Abstract

QCD finite energy sum rules, together with the latest updated ALEPH data on hadronic decays of the tau-lepton are used in order to determine the vacuum condensates of dimension $d = 2$ and $d = 4$. These data are also used to check the validity of the Weinberg sum rules, and to determine the chiral condensates of dimension $d = 6$ and $d = 8$, as well as the chiral correlator at zero momentum, proportional to the counter term of the $\mathcal{O}(p^4)$ Lagrangian of chiral perturbation theory, $\bar{L}_{10}$. Suitable integration kernels are introduced in the sum rules in order to suppress potential quark-hadron duality violations.
1 Introduction

Experimental data on hadronic decays of the $\tau$-lepton [1]-[3] play an essential role in the determination of several QCD quantities [4]. For instance, the $R_\tau$-ratio provides the cleanest determination of the running strong coupling at the scale of the $\tau$-mass. In addition, these data have been used in the past to extract the values of some of the QCD vacuum condensates entering the operator product expansion (OPE) of current correlators at short distances beyond perturbation theory [5]-[11]. This OPE is one of the two fundamental pillars of the method of QCD sum rules, the other being the assumption of quark-hadron duality [12]. The latter allows to relate QCD with hadronic physics by means of Cauchy’s theorem in the complex squared energy plane. A key advantage of hadronic $\tau$-decay data is that it determines both the vector and the axial-vector spectral functions. This feature allows to check the saturation of a variety of chiral sum rules [8], [13]-[14], as well as to determine the chiral correlator at zero momentum [4], [14]-[17], proportional to the counter term of the order $O(p^4)$ Lagrangian of chiral perturbation theory, $\tilde{L}_{10}$. It also allows for a determination of the chiral condensates of dimension $d = 6$ and $d = 8$ [4], [8], [14]-[17].

Most of these past determinations made use of the hadronic spectral functions in the vector and axial-vector channel as measured by the ALEPH Collaboration [2]. This data base was known to be problematic due to the incompleteness of the data correlations [18], thus casting some doubt on the uncertainties in results obtained using these data. A new ALEPH data set has recently become available [3], with the data organized in different bins, and with a corrected error correlation matrix. In this paper we employ these data to revisit the vacuum condensate determinations, the saturation of chiral sum rules, and the determination of $\tilde{L}_{10}$ and the chiral condensates of dimension $d = 6$ and $d = 8$. The procedure is based on finite energy QCD sum rules (FESR), weighted with suitable integration kernels to account for potential duality violations. Our results mostly confirm central values obtained previously using the original ALEPH data base, with uncertainties being slightly higher in some cases, and lower in others.

2 QCD finite energy sum rules and vacuum condensates

We consider the (charged) vector and axial-vector current correlators

$$\Pi^{VV}_{\mu\nu}(q^2) = i \int d^4x \ e^{iqx} \langle 0 | T(V_\mu(x)V_\nu^\dagger(0)) | 0 \rangle$$

$$= (-g_{\mu\nu} q^2 + q_\mu q_\nu) \Pi_V(q^2),$$

$$\Pi^{AA}_{\mu\nu}(q^2) = i \int d^4x \ e^{iqx} \langle 0 | T(A_\mu(x)A_\nu^\dagger(0)) | 0 \rangle$$

$$= (-g_{\mu\nu} q^2 + q_\mu q_\nu) \Pi_A(q^2) - q_\mu q_\nu \Pi_0(q^2),$$

where $V_\mu(x) =: \bar{u}(x)\gamma_\mu d(x) :$, $A_\mu(x) =: \bar{u}(x)\gamma_\mu\gamma_5 d(x) :$, with $u(x)$ and $d(x)$ the quark fields, and $\Pi_{V,A}(q^2)$ normalized in perturbative QCD (PQCD) (in the chiral limit) according to

$$\frac{1}{\pi} \text{Im} \Pi^{PQCD}_V(s) = \frac{1}{\pi} \text{Im} \Pi^{PQCD}_A(s) = \frac{1}{4\pi^2} \left( 1 + \frac{\alpha_s(s)}{\pi} + ... \right),$$

