Inflationary Cosmology with a scalar-curvature mixing term $\xi R\phi^2$

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Abstract

We study the cosmological constraints on the non-minimal scalar-curvature mixing $\xi R\phi^2$ in inflationary cosmology with a class of scalar field inflaton potentials (i) $V = V_0\phi^p e^{-\lambda \phi}$, (ii) $V = V_0(1-\phi^p)e^{-\lambda \phi}$, (iii) $V = V_0(1-\lambda \phi)^p$ and (iv) $V = V_0\frac{\alpha \phi^2}{1+\alpha \phi^2}$. We obtain constraints on the non-minimal coupling parameter $\xi$ for $\phi$ values of the potentials $V$ corresponding to potential parameters $\lambda$, $p$ and $\alpha$ by comparing these with the Planck+WMAP data on spectral index of curvature perturbations generated during inflation. We find that in the parameters space all the above listed potentials can generate the e-fold number $N = 40–60$ necessary for any successful inflation model. The scalar spectral index $n_s$ and the tensor to scalar ratio $r$ are found to lie within $3\sigma$ C.L of Planck 2018 data for the potentials (i) $V \propto \phi^p e^{\lambda \phi}$ with $\lambda = 0$, $p = 2$; $\lambda = 0.01$, $p = 2$, 4; (ii) $V \propto (1-\phi^p)e^{-\lambda \phi}$ for $\lambda = 0.01$, $p = 2$, 4, (iii) $V \propto (1-\lambda \phi)^p$ for $\lambda = \pm 1$, $p = \pm 2$. We also find that the potential $V \propto \frac{\alpha \phi^2}{1+\alpha \phi^2}$ with $\alpha = 1, 2$ can produce the $n_s$ and $r$ values which are consistent with the WMAP3 data.

1 Introduction

In the very early days, the universe was in a state dominated by dark energy with extreme great density according to the Standard Model of Cosmology. It started from a quantum fluctuation of that state of dark energy - it is the repulsive gravity of dark energy that made the universe expand with great acceleration. This state of extremely dense dark energy lasted for about $10^{-33}$ s. The universe has expanded so much by this time that the distances between any two reference points then increased by 40–60 e-folds. This is called the inflationary era of the universe [1–4]. The epoch-making idea of the inflationary expansion has revolutionized our understanding of the early universe cosmology - it not only explain why the universe is isotropic and homogeneous at large scale, but also resolves the cosmological puzzles e.g. the flatness and the horizon problems in an elegant manner. A simple model of dark energy that can drive inflationary expansion requires a positive cosmological constant ($\Lambda$) and the application of general theory of relativity together with the Friedman-Robertson-Walker space-time symmetry lead to the $\Lambda$CDM model - the standard model of cosmological evolution which can cause exponential accelerated expansion. Although it is favoured by the observational data [5], however, it suffers the fine tuning problem [6,7] and the coincidence problem [7,9]. A successful model of inflation requires a
scalar field in a flat potential which rolls so slowly over a sufficiently long period of time to enable the universe to expand by $40 - 60$ e-fold during the inflation period $[1][2][4][10][12]$. Inflation acts as the origin of density fluctuation which later evolves into the large scale structure formation of the universe. The prediction of almost scale-invariant cosmological perturbation by inflation agrees remarkably well with the CMBR observations of COBE $[13]$, WMAP $[14]$, PLANCK $[15]$ and BICEP2 $[16]$. The WMAP-2003 data measures the spectral index of the scalar fluctuations $n_s = 0.99 \pm 0.04$ and put the 95% CL upper limit on the tensor-to-scalar ratio, $r<0.9$. The recent PLANCK-2018 mission measures the scalar spectral index $n_s = 0.9649 \pm 0.0042$ and put the upper limit on $r<0.1$ (at 95% CL), which is further tightened by combining with the BICEP2/Keck Array BK15 data to obtain $r<0.056$.

The simplest extension of the scalar Lagrangian is to include a non-minimal coupling term between the scalar field ($\phi$) and gravity of the form $\xi R \phi^2$ $[17][22]$. The motivations of non-minimal coupling comes from different context. The coupling between scalar field and curvature parameter gives a quantum correction to scalar field in curve space $[23],[24],[25],[26]$, and is required by renormalization process. Non minimal coupling is also interesting in multidimensional theories like super-string theory.

In this paper we have analysed a host of inflationary models with a class of scalar(inflaton) potentials

(i) $V = V_0 \phi^p e^{-\lambda \phi}$, (ii) $V = V_0 (1 - \phi^p) e^{-\lambda \phi}$, (iii) $V = V_0 (1 - \lambda \phi)^p$ and (iv) $V = V_0 \alpha \phi^2 (1+ \alpha \phi^2)$ (where $\lambda, p$ and $\alpha$ are the potential parameters) in the presence of a non-minimal scalar-curvature mixing $\frac{1}{2} \xi R \phi^2$. We have predicted different cosmological quantities e.g. spectral index parameter, tensor to scalar ratio of cosmological perturbation and matches them with the one measured by PLANCK 2018 and WMAP Collaborations.

The paper is organized as follows. In Section 2, we have derived the modified Einstein equations for non-minimal coupling term $\xi R \phi^2$. In Section 3, we have analyzed the inflationary expansion for four potentials. We have derived the slow-roll parameters, spectral index parameters and tensor-to-scalar ratio in our non-minimal model of inflationary expansion and matches them with observed data. In Section 4, we discuss our results and conclude.

