Study of the colour singlet model with $k_T$-factorization in inclusive $J/\psi$ production at CERN LEP2

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Abstract

We calculate the cross section and the transverse momentum distribution of inclusive $J/\psi$ production in $\gamma\gamma$ collisions at CERN LEP2 within the colour singlet model and the $k_T$-factorization approach, including both direct and resolved photon contributions. The unintegrated gluon distribution in the photon is determined, using the Bl"umlein’s prescription for unintegrated gluon distribution in a proton. We compare our theoretical predictions with preliminary data taken by the DELPHI collaboration at LEP2. In addition, we present our predictions for the $J/\psi$ polarization properties.

1 Introduction

Heavy quark and quarkonium production at high energies has been vigorously studied in recent years from both theoretical and experimental viewpoint. At the modern collider conditions these processes are so called semihard ones [1–4]. In such processes by definition the hard scattering scale $\mu \sim m_Q$ is large compare to the $\Lambda_{\text{QCD}}$ parameter but on the other hand $\mu$ is much less than the total center-of-mass energy: $\Lambda_{\text{QCD}} \ll \mu \ll \sqrt{s}$. The last condition implies that the processes occur in small $x$ region: $x \approx m_Q/\sqrt{s} \ll 1$, and that the cross sections of heavy quark and quarkonium production are determined by the behavior of gluon distributions in the small $x$ region.

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It is also known that in the small $x$ region it becomes necessary to take into account the dependences of the subprocess cross section and gluon distribution on a gluon transverse momentum $k_T$ [1–4]. Therefore calculations of heavy quark and quarkonium production cross sections at HERA, LEP/LHC and other collider conditions are necessary to carry out in the so-called $k_T$-factorization [2, 3] (or semihard [1, 4]) QCD approach, which is more preferable for the small $x$ region than standard parton model.

The $k_T$-factorization approach is based on Balitsky, Fadin, Kuraev, Lipatov (BFKL) [5] evolution equations. The resummation of the terms $\alpha^n_S \ln^n(\mu^2/\Lambda^2_{QCD})$, $\alpha^2_S \ln^n(\mu^2/\Lambda^2_{QCD}) \ln^n(1/x)$ and $\alpha^2_S \ln^n(1/x)$ in the $k_T$-factorization approach leads to the unintegrated (dependent from $q_T$) gluon distribution $\Phi(x, q_T^2, \mu^2)$ which determine the probability to find a gluon carrying the longitudinal momentum fraction $x$ and transverse momentum $q_T$ at probing scale $\mu^2$. In contrast with the usual parton model, to calculate the cross section of a physical process the unintegrated gluon distributions have to be convoluted with off mass shell matrix elements corresponding to the relevant partonic subprocesses [1–4].

Nowadays, the significance of the $k_T$-factorization approach becomes more and more commonly recognized. Its applications to a variety of photo-, lepto- and hadroproduction processes are widely discussed in the literature (see [6] and references cited therein).

It is known that $J/\psi$ production at high energies is an intriguing subject in modern physics. It is traces back to the early 1990s, when the CDF data on the $J/\psi$ and $\Upsilon$ hadroproduction cross section revealed a more than an order of magnitude discrepancy with theoretical expectations. This fact has induced extensive theoretical investigations. In particular, it was required to introduce new additional production mechanism, the so-called colour octet (CO) model [7], where $c\bar{c}$-pair is produced in the color octet state and transforms into final colour singlet (CS) state by help soft gluon radiation. Since then, the color octet model has been believed to give the most likely explanation of the quarkonium production phenomena, although there are also some indications that it is not working well.

One of the problems is connected to the $J/\psi$ photo- and leptoproduction data at HERA. For example, the contributions from the color octet mechanism to the $J/\psi$ photoproduction contradict the H1 and ZEUS data for $z$ distribution [8–11]. Also in leptoproduction case the shapes of the $Q^2$, $p_T^2$ and $y^*$ spectra are not reproduced by calculations [12] within the color octet model. The $z$ distributions calculated later in [13] contradict the H1 experimental data, too.

Another difficulty of this model refers to the $J/\psi$ polarization properties in $p\bar{p}$-collisions at Tevatron. If, as expected, the dominant contribution comes from the gluon fragmentation into an octet $c\bar{c}$ pair, the $J/\psi$ mesons must have strong transverse polarization [14–18]. This is in disagreement with the experimental data [19], which point to unpolarized or even longitudinally polarized $J/\psi$ mesons.

