Dc SQUID array with nonlinear inductance

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Abstract. We show that voltage response of dc SQUID can be substantially linearized by the introduction of a nonlinear inductance. The inductance tuning allows us to achieve the response linearity as high as 120 dB. Such a nonlinear inductance can be easily formed using Josephson junction inductance. To obtain high dynamic range commensurate to the high response linearity, one can use series array of the nonlinear inductance dc SQUIDs.

1. Introduction
Dc SQUIDs are known and widely used as extremely sensitive devices, but showing only about linear voltage response. In conventional low-frequency SQUID systems, improved linearity and dynamic range are obtained using an external feedback loop with limited bandwidth. For cases where a SQUID-based system requires a very broad bandwidth, linearization via external feedback is not feasible. Generally, the problem solution may be associated with Josephson-junction array structures which could provide increase in dynamic range with number of elements $N$ as well as highly linear voltage response due to special design of the structures [1], [2].

In this paper, we present the modified dc SQUID directly capable of providing highly linear voltage response. As for dynamic range, one can use a serial array of the SQUIDs to elevate it up to the value compared with the response linearity. In fact, the thermal noise voltage $V_F$ across the serial array of $N$ SQUIDs is proportional to square root of $N$, while the voltage response amplitude $V_{\text{max}}(\Phi)$ and the transfer factor $B = \partial V / \partial \Phi$ both are proportional to $N$. It means that dynamic range $D = V_{\text{max}}(\Phi)/V_F$ increases as $N^{1/2}$.

2. Bi-SQUID
We modified conventional dc SQUID (see figure 1a) by the addition of a nonlinear inductive element shunting the linear inductance of the loop coupling RF magnetic flux into the SQUID [3]. The nonlinear inductive element is a Josephson junction that remains in its superconductive state during operation (see figure 1b). This nonlinear element modifies the nonlinear transfer function of the SQUID to produce higher linearity transfer function, thus increasing the utility of the device as a linear amplifier.

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The nonlinear small-signal shunt inductance is the Josephson-junction inductance

\[ L_J = \Phi_0 \left( \frac{2\pi I_{c3} \sqrt{1 - i^2}}{2} \right)^{-1}, \]

where \( i = I_{sh} / I_{c3} \) is the normalized current passing through the shunt junction. The effective loop inductance is the parallel combination of the main inductance \( L \) and the Josephson inductance \( L_J \). The additional junction and main inductance form a single-junction SQUID. In such a way, the modified dc SQUID can be called as bi-SQUID. Figure 2 shows voltage response of both the conventional dc SQUID (dash line) and bi-SQUID (solid line) with normalized inductance \( l = 2\pi I_c / \Phi_0 = 1 \) at bias current \( I_b = 2I_c \) and \( I_{c3} = 1.15 I_c \). This shows a triangular transfer function where the triangle edges are quite straight. Such a device implementation makes use of a Josephson junction as a nonlinear inductive element that can largely cancel the nonlinearity associated with the SQUID transfer function.
Josephson nonlinear reactance that functions in a similar way would have a similar effect on reducing the transfer function nonlinearity of more complex Josephson systems.

![Figure 3](image)

Figure 3. Dependence of the bi-SQUID voltage response linearity on critical current of the shunt junction for several fixed values of normalized loop inductance $l$ at bias current $I_b = 2I_c$.

The linearity dependence on critical current of the shunt junction $I_{c3}$ at different inductances of the SQUID loop is shown in Figure 3. The linearity is calculated using a single-tone sinusoidal flux input (of amplitude $A/A_{\text{max}} = 0.2$, where $A_{\text{max}}$ corresponds to the flux amplitude $\Phi_0/4$), and measuring the total harmonic distortion in dB. This result shows that the linearity is sharply peaked for each value of $l$, but with different optimized values of $I_{c3}$. Very large values of linearity as high as $\sim 120$ dB are achievable. Figure 4 shows how the linearity parameter varies as a function of the signal amplitude, for other parameters fixed. The linearity decreases as the signal approaches the maximum range.

![Figure 4](image)

Figure 4. Dependence of the response linearity on amplitude of output signal.

Figure 5 shows the measured voltage response with the bi-SQUID bias current slightly above the critical current value. The measured transfer function closely coincides with the one calculated using standard Josephson junction simulation routine PSCAN. As for small hysteresis at the flux value close to $\pm \Phi_0/2$, it indicates that effective inductance parameter of a single-junction SQUID
Formula: \( I^* = \alpha \cdot 2\pi L I c_3 / \Phi_0 \) is more than 1 and hence the static phase diagram becomes hysteretic. Factor \( \alpha \sim 1 \) describes some shunting effect caused by parallel two-junction circuit.

**Figure 5.** The measured voltage response of 12 bi-SQUIDs connected in series at \( I_b = 2.1 I_c \) versus current in magnetic control line. The chip was fabricated with HYPRES’ 4.5 kA/cm² process.

### 4. Conclusion

We have proposed so called bi-SQUID in order to achieve a highly linear voltage response with linearity parameter as high as 120 db and even more for significant loop inductance allowing large coupling to external signals. The device exploits a Josephson junction as a nonlinear inductive element that can largely cancel the nonlinearity of conventional SQUID transfer function.

In practical systems, one would increase the dynamic range by using a series array of such SQUID elements and determining the number of such elements by balancing the required signal amplitude with the required amplifier linearity.

One should note the peak-like dependence of the linearity parameter on the shunt junction critical current. This indicates certain limitations of allowable technological spread in Josephson junction parameters for practical implementation of the high linearity serial array of bi-SQUIDs.

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### References

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