Additive mathematical modeling and development of high-pressure adaptable local ventilation fans

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\textbf{Abstract.} Intensification of mining operations and application of innovation technologies providing efficient mining and processing of mineral materials is constrained by requirements for aerogasdynamic safety systems. The local-ventilation fans distinguished for deficient adaptability and aerodynamic load are one of energy-intensive components of this system. Modified conformal mapping, Chaplygin singular points, and the theory of associated vortices and residues were used as the basis for construction of an additive mathematical model of a rotating circular grating of aerogasdynamic profiles with vortex sources to control circulation and the boundary layer. The aerodynamic scheme and parametric series of adaptive highly loaded fans of VRVP type were developed for airing of dead-end workings of up to 3000 m in length. The combination of the novel fan and gas-sucking fans increases the aerodynamic barrier up to a load of at least 35,000 t/day. The test results of pilot VRVP-6 justified the increase in adaptability by 70% and the aerodynamic load by more than 20%.

1. Introduction

The energy intensity of conventional ecological technologies reaches 25% at gas-hazardous collieries; about 40% of energy consumption is low efficient. The provision of high-class mining productivity and energy efficiency often contradicts with energy intensity of supporting processes assuring ecological safety. The deficient aerogasdynamic safety efficiency restricts introduction of novel technologies composing the integrated innovative mineral deposit management. Growth of loads on stoping workings in combination with the aerogasdynamic safety requirements enhances the actuality of the problem dealing with development of the methodology for design and development of nature-like adaptive mine turbomachines, capable to generate adequate, economic, and wanted pressure drawdown fields in terms of optimal subsoil management conception [1].

High potential to raise aerodynamic loading and adaptability is taken into account in active energy methods designed to control circulation and a boundary layer [1–4]. The research publications lack the data on mathematical turbomachine aerodynamics models taking into account the back relation of dependence of energy parameters of vortex system formed nearby profiles on the characteristics of the external network [5–7].

The present paper is aimed at development of theoretical basis and methodology for design of the principal component of ecotechnology in the mining and metallurgical complex in Russia: energy-efficient nature-like turbomachines with controllable circulation, adapted to performance parameters of the basic technological process.
Considering [8], adaptivity principle and hydrodynamic analogy, the aerogasdynamic profile can be represented as a population of local integrated vortexes, imitating combination of a classical profile and a vortex chamber.

2. Problem statement and its solution
According to the general statement of the problem, \( (n_i + n_s + n_v + 1) = (n' + 1) + n_v = (n' + 1)\)-sheet elliptical contour is associated in a plane case with an elliptical circular grid with \( n_i \) profiles, \( n_s \) source and \( n_v \) local vortexes at every profile. In the first sheet of \((n' + 1)\)-sheet Riemann surface the physical plane comprises the study circular grid of aerogasdynamic profiles, which vortex chambers have inlet and outlet channels through which air streams are capable to flow in and flow out [2, 7–9].

In \( k\)-th sheet \((k = 2, ..., n' + 1)\) of Riemann surface the real channel of vortex chamber is patterned by a jet channel with walls extending to an infinite point \( A_k \). In \( k\)-th sheet \((k = n', ..., n' + 1)\) of Riemann surface at location point of local vortexes the intensity of the source and runoff are equal in magnitude and opposite in sign. The research was conducted at two stages on the basis of a graphic model shown in Figure 1 under assumption that in the entire flow range \( D_z \) on \((n' + 1)\)-sheet Riemann surface the flow is stationary and vortex-free, the fluid is ideal, incompressible, weightless, Bernoulli constant is constant [1, 3–6, 7].