## 2 Non-minimal Inflation in Einstein Frame:

The action for gravity with a non-minimally coupled scalar field($\phi$) in Jordan frame is given by:

$$S_J = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi) + \xi R \phi^2 \right]$$

where $R$ denotes the Ricci curvature of spacetime, $\kappa^2 = 8\pi G_N = 8\pi/M_{pl}^2$, $g$ is the determinant of the metric $g_{\mu\nu}(x)$, $V(\phi)$ is the potential of the scalar field, $\xi$ is the non-minimal coupling of the scalar field $\phi$ with the Ricci scalar $R$. The metric sign convention is chosen to be $(+, -, -, -)$ with spatially flat Friedman-Robertson-walker metric as,

$$ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2)$$

To obtain the fundamental FRW equations in Einstein frame, we need to perform a conformal transformation(Weyl’s rescaling) as $[28],[29],[30],$

$$\hat{g}_{\mu\nu}(x) = \Omega^2(x) g_{\mu\nu}(x)$$

where we have used the hat on variables in Einstein frame. The Christoffel symbols will transform in Einstein frame as follows $[28],[31]$

$$\hat{\Gamma}^\alpha_{\beta\gamma} = \Gamma^\alpha_{\beta\gamma} + \frac{1}{\Omega} \left( \delta^\alpha_\beta \partial_\gamma \Omega + \delta^\alpha_\gamma \partial_\beta \Omega - \partial^\alpha \Omega g_{\beta\gamma} \right)$$

In our calculation we have considered $c = 1, 8\pi G = 1$.
Similarly the transformation of Ricci tensor and Ricci scalar will be (in D dimension),

$$\hat{R}_{\mu\nu} = R_{\mu\nu} + \frac{1}{\Omega^{2}} (2(D-2)\Omega_{\mu}\Omega_{\nu} - (D-3)\Omega_{\alpha}\Omega^{\alpha}g_{\mu\nu}) - \frac{1}{\Omega} ((D-2)\Omega_{\mu\nu} + g_{\mu\nu}\Box \Omega),$$

(5)

$$\hat{R} = \frac{1}{\Omega^{2}} \left( R - \frac{2(D-1)}{\Omega} \Box \Omega - (D-1)(D-4) \frac{1}{\Omega^{2}} g^{\mu\nu}\partial_{\mu}\Omega\partial_{\nu}\Omega \right)$$

(6)

Where $\sqrt{-\hat{g}} = \Omega^{D} \sqrt{-g}$.

Now, considering $\Omega^{2}(x) = 1 + \kappa^{2}\xi \phi^{2}$ (with $\kappa^{2} = 1$), we find the action in Einstein frame

$$S_{E} = \int d^{4}x \sqrt{-\hat{g}} \left( \hat{R}^{2} - F^{2}(\phi) \frac{1}{2} \hat{g}^{\mu\nu}\partial_{\mu}\phi\partial_{\nu}\phi + \hat{V}(\phi) \right)$$

(7)

Where $\frac{d\phi}{d\hat{t}} = F(\phi) = \sqrt{1 + \xi \phi^{2}(1+12\xi)}$ and $\hat{V}(\phi) = \Omega^{-4} V(\phi)$. Note that in Einstein frame, the scalar field $\phi$ is no longer coupled with the Ricci scalar $R$.

The invariance of the 4-dimensional line element under the Conformal Transformations gives

$$ds^{2} = g_{\mu\nu}dx^{\mu}dx^{\nu} = \Omega^{-2} \hat{g}_{\mu\nu}d\hat{x}^{\mu}d\hat{x}^{\nu} = \Omega^{-2} ds^{2} \to d\hat{s}^{2} = d\hat{t}^{2} - \hat{a}^{2}(t)(d\hat{x}^{2} + d\hat{y}^{2} + d\hat{z}^{2})$$

(8)

The energy-momentum tensor in Einstein frame is found to be

$$\hat{T}_{\mu\nu} = \hat{g}_{\mu\nu} \left( -F^{2}(\phi) \frac{1}{2} \hat{g}_{\alpha\beta}\partial_{\alpha}\phi\partial_{\beta}\phi + \hat{V}(\phi) \right) + F^{2}(\phi) \hat{g}_{\mu\nu}\partial_{\mu}\phi\partial_{\nu}\phi$$

(9)

Next, we find the Friedmann equation and the scalar field equation of motion in Einstein frame [29] as,

$$\hat{H}^{2} = \frac{1}{3} \hat{\rho}_{\phi} = \frac{1}{3} \left( \frac{1}{2} \left( \frac{d\phi}{d\hat{t}} \right)^{2} + \hat{V}(\phi) \right)$$

(10)

$$\frac{d^{2}\phi}{d\hat{t}^{2}} + 3\hat{H} \frac{d\phi}{d\hat{t}} + \frac{d\hat{V}}{d\phi} = 0$$

(11)

Using the slow-roll approximations $\frac{d\phi}{d\hat{t}} \ll \hat{V}(\phi), \frac{\dot{\phi}}{\phi} \ll 3\hat{H}$ (where $\dot{\phi} = \frac{d\phi}{d\hat{t}}$ and $\ddot{\phi} = \frac{d^{2}\phi}{d\hat{t}^{2}}$), we obtain the Hubble equation [10] and the scalar field equation [11] as follows

$$\hat{H}^{2} = \frac{1}{3} \hat{V}(\phi),$$

(12)

$$3\hat{H} \frac{d\phi}{d\hat{t}} + \frac{d\hat{V}}{d\phi} = 0$$

(13)

3 \ Analysis of inflationary expansion for a class of inflaton potentials

We consider four type of potentials and show the variation of several cosmological entities like e-fold parameter, spectral index against the scalar field and summarize our findings in Table 1 and Table 2.
3.1 Slow roll parameters and CMB constraints:

