SEARCHING FOR SUPERSYMMETRIC DARK MATTER. THE MODULATION EFFECT DUE TO THE CAUSTIC RINGS.

J. D. VERGADOS
Theoretical Physics Section, University of Ioannina, GR-45110, Greece
E-mail: Vergados@cc.uoi.gr

The detection of the theoretically expected dark matter is central to particle physics and cosmology. Current fashionable supersymmetric models provide a natural dark matter candidate which is the lightest supersymmetric particle (LSP). The theoretically obtained event rates are usually very low or even undetectable. So the experimentalists would like to exploit the modulation effect. In the present paper we study a specific class of non-isothermal models involving flows of caustic rings. We find that the modulation effect arising from such models is smaller than that predicted by the isothermal models.

1 Introduction

In recent years the consideration of exotic dark matter has become necessary in order to close the Universe. Recent data from the Supernova Cosmology Project suggest that the situation can be adequately described by a barionic component \( \Omega_B = 0.1 \) along with the exotic components \( \Omega_{CDM} = 0.3 \) and \( \Omega_\Lambda = 0.6 \) (see also Turner, these proceedings).

Since this particle is expected to be very massive, \( m_\chi \geq 30\text{GeV} \), and extremely non relativistic with average kinetic energy \( T \leq 100\text{KeV} \), it can be directly detected mainly via the recoiling nucleus.

Using an effective supersymmetric Lagrangian at the quark level, see e.g. Jungman et al. and references therein, a quark model for the nucleon and nuclear wave functions one can obtain the needed detection rates. They are typically very low. So experimentally one would like to exploit the modulation of the event rates due to the earth’s revolution around the sun. In our previous work we found enhanced modulation, if one uses appropriate asymmetric velocity distribution. The isolated galaxies are, however, surrounded by cold dark matter which, due to gravity, keeps falling continuously on them from all directions. It is the purpose of our present paper to exploit the results of such a scenario.

2 The Basic Ingredients for LSP Nucleus Scattering

The differential cross section can be cast in the form:

\[
d\sigma(u, v) = \frac{du}{2(\mu r b u)^2} [(\Sigma_S + \Sigma_V \frac{v^2}{c^2}) F^2(u) + \Sigma_{spin} F_{11}(u)]
\] (1)
with

$$\Sigma_s = \sigma_0 \left( \frac{\mu_r}{m_N} \right)^2 \left\{ A^2 \left[ f_S^0 - f_S^1 \frac{A - 2Z}{A} \right]^2 \right\}$$  \hspace{1cm} (2)

The functions $\Sigma_{\text{spin}}$, associated with the spin, and the small coherent term, $\Sigma_V$, associated with the vector contribution, are not going to be discussed further (see our earlier work [8]). In the above expression $m_N$ is the proton mass, $\mu_r$ is the reduced mass, $F(u)$ and $F_{11}(u)$ are the usual and isovector spin nuclear form factors and $u = q^2 b^2 / 2$, with $b$ the harmonic oscillator size parameter and $q$ the momentum transfer to the nucleus. The scale is set by $\sigma_0 = \frac{1}{2\pi} (G_F m_N)^2 \simeq 0.77 \times 10^{-38} \text{cm}^2$ The quantity $u$ is also related to the experimentally measurable energy transfer $Q$ via the relations $Q = Q_0 u$, $Q_0 = [Am_N b^2]^{-1}$ The needed parameters from SUSY are $f_S^0, f_S^1$ (isoscalar and isovector scalar respectively). The differential detection rate for a particle with velocity $v$ and a target with mass $m$ is

$$dR = \rho(0) \frac{m}{m_N} \frac{d\sigma(u, v)}{|v|}$$  \hspace{1cm} (3)

where $d\sigma(u, v)$ is given by Eq. (1). One normally assumes $\rho(0) = 0.3 \text{GeV/cm}^3$ as the LSP density in our vicinity.

3 Convolution of the Event Rate

In this section we will examine the consequences of the earth’s revolution around the sun (the effect of its rotation around its axis is expected to be negligible) i.e. the modulation effect.

