Classical and quantum theory for Superluminal particle

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Abstract

As we all know, when the relative velocity of two inertial reference frames $\Sigma$ and $\Sigma'$ is less than the speed of light, the relations of $x_\mu$ with $x'_\mu$, a particle mass $m$ with its velocity $v$, and a particle mass with its energy are all given by Einstein’s special relativity. In this paper, we will give new relation of $x_\mu$ and $x'_\mu$ when the relative velocity of $\Sigma$ and $\Sigma'$ frame is larger than the speed of light, and also we give the relation of a particle mass $m$ with its velocity $v$, and a particle mass $m$ with its energy $E$ when the particle velocity $v$ is larger than the speed of light.

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1. Introduction

A hundred years ago, Einstein laid the foundation for a revolution in our conception of space and time, matter and energy. Later, special theory of relativity was accepted by mainstream physicists. It is based on two postulates by Einstein [1]:

1. The Principle of Relativity: All laws of nature are the same in all inertial reference frames. In other words, we can say that the equation expressing the laws of nature are invariant with respect to transformations of coordinates and time from one inertial reference frame to another.

2. The Universal Speed of Light: The speed of light in vacuum is the same for all inertia observers, regardless of the motion of the source, the observer, or any assumed medium of propagation.

The invariant principle of the speed of light is right in all inertial reference frames in which their relative velocity $v$ is less than the speed of light $c$. Since the light velocity has no relation with the movement of light source, which has been proved by experiment [2], we can consider a moving light source as a inertial reference frames, and we can obtain the result that the speed of light has nothing to do with the movement speed of the inertial reference frames, i.e., the speed of light is same in all inertial reference frames. It derives that light has no interaction with light source, and the light has no inertia. So, the rest mass of light tend to zero. Recently, a series of experiments have revealed that electromagnetic wave was able to travel at a group velocity faster than $c$. These phenomena have been observed in dispersive media [3-4], in electronic circuits [5], and in evanescent wave cases [6]. In fact, over the last decade, the discussion of the tunnelling time problem has experienced a new stimulus by the results of analogous experiments with evanescent electromagnetic wave packets [7], and the superluminal effects of evanescent waves have been revealed in photon tunnelling experiments in both the optical domain and the microwave range [6]. In nature, maybe there is superluminal phenomena, and the relative velocity $v$ of two inertial reference frames can be larger than $c$ or equal to $c$. The superluminal phenomena can also appear in the progress of light propagation. For example, when a beam of light move at the same direction, all photons relative velocity $v$ is equal to zero and not the speed of light $c$. So, the postulates about the invariant principle of the speed of light is incorrect when the relative velocity of the two inertial reference frames is equal to the speed of light (we regard the light as a inertial reference frames). The result that
the speed of light $c$ is maximum speed is also incorrect, because the relative velocity of two beams of light which move along opposite direction exceeds the speed of light $c$. So, when two inertial reference frames relative velocity $\nu$ is larger than the speed of light or equal to the speed of light, Einstein’s postulation of the invariant principle of the speed of light should be modified. We think that there are two ranges of velocity in nature: One is in the range of $0 \leq \nu < c$, which is suit for special relativity. The other is in the range of $c \leq \nu < c_m$ ($c_m$ is the maximum velocity in nature), which will be researched in this paper.

2. The space-time transformation and mass-energy relation of special relativity

In 1905, Einstein gave the space-time transformation and mass-energy relation which are based on his two postulates. The space-time transformation is

$$
\begin{align*}
    x &= x' + \frac{\nu t'}{\sqrt{1 - \frac{\nu^2}{c^2}}} \\
    y &= y' \\
    z &= z' \\
    t &= t' + \frac{\nu x'}{\sqrt{1 - \frac{\nu^2}{c^2}}},
\end{align*}
$$

