New Indirect Bounds on Lorentz Violation in the Photon Sector

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1. INTRODUCTION

There is a unique Lorentz-violating (LV) modification of the Maxwell theory of photons, which maintains gauge invariance, CPT, and renormalizability. Restricting modified-Maxwell theory to the nonbirefringent sector and adding a standard spin–\(\frac{1}{2}\) Dirac particle with minimal coupling to the nonstandard photon, the resulting modified-quantum-electrodynamics model has 9 dimensionless “deformation parameters.” In this talk, new bounds are presented, which improve significantly upon current laboratory bounds.

The basic idea \cite{1,2} behind these bounds is to consider novel types of particle decays (absent in the Lorentz-invariant theory) and to obtain bounds on the LV parameters from the inferred absence of these decays in air-shower events observed in the Earth’s upper atmosphere.

The present write-up follows the talk given at the conference, but one crucial result obtained afterwards is added: a lower bound on the isotropic LV parameter from TeV gamma-rays (Sec. 4.3). Natural units with \(c = \hbar = 1\) are used throughout.

2. THEORY

2.1. LV Photon Model

For the case of a possible Lorentz noninvariance, the following truism holds with all force: it is difficult to discover or bound what is unknown. Hence, the need for simple concrete models. Consider a LV deformation of quantum electrodynamics (QED):

\[
S_{\text{modQED}} = S_{\text{modM}} + S_{\text{standD}},
\]

with a modified-Maxwell term \cite{3,4} and a standard Dirac term for a spin–\(\frac{1}{2}\) particle with charge \(e\) and mass \(M\):

\[
S_{\text{modM}} = \int_{\mathbb{R}^4} d^4x \left( -\frac{1}{4} \left( \eta^{\mu\rho} \eta^{\nu\sigma} + \kappa^{\mu\nu\rho\sigma} \right) \left( \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) \right) \left( \partial_\rho A_\sigma(x) - \partial_\sigma A_\rho(x) \right) \right),
\]

\[
S_{\text{standD}} = \int_{\mathbb{R}^4} d^4x \bar{\psi}(x) \left( \gamma^\mu \left( i \partial_\mu - e A_\mu(x) \right) - M \right) \psi(x).
\]

The above theory is gauge-invariant, CPT–invariant, and power-counting renormalizable.

In the modified-Maxwell action \cite{2a}, \(\kappa^{\mu\nu\rho\sigma}\) is a constant background tensor with 19 independent components. Ten birefringent parameters are already constrained at the \(10^{-32}\) level \cite{4}. Now, set these 10 birefringent parameters to zero, so that 9 nonbirefringent parameters remain (in the notation of Ref. \cite{5}):

- 3 parity-odd nonisotropic parameters collected in an antisymmetric traceless \(3 \times 3\) matrix \((\tilde{\kappa}_{\alpha+})^{mn}\);
- 5 parity-even nonisotropic parameters collected in a symmetric traceless \(3 \times 3\) matrix \((\tilde{\kappa}_{e-})^{mn}\);
- 1 parity-even isotropic parameter \(\tilde{\kappa}_{tr}\).

The current experimental bounds on these 9 parameters will be summarized in Sec. 3.
2.2. LV Particle Decays

The violation of Lorentz invariance in the modified-QED action (1) leads to modified propagation properties of the photon (denoted $\tilde{\gamma}$). The modified photon propagation, in turn, allows for new types of particle decays.

In this article, we consider two such decay processes (Figs. 1a,b), whose occurrence depends on the signs of the LV parameters, possibly in combination with the flight-direction vector of the initial particle:

\[
(a) : \ p^\pm \rightarrow p^\pm \tilde{\gamma}, \quad (b) : \ \tilde{\gamma} \rightarrow p^- p^+,
\]

where $p^\pm$ stands for the electron/positron particle ($e^-/e^+$ in standard notation) from the original vectorlike $U(1)$ gauge theory, that is, pure QED. It is also possible to take the charged particles $p^+/p^-$ in (3) to correspond to a simplified version of the proton/antiproton (namely, a Dirac particle with partonic effects neglected in first approximation). Process (3a) has been called “vacuum Cherenkov radiation” in the literature and process (3b) “photon decay.” See Ref. [10] for a general discussion of LV decay processes, starting from Lorentz-noninvariant scalar models.

For the vacuum-Cherenkov process (3a) in the full nonbirefringent theory (1)–(2), the square of the threshold energy is given by [11, 12]

\[
\left( E_{\text{thresh}}^{(a)} \right)^2 = \frac{M^2}{R \left[ 2 \kappa_{tr} - \epsilon_{ijk} (\tilde{\kappa}_{o+})^{ij} \tilde{q}^k - (\tilde{\kappa}_{e-})^{j,k} \tilde{q}^j \tilde{q}^k \right]} + O(M^2),
\]

for nonbirefringent LV parameters $|\tilde{\kappa}^{\mu\nu}| \ll 1$ and ramp function $R[x] \equiv (x + |x|)/2$.

