Hydrodynamic effects from ballistic electron jets in GaAs/AlGaAs

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We experimentally study the interaction of a ballistic electron jet with the surrounding electron fluid through nonlocal resistance measurements in a mesoscopic structure fabricated on a high-mobility two-dimensional electron system in a GaAs/AlGaAs heterostructure. We demonstrate that a ballistic jet injected in the vicinity of a lithographic aperture can entrain electrons out of this aperture, leading to nonlocal negative resistance at the aperture. The measurements allow us to characterize the temperature dependence and characteristic length scales of the nonlocal hydrodynamic effects.

Transport regimes can vary in two-dimensional electron systems (2DESs) depending on system parameters and temperature $T$. Ballistic effects dominate the transport when the electron mobility mean-free-path, $\ell_e$ (describing momentum-relaxing collisions) exceeds the characteristic device size $W$, while semiclassical diffusive Drude effects dominate for $\ell_e \ll W$. The mobility $\mu$, however only measures momentum dissipation out of the 2DES to the lattice (e.g. via impurity or phonon scattering), and cannot measure momentum transfer internal to the 2DES. Electron-electron (e-e) interactions, on the other hand, transfer momentum between electron internally to the 2DES and can lead to an effective kinematic viscosity $\nu$, triggering hydrodynamic effects in the system. Hydrodynamic effects can occur when momentum is exchanged predominantly between different parts of the carrier fluid rather than with the lattice, expressed as $\ell_{ee} < \ell_e, W$ where $\ell_{ee}$ denotes the e-e scattering length related to inelastic but momentum-conserving collisions. Recent work on hydrodynamic transport spans materials including GaAs quantum wells, graphene, PdCoO$_2$, Weyl semimetals, and Dirac materials.

Most electron hydrodynamics experiments have been conducted on narrow-channel constricted geometries (small $W$) and not considering the effects of a ballistic carrier jet (a local non-equilibrium current) on the surrounding carrier fluid. In this work, we explore the momentum exchange between a ballistic electron jet and the surrounding electron fluid at zero magnetic field in a more open geometry (large $W$) where the ballistic jet can exist over large distances, allowing resultant hydrodynamic effects to be measurable up to larger distances. In a system with few momentum-relaxing processes like the high-$\mu$ 2DES considered here, an open geometry can enhance hydrodynamic effects such as vortices, while minimizing transport signatures from the ballistic effects which may obscure hydrodynamic effects. Specifically, we experimentally study the nonlocal voltage distribution induced in the surrounding electron fluid by a ballistic electron jet injected from a mesoscopic point contact (PC) $i$, using nonlocal voltage measurements at a distance $L$ from $i$. The ballistic electron jet interacts hydrodynamically via e-e interactions with the surrounding electron fluid by exchanging momentum with it, and gradually dissipates that way before dissipating to the lattice. The importance in high-$\mu$ 2DESs of e-e interactions for ballistic jet dissipation has been recognized earlier, but not from a hydrodynamic perspective. The hydrodynamic aspect of interest in the present work is that as the jet dissipates, it effectuates a reciprocal effect on the electron fluid in its vicinity by imparting momentum to the fluid and concomitantly entraining it. This can lead to vortices and nonlocal currents, as well as nonlocal potential patterns and electron density variations in the fluid. Specifically we study a decrease in electron density in the vicinity of the jet. We observe that the nonlocal voltage measured at a detector PC $d$ placed in-line in the same barrier and at a distance $L$ from $i$, has a negative value in a particular range of $T$. We attribute this to the phenomenon of electron extraction out of $d$ (equivalently hole injection into $d$) due to the local decrease in electron density. We employ the experimentally-determined dependence on $T$ and $L$ of the nonlocal voltage in two configurations, and corroborate our findings using a theoretical model. The extraction effect discussed here differs fundamentally from the Venturi effect in a Fermi liquid. The Venturi extraction effect is based on Bernoulli’s equation (conservation of energy). The present extraction effect on the other hand is understood from viscous entrainment (conservation of momentum, classically the Navier-Stokes equations).

Mesoscopic geometries were patterned on a GaAs/AlGaAs heterostructure containing a 2DES with $\mu$ exceeding 670 m$^2$/s at 4.2 K. The areal electron density is $N_\text{e} \approx 3.4 \times 10^{15}$ m$^{-2}$, corresponding to a Fermi energy $E_F \approx 11.2$ meV and $\ell_e = 64.5$ $\mu$m at 4.2 K. To measure the electron extraction effect experimentally, we fabricate an in-line mesoscopic geometry containing PCs separated by distances $L$ ranging from $1.3$ $\mu$m to $20.5$ $\mu$m on both sides of a multiterminal Hall bar mesa (Fig. 1(b)), the sides being separated by $W \approx 24$ $\mu$m. Each PC can act as a current injector ($i$) or volt-
two current configurations: G1, where the current distribution can be measured at any other PC. We use any PC and the nonlocal voltage induced by the current \( d \).

