A Librational Model for the Propeller Blériot in the Saturnian Ring System

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Abstract

The reconstruction of the orbital evolution of the propeller structure Blériot orbiting in Saturn’s A ring from recurrent observations in Cassini ISS images yielded a considerable offset motion from the expected Keplerian orbit. This offset motion can be composed by three sinusoidal harmonics with amplitudes and periods of 1845, 152, 58 km and 11.1, 3.7, and 2.2 years, respectively. In this paper we present results from N-body simulations, where we integrated the orbital evolution of a moonlet, which is placed at the radial position of Blériot under the gravitational action of the Saturnian satellites. Our simulations yield that, especially the gravitational interactions with Prometheus, Pandora, and Mimas are forcing the moonlet to librate with the right frequencies, but the libration amplitudes are too small to explain the observations. Thus, further mechanisms are needed to explain the amplitudes of the forced librations—e.g., moonlet–ring interactions. Here, we develop a model, where the moonlet is allowed to be slightly displaced with respect to its created gaps breaking the point symmetry and causing a repulsive force in this way. As a result, the evolution of the moonlet’s longitude can be described by a harmonic oscillator. In the presence of external forcing by the outer moons, the libration amplitudes get the more amplified the closer the forcing frequency is to the eigenfrequency of the disturbed propeller oscillator. Applying our model to Blériot, it is possible to reproduce a libration period of 13 years with an amplitude of about 2000 km.

Key words: planets and satellites: individual (Saturn) – planets and satellites: rings

1. Introduction

One of the most puzzling discoveries of the spacecraft Cassini has been the observation of disk-embedded objects orbiting within Saturn’s main rings (Spahn & Schmidt 2006; Tiscareno et al. 2006, 2008; Sremčević et al. 2007). Small sub-kilometer-sized objects (called moonlets) create typical density variations downstream to their orbit by gravitational interactions with the surrounding ring material that resemble a two-bladed propeller, giving the structure its name (Spahn & Sremčević 2000; Sremčević et al. 2002). The largest propeller structure, which is a few thousands km in azimuth, is called Blériot and is caused by a moonlet with a diameter of around 800 m. However, the moonlet is still too small to allow its direct observation by the Cameras aboard the spacecraft Cassini.

Fortunately, the propeller structure permits an orbital tracking of the largest moonlets. The reconstruction of the orbital evolution of Blériot revealed an offset motion with respect to a Keplerian motion of considerable amplitude (Tiscareno et al. 2010).

A satisfying explanation for this excess motion is still lacking (Pan & Chiang 2010, 2012; Rein & Papaloizou 2010; Tiscareno 2013). Up to now the most promising explanation has been proposed by Rein & Papaloizou (2010), who considered a stochastic migration. Pan et al. (2012) showed in N-body simulations that such a mechanism could generate a maximal excess motion of 300 km over a period of four years, which agrees with the findings by Tiscareno et al. (2010).

Figure 1 shows a recent fit of Blériot’s orbital evolution based on a larger set of ISS images taken from 2005 to 2014. The resulting excess motion can be fitted astonishingly well by three harmonic functions with amplitudes and periods of 1845, 152, and 58 km and 11.1, 3.7, and 2.2 years, respectively, where the standard deviation of the remaining residual is about 17 km.

The harmonic behavior of the excess motion might suggest that resonant interactions with the other Saturnian moons serve as a reason for the excess motion. These interactions are known to cause similar excess motions for some of the outer moons of the Saturnian system (Goldreich 1965; Goldreich & Rappaport 2003a, 2003b; Spitale et al. 2006; Cooper et al. 2015).

Following these examples, we perform simulations where the moonlet is perturbed by the moons of Saturn in order to characterize the orbital motion of Blériot (Section 2). It will be shown that this approach can explain the observed frequencies, but not the large libration amplitudes. Thus, we propose a model of ring–moonlet interactions that is capable of explaining the amplification of the perturbed excess motion to observable excursions (Section 3).

2. Test Moonlet Integration

For the numerical integrations, we consider the gravity of 15 Saturnian moons of masses \( m_i \) and the oblate (up to \( J_2 \)) Saturn of mass \( M_S \) and radius \( R_S \) determining the dynamics of the moonlet

\[
\ddot{r} = \nabla \left\{ \frac{G m_e}{r} \left[ 1 - \sum_{j=2}^{\infty} \left( \frac{r_e}{r_j} \right)^2 J_{2j}(\cos \vartheta) \right] \right. \\
- \sum_{i=1}^{N} \frac{G m_i}{|r_i - r|} \left( 1 - \frac{r_i \cdot r}{|r_i|^2} \right). \quad (1)
\]

These moons are: Atlas, Daphnis, Dione, Enceladus, Epimetheus, Hyperion, Iapetus, Janus, Mimas, Pan, Pandora, Prometheus, Rhea, Tethys, and Titan. Their initial phase space vectors have been taken from the SPICE kernels sat375.bsp and sat378.bsp at the initial time 2000-001T12:00:00.000. Our
routine integrates the phase space vectors for the complete system at each time step, where all moons are able to interact with each other gravitationally.

