Dependence of soft pion jet properties in the space of relative four-dimensional velocities on initial energy

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In the paper experimental results obtained by studying of collective and fractal properties of soft pion jets in the space of relative four-dimensional velocities in intermediate energy domain $2 \rightarrow 20$ GeV are presented. Fractional values of cluster dimension are indicated on manifestation of fractal-like properties by pion jets. The changes of the mean square of the distance between secondary particles and jet axis, the mean kinetic energy of particles in jets and the cluster dimension with increasing of collision energy agree with the hypothesis of manifestation of quark degrees of freedom in processes of pion jet production at intermediate energies. For the first time the quantitative estimations are obtained for the low boundary energy at which quark degrees of freedom start to display itself in production of soft pion jets experimentally. The value of estimation for this parameter derived with taking into account of all used collective parameters is $(2.8 \pm 0.6)$ GeV.

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I. INTRODUCTION

At the present time, the decision of the problem of confinement and study of transition from meson-baryon degrees of freedom to quark-gluon ones is one of the most important (and, at the same time, most difficult) task of world research programm in the field of strong interactions. The nature of confinement of color degrees of freedom (quark and gluons) and, correspondingly, the possible phase transitions in strongly interacting matter is not clear in full so far. Observation of hadron jets at high energies is one of the most important and evident experimental manifestation of quark-gluon degrees of freedom. At present, an open question is where is the low boundary on energy starting with which the color degrees of freedom should be taken into account for description of processes of multiparticle production. Obviously, the jet structure of events displays itself more clear at high initial energies $(\sqrt{s})$ than at intermediate ones. But in despite of this feature it seems that the application of collective characteristics of multiparticle final state can be useful in collision energy domain $\sqrt{s} = 2 \rightarrow 20$ GeV both for study of transition from the predominance of meson-baryon degrees of freedom to quark-gluon ones and for qualitative estimation of low boundary (for initial energy) experimental manifestation of quark degrees of freedom in soft jet production. In various fields of physics the onset of manifestation of new degrees of freedom and transition processes is accompanied by the presence of self-affine and fractal properties in collective effects. Therefore, the study of collective and geometric (fractal-like) properties, in particular, of soft pion jets at intermediate energies can give the new important information about hadronization mechanisms, non-perturbative physics and transition to manifestation of quark degrees of freedom in collective phenomena.

The paper is organized as follows. In Sec. 2, definitions of collective variables are described. The Sec. 3 devotes discussion of energy dependence of the properties of soft pion jets in the space of relative four-dimensional velocities, estimations of low boundary energy for quark degrees of freedom experimental manifestation in pion jet production based on empirical approximations of experimental data. Some final remarks and conclusions are presented in Sec. 4.

II. METHOD AND VARIABLES

Traditional collective characteristics used for study of event shape \cite{1} are not relativistically invariant. This introduces some additional kinematic uncertainties, for example, in choice of center-of-mass system for reactions with atomic nuclei. A relativistically invariant method was proposed in \cite{2} for investigation of collective effects in multiparticle production processes of the following type: $b + t \rightarrow 1 + 2 + \ldots$, where $t/b$ – target/beam particle. This method based on the using of new variables allows to obtain relativistically invariant description of excited strongly interacting matter in wide range of initial energies and for different reaction types. Numerous results of study of jet production in soft hadron collisions shown that there are two jets emitted in opposite directions in forward and

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backward hemispheres in these processes (see, for example, [3]). Special features of relativistically invariant method in this case were considered in detail elsewhere [4, 3]. It should be emphasized the method under study is most rigorously justified and applied successfully in the case of two-jet event shape which corresponds to the geometry of final state in the domain of intermediate energies. In the case, the separation of secondary particles on fragmentation regions can be made with the aid of the relativistically invariant variables $X_{b}^{i}$ and $X_{t}^{i}$, which are defined as following [1, 2]:

$$X_{b}^{i} = \frac{m_{i}(U_{i}U_{b})}{m_{a}(U_{b}U_{a})}; \quad \alpha, \beta = t, b; \quad \alpha \neq \beta. \quad \text{Here } m_{i} \text{ is the mass of } i^{\text{th}} \text{ secondary particle, } m_{b} = \text{mass of target / beam particle, } P_{i}, \quad U_{i} = P_{i}/m_{i}; \quad i = b, t, 1, 2, ... - \text{ four-momenta and four-velocities. The variables } X_{t/b}^{i} \text{ characterize the fraction of four-momentum of initial particles } (P_{i}, \quad i = t / b) \text{ carried away by secondary hadrons in the target / beam fragmentation region respectively. Fragmentation regions in the plane } (X_{b}^{i}, X_{t}^{i}) \text{ are defined by the following sets of conditions [3, 2]:}
$$

$$\text{target fragmentation } \begin{cases} X_{t}^{i} \geq \tilde{X}, \\
X_{b}^{i} \leq \tilde{X} \end{cases} \quad \text{beam fragmentation } \begin{cases} X_{t}^{i} \leq \tilde{X}, \\
X_{b}^{i} \geq \tilde{X} \end{cases}$$

