A Standard Siren Cosmological Measurement from the Potential GW190521 Electromagnetic Counterpart ZTF19abanrhr

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ABSTRACT

The identification of the electromagnetic counterpart candidate ZTF19abanrhr to the binary black hole merger GW190521 opens the possibility to infer cosmological parameters from this standard siren with a uniquely identified host galaxy. The distant merger allows for cosmological inference beyond the Hubble constant. Here we show that the three-dimensional spatial location of ZTF19abanrhr calculated from the electromagnetic data remains consistent with the latest sky localization of GW190521 provided by the LIGO-Virgo Collaboration. If ZTF19abanrhr is associated with the GW190521 merger, and assuming a flat $\Lambda$CDM model, we find that $H_0 = 48^{+22}_{-15}$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_m = 0.35^{+0.41}_{-0.26}$, and $\omega = -1.31^{+0.61}_{-0.48}$ (median and 68% credible interval). If we use the Hubble constant value inferred from another gravitational-wave event, GW170817, as a prior for our analysis, together with assumption of a flat $\Lambda$CDM and the model-independent constraint on the physical matter density $\omega_m$ from Planck, we find $H_0 = 68.9^{+8.7}_{-6.0}$ km s$^{-1}$ Mpc$^{-1}$.

Key words: cosmological parameters – gravitational waves – black hole mergers

1 INTRODUCTION

Gravitational waves (GWs) emitted by compact object binaries are self-calibrating standard sirens (Schutz 1986), in that they yield a direct measurement of the source luminosity distance. If the redshift of the source can be estimated by other means, then GWs provide a way to measure cosmological parameters that is entirely independent from classic probes such as those based on standard candles (Riess et al. 2016, 2019; Yuan et al. 2019; Freedman et al. 2019; Pesce 2016; Aghanim et al. 2020) and other methods (Macaulay 2019), the cosmic microwave background (CMB) (Ade et al. 2015; Taylor et al. 2012; Del Pozzo et al. 2017), the two most prominent are the identification of an electromagnetic (EM) counterpart, and a statistical analysis of all galaxies included in the GW uncertainty volume (Schutz 1986; Holz & Hughes 2005; Del Pozzo 2012; Chen et al. 2018). The detection of GWs from the binary neutron star (BNS) merger GW170817 (Abbott et al. 2017a) by the LIGO/Virgo Collaboration (LVC) (Harry 2010; Accornese et al. 2015), together with the observation of EM counterparts at multiple wavelengths (Abbott et al. 2017c) has allowed the first-ever standard siren measurement of the Hubble constant (Abbott et al. 2017b). While the statistical standard siren method has the advantage that it can be applied to all types of compact binary coalescences (CBCs), whether they emit light or not, it is intrinsically less precise, as usually many galaxies are consistent with the GW uncertainty volume (Chen et al. 2018; Soares-Santos et al. 2019; Abbott et al. 2021c). To date, both approaches have been explored (Abbott et al. 2021c,a).

The recent identification of an EM transient at non-negligible redshift ($z \simeq 0.4$) by the Zwicky Transient Facility (ZTF) — ZTF19abanrhr (Graham et al. 2020) — consistent with being a counterpart to the distant ($\sim 4$ Gpc) binary black hole (BBH) GW source GW190521 (LIGO Scientific Collaboration, Virgo Collaboration 2017; Abbott et al. 2020a,b) (see also Ashton et al. 2020; Palmese et al. 2021) for new evaluations of the confidence in this observation), offers the potential to measure cosmological parameters beyond the Hubble constant using GW observations. If indeed a non-negligible fraction of BBHs merge in gas-rich environments such as the disks of active galactic nuclei (AGN) and emit observable EM signals (McKernan et al. 2019; Graham

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et al. 2020), they might contribute significantly to the cosmological inference from standard siren measurements (Chen et al. 2018). Previous cosmological inference from GW observations has been limited to the local Hubble parameter $H_0$, primarily due to the GW detectors’ current limit in their sensitive distance to BNSs, and the number of galaxies consistent with the large BBH uncertainty volumes. Inference on other cosmological parameters was expected to rely on future GW observations at higher redshift (Sathyaprakash et al. 2010; Taylor & Gair 2012; Del Pozzo et al. 2017; Jin et al. 2020; Chen et al. 2021).

