INFRARED EFFECTS AND BUBBLE PROPAGATION AT THE ELECTROWEAK PHASE TRANSITION

R.G. Leigh
Santa Cruz Institute for Particle Physics
University of California, Santa Cruz, CA 95064

ABSTRACT
We discuss aspects of poor infrared behaviour of the perturbation expansion for the effective potential for the Higgs mode near the electroweak phase transition, and enlarge on the discovery that higher order effects weaken the transition. In addition, we outline our recent attempts at understanding the dynamics involved in the propagation of bubbles formed in the first order transition.

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In this talk, I will discuss several issues related to the theory of the electroweak phase transition. A discussion of other aspects which we have recently addressed\(^1\,^2\) can be found in the article by A. Linde in these proceedings.

The theory of phase transitions in the early Universe was pioneered in the seminal work of Kirzhnits and Linde in 1972.\(^3\) Early works claimed that the phase transition corresponding to the breakdown of SU(2) × U(1) was of the second order.\(^4\,^5\,^6\) Finally in 1976, it was shown\(^7\) to be of the first order. This fact essentially lay dormant for many years until it was realized\(^8\) that this character of the phase transition can provide the non-equilibrium environment essential for producing a baryon asymmetry at these scales; the baryon violating process is typified (at temperatures below the critical temperature) by the sphaleron. We now believe that the phase transition proceeds by the nucleation of critical true vacuum bubbles from a rather weakly supercooled state. These bubbles expand under pressure forces and quickly fill space. Baryogenesis typically proceeds in the bubble walls where \(\phi\) is changing,\(^9\,\,14\) or in a thin layer in front of the wall.\(^15\)

It is potentially important for a detailed understanding of the baryon asymmetry, to understand the propagation of the bubble wall, namely its size and shape, and its terminal velocity (if any). In the last half of this presentation, I will discuss our estimates of these quantities, and compare to other recent analyses. Work is continuing on refining the analysis and considering new phenomena that may be important.\(^16\)

In the first section of this presentation, I will review the standard analysis leading to the derivation of the effective potential for the Higgs mode. It has recently been emphasized\(^17\,\,20\) that higher order corrections are potentially important because of infrared divergences and indeed can lead to the breakdown of perturbation theory in relevant regions of parameter space. We discuss several approaches to resuming perturbation theory that soften the poor infrared behaviour. Finally, we outline by powercounting the region of parameter space for which the resummed perturbation theory is good, and comment on what effects the analysis has on various models of baryogenesis at the electroweak scale.

1. The Electroweak Potential

1.1. One-Loop Analysis

The standard approach to equilibrium properties of finite temperature field theories is the imaginary time formalism. In this technique, one continues to Euclidean spacetime, and compactifies the time coordinate on a circle of radius \(h, \beta = T^{-1}\). The resulting path integral is then identical to a Boltzmann sum over states. The fields in the theory may be expanded in Fourier series over a discrete
set of Matsubara frequencies

\[ \psi(\vec{x}, \tau) = \sum_n e^{i\omega_n \tau} \psi_n(\vec{x}). \]  \hspace{1cm} (1.1)

Requiring the fields to satisfy (anti-)periodic boundary conditions on the circle leads to \( \omega_n = (2n + 1)\pi T \) (fermions) or \( \omega_n = 2\pi n T \) (bosons). In this way, the theory can be thought of as an infinite set of coupled three-dimensional fields with a hierarchy of masses \( m^2 = \omega_n^2 + m_o^2 \) where \( m_o \) is the usual zero temperature mass. As we will see later, this viewpoint will be of particular value to us. The quadratic part of the action (for bosons) may be written as

\[ S_\beta = \frac{1}{2} T \sum_n \int d^3x \phi_n (\phi_n - n) \left( -\vec{\nabla}^2 + \omega_n^2 + m_o^2 \right) \phi_n. \]  \hspace{1cm} (1.2)

The effective potential for the Higgs field \( \phi \) can be computed by putting in a source and studying the response of the theory to this source. At the one loop level, this is equivalent to shifting \( \phi \) by its saddle point value; the familiar result is just the determinant of quadratic fluctuations. That is, at lowest order, we can neglect interactions and the effective potential is that of a collection of free particles interacting with a heat bath at temperature \( T \) with masses given by \( \phi \). We have

\[ V_{T,1\text{-}loop} (\phi) = V_0 (\phi) - \frac{1}{2} T \sum_i \ln \det \left[ -\vec{\nabla}^2 + \omega_i^2 + m_i^2 (\phi) \right] \]

\[ = V_{0,1\text{-}loop} + \left( \frac{T^4}{2\pi^2} \right) \sum_i I_{\pm} \left( \frac{m_i (\phi)}{T} \right) \]  \hspace{1cm} (1.3)

where

\[ I_{\pm} (y) = \pm \int_0^\infty dx \ x^2 \ln \left( 1 \mp e^{-\sqrt{x^2 + y^2}} \right). \]  \hspace{1cm} (1.4)

Expanding for small \( y \), \( i.e., \ m(\phi) \ll T \), we find the following potential:

\[ V_T (\phi) = D(T^2 - T_o^2)\phi^2 - E T \phi^3 + \frac{1}{4} \lambda_T \phi^4 \]  \hspace{1cm} (1.5)

where \( D, E \) and \( \lambda_T \) are determined in terms of the gauge and Yukawa couplings of the theory and the Higgs self-coupling. We will assume throughout that the Higgs boson makes only a very small contribution to the potential, \( i.e., \) that it is
sufficiently light. The parameters in the effective potential, Eq. (1.5), are given in many previous works, including Refs. 1,2. For present purposes, we note the following. The parameter $E$ is crucial in making the transition of the first order; when $E$ is non-zero there is a second minimum at positive $\phi$ which first appears at a temperature $T_1$, becomes stable at a temperature $T_c$ and then becomes the true vacuum below the temperature $T_o$. Thus, we expect that supercooling can take place as the Universe cools, the field $\phi$ getting stuck in the false vacuum state at $\phi = 0$ for $T < T_c$. In a second order transition, the parameter $E$ is zero, and there is a smooth transition from $\phi = 0$ to $\phi \neq 0$ with no supercooling. In our context the supercooling is crucial for supplying the non-equilibrium environment necessary for baryogenesis. The transition from false vacuum to true vacuum proceeds by the nucleation of bubbles of true vacuum which then expand (if large enough) to fill the universe, baryons being produced in the vicinity of the advancing bubble walls.

