Interaction model of one jaw of a vibrating jaw crushe with the processed rock, taking into account the properties of the electric motor

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Abstract. Vibratory jaw crushers are modern machines for grinding and rock crushing. Single jaw of vibratory jaw crushe is considered in this paper. Phenomenological model of interaction of the jaw and rock is proposed. Two options for considering the power of an unbalanced drive are considered. In the first, an unbalanced vibrator has a constant angular velocity relative to the bed. In the second, the electromechanical characteristics of the electric motor, which drives the unbalanced vibrator, are taken into account. The impact of crushed material on the vibrational modes of the jaw and the rotation characteristics of the vibrator for each of the vibration excitation variants is considered. The distribution of the technological load obtained based on this model over the jaw is shown.

1. Introduction

Vibratory jaw crushers are common machines for processing and crushing rock. The main feature that distinguishes them from a number of other crushing machines is the lack of rigid and unreliable kinematic bonds that ensure the mutual movement of the jaws of the crusher [1, 2]. The latter is possible due to the phenomenon of self-synchronization, which is that the elements of the system begin to oscillate with matching or multiple frequencies due to the existing links in the system. However, depending on the physical and mechanical properties of the processed rock and the conditions of collision with the jaws may change the type of self-synchronization (phase of mutual oscillations) of vibration exciters, which affects the efficiency of the crushing process.

In order to understand the dynamics of the vibrating jaw crushe, it is important to consider the properties of the processed rock as well as its influence on the crushing process. At the moment, this issue has not been fully resolved. A common approach to the design of vibratory jaw crushers is to include a linear-viscous damper in the dynamic’s equation of the machine [3]. Attempts are being made to develop a more detailed model of the crushed rock [4, 5]. The behavior of the destroyed rock with the help of numerical methods of DEM is studied in the works [6, 7].

This paper proposes a phenomenological model of the crushing process in a vibrating jaw crushe, on the basis of which the influence of the processed material on the fluctuations of the jaw and the rotation of the unbalance has been considered. The two models of unbalance drive accounting are also...
compared. In the first case, the angular rotation speed of the balance with respect to the stationary frame is a constant value. In the second case, the electromechanical characteristics of the motor driving the unbalance vibrator are considered. The load distribution along the jaw is obtained for each case of jaw oscillation.

2. Vibratory Jaw Crusher Model

Due to symmetry, we will only consider one jaw of the Figure 1 vibratory jaw crusher. Consider the jaw as a completely rigid body of a complex box section. The jaw is fixed at point O on the torsional shaft and performs only oscillatory motion around the point of suspension. The centre of the jaw mass is located at point C at a distance of l from the point of suspension. The jaw mass is m0. The length of the jaw is l, and the length of its active part is la. The torsion shaft has a linear angular stiffness k and a damping factor d. To excite the angular vibrations to the jaw at point B, a vibrator with unbalance of md and eccentricity e is connected at a distance l from the suspension point. The unbalance rotates at an angular speed in $\dot{\theta}$ of the drawing plane.

\[ M_r = 2M_{cr} \left( \frac{\omega_{cr}}{\omega_1 - \omega_2} + \frac{\omega_2 - \dot{\theta}}{s_{cr} \cdot \omega_2} \right)^{-1} \]

Here, $M_r$ is a critical moment of the electric motor; $s_{cr}$ - critical sliding of the electric motor, and $\omega_1$ is angular speed of rotation of an electromagnetic field.

Figure 1. Vibratory jaw crusher’s jaw scheme

In this article, two variants of excitation of jaw oscillations have been discussed and compared. In the first of them, the angular velocity of the unbalance is constant $\dot{\theta}=$const. The second variant considers the characteristics of the electric motor, the driving torque of the asynchronous motor is described by the Kloss formula:

\[ M_r = 2M_{cr} \left( \frac{\omega_{cr}}{\omega_1 - \omega_2} + \frac{\omega_2 - \dot{\theta}}{s_{cr} \cdot \omega_2} \right)^{-1} \]
3. Treated Rock Model

The following model is applicable to describe the movement of the processed material and its destruction. The entire working area of the crusher is divided into heights equal to those of Figure 2.

