Model Independent Analysis of the Forward-Backward Asymmetry for the
$B \rightarrow K_1 \mu^+ \mu^-$ Decay

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(Dated: January 12, 2011)

Abstract

The sensitivity of the zero position of the forward backward asymmetry $A_{FB}$ for the exclusive $B \rightarrow K_1(1270) \mu^+ \mu^-$ decay is examined by using most general non-standard 4-fermion interactions. Our analysis shows that the zero position of the forward backward asymmetry is very sensitive to the sign and size of the Wilson coefficients corresponding to the new vector type interactions, which are the counter partners of the usual Standard Model operators but have opposite chirality. In addition to these, the other significant effect comes from the interference of Scalar-Pseudoscalar and Tensor type operators. These results will not only enhance our theoretical understanding about the axial vector mesons but will also serve as a good tool to look for physics beyond the SM.
I. INTRODUCTION

Flavor changing neutral current transitions (FCNC) which generally arise at loop level provides a good testing ground for the Standard Model (SM) \cite{1,2}. Moreover, in such transitions the New Physics (NP) effects can be probed via the loop of the particles that are beyond the spectrum of SM. Therefore, there are solid reasons, both theoretical and experimental, for studying these FCNC transitions. Among all the FCNC processes, the rare $B$ decays are important since one can test both the SM and the possible NP effects by comparing the theoretical results with the current and future experiments.

Some of the radiative and semileptonic decays of $B$ mesons to vector and axial vector mesons, such as $B \rightarrow K^*\gamma$ \cite{3,5}, $B \rightarrow K_1(1270, 1400)\gamma$ \cite{6} and $B \rightarrow K^*(892)e^+e^−(\mu^+\mu^-)$ \cite{7,8} have been observed and for $B \rightarrow K^*(892)e^+e^−(\mu^+\mu^-)$ the measurement of isospin and forward-backward asymmetry at BABAR is also reported \cite{11}. For $B \rightarrow K_1(1270, 1400)\gamma$ the Belle has given the following branching ratios

\begin{equation}
Br(B \rightarrow K_1(1270)\gamma) = (4.28 \pm 0.94 \pm 0.43) \times 10^{-5}
\end{equation}

\begin{equation}
Br(B \rightarrow K_1(1400)\gamma) < 1.44 \times 10^{-5}
\end{equation}

The semileptonic $B$ meson decays, $B \rightarrow (K, K^*)l^+l^− (l = e, \mu, \tau)$ are widely studied in the literature \cite{12} where different physical observables like decay rate, lepton forward-backward asymmetry and lepton polarizations are calculated both in SM and beyond. Among these physical observables, the most interesting one is the lepton forward-backward asymmetry $A_{FB}$ and this lies in the vanishing of $A_{FB}$ at a specific value of dilepton mass in a hadronically clean way \cite{13,15}. This in other words provide a simple relationship between the electric dipole coefficient $C_7$ and $C_9$, which is almost free from the hadronic uncertainties which arises dominantly from the form factors \cite{14}.

The above mentioned decays also open a window to look for new Physics. We know that in SM the decays $B \rightarrow (K, K^*)l^+l^−$ are completely determined by the Wilson coefficients of only three operators $O_7, O_9$ and $O_{10}$ which are evaluated at the scale $\mu = m_b$ \cite{16}. On the other hand the most general analysis of these decays needs other set of new operators which are based on the the general four-fermion interactions. The new structure of effective Hamiltonian \cite{17,18} makes them an ideal platform for the SM, and provide clues for the NP. In the literature, the model independent analysis of the quark level $b \rightarrow s l^+l^−$ decay, in terms of 10 new types of local four fermion interactions, has been performed in Ref. \cite{17} which is then applied to the systematic study of $B \rightarrow (K, K^*)l^+l^−$ \cite{19}. Recently, the discrepancy has been observed in the lepton forward-backward asymmetry in the exclusive $B \rightarrow K^*\mu^+\mu^−$ decay \cite{20,21}. To explain the experimental results, Kumar et al. \cite{22} have done a systematic study $B \rightarrow K^*\mu^+\mu^−$ decay by using the most general model independent Hamiltonian. They have shown that though the scalar and tensor operators are not very important to study the lepton forward-backward asymmetry but the interference of these two is important and is not ignorable which differ from the results given in \cite{19}.

As the radiative decay $B \rightarrow K_1(1270)\gamma$ has already seen by Belle, therefore the related decay with a lepton pair instead of a photon in the final state can also be expected to be seen. Analysis of this decay process will be a useful complement to the widely investigated analysis for the $B \rightarrow K^*l^+l^−$ process, since the analysis probes the effective Hamiltonian in a similar but not identical way. The experimental investigation of this decay will thus provide us independent test of the predication of the SM and also give us the clue for NP.

Like $B \rightarrow K^*l^+l^−$ the semileptonic decay $B \rightarrow K_1(1270)l^+l^−$ is also governed by the quark level transition $b \rightarrow sl^+l^−$. Compared to $B \rightarrow K^*l^+l^−$ the situation is complicated in the decay $B \rightarrow K_1(1270)l^+l^−$, because the axial vector states $K_1(1270)$ and $K_1(1400)$ are the mixtures of ideal $^1P_1(K_{1A})$ and $^3P_1(K_{1B})$ orbital angular momentum states and current limit on the mixing angle is \cite{23}

\begin{equation}
\theta = -(34 \pm 13)°.
\end{equation}

Recently, some studies have been made on $B \rightarrow K_1$ transitions both by incorporating the mixing angle as well as with out it \cite{24}.