Taking into account the above mentioned problems of color octet mechanism, we have considered the inelastic $J/\psi$ meson photo- and leptoproduction at HERA within the colour singlet model with $k_T$-factorization [20]. Our theoretical results agree well with H1 and ZEUS experimental data for all distributions without any additional $J/\psi$ production mechanisms (such as given by the CO model)(see also [21]). Also it was shown that the $k_T$-factorization approach gives a correct description of the $J/\psi$ polarization properties in $ep$ and $p\bar{p}$ interactions [20, 22] at the large transverse momenta $p_T$.

Based on these theoretical results, in this paper we consider the inclusive $J/\psi$ production at CERN LEP2 conditions within the colour singlet model with $k_T$-factorization. There
are several motivations for our study the $J/\psi$ meson production in $\gamma\gamma$ collisions ($e^+e^- \rightarrow e^+e^- + J/\psi + X$). First of all, recently the DELPHI collaboration has presented preliminary data on the inclusive $J/\psi$ production in $\gamma\gamma$ collisions at CERN LEP2 [23], which wait to be confronted with different theoretical predictions. On the one hand it was shown recently that results obtained with account of the color octet contributions in the framework of the NRQCD do not contradict the DELPHI data [24]. On the other hand we know that a description of the inclusive $J/\psi$ production in $p\bar{p}$ collisions at Tevatron in the $k_T$-factorization approach needs some significantly lower values for the color octet matrix elements to fit the Tevatron data [22, 25].

It is also known that the inclusive $J/\psi$ production in photon-photon collisions is dominated by the single-resolved process [26] and therefore it reveals the gluon structure of the photon. However the unintegrated gluon distributions in the photon $\Phi_\gamma(x, q_T^2, \mu^2)$ are poorly known in contrast with the similar distributions in the proton and no attempts have been made to describe them until recently [27, 28]. Their knowledge is in particular necessary for the description of heavy quark and quarkonium production in $\gamma\gamma$ collisions within the semihard QCD approach. First application of the $k_T$-factorization approach for the case of resolved photons in heavy quark production is performed in [29] where the Kimber-Martin-Ryskin (KMR) [30] and Golec-Biernat-Wusthoff (GBW) [31, 32] prescriptions for the unintegrated gluon distribution in the photon were used.

In the present paper we obtain in an independent way the unintegrated gluon distributions in the photon using the method proposed in Ref. [33] by J. Blümlein for the gluon distribution in the proton. We calculate the cross section of inclusive $J/\psi$ production in $\gamma\gamma$ collisions including both direct and resolved photon contributions and compare them with preliminary experimental data [23] taken by the DELPHI collaboration at CERN LEP2. Additionally, we give our theoretical predictions for $J/\psi$ polarization properties at the LEP2 conditions.

The outline of this paper is as follows. In Section 2 we present, in analytic form, the differential cross section for inclusive $J/\psi$ production in $\gamma\gamma$ collisions within the color singlet model with $k_T$-factorization and obtain the unintegrated gluon distributions $\Phi_\gamma(x, q_T^2, \mu^2)$ in the photon. In Section 3 we present the numerical results of our calculations and compare them with the DELPHI data. Finally, in Section 4, we give some conclusions.

2 Theoretical framework

In this section we calculate total and differential cross section for inclusive $J/\psi$ production in $\gamma\gamma$ collisions within the color singlet model with $k_T$-factorization and obtain the unintegrated gluon distributions in the photon.

2.1 Kinematics and cross sections

In $\gamma\gamma$ collisions $J/\psi$ can be produced by one of the three mechanisms: a direct production (Fig. 1a), a photoproduction off a resolved photon (Fig. 1b) and a by a double resolved process (Fig. 1c). We will refer to the direct production, the once- and double resolved photon processes as direct, 1-res and 2-res ones respectively. The appropriate QCD motivated cross section the process $e^+e^- \rightarrow e^+e^- J/\psi X$ is given by

$$
\frac{d\sigma(e^+e^- \rightarrow e^+e^- J/\psi X)}{d\hat{\sigma}(\gamma\gamma \rightarrow J/\psi X)} = \int f_{\gamma/e}(x_1)dx_1 \int f_{\gamma/e}(x_2)dx_2 d\sigma(\gamma\gamma \rightarrow J/\psi X),
$$
where we use the Weizacker-Williams approximation for the bremsstrahlung photon distribution from an electron [34]:

\[ f_{\gamma/e}(x) = \frac{\alpha^2}{2\pi} \left( \frac{1 + (1-x)^2}{x} \ln \frac{Q_{\text{max}}^2}{Q_{\text{min}}^2} + 2m_e^2x \left( \frac{1}{Q_{\text{max}}^2} - \frac{1}{Q_{\text{min}}^2} \right) \right) \]  