1
where \( s \equiv q^2 > 0 \) is the squared energy. These spectral functions and correlators are well-known up to five-loop order \([19]\). Solving the renormalization group equation for the strong coupling, one can express \( \alpha_s(s) \) in terms of the coupling at a given scale \( s_0 \), with the result at six-loop order being \([7]\)

\[
a_s(s) = a_s(s_0) + a_s^2(s_0) \left( \frac{1}{2} \beta_1 \eta \right) + a_s^3(s_0) \left( \frac{1}{2} \beta_2 \eta + \frac{1}{4} \beta_1^2 \eta^2 \right) + a_s^4(s_0) \left[ \frac{1}{2} \beta_3 \eta + \frac{5}{8} \beta_1 \beta_2 \eta^2 + \frac{1}{8} \beta_1^3 \eta^3 \right] + a_s^5(s_0) \left[ -b_3 \eta + \frac{3}{8} \beta_2^2 \eta^2 + \frac{3}{4} \beta_1 \beta_3 \eta^2 + \frac{13}{24} \beta_1^2 \beta_2 \eta^3 + \frac{1}{16} \beta_1^4 \eta^4 \right]
\]

with

\[
\eta = \ln \left( \frac{s}{s_0} \right),
\]

\( \alpha_s(s_0 = M_2^2) = 0.331 \pm 0.013 \) from the latest analysis \([4]\), and

\[
b_3 = \frac{1}{4^4} \left[ \frac{149753}{6} + 3564 \zeta_3 - \left( \frac{1078361}{162} + \frac{6508}{27} \zeta_3 \right) n_F \right. \\
+ \left. \left( \frac{50065}{162} + \frac{6472}{81} \zeta_3 \right) n_F^2 + \frac{1093}{729} n_F^3 \right],
\]

with \( \zeta_3 = 1.202 \).

Non-perturbative contributions are parametrized in terms of the vacuum condensates entering the OPE

\[
\Pi(Q^2)|_{V,A} = \sum_{N=1}^{\infty} \frac{1}{Q^{2N}} C_{2N}(Q^2, \mu^2) \langle O_{2N}(\mu^2) \rangle|_{V,A},
\]

where \( Q^2 = -q^2 \), and \( \mu \) is a renormalization scale separating long distance non-perturbative effects associated with the vacuum condensates \( \langle O_{2N}(\mu^2) \rangle \) from the short distance effects which are encapsulated in the Wilson coefficients \( C_{2N}(Q^2, \mu^2) \). In principle, the lowest dimension is \( d = 4 \) as there are no gauge invariant operators of dimension \( d = 2 \) in QCD. However, the absence of such a condensate will be confirmed by the results of this analysis. At dimension \( d = 4 \), and in the chiral limit, the only contribution is from the (chiral-symmetric) gluon condensate

\[
C_4\langle O_4 \rangle|_{V,A} = \frac{\pi^2}{3} \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu},
\]

where \( \alpha_s \) is the running strong coupling, and in the sequel \( \langle 0|O_{2N}|0 \rangle \equiv \langle O_{2N} \rangle \) is to be understood. This condensate is renormalization group invariant to all orders in PQCD.
Invoking Cauchy’s theorem in the complex squared energy $s$-plane, and assuming (global) quark-hadron duality leads to the FESR

$$\int_{|s|=s_0} ds \, f(s) \, \Pi^{QC D}_{V,A} = \int_0^{s_0} ds \, f(s) \, \frac{1}{\pi} \, \text{Im} \, \Pi(s) |_{V,A}^{HAD} , \quad (9)$$

where $f(s)$ is an integration kernel. Since PQCD is not applicable on the positive real $s$-axis, a very early warning against the unqualified use of sum rules was raised \cite{20} even before the QCD sum rule program was proposed. This is now known as quark-hadron duality violation, which was first addressed in \cite{13,21} by introducing suitable integration kernels in QCD sum rules, $f(s)$, such that they vanish at $s = s_0$, the upper limit of integration. The contour integral in Eq. (9) is usually computed using fixed order perturbation theory (FOPT) or contour improved perturbation theory (CIPT). In the former case the strong coupling is frozen at a scale $s_0$ and the renormalization group (RG) is implemented after integration. In CIPT $\alpha_s(s)$ is running and the RG is used before integrating, thus requiring solving numerically the RG equation for $\alpha_s(s)$ at each point on the integration contour. In the specific case of the determination of the vacuum condensates we found CIPT to be superior to FOPT in that results turn out to be more stable.
as a function of \( s_0 \). To implement CIPT it is convenient to introduce the Adler function

\[
D(s) \equiv -s \frac{d}{ds} \Pi(s) ,
\]

with \( \Pi(s) \equiv \Pi_{V,A}(s) \). Invoking Cauchy’s theorem and after integration by parts the following relation is obtained

\[
\oint_{|s|=s_0} ds \left( \frac{s}{s_0} \right)^N \Pi(s) = \frac{1}{N+1} \left( \frac{1}{s_0} \right)^{N+1} \oint_{|s|=s_0} ds \left( s^{N+1} - s_0^{N+1} \right) D(s) .
\]

