Bi-level Drainage System Design considering Upward Leakage from Semi-impermeable Barrier

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Abstract

Bi-level drainage system, in which alternate shallow and deep subsurface drains are laid, seems to be an economical option as compared to the level drainage system due to substantial reduction in cost of excavation. Mathematical solutions are available to describe fall of water table between two bi-level drains located at some distance above the horizontal impermeable barrier. But, if the barrier is semi-impermeable, little information is available to account for the vertical upward leakage in the soil system and its effect on spacing, fall of water table and discharge of drains. In the present study, a boundary value problem consisting of one dimensional linearized Boussinesq equation incorporating vertical upward leakage from semi-impermeable barrier with appropriate initial and boundary conditions was formulated. Analytical solution of such linearized equation has been obtained after devising a simple transformation through which boundary value problem was transformed to a heat transfer problem for which a solution was available. Spatial and temporal variation of water table between two bi-level drains was obtained employing this solution. A special case of the proposed solution (without vertical leakage) was verified with the existing solution and identical values of water table heights were obtained. The effect of hydraulic resistance of semi-impermeable barrier on spacing, water table heights between two bi-level and level drains and discharge of drains was studied by considering a numerical example based on assumed and experimental data. It was observed that with increase in the value of hydraulic resistance the spacing and fall of water table between two drains increases. Discharge of the drains is also influenced due to vertical upward leakage. An increase in hydraulic resistance, discharge of the drain decreases. The solution can be employed to determine discharge, spacing and fall of water table between two level or bi-level subsurface drains considering the semi-impermeable barrier.

Keywords: Analytical solution; Boussinesq equation; Bi-level drainage; Semi-impermeable barrier; Drain spacing; Drain discharge

Introduction

In India, the extent of waterlogging and soil salinity, as reported by various agencies, varies from 2.5 to 6 Mha and 3.3 to 8.6 Mha, respectively. Subsurface drainage plays an important role in alleviating the problem of waterlogging and soil salinization. It not only helps in providing favourable environment for growth and maturity of plants by maintaining a desirable water table depth and salinity, but also improves soil temperature, soil workability and trafficability, soil structure, tillage condition, biological, microbial, biochemical processes in the soil and hydrological conditions of the soil. Depth of subsurface drains is generally decided on the basis of crop-soil hydrological characteristics, crops to be taken, extent of salinity control, depth to impermeable barrier, availability of gravity outlet, as well as the available capacity of drain laying equipment. Spacing between drains may be computed by adopting a suitable drainage criterion.

Many researchers have developed experimental, analytical or numerical solutions to describe spatial and temporal variation of water table between two level drains located at some distance above the horizontal impermeable barrier. Little information is available on design of bi-level drainage system. De Boer and Chu [1] suggested the use of a bi-level drainage system because, by having alternating deep and shallow drain lines, excavation will be reduced substantially and, in arid and semi-arid regions, will prove more economical compared to a level drainage system. They provided a theoretical analysis for steady state design of bi-level drains located above the impermeable barrier. Chu and De Boer [2] validated their theory using laboratory experiments on a Hele-Shaw viscous flow model. They also compared their theory with field data of bi-level drainage experiments conducted at James Valley Agricultural Research and Extension Centre, Brookings, USA and found a close agreement between predicted and measured results. Sabti [3] obtained analytical solution of linearized and numerical solution of nonlinear Boussinesq equation to describe water table variation between two bi-level drains and concluded that the difference between the two methods increased with an increase in the region of flow. Verma et al. [4] obtained an analytical solution to the linearized Boussinesq boundary value problem involving Laplace transformation. They validated their method with the existing solution and field data reported by Chu and De Boer [2]. Upadhyaya [5] obtained various analytical and numerical solutions to describe spatial and temporal variation of water table between two bi-level and level drains located in a horizontal or sloping aquifer. Upadhyaya and Chuahan [6] obtained analytical solution of linearized Boussinesq equation incorporating depth dependent evapotranspiration from soil and studied the effect of evapotranspiration on fall of water table discharge of drains and spacing between two bi-level and level drains located above the impermeable barrier. Verma et al. [7] developed an analytical solution to design bi-level drainage systems using inverse theorem. The proposed solution was for simple boundary value problem considering impermeable barrier and recommended for simulations of hydraulic heads. However, in practical situation, it is not always possible to find a perfect impermeable barrier but a semi-impermeable barrier commonly exists. None of the above studies describe the water table variation between two bi-level drains when drains are located some distance above the

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semi-impermeable barrier and vertical upward leakage from the semi-impermeable layer is taking place in the soil resting over it. In the present study, an analytical solution of linearized Boussinesq equation incorporating vertical upward leakage from semi-impermeable barrier with appropriate initial and boundary conditions, describing spatial and temporal variation of water table between two bi-level and level drains, has been obtained. Effect of hydraulic resistance on discharge, fall of water table and spacing between two bi-level or level drains has also been studied and discussed.

