Superconformal anomaly free models in $D = 4$

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Abstract
A family of modified $(0,1)$ heterotic string models in $D = 4$ is constructed in which the strings incorporate $R$ flux tubes which may in special cases support a local spacetime superconformal symmetry consistent with quantum mechanics. There is an intrinsic Goldstino multiplet, so that supersymmetry breaking can be driven by any process that generates a non-zero value for the superpotential. The superconformal anomaly freedom of these models may be related to a naturally vanishing $\Lambda$. The $R$ charge of such models may also play a role in producing a generation-ordered fermion spectrum.

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1 Introduction

Our aim in this paper is to argue that strictly 4 dimensional $(0,1)$ heterotic string models have special features that may enable them to provide the propagating states for models with a local space-time superconformal invariance. By strictly 4 dimensional models we mean the sub-family of those studied in [1,2,3] which have a field independent value of $g$, the bare dimensionless 4 dimensional coupling constant. When the effective level zero field theory is
expressed in string tension units, the dilaton is replaced by a residual scalar field $\psi$, with $g$ independent of $\psi$. With suitable twisted boundary conditions, all breathing modes which have an effect on $g$ may be eliminated from the level zero spectrum allowing such models to be constructed. We may now introduce a zero norm chiral scalar superfield $\chi$ (i.e. $\chi$ has no kinetic term) to compensate the string tension in an attempt to make a model with local superconformal invariance. This will be a symmetry of the lifted metric

$$\hat{g}_{\mu\nu} = e^{\text{Re}(\chi)} g_{\mu\nu}$$

(1)

where conformal rescalings of $\hat{g}_{\mu\nu}$ can be absorbed into the compensator $\chi$ with $g_{\mu\nu}$ the metric relative to the string tension. The phase of $\chi$ compensates the phase of mass terms for the massive Ramond sector states, respecting $R$ invariance, contributing a trivial volume factor to the path integral. The spinor components of $\chi$ may be gauged away using $S$ type supersymmetry transformations, as usual for a conformal compensator [4]. The motivation here is to study supersymmetry breaking in the context of superconformal symmetry, as proves useful for conventional supergravity [4]. There is, in general, an obstruction to the validity of superconformal symmetry in conventional field theory in the form of ABJ anomalies [5,6] for $R$ transformations. By identifying $R$ with a suitable ‘handle’ charge in the 2 fermion Lie subalgebra of the Ramond Clifford algebra, it may be possible to arrange for all ABJ anomalies involving $R$ to cancel. This extra requirement would give a powerful constraint on the string models that could consistently be used in this way. A consideration of the non-perturbative defect around a string leads to a proposal that the strings should incorporate intrinsic $R$ flux tubes. This suggests that the strings may be topological features associated with a discrete fibre bundle. A consequence of these intrinsic $R$ flux tubes is that the supermultiplet containing $\psi$ becomes the Goldstone multiplet for local supersymmetry breaking, which will now occur automatically, provided that the superpotential $\mathcal{G}$ takes a non-zero value. If the v.e.v of $\mathcal{G}$ is generated by processes well below the string tension scale, this would lead to a stable realistic supersymmetry breaking mechanism. It may be possible to establish a theorem giving zero cosmological constant in the form of the somewhat stronger statement that the global superconformal invariance of the global (vacuum) state is anomaly free even though it is spontaneously broken. A characteristic feature of this realisation of superconformal invariance is that the antisymmetric tensor counterpart of the graviton is eliminated by local $R$ invariance. The $R$ spectrum of the chiral fermions could provide an
organising principle for the phenomenological generation structure, with extra insertions needed to produce couplings of the Higgs boson to the lower generations.

2 Modified String Model

Consider a string presented in cosmic configuration [7]. A feature of strictly 4 dimensional strings is that they produce a non-perturbative geometric defect, and also a defect in the pseudoscalar $\phi$ dual to $B_{\mu\nu}$ around a loop linking the string worldsheet.

The string tension $T$ and the Regge slope $\alpha'$ for the (0,1) heterotic string are related by [8]

$$T^{-1} = 4\pi\alpha',$$

while $\alpha'$ is related to Newton’s constant $G$ by [8]

$$\alpha' = \frac{16\pi G}{g^2}.$$  

The angular deficit around a string in cosmic configuration is given by [9],

$$\delta = 8\pi GT$$

using the Einstein field equations. This gives a non-perturbative defect around the string site, analogous to the defect around a massive particle in 2+1 dimensions [10]. We now define a modification to the properties of a string by applying the condition for an unbroken $N = 1/2$ supersymmetry around the string site, using the $R$ gauge connection of local superconformal symmetry to compensate for the Lorentz connection defect. This is defined so that the spacetime supersymmetry components right-moving at the string, i.e. those that couple locally to the string, are taken to be unbroken by the net effect of the non-perturbative defects around the string. (This condition is reminiscent of the partial survival of supersymmetry associated with BPS saturation, and may indicate a self-duality property.) This modification would enable supersymmetry breaking to be driven by mechanisms operating at well below the string tension scale, as in the following section. For the Lorentz connection around the string to cancel the $R$ connection on these generators we need
\[ \int_C A_R = \delta \]  

