THE SHAPES, ORIENTATION, AND ALIGNMENT OF GALACTIC DARK MATTER SUBHALOS

MICHAEL KUHLEN
School of Natural Sciences, Institute for Advanced Study, Einstein Lane, Princeton, NJ 08540; mk@ias.edu

JÜRGEN DIEMAND
Department of Astronomy and Astrophysics, University of California, Santa Cruz, CA 95064; diemand@ucolick.org

AND

PIERO MADAU
Department of Astronomy and Astrophysics, University of California, Santa Cruz, CA 95064; madau@ucolick.org

and European Southern Observatory, Karl-Schwarzschild-Strasse 2, D-85748 Garching, Germany; madau@ucolick.org

Received 2007 May 14; accepted 2007 August 13

ABSTRACT

We present a study of the shapes, orientations, and alignments of Galactic dark matter subhalos in the Via Lactea simulation of a Milky Way–size ΛCDM host halo. Whereas isolated dark matter halos tend to be prolate, subhalos are predominantly triaxial. Overall subhalos are more spherical than the host halo, with minor-to-major and intermediate-to-major axis ratios of 0.68 and 0.83, respectively. Like isolated halos, subhalos tend to be less spherical in their central regions. The principal axis ratios are independent of subhalo mass when the shapes are measured within a physical scale such as \( r_{200} \), the radius of the peak of the circular velocity curve. Subhalos tend to be slightly more spherical closer to the host halo center. The spatial distribution of the subhalos traces the prolate shape of the host halo when they are selected by the largest \( V_{\text{max}} \) they ever had, i.e., before they experienced strong tidal mass loss. The subhalos’ orientation is not random: the major axis tends to align with the direction toward the host halo center. This alignment disappears for halos beyond \( 3r_{200} \) and is more pronounced when the shapes are measured in the outer regions of the subhalos. The radial alignment is preserved during a subhalo’s orbit and they become elongated during pericenter passage, indicating that the alignment is likely caused by the host halo’s tidal forces. These tidal interactions with the host halo act to make subhalos rounder over time.

Subject headings: cosmology: theory — dark matter — galaxies: dwarf — galaxies: formation — galaxies: halos — methods: numerical

Online material: color figures

1. INTRODUCTION

Cold dark matter (CDM) halos are not smooth. Early numerical simulations of the formation of CDM halos lacked sufficient resolution to detect much besides the gross features of the mass distribution (Davis et al. 1985; Frenk et al. 1985, 1988; Quinn et al. 1986; Efstathiou et al. 1988). Rapid advances in computational power and in the efficiency of codes have afforded a dramatic reduction in the particle masses and gravitational softening lengths. Multimass techniques (e.g., Katz & White 1993; Bertschinger 2001) allow even further resolution increases in particular areas of interest. The resulting high-resolution maps of the matter distribution in CDM halos have revealed an astonishing abundance of substructure (Klypin et al. 1999; Moore et al. 1999; Diemand et al. 2006, hereafter Paper I). In Via Lactea, the most recent and highest resolution CDM simulation of a Galaxy-scale halo to date, the total number of identifiable subhalos has reached \( \sim 10,000 \), which together account for 5.6% of the total host halo mass (Paper I). Recent progress notwithstanding, the currently achievable mass resolution is orders of magnitude above the true cutoff in the CDM fluctuation power spectrum, and given the observed scale-invariant nature of the subhalo mass function \( (dN/d\ln M) \propto M^{-1} \), it is likely that the total substructure abundance and mass fraction has not yet converged.

CDM halos are not round. Whereas analytical work often treats CDM halos as spherically symmetric mass distributions, it has been known for some time (Barnes & Efstathiou 1987; Efstathiou et al. 1988; Frenk et al. 1988) that in general CDM halo shapes show significant departures from sphericity. There is now a large body of work concerning the shapes of isolated CDM halos (Dubinski & Carlberg 1991; Katz 1991; Warren et al. 1992; Dubinski 1994; Jing et al. 1995; Tormen 1997; Thomas et al. 1998; Bullock 2002; Jing & Suto 2002; Springel et al. 2004; Bailin & Steinmetz 2005; Kasun & Evrard 2005; Hopkins et al. 2005; Allgood et al. 2006; Bett et al. 2007; Hayashi et al. 2007; Macciò et al. 2007) and widespread agreement on a number of findings: CDM halos tend to be prolate, they are more spherical in their outer regions, more massive halos tend to have smaller axis ratios, the moment of inertia is aligned with the shape velocity anisotropy tensor, the smallest principal axis tends to line up with the angular momentum vector, etc. (for a recent summary, see Allgood et al. 2006).

With Via Lactea’s unprecedented resolution, we are now for the first time able to extend this shape analysis to a well-resolved subhalo population. This is interesting from a theoretical point of view because of the important role that tidal interactions play in shaping the properties of the subhalo population. The shapes of

\(^1\) Hubble Fellow.
subhalos are likely to be affected by tidal deformations, and a careful analysis of the subhalo shapes might allow us to better understand the tidal interactions.

Knowledge of subhalo shapes is important for several types of observational studies as well. Weak lensing is becoming a very valuable tool for probing cosmological parameters (Brown et al. 2003; Bacon et al. 2003; Hamana et al. 2003; Jarvis et al. 2003; Hoekstra et al. 2006; Massey et al. 2007) and constraining density profiles of galaxy groups and clusters (Brainerd 2004; Mandelbaum et al. 2006b). Any alignment between intrinsic galaxy ellipticity with other galaxies or the local mass density will introduce a bias into the lensing signal (Hirata & Seljak 2004; Bridle & King 2007). In fact, Lee et al. (2005) found evidence for an intrinsic alignment in subhalos and aligned host halos. From this they concluded that subhalo shapes could be responsible for the observed radial alignment of cluster galaxy isophotes. Our current knowledge of the relationship between the shapes of galaxies and the shapes of their host halos is quite limited, and the true relationship is probably not straightforward. Nevertheless, a better understanding of typical subhalo shapes and their alignment within the host halo might allow a statistical correction of the resulting bias.

