Study on the Influences of Multiple Parameters With Uncertainty in the Clamp-On Ultrasonic Flowmeter

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This work was supported by the National Key Research and Development Project of China under Grant 2019YFB2006601 and Grant 2019YFB2006602.

ABSTRACT For a clamp-on ultrasonic flowmeter based on transit-time differential method, the flow rate calculation model with flow velocity correction is established and verified by experiments carried out in a flow standard facility. Based on established flow rate calculation model, a systematic methodology incorporating the parametric uncertainty distribution is proposed to analyze the influences of multiple parameters with uncertainty on measurement error by establishing the sample-based stochastic model. Monte Carlo sampling (MCS) method is used to randomly select the value of each parameter from its prescribed Gaussian distribution and then combining them together as one sample, and the number of selected samples is determined by conducting stochastic convergence analysis of mean value and the standard deviation of input and output parameters. And finally the influence degrees of multiple parameters are analyzed and compared quantitatively on the same dimension to find out which parameters are influential and which are negligible. The results reveal the different influences of multiple parameters with uncertainty, which can be used as reference for performance improvement and measuring error analysis of the clamp-on ultrasonic flowmeter in industrial local measurement.

INDEX TERMS Clamp-on ultrasonic flowmeter, transit-time differential method, influence of multiple parameters with uncertainty, sample-based stochastic model.

I. INTRODUCTION

Clamp-on ultrasonic flow metering is a popular method by attaching two or more ultrasonic transducers on the surface of the pipeline. It is used for many industrial processes to measure the flow of several liquids like water, oil, acids or other aggressive chemical fluids [1], for the advantages, such as non-invasive, no pressure drop in the pipe, easy to install and adjust on the pipe [2]–[4].

In comparison to ultrasonic inline flowmeter, the accuracy and repeatability of clamp-on ultrasonic flowmeter are relatively low [5]. Especially for low flow rate, the problem of low measurement accuracy is more serious [6]. The sources of measurement errors may come from imperfect measuring tube, installation of transducers, properties of fluid, acoustic characteristics, and so on [7]–[10]. And more specifically, the accuracy of clamp-on ultrasonic flowmeter is affected by some parameters such as: incidence angle of the ultrasonic transducer, inner pipe diameter, sound speed of fluid in the pipe, sound speed in the pipe wall, sound speed inside transducer material, density of fluid, dynamic viscosity of fluid, and so on. Among these parameters, some parameters are coupled with other parameters, some are difficult to be measured directly or accurately, and some are sensitive to the measurement conditions and their values fluctuate. The actual values of these parameters are different to be obtained accurately, and the measurement errors is inevitable. These parameters have a certain degree of uncertainty, and the influence degrees of these uncertain parameters are different. So, measurement errors introduce by different parameters are different too. It is difficult to put these parameters in the same dimension to quantitatively compare their influences and find out which parameters are important and which are negligible for performance improvement and error analysis of clamp-on...
ultrasonic flowmeter. Therefore, a systematic methodology incorporating the parametric uncertainty distribution to analyze the influences of multiple parameters with uncertainty is needed.

H. Peng and Y. Zhang applied a sample-based stochastic model to investigate the influences of different uncertain parameters in the solid-liquid-vapor phase change processes and find the key parameters which have the dominant effects [11]. H. Shi, et al. applied the uncertainty analysis method based on sample-based stochastic model in sensor structure design and optimization [12]–[14]. And the effects of uncertain parameters were also investigated using a sampling-based stochastic model in thermal damage to living biological tissues [15], the optical fiber drawing process [16], [17]. More applications of the stochastic model could be found in non-isothermal flow [18], thermosetting-matrix composites fabrication [19], analysis of complex chemical process [20], and proton exchange membrane (PEM) fuel cells [21]. The uncertainty analyses method reported in these literatures provide a useful tool to investigate the influences of uncertain parameters on measurement error of the clamp-on ultrasonic flowmeter.

In this paper, for a clamp-on ultrasonic flowmeter, the flow rate calculation model is established and verified by experiments. The uncertainty analysis method based on sample-based stochastic model is established using established flow rate calculation model to investigate the influences of multiple parameters on the measurement error when uncertainties of parameters are considered. And the influence degrees of multiple parameters with uncertainty are analyzed and compared quantitatively on the same dimension, so as to find out which parameters are influential and which are negligible.

II. PRINCIPLE OF CLAMP-ON TRANSIT-TIME ULTRASONIC FLOW MEASUREMENT

The principle of the clamp-on ultrasonic flowmeter with V-shaped ultrasonic path is shown as Fig.1. Distance between front end faces of ultrasonic transducer TA and TB is L. Ultrasonic waves are generated by the transducer TA and TB, then transmitted downstream/upstream to the pipe flow by penetrating the surface of the pipe according to Snell’s law [4], [22], [23], and finally received by transducer TB and TA. Based on the principle of transit-time differential method, the transit-time of ultrasonic wave travels downstream along the V-shaped path is different from the transit-time of ultrasonic wave travels upstream along the V-shaped path.

