Parametric Oscillators with gate controlled capacitor-within-capacitor

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Abstract: A Capacitor-within-Capacitor (CWC) is a nested structure that has two components: the cell (e.g., the outer capacitor) and the gate (e.g., the inner capacitor). These designations may be interchanged. We take advantage of a change in capacitance that can reach as much as 50%. Here we demonstrate an efficient parametric oscillator with a CWC interfaced with a diode. An intensity signal-to-noise ratio (SNR) of 40 dB is observed when increasing the input pump power by ca 1 dB from just below to just above threshold. A model and experiments indicate a very large gain that is associated with a frequency shift.

I. Introduction:

In equivalent circuit terms, capacitors may be connected in series (the overall capacitance becomes smaller than that of either capacitor), or in parallel (the overall capacitance is the sum of individual capacitances) [1]. Capacitors may take a simple form, such as two parallel plates, or a more complex structure of interdigitated electrodes [2]. Recently, a third possibility, a capacitor-within-capacitor (CWC), was considered [3]. Specifically, one capacitor, the gate capacitor, electronically controls the capacitance of a cell capacitor. The gate may be nested inside the cell (inner gate structure) or outside it (outer gate structure). Such a concept is general and may be applied to dielectric and super-capacitors alike. This form of coupling is particularly attractive when the gate (or even the cell) electrodes are made of 2-D films, such as graphene; voltage controlled charge doping of either electrode may be achieved in a rather simple manner [4]. Here, we limit ourselves to copper electrodes.

Electrical parametric oscillators have been known for a long time [5]. In an electrical parametric oscillator, a resonating circuit is interfaced with a nonlinear capacitive, or nonlinear inductive [6] element. Modulation of the nonlinear element by a pump source at frequency \( \omega_p \) and above some intensity threshold results in the generation of two frequency components: the signal at \( \omega_s \) and an idler at \( \omega_i \). Conservation of energy dictates that, \( \omega_p = \omega_s + \omega_i \). When the signal and the pump are in phase (or, shifted by \( \pi \) radians), then \( \omega_s = \omega_i \) and the signal oscillates at half-frequency of the pump. Typically, a varactor, or a similar nonlinear capacitive element is used to realize the electrical circuit. Here, we take a somewhat different approach to realize a nonlinear capacitor (Fig. 1).

Consider an inner gate structure with an outer capacitor serving as the cell (Fig. 1a). A diode, placed across the gate capacitor [Ref. 3 and its SI section] provides for a voltage controlled element. From a circuit point of view, the effect of the gate diode on the cell's capacitance is understood as follows: if the diode is reversed biased (namely, the gate is open), then the structure is made of three capacitors connected in series. The capacitance in this configuration is the smallest. If the diode is forward biased, then it shorts the gate and the cell's is made of only two capacitors in series and the cell's capacitance increases. Alternatively, from an artificial dielectric point of view, one may consider the shortened gate as a giant electrical dipole whose increased polarity affects the cell's capacitance via an increase in the effective cell's permittivity. Similar arguments may be made for nonlinear artificial magnetic dipole, formed by the current loop that connects the pump source with the inner gate electrodes, the diode and the inductor and whose effect is to decrease the cell's permittivity value.
II. Experiment:

The CWC is made of 4 copper strips (plates), making an area of 12 mm x 12 mm when put across one another (Fig. 1a). They are separated by dielectric films (pieces of cut paper). The structure is held between two glass slides 25 mm x 25 mm by two clips. The inner gate, which controls the outer cell is operating at resonance with a quality factor of near unity. The capacitance is ca 0.014 nF and ca 0.04 nF between the outer plates and between the inner plates, respectively. The sinusoidal pump frequency is aimed at ca 1.06 MHz; the signal of this degenerate configuration is observed at ca 0.53 MHz (Fig. 2b). The 1N91 1DC723 Ge power diode that connects the gate electrodes has a large reverse breaking voltage rating (VR>75 V) and a negative break-in voltage (Fig. 1b), though with different DC bias. The entire circuit and layout are shown in Fig. 1c-d. A WaveTek frequency generator, an HP spectrum analyzer (SA) and HP oscilloscope are used to assess the input amplitude, the DC offset and the output spectrum.

