A possible new approach of quantum measurements

S. Dumitru
Department of Physics, “Transilvania” University, B-dul Eroilor 29, R-2200 Brasov, Romania
E-mail: s.dumitru@unitbv.ro

Abstract. It is proposed a possible new approach of quantum measurements (QMS), disconnected of the traditional interpretation of uncertainty relations and independent of any appeal to the strange idea of collapse (reduction) of wave functions. The new approach regards QMS as a statistical samplings (but not as simple detection acts) and their description as a distinct task, independent of actual procedures of quantum mechanics. A QMS is described by means of transformations of probability density and probability current, from intrinsic into recorded readings. The quantum observables appear as random variables, described by usual operators and valuable through probabilistic numerical parameters (mean values, correlations and standard deviations). The values of the respective parameters are not the same in the two mentioned readings. Then the measurement uncertainties (errors) are described by means of the changes in the alluded values. The new QMS approach is illustrated through an one-dimensional example.

Submitted to: JPA

PACS numbers: 03.65.Ta, 03.65-w, 03.65.Ca, 01.70+w
1. Introduction

In connection with the foundation and interpretation of quantum mechanics (QM) the description of quantum measurements (QMS) is a problem often considered \[1\] as: “probably the most important part of the theory”. The respective problem germinate from the discussions about the traditional interpretation of uncertainty relations (TIUR). In its essence the mentioned problem refers to the theoretical descriptions of measurements regarding the observables (physical quantities) specific for quantum microparticles. Along the years a large number of works reported approaches of QMS problem (for a significant and updated bibliography see \[1\] and preprints archives \[2,3\]). As a notable aspect today one finds that many of the mentioned approaches are TIUR-connected, because they are founded on conjectures induced (inspired) someway from TIUR. In the main the respective approaches, as well as the TIUR, are centered round the idea that the uncertainty relations (UR) are capital physical formulas with an exclusive quantum (i.e. non-classical) significance.

On the other hand, if it is subjected to a minute re-examination, TIUR proves oneself to be nothing but an incorrect doctrine that must be denied. Such a re-examination was developed progressively in our works \[4–9\] and its essential conclusions, of interest here, can be found in the recent paper \[10\]. Through the mentioned re-examination of TIUR one finds that UR must be be reinterpreted in a more natural manner - i.e as relations belonging to a more general family of formulas (from both quantum and classical physics) which regard the fluctuations of observables with random characteristics. Moreover UR must be deprived of any capital (or extraordinary) attributes usually asserted by TIUR and assumed by the TIUR-connected approaches of QMS.

In the mentioned circumstances as regards the QMS problem becomes of actual interest to search new approaches disconnected from TIUR doctrine. Such an approach is the aim of the present paper. For our aim in the next section we present the main conjectures of the alluded TIUR-connected approaches as well as the corresponding shortcomings. Subsequently, in Sec.3., we present a general schema of a possible new approach of the QMS problem. Our approach is inspired from a view \[8,11,12\] about the measurement of classical (non-quantum) random observables. The general schema from Sec.3 is detailed through a simple exemplification in Sec.4. We end our considerations in Sec.5 with some conclusions.

2. Conjectures and shortcomings

In their essence the alluded TIUR-connected approaches of QMS imply someway one or more of the following conjectures (C):

- **C.1**: The description of QMS must be regarded in an indissoluble association with the UR

\[
\Delta A \cdot \Delta B \geq \frac{1}{2} |\langle [\hat{A}, \hat{B}] \rangle| 
\]

(1)
supposed as a capital physical formula. Consequently QMS approaches must be developed as extensions of TIUR. (Observation: The notations in (1) are the usual ones from QM and they are shortly reminded below in the next section).

- **C.2**: Between quantum and classical measurements there is a fundamental distinction due to the exclusive existence of UR (1) in quantum cases.

- **C.3**: The description of QMS must take into account some non-null jumps in the states of the measured system. The respective jumps are caused by the perturbative action of measuring devices and they are neither avoidable nor negligible.

- **C.4**: A QMS supplies a single value for a measured observable and consequently it must be regarded as a unique (single) detection act representable as a collapse (reduction) of the corresponding wave function.

The mentioned re-examination of TIUR shows [10] that in reality UR (1) are not capital physical formulas. Consequently we can conclude that it is unreasonable to subordinate the QMS approaches to the respective UR. But such a conclusion clearly appears as a true shortcoming for the conjecture C.1. On the other hand within the same re-examination one finds that the UR (1) belongs to a general family of fluctuation relations from both quantum and classical (non-quantum) physics. Then it results that the mentioned UR cannot motivate a distinction between quantum and classical situations. Evidently that such a result leaves without any base the conjecture C.2 and it must be noted as a shortcoming of the respective conjecture.

