QED⊗QCD EXPONENTIATION: SHOWER/ME MATCHING AND IR-IMPROVED DGLAP THEORY AT THE LHC

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We discuss the elements of QED⊗QCD exponentiation and its interplay with shower/ME matching and IR-improved DGLAP theory in precision LHC physics scenarios. Applications to single heavy gauge boson production at hadron colliders are illustrated.

BU-HEPP-06-07, Oct., 2006, presented by B.F.L. Ward at ICHEP06

Given the approaching turn-on of the LHC, the issue of precision predictions for the effects of multiple gluon and multiple photon radiative processes is a more immediate. A problem of some interest is the construction of an event-by-event precision theory of potential luminosity processes such as single heavy gauge boson production processes. Presuming the LHC luminosity experimental error to reach 2% [1], the attendant theoretical precision tag on processes such as single W,Z production should be 2% − 1% so that the theoretical error does not adversely affect the respective luminosity determination and the attendant precision LHC physics⁵. Thus, we have developed [3] a theory of the simultaneous resummation of multiple gluon and multiple photon radiative effects, QED ⊗ QCD exponentiation, to realize systematically the needed higher order corrections on an event-by-event basis, in the presence of parton showers, to the desired accuracy.

We note that in Ref. [3] we have shown that for a process such as $q+Q^c \rightarrow V+n(G)+n'(\gamma)+X \rightarrow \ell\ell' + n(y) + n'(\gamma) + X$ we have the result

$$d\sigma_{\exp} = e^{\text{SUM}^\text{IR}(\text{QCED})} \sum_{n,m=0}^{\infty} \int \prod_{j_1=1}^{n} \frac{d^3k_{j_1}}{k_{j_1}} \prod_{j_2=1}^{m} \frac{d^3k'_{j_2}}{k'_{j_2}} \int \frac{d^4y}{(2\pi)^4} e^{iy(p_1+q_1-p_2-q_2-\sum k_{j_1} - \sum k'_{j_2}) + D_{\text{QCED}}}$$

$$\tilde{\beta}_{n,m}(k_1, \ldots, k_n; k'_1, \ldots, k'_m) \frac{d^3p_2}{p_2^0} \frac{d^3q_2}{q_2^0}.$$  \hspace{1cm} (1)

where the new YFS [6, 7] residuals, defined in Ref. [3], $\tilde{\beta}_{n,m}(k_1, \ldots, k_n; k'_1, \ldots, k'_m)$, with $n$ hard gluons and $m$ hard photons, represent the successive application of the YFS expansion first for QCD and subsequently for QED. The functions $\text{SUM}^\text{IR}(\text{QCED}), D_{\text{QCED}}$ ""
are given in Ref. [3]. The residuals \( \tilde{\beta}_{n,m}(k_1, \ldots, k_n; k'_1, \ldots, k'_m) \) are free of all infrared singularities and the result in (1) is a representation that is exact and that can therefore be used to make contact with parton shower MC’s without double counting or the unnecessary averaging of effects such as the gluon azimuthal angular distribution relative to its parent’s momentum direction.

For our prototypical processes, we have the standard formula

\[
d\sigma_{\text{exp}}(pp \rightarrow V + X \rightarrow \ell \ell' + X') = \sum_{i,j} \int dx_i dx_j F_i(x_i) F_j(x_j) d\sigma_{\text{exp}}(x_i, x_j), \tag{2}
\]

in the notation of Ref. [3]. We will illustrate this formula with semi-analytical methods and the case of single Z production with the structure functions \( \{F_i\} \) from Ref. [8] for definiteness. A MC realization will appear elsewhere [9].

As we have explained in Ref. [10], from the standpoint of using the result (1) with the structure functions \( \{F_i\} \), we intend two possible approaches to the implied shower/ME matching [11]: one is based on \( p_T \) matching and one based on the shower-subtracted residuals \( \tilde{\beta}_{n,m} \). The respective shower can be that from PYTHIA [12], HERWIG [13], or the new shower algorithms in Ref. [14]. This combination of showers and exact ME’s can be improved systematically to arbitrary precision order-by-order in \( (\alpha_s, \alpha) \) in the presence of exact phase space [9].

