Couplings of $N = 1$ chiral spinor multiplets\footnote{Work supported by: The German Science Foundation (DFG) and European RTN Program MRTN-CT-2004-503369.}

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ABSTRACT

We derive the action for $n_L \geq 1$ chiral spinor multiplets coupled to vector and scalar multiplets. We give the component form of the action, which contains gauge invariant mass terms for the antisymmetric tensors in the spinor superfield and additional Green-Schwarz couplings to vector fields. We observe that supersymmetry provides mass terms for the scalars in the spinor multiplet which do not arise from eliminating an auxiliary field. We construct the dual action by explicitly performing the duality transformations in superspace and give its component form.

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1 Introduction

Antisymmetric tensor fields $B_{mn}$ naturally appear in the light sector of all string theories. In four space-time dimensions ($D = 4$) massless antisymmetric tensors are dual to scalar fields while massive tensors are dual to massive vectors. Therefore in the low energy effective action one has the choice to represent these degrees of freedom in either of two dual representations. Depending on the context one formulation might be more convenient than the other and for this reason both formulations have generically been developed.

Recently compactification with background fluxes and/or compactifications on generalized geometries have been studied in detail [1]. One novelty in these compactifications is the appearance of massive antisymmetric tensors [2]. As a consequence their description in terms of appropriate supergravities has been worked out [3]–[9]. In particular in $N = 1$ compactifications of type IIB on Calabi-Yau orientifolds with $O5$- or $O9$- planes a massive antisymmetric tensor appears when both electric and magnetic three-form fluxes are turned on [10]. The corresponding $N = 1$ superspace action was constructed in ref. [5]. Orientifolds of generalized geometries as discussed, for example, in refs. [11, 12] can feature more than one antisymmetric tensor. Therefore it is of interest to generalize the analysis of [5] and discuss the couplings of a set of $n_L$ massive antisymmetric tensors to vector and chiral multiplets. This is the purpose of the present paper.

In $N = 1$ supersymmetry the three form field strength of the antisymmetric tensor is part of a linear multiplet $L$ [13]–[18]. The antisymmetric tensor itself resides in the chiral spinor multiplet $\Phi_\alpha$. Whenever the antisymmetric tensor is massless the supersymmetric action is described in terms of $L$ only. Any mass term for $B_{mn}$ destroys the two-form gauge invariance. However, with the help of appropriate couplings to vector fields gauge invariance can be restored. The resulting Lagrangian is of the Stückelberg type [19] where the vector fields provide the ‘longitudinal’ degrees of freedom to render $B_{mn}$ massive. Put differently, in a unitary gauge the antisymmetric tensor ‘eats’ a vector field and becomes massive. A similar mechanism can be employed for $n_L$ antisymmetric tensors as long as enough ($n_V \geq n_L$) vector fields are coupled. Therefore the first goal of this paper is the derivation of a $N = 1$ superspace action for $n_L$ chiral spinor multiplets $\Phi_I^I, I = 1, \ldots, n_L$ coupled to $n_V$ vector multiplets $V^A, A = 1, \ldots, n_V$. Furthermore the gauge couplings of the vector multiplets are allowed to depend on $n_C$ chiral multiplets $N^i, i = 1, \ldots, n_C$.

As we already stated a massless antisymmetric tensor is dual to a scalar while a massive one is dual to a massive vector. This duality is also manifest at the level of superfields where a linear multiplet is dual to a chiral multiplet while a massive spinor multiplet is dual to a massive vector multiplet. Thus our second aim is to construct the dual theory in superspace.

This paper is organized as follows. In section 2 we introduce the notions of the linear and the chiral spinor multiplet. By means of the Stückelberg mechanism we construct the most general gauge invariant action for $n_L$ massive spinor multiplets and give its corresponding component form. We discuss the resulting scalar potential, which has not the standard $N = 1$ form due to a contribution from the chiral spinor multiplet. In section 3 we perform the duality transformations and rewrite the action in terms of
\(n_V - n_L\) massless and \(n_L\) massive vector multiplets. Finally in the appendix we present the supersymmetry transformations of the chiral spinor multiplet and give a modification of these transformations which preserves the WZ-gauge. This allows us to discuss the order parameters for supersymmetry breaking.

## 2 Spinor superfields coupled to vector and chiral multiplets

In \(N = 1\) supersymmetry an antisymmetric tensor \(B_{mn}\) is part of a chiral spinor superfield \(\Phi_\alpha\) while its three-form field strength \(H_{mnp}\) resides in a linear multiplet \(L\). In this section we consider a set of \(n_L\) linear multiplets \(L^I\) and the corresponding \(n_L\) chiral spinor multiplets \(\Phi^I_\alpha, I = 1, \ldots, n_L\). We review some of their properties and construct a gauge invariant action.

The linear multiplet is a real superfield, defined by the constraint

\[
D^2 L^I = \bar{D}^2 L^I = 0 ,
\]

where \(D_\alpha\) is the superspace covariant derivative.\(^2\) The \(\theta\)-expansion of \(L^I\) reads

\[
L^I = C^I + \theta \eta^I + \bar{\theta} \bar{\eta}^I + \frac{1}{2} \theta \sigma^m \bar{\theta} \epsilon_{mnpq} H^{npqI} - \frac{i}{2} (\theta \bar{\theta}) \bar{\sigma}^m \partial_{m} \eta^I - \frac{i}{2} (\bar{\theta} \theta) \sigma^m \partial_{m} \bar{\eta}^I - \frac{1}{4} \theta \bar{\theta} \bar{\theta} \bar{\theta} C^I .
\]

Here \(C^I\) are real scalars, \(\eta^I\) are Weyl fermions and \(H^{mnp}_{I} = \partial_{[m} B^{I}_{np]}\) are the field strengths of the antisymmetric tensors \(B^{I}_{np}\).

Each antisymmetric tensor \(B^{I}_{mn}\) is contained in a chiral spinor superfield \(\Phi^I_\alpha\) defined by \(^1\)

\[
L^I = \frac{1}{2} (D^\alpha \Phi^I_\alpha + D_\alpha \Phi^{\dagger I}_\alpha), \quad D_\beta \Phi^I_\alpha = 0 .
\]

The \(\Phi^I_\alpha\) enjoy the \(\theta\)-expansion

\[
\Phi^I_\alpha = \chi^I_\alpha - \theta_\gamma \left( \frac{1}{2} \delta^\alpha_\gamma (C^I + i E^I) + \frac{1}{4} (\sigma^m \bar{\sigma}^n)^\gamma_\alpha B^{I}_{mn} \right) + \theta \eta^I_\alpha + i \sigma_\alpha^m \partial_{m} \chi^I_\alpha ,
\]

where \(\chi^I_\alpha\) are additional Weyl fermions and \(E^I\) additional scalars. Due to its definition \(L^I\) the \(L^I\) are invariant under the gauge transformations

\[
\Phi^I_\alpha \rightarrow \Phi^I_\alpha + \frac{i}{8} \bar{D}^2 D_\alpha \Lambda^I ,
\]

where the \(\Lambda^I\) are real superfields. The expressions \(\bar{D}^2 D_\alpha \Lambda^I\) are chiral and we therefore can write\(^3\)

\[
\frac{i}{8} \bar{D}^2 D_\alpha \Lambda^I = - \frac{1}{2} \chi^I_\alpha \left( \partial^\gamma \frac{1}{2} \bar{D}^I + \frac{1}{4} (\sigma^m \bar{\sigma}^n)^\gamma_\alpha \left( \partial_{m} \Lambda^I_n - \partial_{n} \Lambda^I_m \right) \right) \theta_\gamma - \frac{1}{2} \theta n \sigma_\alpha^m \partial_{m} \chi^I_\alpha .
\]

We immediately see that the fields \(\chi^I_\alpha\) and \(E^I\) defined in the \(\theta\)-expansion of the superfield \(\Phi^I_\alpha\) in \((2.4)\) can be gauged away by \(\lambda^I_\alpha\) and \(\bar{D}^I\) using \((2.5)\). This leaves only the physical

\(^2\)Throughout the paper we are using the conventions of ref. \([20]\).

