Probing the QGP with charm at ALICE—LHC∗

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Abstract

The exclusive reconstruction of D0 mesons in the ALICE experiment allows to study the QCD energy loss of charm quarks in the deconfined quark–gluon plasma (QGP) medium expected to be produced in central nucleus–nucleus collisions at the Large Hadron Collider.

1 Introduction

The ALICE experiment [2] at the LHC will study nucleus–nucleus (AA) collisions at a centre-of-mass energy \( \sqrt{s_{NN}} = 5.5 \) TeV (for Pb–Pb) per nucleon–nucleon (NN) pair in order to investigate the properties of QCD matter at energy densities of few hundred times the density of atomic nuclei. In these conditions a deconfined state of quarks and gluons is expected to be formed [3].

Hard partons and heavy quarks, abundantly produced at LHC energies in initial hard scattering processes, are sensitive probes of the medium formed in the collision as they may lose energy by gluon bremsstrahlung while propagating through the medium itself. The attenuation (quenching) of leading hadrons and jets observed at RHIC [4] is thought to be due to such a mechanism. The large masses of the charm and beauty quarks make them qualitatively different probes, since, on well-established QCD grounds, in-medium energy loss off massive partons is expected to be significantly smaller than off ‘massless’ partons (light quarks and gluons). Therefore, a comparative study of the attenuation of massless and massive probes is a promising tool.

∗This talk was presented in the New Talents Session at the “41st International School of Subnuclear Physics, 2003” in Erice (Italy) and selected for publication in the proceedings of the School. The present paper is an extract from Ref. [1], where more details on the subject can be found.
to test the coherence of the interpretation of quenching effects as energy loss in a deconfined medium and to further investigate the properties (density) of such medium.

In the first part of this paper, we shortly summarize one of the widely used models of parton energy loss and we discuss how we used it in our simulation. In the second part, we show that the exclusive reconstruction of $D^0 \rightarrow K^- \pi^+$ decays with ALICE allows to carry out the mentioned comparative quenching studies by measuring the nuclear modification factor of the $D$ mesons transverse momentum ($p_t$) distribution

$$R_{AA}(p_t) \equiv \frac{dN_{AA}/dp_t/\text{binary NN collision}}{dN_{pp}/dp_t},$$

which would be 1 if the AA collision was a mere superposition of independent NN collisions without nuclear or medium effects, and the $D$/charged hadrons ($D/h$) ratio

$$R_{D/h}(p_t) \equiv \frac{R_{D}^{AA}(p_t)}{R_{AA}(p_t)}.$$  

### 2 Parton energy loss and the dead cone effect for heavy quarks

In this work we use the quenching probabilities (or weights) calculated by C.A. Salgado and U.A. Wiedemann [5] in the framework of the ‘BDMPS’ formalism [6], which we summarize in the following. The energy loss obtained with the quenching weights is presented in Section 3.

An energetic parton produced in a hard collision radiates a gluon with a probability proportional to its path length $L$ in the dense medium. Then (Fig. 1, left) the radiated gluon undergoes multiple scatterings in the medium, in a Brownian-like motion with mean free path $\lambda$ which decreases as the density of the medium increases. The number of scatterings of the radiated gluon is also proportional to $L$. Therefore, the average energy loss of the parton is proportional to $L^2$.

The scale of the energy loss is set by the ‘maximum’ energy of the radiated gluons, which depends on $L$ and on the properties of the medium:

$$\omega_e = \hat{q} L^2 / 2,$$

where $\hat{q}$ is the transport coefficient of the medium, defined as the average transverse momentum squared transferred to the projectile per unit path length, $\hat{q} = \langle q_t^2 \rangle_{\text{medium}} / \lambda$ [4].
In the case of a static medium, the distribution of the energy \( \omega \) of the radiated gluons (for \( \omega \ll \omega_c \)) is of the form:

\[
\frac{dI}{d\omega} \sim \frac{2 \alpha_s C_R}{\pi} \sqrt{\frac{\omega_c}{2\omega}},
\]

where \( C_R \) is the QCD coupling factor (Casimir factor), equal to 4/3 for quark–gluon coupling and to 3 for gluon–gluon coupling. The integral of the energy distribution up to \( \omega_c \) estimates the average energy loss of the initial parton:

\[
\langle \Delta E \rangle = \int_{\omega_c}^{\omega_c} \omega \frac{dI}{d\omega} d\omega \propto \alpha_s C_R \omega_c \propto \alpha_s C_R \hat{q} L^2.
\]

