Coherent phenomena in mesoscopic systems *

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A mesoscopic system of cylindrical geometry made of a metal or a semiconductor is shown to exhibit features of a quantum coherent state. It is shown that magnetostatic interaction can play an important role in mesoscopic systems leading to an ordered ground state. The temperature \( T^* \) below the system exhibits long-range order is determined. The self-consistent mean field approximation of the magnetostatic interaction is performed giving the effective Hamiltonian from which the self-sustaining currents can be obtained.

The relation of quantum coherent state in mesoscopic cylinders to other coherent systems like superconductors is discussed.

I. INTRODUCTION

In the light of recent technological advances in nanostructure fabrication there has been a renewed interest in the properties of quasi-one-dimensional (quasi-1D) and two-dimensional (2D) electron systems. The transport properties of mesoscopic metallic or semiconducting samples have been shown to exhibit features characteristic of the quantum coherence of the electronic wave function along the whole sample.

In this paper we want to discuss some of the properties of a quantum coherent state of a mesoscopic system of cylindrical geometry. We show that if we reduce the dimensions of a cylinder made of a normal metal or a semiconductor to mesoscopic dimensions, a system exhibits coherent properties absent in macroscopic samples. We also discuss the possibility of long-range order (LRO) in a set of mesoscopic rings deposited along \( z \) axis.

We study the mesoscopic cylinders made of a normal metal or semiconductor with quasi-1D and 2D conduction and we assume that electrons interact via the magnetostatic (current-current) interaction.

The circumference, the height and the thickness of a cylinder are denoted by \( L_x = 2\pi R \), \( L_z \) and \( d \) respectively. We assume that we have a thin cylinder \( d \ll L_x, L_z \) and that \( L_x \) is of mesoscopic size.

Systems with quasi-1D conduction can be BCC crystals, low dimensional organic conductors, and cylinders made of concentric mesoscopic rings stacked along \( z \) axis.

Cylinders made of materials with layered structure and multiwall carbon nanotubes are the examples of systems with 2D conduction.

II. MAGNETOSTATIC INTERACTION IN MESOSCOPIC CYLINDERS

The properties of quasi-1D mesoscopic rings and 2D cylinders in the presence of a static magnetic field are well known. They exhibit in the presence of a static magnetic flux \( \phi_e \) persistent diamagnetic or paramagnetic currents depending on \( k_F \) and lattice constant \( a \). Moreover the magnetostatic interaction can lead to self-sustaining persistent currents even if we switch the external flux off. Such currents are a hallmark of phase coherence.

In one of our recent papers we derived a microscopic formula for the current-current interaction for a set of \( M_z \) mesoscopic rings stacked along \( z \) axis. Then performing a self-consistent mean field approximation (SMFA) we arrived at the effective Hamiltonian \( H^{MF} \) from which the self-sustaining currents can be obtained.

\[
H^{MF} = \frac{1}{2m_e} \sum_{m/1}^{M_z} \sum_{n/1}^{N_1} (p_{nm} - eA_I)^2 + \frac{\phi_I^2}{2C}.
\]

where for long cylinders \( (L_z \gg R) \) \( L = \mu_0 R^2/2L_z \); \( p_{nm} = p_{nm}^0 - eA_e \), \( p_{nm}^0 \) is a momentum of a \( n \)-th electron in a \( m \)-th channel, \( A_e = \phi_e/2\pi R \), \( \phi_e \) is the external magnetic flux; \( \phi_I = 2\pi RA_I \), \( A_I \) is a vector potential coming from all currents in the system; \( N_1 \) is the number of conducting electrons in a single channel.

\( H^{MF} \) was the basis of our previous investigations of spontaneous self-sustaining currents.

In this paper we want to check whether \( H^{MF} \) given by Eq. (1) is also valid for cylinders with 2D conduction. The general formula for the magnetostatic interaction is of the form:

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\[ H_{\text{mgt}} = -\frac{\mu_0}{8\pi} \int \int d^3r d^3r' \frac{\mathbf{J}(\mathbf{r}) \cdot \mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}, \]  

(2)

where \( \mathbf{J}(\mathbf{r}) = e \mathbf{p}(\mathbf{r})/m_e \), \( \mathbf{J}(\mathbf{r}) \) is the current density, \( \mathbf{p}(\mathbf{r}) \) is the momentum of an electron.

