Students’ difficulties in solving linear equation problems

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Abstract. A linear equation is an algebra material that exists in junior high school to university. It is a very important material for students in order to learn more advanced mathematics topics. Therefore, linear equation material is essential to be mastered. However, the result of 2016 national examination in Indonesia showed that students’ achievement in solving linear equation problem was low. This fact became a background to investigate students’ difficulties in solving linear equation problems. This study used qualitative descriptive method. An individual written test on linear equation tasks was administered, followed by interviews. Twenty-one sample students of grade VIII of SMPIT Insan Kamil Karanganyar did the written test, and 6 of them were interviewed afterward. The result showed that students with high mathematics achievement donot have difficulties, students with medium mathematics achievement have factual difficulties, and students with low mathematics achievement have factual, conceptual, operational, and principle difficulties. Based on the result there is a need of meaningfulness teaching strategy to help students to overcome difficulties in solving linear equation problems.

1. Introduction

Within secondary school mathematics curricula, there are four topics, namely; number, algebra, geometry and measurement, and statistics and probability. Algebra has been widely recognized as one of the most difficult topics [1]. One of algebra materials is linear equation. A linear equation is an algebra material that exists in junior high school to university. A linear equation is important to master more advanced mathematics topics such as derivative in calculus [2, 3], the line of best fit in statistics [4, 5], and to describe nonlinear functions in advanced algebra [6]. Therefore, linear equation material is essential to be mastered. However, students’ ability to solve problems related to linear equation is low. It can be seen from the result of national examination of Indonesia junior high school in 2016. It showed that the result at the national level was only 39.82%. The low result was also shown by students of SMPIT Insan Kamil Karanganyar in solving linear equation problem. The result was only 33.33%.

Studies have found that students experienced various difficulties in a linear equation. Students misunderstand linear equations, graphs, slopes, and are not able to comprehend the connection between slope and the x- and y-intercepts [7, 8]. Research has documented procedural and conceptual students’ difficulties with slope [9]. The misconception and deficiencies of the students in the linear equation also may lead to severe learning difficulties in the subjects of functions, limit, derivation, and integral in high school and the university years. For this reason, it is important to determinate students’ difficulties in solving linear equation problems.
One of the mathematics characteristics is having abstract study objects in the form of facts, concepts, principles, and operations. Objects in mathematics learning such as facts, concepts, operations, and principles influence students in mastering basics competencies of mathematics. Understanding the linear equation is remarkably complex and involves many levels of abstraction. Students’ difficulties in solving linear equation problems can occur due to the lack of understanding of mathematical objects in the form of facts, concepts, operations, and principles. Factual difficulties are the obstacles that students experienced in the mastery of subject matter related to symbols, signs or notations. Conceptual difficulties are the obstacles that students experience in the mastery of subject matter related to the abstract idea used to solve a problem. Operational difficulties are the obstacles that students experience in the mastery of subject matter related to the work of counting and algebra workmanship. Principle difficulties are the obstacles that students experience in the mastery of the subject matter related to the axioms, theorems, relationships between various basic objects of mathematics and formulas of mathematics.

Based on the above explanation, this study is aimed to investigate students’ difficulties in solving linear equation problems, especially students of SMPIT Insan Kamil Karanganyar. Students’ difficulties are classified to factual, conceptual, operational, and principle difficulties. The understanding of the difficulties faced by students can provide a guideline for teachers as well as researchers to plan better approaches. It is necessary to assist the students which will result in more meaningful teaching and learning process.

2. Methods
The researcher conducted a qualitative descriptive study in which an individual written test on linear equation problems was administered, followed by student interviews on the written work. The subjects of the study were 21 grade VIII students of SMPIT Insan Kamil Karanganyar in the school year of 2016-2017. The samples had studied linear equation in its first semester.

First, students were asked to solve a set of linear equation problems with paper and pencil individually for 40 minutes, and students were informed that their solutions would not be graded so they would feel free to use their own solution methods. The goal of this written test was to identify students’ difficulties in solving linear equation problems. Furthermore, the interviewer selected students for the additional individual interviews, which aimed to gather more detailed data on the occurring difficulties. Out of the 21 participating students who did the written test, 6 of them were interviewed. The interviewed subjects consisted of 2 subjects with high mathematics achievement namely SH1 and SH2, 2 subjects with medium mathematics achievement namely SM1 and SM2, and 2 subjects with low mathematics achievement namely SL1 and SL2. Data of students’ mathematics achievement were obtained from the test at the end of the previous semester.