In Fig.1, L1 is wall thickness of round pipe, L2 is the axial projection distance of half path in pipe wall, and L3 is the axial projection distance of half path in fluid. D is outer diameter of round pipe, d is inner diameter of round pipe, θ0 is incidence angle of the ultrasonic transducer, θ1 is incidence angle of the pipe wall, and θ2 is incidence angle of the pipe fluid.

![Figure 1. Clamp-on ultrasonic flowmeter with V-shaped ultrasonic path.](image-url)

A. LINE-AVERAGED FLOW VELOCITY

The transit-time of ultrasonic wave traveling downstream along the V-shaped path is t1, and the ultrasonic wave is generated by transducer TA and received by transducer TB. The transit-time of ultrasonic wave traveling upstream along the V-shaped path is t2, and the ultrasonic wave is generated by transducer TB and received by transducer TA. According to the principle of transit-time differential method, the transit-time difference depends on line-averaged flow velocity along the ultrasonic path.

According to the geometric relationship in Fig.1, the transit-times are

\[
\begin{align*}
  t_1 &= \frac{2L_1}{c_g} \cos \theta_1 + \frac{2L_3}{c_0 + v \sin \theta_2} + \tau_1 \\
  t_2 &= \frac{2L_1}{c_g} \cos \theta_1 + \frac{2L_3}{c_0 - v \sin \theta_2} + \tau_2
\end{align*}
\]

(1)

(2)

where \(v\) is line-averaged flow velocity along the ultrasonic path. \(\tau_1\) and \(\tau_2\) are transducer wedge and circuit delay of TA and TB respectively. It is generally considered that \(\tau_1\) and \(\tau_2\) are the same, because transducer TA and TB are the same. \(c_0\) is sound speed of fluid, \(c_g\) is sound speed in the pipe wall, and \(c_k\) is sound speed inside transducer material. Transit-time difference \(\Delta t\) can be obtained by \(t_2 - t_1\)

\[
\Delta t = \frac{4vL_3}{c_0^2 - v^2 \sin^2 \theta_2}
\]

(3)

When there is flowing water in the pipeline, the flow velocity of water is generally several meters per second while sound speed of water in the pipe is nearly 1500m/s. For example, \(c_0\) is 1497 m/s at 25°C in case of water [24], \(v^2 \sin^2 \theta_2\) is far less than \(c_0^2\) and could be ignored. And \(L_3 = d \tan \theta_2\), so line-averaged flow velocity along the ultrasonic path can be obtained,

\[
v = \frac{\Delta t c_0^2}{4d \tan \theta_2}
\]

(4)

According to Snell’s law, the relationship between different angles in Fig.1 and sound speeds could be obtained,

\[
\begin{align*}
  \sin \theta_0 &= \frac{c_k}{c_0} \\
  \sin \theta_1 &= \frac{c_k}{c_g} \\
  \sin \theta_1 &= \frac{c_0}{c_g} \\
  \sin \theta_2 &= \frac{c_k}{c_0}
\end{align*}
\]

(5)
Eq. (4) is rewritten by eliminating \( \tan \theta \) as
\[
\nu = \frac{\Delta t c_0^2}{4d} \sqrt{\left( \frac{c_k}{c_0 \sin \theta} \right)^2 - 1} \tag{6}
\]

**B. AVERAGE FLOW VELOCITY IN THE PIPE CROSS SECTION**

The velocity directly measured by the ultrasonic flowmeter is the line-averaged flow velocity along the ultrasonic path, and it is different from the average flow velocity of the pipe cross section. In order to make the measured result more accurate, the velocity measured by the ultrasonic flowmeter must be corrected. According to the semi empirical formula of hydrodynamics, the relationship between the line-averaged flow velocity and the average flow velocity of the pipe cross section in smooth circular pipe is express as [25]
\[
\bar{v} = \frac{v}{K} \tag{7}
\]
where \( \bar{v} \) is the average flow velocity of the pipe cross section, and \( K \) is correction factor. The correction factor \( K \) is a function of Reynolds number \( Re \) [25], [26]
\[
K = 1 + 0.01 \sqrt{6.25 + 431 Re^{-0.237}} \tag{8}
\]
Reynolds number of the fluid is
\[
Re = \frac{\bar{v} \rho d}{\mu} \tag{9}
\]
where \( \rho \) is density of fluid, and \( \mu \) is dynamic viscosity of fluid. Both \( \rho \) and \( \mu \) depend on fluid type and temperature. In this paper, water is used as the experiment fluid. There are many researches on the density and viscosity of water, and several models have been established. However, for many kinds of fluids in industrial local measurement, there might be not mathematical model for the density and viscosity. In this case, the values of density and viscosity may directly affect the measured result of flow rate.

