Ground state energy and quasiparticle gaps

in $\nu = \frac{N}{2N \pm 1}$ FQHE states

D.V.Khveshchenko

Physics Department, Princeton University,

Princeton, NJ 08544

and

Landau Institute for Theoretical Physics,

2, st.Kosygina, 117940, Moscow, Russia

Abstract

Applying the transformation of fermion operators to new fermion quasiparticles introduced by Halperin, Lee, and Read we estimate a scaling behavior of the ground state energy and quasiparticle gaps as a function of filling fraction for a "principal sequence" of FQHE $\nu = \frac{N}{2N \pm 1}$ states converging towards the gapless state at half filling. The exponent describing the shape of the cusp $\delta E(\nu) \sim |\delta \nu|^{\eta}$ is found to be greater than one and to depend nontrivially on the interaction potential. The dependence of quasiparticle gaps agrees with the results of recent measurements by R.R.Du et al.
The Fractional Quantum Hall Effect still remains to be among the most intensively studied subjects of the modern Condensed Matter Theory [1]. More generally, it forms part of the problem of the global phase diagram of interacting two-dimensional fermions in an external magnetic field.

Methodologically, understanding this seemingly special issue could, in principal, be crucially important in view of novel scenarios of spontaneous parity- and time-reversal symmetry violation due to strong interactions [2]. The parity-odd ground state resulting from such an instability is supposed to be characterised by a spontaneously generated flux of some fictitious magnetic field associated with a phase of the bi-local operator

$$\Delta(x, x') = \Psi^\dagger(x)\Psi(x')$$ [3].

Recently the theory of FQHE received a novel trend of development with a new look at the nature of states with even denominators [4], and in particular, the $$\nu = \frac{1}{2}$$ state which is of most experimental interest. The results of measurements of resistivity and surface acoustic wave attenuation indicate a presence of a Fermi surface (in the Luttinger sense) consistent with the density of carriers [5].

Because of the intrinsic degeneracy of a partially filled Landau level any analytical calculations in the framework of FQHE are quite complicated. Although numerous symmetry considerations, variational techniques and various phenomenological approaches (see [1] for references) provided an essential progress in understanding of the FQHE phenomena, a consistent method of microscopic calculations is still strongly required. Basically, most of present knowledge about the FQHE concerns those universal properties (such as a quantization of the Hall conductivity, quantum numbers and statistics of quasiparticles) which are independent, to a significant extent, of the details of the interaction. However the latter become very important if one intends to estimate those experimentally measurable characteristics as quasiparticle gaps or to establish the dependence of the ground state energy $$E(\nu)$$ on filling fraction $$\nu$$.

In Ref.[4] a new formalism to treat FQHE fractions with even denominators was proposed. The basic idea first discussed by Jain [6] is to attach to each fermion an even number of
fluxes of some fictitious ("statistical") field. Under this transformation fermions preserve their statistics, but at the mean field level an external magnetic field applied to fermions can be cancelled by an average fictitious one. One then obtains fermions in a zero net field interacting via fluctuations of the fictitious flux.

This procedure can be understood as a particular way to construct "proper zero order eigenstates" which lifts up the Landau level degeneracy as a result of the electron interaction $V(r) = \frac{e^2}{r}$. 

In what follows we shall restrict ourselves to the even-denominator case $\nu = \frac{1}{2}$. It was also proposed in [4] to treat odd-denominator "descendant" $\nu = \frac{N}{2N \pm 1}$ FQHE states within the same framework. Namely, one can keep attaching two flux quanta to each particle, the net field being equal to $B = \frac{2\pi \rho}{N}$, where $\rho$ is a fermion density. Then to find the effect of the original interaction $V(r)$ one has to consider a situation of the IQHE with additional interactions due to fluctuations of the fictitious flux.

