Viscosity is the resistance of a material to continuous deformation exerted by shear force. High viscosity, which is sometimes greater than 1 million mPa s, at the initial reservoir conditions, is a major challenge to recovery, production, and transportation of bitumen. Addition of organic solvents or diluents with bitumen leads to significant viscosity reduction and forms the basis for the steam/solvent-assisted recovery methods of extra-heavy oil and bitumen. Therefore, modeling and predicting viscosity of bitumen–solvent mixture has become an important step in the development of solvent-assisted system. The aim of this article is to present a concise survey of the various viscosity models that have been proposed to predict the viscosity of bitumen–solvent mixtures, and make comparative discussion on their applicability. Available reports revealed that the accuracy of a model to predict the viscosity of bitumen–solvent mixtures depends on various factors including the type and concentration of solvents, and the properties of the bitumen. Thus, no model has been found to have absolute capability to predict the viscosity for all mixtures. Therefore, there is room for further improvement on the viscosity modeling of bitumen–solvent system for wider applications.

**Keywords** Bitumen · Thermophysical properties · Viscosity · Bitumen–solvent mixture · Mixing rules

**Abbreviations**
- \( A, B, C \) System-specific empirical parameter
- \( A_i \) Viscosity interaction parameter
- \( B_{ij} \) Binary viscous interaction term for bitumen diluents mixtures
- \( L \) Viscosity function
- \( P \) Pressure (MPa)
- \( P_c \) Critical pressure (MPa)
- \( P_r \) Reduced pressure (MPa)
- \( Q_n \) Coefficient in MPR viscosity correlation
- \( T \) Temperature (K)
- \( T_c \) Critical temperature of solvent (K)
- \( T_r \) Reduced temperature (K)
- \( T_1 \) NMR spin–lattice relaxation time (ms)
- \( T_2 \) NMR spin–spin relaxation time (ms)
- \( T_{2GM} \) Geometric mean relaxation time
- \( V_S \) Volume fraction of solvent
- \( V_B \) Volume fraction of bitumen
- \( a_1-a_n \) Regression parameter
- \( a, b \) Empirical parameter
- \( b, c \) Fitting parameter
- \( n \) Viscosity reduction parameter
- \( r \) Correlation parameter
- \( v_S \) Kinematic viscosity of solvent (cSt)
- \( v_B \) Kinematic viscosity of bitumen (cSt)
- \( v_{mix} \) Kinematic viscosity of mixture (cSt)
- \( w \) Weight fraction
- \( w_S \) Weight fraction of solvent
- \( w_B \) Weight fraction of bitumen
- \( x_S \) Mole fraction of solvent
- \( x_B \) Mole fraction of bitumen or heavy oil
- \( x_i \) Mole fraction of component \( i \)
- \( x_j \) Mole fraction of component \( j \)
- \( \text{mix} \) Mixture
- \( t, j \) Components

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Introduction

Heavy oil is an important energy resource with commercial availability in several places around the world including Canada, Venezuela, USA, China, etc. (Clark et al. 2007). The generic term heavy oil has been arbitrarily used to describe both the heavy oils that require thermal stimulation of recovery from the reservoir and the bitumen (or extra-heavy oil) in bituminous sand formations from which the heavy bituminous material is recovered by mining operation (Speight 2006). The conventional light crude oil has viscosity as low as 10–100 mPa s at the reservoir temperature and pressure, and greater than 20° API gravity. On the other hand, crude oil is categorized as heavy oil if it has the viscosity between 10^3 and 10^5 mPa s, and lower than 20° API. It is referred to as extra heavy oil or bitumen if it has lower than 10° API with the viscosity of up to 10^6 mPa s at the reservoir conditions (see Fig. 1). Bitumen has been defined as an involatile, adhesive, and waterproofing material derived from crude petroleum (as vacuum residue), or present in natural asphalt, which is completely or nearly completely soluble in toluene, and nearly solid at ambient temperatures (Lesueur 2009; Redelius and Soenen 2015). The total reserves of bitumen and extra-heavy oil in Canada, Venezuela, USA, China, and Nigeria are comparable to that of light crude oil, especially in Middle East, USA, and Russia (Clark et al. 2007). However, because of high viscosity, bitumen recovery, production, transportation, and refining are characterized with several technical challenges including high-energy consumption during recovery, high-pressure drop during flow and transportation, and high energy required for pumping (Alade et al. 2016).

Solvent-assisted methods have been introduced in bitumen recovery and production to serve as alternatives to the thermal recovery methods, which have various disadvantages including high energy, and water consumption, water pollution and emission of greenhouse gas (Azinfar et al. 2015; Haddadnia et al. 2018b). Essentially, addition of diluents and organic solvents with bitumen leads to significant viscosity reduction (Shu 1984; Mehrotra 1990; Miadonye et al. 2001). Better still, injection of steam along with solvent, or combination of both, is an efficient method to extract bitumen. Steam and solvent injection into a bitumen reservoir reduces the viscosity (through solvent dissolution and asphaltene precipitation) and improves the recovery (Imai et al. 2013; Kariznovi et al. 2013; Nourozieh et al. 2015a, b; Azinfar et al. 2017; Nasr and Ayodele 2006; Ardali et al. 2010; Chen et al. 2019; Sherratt et al. 2018). Similarly, co-injection of non-condensable gas, as solvent, with steam causes thermal insulation effect at the top of steam chamber, which subsequently reduces the heat loss during a steam-based system such as SAGD (Azinfar et al. 2018a). Thus, several solvent-based processes and the hybrid ones have been proposed. These include non-thermal...
solvent-based method such as the vapor extraction process (VAPEX), and the thermal-assisted methods including hot VAPEX method (N-Solv), expanding solvent SAGD (ES-SAGD), steam and gas push (SAGP), steam alternating solvent process (SAS) and liquid addition to steam (LASER), etc. (Butler and Mokrys 1992; Etherington and McDonald 2004; Motahhari et al. 2011, 2013; Pathak 2011; Imai et al. 2013; Azinfar et al. 2017, 2018a; Nourozieh et al. 2015a, b, c; Zhang et al. 2019; Nourozieh et al. 2020). Essentially, these processes offered many advantages including improved production rate, in situ upgrading of the bitumen, lower steam requirement, reduced consumption of natural gas as well as reduction in the emission of greenhouse gases (Nourozieh et al. 2020). Typically, in the hot solvent process such as steam/light solvents co-injection in SAGD (see Fig. 2), heat is transferred to the oil concomitantly with mass transfer of lighter components in the oil phase (Pathak 2011). Subsequently, the more-volatile solvent forms a vapor zone between the steam front and the solvent condensation zone. This ultimately increases the mobility and recovery of bitumen.

The adverse effect of injecting solvent as well as other diluents is due to cost, availability, and asphaltene precipitation, which may occur due to disturbance of thermodynamic equilibrium of the system and/or reduction of solubility of asphaltene in the oil phase (Speight 2006; Azinfar et al. 2017). However, proper handling, by choosing a compatible solvent and using right quantity, could minimize this problem. Thus, several investigations focusing on enhancement of the process have been carried out (Marciales and Babadagli 2016; Zirrahi et al. 2017a, b, c; Sherratt et al. 2018). For this purpose, Marciales and Babadagli (2016) investigated selection of optimal solvent using viscosity and asphaltene precipitation parameters. The authors found that solvents with low carbon number yield faster diffusion with low mixing quality. On the other hand, the high carbon number solvents were found to yield higher mixing quality with relatively low asphaltene precipitation. However, the diffusion is low.

Viscosity is the resistance of a material to continuous deformation exerted by shear force and/or resistance to transport of momentum. However, it is one of the important parameters in engineering design and simulation for bitumen recovery, production, and transportation (Shu 1984; Centeno et al. 2011; Zirrahi et al. 2014a; Nourozieh et al. 2015a, b; Mishra and Kumar 2019). Accordingly, several efforts, including characterization of thermodynamic equilibrium properties and determination of solubility parameters of water and solvents in bitumen (Azinfar et al. 2015, 2018b, 2018d; Zirrahi et al. 2014b, 2015, 2017a, b; Haddadnia et al. 2017, 2018a) and model development to predict viscosity of heavy oil/bitumen–solvent mixtures (Shu 1984; Mehrrota 1990; Miadonye and Britten 2001; Miadonye et al. 2001; Wen and Kantzas 2006; Centeno et al. 2011; Motahhari et al. 2011; Nourozieh et al. 2015a, b; Zirrahi et al. 2017b; Azinfar et al. 2018c; Al-Gawfi et al. 2019; Nourozieh et al. 2020), have been made. However, these models vary in different aspects including fundamentals of derivation and application. Hence, a comprehensive review is needed for further improvement. Therefore, this article presents a concise overview of the various viscosity prediction models for bitumen–solvent mixtures that have been reported in the studies, and discusses the mathematical background, flexibility of application, and predictability of these models.

