An indirect measurement process parameterized by a nonunitary system–pointer interaction

A I Trifanov and G P Miroshnichenko

Saint Petersburg National Research University of Information Technologies, Mechanics and Optics 49, Kronverksky Avenue, Saint Petersburg 197101, Russia
E-mail: alextrifanov@gmail.com

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Abstract
An indirect quantum state measurement process is under investigation. Common evolution of the detector’s microscopic part (pointer) and the measured system (target) is considered to be nonunitary due to the interaction between the pointer and the environment. The system of differential equations for superoperators, which map the initial target state into its conditional final state, is obtained. A developed formalism is applied to the problem of cavity quantum mode photodetection to investigate the basic information characteristics of this measurement process.

Keywords: indirect quantum measurement, cavity mode photodetection, atom–EM field interaction, conditional quantum dynamics

(Some figures may appear in colour only in the online journal)

1. Introduction
Correct description of quantum measurements [1] is one of the fundamental problems of quantum physics, which remains an important and difficult research topic at present. The superselection and decoherence process, which is closely related to this subject, controls the flow of information in quantum communication systems [2]. There are at least three formulations, which reflect the detector back action on the state of the measured system [3].

Indirect measurements use interaction between the monitored (target) and ancillary (pointer) system followed by projection measurement on an ancilla. This provides the opportunity of realizing the indirect (and unsharp) detection process with weak disturbance of the initial state of the target. Formally, this process may be described by parameterization \((\rho_p, U_\tau, \{\Pi_i\})\) of the initial projective measurement. Here \(\rho_p\) is the initial pointer state, \(U_\tau\) is the evolution operator, \(\tau\) is the interaction time and \(\{\Pi_i\}\) are the projector valued measures. It may be completed by the canonical Naimark [4] extension. This technique was successfully applied in [5] to describe the Stern–Gerlah experiment with unsharp measurement. Another way is to start from the arbitrary suitable pointer and comparatively simple interaction and construct positive operator valued measures corresponding to this kind of generalized detection. This method was used for the description of the indirect measurement on trapped ions [6] realizing quantum control and for quantum gate realization in cavity quantum electrodynamics [7]. It should be noted that unitary evolution is usually used for parameterization.

In fact, the efficiency of a measurement device is not exactly equal to the identity (to say nothing of the photodetection and particularly the IR region, where this quantity is rather far away from the unity). One can define at least two factors which participate in this process: the first one is a classical and quantum stochastic process, which governs the behavior of the measurement apparatus. This factor may be taken into account by introducing the corresponding phenomenological probability distributions for detection events and errors [8]. The second one is the influence of the environment, which may appear in the initially entangled state with the pointer and/or pointer state decoherence during its interaction with the measured system.

In this paper we investigate the conditional evolution of the monitored system in the case of interaction between the pointer and the environment during the measurement process. Initial states of all subsystems are assumed to be factorized. In section 2 a brief review of the required formalism is presented. Section 3 contains a simple example of the developed theory and considers the case of evolution generated by the superoperator in the Lindblad form. Namely, we deal
with the single cavity quantum mode photodetection with a
two-level atom-pointer in use. In section 4 basic information
characteristics, such as the probability of certain state
detection, information gain and fidelity for this measurement
process are analyzed. Section 5 concludes the paper.

2. Description of parameterized measurement

An interaction between the pointer and the environment leads
to a nonunitary common reduced evolution of the target and
the pointer. To take it into account we will introduce a triplet
\((\rho_p, A_r, \{\Pi_r\})\), where \(A_r = \exp(-i\Delta t)\) and \(\mathcal{L}\) is the evolution generator
\[
\dot{\rho}(t) = \mathcal{L}\rho(t).
\]
(1)

Here \(\rho(t)\) is a common density operator for the system and
the pointer. Let \(\rho_{in} = \rho(0)\) and
\[
\rho_{in} = \rho_S \otimes \rho_p = \rho_S \otimes |in\rangle\langle in|, \quad (2)
\]
where \(|in\rangle\langle in|\) is an initial pure pointer state in the
superoperator representation. The density operator \(A_r\rho_{in} =
\rho_{out}\) describes the common reduced state just after the
interaction and before the projective measurement.

