Gravity Waves and Gravitino Dark Matter in $\mu$-Hybrid Inflation

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We propose a novel reformulation of supersymmetric (more precisely $\mu$-) hybrid inflation based on a local U(1) or any suitable extension of the minimal supersymmetric standard model (MSSM) which also resolves the $\mu$ problem. We employ a suitable Kahler potential which effectively yields quartic inflation with non-minimal coupling to gravity. Imposing the gravitino Big Bang Nucleosynthesis (BBN) constraint on the reheat temperature ($T_r \lesssim 10^6$ GeV) and requiring a neutralino LSP, the tensor to scalar ratio ($r$) has a lower bound $r \gtrsim 0.004$. The U(1) symmetry breaking scale lies between $10^6$ and $10^{12}$ GeV. We also discuss a scenario with gravitino dark matter whose mass is a few GeV.

Minimal supersymmetric hybrid inflation is based on a unique renormalizable superpotential which also resolves the $\mu$ problem. We employ a suitable Kahler potential which effectively yields quartic inflation with non-minimal coupling to gravity. Imposing the gravitino Big Bang Nucleosynthesis (BBN) constraint on the reheat temperature ($T_r \lesssim 10^6$ GeV) and requiring a neutralino LSP, the tensor to scalar ratio ($r$) has a lower bound $r \gtrsim 0.004$. The U(1) symmetry breaking scale lies between $10^6$ and $10^{12}$ GeV. We also discuss a scenario with gravitino dark matter whose mass is a few GeV.

In minimal supersymmetric hybrid inflation the renormalizable superpotential is expressed as

$$W = S \left[ \kappa (\bar{X} X - M^2) \right],$$  \hspace{1cm} (1)

where $X$, $\bar{X}$ denote a pair of conjugate superfields under the symmetry assumed for simplicity to be U(1), $S$ is a singlet under this symmetry, $M$ denotes the symmetry breaking scale, and $\kappa$ denotes a real dimensionless coupling parameter. A U(1)$_R$ symmetry is assumed under which both $W$ and $S$ carry two units of charge and the combination $\bar{X} X$ is invariant. Note that this symmetry prevents the appearance in $W$ of terms such as $S^2$ and $S^3$ that can ruin inflation.

In the supersymmetric vacuum, the underlying symmetry is broken and the scalar component of $S$, denoted with the same symbol, has zero VEV. During inflation the scalar $S$ slowly rolls from sub-Planckian values towards $M$ and finally to the origin (actually to a location displaced from the origin by an amount proportional to $m_{3/2}$, the gravitino mass.) A scalar spectral index $n_s$ lying between 0.96 and 0.97 is realized for $\kappa \approx 10^{-3}$ and $M \approx 10^{15}$ GeV. The tensor to scalar ratio $r$ is tiny but may approach $10^{-6}$ in some cases.

In $\mu$-hybrid inflation the following additional term is included in $W$:

$$\sigma H_u H_d S,$$  \hspace{1cm} (2)

where the dimensionless constant $\sigma > \kappa$, in order to prevent the MSSM doublets from acquiring VEVs of order $M$. Inflation proceeds as in the previous case except that reheating after inflation is determined by the $\sigma$ term above. A careful evaluation reveals that the reheat temperature exceeds $10^{11}$ GeV and the model is compatible with split supersymmetry.
As previously mentioned this conclusion can be circumvented by including higher order terms in the Kahler potential \([11]\). This allows one to work with smaller \(\sigma\) values which, in turn, yield suitably lower values for the reheat temperature that are compatible with TeV scale supersymmetry.

In our reformulation of \(\mu\)-hybrid inflation we propose to modify the Kahler potential in a way that allows us to implement quartic inflation with non-minimal coupling to gravity \([17]\). A suitable combination of \(X, \bar{X}\) fields, and not the field \(S\), drives inflation in this scenario. The inflaton has trans-Planckian values during the slow roll epoch and eventually reaches its present day minimum value \(M\). As we shall see, \(M\) can take values anywhere from \(10^{6}\) GeV to \(10^{13}\) GeV, if the reheat temperature \(T_r \lesssim 10^{6}\) GeV. Somewhat larger \(T_r\) values, say of order \(3 \times 10^{7}\) GeV, will allow \(M\) to reach the GUT scale. The model also predicts observable gravity waves with \(r \gtrsim 0.004\).

