SU(2) lattice gauge theory at non-zero temperature with fixed holonomy boundary condition
*  
E.-M. Ilgenfritz\textsuperscript{a}, B. Martemyanov\textsuperscript{b}, M. Müller-Preussker\textsuperscript{c}\textsuperscript{†}, A. I. Veselov\textsuperscript{b}
\textsuperscript{a}Research Center for Nuclear Physics, Osaka University, Osaka 567-0047, Japan
\textsuperscript{b}Institute for Theoretical and Experimental Physics, Moscow 117259, Russia
\textsuperscript{c}Humboldt-Universität zu Berlin, Institut für Physik, Invalidenstr. 110, D-10115, Germany

We study SU(2) lattice gauge theory at $T > 0$ in a finite box with fixed holonomy value at the spatial boundary. We search for (approximate) classical solutions of the lattice field equations and find in particular the dissociated calorons recently discussed by van Baal and collaborators.

The quark confinement has not yet found a satisfactory explanation. Several models are under consideration. The dual superconductor scenario views confinement as a dual Meissner effect due to the condensation of Abelian monopoles. An alternative promising approach is based on the center-vortex dominance picture. On the other hand there is the semiclassical approach based on instanton solutions. It provides successful phenomenology for many phenomena in hadron physics. Unfortunately, instanton gas or liquid models fail to explain confinement. The question arises, whether other extended classical objects - e.g. monopoles or dyons - could be suited to describe confinement within a semi-classical approach.

We consider SU(2) lattice gauge theory at finite temperature with periodic boundary conditions characterized additionally by a non-trivial holonomy $P(x)$ at $|x| \to \infty$ is admitted. These solutions differ from the 't Hooft periodic instantons employed for the standard semi-classical approach at finite temperatures \[1\]. The latter solutions have trivial holonomy \textit{i.e.} $P(x) \to 1$ for $|x| \to \infty$.

The most interesting feature of the new calorons is the fact that monopole constituents of an instanton can become explicit as degrees of freedom \[2\]. They carry magnetic charge (in fact, they are BPS monopoles \[3\]) and $1/N_{\text{color}}$ units of topological charge. Being part of classical solutions of the Euclidean field equations, one can hope that the instanton constituents can play an independent role in the semiclassical analysis of $T \neq 0$ Yang-Mills theory (and of full QCD).

Here we present an exploratory study where we have searched for characteristic differences between the two phases as far as semiclassical background fields are concerned. The latter become visible in the result of cooling.

We have fixed during the simulation and under cooling the boundary time-like link variables in order to keep a certain value of $P(x) = P_\infty$ everywhere on the spatial surface of the system while conserving periodicity. In this case, the influence of the respective phase, that we want to describe, is twofold:  
(i) the cooling starts from genuine thermal Monte Carlo gauge field configurations,
generated on a $N^3 \times N_t$ lattice; (ii) the value of the holonomy $P_\infty$ was chosen in accordance with the average of $L$, which is approximately vanishing in the confinement phase and nonvanishing but far from unity in the deconfinement phase at not too high temperatures.

For a lattice of size $16^3 \times 4$ we have chosen $\beta = 2.2$ (confinement, $\langle L \rangle \approx 0.$) and $\beta = 2.4$ (deconfinement, $\langle L \rangle = 0.27$), respectively. We freeze the timelike links $U_{x,\mu=4}$ at the spatial boundary equally to each other such that $(U_{x,\mu=4})^{N_t} = P_\infty$. For the holonomy itself, an ‘Abelian’ form $P_\infty = a_0 + i a_3 \tau_3$ was chosen, with $a_0 = \langle L \rangle$ and $a_3 = \sqrt{1-a_0^2}$ in correspondence with the average Polyakov line. In order to search exclusively for objects with low action the criterion for stopping at some cooling step $n$ was that $S_n < 2 S_{\text{inst}}$, the last change of action $|S_n - S_{n-1}| < 0.01 S_{\text{inst}}$, and $S_n - 2 S_{n-1} + S_{n-2} < 0$ ($S_{\text{inst}}$ denoting the action of a single instanton). For each $\beta$-value we have scanned $O(200)$ configurations obtained by cooling.

As can be seen from Table 1 the cooled sample obtained in the confinement phase has a different composition than that of the deconfinement phase.

| Type of solution | $\beta = 2.2$ | $\beta = 2.4$ |
|------------------|--------------|--------------|
| $DD$             | $0.63 \pm 0.08$ | $0.02 \pm 0.01$ |
| $\overline{DD}$ | $0.27 \pm 0.05$ | $0.78 \pm 0.07$ |
| $\text{CAL}$     | $0.02 \pm 0.01$ | $0$          |
| $M$, $2M$        | $0.01 \pm 0.01$ | $0.07 \pm 0.02$ |
| trivial vacuum   | $0.07 \pm 0.03$ | $0.13 \pm 0.03$ |

In the following let us explain these configurations in some detail.