As regards the conjecture C.3 the following facts are notable. The respective conjecture was not inferred directly from the main assertions of TIUR. However, it was promoted adjacently in discussions generated by TIUR. Firstly, it was said that the measurements uncertainties are due to the interactions between measured systems and measuring devices. Secondly, it was added that the respective interactions cause jumps in the states of the measured systems. Then it was accredited the supposition that, in contrast with the classical situations, in QMS the mentioned uncertainties, interactions and jumps have an unavoidable character. Subsequently it was promoted the idea that the alluded measuring jumps must be taken into account in the description of QMS. In spite of its genesis, the above mentioned idea is proved to be incorrect by the following genuine and indubitable opinion [13]: “it seems essential to the notion of a measurement that it answers a question about the given situation existing before measurement. Whether the measurement leaves the measured system unchanged or brings about a new and different state of that system is a second and independent question”. The natural acceptance of the quoted opinion brings the conjecture C.3 in an insurmountable shortcoming.

The conjecture C.4 is contradicted by natural views about random quantities, from both physics and mathematics. From the physics viewpoint, the measurement of a random observable (quantity) must have the same general features, independently of its quantum or classical nature. However, in the classical context (e.g. in the study of fluctuation [11, 12, 14, 15]) the measurement of a macroscopic random observable is
not viewed as a single detection act, associated with some collapse (reduction) of the corresponding probability distribution. More exactly such a measurement is regarded as a statistical sampling, i.e. as an ensemble of great number of individual detection acts. The respective ensemble gives a nontrivial set of values belonging to the spectrum of the considered observable. In addition, from a mathematical viewpoint a random variable (quantity) must be evaluated not by a unique value but through a statistical set of values. Then it directly results that because QMS regards observables with random characteristics they must be viewed as statistical sampling (in the above-mentioned sense). Consequently there are no reasons to represent (describe) a QMS as a collapse (reduction) of a wave function. The mentioned result and consequence incontestably invalidate the conjecture C.4. So one finds a shortcoming for the respective conjecture.

The above mentioned shortcomings of the conjectures C.1-4 have an unsurmountable character because they cannot be combated or avoided by valid arguments derivable from the TIUR doctrine. But such a fact shows that TIUR-connected approaches of QMS are groundless attempts. Then it results that, at least partially, the problem of QMS description is still an open question which requires further investigations. In such a context we think that the new approach that we present in the next sections can be of interest.

3. A new approach

It is known that each approach of QMS description resorts (more or less explicitly) to some conjectures. Then, for the new approach aimed here, we suggest the set of the following reconsidered conjectures (RC):

- **RC.1**: Any measurement searches for information regarding the pre-existent state of the investigated system, independently of the quantum or classical nature of the respective system.
- **RC.2**: Due to the randomness of quantum observables a QMS must consists obligatory in a statistical sampling i.e. in a great number of individual detection acts.
- **RC.3**: A description of QMS must contain some extra-QM elements regarding the measuring devices and procedures, because the mere QM refers only to the intrinsic properties of the considered systems.
- **RC.4**: Because of the fact that in the last analysis, the results supplied by QMS refer to the measured quantum systems they must be evaluated in terms of QM.

In mind with RC.1-4, we develop the announced approach as follows. We consider a spin-less quantum microparticle with own orbital characteristics described by the intrinsic (IN) wave function $\Psi_{IN}$. From a theoretical viewpoint $\Psi_{IN}$ can be regarded as solution of the corresponding Schrödinger equation. In the following probabilistic considerations, the microparticle is regarded as equivalent with a statistical ensemble of its own replica taken at the same instant of time and described by the same wave
function $\Psi_{IN}$. Therefore, for our purposes, the time $t$ appears as a “passive” variable not implied in the randomness of the considered microparticle. That is why $\Psi_{IN}$ will be written as a function only of the radius vector $\vec{r}$, i.e. $\Psi_{IN} = \Psi_{IN}(\vec{r})$. The specific observables $A_j (j = 1, 2, \ldots, n)$ of the microparticle are described by the usual QM operators $\hat{A}_j$ (e.g. $\hat{x}_\mu = x_\mu$ and $\hat{p}_\mu = -i\hbar \frac{\partial}{\partial x_\mu} (\mu = 1, 2, 3)$ for Cartesian coordinate and momenta, $\hat{p} = -i\hbar \nabla$ and $\hat{L} = -i\hbar \vec{r} \times \nabla$ for momentum and angular momentum vectors or $\hat{H} = -\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r})$ for Hamiltonian).

