Comparison of analytical and numerical modeling of distributions of nonequilibrium minority charge carriers generated by a wide beam of medium-energy electrons in a two-layer semiconductor structure

V V Kalmanovich¹,², E V Seregina² and M A Stepovich¹

¹ Tsiolkovsky Kaluga State University, 26 Stepan Razin Street, Kaluga, 248023 Russia
² Bauman Moscow State Technical University (National Research University), Kaluga Branch, 2 Bazhenov Street, Kaluga, 248000 Russia

E-mail: v572264@yandex.ru

Abstract. An analytical matrix method that makes it possible to calculate the distributions of nonequilibrium minority charge carriers generated in multilayer planar semiconductor structures by external influence is described. For the case of generation of nonequilibrium minority charge carriers by a wide electron beam in a two-layer structure, the results of model calculations using the analytical matrix method with the results of calculations using a numerical conservative difference scheme are compared. It is shown that the method allows to carry out calculations of the distributions of nonequilibrium minority charge carriers in a relatively short time with an accuracy sufficient for practical use in electron probe technologies.

1. Introduction

One of the few methods that allow for the implementation of non-contact nondestructive diagnostics of solids are electron probe methods based on the use of beams of kilovolt electrons of low (up to about 8-10 keV) or medium (from 8-10 to 50 keV) energies. Registration of informative signals excited in the target and comparison of experimental data with the mathematical model of the phenomenon under study allow us to identify target parameters that are very difficult or even impossible to determine using other methods [1].

For a wide electron beam, the problem of calculating the distributions of nonequilibrium minority charge carriers (MCC) generated by kilovolt electrons is reduced to one-dimensional. When modeling the processes of interaction of accelerated electrons with a target in a rectangular Cartesian coordinate system, the OX and OY axes are usually located on the flat surface of the target, and the OZ axis is directed deeper into the target [2, 1].

In this paper, we consider some of the possibilities of the analytical matrix method of mathematical modeling of the distributions of nonequilibrium MCC in semiconductor structures. This method as applied to heat conduction problems in composite plates is described in [3]. However, it was not widely used to solve heat and mass transfer problems in multilayer media, possibly due to the fact that the formulas of the analytical solution were extremely complex, symbolic computing systems were just beginning to emerge at that time, and therefore numerical methods were preferred. In [4, 5],
methods similar in concept were applied to the description of transport phenomena on graphs in systems of contacting shells and bodies of revolution, in systems of contacting rods, where heat fluxes are determined by the system’s conductivity matrix. Earlier in our works, the analytical matrix method proposed in [3] was used in conjunction with the apparatus of generalized Bers degrees [6-8], which allowed us to successfully describe the heat and mass transfer process in multilayer media with different geometries: flat, axisymmetric or layers with central symmetry [9-12]. In this paper, we consider some possibilities of using this approach in modeling two-layer semiconductor structures of finite thickness. The analytical results obtained using the matrix method are compared with the results of calculations obtained using the numerical finite difference method. Model calculations were carried out for the electrophysical parameters characteristic of a solid solution of cadmium-mercury-tellurium – cadmium telluride, which is widely used in the manufacture of infrared devices [13, 14].

2. Statement of the problem
In the case of one-dimensional diffusion into the final semiconductor along the axis \( z \) perpendicular to the surface of the two-layer semiconductor structure \( (z \in [0, l]) \), the depth distribution of the MCC is found as a solution of the differential equation

\[
\frac{d}{dz} \left( D(z) \frac{d\rho(z)}{dz} \right) - \frac{\Delta \rho(z)}{\tau(z)} = -\rho(z) \tag{1}
\]

with boundary conditions

\[
D_1 \frac{d\Delta \rho(z)}{dz} \bigg|_{z=0} = v_{s_1} \Delta \rho(0), \quad D_2 \frac{d\Delta \rho(z)}{dz} \bigg|_{z=l} = -v_{s_2} \Delta \rho(l). \tag{2}
\]