Here $\tilde{X} = 0.1 - 0.2$ is some boundary value which is determined empirically. The basic quantities which the probability distributions (cross sections) depend upon are non-dimension positive relativistic invariant quantities $b_{ik} = -(U_{i} - U_{k})^{2}$, where $i, k = b, t, 1, 2, ...$ [2]. As seen the observables $b_{ik}$ mean the squares of relative distances in the four-velocities space. The comparison of this method for distinguishing of some particle groups in the space of four-dimensional velocities with other present non-invariant (traditional) methods allows to name these separate groups as jets [5]. The jet axis in the space under study for certain fragmentation region, $V$, is defined by unit vector of jet four-velocity: $V = U_{J}/|U_{J}|$, where $U_{J} = \sum_{k=1}^{N} U_{k}$, $N$ is the number of particles in considered fragmentation region which are satisfied all cuts and involved in the analysis. One of the most important observables of this method is the square of the distance of the $k^{\text{th}}$ particle from the jet axis [2]:

$$b_{k} = -(V - U_{k})^{2}, \quad k = t, b, 1, 2, ... \quad (1)$$

The study of properties of soft pion jets is fulfilled with the aim of invariant function $F(b_{k})$ introduced for nucleon clusters in [8, 9]. For pion jets this function is re-defined as following [3]:

$$F(b_{k}) = (\lambda/N) dN/db_{k}, \quad \lambda = 4/|m_{\pi}^{2}b_{k}\sqrt{1 + 4/b_{k}}|.$$ 

The investigations of $F(b_{k})$ both for nucleon clusters [6, 3] and for pion jets [3, 10] demonstrated that the $F(b_{k})$ can be approximated by exponent function $F^{s}(b_{k}) = \sum_{i=1}^{m} p_{b}^{s} \exp(-b_{k}/(b_{k}^{s})$ quite reasonably, where $\alpha = t/b, m = 1$ or $m = 2$ depending on certain fragmentation region, reaction type and $\sqrt{s}$. The mean kinetic energy of particles in the jet in its rest frame (it is referred to as "temperature"), $\langle T_{k} \rangle$, is an important characteristic of particles inside jet which is determined on the basis of above approximation of invariant $F(b_{k})$ distributions as following

$$\langle T_{k} \rangle^{2} = m_{\pi}^{2} \langle b_{k} \rangle_{i}^{2} / 2. \quad (2)$$

In [11] it was proposed to study geometric properties of pion jets in the space of relative four-velocities with the aid of the cluster dimension $D$, which is determined by the relation between the number of particles in the jet, $n_{J}(b_{k}) = \int_{0}^{b_{k}} db'_{k} dN/db'_{k}$, and its radius in the space under discussion. Taking into account that [11] is the square of distance in the space of relative four-velocities this relation is $n_{J}(b_{k}) = (ab_{k})^{D/2}$ at $n_{J} \rightarrow \infty$, which is in reasonable agreement with corresponding experimental dependencies [11, 12]. Thus the cluster dimension of pion jet in the space of relative four-velocities can be derived as following

$$D = 2 \ln[n_{J}(b_{k})]/ \ln(ab_{k}). \quad (3)$$

One need to note that the parameter [3] is an integer and coincides with the topology dimension ($D_{T}$) of space under consideration if particle distribution is homogeneous in this space. But $D$ can be non-integer and $D \neq D_{T}$ for particle distribution with highly irregular and complex structure. Thus non-integer value of cluster dimension can be considered as characteristic signature of manifestation of fractal-like properties [13]. For most complex distributions the multi-fractal structure can be appeared and cluster dimension be a function of jet radius for such case: $D = D(b_{k})$. Thus the [3] is the qualitative parameter reflected the features of particle distribution in phase space.

It should be emphasized that in simplest case of lowest order of renormalization group equation (RGE) the following relation can be derived $\alpha S \propto 1/\ln b_{k}$, where $i = t / b$, $\alpha S$ is the renormalized strong coupling constant [3, 2, 14]. It was shown [3, 2, 14] that the $b_{ik}$ values can be used to classify relativistic nuclear interactions: the domain $b_{ik} \sim 10^{-2}$ corresponds to the interaction of nuclei as weakly bound systems of nucleons (domain of classic nuclear physics), the region $b_{ik} > 1$ corresponds to the interaction of hadrons as weakly bound quark systems (domain of QCD). Within the approach under study the range of values $10^{-2} \lesssim b_{ik} \sim 1$ corresponds to the transition from the domain of
dominance of meson-baryon degrees of freedom to the region where the internal structure of colliding particles and, as consequence, quark-gluon degrees of freedom become essential in the processes of secondary particle jet production. It should be emphasized that the estimations for boundaries of various dynamic domains in the space of relative four-velocities indicated above seem valid for $b_k$, i.e. when the jet axis is corresponds to the “reference” $i^{th}$ particle. But the possible exact relation between $b_k$ and $\alpha_S$ is required the additional rigorous substantiation and careful derivation. Here one can note only that jets consist of particles with close masses (pions) in the present study and there is the relation $\langle b_k \rangle = 2(\langle M_J \rangle/(n_3)m_h - 1)$ between the mean square of jet size in the space of relative four-velocities and mean effective mass $\langle M_J \rangle$ of jets of secondary hadrons with identical masses $m_h$ [12, 10]. Here $\langle n_3 \rangle$ is the mean multiplicity of particles inside jet, the averaging is taken over particles in event and over events in sample in the l.h.s.; over event ensemble in the r.h.s. In collider experiments at high energies the jet transverse momentum ($p_T^j$) [17] or average transverse momentum of the two jets leading in $p_T^j$ ($p_T^{j_1, j_2}$) [18] are considered as scale of $Q$ in the RGE. Then one can derive the relation

