On the issue of the solutions existence of the problem of gas hydrate dissociation in a porous medium with the formation of an extended region of phase transitions

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Abstract. Mathematical model of the gas hydrate dissociation in a porous medium with the formation of an extended region of phase transitions is proposed. In a rectilinear-parallel approximation self-similar solutions are constructed that describe the distribution of pressure, temperature, and hydrate saturation in a reservoir. Based on the obtained solutions analysis it was proved that for the problem of injecting warm gas into a hydrate saturated reservoir with piezoconductivity coefficient exceeds the thermal diffusivity coefficient, there are no solutions with an extended region of phase transitions. The existence of such solutions is possible during gas production from a gas hydrate reservoir due to a decrease in pressure at the extraction point.

1. Introduction

A significant part of the world's natural gas reserves is concentrated in gas hydrate deposits [1, 2], which are solid solutions of gas in water with a high relative mass fraction of gas in hydrate \( G \geq 0.1 \) [3-5]. To improve the technology of gas production from hydrate deposits, theoretical studies of the decomposition of gas hydrates in porous structures is necessary. Some features of this process are presented in [6-9]. In these works solutions are constructed for the decomposition of gas hydrates in a porous medium for the case of phase transitions on a frontal surface. Such mathematical description is adequate for a limited range of parameters characterizing the state of the "porous medium + gas hydrate" system.

In this work in a rectilinear-parallel approximation we studied the conditions under which the formation of an extended region of the gas hydrates decomposition in a porous medium is possible.

2. Problem statement and basic equations

Suppose that at the initial time moment a porous reservoir is saturated with methane and its hydrate. The pressure \( p_0 \) and temperature \( T_0 \) in the reservoir initially satisfy the thermodynamic conditions for the stable existence of gas hydrate:

\[
t = 0, \quad x \geq 0: \quad p = p_0, \quad T = T_0.
\]
We assume that the pressure $p_e$ and temperature $T_e$, which correspond to the decomposition conditions of methane hydrate, are instantly established on the left reservoir boundary (in the sequel these values are kept constant):

$$t > 0,\ x = 0: \ p = p_e,\ T = T_e.$$ 

It is known that in order to exert a thermal effect on a hydrate saturated reservoir (for example, when warm gas is injected into the reservoir [10, 11]), the following condition must be fulfilled:

$$p_e > p_0,\ T_e > T_0.$$ 

When the depressive effect on the reservoir is realized, the pressure at the reservoir boundary decreases, i.e. the condition is satisfied: $p_e < p_0$ [7, 12, 13].

At the thermal or depressive effects on a hydrate saturated porous medium, gas hydrate decomposes. Three regions may occur in the reservoir; the location of these regions and the composition of the fluids saturating the porous medium are shown in figure 1.

![Figure 1. Scheme of the problem of the methane hydrate decomposition in a reservoir. The subscripts $s$ and $m$ in brackets refer to the parameters at the boundaries $x = x(s)$ and $x = x(m)$, respectively.](image)

In the second region, where the process of the gas hydrate decomposition occurs, the condition of phase equilibrium (gas, water and gas hydrate) is satisfied. The values of temperature and pressure in this region are related by the condition of phase equilibrium:

$$T = T_0 + T_0 \ln\left(\frac{p}{p_{s0}}\right),$$

where $T_0$ is the initial temperature of the system; $p_{s0}$ is the equilibrium pressure corresponding to the temperature $T_0$; $T_s$ is an empirical parameter depending on the type of gas hydrate and the phases on which it dissociates.

On the surface $x = x(m)$, dividing 2 and 3 regions, the decomposition of methane hydrate begins. And this process ends on the surface $x = x(s)$ (figure 1).

The system of basic assumptions and equations for the presented problem statement can be written on the basis of [9, 12, 14–16]. The dependence of phase permeability for gas $k_g$ on gas saturation $S_g$ is set based on the Kozeny formula: $k_g = k S_g^4$, where $k$ is the porous medium permeability.

