Physiological flow of biomedical compressible fluids inside a ciliated symmetric channel

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Abstract
This article studies the flow induced by cilia for a compressible Jeffrey fluid. The investigation is carried out for electrically conducting magnetohydrodynamic fluid in a space of porous media. The fluid flows inside a two-dimensional symmetric channel. The flow is demonstrated considering small magnetic Reynolds number and velocity slip at the wall. Perturbation technique is used in finding the solution. For different values of parameters, the net axial velocity is computed up to second-order calculations. It is interesting to note that Jeffrey fluid fluctuations decline as it changes from hydrodynamics to hydromagnetic fluid and as a consequence the retardation time turns out to be weak.

Keywords
Cilia, peristaltic flow, compressible fluid, microchannel

Introduction
Cilia are the microscopic, hair-like structure which tend to move around the cell. Cilia are present on maximum number of cells in human physique. Motile and non-motile cilia are two different parts of cilia. All along the surface, the presence of waves spreading is the main aspect of cilia cells. Such type of waves is called metachronal waves, they propagate in all directions, and are observed in the study by Guirao and Joanny.¹ Under the effect of heat transmission, metachronal beating is discussed by Nadeem and colleagues.² Other significant features of cilia are mentioned in previous works.³–⁷

Propagation of waves along elastic walls causes the generation of peristaltic flow. Blood inside the small blood vessels can transfer through peristaltic flow. Collaboration of peristaltic and pulsatile transport under the effect of hall current was discussed by Gad.⁸

Electrically conducting and magnetic properties of a fluid is studied in magnetohydrodynamics (MHD). Many researchers examined MHD flow of a viscous and electrically conducting fluid. Existence of pressure gradient for MHD-bounded sheets are discussed by Gribben.⁹ Continuity equation, temperature equation, Cauchy momentum equation, and Ampere’s Law ignoring displacement current are all MHD equations. Although, for manipulation of a fluid, periodic wave movement might look of small use. Acoustic flowing is the cure of oscillatory fluid movement by inertial nonlinearity.¹⁰–¹³ Ultrasonic radiation on the flow of water or oil was investigated by Chen.¹⁴

Flowing of a fluid through a porous medium is a focus of common attention and has appeared as a
distinct field of examination. A material that contains pores is termed as a porous medium, which is mostly considered by its porosity. Electrical conductivity, permeability, and tensile strength are sometimes obtained by assets of its components. Poromechanics is defined as study of deformation of compact frame and behavior of porous medium. Darcy’s law is used to define the flow that occurred in porous medium. These distinct effects decline the turbulence in the flowing fluid. By reduction of turbulence region, one can easily study the rate of heat transfer and temperature fluid, velocity, and Lorentz forces.\textsuperscript{15–19}

Effects of compressibility in a microchannel that was made by conformity surface acoustic movement with compliant walls were examined by Mekheimer and Abdel-Wahab.\textsuperscript{15}

In microchannels, Knudsen number $K_n$ is the quantity where $K_n = A/l$, which is the relation between mean free path and distance measure. For continuum flows, Knudsen number is very small. Gao and Jian\textsuperscript{16} examined the MHD flow of a Jeffrey fluid in a circular microchannel by considering relaxation time greater than retardation time. The results according to $K_n$ in a microchannel for various flow rules are investigated in Jha et al.\textsuperscript{17} and Schaff and Chambre\textsuperscript{.18}

In porous medium, flow of compressible fluid along peristaltic mechanism is given in Aarts and Ooms.\textsuperscript{11} Effect of weak and non-Newtonian compressible flow with wall slip is studied by Damianou et al.\textsuperscript{19} In microchannel, effects of relaxation time of Maxwell fluid mixed with the magnetic field was studied by Mekheimer and Abdel-Wahab,\textsuperscript{15} which was extended by Mekheimer et al.\textsuperscript{20} Recently, Gao and Jian\textsuperscript{16} considered the circular microchannel and studied the effect of an incompressible, MHD Jeffrey fluid. Peristaltic motion is also considered in human body as an example of tightening and easing of cardiac muscles. The effects of heat transfer on peristaltic transport of a third-grade fluid have been studied by Vafai et al.\textsuperscript{21} Some recent studies on MHD, porous medium, and non-Newtonian fluids are cited in the previous studies.\textsuperscript{22–27}

The purpose of this article is to study the effect on microchannel due to porous medium, magnetic field, and cilia under symmetric boundary conditions. Flow is generated by wavy motion of a compressible Jeffrey fluid in a microchannel under constant magnetic field. We assume that compressible fluid is stationary, which lies inside the microchannel (i.e. the zero-order pressure gradient ignored at the beginning). The analysis is interpreted via graphs.

**Mathematical model**

Consider a compressible, electrically conducting Jeffrey fluid in a two-dimensional symmetric network which is ciliated. Constant magnetic field acting in $y$-direction is analyzed and the induced magnetic field is neglected. Introducing the Cartesian coordinates with $x$-axis along centerline and $y$-axis normal to it as shown in Figure 1. The wall at $y = h(x,t)$ is ciliated and the flow phenomena occur due to the pressure. The fluid medium is considered to be porous.

