Taylor–Dean instability between two concentric rotating spheres

T. Tamsaout and A. Bouabdallah

Thermodynamics and Energetical Systems Laboratory, Faculty of Physics –
University of Sciences and Technology, Algiers – Algeria.

E-mail: ttamsaout@hotmail.com, abouab2002@yahoo.fr

Abstract. There are many theoretical and experimental investigations devoted to study the
Taylor–Dean Instability generated between two horizontal concentric cylinders with a partially
filled gap. We note that, to our best knowledge, the same phenomenon is not studied in case of
spherical Couette–Taylor flow. Our goal consists in analyzing by photometry technique the
appearance of Taylor–Dean instability in laminar–turbulent transition regime. It is shown that
the first instability corresponds to the stationary Taylor–Dean cells and the second corresponds
to the instationary cells. On the basis of these qualitative properties, we impose a controlled
effect on the flow induced by the free surface defined by a given aspect ratio \( \Gamma = \frac{H}{d} \) on the
onset of each instability. Finally, we analyze and discuss the condition of the onset of
instabilities in function of the aspect ratio \( \Gamma \) and the Taylor number \( Ta \).

Keywords: Taylor–Dean instability, Spherical Couette flow, Aspect factor, Taylor number.

1. Introduction

W.R. Dean [1] studied firstly the case where the motion of viscous fluid in the fully filled annulus
of concentric rotating cylinders was driven by an azimuthal pressure gradient as a model for curved
channel flow. The curved channel flow was then called Dean Flow and received much attention in the
past few years [2] [3] [4], so that new ideas have been formulated for the stability of open flows and
shear flows [5].

Up to now, many theoretical and experimental investigations are carried out, without studying, to
our best knowledge the same phenomenon in the case of spherical Taylor–Couette flow. Our objective
consists in analyzing by photometry technique the influence of the aspect factor \( \Gamma \) on the occurrence of
the stationary and instationary Taylor–Dean Cells. On the basis of these results we search to establish
phenomenological laws representing the evolution of the critical Taylor number according to the
influence of the flow control parameter \( \Gamma \).

2. Experiments

The experimental set-up is, however, basically the same or closely related to the system adopted by
many authors [6] [7]. There are a rotating inner sphere of \( R_i = 45 \text{mm} \) radius and spherical outer shell of
\( R_o = 50 \text{mm} \) radius, which is remained at rest. The whole system is made up of Plexiglas from cubic box.
In this way one obtains the dimensionless gap \( \delta = (R_o - R_i)/R_o = 0.11 \) (figure 1). The inner sphere is
driven by a variable-speed electric motor so as to the angular rotation \( \Omega_i \) is controlled without friction
by using a tachymeter and photoelectric measurement. The temperature is measured in order to

© 2008 IOP Publishing Ltd
determine the viscosity of the fluid in annulus by digital thermometer, in practice we adopt the ambient temperature $\theta = 21^\circ\text{C}$.

![Schematic diagram of the flow system](image)

**Figure 1:** Schematic diagram of the flow system

A solution constitutes of small batch of Similli and Vaseline oil (with concentration 80% and 20% respectively) was used as a fluid of visualization. For the flow visualization, a small amount of aluminium flakes (typical mean dimension $17 \mu\text{m}$ and a concentration of about $2 \text{g/l}$) produces good signals in suspension in the fluid; this amount is small enough not to influence the viscosity or the measurements of viscosity $\nu$ is function of $\theta$ showed that the fluid behave as Newtonian law.

The angular velocity is measured with a numerical tachymeter (DT2236), we can evaluate the angular velocity with precision of 1%. By means of digital camera, the images have a dimension of $2592 \times 1944$ pixel, with focal length of $7.2\text{mm}$. This gives a resolution of 5.1 Mega pixels.

The geometrical characteristics are fixed the value of the flow control parameter $Ta$ depends on the velocity of the inner rotating sphere $\Omega_i$, annular space $d=R_1-R_2$ and the kinematics viscosity $\nu$ of the fluid used

$$Ta = Re \sqrt{\delta} \quad \text{with} \quad Re = \frac{V_i d}{\nu} \quad (1)$$

Where $Re$ indicates the Reynolds number, $\delta = d/R_i$ denotes the non–dimensional gap or radial aspect factor, $V_i = R_i \Omega_i$ the linear velocity of the inner sphere.

