8 COMSATS Institute of Information Technology, Lahore, Defence Road, Off Raiwind Road, 54000 Lahore, Pakistan
9 G.I. Budker Institute of Nuclear Physics SB RAS (BINP), Novosibirsk 630090, Russia
10 GSI Helmholtzcentre for Heavy Ion Research GmbH, D-64291 Darmstadt, Germany
11 Guangxi Normal University, Guilin 541004, People’s Republic of China
12 Guangxi University, Nanning 530004, People’s Republic of China
13 Hangzhou Normal University, Hangzhou 310036, People’s Republic of China
14 Helmholtz Institute Mainz, Johann-Joachim-Becher-Weg 45, D-55509 Mainz, Germany
15 Henan Normal University, Xinxiang 453007, People’s Republic of China
16 Henan University of Science and Technology, Luoyang 471003, People’s Republic of China
17 Huangshan College, Huangshan 245000, People’s Republic of China
18 Hunan University, Changsha 410082, People’s Republic of China
19 Indiana University, Bloomington, Indiana 47405, USA
20 (A)INFN Laboratori Nazionali di Frascati, I-00044, Frascati, Italy; (B)INFN and University of Perugia, I-06100, Perugia, Italy
21 (A)INFN Sezione di Ferrara, I-44122, Ferrara, Italy; (B)University of Ferrara, I-44122, Ferrara, Italy
22 Institute of Physics and Technology, Peace Ave. 54B, Ulaanbaatar 13330, Mongolia
23 Johannes Gutenberg University of Mainz, Johann-Joachim-Becher-Weg 45, D-55099 Mainz, Germany
24 Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia
25 Justus-Liebig-Universitaet Giessen, II. Physikalisches Institut, Heinrich-Buff-Ring 16, D-35392 Giessen, Germany
26 KVI-CART, University of Groningen, NL-9747 AA Groningen, The Netherlands
27 Lanzhou University, Lanzhou 730000, People’s Republic of China
28 Liaoning University, Shenyang 110036, People’s Republic of China
29 Nanjing Normal University, Nanjing 210023, People’s Republic of China
30 Nanjing University, Nanjing 210093, People’s Republic of China
31 Nankai University, Tianjin 300071, People’s Republic of China
32 Peking University, Beijing 100871, People’s Republic of China
33 Seoul National University, Seoul, 151-747 Korea
34 Shandong University, Jinan 250100, People’s Republic of China
35 Shanghai Jiao Tong University, Shanghai 200240, People’s Republic of China
36 Shanxi University, Taiyuan 030006, People’s Republic of China
37 Sichuan University, Chengdu 610064, People’s Republic of China
38 Soochow University, Suzhou 215006, People’s Republic of China
39 Southeast University, Nanjing 211100, People’s Republic of China
40 State Key Laboratory of Particle Detection and Electronics, Beijing 100049, Hefei 230026, People’s Republic of China
41 Sun Yat-Sen University, Guangzhou 510275, People’s Republic of China
42 Tsinghua University, Beijing 100084, People’s Republic of China
43 (A)Ankara University, 06100 Tandogan, Ankara, Turkey; (B)Istanbul Bilgi University, 34069 Eyup, Istanbul, Turkey; (C)Uludag University, 16059 Bursa, Turkey; (D)Near East University, Nicosia, North Cyprus, Mersin 10, Turkey
44 University of Chinese Academy of Sciences, Beijing 100049, People’s Republic of China
45 University of Hawaii, Honolulu, Hawaii 96822, USA
46 University of Jinan, Jinan 250022, People’s Republic of China
47 University of Minnesota, Minneapolis, Minnesota 55455, USA
48 University of Muenster, Wilhelm-Klemm-Str. 9, 48149 Muenster, Germany
49 University of Science and Technology Liaoning, Anshan 114051, People’s Republic of China
50 University of South China, Hengyang 421001, People’s Republic of China
51 University of the Punjab, Lahore-54590, Pakistan
52 University of Turin, I-10125, Turin, Italy; (B)University of Eastern Piedmont, I-15121, Alessandria, Italy; (C)INFN, I-10125, Turin, Italy
53 Uppsala University, Box 516, SE-75120 Uppsala, Sweden
54 Wuhan University, Wuhan 430072, People’s Republic of China
55 Zhejiang University, Hangzhou 310029, People’s Republic of China
56 Zhengzhou University, Zhengzhou 450001, People’s Republic of China
57 a Also at Bogazici University, 34342 Istanbul, Turkey
b Also at the Moscow Institute of Physics and Technology, Moscow 141700, Russia
c Also at the Functional Electronics Laboratory, Tomsk State University, Tomsk, 634050, Russia
d Also at the Novosibirsk State University, Novosibirsk, 630090, Russia
e Also at the NRC "Kurchatov Institute", PNPI, 188300, Gatchina, Russia
f Also at Istanbul Arel University, 34295 Istanbul, Turkey
9 Also at Goethe University Frankfurt, 60525 Frankfurt am Main, Germany
h Also at Key Laboratory for Particle Physics, Astrophysics and Cosmology, Ministry of Education; Shanghai Key Laboratory for Particle Physics and Cosmology; Institute of Nuclear and Particle Physics, Shanghai 200240, People’s Republic of China
i Also at Government College Women University, Sialkot - 51310. Punjab, Pakistan.
The charm semileptonic decays $D^+ \rightarrow \eta e^+\nu_e$ and $D^+ \rightarrow \eta' e^+\nu_e$ are studied with a sample of $e^+e^-$ collision data corresponding to an integrated luminosity of 2.93 fb$^{-1}$ collected at $\sqrt{s} = 3.773$ GeV with the BESIII detector. We measure the branching fractions for $D^+ \rightarrow \eta e^+\nu_e$ to be $(10.74 \pm 0.81 \pm 0.51) \times 10^{-4}$, and for $D^+ \rightarrow \eta' e^+\nu_e$ to be $(1.91 \pm 0.51 \pm 0.13) \times 10^{-4}$, where the uncertainties are statistical and systematic, respectively. In addition, we perform a measurement of the form factor in the decay $D^+ \rightarrow \eta e^+\nu_e$. All the results are consistent with those obtained by the CLEO-c experiment.