A ballistic electron jet is injected at \( i \) (Fig. 1 (a), (b)). Calling \( V_d \) the nonlocal voltage measured at \( d \), the results are expressed as nonlocal resistance \( R_d = V_d/I_i \), and \( R_d \) takes the sign of \( V_d \). A variation of G1 with longer \( W \) is also discussed [27]. The lithographic width of PCs is 0.8 \( \mu m \) while side depletion brings the actual conducting width \( w \) down to \( \sim 0.6 \mu m \). Measurements were performed over 4.2 K \( \leq T \leq 40 \) K, using low-frequency AC lock-in techniques under small injection currents (\( \sim 200 \mu A \)) to avoid electron heating.

Measurement results on G1 and G2 are depicted in Fig. 2(a) and (b) respectively, showing the dependence on \( T \) of \( R_d \) for \( L = 7.7 \mu m \) as example. The insets show equivalent results for all values of \( L \). We observe a striking contrast in \( T \) dependence for the configurations. In G1, \( R_d \) shows a non-monotonic dependence on \( T \), initially decreasing as \( T \) increases, crossing to negative values in a particular range of \( T \) (depending on \( L \)) and then increasing towards positive values. Such a non-monotonic \( T \) dependence requires explanation for a metallic system exhibiting standard \( T \) behavior for resistivity [27].

In G2, however, \( R_d \) increases from large negative values at low \( T \) to positive values at higher \( T \). In AC phase-sensitive lock-in voltage detection, a negative \( R_d \) indicates that \( I_i \) injected at \( i \) and \( V_d \) measured at \( d \) are 180° out-of-phase whereas a positive value indicates they are in-phase. Therefore, \( R_d < 0 \) suggests that injection of a jet of electrons from \( i \) (excess electrons in the injector lead) induces a lack of electrons in the lead behind \( d \) or equivalently effective electron extraction from \( d \). The extraction can originate in electron entrainment out of \( d \), resulting from momentum exchange between the ballistic jet and the surrounding fluid (Fig. 1(a)). Hence for both G1 and G2, \( R_d < 0 \) is interpreted as a signature of hydrodynamic effects.

Nonlocal negative resistances can in principle occur as a result of ballistic effects [27] without invoking hydrodynamic effects. Section 3 of Ref. [27] describes the extent of ballistic effects. Performing a Landauer-Büttiker analysis in a geometry similar to G1 shows that negative resistance is not expected at zero magnetic field. Ballistic contributions, such as due to ballistic electron trajectories reflecting off the opposite barrier back into \( d \), are hence expected to lead to only positive \( R_d \). In particular, in the G1 configuration, a region of positive \( R_d \) is observed at low \( T \) except for the lowest distances \( L = 1.3 \mu m \) and \( L = 2.6 \mu m \) (Fig. 2(a)), attributed to a dominance at low \( T \) and higher \( L \) of ballistic reflection effects (which can dominate over the hydrodynamic signal at low \( T \), where \( \ell_c > W \), and at long \( L \), where the hydrodynamic signal is weaker [27]). To verify this ballistic attribution, another device similar to G1 but with larger \( W \) (G1') was characterized and results are reported in Ref. [27]. Section 4 of Ref. [27] describes the differences in ballistic effects between G1 and G2.

The region of negative \( R_d \) extends up to \( L \approx 13 \mu m \) in G1 and 20.5 \( \mu m \) in G2. The interplay between two important scales, namely the vorticity diffusion length \( \ell_V = \sqrt{\nu \tau_e} \) (where \( \tau_e \) represents the scattering time for age detector \( d \)) such that the jet can be injected from any PC and the nonlocal voltage induced by the current distribution can be measured at any other PC. We use two current configurations: G1, where the current \( I_i \) is injected at \( i \) and drained at a faraway counterprobe, and G2, where \( I_i \) is injected at \( i \) and drained at a PC placed across the Hall bar mesa a distance \( W \) removed, opposite
FIG. 2. (a) Dependence of $R_d$ on $T$ for $L = 7.7 \mu m$ in G1 configuration (green circles), exhibiting non-monotonic behavior. Inset: dependence of $R_d$ on $T$ for all $L$. The solid blue line represents $R_d = 0$. (b) Dependence of $R_d$ on $T$ for $L = 7.7 \mu m$ in G2 configuration (green circles). Inset: dependence of $R_d$ on $T$ for all $L$.