The gas saturation in each region ($i = 1, 2, 3$) will be considered a constant value. The equations of piezoconductivity and thermal diffusivity in a rectilinear-parallel approximation can be represented as:

$$\frac{\partial p}{\partial t} = \frac{k_g}{m S_g \mu_g} \frac{\partial}{\partial x} \left( p \frac{\partial p}{\partial x} \right) - \frac{\rho_{g0} G_p}{\rho_{g0} (1 - \nu)} \frac{\partial S_h}{\partial t},$$

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c} \frac{\partial}{\partial x} \left( \frac{\partial T}{\partial x} \right) + \frac{c_s k_g \rho_g}{\rho c \mu_g} \frac{\partial p}{\partial x} \frac{\partial T}{\partial x} + \frac{m p_{h0} L_{h0}}{\rho c} \frac{\partial S_h}{\partial t},$$

$$\rho c = (1 - m) \rho_{sk} c_{sk} + m \sum_{j=g, w, h} \rho_j S_j c_j, \quad \lambda = (1 - m) \lambda_{sk} + m \sum_{j=g, w, h} S_j \lambda_j, \quad \sum_{j=g, w, h} S_j = 1.$$
Here \( p \) is the pressure; \( T \) is the temperature; \( t \) is the time; \( G \) is the mass concentration of gas in methane hydrate; \( \rho_j \) \((j = sk, h, w, g)\) is the true density of the \( j \)-th phase, \( S_j \) \((j = g, w, h)\) is the pore saturation with the \( j \)-th phase; \( v \) is the initial hydrate saturation; \( c_j \) and \( \lambda_j \) \((j = g, w, h)\) are the specific heat and thermal conductivity of the \( j \)-th phase; \( \mu_g \) is the dynamic viscosity of the gas phase; \( m \) is the porous medium porosity; \( L_h \) is the specific heat of the gas hydrate decomposition; the subscripts \( sk, h, w \) and \( g \) refer to the parameters of the porous skeleton, methane hydrate, water, and gas, respectively.

On the surface \( x = x_{(s)} \) a jump in hydrate saturation occurs from \( S_h^- = 0 \) to \( S_h^+ = S_{h(s)} \). From the conditions of the balances of mass and heat on this boundary, taking into account the Darcy law, the equation of state for gas and the condition of phase equilibrium (1), we can write:

\[
\frac{k_1 \partial p(1)}{\partial x} - k_2 \frac{\partial p(2)}{\partial x} = m \mu_g S_{h(s)} \left( \frac{\rho_h G}{\rho_g(s)} + \frac{\rho_h(1 - G)}{\rho_w} - 1 \right) \dot{x}_{(s)},
\]

where \( \dot{x}_{(s)} \) is the velocity of the phase transition boundary; the regions parameters are provided with subscripts in parentheses \( i = 1, 2, 3 \).

On the surface \( x = x_{(m)} \) the value of hydrate saturation is equal to the initial hydrate saturation \( v \). From the conditions of the balances of mass and heat on this boundary, taking into account the Darcy law and the phase equilibrium condition (1), we have the following equations:

\[
\frac{\partial p(2)}{\partial x} = \frac{\partial p(3)}{\partial x}, \quad \frac{\partial T(2)}{\partial x} = \frac{\partial T(3)}{\partial x}, \quad \frac{\partial T(2)}{\partial x} = \frac{T_s}{p(m)} \frac{\partial p(2)}{\partial x}.
\]

On both interphase boundaries, the pressure and temperature will be considered continuous quantities:

\[
x = x_{(s)}: \quad p(1) = p(2) = p(s), \quad T(1) = T(2) = T(s),
\]

\[
x = x_{(m)}: \quad p(1) = p(2) = p(m), \quad T(1) = T(2) = T(m).
\]

