Speedup of the Quantum Adiabatic Algorithm using Delocalization Catalysis

Chenfeng Cao, Jian Xue, Nic Shannon, and Robert Joynt

1 Department of Physics, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, China
2 Institute of Physics, Chinese Academy of Sciences, Beijing, China
3 Theory of Quantum Matter Unit, Okinawa Institute of Science and Technology Graduate University, Onna-son, Okinawa 904-0412, Japan
4 Department of Physics, University of Wisconsin-Madison, 1150 University Avenue, Madison, Wisconsin 53706, USA
5 Kavli Institute for Theoretical Sciences, University of Chinese Academy of Sciences, Beijing, China

(Dated: July 23, 2020)

We propose a method to speed up the quantum adiabatic algorithm using catalysis by many-body delocalization. This is applied to antiferromagnetic Heisenberg spin models. The algorithm is catalyzed in such a way that the evolution approximates such models in the middle of its course, and the model is in a delocalized phase. We show numerically that we can speed up the standard algorithm for finding the ground state of the random-field Ising model using this idea. We can also show that the speedup is due to gap amplification, even though the underlying model is not frustration-free. Our method is verified by experimental results from IBM quantum computer. Even though only relatively small systems can be investigated, the evidence suggests that the scaling of the method with system size is favorable. The cost of the catalytic method compared to the standard algorithm is only a constant factor.

I. INTRODUCTION

In quantum computing, optimization problems play a special role. This is simply because optimization is ubiquitous in all areas of human endeavor. Any significant speedups due to quantum optimization algorithms are guaranteed wide application in computation. The quantum adiabatic algorithm (QAA) is a leading candidate for such a speedup [1]. Adiabatic computation model was proven to be polynomially equivalent to the gate-based model [2]. Multiple tantalizing results continue to make the QAA attractive [3, 4], but provable speedups remain elusive [5]. Small energy gaps that thwart adiabaticity are of course the main issue [6].

One interesting approach to improve the QAA borrows the concept of catalysis from chemistry. When two reactants meet, they may or may not combine to form the stable compound, that is, they may or may not find the global ground state. This process may be compared to the evolution in the QAA, with the meeting of the reactants corresponding to a close avoided level crossing. In chemistry, another species is added to the system. This species allows the reactants to overcome whatever barrier prevents the reaction. In the QAA, we add a catalytic term to the Hamiltonian that is absent at the beginning and end of the evolution, but plays the role of helper at the time of the avoided crossing.

The QAA catalysis idea was proposed rather early in the history of QAA under the rubric of simply changing the path in Hamiltonian space and it was shown to work for an artificially designed problem by Farhi et al. [7]. This possibility also features in the analysis of the universality of the QAA [8]. A more targeted form of QAA catalysis using a new term that suppresses transitions was proposed by ¨Ozg¨uler et al. [9]. However, the speedups achieved were rather small, and improving that method seems to be computationally demanding. The question of the general utility of QAA catalysis remains open.

In this paper we take a different approach based on condensed matter physics ideas. We use spin models that have a transition from a many-body localized (MBL) phase to a many-body delocalized (MBD) phase. The paradigmatic examples, and the one that are best understood [10, 11], are one-dimensional spin models. We use the random-field Ising model (RFIM) as the problem to be optimized and the one-dimensional Heisenberg model as its delocalized counterpart. The catalytic part of the Hamiltonian is designed so that the system approximates an MBD quantum model in the middle of the evolution, while the end point Hamiltonians are the ones chosen in the usual QAA. In the RFIM we have the additional features that there are natural energy gaps due to local spin singlet formation. The ground state of the model in the low-disorder regime has a high degree of local entanglement and is far from the classical Ising-type models that map to interesting classical optimization problems. Hence the RFIM will be the main focus of the paper. It is of course a standard testbed for quantum optimization.

We note as well that one may turn the logic around and use the speedup as a diagnostic for a MBL-MBD transition, i.e., as a tool for investigating whether a given model has such a transition.

Putting the problem into a more general context, however, it is also possible that a suitable catalyst Hamiltonian may be found for other problems by choosing the catalyst such that the eigenstates of the total Hamiltonian change from MBL to MBD. We give some further details of this point of view for the RFIM below.