![Fig. 1. (a) Schematics of the CWC. Four stripes of copper are placed one on top of each other and are separated by dielectric films. (b) I-V curve of Ge and Si diodes used in the experiments. A third Ge diode, whose I-V is similar to the Si diode, also enabled parametric oscillations. (c) A picture of the circuit. (d) The inner gate is excited by a Vin~1 MHz sinusoidal pump source connected to the gate plates, the diode and the inductor. The output voltage, Vout, is assessed with a spectrum analyzer.](image-url)
III. Results and Discussions:

III.a. Experimental Results

As with other parametric oscillators, oscillation occur beyond some pump threshold. In Fig. 2a we show the output signals just below the pump threshold; the pump amplitude is 1.8 V_p-p. The transmitted output is composed of only the pump frequency at 1.06 MHz. By increasing the pump amplitude to V_p-p=2 V (Fig. 2b), two additional peaks appear: at 0.53 MHz and at 1.59 MHz, respectively. The first peak is the parametric oscillation at half the pump frequency; the second is at the pump frequency and the third is for sum frequencies of the pump and the signal, \( \omega_{3p/2} = \omega_p + \omega_s \). The input pump peak intensity was directly measured on the gate (including the inductor and the electrodes' capacitance), as -8.8 dBm for a 2 Vp-p amplitude. At threshold, the transmitted signal intensity at either \( \omega_s \) or \( \omega_{3p/2} \) is ca -40 dB of the direct input pump intensity, or -20 dB of the transmitted pump intensity. An intensity signal-to-noise ratio (SNR) of 40 dB is observed when increasing the input pump power by ca 1 dB from just below \( V_{\text{pump}}=1.8 \text{ V}_p\text{-p} \) to just above threshold \( V_{\text{pump}}=2 \text{ V}_p\text{-p} \). Note a decrease of 2.5 dB in the transmitted peak pump power due to generation of the sidebands (Fig. 1b in comparison to Fig. 1a). Fig. 2c shows that the oscillations are suppressed when reversing the diode connection while retaining the same pump conditions. This means that the polarity of the gate capacitor with respect to the polarity of the cell's capacitor, matters. The oscillations may be recovered with a proper DC bias.
Fig. 2. (a) Just below threshold. The marker is at 1.06 MHz. (b) Above threshold. Seen is the pump at 1.06 MHz, the signal at 0.53 MHz and the sum of the signal and the pump frequencies at 1.59 MHz. Also note that the transmitted pump power was reduced by 2.5 dB due to the generation of sidebands. (c) The diode connection is reversed while retaining the same conditions as in (b). The oscillation may be recovered by applying a proper DC bias.

Parametric oscillations rely on the nonlinearity of the diode resistance. At large DC bias, the diode exhibits a quasi linear I-V curve and its resistance is almost constant (Fig. 1b). Likewise, when reversed biased, the diode exhibits a constant (and a very large) resistance. These two regions are not appropriate for parametric oscillations. The diode’s largest nonlinearity is near the break-in region. For a sinusoidal pump amplitude of <2 V_{p-p}, the DC bias range for the Ge diode shown in Fig. 1b is between ±0.1 V and is optimal for 0 V.

Parametric oscillations may be observed for the above conditions when the pump frequency is scanned between f=0.45 to f=1.25 MHz. Within that bandwidth one may observe a rich spectra of sub-harmonics high pump-frequency’s harmonics, as well as combinations between them all (Fig. 3c). Based on the above bandwidth, the quality factor from the pump point of view, \( Q=\omega_p/\Delta\omega=2\pi f_p/2\pi \Delta f \) is assessed as of order 1 (with \( \omega_p \) chosen at the bandwidth center). The quality factor is also related to the resistance, capacitance and inductance as, \( Q=R\sqrt{C/L} \). From the gate point of view with gate capacitance of ~0.04 nF and inductance of 0.22 mH we assess the resistance as, \( R_{\text{eff}}\sim2.3 \) KOhms. The impedance of the Ge diode in reverse DC bias is \( R_r\sim1.5 \) KOhms.
Fig. 3. (a,b) The DC offset is ±0.1 V and the oscillations are suppressed. (c) Full spectra with harmonics, sub-harmonics and related combinations when the pump frequency is at 0.475 MHz. Note that the pump power is smaller at this lower end of the bandwidth.

What happens if we increase the pump amplitude by 10 times its previous value, say to 20 V\textsubscript{p-p} or, 20 dB larger than the power threshold? Indeed, the transmitted pump intensity increases by 20 dB (from -30 dBm to -10 dBm); yet, the transmitted signal increases by 30 dB (from -50 dBm to -20 dBm (Fig. 4). The SNR of the signal is now 60 dB and its peak intensity is only 10 dB below the transmitted peak pump power.