Experimentally, this decay has not yet been seen, but is expected to be observed at LHC \cite{25} and SuperB factory \cite{26}. In particular LHCb experiment at the LHC where estimates made in \cite{25,27} for LHCb collaboration show that
with an integrated luminosity of $2 fb^{-1}$, one may expect almost 8000 $B \rightarrow K^* l^+ l^-$ events. Although the branching ratio of $B \rightarrow K_1(1270) l^+ l^-$ calculated in $[28]$ is an order of magnitude smaller than the experimentally measured value of $B \rightarrow K^* l^+ l^-$ $[29]$, but still one can expect the significant number of events for this decay and hence making analysis of FB asymmetry for this decay will be experimentally meaningful for comparison with the SM and the theories beyond it.

    In this work, our aim is to analyze the possible new physics effects stemming from the new structures in the effective Hamiltonian $[18]$ to the forward-backward asymmetry for the $B \rightarrow K_1(1270) l^+ l^-$ decay. It has already been mentioned that some experimental analysis for the decay $B \rightarrow K^* \mu^+ \mu^-$ has already been studied in $B$ factories $[20]$, but only the large increase in statistics at LHCb for $B \rightarrow K^* \mu^+ \mu^-$ will make much higher precision measurements possible $[25, 27]$. It is known that the forward-backward asymmetry becomes zero for a particular value of the dilepton invariant mass. In the SM, the zero of the $A_{FB}(q^2)$ appears in the low $q^2$ region, sufficiently away from the charm resonance region and is almost free from the hadronic uncertainties (i.e. the choice of form factors) and so is from the mixing angle. Now this zero position of $A_{FB}$ varies from model to model and this makes it an important tool to search for physics beyond the SM. The organization of the paper is as follows: In section II we introduce the model independent effective Hamiltonian and obtain the transition matrix elements in terms of form factors of the $B \rightarrow K_1(1270) l^+ l^-$. Section III describes the formulas that can be used to determine the zero position of the FBA. In Sec. IV we present our numerical analysis and Sec.V summarizes our conclusion.

II. EFFECTIVE HAMILTONIAN AND MATRIX ELEMENTS

By integrating out the heavy degrees of freedom in the full theory, the general effective Hamiltonian for $b \rightarrow s l^+ l^-$ transitions in the SM can be written as

$$H_{eff} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu),$$

(3)

where $O_i(\mu)$ ($i = 1, \ldots, 10$) are the four-quark operators and $C_i(\mu)$ are the corresponding Wilson coefficients at the energy scale $\mu$ $[16]$. Using renormalization group equations to resum the QCD corrections, Wilson coefficients are evaluated at the energy scale $\mu = m_b$. The theoretical uncertainties associated with the renormalization scale can be substantially reduced when the next-to-leading-logarithm corrections are included.

The explicit expressions of the operators responsible for exclusive $B \rightarrow K_1(1270) l^+ l^-$ transition are given by

$$O_7 = \frac{e^2}{16\pi^2} m_b (\bar{s} \sigma_{\mu \nu} P_R b) F^{\mu \nu},$$

(4)

$$O_9 = \frac{e^2}{16\pi^2} (\bar{s} \gamma_{\mu} P_L b)(\bar{l} \gamma^\mu l),$$

(5)

$$O_{10} = \frac{e^2}{16\pi^2} (\bar{s} \gamma_{\mu} P_L b)(\bar{l} \gamma^\mu \gamma_5 l),$$

(6)

with $P_{L,R} = (1 \pm \gamma_5)/2$. In terms of the above Hamiltonian, the free quark decay amplitude for $b \rightarrow s l^+ l^-$ is:

$$\mathcal{M}_{SM}(b \rightarrow s l^+ l^-) = -\frac{G_F \alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \left\{ C_9^{eff} (\bar{s} \gamma_{\mu} P_L b)(\bar{l} \gamma^\mu l) + C_{10} (\bar{s} \gamma_{\mu} P_L b)(\bar{l} \gamma^\mu \gamma_5 l) - 2m_b C_7^{eff} (\bar{s} \sigma_{\mu \nu} \frac{\sigma}{\pi} P_R b)(\bar{l} \gamma^\mu l) \right\},$$

(7)

where $s = q^2$ and $q$ is the momentum transfer. The operator $O_{10}$ can not be induced by the insertion of four-quark operators because of the absence of the $Z$ boson in the effective theory. Therefore, the Wilson coefficient $C_{10}$ does not renormalize under QCD corrections and hence it is independent of the energy scale. In addition to this, the above quark level decay amplitude can receive contributions from the matrix element of four-quark operators, $\sum_{i=1}^{6} (l^+ l^- s) |O_i|$, which are usually absorbed into the effective Wilson coefficient $C_9^{eff}(\mu)$, that one can decompose
expressions for $Y$ Breit-Wigner formula making use of the vacuum saturation approximation and quark-hadron duality. The manifest cannot be calculated from first principles of QCD and are usually parameterized in the form of a phenomenological $Y$ the perturbative theory. The long-distance contributions $Y_{LD}(z, s')$ from four-quark operators near the $c\bar{c}$ resonance cannot be calculated from first principles of QCD and are usually parameterized in the form of a phenomenological Breit-Wigner formula making use of the vacuum saturation approximation and quark-hadron duality. The manifest expressions for $Y_{SD}(z, s')$ and $Y_{LD}(z, s')$ can be written as

$$Y_{SD}(z, s') = h(z, s')(3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu))$$

$$\frac{-1}{2}h(1, s')(4C_3(\mu) + 4C_4(\mu) + 3C_5(\mu) + C_6(\mu))$$

$$\frac{-1}{2}h(0, s')(3C_3(\mu) + 3C_4(\mu)) + \frac{2}{9}(3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)), \tag{8}$$