\(^3\)The expansion has the same structure as the field strength of the vector multiplet which we introduce in \((2.12)\). To avoid confusions with \((2.12)\) we have hatted the corresponding component fields of \((2.10)\).
degrees of freedom \( C^I, B^I_{mn} \) and \( \eta^I \) in the component expansion of \( \Phi^I_a \). Thus in this WZ-gauge we have

\[
\Phi^I_a = -\theta_\gamma \left( \frac{1}{2}\delta_a^\gamma C^I \right) + \frac{1}{4}(\sigma^m \bar{\sigma}^n)_\alpha \gamma B^I_{mn} \phi^I \, , \tag{2.7}
\]

and the left-over gauge invariance is the standard two-form gauge invariance

\[
B^I_{mn} \rightarrow B^I_{mn} + \partial_m A^I_n - \partial_n A^I_m \, , \quad C^I \rightarrow C^I \, , \quad \eta^I \rightarrow \eta^I \, . \tag{2.8}
\]

The superfields \( \Phi^I \) and \( L^I \) can be used to construct a gauge invariant action. The kinetic term is given by

\[
\mathcal{L}_{\text{kin}} = -\int d^2\theta d^2\bar{\theta} K(L^I) \, , \tag{2.9}
\]

where \( K(L^I) \) is an arbitrary real function of the \( L^I \). In components (2.9) reads

\[
\mathcal{L}_{\text{kin}} = -\frac{i}{4} K_{IJ} \left( \left( \partial_m C^J \right) \left( \partial^m C^I \right) + i(\eta^I \sigma^m \partial_m \bar{\eta}^J + \bar{\eta}^J \sigma^m \partial_m \eta^I) + \frac{3}{2} H^m_{npq} H^I_{mpl} \right) \\
-\frac{1}{8} K_{IJKL} \left( \eta^K \sigma^m \bar{\eta}^L \epsilon_{mpl} H^{npq} \right) - \frac{1}{4} K_{IJKL} \left( \frac{3}{2} \eta^K \eta^L \bar{\eta}^J \right) \tag{2.10}
\]

where we abbreviated

\[
K_{IJ...K} = \frac{\partial^n K(C)}{\partial C^I \partial C^J \ldots \partial C^K} \, . \tag{2.11}
\]

In addition to the kinetic term we can add a mass term for the \( B^I_{mn} \) if we introduce a set of Abelian vector multiplets \( V^A, A = 1, \ldots, n_V \). As we will see they can be used to ensure the gauge invariance (2.8) and they also provide the necessary degrees of freedom in order to render the \( B^I_{mn} \) massive. Let us denote the field strengths of the vector multiplets by \( W^A = -\frac{1}{4} D^2 A V^A \) with the component expansion

\[
W^A_a = -i\lambda^A_a + \left( \delta^\beta_\alpha \beta D^A - \frac{i}{2}(\sigma^m \bar{\sigma}^n)_{\alpha}^\beta F^A_{mn} \right) \theta_\beta + \theta_\alpha \sigma^m \partial_m \bar{\lambda}^A_a \, . \tag{2.12}
\]

Here \( F^A_{mn} = \partial_m v^A_n - \partial_n v^A_m \) are the field strengths of \( n_V \) \( U(1) \) gauge bosons \( v^A \).

The linear combination

\[
2i m^A J \Phi^I_\beta - W^A_\beta \tag{2.13}
\]

is gauge invariant under (2.5) provided we assign the following transformation laws to the \( V^A \)

\[
V^A \rightarrow V^A + m^A J \Lambda^J \, , \quad W^A_\beta \rightarrow W^A_\beta - \frac{i}{4} m^A J D^2 D_\beta \Lambda^J \, . \tag{2.14}
\]

In (2.13) and (2.14) we have introduced the constant coupling matrix \( m^A J \) which we demand to be real. The linear combination (2.13) can be used to build (Lorentz and gauge invariant) mass-terms for \( \Phi^I_\beta \). However this is not the only possible gauge invariant term. Permitting the Lagrangian to be invariant only up to a total derivative we can also add the term \( 2 \int d^2\theta e_{AJ} \Phi^I \left( W^A - im^A J \Phi^I \right) + \text{h.c.} \) where \( e_{AJ} \) is a constant real matrix.

\(^4\) \( W^A_a \) is invariant under the standard \( U(1) \) gauge invariance \( V^A \rightarrow V^A + \Sigma^A + \bar{\Sigma}^A \) where \( \Sigma^A \) are chiral superfields.
Note that in this expression only the symmetric part of the product \( e_{AI}m^A_J \) appears. The gauge invariance of this additional term can be most easily seen by first rewriting the term as

\[ -2 \int d^4 \theta e_{AI} \bar{L}^VI^A - \left( i \int d^2 \theta e_{AI}m^A_J \Phi^I \Phi^J + \text{h.c.} \right), \quad (2.15) \]

where we used \( d^2 \theta = -\frac{i}{2} D^2 \), \[(2.3)\] and the definition of \( W_\alpha \). Using \[(2.5)\] and \[(2.14)\] we perform the gauge transformations on \((2.15)\). For the first term we obtain

\[ \delta \int d^4 \theta (-2e_{AI} \bar{L}^VI^A) = -2 \int d^4 \theta e_{AI}m^A_J \Phi^J \]

as \( \delta \bar{L}^I = 0 \). Transformation of the second term reads

\[ \delta \int d^2 \theta (-i e_{AI}m^A_J \Phi^I \Phi^J) + \text{h.c.} = -i \int d^2 \theta \left( \frac{i}{4} \Phi^I \bar{D}^2 D \Lambda^J + \text{h.c.} \right) \]

\[ = - \int d^4 \theta e_{AI}m^A_J \left( \Phi^I \bar{D} \Lambda^J + \bar{\Phi}^I D \Lambda^J \right) \quad (2.17) \]

\[ = 2 \int d^4 \theta e_{AI}m^A_J \Lambda^J \bar{L}^I + \text{total derivative}, \]

where \[(2.3)\] and the chirality of \( \Phi \) was used. In \[(2.17)\] again only the symmetric part of \( e_{AI}m^A_J \) enters while the variation in \[(2.15)\] contains also the antisymmetric part. Therefore gauge invariance of \[(2.15)\] requires to impose the condition \( e_{AI}m^A_J = e_{AJ}m^A_I \).\(^5\)

Thus the most general gauge invariant action of \( n_L \) massive spinor multiplets coupled to \( n_V \) vector multiplets is given by

\[ \mathcal{L}_m = \frac{1}{4} \int d^2 \theta \left( f_{AB} \left( 2im^A_I \Phi^I - W^A \right) \left( 2im^B_J \Phi^J - W^B \right) \right) + C.I. \quad (2.18) \]

The matrix \( f_{AB} \) is the gauge coupling function of the vector multiplets which can depend holomorphically on additional chiral multiplets which we denote by \( N^i, i = 1, \ldots, n_C \).\(^6\)

The Lagrangian \[(2.18)\] is our first result which coincides with the Lagrangian of ref. \[5\] in the limit of one linear multiplet and was also given previously in ref. \[6\].

