Dynamics of quantum entanglement in Gaussian open systems

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Received 2 June 2010
Accepted for publication 11 June 2010
Published 16 August 2010
Online at stacks.iop.org/PhysScr/82/038116

Abstract

In the framework of the theory of open systems based on completely positive quantum dynamical semigroups, we give a description of the dynamics of entanglement for a system consisting of two uncoupled harmonic oscillators interacting with a thermal environment. Using the Peres–Simon necessary and sufficient criterion for separability of two-mode Gaussian states, we describe the evolution of entanglement in terms of the covariance matrix for a Gaussian input state. For some values of the temperature of the environment, the state keeps for all times its initial type: separable or entangled. In other cases, entanglement generation, entanglement sudden death or a repeated collapse and revival of entanglement takes place. We determine the asymptotic Gaussian maximally entangled mixed states (GMEMS) and their corresponding asymptotic maximal logarithmic negativity.

PACS numbers: 03.65.Yz, 03.67.Bg, 03.67.Mn

(Some figures in this article are in colour only in the electronic version.)

1. Introduction

Quantum entanglement of Gaussian states constitutes a fundamental resource in continuous variable quantum information processing and communication [1]. For the class of Gaussian states, there exist necessary and sufficient criteria of entanglement [2, 3] and quantitative entanglement measures [4, 5]. The quantum information processing tasks are difficult to implement, due to the fact that any realistic quantum system is not isolated and it always interacts with its environment. Quantum coherence and entanglement of quantum systems are inevitably influenced during their interaction with the external environment. As a result of the irreversible and uncontrollable phenomenon of quantum decoherence, the purity and entanglement of quantum states are in most cases degraded. Therefore, in order to describe realistically quantum information processes, it is necessary to take decoherence and dissipation into consideration. Decoherence and dynamics of quantum entanglement in continuous variable open systems have been intensively studied in recent years [6–13].

If two systems are immersed in an environment, then, in addition to and at the same time as the quantum decoherence phenomenon, the environment can also generate a quantum entanglement of the two systems and therefore an additional mechanism to correlate them [9, 14, 15]. In this paper, we study, in the framework of the theory of open systems based on completely positive quantum dynamical semigroups, the dynamics of the continuous variable entanglement of two identical harmonic oscillators coupled to a common thermal environment. We are interested in discussing the correlation effect of the environment; therefore we assume that the two oscillators are uncoupled, i.e. they do not interact directly. The initial state of the subsystem is taken to be of Gaussian form and the evolution under the quantum dynamical semigroup ensures the preservation in time of the Gaussian form of the state. In section 2, we write the Markovian master equation in the Heisenberg representation for two uncoupled harmonic oscillators interacting with a general environment and the evolution equation for the covariance matrix of the considered subsystem. By using the Peres–Simon criterion for separability of two-mode Gaussian states [2, 16], we investigate in section 3 the dynamics of entanglement for this system. For certain values of the environment temperature, the state keeps for all times its initial type: separable or entangled. For other values of the temperature, entanglement generation, entanglement sudden death or a repeated collapse and revival of entanglement takes place. In recent papers,
in a different model, Paz and Roncaglia [17, 18] studied numerically the exact entanglement behavior of two identical oscillators in an infinite bath of other oscillators by using the exact master equation for quantum Brownian motion and showed that the entanglement can undergo three qualitatively different dynamical phases: sudden death, sudden death and revival, and no sudden death of entanglement. Finally, we determine the asymptotic Gaussian maximally entangled mixed states (GMEMS) and the corresponding asymptotic maximal logarithmic negativity, which characterizes the degree of entanglement of these states. A summary is given in section 4.