![Figure 1](image)

Figure 1. Principal scheme of conformal transformation sequence: (a) transformation of \( n_{sh}\)-sheet domain \( D_A \) into \( n_{sh}\)-sheet domain \( D_B \); (b) transformation of \( n_{sh}\)-sheet domain \( D_B \) into \((n' + 1)\)-sheet domain \( D_z \).
Conformal mapping function \( Z(\gamma) \) is [5, 6, 7]:

\[
n_{1}\ln z = \ln \frac{\gamma_{-1} + \Phi}{\gamma} + e^{2i\beta_{1}e^{\rho_{1}}} \ln \frac{\gamma - \Phi_{1}^{-1}e^{i\theta_{1}}}{\gamma - \Phi_{2}^{-1}e^{i\theta_{2}}},
\]

where \( z = re^{i\nu} \), \( \gamma = ce^{i\theta} \) are complex coordinates in domains \( D_z \) and \( D_{\gamma} \), respectively; \( r, \nu \) are radius and polar angle at plane \( z \) respectively; \( \rho, \theta \) are radius and polar angle at plane \( \gamma \) respectively; \( \Phi \) is shape-parameter of an equivalent circular grid of profiles in the form of sections of logarithmic spirals; \( \beta_{1} \) is angle of logarithmic spiral of the equivalent profile grid; \( \gamma_{1} = (1 + \lambda)\Phi_{1}^{-1}e^{i\theta_{1}} \), \( \gamma_{2} = \Phi_{2}^{-1}e^{i\theta_{2}} \), \( K_{\phi} = e^{2i\beta_{1}e^{\rho_{1}}} \) are complex parameters, constituting a profile shape of the initial circular grid of analytical profiles.

To plot complex potential \( F[Z(\gamma)] \) in \( \alpha \)-sheet Riemann surface of appearance of circular singular-radius domain \( D_{\gamma} \) we suggest to use additivity principle and Chaplygin singular points method [7, 8].

The general form of complex potential \( F[Z(\gamma)] \) of current beyond the single-radius circle in \( \alpha \)-sheet Riemann domain \( D_{\gamma} \) can be written as:

\[
F[z(\gamma)] = F_{0}[z(\gamma)] + \sum_{k=1}^{n'} F_{A_{k}}(\gamma) + \sum_{k=n'+1}^{\infty} F_{B_{k}}(\gamma),
\]

where \( F[z(\gamma)] \) is complex potential of the classical conformal mapping problem which expression is described in [5]; \( F_{A_{k}}(\gamma) \) is additional complex potential for a flow, describing the availability of runoff rates \( q_{A_{k}} \) in proper source points:

\[
F_{A_{k}}(\gamma) = \pi^{-1}q_{A_{k}} \ln(\gamma - \tau_{A_{k}}) - 0.5\pi^{-1}[q_{A_{k}} \ln(\gamma^{2} - \Phi)q_{A_{k}} \ln(\gamma^{2} - \Phi^{-2})],
\]

\( F_{B_{k}}(\gamma) \) add-on complex flow potential characterizing local vortexes of \( \Gamma_{B_{k}} \) in intensity at certain points:

\[
F_{B_{k}}(\gamma) = 0.5\pi^{-1}i^{-1}B_{B_{k}} \ln \frac{\gamma^{1} - \Phi_{1}^{-1}}{\gamma - \Phi}.
\]

The solution is unique by the theorem of solution uniqueness for the Dirichlet–Neumann problem [6, 7].

According to (3)–(5) we get a formula for a complex flow rate beyond the circle of single radius of \( \alpha \)-sheet Riemann domain \( D_{\gamma} \):

\[
\frac{dF}{d\gamma} = 0.5\pi^{-1}n_{1}^{-1}(q + i\Gamma_{0})(\gamma + \Phi^{-1}) - (\gamma - \Phi^{-1}) - 0.5\pi^{-1}n_{1}^{-1}(q + i\Gamma_{0})(\gamma + \Phi - (\gamma - \Phi^{-1}) +
\]

\[
+ \frac{(q - n_{1}q_{A_{1}} + i\Gamma_{0})}{2\pi n_{1}(\gamma + \Phi)} + \frac{(q - n_{1}q_{A_{1}} + i\Gamma_{0})}{2\pi n_{1}(\gamma + \Phi^{-1})} + \frac{(\nu_{1}\Gamma_{1} + \nu_{1}\Gamma_{1} + n_{1}q_{A_{1}} - q - i\Gamma_{0})}{2\pi n_{1}(\gamma + \Phi)} -
\]