We apply our integration routine to Blériot, which we model as a test particle, placed in the Saturnian equatorial plane ($i_0 = 0$ deg) on a circular orbit ($e_0 = 0$) at the expected orbital position (initial semimajor axis $a_0 = 134912.125$ km and mean longitude $\lambda_0 = 259.45$ deg, the extrapolation of results from ISS images for the initial time given above).

The numerical deviation of the mean longitude and the semimajor axis of Blériot, calculated after Renner & Sicardy (2006) over a time span of 30 years, are shown in the upper panels of Figure 2. Analyzing the frequency spectrum (seen in the lower panels), a clear librational behavior with different frequencies and amplitudes is visible in the residuals of the mean longitude and semimajor axis. The gravity of the acting 15 moons induce a mean eccentricity of $e_{av} = 2.9 \cdot 10^{-6}$ and an inclination of $i_{av} = 7.5 \cdot 10^{-5}$ deg on the moonlet.

### 2.1. The Dominating Moons

The amount of moons has been decreased systematically in order to identify the ones that cause considerable resonant perturbations on Blériot. As it turns out, the satellites Pandora and Mimas are dominating the resonant behavior, resulting in four characteristic peaks in the Fourier spectrum (FS) of the mean longitude residual (see Figure 3, left panel).

A systematic identification of the resonances causing the libration frequencies has been performed on the basis of the algorithm presented in Lissauer & Cuzzi (1982), calculating the frequencies over the resonance’s distance to the moonlet.

The most dominating influence is found in the 14:13 corotation-eccentricity resonance (CER) of Pandora, which is forcing Blériot to librate with a period of 0.6 years with an amplitude of 5 km. A complete list of the important periods, amplitudes, and resonances causing the librations can be found in Table 1. Note that there is a slight shift in the libration frequencies between Figures 2 and 3, caused by the neglect of large moons such as Titan.

The presence of Mimas results in a three-body resonance between Mimas, Pandora, and Blériot. Mimas and Pandora are known to be in a 3:2 resonance, yielding a libration period of 1.8 years. Considering the resonant arguments (Murray & Dermott 1999) of both resonances (3:2 and 14:13), one can construct related three-body resonances with libration periods of 0.6, 0.8, 2.5, and 14.3 years (compare with Figure 3 and Table 1) by subtracting the corresponding resonant arguments:

$$\varphi_{3:16:13,1} = 3\lambda_{Mim} - 16\lambda_{Pnd} + 13\lambda_{Ble}$$ (2)
Adding Prometheus, Titan, and Tethys (all having influence on the orbital dynamics of Pandora and Mimas) leads to a non-stationary signal in the FS. This could be caused by the chaotic and strong interactions between Prometheus and Pandora, further resulting in time-variable libration frequencies and amplitudes for Blériot. Long-term simulations of the moonlet (considering a time span of up to 100 Saturnian years) show that the 42:40 IVR of Prometheus seems to become more important, forcing the moonlet to librate for a period of about four years, with a radial amplitude of 20 m and an amplitude around 1 km in mean longitude.

Although the libration periods of the moonlet obtained with our simulations agree fairly well with the observational data, the resulting amplitudes are too small.

3. Moonlet–Propeller Interactions

To address the libration amplitude problem, we consider the gravitational interaction between the embedded moonlet with its created gap region. Imagine a non-symmetric propeller structure so that the moonlet gets accelerated by the ring gravity. We will show that the evolution of the related longitudinal residual can be described by a harmonic oscillator, which is periodically forced by the gravity of the outer moons. The amplitudes of the externally driven frequencies get the more amplified the more closely the forcing frequency matches the eigenfrequency of the harmonic oscillator (propeller–moonlet system).

