COSET MODELS IN MONOPOLE BACKGROUNDS

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ABSTRACT: We study a fermionic coset model $G/H$ when the subgroup $H$ is not simply connected. We show that even when the fermionic zero modes impose selection rules which alters the values of the correlators, the Virasoro central charge of the theory results independent of the topological sector.
The $G/H$ coset construction of conformal algebras has proven to be extremely useful in the classification of conformal field theories [GKO]. In particular, it provides a method to obtain new representations of the Virasoro algebra by decomposing the representations of the original group $G$ Kac Moody algebras into representations of a subgroup $H$ Kac Moody algebras. The lagrangian approach for such models, either fermionic or bosonic, is based in the following idea [coset,coset2]: starting from a theory with global symmetry $G$, one constraints the Noether currents in $H$ in a gauge invariant way. In the case of fermionic models [coset] one introduces to this end gauge fields acting as Lagrange multipliers which enforce the constraints. The resulting theory is then bosonized using conventional path integral methods.

An interesting problem is posed when the subgroup $H$ is not simply connected. In this case it is possible to classify topologically inequivalent gauge fields by the fundamental group $\Pi_1(H)$. Important examples of those groups are of the form $H = \tilde{H}/Z$ where $\tilde{H}$ is a compact, connected, simple and simply connected Lie group and $Z$ is a discrete subgroup of the center. For such models the gauge field path integral decomposes into a sum of terms labelled by the fundamental group of $H$ [Bardakci,ellos,cm,yo].

In this work we study a constrained fermionic model with $G = U(NK)$ and $H = SU(N)_K/Z_N$ ($K$ is the Kac Moody level and $Z_N = Z/N$ the center of $SU(N)$). We isolate the topologically non-trivial contribution from the zero charge degrees of freedom of the gauge field and decouple the latter from the fermions. This procedure leads to the factorization of the partition function in three terms: a partition function of fermions coupled to the non-trivial gauge field, a partition function of a gauged Wess-Zumino-Witten (WZW) action and a partition function of a ghost system also coupled minimally to the background monopole. With a convenient election of the monopole background with support at infin-
ity (the north pole), the three subsystems mentioned above become, separately, conformal invariant although the Virasoro generators are affected by the background field. Using the Dotsenko-Fateev [Dotfat] analogy with a Coulomb gas, we show that the modification of the energy-momentum tensor for each subsystem is given by
\[
\Delta T_{zz} \propto \partial_z j^{N-1}
\]
where \( j^{N-1} \) is the current in the direction of the “last” generator of the Cartan subalgebra of \( \hat{su}(N) \). This result is crucial since it implies that the total change of the energy-momentum tensor is proportional to the derivative of the total current \( J^{N-1} \), this being zero due to the imposed constraint. Consequently the modifications of the conformal anomalies for each subsystem cancel each other and the total central charge of the coset system is the same in any topological sector. We prove this statement by computing explicitly the conformal charges of the three subsystems for the general coset \( U(NK)/SU(N)_K \).

SECTION II

A fermionic realization of the coset model \( U(NK)/H \) is constructed starting from a theory of \( N \times K \) free Dirac fermions in which the currents associated to the subgroup \( H \) are constrained to zero \( \text{via} \) lagrange multipliers [coset]. The corresponding Lagrangian in \( R^2 \) is given by
\[
\mathcal{L} = \bar{\psi}_{im}(i\partial_{ij} + A_{ij})\psi_{im} \quad i, j = 1, ..., N; \ m = 1, ..., K
\]
where \( \psi_{im} \) are \( N \times K \) Dirac fermions and the gauge field \( A_\mu \) (which plays the role of a Lagrange multiplier) takes values in the Lie algebra of a compact connected semi-simple Lie group \( H \). The partition function reads
\[
Z = \int D\bar{\psi}D\psi DA \exp\left[\int d^2x \bar{\psi}(i\partial + A)\psi\right].
\]
In order to compactify the two dimensional manifold $R^2$ we impose appropriate boundary conditions at infinity:

$$\lim_{r \to \infty} A \rightarrow ig^{-1}dg$$

(2.3)

where $g(\theta) = \lim_{r \to \infty} g(r, \theta)$ belongs to the loop group $LH$ of $H$.

Topologically non trivial configurations corresponds to elements $g$ not belonging to the identity component $L_0H$ of $LH$. Examples of groups supporting non trivial configurations are [Cachoyyo]

$$H = \tilde{H}/Z$$

(2.4)

with $\tilde{H}$ a compact, connected, simply connected lie group and $Z$ a non trivial subgroup of the center. In particular we take $H = SU(N)/Z_N$ ($Z_N = Z/N$). The fundamental group of $H$ is

$$\Pi_1(SU(N)/Z_N) = Z_N$$

(2.5)

showing that $LH$ has $N$ connected components, each of that defining a different nontrivial configuration of the gauge field $A_\mu$. We will call $A_\mu^{(\alpha)}$ ($\alpha = 0, 1, ..., N - 1$) a gauge field “belonging to the topological class $\alpha$” in the sense of equations (2.3)-(2.5).

The above discussion shows that the functional integral measure in the partition function (equation (2.2)) splits in the sum of $N$ integrals, one for each topological class $\alpha$

$$DA \exp(-S[A]) = \sum_{\alpha=0}^{N-1} DA^{\alpha} \exp(-S[A^{\alpha}]).$$

(2.6)

There is a problem concerning the fermionic integration in equation (2.2). This integral can be computed using well known regularization prescriptions ($\zeta$ function, heat kernel, etc.) which are valid provided the gauge field $A_\mu$ is topologically trivial ($\alpha = 0$ in our case). Otherwise, the integral is zero because of the zero modes of the Dirac operator. There is, however, a way to overcome this difficulty [Bardakci] which we now discuss.
A zero class (trivial) gauge field $a_\mu$ can be written in the form:

$$a_z = i u^{-1} \partial_z u$$

$$a_{\bar{z}} = i v^{-1} \partial_{\bar{z}} v$$

(2.7)

(2.8)

where $z = x + iy$, $\bar{z} = x - iy$ and $u$ and $v$ are single-valued fields taking values in $H$. Then we can write a general gauge field of class $\alpha$ in the manner

$$A_z^{(\alpha)} = a_z + u^{-1} A_z^{(\alpha)} u$$

$$A_{\bar{z}}^{(\alpha)} = a_{\bar{z}} + v^{-1} A_{\bar{z}}^{(\alpha)} v$$

(2.9)