Because $A_j$ have random properties, as in probability theory [17], for practical purposes they are described by means of the so-called numerical parameters (or characteristics). In QM the mostly used such parameters are: the IN-mean-values $\langle A_j \rangle_{IN}$ the IN-correlations $C_{IN}(A_j, A_l)$ respectively the IN-standard-deviations $\Delta_{IN} A_j$. Note that, from a probabilistic perspective, the mentioned numerical parameters are lower order entities. Additionally, as in probability theory [17], the higher order numerical parameters can also be used (e.g. higher order correlations and moments). However, such higher order parameters are not usual in QM literature. As it is known the alluded lower order intrinsic (IN) numerical parameters are defined by the relations:

$$\langle A_j \rangle_{IN} = \left( \Psi_{IN}, \hat{A}_j \Psi_{IN} \right) = \int \Psi_{IN}^* (\vec{r}) \hat{A}_j \Psi_{IN} (\vec{r}) \, d^3 \vec{r} \quad (2)$$

$$C_{IN}(A_j, A_l) = \left( \delta_{IN} \hat{A}_j \Psi_{IN}, \delta_{IN} \hat{A}_l \Psi_{IN} \right), \quad \delta_{IN} \hat{A}_j = \hat{A}_j - \langle A_j \rangle_{IN} \quad (3)$$

$$\Delta_{IN} A_j = \sqrt{C_{IN}(A_j, A_j)} \quad (4)$$

In (2) and (3) $(f_a, f_b)$ denotes the scalar product of the functions $f_a$ and $f_b$.

From a general physical perspective the intrinsic parameters (2)-(4) must be compared with the corresponding recorded (or ascertained) parameters considered as being given by measurements. But if in connection with the measurements, besides the practical experimental actions, one wants to operate with theoretical descriptions the term “recorded” must be regarded in two postures. In one posture it has a significance of “factual records” (FR) and refers to the data supplied by adequate practical experiments. In other posture the respective term has a significance of “prognosticated records” (PR) and refers to the quantities predicted by the considered theoretical description.

For the observables $A_j (j = 1, 2, \ldots, s)$ the alluded FR-parameters can be defined as follows. Because $A_j$ are random variables their measurements must be regarded as statistical samplings (done on statistical replies of a system (microparticle) considered in the same pre-measurement state). By such a sampling, on the recorder of the measuring device, for each observable $A_j$ one obtains a set of experimental values noted as: $a_{j1}, a_{j2}, \ldots, a_{jn}$. Then the ensemble of observables $A_j$ can be characterized by means of the following FR-parameters (defined according to the mathematical statistics rules [17]).

$$\langle A_j \rangle_{FR} = \frac{1}{n} \sum_{k=1}^{n} a_{jk} \quad (5)$$
A new approach of quantum measurements

\[ C_{FR}(A_j, A_l) = \frac{1}{n} \sum_{k=1}^{n} (a_{jk} - \langle A_j \rangle_{FR})(a_{lk} - \langle A_l \rangle_{FR}) \]  

(6)

\[ \Delta_{FR}A_j = \sqrt{C_{FR}(A_j, A_j)} \]  

(7)

For the parameters (5)-(7) can be used the denominations FR-mean-value, FR-correlation respectively FR-standard-deviation.

Evidently that the FR-quantities (5)-(7) depend both on the intrinsic properties of the measured system and on the characteristics of the measuring devices/procedures. That is why the mentioned FR-quantities are significant only if they are considered in connection with the experimental setting which supply the values \( a_{j1}, a_{j2}, \ldots, a_{jn} \).

On the other hand if one wishes to operate with a theoretical description of QMS the IN-parameters (2)-(4) must be compared with corresponding parameters of “prognosticated records” (PR) - type from an adequate mathematical model. For such a model we consider that the respective PR-parameters are defined similarly with (2)-(4) by means of a PR-wave-function \( \Psi_{PR} \) and with the same operators, i.e.

\[ \langle A_j \rangle_{PR} = \left( \Psi_{PR}, \hat{A}_j \Psi_{PR} \right) = \int \Psi_{PR}^* (\vec{r}) \hat{A}_j \Psi_{PR} (\vec{r}) d^3r \]  

(8)

\[ C_{PR}(A_j, A_l) = \left( \delta_{PR} \hat{A}_j \Psi_{PR}, \delta_{PR} \hat{A}_l \Psi_{PR} \right), \quad \delta_{PR} \hat{A}_j = \hat{A}_j - \langle A_j \rangle_{PR} \]  

(9)

\[ \Delta_{PR}A_j = \sqrt{C_{PR}(A_j, A_j)} \]  

(10)

Our above consideration is motivated by the known fact that, in theoretical descriptions, the randomness of a quantum microparticle is incorporated in its wave function but not in operators of its observables. Properties of various states of a microparticle are described by different wave functions but with the same operators. A similar situation exists in the case of classical statistical systems for which the randomness is incorporated in the probability densities but not in the expressions of macroscopic random variables. In the alluded cases also the properties of various states of a system are described with different probability densities but with the same expressions for the macroscopic random variables. In a classical case a measurement is described similarly \([11, 12]\) by appealing to a “recorded” density of probability. The term “recorded” from \([8–12]\) implies the same significance as the here used term “prognosticated records”. Note that in both quantum and classical cases the appeals to “prognosticated records” or “recorded” entities (wave function or probability density) must not be regarded as a description of collapse (reduction) for the corresponding intrinsic entities.