Fig. 4. Higher pump frequencies and broader signals are associated with a larger pump intensities. The SNR of the signal is 60 dB.

Fig. 5 shows a pump peak shift as the pump intensity increases. Clearly, the pump frequency is up-shifting until the point of oscillation. Similarly, upon changing the value of the inductor from L=0.22 mH to L=0.56 mH one may observe a pump frequency down shift (not shown).
Fig. 5. (a-c) As the pump intensity increases throughout the 0-20 V_{pp}, the pump frequency up-shifted.

Fig. 6 shows what happens if the gate is nested within another resonator by adding another inductor (see Fig. 7 below). In this case, the Q factor of the circuit increases and the pump frequency shift is very small. The transmitted signal exceeds the value of the transmitted pump (Fig. 6b).

Fig. 6. Parametric oscillations at pump amplitude of 20 V_{pp}. (a) With reversed Si diode (see Fig. 1b) and changing the gate inductor to L1=0.56 mH. (b) The same as (a) but with additional inductor, L2=1 mH in series with the spectrum analyzer (see Fig. 7b below for circuit model).
Finally, if one connects the first and third (negative) electrodes together for a shared ground keeping all other conditions the same, the resonance frequency shifts due to the 50 Ohms resistance of both the functional generator and the spectrum analyzer. A useful frequency range was experimentally found at lower frequencies (fp~600 KHz and fs~300 KHz).

III.b. Theoretical Considerations and Modeling

Besides nonlinearities near the break-in region, a typical diode circuit exhibits nonlinearity when pumped with either large negative amplitudes (the Zener effect), or large positive amplitudes (saturation effect). The effect at large positive pump amplitudes happens when a small effective resistor, R, is connected in series with a diode. This resistor could be the result of contacts or wires. The current-voltage equation for the circuit current becomes, \( I_d = I_0 \exp \left( \frac{(V-I \cdot R)}{V_q} \right) - 1 \), where \( I_0 \) is the dark current, V is the input voltage and \( V_q = 26 \text{ mV at room temperature} \). For a large input voltage and small resistances, the term \( I \cdot R \) competes with the input V and the current becomes saturated. Thus, the transition from a quasi linear \( I-V \) curve to saturated curve is also a useful parametric oscillation region. When the pump amplitude is increased to \( \sim 20 \text{ V}_{\text{p-p}} \) one can operate at higher frequencies, say at 1.75 MHz, yet, the useful pump bandwidth decreases to 300 KHz. Effectively, this increases the Q factor to \( \sim 6 \). At this point, efficient parametric oscillation is rather sensitive to the pump's DC offset. At pump frequency of \( \sim 1 \text{ MHz} \) and at large pump amplitudes of \( \sim 20 \text{ V}_{\text{p-p}} \), one may observe half-, quarter- and sometimes eighth- of the pump-frequency. One could argue that with large pump amplitudes that covers both the break-in and the saturation regions, there will be an increase in width of the various frequency components (including that of the pump) due to all combinations of shifted signal frequencies as described below. These frequency combinations are attributed to the large nonlinearity of the circuit.

Let us look closely at the nonlinear capacitive response. An equivalent circuit model is shown in Fig. 7. One may assume that the capacitance between nearest pair of plates is the same because the spacing and the dielectric materials are similar. As described before, the maximum nonlinear swing may reach 50%, between cell capacitance values of \( C/3 \) to \( C/2 \) when the gate capacitor is either open or short. By connecting the cell (\( V_{\text{out}} \)) to another inductor one may fabricate a resonator within another resonator (Fig. 7b).
Fig. 7. (a) Equivalent circuit of Fig. 1. C – capacitor; D – diode; L – inductor. C=0.04 nF and L=0.22 mH. (b) A resonator nested within another resonator. L₁=0.561 mH and L₂=1 mH.

We start with the nonlinear equation for a pumped capacitor [7]. We assume that the capacitance of the cell (namely, the outer set of electrodes) depends linearly on the shunt resistance across the gate within some resistance region [3]: \( C(R_{\text{shunt}}) = C_0(1-R_{\text{shunt}}/R_0) \). Here \( C_0 \) is the cell capacitance without the shunt resistance and \( R_0 \) is a constant. From the diode equation for small currents (namely, ignoring the saturation region of the circuit), we can write for the effective diode resistance as a function of \( V_p \equiv V_{\text{pump}} \),

\[
R_{\text{eff}} = (dI/dV_p)^{-1} = (V_q/I_0)\{\exp(-V_p/V_q)\}.
\]

Combining, \( C(V_p) \sim C_0(1-(V_q/R_0I_0)\exp(-V_p/V_q)) \). Finally, \( \Delta C/C_0 \sim (V_q/R_0I_0)\exp[-(V_p/V_q)\sin(\omega_pt)] \). As the voltage increases, the diode resistance decreases and the cell’s capacitance increases.