Theoretical Developments

Boundary value problem

Figure 1 shows a sketch of bi-level and conventional level drainage problem. Initially, the horizontal water table is assumed to be close to the land surface at an elevation of ho above the deep drain. The shallow drain is located at an elevation of hi above the deep drain. The two drains are separated by a distance L. The aquifer is assumed to be homogeneous and isotropic resting on a horizontal semi-impermeable barrier. Considering the deep drain as the origin, the coordinate axes are taken parallel and perpendicular to the semi-impermeable barrier.

The one-dimensional Boussinesq equation incorporating vertical upward leakage through the semi-impermeable base along with appropriate initial and boundary conditions, describes the above physical problem in mathematical terms. The boundary value problem in analysing the fall of water table in a bi-level drainage system is given as physical problem in mathematical terms. The boundary value problem appropriate initial and boundary conditions, describes the above upward leakage through the semi-impervious base along with are taken parallel and perpendicular to the semi-impervious barrier. Considering the deep drain as the origin, the coordinate axes

\[ h(X,0) = h_0 \]
\[ h(0,t) = 0 \quad 0 < t < t_0 \]
\[ h(L,t) = h_0 \]
\[ \frac{\partial h(L,t)}{\partial X} = 0 \]

Here 'h' is height of water table at any time 't' above the deep drain; 'K' is average hydraulic conductivity [LT\(^{-1}\)]; and 'f' is drainable porosity of aquifer (dimensionless); 't_c' is the time when slope of water table at X-L becomes zero. The boundary condition (2d) shows that for \[ t > t_c \] the discharge in the shallow drain becomes zero and the water table will fall below the shallow drain. For time \[ t > t_c \] the shallow drain becomes non-operational. In a bi-level drainage system, the time in which the shallow drain is operational is important, therefore the following solution of this boundary value problem is valid only for \[ t \leq t_c \]. In eq (1) it is well known that the recharge \( q_h \) is given by

\[ q_h = K_s \frac{H_0 - h}{B_s} - \frac{H_0 - h}{c} \]

Where \( K_s \) and \( B_s \) are hydraulic conductivity and thickness of the semi-impervious barrier, respectively. 'c' is hydraulic resistance and \( H_0 \) is the piezometric head in the semi-confined aquifer. When no changes of initial height \( h_0 \) of the ground water table have taken place, then \( H_s = h_0 \), and eq. (3) can be written as

\[ q_h = \frac{h_0 - h}{c} \]

Since eq. (2) is a nonlinear partial differential equation which cannot be solved analytically without linearization, it is linearized by neglecting \((\partial h/\partial X)^2\) \[ \text{and replacing (d + h) with a constant D, average depth of flow, reducing eq (2) to the following form:} \]

\[ \frac{\partial^2 h}{\partial X^2} + \frac{h_0 - h}{KD} = \frac{f}{KD} \frac{\partial h}{\partial t} \]

According to van Schilfgaarde this method of linearization can be assumed if variation in 'h' is very small compared to that in \( \partial h/\partial X \), i.e. change in depth of the unconfined aquifer is small compared to the average depth. This restricts the application of the linearized equation to cases where \( \Delta h << KD \) and also \( KD << L \); these conditions are also needed to satisfy Dupuit Forchheimer assumptions. Here \( \Delta h \) is the change in water table height with time and space, and L is the spacing between two drains in the flow region. 'D', the average depth of flow, used in eq (5) is equal to the sum of depth of the impermeable barrier below the drain and one half the initial water table height above the drains. Taking into account the convergence of flow lines near the drain pipes, the depth of the impermeable barrier below the deep drain is replaced by an equivalent depth, \( d_e \) using the relationship of Hooghoudt [8], such that \( D = d_e + h_0/2 \) as considered by Upadhyaya and Chauhan [6].

