Backstepping active disturbance rejection control for trajectory tracking of underactuated autonomous underwater vehicles with position error constraint

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Abstract
In this article, the three-dimensional trajectory tracking control of an autonomous underwater vehicle is addressed. The vehicle is assumed to be underactuated and the system parameters and the external disturbances are unknown. First, the five degrees of freedom kinematics and dynamics model of underactuated autonomous underwater vehicle are acquired. Following this, reduced-order linear extended state observers are designed to estimate and compensate for the uncertainties that exist in the model and the external disturbances. A backstepping active disturbance rejection control method is designed with the help of a time-varying barrier Lyapunov function to constrain the position tracking error. Furthermore, the controller system can be proved to be stable by employing the Lyapunov stability theory. Finally, the simulation and comparative analyses demonstrate the usefulness and robustness of the proposed controller in the presence of internal parameter uncertainties and external time-varying disturbances.

Keywords
ADRC, backstepping technique, barrier Lyapunov function, trajectory tracking control, underactuated AUV

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Introduction
Autonomous underwater vehicles (AUVs) are widely used in marine scientific investigation, marine mineral exploration, and oceanographic mapping.¹ So the need for AUVs has become increasingly apparent and the research on the trajectory tracking control of AUVs becomes more important. Considering reducing the actuator cost and weight or increasing the reliability of the system in case of actuator failure, most of AUVs have fewer actuators than the number of degrees of freedom.²,³ Nowadays, motion control of underactuated AUVs has been absorbing significant attention of researchers mainly in nonlinear control.

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In past decades, lots of methods have been proposed to track AUVs, such as sliding mode control, neural network control, and so on. There are also several methods combining the backstepping technique and other methods that are proposed to solve trajectory tracking problem of underactuated AUVs in a variety of complex environments. A current observer-based backstepping controller is developed to achieve trajectory tracking control of underactuated AUVs in the presence of unknown current disturbances. The combination of backstepping technique and adaptive sliding mode control enhances the robustness of an AUV in the presence of model parameter uncertainties and external environmental disturbances. Backstepping technique and bio-inspired models are used to increase system robustness and avoid the singularity problem in backstepping control of virtual velocity error.

Active disturbance rejection control (ADRC) was originally proposed by Han. It’s worth noting that nonlinear tracking differentiator (TD) and extended state observer (ESO) are important parts of ADRC. The uniqueness of ADRC is that it treats all factors affecting the plant, such as system nonlinearities, uncertainties, and external disturbances as total disturbances to be observed and compensated by ESO and it has been applied in almost all domains of control engineering. ADRC has been adopted to solve the path following control problem of underactuated AUVs, but it has rarely been used in underactuated AUVs’ trajectory tracking control.

On the other hand, in some cases, it is necessary to constrain the position tracking error of an underactuated AUV within certain given boundary function all the time for safety reasons. It has been proved that the barrier Lyapunov function method and prescribed performance control method are effective solutions to prevent constraint violation. A barrier Lyapunov function is incorporated with the backstepping control scheme to handle the position tracking error constraint. Prescribed performance functions have been adopted to constrain the position and orientation errors of an underactuated AUV which can ensure both the prescribed transient and steady-state performance constraints. But in these studies, point-to-point navigation is used and it is worth noting that the desired yaw angle is not continuously differentiable when the position error equals zero. To avoid this problem, the position tracking error converges to a constant instead of zero.

Underactuated AUVs lack sway and heave propellers, so the transverse and vertical position errors of trajectory tracking are usually eliminated by controlling the yaw and pitch angles, that is also why the backstepping technique is often employed in the trajectory tracking control of underactuated AUVs. However, backstepping controller has two obvious disadvantages. The first one is that the unknown transverse and vertical disturbances are usually omitted when deducing the virtual control variables of yaw angular and pitch angular velocities. As a result, the unknown transverse and vertical disturbances cannot be compensated in time, which always affects the result of trajectory tracking. The other one is that the problem of “explosion of terms” caused by the operation of differentiation always exists. Based on the above analysis, that reduced-order linear extended state observers (RLESOs) are used to estimate and compensate for the total disturbances which can improve the performance of backstepping controller and enhance the robustness of underactuated AUVs. Besides, TD can be used to provide the filtered version of the input signal and its differentiation. So TDs are employed to solve the problem of “explosion of terms.” What’s more, by barrier Lyapunov function, the controller can prevent the violation of the time-varying position error constraint and render the position tracking error converges to zero. In this work, the proposed controller can solve the three-dimensional trajectory tracking control problem of underactuated AUVs in the presence of internal parameter uncertainties and external unknown disturbances.

The rest of this article is organized as follows. The mathematical model of an underactuated AUV system is presented in the second section. In the third section, RLESOs are designed to estimate the total disturbances. The backstepping ADRC controller design procedure for the three-dimensional trajectory tracking of an underactuated AUV is illustrated in the fourth section. The simulation and comparative analyses are provided in the fifth section and some conclusions of this article are brought forward in the sixth section.

