A PERIODIC AND DIFFUSIVE PREDATOR-PREY MODEL WITH DISEASE IN THE PREY

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Abstract. In this paper, we are concerned with a time periodic and diffusive predator-prey model with disease transmission in the prey. First we consider a SI model when the predator species is absent. By introducing the basic reproduction number for the SI model, we show the sufficient conditions for the persistence and extinction of the disease. When the presence of the predator is taken into account, a number of sufficient conditions for the co-existence of the prey and predator species, the global extinction of predator species and the global extinction of both the prey and predator species are given.

1. Introduction. Eco-epidemiology models have become important and efficient tools in analyzing the spread of infectious disease between the interacting species. Hadeler and Freedman [7] developed and analysed a predator-prey model, where both species were involved with parasitism. In order to account for the influence of disease transmission on two interacting species, Venturino [24, 25, 26] proposed and investigated several epidemiology models coupled with Lotka-Volterra model. Since then many elegant works have been dedicated to this direction, see [1, 3, 4, 6, 12, 16, 17, 21, 29, 30, 33, 35] and references therein.

It is well known that spatial heterogeneity of the population habitat environment and spatial-temporal movement of individuals play significant roles in the dynamics of infectious disease (see, e.g., [20, 28]) and the evolution of interacting species (see, e.g., [2]). And not only that, the spread of disease and the growth of population individuals are influenced dramatically on seasonal variation (see e.g., [9, 5, 18, 32]).

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In this paper, we shall incorporate the spatial-temporal factors into the general eco-epidemiology model with disease in the prey:

\[
\begin{align*}
\frac{\partial S(t,x)}{\partial t} &= \nabla \cdot [DS(t,x)\nabla S(t,x)] + a(t,x)S(t,x) \\
&\quad - b(t,x)S(t,x)(S(t,x) + I(t,x)) - \beta(t,x)S(t,x)I(t,x), \\
\frac{\partial I(t,x)}{\partial t} &= \nabla \cdot [DI(t,x)\nabla I(t,x)] - c(t,x)I(t,x) + \beta(t,x)S(t,x)I(t,x) \\
&\quad - f_1(t,x,I(t,x),W(t,x)), \\
\frac{\partial W(t,x)}{\partial t} &= \nabla \cdot [DW(t,x)\nabla W(t,x)] - d(t,x)W(t,x) \\
&\quad + f_2(t,x,I(t,x),W(t,x)).
\end{align*}
\]

In this model, we have two populations: the prey \((S + I)\) and the predator \(W\). Assume that the population habitat \(\Omega\) is a bounded domain in \(\mathbb{R}^n(n \geq 1)\) with a smooth boundary \(\partial \Omega\). We further make the following assumptions:

(H1): In the absence of disease, the growth of prey population obey the logistic type law. Let \(N(t,x)\) be the density of prey population at time \(t\) and location \(x\). Then \(N(t,x)\) satisfies

\[
\frac{\partial N(t,x)}{\partial t} = \nabla \cdot [DN(t,x)\nabla N(t,x)] + a(t,x)N(t,x) - b(t,x)N^2(t,x)
\]

subject to no flux boundary condition

\[
[D_N(t,x)\nabla N(t,x)] \cdot n = 0,
\]

where \(\nabla \cdot [D_N(t,x)\nabla N(t,x)]\) denotes the divergence of \(D_N(t,x)\nabla N(t,x)\), \(D_N(t,x)\) is the diffusion rate at time \(t\) and location \(x\), \(n\) is the outward normal to \(\partial \Omega\).

(H2): In the presence of disease, we assume that the total prey population \(N\) is composed of susceptible class \(S\) and infected class \(I\). Then, the density of total prey population is \(N(t,x) = S(t,x) + I(t,x)\) at time \(t\) and location \(x\).

(H3): Assume that only susceptible prey \(S\) is responsible for reproducing, i.e., the infected prey \(I\) is removed by death or by predator before having the possibility of reproduction (see [24, 30]). However, the infections still contribute to growth of susceptible prey \(N\) according to the logistic growth law.

(H4): The disease spreads only among the prey population and disease is not genetically inherited. The infected individuals do not recover or become immune. The new infections arise from the contact of infected and susceptible individuals and follows the mass action infection mechanism, which leads to the following SI epidemic model:

\[
\begin{align*}
\frac{\partial S(t,x)}{\partial t} &= \nabla \cdot [DS(t,x)\nabla S(t,x)] + a(t,x)S(t,x) \\
&\quad - b(t,x)S(t,x)(S(t,x) + I(t,x)) - \beta(t,x)S(t,x)I(t,x), \\
\frac{\partial I(t,x)}{\partial t} &= \nabla \cdot [DI(t,x)\nabla I(t,x)] - c(t,x)I(t,x) + \beta(t,x)S(t,x)I(t,x),
\end{align*}
\]

where \(\beta\) is the disease transmission rate and \(c\) is death rate of infections. We further assume that both \(\beta\) and \(c\) are temporally-spatially heterogeneous.

(H5): Assume that the predator individuals only feed on the infected prey with general functional response function \(f(t,x,I,W)\) at time \(t\) and location \(x\). We also assume that the natural death rate of predator population is \(d(t,x)\) which depends on time and space variables.
Biologically, (H5) means that the predator only catches the infected prey, this is because the infected individuals are less active or the behavior of the prey individual is modified such that they live in parts of the habitat which are accessible to the predator (see [19]). The goal of the current work is devoted to the dynamics of model (1) with no flux boundary condition, namely,

\[
\begin{align*}
\frac{\partial S(t,x)}{\partial t} &= \nabla \cdot [D_S(t,x)\nabla S(t,x)] + a(t,x)S(t,x) - b(t,x)S(t,x) \left( S(t,x) + I(t,x) \right) - \beta(t,x)S(t,x)I(t,x), \\
\frac{\partial I(t,x)}{\partial t} &= \nabla \cdot [D_I(t,x)\nabla I(t,x)] - c(t,x)I(t,x) + \beta(t,x)S(t,x)I(t,x), \\
\frac{\partial W(t,x)}{\partial t} &= \nabla \cdot [D_W(t,x)\nabla W(t,x)] - d(t,x)W(t,x) + f_2(t,x, I(t,x), W(t,x)), \\
\partial_n S(t,x) &= \partial_n I(t,x) = \partial_n W(t,x) = 0, \quad t > 0, x \in \partial \Omega.
\end{align*}
\]

Here functions \(a(t,x)\) and \(\beta(t,x)\) are Hölder continuous and nonnegative nontrivial on \(\mathbb{R} \times \bar{\Omega}\), and periodic in time with the same period \(T > 0\); the function \(b(t,x), c(t,x)\) and \(d(t,x)\) are Hölder continuous and positive on \(\mathbb{R} \times \bar{\Omega}\) and periodic in time with the same period \(T > 0\); the diffusion coefficients \(D_S(t,x), D_I(t,x)\) and \(D_W(t,x)\) belong to \(C^{\theta/2,\theta}(\mathbb{R} \times \bar{\Omega}, \mathbb{R}_+^+)\) for some \(\theta \in (0,1)\) and are periodic in time with the same period \(T > 0\). Moreover, we assume \(D_i(t,x) \geq D_1 > 0\), for all on \(\mathbb{R} \times \bar{\Omega}\), where \(i = S, I, W\). The functional response functions \(f_i \in C^1(\mathbb{R}^+ \times \bar{\Omega} \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ )\) satisfies the following assumptions (see, e.g., [23])

(F1): \(f_i(\cdot,\cdot,0,\cdot) \equiv 0\) and \(f_i(\cdot,\cdot,\cdot,0,\cdot) \equiv 0, i = 1, 2,\)

(F2): \(f_i(\cdot + T,\cdot,\cdot,\cdot) \equiv f_i(\cdot,\cdot,\cdot,\cdot)\), \(i = 1, 2,\)

(F3): \(\partial_1 f_2(t,x,I,W) := \partial f_2(t,x,I,W)/\partial I > 0\) for all \(t \in [0,T], x \in \bar{\Omega}, I \geq 0\) and \(W > 0,\) and \(\partial_2 f_2(t,x,I,W) := \partial f_2(t,x,I,W)/\partial W > 0\) for all \(t \in [0,T], x \in \bar{\Omega}, I > 0\) and \(W \geq 0,\)

(F4): \(f_2(t,x,I,W) \leq \partial_2 f_2(t,x,I,0)W,\) and there exists a \(k > 0\) such that \(f_2(t,x,I,W) \leq k f_1(t,x,I,W)\) for all \(t \in [0,T], x \in \bar{\Omega}, I \geq 0\) and \(W \geq 0,\)

