Designing learning trajectory for teaching sets at grade 7 using realistic mathematics education approach

Y. Yulia*, A. Fauzan, N. Gustituati, Yerizon
Universitas Islam Negeri Imam Bonjol Padang, Padang, Indonesia

*yuliayo99@ymail.com

Abstract. This paper aimed at designing learning trajectory (LT) for teaching sets using Realistic Mathematics Education (RME) approach and investigating it’s influence on students’ mathematical critical thinking ability. The method used is a design research that contains three stage namely preliminary design, teaching experiment, and retrospective analysis. The research’s subjects at grade 7 students in Padang. Data were collected trough observations, interviews, checklist, videotaping, and analyzing the students’ works. Students have their own strategies in constructing, describing and describing their understanding helping students improve their mathematical critical thinking skills. Finally, we discovered the growth in the students’ mathematical critical thinking ability.

1. Introduction
The topic about set is given at the basic level because it is an important topic for all branches of mathematic, as the basis of more complex mathematics concepts and as the basis of mathematics reasoning [1]. This is similar to that of namely logic and sets of form the foundation of mathematics[2]. Mathematics learning, especially set learning should be able to develop the ability of creative thinking. However, based on interviews with mathematics teachers of MTsN 5 Padang, MTsN 4 Padang and SMP 41 Padang, students are still difficult to understand the set, there are still students who have not been able to understand the set and not the set, difficulty in completing the routine problem, questions that demand students' creative thinking skills such as contextual matters, problem-solving problems related to the set, questions of reasoning, and communication matters. The difficulty is because students are less able to define and clarify information that is on the problem and less able to determine solutions and conclusions. In addition, students are also less able to provide new ideas and provide an alternative answer to the questions given. It shows the students' critical and creative thinking ability is still in low level. It ultimately affects the low student mathematics score[3]. That students are still difficult in learning the topic of set [1].

The teacher has designed the lesson by putting it into the Lesson Plan (RPP) in accordance with the 2013 curriculum, which includes teacher activities and student activities. Seen in the RPP, there has been no prediction or prediction of student responses during the lesson. The lesson plans made by teachers are usually less consideration of the responses given by the students so that the later learning series may not be matched by the different learning trajectory of each student[4]. By knowing the responses of students, the teachers could actually choose the right way to achieve the successful learning process. Learning rarely begins with problems related to the knowledge that students already have. Learning that begins with problems related to the knowledge they already have will allow students to
rediscover the concepts of mathematics that are being studied, will make students more active and more likely to develop their mathematics thinking skills. Those are in line that mathematics learning in schools is still dominated by teachers and the results of mathematics learning in schools have not shown satisfactory results[20]. The students’ learning process and their level of thinking should give the teacher a working framework in order to develop such knowledge about students’ thinking and learning. One of the approaches that can be used in designing HLT is through realistic mathematics education (RME). RME is one of approaches in mathematics learning process that contains constructive, interactive and reflective components. In RME, mathematics is seen as human activities [10][11][21][22]. Mathematics is not transferred to the students as a product, but the students should construct it by themselves through contextual problem solving that is interactive, whether it is formal or informal, so that the students can find it by themselves, without a teacher.

An instructional design which is based on students learning trajectory is known by Hypothetical Learning Trajectory (HLT). HLT is a kind of process that consists of learning goal, learning activity and hypothesis of learning process to predict how students think and comprehend during the context of learning activity[5]. Some researchers showed that hypothetical learning trajectory (HLT) can improve students’ comprehension[6] [7] [8]. The other researches showed that HLT did not only improve students’ comprehension but also improved students’ mathematics thinking ability, just as the research done by Graveimeijer [9] that proves that HLT by using RME helps the teachers in developing a good local instructional theory for a topic lesson and also help students in developing their thinking ability such as analysis and the other mathematics thinking ability. An instructional design in terms of HLT consists of three components; learning goal, task of learning activity and hypothesis of how the students learn and think [5]. Setting the learning activities are based on RME. Those activities give a chance for the students to reinvent mathematics through teacher’s guide [11]. The activity RME-based concerns on RME characteristics, they are: 1) the use of contexts, 2) the use of models, 3) the use of students own production and contructions, 4) the interactive character of teaching process, and 5) the interviewments of various learning strands [11].

In designing and developing HLT RME-based in this research, review of previous researches is needed. The researches related to the development of instructional design in form of HLT RME-based have done previously by many researchers, such as a research done by Web[12]. In this research, Webb designed HLT RME-based to introduce the concept of logarithm. Web used contextual problem in form of height of poni and the growth of E.Coli bacteria. This contextual problem gives a possibility for the students to do informal mathematicsematism process, that later will improve into formal mathematicsematism process by giving problem in mathematics. These informal and formal mathematicsematism processes help the stduents in comprheending the concept of logarithm. The use of RME in Cobb’ research proves that learning RME-based builds students-centered learning.

