NONLINEAR DISTORTION
OF THE FIBER OPTIC MICROPHONE

MUHAMMAD TAHER ABUELMA'ATTI

King Fahd University of Petroleum and Minerals, Box 203,
Dhahran 31261, Saudi Arabia

(Received 7 September 1999; In final form 28 January 2000)

Analytical expressions are obtained for predicting the harmonic and intermodulation
performance of the fiber optic microphone. These expressions are in terms of the ordi-
nary Bessel functions with arguments dependent on the amplitudes of the acoustical ex-
citing signal.

Keywords: Distortion; Microphones; Fiber optic communication

INTRODUCTION

At present, there is a growing interest in designing fiber optic
microphones using different principles [1–6]. The fiber optic micro-
phone offers a simple and cheap solution for a digital telephone line
based entirely on optical fiber support. Moreover, the use of acoustic
vibrations to modulate the light signal, without an electrical inter-
mediate, reduces the power consumption [6]. However, the large sig-
nal performance of this microphone has not yet been investigated.
While it is conjectured that a zone of linearity exists in the static char-
acteristic of the fiber optic microphone reported in Ref. [6], and
shown in Figure 1, no analytical expressions have been obtained for
the harmonic and intermodulation performance of the microphone
under large signal conditions.

The major intention of this paper is, therefore, to present analy-
tical expressions for predicting the harmonic and intermodulation
performance of the fiber optic microphone of Figure 1 when excited by a multisinusoidal acoustical signal. Using these expressions, it is possible to select the parameters of the microphone for a predetermined distortion performance.

ANALYSIS

According to Malki et al. [6], the coupling coefficient of the fiber optic microphone can be expressed as

\[ K(x) = \frac{P_r(x)}{P_e} = \frac{1}{(1 + x)^2} \]  

(1)

where \( P_r \) is the optical power reflected by the membrane, \( P_e \) is the radiated power provided by the fiber, \( x = z/z_0 \), \( z \) is the membrane-fiber distance, \( z_0 = a/2NA \), \( a \) is the core radius of the optical fiber and \( NA \) is the numerical aperture of the fiber.

Figure 2 shows a plot of \( K(x) \) as a function of \( x \). Equation (1) in its present form cannot yield closed-form analytical expressions for the amplitudes of the harmonics and intermodulation products resulting from a multisinusoidal excitation of the fiber optic microphone. Here we propose to approximate the characteristic of Figure 2 by the Fourier-series of Eq. (2).

\[ K(x) \approx F(x) = \sum_{k=1}^{K} B_k \sin \left( \frac{2k\pi}{T} x \right) \]  

(2)
The coefficients $B_k$ can be obtained using the discrete Fourier-transform (DFT) technique. This technique invariably demands a well developed software. Moreover, to reduce the number of multiplications and additions involved in the DFT of a large number of equally-spaced data points, it is essential to organize the problem so that the number of data points can be easily factored, particularly into powers of two [7]. Furthermore, in the DFT technique the number of terms of a Fourier-series function must be less than or equal to the number of data points available. Thus with a limited number of data points, as may be the case, the desired accuracy, in approximating the nonlinear term of Eq. (1) by a Fourier-series, may not be attained.

Alternatively, first we make the characteristic of Figure 2 periodic by removing the offset at $x=0$ and using the resulting curve in mirror image to generate a complete period of the periodic function $f(x) = K(x) - 1$ as shown in Figure 3. Secondly, we choose a number of data points and connect them using straight line segments joined end to end as shown in Figure 3. The $x$-values of the segment joins are termed knots. The number of knots and their positions must generally
be chosen so that closer knots are placed in regions where the function $f(x)$ is changing rapidly. The knots are not necessarily equally-spaced; this allows the choice of a large number of knots to represent the fine details of the function $f(x)$. By denoting the slope of each segment by $\alpha_m$, as shown in Figure 3, it is easy, following the procedure described in [7], to show, without performing any integration, that the coefficients $B_k$ of the Fourier-series of (2) can be expressed by (3).

$$B_k = \frac{-T}{2(k\pi)^2} \left( \sum_{m=1}^{M-2} (\alpha_{m+1} - \alpha_m) \sin \left( \frac{2k\pi}{T} x_{m+1} \right) \right)$$

(3)

where $T$ is the period of the periodic function of Figure 3.

From (3) one can see that calculation of the coefficients $B_k$ requires only simple mathematical operations. Also inspection of (3) suggests that as $k$ becomes infinite, the Fourier-series coefficients $B_k$ always approach zero. In fact, for numerical computation using mainframe or personal computers, there is no reason to avoid increasing the number of terms in (2) until the inclusion of the next term is seen to make a negligible contribution towards a best fit criterion; for example


the minimum relative-mean-square (RRMS) error. Table I shows the coefficients of the first 31 terms for approximating the characteristic of Eq. (1).