In order to implement non-minimal quartic inflation, we employ a Lagrangian with a non-minimal Kahler potential in the superconformal framework of supergravity \([18]\):

\[
\mathcal{L} \supset \int d^4\theta \phi \phi^\dagger (-3M_P^2\Phi),
\]

where \(M_P = 2.44 \times 10^{18}\) GeV is the reduced Planck mass,

\[
\Phi = 1 - \frac{1}{3M_P^2} (|\bar{X}|^2 + |X|^2 + |S|^2)
+ \frac{1}{3M_P^2}\gamma (\bar{X}X + (\bar{X}X)^\dagger) + \cdots,
\]

and \(\phi = 1 + \theta^2 F_\theta\) is the compensating multiplet with a supersymmetry breaking \(\langle F_\theta \rangle = m_{3/2}\) with \(m_{3/2}\) being gravitino mass. Note that \(\Phi\) may include higher order terms for \(S\) (denoted by \(\cdots\)) to stabilize the scalar potential in the \(S\)-direction \([19]\).

The relevant terms in the Jordan frame Lagrangian for inflation are as follows:

\[
\mathcal{L} \supset -\frac{1}{2} M_P^2 \Phi R + (\partial_\mu \bar{X})^\dagger (\partial^\mu \bar{X}) + (\partial_\mu X)^\dagger (\partial^\mu X) \\
+ (\partial_\mu S)^\dagger (\partial^\mu S) - V_{SUSY},
\]

where

\[
V_{SUSY} \supset \kappa^2 |\bar{X}X - M^2|^2 + \kappa^2 |S|^2 (|\bar{X}X|^2 + |X|^2).
\]

Now we consider the inflation trajectory along the \(D\)-flat direction, \(\langle X \rangle = \langle \bar{X} \rangle\), with the inflaton parametrized as \(X = \bar{X} = (1/2)\varphi\). During inflation the scalars \(S, H_u\) and \(H_d\) are at their potential minimum. Along the inflation trajectory for \(\varphi \gg M\), the relevant Lagrangian is simplified to be

\[
\mathcal{L} \supset -\frac{1}{2} M_P^2 (1 + \xi \varphi^2) R + \frac{1}{2} \theta^{\mu\nu}(\partial_\mu \varphi)(\partial_\nu \varphi) - \frac{\kappa^2}{16} \varphi^4,
\]

where the dimensionless parameter \(\xi\) is given by

\[
\xi = \frac{1}{6}(\gamma - 1).
\]

Note that Eq. (7) is nothing but the Jordan frame Lagrangian of \(\lambda\varphi^4\) inflation with non-minimal gravitational coupling (see, for example, Ref. [19]).

After imposing the constraint on the amplitude of the curvature perturbation

\[
\Delta_P = 2.099 \times 10^{-9}
\]

from the Planck measurements \([6]\) with the pivot scale chosen at \(k_0 = 0.002\) Mpc\(^{-1}\) and assuming the number of e-foldings \((N_0)\) to be 50 or 60 as representative values, we obtain the inflationary predictions as a function of \(\xi\). The result is shown in Figure 1 along with the Planck results. The inflationary predictions for various \(\xi\) values are summarized in Table II. To satisfy the limit obtained by the Planck at 95\% confidence level in Figure 1, we find a lower bound on \(\xi = 0.0136\) (\(\xi \geq 0.00526\)), corresponding to \(r \leq 0.0496\) \((r \leq 0.0963)\) for \(N_0 = 50\) \((N_0 = 60)\). This, in turn, yields a lower bound on \(\kappa\), namely

\[
\kappa \geq 2.54 \times 10^{-6} (1.41 \times 10^{-6}).
\]