Survey of viscosity models for bitumen–solvent mixtures

The functional relationship between the viscosity and properties such as temperature, pressure, mixture composition, and velocity variations within the fluid has been expressed using mathematical model. Thus, several viscosity models for predicting viscosity of heavy oil or bitumen–solvent mixtures in the absence of experimental data have been developed. However, determining the suitable one for a specific job is rather challenging (Centeno et al. 2011). Generally, viscosity models that have been used for heavy oil or bitumen–solvent mixtures include the empirical correlations derived from the thermophysical properties of the original oils (i.e., density and viscosity) and the thermodynamic properties of pure heavy oil and solvent components (Zirrahi et al. 2017c; Azinfar et al. 2018a). These models are mainly of two types, viz. empirical methods and semi-theoretical. The empirical models are composition independent, and hence, their reliability is not well defined in the literature (Centeno et al. 2011). On the other hand, semi-theoretical
models have fundamental basis to describe the viscosity as function of pressure and temperature, and contain adjustable parameters determined by fitting the model to experimental data (Mishra and Kumar 2019). Specifically, the models for bitumen–solvent mixtures include those based on mixing rules (Shu 1984; Miadonye and Britten 2001; Centeno et al. 2011; Nourozieh et al. 2015a; b; Zirrahi et al. 2017a, b, c; Azinfar et al. 2018a, b, c, Zirrahi et al. 2020), direct regression modeling (Wen and Kantzas 2006), the corresponding states equations (Pedersen and Fredenslund 1987; Guo et al. 2016), expanded fluid viscosity correlation (Yarranton and Satyro 2009; Motahhari et al. 2011), and the perturbed-chain statistical associating fluid theory—PC-SAFT (Ma et al. 2016), statistical thermodynamics and the perturbed-chain statistical associating fluid theory—PC-SAFT (Ma et al. 2016), expanded fluid viscosity correlation (Yarranton and Satyro 2009; Motahhari et al. 2011), and the NMR-based model (Bryan et al. 2003; Wen and Kantzas 2006). The following sections present basic descriptions of these models.

Models based on mixing rules and their modifications
Models based on mixing rules and their modifications

One of the earliest approaches to estimate the viscosity of multicomponent systems, originally developed by Graham (1846), is the law of summation of partial viscosities of individual components \( \mu_{\text{mix}} = \sum \mu_i \). Susceptibly, due to complex nature of heavy oil/bitumen, the conventional mixing rules for the pure substances might not be generally applicable to predict the viscosity of the mixture. However, following the Graham’s law, several empirical correlations and modifications including the Arrhenius (1887), Bingham (1918), Kendall and Monroe (1917), Cragoe (1933), Lederer (1973), Reid et al. (1977), Shu (1984), Chirinos et al. (1983), and Miadonye et al. (2001) have been developed to predict viscosity of binary hydrocarbon mixtures including bitumen–solvent mixtures. The mixing rules are easy to apply, as they require experimental viscosity of components and composition of mixtures in terms of volume or weight fractions (Centeno et al. 2011).

Arrhenius equation

The log-type, Arrhenius’ mixing rule (Arrhenius 1887) is commonly used in the reservoir simulators to predict the viscosity of oil-blended mixes. The model, like other mixing rules, is based on the volume or weight fractions of each component of the mixtures. The Arrhenius model is given in Eq. 1 as follows:

\[
\ln \mu_{\text{mix}} = x_S \ln \mu_S + (1 - x_S) \ln \mu_B
\]

(1a)

and the natural logarithmic form:

\[
\ln \mu_{\text{mix}} = x_S \ln \mu_S + (1 - x_S) \ln \mu_B
\]

(1b)

Using the Arrhenius mixing rule, Zirrahi et al. (2017c) and Azinfar et al. (2018c) proposed a correlation for the viscosity of bitumen and \( n \)-alkanes as presented in Eqs. 2–4. The dead bitumen viscosity was obtained from the correlation reported by Mehrotra and Srivcek (Mehrotra et al. 1986: Eq. 2).

\[
\ln(\mu_B) = [b_1 + b_2 \ln(T)] + b_3 P
\]

(2)

The parameters \( b_1-b_3 \) are the fitting parameters tuned using the experimental viscosity data. For solvent dissolved in bitumen phase, an effective viscosity was assumed a function of temperature and pressure (Eq. 3):

\[
\mu_S = c_1 + c_2 T + c_3 T^2 + (c_4 + c_5 T) P
\]

(3)

where \( c_1-c_5 \) are the fitting parameters obtained from the experimental viscosity data for solvent-bitumen mixtures.

The final correlation for viscosity of the bitumen–solvent mixture as a function of temperature and pressure is expressed as follows (Eq. 4):

\[
\ln \mu_{\text{mix}} = x_S \ln \left( c_1 + c_2 T + c_3 T^2 + (c_4 + c_5 T) P \right) + x_B \exp\left( [b_1 + b_2 \ln(T)] + b_3 P \right)
\]

(4)

Bingham equation

In order to calculate the viscosity of a binary mixture, the Bingham equation (Bingham 1918) combines the theoretical and experimental bases of viscosity of binary mixtures and discarded the prevailing assumption that viscosities were additive. Equation 5 is the Bingham model.

\[
\frac{1}{\mu_{\text{mix}}} = \frac{V_S}{\mu_S} + \frac{V_B}{\mu_B}
\]

(5)

Kendall and Monroe equation

The Kendall and Monroe’s equation (Kendall and Monroe 1917) predicts viscosity of oil blends by introducing an exponent \( (n = 1/3) \) to the viscosity terms in the mixing rule. As given in Eq. 6, the model was based on measurements of mole fractions of the components.

\[
\mu_{\text{mix}}^{1/3} = x_S \mu_S^{1/3} + x_B \mu_B^{1/3}
\]

(6)

Cragoe equation

Cragoe equation (Cragoe 1933) was developed based on the fact that the mixture viscosity behaviour is not a linear function
of the solvent fraction. The model incorporated the mixing rules or the weighting factors, which combines the degree of liquidity \( L \), as a function of the weight fractions of the oil and solvent (Eqs. 7a, b). A linear form of Cragoe equation is presented in Eq. 7c:

\[
\mu = 5 \times 10^{-4} \exp \left( \frac{1000 \ln 20}{L} \right)
\]

(7a)

The function \( L \) is defined as:

\[
L = w_S\mu_S + w_B\mu_B
\]

(7b)

\[
\frac{1}{\ln (200\mu_{mix})} = \frac{w_s}{\ln (200\mu_S)} + \frac{1 - w_s}{\ln (200\mu_B)}
\]

(7c)

Chirinos equation

The Chirinos equation (Chirinos et al. 1983) is a logarithmic form of the mixing rule based on the experimental viscosity of components and composition of mixtures in terms of volume or weight fractions (Eq. 8).

\[
\log \log (v_{mix} + 0.7) = w_s \left[ \log \log (v_S + 0.7) - \log \log (v_B + 0.7) \right] + \log \log (v_B + 0.7)
\]

(8)

Miadonye correlation

In developing a model to calculate the viscosity of bitumen–solvent mixture, Miadonye et al. (2001) used the viscosities of original bitumen and solvent as endpoints, and the diluent mass fraction was raised to a power of the viscosity reduction parameter (\( \alpha \)), which accounts for the sharp drop in bitumen viscosity with increase in diluent mass fraction. The following equation (Eq. 9a–9c) was proposed:

\[
\mu_{mix} = \exp \left[ \exp \left[ \alpha (1 - x_S^a) + \ln \mu_S - 1 \right] \right]
\]

(9a)

\[
\alpha = \ln \left( \ln \mu_B - \ln \mu_S + 1 \right)
\]

(9b)

\[
n = \frac{\mu_S}{0.9029\mu_S + 0.1351}
\]

(9c)

Reid Equation

As expressed in Eq. 10, the Reid model (Reid et al. 1977) is another mixing rule based on the experimental value of viscosity of components and composition of mixtures in terms of volume or weight fractions.

\[
\mu_{mix} = \frac{(w_S + w_B)\mu_S\mu_B}{w_S\mu_S + w_B\mu_B}
\]

(10)

Lederer equation

The Lederer model (Lederer 1933) is a modified version of classic logarithmic type (Arrhenius equation) of mixing rule, with an adjustable parameter (\( \alpha \)), which can be assigned a value between 0 and 1 (Eqs. 11a–11d).

\[
\ln \mu_{mix} = x_S \ln \mu_S + x_B \ln \mu_B
\]

(11a)

\[
x_S = \frac{aV_S}{aV_B + V_S}
\]

(11b)

\[
x_B = 1 - x_S
\]