Another observational arena dependent on subhalo shapes is stellar kinematical studies of Local Group dwarf galaxies, which are being used to constrain the masses of their dark matter host halos (Wilkinson et al. 2004; Lokas et al. 2005; Muñoz et al. 2005; Walker et al. 2006; Gilmore et al. 2007; Strigari et al. 2007). In almost all cases the analysis is performed assuming spherical symmetry and often also a constant velocity anisotropy. More sophisticated analyses of the dwarf galaxy stellar motions will benefit from firm theoretical expectations of the intrinsic dark matter subhalo shapes and should result in more realistic models of the underlying dark matter mass distribution.

This paper is organized as follows. In §2 we briefly outline the technique we employed to determine the shape of subhalos. We present the shape parameters of the Via Lactea host halo in §3, and move on in §4 to discuss the dependence of the subhalo’s shape parameters on the radius at which they are measured, on their distance from the center of the host, and on the subhalo’s mass. In §5 we present results concerning the spatial distribution of the subhalos within the host halo and the alignment of their ellipsoids toward the host halo center. In §6 we consider the redshift evolution of the shapes of a sample of strongly tidally affected subhalos. Section 7 contains a discussion and summary of our results.

2. SIMULATION AND SHAPE FINDING METHOD

The main sample of subhalos analyzed in this work stems from the $z = 0$ output of the Via Lactea simulation (Paper I). This simulation follows the dark matter substructure of a Milky Way-scale halo ($M_{200} = 1.77 \times 10^{12} M_{\odot}$) with 234 million particles. With a particle mass of $\approx 20,000 M_{\odot}$, the simulation resolves around 10,000 subhalos within $r_{200} = 388$ kpc. The global $z = 0$ properties of the host halo and the substructure population was presented in Paper I, and the joint temporal evolution of host halo and substructure properties, with an emphasis on tidal interactions, was discussed in Diemand et al. (2007, hereafter Paper II). Here we focus on the shapes of the matter distribution in Via Lactea’s host halo and subhalo population.

For the determination of (sub)halo shapes we follow the iterative technique outlined in Allgood et al. (2006). This method is based on diagonalizing the weighted “moment of inertia tensor,”

$$I_{ij} = \sum_n \frac{x_{i,n} x_{j,n}}{r_n^2},$$

where

$$r_n = \sqrt{x_n^2 + (y_n/q)^2 + (z_n/s)^2}$$

is the ellipsoidal distance in the eigenvector coordinate system between the (sub)halo’s center and the $n$th particle, and $q = b/a$ and $s = c/a$ are the intermediate-to-major and minor-to-major axis ratios, respectively ($a \geq b \geq c$). Initially $I_{ij}$ is calculated for all particles within a spherical window of radius $r_0$. In subsequent iterations we fix $a = r_0$ and include only particles with $r_n < r_0$. Iteration continues until $q$ and $s$ change by less than $10^{-3}$. The degree of triaxiality of a halo is quantified by the triaxiality parameter introduced by Frax et al. (1991),

$$T = \frac{1 - q^2}{1 - s^2}.$$

A halo is said to be oblate for $T < \frac{1}{3}$, triaxial for $\frac{1}{3} < T < \frac{2}{3}$, and prolate for $T > \frac{2}{3}$.

For comparison with observational data it is often more desirable to constrain the shape of the potential than the shape of the density distribution. The potential shape has the additional advantage that it is less sensitive to local density variations (from substructure, for example) and is typically smooth and well approximated by concentric ellipsoids (Springel et al. 2004; Hayashi et al. 2007). Instead of measuring the potential shape directly, or by fitting ellipses to the intersections of isopotential surfaces with three orthogonal planes (as advocated by Springel et al. 2004), we diagonalize the unweighted kinetic energy tensor,

$$K_{ij} = \frac{1}{2} \sum_n v_{i,n} v_{j,n},$$

where the $v_{i,n}$ are measured in the rest frame of the subhalo under consideration. The kinetic energy tensor $K_{ij}$ is related to the potential energy tensor $W_{ij} = \sum s_i d\phi/ds_j$ through the tensor virial theorem,

$$\frac{1}{2} \frac{d^2 I_{ij}}{dt^2} = 2K_{ij} + W_{ij}.$$

The internal structure of the Via Lactea host halo remains practically unchanged after $z = 1.7$ (Paper II), and we expect $d^2 I_{ij}/dt^2 = 0$ at $z = 0$. In that case the eigenvectors of $K_{ij}$ should reflect the principal axes of the potential ellipsoid. Note that the assumption of a constant $I_{ij}$ is likely to fail while the host halo is still relaxing and for subhalos that are being tidally stripped, and in this case the principal axes of $K_{ij}$ will not necessarily reflect...
Nevertheless the shape of the velocity ellipsoid is of interest for comparisons with observational data. Instead of the iterative procedure applied for $I_{ij}$, we simply diagonalize $K_{ij}$ for all particles within the moment of inertia ellipsoid at a given radius. In the following we refer to shapes determined by diagonalization of $I_{ij}$ and $K_{ij}$ as the “mass” and “velocity” shapes, respectively.

3. HOST HALO SHAPE

At the present epoch the Via Lactea host halo is prolate. We have determined the principal components of $I_{ij}$ and $K_{ij}$ as a function of major axis radius. The resulting shape ellipsoids are depicted in Figure 1. Neither the shape nor the orientation of the ellipsoids varies much out to $r_{200}$. As a function of radius the major axis eigenvectors for both mass and velocity shape are aligned to within $1°$. As expected the velocity ellipsoid is significantly rounder than the mass distribution, and the major axes of the two are aligned to within $5°$ at all radii. In Figure 2 we plot mass and velocity shape parameters $q$ and $s$ and triaxiality parameter $T$ at $z = 0$, 0.5, and 1 as a function of radius. Recall (from Paper II, Figs. 3 and 4) that the spherically averaged mass distribution of the Via Lactea host halo remains remarkably constant.

