Semileptonic charmed $B$ meson decays in Universal Extra Dimension Model

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Abstract

Form factors parameterizing the semileptonic decay $B_c \rightarrow D_s^* l^+ l^-$ ($l = \mu, \tau$) are calculated using the framework of Ward Identities. These form factors are then used to calculate the physical observables like branching ratio and helicity fractions of final state $D_s^*$ meson in these decay modes. The analysis is then extended to the universal extra dimension (UED) model where the dependence of above mentioned physical variables to the compactification radius $R$, the only unknown parameter in UED model, is studied. It is shown that the helicity fractions of $D_s^*$ are quite sensitive to the UED model especially when have muons as the final state lepton. Therefore, these can serve as a useful tool to establish new physics predicted by the UED model.
I. INTRODUCTION

Living in the LHC era, it is hoped to either verify the Standard Model (SM) or to explore the properties of more accurate underlying theory that describes the theory of weak scale. Flavor Changing Neutral Current (FCNC) decays of B-meson are an important tool to investigate the structure of weak interactions and also provide us a frame work to look for the physics beyond the Standard Model (SM). This lies in the fact that FCNC decays are not allowed at tree level in the SM and occur only at the loop level \[1,3\] and makes them quite sensitive to possible small corrections that may be result of any modification to the SM, or from the new interactions. This gives us solid reason to study these decays both theoretically and experimentally.

Since the CLEO observations of the rare radiative \(b \to s\gamma\) transition \[4\], there have been intensive studies on rare semileptonic, radiative and leptonic decays of \(B_{s,u,d,s}\) mesons induced by FCNC transitions of \(b \to s, d\) \[5\]. The study will be even more complete if one consider the similar decays of the charmed \(B\) mesons (\(B_c\)).

The charmed \(B_c\) meson is a bound state of two heavy quarks, bottom \(b\) and charm \(c\), and was first observed in 1998 at Tevatron in Fermilab \[6\]. Because of two heavy quarks, the \(B_c\) mesons are rich in phenomenology compared to the other \(B\) mesons. At the Large Hadron Collider (LHC) the expected number of events for the production of \(B_c\) meson are about \(10^8 - 10^{10}\) per year \[7,8\] which is a reasonable number to work on the phenomenology of the \(B_c\) meson. In literature, some of the possible radiative and semileptonic exclusive decays of \(B_c\) mesons like \(B_c \to (p, K^*, D_s^*, B_s^*)\gamma\), \(B_c \to l\nu\gamma\), \(B_c \to B_{s}^* l^+ l^-\), \(B_c \to D_s^0 l\nu\), \(B_c \to D_s^0 l^+ l^-\) and \(B_c \to D_{s}^* l^+ l^-\) have been studied using the frame work of relativistic constituent quark model \[9\], QCD Sum Rules and the Light Cone Sum Rules \[10\]. The focus of the present work is the study of exclusive \(B_c \to D_{s}^* l^+ l^-\) decay.

While working on the exclusive \(B\)-meson decays the main job is to calculate the form factors which are the non perturbative quantities and are one consider the similar decays of the charmed \(B\) mesons (\(B_c\)). In this work we calculate the form factors for the above mentioned decay in a model independent way through Ward identities, which was earlier applied to \(B \to \rho, \gamma\) \[12,13\] and \(B \to K_1\) decays \[14\]. This approach enables us to make a clear separation between the pole and non pole type contributions, the former is known in terms of a universal function \(\xi_\perp(q^2) \equiv g_\perp(q^2)\) which is introduced in the Large Energy Effective Theory (LEET) of heavy to light transition form factors \[15\]. The residue of the pole is then determined in a self consistent way in terms of \(g_\perp(0)\) which will give information about the couplings of \(B_s^*(1^-)\) and \(B_{s,A_2}^*(1^+)\) with \(B_c^*\) channel. The above mentioned coupling arises at lower pole masses because the higher pole masses of \(B_c\) meson do not contribute for the decay \(B_c \to D_{s}^* l^+ l^-\). The form factors are then determine in terms of a known parameter \(g_\perp(0)\) and the pole masses of the particles involved, which will then be used to calculate different physical observables like the branching ratio and the helicity fractions of \(D_{s}^*\) for these decays.

At the quark level the semileptonic decay \(B_c \to D_{s}^* l^+ l^-\) is governed by the FCNC transition \(b \to s l^+ l^-\), therefore it is an important candidate to look for physics in and beyond the SM. Many investigations for the physics beyond the SM are now being performed in various areas of particle physics which are expected to get the direct or indirect evidence at high energy colliders such as LHC. During the last couple of years there have been an increased interest in models with extra dimensions, since they solve the hierarchy problem and they can provide the unified framework of gravity and other interactions together with a connection to the string theory \[16\]. Among them the special role plays the one with universal extra dimensions (UED) as in this model all SM fields are allowed to propagate in available all dimensions. The economy of UED model is that there is only one additional parameter to that of SM which is the radius \(R\) of the compactified extra dimension. More above the compactification scale \(1/R\) a given UED model becomes a higher dimensional field theory whose equivalent description in four dimensions consists of SM fields and the towers of KK modes having no partner in the SM. A simplest model of this type was proposed by Appelquist,Cheng and Dobrescu (ACD) \[17\]. In this model all the masses of the KK particles and their interactions with SM particles and also among themselves are described in terms of the inverse of compactification radius \(R\) and the parameters of the SM \[18\].

The most important property of ACD model is the conservation of parity which implies the absence of tree level
contributions of KK states to the low energy processes taking place at scale $\mu << 1/R$. This brings interest towards the FCNC transitions, $b \rightarrow s$ as mentioned earlier that these transitions occur at loop level in SM and hence the one loop contribution due to KK modes to them could in principle be important. These processes are used to constrain the mass and couplings of the KK states, i.e, the compactification radius $1/R$.\[15\ [19].

Buras et al. have computed the effective Hamiltonian of several FCNC processes in ACD model, particularly in $b$ sector, namely $B_{s,d}$ mixing and $b \rightarrow s$ transition such as $b \rightarrow s\gamma$ and $b \rightarrow s\ell^+\ell^-$ decay \[18\]. The implications of physics with UED are examined with data from Tevatron experiments and the bounds on the inverse of compactification radius are found to be $1/R \geq 250 - 300$ GeV \[20\]. There exists some studies in the literature on different $B$ to light meson decays in ACD model, where the dependence of different physical observables like branching ratio, forward-backward asymmetry, lepton polarization asymmetry and the helicity fractions of final state mesons on $1/R$ is examined \[20\ [22\].

In this work we will study the branching ratio and helicity fractions of $D_s^*$ meson in $B_c \rightarrow D_s^*\ell^+\ell^-$ decay both in the SM and ACD model using the framework of $B \rightarrow (K^*, K_1)\ell^+\ell^-$ decays described in refs. \[21\ [22\]. The paper is organized as follows. In Sec. II we present the effective Hamiltonian for the decay $B_c \rightarrow D_s^*\ell^+\ell^-$. Section III contains the definitions as well as the detailed calculation of the form factors using Ward Identities. In Sec. IV we present the basic formulas for physical observables like decay rate and helicity fractions of $D_s^*$ meson where as the numerical analysis of these observables will be given in Section V. Section VI gives the summary of the results.

