A Model of Gravitational Leptogenesis

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Abstract: Gravitational leptogenesis is an elegant way of explaining the matter-antimatter asymmetry in the universe. This paper is a review of the recently proposed mechanism of radiatively-induced gravitational leptogenesis (RIGL), in which loop effects in QFT in curved spacetime automatically generate an asymmetry between leptons and antileptons in thermal quasi-equilibrium in the early universe. The mechanism is illustrated in a simple see-saw BSM model of neutrinos, where the lepton-number violating interactions required by the Sakharov conditions are mediated by right-handed neutrinos with Majorana masses of $O(10^{10})$ GeV. The Boltzmann equations are extended to include new, loop-induced gravitational effects and solved to describe the evolution of the lepton number asymmetry in the early universe. With natural choices of neutrino parameters, the RIGL mechanism is able to generate the observed baryon-to-photon ratio in the universe today.
1 Radiatively-Induced Gravitational Leptogenesis – overview

The origin of the matter-antimatter asymmetry of the universe is one of the most important outstanding issues in cosmology. In this review,\footnote{This paper is an edited arXiv version of a contribution to the Festschrift celebrating the physics career of Peter Suranyi.} based on the recent paper [5], we describe a proposal in which a lepton number asymmetry is generated dynamically in a minimal extension of the standard model by gravitational effects in the expanding universe.

The fundamental mechanism, \textit{radiatively-induced gravitational leptogenesis}, was introduced in the series of papers [1–5]. The key idea is that in a minimal extension of the standard model to include heavy right-handed neutrinos $\nu^\alpha_R$ ($\alpha = 1, 2, 3$), an asymmetry in the propagation of the light neutrinos $\nu^i_L$ ($i = e, \mu, \tau$) and antineutrinos arises due to gravitational tidal effects in loop diagrams in which the heavy neutrinos appear as virtual intermediate states. This induces an effective gravitational interaction in which the rate of change of the Ricci scalar plays the role of a chemical potential for lepton number [6]. In the high temperature environment of the early universe, this generates a lepton number asymmetry in quasi-equilibrium. Later, as the universe expands and cools, this lepton asymmetry is transformed into the observed baryon asymmetry through the well-known sphaleron mechanism [7].

These loop effects violate the strong equivalence principle (SEP) of classical general relativity, and are most usefully expressed in terms of a weak-curvature effective action which includes SEP-violating direct couplings of the curvature tensor to the light lepton fields. The curvature couplings involve inverse powers of the heavy neutrino masses $M_\alpha$, since these set the effective length scale of the virtual loops on which the tidal gravitational forces act.

The necessary conditions for a successful model of baryogenesis, or leptogenesis, were set out long ago in the famous Sakharov conditions [8], according to which the fundamental theory should exhibit (i) baryon or lepton violation, (ii) C and CP violation, and (iii) the mechanism must involve \textit{non-equilibrium} dynamics.

The fundamental BSM theory we consider here is the familiar extension of the standard model with sterile right-handed neutrinos (\textit{i.e.} coupling only to the Higgs field and the light leptons through Yukawa interactions, and neutral under the gauge interactions of the SM). We include Majorana mass terms of $O(10^{10})$ GeV and above for the right-handed neutrinos, which allows the observed light neutrino masses to be
The BSM Lagrangian is therefore

\[ S = \int d^4x \sqrt{-g} \left[ \mathcal{L}_{\text{SM}} + \left( \frac{1}{2} i \bar{\nu}_R \gamma^\mu D_\mu \nu_R - \frac{1}{2} \bar{\nu}_R^c M \nu_R - \bar{\ell}_L \lambda \phi \nu_R + \text{h.c.} \right) \right]. \tag{1.1} \]

Here, \( M_{\alpha \beta} \) is the Majorana mass matrix, which we take to be diagonal. The fields \( \ell^i_L \) \((i = e, \mu, \tau)\) are the SM lepton doublets and \( \phi \) is the Higgs field.\(^2\) The Yukawa couplings \( \lambda_{i \alpha} \) are complex and are responsible for CP violation in this model. These flavour indices are suppressed here and in (1.2), (1.3) below.

The inclusion of Majorana masses for the sterile neutrinos allows the lepton number violating scattering processes \( \nu_L H \leftrightarrow \nu_L^c H \) and \( \nu_L \nu_L \leftrightarrow H H \), where \( H \) is the physical Higgs boson, as required by the first Sakharov condition. As is well-known, the model also exhibits a non-gravitational mechanism for generating a lepton asymmetry – thermal leptogenesis through the \textit{out-of-equilibrium} lepton number violating decays of the heavy neutrinos \( \nu_R \to \nu_L H \) \cite{9}. See, for example, ref. \cite{10} for a review.

In the radiatively-induced gravitational leptogenesis (RIGL) mechanism, the third Sakharov condition is replaced by the time-dependence of the background gravitational field. Tidal effects, arising first at two-loop order in this model, generate the following CP odd effective interaction of the lepton (neutrino) number current \( J^\mu = \bar{\nu}_L \gamma^\mu \nu_L \) with the rate of change of the Ricci scalar,

\[ S_b = \int d^4x \sqrt{-g} \ b \partial^\mu R J^\mu , \tag{1.2} \]

where the coupling \( b \) is \( O(\lambda^4/M^2) \). Evidently, a time-dependence of the Ricci scalar reduces this interaction to \( b \dot{R} Q \), where \( Q \) is the lepton number charge, which is equivalent to adding a chemical potential \( \mu = b \dot{R} \).\(^3\) At finite temperature, this will induce a non-vanishing lepton number density \( n_{eq}^L = \frac{1}{3} b \dot{R} T^2 \) in quasi-equilibrium.

\(^2\) The notation is \( \phi_\tau = \epsilon_{rs} \tilde{\phi}_s^* \) where \( \tilde{\phi} \) is the usual Higgs doublet giving mass to the lower fields in the \( SU(2) \) lepton doublets.

