Quark-hadron crossover equations of state for neutron stars: constraining the chiral invariant mass in a parity doublet model

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We construct an equation of state (EOS) for neutron stars by interpolating hadronic EOS at low density and quark EOS at high density. A hadronic model based on the parity doublet structure is used for hadronic matter and a quark model of Nambu–Jona-Lasinio type is for quark matter. We assume crossover between hadronic matter and quark matter in the the color-flavor locked phase. The nucleon mass of the parity doublet model has a mass associated with the chiral symmetry breaking, and a chiral invariant mass $m_0$ which is insensitive to the chiral condensate. The value of $m_0$ affects the nuclear EOSs at low density, and has strong correlations with the radii of neutron stars. Using the constraint to the radius obtained by LIGO-Virgo and NICER, we find that $m_0$ is restricted as $700\text{ MeV} \lesssim m_0 \lesssim 900\text{ MeV}$.

I. INTRODUCTION

Chiral symmetry and its spontaneous breaking is one of the most important properties in low-energy hadron physics. The spontaneous breaking is triggered by the condensate of quarks and anti-quarks, which generates a part of hadron masses and mass difference between chiral partners.

In case of nucleon, Ref. [1] introduced a notion of the chiral invariant mass in addition to the mass from the spontaneous chiral symmetry breaking using a model based on the parity doublet structure. By regarding $N^*(1535)$ as the chiral partner to the ordinary nucleon and using the decay width, the chiral invariant mass is shown to be smaller than 500 MeV [2]. On the other hand, analysis of nucleon mass at high temperature by lattice simulation [3] suggests a large value of the chiral invariant mass.

There are many works to construct nuclear matter and neutron star (NS) EOS using hadronic models based on the parity doublet structure (see, e.g., Refs. [1–23].) Typical models are $\sigma$–$\omega$ type mean field models which a nucleon acquires the mass from the $\sigma$ condensate, while in parity doublet models (PDMs) nucleons are less sensitive to the details of $\sigma$ due to the presence of the chiral invariant mass.

There have been several refinements in the PDM to account for the nucleon as well as nuclear matter properties. The authors in Refs. [23] and [31] revisited the estimate of the decay width, and found that inclusion of the derivative interactions, not included in Ref. [2], allows larger values of $m_0$, and discussed that relatively large values, $500\text{ MeV} \leq m_0 \leq 900\text{ MeV}$, are more reasonable to explain the saturation properties in nuclear matter. In particular, Ref. [24] showed that inclusion of a $\sigma^6$ term reproduces the incompressibility of the empirical value $K \sim 240\text{ MeV}$, which was much larger in previous analyses. In Ref. [24], the analyses were further extended to NS matter, the chiral invariant mass is restricted to be $m_0 \gtrsim 600\text{ MeV}$ by the tidal deformability estimated from the NS merger GW170817 [32–34].

The previous study in Ref. [24] based on the PDM extrapolates the hadronic equations state to the baryon density $n_B \sim 3n_0$ ($n_0 \approx 0.16\text{ fm}^{-3}$; nuclear saturation density). However, as emphasized in Refs. [35–38], the validity of pure hadronic descriptions at $n_B \gtrsim 2n_0$ are questionable as nuclear many-body forces are very important, and this would imply that we need quark descriptions even before the quark matter formation. In this context it was proposed to construct EOS by interpolating EOS for hadronic matter at $n_B \lesssim 2n_0$ and the one for quark matter in the high-density region, $n_B \gtrsim 5n_0$. For describing the quark matter, the authors adopted a three flavor Nambu–Jona-Lasinio (NJL)-type model which leads to the color-flavor locked (CFL) color-superconducting matter, and examined effective interactions to satisfy the two-solar-mass ($2M_\odot$) constraint. The hadronic EOSs were based on non-relativistic nuclear many-body calculations. In Refs. [21, 24, 25], on the other hand, they construct an effective model combining a PDM and an NJL-type model with two flavors assuming no color-superconductivity.