The governing flow equations for compressible Jeffrey fluid in general form are stated as

$$\frac{\partial p}{\partial t} + (V \cdot \nabla p) + \rho (\nabla \cdot V) = 0 \quad (1)$$

$$\rho \frac{\partial V}{\partial t} + \rho (V \cdot \nabla) V = - \nabla p + \nabla \cdot S + R + J \times B + \frac{\mu}{3} \nabla (\nabla \cdot V) \quad (2)$$

where $\mu, \rho, p, \text{ and } dt$ are the respective dynamic viscosity, density, pressure, and time, whereas, $J$, $V$, and $B$ examines the electric current, velocity vector, and magnetic vector.

![Figure 1. Geometry of the problem.](image-url)
For Jeffrey fluid extra stress tensor $\mathbf{S}$ is defined as

$$
(1 + \lambda_1 \frac{\partial}{\partial t}) \mathbf{S} = \mu \left(1 + \lambda_2 \frac{\partial}{\partial t}\right) \mathbf{A}_1
$$

(3a)

In which $\lambda_1$ and $\lambda_2$ are the relaxation and retardation times, $\mathbf{A}_1$ is the first Rivlin–Ericksen tensor which is given by

$$
\mathbf{A}_1 = (\nabla \mathbf{V}) + (\nabla \mathbf{V})^T
$$

(3b)

Darcy’s resistance $\mathbf{R}$ is defined as

$$
(1 + \lambda_1 \frac{\partial}{\partial t}) \mathbf{R} = -\frac{\mu \phi}{K} \left(1 + \lambda_2 \frac{\partial}{\partial t}\right) \mathbf{V}
$$

(4)

where $0 < \phi < 1$, $K (>) 0$. The conducting fluid passes through uniform magnetic field $\mathbf{B}_0$. For small magnetic Reynold number, induced magnetic field is ignored and body force is $\mathbf{J} \times \mathbf{B} = \sigma (\mathbf{V} \times \mathbf{B}) \times \mathbf{B}$, electric fields are ignored only the magnetic field $\mathbf{B}$ is present, so the current becomes $\mathbf{J} = \sigma (\mathbf{V} \times \mathbf{B})$ ($\sigma$ is the conductivity of electric filed).

Characteristic response of fluid to compression is given by equation

$$
\frac{1}{\rho} \frac{\partial \rho}{\partial t} = k_c
$$

(5)

$k_c$ represents compressibility of the liquid. The solution of equation (5) is

$$
\rho = \rho_0 \exp(k_c (p - p_c))
$$

(6)

Here, $\rho_0$ signify the density and $p_c$ identify the reference pressure.

In component for continuity and momentum, equation for Jeffrey compressible fluid takes the following form

$$
\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho + \nabla \cdot \rho \left(\frac{\partial \mathbf{u}}{\partial x} + \frac{\partial \mathbf{v}}{\partial y}\right) = 0
$$

(7)

$$
\left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \rho \left(\frac{\partial \mathbf{u}}{\partial x} + \frac{\partial \mathbf{v}}{\partial y}\right) + \mathbf{u} \cdot \nabla \rho = 0
$$

(8)

$$
\left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \rho \left(\frac{\partial \mathbf{u}}{\partial x} + \frac{\partial \mathbf{v}}{\partial y}\right) + \mathbf{u} \cdot \nabla \rho
$$

$$
= -\left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \rho \left(\frac{\partial \mathbf{u}}{\partial x} + \frac{\partial \mathbf{v}}{\partial y}\right)
$$

$$
\left[\nabla^2 \mathbf{u} + \frac{1}{3} \frac{\partial}{\partial x} \left(\frac{\partial \mathbf{u}}{\partial x} + \frac{\partial \mathbf{v}}{\partial y}\right)\right]
$$

$$
- \sigma B_0^2 \left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \rho
$$

(9)

Envelope of cilia tips is defined as

$$
y = h(x, t) = \pm d \pm b\phi \cos \left(\frac{2\pi}{\lambda} (x - ct)\right)
$$

(10)

which can be taken as boundary for flow domain. Sleigh observed different patterns of cilia motion, it is assumed that cilia tips can move in elliptical paths, that is, cilia tips in horizontal position is

$$
x = f(x, x_0, t) = x_0 + b\phi \cos \left(\frac{2\pi}{\lambda} (x - ct)\right)
$$

(11)

Here, $x_0$ is the reference position of the particle and $\alpha$ is the measure of elliptical motion. Also, $a$ is amplitude, wave speed is denoted by $c$ and $\lambda$ is wavelength.