**Problem formulation**

This section presents the five degrees of freedom kinematics and dynamics model of an underactuated AUV and then formulates the problem of trajectory tracking.

**AUV modeling**

We define the earth-fixed coordinate system \( \{E\} \) and the body-fixed coordinate system \( \{B\} \) of the AUV as shown in Figure 1, where \( \xi, \eta, \) and \( \zeta \) represent the inertial
coordinates of the vehicle in the earth-fixed frame; \( u, v, \) and \( w \) are the surge, sway, and heave velocities, respectively, defined in the body-fixed frame; \( p, q, \) and \( r \) represent roll angular velocity, pitch angular velocity, and yaw angular velocity; \( \varphi, \theta, \) and \( \psi \) are roll angle, pitch angle, and yaw angle.

The underactuated AUV is assumed to satisfy the following assumptions\(^3\),\(^4\): (1) the center of gravity coincides with the center of buoyancy, (2) the mass distribution is homogeneous, and (3) the hydrodynamic drag terms of order higher than two and roll motion are neglected. Then the five degrees of freedom (DOF) AUV kinematics equations can be written as

\[
\begin{align*}
\dot{u} &= \frac{m_{11} + \Delta m_{11}}{m_{11}} v r - \frac{m_{33} + \Delta m_{33}}{m_{11}} w q - \frac{X_u + \Delta X_u}{m_{11} + \Delta m_{11}} u r - \frac{X_{|u|u} + \Delta X_{|u|u}}{m_{11} + \Delta m_{11}} |u| u_r + \frac{\tau_u + d_1}{m_{11} + \Delta m_{11}}, \\
\dot{v} &= -\frac{m_{11} + \Delta m_{11}}{m_{22}} u r - \frac{m_{33} + \Delta m_{33}}{m_{22}} w r - \frac{X_v + \Delta X_v}{m_{22} + \Delta m_{22}} u v - \frac{X_{|v|v} + \Delta X_{|v|v}}{m_{22} + \Delta m_{22}} |v| v_r + \frac{d_2}{m_{22} + \Delta m_{22}}, \\
\dot{w} &= \frac{m_{11} + \Delta m_{11}}{m_{33} + \Delta m_{33}} u q - \frac{m_{33} + \Delta m_{33}}{m_{33} + \Delta m_{33}} w_r - \frac{X_w + \Delta X_w}{m_{33} + \Delta m_{33}} w r - \frac{X_{|w|w} + \Delta X_{|w|w}}{m_{33} + \Delta m_{33}} |w| w_r + \frac{d_3}{m_{33} + \Delta m_{33}}, \\
\dot{\psi} &= \frac{m_{11} + \Delta m_{11}}{m_{55} + \Delta m_{55}} u w - \frac{m_{33} + \Delta m_{33}}{m_{55} + \Delta m_{55}} v q - \frac{M_{q|q} + \Delta M_{q|q}}{m_{55} + \Delta m_{55}} |q| q_r - \frac{\rho g \sqrt{\mathbf{G} M_L}}{m_{55} + \Delta m_{55}} \sin \theta + \frac{\tau_q + d_5}{m_{55} + \Delta m_{55}}, \\
\dot{\theta} &= \frac{m_{11} + \Delta m_{11}}{m_{66} + \Delta m_{66}} u v - \frac{m_{33} + \Delta m_{33}}{m_{66} + \Delta m_{66}} w r - \frac{N_{r|r} + \Delta N_{r|r}}{m_{66} + \Delta m_{66}} |r| r_r - \frac{\tau_r + d_6}{m_{66} + \Delta m_{66}}.
\end{align*}
\]

The five DOF underactuated AUV dynamics equations, considering the internal parameter uncertainties and external environmental disturbances, can be expressed by the following differential equations:

\[
\begin{bmatrix}
\ddot{x} \\
\ddot{y} \\
\ddot{z} \\
\ddot{\phi} \\
\ddot{\theta} \\
\ddot{\psi}
\end{bmatrix} =
\begin{bmatrix}
\cos \psi \cos \theta & -\sin \psi & \sin \theta \cos \psi & 0 & 0 \\
\sin \psi \cos \theta & \cos \psi & \sin \theta \sin \psi & 0 & 0 \\
-\sin \theta & 0 & \cos \theta & 0 & 0 \\
0 & 0 & 0 & 1 & \sin \theta \\
0 & 0 & 0 & 0 & 1/\cos \theta
\end{bmatrix}
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z} \\
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix}
\begin{bmatrix}
u \\
v \\
w \\
r \\
q \\
r
\end{bmatrix} +
\begin{bmatrix}
\tau_x \\
\tau_y \\
\tau_z \\
\tau_{\phi} \\
\tau_{\theta} \\
\tau_{\psi}
\end{bmatrix} + \begin{bmatrix}
D_x \\
D_y \\
D_z \\
D_{\phi} \\
D_{\theta} \\
D_{\psi}
\end{bmatrix}.
\]
where \( m_{11} = m - X \), \( m_{22} = m - Y \), \( m_{33} = m - Z \), \( m_{55} = I_y - M_q \), and \( m_{66} = I_z - N_r \); \( u_r = u - u_c \), \( v_r = v - v_c \), and \( w_r = w - w_c \) represent the relative velocities of the vehicle with respect to current in the body-fixed frame and \( V_{cx}, V_{cy}, \) and \( V_{cz} \) represent the ocean current velocities in the earth-fixed frame; \( \Delta(\cdots) \) represents the parameter uncertainties in the vehicle model. \( d_1, d_2, d_3, d_5, \) and \( d_6 \) are the external environmental disturbances; \( D_u, D_v, D_w, D_q, \) and \( D_r \) are the total disturbances to be estimated; \( \tau_u, \tau_q, \) and \( \tau_r \) are considered as the available control inputs.

**Assumption 1.** The underactuated AUV’s velocities and control inputs are bounded, that is, \( |\tau_u| \leq \tau_u^*, |\tau_q| \leq \tau_q, |\tau_r| \leq \tau_r, |u| \leq \bar{u}, |v| \leq \bar{v}, |w| \leq \bar{w}, \) \( |\theta| \leq \bar{\theta}, \) where \( \tau_u^*, \tau_q, \tau_r, u, v, w, \bar{\theta}, \) and \( \bar{\theta} \) are the known upper bounds.

**Assumption 2.** For equation (2), the total disturbances and their derivatives are all bounded. There exists:

\[
\left| \frac{d^4 D_j}{dt^4} \right| \leq D_j \quad (3)
\]

where \( D_j(j = u, v, w, q, r) \) are positive constants and \( k = 0, 1. \)

**Control objectives**

In order to facilitate the formulation, \( p = [\xi(t), \eta(t), \zeta(t)]^T \) is defined as the actual coordinate variable and \( p_j = [\xi_j(t), \eta_j(t), \zeta_j(t)]^T \) is a given sufficiently smooth time-varying desired trajectory with its derivative with respect to time bounded.

Considering the kinematics and dynamics equations, design a controller to render the tracking error \( |p - p_d| \) converges to a neighborhood of the origin that can be made arbitrarily small.

**RLESOs design**

According to the design principle of RLESO which is firstly proposed by Huang and Xue,\(^{12} \) RLESOs for the estimations of the total disturbances are designed as follows:

\[
\begin{align}
\dot{p}_1 &= -\beta_1 p_1 - \beta_1^2 u - \beta_1 (f_u + \tau_u/m_{11}) \\
\dot{D}_u &= \beta_1 u + p_1 \\
\dot{p}_2 &= -\beta_2 p_2 - \beta_2^2 v - \beta_2 f_v \\
\dot{D}_v &= \beta_2 v + p_2
\end{align}
\]

where \( \dot{p}_3 = -\beta_3 p_3 - \beta_3^2 w - \beta_3 f_w \)

\[
\begin{align}
\dot{D}_w &= \beta_3 w + p_3 \\
\dot{p}_4 &= -\beta_4 p_4 - \beta_4^2 q - \beta_4 (f_q + \tau_q/m_{55}) \\
\dot{D}_q &= \beta_4 q + p_4 \\
\dot{p}_5 &= -\beta_5 p_5 - \beta_5^2 r - \beta_5 (f_r + \tau_r/m_{66}) \\
\dot{D}_r &= \beta_5 r + p_5
\end{align}
\]

where \( D_u, D_v, D_w, D_q, \) and \( D_r \) are the estimations of \( D_u, D_v, D_w, D_q, \) and \( D_r \); \( \beta_i \) are the observer gains and \( p_i \) (i = 1, 2, 3, 4, 5) are the auxiliary states of the observer.

If Assumption 2 is satisfied, we can obtain\(^{25} \):

\[
||E|| \leq \frac{\max(D_j)}{\min(|\beta_j|)} \quad (5)
\]

where \( E = [\dot{D}_u, \dot{D}_v, \dot{D}_w, \dot{D}_q, \dot{D}_r]^T = [D_u - \dot{D}_u, D_v - \dot{D}_v, D_w - \dot{D}_w, D_q - \dot{D}_q, D_r - \dot{D}_r]^T \) represents the estimation error of the RLESOs and can be tuned arbitrarily small by increasing the observer gains \( \beta_i \).