Let us apply to our system a small magnetic field parallel to the \( z \) axis. The resulting currents will run in the \( x \) direction in \( M_z \) channels.

We obtain

\[ H_{\text{mgt}} = -\frac{1}{2} \sum_{M_z} \sum_{m/1} \sum_{m'/1} \mathcal{L}_{mm'} I_m I_{m'}, \]  

(3)

where \( I_m \) is the current in the \( m \)-th channel,

\[ \mathcal{L}_{mm'} = \frac{\mu_0}{4\pi} \oint_{C_m} \oint_{C_{m'}} \frac{d\xi_m d\xi_{m'}}{|\xi_m - \xi_{m'}|}, \quad \mathcal{L}_{mm'} = \mathcal{L}_{m'm}. \]  

(4)

The interaction constant \( \mathcal{L}_{mm'} \) depends on the sample geometry; here it has to be calculated for the cylinder with channels at distance \( z_{mm'} = z_m - z_{m'} \).

The \( z \) dependence of the coupling constant \( \mathcal{L} \) is presented in Fig. 1. For small \( z \) it falls down slowly proportionally to \( \mu_0 R (\ln 8R/z - 2) \), for large \( z \) it falls down faster proportionally to \( 1/z^3 \). The interaction constant depends only on \( R \) and on the relative distance of the channels. We see that the interaction (3) is a long-range interaction. This indicates in particular that thermodynamic fluctuations of the current will be strongly suppressed.

If we express the currents \( I_m \) via the momenta \( p_m \), \( H_{\text{mgt}} \) can be rewritten in the form:

\[ H_{\text{mgt}} = -\frac{e^2}{2m_e^2} \sum_{M_z} \sum_{m/1} \sum_{m'/1} g_{mm'} p_m p_{m'}, \]  

(5)

where

\[ g_{mm'} = \frac{1}{4\pi^2 R^2} \mathcal{L}_{mm'}, \]  

(6)

\[ I_m = \frac{e}{2\pi R m_e} p_m, \]

\[ p_m = \sum_{n/1} p_{nm}. \]

Let us perform a self-consistent MFA of the interaction (3), such approximation is known to be good for a long-range interaction.

\[ H_{\text{mgt}}^\text{MF} = -\frac{e}{2m_e} \sum_m [2p_m \langle A_I(z_m) \rangle - \langle p_m \rangle \langle A_I(z_m) \rangle], \]  

(7)

where the first term in Eq. (7) has been obtained by use of the symmetry relation (4), \( \langle A_I(z_m) \rangle = (e/2m_e) \sum_m \langle p_m \rangle \langle p_{m'} \rangle \equiv A_I. \)

For the cylinder geometry the formula for \( A_I \) takes the form:

\[ A_I = \mu_0 R \langle \sum_{m/1} I_m \rangle / 2L_z. \]  

(8)

Calculating the current \( \langle \sum_{m/1} I_m \rangle \) with a total vector potential \( A = A_e + A_I \) we obtain after some algebra

\[ eA_I = \frac{\eta}{1 + \eta N_1 M_z} \sum_{m/1} \langle p_m \rangle, \]  

(9)
where
\[ \eta = \frac{\mu_0 e^2}{4\pi L_x m_e}. \]
Inserting Eq. (8) into (5) and adding to \( H_{MF}^{th} \) the kinetic energy term we obtain the Hamiltonian \( H_{MF} \) given by Eq. (4).

Thus we proved that the SMFA of the microscopic Hamiltonian given by Eq. (1) is also valid for systems with 2D conduction. However \( A_I \) and hence \( \phi_I \) has to be calculated in a different way than in the systems with quasi-1D conduction.

The basic difference between systems with quasi-1D and 2D conduction is the formula for the total current \( I \). Systems with quasi-1D conduction have flat Fermi surfaces (FS) perpendicular to \( k_x \) direction. There exists then a largest correlation among the currents from different channels (rings) and the total current \( I \) is the biggest. In the 2D case the current \( I \) depends strongly on the shape of the FS.