By adopting the relations (8)-(10) the task of our approach becomes to express \( \Psi_{PR} \) (or related quantities) in terms of \( \Psi_{IN} \) (or associated entities) and of some elements regarding the measuring devices. For such a task, firstly we show that the parameters (2)-(4) and (8)-(10) can be expressed in terms of certain quantities connected with \( \Psi_Y(Y = IN; PR) \) and having ordinary probabilistic significance in the sense of probability theory \([17]\). So we transcribe \( \Psi_Y \) in the form \( \Psi_Y = |\Psi_Y| \exp(i\Phi_Y) \) where
A ... new approach of quantum measurements

$|Ψ_Y|$ and $Φ_Y$ denote the modulus respectively the argument of $Ψ_Y$. As quantities of the mentioned type we take firstly the probability densities associated with $Ψ_Y$ and defined by

$$ρ_Y = |Ψ_Y|^2$$

(11)

Other quantities with ordinary probabilistic significance are the probability currents (or probability fluxes per unit-area):

$$\vec{J}_Y = -\frac{i\hbar}{2m} (Ψ_Y^* \nabla Ψ_Y - Ψ_Y \nabla Ψ_Y^*) = \frac{\hbar}{m} |Ψ_Y|^2 \cdot \nabla Φ_Y$$

(12)

($m$ denotes the mass of microparticle).

Now let us show that the parameters (2)-(4) and (8)-(10) can be expressed in terms of $ρ_Y$ and $\vec{J}_Y$. Then we observe that if an operator $\hat{A}$ does not depend on $\nabla$, i.e. $\hat{A} = \hat{A}(\vec{r})$ in (2) and (8) can be used the substitutions:

$$Ψ_Y^* \hat{A} Ψ_Y = A(\vec{r}) \rho_Y$$

(13)

On the other hand if $\hat{A}$ depends on $\nabla$, i.e. $\hat{A} = \hat{A}(\nabla)$, by taking $Ψ_Y = |Ψ_Y| \exp(iΦ_Y)$ and using (11)-(12) in (2) and (8) one can resort to the substitutions like:

$$Ψ_Y^* \nabla Ψ_Y = \frac{1}{2} \nabla ρ_Y + \frac{im}{\hbar} \vec{J}_Y$$

(14)

$$Ψ_Y^* \nabla^2 Ψ_Y = ρ_Y^{1/2} \nabla^2 ρ_Y^{1/2} + \frac{im}{\hbar} \nabla \vec{J}_Y - \frac{m^2 \vec{J}_Y^2}{\hbar^2} \rho_Y$$

(15)

The existence of substitutions (13)-(15) suggests that the description of QMS can be completed by adequate considerations about the quantities $ρ_Y$ and $\vec{J}_Y$. As the respective quantities have ordinary probabilistic significance for the alluded completion we resort to the model used \cite{11, 12} in the description of measurements of classical random observables. We also take into account the fact that $ρ_Y$ and $\vec{J}_Y$ refer to the positional respectively motional aspects of probabilities. Or, from an experimental perspective, the two aspects can be regarded as measurable by independent devices and procedures. Then the alluded completion must consist in giving independent relationships between $ρ_{PR}$ and $ρ_{IN}$ on the one hand respectively between $\vec{J}_{PR}$ and $\vec{J}_{IN}$ on the other hand. The mentioned relationships can be expressed formally by the following generic formulas:

$$ρ_{PR} = \hat{G} ρ_{IN}$$

(16)

$$J_{PR,μ} = \sum_{ν=1}^{3} \hat{Λ}_{μ,ν} J_{IN,ν}$$

(17)

($J_{Y,μ}$ with $Y = IN, PR$ and $μ = 1, 2, 3 = x, y, z$ denote the Cartesian components of vectors $\vec{J}_Y$). In (16) and (17) $\hat{G}$ and $\hat{Λ}_{μ,ν}$ signify the measurements operators. They must comprise obligatory characteristics of measuring devices and procedures. So $\hat{G}$ and $\hat{Λ}_{μ,ν}$ must contain some extra-QM elements, i.e. elements that do not belong to the usual QM description of the intrinsic properties of the measured microparticles.
A new approach of quantum measurements

For measuring devices with linear and stationary characteristics, similarly with the classical case \([11, 12]\), the relations (16)-(17) can be written as:

\[
\rho_{PR}(\vec{r}) = \int G(\vec{r}, \vec{r}') \rho_{IN}(\vec{r}') \, d^3\vec{r}'
\]

\[
J_{PR\mu}(\vec{r}) = \sum_{\nu=1}^{3} \int \Lambda_{\mu\nu}(\vec{r}, \vec{r}') \, J_{IN\nu}(\vec{r}') \, d^3\vec{r}'
\]

The kernels \(G(\vec{r}, \vec{r}')\) and \(\Lambda_{\mu\nu}(\vec{r}, \vec{r}')\) are supposed to satisfy the conditions:

\[
\int G(\vec{r}, \vec{r}') \, d^3\vec{r} = \int G(\vec{r}, \vec{r}') \, d^3\vec{r}' = 1
\]

\[
\int \Lambda_{\mu\nu}(\vec{r}, \vec{r}') \, d^3\vec{r} = \int \Lambda_{\mu\nu}(\vec{r}, \vec{r}') \, d^3\vec{r}' = 1
\]