Optimization of the ground state energy in the (RFIM) may be viewed semiclassically as optimizing the positions of domain walls. As the system evolves in the course of
the QAA, domain walls move to keep the energy low. A close avoided crossing is due to a long move. The off-diagonal matrix element of the Hamiltonian between the two levels involved in the crossing is small because the Hamiltonian is local and any perturbation theory expression for this element occurs only at high order. In the catalytic approach, this semiclassical picture breaks down - the domain walls themselves are now delocalized quantum objects because of the additional Heisenberg-like terms in the Hamiltonian. We can then expect to find larger matrix elements for the wall motion, which will result in larger gaps in the energy spectrum. It is evident that this picture is closely related to the thermalization property of MBD systems that distinguishes them from MBL systems.

In one-dimensional Heisenberg models, there is a gap of topological origin for integer spin [12]. This motivated us to do a comparison of spin-1/2 and spin-1 models to see if the presence of this gap would contribute to a speedup of the algorithm.

The simulations are all for the closed QAA. We have not done any detailed analysis of our method in an open quantum system context. However, we have run it on the IBM online system with positive results that we also present. The success of the method in this case suggests that it can also be relevant in the present era of noisy intermediate-scale quantum machines.

A key piece of evidence for the connection of the present method to MBL is the presence of gap amplification. A defining feature of any kind of localization is that there is no correlation between energy eigenvalues for eigenstates that are spatially separated beyond a characteristic length. This leads to level crossings that are not avoided. Gap amplification, though not in the MBL context, has been investigated by numerous authors. The situation at present is that gap amplification that preserves the form of the eigenstates is known to be possible in general for frustration-free optimization problems [13, 14] and not possible for certain specific frustrated models. This is done by a modification of the Hamiltonian. In our work, we also produce gap amplification. This comes from a physically-motivated time-dependent Hamiltonian and the eigenstates change in the course of the evolution. We present strong evidence that this can be achieved for the one-dimensional random-field Ising model, and point to a specific way to do that.

The idea is thus very similar to actual chemical catalysis. The new term in the Hamiltonian makes available paths in phase space from the initial to the final state by increasing the matrix elements that promote motion through bottlenecks. A paper closely related to ours is that of Hormozi et al. [15]. These authors added random nonstoquastic terms to the Hamiltonian that turn on at the initial point and off at the final point of the evolution. The aim is similar: give the system more paths to overcome barriers and find the true ground state near the close avoided crossing. However, the methods are opposite - ours is designed to reduce randomness, giving something close to an ordered model with known properties, in particular, large local gaps. Ref. [15], in contrast, introduces more randomness into the problem. Also, the additional terms in the Hamiltonian are not functions of the original Hamiltonian. In our method, once the original Hamiltonian is specified, the catalyst Hamiltonian is determined. Thus, while we average over the disorder in the original optimization problem to evaluate our method, we do not need to average over many additional terms. We note also that the spin model investigated in Ref. [15] is on a complete graph. We do not know if our method is likely to be useful in such models, since we use features of quantum spins that are only evident when coordination numbers are low (in fact independent of system size). We follow Ref. [15] in that we judge the efficacy of our method by comparison to the results when the catalytic term is absent. The connection of Anderson localization to QAA was first pointed out by Altshuler, Krovi, and Roland, who came to the quite pessimistic conclusion that it rendered the QAA ineffective [16]. One way to get around this was proposed by Dickson [17]. It uses ancilla qubits. The goal of the present work is similar to that of Ref. [17] but the method is completely different.

From the standpoint of computer science, the method we present is entirely heuristic: we offer no speedup proofs. However, it is not unprecedented that successful heuristic methods are later shown to offer certain provable improvements in efficiency. The simplex method is one example in optimization theory.

We describe our calculation method in Sec. II. We apply the method to spin-1/2 models in Sec. III, and to spin-1 models in Sec. IV. In Sec. V we investigate the efficacy of the method as the number of spins is increased. In Sec. VI, we verify our method by experiments on IBM quantum device. The results are summarized and discussed in Sec. VII.

II. CALCULATION METHOD

In the QAA method for this problem, we start a system of $N$ spins at $t = 0$ in the ground state of some simple Hamiltonian $H_0$ whose ground state is easy to prepare. This is $|\psi_0\rangle$. In the absence of catalysis, the state evolves according to the time-dependent Hamiltonian $H_{qaa}$ which is

$$H_{qaa} = f(t)H_0 + g(t)H_f$$

with, for example, $f(t) = 1 - \frac{t}{t_a}$ and $g(t) = \frac{t}{t_a}$.