The density of water \( \rho_{\text{water}} \) depends on temperature. In general, the density of water could be obtained using the formulation derived by the International Association for the Properties of Water and Steam (IAPWS), which is suit for temperature up to 95 °C. For the case of high precision, Tanaka formulation could be used to calculate the density of water, and it is suit for temperature up to 40 °C. The experiment temperature in this paper is room temperature, Tanaka formulation is used [27]
\[
\rho_{\text{water}} = a_5 \left[ 1 - \frac{(T + a_1)^2(T + a_2)}{a_3(T + a_4)} \right] \tag{10}
\]
where, \( T \) is the water temperature, and the coefficients are \( a_1 = 3.983035, a_2 = 301.797, a_3 = 522528.9, a_4 = 69.34881, \) and \( a_5 = 999.974950 \).

For calculation of viscosity of water, Kestin model or IAPWS-95 model are usually used. In temperature range of 0 °C to 85 °C, the error between two models is less than 0.001%, and the uncertainty is estimated to be about 0.5% (Kestin) and 0.3% (IAPWS-95), respectively. Two models have the same mathematical form as [28]
\[
\mu_{\text{water}} = \mu_{20} \exp \left( \sum_{i=1}^{4} C_i (T - 20)^i \right) \tag{11}
\]
where, \( T \) is the water temperature, \( \mu_{20} \) is the viscosity of the water at 20 °C, and \( \mu_{20} = 1.020 \text{mPa} \cdot \text{s} \). In this paper, IAPWS-95 model is used, and the coefficients are \( C_1 = -0.024574314, C_2 = 1.878426 \times 10^{-4}, C_3 = -1.58700 \times 10^{-6}, \) and \( C_4 = 7.849500 \times 10^{-9} \).

**C. VOLUME FLOW RATE OF FULL PIPE FLOW**

According to Eqs. (6) to (9), the volume flow rate of full pipe flow \( Q_{\text{meas}} \) could be calculated as (12), as shown at the bottom of the page.

In order to study the influences of multiple parameters on the measurement error, the flow rate difference between flow rate of full pipe flow \( Q_{\text{meas}} \) and the standard flow rate \( Q_{\text{std}} \) given by the water flow standard facility is used as the measurement error. The expression of the measurement error \( Q_E \) is
\[
Q_E = Q_{\text{meas}} - Q_{\text{std}} \tag{13}
\]
And the relative error between \( Q_{\text{meas}} \) and \( Q_{\text{std}} \) is
\[
Q_{\text{RE}} = \frac{Q_{\text{meas}} - Q_{\text{std}}}{Q_{\text{std}}} \tag{14}
\]

The volume flow rate \( Q_{\text{meas}} \) could be calculated by Eq. (12) after the transit-time \( t_1 \) and \( t_2 \) (\( \Delta t \)) are accurately measured. However, the correction factor \( K \) is still present in the right hand of Eq. (12), and \( K \) is a function of the average flow velocity \( \bar{v} \) which is related to \( Q_{\text{meas}} \). And substituting the expression of \( K \) (Eqs. (6) to (9)) into Eq. (12) will make Eq. (12) more complex, but the correction factor \( K \) still could not be eliminated. So, it is difficult to calculate flow rate \( Q_{\text{meas}} \) directly, and it is also difficult to analyze the influences of parameters by theoretical method.

The accuracy of transit-time difference \( \Delta t \) directly affects the measurement error, and it could also be used as an uncertain parameter for obtaining the flow rate in the pipeline.

\[
Q_{\text{meas}} = \frac{\pi d \Delta t \left[ \left( \frac{c_{k1}}{c_0 \sin \theta_0} \right)^2 - c_0^2 \right]^{0.5}}{16 + 0.16 \left( 6.25 + 431 \left( \frac{\mu_{20} d \Delta t}{4 \pi K} \left( \left( \frac{c_k}{c_0 \sin \theta_0} \right)^2 - 1 \right)^{0.5} - 0.237 \right)^{0.5} \right)} \tag{12}
\]
In addition to $\Delta t$, parameters include incidence angle $\theta_0$, inner diameter $d$, sound speed of fluid $c_0$, sound speed inside transducer material $c_k$, density of fluid $\rho$, and dynamic viscosity of fluid $\mu$ also affect the measurement error.