$$\alpha_S = (b_0t)^{-1}, \quad t \equiv 2\ln \left( \frac{\langle b_k \rangle}{2} + 1 \right) \frac{\langle n_3 \rangle m_h}{\Lambda},$$

if $\langle M_J \rangle$ is chosen as $Q$ for some reaction at fixed $\sqrt{s}$, where $b_0 = (33 - 2n_f)/(12\pi)$ is the one-loop $\beta$-function coefficient, $n_f$ is number of quark flavors considered as light, $\Lambda$ is the QCD parameter. That is why the distributions on $b_k$ are studied usually. Thus the approach under discussion allows to distinguish, at least, at qualitative level different dynamic mechanisms of soft jet production in range of intermediate $\sqrt{s}$. The separation of various dynamics of jet production is a difficult task especially in non-perturbative region of $\sqrt{s}$. The study of traditional collective observables in non-perturbative initial energy domain allows to conclude for event shape mostly [1]. On the other hand investigation of jet properties with the aid of relativistically invariant variables like (1) as well as observables related with $b_k$ and defined above allows to make additional suggestions concerning the dynamic features of mechanism of these jet productions. This is additional improvement of relativistically invariant method which seems (very) important at intermediate $\sqrt{s}$ namely. Therefore approaches for analysis of jet production based on traditional collective characteristics and on relativistically invariant parameters [11, 9] are supplemented each other at intermediate $\sqrt{s}$.

Because of [11] and [13] are dimensionless parameters it seems the use of corresponding non-dimension mean temperature $\langle T_k \rangle \equiv \langle T_k \rangle/\sqrt{s_0}$ is more convenient without loss of generality, where $s_0 = 1$ GeV$^2$. Thus in the present study the set of dimensionless collective observables $G \equiv \{g_i\}_{i=1}^3 = \{\langle b_k \rangle, \langle T_k \rangle, D\}$ characterized the dynamics of creation of final state and its geometry in the space of four-velocities is considered.

### III. RESULTS: ENERGY DEPENDENCE FOR JET CHARACTERISTICS

Taking into account close values of $\langle T_k \rangle$ obtained for secondary $\pi^-$ and $\pi^\pm$ mesons in [3], as well as similar behavior of distributions on $b_k$ observed for $\pi^\pm$ in $\bar{\nu}N$ interactions and for $\pi^-$ in the reactions with hadronic beams at close initial energies $\sqrt{s}$, in present study the values of $\langle T_k \rangle$ and $D$ obtained for $\pi^\pm$ in $\bar{\nu}N$ interactions are considered as estimations of these parameters for $\pi^-$ mesons in the corresponding reactions with $\bar{\nu}$ beam. This suggestion allows to extend the experimental data base for $\langle T_k \rangle$ and $D$ in comparison with [12]. One need to make the two important comments. First of all, the estimations of $\langle T_k \rangle$ and $D$ for $\bar{\nu}N$ interactions agree reasonably both with results of another experiments at close energies and with general trends (see Figs. [11, 9] and detail discussion below). This is additional evidence of applicability of the approach used for estimations of all parameters from $G$ in the case of $\bar{\nu}N$. Second, cluster dimensions for $\bar{\nu}N$ as well as for large number of reactions considered previously in [12] have fractional values for any fragmentation regions. This observation confirms the conclusion from [12] and gives additional grounds to suggest that soft pion jets demonstrate fractal-like properties in wide class of interactions at intermediate energies $\sqrt{s} < 20$ GeV.

In the present study the dependence of all parameters from $G$ on initial energy has is investigated in detail based on the available experimental data. In [3] the visible changing of behavior of dependence $\langle b_k \rangle$ on $\sqrt{s}$ was observed. Taking into account this feature and physical meaning of the parameter [11] the hypothesis about changing of dynamic regimes in processes of soft hadronic jet production at $\sqrt{s} < 3 - 4$ GeV was suggested in [3] and confirmed some later in [12]. In Fig. [1] and Fig. [2] the dependencies of parameters from $G$ on $\sqrt{s}$ are shown for various interactions at boundary values $X = 0.1$ and 0.2 respectively. The data samples from [12] and estimations for parameters $\langle T_k \rangle, D$ in the case of $\bar{\nu}N$ derived on basis of differential distributions $dN/db_k$ from [9] are used. In the last case the mean

\begin{footnote}
One need to note that the boundary values for $b_k$ indicated above are qualitative phenomenological estimations.
\end{footnote}
energy of hadronic final state, $W$, is considered as a initial energy of reaction. Based on the shape of dependencies under discussion and preceding studies [4, 5, 12] various samples of experimental data are fitted by the power function

$$\forall i : \mathcal{G}_i = a_1(\sqrt{s/s_0} - a_2)^{a_3}, \quad \sqrt{s/s_0} \geq a_2$$

(5)

in energy domain $\sqrt{s} < 4$ GeV for hadron-hadron and nucleus-nucleus interactions and by the logarithmic function

$$\forall i : \mathcal{G}_i = a_1 + a_2 \ln(s/s_0).$$

(6)

at higher energies for hadronic and $\bar{\nu}$ beams as well as for hadron-nucleus collisions at any initial energies. One can note that the shapes of energy dependencies for all collective characteristics from $\mathcal{G}$ are similar at qualitative level for fixed value of $\bar{X}$.

