The electroweak phase transition at $m_H \simeq 80$ GeV from $L_t = 2$ lattices

Ferenc Csikor$^a$, Zoltan Fodor$^{b,†}$ and Jochen Heitger$^c$

$^a$ Institute for Theoretical Physics, Eötvös University, H-1088 Budapest, Hungary

$^b$ KEK Theory Group, Tsukuba-shi, Ibaraki 305, Japan

$^c$ Institut für Theoretische Physik I, Universität Münster, D-48149 Münster, Germany

We study the finite-temperature electroweak phase transition by numerical simulations of the four-dimensional SU(2)–Higgs model on anisotropic lattices with temporal extension $L_t = 2$. The physically interesting parameter region of Higgs masses near 80 GeV is reached, and recent results on some thermodynamic quantities are presented.

1. Introduction

Though the scenario of electroweak baryogenesis at a sufficiently strong first order phase transition [1] within the SM seems to be ruled out [2,3], it appears important to quantify its nature and strength at more realistic Higgs masses $m_H \approx 80$ GeV, and to compare with effective 3D–theories claiming an endpoint of the transition line at $m_{H}(\text{crit}) \lesssim 80$ GeV, beyond which the EWPT turns into an analytic crossover [2]. Hence we made numerical simulations of the anisotropic SU(2)–Higgs model, since for weaker transitions at larger $m_H$ one expects the typical excitations $m \ll T$ to require isotropic lattices exceeding most accessible computer resources.

In the following we will focus on interface tension and latent heat from $T > 0$ simulations at $L_t = 2 \ll L_{x,y} \ll L_z$. These, involving a sequence of heatbath and overrelaxation algorithms, were done at HLRZ, Jülich (CRAY-T90), and DESY-IfH, Zeuthen (APE-Quadrics), Germany.

2. Anisotropic SU(2)–Higgs model

The lattice action of the four-dimensional SU(2)–Higgs model on anisotropic lattices reads

$$S[U, \varphi] = \sum_x \left\{ \sum_{i=s,t} \beta_i \sum_{p_i} \left( 1 - \frac{1}{2} \text{Tr} U_{p_i,x} \right) - \kappa_s \sum_{\mu=1}^{3} \text{Tr} \left( \varphi_{x+i \mu}^+ U_{x+i \mu, x} \varphi_x \right) - \kappa_t \text{Tr} \left( \varphi_{x+4}^+ U_{x+4, x} \varphi_x \right) + \frac{1}{2} \text{Tr} \left( \varphi_x^+ \varphi_x \right) + \lambda \left[ \frac{1}{2} \text{Tr} \left( \varphi_x^+ \varphi_x \right) - 1 \right]^2 \right\}$$

in terms of gauge links $U_{x,\mu} \in SU(2)$, space- and timelike plaquettes $U_{p_i, x}$ and $U_{p_{i4}, x}$, site variables $\varphi_x$.  

Figure 1. $\xi$–evaluation at $m_H = 72(5)$ GeV and $g_R^2 = 0.577(15)$, whose equal abscissas are displaced. $\xi_i(\gamma_{\beta})$, as $s$– to $t$–like $m_i$–ratios in the Higgs and vector channels ($i = H, W$) and from a suitable mapping of static potentials ($i = V$), are interpolated to coincide in errors. The error ellipse of the matching point encloses the numerical estimates and the perturbative one [4].
\( \varphi_x = \rho_x \alpha_x, \varphi_x > 0, \alpha_x \in \mathbb{SU}(2) \), and the lattice spacing and coupling anisotropy parameters \( \xi \equiv a_x/a_t, \gamma_\beta \equiv \sqrt{\beta_2/\beta_3} \), and \( \gamma_\kappa \equiv \sqrt{\kappa_t/\kappa_s} \) with \( \beta^2 = \beta_2 \beta_3 \) and \( \kappa^2 = \kappa_s \kappa_t \). The general strategy of \( T > 0 \) studies in \( D = 4 \) is to fix \( T_c = 1/a_i L_i \) at a given temporal extension \( L_i \), to determine the critical hopping parameter \( \kappa_c \), and to calculate in this phase transition point the physical, non-perturbatively renormalized parameters \( R_{HW} \equiv m_H/m_W \) \( m_W^{\text{phys}} = 80 \) GeV and \( g_R^2 \) in simulations on \( T = 0 \) lattices \( [3] \). Then the continuum limit is realized as approach to the scaling region via \( L_i \to \infty \) along lines of constant physics.

In \([5]\) we confirmed the one-loop corrections \( \gamma^{(p)}_\beta = 3.919 \) and \( \xi^{(p)} = 4.052 \) to the tree-level anisotropies \( \gamma_\beta = \gamma_\kappa \equiv \xi \equiv 4 \) non-perturbatively by demanding space-time symmetry restoration (rotational invariance) with correlation lengths in physical units being equal in both directions, see figure \( [4] \). This opens the way to analyze the EWPT for \( m_H \gtrsim 80 \) GeV within the 4D–model in a systematic and fully controllable way.