In components the Lagrangian \[(2.18)\] reads

\[ \mathcal{L}_m = -\frac{1}{4} \text{Re} f_{AB} \bar{F}_{mn}^A \tilde{F}_{nm}^B + \frac{1}{8} \text{Im} f_{AB} e^{klmn} \bar{F}_{kl}^A \tilde{F}_{mn}^B - \frac{1}{16} \epsilon^{klmn} e_{AI} B_{kl}^I \left( \bar{F}_m^I + F_m^I \right) \]

\[ + \frac{i}{2} \text{Re} f_{AB} \bar{D}A \bar{D}B - \frac{1}{2} \left( \epsilon_{AI} + 2 \text{Im} f_{AB} m^B_I \right) C^I D^A - \frac{1}{2} \text{Re} f_{AB} m^A_J m^B_J C^I C^J \]

\[ - \frac{i}{2} f_{AB} \lambda^A \sigma^m \partial_m \bar{\lambda}^B - \frac{i}{2} \bar{f}_{AB} \bar{\lambda}^A \sigma^m \partial_m \lambda^B \]

\[ - \frac{i}{2} \left( \epsilon_{AI} + 2 \text{Im} f_{AB} m^B_I \right) \bar{\eta}^I \lambda^A - \frac{1}{2} \left( -i \epsilon_{AI} + 2 \bar{f}_{AB} m^B_I \right) \bar{\eta}^I \bar{\lambda}^A \]

\[ - \frac{1}{2\sqrt{2}} \partial_i \bar{f}_{AB} (m^A_I C^I - i D^A) \chi^i \lambda^B - \frac{1}{2\sqrt{2}} \partial_i \bar{f}_{AB} (m^A_I C^I + i D^A) \chi^i \bar{\lambda}^B \]

\[ - \frac{1}{2\sqrt{2}} \partial_i \bar{f}_{AB} \bar{F}_{mn}^A \chi^i \sigma^{mn} \lambda^B - \frac{1}{2\sqrt{2}} \partial_i \bar{f}_{AB} \bar{F}_{mn}^A \chi^i \sigma^{mn} \bar{\lambda}^B \]

\[ - \frac{1}{4} \bar{F}_i^A \partial_I f_{AB} \lambda^A \lambda^B - \frac{1}{4} \bar{F}_i^A \partial_I f_{AB} \bar{\lambda}^A \bar{\lambda}^B + \frac{1}{8} \chi^i \lambda^J \partial_i \partial_j f_{AB} \lambda^A \lambda^B + \frac{1}{8} \chi^i \bar{\lambda}^j \partial_i \partial_j f_{AB} \bar{\lambda}^A \bar{\lambda}^B, \]

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\(^5\)We thank U. Theis for discussions on this point.

\(^6\)Of course we also need to add kinetic terms for the \( N^i \) but since they play no role here we omit them in the following.
where we defined

$$\tilde{F}^{A}_{mn} \equiv F^{A}_{mn} - m^{A}B^{I}_{mn},$$

and used as the component expansion of $N^{i}$

$$N^{i} = A^{i} + \sqrt{2}\theta \chi^{i} + \theta F^{i},$$

$$f_{AB}(N) = f_{AB}(A) + \sqrt{2}\theta \chi^{i} \partial_{i}f_{AB}(A) + \theta\left(\tilde{F}^{i}_{\partial_{i}}f_{AB}(A) - \frac{1}{2}\chi^{i} \chi^{j} \partial_{i} \partial_{j}f_{AB}(A)\right).$$

(We abbreviate $\partial_{i} \equiv \frac{\partial}{\partial A^{i}}$.) The auxiliary fields $D^{A}$ may be eliminated by their equations of motion

$$D^{A} = \frac{1}{2}(\text{Re } f)^{-1AB}\left((e_{BI} + 2\text{Im } f_{BCM} C^{I})C^{J} - \frac{4}{\sqrt{2}}(\partial_{i}f_{BC} \chi^{i} \chi^{C} - \partial_{i}\bar{f}_{BC} \chi^{i} \bar{\chi}^{C})\right).$$

Inserting (2.22) into (2.19) we obtain

\[
\mathcal{L}_{m} = -\frac{1}{4}\text{Re } f_{AB}\tilde{F}^{A}_{mn}\tilde{F}^{Bmn} + \frac{1}{2}\text{Im } f_{AE\dot{B}m^{C}I}\tilde{F}^{A}_{kl}\tilde{F}^{B_{klmn}} - \frac{1}{16}\tilde{F}^{A}_{klmn}e_{AIB}B^{I}_{kl}(\tilde{F}^{A}_{mn} + F^{A}_{mn}) - V
\]
\[
-\frac{1}{2}(ie_{AI} + 2f_{ABm^{B}I})\eta^{l}A^{l} - \frac{1}{2}(-ie_{AI} + 2\bar{f}_{ABm^{B}I})\bar{\eta}^{l}\bar{A}^{l}
\]
\[
+ \frac{i}{4\sqrt{2}}(\text{Re } f)^{-1AB}\left(\partial_{l}f_{AG} \chi^{l} \chi^{G} - \partial_{l}\bar{f}_{AG} \chi^{l} \bar{\chi}^{G}\right)(e_{BI} + 2\text{Im } f_{BCm^{C}I})C^{I}
\]
\[
+ \frac{1}{16}(\text{Re } f)^{-1AB}\partial_{k}f_{BC}\left(\partial_{l}f_{AG} \chi^{l} \chi^{G} - \partial_{l}\bar{f}_{AG} \chi^{l} \bar{\chi}^{G}\right)\chi^{k}A^{C}
\]
\[
+ \frac{1}{16}(\text{Re } f)^{-1AB}\partial_{k}\bar{f}_{BC}\left(\partial_{l}\bar{f}_{AG} \chi^{l} \bar{\chi}^{G} - \partial_{l}f_{AG} \chi^{l} \chi^{G}\right)\bar{\chi}^{k}A^{C}
\]
\[
- \frac{1}{2\sqrt{2}}\partial_{k}f_{AB}\tilde{F}^{A}_{mn}\chi^{k}m^{n}A^{B} - \frac{1}{2\sqrt{2}}\partial_{k}\bar{f}_{AB}\tilde{F}^{A}_{mn}\bar{\chi}^{k}m^{n}\bar{A}^{B}
\]
\[
- \frac{1}{4}\bar{F}^{k}\partial_{k}f_{AB}A^{4}A^{B} - \frac{1}{4}\bar{F}^{k}\partial_{k}\bar{f}_{AB}\bar{A}^{4}A^{B}
\]
\[
+ \frac{1}{8}\chi^{k}\chi^{l}\partial_{k}f_{AB}A^{4}A^{B} + \frac{1}{8}\chi^{k}\bar{\chi}^{l}\partial_{k}\bar{f}_{AB}\bar{A}^{4}A^{B},
\]

where the scalar potential is given by

$$V = \frac{1}{8}(\text{Re } f)^{-1AB}(e_{AI} + 2\text{Im } f_{ACm^{C}I})(e_{BJ} + 2\text{Im } f_{BDm^{D}J}) + 4\text{Re } f_{ABm^{A}m^{B}J}C^{I}C^{J}.$$  