In the minimal standard model, the parameter $E$, taking into account the analysis presented above, is given by

$$E = \frac{3}{12\pi v_o^3} \left(2m_W^3 + m_Z^3\right).$$

(1.6)

Only bosonic fields contribute. This is understood in the present context by the appearance of a non-analyticity in the dependence of $I_+$ on $y^2$ coming from the minus sign preceding the exponential in Eq. (1.4). This non-analyticity is absent for $I_-$. It will be of some importance to us to understand the origins of this non-analyticity. As I will now demonstrate, it is due to infrared divergences in loop integrals.

The cubic term in the potential may be derived in several other ways by evaluating Feynman graphs. In order to do this, we will find it convenient to evaluate, instead of the potential directly, the tadpole graphs that contribute to $dV_T/d\phi$. In the minimal standard model at one loop, we then find

$$\frac{dV_T}{d\phi} = \sum_{i,\text{bosons}} (h_i \phi) T \sum_n \int \frac{d^3k}{(2\pi)^3} \frac{1}{\bar{k}^2 + \omega_n^2 + m_i^2(\phi)}$$

$$- \sum_{i,\text{fermions}} T \sum_n \int \frac{d^3k}{(2\pi)^3} \text{tr} \left[h_{f,i} P_1(\bar{k}, \omega_n, \phi)\right]$$

(1.7)

where the $h_i$ are the couplings of $\phi$ to the various particles and $P_1$ is the Euclidean fermion propagator. The trace gives a factor of $m_i(\phi) = h_{f,i}\phi$ for the fermion
contribution. Re-writing $h_i = h_{f,i}^2$ for the fermions, we obtain

$$\frac{dV_T}{d\phi} = \sum_{i, \text{bosons}} (h_i \phi) T \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2 + m_i^2(\phi)}$$

$$+ \sum_i (h_i \phi) \sum_{\omega_n \neq 0} \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2 + \omega_n^2 + m_i^2(\phi)}.$$  \hspace{1cm} (1.8)

I have separated off the $\omega = 0$ terms as these have different behaviour for small $\phi$. Indeed we can perform the above integrations by dimensional regularization and the frequency sums can be zeta-function regulated to obtain, for small $m_i(\phi)/\pi T$, the following:

$$\frac{dV_T}{d\phi} = -\frac{1}{4\pi} \sum_{i, \text{bosons}} h_i T \phi m_i(\phi) + \sum_i c_i h_i T^2 \phi + \ldots$$  \hspace{1cm} (1.9)

where the $c_i$ are $\phi$-independent constants. We note that the first term, which came entirely from the zero frequency mode of the bosons, integrates to give a cubic term in $V_T$, whereas all of the other modes give corrections to quadratic (and higher) terms in $\phi$, and in particular, are analytic in $\phi^2/T^2$. Thus the cubic term is understood as arising from the infrared region of 4-momentum space.

Now, if we go on to powercount higher loop graphs, we find that the infrared behaviour found at the one loop level worsens. Indeed, one finds that there is an effective expansion parameter that is of order $g^2 T^2/m^2(\phi) \simeq (T/\phi)^2$, where $g$ is some generic coupling constant; for concreteness, we take it to be the SU(2) gauge coupling. The appearance of this expansion parameter means that perturbation theory breaks down at $\phi$ less than or of the order of $T$. This is clearly a disaster for an examination of the phase transition, where everything typically happens in this regime.\textsuperscript{23} In what follows, we will consider improvements of perturbation theory obtained by attempting to take into account all of these infrared divergent contributions in some consistent way. To begin, we will assume that the coupling $g$ is sufficiently small that perturbation theory using it as an expansion parameter makes sense. In contrast to ordinary 4-dimensional field theory, we will see that one does not here have the luxury of factors of $4\pi$ in the expansion parameter. For these reasons, much of the powercounting arguments supplied below will be merely formal arguments valid for small $g$; an accurate numerical analysis awaits the future.\textsuperscript{*}

\footnotesize

* In particular, it has been claimed that (light) Higgs bosons may induce numerically large corrections. See Refs. 20.
1. 2. Infrared Improvements

The fact that higher loop graphs are important near the phase transition was first noted by Weinberg\textsuperscript{4} and was also discussed in Refs. 6,22. Indeed in order that a theory develop a second minimum at low temperatures, it must be that the perturbative expansion breaks down because the second minimum is ‘non-perturbatively far away’ from the original vacuum state. More recently, this has been emphasized in the context of the electroweak theory by several authors. In Ref. 17, Brahm and Hsu attempted to take into account some of the higher order graphs but found a large negative linear term $\sim -g^3 T^3 \phi$, leading to the conclusion that the transition is at most very weakly first order. In a similar analysis, Shaposhnikov\textsuperscript{18} found a large positive linear term $\sim g^4 T^3 \phi$, leading him to conclude that the transition could be very strongly first order. In work coincident with our own, Carrington\textsuperscript{19} found corrections to the cubic term, leading her to initially conclude that the transition is more strongly first order. All of these results have now been understood to be incorrect, as I will explain in detail below. In particular I will show that linear terms never arise, but that the cubic term is modified in an important way.

