = -E_0 r/rGM_{bh} \ll 1 \) by condition. Then we get

\[
\rho_i = f_0 B(1/2, 1-\beta) \frac{4^{1-\beta}}{\sqrt{2\pi}} \left( \frac{GM_{bh}}{r} \right)^{3/2} \int_0^{\varepsilon_0} \frac{d\varepsilon}{(1-\varepsilon)^{-\beta/2+1/2}} \varepsilon^{\beta-\beta_0} \varepsilon_0^{\beta_0} \varepsilon_i^{\beta_0}
\]

\[
i = 1, 2,
\]

where limits of integration are given above.

Now there are two cases for \( \rho_1 \): if \( p_1 - \beta > -1 \), then the integrand is finite as \( \varepsilon_0 \to 0 \) and \( \rho_1 \propto r^{-(3/2+p_1)} \). In the opposite case the integrand diverges as \( \varepsilon_0^{\beta+1} \) and \( \rho_1 \propto r^{-(1/2+\beta)} \). The second integral, \( \rho_2 \), is always \( \propto r^{-(1/2+\beta)} \). Hence in the first case \( \rho_1 \gg \rho_2 \) and \( \rho \) is determined by \( \rho_1 \), while in the second case they have the same dependence on \( r \) and \( \rho \propto r^{-(\beta+1/2)} \).

What has this to do with annihilations? In the case \( \beta < -1/2 \) we may get density plateau in the center due to annihilations, which corresponds to \( p_1 = -3/2 \). However, if initially \( \beta \geq -1/2 \), a constant density core cannot develop. Instead a sort of broken power-law density profile will emerge:

\[
\rho \propto \begin{cases} r^{-(\beta+1/2)} & , \ r < r_0 \\ r^{-(p_2+3/2)} & , \ r > r_0 \end{cases}
\]

(8)

The break radius \( r_0 \) is the same as for density plateau, i.e. the density profile outside this radius is \( \rho_0 (r/r_0)^{-(p_2+3/2)} \). Particles outside \( r_0 \) are still not much affected by annihilation, since for them \( \tilde{\rho} \sim \rho(r) < \rho_0 \). Inside \( r_0 \) or, say another way, for \( |E| > |E_0| \), the distribution function is depleted more rapidly than in the case of annihilation core, since the average density that particle “feels” is higher. The boundary \( r_0 \) corresponds to the intermediate area where \( \rho \approx \rho_0(t) \).

FIG. 1: Density profiles after a certain time of evolution \( t_1 \) (see text) for different values of \( \beta \): dotted line – circular orbits \( (\beta = -\infty) \); solid line – \( \beta = -2 \), dotted-dashed line – \( \beta = 0 \). In the latter case a weak cusp \( \rho \propto r^{-1/2} \) develops instead of constant-density plateau. Initial density profile \( \rho \propto r^{-3/4} \) is shown by the long-dashed line. Radius and density are scaled to the black hole influence radius and corresponding density. Annihilation plateau density is \( \rho_a = 10^6 \), corresponding annihilation radius \( r_a = 2.2 \cdot 10^{-3} \) is denoted by the vertical line.

### III. NUMERICAL CALCULATIONS

The above qualitative arguments have to be confirmed by strict calculations. In order to do this, we solve the system of equations (3, 6) by numerical integration of Eq.(3) forward in time on a rectangular grid in \( \{E, R\} \) space, with the density profile recalculated at each time step from Eq.(6). We start from simple power-law distribution function

\[
f(E, R) = \int f_0 R^{-\beta_0}(E/E_0)^p, \quad \frac{\varepsilon^2}{(4GM_{bh})} > |E| > |E_0|
\]

(9)

Here the lower boundary \( E_0 \) defines the energy corresponding to the radius of black hole’s influence, where the gravitational potential becomes dominated by surrounding stars or dark matter rather than black hole itself. The higher boundary is determined by black hole horizon. The range of energies under consideration is large, say, \( 10^7 \), to avoid boundary effects. The corresponding initial density profile is \( \rho_{in} = K r^{-(3/2+p)} \).

We have set \( p = 3/4 \) (\( \rho \propto r^{-9/4} \)), as it corresponds to a NFW halo \( [3] \) adiabatically compressed near a black hole \( [1] \). We scale density and radius to the values at the black hole influence radius \( r_h \), and consider \( r \ll r_h \). The evolution was calculated for \( \beta = -2 \), i.e. dominance of circular orbits, and for \( \beta = 0 \), the case of isotropy.

Results are shown on Fig. 1 for time \( t_1 \) taken so that annihilation plateau density \( \rho_a \) equals \( 10^6 \) in scaled dimensionless units \( [t_1] \) is related to \( \rho_0 \) by Eq.(3). The density profile evolves almost self-similarly if we scale \( r_a \) and \( \rho \) simultaneously to remain on initial profile curve, so the value \( t_1 \) may be taken quite arbitrary.
The results agree well with our preliminary suggestions. For the case $\beta < p + 1$ ($\beta = -2$ in our calculations) a constant-density plateau develops (Fig. 1 solid) with density $\rho_a$ and radius $r_a = (K/\rho_a)^{1/(3/2+p)}$.

In the opposite case a weak cusp $\rho \propto r^{-\gamma_i}, \gamma_i = (1/2 + \beta)$ develops, which extends up to radius $r_a$ and smoothly joins the initial density profile (Fig. 1 dot-dashed). The $R$-averaged distribution function in the latter case is greatly depleted in the region $|E| > |E_a| = GM_{bh}/r_a$ compared to the case of density plateau (Fig. 2). So the cusp is indeed formed by particles with rather low energies (high apocentre radii) and high eccentricities: if the dominance of radial orbits over circular ones is large enough, then the fraction of particles on radial orbits is sufficient to determine the inner density profile. One can see that the $R$-dependence of distribution function in the case $0 < \gamma_i < 1$ is biased towards radial orbits in the region $|E| > |E_a|$ (Fig. 3).

