Validation of Elastic-Brittle, and Elastic-Plastic FEM Model of the Wall Made of Calcium Silicate and AAC Masonry Units

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Abstract Numerous methodologies can be used in numerical analyses of masonry structures. In the engineering applications which allow current design standards linear elastic material models are used. On the other hand, when it comes to the analysis of morphology of cracking and the failure mechanism that form the basis of scientific studies, the choice of use falls on the advanced material models implemented in masonry micro-, macro- or mezzo-models taking into account the fragile properties of masonry based on the Coulomb’s ideas. The Drucker-Prager, Hofman, William-Warnke, Barcelona and the Menétray-Willam (M-W-3) models are used, among others. All advanced material models require the introduction of many mechanical parameters, the determination of which may pose many problems. Many authors use their own test methods, so it is not possible to repeat calculations. The paper presents the author's method of validating numerical masonry models using standard research models and procedures. The elastic-brittle material model (SBeta) and the elastic-plastic model of Menétray–Willam (M-W-3). The basic material parameters of masonry units and interface elements were determined in the tests of specimens of masonry units’ elements or wall specimens. The validation was carried out in the ATENA FEM system using a 2D model of the wall axially and diagonally compressed. The cracking of the wall, values of elastic modulus and destructive stresses as well as the course of the load-displacement dependence were compared.

1. Introduction

The numerical analysis for describing the masonry behaviour can be used to trace its behaviour starting from the linear range, through crack formation until achieving its strength. Non-linear models, being a part of the method of finite elements, are mainly used in numerical analyses. Models based on empirical observations differ in assume constitutive law. Thus, macro-models, mezzo-models and micro-models can be used in the numerical analysis [1,2,3,4]. The macro-model treats the masonry structure as material having the identical mechanical parameters. Such a model is applied in practical computations for large masonry structures (finite elements include even more than one masonry unit). The second commonly used method is the mezzo-model - a variant of the macro-model that is similar to the periodic micro-structure of RVE – Representative Volume Element [5]. And the last commonly used model – the micro-model identifies the masonry structure as a heterogeneous model. Classification of finite elements is made for each material (mortar, masonry unit). This article presents the original approach towards the validation of the material model by using a variant of the mezzo-model based on the empirical homogenization. The validation of the model was performed on two
material models: the elastic-brittle model and the elastic plastic model. These studies use test results for masonry structures described in the paper [6].

2. Strategy of FEM modelling
Numerical calculations for shear walls that are the subject of studies conducted by the author [6] were preceded by the two-stage calibration process (empirical homogenization) performed on smaller models. Calibration was required as mortar layers in bed joints being in direct contact with masonry units were at first replaced with material having mechanical parameters of masonry units. The effect of mortar in bed joints and contact areas between elements in head joints, in which cracks were observed during tests, were simulated by using contact elements of specified shear and tensile characteristics. For the purpose of this article, the modelling strategy included:
a) stage 1 - construction of numerical deep beam micro-models (2D) of walls compressed perpendicularly to the plane of bed joints. Mortar in bed joints of models were replaced with interface elements. Two material models used for brittle materials (rock, concrete) were used for masonry units. The material model applied in calculating walls under diagonal compression was chosen on the basis of calculations.
b) stage 2 - construction of numerical deep beam micro-models of diagonally sheared walls, in which masonry units were modelled using the material model from stage 1. By performing numerical calculations, selected material parameters of contact elements were verified.

3. Applied material models

3.1. The Elastic-Brittle model
Principles of the continuum damage-based method were used to describe the development of damage in the discussed model. Effective stress expressed the non-linear behaviour of the material \( \sigma_{c}^{ef} \). Effective stress should be understood as stress, under which deformations of undamaged body are equal to deformation of body damaged by true stress described as equivalent strain \( \varepsilon^{eq} \) calculated from the following relation [7]:

\[
\varepsilon^{eq} = \frac{\sigma_{ci}}{E_{ci}}.
\]

Effective uniaxial strain is regarded as deformation caused by stress \( \sigma_{ci} \) and modulus of elasticity \( E_{ci} \) in the \( i \)-direction, represented by damage and caused by stress related to stress direction \( \sigma_{ci} \) [7]. A diagram of relationships between equivalent uniaxial stress and strain is shown in Fig. 1.