![Figure 2. Scheme of rock material accounting](image)

In each zone there can be only one elastic-visco-plastic element simultaneously, working only at compression. The first element can be represented as a spring with a stiffness $k_i$, connected in series with a dry friction damper with the ultimate force $F_{lim,i}$ and parallel to them connected linear-viscous damper $d_i$, which allows considering the elastic, viscous and plastic properties, as well as increasing the complexity of crushing in the lower part of the crusher. The approximate diagram of the element is shown in Figure 3.

![Figure 3. Scheme of the element that reproduces the interaction of the rock and jaw](image)

Each element can only withstand a certain amount of jaw compression, after which it is destroyed and removed from the equation system. The properties of each element are determined randomly during its creation. In doing so, new elements appear in the uppermost zone with a certain frequency. All the elements in the crushing chamber are synchronously moved down one level at the lowest compression point. During this movement, the properties of these elements change. This phenomenological model is considered in detail in the article [8].
4. Differential Equations of Jaw Movement

The dynamics of one jaw of a vibrating jaw crusher without considering the characteristics of an electric motor connected to an unbalance vibrator is considered in [8]. The jaw movement equation in this case is also presented there, and the results are presented in detail.

The equations of jaw oscillations considering electromechanical characteristics of the electric motor are as follows:

\[
\mathbf{M} \ddot{\mathbf{x}} = \mathbf{F}(\mathbf{x}, \dot{\mathbf{x}}), \quad \mathbf{x} = \{\varphi, \Theta\}^T
\]

\[
\mathbf{M} = \begin{bmatrix} m_\text{j}l_\text{j}^2 / 4 + m_\text{i}l_\text{i}^2 & m_\text{d}l_\text{e} \sin(\varphi + \Theta - \beta) \\ m_\text{d}l_\text{e} \sin(\varphi + \Theta - \beta) & m_\text{d}e^2 \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}
\]

\[
F_1 = k(\varphi_{st} - \varphi) - d \dot{\varphi} - m_\text{d}l_\text{e} \cos(\varphi + \Theta - \beta) \cdot (\dot{\Theta})^2 - m_\theta g l \sin(\varphi) / 2 - m_\text{d}g l_\text{b} \sin(\varphi - \beta) + Q(u, \dot{u})
\]

\[
F_2 = M_\text{cr} - f \cdot (\dot{\Theta})^2 \text{sign}(\dot{\Theta}) - m_\text{d}l_\text{e} \cos(\varphi + \Theta - \beta) \cdot (\dot{\varphi})^2 - m_\theta g e \cos(\Theta)
\]

\[
Q(u, \dot{u}) = \sum_{i=1}^{N} \left[ F_{\text{limi}} / 2 \cdot (1 + \tanh(s(u_i - F_{\text{limi}} / k_i) / L_i)) + d \dot{u}_i \right] \cdot H(\dot{u}_i)
\]

\(\phi\) is the angular offset of the jaw relative to the vertical; \(\Theta\) is the angle of rotation of the balance relative to the horizontal; \(\dot{\varphi}\) is a time derivative; \(F_i\) is the sum of the moments acting on the jaw, including the moment of elasticity in the torsion, the moment of linear-viscous dissipation in the torsion, the moment of gravity of the jaw and the moment of effects transmitted from the imbalance to the jaw; \(F_2\) is the sum of the moments acting on the unbalance includes the motor torque, the friction torque in the bearing, the moment of gravity of the unbalance, \(\phi_{st}\) is the angular misalignment of the jaw corresponding to the position of the static equilibrium; \(f\) is a friction coefficient in the unbalance drive bearings; \(Q\) is a dynamic response of the medium; \(N\) is the number of elements currently in the crushing chamber; \(F_{\text{limi}}\) is a maximum force of the dry friction damper of the \(i\)-th element; \(s\) is a tilt coefficient of the force characteristic; \(k_i\) is a stiffness coefficient of the \(i\)-th element; \(L_i\) is a length of the \(i\)-th element; \(d_i\) is a dissipation coefficient of the \(i\)-th element; \(u_i\) is a movement of the touch point of the \(i\)-th element and jaw; \(\dot{u}\) - speed of the touch point of the \(i\)-th item and the jaw; \(H(\dot{u})\) - is the Heaviside function ensures that the elements only work when they are compressed.