The average energy loss is therefore: proportional to \( \alpha_s C_R \) and, thus, larger by a factor \( 9/4 = 2.25 \) for gluons than for quarks; proportional to the transport coefficient of the medium; proportional to \( L^2 \); independent of the parton initial energy. The last point is peculiar to the BDMPS model. Other models consider an explicit dependence of \( \Delta E \) on the initial energy \( E \). However, there is always an intrinsic dependence of the radiated energy on the initial energy, determined by the fact that the former cannot be larger than the latter, \( \Delta E \leq E \).

The transport coefficient is proportional to the density of the scattering centres and to the typical momentum transfer in the gluon scattering off these centres. Figure (right) reports the estimated dependence of \( \hat{q} \) on the
energy density $\epsilon$ for different equilibrated media [7]: for cold nuclear matter (marker) the estimate is $\hat{q}_{\text{cold}} \simeq 0.05 \text{ GeV}^2/\text{fm}$; for a QGP formed at the LHC with $\epsilon \sim 50–100 \text{ GeV}/\text{fm}^3$, $\hat{q}$ is expected to be of $\simeq 5–10 \text{ GeV}^2/\text{fm}$.

In Ref. [10] Yu.L. Dokshitzer and D.E. Kharzeev argue that for heavy quarks, because of their large mass, the radiative energy loss should be lower than for light quarks. The predicted consequence of this effect is an enhancement of the ratio of D mesons to pions (or hadrons in general) at moderately large ($5–10 \text{ GeV}/c$) transverse momenta, with respect to what observed in the absence of energy loss (proton–proton collisions).

Heavy quarks with momenta up to 20–30 GeV/c propagate with a velocity which is smaller than the velocity of light. As a consequence, gluon radiation at angles $\Theta$ smaller than the ratio of their mass to their energy $\Theta_0 = m/E$ is suppressed by destructive quantum interference. The relatively depopulated cone around the heavy quark direction with $\Theta < \Theta_0$ is indicated as ‘dead cone’ [11].

In Ref. [10] the dead cone effect is assumed to characterize also in-medium gluon radiation and the energy distribution of the radiated gluons [4], for heavy quarks, is estimated to be suppressed by the factor:

$$\frac{dI}{d\omega} \bigg|_{\text{Heavy}} / \frac{dI}{d\omega} \bigg|_{\text{Light}} = \left[ 1 + \frac{\Theta_0^2}{\Theta^2} \right]^{-2} = \left[ 1 + \left( \frac{m}{E} \right)^2 \sqrt{\frac{\omega^3}{\hat{q}}} \right]^{-2} \equiv F_{\text{H/L}}, \quad (6)$$

where the expression for the characteristic gluon emission angle [10] $\Theta \simeq (\hat{q}/\omega^3)^{1/4}$ has been used. The heavy-to-light suppression factor $F_{\text{H/L}}$ in (6) increases (less suppression) as the heavy quark energy $E$ increases (the mass becomes negligible) and it decreases at large $\omega$, indicating that the high-energy part of the gluon radiation spectrum is drastically suppressed by the dead cone effect.

### 3 Simulation of energy loss

The Salgado–Wiedemann (SW) quenching weight is defined as the probability that a hard parton radiates an energy $\Delta E$ due to scattering in spatially extended QCD matter. In Ref. [5], the weights are calculated on the basis of the BDMPS formalism, keeping into account both the finite in-medium path length $L$ and the dynamic expansion of the medium after the nucleus–nucleus collision. The input parameters for the calculation are the length $L$, the transport coefficient $\hat{q}$ and the parton species (light quark or gluon).