Fig. 1.—Shape of the Via Lactea halo as a function of radius, derived from diagonalizing the moment of inertia tensor $I_{ij}$ (top) and the velocity anisotropy tensor $K_{ij}$ as a proxy for the shape of the potential (bottom). The color indicates radius. The major principal axes at different radii are aligned to within $1°$, and the mass and velocity shapes are aligned to within $5°$. 
in proper coordinates after the last major merger at $z = 1.7$ until today. We now see, however, that the host halo does undergo some adjustments in its shape.

At $z = 0$ the mass distribution becomes slightly less spherical toward the center with axis ratios dropping from $~0.55$ at 200 kpc < $r$ < $r_{200}$ to $~0.45$ at 20 kpc, whereas the velocity shape axis ratios remain constant at around 0.8. The triaxiality parameter remains fairly in the prolate regime ($>\frac{1}{2}$) for both mass and velocity throughout the entire halo. The velocity ellipsoid becomes slightly more prolate in the inner regions. At earlier times, however, we detect some significant changes in the axis ratios, especially in the outer regions. At $z = 0.5$ both $q$ and $s$ are larger by about 0.1–0.15 beyond 100 kpc, and at $z = 1$ $q$ is significantly larger than $s$, resulting in a triaxial, as opposed to prolate, outer region beyond $~200$ kpc. The velocity ellipsoid exhibits variations of comparable magnitude. Note that the host halo accretes some fairly massive subhalos ($M_{\text{sub}}/M_{\text{host}} \sim 0.1$) between $z = 1$ and 0.5. Dynamical friction causes these subhalos to spiral into the center over a few orbits, and they lose most of their mass (>99%) in this process. The associated redistribution of material probably contributes to the observed shape adjustments.

The shape of the Via Lactea host halo is consistent with previous studies of the shapes of dark matter halos, which have generally found them to be mostly prolate (e.g., Allgood et al. 2006 and references therein). Observational studies of the shape of the Milky Way, however, have been much less conclusive. The observed flaring of the Milky Way’s H i disk (Olling & Merrifield 2000) and some models of the Sagittarius tidal stream (Ibata et al. 2001; Majewski et al. 2003; Martínez-Delgado et al. 2004; Fellhauer et al. 2006) suggest that the Milky Way halo is close to spherical ($q \approx s \approx 0.8$) and oblate, whereas some studies of the leading arm of the Sagittarius stream favor a prolate shape with $s = 0.6$ (Helmi 2004; Law et al. 2005).

The collisionless nature of our Via Lactea simulation prohibits a direct comparison between our findings and the observational constraints on the shape of the Milky Way host halo. Previous hydrodynamical numerical studies of galaxy formation have found that the cooling of baryons leads to significantly rounder halos (Dubinski 1994; Kazantzidis et al. 2004; Springel et al. 2004; Bailin et al. 2005), especially in the central regions where $s$ can increase by as much as 0.2–0.3. At the moment it is unclear how much of this effect is due to a potential overcooling problem in these simulations.

### 4. Subhalo Shapes

We now turn to the analysis of the shapes of Via Lactea’s subhalos. Unlike the host halo, which is resolved by over 10 million particles even within $0.1 r_{200}$, the subhalo shape determination is limited by numerical resolution. We restrict our analysis to halos containing at least 5000 particles within their tidal radius, $r_t$. As in Paper I, we define $r_t$ as the point where the subhalo’s spherically averaged density profile drops to twice the local underlying matter density. Since 5000 particles correspond to a tidal mass of $10^8 M_\odot$, we are probing down to the regime of many of the Local Group dwarf galaxies, whose kinematics are being modeled in detail by current observational studies (e.g., Gilmore et al. 2007; Strigari et al. 2007). Via Lactea contains 97 such halos within $r_{200}$, 309 within $3r_{200}$, and 829 in total.

One complication arises from the difficulty in distinguishing between actual subhalo particles and those belonging to the underlying host halo. Our subhalo finder (6DFOF, see Paper I) assigns all particles within the tidal radius to the subhalo, without removing unbound particles. Some contribution to $I_{ij}$ and $K_{ij}$ would thus come from background particles. This contribution is generally negligible for $I_{ij}$, since the background particles are more or less uniformly distributed throughout the subhalo. The host halo particles, however, significantly distort $K_{ij}$ because their velocities are typically much larger and more anisotropic than the subhalo particle velocities, especially for subhalos close to the host halo center. For this reason we have cleaned the subhalos from background host halo particles by comparing the members of each subhalo to those of its progenitor in the penultimate simulation output at $z = 0.005$ (~68.5 Myr before $z = 0$), and retained only those particles that also appear within $r_t$ at the earlier time. This effectively removes all host halo particles from the subhalo and allows an accurate determination of $K_{ij}$. We have confirmed that the shape of $I_{ij}$ is not affected by this cleaning procedure.

The subhalo tidal radius shrinks as subhalos pass through the inner regions of the host halo due to the increasing background density. However, as we showed in Paper II, not all particles beyond this reduced $r_t$ are actually stripped from the subhalo, and $r_t$ reexpands as the subhalo begins its climb out of the host halo potential. Measuring the shape for all particles within the tidal radius thus probes more central regions for subhalos closer to the halo center than for subhalos in the outer regions. To avoid this bias we focus on the subhalo shapes for all particles within the radius of the peak of the circular velocity curve, $r_{V_{\text{max}}}$. This choice comes at the cost of a reduced number of particles, but $r_{V_{\text{max}}}$ does not temporarily decrease at pericenter passage.
Intermediate-to-major axis ratio, $q$, of the Journal for a color energy tensor $I_{ij}$ and by the thin dashed lines for $K_{ij}$. Top: Intermediate-to-major axis ratio, $q = b/a$. Middle: Minor-to-major axis ratio $s = c/a$. Bottom: Triaxiality parameter, $T = (1 - q^2)/(1 - s^2)$. [See the electronic edition of the Journal for a color version of this figure.]