\(^3\) Note that in the original gravitational baryogenesis (leptogenesis) proposal of Davoudiasl \textit{et al.} \cite{6}, the interaction (1.2) with \( J^\mu \) interpreted as the baryon (lepton) number current is postulated essentially \textit{a priori}, with an arbitrary mass-dependent coupling. Here, we show how it is necessarily generated in the effective action of the simple and phenomenologically motivated BSM model with heavy Majorana masses for the right-handed neutrinos. In an earlier proposal of ‘spontaneous baryogenesis’ \cite{11}, an interaction of the form (1.2) but with the Ricci scalar replaced by a time-dependent background scalar field was proposed.
Further, CP even, interactions are generated already at one-loop level and the complete effective action for the light neutrinos is

$$S_{\text{eff}} = \int d^4x \sqrt{-g} \left[ \frac{i}{2} \bar{\nu}_L \gamma^\mu \overleftrightarrow{D}_\mu \nu_L + (a - \frac{1}{2} d) R_{\mu\nu} i \bar{\nu}_L \gamma^\mu \overleftrightarrow{D}_\nu \nu_L + b \partial_\mu R \bar{\nu}_L \gamma^\mu \nu_L \\
+ c R i \bar{\nu}_L \gamma^\mu \overleftrightarrow{D}_\nu \nu_L - d i (D_\mu \bar{\nu}_L) \gamma^\mu \overleftrightarrow{D}_\nu \nu_L \right].$$

(1.3)

Here, $a_{ij}, c_{ij}, d_{ij}$ are coefficients of $O(\lambda^2/M^2)$, and we have suppressed the corresponding $i, j$ flavour indices in the fermion bilinears for simplicity of notation. The effective action is valid to lowest order in $R/M^2$, where $R$ denotes a typical curvature component, and also in the ‘low-energy’ regime $E\sqrt{R}/M^2 \ll 1$ (see ref. [5] for a careful discussion, including the justification for using the effective Lagrangian at temperatures in excess of $M$).

These CP even interactions modify the conservation law of the lepton number current, giving

$$D_\mu J^\mu = -2a R_{\mu\nu} D^\mu J^\nu - 2\hat{b} \partial_\mu R J^\mu,$$

(1.4)

where $\hat{b} = \frac{1}{2} a + c + \frac{1}{4} d$. In a FRW universe, this implies the following equation for the time evolution of the lepton number density $n_L$,

$$\frac{dn_L}{dt} + 3Hn_L + 2a (-3R_{00}^0 + R_{i}^i) Hn_L + 2\hat{b} \dot{R} n_L = 0,$$

(1.5)

where $H$ is the Hubble parameter. This shows that the lepton number density $n_L$ evolves non-trivially in a gravitational background due to quantum loop effects. Whether this effect tends to amplify or suppress the magnitude of $n_L$ depends on the signs and relative magnitudes of the coefficients, especially $a$, which are not arbitrary in RIGL but are determined by the fundamental BSM theory.

Together with the gravitationally-induced equilibrium lepton asymmetry $n_L^{eq}$, these two effects combine to generate a novel dynamical evolution [5] of the lepton number density in the expanding universe. To describe this quantitatively in the high temperature environment of the early FRW universe, we need the corresponding Boltzmann equations. Neglecting for the moment the contribution to the asymmetry from decays of the heavy $\nu_R$ neutrinos, the gravitationally modified Boltzmann equation is

$$\frac{dN_L}{dz} = -W \left( N_L - N_L^{eq} \right) - \mathcal{W} N_L.$$

(1.6)
Here, \( N_L(z) = n_L / n_\gamma \) is the ratio of the number density of light leptons to photons, and we have replaced the time variable with \( z = M_1 / T \), where \( T \) is temperature and \( M_1 \) is the mass of the dominant heavy neutrino.

The new gravitational terms in (1.6) are the equilibrium number asymmetry \( N_L^{eq}(z) \) generated by the interaction (1.2) and the evolution term \( W(z) \) arising from the CP even interactions in (1.3), which reflects the curvature-induced evolution terms in (1.5). \( W(z) = \Gamma / z H \) is the usual factor determined by the interaction rate \( \Gamma(z) \) for the lepton number violating interactions. Without the new gravitational terms, it acts as a ‘washout’ factor – here, it has the different role of driving the lepton number asymmetry towards its \textit{non-vanishing} equilibrium value \( N_L^{eq}(z) \).

![Figure 1](image_url)

**Figure 1.** Illustration of the key stages in the evolution of the lepton asymmetry \( N_L(z) \) with temperature, given by the Boltzmann equation (1.6). The temperature dependence of the normalised rate factor \( W(z) \) for the lepton-violating reactions, which drive the asymmetry towards its equilibrium value \( N_L^{eq}(z) \), is \( W(z) \sim 1 / z^2 \), while both the gravitationally-induced terms \( N_L^{eq}(z) \) and \( W(z) \) fall off as \( 1 / z^5 \).

Putting all this together, the entire evolution predicted by the RIGL Boltzmann equation is shown in Fig. 1. Essentially we find four stages. First, there is a very early high temperature phase (region (a)) in which the new evolution term \( W(z) \) keeps \( N_L \) below equilibrium, followed by a phase where \( W(z) > W(z) \) and \( N_L \) is driven to its gravitationally-induced equilibrium value \( N_L^{eq}(z) \) (region (b)). Next is the vital decoupling transition (region (c)) when \( W(z) \) becomes too weak to hold \( N_L \) to \( N_L^{eq}(z) \) (\textit{i.e.} the \( L \)-violating reactions are too slow compared to the Hubble expansion to main-
tain equilibrium) and it decouples leaving a constant, frozen asymmetry. Finally, there is a late dip in $N_L$ as $W(z)$ develops a resonance peak around $T \sim M_1$, temporarily pushing $N_L$ lower towards the rapidly decreasing $N_L^{eq}(z)$ (region (d)).

The transitions between these stages depend on the dynamical balance between the temperature and curvature dependent rates $W(z)$ and $\mathcal{W}(z)$, and $N_L^{eq}(z)$, as the universe expands and cools. In particular, the sign of $\mathcal{W}(z)$ is key to whether the new evolution term amplifies or suppresses the lepton asymmetry at early times. In the BSM model described here, the sign is such as to suppress the asymmetry, as shown in Fig. 1.

In the remainder of this paper, we describe this scenario in more detail and describe the quantitative predictions for the baryon asymmetry of the universe in this model.

## 2 Matter-Antimatter Asymmetry from Gravitational Interactions

In this section, we explain in principle how gravitational interactions can induce an asymmetry in the dynamics of matter and antimatter, and outline the methods used to obtain the gravitational effective action (1.3) from the fundamental BSM theory (1.1).

