Experimental Realization of Classical $\mathbb{Z}_2$ Spin Liquids in a Programmable Quantum Device

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We build and probe a $\mathbb{Z}_2$ spin liquid in a programmable quantum device, the D-Wave DW-2000Q. Specifically, we observe the classical 8-vertex and 6-vertex (spin ice) states and transitions between them. To realize this state of matter, we design a Hamiltonian with combinatorial gauge symmetry using only pairwise-qubit interactions and a transverse field, i.e., interactions which are accessible in this quantum device. The combinatorial gauge symmetry remains exact along the full quantum annealing path, landing the system onto the classical 8-vertex model at the endpoint of the path. The output configurations from the device allows us to directly observe the loop structure of the classical model. Moreover, we deform the Hamiltonian so as to vary the weights of the 8 vertices and show that we can selectively attain the classical 6-vertex (ice) model, or drive the system into a ferromagnetic state. We present studies of the classical phase diagram of the system as function of the 8-vertex deformations and effective temperature, which we control by varying the relative strengths of the programmable couplings, and we show that the experimental results are consistent with theoretical analysis. Finally, we identify additional capabilities that, if added to these devices, would allow us to realize $\mathbb{Z}_2$ quantum spin liquids on which to build topological qubits.

Introduction – Quantum spin liquids (QSLs) have a long history. They were first proposed in the 1970s by Anderson1 as an alternative to the spin 1/2 Néel antiferromagnetic state, and later as candidates for the insulating parent state of the high-temperature superconductors2. QSLs do not display magnetic symmetry-breaking order, but instead display topological order3. They are also closely related to lattice gauge models in particle physics, also dating to the 1970s4–6. Today, a QSL model known as the toric code is a potential platform for topological computing7. There have been many proposed materials, but to-date gapped QSLs have not been unambiguously observed in nature8–12. Since they are so hard to find in materials, a more recent idea is to build a QSL synthetically out of superconducting circuits13–15. In parallel, another recent idea has emerged to simulate familiar quantum phase transitions on programmable devices16,17. In this paper we take these ideas one step further and show that, in principle, a programmable device can be used to emulate QSL phases so far unreachable by other means, a step towards realizing logical topological qubits in these same devices. While we cannot observe the full quantum regime due to the limitations of the current device, we do observe unmistakable signatures of the phase in its classical limit at the endpoint of the quantum annealing protocol. In the process of doing so, we identify additional features that a programmable device of this sort would need in order to realize QSLs. It is a testament to technological progress that a handful of theorists can observe and experiment with new physics while being equipped only with remote access to a commercial device.

From a theoretical perspective, our framework requires a new general construct: “Combinatorial Gauge Symmetry”18. This is an exact and non-perturbative symmetry that stabilizes the topological phase for a wide range of parameters. From a practical perspective, it enables the programming of QSLs in D-Wave because it requires at most two-body Ising spin interactions. The vast majority of theoretical QSL models rely on pure multi-spin interactions, which heretofore are not attainable. The notable exception is the Kitaev hexagonal model19, but it requires XX, YY and ZZ interactions simultaneously, which have also been unattainable. In our case the model that we program is the $\mathbb{Z}_2$ lattice gauge theory. The toric code7, which is a central model for topological quantum computing, is a special limit of the $\mathbb{Z}_2$ model.

The D-Wave DW-2000Q quantum annealer comprises a superconducting circuit which implements programmable ZZ-couplings between pairs of spins (qubits): $J_{ij}\sigma^z_i\sigma^z_j$. Programmability means that the $J_{ij}$ coefficients can be specified, although the available couplings are constrained by the connectivity of the device. A transverse field $\Gamma$ is applied to each spin, coupling to the $x$-component of the spins, $\sigma^x_i$, inducing spin flips. The machine can be operated with the forward annealing protocol described in the device documentation20, in which all active spins are initialized in the ground state of a Hamiltonian with a high transverse field $\Gamma$ that is gradually decreased to zero, while the $J_{ij}$ couplings are slowly increased from zero to their specified final values. At the endpoint of the protocol the system should land in a ground state of the classical Hamiltonian associated with the $J_{ij}$’s if a dynamical obstruction (such as a glass transition) is not encountered.

