Triply heavy baryon spectroscopy in the relativistic quark model

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Triply heavy baryons are investigated in the framework of the relativistic quark model based on the quark-diquark picture in the quasipotential approach in QCD. Masses of the ground and excited states of the $\Omega_{ccc}$, $\Omega_{bbb}$, $\Omega_{ccb}$ and $\Omega_{cbb}$ baryons are calculated. Orbital and radial excitations between the diquark and quark as well as between quarks inside the diquark are considered. The diquark internal structure is consistently taken into account by the form factor of the diquark-gluon interaction expressed through the overlap integral of the diquark wave functions. The detailed comparison with previous calculations is given.

I. INTRODUCTION

Recently significant experimental progress has been achieved in studying hadrons with heavy quarks. Many new states of heavy mesons as well as of heavy baryons were observed, some of which have properties implying their exotic nature (for recent reviews see, e.g., [1–3] and references therein). A special interest represent the long-awaited discoveries very recently made by the LHCb Collaboration: of the doubly charmed baryon $\Xi^{++}_{cc}$; of the tetraquark $T_{cc\bar{c}\bar{c}}$, composed of two charm quarks and two charm antiquarks, $X(6900)$ [5]; and of the doubly charmed tetraquark, $T_{cc}^+$, with a quark content $cc\bar{c}\bar{d}$ [6]. All these new states require the production of at least two charm quark-antiquark pairs. The next important step forward will be the discovery of the triply heavy baryons, composed only from heavy charm and/or bottom quarks, and, thus, requiring the production of three heavy quark-antiquark pairs. The first observation of the simultaneous production of three $J/\psi$ mesons in proton-proton collisions was very recently presented by the CMS Collaboration [7]. Estimates of the production cross-section of triply heavy baryons in proton-proton [8] and heavy ion [9] collisions indicate that triply charmed $\Omega_{ccc}$ baryons have good chances to be observed at LHC.

In this paper we apply the relativistic quark model based on the quasipotential approach in QCD to calculate the mass spectra of triply heavy baryons. These baryons contain only heavy quarks and in the literature they are usually treated as nonrelativistic systems. However, the investigation of the heavy quark dynamics in heavy quarkonia shows that heavy quarks should be treated relativistically [10]. Indeed, estimates of the charm quark velocity $v$ in charmonia show that it is about one half of the velocity of light $c$, while the bottom quark velocity in bottomonia is about one third of $c$. Our previous investigations of meson [10, 11], baryon [12, 13] and tetraquark [3, 16] properties showed that relativistic effects play a very important role. Thus we treat triply heavy baryons completely relativistically without application of the expansion in heavy quark velocity. To achieve this goal we use the relativistic quark-diquark model which was previously developed and applied for the consideration of heavy [12, 13], doubly heavy [14] and strange [15] baryons. Constructing the triply heavy baryon we assume that two quarks of the same flavor form a doubly heavy diquark and the baryon is a relativistic bound system of this doubly heavy diquark and heavy
quark. The masses and wave functions of diquarks are obtained by solving the relativistic quasipotential equation with the quark-quark interaction, which is one half of the quark-antiquark interaction in mesons. The diquark is considered to be composite, not a point-like object. The diquark internal structure is taken into account by calculating the diquark form factor, which enters the diquark-gluon interaction, and is expressed as the overlap integral of the diquark wave functions. The account of the diquark size softens the diquark-gluon interaction thus increasing the baryon mass. This effect allowed us to get the correct prediction for the doubly charmed baryon $\Xi^{++}$ mass long before its discovery \cite{14}. It is important to point out that consistent treatment of the relativistic quark dynamics permitted us to get predictions for meson \cite{10,11}, baryon \cite{12–15} and tetraquark \cite{3} masses and decays in good agreement with experimental data using the universal set of model parameters, which we keep fixed in the present calculations of the triply baryon spectroscopy. Note that in most quark models the parameters for description of meson and baryon properties are varied. The paper is organized as follows. In Sec. II we briefly describe our relativistic quark-diquark model. The quasipotential equation and quark-quark and quark-diquark interaction potentials are given. Doubly heavy diquarks are considered in Sec. III. Their masses are calculated up to the first radial excitation and second orbital excitation. The form factors entering the diquark-gluon interaction are evaluated and their appropriate parametrization is given. In Sec. IV we calculate the masses of the ground, orbitally and radially excited states of triply heavy baryons and compare our results with previous calculations. Finally, Sec. V contains our conclusions.