The signal of this circuit is \( \omega_s^2 = (1/LC) \). We write, \( C = C_0 + \Delta C = C_0(1+\Delta C/C_0) \) and \( L = L_0 - \Delta L = L_0(1 - \Delta L/L_0) \) with \( \Delta C/C_0 << 1 \) and \( \Delta L/L_0 << 1 \). Here \( L_0 \) and \( C_0 \) are associated with the resonance frequency of the gate. The nonlinear modulation of the gate affects the overall cell’s capacitance and hence its frequency output. Note the choice of a plus sign to the capacitance change; the capacitance increases upon a positive pump swing.

The effect of \( L \) on the effective permittivity of the cell is expected to be smaller than the capacitive effect since magnetic dipole are smaller than electric ones. In addition, the outer electrodes are well separated and form only a partial current loop. We will ignore it for the sake of simplicity and write for the parametrization of the frequency through first order expansion,

\[
\omega_s^2 = (1/LC_0)(1-\Delta C/C_0)(1+\Delta L/L_0) \rightarrow (1/LC_0)(1-(V_q/R_0I_0)\exp[-(V_p/V_q)\sin(\omega_pt)]).
\]

This is to be compared to a more traditional parametric oscillator,

\[
\omega_s^2 \sim (1/L_0C_0)(1 - bV_{\text{p0}}\sin(\omega_pt)),
\]
where, \( b \ll 1 \) is a constant. Eq. (2) collapses to Eq. (3) for small signals. Pumping at twice the
signal frequency, \( \omega_p \sim 2 \omega_{0s} = 2 \sqrt{(1/L_0 C_0)} \sim 2 \omega_s \) leads to a substantial signal gain and eventually to
oscillations at the signal frequency.

In Fig. 6, a second order differential equation for harmonic oscillator was numerically solved with
the parametrized radial frequency of either Eq. 2 or Eq. 3 by using a Mathcad tool. Specifically,

\[
\frac{d^2 y(t)}{dt^2} = -\omega_s^2 y(t) - \kappa \frac{dy(t)}{dt}.
\]  

(4)

Here, the radial frequency is related to the frequency as, \( \omega_s = 2 \pi f_s \) and \( \kappa \) is the loss coefficient. In
this semi ideal case, the loss coefficient in the simulations was small but not zero. Larger loss
coefficients decrease the intensity of the frequency component. The various coefficients for the
simulations were chosen such that Eq. 2 yields the same coefficients as Eq. 3 when expanding
the exponent to first order of approximation. Specifically, when using Eq. 2: \( (V_q/R_{0l_0}) = 0.01 \),
(\( V_{p0}/V_q \))=2, and when using Eq. 3: \( b V_{p0} = 0.02 \). In the case of diode interfaced circuit, higher-order
contributions give rise to a large gain in the signal. Finally, a fast Fourier transform (fft) module
was used to assess the absolute value of the frequency components.

Boundary conditions for the simulations are: \( y(0)=0 \) and \( (dy(t)/dt)_{t=0} = 2 \pi f_0 = 2 \pi 0.55 \). The normalized
pump frequency varies, but it is approximately \( f_p \sim 1.1 \). With Eq. 2, the maximum gain is obtained
with somewhat lower or higher pump frequency than the pump frequency used with Eq. 3.
Specifically, in the case of a parametric oscillator that is driven according to Eq. 3, the maximum
gain to the Fourier component is found with \( f_p = 1.1 \), for which \( f_s = 0.55 \). In the case of a diode
interfaced circuit, the maximum gain to the Fourier component is found with \( f_p = 1.085 \) for which
\( f_s = 0.542 \) (Fig. 8). The experimental peak frequency also shifts as shown in Fig. 5.

![Abs Amplitude vs Normalized Frequency](image)

Fig. 8. Models: the absolute value of the Fourier component vs the normalized frequency, \( f \). (a) When using Eq. 2: \( (V_q/R_{0l_0}) = 0.01 \) and \( (V_{p0}/V_q) = 2 \). (b) When using Eq. 3: \( b V_{p0} = 0.02 \).

