The solution of 4-dimensional Schrodinger equation for Scarf potential and its partner potential constructed By SUSY QM

Wahyulianti, A. Suparmi, C. Cari, and Kristiana N. Wea
Physics Department of Post Graduate Program Sebelas Maret University, Jl. Ir. Sutami 36A Keningan Jebres Surakarta 57126, INDONESIA
E-mail: wahyuliantiyuli@yahoo.com

Abstract. The angular part of 4-dimensional Schrodinger equation for Scarf potential was solved by using the Nikiforov-Uvarov method and Supersymmetric Quantum Mechanic method. The determination of the ground state wave function has been used Nikiforov-Uvarov method and by applying the parametric generalization of the hypergeometric type equation. By using manipulation of the properties and operators of the Supersymmetric Quantum Mechanic method the partner potential was constructed. The ground state wave functions of original Scarf potential is different than the ground state wave functions of the construction result potential.

1. Introduction
The D-dimensional Schrodinger equation is one of the wave equations for higher dimensions in quantum mechanics [1-2]. The D-dimensional Schrodinger equation for some potentials are exactly solved by some methods such as Nikiforov-Uvarov (NU) method [3-6], Supersymmetric Quantum Mechanic (SUSY QM) [4,8], Romanovski polynomial method [9], Asymptotic Iteration Method (AIM) [10] and Factorization methods [11]. The NU method is one of the methods with common application, and NU differential equations have been formulated with general parameters of the general hypergeometric type equation [4]. By applying and manipulating the operators of SUSY QM, the properties of SUSY QM and the ground state wave functions of the original potential, the partner potential was constructed [4].

The purpose of this research is to construct partner potential which is the new potential of the generalized Scarf potential by manipulating the partner potential equations with the ground state wave function of Scarf potential. In this paper, the solution of angular D-dimensional Schrodinger equation for Scarf potential was studied using Nikiforov-Uvarov method with hypergeometric type equations and SUSY QM. The paper is organized as follows. The method is presented in Section 2. The results and discussion are presented in Section 3 and a conclusion in Section 4.

2. Method
In this section, we review the solution of D-dimensional Schrodinger equation with Nikiforov-Uvarov method and Supersymmetry Quantum Mechanics method. The Generalized Schrodinger equation in D-dimensional shown as followed equation [1,12] given as

\[-\frac{\hbar^2}{2\mu} \nabla_D^2 \psi(r, \Omega) + V(r, \Omega) \psi(r, \Omega) = E \psi(r, \Omega) \] (1)
with
$$\psi(r, \Omega) = \frac{1}{r^{d-1}} U(r) Y_{l_1}^{\nu_1 - \lambda_1}(\hat{x}) \quad ; \hat{x} = \theta_1, \theta_2, \ldots, \theta_{d-1}$$
(2)

The 4-dimensional Schrödinger equation obtained by inserted Eq. (2) into Eq. (1) and solve of using variable separation method with $Y_{l_1}^{\nu_1 - \lambda_1} (\theta_1, \theta_2, \theta_3) = P(\theta_1)P(\theta_2)P(\theta_3)$ is given as [13]:

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r^2} U + \frac{2\mu}{h^2} [E - V(r)] U = 0$$
(3)

$$\frac{\partial^2 P(\theta_i)}{\partial \theta_i^2} - V(\theta_i) P(\theta_i) + \lambda_i P(\theta_i) = 0$$
(4)

$$\frac{1}{P(\theta_2)} \left[ \frac{1}{\sin \theta_2} \left( \lambda_3 + \frac{3}{4} \right) - V(\theta_2) \right] - \lambda_2 \sin \theta_2^2 = 0$$
(5)

$$\frac{1}{P(\theta_3)} \left[ \frac{1}{\sin^2 \theta_3} \left( \lambda_3 + \frac{3}{4} \right) - V(\theta_3) \right] = \lambda_3 - \lambda_2 \sin^2 \theta_3 = 0$$
(6)

where $\lambda_1, \lambda_2$ and $\lambda_3$ is variable separation constant.