II. EFFECTIVE HAMILTONIAN AND MATRIX ELEMENTS

At quark level, the semileptonic decay $B_c \rightarrow D_s^*\ell^+\ell^-$ is governed by the transition $b \rightarrow s\ell^+\ell^-$ for which the effective Hamiltonian can be written as

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts} \left[ \sum_{i=1}^{10} C_i(\mu) O_i \right],$$

where $O_i(\mu)$ ($i = 1, \ldots, 10$) are the four quark operators and $C_i(\mu)$ are the corresponding Wilson coefficients at the energy scale $\mu$ \[23\] which was usually take to be the $s$ energy scale $O$. The theoretical uncertainties related to the renormalization scale can be reduced when the next to leading logarithm corrections are included. The explicit form of the operators responsible for the decay $B_c^{-} \rightarrow D_s^{*-}\ell^{+}\ell^{-}$ is

$$O_7 = \frac{e^2}{16\pi^2} m_b(\bar{s}\sigma_{\mu\nu} Rb) F^{\mu\nu}$$

$$O_9 = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu Lb) (\bar{l}\gamma^\mu l)$$

$$O_{10} = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu Lb) (\bar{l}\gamma^\mu \gamma^5 l)$$

with $L, R = (1 + \gamma^5) / 2$.

Using the effective Hamiltonian given in Eq. (1) the free quark amplitude for $b \rightarrow s\ell^+\ell^-$ can be written as

$$\mathcal{M}(b \rightarrow s\ell^+\ell^-) = -\frac{G_F}{\sqrt{2}\pi} V_{tb} V_{ts} \left[ C_9^{\text{eff}}(\mu) (\bar{s}\gamma_\mu Lb)(\bar{l}\gamma^\mu l) + C_{10}(\bar{s}\gamma_\mu Lb)(\bar{l}\gamma^\mu \gamma^5 l) \right.$$

$$\left. -2C_7^{\text{eff}}(\mu) \frac{m_b}{q^2} (\bar{s}\sigma_{\mu\nu} q^\nu Rb)(\bar{l}\gamma^\mu l) \right]$$

where $q^2$ is the square of momentum transfer. Note that the operator $O_{10}$ given in Eq. (3) can not be induced by the insertion of four quark operators because of the absence of $Z$-boson in the effective theory. Therefore, the Wilson coefficient $C_{10}$ does not renormalize under QCD corrections and is independent on the energy scale $\mu$. Additionally the above quark level decay amplitude can get contributions from the matrix element of four quark operators, $\sum_{i=1}^{10} (l^+ l^- s | O_i | b)$, which are usually absorbed into the effective Wilson coefficient $C_9^{\text{eff}}(\mu)$ and can be written as

$$C_9^{\text{eff}}(\mu) = C_9(\mu) + Y_{SD}(z, s') + Y_{LD}(z, s').$$

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where \( z = m_c/m_b \) and \( s' = q^2/m_b^2 \). \( Y_{SD}(z, s') \) describes the short distance contributions from four-quark operators far away from the \( c\bar{c} \) resonance regions, and this can be calculated reliably in the perturbative theory. However, the long distance contribution \( Y_{LD}(z, s') \) cannot be calculated by using the first principles of QCD, so they are usually parameterized in the form of a phenomenological Breit-Wigner formula making use of the vacuum saturation approximation and quark hadron duality. Therefore, one can not calculate them reliably so we will neglect these long distance effects for the case of \( B_c \rightarrow D_s^* l^+ l^- \). The expression for the short distance contribution \( Y_{SD}(z, s') \) is given as

\[
Y_{SD}(z, s') = h(z, s')(3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu))
- \frac{1}{2} h(1, s')(4C_3(\mu) + 4C_4(\mu) + 3C_5(\mu) + C_6(\mu))
- \frac{1}{2} h(0, s')(C_3(\mu) + C_4(\mu)) + \frac{2}{9} (3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)),
\]

with

\[
h(z, s') = -\frac{8}{9} \ln z + \frac{8}{27} + \frac{4}{9} x - \frac{2}{9} (2 + x) |1 - x|^{1/2} \left\{ \ln \frac{\sqrt{1 - x} + 1}{\sqrt{1 - x} - 1} - i\pi \right\} \quad \text{for } x \equiv 4z^2/s' < 1,
\]

\[
h(0, s') = \frac{8}{27} - \frac{8}{9} \ln \frac{m_b}{\mu} - \frac{4}{9} \ln s' + \frac{4}{9} i\pi.
\]

Also the non factorizable effects from the charm Wilson coefficients \( C_7^{ff} \) which then takes the form \( \approx \frac{31}{35} \frac{1}{4\alpha_s(m_W)} \frac{\alpha_s}{\alpha_s(\mu)} \mu \), \( G_1(x_t) = \frac{x_t (x_t^2 - 5x_t - 2)}{8 (x_t - 1)^3} + \frac{3x_t^2 \ln x_t}{4 (x_t - 1)^4} \)

where \( \eta = \alpha_s(m_W)/\alpha_s(\mu) \), \( x_t = m_c^2/m_b^2 \) and \( C_{b\rightarrow s\gamma} \) is the absorptive part for the \( b \rightarrow sc\bar{c} \rightarrow s\gamma \) rescattering.

The new physics effects manifest themselves in rare \( B \) decays in two different ways, either through new contribution to the Wilson coefficients or through the new operators in the effective Hamiltonian, which are absent in the SM. Being minimal extension of SM the ACD model is the most economical one because it has only additional parameter \( R \) i.e. the radius of the compactification leaving the operators basis same as that of the SM. Therefore, the whole contribution from all the KK states is in the Wilson coefficients which are now the functions of the compactification radius \( R \). At large value of \( 1/R \) the new states being more and more massive and will be decoupled from the low-energy theory, therefore one can recover the SM phenomenology.

The modified Wilson coefficients in ACD model contain the contribution from new particles which are not present in the SM and comes as an intermediate state in penguin and box diagrams. Thus, these coefficients can be expressed in terms of the functions \( F(x_t, 1/R), x_t = \frac{m_t^2}{M^2} \), which generalize the corresponding SM function \( F_0(x_t) \) according to:

\[
F(x_t, 1/R) = F_0(x_t) + \sum_{n=1}^{\infty} F_n(x_t, x_n)
\]

with \( x_n = \frac{m_n^2}{M^2} \) and \( m_n = \frac{n}{R} \). The relevant diagrams are \( Z^0 \) penguins, \( \gamma \) penguins, gluon penguins, \( \gamma \) magnetic penguins, Chromomagnetic penguins and the corresponding functions are \( C(x_t, 1/R), D(x_t, 1/R), E(x_t, 1/R), D'(x_t, 1/R) \) and \( E'(x_t, 1/R) \) respectively. These functions are calculated at next to leading order by Buras et al. and can be summarized as:

\[ C_7 \]
In place of $C_7$, one defines an effective coefficient $C_7^{(0)\, eff}$ which is renormalization scheme independent:

$$C_7^{(0)\, eff}(\mu_b) = \eta \frac{16}{3} C_7^{(0)}(\mu_W) + \frac{8}{3} (\eta \frac{16}{3} - \eta \frac{16}{3}) C_8^{(0)}(\mu_W) + C_2^{(0)}(\mu_W) \sum_{i=1}^{8} h_i \eta^{\alpha_i}$$

