Low-complexity Fusion Filtering for Continuous-Discrete Systems

Seokhyoung Lee and Vladimir Shin

Abstract—In this paper, low-complexity distributed fusion filtering algorithm for mixed continuous-discrete multisensory dynamic systems is proposed. To implement the algorithm a new recursive equations for local cross-covariances are derived. To achieve an effective fusion filtering the covariance intersection (CI) algorithm is used. The CI algorithm is useful due to its low-computational complexity for calculation of a big number of cross-covariances between local estimates and matrix weights. Theoretical and numerical examples demonstrate the effectiveness of the covariance intersection algorithm in distributed fusion filtering.

Index Terms—multisensory, fusion filtering, covariance intersection, continuous-discrete system

1 INTRODUCTION

RecenTLY a multisensory data fusion has been an interesting topic to increase the accuracy of parameter estimates or estimates of system states. This interest has been motivated by the increased availability of different types of sensors, and as such fusion estimation has the potential for widespread application, since in a range of scenarios, system states or targets are measured by multisensors.

Consequently several distributed fusion architectures and corresponding techniques were presented in [1-4], especially distributed fusion estimation algorithms for finding the best linear combination of the local estimates. The optimal mean-square linear combinations (fusion formulas) of an arbitrary number of local estimates with explicit/implicit expressions for matrix and scalar weights were reported in [4-9]. Furthermore, an effective suboptimal fusion based on the covariance intersection (CI) algorithm was developed in [10, 11].

However, nowadays, with the increasing sophistication of microprocessors, control schemes are being implemented digitally; i.e., in their practical application, received signal are usually measured at a discrete time instants while real signal is continuous. In this case, the above mentioned fusion formulas can not be effectively applicable, because they still require computation of the matrix weights between discrete measurements. In contrast to the optimal fusion formulas, the CI algorithm is more effective and applicable to continuous-discrete system models due to the fact that it computes the weights only at each discrete time instant. To this end, the main purpose of this paper is to compare the CI algorithm with optimal fusion formulas and verify their effectiveness in real implementation.

2 FUSION FORMULA WITH MATRIX WEIGHTS

Suppose that we have \( N \) local estimates of an unknown random signal \( \mathbf{x}_t \in \mathbb{R}^n \), i.e.,

\[
\hat{x}_t^{(1)}, \ldots, \hat{x}_t^{(N)}.
\]

Next, let us consider a linear combination of the local estimates with matrix weights. We have

\[
\hat{x}_t^{FF} = \sum_{i=1}^{N} C_i^{(i)} \hat{x}_t^{(i)},
\]

where \( C_i^{(i)} \) are \( n \times n \) matrix weights. In addition, from the assumption of an unbiased condition, the constraint of the matrix weights is given by

\[
\sum_{i=1}^{N} C_i^{(i)} = I_n,
\]

where \( I_n \) is an \( n \times n \) identity matrix. We refer to the fusion formula with matrix weights (2) and (3) as FF.

This paper is organized as follows. In Section 2, optimal fusion formula weighted by matrices is presented. A distributed fusion filtering algorithm for mixed continuous-discrete systems is considered in Section 3. In Section 4, the covariance intersection algorithm is presented. Through a theoretical example, accuracy and low-complexity of the covariance intersection algorithm are verified. In Section 5, a numerical example demonstrates the accuracy and computational efficiency of the covariance intersection algorithm are presented. Finally, a brief conclusion is given in Section 6.

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is an important
and corresponding error
sensors. These sensors measure at discrete
times.

Furthermore, the fusion error covariance
is a zero-mean white Gaussian system noise

is a local Kalman filter gain, and the time up-
date equations in (5) and (11), the
takes a lot of time. Therefore, we focus on how t o

Each subsystem with common state
depends on a concrete model of the ran-
tation of

The state-space model of the system takes

with intensity

The fusion error covariance
is obtained by the following differen-
tial equations:

Time-Update Equations Between Measurements:

Using (9) and (10) we have

To compute the matrix weights
in (5) and (11), the

matrix weights can
be obtained by solving the following measurement-
and time-update equations [5, 8]:

where

In the next section, we introduce the covariance in ter-
sion algorithm, which does not consider the cross-
covariances




4 FUSION FILTERING USING COVARIANCE INTERSECTION ALGORITHM

4.1 Covariance Intersection Algorithm

To achieve low-complexity fusion, the covariance inter-
sion (CI) algorithm was proposed for fusion estimation. It
takes the same form as the weighted sum, i.e.,

From (13), only the local error-covariances
are used to calculate the matrix weights
\( W_i^{(c)} \), \( i=1,...,N \) for fusion. Therefore, as will be shown, the CI algorithm has low estimation accuracy, but low computational complexity.

### 4.2 Efficiency of Covariance Intersection Algorithm

Since the CI algorithm is a robust fusion algorithm and provides a bound on the estimation accuracy, sometimes the accuracy of fusion estimation can be low. However, basically, the CI algorithm is low computational complexity because cross-covariances between two local estimates are ignored.

The following example makes it possible to compare the specific accuracies and effectiveness between FF and the CI algorithm.

**Example: Fusion Estimation in a Steady-state Regime**

Let us consider a scalar system with two sensors. Using (7) the system is given by

\[ x(t) = x(t) + w(t), \quad w(t) \sim (0, q), \quad t \geq 0, \]

where \( x(t) \), \( w(t) \) are scalar white Gaussian sequences, \( q \) is the error covariance of the sensor errors with variances \( q_i, i=1,2 \).