For the inner diameter $d$, if the round pipe is too large to be measured, or the fluid produces corrosion or adhesion on the inner surface of the pipe, the accurate value of $d$ could not be obtained. The wall thickness of the pipeline may expand or contract with the change of temperature, which makes the inner diameter $d$ more difficult to be measured accurately too. When ultrasonic transducer $T_A$ and $T_B$ are installed on the pipe, incidence angle of the ultrasonic transducer $\theta_0$ is fixed. When the transducer is installed in full accordance with the designed parameters (mainly distance $L$ between front end faces of two transducers), the V-shaped ultrasonic propagation path is formed as the designed incidence angle $\theta_0$. This propagation path of ultrasonic wave is the designed path under ideal installation. However, when the distance $L$ of two transducers has installation error, the actual path of ultrasonic propagation deviates from the designed path, which make the incidence angle $\theta_0$ of the actual path deviating from the design value. In addition, the actual propagation path of ultrasonic wave and incidence angle also depend on the ultrasonic propagation speed, while sound speed is depending on the propagation materials and temperature, and its exact value is difficult to obtain accurately.

Among these parameters, some parameters are coupled with other parameters, some are difficult to be measured directly or accurately, and some are sensitive to the measurement condition and their values fluctuate. It is difficult to analyze the influences of these parameters and put them in the same dimension to quantitatively compare their influences. Therefore, a methodology of systematic analysis combined with the parametric uncertainty distribution is needed to analyze the effects of multiple parameters.

**III. UNCERTAINTY ANALYSIS METHOD**

In order to investigate the influences of uncertain parameters on the measurement error of clamp-on ultrasonic flowmeter, a sample-based stochastic model is established based on the flow rate calculation model. The specific stochastic modeling procedure is shown in Fig. 2. By introducing the combination of several randomly selected input parameters into the flow rate calculation model, the output parameters are calculated and uncertainty of the output parameters are evaluated. If the uncertainty of the output parameter becomes larger when the uncertainty of one input parameter increases gradually, it indicates that the input parameter has a certain influence on the output parameter. By changing the uncertainty of different input parameters in turn, the uncertainties of selected input uncertain parameters are propagated through deterministic flow rate calculation model previously established, and the variability of output parameters are quantified to evaluate the uncertainty of output parameters. Then the influence of different input parameters on output parameters could be compared, and the parameters with greater influence could be found.

The input uncertain parameters investigated here are: incidence angle $\theta_0$, inner diameter $d$, sound speed of fluid inside $c_0$, sound speed inside transducer material $c_k$, density $\rho$, dynamic viscosity $\mu$, and transit-time difference $\Delta t$. It is not possible to obtain the real value of each uncertain parameter and its distribution, so it is assumed that all of these input parameters obey the Gaussian distribution [11, 13]. In Gaussian distribution of each uncertain input parameter, the mean value $\bar{x}$ is set as parameter value, the standard deviation $\sigma$ shows the uncertainty of this parameter $x$. For each parameter, the coefficient of variance (COV) $\sigma/\bar{x}$ is defined and used to represent the degree of its uncertainty. Monte Carlo sampling (MCS) method is used to randomly select the values of each parameter from its prescribed Gaussian distribution and then combining them together as one sample. With the help of MCS, the combinations of input parameters with the prescribed uncertainty are chosen simplicity and efficiency.

The variability of output parameter depends largely on the number of selected samples [11]. The stochastic convergence analysis is conducted by analyzing the mean value and standard deviation of the input and output parameters with different numbers of samples. When the number of samples increases, the mean value and standard deviation of the input parameters converge to the nominal mean value and standard deviation of the Gaussian distribution and the mean value and standard deviation of the output parameters will also converge within a certain tolerance [11–15]. Through stochastic convergence analysis, the number of selected samples is determined to be appropriate and make sure the selected samples are representative. Deterministic flow rate calculation model established is used to calculate the output parameter for every input sample after sufficient samples are selected. The probability distribution is generated by the combination of output parameter.

In order to verify the applicability of proposed method, three different flow velocities (0.3 m/s, 1 m/s and 3 m/s) are selected. The output parameters of interest in this study are the measurement error $Q_{E0.3}$, $Q_{E1}$ and $Q_{E3}$, and they are flow rate differences between flow rates calculated by...
flow rate calculation model and the standard flow rates given by the water flow standard facility, under three different flow velocities (0.3m/s, 1m/s and 3m/s). Because the input parameters are uncertain, the output parameters are uncertain too. To quantify the uncertainty of the output parameters, the interquartile range (IQR) is defined as the difference between the 25th percentile (P25) and 75th percentile (p75) [11], [15]

\[
IQR = P75 - P25
\]  

(15)

The IQRs of output parameters are function of the COVs of the input parameters. When the COV of one input parameter increases from 0.01 to 0.1, the COVs of other parameters are kept constant at 0.01. In this way, the influences of different input parameters on the output parameters are reflected in the IQR curves. And the influence degrees of different parameters could be compared on the same dimension using the change degrees of IQRs with COVs of the input parameters.

