Mathematical model of allowance removal at nonabrasive
processing of bearing details

M K Reshetnikov, O Y Davidenko, V V Shpilev* and M Y Zakharchenko

Yuri Gagarin State Technical University of Saratov, Saratov, Russian Federation

*Vasya-shpilev@rambler.ru

Abstract. To increase the durability of the bearing, it is necessary to achieve a profile shape of the groove of the bearing close to that obtained during running-in. This effect allows you to achieve a method of non-abrasive tuning of the raceways of the rings of roller bearings. To study the effectiveness of this method, it is necessary at the initial stage to build a mathematical model of metal removal from the treated surface. To derive the mathematical model, we used the energy theory of friction, the theory of elasticity, and the physical theory of similarity. After the conclusion of the model, you can see which factors have the greatest impact on the efficiency of the process of non-abrasive tuning of the raceways of the roller bearing rings.

Obtaining on the working surfaces of the bearing parts of the profile with the optimal geometric shape corresponding to that formed on them as a result of their running-in at the initial stage of operation, increases the durability of the bearing. In the proposed method [1] (Figure 1), this is due to the fact that the processing tool, for example, the raceway of the inner bearing ring 1, in shape and size, corresponds to its outer ring 2 with a separator 4 and a set of rolling bodies 3 (tool rollers). The presence of a certain sag of the tool cage at an angle \( \alpha \), provides the process of profiling the machined surface, on which a convex profile is formed, which allows the bearing to reliably and without critical stresses work in the widest range of operating conditions at various skew angles of the rings that inevitably occurs when mounting bearings in units.

Figure 1. Method for non-abrasive tweaking of raceways of roller bearings rings.
We exploring the mechanism and construct a mathematical model for the removal of allowance with this processing method.

Based on the energy theory of friction [2-6] and the works of N.I. Glagoleva [7-9], in the work [10] the wear value of the workpiece surface was determined during non-abrasive tweaking of the bearing rings (1).

\[ h_{1} = \frac{\varphi \cdot f^{2}(R_{1} + R_{2})}{(\sigma_{-1})^{0.1} \cdot N_{bas1} \cdot C_{K}} \cdot R_{1} \cdot R_{2} \cdot \theta^{0.5} \cdot a^{2} \cdot \sigma_{max} \cdot K_{w} \cdot K_{1} \cdot K_{2} \]  

where

- \( R_{1}, R_{2} \) – workpiece and tool radii;
- \( Q_{1} \) – the proportion of wear of the workpiece material in the total wear of the bodies;
- \( \varphi \) – fraction of friction work spent on wear;
- \( N_{bas} \) – basic number of loading cycles at which the workpiece material is destroyed;
- \( C_{K} \) – coefficient taking into account stress concentration in the presence of defects on the surface of the workpiece;
- \( n_{1} \) – workpiece rotation frequency;
- \( \sigma_{max} \) – maximum tension in the center of the contact area;
- \( \sigma_{-1} \) – symmetrical tensile-compression limited tensile strength for the workpiece;
- \( f \) – friction-slip coefficient;
- \( a \) – half shaft of the elliptical contact area of the workpiece and tool;
- \( t \) – contact time of the workpiece and tool;
- \( \theta = \left( \frac{1 - \mu_{1}^{2}}{E_{1}} \right) + \left( \frac{1 - \mu_{2}^{2}}{E_{2}} \right) \) – coefficient taking into account the mechanical properties of the workpiece and tool.

From this expression it is seen that the amount of wear depends significantly on the mechanical properties of the workpiece material: limited endurance limit \( \sigma_{-1} \), resiliency modulus \( E \), Poisson's ratio. These values are tabular and their values can be found in the reference literature.

The basic number of cycles depends \( N_{bas} \) on the type of material, and for alloy steel [11] is:

\[ N_{bas} = 5 \cdot 10^{6} - 10^{7} \]  

(2)

The sizes of the contact area and contact stresses entering into expression (1) are determined by the formulas of the theory of resiliency.

Upon contact of the workpiece and tool having a cylindrical shape \( K_{2} \) and made of the same material:

\[ a = \frac{8PR_{1}R_{2}\theta}{\pi l (R_{1} + R_{2})} \]  

(3)

\[ \sigma_{max} = \frac{P(R_{1} + R_{2})}{2\pi R_{1}R_{2}\theta} \]  

(4)

where \( l \) – contact strip length of the workpiece and tool.

Or:

\[ a = \frac{4R_{1}R_{2}\theta}{(R_{1} + R_{2})} \]  

(5)

The contact area shape factor \( K_{2} \) will be equal to one.
As can be seen from the expression (6), wear of the surface of the workpiece is largely determined by the magnitude of the contact tensions and springy properties of the material.

