Including of Continuous Model for Discriminating Chromatic and Achromatic Pixels in Cylindrical Distance

HSI (hue, saturation, intensity) color space is widespread used in image processing community. H, S and I components corresponds the way human perceives color and they make possible to separate chromatic (hue and saturation) and achromatic (intensity) information. Cylindrical distance is measure of color difference that takes into account angular differences and it is suitable for color spaces defined in cylindrical coordinate system (HSI, HSV etc.). Because of hue and saturation instability computation of this distance requires discrimination of chromatic and achromatic pixels. For this purpose discrete model based on threshold is used for cylindrical distance.

In this paper we propose new modified cylindrical distance that uses continuous model instead of discrete model for differencing chromatic and achromatic pixels. In continuous model we define transition functions that determines importance (weight) of hue and saturation component, which better model gradual transition from scotopic to photopic vision. Proposed formulation is compared with original formulation of cylindrical distance using human judgment of color difference. Results show that proposed formulation better corresponds to human perception for colors on transition between achromatic and chromatic area.

Key words: HSI, Cylindrical distance, Chromatic pixel, Achromatic pixel

1 INTRODUCTION

Although RGB color space is the most common color representation today, it has some drawbacks which make researchers look to the other color spaces in computer vision tasks. One drawback is high correlation between R, G and B components caused by aliasing of spectral sensitivity curves of three types of cones. Further, RGB components does not correspond the way human perceives and describes colors [1]. For example, it is hard to say, solely looking at the color, how much of R, G and B components comprise the color.

In cylindrical color spaces like HSI color is represented by hue, saturation and intensity (value, brightness). These components are closer to the way human perceives and describes color. Hue, saturation and intensity can also reveal image features that are not so obvious in RGB color space. Also, in HSI color space chromatic (hue and saturation) and achromatic (intensity) information are separated.

When working with the color spaces important question is selection of appropriate color distance measure. RGB and HSI color spaces are perceptually non-uniform. This implies that Euclidean distance between color vectors is not good measure of perceived difference between two colors. Cylindrical distance, introduced in [2], takes into ac-
count angular values and it is appropriate for color spaces defined in cylindrical coordinates. In [3] and [4] cylindrical distance is used as homogeneity criterion in region growing algorithm in HSI color space showing better results than Euclidean and Canberra distance. Application of cylindrical distance in content based sports video analysis is also very interesting. In [5], [6] and [7] playfield is detected in HSI color space as dominant color region using cylindrical distance. Sotelo et al. [8] utilize cylindrical distance for segmentation of un-asphalted roads.

In aforementioned cases cylindrical distance is used for chromatic pixels, that is pixels that carry information about color. For achromatic pixels only relevant attribute is intensity ([2], [9]). In that case cylindrical distance is reduced to intensity difference which means that chromatic difference is discarded. Hence it is necessary to perform distinction of achromatic from chromatic pixels. For this purpose discrete model based on threshold is used with cylindrical distance ([2]-[7]), but such abrupt transition is not in accordance with human perception.

Other authors also noticed this problem and they proposed several continuous models. These models difference chromatic and achromatic pixels using transition function that determines importance of hue or saturation component. In this way gradual transition from scotopic to photopic vision is better represented. It can be noticed that so far continuous models haven’t been used with cylindrical distance. All further described continuous models are applied for other purposes.

In [10] relationship between hue and saturation importance is modeled with arctan function. Drawback of this approach is that intensity is ignored although hue importance is strongly related to it. This weight function is used for calculation of hue difference in edge detection method. Vadivel et al. [11] propose function of saturation and intensity in HSV color space that determines weight (importance) of hue component. This function is applied in histogram generation for content-based image retrieval. Romani et al. [9] developed hue and saturation deviation estimators and define reliability of these components as inverse to their standard deviation. Hue reliability is a function of saturation and intensity, while saturation reliability is a function of intensity. Aptomula et al. [12] propose weighting model for hue in I HLS color space that is defined as double sigmoid function of saturation and luminance. Proposed hue weight function is used in histogram generation for color texture classification.

In this paper we propose new formulation of cylindrical distance that uses continuous model instead of discrete model. Evaluation based on human judgment of color difference shows that proposed approach shows better performance for colors on transition between achromatic and chromatic area.

Paper is organized as follows. In section 2 overview of color distance measures is given. Section 3 discusses models for distinction of achromatic from chromatic pixels. Section 4 introduces modified formulation of cylindrical distance that incorporates continuous model for differentiating chromatic and achromatic pixels. In section 5 results are presented and in section 6 concluding remarks are given.