In this paper the potential energy of Scarf potential generally given as [14]:

$$V_i(\theta) = \left( \frac{A_i}{\sin^2 \theta_i} + \frac{B_i \cos \theta_i}{\sin^2 \theta_i} \right), \text{ for } i = 1, 2 \text{ and } 3$$
(7)

with $A_i = b_i^2 + a_i (a_i - 1)$ and $B_i = -2b_i \left( a_i - \frac{1}{2} \right)$.

### 2.1 Review of NU method

The solution of Schrodinger equation is reduced to the hypergeometric type differential equation by using variable transformation. By using the NU method, the solution of the hypergeometric type differential equation is expressed as

$$\frac{\partial^2 \psi(s)}{\partial s^2} + \frac{\tau(s)}{\sigma(s)} \frac{\partial \psi(s)}{\partial s} + \sigma(s) \psi(s) = 0$$
(8)

where $\tau(s)$ is first order polynomial, $\sigma(s)$ and $\bar{\sigma}(s)$ are mostly second order polynomials. By using variable separation method, substitution equation (4) with $\psi(s) = \phi(s) y(s)$

we obtain hypergeometric type as

$$\sigma \frac{\partial^2 y(s)}{\partial s^2} + \tau(s) \frac{\partial y(s)}{\partial s} + \bar{\sigma}y(s) = 0$$
(10)

where $\phi(s)$ is logarithmic derivative $\frac{\phi'}{\phi} = \pi$ with $\pi(s), \lambda$, and $\tau$ are

$$\pi = \left( \frac{\sigma' - \bar{\tau}}{2} \right) \pm \sqrt{\left( \frac{\sigma' - \bar{\tau}}{2} \right)^2 - \bar{\sigma} + k \sigma}$$
(11)

$$\tau = \bar{\tau} + 2\pi, \quad \lambda = k + \pi \text{ and } \quad \bar{\lambda} = \lambda_n = -n\pi - \frac{n(n-1)}{2} \sigma" \quad n = 0, 1, 2, \ldots$$
(12)

The solution of the part $y_n(s)$ by using Rodrigues relation, is given as
\[ y_n(s) = \frac{C_n}{w(s)} \left( \frac{d^n}{ds^n} (\sigma^n(s)w(s)) \right) \]  

with \( C_n \) is normalization constant, and the weight function \( w(s) \) must satisfy the condition

\[ \frac{\partial}{\partial s} (\sigma w) = \tau(s) w(s) \]  

The parametric generalization of the hypergeometric type equation in equation (8) is written as

\[ \frac{\partial^2 \psi(s)}{\partial s^2} + \frac{(c_1 - c_3)s}{s(1 - c_3)} \frac{\partial \psi(s)}{\partial s} + \frac{(c_1 - c_3 - \varepsilon), c_2 - c_3, c_3}{s^2(1 - c_3)^2} \psi(s) = 0 \]  

By comparing equation (8) and equation (15), we obtained the energy value equation and eigenfunction are respectively given as

\[ c_3n - (2n+1)c_3 + (2n+1)\left(\sqrt{c_9} + c_1\sqrt{c_9}\right) + n(n-1)c_3 + c_7 + 2c_5c_6 + 2\sqrt{c_6c_8} = 0 \]  

and

\[ \psi(s) = N_n \psi^{c_3} \left(1 - c_3s\right)^{-(c_9/c_6)c_2} P_n^{(c_{10},c_1)} (1 - 2c_3s) \]  

where

\[ c_4 = \frac{1}{2}(1 - c_1), c_5 = \frac{1}{2}(c_2 - 2c_3), c_6 = c_3^2 + c_4, c_7 = 2c_5c_6 - c_2, c_8 = c_3^2 + \varepsilon_3 \]
\[ c_9 = c_3, c_7 + c_2^2 + c_8 + c_6, c_{10} = c_4 + 2c_4 + 2\sqrt{c_8}, c_{12} = c_4 + \sqrt{c_8} \]
\[ c_{11} = c_2 - 2c_3 + 2\left(\sqrt{c_9} + c_3\sqrt{c_9}\right), c_{13} = c_5 - \left(\sqrt{c_9} + c_3\sqrt{c_9}\right) \]  