If the detection results are ignored, the state of the
monitored system may be obtained by tracing out ancillary
degrees of freedom
\[
\rho_{S}^\prime = \text{Tr}_A[\rho_{out}].
\]
(3)

In the case of measurements with postselection an outcome
space \(\mathcal{B}_\Lambda\) of the pointer state detector is introduced. Each
measurement result \(r \in \mathcal{B}_\Lambda\) corresponds to the projector
valued measure \(\Pi_r\) on the pointer state space and the
superoperator \(A_r\), which maps the initial system state \(\rho_S\) onto
the conditional final state \(\rho_S^r\)
\[
\rho_S^r = \Lambda_r \rho_S / p_r, \quad (4)
\]
where \(p_r = \text{Tr}_S(\Lambda_r \rho_S)\) is the probability of outcome \(r\) and
\(\text{Tr}_S\) is the trace over the state space of the measured system.
Alternatively, the density operator \(\rho_S^r\) may also be obtained
from
\[
\rho_S^r = \text{Tr}_A[\rho_{out} \Pi_r]. \quad (5)
\]

In the following we will assume that a secular approximation
is established. Namely, a typical time-scale of the intrinsic
evolution of the pointer is large compared to the time over
which its state varies appreciably. In this approach, if \(\Pi_r =
|\psi_r\rangle\langle \psi_r|\), \(\Lambda_r\) may be written as
\[
\Lambda_r = \langle \psi_r | \Pi_r | in\rangle \langle in | \psi_r \rangle. \quad (6)
\]

The secular approximation allows the following decomposition
of the superoperator \(A_r\):
\[
A_r = \sum_{r \in \mathcal{B}_\Lambda} |\psi_r\rangle\langle \psi_r| \mathcal{M}_{r,r}(\tau), \quad (7)
\]
where \(\mathcal{M}_{r,r} = \Lambda_r\) for initial conditions, indexed by \(s \in \mathcal{B}_\Lambda\).
Substituting equation (7) into (1) gives the following system
of differential equations for superoperators \(\mathcal{M}_{r,r}\):
\[
i\mathcal{M}_{r,r} = \sum_{q \in \mathcal{B}_\Lambda} \langle \psi_r | \mathcal{L} | \psi_q \rangle \mathcal{M}_{q,r}. \quad (8)
\]

For every fixed \(s\) we obtain the closed system for \(\mathcal{M}_{r,r}\).
In the next section this formalism will be applied to the indirect
photodetection problem.

3. The indirect photodetection process of the cavity
mode

Indirect cavity mode photodetection is organized as follows.
A two-level atom passes through the cavity and interacts with
an excited quantum mode. Just after the interaction the atomic
state is determined in a selective detector, which returns one
of two possible alternatives: the atom is found in its ground
state \(|g\rangle\Lambda\) or in its excited state \(|e\rangle\Lambda\). The master equation
in this case has the Lindblad form
\[
i\dot{\rho} = \mathcal{L}\rho = [H_{int}, \rho] + i\mathcal{D}_\Lambda \rho, \quad (9)
\]
where \(H_{int}\) is the Jaynes–Cummings Hamiltonian
\[
H_{int} = \Omega \sigma_x e^{i\Delta t} t + \Omega^\ast \sigma_x^\ast e^{-i\Delta t} t \quad (10)
\]
and \(\mathcal{D}_\Lambda\) is the atomic ‘dissipater’
\[
\mathcal{D}_\Lambda \rho = \gamma_{eg}/2 (2\sigma_{-} \rho \sigma_+ - \sigma_{+} \rho \sigma_{-} + \gamma_{eg}/2 \rho) \quad (11)
\]

Here the following notations are used: \(\Omega\) is the coupling
between the atom and the mode, \(\sigma_\pm = |e\rangle \langle g|\) and \(\sigma_\mp =
|g\rangle \langle e|\) are atomic operators, \(a\) is the annihilation operator of the
 cavity mode, \(\Delta\) is the detuning, and \(\gamma_{eg}\) and \(\gamma_{eg}\) are the atomic
population relaxation rates.

