1. Introduction

The TEHL analysis of involute spur gears is a non-stationary problem. In this case, it is not only the load that varies, but entrainment velocity and contact geometry also vary as the gear teeth come into action. Moreover, surface roughness of gear teeth, and high shear stress behavior of solid-liquid lubricant vary significantly along the line of action. So it is very hard to get all these effects into a transient thermo-elastohydrodynamic solution for an involute spur gear.

Gear problem has historically been solved with different simplifications. The numerical solution of elastohydrodynamic lubrication (EHL) problems was described by Dowson and Higginson (1969). Many numerical solutions have been solved in the area ranging from thermo-elastohydrodynamic (TEHD) lubrication problems to transient TEHL problems. Wang and Cheng (1981) made a Grubin-type analysis of involute spur gear transmissions and obtained the minimum film thickness at several points along the line of action. Hua and Khonsari (1995) developed an isothermal full transient solution for involute spur gear by neglecting dynamic load in the model. Later, a full isothermal transient non-Newtonian EHL solution to the Reynolds equation was obtained by Larsson (1997). Wang, Tong and Yang (2004) presented the formulation of the transient TEHL problem in spur gears. The results showed that transient condition affects significantly the film temperature and film thickness of lubricant in the contact region. Mongkolwongrojn and Panichakorn (2012) studied TEHL rough surface spur gears under sudden overloads. The results showed that the effects of rough surface and sudden overloads are significant as well as friction coefficient, film temperature and film thickness of lubricant in the contact region. Sayles and Ioannides (1988) his study showed that particles larger than the oil film thickness and softer than the interacting surfaces would also hurt the surface, and reduce the life of the tribopair. Khonsari, Wang, and Qi (1989) formulated Reynolds equation and energy equation for non-Newtonian solid-liquid lubricants in line contact.
Yousif and Nacy (1994) obtained the improved lubrication characteristics by mixing soft MoS$_2$ and graphite particles with pure oil. Mongkolwongrojn, Aumpornsin and Thammakosol (2006) showed that the solid-liquid non-Newtonian lubricant affects significantly the film temperature and film thickness of lubricant in contact region under sudden load. Lubrecht, Ten Napel, and Bosma (1986) showed that the multigrid algorithm is the most efficient method for solving the highly nonlinear EHL problems with roughness effects. The multigrid technique has been developed to solve transient thermo-elastohydrodynamic lubrication (TEHL) by Osborn and Sadeghi (1992). Ai and Cheng (1994) presented the formulation of the transient rough EHL problem using a multigrid technique. The results showed that surface roughness affects significantly the transient pressure distribution in line contact. Wang, Chen, Hu and Wang (2006) presented a technique to generate non-Gaussian surfaces for mixed lubrication.

In this present research work, rough surface gears under TEHL with solid-liquid non-Newtonian lubricant were analysed based on full transient thermal elastohydrodynamic assumptions. Finite difference multigrid multilevel method with full approximate scheme techniques were implemented to calculate the transient TEHL lubrication of rough surface gear teeth with various percent of particle concentration and various sizes of MoS$_2$ particle for the solid-liquid non-Newtonian lubricants. Minimum film thickness, maximum film temperature and friction coefficient were determined at different contact points along the line of action.