2. Time evolution of the covariance matrix for two harmonic oscillators

We study the dynamics of the subsystem composed of two identical non-interacting oscillators in weak interaction with a thermal environment. In the axiomatic formalism based on completely positive quantum dynamical semigroups, the irreversible time evolution of an open system is described by the following general quantum Markovian master equation for an operator \( A \) in the Heisenberg representation (\( \dagger \) denotes Hermitian conjugation) [19, 20]:

\[
d\frac{dA}{dt} = \frac{i}{\hbar}[H, A] + \frac{1}{2\hbar} \sum_j (2V_j^\dagger AV_j - V_j^\dagger V_j A - AV_j^\dagger V_j).
\]

(1)

Here, \( H \) denotes the Hamiltonian of the open system and the operators \( V_j, V_j^\dagger \), defined on the Hilbert space of \( H \), represent the interaction of the open system with the environment.

We are interested in the set of Gaussian states, therefore we introduce such quantum dynamical semigroups that preserve this set during time evolution of the system. Consequently, \( H \) is taken to be a polynomial of second degree in the coordinates \( x, y \) and momenta \( p_x, p_y \) of the oscillators and \( V_j, V_j^\dagger \) are taken to be polynomials of first degree in these canonical observables. Then, in the linear space spanned by the coordinates and momenta, there exist only four linearly independent operators \( V_j = 1, 2, 3, 4 \) [21]:

\[
V_j = a_{xj}p_x + a_{yj}p_y + b_{xj}x + b_{yj}y,
\]

(2)

where \( a_{xj}, a_{yj}, b_{xj}, b_{yj} \) are complex coefficients. The Hamiltonian \( H \) of the two uncoupled identical harmonic oscillators of mass \( m \) and frequency \( \omega \) is given by

\[
H = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{m\omega^2}{2}(x^2 + y^2).
\]

(3)

The fact that the evolution is given by a dynamical semigroup implies the positivity of the following matrix formed by the scalar products of the four vectors \( a_x, a_y, b_x, b_y \), whose entries are the components \( a_{xj}, a_{yj}, b_{xj}, b_{yj} \), respectively:

\[
\frac{1}{2\hbar} \begin{pmatrix}
(a_x, a_x) & (a_x, b_x) & (a_y, a_y) & (a_y, b_y) \\
(b_x, a_x) & (b_x, b_x) & (b_y, a_y) & (b_y, b_y) \\
(a_x, a_x) & (a_x, b_x) & (a_y, a_y) & (a_y, b_y) \\
(b_x, a_x) & (b_x, b_x) & (b_y, a_y) & (b_y, b_y)
\end{pmatrix}.
\]

(4)

We take this matrix to be of the following form, where all diffusion coefficients \( D_{xx}, D_{xp}, \ldots \) and the dissipation constant \( \lambda \) are real quantities (we set, from now on, \( \hbar = 1 \)):

\[
\begin{pmatrix}
D_{xx} & -D_{xp} & -i\frac{\lambda}{2} & D_{xy} \\
-D_{xp} & D_{pp} & D_{yp} & D_{xp} \\
-i\frac{\lambda}{2} & D_{yp} & D_{yy} & -D_{xp} \\
D_{xp} & D_{yp} & -D_{xp} & 0
\end{pmatrix}.
\]

(5)

It follows that the principal minors of this matrix are positive or zero. From the Cauchy–Schwarz inequality, the following relations hold for the coefficients defined in (5):

\[
D_{xx}D_{pp} - D_{xp}^2 \geq \frac{\lambda^2}{4}, \quad D_{xy}D_{yp} - D_{xp}^2 \geq \frac{\lambda^2}{4},
\]

\[
D_{xx}D_{yy} - D_{xp}^2 \geq 0, \quad D_{xp}D_{yp} - D_{xp}D_{yp} \geq 0, \quad (6)
\]

\[
D_{xx}D_{pp} - D_{xp}^2 \geq 0, \quad D_{xy}D_{yp} - D_{xp}^2 \geq 0.
\]