Let us now look at higher loop graphs in more detail. The graphs of Fig. 1 where the large loop is at $\omega = 0$ and the smaller loops are at $\omega \neq 0$ are the most infrared divergent graphs. These have been referred to in the literature as ‘daisy’ or ‘ring’ diagrams. If we powercount the graphs for $m(\phi) \ll \pi T$, we find that they are proportional to

$$V_{\text{ring},p} \sim \left( \frac{g^2 T^2}{m^2(\phi)} \right)^p \sim \left( \frac{T}{\phi} \right)^{2p}$$

and thus are not formally suppressed by powers of coupling constant. To proceed, we must resum perturbation theory in some way. In the following sections, I will discuss in detail two methods of understanding this resummation.

1.2.1 Schwinger-Dyson Approach

One notes that the diagrams in Fig. 1 correspond to insertions of one-loop propagator corrections on the large loop. This leads us to believe that perturbation theory may be improved if we can take into account these loop corrections to the propagators. In an idea outlined in the book by Kapusta,\textsuperscript{21} and stressed by Carrington\textsuperscript{19} in the context of the Standard Model, one extremizes the effective action, not just with respect to the vacuum value of the Higgs field, but also with respect to the polarization tensors, $\langle \Pi(\omega_n, \vec{k}) \rangle$. Thus we go one step beyond the
mean field approximation. To do this one writes a shifted propagator

\[
D_{0,n}^{-1}(\vec{k}) = \vec{k}^2 + \omega_n^2 + m_0^2
\]

\[
D_n^{-1}(\vec{k}) = D_{0,n}^{-1}(\vec{k}) + \Pi_n(\omega_n, \vec{k})
\]

where we for simplicity give explicit formulae for a scalar boson only. The result can be immediately generalized, and indeed we will do so later for the Standard Model. We now add zero to the Lagrangian in the following way:

\[
L(D_0) = L(D) - \frac{1}{2} T \sum_n \phi_n \Pi_n \phi_n.
\]

We are to do perturbation theory with the first term, and treat the last term as a counterterm. We can then compute the one-loop effective potential, which looks like

\[
V(\phi) = V_{\text{tree}}(\phi) - \frac{1}{2} T \sum_n \int \frac{d^3k}{(2\pi)^3} \left[ \ln(T^2 D_n) - \Pi_n D_n \right] + \sum_{\ell=2}^{\infty} V_\ell(\phi, D) + \text{subtr.}
\]

Note the important correction term to the one-loop logarithm. The integer \(\ell\) counts loops, and one should note that these higher loop diagrams involve corrected propagators. The self-energy is obtained self-consistently by requiring that it extremize the potential. An arbitrary variation of this effective potential can be written

\[
\delta V = \frac{\delta V_{\text{tree}}}{\delta \phi} \delta \phi - \frac{T}{2} \sum_n \int \frac{d^3k}{(2\pi)^3} \left[ \left( \Pi_n(\vec{k}) - 2 \sum_{\ell=2}^{\infty} \frac{\delta V_\ell}{\delta D_n(\vec{k})} \right) \frac{\delta D_n(\vec{k})}{\delta \phi} \right]
\]

\[
+ \left[ \frac{T}{2} \sum_n \int \frac{d^3k}{(2\pi)^3} \frac{\delta D_{0,n}^{-1}(\vec{k})}{\delta \phi} D_n(\vec{k}) + \sum_{\ell=2}^{\infty} \frac{\delta V_\ell}{\delta \phi} \right] \delta \phi,
\]

where \(\phi\) and \(\Pi\) have been varied independently. Thus the requirement on the self-energy turns out to be just the Schwinger-Dyson equation

\[
\Pi_n = 2 \sum_{\ell=2}^{\infty} \frac{\delta V_\ell}{\delta D_n},
\]

given diagrammatically in Fig. 2.
The variation with respect to $\phi$ gives us the equation for the tadpole, shown in Fig. 3, where the internal lines are full propagators, and the vertices are uncorrected. To show that the perturbative series has indeed been improved, we can evaluate, again in just the simple scalar theory (we return to the realistic case presently) this expression at one loop; we find

$$\frac{\delta V}{\delta \phi} \sim \lambda \phi \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2 + m_\phi^2(\phi) + \Pi_\phi(k)}$$

(1.16)

where we have isolated the zero-frequency contribution as above. Taking the limit $k \to 0$ within $\Pi$, we obtain

$$\frac{\delta V}{\delta \phi} \sim -\lambda \phi T \left[ m_\phi^2(\phi) + \Pi_\phi(0) \right]^{1/2}$$

and hence a ‘cubic’ term

$$V \sim -\frac{T}{12\pi} \left[ m_\phi^2(\phi) + \Pi_\phi(0) \right]^{3/2} + \ldots.$$  

(1.17)

As long as $\Pi_\phi(0)$ is not singular in the limit $\phi \to 0$ (it isn’t), we obtain no linear terms in $\phi$. Furthermore, if $\Pi_\phi(0)$ is large compared with $m_\phi^2$ (true for some region of small $\phi$), the cubic term is reduced to essentially zero. This will have important consequences later for the electroweak potential. In the scalar theory, Eq. (1.15) gives us at lowest order

$$\Pi_\phi(\vec{q} \to 0) = 3\lambda T^2 \left\{ \sum_n \int \frac{d^3k}{(2\pi)^3} D_n(\vec{k}) \left[ 1 + 6\lambda \phi^2 D_{-n}(\vec{q} - \vec{k}) \right] \right\}$$

$$= \frac{1}{4} \lambda T^2 + \ldots .$$  

(1.18)

The terms in the ellipsis involve corrections of order $m/T$, and include dependence on $\Pi_n$. The leading term, of order $\lambda T^2$, is unambiguous, and we see that the would-be cubic term is removed for small $\phi$.

