Testing the Landau Fermi liquid description of the fractional quantum Hall effect

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We study theoretically the dispersion of a single quasiparticle or quasihole of the fractional quantum Hall effect, obtained by injecting or removing a composite fermion. By comparing to a free fermion system, we estimate the regime of validity of the Landau-Fermi-liquid-type description and deduce the Landau mass.

During the last decade, a Landau-Fermi-liquid type description of the fractional quantum Hall effect has been investigated in terms of composite fermions, namely electrons binding an even number of quantum mechanical vortices \[ L \]. Numerous studies, both theoretical and experimental, indicate that this is a valid starting point \[ L \]. However, the extent as well as the regime of applicability of the Landau-Fermi liquid ideas is not entirely clear from a theoretical view point. While for interacting electrons at zero magnetic field the Landau-Fermi liquid theory is best justified asymptotically close to the Fermi energy, for the compressible Fermi sea at the half-filled Landau level, a logarithmic divergence of the mass has been predicted at very low energies \[ 2,4 \], suggesting a breakdown of the Landau-type description in the limit of vanishing temperature. At the same time, it is also obvious that the description of the state in terms of composite fermions, and therefore also as a Landau Fermi liquid of composite fermions, must become invalid at sufficiently high energies. Therefore, one may ask what is the range of energies, if any, in which the Landau-Fermi-liquid type description may be valid for composite fermions.

The basic assumption of the Landau Fermi liquid theory is that the low energy excitation spectrum of an interacting system resembles that of a system of free fermions. We consider here the fundamental building block, namely a single quasiparticle or quasihole of the FQHE state, the former obtained by injecting a composite fermion into one of many occupied CF-LLs, and the latter by removing a composite fermion from one of many unoccupied CF-LLs, and the latter by removing a composite fermion from one of many occupied CF-LLs (see Fig. 1). We ask if the energy levels of the particle or hole can be consistently interpreted as the energy levels of a single particle or hole in a free fermion system. In Landau’s Fermi liquid theory, the mass of the quasiparticle is defined by comparing its dispersion to that of a quasiparticle in a free fermion system. By analogy, we interpret the energy level spacing of the composite fermion quasiparticle as an effective cyclotron energy to extract a mass for the composite fermion, which we call the Landau mass, \( m_{L}^{*} \).

Our study shows that the analogy to a free fermion system breaks down beyond a certain energy, which we crudely estimate to be \( \approx 0.1e^{2}/\varepsilon l \), where \( \varepsilon \) is the dielectric constant of the host material and \( l = \sqrt{\hbar e/\varepsilon B} \) is the magnetic length. We also find that the Landau masses of the particle and the hole are in general different, but have substantial filling factor dependence and appear to approach the same value as the half filled Landau level is approached. (We note that because of the existence of a gap at the filling factors considered here, our work will not shed any light on the proposed logarithmic divergence at \( \nu = 1/2 \).)

\[
\begin{array}{ccc}
5 & \text{CF Particle} & \text{CF Hole} & \text{CF Particle-Hole Pair} \\
4 & \text{CF Particle} & \text{CF Hole} & \text{CF Particle-Hole Pair} \\
3 & \text{CF Particle} & \text{CF Hole} & \text{CF Particle-Hole Pair} \\
2 & \text{CF Particle} & \text{CF Hole} & \text{CF Particle-Hole Pair} \\
1 & \text{CF Particle} & \text{CF Hole} & \text{CF Particle-Hole Pair} \\
0 & \text{CF Particle} & \text{CF Hole} & \text{CF Particle-Hole Pair}
\end{array}
\]

FIG. 1. Schematic depiction of a CF particle (in \( \lambda = 4 \) CF-LL), a CF hole (in \( \lambda = 0 \) CF-LL), and a CF particle-hole pair at \( \nu = 3/7 \). The composite fermion is shown as an electron carrying two flux quanta.