**Backstepping ADRC controller design**

**Coordinate transformation**

The desired yaw angle and pitch angle are obtained only based on the reference trajectory

\[
\psi_d = \begin{cases} 
\arctan \left( \frac{\dot{\eta}_d}{\dot{\xi}_d} \right), \dot{\xi}_d \geq 0 \\
\pi + \arctan \left( \frac{\dot{\eta}_d}{\dot{\xi}_d} \right), \dot{\xi}_d < 0 
\end{cases}
\]

\[
\theta_d = -\arctan \left( \frac{\dot{\zeta}_d}{\dot{\xi}_d} \right) 
\]

where \( \nu_p = \sqrt{\dot{\xi}_d^2 + \dot{\eta}_d^2} \) and \( v_i = \nu_p \cos \theta_d = \sqrt{\dot{\xi}_d^2 + \dot{\eta}_d^2} \).

Then the position and attitude error variables \( x_e, y_e, z_e, \psi_e, \theta_e, \) and \( \psi_d \) are defined as

\[
\begin{pmatrix}
x_e \\
y_e \\
z_e \\
\psi_e \\
\theta_e \\
\psi_d
\end{pmatrix} =
\begin{pmatrix}
\cos \psi \cos \theta & \sin \psi \cos \theta & -\sin \theta & 0 & 0 & 0 \\
\sin \psi & \cos \psi & 0 & 0 & 0 & 0 \\
\sin \theta \cos \psi & \sin \theta \sin \psi & \cos \theta & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\xi - \xi_d \\
\eta - \eta_d \\
\zeta - \zeta_d \\
\theta - \theta_d \\
\psi - \psi_d
\end{pmatrix}
\]

which express the errors in the body-fixed frame. \( \xi_d, \eta_d, \zeta_d, \theta_d, \) and \( \psi_d \) are the desired position and attitude variables in the earth-fixed frame.

The derivatives of the position and attitude error variables are described as
\[
\begin{align*}
\dot{x}_e &= u - v_p \cos \theta_e \cos \psi_e - v_p \sin \theta_e \sin \theta + ry_e - qz_e \\
\dot{y}_e &= v + v_p \cos \theta_e \sin \psi_e - rx_e - rz_e \tan \theta \\
\dot{z}_e &= w - v_p \cos \theta_e \sin \psi_e + v_p \sin \theta_e \cos \theta + qx_e + ry_e \tan \theta \\
\dot{\theta}_e &= q - \dot{\theta}_d \\
\dot{\psi}_e &= r / \cos \theta - \dot{\psi}_d 
\end{align*}
\] (9)

### Backstepping controller with RLESOs

This section designs a trajectory tracking controller by combining the backstepping technique and RLESOs and analyzes the stability based on the Lyapunov stability theory.

**Step 1:** A time-varying barrier Lyapunov function is defined as

\[
V_1 = \frac{1}{2} \log(p^2 / (\rho^2 - \rho_e^2))
\] (10)

where \( \rho \) is the position error constraint function and \( \rho_e^2 = x_e^2 + y_e^2 + z_e^2 \). The time derivative of \( V_1 \) yields

\[
\dot{V}_1 = (\rho_d \rho_e - \rho_e^2 \dot{\rho} / \rho) / (\rho^2 - \rho_e^2)
\] (11)

Choose virtual velocity error variables \( \alpha_1, \alpha_2 \)

\[
\begin{align*}
\alpha_1 &= v_p \cos \theta_d \sin \psi_e \\
\alpha_2 &= v_p \sin \theta_e
\end{align*}
\] (12, 13)

In order to make \( \dot{V}_1 \) negative, we choose \( u, \alpha_1, \) and \( \alpha_2 \) as virtual controls, and their desired values are as follows

\[
u_d = v_p \cos \theta_d \cos \psi_e + v_p \sin \theta_d \sin \theta - k_1 x_e + x_e \dot{\rho} / \rho
\] (14)

\[
\alpha_{1d} = -v - k_2 y_e + y_e \dot{\rho} / \rho
\] (15)

\[
\alpha_{2d} = w - v_p \cos \theta_d \sin \theta(\cos \psi_e - 1) + k_3 z_e - z_e \dot{\rho} / \rho
\] (16)

where \( k_1, k_2, \) and \( k_3 \) are positive constants.

Since the virtual variables \( u_d, \alpha_{1d}, \) and \( \alpha_{2d} \) are not true controls, we define error variables

\[
\begin{align*}
u_e &= u - u_d \\
\alpha_{1e} &= \alpha_1 - \alpha_{1d} \\
\alpha_{2e} &= \alpha_2 - \alpha_{2d}
\end{align*}
\] (17, 18, 19)

Substituting equations (14) to (19) into equation (11) yields

\[
\dot{V}_1 = (-k_1 x_e^2 - k_2 y_e^2 - k_3 z_e^2 + u_e x_e + \alpha_{1e} y_e - \alpha_{2e} z_e) / (\rho^2 - \rho_e^2)
\] (20)

**Step 2:** To stabilize the error variable \( u_e \), define

\[
V_2 = V_1 + \frac{1}{2} u_e^2
\] (21)