According to the adiabatic theorem [18], if the minimum spectral gap $\delta_m$ is strictly greater than 0 and the evolution is slow enough, the final state $|\psi_f\rangle$ at $t = t_a$ will have a high fidelity to the ground state $|\psi_{g.s.}\rangle$ of $H_f$.

Our modified method is as follows. We add a catalytic term $H_c$ according to the recipe

$$H_{qaa}(t) = f(t)H_0 + g(t)H_f + h(t)H_c.$$
Here \( f(0) = g(t_a) = 1 \) and \( f(t_a) = g(0) = h(0) = h(t_a) = 0 \).

On quantum annealers, our method can be implemented directly. On gate-based quantum computers, e.g. IBM Q quantum processors, Suzuki-Trotter decomposition may be employed for the original and the catalyzed evolution. Clearly, the increase in cost, measured in the number of quantum gates applied, is a multiplicative constant. In our simulation of such a gate-based machine, we use a fourth-order Runge-Kutta algorithm where the number of time steps is proportional to \( t_a \).

The problem Hamiltonian we choose for our one-dimensional work is the nearest-neighbor RFIM on a ring of \( N \) spins:

\[
H_f = \sum_{k=1}^{N} h_k S_z^k + J \sum_{k=1}^{N} S_z^k S_z^{k+1}
\]

(3)

where \( h_k \) are chosen uniformly from the interval \([-1, 1]\), The catalyst term is

\[
H_c = J \sum_{k=1}^{N} (S_x^k S_x^{k+1} + S_y^k S_y^{k+1}).
\]

(4)

The initial Hamiltonian represents a staggered field:

\[
H_0 = \sum_{k=1}^{N} (-1)^k S_x^k.
\]

(5)

It is very important to choose \( H_0 \) carefully. If \( H_0 \) is not staggered, it commutes with \( H_c \), thus creates symmetry-induced level crossings and associated small gaps.

We use the evolution function

\[
h(t) = 2 \frac{t}{t_a} (1 - \frac{t}{t_a}).
\]

(6)

Thus when \( t = \frac{1}{2} t_a \), the system Hamiltonian is the sum of a nearest-neighbor Heisenberg Hamiltonian and transverse field Hamiltonians. The Heisenberg Hamiltonian can be written as

\[
H_H = J \sum_k S_z^k \otimes S_z^{k+1}
\]

\[
+ \frac{1}{2} J \left( \sum_k S_+^k \otimes S_-^{k+1} + \sum_k S_-^k \otimes S_+^{k+1} \right).
\]

(7)

The second term can interchange neighboring spins. This moves domain walls and provides the system with paths toward the ground state.

The two-dimensional model we use is just the obvious generalization of this and the same remarks concerning domain walls apply, though of course the walls are now one-dimensional objects, not points.

It is known that the Heisenberg model in a random field undergoes a many-body localization at about \( J \approx 1/3 \) [19–21] with the XX and YY terms driving the delocalization. Thus during the evolution, the system is in a delocalized phase for \( J \) sufficiently large and \( t/t_a \) sufficiently close to \( 1/2 \).

In Secs. III and IV we respectively simulate the evolution for spin-1/2 and spin-1 quantum systems. The ground state is calculated by exact diagonalization, and the time-dependent Schrödinger equation is solved numerically as described above. We denote \( \overline{T_{g.s.}} \) as the overlap between the final state \( |\psi_f\rangle \) and the ground state \( |\psi_{g.s.}\rangle \) of the final Hamiltonian. \( \overline{T_{g.s.}} = \langle \psi_f | \psi_{g.s.} \rangle ^2 \).

Each point in Sec. III and Sec. IV is averaged over 32 realizations and each point in Sec. V is averaged over 256 realizations.

III. RESULTS FOR SPIN 1/2

The ground state of the spin-1/2 nearest-neighbor antiferromagnetic Heisenberg chain is not ordered. Instead it is quantum-critical with power-law spin correlations. Due to the low dimensionality, neighboring spins have very strong singlet correlations, and therefore a large amount of entanglement at a local level. Local formation of triplet pairs costs a large energy. Nevertheless, the model does not have a gap due to the Lieb-Schultz-Mattis theorem [22]. The low-energy excitations are spinon pairs on top of a background of resonating valence bonds. The spinons have a large short-range repulsion but are otherwise deconfined.

