Deep Reinforcement Learning with Weighted $Q$-Learning

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Abstract

Reinforcement learning algorithms based on $Q$-learning are driving Deep Reinforcement Learning (DRL) research towards solving complex problems and achieving super-human performance on many of them. Nevertheless, $Q$-Learning is known to be positively biased since it learns by using the maximum over noisy estimates of expected values. Systematic overestimation of the action values coupled with the inherently high variance of DRL methods can lead to incrementally accumulate errors, causing learning algorithms to diverge. Ideally, we would like DRL agents to take into account their own uncertainty about the optimality of each action, and be able to exploit it to make more informed estimations of the expected return. In this regard, Weighted $Q$-Learning (WQL) effectively reduces bias and shows remarkable results in stochastic environments. WQL uses a weighted sum of the estimated action values, where the weights correspond to the probability of each action value being the maximum; however, the computation of these probabilities is only practical in the tabular setting. In this work, we provide methodological advances to benefit from the WQL properties in DRL, by using neural networks trained with Dropout as an effective approximation of deep Gaussian processes. In particular, we adopt the Concrete Dropout variant to obtain calibrated estimates of epistemic uncertainty in DRL. The estimator, then, is obtained by taking several stochastic forward passes through the action-value network and computing the weights in a Monte Carlo fashion. Such weights are Bayesian estimates of the probability of each action value corresponding to the maximum w.r.t. a posterior probability distribution estimated by Dropout. We show how our novel Deep Weighted $Q$-Learning algorithm reduces the bias w.r.t. relevant baselines and provides empirical evidence of its advantages on representative benchmarks.

Keywords: Deep Reinforcement Learning, $Q$-learning, Overestimation bias, Maximum expected value
1 Introduction

Temporal difference (TD) and off-policy learning are the constitutional elements of modern Reinforcement Learning (RL). TD allows agents to bootstrap their current knowledge to learn from a new observation as soon as it is available. Off-policy learning gives the means for exploration and enables experience replay. Q-Learning [13] implements both paradigms. Overestimation of the maximum action value is a well-known problem that hinders Q-Learning performance, leading to suboptimal policies and unstable learning. This overoptimism can be particularly harmful in stochastic environments and when using function approximation [14], notably also in the Deep Reinforcement Learning (DRL) settings [17]. Among possible solutions, the Double Q-Learning [15] algorithm and its DRL variant – Double DQN – tackle the overestimation problem by disentangling the choice of the target action and its evaluation. The resulting estimator, while achieving superior performance in many problems, is negatively biased. In fact, overly pessimistic estimates might undervalue a good course of action and are thus problematic in their own way. In this regard, Weighted Q-Learning (WQL) [4] is among several methods to reduce the bias and shows remarkable empirical results. WQL is based upon the Weighted Estimator (WE), which weights each estimate of the action values, based on an estimated probability of each particular action value being the maximum. However, doing so in high-dimensional environments is not trivial due to the additional challenges imposed by using function approximation. Our objective is, then, to devise an approach to scale WQL to the DRL settings. We do so by using neural networks trained with Dropout as an effective, computationally cheap, Bayesian inference technique [5]. We combine, in a novel way, the dropout uncertainty estimates with the Weighted Q-Learning algorithm, extending it to the DRL settings. The proposed Deep Weighted Q-Learning algorithm, or Weighted DQN (WDQN), leverages an approximated posterior distribution on Q-networks to reduce the bias of deep Q-learning. WDQN bias is neither always positive, nor negative, but depends on the state and the problem at hand. WDQN only requires minor modifications to the baseline algorithm, and its computational overhead is negligible on specialized hardware. In Section 2 we define the problem settings and discuss some related works, then, in Section 3 we present our approach. We present empirical results in Section 4 and draw our conclusions in Section 5.

2 Preliminaries

A Markov Decision Process (MDP) is a tuple $\langle S, A, P, R, \gamma \rangle$ where $S$ is a state space, $A$ is an action space, $P : S \times A \rightarrow S$ is a Markovian transition function, $R : S \times A \rightarrow \mathbb{R}$ is a reward function, and $\gamma \in [0, 1]$ is a discount factor. A sequential decision maker ought to estimate, for each state $s$, the optimal value $Q^*(s, a)$ of each action $a$, i.e., the expected cumulative discounted reward obtained by taking action $a$ in $s$ and following the optimal policy $\pi^*$ afterwards.