2. Nomenclature

| Symbol | Description |
|--------|-------------|
| b      | Semi-width of Hertzian contact under load, m |
| $b = R_0 \left(8W_0^2/\pi\right)^{1/3}$ |
| $C_{UT}$ | Transient dimensionless entrainment velocity |
| $C_{UT} = \bar{u}/u_0$ |
| $E_{1/2}$ | Elastic modulus of pinion/gear, Pa |
| $E_s$ | Elastic modulus of gear surface, Pa |
| $E'$ | Effective elastic modulus, Pa |
| $2/E' = (1 - v_1^2)/E_1 + (1 - v_2^2)/E_2$ |
| $f_s$ | Friction coefficient of solid lubricant particle |
| $h_0$ | Rigid central film thickness, m |
| $H_0$ | Dimensionless rigid central film thickness |
| $k$ | Thermal conductivity of lubricant, W/m-K |
| $k$ | Dimensionless Thermal conductivity of lubricant, $\bar{k} = k/k_0$ |
| $K_{T2}$ | Constant in energy equation |
| $K_{T4}$ | Constant in energy equation |
| $n$ | Power law index |
| $P$ | Dimensionless film pressure, $P = p/P_H$ |
| $r_a$ | Base cycle radii of pinion, m |
| $r_b$ | Base cycle radii of gear, m |
| $R_1$ | Radii of curvature of pinion teeth, m |
| $R_2$ | Radii of curvature of gear teeth, m |
| $S$ | Distance along line of action, m |
| $t$ | Time, s |
| $T$ | Lubricant film temperature, K |
| $T_1$ | Surface temperature of pinion, K |
| $u$ | Lubricant film velocity, m/s |
| $u_1$ | Pinion teeth surface velocity, m/s |
| $u'$ | Transient load of gear teeth, N |
| $x$ | Coordinate, m |
| $z$ | Coordinate, m |
| $z_1$ | Viscosity-Pressure index |
| $C_{RT}$ | Transient dimensionless curvature sum |
| $C_{RT} = R_X/R_0$ |
| $C_{WT}$ | Transient dimensionless load, $C_{WT} = u'/u_0$ |
| $D(X)$ | Dimensionless of combined surface roughness |
| $E_p$ | Elastic modulus of micro-particle, Pa |
| $E_{ps}$ | Equivalent elastic modulus of micro-particle and gear surface, Pa |
| $f$ | Friction coefficient |
| $h$ | Lubricant film thickness, m |
| $H$ | Dimensionless film thickness, $H = (R_0/b^2)h$ |
| $I^*$ | Dimensionless second invariant, $I^* = (S_0/H)^2$ |
| $k_0$ | Thermal conductivity of lubricant at ambient pressure, W/m-K |
| $K_{T1}$ | Constant in energy equation |
| $K_{T3}$ | Constant in energy equation |
| $m_0$ | Apparent viscosity at the shear rate of unit, Pa-s |
| $p$ | Lubricant film pressure, Pa |
| $P_H$ | Maximum Hertzian pressure, Pa |
| $P_H = E'(W_0/2\pi)^{1/2}$ |
| $R_0$ | Pitch circle radii sum, m |
| $1/R_0 = 1/r_a \sin \varphi + 1/r_b \sin \tilde{\varphi}$ |
| $R_X$ | Curvature sum, m, $1/R_X = 1/R_1 + 1/R_2$ |
| $S_0$ | Slip/slide ratio, $S_0 = (u_2 - u_1)/u$ |
| $\tilde{t}$ | Dimensionless time, $\tilde{t} = (u_0/b)\bar{t}$ |
| $T_0$ | Inlet film temperature, K |
| $T_2$ | Surface temperature of gear, K |
| $u_0$ | Reference velocity, m/s |
| $u_2$ | Gear teeth surface velocity, m/s |
| $u_0'$ | Reference load of gear teeth at pitch point, N |
| $X$ | Dimensionless coordinate, $X = x/b$ |
| $Z$ | Dimensionless coordinate, $Z = z/h$ |
| $\beta$ | Coefficient of thermal expansivity |
3. Governing Equations

In this analysis, gear tooth load is assumed to act along the line of action. The gear teeth are assumed to be rigid and the dynamics of the gear are neglected. The pitch error is assumed to be very small and will not influence the load. With these assumptions the load will be constant as long as two pairs of gear teeth carry the total load. For low particle concentration in the solid-liquid lubricant, single-phase flow can be assumed as Khonsari, et al. (1995) and Mongkolwongroj, et al. (2006). The time-dependent thermo-elastohydrodynamic lubrication of rolling/sliding line contacts can be solved simultaneously by using the modified Reynolds, elasticity, and energy equations to obtain pressure, film temperature, film thickness, friction coefficient and powder load carrying.