It is useful to begin by rewriting the action (1.1) explicitly in terms of the fields $\nu_L$ and $\nu_L^c$ corresponding to the light neutrinos and their antiparticles:

$$S = \int d^4x \sqrt{-g} \left[ \mathcal{L}_{SM} + \frac{1}{4} \left( i \nu_R \gamma^\mu \bar{D}^\mu \nu_R + i \bar{\nu}_R \gamma^\mu \nu_R \right) - \frac{1}{2} \left( \bar{\nu}_R^c M \nu_R + \nu_R M \bar{\nu}_R^c \right) - \frac{1}{2} \left( \bar{\nu}_L^c \lambda \bar{\phi} \nu_R + \bar{\nu}_R M \lambda^T \phi \bar{\nu}_L^c + \bar{\nu}_R M^T \lambda^T \bar{\phi} \nu_L^c \right) \right].$$

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4The charge conjugate field is defined as $\nu_L^c \equiv (\nu_L)^c = -i\gamma^2 \nu_L^*$ and is right-handed. With a Higgs VEV $v$, the see-saw mechanism gives rise to three light Majorana neutrinos with mass matrix

$$(m_\nu)_{ij} = \sum_\alpha \lambda_{i\alpha} \frac{1}{M_\alpha} \lambda_{\alpha j}^T v^2,$$

along with the heavy sterile Majorana neutrinos with masses $M_\alpha$. This is diagonalised by the PMNS matrix $U$ such that $U^T m_\nu U = m_{\text{diag}}$, with the corresponding relation between the mass and flavour eigenstates.

5The coupling to gravity is through the connection alone, which is the requirement for the classical Lagrangian to satisfy the strong equivalence principle. The covariant derivative acting on spinors is $D_\mu = \partial_\mu - \frac{i}{4} \omega_{\mu ab} \sigma^{ab}$, where $\omega_{\mu ab}$ is the spin connection and $\sigma^{ab} = \frac{1}{2} [\gamma^a, \gamma^b]$. 

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Figure 2. Feynman diagrams for the $\Delta L = 2$ lepton number violating reactions $\nu_L H \leftrightarrow \nu_c L H$ and $\nu_L \nu_L \leftrightarrow H H$, mediated by the ‘charge violating’ $\langle \nu_R \overline{\nu}_R^c \rangle$ propagator $S^\times_\alpha$. The Majorana mass for the right-handed neutrinos allows the occurrence of both leptonic “charge-conserving” and “charge-violating” propagators:

\[
\langle \nu_R \overline{\nu}_R \rangle = \langle \nu_R^c \overline{\nu}_R^c \rangle = S(x, y),
\]

\[
\langle \nu_R \overline{\nu}_R^c \rangle = \langle \nu_R^c \overline{\nu}_R \rangle = S^\times(x, y).
\]

In flat spacetime, these are

\[
S_\alpha(p) = \frac{i \gamma \cdot p}{p^2 - M_\alpha^2}, \quad S^\times_\alpha(p) = \frac{i M_\alpha}{p^2 - M_\alpha^2}.
\]

The existence of the charge-violating $S^\times(x, y)$ propagator plays a key role in generating the lepton asymmetry. First, it allows the $\Delta L = 2$ scattering reactions $\nu_L H \leftrightarrow \nu_c L H$ and $\nu_L \nu_L \leftrightarrow H H$, illustrated in Fig. 2. These diagrams depend on the Yukawa coupling factor through $\lambda S^\times \lambda^T = \sum_\alpha \lambda_\alpha S^\times_\alpha \lambda^T_\alpha$. This is the source of the lepton number violation required by the first Sakharov condition.

Next, to implement the RIGL mechanism, we need to show that the propagation of leptons and antileptons is different in a gravitational field. Specifically, we find that at two-loop level, the self-energies $\Sigma$ and $\Sigma^c$ for the leptons and antileptons differ when translation invariance no longer holds, leading to distinct dispersion relations.

At one loop, there is a single self-energy diagram for the light neutrinos $\nu_L$ (Fig. 3), which involves the charge-conserving right-handed neutrino propagator $S_\alpha$. (For neu-
trino self-energies in the standard model, see refs. [1, 12]) There is no corresponding one-loop diagram with the charge-violating propagator $S_{\alpha}^\times$. The self-energy is therefore,\[ \Sigma_{ij}(x,y) = \sum_{\alpha} \lambda_{i\alpha} \lambda_{j\alpha}^\dagger G(x,y) S_{\alpha}(x,y) . \] (2.5)

A similar diagram gives the self-energy for the antineutrinos. Since we are interested in the violation of total lepton number, we trace over the light lepton flavours, and find\[ \text{tr} \left( \Sigma_{ij}(x,y) - \Sigma_{ij}^c(x,y) \right) = \sum_{\alpha} \left( \lambda_{i\alpha}^T \lambda_{j\alpha} - \lambda_{i\alpha} \lambda_{j\alpha}^T \right)_{\alpha\alpha} G(x,y) S_{\alpha}(x,y) \]
\[ = 2i \sum_{\alpha} \text{Im} \left( \lambda_{i\alpha}^\dagger \lambda_{j\alpha}^\dagger \right) G(x,y) S_{\alpha}(x,y) = 0 . \] (2.6)

These diagrams therefore imply that the gravitational influence on the propagation of the light neutrinos and antineutrinos is identical. In the effective Lagrangian (1.3), they correspond to the CP even terms. However, they do not contribute to the CP odd term which is responsible for generating the lepton-antilepton asymmetry. For this, we require the two-loop self-energies, which have the $O(\lambda^4)$ dependence necessary to exhibit CP violation.

At two loops, we find the three self-energy diagrams illustrated in Fig. 4. The corresponding self-energies are:
\[ \Sigma_{ij}^{(1)}(x,y) = \int d^4z \sqrt{-g} \int d^4z' \sqrt{-g} \sum_{\alpha,\beta,k} \left( \lambda_{i\alpha}^T \lambda_{k\beta} \lambda_{\beta j} \right) \]
\[ \times G(x,y) G(z,z') S_{\alpha}(x,z) \Delta(z,z') S_{\beta}(z',y) , \] (2.7)
from the ‘nested’ diagram with two $S$ propagators, and
\[ \Sigma_{ij}^{(2)}(x,y) = \int d^4z \sqrt{-g} \int d^4z' \sqrt{-g} \sum_{\alpha,\beta,k} \left( \lambda_{i\alpha} \lambda_{k\beta}^\dagger \lambda_{\beta j}^\dagger \right) \]
\[ \times G(x,y) G(z,z') S_{\alpha}^\times(x,z) \Delta(z,z') S_{\beta}^\times(z',y) , \] (2.8)
Figure 4. Two-loop self-energy diagrams for the light $\nu_L$ neutrinos giving rise to a lepton-antilepton asymmetry in curved spacetime.

and

$$
\Sigma_{ij}^{(3)}(x, y) = \int d^4z \sqrt{-g} \int d^4z' \sqrt{-g} \sum_{\alpha,\beta,k} \left( \lambda_{i\alpha} \lambda_{Tk}\lambda_{k\beta}\lambda_{j}^{\dagger} \right) \\
\times G(x, z') G(z, y) S^x_{\alpha}(x, z) \Delta(z, z') S^x_{\beta}(z', y) ,
$$

(2.9)

for the ‘nested’ and ‘overlapping’ diagrams with two $S^x$ propagators. Note that there is no overlapping-type diagram with two $S$ propagators.