(2.24)

In order to make the contribution from the $D$-terms manifest we can alternatively write the potential as

$$V = \frac{1}{2}\text{Re } f_{AB}D^{A}D^{B} + \frac{1}{2}\text{Re } f_{ABm^{A}m^{B}J}C^{I}C^{J}$$

(2.25)

for $D^{A} = \frac{1}{2}(\text{Re } f)^{-1AB}(e_{BI} + 2\text{Im } f_{BCm^{C}I})C^{I}$. We see that there is a contribution to the mass terms for the scalars $C^{I}$ which does not arise from eliminating an auxiliary field.

For generic charges $e_{BJ}, m^{D}J$ (i.e. non-zero) the minimum of $V$ is at $C^{I} = 0$. This follows from the fact that $\text{Re } f_{AB}$ is the gauge kinetic function and therefore positive definite. As a consequence both terms in (2.24) are manifestly positive.
To close our discussion of the Lagrangian (2.18) let us explicitly display the mass terms for the $B_{mn}^I$. Using (2.20) we can write

$$L_{B}^{B^2} = -\frac{1}{4}(M^2)_{IJ}B^I_{mn}B^J_{mn} + \frac{1}{8}(M^2_T)_{IJ}\varepsilon^{klmn}B^I_{kl}B^J_{mn},$$

$$(M^2)_{IJ} = \text{Re} f_{AB}m^A_I m^B_J,$$

$$(M^2_T)_{IJ} = \text{Im} f_{AB}m^A_I m^B_J + \frac{1}{2}e_{AIM}^J.$$

(2.26)

As we see the action contains an ordinary mass term $M^2$ as well as a topological mass term $M^2_T$. For $m^A_I = 0$ both mass terms vanish and a massless antisymmetric tensor with a Green-Schwarz coupling of the form $e_{AIM}^J$ is left.

### 3 Dual Formulation

So far we discussed the possible couplings of a set of spinor superfields to Abelian vector and chiral multiplets. In components this led to massive antisymmetric tensors possibly with additional Green-Schwarz couplings. It is well known that theories with antisymmetric tensors have an equivalent dual formulation: a massive antisymmetric tensor is dual to a massive vector while a massless antisymmetric tensor is dual to a scalar. The purpose of this section is to derive the dual of the theories discussed in the previous section. More specifically, we perform a duality transformation in superfields and then expand the dual action in components. As a warm-up we first consider the massless case with non-trivial Green-Schwarz couplings ($m^A_I = 0$, $e_{AI} \neq 0$) and then turn to the general case where also $m^A_I \neq 0$.

#### 3.1 Massless tensors with Green-Schwarz couplings

For $m^A_I = 0$ the action given by (2.18) and (2.9) can be rewritten as

$$L = -\int d^4\theta \left(K(L^I) + e_{AI}L^I V^A\right) + \frac{1}{4} \left(\int d^2\tilde{\theta} f_{AB}W^A W^B + \text{h.c.}\right),$$

(3.1)

where we partially integrated using the definition of $W^A_\alpha$, (2.3) and $d^2\tilde{\theta} = -\frac{1}{4}\tilde{D}^2$. We see that the entire action is expressed in terms of linear multiplets only and no mass term for the antisymmetric tensors is present. The Lagrangian (3.1) can be derived from the following first-order Lagrangian

$$L_{\text{first}} = -\int d^4\theta \left(K(V^0 I) + e_{AI}V^0 I V^A + V^0 I (S_I + \bar{S}_I)\right) + \frac{1}{4} \left(\int d^2\tilde{\theta} f_{AB}W^A W^B + \text{h.c.}\right),$$

(3.2)

where $V^0 I$ denote $n_L$ real vector (but not linear) superfields and $S_I$ are $n_L$ chiral superfields. Eliminating the $S_I$ by their equations of motion we find

$$\bar{D}^2 V^0 I = D^2 V^0 I = 0,$$

(3.3)
where we used that a chiral $S_I$ can always be written in terms of an unconstrained superfield $X_I$ via $S_I = \bar{D}^2 X_I$. From (3.3) we learn that $V^{0I}$ is constrained to be a linear superfield and thus can be identified as

$$V^{0I} = L^I .$$

(3.4)

Inserted back into (3.2) using $\int d^4\theta L^I (S_I + \bar{S}_I) = 0$ we finally arrive at (3.1).

If we eliminate the $V^{0I}$ instead we obtain the dual theory in terms of the chiral multiplets $S_I$. The equation of motion for $V^{0I}$ reads

$$\partial_{V^{0K}} K(V^{0I}) + e_{AK} V^A + S_K + \bar{S}_K = 0 .$$

(3.5)

With the help of (3.5) we can express $V^{0K}$ as a function of $e_{AK} V^A + S_K + \bar{S}_K$ and possibly of the other $V^{0I}, I \neq K$. Let us denote this function by $h^K$, i.e.

$$V^{0K} \equiv h^K (V^{0I}, e_{AK} V^A + S_K + \bar{S}_K) .$$

(3.6)

The precise relation will of course depend on the particular form of $K(V^{0I})$. We may rewrite now $K$ in terms of $h^K$ and replace it by its Legendre-transform $\hat{K} - \hat{K}(e_{AI} V^A + S_I + \bar{S}_I) = K(h^J) + (e_{AI} V^A + S_I + \bar{S}_I) h^I$,

(3.7)

which, due to (3.3), is a function of $e_{AK} V^A + S_K + \bar{S}_K$. Inserted into (3.2) we finally arrive at

$$L = \int d^4\theta (\hat{K}(e_{AI} V^A + (S_I + \bar{S}_I)) + \frac{1}{4} \left( \int d^2\theta f_{AB} W^A W^B + \text{h.c.} \right) .$$

(3.8)

$L$ is the dual Lagrangian of (3.1) which is expressed in terms of $n_V$ vector multiplets $V^A$ and $n_L$ chiral multiplets $S_I$.

