Estimation of geometrical shapes of mass-formed nuclei (A=102-178) from the calculation of deformation parameters for two elements (Sn & Yb)

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Abstract. The present research focused on the studying of even-even nuclei forms for elements with mass numbers greater than 100 (A > 100) for (¹⁰²⁻¹³⁴Sn &¹⁵²⁻¹⁷⁸Yb) isotopes. Which included the study of deformation parameters (β²) derived from the Reduced Electric Transition Probability B(E2)↑ based on the energy of the first Excited State (2⁺), and distortion parameter (δ) from Intrinsic Electric Quadrupole Moments (Q₀). Roots Mean Square Radii <r²>↑/2 were also calculated and compared with theoretical values. The diversity of nuclei forms for selected isotopes and their differences was observed by plotting three-dimensional shapes (axially symmetric) in addition to drawing two-dimensional shapes of single element isotopes to distinguish between them by using semi-major (a) and semi-minor (b) axes.

1. Introduction

The atomic nucleus mirrors condition is the protons and neutrons shell structure which are the formation of it with regard that the shells are totally filled, it has been discussed a "magic" ball-shaped nucleus. Most nucleuses may have an orientation for being deformed on the grounds that their shells are partially filled off. The most commonly experienced shapes are elongated (prolate) or prostrated (oblate) as shown in figure (1-1); shapes can change from nucleus to another respectively by aggregation or dislocating a proton or neutron. It is appropriate in some cases to rearrange protons or neutrons at the same nucleus so that the shape can be changed. Thus, nucleus selfsame can suppose several shapes alike different energy states. When the states approach in energy (one thousand of the nucleus's obligated energy), due to quantum mechanics laws, these various shapes can be mixed and nucleus may get along with different shapes.
2. Theoretical

2.1. Nuclear Shape
The nuclear shape is generally spherical when nuclei are stable. This attempt is to lower the surface energy. However, small parts from spheres are observed, such as, in the area $150 < A < 190$. These deformations can only be quantified by using the ratio [1]:

$$\delta = \frac{\Delta R}{R}$$

Where:
$R$ = The nuclear radius average
$\Delta R$ = The difference between semi-minor and semi-major axes.

$$\Delta R = (b - a)$$ (2)

For a sphere $\Delta R = 0$.

2.2. Nuclear Surface Deformations:
The collective motion can be explained as nuclear surface vibrations and rotations in the geometrical collective model that was firstly suggested by Bohr and Mottelson [2], where a nucleus modeled like a charged liquid drop and the moving nuclear surface may be expressed quite generally by an extension in spherical consistent with time-dependent shape parameters that are considered as coefficients [3,4]:

$$R(\theta, \phi, t) = R_{av} [1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu}(t)Y(\theta, \phi)]$$ (3)

Where:
$R(\theta, \phi, t)$: Indicates the nuclear radius in the direction $(\theta, \phi)$ at time $t$ as shown in figure (1.2),
$R_{av}$: The average nucleus radius.
$\alpha_{\lambda\mu}$: Are the deformation variables.
$\lambda$: determines the multipole or mode of nuclear motion.
$\mu$: is the projection of $\lambda$ on the z-axis.
$Y(\theta, \phi)$: is the spherical harmonic.

Figure (1.1). A diagrammatic representation of three of a nuclear shapes (a) Spherical, (b) Oblate, (c) Prolate. The x-axis donates to the symmetry axis of the oblate and prolate shapes.
Figure (1.2). A vibrating nucleus with a spherical equilibrium shape. The time-dependent coordinate $R(t)$ locates a point on the surface in the direction $(\theta, \phi)$[4].

The quadrupole deformation parameter $\beta_2$, is related to the spheroid axes [5]:

$$\beta_2 = \frac{4}{3} \sqrt{\frac{\pi}{5}} \frac{\Delta R}{R_{av}} = 1.06 \frac{\Delta R}{R_{av}}$$

Where:
The average radius $R_{av} = R_0 A^{1/3}$.
$\Delta R$: The difference between both of the semi-major and minor axes. As long as the value of $\beta_2$ is larger, the nucleus becomes more disfigured.