For present study the domain $\sqrt{s} < 4$ GeV is important especially. Therefore results of experiments with different beam types with exception of the hadron-nucleus collisions and corresponding approximations are shown for range $\sqrt{s} < 4$ GeV separately in Figs. 1k, c, e for boundary value $\bar{X} = 0.1$ and in Figs. 2a, c, e at $\bar{X} = 0.2$. The uncertainties of initial momenta are taking into account in the pictures indicated above. At soft cut $\bar{X} = 0.1$ values of $\langle b_k \rangle$ parameter for symmetric nucleus-nucleus collisions are in a good agreement with the general trend (Figs. 1k, b). At initial energies $\sqrt{s} \approx 3$ GeV and boundary cut $\bar{X} = 0.1$ values of $\langle b_k \rangle$ (Figs. 1k, b) and $\langle \bar{T}_k \rangle$ (Figs. 1i, d) are smaller significantly than that in another interactions at slightly larger energies. The behavior of $D(\sqrt{s})$ at these energies does not contradict with that conclusion. At harder cut value of $\bar{X}$ for distinguishing of fragmentation regions all parameters from $\mathcal{G}$ show similar behavior at $\sqrt{s} \approx 3$ GeV. Thus $\forall i = 1 \sim 3$ dependencies $\mathcal{G}_i(\sqrt{s})$ demonstrate the visible changing of its shapes in narrow range $\sqrt{s} \approx 3$ GeV both at $\bar{X} = 0.1$ (Fig. 1) and at $\bar{X} = 0.2$ (Fig. 2). As indicated above the experimental dependencies $\mathcal{G}_i(\sqrt{s})$, $i = 1 \sim 3$ are fitted by power function [5] at initial energies $\sqrt{s} < 4$ GeV. Fit results for samples jointed for different fragmentation regions are shown in Fig. 1 and Fig. 2 by dotted lines, the numerical values of fit parameters are presented in Table I. Results for $\langle b_k \rangle$ are from [5, 12].

In [5, 12, 16] was shown that properties of soft pion jets are depended on fragmentation region in domain $\sqrt{s}$ under consideration. Such dependence results in significant difference of $\langle b_k \rangle$ values for various fragmentation regions in certain reaction. The analogous situation is observed for other parameters from $\mathcal{G}$ here. Therefore the sharp behavior of energy dependencies and spread of experimental points give no way to attain statistically acceptable quality of fits (as usual, $\chi^2$/NDF $\sim 10$). However, as seen from Fig. 1 and Fig. 2 the function (5) is in rather good agreement, at least, at qualitative level with experimental data at all $\bar{X}$ values considered above. It is important to note that, in general case, a power-law behavior is peculiar to the transition region, where new degrees of freedom of the system under study come into relevant and play a role amplified successively [12]. The parameter $a_2$ in (5) can be put in

### Table I: Fit results for $\mathcal{G}_i$ parameters for jointed experimental data samples

| Fit                | $\bar{X} = 0.1$ | $\bar{X} = 0.2$ |
|--------------------|-----------------|-----------------|
|                    | $\langle b_k \rangle$ | $\langle \bar{T}_k \rangle$ | $D$ | $\langle b_k \rangle$ | $\langle \bar{T}_k \rangle$ | $D$ |
| $hh, hA$ interactions ($\sqrt{s} > 8$ GeV) | | | | | | |
| $a_1$              | 3.7 ± 0.2       | 0.108 ± 0.008   | 1.73 ± 0.02 | 4.88 ± 0.05 | - | - |
| $a_2$              | 0.12 ± 0.05     | 0.009 ± 0.002   | 0.0 (fixed) | 0.0 (fixed) | - | - |
| $\chi^2$/NDF       | 7.20            | 1.14            | 0.51        | 1.71        | - | - |
| $hh, \bar{\nu}N$ reactions ($4 \leq \sqrt{s} < 9$ GeV) | | | | | | |
| $a_1$              | 1.85 ± 0.04     | 0.070 ± 0.016   | 0.8 ± 0.2   | 1.94 ± 0.08 | 0.139 ± 0.012 | 1.68 ± 0.13 |
| $a_2$              | 0.53 ± 0.01     | 0.018 ± 0.004   | 0.23 ± 0.06 | 0.68 ± 0.03 | 0.019 ± 0.003 | 0.06 ± 0.04 |
| $\chi^2$/NDF       | 3.00            | 1.39            | 1.65        | 2.18        | 2.52        | 0.13 |
| $hA$ collisions ($\sqrt{s} < 9$ GeV) | | | | | | |
| $a_1$              | -2.56 ± 0.08    | -0.021 ± 0.007  | -0.17 ± 0.11 | -1.17 ± 0.17 | 0.04 ± 0.03 | -0.56 ± 0.14 |
| $a_2$              | 1.58 ± 0.02     | 0.039 ± 0.002   | 0.44 ± 0.03 | 1.39 ± 0.05 | 0.043 ± 0.006 | 0.57 ± 0.04 |
| $\chi^2$/NDF       | 4.30            | 3.17            | 0.33        | 2.68        | 1.76        | 2.33 |
| $hh, AA$ interactions ($\sqrt{s} < 4$ GeV) | | | | | | |
| $a_1$              | 3.69 ± 0.02     | 0.236 ± 0.015   | 1.68 ± 0.05 | 3.63 ± 0.05 | 0.210 ± 0.006 | 1.93 ± 0.05 |
| $a_2$              | 2.76 ± 0.01     | 2.877 ± 0.001   | 2.875 ± 0.001 | 2.51 ± 0.03 | 2.865 ± 0.007 | 2.875 ± 0.001 |
| $a_3$              | 0.49 ± 0.01     | 0.34 (fixed)    | 0.04 ± 0.01 | 0.40 ± 0.01 | 0.080 ± 0.004 | 0.031 ± 0.007 |
| $\chi^2$/NDF       | 133             | 8.42            | 8.21        | 9.25        | 7.17        | 2.84 |
TABLE II: Fit results for $\mathcal{G}_i$ in $\sqrt{s}<4$ GeV energy domain for separated experimental data samples