(2.10)

where $A_z^{(\alpha)}$ is a fixed configuration of class $\alpha$. For convenience we choose this field satisfying the Lorentz gauge condition

$$\partial_\mu A_\mu^{(\alpha)} = 0.$$  

(2.11)

Hence the integral $D A^{\alpha}$ reduces to an integral in the topologically trivial fields $u$ and $v$ over the whole group manifold $H = SU(N)/Z_N$ (there is of course a jacobian which we will consider later). Finally, because of the invariance of $A_\mu^{(\alpha)}$ under gauge transformations in the center $Z_N$ we can express the $H$ integral in terms of the SU(N) variables as follows

$$\int_{SU(N)/Z_N} Dg \exp(-S[g]) = \frac{1}{N} \int_{SU(N)} D\tilde{g} \exp(-S[\tilde{g}]).$$

(2.12)

The final expression for the partition function $Z$ reads

$$Z = \frac{1}{N} \sum_{\alpha=0}^{N-1} \int_{SU(N)} D\tilde{u} D\tilde{v} J[\tilde{u}, \tilde{v}] \det \left( i \tilde{\phi} + A^{(\alpha)} \right)$$

(2.13)

where $A_\mu^{(\alpha)}$ is defined in equations (2.9),(2.10) and $J[\tilde{u}, \tilde{v}]$ is the jacobian

$$DA^{\alpha} = D\tilde{u} D\tilde{v} J[\tilde{u}, \tilde{v}].$$

(2.14)

Using standard methods it is possible to decouple the $\tilde{u}$ and $\tilde{v}$ fields from the fermions in the determinant. In fact the following identity holds [Polyakov]

$$\frac{\det \left( i \tilde{\phi} + A^{(\alpha)} \right)}{\det \left( i \tilde{\phi} + A^{(\alpha)} \right)} = \exp(-S[\tilde{u} \tilde{v}^{-1}, A_\mu^{(\alpha)}]).$$

(2.15)
Here

\[ S[g, A_\mu] = -KI[g] - \frac{K}{\pi} \int d^2 xtr \left( -iA_\bar{z} g^{-1} \partial_z g + iA_z \partial_\bar{z} g g^{-1} - A_\bar{z} g A_\bar{z} g^{-1} + A_z A_\bar{z} \right) \]

(2.16)

and \(-KI[g]\) is the level \(-K\) Wess-Zumino-Witten action. Of course the field \(A_\mu^{(\alpha)}\) cannot be decoupled from the fermions, since such field produces a non trivial kernel to the Dirac operator which prevents the use of the standard methods for computing determinants (but for a carefully treatment of the determinant of the Dirac operator with non trivial kernel see [Solo]).

Let us return to the jacobian in equation (2.14). A direct computation leads to:

\[ J[\tilde{u}, \tilde{v}] = \det \left( i\partial_z + A_\bar{z}^{(\alpha)} \right)_{ADJ} \det \left( i\partial_\bar{z} + A_\bar{z}^{(\alpha)} \right)_{ADJ} \]

(2.17)

where the subscript \(ADJ\) indicate that the Dirac operator acts in the adjoint representation of SU(N). This determinant can be related to the determinant of the same operator in the fundamental (defining) representation by the identity [Polyakov,jacobian](There is a similar equation for the \(\bar{z}\) component)

\[ \frac{\det \left( i\partial_z + A_\bar{z}^{(\alpha)} \right)_{ADJ}}{\det \left( i\partial_\bar{z} + A_\bar{z}^{(\alpha)} \right)_{ADJ}} = \left[ \frac{\det \left( i\partial_z + A_\bar{z}^{(\alpha)} \right)}{\det \left( i\partial_\bar{z} + A_\bar{z}^{(\alpha)} \right)} \right]^{2C_H} \]

(2.18)

where \(C_H\) is the eigenvalue of the Casimir operator in the adjoint representation (in our example \(C_{SU(N)} = N\)). Using equations (2.15)-(2.18) we can write

\[ J[\tilde{u}, \tilde{v}] = \exp(-2NS[\tilde{u}\tilde{v}^{-1}, A_\mu^{(\alpha)}]) \det \left( i\partial_z + A_\bar{z}^{(\alpha)} \right)_{ADJ} \det \left( i\partial_\bar{z} + A_\bar{z}^{(\alpha)} \right)_{ADJ}. \]

(2.19)

Finally, after fixing the trivial gauge \(\tilde{v} = 1\), we can write the partition function as

\[ Z = \frac{1}{N} \sum_{\alpha=0}^{N-1} \int_{SU(N)} D\tilde{g} \det \left( i\phi + A^{(\alpha)} \right) e^{(K+2N)S[\tilde{g}, A^{(\alpha)}_\mu]} \times \]

\[ \det \left( i\partial_z + A_\bar{z}^{(\alpha)} \right)_{ADJ} \det \left( i\partial_\bar{z} + A_\bar{z}^{(\alpha)} \right)_{ADJ}. \]

(2.20)
Note that in virtue of equations (2.3), (2.15) and (2.19), the field $\tilde{g}$ has trivial boundary conditions, \textit{i.e.},

$$
\lim_{r \to \infty} \tilde{g} \to I. \quad (2.21)
$$

\section*{SECTION III}

The equation (2.20) shows that the partition function of the theory is a sum of terms, one for each topological class. Individually each term can be factorized in three: the partition function of $NK$ Dirac fermions coupled to the background field $A_{\mu}^{(\alpha)}$, the partition function of a gauged Wess-Zumino-Witten theory and the partition function of $N^2 - 1$ pair of Fadeev-Popov ghosts also coupled to $A_{\mu}^{(\alpha)}$. As we commented before, the non-trivial topology has a deep impact in the Dirac operator: its kernel becomes non-trivial. In order to study this problem more carefully let us find the square-integrable solutions of the differential equation

$$
\left( i \partial + A^{(\alpha)} \right) \psi = 0, \quad \psi = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{bmatrix} \quad (3.1)
$$

(we are considering for simplicity the case with one flavor, the generalization to the case of $K$ flavors is straightforward). First at all we have to chose a representative background field. In the preceeding section we mapped the problem initially defined on $SU(N)/Z_N$ over its universal covering $SU(N)$. In this manifold the different components of the loop group $L(SU(N)/Z_N)$ are represented by multivalued group-valued functions

$$
g : [0, 2\pi] \to SU(N) \quad / \quad g(0) = h \cdot g(2\pi) \quad (3.2)
$$

where $h$ is an element of the center $SU(N) \ (i.e., h^N = 1)$. Its easy to verify that the more simpler configurations satisfying (3.2) are

$$
g_n(\theta) = exp(i \frac{n}{N} T^{N-1} \theta) \quad (3.3)
$$
where
\[ T^{N-1} = \begin{pmatrix} 1 & 0 & 0 & \ldots & 0 \\ 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & 1 - N \end{pmatrix} \] (3.4)
is the \( N - 1 \) Cartan generator of \( SU(N) \) and \( n \equiv mod(N) \) label the topological sector.