![Figure 1. Equivalent uniaxial stress - strain relationship for elastic model with material degradation [7] (SBeta)](image)

Under tensile state before cracking (range 1 - Fig. 1), material exhibited linear-elastic behaviour. Elasticity modulus \( E_{c} \) is the initial modulus of material elasticity, and effective tensile strength \( f_{c}^{ef} \) was determined on the basis of the criterion of material behaviour under biaxial stress state. The general stress is expressed by the following relationship:
At the post-cracking phase (range 2 - Fig. 1), the material behaviour was described in two ways:

- the model of fictitious cracks resulting from the cracking mechanics and the accepted rule for crack width,
- the model of local deformations of a material point.

In the model of fictitious cracks exponential relationship was applied as shown in Fig. 2.

**Figure 2.** Exponential relationship determining of crack width of SBeta model [8]

Under the compression state (range 3 - Fig. 1), the stress-strain relationships was adopted from CEB-FIP Model Code [9] in the following form:

\[
\sigma_c^{eq} = f_c^{ef} \cdot \frac{kx - x^2}{1 + (k-2)x},
\]

where: \(x = \varepsilon / \varepsilon_c\), \(k = E_o / E_c\), \(\sigma_c^{ef}\) – normal compressive stress in the material, \(f_c^{ef}\) – effective compressive strength of concrete, \(x\) – normalised strains, \(\varepsilon\) – strains, \(\varepsilon_c\) – strains corresponding to effective compressive strength \(f_c^{ef}\), \(k\) – shape coefficient (\(k = 1\) – linear function, \(k = 2\) – parabola), \(E_o\) – initial modulus of elasticity, \(E_c = f_c^{ef} / \varepsilon_c\) – secant modulus of elasticity. The shape of stress-strain relationships used under the compressive state is presented in Fig. 3.

**Figure 3.** Compressive stress - strain relationship

The adopted non-linear relationship stress-strain took into account the material damage developed before maximum stress was reached (as opposed to material with visible cracks). The linear stress-strain relationships (range 4 - Fig. 1) referred to the stadium after achieving maximum compressive stress \(f_c^{ef}\). Displacements after reaching the maximum compressive strength \(f_c^{ef}\) were only observed in the load plane. Displacement was assumed to occur regardless of the specimen size. Such a hypothesis was verified for concrete. The criterion adjusted to results from concrete testing by Kupfer [10] was applied under the biaxial compressive state (Fig. 4). The equivalent compressive strength was expressed with the following equation:

\[
f_c^{ef} = \frac{1 + 3.65a}{(1 + a)^2} f_c^{ef},
\]

where \(a = \sigma_1 / \sigma_2\), \(\sigma_1\) and \(\sigma_2\) are main compressive stresses, and \(f_c^{ef}\) is material strength under uniaxial compressive state.
Figure 4. Failure criterion for material under biaxial stress state [10]

Under biaxial tensile state, tensile was constant and equal to material strength to uniaxial tensile force $f_t$. For elastic material with degradation, the model of smeared cracks developing in uniform direction was applied. The crack was assumed to develop towards main strains $\varepsilon_1$, when $\varepsilon_1 = \varepsilon_t$ corresponding to tensile strength. The direction of crack development did not change during further loading.

3.2. The Menétrey-Willam model

The surface of Menétrey-Willam [11] (M-W-3) is a modified version of the empirical model by Hoek and Brown [12] (developed for rock description) amended by Weihe [13] who introduced the elliptical function of eccentricity $e$ depending on Lode angle $\Theta$. The equations of the three-parameter surface M-W-3 is as follows:

$$f^p(\xi, \rho, \Theta) = \left(1.5 \frac{\rho}{k(\xi)f_c}\right)^2 + m \left(\frac{\rho}{\sqrt{6k(\xi)f_c}} r(\Theta, e) + \frac{\xi}{\sqrt{3k(\xi)f_c}} - c(\kappa)\right) = 0,$$

(5)

where:

$$m = 3 \frac{(k(\xi)f_c)^2 - (\lambda_1 f_1)^2}{k(\xi)f_c \lambda_1 f_1} \frac{e}{e+1} - \text{a parameter equivalent to cohesion},$$

(6)

$$r(\Theta, e) = \frac{4(1-e^2)\cos^2\Theta + (2e-1)^2}{2(1-e^2)\cos\Theta + (2e-1)\sqrt{4(1-e^2)\cos^2\Theta + 5e^2 - 4e}} - \text{elliptical function – Fig. 5},$$

(7)

$e$ – eccentricity of the elliptical function assuming values from the range $e \in (0.5; 1.0)$,

$f_c, f_t$ – uniaxial compressive and tensile strength,

$\lambda_1 \geq 1$ – scaling parameter for M-W-3 surface.