(Deep) Q-Learning A classical approach for solving finite MDPs is the Q-Learning algorithm, an off-policy value-based RL algorithm, based on TD. The popular Deep Q-Network algorithm (DQN) [10] is a variant of Q-Learning designed to stabilize off-policy learning with deep neural networks in high-dimensional state spaces. The two most relevant architectural changes to standard Q-Learning introduced by DQN are the adoption of a replay memory, to learn offline from experience, and the use of a target network, to reduce correlation between the current model estimate and the bootstrapped target value. In practice, a DQN agent interacts with the environment, stores in the replay buffer performed actions and corresponding observations, and learns the Q-values online, training a neural network. The network, with parameters $\theta$, is trained by sampling mini-batches from the memory and using a target network whose parameters $\theta^-$ are updated to match those of the online model every $C$ steps. The model is trained to minimize the loss

$$L(\theta) = \mathbb{E}_{(s_i, a, r_i, s'_i) \sim m} \left[ (y_i^{\text{DQN}} - Q(s_i, a_i; \theta))^2 \right], \quad y_i^{\text{DQN}} = r_i + \gamma \max_a Q(s'_i, a; \theta^-),$$

(1)

where $m$ is a uniform distribution over the transitions stored in the replay buffer and $y_i^{\text{DQN}}$ is the DQN target. Among the many studied improvements and extensions of the baseline DQN algorithm, Double DQN (DDQN) [17] reduces the overestimation bias of DQN with a simple modification of the minimized loss. In particular, DDQN uses the target network to decouple action selection and evaluation, and estimates the target value as

$$y_i^{\text{DDQN}} = r_i + \gamma Q(s'_i, \arg\max_a Q(s'_i, a; \theta); \theta^-).$$

DDQN improves on DQN, converging to a more accurate approximation of the value function, while maintaining the same model complexity and adding minimal computational overhead.

Estimation biases in Q-Learning Choosing a target value for the Q-Learning update rule can be seen as an instance of the Maximum Expected Value (MEV) estimation problem for a set of random variables, here the action values. Q-Learning uses the Maximum Estimator (ME) to estimate the maximum expected return and exploits it for policy improvement. It is well known that ME is a positively biased estimator of MEV [16]. Double Q-Learning [15], on the other hand, learns two value functions in parallel and uses an update scheme based on the Double Estimator (DE). It is shown that DE is a negatively biased estimator of MEV, which helps to avoid catastrophic overestimates of the Q-values. In practice, as also
shown by Lan et al. [8], the overestimation bias of Q-Learning is not always harmful and may also be convenient when the action values are significantly different among each other. Conversely, the underestimation of Double Q-Learning is effective when all the action values are very similar. Unfortunately, prior knowledge about the environment is not always available, and it would be desirable to have an estimator which is not always positively or negatively polarized. Clearly, besides Double Q-Learning several other methods exist to mitigate the overestimation problem (e.g., [2][1][8]), here we mainly focus on Weighted Q-learning and on the two most used algorithms, namely standard Q-Learning and its DE variant.

**Weighted Q-Learning** D’Eramo et al. [4] propose the Weighted Q-Learning (WQL) algorithm, a variant of Q-Learning based on therein introduced Weighted Estimator (WE). WE estimates MEV as the weighted sum of the random variables sample means, weighted according to their probability of corresponding to the maximum. Intuitively, the amount of uncertainty, i.e., the entropy of the WE weights, will depend on the nature of the problem, the number of samples and the variance of the mean estimator (critical when using function approximation). WE bias is bounded by the biases of ME and DE [4]. The target value of WQL can be computed as

\[ y_{t}^{WQL} = r_t + \gamma \sum_{a \in A} w_{a}^{s_t+1} Q(s_{t+1}, a) \quad w_{a}^{s} = P\left(a = \arg\max Q(s, a')\right), \quad (2) \]

where \( w_{a}^{s+i} \) are the WE weights and correspond to the probability of each action value being the maximum. The weights of WQL are estimated in the tabular setting, assuming the sample means to be normally distributed.

