Functional Integral Representation for Relativistic Schrödinger Operator Coupled to a Scalar Bose Field

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Abstract
In this paper the system of a quantum particle interacting with a scalar Bose field is investigated. The particle’s Hamiltonian is given the relativistic Schrödinger operator with potential

\[ H_p = \sqrt{-\triangle + M^2} - M + V \]  

(1)

A scalar Bose field is constructed by infinite dimensional stochastic process. The field operators \( \{ \phi(f) \}_{f \in \mathcal{K}_b} \) are defined by the Gaussian random process indexed by a Hilbert space \( \mathcal{K}_b \) on a probability space \( (Q_b, B_b, P_b) \). The state space is given by \( L^2(Q_b) \) and the free Bose Hamiltonian \( H_b \) is defined by the differential second quantization of \( \omega_b(-i\nabla) \) where \( \omega_b \) is non-negative and continuous function. Physically \( \omega_b(k) \geq 0 \) denotes the one-particle energy of the field with momentum \( k \). Thus the triplet \( (L^2(Q_b), H_b, \{ \phi(f) \}_{f \in \mathcal{K}_b}) \) of the scalar Bose field is defined.

The state space of the interacting system is given by \( \mathcal{H} = L^2(\mathbb{R}^d_x) \otimes L^2(Q_b) \simeq \int_{\mathbb{R}^d} L^2(Q_b) d\mathbf{x} \) where \( \int^{\oplus} \) denotes the fibre direct integral. The free Hamiltonian is defined by \( H_0 = H_p \otimes I + I \otimes H_b \) and the total Hamiltonian by

\[ H_\kappa = H_0 \oplus \kappa \int_{\mathbb{R}^d} P(\rho_\mathbf{x}) d\mathbf{x} \]  

(2)

where \( \oplus \) denotes the form sum, \( P(\lambda) = \sum_{j=1}^{2n} c_j \lambda^j, \) \( c_j \in \mathbb{R}, \) \( j = 1, \ldots, 2n - 1, c_{2n} > 0, \) and the ultraviolet cutoff condition \( \rho_\mathbf{x} \in S^1_{\text{real}} \) for each \( \mathbf{x} \in \mathbb{R}^d \) is supposed.

In the main theorem, the functional integral representation of \( e^{-tH_\kappa} \) is derived.