2 COLOR DISTANCE MEASURES

In order to quantify color difference (distance) or color similarity appropriate measures are needed. Taking into account that colors are usually represented by 3D vectors one solution is to use common and well-established distance measures for m-dimensional vectors.

2.1 Minkowski Metric, Canberra Distance

Generalized weighted Minkowski metric [13] is defined as:

\[ d_p(i, j) = c \left( \sum_{k=1}^{m} \xi_k |x_{ik} - x_{jk}|^p \right)^{\frac{1}{p}} \]  

where \( m \) represents dimension of vector \( \overline{x}_i \) and \( x_{ik} \) is \( k \)-th element of \( \overline{x}_i \). The nonnegative scaling parameter \( c \) determines overall discrimination power, while parameter \( \xi_k \) represents weight of component \( k \) (\( \sum_{k=1}^{m} \xi_k = 1 \)). There are 3 interesting cases for different values of parameter \( p \):

- the city-block distance or Manhattan distance (\( p = 1 \)),
- the Euclidean distance (\( p = 2 \)),
- the Chessboard distance (\( p = \infty \)).

One of the alternatives to Minkowski metric is Canberra distance:

\[ d(i, j) = \sum_{k=1}^{m} \frac{|x_{ik} - x_{jk}|}{x_{ik} + x_{jk}} \]  

where \( m \) represents dimension of vector \( \overline{x}_i \) and \( x_{ik} \) is \( k \)-th element of \( \overline{x}_i \). This distance can be applied for vectors with nonnegative components as it is case for color vectors.

2.2 Angle Distances

Described measures are based on vector component magnitudes, but in the case of color it is interesting to observe angle between the two vectors which can be used as difference measure. Normalized inner product represents...
The cosine of angle between two vectors and defines cosine-based similarity

\[ s(\vec{x}_i, \vec{x}_j) = \frac{\vec{x}_i \cdot \vec{x}_j}{|\vec{x}_i||\vec{x}_j|} = \cos \theta \]  

(3)

where \( \vec{x}_i \) and \( \vec{x}_j \) denote vectors and \( \theta \) denote angle between them. Angle distance can be derived as:

\[ \theta = \cos^{-1}\left(\frac{\vec{x}_i \cdot \vec{x}_j}{|\vec{x}_i||\vec{x}_j|}\right) \]

(4)

It is obvious that color distance or color similarity measure should take into account both the magnitude and the orientation of color vectors. For example, Euclidean distance in RGB color space represents intensity difference, while angle distance computes hue and saturation difference. This complementarity is used for scene text extraction in [14]. Wesolkowski et al. [15] proposed edge detection method that combines angle distance and Euclidean distance in RGB space. Several similarity measures based on both magnitude and orientation of vectors can be found in [16].

In [17] for color based image retrieval 7 different vector distance measures were used: Manhattan distance, Euclidean distance, chessboard distance, Canberra distance, Czekanowski coefficient, angular distance and distance proposed by authors defined as:

\[ d(\vec{x}_i, \vec{x}_j) = 1 - \left[ 1 - \frac{2}{\pi} \cos^{-1}\left( \frac{\vec{x}_i \cdot \vec{x}_j}{|\vec{x}_i||\vec{x}_j|} \right) \right] \left[ 1 - \frac{|\vec{x}_i - \vec{x}_j|}{\sqrt{3} \times 255^2} \right] \]

(5)

According to authors [17] angular distance and distance defined with (5) give results that best match human perception.

Color space is perceptually uniform if Euclidean distance between colors corresponds to perceptual color difference. RGB color space and cylindrical color spaces like HSI, HSV, HSL etc. are perceptually non-uniform. In order to obtain perceptual uniformity CIE defined CIELAB (La*b*) and CIELUV (Lu*v*) color spaces. However, these color spaces are only approximately perceptually uniform and several other color distance measures are proposed for CIELAB color space instead of Euclidean distance: CMC [18], CIE-94 [19], CIEDE2000 [20]. These distances are based on generic formula:

\[ \Delta E = \sqrt{\left( \frac{\Delta L^*}{k_L S_L} \right)^2 + \left( \frac{\Delta C^*}{k_C S_C} \right)^2 + \left( \frac{\Delta H^*}{k_H S_H} \right)^2 + R_T \varphi (\Delta C^* \Delta H^*)} \]

(6)

where \( \Delta L^* \), \( \Delta C^* \) and \( \Delta H^* \) denotes lightness, chroma and hue difference. Scaling parameters \( k_L \), \( k_C \), \( k_H \) are positive, real values depending on the application. \( S_L \), \( S_C \), \( S_H \) are lightness, chroma and hue dependent scaling functions and \( R_T \) is scaling function depending on chroma and hue. \( \varphi () \) is a function of the product of chroma and hue differences.

