SYNTHETIC POST–ASYMPTOTIC GIANT BRANCH EVOLUTION: BASIC MODELS AND APPLICATIONS TO DISK POPULATIONS

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ABSTRACT

We explore the realm of post–asymptotic giant branch (post-AGB) stars from a theoretical viewpoint, by constructing a synthetic population of transition objects, proto–planetary nebulae, planetary nebula nuclei (PNNs), and post–planetary nebula objects. We use the Monte Carlo procedure to filter out the populations according to a given set of assumptions. We explore the parameter space by studying the effects of the initial mass function (IMF), the initial mass–final mass relation (IMFMR), the transition time ($t_t$), the envelope mass at the end of the envelope ejection ($M_{\text{eR}}$), the planetary nebula (PN) lifetime $t_{\text{PN}}$, and the hydrogen- and helium-burning phases of the central stars. The results are discussed on the basis of the H-R diagram distributions, the $M_\text{F} - t$ plane, and mass histograms. We found that (1) the dependence of the synthetic populations on the assumed IMF and IMFMR is generally mild; (2) the $M_{\text{eR}}$ indetermination produces very high indeterminations in the $t_t$ and thus in the resulting post-AGB populations; and (3) the synthetic models give a test check for the ratio of He- to H-burning PNNs. In this paper, disk post-AGB populations are considered. Future applications will include Magellanic Cloud PNs and populations of bulges and elliptical galaxies.

Subject headings: Galaxy: stellar content — stars: AGB and post-AGB — stars: evolution — stars: luminosity function, mass function

1. INTRODUCTION

The post–asymptotic giant branch (post-AGB) phase of evolution, when stars leave the red giant region eventually to become white dwarf (WD) remnants, is characterized by a series of intriguing events and diversities. The beginning of the post-AGB phase is conventionally set at the time of cessation of the high mass-loss rate episode $\dot{M} \gtrsim 10^{-5} M_\odot \text{ yr}^{-1}$, which has almost completely stripped the star of its hydrogen-rich envelope. After this superwind (SW) quenching, the central star shrinks in radius at almost constant luminosity, thus getting hotter and hotter. Initially it is highly obscured by the fossil superwind around it, but in $\sim 100$–1000 yr this circumstellar material becomes optically thin and the star again becomes observable also at optical wavelengths. The so-called proto–planetary nebula (proto-PN) phase begins at SW quenching and ceases when the star becomes hot enough ($T_{\text{eff}} \gtrsim 10,000$ K) to start photoionizing the materials that were ejected at relatively low velocity ($\sim 10$–20 km s$^{-1}$) during the superwind phase. As cascade recombination and free-free emission make the nebular material shining in the optical, the object has turned into a planetary nebula (PN). The hot star is now emanating a very fast (several thousand km s$^{-1}$), radiatively driven wind that dynamically interacts with the fossil superwind, occasionally shaping it in extravagant forms. As nebular expansion continues, the luminosity and surface brightness of the nebula drop as a result of its decreasing emission measure and increasing transparency to the ionizing photons. A time inevitably comes when its surface brightness becomes too dim for the nebulosity to be noticed by terrestrial astronomers and the still hot post-AGB star is now slowly evolving toward its final WD configuration.

Occasionally, however, before becoming a WD a final thermal pulse of the still active helium-burning shell injects enough energy into the outer layers to cause a dramatic expansion of the post-AGB star, sending it back toward the red giant region for a while. Depending on the details of this metamorphosis, this residual hydrogen-rich envelope can be engulfed by the underlying convective helium shell, with its hydrogen being then diluted and burned and the energy thus released causing further expansion. A hydrogen-deficient carbon star, a member of a class of objects including RCrB stars, is thus formed, but even when the ingestion of the hydrogen envelope does not take place, modest wind mass loss suffices to remove virtually all this residual envelope, thus exposing a bare core with a helium, carbon, and oxygen atmosphere emanating a Wolf-Rayet–like spectrum. Therefore, depending on whether or not post-AGB stars miss experiencing such a final thermal pulse, stars enter the WD stage with or without a hydrogen-dominated atmosphere, a condition that certainly has to do with the observed dichotomy of WDs into the DA and non-DA groups (respectively with and without hydrogen). The lifetimes of AGB and post-AGB stars have been extensively reviewed by Iben & Renzini (1983, hereafter IR) and Iben (1993, 1995).

Besides its interest per se, the post-AGB phase in stellar evolution also has a number of interesting connections to other important astrophysical issues. For example, the PN composition provides an essential tool for studying the previous nucleosynthesis that took place during the AGB phase and before, thus offering important input for modeling the chemical evolution of galaxies. Moreover, PNs are

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used as tracers of stellar populations, e.g., for determining the stellar death rates of galaxies, and are being used as standard candles in extragalactic distance determinations. Finally, after neutron stars, post-AGB stars are the hottest objects a stellar population can generate, with the most massive of them coming close to $10^6$ K, though for a short while. Their contribution to the UV and soft X-ray part of the spectrum is therefore of potential importance, especially in old populations.

The broad scenario sketched above for the post-AGB evolution has now gathered fairly general consensus, at least for most of its aspects. This picture has gradually emerged thanks to an extended set of stellar evolutionary calculations, starting with the pioneering work by Paczynski (1971) and continuing with a series of other relevant studies (e.g., Härm & Schwarzschild 1975; Schönberner 1979, 1983; Iben et al. 1983; Iben 1984, 1985, 1987; Iben & Tutukov 1984; Iben & MacDonald 1985, 1986; Wood & Faulkner 1986; Vassiliadis & Wood 1994; Blöcker 1995). Still on the theoretical side, several of the evolutionary transformations mentioned above were first sketched in a scattered series of papers (Renzini 1979, 1981a, 1981b, 1982, 1983, 1989, 1990, hereafter P1–P7), along with a number of conceptual tools that may be useful for their understanding. This paper is now an attempt at presenting these ideas in an orderly and systematic form. To do so we have proceeded with the construction of simulated post-AGB populations, as this technique (as shown by Shaw 1989) offers an opportunity to see in a rather straightforward way what the observable effects of the various theoretical ingredients are and therefore to estimate the relative uncertainties when, proceeding in the opposite direction, one attempts to use evolutionary theory to infer astrophysical quantities from the observational data.

The paper is organized as follows. In § 2 we describe our procedure for the construction of post-AGB synthetic populations, in a way that makes them easily reproducible. We therein also discuss the role of the parameters that are at variation in our population synthesis. In § 3 we present our results: synthetic post-AGB populations are plotted within H-R diagrams and radius–age ($M_V$–PN radius) distributions; mass distributions are also shown. Our conclusions are summarized in § 4, where we also draft future applications. In this paper we deal with disk populations with solar composition. Future work will extend the parameter space to other populations as well.

