Secure Convolutional Neural Networks using FHE

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Abstract
In this paper, a secure Convolutional Neural Network classifier is proposed using Fully Homomorphic Encryption (FHE). The secure classifier provides a user with the ability to outsource the computations to a powerful cloud server and/or setup a server to classify inputs without providing the model or revealing source data. To this end, a real number framework is developed over FHE by using a fixed point format with binary digits. This allows for real number computations for basic operators like addition, subtraction, and multiplication but also to include secure comparisons and max functions. Additionally, a rectified linear unit is designed and realized in the framework. Experimentally, the model was verified using a Convolutional Neural Network trained for handwritten digits. This encrypted implementation shows accurate results for all classification when compared against an unencrypted implementation.

1 Introduction
Privacy and security are at the forefront and keystone problems of cloud computing in today's connected world. Cloud computing allows users to easily outsource their computations but at the cost of losing privacy and security. Maintain the privacy and security of data, particularly personal information, can be prohibitive on the use of a cloud computing resource. Protecting the data and algorithms for usage on cloud computers opens the aperture to many more applications. The protection must provide the method to limit an attacker's ability to use the data. On a parallel tract, machine learning is an important technology in use today. Uses of machine learning algorithms continue to increase, expand and then become vital components of today's world. As a concrete problem to focus on, consider a situation of needing to outsource the classification of a Convolutional Neural Network [LeCun et al. 1995] and/or
Figure 1: Convolutional Neural Network in a Fully Homomorphic Encryption framework

providing a classification from a Convolutional Neural Network to a user without providing the model nor revealing the original source data. In this paper we propose a model for performing a secure realization of a Convolutional Neural Network using FHE.

In the FHE framework, a user processes their private data on an external computationally powerful resource with a level of privacy. In an abstract view of the process, the user encrypts their data, securely transfers it to the computing resource, runs the process, securely transfers back the result, and then decrypts it. Figure [1] depicts this concept in the case of using a Convolutional Neural Network. At the start, the user encrypts their image and securely transfers the image to the server (gray arrow). Without loss of generality, image is used to identify the 2D object that is input to the Convolutional Neural Network. With the encrypted image on the server, the convolution layers of the network are run; followed by the fully connected layer. Despite the simplicity of the CNN architecture considered in this paper, the proposed FHE model can be extended to more complex models. It needs to be noted at this point that any structure of a CNN can be run. After the processing is complete, the user securely transfers the data back for decryption. Decrypting the result provides the user with the classification results from the Convolutional Neural Network. While conceptually this process is very simple, extending the FHE to support the CNN over reals is a nontrivial task.

Understanding what a Fully Homomorphic Encryption scheme provides is critical to understanding the hardness of the problem. The first scheme was developed by [Gentry 2009] and which provided the basic functionality for encrypted data processing using addition and multiplication. This research uses a sequel scheme proposed by Gentry in 2013 [Gentry et al. 2013] which sup-
ports two possible numerical spaces: binary numbers ($\{0, 1\}$) and integers ($\mathbb{Z}_q$ for a modulus $q$ which is an FHE parameter). To support any processing over real numbers, new models are needed that represent real numbers in the FHE framework. As the proposed model provides an approximation model for original reals, there will be errors introduced by the representation. The primary contributions of this research are the ability to perform a Convolutional Neural Network while the data is encrypted using an approximation framework for real numbers, as well as a theoretical bound on the error introduced by the framework. This in turn enables the user with a trade off on time/space complexities for accuracy or vice versa.

The basis of the FHE scheme’s security hardness is the Learning With Errors problem by Regev [2009]. The problem hardness is equivalent to the shortest vector problem in lattices and is considered quantum hard. This hardness result is what allows FHE to process data while encrypted. In this model, the ciphertexts are generally represented as matrices and plaintexts are embedded along the diagonal. A ciphertext can only be decrypted using the secret key which is known as the trap door of the hardness problem. Within this approach, Gentry et al. [2013] supports two plaintext spaces: binary numbers and integers modulo a set value $q$. The scheme provides a single operator for binary numbers: NAND. As Wesselkamper et al. [1975] showed, NAND gates can be used to create all other binary gates. In a general set up, the gate can be computed via matrix-matrix arithmetic $I - C_1 \times C_2$, which requires matrix multiplication. This latter computation is the main source of time and space complexity for the scheme. We will employ the NAND gate to compute fixed point binary numbers in FHE. Lastly, FHE schemes have an additional capability to refresh ciphertexts. This is will be a critical in our approach because the Learning with Errors problem introduces noise into the ciphertext, which is sometimes characterized as error. We will refer to the Learning with Errors error as noise and the numerical introduced by the framework as error to avoid confusion.