Let \(\mathcal{B}_\Lambda = \{g, e\}\) be a set of results of the atomic
state detection. Here we will assume the ideal (projective)
measurement of the pointer state. Using the density operator
decomposition
\[
\rho_{\Lambda \langle f \rangle}(t) = \sum_{\mu, \nu \in \mathcal{B}_\Lambda} |\mu\rangle \langle \nu| \otimes \rho_{\mu \nu}(t) \quad (12)
\]
and secular approximation (in this case the rotating wave
approximation) \(\Gamma \gg \Omega\), where \(\Gamma \equiv (\gamma_{eg} + \gamma_{eg})/2\) is the atomic
decohere rate, gives the following closed system of differential
equations for \(\rho_{eg}(t)\) and \(\rho_{ee}(t)\):
\[
i\dot{\rho}_{gg} = \kappa \Delta [a^\dagger a, \rho_{gg}] + i\kappa \Gamma (2a^\dagger \rho_{ee}a - \rho_{gg} a^\dagger a - a^\dagger a \rho_{gg})
+i (\gamma_{eg}\rho_{ee} - \gamma_{eg}\rho_{gg}), \quad (13a)
\]
\[
i\dot{\rho}_{ee} = \kappa \Delta [a^\dagger a, \rho_{ee}] + i\kappa \Gamma (2a\rho_{ee}a^\dagger - \rho_{gg} a^\dagger a - a^\dagger a \rho_{gg})
+i (\gamma_{eg}\rho_{ee} - \gamma_{eg}\rho_{ee}), \quad (13b)
\]
which may be written as:
\[
\dot{\rho}^\dagger(t) = \mathcal{L}^\dagger \rho^\dagger(t), \quad (14)
\]
where \(\rho^\dagger(t) = \text{diag} \{\rho_{gg}(t), \rho_{ee}(t)\}\) and \(\kappa = |\Omega|^2/(\Gamma^2 + \Delta^2)\).
From equation (8) for \(\mathcal{L}\) two separate systems of differential
equations may be found. One of them corresponds to the atom
prepared in the ground state \(|g\rangle\Lambda\) and another to the excited
state \(|e\rangle\Lambda\). For the first one
\[
\mathcal{M}_{gg} = -(\alpha K_0 + \beta + \gamma_{eg}) \mathcal{M}_{gg} + (\alpha K_+ + \gamma_{eg}) \mathcal{M}_{eg}, \quad (15a)
\]
\[
\mathcal{M}_{eg} = (\alpha K_+ + \gamma_{eg}) \mathcal{M}_{gg} - (\alpha K_0 - \beta + \gamma_{eg}) \mathcal{M}_{eg}. \quad (15b)
\]
Here \(\alpha = \kappa \Gamma, \beta = \kappa (\alpha N - \Gamma/2), \quad K_0 = K_0 - \beta + \gamma_{eg}\) \(\mathcal{M}_{eg}.
\]
\[
\mathcal{M}_{gg} = a^\dagger a^\dagger a^\dagger a^\dagger, \quad N_{gg} = [a^\dagger a, \rho_{gg}] \quad (15b)
\]
Note that,
$K_0, K_+, K_-$ form the full set of $SU(1, 1)$ generators. System (15a–15b) may be solved analytically at least in two approximations: strong ($\gamma_{ge} < \kappa \Gamma \approx \kappa \Delta \ll \gamma_{eg}$) and weak ($\gamma_{ge} \ll \gamma_{eg} \ll \alpha \approx \kappa \Delta$) population relaxation limits. For the following we will use a numerical solution for these extreme cases.

4. Information characteristics of the photodetection process

Here the basic information characteristics of the photodetection process will be investigated in the bounds of the model discussed above. The following quantities are of the greatest interest: the probability of ground or excited atomic state detection $P_r(t) = \text{Tr}_F [M_{gr}(t) \rho_F]$, $r \in \mathbb{B}_A$; information gain $I_r = -\Delta H$ as a measure of entropy change $\Delta H = \rho_{vF}^r \log \rho_{vF}^r - \rho_F^r \log \rho_F^r$ resulting from photodetection; and fidelity $F_r = \sqrt{\rho_{vF}^r \rho_F^r}$, which characterizes the state change caused by the measurement process.