II. RELATIVISTIC QUARK-DIQUARK MODEL

The relativistic quark-diquark model for description of doubly heavy, heavy baryon and hyperon spectroscopy based on the quasipotential approach and quark-diquark picture of baryons was developed and previously used in Refs. \cite{12–15}. Here we apply this model for the consideration of the triply heavy baryon spectroscopy. We use the same assumptions and model parameters. For completeness we give its brief outline. In this approach the complicated relativistic three body problem is reduced to the solution of two more simple relativistic two body problems. First, we introduce diquarks which are considered to be bound states of two quarks. It is assumed that quarks of the same flavor form a diquark. Second, the baryon is considered to be a bound system of a diquark and quark. In quasipotential approach interactions of two quarks in a diquark and of the quark and diquark in a baryon are described by the diquark wave function $\Psi_d$ and by the baryon wave function $\Psi_B$, respectively. These wave functions satisfy the quasipotential equation of the Schrödinger type \cite{10}

$$
\left( \frac{b^2(M)}{2\mu_R} - \frac{p^2}{2\mu_R} \right) \Psi_{d,B}(p) = \int \frac{dq}{(2\pi)^3} V(p, q; M) \Psi_{d,B}(q),
$$

with the relativistic reduced mass given by

$$
\mu_R = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3},
$$

and the center-of-mass system relative momentum squared on mass shell defined by

$$
b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2},
$$
where $M$ is the bound state mass (diquark or baryon), $m_{1,2}$ are the masses of quarks ($q_1$ and $q_2$) which form the diquark or of the diquark ($d$) and quark ($q$) which form the baryon ($B$), and $p$ is their relative momentum.

To construct the kernel $V(p, q; M)$ in Eq. (1), which is the quasipotential operator of the quark-diquark or quark-diquark interactions, it is assumed that the effective interaction is the sum of the usual one-gluon exchange term and the mixture of long-range vector and scalar linear confining potentials [12, 13]. The vector confining potentials contain additional effective Pauli terms, which introduce anomalous chromomagnetic moments of quarks and diquarks.

The quark-quark ($qq$) interaction quasipotential for the diquark is given by

$$ V(p, q; M) = \bar{u}(p)\gamma_\mu u(q), \quad \frac{1}{2} \left[ \frac{4}{3} \alpha_s D_{\mu\nu}(k) \gamma_\mu \gamma_\nu + V^V_{\text{conf}}(k) \Gamma^\mu_1(k) \Gamma_{2;\mu}(-k) + V^S_{\text{conf}}(k) \right], $$

where $\alpha_s$ is the QCD coupling constant, $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge, $k = p - q$; $\gamma_\mu$ and $u(p)$ are the Dirac matrices and spinors, and the effective long-range vector vertex of the quark is defined by [10] \[ \Gamma^\mu_1(k) = \gamma_\mu + \frac{ik}{2m} \sigma_{\mu\nu} \bar{k}_\nu, \quad \bar{k} = (0, k), \] and $\kappa$ is the anomalous chromomagnetic moment of quarks.

The quark-diquark ($qd$) interaction quasipotential in the baryon has the form

$$ V(p, q; M) = \langle d(P) | J_{\mu} | d(Q) \rangle \frac{4}{3} \alpha_s D_{\mu\nu}(k) \gamma_\nu u_q(q) $$
$$ + \psi_d^*(P) \bar{u}_q(p) J_{d;\mu} \Gamma^\mu_1(k) V^V_{\text{conf}}(k) u_q(q) \psi_d(Q) $$
$$ + \psi_d^*(P) \bar{u}_q(p) V^S_{\text{conf}}(k) u_q(q) \psi_d(Q), $$

where $\langle d(P) | J_{\mu} | d(Q) \rangle$ is the vertex of the diquark-gluon interaction which takes into account the diquark internal structure, $J_{d;\mu}$ is the effective long-range vector vertex of the diquark, the diquark momenta are $P = (E_d(p), -p)$, $Q = (E_d(q), -q)$ with $E_d(p) = \sqrt{p^2 + M_d^2}$, and $\psi_d(P)$ is the diquark wave function [12].