The distribution of the in-medium path length in the plane transverse to
Figure 2: Distribution of the path lengths in the transverse plane for partons produced in Pb–Pb collisions with $b < 3.5$ fm (left). Average energy loss as a function of the transport coefficient (right).

the beam line\(^1\) for central Pb–Pb collisions (impact parameter $b < 3.5$ fm, corresponding to the 5% most central collisions) is calculated in the framework of the Glauber model of the collision geometry \[^{12}\]. For a given impact parameter, hard parton production points are sampled according to the density $\rho_{\text{coll}}(x, y)$ of binary nucleon–nucleon collisions in the transverse plane and their azimuthal propagation directions are sampled uniformly. For a parton with production point $(x_0, y_0)$ and azimuthal direction $(u_x, u_y)$, the path length is defined as:

$$L = \frac{\int_0^\infty dl \rho_{\text{coll}}\left(x_0 + l u_x, y_0 + l u_y\right)}{0.5 \int_0^\infty dl \rho_{\text{coll}}\left(x_0 + l u_x, y_0 + l u_y\right)}. \quad (7)$$

Many sampling iterations are performed varying the impact parameter $b$ from 0.25 fm to 3.25 fm in steps of 0.5 fm. The obtained distributions are given a weight $b$, since we verified that $d\sigma_{\text{hard}}/db \propto b$ for $b < 3.5$ fm, and added together. The result is shown in Fig. 2 (left). The average length is 4.5 fm, corresponding to about 70% of the radius of a Pb nucleus and the distribution is significantly accumulated towards low values of $L$ because a large fraction of the partons are produced in the periphery of the superposition region of the two nuclei (‘corona’ effect).

For a given value of the transport coefficient $\hat{q}$ and a given parton species, we use the routine provided in Ref. \[^5\] to get the energy loss probability distribution $P(\Delta E; L)$ for the integer values of $L$ up to 15 fm. Then, these 15

\(^1\)Partons produced at central rapidities propagate in the transverse plane.
distributions are weighted according to the path length probability in Fig. 2 and added together to obtain a global energy loss probability distribution $P(\Delta E)$. The energy loss to be used for the quenching simulation can be directly sampled from the $P(\Delta E)$ distribution corresponding to the chosen $\hat{q}$ and to the correct parton species.

The predicted lower energy loss for charm quarks is accounted for by multiplying the $P(\Delta E)$ distribution for light quarks with the dead cone suppression factor $F_{H/L}$ in (6). It was verified that this approximation is equivalent to recalculate the SW quenching weights with the gluon energy distribution for heavy quarks as given in (6) [13, 14]. Since $F_{H/L}$ depends on the heavy quark energy, the product has to be done for each $c$ quark or, more conveniently, in bins of $p_t$. Figure 2 (right) reports the average energy loss as a function of the transport coefficient for light quarks and for charm quarks ($m_c = 1.2$ GeV) with $p_t = 1–2, 10, 20$ GeV/$c$, as obtained with the described dead cone correction ($p_t$-dependent $P(\Delta E) \otimes F_{H/L}$ product). With $\hat{q} = 4$ GeV$^2$/fm, our estimated transport coefficient for the LHC (see next paragraph), the average energy loss for light quarks is $\langle \Delta E \rangle \approx 35$ GeV (the effective value of $\langle \Delta E \rangle$ is lower by about a factor 2, due to the constraint $\Delta E \leq E$). For $c$ quarks of $1–2, 10, 20$ GeV/$c$ $\langle \Delta E \rangle$ is about 2%, 10% and 20%, respectively, of the average loss for light quarks.

For the estimation of the transport coefficient $\hat{q}$ for our simulation, we consider that it is reasonable to require for central nucleus–nucleus collisions at the LHC a quenching of hard partons at least of the same magnitude as that observed at RHIC. We, therefore, derive the nuclear modification factor $R_{AA}$ for charged hadrons produced at the LHC and we choose the transport coefficient in order to obtain $R_{AA} \approx 0.2–0.3$ in the range $p_t = 5–10$ GeV/$c$ (for RHIC results see e.g. Refs. [11, 15]).

The transverse momentum distributions, for $p_t > 5$ GeV/$c$, of charged hadrons are generated by means of the chain:

1. generation of a parton, quark or gluon, with $p_t > 5$ GeV/$c$, using PYTHIA [16] proton–proton with $\sqrt{s} = 5.5$ TeV and CTEQ 4L parton distribution functions; with these parameters, the parton composition given by PYTHIA is 78% gluons and 22% quarks;

2. sampling of an energy loss $\Delta E$ according to $P(\Delta E)$ and calculation of the quenched transverse momentum of the parton, $p'_t = p_t - \Delta E$ (if $\Delta E > p_t$, $p'_t$ is set to 0);

3. (independent) fragmentation of the parton to a hadron using the leading order Kniehl-Kramer-Pötter (KKP) fragmentation functions [17].
Quenched and unquenched $p_t$ distributions are obtained including or excluding the second step of the chain. $R_{AA}$ is calculated as the ratio of the $p_t$ distribution with quenching to the $p_t$ distribution without quenching. We find $R_{AA} \simeq 0.25$–0.3 in $5 < p_t < 10$ GeV/$c$ for $\hat{q} = 4$ GeV$^2$/fm. This value is reasonable, as it corresponds, using the plot in Fig. 1 to an energy density $\epsilon \simeq 40$–50 GeV/fm$^3$, which is about a factor 2 lower than the maximum energy density expected for central Pb–Pb collisions at the LHC.

