RESEARCH ARTICLE

Thermal performance comparative analysis of nanofluid flows at an oblique stagnation point considering Xue model: a solar application

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Abstract

This exploration aims to study the comparison of heat transfer capabilities of two nanofluid oblique, steady stagnation-point flow combinations comprising single-walled carbon/water (SWCNHs/water) and multiwalled carbon nanotube/water (MWCNTs/water) toward a stretching surface influenced by nonlinear thermal radiation employing the Xue model. This envisaged comparison model is inimitable and still scarce in the literature. Relying on the Tiwari–Das nanofluid model, a mathematical framework is constructed. The system of partial differential equations is converted using suitable transformations into an ordinary differential system of three equations, which is evaluated numerically using the bvp4c method. The physical behavior of significant parameters and their graphical representation are thoroughly examined. The results show that the SWCNHs/water nanofluid outperforms the MWCNTs/water nanofluid. It is further witnessed that SWCNH nanoparticle contained nanofluid has considerably greater thermal radiation than MWCNT nanoparticles. The envisaged model is also validated by comparing it with a published study.

Keywords: oblique stagnation point; Xue model; nonlinear thermal radiation; stretching surface

Nomenclature

| Symbol | Definition |
|--------|------------|
| α | Stretching ratio $(\rho C_p)$ |
| u, v | Velocity components $(\text{m/s})$ |
| N_r | Nonlinear thermal radiation parameter $k_f$ |
| $(\rho C_p) f$ | Effective heat capacity of fluid |
| $f(y)$ | Dimensionless normal velocity |
| $k_f$ | Fluid thermal conductivity |
| $x, y$ | Coordinates axis $(\text{m})$ |
| $k_s$ | Thermal conductivity of particle |
| $\tau$ | Temperature $(\text{K})$ |
| Pr | Prandtl number $N_u$ |
| $\theta_w$ | Surface heating parameter $p$ |
| $C_f$ | Skin friction |
| $N_u$ | Nusselt number |
| $p$ | Pressure $(\text{N/m}^2)$ |

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1. Introduction

As the world’s population grows and traditional energy supplies become depleted, demand for renewable energy technologies, particularly solar thermal, is growing. Solar thermal energy is the process of collecting the sun’s power to produce thermal energy that may be used for a variety of purposes, such as mechanically generating electricity. Currently, solar thermal energy systems suffer from various losses owing to heat transfer processes; as a result, various nanoliquids have been utilized as a way to optimize the working of solar collectors by absorbing direct sunlight. Choi and Eastman (1995) proposed the concept of nanofluids, which are novel revolutionary fluids. Nanofluids have enhanced or improved thermophysical characteristics like thermal conductivity, viscosity, and thermal diffusivity (Wang & Mujumdar, 2007) in comparison to base fluids. Further, the inclusion of nanoparticles to the base fluid can considerably enhance the fluid’s thermophysical (Prasher, 2005), mass diffusivity (Tyagi et al., 2009), and radioactive heat transfer characteristics (Prasher et al., 2006). Because of these intrinsic properties, nanofluids are gaining popularity among scientists, academicians, and engineers who are working to build better systems and technologies employing nanoliquids as heat-conducting and absorption medium. Further, increasing heat generation poses issues in sectors such as cooling and product maintenance. Because of this, scientists are embracing the unique nature and thermal properties of some fluids created by inserting solid particles (on the micrometer and millimeter scales). Anisamaun et al. (2016) explored the 47-nm alumina/H2O nanofluid flow over a surface of a horizontal paraboloid of rotation in the presence of a magnetic field, nonlinear thermal radiation, and an internal heat source. A researcher discovered that when the volume fraction/heat capacity and other properties of 47-nm alumina-water nanofluid are enhanced, the nanofluid generates much more heat energy, which accounts for the overshot observed in the velocity and temperature profiles. Song et al. (2021) examined and highlighted the discussed two combinations of random motion of nanofluid flows over a surface comprising alumina/water and copper/ethylene glycol with convective boundary conditions. Elnaqeeb et al. (2021) studied water colloidal mixed with three distinct nanoparticles of varying densities. The simulation shows that water combined with heavy density nanoparticles (copper oxide, copper, and silver) has increased Nusselt numbers proportionate to heat transfer at all levels of stretching ratio and suction. Anisamaun et al. (2019) investigated the flow of 29-nm CuO–water nanofluid of a paraboloid of revolution over a horizontal surface subjected to Lorentz force and thermoelectric; it was discovered that at higher values of Lorentz force, the optimum temperature distribution with volume fraction is ensured. Additionally, Upreti et al. (2021) analysed the generation of entropy in three-dimensional hybrid nanofluids including CNTS particles flow as a result of the convective surface and the presence of thermal radiation. The authors’ research (Khan et al., 2015; Upreti et al., 2020; Singh et al., 2021) aims to find a more efficient working liquid for the transfer of heat in thermal energy systems.