Another mass for composite fermion has been defined in the past by interpreting the energy gap to creating a particle hole pair of composite fermions as an effective cyclotron energy for the composite fermion \[ 2,5 \]. In order to distinguish it from the Landau mass, it will be referred to it as the activation mass, \( m_{a}^{*} \). The two masses are obviously distinct. The Landau mass may not even be well defined, in general, but when it is, its value is deduced from the energy levels of a pre-existing particle or hole. The activation mass, on the other hand, is defined relative to a state containing no particle or hole, and thus also includes the self energies of the particle and the hole.

In Landau’s theory, the constant self energy contribution is lumped together with the chemical potential and does not contribute to the mass. The Landau and activation masses are relevant for different physical quantities. The former, for example, will be the mass that will appear in the specific heat. The two masses are expected to be of similar magnitude, however.
of the CF particle is \( \Psi \), electron in the \( \lambda \)th LL projection operator. To construct a CF wave function for the ground state at \( \nu = 1/3 \), we work with \( \lambda = 0,1,2, \ldots \), and the state \( |n> \) has all states occupied with \( \lambda \leq n - 1 \). Therefore, for a CF particle, we must have \( \lambda \geq n \). Similarly, the wave function for a CF-hole is given by \( \Psi_{n/(2n+1),\lambda,m}^h = P_{LLL} \Phi_{n,\lambda,m}^h \Phi_1^p \) where \( \Phi_{n,\lambda,m}^p = c_{\lambda,m}^\dagger |n> \), with \( c_{\lambda,m}^\dagger \) creates an electron in the \( \lambda \)th LL in the state labeled by quantum number \( m \). (The Landau level index takes on values \( \lambda = 0,1,2, \ldots \), and the state \( |n> \) has all states occupied with \( \lambda \leq n - 1 \). Therefore, for a CF particle, we must have \( \lambda \geq n \).) Similarly, the wave function for a CF-hole is given by \( \Psi_{n/(2n+1),\lambda,m}^h = P_{LLL} \Phi_{n,\lambda,m}^h \Phi_1^p \), where \( \Phi_{n,\lambda,m}^p = c_{\lambda,m}^\dagger |n> \), with \( \lambda < n \). We will work in the spherical geometry, in which \( N \) electrons move on the surface of a sphere under the influence of a radial magnetic field produced by a monopole of strength \( q \), an integer or a half integer, at the center. The \( \lambda \)th Landau level is the shell with single particle orbital angular momentum \( l = |q| + \lambda \), with single particle degeneracy \( 2l + 1 \). For \( N \) particles, the state \( \Phi_n \) occurs at \( q_n = \frac{N-n^2}{2n} \), which corresponds to a total \( q \) of \( q_n + 2q_1 \), with \( q_1 = (N-1)/2 \), for the ground state \( \Psi_{n/(2n+1)} \). This is a filled shell state, with total orbital angular momentum \( L = 0 \). The state \( \Phi_n^p (\Phi_n^h) \) corresponds to \( q_n^p = \frac{N-1-n^2}{2n} (q_n^h = \frac{N+1-n^2}{2n}) \), because now \( N-1 \) \( (N+1) \) particles completely fill \( n \) Landau levels, giving the total monopole strength of \( q^p = q_n^p + 2q_1 \) (\( q^h = q_n^h + 2q_1 \)) for the state \( \Psi_{n/(2n+1),\lambda,m}^p \) (\( \Psi_{n/(2n+1),\lambda,m}^h \)). It is important to note that the monopole strength is independent of the CF-LL index, \( \lambda \), of the CF particle or hole. The state \( \Psi_{n/(2n+1),\lambda,m}^p \) has total angular momentum \( L = q^p + \lambda \) (\( L = q^h + \lambda \)), \( m \) denoting the \( z \) component of \( L \). Since the energy is independent of \( m \), we work with the state \( m = L \). An advantage of the spherical geometry is that the states with CF-particle or CF-hole in different CF-LLs (i.e., with different \( \lambda \)) are automatically orthogonal on account of their different orbital symmetry. The various Slater determinants \( \Phi_n \) can be readily constructed with the knowledge of the single particle eigenstates. Their explicit forms of the lowest LL projected wave functions has been given previously.