Keywords: BESIII, charm semileptonic decay, form factor

I. INTRODUCTION

Charm semileptonic (SL) decays involve both the c-quark weak decay and the strong interaction. In the Standard Model, the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1] describes the mixing among the quark flavors in the weak decay. The strong interaction effects in the hadronic current are parameterized by a form factor, which is numerically calculable with Lattice Quantum Chromodynamics (LQCD). The differential decay rate for the charm SL decay $D^+ \rightarrow \eta e^+\nu_e$, neglecting the positron mass, is given by

$$\frac{d\Gamma(D^+ \rightarrow \eta e^+\nu_e)}{dq^2} = \frac{G_F^2 |V_{cd}|^2}{24\pi^3} |\vec{p}\eta|^3 |f_+(q^2)|^2,$$  \hspace{1cm} (1)

where $G_F$ is the Fermi constant, $V_{cd}$ is the relevant CKM matrix element, $\vec{p}\eta$ is the momentum of the $\eta$ meson in the $D^+$ rest frame, and $f_+(q^2)$ is the form factor parametrizing the strong interaction dynamics as a function of the squared four-momentum transfer $q^2$, which is the square of the invariant mass of the $e^+\nu_e$ pair. Precise measurements of the SL decay rates provide input to constrain the CKM matrix element $V_{cd}$ and to test the theoretical descriptions of the form factor. LQCD calculations of the form factor can be tested by comparing to the ones determined from the partial branching fraction (BF) measurements, once the CKM matrix element $V_{cd}$ is known.

Moreover, the mixing $\eta-\eta'$ or $\eta'-G$, where $G$ stands for a glueball, is of great theoretical interest, because it concerns many aspects of the underlying dynamics and hadronic structure of pseudoscalar mesons and glueballs [2]. The SL decay $D^+ \rightarrow \eta' e^+\nu_e$ can be used to study the $\eta-\eta'$ mixing in a much cleaner way than in hadronic processes due to the absence of final-state interactions for a glueball, is of great theoretical interest, because it concerns many aspects of the underlying dynamics and hadronic structure of pseudoscalar mesons and glueballs [2].

Based on a data sample with an integrated luminosity of 818 pb$^{-1}$ collected at $\sqrt{s} = 3.77$ GeV, the CLEO collaboration measured the BF for $D^+ \rightarrow \eta e^+\nu_e$ and $D^+ \rightarrow \eta' e^+\nu_e$ to be $B_{\eta e^+\nu_e} = (11.4 \pm 0.9 \pm 0.4) \times 10^{-4}$ and $B_{\eta' e^+\nu_e} = (2.16 \pm 0.53 \pm 0.07) \times 10^{-4}$ [4], respectively. In this paper, we present new measurements of these BFs, using $D\bar{D}$ meson pairs produced near threshold at $\sqrt{s} = 3.773$ GeV with an integrated luminosity of $2.93$ fb$^{-1}$ [5] collected with the BESIII detector [6]. In addition, the modulus of the form factor $f_+(q^2)$ in $D^+ \rightarrow \eta e^+\nu_e$ is measured.