The uncertainty analysis method proposed in this paper is based on the flow rate calculation model, so experimental verification of flow rate calculation model is conducted firstly to ensure the correctness of the analysis results given by uncertainty analysis method.

IV. EXPERIMENTAL VERIFICATION OF FLOW RATE CALCULATION MODEL

A. EXPERIMENT SETUP

In order to verify the flow rate calculation model established in section 2, corresponding experiment were carried out using a flow standard facility. The flow standard facility used in this paper is the water flow standard facility in the Flow Laboratory in National Institute of Metrology of China (NIM), which is used to provide accurate flow rate for calibration of flowmeters [29]. Brief description of the water flow standard facility of NIM is shown in Fig.3, which is mainly composed of storage tank, pump, regulating valve, diverter, weighing system and so on [30].

![FIGURE 3. Brief description of the NIM's water flow standard facility.](image)

After the clamp-on ultrasonic flowmeter is installed on the measuring section of pipe that is made of 304 stainless steel, water is pumped into the pipeline by the pump and enters weighing system through the measuring section of ultrasonic flowmeter at a set flow velocity. In order to ensure the flow stability, two stage pumping systems with double water tanks structure are used. The second water tank is an overflow tank that ensures the inlet pressure of pumps is stable by keeping the water level constant. A centrifugal buffer tank is installed between the pumps and test pipes, which could alleviate the high frequency pulsation caused by the pumps and separate the bubbles from the water. The output results of the weighing system are collected, recorded, and finally compared with the measurement results of the calibrated flow meter, to determine the measurement performance of the calibrated flow meter. The water flow standard facility could provide stable and controllable flow rate with range from 0.03 m$^3$/h to 500 m$^3$/h. The inner diameter of measuring pipe for calibrated flow meter is from 65mm to 200mm. The principle of this facility is static gravimetric method, and its best uncertainty is 0.05%.

The temperature of water in pipeline of the water flow standard facility could be controlled stable within the range 10$^\circ$C~85$^\circ$C using the temperature control system, and the temperature fluctuation is no more than 0.2$^\circ$C. In addition, the water flow standard facility is in laboratory, the indoor environment temperature is stable too (the temperature fluctuation is no more than 1$^\circ$C), and the temperature fluctuation of environment could not cause temperature change of the flowing water.

A commercial clamp-on ultrasonic flowmeter (SCL-83 clamp-on ultrasonic flowmeter) is used, which is manufactured and sold by Tangshan Huizhong Co., Ltd. The measurable velocity range of this clamp-on ultrasonic flowmeter is 0.25$^\circ$~12 m/s, and the best uncertainty is 1%. The diameter range of measurable pipe is DN80 $\sim$ DN1800. The temperature of the measured medium could be 4$^\circ$~90 $^\circ$C, and the temperature of working environment could be $-40\sim+70$ $^\circ$C. In this paper, the inner diameter of measuring section is designed as DN200. The packaged transducers were installed on the outer surface of the pipeline with the coupling agent provided by the manufacturer. Two transducers with central oscillation frequency 1MHz were used in a single channel to form a V-shaped ultrasonic path as shown in Fig.1. The sound speed inside transducer material is 2520 m/s. The designed distance $L$ between front end faces of two transducers is 161.4 mm, and the designed incidence angle in the transducer is 0.733 rad (42$^\circ$). The clamp-on ultrasonic flowmeter is installed on the measuring section of pipe in the water flow standard facility where the installation position of calibrated flowmeter is. There is a long enough straight pipe in front of the measuring pipe section to make the flow at the measurement location being fully developed.

Due to installation error, transducer difference, and differences in the material and size of the pipe at installation location, there is a certain degree of difference between the measured transit-time difference and the transit-time difference under ideal condition, which leads to the measurement error of flow rate. In order to avoid the influence of above aspects as much as possible, four channels were designed and installed with same designed parameters. Schematic diagram and installation photos is shown in Fig.4. Channel #1 and channel #2 were designed and installed symmetrically, and channel #3 and channel #4 were also designed and installed...
symmetrically. By symmetrical design and installation, the influence of the above aspects could well be compared, so as to find out the channel under ideal conditions.