In accordance with the physical theory of similarity, it is more convenient to present equation (6) as a functional dependence between similarity criteria, i.e. dimensionless complexes of physical quantities that determine the wear process. In this case, we find the ratio of wear at any point on the surface over time \( t \) to the path traveled by this point over the same time:

\[
J = \frac{h}{2\pi R_n t}
\]  

Given equality (6), expression (7) takes the form:

\[
J = \frac{8\varphi \cdot Q \cdot f^2 R_2 (1 - \mu^2)^{2-\varepsilon}}{\pi \sigma_{-1p}^2 N_d\varrho C_k (R_1 + R_2)^{3-\varepsilon}} \cdot \frac{\sigma_{\text{max}}^3 \cdot K_u \cdot K_1}{}\]  

In equation (8), several dimensionless parameters can be distinguished that affect the relative wear of surfaces. These include, firstly, the dimensionless parameter characterizing the dimensions of the workpiece and tool:

\[
n_p = \frac{R_2}{R_1 + R_2}
\]  

As can be seen from (9), with increasing size of the workpiece, its wear decreases.

The second important set of parameters characterizes the tensile stress of the contact surface:

\[
n_u = \frac{\varphi^{2-\varepsilon} \sigma_{\text{max}}^{3-\varepsilon} \cdot E_i}{\sigma_{-1p}^2 N_d\varrho C_k}
\]  

As can be seen, with an increase in the reduced modulus of elasticity and a limit of limited endurance, the amount of wear decreases, and with an increase in contact tensions, it increases.

Other dimensionless parameters of expression (8) include: sliding friction coefficient \( f \), sliping coefficient \( K_1 \), roughness coefficient \( \xi \), contact pad shape factor \( K_2 \), coefficient \( Q_1 \), determining the proportion of material wear on the contact surface per workpiece.

The coefficient \( Q_1 \) can be determined on the basis of the data of [6]:

\[
Q_1 = \frac{R_n A_h \cdot E_1}{R_n A_h \cdot E_1 + R_n A_{h2} \cdot E_2}.
\]  

As can be seen from expression (11), the relative wear of the workpiece surface will increase with an increase in its roughness and a decrease in the roughness of the tool surface, with a decrease in the resiliency modulus and a decrease in the specific saturation energy of the material \( A_H \). Since the specific saturation energy of the material is proportional to the limit of cyclic endurance, equality (11) can be represented in another form:

\[
Q_1 = \frac{R_n \sigma_{-1p}^2 \cdot E_2}{R_n \sigma_{-1p}^2 \cdot E_1 + R_n \sigma_{-1p}^2 \cdot E_2}.
\]  

From equalities (11) and (12) it also follows that the intensity of the process of forming the workpiece increases if the tool has a smoother surface, higher cyclic strength and a greater modulus of resiliency.
If the physicomechanical properties and roughness of the workpiece and tool are the same, then $Q_1 = 0.5$.

Then, when imitating non-abrasive tweaking, when the shaft of rotation of the workpiece and tool holder have a crossing angle, expression (6) will take the form:

$$h_1 = \frac{8\rho \cdot z \cdot f^2 \cdot R_1 \cdot R_2 \cdot n_1 \cdot E_1 \cdot t \cdot \vartheta}{\sigma_{1p1} \cdot N_{611} \cdot C_{K1} \cdot (R_1 + R_2)} \sigma_{\max}^3 \cdot K_u \cdot K_1$$

(13)

where

$$\sigma_{\max} = \sqrt{\frac{P(R_1 + R_2) \sin \alpha}{2\pi R_1 R_2 \vartheta}}$$

(14)

$\alpha$ – the angle of intersection of the shaft of rotation of the workpiece and tool holder; $l$ – tool roll forming length, $z$ – the number of rollers in the tool holder.

Expression (13) shows that the physical and mechanical properties of the surfaces of the workpiece and tool, represented in the resulting mathematical model by their hardness, have a significant impact on the efficiency of this process.

With an increase in the hardness of tool rollers, the processing productivity increases, since in this case the microroughnesses of the tool have high strength, wear out little and maintain high durability for a long time. An increase in the surface hardness of the workpiece, on the contrary, leads to a decrease in processing efficiency (Figure 2).

The initial state of the treated surface itself, as can be seen from the expression, also affects the processing performance. The higher the height of the initial microroughness of the treated surface, the greater the volume of metal can be removed during processing.

The increase in the geometric dimensions of the tool rollers also contributes to increased machining efficiency.

With an increase in the number of rollers, the processing efficiency increases (Figure 3).
Figure 3. Dependence of the productivity $t$ of non-abrasive finishing of roller tracks on the number of rollers $Z_p$ in the tool holder.

With an increase in the rotation frequency of the workpiece, the processing productivity increases, this is characteristic of theoretical and experimental dependencies, while the calculation error does not exceed 20% (Figure 4). Research was carried out in the processing of raceways of the inner ring of the roller bearing 42305.

Figure 4. Performance dependence $t$ of non-abrasive tweaking roller tracks on part frequency $n_1$: 1 - theoretical curve; 2 – experimental curve.

The performed analysis suggests that the obtained mathematical model of metal removal from the treated surface during non-abrasive tweaking takes into account a significant number of factors, allows us to study their influence on the processing efficiency, has sufficient accuracy and sensitivity, and can be used to study the laws of the process in a wide range of conditions its implementation.
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