2.3. The Root Mean Square Charge Radius (Isotopes Shift)
The root mean square (rms) nuclear charge radius $R = \langle r^2 \rangle^{1/2}$, with one another nuclear ground-state properties, is considered the key nuclear materials information which refer to stated nuclear structure effectiveness, for instance: shell closures and a deformation starting. [6].

The root mean square (rms) radius, $\langle r^2 \rangle^{1/2}$, is deduced directly from the distribution of scattered electrons; for a uniformly charged sphere, the squared charge distribution radius $\langle r^2 \rangle$ [8,4]:

$$\langle r^2 \rangle = \frac{3}{5} R^2 = \frac{3}{5} R_0^2 A^{2/3} > 100$$

Where:
$A$: Mass number
$R$: is the radius of the sphere
$A$: Mass number
$R = R_0 A^{1/3}$.

2.4. Electric Quadrupole Moment
The charge allocation in a nucleus can be described in terms of electric multipole moments and pursued from the classical electrostatics thoughts [9]. Several nuclei have constant quadrupole
moments which may experimentally be measured. These nuclei are expected to have oval shape with a symmetrical axis. This proposition, has classically led to define the intrinsic quadrupole moment as the following equation [10]:

\[ Q_0 = \int d^3 \rho(r) (3z^2 - r^2) \]  

(6)

Where
\( \rho(r) \) : Radial charge density of the proton.
\( r \): Charge radius.

If \( Q_0 \) is consider to be calculated for a homogeneously charged ellipsoid with charge Ze and semi-axes (a) and (b). With (b) pointing along the z axis, \( Q_0 \) will be[11]:

\[ Q_0 = \frac{2}{5} Z (a^2 - b^2) \]  

(7)

If the deviation from sphericity is not very large, the average radius: \( R = 1/2 (a + b) \) and \( \Delta R = (b - a) \) from equation (2) can be presented and with \( \delta = \Delta R/R \), from equation (1), the quadrupole moment is[11]:

\[ Q_0 = \frac{4}{5} Z R^2 \delta \]  

(8)

The nucleus quadrupole distortion parameter values \( \delta \) calculated from the equation[12]:

\[ \delta = 0.75 Q_0 / (Z(r^2)) \]  

(9)

The semi-axes (a) and (b) are gained from the two following equations [13].

\[ a = \sqrt{\langle r^2 \rangle (1.66 - \frac{2\delta}{0.9})} \]  

(10)

\[ b = \sqrt{5 \langle r^2 \rangle - 2a^2} \]  

(11)

2.5. Quadrupole Deformations

In general, nuclei with Z or N far from a magic number are deformed. The so-called quadrupole is the most ordinary deformations where the nucleus may have a prolate (rugby ball) or oblate (cushion) shape, as shown in ‘figure 1’. A quadrupole deformation holds one symmetry axis (z axis) [14].

It is notorious that the axially symmetric deformed nucleus shape is explained by the deformation parameter \( \beta_2 \) which is connected to the quadrupole moment \( (Q_0) \) and represents the homogeneous charge distribution [15,16]:

\[ \beta_2 = \frac{\sqrt{5\pi} Q_0}{3ZR_0^2} \]  

(12)

Where
\( Z \): The atomic number.
\( R_0 = 1.2 \times A^{1/3} \) fm.
\( (\beta_2) \): The deformation parameter and \( (\beta_2 < 1) \).
2.6. The reduced electric quadrupole transition probability ($E2$)↑
Radioactive electromagnetic transformations between nuclear states are a perfect path to achieve nuclear structure and to experiment nuclear structure models [17]. $B(E2)$ Transmission play a definitive role to determine the lifetimes of nuclear states average, the nuclear deformation parameter $\beta$, the volume of essential electric quadrupole moments and the energy of low-lying nuclei levels. Great quadrupole moments and transmissions forces refer to the collective effects in which many nucleons can participate[18]. From here the reduced electric quadrupole transition likelihood, $B(E2)↑$, from the spin $0^+$ ground state to the first excited spin $2^+$ state is specified by[19]:

$$B(E2 : 0^+ \rightarrow 2^+) = \frac{5}{16\pi} e^2 Q_0^2$$  \hspace{1cm} (13)

Where $B(E2)↑$: reduced electric quadrupole transition probability in the unit of $(e^2 b^2)$. $Q_0$: is intrinsic quadrupole moment in unit of barn (b).

