Phenomenological and semi-microscopic analysis for $^{16}$O and $^{12}$C elastically scattering on the nucleus of $^{16}$O and $^{12}$C at Energies near the Coulomb barrier

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Abstract. The nuclear burning process proceeds from the conservation of the most abundant element hydrogen to helium, then from helium to carbon and oxygen, and then from these to heavier elements. Some of the key reactions for the carbon and oxygen burning stages of the nucleosynthesis are $^{12}$C+$^{12}$C and $^{16}$O+$^{16}$O leading to all possible final states. This paper contains the experimental measurements of $^{12}$C+$^{12}$C and $^{16}$O+$^{16}$O angular distributions performed at the cyclotron DC-60 in Astana, Kazakhstan. The extracted beam of $^{16}$O and $^{12}$C was accelerated up to two energies 1.75 and 1.5 MeV/n and then directed to an Al$_2$O$_3$ target of thickness 20 µg/cm$^2$, and a carbon self-supporting target of thickness 17.4 µg/cm$^2$. The angular distribution calculations were performed using both the phenomenological optical potential (SPI-GENOA) code and the double folding potential (FRESCO) code.

1. Introduction

The optical model, in which the highly complicated nucleus-nucleus interaction is replaced by a complex two body effective potential, plays a central role in the description of nucleus-nucleus scattering [1]. It has recently been found that, at incident energies of a few MeV per nucleon, several light heavy-ion systems, among which $^{12}$C+$^{12}$C and $^{16}$O+$^{16}$O display more transparency than most neighbouring systems, for which absorption at small and intermediate distances is nearly complete. Indeed, their elastic scattering angular distributions reveal unmistakable refractive features, such as rainbow scattering patterns and broad interference minima so called “Airy minima” [2]. These refractive features, can be described consistently by the optical potentials with a deep (several hundreds MeV) real part.

In particular, the clear observation of rainbow scattering features in $^{12}$C+$^{12}$C, $^{16}$O+$^{12}$C, and $^{16}$O+$^{16}$O elastic scattering data has definitely established the fact that: (i) the real part of the light heavy-ion nucleus nucleus optical potential is strongly attractive: the real part of the optical potential is deep. (ii) In some favorable cases (in particular, for the three aforementioned systems), the imaginary part of the potential is weak enough to show some transparency. The combination of these two features—deep real potential and incomplete absorption—makes possible the observation in the elastic scattering data of distinctive refractive effects, like strong Airy minima, superimposed on more classic
diffractive features. This refractive behaviour is conspicuous, e.g., in the systematic analyses carried out for the $^{16}\text{O} + ^{16}\text{O}$ system by Nicoli et al. [3] at incident energies between 75 and 124 MeV and by Khoa et al. [4] between 124 and 1120 MeV, and for the $^{16}\text{O} + ^{12}\text{C}$ system by Nicoli et al. [5] between 62 and 124 MeV and by Ogloblin et al. [6] at 132 MeV. Rainbow scattering and Airy minima have been observed for a long time in medium energy light-ion scattering [7–9], and are also familiar features in atomic and molecular collision processes [10]. In particular, we want to clarify the transition between the region of relatively high incident energies where rainbow scattering has set in and lower energies close to Coulomb barrier energy where rainbow scattering is not yet observed but the nuclear “Airy minima” are clearly present. In our work, it is clearly shown that, refractive features such as; nuclear rainbow phenomenon is not observed in the aforementioned nuclear systems at low energies close to the Coulomb barrier energy. The optical model analysis for these systems at low energies doesn’t require a deep real potential in the comparison with the systematic analysis of these nuclear systems at relatively high energies. The experimental data were fitted within the framework of optical model using SPI-GENOA code [11] and the double folding calculations using FRESCO Code [12].

