LARGE-SCALE STRUCTURE OF THE UNIVERSE AS A COSMIC STANDARD RULER

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ABSTRACT

We propose to use the large-scale structure (LSS) of the universe as a cosmic standard ruler. This is possible because the pattern of large-scale distribution of matter is scale-dependent and does not change in comoving space during the linear-regime evolution of structure. By examining the pattern of LSS in several redshift intervals it is possible to reconstruct the expansion history of the universe, and thus to measure the cosmological parameters governing the expansion of the universe. The features of the large-scale matter distribution that can be used as standard rulers include the topology of LSS and the overall shapes of the power spectrum and correlation function. The genus, being an intrinsic topology measure, is insensitive to systematic effects such as the nonlinear gravitational evolution, galaxy biasing, and redshift-space distortion, and thus is an ideal cosmic ruler when galaxies in redshift space are used to trace the initial matter distribution. The genus remains unchanged as far as the rank order of density is conserved, which is true for linear and weakly nonlinear gravitational evolution, monotonic galaxy biasing, and mild redshift-space distortions. The expansion history of the universe can be constrained by comparing the theoretically predicted genus corresponding to an adopted set of cosmological parameters with the observed genus measured by using the redshift–comoving distance relation of the same cosmological model.

Key words: cosmology: theory – large-scale structure of universe

1. INTRODUCTION

The large-scale distribution of galaxies has long been used to constrain cosmological models through two-point correlation function (CF) and power spectrum (PS) analyses (Davis & Peebles 1983; Maddox et al. 1991; Vogeley et al. 1992; Park et al. 1994; Cole et al. 2005; Tegmark et al. 2006; Percival et al. 2007; Drinkwater et al. 2010) because the shapes of the PS and CF depend on cosmological parameters such as the matter and baryon density parameters ($\Omega_m$, $\Omega_b$) and the primordial spectral index ($n_s$) of the PS.

Recently, it became popular to use the baryon acoustic oscillation (BAO) features in PS or CF as a standard ruler of the universe to constrain the expansion history of the universe (Blake & Glazebrook 2003; Eisenstein et al. 2005; Collett et al. 2005; Tegmark et al. 2006; Percival et al. 2007). Sensitivity of the method to the expansion of the universe appears when the observational data span a range of redshifts. In the linear regime, the shape of PS is maintained in time in comoving space. Comparing among the scales of BAO measured from the PS or CF at different epochs and knowing that the scale is constant in comoving space, one can find how the universe has expanded across the epochs.

On the other hand, it is now well confirmed that the observed galaxy CF and PS have scale-dependent slopes (Maddox et al. 1991; Park et al. 1994; Tegmark et al. 2006). Theoretically, the linear matter density PS of the cold dark matter (CDM) models is expected to have a maximum near the scale corresponding to the epoch of matter-radiation equality, and has a slope approaching $k^{n_s}$ on the largest scales and $k^{-3}$ on the smallest scales, where $k$ is the wavenumber. The power spectra of various cosmological models have their own specific spatial-scale dependences. And just like the BAO, a tiny wiggle on top of the smooth PS, which conveys the scale information embedded in the largescale structure (LSS), the whole shape of the PS also carries the absolute scale information. These absolute scales extracted from the linear-regime PS measured at several redshifts, can be used to constrain the expansion history of the universe and to estimate the cosmological parameters governing the spatial expansion.

The shape of PS or CF is only one of many properties of LSS that can be used as standard rulers, and another is the topology of LSS, the pattern of the large-scale galaxy distribution. Bond et al. (1996) pointed out that the filament-dominated cosmic web is present in embryonic form in the overdensity pattern of initial fluctuations with nonlinear dynamics just sharpening the image. The high-density regions such as galaxy clusters and filaments are mere amplifications of the initial high-density regions. However, it is not just the overdensity regions like filaments but the whole LSS including the cosmic voids that retains its initial pattern. Both high- and low-density regions conserve the initial conditions on large scales. After all, it is the whole cosmic sponge rather than just high-density structures that remains unchanged on large scales throughout the history of the universe.