### 3 Deep Weighted Q-Learning

**Algorithm 1** Deep Weighted Q-Learning

| Input: Q-network parameters \( \theta \), dropout rates \( p_1, \ldots, p_L \), a policy \( \pi \), replay memory \( D \) |
| \( \theta^- \leftarrow \theta \) |
| Initialize memory \( D \). |
| for step \( t = 0, \ldots \) do |
| Select an action \( a_t \) according to some policy \( \pi \) given the distribution over action-value functions \( Q(s_i; \cdot; \theta^\top) \) |
| Execute \( a_t \) and add \( \langle s_t, a_t, r_t, s_{t+1} \rangle \) to \( D \) |
| Sample a mini-batch of transitions \( \{ \langle s_i, a_i, r_i, s_i' \rangle \} \in D \) |
| for \( i = 1, \ldots, M \) do |
| Take \( K \) samples from \( Q(s_i; \cdot; \theta^\top, \omega) \) by performing \( K \) stochastic forward passes |
| Use the samples to compute the WDQN weights (Eq. 5) and targets (Eq. 7) |
| end for |
| Perform an SGD step using \( y_{t}^{WDQN} \) as the target value. |
| Eventually update \( \theta^- \) |
| end for |

A natural way to extend the WQL algorithm to the DRL settings is to consider the uncertainty over the model parameters by using a Bayesian approach. Dropout [13] is a regularization technique used to train large neural networks by randomly dropping units during learning. In recent years, dropout has been analyzed from a Bayesian perspective [5], and interpreted as a variational approximation of a posterior distribution over the parameters of the neural network. In particular, Gal and Ghahramani [5] show how a neural network trained with dropout and weight decay can be seen as an approximation of a deep Gaussian process. A single stochastic forward pass through a neural network trained with Dropout can, in fact, be interpreted as taking a sample from the model’s predictive distribution. This inference technique, known as **Monte Carlo dropout**, can be efficiently parallelized on modern GPUs. This approach has found several applications in RL, e.g., as a practical approach to perform Thompson Sampling [5] and to estimate uncertainty in model-based RL [6]. Here we focus on the problem of action evaluation, and we show how to use approximate Bayesian inference to evaluate WE by introducing a novel approach to exploit uncertainty estimates in DRL. Our method is grounded in theory and simple to implement.

**Weighted DQN** Let \( Q(\cdot; \cdot; \theta, \omega) \) be a neural network with weights \( \theta \) trained with a Gaussian prior and Dropout Variational Inference to learn the optimal action-value function of a certain MDP. We indicate with \( \omega \) the set of random variables that represent the dropout masks, with \( \omega_i \) the \( i \)-th realization of the random variables and with \( \Omega \) their joint distribution:

\[ \omega = \{ \omega_{lk} : l = 1, \ldots, L, k = 1, \ldots, K_l \}, \quad \omega_{lk} \sim \text{Bernoulli}(p_l), \quad \omega_l \sim \Omega(p_1, \ldots, p_L), \quad (3) \]

where \( L \) is the number of weight layers of the network and \( K_l \) is the number of units in layer \( l \). Consider a sample \( q(s,a) \) of the MDP return, obtained taking action \( a \) in \( s \) and following the optimal policy afterwards. Following the Gaussian
process interpretation of Dropout of Gal and Ghahramani [5], we can approximate the likelihood of this observation as a Gaussian such that

\[ q(s, a) \sim \mathcal{N}(Q(s, a; \theta, \omega); \tau^{-1}) , \tag{4} \]

where \( \tau \) is the model precision. We can approximate the predictive mean of the process, and the expectation over the posterior distribution of the Q-value estimates, as the average of \( T \) stochastic forward passes through the network:

\[ \mathbb{E}[Q(s, a; \theta, \omega)] \approx \hat{Q}_T(s, a; \theta) = \frac{1}{T} \sum_{t=1}^{T} Q(s, a; \theta, \omega_t). \tag{5} \]

