Multi-objective Reliability Optimization Design of Planetary Transmission Cutter Mechanism

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Abstract. Traditional planetary cutter mechanism design needs repeated trial and verification to determine a design scheme that meets the given conditions, and the scheme is generally not the best; The conventional single objective optimization design usually only aims at a certain requirement to achieve a certain optimal performance, neglecting the optimal comprehensive performance of the product. In order to solve the above problems, the reliability theory and optimization design technology are applied, and the compact structure, transmission efficiency, transmission stability and bearing capacity of the planetary cutter mechanism are comprehensively considered. Taking the minimum volume, maximum efficiency and maximum coincidence degree of the mechanism as the objective function, a multi-objective optimization design mathematical model different from the existing models is established by using the safety factor method and other constraints, Matlab software is used for optimization calculation. The results show that compared with the conventional design, the volume of the multi-objective optimization design is reduced by 79.97% and the coincidence degree is increased by 2.64% under the premise of almost constant transmission efficiency; Compared with the conventional single objective optimization design, although the single index is not optimal, the indexes are more balanced and the optimization effect is obvious.

1. Introduction
As the direct working part of crop stalk cutting, the cutter of harvester makes reciprocating linear motion, and usually adopts three kinds of transmission forms: crank connecting rod mechanism, swing ring mechanism and planetary gear mechanism[1]. Practice has proved that, compared with crank connecting rod mechanism and swing ring mechanism, planetary gear transmission mechanism can not only realize reciprocating linear motion under certain system parameters, but also can not produce lateral force, so tool wear and vibration are small, cutting speed can be further improved, so it is widely used in harvesters [2]. Li[1] states that the traditional design method is used to design the planetary gear transmission mechanism of combine cutter. However, this traditional design method is a design process in which the designer assumes, calculates, checks and tries again and again according to his own design experience, which costs a lot of time and manpower, The parameter scheme determined by this method is often limited by the designer's experience level. It is only a feasible scheme meeting the basic requirements, not the best design scheme. The reliability optimization design method applies the reliability theory and optimization technology, which can not only meet the reliability of the product in operation, but also optimize the weight, volume, working performance, manufacturing cost and other functional parameters of the product, with significant technical and economic benefits [3]. As an internationally recognized excellent technology application software,
MATLAB can provide optimization solutions for a variety of problems, greatly shorten the product design cycle and improve the design efficiency [4].

In this paper, based on the analysis of the system parameter conditions for the cutter planetary gear mechanism to achieve reciprocating linear motion, the reliability optimization design method is applied to the design of the cutter planetary gear transmission mechanism. The minimum volume, the highest efficiency and the maximum coincidence degree are taken as the design objective functions, and the linear boundary conditions, the maximum contact angle and the maximum contact angle of the planetary transmission are taken as the design objective functions. The mathematical model of multi-objective reliability optimization design, which is different from the existing models and more in line with the engineering practice, is established and solved by MATLAB software.

2. Materials and Methods

2.1 Establishment of mathematical model

2.1.1 Known parameters. Figure 1 shows the structure diagram of the cutter planetary gear transmission system. Li[1] states that, in order to make the cutter realize reciprocating linear motion and meet the ideal harmonic motion function, the system size parameters should meet the requirements that the crank length and the radius of the planetary gear are half of the radius of the fixed ring gear. In this paper, the optimal design is carried out with the objective of minimum volume, maximum transmission efficiency and maximum coincidence. It can be seen from the literature 1 that the labor intensity of the combine cutter transmission mechanism is high in the harvest season, the working time is not less than 16-24h, the input power is 4.5kw, and the input speed is 459r / min.

![Fig1 Planetary transmission cutter mechanism](image)

In the cutter transmission mechanism, the planetary gear and fixed ring gear work in high-speed environment, which requires the parts to have high fatigue strength, high surface hardness, good wear resistance, and good toughness in the center [5]. After multiple consideration, low carbon alloy steel 20CrMnTi can be selected for planetary gear and fixed gear ring, and carburized and quenched. The hardness is 56-62 HRC, $\sigma_{H\text{lim}}=1500\text{MPa}$, $\sigma_{F\text{lim}}=460\text{MPa}$, and the gear accuracy is grade 8.

