An energy balance model for passive solar systems operation

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Abstract. Passive solar systems (such as Trombe-Michel walls and their modifications) are a cost effective technology to reduce the energy demand for buildings’ space heating. In the present paper an energy balance model for the description of the performance of a passive solar system composed of an opaque element with transparent insulation is presented. This energy balance model has the form of a closed system of equations that could easily be solved in an open source software by an engineer.

1. Introduction
Passive solar systems (such as Trombe-Michel walls and their modifications) are a cost effective technology to reduce the energy demand for buildings’ space heating and ventilation [1].

The simulation of these systems’ performance is feasible with the use of transient thermodynamic simulation tools (TTST) e.g. the commercial tool-code TRNSYS. In this context TRNSYS has been used for an hour-by-hour simulation of a Trombe wall performance in order to optimize its design in terms of CO₂ emissions reduction [2]. Another way to study their performance is the full simulation of the transport phenomena involved in such configurations with the use of Computational Fluid Dynamics (CFD) methods, which require also the purchase of special software, along with specialized programming skills and high computational time. Some CFD studies focus on the solar air heater operation [3, 4], while other simulate the whole room equipped with the solar wall [5]. In the last case the computational domain should include both the room and the solar wall with different requirements in the grid. In order to study the internal microclimate of a room equipped with a passive solar system with CFD, without simulating this system, it is necessary to be able to determine the boundary conditions (i.e. temperature or heat flow) as a function of external climatic conditions. Finally, simple models for the operation of solar walls have been presented at various levels of detail [6,7].

The aim of the present work is the development of an energy balance model for the description of the performance of a passive solar system composed of an opaque element with a transparent insulation and heat storage system away from the opening. This energy balance model will have the form of a closed system of equations that could easily be solved in an open source software by an engineer. This model could be used both for the prediction of these systems’ performance and for the design and optimization of the systems and/or in order to specify the boundary conditions of a room equipped with such a system for a specialized CFD study.

2. Mathematical model
The geometry that describes the examined Trombe-Michel wall (in the rest of the text will be referred to as Trombe wall) is given in figure 1a. The examined Trombe wall consists of a transparent glass cover and a massive wall without insulation and with the gap facing surface painted black. The air enters the gap from the holes in the base of the wall, is heated and rises due to thermal buoyancy and finally leaves...
the gap through the upper holes heading for the heated room. In figure 1.b the used nomenclature regarding the temperatures is given. The distance between the lower and upper holes is 2 m and the length of the Trombe wall is 1.5 m, giving an area of 3 m².

**Figure 1.a.** General Trombe-Michel wall configuration, 1.b. Temperature nomenclature in the examined geometry

### 2.1. Basic energy balance equations

The basic equations consist of three energy balance equations [7, 8] corresponding to the energy transport between the internal glass surface and the air in the gap of Trombe wall, the energy transport by the air inside the air gap and the energy transport between the air gap facing wall surface and the air in the gap. All of them describe instantaneous heat transfer considering steady state condition.

The first energy balance equation concerns the air inside the air gap

\[
\dot{m} \cdot C_{p,\text{air}} \cdot (\theta_{\text{out}} - \theta_{f}) = A_{\text{int}} \cdot h_{cv1} \cdot (\theta_{\text{se}} - \theta_{\text{int}}) + A_{\text{int}} \cdot h_{cv2} \cdot (\theta_{\text{si}} - \theta_{\text{int}})
\]  

(1)

Where \(\dot{m}\) [kg/m²] is the air mass flow inside the air gap, \(C_{p,\text{air}}\) [j/kgK] is the air specific heat capacity, \(\theta_{\text{out}}\) [K] is the air temperature with which the air leaves the air gap through the upper holes, \(\theta_{f}\) is the air temperature at which air enters the air gap, \(\theta_{\text{int}}\) [K] is the air temperature inside the air gap, \(A_{\text{int}}\) [m²] is the Trombe wall surface (which is considered equal to the part of the glass surface that participates in the wall operation), \(h_{cv1}\) [W/m²K] is the convective heat exchange coefficient in the glass side of the air gap, \(h_{cv2}\) [W/m²K] is the convective heat exchange coefficient in the wall side of the air gap, \(\theta_{\text{se}}\) [K] is the wall temperature on the air gap side of the glass and \(\theta_{\text{si}}\) is the wall temperature on the air gap side of the Trombe wall.