(11c)

\[
\ln \mu_{mix} = \left( 1 - \frac{aV_B}{aV_B + V_S} \right) \ln \mu_S + \left( \frac{aV_B}{aV_B + V_S} \right) \ln \mu_B
\]

(11d)

Shu Equation

Shu (1984) developed a correlation, which generalized the Lederer equation (Eq. 11). The model requires an auxiliary term incorporating the densities and volume fractions of the pure component (i.e., the adjustable parameter \( \alpha \)). Shu (1984) presented a modified adjustable parameter (\( \alpha' \)) as given in (12b). The Shu model is expressed as follows:

\[
\ln \mu_{mix} = \left( 1 - \frac{aV_B}{aV_B + V_S} \right) \ln \mu_S + \left( \frac{aV_B}{aV_B + V_S} \right) \ln \mu_B
\]

(12a)

\[
\alpha' = \frac{17.04\Delta \rho^{0.5237} \rho_B^{3.2745} \rho_S^{1.6316}}{\ln \left( \frac{\rho_S}{\mu_S} \right)}
\]

(12b)

\[
\Delta \rho = \rho_B - \rho_S
\]

(12c)

Lobe equation

Lobe (1973) developed a correlation for calculating the kinematic viscosity of liquid mixtures, which involves the estimation of parameters from known liquid molar volume of compounds and the molar fraction of the components (Eq. 13a–13e).

\[
v_{mix} = \phi_S v_S \exp \left( \alpha_B \phi_B \right) + \phi_B v_B \exp \left( \alpha_S \phi_S \right)
\]

(13a)
\[
\alpha_s = 1.7 \ln \left[ \frac{V_B}{V_S} \right] \tag{13b}
\]

\[
\alpha_B = 0.27 \ln \left[ \frac{V_B}{V_S} \right] + \left( 1.3 \ln \left[ \frac{V_B}{V_S} \right] \right)^{1/2} \tag{13c}
\]

\[
\phi_S = \frac{m_S v_S}{m_S v_S + m_B v_B} \tag{13d}
\]

\[
\phi_B = \frac{m_B v_B}{m_S v_S + m_B v_B} \tag{13e}
\]

**Power function**

The power law model is a generalized form of Kendall and Monroe model (Eq. 6) in which the viscosity of mixture is directly dependent on the concentration of the components. It simply introduced an exponent in the Kendall and Monroe’s function, as given in Eq. 14.

\[
\mu_m = \left[ x_S \mu_S^n + (1-x_S) \mu_B^n \right]^{1/n} \tag{14}
\]

**The fourth root mixing rule**

The fourth root mixing rule, also known as the quarter-law mixing rule, is a form of the power function with exponent \( n = -0.25 \) which makes use of mass fraction rather than molar fraction as used in power rule. It is extensively used in refinery calculations (Al-Gawfi et al. 2019). The fourth root-mixing rule ((Eq. 15) is expressed as follows:

\[
\mu_{\text{mix}} = \left[ (1-w_S) \mu_B^{-0.25} + w_S \mu_S^{-0.25} \right]^{-4} \tag{15}
\]

Using Eq. 15, Al-Gawfi et al. (2019) proposed a viscosity correlation for bitumen–solvent mixture as a function of temperature and pressure (Eqs. 16a–16c). The parameters \( a_1 - a_6 \) and \( b_1 - b_3 \) are calculated using multiple regression analysis (Al-Gawfi et al. 2019):

\[
\mu_s = 0.0577 \exp \left( 0.0133 T_c - 0.016T \right) \tag{16a}
\]

\[
\mu_B = \exp(222.73 \exp \left( 7.85 \times 10^{-6} T^2 - 0.0158T - 0.000887P^2 + 0.016P + 1.06 \right) \tag{16b}
\]

\[
\mu_m = \left[ (1-w_s) \left( \exp \left( a_1 \exp \left( a_2 T^2 - a_3 T - a_4 P^2 + a_5 P + a_6 \right) \right) \right)^{-0.25} + w_s \left( b_1 \exp \left( b_2 - b_3 T \right) \right)^{-0.25} \right]^{-4} \tag{16c}
\]

**Logarithmic function**

Mehrotra (1990) presented a simple log-type mixing rule to calculate the viscosity of binary or multicomponent mixture of bitumen fractions and toluene. In addition, the author introduced a binary viscous interaction parameter \( B_{ij} \) to improve the performance of the model. The Mehrotra logarithmic function is given in Eq. 17a and 17b.

\[
\log(\mu_{\text{mix}} + 0.8) = \sum_i w_i \log(\mu_i + 0.8) \tag{17a}
\]

\[
\log(\mu_{\text{mix}} + 0.8) = \sum_i w_i \log(\mu_i + 0.8) + \sum_i \sum_j w_i w_j B_{ij} \tag{17b}
\]

**Direct regression model**

Wen and Kantzas (2006) proposed a direct correlation using the multiple regression modeling of bitumen–solvent viscosity with the weight fraction of the bitumen component. Direct correlation of mixture viscosity and the weight fraction of bitumen have been modeled using polynomial equation (Eq. 18). Different values of the coefficients were obtained depending on the bitumen samples and solvents.

\[
\mu_{\text{mix}} = a_n C_B^n + a_{n-1} C_B^{n-1} \cdots + a_2 C_B^2 + a_1 C_B + a_0 \tag{18}
\]

**Expanded fluid viscosity correlation**

Motahhari et al. (2011, 2013) predicted bitumen–solvent viscosity at high temperatures by adapting the expanded fluid-based viscosity correlation previously presented by Yarranton and Satyro (2009). The model correlated hydrocarbon viscosity to the density based on an expanded fluid concept. The inputs to the correlation are the fluid density, the pressure, and the low-pressure gas viscosity. It stemmed from the understanding that the fluidity increases when the fluid expands. Thus, the viscosity (inverse of fluidity) was expressed as a function of expansion as follows (Eq. 19a–19f):

\[
\mu - \mu_o = 0.165 \left( \exp \left( C_2 \beta \right) - 1 \right) \tag{19a}
\]

\[
\mu_B = \exp(222.73 \exp \left( 7.85 \times 10^{-6} T^2 - 0.0158T - 0.000887P^2 + 0.016P + 1.06 \right) \tag{16b}
\]

\[
\mu_m = \left[ (1-w_s) \left( \exp \left( a_1 \exp \left( a_2 T^2 - a_3 T - a_4 P^2 + a_5 P + a_6 \right) \right) \right)^{-0.25} + w_s \left( b_1 \exp \left( b_2 - b_3 T \right) \right)^{-0.25} \right]^{-4} \tag{16c}
\]

\[
\mu_{\text{mix}} = \left[ \left( 1 - w_S \right) \left( \exp \left( a_1 \exp \left( a_2 T^2 - a_3 T - a_4 P^2 + a_5 P + a_6 \right) \right) \right) \right]^{-0.25} + w_S \left( b_1 \exp \left( b_2 - b_3 T \right) \right)^{-0.25} \right]^{-4} \tag{16c}
\]
\[ \beta = \frac{1}{\exp \left\{ \left( \frac{\rho_s^o}{\rho_s^*} \right)^{0.65} - 1 \right\} - 1} \quad (19b) \]

where \( \rho_s^* \) is the pressure dependent compressed state density of the fluid given by:

\[ \rho_s^* = \frac{\rho_s^o}{\exp \left( -C_{3P} \right)} \quad (19c) \]

The correlation parameters are defined as follows:

\[ \rho_{s,mix}^o = \left[ \sum_{j=1}^{nc} W_{ij} \rho_{s,j}^{o,mix} \right]^{-1} \quad (19d) \]

\[ \frac{C_{2,mix}}{\rho_{s,mix}^o} = \sum_{i=1}^{nc} W_i C_{2,i} \quad (19e) \]

\[ C_{3,mix} = \left[ \sum_{i=1}^{nc} W_i C_{3,i} \right]^{-1} \quad (19f) \]

The dilute gas viscosity of the mixture (\( \mu_{o,mix} \)) is related to the mole fraction (\( x \)) and dilute gas viscosity (\( \mu_{o,i} \)) of component \( i \) in the Wilke’s equation (Wilke 1950):

\[ \mu_{o,mix} = \sum_i x_i \mu_{o,i} \quad (19g) \]

The binary weighting factor (\( \varphi_{ij} \)) in Eq. (19g) is expressed as a function of dilute gas viscosity and molecular weight of the component as follows:

\[ \varphi_{ij} = \frac{1 + \left( \frac{\mu_{o,i}}{\mu_{o,j}} \right)^{0.5} \left( \frac{MW_{i}}{MW_{j}} \right)^{0.25}}{8 \left( 1 + MW_{i}/MW_{j} \right)^{0.5}} \quad (19h) \]

