Entropy and temperature from black-hole/near-horizon-CFT duality

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Abstract
We construct a two-dimensional CFT, in the form of a Liouville theory, in the near-horizon limit of four- and three-dimensional black holes. The near-horizon CFT assumes two-dimensional black hole solutions first introduced by Christensen and Fulling (1977 Phys. Rev. D 15 2088–104) and expanded to a greater class of black holes via Robinson and Wilczek (2005 Phys. Rev. Lett. 95 011303). The two-dimensional black holes admit a $\text{Diff}(S^1)$ subalgebra, which upon quantization in the horizon limit becomes Virasoro with calculable central charge. This charge and the lowest Virasoro eigen-mode reproduce the correct Bekenstein–Hawking entropy of the four- and three-dimensional black holes via the known Cardy formula (Blöte et al 1986 Phys. Rev. Lett. 56 742; Cardy 1986 Nucl. Phys. B 270 186). Furthermore, the two-dimensional CFT’s energy–momentum tensor is anomalous. However, in the horizon limit the energy–momentum tensor becomes holomorphic equaling the Hawking flux of the four- and three-dimensional black holes. This encoding of both entropy and temperature provides a uniformity in the calculation of black hole thermodynamic and statistical quantities for the non-local effective action approach.

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1. Introduction

The universally accepted thermodynamic and statistical properties of black holes have provided a unique insight and test for theories of quantum gravity. Though a fully formulated quantum field theory of gravity is lacking, a multitude of candidates exists, with string theory and loop quantum gravity leading in popularity. Despite a variety of theories, any serious candidate should reproduce the correct form of the Bekenstein–Hawking entropy [5]

$$S_{\text{BH}} = \frac{A}{4\hbar G}$$  (1)
and the Hawking temperature \[6, 7\]

\[
T_H = \frac{\hbar \kappa}{2\pi},
\]

(2)

where \(A^1\) is the horizon area and \(\kappa\) is the surface gravity of the black hole. The fact that these quantities depend on both \(\hbar\) and \(G\) is evident of their quantum gravitational origin. For a comprehensive review of approaches to quantum gravity, see \[8–10\].

Effective theories have had much success in reproducing (1) and (2) via analysis of anomalous energy–momentum tensors of non-local effective actions \[11\] and holographic one- and two-dimensional conformal field theories \[9\], respectively. Yet, these methods remain in separate camps and a lack of uniformity in the derivation of both temperature and entropy in one simultaneous effective theory is missing.

We will briefly review some of the pioneering work inspiring this paper.

1.1. Effective action and Hawking temperature

Analysis of an anomalous energy–momentum tensor to compute (2) was first carried out by Christensen and Fulling in \[1\]. Considering the most general solution to the conservation equation

\[
\nabla_\mu T^\mu_\nu = 0,
\]

(3)

they found that by restricting to the \(r-t\) plane of a free scalar field in Schwarzschild geometry the energy–momentum tensor exhibits a trace anomaly leading to the result

\[
\langle T^r_t \rangle = \frac{1}{768\pi G^2 M^2} = \frac{\pi}{12} T_H^2,
\]

(4)

which is exactly the luminosity (Hawking flux, Hawking radiation) of the four-dimensional black hole in units \(\hbar = 1\).

A similar approach was studied in \[12\] where the authors determined the \(s\)-wave contribution of a scalar field to the four-dimensional effective action for an arbitrary spherically symmetric gravitational field. Applying their results to a Schwarzschild black hole, the authors showed the energy–momentum tensor of the non-local effective action to contain the Hawking flux. Other closely related approaches for scalar fields and two-dimensional theories include \[13–17\].

Another method for computing Hawking radiation first introduced by Robinson and Wilczek \[2\] considers a quantum chiral-scalar field theory of two dimensions in the near-horizon limit of a static four-dimensional black hole. A two-dimensional chiral field theory is known to exhibit a gravitational anomaly of the form

\[
\nabla_\mu T^\mu_\nu = \frac{1}{96\pi \sqrt{-g^{(2)}}} \epsilon^{\rho\gamma\rho\lambda} \partial_\rho \Gamma^{(2)}_{\gamma\nu\lambda},
\]

(5)

where \(g^{(2)}_{\mu\nu}\) contains the leftover components of the four-dimensional metric which are not redshifted away in the near-horizon limit of the functional

\[
S_{\text{free scalar}} = \frac{1}{2} \int d^4x \sqrt{-g} \nabla_\mu \psi \nabla^\mu \psi.
\]

(6)

Robinson and Wilczek showed in the near-horizon regime of a Schwarzschild black hole that to ensure a unitary quantum field theory the black hole should radiate as a thermal bath of

\[1\] \(A = \int d^3x \sqrt{|g|}\) where \(g_{ij}\) is obtained by setting \(r = r_s\) for constant time. In the case for Kerr \(\sqrt{|g|} = (r^2 + J^2) \sin \theta\), which deviates from the standard form.

\[2\]
temperature equaling $T_H$. In other words, quantum gravitational effects in the near-horizon regime cancel the chiral/gravitational anomaly [18]. This method has been expanded to include gauge/gravitational anomalies and covariant anomalies [19–22] and has successfully reproduced the correct black hole temperature for charged-rotating black holes [23, 24], dS/AdS black holes [25, 26], rotating dS/AdS black holes [27, 28], black rings and black strings [29, 30], three-dimensional black holes [31, 32] and black holes of non-spherical topologies [33]. This method provides a fundamental reason for black hole thermodynamics based on symmetry principles of a near-horizon quantum field theory. It also provides a two-dimensional analog for higher dimensional black holes besides the Schwarzschild case. This is a rather useful fact since the Ricci tensor in two dimensions is always Einstein with the proportionality constant $c_1$.

\[ (g^{(2)\mu\nu} R^{(2)}_{\mu\nu}) \]

Thus classically, in two dimensions, there are no general relativistic dynamics and any gravitational effects that are present must have quantum gravitational origin with the semi-classical metric $g^{(2)}_{\mu\nu}$.