where $\eta = \frac{\alpha_s(\mu_b)}{\alpha_s(\mu_W)}$, and

$$C_2^{(0)}(\mu_W) = 1, \quad C_7^{(0)}(\mu_W) = -\frac{1}{2} D'(x_t, \frac{R}{1}), \quad C_8^{(0)}(\mu_W) = -\frac{1}{2} E'(x_t, \frac{R}{1});$$

the superscript (0) stays for leading logarithm approximation. Furthermore:

$$\alpha_1 = \frac{14}{23}, \quad \alpha_2 = \frac{16}{23}, \quad \alpha_3 = \frac{6}{23}, \quad \alpha_4 = -\frac{12}{23},$$
$$\alpha_5 = 0.4086, \quad \alpha_6 = -0.4230, \quad \alpha_7 = -0.8994, \quad \alpha_8 = -0.1456,$$
$$h_1 = 2.996, \quad h_2 = -1.0880, \quad h_3 = -\frac{3}{5}, \quad h_4 = -\frac{1}{14},$$
$$h_5 = -0.649, \quad h_6 = -0.0380, \quad h_7 = -0.0185, \quad h_8 = -0.0057.$$

The functions $D'$ and $E'$ are

$$D_0'(x_t) = -\frac{(8x_t^3 + 5x_t^2 - 7x_t)}{12(1 - x_t)^3} \ln x_t,$$
$$E_0'(x_t) = -\frac{x_t(x_t^2 - 5x_t - 2)}{4(1 - x_t)^3} + \frac{3x_t^2}{2(1 - x_t)^3} \ln x_t,$$

$$D_n'(x_t, x_n) = \frac{x_t(-37 + 44x_t + 17x_t^2 + 6x_n^2(10 - 9x_t + 3x_t^2) - 3x_n(21 - 54x_t + 17x_t^2))}{36(x_t - 1)^3} \ln x_n + \frac{x_n(2 - 7x_n + 3x_n^2)}{6} \ln \frac{x_n}{1 + x_n}$$
$$+ \frac{(-2 + x_n + 3x_n)(x_t + 3x_t^2 + x_n^2(3 + x_t) - x_n)(1 + (-10 + x_t)x_t)})}{6(x_t - 1)^4} \ln \frac{x_n + x_t}{1 + x_t},$$

$$E_n'(x_t, x_n) = \frac{x_t(-17 - 8x_t + x_t^2 + 3x_n(21 - 6x_t + x_t^2) - 6x_n^2(10 - 9x_t + 3x_t^2))}{12(x_t - 1)^3} \ln x_n$$
$$+ \frac{-x_n(1 + x_n)(-1 + 3x_n)}{6} \ln \frac{x_n}{1 + x_n}$$
$$+ (1 + x_n)(x_t + 3x_t^2 + x_n^2(3 + x_t) - x_n(1 + (-10 + x_t)x_t))}{2(x_t - 1)^4} \ln \frac{x_n + x_t}{1 + x_t}.$$

Following reference [18], one gets the expressions for the sum over $n$:

$$\sum_{n=1}^{\infty} D_n'(x_t, x_n) = \frac{\pi M_w R}{2} \left[ \int_0^1 dy \frac{1}{6} \left( y^2 + 7y + 1 \right) \coth(\pi M_w R \sqrt{y}) \right. $$
$$+ \left. \frac{-2 + x_t}{6(x_t - 1)^4} \right] J(R, \frac{1}{2})$$
$$\left. - \frac{1}{6(x_t - 1)^4} \right] \left[ x_t(1 + 3x_t) - \left(-2 + 3x_t\right)(1 + (-10 + x_t)x_t) \right] J(R, \frac{1}{2}) $$
$$+ \frac{1}{6(x_t - 1)^4} \left[ (3 + x_t)(3 + x_t) - (1 + (-10 + x_t)x_t) \right] J(R, \frac{3}{2})$$
$$- \frac{1}{6(x_t - 1)^4} \right] J(R, \frac{5}{2}).$$
\[
\sum_{n=1}^{\infty} E'_n(x_t, x_n) = \frac{-x_t(-17 + (-8 + x_t)x_t)}{24(x_t - 1)^3} + \frac{\pi M_w R_t}{2} \int_0^1 dy (y^2 + 2y^3 - 3y^2) \coth(\pi M_w R\sqrt{y})]
\]

\[
\frac{x_t(1 + 3x_t)}{(x_t - 1)^2} J(R, -\frac{1}{2}) + \frac{1}{(x_t - 1)^4} [x_t(1 + 3x_t) - (1 + (-10 + x_t)x_t)]J(R, \frac{1}{2}) - \frac{1}{(x_t - 1)^4} [(3 + x_t) - (1 + (-10 + x_t)x_t)]J(R, \frac{3}{2})
\]

\[
+ \frac{(3 + x_t)}{(x_t - 1)^2} J(R, \frac{5}{2})
\]

(19)

where

\[
J(R, \alpha) = \int_0^1 dy y^\alpha [\coth(\pi M_w R\sqrt{y}) - x_t^{1+\alpha} \coth(\pi m_t R\sqrt{y})].
\]

(20)

• \(C_9\)

In the ACD model and in the NDR scheme one has

\[
C_9(\mu) = P_0^{NDR} + \frac{Y(x_t, \frac{1}{R})}{\sin^2 \theta_W} - 4Z(x_t, \frac{1}{R}) + P_E E(x_t, \frac{1}{R})
\]

(21)

where \(P_0^{NDR} = 2.60 \pm 0.25\) \(\mu_0\) and the last term is numerically negligible. Besides

\[
Y(x_t, \frac{1}{R}) = Y_0(x_t) + \sum_{n=1}^{\infty} C_n(x_t, x_n)
\]

\[
Z(x_t, \frac{1}{R}) = Z_0(x_t) + \sum_{n=1}^{\infty} C_n(x_t, x_n)
\]

(22)

with

\[
Y_0(x_t) = \frac{x_t x_t - 4}{8(x_t - 1)} + \frac{3x_t}{(x_t - 1)^2} \ln x_t
\]

\[
Z_0(x_t) = \frac{18x_t^4 - 163x_t^3 + 259x_t^2 - 108x_t}{144(x_t - 1)^3} + \frac{32x_t^4 - 38x_t^3 + 15x_t^2 - 18x_t}{72(x_t - 1)^2} \ln x_t
\]

(23)

\[
C_n(x_t, x_n) = \frac{x_t}{8(x_t - 1)^2} \left[ x_t^2 - 8x_t + 7 + (3 + 3x_t + 7x_n - x_n x_t) \ln \frac{x_t + x_n}{1 + x_n} \right]
\]

(24)

\[
\sum_{n=1}^{\infty} C_n(x_t, x_n) = \frac{x_t(7 - x_t)}{16(x_t - 1)} - \frac{\pi M_w R x_t}{16(x_t - 1)^2} [3(1 + x_t)J(R, -\frac{1}{2}) + (x_t - 7)J(R, \frac{1}{2})]
\]

(25)

• \(C_{10}\)