\[
- \frac{(\nu_{1}\Gamma_{1} + \nu_{1}\Gamma_{1} + n_{1}q_{A_{1}} - q - i\Gamma_{0})}{2\pi n_{1}(\gamma + \Phi^{-1})} + \frac{\pi^{-1}n_{1}^{-1}q_{A_{1}}}{(\gamma - \tau_{A_{1}})} + V_{0}(\gamma - u_{1}z(\gamma)) \frac{ds}{d\gamma},
\]
where $V_V$, $u_τ$ are tangential components of a plug flow rate and a flow in plane $Z$; $q$ is coefficient of source consumption; $Γ_0$, $Γ_1$ is intensity of vortex in the center of circular grid in plane:

$$DΣ\sum_{k=1}^{n'}q_A_k = qΣ_A, \quad \sum_{k=1}^{n'}Γ_A_k = ΓΣ_A.$$  

Given that vortex source is available at the back critical point of aerogasdynamic profiles, the point of disturbance of conformity mapping of the initial circular grid $θ_{RCP}$ shifts to point of zero potential ambient velocity $θ_{ECP}$, by magnitude $Δθ_{RCP}$ determined by displacement of a vortex source being adaptive to the external network characteristics [10].

With regard to conjugacy property, Zhukovsky–Chaplygin–Kutta postulate, and displacement of the rear critical point (RCP) to the effective critical point (ECP) under hydrodynamic peculiarities, specific for these locations, the formula to calculate circulation $Γ_1$ is:

$$Γ_{dkl} + Γ_{d′kl} - 4q[1 + n_jq'(Φ^2 - 1)^2(Φ^2 + 2cosθ_{0(n′,2)}) + 1] - \frac{Φ(Φ^2 + 1)sinθ_{th(n′,2)}}{n_j(Φ^2 - 1)^2(Φ^2 + 2φcosθ_{0(n′,2)} + 1)} - \frac{2πV'(Φ^2 - 2φcosθ_{0(n′,2)} + 1)}{(Φ^2 - 1)} - \frac{4Γ_0φcosθ_{0(n′,2)}}{n_j(Φ^2 + 2φcosθ_{0(n′,2)} + 1)} + \frac{φsinθ_{0(n′,2)}q_A_k}{1 - cos(θ_{d_k} - θ_{0(n′,2)})} + \frac{q_{0(n′,2)}sinθ_{0(n′,2)} - θ_{0(n′,2)}}{1 - cosθ_{0(n′,2)} - θ_{0(n′,2)}} + \frac{φsinθ_{th(n′,2)} - θ_{th(n′,2)}}{1 - cos(θ_{d_k} - θ_{0(n′,2)})}.$$  

(7)

The resultant mathematical model justifies the above fundamental conclusion on principal specific features of a flow mode in circular grids of aerogasdynamic profiles with vortex chambers and makes it possible to realize the nature-likeness principle for turbomachines.

Figure 2 demonstrates kinematic parameters of a circular grid of profiles adaptive to the external network characteristics [11].

**Figure 2.** Kinematic flow parameters at outlet from the circular grid of profiles: — parameters of the classical grid; — parameters of a circular grid with aerogasdynamic profiles; $C_{V2}$, $β_2$ — peripheral speed coefficient, angle of flow exit from the working wheel considering the effect of adaptive vortex source, respectively; $Δβ_{a}$ — angle of ECP turn relative to RCP under the influence of adaptive vortex source; $C_{V2}$, $C_{V2}$, $β_{a}$ — coefficients of peripheral and radial flow velocity and angle of exit of circular grid profile, respectively.
Hydrodynamic analog $\Delta \beta_a$, being a conformal mapping of angle $\Delta \theta_{\text{skt}}$ on the single radius circle of Riemann domain $D_\gamma$ for domain $D_z$ with account for (1), (7) can be presented in the form:

$$
\Delta \theta_{\text{ECP}} = \theta_a(n_\zeta + 2) - \theta_a(n_\Sigma + 3) = \arcsin \frac{\Gamma_{\text{skl}}(\Phi^2 - 1)}{\Phi}.
$$

Thus, $\Delta \beta_a$ is angle of ECP turn relative to RCP is a function of circulation coefficient $\Gamma_{\text{skl}}$ of a vortex source, which intensity tends to vary adaptively to parameters of ventilation network. The statement is justified by the scheme of kinematic parameter variations in Figure 2.