2. Experimental Details

The experiments were performed using an $^{16}\text{O}$ and $^{12}\text{C}$ beams accelerated in the cyclotron DC-60 INP NNC located in Astana, Kazakhstan, which could accelerate the elements from Lithium to Xenon with an energy range from 0.35 MeV/n to 1.75 MeV/n. Beam current was measured using a Faraday Cup to be nearly 30 nA during these experiments. The dead time was monitored and kept as constant as possible by changing the spectrometer entrance slits and/or the beam intensity. The $^{16}\text{O}$ beam was accelerated up to energies 28 and 24 MeV and then directed to $\text{Al}_2\text{O}_3$ target of thickness 20 µg/cm$^2$. The choice of a light oxide such as $\text{Al}_2\text{O}_3$ as the target has a number of advantages since it allows relatively good spectral separation between the $^{16}\text{O}$ nuclei scattered by $^{27}\text{Al}$ and $^{16}\text{O}$ and also, the forward focusing of the yields from $^{27}\text{Al}$ suppresses their contribution at larger angles. While, $^{12}\text{C}$ beam was accelerated up to energies 21 and 18 MeV and then directed to self supporting carbon target of the thickness 17.4 µg/cm$^2$. The thickness of these targets was determined using the resonance chamber in the linear accelerator UKP-2-1 INP Almaty–Kazakhstan, and also using $\alpha$-particles from $^{241}\text{Am}$ source. The angular distributions were measured for $^{16}\text{O}$ ($^{16}\text{O}$, $^{16}\text{O}$) $^{16}\text{O}$ nuclear system at energies [28, 24] MeV and for $^{12}\text{C}$ ($^{12}\text{C}$, $^{12}\text{C}$) $^{12}\text{C}$ nuclear system at energies [21, 18] MeV, in the 20°–90° range of angles in the centre of mass system with an increment $\Delta \theta = 2^\circ$. To register the scattered $^{16}\text{O}$ and $^{12}\text{C}$ beams, we used a surface-barrier silicon detector from the company ORTEC (diameter of the sensitive area of 8 mm, thickness - 0.2 mm). The detector was located at a distance of 24 cm from the scattering region and had the opportunity to move in the angular range from 10° to 75° in the laboratory frame. The energy resolution of the detector was 250-300 keV, which is mainly determined by the energy spread of the primary beam. The nominal maximum voltage which could be applied on the detector is 30 volts but, during the experiment it was raised up to 20 volt. The $^{16}\text{O}$ and $^{12}\text{C}$ beams were passed through three collimators of 1.5 mm diameter and focused on the target to a spot diameter of $\approx 3.9$ mm. In order to minimize the evaporation of the target, the beam current was limited to 30 nA. Spectrum analysis has been performed using the program MAESTRO [13].

3. Theoretical Analysis

In this work we have assumed, in accordance with previous phenomenological analyses of $^{16}\text{O}+^{16}\text{O}$ and $^{12}\text{C}+^{12}\text{C}$ nuclear systems [14–16], the Woods-Saxon (WS) shape for the potential real part and also WS shape for the imaginary volume part of the potential Thus, the optical potential can be written as:

$$ V(r) = V_0 \left(1 - \frac{r}{a} \right) \left(1 - \frac{r}{b} \right) $$
\[ U(r) = V_c(r) - V(r) - iW_v(r). \]

The first term is the Coulomb potential. Since scattering is not sensitive to a specific form of the charge distribution, and therefore there is no need to consider its diffuse boundary, then the Coulomb potential was assumed to be that between two uniform charge distributions with radii consistent with electron scattering.

\[
\begin{align*}
V_c(r) &= \frac{Z_pZ_ce^2}{2R_c^3} \left( \frac{3 - r^2}{R_c^2} \right) & \text{if } r < R_c \\
V_c(r) &= \frac{Z_pZ_ce^2}{r} & \text{if } r < R_c
\end{align*}
\]

The real part has the following form: \( V(r)f(r, r_s, a_v) = V_0 \left[ 1 + \exp \left( \frac{r - r_v}{a_v} \right) \right]^{-1} \), \( \text{(3)} \)

Imaginary volume part: \( W_v(r)f(r, r_s, a_w) = W_0 \left[ 1 + \exp \left( \frac{r - r_w}{a_w} \right) \right]^{-1} \), \( \text{(4)} \)

So, the Interaction potential can be rewritten as

\[
U(r) = V_c(r) - V_0 \left[ 1 + \exp \left( \frac{r - r_v}{a_v} \right) \right]^{-1} - iW_0 \left[ 1 + \exp \left( \frac{r - r_w}{a_w} \right) \right]^{-1}, \quad \text{(5)}
\]