Using the parameters of Table I and Eq. (2) calculations were made and are shown in Figure 2 from which it is obvious that the proposed Fourier-series accurately represents the nonlinear term of Eq. (1) with RRMS error = 0.0017.

### HARMONIC AND INTERMODULATION PRODUCTS

One of the potential applications of the proposed approximation of (2) is in the prediction of the amplitudes of the harmonic and
intermodulation products resulting from multisinusoidal acoustical excitation of the fiber optic microphone. Thus, assuming that the normalized membrane displacement, resulting from exciting the membrane by a multisinusoidal acoustical signal, can be expressed as

$$x = X_o + \sum_{n=1}^{N} X_n \sin \omega_n t, \quad X_o + \sum_{n=1}^{N} X_n \leq X_{\text{max}}$$

(4)

then combining (2) and (4) and using the trigonometric identities

$$\sin(\beta \sin \omega t) = 2 \sum_{m=1}^{\infty} J_{2m+1}(\beta) \sin(2m+1)\omega t$$

$$\cos(\beta \sin \omega t) = J_0(\beta) + \sum_{m=1}^{\infty} J_{2m}(\beta) \cos 2m\omega t$$

where $J_m(\beta)$ is the Bessel function of order $m$, and after simple mathematical manipulations it is easy to show that the amplitude of a reflected-power component of frequency $\sum_{n=1}^{N} \gamma_n \omega_n$ and order $\sum_{n=1}^{N} |\gamma_n|$, where $\gamma_n$ is a positive, negative integer or zero, will be given by

$$P(\gamma_1, \gamma_2, \ldots, \gamma_n) = 2 \sum_{k=1}^{K} \cos \left(\frac{2k\pi}{T} X_o\right) B_k \prod_{n=1}^{N} J_{|\gamma_n|} \left(\frac{2k\pi}{T} X_n\right)$$

for $\sum |\gamma_n| = \text{odd}$

(5)

and

$$P(\gamma_1, \gamma_2, \ldots, \gamma_n) = 2 \sum_{k=1}^{K} \sin \left(\frac{2k\pi}{T} X_o\right) B_k \prod_{n=1}^{N} J_{|\gamma_n|} \left(\frac{2k\pi}{T} X_n\right)$$

for $\sum |\gamma_n| = \text{even}$

(6)

In general, the Bessel functions can be evaluated using the built-in subroutines in most mainframe computers. However, in some cases, with very small or very large values of $(2k\pi/T)X_n$, simple approximations, suitable for hand calculation, can be used for the Bessel functions [8-10].

Using (5) and (6) the amplitudes of the fundamental, harmonic and intermodulation components of the reflected optical power
The amplitude of the output second-order intermodulation component of frequency $\omega_r - \omega_q$ of the reflected power $P_r(x)/P_e$ will be given by

$$P(0,0,1,0,0,1,0,0,0) = 2 \sum_{k=1}^{K} \cos \left( \frac{2k\pi}{T} X_o \right) B_k J_1 \left( \frac{2k\pi}{T} X \right) \left( J_\alpha \left( \frac{2k\pi}{T} X \right) \right)^{N-2}$$

(8)

and the amplitude of the output third-order intermodulation component of frequency $\omega_r - \omega_3 + \omega_q$ of the electric field $P_r(x)/P_e$ will be given by

$$P(0,0,1,0,0,1,0,0,1,0,0) = 2 \sum_{k=1}^{K} \cos \left( \frac{2k\pi}{T} X_o \right) B_k J_1 \left( \frac{2k\pi}{T} X \right)^2 \left( J_\alpha \left( \frac{2k\pi}{T} X \right) \right)^{N-3}.$$  

(9)

Also, the amplitude of the reflected second-harmonic component of frequency $2\omega_r$ can be expressed as

$$P(0,0,2,0,0,0,0,0,0) = 2 \sum_{k=1}^{K} \sin \left( \frac{2k\pi}{T} X_o \right) B_k J_2 \left( \frac{2k\pi}{T} X \right) \left( J_\alpha \left( \frac{2k\pi}{T} X \right) \right)^{N-1}$$

(10)

and the amplitude of the reflected second-harmonic component of frequency $3\omega_r$ can be expressed as

$$P(0,0,3,0,0,0,0,0,0) = 2 \sum_{k=1}^{K} \cos \left( \frac{2k\pi}{T} X_o \right) B_k J_3 \left( \frac{2k\pi}{T} X \right)^3 \left( J_\alpha \left( \frac{2k\pi}{T} X \right) \right)^{N-1}.$$  

(11)
Using (7)–(11) the relative second-order and third-order harmonic and intermodulation products can be calculated for any number of input signals.