II. THE BESIII DETECTOR

The Beijing Spectrometer (BESIII) detects $e^+e^-$ collisions produced by the double-ring collider BEPCII. BESIII is a general-purpose detector [6] with 93% coverage of the full solid angle. From the interaction point (IP) to the outside, BESIII is equipped with a main drift chamber (MDC) consisting of 43 layers of drift cells, a time-of-flight (TOF) counter with double-layer scintillator in the barrel part and single-layer scintillator in the end-cap part, an electromagnetic calorimeter (EMC) composed of 6240 CsI(Tl) crystals, a superconducting solenoid magnet providing a magnetic field of 1.0 T along the beam direction, and a muon counter containing multi-layer resistive plate chambers installed in the steel flux-return yoke of the magnet. The MDC spatial resolution is about 135 μm and the momentum resolution is about 0.5% for a charged track with transverse momentum of 1 GeV/c. The energy resolution for electromagnetic showers in the EMC is 2.5% at 1 GeV. More details of the spectrometer can be found in Ref. [6].

III. MC SIMULATION

Monte Carlo (MC) simulation serves to estimate the detection efficiencies and to understand background components. High statistics MC samples are generated with a GEANT4-based [7] software package, which includes simulations of the geometry of the spectrometer and interactions of particles with the detector materials. KKMC is used to model the beam energy spread and the initial-state radiation (ISR) in the $e^+e^-$ annihilations [8]. The ‘inclusive’ MC samples consist of the production of $D\bar{D}$ pairs with consideration of quantum coherence for all neutral $D$ modes, the non-$D\bar{D}$ decays of $\psi(3770)$, the ISR production of low mass $\psi$ states, and continuum processes (quantum electrodynamics (QED) and $q\bar{q}$). Known decays recorded by the Particle Data Group (PDG) [9] are simulated with EVTGEN [10] and the unknown decays with LUNDCHARM [11]. The final-state radiation (FSR) of charged tracks is taken into account with the PHOTOS package [12]. The equivalent luminosity of the inclusive MC samples is about 10 times that of the data. The signal processes of $D^+ \rightarrow \eta(1440)e^+\nu_e$ are generated using the modified pole model of Ref. [13].
IV. DATA ANALYSIS

As the ψ(3770) is close to the D̅D threshold, the pair of D⁺D− mesons is produced nearly at rest without accompanying additional hadrons. Hence, it is straightforward to use the D-tagging method [14] to measure the absolute BFs, based on the following equation

\[ B_{\eta'\rightarrow e^+\nu_e} = \frac{n_{\eta'\rightarrow e^+\nu_e,\text{tag}} \cdot \epsilon_{\text{tag}}}{n_{\text{tag}}} \cdot \xi_{\eta'\rightarrow e^+\nu_e,\text{tag}}. \] (2)

Here, \( n_{\text{tag}} \) is the total yield of the single-tag (ST) D− mesons reconstructed with hadronic decay modes, while \( n_{\eta'\rightarrow e^+\nu_e,\text{tag}} \) is the number of the \( D^+ \rightarrow \eta'\nu_e \) signal events when the ST D− meson is detected. \( \epsilon_{\text{tag}} \) and \( \xi_{\eta'\rightarrow e^+\nu_e,\text{tag}} \) are the corresponding detection efficiencies. Note that in the context of this paper, charge conjugated modes are always implied.

A. Reconstruction of the hadronic tag modes

The \( D^- \) decay modes used for tagging are \( K^+\pi^-\pi^- \), \( K^+\pi^-\pi^0 \), \( K^0_S\pi^- \), \( K^0_S\pi^-\pi^0 \), \( K^0_S\pi^+\pi^-\pi^- \) and \( K^+K^-\pi^- \), where \( \pi^0 \to \gamma\gamma \), and \( K^0_S \to \pi^+\pi^- \). The sum of the BFs of these six decay modes is about 27.7%. \( D^- \) tag candidates are reconstructed from all possible combinations of final state particles, according to the following selection criteria.

Moments and impact parameters of charged tracks are measured by the MDC. Charged tracks are required to satisfy \( |\cos \theta| < 0.93 \), where \( \theta \) is the polar angle with respect to the beam axis, and have a closest approach to the interaction point (IP) within ±10 cm along the beam direction and within ±1 cm in the plane perpendicular to the beam axis. Particle identification (PID) is implemented by combining the information of specific energy loss (dE/dx) in the MDC and the time of flight measurements from the TOF into PID likelihoods for the different particle hypotheses. For a charged \( \pi(K) \) candidate, the likelihood of the \( \pi(K) \) hypothesis is required to be larger than that of the \( K(\pi) \) hypothesis.

Photons are reconstructed as energy deposition clusters in the EMC. The energies of photon candidates must be larger than 25 MeV for \( |\cos \theta| < 0.8 \) (barrel) and 50 MeV for \( 0.86 < |\cos \theta| < 0.92 \) (end cap). To suppress fake photons due to electronic noise or beam backgrounds, the shower time must be less than 700 ns from the event start time [15].