In general, for an interaction potential \( U(r) \) between nuclei having nucleon numbers \( A_1 \) and \( A_2 \), the volume integral per interacting nucleon pair, \( J_U \), is defined as

\[
J_U(E) = -\frac{4\pi}{A_1A_2} \left[ U_E(r) r^2 dr \right]
\]

Effective nucleon-nucleon interactions have been used to generate microscopic real potentials which, associated with phenomenological imaginary terms, successfully describe light heavy-ion elastic-scattering data at low and intermediate energies. The degree of success of the model is indicated by the potential renormalization required to give an optimum fit to the measurements. This renormalization should be close to unity. The real part of the optical potential is calculated from a more fundamental basis by the folding method in which the NN interaction \( V_{NN}(r) \), is folded into the densities of both the projectile and target nuclei [17],

\[
V_{DF}(r) = N_s \int \rho_p(r_2) \rho_d(r_1) V_{NN} \left( \sqrt{R + r_2 - r_1} \right) d r_1 d r_2
\]

where \( N_s \) is a free renormalization factor, \( \rho_p(r_1) \) and \( \rho_d(r_2) \) are the nuclear matter density distributions of both the projectile and target nuclei, respectively, and \( V_{NN}(\sqrt{R + r_2 - r_1}) \) is the NN potential. In the case of double folding calculations, the nuclear density distribution for \(^{12}\text{C}\) and \(^{16}\text{O}\) was calculated using the three-parameter Fermi model (3PF), where \( \rho(r) \) was calculated from that relation

\[
\rho(r) = \rho_0 \left( 1 + \frac{wr^2}{c^2} \right) /(1 + \exp((r - c) / z))
\]
With $w = -0.149$, $z=0.5224$ and $c=2.355$ for $^{12}\text{C}$, and $w = -0.051$, $z=0.513$ and $c=2.608$ for $^{16}\text{O}$.

The $NN$ potential was taken in the form of M3Y (Reid-standard)

$$ V(R) = 7999 \frac{\exp(-4R)}{4R} - 2134 \frac{\exp(-2.5R)}{2.5R} - 276 \left(1 - \frac{0.005}{A}E\right) \delta(R) \quad (9) $$

4. Results and discussion

The angular distribution measurements for the elastic scattering process in ($^{12}\text{C}+^{12}\text{C}$ and $^{16}\text{O}+^{16}\text{O}$) nuclear systems had been done at energies 1.5 and 1.75 MeV/n. The systematic description of these nuclear systems at such low energies doesn’t require a deep part and a shallow imaginary part of the nuclear potential. Such a deep real part and shallow imaginary potential is only helpful in the analysis of such nuclear systems at relatively high energies, where refractive features such as, nuclear rainbow and Airy structure are well observed. The optimal parameters obtained using the optical potential code SPI-GENOA for $^{12}\text{C}+^{12}\text{C}$ at energies $E_{\text{lab}}$ 21 and 18 MeV and for $^{16}\text{O}+^{16}\text{O}$ at energies $E_{\text{lab}}$ 28 and 24 are listed in table 1. For $^{12}\text{C}+^{12}\text{C}$ nuclear system, $r_c$, $r_v$, $r_w$ were fixed to 0.95, 1.225, 1.294 fm respectively, and for $^{16}\text{O}+^{16}\text{O}$ nuclear system, $r_c$, $r_v$, $r_w$ were fixed to 0.95, 0.473, 0.871 fm respectively. Figure 1 shows the comparisons between the experimental data and the theoretical predictions for $^{12}\text{C}+^{12}\text{C}$ at $E_{\text{lab}}$ 21 and 18 MeV using both phenomenological (SPI-GENOA Code) and double folding (FRESCO Code), while figure 2 shows the comparisons between the experimental data and the theoretical predictions for $^{16}\text{O}+^{16}\text{O}$ at $E_{\text{lab}}$ 28 and 24 MeV. It is clearly shown that the agreements between the experimental data and calculation predictions are fairly good over the whole angular range.