The Convolutional Neural Networks considered here is based by LeCun LeCun et al. [1995] that reduces the number of trainable parameters in an equivalent fully connected neural network. This in turn reduces the number of trainable parameters by creating layers that use a convolution of an image over a smaller subset of the pixels. The individual convolutions are known as feature maps and are equivalent to a dot product of the following form:

$$\sum_{i=0}^{N} c_i \cdot x_i,$$  \hspace{1cm} (1)

where $c_i$ is the trained constant, $x_i$ is the input, $N$ is the entire size of the convolution, and starting at 0 allows for a bias value to be introduced. The computation also assumes that the two dimensional section is aligned into a single array. As the convolutions are done as a standard neural network node, the additional change is that the activation function is a rectified linear unit.
ReLU:

$$relu(x) = \begin{cases} 
  x & \text{if } x > 0 \\
  0 & \text{otherwise}
\end{cases}$$  \hspace{1cm} (2)

The final step of a convolution layer is subsampling via max pooling of a set of outputs. Max pooling takes in a segment of the image and selects the max value of the segment.

$$\max(x_1, x_2, \ldots, x_n)$$  \hspace{1cm} (3)

A set of convolution layers is used to compute feature maps and reduce the size to a manageable value of trainable parameters for the fully connected layer(s).

The remainder of the paper is organized as follows. Section 2 provides an overview of related work and current research in the realm of FHE, secure processing, and machine learning. Section 3 describes the framework for realizing Convolutional Neural Networks in FHE. Section 4 discusses the experimental results of applying the CNN under FHE framework. Section 5 concludes this paper along with future work.

2 Related Works

This section contains details on related works in FHE, secure processing, and machine learning. As mentioned in the Introduction, the first FHE scheme was developed by Gentry [2009]. This original scheme was limited to binary numbers and was inefficient in comparison to later schemes. Additional significant work continued in the early 2010s time frame to improve the FHE schemes from both functional capabilities and time/space efficiencies (see Brakerski [2012], Brakerski et al. [2012], Brakerski and Vaikuntanathan [2011]). These improvements enabled processing to include the integer ring vice binary. Gentry et al. [2013] was chosen as the FHE scheme for this research because it further improved on the earlier schemes with a simpler conceptual and implementation framework. Additional work in FHE continues, particularly with multi-key implementations of multiple users (see Peikert and Shiehian [2016], Brakerski and Perlman [2016]). While outside the scope of this paper, multi-key research has potential uses when considering training of Convolutional Neural Networks.

FHE is not the only technique for secure processing. Paillier et al. [1999] implemented an encryption scheme that allows for additive homomorphic operations. The Paillier scheme is not an FHE scheme because Paillier’s scheme does not allow for multiplicative homomorphic operations of two ciphertexts. Multiple uses of Paillier have occurred over the years; including for algorithms such as SIFT [Hsu et al., 2011] and SURF [Bai et al., 2014]. While the previous two algorithms are signal processing related, machine learning algorithms have been researched as well. Yu et al. [2006] implemented a privacy-preserving SVM using secure protocol techniques. Laur et al. [2006] also implemented a cryptographically private SVM using secure protocol techniques around the same time as the previous research. Another example of encrypted machine learning classification is Bost et al. [2015], where the authors have implemented a set of
algorithms including SVM, Naive Bayes, Decision Trees, among others; Convolutional Neural Networks was not included among the others. **Yang et al. [2005]** is the last example of classification for Naive Bayes with secure protocol techniques. On the other side of machine learning for training the algorithm(s), **Yuan and Yu [2014]** implemented a neural network training using homomorphic encryption via the BGN scheme.