3.1. Time Dependent Modified Reynolds Equation

The relationship between shear stress and shear rate of non-Newtonian lubricant in this work can be approximated using a power-law viscosity model.

\[
\tau_{xz} = \mu \frac{\partial u}{\partial z} \quad \text{and} \quad \tau_{yz} = \mu \frac{\partial v}{\partial z}
\]

(1)

Where the equivalent viscosity \( \mu = m_0 \mu^*_s \), where \( \mu^*_s = \left( \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 \right)^{\frac{1}{2}} \)

The dimensionless modified Reynolds equation for the transient thermal line contact problem with consideration of the temperature dependency of viscosity and density across the film (Yousif and Nacy, 1994) have been formulated using perturbation technique.

\[
\frac{\partial}{\partial x} \left( \Phi \frac{\partial P}{\partial x} \right) = \Lambda \left\{ C_{UT} (\bar{T}) \frac{\partial}{\partial x} (\bar{\rho} T) + \frac{\partial}{\partial t} (\bar{\rho} T) \right\}
\]

(2)

Where

\[
\Phi = \frac{\bar{\rho} T^2}{\bar{\rho}_U}, \quad \Lambda = \frac{12u_0 \mu_0 R_0^2}{b^3 P_H}, \quad \bar{\mu}_U = \bar{\mu}_R n K_{SH} \left( \bar{T}^* \right)^{\frac{\nu}{\nu + 1}} \quad \text{and} \quad K_{SH} = \frac{m_0}{\mu_0} \left( \frac{u_0 R_0}{b^2} \right)^{\frac{\nu}{\nu + 1}}
\]

The boundary conditions are

\[
P(X_{inlet}; \bar{T}) = P(X_{exit}; \bar{T}) = (\partial P/\partial x)_{X=X_{exit}} = 0
\]

The apparent viscosity in the power-law model needs to be included as a correction factor for viscosity temperature-pressure and the correction factor for solid particles in the lubricants according to Rylander (1966) and Roelands (1969). The dimensionless apparent viscosity can be written as

\[
\bar{\mu} = \frac{\mu}{\mu_0} = \bar{\mu}_R \bar{\mu}_P \bar{\mu}^*_S
\]

(3)

where

\[
\bar{\mu}_R = \exp \left[ \ln \mu_0 + 9.67 \left\{ -1 + \left( 1 + 5.1 \times 10^{-9} P_T P \right) \right\} \right] - \gamma T_0 (\theta - 1)
\]

\[
\bar{\mu}_P = 1 + 2.5 \lambda P_0 / \lambda P_0 + (1 - \lambda) \rho P \quad \text{and} \quad \bar{\mu}^*_S = K_{SH} \left( \bar{T}^* \right)^{\frac{\nu}{\nu + 1}}
\]
According to Dowson and Higginson (1966); the density of the solid-liquid lubricant obeys the following relationship as

\[ \rho = \rho_0 \rho_D \ \text{(4)} \]

where \[ \rho_D = \left[ 1 + 0.6 \times 10^{-9} P_{R0} P_f \left( 1 + 1.7 \times 10^{-9} P_{R0} P_f \right) \right] \left( 1 - \beta T_0 (\theta - 1) \right) \], and \[ \rho_p = 1 - \lambda (1 - \rho_0/\rho_p) \]

The film thickness, including the deformation of surface under line contact, is given as

\[ H = H_0 + \frac{X^2}{2E_{RT}} + D(X) - \frac{1}{\pi} \int_{X_{in}}^{X_{out}} P(X') \ln |X - X'| dX' \ \text{(5)} \]

Where \( D(X) \) is the dimensionless combined surface roughness of gear and pinion. In this calculation, the random roughness distribution was generated numerically and the ratio of the standard deviation to the combined root mean square roughness is constant equal to 0.5.