2.3 Cylindrical Distance

Cylindrical distance, introduced in [2], for two pixels \((H_i, S_i, I_i)\) and \((H_j, S_j, I_j)\) is defined with following expressions:

\[ d_{cylindrical}(i,j) = \sqrt{d_{chroma}(i,j)^2 + d_{intensity}(i,j)^2} \]

(7)

\[ d_{chroma}(i,j) = \sqrt{S_i^2 + S_j^2 - 2S_i S_j \cos \theta} \]

(8)

\[ d_{intensity}(i,j) = |I_i - I_j| \]

(9)

\[ \theta = \left\{ \begin{array}{lr} \Delta & \text{if } \Delta < 180^\circ \\ 360^\circ - \Delta & \text{otherwise} \end{array} \right. \]

(10)

\[ \Delta = |H_i - H_j| \]

(11)

where \( H \in [0^\circ, 360^\circ], S \in [0, 255], I \in [0, 255] \). \( d_{chroma} \) and \( d_{intensity} \) refers to chromatic and intensity difference and \( \theta \) represents hue difference. The value of \( d_{chroma} \) (Fig. 1b) is distance between 2D vectors (hue and saturation) in chromatic plane and it is calculated using cosine law. Cylindrical distance is calculated from right-angled triangle (Fig. 1a).

Fig. 1. a) cylindrical distance b) chromatic distance
This distance takes into account angular values and, as it is shown in [4] and [3], it better correlates with color spaces defined in cylindrical coordinates. Cylindrical distance defined with expressions (7)-(11) is used for chromatic pixels, that is pixels carrying information about color. In the case of achromatic pixels intensity is only relevant attribute and cylindrical distance is reduced on intensity difference (9). This implies that is necessary to perform distinction of achromatic from chromatic pixels when calculating cylindrical distance.

3 DISCRIMINATION OF CHROMATIC AND ACHROMATIC PIXELS

When working with cylindrical color spaces like HSI three well-known properties ([2], [21], [9]) have to be considered:

1. hue is meaningless when intensity is very low,
2. hue is unstable when saturation is very low,
3. saturation is meaningless when intensity is very low.

These properties imply that low saturation increases hue standard deviation, while low intensity increases hue and saturation standard deviations. Hue instability at low saturation is demonstrated in [22] for HCI color space. We performed similar analysis for HSI color space because it is one of the most often used cylindrical color spaces and it is also used in our research.

Figure 2. shows hue distribution in dependence on saturation values. Hue histogram is computed from RGB sample (size 100x100) corrupted with additive white Gaussian noise where every component has the same standard deviation ($\sigma = 3$). The initial hue and intensity values of the sample are 85 and 60. Fig. 2. confirms conclusions from [22]: hue instability increases when saturation decreases.

Results of similar analysis for saturation component in HSI color space is shown in Fig. 3. Saturation histograms are computed from RGB sample corrupted with additive white Gaussian noise with the same standard deviation for every component ($\sigma = 3$). The initial hue and saturation values of the sample are 85 and 100. It can be seen that saturation is unstable for low intensities.

Because of variable stability of hue and saturation components it is usual to perform distinction of achromatic pixels (pixels that do not carry information about color) from chromatic pixels. For achromatic pixels hue and saturation components are unreliable and intensity is the only relevant attribute. For differencing of achromatic and chromatic pixels discrete and continuous models can be used.

3.1 Discrete Models

Discrete models perform discrimination of chromatic and achromatic pixels using the thresholds for saturation and intensity. In [2] for CIELUV color space in cylindrical coordinates achromatic pixels are defined as pixels falling in two zones:

Zone 1:

(\text{intensity} > 95)  
(\text{intensity} < 25)

Zone 2:

(81 < \text{intensity} < 95) \text{ and } (\text{saturation} < 18)  
(61 < \text{intensity} < 80) \text{ and } (\text{saturation} < 20)
Authors use these thresholds when calculating cylindrical distance. Sural et al. [23] define saturation threshold for differencing achromatic and chromatic pixels in HSV color space as function of $V$:

$$th(V) = 1 - \frac{0.8V}{255}$$

where maximum value of $V$ is 255.