We now consider the Standard Model. In this case, we run into additional problems: in a gauge theory, the ‘magnetic mass’ vanishes. That is, if we compute $\Pi_\phi(k \to 0)$ for the transverse gauge bosons, we find there is no term of order $g^2 T^2$. We will see in the following sections that this fact leads to an as yet insurmountable difficulty with the perturbative expansion, at least in some range of $\phi$, and has a direct analogue in finite temperature QCD.
Before discussing this further let us note the changes to the electroweak potential coming from the resummation. The top quark self-energy is $\Pi_{\text{top}} \simeq g_s^2 T^2 / 6$, but being a fermion it has no $\omega = 0$ mode, and so does not contribute to the infrared problem. Higher order graphs lead to numerically small corrections to the $\phi^2$ and $\phi^4$ terms. To discuss the gauge bosons, let us work in Coulomb gauge. In this gauge, we have propagators that mix the transverse gauge fields with the Coulomb mode and the Goldstone mode. At $\omega = 0$, these decouple and we find that the Coulomb and Goldstone mode self-energies have a term of order $g^2 T^2$, as does the physical Higgs, but the two transverse modes have no such term. This is as we discussed above. Referring to the result of Eq. (1.17), we write the corrected cubic term from the $W^\pm$ as

$$V_{\text{cubic}} = -\frac{2}{12\pi} T \left[ (m_W^2 + m_{\text{D}}^2)^{3/2} + 2(m_{W}^2)^{3/2} \right]$$ (1.19)

where $m_D = \sqrt{\Pi_0(k \to 0)} \sim gT$ is the Debye mass. There is also a similar contribution from the $Z^0$. As we will see later, this is reliable for $\phi \gtrsim gT$, and thus we see that the cubic term has been reduced by a factor of $2/3$, as the first term in the brackets of Eq. (1.19) gives (small) corrections to the $\phi^2$ and $\phi^4$ terms. This has important consequences for baryogenesis. We note that the position of the minimum is now given by

$$\frac{\phi}{T} \bigg|_{\text{min}} \simeq \frac{2}{\lambda_e} \left( \frac{2}{3} E \right) (1 + \epsilon)$$

where $\epsilon = (T_c^2 - T^2)/(T_c^2 - T_o^2)$ gives the temperature of nucleation. The position of the minimum, for a given value of $T$ has been moved in towards the origin and hence the transition is less strongly first order (the height of the barrier is correspondingly lower). The bound on the Higgs boson mass, found by requiring that sphalerons not wash out the baryon asymmetry after the transition, is expected to be lowered by a similar factor, down to about 40 GeV. We conclude from this analysis that the minimal standard model is incapable of producing sufficient $n_B/s$ (regardless of the amount of $CP$ violation) because of the LEP bound of $m_H \gtrsim 57$ GeV.

This bound is easily avoided however by extending the model. A multi-Higgs model (two or more doublets) is particularly attractive here, because it can easily avoid the mass bound and there is $CP$ violation at a (nearly) sufficient level. Anderson and Hall have also noted that adding singlet scalars can modify the dependence of the Higgs mass on the self-coupling in such a way that the bound is avoided. As well as raising the theoretical bound on the Higgs mass, an extended model does not suffer from such a restrictive laboratory bound as does the minimal standard model (because of, e.g., $\tan \beta$ dependence, etc.).
1.2.2 Infrared Effective Action

It is also useful to understand these aspects of the electroweak potential in an alternative way. The idea here builds on the fact that the interesting behaviour comes from the infrared region. Since, at weak coupling, we are interested in energy scales much less than $\pi T$, one can first integrate out all modes of the various fields with non-zero Matsubara frequency, thereby obtaining a three-dimensional effective action for the zero-frequency bosonic fields. As we will see below this technique automatically sums the troublesome graphs, and automatically gives the correct combinatorics; we are able to clearly see the absence of linear terms in the potential.

Integrating out all of the heavy modes corresponds to evaluating all Feynman diagrams containing loops of heavy fields, with an arbitrary number of external light legs. Thus we obtain a three dimensional effective action involving an infinite number of operators. In particular, the quadratic terms pick up important contributions. In a scalar theory, this action is given by

\[
\delta L_{\text{QUAD}} = \frac{1}{2} \int d^3 x \phi^2_o \left( \sum_{n \neq 0} \frac{3 \lambda T}{(2\pi)^3} \int \frac{d^3 k}{k^2 + \omega_n^2 + m^2} \right)
\]

\[
= \frac{1}{2} \int d^3 x \phi^2_o \left\{ -3 \lambda T^2 \sum_{n \neq 0} \left[ n^2 + \left( \frac{m}{2\pi T} \right)^2 \right]^{1/2} \right\}
\]

\[
= \frac{1}{2} \int d^3 x \phi^2_o \left\{ -3 \lambda T^2 \zeta(-1) + \ldots \right\}
\]

The quadratic part of the action thus has a term of order $\lambda T^2$ and all corrections to this are analytic in $(m/T)^2$, because the $n = 0$ contribution is absent. The contribution of the $n = 0$ mode corresponds to loop graphs in the infrared effective theory.

Having integrated out the heavy modes (at one loop level), we can now compute the one-loop effective potential in the effective theory. This is just the determinant of Gaussian fluctuations in the low energy theory, and corresponds to the bubble graph of Fig. 4; if one writes this out in terms of the heavy lines that have been integrated out (Fig. 4), we see that we have reproduced the sum of the dangerous ring diagrams. One can easily check that the cubic term is correctly reproduced, as in Eqs. (1.17) and (1.19).