(2)

with \( Q_{\text{min}}^2 = m_e^2x^2/(1-x)^2 \) and \( Q_{\text{max}}^2 = (E\theta)^2(1-x) + Q_{\text{min}}^2 \). Here \( x = E_\gamma/E_e \), \( E = E_e = \sqrt{s}/2 \), \( \theta = 32 \text{ mrad} \) is the angular cut that ensures the photon is real, and \( \sqrt{s} = 197 \text{ GeV} \) [23].

For partonic cross section \( d\sigma(\gamma\gamma \rightarrow J/\psi X) \) we will use expression obtained earlier [20] because the 1-res processes are analogous to those contributing to the lepto- or photoproduction processes at HERA, and the 2-res ones are analogous to the \( J/\psi \) hadroproduction subprocesses at Tevatron. In both cases the unintegrated gluon distributions in the proton should be replaced by ones in the photon.

2.2 Unintegrated gluon distributions in the photon

To obtain the unintegrated gluon distribution in the photon \( \Phi_{\gamma}(x, q_T^2, \mu^2) \) we apply the same method as for the gluon distribution in the proton according to the prescription given in [33]. The proposed method lies upon a straightforward perturbative solution of the BFKL equation where the collinear gluon density \( xG(x, \mu^2) \) is used as the boundary condition. The unintegrated gluon distribution in a photon is calculated as a convolution of collinear gluon distribution \( xG_{\gamma}(x, \mu^2) \) with universal weight factors:

\[ \Phi_{\gamma}(x, q_T^2, \mu^2) = \int_1^0 \varphi(\eta, q_T^2, \mu^2) \frac{x}{\eta} G_{\gamma}(\frac{x}{\eta}, \mu^2) \, d\eta, \]  

(3)

where

\[ \varphi(\eta, q_T^2, \mu^2) = \begin{cases} \frac{\bar{\alpha}_S}{\eta q_T^2} J_0 \left( \frac{2\sqrt{\bar{\alpha}_S \ln(1/\eta) \ln(\mu^2/q_T^2)}}{\eta q_T^2} \right), & \text{if } q_T^2 \leq \mu^2, \\ \frac{\bar{\alpha}_S}{\eta q_T^2} I_0 \left( \frac{2\sqrt{\bar{\alpha}_S \ln(1/\eta) \ln(q_T^2/\mu^2)}}{\eta q_T^2} \right), & \text{if } q_T^2 > \mu^2, \end{cases} \]  

(4)

where \( J_0 \) and \( I_0 \) stand for Bessel function of real and imaginary arguments respectively, and \( \bar{\alpha}_S = 3\alpha_S/\pi \). In the capacity of the boundary condition we used expression for \( xG_{\gamma}(x, \mu^2) \).
from the standard GRV set [35]. The parameter $\bar{\alpha}_S$ is connected with the Pomeron trajectory intercept: $\Delta = 4\bar{\alpha}_S \ln 2 = 0.53$ in the LO and $\Delta = 4\bar{\alpha}_S \ln 2 - N\bar{\alpha}_S^2$ in the NLO approximations, where $N \sim 18$ [36]. The latter value of $\Delta$ have dramatic consequences for high energy phenomenology. However, some resummation procedures proposed in the last years lead to positive value of $\Delta \sim 0.2 - 0.3$ [37, 38]. Therefore in our calculations we will consider $\Delta$ as free parameter varying from 0.2 to 0.53 with a central value at $\Delta = 0.35$.

Figure 2: The unintegrated gluon distribution in the photon $\Phi_\gamma(x, q_T^2, \mu^2)$ as a function of longitudinal momentum fraction $x$ (a) and transverse momenta squared $q_T^2$ (b) of the gluon, for fixed values of $x$ and $q_T^2$. Curves 1 — 3 (Fig. 2a) correspond gluon virtuality $q_T^2 = 1$, 10 and 50 GeV$^2$ as well as curves 1 — 4 (Fig. 2b) correspond longitudinal momentum fraction $x = 10^{-4}$, $x = 10^{-3}$, $x = 10^{-2}$ and $x = 10^{-1}$ respectively. We set here $\mu^2 = q_T^2$.