\(C_{10}\) is \(\mu\) independent and is given by

\[
C_{10} = -\frac{Y(x_t, \frac{1}{R})}{\sin^2 \theta_W}
\]

(26)

The normalization scale is fixed to \(\mu = \mu_b \simeq 5\) GeV.
III. MATRIX ELEMENTS AND FORM FACTORS

The exclusive $B_c \to D_s^* l^+ l^-$ decay involves the hadronic matrix elements which can be obtained by sandwiching the quark level operators given in Eq. (5) between initial state $B_c$ meson and final state $D_s^*$ meson. These can be parameterized in terms of form factors, which are scalar functions of the square of the four momentum transfer ($q^2 = (p - k)^2$). The non-vanishing matrix elements for the process $B_c \to D_s^*$ can be parameterized in terms of the seven form factors as follows

$$\langle D_s^* (k, \varepsilon) | \bar{s} \gamma_\mu b | B_c (p) \rangle = \frac{2 \epsilon_{\mu \nu \alpha \beta} \varepsilon^{* \nu} p^\alpha k^\beta V(q^2)}{M_{B_c} + M_{D_s^*}}$$

$$\langle D_s^* (k, \varepsilon) | \bar{s} \gamma_\mu \gamma_5 b | B_c (p) \rangle = i \left( M_{B_c} + M_{D_s^*} \right) \varepsilon^{* \mu} A_1(q^2) - i \frac{(\varepsilon^* \cdot q)}{M_{B_c} + M_{D_s^*}} (p + k)^\mu A_2(q^2) - i \frac{2 M_{D_s^*}}{q^2} (\varepsilon^* \cdot q) [A_3(q^2) - A_0(q^2)]$$

(27)

where $p$ is the momentum of $B_c$, $\varepsilon$ and $k$ are the polarization vector and momentum of the final state $D_s^*$ meson. Here, the form factor $A_3(q^2)$ can be expressed in terms of the form factors $A_1(q^2)$ and $A_2(q^2)$ as

$$A_3(q^2) = \frac{M_{B_c} - M_{D_s^*}}{2M_{D_s^*}} A_1(q^2) - \frac{M_{B_c} - M_{D_s^*}}{2M_{D_s^*}} A_2(q^2)$$

(28)

with

$$A_3(0) = A_0(0)$$

In addition to the above form factors there are some penguin form factors, which we can write as

$$\langle D_s^* (k, \varepsilon) | \bar{s} \sigma_{\mu \nu} q^\nu b | B_c (p) \rangle = - \epsilon_{\mu \nu \alpha \beta} \varepsilon^{* \nu} p^\alpha k^\beta 2F_1(q^2)$$

$$\langle D_s^* (k, \varepsilon) | \bar{s} \sigma_{\mu \nu} q^\nu \gamma_5 b | B_c (p) \rangle = i \left[ \left( M_{B_c}^2 - M_{D_s^*}^2 \right) \varepsilon_\mu - (\varepsilon^* \cdot q)(p + k)_\mu \right] F_2(q^2)$$

(30)

$$+(\varepsilon^* \cdot q)i \left[ q_\mu - \frac{q^2}{M_{B_c}^2 - M_{D_s^*}^2} (p + k)_\mu \right] F_3(q^2)$$

(31)

with

$$F_1(0) = F_2(0)$$

Now the different form factors appearing in Eqs. (27, 31) can be related to each other with the help of Ward identities as follows:

$$\langle D_s^* (k, \varepsilon) | \bar{s} \sigma_{\mu \nu} q^\nu b | B_c (p) \rangle = (m_b + m_s) \langle D_s^* (k, \varepsilon) | \bar{s} \gamma_\mu b | B_c (p) \rangle$$

$$\langle D_s^* (k, \varepsilon) | \bar{s} \sigma_{\mu \nu} q^\nu \gamma_5 b | B_c (p) \rangle = -(m_b - m_s) \langle D_s^* (k, \varepsilon) | \bar{s} \gamma_\mu \gamma_5 b | B_c (p) \rangle + (p + k)_\mu \langle D_s^* (k, \varepsilon) | \bar{s} \gamma_5 b | B_c (p) \rangle$$

(32)

(33)

By putting Eq. (27, 31) in Eq. (28) and (30) and comparing the coefficients of $\varepsilon^*_\mu$ and $q_\mu$ on both sides, one can get the following relations between the form factors:

$$F_1(q^2) = \frac{(m_b + m_s)}{M_{B_c} + M_{D_s^*}} V(q^2)$$

$$F_2(q^2) = \frac{m_b - m_s}{M_{B_c} + M_{D_s^*}} A_1(q^2)$$

$$F_3(q^2) = -(m_b - m_s) \frac{2M_{D_s^*}}{q^2} \left[ A_3(q^2) - A_0(q^2) \right]$$

(34)

(35)

(36)
The results given in Eqs. (34, 35, 36) are derived by using Ward identities and therefore are the model independent.

The universal normalization of the above form factors at \( q^2 = 0 \) are obtained by defining

\[
\langle D_s^*(k, \varepsilon) | \bar{s}i\sigma_{\alpha\beta}q^5 b \rangle | B_c(p) \rangle = \frac{-i\epsilon_{\alpha\beta\gamma\delta} \varepsilon^\gamma q^\delta[(p + k)^0 g_+ + q^0 g_-] - (\varepsilon^\gamma q^\delta \epsilon_{\alpha\beta\gamma\delta}(p + k)^0 q^\gamma h - i[(p + k)_\alpha \epsilon_{\beta\gamma\delta}(p + k)^0 q^\gamma - \alpha \leftrightarrow \beta \rangle h_1}{(p + k)^0 q^0 g_+ + q^0 g_- - \alpha \leftrightarrow \beta \rangle h_1} \tag{37}
\]

Making use of the Dirac identity

\[
\sigma^{\mu\nu}\gamma^5 = \frac{-i}{2} \epsilon^{\mu\nu\alpha\beta} \sigma_{\alpha\beta}
\]

in Eq.(37), we get

\[
\langle D_s^*(k, \varepsilon) | \bar{s}i\sigma_{\mu\nu}q^\nu q^5 b \rangle | B_c(p) \rangle = \epsilon^\mu \left[(M_{B_c}^2 - M_{D_s^-}^2)g_+ + q^2 g_- \right]
- q \cdot \varepsilon^\nu \left[q^2(p + k)_\mu g_+ - q_\mu g_- \right]
+ q \cdot \varepsilon^\nu \left[q^2(p + k)_\mu - (M_{B_c}^2 - M_{D_s^-}^2)q_\mu \right] h \tag{39}
\]

On comparing coefficients of \( q_\mu, \epsilon^\mu \) and \( \epsilon_{\mu\nu\alpha\beta} \) from Eqs. (30), (31), (37) and (39), we have

\[
F_1(q^2) = [g_+(q^2) - q^2 h_1(q^2)] \tag{40}
\]
\[
F_2(q^2) = g_+(q^2) + \frac{q^2}{M_{B_c}^2 - M_{D_s^-}^2} g_-(q^2) \tag{41}
\]
\[
F_3(q^2) = -g_-(q^2) - (M_{B_c}^2 - M_{D_s^-}^2) h(q^2) \tag{42}
\]