The $B(E2)↑$ values are requisite experiential quantities that have no dependence on nuclear models. A quantity in which the model is thought to be depended on, is perfectly useful as it is the deformation parameter ($\beta_2$). Presuming a uniform charge distribution out to the distance $R(\theta, \phi)$ and zero charge beyond, ($\beta_2$). is associated to $B(E2)↑$ by the formulation [20]

$$\beta_2 = \left(\frac{4\pi}{3Z R_0^2}\right)\left[B(E2)↑/e^2\right]^{\frac{1}{2}}$$  \hspace{1cm} (14)

$$R_0^2 = \left(1.2 \times A^{\frac{1}{3}} fm\right)^2 = 0.0144 A^{2/3} b$$  \hspace{1cm} (15)

In accordance with the global systematic, the energy acknowledgement $E$ (KeV) of the $2^+$ state is whole that is required of creating a prediction for the corresponding $B(E2)↑(e^2 b^2)$ value [18]:

$$B(E2)↑ = 2.6 \times E^{-1} Z^2 A^{-\frac{2}{3}}$$  \hspace{1cm} (16)

3. Calculation and Results

3.1. Deformation Parameters ($\beta_2$)
Deformation Parameters($\beta_2$) derived from Reduced Electric Transition Probability $B(E2)↑$ of even-even nucleus for the ($Sn$ & $Yb$) isotopes were counted using the equation (14). this equation contains many Parameters must be obtained:

3.1.1. Reduced Electric Transition Probability $B(E2)↑: 0^+ \rightarrow 2^+$ from the ground $0^+$ to the first excited $2^+$ states calculated by using equation (16). The energy $E(KeV)$ of the first excited state $2^+$ was obtained from the reference (18).

3.1.2. Average Nuclear Radius $R_0^2$ calculated using equation(15).

3.2. The Deformation Parameters ($\delta$)
The other method for calculation of distortion parameter ($\delta$) is by using the intrinsic quadrupole moments ($Q_0$) Equation (9). To evaluate this, the following variables must be available:
3.2.1. The Mean Square Charge Radius($r^2$) which is obtained from equation (5) for $A > 100$.

3.2.2. Intrinsic Quadrupole Moments ($Q_0$) of nuclei were calculated from the equation (13). These values were compared with the Predicted values of $Q_0$ for SSANM form reference[18]. All these values were tabulated in tables (1), and (2).

3.3. The major axis (a) and minor axis (b) were counted by using Eq. (10) and (11) respectively. The difference $\Delta R$ between (a) and (b) were counted also by using Eq. (1), (2), and (4) respectively. All these values are tabulated in the (3) and (4) tables.

**Table 1.** Isotopes Mass Number (A), Neutron Number (N), Gamma Energy of the First Excited State $2^+(E\gamma)$, Nuclear Average Radius ($R^2_0$), Reduced Electric Transition Probability ($B(E2)$ $\uparrow$ in unit