### 1.2. Holographic CFT and entropy

Applying the seminal work of Brown and Henneaux [34], Strominger showed that the Bekenstein–Hawking entropy for AdS$_3$ black holes arises naturally from microscopic states of an asymptotic CFT via Cardy’s formula [35]. This work has been generalized to a class of rotating and dS/AdS black holes in various dimensions in both the near-horizon regime and asymptotic infinity by Carlip and others [36–41]. Recently, Barnes, Vaman and Wu generalized the method to include four-dimensional charged non-rotating black holes [42] via uplifting the four-dimensional black holes to six-dimensional pure gravitational solutions. Though slightly different in their approaches, the general idea is as follows: there exists a set of diffeomorphism preserving vector fields at either asymptotic infinity or in the near horizon regime of the metric or diffeomorphisms preserving boundary conditions at spacial infinity as in [40]. This set of diffeomorphisms includes a Diff(S$^3$) subalgebra parametrized by some discrete set of vector fields $\xi_n$ for all $n \in \mathbb{Z}$ such that

\[ i\{\xi_m, \xi_n\} = (m - n)\xi_{m+n}. \quad (7) \]

Upon quantization, Brown and Henneaux showed

\[ [\xi_m, \xi_n] = (m - n)\xi_{m+n} + \frac{c}{12} m(m^2 - 1)\delta_{m+n,0} \quad (8) \]

where $c$ is a calculable central extension. The Bekenstein–Hawking entropy is then given by Cardy’s formula [3, 4] in terms of $c$ and $\Delta_0$, where $\Delta_0$ is the eigen-mode of $\xi_0$, via

\[ S = 2\pi \sqrt{\frac{c \cdot \Delta_0}{6}}. \quad (9) \]

Applying the above outline to the two-dimensional dilaton black hole

\[ ds^2 = g^{D\mu\nu} dx^\mu dx^\nu = - \left[ (\lambda x)^2 - \frac{2M}{\lambda} (\lambda x)^3 \right] dt^2 + \left[ (\lambda x)^2 - \frac{2M}{\lambda} (\lambda x)^3 \right]^{-1} dx^2, \quad (10) \]

Cadoni computed the following central extension and zero mode [43]:

\[ c = 48 \frac{M^2}{\lambda^2} \quad \text{and} \quad \Delta_0 = \frac{M^2}{2\lambda^2}. \quad (11) \]

\[ \text{In two dimensions the curvature of any Riemannian manifold is completely characterized by its scalar variant. This is because any 2-form has only one independent component. Thus for any Riemannian–Levi-Civita connection 2-form $\omega_{\mu\nu}, d\omega_{12} = Kvol^2$, where $K = \frac{1}{2} e^{\phi_{(2)}} R^{(2)}$ is the Gauss curvature.} \]

\[ \text{The algebra of the asymptotic symmetry group of three-dimensional gravity is Virasoro with calculable central charge.} \]
Cadoni further showed that by conformally mapping (10) to the s-wave sector of the Schwarzschild metric
\[ g^{(2)}_{\mu\nu} = 2\phi g^D_{\mu\nu} \] (12)
with
\[ \lambda^2 = \frac{1}{G} \] (13)
and
\[ x = \frac{G}{r} \] (14)
(11) and (9) reproduced the correct Bekenstein–Hawking entropy for the respective four-dimensional black hole. In other words, together with Robinson and Wilczek’s results [2] both entropy and temperature of the Schwarzschild black hole induce some semi-classical theory of spacetime
\[ ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1}dr^2. \] (15)

Our goal will be to expound and extend this idea to include a greater class of rotating, charged and other types of black holes.

1.3. Kerr/CFT correspondence

A recent study by Guica, Hartman, Song and Strominger [44] proposed that the near-horizon geometry of an extremal Kerr black hole is holographically dual to a two-dimensional chiral CFT\(^4\) with a non-vanishing left central extension \(c_L\). By constructing the Frolov–Thorne vacuum for generic Kerr geometry [45], which reduces to the Hartle–Hawking vacuum [46] for \(J \to 0\), the authors obtain a non-vanishing left Frolov–Thorne temperature \(T_L\) in the near-horizon, extremal limit. This temperature together with \(c_L\) inside the thermal Cardy formula [47] reproduces the correct Bekenstein–Hawking entropy for the extremal Kerr black hole:
\[ S_{BH} = \frac{\pi^2}{3}c_L T_L. \] (16)

This correspondence has been extended to various exotic black holes in string theory, higher dimensional theories and gauged supergravities to name a few [47–49].

One of the main arguments of the Kerr/CFT correspondence is to apply the rich ideas of holographic duality to more astrophysical objects/black holes, such as the nearly extremal GRS 1915+105, a binary black hole system 11000 pc away in Aquila [50]. In [44] the authors show that GRS 1915+105 is holographically dual to a two-dimensional chiral CFT with \(c_L = (2 \pm 1) \times 10^{99}\) and in the extremal limit the inner most stable circular orbit corresponds to the horizon. Thus, the authors conclude, any radiation emanating from the inner most circular orbit should be well described by the two-dimensional chiral CFT, making the Kerr/CFT correspondence an essential theoretical tool in an astrophysical observation.

Despite the various models observationalists employ they all incorporate four main quantities: the black hole mass \(M\) and the spin \(J\), poloidal magnetic field at the horizon \(B_0\) and (Eddington) luminosity \(L\) for both supermassive [51] and stellar [52] black holes. This provides a new testable playing field for holography, i.e. to use some induced two-dimensional CFT in the near-horizon regime of extremal and non-extremal black holes to model the four main quantities in accordance with observation. In particular, the origin or mechanism of \(B_0\) is

\(^4\) Distinct from the two-dimensional chiral-scalar field theory employed by Robinson and Wilczek.
unclear from a theoretical standpoint, since it must be due to an accreting disk for non-gauged black holes. Yet, it might find its origin in some black hole/CFT duality.

The goal of this paper is to model the near-horizon regime via a two-dimensional CFT as in the Kerr/CFT correspondence. This is done by combining the ideas from an effective action approach and holographic duality and thus encodes both the Hawking temperature and Bekenstein–Hawking entropy. Our final results hold in both extremal and non-extremal cases.