Then, local error-covariances \( P_i^{(l)} \), \( x_i^{(l)} \) and the cross-covariance \( P_{ix}^{(l)} \) in steady-state regime are obtained [12, 13]:

\[ P_i^{(s)} = P_i^{(l)} - P_{ix}^{(l)}, \quad P_{ix}^{(s)} = \frac{2P_i^{(l)}q}{(q+2r_i)(q+2r_2)}, \quad i=1,2. \]

Using (15), we can obtain specific corresponding steady-state values for fusion weights \( C_i^{(s)} \), \( W_i^{(s)} \), \( i=1,2 \):

\[ C_i^{(s)} = \frac{P_{ix}^{(l)} - P_{ix}^{(l)}}{P_i^{(l)} + P_{ix}^{(l)} - 2P_{ix}^{(l)}}, \quad C_i^{(s)} = \frac{P_i^{(l)} - P_{ix}^{(l)}}{P_i^{(l)} + P_{ix}^{(l)} - 2P_{ix}^{(l)}}, \]

\[ W_i^{(s)} = \frac{P_{ix}^{(l)}}{P_i^{(l)} + P_{ix}^{(l)}}, \quad W_i^{(s)} = \frac{P_{ix}^{(l)}}{P_i^{(l)} + P_{ix}^{(l)}}. \]

Let us assume that \( q=1 \), \( r_1=5 \), \( r_2=2 \), and use (6). Then we can take specific fusion errors:

\[ P_i^{(f)} = 0.3896, \quad P_i^{(c)} = 0.3925. \]

In (17), we see that the difference between the fusion error-covariances of FF and CI algorithm is about 0.7%. This means, the accuracy of the CI algorithm is high for this example. Furthermore, as shown in (15) and (16), the calculation of \( P_i^{(l)} \) is not necessary for the weights \( W_i^{(c)} \), \( i=1,2 \), and thus the CI algorithm can have low computational complexity. Therefore, the CI algorithm is considered more effective for implementation.

### 5 NUMERICAL EXAMPLE

In this section we compare the performance of two fusion filters based on FF and CI algorithm through a numerical example.

Let us consider the longitudinal dynamics of an aircraft. This can be approximately represented by the harmonic oscillator system:

\[ \ddot{x}_i = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -2\xi\omega_i \end{bmatrix} x_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w_i, \]

where \( x_i = \begin{bmatrix} 0 \\ \dot{x}_i \end{bmatrix} \) and \( \dot{x}_i \) is pitch angle. \( \omega_i \) and \( \xi \) are natural oscillating frequency and damping ratio, respectively. The process noise \( w_i \) represents wind gusts which influence pitch rate.

In addition, three identical sensors (one of them is a main sensor and the others are reserved) measuring at discrete times \( t_k \) with different measurement errors are given:

\[ y_i^{(l)} = \begin{bmatrix} 1 & 0 \end{bmatrix} x_i + v_i^{(l)}, \quad i=1,...,N = 3, \]

where \( v_i^{(l)}, i=1,2,3 \) are scalar white Gaussian sequences with variances \( r_i^{(l)}, i=1,2,3 \), i.e., \( r_1^{(l)} = 1, \quad r_2^{(l)} = 2, \quad r_3^{(l)} = 3 \).

Let us assume that \( w_i \) is a zero-mean continuous white Gaussian process (system noise) with intensity \( q = 2 \). In addition, we utilize the forth-order Runge-Kutta method to solve differential equations (10) and (12) between measurements with an integration unit time increment \( \Delta t = 0.01 \).

Figure 1 illustrates the mean square errors (MSE) of the fusion estimate \( \hat{\theta}_{k} \) based on FF and CI, i.e.,

\[ P_i^{(f)} = E(\hat{\theta}_k - \hat{\theta}_k^{(f)})^2, \quad P_i^{(c)} = E(\hat{\theta}_k - \hat{\theta}_k^{(c)})^2. \]

Moreover Table 1 shows implemented CPU times of two discussed algorithms using a computer with the following specifications: Intel® Pentium® 4, CPU 3 GHz, 1GB RAM.
As shown in Figure 1, we observe that MSEs corresponding to the CI algorithm and FF are very close, and this point was already verified from the theoretical example in Subsection 4.2. Thus, we can say that the CI algorithm is accurate for FF. Furthermore, the CI algorithm is almost 64 times faster than FF in performing the fusion filtering as shown in Table 1. Therefore, we see that the CI algorithm is a useful algorithm for real-time implementation, especially when applications are modeled by continuous-discrete time systems.

6 CONCLUSION

We presented distributed fusion filtering algorithm for continuous-time dynamic model. This algorithm is based on the optimal fusion formula (FF) and suboptimal CI algorithm. Specific performances of FFS and CI were verified using a theoretical example. Accuracies of the proposed distribution fusion filtering using the FF and CI are very closely. However, the CI algorithm is more effective than the FF with respect to computational complexity, especially on discrete measurements for continuous-time signals. Numerical example demonstrated the effectiveness of the CI algorithm.

Generally when the FF is applied, it takes time progressively longer to fuse the local estimates as the number of sensors increases. In contrast to the use of FF, the weights of the CI only depend on local error covariances, and thus the CI algorithm has lower computational complexity, and can be more easily implemented on discrete measurements in real-time applications.

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