with

$$h(z, s') = -\frac{8}{9}\ln z + \frac{8}{27} + \frac{4}{9}x - \frac{2}{9}(2 + x)|1 - x|^{1/2} \left\{ \ln \frac{\sqrt{x} + 1}{\sqrt{1 - x} - 1} - i\pi \right\} \text{ for } x \equiv 4z^2/s' < 1$$

$$\frac{2\arctan}{\sqrt{x - 1}} \text{ for } x \equiv 4z^2/s' > 1$$

$$h(0, s') = \frac{8}{27} - \frac{8}{9}\ln \frac{m_b}{\mu} - \frac{4}{9}\ln s' + \frac{4}{9}i\pi. \tag{9}$$

and

$$Y_{LD}(z, s') = \frac{3\pi}{\alpha^2}C^{(0)} \sum_{V_i\in\psi_i} \kappa_i \frac{m_{V_i}\Gamma(V_i \rightarrow l^+l^-)}{m_{V_i}^2 - s' - i\Gamma_{V_i}} \tag{10}$$

where $C^{(0)} = 3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6$. The $Y_{LD}(z, s')$ critically depend on the resonance model used to describe these LD contributions and as such they have uncertainties. But these uncertainties will hardly effect the zero position of the FB asymmetry which lies below this charmonium threshold. Keeping in view that there is no experimental data on $B \rightarrow K(1270)l^+l^-$, we have fixed the values of the phenomenological parameters $\kappa_i$ from $B \rightarrow K^*l^+l^-$, which for the resonances $J/\Psi$ and $\Psi'$ are taken to be $\kappa = 1.65$ and $\kappa = 2.36$, respectively.

Apart from this, the non-factorizable effects from the charm loop can bring about further corrections to the radiative $b \rightarrow s\gamma$ transition, which can be absorbed into the effective Wilson coefficient $C_{7}^{eff}$. Specifically, the Wilson coefficient $C_{7}^{eff}$ is given by

$$C_{7}^{eff}(\mu) = C_{7}(\mu) + C_{b\rightarrow s\gamma}(\mu),$$

with

$$C_{b\rightarrow s\gamma}(\mu) = i\alpha_s \left[ \frac{2}{9}\eta^{14/23}(G_1(x_t) - 0.1687) - 0.03C_2(\mu) \right], \tag{11}$$

$$G_1(x) = \frac{x(x^2 - 5x - 2)}{8(x - 1)^3} + \frac{3x^2\ln^2 x}{4(x - 1)^4}, \tag{12}$$

where $\eta = \alpha_s(m_W)/\alpha_s(\mu)$, $x_t = m_t^2/m_W^2$, $C_{b\rightarrow s\gamma}$ is the absorptive part for the $b \rightarrow sc\bar{c} \rightarrow s\gamma$ rescattering and we have dropped out the tiny contributions proportional to CKM sector $V_{ub}V_{us}^*$. In addition to the above mentioned currents, the most general form of the effective Hamiltonian contains 10 local four fermion interactions which can contribute to the $B \rightarrow K(1270)l^+l^-$ decay and these can be written as:
\[ \mathcal{M}_{\text{new}}(b \rightarrow s \ell^+ \ell^-) = \mathcal{M}^{V-A} + \mathcal{M}^{S-P} + \mathcal{M}^T \]

\[ \mathcal{M}^{V-A} = \frac{G_F \alpha}{\sqrt{2} \pi} \bar{V}_{tb} V_{tb} \left\{ C_{LL} \bar{s} \gamma^\mu b l_l \bar{l}_L \gamma^\mu l_L + C_{LR} \bar{s} \gamma^\mu b l_l \bar{l}_R \gamma^\mu l_R + C_{RL} \bar{s} \gamma^\mu b l_l \bar{l}_L \gamma^\mu l_L + C_{RR} \bar{s} \gamma^\mu b l_l \bar{l}_R \gamma^\mu l_R \right\} \]

\[ \mathcal{M}^{S-P} = \frac{G_F \alpha}{\sqrt{2} \pi} \bar{V}_{tb} V_{tb} \left\{ C_{SS} \bar{s} \sigma_{\mu\nu} b l_l \bar{l}_L \sigma^{\mu\nu} l_L + C_{SR} \bar{s} \sigma_{\mu\nu} b l_l \bar{l}_R \sigma^{\mu\nu} l_R + C_{RS} \bar{s} \sigma_{\mu\nu} b l_l \bar{l}_L \sigma^{\mu\nu} l_L + C_{RR} \bar{s} \sigma_{\mu\nu} b l_l \bar{l}_R \sigma^{\mu\nu} l_R \right\} \]

\[ \mathcal{M}^T = \frac{G_F \alpha}{\sqrt{2} \pi} \bar{V}_{tb} V_{tb} \left\{ C_T \bar{s} \sigma_{\mu\nu} b \sigma^{\mu\nu} l + i C_T \epsilon_{\mu\nu\alpha\beta} \bar{s} \sigma^{\mu\nu} l \sigma^{\alpha\beta} b \right\} \]

Thus the explicit form of the free quark amplitude \( \mathcal{M} \) for the \( b \rightarrow s \ell^+ \ell^- \) transition can be written as sum of the SM amplitude (Eq. 7) and of the new physics contributions (Eq. 13), i.e.

\[ \mathcal{M} = \mathcal{M}_{\text{SM}} + \mathcal{M}_{\text{new}} \]