We note that the case of isotropic (and even radially anisotropic) velocity distribution is much more relevant to cosmological dark matter halos than the case of tangential anisotropy. The parameter $\beta$ in the centers of simulated halos is about zero or slightly positive [7], and in most analytical models the situation is the same [8, 9]. Indeed a relation between $\gamma$ and $\beta$ proposed in [10] suggests $\beta >= -0.15$ for all realistic $\gamma >= 0$, which would result in a cusp. However, if a binary black hole was present in the center of a halo, it would destroy the cusp and generate a core with tangential velocity anisotropy [4]. In this case, however, the annihilation plays almost no role because density in the core falls far below $\rho_a$ (the core radius is of order the binary separation radius, which is much greater than typical annihilation radius).

IV. IMPLICATIONS FOR DARK MATTER SEARCH

The annihilation flux $\Phi$ from the direction of the black hole usually is represented as a product of two quantities, the first of them depending on particle physics and the second, called “astrophysical factor” $J$, is related to dark matter spatial density [11]:

$$ J = \frac{1}{R_{\odot} \rho_{\odot}} \int_0^{\Theta_{max}} 2\pi \theta d\theta \int_{-\infty}^{\infty} dl \rho^2 (\sqrt{l^2 + (R_{\odot}/\theta)^2}) . $$

(10)

Here $\Theta_{max}$ is the detector angular resolution (the point-spread function is assumed to be of Heaviside form for simplicity), $R_{\odot}$ is the distance from Sun to the black hole, $\rho_{\odot}$ is the dark matter density near the Sun. The outer integral represents averaging over telescope’s angular resolution, the inner stands for the line-of-sight integration.

We may rewrite this expression in terms of the “vicinity” of the black hole and the “background” from outside this vicin-
Now we note that for present-day observational capabilities we cannot hope to tell cusp from core, and even to resolve the black hole radius of influence. For example, GLAST will have angular resolution $\Omega_{\text{max}}$ of order $0.1^\circ$ [12], which corresponds to spatial distance of 15 pc at the Galactic center ($R_\odot = 8.5$ kpc). The radius of black hole influence $r_h$ in the center of our Galaxy is about 2 pc, and the annihilation radius is even smaller, of order $10^{-3}$ pc [13].

Therefore the integral in (11) is split into three terms: $r < r_a$ is the annihilation plateau or inner cusp, $r_a < r < r_h$ is the black hole domain of influence, and $r_h < r < R_{\text{max}}$ is the rest. At each of these intervals the density is roughly power-law with index $\gamma_k$. It is evident that if $\gamma_k < 3/2$, then the most part of the integral comes from outer boundary of the corresponding region, and if $\gamma_k > 3/2$, the integral is determined by inner boundary.

The density in the case of plateau is given by Eq.(2), and in the case of weak cusp is well approximated by a similar expression:

$$\rho(r) \approx \frac{\rho_0(r)\rho_i(r,t)}{\rho_0 + \rho_i}, \quad \rho_i = \rho_a(t)\left(\frac{r}{r_a}\right)^{-\gamma_i}. \quad (12)$$

$\gamma_i$ is the inner cusp power-law index, and is less than $3/2$. On the other hand, the outer power-law index of the spike is always greater than $3/2$ (in our case it equals $9/4$). Therefore, the flux from the whole black hole domain of influence is determined by $r_a$ only. It appears that the form of transition is significant: if we replace (12) by a simpler formula $\rho(r) = \min(\rho_0, \rho_i)$, we overestimate flux almost twice. But the difference between models with $\beta = -2$ (plateau) and $\beta = 0$ ($r^{-1/2}$ cusp) is less than 10%, which renders the presence of weak cusp almost undetectable. Furthermore, the addition of annihilation outside $r_h$, as well as the “background”, makes the difference even smaller. Nevertheless, we want to stress that if the power-law index everywhere outside $r_h$ is greater than $3/2$, then this area contributes little to the total annihilation flux. It is likely to be the case even for NFW initial density profile, because it should be adiabatically compressed by baryons during the formation of the Galaxy, and the profile becomes steeper than $r^{-3/2}$ [14].

V. CONCLUSION

We have reconsidered the problem of dark matter annihilation around a black hole, for the case of arbitrary velocity anisotropy of dark matter particles. In the case of circular orbits the result is well-known: a constant density core of radius $r_a$ and density $\rho_a$ develops, which smoothly joins the initial profile [Eq.(2)]. However, if the fraction of radially biased orbits is large enough, that is, if the anisotropy coefficient $\beta$ is greater than $-1/2$, a weak cusp is formed inside $r_a$: $\rho \approx \rho_a(r/r_a)^{-(\beta+1)/2}$. The cusp consists of particles which spend most part of their orbital period outside $r_a$, but since their orbits are elongated and their fraction is large, they contribute enough to the density inside $r_a$. The particles with apocentre radii within $r_a$ are annihilated much faster in the latter case, especially on orbits with low eccentricities.

However, unless we have a telescope which can resolve the radius $r_a$, we cannot practically distinguish between a plateau and a weak cusp. We note that other dynamical processes, such as scattering of dark matter particles off stars, significantly affect dark matter density at radii $r \sim r_h$ [13, 15, 16](They tend to decrease the density if it was sufficiently steep initially). So the effect considered in this paper does not seem to play an important role in the evaluation of dark matter annihilation signal from the vicinity of black holes. It is mostly of terminological significance: the term “annihilation plateau” in most cases is misleading, and should be replaced by a “weak cusp”.

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