In the present paper we study a dependence of the ground state energy on $N$ in the vicinity of half filling ($N >> 1$). More concretely, we intend to find the shape of the cusp $\delta E(\nu) \sim |\nu - \nu_c|^\eta$ at $\nu_c = \frac{1}{2}$. We also estimate a scaling behavior of the excitation gaps.

It is widely believed that the function $E(\nu)$ is not differentiable at all rational points and presents an example of a fractal. Evidently, this circumstance makes it impossible to get any analytic formula describing $E(\nu)$ at all values of $\nu$. However one can find the behavior of $E(\nu)$ on some restricted support, for instance, on a sequence of values $\nu = \frac{N}{2N \pm 1}$ and to determine an exponent $\eta$ for this case. It is quite possible, of course, that choosing another sequence one will obtain another value of $\eta$. Nevertheless, from the experimental point of view a determination of $\eta$ for a given sequence of FQHE states still has sense, because not all cusps are equally prominent, hence not all sequences are equally well observed.

We shall demonstrate an importance of fluctuations of the fictitious flux which change the dependence of the ground state energy on the form of interaction potential with respect to the result of the naive Hartree-Fock approximation. However, at least at weak couplings these strong fluctuations don’t make the result universal yet. In particular, we obtain that $\eta$
depends on the exponent \( \alpha \) governing a power-like decay of the bare potential \( V(r) \), although this dependence is quite nontrivial.

To proceed with calculations we use the conventional Lagrangian

\[
L = \int d^2r \Psi^\dagger \left( i \frac{\partial}{\partial t} - \frac{1}{2m} \left( -i \vec{\nabla} - \vec{A} \right)^2 + \mu \right) \Psi + \frac{1}{2} \int d^2r \int d^2r' \Psi^\dagger(r)V(r-r')\Psi(r') \tag{0.1}
\]

where \( \vec{A} \) denotes an external field and a chemical potential \( \mu \) corresponds to \( \nu = \frac{N}{2N \pm 1} \).

By introducing two flux quanta per fermion by means of the well-known "Chern-Simons transformation" one obtains an equivalent Lagrangian in terms of an additional gauge field \((\Phi, \vec{a})\) [4]:

\[
L = \int d^2r \Psi^\dagger \left( i \frac{\partial}{\partial t} - \frac{1}{2m} \left( -i \vec{\nabla} - \vec{A} - \vec{a} \right)^2 + \mu \right) \Psi \\
+ \frac{1}{32\pi^2} \int d^2r \int d^2r' \vec{\nabla} \times \vec{a}(r)V(r-r')\vec{\nabla} \times \vec{a}(r') + \Phi(\Psi^\dagger \Psi - \frac{1}{4\pi} \vec{\nabla} \times \vec{a}) \tag{0.2}
\]

where we identified density and "statistical" (gauge) flux fluctuations \((\delta \rho = \frac{1}{4\pi} \vec{\nabla} \times \delta \vec{a})\) due to the local constraint \( \frac{\partial L}{\partial \Phi} = 0 \). Then one can consider a mean field solution characterised by a uniform total field \( B = \frac{2\pi \rho}{N} \). In fact, the stability of this mean field solution might be crucially dependent on the strength of the interaction potential \( e \). It is unclear now whether such a solution becomes stable at some threshold value of \( e \) or arbitrarily small \( e \) are also acceptable. In what follows we assume that the threshold value (if any) is small enough compared with \( 1/N \).

In the absence of interactions the mean field energy at all \( N \) is, of course, equal to the kinetic energy of free fermions \( E_0(\nu) = \frac{\pi \rho^2}{m} \) (we remind the reader that the energy of 2D fermions occupying an integer number of Landau levels is degenerate with the energy of free fermions with a circular Fermi surface).