A two-mode Gaussian state is completely characterized by its first and second moments of canonical variables. We therefore introduce the following 4 × 4 bimodal covariance matrix, which entirely specifies a two-mode Gaussian state (all first moments have been set to zero by means of local unitary operations which do not affect the entanglement):

\[
\sigma(t) = \begin{pmatrix}
\sigma_{xx}(t) & \sigma_{xp}(t) & \sigma_{xy}(t) & \sigma_{yp}(t) \\
\sigma_{xp}(t) & \sigma_{pp}(t) & \sigma_{yp}(t) & \sigma_{xp}(t) \\
\sigma_{xy}(t) & \sigma_{yp}(t) & \sigma_{yy}(t) & \sigma_{yp}(t) \\
\sigma_{xp}(t) & \sigma_{yp}(t) & \sigma_{xp}(t) & \sigma_{pp}(t)
\end{pmatrix}.
\]

(7)

The problem of solving the master equation for the operators in Heisenberg representation can be transformed into a problem of solving, first order in time, coupled linear differential equations for the covariance matrix elements. Namely, from (1) we obtain the following system of equations for the quantum correlations of the canonical observables, written in matrix form [21] (\( T \) denotes a transposed matrix):

\[
\frac{d\sigma(t)}{dt} = Y\sigma(t) + \sigma(t)Y^T + 2D,
\]

(8)

where

\[
Y = \begin{pmatrix}
-\lambda & 1/m & 0 & 0 \\
0 & -\lambda & 0 & 0 \\
0 & 0 & -\lambda & 1/m \\
0 & 0 & 0 & -\lambda
\end{pmatrix},
\]

(9)

\[
D = \begin{pmatrix}
D_{xx} & D_{xp} & D_{xy} & D_{xp} \\
D_{xp} & D_{pp} & D_{yp} & D_{xp} \\
D_{xy} & D_{yp} & D_{yy} & D_{yp} \\
D_{xp} & D_{yp} & D_{yp} & D_{pp}
\end{pmatrix}.
\]

(10)

The time-dependent solution of equation (8) is given by [21]

\[
\sigma(t) = M(t)\sigma(0) = M(t)(\sigma(\infty) + M(t)^T + \sigma(\infty)),
\]

(11)

where the matrix \( M(t) = \exp(Yt) \) has to fulfill the condition \( \lim_{t \to \infty} M(t) = 0 \). In order that this limit exists, \( Y \) must only have eigenvalues with negative real parts. The values at infinity are obtained from the equation

\[
Y\sigma(\infty) + \sigma(\infty)Y^T = -2D.
\]

(12)
3. Dynamics of two-mode continuous variable entanglement

The characterization of the separability of continuous variable states using second-order moments of quadrature operators was given in [2, 3]. A two-mode Gaussian state is separable if and only if the partial transpose of its density matrix is non-negative (the necessary and sufficient positive partial transpose (PPT) criterion). A two-mode Gaussian state is entirely specified by its covariance matrix (7), which is a real, symmetric and positive matrix with the block structure

\[ \sigma(t) = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}, \]  

where \( A, B \) and \( C \) are \( 2 \times 2 \) Hermitian matrices. \( A \) and \( B \) denote the symmetric covariance matrices for the individual one-mode states, while the matrix \( C \) contains the cross-correlations between modes. Simon [2] derived a PPT criterion for bipartite Gaussian continuous variable states: the necessary and sufficient criterion for separability is \( S(t) \geq 0 \), where

\[ S(t) = \det A \det B + \left( \frac{1}{4} - |\det C| \right)^2 - \text{Tr}[A J C J B J C^T J] - \frac{1}{4}(\det A + \det B) \]  

(14)

and \( J \) is the \( 2 \times 2 \) symplectic matrix

\[ J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \]  

(15)

Since the two oscillators are identical, it is natural to consider environments for which \( D_{xx} = D_{yy}, D_{xp} = D_{yp}, D_{xp, p_x} = D_{yp, p_y}, D_{xp} = D_{yp} \). Then both unimodal covariance matrices are equal, \( A = B \), and the entanglement matrix \( C \) is symmetric.