Thus, cilia velocity components are

$$
u = \frac{\partial h}{\partial t} + \frac{\partial x}{\partial t} = \frac{\partial h}{\partial t} + \frac{\partial x}{\partial t}
$$

(12)

The appropriate boundary conditions for cilia are

$$
u(x, \pm h, t) = \pm \phi \frac{\partial \mathbf{u}}{\partial y} + \frac{1}{\lambda} \left(\frac{2\pi b^{\phi} \cos \left(\frac{2\pi}{\lambda} (x - ct)\right)}{1 - \frac{2\pi b^{\phi} \cos \left(\frac{2\pi}{\lambda} (x - ct)\right)}{1 - \frac{2\pi b^{\phi} \cos \left(\frac{2\pi}{\lambda} (x - ct)\right)}}\right)
$$

(13)

Introducing non-dimensional variables and parameters

$$
u(x, \pm h, t) = \pm \frac{\partial \mathbf{u}}{\partial y} + \frac{\pm \frac{2\pi b^{\phi} \cos \left(\frac{2\pi}{\lambda} (x - ct)\right)}}{1 - \frac{2\pi b^{\phi} \cos \left(\frac{2\pi}{\lambda} (x - ct)\right)}{1 - \frac{2\pi b^{\phi} \cos \left(\frac{2\pi}{\lambda} (x - ct)\right)}}
$$

(14)
\[ \begin{align*}
\dot{x} &= \frac{x}{d}, \dot{y} = \frac{v}{d}, \dot{h} = \frac{u}{c}, \dot{\rho} = \frac{\rho}{\rho_0}, \dot{t} = \frac{ct}{d} \\
\dot{p} &= \frac{p}{\rho_0 C^2}, \dot{\rho_c} = \frac{\rho_c}{\rho_0 C^2}, \dot{\tau} = \frac{\tau}{c}, \dot{L} = -\alpha \phi, \alpha = \frac{2\pi d}{\lambda}, c = \frac{b \phi}{d} \\
\chi &= k_c \rho_0 C^2, R = \frac{\rho_0 c d}{\mu}, M = \frac{dr}{\rho_0 C^2} B^2, K_n = \frac{A}{d}
\end{align*} \]

(16)

These parameters represent the wave number, amplitude ratio, compressibility parameter, Reynolds number, magnetic parameter, and slip parameter.

Using these variables the above equations become

\[ \begin{align*}
\rho &= \exp(\chi(p - p_c)) \\
\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} + \rho \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) &= 0 \\
\left( 1 + \lambda_1 \frac{\partial}{\partial t} \right) \left[ \rho \frac{\partial u}{\partial x} + u \frac{\partial \rho}{\partial x} + \frac{\partial \rho}{\partial t} \right] &= - \left( 1 + \lambda_1 \frac{\partial}{\partial t} \right) \frac{1}{R} \left( 1 + \lambda_2 \frac{\partial}{\partial t} \right) u - \\
\left[ \frac{\nabla^2 u}{3} + \frac{1}{\partial t} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right] - M \left( 1 + \lambda_1 \frac{\partial}{\partial t} \right) u - \frac{1}{K} \left( 1 + \lambda_2 \frac{\partial}{\partial t} \right) u &= 0 \\
\left( 1 + \lambda_1 \frac{\partial}{\partial t} \right) \left[ \rho \frac{\partial v}{\partial y} + u \frac{\partial v}{\partial x} + \frac{\partial \rho}{\partial t} \right] &= - \left( 1 + \lambda_1 \frac{\partial}{\partial t} \right) \frac{1}{R} \left( 1 + \lambda_2 \frac{\partial}{\partial t} \right) v \\
\frac{\nabla^2 v}{3} + \frac{1}{\partial t} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) &= 0 \\
\frac{1}{K} \left( 1 + \lambda_2 \frac{\partial}{\partial t} \right) v &= \pm 1 \pm \eta(x, t)
\end{align*} \]

(17)

(18)

(19)

(20)

(21)

where \( \eta(x, t) = \varepsilon \cos(\alpha(x - t)) \)

\[ \begin{align*}
u &= \mp K_n \frac{\partial u}{\partial y} + \left( \frac{-\alpha^2 \varepsilon \cos(\alpha(x - t))}{1 - \alpha^2 \varepsilon \cos(\alpha(x - t))} \right) \\
\text{at } y = h &= \pm 1 \pm \eta \\
v &= \pm \frac{\partial \eta}{\partial t} \pm \left( \frac{\alpha \varepsilon \sin(\alpha(x - t))}{1 - \alpha^2 \varepsilon \cos(\alpha(x - t))} \right) \\
\text{at } y = h &= \pm 1 \pm \eta
\end{align*} \]

(22)

(23)

\[ \begin{align*}
\frac{d u}{d t} &= \chi p_1 \\
\frac{d^2 u}{d x^2} + \frac{d^2 u}{d y^2} &= 0 \\
\frac{1}{R} \left( 1 + \lambda_2 \frac{\partial}{\partial t} \right) \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{1}{3} \frac{\partial}{\partial t} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right] - M \left( 1 + \lambda_1 \frac{\partial}{\partial t} \right) u_1 - \frac{1}{K} \left( 1 + \lambda_2 \frac{\partial}{\partial t} \right) u_1 &= 0 \\
\frac{d v}{d t} &= \chi p_2 \\
\frac{d^2 v}{d x^2} + \frac{d^2 v}{d y^2} &= 0 \\
\frac{1}{R} \left( 1 + \lambda_2 \frac{\partial}{\partial t} \right) \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{1}{3} \frac{\partial}{\partial t} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right] - \frac{1}{K} \left( 1 + \lambda_2 \frac{\partial}{\partial t} \right) v_1 &= 0
\end{align*} \]

(24)

(25)