In this work, we program the $J_{ij}$ coefficients so that the spectrum represents a system with exact $\mathbb{Z}_2$ gauge symmetry. We observe and study the Hamiltonian in the classical regime at the endpoint of the annealing path, and map out its phases, which include both 8- and 6-vertex
The following Hamiltonian is of the order of four device qubits, so the effective transverse field on an effective gauge qubit, a transverse field must flip a total of four spins. Each gauge qubit is coupled to its neighboring matter qubits with strength $J$ within the unit (Chimera) cell of the device. Second, we utilize four strongly-coupled qubits, $K$ proportional to $J$ within the unit cell of the device. These four spins (two blue and two orange) reside on the bonds. The matter spins $\mu_a$ reside on the vertices (blue dots) of the effective lattice.

The embedding as described above is a realization of the toric code, while the 6-vertex phase is also known as planar spin ice. Both planar and three-dimensional spin ice have been observed; in our construction, we observe planar ice as a special case. In contrast, to our knowledge, neither the classical nor quantum 8-vertex models have been observed. We synthesize them here and probe the classical version.

**Model and Embedding** – The model is based on a square lattice, where we place “gauge” qubits on each link and four “matter” qubits at each vertex. The embedding of this geometry within the available D-Wave architecture is shown in Fig. 1. First, each gauge qubit is coupled to its neighboring matter qubits with strength proportional to $J$ within the unit (Chimera) cell of the device. Second, we utilize four strongly-coupled qubits, with strength $K$, to effectively act as one gauge qubit, which is required because of the constrained coordination number of the Chimera architecture. To flip an effective gauge qubit, a transverse field must flip a total of four device qubits, so the effective transverse field on the gauge qubits is of the order $\Gamma^5 \sim \Gamma^4/K^3$.

The embedding as described above is a realization of the following Hamiltonian

$$H = -\sum_s \left[ J \sum_{a \in s} W_{a} \sigma^x_i \mu^x_a + \Gamma \sum_{a \in s} \mu^x_a \right] - \Gamma \sum_{i} \sigma^x_i,$$

where $\sigma_i$ and $\mu_a$ are Pauli matrices representing the gauge and matter spins, respectively, around each vertex $s$. The $4 \times 4$ interaction matrix,

$$W = \begin{pmatrix} -1 + \eta & 1 & 1 & 1 \\ 1 & 1 - \eta & 1 & 1 \\ 1 & 1 & -1 + \eta & 1 \\ 1 & 1 & 1 & -1 + \eta \end{pmatrix},$$

encodes the ferromagnetic/anti-ferromagnetic interactions between gauge and matter spins that are programmed into the device.

At the special point $\eta = 0$, $W$ is a $4 \times 4$ Hadamard matrix, which has the property that $W^T W = 4I$ and every entry is equal to $\pm 1$. This point is characterized by a local symmetry which is generated by a group of monomial matrices that represent flips of single spins as well as permutations among matter spins on each vertex. This local combinatorial symmetry ensures that the Hamiltonian obeys a $\mathbb{Z}_2$ gauge symmetry for any value of $J$, $\Gamma$ and $\bar{\Gamma}$. At zero-temperature, for $\Gamma$ and/or $\bar{\Gamma}$ small, the model is in its topological phase, as schematically depicted in Fig. 2(a). The energy gap of this QSL phase is on the order of $10^{-3} - 10^{-2}J$, and the temperature of the device renders the gap unobservable. (We note that dynamics in this regime may still reveal coherent propagation of quasiparticles, see Ref. 25 for a theoretical discussion.) While the QSL is inaccessible in the current devices, the annealing cycle in D-Wave, designed to adiabatically transform the system from a region of finite to...
there are two effective ones, $\Gamma$ and $\eta$. The system allows only one transverse field, however Hamiltonian obeys this symmetry along the entire annealing path. Eq. (1) for $\eta = 0$. The annealing path goes from the paramagnetic to the quantum spin liquid with $\mathbb{Z}_2$ symmetry. The Hamiltonian obeys the combinatorial $\mathbb{Z}_2$ symmetry throughout the entire quantum annealing path, minimizing the number of defects at the point of observation. The ground states at zero transverse field and $\eta = 0$ are the same as that of the 8-vertex model$^{27}$, in which the number of up and down gauge qubits around any vertex is an even number (even parity star), thus producing the loop structure associated with the $\mathbb{Z}_2$ spin liquid state. In the Supplementary Material$^{27}$ we show the symmetries and ground state degeneracy explicitly.