| Fit parameter | $\bar{X} = 0.1$ | $\bar{X} = 0.2$ | $\bar{X} = 0.2$ |
|---------------|---------------|---------------|---------------|
| $b_k$         | $\langle T_k \rangle$ | $\chi^2$/NDF | $\langle T_k \rangle$ | $\chi^2$/NDF |
| target fragmentation region |
| $a_1$         | $3.79 \pm 0.03$ | $0.132 \pm 0.019$ | $1.53 \pm 0.05$ | $3.52 \pm 0.07$ | $0.204 \pm 0.006$ | $1.87 \pm 0.04$ |
| $a_2$         | $2.82 \pm 0.02$ | $2.75 \pm 0.04$ | $2.875 \pm 0.004$ | $2.46 \pm 0.04$ | $2.865 \pm 0.007$ | $2.877 \pm 0.001$ |
| $a_3$         | $0.52 \pm 0.05$ | (fixed) | $0.03$ (fixed) | $0.03$ (fixed) | $0.03$ (fixed) | $0.03$ (fixed) |
| $\chi^2$/NDF | $5.70$ | $0.03$ | $3.90$ | $0.00$ | $0.06$ | $0.80$ |

| $\bar{X} = 0.2$ | $\bar{X} = 0.2$ |
|---------------|---------------|
| $\langle T_k \rangle$ | $\chi^2$/NDF |
| beam fragmentation region |
| $a_1$         | $2.65 \pm 0.13$ | $0.142 \pm 0.003$ | $1.79 \pm 0.05$ | $5 \pm 2$ | $-$ | $-$ |
| $a_2$         | $2.43 \pm 0.04$ | $2.877 \pm 0.001$ | $2.78 \pm 0.04$ | $2.5 \pm 0.3$ | $-$ | $-$ |
| $a_3$         | $0.5 \pm 0.4$ | $0.03$ (fixed) | $0.08$ (fixed) | $0.5$ (fixed) | $-$ | $-$ |
| $\chi^2$/NDF | $5.19$ | $0.06$ | $6.43$ | $2.29$ | $-$ | $-$ |