Perturbation solution

To find the solution of above nonlinear equations we use the regular perturbation method. For that we define the perturbed unknown quantities for small values of \( \varepsilon \) in the following form

\[ \begin{align*}
p &= p_0 + \varepsilon p_1 + \varepsilon^2 p_2 + \ldots \\
u &= \omega(x, y, t) + \varepsilon^2 u_2(x, y, t) + \ldots \\
v &= \omega_1(x, y, t) + \varepsilon^2 v_2(x, y, t) + \ldots \\
p &= 1 + \varepsilon p_1(x, y, t) + \varepsilon^2 p_2(x, y, t) + \ldots
\end{align*} \]

(26)

Equating the like powers of \( \varepsilon \), we obtain the following. For \( \varepsilon \)

\[ \begin{align*}
\frac{d p_1}{d t} + \frac{\partial p_1}{\partial x} + \frac{\partial p_1}{\partial y} &= 0 \\
\left( 1 + \lambda_1 \frac{\partial}{\partial t} \right) \frac{\partial u_1}{\partial t} &= - \left( 1 + \lambda_1 \frac{\partial}{\partial t} \right) \frac{\partial p_1}{\partial x} \\
+ \frac{1}{R} \left( 1 + \lambda_2 \frac{\partial}{\partial t} \right) \left[ \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} + \frac{1}{3} \frac{\partial}{\partial t} \left( \frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} \right) \right] - M \left( 1 + \lambda_1 \frac{\partial}{\partial t} \right) u_1 - \frac{1}{K} \left( 1 + \lambda_2 \frac{\partial}{\partial t} \right) u_1 &= 0 \\
\left( 1 + \lambda_1 \frac{\partial}{\partial t} \right) \frac{\partial v_1}{\partial t} &= - \left( 1 + \lambda_1 \frac{\partial}{\partial t} \right) \frac{\partial p_1}{\partial y} \\
+ \frac{1}{R} \left( 1 + \lambda_2 \frac{\partial}{\partial t} \right) \left[ \frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2} + \frac{1}{3} \frac{\partial}{\partial t} \left( \frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} \right) \right] - \frac{1}{K} \left( 1 + \lambda_2 \frac{\partial}{\partial t} \right) v_1 &= 0
\end{align*} \]

(27a)

(27b)

(27c)

(27d)

For \( \varepsilon^2 \)

\[ \rho_2 = \chi p_2 + \frac{1}{2} \chi^2 \rho_1^2 \]

(28a)
\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \rho \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0
\]

(28b)

\[
(1 + \lambda_1 \frac{\partial}{\partial t}) \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{1}{3} \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right)
+ \frac{1}{R} \left( 1 + \lambda_2 \frac{\partial}{\partial t} \right) u_2 = \frac{1}{K} \left( 1 + \lambda_2 \frac{\partial}{\partial t} \right) u_2
\]

(28c)

The solutions of above systems can be obtained with the help of following supposed form of solutions for all the systems

\[
u_1(x, y, t) = U_1(y) e^{i\alpha(x-\xi)} + \overline{U_1(y)} e^{-i\alpha(x-\xi)}
\]

(33)

\[v_1(x, y, t) = V_1(y) e^{i\alpha(x-\xi)} + \overline{V_1(y)} e^{-i\alpha(x-\xi)}\]

\[p_1(x, y, t) = P_1(y) e^{i\alpha(x-\xi)} + \overline{P_1(y)} e^{-i\alpha(x-\xi)}\]

\[\rho_1(x, y, t) = \chi P_1(y) e^{i\alpha(x-\xi)} + \chi \overline{P_1(y)} e^{-i\alpha(x-\xi)}\]

\[u_2(x, y, t) = U_2(y) + U_2(y) e^{i\alpha(x-\xi)} + \overline{U_2(y)} e^{-i\alpha(x-\xi)}\]

(34)

Here, overbar denotes the complex conjugate.

With the help of these solutions, the above boundary value problems take the form

\[V_1' + \alpha U_1 = i\alpha \chi P_1 - \alpha \gamma V_1 - \alpha \gamma' U_1 + \frac{1}{R} \left[ V_1' + \alpha U_1 \right] + \frac{R}{3} \left( \alpha^2 - R \gamma \right) V_1 + \frac{\gamma}{R} \left[ 1 \right] U_1 + \frac{\gamma'}{R} \left[ 1 \right] U_1 + \frac{1}{3R \gamma} \left( \gamma + \gamma' \right) \left( 1 \right) \]

\[V_1(\pm 1) = \mp K_n U_1(\pm 1) - \frac{\alpha^2}{2}
\]

\[V_1(\pm 1) = \mp \frac{\alpha \gamma}{2} + \frac{\alpha}{2}
\]

(35)

\[D_20 = \chi P_2 + \chi^2 \overline{P_1 \overline{P_1}}
\]

\[V_2 = V_2' + \left( \frac{1}{K} + M \right) U_2 + \alpha \chi P_1 U_1 - \alpha \chi \overline{P_1} U_1 + \frac{\gamma}{R} U_1 + \frac{\gamma'}{R} \overline{U_1}
\]

\[U_2(\pm 1) = \mp \frac{1}{2} \left( \overline{U_1}(\pm 1) + U_1'(\pm 1) \right)
\]

\[V_2(\pm 1) = \pm \frac{1}{2} \left( V_1'(\pm 1) + V_1(\pm 1) \right)
\]

(36)

Here, complex parameters are

\[\gamma_1 = 1 - i\alpha \lambda_1, \gamma_2 = 1 - i\alpha \lambda_2\]

\[\gamma' = \frac{\gamma_1}{\gamma_2}\]
Aarts and Ooms\textsuperscript{11} presented the procedure for the solution of equations and Mekheimer and Abdel-Wahab\textsuperscript{15} also followed the respective techniques.