momentum-relaxing collisions derived from $\mu$ and $N_S$ [8, 15, 20, 27], and an effective length $L^*$ (depending on $L$ and $W$) [8, 27] measured from a local current injection point or from a current distribution such as a ballistic electron jet, mark the spatial extent of the hydrodynamic effects. Within $L^* < \ell_V$ hydrodynamic entrainment from $d$ is expected, yet possibly in competition with ballistic effects [27]. For $d$ closer to $i$ ($L = 1.3 \mu m$ and $2.6 \mu m$), the hydrodynamic entrainment is stronger, overpowering the ballistic reflection effects at low $T$ in G1 (Fig. 2(a)) because at low $L$ and low $T$, $L \approx L^* \ll \ell_v$. [8, 27]. Further, at long $L > 12.8 \mu m$ and at higher $T$ the ballistic jet dissipates due to momentum exchange with the electron fluid and the lattice. At long $L$ therefore, Drude effects are expected to dominate over either ballistic or hydrodynamic effects. Hence hydrodynamic effects are missing for $L > 12.8 \mu m$ in G1 and only ballistic effects ($T \lesssim 13 K$) or Drude effects ($T \gtrsim 13 K$) appear. For $T \gtrsim 13 K$, both G1 and G2 follow similar behavior with $R_d$ increasing towards $R_d > 0$, correlating well with increased phonon scattering for $T \gtrsim 13 K$ [27] and hence announcing the disruption of the ballistic jet and the appearance of Drude transport. The advent of Drude transport is also consistent with the observation that for $T \gtrsim 13 K$ we have $\ell_e \lesssim W$, affecting the very existence of a well-defined ballistic jet exchanging momentum with the electron fluid [27]. Regarding hydrodynamic effects specifically from a ballistic electron jet, $\ell_{ee} < W < \ell_e$ and $L^* < \ell_V$ are hence minimal conditions.

Analyzing the dependence on $L$ of $R_d$ provides further insight into interplay between different effects influencing the system. Figures 3(a),(b) depict the dependence on $L$ of $R_d$ for G1 and G2, taking data for $T = 16.5 K$ as
example of $T$ where ballistic effects, if any have already diminished [27]. The insets show equivalent results for all values of $T$. As expected, the electron extraction effect characterized by $R_d < 0$ is stronger at low $L$ and decreases with increasing $L$. A remarkable resemblance is observed between Fig. 3 and Fig. 2 of Ref. [6] (a theoretical model describing injection via Boltzmann transport equations with e-e collision integral) which depicts the carrier density variation profiles plotted vs distance from the injection PC. The carrier density and potential ($\rho_L$ in our case) are related by a linear relation since in a 2DES a local net charge density variation will lead to a local potential variation of the same sign. The regions of negative $R_d$ hence correspond to regions where $\rho_L$ is lowered by the effects of the ballistic jet, and from where electron extraction occurs. Figure 3 corroborates that a ballistic jet can entrain the surrounding fluid and can lead to a depletion of electrons, detected as negative $R_d$.

Beyond consistency with Ref. [6], we further verify our results by using a theoretical model [8] based on Navier-Stokes and continuity equations, capturing the effects of electro-thermal, hydrodynamic and Drude effects dominated transport, vs $T$, for the range of $T$ where the crossovers occur. Solid circles represent experimental data, solid lines represent predictions using Eq. (1) and (2).

\[ R_{d,G1}(L) = -\rho \left\{ \frac{1}{\pi} \ln \left[ 4 \sinh^2 \left( \frac{\pi L}{2W} \right) \right] - \frac{L}{W} \right\} + \pi \left( \frac{\ell_v}{W} \right)^2 \frac{1}{\sin^2 \left( \frac{\pi L}{2W} \right)} \]

(1)

where $\rho$ denotes the 2D resistivity. For G2, using free surface boundary conditions yields:

\[ R_{d,G2}(L) = -\rho \left\{ \frac{1}{\pi} \ln \left[ 4 \tanh^2 \left( \frac{\pi L}{2W} \right) \right] \right\} + 4 \pi \left( \frac{\ell_v}{W} \right)^2 \frac{\cosh(\pi L/W)}{\sin^2(\pi L/W)} \]

(2)