The second energy balance equation is on the internal surface of the glass cover

\[
\frac{\theta_{\text{ope}} - \theta_{\text{se}}}{R_{\text{e}}^*} + N_{i}^* \cdot a_{\text{est}} \cdot I_{\text{t}} = \frac{\theta_{\text{se}} - \theta_{\text{int}}}{R_{cv1}} + \frac{\theta_{\text{se}} - \theta_{\text{si}}}{R_{\text{rd}}}
\]  

(2)

Where, \(\theta_{\text{ope}}\) [K] is the wall temperature of the external surface of the Trombe wall, \(R_{e}^*\) [m²K/W] is the thermal resistance of the glass, \(N_{i}^*\) is the diffusive radiation coefficient [-], \(a_{\text{est}}\) [-] is the glass absorptivity, and \(I_{\text{t}}\) [W/m²] is the total solar radiation incident in the glass surface, \(R_{cv1}\) [m²K/W] is the convective resistance on the glass side of the air gap and \(R_{\text{rd}}\) is the radiative resistance between the two air gap surfaces.

The third energy balance equation is on the gap facing surface of the Trombe wall

\[
\frac{\theta_{\text{int}} - \theta_{\text{si}}}{R_{cv2}} + \frac{\theta_{\text{se}} - \theta_{\text{si}}}{R_{\text{rd}}} + S = \frac{\theta_{\text{se}} - \theta_{\text{opi}}}{R_{i}^*}
\]  

(3)

Where \(S\) [W/m²] is the solar radiation absorbed by the Trombe wall selective surface, \(R_{cv2}\) [m²/WK] is the convective resistance on the wall side of the air gap, \(R_{i}^*\) [m²K/W] is the thermal resistance of the wall and \(\theta_{\text{opi}}\) [K] is the wall temperature on the room facing surface of the Trombe wall.
2.2. Closing relationships

The air mass flow inside the air gap is calculated according to the following relationship [9]

\[
\dot{m} = C_d \cdot \rho \cdot A_{cs} \cdot \sqrt{\frac{2gL}{\text{tn} \cdot \ln(R)}} \frac{\Delta T}{\theta_f} \tag{4}
\]

Where, \(C_d\) is a discharge coefficient taken equal to 0.6, \(\rho [\text{kg/m}^3]\) is the air density, \(A_{cs} [\text{m}^2]\) is the air gap cross section, \(\Delta T [-]\) is the inlet/outlet fraction, \(g [\text{m/s}^2]\) is the gravity acceleration and \(L\) is the distance between the lower and the upper holes.

The convective heat exchange coefficient on the glass side of the air gap is given by

\[
h_{cv1} = \frac{N_{U1} \cdot \kappa_{air}}{L} \tag{5}
\]

Where, \(N_{U1}\) is the Nusselt number and \(\kappa_{air} [\text{W/mK}]\) is the air thermal conductivity. The Nu number is given by the relationship [7]

\[
N_{U1} = \left\{ 0.825 \left[ 1 + \frac{0.387 \cdot R_{L1}}{\left[ 1 + (0.492 / Pr)^{1/6} \right]^{9/16}} \right] \right\}^2 \tag{6}
\]

Where, \(R_{L1}\) is the Rayleigh number and \(Pr\) is the Prandtl number. The \(R_{L}\) number is given by

\[
R_{L1} = \frac{g \beta (\theta_{ê} - \theta_{i})}{v_{air}^2 \cdot Pr} \tag{7}
\]

Where, \(\beta\) is the air thermal expansion coefficient and \(v_{air}\) is the air kinematic viscosity. The thermal expansion coefficient is given by

\[
\beta = \frac{1}{\theta_{i}} \tag{8}
\]

The convective heat transfer coefficient in the wall gap facing surface \(h_{cv2}\) is calculated using the relationships 5 to 7 substituting the temperature difference by \(\theta_{si} - \theta_{i}\). The convective resistances \(R_{cv1}\) and \(R_{cv2}\) are given by

\[
R_{cv1} = \frac{1}{h_{cv1}} \tag{9}
\]