Featuring the influence of centrifugal force on the flow we proceed systematically by increasing speed $\Omega_i$ so as to satisfying the inequality relation,

$$\frac{\Delta\Omega_i}{\Omega_i} \leq 1\% \quad (2)$$

This condition also appears necessary to ensure a good reproductibility of measurements which is besides enough near to experimental uncertainty on the angular velocity $\Omega_i$ estimated of about $1\%$. 
The aspect ratio $\Gamma = H/d$ is defining the axial limitation in such a way the system is filled for a given value of height $H$. Adopted procedure for the various tests is as follows, at the horizontal position, starting from the rest we increase the velocity of the inner sphere gradually with respect to the preceding inequality (2). We stop a few minutes to allowing us to stabilize the flow itself then we observe the onset of instabilities. At fixed $\Gamma = H/d$ and as $Ta$ is increasing the flow system undergoing the effect of aspect ratio $\Gamma$.

For $\Omega$ chosen, characterizing the appearance of a given phenomenon, measurements of characteristics of the structures considered are noted and we take a picture of the state of the flow. The process thus described corresponds to a rigorous thermodynamic procedure that we call quasi-static mode allowing us to observe the conditions of reversibility associated with the movement.

### 3. Results and discussion

At the horizontal position, we increase the angular velocity of the inner sphere $\Omega_1$ as well as Taylor number is increasing. When $\Gamma$ decreases near the critical value of aspect ratio $\Gamma_c = 15$, one observes the appearance of stationary waves, with number of 4 localised in equatorial region. By analogy, these waves correspond to a special case of the primary mode of Taylor–Dean instability in spherical geometry. Owing to its appearance in the boundary layer and the property of stationary waves, one can be identified them as a Görtler instability (Figure 2). On the left, one shows the original photo indicating overall the stationary undulatory structure related to the instability. On the right, one carries out an image processing highlighting the details of the form structural associated to this type of instability. We also check that the cells are parallel in the flow direction and periodical in the transverse direction. The whole of the properties thus enumerated result in making a close analogy with the existence of the standard Görtler waves which one meets within the boundary layer in a concave wall. Further investigations are required to verify this property.

In addition, it is well known [8] that Görtler vortices could be favourable to produce the enhancement in heat transfer and an efficient scalar transporting mechanism as well.

By increasing the Taylor number, the stationary character disappears because of these waves tend to become instationary (figure 3) and move equally from the center to both left and right of the flow. Thereafter, as aspect factor decreases, one notes that the both Taylor–Dean instabilities disappear at $\Gamma = \Gamma_c = 6$. The number of cells remains the same as previously but the form of the structure seems to changed. This one presents an oscillating movement and cell form is not grouped in comparison with the primary mode, as shows it the images having under a treatment.

![Original photo](image1.jpg) ![Photos after image processing](image2.jpg)

**Figure 2:** Stationary Taylor–Dean waves at $\Gamma_c = 6$ and $Ta = 118$

![Original photo](image3.jpg) ![Photos after image processing](image4.jpg)

**Figure 3:** Instationary Taylor–Dean waves at $\Gamma_c = 6$ and $Ta = 148$

We look into the experimental results that enabled us to analyze the effect of the aspect factor $\Gamma$, allowing to establish a phenomenological laws, obeying linear expressions, that one presents in the
following general forms, $T_D(\Gamma) = A \cdot \Gamma + B$. In case of stationary waves $A=-4.2$ and $B=139.8$, and for the instationary waves $A=-6.06$ and $B=183.4$. We note that the Taylor number increases when $\Gamma$ decreases (figure 4), than one can conclude that the most effect of the aspect factor $\Gamma$ leads to delaying the occurrence of the first and second instabilities in such a way the Taylor–Dean instabilities disappear at $\Gamma=\Gamma_c=5$, therefore, the regime and configuration of the flow change.

Figure 4 show up, as $\Gamma$ decrease, the most effect of the aspect factor on the critical Taylor numbers $T_{D1}$ and $T_{D2}$ in cylindrical and spherical case, it leads to delaying the occurrence of the first and second Taylor–Dean instabilities. In addition, we note that the Taylor–Dean instabilities disappear at the same value $\Gamma_c=\Gamma_{c1}=\Gamma_{c2}=6$ in spherical cases. Otherwise, the disappearance of the instabilities proceed with $\Gamma_{c1}=1$ and $\Gamma_{c2}=4.4$ in the cylindrical case.