(1)

where $x, y, z, t$ are space-time coordinates in $\Sigma$ frame, $x', y', z', t'$ are space-time coordinates in $\Sigma'$ frame, $\nu$ is the relative velocity that $\Sigma$ and $\Sigma'$ frame move along with $x$ and $x'$ axes, and $c$ is the speed of light. The velocity transformation is

$$
\begin{align*}
    u_x &= \frac{u_x' + \nu}{1 + \frac{\nu u_x'}{c^2}} \\
    u_y &= \frac{u_y' \sqrt{1 - \frac{\nu^2}{c^2}}}{1 + \frac{\nu u_x'}{c^2}} \\
    u_z &= \frac{u_z' \sqrt{1 - \frac{\nu^2}{c^2}}}{1 + \frac{\nu u_x'}{c^2}},
\end{align*}
$$

(2)

where $u_x, u_y$ and $u_z$ are a particle velocity in $\Sigma$ frames, $u_x', u_y'$ and $u_z'$ are the particle velocity in $\Sigma'$ frames. The relation of a particle mass $m$ with its movement velocity $\nu$ is

$$
m = \frac{m_0}{\sqrt{1 - \frac{\nu^2}{c^2}}},
$$

(3)
with $m_0$ and $m$ being the particle rest mass and relativistic mass. The relation of a particle relativistic energy $E$ with its relativistic mass $m$ is

$$E = mc^2,$$

(4)

and the relation of particle energy $E$ with its momentum $p$ is

$$E^2 = m_0^2c^4 + p^2c^2.$$

(5)

3. The space-time transformation for superluminal reference frames

At some 40 years ago, O.M.P. Bilaniuk, V.K. Deshpande and E.S.G. Sudarshan had researched the space-time relation for superluminal reference frames within the framework of special relativity [8, 9]. They thought the space-time and velocity transformation of special relativity are suit for superluminal reference frames, and they obtained the new results that the proper length $L_0$ and proper time $T_0$ must be imaginary so that the measured quantities length $L$ and time $T$ are real. In the following, we will give the relation of space-time in two inertial reference frames $\Sigma$ and $\Sigma'$ when their relative velocity $v$ is larger than the speed of light $c$. We think even if there is superluminal movement, the movement velocity can not be infinity. So, we can think there is a limit velocity in nature, which is called maximum velocity $c_m$. All particles movement velocity can not exceed the maximum velocity $c_m$ in arbitrary inertial reference frames. In the velocity range of $c \leq v < c_m$, we give two postulates as follows:

1. The Principle of Relativity: All laws of nature are the same in all inertial reference frames.

2. The Universal of Maximum Velocity: There is a maximum velocity $c_m$ in nature, and the $c_m$ is invariant in all inertial reference frames.

From the two postulates, we can obtain the space-time transformation and velocity transformation, which are similar as Lorentz transformation of special relativity:

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c_m^2}}}$$

$$y = y'$$

$$z = z'$$

$$t = \frac{t' + \frac{v}{c_m}x'}{\sqrt{1 - \frac{v^2}{c_m^2}}}.$$  

(6)
and

\[ u_x = \frac{u'_x + v}{1 + \frac{v}{c^2} \frac{u'_y}{c}} \]

\[ u_y = \frac{u'_y \sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{v}{c^2} \frac{u'_y}{c}} \]

\[ u_z = \frac{u'_z \sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{v}{c^2} \frac{u'_y}{c}}, \]

(7)

where \( v (v \geq c) \) is the relative velocity of \( \Sigma \) and \( \Sigma' \) frame. Now, We can discuss the problem of the speed of light from Eq. (7). For two inertial reference frames \( \Sigma \) and \( \Sigma' \), the \( \Sigma' \) frame is a rest frame for light, i.e., their relative velocity \( v \) is equal to \( c \). At the time \( t = 0 \), a beam of light are emitted from origin \( O \). From Eq. (7), we have

\[ u_x = c, \]

(8)

then

\[ u'_x = 0, \]

(9)

and

\[ u_x = -c, \]

(10)

and then

\[ u'_x = -c - c \frac{c}{\sqrt{1 + \frac{c^2}{c^2}}} = -2 \frac{c^2}{c^2 + c^2} > -2c. \]

(11)

It show that the invariant principle of the speed of light is violated in the frame of the speed of light.