In this Letter, we propose to use the topology of LSS as a cosmic ruler, extending the idea of Gott et al. (1986) and Hamilton et al. (1986) who proposed to use the topology of LSS to examine Gaussianity of the primordial density fluctuations as predicted by simple inflationary models (Guth 1981). Compared to the shape of the PS, topology is much less affected by observational systematic effects such as redshift-space distortion, galaxy biasing, and nonlinear gravitational evolution. This is because the intrinsic topology of LSS does not change as structures continuously collapse, expand, or deform without breaking up or connecting with itself or other structures. In the following section, we introduce the genus statistic as a measure of the sponge topology of LSS. In Section 3, we explain how the genus can be used to constrain the expansion history of the universe. Conclusions follow in Section 4.

2. LARGE-SCALE STRUCTURE TOPOLOGY AS A STANDARD RULER

In this Letter, we will adopt the genus statistic (Gott et al. 1986, 1989; Vogeley et al. 1994; Park et al. 2005a, 2005b; James...
et al. 2007, 2009; Choi et al. 2010) as a measure of the LSS topology. The genus is measured in the following way. We treat galaxies as mass points, and smooth them by a Gaussian filter. Then the isodensity contour surfaces of the smoothed galaxy density distribution are searched for a given density threshold or for a volume fraction of high-density regions. The genus is then calculated from the definition:  

\[ G = \text{(number of holes)} - \text{(number of isolated regions)} \]  

enclosed by the contour surfaces (Weinberg 1988).

Being a measure of intrinsic topology, the genus is insensitive to the nonlinear gravitational evolution, galaxy biasing, and redshift-space distortions, particularly when we adopt the volume fraction for the threshold levels to identify the contours. According to the second-order perturbation theory, there is no change in the genus at the median-density threshold due to the weak nonlinear gravitational evolution (Matsubara 1994). Physically speaking, the growth of density in high-density regions and the evacuation of matter from low-density regions do not affect the isodensity contours enclosing a given fraction of sample volume while the gravitational evolution conserves the rank order of density. For the same reason, different amounts of galaxy biasing result in no difference in the isodensity contour surfaces defined by a volume fraction if the galaxy density field is monotonically related to the matter density field. Topology bias does not arise for monotonic galaxy biasing. Continuous radial distortion of the shape of LSS mapped in redshift space also does not change the genus because the number of holes and isolated regions are not affected. It is because of the insensitivity of the genus to these systematics that we adopt the genus topology as a standard ruler.

When the density field is Gaussian, the genus is related to the slope of PS near the smoothing scale. The genus per unit volume for a Gaussian field is given by

\[ G(v) = g(1 - v^2)e^{-v^2/2}, \]  

where \( g = (k^2/3)^{3/2}/2\pi^2 \) is the amplitude of the genus curve per unit volume (genus density) and \( k^2 = \int P_s(k)k^2d^3k/\int P_s(k)d^3k \) depends on the shape of the PS \( P_s(k) \) of the smoothed density field (Hamilton et al. 1986; Doroshkevich 1970). The threshold \( v \) is the density threshold level normalized by the root mean square (rms) density fluctuation, and is related to the volume fraction \( f \) on the high-density side of the density contour surface by

\[ f = \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-x^2/2} dx. \]  

The volume fraction \( f = 50\% \) contour corresponds to the median volume fraction contour \( (v = 0) \). According to most previous studies on non-Gaussianity of the primordial density fluctuations, the large-scale distribution of galaxies is consistent with the cosmological models with initially Gaussian matter density fields (e.g., Gott et al. 2009).

When the density field is not Gaussian, the amplitude \( g \) is not simply related to the shape of the PS. The genus topology of LSS on the linear and weakly nonlinear scales is conserved in the course of time as explained above regardless of whether the initial density field is Gaussian or non-Gaussian. In this sense, the genus topology of LSS is a statistic independent of the PS.

