A New Generation of Standard Solar Models

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Abstract

We compute a new generation of standard solar models (SSMs) that includes recent updates on some important nuclear reaction rates and a more consistent treatment of the equation of state. Models also include a novel and flexible treatment of opacity uncertainties based on opacity kernels, required in light of recent theoretical and experimental works on radiative opacity. Two large sets of SSMs, each based on a different canonical set of solar abundances with high and low metallicity (Z), are computed to determine model uncertainties and correlations among different observables. We present detailed comparisons of high- and low-Z models against different ensembles of solar observables, including solar neutrinos, surface helium abundance, depth of the convective envelope, and sound speed profile. A global comparison, including all observables, yields a p-value of 2.7σ for the high-Z model and 4.7σ for the low-Z one. When the sound speed differences in the narrow region of 0.65 < r/R⊙ < 0.70 are excluded from the analysis, results are 0.9σ and 3.0σ for high- and low-Z models respectively. These results show that high-Z models agree well with solar data but have a systematic problem right below the bottom of the convective envelope linked to steepness of molecular weight and temperature gradients, and that low-Z models lead to a much more general disagreement with solar data. We also show that, while simple parametrizations of opacity uncertainties can strongly alleviate the solar abundance problem, they are insufficient to substantially improve the agreement of SSMs with helioseismic data beyond that obtained for high-Z models due to the intrinsic correlations of theoretical predictions.

Key words: neutrinos – Sun: abundances – Sun: Helioseismology – Sun: interior

1. Introduction

Standard solar models (SSMs) have played a fundamental role in one of the most important discoveries in physics: neutrino flavor oscillations. After several decades, during which the solar neutrino problem puzzled both particle physicists and astrophysicists, helioseismology allowed for the construction of an accurate picture of the solar interior. The agreement between SSMs and helioseismic results was astonishingly good—it provided both strong support to stellar (solar) evolution theory and to a particle physics solution for the problem of the missing neutrinos (Christensen-Dalsgaard et al. 1996; Bahcall et al. 1998, 2001).

The solar surface composition, determined with spectroscopic techniques, is a fundamental constraint in the construction of SSMs. The development of three-dimensional hydrodynamic models of the solar atmosphere, of techniques to study line formation under non-local thermodynamic conditions and the improvement in atomic properties (e.g., transition strengths), since 2001, have led to a complete revision of solar abundances (Asplund et al. 2005, 2009; Caffau et al. 2011). There is no complete agreement among authors, and some controversy still remains as to what the best values for the new spectroscopic abundances are. However, there is consensus in that all determinations of the solar metallicity based on the new generation of spectroscopic studies yield a solar metallicity that is substantially lower than older spectroscopic results (Grevesse & Noels 1993, pp. 15–25; Grevesse & Sauval 1998), in particular, for the volatile and most abundant C, N, and O.

Almost all determinations of element abundances for astronomical objects rely upon solar abundances and the new solar abundances. In particular, those from Asplund et al. (2009), have become a new standard. For modeling of the solar interior, however, they have brought about a series of problems because solar models, in general, and SSMs, in particular, based on the low-Z AGSS09 abundances, fail to reproduce all helioseismic probes of solar properties. This solar abundance problem (Basu & Antia 2004; Bahcall et al. 2005b; Delahaye & Pinsonneault 2006) has defined a complete solution. All proposed modifications to physical processes in SSMs offer, at best, only partial improvements in some helioseismic probes (e.g., Guzik et al. 2005; Castro et al. 2007; Basu & Antia 2008; Guzik & Mussack 2010; Serenelli et al. 2011). An alternative possibility is to consider modifications to the physical inputs of SSMs at the level of the constitutive physics, radiative opacities in particular.

The effective opacity profile in the solar interior results from the combination of the reigning thermodynamic conditions, including composition, and the atomic opacity calculations at hand. Early works (Montalbán et al. 2004; Bahcall et al. 2005a)
already suggested that a localized increase in opacities could solve or, at least, alleviate the disagreement of low-Z solar models with helioseismology. Christensen-Dalsgaard et al. (2009) and Villante (2010) have concluded that a tilted increase in radiative opacities, with a few percent increase in the solar core and a larger (15%–20%) increase at the base of the convective envelope could lead to low-Z SSMs that would satisfy helioseismic probes equally as well as SSMs based on the older, higher, metallicities.

Recent years have seen a surge of activity in theoretical calculations of atomic radiative opacities. Updated calculations (Badnell et al. 2005) by the Opacity Project have led the way, followed by OPAS (Blancard et al. 2012; Mondet et al. 2015), STAR (Krief et al. 2016b), and a new version of OPLIB, the opacities from Los Alamos (Colgan et al. 2016). For conditions in solar interiors, all theoretical opacities agree with each other within 5%. Interestingly, Bailey et al. (2015) have presented the first ever measurement of opacity under conditions very close to those at the bottom of the solar convective envelope. While the experiment has been carried out only for iron, their conclusion is that all theoretical calculations predict a Rosseland mean opacity that is too low, at a level of 7 ± 4%, for the temperature and density combinations realized in the experiment. Krief et al. (2016a) have casted additional doubts on the accuracy of currently available opacity calculations after showing that the approximations done in the modeling of line broadening have a critical impact on the final Rosseland mean opacity.

Parallel to work on radiative opacities, there have been new determinations of important nuclear reaction rates affecting energy and neutrino production in the Sun. These updates introduce differences in model expectations for neutrino fluxes comparable to current experimental uncertainties of the well determined $\Phi(^7\text{Be})$ and $\Phi(^8\text{B})$ fluxes. Additionally, development in GARSTEC (GARching STEllar Code; Weiss & Schlattl 2008) now allows the implementation of an equation of state (EOS) obtained for a mixture of elements that is consistent with the solar composition adopted in the calculation of the SSM.

Motivated by the developments just described, in this work, we present a new generation of SSMs that includes updates to the microscopic input physics of the models. Also, in light of the new results in radiative opacities, we improve on our previous treatment of opacity uncertainties in solar model predictions. The implementation of a new flexible scheme based on opacity kernels (Tripathy & Christensen-Dalsgaard 1998) allows us now to test the impact of any opacity error function without the need to perform lengthy calculations of large sets of solar models. In addition to presenting the models, we carry out an exhaustive comparison of SSMs based on alternative compositions against different ensembles of solar observables. We further test the hypothesis that radiative opacities can in fact offer a solution to the solar abundance problem.

The article is structured as follows. In Section 2, we detail the changes in the input of SSMs with respect to the previous generation of models and discuss the general approach to include input physics uncertainties in model calculations, in particular, the implementation of opacity kernels to treat opacity uncertainties. In Section 3, we present results of the new SSMs for helioseismic probes and solar neutrinos. It also contains a complete discussion of how models of high- and low-Z compare to solar data, together with a discussion of the impact of correlations in models among different observables. This is an important aspect almost always missed in the literature, particularly when helioseismic probes are under consideration (see Villante et al. 2014 for a counterexample). Section 4 gives a full discussion of uncertainties in SSMs: it summarizes our new Monte Carlo (MC) simulations, identifies dominant error sources using power-law expansions and considers the impact of different parametrizations of opacity uncertainties. In Section 5, we summarize our most relevant findings. Finally, all the detailed information, such as the structure of the new solar models, distribution of production of solar neutrinos, results of MC simulations, and updated power-law dependences of neutrino fluxes on solar input parameters, can be found online.\textsuperscript{11}

2. Standard Solar Models

SSMs are a snapshot in the evolution of a $1 M_\odot$ star, calibrated to match present-day surface properties of the Sun. In our models, two basic assumptions are that the Sun was initially chemically homogeneous and that at all moments during its evolution up to the present solar age $\tau_0 = 4.57$ Gyr mass loss is negligible. The calibration is done by adjusting the mixing length parameter (\textit{\alpha}_\textit{MLT}) and the initial helium and metal mass fractions ($Y_\text{ini}$ and $Z_\text{ini}$ respectively) in order to satisfy the constraints imposed by the present-day solar luminosity $L_\odot$, radius $R_\odot$, and surface metal to hydrogen abundance ratio $(Z/X)_\odot$.

In Serenelli et al. (2011), a generation of SSMs was computed using the nuclear reaction rates recommended in the Solar Fusion II paper (Adelberger et al. 2011; hereafter A11). For simplicity, we refer to them as the SFII SSMs. Here we present a new generation of SSMs, Barcelona 2016 or B16 for short. B16 models share with the SFII models much of the physics. However, important changes include updates in some nuclear reaction rates and, most notably, a new treatment of uncertainties due to radiative opacities. The common features between generations of SSMs are described in the paragraph below. In Section 2.1, we detail the changes in the input physics. There, we also discuss new results on important input physics for solar models that have nevertheless not made it into our final choice of SSM inputs, in particular, solar composition and results on the $^3\text{He}(^4\text{He}, \gamma)^7\text{Be}$ rate. In Section 2.2, we discuss the treatment of errors, with special emphasis on radiative opacities.