We now define the potential slow roll parameters for a wide range of inflaton potentials. In Einstein
frame, they can be defined as [29],

$$
\epsilon_v = \frac{1}{2} \left( \frac{\hat{V}'}{\hat{V}} \right)^2, \quad \eta_v = \frac{\hat{V}''}{\hat{V}}
$$

where, $\hat{V}' = \frac{d\hat{V}}{d\phi}$ and $\hat{V}'' = \frac{d^2\hat{V}}{d\phi^2}$. These slow-roll parameters can be related to the CMBR
observables like scalar spectral index $n_s$, tensor spectral index $n_t$ and tensor-to scalar ratio $r$ as,

$$
n_s - 1 = 2\eta_v - 6\epsilon_v, \quad r = 16\epsilon_v, \quad n_t = -2\epsilon_v
$$

The e-fold number($N$) which is defined as the ratio of the final value of the scale factor $a_f$ during the
inflationary era and its initial value $a_i$ can be calculated as,

$$
N = \ln \left( \frac{a_f}{a_i} \right) = -\int_{\phi_{in}}^{\phi_f} \frac{\hat{V}}{\hat{V}'} d\phi = -\frac{1}{\sqrt{2}} \int_{\phi_{in}}^{\phi_f} (\epsilon_v)^{-1/2} \frac{d\hat{\phi}}{d\phi}
$$

Note that $N = 0$ at the end of inflation, so that $N$ counts the number of e-folds until inflation ends and increases as we go backward in time.

3.2 Case 1: $V = V_0 \phi^p e^{-\lambda \phi}$

We start with the potential,

$$
V = V_0 \phi^p e^{-\lambda \phi}
$$

where $V_0$, $p$, $\lambda$ are constants. In Einstein frame, the potential can be written as,

$$
\hat{V}(\phi) = \frac{V_0 \phi^p e^{-\lambda \phi}}{(1 + \xi \phi^2)^2}
$$

In Fig.1 we have plotted this potential for different $\lambda$ and $p$ values. The slow roll parameters for this

![Figure 1](image-url)

Figure 1: (Color online) Inflaton potential $\hat{V}$ is plotted as a function of $\phi$ corresponding to (i) $\xi = 0.005$, $\xi = 0.01$ for $p = 2$ and $\lambda = 0$ (left plot), (ii) $\xi = 0.005$, $\xi = 0.01$ for $\lambda = 0.1$, $p = 2$ (middle plot) and $\xi = 0.005$, $\xi = 0.01$ for $\lambda = 0.01$, $p = 2$ (right plot)

potential are found to be

$$
\epsilon_v = \frac{(p + p\xi \phi^2 - \phi(\lambda + 4\xi \phi + \lambda \xi \phi^2))^2}{2\phi^2(1 + \xi \phi^2 + 12\xi^2 \phi^2)^2}
$$
Primordial density perturbations can be described by three observational parameters such as scalar spectral index $n_s$, tensor to scalar ratio $r$ and tensorial spectral index $n_T$. From Eq. [19], Eq. [20] and Eq. [15] we can calculate the scalar spectral index as,

\[
\eta_v = \frac{1}{\phi^2(1 + \xi \phi^2 + 12 \xi^2 \phi^2)^2} (p + p \xi \phi^2)(1 + \xi \phi^2 + 12 \xi^2 \phi^2) - p(1 + \xi \phi^2)(1 + 9 \xi \phi^2 + 96 \xi^2 \phi^4 + 8 \xi^2 \phi^2(3 + \phi^2) + 2 \lambda(\phi + 2 \xi \phi^3(1 + 6 \xi) + \xi^2 \phi^5(1 + 12 \xi)) + \phi^2(\lambda + \lambda \xi \phi^2)(1 + \xi \phi^2 + 12 \xi^2 \phi^2) + 4 \xi(-1 + 3 \xi \phi^2 + 4 \xi^2 \phi^4 + 48 \xi^3 \phi^4) + \lambda \xi \phi(7 + 84 \xi^3 \phi^4 + 2 \xi(6 + 7 \xi^2) + \xi^2 \phi^2(96 + 7 \xi^2))))
\]

(20)

The observational constraints on the scalar spectral index $n_s$, tensor to scalar ratio $r$ and tensorial spectral index $n_T$ can be obtained as follows,

\[
r = \frac{8(p + p \xi \phi^2 - \phi(\lambda + 4 \xi \phi + \lambda \xi \phi^2))^2}{\phi^2(1 + \xi \phi^2 + 12 \xi^2 \phi^2)},
\]

(22)

and

\[
n_t = -\frac{(p + p \xi \phi^2 - \phi(\lambda + 4 \xi \phi + \lambda \xi \phi^2))^2}{\phi^2(1 + \xi \phi^2 + 12 \xi^2 \phi^2)}
\]

(23)

The e-fold number $N$ for this potential $\tilde{V}(\phi)$ can be calculated from Eq. [16] where we will get the $\phi_f$ by considering $\epsilon_0 = 1$. In Fig. (2), we have plotted the e-fold numbers $N$ as a function of $\phi$ (in units

![Figure 2: (Color online) The e-fold number $N$ is plotted as a function of $\phi$(in units of $M_{Pl}$) corresponding to $\xi = 0.005$ and $V = V_0 e^{-\lambda \phi}$ for $p = 2$, $\lambda = 0$ (left plot), for $\lambda = 0.01$ for $p = 2, 4$ (middle plot) and for $\lambda = 0.1$ for $p = 2, 4$ (right plot) respectively.]

of $M_{Pl}$) for $\xi = 0.005$ corresponding to $p = 2, 4$ and $\lambda = 0, 0.01$ and 0.01, respectively. According to CMBR analysis, a successful inflationary model should produce the necessary e-fold $N$ ranging from 40 to 60 before inflation ends. The green and black dashed horizontal lines correspond to $N = 60$ and $N = 40$ respectively. Note that as $N$ increases from zero (the value of $N$ when inflation ends), $\phi$ increases so we move backward in time. In Table[1] we have tabulated (as shown later) the values of $\phi$ required to produce the e-fold $N = 40$ and $N = 60$, respectively.