Finally our election of the background gauge field is
\[ A^{(n)}_{\mu} = -ia(\rho)g_n^{-1}\partial_\mu g_n = \frac{n}{N}a(\rho)\partial_\mu \theta T^{N-1} \] (3.5)
where \( a(\rho) \) is a function which vanishes at the origin and has the asymptotic behaviour
\[ \lim_{\rho \to \infty} a(\rho) \to 1 \] (3.6)
enforced by the boundary condition equation (2.3). One can verify that this election satisfies the Lorentz gauge condition equation (2.11) and consequently it can be written as a curl
\[ A^{(n)}_{\mu} = -\epsilon_{\mu\nu}\partial_\nu K(\rho)T^{N-1}. \] (3.7)
Here \( K(\rho) \) is defined as
\[ K(\rho) = \frac{n}{N} \int_0^\rho \frac{a(r)}{r} dr \] (3.8)
and has the following behaviour
\[ \lim_{\rho \to \infty} K(\rho) \to \rho^{\frac{n}{N}}. \] (3.9)

Then, in terms of the chiral components \( \psi_L \) and \( \psi_R \), the equation (3.1) takes the form
\[ \partial_\bar{z} \left( e^{-K(\rho)T^{N-1}} \psi_R \right) = 0 \] (3.10)
\[ \partial_\bar{z} \left( e^{K(\rho)T^{N-1}} \psi_L \right) = 0 \] (3.11)
with solutions
\[ \psi_R(x, y) = e^{K(\rho)T^{N-1}} \chi_R(\bar{z}) \] (3.12)
\[ \psi_L(x,y) = e^{-K(r)T^{N-1}} \chi_L(z) \]  

(3.13)

where \( \chi_R (\chi_L) \) is an arbitrary anti-holomorphic (holomorphic) \( N \)-components spinor. However by requiring square-integrability the possible values of \( \chi_R \) and \( \chi_L \) have severe constraints.

There is a subtlety concerning this issue. Because we are dealing with the compactified plane, the inner product of spinors is different from the usual one. In particular the inner product induced by the standard metric of the sphere is [nielsen]

\[ (\psi, \chi) = \int \frac{2}{(1+x^2)} d^2 \bar{x} \psi(\bar{x})^\dagger \chi(\bar{x}). \]  

(3.14)

Writing the \( n \) as

\[ n = r + cN , \quad r, c \in \mathbb{Z} , \quad 0 \leq r < N \]  

(3.15)

\( (r \) is the “topological charge”) and using the equations (3.12), (3.13) and (3.14) we find a set of basis vectors for the kernel of \( \mathcal{D} \) (we consider \( c > 0 \))

Case (I) \( r \neq 0 \)

\[ \psi_{(q)}^L = e^{(1-N)K(r)} \begin{bmatrix} 0 \\ : \\ 0 \\ z^q \end{bmatrix} , \quad q = 0, 1, \ldots, n - c - 1 \]  

(3.16)

\[ \psi_{(i,p)}^R = e^{-K(r)} \begin{bmatrix} 0 \\ : \\ z^p \end{bmatrix} , \quad \{ i = 1, 2, \ldots, N - 1 \text{ (labels the rows)} \} \]

\[ \{ p = 0, 1, \ldots, c - 1, c \}. \]  

(3.17)

Case (II) \( r = 0 \)

\[ \psi_{(q)}^L = e^{(1-N)K(r)} \begin{bmatrix} 0 \\ : \\ 0 \\ z^q \end{bmatrix} , \quad q = 0, 1, \ldots, n - c - 1 \]  

(3.17)

\[ \psi_{(i,p)}^R = e^{-K(r)} \begin{bmatrix} 0 \\ : \\ z^p \end{bmatrix} , \quad \{ i = 1, 2, \ldots, N - 1 \text{ (labels the rows)} \} \]

\[ \{ p = 0, 1, \ldots, c - 1 \}. \]  

(3.18)
Finally we can compute the index of the Dirac operator and we find

\[ \text{index } \mathcal{D} = \dim \ker D_z - \dim \ker D_{\bar{z}} = \begin{cases} (N - 1) - r, & \text{if } r \neq 0 \\ 0, & \text{if } r = 0 \end{cases} \]  

(3.19)

(for \( K \) flavors this value is multiplied by \( K \)).

Note that even when the number of zero modes is \( n \)-dependent, the index only depends on \( n \equiv \text{mod}(N) \), the topological charge.

There are some direct conclusions we can obtain from these results. As we mentioned before, the existence of zero modes of the Dirac operator enforces constraints on the fermionic correlation functions. These features can be explained easily in the path-integral formalism. The fermionic path-integral can be performed by expanding the fermions in a base of eigenvectors of the Dirac operator with Grassmann variables as coefficients, and finally, integrating the Grassmann variables with the Berezin rules. However those Grassmann coefficients associated to the zero modes will be absepts in the fermionic action. Therefore the only non-vanishing correlation functions are those which can provide the necessary number of Grassmann variables to make the integrals non-zero. In general, if we define the following family of “chirality changing” fermionic operators

\[ \alpha^{ij}_R(z) = \bar{\psi}^i_R(z)\psi^j_R(z) \]  

(3.20)

\[ \alpha^{ij}_L(z) = \bar{\psi}^i_L(z)\psi^j_L(z) \]  

(3.21)

we can derive easily the following results. For \( r \neq 0 \) the non-zero correlation functions of \( \alpha \) operators contains a number of \( \alpha^{ij}_R(z) \) bilinears which exceeds in \( N - 1 - r \) the number of \( \alpha^{ij}_L(z) \) bilinears. Moreover the minimum number of \( \alpha^{ij}_L(z) \) operators presents must be \( n - c \). For \( r = 0 \) the non-vanishing correlation functions contains the same number of \( \alpha^{ij}_R(z) \) bilinears than \( \alpha^{ij}_L(z) \) bilinears, with a minimum of \( n - n/N \) of each class.

