Deformation of cavitation bubbles at their strong expansion and collapse in a streamer

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Abstract. Deformation of initially spherical cavitation bubbles is considered in the case of their strong single expansion and subsequent collapse in a streamer consisting of three bubbles located in one straight line. The central bubble is equidistant from the others. The influence of the distance between the bubbles is studied in the range in which the non-sphericity of the central bubble during the collapse does not exceed 20%. The expansion of the bubbles and the most part of their collapse is governed by the ordinary differential equations of the second order in the radii of the bubbles, the positions of their centers on the line of their interaction, and the amplitudes of their axisymmetric non-sphericity in the form of spherical harmonics. In doing so, the interaction between bubbles is taken into account, the assumptions of weak liquid compressibility, homobaricity of bubbles, and their small non-sphericity are used. The final of the collapse of each bubble is governed by the partial differential equations of gas dynamics. In doing so, the interaction of bubbles is ignored, but the axial symmetry of bubbles, liquid compressibility, non-uniformity of vapor, heat conductivity of vapor and liquid are taken into account, wide-range equation of states for vapor and liquid are applied. It is shown that with decreasing the distance between the bubbles, the non-sphericity of the central bubble at the end of its collapse first slowly increases from zero and then quite rapidly grows. Along with that, the central bubble non-sphericity level of 1% is attained at the distance between the bubbles equal to about 20 times their radius at the moment of their maximal expansion. Simulation of the final high-speed stage of the bubble collapse by the model destined to the low-speed stage of expansion and collapse leads to significant overestimation of the non-sphericity of the central bubble at the end of its collapse.

1. Introduction

An important feature of bubble dynamics is a possibility of attaining in the bubbles very high pressures, temperatures, and densities (supercompression) at the end of their strong collapse [1-3]. Theoretical and experimental studies show that the supercompression of the gas-vapor medium may be realized in the better studied case of single bubbles (e.g., in the regime of single bubble sonoluminescence [2, 3]) as well as in the less studied case of bubble cluster (e.g., in the regime of acoustic cavitation of deuterated acetone [1]).

One of important factors restricting the supercompression of the medium in bubbles is instability of the spherical shape at collapse. As a result, initially small bubble sphericity perturbations may become largely grown in the process of collapse up to bubble destruction (i.e., to losing the one-connectedness...
of bubble), which leads to significant reduction in degree of compression of the bubble content. In the case of a single bubble, the issue of its deformation is considered in quite many works [4 - 6]. Unlike this, deformation of bubbles interacting with other bubbles in the mode of their radial oscillations of large amplitude with capability of attaining supercompression of their content still remain comparatively little studied. Some examples of such investigations are given in [7, 8].

The present work studies the dependence of deformations of the central bubbles in a streamer (a group of bubbles located in one straight line) on the distance between the bubbles under their strong single expansion and subsequent collapse. The assumed conditions are close to those used in experiments on acoustic cavitation of deuterated acetone [1]. The employed mathematical model and numerical technique [8] take into account the features of the low-speed stage of the expansion and collapse of bubbles as well as the high-speed stage of their collapse. It should be noted that investigations of bubble deformations in a streamer under similar conditions were also performed in [7]. But the final high-speed stage of bubble collapse was simulated in that work with oversimplification.

2. Problem statement

A single strong expansion and consequent collapse of three cavitation bubbles in liquid acetone is considered under the action of its varying pressure. The bubbles are located in one straight line so that one of them (the central one) is equidistant from the others (the lateral ones). At the beginning of the expansion (at the time moment \( t = 0 \)) all the bubbles are spherical, the radius of each of them is equal to \( R_0 \), the distance between the centers of the neighboring bubbles is equal to \( d \) (figure 1). The liquid pressure \( p_\infty \) changes with respect to the harmonic law

\[
 p_\infty = p_0 - p_a \cos \omega t, \tag{1}
\]

where \( p_0 \) is the static pressure, \( p_a \) is the amplitude, \( \omega \) is the frequency. The initial pressure in the bubbles \( p_{00} \) is the pressure of the saturated vapor of acetone at a temperature of the surrounding liquid.