### 3.2. Load carrying capacity

The total load carrying capacity of the solid-liquid lubricant film consists of two parts: the first load component results from deformation of particles, \( w_p \), and the other load component is due to hydrodynamic pressure, \( w_f \), where

\[ w_f = \int_{X_{in}}^{X_{out}} p(x) dx \ \text{(6)} \]

Consider a spherically shaped particle in the contact region under the action of normal load, the particle may undergo plastic deformation. For a particle that deforms plastically when the mean contact pressure reaches the hardness of particle, \( H_0 \), the load carrying capacity by a plastically deformed was written by Hua and Khonsari (1996).

\[ w_p = \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \left( 9 \pi^3 H_0^3 \frac{d_p}{E_{ps}} + p(x) u_p A_{ipl} \right) \text{ where } \frac{2}{E_{ps}} = \frac{1 - \nu_p^2}{E_p} \frac{1 - \nu_s^2}{E_s} \ \text{(7)} \]

Where \( A_{ipl} \) is the contact area of an individual particle due to plastic deformation. Equations give the load carrying capacity of a single particle due to plastic deformation and \( N_x \) and \( N_y \) are numbers of particles that come in contact with the surface in x-directions and y-directions respectively. The number of particles can be calculated as (Sayles and Ioannides, 1988):

\[ N_j = \left[ \frac{V_f}{l_1 l_2 \pi d_p} \left( \rho_0/\rho_p \right) \frac{1}{1 - \lambda \left( 1 - \rho_0/\rho_p \right)} \right] \]

where \( \lambda \) is the concentration of particles by weight, \( V_f \) is the volume of the solid-liquid lubricant within the contact region and \( l_j \) is the effective length along which the particles deform. For line contact configuration where \( l_y = 1 \), the total load carried by the solid-liquid lubricant can be expressed as

\[ w_T = w_f + w_p \]

The total load carrying capacity of the lubricant is due to hydrodynamic action. The dimensionless form of load balance equation is

\[ \int_{X_{in}}^{X_{out}} P(X) dX = C_{WT} \left( \frac{w_T - w_p}{w_T} \right) \left( \frac{\pi}{2} \right) \ \text{(8)} \]

The friction coefficient on pinion tooth surface is defined as

\[ f = \frac{\mu_0 R_0}{w_0 b C_{WT}} \int_{X_{in}}^{X_{out}} \left( \mu \frac{\partial u^*}{\partial Z} \right)_{Z=0} dX + \frac{f_p E R_0 W_p}{w_0 C_{WT}} \]
3.3. Time Dependent Energy equation

The contact surfaces can be approximated to a semi-infinite solid. From energy equation (Hua and Khonsari, 1996), the time-dependent dimensionless energy equation has been formulated by ignoring the heat conduction in the x and z directions and can be written as

\[
\frac{\partial^2 \theta}{\partial Z^2} = K_{T1} \frac{\hat{H}}{k_p} \left( \frac{\partial \theta}{\partial \bar{X}} + u^* \frac{\partial \theta}{\partial \bar{X}} \right) - K_{T2} \frac{\hat{\mu}}{k_p} \left( \frac{\partial u^*}{\partial \bar{X}} \right)^2 - K_{T3} \frac{\hat{H}^2}{k_p} \left( \frac{\partial \bar{P}}{\partial \bar{X}} + u^* \frac{\partial \bar{P}}{\partial \bar{X}} \right) - K_{T4} \left| S_{0} C_{UT} \right| \frac{H^2 f_{p} W_{p}}{k_p A_h} \tag{10}
\]

where

\[
K_{T1} = \frac{\mu_0 \rho \omega_c \beta_3}{k_0 R_0^2} \quad , \quad K_{T2} = \frac{\mu_0 h_0^2}{k_0 T_0} \quad , \quad K_{T3} = \frac{\beta \mu_0 \beta_3^2 P_h}{k_0 R_0^2} \quad \text{and} \quad K_{T4} = \frac{2 \mu_0 E' b}{k_0 T_0}
\]

The boundary conditions of the energy equation (Carslaw and Jaeger, 1959) are

\[
\theta(X_{inlet}) = 1 \quad \text{and} \quad \theta_{1/2} = 1 \pm \frac{k_0 R_0}{\sqrt{2 \mu_0 \beta_3^2 / k_0 T_0}} \int_{X_{inlet}}^{X} \frac{\hat{K}}{H} \left( \frac{\partial \theta}{\partial Z} \right)_{Z=1/2} dX'
\]

The effect of pressure on thermal conductivity has been implemented in the thermal EHD calculation by Wang, Cusano and Conry (1991).