For the special case of \( N = 2 \) there are only one non-trivial topological sector which we can parametrize with \( n = 1 \). For this case there are two fermionic zero modes, one of left
chirality and the other of the opposite chirality. The non-vanishing correlation functions will contain the same number of $\alpha_{R}^{ij}(z)$ bilinears than the $\alpha_{L}^{ij}(z)$ bilinears.

SECTION IV

The aim of this section is to compute the conformal anomaly of this “twisted” model. We will study the three factors of the partition function separately (i.e., the bosonic, the fermionic and the ghost sector). Of course each of this sectors are not independently conformal invariant for an arbitrary background gauge field. However following Ref. 1,2 we can overcome this difficulty by choosing the arbitrary background field $A_{\mu}$ concentrated at the infinity (with support at the infinity). With this election the three sectors above mentioned becomes separately conformal invariant on $R^{2} = S_{2} - \{\text{north pole}\}$. Moreover we will show that the effect of the background field in the three cases is a modification of the conformal properties as in the Dotsenko-Fateev’s model [Dotfat]. We will also see that due to the constraints of the theory the shifts of the conformal anomalies of the three sectors add to zero, and the total conformal anomaly of the model is independent of the topological sectors [yo].

a) The Case $H = SU(2)/Z_{2}$

Let us first consider the easier case $SU(2)/Z_{2}$. Once solved this problem the generalization to the case $SU(N)/Z_{N}$ is straightforward.

We begin with the analysis of the bosonic action. The bosonic effective action is given by

$$S[g, A_{\mu}] = k I[g] - \frac{k}{\pi} \int d^{2}x \tr \left( -i A_{z} g^{-1} \partial_{z} g + i A_{\bar{z}} \partial_{\bar{z}} g^{-1} - A_{z} g A_{\bar{z}} g^{-1} + A_{z} A_{\bar{z}} \right) \quad (4.1a)$$
where \( kI[g] \) is the level \( k \) Wess-Zumino-Witten action (\( k = -(K + 4) \) in our case) and the background field takes the form

\[
\mathcal{A}_\mu^{(n)} = -\epsilon_{\mu\nu}\partial_\nu K(\rho)\sigma^3. \quad (4.2a)
\]

Although the presence of this field breaks the chiral \( SU(2)_L \times SU(2)_R \) invariance, the symmetries in the Cartan subgroup \( U(1)_L \times U(1)_R \) remains unbroken. The currents associated with this symmetry are

\[
J^3_z = j^3_z + ik \, tr(g^{-1}A_zg\sigma^3 + A_z\sigma^3) \quad (4.3a)
\]

\[
J^3_{\bar{z}} = j^3_{\bar{z}} + ik \, tr(gA_{\bar{z}}g^{-1}\sigma^3 + A_{\bar{z}}\sigma^3) \quad (4.4a)
\]

where

\[
\begin{align*}
    j^3_z &= -k \, tr(g^{-1}\partial_zg\sigma^3) \quad (4.5a) \\
    j^3_{\bar{z}} &= -k \, tr(g\partial_{\bar{z}}g^{-1}\sigma^3) \quad (4.6a)
\end{align*}
\]

and we use the gauge condition equation (2.11). Parallel to the analysis of the abelian case (see Dotsenko-Fateev [Dotfat] and Ref. 1), we easily verify that the second term of each current measures the background charge (in the Cartan algebra direction) created at infinity by the gauge field \( \mathcal{A}_\mu \). In fact, if the operators \( V_{q_i}(z_i) \) creates respectively a \( \sigma^3 \) charge \( q_i \) at the point \( z_i (i = 1, \ldots, n) \), the total \( \sigma^3 \) charge of the state

\[
|\{q_i, z_i\} > = \prod_{i=1}^n V_{q_i}(z_i) |0 > \quad (4.7a)
\]

is computed by

\[
Q^3_z = \frac{1}{4\pi i} \oint dz \, J^3_z |\{q_i, z_i\} > = \frac{1}{4\pi i} \oint dz \, j^3_z |\{q_i, z_i\} > + \frac{1}{2\pi} k \oint d\sigma |\{q_i, z_i\} > = \left( \sum_{i=1}^n q_i + \frac{k}{2} \right) |\{q_i, z_i\} > . \quad (4.8a)
\]
Hence, the neutrality of the charge enforces the following constraint on the vertex operators

$$\sum_{i=1}^{n} q_i = -\frac{k}{2} = \left(\frac{K}{2} + 2\right) \quad \text{for } k = -(K + 4). \quad (4.9a)$$

This relation (equation (4.9a)) changes the conformal dimensions of the vertex operators and consequently the energy-momentum tensor differs from the one of a pure WZW theory. The analogy of this model with the Dotsenko and Fateev’s Coulomb gas suggests that such a modification is given by the addition of a term:

$$\Delta T_{zz} \propto \partial_z j_z^3. \quad (4.10a)$$

Let us now recall the free field representation of the WZW theory [wakimoto]. The kernel of this method is to find a realization of the affine algebra by means of free fields. For the SU(2) group we can write the affine currents in terms of three free fields, $\phi$, $\mu$ and $\nu$, and they read

$$j_z^3 = i\sqrt{2(k + 2)}\partial_z \phi + 2\partial_z \mu \quad (4.11a)$$

$$j_z^+ = \sqrt{2}\partial_z \nu \quad (4.12a)$$

$$j_z^- = \left(-i\sqrt{2}(k + 2)\partial_z \mu + 2\sqrt{k + 2}\partial_z \phi - \sqrt{2}(k + 1)\partial_z \nu\right) e^{-2(\mu - i\nu)} \quad (4.13a)$$

In order to reproduce the conformal anomaly of the WZW model the fields $\phi$, $\mu$ and $\nu$ are coupled to the Gaussian curvature. Their actions can be written as

$$S_\Phi = \frac{1}{2\pi} \int d^2x \left\{ (\partial \Phi)^2 + Q_\Phi R \Phi \right\} \quad \Phi = \phi, \mu, \nu \quad (4.14a)$$

where $R$ is the scalar curvature and the “charges” $Q_\Phi$ takes the values

$$Q_\phi = -\frac{i\sqrt{2}}{\sqrt{k + 2}}, \quad Q_\mu = -1, \quad Q_\nu = i. \quad (4.15a)$$