The vector and scalar confining potentials in the nonrelativistic limit in configuration space are the linear potentials with the mixing coefficient $\varepsilon$:

$$ V^V_{\text{conf}}(r) = (1 - \varepsilon)(Ar + B), \quad V^S_{\text{conf}}(r) = \varepsilon(Ar + B), $$

and their sum is just the usual static Cornell-like potential

$$ V(r) = -\frac{4}{3} \alpha_s \frac{\mu^2}{r} + Ar + B, $$

with the freezing QCD coupling constant

$$ \alpha_s(\mu^2) = \frac{4\pi}{11 - \frac{2}{3} \ln \frac{\mu^2 + M_B^2}{\Lambda^2}}. $$
TABLE I: Masses $M$ and form factor parameters of the doubly heavy diquarks.

| Quark content | State $nl_j$ | $M$ (MeV) | $\xi$ (GeV) | $\zeta$ (GeV$^2$) |
|---------------|-------------|-----------|-------------|-----------------|
| $cc$          | $1s_1$      | 3226      | 1.30        | 0.42            |
|               | $1p_1$      | 3460      | 0.74        | 0.315           |
|               | $2s_1$      | 3535      | 0.67        | 0.19            |
|               | $1d_{1,2,3}$| 3704      | 0.39        | 0.42            |
|               | $2p_1$      | 3712      | 0.60        | 0.155           |
| $bb$          | $1s_1$      | 9778      | 1.30        | 1.60            |
|               | $1p_1$      | 9944      | 0.90        | 0.59            |
|               | $2s_1$      | 10015     | 0.85        | 0.31            |
|               | $1d_{1,2,3}$| 10123     | 0.49        | 0.59            |
|               | $2p_1$      | 10132     | 0.65        | 0.215           |

The scale $\mu = 2\mu_{NR}$ is chosen to be twice the nonrelativistic reduced mass $\mu_{NR} = m_1m_2/(m_1 + m_2)$, the background mass is taken $M_B = 0.95$ GeV, and the parameter $\Lambda = 413$ MeV was fixed from the light meson spectroscopy [11].

All parameters of the model are kept fixed from previous calculations of meson and baryon properties [10, 12, 14]. The constituent heavy quark masses are $m_c = 1.55$ GeV, $m_b = 4.88$ GeV and the parameters of the linear potential are $A = 0.18$ GeV$^2$ and $B = -0.3$ GeV, the value of the mixing coefficient of vector and scalar confining potentials is $\varepsilon = -1$ and the anomalous chromomagnetic quark moment is $\kappa = -1$. Note that the long-range chromomagnetic contribution to the potential, which is proportional to $(1 + \kappa)$, vanishes for the chosen value of $\kappa = -1$.

III. DOUBLY HEAVY DIQUARKS

At first step we consider the doubly heavy diquarks and calculate their masses and form factors. We assume that in triply heavy baryons two quarks of the same flavor form a doubly heavy diquark. We solve the quasipotential equation (1) numerically with the complete relativistic potential (4). Since a diquark is composed of heavy quarks of the same flavor it is necessary to take into account the Pauli principle. The total baryon wave function must be antisymmetric. It is antisymmetric in color, thus the rest of the wave function must be symmetric. The symmetric form of the flavor part of the considered doubly heavy diquark implies that the product of the spin and orbital parts is also symmetric. For the $S$ and $D$ states, which have orbitally symmetric wave functions, the diquark spin wave function must be also symmetric and, thus, the diquark spin is 1. For the $P$ states with antisymmetric orbital wave function the diquark spin wave function must be also antisymmetric and the diquark spin is 0. The resulting value of the total momentum of the diquark is $j = 1$ for the $S$ and $P$ states, while values $j = 1, 2, 3$ are possible for the $D$ states. Note that we do not consider higher orbital excitations of the quarks inside the diquark.

The calculated masses of the ground and excited states of diquarks are presented in Table I. We also give the values of the parameters $\xi$ and $\zeta$. They parameterize with high accuracy the $r$-dependence of the form factor $F(r)$ in the vertex of the diquark-gluon inter-
action \([\mathcal{O}^3]\), which is calculated through the overlap integral of the diquark wave functions \([12]\). It is expressed by
\[
F(r) = 1 - e^{-\xi r - \zeta r^2},
\]
and takes the internal structure of a diquark into account \([12]\) smearing the diquark-gluon interaction. In this Table we use the lowercase letters to denote diquark quantum numbers. This is done to distinguish them from the quark-diquark excitations for which we reserve the uppercase letters. Here \(n = n_r + 1\), where \(n_r\) is the radial quantum number (the number of nodes of the wave function); \(l = s, p, d \ldots\) is the orbital momentum and \(j\) is the total momentum of the diquark. The calculations show that the masses of the \(d\) states with \(j = 1, 2, 3\) differ by less than 1 MeV, thus we consider their masses to be equal and give only one value.