\[
\bar{k}_p = 1 + \frac{\alpha_{K1} P_h P}{1 + \alpha_{K2} P_h P} \tag{11}
\]

3.4. Computational procedure

The dimensionless time dependent Reynolds equation, elasticity equation, energy equation and fluid film load equation of the two surfaces in line contact with micro-particles blended with non-Newtonian lubricant are simultaneously solved using finite difference method and multi-grid multi-level with full approximation scheme technique. The TEHL system is solved on the domain \([X_{inlet}, X_{exit}] = [-5.5, 2.5]\) with four difference level of grid points which are 512, 1024, 2048 and 4096 points respectively. The calculation of correction scheme of pressure, temperature and fluid film load are obtained for each grid point level to improve the convergence rate. The convergence criteria of pressure, temperature and fluid film load are adopted as follows:

\[
\frac{\sum_{i=0}^{N} |P_{i+1}^n - P_i^n|}{\sum_{i=0}^{N} P_i^n} \leq 0.0001 \quad , \quad \frac{\sum_{i=0}^{N} |\bar{P}_{i+1}^n - \bar{P}_i^n|}{\sum_{i=0}^{N} \bar{P}_i^n} \leq 0.0001 \quad \text{and} \quad \left| 1 - \frac{2}{\pi C_{WT}} \int_{X_{inlet}}^{X_{exit}} P(X, \bar{r}) dX \right| \leq 0.0001
\]

4. Results and Discussion

The gear data and the properties of liquid lubricant, solid particle used in the analysis are given in Table 1 and Table 2. These gear data correspond to a commercially available gearbox.

The contact between gear teeth at a distance \( \bar{S} \) from the pitch line in a pair of involute gear teeth having radii \( r_a \) and \( r_b \) and a pressure angle \( \phi \) can be represented by two circular cylinders rotating with the same angular velocity \( \omega_a \) and \( \omega_b \) as the gear teeth themselves. The contact radii vary along the line of the action and tooth surface velocities can be obtained (Mongkolwongrojn and Panichakorn, 2012). Fig. 1 shows the variation of the dimensionless idealized load, slip/slide ratio, dimensionless equivalent curvature and entrainment velocity along the line of action. The dimensionless load \( (C_{WT}) \) is 0.5 at the approach point A and suddenly increased to 1.0 at point B \( (\bar{S} = -1.10 \text{ mm}) \) when the load was carried by one pair of teeth beginning at that moment. The next pair of teeth comes into action at point D \( (\bar{S} = 0.90 \text{ mm}) \) and the load is suddenly decreased to 0.5 again. The amount of sliding reaches its highest level at the approach point. There is no sliding at the pitch point. The slip ratio reaches its highest level at the approach point. The entrainment velocity and the equivalent curvature have their lowest levels at approach point (point A) too. That means approach point might be the most critical point for a lubricant film thickness point of view.

Fig. 2 shows the variation of minimum film thickness along the line of action of the gear tooth for lubricant with smooth surface and with 0.10 \( \mu m \) of combined surface roughness amplitude and the minimum film thickness for solid-liquid lubricant (10% MoS2 by weight) with 0.10 \( \mu m \) of combined roughness. The minimum film thickness of solid-liquid lubricant is higher than the minimum film thickness for liquid lubricant along the line of action because of load...
Table 1  Gear data and lubricant properties

| Gear Material       | UNB C61300 |
|---------------------|------------|
| Number of teeth     | 35:140     |
| Module, mm         | 2          |
| Contact ratio      | 1.786      |
| Pinion speed, rpm  | 1,000      |
| Nominal pressure angle, degree | 20        |
| Teeth width, mm    | 20         |
| Transmitted power, kW | 10        |
| Elastic modulus of pinion and gear, GPa | 117         |
| Density of the teeth of pinion and gear, kg/m³ | 7,950      |
| Poisson ratio of the teeth of pinion and gear | 0.28       |
| Specific heat of the teeth of pinion and gear, J/kg·K | 736        |
| Combined surface roughness amplitude (R_{RMS}), μm | 0.10       |
| Inlet temperature of lubricant, K | 313.15     |
| Ambient density of the lubricant, kg/m³ | 892.8      |
| Ambient viscosity of the lubricant, Pa·s | 0.195      |
| Viscosity-Pressure index (ζ1) | 0.5685     |
| Coefficient of thermal expansivity (βθ), K⁻¹ | 0.00074    |
| Inlet thermal conductivity of lubricant, W/m·K | 0.126      |
| Specific heat of lubricant, J/kg·K | 1,870      |
| Power law index (n) - solid-liquid lubricant | 1.00969    |
| Power law index (n) - liquid lubricant | 1.000      |

