NONLINEAR ANALYSIS OF EAS CLUSTERS

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We apply certain methods of nonlinear time series analysis to the extensive air shower clusters found earlier in the data set obtained with the EAS–1000 Prototype array. In particular, we use the Grassberger–Procaccia algorithm to compute the correlation dimension of samples in the vicinity of the clusters. The validity of the results is checked by surrogate data tests and by some additional quantities. We compare our conclusions with the results of similar investigations performed by the EAS-TOP and LAAS groups.

1 Introduction

We have already studied the distribution of arrival times of extensive air showers (EAS) registered with the EAS–1000 Prototype array both by methods of classical statistics \cite{1,2} and by methods of cluster analysis \cite{3,4}. In particular, we have found EAS clusters—groups of consecutive showers that were registered in time intervals much shorter than expected ones \cite{3,4}. Besides this, we have found that as a rule samples which contain clusters do not allow one to accept a hypothesis that EAS arrival times have an exponential distribution. To the contrary, the vast majority of other sufficiently long samples satisfy the same hypothesis if the barometric effect is taken into account \cite{1,2}. Below we present some results of this investigation.

Recall that modern methods of nonlinear time series analysis are mainly based on the results obtained by Takens \cite{5}, Mañé \cite{6}, and Packard et al. \cite{7}, and an algorithm suggested by Grassberger and Procaccia \cite{8} and modified by Theiler \cite{9}. We are not going to review these results here (see, e.g., \cite{10,11}) but will remind only the basic ideas of this approach. Suppose one studies a scalar time series \{x_i\}_{i=1}^{n} which, for instance, presents experimental results obtained during observations of a (nonlinear) process. It was suggested to study delay vectors \(X_i = (x_i, x_{i+\tau}, x_{i+2\tau}, \ldots, x_{i+(m-1)\tau})\) in order to figure out whether the process possesses chaotic dynamics or not; here \(\tau\) is an arbitrary but fixed parameter, and \(m\) is an integer constant called an embedding dimension. One should compute the number \(K\) of vectors with mutual distance \(\leq \rho\) and such that delay vectors \(X_i\) are shifted by at least \(W\) indices. After this, one calculates the correlation dimension

\[
D_2(\rho) = \frac{d \log C_2(\rho)}{d \log \rho},
\]

where \(C_2 = K/\text{(total number of vectors }X_i)\) is the correlation sum. If the plot of \(D_2\) has a plateau, then it is likely that the process demonstrates chaotic dynamics. A value of \(D_2\) at the plateau is taken as an estimate of the correlation dimension of the attractor underlying the data. This quantity also gives a (lower) estimate for the number of degrees of freedom in the process under consideration.

Investigations in this field have lead to a discovery that a plateau in the plot of the correlation dimension can be observed not only for chaotic deterministic processes but also for certain types of stochastic processes \cite{12,13}. Thus there appeared a problem to distinguish these two classes using experimental data. The problem has occurred to be complicated, and this called an appearance of a number of approaches to its solution. One of the main approaches is the method of surrogate data \cite{14,15}. Other methods are based on certain additional functions. As for surrogate data, they are used both to compute some quantities that allow one to estimate a measure of nonlinearity of the process under consideration and to clarify the “nature” of the plateau in the plot of \(D_2\). Namely, if the plateau is observed for surrogate data, then the process is assessed to be stochastic. Conversely, if the plateau disappears, then the process is deterministic. At the moment, there is a variety of methods for generating surrogate data. One can find a review of the main approaches in \cite{16}.
Still, we should note that to the best of the authors' knowledge there is no test that could automatically and unequivocally distinguish between a stochastic process and deterministic chaos, see, e.g., [12, 13].

2 The Main Results

At the first stage of this investigation the experimental data set (time intervals between arrival times of $1.7 \times 10^6$ EAS registered in the period from August, 1997 to February, 1999) was split into adjacent samples consisting of 128, 256, and 512 elements. For these samples, we computed the Fourier power spectrum, the correlation dimension $D_2$ (for $\tau = 1, W = 1, m = 5 \ldots 12$) and a number of additional quantities. To provide stationarity of the time series under consideration, time intervals between EAS arrival times were adjusted to a joint value of atmospheric pressure equal to 742 mm Hg. To perform calculations, we used GNU Octave [19] running in Mandrake Linux. As one could expect, this analysis has revealed that there is no plateau in the plot of $D_2$ for the vast majority of samples. Still, in the vicinity of some EAS clusters we have found samples that demonstrate signs of chaotic dynamics. Below we shall briefly discuss one of these events, namely a cluster registered on November 11, 1998 between 01:21:17.47 and 01:38:02.27 of Moscow local time [3, 4]. This event consists of three clusters that begin at EAS No. 435, 436, and 437 respectively and end at EAS No. 570. In our opinion, an appearance of these three clusters within one event is an effect of our selection procedure [4] and does not have an astrophysical nature. Thus in what follows we shall only discuss the outer cluster, which consists of 136 EAS.