IV. TRIPLY HEAVY BARYONS

At the second step we calculate the masses of the triply heavy baryons as the bound states of a heavy quark and doubly heavy diquark. Evaluating the baryon masses we treat all relativistic contributions nonperturbatively. The quark-diquark quasipotential contains the relativistic contributions both to the spin-independent \(V_{SI}\) and spin-dependent \(V_{SD}\) parts
\[
V(r) = V_{SI}(r) + V_{SD}(r). \tag{11}
\]
The spin-independent part determines the position of centers of gravity of the baryon levels, while the spin-dependent part is responsible for their fine and hyperfine splittings. These parts are expressed through the static potential and its derivatives. The explicit expressions for these potentials can be found in Refs. \([12, 13]\). It is important to point out that, as it was already noted in the previous section, the diquark form factor \(F(r)\) smears the diquark-gluon interaction, thus, accounting for its internal structure. As a result, in the nonrelativistic limit the one-gluon exchange part of the quark-diquark potential is modified and has the form of the smeared Coulomb-like potential
\[
\hat{V}_{\text{Coul}}(r) = -\frac{4}{3} \alpha_s \frac{F(r)}{r}, \tag{12}
\]
with \(F(r)\) given by Eq. (10).

The spin-dependent part of the quasipotential contains the spin-orbit, tensor and spin-spin interactions. It has the following form \([13]\)
\[
V_{SD}(r) = a_1 \mathbf{L} \mathbf{S}_d + a_2 \mathbf{L} \mathbf{S}_Q + b \left[ -\mathbf{S}_d \mathbf{S}_Q + \frac{3}{r^2} (\mathbf{S}_d \mathbf{r})(\mathbf{S}_Q \mathbf{r}) \right] + c \mathbf{S}_d \mathbf{S}_Q, \tag{13}
\]
where \(\mathbf{L}\) is the orbital angular momentum; \(\mathbf{S}_d\) and \(\mathbf{S}_Q\) are the diquark and quark spin operators, respectively. The coefficients \(a_1, a_2, b\) and \(c\) are expressed through the corresponding derivatives of the smeared Coulomb and confining potentials \([13]\). The smearing of the one-gluon exchange potential \([12]\) naturally softens singularities in the relativistic quasipotential in configuration space and allows us to solve numerically the quasipotential equation in its complete relativistic form. Note that both the one-gluon exchange and confining potentials contribute to the quark-diquark spin-orbit interaction. The presence of the spin-orbit \(\mathbf{L} \mathbf{S}_Q\) and of tensor terms in the quark-diquark potential leads to a mixing of states with the
same total angular momentum $J$ and parity $P$ but different values of the diquark angular momentum ($L+S_d$). We consider such mixing in the same way as in the case of doubly heavy baryons [14].

We calculate masses of all triply heavy baryons: $\Omega_{ccc}$, $\Omega_{bbb}$, $\Omega_{ccb}$ and $\Omega_{cbb}$. Their calculated spectra are given in Tables [14][15]. In the left hand half of these tables we give states with the positive parity and in the right one with the negative parity. We use the standard notations for the baryon states $J^P$, where $J$ and $P$ are the baryon total spin and parity, respectively. The composition of the baryon state is given by $NLnl_P$, where the capital letters denote quantum numbers of the quark-diquark system and the lowercase letters the diquark state. $N$ or $n$ is the radial quantum number (the number of nodes of the wave function) plus one and $L$ or $l$ is the orbital quantum number.

For the baryons composed of identical quarks ($\Omega_{ccc}$ and $\Omega_{bbb}$) there is an additional complication. It is necessary to take into account the Pauli principle not only for the diquark but also for the entire baryon. It requires the total wave function to be antisymmetric. The color wave function of the baryon is antisymmetric. The flavor part is symmetric. This means that the spin-momentum part of the wave function must be fully symmetric.

For the ground 1S1s state, this part is symmetric in momentum and, thus, the spin wave functions must be also fully symmetric, which corresponds to the total baryon spin 3/2. Therefore, only the 3/2$^+$ ground state is possible. The lightest 1/2$^+$ state should contain excitations and has a significantly larger mass. The excited states are combinations of orbital and/or radial excitations of the diquark and/or quark-diquark bound systems with the fully symmetric (3/2) or mixed symmetry (1/2) spin wave functions. Symmetric combinations such as, e.g., $|2S1s\rangle_+ = (|2S1s\rangle + |1S2s\rangle)/\sqrt{2}$, $|1D1s\rangle_+ = (|1D1s\rangle + |1S1d\rangle)/\sqrt{2}$, $|2S2s\rangle$ are combined with fully symmetric spin 3/2 wave function. While antisymmetric combinations such as, e.g., $|2S1s\rangle_+ = (|2S1s\rangle - |1S2s\rangle)/\sqrt{2}$, $|1D1s\rangle_+ = (|1D1s\rangle - |1S1d\rangle)/\sqrt{2}$, $|1P2s\rangle_+ = (|1P2s\rangle - |1S2p\rangle)/\sqrt{2}$, are combined with the mixed symmetry spin 1/2 wave functions. The details can be found in Ref. [17].