When \(a(t,x), b(t,x), c(t,x), d(t,x), \beta(t,x)\) and \(D_i(t,x) (i = S, I, W)\) are positive functions, \(f_1\) and \(f_2\) are independent of time and space variables and chosen as ratio-dependent Michaelis-Menten functional response function \(\frac{m_iW}{aW + m_i},\) the system (2) is reduced to

\[
\begin{align*}
\frac{\partial S(t,x)}{\partial t} &= D_S \Delta S(t,x) + aS(t,x)(1 - bS(t,x) - bI(t,x)) - \beta S(t,x)I(t,x), \\
\frac{\partial I(t,x)}{\partial t} &= D_I \Delta I(t,x) - cI(t,x) + \beta S(t,x)I(t,x) \\
\frac{\partial W(t,x)}{\partial t} &= D_W \Delta W(t,x) - dW(t,x) + \frac{m_I(t,x)W(t,x)}{aW(t,x) + I(t,x)}, \\
\partial_n S(t,x) &= \partial_n I(t,x) = \partial_n W(t,x) = 0, \quad t > 0, x \in \partial \Omega,
\end{align*}
\]

where \(m_2\) and \(m_4\) are positive constants, and \(\partial_n\) is the outward directional derivative normal to \(\partial \Omega\). Li and Gao [12] showed a number of existence and nonexistence results about the non-constant steady states of system (3). They also studied Diffusion-Driven Instability and bifurcation of non-constant solutions. If we further assume that the diffusion rates are identical in system (3), Zhang et
al. [33] achieved the sufficient conditions for the permanence, the global stability of boundary equilibria and the positive equilibrium, and the local stability of the positive equilibrium. In fact, if we ignore the diffusion factor, the model system [1] degenerates into an ODE system, which is formulated and analyzed by Xiao and Chen [30]. It is interesting that in [30], a periodic solution could occur whether the system is permanent or not.

The rest of this paper is organized as follows. In the next section, we show the well-posedness of system (2), that is, the global existence, uniqueness and positivity of solutions of (2), and we further prove the existence of global attractor. In Section 3, under the case where the predator is absence, we study the time-periodicity of solutions of (2), and we further prove the existence of global attractor. In Section 4, we obtain the conditions for the persistence and extinction of system (2).

2. Well-posedness. Let $X := C(\bar{\Omega}, \mathbb{R})$ be the Banach space with the supremum norm $\| \cdot \|_X$. Define $X^+ := C(\bar{\Omega}, \mathbb{R}^+)$. Then $(X, X^+)$ is strongly order space. For any $\varphi \in X^+$, we consider the following reaction-diffusion equation:

$$\begin{cases}
\frac{\partial u(t,x)}{\partial t} = \nabla \cdot (D(t,x)\nabla u(t,x)) - h(t,x)u(t,x), & t > 0, x \in \Omega, \\
D(t,x)\nabla u(t,x) \cdot n = 0, & t > 0, x \in \partial \Omega, \\
u(0,x) = \varphi(x), & x \in \Omega, \varphi \in X^+,
\end{cases}
$$

where $D(\cdot, \cdot) \in C^{\theta/2,\theta}(\mathbb{R} \times \bar{\Omega}, \mathbb{R}^+)$ for some $\theta \in (0, 1)$ is $T$-periodic in $t$, and there is a positive constant $D'$ such that $D(t,x) \geq D', h(t,x)$ is Hölder continuous and non-negative non-trivial on $\mathbb{R} \times \bar{\Omega}$ and $T$-periodic in $t$. By [10, Chapter II], (4) admits an evolution operator $Q(t,s) : X \to X, 0 \leq s \leq t$, which satisfies $Q(t,t) = I$, $Q(t,s)Q(s,\rho) = Q(t,\rho)$ for all $0 \leq \rho \leq s < t$, and $Q(t,0)(\varphi)(x) = u(t,x; \varphi)$ for $t \geq 0, x \in \Omega$ and $\varphi \in X$, where $u(t,x; \varphi)$ is the solution of (4). Since $D(\cdot, \cdot)$ and $h(\cdot, \cdot)$ is periodic in $t$, [3] Lemma 6.1 implies that $Q(t+s+T, t) = Q(t,s)$ holds for $(t,s) \in \mathbb{R}^2$ with $t \geq s$. Moreover, for any $s, t \in \mathbb{R}$ with $s < t$, $Q(t,s)$ is a compact, analytic and strongly positive operator on $X$. In particular, $Q(t,s)(\varphi)(x) > 0$ for $t > s$ and $x \in \Omega$, provided that $\varphi \in X^+$ and $\varphi \neq 0$. In view of [5] Theorem 6.6 with $\alpha = 0$, there exist positive constants $K \geq 1$ and $\omega \in \mathbb{R}$ such that $\|Q(t,s)\| \leq Ke^{-\omega(t-s)}$, $\forall t \geq s, t, s \in \mathbb{R}$.

Consider the periodic parabolic eigenvalue problem:

$$\begin{cases}
\psi_t - \nabla \cdot (D_S(t,x)\nabla \psi(t,x)) - a(t,x)\psi = \mu \psi, & t > 0, x \in \Omega, \\
D_S(t,x)\nabla \psi(t,x) \cdot n = 0, & t > 0, x \in \partial \Omega, \\
\psi(0,x) = \psi(T,x), & x \in \Omega.
\end{cases}
$$

Let $\mu_0$ be the unique principal eigenvalue of (5) (see, e.g., [9]). In view of [9] Theorem 2.1, we have the following observation.

Lemma 2.1. The following equation

$$\begin{cases}
\frac{\partial u(t,x)}{\partial t} = \nabla \cdot (D_S(t,x)\nabla u(t,x)) + a(t,x)u(t,x) - b(t,x)v^2(t,x), & t > 0, x \in \Omega, \\
D_S(t,x)\nabla v(t,x) \cdot n = 0, & t > 0, x \in \partial \Omega.
\end{cases}
$$
admits a positive $T$-periodic solution $S^*$ if and only if $\mu_0 < 0$. In the affirmative case $S^*$ is unique and globally attractive for \( \begin{equation} \end{equation} \) with continuous non-negative non-trivial initial data. Moreover, if $\mu_0 \geq 0$, the trivial solution 0 is globally attractive.

Let $U(t,s), V(t,s)$ and $W(t,s)$ be the evolution operators determined by the following reaction-diffusion equations

\[
\begin{cases}
\frac{\partial u_1(t,x)}{\partial t} = \nabla \cdot [D_S(t,x) \nabla u_1(t,x)], & t > 0, \ x \in \Omega, \\
[D_S(t,x) \nabla u_1(t,x)] \cdot \mathbf{n} = 0, & t > 0, \ x \in \partial \Omega,
\end{cases}
\]

\[
\begin{cases}
\frac{\partial u_2(t,x)}{\partial t} = \nabla \cdot [D_I(t,x) \nabla u_2(t,x)] - c(t,x) u_2(t,x), & t > 0, \ x \in \Omega, \\
[D_I(t,x) \nabla u_2(t,x)] \cdot \mathbf{n} = 0, & t > 0, \ x \in \partial \Omega,
\end{cases}
\]

and

\[
\begin{cases}
\frac{\partial u_3(t,x)}{\partial t} = \nabla \cdot [D_W(t,x) \nabla u_3(t,x)] - d(t,x) u_3(t,x), & t > 0, \ x \in \Omega, \\
[D_W(t,x) \nabla u_3(t,x)] \cdot \mathbf{n} = 0, & t > 0, \ x \in \partial \Omega,
\end{cases}
\]

respectively. Due to the periodicity of $D_i(t,\cdot)(i = S, I, W), c(t,\cdot)$ and $d(t,\cdot)$ in time $t$, it follows from [5, Lemma 6.1] that $U(t+T,s+T) = U(t,s), V(t+T,s+T) = V(t,s)$ and $W(t+T,s+T) = W(t,s)$ for $(t,s) \in \mathbb{R}^2$ with $t \geq s$. Moreover, for $(t,s) \in \mathbb{R}^2$ with $t > s$, $U(t,s) : X \to X, V(t,s) : X \to X$ and $W(t,s) : X \to X$ are compact and strongly positive. Let $Y = C(\overline{\Omega}, \mathbb{R}^3)$ be the Banach space with the supremum norm $\| \cdot \|$. Then $(Y, Y^+)$ is a strongly ordered space, where $Y^+ := C(\overline{\Omega}, \mathbb{R}^3)$. Define $Z(t,s) := \begin{pmatrix} U(t,s) & 0 & 0 \\ 0 & V(t,s) & 0 \\ 0 & 0 & W(t,s) \end{pmatrix}$. Then $Z(t,s) : X \to X$ is an evolution operator for $(t,s) \in \mathbb{R}^2$ with $t \geq s$. For any $\phi = (\phi_1, \phi_2, \phi_3) \in Y^+$, define $F = (F_1, F_2, F_3) : [0, +\infty) \times Y^+ \to Y$ by

