Hot medium effects on pseudotensor \( K_2(1820) \) meson

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**Abstract.** The investigation of mesons in hot medium can give valuable information about the nature of QCD vacuum and deconfinement phase transitions. In this study, thermal properties of pseudotensor \( K_2(1820) \) meson is examined via QCD sum rules at finite temperature. The mass and the decay constant of \( K_2(1820) \) are estimated up to dimension-five by considering the new operators emerging at finite temperature. It is seen that after a certain temperature, the decay constant and the mass decrease significantly due to the hot medium effects.

1 Introduction

Investigating the spectroscopy of pseudotensor mesons which have more complex structures rather than conventional mesons is necessary to understand the QCD dynamics and completion of the hadron spectrum. Also, the behaviour of pseudotensor mesons in hot medium might give us some different clues. It is believed that within the close proximity of a critical temperature, \( T_c = 190 \) MeV, a transition occurs from hadronic matter to Quark Gluon Plasma (QGP) phase [1, 2]. Therefore, thermal QCD studies on the hadronic properties can provide us valuable information on the deconfinement phase transition, which is believed to exist at the very early stage of the Big Bang [3].

The strange pseudotensor states \( K_2(1770) \) and \( K_2(1820) \) were observed by several experiments [4–6], and their vacuum properties were studied in Refs. [7, 8]. However, the thermal properties of pseudotensor states in the strange sector are not investigated. In this work, we use thermal QCD sum rules [9], which is an extension of SVZ sum rules [10] at finite temperature, to construct the mass and the decay constant sum rules of pseudotensor \( K_2(1820) \) meson. Calculations are made up to operator dimension five, and additional operators coming from the hot medium are also considered.

2 Theoretical framework

In order to achieve the mass and the decay constant from sum rules, the interpolating current of the meson under investigation should be chosen carefully. Following Refs. [7, 8], we interpret \( K_2(1820) \) as a pure \( ^3D_2 \) state with \( J^{PC} = 2^{--} \), and chose the interpolating current as

\[
J_{\mu\nu}(x) = \frac{i}{2} [\bar{s}(x)\gamma_\mu \gamma_5 \overleftrightarrow{D}_\nu(x)d(x) + \bar{s}(x)\gamma_\nu \gamma_5 \overleftrightarrow{D}_\mu(x)d(x)],
\]

(1)

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where $\overrightarrow{D}_\mu(x)$ denotes the derivative with respect to four-$x$ acting on left and right simultaneously, and given as

$$\overrightarrow{D}_\mu(x) = \frac{1}{2} \left[ \overleftarrow{D}_\mu(x) - \overrightarrow{D}_\mu(x) \right],$$

$$\overleftarrow{D}_\mu(x) = \partial_\mu(x) - i \frac{q}{2} A_\mu^a(x),$$

$$\overrightarrow{D}_\mu(x) = \partial_\mu(x) + i \frac{q}{2} A_\mu^a(x),$$

(2)

where $A_\mu^a(x)$ and $A_\mu^a(x)$ are the Gell-Mann matrices and gluon fields.

Then, the temperature dependent two point correlation function is written in terms of the interpolating current as

$$\Pi_{\mu\nu,ab}(q, T) = i \int d^4x e^{iq(x-y)} \langle \mathcal{T} \left( J_{\mu\nu}(x) J_{ab}(y) \right) \rangle |_{\mu\nu=0},$$

(3)

where $\mathcal{T}$ is the time ordered product, and $\Omega$ denotes the hot medium.

In thermal QCD sum rules, two point correlation function is calculated with two approaches in two different regions. In physical side (at large distances), it is expressed at the hadronic level, and in terms of hadronic parameters. In QCD side (at short distances), it is evaluated at the quark level, and in terms of QCD parameters such as quark-gluon and mixed condensate via operator product expansion (OPE). To obtain the physical quantities we need to equate both sides using the coefficients of the chosen Lorentz structure. In order to remove the continuum, quark-hadron duality an is employed, and Borel transformation is applied to suppress the unwanted contributions, such as polynomials. Finally, the ground state hadron is isolated from the continuum, and its mass and decay constant can be extracted.