Discrete models based on threshold do not properly model behavior of human visual system. Transition from scotopic vision, where we do not perceive colors, to photopic vision, where we perceive colors, goes gradually through mesopic vision [24]. Nature of this transition can be explained with spectral sensitivity of rod cells (responsible for scotopic vision) and cone cells (responsible for photopic vision) in dependence on luminance levels (Fig. 4). It can be seen that there are no abrupt jumps in rod and cone characteristics: as the luminance rises, the cones become more active and colors gradually become visible.

$$\gamma(S) = \frac{1}{\pi} \left( \frac{\pi}{2} + \arctan(\beta (S - S_0)) \right)$$

where $S$ denotes saturation, while $S_0$ and $\beta$ are parameters for fine tuning ($\beta = 0.07$ and $S_0 = 50$ are suggested
values). This function is used for calculating hue difference that is used for edge detection. In comparison with discrete model this function provides smoother transition between achromatic and chromatic region, but it does not take into account intensity. Consequently, other authors define hue reliability as function of both saturation and intensity. Vadivel et al. [11] propose hue weight function:

\[
w_H(S, I) = \begin{cases} 
S^{r_1(255/I)^2} & \text{for } I \neq 0 \\
0 & \text{for } I = 0 
\end{cases}
\]  
(16)

Intensity weight \(w_I(S, I)\) is defined as:

\[
w_I(S, I) = 1 - w_H(S, I)
\]  
(17)

where \(S \in [0, 1]\), \(I \in [0, 255]\) and \(r_1, r_2 \in [0, 1]\) are experimentally determined. These functions are applied in histogram generation for content-based image retrieval. In proposed solution hue and intensity histogram bin are, for each pixel, increased by their weights (instead of 1).

Romani et al. [9] developed hue and saturation reliability functions:

\[
R_H = \min \left\{ 1, \frac{1}{\sigma_H} \right\} = \min \left\{ 1, \frac{\overline{S I}}{\delta_H \max(S) \sigma_{RGB}} \right\}
\]  
(18)

\[
R_S = \min \left\{ 1, \frac{1}{\sigma_S} \right\} = \min \left\{ 1, \frac{7}{\delta_S \max(S) \sigma_{RGB}} \right\}
\]  
(19)

where \(\sigma_H\) and \(\sigma_S\) are estimated hue and saturation deviation, \(\sigma_{RGB}\) represents deviation of RGB components, \(\delta_H\) and \(\delta_S\) are scaling factors, \(\overline{S I}\) is saturation mean and 7 is intensity mean.

Values of \(\sigma_{RGB}, \delta_H\) and \(\delta_S\) are determined experimentally so that reliability functions can be written as:

\[
R_H = \min \left\{ 1, \frac{\overline{S I}}{20000} \right\}
\]  
(20)

\[
R_S = \min \left\{ 1, \frac{7}{500} \right\}
\]  
(21)

Validity of this approach is shown for 183 color samples. Authors compared estimated reliability values in dependence on real standard deviation of samples. Reliability function show reasonable behavior: small values for large deviation and large values for small deviation.

Hue weight function for IHLS color space that takes into account saturation and luminance is proposed in [12]. Dependency on saturation is defined with sigmoid function:

\[
g(S) = \frac{1}{1 + e^{-k(S - S_0)}} \quad , \quad S \in [0, 1]
\]  
(22)

where \(k = 10\) and \(S_0 = 0.5\). Relationship between hue weight and luminance level is described with:

\[
f(L) = \begin{cases} 
\frac{1}{1 + e^{-k_L(L - L_L)}} & \text{for } L \leq 0.5 \\
\frac{1}{1 + e^{k_L(L - L_U)}} & \text{for } L > 0.5 
\end{cases}
\]  
(23)

where \(k_L = 10, L_L = 0.25, L_U = 0.75\). Final hue weight function is defined as:

\[
\alpha = f(L) \times g(S)
\]  
(24)

This function was applied in texture classification. Author showed that this model obtained best results in comparison with hue weighting scheme presented in [25], linear function of saturation, single sigmoid function of saturation, function defined with (15) and no-weighting scheme.

4 MODIFIED CYLINDRICAL DISTANCE

So far in literature only discrete models for distinction of achromatic and chromatic pixels were used when calculating cylindrical distance. From discussion in previous chapter it is clear that continuous models better model human perception. Because of that we propose new formulation of cylindrical distance that uses continuous model instead of threshold based model for differing of chromatic and achromatic pixels.

In order to include hue reliability function in cylindrical distance it is necessary to define joint hue reliability (i.e. hue difference reliability). For hue reliability function \(\gamma(S)\) described with (15) Carron and Lambert [22] define joint reliability for two pixels \((H_1, S_1, I_1)\) and \((H_2, S_2, I_2)\) as geometrical mean:

\[
w(S_1, S_2) = \sqrt[\gamma(S_1) \gamma(S_2)}
\]  
(25)

Purpose of joint reliability in [22] is to moderate hue difference in cases when it is not reliable:

\[
\Delta H(H_1, H_2) = w(S_1, S_2) \delta(H_1, H_2)
\]  
(26)

where \(\delta(H_1, H_2)\) represents original hue difference. Authors use this difference for hue gradient calculation in
edge detection method. Similar approach is presented in [26] where hue difference is multiplied with mean saturation value:

$$\Delta H (H_1, H_2) = \frac{S_1 + S_2}{2} \theta (H_1, H_2)$$

$$S_1, S_2 \in [0, 1]$$

(27)

In our proposal of modified cylindrical distance joint hue reliability defined as geometrical mean is used (25).