3. Result and Discussion
The instrument used in this research consisted of two linear equations problems that were used for looking students’ difficulties in solving linear equation problems. Students’ difficulties were classified into factual difficulties, conceptual difficulties, operational difficulties, and principle difficulties. The instrument can be seen in Figure 1 and Figure 2.

![Figure 1. The first problem given to students](attachment:image.png)
2. Given the graph as below.

![Graph Image]

If the line \( p \) and \( q \) are perpendicular, determine an equation of the line \( q \)!

**Figure 2.** The second problem given to student

3.1. Subjects with high mathematics achievement

The answer of subjects that have high mathematics achievement can be seen in Figure 3 and Figure 4.

![Figure 3](image1)

**Figure 3.** SH1’s answer

![Figure 4](image2)

**Figure 4.** SH2’s answer

The second problem showed that line \( p \) intercepted \( x \)-axis at the point \((2, 0)\) and \( y \)-axis at the point \((0, -4)\) and line \( q \) intercepted \( y \)-axis at the point \((0, -4)\). It could be seen from Figure 3 and Figure 4 that SH1 and SH2 stated that line \( p \) passed through points \((2, 0)\) and \((0, -4)\) and line \( q \) passed through point \((0, -4)\). SH1 and SH2 didn’t have factual difficulties. SH1 and SH2 stated that line would intercept \( y \)-axis when \( x \) was equal to 0 and would intercept \( x \)-axis when \( y \) was equal to 0. Students
could determine concept of $x$- and $y$-intercepts. SH1 and SH2 could explain about slope. Line $p$ was an increasing line and line $q$ was a decreasing line. They put a positive sign for an increasing line’s slope and negative sign for a decreasing line’s slope. Based on these facts, students could determine concept of slope. SH1 and SH2 did not have conceptual difficulties.

SH1 and SH2 didn’t make a mistake related to algebra workmanship and calculation. They did not have operational difficulties. Figure 3 and Figure 4 showed that SH1 and SH2 used elimination method to determine the point of intersection between $4x - 3y + 1 = 0$ and $x - y - 1 = 0$. They could determine the intersection of lines. SH1 and SH2 could determine the slope of line $2x - 3y + 6 = 0$, their answer was $m = \frac{2}{3}$. SH1 and SH2 used formula $m = -\frac{a}{b}$ to determine the slope of the line $ax + by + c = 0$. They did not have principle difficulties.

3.2. Subjects with medium mathematics achievement

The answer of subjects that have medium mathematics achievement can be seen in Figure 5 and Figure 6.

The second problem showed that line $p$ intercepted $x$-axis at the point $(2,0)$ and $y$-axis at the point $(0,-4)$ and line $q$ intercepted $y$-axis at the point $(0,-4)$ (Figure 5 and Figure 6). Based on the interview, SM1 and SM2 were confused in distinguishing between point and line. SM1 and SM2 had factual difficulties. SM1 and SM2 stated that line would intercept $y$-axis when $x$ was equal to 0 and would intercept $x$-axis when $y$ was equal to 0. The two students could determine concept of $x$- and $y$- interception. SM1 and SM2 could explain about slope. Line $p$ was an increasing line, and line $q$ was a decreasing line. They put a positive sign for an increasing line’s slope and negative sign for a decreasing line’s slope. Based on these facts, the two students could determine concept of slope. SM1 and SM2 did not have conceptual difficulties.

SM1 and SM2 didn’t make a mistake related to algebra workmanship and calculation. They didn’t have operational difficulties. Figure 5 showed that SM1 used elimination method to determine the
point of intersection between $4x - 3y + 1 = 0$ and $x - y - 1 = 0$. Figure 6 showed that SM2 used substitution method to determine the point of intersection between $4x - 3y + 1 = 0$ and $x - y - 1 = 0$. They could determine intersection of lines. SM1 and SM2 could determine the slope of line $2x - 3y + 6 = 0$. They answered that slope of the line $2x - 3y + 6 = 0$ was $m = -\frac{2}{3}$; SM1 and SM2 used formula $m = -\frac{a}{b}$ to determine the slope of the line $ax + by + c = 0$. They did not have principle difficulties.