In the case of the Standard Model at one loop, one finds results similar to Eq. (1.20) for the Coulomb and scalar lines and thus one has the Debye mass

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We have used dimensional regularization.
corrections of order $g^2 T^2$ for these fields, whereas the quadratic term for the transverse gauge bosons goes to zero with $\vec{k}$ and $\phi$, as discussed above. These “mass corrections” are analytic in $|\phi|^2$; in particular, they do not contain linear terms in $\phi$, and thus there are no linear terms in the effective potential, at least at this order. In principal, we should go on to examine higher loop graphs. In the next section, we will discuss the improvements that have been made to perturbation theory by summing the ring diagrams. In the present context, however, we can easily understand the absence of the linear terms at the two-loop level as well. The dangerous graphs are those with a transverse gauge boson in one loop and some other field with a Debye mass $\dagger$ in the other loop. Individual graphs are badly behaved, that is, they apparently give linear terms, but the sum of all such two-loop graphs is not so singular. This can be understood by thinking of the loop containing fields with a Debye mass as a one-loop correction to the propagator of the transverse gauge boson, i.e., by an insertion of the polarization tensor. This has been calculated at one-loop level and contains a momentum factor in the numerator which softens the infrared divergence of the two-loop graphs.$^2$ Thus one sees that there are no linear terms in the effective potential through order $g^4 \phi T^3$.

Additional higher loop graphs are potentially troublesome; these are known in the literature by the name “superdaisy” graphs. In the present context these are taken care of by further integrating out modes. Recall that we have integrated down to a momentum scale of order $\pi T$. In this theory, we have fields with masses of order $g T$, and fields which are lighter. We can consider moving the cutoff lower, below $g T$, whereupon the Coulomb and Goldstone modes are integrated out. As long as the temperature is not too close to $T_o$, we can also integrate out the Higgs boson as well, leaving only the transverse modes. With such a low cutoff, the problems arising from the superdaisy graphs are moot. In the next section, we will study the infrared behaviour of the full theory and the validity of perturbation theory by looking at this low energy theory. At temperatures in the vicinity of the phase transition, additional divergences arise because the Higgs boson is becoming light. In what follows we assume that the Higgs boson mass is at least of order $g T$.

1. 3. How good is perturbation theory now?

We would now like to discuss the improvement of the perturbation expansion in a gauge theory and discuss whether or not we can reliably predict a first-order transition. We have noted that the expansion fails because of the lowest-order vanishing of the magnetic mass of the transverse gauge bosons. We expect that the associated infrared divergences will be cut off in some way. For the purpose

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$\dagger$ Recall that the loops in the effective theory involve only $\omega = 0$ modes; the loop momenta are cut off at the Debye mass.
of the present discussion, let us suppose that this cutoff is provided by a magnetic mass $m_{\text{mag}} \sim g^2 T$. Consider then the theory where we have integrated out all modes except for the transverse gauge bosons (i.e., at scales below $gT$).\footnote{For temperatures very close to the transition, the Higgs will also become light, and the perturbation expansion will break down. We suppose here that we are at sufficiently high $T$ that the Higgs may be integrated out as well.} If we powercount vacuum graphs, we find that they behave like

$$V_p \sim g^6 T^4 \left( \frac{g^2 T}{\sqrt{m_W^2 + m_{\text{mag}}^2}} \right)^p.$$  

(1.21)

If we neglect $m_{\text{mag}}$, the expansion parameter is of order $gT/\phi$, and the expansion is valid for $\phi \gtrsim gT$. This range of $\phi$ is consistent with neglecting $m_W$ in relation to $m_{\text{mag}}$. We can interpret this in another way. In the vicinity of the phase transition, $\phi/T \sim E/\lambda \sim g^3/\lambda$, and so the expansion parameter is of order $\lambda/g^3$. We conclude from this that large $m_H$ is a potential problem for us, at least if we want the minimum of the potential to appear in the perturbative regime.

The typical shape of the potential is shown in Fig. 5. The exact placement of the limit of perturbative applicability and the position of the minimum depend crucially on numerical factors. In any case, for sufficiently strong first order transitions (where $\phi/T \gtrsim 1$) we can reliably predict the presence of a minimum away from the origin, and we expect a first order transition to occur. Further evidence is provided by the $\epsilon$-expansion and the absence of an infrared fixed point.\footnote{28}

We can also worry about the presence of bad behaviour at small $\phi$, and whether or not symmetry is truly restored at high temperatures. Here, the behaviour of higher order graphs is similar to that of QCD.\footnote{29} There, the potential can be written, as in Eq. (1.21), as

$$V = \ldots + T^4 \left\{ O(g^4) + O(g^6 \ln g^2) + O \left[ g^6 \sum_p (g^2 T/m_{\text{mag}})^p \right] \right\}$$

where all of the last terms are formally of the same order. The standard wisdom is that we can compute the small $O(g^4)$ and $O(g^6 \ln g^2)$ corrections, and then hope that all the rest adds up to something small. This is an unsolved problem. In the case of the electroweak theory, we arrive at the same result, but we have additional information. The cutoff at $O(g^2 T)$ implies polynomial behaviour in $\phi$ for small $\phi$
and thus we expect, for example, the quadratic term can be written

\[ V_{\text{quad}} = \phi^2 \left[ -\frac{1}{2} \mu^2 + 2Bv_o^2 + DT^2 + O(g^4 \ln g^2)T^2 + \sum g^4 (g^2 T/m_{\text{mag}})^p \right]. \]

Even though we cannot hope to compute past \( O(g^4 \ln g^2) \) here, we do not expect any bad behaviour at small \( \phi \), and thus we can claim that symmetry is restored at high temperature, regardless of infrared divergences, provided they are cutoff in some way. Furthermore, as above, we conclude that the transition is first order, as long as it is sufficiently strongly so. This includes those theories capable of producing a baryon asymmetry.

2. Bubble Wall Propagation

A detailed study of baryogenesis at the electroweak phase transition necessarily involves an understanding of how and when the bubbles are formed in the first order transition, as well as how they propagate. In this talk we will give only a cursory discussion of the former, noting only those aspects that are of relevance to the propagation of the bubble walls.