Table 2  Physical properties of solid lubricant particle

| Solid particle properties       | MoS₂       |
|---------------------------------|------------|
| Brinell hardness, GPa           | 3.139      |
| Modulus of elasticity, GPa      | 34         |
| Poisson ratio                   | 0.13       |
| Density, kg/m³                  | 4,800      |
| Friction coefficient            | 0.10       |
| Particle diameter, μm           | 2.0        |

Fig. 1  Variation of the load, slip/slide ratio, equivalent curvature and entrainment velocity along the line of action.

carried by the solid particles in lubricant. At the approach point (\(S = -4.97\) mm), the film thickness is at its lowest value; the minimum film is 0.17 μm due to squeeze effect. The transient effect is the most pronounced after points B and D. When the load is suddenly increased at point B (\(S = -0.90\) mm), the minimum film thickness first increases from 0.84 μm to 0.86 μm for the liquid lubricant, and from 0.97 μm to 0.99 μm for solid-liquid lubricant; then it decreases to 0.76 μm for the liquid lubricant, and to 0.90 μm for solid-liquid lubricant, before it increases again. It never reaches an equilibrium state until the load is halved again. When the next pair of teeth comes into action at point D (\(S = 0.40\) mm), the load is suddenly decreased. The minimum film thickness first increases from 0.92 μm to 1.04 μm for the liquid lubricant and 1.05 μm to 1.19 μm for solid-liquid lubricant; then it decreases until it recovers at 1.20 mm approximately, where the minimum film thickness values are 0.83 μm and 0.99 μm for liquid lubricant and solid-liquid lubricant, respectively.

The variation of friction coefficient on pinion teeth and temperature rise of oil film for liquid lubricant and that for solid-liquid lubricant with rough surface gear teeth along the line of action are presented in Fig. 3 and Fig. 4. The friction coefficient and temperature rise of oil film for the solid-liquid lubricant along the line of action are lower than those for liquid lubricant except the contact interface near pitch point. At the approach point (point A, \(S = -4.97\) mm), the friction coefficient and maximum film temperature are 0.164 and 107.71 °C, respectively, for liquid lubricant and 0.158 and 107.37 °C, respectively, for solid-liquid lubricant when contact moves along the line of action. The first peak value of friction coefficient and the maximum film temperature are at \(S = -4.83\) mm and at \(S = -4.63\) mm., where the friction coefficient and maximum film temperature are 0.219 and 238.07 °C for liquid lubricant and 0.203 and 217.47 °C for the
solid-liquid lubricant, respectively and then decreases until the load is suddenly increased. At \( S = -0.40 \) mm, the friction coefficient and maximum film temperature are 0.516 and 254.25 °C, for liquid lubricant and 0.451 and 250.37 °C, for the solid-liquid lubricant respectively and their values decrease to the minimum value at the pitch point. At the pitch point (point C), the friction coefficient and maximum film temperature for solid-liquid lubricant have a higher value than the value for liquid lubricant, due to friction force and friction heating of solid particle. These values are 0.077, 61.73 °C and 0.079, 63.08 °C for liquid lubricant and solid-liquid lubricant, respectively. The friction coefficient is approximately zero and the maximum temperature is near the ambient temperature because there is no sliding. Adjacent to the approach point, the friction coefficient and maximum temperature rise are large because of the high load. The highest temperature rise occurs at the approach point, due to large slip ratio. When the line of action shifts from the pitch point, film temperature and friction coefficient rise up again. At \( S = +0.30 \) mm, the maximum values of friction coefficient and film temperature are 0.342 and 115.85 °C for liquid lubricant, and 0.285 and 105.72 °C for solid-liquid lubricant. When the next pair of teeth comes into action at point D, the film temperature and friction coefficient suddenly decrease and then increase again when contact moves along the line of action. One can see that the maximum temperature rise at the approach point is higher than the maximum temperature at the recess point because the slip ratios at the approach point are larger than those of the recess point.