We now use the Heisenberg antiferromagnetic model to catalyze the evolution from a 1-dimensional antiferromagnet to a 1-dimensional random-field Ising spin chain. The motivation is that when close encounters of energy levels occur, the gap will be increased by the fact that local singlet correlations have built up in the wavefunctions of the ground state and states that are nearby in energy.

The results are shown in Fig. 1 and Fig. 2(a) for a system of \( N = 12 \) spins. In Fig. 1(a) the interaction between neighboring spins is relatively large (i.e. \( J \geq 1 \)). For this case the optimization process can be accurately thought of as the appropriate motion of domain walls, precisely the physical process that the catalyst Hamiltonian \( H_c \) is designed to promote. We see that at \( J = 3 \) the catalysis increases the overlap of the computed ground state and the actual ground state dramatically for any evolution time and the increase is also substantial when \( J = 1 \). In Fig. 1(b) the interaction is relatively small (i.e. \( 0 \leq J \leq 1 \)) and we investigate only shorter times when the overlap is still relatively small. The catalyst Hamiltonian \( H_c \) is still helpful. However, as \( t_a \) increases the catalysis is no longer effective. We can repeat the short-time annealing for many times to find the true ground state.

We define the average overlap of the computed ground state and the actual ground state with catalysis as \( \overline{P^c} \), the average overlap without catalysis as \( P^0 \). Then the speedup \( SP \) is the ratio of \( P^c \) and \( P^0 \), \( SP = P^c/P^0 \). The speedup map for \( 0 < J \leq 1, 0 < t_a \leq 10 \) is shown in
Figure 1. Average ground state probabilities and minimum gaps for spin-1/2 quantum chain with 12 spins. (a)(b) Overlap of the ground state and the final wavefunctions calculated by the catalyzed (full lines) and uncatalyzed (dashed lines) QAA. Shown are the results for $J = 0.1, 0.5, 1, 3$. Catalysis speeds up the algorithm substantially, particularly for $J = 3$, but hardly at all for $J = 0.1$. (c) Average minimum gap with or without catalysis vs. interaction parameter $J$. The abrupt rise in minimum gap coincides with the onset of speedup as $J$ increases.

Figure 2. Speedup SP as a function of $J$ and $t_0$. Darker regions correspond to higher speedups. (a) Speedup for spin-1/2 quantum chain with 12 spins. (b) Speedup for 4 x 3 spin-1/2 grid. (c) The color bar shows the speedup values.

Fig. 2(a). In the weak interaction case the domain wall concept ceases to apply and the optimization becomes rather trivial since the ground state is very close to the state determined entirely by the random field.

In adiabatic quantum computing, the required annealing time of QAA for a fixed error size is given by an expression of the form

$$T = O \left( \frac{\| \tilde{H}(t) \|}{\delta_m^2} \right), \quad (8)$$

where $\delta_m$ is the minimum spectral gap, i.e., the difference between the ground state energy and the first excited state energy minimized over all times $0 \leq t \leq t_a$. It is of great interest to check that the positive effects of $H_c$ do arise from gap amplification. In Fig. 1(c) we
plot $\delta_m$ against $J$ with and without $H_c$. The difference for $J \geq 1/2$ is striking, and the fact that the crossover occurs at $J \approx 1/2$ this point both in the overlap and the minimum gap confirms the overall picture.

Of particular note from the optimization point of view is that the method actually works well in the region of moderate coupling. This is where the optimization problem is the most difficult. Naive methods work well both for $J << 1/2$ and $J >> 1/2$.

From the MBL point of view, the important fact is that the transition from no speedup to speedup around $J = 1/2$ matches approximately the MBL-MBD transition. The latter takes place in the static model at $J \approx 1/3$, which is roughly the average value of $J$ in the course of the QAA evolution. This justifies the idea that speedup can serve as a signal of the many-body transition.

The one-dimensional RFIM is a fairly simple model. We can directly extend the whole method to the two-dimensional square lattice. The initial transverse magnetic field is again staggered in the x-direction. The ground state of the two-dimensional nearest-neighbor Heisenberg model that catalyzes the evolution model is quite different from the one-dimensional version. It is an ordered magnet though with an ordered moment that is substantially reduced by quantum effects [23], spins have a much more classical character. Nevertheless, the concept of domain wall motion remains important, we expect the domains to be delocalized in the MBD phase as they are in one dimension. Indeed, motion of density domain walls in a two-dimensional disordered boson system has been observed [24].