Formally putting \( e = 0 \) we have to recover the energy of a partially filled lowest Landau level \( E_0(\nu) = \frac{\pi \rho^2}{\nu m} \). It can be immediately seen that the role of gauge fluctuations is quite important. In particular, in the mean field picture one-particle excitations have a pseudogap \( \Delta = \frac{B}{m} \) which survives in the absence of any real interaction. This artifact has to disappear, of course, after a proper account of gauge fluctuations.
Although a complete account of those fluctuations doesn’t seem possible, one could assume that to analyze the effects of the interaction $V(r)$ it suffices to take into account the leading contributions corresponding to the conventional RPA. This approximation is supposed to be adequate at least in the vicinity of the half filled case (that is, at $N \gg 1$). We shall also comment on this point below.

In RPA one encounters the problem of calculating the determinant of a quadratic operator which governs gaussian fluctuations of the gauge field

$$K_{\mu\nu}(\omega, k) = \Pi_{\mu\nu}(\omega, k) + \frac{1}{4\pi} \left( \delta_{\mu\nu} \delta_{00} + \delta_{\mu0} \delta_{\nu i} \right) \epsilon_{ij} k_j + \delta_{\mu i} \delta_{\nu j} \frac{1}{(4\pi)^2} V(k)(\delta_{ij} k^2 - k_i k_j)$$  \hspace{1cm} (0.3)

where $\Pi_{\mu\nu}(\omega, k) = \langle J_\mu(-\omega, -k) J_\nu(\omega, k) \rangle$ denotes a fermion polarization operator in a uniform magnetic field $B = \frac{2\pi \rho}{N}$ and $V(k) \sim e^2 k^{n-2}$ stands for a Fourier transform of the interaction potential (in the case of the short-range interaction $V(r) \sim \delta(r)$ we put $\alpha = 2$).

The a priori knowledge of the effects of the gauge interaction at $e = 0$ allows one to use the following normalization of the RPA energy correction

$$E(\nu) = \int \frac{d\omega}{2\pi} \int \frac{d^2k}{(2\pi)^2} \text{Im} \left( \ln \text{Det} \hat{K} - \ln \text{Det} \hat{K}_0 \right)$$  \hspace{1cm} (0.4)

Expanding (4) as a series in $e^2$ we find the lowest order term in the form

$$E_1(\nu) = -\frac{1}{(4\pi)^2} \int \frac{d\omega}{2\pi} \int \frac{d^2k}{(2\pi)^2} \text{Im} \left( \frac{\Pi_{00} V(k) k^2}{\text{Det} \hat{K}_0} \right) = -\frac{1}{(4\pi)^2} \int \frac{d\omega}{2\pi} \int \frac{d^2k}{(2\pi)^2} V(k) k^2 \text{Im} < a_\perp(\omega, k) a_\perp(-\omega, -k) >$$  \hspace{1cm} (0.5)

where we introduced a propagator of the transverse component of the gauge field ($a_\perp = \vec{a} \times \vec{k}$). Notice, in passing, that the absence of poles in the propagator $< a_\perp(\omega, k) a_\perp(-\omega, -k) >$ at $\omega < \frac{B}{m}$ and $k < \frac{B}{\sqrt{\rho}}$ follows from the incompressibility of the mean field state.

To facilitate the calculation of (5) one has to find a treatable representation for the polarization operator $\Pi_{\mu\nu}(\omega, k)$. This subtle problem is a long-standing one. At arbitrary number of occupied Landau levels $N$ one has the following exact formulae ($\omega \neq 0$):

$$\Pi_{\mu\nu}(\omega, k) = \frac{B^2 m}{(2\pi)^2} \sum_{l=N}^{\infty} \sum_{n=0}^{N-1} \frac{(l-n)}{\omega^2 m^2 - B^2(l-n)^2 + i0} < n | J_\mu(k) | l > < l | J_\nu(k) | n > + \delta_{\mu i} \delta_{\nu j} \delta_{ij} \frac{\rho}{m}$$  \hspace{1cm} (0.6)
Three independent components of $\Pi_{\mu\nu}(\omega, k)$ can be expressed in terms of matrix elements of the density $J_0(k)$ and the transverse current $J_\perp(k)$ given by the formulae