Figure 4: Evolution of Critical Taylor number $T_{D1}$ and $T_{D2}$ corresponding to Taylor-Dean instabilities versus aspect factor $\Gamma$

| $\Gamma$ | 15 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 |
|----------|----|----|----|----|----|---|---|---|---|
| $T_{D1}$ | 75 | 85 | 92.4 | 95 | 98.5 | 100 | 105 | 110 | 115 |
| $T_{D2}$ | 86 | 106 | 114.4 | 119 | 126 | 132.2 | 135 | 136 | 146 |

Table 1: Critical values of $T_{D1}$ and $T_{D2}$ versus aspect factor $\Gamma$ (spherical case)

3.1. Comparison between cylindrical and spherical Taylor–Dean Instability

The first instability (primary mode) is characterized by stationary waves in spherical case in contrast with instationary waves in cylindrical case. On the other hand, the second instability (triplet pattern) is the same in both cases.

The variation of critical Taylor number $Ta$ in function of the aspect factor $\Gamma$ is obeying to the linear expressions, that one presents in the following general forms,

$$T_D(\Gamma) = A \cdot \Gamma + B$$

The phenomenological laws can be illustrated in the table 2 by performing the values of $A$ and $B$ in the two cases studied,

| First instability | Second instability |
|-------------------|--------------------|
| **Cylindrical case** | **Spherical case** |
| $A$ | $B$ | $A$ | $B$ |
| $-2.6$ | $154.4$ | $-3.9$ | $194.5$ |
| $-4.2$ | $139.8$ | $-6.06$ | $183.4$ |

Table 2: Comparison between cylindrical and spherical Taylor–Dean Instability

The Slope of the curves representing $Ta=f(\Gamma)$ corresponding to the spherical case is higher that means the Taylor–Dean instabilities are more delayed in comparison with cylindrical case.
3.2. Characteristic parameters

We examine here the effect of the aspect factor $\Gamma$ on the critical wavelength $\lambda$ of the first instability. Figure 5 illustrates that the critical wave-length of the first instability which appear constant as the aspect factor $\Gamma$ decreases.

![Figure 5: Evolution of Critical wavelength $\lambda_C$ of stationary waves (primary mode) versus aspect factor $\Gamma$](image)

4. Conclusion

According to the preceding results it appears that the effect of the aspect factor $\Gamma$ acts considerably on the flow between two coaxial rotating spheres as the cylindrical case (Taylor–Couette flow). Therefore, we show up the existence of two types of instabilities of stationary and instationary nature, corresponding to the critical values of Taylor number $T_{D1}$ and $T_{D2}$ for a given value $\Gamma$. Moreover, we note that in both cases that the values of $T_{D1}$ and $T_{D2}$ increase consequently as $\Gamma$ decrease leading to delaying the appearance of instabilities particularly in the spherical case, the disappearance of primary mode and triplet pattern proceeds at the same critical value of the aspect factor $\Gamma_C$.

Finally, it is interesting to extend these experiments to the study of laminar–turbulent transition regime in order to determine the spatio–temporal characteristics of these instabilities by spectral analysis and to investigate other configurations in small and wide gap configurations.

References

[1] Dean W R (1928) Fluid motion in a curved channel, Proc. R. Soc. Lond. A 121 402-420
[2] Ho C M and Huerre P (1984) Perturbed free shear layers, Ann. Rev. Fluid Mech. 16 365-424
[3] Mutabazi I, Hegseth JJ, Andereck CD, Wesfreid JE (1990) Spatiotemporal pattern modulations in the Taylor-Dean system, Phys. Rev. Lett., 64, 1729-1732
[4] Brancher P and Chomaz J M (1997) Absolute and convective secondary instabilities in spatially periodic shear flows, Phys. Rev. Lett. 78 658-661
[5] Lighthouati Y, Bouaballah A, Mutabazi I, (2005) Evolution des premiers modes d’instabilité dans le système de Taylor–Dean en solution viscoélastique, J. de Rhéologie, 7, 55-60
[6] Wimmer M (1981) Experiments on the stability of viscous flow between two concentric rotating spheres. J. Fluid Mech. 103, 117-131.
[7] Nakabayashi K and Tsuchida Y (1988) Spectral study of the laminar-turbulent transition in spherical Couette flow. J. Fluid Mech., 194, 101-132
[8] Liu J T C (2008) Nonlinear instability of developing streamwise vortices with applications to boundary layer heat transfer intensification through an extended Reynolds analogy, Philosophical Trans. R. Soc. A, 366, 2699–2716