**Chabanne et al. [2017]** also considered the Convolutional Neural Networks classification and FHE. In addition to using the most recent FHE scheme as the underlying security framework, the implementations for the rectified linear unit and the max pooling nodes are implemented using approximations (polynomials and averaging respectively) whereas this research computes the actual functions as defined in the unencrypted domain. **Gilad-Bachrach et al. [2016]** considered CNNs as well using a different FHE scheme known as YASHE **Bos et al. [2013]**. This implementation is similar to **Chabanne et al. [2017]** and has the same similarities and difference with this research. Two other implementations exist using FHE as part of the techniques but additionally used different neural network implementations with binary **Sanyal et al.** and discretized neural networks **Bourse et al. [2017]**. Next, two other implementations exist in terms of using oblivious techniques like garbled circuits. First, **Liu et al. [2017]** created an oblivious neural network (called MiniONN) using garbled circuits combined with homomorphic encryption. Second, **Juvekar et al.** improved on the previous concept for a lower latency framework using a linear algebra kernel in FHE to perform faster computations. Finally, **Meehan et al. [2018]** has developed a concept similar to this research; however, our research shows better performance in accuracy and run time.

Similarly, there is existing FHE research for using binary numbers to implement reals. numbers in FHE. **Shortell and Shokoufandeh [2017]** and **Shortell and Shokoufandeh [2018]** used binary numbers in fixed point format to perform an secure FFT. We improve upon their approach to compute real functions to perform the Convolutional Neural Network in FHE.

### 3 CNN in FHE

This section we present the details of the Convolutional Neural Network in the FHE framework including the numerical error introduced by the implementation. Real numbers are stored in the framework using a fixed point binary format. As mentioned earlier, a Convolutional Neural Network is built from two different types of layers: convolution and fully connected layers. Figure 2 depicts the Convolutional Neural Network structure that is used in this research, which is loosely based on the original network used in **LeCun et al. [1995]**. Overall, there are 3 layers, starting with 2 convolutional layers followed by a fully connected layer.

For the sake of illustration, we assume the input to the network is the handwritten decimal digits in the form of a 28 × 28 image, with 10 output nodes. The first convolution layer reduces the size to 24 × 24 using a 5 × 5 convolution
across the image. Max pooling operates on $2 \times 2$ segments without any overlap causing the feature map to reduce to $12 \times 12$ in size. In the first layer, there are 4 different convolutions that are run; thus the 4 feature maps outputs of the layer. In the second layer, the number of convolutions is 15 with interactions being allowed between the feature maps of the first layer. The second layer uses the same control parameters as the first layer: $5 \times 5$ convolution and $2 \times 2$ max pooling segments. This leads to output feature map size of $4 \times 4$ from the layer. With the completion of the two convolution layers, the fully connected layer takes the 240 ($15 \times 4 \times 4$) outputs of the convolution layer and inputs them directly into 10 output nodes of the fully connected layer. In terms of FHE, the major components that need to be developed are the ability to do a dot product, the ReLU, and max function.

**Fixed Point Binary Format**  We will use the fixed point binary representation to support real-valued computations for FHE. The format is defined by multiplying a real number, $r$, by an integer scaling factor, $\delta$, and then flooring the result. This provides an integer, $z$:

$$|r \cdot \delta| = z.$$  

This number can be converted to binary via:

$$z = \sum_{i=0}^{n} b_i \cdot 2^i.$$  

Figure 2: Specific Convolutional Neural Network Used
Given the number in the binary format, fixed point operations in the integer space are implemented. Basic operations including addition, subtraction, multiplication, and division can be implemented using binary gates starting from the NAND gate to other binary gates to half and full adders. While addition and subtraction are straightforward to implement with full adders, computing generic multiplication requires the use of Wallace Trees [Wallace 1964]. We note that division can be computed as well but this operation is not necessary for the implementation of Convolutional Neural Network. As two’s compliment is being used in the binary format, secure comparisons are done by examining the bits. By subtracting two numbers, the result (in encrypted form as a single bit) can identify equality, less than, greater than, or lack thereof. The single bit has many uses including obfuscation. An example consider

\[ \text{sum} = \text{sum} + b \cdot \text{value}. \]

(6)

that shows conditional operations can be done by using the resultant bit \( b \) in this case) to add \( \text{value} \) to the \( \text{sum} \). We will use this simple but powerful technique to implement the rectified linear unit and the max pooling capabilities.