The other research is conducted by Hidayat and Iksan[13]. This research was aimed to see the effect of RME toward the comprehension of studnets’ linier program concept. It was a kind of quasi experimental non equivalent, pretest and post test control group design. This research had control class and experiment class. The result of this research shows that the use of RME by using contextual problems help the students to have a better comprehension ofliner program concept. The research related to designing HLT is also conducted by Prahmana and Kusumah [14]. This research developed HLT in the research of mathematics education by using research based learning. In developing HLT, the researcher used Graveimeijer ‘s and Cobb’s model. HLT consists of three components; (1) the purpose of mathematics teaching for students, (2) learning activities, device or media used in the learning process, and (3) a conjecture of understanding the process of learning how to learn and strategies students that arise and thrive when learning activities are done in Class [11].

The other research that related to HLT and RME is conducted by Ozkaya and Karaca [15] about The Effect of Realistic Mathematics Education on Students’ Achievements and Attitude In Fifth Grades Mathematics Courses, the research done by Silseth [16] about The Multi-voicedness of game Playing: Exploring the Unfolding of a students’learning trajectory in a gaming context at school, and the research conducted by Rees [17] about developing specialist expertise an unanticipated learning trajectory. The
aim of this research is to improve research ability and academic writing for pre-service teacher of mathematics. To focus this research, we formulated questions of research: what are valid, practical, and effective of learning trajectory criteria for teaching sets using RME approach that extend to improve the students’ mathematical critical thinking ability?

2. Methods

This research is a development research using Gravemeijer & Cobb model[19]. The Gravemeijer and Cobb model consists of three phases: preparing for the experiment phase, conducting the experiment and analysis retrospective. Design research Gravemeijer and Cobb consisted of a cyclic process of preparing for the experiment, conducting the experiment, and retrospective analysis. It can be seen in Figure 1.

![Figure 1. A cyclic process of thought and instruction experiment](image-url)

In preparing for the experiment we determined the end points of the instructions. The goals of our lessons were the students reinvent sets concept, the activity is to collect the data of students at grade VII of junior high school in order to classify object based on such characteristics. The second activity is classifying the students based on their own characteristics. It is aimed to build a problem that can stimulate the students to invent the different among sets and non-sets, blank sets and universal sets, because these problems give a chance for the students to classify the good and bad students. From the first and second activity, the students get the description in grouping the object, stating the reason in grouping the object and stating the group member and non member. After that, the students are asked to solve the problem in which how to add the groups into the general activity of sets concept. At this stage, the students have been familiar to group the object and state it into mathematics.

Second, presenting set, the first activity is collect the data of stationary that usually used by the students in order to stimulate the students to present set by writing the members of sets creator. The second activity, the students are given contextual problem in deciding the animal group based on their characteristics. Those animal groups can be seen as a set that has animals as the members with their own characteristics. For the third activity, the students are given problems related to students’ numbers group that they have commonly learned. Grouping numbers can be seen as a set. All the activities above lead the students to present the set in their real life. Through this activity, the students in the group interact to create a pattern in presenting set based on the first presentation up to the last one.

Third, sets relation, the activity is related to contextual problems about representation of the class to follow the best student’s competition. The students are asked to make the possible formation of students who can represent the class. The formation can also be seen as a set. The second activity is given a challenge for the students to choose a decision. The decision taken is based on their knowledge of set. The problem is designed to help the students invent the concept of the same set and equivalent set. Based on the first and second activity, the student are expected to understand sets relation, they are partial set, equal set, disjoint set sand power of set and equivalent set. The teacher facilitates the students during the class discussion to take a decision and strengthen the conclusion by giving such explanation of set relation. Fourth, set operation, the activity is giving problem related to fruits. The problem stimulates the students to invent the concept of two sets operations by comparing the members of set
that are operated. The second activity is inventing the characteristics of set operation through problems related to extracurricular activity. The problem stimulates the students to invent the principle of inclusion exclusion of two sets operation by comparing the members of each of the groups operated and also the other characteristics that links to each of the set operation.

In the prototyping stage phase, the prototype has been made through an evaluation of formative form which consists of self evaluation conducted by the researcher himself, validation by expert, focus group discussion, one to one, small group, and field test. In the assessment phase, a critical thinking ability test was conducted to grade 7 students of MTsN Padang to see effectiveness. The research data is collected through validation sheet, teacher and student response questionnaire, learning trajectory learning sheet, consciousness sheet and students’ mathematical critical thinking ability test.