In the present analysis, we construct EOS for NSs by interpolating the EOS constructed from the PDM proposed in Ref. [23], and the one from the NJL-type model in Refs. [37, 38]. Through such a construction, we will examine the properties of the PDM, especially the chiral invariant mass. Although the nuclear and quark EOS cover different density domains, in fact they constrain each other as the interpolation of these EOS must satisfy the thermodynamic stability and causality constraints. Our unified EOSs are subject to the following NS constraints: the radius constraint obtained from
the NS merger GW170817 [22, 34], the millisecond pulsar PSR J0030+0451 [39, 40], and the maximum mass constraint obtained from the millisecond pulsar PSR J0740+6620 [41].

In present analyses, the most notable correlations are found between the chiral invariant mass and the radius constraints. In the PDM, for a given $m_0$ we arranged the rest of parameters to fit the nuclear saturation properties, but the density dependence of different sets of parameters can be very different. In particular the choice of $m_0$ affects the balance between the attractive $\sigma$ and repulsive $\omega$ interactions with nucleons; with smaller $m_0$, we need a larger scalar coupling to account for the nucleon mass, while in turn demands a larger $\omega$ coupling for the saturation properties. As the density increases with the chiral restoration, the $\omega$ contributions become dominant, and EOSs for $n_0 \lesssim n_B \lesssim 3n_0$ become stiffer. Too stiff low density EOSs lead to too large NS radii that would contradict with the currently available upperbound. Based on this observation we will find the lowerbound for $m_0$. Meanwhile too large $m_0$ is not allowed by the nucleon mass, the lowerbound of NS radii, and the $2M_0$ constraint.

This paper is organized as follows: In section II we explain the formulation of the present analysis. Main results of the analysis are shown in section III. In section IV we show a summary and discussions.

II. FORMULATION

In this section, we explain our model to determine the EOS for NSs. In the low-density region, we use the parity doublet model to describe the hadronic matter. We use the hidden local symmetry (HLS) [22, 43] to introduce massive vector mesons with chiral symmetry. There are some equivalent method to the HLS [33].

In the high-density region, on the other hand, we follow Refs. [37, 38] and an NJL-type model with additional vector and diquark pairing interactions. We interpolate the resultant hadronic and quark matter EOS assuming a smooth transition between them.

A. Parity Doublet Model

Here we briefly review an effective hadronic model based on the parity doublet structure for nucleons [2, 23].

In our model the excited nucleon $N^*(1535)$ is regarded as a chiral partner to the ordinary nucleon $N(939)$. For expressing these nucleons, we introduce two baryon fields $\psi_1$ and $\psi_2$ which transform under the chiral symmetry as

$$\begin{align*}
\psi_L^L & \rightarrow g_L \psi_L^L, \quad \psi_L^R \rightarrow g_R \psi_L^R, \\
\psi_R^L & \rightarrow g_R \psi_R^L, \quad \psi_R^R \rightarrow g_L \psi_R^R,
\end{align*}$$

where $g_L$ and $g_R$ are the elements of SU(2)$_L$ and SU(2)$_R$ groups, respectively. Two baryon fields $\psi_i^{L,R}$ ($i = 1, 2$) are defined as

$$\begin{align*}
\psi_L^i = \frac{1}{2} (1 - \gamma_5) \psi_i, \quad \psi_R^i = \frac{1}{2} (1 + \gamma_5) \psi_i.
\end{align*}$$

We assign positive parity for $\psi_1$ and negative parity for $\psi_2$:

$$\begin{align*}
\psi_1 & \rightarrow p \gamma_0 \psi_1, \quad \psi_2 \rightarrow -p \gamma_0 \psi_2.
\end{align*}$$

The iso-singlet scalar meson $\sigma$ and the iso-triplet pions are introduced through a $2 \times 2$ matrix field $M$ which transforms as

$$M \rightarrow g_L M g_R^\dagger.$$
hadronic fields. After using the correspondence to constrain the form of the effective Lagrangian, we set the values of the external fields as

\[
\bar{\psi}_\mu = \frac{1}{2} \begin{pmatrix} \mu_Q & 0 \\ 0 & -\mu_Q \end{pmatrix} \delta^0_{\mu}. \tag{10}\]