**Encrypted Dot Product**  Computing a dot product within the framework is crucial to computing the Convolutional Neural Network. A dot product can be defined as:

\[ \sum_{i=0}^{m} c_i \cdot x_i. \]

(7)

As the fixed point binary format provides addition and multiplication, computing the dot product is straightforward. An important final note for Convolutional Neural Networks is that constant multiplication can be used in the case of client classification which the client does not need to know the parameters, i.e. the server is not required to encrypt the parameters.

**Activation Functions - ReLU**  Rectified Linear Unit (ReLU) is required to properly implement the Convolutional Neural Network (see Eq. 2). First we need to determining if the input is greater than or equal to zero. If this result is called \( b \), then the ReLU is computed as:

\[ \text{res} = b \cdot x. \]

(8)

This technique accurately computes ReLU.

**Max Pooling**  Implementing max pooling in FHE is an operation where the max value of the set of inputs is determined; i.e. the max function (Eq. 3). Because a secure comparison can be used to identify the greater than of two ciphertexts, the max function is implementable by running a loop over the inputs and finding the max value. Using obfuscation, the update equation can be stated as:

\[ \text{max} = b \cdot \text{currentmax} + b \cdot \text{next} \]

(9)
where \( b \) and \( \neg b \) identify which of the two values is the max (i.e. between the \text{currentmax} \) and \text{next} \).

Next, we estimate the magnitude of errors introduced during these computations. The main source of the error is due to the use of floor function in Eq. \( 3 \) which increase over iterative computations. We first state the theorem describing the error bound:

**Theorem 1.** Given the FHE implementation of a Convolutional Neural Network with a set of convolutions layers and fully connected layers \( L \), the implementation will result in an error of:

\[
\Delta \cdot \prod_{i \in L} r_i \cdot d_i
\]

where \( \Delta \) is the initial error, \( r_i \) and \( d_i \) are defined per each layer as follows: for a convolution layer, \( r_i \) is square root of convolution size and \( d_i \) is the maximum of the sum of constants for the set of convolutions for the layer. for a fully connected layer, \( r_i \) is the square root of the node input size and \( d_i \) is the maximum of the sum of constants among the outputs of the layer.

Individual lemmas are built up from smaller pieces to larger parts of the Convolutional Neural Network. First we estimate the error bound on the ReLU processing and follow that the max pooling processing. This will be followed by the error bound on the dot product, convolutional, and fully connected layers, respectively.

**Lemma 1.** Given an FHE based ReLU implementation, the computation will not increase the error of the input.

**Proof.** As the ReLU computation only outputs the same value or zero, the only error is the one at the start of the process. Thus the ReLU implementation does not introduce any error. \( \square \)

**Lemma 2.** Given an FHE based max pooling implementation, the computation will not increase the error of the input.

**Proof.** As max pooling takes in a set of values and outputs the max, the only error is the error at the start of the process as this will be continued forward. Thus the max pooling implementation does not introduce any error. \( \square \)

**Lemma 3.** Given an FHE based dot product implementation, the computation will increase the error of the output value by:

\[
\Delta \cdot \sqrt{s} \cdot |c|
\]

where \( \Delta \) is the max introduced error from the inputs, \( s \) is the size of the vector, and \( |c| \) is the magnitude of the convolution constants.
Proof. As the constant will not have any error, the focus is on the input parameters. Using,
\[ \sqrt{\left( \frac{\partial F}{\partial x_1} \delta x_1 \right)^2 + \cdots + \left( \frac{\partial F}{\partial x_n} \delta x_n \right)^2} \] (12)
to propagate the error, the individual partial derivatives will be the individual constants. The \( \delta x_i \) values will be \( \Delta \) as this is the input error. This reduces the equation to:
\[ \sqrt{\sum (c_i^2 \cdot \Delta^2)}. \] (13)
Extracting \( \Delta \) from the sum, the remainder is the magnitude of the constants multiplied by the size of the vector. Thus the error is:
\[ \Delta \cdot \sqrt{s} \cdot |c|. \] (14)
\[ \square \]

Lemma 4. Given the FHE implementation of the convolution layer, the computation will increase the error by
\[ \Delta \cdot \sqrt{s} \cdot |c| \] (15)
where \( \Delta \) is the max input error per input value, \( \sqrt{s} \) is the square of the convolution size, and \( |c| \) is the magnitude of the convolution constants.