3.3. Subjects with low mathematics achievement

The answer of subjects with low mathematics achievement can be seen in Figure 7 and Figure 8.

The second problem showed that line $p$ intercepted $x$-axis at the point $(2,0)$ and $y$-axis at the point $(0,-4)$ and line $q$ intercepted $y$-axis at the point $(0,-4)$. In Figure 7 it could be seen that SL1 stated that line $p$ passed through point $(2,4)$ and line $q$ passed through point $(-4,6)$. In Figure 8 it could be seen that SL2 stated that line $p$ passed through point $(2,-4)$ and line $q$ passed through point $(0,-4)$. SL1 and SL2 showed difficulties in determining points of Cartesian coordinates. This finding was consistent with the finding of Cho and Nagle, students had difficulty in determining points of Cartesian coordinates [9]. SL1 and SL2 had factual difficulties. In Figure 7 and Figure 8 it can be seen that SL1 and SL2 were wrong in determining the point of intersection between $4x - 3y + 1 = 0$ and $x - y - 1 = 0$. SL1 and SL2 could not determine that two lines will intersect if they meet at one point. Students had conceptual difficulty about intersection of lines. SL1 and SL2 did not understand that lines will intercept $y$-axis when $x$ is equal to 0 and will intercept $x$-axis when $y$ is equal to 0. This showed that the students had conceptual difficulty with $x$- and $y$-intercepts. This finding was consistent with the finding of Hattikudur, students had difficulty with the concepts of $y$-intercept [7]. SL1 and SL2 could not explain about slope. Line $p$ was an
increasing line, and line $q$ was a decreasing line. They put a negative sign for an increasing line’s slope and positive sign for a decreasing line’s slope. Students were wrong in determining slope in problem 2. Based on these facts, there were students that had difficulties related to the concept of slope. These findings were relevant to the findings of Birgin, students had difficulties with the concepts of slope [8]. SL1 and SL2 had conceptual difficulties.

SL1 made a mistake related to algebra workmanship. It can be seen in Figure 7, SL1 stated that $y - 1 = -\frac{2}{3} \left(x - \left(-\frac{2}{3}\right)\right)$ into $3(y - 1) = -2x - \frac{12}{3}$. SL2 made mistake related to calculation. It can be seen in Figure 8, SL2 stated that $\frac{9}{-2} = -2$. SL1 and SL2 had operational difficulties. These findings were in line with the findings of previous research, students had difficulties related to applying arithmetic operations [1, 10].

Figure 7 showed that SL1 was wrong in using elimination method to determine the point of intersection between $4x - 3y + 1 = 0$ and $x - y - 1 = 0$. Figure 8 showed that SL2 was wrong in using substitution method to determine the point of intersection between $4x - 3y + 1 = 0$ and $x - y - 1 = 0$. This showed that the students had principle difficulty with intersection of lines. SL1 and SL2 were wrong in determining the slope of line $2x - 3y + 6 = 0$. They stated that the slope of the line $2x - 3y + 6 = 0$ was $m = -\frac{a}{b}$. Students made mistake when they used formula $m = -\frac{a}{b}$ to determine the slope of the line $ax + by + c = 0$. They did not pay attention to coefficients of $x$ and $y$ on the equation, whether they had positive or negative value. It showed that students did not understand how to use the formula to determine the slope of the line $ax + by + c = 0$. SL1 and SL2 had principle difficulties.

4. Conclusion

Based on researcher’s data analysis about students’ difficulties in solving linear equation problems, it can be concluded that students with high mathematics achievement do not have difficulties in solving linear equation problems, students with medium mathematics achievement have factual difficulty related to distinguishing between point and line, and students with low mathematics achievement have factual, conceptual, operational, and principle difficulties in solving linear equation problems. Factual difficulties are related to determining points of Cartesian coordinates. Conceptual difficulties are related to the concept of lines intersection, the concept of $x$- and $y$-intercepts, and the concept of slope. Operational difficulties are related to algebra workmanship and calculation. Principle difficulties are related to determining formula; such as formula of the slope and method to determine the point of lines intersection. Based on the result there is a need of meaningfulness teaching strategy to help students to overcome difficulties in solving linear equation problems.

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