Antiferromagnetic Excitations and Van Hove Singularities in YBa$_2$Cu$_3$O$_{6+x}$

G. Blumberg,1,2 Branko P. Stojković1,3, and M. V. Klein1

1Department of Physics, University of Illinois at Urbana-Champaign, Urbana, IL 61801-3080
2Institute of Chemical Physics and Biophysics, Rävala 10, Tallinn EE0001, Estonia
3Minnesota Supercomputer Institute, Minneapolis, MN 55455

(September 20, 1995; to appear in Phys. Rev. B (Rap. Comm.))

We show that in quasi-two-dimensional $d$-wave superconductors Van Hove singularities close to the Fermi surface lead to novel magnetic quasi-particle excitations. We calculate the temperature and doping dependence of dynamical magnetic susceptibility for YBCO and show that the proposed excitations are in agreement with inelastic neutron scattering experiments. In addition, the values of the gap parameter and in-plane antiferromagnetic coupling are much smaller than usually believed.

PACS numbers: 74.72.Bk, 75.50.Ee, 61.12.Bt

The spin dynamics of the cuprate superconductors is still a subject of controversy. The spin-spin correlation function $\chi_{ph}^0(q,\omega) \equiv \chi' + i\chi''$, $q \equiv (q_x, q_y)$, usually probed by NMR, magnetic Raman scattering or, more directly, by inelastic neutron scattering (INS) experiments, is still not well understood. INS measurements on YBa$_2$Cu$_3$O$_6$ (YBCO) have been performed by a number of groups[4,5] and have lead to a consistent picture of the measured physical susceptibility $\chi_{ph}(q,\omega)$ for $q$ near the antiferromagnetic (AF) wave vector $Q \equiv (\pi, \pi)$. It has been found that the dynamical AF spin fluctuations persist over the whole range of doping ($0.4 \leq x \leq 1$) and the following remarkable features have been reported:

(a) In the superconducting (SC) state $\chi''_{ph}(Q,\omega)$ is limited by an energy gap, $E_g$ (Ref. 3). In heavily doped regime ($0.65 \leq x \leq 0.92$) $E_g$ is roughly proportional to the SC transition temperature $T_c$, such that $E_g \approx 3.4 k_BT_c$. In the weakly-doped regime $E_g$ is very small and it is a much stronger function of $T_c$ (Ref. 5).

(b) In addition, in highly doped YBCO the spin excitation spectrum is characterized by a sharp peak at some energy $E_r > E_g$, e.g., for optimally doped YBa$_2$Cu$_3$O$_{6.92}$, $E_r = 41$ meV (Refs. 6, 7). For under-doped samples a broad resonance has been observed at $20 < E_r < 40$ meV (Refs. 6, 8). The resonance excitation is localized in both energy and wave vector and has $\chi''_{ph}(Q,\omega) \propto \sin^2(q_d/2)$. Here $d$ is the distance between two adjacent CuO$_2$-planes in the unit cell of YBCO.

(c) At $\omega \approx 50$ meV there is a high energy cut-off, i.e., the intensity sharply decreases, regardless of the doping level.

(d) In the normal state $\chi''(Q,\omega)$ is characterized by a broad feature, spread practically over the whole energy range, with a characteristic energy $\omega \sim 30$ meV, approximately independent of the oxygen concentration.

There has been a number of theoretical proposals for the explanation of features (a)-(d). It has been shown by Lu[4] that for $d$-wave SC state $\chi''(q,\omega)$ should exhibit anomalous resonance peaks in $q$ space. Feature (b) has been treated in terms of spin-flip excitations across the SC gap[5] or as collective mode excitations in the particle-particle channel[6].

In this paper we show that the YBCO bilayer structure and 2D band topology, renormalized for AF interactions, with $d$-wave SC order parameter, leads to novel magnetic quasi-particle excitations which explain all of the features (a)–(d), observed in INS.