Its derivative along with equation (17) and equation (2a) becomes

\[
\dot{V}_2 = (-k_1 x_e^2 - k_2 y_e^2 - k_3 z_e^2 + u_e x_e + \alpha_{1e} y_e - \alpha_{2e} z_e) / (\rho^2 - \rho_e^2)
\] (22)

Then the time derivative of \( V_3 \) becomes

\[
\dot{V}_3 = \dot{V}_2 + \alpha_{1e} \dot{\alpha}_{1e} = (-k_1 x_e^2 - k_2 y_e^2 - k_3 z_e^2 - \alpha_{1e} \dot{y}_e - \alpha_{2e} \dot{z}_e) / (\rho^2 - \rho_e^2)
\] (23)

Define

\[
\varpi = -k_2 y_e + y_e \dot{\rho} / \rho
\] (24)

It can be known

\[
\alpha_{1e} = -v + \varpi
\] (25)

\[
\dot{\alpha}_{1e} = -\dot{v} + \dot{\varpi}
\] (26)

The time derivative of equation (18) along with equation (12) and equation (29) becomes

\[
\dot{\alpha}_{1e} = \dot{\alpha}_{1} - \dot{\alpha}_{1d} = \dot{v}_1 \sin \psi_e + v_1 \cos \psi_e (r / \cos \theta - \dot{\psi}_d) + \dot{v} - \dot{\varpi}
\] (27)

(continued)
In order to make $\dot{V}_3$ negative, define $r = r \cos \psi_e$ and its desired value $r_d = r + \psi_d \cos \theta (\cos \psi_e - 1)$. The desired value of $r$ is as follows

$$r_d = \dot{\psi}_d \cos \theta + (-\dot{\psi}_e \sin \psi_e - \dot{\psi}_d + \psi_e' - \psi_e' - \psi_e'/\rho^2 - \rho_e') - k_5 (\alpha_{te}) \cos \theta/v_t$$

(31)

where $\dot{\psi}_d = f_v + \dot{D}_v$ and $k_5$ is a positive constant.

Considering that $r_d$ is not a true control input, we define error variables

$$r_e = r - r_d$$

(32)

$$\dot{r}_e = \dot{r} - \dot{r}_d$$

(33)

We can obtain

$$\dot{r}_e = r_e \cos \psi_e + \delta_1$$

(34)

where $\delta_1 = (r_d - \dot{\psi}_d \cos \theta) (\cos \psi_e - 1)$.

Substituting equations (30) to (34) into equation (26) yields

$$\dot{V}_3 = (-k_1 x_e^2 - k_2 y_e^2 - k_3 z_e^2 - \alpha_{te} z_e) / (\rho^2 - \rho_e^2) - k_4 u_e^2 - k_5 (\alpha_{te})^2 + u_e \dot{D}_u + \alpha_{te} \dot{D}_v + \alpha_{te} \dot{v}_e \cos \theta + r_e (\dot{r} - \dot{r}_d)

+ r_e (f_r + \tau/m_{66} + \dot{D}_r - \dot{r}_d + \alpha_{te} v_e \cos \psi_e \cos \theta + \alpha_{te} \dot{v}_e \delta_1 \cos \theta)$$

(35)

Step 4: To stabilize the error variable $r_e$, define

$$V_4 = V_3 + \frac{1}{2} \dot{r}_e^2$$

(36)

Its derivative along with equation (32), equation (35) and equation (2e) becomes

$$\dot{V}_4 = (-k_1 x_e^2 - k_2 y_e^2 - k_3 z_e^2 - \alpha_{te} z_e) / (\rho^2 - \rho_e^2) - k_4 u_e^2 - k_5 (\alpha_{te})^2 + u_e \dot{D}_u + \alpha_{te} \dot{D}_v + \alpha_{te} \dot{v}_e \cos \theta + r_e (\dot{r} - \dot{r}_d)

+ r_e (f_r + \tau/m_{66} + \dot{D}_r - \dot{r}_d + \alpha_{te} v_e \cos \psi_e \cos \theta + \alpha_{te} \dot{v}_e \delta_1 \cos \theta)$$

(37)

It can be known

$$\alpha_{te} = w - \mu$$

(43)

$$\dot{\alpha}_{te} = \dot{w} - \dot{\mu}$$

(44)

The time derivative of equation (19) along with equations (16) and (44) becomes

$$\dot{\alpha}_{te} = \dot{w} - \dot{\mu} - \dot{\psi}_e \sin \theta_e + \nu_\theta \cos \theta_e (q - \dot{\theta}_d) - \dot{w} + \dot{\mu}$$

(45)

In order to make $\dot{V}_4$ negative, define $q = q \cos \theta_e$ and its desired value $q_d = q_d + \dot{\theta}_d (\cos \theta_e - 1)$. The desired value of $q$ is as follows

$$q_d = \dot{\theta}_d + (-\psi_e \sin \theta_e + \dot{w}_1 - \dot{\mu} + z_e / (\rho^2 - \rho_e^2) - k_7 \alpha_{te}) / \nu_\theta$$

(46)

where $\dot{w}_1 = f_w + \dot{D}_w$ and $k_7$ is a positive constant.