Masses of the ground states of $\Omega_{ccc}$, $\Omega_{bbb}$, $\Omega_{ccb}$ and $\Omega_{cbb}$ baryons were calculated in many papers based on different approaches [17][16]. The predicted masses of the ground state

### Table II: Masses of the $\Omega_{ccc}$ states (in MeV).

| $J^P$ | $NLnl$ | Mass   | $J^P$ | $NLnl$ | Mass   |
|-------|--------|--------|-------|--------|--------|
| $\frac{1}{2}^+$ | 2S1s$-$, 1P1p | 5230   | $\frac{1}{2}^-$ | 1S1p, 1P1s | 5010   |
|       | 1D1s   | 5278   |       |        | 1P2s$-$, 2S1p$-$ | 5370   |
| $\frac{3}{2}^+$ | 1S1s   | 4712   |       | 2S1p+  | 5385   |
|       | 1S2s$+$ | 5137   |       | 1D1p$+$ | 5520   |
|       | 1D1s$-$, 1P1p | 5277   | $\frac{3}{2}^-$ | 1S1p, 1P1s | 5029   |
|       | 2S2s   | 5541   |       | 2S1p+  | 5394   |
| $\frac{5}{2}^+$ | 1D1s$-$, 1P1p | 5278   | $\frac{5}{2}^-$ | 1F1s   | 5519   |
|       | 1D1s$+$ | 5290   |       | 1P1d$+$ | 5523   |
| $\frac{7}{2}^+$ | 1D1s$+$ | 5291   | $\frac{7}{2}^-$ | 1F1s   | 5517   |
|       |        |        |       | 1P1d$+$ | 5526   |
| $J^P$  | $NLnl$ | Mass  | $J^P$  | $NLnl$ | Mass  |
|--------|--------|-------|--------|--------|-------|
| $\frac{3}{2}^+$ | $2S1s_-, 1P1p$ | 14877  | $\frac{1}{2}^-$ | $1S1p, 1P1s$ | 14698 |
|        | $1D1s$  | 14912  |        | $1P2s_-, 2S1p_-$ | 14991 |
| $\frac{3}{2}^+$ | $1S1s$  | 14468  |        | $2S1p_+$ | 15042 |
|        | $1S2s_+$ | 14815  |        | $1D1p_+$ | 15088 |
|        | $1D1s_-, 1P1p$ | 14893  | $\frac{3}{2}^-$ | $1S1p, 1P1s$ | 14702 |
|        | $1D1s_+$ | 14905  |        | $1P2s_-, 2S1p_-$ | 14922 |
| $\frac{5}{2}^+$ | $2S2s$  | 15123  |        | $2S1p_+$ | 15031 |
|        | $1D1s_-, 1P1p$ | 14895  |        | $1D1p_+$ | 15089 |
|        | $1D1s_+$ | 14907  | $\frac{5}{2}^-$ | $1F1s$ | 15081 |
| $\frac{7}{2}^+$ | $1D1s_+$ | 14909  |        | $1P1d_+$ | 15086 |
|        |        |        | $\frac{7}{2}^-$ | $1F1s$ | 15082 |
|        |        |        |        | $1P1d_+$ | 15089 |