$$< n|J_0(k)|l> = \frac{1}{m} e^{-x/2} \sqrt{\frac{Bn!}{l!}} (x) \frac{l-n}{2} L_l^{l-n}(x)$$  \hspace{1cm} (0.7)$$

and

$$< n|J_\perp(k)|l> = \frac{1}{m} e^{-x/2} \sqrt{\frac{Bn!}{l!}} (x) \frac{(l-n-x)L_l^{l-n}(x) + 2x dL_l^{l-n}(x)}{dx}$$  \hspace{1cm} (0.8)$$

where $x = \frac{k^2}{2B}$ and $L_l^{l-n}(x)$ is a Laguerre polynomial. One can easily find the determinant standing at (5) in terms of these components $Det \tilde{K}_0 = \Pi_{\text{od}}^2 - \Pi_{00}\Pi_\perp$.

Notice that to estimate (5) one has to know the behavior of $\Pi_{\mu\nu}(\omega, k)$ at large $\omega$ and $q$. Because of that the expressions (7,8) can be of practical use only at small $N$. In the opposite limit $N \gg 1$ it appears to be possible to use a semiclassical approach elaborated recently by Wiegmann and Larkin [7]. The essence of this method can be demonstrated on an example of the scalar component of the polarization operator $\Pi_{00}(\omega, k)$.

In the mixed representation the one-particle Green function can be written in the form

$$G(\epsilon, \vec{r}) = \frac{B}{2\pi} e^{iBxy - Br^2/4} \sum_{n=0}^{\infty} \frac{L_n(Br^2/2)}{\epsilon + \mu - B(n + 1/2) + i\delta}$$  \hspace{1cm} (0.9)$$

where $\delta = i0sgn\epsilon$. In a weak field (9) can be approximated by the expression

$$G(\epsilon, \vec{r}) \approx \frac{B}{2\pi} e^{iBxy} \sum_{n=0}^{\infty} \frac{J_0(\sqrt{2Br}r)}{\epsilon + \mu - B(n + 1/2) + i\delta}$$  \hspace{1cm} (0.10)$$

Using (10) one can find the following semiclassical formula

$$\Pi_{00}(\omega, k) \approx \frac{1}{4\pi} \sum_{n=0}^{N-1} \sum_{s=-n}^{N-1-n} \frac{sB^3m}{\omega^2m^2 - B^2s^2 + i0}((2nBk^2 - (sB - k^2/2)^2)^{-1/2}$$  \hspace{1cm} (0.11)$$

Comparing this expression with the integral representation of the zero field free fermion polarization

$$\Pi_{00}^{(0)}(\omega, k) = \int_0^{p_F} \frac{d^3p}{(2\pi)^3} \int_{k^2/2-pk}^{p_F^2-p^2} ds \frac{sm}{\omega^2m^2 - s^2 + i0}((p^2k^2 - (s - k^2/2)^2)^{-1/2}$$  \hspace{1cm} (0.12)$$

where $p_F = \sqrt{4\pi\rho}$, we see that (11) can be understood as a result of a ”discretization” of the scattering angle $\theta \rightarrow \cos^{-1} \frac{Bs+k^2/2}{pk}$ as well as energy levels $p^2/2 \rightarrow Bn$ in a weak magnetic field.
It can be readily shown that fluctuations of the gauge field are crucially important for a proper calculation of even the lowest order correction (5) to the ground state energy. Indeed, if one completely discards those fluctuations then the lowest order correction (5) amounts to the exchange energy in a magnetic field

\[ E_{\text{ex}}(N) = -\frac{1}{2} \int \frac{d^2k}{(2\pi)^2} V(k) \int d\omega \Pi_{00}(\omega, k) \]  

(0.13)

The most convenient way to proceed with (13) is to use a real space representation.