Models have been calculated using GARSTEC. Physics included in the models, unless stated otherwise, is as follows. Atomic opacities are from OP Badnell et al. (2005) and are complemented at low temperatures with molecular opacities from Ferguson et al. (2005). Microscopic diffusion of helium and metals is followed according to the method of Thoul et al. (1994). Convection is treated according to the mixing length theory, and the atmosphere is a gray Krishna-Swamy model (Krishna Swamy 1966). For more details about SSMs calibrations and physics included in the models, see Serenelli (2016).

2.1. B16—a New Generation of SSMs

Here we describe in some detail the differences between the older SFII and the new B16 generations of SSMs and discuss

\textsuperscript{11}http://www.ice.cat/personal/aldos/Solar\_Data.html
the reasons why some updates of important SSM inputs have not been included in our new SSM calculations.

Composition: Solar photospheric (surface) abundances of almost all metals can be determined from spectroscopy. In the context of solar models, the only relevant exceptions are Ne and Ar, the latter with a much lesser influence, that have to be determined by other, more indirect, methods (see Asplund et al. 2009 for details). For refractories, however, meteorites offer a very valuable alternative method (see, e.g., Lodders et al. 2009) and, in fact, elemental abundances determined from meteorites have been historically more robust than spectroscopic ones.

In Asplund et al. (2009), mild, but worthy of attention, differences existed between photospheric and meteoritic abundances for some refractory elements. In particular, photospheric Fe and Ca were 0.05 dex higher than their meteoritic counterparts, while for Mg the difference was 0.07 dex. In a recent series of papers, Scott et al. (2015a, 2015b) and Grevesse et al. (2015; for short, AGSS15) have updated previous results from AGSS09 for all but the CNO elements (and Ne, because its abundance is linked to that of O). Interestingly, the photospheric abundances of the discrepant elements just mentioned have shifted toward meteoritic values and with the new AGSS15 recommended photospheric values the differences are now 0.02 dex for Fe, 0.03 dex for Ca, and 0.06 dex for Mg.

In the past, the robustness of meteoritic abundances has made them our preferred choice as the source of solar abundances. Recent changes in the AGSS15 photospheric values compared to AGSS09 give added strength to this preference. Then, in building our SSMs, the sets of solar abundances we use are always composed by meteoritic values. Recent changes in the AGSS15 photospheric abundances of some refractory elements. In particular, photo- represent an obstacle in using the OPAL EOS, which has been, in fact, the most widely used EOS for solar models. However, it is desirable that the EOS offers consistency with the metal mixture used in the calibration of the solar model.

FreeEOS, the EOS developed by A. Irwin (Cassisi et al. 2003), allows us to overcome this difficulty. The source codes are freely available12 and although the running time is too long to allow inline implementation of the EOS during solar model calculations, precomputed tables with any desired solar composition can be computed in advanced. FreeEOS and OPAL EOS are thermodynamically consistent and in good agreement for the solar case. We have performed a comparison between the two EOS, with the composition fixed to the six element mixture hardwired in OPAL EOS, and found differences smaller than 0.1% for the pressure or 0.02% for \( \Gamma_1 \). Differences for the sound speed are about 0.05% at the center and they decrease to about 0.02% in the intermediate region; these differences are well below the sound speed errors considered in this work (see Figure 3). Extensive comparisons between FreeEOS and OPAL EOS are available at http://freeeos.sourceforge.net/documentation.html.

Due to its flexibility and its excellent performance when compared to OPAL EOS, we adopt FreeEOS as our standard EOS for the B16 SSMs. For the first time, EOS tables calculated consistently for each of the compositions used (GS98 and AGSS09met) are used in the solar calibrations. This is a qualitative step forward albeit quantitative differences in the predicted solar properties by use of consistent EOS tables are small and have minimal impact in the production of solar neutrinos or helioseismic diagnostics used in this work. This can be, nevertheless, much more important in the context of abundance determinations from EOS features such as the depression of the adiabatic index \( \Gamma_1 \) (see, e.g., Lin et al. 2007; Vorontsov et al. 2013). In fact, we find that two SSMs calibrated using the same solar mixture but using EOS tables computed with GS98 or AGSS09met mixtures respectively, lead to differences in \( \Gamma_1 \) of the order of \( 2 \times 10^{-4} \) in the region \( r/R_s \), \( >0.7 \). This is comparable to results shown in Lin et al. (2007) using a different EOS, and it is about two times larger than the error in the helioseismic determination of \( \Gamma_1 \). Further study of the impact of the EOS on \( \Gamma_1 \) are beyond the scope of this paper.

\[ \text{Table 1} \]

| Element | GS98  | AGSS09met |
|---------|-------|-----------|
| C       | \( 8.52 \pm 0.06 \) | \( 8.43 \pm 0.05 \) |
| N       | \( 7.92 \pm 0.06 \) | \( 7.83 \pm 0.05 \) |
| O       | \( 8.83 \pm 0.06 \) | \( 8.69 \pm 0.05 \) |
| Ne      | \( 8.08 \pm 0.06 \) | \( 7.93 \pm 0.10 \) |
| Mg      | \( 7.58 \pm 0.01 \) | \( 7.53 \pm 0.01 \) |
| Si      | \( 7.56 \pm 0.01 \) | \( 7.51 \pm 0.01 \) |
| S       | \( 7.20 \pm 0.06 \) | \( 7.15 \pm 0.02 \) |
| Ar      | \( 6.40 \pm 0.06 \) | \( 6.40 \pm 0.13 \) |
| Fe      | \( 7.50 \pm 0.01 \) | \( 7.45 \pm 0.01 \) |

\( (Z/X)_{\odot} \) | 0.02292 | 0.01780 |

Note. Only elements that most strongly contribute to uncertainties in SSM modeling are included.

12 http://freeeos.sourceforge.net/
Nuclear rates: The most relevant changes in the B16 SSMs compared to SFI models arise from updates in the nuclear reaction rates. As usual, the astrophysical S-factor \( S(E) \) is expressed as a Taylor series around \( E = 0 \) (Equation (1), see also Adelberger et al. 2011)

\[
S(E) = S(0) + S'(0) \cdot E + \frac{1}{2} S''(0) \cdot E^2 + \mathcal{O}(E^3).
\]  

(1)

Updates in the reaction rates are generally introduced as changes in \( S(0) \) and, eventually, the first and higher order derivatives. New \( S(0) \) values and errors are summarized in Table 2 together with the fractional changes in \( S(0) \) with respect to A11. Rates not listed in Table 2 are taken from A11 and remain unchanged with respect to the SFI SSMs.

Details about the updated rates are specified below.

1. \( p(p, e^+\nu_e)d \): \( S_{11}(E) \) has been recalculated in Marcucci et al. (2013) by using a chiral effective field theory framework, including the P-wave contribution that had been previously neglected. In addition, they provide fits to \( S(E) \) using the Taylor expansion (Equation (1)). For the leading order they obtain \( S_{11}(0) = (4.03 \pm 0.006) \times 10^{-25} \) MeV b. This is 0.5% higher and with a much smaller error than the recommended value in A11. More recently, and also using chiral effective field theory, Acharya et al. (2016) determined \( S_{11}(E) \), resulting in \( S_{11}(0) = 4.047^{+0.0004}_{-0.0002} \times 10^{-25} \) MeV b. This is in very good agreement with the Marcucci et al. (2013) result. Acharya et al. (2016) have performed a more thorough assessment of the uncertainty source leading to an estimated error of 0.7%, much closer to the 1% uncertainty, which was obtained by A11. Based on larger error estimate by Acharya et al. (2016) and the difference in the central values for \( S_{11}(0) \) and higher order derivatives between both works, we prefer to make a conservative choice and adopt a 1% uncertainty for the \( p+p \) reaction rate.

The evaluation of \( S(E) \) presented by Marcucci et al. (2013) allows a full integration of the \( p+p \) rate, avoiding the Taylor expansion. Moreover, Tognelli et al. (2015) provide a routine\(^{13} \) to directly compute the \( p+p \) rate that we use to implement this rate in GARSTEC. In Figure 1, we plot as a function of temperature the ratio between the newly adopted (Marcucci et al. 2013) and the older A11 reaction rates. For comparison, the rate inferred from Acharya et al. (2016) is also shown. The comparison with the A11 rate shows a larger variation than the 0.5% difference quoted for \( S_{11}(0) \) that is due to changes in the first and the higher order derivatives, as well as to the different integration methods to compute the rate. The solid black line in the plot shows the distribution of \( p+p \) reactions produced per \( \delta \log T \) interval as a function of temperature in SSMs (arbitrary units).

2. \( ^{1}B(p, \gamma)\^{}_{2}B \): A11 recommends \( S_{117}(0) = (2.08 \pm 0.07 \pm 0.14) \times 10^{-5} \) MeV b, where the first error term comes from uncertainties in the different experimental results and the second one from considering different theoretical models employed for the low-energy extrapolation of the rate. Zhang et al. (2015) present a new low-energy extrapolation based on Halo Effective Field Theory, which allows for a continuous parametric evaluation of all low-energy models. Marginalization over the family of continuous parameters then amounts to marginalizing the results over the different low-energy models. They obtain \( S_{117}(0) = (2.13 \pm 0.07) \times 10^{-5} \) MeV b. We note the uncertainty equals that of the experimental uncertainty as given above. In A11, this error source was inflated to accommodate systematic differences seen among different experimental results (see, in particular, their Appendix on Treating Uncertainties). This problem was not found by Zhang et al. (2015) in their analysis. While the different findings by A11 and Zhang et al. (2015) regarding inconsistency in the nuclear data need further study (K. M. Nollett 2016, private communication), we prefer to err on the safe side and adopt an intermediate error between those from Zhang et al. (2015) and A11. Therefore, we finally adopt \( S_{117}(0) = (2.13 \pm 0.1) \times 10^{-5} \) MeV b and add the two error sources quadratically. We have also updated the derivatives by using the recommended values of Zhang et al. (2015).