| (Z) | (A) | (N) | $E\gamma$(KeV) | $B(E2)$ $\uparrow$ | $\beta_2$ for | $R^2_0$ | $B(E2)$ $\uparrow$ | $Q_\gamma(b)$ | $\beta_2$ | $\delta$ |
|-----|-----|-----|----------------|-----------------|--------------|--------|----------------|------------|--------|--------|
|     |     |     |                | ($e^2b^2$)      | (SSANM) (P.w.)|        | ($e^2b^2$)      |            |        |        |
| 102 | 52  | 1472.22 | 0.051 | 0.0602 | 0.3144 | 0.2023 | 1.4260 | 0.1199 | 0.1080 |
| 104 | 54  | 1260.13 | 0.116 | 0.0896 | 0.3185 | 0.2332 | 1.5313 | 0.1271 | 0.1145 |
| 106 | 56  | 1207.75 | 0.195 | 0.1147 | 0.3225 | 0.2403 | 1.5543 | 0.1273 | 0.1147 |
| 108 | 58  | 1206.07 | 0.281 | 0.1360 | 0.3266 | 0.2376 | 1.5457 | 0.1251 | 0.1127 |
| 110 | 60  | 1211.89 | 0.361 | 0.1523 | 0.3306 | 0.2336 | 1.5325 | 0.1225 | 0.1104 |
| 112 | 62  | 1256.85 | 0.407 | 0.1597 | 0.3346 | 0.2226 | 1.4959 | 0.1181 | 0.1064 |
| 114 | 64  | 1299.92 | 0.406 | 0.1577 | 0.3386 | 0.2127 | 1.4622 | 0.1141 | 0.1028 |
| 116 | 66  | 1293.56 | 0.394 | 0.1535 | 0.3425 | 0.2113 | 1.4573 | 0.1124 | 0.1013 |
| 118 | 68  | 1229.66 | 0.379 | 0.1489 | 0.3464 | 0.2197 | 1.4862 | 0.1134 | 0.1021 |
| 120 | 70  | 1171.34 | 0.365 | 0.1445 | 0.3503 | 0.2281 | 1.5143 | 0.1142 | 0.1029 |
| 122 | 72  | 1140.53 | 0.286 | 0.1265 | 0.3542 | 0.2317 | 1.5261 | 0.1138 | 0.1026 |
| 124 | 74  | 1131.73 | 0.190 | 0.1020 | 0.3581 | 0.2310 | 1.5238 | 0.1124 | 0.1013 |
| 126 | 76  | 1141.15 | 0.111 | 0.0771 | 0.3619 | 0.2266 | 1.5094 | 0.1102 | 0.0993 |
| 128 | 78  | 1168.83 | 0.035 | 0.0527 | 0.3657 | 0.2190 | 1.4856 | 0.1072 | 0.0966 |
| 130 | 80  | 1221.26 | 0.037 | 0.0296 | 0.3695 | 0.2074 | 1.4440 | 0.1032 | 0.0930 |
| 132 | 82  | 404.11  | Sph   | ------ | ------ | ------ | ------ | ------ | ------ | ------ |
| 134 | 84  | 752.2   | 0.060 | 0.0344 | 0.3771 | 0.3424 | 1.8553 | 0.1300 | 0.1171 |

**Table 2.** Isotopes Mass Number (A), Neutron Number (N), Gamma Energy of the First Excited State $2^+(E\gamma)$, Nuclear Average Radius ($R^2_0$), Reduced Electric Transition Probability ($B(E2)$ $\uparrow$ in unit

| (Z) | (A) | (N) | $E\gamma$(KeV) | $B(E2)$ $\uparrow$ | $\beta_2$ for | $R^2_0$ | $B(E2)$ $\uparrow$ | $Q_\gamma(b)$ | $\beta_2$ | $\delta$ |
|-----|-----|-----|----------------|-----------------|--------------|--------|----------------|------------|--------|--------|
|     |     |     |                | ($e^2b^2$)      | (SSANM) (P.w.)|        | ($e^2b^2$)      |            |        |        |
| 152 | 82  | 1531.45 | 1.189 | 0.1591 | 0.4101 | 0.2921 | 1.7136 | 0.0789 | 0.0711 |
| 154 | 84  | 821.32  | 1.972 | 0.2031 | 0.4137 | 0.5399 | 2.3298 | 0.1063 | 0.0958 |
| 156 | 86  | 536.41  | 2.566 | 0.2297 | 0.4173 | 0.8196 | 2.8704 | 0.1298 | 0.1170 |
| 158 | 88  | 358.21  | 3.195 | 0.2542 | 0.4209 | 1.2169 | 3.7977 | 0.1569 | 0.1413 |
| 160 | 90  | 243.11  | 3.609 | 0.2679 | 0.4244 | 1.7782 | 4.2280 | 0.1880 | 0.1694 |
| 162 | 92  | 166.85  | 3.982 | 0.2790 | 0.4279 | 2.5694 | 5.0824 | 0.2241 | 0.2020 |
of $e^2 b^2$, Quadrupole Moment ($Q_o$) in unit of barn, and Deformation Parameters ($\beta_2, \delta$) for ($_{50}$Sn).