The observational constraints on the scalar spectral index $n_s$, tensor to scalar ratio $r$ and tensorial spectral index $n_T$ are used to evaluate for those particular value of $\phi$ and the non-minimal coupling $\xi$ in order to have comparison with experimental values. In Fig. (3), the scalar spectral index $n_s$ is plotted as a
Figure 3: (Color online) The variations of the scalar spectral index $n_s$ with $\phi$ values are shown corresponding to $\xi = 0.005$, $p = 2$, $\lambda = 0$ (Left side), $\xi = 0.005$, $p = 2$, $\lambda = 0.1(0.01)$ (middle one) and $\xi = 0.005$, $p = 4$, $\lambda = 0.1(0.01)$ (right side), respectively. The horizontal line on all three plots corresponds to $n_s = 0.9649$ (c.v.) (Planck2018).

...function of $\phi$ corresponding to different choices of $\xi, \lambda$ and $p$ values. On the left (of Fig. (3)), we set $\lambda = 0$, $\xi = 0.005$, $p = 2$. The vertical red dashed lines corresponds to $\phi$ values required to produce $N = 40$(left vertical line) and $N = 60$(right vertical line), respectively. The middle one(of Fig. (3)) shows the variation of $n_s$ with $\phi$ for $p = 2$, $\lambda = 0.1, 0.01$ where the blue and red dashed lines shows the value of $\phi$ required to produce $N = 40$ and $N = 60$, respectively. Finally, the plot on the right side corresponds to $p = 4$, $\lambda = 0.1, 0.01$ with the $\phi$ values for $N = 40$ and $N = 60$ shown in Blue and Red dashed lines. The horizontal lines on all plots correspond to $n_s = 0.9649$ (c.v.) (Planck2018).

3.3 Case 2: $V = V_0(1 - \phi^p)e^{-\lambda \phi}$

We next consider the potential of the form

$$V(\phi) = V_0(1 - \phi^p)e^{-\lambda \phi}$$

In the Einstein frame, it takes the form

$$\hat{V}(\phi) = \frac{V_0(1 - \phi^p)e^{-\lambda \phi}}{(1 + \xi \phi^2)^2}$$

(24)

In Fig[4] we have plotted this potential for different $\lambda$ and $p$ values corresponding to a non-zero $\xi$.

Figure 4: Variation of $\hat{V}(\phi)$ with $\phi$ is shown for $\xi = 0.005$ and $p = 2$ for two different values of $\lambda = 0.1, 0.01$ (left side), while on the right the same for $p = 4$.

The potential slow-roll parameters for the given potential are given as follows:

$$\epsilon_v = \frac{(p\phi^p - \lambda \phi(1 + \xi \phi^2)(-1 + \phi^p) + \xi \phi^2(4 + (-4 + p)\phi^p))}{2\phi^2(1 + \xi \phi^2 + 12\xi^2 \phi^2)(-1 + \phi^p)^2}$$

(25)
\[ \eta_v = -\frac{1}{(\phi + \xi(1 + 12\xi)^3)(1 - \phi^2)}((-1 + p)p\phi^2 + \lambda^2\phi^2(1 + \xi \phi^2)^2(1 + \xi \phi^2 + 12\xi^2 \phi^2)(-1 + \phi^2) \\
+ 12\xi^4 \phi^6(-16 + (-4 + p)^2 \phi^2) + \xi \phi^2(4 + (-4 - 10p + 3p^2) \phi^2) + \xi^3 \phi^4(-16\phi^2 + 24(-5 + p)p\phi^2 + (-4 + p)^2 \phi^{2+p}) - \xi^2(12\phi^4 - 12(-2 + p)p\phi^{2+p} + (-12 + 17p - 3p^2) \phi^{4+p}) - \lambda(1 + \xi \phi^2)(2p\phi^2 + 12\xi^3 \phi^4(7 + (-7 + 2p) \phi^2) + \xi^2 \phi^2(12 + 7\phi^2 + 12(-1 + 2p) \phi^2 + (-7 + 2p) \phi^{2+p}))) \]

(26)

We can obtain the scalar spectral index by using Eq. (15), Eq. (25) and Eq. (26) as:

\[ n_s = 1 - \frac{3(p\phi^2 - \lambda(1 + \xi \phi^2)(-1 + \phi^2) + \xi \phi^2(4 + (-4 + p) \phi^2))^2}{\phi^2(1 + \xi \phi^2 + 12\xi^2 \phi^2)(-1 + \phi^2)^2} \frac{1}{(\phi + \xi(1 + 12\xi)^3)(1 - \phi^2)} \]

(27)

Similarly, the tensor spectral index and tensor-to-scalar ratio can be obtained as:

\[ r = \frac{8(p\phi^2 - \lambda(1 + \xi \phi^2)(-1 + \phi^2) + \xi \phi^2(4 + (-4 + p) \phi^2))^2}{\phi^2(1 + \xi \phi^2 + 12\xi^2 \phi^2)(-1 + \phi^2)^2} \]

(28)

\[ n_t = -\frac{(p\phi^2 - \lambda(1 + \xi \phi^2)(-1 + \phi^2) + \xi \phi^2(4 + (-4 + p) \phi^2))^2}{\phi^2(1 + \xi \phi^2 + 12\xi^2 \phi^2)(-1 + \phi^2)^2} \]

(29)

In Fig[5], we have plotted the e-fold number \( N \) as a function as \( \phi \) for different values of \( \lambda \) and \( p \) corresponding to a non-zero \( \xi \). The two horizontal lines corresponds to \( N = 40 \) (lower one) and \( N = 60 \) (upper one) and their intersection with the curves gives the required \( \phi \) values for producing that number of e-fold. Finally, in Fig[6], we have plotted the scalar spectral-index \( n_s \) against \( \phi \) for different values of \( \lambda \) and \( p \) corresponding to \( \xi = 0.005 \). On the left (right), the plots correspond to \( \lambda = 0.01, 0.1, p = 2(4) \) with \( \xi = 0.005 \). The two vertical pair of blue and red dashed lines corresponds to \( \phi \) values corresponding to \( N = 40 \) and 60, respectively. The horizontal lines on all plots correspond to \( n_s = 0.9649 \) (c.v.) (Planck2018).