Fig. 5 shows the minimum film thickness, friction coefficient and maximum film temperature at the approach point A, at the sudden increased in load, point B and at pitch point C as a function of combined surface roughness for a solid-liquid lubricant. The results show that the combined surface roughness has significant effect on film temperature and friction coefficient in solid-liquid TEHL of spur gears. Minimum film thickness decreases but friction coefficient and maximum
film temperature increase significantly when the combined surface roughness is increased. At approach point (point A), minimum film thickness, friction coefficient and maximum film temperature are 0.32 μm, 0.075 and 59.73 °C, respectively for smooth surface and they are 0.17 μm, 0.158, 107.37 °C and 0.14 μm, 0.183 and 128.81 °C when combined surface roughness are 0.10 μm and 0.12 μm, respectively. At point B, the load suddenly increased, the minimum film thickness, friction coefficient and maximum film temperature are 1.02 μm, 0.118 and 59.09 °C for smooth surface, and they are 0.98 μm, 0.220, 218.27 °C for combined surface roughness are 0.10 μm and 0.97 μm, 0.233 and 254.75 °C for combined surface roughness of 0.12 μm. At pitch point (point C), minimum film thickness, friction coefficient and maximum film temperature are 1.16 μm, 0.008 and 45.41 °C for smooth surface and they are 1.06 μm, 0.079, 63.08 °C and 1.03 μm, 0.089 and 67.35 °C when the amplitudes of combined surface roughness are 0.10 μm and 0.12 μm, respectively.

The variation of friction coefficient of pinion tooth, minimum film thickness and temperature rise of oil film for solid-liquid lubricant along the line of action are presented in Fig. 6. For rough surface gear teeth with liquid lubricant, the minimum film thickness, friction coefficient and maximum film temperature at the approach point (point A) have insignificant affected. At point with load suddenly increased(point B), the minimum film thickness increases when concentration of solid particles increases; 0.86 μm of film thickness for liquid lubricant and 0.99 μm and 1.05 μm of film thickness when the concentrations of solid particle are increased from 10% to 20% by weight, respectively. The friction coefficient and maximum film temperature decrease when concentration of solid particle is increased. Friction coefficient and maximum film temperature are 0.254 and 224.90 °C, respectively, when lubricant is purely liquid lubricant, and they

![Fig. 4 Variation of maximum temperature rise along the line of action for liquid lubricant and solid-liquid lubricant.](image1)

![Fig. 5 Variation of minimum film thickness, friction coefficient and maximum temperature rise with amplitude combined surface roughness for solid-liquid lubricant.](image2)
decrease to 0.220 and $218.27 \degree C$ when the concentration of solid particles is 10% by weight. For concentration of solid particles is 20% by weight, the friction coefficient and the maximum film temperature are 0.194 and $205.48 \degree C$, respectively. The value of friction coefficient and maximum film temperature for solid-liquid lubricant are lower than the values for liquid lubricant of point B because of the effect of the load carrying by particles. At the pitch point (point C) for solid-liquid TEHL in spur gears, the minimum film thickness, friction coefficient and maximum film temperature increase when concentration of solid particle is increased due to pure rolling.

![Graph](image)

Fig. 6 Variation of minimum film thickness, friction coefficient and temperature rise of oil film on pinion surface with powder concentration for solid-liquid lubricant.