By integrating (9) over energy it is easy to show that an equal time polarization operator can be found in the following simple form

\[ \Pi_{00}(r) = G(r)G(-r) = \left(\frac{B}{2\pi}\right)^2 e^{-Br^2/2}  
\]

(0.14)

Although the integral (13) can be found exactly we shall apply the asymptotics of Laguerre polynomials in terms of Airy functions to find the leading term in \( \frac{1}{N} \) expansion

\[ e^{-x^2/2}L_N^1(x) \approx \frac{1}{2(2N)^{1/3}} Ai((4N)^{2/3}(\frac{x}{4N} - 1)) + ... \]  

(0.15)

As a result one arrives at the following formula

\[ E_{\text{ex}}(N) = -\frac{8}{3\pi^2} e^{2(\frac{pF}{2})^{2+\alpha}}(1 + \frac{c_0}{N^{1+\alpha}} + ...) \]  

(0.16)

where \( c_0 \) is positive. This result is in agreement with a known fact that the exchange energy is lower in a magnetic field than without it. Notice, in passing, that this circumstance might play an important role in the context of recent scenarios of a spontaneous gauge flux generation due to strong interactions [3].

However the result (16) is in contrast with a general expectation that the \( \nu = \frac{1}{2} \) state is a local minimum. To obtain a correct sign of the energy correction one has to take into account gauge fluctuations as well.

A lengthy analysis based on the semiclassical formulae of the type (11) leads to a remarkably simple prescription. Namely, it turns out that to estimate the leading term of the \( 1/N \)-expansion one only needs to know \( \Pi_{\mu\nu}(\omega, k) \) at \( \omega > \frac{B}{m} \) and \( k > \frac{B}{\sqrt{p}} \). Moreover in this region one can use the zero field polarization operator...
\[ \Pi_{00} \approx -\kappa + i\gamma \frac{\omega}{k} \]
\[ \Pi_{\perp} \approx \chi k^2 + i\gamma \frac{\omega}{k} \]
\[ \Pi_{\text{odd}} \approx 0 \]  
(0.17)

where \( \kappa \sim m \), \( \chi \sim m^{-1} \) and \( \gamma \sim p_F \). The corrections to the approximate expressions (17) give higher order terms in \( 1/N \).

It can be also shown that the region of small \( x = \frac{\omega m}{B}, y = \frac{k}{\sqrt{B}} \), where the expressions for \( \Pi_{\mu\nu}(\omega, k) \) are given by formulae

\[ \Pi_{00} \approx -N^2my^2(1 + N^2(-x^2 + \frac{3}{8}y^2)) \]
\[ \Pi_{\perp} \approx \frac{N^2}{m}(-x^2 + y^2) \]
\[ \Pi_{\text{odd}} \approx \frac{Ny}{2\pi}(-1 + N^2(-x^2 + \frac{3}{4}y^2)) \],
(0.18)

doesn’t contribute to the leading term. Then the dependence on the field enters only as an effective lower bound of integrations over \( \omega \)’s. Performing this approximate calculation we obtain

\[ E_1(N) \approx \int_B^\mu d\omega \int_0^\infty \frac{kdk}{2\pi} \frac{(\kappa - i\gamma \omega/k)V(k)k^2}{k^2 + (4\pi)^2(\kappa - i\gamma \omega/k)(\chi k^2 + i\gamma \omega/k)} \]
\[ \sim e^2p_F^{2+\alpha}(-1 + \frac{c}{N^{1+\alpha/3}} + ...) \]  
(0.19)

where \( c \) is a positive number or, taking into account that \( \delta \nu = \nu - \nu_c \sim \frac{1}{N} \):

\[ E_1(\nu) - E_1(\nu_c) \sim e^2p_F^{2+\alpha}|\delta \nu|^{1+\alpha/3} \]  
(0.20)

We observe that the result (20) retains an explicit dependence on the from of the interaction potential, although the latter appears to be different form the naive exchange energy (16). Notice that the cusp has a mirror symmetry about \( \nu_c = 1/2 \) required by a particle-hole symmetry for spin-polarized fermions.