Analytical solution

To obtain an analytical solution of eq (5) with initial and boundary conditions, given by eqs (2a)-(2c) a suitable transformation was devised. Eq. (5) is then transformed to a heat transfer boundary value problem for which analytical solutions are available. The transformation is:

\[ \frac{t}{L} = V e^{\psi} + h_0 \]

Applying this transformation in eq. (5) and eqs. (2a-2c) one gets the following boundary value problem:

\[ \frac{\partial^2 V}{\partial X^2} = \frac{1}{a} \frac{\partial V}{\partial t} \]

\[ V(X,0) = 0 \quad 0 < X < L \]

\[ V(0,t) = \left( -h_0 \right) e^{\psi t} = f_1(t) \quad t > 0 \]
\[
V(L,t) = \frac{1}{\epsilon} \left( \frac{L}{2} - h(t) \right) \frac{L}{4} f_2(t) \quad 0 < t < t_0
\]

(8c)

Here \( \alpha = K D / \epsilon \). Ozisik [9] obtained the generalized solution of such a boundary value problem using Duhamal’s theorem. His solution is:

\[
V(X,t) = \sum_{n=1}^{\infty} \int_{0}^{L} \int_{0}^{L} \frac{\sin \beta_n X}{\beta_n} \left( f(t) e^{-\beta_n X} + \left( e^{-\beta_n X} \right) f(t) \right) dX dY
\]

(9)

Here \( \beta_n = n\pi / L \).

Substituting the expressions for \( f(t) \), \( f(t) \), \( f(0) \), \( f(t) \) and \( df(t) \) in eq. (9) and some simplification one gets the following expression for \( V(X,t) \):

\[
V(X,t) = \sum_{n=1}^{\infty} \int_{0}^{L} \int_{0}^{L} \frac{\sin \beta_n X}{\beta_n} \left( -h(t) e^{-\beta_n X} \frac{1}{\epsilon} \frac{h(t)}{1 + a f(t)} \left( e^{\epsilon X} - e^{-\epsilon X} \right) \right) [1 - (-1)^n] + \sum_{n=1}^{\infty} \int_{0}^{L} \int_{0}^{L} \frac{\sin \beta_n X}{\beta_n} \left( \frac{1}{\epsilon} \frac{h(t)}{1 + a f(t)} \right) \left( e^{\epsilon X} - e^{-\epsilon X} \right) [+(-h(t)) e^{\epsilon X} + \frac{h(t)}{1 + a f(t)}]
\]

(10)

Substituting eq. (10) into eq. (6) one gets the following expression for \( h(X,t) \):

\[
h(X,t) = \frac{2h_0}{\pi} \int_{0}^{L} \int_{0}^{L} \frac{\sin \beta_n X}{\beta_n} \left( e^{-\beta_n X} \frac{1}{\epsilon} \frac{h(t)}{1 + a f(t)} \left( e^{\epsilon X} - e^{-\epsilon X} \right) \right) [1 - (-1)^n] + \frac{h_0}{L} X
\]

(11)

**Special cases of the solution**

- If it is assumed that hydraulic resistance, \( \epsilon \) is tending to infinity and barrier becomes impermeable than \( 1/c \) becomes zero in eq. (11), which gives:

\[
h(X,t) = \frac{2h_0}{\pi} \int_{0}^{L} \int_{0}^{L} \frac{\sin \beta_n X}{\beta_n} \left( e^{-\beta_n X} \frac{1}{\epsilon} \frac{h(t)}{1 + a f(t)} \left( e^{\epsilon X} - e^{-\epsilon X} \right) \right) [1 - (-1)^n] + \frac{h_0}{L} X
\]

(12)

- If drains are at the same level and semi-impermeable barrier exists then \( h_0 = 0 \) in eq. (11) and the solution takes the form as:

\[
h(X,t) = \frac{2h_0}{\pi} \int_{0}^{L} \int_{0}^{L} \frac{\sin \beta_n X}{\beta_n} \left( e^{-\beta_n X} \frac{1}{\epsilon} \frac{h(t)}{1 + a f(t)} \left( e^{\epsilon X} - e^{-\epsilon X} \right) \right) [1 - (-1)^n] + \frac{h_0}{L} X
\]

(13)

- If drains are at the same level and hydraulic resistance, \( \epsilon \) tends to infinity, then \( 1/c = 0 \) in eq. (13), which gives:

\[
h(X,t) = \frac{2h_0}{\pi} \int_{0}^{L} \int_{0}^{L} \frac{\sin \beta_n X}{\beta_n} \left( e^{-\beta_n X} \frac{1}{\epsilon} \frac{h(t)}{1 + a f(t)} \left( e^{\epsilon X} - e^{-\epsilon X} \right) \right) [1 - (-1)^n] + \frac{h_0}{L} X
\]

(14)

The nondimensionalized analytical solution obtained by Sabti [3], assuming the initial water table height as a fourth degree parabola, was dimensionilized. If an initially flat water table is assumed, the expression is similar to that of eq. (12), which is a special case of the proposed solution. This partially validates the correctness of the proposed solution.