4.1. Shape versus Radius

We first discuss the dependence of subhalo shape on the radius at which the shapes are measured (see Fig. 3). For this analysis we included the complete sample of 829 halos with more than 5000 particles within their tidal radius. Most of these halos currently lie outside $r_{200}$, but a significant fraction (78% within $2r_{200}$) have passed through the host halo at some earlier time (Paper II) and have experienced tidal interactions. As in the Via Lactea host halo, we find that the subhalos’ mass distributions become slightly less spherical in the inner regions; $\langle q \rangle$ decreases from about 0.87 at $3r_{\text{vir}}$ to 0.68 at 0.2$r_{\text{vir}}$, the innermost point at which we determine subhalo shapes. Similarly, $\langle s \rangle$ decreases from 0.72 to about 0.45. The 68% scatter is about 0.2 for both $q$ and $s$. As expected the velocity ellipsoid shows even less dependence on radius and is almost spherical, with $\langle q \rangle \approx 0.9$ and $\langle s \rangle \approx 0.85$. The velocity axis parameters also have a smaller 68% scatter of about 0.1.

Unlike isolated dark matter halos, which tend to be prolate, we find that subhalos are generally triaxial: $\langle T \rangle$ shows a slight decreasing trend with radius, but remains in the triaxial regime for both mass and velocity shape throughout the range of radii that we probed. Note that the 68% scatter around $\langle T \rangle$ extends into both the prolate and oblate regimes. At $r_{\text{vir}}$ about 25% of all (sub)halos are prolate and an equal number oblate.

4.2. Shape versus Distance

Next we take a look at the dependence of subhalo shapes on the distance from the halo center. For this purpose we use the shapes determined from all particles within $r_{\text{vir}}$. Figure 4 shows that the shape of the mass distribution is close to independent of distance, with a weak but significant trend toward slightly larger axis ratios closer to the host halo center. Within $r_{200}$ the mean axis ratios are $\langle q \rangle = 0.87$ and $\langle s \rangle = 0.74$, and outside $r_{200}$ they are $\langle q \rangle = 0.81$ and $\langle s \rangle = 0.64$. The velocity shape is independent of distance, with $\langle q \rangle = 0.93$ and $\langle s \rangle = 0.86$. The ellipsoids remain predominantly triaxial over the whole range of distance, for both mass and velocity.

One might have expected the reverse trend, with less spherical subhalos closer to the center, since tidal interactions, which are stronger near the center, tend to elongate the mass distribution in the radial direction. The absence of such a trend is likely a consequence of measuring the shapes within $r_{\text{vir}}$, which is deep inside the halo mass distribution and not as strongly affected by tides. The slight increase in axis ratios toward the center is due to the fact that subhalos in the central regions have experienced more extensive tidal interactions with the host halo, which over time tend to make them more spherical (see § 6). When the mass distribution shapes are measured within $r_{i}$ (Fig. 4, dashed lines and open triangles), we do observe a small dip in mean axis ratios at the innermost point. This is partially due to the outer regions of subhalos becoming significantly elongated at pericenter passage (see § 6). An additional contribution to the central dip may have been the result of the recent escape of the subhalo from the host halo.
arise from the temporary decrease of \( r_t \) as the subhalo passes through pericenter and the shape-radius dependence discussed in § 4.1.

4.3. Shape versus \( V_{\text{max}} \)

Lastly, we consider the shape parameters as a function of the subhalo’s \( V_{\text{max}} \), a proxy for mass (Fig. 5. Again we use all 829 subhalos in the simulation. Allgood et al. (2006) found that the axis ratios of isolated field halos measured at 0.3\( r_{\text{vir}} \) decreased with increasing halo mass; i.e., less massive halos are more spherical. In contrast, we find no such trend for subhalos over most of the mass range. With the exception of the highest mass bin, both \( q \) and \( s \) appear to be independent of \( V_{\text{max}} \).

The highest \( V_{\text{max}} \) bin (centered on 35 km s\(^{-1}\)) has \( \langle s \rangle = 0.57 \pm 0.05 \), a bit less than would be expected from an extrapolation of the Allgood et al. (2006) \( \langle s \rangle - M_{\text{halo}} \) relation, which gives \( \langle s \rangle \approx 0.69 \pm 0.04 \) for 10\(^7\)\( M_\odot \) for \( \sigma_8 = 0.74 \). All but one of the six halos in this bin lie within 2\( r_{200} \) and can thus be considered subhalos. In Allgood et al. (2006) shapes were measured at 0.3\( r_{\text{vir}} \), whereas our subhalo shapes are measured at \( r_{\text{vir}} \). We find it difficult to assign a “virial” radius to subhalos (Paper II) and instead use the tidal radius \( r_t \) as the outer “edge” of the subhalo. The median ratio of \( r_{\text{vir}} \) to \( r_t \) in this bin is 0.16, and for a 10\(^7\)\( M_\odot \) field halo with a concentration of 15, \( r_{\text{vir}}/r_{\text{vir}} = 2.163/c = 0.14 \), so it is likely that we are simply measuring the shapes farther in, where axis ratios tend to be smaller (see Fig. 3). For comparison, \( \langle s \rangle = 0.70 \) when measured at \( r_t \).

At any rate, our mean subhalo axis ratios become independent of mass at lower \( V_{\text{max}} \) and thus increasingly discrepant with the Allgood et al. (2006) shape-mass relation for isolated halos. Our lowest \( V_{\text{max}} \) bin corresponds to a tidal mass of \( \sim 2 \times 10^8 \ M_\odot \), for which the Allgood et al. (2006) relation would predict \( \langle s \rangle \) of 0.84 \pm 0.06, whereas we measure \( \langle s \rangle = 0.72 \) at \( r_t \). It is possible that tidal interactions cause this flattening of the subhalo shape-mass relation. Another possibility is that halo shapes (for both subhalos and field halos) are in fact intrinsically independent of mass when measured at a fixed physical scale, such as \( r_{\text{vir}} \). In this case the mass dependence found at 0.3\( r_{\text{vir}} \) would in effect just be combination of radius dependence and the halo mass–concentration relation: 0.3\( r_{\text{vir}} \) is smaller than \( r_{\text{vir}} \) in massive, low-concentration halos. This means that relative to \( r_{\text{vir}} \), the shapes of more massive halos are probed at smaller radii, where axis ratios tend to be smaller. Note also that in this picture the redshift dependence observed by Allgood et al. (2006) might in part be a consequence of a window (0.3\( r_{\text{vir}} \)) which becomes larger with time due to its comoving definition. Further investigations will be necessary to fully address this issue.