The \( \pi^0 \) candidates are selected from pairs of photons of which at least one is reconstructed in the barrel. The two photon invariant mass, \( M(\gamma\gamma) \), is required to lie in the range \( (0.115, 0.150) \) GeV/\( c^2 \). We further constrain the invariant mass of each photon pair to the nominal \( \pi^0 \) mass, and update the four-momentum of the candidate according to the fit results.

The \( K^0_S \) candidates are reconstructed via \( K^0_S \to \pi^+\pi^- \) using a vertex-constrained fit to all pairs of oppositely charged tracks, without PID requirements. The distance of closest approach of a charged track to the IP is required to be less than 20 cm along the beam direction, without requirement in the transverse plane. The \( \chi^2 \) of the vertex fit is required to be less than 100. The invariant mass of the \( \pi^+\pi^- \) pair is required to be within \( (0.487, 0.511) \) GeV/\( c^2 \), which corresponds to three times the experimental mass resolution.

Two variables, the beam-constrained mass, \( M_{BC} \), and the energy difference, \( \Delta E \), are used to identify the tagging signals, defined as follows

\[ M_{BC} = \sqrt{E_{beam}^2/c^4 - |\vec{p}_D|²/c^2}, \]

\[ \Delta E = E_D - E_{beam}. \]

Here, \( \vec{p}_D \) and \( E_D \) are the total momentum and energy of the \( D^- \) candidate in the rest frame of the initial \( e^+e^- \) system, and \( E_{beam} \) is the beam energy. Signals peak around the nominal \( D^- \) mass in \( M_{BC} \) and around zero in \( \Delta E \). Boundaries of \( \Delta E \) requirements are set at \(|\pm 3\sigma|\), except that those of modes containing a \( \pi^0 \) are set as \((|\pm 4\sigma|, +3\sigma)\) due to the asymmetric distributions. Here, \( \sigma \) is the standard deviation from the nominal value of \( \Delta E \). In each event, only the combination with the least \(|\Delta E|\) is kept per \( D^- \)-tagging mode.

| Modes          | \( \Delta E \) (GeV) | \( \epsilon_{\text{tag}} \) (%) | \( n_{\text{tag}} \) |
|----------------|----------------------|-------------------------------|-----------------|
| \( K^+\pi^-\pi^- \) | -0.023, 0.022       | 50.94 ± 0.03                 | 801 283 ± 949  |
| \( K^+\pi^-\pi^0 \) | -0.058, 0.032       | 25.40 ± 0.03                 | 246 770 ± 699  |
| \( K^0_S\pi^- \) | -0.023, 0.024       | 52.59 ± 0.09                 | 97 765 ± 328   |
| \( K^0_S\pi^-\pi^0 \) | -0.061, 0.037      | 28.07 ± 0.03                 | 217 816 ± 632  |
| \( K^0_S\pi^+\pi^- \) | -0.027, 0.025      | 32.28 ± 0.05                 | 126 236 ± 425  |
| \( K^+K^-\pi^- \) | -0.020, 0.019       | 40.08 ± 0.08                 | 69 869 ± 326   |

After applying the \( \Delta E \) requirements in Table I in all the ST modes, we plot their \( M_{BC} \) distributions in Fig. 1. Maximum likelihood fits to these \( M_{BC} \) distributions are performed, where in each mode the signals are modeled with the MC-simulated signal shape convolved with a smearing Gaussian function with free parameters, and the backgrounds are modeled with the ARGUS function [10]. The Gaussian functions are supposed to compensate for the resolution differences between data and MC simulations. Based on the fit results, ST yields of data are given in Table I in the \( M_{BC} \) mass range \([1.86, 1.88] \) GeV/\( c^2 \), along with their MC-determined detection efficiencies.

B. Reconstruction of SL signals

We look for the SL signal of \( D^+ \rightarrow \eta(\phi)\nu_e \) in the events when the ST \( D^- \) mesons are found to satisfy the
FIG. 1. Distributions of $M_{BC}$ for the six ST modes. Data are shown as points with error bars. The solid lines are the total fits to the data.
15% of the total resolution. The background shapes of different \( \eta^{(i)} \) decay modes are modeled with the distributions from backgrounds obtained from the inclusive MC sample. In total, we observe 373 ± 26 signal events for \( D^+ \rightarrow \eta e^+ \nu_e \) and 31.6 ± 8.4 for \( D^+ \rightarrow \eta^0 e^+ \nu_e \). The BF for \( D^+ \rightarrow \eta^{(i)} e^+ \nu_e \) is determined by using Eq. (2) according to the MC-determined efficiencies in Table II, which gives \( B_{\eta e^+ \nu_e} = (10.74 ± 0.81) \times 10^{-4} \), and \( B_{\eta^0 e^+ \nu_e} = (1.91 ± 0.51) \times 10^{-4} \).