Considering that $q_d$ is not a true control input, we define error variables

$$q_e = q - q_d$$

(47)

$$\dot{q}_e = \dot{q} - \dot{q}_d$$

(48)

We can get

$$\dot{q}_e = q_e \cos \theta_e + \delta_2$$

(49)

where $\delta_2 = (q_d - \dot{\theta}_d) (\cos \theta_e - 1)$.
Substituting equations (45) to (49) into equation (41) yields
\[
\dot{V}_5 = (-k_1 x_c^2 - k_2 y_c^2 - k_3 z_c^2)/(\rho^2 - \rho_c^2) - k_4 u_c^2 \\
- k_5 \alpha_1 e^2 - k_6 \alpha_2 e^2 - k_7 \alpha_2 \beta_1^2 + u_c \dot{D}_u + \alpha_1 \dot{D}_e \\
+ r_c \dot{D}_e - \alpha_2 \dot{D}_w + \alpha_2 v_\beta \dot{D}_q + \alpha_1 v_\delta \cos \theta \\
\] (50)

In order to make \( \dot{V}_6 \) negative, control input \( \tau_q \) is chosen as
\[
\tau_q = -m_{55} f_q - m_{55} \dot{D}_q + m_{55} \ddot{q}_d - m_{55} \alpha_2 v_\beta \cos \theta_e - k_{58} m_{55} \dot{q}_e \\
\] (53)
where \( k_{58} \) is a positive constant. Then equation (52) becomes
\[
\dot{V}_6 = (-k_1 x_c^2 - k_2 y_c^2 - k_3 z_c^2)/(\rho^2 - \rho_c^2) - k_4 u_c^2 \\
- k_5 \alpha_1 e^2 - k_6 \alpha_2 e^2 - k_7 \alpha_2 \beta_1^2 + u_c \dot{D}_u + \alpha_1 \dot{D}_e \\
+ r_c \dot{D}_e - \alpha_2 \dot{D}_w + \alpha_2 v_\beta \dot{D}_q + \alpha_1 v_\delta \cos \theta + \alpha_2 v_\beta \delta_2 \\
\] (54)

The complete Lyapunov function is as follows
\[
V_6 = \frac{1}{2} \text{log}(\rho^2/(\rho^2 - \rho_c^2)) + \frac{1}{2} u_c^2 + \frac{1}{2} \alpha_1^2 + \frac{1}{2} r_c^2 \\
+ \frac{1}{2} \alpha_2 e^2 + \frac{1}{2} q_e^2 \\
\] (55)
Here, if we define
\[
\delta = |u_c \dot{D}_u + \alpha_1 \dot{D}_e + v_\delta \cos \theta + r_c \dot{D}_e \\
+ \alpha_2 v_\beta \dot{D}_q| \\
\] (56)
Then with the help of the inequality \( \text{log}(\rho^2/(\rho^2 - \rho_c^2)) < \rho_c^2/(\rho^2 - \rho_c^2) \), we have
\[
\dot{V}_6 \leq -2\gamma V_6 + \delta \\
\] (57)
where \( \gamma = \min\{k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8\} \).
We know from the section “RLESOs design” that \( E = [\dot{D}_u, \dot{D}_c, \dot{D}_w, \dot{D}_q, \dot{D}_\theta]^T \) is bounded, so it is easy to know that \( \delta \) is bounded. According to the comparison principle, the following inequality can be obtained
\[
V_6 \leq V_6(0)e^{-2\gamma t} + \frac{\delta}{2\gamma} \\
\] (58)
Step 6: To stabilize the error variable \( q_e \), define
\[
V_6 = V_5 + \frac{1}{2} \theta_c^2 \\
\] (51)
Differentiating \( V_6 \) with respect to time along with equation (47), equation (50), and equation (2d) yields
\[
\dot{V}_6 = (-k_1 x_c^2 - k_2 y_c^2 - k_3 z_c^2)/(\rho^2 - \rho_c^2) - k_4 u_c^2 \\
- k_5 \alpha_1 e^2 - k_6 \alpha_2 e^2 - k_7 \alpha_2 \beta_1^2 + u_c \dot{D}_u + \alpha_1 \dot{D}_e \\
+ r_c \dot{D}_e - \alpha_2 \dot{D}_w + \alpha_2 v_\beta \dot{D}_q + \alpha_1 v_\delta \cos \theta + \alpha_2 v_\beta \delta_2 \\
\] (52)
Therefore
\[
||z(t)|| \leq ||z(0)||e^{-\gamma t} + \sqrt{\frac{\delta}{\gamma}} t \in [0, +\infty) \\
\] (59)
Equation (59) means that the tracking error signals converge to a compression bounded value near the zero by increasing control gains appropriately and the system is stable.