These quantities are shown in figures 1 and 2 as functions of time interaction $\tau$ for different dimensions of the cavity mode state space. Figure 1 shows the results obtained from the strong relaxation approximation, while figure 2 shows these dependences for the case of weak relaxation. For each approximation two initial states are tested: the completely mixed state $\rho_F = 1/d$ (top) and the Fock state $\rho_F = |n\rangle\langle n|$ (bottom). Here $d$ is the dimension of the cavity mode state space and $n \leq d$ is the number of photons in the mode (cases $d = 2, 4, 6$ and $n = 1, 3, 5$ are depicted).

We investigate the case of conditional density matrix evolution, which corresponds to the detection result $r = g$. The first graph in each row shows the probability $1 - P_g = P_e$ of not detecting the atom in its ground state. In the strong relaxation limit the curves corresponding to interaction with several photons (blue dotted and green chain lines) tend to constant nonzero value due to nonzero atomic spontaneous excitation. In the case of interaction with only one photon in the cavity mode (red solid line) the probability of detecting the atom in its excited state increases with time because of the comparatively small interaction between the atom and the cavity mode. For the weak relaxation limit all curves tend to zero slowly because of the absence of spontaneous excitation.

The second graph in each row shows the information gain as a function of time. The behavior of this dependence for the cases of initially mixed and Fock states are completely different (figure 1). In the first case all curves increase monotonically because the mixed state has maximum entropy. All curves tend to some constant value, which corresponds to the final vacuum field state. In the case of the field prepared in the Fock state the initial entropy is zero and rises due to the common evolution with the atom. Finally, all curves tend to the vacuum state, which has zero entropy, but due to the nonzero spontaneous atomic excitation the resulting entropy has a constant nonvanishing value. The case of weak relaxation (figure 2) may be explained in the same way.

Finally, the third graph in each row of the figures shows the detection fidelity as a function of time. It is interesting to investigate the stationary limit of these dependences in the case of the strong relaxation limit. In this limit all curves decay monotonically which corresponds to the irreversible state change. For the field prepared in a completely mixed state the stationary value is nonzero for all cases because the mixed state and the final vacuum state are nonorthogonal.
Ground state detection: probability, information gain and fidelity as functions of the interaction time: for mixed state $\rho_F = 1/d$ (top) and the Fock state $\rho_F = |n\rangle \langle n|$ (bottom). The results are obtained from the weak relaxation approximation.

For the case of the initially prepared Fock state the curve tends to zero with the increase of the photon number. This may be explained by the orthogonality of the initial and final state. However, due to the spontaneous excitation the probability of detecting the atom in its excited state is nonzero and superoperators of the conditional evolution transform the density matrix into the state, which is not orthogonal to its initial state. This is why all curves tend to a small but nonzero value. In the weak relaxation limit for the considered interaction time stationary limits are not obtained.

For numerical modeling the following parameter values are used (in unit $\gamma_{eg} = 1$): $\gamma_{ge} = 0.1$, $\gamma_{eg} = 1$, $\Gamma = 2$, $\Delta = 0.5$ and $\Omega = 0.7$ (for the strong relaxation limit); $\gamma_{eg} = 0$, $\gamma_{ge} = 0.01$, $\Gamma = 2$, $\Delta = 0.5$ and $\Omega = 0.7$ (for the weak relaxation limit).

5. Conclusion

The parameterization of the measurement process in the presence of non-unitary evolution was investigated. The interaction between the pointer and the environment was discussed and applied to the quantum photodetection problem. Basic characteristics of the measurement process as a function of the interaction time for two approximations were obtained. Properties of superoperators corresponding to different measurement results may be used for finding special regimes of detection: measurement without state or entropy change on the one hand and detection with best information gain on the other hand. The generalization of the previous description beyond secular approximation (quantum Brownian motion) will be considered in future. It is attractive to obtain detector superoperators from the evolution generator, although it requires a deep analysis of their algebraic properties.

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