3. \( ^{14}N(p, \gamma)\^{}_{15}O \): Marta et al. (2011) present new cross-section data for this reaction obtained at the Laboratory for Underground Nuclear Astrophysics experiment. With the new data and using R-matrix analysis they recommend a new value for the ground-state capture of \( S_{G_{5}}(0) = (0.20 \pm 0.05) \times 10^{-3} \) MeV b, down from the

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\( ^{13} \) http://astro.df.unipi.it/stellar-models/pprate/
previously recommended value of $0.27 \times 10^{-3}$ MeV b (A11). Combined with other transitions (see Table XI in that work) this leads to $S_{144}(0) = (1.59 \times 10^{-3})$ MeV b, about 4% lower than the previous A11 recommended value. The derivatives and the errors remain unchanged.

4. $^{4}$He($^3$He, $^5$He)$^7$Be: deBoer et al. (2014) combine recent (post 2004) experimental results including those in A11 but also newer data at medium and higher energies, from 300 up to 3500 keV (see references in their work). They perform a global $R$-matrix fit to determine the extrapolated $S_{14}(0)$ value and an MC analysis of the $R$-matrix fit to determine the uncertainties in the rate. This is a very different approach to that used in A11, where microscopic models were used to determine $S_{14}(0)$ for four independent data sets and then combined statistically to provide a final result for $S_{14}(0)$. The final value reported by deBoer et al. (2014) is $S_{14}(0) = (5.42 \pm 0.23) \times 10^{-4}$ MeV b. The central value is $\sim 3\%$ lower than the previous A11 recommended value. It should be pointed out that this value is systematically lower than three out of the four results used in A11 and very similar to the fourth one. The underlying reasons for these differences are not discussed in deBoer et al. (2014).

More recently, Iliadis et al. (2016) performed a global Bayesian estimation of $S_{14}$ using the same data as in A11, but extended up to 1.6 MeV instead of 1 MeV, and found $S_{14}(0) = (5.72 \pm 0.12) \times 10^{-4}$ MeV b. This is $2\%$ higher than A11 and almost $6\%$ larger than deBoer et al. (2014). The use of larger energy ranges in deBoer et al. (2014) and Iliadis et al. (2016) compared to what is required for an accurate determination of $S_{14}(E)$ in the energy range required for solar neutrino calculations (see A11) makes us wary of the impact that derivatives of $S(E)$ could have on the expansion.

Given the reasons above, and that deBoer et al. (2014) and Iliadis et al. (2016) results bracket that from A11, we have decided to keep the latter as our preferred choice in B16 SSMs. We point out, however, that a reduction in $S_{14}(0)$ such as that claimed by deBoer et al. (2014) would have an impact on the comparison between solar neutrino data and SSMs built with the GS98 or AGSS09met compositions for the $\Phi(8B)$ and $\Phi(7Be)$ fluxes. We briefly touch upon this in Section 3.2.

We use Salpeter’s formulation of weak screening Salpeter (1954). The validity of this formulation for solar conditions, where electrons are only weakly degenerate, has been discussed in detail in Gruzinov & Bahcall (1998), where a more sophisticated approach was shown to lead, to within differences of about 1%, to Salpeter’s result. Other proposed deviations from this formulation have been discussed at length in Bahcall et al. (2002), including different approaches to dynamic screening, and shown to be flawed or not well physically motivated. We therefore assume that uncertainties associated to electron screening are negligible in comparison to others entering SSM calculations. We are aware, however, that more recent calculations of dynamic screening (Mao et al. 2009; Mussack & Däppen 2011) still leave some room for discussion on this topic.

To conclude this section, we present in Table 3 a summary of other main cross-sections and input parameters used to construct the SSMs.

### Table 3

| Qnt. | Central value  | $\sigma$ (%) | References |
|------|----------------|--------------|------------|
| $^8$He($^3$He, $^5$He)$^7$Be | $5.21$ MeV b | $5.2$ | 1 |
| $^8$He($^3$He, $^4$He)$^7$Li | $5.6 \times 10^{-4}$ MeV b | $5.2$ | 1 |
| $^8$Be($^3$Li, $^6$Li)$^7$Be | Eq (40) SFII | $2.0$ | 1 |
| $^7$He($e^-, e^-)$He | $8.6 \times 10^{-20}$ MeV b | $30.2$ | 1 |
| $^7$Be($e^-, e^-)$Be | $1.06 \times 10^{-2}$ MeV b | $7.6$ | 1 |
| $T_7$ | $4.57 \times 10^8$ yr | $0.44$ | 2 |
| diffusion | $1.0$ | $15.0$ | 2 |
| $L_3$ | $3.8412 \times 10^{33}$ erg s$^{-1}$ | $0.4$ | 2 |

References. (1) A11, (2) Bahcall et al. (2006).

### 2.2. Treatment of Model Uncertainties

Within the framework defined by SSMs, the treatment of model uncertainties is generally simple. Most of the input physics in the models can be characterized by simple numbers such as the astrophysical factors or the surface abundance of a given element, as discussed in the previous section. Tables 1–3 list the uncertainties we adopt for each of the input quantities that allow such simple parametrizations. The adopted uncertainty for the microscopic diffusion coefficients deserves a special comment, as the coefficients cannot be obtained experimentally. The quoted 15% uncertainty, the same used in previous SSM calculations by our group, stems from results presented in Thoul et al. (1994), where the complete solution of the multilow Burgers equations was initially presented. As discussed in that work, the uncertainty in the diffusion coefficients comes from the calculation of the Coulomb collision integrals. The comparison of their results with equivalent calculations available in the literature (Proffitt & Michaud 1991) yielded a <15% difference in the diffusion coefficients for all relevant elements in the solar interior. The adopted 15% uncertainty is therefore conservative in more than one aspect: it is based on the difference between calculations (we could equally well define 1σ as half the difference between calculations), and it reflects the largest difference between different calculations for all the solar interior and all relevant chemical elements. Later works showed that the inclusion of radiative levitation, for instance (Turcotte et al. 1998), or quantum corrections to the collision integrals Schlattl & Salaris (2003), have very minor effects on solar model calculations, which are well within the adopted uncertainties.

A fundamentally important physical ingredient in solar models that cannot be quantified by just one parameter is the radiative opacity, which is a complicated function of temperature ($T$), density ($\rho$), and chemical composition ($X_i$) of the solar plasma. The magnitude and functional form of its uncertainty is currently not well constrained in available opacity calculations. As a result, representation of the uncertainty in radiative opacity by a single parameter (Serenelli et al. 2013) or by taking the difference between two alternative sets of opacity calculations (Bahcall et al. 2006; Villante et al. 2014) are strong simplifications, at best. In this paper, instead, we choose to follow a general and flexible approach based on opacity kernels originally developed by Tripathy & Christensen-Dalsgaard (1998) and later on by Villante (2010), which we describe next.
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The reference opacities \( \kappa(p, T, X_i) \) in SSMs can be modified by a generic function of \( T, p, \) and \( X_i \). For simplicity, we assume that opacity variations are parametrized as a function of \( T \) alone, such that

\[
\kappa(p, T, X_i) = [1 + \delta\kappa(T)] \pi(p, T, X_i)
\]

(2)

where \( \delta\kappa(T) \) is an arbitrary function. The Sun responds linearly even to relatively large opacity variations \( \delta\kappa(T) \) as shown by Tripathy & Christensen-Dalsgaard (1998) and Villante (2010). Thus, the fractional variation of a generic SSM prediction

\[
\delta Q \equiv Q/\bar{Q} - 1
\]

(3)

where \( Q, \bar{Q} \) corresponds to the modified (reference) value, can be described as

\[
\delta Q = \frac{\int dT}{T} K_Q(T) \delta\kappa(T)
\]

(4)

by introducing a suitable kernel \( K_Q(T) \) that describes the response of the considered quantity to changes in the opacity at a given temperature. We determine the kernels \( K_Q(T) \) numerically by studying the response of solar models to localized opacity changes, as it was done in Tripathy & Christensen-Dalsgaard (1998). Our results agree very well in all cases except for variations in the chemical composition because our models include gravitational settling. However, this has a negligible effect for the calculation of opacity uncertainties.

The evaluation of \( \delta Q \) is subject to the choice we make for \( \delta\kappa(T) \). In Haxton & Serenelli (2008) and Serenelli et al. (2013), the opacity error was modeled as a 2.5% constant factor at 1σ level, comparable to the maximum difference between the OP and OPAL (Iglesias & Rogers 1996) opacities in the solar radiative region. Villante (2010) showed that this prescription underestimates the contribution of opacity uncertainty to the sound speed and convective radius error budgets because the opacity kernels for these quantities are not positive definite and integrate to zero for \( \delta\kappa(T) = \text{const.} \). Later on, Villante et al. (2014) considered the temperature-dependent difference between OP and OPAL opacities as 1σ opacity uncertainty. However, it is by no means clear that this difference is a sensible measure of the actual level of uncertainty in current opacity calculations.