The boundary surface M-W-3 in deviatoric cross-section is composed of three tangential curves along the compressive meridians – Fig. 5 whose shape is affected by the assumed eccentricity $e$ of the elliptical function. When eccentricity $e$ is equal to 0.5, the deviatoric cross-section of failure surface is in the shape of an equilateral triangle, meanwhile, for $e = 1.0$, curves forming the deviatoric cross section take on a shape of circle. A curve, whose shape is similar to ellipse in the zone of biaxial compression values $\sigma_1 = \sigma_2, \sigma_3 = 0$, is a track of boundary surface in the plane of principal stresses. In the axial cross section, the surface is formed by parabolic meridians intersecting at the tension point corresponding to triaxial tension. The ellipse extreme corresponds to material strength to biaxial compression $f_{bc}$. Concrete strength to biaxial stress was empirically determined as $f_{bc} = 1.14f_c$, and the corresponding eccentricity of elliptical function was $e=0.52$. For masonry units the majority of tests involved solidbrick [14]. The obtained values of solid brick strength to biaxial compression $f_{bc}$ were
within the range 1.02–1.14\(f_c\), and the corresponding eccentricity values were \(e = 0.501–0.511\). In tests [6, 15, 16] conducted by the author on silicate and AAC masonry units, the following \(e\) values were obtained: \(e = 0.504\) (Ca-Si masonry units) and \(e = 0.52\) AAC masonry units.

**Figure 5.** The Menétrey–Willam surface in the Haigh–Westergaard space: a) the view from the space of principal stresses, b) axial cross-section view, c) deviatory cross-section view

The parameter of surface adjustment \(\lambda_t > 1\) determined the position of M-W-3 surfaces to the Rankine failure surface. At the value of \(\lambda_t = 1\) plasticity surface of M-W-3 was always within the Rankine pyramid, and at \(\lambda_t = 2\) surfaces intersected at the plane of hydrostatic tension and minor compression.

### 4. Validation of numerical FEM models

According to the adopted strategy of masonry units, numerical models of walls subjected to compression in accordance with the standard PN-EN 1052-1:2000 [16] were used to calibrate material parameters of silicate and AAC masonry units. Initial parameters from tests were attributed to finite elements representing masonry units [6]. Mortar layers in bed joints, which were replaced with contact elements, were neglected. Contact elements were also used in head joints. Contact elements were used on the contact area between the wall and steel parts of the strength testing machine. Friction coefficient for those contact elements was \(\tan \alpha = 0.1\) and cohesion \(f_{co} = 0\). Element thickness was equal to the wall thickness of 180 mm, and thickness of steel parts of the strength testing machine was 500 mm. Models were loaded with vertical displacement of the upper part of the strength testing machine equal to 10 mm. Numerical models of wall subjected to compression are shown in Fig. 6. Numerical calculations were used to determine the relationship between the standard stress \(\sigma_y\) and horizontal strain \(\varepsilon_x\) and vertical strain \(\varepsilon_y\) illustrated in Fig. 7 and compared in Table 1. Using parameters of walls with silicate and AAC masonry units (specified in, inter alia, the papers [6, 15, 17]) obtained from tests on standard models, delivered results were consistent with results from tests on modulus of longitudinal elasticity of the wall, however obtained values of maximum compressive stress were greater by 60% than those for the elastic-based degradation model, and by 45% compared to walls with AAC masonry units.