B. EXPERIMENT AND RESULTS

In order to verify the flow calculation model, experiments at velocity of 0.3 m/s, 1 m/s and 3 m/s were conducted, and only the transit-time difference and sound speed of water in the pipe measured by the selected ultrasonic flowmeter are used to calculate the flow rate by the established flow calculation model. For each flow velocity, the experiment begins after the water flow standard facility was running for a period of time and reached steady state. The temperature control system of the water flow standard facility is used to keep the water temperature stable at 26°C, and the indoor environment temperature is 28°C. Three experiments were carried out at each flow velocity, and the time interval between two experiments was 5 minutes. Through the relationship between flow velocity and flow rate in the pipe, the flow rate corresponding to different flow velocity were set to ensure that the facility provides accurate flow rate. For velocity of 0.3 m/s, 1 m/s and 3 m/s, the set flow rates are 33.94 m³/h, 113.13 m³/h and 339.39 m³/h respectively, and corresponding Reynolds numbers are 72074, 240250 and 720740. In each experiment, experiment data of the clamp-on ultrasonic flowmeter was recorded once every 4 seconds, and a total of 40 data were recorded. And the standard flow rate given by the water flow standard facility was also recorded at the same time. Experiment results were recorded, processed, and plotted using MATLAB software.

Based on the experiment results, the average flow rate of each experiment was calculated and used as the flow rate $Q_{\text{mean}}$, and the flow rate given by the water flow standard facility was used as the standard flow rate $Q_{\text{std}}$. And the relative errors of the average flow rates of each experiment with the standard flow rates were also calculated. The relative errors calculated using Eq.(14) are show in Fig.5. For low flow velocity 0.3 m/s, the relative errors of channel #1 and channel #3 are relatively large as −3.05% and 3.29% respectively, while the relative errors of three experiments are in good agreement. For flow velocity 1 m/s, the relative errors of different channels are within −2 % ~ 2 %, such as the relatively large errors are −1.41% from channel #1 and 1.69% from channel #3, and relative errors of the three experiments are very close. For flow velocity 3 m/s, the relative errors of different channels are within −1.2 % ~ 1.2 %, such as the relatively large errors are -0.86% from channel #1 and 1.06% from channel #2.

For different channels, it can be found that the relative errors of channel #2 and channel #4 are relatively small. For channel #1(channel #3) and channel #2 (channel #4) were designed and installed symmetrically, the result difference of channel #1(channel #3) and channel #2 (channel #4) may be caused by the installation error, transducer difference, and/or differences in material and size of the pipe at installation location. For different channels, average the three groups of measurement results, and then the average relative errors are shown in Table. 1. It could be found that the results of channel #2 and channel #4 are good, especially when the velocity is high, and the flow rate can be accurately obtained by the calculation model established. Through experiments, the flow rate calculation model established is verified.

There is difference in measurement results between two symmetrical channels, which may be caused by the installation error, transducer difference, and/or differences in material and size of the pipe at installation location. When the transducers are installed and working under ideal conditions, the flow rate obtained by the flow rate calculation model is
accurate and reliable. At the same time, the influence of above aspects on the results of the uncertainty analysis method proposed is minimized. Therefore, after the comprehensive comparison, it could be considered that channel #2 works in a relatively ideal state, and the transit-time difference and sound speed of water obtained by channel #2 is selected as the parameter in the uncertainty analysis method.

V. RESULTS AND DISCUSSIONS OF UNCERTAINTY ANALYSIS

The output parameter distributions are obtained using the sample-based stochastic model that previously established and the distribution of input parameter \( \theta_0, d, c_0, c_k, \rho, \mu, \) and \( \Delta t \). In order to get the real distribution of the output parameter, a lot of input samples are needed, while large numbers of input samples may lead to massive computation and inefficiency. Convergence analysis is conducted to select a minimum quantity of input samples which can represent the input sample distribution and guarantee steady output distribution.

In the process to obtain the number of input samples \( N_i \), the values of the input parameters used in experiments are set as their nominal mean values: \( \theta_0 = 0.733 \text{ rad}, d = 200 \text{ mm}, c_k = 2520 \text{ m/s}, \rho = 995.95 \text{ kg/m}^3, \mu = 8.292 \times 10^{-4} \text{ pa s}, c_0 \) and \( \Delta t \) are calculated according to the transit time between \( T_A \) and \( T_B \) measured under different velocities, and average value of measurement results is used. For different flow velocities, the measurement results of \( c_0 \) are basically the same as \( 1496.52 \text{ m/s} \), while \( \Delta t \) is different: for \( 0.3 \text{ m/s}, \Delta t_{0.3} \) is \( 5.14 \times 10^{-8} \text{ s} \); for \( 1 \text{ m/s}, \Delta t_1 \) is \( 1.68 \times 10^{-7} \text{ s} \); for \( 3 \text{ m/s}, \Delta t_3 \) is \( 5.09 \times 10^{-7} \text{ s} \).

Due to lack of direct measuring methods or insufficient measurement accuracy, the values of these parameters are difficult to be obtained accurately. The general distribution of each input parameter could be estimated and as reference for setting the COV. According to the general distribution of the parameters, the COV of each input parameter is set to be 0.04.