Thus, by omitting the lengthy calculations solution of first-order equations for velocity and pressure, one have

\[ V_1(y) = C_1 \sinh a_1 y + C_2 \sinh a_2 y \]
\[ U_1(y) = b_1 C_1 \cosh a_1 y + b_2 C_2 \cosh a_2 y + C_3 \chi \]
\[ P_1(y) = C_1 \left( \frac{a_1^2 - \beta_1^2}{\gamma a_1} \right) \cosh a_1 y + C_2 \left( \frac{a_2^2 - \beta_1^2}{\gamma a_2} \right) \cosh a_2 y + C_3 \]

(37)

And solution of second-order equation (35) is

\[ V_{20} = -\chi (V_1 \overline{r}_1 + P_1 \overline{r}_1) + D_1, \]
\[ U_{20}(y) = D_2 \cosh \delta y + D_3 \sinh \delta y + E(y), \]
\[ P_{20}(y) = D_4 - \frac{4\chi}{3\gamma} \overline{H}(y) = \frac{1}{K} \int_{-1}^{y} V_{20}(r) dr - \int_{-1}^{y} F(r) dr \]

Net axial velocity is defined as

\[ \langle g \rangle = \frac{1}{T} \int_{0}^{T} g(x, y, t) dt \]
\[ T = \frac{2\pi}{\alpha} \]

Mean axial velocity is

\[ \langle u \rangle = \varepsilon^2 U_{20}(y) \]

Perturbation function of mean velocity is defined as

\[ G(y) = \frac{200}{\alpha^2 R^2} [E(y) - E(1)] \]

(41)

All the constants appear in above equations are defined in Appendix.

**Results and discussion**

Figure 2(a)–(e) gives the change in behavior of $G(y)$ with respect to $y$ which occurs due to change in conduct or character of parameters. For MHD perturbation function, various values of $M$ are considered. Figure 2(a) shows that when magnetic field is increased then $G(y)$ is reduced, reason being the current $J \times B$ defined in equation (2), by increasing $M$ by means of which the perturbed velocity becomes flat. Figure 2(b) shows the increasing effect of $K$ on $G(y)$. It is seen that $G(y)$ moves toward the wall when $K$ is varied. Figure 2(c) and (d) displays the behavior of perturbation function $G(y)$ for retardation and relaxation times when $M = 0$. Figure 2(c) shows that $\lambda_1$ has increasing effect for perturbation function for $M = 0$. Also, combined effects of $M$ and $\lambda_2$ are shown in Figure 2(d). Figure 2(d) shows that $\lambda_2$ has decreasing effect on perturbation function for $M = 0$. Figure 2(e) shows effect of compressibility parameter on perturbation function. By increasing $\chi$, graph of velocity perturbation decreases.

Figure 3(a)–(f) shows the change in behavior of mean velocity with $y$ for various values of $M$, $\lambda_1$, $\lambda_2$, $K$, $K_m$, and $\chi$. Figure 3(a) shows that for $R = 1$, magnetic number is increased. Mean flow distribution decreases by increasing the magnetic parameter and obtain a backward flow. It is interesting to note that backward flow for non-Newtonian fluid is less than for Newtonian fluid. In Figure 3(b), it is to be noted that by increasing the values of $K$ mean flow decreases, this is because the permeability parameter allowed more fluid to pass through the pores, but when $K \approx 2$ mean flow increases, thus velocity of the fluid is more for large permeability parameter. Figure 3(c) shows that for an increasing $\lambda_1$ backward flow occurs in neighboring focus line, and the mean axial velocity increases. Figure 3(d) and (e) shows the combined consequence of $K_m$, $\chi$ on mean velocity spreading, for $\chi < 0.5$ backward flow occurs on center line, and by increasing $K_m$ mean axial velocity also increases. Figure 3(d) shows that by increasing the value of $\chi$ there occurred decreasing effect for mean axial velocity. Figure 3(e) shows that for increasing Kundsen number $K_m$, the mean axial velocity also increases. Figure 3(f) shows the compressibility effect that by increasing the values of $\chi$ ($\chi > 0.5$) and $K_m$ mean flow distribution increases.

Figure 4(a)–(d) shows that changes occurred in $D_{wall}$ where $D_{wall} = U_{20}(1)$ with respect to wavenumber $\alpha$ for different values of $\chi$, $K$, $\lambda_2$, and $\lambda_1$. Figure 4(a) shows that for any value of $\alpha$, $D_{wall}$ decreases by increasing the values of $\chi$. Figure 4(b) depicted that $D_{wall}$ decreases by increasing values of $K$ for any value of $\alpha$. It is shown by Figure 4(c) that $D_{wall}$ is much greater for Jeffrey fluid than that for Maxwell fluid. It is also observed that values of $D_{wall}$ increase rapidly, which depicts that effects of viscoelasticity are more evident for larger values of $\lambda_2$. Figure 4(d) shows the variations of $\lambda_1$ in presence of $M$. It is to be noted that for $M = 2$ effects on $D_{wall}$ increase by increasing values of $\lambda_1$.