| $J^P$  | $NLnl$ | Mass  | $J^P$  | $NLnl$ | Mass  |
|--------|--------|-------|--------|--------|-------|
| $\frac{1}{2}^+$ | $1S1s$  | 7984  | $\frac{1}{2}^-$ | $1P1s$ | 8250 |
|        | $1S2s$  | 8361  |        | $1S1p$ | 8266 |
|        | $2S1s$  | 8405  |        | $1P1s$ | 8268 |
|        | $1D1s$  | 8472  |        | $1S2p$ | 8550 |
|        | $1P1p$  | 8505  |        | $2P1s$ | 8538 |
|        | $1P1p$  | 8511  |        | $2P1s$ | 8591 |
|        | $1S1d$  | 8531  |        | $1P2s$ | 8592 |
| $\frac{3}{2}^+$ | $1S1s$  | 7999  |        | $1P2s$ | 8595 |
|        | $1S2s$  | 8366  | $\frac{3}{2}^-$ | $1P1s$ | 8262 |
|        | $2S1s$  | 8412  |        | $1P1s$ | 8268 |
|        | $1D1s$  | 8474  |        | $1S1p$ | 8273 |
|        | $1D1s$  | 8476  |        | $1S2p$ | 8554 |
|        | $1P1p$  | 8506  |        | $2P1s$ | 8587 |
|        | $1P1p$  | 8510  |        | $2P1s$ | 8591 |
|        | $1S1d$  | 8534  |        | $1P2s$ | 8591 |
| $\frac{5}{2}^+$ | $1D1s$  | 8473  |        | $1P2s$ | 8594 |
|        | $1D1s$  | 8476  | $\frac{5}{2}^-$ | $1P1s$ | 8267 |
|        | $1P1p$  | 8508  |        | $2P1s$ | 8590 |
|        | $1S1d$  | 8536  |        | $1P2s$ | 8592 |
| $\frac{7}{2}^+$ | $1D1s$  | 8473  | $\frac{7}{2}^-$ | $1F1s$ | 8647 |
|        | $1S1d$  | 8538  |        |        |      |
TABLE V: Masses of the $\Omega_{cbb}$ states (in MeV).

| $J^P$ | $Nnl$ | Mass  | $J^P$ | $Nnl$ | Mass  |
|-------|-------|-------|-------|-------|-------|
| $\frac{1}{2}^+$ | 1S1s  | 11198 | $\frac{3}{2}^-$ | 1P1s  | 11414 |
|       | 1S2s  | 11507 |       | 1S1p  | 11506 |
|       | 1S1d  | 11622 |       | 1P1s  | 11540 |
|       | 2S1s  | 11690 |       | 1S2p  | 11654 |
|       | 1P1p  | 11692 |       | 1P2s  | 11778 |
|       | 1P1p  | 11714 |       | 1P2s  | 11796 |
|       | 1D1s  | 11796 |       | 2S1p  | 11893 |
| $\frac{3}{2}^+$ | 1S1s  | 11217 | $\frac{3}{2}^-$ | 1S1p  | 11424 |
|       | 1S2s  | 11515 |       | 1P1s  | 11535 |
|       | 1S1d  | 11629 |       | 1P1s  | 11541 |
|       | 2S1s  | 11700 |       | 1S2p  | 11660 |
|       | 1P1p  | 11707 |       | 1P2s  | 11788 |
|       | 1P1p  | 11717 |       | 1P2s  | 11795 |
|       | 1D1s  | 11797 |       | 2S1p  | 11897 |
|       | 1D1s  | 11807 | $\frac{5}{2}^-$ | 1P1s  | 11543 |
| $\frac{5}{2}^+$ | 1S1d  | 11632 |       | 1P2s  | 11795 |
|       | 1P1p  | 11715 | $\frac{7}{2}^-$ | 1P1d  | 11903 |
|       | 1D1s  | 11806 |       |       |       |
|       | 1D1s  | 11807 |       |       |       |

$3/2^+ \Omega_{ccc}$ baryons range from 4670 to 4990 MeV and masses of the $3/2^+ \Omega_{cbb}$ baryons range from 13280 to 14834 MeV. Our predictions for these masses: 4712 MeV and 14468 MeV, respectively, are well inside both ranges.

Excited states received significantly less attention. In Tables VI-X we compare our predictions with previous calculations [17–28] for the masses of $\Omega_{ccc}$, $\Omega_{cbb}$, $\Omega_{ccb}$ and $\Omega_{cbb}$ baryons. Masses of the triply heavy baryons were calculated using lattice QCD with dynamical light quark fields in Refs. [18, 24–26]. Our predictions for the masses of the ground and excited states of the $\Omega_{ccc}$ baryon are lower than lattice [18] results by about 50–150 MeV, however the structure of our excited spectrum is close to the lattice one. For the $\Omega_{cbb}$ baryon the agreement of our predictions with lattice results [24] is even better. Masses of only ground states $1/2^+$ and $3/2^+$ of $\Omega_{ccb}$ and $\Omega_{cbb}$ baryons were calculated on the lattice [24, 26]. They agree well with our results. The constituent quark model, which employs the Gaussian expansion method and the variational principle to solve the nonrelativistic three-body problem, was used in Ref. [19] to compute the mass spectra of triply heavy baryons. The renormalization group procedure for effective particles was applied for studying baryons with heavy quarks in Ref. [17]. The hypercentral constituent quark model was employed in Refs. [20, 27, 28]. For the calculation of the triply heavy baryon masses Refs. [21, 22] used the nonrelativistic quark model with the harmonic oscillator wave functions and perturbative account of the relativistic corrections. The relativistic Faddeev equation with the rainbow-ladder truncated kernel was employed in Ref. [23].
TABLE VI: Comparison with previous theoretical predictions for the masses of the $\Omega_{ccc}$ states (in MeV).  

