Measurement of the strong coupling $\alpha_S$ in $e^+e^-$-annihilation using the three-jet rate

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Abstract
We present a measurement of the strong coupling $\alpha_S$ using data collected with the JADE detector at centre-of-mass energies between 14 and 44 GeV. The three-jet rate as a function of the transition parameter $y_{\text{cut}}$ is determined using the Durham jet algorithm and the distribution is compared to QCD predictions. Recent theoretical calculations predict the three-jet rate at next-to-next-to-leading order. For the first time a measurement of $\alpha_S$ is presented using QCD predictions at next-to-next-to-leading order matched to predictions at next-to-leading logarithmic approximation, with subleading terms being included as well. We obtain $\alpha_S(M_Z^0) = 0.1199 \pm 0.0010(\text{stat.}) \pm 0.0021(\text{exp.}) \pm 0.0054(\text{had.}) \pm 0.0007(\text{theo.})$, being consistent with the world average value of $\alpha_S$.

Keywords: QCD, strong coupling, three-jet rate, NNLO, NLLA

1. Introduction
The strong coupling $\alpha_S$ is one of the fundamental parameters of the Standard Model of particle physics. Several possibilities exist to determine this parameter, one method being presented in this paper. In $e^+e^-$-annihilation with quarks in the final state hard gluons can be emitted from the quark with the emission probability being proportional to the strong coupling $\alpha_S$. Events with a hard gluon are identified by observing at least three particle jets in the detector. The theory of strong interaction, Quantum Chromo Dynamics (QCD), allows to predict the rate of three-jet events as a function of a single parameter, the strong coupling $\alpha_S$. Recent theoretical improvements \cite{1-4} allow to calculate the three-jet rate at next-to-next-to-leading order (NNLO) and a first measurement with data collected by the ALEPH detector at 91 GeV using the three-jet rate at a single resolution parameter was published \cite{5}.

In this paper we present the measurement of $\alpha_S$ using data collected with the JADE detector between the years 1979 and 1986. For the first time NNLO predictions are matched with next-to-leading logarithmic approximations (NLLA), including subleading soft logarithms, to measure $\alpha_S$ with the three-jet rate. The complete description of the analysis can be found in \cite{6}.

2. The three-jet rate determined with the Durham clustering scheme

In order to count the number of jets in the final state the reconstructed particle tracks and neutral clusters have to be clustered to jets. Several clustering schemes are available and we use the Durham clustering algorithm \cite{7}. The number of jets depends on the resolution parameter $y_{\text{cut}}$, a measure indicating the separation between the second and the third reconstructed jet. The fixed order prediction of the three-jet rate determined with the Durham clustering scheme is matched to NLLA predictions using the $R$-matching scheme. Parts of the subleading soft logarithms, which are not included in the NLLA calculations, can be systematically controlled and included by adding a renormalisation scheme dependent $K$-term \cite{8}. Including the $K$-term significantly improves the description of the data. QCD predictions of the three-jet rate, without the expected contribution from hadronisation effects, are shown in Fig. \cite{1}.

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Preprint submitted to Nuc. Phys. (Proc. Suppl.)

August 23, 2012
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Figure 1: QCD predictions for partons clustered to three jets using the Durham algorithm evaluated at a centre-of-mass energy of 35 GeV and the strong coupling being set to $\alpha_s(M_Z^0)=0.118$. The predictions do not contain corrections expected from hadronisation effects. The figure shows the fixed order prediction (NNLO) together with the fixed order prediction matched to NLLA (NNLO+NLLA) and the matched predictions taking subleading soft logarithms into account (NNLO+NLLA+K). The band reflects the theoretical uncertainty estimated by setting the renormalisation scale factor $x_\mu$ to 0.5 and 2.

4. Measurement of the strong coupling $\alpha_s$

The measured and corrected three-jet rate together with the matched NNLO+NLLA+K QCD predictions is used to determine the strong coupling $\alpha_s$. A $\chi^2$-value, obtained from the difference between the measured three-jet rate and the QCD predictions, is calculated and minimised. The QCD predictions are applied with the renormalisation scale factor set to the natural choice $x_\mu=1$. Events simulated with the PYTHIA event generator are used to correct the predicted three-jet rate for hadronisation effects. The fit range is determined by requiring the correction from hadronisation and detector effects to be small. In addition the leading log contribution in the QCD prediction is required to be well below unity. The correlations between the different $y_{\text{cut}}$ bins are determined using MC events. The result obtained at a centre-of-mass energy of 35 GeV from the fit to the three-jet rate is shown together with the three-jet rate distribution in Fig. 3.

4.1. Systematic Uncertainties

Several sources of systematic uncertainties are considered for the measurement of the strong coupling $\alpha_s$. The fit is repeated and the difference between the variation and the default analysis is taken as the symmetric systematic uncertainty. The different sources of systematic uncertainties are arranged in three categories:
from the fit to the data taken at 14 GeV is not included in the combination. The combined value for $\alpha_S$ evolved to the energy $M_Z$ is $\alpha_S(M_Z) = 0.1199 \pm 0.0010$(stat.)$\pm 0.0021$(exp.)$\pm 0.0054$(had.)$\pm 0.0007$(theo.), consistent with the world average value of 0.1184 $\pm 0.0007$.