To illustrate the size of the interplay between QED and QCD in the threshold region, we compute, with and without the QED, the respective ratio \( r_{\text{exp}} = \sigma_{\text{exp}} / \sigma_{\text{Born}} \), with the result

\[
r_{\text{exp}} = \begin{cases} 1.1901, & \text{QCD} \equiv \text{QCD} + \text{QED}, \text{ LHC} \\ 1.1872, & \text{QCD}, \text{ LHC} \\ 1.1911, & \text{QCD} \equiv \text{QCD} + \text{QED}, \text{ Tevatron} \\ 1.1879, & \text{QCD}, \text{ Tevatron}. \end{cases}
\tag{3}
\]

Here, we have used from (1) the \( \tilde{\beta}_{0,0} \)-level result

\[
\hat{\sigma}_{\text{exp}}(x_1 x_2 s) = \int_0^{v_{\text{max}}} dv \gamma_{\text{QCED}} v^{\gamma_{\text{QCED}} - 1}
\]

\[
F_{YFS}(\gamma_{\text{QCED}}) e^{\delta_{YFS} \delta_{\text{Born}}} ((1 - v)x_1 x_2 s) \tag{4}
\]

where we intend the well-known results for the respective parton-level Born cross sections and the value of \( v_{\text{max}} \) implied by the experimental cuts under study. The value for the QED\(\otimes\)QCD exponent is

\[
\gamma_{\text{QCED}} = \left\{ 2Q_f^2 \frac{\alpha_s}{\pi} + 2C_F \frac{\alpha_s}{\pi} \right\} L_{\text{nl}s}
\]

where \( L_{\text{nl}s} = \ln x_1 x_2 s / \mu^2 \) when \( \mu \) is the factorization scale. The functions \( F_{YFS}(\gamma_{\text{QCED}}) \) and \( \delta_{YFS}(\gamma_{\text{QCED}}) \) are well-known [7] as well:

\[
F_{YFS}(\gamma_{\text{QCED}}) = \frac{e^{-\gamma_{\text{QCED}} r_{\text{QED}}}}{\Gamma(1 + \gamma_{\text{QCED}})}, \tag{5}
\]

with \( \delta_{YFS}(\gamma_{\text{QCED}}) = \frac{1}{2} \gamma_{\text{QCED}} + Q_f^2 \frac{\alpha_s}{\pi} + C_F \frac{\alpha_s}{\pi} (2\zeta(2) - \frac{1}{2}) \), where \( \zeta(2) \) is Riemann’s zeta function of argument 2, i.e., \( \pi^2 / 6 \), and \( \gamma_E \) is Euler’s constant, i.e., 0.57727... .

We see that the size of the QED effects, \( \sim 0.3\% \), is appreciable in a 1% precision tag budget and these effects are comparable to the QED effects found for the structure function evolution itself in Refs. [15–19].

We also see that our soft, QCD correction is entirely consistent with the exact results in Ref. [20–22] and that our QED effects are consistent with the exact \( \mathcal{O}(\alpha) \) results in Refs. [23–25]. Such cross-checks are essential in establishing a rigorous precision tag on the theoretical predictions.

With an eye toward the 1% precision tag for the single heavy gauge boson production at the LHC, we have analyzed [26] the IR limit of the DGLAP [27] kernels themselves. The effects which we address are illustrated in Fig. 1. We apply the QCD exponentiation master formula from Ref. [4], the analog of (1) for just QCD, to the gluon emission transition that corresponds to \( P_{q\bar{q}}(z) \), i.e., to the squared amplitude for \( q \to q(z) + G(1 - z) \); as we show in Ref. [26], this allows us to get the following IR-improved DGLAP kernels

\[
P_{q\bar{q}}^{\text{exp}}(z) = C_F F_{YFS}(\gamma_q) e^{K_8 \frac{1 + z}{1 - z} (1 - z)^{\gamma_q}}
\]

\[- f_0(\gamma_q) \delta(1 - z) \{ \}
\]

(6)
Figure 1. In (a), we show the usual process $q \rightarrow q(1 - z) + G(z)$; in (b), we show its multiple gluon improvement $q \rightarrow q(1 - z) + G_1(\xi_1) + \cdots + G_n(\xi_n)$. $z = \sum_j \xi_j$.