Figure 5: Variation of e-fold number \( N \) is plotted against \( \phi \) for fixed \( \xi(= 0.005) \) and \( \lambda = 0.1, 0.01 \) corresponding to \( p(= 2) \) (left side), while the same for \( p = 4 \) on the right side.
The scalar spectral index \( n_s \) potential is shown for fixed \( \xi = 0.005 \) and \( p = 4 \) for two different values of \( \lambda = 0.1, 0.01 \). The horizontal line on both plots corresponds to \( n_s = 0.9649 \) (c.v.) (Planck2018)

### 3.4 Case 3: \( V = V_0(1 - \lambda \phi)^p \)

Next we analyse the potential \( V = V_0(1 - \lambda \phi)^p \), which in Einstein frame, takes the form:

\[
\hat{V}(\phi) = \frac{V_0(1 - \lambda \phi)^p}{(1 + \xi \phi^2)^2}
\]

In Fig.7 we have plotted \( V(\phi) \) against \( \phi \) corresponding to \( p = 2 \) (left plot) and \( p = -2 \) (right plot) The potential slow roll parameters are calculated as

\[
\epsilon_v = \frac{(-4\xi \phi(-1 + \lambda \phi) + p(\lambda + \lambda \xi \phi^2))^2}{2(1 - \lambda \phi)^2(1 + \xi(1 + 12\xi)\phi^2)}
\]

\[
\eta_v = (p^2(\lambda + \lambda \xi \phi^2)^2(1 + \xi \phi^2 + 12\xi^2 \phi^2)) + 4\xi(-1 + \lambda \phi)^2(-1 + 3\xi \phi^2 + 4\xi^2 \phi^4 + 48\xi^3 \phi^4) - p\lambda(1 + \xi \phi^2)(\lambda(1 + 9\xi \phi^2 + 96\xi^3 \phi^4 + 8\xi^2 \phi^2(3 + \phi^2)) - \xi \phi(7 + 84\xi^2 \phi^2 + \xi(12 + 7\phi^2))))/((-1 + \lambda \phi)^2(1 + \xi \phi^2 + 12\xi^2 \phi^2)^2)
\]

The scalar spectral index \( n_s \) can be calculated as

\[
n_s = -(p^2(\lambda + \lambda \xi \phi^2)^2(1 + \xi \phi^2 + 12\xi^2 \phi^2) + (-1 + \lambda \phi)^2(-1 - \xi^2 \phi^4 - 8\xi^3 \phi^4 + 48\xi^4 \phi^4)
- 2\xi(-4 + \phi^2)) - 2p\lambda(1 + \xi \phi^2)(\lambda(1 - 3\xi \phi^2 + 48\xi^2 \phi^4 + 4\xi^2 \phi^2(-6 + \phi^2)) - \xi \phi(5 + 60\xi^2 \phi^2)
+ \xi(-12 + 5\phi^2)))/((-1 + \lambda \phi)^2(1 + \xi \phi^2 + 12\xi^2 \phi^2)^2)
\]
and the tensor spectral index $n_t$,
\[
    n_t = -\frac{(-4\xi\phi(-1 + \lambda\phi) + p(\lambda + \lambda \xi \phi^2))^2}{(-1 + \lambda\phi)^2(1 + \xi(1 + 12\xi)\phi^2)} \tag{34}
\]
The tensor to scalar ratio will be,
\[
    r = \frac{8(-4\xi\phi(-1 + \lambda\phi) + p(\lambda + \lambda \xi \phi^2))^2}{(-1 + \lambda\phi)^2(1 + \xi(1 + 12\xi)\phi^2)} \tag{35}
\]
In Fig.8, we have plotted the e-fold $N$ against the field $\phi$ corresponding to $p = 2$(left plot) and $p = -2$(right plot). We have also shown the spectral index $n_s$ against $\phi$ for $\xi = 0.005$ corresponding to $p = 2$(left plot) and $p = -2$(right plot) for $\lambda = 1, -1$ respectively.

To $p = 2$(left plot) and $p = -2$(right plot) for $\lambda = 1$ and $-1$, respectively.

The potential slow-roll parameters in this case are found to be

Figure 8: The e-fold number $N$ is plotted against $\phi$ for fixed $\xi = 0.00005$ and $p = 2$(left plot) and $p = -2$ (right plot) for $\lambda = 1, -1$.

Figure 9: The scalar spectral index $n_s$ is plotted against $\phi$ for $\xi = 0.00005$ and $p = 2$(left plot) and $p = -2$(right plot) with $\lambda = 1, -1$, respectively. The horizontal line on both plots corresponds to $n_s = 0.9649$ (c.v.) (Planck2018)

on both plots corresponds to $n_s = 0.9649$ (c.v.) (Planck2018).