By using equations (16) and (17) for \( n = 0 \) the ground state energy and wave function are determined [3-6].

### 2.2 Review of SUSY QM

Witten defined two charge operators are commute with Hamiltonian (\( H_0 \)) of the supersymmetry quantum system [4,8] is given as

\[ H_s = \begin{pmatrix} -\frac{d^2}{dx^2} + \frac{d\phi(x)}{dx} + \phi^2(x) & 0 \\ 0 & -\frac{d^2}{dx^2} - \frac{d\phi(x)}{dx} + \phi^2(x) \end{pmatrix} = \begin{pmatrix} H_+ & 0 \\ 0 & H_- \end{pmatrix} \]  

with partner Hamiltonian \( H_- = H_1 \) and \( H_+ = H_2 \). By setting the new SUSY operators for raising operator \( A^+ = -\frac{d}{dx} + \phi(x) \) and lowering operator \( A = \frac{d}{dx} + \phi(x) \).

The SUSY Hamiltonians of equation (19) are

\[ H_-(x) = H_1 = A^+ A_t \text{ and } H_+(x) = H_2 = AA^+ \]  

and

\[ A\psi_0^{(\pm)} = A\psi_0^{(0)} = 0 \]  

The partners of potential in the SUSY QM method is \( V_+ = V_1 \) and \( V_+ = V_2 \) are

\[ V_-(x) = V_1 = \phi^2(x) - \phi'(x) \text{ and } V_+(x) = V_2 = \phi^2(x) + \phi'(x) \]  

where \( \phi \) is superpotential. The partner potential \( V_1 \) is determined as

\[ V_-(x,a_0) = V_1(x,a_0) = V_{e_0}(x) - E_0 \]  

with \( V_{e_0} \) is effective potential and \( E_0 \) is the ground state energy.
By manipulating equation (22) and by applying equation (17) and equation (23), we construct the new partner potential \( V_2 \) as

\[
V_2(x) = V_{sf}(x) - E_0 - 2 \frac{d}{dx} \left( \frac{d\psi_0}{dx} \right)
\]

where \( \psi_0 \) is the ground state wave function [4,8]. Then the new eigen function of new partner potential is also determined by using Nikiforov-Uvarov method.

3. Results and Discussion
In this section, the solution of the Schrödinger equation in 4-dimensional with Nikiforov-Uvarov method and Supersymmetry Quantum Mechanics method.

The solution of the Schrödinger equation in 4-dimensional for \( \theta_1 \) obtained by inserting Eq. (7) to Eq. (4) is given as

\[
\frac{\partial^2 P(\theta_1)}{\partial \theta_1^2} - \left( \frac{A_1}{\sin^2 \theta_1} + \frac{B_1 \cos \theta_1}{\sin^2 \theta_1} \right) P(\theta_1) + \lambda_1 P(\theta_1) = 0
\]

By substituting Eq. (25) with \( \cos \theta_1 = (1 - 2s) \) we obtain

\[
d^2P(s) + \left( 1 - \frac{s}{s(1-s)} \right) ds + \frac{1}{s^2(1-s)} \left[ - \left( \lambda_1 \right) s^2 + \left( \lambda_1 + \frac{B_1}{2} \right) s - \frac{A_1 + B_1}{4} \right] P(s) = 0
\]