In order to get an idea about the behavior of the correlation dimension in the vicinity of the cluster, let us consider three adjacent samples, each consisting of 256 time intervals. Fig. 1 depicts the correlation dimension for a sample that ends up in 2.5 minutes before the cluster, a sample that contains the cluster, and a sample that begins in 23 minutes after the cluster. The curves were computed for $\tau = 1, W = 1, m = 1 \ldots 12$. The upper curves correspond to larger values of the embedding dimension $m$. The maximum norm was used to compute mutual distances between delay vectors.

As one can see from the figure, the plot of the correlation dimension obtained for the sample that contains the cluster has a clear plateau with $D_2 \approx 2.5$. On the contrary, no plateau is observed for two other samples. Thus we conclude that the sample with the cluster demonstrates signs of chaotic dynamics with the (fractal) dimension of an attractor approximately equal to 2.5. At the same time, the Fourier power spectrum of this sample does not considerably differ from a broadband spectrum, which can be observed, e.g., for random noises. We have employed the surrogate data method in order to figure out whether the sample with the cluster represents a deterministic chaotic process or a stochastic process. To make surrogate data, we

![Figure 1: The correlation dimension $D_2$ for samples that consist of EAS No. 168–424, 425–681, and 682–938 (from top to bottom).](image-url)
used two different approaches: a random shuffling time delays that constitute the sample, and the amplitude adjusted Fourier transform method suggested in [16]. Both methods preserve the distribution of the original data set. Besides this, the second method preserves the Fourier power spectrum. We used the TISEAN package [20] to prepare Fourier-based surrogate data.

An analysis of the surrogate data made by both methods for the sample that contains the cluster has revealed that plots of the correlation dimension do not contain a plateau. This gives an argument in favor of the hypothesis for the deterministic nature of the original sample since the order of time delays that constitute the sample occurs to be important for an appearance of the plateau. A similar conclusion can be made on the basis of an analysis of certain other quantities, e.g., a maximum likelihood estimator for the correlation dimension introduced in [2], and a function suggested in [23].

On the other hand, we must note that some other tests has lead to opposite results. For instance, one of the tests for nonlinearity is based on a measure for the time-reversibility of a time series [24]. An application of this test to the sample that contains the cluster and to the surrogates has revealed that the null hypothesis for the linear structure of the time series cannot be rejected. A possibly stochastic nature of the observed behavior of the correlation dimension was also revealed by the analysis of the normalized slope, introduced in [25].

It is worth mentioning that a plateau in the plot of $D_2$ for samples that partially or completely include the cluster can be observed in a wide range of sample lengths (from $N \sim 100$ up to $N \sim 500$) and values of the Theiler window $W$. The value of the correlation dimension varies depending on $N$ and $W$ and the position of the cluster inside a sample. For example, for the same sample with $N = 256$, $D_2 \approx 3$ for $W \geq 7$.

### 3 Discussion

The results presented above demonstrate that one can observe an unusual dynamics of EAS arrival times in the vicinity of certain clusters of EAS with the electron number of the order of $10^5$. Still it is rather difficult to make a final conclusion on the nature of this phenomenon: Does it represent deterministic chaos or a special type of a stochastic process? In our opinion, the majority of the tests performed witness in favor of the first of these two alternatives. On the other hand, it is not easy to suggest an astrophysical model that could explain chaotic dynamics in EAS arrival times. Thus it is interesting to compare our results with the conclusions of similar investigations performed by other research groups.

In a considerable number of articles devoted to the nonlinear time series analysis, one can find a comprehensive investigation of EAS arrival times registered with the EAS-TOP array [26]. Basing on a detailed study of the available experimental data set and the results obtained with the underground muon monitor [27] the authors of this work made a conclusion that though the existence of deterministic chaotic effects in cosmic ray time series cannot be completely excluded, cosmic ray signals are all color random noise, independently of the nature of the secondary particle and of the primary parent particle. It was also demonstrated in one of the following articles that an impact of background noise brings additional difficulties to the problem of distinguishing between chaotic and stochastic dynamics [28].

Besides this, a whole series of investigations devoted to the nonlinear analysis of EAS time series is carried out in Japan beginning from early nineties at the experimental arrays that now constitute the LAAS network, see [24] and references therein. The authors of these investigations presented several dozens of events that demonstrate chaotic dynamics. More than this, it was conjectured that the observed dynamics may be due not only to the chaotic structure of the medium through which particles have traversed but also to the nature of the primary particles [31]. Later on, there was suggested a model according to which chaotic events may be generated by cosmic rays that have a structure of a fractal wave arriving from a nonlinear accelerator like a supernova remnant [31]. This model needs to be studied in details, but seems to be promising.

Thus, the results obtained during our analysis do not contradict the conclusions of similar investigations performed at other EAS arrays. It seems to be necessary to continue the work in this area and to involve some other methods of nonlinear time series analysis.

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Only free, open source software was used for this investigation.
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