These conditions show the one-to-one probabilistic correspondence between the intrinsic quantities \(\rho_{IN}\) and \(J_{IN}\) respectively the recorded ones \(\rho_{PR}\) and \(J_{PR}\). Parameters \((8)-(10)\), evaluated by means of the relations \((13)-(15)\) and \((16)-(19)\), incorporate randomness of both intrinsic and extrinsic nature, corresponding to the own properties of the investigated microparticle respectively to the measuring devices. As evaluated the mentioned parameters have a theoretical significance. Their adequacy must be tested by comparing with the corresponding FR-parameters \((3)-(7)\) obtained by statistical processing of the real experimental data. If the test is affirmative both descriptions, of intrinsic QM properties respectively of QMS, can be accepted as adequate. However, if the test invalidates the theoretical results, at least one of the respective descriptions must be regarded as inadequate.

From the origins of their history, the QMS approaches are concerned with the problem of quantitative evaluation for measuring uncertainties (i.e. for errors induced by the measurements in the values of the measured quantum observables). That is why it is of interest to discuss the respective problem in connection with the here promoted approach. Our discussion starts by pointing out the fact that quantum observables have a random character. Consequently, the uncertainties of such an observable must be evaluated through indicators, which comprise information from the whole its spectrum. It is easy to see that indicators of the alluded kind can be introduced by means of the numerical parameters defined by relations \((5)-(7)\) and \((8)-(10)\). That is why we suggest that, conjointly with the above-presented approach of QMS, the measuring uncertainties to be evaluated through the following uncertainty (or error) indicators of FR-type respectively PR-type:

\[
\delta_{FR}(\langle A_j \rangle) = |\langle A_j \rangle_{FR} - \langle A_j \rangle_{IN}| \tag{22}
\]

\[
\delta_{FR}(C(A_j, A_l)) = |C_{FR}(A_j, A_l) - C_{IN}(A_j, A_l)| \tag{23}
\]

\[
\delta_{FR}(\Delta A_j) = |\Delta_{FR} A_j - \Delta_{IN} A_j| \tag{24}
\]

\[
\delta_{PR}(\langle A_j \rangle) = |\langle A_j \rangle_{PR} - \langle A_j \rangle_{IN}| \tag{25}
\]
A new approach of quantum measurements

\[
\delta_{PR}(\mathcal{C}(A_j, A_l)) = |C_{PR}(A_j, A_l) - C_{IN}(A_j, A_l)|
\]  

(26)

\[
\delta_{PR}(\Delta A_j) = |\Delta_{PR}A_j - \Delta_{IN}A_j|
\]  

(27)

The above defined uncertainty indicators of FR-type \((22)-(24)\) respectively of PR-type \((25)-(27)\) are significant for a given practical measurement respectively for a considered theoretical description of QMS. The concordance degree between the two types of indicators shows the level of adequation of the theoretical description in respect with the considered measurement.

The uncertainty indicators of PR-type \((25)-(27)\) have a restricted significance for a system (microparticle), because they refer to some particular observables of the respective system. A more generic uncertainty indicators, also of PR-type but regarding a system in the whole, can be introduced by means of the following informational entropies of Shannon type:

\[
\mathcal{H}_Y = - \int \rho_Y \ln \rho_Y \, d^3 \vec{r}
\]

\[
\tau_Y = - \int |\vec{J}_Y| \ln |\vec{J}_Y| \, d^3 \vec{r}
\]

(28)-(29)

Here \(\mathcal{H}_Y\) and \(\tau_Y\) can be called positional respectively motional informational entropies. Then the alluded generic uncertainty indicators can be defined as

\[
\delta_{PR}\mathcal{H} = \mathcal{H}_{PR} - \mathcal{H}_{IN}
\]

\[
\delta_{PR}\tau = \tau_{PR} - \tau_{IN}
\]

(30)-(31)

It is interesting to note the fact that within the above-presented description of QMS the indicator \(\delta_{PR}\mathcal{H}\) is a nonnegative quantity (i.e. \(\delta_{PR}\mathcal{H} \geq 0\)). The respective fact can be proved, similarly with the classical situation \([11,12]\), by means of the relations \((18)\) and \((20)\). So by taking into account the respective relations, the normalization of both \(\rho_{IN}\) and \(\rho_{PR}\), and the evident formula \(\ln y \leq y - 1\) \((y > 0)\) one can write:

\[
\delta_{PR}\mathcal{H} = \mathcal{H}_{PR} - \mathcal{H}_{IN} = - \int d^3 \vec{r} \int d^3 \vec{r}' G(\vec{r}, \vec{r}') \rho_{IN}(\vec{r}')^2 \ln \frac{\rho_{PR}(\vec{r})}{\rho_{IN}(\vec{r}')} \geq - \int d^3 \vec{r} \int d^3 \vec{r}' G(\vec{r}, \vec{r}') \rho_{IN}(\vec{r}') \left[ \frac{\rho_{PR}(\vec{r})}{\rho_{IN}(\vec{r}')} - 1 \right] = 0
\]

(32)

The above considerations give a genuine description of QMS in which one finds, in adequate positions, all the essential elements. The respective elements include: (i) the intrinsic numerical parameters \((2)-(4)\), (ii) the model represented by \((16)-(21)\) for describing the influences of measuring devices, (iii) the recorded numerical parameters \((8)-(10)\) and (iv) the uncertainties indicators of PR-type \((25)-(27)\) or \((30)-(31)\).