$$P_{Gq}(z)^\exp = C_F F_Y F_S(\gamma_q) e^{\frac{1}{2} \delta_2} \frac{1 + (1 - z)^2}{z} z^{\gamma_q},$$

$$P_{GG}^\exp(z) = 2 C_G F_Y F_S(\gamma_G) e^{\frac{1}{2} \delta_2} \left\{ \frac{1 - z}{z} z^{\gamma_G} + \frac{z}{1 - z} (1 - z)^{\gamma_G} + \frac{1}{2} (e^{1 - \gamma_G} - 1) (1 - z) + z (1 - z)^{1 + \gamma_G} \right\}.$$

$$P_{Gq}^\exp(z) = F_Y F_S(\gamma_G) e^{\frac{1}{2} \delta_2} \left\{ \frac{1}{2} (z (1 - z)^{\gamma_G}) + (1 - z)^2 z^{\gamma_G} \right\},$$

where from the standard DGLAP methodology [26] we have $f_q(\gamma_q) = \frac{2}{\gamma_q + \frac{2}{\gamma_2} + \frac{1}{2} \gamma_1}$, $\gamma_q = C_F \alpha_s \pi t = C_F \frac{\alpha_s \pi t}{\gamma_q}$, $\gamma_G = C_G \alpha_s \pi t = C_G \frac{\alpha_s \pi t}{\gamma_G}$, and $f_G(\gamma_G) = \frac{n_f}{C_G (1 + \gamma_G)(2 + \gamma_G)(3 + \gamma_G)} + \frac{1}{\gamma_G(1 + \gamma_G)(2 + \gamma_G)} + \frac{1}{(1 + \gamma_G)(2 + \gamma_G)} + \frac{1}{(2 + \gamma_G)(3 + \gamma_G)(4 + \gamma_G)}.$

$A_n^{NS}$ becomes, from the exponentiated kernels,

$$A_n^{NS} = C_F F_Y F_S(\gamma_q) e^{\frac{1}{2} \delta_2} \left[ B(n, \gamma_q) + B(n + 2, \gamma_q) - f_q(\gamma_q) \right]$$

where $B(x, y)$ is the beta function given by $B(x, y) = \Gamma(x) \Gamma(y)/\Gamma(x + y)$. Compare the usual result

$$A_n^{NS} = C_F \left[ -\frac{1}{2} + \frac{1}{n(n + 1)} - 2 \sum_{j=2}^{n} \frac{1}{j} \right].$$

The IR-improved n-th moment goes for large $n$ to a multiple of $-f_q$, consistent with $\lim_{n \rightarrow \infty} z^{n - 1} = 0$ for $0 \leq z < 1$; the usual result diverges as $-2C_F \ln n$. The two results differ for finite $n$ as well: we get, for example, for $\alpha_s \equiv .18$, $A_2^{NS} = C_F (-1.33), C_F (-0.966)$ for (12) and (11), respectively. See Refs. [26, 28] for further discussion.

Contact with the exact 2-loop and 3-loop results in Refs. [29, 30] has also been made [26]; for the non-singlet case, we have, in the notation of Ref. [30], the exact 3-loop IR-improved result

$$P_{n+q}^{\exp}(z) = -\frac{\alpha_s}{4\pi} \left[ f_q^\exp(z) + f_G^\exp(z) \right]$$

$$= \left\{ \frac{\alpha_s}{4\pi} \right\}^2 \left[ (1 - z)^{\gamma_q} P_{ns}^{(1)+}(z) + B_2 \delta(1 - z) \right]$$

$$+ \left( \frac{\alpha_s}{4\pi} \right)^3 \left[ (1 - z)^{\gamma_q} P_{ns}^{(2)+}(z) + B_3 \delta(1 - z) \right]$$

where $P_{qq}^\exp(z)$ is given in (9) and the resummed residuals $P_{ns}^{(i)+}, i = 1, 2$ are related to the exact results [29, 30] for $P_{ns}^{(i)+}, i = 1, 2$ via

$$P_{ns}^{(i)+}(z) = P_{ns}^{(i)+}(z) - B_{1+i} \delta(1 - z) + \Delta_{ns}^{(i)+}(z),$$

where the $B_i$ [29, 30] and the $\Delta_{ns}^{(i)+}(z)$ are given explicitly in Ref. [26]. The detailed phenomenological consequences of the fully exponentiated 2- and 3-loop DGLAP kernel set will appear elsewhere [9].

In summary, the methods illustrated and presented herein are all under investigation and implementation as we prepare for the era of 1% precision LHC theoretical predictions.
on the necessary processes. Finally, one of us (B.F.L.W.) thanks Prof. W. Hollik for the support and kind hospitality of the MPI, Munich, while a part of this work was completed. We also thank Prof. S. Jadach for useful discussions. This work is partly supported by US DOE grant DE-FG02-05ER41399 and by NATO grant PST.CLG.980342.

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