For the photon-decay process (3b) in the restricted isotropic theory with $\kappa_{tr} < 0$ and $(\tilde{\kappa}_{o+})^{mn} = (\tilde{\kappa}_{e-})^{mn} = 0$, the square of the energy threshold is given by [14]

\[
\left( E_{\text{thresh}}^{(b)} \right)^2 = \frac{1 - \kappa_{tr} \kappa_{tr}}{2 \kappa_{tr}} 2 M^2 = \frac{2 M^2}{-\kappa_{tr}} + 2 M^2,
\]

where the last expression holds for all $\kappa_{tr} \in [-1,0)$ for which the decay rate is well-behaved (see Ref. [14] for details).

3. CURRENT LABORATORY BOUNDS

The current laboratory bounds on the nonbirefringent parameters of modified-Maxwell theory are as follows:

- direct bounds at the $10^{-12}$ level [6] for the three nonisotropic parameters in $\tilde{\kappa}_{o+}$;
- direct bounds at the $10^{-14}$ to $10^{-16}$ levels [6] for the five nonisotropic parameters in $\tilde{\kappa}_{e-}$;
- direct bound at the $10^{-7}$ level [7] for the single isotropic parameter $\kappa_{tr}$;
- indirect bound at the $10^{-8}$ level [8] for $\kappa_{tr}$ from the measured value of the electron anomalous moment;
- indirect bound at the $10^{-11}$ level [9] for $\kappa_{tr}$ from particle colliders (LEP and Tevatron).
4. NEW INDIRECT BOUNDS

4.1. Threshold Conditions

A remarkable suggestion \cite{1,2} has been made for a way to obtain bounds on nonstandard parameters, e.g., the 9 Lorentz-violating parameters from Sec. 2.1 (here, occasionally written as \( \tilde{\kappa} \)). The argument proceeds in three steps:

- if vacuum Cherenkov radiation or photon decay has a threshold energy \( E_{\text{thresh}}(\tilde{\kappa}) \), then UHECRs or TeV gamma-rays with \( E > E_{\text{thresh}} \) cannot travel far, as they rapidly radiate away their energy or simply disappear;
- this implies that, if an UHECR or TeV gamma-ray of energy \( E \) is detected, its energy must be at or below threshold, \( E \leq E_{\text{thresh}}(\tilde{\kappa}) \);
- the last inequality gives, using the thresholds (4ab) for processes (3ab), an upper bound on the LV parameters,

\[
(a) : \quad R \left[ 2 \tilde{\kappa}_{\text{tr}} - \epsilon_{ijk} (\tilde{\kappa}_{o+})^{ij} \hat{q}^k - (\tilde{\kappa}_{e-})^{jk} \hat{q}^j \hat{q}^k \right] \leq M^2/E^2, \quad (b) : \quad -\tilde{\kappa}_{\text{tr}} \leq 2 M^2/E^2, \quad (5)
\]

with the energy \( E \) of the primary, its flight direction \( \hat{q} \), and the mass \( M \) of the Dirac particle involved as input.

These UHECR/gamma-ray bounds on \( \tilde{\kappa} \) depend on energies and flight directions of the charged/neutral primaries at the top of the Earth’s atmosphere. Hence, they are “terrestrial” bounds, rather than “astrophysical” bounds.

4.2. Terrestrial UHECR Bounds

From the absence of the vacuum-Cherenkov process (3b) in 29 selected UHECR events with primary energies above 57 EeV = \( 5.7 \times 10^{19} \) eV (27 events from Auger, 1 event from AGASA, and 1 event from Fly’s Eye), the following two–\( \sigma \) bounds have been obtained \cite{13}:

\[
(ij) \in \{ (23), (31), (12) \} : \quad |(\tilde{\kappa}_{o+})^{ij}| < 2 \times 10^{-18}, \quad (6a)
\]

\[
(kl) \in \{ (11), (12), (13), (22), (23) \} : \quad |(\tilde{\kappa}_{e-})^{kl}| < 4 \times 10^{-18}, \quad (6b)
\]

\[
\tilde{\kappa}_{\text{tr}} < 1.4 \times 10^{-19}, \quad (6c)
\]

for a conservative value \( M = 56 \) GeV in the threshold condition (5b).

4.3. Terrestrial TeV Gamma-Ray Bound

From the absence of photon-decay process (3b) for \( E_\gamma = 30 \) TeV = \( 3.0 \times 10^{13} \) eV gamma-ray photons from a particular supernova remnant observed by the HESS atmospheric Cherenkov telescopes, the following two–\( \sigma \) bound has been obtained \cite{14}:

\[-9 \times 10^{-16} < \tilde{\kappa}_{\text{tr}}, \quad (6d)\]

for an electron mass value \( M = 511 \) keV in the threshold condition (5b). This lower bound obtained from a neutral primary nicely complements the upper bound (6c) obtained from a charged primary.