The statistics of \( D^+ \rightarrow \eta e^+ \nu_e \) allows to determine \( f_+(q^2) \), as defined in Eq. (1). Hence, a fit is implemented to the partial BFs in the three \( q^2 \) bins used in Fig. 2. By introducing the life time \( \tau_{D^+} = (1040 ± 7) \times 10^{-15} \) s from PDG [9], we construct \( \chi^2 = \Delta \gamma^2 V^{-1} \Delta \gamma \), where \( \Delta \gamma = \Delta \Gamma_m - \Delta \Gamma_p \) is the vector of differences between the measured partial decay widths \( \Delta \Gamma_m \) and the expected partial widths \( \Delta \Gamma_p \) integrated over the different \( q^2 \) bins, and \( V \) is the total covariance matrix consisting of the statistical covariance matrix \( V_{\text{stat}} \) and the systematic covariance \( V_{\text{syst}} \). The statistical correlations among the different \( q^2 \) bins are negligible. We list the elements of the total covariance matrix \( V \) in Table III.

| Modes | \( D^+ \rightarrow \eta e^+ \nu_e \) | \( D^+ \rightarrow \eta^0 e^+ \nu_e \) |
|-------|-----------------|-----------------|
| Sub-decay modes | \( \gamma \gamma \) | \( \eta^0 \rightarrow \gamma \gamma \) |
| \( K^+ \pi^+ \pi^- \) | 23.58 ± 0.09 12.65 ± 0.07 | 8.50 ± 0.09 2.41 ± 0.05 11.68 ± 0.11 |
| \( K^+ \pi^+ \pi^- \pi^0 \) | 9.77 ± 0.07 4.75 ± 0.05 | 3.48 ± 0.06 0.82 ± 0.03 4.96 ± 0.07 |
| \( K^0_s \pi^- \) | 25.23 ± 0.09 13.45 ± 0.08 | 9.23 ± 0.09 2.29 ± 0.05 12.47 ± 0.11 |
| \( K^0_s \pi^- \pi^0 \) | 9.82 ± 0.07 5.40 ± 0.05 | 4.60 ± 0.07 0.83 ± 0.03 5.83 ± 0.08 |
| \( K^0_s \pi^+ \pi^- \pi^0 \) | 13.98 ± 0.08 6.24 ± 0.05 | 4.09 ± 0.06 0.82 ± 0.03 5.87 ± 0.08 |
| \( K^+ K^- \pi^- \) | 18.41 ± 0.09 9.93 ± 0.07 | 6.28 ± 0.08 1.52 ± 0.04 8.18 ± 0.09 |

where \( m_{\text{pole}} \) is fixed at the mass of \( D^{*+} \) and \( \alpha \) is a free parameter to be determined. The third is a general series parametrization with \( z \)-expansion, which is formulated as

\[
f_+(q^2) = \frac{1}{P(q^2)\phi(q^2, t_0)} \sum_{k=0}^{\infty} a_k(t_0)[z(q^2, t_0)]^k.
\]

Here, \( t_0 = t_+ (1 - \sqrt{1 - t_+/t_+}) \) with \( t_\pm = (m_{D^{*\pm}} + m_\eta)^2 \) and \( a_k(t_0) \) are real coefficients. The functions \( P(q^2) \), \( \phi(q^2, t_0) \) and \( z(q^2, t_0) \) are formulated following the definitions in Ref. [17]. In the fit, the series is truncated at \( k = 1 \).

Three separate fits to data are implemented, based on the three form-factor models. Their fit curves are plotted in Fig. 3. We determine the values of \( f_+(0)|V_{\text{cd}}| \) in all three scenarios, as listed in Table IV. We observe that the results of \( f_+(0)|V_{\text{cd}}| \) in the three fits are consistent and the fit qualities are good.