One can see from Eq. (40) and Eq. (41) that at \( q^2 = 0 \), \( F_1(0) = F_2(0) \). The form factors \( V(q^2), A_1(q^2) \) and \( A_2(q^2) \) can be written in terms of \( g_+, g_- \) and \( h \) as

\[
V(q^2) = \frac{M_{B_c}^2 + M_{D_s^-}^2}{m_b + m_s} [g_+(q^2) - q^2 h_1(q^2)] \tag{43}
\]
\[
A_1(q^2) = \frac{M_{B_c}^2 + M_{D_s^-}^2}{m_b - m_s} \left[g_+(q^2) + \frac{q^2}{M_{B_c}^2 - M_{D_s^-}^2} g_-(q^2) \right] \tag{44}
\]
\[
A_2(q^2) = \frac{M_{B_c}^2 + M_{D_s^-}^2}{m_b - m_s} [g_+(q^2) - q^2 h_1(q^2)] - \frac{2}{M_{B_c}^2 - M_{D_s^-}^2} A_0(q^2) \tag{45}
\]

By looking at Eq. (43) and Eq. (44) it is clear that the normalization of the form factors \( V \) and \( A_1 \) at \( q^2 = 0 \) is determined by a single constant \( g_+(0) \), where as from Eq. (44) the form factor \( A_2 \) at \( q^2 = 0 \) is determined by two constants i.e. \( g_+(0) \) and \( A_0(0) \).

### A. Pole Contribution

In \( B_c \to D_s^{(*)} l^+ l^- \) decay, there will be a pole contribution to \( h_1, g_-, h \) and \( A_0 \) from \( B_s^*(1^-), B_s^{*A}(1^+) \) and \( B_s(0^-) \) mesons which can be parameterized as

\[
h_1|_{pole} = -\frac{1}{2} \frac{g B_s^{*B_c D_s^-}}{M_{B_s^*}^2} \frac{f_{B_s^*}}{1 - q^2/M_{B_s^*}^2} = \frac{R_V}{M_{B_s^*}^2} \frac{1}{1 - q^2/M_{B_s^*}^2} \tag{46}
\]
\[
g_-|_{pole} = -\frac{9}{2} \frac{g B_s^{*B_c D_s^-}}{M_{B_s^*}^2} \frac{f_{B_s^*}}{1 - q^2/M_{B_s^*}^2} = \frac{R_A}{M_{B_s^*}^2} \frac{1}{1 - q^2/M_{B_s^*}^2} \tag{47}
\]
\[
h|_{pole} = \frac{1}{2} \frac{f_{B_s^{*B_c D_s^-}}}{M_{B_s^*}^2} \frac{f_{B_s^*}}{1 - q^2/M_{B_s^*}^2} = \frac{R_A}{M_{B_s^*}^2} \frac{1}{1 - q^2/M_{B_s^*}^2} \tag{48}
\]
\[
A_0(q^2)|_{pole} = \frac{9}{2} \frac{g B_s^{*B_c D_s^-}}{M_{B_s^*}^2} \frac{f_{B_s^*}}{1 - q^2/M_{B_s^*}^2} = \frac{R_0}{M_{B_s^*}^2} \frac{q^2/M_{B_s^*}^2}{1 - q^2/M_{B_s^*}^2} \tag{49}
\]
where the quantities $R_V, R_A^S, R_A^D$ and $R_0$ are related to the coupling constants $g_{B^*_cB_cD^*_c}, g_{B^*_cA_cB_c}, g_{B^*_cA_cD^*_c}$ and $g_{B^*_cA_cD^*_c}$, respectively. Here we would like to mention that the above mentioned couplings arises as the lower pole mass, because the higher pole masses of $B_c$ meson do not contribute for the $B_c \to D_s^0 l^- \nu$ decay. The form factors $A_1(q^2), A_2(q^2)$ and $V(q^2)$ can be written in terms of these quantities as

$$V(q^2) = \frac{M_{B_c^*} + M_{D^*_c}}{m_b + m_s} \left[ g_+(q^2) - \frac{R_V}{M_{B_c^*}} \frac{q^2}{1 - q^2/M_{B_c^*}^2} \right]$$ (50)

$$A_1(q^2) = \frac{M_{B_c^*} - M_{D^*_c}}{m_b - m_s} \left[ g_+(q^2) + \frac{q^2}{M_{B_c^*} - M_{D^*_c}} \tilde{g}_-(q^2) + \frac{R_A^S}{M_{B^*_c A_c}^*} \frac{q^2}{1 - q^2/M_{B^*_c A_c}^*} \right]$$ (51)

$$A_2(q^2) = \frac{M_{B_c^*} + M_{D^*_c}}{m_b - m_s} \left[ g_+(q^2) - \frac{R_A^D}{M_{B^*_c A_c}^*} \frac{q^2}{1 - q^2/M_{B^*_c A_c}^*} \right] - 2M_{D^*_c} - A_0(q^2)$$ (52)

Now, the behavior of $g_+(q^2), \tilde{g}_-(q^2)$ and $A_0(q^2)$ is known from LEET and their form is 

$$g_+(q^2) = \frac{\xi_{\perp}(0)}{(1 - q^2/M_{B_c^*}^2)^2} = -\tilde{g}_-(q^2)$$ (53)

$$A_0(q^2) = \left( 1 - \frac{M_{D^*_c}^2}{M_{B_c^*}^2 D_{D^*_c}^*} \right) \xi_{\parallel}(0) + \frac{M_{D^*_c}^2}{M_{B_c^*}^2} \xi_{\perp}(0)$$ (54)

$$E_{D^*_c} = \frac{M_{B_c^*}}{2} \left( 1 - \frac{q^2}{M_{B_c^*}^2} + \frac{M_{D^*_c}^2}{M_{B_c^*}^2} \right)$$ (55)

$$g_+(0) = \xi_{\perp}(0)$$ (56)

The pole terms given in Eqs. (50, 52) dominate near $q^2 = M_{B_c^*}^2$ and $q^2 = M_{B^*_c A_c}^2$. Just to make a remark that relations obtained from the Ward identities can not be expected to hold for the whole $q^2$. Therefore, near $q^2 = 0$ and near the pole following parametrization is suggested [12]

$$F(q^2) = \frac{F(0)}{(1 - q^2/M^2) (1 - q^2/M^2)}$$ (57)

where $M^2$ is $M_{B_c^*}^2$ or $M_{B^*_c A_c}^2$, and $M'$ is the radial excitation of $M$. The parametrization given in Eq. (57) not only takes into account the corrections to single pole dominance suggested by the dispersion relation approach but also give the correction of off-mass shell-ness of the couplings of $B_c^*$ and $B^*_cA_c$ with the $B_c D_{s0}^*$ channel.