The energies of the CF particle and the CF hole can be expressed as multidimensional integrals that are evaluated by Monte Carlo requiring up to \( 10^5 \) steps. Particle and hole occupying CF Landau levels up to \( \lambda \leq 6 \) are considered. In each case, the thermodynamic limit has been extracted by studying systems with up to 46 particles. The results for up to \( \nu = 5/11 \) are shown in Fig. 3. The analogous plot of the energy levels of a free fermion system will have straight lines (with slope equal to the cyclotron energy), so a deviation from a straight line indicates a breakdown of the free fermion description. At \( \nu = 1/3 \), the cyclotron gap changes even for the lowest energy levels, which implies that the Landau Fermi liquid description is not valid here and no mass can be defined. It would be troublesome if the energy levels continued to behave similarly at other fractions. However, for other fractions, especially for \( n \geq 3 \), the cyclotron energy becomes reasonably constant close to the Fermi energy. Taken all together, the results suggest that the

![Graph](image-url)
free fermion description of the system is valid up to an energy $E \sim 0.1e^2/\ell$, but breaks down beyond that. This conclusion obviously extends to states with a sufficiently dilute concentration of particles and holes, when the interaction between them is negligible.

TABLE I. The effective cyclotron energy, $h\omega_c$, (in units of $e^2/\ell_0$) for the CF particle and the CF hole at several filling factors. Also given is the normalized Landau mass $m_{L,nor}$, defined by the equation $m_{L,nor}^2 = m_L^2/\sqrt{B|I|}$.

| $\nu$ | CF particle | CF hole |
|-------|-------------|---------|
|       | $h\omega_c$ | $m_{L,nor}^2$ | $h\omega_c$ | $m_{L,nor}^2$ |
| 2/5   | -           | 0.111(5)  | 0.075(4)  | 0.073(5)  |
| 3/7   | 0.0356(40)  | 0.106(12) | 0.0465(30) | 0.081(5)  |
| 4/9   | 0.0255(40)  | 0.115(18) | 0.0351(47) | 0.084(11) |
| 6/11  | 0.0208(33)  | 0.119(18) | 0.0248(97) | 0.087(14) |

We determine the particle and hole masses from the slope of the points below $E \sim 0.1e^2/\ell$ for several fractions. The results are given in Table I. The masses are different for the particle and the hole. They are in general also filling factor dependent, but seem to be approaching the same value in the limit of $\nu = 1/2$, as expected. A simple extrapolation produces a CF mass of $m_{L,nor}^2 \approx 0.11 - 0.13$ at the half-filled Landau level. Such an extrapolation must obviously be treated with the understanding that it does not capture a logarithmic divergence of the mass at $\nu = 1/2$ predicted in other approaches or a much stronger divergence indicated by experiment. The extrapolation in the present work may give the mass at an intermediate range of temperatures, though. The theoretical value for the activation mass (for Coulomb interaction) for various FQH states is smaller but of similar magnitude ($m_{L,nor}^2 \approx 0.08$).

The existence of a well defined Landau mass is nontrivial for composite fermions. The mass of the quasiparticle in ordinary Fermi liquids is subject to perturbative corrections due to interparticle interactions. In contrast, even though it is natural to attribute a mass to composite fermions, given that we speak of their Fermi seas and Landau levels, the situation is subtle here because the mass is a non-perturbative effect: the original problem, namely interacting electrons in the lowest Landau level, has no mass parameter, and the mass of the composite fermion is generated entirely from the inter-electron interactions. There is no simple, solvable limit in which the mass of the composite fermion is known.