1.4. Outline of main idea

We will model the near-horizon regime with a two-dimensional Liouville-type quantum field theory, which is well understood [53]:

\[
S_{\text{Liouville}} = \frac{1}{96\pi} \int d^2x \sqrt{-g^{(2)}} \left\{ \Phi \Box_{g^{(2)}} \Phi + 2 \Phi R^{(2)} \right\}.
\]  

(17)

We make this choice based on the fact that in this regime all mass and angular terms of (6) fall off exponentially fast upon transformation from \( r \rightarrow r^* \), we have \( \partial r / \partial r^* = f(r) \) [2] and \( g^{(2)}_{tt} = -f(r) \). This leaves us with an infinite collection of two-dimensional free scalars in a spherically symmetric spacetime \( g^{(2)}_{\mu \nu} \). The effective action of two-dimensional free scalars is given by the Polyakov action [54, 55]

\[
\Gamma_{\text{Polyakov}} = \frac{1}{96\pi} \int d^2x \sqrt{-g^{(2)}} R^{(2)} \left\{ \frac{1}{4} \Box_{g^{(2)}} R^{(2)} \right\}
\]  

(18)

and integrating out \( \Phi \) in \( S_{\text{Liouville}} \) yields \( \Gamma_{\text{Polyakov}} \). In the case where the original four-dimensional metric is not spherically symmetric [23, 24], a \( U(1) \) gauge sector appears in addition to (18), which adds a gauge anomaly to Robinson and Wilczek’s method for computing Hawking radiation. Yet in our analysis, in addition to the Hawking flux, the presence of the \( U(1) \) field only contributes \( \sim e^{2 \pi \Lambda^2 \mid_{\text{Horizon}}} \) to the holomorphic energy–momentum tensor, which is the total flux of the gauge field. Thus it will suffice to only consider the field theory of (17) since we are interested in Hawking effects.

The energy–momentum tensor for (17) is defined as

\[
\langle T_{\mu \nu} \rangle = -\frac{2}{\sqrt{-g^{(2)}}} \frac{\delta S_{\text{Liouville}}}{\delta g^{(2)\mu \nu}}
\]  

and the equation of motion for the auxiliary scalar \( \Phi \) is

\[
\Box_{g^{(2)}} \Phi = R^{(2)}.
\]  

(20)

As an ansatz for the two-dimensional metric \( g^{(2)}_{\mu \nu} \), we choose the dimensional-reduced spacetimes obtained via Robinson and Wilczek. Thus given a \( g^{(2)}_{\mu \nu} \) we are free to solve (20) and (19) up to integration constants. Finally adopting Unruh vacuum boundary conditions [56]

\[
\begin{align*}
T_{++} &= 0 \quad r \rightarrow \infty, l \rightarrow \infty \\
T_{--} &= 0 \quad r \rightarrow r_+
\end{align*}
\]  

(21)

where \( x^\pm = t \pm r \) are light-cone coordinates, \( r_+ \) is the horizon radius defined as the largest real root of \( f(r) = 0 \) and \( l \) is the de Sitter radius, all relevant integration constants are determined. At the horizon and asymptotic infinity for \( (\Lambda = \pm \frac{1}{l^2}) = 0 \) and at the horizon

\footnote{\( g^{(2)}_{\mu \nu} \) may always be assumed spherically symmetric since any Riemannian space in two dimensions is conformally flat.}
only for $\Lambda \neq 0$, (19) will be dominated by one holomorphic component. This component equals the Hawking flux of the four- and three-dimensional black holes, which determines the Hawking temperature.

The entropy will be determined by counting the horizon microstates of $g^{(2)}_{\mu\nu}$ via the Cardy formula (9). Following the outline proposed in [38] we construct a near-horizon $\text{Diff}(S^1)$ subalgebra satisfying (7) based on the isometries of $g^{(2)}_{\mu\nu}$. In the horizon limit ($I^+$ boundary) the $\text{Diff}(S^1)$ subalgebra takes the form

$$i2\{\xi^+_m, \xi^+_n\} = (m-n)\xi^+_{m+n},$$

(22)

where the factor 2 comes from neglecting the asymptotic infinity limit ($I^-$ boundary). On the $I^+$ boundary the energy–momentum tensor is holomorphic given by the $\langle T^{++}\rangle$ component. Next, we define the charge on the $I^+$ boundary

$$Q_n = \frac{3A}{\pi G} \int d^2x \langle T^{++}\rangle \xi^+_n,$$

(23)

where $A$ is the horizon area of the higher dimensional black hole. The coefficient on the integral of $Q_n$ is chosen such that in the case when the higher dimensional black hole is Schwarzschild $Q_n = 1/16\pi G \int d^2x \langle T^{++}\rangle \xi^+_n$, where we have normalized the units of the energy–momentum tensor. For a one-dimensional CFT with holomorphic energy–momentum tensor $T(z)$ we have [57]

$$\delta z T(z) = z T' + 2T z' + \frac{k}{24\pi} z^{iii},$$

(24)

where $k$ is the central extension associated with the CFT (17). In the case for two-dimensional quantum scalar, $k = 1$ in agreement with (35). Thus, given the transformation (24) and compactifying the $I^+$ boundary to a circle with period $(1/2 \cdot 1/T_H)$, where $1/2$ takes the $I^-$ boundary into account, we obtain the following charge algebra:

$$[Q_m, Q_n] = (m-n)Q_n + \frac{c}{12} m(m^2 - 1)\delta_{m+n,0},$$

(25)

where $c$ is the central extension associated with the horizon microstates of $g^{(2)}_{\mu\nu}$. The Bekenstein–Hawking entropy of the four- and three-dimensional black holes is then given by $Q_0$ and $c$ via (9).

Finally we compare our results to [43] by conformally mapping $g^{D}_{\mu\nu}$ to $g^{(2)}_{\mu\nu}$

$$g^{(2)}_{\mu\nu} = 2\phi g^{D}_{\mu\nu},$$

(26)

for some conformal factor $2\phi = (\lambda x)^{-2}$, where (11) is assumed invariant under conformal transformations [16, 58]. We should note that the dimensionally reduced spacetimes satisfy the equations of motion for two-dimensional dilaton gravity in the Schwarzschild case only. This is due to the fact that two-dimensional dilaton gravity is a dimensionally reduced theory of the pure Einstein–Hilbert action with metric ansatz:

$$ds^2 = (2a^2 + 2^{2}\phi d\Omega^2_{(2)}),$$

(27)

where $ds^2_{(2)}$ is as in (10), with no additional matter sources. Now, since the above metric ansatz does not incorporate axisymmetric solutions and the Reissner–Nordström solves Einstein–Hilbert–Maxwell theory, it is clear why their two-dimensional counterparts do not solve the same field equations. Despite this shortcoming, we conjecture that the conformal map (26) still encodes the correct microstates for $g^{(2)}_{\mu\nu}$ based on comparisons of respective entropies obtained from (25) and (26) applied to the Reissner–Nordström case. This may be since

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6 $Q_n$ is only conserved on the $I^+$ boundary.
in two dimensions there are no classical general relativistic dynamics, as mentioned in the introduction, and $g^{(2)}_{\mu\nu}$ might be a dimensionally reduced effective metric of an ultraviolet complete four-dimensional theory of gravity.