2. SYNTHETIC POST-AGB POPULATIONS: INGREDIENTS

2.1. Interpolation of the Evolutionary Tracks

The stellar models used in this paper are interpolations of the hydrogen-burning evolutionary tracks of Vassiliadis & Wood (1994, hereafter VW), calculated for post-AGB masses between 0.569 and 0.9 $M_\odot$ with solar composition (from here onward, $M$ indicates the post-AGB stellar mass). To extend the baseline of our population synthesis, we extrapolate to lower and higher masses by using, as templates, the tracks for $M \approx 0.546$ and 1.2 $M_\odot$, respectively, from Schönberner (1983) and Paczynski (1971). We choose the VW database for its homogeneity and wide mass range. The VW data set includes also some lower metallicity tracks, which will be used in a future paper for simulating the Magellanic Cloud post-AGB populations.

In Figures 1 and 2 we plot effective temperature and luminosity versus time, as read directly from the VW tracks. These tracks do not support straightforward interpolation since they cross one another at several points. We divide each track into parts roughly representing physical phases, and then we interpolate the appropriate normalized functions for each phase.

The effective temperature–age curves have been interpolated as follows. The six log $T_{\text{eff}}$–log $t$ tracks have been

3 In the VW database, all log $T_{\text{eff}}$–log $t$ evolutionary tracks start at $t = 0$, corresponding to log $T_{\text{eff}} = 10,000$ K. The curves in Fig. 1 start from the second evolutionary data point for clarity.
divided into four parts, which hereafter are called (1) H-burning phase, (2) quenching phase, (3) cooling phase, and (4) white dwarf phase. In Table 1 we list the characteristics of the VW models. Column (1) gives the phase, column (2) the mass of the evolutionary models, and column (3) the characteristic time of each phase and mass. Note that each phase starts at the end of the previous one and that the H-burning phase starts at \( t = 0 \). Table 1 also indicates the values of \( \log T_{\text{eff}} \) at the end of each phase, in columns (4) and (5). From here on, the capital letters H, Q, C, and W will flag the physical variables at the ends, respectively, of the hydrogen-burning, quenching, cooling, and white dwarf phases [e.g., \( t_{\text{H}}(0.569M_\odot) = 3.215 \times 10^4 \) yr]. For each phase, which corresponds to a well-defined age interval, we have studied the best way to normalize the \( \log T_{\text{eff}} - \log t \) function to eliminate the crossing of the tracks of contiguous masses.

In Figure 3 we show the temperature evolution in the hydrogen-burning phase. Here \( T_{\text{eff}} \) has been plotted as a function of \( t_{\text{nn}} \). In Figure 4 we show the temperature evolution in the quenching phase. In Figure 5 we show the temperature evolution in the cooling phase.

| Phase       | \( M \) (\( M_\odot \)) | \( t_{\text{end}} \) (yr) | \( \log T_{\text{eff,end}} \) (K) | \( \log L_{\text{end}} \) (\( L_\odot \)) |
|-------------|-----------------|------------------|----------------|----------------|
| H burning   | 0.569 \( \times 10^4 \) | 4.907            | 1.47           |
|             | 0.597 \( \times 10^4 \) | 5.153            | 3.19           |
|             | 0.633 \( \times 10^4 \) | 5.238            | 3.348          |
|             | 0.677 \( \times 10^4 \) | 5.292            | 3.474          |
|             | 0.754 \( \times 10^4 \) | 5.39             | 3.699          |
|             | 0.9 \( \times 10^4 \)  | 5.581            | 4.02           |
| Quenching   | 0.569 \( \times 10^4 \) | 5.038            | 2.579          |
|             | 0.597 \( \times 10^4 \) | 5.071            | 2.458          |
|             | 0.633 \( \times 10^4 \) | 5.112            | 2.424          |
|             | 0.677 \( \times 10^4 \) | 5.162            | 2.527          |
|             | 0.754 \( \times 10^4 \) | 5.315            | 3.033          |
|             | 0.9 \( \times 10^4 \)  | 5.403            | 2.953          |
| Cooling     | 0.569 \( \times 10^5 \) | 4.907            | 1.47           |
|             | 0.597 \( \times 10^5 \) | 4.906            | 1.203          |
|             | 0.633 \( \times 10^5 \) | 4.911            | 1.125          |
|             | 0.677 \( \times 10^5 \) | 4.91             | 0.919          |
|             | 0.754 \( \times 10^5 \) | 4.952            | 0.855          |
|             | 0.9 \( \times 10^5 \)  | 5.043            | 0.855          |
| White dwarf | 0.569 \( \times 10^6 \) | 4.659            | 0.112          |
|             | 0.597 \( \times 10^6 \) | 4.62             | -0.127         |
|             | 0.633 \( \times 10^6 \) | 4.596            | -0.299         |
|             | 0.677 \( \times 10^6 \) | 4.583            | -0.411         |
|             | 0.754 \( \times 10^6 \) | 4.547            | -0.683         |
|             | 0.9 \( \times 10^7 \)  | 4.545            | -0.877         |

It is understood that all temperatures are in kelvins, all masses in \( M_\odot \), the stellar luminosities in \( L_\odot \), and the ages in years, unless otherwise noted.
against the normalized age

\[ t_{nn} = t/t_H \]  

the dots represent the normalized evolutionary data points. The interpolation of the normalized function is straightforward in this phase. Similarly, Figures 4, 5, and 6 show, respectively, the log \( T_{\text{eff}} \) curves for the quenching, cooling, and white dwarf phases. In the quenching phase (Fig. 4), it is possible to interpolate directly log \( T_{\text{eff}} \) versus \( t_{nn} \). The normalized age in the quenching phase is

\[ t_{nn} = (t - t_H)/(t_Q - t_H) . \]  

In the cooling phase (Fig. 5), we use the interpolating function

\[ T_{nn} = \log T_{\text{eff}}/\log T_{\text{eff},Q} \]  

with normalized age

\[ t_{nn} = (t - t_Q)/(t_C - t_Q) . \]  

In the last branch of the evolutionary track, the white dwarf phase (Fig. 6), we normalize the effective temperature as

\[ T_{nn} = \log T_{\text{eff}} - \log T_{\text{eff},C} , \]

where the normalized age assumes the values of

\[ t_{nn} = (t - t_C)/(t_W - t_C) \]  

(note that \( T_{nn} \) of eqs. [3] and [5] do not correspond to an actual model temperature).