**Backstepping controller with TDs and RLESOs**

TD can be used to provide the filtered version of the input signal and its differentiation as fast as possible. And it is well known that the operation of differentiation always causes the problem of “explosion of terms” when the traditional backstepping technique is employed. In order to solve this problem, we add TDs to the above-mentioned controller and the TDs are as follows

\[
\begin{align*}
fh &= \text{fhan}(u_c(k) - u_d(k), R_1, h) \\
u_c(k + 1) &= u_c(k) + h \cdot \dot{u}_c(k) \\
\dot{u}_c(k + 1) &= \dot{u}_c(k) + h \cdot \ddot{u}_c(k) \\
r_c(k + 1) &= r_c(k) + h \cdot \dot{r}_c(k) \\
\dot{r}_c(k + 1) &= \dot{r}_c(k) + h \cdot \ddot{r}_c(k) \\
q_c(k + 1) &= q_c(k) + h \cdot \dot{q}_c(k) \\
\dot{q}_c(k + 1) &= \dot{q}_c(k) + h \cdot \ddot{q}_c(k) \\
\end{align*}
\] (60a)

where \( R_1, R_2, \) and \( R_3 \) are the acceleration factors to be adjusted and \( h \) is the sampling period. The function \text{fhan}() can be found in Han\(^{11}\) and the following statement can be found in Miao et al.\(^{13}\)

Corollary: Considering the TDs described by equation (60), if the input signals \( u_d, q_d, \) and \( r_d \) are differentiable and
bounded, and if there exist arbitrarily small values a, b, c, 
\(\varepsilon_k (k = 1, 2, \cdots, 6)\), then the solution of the considered TDs are

\[
\begin{align*}
\lim_{R_1 \to 0} |u_c - u_d| &\leq \varepsilon_1 t \in [a, \infty) \\
\lim_{R_2 \to 0} |\dot{u}_c - \dot{u}_d| &\leq \varepsilon_2 t \in [b, \infty) \\
\lim_{R_3 \to 0} |r_c - r_d| &\leq \varepsilon_3 t \in [c, \infty) \\
\lim_{R_4 \to 0} |q_c - q_d| &\leq \varepsilon_4 t \in [d, \infty) \\
\lim_{R_5 \to 0} |\dot{q}_c - \dot{q}_d| &\leq \varepsilon_5 t \in [e, \infty) \\
\lim_{R_6 \to 0} |\dot{r}_c - \dot{r}_d| &\leq \varepsilon_6 t \in [f, \infty) 
\end{align*}
\]  

(61)

Using \(u_c, q_c, r_c\) and \(\dot{u}_c, \dot{q}_c, \dot{r}_c\) produced by TDs, the controller can be rewritten as

\[
\begin{align*}
\delta &= |u_c| + (\dot{u}_c - \dot{u}_d) + k_9(u_c - u)\right] + \alpha_1(\dot{v}_c + \delta v_1\cos\theta)\right) + \alpha_2(\dot{v}_c + \delta v_2 - \dot{D}_w) \\
&+ r_c(\dot{r}_c - \dot{r}_d) + k_{10}(r_c - r)\right] + q_c(\dot{q}_c - \dot{q}_d) + k_{11}(q_c - q)\right] 
\end{align*}
\]  

(63)

where \(\delta\) is still bounded. And the system is still stable.

**Simulation and discussion**

In this section, the simulation and comparative analyses demonstrate the usefulness and robustness of the proposed controller. The simulation is performed in MATLABR2014a/ Simulink and the following two controllers are compared.

1. Backstepping sliding mode controller.

2. Backstepping ADRC controller combining the backstepping technique, TDs, and RLESOs (equation (62)), is marked as backstepping ADRC in the simulation figures. The time-varying error constraint is chosen as \(\rho = 3\exp(-0.3t) + 0.5\) and the control gains are chosen as \(k_1 = 0.2, k_2 = 0.1, k_3 = 0.1, k_4 = 1, k_5 = 1, k_6 = 2, k_7 = 1, k_8 = 3, k_9 = 2, k_{10} = 1, k_{11} = 10, k_{12} = 50, k_{13} = 50, k_{14} = 40, k_{15} = 30, R_1 = 15, R_2 = 8, R_3 = 9, h = 0.01\).

We conduct simulation on the underactuated AUV WeiLong-II (Figure 2) developed by Harbin Engineering University in China. The nominal values of model parameters are \(m = 45\) kg, \(I_x = 8.09\) kg·m\(^2\), \(I_z = 8.067\) kg·m\(^2\), \(X_a = -2.52\) kg, \(Y_a = -49.05\) kg,
Figure 3. (a to c) Desired trajectory and vehicle actual trajectory.