3. Self-similar solution

We introduce a self-similar variable \( \xi = x/\sqrt{x^{(T)} t} \). Then the system of equations (2) based on the Leibenzon linearization method can be represented as a system of ordinary differential equations of the second order:

\[
-\frac{\xi}{2} \frac{dp^2}{d\xi^2} = \eta \frac{d}{d\xi} \left( \frac{dp^2}{d\xi} \right) + \phi_h p^2 \frac{dS_h}{d\xi},
\]

\[
-\frac{\xi}{2} \frac{dT}{d\xi} = \frac{\xi}{2} \frac{d}{d\xi} \left( \frac{dT}{d\xi} \right) + \frac{Pe}{2 \nu_0} \frac{dp^2}{d\xi} \frac{dT}{d\xi} - \frac{\Delta T}{2} \frac{dS_h}{d\xi} + \phi_h \frac{dS_h}{d\xi},
\]

\[
\eta = \frac{x^{(p)}}{x^{(T)}}, \quad \chi^{(T)} = \frac{\lambda}{\rho c}, \quad \chi^{(p)} = \frac{k_g p_0}{m(1 - v) \mu_g}, \quad \phi_h = \frac{\rho_h G}{\rho_g(1 - v)}, \quad \Delta T = \frac{m \rho_h L_h}{\rho c}, \quad Pe = \frac{\rho_g \nu_0 c_k g \xi}{\mu_g \lambda}.
\]
In self-similar variables relations (3) can be written as:

\[
 k_1 \frac{dp_{(1)}^2}{d\xi} - k_2 \frac{dp_{(2)}^2}{d\xi} = K S_{h(s)} \xi, \\
(6)
\]

\[
 \frac{dT_{(2)}}{d\xi} - \frac{dT_{(1)}}{d\xi} = \frac{\Delta T}{2} S_{h(s)} \xi, \\
\]

\[
 \frac{dT_{(2)}}{d\xi} = T_s \frac{dp_{(2)}^2}{d\xi}, \\
\]

where \( K = m u g \xi^2 \) \( \rho \) \( \left( \frac{\rho_h G}{\rho_h (1-G)} - 1 \right) \).

Similarly, in self-similar variables based on (4) we get:

\[
 \frac{dp_{(2)}^2}{d\xi} = \frac{dp_{(3)}^2}{d\xi}, \quad \frac{dT_{(2)}}{d\xi} = \frac{dT_{(3)}}{d\xi}, \quad \frac{dT_{(2)}}{d\xi} = \frac{T_s}{2} \frac{dp_{(2)}^2}{d\xi}. \\
(7)
\]

Based on (5), distributions for pressure, temperature, and hydrate saturation in each region can be found:

- in the first region \((0 < \xi < \xi_{(1)})\):

\[
 p_{(1)} = p_{(1)} + (p_{(2)} - p_{(3)}) \frac{\xi_{(1)}}{\xi} \int_0^{\xi_{(1)}} \exp \left( -\frac{\xi^2}{4\eta_{(1)}} \right) d\xi, \\
 T_{(1)} = T_s + (T_{(2)} - T_{(3)}) \frac{\xi_{(1)}}{\xi} \int_0^{\xi_{(1)}} \exp \left( -\frac{\xi^2}{4\eta_{(1)}} \right) d\xi, \\
 S_{h(1)} = 0, \\
(8)
\]

- in the second region \((\xi_{(1)} < \xi < \xi_{(2)})\):

\[
 p_{(2)} = p_{(2)} + \left( p_{(3)} - p_{(2)} \right) \frac{\xi_{(2)}}{\xi} \int_0^{\xi_{(2)}} \exp \left( -\frac{\xi^2}{4\eta_{(2)}} \right) d\xi, \\
 T_{(2)} = T_s + T_s \ln \left( \frac{p_{(2)}}{p_{(3)}} \right), \\
 S_{h(2)} = \frac{T_s \left( \eta_{(2)} - 1 \right)}{2 \phi_h T_s + \eta_{(2)} \Delta T} \ln \left( \frac{p_{(2)}}{p_{(m)}} \right) + \nu, \\
(9)
\]

- in the third region \((\xi_{(2)} < \xi < \infty)\):

\[
 p_{(3)} = p_{(3)} + \left( p_{(m)} - p_{(3)} \right) \frac{\xi_{(m)}}{\xi} \int_0^{\xi_{(m)}} \exp \left( -\frac{\xi^2}{4\eta_{(3)}} \right) d\xi, \\
 T_{(3)} = T_0 + (T_{(m)} - T_0) \int_0^{\xi_{(m)}} \exp \left( -\frac{\xi^2}{4\eta_{(3)}} \right) d\xi, \\
(10)
\]
\[ S_{h(3)} = \nu, \]

where \( \chi = \frac{\phi h T_\ast + \eta(2) \Delta T}{\phi h T_\ast + \Delta T} \).