The dependence of deformations of the central bubble on the distance between the bubbles is studied in the case the liquid temperature is \( T_0 = 273 \) K, the static liquid pressure is \( p_0 = 1 \) bar, the amplitude of variation of the liquid pressure is \( p_a = 15 \) bar, the frequency of its variation is \( \omega = 2\pi \times 19.3 \) kHz, the initial radius of the bubbles is \( R_0 = 5 \) \( \mu \)m, their internal pressure is \( p_{00} = 0.09 \) bar. The initial distance between the neighboring bubbles is varied in the range \( d \geq 9R_{max} \), where \( R_{max} = 450 \) \( \mu \)m is the radius of the bubbles at the moment of their maximal expansion under \( d \to \infty \). In this range, the non-sphericity of the central bubble grows to the end of its collapse comparatively not much (no more than 20%).

![Figure 1. A linear streamer of initially identical equidistant bubbles (\( R \) is the radius of the central bubble, \( d \) is the distance between the bubble centers).](image)

It should be noted that the assumed values of the liquid temperature \( T_0 \), the static liquid pressure \( p_0 \), the amplitude of variation of the liquid pressure \( p_a \), the frequency of its variation \( \omega \), and the radius of the bubbles \( R_{max} \), they attain at the moment of their maximal expansion, are close to those realized under conditions of the experiments on acoustic cavitation of deuterated acetone [1].

With the liquid pressure varying according to (1) with the amplitude much larger than the static pressure, the liquid pressure \( p_\infty \) in the course of the first semi-period of its variation constantly grows from its minimal negative value (–14 bar) to its maximal positive value (16 bar). This results in that
the bubbles first strongly expand and then strongly collapse. In doing so, the lateral bubbles displace with the velocities equal in magnitude and opposite in direction, whereas the central bubble remains motionless. This results from the symmetrical configuration of the streamer with respect to the central bubble. It should be noted that the bubbles in the central region of cluster in the experiments [1] might also stay motionless.

3. Mathematical model and numerical technique

A detailed description of the model and the numerical technique of the present work, as well as the results of their validation can be found in [8]. These model and numerical technique are in fact combinations of a model and a technique of calculating the dynamics of interacting slightly-nonspherical bubbles in a streamer [7] and a model and a technique of calculating the dynamics of a single axisymmetric bubble [8]. The first ones are used at the low-speed stage including the whole expansion of the bubbles and the initial longer-lasting part of their collapse. Taking into account the features of this stage, the model of the dynamics of interacting bubbles in a streamer assumes that the liquid is weakly compressible, its motion is potential, the vapor in the bubbles is always in the saturation state at a temperature of the surrounding liquid with the corresponding pressure. The bubble sphericity distortions are assumed small. In this case, the equation of the \( i \)th bubble surface in the spherical coordinate system \( r_i, \theta_i, \phi_i \) with the origin at the center of this bubble can be written as follows

\[
r_i(\theta_i,t) = R_i(t) + \sum_{n=2}^{N} a_{n,i}(t) P_n(\cos \theta_i).
\]

Here \( a_{n,i} \) is the amplitude of non-sphericity of the \( i \)th bubble in the form of the spherical harmonic defined by the Legendre polynomial \( P_n(\cos \theta_i) \) of degree \( n \), \( n=2, \ldots, N \). The non-sphericity in the form of \( P_n(\cos \theta_i) \) is characterized by a dimensionless amplitude \( \varepsilon_{n,i} = a_{n,i}/R_i \). It is assumed that \( |\varepsilon_{n,i}| << 1 \) for each \( n \). The bubble dynamics is governed by a system of ordinary differential equations in the radii of the bubbles, the positions of their centers on the line of their interaction, and the amplitudes of their non-sphericity in the form of spherical harmonics.

The results presented below are related to only the central bubble. Therefore, the bubble number index \( i \) is omitted.

In the high-speed stage of collapse, the interaction between the bubbles becomes insignificant. However, it becomes necessary to correctly describe the vapor non-uniformity in the bubbles, the vapor and liquid heat-conductivity, the non-equilibrium evaporation and condensation at the bubble surfaces, the bubble surface non-sphericity. Taking this into account, a hydrodynamic model of the dynamics of a single axisymmetric bubble is applied to each bubble. In this model, the vapor in the bubble and the surrounding liquid is governed by the gas dynamics equations presented in moving coordinates related to the bubble surface.