Charm quarks are generated using PYTHIA, tuned in order to reproduce the single-inclusive $c$ (and $\bar{c}$) $p_t$ distribution predicted by the pQCD program HVQMNMR [18] with $m_c = 1.2$ GeV and $\mu_{\text{Fact.}} = \mu_{\text{Renorm.}} = 2 \cdot m_t \equiv 2 \sqrt{m_c^2 + p_t^2}$ (the details on this tuning can be found in Ref. [19]). We use the CTEQ 4L parton distribution functions including the nuclear shadowing effect by means of the EKS98 parameterization [20] and the parton intrinsic transverse momentum broadening as reported in Ref. [19]. Energy loss for charm quarks is simulated following a slightly different procedure with respect to that for light quarks and gluons. Since the total number of $c\bar{c}$ pairs per event has to be conserved, in the cases where the sampled $\Delta E$ is larger than $p_t$, we assume the $c$ quark to be thermalized in the medium and we give it a transverse momentum according to the distribution $dN/dm_t \propto m_t \exp(-m_t/T)$. We use $T = 300$ MeV as the thermalization temperature. The other difference with respect to the previous case is that we use the standard string model in PYTHIA for the $c$ quark fragmentation.

4 Charm reconstruction with ALICE

The transverse momentum distribution of charm mesons produced at central rapidity, $|y| < 1$, can be directly measured with ALICE from the exclusive reconstruction of $D^0 \rightarrow K^-\pi^+$ (and charge conjugates). The displaced vertices of $D^0$ decays ($c\tau = 124$ $\mu$m) can be identified in the ALICE Inner Tracking System, that provides a measurement of the track impact parameters to the collision vertex with a resolution better than 50 $\mu$m for $p_t > 1$ GeV/$c$. The low value of the magnetic field (0.4 T) and the K/$\pi$ separation in the ALICE Time of Flight allow to extend the measurement of the $D^0$ production cross section down to almost 0 transverse momentum. The strategy for this analysis and the selection cuts to be applied were studied with a realistic and detailed simulation of the detector geometry and response, including the main background sources [14, 21].

The expected performance for central Pb–Pb ($b < 3.5$ fm) at $\sqrt{s_{\text{NN}}} = 5.5$ TeV and pp collisions at $\sqrt{s} = 14$ TeV, as obtained using the input production yields $N_{\text{Pb–Pb}} = 115$ and $N_{\text{pp}} = 0.16$ (see Ref. [19]), is summarized in
Figure 3: Double differential cross section per nucleon–nucleon collision for $D^0$ production as a function of $p_t$, as it can be measured with $10^7$ central Pb–Pb events (left) and $10^9$ pp minimum-bias events (right). Statistical (inner bars) and $p_t$-dependent systematic errors (outer bars) are shown. A normalization error of 11% for Pb–Pb and 5% for pp is not shown.

Fig. 3 The accessible $p_t$ range is 1–14 GeV/c for Pb–Pb and 0.5–14 GeV/c for pp. In both cases the statistical error (corresponding to 1 month of data-taking for Pb–Pb and to 9 months for pp) is better than 15–20% and the systematic error (acceptance and efficiency corrections, subtraction of the feed-down from $B \to D^0 + X$ decays, cross section normalization, centrality selection for Pb–Pb) is better than 20%. More details are given in Ref. [14].

5 Results: $R_{AA}$ and $R_{D/h}$

The nuclear modification factor for $D^0$ mesons is reported in Fig. 3. Nuclear shadowing, parton intrinsic transverse momentum broadening and energy loss are included. The dead cone effect is not included in the left-hand panel and included in right-hand panel. Different values of the transport coefficient are used for illustration; we remind that the value expected on the basis of the pion quenching observed at RHIC is $\hat{q} = 4$ GeV$^2$/fm. The reported statistical (bars) and systematic (shaded area) errors are obtained combining the previously-mentioned errors in Pb–Pb and in pp collisions and
considering that the contributions due to cross section normalization, feeddown from beauty decays and, partially, acceptance/efficiency corrections will cancel out in the ratio. An uncertainty of about 5% introduced in the extrapolation of the pp results from 14 TeV to 5.5 TeV by pQCD is also accounted for (see Ref. [14]).