5. SUBHALO DISTRIBUTION AND SHAPE ALIGNMENT

We have discussed the dependence of subhalo shapes on their properties, so we now consider the spatial distribution and alignment within the host halo.

5.1. Spatial Distribution

The spatial distribution of subhalos depends sensitively on the sample selection criterion. In Paper II we showed that the \( z = 0 \) distribution of Via Lactea subhalos is antibiased compared to the mass distribution of the host halo. For subhalos selected to have a \( z = 0 \) mass greater than \( 4 \times 10^6 \ M_\odot \), the ratio of subhalo number density to host halo mass density is simply proportional to radius, whereas for subhalos selected to have a \( z = 0 \) \( V_{\text{max}} \) greater than 5 km s\(^{-1}\), this ratio scales as the enclosed mass. Previous studies have found that when subhalos are selected by their mass or \( V_{\text{max}} \) at the time of accretion, this bias is substantially reduced, and the resulting subhalo number density profile more closely traces the underlying mass distribution (Nagai & Kravtsov 2005; Faltenbacher & Diemand 2006).

Here we extend this analysis to a full three-dimensional spatial distribution of the subhalos. In addition to the two samples considered in Paper II, we include three samples selected by the highest \( V_{\text{max}} \), a subhalo reaches throughout its lifetime, a quantity we refer to as \( V_{\text{max, p}} \). These three samples are limited to the 100, 500, and 1000 subhalos with the largest \( V_{\text{max, p}} \). This type of selection is designed to remove the bias introduced by tidal interactions, since the selection is performed on subhalo properties prior to their interaction with the host. For each sample of subhalos we diagonalize a weighted “moment of inertia” tensor (eq. [1]) constructed from the \( z = 0 \) positions of the subhalos, without regard for their masses. We then calculate the axis ratios of the resulting ellipsoids and the angles between their principal axes and the principal axes of the host halo’s mass distribution measured at \( r_{200} \).

The spatial distribution of all subhalo samples is triaxial, with \( s \) comparable to the underlying host halo density distribution and slightly larger \( q \). Not all of the ellipsoids, however, are aligned with the host halo. The major axis of the ellipsoid defined by all subhalos with \( M_t > 4 \times 10^6 \ M_\odot \) (\( V_{\text{max}} > 5 \) km s\(^{-1}\)) is tilted by 10.1° (13.5°) with respect to the host halo’s major axis. In general, the subhalo samples selected by \( V_{\text{max, p}} \) are more closely aligned with the host halo. The best alignment is found for the sample...
consisting of the 500 subhalos with the largest $V_{\text{max},p}$, whose principal axes are tilted by less than 5° from the host halo’s. The results for all samples are summarized in Table 1. Note that Zentner et al. (2005), Libeskind et al. (2005, 2007), and Kang et al. (2007) also find that the substructure distribution is well aligned with the host halo orientation.

The radial dependence of the subhalo number density is presented in Figure 6, where we plot the ratio of the subhalo number density to the host halo’s mass density as a function of $r_e$, the ellipsoidal radius (eq. [2]). The ratio has been normalized to unity at $r_e = r_{200}$, in order to highlight the radial dependence in the interior of the host halo. The top panel clearly shows that the radial dependence published in Paper II also holds for ellipsoidal binning: subhalo samples selected according to their present mass or $V_{\text{max}}$ are antibiased with respect to the host halo mass distribution, with the former scaling as $n_{\text{sub}}/\rho \propto r$ down to $\sim 60$ kpc and the latter as $n_{\text{sub}}/\rho \propto M(< r)$ for the entire range of radii probed. The bottom panel shows that a selection based on $V_{\text{max},p}$ removes a lot of this antibias. All six samples have a radial number density dependence closer to the host halo mass distribution than the samples selected according to $z = 0$ properties. The more restrictive the selection criterion is, the larger $n_{\text{sub}}/\rho$ becomes. For some of the selections the ratio even exceeds unity from 80 kpc to $r_{200}$, indicating that these samples are spatially biased with respect to the host halo; i.e., their abundances falls off with radius faster than the underlying density distribution. This bias is probably a consequence of dynamical friction. The subhalos with the largest $V_{\text{max},p}$ were massive enough to experience some degree of dynamical friction and spiraled in toward the center. Along the way they lost mass, reducing the dynamical friction and preventing them from completely merging with the host. This process would preferentially remove such subhalos from the outer regions.

The “top 500 $V_{\text{max},p}$” sample comes closest to tracing $\rho(r)$ (both in radial dependence and in the orientation of the shape ellipsoid), and we have plotted it in the top panel for direct comparison with the samples discussed in Paper II. The decrease in $n_{\text{sub}}/\rho$ below $r_e = 70$ kpc is probably at least partially due to numerical resolution, since the decrease is smaller for larger, and therefore better resolved, subhalos.

### 5.2. Radial Alignment

Via Lactea’s very high numerical resolution allows us to investigate the orientation of dark matter subhalos with respect to the host halo center. We find strong evidence for preferential radial alignment of the subhalo triaxial mass distribution.

In the top panel of Figure 7 we plot the distribution of $|\cos \theta|$, the absolute value of the cosine of the angle between each of the principal axes of the subhalo’s triaxial ellipsoid and the radius vector from the host halo center, for the 97 Via Lactea subhalos within $r_{200}$. If the subhalo ellipsoids were randomly distributed with respect to the halo center, this distribution would be flat. Instead we see that the major axis distribution is strongly peaked toward large values of $|\cos \theta|$, indicating that the major axis preferentially points toward the halo center. Correspondingly, the intermediate and minor axes are biased toward low values of $|\cos \theta|$.