Through the convergence analysis, it is found that when 1000 samples are used, the mean and the standard deviation of input/output parameters are basically stable, and the mean and the standard deviation of input/output parameters at sample number 1000 is used as the nominal mean value and standard deviation. The relative error between the calculated results (the mean and the standard deviation of input/output parameters) at different sample numbers and the stable value are regarded as fluctuating amplitudes. It could be considered that when the fluctuating amplitude is no more than 2%, the mean and the standard deviation of input/output parameters have converged.

The mean value and the standard deviation of input uncertain parameters are analyzed through the stochastic convergence analysis, and the results are shown in Fig. 6. It could be seen from Fig. 6 that: the fluctuation of mean values decreases very fast, and the mean values converge to be within 1% fluctuating amplitude after \( N_i = 400 \); even there are more than 400 input samples, the standard deviation still fluctuates significantly, because the standard deviation is a higher-order moment and the convergence speed is much slower than the mean values [11], and the standard deviation of \( \theta_0, d, c_0, c_k, \rho, \mu, \Delta t_{0.3}, \Delta t_1 \) and \( \Delta t_3 \) converges within \( 1.06\%, 1.27\%, 0.38\%, 0.38\%, 1.04\%, 1.85\%, 0.02\%, 0.06\% \) and 0.11%, respectively when the sample number \( N_i = 600 \).

The mean value and the standard deviation of output parameters are analyzed through the stochastic convergence analysis, and the results are shown in Fig. 7. It could be seen from Fig. 7 that: the fluctuation of mean values decreases very fast, and the mean values converge to be within 1% fluctuating amplitude after \( N_i = 500 \); the standard deviation converges to be within 2.13% for \( Q_{E0.3}, 2.07\% \) for \( Q_E \) and 2.23% for \( Q_E^3 \) when \( N_i = 500 \), and it is 1.09% for \( Q_{E0.3}, 1.39\% \) for \( Q_E \) and 1.66% for \( Q_E^3 \), within 2% for all output parameters when 600 samples are used. According to stochastic convergence analysis, the number of samples 600 is used and 600 samples is used to conduct IQR analyses.

Typical distributions of the output parameters \( Q_{E0.3}, Q_E, Q_E^3 \) are shown in Fig. 8, and the histograms are presented, which are corresponding to the sample-based stochastic model with 600 samples. According to Eq.(15), the interquartile range (IQR) is defined as the difference between the 25th percentile (P25) and 75th percentile, and it can be found that the variability of the output parameter has the same trend to the IQR. P5, P25, P50, P75, and P95 are used to represent the 5th, 25th, 50th, 75th, and 95th percentiles in the histograms respectively.

The IQRs of \( Q_{E0.3}, Q_E^1 \) and \( Q_E^3 \) as function of the COVs of the input parameters \( \theta_0, d, c_0, c_k, \rho, \mu, \Delta t_{0.3}, \Delta t_1 \) and \( \Delta t_3 \) are presented in Fig. 9, Fig. 10 and Fig. 11. When the COV of one input parameter increases from 0.01 to 0.1, the COVs of other parameters are kept constant as 0.01. In the IQR analysis of \( Q_{E0.3} \) as shown in Fig. 9, the IQR of \( Q_{E0.3} \) significantly increases from 0.911 \text{ m}^3/\text{h} to 6.288 \text{ m}^3/\text{h} when the COV of \( c_k \) increases from 0.01 to 0.1, which shows \( c_k \) has the greatest influence on \( Q_{E0.3} \) compared with other parameters. The IQR of \( Q_{E0.3} \) increases from 0.911 \text{ m}^3/\text{h} to 4.688 \text{ m}^3/\text{h} when the COV of \( d \) increases from 0.01 to 0.1, and the IQR of \( Q_{E0.3} \) increases from 0.911 \text{ m}^3/\text{h} to 4.303 \text{ m}^3/\text{h} when the COV of \( \theta_0 \) increases from 0.01 to 0.1, which shows that \( d \) and \( \theta_0 \) have almost the same influence, and their influences are smaller than that of \( c_k \). The IQR of \( Q_{E0.3} \) increases from 0.911 \text{ m}^3/\text{h} to 4.233 \text{ m}^3/\text{h} when the COV of \( c_0 \) increases from 0.01 to 0.1, which shows that \( c_0 \) could also affect \( Q_{E0.3} \) greatly, and the influence of \( c_0 \) is slightly smaller than influences of \( d \) and \( \theta_0 \). On the contrary, the COVs of other input parameters such as \( \rho, \mu \) and \( \Delta t_{0.3} \) are relatively less important to IQR of \( Q_{E0.3} \) which indicates that these parameters have relatively
FIGURE 6. Stochastic convergence analysis of mean value and standard deviation of input parameters: (a) $\theta_0$; (b) $d$; (c) $c_0$; (d) $c_k$; (e) $\rho$; (f) $\mu$; (g) $\Delta t_{0.3}$; (h) $\Delta t_1$; (i) $\Delta t_5$. 
less influence on $Q_{E0.3}$. Similarly, in the IQR analysis of $Q_{E1}$ and $Q_{E3}$ shown in Fig.10 and Fig.11 respectively, it could obtain the same result as the IQR analysis of $Q_{E0.3}$.