4.1 Including Hue Reliability in Cylindrical Distance Formulation

Substitution of discrete model with continuous model is performed by including of hue reliability function in cylindrical distance formulation. For this purpose we apply addition-based approach for distance measure combination:

$$C = a_1 D_1 + a_2 D_2 + \ldots + a_n D_n$$

(28)

where $C$ represents combined distance, $D_i$ is $i$-th distance measure ($1 \leq i \leq n$) and $a_i$ is its weight factor. $D_i$ can be result of applying of distance measure to the different color spaces or the result of applying different distance measures to the same color space.

In order to define addition-based formulation that includes hue weight it is necessary to observe chromatic distance values for 2 border cases: hue difference $\theta$ is absolutely reliable ($w_\theta = 1$, Fig. 5.a) and hue difference is absolutely unreliable ($w_\theta = 0$, Fig. 5.b).

For absolutely reliable hue difference ($w_\theta = 1$) $d_{chroma}$ is defined with (8). It can be written:

$$d_{chroma,w_\theta=1} (i, j) = \sqrt{S_i^2 + S_j^2 - 2S_iS_j \cos \theta}$$

(29)

According to original formulation, hue difference is unreliable for two achromatic pixels and in that case chromatic distance is set to zero reducing cylindrical distance to intensity difference. We propose that for fully unreliable hue difference ($w_\theta = 0$) $\theta$ is discarded ($\theta = 0$) and $d_{chroma}$ is equal to saturation difference (Fig. 5.b):

$$d_{chroma,w_\theta=0} (i, j) = |S_i - S_j|$$

(30)

Note that in the case of unreliable hue difference $d_{chroma}$ is not set to zero like in original formulation because saturation difference can also play important role in color differing.

Also, we take into account combination where one pixel has reliable hue (high saturation-chromatic pixel) and another has unreliable hue (low saturation-achromatic pixel) what implies that hue difference is also unreliable. Despite that original formulation treats this situation in the same way as for two chromatic pixels leaving $d_{chroma}$ unchanged.

According to addition-based model (28), when $0 < w_\theta < 1$ chromatic distance can be formulated as weighted sum of solutions for $w_\theta = 1$ (29) and $w_\theta = 0$ (30):

$$d_{chroma,w_\theta} (i, j) = a_1 d_{chroma,w_\theta=1} (i, j) + a_2 d_{chroma,w_\theta=0} (i, j)$$

(31)

Weight factors $a_1$ and $a_2$ are defined using hue difference reliability $w_\theta$:

$$a_1 = w_\theta$$
$$a_2 = 1 - w_\theta$$

(32)

Equation (31) can be written as:

$$d_{chroma,w_\theta} (i, j) = w_\theta d_{chroma,w_\theta=1} (i, j) + (1 - w_\theta) \frac{d_{chroma,w_\theta=0} (i, j)}{w_\theta \sqrt{S_i^2 + S_j^2 - 2S_iS_j \cos \theta} + (1 - w_\theta) |S_i - S_j|}$$

(33)

Taking into account that hue difference is unreliable if hues of one or both pixels are unreliable $w_\theta$ is defined as in [22]:

$$w_\theta = \sqrt{w_H (i) w_H (j)}$$

(34)

where $w_H (i)$ and $w_H (j)$ denote hue reliability of pixel $i$ and pixel $j$. Vadivel hue reliability $w_H$ (16) was used in experiments.

Formulation described with (33) includes hue reliability function and models gradual transition between achromatic and chromatic area. When hue difference reliability increases more weight is given to original formulation of $d_{chroma}$ that takes into account hue and saturation. With
decreasing hue difference reliability more weight is given to saturation difference.

Reducing \( d_{\text{chroma}} \) to saturation difference for unreliable hue difference is reasonable if saturation component is reliable. Equation (33) does not cover case when saturation difference is unreliable. Consequently, it is necessary to include saturation reliability in proposed formulation.

4.2 Including Saturation Reliability in Cylindrical Distance Formulation

In order to include saturation reliability in cylindrical distance formulation it is necessary to define saturation difference reliability.