3. Result and Discussion

Based on the results of the preliminary analysis an HLT was developed to teach the topic set. HLT is validated by RME mathematicians, linguists and learning design experts. The result showed that the HLT met the criteria of validity with the characteristics; the activities of solving contextual problems in the HLT were potential to facilitate the students to reinvent the concepts in sets, the activities were well sequenced, the HLT suit the key principles and characteristics of RME; and the components in the HLT were well designed and consistent between one and another. The HLT also satisfied the criteria of practicality, in which it worked as intended during the try out. The students understood the contextual problems and they conducted ‘doing math’ activities without major obstacle. The probing questions that were prepared as the anticipations of students’ thinking and solutions also helped the students to achieved the goals of the activities. In addition, the time provided for doing the activities of solving contextual problems was well planned.

The findings of the concept of the set in this study were conducted through the provision of activities to determine student data in the class. Various solutions provided by students in solving this problem are:

![Figure 2. Student’s Answer in Concept of Set](image)

Students present in list form, this becomes the beginning for students in understanding the set as a collection of objects. The teacher asks whether students from other classes go into the data students create, whether students can register members according to the given problem. With that question the student understands that the name written is the name of a classmate, according to what is requested. This is the beginning of the students understanding that the concept of the set as a set of clearly defined objects. To find a concept rather than a set, it is given the problem of determining a group member of a set. Each student has a different answer. This is the beginning of students’ understanding of non-sets. The teacher asks the students to compare with the answer to the previous problem, asking the question why the answers among the students vary, in contrast to the previously created set where all students give the same answer. Students respond because according to their judgment, beautiful and handsome students vary. This will make students understand that a collection of objects is called a set if clearly
defined. From some conclusions who made by students, students have been able to make a conclusion related to the concept of the set. The teacher makes an agreement with the students regarding the set concept, starting from the definition, how to write the set, symbol set, member set, number of members, not set and empty set. Retrospective analysis of this problem is that students use their thinking skills in finding the concept of members, not members, sets and not sets. Students know why a collection of objects is said to be a set, not all collections of objects are said to be a set, and when a set is called an empty set.

After understanding the concept of the set, then students are invited to present the set into various representations. The problem given is to mention the stationery needed by a student.

Figure 3. Student’s Answer in representation of set

Students have been able to make a set and mention members who belong to the set. Students have been able to write the set correctly. The teacher and students make an agreement that the way to present the first set is to mention the members of the set. Students can use their knowledge in science lessons about animal grouping. In general, students in all schools of research subjects are able to provide answers. Here are example of student answers:

Figure 4. Student’s Answer in representation of set

Figure 4 showed that students have the ability to think, students have other knowledge they can use in finding mathematical concepts. After students make the presentation of the set, the teacher invites students to check the name that matches the given animal group. To understand Venn diagrams, problems are drawn from the activities students perform in presenting the set. Students are asked to restate some sets that have the same members into a diagram.

Figure 5. Student’s Answer

Figure 5 showed the student's answer it appears that the students start to develop their thinking ability in making a diagram, initially the students make diagrams that are not yet compatible with the Venn diagram, but they understand the intended purpose. After the teacher gives directions how to make a better diagram, students can already improve to the better. Teachers with students make agreement on how to make the right Venn diagram and what elements of the Venn diagram. To determine the Principles and the properties of the set operation are given problems about student's passion for sports. Based on the survey results students are asked to answer some questions. From the answers given by the students it appears that the students have used Venn diagrams to determine the answers to the questions asked. But not all students are using this Venn diagram. There are still students who decide by using ordinary operations. The teacher reminds students who have not used Venn diagrams about the previous lesson. Teachers with students make agreement on the nature of set operations.
The student’s answers presented above indicates that the didactical phenomenology, as a key principle of RME, worked as intended in our experiment. The students used the context in the problem to reinvent the concepts. Besides, the contexts in the problems were facilitated the students to used their own strategies, which met one of the characteristics of RME namely ‘students’ free production’. This situation helped the students building their confidence in learning mathematics. Finally, we observed the development in students’ mathematical critical thinking ability. They started giving an argument, a reason, or an explanation when solving the contextual problems. The students’ mathematical critical thinking ability test that we gave at the end of the experiment showed that 78.1% of the students achieved the score greater than 79.35. This finding confirmed that LT and RME approach are potential to improve students’ mathematical critical thinking ability.