**Discharge equation for bi-level drains**

Discharge of the drains can be computed using Darcy’s Law, which states that the quantity of water passing a unit cross-section of soil is proportional to the gradient of hydraulic head. The expression for the gradient of hydraulic head is obtained by differentiating eq. (11) with respect to \( X \). Discharge per unit length of the drains is obtained by multiplying the first derivative of \( h(X,t) \) at \( X = 0 \) and \( X = L \) with - KD. Final expressions for discharge of deep and shallow drains are as follows:

\[
Q_d = \frac{2h_0}{\pi} \int_{0}^{L} \int_{0}^{L} \frac{\sin \beta_n X}{\beta_n} \left( e^{-\beta_n X} \frac{1}{\epsilon} \frac{h(t)}{1 + a f(t)} \left( e^{\epsilon X} - e^{-\epsilon X} \right) \right) [1 - (-1)^n]
\]

(15)

\[
Q_s = \frac{2h_0}{\pi} \int_{0}^{L} \int_{0}^{L} \frac{\sin \beta_n X}{\beta_n} \left( e^{-\beta_n X} \frac{1}{\epsilon} \frac{h(t)}{1 + a f(t)} \left( e^{\epsilon X} - e^{-\epsilon X} \right) \right) [1 - (-1)^n - 1]
\]

(16)

Here \( Q_d \) and \( Q_s \) denote discharge from deep and shallow drains. Total discharge will be the algebraic sum of absolute values of discharge from both the bi-level drains.

**Results and Discussion**

To validate the proposed analytical solution the field data of the conventional level drainage system at the Sampla (India) drainage site, as reported by Verma et al. [4], were utilized (Table 1). Some data like thickness of semi-impervious barrier was assumed as 2 m and hydraulic conductivity of semi-impervious barrier as 0.1, 0.05, 0.01, 0.005, 0.001 and 0.00001 ≈ 0.0 m/day. Spacing between two drains was computed employing the drainage criteria of lowering the water table by 30 cm in 2 days from the initially assumed water table at the soil surface.

**Effect of hydraulic conductivity of semi impervious barrier on water table elevations above the deep drain level**

Water table elevations above the deep drain level (1.8 m below the soil surface) between two bi level drains separated by a distance of 50 m were computed by the proposed analytical solution (considering hydraulic conductivity of semi-impervious barrier varying from 0.1 m/day to 0.0 m/day) at \( X = 15.0 \) and 35.0 m away from the deep drain (Table 2). In the proposed solution, the values of water table elevation

| Parameter                  | Value   |
|----------------------------|---------|
| Hydraulic conductivity (m/day) | 3.00    |
| Drainable porosity (dimensionless) | 0.14    |
| Thickness of the envelope material (m) | 0.10    |
| Tile radius (m)            | 0.05    |

Table 1: Parameters of the conventional level drainage system at the Sampla (India) drainage site and some assumed data considered for analysis.
were found to converge for number of terms, (n=5) So only five terms were retained in the solution.

It may be observed from Table 2 that water table elevations at X=15.0 m and X=35.0 m decrease with decrease in hydraulic conductivity of semi-impervious barrier. It is because of the fact that with decrease in hydraulic conductivity of semi-impervious barrier the contribution of vertical upward leakage from this barrier reduces resulting in faster fall of water table between two drains. Water table elevations at X=35.0 m are comparatively higher than at X=15.0 m because in bi-level drainage systems considering semi impervious or impervious barrier.

**Spatial distribution of water table elevations**

Spatial distribution of water table elevations between two bi-level drains spaced at 50 m distance considering Ks varying from 0.1 m/day to 0.0 m/day, initial water table, h0=1.8 m and shallow drain located at h1=0.6 m (all heights measured above the deep drain level) for 2, 6 and 12 days is shown in Figure 2. It may be observed from the figure that water table elevations at all the points in space are lower for the case when barrier is impervious compared to the case when barrier is semi-impervious. With increase in time fall of water table is comparatively slower for the case of semi-impervious barrier as compared to impervious barrier.

**Temporal variation of maximum height of the water table**

Temporal variation of maximum height of the water table for both bi-level and level drainage systems (drains separated by a distance of 50 m) considering Ks as 0.1 and 0.0 m/day is shown in Figure 3. There was a slower rate of decline of the maximum water table in the bi-level drainage system compared to the level drainage system with the same drain-spacing. In the case of impervious barrier the difference in the maximum water table elevations between two bi-level and level drains was initially smaller, but gradually increased with time. For the case of semi-impervious barrier the difference in maximum hydraulic head between bi-level and level drains was very less as compared to the case of impervious barrier.