Saturation difference reliability is included in cylindrical distance formulation using addition-based distance measure combination:

\[
d_{\text{chroma}_{w_0}w_{\Delta S}} = w_{\Delta S}d_{\text{chroma}_{w_0}w_{\Delta S}=1} + (1 - w_{\Delta S}) d_{\text{chroma}_{w_0}w_{\Delta S}=0} = 0
\]

(35)

where \( d_{\text{chroma}_{w_0}w_{\Delta S}=1} \) and \( d_{\text{chroma}_{w_0}w_{\Delta S}=0} \) represent chromatic distance from (33) when saturation difference is fully reliable \( (w_{\Delta S} = 1) \) and fully unreliable \( (w_{\Delta S} = 0) \). For \( w_{\Delta S} = 1 \) chromatic distance remains unchanged and it is defined with (33). Saturation difference is fully unreliable \( (w_{\Delta S} = 0) \) if intensity of one or both pixel is equal to zero because saturation is unstable for low intensity. Low intensity also implies hue instability. In this case hue and saturation components are not reliable and chromatic distance can be discarded, that is \( d_{\text{chroma}_{w_0}w_{\Delta S}=0} = 0 \). From (35) next expression can be written:

\[
d_{\text{chroma}_{w_0}w_{\Delta S}} = w_{\Delta S}d_{\text{chroma}_{w_0}} + (1 - w_{\Delta S}) d_{\text{chroma}_{w_0}w_{\Delta S}=0} = 0
\]

(36)

Saturation difference \( w_{\Delta S} \) is unreliable if saturation of one or both pixels are unreliable. We propose next definition of \( w_{\Delta S} \):

\[
w_{\Delta S} = \min (w_S(i), w_S(j))
\]

(37)

where \( w_S(i) \) and \( w_S(j) \) denotes saturation reliability for pixel \( i \) and pixel \( j \). Instead of saturation reliability that linearly depends on intensity (21) we used sigmoid saturation reliability function:

\[
w_S(I) = \frac{1}{1 + e^{-k(I-I_0)}}
\]

(38)

Fig. 6. Distribution of a) hue and b) saturation in dependence on intensity level

where \( I_0 = 0.2 \) and \( k = 20 \)

In comparison with hue difference reliability where geometrical mean is used, \( \min \) function results with lower weight in case when one pixel has unreliable saturation. Such choice can be explained if we compare degree of instability of hue and saturation at same intensity level. From Fig. 6, it is obvious that saturation shows higher instability.

Finally, with equation (36) modified formulation of cylindrical distance is given by:

\[
d_{\text{cylindrical}}(i, j) = \sqrt{d_{\text{chroma}_{w_0}w_{\Delta S}}^2(i, j) + d_{\text{intensity}}^2(i, j)} = w_{\Delta S}(w_g \sqrt{S_i^2 + S_j^2 - 2S_iS_j \cos \theta + (1 - w_g)|S_i - S_j|})
\]

(39)

For two fully chromatic pixels \( (w_g = 1, w_{\Delta S} = 1) \) proposed expression reduces to standard formulation (7)-(11). Modified formulation differs from original cylindrical distance for pixels on the transition between achromatic and chromatic area.
5 EVALUATION AND RESULTS

Evaluation of modified cylindrical distance is performed using method from [27] where color similarity ranking obtained with color distance measure is compared with ranking created by human observer. This approach shows how close is selected color distance to human perception. Between several different cylindrical color spaces (HSI, HSV, HSL etc.) HSI color space is selected for testing because of its widespread use and easier comparison with original formulation of cylindrical distance: in [4] saturation and intensity thresholds are defined in HSI color space and they are used for cylindrical distance calculation.

In order to calculate modified cylindrical distance it is necessary to select hue and saturation reliability function \((w_H \text{ and } w_S)\). Between hue reliability functions described in 3.2 we selected those proposed by Vadivel (equation (16) - Fig. 7.a) because it is defined for HSI color space. Parameter values are set to \(r_1 = 0.1\) and \(r_2 = 0.85\) as suggested by authors. We also used saturation reliability function defined with equation (38) and showed in Fig. 7.b.

Modified cylindrical distance should exhibit difference in relation to original formulation for pixels on the border between achromatic and chromatic area. For chromatic area \((w_A = 1, w_g = 1)\) modified cylindrical distance equals to original formulation. In order to encompass this transition we select intensity and saturation values:

1. \(S = 20, 50, 75, 100 \quad S \in [0, 255]\)
2. \(I = 20, 50, 75, 100 \quad I \in [0, 255]\)

For this values test set with 10 reference colors is generated (hue is changed with step 30).

Using cylindrical and modified cylindrical distance each reference color is compared with set of 15625 colors generated by sampling RGB color space with step equal to 10 for each component. Two similarity rankings are formed using distance values of cylindrical and modified cylindrical distance. Evaluation is based on comparison of these two rankings with ranking generated by human judgement.