In the end of this section we note that the description of QMS presented here, as well as the one discussed in \([11,12]\) for classical measurements, can be regarded formally from the perspective of information theory. In such a perspective, a measurement appears
as a process of information transmission. The source of information is the measured system and the intrinsic values of its observables represent the input information. The chain of measuring devices plays the role of channel for information transmission. The recorded data about the measured observables represent the output information. Then the measurement uncertainties can be regarded as alterations of the transmitted information.

4. A simple exemplification

To illustrate the above-introduced QMS approach let us refer to the following simple exemplification. We consider a quantum microparticle in a one-dimensional motion along the x-axis. Its own properties are supposed to be described by the intrinsic wave function \( \Psi_{IN}(x) = |\Psi_{IN}(x)| \exp(i \Phi_{IN}(x)) \) with:

\[
|\Psi_{IN}(x)| = \left( \alpha \sqrt{2\pi} \right)^{-1/2} \exp\left\{ -\frac{(x - x_0)^2}{4\alpha^2} \right\}, \quad \Phi(x) = kx
\]  

(33)

Then the intrinsic probability density and current defined by (11) and (12) are:

\[
\rho_{IN}(x) = \frac{1}{\alpha \sqrt{2\pi}} \exp\left\{ -\frac{(x - x_0)^2}{2\alpha^2} \right\}
\]  

(34)

\[
J_{IN}(x) = \frac{\hbar k}{m\alpha \sqrt{2\pi}} \exp\left\{ -\frac{(x - x_0)^2}{2\alpha^2} \right\}
\]  

(35)

So the intrinsic characteristics of the microparticle are described by the parameters \( x_0, \alpha \) and \( k \).

Considering that the errors of QMS are small in (18) and (19), one can operate with the one-dimensional kernels of Gaussian forms given by:

\[
G(x, x') = \frac{1}{\sigma \sqrt{2\pi}} \exp\left\{ -\frac{(x - x')^2}{2\sigma^2} \right\}
\]  

(36)

\[
\Lambda(x, x') = \frac{1}{\lambda \sqrt{2\pi}} \exp\left\{ -\frac{(x - x')^2}{2\lambda^2} \right\}
\]  

(37)

Here \( \sigma \) and \( \lambda \) describe the error characteristics of the measuring devices (see below).

By using (36)-(37) in the one-dimensional versions of the relations (18)-(19) one finds:

\[
\rho_{PR}(x) = \frac{1}{\sqrt{2\pi(\alpha^2 + \sigma^2)}} \exp\left\{ -\frac{(x - x_0)^2}{2(\alpha^2 + \sigma^2)} \right\}
\]  

(38)

\[
J_{PR}(x) = \frac{\hbar k}{m\sqrt{2\pi(\alpha^2 + \lambda^2)}} \exp\left\{ -\frac{(x - x_0)^2}{2(\alpha^2 + \lambda^2)} \right\}
\]  

(39)

One can see that in the case when \( \sigma \to 0 \) and \( \lambda \to 0 \) the kernels \( G(x, x') \) and \( \Lambda(x, x') \) degenerate into the Dirac function \( \delta(x - x') \). Then \( \rho_{PR}(x) \to \rho_{IN}(x) \) and \( J_{PR}(x) \to J_{IN}(x) \). Such a case corresponds to an ideal measurement. Alternatively the cases with \( \sigma \neq 0 \) and/or \( \lambda \neq 0 \) are associated with non-ideal measurements.
As observables of interest, we consider the coordinate $x$ and momentum $p$ described by the operators $\hat{x} = x$· and $\hat{p} = -i\hbar \frac{\partial}{\partial x}$. Adequately we use the expressions (34)-(35) and (38)-(39) in the relations (2)-(4) and (8)-(10). Then, by using (13)-(15), for the mentioned observables one finds the following numerical parameters of IN-type respectively of PR-type.

