Probing Matter-Field and Atom-Number Correlations in Optical Lattices by Global Nondestructive Addressing

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We show that light scattering from an ultracold gas reveals not only density correlations, but also matter-field interference at its shortest possible distance in an optical lattice, which defines key properties such as tunneling and matter-field phase gradients. This signal can be enhanced by concentrating probe light between lattice sites rather than at density maxima. As addressing between two single sites is challenging, we focus on global nondestructive scattering, allowing probing order parameters, matter-field quadratures and their squeezing. The scattering angular distribution peaks even if classical diffraction is forbidden and we derive generalized Bragg conditions. Light scattering distinguishes all phases in the Mott insulator - superfluid - Bose glass phase transition.

The modern field of ultracold gases is successful due to its interdisciplinarity [11,2]. Originally condensed matter effects are now mimicked in controlled atomic systems finding applications in areas such as quantum information processing (QIP). A really new challenge is to identify novel phenomena which were unreasonable to consider in condensed matter, but will become feasible in new systems. One such direction is merging quantum optics and many-body physics [3,4]. The former describes delicate effects such as quantum measurement and many-body physics [3,4]. The former describes delicate effects such as quantum measurement and many-body correlations (e.g. atomic ensembles). In the latter, decoherence destroys these effects in conventional condensed matter. Due to recent experimental progress, e.g. Bose-Einstein condensates (BEC) in cavities [5,7], quantum optics of quantum gases can close this gap.

The atomic operator is ˆ\( a_i \) and chemical potential \( \mu \). The Hamiltonian is

\[
\hat{H} = \hat{H}_{BH} + \sum_{l} \hbar \omega_l a_l^\dagger a_l + \hbar \sum_{l,m} U_{lm} a_l^\dagger a_m \hat{F}_{lm},
\]

\[
\hat{H}_{BH} = -J^d \sum_{(i,j)}^M b_i^\dagger b_j + \frac{U}{2} \sum_{i}^M \hat{n}_i (\hat{n}_i - 1) - \mu \sum_{i}^M \hat{n}_i,
\]

(where \((i,j)\) gives summation over nearest sites, \(a_l (l = 0,1)\) are the annihilation operators of the light modes with frequency \(\omega_l\), atom-light coupling constant \(g_l (U_{lm} = g_l g/m_\lambda)\), light-atom detuning \(\Delta_\lambda = \omega_l - \omega_a\). \(b_i (\hat{n}_i)\) is the atom annihilation (number) operator with hopping amplitude \(J^d\), interaction strength \(U\), and chemical potential \(\mu\).

The atomic operator is \(\hat{F}_{lm} = \hat{D}_{lm} + \hat{B}_{lm}\). \n
\[
\hat{D}_{lm} = \sum_{i=1}^K J_{i,i}^{lm} \hat{n}_i, \quad \hat{B}_{lm} = \sum_{(i,j)}^K J_{i,j}^{lm} b_i^\dagger b_j,
\]

(1)

FIG. 1: Setup. Atoms in OL are illuminated by a probe beam; scattered light is measured by a detector.
comes from overlaps of light mode functions \(u_l(\mathbf{r})\) and density operator \(\hat{n}_i(\mathbf{r}) = \hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}(\mathbf{r})\), after
the matter-field operator is expressed via Wannier functions: \(\hat{\Psi}(\mathbf{r}) = \sum_i b_i w(\mathbf{r} - \mathbf{r}_i)\). \(\hat{D}_m\) sums
the density contributions \(\hat{n}_i\), while \(\hat{B}_{lm}\) sums the matter-field interference terms. \(\hat{J}_{lm} = \int w(\mathbf{r} - \mathbf{r}_i)u_l^\dagger(\mathbf{r})u_m(\mathbf{r})w(\mathbf{r} - \mathbf{r}_j)\,d\mathbf{r}\) are the convolutions of Wannier and light mode functions. The light-atom
coupling via operators assures the dependence of light on the atomic quantum state.