We stress that the exponent in (20) is greater than one which implies that states in the vicinity of half filling are closer in energy to the \( \nu = \frac{1}{2} \) state than one could expect on general
grounds (linear cusp). It is also consistent with the fact that the ground state at \( \nu = 1/2 \) is compressible.

The present approach also makes it possible to get an estimate of quasiparticle energy gaps. In the system with fixed number of particles the lowest neutral excitation ("exciton") can be associated with the pole of the charge-density response function \( \chi_\rho(\omega, \vec{k}) \) [4]. A spatial separation of a particle and a hole (\( \delta r \sim k/B \)) constituting the exciton increases with \( k \). At \( k \to \infty \) one obtains a pair of distant charged quasihole and quasiparticle with a total energy
\[
\Delta = \omega(k = \infty) = \epsilon_p + \epsilon_h.
\]

A quasihole (quasiparticle) excitation can be viewed as an addition (removal) of \( 1/N \) flux unit of an external magnetic field to the system. A corresponding perturbation of incompressible electron fluid causes a local density distortion which carries fractional electric charge \( \pm \frac{e}{2N+1} \) and obeys anyonic statistics \( \theta = \pi(\frac{2N-1}{2N+1}) \) as a result of the fermion liquid polarization. More precisely, each fractionally charged quasiparticle in the bulk is accompanied by a complementary charge \( \pm e(1 - \frac{1}{N}) \) located on the boundary. In the case of a neutral bulk excitation a total charge on the boundary cancels out.

To estimate a scaling behavior of the gap function \( \Delta(\nu) \) we proceed with a consideration close to that of the Ref. [8] where dispersions of collective excitations were found in the context of the Integer Quantum Hall Effect. In this case the exciton is merely a transition of a fermion from the \( N^{th} \) Landau level to the \( (N+1)^{th} \) one. In contrast to the case of IQHE one has to start from the bare dispersion which is completely degenerate \( (\omega(k) = 0) \). This zero order approximation is consistent with a proper account of gauge fluctuations in the limit of arbitrarily small interaction strength \( e \to 0 \).

The only contribution to the exciton dispersion which remains finite at large \( k \) comes from the exchange self-energy corrections \( \Sigma_{N,N+1} \) to the \( N^{th} \) and \( (N+1)^{th} \) mean field Landau levels. The other terms which can be understood as a particle-hole binding energy decay as \( \delta \omega(k) \sim -e^2(B/k)^\alpha \).

In the first order approximation the exchange self-energy associated with the \( n^{th} \) Landau
level is a real constant which can be written in the form

\[ \Sigma_n = \int \frac{d\omega d\vec{k}}{(2\pi)^3} \sum_{l=0}^{n} |< n|J_0(\vec{k})|l>|^2 G_l(\omega) < a_0(\omega, \vec{k})a_0(-\omega, -\vec{k}) > \]  

(0.21)

where \( G_l(\omega) = (B(n - l) - \omega + i\delta)^{-1} \) and the propagator of the temporal component of the gauge field \(< a_0(\omega, \vec{k})a_0(-\omega, -\vec{k}) >\) can be obtained by inverting (3).