Based on the previous reasons, here we follow a different approach inspired by the most recent experimental and theoretical results and some simple assumptions. The contribution of metals to the radiative opacity is larger at the bottom of the convective envelope (∼70%) than at the solar core (∼30%). Also, Krief et al. (2016a), in a recent theoretical analysis of line broadening modeling in opacity calculations, have found that uncertainties linked to this are larger at the base of the convective envelope than in the core. These arguments suggest that opacity calculations are more accurate at the solar core than in the region around the base of the convective envelope. It is thus natural to consider error parameterizations that allow opacity to fluctuate by a larger amount in the external radiative region than in the center of the Sun.

Taking all this into account, we consider the following parameterization for the opacity change \( \delta\kappa(T) \):

\[
\delta\kappa(T) = a + b \frac{\log_{10}(T_C/T)}{\Delta}
\]

(5)

where \( \Delta = \log_{10}(T_C/T_{CZ}) = 0.9 \). \( T_C \) and \( T_{CZ} \) are the temperatures at the solar center and at the bottom of the convective zone respectively. This equation is applied only to the lower regions of the convective envelope, where convection is adiabatic and changes in the opacity are irrelevant. Changes in the opacity in the uppermost part of the convective envelope and atmosphere are absorbed in the solar calibration by changes in the mixing length parameter and in sound speed inversions by the surface term and, in the context of SSMs, will not produce changes in the solar properties considered in the present work. By changing the parameters \( a \) and \( b \), one is able to rescale and tilt the solar opacity profile by arbitrary amounts. We consider them as independent random variables with mean equal to zero and dispersions \( \sigma_a \) and \( \sigma_b \), respectively. This corresponds to assuming that the opacity error at the solar center is \( \sigma_{in} = \sigma_a \), while it is given by \( \sigma_{out} = \sqrt{\sigma_a^2 + \sigma_b^2} \) at the base of the convective zone. We fix \( \sigma_{in} = \sigma_a = 2\% \) which is the average difference of the OP and OPAL opacity tables. This is also comparable to differences found with respect to the new Los Alamos OPLIB opacity tables (Colgan et al. 2016). For \( \sigma_{out} \), we choose 7% (i.e., \( \sigma_b = 6.7\% \)), motivated by the recent experimental results of Bailey et al. (2015) that have measured the iron opacity at conditions similar to those at the base of the solar convective envelope and have found a 7% ± 4% increase with respect to the theoretical expectations. We note that the adopted functional form, see Equation (5), for the opacity error function is a simplified parametric description of a more complex (and unknown) behavior. Our choice is motivated by practical reasons and by the important fact that Christensen-Dalsgaard et al. (2009) and Villante (2010) have shown that an opacity solution to the solar abundance problem requires a tilt of opacity profile of the Sun by increasing opacity by a few percent at the solar center and a much larger increase (up to 15 to 20%) at the base of the convective region, i.e., the kind of behavior for \( \delta\kappa(T) \) described by Equation (5) when \( b = 0 \).

The adopted functional form can mimic, moreover, the uncertainty in theoretical calculations introduced by line broadening modeling discussed by Krief et al. (2016a).

---

**Table 4**

| Qnt. | B16-GS98       | B16-AGSS09met | Solar |
|------|----------------|---------------|-------|
| \( Y_s \) | 0.2426 ± 0.0059 | 0.2317 ± 0.0059 | 0.2485 ± 0.0035 |
| \( R_{CZ}/R_s \) | 0.7116 ± 0.0048 | 0.7223 ± 0.0053 | 0.713 ± 0.001 |
| \( \langle k_c/e \rangle \) | 0.0005 ± 0.000002 | 0.0021 ± 0.001 | 0.0 |

Note. The observational values of \( Y_s \) are taken from Basu & Antia (2004) and \( R_{CZ} \) from Basu & Antia (1997).

\( * \) The solar value is zero, by construction of \( k_c/e \).
Table 5
Comparison of B16 SSMs against Different Ensembles of Solar Observables

| Case            | dof | $\chi^2$ | p-value ($\sigma$) | $\chi^2$ | p-value ($\sigma$) |
|-----------------|-----|----------|---------------------|----------|---------------------|
| $\nu_g + R_{CZ}$ only | 2   | 0.9      | 0.5                 | 6.5      | 2.1                 |
| $\dot{c}/c$ only | 30  | 58.0     | 3.2                 | 76.1     | 4.5                 |
| $\dot{c}/c$ no-peak | 28  | 34.7     | 1.4                 | 50.0     | 2.7                 |
| $\Phi(\alpha) + \Phi(\alpha)$ | 2   | 0.2      | 0.3                 | 1.5      | 0.6                 |
| All $\nu$-fluxes | 8   | 6.0      | 0.5                 | 7.0      | 0.6                 |
| Global          | 40  | 65.0     | 2.7                 | 94.2     | 4.7                 |
| Global no-peak  | 38  | 40.5     | 0.9                 | 67.2     | 3.0                 |

3. Results

Here we present the main results of the B16 SSMs for GS98 and AGSS09met compositions and discuss differences with respect to our previous SFII models. Table 4 presents a summary of the most relevant quantities linked to the calibration of SSMs and helioseismic quantities. In Table 5, we quantify the agreement between SSMs and solar data and Table 6 gives results for solar neutrino fluxes. Model errors and theoretical correlations among observable quantities have been obtained from MC simulations that are discussed in some detail in Section 4.

3.1. Helioseismology

Two helioseismic quantities widely used in assessing the quality of SSMs are the surface helium abundance $Y_S$ and the location of the bottom of the convective envelope $R_{CZ}$. Both are listed in Table 4 together with the corresponding seismic values. The model errors associated with these quantities are larger in B16 models than previously computed Bahcall et al. (2006) generations of SSMs because of the different treatment of uncertainties in radiative opacities (see Section 4 for details). Compared to SFI models, we find a small decrease in the predicted $Y_S$ by 0.0003 for both compositions and a decrease in the theoretical $R_{CZ}$ by 0.0007 $R_\odot$, also for both compositions. The Pearson correlation between these quantities in SSMs is $\rho(Y_S, R_{CZ}) = -0.35$ and $0.41$ for B16-GS98 and B16-AGSS09met models, respectively, as obtained from the MC calculations. A comparison of models and data for these two quantities yields $\chi^2 = 0.91$ and $\chi^2 = 6.45$ for GS98 and AGSS09met compositions that translate into 0.5$\sigma$ and 2.1$\sigma$ differences between models and data. This is summarized in Table 5.

Figure 2 shows the fractional sound speed difference as a function of solar radius. The solar sound speed differences have been obtained for each of the two SSMs by performing new sound speed inversions, using the appropriate reference solar model, based on the BiSON-13 data set (a combination of BiSON+MDI data) as described in Basu et al. (2009). The resulting $\dot{c}/c$ curves are not too different with respect to SFII models. This is expected because the differences between the two generations of models are mostly due to changes in the nuclear reaction rates. All rates have a negligible impact on the solar sound speed profile except for the $p+p$, $e^+e^-$ rate. It is the newly adopted rate for this reaction that introduces the small differences with respect to the older generation of SSMs. This is shown in Figure 2 by including the sound speed difference for the previous SFII-GS98 SSM as a dashed line. A change in the $p+p$ rate leads to structural changes in the structure of the model that are non-local (Villante 2010) due to the constraints imposed in building an SSM, in particular, $R_\odot$ and $L_\odot$, so the model sound speed is also affected at larger radii, where nuclear burning is negligible.

A quantitative assessment of the agreement between model and solar sound speeds is not straightforward. It requires a proper evaluation of model errors and correlations. Also, given a set of observed frequencies, extraction of the sound speed profile is sensitive to uncertainties in the measured frequencies, numerical parameters inherent to the inversion procedure and the solar model used as a reference model for performing the inversion. Such detailed analysis was carried out in Villante et al. (2014), in which the SSM response to varying input parameters was modeled using power-law expansions and the three uncertainties related to the extraction of $\dot{c}/c$ from observed data were taken directly from Degl’Innocenti et al. (1997).

In this work, we use large MC sets of SSMs (Section 4) to account for model errors and correlations instead of using power-law expansions around a reference model. The total...
error from all input parameters in SSMs is illustrated in Figure 2 as the shaded area embracing the B16-AGSS09met curve. Note that in comparison to previous estimates, e.g., Villante et al. (2014), errors are larger due to the adoption of the larger opacity uncertainty. It should also be noted that model errors are strongly correlated across the solar radius.