The paper is organized as follows: In Section 2 we describe the spatial correlation between the ZTF and GW events. We then lay out the framework of our cosmological inference and present our results under different priors and assumptions in Section 3. We conclude with our outlook in Section 4.

2 ZTF19abanrhr ASSOCIATION IN 3D LOCALIZATION

In Figure 1(a) we show the three-dimensional localization uncertainty volume of GW190521 assuming a uniform prior in luminosity volume ($\propto D_L^3$). Using a Planck 2018 cosmology (Aghanim et al. 2020), we also mark the location of ZTF19abanrhr. We found that ZTF19abanrhr lies at a 67% credible level of the GW190521 localization volume.

The credible level at which the counterpart lies in the localization of GW190521 depends on the assumed prior distribution of GW sources. Figure 1(b) shows the posterior distribution of luminosity distance along the line of sight to ZTF19abanrhr for several different choices of prior; in all cases the luminosity distance to ZTF19abanrhr computed from a reasonable cosmology (Aghanim et al. 2020) is found well within the bulk of this conditional distance distribution. For the primary estimate of the distance marginal used in this study, we rely on a parameter estimation analysis conditional on J1249 + 3449, the sky location of ZTF19abanrhr (Isi 2020; Abbott et al. 2021b), and otherwise matching the preferred analysis from Abbott et al. (2020b); LIGO Scientific Collaboration and Virgo Collaboration (2020); Varma et al. (2019).

3 COSMOLOGICAL INFERENCE

The mathematical and statistical background behind a standard siren measurement of the Hubble constant has already been presented in the literature (Schutz 1986; Holz & Hughes 2005; Abbott et al. 2017b; Chen et al. 2018; Fishbach et al. 2019; Gray et al. 2020). In this letter, we follow the same framework to infer the Hubble constant $H_0$, the matter density of the Universe $\Omega_m$, and the dark energy equation of state (EoS) parameter $w_0$.

Given a set of GW data $D_{GW}$ and EM data $D_{EM}$ corresponding to a common observation, the joint posterior of

$$p(D_L) \propto \frac{1}{D_L^3} \left( \frac{2\pi}{3} \right)^{3/2} \frac{1}{\sigma^2} e^{-\frac{1}{2\sigma^2} \left( D_L - D_{\text{obs}} \right)^2}$$

Figure 1. Panel (a): The 3D localization of GW190521 presented in a Cartesian luminosity distance coordinates, centered on the Earth marked with a black +. Here we use the localization inferred by the NRSur analysis from Abbott et al. (2020b); LIGO Scientific Collaboration and Virgo Collaboration (2020) which applied a uniform prior in luminosity volume. The size and hue of each point is weighted by the logarithm of its posterior probability. The location of ZTF19abanrhr, assuming the Planck 2018 cosmology (Aghanim et al. 2020), is shown by the orange star. Panel (b): The 1D $D_L$ posterior for GW190521 along the line of sight to ZTF19abanrhr under four different prior assumptions for the luminosity distance $D_L$ (Isi 2020). The location of ZTF19abanrhr, assuming the Planck 2018 cosmology (Aghanim et al. 2020), is shown by the orange line. The priors are (solid blue line) uniform in luminosity distance(i.e. proportional to the conditional distance likelihood); uniform in luminosity volume (dashed blue line); uniform in the comoving frame (dotted blue line); and tracing the star formation rate (Madau & Dickinson 2014) (dash-dotted blue line).

\footnote{We approximate the full three-dimensional probability density using the clustered-KDE method in (Farr 2020). Using this KDE, we evaluate the posterior probability at the location of ZTF19abanrhr and find its credible level as the fraction of samples in the GW posterior that have larger posterior probability values.}
The first term is the marginalized GW likelihood evaluated at the right ascension $\alpha_{\text{EM}}$, declination $\delta_{\text{EM}}$, and luminosity distance implied by the redshift of ZTF19abanrhr given cosmological parameters $H_0$, $\Omega_m$, and $w_0$; this function is shown by the solid blue curve in Figure 1(b). The next term accounts for selection effects and the assumed GW source population and involves the ratio of the (normalized) population density at the ZTF19abanrhr redshift and the fraction of the (normalized) population that is jointly detectable in GW and EM emission as described above (in the local universe the effect of this term is to introduce a factor $1/H_0^2$ (Abbott et al. 2017b; Chen et al. 2018; Fishbach et al. 2019) but at $z \geq 0.4$ cosmological effects weaken the dependence on $H_0$ substantially (Farr 2020)). The third term is the prior on cosmological parameters. We impose several different priors incorporating various additional cosmological measurements in the following.