With \(r \gtrsim 0.004\) \((0.003)\) for \(N_0 = 50\) \((60)\), primordial gravity waves according to this scenario lie in the observable range.
This is in sharp contrast to standard hybrid inflation which yields $r \lesssim 10^{-4}$. In order to discuss inflaton decay and subsequent reheating, we recall that the VEV of $S$ in the presence of the supersymmetry breaking $(\langle F_S \rangle = m_{3/2}/2\kappa)$ is given by $\langle S \rangle \simeq m_{3/2}/\kappa$, for $m_{3/2} \ll M$ [9]. In turn, this generates the MSSM $\mu$ term:

$$\mu = \frac{\sigma}{\kappa} m_{3/2}. \quad (11)$$

The inflaton field $\varphi$ can decay, in the exact SUSY limit, into the MSSM Higgs fields via the $F$-term ($F_S$) of the superfield $S$ between the superpotential terms $\kappa S\tilde{X} X$ and $\sigma SH_uH_d$ [20]. The decay width is given by

$$\Gamma = \frac{\sigma^2}{8\pi} m_\varphi = \frac{\sqrt{2}}{8\pi} \kappa^3 \left( \frac{\mu}{m_{3/2}} \right)^2 M, \quad (12)$$

where $m_\varphi = \sqrt{2}\kappa M$ is the inflaton mass at the potential minimum, and we have assumed, for simplicity, that the inflation can decay into all the Higgs bosons in the MSSM. We estimate the reheat temperature $T_r$ from the relation,

$$\Gamma = H(T_r) = \left( \frac{\pi^2}{90} g_* \right)^{1/2} \frac{T_r^2}{M_P}. \quad (13)$$

Setting $g_* \simeq 200$ for the MSSM, we find in units of GeV

$$T_r \simeq 1.71 \times 10^8 \kappa^{3/2} \left( \frac{\mu}{m_{3/2}} \right) M^{1/2}. \quad (14)$$

Here, we comment on a justification of our estimate for the reheat temperature in Eq. (14). In $\lambda\varphi^4$ inflation with non-minimal gravitational coupling, in particular the Higgs inflation, the reheat temperature is expected to be very high, $T_r \geq 10^{13}$ GeV, because of the parametric resonance effects [22, 23]. However, we notice that the parametric resonance effects discussed in Refs. [22, 23] are important for a limited case, namely, $\xi \gg 1$. From Figure 2 we have an upper bound on $\kappa \lesssim 10^{-4}$, which means $\xi$ is not large: $\xi \lesssim 10$ from Table I. As previously studied in, for example, Ref. [24], even if a non-minimal gravitational coupling is zero, an inflaton oscillating in its quartic potential can give rise to resonant production of a particle (Higgs bosons in our case) through its coupling. To see if the parametric resonance effect is important in our model, we follow the analysis in Ref. [25]. From our superpotential, we have a coupling between the inflaton and the Higgs doublets in the scalar potential given by

$$V_{\text{SUSY}} \supset \frac{1}{4} \kappa \sigma \varphi^2 (H_uH_d + h.c.). \quad (15)$$

For inflaton values $|\varphi| \gg M$ during its oscillation in the quartic potential $\kappa^2 \varphi^4/16$, the Higgs doublets acquire an effective mass $m_H \simeq \sqrt{\kappa}\sigma|\varphi|$. This effective mass is to be compared with the effective frequency of the oscillating inflaton, $\omega \simeq \kappa|\varphi|$. Since we take $\sigma > \kappa$, the effective mass is larger than the frequency (for $|\varphi| \gg M$), so that the resonant production of Higgs doublets is not effective. When $m_H$ becomes smaller than $\omega$, the resonant production may begin. However, the condition $m_H < \omega$ is satisfied when the inflaton gets close to the potential minimum, $|\varphi| \simeq M$. In this case, the inflaton is oscillating around the potential minimum with $\omega \simeq m_\varphi$, and hence, our estimate of the reheat temperature by the inflaton decay width is reasonable, neglecting the parametric resonance effects as discussed above.