The transition from the low-speed stage of the bubble dynamics to its high-speed stage is made when the interaction of bubbles becomes insignificant.

4. Results

Figure 2 illustrates main features of evolution of the radius of the central bubble in the streamer and deformation of the surface of this bubble in the process of its expansion and consequent collapse in the case the initial distance between the bubble centers is \( d = 15R_{\text{max}} \).

One can see that the bubble first strongly expands because of the large negative pressure of the surrounding liquid and then strongly collapse owing to the also large but now positive pressure of the surrounding liquid (figure 2 (a)). In the process of expansion and collapse of the bubble, its initially spherical shape significantly changes (figures 2 (b), (c)) by reason of the influence of the neighboring bubbles. Figures 2(b), (c) show only the deformations in the form of the second spherical harmonics \( n = 2 \) since the deformations in the form of other even harmonics \( (n = 4, 6, \ldots) \) appear to be much smaller whereas the deformations in the form of odd harmonics \( (n = 3, 5, \ldots) \) do not arise at all owing
to the symmetry of the streamer configuration about the central bubble. It follows from figures 2(b),(c) that in the course of expansion and collapse of the bubble, its non-sphericity changes in the form of oscillations with growing amplitude. In particular, during expansion, the amplitude $|\varepsilon_2|$ monotonically increases to reach its maximum shortly before the end of expansion. Soon after the beginning of collapse, the bubble transforms into a spherical one and then deforms again. In the final stage of collapse (figure 2(c)), the bubble again transforms into a spherical one and after that its non-sphericity grows very rapidly (figure 2(c)). It should be noted that the simplified simulation of the high-speed stage of the bubble dynamics (red line in figure 2(c)) using the model destined for the low-speed stage leads to significant errors in resolving the evolution of the amplitude $|\varepsilon_2|$, including rather large overestimation of its maximal value at the end of collapse.

**Figure 2.** Evolution of the radius of the bubble in the process of its expansion and whole collapse (a) and deformation of the bubble surface in the form of the second spherical harmonic at the low-speed stage, including the expansion of the bubble and the beginning of its collapse, (b) and at the final high-speed stage of collapse (c), $d/R_{\text{max}}=15$. Dots mark the boundary between the mentioned stages, circles show the boundary between the bubble expansion and collapse. Red lines represent the results of the model of interacting bubbles, blue lines correspond to the results of the model of a single bubble (the first model is adequate to the low-speed stage, the second one is adequate to the high-speed stage).

Computations show that most of the above-mentioned features of deformation of the central bubble in the streamer in the case of $d = 15R_{\text{max}}$ remain the same under other distances between the bubbles in the whole range $d \geq 9R_{\text{max}}$. Variation in the distance leads only to changes in the numerical values of the amplitude $|\varepsilon_2|$.

Figure 3 illustrates the influence of the distance between the bubbles in the streamer on the maximal deformations of the central bubble in the course of its expansion and collapse. It is seen (figure 3, blue line) that with increasing $d$, the magnitude of the maximal non-sphericity of the central bubble in the form of the second spherical harmonic first sharply decreases in the region from $9R_{\text{max}}$ to $20R_{\text{max}}$ and then slowly tends to zero. At the bubble spacing $d > 20R_{\text{max}}$, the bubble keeps being quite close to the spherical one (with maximal non-sphericity of less than 1%).

It should be noted that under simplified simulation of the high-speed stage using the model of interacting bubbles adequate to the low-speed stage (figure 3, red line), the behavior of the dependence of the maximal amplitude of the central bubble non-sphericity in the form of the second harmonic on the distance between the bubbles remains the same. At the same time, the magnitude of the maximal amplitude is appreciably overestimated.
Figure 3. Influence of the initial distance between the bubbles in the streamer on the magnitude of the maximal amplitude of the central bubble non-sphericity in the form of the second spherical harmonic in the process of expansion and collapse of this bubble. Blue line corresponds to the results of applying the model of dynamics of a single bubble adequate to the high-speed stage of collapse, red line represents the results of simplified simulation of that stage, using the model of interacting bubbles adequate to the low-speed stage.

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