**Effect of Ks on spacing**

The effect of vertical upward leakage from semi-impervious barrier on spacing between two bi-level or conventional level drains was studied by varying the values of Ks from 0.1 to 0.0000 1 m/day. In the present numerical example it was observed that spacing between two bi-level or level drains increases with decrease in hydraulic conductivity of semi-impervious barrier (Table 3). In other words one can say that if spacing is computed assuming an impervious barrier in place of a semi-impervious barrier with Ks=0.1 m/day, there is a chance of overestimation of spacing to the tune of 17.77%.

**Variation of discharge with time**

Temporal variations of discharge in deep and shallow drains of bi-level drainage systems considering semi impervious or impervious barrier are shown in Figure 4. It can be observed that discharge of the deep drain is always greater than the discharge of the shallow drain. Therefore, for the shallow drainage systems considering semi impervious or impervious barrier.

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**Table 2: Water table elevations (m) above the deep drain level at X=15.0 and 35.0 m with Ks=0.1 and 0.0 m/day.**

| Time (Days) | Ks=0.1 m/d | Ks=0.0 m/d | Ks=0.1 m/d | Ks=0.0 m/d |
|-------------|------------|------------|------------|------------|
| 1           | 1.64       | 1.60       | 1.59       | 1.87       |
| 2           | 1.48       | 1.33       | 1.58       | 1.48       |
| 3           | 1.39       | 1.12       | 1.51       | 1.32       |
| 4           | 1.35       | 0.96       | 1.47       | 1.18       |
| 5           | 1.32       | 0.83       | 1.45       | 1.06       |
| 6           | 1.30       | 0.73       | 1.43       | 0.96       |
| 7           | 1.30       | 0.64       | 1.42       | 0.88       |
| 8           | 1.29       | 0.57       | 1.42       | 0.81       |
| 9           | 1.29       | 0.50       | 1.42       | 0.74       |
| 10          | 1.29       | 0.45       | 1.41       | 0.69       |
| 11          | 1.29       | 0.41       | 1.41       | 0.65       |
| 12          | 1.29       | 0.37       | 1.41       | 0.61       |
| 13          | 1.29       | 0.34       | 1.41       | 0.58       |

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**Table 3: Effect of hydraulic conductivity of semi-impervious barrier on spacing.**

| Ks | Computed spacing (m) |
|----|----------------------|
|    | Bi-level drainage system | Level drainage system |
| 0.1 | 36.58 | 39.66 | 42.27 | 42.61 | 42.88 | 42.94 |
| 0.05 | (8.42) | (15.55) | (16.48) | (17.22) | (17.39) |
| 0.01 | 34.16 | 37.09 | 39.59 | 39.92 | 40.17 | 40.28 |
| 0.005 | (8.58) | (15.90) | (16.86) | (17.59) | (17.77) |
| 0.001 | 39.58 | 42.63 | 45.21 | 45.54 | 45.80 | 45.87 |
| 0.0 | (7.71) | (14.22) | (15.06) | (15.72) | (15.89) |

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Figure 2: Variation of water table with increase in horizontal distance from deep drain from different values of time.

Figure 3: Variation of maximum hydraulic head with time for level and bi-level drainage systems considering semi impervious or impervious barrier.
drain, a pipe of smaller diameter than the deep drain pipe may be required. The discharge of the shallow drain becomes zero on the 13th day and boundary condition (3d) becomes effective. It indicates that the proposed solution for the bi-level drainage system is valid only up to $t_0=13$ days.

**Field application**

The proposed analytical solution is a generalized method, which can be used for designing bi-level or level subsurface drainage systems in the regions where horizontal semi-impervious or impervious barrier exists at some finite depth below the land surface. In practice, it is difficult to get an impervious barrier so consideration of vertical upward leakage from semi-impervious barrier gives an opportunity to analyze the field problem in a better way.

**Conclusion**

An analytical solution to the linearized Boussinesq equation for bi-level drainage system design considering vertical upward seepage from semi-impermeable barrier was obtained. The analytical solution was obtained after devising a simple transformation through which the boundary value problem was transformed to a heat transfer problem for which a solution was available. This technique is relatively easy and yields a simple expression of $h(X, t)$.

The effect of hydraulic conductivity of semi-impervious barrier or hydraulic resistance on drain spacing is significant. Discharge of the shallow drain becomes zero on the 13th day. This indicates that the proposed solution for a bi-level drainage system is valid only up to 13 days.

It may be concluded that incorporating vertical upward leakage from semi-impervious barrier in a bi-level or level drainage system improves the design and helps in analyzing the realistic field problem with more accuracy in computation of spacing between drains and fall of water table.

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