Proof. To prove this lemma, it is necessary to use Lemmas 1, 2, and 3. The first step of the convolution layer is the dot product, which will introduce an error of \( \Delta \cdot |c| \). This was proved by Lemma 3. The second step is the activation function post the dot product which is an ReLU, which as Lemma 1 proved introduces no additional error; so error is \( \Delta \cdot \sqrt{s} \cdot |c| \). The final step is the max pooling part. Lemma 2 proved that no additional error is introduced. Thus, the error introduced by a single convolution layer is \( \Delta \cdot \sqrt{s} \cdot |c| \).  
\[ \square \]

Lemma 5. Given the FHE implementation of the fully connected layer, the implementation will increase the error by
\[ \Delta \cdot \sqrt{s} \cdot |f| \] (16)
where \( \Delta \) is the max input error per input value, \( \sqrt{s} \) is the square of the inputs to the layer, and \( |f| \) is the magnitude of the fully connected layer constants.

Proof. In this context, the fully connected layer is a dot product with an activation function that is linear. With \( \Delta \) as the input error, Lemma 3 proves this error will be:
\[ \Delta \cdot \sqrt{s} \cdot |f| \] (17)
where \( |f| \) has been substituted for \( |c| \). As a linear function is simply an identity function which will not introduce any error beyond the error already included.  
\[ \square \]
Theorem 1 can be proved using the results of Lemma 4 and Lemma 5.

Proof. Given the structure of the Convolutional Neural Network as $L$, each layer is input to the next layer. This means the error will propagate over time from layer to layer. Lemmas 4 and 5 provide the error contributed by a single layer for convolutions and fully connected respectively. As the error is propagated from layer to layer, the maximum error is what should be retained to the next layer. For the convolutional layer (Lemma 4), this will be the value $|c|$. For the fully connected layer (Lemma 5), this will be the value $|f|$. The initial error, $\Delta$, can be pulled out at each layer. By designating the individual $|c|$ and $|f|$ as $d_i$ for the individual layers and $\sqrt{s}$ as $r_i$ per layer, the error is increased from the original error to the final error by the product of each individual layer. This equates to the following equation:

$$\Delta \cdot \prod_{i \in L} r_i \cdot d_i$$

(18)

Consider the CNN in Figure 2, the following corollary shows the specific situation for that structure as an example of applying Theorem 1 to a specific situation:

Corollary 1. For a CNN with 2 convolutional layers and a fully connected layer, the total error will be:

$$\Delta \cdot 25 \cdot \sqrt{240} \cdot d_1 \cdot d_2 \cdot d_3$$

(19)

where the $r_i$ values have been filled in. $d_i$ values are dependent on the specific training.

4 Experimental Results

To verify Theorem 1, an implementation of the CNN is necessary. The code was developed in C++ using a home grown FHE implementation supported by FFLAS-FFPACK (The FFLAS-FFPACK Group [2018]) and HDF5 (The HDF Group [2018]). Input values are real numbers between -1 and 1. Fixed point binary format is configured to be 32 total bits with 16 bits being the fractional space; requiring a fixed point scale of 65536. The actual CNN used is based on the original CNN paper LeCun et al. [1995]. Input images are handwritten numerical digits from 0 to 9. The same data set as LeCun et al. [1995] was used. Training was done offline from the FHE implementation; training was not part of the FHE implementation. Tiny CNN (Pearlman [2018]) was used to train the network using the original training set. Individual images from the test set were run against the implementation. The ten outputs between unencrypted and encrypted implementation are compared to verify the encrypted Convolution Neural Network performs as well as the unencrypted implementation. These results are discussed in the next paragraph.
Using a set 20 images from the original test set, each image was encrypted, run through the Convolutional Neural Network implementation and the classifications decrypted for analysis. All encrypted results matched the unencrypted recommended classifications exactly. Numerical errors where contained within the $10^{-3}$ region. On average, the individual error was 0.00085 with a standard deviation of 0.00022. These results are aligned with Theorem I and Corollary I, clearly showing that the Convolutional Neural Network implementation successfully works in the encrypted domain.