For $\eta \neq 0$, the $W$ matrix is deformed away from the point of combinatorial gauge symmetry, allowing us to access other states. Non-zero $\eta$ splits the 8 vertices into groups of 2 and 6 vertices, as shown in Fig. 2(b), where up and down gauge qubits are pictured as black and gold links, respectively (the configurations of the matter qubits are tethered to those of the gauge qubits in the ground state). The energy separation is $\delta = 4J\eta$; $\eta < 0$ favors the 6-vertex model and $\eta > 0$ favors the ferromagnetic states, where all spins in a star are up or down. The topological phase $\eta = 0$ sits at a zero-temperature critical point.

At non-zero temperature, vertices with odd parity are allowed but exponentially suppressed at low temperatures. This enables us to study the phases of the classical 8-vertex model, whose theoretical phase diagram is shown in Fig. 2(c). In our programmed Hamiltonian we vary $\eta$ and $J$, but since the operating temperature of the device is fixed, we are effectively varying $\eta$ and $J/T$. This corresponds to varying the relative Boltzmann weight (fugacity) between the 6- and 2-vertex states, $d = e^{\delta/T} = e^{4J\eta/T}$. The 8-vertex spin liquid phase is stable in the range $0 < d < 3^{28,29}$. In this region there is a continuously varying imbalance between the 6- and 2-vertex types. The system transitions to an ordered ferromagnet at $d = 3$, and for $d > 3$ it remains magnetized, settling to one out of the two 2-vertex states. Strictly speaking, the $d = 0$ limit exists at zero temperature only, where $\eta < 0$ and we have the 6-vertex, or spin ice, phase. In the experiments, however, there is a small range of temperatures where it occurs because the system size is finite.

**Results** – The experiments were run on the D-Wave DW-2000Q machine at Los Alamos National Laboratory (LANL). The $J$ and $K$ couplings are programmed into the D-Wave’s Chimera architecture according to the embedding in Fig. 1. The ratio $J/K$ is fixed to 1/2 in the experiments, which allows the ghost qubits to be coupled strongly while allowing sufficiently broad range for varying $J$. The operating temperature of the device is approximately 12 mK, but by scaling $J$ and $K$ (by the same factor) we effectively control the ratio $T/J$. The lowest temperature in our measurements corresponds to choosing the value $J_{\text{API}}^{\text{max}} = 1/2$ in the Application Programming Interface (API) for the D-Wave DW-2000Q$^{20}$, by decreasing the input variable to $J_{\text{API}}^{\text{max}} = J_{\text{max}}/\alpha$, where $\alpha$ is the factor. The factor can be expressed as $\alpha = \alpha T/J$, where $\alpha$ is the dimensionless ratio between the maximum physical coupling $J_{\text{max}}$ (in units of mK) to the device temperature when $J_{\text{API}}^{\text{max}} = 1/2$. We calibrate $\alpha$ by fitting the spectrum of the Hamiltonian for independent stars, and find it to be approximately 15 (see Supplementary Material$^{27}$).

The total annealing time in the experiments is chosen to be 2000 $\mu$s, with a pause from 20 $\mu$s to 1000 $\mu$s...
FIG. 3. Four typical configurations of final states returned by the device at the endpoint of the annealing cycle, obtained at base temperature ($\tau = 1$) for different values of the parameter $\eta$. The color of the bonds represent the state of the gauge spin (associated to that of the four strongly coupled qubits on the bonds, see Fig. 1(c)): up gauge spins are colored in gold, and down spins are colored in black. The matter spins, which are tethered to the gauge spins at low energies, are omitted in the picture. The blank areas indicate punctures on the lattice due to the inactive qubits in the machine. (a) at $\eta = 0$, a $\mathbb{Z}_2$ spin liquid state is observed where closed loops or connected strings of either spin up or down are formed, and can intersect; (b) at $\eta = -0.2$, a spin ice state (or 6-vertex model) is obtained where similar loops and strings are formed, but do not intersect. (c) at $\eta = 0.2$, system fully magnetizes; and (d) at $\eta = 0.2$, two ferromagnetic domains with opposite magnetization are separated by a domain wall.

FIG. 4. Densities of all 8-vertex types for $\eta$ varying from $-0.25$ to $0.25$, at base temperature ($\tau = 1$). The data clusters according to three different families of stars, indicated next to the curves. At $\eta = 0$ all 8-vertex types appear with equal densities, consistent with the presence of the combinatorial gauge symmetry. The 6-vertex types are favored for $\eta < 0$, and the 2-vertex types for $\eta > 0$.