The measurements of $\alpha_S$ at the various energy points is shown in Fig. 4. The evolution of $\alpha_S$ measured with the three-jet rate as a function of the centre-of-mass energy is consistent with the expectation from QCD.

### 4.3. Study of the renormalisation scale dependence

Besides fixing the renormalisation scale $x_{\mu}$ to the natural choice the fit is repeated with $x_{\mu}$ and $\alpha_S$ being free in the fit. The renormalisation scale obtained by the fit is consistently below unity, but with large statistical uncertainties and being consistent with one. The large uncertainty can be understood by looking in more detail to the variation of $\alpha_S$ with respect to $x_{\mu}$. The change in $\alpha_S$ is rather flat, leading to small theoretical uncertainties and to large statistical uncertainties for the renormalisation scale $x_{\mu}$. The results obtained from the various energy points are combined to a single value, leading to $\alpha_S(M_Z) = 0.1204 \pm 0.0009$(stat.)$\pm 0.0021$(exp.)$\pm 0.0059$(had.)$\pm 0.0008$(theo.), again being consistent with the world average value.

### 4.4. Measurement of $\alpha_S$ using fixed order predictions

The fit is repeated with fixed order predictions only and $x_{\mu}$ being set to one or leaving $x_{\mu}$ as free parameter.

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**Figure 3:** Result of the fit from data taken at a centre-of-mass energy of 35 GeV. The insert reflects the difference between the QCD prediction calculated with the strong coupling returned from the fit and the measured distribution, normalized to the combined statistical and experimental uncertainty.

**Figure 4:** Measurements of the strong coupling $\alpha_S$ determined with the three-jet rate using the Durham jet-finder algorithm and NNLO order predictions. For the measurements between 14 and 44 GeV the QCD predictions are matched with NLLA including soft logarithms. The energy points at 34.6 and 35 GeV are combined to a single entry. The line corresponds to the world average value with the corresponding uncertainty.
ter in the fit. For the fit with $x_p$ being set to the natural choice the $\chi^2$/d.o.f. is considerably worse than the one obtained with matched QCD predictions (almost a factor four in the $\chi^2$/d.o.f. for the fit to the data collected at 35 GeV). This points to missing higher order terms in the QCD prediction. In addition the fit result also shows an increased sensitivity to the fit range. Using matched NNLO+NNLLA+K QCD predictions the dependency on the fit range is almost negligible, while for NNLO predictions the choice of the lower $y_{\text{cut}}$ value in the fit range has a significant impact on the result.

Leaving $\alpha_S$ as well as $x_p$ free in the fit leads to a significantly improved description of the fit with $\chi^2$/d.o.f. values similar to the one obtained from the fit with matched QCD predictions. However, the value obtained for the scale $x_p$ is rather low (around 0.2) and within the statistical uncertainty inconsistent with being one. The uncertainty on the fitted scale parameter $x_p$ is about one order of magnitude smaller than the uncertainty on the scale parameter obtained with matched predictions. This indicates that the sensitivity of $\alpha_S$ with respect to the renormalisation scale $x_p$ is increased compared to the fit with matched QCD predictions.

5. Conclusions

We present the first measurement of $\alpha_S$ determined with the Durham three-jet rate using matched NNLO+NNLLA+K QCD predictions. For this measurement hadronic events from e+e- annihilation taken at centre-of-mass energies between 14 and 44 GeV are used. The result $\alpha_S(M_Z) = 0.1199 \pm 0.0010(\text{stat.}) \pm 0.0021(\exp.) \pm 0.0054(\text{had.}) \pm 0.0007(\text{theo.})$ is consistent with the world average value. The largest uncertainty originates from modeling the transition from parton level, as predicted from the perturbative QCD calculations, to the hadron level, as observed by the experiment. The increased precision of perturbative QCD predictions, together with large hadronic e+e- event samples, will allow to scrutinize MC generators modeling the transition from partons to hadrons. A measurements of $\alpha_S$ using fixed order predictions only cannot describe the data satisfactorily. A fit leaving $\alpha_S$ as well as $x_p$ free returns a good description of the data, but the fitted value of $x_p$ is inconsistent with one. Fig. 5 summarizes the result obtained here to previous analyses using jet rates and differential jet rates. The value obtained for $\alpha_S$ using a fit to the $y_{\text{jet}}$ distribution returns the almost identical result for $\alpha_S$ as measured with this analysis. The measurements obtained at higher centre-of-mass energies return smaller uncertainties, since the size of the hadronisation correction decreases with increasing centre-of-mass energy. The measurement of $\alpha_S$ using the four-jet rate and data taken between 14 and 44 GeV returns a smaller hadronisation uncertainty, leading to a decreased overall uncertainty.

![Figure 5: Summary of $\alpha_S$ measurements using the three-jet rate, the four-jet rate and the differential $y_{\text{jet}}$ distribution applied to e+e- data taken between 14 and 91 GeV. The first line corresponds to the measurement summarized in this paper, the second line to [5], the third line to [10], the fourth line to [12] and the last line to [8]. The vertical band indicates the world average value of $\alpha_S$.](image)

### Acknowledgements

This research was supported by the DFG cluster of excellence "Origin and Structure of the Universe" (www.universe-cluster.de).

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