| TABLE II. SL signal detection efficiencies for the different different ST tag modes in percent. The errors are all statistical. |
|---|---|---|
| Modes | \( D^+ \rightarrow \eta e^+ \nu_e \) | \( D^+ \rightarrow \eta^0 e^+ \nu_e \) |
| Sub-decay modes | \( \pi^+ \pi^- \pi^0 \) | \( \pi^+ \pi^- \pi^0 \) | \( \gamma \rho^0 \) |
| \( K^+ \pi^+ \pi^- \) | 23.58 ± 0.09 12.65 ± 0.07 | 8.50 ± 0.09 2.41 ± 0.05 11.68 ± 0.11 |
| \( K^+ \pi^+ \pi^- \pi^0 \) | 9.77 ± 0.07 4.75 ± 0.05 | 3.48 ± 0.06 0.82 ± 0.03 4.96 ± 0.07 |
| \( K^0_s \pi^- \) | 25.23 ± 0.09 13.45 ± 0.08 | 9.23 ± 0.09 2.29 ± 0.05 12.47 ± 0.11 |
| \( K^0_s \pi^- \pi^0 \) | 9.82 ± 0.07 5.40 ± 0.05 | 4.60 ± 0.07 0.83 ± 0.03 5.83 ± 0.08 |
| \( K^0_s \pi^+ \pi^- \pi^0 \) | 13.98 ± 0.08 6.24 ± 0.05 | 4.09 ± 0.06 0.82 ± 0.03 5.87 ± 0.08 |
| \( K^+ K^- \pi^- \) | 18.41 ± 0.09 9.93 ± 0.07 | 6.28 ± 0.08 1.52 ± 0.04 8.18 ± 0.09 |

V. SYSTEMATIC UNCERTAINTIES

With the double-tag technique, the systematic uncertainties in detecting the ST \( D^- \) mesons in the BF measurements mostly cancel as shown in Eq. (2). For the SL signal side, the following sources of systematic uncertainties are studied, as summarized in Table V. All of these contributions are added in quadrature to obtain the total systematic uncertainties on the BFs.

The uncertainties of tracking and PID efficiencies for \( \pi^\pm \) are studied with control samples of \( D \bar{D} \) Cabibbo favored ST decays [18]. The uncertainties in \( \epsilon^\pm \) tracking and PID efficiencies are estimated with radiative Bhabha events, taking account of the different tracking and PID efficiencies in different \( \cos \theta \) and momentum distributions of \( \epsilon^\pm \).

The uncertainty due to the \( \pi^0 \) and \( \eta \) reconstruction efficiency is estimated with a control sample using \( D^0 \rightarrow K^- \pi^+ \pi^- \) selected without requiring the \( \pi^0 \) meson. The uncertainties associated with the \( \eta \) and \( \eta^0 \) invariant mass requirements are estimated by changing the requirement boundaries and taking the maximum variations of the resultant BFs as systematic uncertainties. The uncertainty due to the extra shower veto is studied with doubly tagged hadronic events, and is found to be negligible.
FIG. 2. Distributions of $U_{\text{miss}}$ for the different signal modes. Data are shown as points with error bars. The solid lines are the total fits and the dashed lines are the background contributions. Data for $D^+ \rightarrow \eta e^+ \nu_e$ are plotted in 3 bins of $0.0 < q^2 < 0.6$ GeV$^2/c^4$ (a, d), $0.6 < q^2 < 1.2$ GeV$^2/c^4$ (b, e) and $q^2 > 1.2$ GeV$^2/c^4$ (c, f).

TABLE IV. The fit results of the form-factor parameters. For simple pole and modified pole parameterizations, shape parameters denote $m_{\text{pole}}$ and $\alpha$, respectively. For the series parametrization, we provide results of $f_3(0)|V_{cd}|$, $r_1 = a_1/a_0$ (shape parameter). The correlation coefficients $\rho$ between fitting parameters and the reduced $\chi^2$ are given.

| Fit parameters                        | Simple pole     | Modified pole   | Series expansion |
|---------------------------------------|-----------------|-----------------|-----------------|
| $f_3(0)|V_{cd}| \times 10^{-2}$       | 8.15 $\pm$ 0.45 $\pm$ 0.18 | 8.24 $\pm$ 0.51 $\pm$ 0.22 | 7.86 $\pm$ 0.64 $\pm$ 0.21 |
| Shape parameter                       | 1.73 $\pm$ 0.17 $\pm$ 0.03 | 0.50 $\pm$ 0.54 $\pm$ 0.08 | -7.33 $\pm$ 1.69 $\pm$ 0.40 |
| $\rho$                                | 0.80            | -0.85           | 0.90            |
| $\chi^2/\text{ndf}$                    | $0.1/(3 - 2)$   | $0.3/(3 - 2)$   | $0.5/(3 - 2)$   |

The uncertainties of the radiative $\gamma$ selection in $\eta' \rightarrow \gamma \rho^0$ are studied using a control sample from $D^0 \bar{D}^0$ decays where the $D^0$ meson decays to $K_S^0 \eta'$, $\eta' \rightarrow \gamma \rho^0$ and the $\bar{D}^0$ decays to Cabibbo favored ST modes. We impose the same selection criteria on the radiative photon to the control sample, and the difference of signal survival rates between data and MC simulations is found to be 3.1%. The uncertainty due to the $\rho$ invariant mass requirement is also estimated with this control sample. The difference of signal survival rates between data and MC simulations is found to be 0.6%.