Fig. 2(b) shows the speedup map for the 4 x 3 spin-1/2 grid. The effect of the 2-dimensional catalyst Hamiltonian is quite similar to the 1-dimensional case. For small $J$, the lack of speedup is analogous to what happens in one dimension, while for larger $J$, the speed up is obvious since the minimum gap can be efficiently amplified.

The ground state of the spin-1 antiferromagnetic Heisenberg chain is also not ordered, but its spin correlations are exponential, not power-law. It exhibits the Haldane gap [12]. One may think of the ground state as being effectively spontaneously dimerized, as shown by the Affleck-Kennedy-Lieb-Tasaki (AKLT) construction [25] for the ground state of a closely related Hamiltonian also to be used below. The low-energy excitations are massive spin waves. However, disorder can also induce localized fractional (spin-1/2) excitations, which also appear at the boundaries if open boundary conditions are employed.

This is the second model that we use to catalyze the evolution from a 1-dimensional nearest-neighbor antiferromagnet to a 1-dimensional nearest-neighbor RFIM spin chain. It is easy to show that finding the ground state of the spin 1 model also gives the solution to the problem of

IV. RESULTS FOR SPIN 1

The ground state of the spin-1 antiferromagnetic Heisenberg chain is also not ordered, but its spin correlations are exponential, not power-law. It exhibits the Haldane gap [12]. One may think of the ground state as being effectively spontaneously dimerized, as shown by the Affleck-Kennedy-Lieb-Tasaki (AKLT) construction [25] for the ground state of a closely related Hamiltonian also to be used below. The low-energy excitations are massive spin waves. However, disorder can also induce localized fractional (spin-1/2) excitations, which also appear at the boundaries if open boundary conditions are employed.

This is the second model that we use to catalyze the evolution from a 1-dimensional nearest-neighbor antiferromagnet to a 1-dimensional nearest-neighbor RFIM spin chain. It is easy to show that finding the ground state of the spin 1 model also gives the solution to the problem of

Figure 3. Average ground state probabilities and minimum gaps for spin-1 quantum chain with 8 spins. (a) Overlap of the ground state and the final wavefunctions calculated by the catalyzed (full lines) and uncatalyzed (dashed lines) QAA. Shown are the results for $J = 0.1, 0.5, 1, 3$. Catalysis speeds up the algorithm substantially, particularly for $J = 3$, but hardly at all for $J = 0.1$. (c) Average minimum gap with or without catalysis vs. interaction parameter $J$. The catalysis gives very considerable amplification of the minimum gap and this is closely associated with the speedup of the algorithm.
finding the RFIM ground state. Again the expectation is that local correlations will gap out close avoided level crossings. We wish to test whether the existence of the Haldane gap will increase this effect or not.

MBL also occurs in the spin-1 chain [26], but it is important to note that the promotion to spin-1 from spin-1/2 also has several clear disadvantages. The most obvious is that the computation time for fixed number of spins increases. Also, the energy level density is increased by a factor of \((3/2)^N\), where \(N\) is the number of spins, with a corresponding decrease in the average separation between energy levels. Finally, as the spin \(S\) increases, the spins become more classical in the sense that the large energy difference between the singlet and triplet energies for a pair of spins becomes less evident with increasing \(S\). This is clearly not in line with what we believe to be the advantages of our method.

The numerical results for the spin-1 quantum chain are shown in Fig. 3(a) for \(J = 1\) and \(J = 3\). The speedup is very similar to the spin-1/2 case. Again the catalyst Hamiltonian can speed up the evolution significantly. In Fig. 3(b), we show that even for weak interactions there is some speedup as long as the evolution time is short. Thus, as we saw in the spin-1/2 case, the size of the speedup increases as the interaction becomes stronger. Gap amplification is present as well, as shown in Fig. 3(c). It is slightly less strong than in the spin-1/2 case, reflecting the compression of energy levels. The initial spectral gap is an upper bound of the minimum gap \(\delta_m\).

For spin-1/2 quantum chain, the initial spectral gap is 2, but for spin-1 quantum chain, the initial gap is 1.

At present, we do not know how to disentangle any effects of the Haldane gap from the other effects we showed to exist in the spin-1/2 case. Hence the effects of topology remain an open question.

V. SCALING

In this section, we investigate the scaling properties of the method. We fix a short annealing time \((t_a = 1)\) and measure the speedup by \(SP = P^c/P^0\), where \(P^c\) is the average overlap of the computed ground state and the actual ground state with catalysis, \(P^0\) is the average overlap without catalysis.