Furthermore, in order to correct the non-idealities in the prediction due to variation of viscosity and density of the fluids, Motahhari et al. (2013) introduced generalized viscosity binary interaction parameter between solvent and bitumen (\( \beta_{S-B} \)). Thus, the viscosity binary interaction parameter is expressed as a function of the density difference (normalized density difference: \( \Delta SG_{norm} \)) between the components as follows:

\[ \beta_{S-B} = 0 \quad \text{for } \Delta SG_{norm} \leq 0.355 \quad (19i) \]

\[ \beta_{S-B} = 0.055 - 0.155 \Delta SG_{norm} \quad \text{for } \Delta SG_{norm} > 0.355 \quad (19j) \]

\[ \Delta SG_{norm} = \frac{2|SG_B - SG_S|}{(SG_B + SG_S)} \quad (19k) \]

### Corresponding states equations

The understanding of geometric similarity of the P–V–T (pressure–volume–temperature) and T–\( \mu \)-\( p \) (temperature–viscosity–pressure) diagrams has been employed to develop an equation of state (EOS)-based viscosity model heavy oils and solvent mixtures (Guo et al. 2001; Mishra and Kumar 2019). With reference to the Peng–Robinson EOS (Peng and Robinson 1976), this model has been presented as follows (Eq. 20a–t):

\[ T = \frac{rP}{\mu - b} - \frac{a}{\mu(\mu + b) + b(\mu - b)} \quad (20a) \]

The correlation parameters are defined as follows:

\[ a' = 0.457724 \frac{r^2 P^2_c}{T_c} \quad (20b) \]

\[ b' = 0.07780 \frac{r^2 P^2_c}{T_c} \quad (20c) \]

\[ r = r_c \tau(T_r, P_r) \quad (20d) \]

\[ r_c = \frac{\mu_c T_c}{P_c T_c} \quad (20e) \]

\[ \mu_c = 7.7T_c^{-1/6}M^{1/2}P_c^{1/3} \quad (20f) \]

\[ \tau(T_r, P_r) = \left[ 1 + Q_1 \left( (T_r P_r)^{0.5} - 1 \right) \right]^{-2} \quad (20g) \]

\[ b' = b\phi(T_r, P_r) \quad (20h) \]

\[ \phi(T_r, P_r) = \exp \left[ Q_2 \left( \sqrt{T_r} - 1 \right) \right] + Q_3 \left( \sqrt{P_r} - 1 \right)^2 \quad (20i) \]

\[ \omega < 0.3 \]

\[ Q_1 = 0.829599 + 0.350857\omega - 0.747680\omega^2 \quad (20j) \]

\[ Q_2 = 1.94546 - 3.19777\omega + 2.80193\omega^2 \quad (20k) \]

\[ Q_3 = 0.299757 - 2.20855\omega + 6.64959\omega^2 \quad (20l) \]
For a mixture:

\[ a_m = \sum_i x_i a_i \]  

(20p)

The correlation parameters \( a_m, b_m, b'_m, \) and \( r_m \) are defined by mixing rules as follows:

\[ a_m = \sum_i x_i a_i \]  

(20q)

\[ b_m = \sum_i x_i b_i \]  

(20r)

\[ b'_m = \sum_i \sum_j x_i x_j \sqrt{b_i b_j} (1 - k_{ij}) \]  

(20s)

\[ r_m = \sum_i x_i r_i \]  

(20t)

Equation 21 is another form of corresponding state model proposed by Pedersen and Fredenslund (1987). This has been used to calculate the viscosity of mixtures of bitumen and different solvents such as dimethyl ether (DME), propane, and butane. The Pederson corresponding state equation has been expressed as follows (Haddadnia et al. 2018c):

\[
\frac{\mu_{\text{mix}}(P, T)}{\mu_o(P_o T_o)} = \left( \frac{T_{c, \text{mix}}}{T_{c, o}} \right)^{-1/6} \left( \frac{P_{c, \text{mix}}}{P_{c, o}} \right)^{2/3} \left( \frac{\text{MW}_{\text{mix}}}{\text{MW}_o} \right)^{1/2} \left( \frac{a_{\text{mix}}}{a_o} \right)
\]

(21a)

\( \text{MW}_{\text{mix}} \) is the molecular weight of the mixture:

\[
\text{MW}_{\text{mix}} = b_1 (\text{MW}_{w}^{b_2} - \text{MW}_{n}^{b_2}) + \text{MW}_n
\]

(21b)

\[ \alpha' = 1 + 7.378 \times 10^{-3} \rho_\tau^{1.847} \text{MW}^{0.5173} \]  

(21c)

**NMR-based model for bitumen–solvent mixture**

Nuclear magnetic resonance (NMR) of hydrogen nuclei or proton NMR is an effective tool for fluid typing of any hydrogen-bearing fluids (Kleinberg and Vinegar 1996; Lawal et al. 2020). The hydrogen protons are present in oil and water, and have strong relaxation response to impose magnetic signal pulses (Bryan et al. 2003; Yang et al. 2018; Lawal et al. 2020) which is quantified by the low-field NMR. The NMR relaxometry is a novel and versatile method for viscosity measurement with broader applications for fluids in the subsurface. Experimental insight into Debye’s theory by Bloembergen et al. (1948) laid the foundation for predicting viscosity from NMR relaxation time (Kleinberg and Vinegar 1996; Wen and Kantzas 2006). Fluid types can be differentiated based on their respective HI using low or high magnetic field measurement (Kleinberg and Vinegar 1996). With respect to HI of unity for water and \( n \)-octane, saturated hydrocarbons (straight chain and branched chain) have at most hydrogen index of 0.95 (see Fig. 3). The HI is the amount of hydrogen relative to water of 0.11 mole/cm\(^3\). It is defined as a function of number of hydrogen (\( n_h \)), molecular weight (\( M_w \)), and density (\( \rho \)) of the substance, as expressed in Eq. 22a.

\[
\text{HI} = \frac{\rho n_{HI}}{(0.11 M_w)}
\]

(22a)

However, unlike pure hydrocarbons, information on the exact chemical formula for mixture of hydrocarbon is usually scarce. Thus, Eq. 22a is rarely used for calculating the HI of crude oil. Alternatively, empirical correlation (Eq. 22b) is used for estimating the HI of crude oil as a function API gravity. The HI of heavy oil and extra heavy oil bitumen can be approximated as a quadratic function of API gravity.

\[
\text{HI} = 0.1278 + 0.0725 (\text{API}) - 0.0015 (\text{API})^2
\]

(22b)

Comprehensive discussions on the fundamentals of viscosity measurements using NMR have been presented.
elsewhere in the studies (Kleinberg and Vinegar 1996; Morriss et al. 1997; LaTorresca et al. 1999; Galford and Marschall 2000; Lo et al. 2002; Wen and Kantzas 2006; Yang et al. 2018; Kausik et al. 2019; Singer et al. 2020). Specifically, LaTorresca et al. (1999) and Galford and Marschall (2000) presented models to predict heavy oil viscosity using NMR logging tools. In addition, Bryan et al. (2003) established the fact that there is a strong relationship between the amplitude index and the fluid viscosity. Subsequently, the authors proposed NMR viscosity model, which has wider coverage for both conventional oil, heavy oil, and bitumen. The model essentially relates the viscosity of bitumen with the NMR relative hydrogen index (RHI), and the oil geometric mean relaxation time ($T_{2GM}$) as follows (Eq. 23a):

$$\mu = \frac{\alpha'}{RHI^{\beta} T_{2gm}}$$ (23a)

The empirical parameters $\alpha'$ and $\beta$ were obtained from 26 different oil samples (Bryan et al. 2003) as 1.15 and 4.55, respectively. The relative hydrogen index (RHI) is related to the amplitude index (AI) parameter of NMR as follows (Eqs. 23b–23d):

$$RHI = \frac{AI_{bitumen}}{AI_{water}}$$ (23b)

$$AI = \frac{A_{bitumen(or water)}}{mass}$$ (23c)

The geometric mean mean relaxation time ($T_{2GM}$) is given as:

$$T_{2GM} = \exp \left[ \frac{\sum \theta_i \ln T_{2i}}{\sum \theta_i} \right]$$ (23d)

Wen and Kantzas (2006) successfully fitted experimental data obtained from mixtures of different bitumen and solvent samples to the Bryan model (Eq. 23).