\[
F_1(t,\phi) := \alpha(t,\cdot) \phi_1(\cdot) - \beta(t,\cdot) \phi_1(\cdot) + \phi_2(\cdot) - \phi_3(\cdot), \\
F_2(t,\phi) := \beta(t,\cdot) \phi_1(\cdot) + \phi_2(\cdot) - f_1(t,\cdot) \phi_2(\cdot), \\
F_3(t,\phi) := f_2(t,\cdot) \phi_2(\cdot) - f_3(t,\cdot) \phi_3(\cdot)
\]

for $t \geq 0$. Then \( \begin{equation} \end{equation} \) becomes

\[
\begin{aligned}
\frac{\partial u(t,x)}{\partial t} &= \mathbb{A}(t) u(t,x) + F(t,u), & t > 0, \ x \in \Omega, \\
u(0,x) &= \phi(x), & x \in \Omega,
\end{aligned}
\]

where $u(t,x) = (S(t,x), I(t,x), W(t,x)), \mathbb{A}(t) := \begin{pmatrix} A_1(t) & 0 & 0 \\ 0 & A_2(t) & 0 \\ 0 & 0 & A_3(t) \end{pmatrix}, A_1(t), A_2(t)$ and $A_3(t)$, respectively, are defined by

\[
\begin{cases}
D(A_1(t)) = \{ \varphi \in C^2(\overline{\Omega}) \mid [D_S(t,\cdot) \nabla \varphi] \cdot \mathbf{n} = 0 \text{ on } \partial \Omega \}, \\
A_1(t) \varphi = \nabla \cdot [D_S(t,\cdot) \nabla \varphi], \forall \varphi \in D(A_1(t)),
\end{cases}
\]

\[
\begin{cases}
D(A_2(t)) = \{ \varphi \in C^2(\overline{\Omega}) \mid [D_I(t,\cdot) \nabla \varphi] \cdot \mathbf{n} = 0 \text{ on } \partial \Omega \}, \\
A_2(t) \varphi = \nabla \cdot [D_I(t,\cdot) \nabla \varphi] - c(t,\cdot) \varphi, \forall \varphi \in D(A_2(t)),
\end{cases}
\]

and

\[
\begin{cases}
D(A_3(t)) = \{ \varphi \in C^2(\overline{\Omega}) \mid [D_W(t,\cdot) \nabla \varphi] \cdot \mathbf{n} = 0 \text{ on } \partial \Omega \}, \\
A_3(t) \varphi = \nabla \cdot [D_W(t,\cdot) \nabla \varphi] - d(t,\cdot) \varphi, \forall \varphi \in D(A_3(t)).
\end{cases}
\]
Rewrite the abstract equation (7) as an integral equation

\[ u(t, \phi) = Z(t, 0)\phi + \int_0^t Z(t, s)F(s, u)ds, \ t \geq 0, \phi \in Y^+. \] (8)

We regard the solution of (8) as a mild solution of (7). Clearly, \( F \) is locally Lipschitz continuous. It is not difficult to verify that \( F \) is quasi-positive (see, e.g., [15 Remark 1.1]). By [15 Theorem 1.1 and Remark 1.1], it then follows that for any \( \phi \in Y^+ \), system (2) admits a unique non-continuable mild solution satisfying \( u_0 = \phi \) and \( u(t, \phi) \in Y^+ \) for any \( t \) in its maximum interval of existence \([0, \sigma_0)\). Moreover, by the analyticity of \( Z(t, s), s, t \in \mathbb{R}, s < t, u(t, x; \phi) \) is a classical solution when \( t > 0 \).

Recall that a family of operators \( \{\Phi_t\}_{t \geq 0} \) is an \( \omega \)-periodic semiflow on a metric space \((G, \rho)\) with the metric \( \rho \), provided that \( \{\Phi_t\}_{t \geq 0} \) satisfies: (i) \( \Phi_0(v) = v, \forall v \in G; \) (ii) \( \Phi_t(\Phi_s(v)) = \Phi_{t+s}(v), \forall t, s \in [0, \infty) \); (iii) \( \Phi_t \) is continuous in \((t, v)\) on \([0, \infty) \times G\). We are in position to show that the solutions of system (2) exist globally on \([0, \infty)\).

**Theorem 2.2.** Let (F1)-(F4) hold. For any \( \phi \in Y^+ \), system (2) admits a unique solution \( u(t, \phi) \in Y^+ \) on \([0, \infty) \) with \( u_0 = \phi \). Furthermore, system (2) generates a \( T \)-periodic semiflow \( \Phi(t) := u(t, \cdot) : Y^+ \to Y^+ \), i.e., \( \Phi(t)(\phi)(x) = u(t, t; \phi), \forall \phi \in Y^+, t \geq 0, x \in \Omega \), and \( \Phi(T) : Y^+ \to Y^+ \) has a global compact attractor in \( Y^+ \).

**Proof.** In view of equation (6) and Lemma 2.1 it follows that either the zero solution of (6) or the unique positive \( T \)-periodic solution \( S^* \) of (6) in \( X^+ \) is globally attractive. Since the first equation in (2) is dominated by (6), the comparison principle implies that \( S(t, \cdot; \phi) \) is bounded on \([0, \sigma_0)\). Since \( f_1 \) and \( f_2 \) satisfies (F2)-(F3), we see that the equations for \( I \) and \( W \) are dominated by a scalar linear reaction-diffusion equation, respectively. It then follows that \( \sigma_0 = \infty \) for each \( \phi \in Y^+ \). Thus, there exists a constant \( M_1 > 0 \) such that for any \( \phi \in Y^+ \), we can find a positive integer \( l_1 = l_1(\phi) > 0 \) satisfying \( S(t, x; \phi) \leq M_1 \) for all \( t \geq l_1T \) and \( x \in \Omega \).

Let

\[
\begin{align*}
\hat{a} &:= \max_{t \in [0,T], x \in \Omega} a(t, x), \quad \hat{c} := \min_{t \in [0,T], x \in \Omega} c(t, x), \\
\hat{d} &:= \min_{t \in [0,T], x \in \Omega} d(t, x), \quad \hat{\beta} := \max_{t \in [0,T], x \in \Omega} \beta(t, x).
\end{align*}
\]

For any \( \phi \in Y^+ \), let \( (S(t, x), I(t, x), W(t, x)) := (S(t, \phi)(x), I(t, \phi)(x), W(t, \phi)(x)), \ t \geq 0, x \in \Omega \) and \( S(t) = \int_\Omega S(t, x)dx, I(t) = \int_\Omega I(t, x)dx, W(t) = \int_\Omega W(t, x)dx \). An integration of the first equation and Green’s formula imply

\[
\frac{dS(t)}{dt} \leq \hat{a}S(t) - \int_\Omega \beta(t, x)S(t, x)I(t, x)dx,
\]

that is,

\[
\int_\Omega \beta(t, x)S(t, x)I(t, x)dx \leq \hat{a}S(t) - \frac{dS(t)}{dt}, \quad \forall t > 0.
\] (9)

By using the above inequality (9) and Green’s formula, integrating the equation for \( I \) in (2) yields

\[
\frac{dI(t)}{dt} \leq -\hat{c}I(t) + \hat{a}S(t) - \frac{dS(t)}{dt}, \quad \forall t > 0,
\]