### 3.5 Case 4: $V = V_0 \frac{\alpha \phi^2}{1 + \alpha \phi^2}$

Next we consider the potential $V = V_0 \phi^p e^{-\lambda \phi}$, which in Einstein frame can be written as,
\[
    \hat{V}(\phi) = \frac{V_0 \alpha \phi^2}{(1 + \alpha \phi^2)(1 + \xi \phi^2)^2} \tag{36}
\]
The potential slow-roll parameters in this case are found to be
\[ \epsilon_V = \frac{2(-1 + \xi(\phi^2 + 2\alpha\phi^4))^2}{(1 + \xi\phi^2 + 12\xi^2\phi^2)(\phi + \alpha\phi^3)^2} \] 

and,

\[ \eta_V = \frac{1}{\phi^2(1 + \alpha\phi^2)^2(1 + \xi\phi^2 + 12\xi^2\phi^2)^2}(2(1 - 6\xi\phi^2 - 5\xi^2\phi^4 + 24\xi^4\phi^6 + 2\xi^3\phi^4(-36 + \phi^2) + 2\alpha^2\phi^6(-1 + 3\xi\phi^2 + 4\xi^2\phi^4 + 48\xi^3\phi^4) + \alpha^2\phi^2(-3 - 20\xi\phi^2 + 72\xi^4\phi^6 + 6\xi^3\phi^4(-28 + \phi^2) - \xi^2\phi^2(48 + 11\phi^2)))) \]

Using Eq.(15), we get the scalar spectral index as,

\[ n_s = \frac{1}{\phi^2(1 + \alpha\phi^2)^2(1 + \xi\phi^2 + 12\xi^2\phi^2)^2}((-8 - (-1 + 12\alpha + 12\xi + 144\xi^2)\phi^2 + 2\phi^4(\alpha + \xi - 16\alpha\xi + 8\xi^2 - 96\alpha\xi^2) + \alpha^2(1 - 8\xi) + 4\xi\alpha(1 + \xi - 24\xi^2) + \xi^2(1 + 20\xi + 96\xi^2))\phi^6 + 2\alpha\xi(\alpha + \xi + 12\xi^2)\phi^8 + \alpha^2\xi^2\phi^{10}(1 + 8\xi - 48\xi^2)) \]

and the tensor-to scalar ratio and tensor spectral index as follows

\[ r = \frac{32(-1 + \xi(\phi^2 + 2\alpha\phi^4))^2}{(1 + \xi\phi^2 + 12\xi^2\phi^2)(\phi + \alpha\phi^3)^2}, \quad n_t = -\frac{4(-1 + \xi(\phi^2 + 2\alpha\phi^4))^2}{(1 + \xi\phi^2 + 12\xi^2\phi^2)(\phi + \alpha\phi^3)^2} \]

As before, we can estimate the e-fold number(N) from Eq. (16) where \( \phi_f \) will be obtained by taking

**Figure 10:** (Color online) Plot between potential and \( \phi \) corresponding to \( \xi = 0.002, 0.02 \) for \( \alpha = 1 \) (Left side) and for \( \alpha = 2 \) respectively. (right side )

**Figure 11:** (Color online) Plot between e-fold number(N) and \( \phi \) corresponding to \( \xi = 0.002 \) for \( \alpha = 1, 2 \) (Left side) and scalar spectral index \( n_s \) with \( \phi \) for \( \alpha = 1, 2 \) respectively(right side ). The horizontal line on the r.h.s plot corresponds to \( n_s = 0.9649 \) (c.v.) (Planck2018)

\( \epsilon_V = 1 \). A plot of \( N \) against \( \phi \) is shown is Fig.11I- left plot. The two horizontal lines correspond to \( N = 40 \) and \( N = 60 \), the necessary \( \phi \) values required to trigger the inflation are obtained from the intersection of these horizontal lines with the curves corresponding to \( \alpha = 1 \) and \( \alpha = 2 \). On the right side \( n_s \) is plotted against \( \phi \) corresponding to \( \alpha = 1 \) and 2, respectively. The two pair of vertical lines, blue and red dashed lines, corresponds to \( \phi \) values required for generating the e-fold \( N = 40 \) and \( N = 60 \). The horizontal line on the r.h.s plot corresponds to \( n_s = 0.9649 \) (Planck2018).
4 Analysis and discussion

We have obtained the value of the field $\phi$ corresponding to $N = 40 - 60$ where $\phi_f$ is evaluated by taking $\epsilon_v = 1$ for a particular choice of parameters of the potential and in the presence of the non-minimal scalar-curvature mixing term $\xi$. The results are shown in Table 1 for different values of the potential parameters $\lambda$ and $p$ and the non-minimal parameter $\xi$. For a given $\xi$, $\lambda$ and $p$ values, we see that higher e-fold ($N$) requires larger $\phi$ value. For $\xi = 0.005$ and $p = 2$, as $\lambda$ decreases, a larger $\phi$ is required to generate the same e-fold, say e.g. $N = 60$. Also for a given $\lambda$ and $\xi$ value, as $p$ increases from 2 to 4, to produce the same e-fold, larger $\phi$ is required. As an example, for $V = V_0\phi^p e^{-\lambda\phi}$ with $\xi = 0.005, \lambda = 0.1$ one requires $\phi = 8.5(13.7)$ to produce the e-fold $N = 60$ corresponding to $p = 2(4)$. Similarly, for $V = V_0\alpha\phi^2$ with $\alpha = 1(say)$ and $\xi = 0.002$, one requires $\phi = 3.6$ and 3.8 necessary to

