LATEST RESULTS FROM LATTICE QCD FOR EXOTIC HYBRID MESONS.

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ABSTRACT
I review the results from lattice gauge theory for the masses of exotic hybrid mesons.

1 Introduction
The quark model predicts that the charge conjugation (C) and parity (P) of a meson with spin $S$ and orbital angular momentum $L$ is
\[ P = (-1)^{L+1}, \quad C = (-1)^{L+S} \] (1)
States with quantum numbers not produced by eq. (1), such as
\[ J_{exotic}^{PC} = 1^{--}, 0^{+-}, 2^{+-}, 0^{--} \] (2)
are known as exotics (\[\square\]). Exotic states are allowed by QCD.
There are a number of different possibilities for the structure of an exotic state. An exotic signal could be: a hybrid meson, which is quark and anti-quark and excited glue, or bound state of two quarks and two anti-quarks ($\overline{QQ}QQ$). The two most popular guesses for the structure of the ($\overline{QQ}QQ$) state are either a molecule of two mesons or diquark anti-diquark bound state. In this paper I review the latest lattice results for the masses of exotic hybrid mesons, concentrating on the $1^{--}$ state, obtained from lattice QCD.
2 Lattice simulations of exotic mesons

Many numerical predictions of QCD can be determined from the path integral

$$c(t) \sim \int dU \int d\psi \int d\bar{\psi} \sum_{x} O(0,0)O(x,t)^{\dagger}e^{-S_{F}-S_{G}}$$  \hspace{1cm} (3)$$

where $S_{F}$ is the fermion action (some lattice discretization of the Dirac action) and $S_{G}$ is the pure gauge action. The path integral in eq. (3) is put on the computer using a clever finite difference formalism [2], due to Wilson, that maintains gauge invariance. The physical picture for eq. (3) is that a hadron is created at time 0, from where it propagates to the time $t$, where it is destroyed. The fermion integration can be done exactly in eq. (3) to produce the fermion determinant. Simulations that include the effect of the determinant are very expensive computationally, so typically it is not included in the simulation (the quenched approximation). However there has been some recent work that includes the effect of the determinant [3] on the light exotic spectrum.

The standard interpolating operator for the pion, which can be used in eq. (3), is

$$O_{\pi}(x,t) = \bar{\psi}(x,t)\gamma_{5}\psi(x,t)$$  \hspace{1cm} (4)$$

which has the correct $J^{PC} = 0^{-+}$ quantum numbers. One possible interpolating operator [4] for an exotic $1^{-+}$ particle is

$$O_{1^{-+}}(x,t) = \bar{\psi}(x,t)\gamma_{j}F_{ij}(x,t)\psi(x,t)$$  \hspace{1cm} (5)$$

where $F$ is the QCD field strength tensor. It is essential to use operators that have some spatial separation between the quarks in the meson to get a good signal. Recently the MILC collaboration has attempted to measure the “wave function” of the $1^{-+}$ hybrid meson in coulomb gauge [5]. Unfortunately the operator used did not have the correct charge conjugation quantum number, so the published wave function [5] is incorrect.

In this formalism a gauge invariant interpolating operator, for any possible exotic hybrid particle or four particle state can be constructed. The dynamics then determines whether the resulting state has a narrow decay width, which can be detected experimentally. In the large $N_{c}$ (number of colours) limit [1, 6] both exotic hybrid mesons and non-exotic mesons have widths vanishingly small compared to their masses.

The data from the simulation is extracted using a fit model [4]:

$$c(t) = a_{0}exp(-m_{0}t) + a_{1}exp(-m_{1}t) + \cdots$$  \hspace{1cm} (6)$$
where $m_0 (m_1)$ is the ground (first excited) state mass and the dots represent higher excitations. Although in principle excited state masses can be extracted from a multiple exponential fit, in practice this is numerically non-trivial because of the noise in the data from the simulation. Simulations that involve the calculation of the properties of exotic hybrid mesons are harder than those that concentrate on the non-exotic hadrons, because the signal to noise ratio is worse for exotic mesons than for $Q\bar{Q}$ mesons.

In an individual lattice simulation there are errors from the finite size of the lattice spacing and the finite lattice volume. State of the art lattice simulations in the quenched theory, run at a number of different lattice spacings and physical volumes and extrapolate the results to the continuum and infinite volume. For the exotic mesons, this is done for heavy quarks (see section 4), but as yet, the continuum extrapolation has not been done for light exotic mesons.

How successful is lattice QCD in practice? One way to tell is to compare the predictions for the masses from lattice QCD for well known particles (proton, $\rho$, etc.) with experiment. The most accurate quenched calculation to date has recently been completed by the CP-PACS collaboration 7). From the masses of 11 light hadrons, they conclude that the quenched approximation disagrees with experiment by at most 11%. The comparison of results from simulations that include dynamical fermions with experiment is less clear, because of their high computational cost (see Kenway 8) for a review of the latest results).