Through the comparison of the IQR analysis results of different flow velocities, it is found that the analysis results for three flow velocities are consistent, and there is no difference in the results due to change of flow velocity.

According to the IQR analysis results of the uncertain parameters, it could be found that: sound speed inside transducer material $c_k$, incidence angle $\theta_0$, inner diameter $d$ and sound speed of fluid $c_0$, have relatively large influence, and the deviation of these parameters may lead to large measurement error. Among these influential parameters, the impact of $c_k$ is the largest; $d$ and $\theta_0$ have almost the same impact, and their impacts are smaller than that of $c_k$; the impact of $c_0$ is slightly smaller than impacts of $d$ and $\theta_0$. On the contrary, influence of parameter $\rho$, $\mu$ and $\Delta t$ are relatively small. For the velocity of 0.3 m/s, 1 m/s and 3 m/s, the results of IQR analysis are consistent.

According to the IQR analysis results, influential parameters $c_k, \theta_0, d, c_0$ are studied by the flow rate calculation model,
and the measurement error caused by the deviation of single parameter is shown in Fig. 12.

With the increase of \( c_k \) deviation, the measurement error increases nearly linearly. The error caused by deviation of 1 m/s (0.04% of designed value) is 0.017 m\(^3\)/h, 0.055 m\(^3\)/h, 0.168 m\(^3\)/h when the velocity is 0.3 m/s, 1 m/s and 3 m/s respectively. The error introduced by \( c_k \) deviation is relatively large when the velocity is larger. Therefore, the acoustic material in the transducer must have good acoustic stability, otherwise it may cause large \( c_k \) deviation and then cause large measurement error.

With the increase of the \( \theta_0 \) deviation, the measurement error tends to decrease linearly, and the error caused by deviation of 0.1 °C (0.24% of designed value) is −0.083 m\(^3\)/h, −0.271 m\(^3\)/h, −0.826 m\(^3\)/h for velocity 0.3 m/s, 1 m/s and 3 m/s respectively. The deviation of incidence angle \( \theta_0 \) depends on the installation error of the distance \( L \) and the stability of the sound speeds. Therefore, the installation distance error should be minimized while the sound speeds keep stable.

For the inner diameter \( d \), the measurement error linearly increases with the increase of the inner diameter deviation, and the error caused by deviation of 1 mm (0.50% of designed value) is 0.178 m\(^3\)/h, 0.586 m\(^3\)/h, 1.783 m\(^3\)/h for velocity 0.3 m/s, 1 m/s and 3 m/s respectively. So the measuring section of the pipeline should be convenient for accurate measurement of the inner diameter, the thermal expansion of the measuring pipe material should be small, and the solid particles in the liquid medium should not be adsorbed on the inner surface of the pipeline.

The change of sound speed \( c_0 \) will also cause large measurement error, and the error caused by deviation of 1 m/s (0.07% of designed value) is 0.019 m\(^3\)/h, 0.063 m\(^3\)/h, 0.193 m\(^3\)/h for velocity 0.3 m/s, 1 m/s and 3 m/s respectively. So it is necessary to measure the sound speed of fluid accurately and the temperature of the measured fluid should not change too much to keep the sound speed of fluid stable.

VI. CONCLUSION

In this paper, for a clamp-on ultrasonic flowmeter with V-shaped ultrasonic path, the flow rate calculation model with line-averaged flow velocity correction is established and verified by experiments, and a methodology of systematic analysis incorporating the parametric uncertainty distribution is proposed to analyze the influences of multiple parameters.
with uncertainty on measurement error by establishing the sample-based stochastic model. It could be found from the results that sound speed inside transducer material, incidence angle of the ultrasonic transducer, inner diameter of pipe, sound speed of fluid in the pipe, have relatively large influence, and the deviations of these parameters may lead to large measurement error. On the contrary, influences of density of fluid, dynamic viscosity of fluid and transit-time difference are relatively small.

Among the influential parameters, the influence of sound speed inside transducer material is the largest, and sound speed should keep stable by using material with good acoustic stability; the installation distance error should be minimized to make the deviation of incidence angle small; the inner diameter of the measuring pipe section should be stable and easy to be measured accurately; the temperature of the measured fluid should not change too much that may result in great change in the sound speed of fluid. The results reveal the different influences of multiple parameters with uncertainty, which could be used as reference for performance improvement and error analysis of clamp-on ultrasonic flowmeter in industrial local measurement.

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