4. The relation of mass with velocity for superluminal particle

In Refs. [8, 9], the authors thought that the relation of particle’s mass with its velocity and energy with its mass in special relativity are also suit for superluminal particles, and they obtained the interesting result that the rest mass of particle \( m_0 \) must be imaginary so that the the particle’s energy and momentum are real. In the following, we will give the new relation of particle mass \( m \) with its velocity \( v \). We can consider the collision between two identical particle. It is shown in Fig. 1
The $\Sigma$ is laboratory system, and $\Sigma'$ is mass-center system of tow particles. In $\Sigma$ system, the velocity of tow particles $m_1$ and $m_2$ are $\vec{v}_1$ and $\vec{v}_2$ ($v_1 > v_2 \geq c$), which are along with $x(x')$ axis, and they are $v'$ and $-v'$ in $\Sigma'$ system. After collision, the velocity of tow particles are all $v$ ($v \geq c$) in $\Sigma$ system. Momentum was conserved in this process:

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v. \quad (12)$$

According to equation (7),

$$v_1 = \frac{v' + v}{1 + \frac{vv'}{c^2 m}},$$

$$v_2 = \frac{-v' + v}{1 - \frac{vv'}{c^2 m}}. \quad (13)$$

From equations (12) and (13), we get

$$m_1(1 - \frac{vv'}{c^2 m}) = m_2(1 + \frac{vv'}{c^2 m}), \quad (14)$$

from equation (7), we can obtain

$$1 + \frac{u'_x v}{c^2 m} = \frac{\sqrt{1 - \frac{u'^2}{c^2 m}} \sqrt{1 - \frac{v^2}{c^2 m}}}{\sqrt{1 - \frac{v'^2}{c^2 m}}}, \quad (15)$$

for the particle $m_1$, the equation (15) is

$$1 + \frac{v' v}{c^2 m} = \frac{\sqrt{1 - \frac{v'^2}{c^2 m}} \sqrt{1 - \frac{v^2}{c^2 m}}}{\sqrt{1 - \frac{v^2}{c^2 m}}}, \quad (16)$$

for particle $m_2$, the equation (15) is

$$1 - \frac{v' v}{c^2 m} = \frac{\sqrt{1 - \frac{v'^2}{c^2 m}} \sqrt{1 - \frac{v^2}{c^2 m}}}{\sqrt{1 - \frac{v^2}{c^2 m}}}. \quad (17)$$
On substituting equations (16) and (17) into (14), we get

\[ m(v_1)\sqrt{1 - \frac{v_1^2}{c_m^2}} = m(v_2)\sqrt{1 - \frac{v_2^2}{c_m^2}} = m(c)\sqrt{1 - \frac{c^2}{c_m^2}} = \text{constant}, \]  

(18)

where \( m(c) \) is the particle mass when its velocity is equal to \( c \).

For velocity \( v \) \((v > c)\), we have

\[ m(v)\sqrt{1 - \frac{v^2}{c_m^2}} = m(c)\sqrt{1 - \frac{c^2}{c_m^2}}, \]  

(19)

and hence

\[ m(v) = m_c\sqrt{\frac{c_m^2 - c^2}{c_m^2 - v^2}}. \]  

(20)

with \( m_c = m(c) \). The equation (20) is the relation of superluminal particle mass \( m \) with its velocity \( v \) \((v \geq c)\).