\( \hat{Q}_T(s, a; \theta) \) is the estimate of the action values associated with state \( s \) and action \( a \). We can estimate the probability required to calculate WE similarly, by again exploiting a Monte Carlo estimator. Given an action \( a \), the probability that \( a \) corresponds to the maximum expected action value can be approximated as the number of times in which, given \( T \) samples, the sampled action value of \( a \) is the maximum over the number of samples

\[ w^a_\theta = P \left( a = \arg\max_{a'} Q(s, a'; \theta, \omega) \right) \approx \frac{1}{T} \sum_{t=1}^{T} \left[ a = \arg\max_{a'} Q(s, a'; \theta, \omega_t) \right] , \tag{6} \]

where \([ \ldots ]\) are the Iverson brackets (\([P]\) is 1 if \( P \) is true, 0 otherwise). The weights can be efficiently inferred in parallel with no impact in computational time. We can define the WE target given the target Q-network estimates by using the obtained weights as:

\[ y^W_{DQN} = r + \gamma \sum_{a \in A} w^a_\theta \hat{Q}_T(s', a; \theta). \tag{7} \]

For completeness, the complete WDQN algorithm is reported in Algorithm 1. Dropout probabilities are variational parameters and influence the quality of the approximation. Ideally, they should be tuned to maximize the log-likelihood of the observations using a validation method. This is clearly not possible in RL where the available samples and the underlying distribution generating them, change as the policy improves. In fact, using dropout with a fixed probability might lead to poor uncertainty estimates [11,12,7]. Concrete Dropout [7] mitigates this problem by using a differentiable continuous relaxation of the Bernoulli distribution and learning the dropout rate from data. In practice, then, we substitute the standard Dropout with its Concrete variant in our WDQN implementation.

4 Experiments

Figure 1: Learning curves on Atari games. The curves on the left of each subfigure show the evaluation scores. The curves on the right, conversely, report the estimate of the expected return w.r.t. the starting screen of the game; the dashed curves indicate the real discounted return for each agent. Each epoch corresponds to 1M frames. The results shown here are the average of 10 independent runs and the shaded areas represent 95% confidence intervals. The curves are smoothed using a moving average of 10 epochs to improve readability.

In this section, we compare WDQN against the standard DQN algorithm and its Double DQN variant in the Arcade Learning Environment (ALE) [2]. In particular, we choose environments where DQN is known to overestimate action values and exhibit unstable learning [17]. We use the same neural network and hyperparameters of Mnih et al. [10], for WDQN we use Concrete Dropout only in the fully connected layer after the convolutional block. We found WDQN to be robust to the number of dropout samples used to compute the WE (e.g., \( T \geq 30 \)), while being more sensitive to the Concrete Dropout regularization coefficient. For more in-depth results and discussion, we refer to our journal paper [3].

Figure 1 shows the result of the comparison in terms of the average reward and prediction accuracy of WDQN against DQN and DDQN. Here, WDQN stabilizes learning in both scenarios, but it also outperforms DDQN. Conversely, DDQN
manages to stabilize the learning performance in Asterix, but, in our setting, is not able to yield satisfactory performance in Wizard of Wor differently from what observed by Van Hasselt et al. [17], probably due to minor implementation differences. However, it is worth mentioning that DDQN would most likely improve in this scenario by reducing the frequency of updating the target network’s parameters. WDQN, on the other hand, manages to control the bias and the learning instability in both cases, achieving a comparable performance and prediction accuracy w.r.t. DDQN in Asterix and outperforming the other two baselines on Wizard of Wor. Again, we refer to the journal paper for additional results.

5 Conclusion and future works

We present WDQN, a new value-based Deep Reinforcement Learning algorithm that extends the Weighted Q-Learning algorithm to work in environments with a high-dimensional state representation. WDQN is a principled and robust method to exploit uncertainty in DRL to accurately estimate the maximum action value in a given state. Our results corroborate the findings of previous works [5, 7], confirming that dropout can be used successfully for approximate Bayesian inference in DRL. Future works may explore the combination of WDQN with other orthogonal DQN extensions and may attempt to adapt the WDQN approach to other techniques modeling uncertainty in deep neural networks.