We start by calculating $\chi''_{ph}(q,\omega)$ for YBCO: we consider a system of two 2D planes, corresponding to the Cu$_2$O$_4$ bilayer in a unit cell of YBCO, with an effective band dispersion: $E^g(\pi) = -2t\cos(k_x) - 4t'\cos(k_x)\cos(k_y) - 2t''\cos(2k_x)\cos(2k_y)$. Here $t''$ corresponds to the electron bands with odd $(a)$ and even $(c)$ wave functions with respect to the mirror plane between the adjacent planes. We choose the parameters $t$, $t'$, $t''$, $t_\perp$, and $\mu$ by requiring that the shape and depth of the band and the size of the Fermi surface (FS) are approximately equal to those observed in ARPES measurements. According to Ref. 21, the odd band is nearly half filled regardless of the oxygen content, while at optimal doping the odd band is shifted upwards by $2t_\perp \approx 100$ meV and thus it is far away from half filling. This yields band parameters $t = 115$ meV, $t'/t = -0.2$, $t''/t = 0.25$, and $\mu = -228$ meV (density of electrons $n_0 = 0.55$ and $n_0' = 0.9$). Although one usually interprets the parameters $t$, $t'$, and $t''$ in terms of tight-binding overlap integrals, here they should be regarded as results of our fits to the experimental data and as such, they should include some of the effects of mass renormalization. The odd band has a bifurcated saddle point at $q = (\pi, 0)$, with $\epsilon^{(o)}_{\pi,0} = 30$ meV and $\epsilon^{(o)}_{0,\pi,0} = 10$ meV (Ref. 21); however, none of our conclusions are altered if one assumes an extended Van Hove singularity (VHS) instead. The FS of the odd and even bands are shown in Fig. 1.

In BCS theory the 2D Fermi liquid spin susceptibility is given by[22]

$$\chi_0^{(ij)}(q,\omega) = \sum_k E_{k+q}^{(ij)} f(E_k^{(ij)}) - f(E_k^{(ij)}) \frac{E_k^{(ij)}}{E_{-k+q}^{(ij)}}$$
\[ + \sum_{k} \frac{p_{k+q_k}^{(ij)^2} E_{ij}^{(j)}(k, q) - f(E_{k+q_k}^{(j)}) - f(E_{k}^{(j)})}{E_{ij}^{(j)^2} - (\omega - i\Gamma)^2}, \]

where \( E_{ij}^{(j)}(k, q) \) is the quasiparticle dispersion law in the SC state, \( f(E_{j}^{(j)}) = 1/[\exp(E_{k}^{(j)}/T) + 1] \) is the Fermi function and the coherence factors are given by

\[ p_{k+q_k}^{(ij)^2} = \frac{1}{2} \left( 1 - \frac{e^{(j)}_{k+q_k} + \Delta^{(j)}_{k+q_k}}{E_{k}^{(j)}} \right), \]

\[ i_{k+q_k}^{(ij)^2} = \frac{1}{2} \left( 1 + \frac{e^{(j)}_{k+q_k} + \Delta^{(j)}_{k+q_k}}{E_{k}^{(j)}} \right). \]

We assume that the SC gap parameter \( \Delta^{(j)}_k \) has a simple form \( \Delta^{(j)}_k = \Delta_{\text{max}} (\cos(k_x) - \cos(k_y))/2 \), although self-consistent numerical solutions of the gap equation suggest that \( \Delta^{(j)}_k \) may have a somewhat different shape. In Eq. (1) \( \Gamma \) is the phenomenological damping parameter. Accounting for the interaction between the susceptibility bubbles, we calculate the susceptibility renormalized in the RPA manner

\[ \chi^{(i,j)}(q, \omega) = \chi_0^{(i,j)}(q, \omega) / [1 + J^{(i,j)}(q) \chi_0^{(i,j)}(q, \omega)], \]

where \( J^{(e,c)}(q) = J_{\parallel} + J_{\perp}, J^{(o,o)}(q) = J_{\parallel} - J_{\perp}, J^{(o,e)}(q) = J^{(o,c)}(q) = 0, J^{(i)}(q) \) is the in-plane magnetic interaction and \( J^{(j)}(q) \) is the interplane magnetic coupling. We do not take into account the vertex and self-energy corrections which would further renormalize \( \chi^{(i,j)}(q, \omega) \).

For the bilayer case the physical susceptibility, proportional to the INS measured intensity, is given by

\[ \chi_{PH}(q, z, \omega) = 2[\chi^{(e,c)}(q, \omega) \cos^2(q_g d/2) + \chi^{(o,o)}(q, \omega) \sin^2(q_g d/2)] \]

It is clear from Eq. (4) that the experimentally observed \( \sin^2(q_g d/2) \) modulation of the INS (b) must originate from the odd band, provided the coupling between bilayers is incoherent.