Following Sikivie we will consider $2 \times N$ caustic rings, (i,n) , i=(+,-) and n=1,2,...N (N=20 in the model of Sikivie et al), each of which contributes to the local density a fraction $\rho_n$ of the the summed density $\rho$ of each type $i = +,-$. and has velocity $y_n = (y_{nx}, y_{ny}, y_{nz})$, in units of $v_0 = 220 \text{Km/s}$, with respect to the galactic center.

We find it convenient to choose the z-axis in the direction of the motion of the the sun, the y-axis is normal to the plane of the galaxy and the x-axis is in the radial direction. The needed quantities are taken from the work of Sikivie (table 1 of last Ref. [10]) by the definitions $y_n = v_n / v_0, y_{nx} = v_{nx} / v_0, y_{nx} = v_{nr} / v_0, y_{ny} = v_{nz} / v_0$. This leads to a velocity distribution of the form:

$$f(v') = \sum_{n=1}^{N} \delta(v' - v_0 y_n)$$  \hspace{1cm} (4)
The velocity of the earth around the sun is given by:

\[
\upsilon_E = \upsilon_0 + \upsilon_1 = \upsilon_0 + \upsilon_1 \left( \sin\alpha \hat{x} - \cos\alpha \cos\gamma \hat{y} + \cos\alpha \sin\gamma \hat{z} \right)
\] (5)

where \(\alpha\) is the phase of the earth’s orbital motion, \(\alpha = 0\) around second of June. In the laboratory frame we have:

\[v = v' - \upsilon_E\]

4 Undirectional Event Rates

Integrating Eq. (3) we obtain for the total undirectional rate:

\[
\bar{R} = \bar{\bar{R}} t \frac{\bar{\rho}}{\rho(0)} [1 - h(a, Q_{\text{min}}) \cos\alpha]
\] (6)

where \(Q_{\text{min}}\) is the energy transfer cutoff imposed by the detector and \(a = \sqrt{2\mu_{\text{LSP}}/m}\)^{-1}. Also \(\rho_n = d_n/\bar{\rho}, \bar{\rho} = \sum_{n=1}^{N} d_n\) (for each flow +,-). In the Sikivie model \(2\bar{\rho}/\rho(0) = 1.25\). In the above expressions \(\bar{R}\) is the rate obtained by neglecting the folding with the LSP velocity and the momentum transfer dependence, i.e. by

\[
\bar{R} = \frac{\rho(0)}{m_X A m_N} \sqrt{\langle v^2 \rangle} \bar{\Sigma}_S + \bar{\Sigma}_\text{spin} + \frac{\langle v^2 \rangle}{c^2} \bar{\Sigma}_V
\] (7)

and it contains all SUSY parameters except \(m_{\text{chi}}\). The modulation is described in terms of the parameter \(h\). The effect of folding with LSP velocity on the total rate is taken into account via the quantity \(t\).

We like to stress that it is a common practice to extract the LSP nucleon cross section from the the measured event rates in order to compare with the SUSY predictions. In such analyses the factor of \(t\) is commonly omitted. It is clear, however, that, in going from the data to the cross section, one should divide by \(t\).

The undirectional differential rate takes the form:

\[
\langle \frac{dR}{du} \rangle = \bar{R} \frac{2\bar{\rho}}{\rho(0)} T(u)[1 - \cos\alpha H(u)]
\] (8)

The factor \(T(u)\) takes care of the \(u\)-dependence of the unmodulated differential rate. It is defined so that

\[
\int_{u_{\text{min}}}^{u_{\text{max}}} du T(u) = 1.
\] (9)

i.e. it is the relative differential rate. \(u_{\text{min}}\) is determined by the energy cutoff due to the performance of the detector. \(u_{\text{max}}\) is determined by the escape velocity \(v_{\text{esc}}\) via the relations: \(u_{\text{max}} = max(n_{\text{esc}}/\alpha, n = 1, 2, ..., N)\). On the other hand \(H(u)\) gives the energy transfer dependent modulation amplitude (relative to the unmodulated one).
5 Discussion of the Results and Conclusions

We have calculated the total event rates for elastic LSP-nucleus scattering including realistic nuclear form factors. We focused our attention on those aspects of the problem, which do not depend on the parameters of supersymmetry other than the LSP mass. The parameter $\bar{R}$, normally calculated in SUSY theories, was not considered in this work. The interested reader is referred to the literature for a review and references therein and, in our notation, to our previous work.