In the fit to the $U_{\text{miss}}$ distribution, the uncertainty due to the parametrization of the signal shape is estimated by introducing a Gaussian function to smear the MC-simulated signal shape and varying the parameters of the smearing Gaussian. The uncertainty due to the background modeling is estimated by changing the background model to a 3rd degree Chebychev polynomial. The uncertainty due to the fit range is estimated by repeating the fits in several different ranges. The uncer-
tainties of the input BFs and the limited MC statistics are also taken into account.

We also study the $\Delta E$ and $M_{BC}$ requirements by varying the ranges and compare the efficiency-corrected tag yields. The resultant maximum differences are taken as systematic uncertainties. The SL signal model for $D^+ \to \eta e^+\nu_e$ is simulated according to the form factor measured in this work and the variations within one standard deviation are studied. For $D^+ \to \eta' e^+\nu_e$, since there is no available form-factor data, we take the form factor of $D^+ \to \eta e^+\nu_e$ and evaluate the systematic uncertainty as we do for $D^+ \to \eta e^+\nu_e$.

Table V. Relative systematic uncertainties in the BF measurements (in %). The lower half of the table presents the common uncertainties among the different channels.

| Source | $D^+ \to \eta e^+\nu_e$ | $D^+ \to \eta' e^+\nu_e$ |
|--------|------------------------|--------------------------|
| $\pi^+$ tracking and PID | 2.8 | 4.1 |
| $\pi^0/\eta$ reconstruction | 2.0 | 2.2 |
| Input BF | 0.3 | 1.7 |
| $\rho$ mass window | 0.6 | 2.0 |
| Radiative $\gamma$ | 3.1 | 1.7 |
| $\eta'$ mass window | 1.8 | 1.6 |
| $e^+$ tracking and PID | 1.1 | 3.7 |
| $\eta$ mass window | 2.4 | 2.4 |
| $U_{miss}$ fit | 2.1 | 1.0 |
| $\Delta E/M_{BC}$ window | 0.9 | 0.9 |
| MC statistics | 0.2 | 0.5 |
| SL signal model | 0.9 | 0.9 |
| Total | 4.7 | 6.9 |

Systematic uncertainties of the partial decay widths of $D^+ \to \eta e^+\nu_e$ to calculate $V_{\text{syst}}$, are studied following the same procedure mentioned above. For most of the common systematics, we quote the values from the total BF measurements in Table V. For charged pion tracking and PID, we evaluate the uncertainty averaged over the two $\eta$ decay modes according to their relative yields. For $e^+$ tracking and PID, we reweight the systematic uncertainties in each $q^2$ bin. All these items are summarized in Table VI. For the systematics of $\eta$ mass window and fitting procedure, we re-fit the $U_{miss}$ distribution after varying the $\eta$ mass window and changing fitting region and compare the refitting results of the form factors. The maximum deviations from the nominal results are calculated to be $1.3\%$ and $0.4\%$ for the $f_+(0)\{|V_{cd}|^2$ and shape parameter and are considered as systematic uncertainties. The sum of the systematic uncertainties is given in Table IV.

| Source | $q^2$ (GeV$^2$/c$^4$) |
|--------|----------------------|
| $e^+$ tracking and PID | 1.4 |
| $\pi^0$ tracking and PID | 1.7 |
| $\pi^0/\eta$ reconstruction | 2.0 |
| $\Delta E/M_{BC}$ window | 0.9 |
| MC statistics | 0.2 |
| SL signal model | 0.9 |
| Input BF | 0.3 |
| $D^+$ lifetime | 0.7 |
| Total | 3.3 |

Table VI. Relative systematic uncertainties (in %) of the measured partial decay widths of $D^+ \to \eta e^+\nu_e$ used to obtain $V_{\text{syst}}$. $D^+ \to \eta' e^+\nu_e$ to be $B_{\eta' e^+\nu_e} = (1.91 \pm 0.51 \pm 0.13) \times 10^{-4}$, where the first and second uncertainties are statistical and systematic, respectively, and $D^+ \to \eta e^+\nu_e$ based on three form-factor models, whose results are given in Table IV. This helps to calibrate the form-factor calculation in LQCD. All these results are consistent with the previous measurements from CLEO-c [4]. Our precision is only slightly better than CLEO-c’s, because our limitations on PID and low-momentum tracking efficiency hinder to adopt CLEO-c’s generic $D$-tagging method [4]. We average the results of $B_{\eta e^+\nu_e}$ and $B_{\eta' e^+\nu_e}$ in the two experiments to be $(11.04 \pm 0.60 \pm 0.33) \times 10^{-4}$ and $(2.04 \pm 0.37 \pm 0.08) \times 10^{-4}$, respectively. Using the input value recommended by Ref. [2], the $\eta - \eta'$ mixing angle $\phi_P$ is determined to be $\left(40 \pm 3 \pm 3\right)^\circ$, where the first uncertainty is experimental and the second theoretical, in agreement with the results obtained by Ref. [2]. However, the current precision for $D^+ \to \eta' e^+\nu_e$ is not enough to provide meaningful constraints on the $\eta - \eta'$ mixing parameters.