For fixed \( q \) the quantity \( E^{i(j)}(k, q) \) is a function of 2D vector \( k \) and we denote its minima and saddle points by \( E_{min}^{(i)}(k, q) \) and \( E_{sp}^{(i)}(k, q) \) respectively. In a 2D system and in the \( \Gamma, T \to 0 \) limit these extrema produce logarithmic singularities in either \( \chi_0^{(i,j)}(q, \omega) \) or \( \chi_0^{(i,j)}(q, \omega) \) (see Eq. (4)). For example, for \( p_{k+q_k}^{(ij)^2} \neq 0, \chi_0^{(i,j)}(q, \omega) \) diverges at \( \omega \approx E_{min}^{(i)}(q) \), while \( \chi_0^{(i,j)}(q, \omega) \) behaves like a step function. Alternatively, \( \chi_0^{(e)}(q, \omega) \) diverges at the saddle point energies, \( E_{sp}^{(i)}(q) \), while \( \chi_0^{(i,j)}(q, \omega) \) has a

“kink”. Provided that \( J_{\parallel} \) and \( J_{\perp} \) are not too large, the renormalized susceptibility (5) has poles (resonances) at \( \omega \) close to both \( E_{min}^{(i)}(q) \) and \( E_{sp}^{(i)}(q) \). Physically, these poles in \( \chi^{(i,j)}(q, \omega) \) describe spin-flip quasi-particle excitations (magnons). The self-energy corrections shift the position of the poles to the complex plane, producing the finite lifetime of the excitations, and more importantly, the logarithmic divergences become finite peaks, introducing a minimal value of \( J^{(i,j)}(q) \) for the creation of a quasi-particle. For quasi-2D system the resonances become broader due to the weak interaction between the bilayers.

Low energy excitations at \( q = Q \) correspond to particles and holes near the intercepts of the FS and the magnetic Brillouin zone (e.g., points A and B in Fig. 1). The SC gap at these points \( \Delta^{(o)}_{kA} = \Delta^{(o)}_{kB} \) is close to \( \Delta_{\text{max}} \), and thus the lowest energy excitation [feature (a) in INS] is \( E_{g} = \Delta^{(o)}_{kA} + \Delta^{(o)}_{kQ} = E_{min}^{(o)}(Q) \). On the other hand, holes near VHSs and particles above the SC gap produce \( E_{sp}^{(o)}(q) \) resonances. At \( q = Q \) this yields the excitation energy \( E_{r} = E_{sp}^{(o)}(Q) = \Delta^{(o)}_{kC} + E_{sp}^{(o)} \), where C and D are points depicted in Fig. 1 [feature (b)]. In Fig. 2 we present the energy dispersion of the quasi-particles, originating from both \( E_{min}^{(o)}(q) \) and \( E_{sp}^{(o)}(q) \), for \( q \) along the high symmetry lines leading to \( Q \), and assuming \( \Delta_{\text{max}} = 15 \) meV. Away from \( Q \) the excitations clearly split into several bands, whose dispersion depends on the shape of VHSs, and \( E_{sp}^{(o)}(q) \) reaches approximately 0.3 eV near the magnetic Brillouin zone boundary. Of course, the excitations (resonances) shown in Fig. 2 may have very different lifetimes as well as intensities, depending not only on the FS geometry, but also on the appropriate coherence factors.

Returning to the INS results, both \( E_{g} \) and \( E_{r} \) can be observed experimentally only if damping is sufficiently small. We calculated the single electron (hole) magnetic relaxation rate \( \gamma^{-1}_k \) (see Eq. (9b) in Ref. 25) and found that in the SC state the relaxation is negligible for \( k \) near \( (\pi, 0) \) and energies below \( \sim 2.4 \Delta_{\text{max}} \), in contrast with ordinary superconductors where \( \gamma^{-1}_k (\omega, T) \) vanishes for \( \omega < 3\Delta \) (Ref. 25). As a result, any spin-flip excitation in \( \chi^{(i,j)}(Q, \omega) \), will not be damped for \( \omega \) smaller than approximately \( 3\Delta_{\text{max}} \) even in d-wave superconductors. Thus, both \( E_{g} \) and \( E_{r} = \Delta^{(o)}_{kD} + \Delta^{(o)}_{kC} \) should be observed experimentally. On the other hand, in the normal state the \( E_{r} \) excitation becomes heavily damped and is sometimes referred to as the spin-pseudogap. We note that for the even band the saddle point energy lies far below the FS and the corresponding resonance excitation is overdamped even in the SC state and contributes only to a structureless background.