Tracing over the light flavours as before, we find the following structure of the Yukawa couplings for the three diagrams:

$$
\text{tr} \left( \Sigma_{ij}^{(1)} - \Sigma_{ij}^{(1)c} \right) = 2i \sum_{\alpha,\beta} \text{Im} \left[ \left( \lambda^{\dagger}\lambda \right)_{\beta\alpha} \left( \lambda^{\dagger}\lambda \right)_{\alpha\beta} \right] I_{\alpha\beta}^{(1)} = 0 ,
$$

(2.10)

whereas

$$
\text{tr} \left( \Sigma_{ij}^{(2)} - \Sigma_{ij}^{(2)c} \right) = 2i \sum_{\alpha,\beta} \text{Im} \left[ \left( \lambda^{\dagger}\lambda \right)_{\beta\alpha} \left( \lambda^{T}\lambda^{*} \right)_{\alpha\beta} \right] I_{\alpha\beta}^{(2)} \\
= 2i \sum_{\alpha,\beta} \text{Im} \left[ \left( \lambda^{\dagger}\lambda \right)_{\beta\alpha} \left( \lambda^{\dagger}\lambda \right)_{\beta\alpha} \right] I_{\alpha\beta}^{(2)} ,
$$

(2.11)

and

$$
\text{tr} \left( \Sigma_{ij}^{(3)} - \Sigma_{ij}^{(3)c} \right) = 2i \sum_{\alpha,\beta} \text{Im} \left[ \left( \lambda^{\dagger}\lambda \right)_{\beta\alpha} \left( \lambda^{\dagger}\lambda \right)_{\beta\alpha} \right] I_{\alpha\beta}^{(3)} .
$$

(2.12)
have the same dependence on the $\lambda_\alpha$. Note that $\text{Im} \left[ (\lambda^\dagger \lambda)_{\beta\alpha} (\lambda^\dagger \lambda)_{\beta\alpha} \right]$ is antisymmetric in $\alpha, \beta$, so only the antisymmetric part of the dynamical factors $I^{(2)}_{[\alpha\beta]}$ and $I^{(3)}_{[\alpha\beta]}$ contributes to the lepton-antilepton asymmetry.

Now, we can prove in general that CPT invariance and Poincaré invariance together imply that the propagation of particles and antiparticles is identical [3]. To see how this is realised in this context, note that in flat spacetime translation invariance implies that the propagators are functions only of the difference in coordinates, i.e. $\Delta(x, y) \to \Delta(x - y)$, etc. But given this, we can readily show that the factors $I^{(2)}_{\alpha\beta}$ and $I^{(3)}_{\alpha\beta}$ are symmetric in $\alpha, \beta$. For example, for diagram (2), translation invariance implies

$$I^{(2)}_{\alpha\beta}(x, y) = \int d^4 z \int d^4 z' G(x - y) G(z - z') S^x_\alpha(x - z) \Delta(z - z') S^x_\beta(z' - y)$$

$$= \int d^4 u \int d^4 u' G(x - y) G(u - u') S^x_\beta(x - u) \Delta(u - u') S^x_\alpha(u' - y)$$

$$= I^{(2)}_{\beta\alpha}(x, y), \quad (2.13)$$

under the change of dummy variables $u = x + y - z'$ and $u' = x + y - z$.

This is no longer necessarily true in curved spacetime, and indeed we find by explicit calculation that, provided the Majorana masses of the sterile neutrinos are non-degenerate, the diagrams with the charge-violating propagators $S^x_\alpha$ have non-vanishing antisymmetric factors $I^{(2)}_{\alpha\beta}$ and $I^{(3)}_{\alpha\beta}$. We therefore find a difference in the self-energies of the light leptons and antileptons given by

$$\text{tr} \left( \Sigma_{ij} - \Sigma^c_{ij} \right) = 2i \sum_{\alpha, \beta} \text{Im} \left[ (\lambda^\dagger \lambda)_{\beta\alpha} (\lambda^\dagger \lambda)_{\beta\alpha} \right] \left( I^{(2)}_{[\alpha\beta]} + I^{(3)}_{[\alpha\beta]} \right). \quad (2.14)$$

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6In QFT in curved spacetime, we regard the discrete symmetries C, P and T as defined with respect to the local Minkowski spacetime that exists, due to its pseudo-Riemannian nature, at each point in spacetime. (See, for example, ref. [1] for a detailed discussion.) This is also true of the quantum fields themselves, as evident in the definition of spinor fields as representations of the SL(2,C) symmetry of the local Minkowski spacetime.

With this understanding, CPT symmetry and its standard consequences in QFT for the properties of particles and antiparticles, the spin-statistics theorem, etc. holds independently of the curvature of the background spacetime, which is not required to be P or T symmetric.

This may be contrasted with an interesting recent proposal for a “CPT symmetric” cosmology [13], in which the gravitational background itself – an extension to $t < 0$ of FRW spacetime – is taken to be time-reversal symmetric about a single privileged point, interpreted as a bounce or creation event. In our interpretation, this “CPT” would be viewed as an environmental symmetry of the background fields (in this case gravitational), rather than the fundamental CPT symmetry of the QFT itself. Similar remarks apply to the use of “CPT” in the ‘spontaneous’ and gravitational baryogenesis papers in refs. [6, 11]
This is the fundamental observation allowing the generation of a lepton-antilepton asymmetry in curved spacetime through two-loop contributions to the light neutrino propagators in this CP-violating BSM model.