The exclusive \( B \rightarrow K_1(1270)\ell^+ \ell^- \) decay involves the hadronic matrix elements of quark operators given in Eq. 7 and Eq. 13 which one can be parametrize in terms of the form factors as follows:

\[ \langle K_1(k, \varepsilon) | V_\mu | B(p) \rangle = \varepsilon^*_\mu (M_B + M_{K_1}) V_1(s) \]

\[ \quad - (p + k)_\mu (\varepsilon^* \cdot q) \frac{V_2(s)}{M_B + M_{K_1}} \]

\[ \quad - q_\mu (\varepsilon \cdot q) \frac{2M_{K_1}}{s} [V_3(s) - V_0(s)] \]

\[ \langle K_1(k, \varepsilon) | A_\mu | B(p) \rangle = \frac{2i \epsilon_{\mu\nu\alpha\beta} \varepsilon^{\alpha\nu} p^\beta A(s)}{M_B + M_{K_1}} \]

where \( V_\mu = \bar{s} \gamma_\mu b \) and \( A_\mu = \bar{s} \gamma_\mu \gamma_5 b \) are the vectors and axial vector currents respectively. Also \( p(k) \) are the momentum of the \( B(K_1) \) meson and \( \varepsilon^*_\mu \) is the polarization of the final state axial vector \( K_1 \) meson. In Eq. 15 we have

\[ V_3(s) = \frac{M_B + M_{K_1}}{2M_{K_1}} V_1(s) - \frac{M_B - M_{K_1}}{2M_{K_1}} V_2(s) \]

\[ V_5(0) = V_0(0) \]

In addition to the above, there is also a contribution from the Penguin form factors that can be written as

\[ \langle K_1(k, \varepsilon) | \bar{s} i \sigma_{\mu\nu} q^\nu b | B(p) \rangle = \left[ (M_B^2 - M_{K_1}^2) \varepsilon_\mu - (\varepsilon \cdot q)(p + k)_\mu \right] F_2(s) \]

\[ + (\varepsilon^* \cdot q) \left[ q_\mu - \frac{s}{M_B^2 - M_{K_1}^2} (p + k)_\mu \right] F_3(s) \]

\[ \langle K_1(k, \varepsilon) | \bar{s} i \sigma_{\mu\nu} q^{\prime \nu} \gamma_5 b | B(p) \rangle = -i \epsilon_{\mu\nu\alpha\beta} \varepsilon^{\alpha\nu} p^\beta F_1(s) \]

\[ \langle K_1(k, \varepsilon) | \bar{s}(1 \pm \gamma_5) b | B(p) \rangle = \frac{1}{m_b + m_s} \left\{ \mp 2iM_{K_1}(\varepsilon^* \cdot q)V_0(s) \right\} \]
The form factors for $B \to K_1(1270)$ transition are the non-perturbative quantities and are needed to be calculated using different approaches (both perturbative and non-perturbative) like Lattice QCD, QCD sum rules, Light Cone sum rules, etc. As the zero position of the forward-backward asymmetry depends on the short distance contribution i.e. the Wilson coefficients and is not very sensitive to the long distance contribution (Form factors) [28] and consequently on the mixing angle between $^1P_1$ and $^3P_1$ states. As such we will consider the form factors that were calculated using Ward Identities in Ref. [28] which can be summarized as follows:

\[
A(\hat{s}) = \frac{A(0)}{1 - \hat{s}} (1 - \frac{M_B^2}{M_{B^*}^2}) \\
V_1(\hat{s}) = \frac{V_1(0)}{1 - \hat{s}} (1 - \frac{M_B^2}{M_{B^*}^2}) \left( 1 - \frac{\hat{s}}{1 - M_{K_1}^2} \right) \\
V_2(\hat{s}) = \frac{V_2(0)}{1 - \hat{s}} (1 - \frac{M_B^2}{M_{B^*}^2}) - 2\hat{M}_{K_1} \frac{V_0(0)}{1 - M_{K_1} (1 - \hat{s}) (1 - \frac{M_B^2}{M_{B^*}^2})}
\]

with

\[
V_0(0) = 0.36 \pm 0.03 \\
A(0) = -(0.52 \pm 0.05) \\
V_1(0) = -(0.24 \pm 0.02) \\
\tilde{V}_2(0) = -(0.39 \pm 0.05)
\]

III. FORWARD BACKWARD ASYMMETRY FOR $B \to K_1(1270)l^+ l^-$

In this section, we are going to perform the calculation of the forward-backward asymmetry. From Eq. (23), it is straightforward to obtain the decay amplitude for $B \to K_1(1270)l^+ l^-$ as

\[
M_{B \to K_1(1270)l^+ l^-} = \frac{G_F \alpha}{4\sqrt{2}\pi} V_{tb} V_{ts} M_B \left\{ T^1_{\mu} \gamma^\mu l + T^2_{\mu} \gamma^\mu \gamma^5 l + T^3 \bar{U} l + T^4 \bar{U} \gamma^5 l \\
+ 8C_T (\sigma_{\mu\nu} l) (-2F_1(\hat{s}) \varepsilon^{*\mu} (\hat{p}_B + \hat{p}_{K_1})^\mu + J_1 \varepsilon^{*\mu} \hat{q}^\mu - J_2 (\varepsilon^{*\alpha} \hat{q}^\beta - J_2 (\varepsilon^{*\beta} \hat{q}^\alpha)) + 2iC_T E_{\mu\alpha\beta} (\sigma_{\mu\nu} l) (-2F_1(\hat{s}) \varepsilon^{*\alpha} (\hat{p}_B + \hat{p}_{K_1})^\beta + J_1 \varepsilon^{*\alpha} \hat{q}^\beta - J_2 (\varepsilon^{*\beta} \hat{q}^\alpha)) \right\}
\]

where the functions $T^1_{\mu}, T^2_{\mu}, T^3$ and $T^4$ in terms of auxiliary functions are given by