2. Examples from four- and three dimensions

We will now apply the method, outlined above, to various four- and three-dimensional black holes with zero and non-zero cosmological constants and construct their Hawking flux, associated entropy and temperature.

2.1. Spherically symmetric solutions

In this class we will consider the Schwarzschild (SS) and Reissner–Nordström (RNS) black holes. Their two-dimensional analogs have the form [2, 59]

$$g^{(2)}_{\mu\nu} = \begin{pmatrix} -f(r) & 0 \\ 0 & \frac{1}{f(r)} \end{pmatrix},$$

where

$$f_{SS}(r) = 1 - \frac{2GM}{r}$$

and

$$f_{RNS}(r) = 1 - \frac{2GM}{r} + \frac{Q^2}{r^2}.$$  

Next, using the above ansatz and solving (20) we get

$$\Phi_{SS} = C_2 t + C_1 r + \ln r - \ln (1 - 2GM) + C_3$$

and

$$\Phi_{RNS} = C_2 t + C_1 r + \frac{C_1 \sqrt{G(2GM^2 - Q^2)}}{\sqrt{G^2M^2 - Q^2}} \arctan \left( \frac{GM - r}{\sqrt{G^2M^2 - GQ^2}} \right)$$

$$+ 2 \ln (r - 2GM) + C_3.$$  

Using these auxiliary fields in (19) and transforming to light cone coordinates we obtain

$$\langle T_{SS++}^{SS} \rangle = \frac{-r^4(C_1 + C_2)^2 + 8rGM - 12G^2M^2}{192\pi r^4}$$

$$\langle T_{SS--}^{SS} \rangle = \frac{-r^4(C_1 - C_2)^2 + 8rGM - 12G^2M^2}{192\pi r^4}$$

$$\langle T_{SS+}^{SS} \rangle = \langle T_{SS-}^{SS} \rangle = \frac{GM(r - 2GM)}{24\pi r^4}$$

and

$$\langle T_{RNS++}^{RNS} \rangle = \frac{-r^6(C_1 + C_2)^2 + 8r^3GM - 12r^2G(M^2 + Q^2)}{192\pi r^4}$$

$$+ 24rG^2MQ^2 - 8G^2Q^4/[192\pi a^6]$$

$$\langle T_{RNS--}^{RNS} \rangle = \frac{-r^6(C_1 - C_2)^2 + 8r^3GM - 12r^2G(M^2 + Q^2)}{192\pi r^4}$$

$$+ 24rG^2MQ^2 - 8G^2Q^4/[192\pi a^6]$$

$$\langle T_{RNS+}^{RNS} \rangle = \langle T_{RNS-}^{RNS} \rangle = \frac{G(2rM - 3Q^2)(r^2 - 2rGM + GQ^2)}{48\pi r^5}.$$
The fact that both energy–momentum tensors are not holomorphic/anti-holomorphic signals the existence of a conformal anomaly taking the form

$$\langle T_{\mu}^{\mu} \rangle = -\frac{1}{24\pi} R^{(2)}$$

(35)

due to the trace anomaly [57]. Imposing (21) to eliminate $C_1$ and $C_2$ our final steps are to analyze $\langle T_{\mu\nu} \rangle$ at the horizon and construct the conformal map (26). At the horizon the energy–momentum tensors are dominated by one holomorphic component $\langle T^{++} \rangle$ given by

$$\langle T^{SS} \rangle = 1768\pi G M^2 = \frac{\pi}{12} (T_H)^2$$

(36)

and

$$\langle T^{RNS} \rangle = \frac{G^2(GM^2 - Q^2)(2M\sqrt{G(GM^2 - Q^2)} + 2GM^2 - Q^2)}{48\pi(\sqrt{G(GM^2 - Q^2)} + GM)^6}$$

$$= \frac{\pi}{12} (T_H)^2,$$

(37)

which are in agreement with Hawking’s original results [6, 7].

Following [38], we compute the near-horizon diffeomorphisms satisfying (7). We get

$$\xi^{SS}_n = 4GM e^{i\kappa n} \partial_\theta + \frac{in(r - 2GM)^2 e^{i\kappa n}}{GM} \partial_r$$

(38)

and

$$\xi^{RNS}_n = \frac{\sqrt{G(GM^2 - Q^2) + GM}e^{i\kappa n}}{G(M\sqrt{G(GM^2 - Q^2) + GM^2 - Q^2})} \partial_\theta + \frac{in e^{i\kappa n}(r^2 - 2GM + GQ^2)^2}{rG(rM - Q^2)} \partial_r,$$

(39)

where $\kappa$ is the horizon surface gravity. Transforming to $x^\pm$ coordinates and taking the horizon limit, we obtain the $I^+$ boundary charge algebras

$$[Q_n, Q_m]_{SS} = (m - n)Q_n + 2GM^2m(m^2 - 1)\delta_{m+n,0},$$

(40)

and

$$[Q_m, Q_n]_{RNS} = (m - n)Q_n$$

$$+ \left(M\sqrt{G(GM^2 - Q^2) + GM^2 - \frac{Q^2}{2}}m(m^2 - 1)\delta_{m+n,0},$$

(41)

which imply

$$Q^0_{0SS} = GM^2$$

(42)

$$c^{SS} = 24GM^2$$

(43)

and

$$Q^0_{0RNS} = \frac{(\sqrt{G(GM^2 - Q^2) + GM})^2}{4G}$$

(44)

$$c^{RNS} = \frac{6(\sqrt{G^2M^2 - GQ^2} + GM)^2}{G}$$

(45)

and using these values in (9) we get

$$S_{SS} = 4\pi GM^2 = \frac{A}{4G}$$

(46)
and

\[ S_{\text{RNS}} = \frac{\pi (\sqrt{G(GM^2 - Q^2)} + GM)^2}{G} = \frac{A}{4G}. \]  

Finally, conformally mapping \( g^{(2)}_{\mu\nu} \) to \( g^{(D)}_{\mu\nu} \) implies

\[ \lambda_{SS}^2 = \frac{1}{G} \]  
\[ x_{SS} = \frac{G}{r} \]  
\[ c_{SS} = 48GM^2 \]  
\[ \Delta_{0}^{SS} = \frac{GM^2}{2} \]

as in [43] and

\[ \lambda_{\text{RNS}}^2 = \frac{4GM^2}{(\sqrt{G^2M^2 - GQ^2 + GM})^2} \]  
\[ x_{\text{RNS}} = \frac{G(2rM - Q^2)}{2r^2M} \]  
\[ c_{\text{RNS}} = \frac{12(\sqrt{G^2M^2 - GQ^2 + GM})^2}{G} \]  
\[ \Delta_{0}^{\text{RNS}} = \frac{(\sqrt{G^2M^2 - GQ^2 + GM})^2}{8G} \]

where \( \lambda_{\text{RNS}} \) and \( x_{\text{RNS}} \) are such that

\[ \lim_{Q \to 0} \lambda_{\text{RNS}} = \lambda_{SS} \quad \text{and} \quad \lim_{Q \to 0} x_{\text{RNS}} = x_{SS}. \]  