Table 1. Optical potential parameters used in the analysis of the elastic $^{12}\text{C}+^{12}\text{C}$ at energies 18 and 21 MeV and $^{16}\text{O}+^{16}\text{O}$ at energies 24 and 28 MeV. Folding parameters with normalization coefficient $N_r$ nearly close to unity are listed. The * parameters were fixed during the search, $R=r_c(A^{1/3} + A_p^{1/3})$.

| Nuclear System | $E_{\text{lab}}$ (MeV) | Potential | $V_0$ (MeV) | $r_c$ (fm) | $w_v$ (fm) | $N_r$ | $J_V$ (MeV fm$^3$) | $W_0$ (MeV) | $r_w$ (fm) | $a_w$ (fm) | $J_W$ (MeV fm$^3$) |
|----------------|-----------------|------------|-------------|------------|-----------|------|-----------------|-------------|------------|-----------|-----------------|
| $^{12}\text{C}+^{12}\text{C}$ | 18   | WSI       | 96.98       | 1.255*     | 0.44      | 566.97   | 31.92           | 1.294*      | 0.30       | 198.02    |                 |
|                 |      | Folding   |             |            |           |        |                 |             |            |           |                 |
|                 | 21   | WSI       | 85.98       | 1.255*     | 0.443     | 502.53   | 29.64           | 1.294*      | 0.292      | 183.64    |                 |
|                 |      | Folding   |             |            |           |        |                 |             |            |           |                 |
| $^{16}\text{O}+^{16}\text{O}$ | 24   | WSI       | 141.6       | 0.473*     | 1.124     | 102.62   | 5.304           | 0.871*      | 1.074      | 11.69     |                 |
|                 |      | Folding   |             |            |           |        |                 |             |            |           |                 |
|                 | 28   | WSI       | 144.62      | 0.473*     | 1.077     | 98.56    | 2.39            | 0.871*      | 2.354      | 13.20     |                 |
Fig. 1: Differential cross sections of $^{12}$C elastically scattering on $^{12}$C at energies 18 and 21 MeV. The open circles represent experimental data; solid red lines represent the calculation using SPI-GENOA code, and dotted blue lines represent double folding calculations using Code FRESCO.

Fig. 2: Differential cross sections of $^{16}$O elastically scattering on $^{16}$O at energies 24 and 28 MeV. The open circles represent experimental data; solid red lines represent the calculation using SPI-GENOA code, and dotted blue lines represent double folding calculations using Code FRESCO.
In the case of identical particles elastic scattering such as $^{12}\text{C}+^{12}\text{C}$ and $^{16}\text{O}+^{16}\text{O}$, due to symmetry under the interchange of spatial coordinates of the two particles, the differential cross section contains an interference term, which has no classical analogy, and is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{identical}} = |f(\theta) + f(\pi - \theta)|^2.$$ 

Where, $f(\theta) = f_c(\theta) + f_n(\theta)$, $f(\pi - \theta) = f_c(\pi - \theta) + f_n(\pi - \theta)$,

with $f_c$ is the Coulomb scattering amplitude and $f_n$ is the nuclear scattering amplitude. The interference term peaks at $\theta_{CM} = 90^\circ$ as can be seen in figure 1. It should be noted that the beam energy of 18 and 21 MeV is somewhat higher than the Coulomb barrier of $^{12}\text{C}+^{12}\text{C}$ nuclear system which nearly equals 17.44 MeV, and hence the nuclear forces between the two colliding nuclei cannot be neglected. The same situation also could be observed in $^{16}\text{O}+^{16}\text{O}$ nuclear system. While, classically the differential scattering cross section for distinguishable particle in central interaction potential is defined as $\left(\frac{d\sigma}{d\Omega}\right)_{\text{distinct}} = |f(\theta)|^2 + |f(\pi - \theta)|^2$ which is the sum total of the differential scattering cross section of both the nuclei.

5. Conclusion

Elastic scattering measurements for $^{12}\text{C}+^{12}\text{C}$ angular distribution at $E_{lab}=18$, 21 MeV and for $^{16}\text{O}+^{16}\text{O}$ angular distribution at $E_{lab} = 24$, 28 MeV has been performed. Both phenomenological and microscopic analyses were performed within the framework of SPI-GENOA and FRESCO Codes. The fitted values of optical model are nearly in a good agreement with the experimental data.

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