The total error due to the three error sources linked to $\delta c/c$ inversion is shown in Figure 2 as the gray shaded area around 0. We have improved the calculation of two of these error sources in comparison to results in Degl’Innocenti et al. (1997). The first one is the error in $\delta c/c$ resulting from propagating the errors in the observed frequencies. This is now done on the basis of the BISON-13 data set, a much more modern data set with smaller frequency errors. This is not a dominant error source at any location in the Sun. More importantly, however, is the dependence of the solar sound speed on the reference model employed for the inversion. Previously (Degl’Innocenti et al. 1997; Basu et al. 2000), this dependence was estimated by performing sound speed inversions for a few solar models with different input physics, but with fixed solar composition. Here, instead, we resort to using two sets of 1000 SSMs originally computed by Bahcall et al. (2006), with one set based on GS98 and the other one on AGS05 (Asplund et al. 2005) solar compositions. In both cases, composition uncertainties used for those data sets correspond to the so-called “conservative” uncertainties and are, in fact, about twice as large, or more, as those quoted in the corresponding spectroscopic results. In addition, all other input parameters in SSM calculations have been varied. For these 2000 models, inversions have been carried to determine the solar sound speed profiles. The dispersion of the results, as a function of radius, have been used to derive the dependence of inferred solar sound speed on the inversion reference model. An alternative, and more consistent approach, would be to perform inversions for all the models in our MC simulations, as was done in Bahcall et al. (2006). This is a very time consuming procedure because it is not fully automated and we decided not to repeat it in the present paper. However, our approach, just described, makes use of a broad range of SSMs and ensures a conservative estimate of this error source. A comparison of our current estimates of uncertainties with respect to previous estimates is shown in Figure 3, where solid and dashed lines depict currently adopted and older errors respectively.

Using model and inversion uncertainties as described above, we compare how well the predicted sound speed profiles of B16-GS98 and B16-AGSS09met agree with helioseismic inferences. For this, we use the same 30 radial points employed in Villante et al. (2014). We use the models in the MC simulations to obtain the covariance matrix for these 30 points and assume inversion uncertainties at different radii as uncorrelated. We acknowledge the latter is an assumption and we expect to improve on it in the future. Results are shown in the second row of Table 5. For 30 degrees-of-freedom (df), B16-GS98 gives $\chi^2 = 58$, or a $3.2\sigma$ agreement with data. For B16-AGSS09met results are $\chi^2 = 76.1$, or $4.5\sigma$. Below we analyze in some detail the significance of these results.

It is apparent from Figure 2 that, at almost all radii, the sound speed profile of B16-GS98 fits well within the 1$\sigma$ uncertainties. This is true even for the peak right below the CZ at $r/R_\odot \approx 0.6-0.7$. However, looks can be deceiving. The difference between B16-GS98 and the Sun is dominated by wiggles of relatively small amplitude. However, changes in input quantities, including radiative opacities, do not lead to variations in SSM sound speeds on small radial scales, so values of the sound speed at different radii in solar models are strongly correlated. Including these correlations by means of a covariance matrix in the calculation of $\chi^2$ explains why the large value $\chi^2 = 58$ is obtained for the B16-GS98, which apparently fits well within 1$\sigma$ contours. This result reflects the fact that, within the framework of SSMs and our treatment of uncertainties, particularly of opacities, it is not possible to find a combination of input parameters that would make the wiggles go away. To confirm this, we checked that when the covariance matrix is assumed diagonal, i.e., correlations are neglected, $\chi^2 = 23.6$ for the sound speed profile of B16-GS98, well within a 1$\sigma$ result as expected by a naive look at Figure 2.

For B16-AGSS09met, the discrepancy with the solar sound speed is dominated by the large and broad peak in $0.35 < r/R_\odot < 0.72$. In this case, correlations in the model sound speed decrease the level of disagreement with the data. Variations in the model leading to improvements in the sound speed profile will do so at a global scale. If, as a test, sound speed correlations are neglected for this model, we obtain a larger $\chi^2 = 100.4$, i.e., the opposite behavior than for B16-GS98.

It is important to notice that in the case of B16-GS98, the largest contribution to the sound speed $\chi^2$ comes from the narrow region $0.65 < r/R_\odot < 0.70$ that comprises 2 out of all the 30 points. If these two points are removed from the analysis, $\chi^2$ is reduced from 58 to 34.7, equivalent to a $1.4\sigma$ agreement with the solar sound speed (entry identified as $\delta c/c$ no-peak in Table 5). For B16-AGSS09met, this test leads to a $2.7\sigma$ result. This exercise highlights the qualitative difference between SSMs with different compositions; it shows that for GS98 the problem is highly localized whereas for AGSS09met the disagreement between SSMs and solar data occurs at a global scale, i.e., the solar abundance problem.

The result of removing two points from the sound speed analysis is reassuring in that the GS98 composition leads to an SSMs that is in quite good agreement with solar data. However, it also highlights limitations of SSMs in providing an accurate description of the solar region just below the
convective envelope, a fact known for a long time, e.g., Antia & Chitre (1998) and Christensen-Dalsgaard et al. (2011).

Table 4 lists additional quantities and their model uncertainties associated with the solar calibration: mixing length parameter ($\alpha_{\text{MLT}}$), initial helium $Y_{\text{ini}}$, and metallicity $Z_{\text{ini}}$ and the corresponding surface ($Y_0$ and $Z_0$) and central $Y_0$ and $Z_0$ quantities, $R_{CZ}$ as stated above and the average rms difference of the relative sound speed difference shown in Figure 2. As discussed in this section, this rms value is only indicative of the quality of the models because it neglects correlations in the models.

3.2. Neutrino Fluxes

The most relevant updates in the B16 SSMs are related to updates in several key nuclear reaction rates (see Section 2.1) that have a direct effect on the predicted solar neutrino fluxes. The detailed results for all the neutrino fluxes are summarized in Table 6.

Currently, $\Phi(\overline{\text{B}})$ and $\Phi(\overline{\text{Be}})$ are the fluxes most precisely determined experimentally, and can be used to perform a simple test of the models. Furthermore, these are also the two fluxes from the pp-chains that are most sensitive to temperature, i.e., to the conditions in the solar core and the inputs in solar models. In Serenelli et al. (2011), the agreement between SFII-GS98 and SFII-AGSS09met with solar fluxes was virtually the same. The solar values determined from experimental data in that work were $\Phi(\overline{\text{B}})=5 \times 10^6 \text{cm}^2 \text{s}^{-1}$ and $\Phi(\overline{\text{Be}})=4.82 \times 10^8 \text{cm}^2 \text{s}^{-1}$ with 3% and 4.5% uncertainties respectively. SFII-GS98 yields $\Phi(\overline{\text{B}})=5.58(1 \pm 0.14) \times 10^6 \text{cm}^2 \text{s}^{-1}$ and $\Phi(\overline{\text{Be}})=5.00(1 \pm 0.07) \times 10^8 \text{cm}^2 \text{s}^{-1}$, while SFII-AGSS09met gives $\Phi(\overline{\text{B}})=4.59 \times 10^6 \text{cm}^2 \text{s}^{-1}$ and $\Phi(\overline{\text{Be}})=4.56 \times 10^8 \text{cm}^2 \text{s}^{-1}$ with same fractional errors as SFII-GS98. Experimental results for both $\Phi(\overline{\text{B}})$ and $\Phi(\overline{\text{Be}})$ were right in between the predictions for the two SSMs.

The new B16 generation of solar models, together with the recent determination of solar fluxes by Bergström et al. (2016) included in Table 6, leads to some changes in this stalemate. On one hand, SSM predictions for $\Phi(\overline{\text{B}})$ and $\Phi(\overline{\text{Be}})$ are reduced for both GS98 and AGSS09met compositions by about 2% with respect to previous SFII SSM due to the larger $p(\text{p}, e^-\overline{\text{e}})d$ rate (for $\Phi(\overline{\text{B}})$ this is partially compensated by the increase in the $\overline{\text{Be}}(p,\gamma)\overline{\text{B}}$ rate). On the other hand, solar fluxes determined by Bergström et al. (2016) result in a solar $\Phi(\overline{\text{B}})$ that is about 3% higher than the value used in Serenelli et al. (2011). Bergström et al. (2016) include in their analysis the Borexino Phase-2 data (BOREXINO Collaboration et al. 2014), the combined analysis of all three SNO phases (Aharmim et al. 2013), and results from phase IV of Super-Kamiokande (Super-Kamiokande Collaboration et al. 2016), all of which were not available in 2011. Figure 4 reflects the updated state of matters and shows model and experimental results normalized to the newly determined solar values. Central values of B16-GS98 are closer to the solar values, both for $\Phi(\overline{\text{B}})$ and $\Phi(\overline{\text{Be}})$, than B16-AGSS09met fluxes. Both solar compositions lead to SSMs, however, that are consistent with solar neutrino fluxes within 1σ. A comparison between models and solar data for these two fluxes yields $\chi^2(\text{GS98})=0.2$ and $\chi^2(\text{AGSS09met})=1.45$ (see Table 5). This calculation includes model correlations obtained from the MC simulations and the distribution of solar fluxes from Bergström et al. (2016).