Using (9) we find the respective entropies:

\[ S_{SS} = 4\pi GM^2 = \frac{A}{4G} \]  

and

\[ S_{\text{RNS}} = \frac{\pi (\sqrt{G(GM^2 - Q^2)} + GM)^2}{G} = \frac{A}{4G}. \]  

We see that our central extension and zero mode relate to Cadoni’s via

\[ c = \frac{c_c}{2} \]  

\[ Q_0 = 2\Delta_0. \]

Yet, their respective products are equal and produce entropies in agreement with the Bekenstein–Hawking area law [5] for \( \hbar = 1 \). Thus in accord with our conjecture at the end of section 1.4, we choose to conformally map into Cadoni’s solution for all \( g^{(2)}_{\mu\nu} \) for calculational simplicity.

We will proceed to solidify our main argument by applying the methods of section 1.4 to several more black hole solutions of various types.
2.2. Axisymmetric solutions

For this class we analyze the Kerr (K) and Kerr–Newman (KN) black holes with two-dimensional analogs \[23, 59\]

\[
g^{(2)}_{\mu \nu} = \left( \begin{array}{cc} -f(r) & 0 \\ 0 & \frac{1}{f(r)} \end{array} \right),
\]

where

\[ f(r) = \frac{\Delta}{r^2 + J^2} \]

and

\[ \Delta = \begin{cases} r^2 - 2rGM + J^2 & \text{K} \\ r^2 - 2rGM + GQ^2 + J^2 & \text{KN}. \end{cases} \]

The auxiliary scalars read

\[
\Phi_K = rC_1 + tC_2 + (C_1GM - 1) \log(r^2 - 2rGM + J^2) + \log(r^2 + J^2) + \frac{2C_1G^2M^2}{\sqrt{J^2 - G^2M^2}} \arctan \left( \frac{r - GM}{\sqrt{G^2M^2} + J} \right) + C_3
\]

and

\[
\Phi_{KN} = rA + tC_2 + (C_1GM - 1) \log(r^2 - 2rGM + GQ^2 + J^2) + C_1G(2GM^2 - Q^2) \arctan \left( \frac{r - GM}{\sqrt{G^2M^2} + J} \right) + C_3
\]

from which we obtain the energy–momentum tensors

\[
\left\langle T^{K}_{++} \right\rangle = -\left[ r^8(C_1 + C_2)^2 + 4r^6J^2(C_1 + C_2)^2 - 8r^5GM + 6r^4 \\
\times \left( C_1^2J^4 + 2C_1J^4C_2 + 2G^2M^2 + J^4C_2^2 \right) + 16r^3GJ^2M + 4r^2 \\
\times \left( C_1^2J^6 + 2C_1J^6C_2 - 10G^2J^2M^2 + J^6C_2^2 \right) + 24rGJ^4M + C_2J^8 \\
+ 2C_1J^8C_2 - 4G^2J^4M^2 + J^8C_2^2 \right]/\left[ 192\pi(r^2 + J^2)^4 \right]
\]

\[
\left\langle T^{K}_{--} \right\rangle = -\left[ r^8(C_1 - C_2)^2 + 4r^6J^2(C_1 - C_2)^2 - 8r^5GM + 6r^4 \\
\times \left( C_1^2J^4 - 2C_1J^4C_2 + 2G^2M^2 + J^4C_2^2 \right) + 16r^3GJ^2M + 4r^2 \\
\times \left( C_1^2J^6 - 2C_1J^6C_2 - 10G^2J^2M^2 + J^6C_2^2 \right) + 24rGJ^4M + C_2J^8 \\
- 2C_1J^8C_2 - 4G^2J^4M^2 + J^8C_2^2 \right]/\left[ 192\pi(r^2 + J^2)^4 \right]
\]

\[
\left\langle T^{K}_{+-} \right\rangle = \left\langle T^{K}_{-+} \right\rangle = \frac{rGM(r^2 - 3J^2)(r^2 - 2rGM + J^2)}{24\pi(r^2 + J^2)^4}
\]
and
\[ T^{KN}_{++} = -\left[r^2 (C_1 - C_2)^2 + 4r^6 J^2 (C_1 + C_2)^2 - 8r^5 GM + 6r^4 \times (C_1 J^4 + 2C_1 J^4 C_2 + 2G^2 M^2 + 2G Q^2 + J^4 C_2^2) - 8r^3 GM \times (3G Q^2 - 2J^2) + 4r^3 (C_1 J^6 + 2C_1 J^6 C_2 + 2G^2 (Q^4 - 5J^2 M^2) + 2G J^2 Q^2 + J^6 C_2^2) + 24r G J^2 M (G Q^2 + J^2) + C_1 J^8 + 2C_1 J^6 C_2 - 4G^2 J^4 M^2 - 4G J^2 Q^4 - 4G J^4 Q^2 + J^8 C_2^2 \right] \]
\[ T^{KN}_{-+} = -\left[r^2 (C_1 - C_2)^2 + 4r^6 J^2 (C_1 - C_2)^2 - 8r^5 GM + 6r^4 \times (C_1 J^4 - 2C_1 J^4 C_2 + 2G^2 M^2 + 2G Q^2 + J^4 C_2^2) - 8r^3 GM \times (3G Q^2 - 2J^2) + 4r^3 (C_1 J^6 - 2C_1 J^6 C_2 + 2G^2 (Q^4 - 5J^2 M^2) + 2G J^2 Q^2 + J^6 C_2^2) + 24r G J^2 M (G Q^2 + J^2) + C_1 J^8 - 2C_1 J^6 C_2 - 4G^2 J^4 M^2 - 4G J^2 Q^4 - 4G J^4 Q^2 + J^8 C_2^2 \right] \]
\[ T^{KN}_{++} = \frac{\pi (G^2 M^2 - J^2)}{12(4\pi GM \sqrt{G^2 M^2 - J^2} + 4\pi G^2 M^2)^2} = \frac{\pi}{12} (T_H)^2 \]
(68)