Let us explore the luminosity tracks, for which we use a similar interpolation approach. The luminosities against normalized ages are plotted, for phase H, in Figure 7. The luminosity interpolation of this phase is evidently straightforward. The quenching phase is very well interpolated by a straight line, calibrated for each mass as

\[ \log L_{nn} = A + B[(t - t_H)/(t_Q - t_H)] , \]

with \( A = 0.041 \) and \( B = 1.02 \). The cooling phase normalized luminosity is plotted in Figure 8 for tracks corresponding to masses between 0.569 and 0.754 \( M_\odot \). For larger masses, a better interpolation is achieved by using the power function, with exponent \( \alpha = 0.23 \) for 0.754 \( M_\odot \leq M \leq 0.8 M_\odot \) and \( \alpha = 0.26 \) for \( M \geq 0.8 M_\odot \). Figure 9 shows the white dwarf phase of the luminosity (vs. time) curves, where log \( L \) has been plotted against the normalized time,

\[ t_{nn} = (t - t_C)/(t_W - t_C) . \]

Even in this case, the interpolation is straightforward. After breaking the log \( T_{\text{eff}} \) and log \( L \) tracks into phases and constructing the normalized functions as described
above, our procedure includes the following steps, for each given mass and age. (1) We identify the mass interval to be used. For example, if $M = 0.8 M_\odot$, we will use the two enclosing tracks for the interpolation, corresponding to 0.754 and 0.9 $M_\odot$. (2) We identify the phase of the evolution to consider, given the age and mass. To do so, we evaluate $t_H$, $t_Q$, $t_C$, and $t_W$ for the given mass. Let us suppose, in our example, that $t = 400$ yr. We obtain, respectively, 775, 859, and 29,689 yr for the H, Q, and C characteristic times. This means that at $t = 400$ our 0.8 $M_\odot$ star is in the H-burning phase. (3) We calculate the normalized time, $t_{nn}$. In our example, $t_{nn} = 0.52$. (4) On the appropriate tracks (in our example, Figs. 3 and 7) we read off and log $T_{eff}$ values from the two upper tracks, the four points immediately before and after the vertical line. (5) We interpolate, on the higher and lower mass tracks, and find the log $T_{eff}$ and log $L$ that intersect the $x = t_{nn}$ vertical line; and (6) we interpolate vertically versus log $M$, where $M$ is the given mass.

When evaluated on the masses of the original VW evolutionary tracks, the synthetic temperature and luminosity curves are indistinguishable from those plotted in Figures 1 and 2. We have studied the interpolation offsets in detail. The relative errors in the linear temperatures, $(T_{eff, syn} - T_{eff})$,
- $T_{\text{eff},\text{VW}}/T_{\text{eff},\text{VW}}$ result in being collimated between $-0.005$ and 0.005 in most evolutionary phases, with the exception of the very fast quenching phase, where the linear errors on the temperature are rather between $-0.02$ and 0.02 for the lower five mass tracks and between $-0.02$ and 0.1 for the 0.9 track. The relative errors in the luminosities, $(L_{\text{syn}} - L_{\text{VW}})/L_{\text{VW}}$, are between $-0.01$ and 0.01 in most evolutionary phases, with the exception of the quenching phase, where the linear errors on the luminosity are between $-0.15$ and 0.15 for the lower five mass tracks and between $-0.15$ and 0.7 for the 0.9 track.

When using the interpolated values in lieu of the evolutionary ones, we obtain a perfect substitution. In the quenching phase, the substitution would displace the temperatures by a marginal fraction, and the misplacement of the luminosity curves would propagate in only a few percent error on log $L$. In all other cases, the substitution results in virtually no errors. We conclude that the interpolations are excellent and can be reliably used as evolutionary tracks for all stellar masses between 0.569 and 0.9 $M_\odot$.

We obtain extrapolations to higher and lower masses using the $T_{\text{eff}}$ templates of 0.546 and 1.2 $M_\odot$ from Schönberner (1983) and Paczynski (1971). Only the shape of the $T_{\text{eff}}$–log $L$ relation is used here, not the actual evolution of the physical parameters, which are instead extrapolated directly from the 0.596 and 0.9 $M_\odot$ tracks by VW, for homogeneity.

### 2.2. The Initial Mass Function

The synthetic distribution of post-AGB stars (e.g., on the H-R diagram) depends on the adopted IMF and history of star formation. Note that a given distribution of post-AGB stars obtained with a certain IMF and star formation (SF) history can also be reproduced with a different IMF provided a properly tuned SF history is adopted. We have renounced playing with two independent parameters, such as the slope of the IMF and the $e$-folding time of the SF rate, and have exclusively explored the effect of changing the slope of the actual initial mass distribution $\psi(M) \propto M^{-x}$. Therefore, $x$ is effectively the slope of the IMF only if a constant star formation rate is assumed. This may not be a bad approximation for the Galactic disk (Scalo 1998). In our simulations it is supposed that only stars in the mass range $0.85 M_\odot < M < 9 M_\odot$ experience the thermal pulses on the AGB (TP-AGB) and post-AGB phases. The upper limit of the mass domain has been chosen in the light of the carbon ignition limits described in Iben (1995). We include in our investigation the classic IMF of Salpeter (1955), with constant index for each mass range. Furthermore, we inspect the effects of an IMF with variable
The Initial Mass–Final Mass Relation

The initial mass–final mass relation (IMFMR) may have a major impact on the resulting synthetic distributions. To illustrate the case we have constructed simulations adopting four different IMFMRs, either from theoretical calculations or from observations: (1) the old theoretical Renzini & Voli (1981) relation as analytically approximated by IR;\(^6\) (2) a more recent theoretical IMFMR introduced by Ciotti et al. (1991), which is an attempt at incorporating new important results of evolutionary calculations; (3) the classic empirical IMFMR proposed by Weidemann (1987) on the basis of WD masses observed in open clusters; and (4) a new empirical relation presented by Herwig (1995) that includes new observations of cluster white dwarfs in the Pleiades, the Hyades, and NGC 3451. Figure 10 shows the four IMFMRs (note that in this paper we use \(M = M_f\)).

A brief justification for the most recent theoretical relation (Ciotti et al. 1991) is appropriate. The old theoretical IMFMR (IR) was obtained by interpolating on an insufficient grid of stellar models and assumed a universal core mass–luminosity relation. Lattanzio (1989) pointed out that the straight-line approximation for the core mass at the first AGB thermal pulse (as a function of \(M_f\) and for \(M_i < 3 M_\odot\)) was much steeper than that resulting from actual evolutionary calculations. Furthermore, Blöcker & Schönberner (1991) have found a major breakdown of the core mass–luminosity relation (a key ingredient in theoretical IMFMRs) for those stellar models in which the so-called envelope-burning process is activated, i.e., for \(M_i \gtrsim 3 M_\odot\) (cf. Renzini & Voli 1981). In practice the former effect results in a flattening of the IMFMR below \(~ 3 M_\odot\) the second one in a flattening of the IMFMR above \(~ 3 M_\odot\).