In our most generic analysis, we use flat priors in the ranges $H_0 = [35, 140] \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_m = [0, 1]$, and $w_0 = [-2, -0.33]$. The result is presented in Figure 2. We find a broad posterior for $H_0$ with a median and 68% credible interval of $H_0 = 48^{+23}_{-35} \text{ km s}^{-1} \text{ Mpc}^{-1}$, with a peak below the maximum likelihood Planck 2018 value (Aghanim et al. 2020) (as well as the SH0ES estimate (Riess et al. 2016), reported with a yellow (pink) solid line. The Planck and SH0ES estimates are contained within the 90% credible regions of our measurements. The posteriors for $\Omega_m$ and $w_0$ are nearly uninformative with $\Omega_m = 0.35^{+0.41}_{-0.26}$, and $w_0 = -1.31^{+0.61}_{-0.45}$. Nevertheless, given the large inferred distance of GW190521, they are mildly correlated with $H_0$, and must be included in the analysis.

To evaluate Eq. (1) we need to specify the distribution for the parameters of the underlying population of BBH mergers with counterparts, $p_{\text{pop}}(\tilde{\Theta}|H_0, \Omega_m, w_0)$, as well as the EM likelihood $p_{\text{EM}}(\tilde{\Theta}|H_0, \Omega_m, w_0)$, $\beta$, and $\alpha_{\text{EM}}$. The result is shown in the dark blue curve. We find $H_0 = 48.3^{+2.6}_{-1.1} \text{ km s}^{-1} \text{ Mpc}^{-1}$ with this assumption and the new $w_0$ prior. Next, we apply the $H_0$ likelihood from GW170817 (Abbott et al. 2017b, 2021c) as a prior on $H_0$. The result is shown in the dark blue curve in Figure 3. The joint measurement is narrower than either measurement alone, with a median and 68% credible interval of $H_0 = 48.0^{+2.0}_{-1.3} \text{ km s}^{-1} \text{ Mpc}^{-1}$ and a clear peak consistent with estimates using observations from both the CMB (Aghanim et al. 2020) and the local distance ladders (Riess et al. 2016, 2019; Macaulay et al. 2019; Yuan et al. 2019; Freedman et al. 2019; Pesce et al. 2020); GW190521 rules out some large $H_0$ values that are permitted from GW170817.

Finally, in Figure 4 we present the measurements on $\Omega_m$, $w_0$ with both of the GW events and Planck’s prior on $w_0$ in a flat $w$CDM cosmology. We find that the $\Omega_m$ posterior now shows a departure from its prior, and features a peak with a median and 68% credible interval of $\Omega_m = 0.298^{+0.061}_{-0.064}$. To a lesser extent, the same is true for $w_0$ now estimated as $w_0 = -1.33^{+0.63}_{-0.47}$. 

\[ \beta (H_0, \Omega_m, w_0) \equiv \int P_{\text{det}}(\tilde{\Theta}) p_{\text{pop}}(\tilde{\Theta}|H_0, \Omega_m, w_0) d\tilde{\Theta} \] (2)

where $\tilde{\Theta}$ represents all the binary parameters, such as the masses, spins, luminosity distance, sky location, orbital inclination etc. $p(\tilde{\Theta})|H_0, \Omega_m, w_0)$ is the distribution of the population of binaries with parameters $\tilde{\Theta}$ in the Universe. The denominator, $\beta$, is the fraction of the population of events that would pass detection thresholds (Loredo 2004; Abbott et al. 2017b; Mandel et al. 2019; Fishbach et al. 2019; Vitale 2020; Farr & Gair 2020):

\[ \beta (H_0, \Omega_m, w_0) \equiv \int P_{\text{det}}(\tilde{\Theta}) p_{\text{pop}}(\tilde{\Theta}|H_0, \Omega_m, w_0) d\tilde{\Theta} \] (2)

where $P_{\text{det}}(\tilde{\Theta}) \equiv \int \int \int \int \int \frac{d\mathbf{D}_{\text{GW}}}{d\mathbf{D}_{\text{EM}}} p(\mathbf{D}_{\text{GW}}|\mathbf{D}_{\text{EM}}|\tilde{\Theta})d\mathbf{D}_{\text{GW}}d\mathbf{D}_{\text{EM}},$ (3)