and
\[ T^{KN}_{-+} = -\frac{G (Q^2 - GM^2) + J^2}{48\pi ((\sqrt{G^2 M^2 - G Q^2 - J^2} + GM)^2 + J^2)^2} = \frac{\pi}{12} (T_H)^2 \]
(69)

agreeing with Hawking’s result [6, 7]. Next, from (26) and applying similar boundary conditions as in (56) we obtain

\[ \lambda^2_K = \frac{4GM^2}{(\sqrt{G^2 M^2 - J^2} + GM)^2 + J^2} \]
(70)

\[ x_K = \frac{r G}{r^2 + J^2} \]
(71)

\[ c_K = \frac{12((\sqrt{G^2 M^2 - J^2} + GM)^2 + J^2)}{G} \]
(72)

\[ \Delta^K_{ij} = \frac{(\sqrt{G^2 M^2 - J^2} + GM)^2 + J^2}{8G} \]
(73)

and

\[ \lambda^2_{KN} = \frac{4GM^2}{(\sqrt{G^2 M^2 - G Q^2 - J^2} + GM)^2 + J^2} \]
(74)

\[ x_{KN} = \frac{2\alpha GM - G Q^2}{2\alpha^2 M + 2J^2 M} \]
(75)

\[ c_{KN} = \frac{12((\sqrt{G^2 M^2 - G Q^2 - J^2} + GM)^2 + J^2)}{G} \]
(76)
Their two-dimensional analogs \[25, 32\] are in (61) where
\[
\langle S\rangle = \langle 4G\rangle \quad \text{(77)}
\]
which give the respective entropies:
\[
S_K = 2\pi M(\sqrt{G^2 M^2 - J^2} + GM) = \frac{A}{4G}
\quad \text{(78)}
\]
and
\[
S_{KN} = \pi (2M(\sqrt{G(G M^2 - Q^2) - J^2} + GM)) = \frac{A}{4G}
\quad \text{(79)}
\]
reproducing the Bekenstein–Hawking area law [5] and continuing the trend of section 2.1.

2.3. Spherically symmetric SSdS and rotating BTZ

Now, we turn our attention to black holes with non-zero cosmological constant:
\[
\Lambda = \begin{cases} 
\frac{1}{l^2} & \text{dS} \\
-\frac{1}{l^2} & \text{AdS}, 
\end{cases}
\quad \text{(80)}
\]
where \(l\) is the de Sitter radius. In this black hole class we consider the spherically symmetric dS (SSdS) with line element
\[
ds^2 = -\left(1 - \frac{2GM}{r} - \frac{r^2\Lambda}{3}\right)dr^2 + \left(1 - \frac{2GM}{r} - \frac{r^2\Lambda}{3}\right)^{-1}d\Omega^2 + r^2d\Omega^2
\quad \text{(81)}
\]
and the three-dimensional BTZ black hole with line element
\[
ds^2 = -\left(-8GM + \frac{r^2}{l^2} + \frac{16GJ^2}{r^2}\right)dr^2 + \left(-8GM + \frac{r^2}{l^2} + \frac{16GJ^2}{r^2}\right)^{-1}dr^2 + r^2\left(d\phi - \frac{4GJ}{r^2}dr\right)^2.
\quad \text{(82)}
\]
Their two-dimensional analogs [25, 32] are as in (61) where
\[
f(r) = \begin{cases} 
1 - \frac{2GM}{r} - \frac{r^2\Lambda}{3} & \text{SSdS} \\
-8GM + \frac{r^2}{l^2} + \frac{16GJ^2}{r^2} & \text{BTZ}, 
\end{cases}
\quad \text{(83)}
\]
Following the steps outlined in section 1.4 we obtain the energy–momentum tensors
\[
\begin{align*}
\langle T^{\text{SSdS}}_{++} \rangle &= -\left[r^4(3C_1^2 + 6C_1C_2 + 3C_2^2 - 4\Lambda) + 24r^3GM\Lambda - 24rG M + 36G^2M^2]\right]/[576\pi r^4] \\
\langle T^{\text{SSdS}}_{--} \rangle &= \left[r^4(-3C_1^2 + 6C_1C_2 - 3C_2^2 + 4\Lambda) - 24r^3GM\Lambda + 24rG M - 36G^2M^2]\right]/[576\pi r^4] \\
\langle T^{\text{SSdS}}_{+-} \rangle &= \langle T^{\text{SSdS}}_{-+} \rangle = -\left[r^6\Lambda^2 - 3r^4\Lambda + 12r^3GM\Lambda - 18r GM + 36G^2M^2\right]/[432\pi r^4]
\end{align*}
\quad \text{(84)}
\]
and
\[
\begin{align*}
\langle T^{\text{BTZ}}_{++} \rangle &= -\left[r^6(C_1^2 + 2C_1l^2C_2 - 32G M + l^2C_2^2) + 384r^4G^2 J^2 - 1536r^2G^3 J^2 l^2 M + 2048G^4 J^4 l^2\right]/[192\pi r^6 l^2] \\
\langle T^{\text{BTZ}}_{--} \rangle &= \left[-r^6(C_1^2 - 2C_1l^2C_2 - 32G M + l^2C_2^2) - 384r^4G^2 J^2 - 1536r^2G^3 J^2 l^2 M - 2048G^4 J^4 l^2\right]/[192\pi r^6 l^2] \\
\langle T^{\text{BTZ}}_{+-} \rangle &= \langle T^{\text{BTZ}}_{-+} \rangle = \left[-r^4(48G^2 J^2 l^2) - (r^4 - 8r^2G^2 M + 16G^2 J^2 l^2)\right]/[48\pi r^6 l^4]
\end{align*}
\quad \text{(85)}
\]