For a two-layer structure, we introduce the following notation: \( z_1 \) – the coordinate of the interface between the first and second layers; \( D_1, D_2, L_1, L_2, \tau_1, \tau_2 \) – electrophysical parameters of the first and second layers: diffusion coefficients, diffusion lengths and lifetimes of the MCC, respectively, \( a \) and \( S_1 \) and \( S_2 \) the reduced surface recombination rates, respectively, on the surfaces of the first (at \( z = 0 \) ) and second (at \( z = l \)) materials. In this case \( L_1 = (D_1 \tau_1)^{1/2}, \quad L_2 = (D_2 \tau_2)^{1/2} \) and \( S_1 = L_1 v_{s_1}/D_1, \quad S_2 = L_2 v_{s_2}/D_2 \) where \( v_{s_1} \) and \( v_{s_2} \) are the surface recombination rates of the MCC on surfaces of the first and the second layers, respectively. The function \( \Delta \rho(z) \) describes the depth distribution of nonequilibrium MCC generated by external energy exposure after their diffusion in the semiconductor, in this case, \( z \) – the coordinate measured from the flat surface of the irradiated target deep into the semiconductor. The function \( \rho(z) \) is the dependence on the coordinate of the density of the MCC generated by the electron beam in the semiconductor target. For a wide electron beam \( \rho(z) \) can be found from the expression for the energy density of the electron beam \( \rho^*(z) \)

\[
\rho^*(z) = \frac{1.085(1-\eta)E_0}{\sqrt{\pi}z_{ms}(1-\eta+\eta z_{ss}/z_{ms})} \left[ \exp \left[ -\left( \frac{z-z_{ms}}{z_{ms}} \right)^2 \right] + \frac{\eta}{1-\eta} \exp \left[ -\left( \frac{z-z_{ss}}{z_{ss}} \right)^2 \right] \right],
\]

released in the target per unit time before the start of the diffusion process [15-17, 1], by dividing \( \rho^*(z) \) by the energy of formation of an electron-hole pair. Here \( E_0 \) is the energy of the electron beam scattered in the target per unit time, \( z_{ms} \) is the depth of the maximum energy loss by the primary electrons, experienced small-angle scattering and absorbed by the target; \( z_{ss} = Z^{-1/3}z_{ms} \) – the depth of
the maximum energy loss by backscattered electrons that experienced scattering at large angles and exiting the target, $Z$ is the core charge of the target substance; $\eta$ is the backscattering coefficient of the beam electrons, $\eta = 0.024eZ^{1.67}/A$, and $A$ is the atomic weight of the target substance. Note that the value $z_{ms}$ for electrons with energy $E_0$ [keV] and density target $\rho_0$ [g/cm$^3$] can be determined from the diffusion model [18]:

$$z_{ms} \, [\mu m] = R^* \left[ 1 + \left( \frac{C \gamma}{1 + \gamma} \right)^2 \right], \quad R^* \, [\mu m] = \frac{2.76 \cdot 10^{-2} AE_0^{1/3}}{\rho_0 Z^{1/2}} \frac{(1 + 0.978 \cdot 10^{-3} E_0)^{1/3}}{(1 + 1.957 \cdot 10^{-3} E_0)^{1/3}}.$$  

Here $R^*$ is the total path of the beam electrons in the target.

3. Analytical solution

Let us briefly describe the essence of the joint application of the apparatus of generalized Bers degrees and the analytic matrix method.

In general, a one-dimensional stationary process of heat and mass transfer in a multilayer medium can be defined by the equation

$$D^{(i)}_2 \Phi^{(i)}(z) - m^2 \Phi^{(i)}(z) = \mu^{(i)}(z), \quad i = 1, n,$$

where the potential $\Phi^{(i)}(z)$ is the desired function, $D^{(i)}_1 = a^{(i)}_1(z) d/dz$ and $D^{(i)}_2 = a^{(i)}_2(z) d/dz$ are the differential operators, $a^{(i)}_1(z)$ and $a^{(i)}_2(z)$ are the positive functions determined by the physical and geometric parameters of the layers, $\mu^{(i)}(z)$ is a value proportional to the volumetric energy density of the sources, $i$ is the number of the layer. The flow is given by the formula $J^{(i)}(z) = -D^{(i)}_1 \Phi^{(i)}(z)$. At the contact point of the layers, conditions such as ideal contact are satisfied, i.e. the conditions of continuity of potential and flow are valid:

$$\Phi^{(i)}(z_{i+1}) = \Phi^{(i+1)}(z_{i+1}), \quad J^{(i)}(z_{i+1}) = J^{(i+1)}(z_{i+1}), \quad i = 1, n-1.$$

We set the potential $\Phi^{(i)}(z_i)$ and flow $J^{(i)}(z_i)$ values, i.e. set the Cauchy problem for the first layer.