Studies of the nucleation event\(^2,27\) indicate that bubbles are formed at a temperature just below \( T_c \), but sufficiently far below this that the thin-wall approximation is not in all respects valid. Using the improved potential evaluated at a Higgs boson mass of 35 GeV,\(^*\) one finds that the nucleation occurs at a temperature given by \( \epsilon \sim \frac{1}{4} \) where \( \epsilon = (T_c^2 - T^2)/(T_c^2 - T_o^2) \). Because the expansion of the Universe is so slow at this temperature, a typical bubble grows to a macroscopic size before colliding with other bubbles. The fractional change in the temperature of the Universe during the period of expansion is found to be roughly \( \delta T/T \sim 10^{-5} \). It is thus a good approximation to ignore the expansion of the Universe in our calculations.

In the first scenarios proposed for the formation of the asymmetry, baryon number was produced in the bubble wall.\(^9-12,14\) This mechanism, at best, is not terribly efficient, because the baryon number violating processes turn off rapidly as the scalar field expectation value turns on. The resulting asymmetry is sensitive to the speed and thickness of the bubble. The most effective scenarios for electroweak baryogenesis have the baryons produced in front of the wall, in the symmetric phase.\(^13,15\) In this picture, scattering, for example, of top quarks from the bubble

\(^*\) We choose this value here and throughout this section because it represents roughly the largest value that would be compatible with baryogenesis. We emphasize that the minimal standard model is being taken here as a toy model. The strength of \( CP \) violation is of course another issue of importance for baryogenesis, but of no importance for the propagation of the bubble wall.
wall leads to an asymmetry in left vs. right-handed top quarks in a region near the wall. This asymmetry, resulting from an asymmetry between reflection and transmission of different quark helicities at the wall, biases the rate of baryon number violation in the region in front of the wall; the resulting value of $n_b/n_\gamma$ can be as large as $10^{-5}$. However, the authors of Ref. 15 assumed that the wall was rather thin, with a thickness of order $T^{-1}$. For thicker walls, the asymmetry goes rapidly to zero.\(^\dagger\) This can easily be understood. In order to have an asymmetry in reflection coefficients, the top quarks must have enough energy to pass through the wall. For $m_t \sim 120$ GeV, this means typically the energy must be greater than about $T/2$. If the wall is very thick compared to this scale, the motion of the top quarks is to a good approximation semiclassical, and the reflection coefficient is exponentially suppressed. The analyses of other authors also exhibit sensitivity to the wall shape and velocity.

Clearly, then, it is important to understand how the bubble propagates after its initial formation. A complete description of the wall evolution is rather complicated. We will see, however, that in certain limits it is not too difficult to determine how the velocity and thickness of the wall depend on the underlying model parameters.

We begin with a history of calculations of the bubble velocity. In the 1970’s Coleman\(^31\) studied the zero temperature case and found that the bubbles quickly accelerated to the speed of light. At high temperatures however the Universe is filled with a plasma and one expects that it is important to consider the effects of this plasma on the propagation of the wall. There have been a number of studies of the kinematics of the process.\(^32,33\) We will assume in this analysis that a steady state is reached after a sufficient time such that the wall propagates with a constant terminal velocity. In this case we can boost to the wall frame, in which the plasma on either side of the wall is characterized by a mean velocity $v_{\text{in(out)}}$ and temperature $T_{\text{in(out)}}$.\(^*\) Applying conservation of energy-momentum to the plasma far on either side of the wall, one obtains two equations relating the parameters above (from $T^{zz}$ and $T^{0z}$). The result of this analysis is that essentially two different types of propagation are possible: detonations, characterized by $v_{\text{in}} < v_{\text{out}}$ and $T_{\text{in}} > T_{\text{out}}$, and deflagrations characterized by $v_{\text{in}} > v_{\text{out}}$ and $T_{\text{in}} < T_{\text{out}}$.

In order to fully determine the velocities and temperatures, one needs additional dynamical information. One attains this by studying the interaction of the plasma with the wall. In the present case, the interaction is provided by the increase in mass of the various particles as they traverse the wall. It is clear that the most important particles here are the heaviest, namely the top quark and the

\(^\dagger\) It may be possible to construct extended models for which this statement is relaxed.\(^30\)

\(^*\) The label ‘in’ refers to the broken phase, and ‘out’ to the symmetric phase.
$\long W^\pm$ and $Z^0$. The lighter degrees of freedom essentially pass through the wall untouched; however these degrees of freedom constitute approximately 80\% of the plasma and are important in thermalizing the distributions of heavy particles. Because of the preponderance of light species, it is natural to suppose that the changes in temperature and velocity across the wall are small. This assumption allows us to write a simple equation for the mean velocity of the wall. The smallness of the variations in velocity and temperature will be verified \textit{a posteriori}. With these assumptions, the wall velocity is computed by summing up all of the forces on the wall and then solving for the velocity. In our discussion, we will assume that the velocity is non-relativistic\footnote{This assumption is not operationally crucial, but is merely convenient.} and expand the force in small velocity. By looking at energy-momentum conservation in the wall frame, we can write

$$\partial_\mu T_\phi^{\mu z} = \sum_i F_z^{(i)}$$  \hfill (2.1)$$

where the left hand side contains the (zero temperature) energy-momentum tensor of the scalar field, and the right hand side is the sum of forces from the $i^{th}$ species. At zero velocity, as we will see below, the right hand side is just the difference in pressure of the plasmas on either side of the wall, and thus the velocity independent part of the force is just the difference in the effective potential on either side of the wall.

There have been a number of attempts in the past to compute the wall velocity. In Ref. 34 a simple formula for the wall velocity was given, based on a semiclassical picture in which a species of particle gains a large mass $M \gg T$ as it passes through the wall. Balancing the force on the wall due to these particles with the pressure difference between the two phases gives a relation of the form

$$v = \frac{\Delta p}{\Delta \rho}.$$  \hfill (2.2)$$

where $p$ is the pressure and $\rho$ is the internal energy.