References

[1] Oron Anschel, Nir Baram, and Nahum Shimkin. Averaged-DQN: Variance reduction and stabilization for deep reinforcement learning. In Proceedings of the 34th International Conference on Machine Learning-Volume 70, 2017.
[2] M. G. Bellemare, Y. Naddaf, J. Veness, and M. Bowling. The arcade learning environment: An evaluation platform for general agents. Journal of Artificial Intelligence Research, 47:253–279, jun 2013.
[3] Carlo D’Eramo, Andrea Cini, Alessandro Nuara, Matteo Pirotta, Cesare Alippi, Jan Peters, and Marcello Restelli. Gaussian approximation for bias reduction in Q-learning. Journal of Machine Learning Research, 2021.
[4] Carlo D’Eramo, Marcello Restelli, and Alessandro Nuara. Estimating maximum expected value through Gaussian approximation. In International Conference on Machine Learning, pages 1032–1040, 2016.
[5] Yarin Gal and Zoubin Ghahramani. Dropout as a Bayesian approximation: Representing model uncertainty in deep learning. In Proceedings of The 33rd International Conference on Machine Learning, 2016.
[6] Yarin Gal, Rowan Thomas McAllister, and Carl Edward Rasmussen. Improving PILCO with Bayesian neural network dynamics models. In Data-Efficient Machine Learning workshop, ICML 2016, 2016.
[7] Yarin Gal, Jiri Hron, and Alex Kendall. Concrete Dropout. In Advances in Neural Information Processing Systems 30, pages 3581–3590. Curran Associates, Inc., 2017.
[8] Qingfeng Lan, Yangchen Pan, Alona Fyshe, and Martha White. Maxmin Q-learning: Controlling the estimation bias of Q-learning. In International Conference on Learning Representations, 2020.
[9] Donghun Lee, Boris Defourny, and Warren B Powell. Bias-corrected Q-learning to control max-operator bias in Q-learning. In 2013 IEEE Symposium on Adaptive Dynamic Programming and Reinforcement Learning (ADPRL), pages 93–99. IEEE, 2013.
[10] Volodymyr Mnih, Koray Kavukcuoglu, David Silver, Andrei A Rusu, Joel Veness, Marc G Bellemare, Alex Graves, Martin Riedmiller, Andreas K Fidjeland, and Georg Ostrovski. Human-level control through deep reinforcement learning. Nature, 518(7540):529, 2015.
[11] Ian Osband, Charles Blundell, Alexander Pritzel, and Benjamin Van Roy. Deep exploration via bootstrapped DQN. In Advances in Neural Information Processing Systems, pages 4026–4034, 2016.
[12] Ian Osband, John Aslanides, and Albin Cassirer. Randomized prior functions for deep reinforcement learning. In Advances in Neural Information Processing Systems 31, pages 8617–8629. Curran Associates, Inc., 2018.
[13] Nitish Srivastava, Geoffrey Hinton, Alex Krizhevsky, Ilya Sutskever, and Ruslan Salakhutdinov. Dropout: A simple way to prevent neural networks from overfitting. Journal of Machine Learning Research, 15:1929–1958, 2014.
[14] Sebastian Thrun and Anton Schwartz. Issues in using function approximation for reinforcement learning. In Proceedings of the 1993 Connectionist Models Summer School, pages 255–263. Lawrence Erlbaum, 1993.
[15] Hado Van Hasselt. Double Q-learning. In Advances in Neural Information Processing Systems, 2010.
[16] Hado Van Hasselt. Estimating the maximum expected value: an analysis of (nested) cross-validation and the maximum sample average. arXiv preprint arXiv:1302.7175, 2013.
[17] Hado Van Hasselt, Arthur Guez, and David Silver. Deep reinforcement learning with double Q-learning. In AAAI, volume 16, pages 2094–2100, 2016.
[18] Christopher John Cornish Hellaby Watkins. Learning from Delayed Rewards. PhD thesis, King’s College, 1989.