When the probe has a negligible effect on atomic dynamics, the stationary light amplitude \(a_1\) is given by \(\hat{F}_{10}\) (we drop the subscripts in \(\hat{F}\), \(\hat{D}\), and \(\hat{B}\)). In a cavity with the decay rate \(\kappa\) and probe-cavity detuning \(\Delta_p\), \(a_1 = C\hat{F}\), where
\(C = iU_{10}a_0/(i\Delta_p - \kappa)\). In free space, the electric field operator in the far-field point \(\mathbf{r}\) is \(\hat{E}_1 = C_E\hat{F}\) with \(C_E = \omega_0^2d^2E_0/(8\pi^2\epsilon_0c^2\Delta_o\mathbf{r})\) (\(d\) is the dipole moment, \(E_0\) is probe electric field) [21].

The light quadrature operators \(\hat{X}_\phi = (a_1 e^{-i\phi} + a_1^\dagger e^{i\phi})/2 = |C|\hat{X}_\phi^F = |C|\hat{F} e^{-i\beta} + \hat{F}^\dagger e^{i\beta}/2\) are expressed via \(\hat{F}\) quadratures \(\hat{X}_\phi^F\) (\(\beta = \phi - \phi_C\), \(C = |C|\exp(i\phi_C)\)), where \(\phi\) is the local oscillator phase. The means of amplitude and quadrature, \(\langle a_1 \rangle\) and \(\langle \hat{X}_\phi \rangle\), only depend on atomic mean values. In contrast, the mean intensity \(\langle a_1^\dagger a_1 \rangle = |C|^2\langle \hat{F}^\dagger \hat{F} \rangle\) and quadrature variance \((\Delta X_\phi)^2 = \langle \hat{X}_\phi^2 \rangle - \langle \hat{X}_\phi \rangle^2 = 1/4 + |C|^2(\Delta X_\phi^F)^2\) reflect atomic correlations and fluctuations, which is our main focus. For intensity, the role of \((\Delta X_\phi^F)^2\) is played by \(R = \langle \hat{D}^* \hat{D} \rangle - \langle \hat{D} \rangle^2\), which is the "quantum addition" to light due atom quantum fluctuations (classical diffraction signal is subtracted).

![FIG. 2: Light intensity scattered from a SF in a 3D lattice (units of \(R/N_K\)). Arrows denote incoming probes. The Bragg condition is not fulfilled, so classically there is no diffraction, but intensity still shows peaks, whose heights are tunable: (a) \(\varphi_1 = 0\); (b) \(\varphi_1 = \pi/2\).](image)

Typically, the dominant term in \(\hat{F}\) is the density-term \(\hat{D}\), rather than inter-site matter-field interference \(\hat{B}\) [22, 24], because the Wannier functions' overlap is small. Our aim is to enhance the \(\hat{B}\)-term in light scattering by suppressing the density signal. To clarify typical light scattering, we start with a simpler case when scattering is faster than tunneling and \(\hat{F} = \hat{D}\). This corresponds to a QND scheme [8,11]. The density-related measurement destroys some matter-phase coherence in the conjugate variable [25, 27] \(b_i^\dagger b_{i+1}\), but this term is neglected.

In a deep lattice, \(\hat{D} = \sum_{l}^K u_l(\mathbf{r}_i)u_{0}(\mathbf{r}_i)\hat{n}_i\), which for traveling \([u_l(\mathbf{r}) = \exp(\mathbf{i}k\mathbf{r} + \varphi_l)]\) or standing \([u_l(\mathbf{r}) = \cos(\mathbf{k}\cdot\mathbf{r} + \varphi_l)]\) waves is just a density Fourier transform at one or several wave vectors \(\pm(\mathbf{k}_l \pm \mathbf{k}_0)\). The quadrature for two traveling waves is reduced to \(X_\phi^F = \sum_{l}^K \hat{n}_i\cos(\mathbf{k}_l \cdot \mathbf{r}_i - \beta)\).