In the QCD side, contributions coming from the short distances can be written as a dispersion relation

$$\Pi^{\text{QCD}}(q, T) = \int ds \rho^{\text{QCD}}(s) \frac{1}{s - q^2},$$

(4)

where $\rho^{\text{QCD}}(s)$ is the spectral density which is obtained from imaginary part of the selected coefficient via $\rho^{\text{QCD}}(s) = \text{Im}[\Pi^{\text{QCD}}(s, T)]/\pi$. In order to get the spectral density, we use the thermal light quark propagators as

$$S_{ij}(x - y) = i \frac{k - \bar{y}}{2\pi^2(x - y)^4} \delta_{ij} - \frac{m_q}{4\pi^2(x - y)^2} \delta_{ij} - \frac{\bar{q}q_T}{12} \delta_{ij} - \frac{(x - y)^2}{192} m_0^2(\bar{q}q)_T$$

$$\times \left[ 1 - i \frac{m_q}{6} (k - \bar{y}) \delta_{ij} + \frac{i}{3} [(k - \bar{y})](m_q^2(\bar{q}q)_T - \frac{1}{12} \langle u\Theta' u \rangle) + \frac{1}{3}(u \cdot (x - y)) \right]$$

$$\times \frac{G_{ij}^a(k - \bar{y})}{32\pi^2(x - y)^2} G_{\mu\nu}(k - \bar{y}) \sigma^{\mu\nu} + \sigma^{\mu\nu}(k - \bar{y}) \delta_{ij},$$

(5)

where $u_\mu$ is four-velocity of the heat bath and $\Theta'_{\mu}$ is the fermionic part of the energy momentum tensor. After standard but lengthy calculations, perturbative contributions to QCD side are obtained as

$$\rho_{K}(s) = \frac{3 s^2 - 10 s m_d m_s}{80\pi^2}.$$ 

(6)

where only the coefficients of chosen Lorentz structure, which is $(g_{\mu\nu} g_{ij} + g_{\mu\nu} g_{ij})/2$ in this work, are shown explicitly. The explicit forms of non perturbative contributions are not given here for brevity.
In the physical side, a complete set of states with the same quantum numbers as $K_2(1820)$ are inserted in Eq. (3), and decay constants are related to the matrix elements via relation $\langle \Omega | J_{\mu \nu}(0) | K_2 \rangle = f_{K_2}(T) m_{K_2}^2(T) \epsilon_{\mu \nu}$, where $\epsilon_{\mu \nu}$ is the polarization tensor. After performing the integral over four-$x$, final expression in the physical side is evaluated as

$$\Pi_{\mu \nu, \alpha \beta}^{phys}(q, T) = \frac{f_{K_2}(T) m_{K_2}^0(T)}{m_{K_2}^2(T) - q^2} + \ldots,$$

(7)

here dots denotes the contribution of continuum and higher states. After employing Borel transformation with respect to $q^2$, to both physical and QCD sides, sum rules are obtained as

$$f_{K_2}^2(T) m_{K_2}^6(T) e^{-m_{K_2}^2(T)/M^2} = \int_{s_{\text{min}}}^{s_0(T)} ds \rho(s) e^{(-s)/M^2} + \hat{B} \Pi^{\text{non-pert}}(q, T).$$

(8)

In order to extract mass from the sum rules given in Eq. (8), $f_h$ dependence of the expression is removed by standard manipulations and the temperature dependent mass sum rules is obtained as

$$m_{K_2}^2(T) = \frac{\int_{s_{\text{min}}}^{s_0(T)} ds \rho(s) s e^{(-s)/M^2} + \frac{d}{dT} \Pi^{\text{non-pert}}(q, T)}{\int_{s_{\text{min}}}^{s_0(T)} ds \rho(s) e^{(-s)/M^2} + \hat{B} \Pi^{\text{non-pert}}(q, T)}.$$ 

(9)

In the above expression $M^2$ is the Borel parameter and $s_0(T)$ is the thermal continuum threshold.