The corresponding energy-momentum tensors and conformal charges are

$$T_\Phi = -2(\partial_z \Phi)^2 + Q_\Phi \partial_z^2 \Phi \quad (4.16a)$$
\[ c_\Phi = 1 + 3Q_\Phi^2. \]  \hspace{1cm} (4.17a)

Returning to our model, a WZW model with a minimal coupling to a background field located at infinity, it is now easy to describe the effect of the background field in terms of the free field realization. In fact, we can show [Dotfat, yo] that, instead of working with the currents \( J^3_z \) and \( J^3_{\bar{z}} \) (equations (4.3a) and (4.4a)) we can use the free currents \( j^3_z \) and \( j^3_{\bar{z}} \) but defining a “new” vacuum state \( |k> \) as

\[ |k> = e^{-i\sqrt{2(k+2)}\phi(\infty)}e^{-2\mu(\infty)}|0>. \]  \hspace{1cm} (4.18a)

This vertex insertion creates a charge \( k \) at the infinity and therefore reproduces the contribution of the background field in equation (4.8a). Moreover, we can include this insertion in the actions (4.14a) by a simple redefinition of the charges \( Q_\Phi \) which takes the form

\[ Q_\phi = \frac{ik}{\sqrt{2(k+2)}}, \quad Q_\mu = 0, \quad Q_\nu = i. \]  \hspace{1cm} (4.19a)

Note that this procedure is equivalent to add to the energy-momentum tensor the term

\[ \Delta T(z)_B = \frac{1}{2} \partial_z j^3_{\bar{z}}(z) \]  \hspace{1cm} (4.20a)

in accordance with equation (4.10a). Finally using equation (4.17a) we find the conformal charge of this model

\[ c_B = -\frac{3k^2}{2(k+2)} = \frac{3(N+4)^2}{2(N+2)} \quad \text{for} \quad k = -(N+4). \]  \hspace{1cm} (4.21a)

We can also study the fermionic action in a close related way. With the election (4.2a) of the gauge field (and after a harmless rescaling of the fermions) the fermionic action is the sum of \( K \) terms of the form

\[ S_F = \frac{1}{4\pi} \int d^2x \bar{\psi}^{(1)} (i\partial + A) \psi^{(1)} + \frac{1}{4\pi} \int d^2x \bar{\psi}^{(2)} (i\partial - A) \psi^{(2)} \]  \hspace{1cm} (4.22a)

where

\[ A_\mu = -\epsilon_{\mu\nu}\partial_\nu K(\rho) \]  \hspace{1cm} (4.23a)
and the two SU(2) color components decouple. We can then analyze each color separately. (We will analyze the action corresponding to \( \psi^{(1)} \), all the results are valid for \( \psi^{(2)} \) replacing \( A_\mu \rightarrow -A_\mu \)). For the first action we have a conservation equation (vectorial symmetry) and an anomalous equation (chiral symmetry)

\[
\partial_\mu j_\mu = 0, \quad j_\mu = \bar{\psi} \gamma_\mu \psi \tag{4.24a}
\]

\[
\partial_\mu j_5^\mu = -2\epsilon_{\mu\nu} F_{\mu\nu}, \quad j_5^\mu = \bar{\psi} \gamma_\mu \gamma^5 \psi. \tag{4.25a}
\]

However, because the gauge field satisfies the Lorentz gauge condition, we can construct a new pair of conserved currents, one holomorphic and the other anti-holomorphic

\[
\partial \bar{z} J_z = 0, \quad J_z = i \bar{\psi} R \psi_L + 4A_z \tag{4.26a}
\]

\[
\partial \bar{z} J_{\bar{z}} = 0, \quad J_{\bar{z}} = i \bar{\psi} R \psi_L - 4A_{\bar{z}}. \tag{4.27a}
\]

Note the similarity of this currents with the bosonic currents \( J_z^3 \) and \( J_{\bar{z}}^3 \) (equations (4.3a) and (4.4a)). In fact we can proceed in the same way we derive the equations (4.7a)-(4.10a) and show that the effect of the “anomalous” term in the currents \( J_z \) and \( J_{\bar{z}} \) is to create a Dotsenko and Fateev’s charge \( \frac{i}{2} \) at infinity [yo]. As in the bosonic case we can incorporate this effect in the energy-momentum tensor by adding a term

\[
\Delta T_F \propto \partial \bar{z} j_{z \ F} \tag{4.28a}
\]

where \( j_{z \ F} = i \bar{\psi} R \psi_L \) is the left “number” current. Finally using the rules of abelian bosonization [FMS,cuerdistas] we can compute the changes in the Virasoro central charge employing again the equation (4.17a) with \( Q = \frac{i}{2} \). Finally adding the contribution of the fermion \( \psi^{(2)} \) (replacing \( Q \rightarrow -Q \)) and summing over the \( K \) flavors we find that the conformal charge for the fermionic action is

\[
c_F = K \left( \frac{1}{4} + \frac{1}{4} \right) = \frac{K}{2}. \tag{4.29a}
\]
The former argument can be easily extended to study the contribution to the conformal anomaly of the ghost action in equation (2.20). The determinant in the adjoint representation can be written as the path-integral

\[ \det \left( i \partial_z + A_\alpha^z \right)_{ADJ} = \int D\bar{\xi}D\xi e^{-\frac{1}{2\pi} \int tr(\bar{\xi}D\xi)} \]

where \( \bar{\xi}, \xi \) are a pair ghost-antighost taking values in the Lie algebra of \( SU(2) \) and \( D_z \) is defined by

\[ D_z\xi = i\partial_z\xi + [A_z, \xi] \]

We can expand the ghosts fields in the Cartan-Weyl basis \( (\sigma^+, \sigma^-, \sigma^3) \) introducing the three pairs of ghosts \((b^+, c^+)\), \((b^-, c^-)\) and \((b^3, c^3)\)

\[ \bar{\xi} = b^+ \sigma^- + b^- \sigma^+ + b^3 \sigma^3 \sqrt{2} \quad (4.32a) \]

\[ \xi = c^+ \sigma^- + c^- \sigma^+ + c^3 \sigma^3 \sqrt{2} \quad (4.33a) \]

(the generators are normalized to \( tr (\sigma^+ \sigma^-) = tr (\sigma^3)^2 = 1 \) and any other combination equal to zero).