The radiative resistance between the two air gap surfaces is given by

\[
R_{rd} = \frac{\sigma (\theta_{ê} + \theta_{i}) (\theta_{ê} + \theta_{si})}{\frac{1}{\varepsilon_g} + \frac{1}{\varepsilon_w} - 1} \tag{10}
\]

Where, \(\sigma = 5.67 \times 10^{-8} [\text{W/m}^2\text{K}^4]\) is the Stefan-Boltzmann constant, \(\varepsilon_g\) is the glass emissivity and \(\varepsilon_w\) is the wall emissivity. In order to close the equation system two more energy balance equations are used. The energy balance in the room facing surface of the wall and the energy balance on the external glass surface.

\[
\frac{\theta_{si} - \theta_{o*pi}}{R_i} = \frac{\theta_{o*pi} - \theta_{i}}{R_i} \tag{11}
\]

Where, \(\theta_{o*pi} [\text{K}]\) is the room facing wall temperature, \(R_i [\text{m}^2\text{K}/\text{W}]\) is the wall thermal resistance and \(R_i\) is the convective thermal resistance of the internal wall.

\[
(h_{rs} + h_{wind}) \cdot (\theta_{ext} - \theta_{o*pi}) = \frac{\theta_{o*pi} - \theta_{si}}{R_i} \tag{12}
\]

Where, \(h_{rs} [\text{W/m}^2\text{K}]\) is the radiation heat exchange coefficient between the external glass surface and the sky, \(\theta_{ext}\) is the external air temperature and \(h_{wind} [\text{W/m}^2\text{K}]\) the convective heat exchange coefficient on the external glass surface. The radiation heat exchange coefficient is given by
\[ h_{rs} = \varepsilon_g \cdot \sigma \cdot (\theta_{\text{ope}} - \theta_{\text{sky}}) \cdot (\theta_{\text{ope}}^2 + \theta_{\text{sky}}^2) \cdot \frac{\theta_{\text{ope}} - \theta_{\text{sky}}}{\theta_{\text{ope}} - \theta_f} \]  \hspace{1cm} (13)

Where, \( \theta_{\text{sky}} \) [K] is the equivalent sky temperature given by

\[ \theta_{\text{sky}} = 0.0552 \cdot \theta_{\text{ext}}^{1.5} \]  \hspace{1cm} (14)

2.3. Assumptions

It is assumed that the air temperature inside the air gap is the average value of the inlet and outlet air temperatures.

\[ \theta_{\text{int}} = \frac{\theta_f + \theta_{\text{out}}}{2} \]  \hspace{1cm} (15)

It is assumed that the room temperature is always kept constant at the desired design temperature using an appropriate mechanical heating system. The air thermophysical properties are considered also constant and the wall thermal resistance of the room facing wall surface is taken constant from the EN-ISO-6946. Thus a system with 19 equations and 19 unknowns is created.

2.4. Auxiliary relationships

Some auxiliary relationships are used in order to calculate variables which are not coupled with the above equations. The convective heat exchange coefficient on the external glass surface is given by

\[ h_{\text{wind}} = 5.7 + 3.8w \]  \hspace{1cm} (16)

Where, \( w \) [m/s] the external air velocity.

The solar radiation absorbed by the painted Trombe wall surface is given by [10] for a vertical wall facing south.

\[ S = I_b R_b (\tau \alpha)_b + \frac{I_d (\tau \alpha)_d}{2} + \rho_g \frac{I (\tau \alpha)_g}{2} \]  \hspace{1cm} (17)

Where, \( I \) [W/m²] is the total solar irradiance on the horizontal surface, \( I_b \) [W/m²] is the beam solar irradiance, \( I_d \) [W/m²] is the diffusive solar irradiance, \( \rho_g \) is the ground reflectance and \( R_b \) [-] is the fraction of the irradiation on tilted surface to the irradiation on horizontal surface. The transmittance-absorptance product \( (\tau \alpha) \) is calculated by the cover and wall properties for normal incident radiation and then it is modified according to the solar angle of incidence.

\[ (\tau \alpha)_n = \frac{\tau \alpha_n}{1 - (1 - \alpha_n) \rho_d} \]  \hspace{1cm} (18)

Where, \( \tau \alpha_n \) is the glass transmittance, \( \alpha_n \) is the wall absorptance and \( \rho_d \) the internal glass surface reflectance. The thermal resistances of glass and wall are calculated according to the EN-ISO-6946.