The correspondence to the energy (in GeV), $\sqrt{s}$, at which the internal structure of interacting hadrons, i.e., quark-gluon degrees of freedom, begins to manifest itself experimentally. The values of dimensionless parameter $a_2$, i.e., values of low boundary energy $\sqrt{s}$ in GeV for color degrees of freedom experimental appearance in jet production, are shown in Table II for all members of set $\mathcal{G}$ at various $\bar{X}$. One can see that the values of $a_2$ derived for $\langle T_k \rangle$ and $D$ agree with each other and are some larger than values of the fit parameter obtained for energy dependence of $\langle b_k \rangle$ at fixed $\bar{X}$. The such disagreement can be caused by some physical reasons which can result in more sharp increasing of $\langle T_k \rangle$ and $D$ in comparison with $\langle b_k \rangle$ as well as some methodological features, for example, smaller volumes of experimental samples in the case of former two parameters. Additional new experimental data in initial energy domain $\sqrt{s} \leq 4$ seems important for detail investigation of behavior of dependencies $\mathcal{G}_i(\sqrt{s})$, $i = 1 - 3$ and for decreasing of uncertainties in physical conclusions. In the energy range under consideration available data samples were fitted by function (4) for the region of target fragmentation (curves 1 in Figs. 1h, c, e and in Figs. 2h, c, e) and beam fragmentation (curves 2 in Figs. 1h, c, e and in Fig. 2h) individually for various values of cut $\bar{X}$. The numerical values of fit parameters are shown in Table III. Approximations for parameters $\langle T \rangle$ and $D$ in the case of beam fragmentation at $\bar{X} = 0.2$ are impossible because of absent of necessary amount of data points (Figs. 2, e). As seen there is significant improvement of fit qualities for any fragmentation region and value of $\bar{X}$ ($\chi^2$/NDF $\sim 3 - 5$), moreover statistically acceptable values of $\chi^2$/NDF are succeeded for 50% of samples under study. Taking into account relatively small volumes of available experimental data samples for $\langle T_k \rangle$ and $D$, especially at $\bar{X} = 0.2$, and shapes of $\langle T_k \rangle/\sqrt{s}$ and $D/\sqrt{s}$ the energy dependencies for these collective parameters were fitted by function (5) in the full energy domain $\sqrt{s} > 2$ GeV. Obviously the sample volumes increase significantly for such energy domain with respect to the samples fitted for separate energy ranges with results shown in Tables II, III. The approximations for $\sqrt{s} > 2$ GeV were made for samples jointed by fragmentation regions as well as for target and beam fragmentation regions separately. The fits of $\langle T_k \rangle/\sqrt{s}$ in the first case are shown by dashed lines in Fig. 1H at $\bar{X} = 0.1$ and for harder kinematic cut – in Fig. 2H; numerical values of fit parameters at $\bar{X} = 0.1(0.2)$ are following: $a_1 = 0.1168 \pm 0.0005(0.1946 \pm 0.0015)$, $a_2 = 2.854 \pm 0.001(2.854 \pm 0.001)$, $a_3 = 0.131 \pm 0.002(0.071 \pm 0.004)$, $\chi^2$/NDF = 266(3.50). As seen the only qualitative agreement is observed between fit function (5) and experimental data at $\bar{X} = 0.1$ (Fig. 1H) so long as fit quality is significantly poorer than that at $\bar{X} = 0.2$ (Fig 2H). The fits of $D/\sqrt{s}$ for the jointed samples are shown by dashed lines in Figs. 1H, 2H at $\bar{X} = 0.1$ and 0.2 respectively; numerical values of fit parameters at $\bar{X} = 0.1(0.2)$ are following: $a_1 = 1.539 \pm 0.007(1.71 \pm 0.02)$, $a_2 = 2.60 \pm 0.03(2.68 \pm 0.03)$, $a_3 = 0.059 \pm 0.003(0.041 \pm 0.005)$, $\chi^2$/NDF = 28(32). For this collective parameter the fit qualities allow to consider qualitative agreement only between function (5) and experimental data (Figs. 1H, 2H). Separate approximations by (5) of $\langle T_k \rangle/\sqrt{s}$ result in the following results at $\bar{X} = 0.1(0.2)$ for target fragmentation $a_1 = 0.1014 \pm 0.0007(0.189 \pm 0.002)$, $a_2 = 2.853 \pm 0.001(2.854 \pm 0.001)$, $a_3 = 0.200 \pm 0.003(0.070 \pm 0.005)$, $\chi^2$/NDF = 10(2.04) and for beam fragmentation $a_1 = 0.1403 \pm 0.0012(0.204 \pm 0.003)$, $a_2 = 2.877 \pm 0.001(2.854 \pm 0.001)$, $a_3 = 0.033 \pm 0.005(0.062 \pm 0.006)$, $\chi^2$/NDF = 8.36(2.71). As expected the fit qualities improve significantly with respect to the fits of jointed samples, the values of $\sqrt{s}$ are equal within errors for various approximation approaches with exception of beam fragmentation at $\bar{X} = 0.1$. The shape of $D/\sqrt{s}$ (Figs. 1H, 2H) and detail study show that the presence of experimental data for $\pi^+\text{Ne}$ interactions at initial momentum $p_0 = 6.2$ GeV/c in sample under fit leads to significant poor fit quality and to weak changing of $\sqrt{s}$ value at the same time. Therefore the separate fits of $D/\sqrt{s}$ in full energy domain were made for samples with excepted of experimental
points for $\pi^-\text{Ne}$ reaction. The following results are obtained at $\tilde{X} = 0.1(0.2)$ for target fragmentation $a_1 = 1.559 \pm 0.014(1.78 \pm 0.02)$, $a_2 = 2.854 \pm 0.004(2.858 \pm 0.001)$, $a_3 = 0.051 \pm 0.003(0.043 \pm 0.004)$, $\chi^2/\text{NDF} = 3.98(1.44)$ and for beam fragmentation $a_1 = 1.658 \pm 0.001(1.89 \pm 0.04)$, $a_2 = 2.9 \pm 0.4(2.874 \pm 0.001)$, $a_3 = 0.029 \pm 0.002(0.013 \pm 0.016)$, $\chi^2/\text{NDF} = 3.08(1.78)$. As seen the fit qualities are improved significantly, moreover statistically acceptable values of $\chi^2/\text{NDF}$ are succeeded for samples under study at $\tilde{X} = 0.2$. Therefore the approximations of experimental samples for $\langle T_k \rangle$ and $D$ in full energy domain allows to get, in particular, the estimations for $a_2$ for any fragmentation regions and values of $\tilde{X}$ considered here.