In this representation the Lagrangian in (4.30a) is diagonal in the ghost fields and takes the form

\[ S[\bar{b}, \bar{c}] = \frac{1}{2\pi} \int d^2x \left\{ b^+_z(i\partial_z + 2A_z)c^-_z + b^-_z(i\partial_z - 2A_z)c^+_z + b^3_z(i\partial_z)c^3_z \right\} \]

(4.34a)

(Of course there is an identical expression for the \( \det \left( i \partial_z + A_\alpha^z \right)_{ADJ} \)). The fields \( A_z \) and \( A_{\bar{z}} \) are the same defined in equation (4.23a) and \((b^i_z, c^i_z)(i = +, -, 3)\) are ghosts of conformal dimensions 1 and 0 respectively. There is a fundamental difference between this system and the fermionic one (equation (4.22a)). Owing to the tensorial character of the ghosts they have a non-trivial coupling with the metric. In fact although the gauge current in both systems are conserved the chiral current for the ghosts suffers two types of anomalies, the usual chiral anomaly and a gravitational anomaly proportional to the
Gaussian curvature of the two-manifold [Fujikawa,FMS,cuerristas]. For example for the system \((b^+, c^-)\) we have

\[
\partial_\mu j^\mu = 0, \quad \vec{j} = (-b^+_z c^-_z - b^+_z c^-_z, -ib^+_z c^-_z - ib^+_z c^-_z) \quad (4.35a)
\]

\[
\partial_\mu j^5_\mu = -4\epsilon_{\mu\nu}F_{\mu\nu} - 2\sqrt{g}R, \quad \vec{j}^5 = (-b^+_z c^-_z + b^+_z c^-_z, -ib^+_z c^-_z + ib^+_z c^-_z). \quad (4.36a)
\]

In a conformally flat metric \(g_{\mu\nu} = e^{4\eta}\delta_{\mu\nu}\) we can also write a new pair of conserved currents as in equations (4.26a) and (4.27a)

\[
\partial_z J_z = 0, \quad J_z = ib^+_z c^-_z + 8A_z + 4i\partial_z \eta \quad (4.37a)
\]

\[
\partial_{\bar{z}} J_{\bar{z}} = 0, \quad J_{\bar{z}} = ib^+_z c^-_z - 8A_z - 4i\partial_{\bar{z}} \eta \quad (4.38a)
\]

With the help of the Riemann-Roch theorem we can prove that the anomalous terms in the currents alter the balance of the ghosts charge by creating a background charge \([yo]\) \(Q_1 = i + i\) (from the gauge field and from the Gaussian curvature respectively). The analysis of the system \((b^-, c^+)\) is similar. In virtue of the opposite sign of the gauge field in equation (4.34a) the background charge in this case is \(Q_2 = i - i\). Finally for the free ghost system \((b^3, c^3)\) we have \(Q^3 = 0 + i\). Once again we can realize these effects by adding to the energy-momentum tensor a term

\[
\Delta T_G \propto \partial_z j^3_{z, G} \quad (4.39a)
\]

where \(j^3_{z, G} = i( 2b^+_z c^-_z - 2b^-_z c^+_z + b^3_z c^3_z )\). Using the equation (4.17a) with the above mentioned values of the charges we obtain the total ghost central charge

\[
c_G = -12. \quad (4.40a)
\]

The total conformal charge of this coset model is the sum of the values (4.21a), (4.29a) and (4.40a) and the answer is

\[
c = \frac{2(K - 1)^2}{K + 2} + 1 \quad (4.41a)
\]
which corresponds exactly to the usual value of the central charge of the coset model 
\( U(2K)/SU(2)_K \sim SU(K)_2 \times U(1) \). That is we find that the conformal anomaly is independent of the topological background charge.

At the light of the above discussion this result became expected. In fact, we have proved that for each subsystem \( i.e. \) bosonic, fermionic or ghost, the effect of the monopole is a shift of the energy-momentum tensor in a quantity

\[ \Delta T_{B,F,G} \propto \partial z j^3_{B,F,G} \]  

(4.42a)

where \( B, F, G \) stands for bosons, fermions and ghost respectively. Thus the change of the total energy-momentum tensor of the theory is proportional to the derivative of the total current in the direction \( \sigma^3 \). But this is one of the currents we are constraining to vanish with the Lagrange multiplier \( i.e. \), \( j^3_{\text{total}} = 0 \). Hence once proved that the modification of the energy-momentum due to the presence of the monopole is of the form (4.42a), the result (4.41a) is natural. Note that this result, the independence of the conformal anomaly on the topological sector, is a property of the whole theory but each subsystem separately suffers a change. In particular the primary fields of each subsystem have topology-dependent conformal dimensions. Moreover the vacuum expectation values of the original fermionic fields depends on the topological charge as we mentioned in the previous section.

In the next subsection we will show that these results are also valid for the general coset \( U(NK)/SU(N)_K \sim SU(K)_N \times U(1) \).

**b) The Case** \( H = SU(N)/Z_N \)

Now we can generalize the former results to the case \( H = SU(N)/Z_N \). In this case there are \( N - 1 \) different topological sectors and not just one as in the \( SU(2)/Z_2 \). We begin with the study of the gauged WZW action (2.16). First we need to recall, as in the
earlier case, the free field realization of the affine Kac-Moody algebra associated to $\hat{su}(N)$ [wakimoto].

Let $\{h_i, e_i, f_i; \ i = 1, \cdots, N - 1\}$ the Chevalley basis of $\hat{su}(N)$ defined by the relations

$$
[e_i, f_j] = \delta_{ij} h_i \\
[h_i, e_j] = a_{ij} e_j \\
[h_i, f_j] = -a_{ij} f_j
$$

where

$$a_{ij} = \frac{2(\alpha_i \cdot \alpha_j)}{(\alpha_j \cdot \alpha_j)}$$

(4.2b)

is the Cartan matrix. Our convention for the simple roots $\alpha_j$ is [georgi]

$$\alpha_j = -\sqrt{\frac{j - 1}{2j}} \hat{e}_{j-1} + \sqrt{\frac{j + 1}{2j}} \hat{e}_j$$

(4.3b)

where $\{\hat{e}_1, \cdots, \hat{e}_{N-1}\}$ is the canonical base of $\mathcal{R}^{N-1}$. The simple roots are normalized to

$$2(\alpha_i \cdot \alpha_j) = 2\delta_{ij} - \delta_{i+1j} - \delta_{ij+1}.$$  

(4.4b)

Then the affine Kac-Moody algebra associated to $\hat{su}(N)$ is defined by the following OPE

$$H_i(z) H_j(\omega) = 2(\alpha_i \cdot \alpha_j) \frac{k}{(z-\omega)^2} + \cdots$$

$$H_i(z) E_j(\omega) = \frac{1}{z-\omega} a_{ij} E_j(\omega) + \cdots$$

$$H_i(z) F_j(\omega) = -\frac{1}{z-\omega} a_{ij} F_j(\omega) + \cdots$$

$$E_i(z) F_j(\omega) = \frac{k}{(z-\omega)^2} \delta_{ij} + \frac{1}{z-\omega} \delta_{ij} H_j(\omega) + \cdots$$

(5.5b)

where $k$ is the Kac-Moody level.