2.1.2 Selection of design variables. Cutter planetary gear mechanism is mainly composed of fixed ring gear and planetary gear, and its volume is closely related to the size of fixed ring gear [6]. Therefore, the volume of fixed ring gear root cylinder is taken as the first objective function of optimization design, the expression of the optimal target function is as follows:

$$f_1(x) = \frac{\pi}{4} m^2 (z_2 + 2.5)^2 b$$

Where m is the gear module, Z2 is the number of teeth of the inner ring gear, b is the width of the inner ring gear.

In the efficiency calculation of fixed axis gear mechanism, meshing loss and bearing friction loss are generally considered in foreign countries, but only meshing loss is calculated in China [7]. In order
to improve the transmission efficiency of cutter planetary gear mechanism and reduce the meshing loss, the transmission efficiency is selected as the second objective function:

$$f_2(x) = 1 - \frac{\psi}{1 + \frac{\mu}{i_{31}^H}} = 1 - f\mu\left(\frac{1}{z_1} - \frac{1}{z_2}\right)$$

Where \(z_1\) is the number of planetary gear teeth, \(f\) is the coefficient, which is related to the top height of two teeth, we can take \(f = 2.3\), \(\mu\) is the friction coefficient of meshing contact, 0.06-0.01 is recommended, \(i_{31}^H\) is the transmission ratio between the sun gear and the inner ring gear in the conversion gear train.

Coincidence degree is an important index to measure the bearing capacity and stability of gears. In order to ensure the bearing capacity, continuity and stability of gear transmission, the necessary coincidence degree must be ensured in gear design [8]. The larger the coincidence degree is, the higher the bearing capacity is, and the more stable the transmission is, so the coincidence degree is chosen as the third objective function:

$$f_3(x) = \left[z_1(tg\alpha_{a1} - tg\alpha') - z_2(tg\alpha_{a2} - tg\alpha')\right]$$

$$\alpha_{a1} = \arccos\frac{mz_1\cos\alpha}{m(z_1 + 2)} = \arccos\frac{z_1\cos20^\circ}{z_1 + 2}$$

$$\alpha_{a2} = \arccos\frac{mz_2\cos\alpha}{m(z_2 - 2)} = \arccos\frac{z_2\cos20^\circ}{z_2 - 2}$$

Where \(\alpha_{a1}\) is the pressure angle of planetary gear addendum circle, \(\alpha_{a2}\) is the tip pressure angle of the fixed gear ring. \(\alpha'\) is the meshing angle between planetary gear and fixed ring gear, which is equal to the pressure angle in standard installation, \(\alpha\) is the pressure angle, the standard value is 20 degrees.

Because of the particularity of cutter planetary gear transmission system, the number of planetary gear teeth is half of that of fixed ring gear. Considering the design parameters of three sub objective functions, the optimal design variables can be selected:

$$X = [m, z_1, b]^T = [x_1, x_2, x_3]^T$$

2.1.3 Establish objective function. When the volume of fixed ring gear root cylinder is expressed by design variable, the first sub objective function can be expressed as:

$$f_1(x) = \frac{\pi}{4} x_1^2 (2x_2 + 2.5)^2 x_3$$

When the transmission efficiency is expressed by design variables, the second objective function can be expressed as:

$$f_2(x) = 1 - \frac{0.115}{x_2}$$

When the coincidence degree is expressed as a design variable, the third objective function can be expressed as:

$$f_3(x) = \frac{x_3}{2\pi} (tg\alpha_{a1} - 2tg\alpha_{a2} + 0.364) \quad \alpha_{a1} = \arccos\frac{0.9397x_3}{x_2 + 2} \quad \alpha_{a2} = \arccos\frac{0.9397x_2}{x_2 - 1}$$

The unified objective function is:

$$f(x) = \sum_{i=1}^{n} \left[\frac{f_i(x) - f_i^0}{f_i^0}\right]^2$$

Where \(f_i^0\) is the ideal value of each sub objective function.
2.2 Determine constraints

