Numerical Modeling of Complex Porous Media For Borehole Applications

NMR-response and Transport in Carbonate and Sandstone Rocks

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ABSTRACT. The diffusion/relaxation behavior of polarized spins of pore filling fluid, as often probed by NMR relaxometry, is widely used to extract information on the pore-geometry. Such information is further interpreted as an indicator of the key transport property of the formation in the oil industry. As the importance of reservoirs with complex pore geometry grows, so does the need for deeper understanding of how these properties are inter-related. Numerical modeling of relevant physical processes using a known pore geometry promises to be an effective tool in such endeavor. Using a suite of numerical techniques based on random-walk (RW) and Lattice-Boltzmann (LB) algorithms, we compare sandstone and carbonate pore geometries in their impact on NMR and flow properties. For NMR relaxometry, both laboratory measurement and simulation were done on the same source to address some of the long-standing issues in its borehole applications. Through a series of “numerical experiments” in which the interfacial relaxation properties of the pore matrix is varied systematically, we study the effect of a variable surface relaxivity while fully incorporating the complexity of the pore geometry. From combined RW and LB simulations, we also obtain diffusion-convection propagator and compare the result with experimental and network-simulation counterparts.

KEYWORDS: NMR, Lattice-Boltzmann, Randomwalk, Tomogram, Carbonate, Sandstone Rocks
1. Introduction

Despite the long history of research on porous media, there remain open issues which critically affect various industrial endeavor such as oil/gas exploration, CO₂ sequestration, water management, storage and migration of toxic waste. A wide range of scales permeates through these disciplines, but pore-scale physical processes remain their common denominator. In this work, we report our recent effort on the pore-level modeling based on micro-tomograms in the context of a borehole application.

Figure 1 – Pore-grain interface morphology for a typical sandstone (top) and a carbonate rock (bottom panel). Also shown are the radial pore-to-pore auto-correlation function $g_p(r)$ at various porosity values (top: three distinct sandstones; bottom: different thresholding of the original carbonate tomogram). The insets show scaled correlation functions $h_p(r)$ which largely collapse into a universal form.

Several techniques (e.g. [Auz 96]) utilizing detailed 3D pore geometry have reached maturity in recent decade thanks to the affordable computing resource, imaging techniques, and parallelized simulations. These numerical results are in good standing for a class of porous media such as bead packs and sandstones. Here, we focus on aspects of more challenging situations involving carbonates in the oil field.
Figure 1 shows two contrasting images of the pore-grain interface for a Fontainebleu sandstone and a carbonate rock. While quasi-periodicity is clearly visible in the former, the latter displays pronounced heterogeneity. This is quantified in the right column where the radial pore-to-pore autocorrelation function \( g(|r_2 - r_1|) \equiv <\phi(r_1)\phi(r_2) > \) \((\phi(r) = 1 \text{ in pore, } 0 \text{ in grains})\) is plotted for different values of porosity. Note that in both cases, the curves collapse to a generic form (insets) \( h_p(r) \equiv (g(r) - \phi^2)/(\phi(1 - \phi)) \) with \( \phi = <\phi(r) > \), average porosity. Quasi-periodicity of the sandstone is manifest in the form of mild bumps in \( g(r) \), reminiscent of the classic density correlation in a simple liquid, while the carbonate sample displays a monotonic, featureless profile, which actually arises from preponderance of many competing length scales. In many carbonate rocks, the complexity extends even further than suggested in the figure, as there exist structures on finer scales beyond the tomographic resolution, as well as on scales much beyond the typical sample sizes. Quantitative elucidation of both these aspects remain an open challenge which we aim to address via further extending the steps described in the following.