More recently, Turok has argued\textsuperscript{35} that this type of analysis is incorrect. He suggests that reflection of particles from the wall does not slow the wall at all. The crucial step in Turok’s original analysis is the assumption that equilibrium distributions appropriate to the value of the mass are maintained locally. Under
this assumption, the force on the wall can be written

\[ F_z = \int dz \int \frac{d^3k}{(2\pi)^3} n(\vec{k}, \vec{x}) v_z \frac{\Delta p_z}{\Delta z} \]

\[ = \int dz \frac{\partial m^2}{\partial z} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2k_o} n(\vec{k}) \]

\[ = \text{velocity} - \text{independent}. \quad (2.3) \]

The distribution \( n \) is taken to be the boosted Bose-Einstein (or Fermi-Dirac) distribution. The second line follows by writing the change in momentum in terms of the change in mass as the particle traverses the wall (using energy conservation). The last line follows since the integral over momentum involves Lorentz-invariant quantities only, and thus the velocity dependence may be removed by a change of variables. With this analysis one concludes that the wall accelerates relentlessly, at least until some other effect takes over. This analysis is indeed correct; it is the assumption of equilibrium that is physically unmotivated. We will see shortly that the most important effects arise because of small, velocity-dependent departures from equilibrium.

Before going on to the more realistic case hinted at in the last paragraph, we will improve on the analysis of Ref. 34; in the course of doing this we will identify some of the shortcomings of this analysis.

2. 1. Thin Wall Analysis

We will now make the following assumptions: we suppose that the individual particles of the plasma, while interacting with the wall, do not interact among themselves. We thus may follow individual particles as they traverse the wall and compute the force accordingly. This assumption obviously corresponds to assuming that any relevant mean free path is long compared with a scale set by the size of the wall. This, it will turn out, is an implicit assumption in the estimate of Ref. 34. However, we can improve on that analysis by noting that in reality there are no particles of relevance with mass much larger than the temperature. Depending on the momentum, a certain fraction will be reflected from the wall, but a significant fraction is also able to pass through the wall in either direction. If we then make the simple assumption that the incoming particles have a roughly thermal distribution, we can compute the total force on the wall by carefully adding up the contributions from each type of event noted above. The details of this calculation are given in Ref. 2, and we will simply quote the result here. If we write the force as

\[ 0 = \Delta V_T + v \sum_i \mathcal{E}_i + O(v^2) \]
we obtain, in the thin wall case,

\[ E_i = \rho(o, T) - \rho(m_i, T) - \frac{m_i^2}{4\pi^2} \int_{m_i}^{\infty} E \, n_o(E) dE. \]  

(2.4)

We note that in the limit \( m \to \infty \), the result of Ref. 34 is recovered. If we expand this expression in powers of \( m/T \), we find the following result. For top quarks, we have

\[ E_{f \text{ thin}} \simeq -\frac{3\phi^4}{16\pi^2 v_o^4} m_t^4 \left[ \ln \frac{m_t^2 \phi^2}{a_F v_o^2 T^2} - \frac{7}{2} \right] + O \left( \left( \frac{m}{T} \right)^5 \right) \]  

(2.5)

and for \( W \) and \( Z \) bosons, we have

\[ E_{b \text{ thin}} \simeq \frac{3}{\pi} E T \phi^3 + \frac{3\phi^4}{64\pi^2 v_o^4} \left[ 2m_W^4 \left( \ln \frac{m_W^2 \phi^2}{a_B v_o^2 T^2} - \frac{7}{2} \right) + m_Z^4 \left( \ln \frac{m_Z^2 \phi^2}{a_B v_o^2 T^2} - \frac{7}{2} \right) \right]. \]  

(2.6)

If we ignore the top quark, the velocity turns out to be a pure number times a simple function of \( \epsilon \),

\[ v \simeq \frac{\pi}{6} \frac{\epsilon}{1 + \epsilon} \quad (\sim 0.1 \text{ for } m_H = 35 \text{ GeV}) \]

valid for small \( \epsilon \). Putting back in the top quark, we find that the velocity, in this approximation, is diminished. For \( m_H = 35 \text{ GeV} \) and \( m_t = 120 \text{ GeV} \), one finds \( v \sim 0.07 \).

We may now go back and check the validity of our assumptions. First, the working assumption of non-relativistic velocities clearly holds. Secondly, if we go back to the kinematics and compute the variations in velocity and temperature consistent with energy-momentum conservation we find

\[ \frac{\delta v}{v} \sim +10^{-2} \]
\[ \frac{\delta T}{T} \sim -10^{-4}. \]

This result is clearly consistent with our assumption of slowly varying temperature and velocity. Furthermore, the signs of these quantities are such that we identify the process as a (weak) deflagration.

* For the sake of this discussion, we are assuming that the expansion is valid for top quarks.
2. 2. Parameters of Interest

In order to proceed, we now investigate various distance scales relevant at this phase transition. To get an idea of the thickness of the wall, we note that at $T_c$ there is an exact kink solution known of the form $\phi = \phi_c / [2(1 + \tanh(2z/\delta))]$ and $\delta$ characterizes the wall thickness. In our case, we find

$$\delta \approx \frac{2\sqrt{2\lambda}}{E} T^{-1} \sim 40 T^{-1}.$$  

The value quoted is for $m_H = 35$ GeV. From numerical calculations, this width is accurate at the actual nucleation temperature as well, and thus we see that the wall is rather thick.†

It is useful to compare this number with the mean free paths for various processes. In considering the properties of the bubble wall, the relevant mean free paths are those for particles which interact with the wall, i.e., principally top quarks, $W$'s and $Z$'s. The processes with the shortest mean free paths are elastic scatterings. These exhibit the characteristic singularities of Coulomb scattering at small angles. What actually interests us, however, is the momentum and energy transfer in these collisions. This is a problem which has been extensively studied, and we can borrow the relevant results. One finds that the momentum loss per unit length due to scattering is in all cases much larger than that due to the wall. This result may be understood in an alternative way. The elastic scattering cross section diverges at small angles in empty space. In the plasma, we expect that this divergence is cut off, essentially by a temperature factor. Examining the expression for the elastic scattering cross section, one obtains an estimate for the mean free paths of order $\ell \sim 4 T^{-1}$ for quarks, and $\ell \sim 12 T^{-1}$ for $W$’s and $Z$’s. We will use these estimates in what follows but it should be kept in mind that a more complete analysis of the mean free paths is desirable.