The next step is to determine the coefficients of the effective Lagrangian (1.3) by explicit evaluation of these diagrams in curved spacetime. Since we only need $L_{\text{eff}}$ in the weak curvature regime $\mathcal{R}/M_\alpha^2 \ll 1$, the quickest way is to expand the metric around Minkowski spacetime as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ in (1.3) and evaluate the corrections to the self-energies to $O(h)$. We then match the coefficients of the corresponding expansions with explicit calculations of the Feynman diagrams above in flat spacetime but with a single graviton insertion in the propagators or vertices. Especially at two-loops, these calculations are highly non-trivial, so here we simply refer to the original papers [1–5] and quote the results:

\[
(a_{ij}, c_{ij}, d_{ij}) = \left( \frac{1}{12} \right) \sum_\alpha \lambda_{\alpha j}^\dagger \lambda_{\alpha i} \frac{1}{M_\alpha^2} \left( -\frac{4}{3}, \frac{3}{4}, -3 \right).
\]

and

\[
b_{ij} = \left( \frac{1}{9} \right) \sum_{\alpha, \beta, k} \lambda_{\beta j}^\dagger \lambda_{\alpha i} \lambda_{\beta k}^\dagger \lambda_{k\alpha} \frac{1}{M_\alpha M_\beta} I_{[\alpha\beta]} ,
\]

where

\[
I_{[\alpha\beta]} \sim \left( \frac{M_\beta}{M_\alpha} \right)^{2p} \log \left( \frac{M_\beta}{M_\alpha} \right) ,
\]

up to an $O(1)$ numerical factor.

The question of whether the hierarchy parameter $p$ is 0 or 1 was left unresolved in ref. [3]. The most natural result, which holds in all the diagrams we calculated to a conclusion, is $p = 0$. This would accord with the expected decoupling of heavy mass intermediate states in the Feynman diagram. Nevertheless, in calculating the arbitrary-momentum, two-loop triangle diagram involved – which appears to be at the limit of known techniques – we found contributions with $p = 1$ which would require a remarkable cancellation if they were to be absent in $I_{[\alpha\beta]}$. For this reason, we retain the possibility that $p$ could be 1 in what follows since, as we see in the cosmological scenarios, the presence of a large sterile mass hierarchy dependence could significantly enhance the ultimate prediction for the baryon-to-photon ratio in this BSM model.
3 Gravity-Extended Boltzmann Equations in FRW Spacetime

At this point, we have established that loop effects in the BSM model in a gravitational field may be encoded in the effective Lagrangian (1.3), with dynamically generated couplings $a_{ij}, \ldots, d_{ij}$. The next step is to use this effective action to describe the gravitational generalisation of the Boltzmann equations which determine the evolution of the lepton number density in a finite-temperature, expanding universe.

Using the standard Noether procedure with the effective action, we find that the lepton number current $J^\mu = \sum_i \overline{\nu}_L^i \gamma^\mu \nu_L^i$ for the light neutrinos satisfies the broken conservation equation,

$$ (1 + 2cR) D_\mu J^\mu + 2a R_{\mu\nu} D^\mu J^\nu + \left( a + 2c + \frac{1}{2}d \right) \partial_\mu R J^\mu + 2d \left( \overline{\nu}_L^i D^2 \gamma^i D_{\nu L}^i + \overline{\nu}_L^i \gamma^i \overleftarrow{D} D^2 \nu_L^i \right) \sim 0 \ , \quad (3.1) $$

with the same suppression of flavour indices $i, j$ in the couplings, currents and bilinears employed in (1.3). Since $D_\mu J^\mu \sim O(\lambda^2)$, the pre-factor $2cR$ of the $D_\mu J^\mu$ term must be omitted to consistent perturbative order so, up to terms vanishing by the equation of motion, we find simply,\footnote{From this point, we disregard any light flavour dependence for simplicity and take (3.2) as an equation with all the $J^\mu$ being flavour singlet currents and with coefficients $a \sim \text{tr} a_{ij}$, etc. depending on the Yukawa couplings through $(\lambda^\dagger \lambda)_{\alpha\alpha}$.}

$$ D_\mu J^\mu + 2a R_{\mu\nu} D^\mu J^\nu + 2\hat{b} \partial_\mu R J^\mu \sim 0 \ , \quad (3.2) $$

defining $\hat{b} = \frac{1}{2}a + c + \frac{1}{4}d$. Note that (3.2) involves only the CP even terms – the CP odd term in the effective action with coupling $b_{ij}$ does not contribute to the current conservation equation.

For applications in cosmology, we need to evaluate this in a FRW spacetime. We consider the spatially flat FRW metric,

$$ ds^2 = dt^2 - a(t)^2 \delta_{ij} dx^i dx^j \ , \quad (3.3) $$

with non-vanishing Christoffel symbols $\Gamma^0_{ij} = -H \delta_{ij}$, $\Gamma^i_{00} = H \delta^i_j$ and Ricci tensor components $R_{00} = 4\pi G \rho(1+3w)$, $R_{0i} = 0$, $R_{ij} = -4\pi G \rho(1-w)\delta_{ij}$. The Ricci scalar is $R = -8\pi G \rho(1-3w)$ and $\dot{R} = 8\pi G \rho^3 H(1-3w)(1+w)$. Here, $\rho$ is the energy density and the equation of state is $p = w\rho$. The Hubble constant $H = a/\dot{a}$ is related

\footnote{See also ref. [14] for related ideas.}
to $\rho$ through the Friedmann equation $3H^2 = \rho/M_p^2$, where $M_p$ is the reduced Planck mass, $8\pi G = 1/M_p^2$.

Evaluating (3.2) in this background, and identifying the current component $J^0$ as lepton number density $n_L = n_\nu - n_{\bar{\nu}}$, gives the following equation for the time evolution of $n_L$,

$$
(1 + 2aR_{00})\frac{dn_L}{dt} + 3Hn_L + 2aR^i_iHn_L + 2\dot{b}\dot{R}n_L = 0 .
$$

Writing this to consistent $O(\lambda^2)$ perturbative order, we find the form quoted in (1.5) and substituting for the curvatures gives finally,

$$
\frac{dn_L}{dt} + 3Hn_L = 3H \frac{\rho}{M_p^2} (1 + w) [2a - 2\dot{b}(1 - 3w)] n_L .
$$

To develop this into the full Boltzmann equation, we also need to take account of the lepton number violating interactions. This is where the non-vanishing equilibrium density $n_L^{eq}$ induced by the CP odd term in the effective Lagrangian enters. The $\Delta L = 2$ reactions are shown in Fig. 2, and the model also allows contributions from the ‘inverse decays’ $\nu_LH \rightarrow \nu_R$ at finite temperature. (We neglect $\Delta L = 1$ scattering reactions involving other SM particles here for simplicity [10].) Collecting these effects into a single rate factor $\Gamma = \Gamma_{ID} + 2\Gamma_{\Delta L=2}$, a standard kinetic theory analysis shows that (3.5) should be extended to [5]

$$
\frac{dn_L}{dt} + 3Hn_L = 3H \frac{\rho}{M_p^2} (1 + w) [2a - 2\dot{b}(1 - 3w)] n_L + \Gamma (n_L - n_L^{eq}) ,
$$

with $n_L^{eq} = \frac{1}{3}b \dot{R} T^2$.