2. Pore geometry and open issues for the carbonate

Figure 2 – Low-field experimental data (solid zagged curve) and the best simulation match (solid curve) with \( \rho = 500 \mu m/s \). Simplistic model results for NMR response at various strengths of surface relaxivity \( \rho \) and pore coarsening (colored broken lines, four curves for each value of \( \rho \)) are also shown for comparison. The inset shows the porosity and \( S/V_p \) distribution at the five stages of coarse graining. Shown on the right panel are the local magnetization density evolution at various stages of the simulation for \( \rho = 500 \mu m/s \) which yields results matching the experiment.
Figure 3 – The internal magnetic field of a carbonate packstone numerically obtained for the entire tomogram \((1.5^3\text{cm}^3)\) of the carbonate rock. Shown on the right are the slice-cut views of the field and the probability distribution of its strength for a sub-block of \(512^3\) voxels at the center rendered separately for the pore (filled black) and grain (gray) space. The curves represent the best Lorentzian (broken lines) and Gaussian (solid lines) fits. The latter works better for the pore portion, while neither of the methods successfully fits the grain portion.

3. Numerical NMR relaxometry

Simulations based on realistic 3D pore allows us to address some of the long-standing issues in probing its geometry. One surrounds the validity of the conventional mapping between the NMR relaxation spectrum and the pore-size distribution\([\text{KLE 99}\,\,\,\,\text{GRE 07}]\) of carbonate rocks in the possible presence of haphazard heterogeneity in their interfacial properties.\([\text{RYU 09b}]\) The low field NMR data of the carbonate sample of Fig.1 (black zagged line in Fig.2 for which bulk relaxation rate of \(1/T_b = 0.67\text{s}^{-1}\) was used and 5-points moving average was taken), shows the typical multi-exponential characteristics. This often invites an interpretation in terms of a broad pore size (more precisely, the surface-to-volume (S-V) ratios) distribution. In this scenario, the pore space is approximated as a collection of isolated pores of varying sizes, all in the so-called fast diffusion limit\([\text{BRO 79}]\). From the tomogram of the same piece, we obtain the distribution \(P\) of local \(\{S_i/V_i\}\) and porosity \(\{V_i\}\) (shown in the inset of Figure 2) at various stages of coarse-graining \((i\text{ labeling sub-blocks of linear dimension } L)\). Attempts to fit experimental data using such recipe neglect the diffusive coupling between pores as shown by series of curves in the figure with a range of controlling surface relaxivity parameter, \(\rho\), values \([\text{KLE 99}]\) (broken lines with \(\rho = 20, 60, 120\mu\text{m/s}\)) and they fail to work: if \(\rho\) is chosen to match the initial slope (red curves), it fails to match the experimental data at long times, and vice versa (blue lines). This suggests that the model neglecting the extended nature of the pore and the heterogeneous diffusive-coupling among its constituents has limited validity in these types of rocks. The role of the latter had been previously considered in simple 1D
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Random-walk based simulation [RYU 08] on the 3D tomographic pore yields an excellent overall agreement (solid black curve) with the choice of 500 µm/s for the single parameter. The large $\rho$ value thus inferred partly accounts for the fact that at the resolution of 17 µm per voxel, the digital representation of the interface significantly under-estimates the surface area. The effective $\tilde{\rho}$, which represents the combination of raw $\rho$ value and the actual surface-area, dictates the long time scale dynamics responsible for the good agreement with the entirety of data.

These numerics can be extended for sophisticated NMR probes [SON 00, SON 08]. The internal field arising from the weak susceptibility contrast between the rock matrix and fluid, often a nuisance for NMR probes, offers a way to probe length scales thanks to the close geometrical correlation between its spatial profile and the geometry of the matrix. [SON 00] Figure 3 shows an example of the internal field profile in the carbonate rock calculated using a weak dipole field ansatz, a method verified via direct imaging on a pack of cylindrical tubes. [CHO 09] We further derive the local field gradients that play a critical role in stimulated-echo probes [SON 00] as well as NMR at high fields. [ANA 07, RYU 09a] We find that the field inside the 3D pore space can be better approximated by a Gaussian distribution rather than a Lorentzian as reported by Chen et al. [CHE 05] A detailed study on the statistics of internal field and their gradients inside various types of rocks is under way, as well as their effect on the NMR response.

Figure 4 – Local flow speed through the pore matrix (Fontainebleu sandstone (left) and a carbonate packstone (right)), driven from left to right. The color represents the local speed and its scheme was chosen to enhance its features. In both cases, the voxels with less than 2 % of the maximum speed are omitted from view for enhanced views.