2.2.1 Linear inequality constraints.
(1) Generally, the module of transmission power gear should be greater than 1.5mm ~ 2mm, and the commonly used module is 2 ~ 6mm, so the limit is:

\[ g_1(X) = 2 - x_1 \leq 0 \quad g_2(X) = 6 - x_1 \geq 0 \]

(2) In order to avoid undercutting, the number of teeth should not be less than 17. At the same time, in order to make the gear structure compact, the number of teeth should not be too many:

\[ g_3(X) = 17 - x_2 \leq 0 \quad g_4(X) = 30 - x_2 \geq 0 \]

(3) Under the action of a certain load, increasing the tooth width can reduce the gear diameter and transmission center distance, making the structure compact. But at the same time, the larger the tooth width is, the more uneven the load distribution is, so:

\[ g_5(X) = 10 - x_3 \leq 0 \quad g_6(X) = 100 - x_3 \geq 0 \]

2.2.2 Nonlinear inequality constraints.
(1) In the design of planetary gear train, in order to ensure the reliability of gear transmission performance, it is necessary to reasonably select the tooth width coefficient and limit the ratio of tooth width and modulus, common tooth width \( b = (5-17) \text{mm} \):

\[ g_7(X) = x_3 - 5x_1 \geq 0 \quad g_8(X) = x_3 - 17x_1 \leq 0 \]

(2) Reliability constraint of tooth surface contact strength: when the contact stress exceeds the contact fatigue limit of tooth material, pitting corrosion can occur on tooth surface, which will increase the transmission vibration, increase the noise, reduce the bearing capacity, and eventually lead to transmission failure. Therefore, the safety factor of actual tooth surface contact fatigue strength should be greater than the minimum allowable safety factor, that is:

\[ S_H = \frac{[\sigma_H]}{\sigma_H} \geq S_{H,\text{lim}} \quad \sigma_H = 3.52Z_E \sqrt{\frac{KT(u \pm 1)}{bd^2u}} \]

Where \( S_H \) is the actual contact safety factor, \( S_{H,\text{lim}} \) is the minimum allowable contact safety factor, \( S_{H,\text{lim}} = 1.3 \), \( Z_E \) is the elastic coefficient of the material, \( Z_{E} = 189.8 \sqrt{MPa} \), \( K \) is the load factor, \( K = 2 \), \( u \) is the tooth ratio, \( u = 2 \), \( \pm \) is used for external engagement and internal engagement respectively.

The constraint function is obtained by substituting the above formula into the relevant data:

\[ g_9(X) = 1.3 - 0.0082x_1x_2\sqrt{x_3} \leq 0 \]

(3) Reliability constraint of gear bending strength: when the stress value exceeds the bending fatigue limit of the material, fatigue cracks will occur at the root of the tooth, which may lead to the fracture of the tooth, which will make the gear unable to work normally. Therefore, the safety factor of actual root bending fatigue strength should be greater than the minimum allowable safety factor, that is:

\[ S_F = \frac{[\sigma_F]}{\sigma_F} \geq S_{F,\text{min}} \quad \sigma_F = \frac{2KT}{bm^2z_i^2}Y_F Y_S \]

Where \( S_F \) is the actual bending safety factor, \( S_{F,\text{min}} \) is the allowable minimum bending safety factor, \( S_{F,\text{min}} = 1.6 \), \( K \) is the load factor, because the load characteristic of gear is large, so \( K = 1.6 \), \( Y_F = 2.97 \), \( Y_S \) is the stress correction factor, \( Y_S = 1.53 \).