Using the new-proposed additive mathematical model of circular grids of aerogasdynamic profiles, the researchers developed and experimentally tested the radial aerodynamic scheme Ts149-20 and proposed the parametric line of local-ventilation fans of VRVP (Table 1).

Table 1. Parameters of type-size line of local-ventilation fans as compared to the latest fans of opponents.

| Parameters                          | VMEV-8 | VRVP-8 | VME-2-10 | MS760-22 (Germany) | VRVP-10 | Parameter variation index |
|-------------------------------------|--------|--------|----------|--------------------|---------|--------------------------|
| Rated capacity, $Q$, m$^3$/s        | 13.5   | 14     | 16       | 17                 | 19      | 1.13                     |
| Complete pressure, $P$, Pa          | 800    | 890    | 380      | 420                | 510     | 1.23                     |
| Depth of economic adjustment, $N$, kW | 0.44   | 0/78   | 0.45     | 0.55               | 0.78    | 1.42                     |
| Specific energy consumption         | 1.67   | 1.22   | 1.64     | 1.41               | 1.22    | 1.22                     |
| Working length, $L_{w,m}$           | 2000   | 3500   | 1500     | 1700               | 2000    | 1.75                     |

The pilot prototype of VRVP-8 passed tests in the certified laboratory SMK-Center. The commercial tests of the device are held at INTEKS. The design parameters calculated based on the novel mathematical model and the full-scale test results on VRVP-8 differ by 7–10%, thus justifying the acceptable reliability of the new-proposed model.

3. Conclusions
1. The novel parametric line of local-ventilation fans improves the fan adaptability by more than 70% and boosts the developed pressure by 20% to provide efficient blowing of dead-end workings of more than 1500 m in extension.
2. The application of the proposed parametric fan line in combination with gas-suction fans enables to higher the aerogasdynamic barrier of the stoping working load up to at least 35 000 t/day.

References
[1] Makarov VN, Patrakeeva IY and Makarov NV 2016 Genesis of perfection of fans of local airing GIAB No 4 pp 54–61
[2] Pat. 2543638 RF 2015 A way of increase of pressure and profitability of bladed turbomachines of radial type, NP Kosarev, NV Makarov, VN Makarov Byull. Izobret. No 7
[3] Li H, Song L, Li Y, Feng, Z 2011 2D viscous aerodynamic shape design optimization for turbine blades based on adjoint method Journal of Turbomachinery Vol 133 No 3 Article ID 031014
[4] Rao RS, Kuma CG, Prakasham RS, Hobbs, PJ 2008 The Taguchi methodology as a statistical tool for biotechnological applications: a critical appraisal Biotechnology Journal Vol 3 No 4 pp 10–523
[5] Loitsyan D 1978 Mechanics of Liquid and Gas Moscow: Nauka (in Russian)
[6] Smirnov VI 1974 Course of Higher Mathematics Moscow: Nauka Vol 3.4.2 (in Russian)
[7] Amaral S, Verstraete T, Van den Braembussche R and Arts T 2010 Design and optimization of the internal cooling channels of a high pressure turbine blade. Part I: Methodology. *Journal of Turbomachinery* Vol 132 No 2 Article ID 021013

[8] Gostelow DZh 1987 *Aerodynamics of Lattices of Turbomachines* Moscow: Mir (in Russian)

[9] Shi W, Zhou L, Lu, W, Pei B and Lang T 2013 Numerical prediction and performance experiment in a deep-well centrifugal pump with different impeller outlet width *Chinese Journal of Mechanical Engineering* Vol 26 No 1 pp 46–52

[10] Shojaeeffard MH, Tahani M, Ehghaghi MB, and Fallahian MA Beglari 2012 Numerical study of the effects of some geometric characteristics of a centrifugal pump impeller that pumps a viscous fluid *Computers and Fluids* Vol 60 pp 61–70

[11] Ivanov OP and Manchenko VO 1986 *Aerodynamics and Fans* Leningrad: Mashinostroenie (in Russian)