In fluid dynamics, when fluid hits a hard surface, its velocity ultimately drops to zero at a point known as the stagnation point. Stagnation-point flow signifies the flow in the neighborhood of a stagnation point. Orthogonal stagnation-point flow has been an important study area for researchers, although oblique stagnation-point flows have received less attention. When a liquid collides with a solid surface at an arbitrary angle, oblique stagnation-point flow develops. Stuart (1959) pioneered the study of the viscous, steady stagnation-point flow of a Newtonian liquid obliquely impinging on an inclined surface. He employed similarity transformation to transform the problem into a system of ordinary differential equations and subsequently found the exact solution. Following Stuart, Tamada (1979) studied stagnation-point flow (2D) obliquely impinging over the planar wall. He discovered an analytical solution to the problem and gave a comparison with existing answers. He also drew streamlines to forecast fluid flow behavior. Later, Dorrrepaal (1986) examined Stuart’s and Tamada’s work and discovered the 2D similarity solution for oblique stagnation-point flow. He assumed the stream function in terms of impinging angle α in his work, and for α = 0, the model resolved to orthogonal stagnation-point flow. Further, Yacob et al. (2011) exhibited boundary layer stagnation-point flow of a Micropolar liquid impinging on shrinking as well as stretching surface along with the influence of melting heat transfer. Attia (2007) reported the flow of a three-dimensional liquid (viscous) impinging on a stretched surface (permeable) with heat absorption/generation in a laminar axisymmetric stagnation point. It is discovered that velocity components escalate by raising the stretching velocity but dwindled the thickness of the boundary layer of velocity. Singh et al. (2010b) investigated the impact of chemical reactions on the MHD stagnation-point flow of micropolar fluid in a stretched sheet with slip and convective boundary conditions. Following this, Singh et al. (2010a) scrutinized the Micropolar fluid flow over an extended surface in the vicinity of the stagnation point with nonuniform heat source/sink and melting heat in a permeable medium. Using a vertical stretching surface as a starting point, Nadhirah and Noor (Abdul Halim et al., 2021) investigated the mixed convection flow of Powell–Eyring nanofluid at a stagnation point. It has been discovered that stagnation has a greater impact on increasing the heat transfer rate when the fluid is under passive control as opposed to when the fluid is under active control. Nadhirah et al. (Halim et al., 2017) studied numerically the continuous Williamson nanofluid flow over an extended stretching/shrinking surface with active/passive...
wall mass flux controls near a stagnation point. It was understood that the temperature and nanoparticle volume fraction decrease with the stagnation parameter. Additional information on stagnation-point flow is available in El-Hakiem et al. (2016) and Bilal et al. (2021).

In high-temperature industrial processes, the role of radiation heat transfer is superficial. Ghaﬀari et al. (2016) scrutinized the influence of radiations on fluid that is non-Newtonian in the vicinity of oblique stagnation-point flow in spongy media. Numerous scholars have carried out a significant number of studies on the radiation effect on the oblique fluid flow (Rasheed et al., 2021; Raza et al., 2021). The linearized Rosseland approximation has been studied for the radiation effect in the majority of the above-mentioned literature. The dimensionless Radiation parameter and Prandtl number are used in this form of approximation, and they are only viable if the temperature gradient between the plate and far from the plate is minimal. However, for higher temperature differences, the Rosseland approximation (nonlinear) probably applies. Ghaﬀari et al. (2018) address the oblique stagnation-point flow of Maxwell liquid with the additional impact of nonlinear thermal radiation. It was discovered that when the upsurge in the values of the radiation parameter occurs, the temperature of the liquid increases. Das et al. (2015) deliberated the stagnation-point flow of a Cu–H₂O nanofluid with the addition of a chemical reaction and nonlinear thermal radiation. With nonlinear thermal radiation, Sreelakshmi and Sarojamma (2018) investigated heat transfer in the nonorthogonal stagnation-point flow of Maxwell liquid. Babu and Sandeep (2016) tested the influence of changing viscosity and nonlinear heat radiation on oblique bio convective flow.