While Eq. (1) and (2) contain several assumptions and hence can be applied to trends of our experimental data but not to predict exact values, Fig. 3(a) and (b) show that the experimental values for $R_d$ for both G1 and G2 follow the theoretical trends of Eq. (1) and (2), respectively. The crossover from negative to positive $R_d$ vs $L$ is observed in Figs. 3(a) and (b) (marked by red and black arrows), due to $L \approx < \ell_v$ being no longer satisfied. Physically, the crossover signifies a transition from transport dominated by hydrodynamic effects to that by Drude effects (incipient contributions from Drude effects start around $T = 13$ K [27] and become more profound for $T \gtrsim 16$ K) and depends on the ratio $\rho/\nu$ [9]. However, at sufficiently small $L \approx 1.3 \mu m$, we still have $L^* < \ell_v$ even in a region of high $\rho$ and low $\nu$ ($T \gtrsim 25 K$), and hence hydrodynamic effects still prevail. From Eq. (1) and (2), the theoretical values of the crossover lengths, $L_{cr}$, can be graphed vs $T$ and compared with the experimental values. Figure 4 shows that Eq. (1) and (2) are able to predict the experimental crossover lengths within the limits of experimental uncertainty for both G1 and G2. Therefore, for $T \gtrsim 16$ K, $L < L_{cr}$, is a sufficient condition for observing hydrodynamic extraction effects. In G1 however (Fig. 3(a) inset), we also witness a crossover at $L \approx 4 \mu m$ at low $T$ ($4 K < T < 10 K$). At such lower $T$, the rarity of momentum-relaxing processes renders a crossover from predominantly hydrodynamic to Drude transport unlikely. Yet, as discussed under Fig. 2(a), in G1 a gradual transition from the transport dominated by ballistic effects to that by hydrodynamic effects occurs at low $T$. Hence, the crossover observed for G1 in Fig. 3(a) at low $T$ at $L \approx 4 \mu m$ corresponds to a crossover into transport dominated by ballistic effects. We note that Eq. (1) cannot capture this ballistic crossover because the theoretical model does not account for ballistic effects, which are prevalent at low $T$ in G1 [27].

Based on the experimental $R_d$, we can now identify different transport effects depending on values of $T$ and $L$. We categorize $T$ and $L$ values as low, intermediate and high, with values listed in Table I. The table emphasizes that in both G1 and G2, hydrodynamic effects can be accessed by choosing appropriate ranges of $T$ and $L$.

In conclusion, e-e interactions can lead to momentum exchange between a ballistic electron jet and the surrounding electron fluid, resulting in a condition where
TABLE I. Transport effects depending on $T$ and $L$

|          | Low $L$       | Int. $L$       | High $L$        |
|----------|---------------|---------------|-----------------|
|          | (1-4 $\mu$m)  | (4-13 $\mu$m) | (13-20.5 $\mu$m) |
| $G_1$    | Hydrodynamic  | Ballistic     | Ballistic       |
| Low $T$  | (4 - 10K)     | (10 - 25K)    | (25 - 40K)      |
| $G_2$    | Hydrodynamic  | Hydrodynamic  | Hydrodynamic    |
| Low $T$  | (4 - 10K)     | (10 - 25K)    | (25 - 40K)      |

hydrodynamic effects and ballistic transport can coexist. Hydrodynamic entrainment of electrons out of a mesoscopic aperture by the nearby ballistic electron jet effectively leads to electron extraction out of the aperture and to nonlocal negative resistances detected at the aperture. The dependence of the nonlocal resistance on temperature and on distance to the jet injection point can help distinguish between different transport signatures and can provide a guide for future experiments on electron hydrodynamics. While existing theory distinguishes well between hydrodynamic and Drude effects, a deeper description of the interface between the ballistic and hydrodynamic effects may need further attention. The authors acknowledge support by the U.S. Department of Energy, Office of Basic Energy Sciences, Division of Materials Sciences and Engineering under award DE-FG02-08ER46532 (JJH) and DE-SC-0006671 (MJM).

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S1. Device fabrication and materials properties

The devices were fabricated from the GaAs/AlGaAs MBE-grown material hosting the two-dimensional electron system (2DES) (Fig. 1(b) main text). The mesoscopic geometries were patterned by electron beam lithography and wet etching of the barriers, using PMMA as the etching mask. The geometries feature mesoscopic apertures (point contacts, PCs) separated by various distances on both sides of the Hall mesa. Different L are chosen such that a wide range of distances L exists, from 1.3 μm to 20.5 μm, between injector and detector PCs.