We would like to note that intercept parameter $\Delta = 0.35$ was obtained from the description of $p_T$ spectrum of $D^*$ meson electroproduction at HERA [39]. This value of $\Delta$ also was used at the analysis of experimental data on inelastic $J/\psi$ photo- and leptoproduction at HERA [20], the $b\bar{b}$ quark production at Tevatron [40] and in the deep inelastic structure function $F_2^c$, $F_L^c$ and $F_L$ description at small $x$ region [41, 42].

In Fig. 2 the unintegrated gluon distribution in the photon (3) — (4) (so called JB parametrization) is plotted as a function of longitudinal momentum fraction $x$ (Fig. 2a) and transverse momenta squared $q_T^2$ (Fig. 2b) of the gluon, for fixed values of $x$ and $q_T^2$. Curves 1 — 3 plotted on the Fig. 2a correspond gluon virtuality $q_T^2 = 1$, 10 and 50 GeV$^2$ as well as curves 1 — 4 plotted on the Fig. 2b correspond longitudinal momentum fraction $x = 10^{-4}$, $x = 10^{-3}$, $x = 10^{-2}$ and $x = 10^{-1}$, respectively.

Integrating expression (3) — (4) over the transverse momenta, we can obtain effective gluon distribution in photon and compare it to experimental data [42] taken by the H1 collaboration at HERA. The results of such calculations are shown in Fig. 3. Curves 1 and 2 correspond to the gluon density in photon from the standard GRV set [35] and the effective gluon density obtained from JB parametrization, respectively. One can see that both curves agree very well with the H1 data and at $x > 10^{-2}$ they practically coincide.
Figure 3: Effective gluon distribution in photon at $Q^2 = 74$ GeV$^2$. Curves 1 and 2 correspond to the gluon density from the standard GRV set [35] and the effective gluon density obtained from JB parametrization, respectively. Experimental data are from H1 [43] collaboration •.

3 Numerical results

In this section we present the theoretical results in comparison with preliminary experimental data [23] taken by the DELPHI collaboration at CERN LEP2.

There are three parameters which determine the common normalization factor of the cross section under consideration: $J/\psi$ meson wave function at the origin $\Psi(0)$, charmed quark mass $m_c$ and factorization scale $\mu$. The value of the $J/\psi$ meson wave function at the origin may be calculated in a potential model or obtained from the well known experimental decay width $\Gamma(J/\psi \rightarrow \mu^+ \mu^-)$. In our calculation we used $|\Psi(0)|^2 = 0.0876$ GeV$^3$ as in Refs. [15, 20].

Concerning a charmed quark mass, the situation is not clear: on the one hand, in the nonrelativistic approximation one has $m_c = m_\psi/2 = 1.55$ GeV, but on the other hand there are many examples when smaller value of a charm mass is used in $J/\psi$ production processes at high energies, for example, $m_c = 1.4$ GeV [12–15, 24]. However, in our previous papers [20] we analyzed in detail the influence of the charm quark mass on the theoretical results. We found that the main effect of a change of the charm quark mass connects with the final phase space of the $J/\psi$ meson, and in the subprocess matrix elements this effect is negligible. Taking into account that the value of $m_c = 1.4$ GeV corresponds to the unphysical phase space of the $J/\psi$ state, in the present paper we will use the value of the charm mass $m_c = 1.55$ GeV only.

Also the most significant theoretical uncertainties come from the choice of the factorization scale $\mu_F$ and renormalization one $\mu_R$. One of them is related to the evolution of the gluon distributions $\Phi_\gamma(x, q_T^2, \mu_F^2)$, the other is responsible for strong coupling constant $\alpha_s(\mu_R^2)$. As often one done in literature, we set them equal $\mu_F = \mu_R = \mu$. In the present paper we used the following choice $\mu^2 = q_T^2$ as in Refs. [4, 20].

The results of our calculations are shown in Fig. 4—6. Fig. 4 displays the $J/\psi$ meson transverse momentum distribution in comparison with DELPHI preliminary data [22] at $\sqrt{s} = 197$ GeV and $|y_\psi| < 2$. The solid lines 1 and 2 correspond to the standard parton
model (SPM) calculations at the leading order approximation with the GRV (LO) gluon density (at $m_c = 1.55$ GeV) and the $k_T$-factorization results with the JB unintegrated gluon distribution (at $m_c = 1.55$ GeV and $\Delta = 0.35$). The shaded band of the $k_T$-factorization predictions connected with the value of pomeron intercept $\Delta$: the lower border corresponds to $\Delta = 0.2$ and the upper one corresponds to $\Delta = 0.53$. One can see that the behavior of the transverse momentum distribution within the experimental and theoretical uncertainties can be explained by the color singlet model with $k_T$-factorization\(^3\), while the collinear approximation prediction are lower than data by an order of magnitude.