Clear differences in maximum compressive stresses were caused by the obvious impact of neglecting bed joints and replacing them with contact elements, and preliminary acceptance of limit damage surface and weakening parameters corresponding to masonry units, and not the limit surface of the wall.
Figure 6. Numerical models of compressed masonry: a) masonry with calcium silicate masonry units, b) masonry with AAC masonry units; 1 – homogeneous calcium silicate masonry units, 2 – homogeneous AAC masonry units, 3 – contact elements representing bed joints, 4 – contact elements representing head joints, 5 – contact elements representing connection of masonry to steel elements of strength testing machine, 6 – steel elements of strength testing machine

Table 1. Comparison of test results and numerical calculations for compressed walls

| Wall type                        | Test results | Results from numerical calculations | Before calibration | After calibration |
|----------------------------------|--------------|--------------------------------------|--------------------|------------------|
|                                  |              |                                      | SBeta model        | M-W-3 model      |
|                                  |              |                                      | σ_y,max N/mm²      | E N/mm²          |
|                                  |              |                                      | σ_y,max N/mm²      | E N/mm²          |
|                                  |              |                                      | σ_y,max N/mm²      | E N/mm²          |
| Wall with silicate masonry units| 11.29 7833  | 17.82 7181 18.08 7047 11.61 7610 11.51 7583 | 17.82 7181 18.08 7047 11.61 7610 11.51 7583 | |
| Wall with AAC masonry units     | 2.97 2040   | 4.27 2021 4.3 2014 2.99 2124 3.04 2175 | 4.27 2021 4.3 2014 2.99 2124 3.04 2175 | |

Material parameters of walls were calibrated - empirical homogenization, to achieve numerical calculations consistent with results obtained from tests. For that purpose, compressive and tensile strength of masonry units, their resistance to maximum and plastic deformations were corrected by using the reduction coefficient defined as ratio of compressive strength of wall and masonry unit \( f_{cm}/f_b \). For silicate masonry units, the calibration coefficient was \( f_{cm}/f_b = 11.29/17.7 = 0.64 \), and for AAC masonry units that coefficient was equal to \( f_{cm}/f_b = 2.97/4.25=0.70 \). Calculated results for walls subjected to compression and corrected material parameters are compared in Table 1 and on Fig. 7.

The best conformity between calculated and empirical values of failure stress was found after calibration for elastic-plastic models, in which the difference in maximum stress values did not exceed 2%. And for elastic-based degradation models, the maximum difference in calculated and empirical failure stress was ca. 3% The best conformity between empirical and calculated results for the modulus of longitudinal elasticity was achieved for elastic-based degradation models. Apart from strength parameters, also wall deformability after at the moment of reaching the highest compressive stress and after that time was analysed. For elastic-based degradation models, values of vertical and horizontal deformations for both types of walls were considerably smaller than test results, and nearly brittle failure was observed in accordance with the material model.
In case of elastic-plastic based models, values of vertical and horizontal deformations were similar to empirical values. Clear weakening and plastic deformations were observed when maximum compressive stress was achieved. The last analysed factor included images of the element cracking under the highest compressive stresses. Compared results from tests and calculations are shown in Fig. 8. For elastic-based degradation models, crack distribution was uniform in the central part of the wall and developed towards corner, covering whole masonry units. And the widest cracks in elastic-plastic based model were running from corners to the central part of the wall, almost as during tests. Because of high conformity between calculated and empirical results with reference to failure stress and modulus of elasticity, the most satisfactory results from analyses were obtained for the elastic-plastic based model. Also strains under the highest compressive stress and after achieving those values and the morphology of cracking demonstrated the highest conformity with test results when the elastic-plastic based model with M-W-3 failure surface was used. Therefore, to accomplish the purpose of this article, that is, to predict significant mechanical parameters of walls using simplified methods, the elastic-plastic based model with M-W-3 failure surface was the most reliable with reference to FEM. In stage 1, empirical homogenization was conducted for models of compressed walls, in which contact effects were not very significant. In stage 2, the correctness of accepted material parameters of contact elements in bed and head joints were verified with reference to possible calibration. Numerical models representing tested elements were used for calculations. Those elements were required to determine the modulus of strain deformation during the test of diagonal compression in accordance with the USA standard ASTM E519-81 [19] – Fig. 9. Deep beam 2D models (flat state of stress) of diagonally sheared walls, in which finite elements representing silicate and AAC masonry units were attributed parameters of homogenized material with M-W-3 failure surface that was properly calibrated at 1st stage of calculations.