Since $g_+(0)$ and $\tilde{g}_-(q^2)$ have no pole at $q^2 = M_{B_c^*}^2$, hence we get

$$V(q^2) (1 - \frac{q^2}{M_{B_c^*}^2}) |_{q^2=M_{B_c^*}^2} = -R_V \left( \frac{M_{B_c^*} + M_{D^*_c}}{m_b - m_s} \right)$$

This becomes

$$R_V \equiv -\frac{1}{2} g_{B_c^* B_c D^*_c} f_{B_c^*} = -\frac{g_+(0)}{1 - M_{B_c^*}^2/M_{B_c^*}^2}$$ (58)

and similarly

$$R_A^D \equiv \frac{1}{2} f_{B^*_c A_c B_c} f_{B^*_c A_c} = -\frac{g_+(0)}{1 - M_{B^*_c A_c}^2/M_{B^*_c A_c}^2}$$ (59)

We cannot use the parametrization given in Eq. (57) for the form factor $A_1(q^2)$, since near $q^2 = 0$, the behavior of $A_1(q^2)$ is $g_+(q^2) \left[ 1 - q^2 / \left( M_{B_c^*}^2 - M_{D^*_c}^2 \right) \right]$, therefore we can write $A_1(q^2)$ as follows

$$A_1(q^2) = \frac{g_+(0)}{\left( 1 - q^2/M_{B_c^*}^2 \right) \left( 1 - q^2/M_{B^*_c A_c}^2 \right)} \left( 1 - \frac{q^2}{M_{B_c^*}^2 - M_{B_c^*}^2} \right)$$ (60)
TABLE I: Values of the form factors at \( q^2 = 0 \).

| \( V(0) \) | \( A_1(0) \) | \( A_2(0) \) | \( A_0(0) \) |
|----------------|----------------|----------------|----------------|
| 0.51 \( \pm 0.17 \) | 0.28 \( \pm 0.08 \) | 0.22 \( \pm 0.07 \) | 0.35 \( \pm 0.11 \) |

The only unknown parameter in the above form factors calculation is \( g_+(0) \) and its value can be extracted by using the central value of branching ratio for the decay \( B_c^{-} \rightarrow D_s^{-}\gamma \) \[39\]. From the formula of decay rate

\[
\Gamma (B_c^{-} \rightarrow D_s^{-}\gamma) = \frac{G_F^2 \alpha}{32\pi^3} |V_{tb}V_{ts}^*|^2 \frac{m_b^2 M_{B_c^{-}}}{M_{B_{s}^{-}}} \times \left( 1 - \frac{M_{D_s^{-}}^2}{M_{B_{s}^{-}}^2} \right) |C_{eff}^{cf}|^2 |g_+(0)|^2
\]

and by putting the values of everything one can find the value of unknown parameter \( g_+(0) = 0.32 \pm 0.1 \). In the forthcoming analysis we use the value of \( g_+(0) = 0.42 \) which was calculated in ref. \[39\].

Using \( f_{B_c} = 0.35 \text{ GeV} \) we have prediction from Eq. (58) that

\[
g_{B_s^{-}B_s^{-}D_s^{-}} = 10.38 \text{ GeV}^{-1}
\]

Similarly the ratio of \( S \) and \( D \) wave couplings are predicted to be

\[
\frac{g_{B_s^{-}B_s^{-}D_s^{-}}}{f_{B_s^{-}B_s^{-}D_s^{-}}} = -0.42 \text{ GeV}^2
\]

The different values of the \( F(0) \) are

\[
V(0) = \frac{M_{B_c^{-}} + M_{D_s^{-}}}{m_b + m_s} g_+(0)
\]

(64)

\[
A_1(0) = \frac{M_{B_c^{-}} - M_{D_s^{-}}}{m_b - m_s} g_+(0)
\]

(65)

\[
A_2(0) = \frac{M_{B_c^{-}} + M_{D_s^{-}}}{m_b - m_s} g_+(0) - \frac{2M_{D_s^{-}}}{M_{B_c^{-}} - M_{D_s^{-}}} A_0(0)
\]

(66)

The calculation of the numerical values of \( V(0) \) and \( A_1(0) \) is quite trivial but for the value of \( A_2(0) \), the value of \( A_0(0) \) has to be known. Although LEET does not give any relationship between \( \xi_{\parallel}(0) \) and \( \xi_{\perp}(0) \), but in LCSR \( \xi_{\parallel}(0) \) and \( \xi_{\perp}(0) \) are related due to numerical coincidence \[40\].

\[
\xi_{\parallel}(0) \simeq \xi_{\perp}(0) = g_+(0)
\]

(67)

From Eq. 54 we have

\[
A_0(0) = 1.12 g_+(0)
\]

The value of the form factors at \( q^2 = 0 \) is given in Table-1 and can be extrapolated for the other values of \( q^2 \) as follows:

\[
V(q^2) = \frac{V(0)}{(1 - q^2/M_{B_{s}^{-}}^2)(1 - q^2/M_{B_{s}^{-}}^2)}
\]

(68)

\[
A_1(q^2) = \frac{A_1(0)}{(1 - q^2/M_{B_{s}^{-}}^2)(1 - q^2/M_{B_{s}^{-}}^2)}
\]

(69)

\[
A_2(q^2) = \frac{A_2(0)}{(1 - q^2/M_{B_{s}^{-}}^2)(1 - q^2/M_{B_{s}^{-}}^2)} - \frac{2M_{D_s^{-}}}{M_{B_c^{-}} - M_{D_s^{-}}}(1 - q^2/M_{B_{s}^{-}}^2)(1 - q^2/M_{B_{s}^{-}}^2) A_0(0)
\]

(70)
The behavior of form factors $V(q^2)$, $A_1(q^2)$ and $A_2(q^2)$ are shown in Fig. 1.

![Graphs showing form factors](image)

**FIG. 1**: Form factors are plotted as a function of $q^2$. Solid line, dashed line and long-dashed line correspond to $q_0(0)$ equal to 0.42, 0.32 and 0.22 respectively.

### IV. PHYSICAL OBSERVABLES FOR $B_c \to D_s^* l^+ l^-$

In this section we will present the calculations of the physical observables like the decay rates and the helicity fractions of $D_s^*$ meson. From Eq. (5) it is straightforward to write

$$\mathcal{M}_{B_c \to D_s^* l^+ l^-} = \frac{G_F \alpha}{2\sqrt{2} \pi} V_{td} V_{ts}^* \left[ T_1^1 (\bar{l} \gamma^\mu l) + T_1^2 (\bar{l} \gamma^\mu \gamma^5 l) \right]$$

where

$$T_1^1 = f_1(q^2) \epsilon_{\mu \alpha \beta \gamma} \epsilon^{\star \nu \rho \delta} k^\beta + i f_2(q^2) \epsilon_\mu^{\star} + i f_3(q^2)(\epsilon_\mu^{\star} \cdot q) P_\mu$$

$$T_1^2 = f_4(q^2) \epsilon_{\mu \alpha \beta \gamma} \epsilon^{\star \nu \rho \delta} k^\beta + i f_5(q^2) \epsilon_\mu^{\star} + i f_6(q^2)(\epsilon_\mu^{\star} \cdot q) P_\mu$$