Time and space complexity are an important consideration in FHE. FHE’s basic time and space complexity have been considered inefficient. A single FHE ciphertext is a matrix $O(\hat{n}^2)$; this is a single bit in the fixed point binary format. $\hat{n}^2$ is the size of the FHE ciphertext matrix and worse case operations are $O(\hat{n}^3)$. All algorithm complexity is multiplied by this.

From a space complexity perspective, there will be interim space needed during the computations and output space for the result. Starting with the interim space, a convolution layer has $l$ feature maps with $m^2$ as the individual feature map size. Combining, this is bounded by $O(l \cdot m^2 \cdot f \cdot \hat{n}^2)$ where $f$ is used for the number of fixed point binary digits. As this is for a single layer, each layer will have different parameters. Considering $l$ and $m$ as the max values from all the layers, the asymptotic bound for the interim space complexity can include the number of layers, $n_l$, thus $O(n_l \cdot l \cdot m^2 \cdot f \cdot \hat{n}^2)$. Looking at the output space complexity, the fully connected layer will output a set number of classifications, designated by $c$. As these are individual ciphertexts of the format, the asymptotic bound will be the combination: $O(c \cdot f \cdot \hat{n}^2)$.

We note that the NAND gate that is the basis of the FHE operations is $O(\hat{n}^3)$. For a single convolutional layer, there are $l$ feature maps of size $m^2$ that each have a convolution of a $5 \times 5$ size. The convolution is effectively 25 multiplications and additions (designated $25am$). So far, this time complexity is $O(l \cdot m^2 \cdot (25am))$. Next, the $25am$ needs to be converted into individual NAND gates. Multiplications in the fixed point format start with a $4f$ operation (AND gates of all digits) and then a log $f$ operation of $2f$ full adders. Full adders need a total of 36 NAND gates. Combining these terms becomes $O(f^2 \log f)$. Addition takes $f$ full adders; thus $36f$ NAND gates. Thus a single layer’s asymptotic time complexity is $O(l \cdot m^2 \cdot f^2 \log f \cdot \hat{n}^3)$. A fully connected layer is similar by changing to the number of classifications and the proper inputs. Becoming $O(c \cdot m^2 \cdot f^2 \log f \cdot \hat{n}^3)$. Considering a finite number of convolution layers, $n_l$, and fully connected layers, $n_c$, the asymptotic time complexity is $O(n_l \cdot l \cdot m^2 \cdot f^2 \log f \cdot \hat{n}^3 + n_c \cdot c \cdot m^2 \cdot f^2 \log f \cdot \hat{n}^3)$. This asymptotic time complexity is worst case because after the first layer the number $m^2$ drops.

It is worth noting that parallel processing techniques are available to improve actual running time. The current implementation used a GPU to increase efficiency of the matrix-matrix multiplication. Additionally, the individual convolutions in a layer do not depend on each other, thus these can be run in parallel. For concrete numbers, the implementation was able to compute the classifications in two days using a four CPU desktop computer with two GPUs and total space of about one GB. As the desktop computer used is not a real
cloud server, greater efficiency could be gained by using more CPUs and GPUs to bring the two day computation down to a user tolerable time.

5 Conclusion

As has been shown in the preceding sections, Convolutional Neural Network classifications can be computed over FHE. This enables a user to offload their computations to a cloud computer or for a client/server model of protecting privacy of the individual data. An open problem to be considered is potentially using the encrypted classification for other algorithms post classification. An argmax function is easy to implement in FHE given the work done in Section 3 (basically an extension of the max function). Argmax allows for selecting the classification for other uses.

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