with conformal anomaly (35). Applying (21) we obtain the holomorphic piece
\[
\langle T_{++} \rangle = \frac{\pi}{12} (\mathcal{H})^2
\]
for both spacetimes in their respective horizon limits and agreeing as before with Hawking’s results [6, 7]. Next, their respective entropies are computed via (9), (26),
\[
\lambda_{SSdS}^2 = -[16G M^2 \Lambda^2 (\sqrt{\Lambda^3 (9G^2 M^2 \Lambda - 1) - 3G M \Lambda^2})^{2/3}] / \\
[ (\sqrt{3} - i)(\sqrt{\Lambda^3 (9G^2 M^2 \Lambda - 1) - 3G M \Lambda^2})^{2/3} \Lambda^2 + (\sqrt{3} - i) \Lambda^2 ]
\]
\[
x_{SSdS} = \frac{r^3 \Lambda + 6GM}{6r M}
\]
\[
c_{SSdS} = -[3(\sqrt{3} - i)(\sqrt{\Lambda^3 (9G^2 M^2 \Lambda - 1) - 3G M \Lambda^2})^{2/3} + (\sqrt{3} - i) \Lambda^2] / \\
[ GA^2 (\sqrt{\Lambda^3 (9G^2 M^2 \Lambda - 1) - 3G M \Lambda^2})^{2/3} ]
\]
\[
\Delta_0^{SSS} = -[3(\sqrt{3} - i)(\sqrt{\Lambda^3 (9G^2 M^2 \Lambda - 1) - 3G M \Lambda^2})^{2/3} + (\sqrt{3} - i) \Lambda^2] / \\
[32G \Lambda^2 (\sqrt{\Lambda^3 (9G^2 M^2 \Lambda - 1) - 3G M \Lambda^2})^{2/3} ]
\]
and
\[
\lambda_{BTZ}^2 = \frac{4G M^2}{\sqrt{G^2 (l^2 M^2 - J^2) + G l^2 M}}
\]
\[
x_{BTZ} = \frac{(-r^4 + r^2 J^2 + 16G^2 J^2 J^2 + 8r^2 G l^2 M)}{(2r^2 l^2 M)}
\]
\[
c_{BTZ} = \frac{12 \sqrt{G^2 (l^2 M^2 - J^2) + G l^2 M}}{G}
\]
\[
\Delta_0^{BTZ} = \frac{\sqrt{G^2 (l^2 M^2 - J^2) + G l^2 M}}{8G}
\]
reproducing the Bekenstein–Hawking area law [5]
\[
S = \frac{A}{4G}
\]
in both cases via (9). Thus by modeling the near-horizon regime with a two-dimensional CFT, we have computed both entropy and temperature of four-dimensional Schwarzschild, Reissner–Nordström, Kerr, Kerr–Newman, spherically symmetric dS and of the three-dimensional BTZ black hole.

3. Conclusion

To conclude, we have analyzed quantum black hole properties via non-local effective action in the near-horizon regime. For a relatively large class of black holes, including dS and AdS solutions in three- and four dimensions, both entropy and temperature are computed from two-dimensional conformal field theory techniques in the near-horizon regime. The two-dimensional CFT was modeled via a Liouville action with two-dimensional black hole solutions given by Robinson and Wilczek’s dimensional reduction first discussed in [2]. These
two-dimensional black holes exhibit a Diff($S^1$) subalgebra, up to conformal transformation, first discovered by Cadoni [43] for the s-wave sector of a Schwarzschild black hole. Analysis of the anomalous energy–momentum tensor of the Liouville theory and the Diff($S^1$) subalgebra reproduces the Hawking temperature and Bekenstein–Hawking entropy for the respective four- and three-dimensional black holes.

The anomalous contribution (35) signals interesting physics as Christensen and Fulling showed [1]. In this case the anomaly relates to quantum black hole physics and in fact for Schwarzschild

$$\frac{1}{16} \langle T_{\mu}^{\mu} \rangle = -\frac{\pi}{12} (T_H)^2,$$

which is not the case for any other spacetime considered above except in their respective holomorphic limits. Though the two-dimensional trace anomaly did not factor much into the main calculations of this paper, it remains an interesting feature to interpret especially for the non-Schwarzschild cases.

The methods outlined in section 1.4 seem to be universal at least in three- and four dimensions. It remains interesting, for future work, to generalize (23) and (25) to general black holes/strings in arbitrary dimensions as Peng, Wu [30] and Xu, Chen [28] have done for Robinson and Wilczek’s gauge gravitational anomaly cancellation method.

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References

[1] Christensen S M and Fulling S A 1977 Trace anomalies and the Hawking effect Phys. Rev. D 15 2088–2104
[2] Robinson S P and Wilczek F 2005 Relationship between Hawking radiation and gravitational anomalies Phys. Rev. Lett. 95 011303
[3] Blöte H W J, Cardy J A and Nightingale M P 1986 Phys. Rev. Lett. 56 742
[4] Cardy J A 1986 Nucl. Phys. B 270 186
[5] Bekenstein J D 1973 Black holes and entropy Phys. Rev. D 7 2333–46
[6] Hawking S W 1975 Particle creation by black holes Commun. Math. Phys. 43 199–220
[7] Hawking S W, Bardeen J M and Carter B 1973 The four laws of black hole mechanics Commun. Math. Phys. 31 161–70
[8] G K AU 1995 The quest for quantum gravity arXiv:gr-qc/9506001v1
[9] Carlip S 2005 Horizon constraints and black hole entropy arXiv:gr-qc/0508071v1
[10] Rovelli C 1998 Strings, loops and others: a critical survey of the present approaches to quantum gravity arXiv:gr-qc/9803024v3
[11] Wipf A 1998 Quantum fields near black holes arXiv:hep-th/9801025v1
[12] Mukhanov V, Wipf A and Zelnikov A 1994 On 4d-hawing radiation from effective action Phys. Lett. B 332 283–91 (arXiv:hep-th/9403018v1)
[13] Balbinot R and Fabbri A 1999 4d quantum black hole physics from 2d models? Phys. Lett. B 459 112–8 (arXiv:gr-qc/9904034v2)
[14] Balbinot R and Fabbri A 1999 Hawking radiation by effective two-dimensional theories Phys. Rev. D 59 044031 (arXiv:hep-th/9807123v1)
[15] Cadoni M 1996 Trace anomaly and Hawking effect in generic 2d dilaton gravity theories *Nucl. Phys. B Proc. Suppl.* **57** 188–91 (arXiv:gr-qc/9510012v1)

[16] Cadoni M 1996 Trace anomaly and Hawking effect in 2d dilaton gravity theories arXiv:gr-qc/9612041v1

[17] Wu S-Q, Peng J-J and Zhao Z-Y 2008 Anomalies, effective action and Hawking temperatures of a anomalies, effective action and Hawking temperatures of a Schwarzschild black hole in the isotropic coordinates *Class. Quantum Grav.* **25** 135003 (arXiv:0803.1338v5)

[18] Das S, Robinson S P and Vagenas E C 2008 Gravitational anomalies: a recipe for Hawking radiation *Int. J. Mod. Phys.* D **17** 533–9

[19] Banerjee R 2009 Covariant anomalies, horizons and Hawking radiation *Int. J. Mod. Phys.* D **17** 2539–42 (arXiv:0807.4637v1)