(5)

where \( \delta \) is the deficit angle around the string. The physical interpretation of eq.(5) is that the strings incorporate a flux tube of strength \( \delta \) for the magnetic field of \( A_R \). The field strength \( H \) around a cosmic configuration string may be interpreted as follows. Introducing cylindrical coordinates \((t, r, \theta, z)\) centred on the string, \( H \) is parallel to the surfaces of constant \( \theta \). The coupled equations for gravity and \( H \) are equivalent to an Einstein-Cartan system with \( H \) representing a totally antisymmetric torsion which Ricci flattens the connection on surfaces of constant \( \theta \). The transverse metric on surfaces with \( r, t \) constant remains flat with azimuthal defect angle \( \delta \). (Although a classical static solution is singular at the string, the quantum string theory is, as usual, finite.)

As regards \( \phi \), a straightforward calculation gives [7]

\[ \Delta \equiv \frac{1}{g} \int_C *H = -\frac{g}{8}. \]  

(6)

Suppose now that the dual field \(*H\) can be described in terms of the pseudoscalar field \( \phi \) as

\[ *H = d\phi + C\omega_R^{(1)} = d\phi + CA_R. \]  

(7)

Here we have extended the global gauge invariance of \( \phi \), which arises since the absolute value of \( \phi \) is irrelevant, to a local symmetry by using the axial vector \( U(1) \) gauge field \( A_R \), which appears as the axial auxiliary field in conventional \( N = 1 \) supergravity [4]. By introducing this gauge compensation of \( \phi \) by \( A_R \) it is now possible for \( \phi \) to be single valued around a string, repairing the defect noted in [7], giving \( C = g\Delta/\delta = -g^2/8\delta \), and we may make a natural gauge choice \( \phi = 0 \). This means that local \( R \) gauge symmetry now removes \( \phi \) (and thus \( B_{\mu\nu} \)) from the physical spectrum.

We now face a problem with the consistency of \( R \) symmetry at the quantum level due to ABJ anomalies [5,6]. Diagrams with one \( R \) plus two Yang-Mills or two Lorentz currents, or three \( R \) currents are potentially anomalous. To address this problem we need a suitable realisation of \( R \) symmetry within our model. The \( R \) transformation produces a \( \gamma_5 \) ‘rotation’ on the supersymmetry generator \( Q_\alpha \) which must be realised on the right-moving current algebra. One way would be to realise \( \gamma_5 \) directly, using the Clifford algebra.
generated by $DX^\mu$. However, all the fermions would then contribute with the same sign to the potentially anomalous diagrams. In any event, all the gauge fermions will contribute with the same sign. This suggests the possibility that the $R$ anomalies are cancelled by the contribution of the generation and Higgs fermions. Another possibility would be to invoke a Green-Schwarz cancellation mechanism [11], but this may not work as $B_{\mu\nu}$ is removed from the spectrum. As well as the 4 spacetime coordinate superfields on the string worldsheet, there will be 6 internal superfields $Y^m$. The fermion components $DY^m$ generate a Clifford algebra, which includes a Lie subalgebra $T$ generated by $[DY^m, DY^n]$. The $R$ charge may be realised as a ‘handle’ charge within this algebra. This requires that $R$ acts appropriately on the gravitino $\psi_\mu$. The right-moving factor of $\psi_{\mu
u}$ is a spinor $\psi_R$ in the Ramond ground state, which carries $T$ charges, so we can take $R \in T$. Let $R_0$ be the weight of $\psi_R$ in $T$. The most obvious choice would be to set $R = R_0$. However, it seems unlikely that this gives enough freedom to arrange for all the anomalies to cancel. However, there will be a $SU(3)$ subalgebra $S$ of $T$ which stabilises $\psi_R$, and we may take $R = R_0 + R_1$ with $R_1 \in S$. It is now possible for the spectrum of $R$ on the non-gauge fermions to be both biased and dispersed, so that a complete cancellation of $R$ anomalies may be possible for some special choices of twisted boundary condition sets for the strings. The group $U$ generating the twisted boundary conditions would commute with $R$. Because $R$ lies within the right-moving current algebra, $R$ will be defined on all states, with chiral representations (and thus contributions to the anomalies of $R$) restricted to level zero. Note also that the scalar component of the conformal compensator $\chi$ has $R = 0$ and so leaves the breaking of $R$ to $\phi$. A by-product of this construction is that a realistic generation structure may be organised by the need for $R$ breaking string-scale insertions of $\exp(i\phi)$ for Higgs couplings, leading to generation indexed coupling matrices of the form

$$M_{jk} = O(I^{R(j) + R(k)})g$$

where $I$ represents an insertion factor. The strings in such a superconformal model may have a topological significance. Express the fractional deficit angle as

$$\frac{\delta}{2\pi} = \frac{1}{\nu}.$$  

If $\nu = N$, an integer, this suggests an interpretation of the string worldsheet as a branching site for a $Z_N$ bundle, just as a set of branch points defines
a Riemann surface. This interpretation would be relevant to amplitudes for processes involving linked strings, possibly selecting the $\Gamma_N$ subgroup as the relevant modular invariance group in situations where only the $N$th power of a Dehn twist would be $\mathbb{Z}_N$ trivial - otherwise, modular invariance in the presence of a background string could impose further restrictions on the choice of twisted boundary conditions. Note that $N$ would determine the value of the dimensionless bare coupling constant $g$, which can otherwise take an arbitrary value up to $4\pi$, corresponding to $\delta = 2\pi$.