The new theoretical IMFMR incorporates these findings while keeping the same parameterization of mass-loss processes (wind and superwind) as adopted by Renzini & Voli, with \(\eta = 0.5\) and \(b = 1\).

On the empirical side, the new relation by Herwig (1995) is not very different from the new theoretical relation. In addition, we should mention that new data points by Jeffries (1997) of white dwarfs in NGC 2516 also lie very close to Herwig’s relation. Furthermore, it should be noted that more recent calculations presented in conferences (e.g., Lattanzio & Forestini 1999; Blöcker 1999) show that the envelope burning occurs for \(M > 4 M_\odot\), making the theoretical IMFMR hardly distinguishable from that of Herwig (1995).

2.4. The Transition Time and the Post-AGB Envelope Mass

At the TP-AGB phase, the red giant envelope is ejected by the superwind. The subsequent evolution of the star to planetary nebula nucleus (PNN) depends in a substantial way on the amount of envelope mass left on the star at the SW quenching, \(M^*\). No matter how the transition between the AGB and the PN illumination occurs, the remnant envelope mass at the end of the SW plays a central role. This is just unfortunate when trying to simulate this evolutionary phase: in fact, \(M^*\) is not defined by stellar evolution (given the hydrodynamical nature of the superwind), and only approximations or guesses can be made about its entity and its possible relation to the physical parameters. \(M^*\) can indeed be considered a free parameter, in the sense that there is no theoretical nor observational constraint to fit it nor to give it an exact dependence on any nebular or stellar parameter.

In principle, the higher the core mass, the higher the stellar luminosity and the lower the envelope mass left on the AGB star. However, this also depends on the thermal pulses that occur at the TP-AGB, and it is hard to know from first principles how the superwind ejection changes the stellar structure. We examine later in this section how we choose \(M^*\) for our simulation.

To understand the transition between the AGB and the PN phases, we should introduce the timescales involved in the different phases. Immediately after the superwind quenching, the mass loss due to stellar wind dominates, until the star detaches itself from the AGB. The timescale at which this phase occurs is the wind timescale and can be written as

\[
t_w = \frac{\Delta M_e}{M},
\]

where \(\Delta M_e\) is the difference between the residual envelope mass at the superwind quenching, \(M^*_e\), and the envelope mass at the detachment from the AGB (i.e., at a later phase), \(M^*_D\). The wind mass-loss rate (MLR) is taken from Reimers (1975), evaluated at the AGB temperature of 5000 K. Reimers’ approximation seems reasonable in the considered evolutionary phase, that is, after the superwind quenching and before the onset of the radiation-driven wind (see Blöcker 1995). In effect, the MLR right after the detachment from the AGB declines as the stellar temperature increases (Blöcker 1995); however, in the present application, i.e., to evaluate the transition time, this choice does not affect the results.

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\(^6\) In this paper we flag this IMFMR with “IR.”
For larger temperatures, in the blueward runaway, the mass consumption occurs at a nuclear timescale, following the equation
\[ t_n = \frac{X_e \Delta M_e E_{H}}{L} . \] (10)

\( \Delta M_e \) is now the difference between the smaller of \( M_e^R \) and \( M_e^D \) and the amount of envelope mass remaining at the illumination of the nebula, \( M_e^N \); the luminosity in the equation is the plateau luminosity, \( L^* \) and the variables \( X_e \) and \( E_{H} \) are the envelope hydrogen abundance and the energy released by the nuclear burning of 1 g of hydrogen, respectively.

Finally, the transition time will scale with the shorter among the wind and the nuclear timescales. We set the reciprocal of the transition time to be the sum of the reciprocals of the wind and the nuclear times.

The transition occurs on a thermal timescale in the case in which \( M_e^R \leq M_e^N \), thus in the case in which either the nuclear or the wind time is zero. The formula to be used then is
\[ t_{th} = \frac{G M e^R}{L R} , \] (11)
where \( L \) and \( R \) are the stellar luminosity and radius (see also P6).

It is worth recalling that the synthetic tracks obtained in § 2.1 have their zero-age points at log \( T_{eff} = 4.0 \), which are approximately reached when the transition has been completed. At 10,000 K, the star is able to ionize hydrogen, although the complete nebular transparency at the optical wavelengths occurs generally at slightly higher temperatures (Käufl, Renzini, & Stanghellini 1993). In all events, we use the synthetic tracks from their natural zero point.

Evolutionary models for post-AGB stars predict the amount of mass available in the stellar envelope as a function of the effective temperature (Schönberner 1983; Paczyński 1971). We use Schönberner’s models (\( M < 0.65 \)) to obtain \( M_e^D \) (envelope mass at the detachment) and \( M_e^N \) (envelope mass at nebular illumination, when \( T_{eff} = 10,000 \) K) as functions of \( M \), by linear interpolation. For \( M > 0.65 \) \( M_\odot \), we obtain \( M_e^D \) and \( M_e^N \) by scaling Paczyński’s 1.2 \( M_\odot \) model to Schönberner’s values.

As discussed at the beginning of this section, \( M_e^R \) is quite unconstrained, and its value affords unpredictable variations. We set ourselves to explore a wide parameter space with three different assumptions for \( M_e^R \).

First, since the evolutionary models have shown that the envelope mass at several evolutionary phases in the post-AGB is inversely proportional to the core mass, we could envision the possibility that also at the superwind quenching \( M_e^R \) has an inverse dependence on \( M \). We parameterize this case following the usual models as guidelines, by taking \( M_e^R \) as the envelope mass at the onset of the post-AGB models. Figure 11 illustrates how the characteristic timescales vary if \( M_e^R = f(M) \). The transition time peaks for the very small mass models, and then it declines for larger masses.

Second, we chose a constant \( M_e^R \), independent of mass. In Figures 12 and 13 we analyze the consequences of assuming two values of \( M_e^R \), \( M_e^R = 3 \times 10^{-4} \) and \( M_e^R = 5 \times 10^{-3} \). In the first case (Fig. 12), the transition time follows the thermal timescale closely then the nuclear timescale for \( m > 0.85 M_\odot \), and declines for larger masses. In the second case (Fig. 13), the transition time rises to almost 6000 yr for \( m \sim 0.55 M_\odot \). These two cases with constant \( M_e^R \) are illustrated only for the sake of showing extreme cases, but it is very unrealistic that all stars end the unstable SW phase with exactly equal \( M_e^R \).