is the probability of detecting a source with parameters $\tilde{\Theta}$ in both GW and EM emission. This latter integration should be carried out over data above the GW and EM detection thresholds, $\mathbf{D}_{\text{GW}}$, and $\mathbf{D}_{\text{EM}}$. We assume that the counterparts to systems like GW190521 can be observed by ZTF and other telescopes far beyond the distance at which GW observatories can detect them (ZTF19abanrhr was $\sim 18.8$ mag in g-band at $z=0.438$), so the integral’s domain is truncated by GW selection effects.

To evaluate Eq. (1) we need to specify the distribution for the parameters of the underlying population of BBH mergers with counterparts, $p_{\text{pop}}(\tilde{\Theta}|H_0, \Omega_m, w_0)$. Since the astrophysical rate of GW190521-like BBHs is still uncertain, we assume their redshift distribution follows the star formation rate (SFR) as modeled by Madau & Dickinson (2014). We adopt the default assumptions of Abbott et al. (2020b) that the population is flat in the detector frame masses and spin magnitudes and isotropic over binary and spin orientations.

Given the small uncertainty in the redshift and counterpart sky location measured in ZTF19abanrhr, we treat the EM likelihood in Eq. (1) as a $\delta$-function at these measurements. Performing the integral over $\tilde{\Theta}$, Eq. (1) becomes

\[ p(H_0, \Omega_m, w_0 | \mathbf{D}_{\text{GW}}, \mathbf{D}_{\text{EM}}) \propto p(\mathbf{D}_{\text{GW}} | D_L (z_{\text{EM}} | H_0, \Omega_m, w_0), \alpha_{\text{EM}}, \delta_{\text{EM}}) \times p_{\text{pop}}(\tilde{\Theta}|H_0, \Omega_m, w_0) \beta (H_0, \Omega_m, w_0) p(H_0, \Omega_m, w_0). \] (4)

The priors are uniform on the component masses in the detector frame from $[30, 200] M_\odot$. The mass priors are further restricted such that the total mass must be greater than $200 M_\odot$, and the chirp mass to be between 70 and $150 M_\odot$, both in the detector frame. The mass ratio between the lighter and heavier objects is restricted to be $> 0.17$. These priors are chosen to match those used in the analysis from (Abbott et al. 2020b), to comply with the parameter space supported by the NRSкри7Dq4 waveform model (Varma et al. 2019) used for our flagship results and to not impose preference for a specific astrophysically informed BBH population model.
The choice of waveform models for GW data analysis can contribute to the systematic uncertainty of the standard siren measurement via the luminosity distance estimate. In Abbott et al. (2020b), the LVC estimated the parameters of GW190521 with three different waveform models (Varma et al. 2019; Khan et al. 2020; Ossokine et al. 2020). We use a clustering decomposition followed by a kernel density estimate within clusters (Farr 2020) to estimate the marginal posterior probability distribution of $D_L$ along the line-of-sight to ZTF19abanrhr (Graham et al. 2020) from these analyses. In Figure 5 we show the $H_0$ inference with the three waveform models using the GW170817 prior on $H_0$ and Planck’s prior on $\omega_m$ in a $\Lambda$CDM cosmology. The strong prior on $H_0$ dominates over the slight difference between $D_L$ estimates from different models, and they all yield a very similar posterior on $H_0$. Similar to Abbott et al. (2020b), our flagship results are inferred using the NRSur (NSRSur7dq4) waveform model (Varma et al. 2020) only, as it has been shown to be the most faithful against numerical relativity (NR) simulations in the parameter space relevant for GW190521.

### 4 DISCUSSION

The EM transient ZTF19abanrhr (Graham et al. 2020) could be associated with the BBH merger GW190521. We find that ZTF19abanrhr lies at the 67% credible level of the
GW190521 three-dimensional localization volume under a default luminosity-distance prior and assuming the Planck 2018 cosmology (Aghanim et al. 2020). Assuming the GW–EM association is true, we report a standard-siren measurement of cosmological parameters from these transients. The large inferred distance of GW190521 enables probing $H_0$ and additional cosmological parameters $\Omega_m$ and the dark energy EoS parameter $\omega$.