In the middle panel of Figure 7 we plot the same distribution for all halos outside $3r_{200}$. The distribution is flat, showing no evidence for alignment of any of the principal axes. The alignment effect appears to only be present for subhalos physically associated with the Via Lactea host halo, and this is one piece of evidence for a tidal origin of this radial alignment.

For comparison we have plotted (Fig. 7, bottom) the same distribution for the subhalos of a galaxy-cluster-scale dark matter halo, the 14 million particle “D12” cluster discussed in Diemand et al. (2004). The same subhalo shape alignment is present here, too. Ragone-Figueroa & Plionis (2007) and Faltenbacher et al.
that the subhalo spatial distribution traces the host halo’s mass distribution together imply that our subhalo sample should in principle also exhibit a weak secondary alignment of the subhalo major axis with the host halo’s major axis. Perhaps a larger sample of high-resolution host halos and their subhalo population would allow us to detect evidence for such a direct alignment.

A possible mass dependence in the strength of this direct alignment signal could arise from the fact that more massive isolated subhalos, we find no evidence for such a direct alignment. If tidal interactions are indeed responsible for the radial alignment, then it may be expected that the alignment would be more pronounced in the outer regions of the subhalo, where tidal effects are strongest. Indeed we observe just such a trend when we look at the mean of $\mid \cos \theta \mid$ and $f(\cos \theta > 0.8)$, the fraction of subhalos with $\mid \cos \theta \mid > 0.8$, versus subhalo radius (see Fig. 8). For this analysis we restrict ourselves to subhalos within $r_{200}$ and consider radii as a fraction of both $r_s$ and $r_{\text{max}}$. When measured at $r_s$, about 55% of all subhalos have a major axis that is aligned to within 35° of the direction toward the halo center and $\langle \mid \cos \theta \mid \rangle = 0.75$. Both $\langle \mid \cos \theta \mid \rangle$ and $f(\cos \theta > 0.8)$ decrease monotonically toward smaller $r/r_s$. The mean tidal radius for the 97 subhalos in this sample is $r_s = 10.6$ kpc, and $r_{\text{max}} = 2.31$ kpc. Not surprisingly, the alignment signal is less pronounced when shapes are measured at $r_{\text{max}}$. However, even in this case the alignment is significant: $\langle \mid \cos \theta \mid \rangle = 0.58$, with an uncertainty (= $\text{var} (\mid \cos \theta \mid)$/$N_{\text{sub}}^{1/2}$) of 0.03, which corresponds to a $\sim 2.5 \sigma$ significance.

Note that while Lee et al. (2005) find that the subhalo minor axes are preferentially perpendicular to the host halo’s major axis in a study of cluster-scale subhalos, and Faltenbacher et al. (2007a) find a similar but weaker alignment signal in group-scale subhalos, we find no evidence for such a direct alignment in the Via Lactea subhalo population. None of the principal axes of our subhalo population are significantly correlated with any of the host halo’s principal axes. Of course the facts that the major axes of our Via Lactea subhalos point toward the host halo center and that the subhalo spatial distribution traces the host halo’s mass distribution together imply that our subhalo sample should in principle also exhibit a weak secondary alignment of the subhalo...
Halos tend to be less spherical. The subhalo population in a cluster-scale halo would thus trace a more elongated host halo mass distribution, and a larger fraction of subhalos would be found close to the host halo major axis. Assuming radial alignment, averaging over all subhalos would thus lead to a stronger direct alignment signal for clusters than in the case of a group- or galaxy-scale halo, for which the subhalo spatial distribution would be more spherical.

6. REDSHIFT EVOLUTION

The fact that the radial subhalo alignment is stronger for subhalos closer to the host halo center (see Fig. 7) and that the alignment signal is more pronounced when the shapes are measured in the outer regions of the subhalo (see Fig. 8) is suggestive of a tidal origin of the alignment. In this section we present additional support for this hypothesis by looking at the temporal evolution of the shapes and orientations of a small sample of subhalos, chosen according to the following criteria.

1. The subhalos must lie within $r_{200}$ at $z = 0$.
2. They must have undergone at least three pericenter passages since $z = 1.7$.
3. They must have experienced significant tidal mass loss, $\Delta M/M = 1.0 - M(z = 0)/M(z = 1.7) > 0.4$.
4. They must contain more than 4000 particles at $z = 0$ ($M(z = 0) > 8 \times 10^7 M_\odot$).

These constraints result in a sample of 19 well-resolved subhalos that have experienced significant tidal interactions with the host halo.

In Figure 9 we show the temporal evolution of $\langle q \rangle$, $\langle s \rangle$, and $\langle V_{\text{max}} \rangle$ for this subsample. The $\langle V_{\text{max}} \rangle$ curve shows that these subhalos experience most of their mass growth early on and then continually lose mass due to tidal interactions with the host halo (see Paper II for a more extensive discussion). Here we show that this tidal interaction also leads to subhalos becoming rounder with time on average. At formation they have $\langle q \rangle \approx 0.65$ and $\langle s \rangle \approx 0.5$, but these grow as they are captured and begin to feel the host halo’s tidal field. After $z = 1$ $\langle q \rangle$ remains stable at about 0.9, but $\langle s \rangle$ is still increasing slightly, reaching $\sim 0.85$ at $z = 0$.