The solution to the Cauchy problem for equation (3) for the first layer has the form

$$\Phi^{(1)}(z) = \left( \Phi^{(1)}(z_i) - w^{(1)}(z_i) \right) \text{ch} mX_1(z, z_i) - \frac{1}{m} \left( J^{(1)}(z_i) + D^{(1)}_1 w^{(1)}(z_i) \right) \text{sh} mX_1(z, z_i) + w^{(1)}(z), \quad i = 1, n-1.$$

$$J^{(1)}(z) = -\left( \Phi^{(1)}(z_i) - w^{(1)}(z_i) \right) m \text{sh} mX_1(z, z_i) + \left( J^{(1)}(z_i) + D^{(1)}_1 w^{(1)}(z_i) \right) \text{ch} mX_1(z, z_i) - D^{(1)}_1 w^{(1)}(z), \quad i = 1, n-1.$$

where $w^{(1)}(z)$ is a particular solution of equation (3) for the first layer.

We introduce the column vectors $V, W$ and the matrix $K$

$$V^{(1)}(z) = \begin{pmatrix} \Phi^{(1)}(z) \\ J^{(1)}(z) \end{pmatrix}, \quad W^{(1)}(z) = \begin{pmatrix} w^{(1)}(z) \\ -D^{(1)}_1 w^{(1)}(z) \end{pmatrix},$$

$$K^{(1)}(z, z_i) = \begin{pmatrix} \text{ch} mX_1(z, z_i) & -m^{-1} \text{sh} mX_1(z, z_i) \\ -m \text{sh} mX_1(z, z_i) & \text{ch} mX_1(z, z_i) \end{pmatrix}.$$
Here \( w^{(i)}(z) \) is a particular solution to Eq. (3) for the \( i \)th layer, \( X_i(z, z_i) \) and \( \bar{X}_i(z, z_i) \) are the generalized Bers degree and the adjoint generalized Bers degree on the interval \( (z_i, z_{i+1}) \), respectively. Then we write relations (5) and (6) in matrix form

\[
V^{(i)}(z) = K^{(i)}(z, z_i)\left(V^{(i)}(z_i) - W^{(i)}(z_i)\right) + W^{(i)}(z).
\]

For \( z = z_2 \) we have

\[
V^{(1)}(z_2) = K^{(1)}(z_2, z_1)\left(V^{(1)}(z_1) - W^{(1)}(z_1)\right) + W^{(1)}(z_2).
\]

For the second layer, acting similarly, we write

\[
V^{(2)}(z) = K^{(2)}(z, z_2)\left(V^{(2)}(z_2) - W^{(2)}(z_2)\right) + W^{(2)}(z).
\]

(7)

In the general case, on the \( i \)th layer we have

\[
V^{(i)}(z) = K^{(i)}(z, z_i)\left(V^{(i)}(z_i) - W^{(i)}(z_i)\right) + W^{(i)}(z),
\]

and for endpoints of the \( i \)th layer point

\[
V^{(i)}(z_{i+1}) = K^{(i)}(z_{i+1}, z_i)\left(V^{(i)}(z_i) - W^{(i)}(z_i)\right) + W^{(i)}(z_{i+1}).
\]

Perfect contact condition at a point \( z_2 \) means

\[
V^{(i)}(z_2) = V^{(2)}(z_2)\tag{8}
\]

Substitute, according to (8), in the formula (7)

\[
V^{(2)}(z) = K^{(2)}(z, z_2)K^{(1)}(z_2, z_1)\left(V^{(1)}(z_1) - W^{(1)}(z_1)\right) + K^{(2)}(z, z_2)\left(W^{(1)}(z_2) - W^{(2)}(z_2)\right) + W^{(2)}(z).
\]

At the end point of the second layer we get

\[
V^{(2)}(z_3) = K^{(2)}(z_3, z_2)K^{(1)}(z_2, z_1)\left(V^{(1)}(z_1) - W^{(1)}(z_1)\right) + K^{(2)}(z_3, z_2)\left(W^{(1)}(z_2) - W^{(2)}(z_2)\right) + W^{(2)}(z_3).
\]