We would like to note that there is a difference in the shapes between curves obtained using the $k_T$-factorization approach and the collinear parton model. This difference manifests the $p_T$-broadening effect which is connected to the initial gluon transverse momentum and it is usual for the $k_T$-factorization approach \([20, 39, 40]\).

Fig. 5 shows the transverse momenta and rapidity distributions of the inclusive $J/\psi$ meson production at $\sqrt{s} = 197$ GeV, where all contribution (direct production, 1-res and 2-res processes) are plotted separately. Curves 1 and 2 and shaded bands are the same as in Fig. 4. One can see that 1-res contribution dominates, while the direct and 2-res contributions are only a small corrections to the 1-res one. This is contrast to the inelastic $J/\psi$ production case at HERA, where the resolved photon contribution is a small fraction of the direct one. Also one can see that 2-res contributions is more sensitive to various theoretical uncertainties.

As it mentioned above, the main difference between $k_T$-factorization and other approaches connects with polarization properties of the final particles. The "nonsense" polarization of

\(^3\)In a Ref. \([24]\) it was claimed about some evidence for color octet mechanism of NRQCD from these DELPHI experimental data \([23]\). However in a recent paper \([44]\) the Belle collaboration has published results on prompt $J/\psi$ and double $c\bar{c}$ quark production in $e^+e^-$-annihilation, where the additional $c\bar{c}$ pair fragments into either charmonium or charmed mesons. It was shown that $J/\psi$ signal predicted in color octet model is not observed and that large fraction of prompt $J/\psi$ events is due to the $e^+e^- \to J/\psi c\bar{c}$ process \([44]\). We would like to note it is possible that in the DELPHI data at LEP2 the contributions of $J/\psi D^{*\pm}$ associated production are presented too. The calculations of these processes are in progress.
Figure 5: The transverse momenta and rapidity distributions of the inclusive $J/\psi$ meson production at $\sqrt{s} = 197$ GeV, where all contribution (direct production, 1-res and 2-res processes) are plotted separately. Curves 1 and 2 are the same as in Fig. 4.

The initial BFKL gluons should result in observable spin effects of final $J/\psi$ mesons [45]. In the present paper we calculate the $p_T^2$ dependence of the spin alignment parameter $\alpha$:

$$\alpha(p_T^2) = \frac{d\sigma/dp_T^2 - 3 d\sigma_L/dp_T^2}{d\sigma/dp_T^2 + d\sigma_L/dp_T^2},$$

(5)

where $\sigma_L$ is the production cross section for the longitudinally polarized $J/\psi$ mesons. The parameter $\alpha$ controls the angular distribution for leptons in the decay $J/\psi \to \mu^+ \mu^-$ (in the $J/\psi$ meson rest frame):

$$\frac{d\Gamma(J/\psi \to \mu^+ \mu^-)}{d\cos \theta} \sim 1 + \alpha \cos^2 \theta.$$

(6)

The cases $\alpha = 1$ and $\alpha = -1$ correspond to transverse and longitudinal polarizations of the $J/\psi$ meson, respectively.
The results of our calculations are shown in Fig. 6. Curves 1 and 2 are the same as in Fig. 4. As in inelastic $J/\psi$ photoproduction at HERA [20], we have large difference between predictions of the leading order of collinear parton model and the $k_T$-factorization approach. It was shown that account of the color octet contributions do not change a predictions of $k_T$-factorization approach for the $J/\psi$ polarization properties in $p\bar{p}$ and $ep$ collisions (see [22]). Therefore experimental study the polarized $J/\psi$ production at CERN LEP2 will be an additional test of BFKL gluon dynamics.

4 Conclusions

In this paper we considered the inclusive $J/\psi$ meson production at CERN LEP2 within the colour singlet model and the $k_T$-factorization approach, including both direct and resolved photon contributions. The unintegrated gluon distribution in the photon is determined, using the Blümlein’s prescription for unintegrated gluon distribution in a proton. We compared the theoretical results with preliminary experimental data taken by the DELPHI collaboration at CERN LEP2. We find that the $k_T$-factorization results (in contrast with the SPM ones) with the JB unintegrated gluon distribution in the photon agree with data within the experimental and theoretical uncertainties. Finally, it is shown that experimental study of a polarization of $J/\psi$ meson at LEP2 should be additional test of BFKL gluon dynamics.

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