After RG improvement, the perturbative expansion of the Adler function becomes

\[
D(s) = \frac{1}{4 \pi^2} \sum_{m=0} K_m \left( \frac{\alpha_s(-s)}{\pi} \right)^m ,
\]

where [19] \( K_0 = K_1 = 1, \ K_2 = 1.6398 , \ K_3 = 6.3710, \) for three flavours, and \( K_4 = 49.076 \) [22].
The vacuum condensates are then determined from the pinched FESR

\[ C_{2N+2}(O_{2N+2}) = (-)^{N+1} 4\pi^2 s_0^N \int_0^{s_0} ds \left[ 1 - \left( \frac{s}{s_0} \right)^N \right] \left( \frac{1}{\pi} \right) \text{Im} \Pi(s)^{HAD} \]

\[ + (-)^{N+1} s_0^{N+1} [M_0(s_0) - M_N(s_0)], \]

where the moments \( M_N(s_0) \) are given by

\[ M_N(s_0) = \frac{1}{2\pi(N+1)} \sum_{m=0}^{K} K_m [I_{N+1,m}(s_0) - I_{0,m}(s_0)], \]

with

\[ I_{N,m} \equiv i \int_{|s|=s_0} ds \left( \frac{s}{s_0} \right)^N \left[ \frac{\alpha_s(-s)}{\pi} \right]^m. \]

The latest ALEPH data compilation \([3]\) includes the vector and axial-vector channels separately, as well as their sum. Proceeding with the determination of a potential \( d = 2 \) condensate (presumably chiral-symmetric) we have used the data base for \( V + A \) in the FESR and divided the answer by a factor two. In Fig. 1 we show the result in the stability region. The solid dots correspond to the minimum value of \( \alpha_s \) and the open squares to its maximum value. As expected, this \( d = 2 \) term is consistent with zero. Notice that in this case there is no pinching integration kernel as \( N = 0 \) in Eq.(13). Next, we make use of this result and consider the pinched FESR, Eq.(13), with \( N = 1 \). The condensate of \( d = 4 \) is shown in Fig.2, and its value is

\[ C_4(O_4) = (0.019 \pm 0.019) \text{ GeV}^4, \]

where this value is obtained by reading results in the range \( s_0 \simeq (2.00 - 2.35) \text{ GeV}^2 \), i.e. the region of better accuracy, but consistent within errors with the rest of the points. This value agrees with a previous determination \([11]\) using the original ALEPH data base \([2]\). However, the uncertainty is now larger due to the new ALEPH error correlation matrix. If one were to use the data separately in the vector and the axial-vector channel, then the result from the former would be \( C_4(O_4) = (0.020 \pm 0.014) \text{ GeV}^4 \). However, the axial-vector data on its own is not particularly stable, thus leading to a result with a much larger uncertainty. This behaviour is understood from the fact that the \( \rho(770) \)-meson is a narrow resonance at a lower energy than the much broader \( a_1(1260) \), leading to a different saturation of the hadronic integrals in the FESR.

The next condensates, i.e. with dimension \( d = 6 \), in the vector and the axial-vector channels do not show a stability region. This type of FESR is not suited to extract higher dimensional condensates because the power weight in the FESR increasingly emphasizes the low energy region, where the strong coupling becomes large. In other words, the condensates are the result of a fine balanced cancellation between the hadronic integral and the PQCD moments, with a marginally meaningful result at \( d = 4 \), but not beyond. In the next section we shall determine the chiral condensates of dimension \( d = 6 \) and \( d = 8 \), which do not suffer from this handicap as the perturbative contribution cancels exactly (in the chiral limit).
3 Chiral sum rules and chiral vacuum condensates