For each positive root $\alpha_{pp+q}$ ($\alpha_{pp+q} = \alpha_p + \alpha_{p+1} + \cdots + \alpha_{p+q-1}; \ p + q \leq N$) we introduce [wakimoto] a pair of scalar bosons $\mu_\alpha, \nu_\alpha$ whose dynamics is governed by the Liouville action

$$S_\alpha = \frac{1}{2\pi} \int \{ (\partial \mu_\alpha)^2 - R \mu_\alpha \} d^2 x + \frac{1}{2\pi} \int \{ (\partial \nu_\alpha)^2 + i R \nu_\alpha \} d^2 x.$$  

(4.6b)
The OPE of these fields is given by

$$\mu_\alpha(z)\mu_\beta(\omega) = \nu_\alpha(z)\nu_\beta(\omega) = -\frac{1}{4}\delta_{\alpha\beta}\ln(z-\omega). \quad (4.7b)$$

We also introduce another set of scalar bosons $\phi_i, \ i = 1, \ldots, N - 1$, one for each Cartan generator, whose action is also a Liouville action

$$S_\phi = \frac{1}{2\pi} \int d^2x \left\{ \partial\vec{\phi} \cdot \partial\vec{\phi} - \frac{2\sqrt{2}}{\sqrt{K + N}} \rho \cdot \vec{\phi} \right\} \quad (4.8b)$$

where $\rho = \frac{1}{2} \sum_{\alpha > 0} \alpha$. The OPE of these fields is given by

$$\phi_i(z)\phi_j(\omega) = -\frac{1}{4}\delta_{ij}\ln(z-\omega). \quad (4.9b)$$

Hence we can realize the entire affine current algebra in terms of the fields $\mu_\alpha, \nu_\alpha, \phi_i$. For example the currents in the Cartan subalgebra (luckily these are the only currents we need) can be written as

$$H_i(z) = \sum_{\alpha > 0} 4(\alpha_i \cdot \alpha) \partial_z \mu_\alpha + i2\sqrt{2(K + N)}\alpha_i \cdot \vec{\phi}. \quad (4.10b)$$

The energy momentum tensor of this theory is given by

$$T(z) = \sum_{\alpha > 0} T_\alpha(z) + T_{\vec{\phi}}(z)$$

$$= \sum_{\alpha > 0} \left\{ -2(\partial_z \mu_\alpha)^2 - \partial_z^2 \mu_\alpha - 2(\partial_z \nu_\alpha)^2 + i\partial_z^2 \nu_\alpha \right\} +$$

$$+ \left\{ -2\partial_z \vec{\phi} \cdot \partial_z \vec{\phi} - i\frac{2\sqrt{2}}{\sqrt{K + N}} \rho \cdot \vec{\phi} \right\} \quad (4.11b)$$

which satisfies a Virasoro algebra with a central charge

$$c = \frac{k(N^2 - 1)}{k + N}. \quad (4.12b)$$

It is important to mention that this energy-momentum tensor, computed by varying the actions (4.6b), (4.8b) respect of the metric is the same one obtain in the Sugawara construction with the currents $H_i, \ E_\alpha, \ F_\alpha$. Hence we conclude that because this model has
the same current algebra and the same Virasoro algebra that the \( SU(N)_k \) WZW theory, both theories are equivalents.

Now we can return to our problem: How the presence of the topological background field \( A_\mu \) in the action (2.16) affects the conformal anomaly. The answer is not difficult. By the same arguments we use for the case \( N=2 \) is easy to see that the effect of the background field is the appearance in the energy-momentum tensor of an additional term

\[
\Delta T(z) = \frac{n}{N} \partial_z j_z^{N-1} \tag{4.13b}
\]

where \( j_z^{N-1} = \text{tr} \left( k g^{-1} \partial_z g T^{N-1} \right) \) is the Kac-Moody current in the direction \( T^{N-1} \), \( k = -(M + 2N) \) is the level and \( n = 1, \ldots, N-1 \) is the topological charge. We can express this quantity in terms of the free fields \( \mu_\alpha, \nu_\alpha, \phi_i \) and add the result to the free field representation of the energy-momentum tensor equation (4.11b). In fact noting that

\[
T^{N-1} = \sum_{m=1}^{N-1} m h_m \tag{4.14b}
\]

we can write the current \( j_z^{N-1} \) as

\[
j_z^{N-1}(z) = \frac{1}{2} \sum_{j=1}^{N-1} j H_j(z). \tag{4.15b}
\]

And hence using the representation (4.3b) for the roots we find

\[
j_z^{N-1}(z) = N \sum_{j=1}^{N-1} \partial_z \mu_{iN} + i2 \sqrt{(k + N)N(N - 1)} \partial_z \phi_{N-1}. \tag{4.16b}
\]

Finally the energy-momentum tensor of the “twisted” theory takes the form

\[
T^{(n)}_B = \sum_{i<j<N} \left\{ -2(\partial_z \mu_{ij})^2 - \partial^2_{z} \mu_{ij} - 2(\partial_z \nu_{ij})^2 + i\partial^2_{z} \nu_{ij} \right\} + \\
+ \sum_{i=1}^{N-1} \left\{ -2(\partial_z \mu_{iN})^2 - (1 - n)\partial^2_{z} \mu_{iN} - 2(\partial_z \nu_{iN})^2 + i\partial^2_{z} \nu_{iN} \right\} + \\
-2(\partial_z \vec{\phi} \cdot \partial_z \vec{\phi}) - i \frac{2\sqrt{2}}{\sqrt{k + N}} \left( \rho - n(K + N)\sqrt{\frac{N-1}{2}} \hat{e}_N \right) \cdot \partial^2_{z} \vec{\phi}^{N-1}. \tag{4.17b}
\]
The conformally anomaly can be easily computed using equation (4.17a) and we obtain the result (for $k = -(K + 2N)$)

$$c_B^{(n)} = \frac{(K + 2N)(N^2 - 1)}{K + N} + 3n^2 \frac{(K + 2N)(N - 1)}{N}. \quad (4.18b)$$