\[
\langle x \rangle_{IN} = \langle x \rangle_{PR} = x_0, \quad \langle p \rangle_{IN} = \langle p \rangle_{PR} = \hbar k \tag{40}
\]

\[
C_{IN}(x, p) = C_{PR}(x, p) = \frac{i\hbar}{2} \tag{41}
\]

\[
\Delta_{IN}x = \alpha, \quad \Delta_{PR}x = \sqrt{\alpha^2 + \sigma^2} \tag{42}
\]

\[
\Delta_{IN}p = \hbar \frac{1}{2\alpha} \tag{43}
\]

\[
\Delta_{PR}p = \hbar \sqrt{\frac{k^2(\alpha^2 + \sigma^2)}{\sqrt{\alpha^4 - \lambda^4 + 2\sigma^2(\alpha^2 + \lambda^2)}} - k^2 + \frac{1}{4(\alpha^2 + \sigma^2)}} \tag{44}
\]

Then for the considered observables $x$ and $p$ the uncertainty (error) indicators of PR-type (25)-(27) become:

\[
\delta_{PR}(\langle x \rangle) = 0, \quad \delta_{PR}(\langle p \rangle) = 0, \quad \delta_{PR}(C(x, p)) = 0 \tag{45}
\]

\[
\delta_{PR}(\Delta x) = \sqrt{\alpha^2 + \sigma^2} - \alpha \tag{46}
\]

\[
\delta_{PR}(\Delta p) = \hbar \left\{ \frac{k^2(\alpha^2 + \sigma^2)}{\sqrt{\alpha^4 - \lambda^4 + 2\sigma^2(\alpha^2 + \lambda^2)}} - k^2 + \frac{1}{4(\alpha^2 + \sigma^2)} - \frac{1}{2\alpha} \right\} \tag{47}
\]

These relations show that for the considered association microparticle-QMS the numerical parameters $\langle x \rangle$, $\langle p \rangle$ and $C(x, p)$ are not affected by uncertainties (errors). However, for the same association the parameters $\Delta x$ and $\Delta p$ are troubled by the measurement, the corresponding non-null uncertainty (error) indicators of PR-type being given by (46)-(47).

Now, for the here discussed model of QMS description, let us search the entropic error indicators of PR-type defined by the relations (28)-(31). By using the expressions (33)-(35) and (38)-(39) one finds:

\[
\delta_{PR}\mathcal{H} = \frac{1}{2} \ln \left( 1 + \frac{\sigma^2}{\alpha^2} \right) \tag{48}
\]

\[
\delta_{PR}\tau = \frac{\hbar k}{2m} \ln \left( 1 + \frac{\lambda^2}{\alpha^2} \right) \tag{49}
\]

If in (33) we restrict to the values $x_0 = 0$, $k = 0$ and $\alpha = \sqrt{\frac{\hbar}{2m\omega}}$, our system is just a quantum oscillator with mass $m$ and pulsation $\omega$ situated in its ground state.

The corresponding numerical parameters and error indicators for observables $x$ and $p$ can be obtained from (40)-(44) respectively (15)-(17) by means of the mentioned
restrictions. However, in the case of oscillator it is interesting to point out the measuring characteristics for another observable, namely for energy described by the Hamiltonian:

$$\hat{H} = \frac{1}{2m} \hat{p}^2 + \frac{m\omega^2}{2} \hat{x}^2$$  \hspace{1cm} (50)

Then, for the probabilistic numerical parameters of oscillator energy one finds:

$$\langle H \rangle_{IN} = \frac{\hbar \omega}{2}, \quad \Delta_{IN}H = 0 \hspace{1cm} (51)$$

$$\langle H \rangle_{PR} = \frac{\omega \left[ \hbar^2 + (\hbar + 2m\omega \sigma^2)^2 \right]}{4(\hbar + 2m\omega \sigma^2)} \hspace{1cm} (52)$$

$$\Delta_{PR}H = \frac{\sqrt{2}m\omega \sigma^2(\hbar + m\omega \sigma^2)}{(\hbar + 2m\omega \sigma^2)} \hspace{1cm} (53)$$

The corresponding PR-uncertainty (error) indicators are:

$$\delta_{PR}(\langle H \rangle) = \frac{\omega \left[ \hbar^2 + (\hbar + 2m\omega \sigma^2)^2 \right]}{4(\hbar + 2m\omega \sigma^2)} - \frac{\hbar \omega}{2} \hspace{1cm} (54)$$

$$\delta_{PR}(\Delta H) = \frac{\sqrt{2}m\omega \sigma^2(\hbar + m\omega \sigma^2)}{(\hbar + 2m\omega \sigma^2)} \hspace{1cm} (55)$$

5. Conclusions

The problem of QMS description persists in our days as an open and disputed question. Many of its approaches are TIUR-connected because they are founded on conjectures mainly inspired from TIHR. But indubitable facts \[10\] show that TIUR is an incorrect doctrine. Consequently all arguments founded on TIUR imply important deficiencies. Particularly in the case of TIUR-connected approaches of QMS the main conjectures are affected by insurmountable shortcomings. Such a finding motivates our interest for a possible new approach of QMS, based on a set of reconsidered and natural conjectures.

We propose a set of four such conjectures and develop an adequate approach of QMS.

