Applications of semiempirical shell model masses based on magic proton number $Z = 126$ to superheavy nuclei

S. Liran*, A. Marinov and N. Zeldes

The Racah Institute of Physics, The Hebrew University of Jerusalem, Jerusalem 91904, Israel

Abstract

A recently published highly extrapolatable semiempirical shell model mass equation is shown to describe rather well the energies of several seemingly well identified $\alpha$-decay chains with known end product nuclei observed in superheavy elements research. The equation is also applied to the interpretation problem of some recent hot fusion-evaporation experiments with unknown end products and several conceivable reaction channels. Some plausible interpretations are indicated.

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I. INTRODUCTION

Recently [1,2] a semiempirical shell model mass equation (SSME) based on proton magic number $Z = 126$ [3] was shown to have a high predictive power in the interior of major shell regions beyond lead. The equation has been proposed as a substitute inside the above regions for the previous SSME [4] which is based on $Z = 114$ as the highest proton magic number and it stops there. Tabulated masses with separation and decay energies for both regions are available [5].

In the present work the predicted masses (eq. (1) of ref. [1], referred to as eq. (1) in the sequel, and table B of ref. [3]) are applied to the results of recent superheavy elements (SHE) experiments. We first address the smoothness of the predictions and their overall agreement with $Q_\alpha$ systematics. Then we test the agreement of the equation with the data observed in several recent fusion-evaporation SHE experiments where the produced nuclei are identified by their decay ties to known daughter isotopes. Finally we address the interpretation problem in some recent hot fusion experiments where the observed decay chains do not connect the parent to a known daughter, and several reaction channels are in principle possible.

*Present address: Kashtan 3/3, Haifa 34984, Israel.

1There are two such regions, with $82 \leq N \leq 126$ and with $126 \leq N \leq 184$. We refer to them respectively as region A and region B, like in refs. [1,2].
II. OVERSmoOTHNeSS OF THE EQUATIoN INSIDE ReGIoN B

Inside a shell region the mass surface predicted by eq. (1) is smoother than the empirical surface and does not account for fine structure effects.

This is illustrated in fig. 1 showing $Q_\alpha$ systematics for the heaviest $N \geq 140$ even-Z nuclei from Pu through $Z = 110$ [3]. Respective full and empty circles denote experimental values and values estimated from systematics. The small circles connected by thin lines show the predictions of eq. (1).

As a rule both the experimental and the predicted isotopic lines show similar negative trends when $N$ increases, and they shift upwards rather uniformly when $Z$ increases. For the experimental lines, though, this regular pattern breaks down for nuclei in the vicinity of the deformed doubly submagic nucleus $(N_0, Z_0)$ $^{252}\text{Fm}$ $(N_0 = 152, Z_0 = 100)$, and presumably even more so near $^{270}\text{Hs}$ $(N_0 = 162, Z_0 = 108)$ [7–10] (see also fig. 5). In these neighborhoods the trend of isotopic lines between $N = N_0$ and $N = N_0 + 2$ is positive, and the vertical distance between isotopic lines with $Z = Z_0$ and $Z = Z_0 + 2$ for $N \approx N_0$ is larger than for other $Z$ values.

None of these submagic number effects is shown by the predicted thin lines systematics. (See also refs. [1,4,11].)

In the SSME non-smooth abrupt local changes, which are associated with subshell and deformation effects, are assumed to have been smoothed out by configuration interaction (eq. (4) of ref. [1]), and the mass equation describes a smooth surface representing their average. The deviations from the average are mostly small, though, resulting in the above mentioned overall high-quality predictive power. For the 57 experimental $Q_\alpha$ values measured in region B after the equation was adjusted the respective average deviation ($\delta_{av}$) and rms deviation ($\delta_{rms}$) of the predicted values from the data are $-8$ and $220$ keV (table II of ref. [1]). (For all the 115 data points shown in fig. 1, both experimental and estimated from systematics, the corresponding values are $-33$ and $214$ keV.)