\[
T^1_{\mu} = iA'(\hat{s}) \varepsilon_{\mu\rho\sigma \beta} \varepsilon^{*\rho} \hat{p}_{K_1}^\sigma \hat{p}_B^\beta - B'(\hat{s}) \varepsilon^{*\mu} (\hat{p}_B + \hat{p}_{K_1}) \bar{\nu}_{\mu} + D'(\hat{s}) (\varepsilon^{*\mu} \hat{q}) \bar{\nu}_\mu \\
T^2_{\mu} = iE'(\hat{s}) \varepsilon_{\mu\rho\sigma \beta} \varepsilon^{*\rho} \hat{p}_{K_1}^\sigma \hat{p}_B^\beta - F'(\hat{s}) \varepsilon^{*\mu} (\hat{p}_B + \hat{p}_{K_1}) \bar{\nu}_\mu + H'(\hat{s}) (\varepsilon^{*\mu} \hat{q}) \bar{\nu}_\mu \\
T^3 = iI' (\varepsilon^{*\cdot} \hat{q}) \\
T^4 = iJ' (\varepsilon^{*\cdot} \hat{q})
\]

where $\hat{s} = s/M_B^2, \hat{p}_{K_1} = p_{K_1}/M_B, \hat{p}_B = p_B/M_B, \hat{m}_b = m_b/M_B$ and $\hat{M}_{K_1} = M_{K_1}/M_B$.

Defining the combinations

\[
C_{RR}^{(+)} = C_{RR} + C_{RL}, \quad C_{RR}^{(-)} = C_{RR} - C_{RL}, \\
C_{LL}^{(+)} = C_{LL} + C_{LR}, \quad C_{LL}^{(-)} = C_{LL} - C_{LR}, \\
C_{RLLR}^{(+)} = C_{RLLR} + C_{RLRL}, \quad C_{RLLR}^{(+)} = C_{RLLR} + C_{RLRL}, \\
C_{RLLR}^{(-)} = C_{RLLR} - C_{RLRL}, \quad C_{RLLR}^{(-)} = C_{RLLR} - C_{RLRL},
\]

(28)
the auxiliary functions appearing in Eq. (27) can be written as follows:

\[
A'(\hat{s}) = -\frac{2}{1 + M_{K_1}}[C_{9eff} + \frac{1}{2}(C_{RR}^{(+)} + C_{LL}^{(+)} + C_{RR}^{(-)})]A(\hat{s}) + \frac{2\hat{m}_b}{\hat{s}} C_{7eff} F_1(\hat{s})
\]

\[
B'(\hat{s}) = (1 + \hat{M}_{K_1})(C_{9eff} + \frac{1}{2}(C_{LL}^{(-)} - C_{RR}^{(-)}))V_1(\hat{s}) + \frac{2\hat{m}_b}{\hat{s}}(1 - \hat{M}_{K_1})C_{7eff} F_2(\hat{s})
\]

\[
C'(\hat{s}) = \frac{1}{(1 - \hat{M}_{K_1})} \left[ ((1 - \hat{M}_{K_1})(C_{9eff} + \frac{1}{2}(C_{LL}^{(-)} - C_{RR}^{(-)})))V_2(\hat{s}) + 2\hat{m}_b C_{7eff} (F_3(\hat{s}) - (1 - \hat{M}_{K_1})/\hat{s}) F_2(\hat{s}) \right]
\]

\[
D'(\hat{s}) = \frac{1}{\hat{s}} \left[ ((1 - \hat{M}_{K_1})V_1(\hat{s}) - (1 - \hat{M}_{K_1})V_2(\hat{s}) - 2\hat{M}_{K_1}V_0(\hat{s})) (C_{9eff} + \frac{1}{2}(C_{LL}^{(+)} - C_{RR}^{(+)})) - 2\hat{m}_b C_{7eff} F_3(\hat{s}) \right]
\]

\[
E'(\hat{s}) = -\frac{2}{1 + \hat{M}_{K_1}}(C_{10} + \frac{1}{2}(C_{RR}^{(-)} - C_{LL}^{(-)})) A(\hat{s})
\]

\[
F'(\hat{s}) = (1 + \hat{M}_{K_1})(C_{10} - \frac{1}{2}(C_{RR}^{(-)} - C_{LL}^{(-)})) V_1(\hat{s})
\]

\[
G'(\hat{s}) = -\frac{1}{(1 + \hat{M}_{K_1})} (C_{10} - \frac{1}{2}(C_{LL}^{(+)} + C_{RR}^{(+)})) V_2(\hat{s})
\]

\[
H'(\hat{s}) = \frac{1}{\hat{s}} \left[ ((1 - \hat{M}_{K_1})V_2(\hat{s}) - (1 + \hat{M}_{K_1})V_1(\hat{s}) + 2\hat{M}_{K_1}V_0(\hat{s})) (C_{10} - \frac{1}{2}(C_{RR}^{(-)} + C_{LL}^{(-)})) \right]
\]

\[
I'(\hat{s}) = \frac{2\hat{M}_{K_1}}{\hat{m}_b} V_0(\hat{s}) (C_{RLLR}^{(+)} + C_{LRRL}^{(+)})
\]

\[
J'(\hat{s}) = \frac{2\hat{M}_{K_1}}{\hat{m}_b} V_0(\hat{s}) (C_{RLLR}^{(+)} - C_{LRRL}^{(+)})
\]

\[
J'_1(\hat{s}) = 2 \left( 1 - \frac{\hat{M}_{K_1}^2}{\hat{s}} \right) \left( F_1(\hat{s}) - F_2(\hat{s}) - \frac{\hat{s}}{1 - \hat{M}_{K_1}^2} F_3(\hat{s}) \right)
\]

\[
J'_2(\hat{s}) = \frac{4\hat{M}_{K_1}^2}{\hat{s}} \left( F_1(\hat{s}) - F_2(\hat{s}) - \frac{\hat{s}}{1 - \hat{M}_{K_1}^2} F_3(\hat{s}) \right)
\]

(29)

where, $A'$, $B'$, $C'$, $D'$, $E'$, $F'$, $G'$, $H'$ corresponds to $VA$ interactions where as $I'$, $J'$, $J'_1$, $J'_2$ are relevant for $SP$ and $T$ interactions.