3.1. Time evolution of entanglement

In order to describe the dynamics of entanglement, we use the PPT criterion [2, 16] according to which a state is entangled if and only if the operation of partial transposition does not preserve its positivity. Concretely, we have to analyze the time evolution of the Simon function \( S(t) \) (14). For a thermal environment characterized by the temperature \( T \), we consider such diffusion coefficients, for which

\[ m_0 D_{xx} = \frac{D_{pp_x}}{m_0} = \frac{\lambda}{2} \coth \frac{\omega}{2kT}, \quad D_{xp} = 0, \]  

(16)

\[ m^2 \omega^2 D_{yy} = D_{pp_y}. \]  

(17)

This corresponds to the case when the asymptotic state is a Gibbs state [20]. We consider two cases, according to the type of the initial Gaussian state: (1) separable and (2) entangled.

(1) To illustrate a possible generation of the entanglement, we represent in figure 1 the dependence of function \( S(t) \) on time \( t \) and temperature \( T \) for a separable initial Gaussian mixed state. We note that, according to the Peres–Simon criterion, for relatively small values of the temperature \( T \), the initial separable state \( S(t) = 0 \) becomes entangled shortly after the initial moment of time \( t = 0 \). For relatively large values of \( T \), \( S(t) \) is strictly positive and the state remains separable for all times.

Depending on the environment temperature, there are three situations in the case of a generated entanglement [22]:

(i) entanglement may persist forever, including the asymptotic final state;
(ii) there exist repeated collapse and revival of entanglement;
(iii) the entanglement is created only for a short time, then it disappears and the state becomes again separable.

The entanglement of the two modes can be generated from an initial separable state during the interaction with the environment only for certain values of the mixed diffusion coefficient \( D_{pp} \) and the dissipation constant \( \lambda \).

(2) An example of the evolution of an entangled initial state is shown in figure 2, where we present the dependence of the function \( S(t) \) on time \( t \) and temperature \( T \) for...
an entangled initial Gaussian mixed state. For relatively small values of $T$, the initial entangled state may remain entangled for all times [22]. For relatively large values of the temperature $T$, at some finite moment of time, $S(t)$ takes non-negative values and therefore the state becomes separable. This is the so-called phenomenon of entanglement sudden death. This phenomenon is in contrast to the loss of quantum coherence, which is usually gradual [12]. Depending on the values of the temperature, it is also possible to have a repeated collapse and revival of the entanglement.

3.2. Logarithmic negativity

For Gaussian states, the measures of entanglement of bipartite systems are based on some invariants constructed from the elements of the covariance matrix [6, 10]. The Heisenberg uncertainty principle can be expressed as a constraint on the global symplectic invariants $\Delta \equiv \det A + \det B + 2 \det C$ and $\det \sigma$ [2]:

$$\Delta \leq \frac{1}{4} + 4 \det \sigma,$$

(18)

and the symplectic eigenvalues $\nu_\pm$, which form the symplectic spectrum of covariance matrix $\sigma$, are determined by

$$2 \nu_\pm^2 = \Delta \mp \sqrt{\Delta^2 - 4 \det \sigma}.$$  

(19)

In terms of $\nu_\pm$, relation (18) takes the form $\nu_- \geq 1/2$.

In order to quantify the degrees of entanglement of the infinite-dimensional bipartite system states of the two oscillators, it is appropriate to use the logarithmic negativity. For a Gaussian density operator, the logarithmic negativity is completely defined by the symplectic spectrum of the partial transpose of the covariance matrix. It is given by $E_N = \max \{0, - \log_2 2\nu_-\}$, where $\nu_-$ is the smallest of the two symplectic eigenvalues of the partial transpose $\tilde{\sigma}$ of the two-mode covariance matrix $\sigma$:

$$2 \nu_+^2 = \Delta \mp \sqrt{\Delta^2 - 4 \det \sigma}.$$  

(20)

Here $\Delta$ is given by $\Delta = \det A + 2 \det C$. A state is separable if and only if $\nu_- \geq 1/2$  

(21)

and logarithmic negativity quantifies the violation of inequality (21).