The values of $\sqrt{s_{cc}}$ corresponded to the approximations of energy dependence of parameters from $G$ with accounts of optimal combinations of fit qualities and volumes of available experimental data samples are summarized in the Table III.

| Parameter from $G$ | target fragmentation, $\tilde{X}$ | beam fragmentation, $\tilde{X}$ |
|-------------------|-----------------------------|-----------------------------|
| $\langle b_k \rangle$ | $2.82 \pm 0.02$ | $2.46 \pm 0.04$ |
| $\langle T_k \rangle$ | $2.853 \pm 0.001$ | $2.854 \pm 0.001$ |
| $D$ | $2.854 \pm 0.002$ | $2.858 \pm 0.001$ |

As seen from Table III the values of $\sqrt{s_{cc}}$ agree reasonably for different kinematic cuts ($\tilde{X}$) and fragmentation regions. It should be emphasized that the results obtained by study of fractal properties of pion jets are in a good agreement with results of physical analysis of energy dependencies of parameters $\langle b_k \rangle$ and $\langle T_k \rangle$. Thus this study extended on the full parameter set $G$ increases the degree of confidence of results obtained earlier \cite{5,12} and gives the additional evidences in favor of the suggested hypothesis concerned the changing of dynamic regimes in multiparticle production processes caused by the onset of experimental manifestation of quark degrees of freedom in soft jet production at $\sqrt{s} \sim 3$ GeV and the corresponding transition from description of strong interaction with the aid of meson-baryon degrees of freedom to the using of quark-gluon ones. For the first time the quantitative estimation is obtained for the low boundary on energy at which the quark degrees of freedom start to manifest itself experimentally in production of soft pion jets. The value of estimation of parameter under discussion matched with taking into account of results for all collective characteristics from $G$ is $\sqrt{s_{cc}} = (2.8 \pm 0.6)$ GeV. In \cite{19} the following qualitative estimation was obtained for universal low boundary of experimental manifestation of jet shape of final state in multiparticle production processes: $\sqrt{s_{\text{min}}} \sim 3$ GeV. This qualitative estimation is equal to $\sqrt{s_{cc}}$ within errors. It should be noted that in despite of possibility for production of localized and separated groups of secondary particles in the framework of models corresponded to meson-baryon level of matter the conception of "hadron jet" itself is deeply relates with QCD because the jet is usually defined as group of particles produced due to hadronization of common color charge. Therefore it can not be excluded the possibility that the parameters $\sqrt{s_{\text{min}}}$ and $\sqrt{s_{cc}}$ characterize the united physical effect – the onset of experimental manifestation of quark degrees of freedom in soft processes of multiparticle production and, as consequence, jet event shape. Therefore the results obtained by using of traditional and relativistically invariant collective observables agree and add each other.

Earlier in \cite{6}, a statistically reasonable description by function $\xi_{b_k}(\sqrt{s})$ was obtained for experimental dependencies $\langle b_k \rangle(\sqrt{s})$ in the case of joined samples for hadron-hadron and $\pi N$ reactions (solid lines in Figs. 1b, 2b), as well as for hadron-nucleus interactions (solid thick lines in Figs. 1b, 2b) in the range $3.5 < \sqrt{s} < 9$ GeV; for hadron-hadron and hadron-nucleus collisions at higher energies (solid thin lines in Figs. 1b, 2b). Additionally in the present study it is obtained that quality of approximation of dependence $\langle b_k \rangle(\sqrt{s})$ by generalized logarithmic function $a_1 + a_2 \ln(\sqrt{s}/s_0)$ improves at fixed value $a_3 = 1.0$ for fitting procedure in both energy domains $3.5 < \sqrt{s} < 9$ GeV and $\sqrt{s} > 8$ GeV. Thus, the dependence $\langle b_k \rangle(\sqrt{s})$ admits a universal approximation by $\xi_{b_k}(\sqrt{s})$ in domain $\sqrt{s} > 3.5$ GeV for a wide class of interactions at any $\tilde{X}$ values studied here. The results of quantitative analysis of $\langle b_k \rangle$ agree with qualitative hypothesis about slow (logarithmic) growth of jet size and estimation of $\langle b_k \rangle$ at $\sqrt{s} > 10$ GeV indicated in \cite{15}. Taking into account qualitative shapes of dependencies $\langle T_k \rangle(\sqrt{s})$ and $D(\sqrt{s})$ in energy ranges under consideration the experimental samples joined on fragmentation regions for these members of $G$ were fitted by function $\xi_{T_k}(\sqrt{s})$. The fit results are presented in Table III. As seen both $\langle b_k \rangle$ and other parameters from $G$ increase faster for hadron-nucleus reactions than that for nucleonic target in the range of initial energies under study. Results for $\langle T_k \rangle$ and $D$ obtained for hadron-nucleus collisions at $\sqrt{s} \sim 3 - 5$ GeV can be considered as indication on sensitivity of nuclear target on dynamic (mean temperature) and geometric (cluster dimension) parameters of hadronic system passes through nucleus. This observation allows to generalize the conclusion about influence of nuclear matter on size of soft pion jets in full set of collective parameters $G$. This assumption is in good agreement with estimations both for fragmentation length.
at $\sqrt{s} \sim 3 \text{–} 5 \text{ GeV}$ \cite{4} and for color field formation length $L_{p.f.}^c \sim R_{b} / 2$, where $R$ is the radius of target nucleus \cite{9}. In particular, the last relation indicates that $L_{p.f.}^c \lesssim R$ and nucleus can influence on jet properties at values $b_k \lesssim 2$ which correspond the hadron-nuclear reactions in energy domain $\sqrt{s} \sim 3 \text{–} 5 \text{ GeV}$ (Figs. 1b, 2b).