4.4. Combined Terrestrial and Astrophysical Bound

From previous astrophysical bounds \cite{5} on the 10 birefringent modified-Maxwell-theory parameters at the \( 10^{-32} \) level and the “terrestrial” UHECR/gamma-ray bounds \cite{8} on the 9 nonbirefringent parameters, the following two–\( \sigma \) bound is obtained for each component of the background tensor \( \kappa^{\mu\nu\rho\sigma} \) in the general modified-Maxwell action (2a):

\[
\max_{\{\mu,\nu,\rho,\sigma\}} |\kappa^{\mu\nu\rho\sigma}| < 5 \times 10^{-16}, \quad (7)
\]

where the fact has been used that the largest entry of \( |\kappa^{\mu\nu\rho\sigma}| \) has a value \( (1/2) |\tilde{\kappa}_{\text{tr}}| \) if the other 18 parameters are negligibly small. Remark that the indirect bound (7) holds in a Sun-centered, nonrotating frame of reference.
5. DISCUSSION

Explicit calculations \[15\] of standard photons and standard Dirac particles propagating over simple classical spacetime-foam manifolds reproduce, in the large-wavelength limit, a restricted (isotropic) version of model \(1\)–\(2\):

\[
2 \kappa_{tr} = \left( \frac{\tilde{b}}{\tilde{l}} \right)^4, \quad (\tilde{\kappa}_{0+})^{mn} = (\tilde{\kappa}_{0-})^{mn} = 0, \tag{8}
\]

for randomly orientated “defects” with an effective size \(\tilde{b}\) and an average separation \(\tilde{l}\) (cf. Fig. 2). The heuristics of the result is well understood, as the type of Maxwell solution found for the classical spacetime foam is analogous to the solution from the so-called “Bethe holes” for waveguides \[16\]. In both cases, the standard Maxwell plane wave is modified by the radiation from fictitious multipoles located in the holes or defects. But there is a difference: for Bethe, the holes are in a material conductor, whereas for us, the defects are holes in space itself.

The UHECR bound \[6c\] implies that a single-scale \((\tilde{b} \sim \tilde{l})\) classical spacetime foam is ruled out. This conclusion holds, in fact, for arbitrarily small defect size \(\tilde{b}\), as long as a classical spacetime makes sense. That is, down to distances at which the classical-quantum transition occurs, possibly of order \(l_{\text{Planck}} \equiv \sqrt{\hbar G N / c^3} \approx 1.6 \times 10^{-35}\) m.

This result is really like having a null experiment and there is an analogy with the Michelson–Morley experiment \[17\]: theorists expect novel effects which are not seen by experimentalists.

In turn, this suggests the need for radically new concepts. Then, there was the “relativity of simultaneity” introduced by Einstein \[18\]. Now, for the quantum origin of spacetime, there is . . . (alas, the margin is too narrow!)

References

[1] E.F. Beall, Phys. Rev. D 1, 961 (1970), Sec. III A, Cases 1 and 2.
[2] S.R. Coleman and S.L. Glashow, Phys. Lett. B 405, 249 (1997), arXiv:hep-ph/9703240.
[3] S. Chadha and H.B. Nielsen, Nucl. Phys. B 217, 125 (1983).
[4] D. Colladay and V.A. Kostelecký, Phys. Rev. D 58, 116002 (1998), arXiv:hep-ph/9809521.
[5] V.A. Kostelecký and M. Mewes, Phys. Rev. D 66, 056005 (2002), arXiv:hep-ph/0205211.
[6] H. Müller et al., Phys. Rev. Lett. 99, 050401 (2007), arXiv:0706.2034.
[7] S. Reinhardt et al., Nature Phys. 3, 861 (2007).
[8] C.D. Carone, M. Sher, and M. Vanderhaeghen, Phys. Rev. D 74, 077901 (2006), arXiv:hep-ph/0609150.
[9] M.A. Hohensee, R. Lehner, D.F. Phillips, and R.L. Walsworth, arXiv:0809.3442.
[10] C. Kaufhold and F.R. Klinkhamer, Nucl. Phys. B 734, 1 (2006), arXiv:hep-th/0508074.
[11] B. Altschul, Phys. Rev. Lett. 98, 041603 (2007), arXiv:hep-th/0609030.
[12] C. Kaufhold and F.R. Klinkhamer, Phys. Rev. D 76, 025024 (2007), arXiv:0704.3255.
[13] F.R. Klinkhamer and M. Risse, Phys. Rev. D 77, 117901 (2008), arXiv:0806.4351.
[14] F.R. Klinkhamer and M. Schreck, to appear in Phys. Rev. D, arXiv:0809.3217.
[15] S. Bernadotte and F.R. Klinkhamer, Phys. Rev. D 75, 024028 (2007), arXiv:hep-ph/0610216.
[16] H.A. Bethe, Phys. Rev. 66, 163 (1944).
[17] A.A. Michelson and E.W. Morley, Am. J. Sci. 34, 333 (1887).
[18] A. Einstein, Ann. Phys. (Leipzig) 17, 891 (1905) [reprinted ibid. 14, S1, 194 (2005)].