which is equivalent to

\[
\frac{d}{dt} (\hat{S}(t) + \hat{I}(t)) \leq -\hat{c} (\hat{S}(t) + \hat{I}(t)) + (\hat{a} + \hat{c}) \hat{S}(t), \quad \forall t > 0.
\]
Thus, for \( t \geq l_1 T \), we have
\[
\frac{d}{dt} \left[ \bar{S}(t) + \bar{I}(t) \right] \leq -\bar{c} \left[ \bar{S}(t) + \bar{I}(t) \right] + M_1 |\Omega| (\bar{a} + \bar{c}) ,
\]
which yields \( \bar{S}(t) + \bar{I}(t) \leq \frac{\bar{M}_1}{\bar{c}} + 1 \) for \( t \geq l'_1 T \), where \( \bar{M}_1 := M_1 |\Omega| (\bar{a} + \bar{c}) \) and \( l'_1 > l_1 \) is some positive integer, and hence,
\[
\bar{I}(t) = \| \bar{I}(t, \cdot) \|_{L^1(\Omega)} \leq \frac{\bar{M}_1}{\bar{c}} + 1, \quad \forall t \geq l'_1 T.
\]
Integrating the second equation of (2) and using Green’s formula, we have that
\[
\frac{d\bar{I}(t)}{dt} = -\int_{\Omega} c(t,x) I(t,x) dx + \int_{\Omega} \beta(t,x) S(t,x) I(t,x) dx
\]
\[
- \int_{\Omega} f_1(t,x, I(t,x), W(t,x)) dx
\]
\[
\leq -\bar{c}\bar{I}(t) + \hat{\beta} M_1 \bar{I}(t) - \int_{\Omega} f_1(t,x, I(t,x), W(t,x)) dx, \quad t \geq l_1 T,
\]
that is,
\[
\int_{\Omega} f_1(t,x, I(t,x), W(t,x)) dx \leq -\bar{c}\bar{I}(t) + \hat{\beta} M_1 \bar{I}(t) - \frac{d\bar{I}(t)}{dt} . \quad (10)
\]
By (2) and Green’s formula gives
\[
\frac{d\bar{W}(t)}{dt} = -\int_{\Omega} d(t,x) W(t,x) dx + \int_{\Omega} f_2(t,x, I(t,x), W(t,x)) dx,
\]
and hence, (10) and (F3) imply that there exists a positive number \( k_1 \) independent of \( \phi \) such that
\[
\frac{d}{dt} \left[ \bar{W}(t) + k\bar{I}(t) \right] \leq -\bar{d} \left[ \bar{W}(t) + k\bar{I}(t) \right] + (\bar{d}k + k_1)\bar{I}(t)
\]
\[
\leq -\bar{d} \left[ \bar{W}(t) + k\bar{I}(t) \right] + M_2, \quad t \geq l'_1 T,
\]
where \( M_2 := (\bar{d}k + k_1) \cdot (\bar{M}_1 / \bar{c} + 1) \). This indicates that \( \bar{W}(t) \leq \frac{M_2}{\bar{d}} + 1 \) for \( t \geq l_2 T \), where \( l_2 > l_1 \) is some integer, and hence
\[
\bar{W}(t) = \| \bar{W}(t, \cdot) \|_{L^1(\Omega)} \leq \frac{M_2}{\bar{c}} + 1, \quad \forall t \geq l_2 T.
\]
In view of [13, Lemma 3.1] (see also [11, Theorem 1 and Corollary]), we can conclude that there exists a positive constant \( K \) independent of \( \phi \) such that
\[
\| I(t, \cdot) \|_{L^\infty(\Omega)} \leq K \| W(t, \cdot) \|_{L^\infty(\Omega)} \leq K, \quad t \geq l_2 T,
\]
and hence, \( S(t,x) \), \( I(t,x) \) and \( W(t,x) \) are ultimately bounded.

Define a family of operators \( \{ \Phi_\lambda \}_{\lambda \geq 0} \) on \( Y^+ \) by \( \Phi(\lambda)(\phi)(x) = u(t,x; \phi) \) for \( t \geq 0, x \in \Omega \) and \( \phi \in Y^+ \). By similar arguments to those in [32, Lemma 2.1], it follows that \( \{ \Phi_\lambda \}_{\lambda \geq 0} \) is an \( T \)-periodic semiflow on \( Y^+ \). The above analysis shows that \( \Phi(\lambda) : Y^+ \to Y^+ \) is point dissipative. In addition, \( \Phi(t) : Y^+ \to Y^+, t > 0 \) is compact. It then follows from [33, Theorem 1.1.2] that \( \Phi(T) : Y^+ \to Y^+ \) admits a global compact attractor. This completes the proof. \( \square \)
3. The SI model. When the predator population is absent, the system (2) becomes the following periodic and reaction-diffusion SI epidemic model:

\[
\begin{aligned}
\frac{\partial S(t,x)}{\partial t} &= \nabla \cdot [D_S(t,x)\nabla S(t,x)] + a(t,x)S(t,x) - b(t,x)S(t,x)I(t,x) - \beta(t,x)S(t,x)I(t,x), && t > 0, x \in \Omega, \\
\frac{\partial I(t,x)}{\partial t} &= \nabla \cdot [D_I(t,x)\nabla I(t,x)] - c(t,x)I(t,x) + \beta(t,x)S(t,x)I(t,x), && t > 0, x \in \Omega, \\
[D_S(t,x)\nabla S(t,x)] \cdot n &= [D_S(t,x)\nabla S(t,x)] \cdot n = 0, && t > 0, x \in \partial \Omega,
\end{aligned}
\]  

(11)

Define \( Z := C(\Omega, \mathbb{R}^2) \) and \( Z^+ := C(\Omega, \mathbb{R}^{2+}) \). Then \((Z, Z^+)\) is a strongly ordered Banach space with supremum norm \( \| \cdot \|_Z \). In view of Theorem 2.2, it is easy to show the conclusion on the well-posedness of system (11).

**Theorem 3.1.** For any \( \varphi = (\varphi_1, \varphi_2) \in Z^+ \), system (11) admits a unique solution \( v(t, \varphi) = (S(t, \varphi), I(t, \varphi)) \) on \([0, \infty)\) with \( v_0 = \varphi \). Furthermore, system (11) generates an \( T \)-periodic semiflow \( \Psi(t) := v(t, \cdot) : Z^+ \rightarrow Z^+ \), i.e., \( \Psi(t)(\varphi)(x) = v(t, x; \varphi), \forall \varphi \in Z^+, t \geq 0, x \in \Omega, \) and \( \Psi(T) : Z^+ \rightarrow Z^+ \) has a global compact attractor in \( Z^+ \).

In the following, motivated by the idea in [27] (see also [18, 32]), we introduce the basic reproduction number \( R_0 \) for the model (11). Let \( C_T(\mathbb{R}, X) \) be the ordered Banach space consisting of all \( T \)-periodic and continuous functions from \( \mathbb{R} \) to \( X \), where \( \| \psi \|_{C_T(\mathbb{R}, X)} := \max_{t \in [0,T]} \| \psi(t) \|_X \) for any \( \psi \in C_T(\mathbb{R}, X) \). Define

\[
C_T^+(\mathbb{R}, X) := \{ \psi \in C_T(\mathbb{R}, X) : \psi(t)(x) \geq 0, \forall t \in \mathbb{R}, x \in \Omega \}.
\]

Letting \( I = 0 \) in (11), we obtain the equation for \( S(t,x) \) as follows:

\[
\begin{aligned}
\frac{\partial S(t,x)}{\partial t} &= \nabla \cdot [D_S(t,x)\nabla S(t,x)] + a(t,x)S(t,x) - b(t,x)S^2(t,x)S(t,x) \quad t > 0, x \in \Omega, \\
[D_S(t,x)\nabla S(t,x)] \cdot n &= 0, \quad t > 0, x \in \partial \Omega.
\end{aligned}
\]  

(12)

By means of Lemma 2.1, it follows that (12) admits a unique positive \( T \)-periodic solution \( S^+(t, x) \) and it attracts the solutions of (12) with continuous non-negative non-trivial initial data if the principal eigenvalue \( \mu_0 \) of (5) satisfies

\( \text{(P): } \mu_0 < 0. \)

Then we have the following observation.

**Theorem 3.2.** Let \( \text{(P) hold. Then (11) admits a unique disease-free periodic solution \( (S^+, 0) \).} \)

Linearizing system (11) at \((S^+, 0)\), we obtain the following equation for the infection component:

\[
\begin{aligned}
\frac{\partial v(t,x)}{\partial t} &= \nabla \cdot [D_I(t,x)\nabla v(t,x)] - c(t,x)v(t,x) + \beta(t,x)S^+(t,x)v(t,x), \quad t > 0, x \in \Omega, \\
[D_I(t,x)\nabla v(t,x)] \cdot n &= 0, \quad t > 0, x \in \partial \Omega.
\end{aligned}
\]  

(13)

From the discussion in Section 2, it follows that there exist positive constants \( H \geq 1 \) and \( \omega \in \mathbb{R} \) such that

\[ \| V(t, s) \| \leq He^{\omega(t-s)}, \quad \forall t \geq s, t, s \in \mathbb{R}. \]
Consider the following periodic-parabolic eigenvalue problem

\[
\begin{cases}
\psi_t - \nabla \cdot [D_I(t,x)\nabla \psi] = -c(t,x)\psi + \beta(t,x)S^*(t,x)\psi + \lambda \psi, & t > 0, x \in \Omega, \\
[D_I(t,x)\nabla \psi] \cdot n = 0, & t > 0, x \in \partial \Omega, \\
\psi(0,x) = \psi(T,x), & x \in \Omega.
\end{cases}
\] (14)

Referring to [9], we know that (14) has a unique principal eigenvalue \( \rho_0 \) with positive periodic eigenfunction.