We observe the $\mathbb{Z}_2$ spin liquid in the expected range of parameters, with an example at $\eta = 0$ in Fig. 3(a). At each vertex, or star, the parity of the qubits at the links is even, and each of the allowed 8-vertex configurations occurs with equal proportion (see Fig. 4). Strings are visible in Fig. 3(a) by following a gold or black line. These strings either form closed loops in the bulk, or are open but terminate either at the external boundary or at the internal holes formed by the cluster of inactive qubits. We also observe the spin ice, or 6-vertex model, when $\eta < 0$, as shown in Fig. 3(b). The parity constraints are satisfied but the 2-vertex configurations are suppressed resulting in non-intersecting strings. Figs. 3(c) and (d) show ferromagnetic configurations, where the up/down symmetry is broken and there is an imbalance between the two 2-vertex types. In (c) the fully magnetized state is reached, while in (d) the system breaks into two domains with opposite magnetization and separated by a domain wall.

We gather 1000 sample configurations for each of 51 values of $\eta$ equally spaced in the window $-0.25 \leq \eta \leq 0.25$ and 31 values of $\tau$ equally spaced in the range $1 \leq \tau \leq 2.5$. The data accumulated over these $51 \times 31 \times 1000 = 1581000$ samples can be aggregated to yield evidence that the 8-vertex constraint is satisfied in this window of the $\eta - \tau$ plane. Violations of the 8-vertex constraints take two forms in this embedding: (a) an odd parity star, or (b) a broken gauge link where the four physical qubits representing it are not all aligned. Out of the $51 \times 31 \times 1000 = 1581000$ samples gathered, 1007751 samples (or 63.7%) had no defects in any of the stars or links comprising the system.
FIG. 5. Heat plots (a) and (b) show the observables $\rho_2$ and $m^2$ for $\eta$ varying from $-0.25$ to $0.25$ in intervals of $0.01$, and $\tau$ varying from $1$ to $2.5$ in intervals of $0.05$. The dependence of $\rho_2$ on the slopes $\tau/\eta$ of lines on the $\eta - \tau$ plane can be observed in (a). Map (b) shows that the magnetization vanishes in the spin liquid region (black region), and continuously grows past the transition to the ferromagnetic state (purple and yellow regions). (c) shows $\rho_2$ and $m^2$ as a function of $\eta/\tau$. The collapse of the data shows that $\rho_2$ and $m^2$ are controlled by a single parameter, the ratio $\eta/\tau$.
The gauge symmetry of the Hamiltonian in Eq. (1) at \( \eta = 0 \) is the result of the following property of the matrix \( W \):

\[
L^{-1} W R = W, \tag{3}
\]

where \( R \) and \( L \) are monomial matrices. \( R \) and \( L \) can be viewed as transforming the gauge and matter spins, respectively, on each vertex. For example, the following pair satisfies Eq. (3) on each site:

\[
L = \begin{pmatrix}
0 & 1 & 0 & 0 \\
+1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 \\
0 & 0 & -1 & 0
\end{pmatrix}, \\
R = \begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & +1 & 0 \\
0 & 0 & 0 & +1
\end{pmatrix}. \tag{4}
\]

The two key points are that the matrix transformation \( R \) is diagonal with an even number of \(-1\)'s, and that \( L \) is uniquely determined by \( R \) as a result of the constraint Eq. (3). This symmetry holds for any transverse field \( \Gamma \), i.e., throughout the entire annealing cycle. In the special case when the system is in the classical limit, at the end of the annealing cycle (\( \Gamma = 0 \)), the 8 degenerate ground states are shown in Table I. Each of these states has energy \(-8J\).

| Index | \( \sigma_1^z \) | \( \sigma_2^z \) | \( \sigma_3^z \) | \( \mu_1^z \) | \( \mu_2^z \) | \( \mu_3^z \) |
|-------|-----------------|-----------------|-----------------|-------------|-------------|-------------|
| 1     | -1              | -1              | 1               | 1           | 1           | -1          |
| 2     | -1              | 1               | -1              | 1           | 1           | -1          |
| 3     | -1              | 1               | -1              | 1           | 1           | -1          |
| 4     | 1               | -1              | -1              | 1           | 1           | 1           |
| 5     | 1               | -1              | -1              | -1          | 1           | 1           |
| 6     | 1               | 1               | -1              | -1          | 1           | 1           |
| 7     | 1               | 1               | 1               | 1           | 1           | 1           |
| 8     | -1              | -1              | -1              | -1          | -1          | -1          |

TABLE I. The 8 degenerate ground states at \( \eta = 0 \) and \( \Gamma = 0 \) at each vertex in the lattice. The matter spins are tethered to the gauge spins. Note the non-trivial structure; when all gauge spins are up/down the matter spins are in the same direction as the gauge spins, but when only two gauge spins are up/down the matter spins are opposite.