The temperature dependence of the calculated odd band susceptibility \( \chi^{(o,o)}(Q, \omega) \) is presented in Fig. 3a. We assume a BCS temperature dependence of the gap parameter, although, admittedly, \( \Delta_{\text{max}}(T) \) can be regarded
as a fitting parameter in order to match the experimental INS spectra. Single band fits to NMR might shift suggest a larger value of SC gap parameter than that we use here ($\Delta_{\text{max}}^{(o)} = 15 \text{ meV}$ at $T = 0$). In the bilayer case only even band contributes to $\chi_{ph}^{(o), (0,0,0)}$ [Ref. 28], i.e., $\Delta_{\text{max}}^{(i)}$ may be larger than $\Delta_{\text{max}}^{(o)}$ (Ref. 29). In addition, a different form of $\Delta_{\text{max}}^{(i)}$ may alter $\Delta_{\text{max}}^{(i)}$ necessary to fit experiments. We assume $J^{(o,o)}(Q) = -120 \text{ meV}$. The calculated spectra reproduce all of the observed INS features in the SC state: the gap originating from the SC gap parameter (a); the sharp resonance peak at 41 meV at low temperatures, due to spin-flip quasi-particle excitations related to the VHS in the band structure (b); the sharp drop of the signal above $45 \text{ meV}$, also due to the band topology and the effect of renormalization (c). The overall enhancement of $\chi_{ph}^{(o,o)}(Q, \omega)$ is due to the non-vanishing coherence factor $p_{(o,o)}^{(i)}$, present in d-wave superconductors ([28]) ($\Delta_{\text{max}}^{(i)} < 0$). At temperatures above $T_c$ the magnetic scattering is depressed. However, in agreement with the experimental data (d), a broad peak at $\omega \approx 20-30 \text{ meV}$ remains (spin-pseudogap). Its origin is the same as that of $E_c$ resonance in the SC state (VHS); its position is shifted due to the SC gap vanishing.

The doping dependence of $\chi_{ph}^{(o,o)}(Q, \omega)$ is shown in Fig. 3b. For the underdoped case we applied the same criteria as for the optimally doped, i.e., the band parameters are chosen in accordance with ARPES measurements, which show that the VHS at $(\pi, 0)$ remains approximately at the same energy below the FS, yet the filling is, obviously, much smaller. Therefore, in order to fit the experimental data we assumed, for the underdoped case, that $t = 106 \text{ meV}$, $t'/t = -0.18$, $t''/t = 0.18$ and $\mu = -125 \text{ meV}$ ($\mu^{(o)} = 0.7$), with $\Delta_{\text{max}}^{(o)} = 5 \text{ meV}$ and $J^{(o,o)}(Q) = -200 \text{ meV}$, and for the overdoped case we assumed $t = 124 \text{ meV}$, $t'/t = -0.22$, $t''/t = 0.24$ and $\mu = -204 \text{ meV}$ ($\mu^{(o)} = 0.5$) with $\Delta_{\text{max}}^{(o)}$ equal to that in the optimally doped case and $J^{(o,o)}(Q) = 0$. The calculated result is qualitatively similar to the experimental one. Most importantly, since the value of $\Delta_{\text{max}}^{(o)}$ is smaller for the underdoped materials, but the distance from the VHS at $Q = (\pi, 0)$ to the FS remains the same as in the optimally doped case, the resonance $E_c$ must become broad at some doping level when this distance is approximately equal to $2.4\Delta_{\text{max}}^{(o)}$, and this is what the experiment suggests.

In conclusion, we have shown that the Van Hove singularities, observed in ARPES at approximately $30 \text{ meV}$ below the Fermi level, lead to a novel, well-defined magnetic quasi-particle excitations in the SC state, for quasi-2D systems and d-wave gap symmetry. Our model calculations of $\chi_{ph}^{(o,o)}(Q, \omega)$ are consistent with all major features observed in INS experiments. The model suggests a relatively small value for $\Delta_{\text{max}}^{(o)} \approx 1.9k_BT_c$, and a relatively weak Heisenberg interaction for the odd band of optimally doped YBCO. In addition, our results show that the comparison of INS data and other magnetic probes, such as NMR, where both odd and even bands must be taken into account, may not be as straight-forward as previously believed.

We are indebted to V. Barzykin, A. J. Leggett, D. Pines and O. T. Valls for valuable discussions. This work has been supported in part by NSF cooperative agreement DMR 91-20000 through the STCS.

