Paper Laser: a step towards a time scale generation from an ensemble of optical clocks

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Abstract. In this paper a simple and innovative technique to combine n optical frequencies with the aim to produce a virtual laser with superior metrological characteristics is introduced. The algorithms to combine a number of clocks to produce a virtual clock, which is also referred as paper clock, are well known. An example of this is the statistical generation of the UTC time scale by the Bureau International des Poids et Mesures (BIPM) using a recursive algorithm (ALGOS). A similar algorithm to combine n optical frequencies, all of them with same nominal value, to produce a “paper laser” whose frequency is known through its difference with respect to the optical frequencies of the ensemble is proposed here. As a demonstration of this, three optical frequencies stabilized to the D2 Cs-133 line, all of them with similar frequency stability were experimentally combined. A paper laser has been produced during hours whose frequency stability is about 3^{-1/2} times with respect to the original optical frequencies. This technique can be applied to combine ultra-stable optical frequencies to produce a paper laser that can be materialized by correcting one of the real optical frequencies of the ensemble. The robustness and stability of a paper laser is very attractive to produce a time scale from its operation.

1. Introduction

In time and frequency metrology, optical frequencies present a lot of advantages in comparison with radio frequencies (RF) based on quartz or sapphire crystals. Some optical oscillators are able to have quality factors (Q) ten thousand times bigger than RF oscillators due to the ratio between the central frequency (10^{14} Hz) and its linewidth (around few Hz). State-of-the-art laser stabilization usually involves phase-locking to a single mode of a passive ultra-stable Fabry-Pérot (FP) cavity. That has allowed a vertiginous advance of the optical clocks during the past three decades and that, in turn, has motivated an international debate about the need to redefine the SI second in terms of an atomic transition in the optical region in order to take advantage of such a grade of stability and accuracy [1]. Also, there are a variety of fundamental physics tests that could be implemented as application of ultra-stable and accurate optical frequencies to increase our understanding of different principles of nature, which remains unexplored.

In addition, using frequency combs, the stability of ultra-stable optical oscillators can be transferred to a wide range of frequencies in the electromagnetic spectrum ranging from RF to the visible [2-5] achieving an outstanding stability. In this paper we propose a simple and innovative technique (similar
to those used in the generation of averaged time scales) to improve the stability of an optical oscillator combining n lasers whose frequencies are previously stabilized to an optical reference.

1.1 Motivations

The production of time scales based on the operation of optical clocks is one of the requirements to redefine the SI second in terms of an optical frequency. In computation of time scales, it is desirable to operate simultaneously n different optical clocks. Even more, as in the microwave (MW) clocks case, it is certainly an interesting idea to combine an ensemble of optical clocks to produce a unique virtual optical clock with superior metrological characteristics compared to any member of the ensemble. Based on that, we explore the idea of combining an ensemble of optical frequencies to produce a unique virtual frequency. As a first approach, and to demonstrate the feasibility of this proposal, a simple algorithm has been developed and implemented experimentally to incorporate the frequency differences between three optical oscillators into a mathematical entity, named “paper laser” (PL).

2. The “paper laser” concept

In order to calculate a virtual optical oscillator, an ensemble of n optical oscillators with similar metrological characteristics is required and the frequency differences between them should be determined to feed a recursive algorithm. The PL calculated can be materialized using a member of the ensemble to correct its frequency in almost real time by using an acousto-optical modulator (AOM). In figure 1 a general simplified diagram of a PL generation and materialization is presented. Below the particular algorithm used in this work is described briefly.

![Figure 1. Simplified diagram of generation and materialization of a PL using an ensemble of n independent optical oscillators. AOM: acousto-optical modulator; DDS: direct digital synthesizer.](image)

2.1 Algorithm

Using data obtained from measurements of frequency differences; a simple model, represented by the equation (1), is used to combine the optical frequencies into a single virtual optical frequency $v_{PL}$. The frequency difference $f_k(t+\tau)$ at time $t+\tau$ between the PL optical frequency $v_{PL}$ and the optical frequency $v_k$ of the k-th member of the ensemble is given by the recursive equation:
where \( f_{jk} \) is the frequency difference between the optical frequencies of lasers \( j \) and \( k \) and \( \omega_j \) represents the weight of the \( j \)-th optical frequency of the ensemble. Of course, the condition of normalized weights is given by \( \sum \omega_j = 1 \). For simplicity, and due to characteristics of our experimental setup, the weights are all equal to \( 1/3 \) \( (\omega_1 = \omega_2 = \omega_3 = 1/3) \). Moreover, the initial conditions on the optical frequency differences are fixed to zero, that is \( f_j(t = 0) = f_j(t = 0) = f_j(t = 0) = 0 \) Hz.

3. Experimental setup

3.1 The optical frequency ensemble

Similarly to the case of MW clocks, an algorithm to combine optical frequencies could produce better results when clocks of the ensemble have similar metrological characteristics. With that in mind, this experiment started with a laser coupled to an ultra-low expansion (ULE) cavity to produce an ultra-stable optical frequency. The master laser is a commercial AlGaAs which is an extended cavity diode laser (ECDL) equipped with a low loss interference filter, 2 W maximum output power, 852 nm wavelength (near to Cs-133’s D2 line) and 20 kHz linewidth. Additionally, a commercial optical FP cavity made of ULE glass is used. The ULE cavity has a free spectral range of 1.49 GHz and a linewidth less than 2.3 kHz. In order to stabilize the ECDL to the optical cavity, the Pound-Drever Hall technique is used [6]. In figure 2, the experimental setup of the ultra-stable laser (USL) is presented.