The aforementioned study’s primary motivation is to scrutinize the thermal performance of the stagnation (oblique) flow of water-based nanofluids containing SWCNH and MWCNT nanoparticles impinging on a stretching surface. This type of research could help to increase the thermal performance of solar collectors and solar energy. To improve the usage of solar nanofluids systems, it must be characterized and studied to determine which one is the best. So, in this study, two distinct nanofluids are formed and assessed using carbon-based thermal conductivity model, i.e. the Xue model. It is pertinent to mention that the Xue model is dedicated to studying the carbon nanotubes submerged nanofluids flow. No such study is done so far on the oblique stagnation point that contains a mix of the abovedescribed factors. Using proper similarity transformations, the obtained system of a highly nonlinear system may be numerically determined. Table 1 shows the precise uniqueness of the provided model in comparison to the previously available literature. The findings are presented using graphs. While evaluating this investigation, the objective of this study is to answer the following questions:

1. How do velocity ratio parameter influence velocity and temperature profiles? Is stretching suitable for the enhancement of temperature in solar thermal systems?
2. How nonlinear thermal radiation will affect the heat transfer of SWCNH and MWCNT nanoparticles? Which nanofluid will perform best in solar thermal systems?
3. What influence does the temperature ratio parameter have on the temperature profile in a water-based fluid comprising SWCNH and MWCNT nanoparticles? Concerning solar thermal systems, which nanofluid will be accountable for the increased heating of the fluid?
4. What influence do the velocity ratio parameter and shear stream have on the skin friction coefficient?
5. In a solar energy system, which nanoparticle has the best performance?

### 2. Research Methodology

Here, we consider the 2D steady and incompressible stagnation-point flow of two different nanofluids impinging on a surface that is stretching. Along the x-axis, the two opposite and equal forces are applied in company with the velocity u = ex, for the purpose of the surface to be stretched maintaining the origin fixed and y is in the normal direction of flow as portrayed in the following Figure 1. We further assume that the temperature of the surface is symbolized as T∞ and liquid having temperature, which is far from the surface, is given as T∞, where T∞ > T∞. The fluid employed in this research is a nanofluid comprising two types of nanoparticles, SWCNH and MWCNT along with water as a working base liquid. It is also included that the liquid and nanomaterials are in (thermal) energy balance and that there is no slip between them. Further, Table 2 includes thermophysical characteristics.

Taking into account the aforementioned conditions, the following mathematical model depicts the provided scenario in the context of nonlinear radiation and the idea of the boundary layer along with the Tiwari and Das model (Tiwari & Das, 2007; Nadeem et al., 2014; Bashia et al., 2019):

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} = \frac{v_{nf}}{\rho_{nf}} (\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}), \quad (2)
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{1}{\rho_{nf}} \frac{\partial p}{\partial y} = \frac{v_{nf}}{\rho_{nf}} (\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}), \quad (3)
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = a_{nf} (T_{xx} + T_{yy}) - \frac{(qr)_{y}}{\sqrt{C_{nf}}}, \quad (4)
\]

where qr is the radiative heat flux and is stated as

\[
qr = \frac{4\sigma^{*} \frac{\partial T}{\partial y}}{3k^{*}} = -\frac{16\sigma^{*} - \frac{3}{3k^{*}} \frac{\partial T}{\partial y}}{3k^{*}} \quad (5)
\]
Thermal performance comparative analysis of nanofluid flows

We begin by introducing the following relevant quantities given

2.1 Simplification of mathematical analysis

Boundary conditions of the above scenario are depicted as

\[ u = 0 \quad \text{at} \quad y = 0, \quad v = \frac{1}{\sqrt{\nu B}}, \quad T = T_0 \quad \text{at} \quad y = \infty. \] (6)

In preceding equations, constants are a, b, and e with dimensions of each are expressed as \( s^{-1} \).