Suppose that \( \varphi(s,x) = \varphi(s)(x) \in C_T(\mathbb{R}, X) \) is the initial distribution of infectious individuals at time \( s \in \mathbb{R} \) and spatial location \( x \in \Omega \). Then the term \( (F(s)\varphi(s))(x) = \beta(s,x)S^*(s,x)\varphi(s,x) \) denotes the density distribution of the new infections produced by the infected individuals who were introduced at time \( s \). Thus, for given \( t \geq s \), \( (V(t,s)F(s)\varphi(s))(x) \) represents the density distribution at location \( x \) of those infected individuals who were newly infected at time \( s \) and remain infected at time \( t \). Consequently, the term

\[
\int_{-\infty}^{t} V(t,s)F(s)\varphi(s)ds = \int_{0}^{\infty} V(t,t-\sigma)(\beta(t-\sigma,.)S^*(t-\sigma,.)\varphi(t-\sigma, .))d\sigma
\]

measures the density distribution of the accumulative new infections at time \( t \) and location \( x \) produced by all those infected individuals \( \varphi(s,x) \) introduced at all the previous time to \( t \). As a result, we can define the next generation operator \( L \) as

\[
L(\varphi)(t) := \int_{0}^{\infty} V(t,t-\sigma)F(t-\sigma)\varphi(t-\sigma)d\sigma, \quad \forall t \in \mathbb{R}, \varphi \in C_T(\mathbb{R}, X).
\]

Based on the assumptions on \( \beta \) and \( c \), it is not difficult to see that \( L \) is a positive and bounded linear operator on \( C_T(\mathbb{R}, X) \). We define the spectral radius of \( L \) to be the basic reproduction number for the model (11), namely,

\[
\mathcal{R}_0 := r(L).
\]

As a straight consequence of [22, Theorem 5.7] (see also [18, Lemma 2.2]), we have the following lemma.

**Lemma 3.3.** \( 1 - \mathcal{R}_0 \) has the same sign as \( \rho_0(S^*) \).

Similar to [18], in order to obtain an equivalent characterization of \( \mathcal{R}_0 \), we first consider a periodic-parabolic eigenvalue problem:

\[
\begin{cases}
\psi_t - \nabla \cdot [D_I(t,x)\nabla \psi] = -c(t,x)\psi + \frac{\beta(t,x)S^*(t,x)}{\rho} \psi, & t > 0, x \in \Omega, \\
[D_I(t,x)\nabla \psi] \cdot n = 0, & t > 0, x \in \partial \Omega, \\
\psi(0,x) = \psi(T,x), & x \in \Omega.
\end{cases}
\] (15)

According to [9, Theorem 16.3], we see that (15) admits a unique and positive principal eigenvalue \( \rho_0 \) associated with an eigenfunction \( \psi_0 \in C_T(\mathbb{R}, X) \) and \( \psi_0 > 0 \) on \( \mathbb{R} \times \Omega \). By similar arguments to [18, Lemma 2.1], we show the conclusion as follows.

**Lemma 3.4.** \( \mathcal{R}_0 = \rho_0 > 0 \).

We are in position to establish the threshold dynamics of (11) in terms of \( \mathcal{R}_0 \).

**Theorem 3.5.** Assume that (P) holds. Let \( v(t, x; \varphi) \) be the solution of (11) with \( v_0 = \varphi \in \mathbb{Z}^+ \). Then the following statements hold:
(i): If $R_0 < 1$, then for any $\varphi = (\varphi_1, \varphi_2) \in Z^+$ with $\varphi_1(\cdot) \neq 0$, the disease-free $T$-periodic solution $(S^*(t,x),0)$ is globally attractive in $Z^+ \setminus \{0\}$.

(ii): If $R_0 > 1$, then system (11) admits at least one positive $T$-periodic solution $(\tilde{S}(t,x), \tilde{I}(t,x))$ and there exists an $\eta > 0$ such that for any $\varphi \in Z^+$ with $\varphi_1(\cdot) \neq 0$ and $\varphi_2(\cdot) \neq 0$, we have

$$\lim_{t \to \infty} (S(t,x;\varphi), I(t,x;\varphi)) \geq (\eta, \eta)$$

uniformly for all $x \in \Omega$.

Proof. (i). In the case where $R_0 < 1$, Lemma 3.3 implies that $\lambda_0(S^*) > 0$. Since $\lim_{\epsilon \to 0+} \lambda_0(S^* + \epsilon) = \lambda_0(S^*) > 0$, there exists a $\epsilon > 0$ such that $\lambda_0(S^* + \epsilon) > 0$. From the first equation in system (11), it is easy to see that

$$\begin{cases}
\frac{\partial S(t,x)}{\partial t} \leq \nabla \cdot [D_S(t,x)\nabla S(t,x)] + a(t,x)S(t,x) - b(t,x)S^2(t,x) & t > 0, x \in \Omega, \\
[D_S(t,x)\nabla S(t,x)] \cdot n = 0, & t > 0, x \in \partial \Omega.
\end{cases}$$

By the comparison principle and Lemma 2.1, it follows that there exists an integer $q > 0$ such that $S(t,x;\varphi) \leq S^*(t,x) + \epsilon, \forall t \geq qT, x \in \Omega$. Thus, for all $t \geq qT$ and $x \in \Omega$, we have

$$\begin{cases}
\frac{\partial I(t,x)}{\partial t} \leq \nabla \cdot [D_I(t,x)\nabla I(t,x)] - c(t,x)I(t,x) \\
+ \beta(t,x)(S^*(t,x) + \epsilon)I(t,x), & t > qT, x \in \Omega,
\end{cases}$$

$$[D_I(t,x)\nabla I(t,x)] \cdot n = 0, \quad t > qT, x \in \partial \Omega.$$

We consider a periodic and reaction-diffusion equation with $\epsilon > 0$:

$$\begin{cases}
\frac{\partial v^\epsilon(t,x)}{\partial t} = \nabla \cdot [D_I(t,x)\nabla v^\epsilon(t,x)] - c(t,x)v^\epsilon(t,x) \\
+ \beta(t,x)(S^*(t,x) + \epsilon)v^\epsilon(t,x), & t > 0, x \in \Omega,
\end{cases}$$

$$[D_I(t,x)\nabla v^\epsilon(t,x)] \cdot n = 0, \quad t > 0, x \in \partial \Omega.$$

Clearly, $v^\epsilon(t,x) = e^{-\lambda_0(S^*)t}\psi_\epsilon(t,x)$ is a solution of the above equation, where $\psi_\epsilon \in C_T(\mathbb{R}, X)$ is the positive eigenfunction corresponding to $\lambda_0(S^*)$. Since $I(t,x;\varphi)$ is globally bounded for any $\varphi \in Z^+$, we can find an $\alpha > 0$ such that $I(qT,x;\varphi) \leq \alpha \psi_0(0,x), x \in \Omega$. Thus, the standard comparison theorem indicates that for any $t \geq qT$ and $x \in \Omega$, $I(t,x;\varphi) \leq \alpha e^{-\lambda_0(S^*)qT}\psi_\epsilon(t,x)$, and hence,

$$\lim_{t \to \infty} I(t,x;\varphi) = 0 \quad \text{uniformly for } x \in \Omega.$$

Then the equation for $S$ is asymptotic to the equation (6). Note that $S^*(t,x)$ is a global attractive solution of (6). We shall use the theory of internally chain transitive sets (see, e.g., [10] or [34]) to prove that $\lim_{t \to \infty} (S(t,x;\varphi) - S^*(t,x)) = 0$ uniformly for $x \in \Omega$.

For any given $\varphi \in (X^+ \setminus \{0\}) \times X^+$, let $J$ be the omega limit set of $\{\Psi_k^\epsilon(\varphi)\}_{n \geq 1}$ and $\Psi_k^\epsilon(\varphi) = \Psi(nT)(\varphi)$. It then follows from [10] Lemma 2.1 (see also [34] Lemma 1.2.1) that $J$ is an internally chain transitive set for $\Psi(T)$, since $I(t,\cdot;\varphi) \to 0$ as $t \to \infty$, there is a subset $J_1$ of $X^+$ such that $J = J_1 \times \{0\}$. It is easy to see that $\Psi(T)(J_1 \times \{0\}) = J_1 \times \{0\}$, where $\{Q_t\}_{t \geq 0}$ is the periodic semiflow associated with equation (6) on $X^+$, and hence, $Q(T)(J_1) = J_1$. It then follows that $J$ is an internally chain transitive set for $Q(T)$. Since (P) holds, Lemma 2.1 implies that $S^*(0,\cdot)$ is globally attractive in $X^+ \setminus \{0\}$. Consequently, we can conclude from [10] Theorem 3.1 (see also [34] Theorem 1.2.1) that $J = \{(S^*(0,\cdot),0)\}$. This implies that the statement (i) is valid.
(ii). In the case where $R_0 > 1$, Lemma 3.3 implies that $\lambda_0(S^*) < 0$. Let
$$\mathcal{W}_0 = \{ \varphi \in Z^+ : \varphi_i(\cdot) \neq 0, \ i = 1, 2 \}$$
and
$$\partial \mathcal{W}_0 := Z^+ \setminus \mathcal{W}_0 = \{ \varphi \in Z^+ : \varphi_1(\cdot) \equiv 0 \text{ or } \varphi_2(\cdot) \equiv 0 \}.$$ 
Define
$$M_\delta := \{ \varphi \in \partial \mathcal{W}_0 : \Psi(nT) \varphi \in \partial \mathcal{W}_0, \ \forall n \in \mathbb{N} \}.$$ 
Let $E_1 := \{(0,0)\}$, $E_2 := \{(S^*(0),0)\}$. It is easy to see that $\Psi(nT)\mathcal{W}_0 \subset \mathcal{W}_0, \forall n \in \mathbb{N}$. Furthermore, we have the following three claims.