From the above discussion we see that the mean free paths for thermalization processes are typically a significant fraction of the wall thickness.‡ We thus expect that a more realistic analysis of the bubble wall velocity lies somewhere in between the thin-wall analysis given in the previous section and the equilibrium analysis of Ref. 35. There have been two attempts at this in the literature. The first builds on the thin-wall analysis, whereas the second perturbs around the equilibrium.

† We make the assumption that the thickness of the wall remains constant throughout the expansion.

‡ Also of interest are mean free paths for processes that change some approximately conserved quantum number. It has been speculated that a “snowplow” effect may occur in front of the wall which may further act to decrease the wall velocity. Estimates given elsewhere indicate that this effect is at most of approximately the same strength as the effects discussed herein.
picture. Both approaches show qualitative agreement that the wall velocity is non-relativistic. We will briefly discuss these two approaches in the following two sections.

2.3. DLHLL Thick Wall Analysis

To get an idea of how finite elastic scattering lengths affect the velocity of the bubble wall, we assume that particles propagate freely over distances of order a mean free path, \( \ell \). As shown in the previous section this is typically shorter than the thickness of the wall and so we view the bubble wall as a succession of slices with thickness of order \( \ell \), and for each of these we repeat the thin wall analysis. Thus we make the simple assumption that the distributions are given roughly by equilibrium distributions appropriate to a given point within the wall, and then follow that distribution over a length of order \( \ell \). On general grounds, one expects that the result may be written in the following form:

\[
E = S_b E_{\text{thin}}^b + S_f E_{\text{thin}}^f,
\]

where \( S \) are suppression factors dependent on \( \ell \). One expects them to have several limiting behaviours. When \( \ell/\delta \to 0 \) they should approach zero, reproducing the equilibrium analysis in which the velocity dependence goes away. When \( \delta < \ell \), the thin wall analysis is recovered. Numerically, one finds that the suppression factors are not very sensitive to \( m_t \) and \( m_H \), and are well fit by

\[
S \simeq \frac{\sqrt{2}(\ell/\delta)}{\sqrt{1 + (\ell/\delta)^2}}.
\]

Using the values quoted above for the mean free paths, the equations for \( E \) from the previous section and assuming \( m_H \sim 35 \text{ GeV} \) \((\delta \sim 40 T^{-1})\) and \( m_t \sim 120 \) GeV, we find a velocity of about \( v \sim 0.2 \). Fig. 6 illustrates the velocity for a range of top masses, for two values of the Higgs boson mass. Also included on this plot for comparison is the thin wall result for these Higgs boson masses.

2.4. LMT Thick Wall Analysis

An alternative analysis of the thick wall velocity has recently been performed in Ref. 36. These authors study linear perturbations of the distribution function \( n \), determined by the relativistic Boltzmann equation

\[
p_z \partial_z n + m \gamma F_z \partial_{k_z} n = C(n).
\]  

(2.7)

Here the force term is given by the change in mass of the particles in the wall \( F_z = -1/2k_o \partial_z m^2 \).
Let us now make the relaxation time approximation, whereby the collision term is replaced by $C = \delta n/\ell$ where $\ell$ is again the relevant mean free path. One can then solve for $\delta n = n - n_o$ in terms of the equilibrium distribution function and the velocity of the wall. Balancing the forces on the wall one obtains the result of Turok, with an additional velocity dependent term

$$\Delta V_T = \gamma v \frac{\ell E}{\delta \pi} T \phi^3.$$ 

Solving for the velocity, one obtains

$$\gamma v \sim c \frac{\pi}{6} \frac{\delta}{\ell} \frac{\epsilon}{1 + \epsilon},$$

where $c$ is of order unity, a result which obviously agrees very well with our result for very small $\ell/\delta$.

The authors of Ref. 36 however perform a more detailed analysis, whereby they solve Eq. (2.7) directly without making the relaxation time approximation. The result of this analysis differs from the above in some respects, but within the range of parameters of interest agrees qualitatively with the above result. One must conclude from these analyses that the simple analysis of Ref. 2 is equivalent to the relaxation time approximation, but extends the analysis beyond very small $\ell/\delta$. The conclusion in all cases is that the velocity of the bubble walls is non-relativistic.

3. Conclusions and Acknowledgments

In this talk, we have discussed the recently improved understanding of the effective potential describing the electroweak phase transition of the minimal standard model, and outlined several ways of understanding the resummation necessary to improve perturbation theory. We have also discussed recent calculations of bubble wall properties, understood how they relate to one another, and noted that all calculations are in qualitative agreement that the wall velocity is non-relativistic.

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**FIGURE CAPTIONS**

1) Infrared divergent ring diagrams.
2) Schwinger-Dyson equation.
3) Tadpole graphs through two-loop order in Schwinger-Dyson approach.
4) One-loop diagram in infrared effective theory, showing expansion in terms of ring diagrams.
5) Typical electroweak potential showing hypothetical region of validity of expansion.
6) Plot of velocity versus $m_t$ for several values of $m_H$ for thick and thin wall approximations.