Calculations were made in sequence, that is, at first they were made for walls taking into account self-weight of walls and steel elements of supports. The model was loaded by applying displacement compression of 10 mm to upper supports. Numerical calculations were used to determine the following relationships: shear stress $\tau$ - shear strain, shear stress at cracking $\tau_{cr}$ and failure $\tau_u$, and the corresponding angles of shear strain $\Theta_{cr}$ and $\Theta_u$. Compared results from tests and calculations are shown in Fig. 10 and Table 2. Taking into considerations parameters of walls with silicate and AAC masonry units corrected during calculations in the stage 1, the relationship shear stress-shear strain conformed to test results until maximum values of shear stress were achieved.
Figure 8. Maps of horizontal strains and cracking patterns of numerical models after calibration: 
a) masonry with calcium silicate units – SBeta model, b) masonry with calcium silicate units – M-W-3 model, 
c) masonry with autoclaved aerated concrete – SBeta model, d) masonry with autoclaved aerated concrete 
masonry units – M-W-3 model

Figure 9. Numerical models of masonry under diagonal compression: a) masonry with calcium 
silicate masonry units, b) masonry with AAC masonry units; 1 – homogeneous calcium silicate 
masonry units, 2 – homogeneous AAC masonry units, 3 – contact elements representing bed joints, 4 – contact 
elements representing head joints, 5 – contact elements representing connection of masonry 
to steel elements of test stand, 6 – steel elements of strength testing machine

For AAC walls, calculated values of cracking and failure stress did not differ by more than 5%, and 
values of shear strain angle corresponding to shear stress differed from empirical values by ca. 8%. 
And for AAC walls, the maximum difference in calculated and empirical shear stress was ca. 7%. A 
similar situation was observed for angles of shear strain at cracking and failure, where differences 
between empirical and calculated values did not exceed 8%. Those differences justified an additional 
correction of parameters of contact elements. Cracks in walls under maximum shear stress were also 
analysed (Fig. 11). The wall with calcium silicate masonry units had a vertical crack developed during 
tests that was running near a vertical diagonal. Besides, cracks were formed in bed joints, at mid-
height of the wall. Numerical calculations did not indicate the formation of the main vertical crack, but 
clear loosening in layers of masonry units was found nearly for the whole wall area (Fig. 11a). In AAC 
models, at failure the crack was running vertically through layers of bed joints, and when close to 
supports, also through masonry units (Fig. 11b). A similar result was obtained using the numerical 
model; however, there were also vertical cracks running through masonry units in the central part of 
the wall.
Figure 10. Comparison of shear stress–shear strain relationship from tests and numerical calculations: a) masonry with calcium silicate masonry units, b) masonry with AAC masonry units

Table 2. Comparison of test results and numerical calculations for diagonally sheared walls using the M-W-3 model

| Type of masonry unit | Test results | Results from numerical calculations for elastic-plastic model |
|----------------------|--------------|-------------------------------------------------------------|
|                      | $\tau_{cr,mv}$ N/mm$^2$ | $\theta_{cr,mv}$ mrad | $\tau_{u,mv}$ N/mm$^2$ | $\theta_{u,mv}$ mrad | $\tau_{cr,cal}$ N/mm$^2$ | $\theta_{cr,cal}$ mrad | $\tau_{u,cal}$ N/mm$^2$ | $\theta_{u,cal}$ mrad |
| Silicate units       | 0.10         | 0.617            | 0.12                 | 1.12                 | 0.103         | 0.671            | 0.115                 | 1.033                 |
| AAC units            | 0.192        | 0.587            | 0.196                | 0.632                | 0.179         | 0.543            | 0.185                 | 0.763                 |

Figure 11. Cracking patterns of research models and maps of horizontal strains with cracks found in numerical models under diagonal compression: a) masonry with calcium silicate masonry units – M-W-3 model, b) masonry with autoclaved aerated concrete masonry units – M-W-3 model
5. Conclusions

Using in numerical models the method of empirical homogenization and finite contact elements replacing bed and head joints, the FEM calculations can lead to following conclusions:

At the stage of selecting the strength criterion, the best conformity between calculated and empirical results for compressed walls in terms of the stress-strain relationship, cracking and failure was observed using the elastic-plastic based model and the M-W-3 model. For an alternative elastic-based degradation model, values of failure stress and $\sigma - \varepsilon$ relationship were similar, but results significantly differed from test results.

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