Note that different light quadratures are differently coupled to the atom distribution, hence varying local oscillator phase and detection angle, one scans the coupling from maximal to zero. An identical expression exists for \(\hat{D}\) for a standing wave, where \(\beta\) is replaced by \(\varphi_1\), and scanning is achieved by varying the position of the wave with respect to atoms. Thus, variance \((\Delta X_\phi^F)^2\) and quantum addition \(R\), have non-trivial angular dependence, showing more peaks than classical diffraction and the peaks can be tuned by the light-atom coupling.

Fig. 2 shows the angular dependence of \(R\) for standing and traveling waves in a 3D OL. The isotropic background gives the density fluctuations \(R = K(\langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2)/2\) in MF with inter-sites correlations neglected. The radius of the sphere changes from zero (MI with suppressed fluctuations) to half the atom number at \(K\) sites, \(N_K/2\), in the deep SF. There exist peaks at angles different than the classical Bragg ones and thus, can be observed without being masked by classical diffraction. Interestingly, even if 3D diffraction is forbidden (Fig. 19), the peaks are still present. As \((\Delta X_\phi^F)^2\) and \(R\) are squared variables, the generalized Bragg conditions for the peaks are \(2\mathbf{G} = \mathbf{G}\) (\(\Delta \mathbf{k} = \mathbf{k}_0 - \mathbf{k}_1\) and \(\mathbf{G}\) is the reciprocal lattice vector) for quadratures of traveling waves, and \(2\mathbf{k}_1\) for standing wave \(a_1\) and traveling \(a_0\), which is clearly different from the classical Bragg condition \(\Delta \mathbf{G}\) = \(\mathbf{G}\). The peak height is tunable by the local oscillator phase or standing wave shift, see Fig. 2b.

We estimate the mean photon number per second integrated over the solid angle \(n_\phi = (\Omega_0/\Delta_\phi)^2\Gamma K(\langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2)/8\) (\(\Omega_0 = dE_0/h\) and \(\Gamma\) is the atomic relaxation rate) for the only two experiments so far on light diffraction from truly ultracold bosons where the measurement object was light. The background signal should reach \(n_\phi \approx 10^6 \text{s}^{-1}\) in Ref. [18] (150 atoms in 2D), and \(n_\phi \approx 10^{11} \text{s}^{-1}\) in Ref. [19] (10^5 atoms in 3D).

Now we focus on enhancing the interference term \(\hat{B}\) in the operator \(\hat{F}\). \(\hat{D}\) [1] is given by the convolution with on-site functions \(W_0(\mathbf{r}) = w^2(\mathbf{r})\), while \(\hat{B}\) is with that of overlap \(W_1(\mathbf{r}) = w(\mathbf{r} - d/2)w(\mathbf{r} + d/2)\). (Fig. 3 \(d\) is the lattice period). Therefore, we need to concentrate light between the sites rather than at atom positions. Ideally, one could measure between two sites similar to single-site addressing [28, 29], which would measure a single term \((b_i^\dagger b_{i+1} + b_i^\dagger b_{i+1})\), e.g., by ramping up an ad-
amplitude-squeezed state, Fig. 4(a,b). We prove that all states can be probed in-situ by light. The sub-Poissonian density fluctuations can be probed as well (Fig. 2).

For two standing waves

$$\dot{D} = \frac{1}{2} [\mathcal{F}[W_0](k_-) \sum_m \hat{n}_m \cos(k_x m + \varphi_-) + \mathcal{F}[W_0](k_+) \sum_m \hat{n}_m \cos(k_x m + \varphi_+)],$$

$$\dot{\hat{B}} = \frac{1}{2} [\mathcal{F}[W_1](k_-) \sum_m \hat{\hat{B}}_m \cos(k_x m + \frac{k_d}{2} - \varphi_-) + \mathcal{F}[W_1](k_+) \sum_m \hat{\hat{B}}_m \cos(k_x m + \frac{k_d}{2} + \varphi_+)],$$

where $\hat{\hat{B}}_m = \hat{b}_m^\dagger \hat{b}_{m+1} + b_m \hat{b}_{m+1}^\dagger$, $\varphi_\pm = \varphi_0 \pm \varphi_1$; $\hat{D}$ and $\hat{\hat{B}}$ have different phase shifts, allowing to decouple them (although at specific angles only).