| $J^P$ | Our   | [18] | [19] | [17] | [20] | [21] | [22] | [23] |
|-------|-------|------|------|------|------|------|------|------|
| $\frac{1}{2}^+$ | 5230  | 5397(13) | 5376 | 5358 | 5473 | 5325 | 5352 |
|       |       | 5278  | 5403(14) |      |      | 5332 | 5373 |
| $\frac{3}{2}^+$ | 4712  | 4761(6) | 4798 | 4797 | 4806 | 4965 | 4828 | 4760 |
|       | 5137  | 5315(31) | 5286 | 5309 | 5300 | 5313 | 5285 | 5150 |
|       | 5267  | 5428(13) | 5376 | 5358 | 5448 | 5368 |      |      |
|       | 5277  | 5463(13) |      |      |      |      | 5412 |      |
| $\frac{5}{2}^+$ | 5278  | 5404(15) | 5376 | 5358 | 5416 | 5329 | 5392 |
|       | 5290  | 5462(15) |      |      |      | 5343 | 5433 |
| $\frac{7}{2}^+$ | 5291  | 5395(49) | 5376 | 5358 | 5375 | 5331 | 5418 |
|       | 5010  | 5118(9) | 5129 | 5103 | 5012 | 5155 | 5142 |
|       | 5370  | 5610(31) | 5525 |      | 5607 |      |      |
|       | 5385  | 5629(43) |      |      |      |      |      |
| $\frac{1}{2}^-$ | 5029  | 5122(13) | 5129 | 5103 | 4991 | 5160 | 5162 | 5027 |
|       | 5379  | 5660(31) | 5525 |      | 5584 |      |      |
|       | 5394  | 5722(44) |      |      |      |      |      |
| $\frac{3}{2}^-$ | 5519  | 5514(64) | 5558 |      | 4965 |      |      |
|       | 5523  | 5707(25) |      |      | 5584 |      |      |
|       | 5517  | 5679(28) |      |      | 5829 |      |      |

TABLE VII: Comparison with previous theoretical predictions for the masses of the $\Omega_{bbb}$ states (in MeV).  

| $J^P$ | Our   | [24] | [19] | [17] | [20] | [21] | [22] | [23] |
|-------|-------|------|------|------|------|------|------|------|
| $\frac{1}{2}^+$ | 14877 | 14938(18) | 14894 | 14896 | 15306 | 15097 | 14971 |
|       | 14912 | 14953(17) |      |      | 15102 | 14959 |      |
| $\frac{3}{2}^+$ | 14468 | 14371(12) | 14396 | 14347 | 14496 | 14834 | 14432 | 14370 |
|       | 14815 | 14840(14) | 14805 | 14832 | 15154 | 15089 | 14848 | 14980 |
|       | 14893 | 14958(18) | 14894 | 14896 | 15300 |      | 14975 |
|       | 14905 | 15005(20) |      |      |      |      | 15016 |
| $\frac{5}{2}^+$ | 14895 | 14964(18) | 14894 | 14896 | 15293 | 15101 | 14981 |
|       | 14907 | 15007(20) |      |      | 15109 | 15022 |      |
| $\frac{7}{2}^+$ | 14909 | 14969(17) | 14894 | 14896 | 15286 | 15101 | 14988 |
|       | 14698 | 14706(9) | 14688 | 14645 | 14944 | 14975 | 14773 |
|       | 14991 |      |      | 15016 |      |      |      |
| $\frac{1}{2}^-$ | 14702 | 14714(9) | 14688 | 14645 | 14937 | 14976 | 14779 | 14771 |
|       | 14922 |      |      | 15016 |      |      |      |
| $\frac{3}{2}^-$ | 15081 |      | 15038 |      | 14931 |      |      |      |
|       | 15082 |      |      | 15641 |      |      |      |      |
TABLE VIII: Comparison with previous theoretical predictions for the masses of the $\Omega_{ccb}$ states (in MeV).