We specialized our results for the target $^{127}I$. We considered the effects of the detector energy cutoff, by considering two typical cases $Q_{\text{min}} = 10$ and $Q_{\text{min}} = 20$ KeV. We assumed that the LSP density in our vicinity and the velocity spectrum is that of caustic rings of Sikivie et al.

The total rates are described in terms of $t$ and $h$. In TABLE I we show how they vary the detector energy cutoff and the LSP mass.

The parameters $T(u)$ and $H(u)$ entering the differential amplitude are shown in Fig. 1. The shape of $T(u)$ is analogous to that of the isothermal models except that the maximum occurs at $u = 0.0$, rather than at $u = 0.1$. The function $H(u)$ shows oscillations, which result in a smaller total modulation. Another way of understanding how the cancellations arise is to note that for some rings $y_{nz} > 1$, while for others $y_{nz} < 1$.

The maximum occurs around the 2nd of December, something already noticed by Sikivie et al. Furthermore the modulation is small, $h = 0.25$, i.e. a 5% difference between the maximum and the minimum (see TABLE I). It is a bit smaller than that of the symmetric models, but a lot smaller than that predicted by the asymmetric ones, i.e. $h = 0.46$. 

![Figure 1: The quantities $T(u)$ and $H(u)$ entering the differential amplitude. Thick solid line corresponds to $m_{\chi} = 30$ GeV, the intermediate thickness line to $m_{\chi} = 80$ GeV, the fine line to $m_{\chi} = 100$ GeV. The rest correspond to larger LSP masses and fall on top of each other.](image)
Table 1: The quantities $t$ and $h$ in the case of the target $^{53}I^{127}$ for various LSP masses and $Q_{min}$ in KeV (for definitions see text).

| Quantity | $Q_{min}$ | 10  | 30  | 50  | 80  | 100 | 125 | 250 |
|----------|-----------|-----|-----|-----|-----|-----|-----|-----|
| $t$      | 0.0       | 1.451 | 1.072 | 0.751 | 0.477 | 0.379 | 0.303 | 0.173 |
| $h$      | 0.0       | 0.022 | 0.023 | 0.024 | 0.025 | 0.026 | 0.026 | 0.026 |
| $t$      | 10.0      | 0.000 | 0.226 | 0.356 | 0.265 | 0.224 | 0.172 | 0.098 |
| $h$      | 10.0      | 0.000 | 0.013 | 0.023 | 0.025 | 0.025 | 0.026 | 0.026 |
| $t$      | 20.0      | 0.000 | 0.013 | 0.126 | 0.139 | 0.116 | 0.095 | 0.054 |
| $h$      | 20.0      | 0.000 | 0.005 | 0.017 | 0.024 | 0.025 | 0.026 | 0.026 |

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References

1. For a recent review see e.g. G. Jungman et al., Phys. Rep. 267, 195 (1996).
2. R.S. Somerville, J.R. Primack and S.M. Faber, astro-ph/9806228; Mon. Not. R. Astron. Soc. (in press).
3. S. Perlmuter et al, Astrophys.J., in press [astro-ph/9812133];
   S. Perlmuter, M.S. Turner and M. White, Phys. Rev. Let. 83,670 (1999).
4. J.D. Vergados, J. of Phys. G 22, 253 (1996).
5. T.S. Kosmas and J.D. Vergados, Phys. Rev. D 55, 1752 (1997).
6. M. Drees and M.M. Nojiri, Phys. Rev. D 48, 3843 (1993); Phys. Rev. D 47, 4226 (1993).
7. J.D. Vergados, Phys. Rev. Let. 83, 3597 (1999).
8. J.D. Vergados, Phys. Rev. D62 (2000) 0235XX-1 [astro-ph/0001190]
9. T.P. Cheng, Phys. Rev. D 38 2869 (1988); H -Y. Cheng, Phys. Lett. B 219 347 (1989).
10. P. Sikivie, I. Tkachev and Y. Wang Phys. Rev. Let. 75, 2911 (1995; Phys. Rev. D 56, 1863 (1997); P. Sikivie, Phys. Let. b 432, 139 (1998); astro-ph/9810286