**Defect Analysis**

Two types of defects can occur in our embedding of the 8-vertex model: (a) a negative parity star where the constraints imposed by the \( J \) coupling and \( W \) matrix are not respected (blue lines in Fig. 1 (a)); or (b) a broken gauge link where the four physical qubits representing a single gauge qubit do not align (red lines in Fig. 1 (a)). The measurements were taken at each bin across the entire phase diagram in Fig. 5, with \( \tau \) swept from 1 to 2.5 in intervals of 0.05 and \( \eta \) from \(-0.25\) to 0.25 in intervals of 0.01. For each bin, we collect 1000 samples and determine the fraction of samples that have: 1 or more defects; 2 or more defects; and 3 or more defects. The data for these three cases are shown as heat plots in Fig. 6. In our experiments, we applied spin reversal transformations on a set of spins to reduce the systematic biases in the D-Wave superconducting qubits. The total number of active stars and gauge links in one sample are \( N_{\text{stars}} = 114 \) and \( N_{\text{links}} = 183 \).

In the \( \eta < 0 \) region, in the low temperature regime, we rarely observe any defect. This is the region of the phase diagram where the spin liquid is observed. At high temperatures, the \( \eta < 0 \) region has at most 58% defects, and this fraction of samples with defects mostly has a single defect. The \( \eta > 0 \) region contains higher fractions of defects, which we attribute to the formation of domain walls in the ferromagnetic state. The highest defect fraction occurs in the \( \eta > 0 \) region at high temperatures, reaching as high as 84% of the samples. Other than in this corner of the plot, most samples rarely have more than 2 defects.

If we aggregate all bins, as stated in the main text, there are in total \( 31 \times 51 \times 500 = 1581000 \) samples, out of which 1007751 (63.7%) have 0 defects, 374217 (23.7%) have 1 defect, 146865 (9.3%) have 2 defects, and 38857 (2.5%) have 3 defects.
FIG. 6. Heat plot of the measured fraction of the 500 samples in each bin in the $\eta - \tau$ plane that have: (a) 1 or more defects; (b) 2 or more defects; and (c) 3 or more defects. The bins were constructed with $\tau$ varying from 1 to 2.5 in intervals of 0.05 and $\eta$ varying from $-0.25$ to $0.25$ in intervals of 0.01.

Temperature Calibration

Although the operating temperature in D-Wave is approximately 12 mK, by scaling the coupling $J$ (with $K/J = 1/2$ fixed) we effectively control the ratios $T/J$ and $T/K$ in the experiments. The lowest temperature corresponds to choosing the value $J^\text{API}_{\text{max}} = 1/2$, and the temperature is increased by decreasing the input value $J^\text{API} = J^\text{API}_{\text{max}}/\tau$, where $\tau$ increases from 1 to 20 in intervals of 1 in the temperature calibration experiments. This factor $\tau$ can be expressed as

$$\tau = \frac{J^\text{API}_{\text{max}}}{J^\text{API}} = \frac{T/J}{T/J^\text{max}} = \frac{1}{T/J^\text{max}} \frac{T}{J} = \frac{\alpha T}{J},$$

where $\alpha$ is a dimensionless ratio between the physical $J^\text{max}$ (in units of mK) to the device temperature when $J^\text{API}_{\text{max}} = 1/2$. The value of $\alpha$ can be pinpointed by calibrating the experimental density of defects $\rho(\tau)$ with the analytical probability of defects $p(T/J)$ calculated directly from the model. We calibrate $\alpha$ with two variations of the model where the theoretical probability of defects is easy to calculate: (a) independent stars where $J^\text{API} = J^\text{API}_{\text{max}}$ and $K^\text{API} = 0$ (or $J = J^\text{max}$ and $K = 0$); and (b) independent gauge links where $J^\text{API} = 0$ and $K^\text{API} = K^\text{API}_{\text{max}} = 1$ (or $J = 0$ and $K = K^\text{max}$).