The functions $f_1$ to $f_6$ in Eq. (72) and Eq. (73) are known as auxiliary functions, which contains both long distance (Form factors) and short distance (Wilson coefficients) effects and these can be written as

$$f_1(q^2) = 4(m_b + m_s) \frac{C_{7}^{\epsilon \gamma}}{q^2} F_1(q^2) + C_{9}^{\epsilon \gamma} \frac{V(q^2)}{M_{B_c} + M_{D_s^*}}$$

$$f_2(q^2) = \frac{C_{7}^{\epsilon \gamma}}{q^2} 4(m_b - m_s) F_2(q^2) \left( M_{B_c}^2 - M_{D_s^*}^2 \right) + C_{9}^{\epsilon \gamma} A_1(q^2) (M_{B_c} + M_{D_s^*})$$

$$f_3(q^2) = - \left[ C_{7}^{\epsilon \gamma} 4(m_b - m_s) \left( F_2(q^2) + q^2 \frac{F_3(q^2)}{M_{B_c}^2 - M_{D_s^*}^2} \right) + C_{9}^{\epsilon \gamma} A_2(q^2) \right] \frac{A_2(q^2)}{M_{B_c} + M_{D_s^*}}$$

$$f_4(q^2) = C_{10} \frac{V(q^2)}{M_{B_c} + M_{D_s^*}}$$

$$f_5(q^2) = C_{10} A_1(q^2) (M_{B_c} + M_{D_s^*})$$

$$f_6(q^2) = - C_{10} \frac{A_2(q^2)}{M_{B_c} + M_{D_s^*}}$$

$$f_6(q^2) = C_{10} A_0(q^2)$$

The next task is to calculate the decay rate and the helicity fractions of $D_s^*$ meson in terms of these auxiliary functions.
A. The Differential Decay Rate of $B_c \rightarrow D_s^* l^+ l^-$

In the rest frame of $B_c$ meson the differential decay width of $B_c \rightarrow D_s^* l^+ l^-$ can be written as

$$\frac{d\Gamma(B_c \rightarrow D_s^* l^+ l^-)}{dq^2} = \frac{1}{(2\pi)^3} \frac{1}{32M_{B_c}^3} \int_{-u(q^2)}^{+u(q^2)} du \left| M_{B_c \rightarrow D_s^* l^+ l^-} \right|^2$$

(75)

where

$$q^2 = (p_{l+} + p_{l-})^2$$

(76)

$$u = (p - p_{l-})^2 - (p - p_{l+})^2$$

(77)

Now the limits on $q^2$ and $u$ are

$$4m_l^2 \leq q^2 \leq (M_{B_c} - M_{D_s})^2$$

(78)

$$-u(q^2) \leq u \leq u(q^2)$$

(79)

with

$$u(q^2) = \sqrt{\lambda \left( 1 - \frac{4m_l^2}{q^2} \right)}$$

(80)

where

$$\lambda \equiv \lambda(M_{B_c}^2, M_{D_s}^2, q^2) = M_{B_c}^2 + M_{D_s}^2 + q^4 - 2M_{B_c}^2 M_{D_s}^2 - 2M_{D_s}^2 q^2 - 2q^2 M_{B_c}^2$$

The decay rate of $B_c \rightarrow D_s^* l^+ l^-$ can easily obtained in terms of auxiliary function by integrating on $u$ (c.f. Eq. 75) as

$$\frac{d\Gamma(B_c \rightarrow D_s^* l^+ l^-)}{dq^2} = \frac{G_F^2 |V_{ub} V_{us}^*|^2 \alpha^2}{2^{11} \pi^5 (3M_{B_c}^2 M_{D_s}^2 q^2)} q^2 u(q^2) \left[ 24 \left| f_0(q^2) \right|^2 m_l^2 M_{D_s}^2 \lambda \right. + 8M_{D_s}^2 q^2 \lambda(2m_l^2 + q^2) \left| f_1(q^2) \right|^2 - (4m_l^2 - q^2) \left| f_4(q^2) \right|^2 \left. + \lambda(2m_l^2 + q^2) \left| f_2(q^2) + (M_{B_c}^2 - M_{D_s}^2 - q^2) f_3(q^2) \right|^2 \right. \right.$$  

$$- (4m_l^2 - q^2) \left| f_5(q^2) + (M_{B_c}^2 - M_{D_s}^2 - q^2) f_6(q^2) \right|^2 \left. + 4M_{D_s}^2 q^2 (2m_l^2 + q^2) \left( 3 \left| f_2(q^2) \right|^2 - \lambda \left| f_3(q^2) \right|^2 \right) \right. \right.$$  

$$- (4m_l^2 - q^2) \left( 3 \left| f_5(q^2) \right|^2 - \lambda \left| f_6(q^2) \right|^2 \right) \right]$$

(81)

B. HELICITY FRACTIONS OF $D_s^*$ IN $B_c \rightarrow D_s^* l^+ l^-$

We now discuss helicity fractions of $D_s^*$ in $B_c \rightarrow D_s^* l^+ l^-$ which are interesting variable and are as such independent of the uncertainties arising due to form factors and other input parameters. The final state meson helicity fractions were already discussed in literature for $B \rightarrow K^*(K_1) l^+ l^-$ decays [21, 22]. Even for the $K^*$ vector meson, the longitudinal helicity fraction $f_L$ has been measured by Babar collaboration for the decay $B \rightarrow K^* l^+ l^- (l = e, \mu)$ in two bins of momentum transfer and the results are [44]

$$f_L = 0.77^{+0.63}_{-0.30} \pm 0.07, \quad 0.1 \leq q^2 \leq 8.41 \text{GeV}^2$$

(82)

$$f_L = 0.51^{+0.22}_{-0.25} \pm 0.08, \quad q^2 \geq 10.24 \text{GeV}^2$$
been mentioned that besides the contribution in the C resonances like J/ψ momentum transfer effects. One can see that there is a significant enhancement in the decay rate due to KK-contribution for $1$ using the Ward identities and their dependence on momentum transfer which are non-perturbative quantities and are the major source of uncertainties. Here we calculated the form factors V. NUMERICAL ANALYSIS.

In this section we present the numerical analysis of the branching ratio and helicity fractions of $D_s^+$ meson in $B_c \rightarrow D_s^* l^+ l^− (l = \mu, \tau)$ both in the SM and in ACD model. One of the main input parameters are the form factors which are non-perturbative quantities and are the major source of uncertainties. Here we calculated the form factors using the Ward identities and their dependence on momentum transfer $q^2$ is given in Section III. We have used next-to-leading order approximation for the Wilson Coefficients at the renormalization scale $\mu = m_b$. It has already been mentioned that besides the contribution in the $C_{ij}^{ff}$, there are long distance contributions resulting from the $c\bar{c}$ resonances like $J/\psi$ and its excited states. For the present analysis we do not take into account these long distance effects.