Third, we use random values of \( M_e^R \) for our calculations. In Figure 14 we show the transition, wind, and nuclear timescales for random \( M_e^R \), with maximum equal to \( M_e^R = 5 \times 10^{-3} \). The transition time peaks at about 0.6 \( M_\odot \), then declines. The same simulation with different maximum \( M_e^R \) gives similar results, except for the vertical scale. Naturally, given the randomness of the extraction, the population plotted in Figure 14 is one of the infinite possible extractions with random \( M_e^R \) less than \( 5 \times 10^{-3} M_\odot \).

2.5. The Final Helium-Shell Flash

During the post-AGB phase hydrogen is burned in a shell, and therefore the mass of the helium buffer zone between the C-O core and the hydrogen shell keeps increasing. In some cases the increase of the buffer mass can lead to a last thermal pulse (also called flash) of the helium-burning shell (Schönberner 1979). This happens if the star left the AGB with a sufficiently massive buffer zone, so that its further increase during the post-AGB allows it to reach the critical value for a last flash to erupt. This chance is therefore related to the phase in the AGB thermal pulse cycles at which the SW envelope ejection takes place. Detailed model calculations show that as a response to the final flash stars undergo an extended loop in the H-R diagram. The model star would expand back to the AGB very rapidly; then it would evolve again as a post-AGB star, powered by helium.
burning; the duration of the He-burning phase is about 3
times longer than the fading time of hydrogen-burning
post-AGB stars (Iben 1984). These calculations also show
that during the power-down phase the rate of luminosity
decline is nearly constant, as opposed to the case of
hydrogen-burning post-AGB stars in which an abrupt drop
follows the plateau phase.

To incorporate the effect of the final helium-shell flash
(FF) we have proceeded as follows: we assume that such
stars return to \((L, T_{\text{eff}}) = (L_{\text{PL}}, 10,000 \text{ K})\) upon a final flash
and their luminosity fades then linearly with time to \(L_F\) in a
time \(3 \times t_F\) (where \(t_F\) is the total fading time for the H-
burning star, see § 2.7.2, and \(L_F\) is the luminosity of the
 corresponding H burner); i.e.,

\[
L = L_{\text{PL}} - \frac{L_{\text{PL}} - L_F}{3 \times t_F} \times t'.
\]

We then locate the star on the \(\log L - \log T_{\text{eff}}\) curves as
if the evolution were 3 times as slow as H-powered evolution.
This provides a fairly good approximation of the
behavior of the models constructed by Iben (1984). The
fraction of stars experiencing a final flash is not strictly
determined by theory. Iben has calculated the probability
that a star ignites helium in various post-AGB phases (Iben
1984, Table 2), obtaining different guesses for FF
occurrence in 10%–21% of all stars in this mass range and
up to about 60% when he includes stars that leave the AGB
burning helium. Our parameterization is comparable to
Iben’s prediction. In fact, if we were to choose that 60% of
all stars (in this mass range) have the chance to experience a
final flash, then about 12% of the PNNs would be in the
He-burning phase as observed in a synthetic H-R diagram.

2.6. The Duration of the Planetary Nebula Phase
Following common wisdom, the duration of the PN
phase is \(\sim 30,000 \text{ yr}\) (e.g., Phillips 1989, and references
therein). This is derived from the size of the largest observed
PNs (\(\sim 0.7 \text{ pc}\)) coupled with a typical nebular expansion
velocity of \(\sim 25 \text{ km s}^{-1}\). In practice this assumes that all
PNs remain visible as such until the expanding nebula has
reached the maximum observed size or, equivalently, that

Fig. 18.—Synthetic post-AGB populations on the H-R diagram. The IMFs are from (a) Salpeter (1955), (b) Miller & Scalo (1979), (c) Kroupa et al. (1991),
and (d) Scalo (1998). Other parameters are as in the basic model, which is panel (a). Symbols and tracks as in Fig. 15.
all PNs evolve in nearly the same way. On the other hand, the mere fact that PNs are produced by precursor stars in such a very extended range of initial masses (0.85 \(M_\odot\) \(\lesssim M_i \lesssim 9 \ M_\odot\)) makes it most unlikely that all PNs have the same lifetime. Simple arguments suggest that the time \(t_{\text{max}}\) after the cessation of the superwind during which a PN remains observable scales as \(t_{\text{max}} \propto M_{\text{PN}}^{2/5} S_{\text{min}}^{-1/5}\), where \(M_{\text{PN}}\) is the nebular mass and \(S_{\text{min}}\) is the minimum surface brightness for a PN to be detected (P2, P5). In the adopted mass-loss parameterization, \(M_{\text{ejected}}\) corresponds to the mass ejected during the AGB superwind phase and ranges from \(\sim 0.02 \ M_\odot\) for \(M_i = 0.85 \ M_\odot\) to over \(1 \ M_\odot\) for \(M_i = 8 \ M_\odot\). This factor of \(\sim 50\) in PN mass therefore translates into at least a factor \(\sim 5\) in \(t_{\text{max}}\). To obtain the actual duration of the PN phase we must subtract from \(t_{\text{max}}\) the AGB to PN transition time, during which the object is a proto-PN; i.e., \(t_{\text{PN}} = t_{\text{max}} - t_{\text{tr}}\). If \(t_{\text{tr}} > t_{\text{max}}\), then when the central star has reached 10,000 K the nebular material has already dispersed, no observable PN is produced, and one has a so-called lazy post-AGB remnant (P2). To explore the effects of this subtle interplay between central star and (partially decoupled) nebular evolution, we have explored the case of mass-dependent \(t_{\text{max}}\), scaled with \(M_{\text{PN}}\). The nebular mass was derived by imposing its dependence on the plateau luminosity since it is at the reach of a critical luminosity that the shell is ejected. The parameterization of \(M_{\text{PN}}\) and \(t_{\text{max}}\) are described in the Appendix (eqs. [A1] and [A2]). Simulations of post-AGB populations with constant maximum PN ages will be also shown in this paper.

2.7. The Monte Carlo Procedure

Our population synthesis code starts with the option for a time-limited or luminosity-limited sample. The former option is to explore post-AGB populations within a fixed time range (we are talking here about the stellar evolutionary time, not the duration of the PN phase, described in § 2.6). This option is used to deduce the mass distribution of the PNN as derived from the synthetic population analysis.
The latter option, the luminosity-limited sample, is used directly to compare synthetic and observed diagrams. All stars in the synthetic population would have \( L \gtrsim L_F \) (we assume \( L_F = 1.0 \, L_\odot \)).

To build the synthetic population, we proceed as follows.