![Figure 2](image)

Figure 2. ECDL stabilized to an optical cavity made of ULE glass (USL). Red solid lines represent laser's light and black dashed lines represent electrical connections. PBS: polarizing beamsplitter; M: mirror; \( \lambda/2 \): half-wave plate; \( \lambda/4 \): quarter-wave plate; L: lens; AOM: acousto-optical modulator; SLF: servo loop filter. Colour online.

Then, as shown in figure 3, the USL is used to monitor the most probable \( D_2 \) Cs-133 transition \( \left[ \left[ 6^2 S_{1/2}, F = 4 \right] \rightarrow \left[ 6^2 P_{3/2}, F' = 5 \right] \right] \). Using independent AOMs in cat’s eye configuration [7], three independent laser beams (obtained by splitting the USL beam) are “re-locked” to the cesium \( D_2 \) line using the polarized spectroscopy technique [8]. To avoid magnetic interferences from external fields, the cesium’s cells are located inside a double-layered \( \mu \)-metal magnetic shield. The process of this frequency locking degrades the original frequency stability of the ultra-stable optical frequency but it results in three independent optical frequencies \( (\nu_1, \nu_2 \) and \( \nu_3 \) ) with similar metrological characteristics.
3.2 Frequency measurements

Variations of frequency $RF_i$ fed into the i-th AOM ($i = 1, 2, 3$) are measured (figure 3) to monitor the time variations of the frequency difference $\nu_i - \nu_{USL} = 2RF_i$. Due to the fact that $\nu_{USL}$ is an ultra-stable frequency, the variations of $RF_i$ allow us to measure the short-term stability of the $\nu_i$ optical frequency ($\delta\nu_i \approx 2\delta RF_i$). The factor of 2 is associated to the double-pass configuration. In order to have simultaneous $RF_i$ measurements, an external trigger signal is sent to frequency counters each second and the readings are sent to a PC before the next trigger pulse.

4. Results and discussion

4.1 Stability analysis

Measurements were taken during 7 days (8 hours per day) to analyze a significant set of data in order to evaluate different kinds of effects affecting the experiment. The long-term drift of data, which is associated to frequency fluctuations of the ULE optical cavity, is suppressed when frequency measurements of each polarization spectroscopy experiment are subtracted given the fact that drift is common. Then, the algorithm uses these data and the PL is mathematically calculated every second. In order to evaluate the frequency stability of $f_k = \nu_{PL} - \nu_k$, the Allan deviation (ADEV) of the PL with respect to members of the ensemble is calculated. Preliminary results in figure 4 are promising because the stabilities of $f_k$ are smaller than the stabilities of the $\nu_k$ frequencies. That is, the stability of the PL is superior to the stability of any member of the ensemble. Also for completeness, the relative stabilities between USL and PL and the relative stabilities between USL and the optical frequencies.
\( (\nu_1, \nu_2 \text{ and } \nu_3) \) are shown. Therefore, a virtual optical oscillator with superior stability features has been created.

![Allan deviation of frequency differences between optical frequencies of the ensemble \( (f_{12}, f_{13}, \text{ and } f_{23}) \), between optical frequencies and the virtual optical frequency \( (f_1, f_2, \text{ and } f_3) \), between PL and USL, and between optical frequencies \( (\nu_1, \nu_2 \text{ and } \nu_3) \) and the USL.](image)

**Figure 4.** Allan deviation of frequency differences between optical frequencies of the ensemble \( (f_{12}, f_{13}, \text{ and } f_{23}) \), between optical frequencies and the virtual optical frequency \( (f_1, f_2, \text{ and } f_3) \), between PL and USL, and between optical frequencies \( (\nu_1, \nu_2 \text{ and } \nu_3) \) and the USL.

4.2 Noise analysis

It is well known that the power law noise implies particular slopes \( \mu \) in the Allan deviation plots. The stability for a generic atomic clock limited by Poisson noise is given by [9]:

\[
\sigma(\tau) \approx \frac{\delta \nu}{\nu_0} \frac{1}{\sqrt{N \tau}}
\]

(2)

where \( \delta \nu \) is the linewidth of the transition, \( \nu_0 \) is the clock frequency, \( N \) is the number of detected atoms per second and \( \tau \) is the averaging time. Analyzing the result of the best fit of the relative stability between the PL and the USL (Figure 5) by using an Allometric function \( \sigma(\tau) \sim A \tau^{-\mu/2} \), it is found that \( \mu = 0.37 \) and, according to equation (2), the stability of an optical oscillator, in the shot-noise limit, is proportional to \( \tau^{-\mu/2} \). As can be noticed in figure 5, there is an additional contribution to frequency noise that can be attributed to the spectroscopy technique used. In this experimental setup the changes in the offset of the error signal are associated to beam misalignments produced by temperature fluctuations, vibrations during the data acquisition process and alterations in the polarization of the laser beams. Those drifts are erroneously interpreted by servos as frequency changes impacting the values of the relative stability between oscillators. Therefore, \( \mu \) is smaller than the expected value \( (\mu = 1) \). As a perspective to avoid that added noise, modulation transfer (MT) spectroscopy will be used in future experiments. In that case, it is expected that PL continues to have the best metrological features and the stability should be better with respect to results presented in this paper.
Finally, as can be noticed from stability data of figure 4, short-term stability of the virtual optical frequencies is $3^{-1/2}$ times with respect to the optical frequencies of the ensemble as expected, in concordance with equation (2).

5. Conclusions
In this paper a simple and innovative technique to combine optical frequencies into a virtual optical frequency (also called Paper Laser) has been introduced. The power of this technique is that by using a mathematical algorithm, a virtual optical oscillator with superior frequency stability could be generated. In this paper it has been demonstrated that, under particular conditions, the resulting PL has superior frequency stability than those of the optical frequencies of the ensemble, as expected.

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