In the original formulation given in (3.1) the gauge invariance of the vector multiplets

$$V^A \rightarrow V^A + \Sigma^A + \bar{\Sigma}^A , \quad \bar{D}_a \Sigma^A = 0 ,$$

(3.9)

is manifest since the entire action is expressed in terms of the gauge invariant field strength $W^A$. In the dual formulation (3.9) has to be accompanied by a shift of the chiral multiplets

$$S_I \rightarrow S_I - e_{AI} \Sigma^A .$$

(3.10)

We see that the $S_I$ play the role of Goldstone supermultiplets which are necessary in order to maintain the $U(1)$ gauge invariance. Thus the first term in (3.8) corresponds to a mass term for the vector fields while the second term is the standard kinetic term. In order to see this more explicitly let us expand the Lagrangian (3.8) in components. We take $V^A$ in a Wess-Zumino gauge and expand accordingly

$$V^A = -\theta \sigma^m \bar{\nu}_m^A + i \theta \theta \bar{\lambda}^A - i \bar{\theta} \theta \lambda^A + \frac{1}{2} \theta \theta \bar{\theta} \bar{\theta} D^A ,$$

$$S_I = \frac{1}{2} E_I + \sqrt{2} \theta \psi_I + \theta \theta F_I .$$

(3.11)
In this case we start from the first-order Lagrangian
\[ L = -\frac{1}{4} \text{Re} f_{AB} F^A_{mn} F^B_{mn} + \frac{1}{8} \text{Im} f_{AB} \epsilon^{mnpq} F^A_{mn} F^B_{pq} + \frac{1}{2} \text{Re} f_{AB} D^A D^B \]
\[ -\frac{i}{2} \left( f_{AB} \sigma^m \partial_m \lambda^B - \bar{f}_{AB} \partial_m \lambda^A \sigma^m \bar{\lambda}^B \right) + \frac{i}{2\sqrt{2}} \partial_i \bar{f}_{AB} D^A \bar{\chi}^i \lambda^B - \frac{i}{2\sqrt{2}} \partial_i \bar{f}_{AB} D^A \bar{\chi}^i \bar{\lambda}^B \]
\[ -\frac{1}{4} F_i^i \bar{f}_{AB} D^A \lambda^B - \frac{1}{4} F^i \partial_i \bar{f}_{AB} \lambda^A \bar{\lambda}^B \]
\[ + \frac{1}{8} \chi^i \chi^j \partial_i \partial_j \bar{f}_{AB} \lambda^A \lambda^B + \frac{1}{8} \chi^i \chi^j \partial_i \partial_j \bar{f}_{AB} \bar{\lambda}^A \bar{\lambda}^B \]
\[ + \frac{1}{2} K_J e_{AI} D^A + \frac{1}{4} K_{JKL} \psi^I \bar{\psi}^J \bar{\psi}^K \bar{\psi}^L \]
\[ + \bar{K}_{IJ} \left( - \frac{1}{2} \partial^m (\text{Re} E_I) \partial_m (\text{Re} E_J) - \frac{1}{2} (\partial^m (\text{Im} E_I) + e_{AI} \epsilon^{Amn}) (\partial^m (\text{Im} E_J) + e_{BJ} \epsilon^{Bmn}) \right) \]
\[ + \frac{i}{2} e_{AI} (\psi_J \partial^A \bar{\psi}^I - \chi_i \lambda^A) - \frac{i}{2} (\chi_i \sigma^m \partial_m \bar{\psi}^I + \psi_J \sigma^m \partial_m \psi_I) + F_I \bar{F}_J \}
\[ + \frac{1}{2} K_{JKL} \left\{ - \psi_I \sigma^m \bar{\psi}^J \epsilon^{Amn} + \partial^m (\text{Im} E_K) \right\} - (\psi_I \psi_J) F_K - (\bar{\psi}_I \bar{\psi}_J) F_K \right\} , \quad (3.12) \]
where we abbreviate \( \bar{K}_I = \partial_{\text{Re} E_I} \bar{K}_I \). As promised we see that the real scalars \( (\text{Im} E_K) \) play the role of \( n_L \) Goldstone bosons which render the linear combinations \( e_{AI} \epsilon^{Amn} \) of the \( n_L \) vector fields \( v^A_m \) massive.

Eliminating the auxiliary fields \( F_I \) and \( D^A \) by their equations of motion we arrive at the following bosonic action
\[ L_b = -\frac{1}{4} \text{Re} f_{AB} F^A_{mn} F^B_{mn} + \frac{1}{8} \text{Im} f_{AB} \epsilon^{mnpq} F^A_{mn} F^B_{pq} \]
\[ - \frac{1}{4} \bar{K}_{IJ} \left( \partial^m (\text{Re} E_I) \partial_m (\text{Re} E_J) + e_{AI} e_{BJ} \epsilon^{Amn} v^B_m \right) - V , \quad (3.13) \]
where we have chosen the unitary gauge and absorbed \( \text{Im} E_K \) into a redefinition of \( v^A_m \). The scalar potential is of the standard \( N = 1 \) form and given by
\[ V = \frac{1}{2} \text{Re} f_{AB} D^A D^B = \frac{1}{8} (\text{Re} f)^{-1} e^{CD} e_{CIE} e_{DJ} \bar{K}_I \bar{K}_J . \quad (3.14) \]
This potentials agrees with the one given in (2.25) for \( m^A_I = 0 \) if one also identifies \( \bar{K}_I = -C^I \). For this substitution also the kinetic terms of the scalars \( C^I \) and (Re \( E_I \)) agree. Indeed starting from \( -\frac{1}{4} \bar{K}_{IJ} (\partial_m C^J) (\partial^m C^I) \) and using the above identification we arrive at \( -\frac{1}{4} \bar{K}_{IJ} (\partial^m \text{Re} E_I) (\partial_m \text{Re} E_J) \), taking into account that due to (3.7) we have \( \partial_{\text{Re} E_I} \bar{K}_J = -\delta^I_J \).

### 3.2 Massive antisymmetric tensors

Let us now turn on the couplings \( m^A_I \) and repeat the analysis of the previous section. In this case we start from the first-order Lagrangian
\[ L_{\text{first}} = \int d^4 \theta \left\{ \hat{K} (\bar{V}^I - \frac{1}{2} \bar{V}^I (D^a \Phi^I - D_a \bar{\Phi}^I)) \right\} + L_m \quad (3.15) \]
where \( L_m \) is given in (2.18). \( \hat{K} \) is a real function of the vector multiplets \( \bar{V}^I \) which will turn out to be the Legendre transform of \( K \).
Let us first show that from (3.15) one can derive the Lagrangian for \( n_L \) massive linear multiplets as given by the sum of (2.18) and (2.23). To do so we vary (3.15) with respect to \( \tilde{V}^J \) and obtain
\[
\partial_{\tilde{V}^J} \hat{K}(\tilde{V}^I) = \frac{1}{2} (D^a \Phi^I_\alpha + \bar{D}_\alpha \Phi^J_\dot{\alpha}) = L^J .
\]
(3.16)
For appropriate \( \hat{K} \) (3.16) may be solved giving \( \tilde{V}^J \) as a function of \( L^J \) and \( \tilde{V}^I, I \neq J \). We shall denote this function by \( h^K = h^K(L^K, \tilde{V}^I) \equiv \tilde{V}^K \). As in the massless case we can express \( \tilde{K} \) in terms of the \( h^K \). Together with (3.16) this leads us from (3.15) to
\[
\mathcal{L} = \int d^4 \theta \left\{ \hat{K}(h^K) - h^I L^I \right\} + \mathcal{L}_m .
\]
(3.17)
Due to (3.16) the expression \(-K(L^I) := \hat{K}(h^K) - h^I L^I \) is the Legendre transform of \( \hat{K}(h^K) \) with respect to all \( h^I \), i.e. a function depending only on the \( L^I \). Substituting \( K(L^I) \) into (3.17) we have arrived at the Lagrangian for \( n_L \) massive linear multiplets as stated above.