**Conclusion**

The research work of peristalsis and cilia on the flow of MHD compressible Jeffrey fluid made by surface acoustic wave through micro-parallel plates over a porous medium is analyzed, the main points of the problem areas are as follows:

- For no-slip fluid, velocity perturbations $G(y)$ are large.
- The permeability parameter has growing result on $G(y)$ and has reducing effect on velocity at boundaries.
With the existence of retardation time, the flow becomes slow.

Oscillations decompose with an increase in magnetic parameter, and retardation effect becomes weak.

Perturbed velocity becomes flat around centerline for high values of magnetic field and Reynolds numbers.

Occurrence of porous medium in symmetric channel reduces the flow in case of no-slip condition with peristalsis effect associated with ciliated effect.

For Newtonian fluid, magnetic field is higher and for micropolar fluid magnetic fluid is small, also magnetic field is smaller as transverse magnetic field increases.

Figure 2. Perturbation function $G(y)$ for variation of mean velocity: (a) $K = 0.5, K_n = 0.15, \gamma = 0.1, \alpha = 0.1, \lambda_1 = 0.7, \lambda_2 = 0.4$, $R = 10$, (b) $M = 0, R = 15, K_n = 0.15, \gamma = 0.1, \alpha = 0.1, \lambda_1 = 0.7, \lambda_2 = 0.4$, (c) $\alpha = 0.4, K = 1.5, M = 0$, $R = 10, \lambda_2 = 0.6, K_n = 0.15$, (d) $\alpha = 0.1, M = 0, R = 10, K = 0.5, K_n = 0.5, \lambda_1 = 0.7$, and (e) $\alpha = 0.5, M = 2.0, R = 100$, $K = 1.5, K_n = 0.15, \lambda_1 = 1.7, \lambda_2 = 0.9$. 
Figure 3. The mean-axial velocity distribution for: (a) $\alpha = 0.1$, $R = 1$, $K = 1$, $\lambda_1 = 1.2$, $\lambda_2 = 0.4$, $\chi = 0.04$, $K_n = 0.15$, (b) $M = 0.2$, $\alpha = 0.1$, $R = 5$, $\lambda_1 = 0.7$, $\lambda_2 = 0.4$, $\chi = 0.9$, $K_n = 0.15$, (c) $\alpha = 0.5$, $R = 1$, $K = 1.3$, $\lambda_2 = 1.5$, $\chi = 0.04$, $K_n = 0.15$, (d) $\alpha = 1.5$, $R = 10$, $K = 3.3$, $\lambda_1 = 0.7$, $\lambda_2 = 0.4$, $\chi = 0.1$, $M = 0.01$, (e) $\alpha = 0.4$, $R = 10$, $K = 1.3$, $\lambda_1 = 0.9$, $\lambda_2 = 0.4$, $\chi = 0.9$, $M = 0.03$, and (f) $M = 0.01$; $K = 3.3$, $K_n = 0.15$, $\alpha = 0.5$, $R = 10$, $\lambda_1 = 1.2$, $\lambda_2 = 0.5$, $\chi = 0.9$. 
Declaration of conflicting interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding
The author(s) received no financial support for the research, authorship, and/or publication of this article.

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References
1. Guirao B and Joanny JF. Spontaneous creation of macroscopic flow and metachronal waves in an array of cilia. *Biophys J* 2007; 92: 1900–1917.
2. Akbar NS, Khan ZH and Nadeem S. Metachronal beating of cilia under influence of Hartmann layer and heat transfer. *Eur Phys J Plus* 2014; 129: 176.
3. Cordero JRV and Lauga E. Waving transport and propulsion in a generalized Newtonian fluid. *J Nonnewton Fluid Mech* 2013; 199: 37–50.
4. Gueron S and Gurevich KL. Energetic considerations of ciliary beating and the advantage of metachronal coordination. *Proc Natl Acad Sci USA* 1999; 96: 12240–12245.
5. Shapiro EM, Sharer K, Skrtic S, et al. In vivo detection of single cells by MRI. *Magn Reson Med* 2006; 55: 242–249.
6. Misra JC and Maiti S. Peristaltic pumping of blood in micro-vessels of non-uniform cross-section. *J Appl Mech* 2010; 79: 1–28.
7. Khaderi SN, den Toonder JMJ and Onck PR. Fluid flow due to collective non-reciprocal motion of symmetrically-beating artificial cilia. *Biomicrofluidics* 2012; 6: 014106.
8. Gad NS. Effect of Hall currents on interaction of pulsatile and peristaltic transport induced flows of a particle-fluid suspension. *Appl Math Comput* 2011; 217: 4313–4320.
9. Gribben RJ. The magnetohydrodynamic boundary layer in the presence of a pressure gradient. *Proc R Soc Lond A Math Phys Sci* 1965; 287: 123–141.
10. Tsiklauri D and Beresnev I. Non-Newtonian effects in the peristaltic flow of a Maxwell fluid. Phys Rev E 2001; 64: 036303.