The total current \( I \) in the system can be written as a Fourier series:
\[ I(\phi) = M_r \sum_{m/1}^{M} \sum_{g/1}^{\infty} \frac{4I_0(m)}{\pi} \left( \frac{L_x}{2\gamma} + \frac{2\pi^2 k_B T}{\Delta_0} \right) \exp \left[ -\frac{g}{\gamma} \left( \frac{L_x}{2} + \frac{2\pi^2 k_B T}{\Delta_0} \right) \right] \cos \left[ g k_{Fz}(m) L_x \right] \sin \left( 2\pi g \frac{\phi}{\phi_0} \right), \tag{10} \]
where \( \phi_0 = h/e, \) \( 1/\gamma \) is a disorder parameter, \( \Delta_0 = h^2 N_1/(2m_e R^2) \) is the quantum size energy gap at the FS for electrons running in the \( x \) direction, \( I_0(m) = e\hbar k_{Fz}(m)/m_e L_x, k_{Fz}(m) = k_F \left[ 1 - (k_F(m)/k_F)^u \right]^{1/2}, k_F(m) = m\pi/L_z, M \) is the number of channels in the \( k_z \) direction, \( M_r \) is the number of channels on the thickness \( z \), \( u \) is the parameter used to model different shapes of the FS. Changing \( u \) from 2 to \( u \gg 2 \) we can investigate the shape of the FS from circular to rectangulal with rounded corners, with different amount of flat regions. Currents given by Eq. (10) are persistent at \( k_B T \ll \Delta_0 \) because the energy gap prevents scattering.

The flux \( \phi \) which drives the current is the sum of the external flux \( \phi_e \) and the flux \( \phi_I \) from the current itself,
\[ \phi = \phi_e + \phi_I, \quad \phi_I = LI. \tag{11} \]

The self-consistent solutions of Eqs. (10) and (11) at \( \phi_e = 0 \) give the values of self-sustaining flux. They are presented as circles in Figs. 2, 3 for a quasi-1D system and in Fig. 4 for a 2D system.

In Fig. 2 the case of a system running paramagnetic persistent currents is considered. The self-sustaining solutions correspond to spontaneous fluxes and are obtained for temperature \( T < T_c = 3.5 \) K, \( T_c \) is the transition temperature to an ordered state.

The system discussed in Fig. 3 exhibits strong (Meissner-like) diamagnetic reaction at small fluxes. The self-sustaining solutions correspond to trapped flux (for \( T < T_c = 0.6 \) K). We see that the coherent state of a system presented in Fig. 3 posses features characteristic of superconductors.

We notice that the spontaneous flux solution can be obtained at higher temperatures than the flux trapped solution because the curves from Eqs. (10) and (11) have to cross in the first and the second half of the \( \phi/\phi_0 \) period, respectively.

The current-flux characteristics for 2D cylinders with different shapes of the FS are presented in Fig. 4. We see that the self-sustaining current solutions can be obtained only for \( u \geq 5 \), i.e. for systems with the FS having substantial flat regions. Such FS are frequently met, e.g. in high temperature superconductors.

### III. EFFECTIVE LONG-RANGE ORDER

Having the microscopic \( H \) of the magnetostatic interaction [Eq. (3)] we can discuss the concept of effective long-range order (LRO) and phase transitions in finite quasi-1D systems. Let us focus on a set of stacked rings. As the currents \( I_m \) can run only in the clockwise or anticlockwise direction the Hamiltonian (3) has the form of the Ising-like Hamiltonian. It has been well established that 1D systems with short range forces cannot have a phase transition at finite in the thermodynamic limit. For a system described by an Ising Hamiltonian \( H_I \) the proof goes as follows
\[ H_I = -\sum_{m/2}^{M} \sum_{n/1}^{m-1} J(m-n) S_m S_n \tag{12} \]
In the ground state all ”spins” are parallel ($S_n S_n = \pm 1$). The change in the free energy $\Delta F$ connected with reversing the direction of $L$ ”spins” ($L \ll M_z$) in $p$ ($1 \leq p \ll M_z$) different places is

$$\Delta F = p \Delta E_L - T \Delta S = p (\Delta E_L - k_B T \ln M_z),$$

where $\Delta E_L$ is the change in the internal energy.