2.7.1. Time-limited Sample

1. First, a value of the initial mass \( M_i \) is extracted, following the distribution

\[
\psi(M_i) \propto M_i^{-(1+x)}. \tag{13}
\]

2. The extracted value of \( M_i \) is entered into the initial mass–final mass relation to get the mass \( M \) of the post-AGB object.

3. To proceed, we need to evaluate \( M_F^e \) (see § 2.4), and correspondingly the wind timescale can be calculated (see eq. [9]).

4. A random time is then extracted, within a chosen range \( 0 \leq t \leq t_{\text{lim}} \).

5. If the extracted time \( t \) is larger than the wind timescale, we set \( t = t - t_w \), and this value of \( t \) is entered into the routines of § 2.1 thus getting the corresponding location in the H-R diagram. If \( t \) is smaller than \( t_w \), the star is still in the wind phase and we assign \( T_{\text{eff}} = 3.7 \) and \( L = L_{\text{PL}} \). We call the resulting simulated star a wind object.

6. Should we consider the effects of a final helium flash, another value of time \( t_{\text{FF}} \) is extracted, with \( 0 \leq t_{\text{FF}} \leq t_{\text{lim}} \), so

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\( ^8 \) Naturally, the luminosity limit can be varied according to the observed situation we may want to reproduce.

\( ^9 \) Given our assumption on the MLR, we distinguish between wind objects and proto-PNNs (in [9], below); wind objects are those transition objects whose evolution is wind dominated; proto-PNNs are all the other transition objects, located beyond the AGB. The assigned temperature for the wind object is rather arbitrary but homogeneous to our mass-loss choices. Even if we were to locate the wind objects within the transition area, the results would not be greatly affected, given the paucity of the samples. In a future paper, we will use different mass-loss choices to analyze the possible differences. Observationally, it may be hard to distinguish between these two types of objects.
as to get a random value for the time at which the flash takes place.\textsuperscript{10}

7. If $t < t_{\text{FF}}$, then this $t$ is entered into the routines of §2.1 to get the luminosity and temperature of the star (in this case we have extracted a post-AGB star that will experience a final flash, but the flash has not taken place yet). If $t > t_{\text{FF}}$, then the time elapsed after the flash ($t' = t - t_{\text{FF}}$) is entered into equation (12) to get the luminosity of the post-FF star. To calculate the temperature we use the routines of §2.1.

8. The transition time $t_{\text{tr}}$ is obtained following the prescriptions in §2.4.

9. The object is finally classified into one of the following classes and plotted with a different symbol for each class. The object is classified as a proto–planetary nebula nucleus (proto-PNN) if $t < t_{\text{tr}}$. It is classified as a PNN if $t + t_{\text{tr}} < t_{\text{max}}$ or $t' + t_{\text{FF}} + t_{\text{tr}} < t_{\text{max}}$ with the further distinction between objects that have experienced a final flash (PNNHe) and those that have not (PNNH). It is classified as a post-PNN object otherwise, again distinguishing post-FF and non–post-FF objects.

2.7.2. Luminosity-limited Sample

To build a luminosity-limited sample, we follow a similar procedure to that in the time-limited sample, except in (4), the extraction of a random time, which is chosen in the range $0 \leq t \leq t_{\text{FF}}(M) + t_{\text{tr}}$, where $t_{\text{FF}}$ is the fading time to the minimum luminosity, $L = 1.0 L_\odot$ (see eq. [A3]).

Then we proceed as in points (5)–(8) of the procedure in §2.7.1, with the difference that $t_{\text{FF}}$ is extracted within the new time limits.

3. RESULTS

Our code is flexible, suitable for producing post-AGB population synthesis for many applications and many sets of parameters. Far from being exhaustive, this paper includes only a small part of the possible applications. We have explored the issues and questions that we thought to

\textsuperscript{10} Note that Iben (1984) has calculated slightly different probabilities for FF occurrence at different phases, but we use a flat probability for simplicity.
be of wide interest and allow us to advance in our post-AGB evolution knowledge. Other applications will be developed in the future.

3.1. Synthetic Diagrams and Mass Histograms: The Basic Model

The first synthetic population shown here is a luminosity-limited sample of post-AGB stars. We extract 1500 objects, and we separate the wind objects, the PNNs, the proto-PNNs, and the post-PNNs following the prescriptions of § 2. In Figure 15 we show the synthetic H-R diagram, where 313 objects of the 1500 extracted are in the PNN phase, 21 are in the wind phase, and nine are proto-PNNs. The remaining objects are, following our prescription, already in the post-PNN phase.

In the simulation of Figure 15 the residual envelope mass, $M_{eR}$, is chosen to be a function of the core mass, as described in § 2.4; we have used Weidemann's IMFMR, Salpeter's IMF, and no FF objects (i.e., all PNN are hydrogen burners). The maximum PN time has been set following equation (A2), with $K = 4 \times 10^5$, a relatively long time for PNs to disappear. We use this time through § 3.5, considering further the effects of the maximum PN age in § 3.6. We define the simulation plotted in Figure 15 as basic, and we will explore the parameter space by changing one parameter at a time with respect to this basic population.

As expected, the synthetic stars cluster toward the lower masses, only the very low luminosity part being populated by higher masses. This is exactly the effect we see in all complete Galactic PNN samples.

For more insight, let us plot the corresponding distribution of the visual magnitudes versus evolutionary timescale (Fig. 16). We translate temperatures and luminosities into visual magnitudes by following the bolometric correction from Code et al. (1976). As already evident in the previous plot, there are two mainly populated loci of the $M_v - t$ plane as well, one corresponding to plateau luminosity of the low-mass objects and the other to the stellar crowding toward low luminosities of the intermediate- and high-mass objects. The large fraction of PNNs that have decayed to...
post-PNNs at late evolutionary times is very evident in this figure. This effect is mass dependent, and smaller mass PNNs disappear earlier in their lives.

In Figure 17 we show the (logarithmic) mass distribution for the basic synthetic population. This distribution has been obtained by running a sample of 1500 post-AGB stars with maximum H-R diagram lifetime of 30,000 yr. The logarithmic scale has been chosen for a better view of the distribution in the whole mass range. The mass distribution of Figure 17 shows the clear clustering of model post-AGB stars around \( M_0 \approx 0.6 \) and a gradual spread to higher masses.

Because of the particular choice of the time interval explored (\( t < 30,000 \) yr), we do not obtain post-PNNs with this time-limited selection. In Table 2 we show the composition of a selection of the synthetic populations shown in this paper. The columns list, respectively, the number of PNNs, wind objects, post-PNNs, and proto-PNN stars for each extraction. The composition of typical populations may vary from one random extraction to another. From Table 2 we can directly compare the time-limited with the luminosity-limited samples.