5. The relation of energy with mass for superluminal particle

In the following, we define a 4-vector of space-time

\[ x_\mu = (x_1, x_2, x_3, x_4) = (x, y, z, ic_m t). \]  

(21)

The invariant interval \( ds^2 \) is given by the equation

\[ ds^2 = -dx_\mu dx_\mu = c_m^2 dt^2 - (dx)^2 - (dy)^2 - (dz)^2 = c_m^2 d\tau^2, \]  

(22)

we get

\[ d\tau = \frac{1}{c_m} ds, \]  

(23)

where \( d\tau \) is proper time, the 4-velocity can be defined by

\[ U_\mu = \frac{dx_\mu}{d\tau} = \frac{dx_\mu}{dt} \frac{dt}{d\tau} = \gamma_\mu (\vec{v}, ic_m), \]  

(24)

where \( \gamma_\mu = \frac{1}{\sqrt{1 - \frac{v^2}{c_m^2}}} \), \( \vec{v} = \frac{dx}{dt} \) and \( \frac{dt}{d\tau} = \gamma_\mu \). We can define 4-momentum as

\[ p_\mu = m_c U_\mu = (\vec{p}, ip_4), \]  

(25)

with \( \vec{p} = \frac{m_c \vec{v}}{\sqrt{1 - \frac{v^2}{c_m^2}}}, \) \( p_4 = \frac{m_c c_0}{\sqrt{1 - \frac{v^2}{c_m^2}}} \). We can define a particle energy as

\[ E = \frac{m_c c_m^2}{\sqrt{1 - \frac{v^2}{c_m^2}}}, \]  

(26)
then
\[ p_\mu = (\vec{p}, \frac{i}{c_m} E), \]  
(27)
the invariant quantity constructed from this 4-vector is
\[ p_\mu p_\mu = p^2 - \frac{E^2}{c_m^2} = -m_c^2 c_m^2, \]  
(28)
i.e.,
\[ E^2 - p^2 c_m^2 = m^2 c_m^4. \]  
(29)
The equation (29) is the relation of superluminal particle mass \( m \), momentum \( \vec{p} \) and energy \( E \).

On substituting (20) into (26), we can obtain the relation of mass-energy for superluminal particle.
\[ E = \frac{m(v)}{\sqrt{c_m^2 - c^2 c_m^2}}. \]  
(30)

In the following, we research the problem of photon mass. From Eq. (26), we can obtain the mass of photon when the photon velocity is \( c \).
\[ E_\nu = \frac{m_{cv} c_m^2}{\sqrt{1 - \frac{c^2}{c_m^2}}} = h\nu, \]  
(31)
i.e.,
\[ m_{cv} = \frac{h\nu}{\sqrt{1 - \frac{c^2}{c_m^2}}}. \]  
(32)
If there is a superluminal photon, we can calculate its mass and energy. On substituting equation (32) into (20), we can obtain the mass of the superluminal photon
\[ m_\nu = m_{cv} \sqrt{\frac{c_m^2 - c^2}{c_m^2 - v^2}} = \frac{h\nu}{c_m^3} \sqrt{\frac{c_m^2 - c^2}{c_m^2 - v^2}} \]  
(\( v > c \)),
(33)
from equations (30) and (33), we can obtain the energy of the superluminal photon
\[ E = \frac{m_\nu c_m^3}{\sqrt{c_m^2 - c^2}} = h\nu \sqrt{\frac{c_m^2 - c^2}{c_m^2 - v^2}} = h\nu', \]  
(34)
where \( \nu' \) is the frequency of superluminal photon. It is
\[ \nu' = \nu \sqrt{\frac{c_m^2 - c^2}{c_m^2 - v^2}} > \nu. \]  
(35)
It is shown that the frequency of superluminal photon is larger than the frequency of light velocity photon.