In initial energy domain $\sqrt{s} > 8 \text{ GeV}$ the samples of available experimental data for $\langle b_k \rangle$, $i = 1 \text{–} 3$ have a significantly smaller volumes than that at $\sqrt{s} \leq 8 \text{ GeV}$. Therefore the experimental points at $p_0=40 \text{ GeV}/c$ are used as boundary for "linkage" of fit results both for energies $\sqrt{s} > 8 \text{ GeV}$ and for lower energies. Accounting for suggestions about universal jet properties in hadron-hadron and hadron-nucleus interactions \cite{9} the experimental data both for $\pi^+ \text{–} p$ and for $\pi^- \text{–} C$ reaction at $p_0=40 \text{ GeV}/c$ are included in the sample under fit. In consequence of absent of experimental data the approximations of $\langle T_k \rangle(\sqrt{s})$ and $D(\sqrt{s})$ in separate energy domain $\sqrt{s} > 8 \text{ GeV}$ are possible at soft cut for $X$ only (Figs. 1b, f). Fit results are shown in Table I. The value of $\alpha_2$ parameter in (6) is equal to zero within statistical errors for fit of $D(\sqrt{s})$ in energy domain under study. Therefore the experimental sample is fitted at fixed value $\alpha_2 = 0$ and results are shown in Table I. On the other hand the slow logarithmic growth for $D$ in accordance with \cite{3} can not be excluded unambiguously because of small volume of sample of available experimental data.

Therefore $\forall i = 1 \text{–} 3$ dependencies $\mathcal{G}_i(\sqrt{s})$ admit a universal approximation in the form (6) at $\sqrt{s} > 3.5 \text{ GeV}$ for a wide class of interactions at any $X$ values considered here.

The comparative analysis collective properties ($\langle b_k \rangle$, $\langle T_k \rangle$) of soft pion jets and proton clusters \cite{8,10} at intermediate energies allows to get the following conclusions. The proton clusters in hadron-nucleus and nucleus-nucleus collisions are characterized significantly smaller values of $\langle b_k \rangle$ than pion jets. The mean temperature of particles in clusters is $\langle T_k \rangle \sim 65 \text{ MeV}$ for target fragmentation in $pC$, $dC$ and CC (clusters of the first type) collisions at $p_0=4.2 \text{ A GeV}/c$ \cite{8,9} which is comparable with $\langle T_k \rangle$ of pion jets in this fragmentation region for $\pi^+ \text{–} p$ reactions at the correspondent initial momentum $p_0=4.2 \text{ GeV}/c$ \cite{3}. The temperature of second type clusters produced in CC collisions due to multi-nucleon interactions in comparison with $\pi^+ \text{–} p$ interactions \cite{3} is significantly larger than that for pion jets in target fragmentation region and is equal to the $\langle T_k \rangle$ of pion jets for beam fragmentation within errors. These relations between mean temperatures of pion jets and proton clusters can be dominated, in particular, significantly larger proton mass ($m_p$) that pion one and definition \cite{2} re-written for proton clusters \cite{8,9}.

IV. SUMMARY

The following conclusions can be obtained by summarizing of the basic results of the present study.

The qualitative relation between jet size in the space of relative four-velocities and strong coupling constant is derived in the lowest order of RGE.

The energy dependencies for all parameters from set $\mathcal{G}$ show the similar behaviors at qualitative level. These dependencies are described by power function at $\sqrt{s} \leq 4 \text{ GeV}$ and logarithmic function at $\sqrt{s} > 3.5 \text{ GeV}$ reasonably.

The behavior of dependencies of $\langle b_k \rangle$, $\langle T_k \rangle$ and $D$ on initial energy at $\sqrt{s} \sim 3 \text{ GeV}$, possibly, is dominated by the onset of experimental manifestation of quark degrees of freedom in production of soft pion jets and corresponding transition from the description of these processes in terms of meson-baryon degrees of freedom to using of color (quark-gluon) degrees of freedom. For the first time the quantitative estimations are obtained for the low boundary energy at which quark degrees of freedom start to display itself in production of soft pion jets experimentally.

The influence of presence of nuclear matter on dynamic and geometric characteristics of soft pion jets in the space of relative four-dimensional velocities is observed at $\sqrt{s} \sim 3 \text{–} 5 \text{ GeV}$.

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FIG. 1: Dependencies of parameters from the set $G$ on $\sqrt{s}$ at $\bar{X} = 0.1$ in the target (beam) fragmentation region for various interactions. Experimental data are from [12] excepting values of $\langle T_k \rangle$ and $D$ for $\tilde{\nu}N$. The curve 1 (2) corresponds to the fit by power function of data samples for target (beam) fragmentation region. Approximations for joint samples for target and beam fragmentation regions are shown by dotted line for experimental data on hh, $\tilde{\nu}N$ and AA at $\sqrt{s} < 4$ GeV, by solid line – for hh and $\tilde{\nu}N$ reactions at $4 \leq \sqrt{s} < 9$ GeV, by solid thick line – for hA collisions, by solid thin line – for hh and hA interactions at $\sqrt{s} > 8$ GeV. Fits by function (5) of all available experimental data for $\langle T_k \rangle$ (d) and $D$ (f) are shown by dashed lines.
FIG. 2: Dependencies of parameters from the set \( G \) on \( \sqrt{s} \) at \( \tilde{X} = 0.2 \) in the target (beam) fragmentation region for various interactions. Experimental data are from [12] with exception of the values of \( \langle T_k \rangle \) and \( D \) for \( \bar{\nu}N \). Notations for the experimental points and fit curves are identical to that in Fig. 1.