The electron transport properties of the unpatterned 2DES were characterized by the van der Pauw method. We use the value of 2D resistivity $R_\square$ from this method, and areal electron density $N_S$ from Hall measurements on the fabricated device to obtain electron mobility $\mu$. At temperature $T = 4.2$ K, we obtain $N_S \approx 3.4 \times 10^{15}$ m$^{-2}$, and $R_\square = 2.74 \ \Omega/\square$, yielding $\mu \approx 670$ m$^2$/s (confirming the quality of the material).

Figure S1 | Transport characteristics. (a) Carrier density $N_S$ vs $T$. (b) 2D resistivity $R_\square$ vs $T$. (c) $\mu$ vs $T$ with inset depicting $\mu$ vs $T$ on log-log scale, highlighting the change in slope. (d) $1/\mu$ vs $T$. From the fit (red curve), we extract the fitting parameters $1/\mu_0 = 1.2 \times 10^{-4}$ m$^2$V$^{-1}$s and $\alpha = 2.7 \times 10^{-4}$ m$^2$V$^{-1}$sK$^{-1}$. 
Drude (mobility) mean-free-path $\ell_e \approx 64.5 \, \mu\text{m}$ and Fermi energy, $E_F \approx 11.2 \, \text{meV}$ (equivalent to $\sim 130 \, \text{K}$). Here, $\ell_e = v_F \tau_e$ with $v_F$ the Fermi velocity and $\tau_e$ the Drude momentum relaxation time derived from $\mu = e\tau_e/m^*$, where $m^*$ denotes the electron effective mass ($0.067 \, m_e$ with $m_e$ the free-electron mass) and $e$ the electron charge. Non-parabolicity of the band structure is accounted for in calculating the transport properties [27, 28]. $N_S$ (Fig. S1(a)) and $R_S$ (Fig. S1(b)) are observed to increase with increasing $T$, while $\mu \sim 1/T$ (Fig. S1(c)), demonstrating that as expected $\mu$ is limited by scattering with acoustic phonons. However, we observe a slight change in slope of $\mu$ vs $T$ for $T \approx 13 \, \text{K}$ (inset in Fig. S1(c)), attributed to incipient scattering by LO phonons for $T > 13 \, \text{K}$. In 2DESs of lower $\mu$, the contribution to scattering by optical phonons is only apparent at higher $T$. In a high-$\mu$ 2DES like the present, LO phonon scattering can start to be observed even at $T \approx 13 \, \text{K}$ since the lack of residual scattering does not mask their effect. As a result, the dependence on $T$ of $\mu$ is affected slightly. Fig S1 (d) depicts $1/\mu$ vs $T$, indicating that $\frac{1}{\mu(T)} = \frac{1}{\mu_0} + aT$ is closely followed, where $\mu_0$ denotes $\mu$ limited by impurity scattering and $aT$ describes the linear dependence on $T$ due to (predominantly) acoustic phonons [29]. The exponent of $T$ changes from 1 to 1.2 at $T > 13 \, \text{K}$, but since the deviation due to optical phonons is small, the dependence due to acoustic phonons is a good approximation. The Fermi wavelength $\lambda_F = 43 \, \text{nm}$. Since the conducting aperture width $w \approx 0.6 \, \mu\text{m}$, it is expected that $w/(\lambda_F/2) \approx 28$ modes contribute to transport. This number implies that quantized transport through the apertures can be disregarded.

**S2. Determination of different length scales**

In general a hydrodynamic effects occur when momentum is exchanged predominantly internal to the carrier fluid via electron-electron (e-e) interactions (conserving momentum within the fluid) rather than with external entities (the lattice, e.g. phonon baths or impurities) [1, 3, 6-12]. The length scale associated with momentum exchange within the fluid is $\ell_{ee}$, where $\ell_{ee} = v_F \tau_e$ with $\tau_e$ the inelastic e-e scattering time. Momentum relaxation out of the carrier fluid occurs over the length scale of the Drude mean-free-path $\ell_e = v_F \tau_e$ (cfr S1). Momentum exchange between layers of the carrier fluid leads to an effective kinematic viscosity, $\nu$ [6-9, 12]. In the kinetic theory of gases, $\nu$ is expressed as $\nu = (1/3)v_\lambda$ (where $v_\lambda$ is the average thermal velocity and $\lambda$ is the molecule mean-free-path) [30, 31]. An analogous expression can be derived for a 2DES where $v_\lambda$ can be replaced with the Fermi velocity $v_F$ of electrons and $\lambda$ by $\ell_{ee}$ signifying the mean-free-path between momentum exchange events within the fluid. Drawing the analogy, we obtain, $\nu = v_F \ell_{ee} / 2$ [8]. Two important length scales that mark the spatial scale up to which hydrodynamic effects can be felt are the vorticity diffusion length $\ell_V = \sqrt{\nu \tau_e}$ [6-8, 15, 16] which depends on momentum exchange internal to the fluid through $\nu$ and momentum exchange external to the fluid through $\tau_e$ and an effective length, $L^*$ [7] which is a function of the injector to detector distance $L$ and the characteristic device size $W$. The interplay between $\ell_V$ and $L^*$ can give the conditions necessary for observing hydrodynamic effects.