Alternatively we can eliminate the \( \Phi^I_\alpha \) multiplets and this yields the desired dual action. To do so let us first rewrite (3.15) as
\[
\mathcal{L}_{\text{first}} = \int d^4 \theta \hat{K}(\tilde{V}^I) + \frac{1}{4} \int d^2 \theta f_{AB} W^A W^B + \text{h.c.}
\]
(3.18)
\[
\quad + \int d^2 \theta \left\{ \Phi^I \left( \frac{1}{2} \tilde{W}^I + \frac{1}{2} \varepsilon_{AI} W^A - i f_{AB} m^B W^A \right) - \mu^2_{IJ} \Phi^I \Phi^J \right\} + \text{h.c.},
\]
where \( \tilde{W}^J = -\frac{1}{4} \bar{D}^2 D \tilde{V}^J \) is the field strength of \( \tilde{V}^J \) and we have performed a partial integration. We also introduced the mass matrix
\[
\mu^2_{IJ} := (M^2)_{IJ} + (M^2_T)_{IJ} ,
\]
(3.19)
with \( M^2 \) and \( M^2_T \) being defined in (2.26). The equation of motion for \( \Phi^I_\alpha \) can be obtained from (3.18) by using again \( \Phi^I_\alpha = \bar{D}^2 X^I_\alpha \). Demanding \( \mu^2_{IJ} \) to be invertible we arrive at
\[
\Phi^I_\alpha = \frac{1}{2} (\mu^2)^{-1/(2)} (\tilde{W}^K_{\hat{A}} + \frac{1}{2} \varepsilon_{AK} W^A_{\hat{A}} - i f_{AB} m^B_{\hat{A}} W^A_{\hat{A}}) .
\]
(3.20)
Inserting back into (3.15) we obtain
\[
\mathcal{L} = \int d^4 \theta \hat{K}(\tilde{V}^I) + \frac{1}{4} \int d^2 \theta f_{\hat{A} \hat{B}} \tilde{W}^\hat{A} \tilde{W}^\hat{B} + \text{h.c.},
\]
(3.21)
where we have introduced \( \tilde{W}^\hat{A} := \left( -\frac{1}{2} \tilde{W}^I_{\hat{A}}, W^A \right) \). So the index \( \hat{A} \) takes values \( \hat{A} = (I, A) = (1, \ldots, n_L, n_L + 1, \ldots, n_L + n_V) \). Furthermore the \( (n_V + n_L) \times (n_V + n_L) \) - dimensional gauge coupling matrix \( \tilde{f}_{\hat{A} \hat{B}} \) is given by
\[
\tilde{f}_{\hat{A} \hat{B}} = \left( \begin{array}{cc} \hat{f}_{IJ} & \hat{f}_{IB} \\ \hat{f}_{AJ} & \hat{f}_{AB} \end{array} \right) ,
\]
(3.22)
plets. Thus the Lagrangian (3.21) appears to depend on $n$ while $\text{Im } \hat{V}$ can be seen from the fact that the gauge coupling matrix $\text{Re } \hat{V}$ is zero. Inserted into (3.21) we arrive at

$$\hat{f}_{IB} m^B_K = i\delta_{IK}, \quad \hat{f}_{AB} m^B_K = -i e_{AK}. \quad (3.24)$$

This shows that the $n_L$ vectors $(0, \ldots, 0, m^B_K)$ are eigenvectors of $\text{Re } \hat{f}_{AB}$ with eigenvalue zero.

In order to display the physical components of $\hat{V}$ we decompose it into a vector multiplet $V^0I$ in the WZ-gauge and the real part of a chiral superfield $S^I$

$$\hat{V}^I := V^0I + S^I + \bar{S}^I. \quad (3.25)$$

The component form of (3.21) can then be obtained by inserting the Wess-Zumino gauge (3.11) for $V^0I$ while for the chiral multiplets $S^I$ we use

$$S_I = \frac{1}{2} A_I + \sqrt{2} \bar{\theta} \psi_I + \frac{i}{2} \theta \bar{\sigma}^m \partial_m A_I + \theta \bar{\theta} F_I - \frac{i}{4} \theta \partial_m \psi_I \bar{\sigma}^m \bar{\theta} + \frac{i}{4} \theta \bar{\theta} \bar{\theta} \square \frac{1}{2} A_I. \quad (3.26)$$

Inserted into (3.21) we arrive at

$$\mathcal{L} = -\frac{1}{4} \text{Re } \hat{f}_{AB} F^A_{mn} F^B_{mn} + \frac{1}{8} \text{Im } \hat{f}_{AB} \varepsilon_{klmn} F^A_{kl} F^B_{mn} + \frac{1}{2} \text{Re } \hat{f}_{AB} D^A D^B + \frac{1}{2} \hat{K}_I D^0I - \frac{1}{4} \hat{K}_{IJ} V^0I_m v^0_m - \frac{1}{4} \hat{K}_{IJ} \bar{\sigma}^m \text{Re } A_I \partial_m \text{Re } A_J - \hat{K}_{IJ} F_I \bar{F}_J - i \left( \hat{f}_{A\tilde{B}} \bar{\lambda}^{A'} \bar{\sigma}^k \partial_k \bar{\lambda}^{B'} + \bar{\hat{f}}_{A\tilde{B}} \bar{\lambda}^{A'} \bar{\sigma}^k \partial_k \lambda^{B'} \right) + \frac{i}{2} \hat{K}_{IJ} \left\{ i \bar{\partial} \bar{\psi}_I \lambda^0I - \bar{\psi}_J \lambda^0I \right\} - i \left( \psi_J \bar{\sigma}^m \partial_m \bar{\psi}_I + \bar{\psi}_J \bar{\sigma}^m \partial_m \psi_I \right) + \frac{1}{4} \hat{K}_{IJK} \left\{ -\psi_I \sigma^m \bar{\psi}_J v^0_k - (\psi_I \sigma^m \bar{\psi}_J) \bar{F}_K - (\bar{\psi}_I \bar{\psi}_J) F_K \right\} + \frac{1}{4} \hat{K}_{IJKL} \bar{\psi}_I \psi_J \bar{\psi}_K \bar{\psi}_L + \ldots, \quad (3.27)$$

where $\partial_{\text{Re } A_I} \hat{K}$ was used and terms proportional to $\partial_\alpha \hat{f}_{AB}$ have been neglected.