Acting in a similar way, further for the \( i \)th layer we find

\[
V^{(i)}(z) = K^{(i)}(z, z_i)K^{(i-1)}(z_i, z_{i-1}) \cdots K^{(1)}(z_{i+1}, z_i)\left(V^{(1)}(z_i) - W^{(1)}(z_i)\right) \\
+ K^{(i)}(z, z_i)K^{(i-1)}(z_i, z_{i-1}) \cdots K^{(2)}(z_3, z_2)\left(W^{(1)}(z_2) - W^{(2)}(z_2)\right) + \cdots \tag{9}
\]

\[
+ K^{(i)}(z, z_i)\left(W^{(i-1)}(z_i) - W^{(i)}(z_i)\right) + W^{(i)}(z).
\]

We denote

\[
L^{(i,j)}(z, z_i) = K^{(i)}(z, z_i)K^{(i-1)}(z_i, z_{i-1}) \cdots K^{(j)}(z, z_j), \ i \geq j.
\]

Then formula (9) can be written

\[
V^{(i)}(z) = L^{(i,2)}(z, z_i)V^{(1)}(z_i) + L^{(i,1)}(z, z_i)\left(W^{(1)}(z_i) - W^{(1)}(z_i)\right) \\
+ L^{(2,1)}(z, z_2)\left(W^{(1)}(z_2) - W^{(2)}(z_2)\right) + \cdots \\
+ L^{(i,1)}(z, z_i)\left(W^{(i-1)}(z_i) - W^{(i)}(z_i)\right) + W^{(i)}(z)
\]

or
\[ V^{(i)}(z) = L^{(i)}(z, z_{i})V^{(i)}(z_{i}) + \sum_{j=1}^{i-1} L^{(i)}(z, z_{j})(W^{(j-1)}(z_{j}) - W^{(j)}(z_{j})) + W^{(i)}(z), \]

where \( W^{(0)}(z_{i}) = 0. \)

It was shown in [12] that the solution of problem (3) in matrix form on the \( i \)th layer, taking into account the matching conditions (4), has the form

\[ V^{(i)}(z) = L^{(i)}(z, z_{i})V^{(i)}(z_{i}) + \sum_{j=1}^{i-1} L^{(i)}(z, z_{j})(W^{(j-1)}(z_{j}) - W^{(j)}(z_{j})) + W^{(i)}(z), \quad (10) \]

where

\[ W^{(0)}(z_{i}) = 0, \quad L^{(i, j)}(z, z_{j}) = K^{(i)}(z, z_{j})K^{(i-1)}(z_{j}, z_{j-1}) \ldots K^{(j)}(z_{j+1}, z_{j}), \quad i \geq j, \quad z_{i} \leq z \leq z_{i+1}. \]

Formula (10) gives an exact analytical solution to the Cauchy problem for equation (3) for an arbitrary number of layers.

At the end point of the system of layers, by (10), we obtain

\[ V^{(i)}(z_{n+1}) = L^{(i)}(z_{n+1}, z_{i})V^{(i)}(z_{i}) + \sum_{j=1}^{i-1} L^{(i)}(z_{n+1}, z_{j})(W^{(j-1)}(z_{j}) - W^{(j)}(z_{j})) + W^{(n)}(z_{n+1}). \quad (11) \]

Formula (11) relates the potential and flow values at the first and last points of the layer system, which allows in the general case to reduce the solution of the boundary value problem of the first, second, or third type for any finite number of layers to the solution of a system of two linear equations without unknowns.

Thus, the analytical matrix method can be applied to solving boundary value problems of heat and mass transfer of the first, second, or third type for system (3) in a multilayer medium with any finite number of layers.

Using this method to solve differential equation (1), (2), which describes the diffusion of MCC generated by a wide electron beam in a multilayer semiconductor target, we get that \( a^{(i)}_{1}(z) = D^{(i)} \) and \( a^{(i)}_{2}(z) = \tau^{(i)} \) in the \( i \)th layer, \( m = 1 \) and \( \mu^{(i)}(z) = -\tau^{(i)} \times \rho^{(i)}(z) \), where \( \rho^{(i)}(z) \) is the density of the MCC generated in the semiconductor in the \( i \)th layer. For constant coefficients of the equation on the \( i \)th layer, the matrix \( K \) takes the form

\[ K^{(i)}(z, z_{j}) = \begin{bmatrix} \text{ch} & \frac{z - z_{j}}{\sqrt{D^{(i)}\tau^{(i)}}} & -\frac{\tau^{(i)}}{D^{(i)}} \text{sh} & \frac{z - z_{j}}{\sqrt{D^{(i)}\tau^{(i)}}} \\ \frac{D^{(i)}}{\tau^{(i)}} \text{sh} & \frac{z - z_{j}}{\sqrt{D^{(i)}\tau^{(i)}}} & \text{ch} & \frac{z - z_{j}}{\sqrt{D^{(i)}\tau^{(i)}}} \end{bmatrix}. \]