We consider a moonlet located at \((x_m, y_m)\) as illustrated in Figure 4. Interacting with the surrounding viscous ring material the propeller moonlet gravitationally scatters ring particles to larger and smaller orbits and creates two gaps in its vicinity in the ring material (regions of reduced surface mass density) that are decorated by two density-enhanced regions pairwise downstream of its orbit (Spahn & Wiebicke 1989). Viscous diffusion of the ring material counteracts this gap-creation, smoothing out the structure with growing azimuthal distance downstream to the moonlet (Spahn & Sremčević 2000; Sremčević et al. 2002). For simplicity, we assume the gap shape to be a rectangular area with reduced density \(\sigma_{g}\), illustrated by the shaded regions at \(|x| = x_g\) and width \(\Delta\) in Figure 4.

The diffusion process defines the length \(L = y_m\) of the gaps, while the radial propeller structure is mainly defined by the scattering of ring particles by the moonlet and scales with the...
where \( \Delta = 2h \), which is consistent with radial gaps to the moonlet. Thus, the gravity of Prometheus results in a non-stationary signal in the FS. Thus, the libration frequencies are changing due to the interaction between Pandora and Prometheus.

After splitting the integrals, subtracting the unperturbed term in Equation \( (10) \) has been neglected, because it is at least one order of magnitude smaller than \( F_{g,y} \) for \( e < 10^{-6} \). Inserting \( F_{g,y} \) from Equation \( (8) \) results in an equation of a harmonic oscillator

\[
\frac{d^2 \delta \lambda}{dt^2} = -\omega^2 \delta \lambda, \tag{11}
\]

where \( \omega^2 = 6G(-\delta \sigma) \Delta / L^2 \) is the libratin frequency and \( T = 2\pi \omega^{-1} \) is the libration period. To arrive at Equation \( (11) \) we neglected higher orders in eccentricity and used the relation \( \delta \sigma = \gamma \omega a^{-1} \) for the moonlet’s mean longitude residual. Here, the eigenfrequency contains the properties of the propeller feature as the radial width \( \Delta \) and the azimuthal length \( L \), but also the drop in the surface mass density \( \delta \sigma \).

3.2. Application to Blériot

To find the eigenlibration period for Blériot, we first estimate the gap length \( L \) in the simplified model (Figure 4).

Sremčević et al. (2002) have shown that \( aK = \Omega h^3/(2 \nu) \) is the characteristic azimuthal length of the viscous mass diffusion process. Here, \( \nu \) and \( \Omega \) denote the viscosity and the

\[
\frac{ds}{dt} = \frac{m}{3m_c} \hat{\sigma}, \tag{6}
\]

(Sremčević et al. 2002; Seiß et al. 2005).

In the symmetric case \((x_m = 0, y_m = 0)\) the torque on the moonlet due to the gravitational interaction with the gap is zero because of point symmetry. When the moonlet is leaving this position, the gravitational force \( F_g \) of the ring material on the moonlet is

\[
F_g(r_m) = -G \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \sigma(r) \frac{r_m - r}{|r_m - r|^3},
\tag{7}
\]

where \( \sigma(r) = \sigma_x(x, y)^T \) and \( r_m = (x_m, y_m)^T \) are the surface mass density and the position vectors of the ring material and the moonlet.

After splitting the integrals, subtracting the unperturbed background density \( \sigma_0 \), and evaluating the integrals up to first order in \( x_m \) and \( y_m \), the azimuthal force acting on the moonlet reads

\[
F_{g,y}(x_m, y_m) \approx -2G\delta \sigma \Delta \left( \frac{x_m}{x_g^2} + \frac{y_m}{L^2} \right), \tag{8}
\]

where \( \delta \sigma = \sigma - \sigma_0 \). To arrive at Equation \( (8) \), we assume that the azimuthal excursions of the moonlet are negligibly small compared to the azimuthal extent of the propeller structure \((y_m \ll L)\). We also assume that the radial displacement of the moonlet is very small compared to the radial distance of the gaps to the moonlet \((x_m \ll x_g)\).
Table 1

| $f$ (years$^{-1}$) | $T$ (years) | $FS(a\Delta) \, (\text{km})$ | $FS(a\Delta n) \, (\text{m s}^{-1})$ | $FS(\Delta n) \, (\text{m})$ | Resonance |
|------------------|-------------|-------------------------------|---------------------------------|-----------------|------------|
| 1.7              | 0.6         | 4.6                           | 1.5 $\cdot$ $10^{-3}$           | 8               | 14:13 CER Pnd-Ble |
| 1.25             | 0.8         | 1.0                           | 2.2 $\cdot$ $10^{-4}$           | 1.2             | 3:16:13 Mim-Pnd-Ble ($\frac{\Delta}{n}$,16:13,2) |
| 0.4              | 2.5         | 0.15                          | 4.3 $\cdot$ $10^{-5}$           | 0.5             | 3:16:13 Mim-Pnd-Ble ($\frac{\Delta}{n}$,16:13,1) |
| 0.07             | 14.3        | 2.3                           | 3 $\cdot$ $10^{-5}$             | 0.15            | 3:16:13 Mim-Pnd-Ble ($\frac{\Delta}{n}$,16:13,3) |

Note. Additionally, the amplitudes in mean motion, as well as the semimajor axis and corresponding resonance, are given. In addition to the 14:13 CER of Pandora, three-body resonances between Mimas, Pandora, and Blériot seem to play an important role.