**Claim 1.** $\cup_{\varphi \in M_\delta} \mathcal{J}(\varphi) = E_1 \cup E_2$.

Assume that $\varphi \in M_\delta$. It then follows from the maximum principle that either $S(t,x;\varphi) := S(t;\varphi)(x) \equiv 0$ or $I(t,x;\varphi) := I(t;\varphi)(x) \equiv 0$ on $(t,x) \in \mathbb{R}^+ \times \Omega$. If $S(t,x;\varphi) \equiv 0$ on $(t,x) \in \mathbb{R}^+ \times \Omega$, then $\lim_{t \to \infty} I(t,\cdot;\varphi) \equiv 0$. If $I(t,\cdot;\varphi) \equiv 0$, then
$$S(\cdot,\cdot;\varphi) \equiv v(\cdot,\cdot;\psi),$$
where $v(\cdot,\cdot;\psi)$ is the solution of (10) with $\psi_1(x) = \varphi(x)$ for all $x \in \Omega$. Therefore, $\cup_{\varphi \in M_\delta} \mathcal{J}(\varphi) = E_1 \cup E_2$, which proves Claim 1.

Consider the following periodic-parabolic eigenvalue problem with $\delta > 0$ :
$$\begin{align*}
\frac{\partial \psi_\delta}{\partial t} &= \nabla \cdot [DS(t,x)\nabla \psi_\delta] - [2b(t,x)\delta + \beta(t,x)\delta] \psi_\delta \\
&\quad + a(t,x)\psi_\delta + \mu \psi_\delta, \quad t > 0, \ x \in \Omega, \\
[DS(t,x)\nabla \psi_\delta] \cdot n &= 0, \quad t > 0, \ x \in \partial \Omega, \\
\psi_\delta(0,x) &= \psi_\delta(T,x), \quad x \in \Omega.
\end{align*}$$

(16)

Since (16) has a principal eigenvalue $\mu_0 < 0$, there is a sufficiently small $\delta_0 > 0$ such that for all $\delta \in (0,\delta_1)$, (16) admits a principal eigenvalue $\mu_\delta < 0$ with a positive eigenfunction $\psi_\delta \in C_T(\mathbb{R},X)$. Let $\delta \in (0,\delta_1)$ be fixed. There exists a $0 < \rho_\delta < \delta$ such that $S^2 < \delta S$ for $\delta \in (0,\rho_\delta)$.

**Claim 2.** $E_1$ is a uniform weak repellor for $\mathcal{W}_0$ in the sense that
$$\lim_{n \to \infty} \sup \| \Psi^n_T(\varphi) - E_1 \| \geq \rho_\delta \text{ for all } \varphi \in \mathcal{W}_0.$$ 

Suppose, by contradiction, there exists $\varphi_0 \in \mathcal{W}_0$ such that
$$\lim_{n \to \infty} \sup \| \Psi^n_T(\varphi_0) - E_1 \| < \rho_\delta.$$ 

Then there exists $q' \in \mathbb{N}$ such that
$$S(t,\cdot;\varphi_0) < \rho_\delta, \ I(t,\cdot;\varphi_0) < \rho_\delta \text{ for all } t \geq q'T.$$ 

Hence, $S(t,x;\varphi_0)$ satisfies
$$\frac{\partial S(t,x)}{\partial t} > \nabla \cdot [DS(t,x)\nabla S(t,x)] + a(t,x)S(t,x) - (2b(t,x)\delta + \beta(t,x)\delta)S(t,x) \quad (17)$$
for $t \geq q'T, \ x \in \Omega$. Since $S(t,x;\varphi_0) > 0$ for all $t > 0$ and $x \in \Omega$, there exists $\zeta > 0$ such that
$$S(q'T,x;\varphi_0) \geq \zeta \psi_\delta(0,x), \ \forall x \in \Omega.$$ 

By the inequality (17) and the comparison principle, we obtain
$$S(t,x;\varphi_0) \geq \zeta e^{-\mu_\delta(t-q'T)}\psi_\delta(t,x), \ \forall t \geq q'T, \ x \in \Omega.$$ 

Since $\mu_\delta < 0$, it follows that $S(t,x;\varphi_0)$ is unbounded, a contradiction. This implies the claim 2 is true.

Since $\lim_{\nu \to 0^+} \lambda_0(S^* - \nu) = \lambda_0(S^*) < 0$, there exists a sufficiently small positive number $\nu$ such that $\lambda_0(S^* - \nu) < 0$. 

A PERIODIC AND DIFFUSIVE PREDATOR-PREY MODEL 455
Claim 3. $E_2$ is a uniform weak repellor for $\mathcal{W}_0$ in the sense that
\[
\limsup_{n \to \infty} \|\Psi(T^n) - E_2\| \geq \nu \quad \text{for all } \varphi \in \mathcal{W}_0.
\]

Suppose, by contradiction, there exists $\varphi'_0 \in \mathcal{W}_0$ such that
\[
\limsup_{n \to \infty} \|\Psi(T^n)\varphi'_0 - E_2\| < \nu.
\]
Then there exists an integer $q_1 > 0$ such that
\[
S(t;x;\varphi'_0) > S^*(t,x) - \nu, \quad I(t;x;\varphi'_0) < \nu, \quad \forall t \geq q_1T, \ x \in \overline{\Omega}.
\]
Hence, for $t \geq q_1T$ and $x \in \Omega$, $I(t;x;\varphi'_0)$ satisfies
\[
\frac{\partial I(t,x)}{\partial t} \geq \nabla \cdot [D_I(t,x)\nabla I(t,x)] - c(t,x)I(t,x) + \beta(t,x)(S^*(t,x) - \nu)I(t,x). \quad (18)
\]
Since $I(t;x;\varphi'_0) > 0$ for all $t > 0$ and $x \in \Omega$, there exists $\xi > 0$ such that
\[
I(q_1T;T;x;\varphi_0) \geq \xi e^{-\lambda_0(S^* - \nu)q_1T} \psi_0(q_1T,x), \quad \forall x \in \Omega,
\]
where $\psi_0 \in C_T(\mathbb{R},X^+)$ is the eigenfunction corresponding to $\lambda_0(S^* - \nu)$. Since $\lambda_0(S^* - \nu) < 0$, it then follows from the inequality (18) and the comparison principle that $I(t;x,\varphi'_0)$ is unbounded, which is a contradiction. This completes the proof of Claim 3.

From the above claims, we can conclude that $E_i, i = 1, 2$ are isolated invariant sets for $\Psi(T)$ in $\mathcal{W}_0$ and $W^*(E_i) \cap \mathcal{W}_0 = \emptyset$, where $W^*(E_i)$ is the stable set of $E_i, i = 1, 2$. By means of the acyclicity theorem on uniform persistence for maps (see, e.g. [34] Theorem 1.3.1 and Remark 1.3.1)), it follows that $\Psi(T) : Z^+ \to Z^+$ is uniformly persistent with respect to $(\mathcal{W}_0, \partial \mathcal{W}_0)$, and hence, [34] Theorem 3.1.1] implies that $\Psi(t) : Z^+ \to Z^+$ is also uniformly persistent with respect to $(\mathcal{W}_0, \partial \mathcal{W}_0)$. Since, in addition, $\Psi(T)$ is compact, it follows from [13] Theorem 4.5] that $\Psi(T) : \mathcal{W}_0 \to \mathcal{W}_0$ has a global attractor $\mathcal{A}_0$ and system (11) admits a $T$-periodic solution $(\tilde{S}(t,\cdot), \tilde{I}(t,\cdot)) \in \mathcal{W}_0$.