2.1 Simplification of mathematical analysis

We begin by introducing the following relevant quantities given in Eqs. (7) and (8) (Nadeem et al., 2014):

\[ \begin{align*}
    x &= x \sqrt{\frac{\nu}{v}}, \quad y = \frac{\sqrt{\frac{\nu}{v}}}{v}, \quad p = \frac{1}{\mu_{nf} \rho_p}, \quad \rho = \frac{1}{\mu_{nf} \rho_p} p, \\
    u &= \frac{1}{\sqrt{\nu B}}, \quad v = \frac{1}{\sqrt{\nu B}}, \quad T = \frac{T - T_0}{T_0 - T_0}.
\end{align*} \] (8)

Equations (1)–(6) are presented as follows:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \] (9)

\[ \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \frac{\nu_{nf}}{\nu f} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{1}{\rho_{nf}} \frac{\partial p}{\partial x}, \] (10)

\[ \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} = \frac{\nu_{nf}}{\nu f} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{1}{\rho_{nf}} \frac{\partial p}{\partial y}. \] (11)

Consider the following relations of stream equation:

\[ u = \psi_x, \quad v = -\psi_y. \] (13)

Substitution of equation (13) into equations (9)–(12) and using \( \rho_{nf} = \rho_{sf} \), to eliminate the pressure term:

\[ \frac{\nu_{nf}}{\nu f} (\nabla^2 \psi) + \frac{\partial \psi}{\partial (x, y)} = 0, \] (14)

\[ \frac{\nu_{nf}}{\nu f} \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) - \frac{16 \nu_{sf}}{3 \kappa_t (\mu C_{pf})_{nf}} \frac{\partial \psi}{\partial y} \left( \frac{\partial^2 \psi}{\partial y^2} \right) = 0. \] (15)

The following are the related boundary conditions:

\[ \psi = 0, \quad \psi = x, \quad T = 1. \] (16)

Here, shear in the stream is given by \( \gamma = \frac{\partial u}{\partial y} \). We obtain the solution of equations (14)–(16) of the form:

\[ \psi = x f(y) + g(y). \quad T = 0 \quad \text{as} \quad y \rightarrow \infty. \] (17)

By comparing the powers of \( x \) and the appropriate boundary conditions, we obtain the following ODEs by substituting equation (17) into equations (14)–(16) after simplification:

\[ \frac{1}{(1 - \phi)^2} f'' + \left( 1 - \phi + \phi \left( \frac{\nu_{nf}}{\nu f} \right) \right) \left[ f f'' - f^2 + B_1 \right] = 0. \] (18)

\[ \frac{1}{(1 - \phi)^2} g'' + \left( 1 - \phi + \phi \left( \frac{\nu_{nf}}{\nu f} \right) \right) \left[ f g'' - f g' + B_2 \right] = 0. \] (19)

Table 2: Physical and thermic characteristics of water (base liquid), SWCNH, and MWCNT(nanomaterials) (Ahmed et al., 2018; Kumar et al., 2019).

| Base fluid/nanoparticles | \( \rho(\frac{kg}{m^3}) \) | \( C_p(\frac{J}{kg K}) \) | \( k(\frac{W}{m K}) \) |
|-------------------------|-----------------|-----------------|-----------------|
| SWCNH                   | 1100            | 4179            | 0.6130          |
| Water                   | 1600            | 796             | 6000            |
| MWCNT                   | 1600            | 796             | 6000            |

Figure 1: Flow model.
\[ \theta^0 \left[ \frac{k_{bf}}{k_f} + \frac{4}{3} N_i (1 + (\theta_s - 1) \theta^0) \right] + 4 N_i \theta^2 (\theta_s - 1) (1 + (\theta_s - 1) \theta^0) + \frac{\rho f}{\rho_j} \rho f \theta^0 = 0. \]

Employing the limit \( y \to \infty \) in equation (18) along with the boundary condition \( f'(\infty) = \frac{v}{e} \), we get \( B_1 = \frac{v}{e} \). Further, by applying the limit \( y \to \infty \) in (19) equation and using the BC \( g'(\infty) = \gamma \), we attain the value of constant as \( B_2 = -A \phi \). As a result, equations (18)–(20) assume the following form:

\[ \frac{1}{(1 - \phi)^{1/2}} \left[ f' + \left( 1 - \phi + \frac{\rho f}{\rho_j} \right) \right] \left[ f' + \frac{1}{(1 - \phi)^{1/2}} \right] = 0, \]

\[ \frac{1}{(1 - \phi)^{1/2}} \left[ f' + \left( 1 - \phi + \frac{\rho f}{\rho_j} \right) \right] \left[ f' + \frac{1}{(1 - \phi)^{1/2}} \right] = 0. \]

\[ \theta^0 \left[ \frac{k_{bf}}{k_f} + \frac{4}{3} N_i (1 + (\theta_s - 1) \theta^0) \right] + 4 N_i \theta^2 (\theta_s - 1) (1 + (\theta_s - 1) \theta^0) + \frac{\rho f}{\rho_j} \rho f \theta^0 = 0. \]