For spin-1/2 and spin-1 quantum chain with different sizes, the speedup \(SP\) is shown in Fig. 4. as a function of the interaction strength \(J\) and the number of spins \(N\). As \(N\) increases, the speedup increases exponentially in this range of parameters. This analysis is limited to a small number of spins and a short evolution time, but there is a strong suggestion that the method works better for larger systems: the positive results reported above are not an artifact of small \(N\). For spin-1/2 case, we verified the existence of speedup and gap amplification up to \(N = 20\) spins with DMRG algorithm.

VI. EXPERIMENTS ON AN IBM QUANTUM COMPUTER

In this section, we run QAA algorithm on a real IBM quantum computer to test our method in the context of a real system with errors. The device we use is ibmq_5_yorktown-ibmqx2 [27], which is a gate-based programmable quantum computer with 5 qubits. The structure of the quantum computer is shown in Fig. 5.

We run our circuits on a 5-qubit chain ordered by qubit-0, 1, 2, 3, 4. The staggered initial Hamiltonian is

\[
H_0 = \sum_{k=1}^{5} (-1)^k \sigma_x^k. \tag{9}
\]
The final Hamiltonian is
\[ H_f = -\frac{1}{2}(\sigma_x^1 + \sigma_z^3 + \sigma_z^5) + J \sum_{k=1}^{4} \sigma_z^k \sigma_{z}^{k+1}. \]  
(10)

The catalyst Hamiltonian is
\[ H_c = J \sum_{k=1}^{4} (\sigma_x^k \sigma_{x}^{k+1} + \sigma_y^k \sigma_{y}^{k+1}). \]  
(11)

The ground state of the final Hamiltonian is \(|01010\rangle\). Since ibmq_5_yorktown is not a quantum annealer, we need to do Trotter decomposition to implement the evolution [28]. First, we discretize the evolution time \(t_a\) so that the evolution operator \(U(t)\) becomes a product of discrete interval operators \(\{U(\Delta t)\}\), then we approximate these operators with CNOT and single-qubit gates.

Operators \(e^{-i\sigma_z \Delta t}\) and \(e^{-i\sigma_x \Delta t}\) can be implemented by single-qubit rotation gates \(R_x\) and \(R_z\) directly. Operator \(N(\alpha, \beta, \gamma) = \exp(i(\alpha \sigma_x \otimes \sigma_x + \beta \sigma_y \otimes \sigma_y + \gamma \sigma_z \otimes \sigma_z))\) can be decomposed by the circuit in Fig. 6, which has been proven to be optimal [29]. With Trotterization and gate decomposition, we can simulate a continuous time evolution with ibmq_5_yorktown.

\[ R_y(\frac{\alpha}{2} - \beta) \quad R_x(\frac{x}{2} - 2\gamma) \quad R_x(\frac{x}{2}) \quad R_y(\frac{2\alpha}{2} - 2\beta) \]

Figure 6. Optimal decomposition of \(N(\alpha, \beta, \gamma)\)

Since the CNOT error rate of ibmq_5_yorktown is not negligible, which ranges from \(1.37 \times 10^{-2}\) to \(1.79 \times 10^{-2}\), we can only simulate a short-time evolution. For annealing times \(t_a = 0.2, 0.4, 0.6, 0.8, 1\), the probabilities of the ground state are shown in Fig. 7. The catalyst term can speed up the evolution efficiently for large interaction parameter \(J\), which is consistent with the theoretical expectations and the earlier simulation results.

VII. CONCLUSIONS AND FUTURE DIRECTIONS

A great deal is known about low-dimensional spin systems in condensed matter physics. They show particularly strong quantum effects. The method described here is an attempt to exploit these effects for the purpose of improving quantum adiabatic optimization. Our simulations are necessarily limited to quite small systems, but the results are extremely encouraging. In particular, there are indications from scaling arguments that the speedups are not limited to small systems.

We have investigated only the RFIM problem. The necessary quantum effects will likely be useful for other models that can be delocalized. It is possible it will work only on models defined on relatively sparse graphs. When the coordination number increases, the spins become effectively more classical. However, even in two dimensions we still found a speedup.