By comparing Eq. (26) and Eq. (15) and then by applying Eq. (16), Eq. (17) and Eq. (18) we obtained the \( \lambda_1 \) and eigen function are respectively given as

\[
\lambda_1 = \frac{1}{2} \left( 3 + A_1 \right) + n(n+1) + (2n+1) \left( \frac{1}{2} \sqrt{\frac{A_1 + B_1}{4}} \right) + \frac{1}{2} \sqrt{\left( \frac{A_1 + B_1}{4} \right)} \left( \frac{1}{4} + A_1 - B_1 \right)
\]

and

\[
P(\theta_1) = N_o \left( \frac{1 - \cos \theta_1}{2} \right) \frac{1}{2} \sqrt{\frac{A_1 + B_1}{4}} \left( \frac{1 + \cos \theta_1}{2} \right) \frac{1}{2} \sqrt{\frac{A_1 - B_1}{4}} P_{n1} \left( \frac{1}{\sqrt{4}} \frac{1}{A_1 + B_1} \sqrt{1 + \frac{A_1 - B_1}{4}} \right)
\]

with

\[
c_1 = \frac{1}{2}, \quad c_2 = c_3 = 1, \quad \epsilon_1 = \lambda_1, \quad \epsilon_2 = \lambda_1 + \frac{B_1}{2}, \quad \epsilon_3 = \frac{1}{4} (A_1 + B_1)
\]

The solution of Eq. (27) and Eq. (28) for the ground state \( n = 0 \), we obtained

\[
\lambda_{01} = \frac{1}{2} \left( 3 + A_1 \right) + \frac{1}{2} \sqrt{\frac{1}{4} + A_1 - B_1 + \frac{1}{2} \sqrt{\frac{1}{4} + A_1 + B_1 + \frac{1}{2} \sqrt{\frac{1}{4} + A_1 + B_1} \left( \frac{1}{4} + A_1 - B_1 \right)}}
\]

and

\[
P_0(\theta_1) = N_o \left( \frac{1 - \cos \theta_1}{2} \right) \frac{1}{2} \sqrt{\frac{1}{4} + \frac{1}{2} \sqrt{\frac{A_1 + B_1}{4}} \left( \frac{1 + \cos \theta_1}{2} \right) \frac{1}{2} \sqrt{\frac{A_1 - B_1}{4}}}
\]

The partner potential \( V_2 \) as new potential constructed from Eq. (31) and Eq. (24) is
\[ V_2(\theta_1) = \frac{A}{\sin^2 \theta_1} + \frac{B \cos \theta_1}{\sin^2 \theta_1} - E_{01} + \frac{c_{12}}{2} - \frac{c_{14}}{2} \]  

(32)

with

\[ E_{01} = \lambda_{01}, \quad c_{12} = \frac{1}{4} + \frac{1}{2} \sqrt{\frac{1}{4} + A_1 + B_1} \quad \text{and} \quad c_{14} = \frac{1}{4} + \frac{1}{2} \sqrt{\frac{1}{4} + A_1 - B_1} \]  

(33)

The solution of the Schrödinger equation in 4-dimensional for \( \theta_1 \) partner potential obtained by inserting Eq. (32) to Eq. (4) is given as

\[ \frac{\partial^2 P(\theta_1)}{\partial \theta_1^2} - \left( \frac{A}{\sin^2 \theta_1} + \frac{B \cos \theta_1}{\sin^2 \theta_1} - E_{01} + \frac{c_{12}}{2} - \frac{c_{14}}{2} \right) P(\theta_1) + \lambda_{01} P(\theta_1) = 0 \]  

(34)

By substituting Eq. (34) with \( \cos \theta_1 = (1 - 2s) \) we get

\[ \frac{d^2 P(s)}{ds^2} + \left( \frac{1 - s}{s(1-s)} \right) \frac{dP(s)}{ds} + \frac{1}{s(1-s)^2} \left[ -\left( E_{01} + \lambda_{01} \right) s^2 + \left( E_{01} + \lambda_{01} + c_{14} + c_{12} + \frac{B_1}{2} \right) s \right] P(s) = 0 \]  