The two Weinberg sum rules (WSR) [23] were first derived in the framework of chiral $SU(2) \times SU(2)$ symmetry and current algebra, retaining their validity in QCD in the chiral limit, and read

$$W_1 \equiv \int_0^\infty ds \frac{1}{\pi} \left[ \text{Im}\Pi_V(s) - \text{Im}\Pi_A(s) \right] = 2 f_\pi^2,$$

$$W_2 \equiv \int_0^\infty ds \frac{s}{\pi} \left[ \text{Im}\Pi_V(s) - \text{Im}\Pi_A(s) \right] = 0,$$

where $f_\pi = 92.21 \pm 0.14 \text{MeV}$ [24]. The integration region can be split into two parts, one in the range $0 - s_0$ and the other in $s_0 - \infty$. Since the spectral function difference vanishes in PQCD for $s_0$ sufficiently large, these sum rules effectively become FESR.

However, as pointed out long ago [13]-[14], the original $\tau$-decay ALEPH data [2] does not saturate these integrals up to the kinematic end point $s_0 \simeq M_\tau^2$. This also holds for the updated
ALEPH data [3]. Saturation is achieved, though, after introducing a simple pinched kernel and combining the two sum rules into one

$$W_{1P}(s_0) \equiv \int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right) \frac{1}{\pi} \left[ \text{Im} \Pi_V(s) - \text{Im} \Pi_A(s) \right] = 2 f_\pi^2. \quad (19)$$

The result is shown in Fig. 3, indicating a very good saturation of the pinched sum rule. This supports the use of simple integration kernels, although duality violations could be channel or application dependent.

![Graph showing the CHPT constant $-L_{10}$ obtained from the pinched chiral sum rule for $\Pi(0)$ Eq.(21).](image)

Next we consider the chiral correlator $\Pi(Q^2)|_{V-A}$, and absorb the Wilson coefficients entering Eq.(7) into the operators, renaming them $O_N$ to conform with a usual convention in the literature,

$$\Pi(Q^2)|_{V-A} = \sum_{N=1}^{\infty} \frac{1}{Q^{2N+4}} \langle O_{2N+4} \rangle, \quad (20)$$
with the first two chiral condensates being $\langle \mathcal{O}_6 \rangle$ and $\langle \mathcal{O}_8 \rangle$. Dropping the label $V-A$, the finite remainder of this chiral correlator at zero momentum, $\Pi(0)$, is given by

$$\Pi(0) = \int_{0}^{s_0} \frac{ds}{s} \left[ \text{Im}\Pi_V(s) - \text{Im}\Pi_A(s) \right], \tag{21}$$

where $\text{Im}\Pi_A(s)$ does not include the pion pole. The correlator $\Pi(0)$ is proportional to the counter term of the order $\mathcal{O}(p^4)$ Lagrangian of chiral perturbation theory, $\bar{L}_{10}$ [4]

$$\Pi(0) = -8 \bar{L}_{10} = 2 \left( \frac{1}{3} f_\pi^2 \langle r_\pi^2 \rangle - F_A \right), \tag{22}$$

where $\langle r_\pi^2 \rangle = 0.439 \pm 0.008 \text{ fm}^2$ is the electromagnetic radius of the pion [25], and $F_A = 0.0059 \pm 0.009$ is the radiative pion decay constant [24]. Using these values in Eq.(22) we find $\bar{L}_{10} = -6.5 \times 10^{-3}$.

Figure 5: The chiral condensate of dimension $d = 6$ from the pinched chiral sum rule Eq.(25).
Considerable improvement on the stability of the result for \( \bar{\Pi}(0) \) is obtained from the pinched sum rule \([14]\)

\[
\bar{\Pi}(0) = 4 \frac{F^2}{s_0} + \int_0^{s_0} \frac{ds}{s} \left(1 - \frac{s}{s_0}\right)^2 \left[\text{Im}\Pi_V(s) - \text{Im}\Pi_A(s)\right],
\]

which gives

\[
\bar{L}_{10} = -(6.5 \pm 0.1) \times 10^{-3}.
\]  

(24)

This result is in very good agreement with an early determination based on the original ALEPH data base \([14]\), \( \bar{L}_{10} = -(6.43 \pm 0.08) \times 10^{-3} \), as well as with more recent results using more involved methods to deal with duality violation, e.g. \( \bar{L}_{10} = -(6.46 \pm 0.15) \times 10^{-3} \) from \([15]\), and \( \bar{L}_{10} = -(6.52 \pm 0.14) \times 10^{-3} \) from \([16]\). It also agrees with lattice QCD determinations within their larger uncertainties \([26]\).