| $J^P$ | Our     | [25]  | [26]  | [19]  | [17]  | [27]  | [21]  | [23]  |
|-------|---------|-------|-------|-------|-------|-------|-------|-------|
| $\frac{7}{2}^+$ | 7984    | 8005(17) | 8007(29) | 8004 | 8301 | 8005 | 8245 | 7867 |
|       | 8361    | 8455  | 8647 | 8621 | 8537 | 8337 |
|       | 8405    | 8536  |       | 8848 |
| $\frac{3}{2}^+$ | 7999    | 8026(18) | 8037(29) | 8023 | 8301 | 8049 | 8265 | 7963 |
|       | 8366    | 8468  | 8600 | 8637 | 8553 | 8427 |
|       | 8412    | 8536  | 8647 | 8831 |
| $\frac{5}{2}^+$ | 8473    | 8536  | 8647 | 8808 | 8568 |
|       | 8476    |       |       | 8571 |
| $\frac{7}{2}^+$ | 8473    | 8538  | 8647 | 8780 | 8568 |
|       | 8538    |       |       | 8653 |
| $\frac{1}{2}^-$ | 8250    | 8306  | 8491 | 8400 | 8418 | 8164 |
|       | 8266    |       |       | 8422 |
| $\frac{3}{2}^-$ | 8262    | 8306  | 8491 | 8383 | 8420 | 8275 |
|       | 8268    |       |       | 8422 |
| $\frac{5}{2}^-$ | 8267    | 8311  | 8491 | 8365 | 8432 | 8422 |

V. CONCLUSIONS

In this paper we applied the relativistic quark model based on the quasipotential approach in QCD for the calculation of the mass spectra of triply heavy baryons. The relativistic quark-diquark approximation was used to reduce a very complicated relativistic three-body problem for the subsequent solution of two more simple two-body problems: first, calculation of the diquark properties and then considering baryon as a quark-diquark bound system. Such an approach was previously successfully applied for the calculation of the masses of the ground and excited states of doubly heavy [14], heavy [13] and strange baryons [15]. It is important to emphasize that all parameters of the model were kept fixed from the previous calculations and no new parameters were introduced. We assumed that two identical heavy quarks form a doubly heavy diquark. Masses and wave functions of the ground and excited states of such diquarks were calculated. The internal structure of the diquark was taken into account by the form factor of the diquark-gluon interaction which was calculated as the overlap integral of the diquark wave functions. The internal structure of the diquark was found to be very important for obtaining the correct prediction for the mass of the doubly heavy baryon $\Xi_{cc}$ [14]. It also allows to obtain local completely relativistic quark-diquark quasipotential without fictitious singularities.

We solved numerically the corresponding quasipotential equation and obtained the masses of the ground and excited states of $\Omega_{ccc}$, $\Omega_{bbb}$, $\Omega_{ccb}$ and $\Omega_{cbb}$ baryons. Excited states with total spin up to $J = 7/2$ both with positive and negative parity were considered. The calculated masses were compared with previous lattice QCD [18, 24, 26] and quark model calculations. Reasonable agreement with lattice results was found.
TABLE IX: Comparison with previous theoretical predictions for the masses of the Ω_{cbb} states (in MeV).

| \( J^P \) | Our  | [25]  | [26]  | [19]  | [17]  | [28]  | [21]  | [23]  |
|---------|------|-------|-------|-------|-------|-------|-------|-------|
| \( {}^1_2^+ \) | 11198 | 11194(17) | 11195(28) | 11200 | 11218 | 11231 | 11535 | 11077 |
|         | 111507 | 11607 | 11585 | 11757 | 11787 | 11603 |
|         | 11622 | 11677 | 11626 | 11934 |
| \( {}^3_2^+ \) | 11217 | 11211(18) | 11229(28) | 11221 | 11218 | 11296 | 11554 | 11167 |
|         | 111515 | 11622 | 11585 | 11779 | 11798 | 11703 |
|         | 11629 | 11677 | 11626 | 11928 |
| \( {}^5_2^+ \) | 11632 | 11677 | 11626 | 11919 | 11823 |
|         | 11715 | 11831 |
| \( {}^3_4^+ \) | 11635 | 11688 | 11626 | 11909 | 11810 |
| \( {}^3_5^+ \) | 11414 | 11482 | 11438 | 11573 | 11710 | 11413 |
| \( {}^7_5^+ \) | 11566 | 11535 | 11759 |
| \( {}^3_7^+ \) | 11424 | 11482 | 11438 | 11566 | 11711 | 11523 |
| \( {}^3_9^+ \) | 11543 | 11569 | 11601 | 11558 | 11762 |

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