(35)

By comparing Eq. (35) and Eq. (15) and then by applying Eq. (16), Eq. (17) and Eq. (18) we obtained \( \lambda_{01} \) and eigen function are respectively given as

\[ \lambda_{01} = \frac{3}{8} - E_{10} - c_{14} + \frac{1}{2} (A_1 + 2c_{12}) + \frac{1}{2} \sqrt{\frac{1}{4} + A_1 - B_1 - 4c_{14}} + \frac{1}{2} \sqrt{\frac{1}{4} + A_1 + B_1 + 4c_{14}} \]  

(36)

\[ \text{and} \]

\[ P_0(\theta_1) = N_{01} \left( \frac{1 - \cos \theta_1}{2} \right)^{1/4} \left( 1 + \frac{1}{2} \sqrt{\frac{1}{4} + A_1 + B_1 + 4c_{12}} \right) \left( 1 + \frac{1}{2} \sqrt{\frac{1}{4} + A_1 - B_1 - 4c_{14}} \right) \]  

(37)

Secondly, the similarly with the steps of \( \theta_1 \) for solution of \( \theta_2 \) by applying Eq. (5) as the Schrödinger equation. The solution of \( \theta_2 \) for ground state \( n = 0 \), we obtained \( \lambda_{02} \) and eigen function are respectively given as:

\[ \lambda_{02} = \frac{1}{2} (A_2 + \lambda_{01}) + \frac{1}{2} \sqrt{A_2 - B_2 + \lambda_{01}} + \frac{1}{2} \sqrt{A_2 + B_2 + \lambda_{01}} + \frac{1}{2} \sqrt{(A_2 + B_2 + \lambda_{01})(A_2 - B_2 + \lambda_{01})} \]  

(38)
By inserting Eq. (39) to Eq. (24) We get the partner potential \( \theta_2 \) as

\[
V_2(\theta_2) = \frac{A_2}{\sin^2 \theta_2} + \frac{B_2 \cos \theta_2}{\sin^2 \theta_2} + \frac{\lambda_{01} - \frac{\sqrt{A_2 + B_2 + \lambda_{01}}}{2}}{1 + \cos \theta_2} + \frac{c_{12}}{1 - \cos \theta_2} - \frac{c_{14}}{1 + \cos \theta_2}
\]

with

\[
E_{02} = \lambda_{02} + \frac{1}{4}, \quad c_{12} = \frac{1}{4} + \frac{1}{2} \sqrt{A_2 + B_2 + \lambda_{01}} \quad \text{and} \quad c_{14} = -\frac{1}{4} - \frac{1}{2} \sqrt{A_2 + B_2 + \lambda_{01}}
\]

By substituting Eq. (40) to Eq. (5) and then by applying Eq. (15) - Eq. (18) we obtain

\[
\lambda_{03} = c_{12} - c_{14} - E_{02} + \left( A_2 + \lambda_{01} + \lambda_{02} - \frac{1}{4} \right) + \frac{1}{2} \sqrt{A_2 + B_2 + \lambda_{01} + \lambda_{02} - 4c_{14} - \frac{1}{4}} + \frac{1}{2} \sqrt{A_2 + B_2 + 4c_{12} + \lambda_{01} + \lambda_{02} - \frac{1}{4}} \left( A_2 + B_2 + \lambda_{01} + \lambda_{02} - 4c_{14} - \frac{1}{4} \right)
\]

and

\[
P_0(\theta_3) = N_0 \left( 1 - \cos \frac{\theta_3}{2} \right)^{\frac{1}{2} + \frac{1}{4} \sqrt{A_2 + B_2 + \lambda_{01} + \lambda_{02} - \frac{1}{4}}} \left( 1 + \cos \frac{\theta_3}{2} \right)^{\frac{1}{2} + \frac{1}{4} \sqrt{A_2 + B_2 + \lambda_{01} - \lambda_{02} - \frac{1}{4}}}
\]