In this paper, we found that the lowest film thickness occurs at the approach point(point A) that means the approach point might be the most critical point in gear design under TEHL. The load of gear teeth may be increased more than the normal operating condition due to a sudden overload or impact of gear teeth. The characteristics of spur gears were investigated for sudden overload at 125% of normal load with liquid lubricant and solid-liquid lubricant as shown in Fig 7. During sudden overload or impact load at the approach point (point I) for 0.10 $\mu m$ of combined surface roughness, the minimum film thickness for liquid lubricant and for solid-liquid lubricant are close to that for normal operating conditions because the minimum film thickness depends on the mechanism of the squeeze effect; the minimum film thickness at the approach point are 0.169 $\mu m$, 0.167 $\mu m$ and 0.168 $\mu m$ for normal load conditions, sudden overload with liquid lubrication and sudden overload with solid-liquid lubricant, respectively. The friction coefficients and maximum film temperatures with sudden overload at the approach point are greater than that for normal load condition; they are 0.164 and 107.71 $\degree C$, 0.200 and 124.84 $\degree C$ and 0.172 and 119.18 $\degree C$ for normal load condition and sudden overload with liquid lubrication and for sudden overload with solid-liquid lubricant, respectively. When the contact moves forward along the line of action, the
minimum film thickness, friction coefficient and maximum film temperature rapidly increase. At point J, the minimum film thickness, friction coefficient and maximum film temperature for normal load condition with liquid lubricant, sudden overload condition with liquid lubricant and sudden overload operating condition with solid-liquid lubricant are 0.586 μm, 0.184 and 174.30 °C, 0.522 μm, 0.213 and 215.32 °C and 0.621 μm, 0.195 and 200.31 °C, respectively. When the load is suddenly decreased at point K, the minimum film thickness for sudden overload with liquid lubricant and with solid-liquid lubricant rapidly increase to 0.654 μm and 0.773 μm, respectively but the friction coefficient and maximum film temperature for liquid lubricant and with solid-liquid lubricant rapidly decrease to 0.175 and 158.61 °C and 0.157 and 141.79 °C, respectively as shown in Fig 8, Fig. 9 and Fig. 10.

For sudden overload, the minimum film thickness for solid-liquid lubricant is larger than the minimum film thickness for liquid lubricant but friction coefficients and maximum film temperatures for solid-liquid lubricant are smaller than the friction coefficients and maximum film temperatures for liquid lubricant because of the load carried by solid particle in the lubricant and the change of lubricant properties of non-Newtonian fluid.

Fig. 11 shows the variation of friction coefficient on pinion tooth, minimum film thickness and temperature rise for solid-liquid lubricant gear teeth along the line of action under sudden overload at the approach point(point I). At the approach point, the concentration of solid particle has almost no effect on the minimum film thickness and maximum film temperature, but the friction coefficient first decreases until concentration of solid particle is more than 10% by weight, then it increases again. When the contact moves forward along the line of action to point J and point K, the minimum
5. Conclusions

The time dependent modified Reynolds equation, energy equation and particle load carrying capacity equation were formulated for spur gear with solid-liquid lubricant to obtain minimum film thickness, maximum film pressure, friction coefficient and maximum film temperature in the contact region for rough involute spur gear. The characteristics of spur gear under TEHL were examined for both liquid lubricant and liquid lubricant mixed with micro particles. The load is kept constant as long as two pair of teeth carried the total load and it is doubled when only one pair is in action. The following can be concluded as:

1. For solid-liquid lubricant, the minimum film thickness is larger than the minimum film thickness for liquid lubricant along the line of action, but friction coefficient and maximum film temperature are lower value than that for liquid lubricant except the values of friction coefficient and maximum film temperature at pitch point.

2. For rough surface spur gear, the minimum film thickness is very small but the maximum film temperature and friction coefficient are large as the amplitude of combined surface roughness increased. Therefore, the surface roughness of gear teeth have significantly affect and should be considered in gear design application.
3. When concentration of solid particle is increased, the minimum film thickness increases but friction coefficient and maximum film temperature decrease. The sizes of solid particles in solid-liquid lubricant have no effect for rough solid-liquid TEHL in spur gear.

4. For spur gears under sudden overload with solid-liquid lubricant at the approach point, the film temperature and friction coefficient decrease but minimum film thickness slightly increases, when compared with the minimum film thickness for liquid lubricant that means solid-liquid lubricant can be protected the damage of gear teeth under severe operating condition.

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