In the case of IQHE considered in [8] there is a nonzero energy gap \( \Delta_0 = \frac{B}{m} \) at \( e \to 0 \) and therefore one may use an unambiguous prescription to evaluate corrections as \( \delta \Delta = \Sigma_N - \Sigma_{N+1} \). In our problem a spurious cyclotron frequency disappears under normalizing the exciton dispersion \( \omega(k) \) with respect to the case \( e = 0 \) when it becomes flat. Then one can no longer use the above definition for \( \Delta \) in terms of \( \Sigma_n \). However we consider \( \Delta \sim \text{max}(\Sigma_N; \Sigma_{N+1}) \) as a reasonable estimate although it may also overestimate the correct result. Then using (21) we get the following relation

\[ \Delta(N) \approx \int_{0}^{\infty} d\omega \int_{0}^{\infty} d\vec{k} \sum_{l=0}^{N-1} |< N|J_0(\vec{k})|l>|^2 G_l(\omega) V(k)k^2 \]  

(0.22)

On neglecting the fermion polarization operator in the denominator of (22) one obtains a naive result \( \Delta(N) \approx V(\sqrt{B}) \sim e^2 p_F^2 \alpha F/N^{\alpha/2} \) which coincides with the mean field exchange energy in the absence of gauge fluctuations [9].

To perform a more complete account of gauge fluctuations we must keep \( \Pi_{\mu\nu}(\omega, \vec{k}) \) in \( \det \hat{K}_0 \). Again using the semiclassical approach [7] and integrating over continuous variable \( \xi \) substituting \( B(N - l) \) we arrive at the integral

\[ \Delta(N) \approx \int_{0}^{\infty} d\omega \int_{0}^{\infty} dk \sum_{l=0}^{N-1} \frac{V(k)k^2}{(2\pi)^2} Im \left( \frac{k^2 + (4\pi)^2(\kappa - i\gamma \omega/k)(\chi k^2 + i\gamma \omega/k)}{k^2} \right) \]  

(0.23)

Assuming that away from hal filling the quasiparticle spectrum immediately develops a gap we subtract from (23) its value at \( B = 0 \) which yields a renormalization of the chemical potential. The resulting quasiparticle gaps obey the scaling law

\[ \Delta(\nu) \sim e^2 p_F^2 |\delta\nu|^{\frac{\alpha}{2+\alpha}} \]  

(0.24)

To implement this estimate one has to take into account that the physical interaction potential \( V(k) \) may have different values of \( \alpha \) at different scales of \( k \). As a most physically relevant
example, the bare Coulomb potential has $\alpha = 1$ but it changes to $\alpha = 2$ at $k < \kappa_D \sim \frac{e^2 m}{\varepsilon_0}$ due to the Debye screening (here $\varepsilon_0$ is the background dielectric constant). Thus the Fourier transform of the Coulomb potential has to be described by different values of $\alpha$ depending on the relation between relevant momenta $k \sim p_F/N$ and $\kappa_D$. Then one easily obtains that the dependence $\Delta(\nu) \sim |\delta \nu|$ holds for $\delta \nu > \frac{e^2 m}{\varepsilon_0 p_F}$ while at smaller $\delta \nu$ it changes to $\Delta(\nu) \sim |\delta \nu|^{1/3}$.

This crossover behavior appears to be in a qualitative agreement with recent quasiparticle gap measurements [10]. It was found in [10] that $\Delta(\nu)$ grows linearly with deviation from $\nu_c = 1/2$ but an extrapolation of the linear plot to half filling yields a negative intercept which can be interpreted as a sign of an essentially nonlinear dependence of $\Delta(\nu)$ at small $\delta \nu$.

However, as an alternative explanation of these experiments one could suggest that the gap simply remains zero in some interval of fractions. In fact, our present consideration doesn’t allow us to answer the question whether a gap opens at some small deviation from half filling or whether it happens only at some finite $\delta \nu_c$. In the latter case we expect that the scaling behavior (24) can only take place at $\delta \nu_c << \delta \nu << 1$.