4. Diffusion-Flow propagators

One of the objectives of our numerical modeling is to improve the link between the static pore geometry (as inferred in situ from borehole measurements) and its transport, since the latter ultimately controls viability of a hydrocarbon reservoir. The stark
contrast between the sandstone and the carbonate pore geometry (Fig.1) underlies
the distinct way how the flow is distributed. Figure 4 shows the local flow speed
distribution for the sub-volume of the rocks obtained from the Lattice Boltzmann
simulations. Clearly, the inhomogeneous flux distribution and extreme tor-
tuosity as apparent in the carbonate sample hints at why any framework based soley
on parallel channels of effective hydraulic radii should fail.

![Subvolume of network](image1)

![Snapshots from LB-RW simulations](image2)

Figure 5 – The first panel shows the carbonate rock as used in the network simulation. The second panel shows the random-walk-LB based propagator simulation on the sub volume of the same rock at two different stages of mean displacement.

The flow propagator \[ P(\zeta, \zeta_0) \] which stands for the probability distribution of tagged particle displacements at two different wait-times (as indicated by the average displacement \( \zeta_0 \)), provides further details on the interplay between the geometrical restriction and the flow, and can be measured by NMR \[ \text{HUL 91, LEB 96, SCH 04} \]. We simulate its process using two complementary techniques: one based on network reduction, the other through a combination of random walk and lattice-Boltzmann. (See Figure 5) The former addresses much bigger volumes, while the latter incorporates detailed diffusion/fluid dynamics at fi-
nier length scales. The top-left panel of Figure 6 shows the experimental data on a clean dolomite rock. (The rock has a much simpler pore geometry than the carbonate rock used in Figs.1-4.) From the tomogram of the same source rock, we first ran a network-model simulation. \[ \text{ZHA 09} \] The result (lower-left panels) under-represents the sharp peak near the origin present in the experimental data. An ad-hoc time de-
lay factor was introduced, which to a certain degree enhances the weight near zero displacement \[ \text{ZHA 09} \]. Questions arise as to what degree one requires the presence of sub-micron-pores which lie beyond the resolution of the tomogram, and how large-
scale heterogeneity and a finite \( \rho \) affect \( P(\zeta, \zeta_0) \). To clarify these, we developed an al-
ternative method by allowing the random walkers to ride on the background flow from an LB run. (the second panel of Fig5) Preliminary results (the 2nd and 3rd columns for \( \rho = 0 \) and \( \rho = 40 \mu m/s \)) based on the resolution \( 3 \mu m/vox \) seem to capture the most salient features of the data. This improved agreement provides a valuable insight on how one should attribute such experimental features to the heterogeneity at extreme length scales. More quantitative inquiry into this issue is in progress.

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Figure 6 – Top panel on the first column shows experimental $P(\zeta, \zeta_0)$ at two different stages. The lower panels show the result of a network model simulation with and without the ad hoc time dealy. The middle column shows the combined RW-LB simulated propagator with $\rho = 0$ at three different stages (parametrized by the average displacement $\zeta_0$). The third column shows the same but with $\rho = 40 \mu m/s$. The actual $\rho$, from more recent NMR relaxometry, is estimated to be about 20 $\mu m/s$. Also indicated for reference is the diffusion length $\sigma_t \equiv \sqrt{2Dt}$ for each stage.

5. Bibliographie

[ANA 07] ANAND V., HIRASAKI G., « Paramagnetic relaxation in sandstones : distinguishing T1 and T2 dependence on surface relaxation, internal gradients and dependence on echo spacing », SPWLA 48th Annual Logging Symposium, vol. 446359V, 2007.

[ARO 84] ARONOVITZ J., NELSON D., « Anomalous diffusion in steady fluid flow through a porous medium », Phys. Rev. A, vol. 30, n° 4, 1984, page 1948.

[AUZ 96] AUZERAI F., DUNSMUIR J., FERREOL B., MARTYS N., OLSON J., RAMAKRISHNAN T., ROTMAN D., SCHWARTZ L., « Transport in sandstone : A study based on three dimensional microtomography », Geo. Phys. Lett., vol. 23, 1996, page 705.

[BRO 79] BROWNSTEIN K., TARR C., « Importance of classical diffusion in NMR studies of water in biological cells », Phys. Rev. A, vol. 19, 1979, page 2446.