[20] Banerjee R and Kulkarni S 2008 Hawking radiation and covariant boundary conditions *Phys. Lett.* B **659** 827–31 (arXiv:0709.3916v3)

[21] Banerjee R and Kulkarni S 2009 Hawking radiation, covariant boundary conditions and vacuum states *Phys. Rev.* D **79** 084035 (arXiv:0810.3683v2)

[22] Banerjee R and Kulkarni S 2008 Hawking radiation, effective actions and covariant boundary conditions *Phys. Lett.* B **647** 200–6

[23] Xu Z and Chen B 2007 Hawking radiation from general Kerr–(anti)de Sitter black holes *Phys. Rev.* D **75** 024041 (arXiv:hep-th/0606069v2)

[24] Gangopadhyay S 2010 Hawking radiation from black holes in de Sitter spaces via covariant anomalies *Gen. Rel. Grav.* **42** 1183

[25] Jiang Q-Q 2007 Hawking radiation from black holes in de Sitter spaces *Class. Quantum Grav.* **24** 4391–406

[26] Jiang Q-Q and Wu S-Q 2007 Hawking radiation from rotating black holes in anti-de Sitter spaces via gauge and gravitational anomalies *Phys. Lett.* B **647** 200–6

[27] Xu Z and Chen B 2007 Hawking radiation from general Kerr–(anti)de Sitter black holes *Phys. Rev.* D **75** 024041 (arXiv:hep-th/0612261v1)

[28] Chen B and He W 2008 Hawking radiation of black rings from anomalies *Class. Quantum Grav.* **25** 135011 (arXiv:0705.2984v2)

[29] Peng J-J and Wu S-Q 2008 Covariant anomalies and Hawking radiation from charged rotating black strings in anti-de Sitter spacetimes *Phys. Lett.* B **661** 300–6

[30] Nam S and Park J-D 2009 Hawking radiation from covariant anomalies in (2+1)-dimensional black holes *Class. Quantum Grav.* **26** 15 pp

[31] Setare M R 2006 Gauge and gravitational anomalies and Hawking radiation of rotating BTZ black holes *Eur. Phys. J.* C **49** 865–8 (arXiv:hep-th/0608080v1)

[32] Papantonopoulos E and Skamagoulis P 2009 Hawking radiation via gravitational anomalies in non-topological topologies *Phys. Rev.* D **79** 084022 (arXiv:0812.1759v3)

[33] Brown J D and Henneaux M 1986 J. Math. Phys. **27** 489

[34] Strominger A 1998 Black hole entropy from near-horizon microstates *J. High Energy Phys.* JHEP02(1998)009 (arXiv:hep-th/9712251v3)

[35] Carlip S 1998 Black hole entropy from conformal field theory in any dimension *Phys. Rev. Lett.* **82** 2828–31 (arXiv:hep-th/9812013v3)

[36] Carlip S 2005 Conformal field theory, (2+1)-dimensional gravity, and the BTZ black hole *Class. Quantum Grav.* **22** R85–123 (arXiv:gr-qc/0503022v4)

[37] Carlip S 1999 Entropy from conformal field theory at Killing horizons *Class. Quantum Grav.* **16** 3327–48 (arXiv:gr-qc/9906126v2)

[38] Kanga G, Kobag J I and Park M I 2004 Near-horizon conformal symmetry and black hole entropy in any dimension *Phys. Rev.* D **70** 024005 (arXiv:hep-th/0402113v1)

[39] Silva S 2002 Black hole entropy and thermodynamics from symmetries *Class. Quantum Grav.* **19** 3947–61 (arXiv:hep-th/0204179)

[40] Barnes E, Vaman D and Wu C 2010 All 4-dimensional static, spherically symmetric, 2-charge Abelian Kaluza–Klein black holes and their CFT duals *Class. Quantum Grav.* **27** 095019 (arXiv:0908.2425v2)

[41] Cadoni M 2006 Statistical entropy of the Schwarzschild black hole *Modern Phys. Lett.* A **21** 1879–87

[42] Guica M, Hartman T, Song W and Strominger A 2009 The Kerr/CFT correspondence *Phys. Rev.* D **80** 124008 (arXiv:0809.4266)
[45] Frolov V P and Thorne K S 1989 Renormalized stress-energy tensor near the horizon of a slowly evolving, rotating black hole Phys. Rev. D 39 2125–54
[46] Hartle J B and Hawking S W 1976 Path-integral derivation of black-hole radiance Phys. Rev. D 13 2188–2203
[47] Chow D D K, Cvetić M, Lü H and Pope C N 2009 Extremal black hole/CFT correspondence in (gauged) supergravities Phys. Rev. D 79 084018 (arXiv:0812.2918)
[48] Azeyanagi T, Ogawa N and Terashima S 2009 The Kerr/CFT correspondence and string theory Phys. Rev. D 79 106009
[49] Lü H, Mei J and Pope C N 2009 Kerr-AdS/CFT correspondence in diverse dimensions J. High Energy Phys. JHEP04(2009)054 (arXiv:0811.2225)
[50] McClintock J E, Shafee R, Narayan R and Remillard R A 2006 The spin of the near-extreme Kerr black hole GRS 1915+105 Astrophys. J. 652 518–39 (arXiv:astro-ph/0606076)
[51] Daly R A 2009 Bounds on black hole spins Astrophys. J. 696 L32–6
[52] Remillard R A and McClintock J E 2006 X-ray properties of black-hole binaries Annu. Rev. Astron. Astrophys. 44 49–92 (arXiv:astro-ph/0606352)
[53] Seiberg N 1990 Notes on quantum Liouville theory and quantum gravity Prog. Theor. Phys. Suppl. 102 319–49
[54] Mukhanov V and Winitzki S 2007 Introduction to Quantum Effects in Gravity (Cambridge: Cambridge University Press)
[55] Polyakov A M 1981 Quantum geometry of bosonic string Phys. Lett. B 103 207
[56] Unruh W G 1976 Notes on black-hole evaporation Phys. Rev. D 14 870–892
[57] Francesco P D, Mathieu P and Sénéchal D 1997 Conformal Field Theory (Berlin: Springer)
[58] Cadoni M and Mignemi S 1995 On the conformal equivalence between 2d black holes and Rindler spacetime Phys. Lett. B 358 217–22 (arXiv:gr-qc/9505032v1)
[59] Iso S, Umetu H and Wilczek F 2006 Hawking radiation from charged black holes via gauge and gravitational anomalies Phys. Rev. Lett. 96 151302 (arXiv:hep-th/0602146v3)