### Prediction of bitumen–solvent viscosities

One or more of the models discussed above have been evaluated for mixtures of different heavy oil or bitumen samples and solvents (Miadonye and Britten 2001; Miadonye et al. 2001; Ma et al. 2016; Wen and Kantzas 2006; Centeno et al. 2011; Motahhari et al. 2011; Kariznovi et al. 2013; Nourozieh et al. 2015a, b; Azinfar et al. 2017). Different solvents varying from the light solvents (such as ethane, propane), to heavier ones (such as n-pentane, n-heptane, to n-octane), field solvents (such as condensate and naphtha), and aromatic solvents (such as xylene and toluene), have been tested with different bitumen samples. Extensive review of the mathematical structure and predictability of the models exist in the previous studies (Miadonye et al. 2001; Wen and Kantzas 2006; Centeno et al. 2011; Motahhari et al. 2011). However, no mixing rule has been found to have absolute capability to predict the viscosity for all mixtures (Centeno et al. 2011). The reason is susceptibly due to the complex nature of crude oil, most especially, heavy oil/bitumen. In addition, complications may arise because of tendency of asphaltene precipitation and/or compatibility of a particular solvent, which occurs due to change in the thermodynamic equilibrium of the system (Speight 2006). Furthermore, other properties such as molecular weight, density (or API gravity), and the rheological behaviour of the fluid may affect the performance of the model. In summary, the mixing rules without auxiliary parameters appear relatively easy to apply compared to those that require auxiliary terms. However, the former might result in larger deviation from the experimental values of viscosities, while the later may yield better results but with further mathematical rigor to calculate the extra parameters. This may also result in computational errors in predicting the viscosity (Miadonye et al. 2001; Wen and Kantzas 2006; Centeno et al. 2011).

Available reports on prediction of viscosity of heavy oil or bitumen mixtures indicate that the performance of the models varies along different trends including types of solvents, concentration of solvent, the API gravity of heavy oil, and the temperature and pressure of measurement. Detailed quantitative information on the performance of the models is summarized in Tables 1 and 2.

Centeno et al. (2011) tested the performance of sixteen mixing rules to predict the viscosities of four different crude oils having 21.31, 15.93, 12.42, and 9.89 °API mixed with 5–95 volume (%) diesel blends. The authors concluded that the predictability decreased as the API gravity of the fluid decreased, with the exception that the performance improved at high temperature of analysis. Miadonye et al. (2001) proposed a correlation, which was compared with those of Chirinos and Cragoe. The model was verified for viscosity ratios up to 1e6 and requires no experimental data to evaluate the parameters (Miadonye et al. 2001). The authors claimed that the best prediction was attained using the model with overall average absolute deviations of 12% for viscosity predictions, compared to Chirinos (17%), and Cragoe (23%). In addition, the performance with data not used in developing the model showed an excellent match between experimental and predicted values, with an overall average absolute deviation of below 10% for viscosities of mixtures at 25° C, 60.3° C, and 82.6° C. Likewise, Kariznovi et al. (2013) and Nourozieh et al. (2015a, b, c) evaluated the performance of seven selected mixing rules namely Arrhenius’, Cragoe’s, Shu’s, Lobe’s, double-log, Lederer’s, and power law functions to predict the viscosity of mixtures of bitumen and different solvents, viz. n-Tetradecane (Kariznovi et al. 2013), n-hexane (Nourozieh et al. 2015a), and n-heptane
Similar trends varied along the solvent types were observed. Specifically, Nourozieh et al. (2015a, b) reported that among the tested equations, the power law and Cragoe's models (using the volume fractions) better represented the viscosity data. However, when the mole fractions of the solvent were considered in the calculations, the authors observed that Arrhenius' model gave better results compared to the Cragoe's and the power law models. Within the tested viscosity range, the authors claimed that Cragoe and power law yielded the most reliable prediction. In addition, the power law model predicts the viscosity well at temperatures higher than 50 °C while at slight deviation in viscosities (under-prediction of data) are observed at 28 °C and low solvent weight fractions. In general, the authors observed that over-prediction error occurred when weight and mole fractions were used in the calculation, and that the mole fraction gives smaller viscosity values compared to experimental data. Furthermore, the fourth root-mixing rule was found to provide superior results when compared to the typically used logarithmic-based mixing rule with 6% margin of error when validated against a large list of experimental viscosity measurements (Al-Gawfi et al. 2019). The pressure and temperature ranges were 322–463 K and 0.5–8.2 MPa, respectively. However, the correlation is only valid for the temperature and pressure at which the solvent is in the gaseous state (i.e., $T > T_{sat}$ and $P < P_{sat}$).

Motahhari et al. (2011) tested the applicability of the expanded fluid viscosity model using diluted dead and live Athabasca bitumen at temperatures from 20 to 175 °C and pressures up to 10 MPa. The correlation yielded average absolute relative deviation (AARD) of 4.8, 6.9, and 16 for the dead bitumen–condensate mixture at 3, 6, and 30 wt.%, respectively. From the live bitumen–condensate mixtures of 3 and 5.9 wt%, the AARD were reported as 13 and 55 wt.%, respectively.

**Table 1** Evaluated viscosity models for bitumen–solvent mixtures

| Bitumen/heavy oil $\mu_B$ | $\rho_B$ (kg/m$^3$) | MW (g/mol) | °API | Solvent Type | Conc | $T$ (°C) | $P$ (MPa) | Model (Eqn.) | References |
|---------------------------|--------------------|------------|------|--------------|------|---------|----------|-------------|------------|
| 2.97e3–1.57e5 at 30 °C (mm$^3$/s) | iv, v, vi, vii | 5–75 wt% | 25–82.6 | 7, 8, 9 | Miadonye et al. (2001) |
| 6e3–6.7e5 at 25 °C (mPa s) | i, ii, v, vi, vii, viii, ix | 0–100 wt% | 7, 12, 18, 23 | Wen and Kantzas (2006) |
| 8.2e2 at 50 °C (mPa s): live bitumen* | iv | 3, 5.9 wt% | 20–175 | 0–10 | 19 | Motahhari et al. (2011) |
| 3.18e3 at 50 °C (mPa s): dead bitumen** | iv | 3–30 wt% | 20–175 | 0–10 | 19 | Motahhari et al. (2011) |
| Alberta bitumen* | 7.4 | i | 15, 30 | 20–175 | 0–10 | 19, 19* | Motahhari et al. (2013) |
| Alberta bitumen** | 7.2 | ix, xi, xii, xiii | 5.1–30 | 20–175 | 0–10 | 19, 19* | Motahhari et al. (2013) |
| Athabasca bitumen | 512 | ii | 5–50 wt% | 27–70 | 0–10 | 1, 7, 11, 12, 14 | Karzinov et al. (2013) |
| Athabasca bitumen | 1009 | 539.2 ± 7.9 | ii | 5–50 wt% | 25–190 | 2–10 | 1, 7, 11, 12, 13, 14 | Nourozieh et al. (2015a) |
| Athabasca bitumen | 539.2 ± 7.9 | i | 5–50 wt% | 25–190 | 2–10 | 1, 7, 11, 12, 13, 14 | Nourozieh et al. (2015b) |
| Athabasca bitumen | 539.2 ± 7.10 | ix | 5–50 wt% | 25–190 | 2–10 | 1, 7, 11, 12, 13, 14 | Nourozieh et al. (2015c) |
| Athabasca bitumen | v | 5–70 wt% | 25–190 | 2–10 | 1, 7, 11, 12, 13, 14 | Nourozieh et al. (2016) |
| Athabasca bitumen | iv | 5–50 wt% | 25–190 | 2–10 | 1, 7, 11, 12, 13, 14, 19 | Nourozieh et al. (2017) |
| Athabasca bitumen | 512.5 ± 6.9 | i, ii | 5–50 wt% | 30–190 | 2–8 | 12, 14 | Haddadnia et al. (2018a, b, c) |
| 1.84e2–1.66e5 (mm$^3$/s) | 9.89–21.31 | vii | 5–95 vol% | 25–54.4 | 1, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 | Ceteno et al. (2019) |