The scattering rate from inelastic e-e scattering, $1/\tau_{ee}$, is expressed as [3, 32]:

$$\frac{1}{\tau_{ee}(T)} = \frac{(k_BT)^2}{hE_F} \left[ \ln \frac{E_F}{k_BT} + \ln \frac{4}{a_F^e k_F} + 1 \right] \quad (S1)$$

where $E_F$, $k_F$, $a_F^e$ and $k_B$ represent the Fermi energy, the Fermi wave vector, the effective Bohr radius and the Boltzmann constant respectively.

The rate of momentum relaxation out of the carrier fluid, $1/\tau_e$, can be approximated as [9, 29] (cfr S1):
\[
\frac{1}{\tau_e} = \frac{1}{\tau_{ph}(T)} + \frac{1}{\tau_{e,o}} \approx A_{ph} T + \frac{1}{\tau_{e,o}} \quad (S2)
\]

with \(1/\tau_{ph}\), the phonon scattering rate, \(A_{ph}\), a phonon scattering coefficient, and \(1/\tau_{e,o}\), the residual scattering rate due to impurities [29].

In Fig. S2(a) \(\ell_e, \ell_{ee}, \ell_V\) and \(W\) are depicted vs \(T\) as obtained on the actual devices, with \(\ell_e\) calculated from measurements of \(N_S\) and \(R_o\) (cfr S1) and \(\ell_{ee}\) from Eq. S1 using the measurements of \(N_S\). In Fig. S2(b), \(L^*\) is plotted vs \(L\) using Eq. 43 and Eq. 33 from Ref. [7] for G1 and G2 configurations respectively. For a particular \(L\), the condition \(L^* < \ell_V\) becomes an important condition to observe the hydrodynamic phenomena, yet in competition with ballistic effect as discussed in the next section. Since \(\ell_V\) is a decaying function of \(T\), this condition is not satisfied for longer \(L\) at high \(T\) and we see an upper limit in \(T\) to hydrodynamic effects, physically caused by Drude classical transport dominating at higher \(T\), particularly due to phonon scattering, as discussed in the last section and the main text. At low \(L\) (1.3 \(\mu m\)), \(L \approx L^*\) and the condition \(L^* < \ell_V\) is satisfied for almost the entire range of \(T\) as seen in Fig. 2(a) and 2(b) of main text, indicating that in close proximity to the injector the hydrodynamic effects like electron extraction are the strongest [3,7].

Figure S2(c) shows that Eq. S2 describes not only the dependence on \(T\) of \(1/\tau_e\), but also the dependence on \(T\) of \(1/\ell_e\) well (cfr S1). Acoustic phonon scattering dominates in the range of \(T\) of the experiments, with incipient contributions from optical phonons above \(T \approx 13\) K. From Fig. S2(c) and Eq. S2 we extract \(A_{ph} \approx 5.2 \times 10^8 s^{-1} K^{-1}\) and \(\tau_{e,o} \approx 6.4 \times 10^{-10}\) s. In contrast to the role of e-e scattering in dissipating a ballistic carrier jet, the electron transport data confirms that e-e scattering does not play a role in determining \(\mu\) in the 2DES. That is because e-e scattering conserves the total momentum of the carrier fluid and merely causes a redistribution of momentum internally to the fluid. The loss of total momentum of the carrier fluid to the lattice is what is quantified by \(\mu\).