4. Conclusion
The learning trajectory for teaching sets using RME approach developed in this research met the criteria of validity, practicality, and effectiveness. Design research approach that we used to develop the learning trajectory was very helpful in reaching our goal. The learning trajectory for teaching sets reflected the state of the art of RME and it worked as intended in the classroom. Moreover, the learning trajectory could help the students to reinvent the concepts in sets. The students had more confidence to use their own strategies in solving contextual problems. The most important thing, we discovered the growth in the students’ mathematical critical thinking ability.

References
[1] Lee Peng Yee & Lee Ngan Hoe. Teaching Secondary School Mathematics. A Resource book (2nd ed). Singapore: McGraw-Hill Education. (2009)
[2] Barnet, R. A., Ziegler, M. R &Byleen, K. E. Finite Mathematics For Business, Economics, Life Sciences, and Social Sciences. New Jersey; Pearson Education Inc. (2008)
[3] Asui Nelson Chukwuyenum., Impact of Critical Thingking on Performance in Mathematics among Senior Secondary School Students in Lagos State. IOSR Journal of Research & Method in Education. Volume 3 Issues 5. 18-25. (2013)
[4] Suryadi, D. Didactical Design Research (DDR) dalamPengembanganPembelajaranMatematikaIn Bandung: Seminar NasionalPembelajaran MIPA di UM Malang. (2010).
[5] Simon, Martin A. Reconstructing Mathematics Pedagogy From A Constructivist Perspective. Journal of Research in Mathematics Education. Volume 26, No.2, 135-137. (1995)
[6] Baker, Arthur. Design Research on How IT May Support the Development of Symbols and Meaning in Mathematics Education. Freudenthal Institute, Utrecht University, (2003)
[7] Bardsley M. E.,Pre-Kindergarten Teachers’ and Understanding of Hypothetical Learning Trajectories in Mathematics Education. Utrecht: University of Utrecht, (2006)
[8] Hadi, Sutarto. Pendidikan Matematika Realistik. Tulip Banjarmasin, (2005)
[9] Graveimejer. K.P.E. Local Instruction Theories as Means of Support for Teachers in Reform Mathematics Education. Mathematics Thinking and Learning, 6(2), 125-127. Copyright 2004. Lawrence Erlbaum Associates. Inc, (2004)
[10] Freudhental. H. Revisiting Mathematics Education. Utrecht, The Nederland: Kluwer Academic,(1991)
[11] Graveimejer. K.P.E.,Developing Realistic Mathematics Education. Freudenthal Institute, Utrecht,(1994)
[12] Webb, David C. Design Research in The Netherlands: Introducing Logarithms Using ReralisticMathematics Education, Journal of Mathematics Education at Teachers College, Spring-Summer 2011, Vol 2,47-52, (2011)
[13] Hidayat, Riyan&Iksan, Zanaton H. The effect of Realistic Mathematics Education Students’ Conceptual Understanding of Linear Programing. Creative Education, 2015,6,2438-2445. http://www.scrip.org/journal/ce, (2015)
[14] Prahmana, Rully C.I &Kusumah, Yaya. S. The Hypotetical Learning Trajectory on Research in Mathematics Education Using Research-Based Learning.Pedagogika/ Pedagogy. 2016, t123,Nr.p.42-54/Vol.123. No.3 pp42-54,DOI:http://dx.doi.org/10.15823/p.2016.32. (2016)
[15] Ozkaya, Ali & Karaca, Sebahat Yetim. The Effect of Realistic Mathematics Education On Students’ Achievements Attitude in fifth Grades Mathematics Course. IOJET 2017, 4(2), 185-197. (2017)

[16] Silseth, Kenneth. The Multivoicedness of game Playing: Exploring the Unfolding of a students’ learning trajectory in a gaming context at school. Computer-Supported Collaborative Learning (2012) 7:63–84 DOI 10.1007/s11412-011-9132-x. (2012).

[17] Rees, Mary E. Developing specialist expertise: an unanticipated learning trajectory. Teacher Development Vol. 13, No. 4, November 2009, 373–383. DOI: 10.1080/13664530903578272 http://www.informaworld.com. (2009)

[18] Plomp, T dan N. Nieveen. Educational Design Research. Enshede: Netherlands Institute for Curriculum Development (SLO), (2013)

[19] Gravemeijer, Koeno and Cobb, Paul. Design research from the Learning Design Perspective. Dalam Jan Ven Den Akker, et. al. Educational Design Research. London: Routledge, (2006)

[20] Turmudi. (2010). Pembelajaran Matematika Kinidan Kecenderungan Masa Mendatang. Bandung: JICA FPMIPA UPI.

[21] Treffers A 1991 Three Dimensions: A Model of Goal and Theory Description in Mathematics Education (Dordrecht: Reidel)

[22] de Lange J 1987 Mathematics, Insight, and Meaning (Utrecht: OW & OC)