As discussed in Section 2.1, recent determinations of $S_{\Delta}(\overline{\text{B}})$ range between $5.42 \times 10^{-4} \text{MeV b}^{-1}$ and $5.72 \times 10^{-4} \text{MeV b}^{-1}$. We speculate here on the impact of adopting the slightly lower $S_{\Delta}(\overline{\text{B}})$ value such as that determined by deBoer et al. (2014); see Section 2.1). A 3.2% reduction in $S_{\Delta}(\overline{\text{B}})$ leads to a decrease in $\Phi(\overline{\text{B}})$ and $\Phi(\overline{\text{Be}})$ of about 2.7% and 2.8% respectively. This change leads to $\chi^2(\text{GS98})=0.13$ (0.1σ) and $\chi^2(\text{AGSS09met})=2.4$ (1.0σ). In this hypothetical scenario, the agreement between B16-AGSS09met and data is slightly larger than 1σ. Although this would still be far from being too useful as a discrimination test between solar models, this exercise helps by showing that the few percent systematics present in the determination of nuclear reaction rates can still be a relevant source of difficulty in using neutrino fluxes as constraints to solar model properties.

The most important changes in the neutrino fluxes occur for $\Phi(\overline{\text{N}})$ and $\Phi(\overline{\text{O}})$, in the CN-cycle. These fluxes are potentially excellent diagnostics of properties of the solar core. In particular, their dependence on the metallicity is two-fold: through opacities much in the same manner as pp-chain fluxes, and also through the influence of the added C+N abundance in the solar core. This latter dependence makes these fluxes a unique probe of the metal composition of the solar core. The expectation values in the B16 SSMs are about 6% and 8% lower than for the previous SFII models for $\Phi(\overline{\text{N}})$ and $\Phi(\overline{\text{O}})$ respectively. This results from the combined changes in the $p+p$ and $^{14}\text{N}+p$ reaction rates (Table 2).

CN fluxes have not yet been determined experimentally. The global analysis of solar neutrino data performed by Bergström et al. (2016) yields the upper limits that we include in Table 6. The Borexino collaboration, based on a different analysis of Borexino data alone, has reported an upper limit for the added fluxes $\Phi(3\text{N})+\Phi(3\text{O})<7.7 \times 10^8 \text{cm}^2 \text{s}^{-1}$ (Bellini et al. 2012).

We close this section with a comparison of models and solar data for all neutrino fluxes. $\chi^2$ values are 6.01 and 7.05 for B16-GS98 and B16-AGSS09met models, respectively, and are also included in Table 5. This global comparison is clearly dominated by the $\Phi(\overline{\text{B}})$ and $\Phi(\overline{\text{Be}})$ fluxes. It is evident that the current determination of solar neutrino fluxes is well described by models with any of the two solar compositions.
3.3. Global Analysis

What is the performance of both B16 SSMs when all the observables discussed before are used for the comparison? Results are summarized in the last two rows of Table 5 when all the sound speed profile is used or the two points in the region \(0.65 < r/R_\odot < 0.70\) are excluded. Global \(\chi^2\) is not strictly the sum of the individual contributions because of correlations between, e.g., \(R_{\text{CZ}}\) and the sound speed profile. However, deviations are small.

Final \(\chi^2\) values are dominated by the sound speed for both models, though \(Y_\text{e}\) and \(R_{\text{CZ}}\) are also relevant for B16-AGSS09met. The global analysis yields a poor 2.7\(\sigma\) result for B16-GS98. However, this is strongly linked to the behavior of the sound speed profile right below the convective zone, as explained in Section 3.1. Excluding two points in the sound speed lead to an overall, comforting, 0.9\(\sigma\) agreement of this model with solar data. In the case of B16-AGSS09met, the overall agreement with the data is quite poor, at 4.7\(\sigma\), which improves to only 3.0\(\sigma\) if the critical points in the sound speed profile are excluded. This is still a poor agreement with data.

It is interesting here to consider the impact of radiative opacity in the results we obtain. We have assumed a 7\% uncertainty for the opacity at the base of the convective zone. Different authors have estimated that changes between 15\% and 20\% at that location are required to solve the solar abundance problem. Therefore, it may seem somewhat surprising that AGSS09met yields a much larger, 4.7\(\sigma\), disagreement. Naively, we would expect a disagreement at approximately 3\(\sigma\) or a smaller level provided the level of uncertainty we adopt for opacity. It should be noted, however, that the linear behavior for the opacity error function (Equation (5)) permits to compensate the differences between the AGSS09met and GS98 SSMs, but it is not flexible enough (for both compositions) to accommodate a better fitting sound speed profile.

A more detailed analysis of the error function of the opacity requires a more flexible implementation. Because the shape of the error function is itself unknown, a non-parametric approach, where no a priori assumptions are made about this shape, is a promising avenue to treat this problem. This is beyond the scope of this paper, but it is work in progress and will be reported in a forthcoming publication (N. Song et al. 2017, in preparation).

4. Errors in SSMs

Model uncertainties for all solar quantities \(Q\) of interest are listed in Tables 4 and 6 and correspond to 68\% C.L.

We base our model error determination in a MC approach, as introduced in Bahcall & Ulrich (1988) and later used in Bahcall et al. (2006). This is described in Section 4.1. The use of MC simulations has the advantage that it does not require the assumption of linear response of the models to changes in the input parameters. Linearity of solar models is usually a very good approximation because the uncertainties of most of the input parameters are small. However, deviations from a linear response are seen when \(2\sigma\) variations of diffusion coefficients and abundances of elements such as C, N, and O are considered. MC calculations ensure, then, a consistent assessment of errors. It is true, however, that MC hardwires uncertainties and makes it more difficult to single out relevant individual uncertainty sources for specific observables. Here, power-law expansions are very useful and we in fact resort to them to identify dominant sources of uncertainty for neutrino fluxes and helioseismic probes (Section 4.2). We close the discussion on errors by considering in some detail the impact of our specific choice of opacity uncertainty (Equation (5)) in comparison to the previously adopted error measure (OP-OPAL difference; Bahcall et al. 2006; Villante et al. 2014).

4.1. MC Simulations

We construct MC simulations of SSMs following the procedure used in Bahcall et al. (2006). A large set of 10,000 SSMs is constructed where, for each model, the values of the input quantities \(i\) are chosen randomly from their respective distributions. Here, \(\{i\}\) is the set of input parameters including composition of chemical elements, diffusion rate, \(L_\odot\), \(\tau_\odot\) and nuclear cross-section parameters. They are treated as follows.

4.1.1. Treatment of Uncertainties

Composition parameters—Two different MC sets are computed, one per reference composition, i.e., GS98 and AGSS09met (Table 1). In each set, the jth SSM is calculated by assuming

\[
\epsilon_{ij} = \tau_i + C_{ij} \cdot \sigma_i, \tag{6}
\]

where \(\tau_i\) and \(\sigma_i\) are the central value and error for each element \(i\) (different for each reference composition) and the factors \(C_{ij}\) are sampled from independent univariate Gaussian random distributions. Note that \(\tau_i\) is a logarithmic measure of abundance.

Other input parameters—In Tables 2 and 3, we give the central values \(\bar{I}\) and the fractional uncertainties \(\sigma_i\) of the 11 non-composition input parameters that are varied in our MC runs. The jth SSM is calculated by assuming

\[
I_j = \bar{I} \left(1 + A_{i,j} \cdot \sigma_i\right), \tag{7}
\]

where the factors \(A_{i,j}\) are sampled from independent univariate Gaussian random distributions. For the case of diffusion, the central value “1” in Table 3 refers to the standard coefficients used in GARSTEC, computed following the method of Thoul et al. (1994).

Opacities—As described in Section 2.2, the contribution of opacity uncertainty to all quantities \(Q\) are included a posteriori by means of Equation (5). The linear response of solar models to opacity variations ensures that this procedure is sufficiently accurate. In return, our set of MC calculations can be used to test other opacity error functions, e.g., the OP-OPAL difference (Section 4.3) or others motivated by future theoretical or experimental work on opacities.

With the opacity kernels in hand, our implementation of opacity uncertainties is very simple. After the jth SSM is calculated, a change of the opacity profile \(\delta \kappa(T)\) is modeled using Equation (5), with the coefficients \(a_j\) and \(b_j\) being extracted from independent Gaussian distributions with zero means and dispersions \(\sigma_{a_j} = 2\%\) and \(\sigma_{b_j} = 6.7\%\), respectively, values that reflect our estimates of the magnitude of opacity uncertainties (Section 2.2). The effect of the opacity variation on the various SSM predictions \(Q\) is then calculated by using the kernels \(K_{\sigma}(T)\) and the estimated change \(\delta Q\) then added to the SSM prediction.
4.1.2. Monte Carlo Results

The distributions of important quantities resulting from the MC simulations are presented in Figure 5 for GS98 and AGSS09met compositions. We have used these distributions to compute the uncertainties $\delta Q_i$ (68.3% C.L.) for model predictions of all quantities $Q$ given in this work, in particular, results reported in Tables 4 and 6. Plots for $Y_s$, $R_{cz}$, $\Phi(^{13}\text{N})$, and $\Phi(^{15}\text{O})$ show the distributions neglecting errors from solar composition. Vertical black dotted lines show the observational results and the gray shaded region shows the associated errors. Units for neutrino fluxes are as in Table 6.

![Figure 5](https://example.com/figure5.png)

**Figure 5.** Monte Carlo results for neutrino fluxes and main properties of SSMs. Blue shows results for GS98 and red for AGSS09met. Black solid lines show the associated Gaussian distributions and the dashed lines, for $\Phi(^{13}\text{N})$ and $\Phi(^{15}\text{O})$, show the distributions neglecting errors from solar composition. Vertical black dotted lines show the observational results and the gray shaded region shows the associated errors. Units for neutrino fluxes are as in Table 6.