3 Supersymmetry Breaking

Since we have selected models with $g$ field independent, the gaugino condensation mechanism for supersymmetry breaking [12,13] will not work. Consider, however, the supersymmetry variation of the gravitino field $\psi$ which is given by

$$\delta \psi = D\epsilon = d\epsilon + A_R \gamma_5 \epsilon.$$  \hspace{1cm} (10)

With the natural gauge choice $\phi = 0$, we have $A_R = \pi^{-1}D\phi$ and eq.(10) becomes

$$\delta \psi = d\epsilon + \frac{1}{\pi} \gamma_5 \epsilon D\phi$$ \hspace{1cm} (11)

so that the global ($d\epsilon = 0$) variation of the Goldstino fermion $\chi$ is given by

$$\delta \chi = \gamma \cdot \delta \psi = \frac{1}{\pi} \gamma \cdot (D\phi) \gamma_5 \epsilon = \frac{1}{\pi} \delta \zeta$$ \hspace{1cm} (12)

where $\zeta$ is the superpartner of $\phi$, so that we may identify $\chi = \pi^{-1}\zeta$, giving a natural candidate for the Goldstino multiplet in the vacuum state. The Goldstone supermultiplet and the graviton multiplet together are thus given by the spin $(1)_L \otimes (1 \oplus 1/2)_R$ heterotic level zero fields [14]. The scalar/pseudoscalar part is described by the complex field $z = \psi + i\phi$. Supersymmetry breaking is characterised by the superpotential $\mathcal{G}$. Note that the gaugino condensation mechanism is ruled out in strictly $D = 4$ models by the field independence of $g$. The normalization of the kinetic term of $\phi$ through eq.(??), together with the fact that $\mathcal{G}$ should be independent of $\phi$, which can be gauged away, gives

$$\mathcal{G} = \frac{1}{2g^2} \psi^2 + \mathcal{G}_1$$ \hspace{1cm} (13)
with $G_1$ independent of $z$. This gives the $z$ space a flat, cylindrical Kahler geometry. $G$ has a maximum at $\psi = 0$, with $m_\psi = m_{3/2}$. We may make a comparison with the situation in conventional $N = 1$ supergravity where the potential depends on scalar fields through

$$V = e^{-G}(\text{Tr}[(\partial \partial^* G)^{-1} \partial G \partial^* G] - 3) \quad (14)$$

where the (anti-)holomorphic differentials $(\partial^*, \partial)$ are defined on chiral field space. For a given gravitino mass $m_{3/2}$ a pure supergravity theory is obtained in anti-deSitter space with curvature $-3m_{3/2}^2$ and a flat spacetime is only obtained with the bracing effect of a nonzero $\partial G$, with the direction of $\partial G$ in field space identifying the Goldstino multiplet. Our model uses the zero norm tension compensator $\chi$ and the usual negative norm scalar compensator for conformal symmetry in conventional $N = 1$ supergravity is not present. The Goldstino can be identified in our model even at a stationary value of $G$, where it becomes $\zeta$. Since now $\partial G = 0$ at the potential minimum, no cancelling negative term is required for $\Lambda = 0$, and indeed without the negative norm compensator, eq. (14) takes the form

$$V = e^{-\psi}\text{Tr}[(\partial \partial^* G)^{-1} \partial G \partial^* G] \quad (15)$$

and the balance between large terms of opposite sign is no longer needed. The gravitino mass in the vacuum state will be given by

$$m_{3/2}^2 = e^{-\psi_0} \quad (16)$$

where $G_0$ is the maximum value of $G$. This gives stable supersymmetry breaking, which would be at a realistic scale for a suitable value of $G_0$. Such a value might be generated by renormalization group effects. The cosmological constant may vanish naturally in this model, due to global superconformal symmetry. The global superconformal invariance of the action under finite rescalings of $g_{\mu\nu}$ rather than of $\hat{g}_{\mu\nu}$ would be used to demonstrate this. The same brane/flux tube mechanism that cancels the anomalies of $R$ invariance may also make this symmetry viable at the quantum level. For this strategy to work, it would be necessary to apply the principle of global superconformal invariance to show that the maximum of $G$ lies on the surfaces $D^\alpha = 0$ for all Yang-Mills generators $\alpha$, as without $D$ terms the potential eq. (15) would already give $\Lambda = 0$. 

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