Our effective Lagrangian for hadrons consists of a nucleon part and a meson part,

\[
\mathcal{L}_{\text{PDM}} = \mathcal{L}_N + \mathcal{L}_M. \tag{11}\]

The nucleon part is given by

\[
\mathcal{L}_N = \sum_{i=1,2} \bar{\psi}_i \gamma^\mu D_\mu \psi_i \\
- g_1 \left( \bar{\psi}_L^i M \psi_R^i + \bar{\psi}_R^i M^\dagger \psi_L^i \right) \\
- g_2 \left( \bar{\psi}_L^i M \psi_L^i + \bar{\psi}_R^i M \psi_R^i \right) \\
- m_0 \left( \bar{\psi}_L^i M - \bar{\psi}_R^i M \right) - \bar{\psi}_L^i \left( \bar{\psi}_L^i M + \bar{\psi}_R^i M \right) + a_{VN N} \left[ \bar{\psi}_L^i \xi_{\gamma}^\mu \hat{\alpha}^\mu \psi_L^i + \bar{\psi}_R^i \xi_{\gamma}^\mu \hat{\alpha}^\mu \xi_R^i \psi_R^i \right] \\
+ a_{VN N} \left[ \bar{\psi}_L^i \xi_{\gamma}^\mu \hat{\alpha}^\mu \xi_L^i \psi_L^i + \bar{\psi}_R^i \xi_{\gamma}^\mu \hat{\alpha}^\mu \xi_R^i \psi_R^i \right] \\
+ a_{0NN} \sum_{i=1,2} \bar{\psi}_i \gamma^\mu \left( \hat{\alpha}^\mu \right) \bar{\psi}_i \gamma^\mu \left( \hat{\alpha}^\mu \right) \psi_i, \tag{12}\]

where the covariant derivatives on the nucleon fields are defined as

\[
D_\mu \psi_{1,2}^i = (\partial_\mu - iV_\mu) \psi_{1,2}^i, \tag{13}\]

with

\[
V_\mu = \begin{pmatrix} \mu_B + \mu_Q & 0 \\ 0 & \mu_B \end{pmatrix} \delta^0_{\mu}. \tag{14}\]

The meson part is given by

\[
\mathcal{L}_M = \mathcal{L}_{\text{kin}}^M - V_M - V_{\text{SB}} + \mathcal{L}_{\text{vector}}^M, \tag{15}\]

where \(\mathcal{L}_{\text{kin}}^M\), \(V_M\) and \(V_{\text{SB}}\), are the kinetic term, the chiral symmetric potential and the potential including the explicit chiral symmetry breaking for the scalar and pseudo-scalar mesons, respectively, and \(\mathcal{L}_{\text{vector}}^M\) includes the kinetic and mass terms for vector mesons. The kinetic and potential terms for the scalar and pseudo-scalar mesons are expressed as \(\mathcal{L}_{\text{kin}}^S\) and \(\mathcal{L}_{\text{kin}}^P\) as

\[
\mathcal{L}_{\text{kin}}^S = \frac{1}{4} \text{tr} \left[ D_\mu M D^\mu M \right] = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} \partial_\mu \hat{\alpha}_\mu \partial_\mu \hat{\alpha}_\mu, \tag{16}\]

\[
V_M = - \frac{1}{4} \bar{\psi}_\mu \psi^\mu \left( MM^\dagger \right) + \frac{1}{16} \lambda_4 \left( \text{tr} \left[ MM^\dagger \right] \right)^2 \\
- \lambda_6 \frac{1}{48} \left( \text{tr} \left[ MM^\dagger \right] \right)^3, \tag{17}\]

\[
V_{\text{SB}} = - \frac{1}{4} m^2_\pi f_\pi \text{tr} \left[ M + M^\dagger \right]. \tag{18}\]