Does the above insight into the energy levels of a single particle or hole relate in any way to the structure of the energy spectrum of more complicated excitations, for example a particle hole excitation (PHE) of the incompressible state at $\nu = n/(2n + 1)$? Motivated by this question, we proceed to study the PHE spectrum, also known as the “single particle excitation spectrum,” of composite fermions, which is the allowed range of energy for the excitation of a single composite fermion out of the ground state, leaving behind a hole. The wave function for such a state is given by $P_{LL}\Phi_{n,\lambda,m,\lambda',m'}^{h-h}$ where $\Phi_{n,\lambda,m,\lambda',m'}^{h-h} = c_{\lambda,m}c_{\lambda',m'}|n\rangle$. The situation is now significantly more complicated for two reasons. First, these states do not automatically have the correct symmetry, because they are not the eigenstates of $L$ and $L_z$. This can be taken care of straightforwardly by working instead with appropriate linear combinations, $\sum_{m,m'} < LM|q_n + \lambda,m; q_n + \lambda',m' > \Phi_{n,\lambda,m,\lambda',m'}^{h-h}$. Now, we must diagonalize the Coulomb Hamiltonian in the Hilbert space defined by these states. The number of states at a given $L$ is finite because the minimum $L$ of a particle hole pair is $\Delta \lambda = |\lambda - \lambda'|$. However, these states are in general not orthogonal. We therefore first obtain an orthogonal basis according to the Gram-Schmidt procedure. For this purpose, we need to evaluate the off-diagonal projections $\langle u_r|u_s \rangle$, where $\{|u_r\rangle\}$ denotes the set of non-orthogonal basis states. Since the Monte Carlo method is most efficient when the integrand is positive definite, we determine $\langle u_r|u_s \rangle$ ($r \neq s$) by using the relation:

$$\langle u_r|u_s \rangle = \frac{\langle u_r + u_s|u_r + u_s \rangle - \langle u_r|u_r \rangle - \langle u_s|u_s \rangle}{2}.$$
roduces the energy eigenvalues in the restricted basis of states containing a single CF particle-hole pair. The results are in principle exact in the subspace of states containing only a single particle hole pair; the statistical uncertainty is estimated to be smaller than the symbols used in Fig. 3.

Fig. 3 shows the PHE spectrum of composite fermions at \( \nu = 1/3 \) and \( \nu = 2/5 \) for \( N = 10 \) particles, along with the PHE spectrum at \( \nu^* = 1 \) and \( \nu^* = 2 \), which is what one would expect in a model of non-interacting composite fermions with a well defined mass. Fig. 4 shows the PHE spectrum at \( \nu = 1/3 \) for a much bigger system \([1] \). Again, the basic message is that the analogy to free fermions breaks down above \( \sim 0.1e^2/\ell_0 \). Certain qualitative features of the PHE spectrum, which might appear puzzling at first sight, can be understood in light of our earlier results. For example, at \( \nu = 1/3 \), the first PHE branch is well defined, but the higher branches merge into one another (Figs. 3 and 4), which can be understood as a consequence of the fact that at \( \nu = 1/3 \), the effective cyclotron energy of composite fermions decreases rapidly as the CF-LL index increases. At \( \nu = 2/5 \), the effective cyclotron energy is better defined (Fig. 3), consistent with a more spread out PHE spectrum.

As expected from previous work \([12] \), there are fewer states in the actual PHE spectrum of composite fermions than what one would expect from the naive model. The reason is that many states at \( \nu^* = n \) are annihilated upon multiplication by \( \mathcal{F}^2 \) followed by lowest Landau level projection. The naive independent CF model thus contains spurious states that have no counterpart in the exact spectrum. The number of spurious states in the independent CF model increases with energy, again indicating that the model may be valid only for low energy states. The effective independent fermion description appears to be valid only for the low energy solutions, namely the lowest band, and also the first excited band (providing one neglects the thermodynamically insignificant spurious state at \( L = 1 \)).

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FIG. 4. The single particle excitation spectrum for composite fermions at \( \nu = 1/3 \) for \( N = 30 \) particles.