| potential          | $\lambda$ | $p$ | $\xi$ | $\phi_f$ | e-fold number($N$) | $\phi$ |
|---------------------|-----------|-----|-------|----------|--------------------|-------|
| $V = V_0\phi^p e^{-\lambda\phi}$ | 0 | 2  | 0.005 | 1.39334  | 40                 | 10.490 |
|                     |           |     |       |          | 60                 | 11.797 |
|                     | 0.01      | 2  | 0.005 | 1.38379  | 40                 | 10.204 |
|                     |           |     |       |          | 60                 | 11.382 |
|                     | 0.1       | 2  | 0.005 | 1.3034   | 40                 | 8.051  |
|                     |           |     |       |          | 60                 | 8.504  |
|                     | 0.01      | 4  | 0.005 | 2.75344  | 40                 | 17.380 |
|                     |           |     |       |          | 60                 | 20.857 |
|                     | 0.1       | 4  | 0.005 | 2.5931   | 40                 | 13.747 |
|                     |           |     |       |          | 60                 | 15.118 |
| $V = V_0(1 - \phi^p)e^{-\lambda\phi}$ | 0.01 | 2  | 0.005 | 1.90461  | 40                 | 10.375 |
|                     |           |     |       |          | 60                 | 11.517 |
|                     | 0.1       | 2  | 0.005 | 1.842141 | 40                 | 8.200  |
|                     |           |     |       |          | 60                 | 8.622  |
|                     | 0.01      | 4  | 0.005 | 2.79849  | 40                 | 17.311 |
|                     |           |     |       |          | 60                 | 20.687 |
|                     | 0.1       | 4  | 0.005 | 11.10854 | 40                 | 31.120 |
|                     |           |     |       |          | 60                 | 38.750 |
| $V = V_0(1 - \lambda\phi)^p$ | 1 | 2 | 0.00005 | 2.41394 | 40 | 13.703 |
|                     |           |     |       |          | 60                 | 16.510 |
|                     | -2        | 0.00005 | 2.4149 | 40 | 13.817 |
|                     |           |     |       |          | 60                 | 16.716 |
|                     | -1        | 0.00005 | 0.414137 | 40 | 11.705 |
|                     |           |     |       |          | 60                 | 14.510 |
|                     | -2        | 0.00005 | 0.414302 | 40 | 11.795 |
|                     |           |     |       |          | 60                 | 14.683 |
| $V = V_0\alpha\phi^2$ | 1 | 0.002 | 0.832199 | 40 | 3.592 |
|                     |           |     |       |          | 60                 | 3.765  |
|                     | 2 | 0.002 | 0.705871 | 40 | 3.038 |
|                     |           |     |       |          | 60                 | 3.180  |

Table 1: Table shows the values of initial field values $\phi$ (in units of $M_{Pl}$) which corresponds to $N = 40 - 60$ for different potential and for particular value of $\xi$. Here $\phi_f$ we have calculated by taking $\epsilon_v = 1$ for each potential.
produce \( N = 40 \) and \( N = 60 \), respectively. Using the field values presented in Table 1 corresponding to \( N = 40 - 60 \) as an input, we have calculated the scalar spectral index \( n_s \), tensor to scalar ratio \( r \) and tensor spectral index \( n_t \) for specific choice of coupling constant \( \xi \) and scalar field \( \phi \) which corresponds to \( N = 40 - 60 \) and presented those in Table 2. From Table 2 we can see that \( n_s \) lies within \( \pm 3 \sigma \) C.L and \( r \)

| potential | \( \xi \) | \( N \) | \( \phi \) | \( n_s \) | \( r \) | \( n_t \) |
|-----------|--------|-----|-----|-----|-----|-----|
| \( V = V_0 \phi^2 \) | 0.005 | 40 | 10.490 | 0.934282 | 0.0121734 | -0.00465 |
| | | 60 | 11.797 | 0.947277 | 0.0122418 | -0.00153 |
| \( V = V_0 \phi^2 \text{e}^{-0.01\phi} \) | 0.005 | 40 | 10.204 | 0.931388 | 0.03196 | -0.00399 |
| | | 60 | 11.382 | 0.943678 | 0.00978 | -0.00122 |
| \( V = V_0 \phi^4 \text{e}^{-0.1\phi} \) | 0.005 | 40 | 8.051 | 0.900453 | 0.00749 | -0.00094 |
| | | 60 | 8.504 | 0.908093 | 0.00113 | -0.00014 |
| \( V = V_0 \phi^4 \text{e}^{-0.01\phi} \) | 0.005 | 40 | 17.380 | 0.940901 | 0.129319 | -0.01616 |
| | | 60 | 20.857 | 0.960301 | 0.0619802 | -0.00775 |
| \( V = V_0 \phi^4 \text{e}^{-0.1\phi} \) | 0.005 | 40 | 13.747 | 0.920675 | 0.037206 | -0.00465 |
| | | 60 | 15.118 | 0.932107 | 0.0091562 | -0.00114 |
| \( V = V_0 (1 - \phi^2) \text{e}^{-0.01\phi} \) | 0.005 | 40 | 10.375 | 0.931805 | 0.0297484 | -0.0037186 |
| | | 60 | 11.517 | 0.943719 | 0.0091122 | -0.001139 |
| \( V = V_0 (1 - \phi^2) \text{e}^{-0.1\phi} \) | 0.005 | 40 | 8.20 | 0.899642 | 0.0065 | -0.008128 |
| | | 60 | 8.622 | 0.906991 | 0.000957 | -0.001196 |
| \( V = V_0 (1 - \phi^4) \text{e}^{-0.01\phi} \) | 0.005 | 40 | 17.311 | 0.940149 | 0.1247 | -0.015588 |
| | | 60 | 20.687 | 0.954921 | 0.05788 | -0.007235 |
| \( V = V_0 (1 - \phi^4) \text{e}^{-0.1\phi} \) | 0.005 | 40 | 31.120 | 0.947723 | 0.214131 | -0.026767 |
| | | 60 | 38.750 | 0.965824 | 0.142278 | -0.01778 |
| \( V = V_0 (1 - \phi)^2 \) | 0.00005 | 40 | 13.703 | 0.950633 | 0.193323 | -0.024165 |
| | | 60 | 16.510 | 0.966951 | 0.128108 | -0.016013 |
| \( V = V_0 (1 - \phi)^{-2} \) | 0.00005 | 40 | 13.817 | 0.963742 | 0.14124 | -0.017655 |
| | | 60 | 16.716 | 0.974693 | 0.09743 | -0.012178 |
| \( V = V_0 (1 + \phi)^2 \) | 0.00005 | 40 | 11.705 | 0.950618 | 0.193749 | -0.024219 |
| | | 60 | 14.510 | 0.966926 | 0.128502 | -0.0160127 |
| \( V = V_0 (1 + \phi)^{-2} \) | 0.00005 | 40 | 11.795 | 0.998876 | 0.202716 | -0.02534 |
| | | 60 | 14.683 | 0.998056 | 0.137566 | -0.017195 |
| \( V = V_0 \phi^2 \text{e}^{-\phi} \) | 0.002 | 40 | 3.592 | 0.91954 | 0.001188 | -0.000048 |
| | | 60 | 3.765 | 0.930133 | 0.000268 | -0.000034 |
| \( V = V_0 \phi^2 \text{e}^{-2\phi} \) | 0.002 | 40 | 3.038 | 0.918955 | 0.0008 | -0.0001 |
| | | 60 | 3.180 | 0.929455 | 0.00018 | -0.00002 |