The effect of shadowing, clearly visible for \( \hat{q} = 0 \) (no energy loss) as a suppression of \( R_{AA} \), is limited to \( p_t < 6-7 \) GeV/c. Above this region only (possible) parton energy loss is expected to affect the nuclear modification factor of D mesons.

For \( \hat{q} = 4 \) GeV\(^2\)/fm and no dead cone, we find \( R_{AA} \) reduced, with respect to 1, by a factor about 3 and slightly increasing with \( p_t \), from 0.3 at 6 GeV/c to 0.4 at 14 GeV/c. Even for a transport coefficient lower by a factor 4,
\( \hat{q} = 1 \text{ GeV}^2/\text{fm} \), \( R_{AA} \) is significantly reduced (0.5–0.6). When the dead cone effect is taken into account, the \( R_{AA} \) reduction due to quenching is found to be lower by about a factor 1.5–2.5, depending on \( \hat{q} \) and \( p_t \).

We point out that the estimated systematic uncertainty of about 18\% may prevent from discriminating between a scenario with moderate quenching and negligible dead cone effect (e.g. \( \hat{q} = 1 \text{ GeV}^2/\text{fm} \) in the left-hand panel of Fig. 4) and a scenario with large quenching but also strong dead cone effect (e.g. \( \hat{q} = 4 \text{ GeV}^2/\text{fm} \) in the right-hand panel).

The comparison of the quenching of charm-quark-originated mesons and massless-parton-originated hadrons will be the best suited tool to disentangle the relative importance of energy loss and dead cone effects. The \( D/\text{charged hadrons} \) (\( D/h \)) ratio, defined as in (2), is presented in Fig. 5 for the range \( 5 < p_t < 14 \text{ GeV}/c \). We used \( R_{AA}^{ch} \) calculated as previously described and \( R_{AA}^{Dh} \), without and with dead cone, as reported in Fig. 4. Being essentially a double ratio \( \text{Pb–Pb}/\text{Pb–Pb} \times \text{pp}/\text{pp} \), this observable is particularly sensitive, as many systematic uncertainties cancel out (centrality selection and, partially, acceptance/efficiency corrections and energy extrapolation by pQCD). The residual systematic error is estimated to be of about 10–11\%.

We find that, if the dead cone correction for \( c \) quarks is not included, \( R_{D/h} \) is essentially 1 in the considered \( p_t \) range, independently of the value of the transport coefficient, i.e. of the magnitude of the energy loss effect. When the dead cone is taken into account, \( R_{D/h} \) is enhanced of a factor strongly dependent on the transport coefficient of the medium: e.g. 2–2.5 for \( \hat{q} = 4 \text{ GeV}^2/\text{fm} \) and 1.5 for \( \hat{q} = 1 \text{ GeV}^2/\text{fm} \). The enhancement is decreasing with \( p_t \), as expected (the \( c \) quark mass becomes negligible).

The \( R_{D/h} \) ratio is, therefore, found to be enhanced, with respect to 1, only by the dead cone and, consequently, it appears as a very clean tool to investigate and quantify this effect.

Since hadrons come mainly from gluons while \( D \) mesons come from (c) quarks, the \( D/h \) ratio should, in principle, be enhanced also in absence of dead cone effect, as a consequence of the larger energy loss of gluons with respect to quarks. Such enhancement is essentially not observed in the obtained \( R_{D/h} \) because it is ‘compensated’ by the harder fragmentation of charm quarks with respect to light quarks and, particularly, gluons. With \( z \) the typical momentum fraction taken by the hadron in the fragmentation, \( p_t^{\text{hadron}} \) and \( \Delta E \) the average energy loss for the parton, \( (p_t^{\text{parton}})' = p_t^{\text{parton}} - \Delta E \), we have