In our model, the logarithmic negativity is calculated as

$$E_N(t) = -\frac{1}{2} \log_2 \{4 f(\sigma(t))\},$$

(22)

where

$$f(\sigma(t)) = \frac{1}{2} (\det A + \det B) - \det C - \left(\frac{1}{2} (\det A + \det B) - \det C\right)^{1/2}.$$

(23)

It determines the strength of entanglement for $E_N(t) > 0$, and if $E_N(t) \leq 0$, then the state is separable.

As expected, the logarithmic negativity has a behavior similar to that of the Simon function in what concerns the characteristics of the state of being separable or entangled [23–26].

3.3. Asymptotic entanglement

From (12), (16) and (17) we obtain the following elements of the asymptotic matrices $A(\infty) = B(\infty)$:

$$m \sigma_{xx}(\infty) = \frac{\sigma_{pp,2}(\infty)}{m \omega} = \frac{1}{2} \coth \frac{\omega}{2kT}, \quad \sigma_{sp,2}(\infty) = 0$$

(24)

and of the entanglement matrix $C(\infty)$:

$$\sigma_{xy}(\infty) = \frac{m^2 (\lambda^2 + \omega^2) D_{xy} + m \lambda D_{xp}}{m^2 (\lambda^2 + \omega^2)},$$

(25)

$$\sigma_{sp,2}(\infty) = \lambda D_{xp},$$

(26)

$$\sigma_{pp,2}(\infty) = \frac{m^2 \omega^2 (\lambda^2 + \omega^2) D_{xy} - m \omega^2 \lambda D_{xp}}{\lambda (\lambda^2 + \omega^2)}.$$  

(27)

The mixedness of a quantum state $\rho$ is characterized by its purity $\mu \equiv \text{Tr} \rho^2$. For a two-mode Gaussian state, the purity is given by $\mu = 1/4\sqrt{\det \sigma}$. The marginal purities $\mu_i$ ($i = 1, 2$) of the reduced states in mode $i$ are given by $\mu_1 = 1/2\sqrt{\det A}$ and $\mu_2 = 1/2\sqrt{\det B}$. The global and marginal purities range from 0 to 1, and they fulfill the constraint $\mu \geq \mu_1 \mu_2$, as a direct consequence of the Heisenberg uncertainty relations. In [27], the following upper and lower bounds on the invariant $\Delta$ have been obtained in terms of global and marginal purities:

$$\frac{1}{2\mu} + \frac{(\mu_1 - \mu_2)^2}{4\mu_1^2 \mu_2^2} \leq \Delta \leq \min \left\{ \frac{(\mu_1 + \mu_2)^2}{4\mu_1^2 \mu_2^2} - \frac{1}{2\mu} \frac{1}{4} \left(1 + \frac{1}{\mu^2}\right) \right\}.$$  

(28)

The invariant $\Delta$ has a direct physical interpretation [27]: at given global and marginal purities, $\Delta$ determines the amount of entanglement of the state and the smallest symplectic eigenvalue $\nu_-$ of the partially transposed state is strictly monotone in $\Delta$. Consequently, the entanglement of a Gaussian state with fixed global purity $\mu$ and marginal purities $\mu_1, \mu_2$ is strictly increasing with decreasing $\Delta$. According to double inequality (28), giving lower and upper bounds on $\Delta$, there exist both maximally and minimally entangled Gaussian states.

We analyze the existence of the entanglement in the asymptotic regime in the symmetric situation $A = B$. First we consider the particular case $D_{xy} = 0$. In the limit of long times, we obtain the following quantities:

$$\det A = \det B = \frac{C_T^2}{4}, \quad \det C = -\frac{d^2}{\Lambda^2}, \quad \det \sigma = \frac{(C_T^2 - d^2)^2}{4\Lambda^2},$$

(29)

where we have used the notations:

$$C_T \equiv \coth \frac{\omega}{2kT}, \quad d \equiv D_{xp}, \quad \Lambda^2 \equiv \omega^2 + \lambda^2.$$  

(30)