Our approach is founded on the usual probabilistic conception of QM. Therefore, for a quantum microparticle we operate with probabilistic entities (probability density and probability current) respectively with QM operators. We opine that, because in practice a correct QMS must consist in a statistical sampling, from a theoretical viewpoint a QMS must be represented as processing of the mentioned probabilistic entities while the quantum operators remain unchanged. Similarly, with the description of classical (non-quantum) measurements, the alluded processing must be pictured as changes of the respective entities. We opine that for a wide class of situations such changes can be modeled as linear integral transforms. Therefore, both probability density and probability current appear in intrinsic respectively “prognosticated records” posture. In the first posture, they regard the own characteristics of the measured microparticle, while in the second posture they comprise information related both to the respective
microparticle and to the measuring devices. The information regarding the measuring devices is introduced theoretically through the adopted model in description of QMS.

Together with the mentioned features of QMS the quantum observables must be naturally evaluated through the probabilistic numerical parameters such as mean values, correlations and standard deviations. Within the discussed approach, the respective parameters are characterized by intrinsic (IN) respectively “prognosticated records” (PR) values. Such values are calculable by means of QM operators but with IN-respectively PR-expressions for probability density and probability current. Then a natural description of measuring uncertainties for quantum observables is expressible in terms of differences between the mentioned “prognosticated records” and intrinsic values. Another description of measurements uncertainties, more generic (i.e. not associated with some particular observables), can be done in terms of informational entropies of Shannon type.

The here recapitulated features of our QMS approach are detailed from a general perspective in Sec.3, while in Sec.4 they are illustrated by means of a simple exemplification.

We remind here that our QMS approach is quite different from the TIUR-connected approaches (founded on (or inspired from) TIHR). The difference is evidenced on the one hand by the idea that QMS must be regarded as statistical samplings but not as individual detection acts. Consequently, we can avoid completely the controversial conception of wave function collapse (reduction). On the other hand, the alluded difference is pointed out by our presumption that the description of QMS must be regarded as a distinct and independent task comparatively with the usual QM procedures. Accordingly, with the respective regards the description of measurement must be considered and discussed as a scientific branch self-determined and additional comparatively with the quantum or classical chapters of physics. The mentioned chapters, as in fact is well established by the scientific practice, investigate only the intrinsic properties of the physical systems.

Acknowledgments

The above-presented ideas about QMS were announced preliminarily in the e-print [18]. Now they are reported here in a somewhat revised form in order to offer a possible completion for our opinions about TIUR, reiterared and consolidated in a more recent text [10].

I wish to express my deep gratitude to the World Scientific Publishing Company for putting at my disposal a copy of the monumental book [1].

The work reported in the present text benefited partially of some facilities offered by grants from the Romanian Ministry of Education and Research.

References
A ... new approach of quantum measurements

[1] Auletta G 2000 Foundations and Interpretation of Quantum Mechanics (World Scientific, Singapore)
[2] Lists New Preprints and Reports in the CERN Library http://weblib.cern.ch/
[3] arXiv.org e-Print archive - Quantum Physics: http://mentor.lanl.gov/archive/quant-ph
[4] Dumitru S 1977 Epistemological Letters 15 1
[5] Dumitru S 1980 CERN Library Preprint PRE-24165
[6] Dumitru S 1984 Microphysics (Solved Problems and a Critical Analysis of the Question of Significance of the Uncertainty Relations) (Cluj-Napoca: Dacia) (in Roumanian)
[7] Dumitru S 1987 in Recent Advances in Statistical Physics Ed. Datta B and Dutta M (Singapore: World Scientific)
[8] Dumitru S 1988 Rev. Roum. Phys. 33 11
[9] Dumitru S 2000 Preprint arXiv quant-ph/0004013
[10] Dumitru S 2002 Preprint arXiv quant-ph/0206009
[11] Dumitru S 1974 Phys. Lett. A 48 109
[12] Dumitru S 1999 Optik (Stuttgart) 110 110
[13] Albertson J 1963 Phys. Rev. 129 940
[14] Landau L Lifchitz E 1984 Physique Statistique (Mir: Moscou)
[15] Diu B Guthmann C Lederer D and Roulet B 1995 Elements de Physique Statistique (Hermann: Paris)
[16] Shilling H 1972 Statistische Physik in Beispielen (Veb Fachbucherverlag, Leipzig)
[17] Korn G A Korn T M 1968 Mathematical Handbook (Mc Graw Hill, New York) (Russian version 1977 (Nauka: Moscow))
[18] Dumitru S 2001 Preprint arXiv quant-ph/0111143

List of abbreviations

\[\begin{align*}
C & = \text{conjectures} \\
FR & = \text{“factual records”} \\
IN & = \text{intrinsic} \\
PR & = \text{“prognosticated records”} \\
QM & = \text{quantum mechanics} \\
QMS & = \text{quantum measurements} \\
RC & = \text{reconsidered conjectures} \\
TIUR & = \text{traditional interpretation of uncertainty relations} \\
UR & = \text{uncertainty relations}
\end{align*}\]