For short-range (n.n) interactions $\Delta E_L = 2J$ is independent of $M_z$ and $L$ and $\Delta F < 0$ for $M_z \to \infty$. It means that the configuration with domain structure has always lower energy in the thermodynamic limit and there is no LRO. However for finite $M_z$ long-range order is effectively obtained if $\Delta F > 0$. We can define a correlation range

$$\xi_T = e^{\Delta E_L / k_B T},$$

and we see from Eq. (13) that the system has an ordered ground state if $\xi_T > M_z$, i.e. if the correlations extend over all length.

The microscopic Hamiltonian of the magnetostatic interaction given by Eq. (3) is of the form of an Ising Hamiltonian but the situation is much more interesting here because the interaction is long-range. The energy change when reversing a direction of $L$ currents is given by

$$\Delta E_L(M_z) = \frac{1}{2} \left( \frac{e \hbar N_1}{2 \pi m_e R^2} \right)^2 \sum_{n=1}^{M_z} \sum_{m=0}^{M_z-n} \mathcal{L}(m - m').$$

The inspection of $\Delta E_L(M_z)$ for different values of $b = z_{n+1} - z_m$ shows that it first increases with increasing both $L$ and $M_z$ and then saturates. From $\Delta F = 0$ we can calculate the temperature $T^*$ at which the crossover from an ordered to disordered state occurs

$$T^*_L = \frac{\Delta E_L(M_z)}{k_B \ln M_z}.$$  

We can investigate now the possibility of LRO in a set of $M_z$ ($M_z \gg 1$) stacked rings. We discuss different possibilities.

At first we can consider the instability against the formation of long domains ($L \gg 1$) of rings with currents running in the opposite directions. Taking e.g. $L = 1000$ and $b = 60$ Å, $R = 5000$ Å, $M_z = 10^4$ we get from Eqs. (15) and (16) $T_{1000}^* \sim 1.5$ K.

Thus at $T < 1.5$ K the instability implied by Eq. (16) excludes the existence of disorder in the form of long alternating domains with opposite currents. However short-range disorder with small number of rings with reversed currents is not excluded.

In order to get such long-range stable domains one should need some additional interaction which should stabilize long domains against short-range fluctuations. Then with such additional mechanism included, one could say that at $T < T^*$ the system exhibits long range order.

Let us discuss now the instability against short-range disorder. The smallest energy change is obtained when reversing a direction of a single current in several places. Taking $L = 1$ and other parameters the same as in previous example we obtain from Eqs. (15) and (16)

$$T_{1}^* \sim 2.2 \times 10^{-2} \text{ K}.$$  

However we can also assume that stacked rings have finite but small thickness $d$. Then the number of channels in each ring increases what results in increasing $\Delta E_L$ and hence $T_{1}^*$.

Taking e.g. $d = 10$ Å, $b = 60$ Å, $R = 5000$ Å, $L = 1$, $M_z = 10^4$ we obtain from Eq. (16) $T_{1}^* \sim 0.45$ K, and for $d = 20$ Å, $b = 110$ Å, other parameters as above, we obtain $T_{1}^* \sim 0.98$ K.

Finite values of transition temperatures are related to finite size of the considered samples. Indeed if we put $M_z \to \infty$ then due to the fact that $\Delta E_L(M_z)$ saturates for large $M_z$, $T_{L}^* \to 0$.

It has been proved many years ago that for the interaction $J(m - n) = J/|m - n|^\alpha$, $\alpha > 2$ in Eq. (12) the model exhibits no phase transition at finite temperature. In our case $\mathcal{L}(m - n) \sim 1/|m - n|^3$ for large $|m - n|$ and we do not expect LRO in the thermodynamic limit. So our system is an example when large but finite system can show interesting effects which will be wiped away in the thermodynamic limit.