3. Thermodynamic quantities and results

In view of the large lattices to be used, the interface tension \( \sigma \) has been determined by employing the two-coupling method \( [6] \) in \( \kappa \), which in previous investigations of the SU(2)–Higgs model \( [7] \) turned out to be quite robust and, at the same time, most economic among the other methods at disposal \( [8] \). After enforcing an interface pair perpendicular to the \( z \)-direction by dividing the lattice volume in symmetric and Higgs phases with \( (\kappa_i < \kappa_c \colon z \leq L_z/2, \kappa_2 > \kappa_c \colon z > L_z/2) \), the related additional free energy \( \Delta F \) yields for \( \Delta \kappa \equiv \kappa_2 - \kappa_1 \ll 1 \) the estimator \( \[3\] \)

\[
\sigma^2_{\kappa} \equiv \frac{1}{2} \lim_{\Delta \kappa \to 0} \left\{ \Delta \kappa \cdot L_z \cdot \left[ L^{(1)}_{\varphi} - L^{(2)}_{\varphi} \right] \right\}, \quad (2)
\]

\( L^{(j)}_{\varphi} = L^{(j)}_{\varphi}(\kappa_1, \kappa_2) \) denotes the expectation value of the \( \varphi \)-link operator \( L_{\varphi;x\mu} = \frac{1}{2} \text{Tr} (\varphi^+_x U_{x\mu} \varphi_x) \) in the respective phases, and, since \( \Delta F \simeq O(\Delta \kappa) \), the \((N + 2)\)-parametric Laurent ansätze

\[
L^{(j)}_{\varphi} = -\frac{c_i}{\kappa_i - \kappa_c} + \sum_{j=0}^{N} \gamma^{(j)}_{i} (\kappa_i - \kappa_c)^j + \cdots \quad (3)
\]

give \( \hat{\sigma}/T_{c}^3 = L_{t}^2 L_{c}(c_1 + c_2)/\xi^2 \). As exemplarily displayed in figures \( [3] \) and \( [4] \), we performed such fits to sets of 2–\( \kappa \) data at \( L_t = 2 \) with simulation parameters corresponding to \( m_H = 78(4) \) GeV pole mass and \( g_R^2 = 0.539(16) \). For the

![Figure 2. Four-parameter \( \chi^2 \)-fit of \( L^{(i)}_{\varphi} \), \( i = 1, 2 \).](image)
Figure 3. As in figure 2 but for $\Delta L_\phi \equiv L_\phi^{(2)} - L_\phi^{(1)}$.

Table 1. Lattice results at $L_t = 2$ and, preliminarily, at $L_t = 3$. The transition points lie at $\kappa_c = 0.107791(3)$ and $\kappa_c = 0.10703(3)$.

| $L_t$ | $T_c/m_H$ | $10^3 \Delta \sigma/T_c^3$ | $10^4 \Delta \epsilon/T_c^4$ |
|-------|-----------|-----------------|-----------------|
| 2     | 1.86(2)   | 6(4)            | 0.37(16)        |
| 3     | 1.8(2)    | —               | —               |

4. Discussion and outlook

$\hat{\sigma}/T_c^3$ and $\Delta \epsilon/T_c^4$ for $m_H \approx 80$ GeV are substantially smaller than perturbatively ($\sigma/T_c^3 \approx 0.002$ [8]). They are even consistent with a no first order phase transition scenario approximately on the 1–σ level. The fact that this result deviates from those of the 3D–investigations [2] should be clarified in future. However, a temporal lattice extension of $L_t = 2$ may be still too far from continuum physics, and at least the knowledge of the behaviour at $L_t = 3$ seems necessary to draw a final conclusion.

REFERENCES

1. D. A. Kirzhnitz, JETP Lett. 15 (1972) 529; D. A. Kirzhnitz, A. D. Linde, Phys. Lett. B72 (1972) 471; Ann. Phys. 101 (1976) 195; V. A. Kuzmin, V. A. Rubakov, M. Shaposhnikov, Phys. Lett. B155 (1985) 36; M. Shaposhnikov, Nucl. Phys. B287 (1987) 757.
2. K. Kajantie et al., Phys. Rev. Lett. 77 (1996) 2887; Nucl. Phys. B439 (1995) 147; F. Csikor et al., Nucl. Phys. B474 (1996) 421.
3. Z. Fodor et al., Nucl. Phys. B439 (1995) 147; F. Csikor, Z. Fodor, J. Hein, J. Heitger, in preparation.
4. F. Csikor, Z. Fodor, Phys. Lett. B380 (1996) 113; F. Csikor, Z. Fodor, J. Heitger, in preparation.
5. J. Potvin, C. Rebbi, Phys. Rev. Lett. 62 (1989) 3062; S. Huang et al., Phys. Rev. D42 (1990) 2864; Phys. Rev. D43 (1991) 2056 [E].
6. F. Csikor, Z. Fodor, J. Hein, J. Heitger, Phys. Lett. B357 (1995) 156; J. Hein, J. Heitger, Phys. Lett. B385 (1996) 242.
7. K. Binder, Z. Phys. B43 (1981) 119; Phys. Rev. A25 (1982) 1699; K. Jansen et al., Nucl. Phys. B322 (1989) 698; M. Gürtler, E.-M. Ilgenfritz, A. Schiller, UL-NTZ-08-97 [hep-lat/9702020], 1997 and these proceedings.
8. Z. Fodor, A. Hebecker, Nucl. Phys. B447 (1994) 127; W. Buchmüller, Z. Fodor, A. Hebecker, Phys. Lett. B331 (1994) 131; Nucl. Phys. B447 (1995) 317.