6. The relativistic dynamics for superluminal particle

In the following, we research the relativistic dynamics for superluminal particle. We define a 4-force as:

\[ K_\mu = \frac{dp_\mu}{d\tau}, \tag{36} \]

from equation (27), we have

\[ K_\mu = (\vec{K}, iK_4), \tag{37} \]

the "ordinary" force \( \vec{K} \) is

\[ \vec{K} = \frac{d\vec{p}}{d\tau} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \frac{d\vec{p}}{dt}, \tag{38} \]

while the fourth component

\[ K_4 = \frac{dp_4}{d\tau} = \frac{1}{c_m} \frac{dE}{d\tau} \]
\[ = \frac{1}{c_m} \frac{d}{d\tau} \sqrt{m^2 c_m^4 + p^2 c_m^2} \]
\[ = c_m \frac{1}{E} \vec{p} \cdot \frac{d\vec{p}}{d\tau} \]
\[ = \frac{1}{c_m} \vec{v} \cdot \vec{K}, \tag{39} \]

and so

\[ K_\mu = (\vec{K}, \frac{i}{c_m} \vec{v} \cdot \vec{K}), \tag{40} \]

the covariant equation for the superluminal particle are

\[ \vec{K} = \frac{d\vec{p}}{d\tau}, \tag{41} \]
\[ \vec{K} \cdot \vec{v} = \frac{dE}{d\tau}, \tag{42} \]

the equations (36) and (37) can be written as:

\[ \vec{K} = \frac{d\vec{p}}{d\tau} = \frac{d\vec{p}}{dt} \frac{dt}{d\tau} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \frac{d\vec{p}}{dt}, \tag{43} \]
\[ \sqrt{1 - \frac{v^2}{c_m^2}} \vec{K} = \frac{d\vec{p}}{dt}, \tag{44} \]
we define force $\vec{F}$ as

$$\vec{F} = \sqrt{1 - \frac{v^2}{c^2}} \vec{K}.$$  

(46)

The relativistic dynamics equation for the superluminal particle are

$$\vec{F} = \frac{d\vec{p}}{dt},$$  

(47)

$$\vec{K} \cdot \vec{v} = \frac{dE}{dt}.$$  

(48)

7. The quantum wave equation for superluminal particle

In the following, we will give the quantization wave equation for a superluminal particle.

Form equation (29), we express $E$ and $\vec{p}$ as operators:

$$E \rightarrow i\hbar \frac{\partial}{\partial t},$$

$$\vec{p} \rightarrow -i\hbar \vec{\nabla},$$  

(49)

then we obtain the quantum wave equation of superluminal particle

$$\left[ \frac{\partial^2}{\partial t^2} - c^2_m \nabla^2 + \frac{m^2 c^4}{\hbar^2} \right] \Psi(\vec{r}, t) = 0.$$  

(50)

The equation is similar as the K-G equation. At $m_c = 0$, we have

$$\left[ \frac{\partial^2}{\partial t^2} - c^2_m \nabla^2 \right] \Psi(\vec{r}, t) = 0.$$  

(51)

The equation is similar as the wave equation of photon.

We know that the particle wave equation of spin $\frac{1}{2}$ is Dirac equation

$$i\hbar \frac{\partial}{\partial t} \Psi = [-i\hbar \alpha \cdot \vec{\nabla} + mc^2 \beta] \Psi,$$  

(52)

where $\alpha$ and $\beta$ are matrixes

$$\alpha = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix},$$

and

$$\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix},$$

(52)
where $\vec{\sigma}$ are Pauli matrixes, and $I$ is unit matrix of $2 \times 2$.

We can give the superluminal particle wave equation for spin $\frac{1}{2}$, which it is similar as Dirac equation

$$i\hbar \frac{\partial}{\partial t} \psi = [-i\hbar c_m \vec{\alpha} \cdot \vec{\nabla} + m c^2 \beta] \psi.$$  (53)

8. Conclusion

In conclusion, we think there may be two kinds of motion phenomena in nature: One is lowerlumial phenomena ($0 \leq v < c$), the other is superlumial phenomena ($c \leq v < c_m$). In this paper, we research the classical and quantum theory for superlumial particle, which ia based on two postulates: The principle of relativity and the universal of maximum velocity. We obtain all the results for superluminal theory should be tested by the future experiment.
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