3. Study case

The operation of the Trombe-Michel wall presented in figure 1 was examined using the above developed model considering the 15th of March for a site with 39.39° latitude and 22.75° longitude. The monthly average daily total radiation on horizontal surface taken was equal to 3629 Wh/m² and the diffusive 1,584 Wh/m². The monthly average external temperature taken was 11.3 °C undertaking a sinusoidal variation during the day [11] with a maximum temperature of 14.4 °C and a minimum of 4.9 °C. The external air velocity is considered 2.7 m/s. The cover is considered to be a low iron and low-e 4-6-4 double glass and the wall is made of armed concrete with the gap facing surface painted black. The considered properties are summarized in the following Table 1. The design room temperature taken is a constant 20 °C and the convective thermal resistance of the internal wall taken is 0.1 m²K/W.

| Property                  | Unit  | Value |
|---------------------------|-------|-------|
| Specific heat capacity of air | J/KgK | 1006  |
| Property                                      | Unit          | Value     |
|----------------------------------------------|---------------|-----------|
| Thermal conductivity of air                  | W/mK          | 0.0242    |
| Prandtl number of air                        | [-]           | 0.73      |
| Kinematic viscosity of air                   | m²/s          | 1.51E-05  |
| Glass emissivity                             | [-]           | 0.1       |
| Density of wall                              | kg/m³         | 2000      |
| Specific heat capacity of wall               | J/kgK         | 1000      |
| Wall emissivity                              | [-]           | 0.97      |
| Convective thermal resistance of internal wall| m²/WK         | 0.1       |
| Inlet/outlet fraction                        | [-]           | 1         |
| Cross section of the gap                     | m²            | 0.15      |
| Thermal resistance of cover                  | m²/kW         | 0.267     |
| Thermal resistance of the wall               | m²/kW         | 0.148148  |
| Density of air                               | [kg/m³]       | 1.225     |
| Glass absorptivity                           | [-]           | 0.05      |
| Glass transmissivity                         | [-]           | 0.85      |
| Glass reflectivity                           | [-]           | 0.1       |
| Wall absorptivity                            | [-]           | 0.97      |
| Ground reflectance                           | [-]           | 0.2       |

4. Results

In figure 2 the daily hour-by-hour variation of the temperatures on the Trombe wall surfaces is given along with the air temperatures in the air gap and the external air temperature. The temperature nomenclature is the one given in figure 1.b and described in paragraph 2. The Trombe wall operation is the one given in figure 1.a and the heat transfer mechanisms are described in paragraph 2. There are two basic mechanisms through which the Trombe wall heats the room: a) by conduction-convection across the Trombe wall with the wall surface temperature facing the room, $\theta_{opi}$, varying from 30 to 50 °C and b) by the convection of the air entering the room through the upper holes at temperature, $\theta_{out}$, varying from 28 to 39 °C. The developed temperatures mainly depend on the incident radiation and not on the external temperature.

![Figure 2. Daily temperature evolution](image-url)
In figure 3 the air mass flow during the day is given. It must be noted that the hour is the solar hour. Since there is no mechanical ventilation, the air mass flow is only due to buoyancy and reaches the maximum when the incident solar radiation is maximum during the solar noon. For the specific geometry, in the examined place and date the air mass flow varies from 0.04 to 0.062 kg/s.

![Figure 3. Air mass flow](image)

5. Conclusions
In the present paper a closed energy balance model was developed for the instantaneous simulation of a Trombe wall operation. The developed energy balance simplified model was used for the calculation of a Trombe wall on a March day. It is proved that on a rather ‘cold’ day the air temperature inside the air gap can be increased from 7 to 19 degrees while a considerable amount of heat is also transported into the room through convection since the roof facing surface temperature is increased from 10 to 30 degrees. An important issue in the development of Trombe wall models is the calculation of the air mass flow since the majority of existing models, like the model suggested by the EN-ISO-13790 consider only mechanically ventilated configurations.

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