Introducing

\[ g'(y) = \gamma h(y). \]

Using equation (25) in equation (23), we have

\[ \frac{1}{(1 - \phi)^{1/2}} h' + \left( 1 - \phi + \frac{\rho f}{\rho_j} \right) \left[ f' - h' - A \right] = 0. \]

With depicted conditions at the boundary:

\[ h(0) = 0, \quad \text{and} \quad h'(\infty) = 1. \]

The dimensionless parameters that result from the aforementioned equations are given in Eq. (28):

\[ \frac{\rho f}{\rho_j} = \frac{1 - \phi + \frac{\rho f}{\rho_j}}{\rho f}, \quad \frac{\rho f}{\rho_j} = \frac{1 - \phi + 2 \phi \frac{\rho f}{\rho_j}}{\rho f}, \quad \frac{k_{bf}}{k_f} = \frac{\rho f}{\rho_j} \frac{k_{bf}}{k_f}, \quad \frac{\rho f}{\rho_j} = \frac{1 - \phi + \frac{\rho f}{\rho_j}}{\rho f}, \quad \frac{k_{bf}}{k_f} = \frac{\rho f}{\rho_j} \frac{k_{bf}}{k_f}. \]

### 2.2 Thermal and physical characteristics of nanofluids

#### 2.2.1 Density (\( \rho \))

The density has an impact on the heat transfer characteristics. Pak and Cho (1998) provide the following model Eq. (29), which was applied in this study:

\[ \rho_{nf} = \varphi \rho_f + (1 - \varphi) \rho_j. \]

#### 2.2.2 Viscosity (\( \mu \))

The viscosity of fluid has been discovered to be an important factor in affecting the convective heat transfer coefficient. Brinkman (1952) describes the model employed in this investigation as given in Eq. (30):

\[ \frac{\mu_f}{(1 - \phi)^{1/2}}. \]

### 2.2.3 Specific capacity of heat (\( C_p \))

The potential of a substance to acquire (absorb) energy in the form of heat and transfer it if a temperature gradient occurs is measured by its specific heat capacity (Teng et al., 2011). The model used in this study has the property that nanoparticles and working liquid are in a thermal state of equilibrium (Xuan & Roetzel, 2000) and is given mentioned in Eq. (31):

\[ (1 - \phi) (\rho C_p) f + \phi (\rho C_p) j. \]

### 2.2.4 Thermal conductivity model (\( k_{bf} \))

It is one of the most essential thermophysical features of a nanoliquid that helps to optimize the heat transfer mechanism. The following is the carbon-based thermal conductivity model that is employed in this study.

Xue model. Jiang et al. (2015) developed a model for CNT nanofluids that is centered on the Maxwell model and incorporates the influence of the CNT axial ratio and spatial distribution. Equation (32) defines this model, which is based on the thermal conductivity of nanomaterials, volume fraction, in addition to the base liquid. In this research, the Xue model is developed to study the thermal conductivity of SWCNT/H2O.

\[ k_{nf} = \frac{1 - \phi + 2 \phi \kappa_{nf} \ln \frac{k_{nf}}{k_f}}{1 - \phi + 2 \phi \kappa_{nf} \ln \frac{k_{nf}}{k_f}}. \]

The (effective) thermal conductivity improvement is only related to the CNT volume fraction in Xue’s model. In this model, additional parameters such as nanoparticle shape and interfacial nanolayer were ignored.

### 3. Physical Quantities

The dimensional expression of skin friction coefficient \( C_f \) and \( N_u \) Nusselt number are appended in Eqs. (33) and (34):

\[ C_f = \frac{r_e}{\frac{1}{2} \rho_f (E X)^{2}}, \quad \text{where shear stress} \quad r_e = \mu f \frac{\partial u}{\partial y} \bigg|_{y=0}. \]

\[ N_u = \frac{x_{du}}{k_f (T_s - T_o)} \quad \text{exhibiting heat flux} \quad q_n = -\frac{\partial T}{\partial y} \bigg|_{y=0} + q_f. \]

Dimensionless expression of the above-mentioned quantities is mentioned in Eq. (35).

\[ \frac{1}{2} \operatorname{Re} C_f = \frac{\mu f}{\mu_{nf}} \left[ \gamma h'(0) + \sqrt{\operatorname{Re}} f''(0) \right], \]

\[ \frac{N_u}{\sqrt{\operatorname{Re}}} = -\theta'(0) \left[ \frac{k_{nf}}{k_f} + \frac{4}{3} N_i (1 + (\theta_s - 1) \theta^0)^3 \right]. \]

where \( \operatorname{Re} = \frac{uv}{v} \) is referred to as local Reynold number.