Finally, this model – which after all is a conventional BSM theory for thermal leptogenesis – also includes the standard mechanism for generating a lepton asymmetry through the out-of-equilibrium decays of the heavy sterile neutrinos, $\nu_R^a \rightarrow \nu_L^i H$ and $\nu_R^a \rightarrow \nu_L^i H$, shown in Fig. 5. These display an asymmetry dependent on the CP-violating combination of Yukawa couplings [15],

$$
\varepsilon_\alpha \approx \frac{3}{16\pi} \sum_{\beta \neq \alpha} \frac{\text{Im}((\lambda^T\lambda)^2_{\alpha\beta})}{(\lambda^T\lambda)_{\alpha\alpha}} \frac{M_\alpha}{M_\beta} .
$$

arising from the interference of the tree and one-loop diagrams for the decays in Fig. 5. Note that the charge-violating propagators $S^x_\alpha$ are crucial in establishing the lepton-antilepton asymmetry in these diagrams.
Figure 5. Diagrams for the decay $\nu_R^i \rightarrow \nu_L^j H$ which contribute to the decay rate asymmetry factor $\varepsilon_\alpha$. At $O(\lambda^4)$ the relevant contribution to the $\Gamma(\nu_R \rightarrow \nu_L H)$ decay rate arises from the interference of the tree and one-loop diagrams shown. Similar diagrams, where the $S_\alpha$ and $S_\beta \times \alpha$ type $\nu_R$ propagators are switched compared to the figure, give the decay rate for $\nu_R^i \rightarrow \nu_L^j c H$.

It is usual in discussing leptogenesis to express the evolution in terms of temperature rather than time, and as in section 1 we introduce the variable $z = M_1/T$ and express the Boltzmann equation in terms of the lepton-to-photon ratio $N_L(z) = n_L/n_\gamma$. So in terms of these variables, the final coupled Boltzmann equations for $N_L$ and $N_{\nu_R}$ read,

$$\frac{dN_{\nu_R}}{dz} = - D (N_{\nu_R} - N_{\nu_R}^{\text{eq}})$$  \hspace{1cm} (3.8)

$$\frac{dN_L}{dz} = - D \varepsilon_1 (N_{\nu_R} - N_{\nu_R}^{\text{eq}}) - W (N_L - N_L^{\text{eq}}) - WN_L,$$  \hspace{1cm} (3.9)

where recall that $W(z) = \Gamma/zH$, where $\Gamma(z)$ is the finite-temperature rate factor appearing in (3.6) and we similarly define $D(z) = \Gamma(\nu_R \rightarrow \nu_L H)/zH$.

The gravitational terms $N_L^{\text{eq}}(z)$ and $W(z)$ in (3.9) can be read off in terms of the radiatively-generated couplings from (3.6). We find,

$$W(z) = - 3(1 + w) \left[2a - 2b(1 - 3w)\right] \frac{\rho}{M_p^2} \frac{1}{z^2},$$  \hspace{1cm} (3.10)

and

$$N_L^{\text{eq}}(z) = (1 + w)(1 - 3w) b M_p^2 H \frac{\rho}{n_\gamma M_p^2} \frac{1}{z^2}.$$  \hspace{1cm} (3.11)

Note here that for a radiation-dominated FRW universe, $w \simeq 1/3$, with the deviation from this value arising purely from the beta functions characterising the

\footnote{Since $n_\gamma \sim T^3$ and $T \sim 1/a$, we have}

$$\frac{dN_L}{dt} = \frac{1}{n_\gamma} \left(\frac{dn_L}{dt} + 3Hn_L\right),$$

and the change of variables gives $d/dz = (1/Hz)d/dt$. 

\hspace{1cm} – 14 –
energy-momentum trace anomaly, $T^\mu_\mu \neq 0$. With standard model fields, this gives $(1 - 3w) \simeq 0.1$. This quantum deviation from the classical conformal symmetry value is essential to realising a non-vanishing $N^{eq}_L(z)$ in (3.11). Moreover, in the radiation-dominated case, $H \sim 1/z^2$ while $\rho \sim 1/z^4$, so both the gravitational terms $N^{eq}_L(z)$ and $\mathcal{W}(z)$ fall sharply as $1/z^5$ as the universe cools.

From (3.10) we see that whether the radiative curvature corrections encoded in $\mathcal{W}(z)$ act to amplify or reduce the lepton number density as the universe evolves depends on the sign of the combination $[2a - 2\hat{b}(1 - 3w)]$. Since $(1 - 3w)$ is small, this essentially depends just on the sign of the coupling $a$; negative $a$ tends towards damping (washout) of the lepton number with time, while a positive $a$ would imply an amplification. In the BSM model studied here we found in (2.15) that $a$ is negative. This was assumed in the picture sketched in Fig. 1, where it controls the early, high-temperature evolution in region (a).

4 Gravitational Leptogenesis in the Early Universe

In this final section, we solve these coupled Boltzmann equations in the BSM model with physical neutrino parameters in realistic cosmological settings and discuss the viability of the gravitational leptogenesis mechanism for generating the observed baryon asymmetry in the universe today.

First we need expressions for the conventional finite-temperature rate factors $W(z)$ and $D(z)$ in (3.8), (3.9). The necessary results are quoted in refs. [4, 5] and are discussed in detail in, for example, the review [10]. With standard definitions\(^{10}\) for parameters

\[^{10}\text{The definitions of the neutrino parameters and notation used here are explained in refs.}[5, 10]\.