Then we obtain

$$\frac{1}{\mu} = C_T^2 - 4 \frac{d^2}{\Lambda^2}, \quad \frac{1}{\mu_1} = \frac{1}{\mu_2} = C_T, \quad \Delta = 2\sqrt{\det \sigma} = \frac{1}{2\mu}.$$  

(31)
These values saturate the lower bound in inequalities (28) and this situation entails a maximal entanglement. Consequently, the corresponding states are GMEMS. They are thermal squeezed states with the squeezing parameter \( r \) given by \( \tanh 2r = d / \Lambda C_T \). In the pure case (det \( \sigma = 1/16 \)), these states are equivalent to two-mode squeezed vacua with the squeezing parameter determined only by the temperature of the thermal environment: \( \tanh 2r = \sqrt{C_T^2 - 1/2C_T} \).

According to [27], these states are separable in the range
\[
\mu \leq \frac{\mu_1 \mu_2}{\mu_1 + \mu_2 - \mu_1 \mu_2}.
\]
(32)

Then, for a given temperature \( T \), we obtain that the asymptotic final state is separable for the following range of positive values of the mixed diffusion coefficient \( d \):
\[
2d \Lambda \leq C_T - 1.
\]
(33)

We recall that, according to inequalities (6), the coefficients have to fulfill also the constraint \( \lambda C_T / 2 \geq d \). For a given temperature of the environment and for this range of mixed diffusion coefficients, no entanglement can occur for these states. Outside this region, i.e. for a temperature and diffusion coefficient satisfying
\[
C_T - 1 \leq 2d \Lambda \leq C_T + 1,
\]
(34)

they are GMEMS.

The asymptotic logarithmic negativity has the form
\[
E_N(\infty) = - \log_2 \left( C_T - \frac{2d}{\Lambda} \right).
\]
(35)

Outside the separable region, this is the maximum possible value of the logarithmic negativity, attained by GMEMS. It depends only on the mixed diffusion coefficient, dissipation constant and temperature and does not depend on the initial Gaussian state.

When \( d = 0 \), but \( D_{xy} \neq 0 \), then det \( C > 0 \) and the asymptotic final state is always separable. If both these diffusion coefficients are non-zero, \( d \neq 0 \), \( D_{xy} \neq 0 \), then the lower bound in inequalities (28) is not anymore saturated and, consequently, the corresponding states are Gaussian non-maximally entangled mixed states.

We note that the asymptotic persistence of entanglement is the result of the competition between thermal decoherence, determined by the uni-modal diffusion coefficients (16), and the statistical coupling of the two modes, due to the presence of position–position and momentum–momentum diffusion coefficients, which are chosen to be of equal relevance in our model \( (m^2 \omega^2 D_{xy} = D_{pp}) \) and position–momentum diffusion coefficients \( D_{xy} = D_{xy} \equiv d \).

4. Summary

In the framework of the theory of open quantum systems based on completely positive quantum dynamical semigroups, we investigated the Markovian dynamics of the quantum entanglement for a subsystem composed of two non-interacting modes embedded in a thermal environment. By using the Peres–Simon necessary and sufficient criterion for separability of two-mode Gaussian states, we have described the evolution of entanglement in terms of the covariance matrix for Gaussian input states. The dynamics of the quantum entanglement is sensitive to the initial states and the parameters characterizing the environment (diffusion and dissipation coefficients and temperature). For some values of these parameters, the state keeps for all times its initial type: separable or entangled. In other cases, entanglement generation or entanglement suppression (entanglement sudden death) takes place, or one can even notice repeated collapse and revival of entanglement. We have also shown that, independent of the type of the initial state—separable or entangled, for certain values of temperature, the initial state evolves asymptotically to an equilibrium state that is entangled, whereas for other values of temperature, the asymptotic state is separable. For a given temperature, we calculated the range of mixed diffusion coefficients that determine the existence of asymptotic GMEMS. We obtained also the expression for the maximal logarithmic negativity, which characterizes the degree of entanglement of these states.

Acknowledgments

The author acknowledges financial support from the projects CNCSIS-IDEI 497/2009 and PN 09 37 01 02/2009.

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