To calculate the forward-backward asymmetry of the final state leptons, one needs to know the differential decay width of $B \rightarrow K_1(1270)l^+l^-$, which in the rest frame of $B$ meson can be written as

\[
\frac{d\Gamma(B \rightarrow K_1(1270)l^+l^-)}{ds} = \frac{1}{(2\pi)^3 32M_B} \int_{u_{\min}}^{u_{\max}} |M_{B \rightarrow K_1(1270)l^+l^-}|^2 du,
\]

(30)

where $u = (k + p_{l^-})^2$ and $s = (p_{l^+} + p_{l^-})^2$; $k$, $p_{l^+}$ and $p_{l^-}$ are the four-momenta vectors of $K_1(1270)$, $l^+$ and $l^-$ respectively; $|M_{B \rightarrow K_1(1270)l^+l^-}|^2$ is the squared decay amplitude after integrating over the angle between the lepton $l^-$ and $K_1(1270)$ meson. The upper and lower limits of $u$ are given by

\[
u_{\max} = (E_{K_1}^{*}(1270)+E_{l^-}^{*})^2 - (E_{K_1}^{*2}(1270) - M_{K_1}^2(1270)) - E_{l^+}^{2} - m_{l^-}^2;
\]

\[
u_{\min} = (E_{K_1}^{*}(1270)+E_{l^-}^{*})^2 - (E_{K_1}^{*2}(1270) - M_{K_1}^2(1270)) + E_{l^+}^{2} - m_{l^-}^2;
\]

(31)

where $E_{K_1}^{*}(1270)$ and $E_{l^-}^{*}$ are the energies of $K_1(1270)$ and $l^-$ in the rest frame of lepton pair and can be determined as

\[
E_{K_1}^{*}(1270) = \frac{M_B^2 - M_{K_1}^2(1270) - s}{2\sqrt{s}}, \quad E_{l^-}^{*} = \frac{s}{2\sqrt{s}}.
\]

(32)

The differential FBA of final state lepton for the said decay can be written as

\[
\frac{dA_{FB}(s)}{ds} = \int_{0}^{1} d\cos \theta \frac{d^2\Gamma(s, \cos \theta)}{ds d\cos \theta} - \int_{-1}^{0} d\cos \theta \frac{d^2\Gamma(s, \cos \theta)}{ds d\cos \theta}
\]

(33)

and

\[ A_{FB}(s) = \frac{1}{f_0} \frac{d}{d\cos \theta} \frac{d\Gamma(s, \cos \theta)}{d\cos \theta} - \frac{1}{f_0} \frac{d}{d\cos \theta} \frac{d\Gamma(s, \cos \theta)}{d\cos \theta} + \frac{1}{f_0} \frac{d}{d\cos \theta} \frac{d\Gamma(s, \cos \theta)}{d\cos \theta} \]  

(34)

Now putting everything together in hat notation we have

\[ \frac{dA_{FB}}{ds} = \frac{G_F^2 m_B^5}{2 \pi s} |V_{tb}^* V_{tb}|^2 u(\hat{s}) \left[ X_{VA} + X_{SP} + X_T + X_{VA-SP} + X_{VA-T} + X_{SP-T} \right] \]  

(35)

where

\[ u(\hat{s}) = \sqrt{\lambda(1, \hat{M}_{K_1}, \hat{s})(1 - \frac{\hat{m}_t^2}{\hat{s}})} \]

\[ \lambda(1, \hat{M}_{K_1}, \hat{s}) = 1 + \hat{M}_{K_1}^2 + \hat{s}^2 - 2\hat{s} - 2\hat{M}_{K_1}^2(1 + \hat{s}) \]

and

\[ X_{VA} = m_B \hat{s} \hat{M}_{K_1} \Re[A^* F' + B^* E'] \]

\[ X_{SP} = 0 \]

\[ X_T = 0 \]

\[ X_{SP-V A} = \hat{m}_t \left[ \left( \hat{M}_{K_1}^2 + \hat{s} - 1 \right) \Re(B^* I') + \hat{M}_B^2 \lambda \Re(I'^* C') \right] \]

\[ X_{SP-T} = m_B \hat{M}_{K_1}^2 \Re(2I'^* C_T + J'^* C_TE) \left( 2J'_1(M_{K_1}^2 + \hat{s} - 1) + J'_2 \hat{M}_B^2 \lambda + F_1(\hat{s}) (3\hat{M}_{K_1}^2 - \hat{s} + 1) \right) \]

(36)

\[ X_{VA-T} = \hat{m}_t [2\Re(F'^* C_TE) \left( 2J'_1(M_{K_1}^2 + \hat{s} - 1) + J'_2 \hat{M}_B^2 \lambda + F_1(\hat{s}) (4\hat{M}_{K_1}^2 - 4\hat{s} + 4) \right) \]

\[ -2\Re(G'^* C_TE) \hat{M}_B^2 \left( 2J'_1(M_{K_1}^2 + \hat{s} - 1) + J'_2 \hat{M}_B^2 \lambda + F_1(\hat{s}) (3\hat{M}_{K_1}^2 - 3\hat{s} - 3\hat{m}_t^2 + 3\lambda - 4) \right) \]

\[ +2\Re(H'^* C_TE) \hat{M}_B \hat{M}_{K_1} \left( 2J'_1(M_{K_1}^2 + \hat{s} - 1) + J'_2 \hat{M}_B^2 \lambda + F_1(\hat{s}) (3\hat{M}_{K_1}^2 - \hat{s} + 1) \right) \]

\[ -64\Re(E'^* C_TE) \hat{M}_B^2 \left( J_1 \hat{M}_{K_1}^2 \hat{s} + 2F_1(\hat{s}) \left( \hat{M}_{K_1}^2 \hat{s} + \hat{s} - (\hat{s} - 1)^2 + \lambda \right) \right) \]

From experimental point of view the normalized forward-backward asymmetry is more useful, i.e.

\[ \frac{d\hat{A}_{FB}}{ds} = \frac{dA_{FB}}{ds} / \frac{d\Gamma}{ds} \]

IV. NUMERICAL ANALYSIS

In the following section, we examine the lepton forward-backward asymmetry and study the sensitivity of its zero position to New Physics operators. We consider different Lorentz structures of NP, as well as their combinations and take all the NP couplings to be real.