4.2. Dominant Sources of Errors

In order to estimate the different contributions $\delta Q_{I}$ to the total error of the quantity $Q$, we follow the standard approach and calculate

$$\delta Q_{I} = \alpha_{Q,I} \sigma_{I}$$ (8)

where $\sigma_{I}$ is the 1σ fractional uncertainty of the $I$ input parameter and $\alpha_{Q,I}$ is defined by

$$\alpha_{Q,I} = \frac{\partial \ln Q}{\partial \ln I}$$ (9)

The log-derivatives $\alpha_{Q,I}$ are calculated numerically by varying the various input parameters, one at a time, over a range typically larger than their respective 3σ uncertainty. The values obtained using the new SSM calculations are similar but update previous determinations (Serenelli et al. 2013).

The opacity error is described in terms of the two independent parameters $a$ and $b$, Equation (5), that fix the scale and the tilt of the opacity profile. The derivative of a given quantity $Q$ with respect to these parameters can be calculated from Equation (13) as

$$\alpha_{Q,a} = \frac{\partial \ln Q}{\partial a} = \int \frac{dT}{T} K_Q(T)$$

$$\alpha_{Q,b} = \frac{\partial \ln Q}{\partial b} = \int \frac{dT}{T} K_Q(T) \frac{\log_{10}(T/T_0)}{\Delta}$$ (10)
The total error due to opacity is estimated by combining the two contributions in quadrature, i.e.,

$$\delta Q_\kappa = \sqrt{(\alpha_{Q,a} \sigma_a)^2 + (\alpha_{Q,b} \sigma_b)^2}$$  \hspace{1cm} (11)$$

Results obtained with Equations (8) and (11) are presented for the dominant error sources in Table 7 for all relevant solar quantities. Dominant error sources can be roughly grouped as composition, nuclear, and stellar physics, the latter dominated by opacity and microscopic diffusion.

**Composition**—Errors from composition are dominated by the C, O, and Ne. This is not related to the solar composition problem, however, but just to the fact that even the most optimistic spectroscopic determinations of solar abundances have a level of uncertainty of about 10%–12% that is very difficult to beat. Refractories, on the other hand, are more precisely measured from meteorites so their contribution to uncertainties in solar quantities is currently minimal. Clearly, CNO neutrino fluxes are directly affected by these uncertainties, which are, in fact, the dominant error sources. For the same reason, $Z_{\text{ini}}$ (and $Z_5$) error is also dominated by uncertainties in these elements. For helioseismic quantities, $O$ affects $R_{CZ}$ because it is a dominant contributor to opacity at the base of the convective envelope. On the other hand, $Y_{\text{ini}}$ and $Y_5$ depend more strongly on Ne due to a combination of its large abundance, impact on opacity at deeper layers and larger error.

**Nuclear reactions**—Nuclear rates are still an important uncertainty source for neutrino fluxes despite big progress in the field. In particular, errors in $S_{33}$ and $S_{17}$ are still comparable or larger than the uncertainties in the experimental determinations of $\Phi(^8B)$ and $\Phi(^7Be)$. As discussed in Section 3.2, the ability of solar neutrinos linked to pp-chains to play a significant role in constraining condition in the solar interior depends, though it is not the only factor, on pinning down errors of nuclear reaction rates to just ~2%. For CN fluxes, $S_{114}$ is the dominant error source if composition is left aside. Assuming a precise measurement of CN fluxes becomes available in the future, right now $S_{114}$ is the limiting factor in using such a measurement as a probe of the solar core C+N abundance (Serenelli et al. 2013).

**Opacity and microscopic diffusion**—These are the dominant sources of errors not linked to composition or nuclear reactions. For solar neutrinos, our estimate of the contribution of opacity to the total error is similar to previous calculations (Serenelli et al. 2013) despite the different treatment given to opacity errors. In this work, we assume a 2% uncertainty in the center that increases linearly outwards. Because neutrinos are produced in a localized region, our results are not too different from assuming a constant 2.5% fractional opacity variation, which was the previous choice. Opacity is the dominant error source for $\Phi(^8B)$, and the second one for $\Phi(^7Be)$. For these fluxes, it is important that opacities in the solar core be known to a 1% level of uncertainty. Current theoretical work shows variations of about 2% (Krief et al. 2016a) and experimental measurements are notoriously difficult due to the combination of high temperatures and densities involved.

Opacity is a dominant uncertainty error source for helioseismic quantities, most notably $R_{CZ}$ and $Y_5$, with our new treatment of uncertainties. A 7% opacity uncertainty at the base of the convective envelope implies a 0.6% change in $R_{CZ}$. This is larger than all other uncertainty sources combined and explains the substantially larger error in $R_{CZ}$ given in this work, 0.7% (Table 4), compared to 0.5% previously determined (Bahcall et al. 2006). We observe a similar impact on $Y_5$, for which we estimate now a total 2.5% uncertainty compared to 1.5% from previous estimations.

At first glance, the change in uncertainties for $R_{CZ}$ and $Y_5$ might seem small but in fact model uncertainties are now substantially larger than helioseismically inferred ones. Moreover, the larger uncertainties lead to a formally better agreement between solar data and SSMs based on AGSS09met composition, which is now placed at about 2.1$\sigma$ level when $R_{CZ}$ and $Y_5$ are considered, whereas before this was closer to 3.5$\sigma$.

Microscopic diffusion is typically a smaller source of uncertainty than radiative opacities. However, for CN neutrino fluxes, its contribution is larger, only after $S_{114}$ and C. The reason is that accumulation of metals in the solar core due to gravitational settling increases CN fluxes both because it leads to a larger opacity in the solar core but also because the increase in the C+N abundance directly affects the efficiency of the CN-cycle.

### 4.3. Opacity Uncertainties: Effect of Different Parametrizations

As discussed before, there is a certain level of arbitrariness in the choice of the error function for opacity. Our standard choice (Section 2.2) stems from a comparison of available opacity data from both theoretical and experimental sources but also from an attempt not to being too aggressive (optimistic) in our choice.

A viable alternative is using the OP-OPAL difference as a 1$\sigma$ measure of the true opacity error (Bahcall et al. 2006; Villante 2010). Using the opacity kernels, it is straightforward to evaluate the error in solar quantities $Q$ if this opacity error function is used. In Table 8, we compare the fractional errors of solar quantities $Q$ for the linear and the OP-OPAL error functions. This comparison highlights the enhanced impact of opacity errors on solar quantities following our new approach, which we believe better reflects the current level of uncertainty in stellar opacities.

Finally, Figure 6 compares the sound speed uncertainties corresponding to the linear error function (indicated as the 7% curve in the plot) and the OP-OPAL error function. The former...
Table 8
Fractional Error Contribution of Opacity to the Neutrino Fluxes and Helioseismic Quantities SSMs for Different Choices of Opacity Variations, as Described in the Text

| Quant.       | \( \sigma_{\text{OP-OPAL}} \) | \( \sigma_{\text{out}} = 7\% \) |
|--------------|-------------------------------|-------------------------------|
| \( \Phi(\text{pp}) \)   | 0.001                         | 0.002                         |
| \( \Phi(\text{pep}) \)  | 0.001                         | 0.005                         |
| \( \Phi(\text{bep}) \)  | 0.002                         | 0.011                         |
| \( \Phi(\text{Be}) \)   | 0.009                         | 0.038                         |
| \( \Phi(\text{B}) \)    | 0.021                         | 0.074                         |
| \( \Phi(\text{N}) \)    | 0.012                         | 0.039                         |
| \( \Phi(\text{O}) \)    | 0.017                         | 0.055                         |
| \( \Phi(\text{F}) \)    | 0.019                         | 0.061                         |
| \( \gamma_S \)          | 0.004                         | 0.021                         |
| \( z_S \)               | 0.001                         | 0.007                         |
| \( R_{CZ} \)            | 0.001                         | 0.005                         |

Figure 6. Fractional sound speed variation resulting from different assumptions for the opacity error (see the text). For comparison, the uncertainty due to all other model inputs and the total uncertainty (opacity + other model sources) are also shown.

leads to larger uncertainties at almost all radii. Also, the plot includes the total contribution to the sound speed uncertainty from all (model) sources other than radiative opacity. The total uncertainty of the sound speed is also included and it corresponds to the model error budget shown as a pink band in Figure 2. The opacity contribution to the total sound speed error matches that of all other uncertainty sources combined, including those from composition errors.