Figure 4 shows the results obtained for an exciting signal resulting in a membrane displacement of the form

\[ x = X \left( 1 + \frac{1}{2} \sin \omega_1 t + \sin \omega_2 t \right). \]  

Equation (12) implies that the amplitude of the membrane displacement at frequencies \( \omega_1 \) and \( \omega_2 \) is equal to the bias. From Figure 4 it can be seen that the second-order intermodulation is dominant.

Figure 5 shows the results obtained for an exciting signal resulting in a membrane displacement of the form

\[ x = \frac{1}{2} X_{\text{max}} \left( 1 + \alpha \sin \omega_1 t + \sin \omega_2 t \right), \]  

FIGURE 4 Variation of the harmonic and intermodulation products as a function of \( X \) with a membrane-displacement of the form of Eq. (12). a: intermodulation product \( \omega_1 \pm \omega_2 \), b: second-harmonic product \( 2\omega_1 \) (or: \( 2\omega_2 \)), c: intermodulation product \( 2\omega_1 \pm \omega_2 \) (or: \( 2\omega_2 \pm \omega_1 \)), d: third-harmonic product \( 3\omega_1 \) (or: \( 3\omega_2 \)).
FIGURE 5 Variation of the harmonic and intermodulation product as a function of $\alpha$ with a membrane-displacement of the form of Eq. (13). a: intermodulation product $\omega_1 \pm \omega_2$, b: second-harmonic product $2\omega_1$ (or: $2\omega_2$), c: intermodulation product $2\omega_1 \pm \omega_2$ (or: $2\omega_2 \pm \omega_1$), d: third-harmonic product $3\omega_1$ (or: $3\omega_2$).

for different values of $\alpha$. Equation (13) implies that the amplitude of the membrane displacements at frequencies $\omega_1$ and $\omega_2$ is a fraction of the fixed bias.

From Figures 4 and 5 it can be seen that the amplitudes of the third-order harmonic and intermodulation products are smaller than the amplitudes of the second-order harmonic and intermodulation products. Moreover, the second-order intermodulation component of the form $\omega_r - \omega_q$ is dominant.

CONCLUSION

By approximating the coupling coefficient of Eq. (1), using a Fourier-series, analytical expressions can be obtained for the harmonic and
intermodulation performance owing to an acoustical signal resulting in multisinusoidal membrane displacement of a fiber optic microphone of Figure 1. The Fourier-series coefficients can be evaluated using simple calculations without recourse to numerical integration or DFT techniques. The analytical expressions obtained for the harmonic and intermodulation products are in terms of the ordinary Bessel functions and can be easily evaluated using programmable hand calculators.

The results show that the second-order intermodulation component is the dominant nonlinear distortion component. The results obtained can be used for selecting the parameters of the fiber optic microphone to meet a prespecified large signal performance under multisinusoidal acoustical excitation.

It is worth mentioning here that the accuracy of the results predicted using the present analysis depends on the accuracy of Eq. (1). It is known, however, that from the physical point of view, Eq. (1) is an approximation based upon several assumptions, for example uniform modal distribution. Thus, while in principal, the analysis presented here is correct, the discrepancies between theoretical and analytical results is due mainly due the accuracy by which Eq. (1) represents the operation of the fiber optic microphone of Figure 1.

References

[1] Ziming, H., Jing, C. and Quiojing, Q. (1990). Fiber optic microphone, Proceedings SPIE, 1230, 551–552.
[2] Garthe, D. (1991). A fibre-optic microphone, Sensors and Actuators, A26, 341–345.
[3] Garthe, D. (1993). Fiber and integrated-optical microphones based on intensity modulation by beam deflection at a moving membrane, Sensors and Actuators, A37–A38, 484–488.
[4] Hu, A., Cuomo, F. W. and Zuckerwar, A. J. (1993). Theoretical and experimental study of a fibre optic microphone, Journal of the Acoustic Society of America, 91, 3049–3056.
[5] Bush, J., McNair, F. and DeMetz, F. (1994). Low-cost fibre optic interferometric microphones and hydrophones, Proceedings SPIE, 2292, 83–93.
[6] Malik, A., Gafsi, R., Lecoy, P. and Mevel, Y. (1996). Fibre-optic microphone for optical communication systems, Measurement Science and Technology, 7, 908–910.
[7] Abuelma'atti, M. T. (1994). Simple method for calculating Fourier coefficients of experimentally obtained waveforms, IEE Proceedings, Science, Measurement and Technology, 141, 177–178.
[8] Abramowitz, M. and Stegun, I. A., Handbook of Mathematical Functions, National Bureau of Standards, Washington D.C., 1964.
[9] Blachman, N. M. and Mousavinezhad, S. H. (1986). Trigonometric approximations for Bessel functions, *IEEE Transactions on Aerospace and Electronic Systems, AES-22*, 2–7.

[10] Waldron, R. A., Formulas for computation of approximate values of some Bessel functions, *Proceedings IEEE*, 69(1981), 1586–1588.
Submit your manuscripts at
http://www.hindawi.com