Procedure consists of next steps:

1. Generation of similarity rankings of first \(n\) colors (of 15625 colors) that are most similar to reference color according to cylindrical \((\text{ranking}\_\text{cyl})\), Fig.8.a) and modified cylindrical distance \((\text{ranking}\_\text{mod})\) - Fig.8.b).

2. Generation of combined color set in which ranking will be formed based on human judgment. This set is formed as union of \(\text{ranking}\_\text{cyl}\) and \(\text{ranking}\_\text{mod}\). If same color occurs in \(\text{ranking}\_\text{cyl}\) and \(\text{ranking}\_\text{mod}\), in combined set it is represented with one instance so that all colors are different.

3. Sorting of first \(n\) most similar colors from combined set using human judgment. In this way perceptual ranking \(\text{ranking}\_\text{per}\) (Fig.8.c) is formed and it is taken as ground truth.

4. Comparison of \(\text{ranking}\_\text{cyl}\) and \(\text{ranking}\_\text{mod}\) with \(\text{ranking}\_\text{per}\) using selected measures.

Number of colors \(n\) is set to 10. Each human observer sorts first 10 colors from combined set in order to obtain \(\text{ranking}\_\text{per}\). For this purpose we adapted Heap Sort algorithm where comparison operator ("<", ">") is substituted with human judgment of color similarity. Heap Sort algorithm is used because of execution time \(\theta (n \log n)\): it requires lesser comparisons for sorting than for example Bubble sort \(\theta (n^2)\). With HeapSort observer needed about 30 minutes to complete all experiments.

Fifteen observers from faculty staff participated in experiments. There were 4 female and 11 male subjects with ages from 23 to 40. The experiments were conducted in darkened room. Color samples were presented

| Color | HSI values | \(L^* a^* b^*\) values (D65) |
|-------|------------|---------------------------------|
| 1     | 0, 20, 20  | 9.87, 2.84, -6.08               |
| 2     | 30, 20, 50 | 12.51, 12.37, -17.23            |
| 3     | 60, 20, 75 | 38.58, 0.50, 3.33               |
| 4     | 90, 50, 20 | 13.79, -0.67, 11.11             |
| 5     | 120, 50, 50| 31.89, -16.06, 11.93            |
| 6     | 150, 50, 75| 40.95, -12.71, 3.26             |
| 7     | 180, 75, 20| 12.83, -1.29, -3.89             |
| 8     | 210, 75, 50| 25.59, 0.56, -13.12             |
| 9     | 240, 75, 75| 31.63, 21.21, -39.25            |
| 10    | 270, 100, 100| 39.77, 37.79, -64.71            |

Table 1. Test set with 10 reference colors

Fig. 7. a) hue reliability function b) saturation reliability function
Fig. 8. a) similarity ranking of first 10 colors according to cylindrical distance b) similarity ranking of first 10 colors according to modified cylindrical distance c) similarity ranking of first 10 colors according to human observers. First cell is reference color with HSI (0,20,20). Similarity is decreasing from left to the right.

Fig. 9. Display configuration used for color comparison

on CRT monitor Samsung SyncMaster 755 DX which was calibrated and characterized using Spyder2Pro colorimeter. Measured colorimetric values of reference colors are calibrated and characterized using Spyder2Pro colorimeter. Samples were arranged according to guidelines from [28] (Fig. 9.). Upper color samples in both color pairs are same and they represent reference color. Lower samples represent two colors that are compared with reference color. Observer judges whether left or right color is more similar to reference. Each sample has size of $4 \times 4.7$ cm subtending visual angle $4.5^\circ \times 5.5^\circ$ at a viewing distance of 50 cm. Samples were framed with 1 pixel black line. Background was set to $L^*=60.56$, $a^*=-1.74$, $b^*=-0.97$ and surrounded with 1 cm ($1^\circ$ visual angle) white border close to D65 ($X = 121.38$, $Y = 125.622$, $Z = 135.142$).