Table 2: CMB parameters are evaluated for particular choice of \( \xi \) and \( \phi \) consistent with \( N = 40 - 60 \) (from tabl.1) for different potentials. Values quoted lies within the limit of \( N = 40 - 60 \) and the spectral index \( n_s = 0.9649 \pm 0.0042 \) upto 3\( \sigma \) C.L. Except few case most values correspond to \( r < 0.106 \).

agrees with PLANCK 2018 Data for potentials \( V \propto \phi^2 \), \( V \propto \phi^2 \text{e}^{-0.01\phi} \), \( V \propto \phi^4 \text{e}^{-0.01\phi} \) corresponding to \( N = 60 \). Similarly for potentials \( V \propto e^{-0.01\phi}(1 - \phi^4) \), \( V \propto e^{-0.1\phi}(1 - \phi^4) \), \( V \propto (1 - \phi)^2 \) and \( V \propto (1 + \phi)^2 \), the \( n_s \) value lies within \( \pm 3 \sigma \) C.L of PLANCK 2018 data. We can see that tensor to scalar ratio \( r \) lies within experimental data \( r < 0.106 \) (PLANCK+BAO) and scalar spectral index \( n_s = 0.9649 \pm 0.0042 \) (PLANCK 2018) upto 3\( \sigma \) C.L for some potentials discussed so far and e-fold number \( N = 60 \). For the rest of potentials, \( n_s \) and \( r \) lies within WMAP3 data [14].

Finally, in Fig. [12] we have plotted the variation of tensor to scalar ratio \( r \) with scalar spectral index \( n_s \) for all potential discussed above. In Fig. [12], the blue and red shaded regions correspond to WMAP data upto 95% and 68% C.L whereas grey, green and purple shaded regions correspond to PLANCK, PLANCK+BK15, PLANCK+BK15+BAO upto 95% and 68% C.L. On the left side of Fig. [12] the
Figure 12: (Color online) Constraints on $n_s$ and $r$ form CMB measurements of different potential. Shaded regions are allowed by WMAP5 measurements, PLANCK alone, PLANCK+BK15, PLANCK+BK15+BAO to 68% and 95% confidence. Models plotted in the left side figure are for scalar-field potential $V \propto \phi^p e^{-\lambda \phi}$ for $p = 2, \lambda = 2, p = 2 \lambda = 0.1, 0.01$ and $p = 4, \lambda = 0.1, 0.01$ and $V \propto \frac{\alpha \phi^2}{1+\alpha \phi^2}$ for $\alpha = 1, 2$. Other potentials like $V \propto e^{-\lambda \phi}(1 - \phi^p)$ for $\lambda = 0.01, 0.1, p = 2, 4$ and $V \propto (1 - \lambda \phi)^p$ for $\lambda = \pm 1, p = \pm 2$ are shown in the right figure.

e-fold numbers $N = 40 - 60$ corresponds to potentials $V = V_0 \phi^p e^{-\lambda \phi}$ for (i) $p = 2, \lambda = 0$, (ii) $p = 2, \lambda = 0.1, 0.01$, (iii) $p = 4, \lambda = 0.1, 0.01$, respectively, which are represented by black, green, blue, cyan and yellow lines and $V = V_0 \frac{\phi^2}{1+\alpha \phi^2}$ for $\alpha = 1, 2$ shown in purple, red lines respectively. Similarly on the right side of Fig. (12) we have shown potentials $V = V_0 e^{-\lambda \phi}(1 - \phi^p)$ for (i) $p = 2, \lambda = 0.01, 0.1$ and (ii) $p = 4, \lambda = 0.01, 0.1$ as in green, cyan, orange and pink lines and $V = V_0 (1 - \lambda \phi)^p$ for $\lambda = \pm 1, p = \pm 2$ in yellow, red, blue and purple respectively.

Thus we find that in the non-minimal scenario with a non-zero $\xi R \phi^2$ term in the Einstein-Hilbert action, the set of potentials (i) $V \propto \phi^p e^{-\lambda \phi}$, (ii) $V \propto e^{-\lambda \phi}(1 - \phi^p)$, (iii) $V \propto (1 - \lambda \phi)^p$ and (iv) $V \propto \frac{\alpha \phi^2}{1+\alpha \phi^2}$ can predict the correct number of e-fold $N = 40 - 60$ in their parameter space. We also estimate the scalar spectral index $n_s$ and tensor-to-scalar ratio $r$ using these four potentials which lie well within the $\pm 3\sigma$ of the Planck 2018 data and WMAP5 measurements.

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Yellow and blue lines have been merged for the potential $V = (1 - \phi)^2$ and $V = (1 + \phi)^2$ on right side of Fig. (3). As we can see from Table we that scalar spectral index $n_S$ and $r$ values are almost same for those two potentials.

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