In the simulations, a bandwidth of 0.4% of the central pump frequency, or Q~250 is noted. The
effective parameters, chosen for the comparison between Eqs. 2 and 3 favor a relatively narrow
band; these parameters are translated to a large effective circuit resistance, \( R_0 \).

Instead of looking at the leading Fourier component it might be instructive to watch the time
evolution of the signal. A typical parametric oscillation, which is driven along with Eq. 3 would
exhibit a monotonous, exponentially growing amplitude (Fig. 9b). In the case of a diode interfaced circuit, higher orders affect the kinetics of the amplitude. The amplitude of the signal first decreases and then increases (Fig. 9a). Eventually, and as observed from Fig. 6a its signal grows much faster to provide a larger gain. Simulations also indicate that the amplitude minimum is shifting towards earlier times as the parameter V_q/R_0I_0 increases while keeping V_p0/V_q constant. Similar trend is exhibited if we keep V_q/R_0I_0 constant while increasing V_p0/V_q; for example by increasing the pump amplitude, or by decreasing the temperature (and, thus decreasing V_q=k_BT/q, with k_B T - the thermal energy and q - the charge).

![Normalized Amplitude vs Normalized Time](a)

![Normalized Amplitude vs Normalized Time](b)

Fig. 9. Normalized amplitudes of the signal as a function of normalized time for: (a) parametric frequency of Eq. 2 and with parametric frequency of Eq. 3 (b). In the harmonic oscillator equation (Eq. 4), the Normalized Amplitude, y multiplies the square of normalized radial frequency of either Eq. 2, or Eq. 3. The normalized time is in units of 1/2\pi f_p.

If the capacitance decreases upon a positive pump swing, we may choose a negative sign for the capacitance change. This is the case near the saturation region or when the diode direction is reversed. Hence, C=C_0-\Delta C=C_0(1-\Delta C/C_0) and Eq. 2 now reads,

$$\omega_s^2=\frac{1}{LC_0}(1+(V_q/R_0I_0)\exp[-(V_p0/V_q)\sin(\omega_p t)]).$$

For the simulations, parameters were selected as before: (V_q/R_0I_0)=0.01 and (V_p0/V_q)=2. Maximum gain is achieved with a slightly larger frequency than resonance, f_p=1.115, and the signal is exhibited at f_s=0.558. The Fourier component is now 5 time larger than the one depicted in Fig. 8a and the minimum in the temporal growth has shifted to t=0 (Fig. 10a).
IV. Conclusions:

A simple and efficient parametric oscillator was built out of a nested structure, a capacitor-within-capacitor, a diode and an inductor. Higher orders and multiples of frequencies’ sum and difference are exhibited due to large nonlinearities [8]. These could be, in principle, suppressed by building a resonator-within resonator circuit. Such combination could be useful for graphene electrodes interfaced with a diode-like or transistor-like structures where a voltage bias shifts the Fermi level and hence the electrode’s doping.

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1. L.D. Landau & E.M. Lifshitz, Electrodynamics of Continuous Media, (Volume 8 of A Course of Theoretical Physics), Pergamon Press 1960
2. Lei Zhu and Ke Wu, Accurate Circuit Model of Interdigital Capacitor and Its Application to Design of New Quasi-Lumped Miniaturized Filters with Suppression of Harmonic Resonance, IEEE Trans. Microwave Theory and Techniques, 48 (2000) 347–356.
3. Grebel, H., Capacitor within capacitor, SN Appl. Sci., 1 (2019) 48. doi.org/10.1007/s42452-018-0058-z
4. Inanc Meric, Melinda Y. Han, Andrea F. Young, Barbaros Ozyilmaz, Philip Kim, and Kenneth L. Shepard, Current saturation in zero-bandgap, top gated graphene field-effect transistors, Nature Nanotechnology, 3 (2008) 654-659.
5. Roer, T.G., Microwave Electronic Devices. Springer Science and Business Media, (2012) ISBN 978-1461525004.
6. K. D. Irwin and M. E. Huber, SQUID Operational Amplifier, Applied Superconductivity 2000, Sept. 17-22, 2000, Virginia Beach, Virginia.
7. A. Yariv, Quantum Electronics, 3rd edition, John Wiley and Sons, 1989.
8. Samuel Boutin, David M. Toyli, Aditya V. Venkatramani, Andrew W. Eddins, Irfan Siddiqi, and Alexandre Blais, “Effect of Higher-Order Nonlinearities on Amplification and Squeezing in Josephson Parametric Amplifiers”, Phys. Rev. Appl. 8, (2017) 054030.