Figure 1 shows the amplitude of the genus curve per unit smoothing volume, \( g R_G \), as a function of the Gaussian smoothing length, \( R_G \), for a series of CDM models with the WMAP five-year cosmological parameters (Dunkley et al. 2009) except for the density parameter \( \Omega_m \). Each curve, labeled by \( \Omega_m \), has a characteristic shape reflecting the different shapes of the CDM power spectra. The thick line is for \( \Omega_m = 0.26 \), and the boundaries of the shaded region correspond to \( \Omega_m = 0.29 \) and 0.23, which are chosen from the mean and the 1σ uncertainty limits of the WMAP five-year data. The data points are from the SDSS DR4 plus LRG sample measured with smoothing lengths of 21 and 34 \( h^{-1} \) Mpc (Gott et al. 2009).

### 3. Constraining the Expansion History of the Universe

There are now many deep and wide-angle redshift surveys of LSS that are being made or planned in the near future (Kovac et al. 2010; Percival et al. 2010; Schlegel et al. 2007). In this section, we will describe how an observational sample of LSS covering a range of redshift can be used to constrain the expansion history of the universe.

Figure 2 illustrates what happens to the analysis when wrong \( r(z) \) relations (wrong cosmologies) are adopted. Suppose one is trying to constrain the dark energy equation of state parameter \( w \) while other parameters are fixed. The box on the left shows LSS at \( z = 0 \) in a square region of the universe with \( w = -1 \). The error bar indicates the smoothing scale. If one transforms the redshifts of distant galaxies to comoving distances using the \( r(z) \) relation adopting the correct value of \( w \), a region constrained to have the same comoving size at high \( z \) will enclose statistically the same amount of LSS as in the \( z = 0 \) case and the smoothing length will correspond to the scale equal to that at \( z = 0 \) (the middle box on the right). But if \( w = -0.5 \) is chosen, the universe is mistakenly treated as expanding slower than reality. As a result of the wrong \( r(z) \) transformation, a “unit” volume (the dashed
in the following way. The volume factor at redshift $z$ compared to that at low redshift $z_0$ is adopted while the true cosmology is a cosmology is given by $V_{X}$ expands faster than the real one ($w = -1$). When one adopts a universe that is scale-free. When one adopts a universe that expands faster than the real one ($w = -1.5$, for example), the “unit” box at high redshift will contain a smaller amount of LSS compared to that at $z = 0$, and the smoothing scale corresponds to a physical scale smaller than that at low $z$. Because the box of a unit comoving volume contains more LSS but the smoothing is made over a larger scale, their effects on the genus partially cancel with each other but there remains some net effect unless the density fluctuations are scale-free. When one adopts a universe that expands faster than the real one ($w = -1.5$, for example), the “unit” box at high redshift will contain a smaller amount of LSS compared to that at $z = 0$, and the smoothing scale corresponds to a physical scale smaller than what is intended.

The genus that will be obtained when a wrong cosmology “Y” is adopted while the true cosmology is “X” can be calculated in the following way. The volume factor at redshift $z$ in a cosmology is given by $V = D_A^2/H(z)$ (Peebles 1993), where $D_A = r(z)/(1 + z)$ is the angular diameter distance and $H(z)$ is the Hubble parameter. When a wrong cosmology is adopted, the fractional change in the amount of LSS is $V_X(z)/V_Y(z)$ when the samples are constrained to have the same comoving volume under the assumed cosmology. (Due to the Alcock–Paczynski effect (Alcock & Paczynski 1979), the amounts of radial and tangential length variations are slightly different from each other. In the present treatment, we average structures over angles and consider only the volume effect.) On the other hand, the smoothing scale changes by a factor $\lambda_{XY}(z) = (V_X(z)/V_Y(z))^{1/3}$. Therefore, the amplitude of the genus curve measured at the smoothing scale $R_G$ when the wrong cosmology $Y$ is adopted in converting redshift to comoving distance, becomes

$$g_Y(z; R_G) = g_X(R_G^3)V_X(z)/V_Y(z),$$

where $R_G^3 = \lambda_{XY}(z)R_G$. This formula is equivalent to $g_Y(z; R_G)R_G^3 = g_X(R_G^3)/V_Y(z)$.