Figure S2] Relevant length scales. (a) \(\ell_{ee}, \ell_e, \ell_V\) plotted vs \(T\) for \(W = 24\) \(\mu m\). (b) \(L^*\) plotted vs \(L\) for G1 and G2. The solid blue line depicts \(L\), plotted in comparison to \(L^*\). (c) \(\ell_e^{-1}\) plotted vs \(T\) showing that \(\ell_e\) can indeed be considered \(~ T^{-1}\).
S3. Ballistic effects and additional geometry G1’

1) We use inline mesoscopic geometries containing PCs on both sides of a Hall Bar mesa. Effectively, a channel of width \( W \) is formed. Ballistic electrons injected from a PC on one side can reflect from a mesa wall on the opposite side, to reach a detector PC if \( \ell_e \) is comparable to \( 2W \). This ballistic trajectory may result in voltage signals at the detector PC. If these ballistic effects produce a nonlocal detector voltage (or nonlocal resistance \( R_d \) after normalizing to injected current) that is negative (\( R_d < 0 \)), then they may be confused with the hydrodynamic extraction signals of interest. We therefore performed a Landauer-Büttiker analysis [33] in a configuration similar to G1 and found only positive \( R_d \) for ballistic effects for a realistic geometry. The analysis is standard, so we do not report the calculations here. Hence in our geometries, ballistic reflections, if any, should result in positive \( R_d \).

2) In the G1 configuration, a region of positive \( R_d \) is observed at low \( T \) except for the shortest distances \( L = 1.3 \) \( \mu \)m and \( L = 2.6 \) \( \mu \)m. We attribute this positive \( R_d \) observed at low \( T \) to a dominance for the longer \( L \) of the effects of ballistic electron trajectories reflecting back into the detector PC, over hydrodynamic effects which are weaker at longer \( L \). Also, at lower \( T \), \( \ell_e \) is longer, promoting ballistic effects. A ballistic effect is consistent with the observation that at \( T = 4.2 \) K, \( \ell_e \approx 64.5 \) \( \mu \)m > \( 2W = 48 \) \( \mu \)m. To further confirm that the positive \( R_d \) at low \( T \) indeed originates in ballistic reflection, we fabricated another wider device with \( W = 37 \) \( \mu \)m and call it G1’ (Fig. S3). The probability of the electrons reflecting back to the injection side is exponentially dependent on \( W \) and \( \ell_e \), following \( \sim \exp(-2W/\ell_e) \). \( W \) is varied by design, and \( \ell_e \) increases with decreasing \( T \). Increasing \( W \), we should observe a ballistic reflection signal shifting to lower \( T \) and hence observe the ballistic to hydrodynamic signal crossover at lower \( T \). Figure S4(a) compares the \( T \)-dependence of \( R_d \) in G1 and G1’ for \( L = 5 \) \( \mu \)m (in G1, \( L = 5.12 \) \( \mu \)m, but this difference in \( L \) is too small to affect the transport signal). The crossover in G1’ takes place around \( T = 6 \) K while in G1, around \( T = 12 \) K, strongly suggesting that the positive \( R_d \) at low \( T \) indeed originates in reflected ballistic trajectories. We next compare the probabilities of ballistic reflected electrons reaching the detector in G1 and G1’ (Fig. S4(b)). Figure S4(b) depicts the probability \( P = \exp(-2W/\ell_e) \) for G1 and G1’ vs \( T \) using \( \ell_e \).

Figure S3| Wider device G1’. Optical micrograph of device G1’. The left micrograph (rotated by 90°) depicts the measurement setup. The right micrograph provides a zoomed-in view.
For G1 the crossover point in $T$ between the low-$T$ ballistic effect and the higher-$T$ hydrodynamic effect occurs at $T \approx 11.7$ K, according to Fig. S4(a). From Fig. S4(b), the crossover for G1 corresponds to $P = 0.25$. From Fig. S4(b), the crossover at the same $P = 0.25$ for G1’ is then expected at $T = 6$ K. In Fig. S4(a) we indeed find the corresponding crossover for G1’ at $T \approx 6$ K. The consistency validates the origin of the positive $R_d$ at low $T$ in G1 as due to ballistic electrons reflecting back from the opposite barrier. This ballistic reflection is not observed in G2, as explained in S4, because of an electric field on the current path effectively preventing the carriers from reflecting back to the injection side.