19* is the EF model using the generalized viscosity binary interaction parameter($\beta_{S-B}$) and normalized density difference ($\Delta SG_{norm}$)
| References                  | Model | Solvent | AARD % (Eqn) | $R^2$ |
|-----------------------------|-------|---------|---------------|-------|
|                            |       |         | $x_i$ | $v_i$ | $w_i$ |        |
| Miadonye et al. (2001)      | 7     | xiv     | 21.9–30.1 |       |       |       |
|                            |       | xv      | 18.5–39.1 |       |       |       |
|                            |       | v       | 23.9–46.6 |       |       |       |
|                            |       | vi      | 10.4–19.2 |       |       |       |
|                            |       | vii     | 9.9–18.4  |       |       |       |
|                            | 8     | xiv     | 18.4–32.2 |       |       |       |
|                            |       | xv      | 9.98–26.8 |       |       |       |
|                            |       | v       | 9.19–22.7 |       |       |       |
|                            |       | vi      | 4.7–17.2  |       |       |       |
|                            |       | vii     | 11.8–19.6 |       |       |       |
|                            | 9     | xiv     | 6.1–19.3  |       |       |       |
|                            |       | xv      | 11.8–14.7 |       |       |       |
|                            |       | v       | 2.91–13.5 |       |       |       |
|                            |       | vi      | 5.15–13.9 |       |       |       |
|                            |       | vii     | 16.8–24.9 |       |       |       |
| Wen and Kantzas (2006)      | 7     |         | 0.98–1    |       |       |       |
|                            | 12    |         | 0.97–0.99 |       |       |       |
|                            | 18    |         | 0.99      |       |       |       |
|                            | 23    |         | 0.99      |       |       |       |
| Motahhari et al. (2011)     | 20a   | iv      | 13, 55*   |       |       |       |
|                            |       | iv      | 4.8–16**  |       |       |       |
| Motahhari et al. (2013)*    | 20a   | ii     | 4, 6      |       |       |       |
|                            | 20b   | ii     | 3, 3      |       |       |       |
|                            | 20a   | ix     | 9, 22     |       |       |       |
| Motahhari et al. (2013)**   | x     |         | 25        |       |       |       |
|                            | xi    |         | 30, 38    |       |       |       |
|                            | xii   |         | 15        |       |       |       |
|                            | xiii  |         | 4         |       |       |       |
| Motahhari et al. (2013)**   | 20b   | ix     | 7, 10     |       |       |       |
|                            | x     |         | 6         |       |       |       |
|                            | xi    |         | 7, 16     |       |       |       |
|                            | xii   |         | 4         |       |       |       |
|                            | xiii  |         | 4         |       |       |       |
| Nourozieh et al. (2015a)    | 1     | ii     | 46.6      | 1329  | 4388  |       |
|                            | 7     | ii     | 82.9      | 30.3  | 189   |       |
|                            | 11    | ii     | 44.5      |       |       |       |
|                            | 12    | ii     | 60.4      |       |       |       |
|                            | 13    | ii     | 107       |       |       |       |
|                            | 14    | ii     | 43.6      | 12.3  | 28.8  |       |
| Nourozieh et al. (2015b)    | 1     | i      | 43.6      | 348   | 886   |       |
|                            | 7     | i      | 77.4      | 13.6  | 97.6  |       |
|                            | 11    | i      | 30.3      |       |       |       |
|                            | 12    | i      | 32.8      |       |       |       |
|                            | 13    | i      | 27.8      |       |       |       |
|                            | 14    | i      | 31.8      | 7     | 21    |       |
|                            | 17    | i      | 64.2      |       |       |       |
| Nourozieh et al. (2015c)    | 1     | ix     | 54.1      | 1626  | 6551  |       |
|                            | 7     | ix     | 85.8      | 26.1  | 211   |       |
|                            | 11    | ix     | 43.2      |       |       |       |
respectively. Similarly, Motahhari et al. (2013) tested the model on viscosity data for gas-free bitumen diluted with carbon dioxide (5.2 wt%), ethane (5.1 wt%), propane (7.6 and 16 wt%), n-butane (14.5 wt%), n-pentane (15 and 30 wt%) and n-heptane (15 and 30 wt%) at temperatures from 20 to 175 °C and pressures up to 10 MPa. Their results show that the viscosity of the diluted bitumen mixtures was predicted with an overall AARD of 17% using the EF model. The performance was enhanced to an AARD of 7% using generalized viscosity binary interaction parameters. In addition, their findings show that the solubility of the solvent in bitumen was the key factor controlling the mixture viscosity. The authors reported that greater viscosity reduction was obtained using less volatile the solvent at a given pressure and temperature. Accordingly, the solubility of a given solvent increases at higher saturation pressure as the process temperature increases, thus causing higher viscosity reduction.

In their efforts, Wen and Kantzas (2006) proposed the direct regression modeling, as well as the NMR-based model to predict the viscosity of bitumen–solvent mixtures. They tested their models alongside the Cragoe and Shu models. Using four heavy oil/bitumen samples mixed with six solvents in different ratios, the authors reported that the

| References                        | Model (Eqn) | Solvent | AARD % | $R^2$ |
|----------------------------------|-------------|---------|--------|-------|
|                                  |             | xs vs ws | $x_i$   | $v_i$ | $w_i$ |
| Nourozieh et al. (2016)          | 12 ix       |         | 50.7   |       |       |
|                                  | 13 ix       |         | 110    |       |       |
|                                  | 14 ix       |         | 43.2   | 13.8  | 30.7  |
| Nourozieh et al. (2017)          | 1 iv        |         | 48.6   | 450   | 660   |
|                                  | 7 iv        |         | 71.4   | 36.3  | 77.7  |
|                                  | 11 iv       |         | 28.3   |       |       |
|                                  | 12 iv       |         | 74.7   |       |       |
|                                  | 13 iv       |         | 38.9   |       |       |
|                                  | 14 iv       |         | 10.4   | 33    | 16.9  |
|                                  | 19 iv       |         | 48.3   |       |       |
| Kariznovi et al. (2013)          | 1 ii        |         | 49.4   | 445   | 851   |
|                                  | 11 ii       |         | 4.3    | 15.6  | 22.9  |
|                                  | 4 ii        |         | 61.2   | 23.6  | 110   |
|                                  | 8 ii        |         | 7.4    |       |       |
|                                  | 9 ii        |         | 38     |       |       |
| Haddadnia et al. (2018a, b, c)   | 12 ii       |         | 40.7   | 10.1  | 25.9  |
|                                  | ix         |         | 41.8   | 16.2  | 24.7  |
|                                  | 14 ii       |         | 9      |       |       |
|                                  | ix         |         | 9.9    |       |       |
| Ceteno et al. (2019)             | 1           |         | 3.8e1–1.1e5 |     |
|                                  | 2           |         | 5.1e2–1.2e5 |     |
|                                  | 3           |         | 2.8e1–1.2e5 |     |
|                                  | 4           |         | 7.7e2–1.2e5 |     |
|                                  | 7           |         | 1.4e3–1.2e5 |     |
|                                  | 5           |         | 3.5e2–4.2e5 |     |
|                                  | 8           |         | 4.5e1–1.2e5 |     |
|                                  | 9           |         | 1.3e1–3.7e5 |     |
|                                  | 10          |         | 7.1e1–1.2e5 |     |
|                                  | 11          |         | 1.6e1–1.1e7 |     |
NMR-based predictions are found to be similar to those of the Shu’s model and superior to the predictions of the Cragoe’s model. In addition, the proposed regression model was admitted to have limitation being suitable for only certain heavy oil and solvent mixtures, as used in the study. The authors also observed different results from different bitumen and solvent types, and ratios. Their results indicated that both Cragoe and Shu models show better prediction with low viscosity ratios than with high viscosity ratios. In comparison, however, the Shu’s model yielded better results with heavy oil/bitumen–solvent mixtures for wide range of blending data at higher viscosity ratio. As stated earlier, susceptibility due to the complex structure of heavy oil/bitumen, and the chemical properties of the solvents (tendency of asphaltene precipitation), the predictive performance of the model differs. In other words, the models and/or model parameters are sensitive to the conditions of the system under consideration. Another notable observation, in the present case, is that the adjustable parameter in the Shu’s model incorporates additional property, viz. the density and density difference of the components. Hence, it gave better overall performance compared to the Cragoe’s model.

Moreover, Wen and Kantzas (2006) is, probably, the only available effort, which explicitly applied the low field nuclear magnetic resonance (NMR) relaxometry to predict the viscosity of bitumen–solvent mixtures. The authors seized the opportunity that the NMR gave different spectrum with a mixture of oil and solvent than that of the oil or the solvent, individually. More interestingly, they reported that the spectrum changes in accordance with the concentration of the solvent. Then, using this change in the NMR response of the mixture, the overall prediction of the NMR was found to be fairly compared to those of Cragoe and Shu’s models. Moreover, the NMR approach is more attractive than other methods (Cragoe and Shu) since it is not limited to the viscosity and density data of the components. In addition, the performance of the Cragoe and Shu’s models (including other ones developed based on the mixing rules) could be affected by solubility and precipitation of asphaltene. Other potential advantages of the NMR included application to wider range of oils (conventional and heavy oils) without reference viscosity or density data, ability to detect precipitation of asphaltene, and application in in situ measurements (Wen and Kantzas 2006).

Conclusion

Solvent-based process is an attractive technology to improve recovery and production of the highly viscous oil such as bitumen. Viscosity is one of the important thermophysical properties, which is required in the process design, modeling and simulation of the recovery, production and transportation of bitumen–solvent system. Thus, viscosity models provide handy information when laboratory data are not readily available. Several empirical and semiempirical equations have been proposed for mixture of heavy oil/bitumen and solvents. However, no viscosity model has the absolute capability to predict the viscosity for all bitumen. In summary, the mixing rules without auxiliary parameters appear relatively easy to apply compared to those that require auxiliary terms. However, the former might result in larger deviation from the experimental values of viscosities, while the later may yield better results but with further mathematical rigors to calculate the extra parameters. This may also result in computational errors when predicting the viscosity. Hitherto, available reports on the prediction of viscosity of heavy oil/bitumen–solvent mixtures show that the accuracy of these models is difficult to generalize. It strongly depends on the environment under which a particular model was developed. Such may include the type and concentration of solvent, the properties of the bitumen, and the temperature and pressure of the experiment or tests. With reference to the available reports, the NMR modeling approach appears to be more attractive compared to the others. However, the NMR measurement of the bitumen–solvent mixture has not received adequate attention. Therefore, there is room for further improvement on the viscosity modeling of bitumen–solvent system for wider applications. This would be necessary due to increasing advances in solvent-assisted technology for bitumen recovery and production.