Figure 3 shows how the genus is used to measure the cosmological parameters that govern the expansion history of the universe. The left panel shows the theoretical amplitude of the genus curve per unit smoothing volume when $R_G = 15 h^{-1}$ Mpc for five flat cosmological models with different set of $(\Omega_m, w)$. The mean galaxy separation of the SDSS-III survey is expected to be $15 h^{-1}$ Mpc (Schlegel et al. 2007). The right panel shows the genus that will be actually measured when five different sets of the cosmological parameters are assumed in the $r-z$ transformation even though the correct one is $(0.26, -1.0)$. (A color version of this figure is available in the online journal.)
In the right panel, the true cosmology is the ΛCDM model with $\Omega_m = 0.26$ and $\Omega_E = 0.74$ with $w = -1$. The lines are the genus that will be actually measured when five different cosmologies are used in the $r-z$ transformation. The figure demonstrates that the genus depends on redshift differently depending on the values of $\Omega_m$ and $w$ used for the transformation. The correct choice of (0.26, −1.0) results in an agreement (black solid line) with the theoretical prediction for the model, the horizontal line with $g_{R_0} = 0.00470$. But when (0.26, −0.5) is mistakenly adopted, for example, the wrong $r(z)$ relation makes the genus amplitude overestimated. This means that the volume factor dominates over the smoothing effect in Equation (2). At $z = 0.5$, the measured value of $g$ will be 5.3% larger than that at $z = 0$. When one adopts a model with $w$ more negative than −1, the measured genus amplitude falls below the value at $z = 0$. (See the (0.26, −1.5) case.)

Effects of $\Omega_m$ and $w$ on the genus have some degeneracy as they both affect the expansion of the universe. $\Omega_m$ determines the shape of the PS at a given epoch. Therefore, in the case of Gaussian models the genus measured at low redshifts constrains $\Omega_m$. But at high redshifts, the genus depends on $\Omega_E$ and $w$ as well as $\Omega_m$. Therefore, $\Omega_m$ can be estimated by the genus measured at low redshifts, and the other parameters are constrained by the genus of high-redshift samples.

4. SUMMARY

The key points of this Letter can be summarized as follows.

1. We point out that the pattern of LSS is conserved in time and scale-dependent, and can be used as a cosmic standard ruler.

2. We propose the use of the topology of LSS as a robust standard ruler because it is insensitive to various linear and nonlinear systematic effects.

Recently, Gott et al. (2009) measured the genus amplitude with 4% error using the LRG sample of the SDSS DR4plus smoothed with $R_g = 21h^{-1}$ Mpc. The error included the uncertainty due to the cosmic variance expected in the ΛCDM universes. The final Sloan Digital Sky Survey Data Release 7 data and the future LSS surveys are expected to significantly reduce the uncertainty.

To enhance the power of the standard rulers, it is necessary to have accurate knowledge on the systematic effects such as nonlinear gravitational evolution, scale-dependent galaxy biasing, and redshift-space distortion (Park et al. 2005a, 2005b; James et al. 2007, 2009). Effects of these systematics on the BAO scale have been extensively studied by Kim et al. (2009), for example. Estimates of the systematic effects on the genus and various constrains on $w$ from existing and future surveys will be presented in our subsequent papers. However, in the case of the flat ΛCDM model with the WMAP five-year cosmological parameters, our $N$-body simulations show that the rms displacements of matter particles from their initial comoving space positions until redshifts $z = 0.5$ and 0 are 7.7 and 9.7$h^{-1}$ Mpc, respectively. On smoothing scales much larger than these distances all nonlinear effects will be small, and the observed topology of LSS should follow that of the initial density fluctuations accurately.

In addition to the genus in three-dimensional space, one can also use the scale dependence of the one-dimensional level-crossing statistic (Longuet-Higgins 1957; Ryden 1988) or the two-dimensional genus of LSS (Melott et al. 1989) as standard rulers. This is possible because the statistics, as measured by the number of galaxy number density level crossings per unit comoving length or by the two-dimensional genus of the galaxy distribution per unit comoving area, should be the same at different redshifts on large scales if the $r-z$ transformation is correctly made in the calculation of these statistics. We are currently exploring the usefulness of these statistics in constraining cosmological parameters.

C.B.P. acknowledges the support of the Korea Science and Engineering Foundation (KOSEF) through the Astrophysical Research Center for the Structure and Evolution of the Cosmos (ARCEC).

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