III. FUSION-EVAPORATION SEEMINGLY-WELL-IDENTIFIED $\alpha$-DECAY CHAINS OF ELEMENTS 106-112

We now address the agreement of the equation with $\alpha$-decay energies observed in the cold fusion production experiments of elements 107-112 [11–18] (see also refs. [19] and [20]).

Figs. 2-5 show the measured and the predicted $Q_\alpha$ values, assuming that the decays go through or near the g.s. Fig. 5 shows as well the $Q_\alpha$ values observed in hot fusion experiments producing $^{265}\text{Sg}$ [21][22] and $^{267}\text{Bh}$ [23][24], where in refs. [23] and [24] the $\alpha$-chains were studied after chemical separation of the recoiling evaporation residue (EVR) parent, leading to chemical identification.

In the Introduction of ref. [5] we were unaware of ref. [24], and ref. [23] was considered among those with an unidentified evaporation residue parent (like in sect. IV). We are indebted to Dr. A. Türler for clarification of the situation.
The figures show as well the $\delta_{av}$ and $\delta_{rms}$ values for the plotted data points. On the whole they are not large, with $\delta_{rms}$ values compatible with the above quoted 220 keV value from table II of ref. [1]. The respective overall $\delta_{av}$ and $\delta_{rms}$ values for all the 32 data points in figs. 2-5 are 86 and 337 keV.

The largest individual deviations occur for $Z = 107, 108$ and 110 in fig. 5, with respective values of 659, 712 and $-605$ keV. These seem to be too large for the above quoted value of 220 keV, and they correspond to a kink observed in the $^{277}112 Q_\alpha$ chain at $Z = 108$. If this kink is a genuine submagic number effect due to the crossing of $N = 162$ on the way from $Z = 108$ to $Z = 110$ [17][19], then fig. 5 illustrates the oversmoothness of the predicted masses as compared to the empirical data with its discontinuities in the vicinity of doubly submagic nuclei. (See sect. II). The pertinent submagic numbers in the present case are $N_0 = 162$ and $Z_0 = 108$.

Closer scrutiny of figs. 1 and 5 reveals stronger magicity of $^{270}Hs$ as compared to $^{252}Fm$.

IV. THE INTERPRETATION PROBLEM FOR SOME RECENT HOT FUSION EXPERIMENTS WITH NOT-WELL-IDENTIFIED $\alpha$-DECAY CHAINS

In hot fusion-evaporation SHE experiments, when the compound nucleus (CN) is produced at a sufficiently high excitation energy ($E_{cn}^x$), there are often several conceivable open emission channels for forming the parent, which would lead to different observable $\alpha$-decay chains. When the $\alpha$-decay chain starting from the EVR parent nucleus does not connect to a known daughter isotope, comparing different predictions to the observed data might facilitate the choice between different conceivable interpretations. We consider from this point of view some recent SHE experiments [25–28].

In refs. [25,27] a $^{248}Cm$ target was bombarded by 240 MeV $^{48}Ca$ ions. Three observed three-members $\alpha$-decay chains are assigned to the nuclide $^{292}116$ and its sequential decay down to $^{280}110$.

The CN formed in the reaction is $^{296}116$ at $E_{cn}^x \approx 27$ MeV obtained from the Cm and Ca masses [3] and the predicted mass of the CN [14]. At this higher energy more channels for particle emission might be open than in the cold fusion experiments considered in sect. III, including up to $3n$ and also $p$ or $\alpha$ emission. Four conceivable EVR parents in addition to the assigned parent $^{292}116$ are considered in table I. Their corresponding formation channels, the estimated values of their excitation energies ($E_{x}^{cvr}$) when formed, and the deviations

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3 We do not consider here the experiment described in ref. [29] which has not yet been confirmed. It has been considered from the present point of view in the Introduction of ref. [5].