When carrying out calculations by the matrix method in the problem under consideration, the following particular solution was used for the diffusion equation of the MCC (1):

\[ W^{(i)}(z) = K_{1} \exp \left( \frac{z_{ms}^{2}}{4L^{2}} \right) \exp \left( \frac{z}{z_{ms}} - 1 + \frac{z_{ms}}{2L} \right) \text{erf} \left( \frac{z}{z_{ms}} - 1 + \frac{z_{ms}}{2L} \right) - \exp \left( -\frac{z - z_{ms}}{L} \right) \text{erf} \left( \frac{z}{z_{ms}} - 1 + \frac{z_{ms}}{2L} \right) \]

\[ + K_{2} \exp \left( \frac{z_{ss}^{2}}{4L^{2}} \right) \exp \left( \frac{z - z_{ss}}{L} - 1 + \frac{z_{ss}}{2L} \right) - \exp \left( -\frac{z - z_{ss}}{L} \right) \text{erf} \left( \frac{z}{z_{ss}} - 1 + \frac{z_{ss}}{2L} \right), \]

where
\[ K_1 = \frac{-1.085(1-\eta)E_0L}{4D(1-\eta + \eta \frac{z_{ss}}{z_{ms}})}, \quad K_2 = \frac{-1.085z_{ss}E_0L\eta}{4Dz_{ms}(1-\eta + \eta \frac{z_{ss}}{z_{ms}})}. \]

4. Numerical solution

For the differential problem (1), (2), a conservative difference scheme [19] is constructed on a uniform grid \( \bar{D}_n = \{ z_i = ih, \ i = 0, \ldots, N, \ h = l/N \} : \)

\[
\frac{1}{h}\left( a_{i+1} \frac{\Delta p_{i+1} - \Delta p_i}{h} - a_i \frac{\Delta p_i - \Delta p_{i-1}}{h} \right) - d_i \Delta p_i = -\varphi_i, \quad 1 \leq i \leq N - 1,
\]

\[
\left( 1 + \frac{h^2}{2L_1} + \frac{S_1}{L_1} \right) \Delta p_0 - \Delta p_1 \frac{\tau_1 h^2 \rho_0}{2L_1}, \quad \left( 1 + \frac{h^2}{2L_2} + \frac{S_2}{L_2} \right) \Delta p_N - \Delta p_{N-1} \frac{\tau_2 h^2 \rho_N}{2L_2},
\]

(12)

where

\[
a_i = \left[ \frac{1}{h} \int_{z_{i-1}}^{z_i} \frac{dz}{D(z)} \right]^{-1} = \left[ \frac{1}{h} \int_{-1}^{0} \frac{dx}{D(z_i + sh)} \right]^{-1}, \quad d_i = \int_{-0.5}^{0.5} \rho(z_i + sh) ds, \quad \varphi_i = \int_{-0.5}^{0.5} \rho(z_i + sh) ds.
\]

Here \( \Delta p_i \), the approximate value of the exact solution \( \Delta p(z_i) \), \( \rho_i = \rho(z_i) \), and the integrals are replaced by their approximate expressions:

\[
\frac{1}{h} \int_{z_{i-1}}^{z_i} \frac{dz}{D(z)} - \frac{1}{D_{i-1/2}}, \quad d_i = \frac{1}{\tau_i}, \quad \varphi_i = \rho_i.
\]

System (12) was solved by the sweep method.

5. Calculation results

Comparison of analytical and numerical modeling of distributions of nonequilibrium minority charge carriers generated by a wide beam of medium-energy electrons was made for a two-layer semiconductor structure \( \text{Cd}_{0.2}\text{Hg}_{0.8}\text{Te}/\text{CdTe} \) which is widely used in the manufacture of infrared devices [13, 14]. The thickness of the structure was 0.4 \( \mu \text{m} \) (0.1 \( \mu \text{m} \) for \( \text{Cd}_{0.2}\text{Hg}_{0.8}\text{Te} \) and 0.3 \( \mu \text{m} \) for \( \text{CdTe} \)). Numerical calculations were performed by the finite difference method [19].