1. J. Rossat-Mignod et al., Physica B 180 & 181, 383 (1992); M. Sato et al., Sternelle, J. Phys. Soc. Jpn. 62, 263 (1993); B. J. Sternlieb et al., Phys. Rev. B 47, 5320 (1993); S. Shamamoto et al., Phys. Rev. B 48, 13817 (1993).
2. J. M. Tranquada et al., Phys. Rev. B 46, 5561 (1992).
3. P. Bourges et al., Proceedings of the symposium Jean Rossat-Mignod, Grenoble, France, 4 July 1994, and references there.
4. L. P. Regnaul et al., Physica C 235-240, 59 (1994) and references therein.
5. J. Rossat-Mignod et al., Physica C 185-189, 86 (1991); J. of Magnetism and Magnetic Materials 116, 336 (1992).
6. H. A. Mook et al., Phys. Rev. Lett. 70, 3490 (1993).
7. Hung Fu Fong et al., Phys. Rev. Lett. 75, 316 (1995).
8. J. P. Lu, Phys. Rev. Lett. 68, 125 (1992).
9. N. Bulut, D. J. Scalapino, and R. T. Scalettar, Phys. Rev. B 45, 5577 (1992).
10. S. Charfi-Kaddour, R. J. Tarento, and M. Héritier, J. Physique I 2, 1853 (1992).
11. N. Bulut and D. J. Scalapino, Phys. Rev. B 47, 3419 (1993).
12. P. Monthoux and D. J. Scalapino, Phys. Rev. Lett. 72, 1874 (1994).
13. K. Maki and H. Won, Phys. Rev. Lett. 72, 1758 (1994); Phys. Rev. B 49, 15305 (1994).
14. G. Stemmann, C. Pépin, and M. Lavagna, Phys. Rev. B 50, 4075 (1994).
15. H. Fukuyama, H. Kohno, and T. Tanamoto, J. Low Temp. Phys. 95, 309 (1994).
16. M. Lavagna and G. Stemmann, Phys. Rev. B 49, 4235 (1994).
17. F. Onufrieva and J. Rossat-Mignod, Physica C 235-240, 1687 (1994); F. Onufrieva, preprint cond-mat / 9502025.
18. I. I. Mazin and V. M. Yakovenko, e-print cond-mat / 9502060.
19. E. Demler and S.-C. Zhang, e-print cond-mat/9502060.
20. B. W. Veal and Chun Gu, J Electron Spectrosc. Relat. Phenom. 66, 321 (1994); also private communications.
21. O. K. Andersen et al., Phys. Rev. B 49, 4145 (1994); cond-mat/9509044; A. I. Liechtenstein et al., cond-mat/9509101.
22. J. R. Schrieffer, Theory of Superconductivity (Addison-Wesley, New York, 1983), Sec. 8-4.
23. M. U. Ubbens and P. A. Lee, Phys. Rev. B 50, 438 (1994).
24. D. Branch, J.P. Carbotte, Phys. Rev. B 52, 603 (1995).
25. M. Sato et al., Phys. Rev. Lett 61, 1317 (1988).
26. Q. Si et al., Phys. Rev. B 47, 9055 (1993).
FIG. 1. Fermi surfaces of odd (solid line) and even bands (dashed line) in YBCO. The parameters are given in the text. The dash-dotted line shows the magnetic Brillouin zone. Points A and B (C and D) can be connected by the AF wave vector $Q \equiv (\pi, \pi)$ and correspond to $E_g$ ($E_r$) resonances observed in INS measurements (see text).

FIG. 2. Magnetic quasi-particle dispersion for $q$ along lines of high symmetry, in the vicinity of $Q$. The values are obtained from the minima (triangles) and saddle points (solid circles) of the particle-hole energy $E_p(k, q)$ [see Eq. (1)] at fixed momentum transfer $q$.

FIG. 3. The odd band susceptibility $\chi^{(o,o)}(Q, \omega),$ defined in Eq. (3) and proportional to the INS measured intensity, as a function of energy. No finite lifetime effects are included (in both normal and SC state $\Gamma = 0.5$ meV). $\chi^{(o,o)}(Q, \omega)$ is gaped in the SC state and has a peak at $E_{sp}^{(o,o)}(q)$ (see text) due to the VHS below the FS. (a) The results for optimally doped YBCO corresponding to $T = 0, 80, 100, 200, 300$ K (top to bottom). (b) The results for different doping levels and at $T = 0$ K.
$E(q, \omega)$ (meV)

$q$ (a$^{-1}$)

(\pi/2, \pi) (\pi, \pi) (\pi, \pi) (\pi/2, \pi/2)
Figure (a): Graph showing the dependence of \( \chi''_{ph} \) on \( \omega \) for different doping levels.

Figure (b): Graph showing the variation of \( \omega \) (meV) with different states: Underdoped, Optimally doped, and Overdoped.