### 3.2. Effects of the Initial Mass Function

To test the variations of post-AGB distribution for different initial mass functions we ran synthetic populations with the following IMFs (see § 2.2 for the meaning of the variable \( x \) and for SF choice): (1) the standard, the IMF of Salpeter (1955), with \( (1 + x) = 2.35 \) in the whole mass range considered; (2) the IMF of Miller & Scalo (1979), with \( (1 + x) = 1.4 \) for \( M < 1 \) and \( (1 + x) = 2.5 \) for \( M > 1 \); (3) the IMF by Kroupa et al. (1991), with \( (1 + x) = 1.85 \) for \( M < 1 \) (this distribution is not defined for larger masses); and (4) an updated empirical parameterization by Scalo (1998) with \( (1 + x) = 1.2 \) for \( M < 1 \) and \( (1 + x) = 2.7 \) for \( M > 1 \).

In Figure 18 we show H-R distributions accounting for IMF variations, when the IMF is the only parameter changing across the panels. In each case, we have extracted a luminosity-limited sample of 1500 post-AGB stars, with Weidemann's IMF and mass-dependent residual envelope mass. The stars do not experience a final helium shell flash. In the case of Kroupa et al.'s low-mass IMF, we use the IMF of Scalo (1998) for masses larger than solar.
By examining Figure 18\(^\text{11}\) we see very little change on the \(\log L - \log T_{\text{eff}}\) plane for different IMFs in the mass range considered. The PNN populations in the case of variable IMF index (Figs. 18b, 18c, and 18d) are larger for low masses with respect to the basic model. The proto-PNN population changes slightly (we are dealing with low-number statistics, anyway), but overall the distributions are similar in all four cases of Figure 18 (see also Shaw 1989). It would be better to say that the H-R diagram is not the ideal way to pick up these differences, occurring in this case at low luminosities, in a crowded part of the diagram. Observations of real situations such as those of the four panels of Figure 18 would not be distinguishable from one another.

In Figure 19 we illustrate the mass distribution for the synthetic populations corresponding to Figure 18, each panel showing a different IMF. Naturally, time-limited samples have been used. It is worth noting that peaks in the distribution of less than \(\sigma\) can be produced by the randomness of the simulation.

The effects of the IMF are noticeable given the large sample of model stars. For example, Scalo's IMF (Fig. 19d) would produce a broader distribution toward the higher masses. Note that the logarithmic scale partially hides the fact that all these mass distributions are extremely peaked around 0.55 \(M_\odot\).

3.3. Effects of the Initial Mass–Final Mass Relation

As introduced in § 2.3, we will use four different IMF-MRs and test their effects on the post-AGB populations. In Figures 20 and 21 we plot the results relative to the IMF-MRs of (Figs. 20a and 21a) Weidemann, (Figs. 20b and 21b) Ciotti et al., (Figs. 20c and 21c) Herwig, and (Figs. 20d and 21d) IR. Figure 20 shows the distributions on the H-R diagram. On this plane, the sample with IR's IMF-MR stands aside since the IR prescription produces more massive post-AGB.

The differences among the other distributions are more evident in Figure 21, where we plot the visual magnitude versus evolutionary time of the synthetic stars. By compar-

\(^{11}\) In Fig. 18 and after, we use the following conventional nomenclature for figure panels: (a) = lower left, (b) = upper left, (c) = lower right, and (d) = upper right. Panel (a) always shows the basic sample.
Fig. 25.—Effects of a final helium-shell flash during cooling on the H-R diagram. (a) Basic population. (b) Post-FF stars. (c) H-burning or post H-burning stars. (d) Population of post-AGB stars with 20% objects in post-FF phase [i.e., (b) + (c) populations]. All other parameters are as in the basic model. Symbols and tracks as in Fig. 15.

The residual envelope mass

The envelope mass that is left on the star after the envelope ejection, $M_e^R$, plays a fundamental role in the following stellar fate. Our basic model uses $M_e^R = f(M)$, with an ad hoc parameterization (see § 2.4). Since the value of $M_e^R$ is undetermined, as is its distribution with respect to the other physical parameters, we chose to explore scenarios of post-AGB evolution with both variable and constant $M_e^R$. We also produce populations with random residual envelope masses, to represent the extreme (but not unrealistic) indetermination of $M_e^R$ and consequently of the transition time.

In Figure 23 we show the results with the different scenarios: the basic model (Fig. 23a) has mass-dependent $M_e^R$, model B (Fig. 23b) has random $M_e^R$ (with $M_e^R < 0.1 M_\odot$), model C (Fig. 23c) also has random $M_e^R$ (with $M_e^R < 1 \times 10^{-4} M_\odot$), and model D (Fig. 23d) has constant $M_e^R = 5$
The effects of the indetermination of the residual envelope mass on the resulting post-AGB populations are striking. A complete description of mass loss for low- and intermediate-mass stars does not exist to date. Our experiments (e.g., Fig. 23) represent a way to determine the effects of mass dependence on the MLR in observable sets of PNs and related objects. Once again, in this paper we do not compare these effects with the observed stars, we just set the stage for future comparisons. When using $M^R = 5 \times 10^{-3} M_\odot$, the distribution is very similar to the basic case.

We use the random $M^R$ to further analyze the effects of mass loss on observed populations. The distributions that we obtain as a result of the simulations with random $M^R$ are very different depending on the upper limits that we set for $M^R$. For instance, for $M^R < 1 \times 10^{-4} M_\odot$, neither wind nor proto-PN is created. Furthermore, very few high-luminosity PNs are available with this low-limit random $M^R$ (see also Table 2). These effects could be observed in PNs and proto-PNs at known distances. At lower luminosities, the two distributions look similar, especially if we take into account observing errors.

The mass distributions of post-AGB populations with different choices of $M^R$ are shown in Figure 24. The effects of changing $M^R$ are even more extreme in these time-limited samples. Let us examine Figures 24b and 24c (upper-left and lower-right panels). In both panels, $M^R$ has random values, the difference being the maximum allowed $M^R$. In the simulation of Figure 24b, where the maximum residual envelope mass is $0.1 M_\odot$, the distribution is dominated by the objects in the wind phase and the lack of PNs (actually, the simulation produces two PNs, hidden in the logarithmic representation). The most notable characteristics of Figure 24c are the lack of wind objects or proto-PNs. The very low values of $M^R$ make the transition time very short, and, as a consequence, there is a lack of proto-PNs. Studies of a sample including OH/IR and other transition objects and PNs within the same environment should reflect the mean value of $M^R$ in the sample.