At what follows, by using the arguments similar to [13] Theorem 4.1] (see also [32] Theorem 4.3(ii))], we are ready to prove the practical uniformly persistence. Define a continuous function $p : Z^+ \to [0, \infty)$ by
\[
p(\varphi) := \min\{\varphi_1(x), \varphi_2(x)\} \quad \forall \varphi = (\varphi_1, \varphi_2) \in Z^+, \ x \in \overline{\Omega}.
\]
Since $\mathcal{A}_0 = \Psi(T)\mathcal{A}_0$, it follows that $\varphi_1(\cdot) > 0$ for all $\varphi \in \mathcal{A}_0$. Let $B_0 := \cup_{t \in [0,T]} \Psi(t)\mathcal{A}_0$. It then follows that $B_0 \subset \mathcal{W}_0$ and $\lim_{t \to \infty} d(\Psi(t)\varphi, B_0) = 0$ for all $\varphi \in \mathcal{W}_0$. Due to the compactness of $B_0$ for $\Psi(t)$ in $\mathcal{W}_0$, it is easy to see that there exists $\eta > 0$ such that $\liminf_{t \to \infty} \Psi(t)\varphi \geq (\eta, \eta)$ for all $\varphi \in \mathcal{W}_0$. This complete the proof. \qed

4. Dynamics of system (2). In this section, we are going to establish the uniform persistence and extinction for model (2).

4.1. Uniform Persistence. Consider the following periodic-parabolic eigenvalue problem with $\eta > 0$ determined in Theorem 3.5(ii):
\[
\begin{aligned}
\psi_t - \nabla \cdot [D_W(t,x)\nabla \psi] + d(t,x)\psi - \partial_1 f_2(t,x,\eta,0)\psi &= \lambda \psi, \quad t > 0, \ x \in \Omega, \\
[D_W(t,x)\nabla \psi] \cdot \mathbf{n} &= 0, \quad t > 0, \ x \in \partial \Omega, \\
\psi(0,x) &= \psi(T,x), \quad x \in \Omega.
\end{aligned}
\]
Let $\lambda_0(D_W, \partial_1 f_2(t,x,\eta,0))$ be the principal eigenvalue (see, e.g., [9]).
Theorem 4.1. Assume that (P) and (F1)-(F3) hold. Let
\[ u(t, x; \phi) = (S(t, x; \phi), I(t, x; \phi), W(t, x; \phi)) \]
be the solution of (1) with \( u_0 = \phi = (\phi_1, \phi_2, \phi_3) \in Y^+ \). If \( R_0 > 1 \) and
\[ \lambda_0(D_W, \partial_1 f_2(t, x, \eta, 0)) < 0, \]
then the system (1) admits at least one positive \( T \)-periodic solution, and there exists an \( \eta^* > 0 \) such that for any \( \phi = (\phi_1, \phi_2, \phi_3) \in Y^+ \) with \( \phi_i(\cdot) \neq 0, i = 1, 2, 3 \), we have
\[ \liminf_{t \to \infty} (S(t, x; \phi), I(t, x; \phi), W(t, x; \phi)) \geq (\eta^*, \eta^*, \eta^*) \]
uniformly for \( x \in \Omega \).

Proof. Motivated by [31] Theorem 4.1, in the following, we prove the conclusion stated as the above. Since (P) holds and \( R_0 > 1 \), Theorem 3.5(ii) implies that
\[ \liminf_{t \to \infty} (S(t, x; \varphi), I(t, x; \varphi)) \geq (\eta, \eta) \tag{20} \]
uniformly for all \( x \in \Omega \), where \( \varphi = (\varphi_1, \varphi_2) = (\phi_1, \phi_2) \in Z^+ \). Recall that the semiflow \( \Psi(t) \) of (11) is defined by \( (\Psi(t) \varphi)(x) := v(t, x; \varphi) = (S(t, x; \varphi), I(t, x; \varphi)) \) for \( t > 0, x \in \Omega \) and \( \varphi = (\varphi_1, \varphi_2) = (\phi_1, \phi_2) \in Z^+ \), and \( \Psi(T) : Z^+ \to Z^+ \) admits a positive global attractor \( A_0 \). Let \( B_0 = \bigcup_{t \in [0,T]} \Psi(t) A_0 \). Then \( B_0 \subset Z^+ \) and \( \lim_{t \to \infty} d(\Psi(t) \varphi, B_0) = 0 \) for all \( \varphi \in Z^+ \) with \( \varphi_1(\cdot) \neq 0 \). By (20), it follows that
\[ (\chi_1(x), \chi_2(x)) \geq (\eta, \eta), \quad \forall (\chi_1, \chi_2) \in B_0, x \in \Omega. \]
For any \( \chi := (\chi_1, \chi_2) \in B_0 \), it is easy to see that the solution \( (S(t, x; \chi), I(t, x; \chi)) \) is well defined for \( t \in \mathbb{R} \) and \( (S(t, \cdot; \chi), I(t, \cdot; \chi)) = (\Psi(t) \chi)(\cdot) \in B_0 \) for all \( t \in \mathbb{R} \). Thus, \( (S(t, x; \chi), I(t, x; \chi), 0) \) is also a solution of (2) defined for all \( t \in \mathbb{R} \). Define
\[ B_0 = \{ (\chi_1, \chi_2) \in Z : (\chi_1, \chi_2) = (S(t, \cdot; \chi), I(t, \cdot; \chi)), \forall t \in \mathbb{R}, \chi \in B_0 \}. \]
Let
\[ V_0 = \{ \phi \in Y^+ : \phi_i(\cdot) \equiv 0, i = 1, 2, 3 \} \]
and
\[ \partial V_0 := Y^+ \setminus V_0 = \{ \phi \in Y^+ : \phi_1(\cdot) \equiv 0 \text{ or } \phi_2(\cdot) \equiv 0 \text{ or } \phi_3(\cdot) \equiv 0 \}. \]
Define
\[ N_\delta := \{ \phi \in \partial V_0 : \Phi(nT) \phi \in \partial V_0, n \in \mathbb{N} \}. \]
Let \( \tilde{E}_1 := \{ (0, 0, 0) \}, \tilde{E}_2 := \{ (S^*(0), 0, 0) \}, \tilde{E}_3 := \{ (\phi_1, \phi_2, 0) : (\phi_1, \phi_2) \in E_0 \} \) and \( \tilde{J} \) be the omega limit set of \( \{ \Phi^n \phi \}_{n \geq 1} \). By similar arguments to the proof of Theorem 3.5(ii), we have the following two claims and omit the proofs.

Claim 1. \( \cup_{\phi \in N_0} \tilde{J}(\phi) = \tilde{E}_1 \cup \tilde{E}_2 \cup \tilde{E}_3 \).

Claim 2. \( E_i, i = 1, 2 \) are uniform weak repellers for \( V_0 \) in the sense that
\[ \limsup_{n \to \infty} ||\Phi^n_T(\phi) - E_i|| = \rho_i, i = 1, 2, \text{ for all } \phi \in V_0. \]

Since \( \lambda_0(D_W, \partial_1 f_2(t, x, \eta, 0)) < 0 \), there exists a sufficiently small positive number \( \varrho > 0 \) such that \( \lambda_0(D_W, \partial_1 f_2(t, x, \eta - \varrho, 0)) < 0 \).

Claim 3. \( E_3 \) is a uniform weak repeller for \( V_0 \) in the sense that
\[ \limsup_{n \to \infty} \inf_{\varphi \in E_3} ||\Phi^n_T(\phi) - \varphi|| \geq \rho \text{ for all } \phi \in V_0. \]

Suppose, by contradiction, there exists \( \phi_0 \in W_0 \) such that
\[ \limsup_{n \to \infty} \inf_{\varphi \in E_3} ||\Phi^n_T(\phi_0) - \varphi|| < \rho. \]
Then there exists an integer $q > 0$ such that
\[ \inf_{\vartheta \in \mathcal{E}_3} \| \varphi_0^q (\vartheta) - \vartheta \| < q \quad \forall n > q. \]

Due to the compactness of $\mathcal{E}_3$, we can find that there exists a $\vartheta^* \in \mathcal{E}_3$ such that
\[ \| \varphi_0^q (\vartheta) - \vartheta^* \| \leq q \quad \forall n > q. \]

Thus, we have $I(t, x; \vartheta_0) > \eta - q$ and $W(t, x; \phi) \leq q$ for all $t \geq qT$ and $x \in \Omega$. In view of (F3), it is easy to see that $W(t, x; \vartheta_0)$ satisfies
\[
\frac{\partial W(t, x)}{\partial t} \geq \nabla \cdot [D_W(t, x) \nabla W(t, x)] - d(t, x) W(t, x) + \partial_1 f_2(t, x, \eta - q, 0) W(t, x) \quad (21)
\]
for $t > qT$ and $x \in \Omega$. Since $W(t, ; \vartheta_0)$ is positive for all $t > 0$, there exists $c > 0$ such that
\[ W(qT, ; \vartheta_0) \geq c e^{-\lambda_0(D_W, \partial f_2(t, x, \eta - q, 0)) qT} \psi_0^q(qT, \cdot), \]
where $\psi_0^q \in C_T(\mathbb{R}, X)$ is the positive eigenfunction corresponding to $\lambda_0(D_W, \partial f_2(t, x, \eta - q, 0))$. It then follows from the inequality (21) and the comparison principle that
\[ W(t, x; \vartheta_0) \geq c e^{-\lambda_0(D_W, \partial f_2(t, x, \eta - q, 0)) qT} \psi_0^q(t, x), \quad \forall t > qT, \ x \in \overline{\Omega}, \]
which implies that $W(t, x; \vartheta_0)$ is bounded because $\lambda_0(D_W, \partial f_2(t, x, \eta - q, 0)) < 0$. This is a contradiction. Thus, we complete the proof of Claim 3.