[CHE 05] CHEN Q., MARBLE A., COLPITTS B., BALCOM B., « The internal magnetic field distribution, and single exponential magnetic resonance free induction decay, in rocks », Journal of Magnetic Resonance, vol. 175, n° 2, 2005, p. 300–308.

[CHO 09] CHO H., RYU S., ACKERMAN J., SONG Y.-Q., « Visualization of inhomogeneous local magnetic field gradient due to susceptibility contrast », J. Mag. Res., vol. 198, 2009.
page 88.

[GEN 83] DE GENNES P., « Hydrodynamic dispersion in unsaturated porous media », *Journal of Fluid Mechanics Digital Archive*, vol. 136, n° 1, 1983, p. 189–200, 10.1017/S0022112083002116.

[GRE 07] GREBENKOV D., « NMR survey of reflected Brownian motion », *Rev. Mod. Phys.*, vol. 79, 2007, p. 1077–61.

[HUL 91] HULIN J., GUYON E., CHARLAIX E., LEROY C., MAGNICO P., « Abnormal diffusion and dispersion in porous media », *Physica Scripta*, vol. 1991, n° T35, 1991, page 26.

[KLE 99] KLEINBERG R., « Methods in the physics of porous media », WONG P., Ed., *Nuclear Magnetic Resonance*, vol. 35, Academic Press, 1999.

[LEB 96] LEBON L., OGER L., LEBLOND J., HULIN J., MARTYS N., SCHWARTZ L., « Pulsed gradient NMR measurements and numerical simulation of flow velocity distribution in sphere packings », *Physics of Fluids Fluids Fluids*, vol. 8, n° 2, 1996, p. 293–301.

[RAM 99] RAMAKRISHNAN T. S., SCHWARTZ L. M., FORDHAM E. J., KENYON W. E., WILKINSON D. J., « Forward Models for Nuclear Magnetic Resonance in Carbonate Rocks », *The Log Analyst*, vol. 40, 1999, page 260.

[RYU 08] RYU S., « Effects of Spatially Varying Surface Relaxivity and Pore Shape on NMR Logging », *SPWLA Proceedings of the 49th Annual Logging Symposium, SPWLA*, 2008, page 737008 BB.

[RYU 09a] RYU S., « Effect of inhomogeneous surface relaxivity, pore geometry and internal field gradient on NMR logging : exact and perturbative theories and numerical investigations », *SPWLA 2009*, 2009, page JJJJ.

[RYU 09b] RYU S., JOHNSON D., « Aspects of diffusive-relaxation dynamics with a non-uniform, partially absorbing boundary in general porous media », *Phys. Rev. Lett.*, vol. in press, 2009.

[SCH 04] SCHEVEN U., SELAND J., CORY D., « NMR propagator measurements on flow through a random pack of porous glass beads and how they are affected by dispersion, relaxation, and internal field inhomogeneities », *Phys. Rev. E*, vol. 69, 2004, page 021201.

[SCH 08] SONG Y.-Q., RYU S., SEN P., « Determining multiple length scales in rocks », *Nature*, vol. 406, 2000, page 178.

[SCH 08] SONG Y.-Q., ZIELINSKI L., RYU S., « Two-Dimensional NMR of Diffusion Systems », *Phys. Rev. Lett.*, vol. 100, 2008, page 248002.

[STA 84] STANLEY H. E., CONIGLIO A., « Flow in Porous Media : The ‘Backbone’ Fractal at the Percolation Threshold », *Physical Review B*, vol. 29, 1984, page 522.

[SUC 01] SUCCI S., *The Lattice Boltzmann Equation for Fluid Dynamics and Beyond*, Oxford University Press, 2001.

[ZHA 09] ZHAO W., PICARD G., LEU G., SINGER P., « Characterization of single phase flow through carbonate rocks : Quantitative comparison of NMR flow propagator measurements with a realistic pore network model », *Trans. Porous. Media*, 2009.

[ZIE 02] ZIELINSKI L., SONG Y.-Q., RYU S., SEN P., « Characterization of coupled pore systems from the diffusion eigenspectrum », *J. of Chem. Phys.*, vol. 117, n° 11, 2002, p. 5361–5365.