Finally, the similarly with the steps of \( \theta_1 \) for solution of \( \theta_3 \) by applying Eq. (6) as the Schrodinger equation. Then \( \lambda_{03} \) and eigen function for ground state \( n = 0 \) are respectively given as:

\[
\lambda_{03} = -\frac{5}{8} + \frac{1}{2} \left( \lambda_{02} + A_1 \right) + \frac{1}{2} \sqrt{\frac{1}{4} + \lambda_{02} + A_1 - B_3} + \frac{1}{2} \sqrt{\frac{1}{4} + \lambda_{02} + A_1 + B_3}
\]

and

\[
P_0(\theta_3) = N_0 \left( 1 - \cos \theta_3 \right)^{\frac{1}{2} + \frac{1}{4} \sqrt{A_2 + B_2 + \lambda_{01} + A_3 - B_3}} \left( 1 + \cos \theta_3 \right)^{\frac{1}{2} + \frac{1}{4} \sqrt{A_2 + B_2 + \lambda_{01} - A_3 - B_3}}
\]

By Substituting Eq. (45) to Eq. (24) we obtain the partner potential \( \theta_3 \) as

\[
V_2(\theta_3) = \frac{A_3}{\sin^2 \theta_3} + \frac{B_3 \cos \theta_3}{\sin^2 \theta_3} + \frac{\lambda_{02}}{1 - \cos \theta_3} + \frac{c_{12}}{1 - \cos \theta_3} - \frac{c_{14}}{1 + \cos \theta_3}
\]

with

\[
E_{03} = \lambda_{03} + 1, \quad c_{12} = \frac{1}{4} + \frac{1}{2} \sqrt{\frac{1}{4} + \lambda_{02} + A_3 + B_3} \quad \text{and} \quad c_{14} = -\frac{1}{4} - \frac{1}{2} \sqrt{\frac{1}{4} + \lambda_{02} + A_3 - B_3}
\]
By substituting Eq. (46) to Eq. (6) and then by applying Eq. (15) - Eq. (18) we get:

\[
\lambda_{03} = \frac{3}{4} - E_{03} - \frac{B_3}{2} - c_{12} - c_{14} + \frac{1}{4} \sqrt{4 A_3 - B_3 + \lambda_{03} + \lambda_{02} - 4 c_{14} + \frac{1}{4} A_3 + B_3 + \lambda_{03} + \lambda_{02} + 4 c_{14}} \]

and

\[
P_{n}^{(2)}(\theta) = N_{n}^{(2)} \frac{1 - \cos \theta_{n}}{\sin \theta_{n}} \left( \frac{1}{2} \left( \frac{1}{2} \sqrt{4 A_n + B_n + \lambda_{03} + \lambda_{02} + 4 c_{14}} \right) + \frac{1}{2} \frac{1}{2} \sqrt{4 A_n + B_n + \lambda_{03} - \lambda_{02} + 4 c_{14}} \right)
\]

The generalized wave functions are

\[
P(\theta) = N_{n_1} \frac{1 - \cos \theta_{n_1}}{\sin \theta_{n_1}} \left( \frac{1}{2} \sqrt{4 A_{n_1} + B_{n_1} + \lambda_{03} + \lambda_{02} + 4 c_{14}} \right) P_{n_1}(\cos \theta)
\]

\[
P(\theta) = N_{n_2} \frac{1 - \cos \theta_{n_2}}{\sin \theta_{n_2}} \left( \frac{1}{2} \sqrt{4 A_{n_2} + B_{n_2} + \lambda_{03} - \lambda_{02} + 4 c_{14}} \right) P_{n_2}(\cos \theta)
\]

and

\[
P(\theta) = N_{n_3} \frac{1 - \cos \theta_{n_3}}{\sin \theta_{n_3}} \left( \frac{1}{2} \sqrt{4 A_{n_3} + B_{n_3} + \lambda_{03} + \lambda_{02} - 4 c_{14}} \right) P_{n_3}(\cos \theta)
\]