The next step is to eliminate the auxiliary fields from the Lagrangian. The equations of motion for $F_I$ can be determined in a straightforward manner to be

$$F_I = \frac{1}{2} \hat{K}^{-1}_{IL} \hat{K}_{LJK} \bar{\psi}_J \psi_K. \quad (3.28)$$

For $D^A$ however the situation is more difficult since some of the vector multiplets are unphysical. In order to remove the unphysical degrees of freedom we fix the gauge invariance of (3.20).
To this aim let us rewrite \((3.20)\) in the following way
\[
\Phi_I^I = -\frac{1}{2} R_{IK} \left\{ -\frac{1}{2} \tilde{W}^K + \left( \frac{1}{2} e_{AK} + \text{Im} \, f_{AB} m^B_K \right) W^A + R_{KL}^{-1} I_{LJ} \text{Re} \, f_{AB} m^B_J W^A \right\}
+ \frac{1}{2} I_{IK} \left\{ \frac{1}{2} \tilde{W}^K + \left( \frac{1}{2} e_{AK} + \text{Im} \, f_{AB} m^B_K \right) W^A - I_{KL}^{-1} R_{LJ} \text{Re} \, f_{AB} m^B_J W^A \right\},
\]
where we divided the coupling matrices of \((3.20)\) into their real and imaginary parts and abbreviated
\[
R_{IK} = [\text{Re} \left( (\mu^2)^{-1} \right)]_{IK} \quad \text{and} \quad I_{IK} = [\text{Im} \left( (\mu^2)^{-1} \right)]_{IK}. \tag{3.30}
\]
Going to the WZ-gauge \((2.7)\) for \(\Phi^I\) we see that the \(\theta\)-component of the imaginary part of the right hand side of \((3.29)\) has to vanish. This implies
\[
D^{0K} = - \left( e_{AK} + 2 \text{Im} \, f_{AB} m^B_K \right) D^A + 2 I_{KL}^{-1} R_{LJ} \text{Re} \, f_{AB} m^B_J D^A + \ldots, \tag{3.31}
\]
where we have omitted the fermionic terms. We can now use the constraint \((3.31)\) to eliminate the \(D^{0K}\) from the Lagrangian. Let us concentrate on the bosonic terms which we read off from \((3.27)\) to be
\[
V = -\frac{1}{2} \text{Re} \, f_{AB} \tilde{D}^A \tilde{D}^B - \frac{1}{2} \hat{K}_I D^{0I}. \tag{3.32}
\]
Inserting \((3.23), (3.31)\) and using \(D^A = (-\frac{1}{2} D^{0K}, D^A)\) we obtain the following equation of motion for \(D^A\)
\[
\{ R_{IJ} I_{JK} I_{KN} R_{KL} R_{NS} f_{AC} R_{CS} f_{BD} m^C_L m^D_S + R_{IJ} R_{AC} R_{CS} f_{BD} m^C_I m^D_J + \text{Re} \, f_{AB} \} D^B
= -\hat{K}_K \left\{ - \left( \frac{1}{2} e_{AK} + \text{Im} \, f_{AC} m^C_K \right) + I_{KL}^{-1} R_{LJ} \text{Re} \, f_{AC} m^C_J \right\}. \tag{3.33}
\]
The inverse of the matrix multiplying \(D^B\) is found to be
\[
(\text{Re} \, f)^{-1E_A} - R_{TU} m^E_T m^A_U, \tag{3.34}
\]
which implies
\[
D^E = \hat{K}_K (\text{Re} \, f)^{-1E_A} \left( \frac{1}{2} e_{AK} + \text{Im} \, f_{AC} m^C_K \right). \tag{3.35}
\]
Inserting \((3.28)\) and \((3.35)\) back into \((3.27)\) we arrive at
\[
\mathcal{L} = -\frac{1}{4} \text{Re} \, f_{AB} F_{AB} \hat{F}_{AB} + \frac{1}{4} \text{Im} \, f_{AB} \varepsilon^{klmn} F_{kl} F_{mn} - \frac{1}{4} \hat{K}_{IJ} \psi^0_I \bar{\psi}^0_J
- \frac{1}{4} \hat{K}_{IJ} \theta^0_I \partial_m (\text{Re} \, A_I) \partial_m (\text{Re} \, A_J) - \frac{i}{2} \left( f_{AB} \lambda^A \bar{\lambda}^B + f_{AB} \bar{\lambda}^A \lambda^B \right)
+ \frac{1}{2} \hat{K}_{IJ} \left\{ i \sqrt{2} (\psi_I \lambda^0_J - \bar{\psi}_I \bar{\lambda}^0_J) \right. \left. - i (\psi_J \sigma^m \partial_m \bar{\psi}_I + \bar{\psi}_J \bar{\sigma}^m \partial_m \psi_I) \right\}
- \frac{1}{2} \hat{K}_{IJ} \psi_I \bar{\psi}_J \lambda^0_K
+ \frac{1}{4} \left( \hat{K}_{IJKL} - \hat{K}_{IKMS} \hat{K}_{MNS} \hat{K}_{NLS} \right) \psi_I \psi_J \bar{\psi}_K \bar{\psi}_L - V + \ldots, \tag{3.36}
\]
where the scalar potential is given by
\[
V = \frac{1}{8} \left\{ (e_{AI} + 2 \text{Im} \, f_{AC} m^C_I) \text{Re} \, f^{-1AB} (e_{BJ} + 2 \text{Im} \, f_{BD} m^D_J) + 4 \text{Re} \, f_{AB} m^A_I m^B_J \right\} \hat{K}_I \hat{K}_J. \tag{3.37}
\]
This potential indeed coincides with \((2.23)\) for \(C^I = \hat{K}^I\). For this identification also the kinetic terms agree which is expressing simply the fact that \(K\) and \(\hat{K}\) are related to each other by a Legendre transformation. Thus \((3.36)\) is the desired dual action of \((2.23)\).
4 Conclusion

Let us summarize our results. We proposed an $N = 1$ superfield action for $n_L$ chiral spinor superfields coupled to $n_V$ vector and $n_C$ chiral multiplets. The component form of this action was given and shown to contain gauge invariant mass terms for $n_L$ antisymmetric tensors. In addition the action also features Green-Schwarz couplings to the $n_V$ vector fields. Supersymmetry gives a mass to the supersymmetric partners $C^I$ of the antisymmetric tensors with the peculiarity that these mass terms do not arise from eliminating an auxiliary field. Indeed the supersymmetry transformation laws show that any Lorentz invariant ground state of the spinor superfield preserves supersymmetry. Instead the supersymmetry transformations of the vector multiplets are modified and a vacuum expectation value of the scalars $C^I$ can break supersymmetry by generating a non-vanishing gaugino transformation.

We also constructed the dual action in terms of $n_L$ massive and $n_V - n_L$ massless vector multiplets by explicitly performing the duality transformations in superspace. We gave the component form of the dual action and showed that the scalar potentials in both formulations coincide.

For one chiral spinor superfield the action agrees with the action given in [5] which also appeared in Kaluza-Klein reduction of type IIB string theory compactified on Calabi-Yau orientifolds [10].

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A Modified SUSY-transformations of the chiral spinor superfield

In this appendix we derive the supersymmetry transformation laws of a chiral spinor superfield in the WZ-gauge. The main motivation for this exercise is to identify the order parameters for spontaneous supersymmetry breaking. For simplicity we perform this analysis for a single $\Phi_\alpha$.