Now let us analyze the fermionic system. The action can be written as

$$S_F = \sum_{j=1}^{N-1} \sum_{m=1}^{M} \frac{1}{4\pi} \int d^2 \bar{\psi}_j(i\partial + A)\psi_j + \sum_{m=1}^{M} \frac{1}{4\pi} \int d^2 \bar{\psi}_m(i\partial + (1 - N)A)\psi_m^N \quad (4.19b)$$

where $A_\mu$ is a gauge field with a fractional topological charge $q = n/N$. As in the case $N=2$ this model reduces to the case of $KN$ abelian fermionic system in presence of a fractionally charged topological field. Taking into account the chiral anomaly and using the Lorentz gauge condition (2.11) we can write $N$ pairs of conserved currents, one for each color

$$\partial \bar{z} j^j_F = 0 \quad j = 1, \cdots, N \quad (4.20b)$$

$$\partial z j^j_F = 0 \quad j = 1, \cdots, N \quad (4.21b)$$

where

$$j^j_F = i\bar{\psi}_R^j \psi_L^j \quad j = 1, \cdots, N - 1$$

$$j^N_F = i(1 - N)\bar{\psi}_R^N \psi_L^N$$

$$j^j_F = i\bar{\psi}_L^j \psi_R^j \quad j = 1, \cdots, N - 1$$

$$j^N_F = i(1 - N)\bar{\psi}_L^N \psi_R^N. \quad (4.22b)$$

Hence we can compute the conformal anomaly using again formula (4.17a) with the values of the charges $Q^i_m = q, \ i < n$ and $Q^N_m = q(1 - N) \ (m = 1, \cdots, M)$. The result is

$$c_F^{(n)} = KN - 3n^2 \frac{K(N - 1)}{N}. \quad (4.23b)$$

Finally we study the ghost action which appears when we exponentiate the determinant of the Dirac operator in the adjoint representation using Grassmann variables. To do this we introduce for each root $\alpha$ and for each Cartan generator $h^i$ of $\hat{su}(N)$ a ghost system,
\((b^\alpha, c^\alpha)\) and \((b^i, c^i)\) respectively, of conformal dimensions \((1,0)\). Using the commutation relations

\[
[T^{N-1}, E_{ij}] = \begin{cases} 0 & i, j < N \\ -N & i < j = N \\ N & j < i = N \\ \end{cases} \tag{4.24b}
\]

\[
[T^{N-1}, h_i] = 0 \quad i = 1, \ldots, N - 1
\]

we can write the ghost action as

\[
S_{Gh} = \sum_{i=1}^{N-1} b_{N_i}(i\partial_z + N A_z)c_{N_i} + \sum_{i=1}^{N-1} b_{iN}(i\partial_z - N A_z)c_{iN} + (N-1)^2 \sum_{p=1}^{i} b_p i\partial_z c_p + h.c. \tag{4.25b}
\]

Once more is easy to follow the steps of the case \(N=2\) to compute the conformal anomaly. We use again the formula (4.17a) with the values of the background charges \(Q_{jN} = i(1+n)\), \(Q_{Nj} = i(1-n)\) and \(Q_p = i\) for the ghost systems \((b_{Nj}, c_{Nj})\), \((b_{jN}, c_{jN})\) and \((b_p, c_p)\) respectively \((j = 1, \ldots, N = 1; \ p = 1, \ldots, (N-1)^2)\) and we obtain the result

\[
c_G^{(n)} = -2(N^2 - 1) - 6n^2(N - 1). \tag{4.26b}
\]

The central charge of the coset model \(U(NK)/SU(N)_K \sim SU(K)_N \times U(1)\) is the sum of the three values (4.18b), (4.23b) and (4.26b). The final result is

\[
c_{U(NK)/SU(N)_K} = \frac{N(K^2 - 1)}{N + K} + 1 \tag{4.27b}
\]

independent of the topological charge as we explained at the end of the previous subsection. Of course the result (4.27b) is the usual value given by the coset construction [GKO]

\[
c_{U(NK)/SU(N)_K} = c_{U(NK)} - c_{SU(N)_K}. \tag{4.29b}
\]

**CONCLUSIONS**

We have studied a fermionic coset model \(G/H\) with subgroup \(H\) admitting non-trivial topology \((\Pi_1(H) \neq 0)\). The partition function of this theory corresponds to a sum over
the different topological sectors. For each sector we showed that the partition function can be factorized in three factors: a partition function for fermions coupled to a non-abelian monopole, a partition function of a gauged WZW theory and a partition function for a ghost system minimally coupled to the same monopole field. We computed the fermionic zero modes produced by the monopole background and showed the dependence of the index of the Dirac operator with the topological sector. With the election of a particular monopole background (a non-abelian gauge field with support at infinity) the three subsystems become conformal invariant; moreover, the presence of the monopole alters the value of the conformal anomaly of each subsystem. This change can be computed exactly using an analogy with the Dotsenko and Fateev’s Coulomb Gas approach to CFT. We showed that, for each subsystem, the effect of the monopole is to create a background charge in the direction of the Cartan subalgebra which modifies the energy momentum tensor of the theory. This modification has the general form

$$\Delta T_{zz} \propto \partial_z J^{N-1}_{total}$$

where $J^{N-1}_{total}$ is the total current in the direction $T^{N-1}$. However, since in coset models this current is constrained to zero by the Lagrange multiplier, the total energy-momentum tensor does not suffer any change due to the topology. The variations of the conformal anomaly of the three subsystems adds up to zero. We proved this result explicitly by computing the conformal central charge for the three subsystems in the general coset $U(NK)/SU(N)K$.

Finally let us mention that this result does not imply the independence of the model on the topology of the gauge field. In fact, we have showed explicitly that the dependence of the effective action on the monopole charge affects the conformal dimensions of the primary fields of each subsystem. Moreover as we mentioned in section III, the existence of fermionic zero modes impose selection rules over the vacuum expectation values of the
original fermionic fields which consequently have a strong dependence on the topological charge [Bardakci,ellos,yo].

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