From the see-saw mechanism we have the sum of the light neutrino masses,

$$m^2 = v^4 \sum_{\alpha,\beta} \frac{1}{M_\alpha M_\beta} \text{Re}(\lambda^\dagger \lambda)^2_{\alpha\beta} \simeq \Delta m^2_{\text{sol}} + \Delta m^2_{\text{atm}},$$

where the ‘solar’ and ‘atmospheric’ masses are $\Delta m^2_{\text{sol}} = 7.53 \times 10^{-5} \text{eV}^2$ and $\Delta m^2_{\text{atm}} = 2.44 \times 10^{-3} \text{eV}^2$. A useful mass scale is set by $m_\ast = 8\pi (G/3)^{1/2} v^2 / M_p = 1.08 \times 10^{-3} \text{eV}$, where $v = 174 \text{GeV}$ is the electroweak scale, $M_p = 2.4 \times 10^{18} \text{GeV}$ is the reduced Planck mass, where $8\pi G = 1/M_p^2$, and $\sigma = \pi^2 g_*/30$ is the constant appearing in the radiation energy density $\rho = \sigma T^4$, where $g_*$ is the effective number of relativistic degrees of freedom at temperature $T \sim M_1$. A key parameter is the combination

$$K = \frac{v^2}{M_1 m_\ast} (\lambda^\dagger \lambda)_{11}.$$
related to the light neutrino masses, we have the following expressions for the lepton number violating rate factor $W(z)$. For temperatures well above $T \simeq M_1$,

$$W(z \ll 1) \simeq \frac{12}{\pi^2} \frac{m_* M_1}{v^2} \left( \frac{\bar{m}^2}{m_*^2} + K^2 \right) \frac{1}{z^2}, \quad (4.1)$$

with the same formula without the $K^2$ term giving $W(z \gg 1)$. Near $z \simeq 1$, $W(z)$ shows a resonance enhancement from the intermediate $\nu^1_\text{R}$ state, as is evident from Fig. 2. This is illustrated in Fig. 6. The key features are the asymptotic behaviour $W(z) \sim 1/z^2$, which is a much weaker $z$-dependence than the $1/z^5$ of the gravitational terms $W(z)$ and $N_{\nu_{\text{eq}}}(z)$, and the resonance around $z = 1$.

![Figure 6](image_url)

**Figure 6.** This shows the dependence on $z = M_1/T$ of the coefficient $W(z)$ in the Boltzmann equation for $K = 1$ and $K = 10$, with $M_1 = 5 \times 10^{10}$ GeV and the neutrino parameters in the text. The resonance peak around $T \simeq M_1$ increases with $K$.

Next, the sterile neutrino decay rate factor is given in terms of the Yukawa couplings and Bessel functions by

$$D(z) = K z \frac{K_1(z)}{K_2(z)}, \quad (4.2)$$

and has an equilibrium value at finite temperature, $N_{\nu_{\text{eq}}}(z) = \frac{3}{8} z^2 K_2(z)$.

For convenience, we also quote here the formulae for the gravitational terms in a radiation-dominated universe with $w \simeq 0.3$ and with the radiatively-induced couplings $a, \ldots d$ found in (2.15), (2.16).
These are,
\[ \mathcal{W}(z) \simeq 0.075 \sigma^{3/2} K \left( \frac{M_1}{M_p} \right)^3 \frac{1}{z^5}, \]  
and
\[ N_L^\text{eq}(z) \simeq 0.034 \sigma^{3/2} \left( \frac{M_1}{M_p} \right)^3 \sum_{\beta \neq 1} \frac{\text{Im}(\lambda^\dagger \lambda)^2}{(4\pi)^4} \left( \frac{M_\beta}{M_1} \right)^{2p-1} \log \left( \frac{M_\beta}{M_1} \right) \frac{1}{z^5}. \]  

We have now collected all the ingredients to solve the Boltzmann equations and determine the evolution of the lepton number asymmetry \( N_L(z) \) with temperature in this BSM model, assuming a radiation-dominated background spacetime.

First, recall that in leptogenesis models, we assume that at much lower temperatures around the electroweak scale, the lepton number asymmetry generated in the early universe is converted through sphaleron processes into a baryon asymmetry. Defining \( \eta = n_B/n_\gamma \) as the baryon-to-photon ratio, the sphaleron conversion in this model gives \( \eta \simeq 0.02 |N_L| \) \cite{16}. Since the observed value is \( \eta \simeq 6 \times 10^{-10} \), we see that successful leptogenesis requires a final value of \( N_L(z) \) for \( z \gg 1 \) of \( |N_L| \simeq 10^{-8} \).

![Figure 7](image)

**Figure 7.** The left-hand diagram shows the evolution of the sterile neutrino density starting from an initial condition with \( N_{\nu_R}(z \ll 1) = 0 \). The right-hand figure shows the corresponding absolute value of the lepton asymmetry induced by the out-of-equilibrium decays of \( \nu_R^1 \). The cusp in the plot indicates that the asymmetry \( N_L(z) \) changes sign as the sterile neutrino density \( N_{\nu_R}(z) \) switches from under to just over its equilibrium value. The parameters here are \( M_1 = 5 \times 10^{10} \text{GeV} \) and \( K = 5 \), with \( \varepsilon_1 = 10^{-6} \).

Now consider the Boltzmann equation (3.8) for \( N_{\nu_R}(z) \). Fig. 7 shows \( N_{\nu_R}(z) \) rising from an initial condition \( N_{\nu_R}(z \ll 1) = 0 \) at early times, then slightly overshooting its equilibrium value around \( z \simeq 1 \) before rapidly being driven back to equilibrium.
Ignoring all other effects in the Boltzmann equation (3.9) for $N_L(z)$, we see that during this out-of-equilibrium phase, the lepton number violating decays $\nu_R \to \nu_L H$ generate the lepton asymmetry shown in the right-hand plot in Fig. 7. This is of course the familiar mechanism of *thermal leptogenesis* through out-of-equilibrium decays, satisfying the third Sakharov condition. Note that the size of the induced asymmetry depends on the decay rate $D(z)$ and, crucially, on the CP violating parameter $\varepsilon_1$ which is itself sensitive to the phases of the Yukawa couplings.