3) In G2, a region of negative $R_d$ is observed at low $T$. In the main text we state that this negative $R_d$, observed at low $T$ originates in hydrodynamic effects, namely electron extraction out of the detector PC. In fact this observation is in agreement with the hydrodynamic condition discussed in S2, $L^* < \ell_V$ being satisfied for the entire range of $L$ at low $T$. It is still important to rule out a ballistic origin for this negative $R_d$. Specifically, we rule out ballistic bend resistance [34, 35], a prominent effect known to produce a nonlocal negative $R_d$ at low $T$. In a bend resistance 4-probe geometry [34, 35], a voltage-probe detector is placed adjacent to the current-probe injector along a bend, while the current is drained to a probe situated on another bend opposite the detector probe, and the voltage-counterprobe is situated opposite the current injector. A magnetic field $B$ is applied perpendicularly to the 2DES. At $B \approx 0$, the ballistic jet travels straight on, fails to negotiate either bends and fails to transmit to the detector. This results in a sharp minimum in $R_d$ at $B = 0$. We note that G2 differs from a bend resistance setup since in G2 the detector is not placed on a bend relative to the injector. Yet, to rule out the observation of bend resistance in both G2 and G1, we measure $R_d$ vs $B$ in G1 and G2 for $L = 7.7 \mu$m at $T = 4.2$ K, as depicted in Figs. S5(a),(b). The oscillatory behavior of $R_d$ vs $B$ in Figs. S5(a),(b) is due to transverse magnetic focusing (in fact confirming the long $\ell_e$ in our 2DES) [4, 36]. A bend resistance would appear as a negative $R_d$ with minimum at $B = 0$.
and symmetrically extending to $|B| \sim 0.1$ T for both $B < 0$ and $B > 0$. For neither G2 nor G1 is such a bend resistance observed.

4) We measured the $T$ and $L$ dependence of $R_d$ in G1’ similarly to the main text, for $4.2 \text{ K} < T < 26 \text{ K}$ and $3 \mu\text{m} < L < 15 \mu\text{m}$, with results in Figs. S6(a),(b). Traces of $R_d$ vs $T$ (Fig. S6(a)) and $R_d$ vs $L$ (Fig. S6(b)) are similar to the traces for G1 in Fig. 2(a) and Fig. 3(a) respectively, attesting to the robustness of the observations. We observe the crossover from hydrodynamic effects to the Drude effects dominated transport at lower $T$ as compared to G1. The crossover between ballistic and hydrodynamic effects also

and symmetrically extending to $|B| \sim 0.1$ T for both $B < 0$ and $B > 0$. For neither G2 nor G1 is such a bend resistance observed.

Figure S5| Dependence on perpendicular magnetic field $B$ of $R_d$ for G1 and G2. (a) $R_d$ vs $B$ plotted in G1 configuration for $L = 7.7 \mu\text{m}$ (b) $R_d$ vs $B$ plotted in G2 configuration for $L = 7.7 \mu\text{m}$. Both measurements yield transverse magnetic focusing spectra. No qualitative differences are observed between both spectra, and neither shows negative bend resistance.

Figure S6| Dependence of $R_d$ on $T$ and $L$ in G1’. (a) $R_d$ vs $T$ plotted in G1’ at different $L$. (b) $R_d$ vs $L$ plotted in G1’ at different $T$. The general trend is similar to G1 and to the theoretical predictions, however, the crossover points between different effects (ballistic, hydrodynamic, Drude transport) have shifted compared to G1 due to the larger $W$. 

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occurs at lower $T$, as explained above. We attribute the shift to lower $T$ to the longer $W$ in G1’. G1 and G1’ demonstrate that $W$ forms an important length scale in the crossover between different transport effects.

**S4. Differences between G1 and G2**

In the G1 configuration, a region of positive $R_d$ is observed at low $T$ except for the lowest distances $L = 1.3 \, \mu m$ and $L = 2.6 \, \mu m$, attributed to a dominance at higher $L$ of ballistic electron trajectories reflecting back into the detector PC (Fig. 2(a)). This ballistic reflection is not observed in G2 and hence fails to dominate over hydrodynamic effects in G2 (Fig. 2(b)). We attribute the contrasting behavior of G1 and G2 at low $T$ to differences in the current drain configurations (Fig. 1), qualitatively understood as follows: if current $I_i$ is injected at PC $i$ at one side of a channel of width $W$ and drained to ground at the opposite side as in G2, then an electric field $\approx V_i/W$ is created with a main component spanning opposite sides, where $V_i$ denotes the injection voltage ($\approx$ few $\mu V$ vs ground). This electric field attracts carriers to the drain side, and discourages carriers reflected from this drain side from reaching apertures $d$ at the injection side. In G1, $I_i$ is drained far off along the injection-side barrier $i-d$, and an electric field profile predominantly spanning the opposite sides is not similarly created. The electric field components created in G1 are less effective in repelling reflected carriers reaching apertures $d$ at the injection side. Hence G1 is more prone to ballistic signals due to reflected carriers. The configuration of current injection and drain also affects the formation of vortices, which affect the generation of the nonlocal resistances [7, 8, 15].