First measure used for evaluation is retrieval rate:

$$D = \frac{n_b}{n}$$  \hspace{1cm} (40)

where $n_b$ denotes number of colors from ranking_cyl$n$ (ranking_mod$n$) that are also found in ranking_per$n$; $n$ denotes number of colors in ranking_cyl$n$ (ranking_mod$n$). Color distance that in first $n$ colors returns more colors from perceptual ranking is closer to human perception. Similar criterion is used in [17] for evaluation of color distance measures in color based image database retrieval. Drawback of aforementioned measure is that it does not take into account color similarity ranking. Hence we used effectiveness measure proposed in [29]:

$$Eff = \frac{1}{1 + \log \left( \frac{n}{n_b} \right)} \sum_{i=1}^{n_b} \sum_{i=1}^{n_b} |i - r_i|$$  \hspace{1cm} (41)

where $n_b$ denotes number of colors from ranking_cyl$n$ (ranking_mod$n$) that are also found in ranking_per$n$; $n$ is number of colors in ranking_cyl$n$ (ranking_mod$n$), $i$ is color position in ranking_cyl$n$ (ranking_mod$n$) and $r_i$ is color position in ranking_per$n$. Deviation of ranking_cyl$n$ (ranking_mod$n$) from ranking_per$n$ is taken into account with sum of differences $\sum_{i=1}^{n_b} |i - r_i|$. Evaluation results obtained for 15 observers are shown in Table 2 and Fig. 10. For both measures and all observers proposed formulation shows better performance. Retrieval and effectiveness characteristic is relatively constant (Fig. 10.) confirming consistency of subjective judgment for different observers.

Table 2. Results for cylindrical and modified cylindrical distance in dependence on observer

| Observer | $D_{cyl}$ ($\%$) | $D_{mod}$ ($\%$) | $Eff_{cyl}$ ($\%$) | $Eff_{mod}$ ($\%$) |
|----------|---------------|---------------|----------------|----------------|
| 1        | 63            | 80            | 47             | 56             |
| 2        | 58            | 82            | 44             | 59             |
| 3        | 55            | 83            | 48             | 55             |
| 4        | 61            | 78            | 44             | 59             |
| 5        | 61            | 80            | 42             | 54             |
| 6        | 59            | 79            | 44             | 57             |
| 7        | 59            | 80            | 41             | 59             |
| 8        | 63            | 78            | 45             | 57             |
| 9        | 63            | 82            | 44             | 58             |
| 10       | 61            | 78            | 44             | 56             |
| 11       | 61            | 79            | 46             | 55             |
| 12       | 62            | 81            | 46             | 55             |
| 13       | 59            | 82            | 42             | 59             |
| 14       | 59            | 81            | 40             | 56             |
| 15       | 59            | 80            | 41             | 57             |
| Overall  | 61            | 80            | 44             | 57             |

Table 3. and Fig. 11. show results for each reference color. For color 1, 4 and 7 modified cylindrical distance
demonstrates significant improvement. These 3 colors (Table 1.) have intensity equal to 20. According to discrete model from [4], these colors are classified as achromatic meaning that original cylindrical distance is reduced on intensity difference. For these cases modified cylindrical distance takes into account chromatic difference with appropriate weight and results with better performance.

These results are obtained at the expense of higher computational complexity. Execution time needed for calculation of distance between 15625 color pairs is measured on PC with matlab 7.4, Core 2 Duo 2x3GHz, and 2GB RAM: modified cylindrical distance requires 1.27 seconds and cylindrical distance requires 1.05 seconds.

6 CONCLUSION

Problems concerning hue and saturation instability in cylindrical distance calculation have been studied in this paper. So far discrete model for differencing chromatic and achromatic have been used for cylindrical distance although it does not correspond to the behavior of human visual system. We proposed modified formulation of cylindrical distance that includes continuous model for discrimination of chromatic and achromatic pixels. Modification consists of two steps: including of hue reliability and including of saturation reliability.

Evaluation results based on color difference judgment for 7 observers and 10 reference colors show that modified cylindrical distance gives better effectiveness and retrieval rate meaning that it is closer to human perception. Most obvious improvement is noticed for colors that are, according to the discrete model, just below threshold and therefore classified as achromatic. For such colors original formulation of cylindrical distance takes only intensity difference, while our formulation includes chromatic difference with appropriate weight.

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Table 3. Results for cylindrical and modified cylindrical distance in dependence on color

| Color | \(D_{cyl}\) (%) | \(D_{mod}\) (%) | \(Eff_{cyl}\) (%) | \(Eff_{mod}\) (%) |
|-------|-----------------|-----------------|-----------------|-----------------|
| 1     | 14              | 86              | 13              | 60              |
| 2     | 73              | 64              | 61              | 45              |
| 3     | 60              | 73              | 43              | 55              |
| 4     | 29              | 81              | 17              | 57              |
| 5     | 60              | 74              | 51              | 55              |
| 6     | 80              | 89              | 60              | 61              |
| 7     | 33              | 77              | 22              | 50              |
| 8     | 76              | 76              | 44              | 54              |
| 9     | 76              | 71              | 54              | 52              |
| 10    | 100             | 100             | 63              | 64              |
| Overall | 61              | 79              | 43              | 55              |

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