It is important to point out that the concepts are not limited to the RFIM. The method is based on identifying the paths that the system needs to follow in order to optimize its configuration efficiently. In the RFIM, this is domain wall motion. Then one chooses a catalyst that delocalizes the degrees of freedom for these paths. We conjecture that if a catalyst Hamiltonian can be found that changes the model from one that is MBL to one that is MBD, then gap amplification and speedup of the QAA can be achieved, even in the degree of freedom is not a domain wall.

In future work, one may reverse the logic to use the speedup in condensed-matter research to investigate the existence of the MBL-MBD transition in specific models.

Also, we note that the quantum optimization results may be improved by increasing the number of parameters.
and classically optimizing over different catalyst terms and the schedule of the catalysis.

ACKNOWLEDGMENTS

We thank B. Ozguler, M. G. Vavilov and Tao Xiang for useful discussions. We acknowledge the use of IBM Quantum services for this work.

[1] E. Farhi, J. Goldstone, S. Gutmann, J. Lapan, A. Lundgren, and D. Preda, Science 292, 5516 (2001).
[2] D. Aharonov, W. Van Dam, J. Kempe, Z. Landau, S. Lloyd, and O. Regev, SIAM review 50, 755 (2008).
[3] A. Mott, J. Job, J.-R. Vlimant, D. Lidar, and M. Spiropulu, Nature 550, 375 (2017).
[4] Y. Subasçı, R. D. Somma, and D. Orsucci, Physical review letters 122, 060504 (2019).
[5] T. Albash and D. Lidar, Rev. Mod. Phys. 90, 015002 (2018).
[6] A. P. Young, S. Knysh, and V. N. Smelyanskiy, Phys. Rev. Lett. 101, 170503 (2008).
[7] E. Farhi, J. Goldstone, and S. Gutmann, arXiv:quant-ph/0208135 101, 170503 (2008).
[8] D. Aharonov, W. van Dam, J. Kempe, Z. Landau, S. Lloyd, and O. Regev, SIAM J. Comput. 37, 170503 (2007).
[9] A. Özgüler, R. Joynt, and M. Vavilov, Phys. Rev. A 98, 062311 (2018).
[10] R. Nandkishore and D. Huse, Ann. Rev. of Cond. Matt. Phys. 6, 15 (2015).
[11] J. Imbrie, J. Stat. Phys. 163, 988 (2016).
[12] F. Haldane, Phys. Rev. Lett. 50, 1153 (1983).
[13] R. Somma, S. Boixo, H. Barnum, and E. Knill, Phys. Rev. Lett. 101, 130504 (2008).
[14] R. Somma and S. Boixo, SIAM J. Comput. 37, 593 (2013).
[15] L. Hormozi, E. W. Brown, G. Carleo, and M. Troyer, Phys. Rev. B 95, 184416 (2017).
[16] B. Altshuler, H. Krovi, and J. Roland, Proc. Nat. Acad. Sci. 107, 12446 (2010).
[17] N. Dickson, New Journ. Phys. 13, 073011 (2011).
[18] M. Born and V. Fock, Zeitschrift für Physik 51, 165 (1928).
[19] A. Pal and D. A. Huse, Phys. Rev. B 82, 174411 (2010).
[20] D. J. Luitz, N. Laflorencie, and F. Alet, Phys. Rev. B 91, 081103(R) (2015).
[21] A. Özgüler, C. Xu, and M. G. Vavilov, Phys. Rev. B 101, 024204 (2020).
[22] L. E., T. Schultz, and D. Mattis, Annals of Physics 16, 407 (1961).
[23] S. Chakravarty, B. Halperin, and D. Nelson, Phys. Rev. B 39, 2344 (1988).
[24] J. Choi, S. Hild, J. Zeiher, P. Schauss, A. Rubio-Abadal, T. Yefsah, V. Khemani, D. A. Huse, I. Bloch, and C. Gross, Science 352, 1547 (2016).
[25] I. Affleck, T. Kennedy, E. H. Lieb, and H. Tasaki, Phys. Rev. Lett. 59, 799 (1987).
[26] A. Chandran, V. Khemani, C. Laumann, and S. Sondhi, Phys. Rev. B 89, 144201 (2014).
[27] 5-qubit backend: IBM Q team, “IBM Q 5 Yorktown backend specification V2.0.6,” (2020). Retrieved from https://quantum-computing.ibm.com.
[28] A. Smith, M. Kim, F. Pollmann, and J. Knolle, npj Quantum Information 5, 1 (2019).
[29] F. Vatan and C. Williams, Physical Review A 69, 032315 (2004).