11. Aarts ACT and Ooms G. Net flow of compressible viscous liquids induced by travelling waves in porous media. J Eng Math 1998; 34: 435–450.

12. Qi Q, Johnson RE and Harris JG. Boundary layer attenuation and acoustic streaming accompanying plane-wave propagation in a tube. J Acoust Soc Am 1995; 97: 1499–1509.

13. Beil FW, Wixforth A and Blick RH. Investigation of nano-electromechanical-systems using surface acoustic waves. Physica E: Low Dimens Syst Nanostruct 2002; 13: 473–476.

14. Chen WI. Influence of ultrasonic energy upon the rate of flow of liquids through porous media. PhD Thesis, West Virginia University, Morgantown, WV, 1969.

15. Mekheimer KS and Abdel-Wahhabi AN. Effect of wall compliance on compressible fluid transport induced by a surface acoustic wave in a microchannel. Numer Methods Partial Differ Equ 2011; 27: 621–636.

16. Gao C and Jian Y. Analytical solution of magnetohydrodynamic flow of Jeffrey fluid through a circular microchannel. J Mol Liq 2015; 211: 803–811.

17. Jha BK, Aina B and Ajiya AT. MHD natural convection in a vertical parallel plate microchannel. Ain Shams Eng J 2015; 6: 289–295.

18. Schaaf SA and Chambre PL. Flow of rarefied gases (Princeton aeronautical paperbacks), vol. 8. Princeton, NJ: Princeton University Press, 1961.

19. Damianou Y, Georgiou GC and Moulitsas I. Combined effects of compressibility and slip in flows of a Herschel-Bulkley fluid. J Nonnewton Fluid Mech 2013; 193: 89–102.

20. Mekheimer KS, Komy SR and Abdelsalam SI. Simultaneous effects of magnetic field and space porosity on compressible Maxwell fluid transport induced by a surface acoustic wave in a microchannel. Chin Phys B 2013; 22: 124702.

21. Vafai K, Khan AA, Sajjad S, et al. The study of peristaltic motion of third grade fluid under the effects of Hall current and heat transfer. Z Naturforsch A 2015; 70: 281–293.

22. Mekheimer KS and Wahab AN. Net annulus flow of a compressible viscous liquid with peristalsis. J Aerospace Eng 2011; 25: 660–669.

23. Selimefendigil F, Öztop HF and Chamkha AJ. Mixed convection of pulsating ferrofluid flow over a backward-facing step. Iran J Sci Technol Trans Mech Eng 2019; 43: 593–612.

24. Kumar B, Seth GS, Nandkeolyar R, et al. Outlining the impact of induced magnetic field and thermal radiation on magneto-convection flow of dissipative fluid. Int J Therm Sci 2019; 146: 106101.

25. Menni Y, Azzi A, Chamkha A, et al. Effect of wall-mounted V-baffle position in a turbulent flow through a channel: analysis of best configuration for optimal heat transfer. Int J Numer Methods Heat Fluid Flow 2019; 29: 3908–3937.

26. Ghalambaz M, Zadeh SMH, Mehryan SAM, et al. Analysis of melting behavior of PCMs in a cavity subject to a non-uniform magnetic field using a moving grid technique. Appl Math Model 2020; 77: 1936–1953.

27. Mehryan SAM, Tahmasebi A, Izadi M, et al. Melting behavior of phase change materials in the presence of a non-uniform magnetic-field due to two variable magnetic sources. Int J Heat Mass Transf 2020; 149: 119184.

28. Sleigh M. An example of mechanical co-ordination of cilia. Nature 1961; 191: 931–932.

Appendix I

Variable and parameters for first-order solution are

\[ \gamma = \gamma^* R - \frac{\alpha \chi}{3} \beta^2 = \alpha ^2 - \alpha \gamma^* R + \frac{R}{K} + MR \gamma^* \]

\[ \beta_1 = \alpha ^2 - \alpha \gamma^* R + \frac{R}{K} \]

\[ a_i^2 = \frac{(\beta^2 + \nu^2) + \sqrt{(\beta^2 + \nu^2)^2 - 4\beta_1^2 \nu^2}}{2} \]

\[ a_i^2 = \frac{(\beta^2 + \nu^2) - \sqrt{(\beta^2 + \nu^2)^2 - 4\beta_1^2 \nu^2}}{2} \]

\[ b_1 = \chi \left( \frac{a_i^2 - \beta_1^2}{a_i} \right)^2 a_1 = \frac{\chi (a_i^2 - \beta_1^2)}{a_i} + \frac{\chi a_2}{a} \]

\[ C_1 = \frac{\chi a_2 b_2 g_2}{2 G} - \frac{\alpha}{2} \frac{(b_2 g_2 + a_2 \sinh a_2)}{G} \]

\[ C_2 = -\frac{\chi}{2} \frac{1}{\sinh a_2} \left[ 1 + \frac{b_2 g_2}{G} \sinh a_1 \right] + \frac{\alpha}{2} \frac{1}{\sinh a_2} \left[ 1 + \frac{b_2 g_2}{G} \sinh a_1 \right] + \frac{\alpha^2 \sinh a_1}{2} \frac{1}{G} \]