We want to stress that such conclusions are valid only if the systems can be treated with good approximation as 1D systems. The systems considered by us in the previous section were 2D systems and they should be described by a 2D Ising model which has a phase transition to an ordered state in the thermodynamic limit. Such consideration will be a subject of our further work.
IV. CONCLUSIONS

In the presented paper we discussed some aspects of quantum coherence in mesoscopic systems. One of the basic thermodynamical properties of a macroscopic metal or semiconductor is a very weak reaction to a static magnetic field (weak Landau diamagnetism).

Whether or not a sample of a given size can be considered as a macroscopic one, depends on temperature. It turns out that if \( k_B T < \Delta_0 \) the system exhibits mesoscopic orbital magnetism\(^{13}\), which manifests itself in strong paramagnetic or diamagnetic persistent currents in the presence of a static magnetic field. What is more, if the magnetic response is sufficiently large one should account also for the field produced by the orbital currents and solve the entire problem self-consistently.

To treat this problem we considered the long-range magnetostatic interaction. Performing the SMFA we arrived at the effective Hamiltonian, valid both in systems with quasi-1D and 2D conduction which leads to self-consistent equations for the current.

The self-sustaining solutions (at \( \phi_e = 0 \)) for samples exhibiting paramagnetic behavior at low \( \phi \) are spontaneous currents manifesting a break of time reversal symmetry.

The self-sustaining solutions (at \( \phi_e = 0 \)) for samples exhibiting diamagnetic behavior at low \( \phi \) are equivalent to flux trapped in the systems. Such behavior was previously attributed solely to superconductors, we show here that it can be obtained in a coherent state in mesoscopic cylinders.

The self-sustaining currents present in mesoscopic systems with low-dimensional conduction, are a hallmark of a quantum coherence. The amount of coherent electrons running persistent currents is very sensitive to the shape of the FS. The most favorable situation is for the system with flat FS where the correlation of currents from different channels is the strongest.

The microscopic \( H \) of the magnetostatic interaction for a cylinder made of a set of stacked rings has the form of an Ising Hamiltonian with the long-range interaction. Using it we have discussed the effective long-range order for large but finite systems. We calculated the temperature \( T^* \) below which the system is completely ordered in a sense that the correlation range extends over its length. Most of the theorems discussing LRO are valid only in the thermodynamic limit. They therefore miss some interesting properties of the system in question.

The properties of such quasi-1D systems are interesting by itself but can also be used to study phase transitions in materials which can be viewed as arrays of quasi-1D systems. This will be studied in a forthcoming paper.

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Figure captions

FIG. 1. The interaction constant $\mathcal{L}$ as a function of distance $z$ between channels.

FIG. 2. Paramagnetic persistent currents $I/I_0$ as a function of a magnetic flux $\phi/\phi_0$ at $\phi_e = 0$ and different $T$. Spontaneous flux solutions are denoted by circles.

FIG. 3. Diamagnetic persistent currents $I/I_0$ as a function of a magnetic flux $\phi/\phi_0$ at $\phi_e = 0$ and different $T$. Trapped flux solutions are denoted by circles.

FIG. 4. Persistent currents $I/I_0$ as a function of magnetic flux $\phi/\phi_0$ at $\phi_e = 0$ in 2D mesoscopic cylinders with different shapes of the Fermi surfaces. Self-sustaining currents are denoted by circles. $N$ is a number of conducting electrons in a single cylinder.
1) $T = 0 \, \text{K}$
2) $T = T_c = 0.6 \, \text{K}$
3) $T = 1.5 \, \text{K}$
4) $T = 3 \, \text{K}$
5) $T = 5 \, \text{K}$

$N_1 = 15 \, 709$, $M_z = 10 \, 000$, $M_r = 20$

$\gamma = 500 \, 000 \, \text{Å}$
1) \( u = 20 \) \( T_c = 2.2 \) K
2) \( u = 6 \) \( T_c = 2.0 \) K
3) \( u = 5 \) \( T_c = 0.9 \) K
4) \( u = 4 \)
5) \( u = 3 \)
6) \( u = 2 \)

\[ N = 15709 \times 10^4, M_r = 40 \]
\[ T = 1 \text{mK}, \gamma = 500 \text{ 000 } \text{Å} \]