Switching off all New Physics Operators

By switching off all the new physics operators one will get the SM result of the lepton forward-backward asymmetry for \( B \rightarrow K_1(1270)\mu^+\mu^- \) which was earlier calculated by Paracha et al. [28] and has been shown by solid line in all the figures shown below. The zero position lies at \( \hat{s} = 0.16 \) (\( s = 4.46 \text{ GeV}^{-2} \)) and is almost independent of the choice of form factors and also from the uncertainties arising from different input parameters like form factors, CKM matrix elements, etc. In the subsequent analysis we will ignore these uncertainties.

In case of \( B \rightarrow K^* \), Arda et. al. have shown [12] that the presence of the tensor and the scalar type interactions have very mild effect on the zero position of forward-backward asymmetry (\( A_{FB} \)) and they have ignored it in their analysis. However, recently the discrepancy has been observed in the lepton forward-backward asymmetry in the exclusive \( B \rightarrow K^*\mu^+\mu^- \) decay [20][21]. To explain the experimental results, Kumar et al. [22] have done a systematic
study of $B \to K^* \mu^+ \mu^-$ decay by using the most general model independent Hamiltonian. They have shown that though the scalar and tensor operators are not important to study the lepton forward-backward asymmetry but the interference of these two is important and is not ignorable. Therefore, keeping this in view we will not ignore these scalar and tensor type couplings in our analysis of $B \to K_1(1270)$ decay. In order to see the effect of the new vector type Wilson coefficients ($C_X = C_{LL}, C_{LR}, C_{RR}, C_{RL}, C_{LRLR}, C_T, C_{TE}$), we have plotted the dependence of $A_{FB}$ on $\hat{s}$ by using different values of $C_X$, which can be summarized as follows.

**Switching on only $C_{LL}$ and $C_{LR}$ along with SM operators**

Considering the constraints provided by Kumar et al. [22] we took broad range of the values of different VA couplings. Fig. 1(a, b) shows the dependence of $A_{FB}$ on $\hat{s}$ when all the $C_{LL}$ and $C_{LR}$ are present. When $C_{LL(LR)} = -C_{10}$, $C_{LL(LR)} = C_{10}$, $C_{LL(LR)} = -0.7 \times C_{10}$, $C_{LL(LR)} = 0.7 \times C_{10}$ (and all other Wilson coefficients are set to zero) we denote the curves of $A_{FB}$ by dashed double dotted, dashed triple dotted, dashed and dashed dotted lines respectively. The solid line corresponds to the SM result. One can deduce from here that there is a significant shift in the zero position of the forward-backward asymmetry and the position of zero is gradually shifted to the left for positive values of $C_{10}$ and to the right for negative values of $C_{10}$ compared to the SM value. This is contrary to the $B \to K^* \mu^+ \mu^-$ decay process where for the positive values of $C_{LL(LR)}$ the zero position of $A_{FB}$ shifts to the right and for negative value of these new coefficients the shift in the zero position is to the left [43]. This difference is due to the axial vector nature of the $K_1(1270)$. For different values of NP coefficients, the location of the zero of the $A_{FB}$ varies from $\hat{s} = 0.12$ to 0.23.

![Fig. 1: Forward-backward asymmetry for the $B \to K_1(1270)$ decay as functions of $\hat{s}$ for different values of $C_{LL(LR)}$. Solid line correspond to SM value,dashed line is for $C_{LL(LR)} = -C_{10}$, dashed-dot-dot is for $C_{LL(LR)} = -0.7C_{10}$, dashed dotted line is for $C_{LL(LR)} = C_{10}$, dashed-triple-dotted is for $C_{LL(LR)} = 0.7C_{10}$. The coefficients of the other interactions are all set to zero.](image)

**Switching on $C_{RR}$ and $C_{RL}$ along with SM operators**

In Fig. 2(a, b) we have shown the dependence of forward-backward asymmetry on $C_{RR}$ and $C_{RL}$. Fig. 2a give the plot of the $A_{FB}$ with $\hat{s}$ by using different values of $C_{RR}$ and setting all the other Wilson Coefficients to zero. By varying the $C_{RR}$ from $-C_{10}$ to $C_{10}$ in the same way as we did for the $C_{LL}$ in Fig. 1, we have plotted the $A_{FB}$ with $\hat{s}$ in Fig. 2a where, the legends of the curves are the same as in Fig. 1. One can clearly see that the zero position of the forward-backward asymmetry is less sensitive to $C_{RR}$ compares to the $C_{LL}$ and $C_{LR}$ and the position of the zero shifts left to the SM value from $\hat{s} = 0.16$ to 0.12 when $C_{RR}$ is changed from $-C_{10}$ to $C_{10}$. Again this is contrary to the $B \to K^* \mu^+ \mu^-$ case where is the shift of zero position of $A_{FB}$ is on the other way.