The vector mesons part \(\mathcal{L}_{\text{vector}}^V\) is given by

\[
\mathcal{L}_{\text{vector}}^V = - \frac{1}{4} \omega_{\mu \nu} \omega^{\mu \nu} + \frac{m_\omega^2}{2g_\omega^2} \text{tr} \left[ \hat{\alpha}_\mu \right] \text{tr} \left[ \hat{\alpha}_\mu \right] \\
- \frac{1}{2} \text{tr} \left[ \rho_\mu \rho^\mu \right] \\
+ \frac{m_\rho^2}{g_\rho^2} \left( \text{tr} \left[ \hat{\alpha}_\mu \hat{\alpha}_\mu \right] - \frac{1}{2} \text{tr} \left[ \hat{\alpha}_\mu \right] \text{tr} \left[ \hat{\alpha}_\mu \right] \right), \tag{19}\]

where \(m_\omega\) and \(m_\rho\) are the masses of \(\omega\) and \(\rho\) mesons, and \(\omega_{\mu \nu}\) and \(\rho_{\mu \nu}\) are the field strengths of \(\omega^{\mu \nu}\) and \(\rho^{\mu \nu}\) respectively. The second and fourth terms include the mass terms for \(\omega^{\mu \nu}\) and \(\rho^{\mu \nu}\) as

\[
\text{tr} \left[ \hat{\alpha}_\mu \right] \text{tr} \left[ \hat{\alpha}_\mu \right] = g_\omega^2 \omega^{\mu \nu} \omega_{\mu \nu}, \\
\text{tr} \left[ \hat{\alpha}_\mu \hat{\alpha}_\mu \right] - \frac{1}{2} \text{tr} \left[ \hat{\alpha}_\mu \right] \text{tr} \left[ \hat{\alpha}_\mu \right] = \frac{1}{2} g_\rho^2 \rho^{\mu \nu} \rho_{\mu \nu} + \cdots, \tag{20}\]

where “\(\cdots\)” stands for interaction terms.

In the present analysis, we calculate the thermodynamic potential in the mean field approximation as

\[
\langle \sigma \rangle = \sigma, \quad \langle \omega^{\mu \nu} \rangle = \omega^{\mu \nu}_0, \quad \langle \rho^{\mu \nu} \rangle = \left( \rho - \frac{\mu_\rho}{g_\rho} \right) T_j \delta^{\mu \nu}_0. \tag{21}\]

Each mean field is assumed to be independent of the spatial coordinates. Mean field \(\rho\) is defined in such a way that \(\mathcal{L}_M\) does not explicitly include \(\mu_Q\).

It is convenient to introduce the effective chemical potentials of protons and neutrons as

\[
\mu_p^* = \mu_p + \mu_B - g_{\omega NN} \omega - \frac{1}{2} g_{\rho NN} \rho, \\
\mu_n^* = \mu_B - g_{\omega NN} \omega + \frac{1}{2} g_{\rho NN} \rho, \tag{22}\]

where

\[
g_{\omega NN} = (a_{VN N} + a_{0NN N}) g_\omega, \\
g_{\rho NN} = a_{VN N} g_\rho. \tag{23}\]

The thermodynamic potential in the hadronic matter is calculated as

\[
\Omega_{\text{PDM}} = V(\sigma) - V(f_\pi) - \frac{1}{2} m_\omega^2 \omega^2 - \frac{1}{2} m_\rho^2 \rho^2 \\
- 2 \sum_{i=1,2} \sum_{\alpha=p,n} \int \frac{d^3p}{(2\pi)^3} \left( \mu_\alpha^* - E_p^\alpha \right), \tag{24}\]

where \(i = 1\) labels the ordinary nucleon \(N(939)\) and \(i = 2\) the excited nucleon \(N^*(1535)\). \(E_p^\alpha = \sqrt{p^2 + m_\alpha^2}\) is the energy of relevant particle with mass \(m_\alpha^*\) and momentum \(p\).

In the integration above, the integrals region is restricted as \(|p| < k_F\) where \(k_F = \sqrt{(\mu_p^*)^2 - m_p^2}\) is the fermi momentum for the relevant particle. We notice that we use the so called no sea approximation, assuming that the
structure of the Dirac sea remains the same for the vacuum and medium. \( V(\sigma) \) is the potential of \( \sigma \) mean field,
\[
V(\sigma) = -\frac{1}{2} \mu^2 \sigma^2 + \frac{1}{4} \lambda_4 \sigma^4 - \frac{1}{6} \lambda_6 \sigma^6 - m^2 f_x \sigma .
\]
(25)
In Eq. (24) we subtracted the potential in vacuum \( V(f_x) \), with which the total potential in vacuum is zero.