Table 3. Mass number (A), Neutron Number (N), Root Mean Square Radii $<r^2>^{1/2}$, Major and minor axes (a,b) and the difference between them ($\Delta R$) by two method for ($_{50}$Sn) Isotopes.

| (Z) | (A) | (N) | Theoretical Value | Present Work |
|-----|-----|-----|-------------------|--------------|
|     |     |     | $(r^2)^{1/2}$ fm | $(r^2)^{1/2}$ fm | a (fm) | b (fm) | $\Delta R_1$ | $\Delta R_2$ | $\Delta R_3$ |
| 102 | 52  |     | 4.4503            | 2.5197        | 3.0908 | 0.5400 | 0.5711        | 0.6358        |
| 104 | 54  |     | 4.4792            | 2.5151        | 3.1216 | 0.5761 | 0.6065        | 0.6784        |
| 106 | 56  |     | 4.5077            | 2.5226        | 3.1323 | 0.5810 | 0.6097        | 0.6842        |
| 108 | 58  |     | 4.5605            | 2.5345        | 3.1355 | 0.5742 | 0.6010        | 0.6762        |
| 110 | 60  |     | 4.5785            | 2.5469        | 3.1377 | 0.5659 | 0.5907        | 0.6664        |
| 112 | 62  |     | 4.5948            | 2.5624        | 3.1343 | 0.5490 | 0.5719        | 0.6465        |
| 114 | 64  |     | 4.6099            | 2.5772        | 3.1318 | 0.5335 | 0.5546        | 0.6283        |
| 116 | 66  |     | 4.6250            | 2.5877        | 3.1358 | 0.5287 | 0.5481        | 0.6225        |
| 118 | 68  |     | 4.6393            | 2.5934        | 3.1476 | 0.5361 | 0.5541        | 0.6313        |
| 120 | 70  |     | 4.6519            | 2.5991        | 3.1589 | 0.5431 | 0.5598        | 0.6396        |
| 122 | 72  |     | 4.6634            | 2.6070        | 3.1665 | 0.5444 | 0.5596        | 0.6411        |
| 124 | 74  |     | 4.6735            | 2.6166        | 3.1709 | 0.5406 | 0.5543        | 0.6366        |
| 126 | 76  |     | 4.6833            | 2.6277        | 3.1727 | 0.5327 | 0.5450        | 0.6273        |
| 128 | 78  |     | 4.6921            | 2.6401        | 3.1719 | 0.5208 | 0.5318        | 0.6133        |
| 130 | 80  |     | 4.7019            | 2.6541        | 3.1681 | 0.5043 | 0.5140        | 0.5938        |
| 132 | 82  |     | 4.7093            | 2.7459        | 3.0279 | 0.2744 | 0.2820        | 0.3231        |
| 134 | 84  |     | 4.8740            | 2.6181        | 3.2651 | 0.6414 | 0.6470        | 0.7553        |

Table 4. Mass number (A), Neutron Number (N), Root Mean Square Radii $<r^2>^{1/2}$, Major and minor axes (a,b) and the difference between them ($\Delta R$) by two method for ($_{70}$Yb) Isotopes.