Similarly Fig. 2b shows the dependency of the zero position of forward backward asymmetry on different values of $C_{RL}$. It can be seen that when $C_{RL}$ vary from $-C_{10}$ to $C_{10}$, the zero position of the $A_{FB}$ shifts gradually right to the SM value from $\hat{s} = 0.16$ to 0.21.
Switching only Scalar- Pseudoscalar \((C_{LRLR}, C_{RLLR}, C_{LRRL}, C_{RLRL})\) operators along with SM operators

Fig. (3) shows the behavior of the lepton forward-backward asymmetry for different NP scalar operators. In the graph we have chosen the value of the scalar and pseudoscalar operators such that they satisfy the constraint
\[
R \equiv |C_{LRLR}^{(+)} - C_{RLLR}^{(+)}|^2 + |C_{LRRL}^{(-)} - C_{RLRL}^{(-)}|^2 \leq 0.44
\]
as provided by the \(B^0_s \rightarrow \mu^+\mu^-\) decay \[22\]. It can be seen from the Eq. \[31\] that the contribution from the scalar operators alone is zero. This is quite clear in the graph where the value of \(A_{FB}\) overlap with that of the SM value and this is due to the interference between the NP scalar operators and that of the SM operators (i.e their coefficients).

Switching on only Tensor-Axial Tensor\((C_T, C_{TE})\) operators along with SM operators

This is the case where only NP tensor operators are added. It is expected from Eq. \[38\] that the contribution alone from the tensor operators to \(A_{FB}\) is zero and Fig. (4) reflects this scenario. Just like the scalar operators, the non zero value of the forward-backward asymmetry is due to the interference between the tensor type operator and
FIG. 4: Forward-backward asymmetry for the $B \to K_1 \mu^+ \mu^-$ decays as functions of $\hat{s}$ for different values of Scalar and Pseudoscalar operators. Solid line correspond to SM value, dashed line is for $|C_T|^2 + 4 |C_{TE}|^2 = 1.3$, dashed-dot is for $|C_T|^2 + 4 |C_{TE}|^2 = 0.9$. The coefficients of the other NP interactions are all set to zero.

of the SM operators and these are $\hat{m}_l$ suppressed (c.f. Eq. (36)). The allowed values of new tensor type operators are restricted to be $\leq 1.3$ (37)

$$|C_T|^2 + 4 |C_{TE}|^2 \leq 1.3$$

In Fig. 4 one can see the $\hat{m}_l$ suppression (which is not negligible) for the value of $A_{FB}(\hat{s})$ in the low $\hat{s}$ region. Though the value is suppressed but still the shift in the zero position is quite significant in the low $\hat{s}$ region, which is due to the mixing of Tensor and SM interactions.

Combination of SP, VA and T operators

FIG. 5: Forward-backward asymmetry for the $B \to K_1 \mu^+ \mu^-$ decays as functions of $\hat{s}$ for different values of Vector and Axialvector operators. Solid line correspond to SM value, dashed line is for VA couplings equal to $-0.3C_{10}$ and dashed-dot-dot lines are for VA equal to $0.3C_{10}$. Here took the value of SP operators such that they satisfy $R=0.44$. The coefficients of the other NP interactions are all set to zero.

Apart from the individual contribution of NP operators and their interference with the SM operators there is a
also a mixing between NP operators by itself. By looking at the term $X_{SP-VA}$ in Eq. (36) one can see that it is $\tilde{m}_l$ suppressed but with the second term there is a factor of $M_B^2$ which will overcome this suppression. This will not only change the zero position of $A_{FB}$ but also increases or decreases its value compared to SM value depending on the size and sign of NP couplings. In Fig. 5, we took $R=0.44$ and the values of NP vector type operators is taken to be $0.3C_{10}$ or $-0.3C_{10}$.

Among different mixing terms the most important is the SP and T term. Though the individual contribution of SP and T to the $A_{FB}$ are not very significant but their interference term is quite promising. One can see it from $X_{SP-T}$ term in Eq. (36) in which there is no lepton mass suppression. In Fig. 6, we have shown the dependencies of the zero position of forward-backward asymmetry for different values of SP couplings. The value of tensor couplings is chosen to be $|C_T|^2 + 4|C_{TE}|^2 \leq 1.3$.

Finally, the contribution from the mixing terms of VA and T is suppressed by $\tilde{m}_l$ which can be seen in $X_{VA-T}$ term of Eq. (36).

![Graph](image)

**FIG. 6:** Forward-backward asymmetry for the $B \rightarrow K^* l^+ l^-$ decays as functions of $\hat{s}$. Solid line correspond to SM value, dashed line is for $|C_T|^2 + 4|C_{TE}|^2 = 1.3$ and dashed-dot-dot is for $|C_T|^2 + 4|C_{TE}|^2 = 0.9$. Here we kept $R=0.44$ and the coefficients of other VA NP interactions are all set to zero.

**V. CONCLUSION:**

The sensitivity of the zero position of the forward backward asymmetry to the new physics effects is studied here. We showed that the position of the zero of the forward backward asymmetry shifts significantly from its Standard Model value both for the size and sign of the vector-vector new physics operators which are the opposite chirality part of the corresponding SM operators. The scalar-scalar four fermion interactions have very mild effects on the zero of the forward-backward asymmetry. The tensor type interactions shifts the zero position of the forward-backward asymmetry but these are $\tilde{m}_l$ suppressed. However, the interference of SP and T operators gives significant change in the zero position of $A_{FB}$.

In short, our results provide, just as in case of the $B \rightarrow K^* l^+ l^-$ process, an opportunity for the straightforward comparison of the basic theory with the experimental results, which may be expected in near future for this process.
Acknowledgements

The authors would like to thank Profs. Riazuddin and Fayyazuddin for their valuable guidance and helpful discussions. The authors M. A. P. and M. J. A. would like to acknowledge the facilities provided by National Centre for Physics during this work.

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