Now consider the evolution of the lepton number asymmetry including the gravitational effects encoded in the full Boltzmann equation (3.9). This is shown, in the case of a hierarchy enhancement $p = 1$, in Fig. 8. From an initial condition $N_L(z) \ll 1$ at ultra-high temperatures, $N_L$ is initially driven towards its equilibrium value $N_L^{eq}(z) \sim 1/z^5$ by the rate factor $W(z) \sim 1/z^2$ for the $\Delta L \neq 0$ reactions. However, at these early times, the gravitational ‘washout’ factor $W(z) \sim 1/z^5$ dominates over $W(z)$ and delays the approach to equilibrium (labelled as region (a) in Fig. 1). $N_L$ then follows its equilibrium trajectory (region (b)) until at some lower temperature ($z \simeq 10^{-3}$ with the parameters shown), the $\Delta L \neq 0$ reaction rate falls below the Hubble expansion rate, *i.e.* $W(z) \simeq 1$, and can no longer maintain $N_L$ in equilibrium. It then decouples and is frozen out at a constant value (region (c)) as

![Figure 8](image-url)
the universe continues to expand and cool. Much later, when $W(z)$ grows around the resonance at $z \approx 1$, it is strong enough to temporarily force $N_L$ back towards the equilibrium value, which is falling away extremely rapidly, resulting in a late dip (region (d)) before settling to its final asymptotic value $|N_L| \approx 10^{-8}$.

The ultimate value of the residual asymmetry $|N_L|$ for temperatures below the Majorana mass scale, $z \gg 1$, is therefore the result of a competition between the gravitational leptogenesis mechanism and thermal leptogenesis. Fig. 8 shows the case where $\varepsilon_1$ is small enough and the gravitational leptogenesis mechanism dominates.

![Figure 9](image_url)

**Figure 9.** Dynamical evolution of the lepton number asymmetry $N_L(z)$ in the case of no hierarchy enhancement, $p = 0$. Here, the final lepton asymmetry is determined by the out-of-equilibrium $\nu_R^1$ decays, with the gravitational mechanism producing a steep rise in the asymmetry at earlier times for temperatures $z \lesssim 0.01$. Here, $M_1 = 10^{10}$ GeV, $K = 5$, with $M_2 = 10^{12}$ GeV, $\text{Im}(\lambda^\dagger\lambda)^2/(4\pi)^2 = 5 \times 10^{-4}$ and $\varepsilon_1 = 10^{-7}$.

In contrast, Fig. 9, where we have assumed no hierarchy enhancement, $p = 0$, and chosen a larger CP violating factor $\varepsilon_1$, shows a scenario in which the thermal leptogenesis mechanism determines the final value of the asymmetry $|N_L|$. Even in this case, however, the gravitational effects describe the early-time evolution of $N_L(z)$ and predict a large lepton number asymmetry in the early universe.

We therefore see from these quantitative examples that radiatively-induced gravitational leptogenesis provides a mechanism which can generate the observed baryon-to-photon ratio in the universe today.
As we have shown, while the RIGL mechanism always determines the evolution of the lepton asymmetry in the early universe above \( T \approx M_1 \), whether the gravitational or sterile neutrino decay mechanisms determine the final value of \( |N_L| \) depends on the masses and Yukawa couplings in the BSM model. A comprehensive investigation of the parameter space of this model, incorporating flavour effects and varying the sterile neutrino mass spectrum, has been given in an interesting recent paper [17]. In particular, it is shown that exploiting flavour effects, the RIGL mechanism is naturally able to achieve successful leptogenesis for ranges of the sterile neutrino masses which are not compatible with standard thermal leptogenesis. This paper also analyses the implications of the model parameters favouring gravitational leptogenesis for low-energy neutrino phenomenology.

![Figure 10. Dynamical evolution of the gravitationally-induced lepton number asymmetry \( N_L(z) \) for effective equation of state parameters \( w = 0.3 \) and 0.5, with \( z_* = 0.1 \). The corresponding equilibrium values \( N_L^{eq} \) are shown as the dashed lines. Decoupling from \( N_L^{eq} \) occurs earlier with higher values of \( w \), resulting in a larger final asymmetry. The parameters here are \( M_1 = 10^{10} \) GeV, \( K = 5 \), with \( M_2 = 10^{12} \) GeV, \( \text{Im}(\lambda_1\lambda_2^*)^2/(4\pi)^2 = 5 \times 10^{-4} \) and no hierarchy enhancement, \( p = 0 \).](image-url)

Finally, we describe briefly the implementation of the RIGL mechanism in an alternative cosmological scenario [4–6]. This picture is especially economical in the sense of minimising the introduction of new fields and interactions, relying on gravitational effects to drive the dynamical evolution of the early universe. We therefore suppose that in place of the usual post-inflationary reheating phase, the relativistic particles giving rise to the entropy of the universe arise through gravitational particle creation.
at the transition from the de Sitter inflationary vacuum to the FRW vacuum at the end of inflation [18, 19]. This removes the need for an unknown direct coupling of the inflaton to the standard model fields to induce reheating. Instead, immediately after inflation the energy density is dominated by the inflaton component with equation of state $w > 1/3$, which gradually dilutes relative to the thermalised relativistic particles as the universe expands, eventually leading, beyond a crossover temperature $z_*$, to a conventional radiation-dominated FRW universe. The stiff equation of state $w > 1/3$ is achieved if the potential at the end of inflation is sufficiently steep [20], the limiting value $w = 1$ in which the kinetic energy dominates being known as ‘kination’. If the temperature during this phase is greater than the lightest sterile neutrino mass, $T > M_1$, then gravitational leptogenesis occurs during this era.

It is relatively straightforward to trace the dependence on $w$ through the various terms in the Boltzmann equations and investigate their solutions. In particular, the power dependence of the gravitational terms becomes $W(z) \sim 1/z^{5-(1-3w)}$ while $N_{\text{eq}}^L(z) \sim 1/z^{5-3(1-3w)}$, so for higher $w$ the fall-off of $N_{\text{eq}}^L$ is steeper, but decoupling occurs earlier giving a higher final value for $|N_L|$. In Fig. 10 we show the dynamical evolution of $N_L(z)$ in an illustrative scenario with no hierarchy enhancement but with $w = 0.5$, which gives the required asymptotic asymmetry $|N_L| \simeq 10^{-8}$, compared to the radiation-dominated case $w \simeq 0.3$ with the same parameters.

Clearly there are many avenues still to explore in developing RIGL as a potential explanation of the matter-antimatter asymmetry of the universe, both in its implementation in a variety of BSM models and in different cosmological scenarios. More generally, this work once again illustrates the importance of loop effects in QFT in curved spacetime in determining the physics of the early universe.

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