Appendix

**Diluents/solvents**

| Name        | Code |
|-------------|------|
| Pentane     | ix   |
| Hexane      | ii   |
| Heptane     | i    |
| n-Tetradecane | iii  |
| Condensate  | iv   |
| Naphtha     | vi   |
| Toluene     | v    |
| Diesel      | vii  |
| Kerosene    | viii |
| Methane     | x    |
| Propane     | xi   |
| Butane      | xii  |
Statistical error analysis

i. Arithmetic average of the absolute values of the relative errors (AARD).

ii. Standard error (SE).

iii. Coefficient of determination ($R^2$).

Acknowledgements The authors of this article highly appreciate and acknowledge the supports provided by the King Fahd University of Petroleum and Minerals (KFUPM) and Saudi Aramco through the Funded Project No. CIPR 2330.

Funding Saudi Aramco through the Funded Project No. CIPR 2330.

Compliance with ethical standards

Conflict of interest The authors declare no conflict of interests.

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References

Alade OS, Ademod B, Sasaki K, Sugai Y, Kumasaka J, Ogunsala AS (2016) Development of models to predict the viscosity of a compressed Nigerian bitumen and rheological properties of its emulsions. J Pet Sci Eng 145:711–722

Al-Gawfi A, Zirrahi M, Hassanzadeh H, Abedi J (2019) Development of generalized correlations for thermophysical properties of light hydrocarbon solvents (C1–C5)/bitumen systems using genetic programming. ACS Omega 4(4):6955–6967

Ardali M, Mamora D, Barrufet M (2010) A comparative simulation study of addition of solvents to steam in SAGD process. Paper CSUG/SPE 138170 presented at the Canadian Unconventional Resources and International Petroleum Conference, Calgary, Alberta, Canada, 19–21 October

Arrhenius SA (1887) Über die Dissociation der in Wasser gelösten Stoffe. Z Phys Chem 1:631–648

Azinfar B, Haddadnia A, Zirrahi M, Hassanzadeh H, Abedi J (2015) A method for characterization of bitumen. Fuel 153:240–248

Azinfar B, Haddadnia A, Zirrahi M, Hassanzadeh H, Abedi J (2017) Effect of asphaltene on phase behavior and thermophysical properties of solvent/bitumen systems. J Chem Eng Data 62:547–557

Azinfar B, Haddadnia A, Zirrahi M, Hassanzadeh H, Abedi J (2018a) Generalized approach to predict K-values of hydrocarbon solvent/bitumen mixtures. SPE-189744-MS

Azinfar B, Zirrahi M, Hassanzadeh H, Abedi J (2018b) Characterization of heavy crude oils and residues using combined gel permeation chromatography and simulated distillation. Fuel 233:885–893

Azinfar B, Haddadnia A, Zirrahi M, Hassanzadeh H, Abedi J (2018c) Phase behaviour of butane/bitumen fractions: Experimental and modeling studies. Fuel 220:47–59

Azinfar B, Haddadnia A, Zirrahi M, Hassanzadeh H, Abedi J (2018d) A thermodynamic model to predict propane solubility in bitumen and heavy oil based on experimental fractionation and characterization. J Pet Sci Eng 168:156–177

Bingham EC (1918) The variable pressure method for the measurement of viscosity. In: Proceeding of American society for testing materials 18 (Part II):10

Bloembergen N, Purcell EM, Pound RV (1948) Relaxation effects in nuclear magnetic resonance absorption. Phys Rev 73(7):679–712

Bryan J, Mirotchnik K, Kantzas A (2003) Viscosity determination of heavy oil and bitumen using NMR relaxometry. J Can Pet Technol 42:29–34

Butler AM, Mokrys JJ (1992) Recovery of heavy oils using vaporized hydrocarbon solvents: further development of the vapex process. Pet Soc Canada. https://doi.org/10.2118/SS-92-7

Centeno G, Sanchez-Reyna G, Ancheyta J, Muñoz JAD, Cardona N (2011) Testing various mixing rules for calculation of viscosity of petroleum blends. Fuel 90(12):3561–3570

Chen Z, Li X, Yang D (2019) Quantification of viscosity for solvents−heavy oil/bitumen systems in the presence of water at high pressures and elevated temperatures. Ind Eng Chem Res 58:1044–1054. https://doi.org/10.1021/acs.iecr.8b04679

Chirinos ML, Gouzalez J, Layrisse I (1983) Rheological properties of crude oils from the orinoco oil belt and the mixtures with diluents. Rev Teicina Intevyp 3(2):103–115

Clark B, de Cardenas JL, Peats A (2007) Working document of the NPC global oil and gas study: heavy oil, extra-heavy oil and bitumen, unconventional oil. Retrieved (March 2011) from www.npc.org

Cragoe CS (1933) Changes in the viscosity of liquids with temperature, pressure and composition. Proc World Pet Congr Lond 2:529–541

Etherington JR, McDonald IR (2004) Is bitumen a petroleum reserve? SPE annual technical conference and exhibition, 26–29 September 2004, Houston, Texas. https://doi.org/10.2118/90242-MS

Galford JE, Marschall DM (2000) Combining NMR and conventional logs to determine fluid volumes and oil viscosity in heavy-oil reservoirs. Society of Petroleum Engineers. https://doi.org/10.2118/90242-MS

Guo XQ, Sun CY, Rong SX, Chen GJ, Guo TM (2001) Equation of state analog correlations for the viscosity and thermal conductivities of hydrocarbons and reservoir fluids. J Pet Sci Eng 30:15–27

Graham T (1846) On the motion of gases. Philos Trans 136:537–631

Haddadnia A, Zirrahi M, Hassanzadeh H, Abedi J (2017) Solubility and thermo-physical properties measurement of CO2-and N2-athabasca bitumen systems. J Pet Sci Eng 154:277–283

Haddadnia A, Azinfar B, Zirrahi M, Hassanzadeh H, Abedi J (2018) New solubility and viscosity measurements for methane-, ethane-, propane-, and butane-Athabasca bitumen systems at high temperatures up to 260°C. J Chem Eng Data 63(9):3566–3571