In the diffraction maximum (e.g. modes cross at opposite angles $k_{0x} = k_{1x} = \pi/d$, $D = 0$ (totally suppressed) and $\hat{B} = \mathcal{F}[W_1](2\pi/d) \sum_m \hat{\hat{B}}_m/2$ (reaches its maximum) for a nontrivial choice $\varphi_0 = -\varphi_1 = \xi = \arccos[-\mathcal{F}[W_0](2\pi/d)/\mathcal{F}[W_0](0)]/2$ (Fig. 3). Hence, measuring the light quadrature we probe the kinetic energy and, in MF, matter-field amplitude (order parameter) $\Phi$: $\langle X_{\beta=0}^F \rangle = |\Phi|^2 \mathcal{F}[W_1](2\pi/d)(K-1)$. Note that the density signal is not suppressed when the atoms are in the standing wave nodes, but when the waves interfere to compensate for a spread of Wannier functions.

$\hat{B}$-variance is probed in a diffraction minimum ($k_{0x} = 0$, and $k_{1x} = \pi/d$). Varying $\varphi_{0,1}$ one chooses again, what to probe, $\hat{D}$ or $\hat{\hat{B}}$: $\dot{\hat{D}} = \mathcal{F}[W_0](\pi/d) \sum_m (-1)^m \hat{n}_m \cos \varphi_0 \cos \varphi_1$, $\dot{\hat{\hat{B}}} = -\mathcal{F}[W_1](\pi/d) \sum_m (-1)^m \hat{\hat{B}}_m \cos \varphi_0 \sin \varphi_1$ (Fig. 3d). For $\varphi_1 = \pm \pi/2$, $D = 0$, which is intuitive as that is the node, but the light amplitude is also zero, $\langle \hat{B} \rangle = 0$. However, the intensity $\langle a_1^\dagger a_1 \rangle = |C|^2 \langle B^2 \rangle \neq 0$. Assuming $\Phi$ is real in MF:

$$\langle a_1^\dagger a_1 \rangle = 2|C|^2(K-1) \mathcal{F}[W_1](\pi/d) \times [\langle \hat{b}^2 \rangle - \Phi^2] + (n - \Phi^2)(1 + n - \Phi^2).$$

Thus, the intensity measures $\langle \hat{b}^2 \rangle - \Phi^2$, which defines the quadrature variances of matter-filed ($\Delta X_{0,\pi/2}^2 = 1/4 + 1/2(n - \Phi^2) \pm (\langle \hat{b}^2 \rangle - \Phi^2)$).

In the diffraction minimum, one can use just two traveling waves and measure the light quadrature variance. Similarly, $X_{\beta}^F = D \cos \beta + \hat{B} \sin \beta$, and by varying the local oscillator phase, one chooses which conjugate operator to measure. For $\beta = \pi/2$, ($\Delta X_{\pi/2}^F$) looks identical to Eq. (3).

Access to $\hat{B}^2$ probes kinetic energy fluctuations with 4-point correlations (in pairs). Measuring the photon number variance, which is standard in quantum optics, will lead up to 8-point correlations similar to 4-point density correlations [9].
Surprisingly, inter-site terms scatter more light from a MI than a SF [3]. Fig. 4, although the mean inter-site density ($\langle n(r) \rangle$) is tiny in a MI. This reflects a fundamental effect of the boson interference in Fock states. It indeed happens between two sites, but as the phase is uncertain, it results in the large $\langle n^2(r) \rangle$-variance captured by light $[3]$. Our method enables observation of this, in microscopic scale in-situ, in contrast to macroscopic BECs [31].