The scaling behavior (24) must be compared with the predictions made in ref.[4]

$$\Delta(\nu) \sim \frac{\rho}{N m^*}$$

where $m^*$ is an effective mass of a quasiparticle. The authors of ref.[4] argued that gauge fluctuations can be accounted for by including the effective mass renormalization. This was found in the form $m^* \sim (\frac{\kappa_D}{\pi})^{1/3}$ in the case of short-range interactions ($\alpha = 2$) versus $m^* \sim \ln \frac{\kappa_D}{\pi}$ for the Coulomb interaction ($\alpha = 1$). Then (25) yields $\Delta(\nu) \sim |\delta \nu|^{3/2}$ for the case of short-range interactions and $\Delta(\nu) \sim \frac{|\delta \nu|}{C + \ln |\delta \nu|}$ for the Coulomb interaction.

At the same time it was mentioned in [4] that the available results for the Coulomb interaction at small $N$ obtained from a diagonalization of small systems [11] can be better fitted by the linear dependence $\Delta(\nu) \sim |\delta \nu|$.

In fact, the statements of the ref.[4] were made on the basis of some ansatz for the one-particle self-energy caused by infrared divergent corrections due to the long-range transverse
gauge field $a_\perp$. At $\nu = 1/2$ the lowest order contribution was found in the form

$$\Sigma_1(\epsilon) \approx -g^2 \frac{p_F}{m\chi^{2/3}\gamma^{1/3}}(i\epsilon)^{2/3}$$

The ansatz proposed in [4] to account for all higher order corrections is equivalent to the statement that both real and imaginary parts of the exact $\Sigma(\epsilon)$ are proportional to $\epsilon$ at $\epsilon << \mu|p-p_F|^3$.

On the other hand, a recent investigation of this problem in the framework of the eikonal approximation [12] which is supposed to be adequate for singular fermion interactions leads to different conclusions. Namely, it was found that transverse gauge fluctuations completely destroy a pole structure of the bare one-particle Green function. The asymptotical behavior near the (Luttinger) Fermi surface found in the eikonal approximation is essentially nonpole-like

$$G(\epsilon, p \approx p_F) \sim \frac{\mu^{1/4}}{\epsilon^{5/4}} \exp\left(-\frac{\mu^{1/2}}{\epsilon^{1/2}}\right)$$

A similar form of the one-particle Green function was also obtained in [13] by means of a different method.

An additional study shows that due to the intrinsic Ward identities the effect of infrared singularities manifested in (27) on longwavelength asymptotics of $\nu = 1/2$ response functions is not very prominent. It also turns out that the exponential singularity revealed in (27) makes $\nu = 1/2$ response functions regular at momenta close to $2p_F$ [14]. It would be interesting to find out whether an analogous (partial) cancellation between self-energy and vertex corrections at $\nu = \frac{N}{2N+1}$ results to electromagnetic response functions satisfying the $f$-sum rule and the Kohn theorem whose crucial importance for a correct description was stressed in [15].

Moreover, on the basis of the results of the eikonal approximation an effective free boson description of the gauge dynamics in $\nu = \frac{1}{2}$ state was constructed [13,14]. An attempt to estimate temperature $(\frac{T}{\epsilon_F})$ corrections to the ground state energy $E(\nu = \frac{1}{2}, T)$ performed in the framework of this effective representation yields a result consistent with RPA [14].
This fact can be considered as an implicit argument in favor of the hypothesis that RPA is capable to give leading $1/N$-corrections in a weak magnetic field as well.

In addition, it was shown by Wiegmann and Larkin [7] that the three-loop ”beyond RPA” contributions to the ground state energy of 2D fermions interacting via transverse gauge field cancel each other up to irrelevant terms. We consider this fact as another argument in favor of our conjecture that RPA formulae (4),(22) capture relevant physics.

In conclusion, we study the dependence of the ground state energy as well as quasiparticle gaps on filling fraction in the vicinity of $\nu = 1/2$. In the lowest order in interaction strength we find $\delta E(\nu) \sim |\delta \nu|^{1+\alpha/3}$ and $\Delta(\nu) \sim |\delta \nu|^{3(2+\alpha)}$ where the exponent $\alpha$ describes a power-like decay of the effective interaction potential $V(r) \sim \frac{1}{r^\alpha}$.

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