The numerical results for the decay rates and helicity fractions of $D_s^*$ for the decay mode $B_c \rightarrow D_s^* l^+ l^−$ both for the SM and ACD model are depicted in Figs. 2-4. Figs. 2 (a, b) shows the differential decay rate of $B_c \rightarrow D_s^* l^+ l^− (l = \mu, \tau)$. One can see that there is a significant enhancement in the decay rate due to KK-contribution for $1/R = 300$ GeV, whereas the value of the decay rate is shifted towards the SM at large value of $1/R$, both in small and large value of momentum transfer $q^2$.

while the average value of $f_L$ in full $q^2$ range is

$$f_L = 0.63^{+0.18}_{-0.19} \pm 0.05, \quad q^2 \geq 0.1 GeV^2 \tag{83}$$

The explicit expression of the helicity fractions for $B_c \rightarrow D_s^* l^+ l^−$ decay can be written as

$$\frac{d\Gamma_L(q^2)}{dq^2} = \frac{G_F^2 |V_{cb}V_{tb}^*|^2 \alpha^2 u(q^2)}{2^{11} \pi^5} \times \left( \frac{1}{3 q^2 M_{D_s^*}} \left[ 24 \frac{f_0(q^2)}{q^2} \left( m_{D_s^*}^2 M_{D_s^*}^2 \lambda + (2m_{D_s^*}^2 + q^2) \left( M_{D_s^*}^2 - M_{D_s^*}^2 - q^2 \right) f_2(q^2) + \lambda f_3(q^2) \right)^2 + (q^2 - 4m_{D_s^*}^2) \left( M_{D_s^*}^2 - M_{D_s^*}^2 - q^2 \right) f_5(q^2) + \lambda f_6(q^2) \right] \right)$$

$$\frac{d\Gamma_+(q^2)}{dq^2} = \frac{G_F^2 |V_{cb}V_{tb}^*|^2 \alpha^2 u(q^2)}{2^{11} \pi^5} \times \left( \frac{4}{3} \left[ (q^2 - 4m_{D_s^*}^2) f_5(q^2) - \sqrt{\lambda} f_4(q^2) \right]^2 + (q^2 + 2m_{D_s^*}^2) \left( f_2(q^2) - \sqrt{\lambda} f_1(q^2) \right)^2 \right) \tag{84}$$

$$\frac{d\Gamma_-(q^2)}{dq^2} = \frac{G_F^2 |V_{cb}V_{tb}^*|^2 \alpha^2 u(q^2)}{2^{11} \pi^5} \times \left( \frac{4}{3} \left[ (q^2 - 4m_{D_s^*}^2) f_5(q^2) + \sqrt{\lambda} f_4(q^2) \right]^2 + (q^2 + 2m_{D_s^*}^2) \left( f_2(q^2) + \sqrt{\lambda} f_1(q^2) \right)^2 \right) \tag{85}$$

where the auxiliary functions and the corresponding form factors are given in Eq. (74) and Eqs. (68-70). Finally the longitudinal and transverse helicity amplitude becomes

$$f_L(q^2) = \frac{d\Gamma_L(q^2)/dq^2}{d\Gamma(q^2)/dq^2}$$

$$f_\pm(q^2) = \frac{d\Gamma_\pm(q^2)/dq^2}{d\Gamma(q^2)/dq^2}$$

$$f_T(q^2) = f_+(q^2) + f_-(q^2) \tag{86}$$

so that the sum of the longitudinal and transverse helicity amplitudes is equal to one i.e. $f_L(q^2) + f_T(q^2) = 1$ for each value of $q^2$[21].

V. NUMERICAL ANALYSIS.
In general the sensitivity on $1/R$ is usually masked by the uncertainties which arises due to the number of sources. Among them the major one lies in the numerical analysis of $B_c \to D_s^* l^+ l^-$ decay originated from the $B_c \to D_s^*$ transition form factors calculated in the present approach as shown in Table I, which can bring about almost 40% errors to the differential decay rate of above mentioned decay, which showed that it is not a very suitable tool to look for new physics. The large uncertainties involved in the form factors are mainly from the variations of the decay constant of $B_c$ meson and also there are some uncertainties from the strange quark mass $m_s$, which are expected to be very tiny on account of the negligible role of $m_s$ suppressed by the much larger energy scale of $m_b$. Moreover, the uncertainties of the charm quark and bottom quark mass are at the 1% level, which will not play significant role in the numerical analysis and can be dropped out safely. It also needs to be stressed that these hadronic uncertainties almost have no influence on the various asymmetries including the polarization asymmetries of final state meson on account of the serious cancelation among different polarization states and this make them one of the best tool to look for physics beyond the SM.

Figs. 3 (a,b) shows the longitudinal and transverse helicity fractions of $D_s^*$ for the decay $B_c \to D_s^* \mu^+ \mu^-$ where we have used the central value of the form factors which we have calculated in Section III. Choosing the different values of compactification radius $1/R$, one can see from the graphs that the effect of extra dimensions are quite significant at a particular region of $q^2$. These effects are constructive for the case of transverse helicity fraction and destructive for the case of longitudinal helicity fraction.

Similarly, Figs. 4 (a,b) show the helicity fraction of $D_s^*$ for the decay $B_c \to D_s^* \tau^+ \tau^-$ where one can see that the effects of the extra dimensions are mild as compared to the case of $B_c \to D_s^* \mu^+ \mu^-$. Moreover from Figs.2-4 it is clear that each value of momentum transfer $q^2$ the sum of the longitudinal and transverse helicity fractions are equal to one, i.e. $f_L(q^2) + f_T(q^2) = 1$.

VI. CONCLUSION:

We investigated the semileptonic decay $B_c \to D_s^* l^+ l^-$ ($l = \mu, \tau$) using the Ward identities. The form factors have been calculated and we found that the normalization of the form factors in terms of a single universal constant $g_+(0)$. The value of $g_+(0) = 0.42$ is obtained from the decay $B_c \to D_s^* \gamma$\cite{33}. Considering the radial excitation at lower pole masses $M$ (where $M = M_{B_c}$ and $M_{B^*_c}$) one can predict the coupling of $B_c^*$ with $B_c D_s^*$ channel as indicated in Eq.\cite{42} which is $g_{B_c^*,B_c D_s^*} = 10.38 \text{ GeV}^{-1}$. Also we predicted the ratio of $S$ and $D$ wave couplings $\frac{g_{B_c^*,B_c D_s^*}}{f_{B_c^*,B_c D_s^*}} = -0.42$.
FIG. 3: Longitudinal Lepton polarization Fig.1a and Transverse Lepton polarization Fig.2b for the $B \to D_s^* \mu^+ \mu^-$ decays as functions of $q^2$ for different values of $1/R$. Solid line correspond to SM value, dotted line is for $1/R = 300$, dashed is for $1/R = 500$, long dashed line is for $1/R = 700$.

FIG. 4: Longitudinal Lepton polarization Fig.1a and Transverse Lepton polarization Fig.2b for the $B \to D_s^* \tau^+ \tau^-$ decays as functions of $q^2$ for different values of $1/R$. Solid line correspond to SM value, dotted line is for $1/R = 300$, dashed is for $1/R = 500$, long dashed line is for $1/R = 700$.

$GeV^2$ given in Eq. (63). The form factors are summarized in Eqs. (68-70) and their values at $q^2 = 0$ are given in Table-I. Using these form factors we studied the observables, i.e. the branching ratio and helicity fraction of $D_s^*$ in the decay $B_c \to D_s^* l^+ l^-$ ($l = e, \mu$) both in SM and in ACD model, which has one additional parameter i.e. the inverse compactification radius $1/R$. The effects of extra dimensions to the helicity fraction of $D_s^*$ is very mild for the case when the tauon ($\tau$) is taken as a final state lepton as shown in fig 3, however the effects of extra dimensions are quite significant for the case when muon ($\mu$) is taken as a final state lepton as shown in fig 2. In near future when LHC is fully operational where more data is available, will put a stringent constraint on compactification radius $R$ and gives us a deep understanding of $B$ Physics.
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