The total thermodynamic potential of the hadronic matter in NSs is obtained by including the effects of leptons as
\[
\Omega_H = \Omega_{\text{PDM}} + \sum_{l=e,\mu} \Omega_l ,
\]
(26)
where \( \Omega_l \) (\( l = e, \mu \)) are the thermodynamic potentials for leptons given by
\[
\Omega_l = -2 \int^{k_F} d^3p (\mu_l - E^l_p) .
\]
(27)

Here, the mean fields are determined by the following stationary conditions:
\[
0 = \frac{\partial \Omega_H}{\partial \sigma} , \quad 0 = \frac{\partial \Omega_H}{\partial \omega} , \quad 0 = \frac{\partial \Omega_H}{\partial \rho} .
\]
(28)
In NSs, we impose the beta equilibrium and the charge neutrality condition represented as
\[
\mu_e = \mu_\mu = -\mu_Q ,
\]
(29)
\[
\frac{\partial \Omega_H}{\partial \mu_Q} = n_p - n_l = 0 .
\]
(30)
Finally, we obtain the pressure in the hadronic matter as
\[
P_H = -\Omega_H .
\]
(31)

In the present analysis, following Ref. [23], we determine the model parameters from the following physical inputs for fixed values of the chiral invariant mass \( m_0 \): five masses of the relevant hadrons and the pion decay constant in vacuum as listed in Table I; saturation properties of nuclear matter at the saturation density as in Table II. We show the values of model parameters for several typical choices of \( m_0 \). The values of the slope parameter is also shown as output.

for smaller chiral invariant mass, which leads to stronger attractive force mediated by \( \sigma \) contribution. The \( \omega \) contribution causing the repulsive force is also larger to satisfy the saturation properties at saturation density. This \( \omega \) contribution becomes larger in the high density region, while the \( \sigma \) contribution becomes smaller. The resulting large repulsive force makes the EOS stiff.

### B. Color-Superconductivity

Following Ref. [23], we use an NJL-type effective model of quarks including the 4-Fermi interactions which cause the spontaneous chiral symmetry breaking and the color-superconductivity. The Lagrangian is given by
\[
\mathcal{L}_{\text{CSC}} = \mathcal{L}_0 + \mathcal{L}_\sigma + \mathcal{L}_d + \mathcal{L}_{\text{KMT}} + \mathcal{L}_{\text{vec}} ,
\]
(32)
where
\[
\mathcal{L}_0 = \bar{q} i \gamma^\mu \partial_\mu - m_q + \gamma_5 \hat{A}_\mu q ,
\]
(33)
\[
\mathcal{L}_\sigma = G \sum_{A=0}^8 \left[ (\bar{q} \tau_A q)^2 + (\bar{q} i \gamma_5 \tau_A q)^2 \right] ,
\]
(34)
\[
\mathcal{L}_d = H \sum_{A,B=2.5,7} \left[ (\bar{q} \tau_A B \bar{C} \tau_B q)(q^i C \tau_A \lambda_B q) + (\bar{q} i \gamma_5 \tau_A B \bar{C} \tau_B q)(q^i C i \gamma_5 \tau_A \lambda_B q) \right] ,
\]
(35)
\[
\mathcal{L}_{\text{KMT}} = -K \left[ \det \bar{q}(1 - \gamma_5)q + \det \bar{q}(1 + \gamma_5)q \right] ,
\]
(36)
\[
\mathcal{L}_{\text{vec}} = -g_V (\bar{q} i \gamma_5 q)(\bar{q} \gamma_\mu q) ,
\]
(37)
and $\hat{A}^\mu$ is the external field. The chemical potentials are introduced in the same way as the hadronic case by

$$\hat{A}^\mu = (\mu_q + \mu_3\lambda_3 + \mu_8\lambda_8 + \mu_Q Q) \delta_0^\mu,$$  