ACKNOWLEDGMENTS

The BESIII collaboration thanks the staff of BEPCII, the IHEP computing center and the supercomputing center of USTC for their strong support. This work is supported in part by National Key Basic Research Program of China under Contract No. 2015CB856700; National Natural Science Foundation of China (NSFC) under Contracts Nos. 11625523, 11635010, 11735014, 11475164, 11475169, 11605196, 11625523, 11635010, 11735014; the Chinese Academy of Sciences (CAS) Large-Scale Scientific Facility Program; the CAS Center for Excellence in Particle Physics (CCEPP); Joint Large-Scale Scientific Facility Funds of the NSFC and CAS under Contracts Nos. U1532102, U1532257, U1532258, U1732263; CAS Key Research Program of Frontier Sciences under Contracts Nos. QYZDJ-SSW-SLH003, QYZDJ-SSW-SLH040; 100 Talents Program of CAS; INPAC and Shanghai Key Laboratory for Particle

VI. SUMMARY

We exploit a double-tag technique to analyze a sample of $2.93 \text{ fb}^{-1} e^+e^- \to D^+D^- \sqrt{s} = 3.773 \text{ GeV}$. The BF for the SL decay $D^+ \to \eta e^+\nu_e$ is measured to be $B_{\eta e^+\nu_e} = (10.74 \pm 0.81 \pm 0.51) \times 10^{-4}$, and for
Physics and Cosmology; German Research Foundation DFG under Contracts Nos. Collaborative Research Center CRC 1044, FOR 2359; Istituto Nazionale di Fisica Nucleare, Italy; Koninklijke Nederlandse Akademie van Wetenschappen (KNAW) under Contract No. 530-4CDP03; Ministry of Development of Turkey under Contract No. DPT2006K-120470; National Science and Technology fund; The Swedish Research Council; U. S. Department of Energy under Contracts Nos. DE-FG02-05ER41374, DE-SC-0010118, DE-SC-0010504, DE-SC-0012069; University of Groningen (RuG) and the Helmholtzzentrum fuer Schwerionenforschung GmbH (GSI), Darmstadt.

[1] N. Cabibbo, Phys. Rev. Lett. 10 (1963) 531; M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49 (1973) 652.
[2] H. W. Ke, X. Q. Li and Z. T. Wei, Eur. Phys. J. C 69 (2010) 133; H. Li, Nucl. Phys. Proc. Suppl. 162 (2006) 312.
[3] C. Di Donato, G. Ricciardi and I. Bigi, Phys. Rev. D 85 (2012) 013016.
[4] J. Yelton et al. [CLEO Collaboration], Phys. Rev. D 84 (2011) 032001.
[5] M. Ablikim et al. [BESIII Collaboration], Chin. Phys. C 37 (2013) 123001; M. Ablikim et al. [BESIII Collaboration], Phys. Lett. B 753 (2016) 629.
[6] M. Ablikim et al. [BESIII Collaboration], Nucl. Instrum. Meth. A 614 (2010) 345.
[7] S. Agostinelli et al. [GEANT4 Collaboration], Nucl. Instrum. Meth. A 506 (2003) 250.
[8] S. Jadach, B. F. L. Ward and Z. Was, Phys. Rev. D 63 (2001) 113009.
[9] C. Patrignani et al. [Particle Data Group], Chin. Phys. C 40 (2016) 100001.
[10] D. J. Lange, Nucl. Instrum. Meth. A 462 (2001) 152; R. G. Ping, Chin. Phys. C 32 (2008) 509.
[11] J. C. Chen et al., Phys. Rev. D 62 (2000) 034003.
[12] E. Richter-Was, Phys. Lett. B 303 (1993) 163.
[13] D. Becirevic and A. B. Kaidalov, Phys. Lett. B 478 (2000) 417.
[14] R. M. Baltrusaitis et al. [MARK-III Collaboration], Phys. Rev. Lett. 56 (1986) 2140; J. Adler et al. [MARK-III Collaboration], Phys. Rev. Lett. 60 (1988) 89.
[15] Xiang Ma et al., Chin. Phys. C 32 (2008) 744; Y. Guan, X.-R. Lu, Y. Zheng and Y.-F. Wang, Chin. Phys. C 38 (2014) 016201.
[16] H. Albrecht et al. [ARGUS Collaboration], Phys. Lett. B 241 (1990) 278.
[17] T. Becher and R. J. Hill, Phys. Lett. B 633 (2006) 61.
[18] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. D 92 (2015) 072012.