\[
\begin{align*}
\text{FIG. 5: (a) Angular dependence of scattered light } R & \text{ for SF (thin purple, left scale, } U/2J^f = 0 \text{) and MI (thick blue, right scale, } U/2J^f = 10 \text{). Light scattering maximum } R_{\text{max}} \text{ (b, d) and width } W_R \text{ (c, e). Varying chemical potential } \mu \text{ or density } n, \text{ the SF-MI transition beyond MF is well visible in both quantities. (b) and (c) are cross-sections of phase diagrams (d) and (e) at } U/2J^f = 2 \text{ (thick blue), } 3 \text{ (thin purple), and } 4 \text{ (dashed). Insets show density dependencies for the } U/(2J^f) = 3 \text{ line. } K = M = N = 25. 
\end{align*}
\]

To probe beyond MF, we consider a 1D lattice, where MF fails [32,33]. Observing the transition in 1D by light at fixed density was considered to be difficult [11] or even impossible [37]. By contrast, here we propose to vary the density or chemical potential, which sharply identifies the transition. We calculate the ground state of the BH model using DMRG [38]. For two traveling waves maximally coupled to the density ($\langle F = D \rangle$), the quantum addition is $R = \sum_{i,j} \langle \exp[i(k_i - k_0)(r_i - r_j)] (\delta n_i \delta n_j) \rangle \delta n_i = \tilde{n}_i - \langle \tilde{n}_i \rangle$, which is the structure factor.

The angular dependence of $R$ for a MI and a SF is shown in Fig. 3, and there are two variables distinguishing the states. Firstly, maximal $R$, $R_{\text{max}} \propto \sum_i (\delta \tilde{n}_i^2)$, probes the fluctuations and compressibility $\kappa'$ ($\langle \delta n_i^2 \rangle \propto \kappa' \langle \tilde{n}_i \rangle$). Secondly, being a Fourier transform, the width $W_R$ of the dip in $R$ is a direct measure of the correlation length $l$, $W_R \propto 1/l$. From the MI to the SF, correlation decay changes from exponential to algebraic, increasing $l$ and decreasing $W_R$ (Fig. 2a). The diffraction peak (subtracted from $R$) has a much smaller width $\propto 1/M$.

Fig. 5 shows the MI-SF phase diagram. Both $R_{\text{max}}$ and $W_R$ display the transition sharply. Scattering indeed shows features beyond MF, as MF prediction for the tip is beyond this range ($U/2J^f \approx 5.8$). Here both variables provide similar information. Next, we present a case where it is very different.

BG is a localized phase with exponentially decaying correlations but large compressibility and on-site fluctuations in a disordered OL. Measuring both $R_{\text{max}}$ and $W_R$ will distinguish all the phases. In BG: large $R_{\text{max}}$ and large $W_R$; in SF: large $R_{\text{max}}$ and small $W_R$; in MI: small $R_{\text{max}}$ and large $W_R$. We confirm this in Fig. 6 for simulations with the ratio of superlattice- to trapping lattice-period $r \approx 0.77$ for various disorder strengths $V$ [39]. Only recently [20] BG was studied by combined measurements of coherence, transport, and excitation spectra. Our method is simpler as it only requires measurement of the quantity $R$.

In summary, we proposed a nondestructive method to probe quantum gases in an OL. Firstly, we showed that the density-term in scattering has an angular distribution richer than classical diffraction, derived generalized Bragg conditions, and estimated parameters for the only two relevant experiments to date [18,19]. Secondly, we proposed how to measure the matter-field interference by concentrating light between the sites. This corresponds to interference at the shortest possible distance in an OL. By contrast, standard destructive time-of-flight measurements deal with far-field interference and a relatively near-field one was used in Ref. [19]. This defines most processes in OLs. E.g. matter-field phase changes may happen not only due to external gradients, but also due to intriguing effects such quantum jumps leading to phase flips at neighboring sites and sudden cancellation of tunneling [10], which should be accessible by our method. In MF, one can measure the matter-field amplitude (order parameter), quadratures and squeezing. This can link atom optics to areas where quantum optics has already made progress, e.g., quantum imaging [16,17], using an OL as an array of multimode nonclassical matter-field sources with a high degree of entanglement for QIP. Thirdly, we demonstrated how the method accesses effects beyond MF and distinguishes all the phases in the MI-SF-BG transition, which is currently a challenge [20].

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