where $\lambda_a$ are Gell-Mann matrices in color space and $Q = \text{diag}(2/3, -1/3, -1/3)$ is a charge matrix in flavor space. We introduce the mean fields as

$$\sigma_f = \langle \hat{q}^\dagger f q_f \rangle, \quad (f = u, d, s),$$  
$$d_j = \langle \hat{q}^\dagger C \gamma \gamma_j R_q q \rangle, \quad (j = 1, 2, 3),$$  
$$n_q = \sum_{f=u,d,s} \langle \hat{q}^\dagger f q_f \rangle,$$

where $(R_1, R_2, R_3) = (\tau_7 \lambda_7, \tau_5 \lambda_5, \tau_2 \lambda_2)$. Then, the thermodynamic potential is calculated as

$$\Omega_{\text{CSC}} = \Omega_s - \Omega_s [\sigma_f = \sigma_f^0, d_j = 0, \mu_q = 0] + \Omega_c - \Omega_c [\sigma_f = \sigma_f^0, d_j = 0],$$

where

$$\Omega_s = -2 \sum_{i=1}^{18} \int_{-\Lambda}^\Lambda d^3p \frac{\varepsilon_i}{(2\pi)^3},$$
$$\Omega_c = \sum_j (2G\delta_i^2 + e_i) - 4K\sigma_a \sigma_d - 4\varepsilon_i n_i.$$

In Eq. (43), $\varepsilon_i$ are energy eigenvalues obtained from the following inverse propagator in Nambu-Gorkov basis

$$S^{-1}(k) = \left( \gamma_\mu k^\mu - M + \gamma^0 \hat{\mu} + \gamma_5 \sum_i \Delta_i R_i - \gamma_5 \sum_i \Delta_i R_i, \gamma_\mu k^\mu - M - \gamma^0 \hat{\mu} \right),$$

where

$$M_i = m_i - 4G\sigma_i + K \epsilon_{ijk} \sigma_j \sigma_k,$$  
$$\Delta_i = -2H d_i,$$  
$$\hat{\mu} = \mu_q - 2g_\nu s_q + \mu_3\lambda_3 + \mu_8\lambda_8 + \mu_Q Q.$$

$S^{-1}(k)$ in Eq. (45) is $72 \times 72$ matrix in terms of the color, flavor, spin and Nambu-Gorkov basis, which has $72$ eigenvalues. Note that the matrix does not depend on the spin, and that the charge conjugation invariance relates two eigenvalues. Then, there are $18$ independent eigenvalues at most.

The total thermodynamical potential is

$$\Omega_Q = \Omega_{\text{CSC}} + \sum_{i=\mu} \Omega_i,$$

where $\Omega_i$ is the thermodynamic potential for leptons given in Eq. (27). The chiral condensates $\sigma_j$ and the diquark condensates $d_j$ are determined from the gap equations,

$$0 = \frac{\partial \Omega_Q}{\partial \sigma_i} = \frac{\partial \Omega_Q}{\partial d_i}.$$

To determine the relevant chemical potentials other than the baryon number density, we use the beta equilibrium condition given in Eq. (29), and the conditions for electromagnetic charge neutrality and color charge neutrality expressed as

$$n_j = -\frac{\partial \Omega_Q}{\partial \mu_j} = 0,$$

where $j = 3, 8, Q$. The baryon number density $n_B$ is three times of quark number density determined as

$$n_q = -\frac{\partial \Omega_Q}{\partial \mu_q},$$

where $\mu_q$ is $1/3$ of the baryon number chemical potential. Substituting the above conditions, we obtain the pressure of the system as

$$P_Q = -\Omega_Q.$$

C. Interpolation of EOS

Here, we consider interpolation of two EOSs for hadronic matter and quark matter which are constructed in previous subsections. Following Ref. 37, we assume that hadronic matter is realized in the low density region $n_B < 2n_0$, and use the pressure constructed in Eq. (31). In the high density region $n_B > 5n_0$, on the other hand, the pressure given in Eq. (33) of quark matter is used. In the intermediate region $2n_0 < n_B < 5n_0$, we assume that the pressure is expressed by a fifth order polynomial of $\mu_B$