Figure 4. (a to d) Position tracking errors.

Figure 5. (a to e) Velocity errors and virtual variable errors.
Zw = −49.12 kg, Mq = −5.43 kg \cdot m^2, Nr = −5.31 kg \cdot m^2, 
Xu = 13.5 kg/s, Xu|\| = 6.44 kg/m, Yv = 44.96 kg/s, 
Yv|\| = 194.77 kg/m, Zw = 46.67 kg/s, Zw|\| = 141.66 
kg/m, Mq = 23.76 kg \cdot m^2/(s \cdot rad), Mq|\| = 23.76 
kg \cdot m^2/rad^2, Nv = 27.2 kg \cdot m^2/(s \cdot rad), and Nr|\| = 
4.09 kg \cdot m^2/rad^2. We assume that parameter perturbations 
exist and all the model parameters are increased by 20% 
with respect to their respective nominal values. The 
ocean current disturbances are set as \(V_{cx} = 0.3\)m/s, 
\(V_{cy} = 0.2\)m/s, and \(V_{cz} = 0.1\)m/s. The time-varying dis-
turbances are given as

\[
\begin{align*}
\xi_d &= 0.4\sin(0.2t) \quad \text{N} \\
\eta_d &= 0.4\sin(0.2t) \quad \text{Nm} \\
\zeta_d &= 0.4\sin(0.2t) \quad \text{Nm} \\
\end{align*}
\]

The reference trajectory is given as follows

\[
\begin{align*}
\xi_d &= 10\sin(0.1t) \quad \text{m} \\
\eta_d &= 10\cos(0.1t) \quad \text{m} \\
\zeta_d &= -0.1t \quad \text{m}
\end{align*}
\]
The AUV initial conditions are set as follows: $x_0 = 2\text{m}$, $y_0 = 8\text{m}$, $z_0 = 0\text{m}$, $\theta_0 = 0\text{rad}$, $\psi_0 = \pi/3\text{rad}$, $u_0 = 0.3\text{m/s}$, $v_0 = 0\text{m/s}$, $w_0 = 0\text{m/s}$, $q_0 = 0\text{rad/s}$, and $r_0 = 0\text{rad/s}$. It can be seen that $\rho_e(0) < \rho(0)$.

The simulation results are shown in Figures 3 to 8. As can be seen in Figure 3, the vehicle actual trajectory fluctuates around the desired trajectory under the backstepping sliding mode controller which is induced by the model uncertainties and disturbances, but the backstepping ADRC controller shows perfect performance. Clearly, the performance of the backstepping ADRC control laws is better than that of the backstepping sliding mode control laws in the presence of both internal parameter uncertainties and external time-varying disturbances.

The position error $\rho_e$ and $x_e$, $y_e$, and $z_e$ are displayed in Figure 4. It shows the proposed control laws can constrain the position tracking error of an underactuated AUV within the given boundary function all the time by using a time-varying barrier Lyapunov function and make position tracking errors converge to zero while backstepping sliding mode control method results in large tracking error.

The responses of velocity errors and virtual variable errors are shown in Figure 5. The velocity errors and virtual variable errors converge to zero rapidly and smoothly under the proposed controller, but the velocities and virtual variables fluctuate around desired values with the disturbances under the backstepping sliding mode controller. The results clearly demonstrate the effectiveness of the designed RLESOs.

The responses of actual control inputs are displayed in Figure 6. The actual values of the total disturbances and the estimation values of the RLESOs are shown in Figure 7 which demonstrates that the designed RLESOs can quickly and accurately estimate the total disturbances $D_u$, $D_v$, $D_w$, $D_q$, and $D_r$.

In addition, Figure 8 shows $u_c, r_c, q_c$ and $u_d, r_d, q_d$ produced by TDs, which can rapidly converge to $u_d, r_d, q_d$.

**Conclusion**

In this work, a backstepping ADRC controller based on the time-varying barrier Lyapunov function is developed to achieve three-dimensional trajectory tracking control of underactuated AUVs in the presence of internal parameter uncertainties and external time-varying disturbances and guarantee the satisfaction of predefined performance requirements. RLESOs are used to estimate and compensate for the total disturbances and TDs are employed to calculate the derivative of virtual control commands. From the simulation results, the proposed controller can remain satisfactory performance of an underactuated AUV in a complex environment and the simulation shows high accurate tracking capacity which demonstrates that using RLESOs can improve the performance of backstepping controller and using TDs can make the calculation of backstepping control laws simplified. Conclusions can be drawn that the backstepping ADRC controller has the ability to reject both internal parameter uncertainties and external
time-varying disturbances. Experiments will be conducted to demonstrate the effectiveness of the proposed control laws and the effect of actuator saturation and faults will be studied in the near future.

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