As a result, we estimate an eigenlibration period of that ring–moonlet harmonic oscillator:

$$T = \frac{2\pi L}{\sqrt{6G(\delta\sigma)}} \approx 13 \text{ yr}$$

Assuming a Hill radius of $h = 430 \text{ m}$ (recent reported values range from 300 to 600 m; see Hoffmann et al. 2016; Seiß et al. 2017) leads to a libration period of $T = 13 \text{ years}$. The Hill radius of the propeller moonlet has a strong influence on the gap length $L \propto h^3 \sim m$ and thus on the libration period. For example, varying $h$ by 10 percent changes $T$ by about 25 percent (about three years).

This oscillating system (moonlet–propeller) is steadily driven by the outer satellites. When a forcing frequency closely matches the eigenfrequency, a large gain in the resulting amplitude is possible. For example, if the period of the resonant interaction and the eigenperiod of the oscillator differ by one-third of a year ($\approx 2\%$), the amplitude is amplified by two orders of magnitude. The closer the forcing frequency is to the eigenfrequency, the larger the resulting amplitudes. Formally, to get to the observed amplitudes, which are about three orders of magnitude larger than the values from Table 1, the difference of the periods must be smaller than 0.03 years ($< 0.2\%$).

4. Conclusion and Discussion

Our simulations have shown that gravitational interactions with mainly Prometheus, Pandora, and Mimas are forcing the propeller Blériot to librate with frequencies similar to those seen in observations of the excess motion in Cassini ISS images (Figure 1). Unfortunately, the corresponding amplitudes are too small.

The amplitude problem has been approached by introducing a simplified propeller–moonlet interaction model. Our model considers the gravitational back-reaction of the induced gaps onto a moonlet which is slightly displaced from its mean position. This results, to lowest order, in a harmonic oscillator equation for the moonlet’s mean longitude. The eigenfrequency of this oscillating system contains key properties of the propeller structure. Being periodically driven externally by the Saturnian moons, this harmonic oscillator amplifies certain modes, where the amplification depends on the match of the forcing frequency and the eigenfrequency of the oscillator.

By applying our model to Blériot, reasonable results have been obtained for the libration period, which fit the observations fairly well. Combining our model with the simulation results, we are able to reproduce the largest observed mode for...
the libration of Blériot. The smaller observed modes are not reproduced, but may evolve if one regards processes of nonlinear mode coupling in our model, which we have not yet considered.

In addition to the strong influence of the moonlet’s Hill radius on the oscillating system’s eigenfrequency, the large range of reported outer A-ring viscosity values also adds to its radius on the oscillating system considered.

nonlinear mode coupling in our model, which we have not yet

One should note that our simple model is not fully consistent, because the libration amplitude \( y_m \) is actually larger than the length \( L \) that is used in the rectangular simplification of the gaps. Therefore, the condition to approximate the force for small amplitudes is violated. However, in reality the gap extends for several thousand kilometers (the expected gap length for Blériot is 6500 km, see Footnote 3) and \( L \) is rather an effective length of the gap.

A further simplification is that the gap ends do not follow the librating moonlet. In case of Blériot, ring particles in the gap regions need about 1.1 years to azimuthally travel 6500 km relative to the mean moonlet position, which is shorter than the largest libration period. However, the gap ends do not instantaneously follow the moving moonlet either. Instead, there will be a retardation and the mass distribution in both gaps will differ significantly, leading to the asymmetry needed for the oscillatory behavior. Therefore, our simplified moonlet–ring interaction model demonstrates that moonlet–gap interactions form a physical oscillating system that is able to substantially amplify gravitationally induced moonlet libration amplitudes. A more complex model should remove these inconsistencies.

We plan to address such a comprehensive nonlinear model in the future by performing hydrodynamical propeller simulations with a freely moving moonlet. In addition to improving the propeller model, we also plan to apply the model to the other propeller structures located in the outer A ring.

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