The constraint function is obtained by substituting the above formula into the relevant data:

\[ g_{10}(X) = 1.6 - \frac{x_1^2x_2x_3}{330.56} \leq 0 \]

After analysis, it is a three-dimensional nonlinear optimization design problem with 10 independent constraints.
2.3 Using fmincon function to solve the problem

2.3.1 Write the object sub function file gd_f.m

```matlab
function f=jsq_f(x);
a1=acos(0.9397*x(2)/(x(2)+2));
daeta=2.7475*x(1)/x(2);
da2=x(1)*(2*x(2)-2);
a2=acos(1.8794*x(1)*x(2)/(deta+da2));
d1=pi*x(1)^2*(2*(x(2)+2.5)^2*x(3)/4;
d2=-1+0.115/x(2);
d3=x(2)^2*(tan(a1)-2*tan(a2)+0.3640)/(2*pi);
f=((d1-87032)/87032)^2+((d2+0.9962)/0.9962)^2+
((d3+2.1028)/2.1028)^2;
```

2.3.2 Write nonlinear inequality constraint sub function file gd_y.m

```matlab
function [c,ceq]=jsq_y(x);
c(1)=5*x(1)-x(3);
c(2)=x(3)-17*x(1);
c(3)=1.6-x(1)^2*x(2)*x(3)/330.56;
c(4)=1.3-0.0082*x(1)*x(2)*x(3)^(1/2);
ceq=[];
```

2.3.3 Write the main function file of calling sub function gd_fy.m

```matlab
x0=[2;30;10];
lb=[2;17;10];
ub=[6;30;100];
a=zeros(6,3);
a(1,1)=-1;a(2,1)=1;
a(3,2)=-1;a(4,2)=1;
a(5,3)=-1;a(6,3)=1;
b=[-2;6;-17;30;-10;100];
[x,fval]=fmincon(@(jsq_f,x0,a,b,[],[],lb,ub,@jsq_y)
```

3. Results & Discussion

The optimized design results are shown in Table 1. It can be concluded from Table 1 that compared with the conventional design, the transmission efficiency of multi-objective optimization is almost unchanged, while the volume is greatly reduced by 79.97%, and the coincidence degree is slightly increased by 2.64%. Compared with volume single objective optimization, the volume of multi-objective optimization increases by 5.03%, and the coincidence degree increases by 5.41%; Compared with the single objective optimization of transmission efficiency, the transmission efficiency is only reduced by 0.26%, but the volume is reduced by 25.89%, and the coincidence degree is increased by 6.36%; Compared with the single objective optimization of coincidence degree, the transmission efficiency is almost the same, the coincidence degree is slightly reduced by 2.22%, but the volume is greatly reduced by 25.71%.

Therefore, compared with the conventional design, the multi-objective optimization has significant optimization effect. Compared with the single objective optimization, although it does not achieve the best in a single index, the indexes are more balanced, the comprehensive performance of the mechanism is better, and the optimization effect is obvious.
Table 1 Comparison of optimized design and conventional design results

| Programme                             | m  | Z1 | b  | V     | η    | ε       |
|---------------------------------------|----|----|----|-------|------|---------|
| Conventional design                   | 4  | 20 | 20 | 4.5396×10^5 | 99.43 | 2.0032  |
| Volume optimization                   | 2  | 25 | 10 | 86590 | 99.54 | 1.9505  |
| Efficiency optimization               | 2  | 30 | 10 | 1.2272×10^5 | 99.62 | 1.9331  |
| Coincidence degree optimization      | 3  | 17 | 13 | 1.2242×10^5 | 99.32 | 2.1028  |
| Multi objective optimization         | 2.5| 18 | 12.5 | 90950  | 99.36 | 2.0561  |

Note: V= volume (mm³), η= transmission efficiency (%), ε= coincidence degree.

4. Conclusions

By analyzing the structural characteristics and motion parameters of planetary transmission cutter mechanism, a reliability optimization design method is proposed, which takes the minimum volume, the highest transmission efficiency and the maximum coincidence as the optimization design objectives, and can effectively solve the design problems of planetary transmission cutter mechanism. The optimization results show that the established mathematical model of reliability optimization design is more in line with the requirements of cutter design, and the data of multi-objective optimization design is more ideal and balanced than the traditional conventional design and single objective optimization design. It is an effective optimization design method for cutter mechanism design, and the optimization design using fmincon function is simple in programming and reliable in algorithm, It can effectively improve the design efficiency.

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