Haddadnia A, Azinfar B, Zirrahi M, Hassanzadeh H, Abedi J (2018) Thermo-physical properties of n-pentane/bitumen and n-hexane/bitumen mixture systems. Can J Chem Eng 96(1):339–351
Haddadnia A, Azinfar B, Zirrahi M, Hassanzadeh H, Abedi J (2018) Thermophysical properties of dimethyl ether/Athabasca bitumen system. Can J Chem Eng 96(2):597–606
Imai M, Nishioka I, Nakano M, Kaneko M (2013) How heavy gas solvents reduce heavy oil viscosity. SPE 165451
Kariznovi M, Nourozieh H, Guan JG, Abedi J (2013) Measurement and modelling of density and viscosity for mixtures of Athabasca bitumen and heavy n-alkane. Fuel 112:83–95
Kausik R, Freed D, Fellah K, Feng L, Ling Y, Simpson G (2019) Frequency and temperature dependence of 2D-NMR T1–T2 maps of shale. Petrophysics 60(1):37–49
Kendall J, Monroe K (1917) The viscosity of liquids II. The viscosity-composition curve for ideal liquid mixtures. J Am Chem Soc 39(9):1787–1802
Kleinberg RL, Vinegar HJ (1996) NMR properties of reservoir fluids. Society of petrophysicists and well-log analysts
LaToraca GA, Dunn KJ, Webber PR, Carson RM, Stonard SW (1999) Heavy oil viscosity determination using NMR logs. Society of petrophysicists and well-log analysts
Lawal OL, Adebayo AR, Mahmoud M, Dia BM, Sultan AS (2020) A novel NMR surface relaxation measurements on rock cuttings for conventional and unconventional reservoirs. Int J Coal Geol 231:103605. https://doi.org/10.1016/j.coal.2020.103605
Lederer EL (1933) Viscosity of mixtures with and without diluents. Proc World Pet Congr Lond 2:526–528
Lesueur D (2009) The colloidal structure of bitumen: Consequences on the rheology and on the mechanisms of bitumen modification. Adv Colloid Interface Sci 145(1–2):42–82
Lo SW, Hirasaki GJ, House WV, Kobayashi R (2002) Mixing rules and correlations of NMR relaxation time with viscosity, diffusivity, and gas/oil ratio of methane/hydrocarbon mixtures. Soc Pet Eng J 7(1):24–34
Lobe VM (1973) A model for the viscosity of liquid–liquid mixtures. (MSc. Thesis) University of Rochester, Rochester, New York, USA
Ma M, Chen S, Abedi J (2016) Modeling the density, solubility and viscosity of bitumen/solvent systems using PC-SAFT. J Pet Sci Eng 139:1–12
Marc GHP (2020) Elucidating the 1H NMR relaxation mechanism in polydispersed polymers and bitumen using measurements, MD simulations, and models. arXiv:2003.02370 [physics.chem-ph]
Marciales A, Babadagli T (2016) Selection of optimal solvent type for high temperature solvent applications in heavy-oil and bitumen recovery. Energy Fuels 30(8):4243–4253
Lo SW, Hirasaki GJ, House WV, Kobayashi R (2002) Mixing rules and correlations of NMR relaxation time with viscosity, diffusivity, and gas/oil ratio of methane/hydrocarbon mixtures. Soc Pet Eng J 7(1):24–34
Lobe VM (1973) A model for the viscosity of liquid–liquid mixtures. (MSc. Thesis) University of Rochester, Rochester, New York, USA
Ma M, Chen S, Abedi J (2016) Modeling the density, solubility and viscosity of bitumen/solvent systems using PC-SAFT. J Pet Sci Eng 139:1–12
Marc GHP (2020) Elucidating the 1H NMR relaxation mechanism in polydispersed polymers and bitumen using measurements, MD simulations, and models. arXiv:2003.02370 [physics.chem-ph]
Marciales A, Babadagli T (2016) Selection of optimal solvent type for high temperature solvent applications in heavy-oil and bitumen recovery. Energy Fuels 30(8):4243–4253
Miadonye A, Britten AJ (2001) Generalized equation predicts viscosity of heavy oil-solvent mixtures. In: Transactions on modelling and simulation, vol 30. WIT Press, ISSN: 1743-355X
Miadonye A, Doyle NL, Britten A, Latour N, Puttagunta VR (2001) Modelling viscosity and mads fraction of bitumen–diluent mixtures. J Can Pet Technol 40(7):52–57
Mehrotra AK (1990) Development of mixing rules for predicting the viscosity of bitumen and its fractions blended with toluene. Can J Chem Eng 68:839
Mishra AK, Kumar A (2019) Modified Guo viscosity model for heavier hydrocarbon components and their mixtures. J Pet Sci Eng 182:106248
Morriss C, Rossini D, Straley C, Tutunjian P, Vinegar H (1997) Core analysis by low-field Nmr. Society of petrophysicists and well-log analysts
Motahhari H, Schoeggi FF, Satyro MA, Yarranton HW (2011) Prediction of the viscosity of solvent diluted live bitumen at temperatures up to 175 °C. CSUG/SPE 149405.
Motahhari H, Schoeggi FF, Satyro MA, Yarranton HW (2013) The effect of solvents on the viscosity of an Alberta bitumen at in situ thermal process conditions. Paper presented at the SPE heavy oil conference-Canada, Calgary, Alberta, Canada, June 2013. https://doi.org/10.2118/165548-MS
Nasr TN, Ayodele OR (2006) New hybrid steam–solvent processes for the recovery of heavy oil and bitumen. In: Paper SPE 101717 presented at Abu Dhabi international petroleum exhibition and conference, 5–8 November. Abu Dhabi, UAE
Nourozieh H, Kariznovi M, Abedi J (2015a) Viscosity measurement and modelling for mixtures of Athabasca bitumen/hexane. J Pet Sci Eng 129:159–167
Nourozieh H, Kariznovi M, Abedi J (2015b) Modeling and measurement of thermophysical properties for Athabasca bitumen and n-heptane mixtures. Fuel 157:73–81
Nourozieh H, Kariznovi M, Abedi J (2015c) Viscosity measurement and modeling for mixtures of athabasca bitumen/n-pentane at temperatures up to 200 °C. SPE J 20(2015):226–238. https://doi.org/10.2118/170252-PA
Nourozieh H, Kariznovi M, Abedi J (2016) Measurement and evaluation of bitumen/toluene-mixture properties at temperatures Up to 190 °C and pressures up to 10 MPa. SPE J 21(2016):1705–1720. https://doi.org/10.2118/180922-PA
Nourozieh H, Kariznovi M, Abedi J (2017) Solubility of n-butane in Athabasca bitumen and saturated densities and viscosities at temperatures Up to 200 °C. SPE J 22(2018):94–102. https://doi.org/10.2118/180927-PA
Nourozieh H, Ranjarb E, Kumar A, Forrester K, Mohns S (2020) Nonlinear Double-Log Mixing Rule for Viscosity Calculation of Bitumen/Solvent Mixtures Applicable for Reservoir Simulation of Solvent-Based Recovery Processes. SPE J 25(2020):2648–2662. https://doi.org/10.2118/203823-PA
Pathak V (2011) Heavy oil and bitumen recovery by hot solvent injection: an experimental and computational investigation. In: SPE annual technical conference and exhibition, 30 October–2 November 2011, Denver, Colorado, USA. https://doi.org/10.2118/15237-4STU
Pedersen KS, Fredenslund A (1987) An improved corresponding states model for the prediction of oil and gas viscosities and thermal conductivities. Chem Eng Sci 42(1):182–186. https://doi.org/10.1016/0009-2509(87)80225-7
Peng D-Y, Robinson DB (1976) A New Two-Constant Equation of State. Ind Eng Chem Fundam 15(1):59–64. https://doi.org/10.2118/160057a011
Redelius P, Soenen H (2015) Relation between bitumen chemistry and non-linear conductivities. Chem Eng Sci 42(1):182–186. https://doi.org/10.1016/0009-2509(87)80225-7
Sherratt J, Haddad AS, Rafati R (2018) Hot Solvent-Assisted Gravity Drainage in Naturally Fractured Heavy Oil Reservoirs: A New Model and Approach to Determine Optimal Solvent Injection Temperature. Ind Eng Chem Res 57(8):3043–3058. https://doi.org/10.1021/acs.iecr.7b04862
Shu WR (1984) A viscosity correlation for mixtures of heavy oil, bitumen, and petroleum fractions. SPE 1984:277–282
Singer PM, Parambathu AV, Wang X, Asthagiri D Chapman WG, Hirasaki GJ, Fleury M (2020) J Phys Chem B 124(20):4222–4233. https://doi.org/10.1021/acs.jpcb.0c01941
Speight JG (2006) The Chemistry and Technology of Petroleum, 5th edn. CRC Press Taylor & Francis Group, Boca Raton, FL
Wen Y, Kanzas A (2006) Evaluation of heavy oil/bitumen–solvent mixture viscosity models. J Can Pet Technol 45(4):36–43
Wilke CR (1950) A viscosity equation for gas mixtures. J Chem Phys 18:517–519. https://doi.org/10.1063/1.1747673
Yang ZM, Ma ZZ, Luo YT, Zhang YP, Guo HK, Lin W (2018) A measured method for in situ viscosity of fluid in porous media by nuclear magnetic resonance. Geofluids 2018:9542152. https://doi.org/10.1155/2018/9542152
Yarranton HW, Satyro MA (2009) Expanded fluid-based viscosity correlation for hydrocarbons. Ind Eng Chem Res 48:3640–3648
Zhang K, Zhou X, Peng X, Zeng F (2019) A comparison study between N-Solv method and cyclic hot solvent injection (CHSI) method. J Pet Sci Eng 173(2019):258–268
Zirrahi M, Hassanzadeh H, Abedi J (2014) Prediction of bitumen and solvent mixture viscosity using thermodynamic perturbation theory. J Can Pet Technol 53:48–54
Zirrahi M, Hassanzadeh H, Abedi J, Moshfeghian M (2014) Prediction of solubility of CH₄, C₂H₆, CO₂, N₂ and CO in bitumen. Can J Chem Eng 92(3):563–572
Zirrahi M, Hassanzadeh H, Abedi J (2015) Prediction of water solubility in petroleum fractions and heavy crudes using cubic-plus-association equation of state (CPA-EoS). Fuel 159:894–899
Zirrahi M, Hassanzadeh H, Abedi J (2017a) Experimental and modeling studies of water, light n-alkanes and MacKay River bitumen ternary systems. Fuel 196:1–12
Zirrahi M, Hassanzadeh H, Abedi J (2017b) Experimental and modeling studies of MacKay River bitumen and water. J Pet Sci Eng 151:305–310
Zirrahi M, Hassanzadeh H, Abedi J (2017c) Experimental and modeling studies of MacKay River bitumen and light n-alkane binaries. Can J Chem Eng 95(7):1417–1427

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