The same analysis as the end of the proof of Theorem 3.5 we can prove the conclusion stated in the theorem.

4.2. Global extinction. In this section, we focus on the global extinction of the predator species, and global extinction of both prey and predator species. Furthermore, the extinction of the disease for prey species is also discussed.

Theorem 4.2. Assume that (P) and (F1)-(F3) hold. Set $w^* = (w_1^*, w_2^*) \in \text{int}(Z^+)$ be fixed and satisfies $w^* (x) \geq \iota(x)$ for all $x \in \Omega$ and $\iota \in \mathcal{C}_0$, where $\mathcal{C}_0$ is defined in Theorem 4.7. Let $u(t, x; \phi) = (S(t, x; \phi), I(t, x; \phi), W(t, x; \phi))$ be the solution of (2) with $u_0 = (\phi_1, \phi_2, \phi_3) \in Y^+$. Then the following statements are valid:

(i): If $\mathcal{R}_0 < 1$, then for any $\phi = (\phi_1, \phi_2, \phi_3) \in Y^+$ with $\phi_1(\cdot) \neq 0$, we have
\[ \lim_{t \to \infty} ((S(t, ; \cdot), I(t, ; \cdot), W(t, ; \cdot)) - (S^*(t, ; \cdot), 0, 0)) = 0. \]

(ii): If $\mathcal{R}_0 > 1$ and $\lambda_0(D_W, \partial f_2(t, x, w^*(\cdot), 0)) > 0$, then there exists an $\eta^* > 0$ such that for any $\phi = (\phi_1, \phi_2, \phi_3) \in Y^+$ with $\phi_1(\cdot) \neq 0$ and $\phi_2(\cdot) \neq 0$, we have
\[ \liminf_{t \to \infty} S(t, : \cdot, \phi) > (\eta^*, \eta^*) \quad \text{and} \quad \lim_{t \to \infty} W(t, ; \cdot, \phi) = 0. \]

Proof. The conclusion (i) can follow from the similar arguments to the proof of Theorem 3.5.

(ii) Since
\[ \lambda_0(D_W, \partial f_2(t, x, w^*(\cdot) + \varepsilon, 0)) = \lambda_0(D_W, \partial f_2(t, x, w^*(\cdot), 0)) > 0, \]
there exists some $\varepsilon > 0$ such that $\lambda_0(D_W, \partial f_2(t, x, w^*_2(\cdot) + \varepsilon, 0)) > 0$. It is clear that ($S(t, x), I(t, x))$ satisfies
\[
\frac{\partial S(t, x)}{\partial t} = \nabla \cdot [D_S(t, x) \nabla S(t, x)] + a(t, x) S(t, x) - b(t, x) S(T, x) + I(t, x) - \beta(t, x) S(t, x) I(t, x),
\]
\[
\frac{\partial I(t, x)}{\partial t} \leq \nabla \cdot [D_I(t, x) \nabla I(t, x)] - c(t, x) I(t, x) + \beta(t, x) S(t, x) I(t, x). \]
Thus, \( (S(t,x), I(t,x)) \) can be controlled by the system which generates a \( T \)-periodic semiflow \( \{\Psi(t)\}_{t \geq 0} \). Since \( (P) \) hold and \( R_0 > 1 \), it follows from Theorem 3.5(ii) that \( \lim_{t \to \infty} \partial t(x) = 0 \) for \( \partial t(x) \in Z^+ \) with \( \partial t(x) \neq 0, i = 1, 2 \), and hence, there exists an positive integer \( m = m(\partial t(x)) \) such that

\[
(S(t,x), I(t,x)) \leq (w^*_1(x) + \varepsilon, w^*_2(x) + \varepsilon), \quad \forall t \geq mT, x \in \Omega.
\]

By (F2)-(F3), \( W(t,x) \) satisfies

\[
\frac{\partial W(t,x)}{\partial t} \leq \nabla \cdot [D_W(t,x)\nabla W(t,x)] - d(t,x)W(t,x) + \partial t f_2(t, x, w^*_2(x) + \varepsilon, 0)W(t,x)
\]

for \( t \geq mT \) and \( x \in \Omega \). Since \( \lambda_0(D_W, \partial t f_2(x, w^*, 0)) > 0 \), we can conclude from the comparison principle that \( \lim_{t \to \infty} W(t, \cdot) = 0 \). As a consequence, the equations for \( S \) and \( I \) are asymptotic to system (11). By virtue of \( (P) \) and \( R_0 > 1 \), it follows from Theorem 3.5 that \( \Psi(T) \) admits a positive global attractor \( A_0 \) in \( \mathbb{W}_0 \). Recall that \( \bar{J} \) is the omega limit set of \( \{\xi_n \}_{n \geq 1} \). Then similar arguments to the proof of Theorem 3.5(i) yield that \( \bar{J} = \bar{J}_1 \times \{0\} \) and \( \Psi(T)\bar{J}_1 = \bar{J}_1 \). As a result, \( \bar{J}_1 \) is an internally chain transitive set for \( \Psi(T) \). It then follows from [10, Theorem 3.1] (see also [34, Theorem 1.2.1]) that \( \bar{J}_1 \subset A_0 \). Since \( A_0 = \bigcup_{\phi \in [0,T]} \Psi(t,A_0) \), we have \( \lim_{t \to \infty} \partial t(x) = 0 \) for \( \phi(t) \neq 0, i = 1, 2 \), and hence, there exists an \( \eta' > 0 \) such that for any \( \phi \in Y^+ \) with \( \phi(t) \neq 0, i = 1, 2 \), we have

\[
\lim \inf_{t \to \infty} (S(t, \cdot), I(t, \cdot), W(t, \cdot)) \geq (\eta', \eta', \eta).
\]

This completes the proof. 

The following result asserts that when the susceptible prey cannot survive, the disease goes to extinction and the predator species dies out.

**Theorem 4.3.** Let (F1)-(F3) hold. Assume that \( \mu_0 \geq 0 \). Then for any \( \phi = (\phi_1, \phi_2, \phi_3) \in Y^+ \), the solution \( u(t, x; \phi) = (S(t, x; \phi), I(t, x; \phi), W(t, x; \phi)) \) of system (1) satisfies

\[
\lim_{t \to \infty} (S(t, \cdot; \phi), I(t, \cdot; \phi), W(t, \cdot; \phi)) = (0, 0, 0).
\]

**Proof.** It is easy to see that \( S(t, x) \) satisfies that

\[
\frac{\partial S(t,x)}{\partial t} \leq \nabla \cdot [D_S(t,x)\nabla S(t,x)] + a(t,x)S(t,x) - b(t,x)S^2(t,x)
\]

for \( t > 0 \) and \( x \in \Omega \). By \( \mu_0 \geq 0 \), it follows from Lemma 2.1 that \( \lim_{t \to \infty} S(t, \cdot) = 0 \). Then we regard \( I(t, x) \) as a solution of the following non-autonomous reaction-diffusion equation

\[
\begin{cases}
\frac{\partial I(t,x)}{\partial t} = \nabla \cdot [D_I(t,x)\nabla I(t,x)] - c(t,x)I(t,x) \\
+ \beta(t,x)S(t,x)I(t,x), \quad t > 0, x \in \Omega,
\end{cases}
\]

\[
[D_I(t,x)\nabla I(t,x)] \cdot n = 0, \quad t > 0, x \in \partial \Omega.
\]

Since \( I(t,x) \) is bounded and \( \lim_{t \to \infty} S(t, \cdot) = 0 \), we see that the above equation is asymptotic the periodic reaction-diffusion equation as follows:

\[
\begin{cases}
\frac{\partial v(t,x)}{\partial t} = \nabla \cdot [D_I(t,x)\nabla v(t,x)] - c(t,x)v(t,x), \quad t > 0, x \in \Omega, \\
[D_I(t,x)\nabla v(t,x)] \cdot n = 0, \quad t > 0, x \in \partial \Omega.
\end{cases}
\]

(22)

It is obvious that \( v^* = 0 \) is globally attractive for (22) in \( X^+ \) (see [32, Lemma 2.1]). By [34, Proposition 3.2.1], \( \lim_{t \to \infty} I(t,x) = 0 \) uniformly for \( x \in \Omega \). The same
arguments as above can lead to \( \lim_{t \to \infty} W(t, x) = 0 \) uniformly for \( x \in \Omega \). This proof is completed.

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