From the third equation of the system (9) we obtain the value of hydrate saturation on the boundary \( \xi = \xi_{(2)} \) (from the second area side):

\[ S_{h(3)} = \frac{T_\ast (\eta(2) - 1)}{2(\phi h T_\ast + \eta(2) \Delta T)} \ln \frac{p^2_{(s)}}{p^2_{(m)}} + \nu. \]  \hspace{1cm} (11)

4. Analysis of the obtained solutions

For real natural reservoirs, the reservoir piezococonductivity coefficient \( \chi^{(p)} \) significantly exceeds the thermal diffusivity coefficient \( \chi^{(T)} \), therefore we assume that the following condition is satisfied:

\[ \chi^{(p)} > \chi^{(T)} . \]

In this case \( \eta(2) = \frac{\chi^{(p)}_{(2)}}{\chi^{(T)}} > 1 \) and therefore, the multiplier for the logarithmic function in formula (11) is a positive quantity:

\[ \frac{T_\ast (\eta(2) - 1)}{2(\phi h T_\ast + \eta(2) \Delta T)} > 0 . \]

Since in the initial state the porous medium was saturated only with gas and hydrate, and hydrate and water are assumed to be immovable, the hydrate saturation value in the second region cannot exceed the initial hydrate saturation value \( \nu \). In this case, from the relation (11) for the existence of a volumetric region of hydrate decomposition, the following condition must be satisfied:

\[ p_{(s)} < p_{(m)} \]  \hspace{1cm} (12)

Substituting the solutions for the pressure distribution in the first and second regions (8) and (9) into the first equation of system (6), we obtain the following relation on the boundary \( \xi = \xi_{(1)} \):

\[ k_{(1)} \frac{p^2_{(s)} - p^2_{e}}{\exp \left( -\frac{\xi^2}{4\eta(1)} \right)} - k_{(2)} \frac{p^2_{(m)} - p^2_{(s)}}{\exp \left( -\frac{\xi^2}{4\chi} \right)} = K S_{h(s)} \frac{\xi}{\xi_{(s)}}, \]  \hspace{1cm} (13)

Similarly, substituting the solutions for the pressure distribution in the second and third regions (9) and (10) into the first equation (7), we obtain the following relation on the boundary \( \xi = \xi_{(m)} \):

\[ \frac{\left( \frac{p^2_{(m)} - p^2_{(s)}}{\exp \left( -\frac{\xi^2}{4\eta(3)} \right)} \right)}{\xi_{(m)}} = \frac{\left( \frac{p^2_{0} - p^2_{(m)}}{\exp \left( -\frac{\xi^2}{4\chi} \right)} \right)}{\xi_{(m)}}, \]  \hspace{1cm} (14)

In Eq. (13), the right side is a positive quantity, therefore, taking into account condition (12), we have: \( p_e < p_{(s)} \). From here we can write the following inequality:

\[ p_e < p_{(s)} < p_{(m)}. \]  \hspace{1cm} (15)
Given this inequality, it can be shown that the left side of relation (14) is a positive quantity. The right side of this relation is greater than zero when the following inequality satisfied:

From here we can write the following inequality:

$$p_{(m)} < p_0$$  \hspace{1cm} (16)$$

Based on inequalities (15) and (16), we can write the necessary condition for the existence of a volume region of gas hydrate decomposition:

$$p_e < p_{(1)} < p_{(m)} < p_0$$  \hspace{1cm} (17)$$

5. Conclusions

It follows from the last inequality that the extended decomposition region of gas hydrate is realized when the pressure on the left reservoir boundary decreases, i.e. in the implementation of depressive effects on a hydrate saturated reservoir. If warm gas is injected into a reservoir with pressure $p_e$ and temperature $T_e$ corresponding to the decomposition conditions of methane hydrate, then the phase transition will occur only on the frontal surface.

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