| (Z) | (A) | (N) | Theoretical Value | Present Work |
|-----|-----|-----|-------------------|--------------|
|     |     |     | $(r^2)^{1/2}$ fm | $(r^2)^{1/2}$ fm | a (fm) | b (fm) | $\Delta R_1$ | $\Delta R_2$ | $\Delta R_3$ |
| 152 | 82  |     | 5.0423            | 5.0831        | 2.7693 | 3.1745 | 0.4058        | 0.4051        | 0.4778        |
| 154 | 84  |     | 5.0875            | 5.1053        | 2.7244 | 3.2683 | 0.5493        | 0.5439        | 0.6468        |
| 156 | 86  |     | 5.1219            | 5.1274        | 2.6856 | 3.3483 | 0.6738        | 0.6627        | 0.7935        |
| 158 | 88  |     | 5.1498            | 5.1492        | 2.6391 | 3.4375 | 0.8176        | 0.7985        | 0.8628        |
| 160 | 90  |     | 5.1781            | 5.1708        | 2.5829 | 3.5372 | 0.9842        | 0.9543        | 1.1589        |
| 162 | 92  |     | 5.2054            | 5.1923        | 2.5146 | 3.6490 | 1.1782        | 1.1344        | 1.3874        |
| Neutron Number (N) | \( \beta_2 \) for \( \frac{50}{70} \)Sn | \( \beta_2 \) for \( \frac{70}{70} \)Yb |
|-------------------|------------------|------------------|
| 164 | 94 | 5.2307 | 5.2135 |
| 166 | 96 | 5.2525 | 5.2346 |
| 168 | 98 | 5.2702 | 5.2556 |
| 170 | 100 | 5.2853 | 5.2764 |
| 172 | 102 | 5.2995 | 5.2970 |
| 174 | 104 | 5.3108 | 5.3174 |
| 176 | 106 | 5.3215 | 5.3377 |
| 178 | 108 | 5.3579 | 2.4177 |

**Figure 3.** Deformation Parameter (\( \beta_2 \)) values as a function of neutron Number (N) for the \( \frac{50}{70} \)Sn and \( \frac{70}{70} \)Yb Isotopes.
**Figure 4.** Shapes of axially symmetric quadrupole deformation for $^{50}$Sn isotope from major (a) and minor (b) axes.

**Figure 5.** Shapes of axially symmetric quadrupole deformation for $^{70}$Dy isotope from major (a) and minor (b) axes.
4. Discussion
From observation the values of the electric quadrupole moments $B(E2)$ of selected elements, tables (1-1) & (1-2), we found that these values vary according to their mass numbers (number of protons and
neutrons), and when we approaching to the magic numbers of protons and/or neutrons, the values of \( B(E2) \uparrow \) become less than those of the other isotopes of the same element, in other words the values of deformation \( (\beta_2) \) become as low as possible and, therefore, this isotope with magic numbers is more stable than others.

On the other hand, we also found when the mass numbers are less than 150 (\( A < 150 \)), the values of intrinsic quadrupole moments are seems to be less than those of with mass number between 150 and 180 (\( 150 < A < 180 \)), this is belonged to collective behavior (vibrational and rotational) of nucleons.

Also in the even-even nuclei that appear collective behavior. The energy of the first excited state \( (2^+) \) appears to be decrease sort of smoothly as a function of \( A \) (except the regions near closed shells).

From observable values of the root mean square charge radii \((r^2)_{1/2}\), table (1.3) to (1.4), we found that these values increased as the mass number \( A \) increasing. For comparison purposes, it was found that the calculated values of \((r^2)_{1/2}\) (P.w.) correspond well to the experimental values of \((r^2)_{1/2}\) from references [].

What has been mentioned above can be explained in detail in the following paragraphs.