$$P_i(\mu_B) = \sum_{i=0}^{5} C_i \mu_B^i,$$

where $C_i$ are six free parameters to be determined from the following boundary conditions,

$$\left| \frac{d^n P_i}{d\mu_B^n} \right|_{\mu_B = 0} = \left| \frac{d^n P_H}{d\mu_B^n} \right|_{\mu_B = 0},$$
$$\left| \frac{d^n P_i}{d\mu_B^n} \right|_{\mu_B = 5n_0} = \left| \frac{d^n P_Q}{d\mu_B^n} \right|_{\mu_B = 5n_0} (n = 0, 1, 2)$$

where $\mu_{BL}$ is the chemical potential corresponding to $n_B = 2n_0$ and $\mu_{BU}$ to $n_B = 5n_0$.

To determine the parameter, we set $n_B = \frac{dP}{d\mu_B}$ and $\chi_B = \frac{dP}{d\mu_B^2}$ in Fig. 2. We see that, although both plots in Fig. 2(a) and 2(b) in Fig. 1 are smooth, Fig. 2 shows that the parameter set (b) violates causality. In this way, the parameter choice
(H/G, gV/G) = (1.45, 1.2) in quark matter is excluded when m0 = 800 MeV in hadronic matter.

Figure 3 shows allowed combinations of (H, gV) for several choices of m0. In all cases, the allowed values of H and gV have a positive correlation. When m0 is increased, larger H and smaller gV are needed.

III. MASS-RADIUS RELATION

In this section, we calculate mass-radius relations of NSs using the Tolman-Oppenheimer-Volkoff (TOV) equation \[ \frac{dP}{dr} = -G(\varepsilon + P)(m + 4\pi r^3 P) \cdot \frac{1}{r^2 - 2Gmr} \] \[ \frac{dm}{dr} = 4\pi r^2 \varepsilon , \] where G is the Newton constant, r is the distance from the center of an NS, P, m and \( \varepsilon \) are the pressure, mass, and energy density as functions of r:

\[ P = P(r), \quad m = m(r), \quad \varepsilon = \varepsilon(r) . \] (58)

To correctly estimate NS radii, we need to include the crust equations. We use the EOS constructed in Ref. \[ [46] \] in the low density region, \( n_B \leq 0.1 \text{ fm}^{-3} \), and beyond which we use our unified EOS.

Given the central density as an initial value, the corresponding radius R and mass M of NS are obtained. The radius is determined by the condition that the pressure vanishes: \( P(R) = 0 \), and the mass is the value of m at the radius: \( M = m(R) \).

We show the resultant mass-radius relations in Fig. 4 and relation between mass and central density in Fig. 5. Five panels in Figs. 4 and 5 correspond to five typical choices of m0.

In each panel of Figs. 4 and 5, different curves are drawn for different combinations of (H, gV) indicated by

FIG. 1: Pressure \( P(\mu_B) \) of the PDM and the unified equations of state. For the PDM we chose \( m_0 = 800 \text{ MeV} \), and for quark models we used \( (H/G, gV/G) = (1.45, 1.0) \) and \( (1.45, 1.2) \). The thick curves in the unified equations of state are used to mark the pure hadronic and quark parts.

FIG. 2: Squared speed of sound \( c_s^2 \) for \( (H/G, gV/G) = (1.45, 1.0) \) and \( (1.45,1.2) \). Curves are same as in Fig. 1.

FIG. 3: Allowed combinations of \( (H, gV) \) for several choices of \( m_0 \). In all cases, the allowed values of H and gV have a positive correlation. When \( m_0 \) is increased, larger H and smaller gV are needed.
circles in Figs. 3. Thick curves in the low-mass region in Figs. 4 and 5 indicate that central density of the NS is smaller than \(2\rho_0\), and that the NS is made only from hadronic matter. Thick curves in high-mass region, on the other hand, imply that central density is larger than \(5\rho_0\), and that core of the NS includes quark matter. Thin curves show that the core is in the crossover domain.