### 4. Numerical Solution

Transformed ODEs (22), (23), (24), and (26) and boundary conditions (21) and (27) the MATLAB, bvp4c approach is applied, which incorporates the Collocation technique, is employed to get numerical estimates. Further, it is crucial that values that are finite of \( \eta_{nf} \) must be determined. For this computational objective, the boundary conditions at \( \eta_{nf} \) for a given case are confined to \( \eta = 4 \) and 7, which is required to illustrate the behavior of the required equations’ asymptotic solution. This means that we are using [0, 7] as the domain of the problem rather than [0, \( \infty \)] because for \( \eta > 0 \) there is no substantial variance in the results. The
computing technique necessitates the translation of differential higher order equations into equations of order one. The numerical system works as given in Eqs. (37)–(39):

\[ f = y(1), f' = y(2), f'' = y(3), f''' = yy_1, h = y(4), h' = y(5), \]

\[ h'' = yy_2, \theta = y(6), \theta' = y(7), \theta'' = yy_3. \]  

(36)

\[ yy_1 = -(1 - \phi)^{2.5} \left( (1 - \phi) + \phi \left( \frac{\rho_s}{\rho_f} \right) \right) \]

\[ \times \left[ y(1) y(3) - y(2) y(2) + \left( \frac{a e}{\eta} \right) \right]; \]  

(37)

\[ yy_2 = -(1 - \phi)^{2.5} \left( (1 - \phi) + \phi \left( \frac{\rho_s}{\rho_f} \right) \right) \]

\[ \times \left[ y(1) y(5) - y(4) y(2) - A \right]; \]  

(38)

\[ yy_3 = \frac{4N_r y(7) y(7)(\theta_w - 1)(1 + (\theta_w - 1) y(6))^2 + \left( \frac{C_p}{\rho_f} \right) P r y(1) y(7)}{\left[ \frac{\eta_f}{T} + \frac{4}{T} N(1 + (\theta_w - 1) y(6))^2 \right]}. \]  

(39)

With transformed boundary conditions as given in Eq. (40).

\[ y(1) = 1, y(2) = 1, y(4) = 1, y(6) = 1, y_{inf}(2) = \frac{a e}{\epsilon}, y_{inf}(5) = 1, y_{inf}(6). \]  

(40)

The bvp4c function requires a starting presumption for the explanation, with a tolerance of \( 10^{-6} \) for the problem under discussion. Step size is taken as \( \Delta n = 0.08 \), for obtaining the numerical results. Asymptotically, the chosen starting estimate must be associated with both the boundary condition and the solution. The Nusselt number is also subjected to the grid independence test using the MATLAB software’s bvp4c function. As shown in Table 3, grid size \( 15 \times 15 \) is sufficient for a grid-free system. Following that, it appears as though the values of the Nusselt number are independent of the grids. Flow chart of the numerical solution is given in Fig. 2.

### 5. Analysis and Discussion of Results

The essence of this section is to evaluate the fluctuations of diverse parameters displayed in graphical representations. The values allocated to the dimensionless parameters used in calculations are as follows: \( \tau = 0.6, A = 0.6, \gamma = 0.65, Pr = 6.2, \phi = 0.05, \theta_w = N_r = 0.5 \). All of the graphs show SWCNH and MWCNT nanoparticle suspensions in a water-based fluid.

#### 5.1 Analysis of results

This section examines the behavior of normal and tangential velocities, as well as the temperature profile, in relation to the effects of various factors. Figures 3, 3(A), and 3(B) are drawn for various values of \( \tau \) on SWCNH/H\(_2\)O and MWCNT/H\(_2\)O nanofluids for normal, tangential velocities, and temperature profiles. It is determined that the normal velocity \( f'(y) \) profile in Fig. 3 is found to escalate with enhancement in the values of

| S. # | Grid size | Nu\(\sqrt{Re}\) |
|------|-----------|----------------|
| 1    | 5 \times 5 | 0.8743        |
| 2    | 10 \times 10 | 0.8485      |
| 3    | 15 \times 15 | 0.8484      |
| 4    | 20 \times 20 | 0.8484      |
| 5    | 25 \times 25 | 0.8484      |

Table 3: Grid point analysis for \( Nu\sqrt{Re} \).