4 The estimated values of $E_{x}^{cvr}$ in tables I and II are obtained from the kinematics of the reactions assuming that the evaporated neutrons have zero kinetic energy and the evaporated charged particles ($p$ and $\alpha$) have a kinetic energy which is equal to their potential energy at the top of the Coulomb barrier. Higher kinetic energies of the evaporated particles would reduce the estimates given in the tables. This is the case for the $xn$ channels, with an average C.M. neutron kinetic energy of about 1 MeV per neutron [30].
\( \delta_{av} \) and \( \delta_{rms} \) from the data of the corresponding predicted \( Q_\alpha \) values, assuming that the \( \alpha \)-decays go through or near the g.s., are given in the table.

For the \( 4n \) formation channel assigned by the authors the estimated \( E_{x}^{\text{evr}} \) value of the parent nucleus \( ^{292}116 \) is too low (less than 2 MeV) to have a reasonable production cross section. Moreover, the 758 keV \( \delta_{rms} \) value of the predicted \( Q_\alpha \) values is too large. On the other hand, for the \( 2n \) channel leading to the EVR parent \( ^{294}116 \) the corresponding estimated \( E_{x}^{\text{evr}} \) value and the value of \( \delta_{rms} \) are \( \leq 14 \) MeV and 466 keV which are reasonable. This might lend support to a scenario based on the \( 2n \) (and possibly also \( 3n \)) channel.

The last two members of the \( \alpha \)-decay chains seen in this experiment agree with the two-members \( \alpha \)-decay chains observed before in a \( Z = 114 \) experiment \[26\]. If they are the same, a formation of the present \( Z = 116 \) parent by \( 2n \) (or \( 3n \)) emission would imply the same formation channel for the \( Z = 114 \) EVR parent in ref. \[26\], rather than the assigned \( 4n \) channel.

Fig. 6 compares the experimental \( Q_\alpha \) values with the predictions of eq. (1) for assumed \( 2n, 3n \) and \( 4n \) evaporation channels. The improvement when going from \( 4n \) to \( 2n \) is clear.

Likewise, fig. 10 of ref. \[31\] shows the improved agreement with macroscopic-microscopic \( Q_\alpha \) systematics \[9\] achieved by adopting a \( 2n \) assignment as compared to \( 4n \).

As a second example we consider the experiment reported in ref. \[28\]. A \( ^{244}Pu \) target was bombarded by 236 MeV \( ^{48}Ca \) ions. An observed three-members \( \alpha \)-decay chain is considered a good candidate for originating from the parent \( ^{289}114 \) and its sequential decay down to \( ^{277}Hs \) (\( Z = 108 \)).

The CN formed in the reaction is \( ^{292}114 \) at \( E_{x}^{\text{en}} \approx 27 \) MeV \[1,5,6\]. Details for four conceivable EVR parents are given in table II.

For the \( 3n \) formation channel assigned by the authors, leading to the EVR parent \( ^{289}114 \), the respective estimated \( E_{x}^{\text{evr}} \) value and the \( \delta_{rms} \) value are \( \leq 9 \) MeV and 905 keV. The \( \delta_{rms} \) value is too large. If the assignment of the authors is confirmed this might indicate that the decay chain does not proceed through levels in the vicinity of the g.s. On the other hand, for the \( p \) or \( \alpha \) channels, leading to the respective EVR parents \( ^{291}113 \) or \( ^{288}112 \), the respective \( E_{x}^{\text{evr}} \) and \( \delta_{rms} \) values are 8 or 9 MeV and 414 or 363 keV. This might lend some support to scenarios based on the \( p \) or \( \alpha \) evaporation channels.

Fig. 7 compares the experimental \( Q_\alpha \) values with the predictions of eq. (1) for assumed \( 2n, 3n, p \) and \( \alpha \) evaporation channels. The advantage of the \( p \) or \( \alpha \) assignments over \( 3n \) and \( 2n \) is obvious.