| Subhalo ID | $M_t (M_\odot)$ | $V_{\text{max}}$ (km s$^{-1}$) | $M_t (M_\odot)$ | $V_{\text{max}}$ (km s$^{-1}$) | $\Delta M/M$ |
|------------|-----------------|-------------------------------|-----------------|-------------------------------|-------------|
| 04242...... | $2.4 \times 10^8$ | 15.4                          | $3.7 \times 10^8$ | 28.2                          | 0.93        |
| 10876...... | $1.6 \times 10^8$ | 14.6                          | $4.6 \times 10^8$ | 18.8                          | 0.66        |
| 13351...... | $4.4 \times 10^8$ | 18.8                          | $2.6 \times 10^8$ | 66.9                          | 0.84        |
| 13467...... | $1.2 \times 10^8$ | 14.3                          | $2.8 \times 10^8$ | 28.6                          | 0.96        |
| 18412...... | $1.3 \times 10^8$ | 12.5                          | $4.0 \times 10^8$ | 32.5                          | 0.97        |
| 21500...... | $8.4 \times 10^7$ | 11.9                          | $1.5 \times 10^8$ | 13.4                          | 0.45        |

Fig. 10.—Proper space orbits and shapes of six subhalos (see text for a discussion of their selection) as function of time. Colors indicate the distance in proper kiloparsecs from the host halo center, which is indicated by the red icosahedron. The symbols depict ellipsoids of a constant size (major principal axis of 10 kpc) whose axis ratios and orientations are determined from all particles within the subhalo’s tidal radius. Ellipsoids are plotted for outputs between $z = 2$ and 0 with a stride of 4 outputs, corresponding to a time interval of 274 Myr. The major principal axes are indicated by short solid lines.
For comparison we also show the $z = 0$ mean axis ratios of the 520 halos outside $3r_{200}$. Owing to their large distance from the host halo, these halos have experienced weaker tidal interactions. As expected these halos are less spherical than the tidally stripped sample: $\langle q \rangle = 0.85$ and $\langle s \rangle = 0.72$. This further supports the notion that tidal interactions tend to make subhalos rounder.

In the following we further restrict our sample and look in more detail at the orbits of individual subhalos and the time dependence of their shapes and alignments. For this purpose we hand-selected five illustrative examples of subhalos with small pericenters ranging from 9.6 to 32 kpc. For comparison we also included one subhalo (No. 21500) with a higher angular momentum orbit and a pericenter of 69 kpc. The properties of the selected subhalos are summarized in Table 2.

In Figure 10 we present a three-dimensional visualization of the orbits for the six subhalos in this sample. Starting with the $z = 0$ output and for every fourth output thereafter (corresponding to a time interval of 274 Myr) up to $z = 2$, we have plotted at the subhalo’s center of mass location an ellipsoid whose orientation and axis ratios are determined from all particles within the subhalo’s tidal radius. The major axis of each ellipsoid is indicated by a short solid line. For clarity we used a fixed major axis length of 10 kpc, although the tidal radii vary along the orbit. The orbits are shown in proper coordinates in the rest frame of the host halo, and the sides of the box range from $-300$ to $+300$ kpc.

The most striking feature of these plots is that the radial alignment of the subhalo is preserved throughout the majority of its orbit. The subhalos’ ellipsoids perform a near-perfect figure rotation such that the major axis continually points close to the host halo’s center. This figure rotation, however, is not seen in subhalo 21500, the one with the higher angular momentum orbit. Its orientation is almost independent of its orbital position.

We have quantified these trends in Figure 11, where we plot each subhalo’s radius, its axis ratios $q$ and $s$, and its $|\cos \theta|$ as a function of time. The evolution of $q$ and $s$ very closely tracks the orbit of the subhalo. Every time a subhalo passes through pericenter its axis ratios decrease. Note that part of this decrease can be attributed to the fact that the tidal radii shrink during pericenter passage, and so we are effectively measuring the subhalo shapes farther in, where they are intrinsically less spherical (see § 4.1). However, $r_t$ drops below $r_{\text{tidal}}$ only for one or two outputs close to pericenter and for most of the orbit remains in the regime where the axis ratios are almost independent of radius. Thus, we conclude that the temporary decrease in $r_t$ at pericenter is not

![Figure 11](image-url)
sufficient to explain the full extent of the correlation between axis ratios and orbital position. Furthermore, we see from the plot of $|\cos \theta|$ that the subhalos are indeed pointing close to the host halo center for the majority of their orbits. During the fast pericenter passages $|\cos \theta|$ drops significantly, but remains close to unity almost everywhere else. There are a few exceptions, where $|\cos \theta|$ drops below unity even away from pericenter, for example at $z = 0.7$ for subhalo 04242 and at $z = 0.4$ for 10876 and 13467, and these are most likely caused by close passages to other massive subhalos.

Together these trends provide strong evidence for a tidal origin of the radial alignment of subhalos. As they orbit the host halo’s center of mass, tidal forces continually distort the subhalos’ mass distribution, stretching them along the radial direction and compressing them in the perpendicular directions. This tidal distortion is stronger the closer the subhalo gets to the host halo center, but the pericenter passage occurs so quickly that the subhalo does not have enough time to adjust its orientation to point toward the center. Note that owing to their complicated orbits the subhalos’ actual orientation with respect to the host halo center is likely to change between subsequent pericenter passages. This means that a given subhalo will be stretched in different directions at each pericenter passage, and averaged over time this may cause them to become more spherical.

Our subsample, of course, was chosen to have experienced strong tidal interactions by requiring $\Delta M/M > 0.4$ and at least three pericenter passages. However, as we showed in Paper II, more than half of all subhalos lose more than 50% of their mass from $z = 1$ to 0, and so tidally caused radial alignment is expected for most subhalos. As mentioned above, subhalo 21500 is one example of a subhalo that does not appear to experience much radial alignment. We found that of the 19 subhalos for which we performed a time-dependent analysis, only three subhalos exhibit a similar lack of alignment-orbit correlation.

7. CONCLUSION

The main conclusions of this paper are summarized as follows.

1. The shape of the Via Lactea host halo is prolate and, slightly less spherical (lower axis ratios) in the central regions. The shape of the velocity ellipsoid ($q \approx s \approx 0.8$) is significantly rounder than the mass distribution ($q \approx s \approx 0.4-0.55$).