The general supersymmetry transformation of $\Phi_\alpha$ reads

$$\Phi_\alpha \to \Phi'_\alpha = \Phi_\alpha + \delta \xi \Phi_\alpha = \Phi_\alpha + (\xi Q + \bar{\xi} \bar{Q}) \Phi_\alpha ,$$

(A.1)

where $Q$ and $\bar{Q}$ are the supersymmetry generators. In terms of the component expansion
In the WZ-gauge we choose 
\[ \chi \]
\[ \chi \]
We see that 
\[ \text{the vector multiplet which without couplings to a spinor superfield reads} \]
\[ \text{charged multiplets also change. For the case at hand these are the transformations of} \]
\[ \text{We see that in a Lorentz invariant ground state supersymmetry cannot be broken by} \]
\[ \text{(2.4) we have} \]
\[ \delta \xi \chi = - \xi_\gamma \left( \frac{1}{2} \delta_\alpha \gamma (C + iE) + \frac{1}{4} (\sigma^m \sigma^n)_{\alpha} \gamma B_{mn} \right), \]
\[ \delta \xi C = \xi^\alpha \eta_\alpha + \bar{\xi}_\alpha \bar{\eta}^\alpha, \]
\[ \delta \xi E = - i \xi_\alpha \eta_\alpha + i \bar{\xi}_\alpha \bar{\eta}^\alpha + 2 \left( \xi^\beta \sigma^m_{\beta \alpha} \partial_m \chi^\alpha - \bar{\xi}_\alpha \sigma^{\bar{m} \beta} \partial_m \chi_\beta \right), \]
\[ \delta \xi B^{mn} = 2 \eta^\alpha (\sigma_{mn})_{\alpha} \beta \xi_\beta + 2 \bar{\eta}_\alpha (\sigma_{mn})^\alpha \beta \bar{\xi}^\beta + 2i (\xi^\beta \sigma^{m \bar{m}_\alpha} \partial^m \chi^\alpha - \bar{\xi}_\alpha \sigma^{n \bar{m} \beta} \partial^m \chi_\beta), \]
\[ \delta \xi \eta_\alpha = i \sigma_{\alpha \alpha} \xi_\alpha \partial_k C - \frac{1}{2} \varepsilon^{kmnr} \sigma_{r\alpha \alpha} \xi_k \partial_k B_{mn}. \]

In the WZ-gauge we choose \( \chi_\alpha = 0 \) and \( E = 0 \) and thus (A.2) becomes
\[ \delta \xi_{WZ} \chi_\alpha = - \xi_\gamma \left( \frac{1}{2} \delta_\alpha \gamma C + \frac{1}{4} (\sigma^m \sigma^n)_{\alpha} \gamma B_{mn} \right), \]
\[ \delta \xi_{WZ} C = \xi \eta + \bar{\xi} \bar{\eta}, \]
\[ \delta \xi_{WZ} E = - i \xi \eta + i \bar{\xi} \bar{\eta}, \]
\[ \delta \xi_{WZ} B^{mn} = 2 \eta \sigma^{mn} \xi + 2 \bar{\eta} \sigma^{mn} \bar{\xi}, \]
\[ \delta \xi_{WZ} \eta_\alpha = i (\sigma^k \xi_\alpha) \partial_k C - \frac{1}{2} \varepsilon^{kmnr} (\sigma_r \xi)_\alpha \partial_k B_{mn}. \]

We see that \( \chi_\alpha \) and \( E \) do not transform to zero and therefore one needs a compensating gauge transformation to stay in the WZ-gauge. These are given in (2.5) and (2.6) and so we are led to choose
\[ \chi_\alpha^e = - \xi_\gamma \left( \frac{1}{2} \delta_\alpha \gamma C + \frac{1}{4} (\sigma^m \sigma^n)_{\alpha} \gamma B_{mn} \right), \]
\[ D^e = i \xi \eta - i \bar{\xi} \bar{\eta}. \]

This ensures \( (\delta \xi_{WZ} + \delta_{\text{gauge}}) \chi = 0 = (\delta \xi_{WZ} + \delta_{\text{gauge}}) E \) and modifies the transformations of the physical fields according to
\[ (\delta \xi_{WZ} + \delta_{\text{gauge}}) C = \xi \eta + \bar{\xi} \bar{\eta}, \]
\[ (\delta \xi_{WZ} + \delta_{\text{gauge}}) B^{mn} = 2 \eta \sigma^{mn} \xi + 2 \bar{\eta} \sigma^{mn} \bar{\xi} + \partial^m \Lambda^e_n - \partial^n \Lambda^em, \]
\[ (\delta \xi_{WZ} + \delta_{\text{gauge}}) \eta_\alpha = i (\sigma^k \xi_\alpha) \partial_k C - \frac{1}{2} \varepsilon^{kmnr} (\sigma_r \xi)_\alpha \partial_k B_{mn}. \]

We see that in a Lorentz invariant ground state supersymmetry cannot be broken by any of these transformations. However, in a WZ-gauge the transformation laws of the charged multiplets also change. For the case at hand these are the transformations of the vector multiplet which without couplings to a spinor superfield read
\[ \delta \xi F_{mn} = i \left[ (\xi \sigma^n \partial_m \lambda + \bar{\xi} \sigma^n \partial_m \lambda) - (\xi \sigma^m \partial_n \lambda + \bar{\xi} \sigma^m \partial_n \lambda) \right], \]
\[ \delta \xi \lambda_\alpha = i \xi_\alpha D + (\sigma^{mn})_{\alpha} F_{mn}, \]
\[ \delta \xi D = \bar{\xi} \sigma^m \partial_m \lambda - \xi \sigma^m \partial_m \lambda. \]
Gauge invariance of the couplings to the spinor superfield forces the gauge fields to transform according to (2.14). Thus with the special choice (A.4) we obtain for the combined supersymmetry and gauge transformations

\[
(\delta_\xi + \delta_{\text{gauge}}) \lambda_\alpha = - m_\xi \gamma (\delta_\alpha \gamma C + \frac{1}{2}(\sigma^m \bar{\sigma}^n)_{\alpha \gamma} B_{mn}) + i \xi_\alpha D + (\sigma^m \xi)_\alpha F_{mn},
\]

\[
(\delta_\xi + \delta_{\text{gauge}}) D = m(i \xi \eta - i \bar{\xi} \bar{\eta}) + \bar{\xi} \bar{\sigma}^m \partial_m \lambda - \xi \sigma^m \partial_m \bar{\lambda},
\]

\[
(\delta_\xi + \delta_{\text{gauge}}) F_{mn} = i \left[ (\xi \sigma^n \partial_m \bar{\lambda} + \bar{\xi} \bar{\sigma}^n \partial_m \lambda) - (\xi \sigma^m \partial_n \bar{\lambda} + \bar{\xi} \bar{\sigma}^m \partial_n \lambda) \right] + m F^e_{mn}.
\]

As one can see supersymmetry can be broken in (A.7) if either \(C\) or \(D\) acquire a vacuum expectation value that is different than zero.

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