### 3 Numerical analysis

To test the reliability of the obtained sum rules, first the numerical analysis is done at $T = 0$, and the working regions of parameters $s_0$ and $M^2$ are determined by checking the requirements of OPE convergence and pole dominance. Working regions are obtained as given in Table 1. Denote that, to satisfy pole dominance, $M^2$ should also satisfy the condition given in column four. The values of the mass and the decay constant of $K_2(1820)$ are also given in columns four and five of Table 1, with corresponding uncertainties. The obtained results at vacuum are in good agreement with those of Refs. [7, 8]. Our estimations for the mass and the decay constant exhibit very low dependence on $s_0$ and $M^2$.

| $M^2$ (GeV$^2$) | $s_0$ (GeV$^2$) | $M^2 \leq$ (GeV$^2$) | $m$ (GeV) | $f$ ($10^{-2}$) |
|-----------------|-----------------|------------------|----------|------------|
| $K_2(1820)$     | 1.8 - 2.2       | 4.5 - 5.4        | 0.5$s_0^2$ - 0.5 | 1.805$^{+0.048}_{-0.049}$ | 6.60$^{+0.08}_{-0.06}$ |

Table 1: For $K_2(1820)$, working ranges of $M^2$ and $s_0$ (columns 2-4), and vacuum values of the mass (column 5) and the decay constant (column 6).

In addition to analysis at $T = 0$, we also extended the extracted sum rules to finite temperatures. The thermal effects on the condensates are also considered. Concretely, thermal behaviours of the mass and the decay constant of pseudotensor $K_2(1820)$ are obtained. We present the dependence of the mass and the decay constant of $K_2(1820)$ in Figure 1. Details of this analysis will be presented in a future work. It is seen from Figure 1 that the mass and the decay constant of $K_2(1820)$ remain unchanged up to $T \approx 0.17$ GeV. However after this temperature, they begin to diminish rapidly with increasing temperature. Near the critical
Figure 1: The thermal behavior of the mass (left) and the decay constant (right) of pseudotensor $K_2(1820)$ meson.

At the critical temperature, the mass of $K_2(1820)$ reduces to 54% of its value in vacuum, while its coupling constant decreases to 26% of the vacuum value. These patterns indicate that the mass and the decay constant of strange pseudotensor mesons also dissolve at the critical temperature. These results can be tested in future, both by theoretical and experimental investigations, and might help us to understand the nature of QCD at finite temperatures.

References

[1] K. Yagi, T. Hatsuda, Y. Miake, Quark-Gluon Plasma (Cambridge University press, 2005)
[2] J. Letessier, J. Rafelski, Hadrons and Quark-Gluon Plasma (Cambridge University press, 2002)
[3] T. Matsui, H. Satz, Phys. Lett. B 178, 416 (1986)
[4] C. Daum et al. (ACCMOR Collaboration), Nucl. Phys. B 187, 1-41 (1981)
[5] D. Aston et al., Phys. Lett. B 308, 186-192 (1993)
[6] R. Aaij et al. (LHCb Collaboration), Phys. Rev. Lett. 118, no.2, 022003 (2017)
[7] W. Chen, Z. X. Cai, S. L. Zhu, Nucl. Phys. B 887, 201 (2014)
[8] T. M. Aliiev, S. Bilmis, K. C. Yang, Nucl. Phys. B 931, 132-145 (2018)
[9] A. I. Bochkarev, M. E. Shaposhnikov, Nucl. Phys. B 268, 220 (1986)
[10] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B 147, 385 (1979)