4-1 Strontium Isotopes \( ^{102-134}_{50}\text{Sn} \)

Clearly from table (1 - 1), that the lowest value of the deformation Parameter is for the \( (^{132}_{50}\text{Sn}) \) equal to \( (\beta_2 = 0.0559) \) and the largest value of deformation parameter is for \( (^{134}_{50}\text{Sn}) \) \( (\beta_2 = 0.1300) \). The remaining values of \( \beta_2 \) are ranging between these two values. This is due to the fact that the nucleus of the \( (^{180}_{50}\text{Sn}) \) is one of the nucleus with double magic numbers \((Z = 50, N = 82)\), and therefore this nucleus is more stable than others. Furthermore, the energy level of the first excited state \( 2^+ \) is very high \((E_\gamma = 4041.1 \text{KeV})\), (the gap is large between the ground and the first excited states, thus the hardness of transfer nucleons between these two states), compared with the energy levels of the same states for others. This means that the nucleus of \( ^{132}_{50}\text{Sn} \) isotope has closed shell, spherically Symmetric, and be especially stable.

More nucleons are added outside the closed shell in the \( ^{134}_{50}\text{Sn} \) isotope and the energy level of the first excited state \( 2^+ \) is \( (E_\gamma = 725 \text{KeV}) \). All these factors are encouraging the small deformation of this nuclide.

These results are confirmed in Figure (1-3), which shows the relationship between deformation parameters \( (\beta_2) \) as a function of neutrons numbers \( (N) \). It is clear that the distortion of nuclides decreases as neutron numbers close to the magic number of \( (82) \). Then the value of \( (\beta_2) \) begins to increase thereafter as the \( (N) \) increased, which mean increase of nucleons outside closed shell.

Generally speaking all nuclides of the isotopes of \( (^{50}\text{Sn}) \) show a small deviation from the spherical shape, with the exception of the isotope \( (^{132}_{50}\text{Sn}) \) as shown in the figure 4’. Also figure 6’ show the 3-D shapes of the smallest and highest values of deformation Parameter of \( ^{50}\text{Sn} \) Isotopes

4-2 Ytterbium Isotopes \( ^{152-178}_{70}\text{Yb} \)

From observable table 2, we find that it starts with \( ^{152}_{70}\text{Yb} \), where the number of neutrons represent a magic number \( (N = 82) \) and the number of protons \( (Z = 70) \). The energy of the first excited state \( 2^+ \) \((E_\gamma = 1531.4 \text{KeV})\) (the gap is large between the ground and the first excited states, thus the hardness of transfer nucleons between these two states), So that the reduced electric transmission probability \( B(E2) \uparrow \) is low and therefore the \( \beta_2 \) will be at its minimum value \( (\beta_2 = 0.0789) \), this will lead that the nuclide of this isotope is more stable, almost spherical and the most tightly bound shape.

From same table, values of \( (\beta_2) \) will increase with increase \( (N) \) until reach to the confined area between \( (92 \leq N \leq 108) \), deformation values are approximately equal and ranging from \( (\beta_2 = 0.2241) \) with \( (E_\gamma = 166.85 \text{KeV}) \) for \( ^{162}_{70}\text{Yb} \) to \( (\beta_2 = 0.2875) \) with \( (E_\gamma = 84 \text{KeV}) \) for \( ^{178}_{70}\text{Yb} \).
the maximum value of \( \beta_2 = 0.3083 \) for \(^{162}\text{Yb}\), this is due to the low energy value of the first excited state \( (E_x = 76 \text{ KeV}) \) which is in turn leads to maximum value of \( B(E2) \) and then the highest value of deformation. This seems to be clear in the 'figure 3' which shows the relationship between \( \beta_2 \) as a function of the neutrons number \((N)\). As a result, these nuclei will be less stable, non-spherical shape and will be more elongated.

On the other hand from observable table 2 we find the distortion values \((\delta)\) derived from \( Q_0 \), become as low as possible because it started with magic number \((N = 82)\) and the values of the intrinsic electric quadrupole moment become on its minimum value. When add more nucleons in the shell or sub-shell outside close shell this will lead to restrict the vibrations of wholly or partially to one direction (polarization the core), and the nucleus can get a permanent deformation.

'Figure 5' shows the differences between these values of deformation Parameters \((\beta_2)\) based on the values of major and minor axes \((a, b)\) respectively. Also 'figure 7' show the 3-D shapes of the smallest and highest values of deformation Parameter of \(^{70}\text{Yb}\) Isotopes.

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