\[ C_3 = 0 \]

\[ \nu^2 = -\frac{4}{3} \chi \alpha^2 + \alpha^2 \gamma^* R (1 - \chi) - \frac{\alpha \alpha R}{K} \]

\[ \nu_A^2 = -\frac{4}{3} \chi \alpha^2 + \alpha^2 \gamma^* R (1 - \chi) - \frac{\alpha \alpha R (M \gamma^* + \frac{1}{K})}{K} \]

\[ g_1 = \cosh a_1 + K_a a_1 \sinh a_1, g_2 = \cosh a_2 + K_a a_2 \sinh a_2 \]

\[ G = b_1 g_1 \sinh a_2 - b_2 g_2 \sinh a_1 \]

Using boundary conditions, the complex constants for second-order solutions are
\[ D_1 = \frac{\alpha}{2} (U_1(1) - U_1'(1)) + \frac{\chi_2}{2} (P_1(1) + P_1'(1)) \]
\[ D_4 = P_{20}(-1) + \frac{4\chi}{3R} H(-1) \]
\[ D_2 = \frac{-1}{2(\cosh \delta + K_2 \sinh \delta)} \left[ K_2 (E'(1) - E'(-1)) + K_2 (\beta_4 - \beta_3) + \beta_2 + \beta_3 + E(1) + E(-1) - \alpha^t \right] \]
\[ D_3 = \frac{-1}{2(\sinh \delta + K_2 \cosh \delta)} \left[ K_2 (E'(1) + E'(-1)) + K_2 (\beta_4 + \beta_3) + \beta_2 - \beta_3 + E(1) - E(-1) \right] \]
\[ F = \alpha_1 P_1 V_1' - \alpha_2 P_1 V_1 + V_1 V_1' + V_1 V_1' - \alpha U_1 V_1 - \alpha U_1 V_1 \]
\[ H = \frac{d}{dy} (P_1 V_1 + P_1 V_1) \]
\[ \beta_2 = \frac{1}{2} \left( U_1'(1) + U_1'(1) \right), \beta_3 = -\frac{1}{2} \left( U_1'(1) + U_1'(1) \right) \]
\[ \beta_4 = \frac{1}{2} \left( U_1''(1) + U_1''(1) \right), \beta_5 = -\frac{1}{2} \left( U_1''(1) + U_1''(1) \right) \]
\[ E(y) = R \left( \frac{j_1}{(a_1 + a_1^2)^2 - \delta^2} \cosh(a_1 + a_1) y + \frac{j_{11}}{(a_1 - a_1)^2 - \delta^2} \cosh(a_1 - a_1) y \right) \]
\[ + \frac{j_2}{(a_1 + a_1^2)^2 - \delta^2} \cosh(a_1 + a_1) y + \frac{j_{22}}{(a_1 - a_1)^2 - \delta^2} \cosh(a_1 - a_1) y \]
\[ + \frac{j_3}{(a_2 + a_2^2)^2 - \delta^2} \cosh(a_2 + a_2) y + \frac{j_{33}}{(a_2 - a_2)^2 - \delta^2} \cosh(a_2 - a_2) y \]
\[ + \frac{j_4}{(a_2 + a_2^2)^2 - \delta^2} \cosh(a_2 + a_2) y + \frac{j_{44}}{(a_2 - a_2)^2 - \delta^2} \cosh(a_2 - a_2) y \]
\[ j_1 = \frac{1}{2} (a_1 C_1 b_1 C_1 + a_1 C_1 b_1 C_1 + a_1 b_1 C_1 C_1 + a_1 b_1 C_1 C_1) \]
\[ j_2 = \frac{1}{2} (a_1 C_1 b_1 C_2 + a_1 C_1 b_1 C_2 + a_1 b_1 C_1 C_2 + a_1 b_1 C_1 C_2) \]
\[ j_3 = \frac{1}{2} (a_2 C_1 b_2 C_1 + a_2 C_1 b_2 C_1 + a_2 b_1 C_1 C_2 + a_1 b_1 C_1 C_2) \]
\[ j_4 = \frac{1}{2} (a_2 C_2 b_2 C_2 + a_2 C_2 b_2 C_2 + a_2 b_2 C_2 C_2 + a_2 b_2 C_2 C_2) \]
\[ j_{11} = \frac{1}{2} (a_1 C_1 b_1 C_1 + a_1 C_1 b_1 C_1 - a_1 b_1 C_1 C_1 + a_1 b_1 C_1 C_1) \]
\[ j_{22} = \frac{1}{2} (a_1 C_1 b_2 C_2 + a_2 C_2 b_1 C_1 - a_2 b_1 C_2 C_1 + a_1 b_1 C_1 C_2) \]
\[ j_{33} = \frac{1}{2} (a_2 C_2 b_1 C_1 + a_1 C_1 b_2 C_2 + a_1 b_1 C_1 C_2 - a_2 b_2 C_2 C_1) \]
\[ j_{44} = \frac{1}{2} (a_2 C_2 b_2 C_2 + a_2 C_2 b_2 C_2 - a_2 b_2 C_2 C_2 + a_2 b_2 C_2 C_2) \]