By using the same value of potential parameters, the wave functions for the original Scarf potential and the partner potential as potential construction result are different. For details of the wave function of Equation (50) and Equation (51) can be shown Table 1.
Table 1 Three-dimensional representations of the angular wave function for variation of $n_l$, $a$ and $b$.

| $a$ | $b$ | $n_l$ | $P(\theta_2)$ | $P'(\theta_2)$ |
|-----|-----|-------|----------------|----------------|
| 4   | 4   | 0     | ![Image](image1.png) | ![Image](image2.png) |
| 4   | 4   | 1     | ![Image](image3.png) | ![Image](image4.png) |
| 6   | 6   | 1     | ![Image](image5.png) | ![Image](image6.png) |
| 8   | 8   | 1     | ![Image](image7.png) | ![Image](image8.png) |
From Table 1, we have wave function for variations of $a$, $b$, and $n_l$. The increases of $n_l$ value is given the effect in form of wave functions and the motion range of particle. The wave functions for the original Scarf potential and its the partner potential are different. There is a decrease in the amplitude value at $P'(\theta_2)$ beside $P(\theta_2)$. An increase in the value of $n_l$ causes a decrease in the amplitude value. The increase of $a$ and $b$ values causes a decrease in the amplitude value.

4. Conclusion
In this paper, we have presented the solution of 4-dimensional Schrodinger equation of angular part for Scarf potential and partner potential using Nikiforov-Uvarov method and Supersymmetric quantum mechanics method. We obtained the wave functions from angular part solution, which the wave functions depend on the parameters of all components of the composed potential. The partner potential have different wave functions compared to the ground state wave functions of the original potential. This partner potential was considered to be new potential.

Acknowledgement
This research was partly supported by PUT-MRG of Sebelas Maret University

References
[1] Shi-Hai Dong 2011 Wave Equations in Higher Dimensions Springer New York P97
[2] Wahyulianti, Suparmi, Cari, and Fuad Anwar 2017 IOP Conf. Series: Journal of Physics: Conf. Series 795 012022
[3] B H Yazarloo, L Lu, G Liu, S Zarrinkamar, and H Hassanabadi 2013 Advances in High Energy Physics 2013 5
[4] A. Suparmi, C. Cari, Beta Nur Pratiwi, and Dewanta Arya Nugraha 2017 IOP Conf. Series: Journal of Physics: Conf. Series 820 012023
[5] A. N. Ikot 2013 Communications in Theoretical Physics 59 268-272
[6] Benedict Iserom Ita, Alexander Immaanyikwa Ikeuba 2013 Applied Mathematics 4 1-6
[7] D. Agboola 2009 Phys. Scr. 80 065304
[8] Suparmi dan Cari 2014 J. Math. Fund. Sci. 46 205
[9] A Suparmi, C Cari, and U A Deta 2014 Chin. Phys. B 23 090304
[10] R A Sari, A Suparmi, and C Cari 2016 Chin. Phys. B 25 010301
[11] Ituen B Okon, Eno E Ituen, Oyebola Popoola, and Akaninyene D Antia 2013 International Journal of Recent Advances in Physics (IJRAP) 2 1
[12] Akpan N Ikot, O A Awoga, and A D Antia 2013 Chin. Phys. B 22 020304
[13] A Suparmi, C Cari, Beta Nur Pratiwi, and Dewanta Arya Nugraha 2017 IOP Conf. Series: Journal of Physics: Conf. Series 795 012001
[14] Luqman H, Suparmi, Cari, and U.A. Deta 2014 IOP Conf. Series: Journal of Physics: Conference Series 539 012019