Figure 1 shows the results calculations of the relative density of energy loss by electron beam on depth. The calculations were performed for homogeneous single-crystals \( \text{Cd}_{0.2}\text{Hg}_{0.8}\text{Te} \) (curve 1) and \( \text{CdTe} \) (curve 2) at an electron beam energy of 20 keV. For heavier material (cadmium-mercury-tellurium), the energy loss region is closer to the target surface. At a depth of \( z > 0.25 \mu \text{m} \), an increase in the energy loss density due to backscattered electrons is seen for this material [1, 15-17].

Figure 2 shows the distributions of the MCC generated by the electron beam in the semiconductor structure \( \text{Cd}_{0.2}\text{Hg}_{0.8}\text{Te}/\text{CdTe} \) for various beam electron energies: 5 (curve 1), 10 (2), 15 (3), 20 (4), 25 (5), and 30 keV (6). The following parameter values were used: \( L_1 = 35 \mu \text{m} \), \( \tau_1 = 10^{-6} \text{s} \), \( S_1 = 0.0857 \) for \( \text{Cd}_{0.2}\text{Hg}_{0.8}\text{Te} \) and \( L_2 = 30 \mu \text{m} \), \( \tau_2 = 10^{-5} \text{s} \), \( S_2 = 10 \) for \( \text{CdTe} \).
Figure 1. Depth distribution of relative values of the energy loss density of the electron beam, calculated for homogeneous single crystal Cd$_{0.2}$Hg$_{0.8}$Te (solid curve) and CdTe (dashed curve). The electron energy of the beam is 20 keV.

The results of calculations by the analytical method and the numerical method of finite differences practically coincide and therefore are not highlighted in the figure. Highlighting the results for these two methods is presented in figure 3. Both methods give almost the same results.

Figure 2. Distributions of minority charge carriers generated by an electron beam in a two-layer semiconductor structure Cd$_{0.2}$Hg$_{0.8}$Te/CdTe 0.4 μm thick (0.1 μm – Cd$_{0.2}$Hg$_{0.8}$Te and 0.3 μm – CdTe) for different energies of electron beam: 5 (curve 1), 10 (2), 15 (3), 20 (4), 25 (5) and 30 (6) keV. The results of calculations by the analytical method and the numerical method practically coincide and therefore are not separately highlighted in the figure.
Figure 3. Distribution of minority charge carriers generated by an electron beam in a semiconductor structure Cd$_{0.2}$Hg$_{0.8}$Te/CdTe, calculated by the analytical method (continuous line) and the numerical method (marked by circles). The thickness of the first layer (cadmium-mercury-tellurium) is 0.1 μm, the beam electron energy is 20 keV.

The calculations were performed using the mathematical package Matlab (MathWorks, Inc.) and Maple on personal computer with the following characteristics: an Intel Pentium E5400 processor (2×2.70 GHz, 2MB Cache) and the RAM memory space was 2 GB. The numerical solution is calculated on a grid of 450 cells with a step of $h = 8.8889 \times 10^{-4}$ μm. The following estimate of the relative error in the norm of space $C$ is obtained:

$$\Delta_i \left( \frac{\Delta p(z_i)}{\Delta r_i} \right) = \frac{\| \Delta p(z_i) - \Delta r_i \|_C}{\| \Delta r_i \|_C} \cdot 100\% \approx 0.055\%.$$ 

We note that the error in the results is not high, and in the figures on the chosen scale the solutions obtained practically coincide. Machine time for calculating the distributions of MCC by analytical and numerical methods was about 2 s, which indicates the practical applicability of the proposed matrix analytical method for solving the problem under consideration.

6. Conclusions

An analytical matrix method that allows solving heat and mass transfer problems in multilayer planar structures with an arbitrary number of layers is described. For electrophysical parameters characteristic for a two-layer semiconducting structure Cd$_{0.2}$Hg$_{0.8}$Te/CdTe, the calculation results using the analytical matrix method and calculation results using a numerical conservative difference scheme are compared. It is shown that the proposed matrix method makes it possible to calculate the
distribution of nonequilibrium minority charge carriers in a relatively short time with an accuracy sufficient for practical use in electron-probe technologies.