2. Whereas isolated halos tend to be prolate, we find that subhalos are predominantly triaxial. Overall subhalos are more spherical than the host halo. The mass shape has $\langle q \rangle = 0.83$ and $\langle s \rangle = 0.68$, averaged over all subhalos within $3 r_{200}$. The velocity ellipsoid for these subhalos is very close to spherical, with $\langle q \rangle = 0.93$ and $\langle s \rangle = 0.86$. Like isolated halos, subhalos tend to be slightly less spherical in the central regions.

3. We find a weak trend toward larger axis ratios for subhalos closer to the host halo center. Within $r_{200}$, $\langle q \rangle = 0.87$ and $\langle s \rangle = 0.74$, compared with $\langle q \rangle = 0.82$ and $\langle s \rangle = 0.65$ for all subhalos outside $r_{200}$.

4. For subhalos with $V_{\text{max}} < 30 \text{ km s}^{-1}$ the axis ratios are independent of $V_{\text{max}}$. At higher $V_{\text{max}}$ they are slightly lower.

5. The spatial distribution of subhalos matches the prolate shape of the host halo when subhalos are selected by $V_{\text{max}, p}$, the largest $V_{\text{max}}$ they ever had during their lifetime. This type of selection also results in an ellipsoidal radius dependence of the subhalo abundance that more closely follows the mass distribution of the host halo than the unbiased distributions from selections based on $z = 0$ subhalo properties such as $M_t$ and $V_{\text{max}}$.

6. The orientation of subhalo shape ellipsoids is not random. The major principal axis of the subhalo mass distribution tends to align with the direction toward the halo center. This alignment disappears for subhalos beyond $\sim 3 r_{200}$ and is more pronounced when the shape is measured in the outer regions of the subhalo.

7. Tidal interactions with the host halo tend to make the subhalos rounder over time.

8. For the majority of subhalos whose temporal evolution we studied here in detail, the radial alignment is preserved during the subhalo’s orbit, and the axis ratios decrease during pericenter passage. We conclude that the radial subhalo alignment is likely caused by tidal interactions with the host halo.

Support for this work was provided by NASA grants NAG5-11513 and NNG04GK85G. M. K. gratefully acknowledges support from the Institute for Advanced Study. J. D. acknowledges support from NASA through Hubble Fellowship grant HST-HF-01194.01. The Via Lactea simulation was performed on NASA’s Project Columbia supercomputer system. The visualizations in Figures 1 and 10 were created with VisIt (http://www.llnl.gov/visit/), a visualization tool developed by the Department of Energy’s Advanced Simulation and Computing Initiative.

REFERENCES

Agustsson, I., & Brainerd, T. G. 2006, ApJ, 644, L25
Allgood, B., Flores, A. R.,Primack, J. R.,Kravtsov, A. V., Wechsler, R. H., Faltenbacher, A., & Bullock, J. S. 2006, MNRAS, 367, 1781
Bacon, D. J., Maussey, R. J., Refregier, A. R., & Ellis, R. S. 2007, MNRAS, 384, 673
Bailin, J., & Steinmetz, M. 2005, ApJ, 627, 647
Bailin, J., et al. 2005, ApJ, 627, L17
Barnes, J., & Efstathiou, G. 1987, ApJ, 319, 575
Bertschinger, E. 2001, ApJS, 137, 1
Bett, P., Eke, V., Frenk, C. S., Jenkins, A., Helly, J., & Navarro, J. 2007, MNRAS, 376, 215
Brauder, T. G. 2004, in AIP Conf. Proc. 743, The New Cosmology, ed. R. E. Allam, D. V. Nanopolou, & C. N. Pope (Melville: AIP), 129
Bridle, S., & King, L. 2007, New J. Phys., submitted (arXive:0705.0166)
Brown, M. L., Taylor, A. N., Bacon, D. J., Gray, M. E., Dye, S., Meisenheimer, K., & Wolf, C. 2003, MNRAS, 341, 100
Bullock, J. S. 2002, in The Shapes of Galaxies and Their Dark Halos, ed. P. Natarajan (River Edge: World Scientific), 109
Ciotti, L., & Dutta, S. N. 1994, MNRAS, 270, 390
Diemand, J., Kuhlen, M., & Madau, P. 2006, ApJ, 649, 1 (Paper I)
———. 2007, ApJ, 667, 859 (Paper II)
Diemand, J., Moore, B., & Stadel, J. 2004, MNRAS, 353, 624
Dubinski, J. 1994, ApJ, 431, 617
Dubinski, J., & Carlberg, R. G. 1991, ApJ, 378, 496
Efstathiou, G., Frenk, C. S., White, S. D. M., & Davis, M. 1988, MNRAS, 235, 715
Faltenbacher, A., & Diemand, J. 2006, MNRAS, 369, 1698
Faltenbacher, A., Jing, Y. P., Li, C., Mao, S., Mo, H. J., Pasquali, A., & van den Bosch, F. C. 2007a, ApJ, submitted (arXive:0706.0262)
Faltenbacher, A., Li, C., Mao, S., van den Bosch, F. C., Yang, X., Jing, Y. P., Pasquali, A., & Mo, H. J. 2007b, ApJ, 662, L71
Fellhauer, M., et al. 2006, ApJ, 651, 167
Franx, M., Illingworth, G., & de Zeeuw, T. 1991, ApJ, 383, 112
Frenk, C. S., White, S. D. M., Davis, M., & Efstathiou, G. 1988, ApJ, 327, 507
Frenk, C. S., White, S. D. M., Efstathiou, G., & Davis, M. 1985, Nature, 317, 595
Gilmore, G., Wilkinson, M. I., Wyse, R. F. G., Klyemax, T. J., Koch, A., & Evans, N. W. 2007, ApJ, 663, 948
Hamana, T., et al. 2003, ApJ, 597, 98
Hayashi, E., Navarro, J., & Springel, V. 2007, MNRAS, 377, 50
Helfri, A. 2004, ApJ, 610, L97
Hirata, C. M., & Seljak, U. 2004, Phys. Rev. D, 70, 063526
Hoekstra, H., et al. 2006, ApJ, 647, 116