In the confinement phase clearly dominate ‘dyon-antidyon’ pairs ($DD$) reminiscent of the new caloron solutions. In Figs. 3 we show, projected onto the $x_1-x_2$-plane (i.e. summed over $x_3$, $x_4$ or $x_3$, resp.), the topological charge and the Polyakov line, respectively, of such a ‘dyon’ pair. Notice the opposite sign of the Polyakov line near the maxima of the two bumps of topological charge.

Other selfdual objects, having a rather rotationally invariant distribution of action and topological charge, are frozen out relatively infrequently. They resemble the ’t Hooft periodic instanton. We call them caloron ($\text{CAL}$). Under the specific boundary conditions, however, the Polyakov line distribution around the caloron exhibits opposite peaks. Thus, this type of configurations appears to be a limiting case of the ‘dyon-antidyon’ pairs.

Both $DD$ and $\text{CAL}$ objects can be well fitted by the analytically known solutions 8. In Fig. 2 we show measured and fitted action density profiles for a typical $DD$ configuration found in the confinement phase.

The frequency distribution of the scale sizes $\rho$ obtained by these fits is shown in Fig. 3.

Mixed configurations with two lumps of opposite topological charge are found in a quarter of the configurations. We call them ‘dyon-antidyon’ pairs ($\overline{DD}$). In these configurations the Polyakov line has a same-sign maximum on top of the opposite-sign topological charge lumps. Besides of this, the two sums $Q_+ = \sum_x q(x) \Theta(q(x))$ and $Q_- = \sum_x q(x) \Theta(-q(x))$ are almost equal to $+\frac{1}{2}$ and $-\frac{1}{2}$, respectively, which supports an interpretation as half-instanton and half-antiinstanton. With respect to cooling these semi-classical objects are as quasistable as the $DD$ configurations. Therefore, one is tempted to interpret the $\overline{DD}$ pair as a solution of the field equation of motion, too. But so far we do not have a clear understanding of these objects.

It is remarkable that selfdual or antiselfdual $DD$ configurations are very rare in the deconfinement phase. $\overline{DD}$ mixed configurations are typical for this phase.

In the deconfined phase the next important type of cooled configurations are purely magnetic ones ($S_{\text{magnetic}} >> S_{\text{electric}}$) with quantized ac-
Figure 1. The 2d projected distributions of topological charge (a) and Polyakov line (b) for a self-dual DD pair.

Figure 2. Action density profiles $s(x), s(y), s(z)$ of a DD event. Dashed lines correspond to a fit with van Baal’s solution (scale size $\rho \cdot T = 0.63$).

Figure 3. $\rho$ distribution of 94 DD and CAL events, resp.
tion in units of \( S_{\text{inst}}/2 \). We call them \( M \) configurations. With a lower probability also magnetic configurations with twice as large action (2\( M \) type configurations) are found. After fixing the maximally Abelian gauge these configurations turn out to be completely Abelian. We can identify them as pure equally distributed magnetic fluxes related to world-sheets of Dirac strings (‘Dirac sheet’) on the dual lattice. With some rate they also emerge in the result of further cooling of \( D\bar{D} \) configurations.

Concluding we can say that the environment considered with fixed holonomy at spatial boundaries provides an interesting pattern of semiclassical objects characteristic for the confinement as well as for the deconfinement phase. We have no evidence so far, that the finite temperature gauge fields in large volumes can be understood in terms of quantum fluctuations around calorons with non-trivial holonomy. This question is under consideration at present. Anyway, we feel that the development of a semiclassical approach based on solutions with non-trivial holonomy might have a chance to shed more light on the mechanisms of the deconfinement transition.

REFERENCES

1. E.-M. Ilgenfritz, M. Müller-Preussker, and A.I. Veselov, Proceedings NATO Advanced Workshop *Lattice Fermions and Structure of the Vacuum*, Dubna, 1999; e-Print Archive: hep-lat/0003025.

2. For a recent review see P. van Baal, Proceedings NATO Advanced Workshop *Lattice Fermions and Structure of the Vacuum*, Dubna, 1999; e-Print Archive: hep-th/9912035.

3. G. ’t Hooft (1976), unpublished; see R. Jackiw, C. Nohl, and C. Rebbi, Phys. Rev. **D15** (1977) 1642.

4. B.J. Harrington and H.K. Shepard, Phys. Rev. **D17** (1978) 2122; Phys. Rev. **D8** (1978) 2990.

5. D.J. Gross, R.D. Pisarski, and L.G. Yaffe, Rev. Mod. Phys. **53** (1983) 43.

6. M.K. Prasad and C.M. Sommerfield, Phys. Rev. Lett. **35** (1975) 760; E.B. Bogomol’nyi, Sov. J. Nucl. Phys. **24** (1976) 449.

7. T.C. Kraan and P. van Baal, Phys. Lett. **B435** (1998) 389.

8. M. Garcia Perez, A. Gonzalez-Arroyo, A. Montero, and P. van Baal, JHEP **06** (1999) 001; e-Print Archive: hep-lat/9903022.