The experimental density of defects $\rho(\tau)$ is defined as the number of defective independent stars/links divided by the total number of stars/links in a sample, averaged over 500 samples. The analytical probability of defects is
given by \( p(T/J) = \frac{Z_{\text{defects}}}{Z} \), where \( Z \) is the partition function, and \( Z_{\text{defects}} \) is the partition function computed using only the defective states. An independent star, built out of 4 gauge spins and 4 matter spins, has \( 2^8 \) possible spin configurations with energies determined by the Hamiltonian \( H_s = \frac{1}{2} \sum_{a=1}^{4} \sum_{i=1}^{4} W_{ai} \sigma_i^a \sigma_i^b \), where \( W_{ai} \) is the interaction matrix between gauge spins \( \sigma_i \) and matter spins \( \mu_a \) defined in Eq. (2). From the single star (classical) Hamiltonian \( H_s \) we find the energy levels for all the \( 2^8 \) spin configurations. We then construct the partition function and compute the \( T/J \) dependence of the probability that a star has negative parity given the four gauge spins, for \( \eta = 0, 0.4, \) and \( -0.4 \). We compare the theoretical functions \( p(T/J) \) to the experimental data \( \rho(\tau) \) for \( \tau \) varying from 1 to 20, shown in Figs. 7 (a), (b), and (c). To illustrate this calculation, consider the case \( \eta = 0 \). The full spectrum of the Hamiltonian in Eq. (1) with \( K = \Gamma = \tilde{\Gamma} = 0 \) is straightforward to compute, and is shown in Table II: The corresponding density of negative parity stars is given by:

\[
p(T/J) = \frac{8e^{4J/T} + 8e^{-4J/T}}{e^{8J/T} + 12e^{4J/T} + 6 + 12e^{-4J/T} + e^{-8J/T}}.
\]

The other cases are calculated similarly. Using a least squares fit for the parameter \( \alpha \), we find that \( \alpha = 15.5 \) for \( \eta = 0 \), \( \alpha = 14.9 \) for \( \eta = 0.4 \) and \( \alpha = 15.5 \) for \( \eta = -0.4 \).

A similar analysis can be done for the case of independent gauge links. An independent gauge link consists of 4 physical spins interacting via an Ising-type Hamiltonian \( H_l = -1 \sum_{i=1}^{3} s_i s_{i+1} \). Among \( 2^4 \) states in total, the defective states correspond to the configurations where the four spins are not all aligned. We again construct the \( T/J \) dependence of the probability that a gauge link is defective, and find that \( \rho(\tau) \) fits \( p(T/J) \) for \( \alpha = 15.2 \), shown in Fig. 7 (d).

In conclusion, we fitted the defects density of the D-Wave data to the theoretical defects probability calculated directly from the model, and calibrated the value of \( \alpha \) that relates the experimental temperature control parameter \( \tau \) to the physical ratio \( T/J \). The fact that \( \alpha \) is close and approximately the same in all the cases above serves as another indication of robustness in our measurements.

| Energy | States (P=+1) | States (P=-1) | Total No. of States |
|--------|---------------|---------------|---------------------|
| 8J     | 8             | 0             | 8                   |
| 4J     | 32            | 64            | 96                  |
| 0J     | 48            | 0             | 48                  |
| -4J    | 32            | 64            | 96                  |
| -8J    | 8             | 0             | 8                   |

TABLE II. The full spectrum and degeneracies of the Hamiltonian in Eq. (1) with \( K = \Gamma = \tilde{\Gamma} = 0 \) for the two different parities. The total number of states is \( 256 = 2^8 \).

FIG. 7. Temperature calibration for two variations of the model. In (a), (b), and (c) we disconnect all the stars and form a configuration of 114 independent stars by setting \( K^{API} = 0 \). The value of \( \eta \) is set to 0 in (a), 0.4 in (b), and -0.4 in (c). In (d) we turn off the coupling \( J \) by setting \( J^{API} = 0 \), and analyze the ferromagnetic chain of 4 physical qubits forming a gauge spin. We use the least squares to fit the analytical probability of defects \( p(T/J) \) to the experimental density of defects \( \rho(\tau) \), and extract the value of \( \alpha \) that relates the experimental temperature control parameter \( \tau \) to the physical ratio \( T/J \).