![Figure 2: Flow chart of the numerical solution.](https://www.oup.com/us/product/9781626763979)
Figure 3: Normal velocity against varied values of $\xi$.

Figure 3A: Tangential velocity against varied values of $\xi$. 
Figure 3B: Temperature against varied values of $\alpha$.

Figure 4A: Normal velocity against varied values of $\phi$. 
Figure 4B: Temperature against varied values of $\phi$.

Figure 5: Temperature against varied values of $N_r$. 
Figure 6: Temperature against varied values of $\theta_w$.

Figure 7: The consequence of $a/e$ and $N_r$ on Nusselt number.
Figure 8: Streamline layout for oblique flow for $\gamma = 0.5$.

Figure 9: Streamline layout for oblique flow for $\gamma = -0.5$. 
stretching ratio parameter $\frac{a}{e}$. On the other hand, the tangential profile in Fig. 3(A) decreases by enhancing the values of stretching ratio parameter $\frac{a}{e}$. Further temperature profile in Fig. 3(B) also declines by increasing stretching ratio parameter $\frac{a}{e}$. Figures 4(A) and 4(B) are drawn for diverse values of $\phi$ versus normal velocity and temperature profile for two different nanofluids namely SWCNH/H$_2$O and MWCNT/H$_2$O nanofluids. It can be seen that normal velocity decreases by enhancing volume fraction $\phi$ while temperature profile enhances for growing values of $\phi$. The illustration of the radiation parameter on the temperature profile is seen in Fig. 5. It is observed that by mounting values of $N_r$, the temperature profile increases. The influence of the surface heating parameter $\theta_w$ on fluid temperature is seen in Fig. 6. The temperature rises as the values of surface heating parameter are increased. The consequences of the velocity ratio parameter $\frac{a}{e}$ and the nonlinear thermal radiation parameter $N_r$ on the Nusselt number are depicted in Fig. 7, for both SWCNH/H$_2$O and MWCNT/H$_2$O nanofluids. It is depicted that the local Nusselt number is enhancing function for both $\frac{a}{e}$ and $N_r$. The streamline patterns for the oblique flows are displayed in Figs 8 and 9, for varied values of $\gamma$. It is observed that the point of stagnation for a positive value of $\gamma$ is to the left of the origin, whereas for a negative value of $\gamma$ it is to the right of the origin.

5.2 Discussion of results

This section will validate all of the preceding observations of governing parameters on velocities and thermal profiles reported in Section 5.1. The stretching ratio parameter is defined as free stream velocity to the stretching velocity. So, the main reason for the enhancement of normal velocity in Fig. 3 is that, for a constant value of $\frac{a}{e}$ related to the stretching of the employed surface, the upsurge in $a$ in proportion to $e$ implies an enhancement in the straining motion near the stagnation region that might upsurge the external stream’s acceleration. On the other hand, the tangential profile in Fig. 3A exhibits the opposite tendency since it is parallel to the x-axis and stretching parallel to the x-axis is $e$, which is in the denominator of the stretching ratio parameter $\frac{a}{e}$. As the stretching parameter grows, the tangential velocity diminishes. Further temperature profile also declines by increasing stretching ratio parameter $\frac{a}{e}$ in Fig. 3B. The temperature profile of MWCNT/H$_2$O nanofluid exhibits a more pronounced tendency toward decrease. Due to the substantially reduced temperature profile of the MWCNT/H$_2$O nanofluid, it will be inappropriate for future solar thermal systems. Further, normal velocity decreases by enhancing volume fraction $\phi$ in Fig. 4A while temperature profile enhances for increasing values of $\phi$ in Fig. 4B. Physically, increasing the volume fraction leads to the development of viscous forces inside the nanofluid. As a result of the resistance engendered between the fluid particles, the velocity of the fluid particles lowers, and the temperature profile rises. It is further revealed that by enhancing the values of particle volume fraction $\phi$ (SWCNH and MWCNT) nanoparticles, the temperature of nanofluid is greater for SWCNH/H$_2$O than for the MWCNT/H$_2$O, which will be favorable for thermal performance in solar systems. For mounting values of $N_r$, the temperature profile increases in Fig. 5. This is because increasing the value of $N_r$ tends to increase the conduction effect and the thermal boundary layer. This component raises the temperature at every position distant from the surface. In other words, by enhancing nonlinear radiation on the surface, heat transmission can be regulated. The temperature profile in Fig. 6 rises as the values of surface heating parameter $\theta_w$ are increased. For both temperature profiles, it was found that for SWCNH/H$_2$O
nanofluid, the growing trend of fluid temperature was more pronounced than for MWCNT/H2O nanofluid, indicating that this nanofluid will be more appropriate for increasing the temperature of the fluid in solar thermal systems. In Fig. 7, it is depicted that the local Nusselt number is enhancing function for both $\frac{a}{e}$ and $N_r$. Remarkably, for MWCNT nanoparticles, this rise is higher than that for SWCNH nanoparticles.