For each combination of \((H, g_V)\), the maximum mass of a NS is determined, which are indicated by the color in Fig. 3. This shows that a larger \(g_V\) or a smaller \(H\) leads to a larger maximum mass.

In this paper we use the mass of the millisecond pulsar PSR J0740+6620 \([41]\)

\[
M_{\text{TOV}}^{\text{lowest}} = 2.14^{+0.10}_{-0.09} M_\odot ,
\]

as the lowest maximum mass, which is shown by gray-shaded area in Figs. 4 and 5. Each red solid curve in these figures exhibits the mass-radius relation for which maximum mass is larger than the above lowest maximum mass, while the maximum masses for mass-radius relations by blue dashed curves do not exceed the lowest maximum mass. We also show the constraint to the radius obtained from the LIGO-Virgo \([23, 24]\) and the NICER \([39, 40]\) by thin solid lines. These values are summarized in Table IV.

Figure 4(b) shows that for \(m_0 = 600\) MeV, mass-radius relations by red solid curves do not satisfy the radius constraint for NS with \(1.4 M_\odot\), and that for these stars the cores are entirely made from hadronic matter. This implies that \(m_0 = 600\) MeV is excluded by the constraint independently of the parameter choice in quark matter. For \(m_0 = 700\) MeV, 800 MeV and 900 MeV, we can see that the relations except one relation for \(m_0 = 900\) MeV satisfy the radius constraints. We conclude that the chiral invariant mass is constrained as

\[
700 \text{ MeV} \lesssim m_0 \lesssim 900 \text{ MeV} .
\]

We also note that the larger \(m_0\) leads to smaller slope parameter for the symmetry energy, 80.08 \(\lesssim L_0 [\text{MeV}] \lesssim 86.24\), as one can read off from Table. [11]
FIG. 4: Several choices of mass-radius relations for each $m_0$. (See main texts for detail.)

FIG. 5: Several choices of relations between mass and central density for each $m_0$. (See main texts for detail.)

IV. SUMMARY AND DISCUSSIONS

We construct EOS for NS matter by interpolating the EOS obtained in the PDM and the one in the NJL-type model. We obtain constraints to the model parameters from thermodynamic stability, causality and the constraints on $M$-$R$ curves.

Our primary purpose was to examine how neutron star observations constrain a hadronic EOS and the micro-

stiff quark EOS, setting the upperbound $m_0 \lesssim 900$ MeV. This upperbound is close to the total nucleon mass $m_N \sim 939$ MeV, and hence is not as remarkable as the radius constraint.

We would like to note that, as one can see in Fig. 6, the cores of heavy NSs with $M \sim 2M_\odot$ includes quark matter as shown by thick curves in the heavy-mass region. On the other hand, the core of $1.4M_\odot$ NS is in the crossover domain of quark and hadronic matter. As a result, variations in the radii of $1.4M_\odot$ NS are rather small, $\Delta R \lesssim 0.5$ km, in our crossover construction of unified EOS.

In this analysis we assumed crossover between hadronic matter and quark matter. As we see in Fig 3, our result showed that the coupling $H$ needs to be sufficiently large to satisfy the causality for smooth connection, as in Ref. [37]. Such large $H$’s ($\gtrsim 1.4G$) is consistent with the $N$-$\Delta$ splitting [48], and lead to the CFL phase for $n_B \gtrsim 5n_0$.

We note that, the previous studies as in Ref. [83] primarily referred to the constraint $R \lesssim 13$ km from GW170817, but then new NICER results appear, favoring the radii $\sim 13$ km, and hence relaxing the condition on low density EOS. Allowing stiffer hadronic EOS broadens the possibility of the first order phase transitions. In this respect, it is interesting to explicitly implement the first order transition in the interpolated domain, as in Refs. [20, 21], while taking quark and hadronic EOS as boundary conditions.

The predicted values of the slope parameter $L_0$ shown in Table III are larger than some constraints, e.g. the one given in Ref. [37]. This can be adjusted by adding e.g. a term proportional to $\omega^2 \rho^2$ into the hadronic part. Such modification is expected to relax the lower bound of $m_0$ in Eq. (60).

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