To proceed further we first assume that the gravitino is not the LSP. Imposing the constraint $T_r \leq T_{\text{max}} \simeq 10^9 - 10^{10}$ GeV for $m_{3/2} = 1 - 10$ TeV in order to avoid the cosmological gravitino problem [21], we find

$$\kappa \leq 0.0324 \left( \frac{T_{\text{max}}}{10^9 \text{GeV}} \right)^{2/3} \left( \frac{m_{3/2}}{\mu} \right)^{2/3} \left( \frac{1 \text{ GeV}}{M} \right)^{1/3}. \quad (16)$$

In order to keep the inflaton decay channel open into all the Higgs bosons in the MSSM, we impose a lower bound on the inflaton mass as $m_\varphi \geq M_H$ and hence,

$$\kappa \geq \frac{1}{\sqrt{2}} \left( \frac{M_H}{M} \right), \quad (17)$$

where $M_H$ is the mass scale of the heavy Higgs bosons in the MSSM. Varying $\mu/m_{3/2}$ from 1 to 500, a plot of $\kappa$ versus $M$ is shown in Figure 2. Corresponding to a low reheating temperature $T_r \leq 10^6$ GeV, we have set $N_0 = 50$ [26]. Note that a solution exists only for $\mu/m_{3/2} \leq 86$ (8.6) for $M_H = 1$ (100) TeV. In summary, for $T_r \leq 10^6$ GeV and $M_H = 1$ TeV, the symmetry breaking scale $M$ is constrained as follows:

$$2.8 \times 10^8 \leq M [\text{GeV}] \leq 2.1 \times 10^{12}. \quad (18)$$
Note that \( M \) values of order \( M_{GUT} = 2 \times 10^{16} \text{ GeV} \) are realized if we set \( T_r \approx 10^8 \text{ GeV} \). These results should be contrasted with conventional \( \mu \)-hybrid inflation \([1, 9, 10]\), which yield \( M \) values of the order of \( 10^{15} - 10^{16} \text{ GeV} \). Using a one-to-one correspondence between the \( \kappa \) value and the inflationary predictions, we show in Figure 3 a plot of \( r \) versus \( M \) for \( \mu/m_{3/2} = 1 \).

Next let us explore the gravitino LSP scenario. The gravitino relic density from thermal production is given by \([27]\)

\[
\Omega h^2 \approx 1.8 \times 10^{-8} \left( \frac{T_r}{m_{3/2}} \right) \left( \frac{M_3}{3 \text{ TeV}} \right)^2,
\]

where \( M_3 \) is a running gluino mass. In order to reproduce the observed dark matter relic abundance, \( \Omega h^2 \approx 0.1 \) \([28]\), we find

\[
\kappa = 0.102 \left( \frac{m_{3/2} \text{[GeV]}^2}{\mu \text{[GeV]} \sqrt{M \text{[GeV]}}} \right)^{2/3} \left( \frac{3 \text{ TeV}}{M_3} \right)^{4/3}.
\]

In the gravitino LSP scenario, we also have the BBN constraint on the next LSP (NLSP) lifetime given by \([29]\)

\[
\tau_{NLSP} \approx 10^4 \text{ sec} \left( \frac{100 \text{ GeV}}{m_{NLSP}} \right)^5 \left( \frac{m_{3/2}}{1 \text{ GeV}} \right)^2 < 1 \text{ sec}.
\]

As an example, let us consider \( \mu \approx 1 \text{ TeV} \approx m_{NLSP} \), which yields

\[
m_{3/2} \approx 3.2 \text{ GeV}
\]

from the BBN constraint. In Figure 4 we plot \( \kappa \) versus \( M \) for various \( m_{3/2} \) values. A gravitino dark matter scenario is viable with \( m_{3/2} > 1.45 \text{ GeV} \).

A discussion about inflation is not complete without at least mentioning how the observed baryon asymmetry is realized.
In our case this can be achieved via thermal \(10\) or non-thermal \(10\) leptonogenesis by introducing right handed neutrinos. For instance, identifying \(U(1)\) with a local \(U(1)_{B-L}\) symmetry requires three right handed neutrinos, one per family, to avoid gauge anomalies, which have Yukawa couplings with the inflaton system. With at least one of the right handed neutrinos lighter than the inflaton, the decay products of the latter would include this neutrino and leptogenesis becomes possible.

In summary, the inflationary scenario we have described can be based on a fairly minimal, namely, a local \(U(1)\) extension of the MSSM or a more elaborate one such as grand unification. If the gravitino is not the LSP, the BBN constraint leads to a lower bound on the tensor to scalar ratio of \(r \gtrsim 0.004\) and the \(U(1)\) symmetry breaking scale lies in the range of \(10^8\)–\(10^{12}\) GeV. A LSP gravitino with mass of order \(1\) GeV is a viable dark matter candidate.

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