Calculations were performed numerically in Matlab using the following parameter values: \(m_0 = 3250 \text{ kg}, m_d = 170 \text{ kg}, l = 1.6 \text{ m}, e = 0.1 \text{ m}, \beta = 0.472 \text{ rad}, \phi_{st} = 0.2618 \text{ rad}\)

\(k = 6.7 \cdot 107 \text{ Nm rad}^{-1}, d = 3.8 \cdot 104 \text{ Ns m}^{-1}, M_{\text{cr}} = 600 \text{ N m}, s = 0.12, f = 0.001, \omega_1 = 157 \text{ rad c}^{-1}\).

The properties of the elements are set randomly with equal probability in the interval: \(F_{\text{limi}} = 3 \pm 0.6 \text{ KN}, k_i = 45 \pm 0.0 \text{ KN m}^{-1}, d_i = 57 \pm 11.4 \text{ KN s m}^{-1}\).

5. Results
The Figure 4 shows the jaw oscillations for two cases of motor power accounting:
• the case when the unbalance speed \(\dot{\Theta}\) relative to the frame is constant;
• the characteristics of the electric motor are considered.

In this case, the presence of the material and its random properties create a polyharmony character of jaw oscillations. In both cases, oscillations occur around the static equilibrium position.

The vibration amplitude depends on the number of elements in the crushing chamber. When a large number of elements approach the lower part of the jaw, the resistance of the material to crushing increases due to the increase in the parameters of the elements of the phenomenological model and the...
increase in the force arm with which one element acts on the jaw. This leads to a gradual decrease in amplitude. After one of the lower elements with a high value of force breaks down, the total force acting on the jaw sharply decreases, which causes jumps and increases the amplitude.

When considering the characteristics of the motor, it takes time at the beginning for the unbalance to swing and move the jaw (Figure 4, b).

Figure 5 shows a graph of the unbalance speed $\dot{\Theta}(t)$ when the characteristics of the electric motor are considered. After entering the operating mode, the speed of the electric motor varies according to the polyharmonic law. This is due to the reverse effect of the material on the jaw and imbalance.

Figure 6 - Figure 7 shows the average distribution of the process load along the jaw for four-time moments for two excitation options. A total of 120 calculations were made and the results averaged. Black squares represent the mean values of the element strength in the j zone, and thin horizontal lines represent the standard deviation for the j-q zone. Distribution for the second case of excitation of fluctuations of a jaw has more chaotic character that speaks about strong enough influence of processed breed on movement of a jaw.

Figure 4. Jaw oscillations.  
(a) Constant angular velocity of the balance;  
(b) Motor characteristics are considered

Figure 5. Debalance speed graph
6. Conclusions

Two calculations of vibrations of one jaw of a vibrating jaw crusher interacting with the crushed rock were made. The processed material is considered as a phenomenological model reflecting elastic, linear-viscous and plastic properties. In addition, this model allows for consideration of the effect of material movement on the crushing chamber during crushing.

In the first calculation, the motor characteristics were not considered. In this case, the consideration of the rock in the proposed way leads to a change in the amplitude of oscillations over time. The distribution of load on the jaw changes little over time, with the maximum load on the lower third of the jaw.

Figure 6. Load distribution by crushing chamber height for constant unbalance speed

Figure 7. Load distribution by height of crushing chamber for the case of electric motor characteristics
The second calculation was made considering the characteristics of the electric motor that drives the jaw. The real characteristics of the electric motor have led to a reduction in the maximum amplitude of the jaw oscillation. It takes some time for the unbalance vibrator to reach the operating mode and start to vibrate the jaw. The plot of unbalance speed fluctuations shows the effect of the treated rock on the jaw oscillations and the movement of the electric motor-driven unbalance. The rate of imbalance is not constant, but fluctuates in a polyharmony way.

Later, considering the complete model of the crusher, including the second jaw, as well as the body of the vibrating jaw crushed, and improving the model of the medium so that it gives an asymmetric distribution of load on both jaws, it is possible to obtain a non-anti-phase rotation of vibrators, which leads to a deterioration in the quality of crushing. Using the experimental data at crushing and known properties of the processed rock, it is possible to estimate the values of parameters of the phenomenological model of crushing.

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