We find $H_0 = 68.9^{+8.7}_{-6.0} \text{ km s}^{-1} \text{ Mpc}^{-1}$ from the associated ZTF19abanhr–GW190521 and the kilonova AT 2017gfo–GW170817 observations assuming a model-independent constraints on the physical matter density $\omega_m$ from the Planck observations (Aghanim et al. 2020) in a flat $\Lambda$CDM cosmology. The same measurement yields $\Omega_m = 0.298^{+0.061}_{-0.064}$ and $\omega = -1.33^{+0.63}_{-0.41}$ in a flat $w$CDM cosmology. Since there is only one standard siren measurement at higher redshift, the inference on $\Omega_m$ mainly relies on the prior from GW170817 and Planck. The strong prior on $H_0$ from GW170817 dominates the $H_0$ measurement. When GW170817 is combined with the Planck prior on $\omega_m$, $\Omega_m$ is constrained to $\sim 20\%$. On the other hand, without any informative priors, $\omega_0$ is only marginally confined even when both GW170817 and GW190521 are included in the analysis.

We find that the choice of GW waveform for the estimation of luminosity distance and the assumption of BBH population for the evaluation of selection effect do not introduce noticeable difference in our results. However, when more events are combined in the future and the cosmological parameters are confined more precisely, the systematic uncertainties arising from waveform and selection effect will have to be investigated more carefully. For example, a joint inference of the BBH population and the cosmological parameters will help reduce bias from unrealistic population assumptions.

We note that different choices of priors on the cosmological parameters naturally lead to different results. For example, Mukherjee et al. (2020) chose a wider $H_0$ prior and found slightly more support at lower $H_0$ value. Using different physical assumptions of the binary systems or external information about relevant physical parameters also affect the $H_0$ inference. Gayathri et al. (2020) explored the assumption that GW190521 was an eccentric binary merger, Calderón Bustillo et al. (2021) applied a more unequal mass prior, and Mukherjee et al. (2020) introduced additional constraint on the binary inclination angle inferred from Very Large Baseline Interferometry observations. These assumptions and the external information induced different levels of deviation from our measurements. The dependence on the specific GW models used or on the assumed astrophysical constraints further highlights the need for high-accuracy GW models covering a wider and expanded set of BBH parameter configurations in order to avoid systematic bias on the standard siren measurements, especially so for a potential future population of BBH–EM observations. In this study, we have selected to follow the analysis choices from Abbott et al. (2020b) as the NRSub GW model was shown to be the most accurate for the parameter space associated with GW190521. Similarly, although external information on the binary inclination angle can significantly reduce the $H_0$ statistical uncertainty, the systematic uncertainty of the inclination angle estimate will still need to be carefully addressed to ensure the accuracy of the standard siren measurements (Chen 2020).

Continued followup searches for EM counterparts of BBHs will help establishing or diminishing the association between BBH events and their EM counterpart candidates, providing a better estimate of the chance coincident rate and allowing for thorough mitigation of the systematic uncertainty originating from false EM emission association for the standard siren measurement (Palmese et al. 2021).

In the next five years, LIGO, Virgo and KAGRA are predicted to detect hundreds of BBHs per year (Abbott et al. 2018). If indeed ZTF19abanhr is the counterpart of GW190521, we should see more BBHs accompanied by EM counterparts. Owing to their generally larger distances, compared to standard BNS bright sirens, these have a significant potential of yielding an interesting GW measurement of $\Omega_m$ and $\omega_0$.

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Figure 5. The posterior PDF of $H_0$ for the associated GW190521–ZTF19abanhr observations using $\omega_m$ constraints from Planck 2018 (Aghanim et al. 2020), a prior on $H_0$ from GW170817 (Abbott et al. 2017b, 2021c) (shown in grey) and a flat $\Lambda$CDM cosmology. We show estimates on $H_0$ using all three waveform analyses from Abbott et al. (2020b); LIGO Scientific Collaboration and Virgo Collaboration (2020). The yellow (pink) solid lines report the Planck 2018 (Aghanim et al. 2020) (SHOES Riess et al. (2016)) cosmology, with shaded regions representing their respective 68% credible interval.
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## DATA AVAILABILITY

The data behind Figures 1(b), 2, 3, 4, and 5 are publicly available at Chen et al. (2020).

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