6. Comparisons of Numerical Results

Figures 10 and 11 demonstrate the numerical plots of velocity profile with available literature for diverse values of stretching ratio parameter $a/e$. A good association with Nadeem et al. (2015) results between the graphs is noticed. In order to make the comparison more obvious, the temperature profile is computed for various values of $y$ and provided in Table 4. It can be evident in this table that the temperature profile of SWCNH/H2O nanofluid is higher than the temperature profile of MWCNT/H2O nanofluid. Further, to ensure the validity of the current calculated findings in comparison to existing published data, a comparison between the current computed results and available literature is done in limiting cases. Table 5 is erected to check the validity of the presented mathematical model by comparing

| $a/e$ | Present | Nadeem et al. (2015) | Labropulu et al. (2010) | Present | Nadeem et al. (2015) | Labropulu et al. (2010) |
|-------|---------|----------------------|------------------------|---------|----------------------|------------------------|
| 0.1   | -0.96930 | -0.96938             | -0.96938               | 0.26339 | 0.26341              | 0.26278                |
| 0.3   | -0.84938 | -0.84942             | -0.84942               | 0.60629 | 0.60631              | 0.60573                |
| 0.8   | -0.29935 | -0.29938             | -0.29938               | 0.93471 | 0.93473              | 0.93430                |
| 1.0   | 0.0       | 0.0                   | 0.0                    | 1.0     | 1.0                  | 1.0                    |

Table 4: Comparison of temperature profile $\theta(y)$ against $y$ for tabular form of different nanofluids.

| Parameters | $\theta(y)$ | $\phi(y)$ |
|------------|-------------|-----------|
| $\phi=0.03$, $y=2$ | 0.20 | 0.18 |
| $\phi=0.05$, $y=3.5$ | 0.10 | 0.09 |
| $\phi=0.07$, $y=4$ | 0.09 | 0.08 |
it with Nadeem et al. (2015) and Labropulu et al. (2010) for numerous values of $\alpha$ in limiting case. An excellent correlation between the values is found.

7. Concluding Remarks

With nonlinear thermal radiation, we have scrutinized the heat transfer analysis of steady oblique stagnation-point flow of SWCNH/H$_2$O and MWCNT/H$_2$O nanofluids toward a stretched surface. To evaluate the thermal performance of nanofluids, the Xue model of thermal conductivity is adopted. Furthermore, consequences of diverse parameters on various profiles are represented and portrayed. The signified important consequences of the situation are listed below:

1. Velocity ratio parameter $\frac{a}{e}$ has increased for normal velocity profile but decreased for tangential and temperature profiles. Stretching causes a reduction in the temperature profile, which demonstrates that the surface of solar thermal systems should not be stretched to maximize efficiency.

2. In terms of the temperature profile, SWCNH nanoparticle contained nanofluid has considerably greater thermal radiation $N$, than MWCNT nanoparticles. The lower absorptivity of SWCNH nanofluid indicates that it will perform better in solar thermal systems to boost the temperature of the fluid, which is a positive idea.

3. The temperature profile improves when the surface heating parameter $\theta_0$ is increased. This property is more noticeable in SWCNH nanofluid than in MWCNT nanofluid. SWCNH nanofluid outperforms MWCNT nanofluid when used for heating purposes in solar thermal systems.

4. The snowballing estimates of the velocity ratio parameter $\frac{a}{e}$ and $\gamma$ enhances the coefficient of skin friction.

5. According to the current model, the incorporation of SWCNH nanoparticles can result in improved performance in an energy solar system when compared to the suspension of MWCNT nanoparticles. This is because thermal performance in SWCNH nanofluid was consistently excellent, whereas thermal performance in MWCNT nanofluid was inadequate.

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Conflict of interest statement

None declared.

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