Acknowledgments
This work was supported by the Russian Foundation for Basic Research, project no. 19-03-00271, and by the Russian Foundation for Basic Research and the Government of Kaluga Oblast, project no. 18-41-00001.

References
[1] Stepovich M A 2003 Quantitative cathodoluminescence microscopy of direct-gap materials of semiconductor optoelectronics Thesis Dr. Phys. and Math. Sci. (Bauman Moscow State Technical University, Moscow) 351 pp. [in Russian]
[2] Wittry D B and Kyser D F 1967 Measurements of diffusion lengths in direct-gap semiconductors by electron beam excitation J. Appl. Phys. 38 375
[3] Carslaw H S and Jaeger J C 1959 Conduction of Heat in Solids (Oxford University Press, Oxford) 517 pp.
[4] Afanasenkov Yu V and Gladyshev Yu A 2013 On the use of the flow matrix for solving boundary value problems on a graph Bulletin of the Saratov University. New series. Mathematics. Mechanics. Computer science 13 11
[5] Ginzgemyer S A 2006 Mathematical modeling of heat transfer processes in systems of contacting rods Thesis Ph. D. Phys. and Math. Sci. (Tsio1kovsky Kaluga State Pedagogical University, Kaluga) 163 pp. [in Russian]
[6] Bers L and Gelbart A 1944 On a class of functions defined by partial differential equations Transactions of the American Math. Society 56 67
[7] Gladyshev Yu A 1994 On a sequence of generalized Bers exponential functions with interior structure Mathematical Notes 55 21
[8] Gladyshev Yu A 2011 The method of generalized degrees of Bers and its applications in mathematical physics (Tsio1kovsky Kaluga State University, Kaluga) 204 pp. [in Russian]
[9] Gladyshev Yu A, Kalmanovich V V and Stepovich M A 2017 On the possibility of applying the Bers apparatus to modeling the processes of heat and mass transfer caused by electrons in a planar multilayer medium Journal of Surface Investigation. X-ray, Synchrotron and Neutron Techniques 11 1096
[10] Kalmanovich V V and Stepovich M A 2018 On the joint application of the matrix method and the apparatus of generalized Bers degrees for mathematical modeling of heat and mass transfer processes in semiconductor materials of electronic technology Problems of Developing Promising Micro- and Nanoelectronic Systems-2018: Proceedings (Ed. Academician of the RAS Stempkovsky A L, Institute for Design Problems in Microelectronics of Russian Academy of Sciences, Moscow) Is, III 194 [in Russian]
[11] Gladyshev Yu A, Kalmanovich V V, Seregina E V and Stepovich M A 2018 On the possibility of joint application of the matrix method and the apparatus of generalized Bers degrees for mathematical modeling of heat transfer in objects with cylindrical symmetry Questions of Atomic Science and Technology. Nuclear and Reactor Constants 3 158 [in Russian]
[12] Kalmanovich V V, Seregina E V and Stepovich M A 2019 On the possibility of a numerical solution of the heat and mass transfer problem with the combined matrix&generalized powers of Bers method J. of Physics: Conf. Series 1163 012012
[13] Filachev A M, Taubkin II and Trishenkov M A 2011 Solid State Photoelectronics. Photodiodes (Fizmatkniga, Moscow) 448 p. [in Russian]
[14] Filachev A M, Taubkin II and Trishenkov M A 2012 Solid State Photoelectronics. Photoresistors and photodetectors (Fizmatkniga, Moscow) 368 p. [in Russian]
[15] Mikheev N N, Nikonorov I M, Petrov V I and Stepovich M A 1990 Determining the electrophysical parameters of semiconductors in a raster electron microscope by the induced-
current and cathodoluminescence methods *Bulletin of the Academy of Sciences of the USSR. Physical Series* **54** 82

[16] Mikheev N N, Petrov V I and Stepovich M A 1991 Quantitative analysis of semiconductor optoelectronic materials by raster electron microscopy *Bulletin of the Academy of Sciences of the USSR. Physical Series* **55** 1

[17] Mikheev N N and Stepovich M A 1996 Distribution of energy losses in interaction of an electron probe with material *Industrial Laboratory* **62** 221

[18] Kanaya K and Okayama S 1972 Penetration and energy-loss theory of electrons in solid targets *J. Phys. D: Appl. Phys* **5** 43

[19] Samarsky A A 1977 *Theory of difference schemes* (Nauka, Moscow) 656 p. [in Russian]