(8) \( (p_t^{\text{hadron}})' = p_t^{\text{hadron}} - z \Delta E \),

meaning that the energy loss observed in the nuclear modification factor is,
indeed, \( z \Delta E \). We have, thus, to compare \( z_{c \rightarrow D} \Delta E_c \) to \( z_{\text{gluon} \rightarrow \text{hadron}} \Delta E_{\text{gluon}} \). With \( z_{\text{gluon} \rightarrow \text{hadron}} \approx 0.4 \), \( z_{c \rightarrow D} \approx 0.8 \) for \( p^D_{t,h} > 5 \text{ GeV/c} \) and \( \Delta E_c = \Delta E_{\text{gluon}}/2.25 \) (without dead cone), we obtain

\[
z_{c \rightarrow D} \Delta E_c \approx 0.9 z_{\text{gluon} \rightarrow \text{hadron}} \Delta E_{\text{gluon}}.
\] (9)

This simple estimate confirms that the quenching for D mesons is almost the same as for (non-charm) hadrons, if the dead cone effect is not considered.

The errors reported in Fig. 5 show that ALICE is expected to have good capabilities for the study of \( R_{D/h} \): in the range \( 5 < p_t < 10 \text{ GeV/c} \) the enhancement due to the dead cone is an effect of more than 3 \( \sigma \) for \( \hat{q} > 1 \text{ GeV}^2/\text{fm} \). The comparison of the values for the transport coefficient extracted from the nuclear modification factor of charged hadrons and, independently, from the \( D/\text{charged hadrons} \) ratio shall provide an important test for the coherence of our understanding of the energy loss of hard probes propagating in the dense QCD medium formed in Pb–Pb collisions at the LHC.

Acknowledgements

I am grateful to F. Antinori, A. Morsch, G. Paic, K. Šafařík, C.A. Salgado and U.A. Wiedemann for many fruitful discussions and to Prof. A. Zichichi and Prof. G. ’t Hooft for providing me with the opportunity to present my work in the New Talents session of the 41st International School of Subnuclear Physics in Erice.

References

[1] A. Dainese, submitted to Eur. Phys. J. [arXiv:nucl-ex/0312005].
[2] ALICE Technical Proposal, CERN/LHCC 95-71 (1995).
[3] F. Karsch, these proceedings.
[4] T.W. Ludlam, these proceedings.
[5] C.A. Salgado and U.A. Wiedemann, Phys. Rev. D68 (2003) 014008 [arXiv:hep-ph/0302184]; http://csalgado.home.cern.ch/csalgado
[6] R. Baier, Yu.L. Dokshitzer, A.H. Mueller, S. Peigné and D. Schiff, Nucl. Phys. B483 (1997) 291; ibidem B484 (1997) 265.
[7] R. Baier, Nucl. Phys. A715 (2003) 209c.
[8] M. Gyulassy and X.N. Wang, Nucl. Phys. B420 (1994) 583
[arXiv:nucl-th/9306003].

[9] M. Gyulassy, P. Lévai and I. Vitev, Nucl. Phys. B571 (2000) 197; Phys.
Rev. Lett. 85 (2000) 5535; Nucl. Phys. B594 (2001) 371.

[10] Yu.L. Dokshitzer and D.E. Kharzeev, Phys. Lett. B519 (2001) 199
[arXiv:hep-ph/0106202].

[11] Yu.L. Dokshitzer, V.A. Khoze and S.I. Troyan, J. Phys. G17 (1991)
1602.

[12] R.J. Glauber and G. Matthiae, Nucl. Phys. B21 (1970) 135.

[13] C.A. Salgado and U.A. Wiedemann, private communication.

[14] A. Dainese, Ph.D. Thesis, [arXiv:nucl-ex/0311004].

[15] J. Jia et al., PHENIX Coll., Nucl. Phys. A715 (2003) 769c.

[16] T. Sjöstrand et al., Computer Phys. Commun. 135 (2001) 238
[arXiv:hep-ph/0010017].

[17] B.A. Kniehl, G. Kramer and B. Pötter, Nucl. Phys. B582 (2000) 514
[arXiv:hep-ph/0010289].

[18] M. Mangano, P. Nason and G. Ridolfi, Nucl. Phys. B373 (1992) 295.

[19] N. Carrer and A. Dainese, ALICE Internal Note, ALICE-INT-2003-019
[arXiv:hep-ph/0311225].

[20] K.J. Eskola, V.J. Kolhinen, C.A. Salgado, Eur. Phys. J. C9 (1999) 61.

[21] N. Carrer, A. Dainese and R. Turrisi, J. Phys. G29 (2003) 575.