Figure 6: The chiral condensate of dimension \( d = 8 \) from the pinched chiral sum rule Eq.(28).
Turning to the chiral condensates, for dimension \( d = 6 \) we use the following pinched FESR \[14\]

\[
\langle O_6 \rangle = -2 f_\pi^2 s_0^2 + s_0^2 \int_0^{s_0} ds \left( 1 - \frac{s}{s_0} \right)^2 \left[ \text{Im} \Pi_V(s) - \text{Im} \Pi_A(s) \right].
\]

(25)

The result is shown in Fig.5 which gives

\[
\langle O_6 \rangle = -(5.0 \pm 0.7) \times 10^{-3} \text{ GeV}^6.
\]

(26)

This value agrees with \[14\] obtained from the same sum rule, Eq.(25), but using the original ALEPH data, i.e. \( \langle O_6 \rangle = -(4.0 \pm 1.0) \times 10^{-3} \text{ GeV}^6 \). It also agrees with \[15\], i.e. \( \langle O_6 \rangle = -(4.3 \pm 0.9) \times 10^{-3} \text{ GeV}^6 \), as well as with \[16\] \( \langle O_6 \rangle = -(6.6 \pm 1.1) \times 10^{-3} \text{ GeV}^6 \). In addition, this value agrees within errors with the four-quark condensate in the vacuum-saturation approximation \[27\]

\[
\langle O_6 \rangle|_{VS} = -\frac{64 \pi}{9} \alpha_s \langle \bar{q}q \rangle^2 \left[ 1 + \frac{247}{48 \pi} \alpha_s(s_0) \right] \simeq -4.6 \times 10^{-3} \text{ GeV}^6.
\]

(27)

This agreement is in contrast with the result of subtracting separate values of the dimension \( d = 6 \) condensates in the vector and the axial-vector channels. In fact, the analysis of \[11\] found that the sign of the four-quark condensate from the axial-vector channel was opposite from that expected from vacuum saturation.

Finally, we determine the \( d = 8 \) chiral condensate from the pinched sum rule \[14\]

\[
\langle O_8 \rangle = 16 f_\pi^2 s_0^3 - 3 s_0^4 \bar{\Pi}(0) + s_0^3 \int_0^{s_0} \frac{ds}{s} \left( 1 - \frac{s}{s_0} \right)^3 (s + 3 s_0) \left[ \text{Im} \Pi_V(s) - \text{Im} \Pi_A(s) \right].
\]

(28)

The result is shown in Fig.6, which leads to

\[
\langle O_8 \rangle = -(9.0 \pm 5.0) \times 10^{-3} \text{ GeV}^8,
\]

(29)

a considerably more accurate value than that of \[14\], \( \langle O_8 \rangle = -(1.0 \pm 6.0) \times 10^{-3} \text{ GeV}^8 \), as well as that of \[16\] \( \langle O_8 \rangle = (5.0 \pm 5.0) \times 10^{-3} \text{ GeV}^8 \). The present result does agree within errors with that of \[15\] \( \langle O_8 \rangle = -(7.2 \pm 4.8) \times 10^{-3} \text{ GeV}^8 \).
4 Conclusion

The new ALEPH data base [3] has been used together with QCD FESR to redetermine a potential dimension $d = 2$ term in the OPE, as well as the dimension $d = 4$ vacuum condensate, i.e. the gluon condensate in the chiral limit. The former term is consistent with zero, thus confirming expectations [10], as well as previous results [11], while the latter is affected by a larger uncertainty than the result from the original ALEPH data base [11]. It is important to notice that the current uncertainty in the strong coupling (at the scale of the $\tau$-lepton) dominates over the data errors in the final uncertainty in the condensates as obtained from FESR.

The two Weinberg sum rules are not well saturated by the data, perhaps an indication of duality violations. However, a simple pinched combination of the sum rules is well satisfied. Similar pinched integration kernels were then used to determine the chiral correlator at zero momentum, as well as the chiral condensates of dimension $d = 6$ and $d = 8$. In comparison with results using the original ALEPH data base, the major changes are in the values of the gluon condensate and of the chiral condensates.

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