In Figure 24d most post-AGB stars belong to the pre-planetary nebula (wind and proto-PNN) phases. Only a few percent of the stars are PNs. The situation of Figure 24d has been produced only to show an extreme case since it is highly unrealistic that all stars have the same residual envelope mass, independent of their initial mass on the main sequence and of their core mass.

### 3.5. The Final Helium-Shell Flash

In this paper we did not use the helium-burning (nor a combination of hydrogen and helium burning) stellar models to determine the effect of a helium-burning PNN population. The parameterization that we use is described in § 2.5, and it agrees with observations. In the following, we show how the presence of helium-burning stars can affect the observable post-AGB populations. In Figure 25 we show the synthetic population on the H-R diagram for a sample of 1500 post-AGB stars in which 20% of the post-AGB stars have experienced a final flash. The fraction of stars in post-FF is chosen in agreement with the guesses based on the evolutionary models by Iben (1984). We compare this population with the basic model. We note, from Table 2, that the overall composition of the sample in terms of ratio of PNs to other components is (statistically) very similar to the basic sample.

In Figure 25 we have separated the He- and H-burning stars. Figure 25a shows the basic model (1500 objects with Salpeter’s IMF, Weidemann’s IMFMR, $M^R = f(M)$, and non–post-FF). In the other panels, we show (Fig. 25b) the post-AGB stars having experienced an FF (20% of the sample); (Fig. 25c) stars that are in H-burning, post-H-burning, and/or will experience FF (the latter group of stars is observationally indistinguishable from H-burning stars);
and (Fig. 25d) the composite population of Figures 25b and 25c, thus the observable post-AGB population. Figure 26 shows the same populations as Figure 25 in the $M_V - t$ plane. The simulated He-burning populations stand out for lack of low-brightness PNNs.

It is not easy to discern observationally among H- and He-burning stars. The problem is that the stellar abundances are not easy to measure in PNNs. In general, H-depleted post-AGB stars are believed to be He burners, but it is not so easy to single out the H-burning stars, as H-rich post-AGB stars could indeed be He burners as well as H burners. The simulation of Figures 25 and 26 can help us in this respect. In fact, the ratio of PNNs brighter than, say, 5 mag in the basic sample is about one-third of the total sample, while in the composite population (Fig. 26c) this ratio goes up to one-half the sample. A homogeneous, complete observed PNN sample should show this kind of discrepancy.

3.6. The Planetary Nebula Lifetime

Throughout this paper we have assumed that the maximum age for a PN depends on the PN mass, as in equation (A2), with constant $K = 4 \times 10^5$. We have also produced other synthetic populations by keeping the nebular mass dependence and changing the constant. If we keep all other parameters as in the basic model, we obtain that the number of PNNs goes to zero as $K$ is lowered to about $5 \times 10^3$. This happens because the shorter lifetime of PNs makes most of the post-AGB population be in a post-PNN phase. In this case, a large number of lazy AGB stars are also produced. In general, lazy AGB stars are produced by running the basic model with $K < 8 \times 10^4$.

If we were to eliminate the nebular mass dependence from $t_{\text{max}}$, the situation would change noticeably. In Figure 27 we show a simulated population with all parameters identical to the basic model, except we have fixed the PN lifetime to 30,000 yr for all nebulae. The result is remarkably different from the basic model (see Fig. 15 and Table 2). Most low-mass post-AGB stars are in the post-PNN phase. If we were to lower the maximum PN lifetime to 10,000 yr (not an unreasonable choice), we would obtain that most PNNs in the diagram have a high mass. This is in contrast with the observations; thus our simulations lead us to believe that the lifetime of PNs depends on the nebular
mass. Detailed comparison with a homogeneous data set should be used to confirm this inclination.

4. SUMMARY AND FUTURE WORK

We have used up-to-date evolutionary tracks as templates to build a code for post-AGB population synthesis. Our models aim at understanding the fine tuning of post-AGB evolution, including the consequences of IMF and IMFMR, the transition time and its correlation with the residual envelope mass, the actual duration of the PN phase, the relative population of proto-PNNs, wind objects, and post-PNN stars, and the occurrence and timing of the FF phase. Our synthetic tracks, available to obtain $\log T_{\text{eff}}$ and $\log L$ for post-AGB stars of any stellar mass in the range $0.85 M_\odot \lesssim M_\star < 9 M_\odot$, are obtained with very high precision in reproducing the actual evolutionary tracks. The interpolation method is simple and fully explained in this paper, so that the readers can reproduce the synthetic populations included here.

In this paper we have shown a sample of the possible applications, without going into detailed comparisons with the observed data. Among the results shown here, we found that (1) the dependence of the synthetic populations on the assumed IMF and IMFMR is mild; the post-AGB populations are not ideal indicators of the IMF. (2) The residual envelope mass, after the envelope ejection, has a strong effect in determining the subsequent post-AGB evolution; its indetermination produces very high indetermination of the transition time and ultimately of the resulting post-AGB populations. (3) The ratio of He- to H-burning PNNs can be reproduced with population synthesis.

The central importance of this paper consists in showing that the many fundamental variables of stellar evolution have a major role in determining post-AGB populations. The variation and indetermination of these parameters should not be overlooked in comparing data and theory.

The theoretical work contained in this paper was implemented with the goal of being versatile and useful for a full host of applications. Among other possible uses of these synthetic tracks are studies of evolutionary effects on the planetary nebula luminosity function (PNLF) and how those translate in the variation of the extragalactic distance scale, as derived from the PNLF. Stanghellini (1995) has shown in a preparatory work that the PNLF is at variation with respect to the transition time. The updated models presented here allow the most detailed study of the effects of the evolutionary parameters on the PNLF.

Future applications also include simulations of bulge, elliptical galaxy, and Magellanic Cloud populations, where a different treatment of the SF/IMF should be used and possibly different stellar chemistry.

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APPENDIX

In order to parameterize the duration of the PN phase, we assume that the nebular mass is $0.02 M_\odot$ for $M = 0.55 M_\odot$ (core mass) and $1.5 M_\odot$ for $M = 1.2 M_\odot$. The planetary nebula mass can be described by the following correlation between nebular mass and plateau luminosity:

$$M_{\text{PN}} = 4.495 \times 10^{-8} L_{\text{PL}}^{6.36} ,$$

and

$$t_{\text{max}} = K M_{\text{PN}}^{5/2} .$$  \hspace{1cm} \text{(A2)}

To calculate the fading time to $L = 1.0 L_\odot$, $t_F$, we use the VW tracks and we interpolate versus the (post-AGB) mass, obtaining

$$t_F(M) = 7.0957 - 2.1441 M .$$  \hspace{1cm} \text{(A3)}
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