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TABLES

TABLE I. Conceivable EVR parents of the $\alpha$-decay chain [25,27] with their formation channels, their estimated values of $E_{\text{evr}}^x$, and the deviations $\delta_{\text{av}}$ and $\delta_{\text{rms}}$ from the data of the corresponding predicted $Q_\alpha$ values.

| EVR parent | Evaporation channel | Estimated $E_{\text{evr}}^x$ (MeV) | $\delta_{\text{av}}$ (MeV) | $\delta_{\text{rms}}$ (MeV) |
|------------|---------------------|----------------------------------|--------------------------|-----------------------------|
| $^{294}\text{116}$ | $2n$ | 14 | 0.177 | 0.466 |
| $^{293}\text{116}$ | $3n$ | 7 | 0.423 | 0.577 |
| $^{292}\text{116}$ | $4n$ | 2 | 0.669 | 0.758 |
| $^{295}\text{115}$ | $p$ | 7 | -0.767 | 0.905 |
| $^{292}\text{114}$ | $\alpha$ | 8 | -0.590 | 0.699 |

$^a$See footnote 4.

TABLE II. Conceivable EVR parents of the $\alpha$-decay chain [28] with their formation channels, their estimated values of $E_{\text{evr}}^x$, and the deviations $\delta_{\text{av}}$ and $\delta_{\text{rms}}$ from the data of the corresponding predicted $Q_\alpha$ values.

| EVR parent | Evaporation channel | Estimated $E_{\text{evr}}^x$ (MeV) | $\delta_{\text{av}}$ (MeV) | $\delta_{\text{rms}}$ (MeV) |
|------------|---------------------|----------------------------------|--------------------------|-----------------------------|
| $^{290}\text{114}$ | $2n$ | 16 | 0.643 | 0.720 |
| $^{289}\text{114}$ | $3n$ | 9 | 0.847 | 0.905 |
| $^{291}\text{113}$ | $p$ | 8 | -0.241 | 0.414 |
| $^{288}\text{112}$ | $\alpha$ | 9 | -0.181 | 0.363 |

$^a$See footnote 4.
FIG. 1. $Q_\alpha$ systematics of even-Z elements from Pu through $Z = 110$ for $N \geq 140$. Experimental values and values estimated from systematics are taken from ref. [6] and predictions from refs. [1,5].
FIG. 2. Experimental $Q_\alpha$ values of the neutron excess $I = 48$ $\alpha$-decay chains starting from the parent nuclei $^{262}\text{107}$ [11] and $^{266}\text{109}$ [13,14]. The predictions are taken from refs. [1,5].

FIG. 3. Like fig. 2 for the $I = 49$ $\alpha$-decay chains starting from the parent nuclei $^{265}\text{108}$ [12] and $^{269}\text{110}$ [15].
FIG. 4. Like fig. 2 for the $I = 50$ $\alpha$-decay chain starting from the parent $^{272}_{111}$ [16,18].

FIG. 5. Like fig. 2 for the $I = 53$ $\alpha$-decay chains starting from the deformed g.s. of $^{277}_{112}$ [17-19], from $^{265}_{106}$Sg [21,22] and from $^{267}_{107}$Bh [23,24].
FIG. 6. Experimental [25,27] (full circles) and predicted [1,5] $Q_{\alpha}$ values for assumed 2$n$ (empty circles), 3$n$ (triangles) and 4$n$ (squares) evaporation channels in the $^{48}$Ca on $^{248}$Cm reaction leading to the $^{296}$116 CN.

FIG. 7. Experimental [28] (full circles) and predicted [1,5] $Q_{\alpha}$ values for assumed 2$n$ (empty circles), 3$n$ (triangles), $p$ (squares) and $\alpha$ (diamonds) evaporation channels in the $^{48}$Ca on $^{244}$Pu reaction leading to the $^{292}$114 CN.