5. Summary and Conclusions
We have presented B16-GS98 and B16-AGSS09met, a new generation of SSMs. They have been computed using the new release of GARSTEC that includes the possibility of using an EOS consistent with the composition used in the SSM calibration. We have also incorporated the most recent values for the nuclear reactions \( p(e^-\nu_e)d, \) \( ^3\text{He}(^4\text{He}, \gamma)^7\text{Be}, \) and \( ^{14}\text{N}(p, \gamma)^{15}\text{O}. \) With respect to previous works, we have implemented a flexible treatment of opacity uncertainties based on opacity kernels that allows testing any arbitrary modification to, or error function of, the radiative opacity profile. Based on current theoretical and experimental results on solar opacities, we have adopted an opacity error that increases linearly from the solar core, where it is 2%, toward the base of the convective envelope where we assume a 7% uncertainty. However, we have also tested the difference between OP and OPAL as the opacity error function. The estimation of central values of solar observables, their uncertainties and model correlations have been obtained from large MC sets of simulations of SSMs that comprise 10,000 SSMs per reference composition (either GS98 or AGSS09met). SSMs have been compared against different ensembles of solar observables: \( \gamma_S \) and \( R_{CZ}, \) sound speed profile, solar neutrinos, and a global comparison that includes the three classes of observables.

We summarize our most important findings here.

1. Central values for \( \Phi(\text{Be}) \) and \( \Phi(\text{B}) \) in B16 SSMs are reduced by about 2% with respect to the previous generation of models that were based completely on the A11 nuclear reaction rates. \( \Phi(13\text{N}) \) and \( \Phi(15\text{O}), \) the CN-cycle fluxes, are reduced by 6% and 8% respectively.
2. Solar neutrino fluxes (Bergström et al. 2016) are reproduced almost equally well by both B16-GS98 and B16-AGSS09met, with only a very minor preference for B16-GS98 (\( \chi^2 = 6.01 \) versus 7.05). If only \( \Phi(\text{Be}) \) and \( \Phi(\text{B}) \) are considered, then \( \chi^2 = 0.21 \) and 1.50 for B16-GS98 and B16-AGSS09met models respectively.
3. Helioseismic properties of B16 models are almost unchanged with respect to SFII models. However, our estimation of errors is larger due to our more pessimistic assumption of a 7% uncertainty in the radiative opacity at the base of the convective envelope. Comparison of models against \( \gamma_S \) and \( R_{CZ} \) yields a very good agreement for B16-GS98 (0.5\( \sigma \)) and a poor one (2.1\( \sigma \)) for B16-AGSS09met.
4. We have reevaluated some of the sources of uncertainty associated with solar sound speed inversion. This, together with our new adoption of larger opacity uncertainties lead to B16-GS98 and B16-AGSS09met to an agreement with data at the level of 3.16 and 4.5\( \sigma \) respectively.
5. The seemingly, and surprising, bad performance of the sound speed profile of B16-GS98 is caused almost exclusively by the large sound speed difference in the region 0.65 < \( r/R_\odot \) < 0.70. It is well known that the structure of the Sun in this narrow range of radius is not well reproduced by standard models. There is a long list of possibilities suggested to explain this deficit in SSMs: a smoother chemical profile as claimed by Antia & Chitre (1998) due to, e.g., turbulent mixing (Proffitt & Michaud 1991; Christensen-Dalsgaard & Di Mauro 2007), a smoother transition between an adiabatic and radiative temperature gradient Christensen-Dalsgaard et al. (2011) due to overshooting, dynamic effects at the tachocline (Brun et al. 2011), among others. Removing this region from the analysis brings the agreement of B16-GS98 and the solar sound speed to a comforting 1.4\( \sigma \). However, this discrepancy cannot be ignored and deserves further work.
6. For B16-AGSS09met, the mismatch with the solar sound speed profile is global. Removing the region 0.65 < \( r/R_\odot \) < 0.70 leads to a 2.7\( \sigma \) discrepancy with the solar sound speed profile.
7. The comparison of models with all data yields \( \chi^2 = 65 \) for B16-GS98 (40 dof) but only 40.5 (38 dof) when the region 0.65 < \( r/R_\odot \) < 0.70 is removed. This is equivalent to a very good 0.9\( \sigma \) result. For B16-AGSS09met, results are \( \chi^2 = 94, \) i.e., 4.7\( \sigma \) (or \( \chi^2 = 67 \) and 3\( \sigma \) without the problematic region). B16-GS98 is a better
model than B16-AGSS09met at a statistically significant level.

8. The estimated increase in opacity required to solve the solar abundance problem is 15% to 20% at the base of the convective envelope. By assuming a 7% opacity uncertainty in that region, it would be naively expected that B16-AGSS09met is discrepant with solar data at a 2–3σ level. The much larger, 4.7σ discrepancy is due to the fact that the adopted opacity error function Equation (5) allows us to compensate the differences between AGSS09met and GS98 SSMs but is not flexible enough (for both compositions) to accommodate a better fitting sound speed profile. This is in qualitative agreement with Villante et al. (2014).

9. We have identified the dominant uncertainty sources for neutrino fluxes. Radiative opacity is the dominant source among the stellar physics quantities, followed by the microscopic diffusion rates. Among nuclear reaction rates, the astrophysical factors $S_{34}, S_{17}$, and $S_{114}$ should have their uncertainties reduced to allow more precise tests of solar physics based on solar neutrino experiments. A smaller uncertainty for $S_{114}$ will be crucial in determining the abundance of C–N in the solar core when a precision measurement of the $\Phi(^{14}N)$ and $\Phi(^{16}O)$ fluxes becomes available.

10. For helioseismic quantities, opacity and diffusion are the dominant stellar uncertainty sources. Volatile elements, particularly O and Ne, also play an important role.

A novel and important aspect of our work is the implementation of opacity kernels to generalize the treatment of opacity uncertainties. This opens up the possibility to use the large sets of models computed as part of our MC simulations to study the impact of any arbitrary modification to the solar opacity profile on all solar observables. However, the shape of the profile of opacity uncertainties across the solar interior is unknown. As part of follow-up work, we are currently implementing an approach to treat opacity modifications based on a non-parametric Gaussian process approach that does not assume a specific shape for this error function (N. Song et al. 2017, in preparation). This will allow us to test the best possible solutions that opacity variations can offer not only to the solar abundance problem but also to how well any SSM can fit available solar observables.

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Appendix

Opacity Kernels

The calculation of the opacity kernels used to evaluate the contribution of opacity to the uncertainties of the solar properties has been done following the procedure presented in Tripathy & Christensen-Dalsgaard (1998). We summarize it in this Appendix.

First, we assume that in the region of the parameter-space $(\rho, T, X_i)$, which describes the solar plasma during the Sun evolution, the variation of the opacity can be approximately described as a function of the temperature only, i.e., the modified opacity table $\kappa(\rho, T, X_i)$ is related to the reference opacity table $\kappa(\rho, T, X_i)$ by

$$\kappa(\rho, T, X_i) = [1 + \delta \kappa(T)] \kappa(\rho, T, X_i) \quad (12)$$

where $\delta \kappa(T)$ is an arbitrary function. If the changes of opacity are small enough, i.e., $\delta \kappa(T) \ll 1$, the model responds linearly to these perturbations and the fractional variation of a generic observable $Q$ can be expressed as

$$\delta Q = \int \frac{dT}{T} K_Q(T) \delta \kappa(T). \quad (13)$$

In the above equation, the kernel $K_Q(T)$ describes the response of the considered quantity to changes of the opacity at a given temperature.

Our goal is to obtain these kernels and to use them in order to study the effects produced by arbitrary opacity changes. To do so, we need to determine the response of SSMs when opacity is changed in a thin shell whose position is determined by the temperature. This narrow perturbation is ideally represented as delta function:

$$\delta \kappa(T) = C \delta (\ln T - \ln T_0). \quad (14)$$

In numerical calculations, the $\delta$-function is approximated by a Gaussian, i.e.,

$$\delta \kappa(T) = \frac{C}{\sqrt{2\pi} \sigma} \exp \left[- \frac{(\ln T - \ln T_0)^2}{2\sigma^2} \right]. \quad (15)$$

where we require $\sigma \ll 1$, so that the opacity variations are sufficiently localized and $C/\sigma \ll \sqrt{2\pi}$ in order to avoid non-linear effects. We choose $\sigma = 0.03$ and $C = 0.004$. We have calculated a set of SSMs with opacity perturbations located in the temperature range of $\log_{10} T_0 = 6.3$–7.2, where radiative transport takes place and thus, the uncertainties of the radiative opacities play an important role. Finally, the opacity kernels are calculated by using the outputs of these models and normalizing the obtained variations $\delta Q$ according to

$$K_Q(T_0) = \frac{\delta Q}{C} \quad (16)$$

as is prescribed by Equation (13).

In Figure 7, the kernels are plotted as function of the temperature for the different relevant outputs of the system.
Comparing our results with the ones presented in Tripathy & Christensen-Dalsgaard (1998) and Villante (2010), we find a very good agreement with the present calculations. The only noticeable difference with Tripathy & Christensen-Dalsgaard (1998) is found for the kernel of the hydrogen abundance. Note, however, that our SSMs are calculated using diffusion while it was not taken into account in Tripathy & Christensen-Dalsgaard (1998). In Figure 8, we present the relative changes for the hydrogen abundance profile when the opacity is perturbed at $\log_{10} T_0 = 7$ with and without diffusion included.

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\[ \text{Figure 8. Relative changes for the hydrogen abundance profile when an opacity perturbation at } \log_{10} T_0 = 7 \text{ (corresponding to } r = 0.18 R_\odot \text{) is introduced. The solid red line corresponds to an SSM model with diffusion and the solid blue line corresponds to an SSM without diffusion.} \]
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