The results from lattice QCD also provide insight into the underlying dynamics of light hadrons. Lattice QCD simulations can test the various assumptions made in models of the QCD dynamics. For example there are a number of models of exotic states based on the idea of a bound diquark anti-diquark pair 9). A critical assumption in diquark models is that two quarks actually do cluster to form a diquark. This assumption has recently been tested in a lattice gauge theory simulation by the Bielefeld group 10), where they found no deeply bound diquark state in Landau gauge.

3 Results for light exotic mesons

In the last year the MILC collaboration have repeated 5) their initial simulations 4) using the clover fermion action. The clover action is “closer” to the continuum than the Wilson fermion action, because it has the leading order lattice spacing terms removed. There are also new results for the hybrid masses from the SESAM collaboration (reported by Lacock and Schilling) 3), that include some effects from
Table 1: Masses of the light $1^{-+}$ hybrid from lattice gauge theory.

| Simulation | Group              | mass GeV |
|------------|--------------------|----------|
| A          | UKQCD              | 2.0(2)   |
| B          | MILC               | 2.0(1) ± sys |
| C          | MILC               | 2.1(1) ± sys |
| D          | Lacock and Schilling | 1.9(2)  |

Table 2: Parameters of light exotic meson simulations,

| Simulation | Action | fermions | length (fm) | $a^{-1}$ GeV | $M_{PS}/M_V$ |
|------------|--------|----------|-------------|--------------|--------------|
| A          | clover | quenched | 1.6         | 2.0          | 0.76         |
| B          | Wilson | quenched | 2.3         | 2.8          | 0.96, 0.93, 0.88, 0.77 |
| C          | clover | quenched | 2.3         | 2.8          | 0.94, 0.90, 0.72 |
| D          | Wilson | dynamical | 1.4         | 2.3          | 0.83, 0.81, 0.76, 0.69 |

dynamical sea fermions.

The results for the mass of the $1^{-+}$ exotic state are summarised in table 1. All the results from the various simulations are essentially consistent with the mass of the $1^{-+}$ state around 2 GeV. In table 2 we show the physical parameters for each of the simulations. The interpolating operators used to create the exotic meson states in the MILC calculations are different to those used in the UKQCD and SESAM simulations.

The observation that the results for the mass of the $1^{-+}$ hybrid meson are consistent for very different simulations gives us confidence in the final result. Although I would prefer to see simulation results at lighter quark masses. Simulations at the point where the ratio of the pseudoscalar mass to vector mass ($M_{PS}/M_V \sim 0.5$) are possible with current algorithms and computers.

4 Results for heavy exotic mesons

There has been a lot of work on calculating the spectroscopy of $\bar{c}c$ and $\bar{b}b$ mesons from lattice gauge theory (see Davies for a review). The main technical complication in heavy quark simulations is that the lattice spacing of current simulations is not smaller than the heavy quark mass. So various effective field theory Lagrangian approximations to QCD are simulated.

The NRQCD (nonrelativistic QCD) Lagrangian is one such effective field theory approximation to QCD, with the expansion parameter equal to the veloc-
Table 3: Mass splitting between the $\bar{b}b$ $1^{--}$ hybrid and the $\bar{b}b$ 1S state.

| Group           | comments                      | mass GeV   |
|-----------------|-------------------------------|-----------|
| UKQCD I         | $O(Mv^4)$ errors              | 1.68(10)  |
| CP-PACS I       | Asymmetric, $O(Mv^4)$ errors  | 1.542(8)  |
| Juge et al. I    | Asymmetric, $O(Mv^4)$ errors  | 1.49(2)(5)|

NRQCD has been particularly successful in simulating the Upsilon spectrum [12], but is less well converged for charmomium, (particularly for spin splittings), because the charm quarks move with higher velocity [12]. The NRQCD Langrangian correct up to $O(Mv^2)$ is

$$\mathcal{L}^{NRQCD} = \overline{\psi} \left( -\frac{\Delta^2}{2M} - \frac{c_0 \Delta^4}{8M^2} - \frac{c_1 \sigma B}{2M} \right) \psi$$  \hspace{1cm} (7)$$

where $c_0$ and $c_1$ are coefficients obtained by a perturbative matching procedure to QCD. In table 3 the results of all the recent NRQCD simulations of the $\bar{b}b\bar{g}$ hybrids in the quenched approximation are compiled. In the hybrid meson simulations no spin terms are included in the Lagrangian ($c_1 = 0$ in eq. 7), so the $1^{--}$, $0^{+-}$, and $2^{+-}$ states are degenerate. Both the results from the CP-PACS collaboration [13] and from Juge, Kuti and Morningstar [14] were shown to be independent of the lattice spacing and lattice volume. For example, the CP-PACS collaboration [13] found that the masses of the hybrids were independent of the box size above 1.2 fm.

The “asymmetric” comment in table 3 refers to the technique of treating space and time asymmetrically. A smaller lattice spacing was used in the time direction than in the space direction, which allowed the signal to be seen for further, for a given spatial volume. This technique has helped to reduce the errors. The practicalities of this idea were demonstrated by Morningstar and Peardon [15] for the glueball spectrum.

In table 4 the results of the mass splitting of the $1^{--}$ states and the 1S state are shown in the charmonium system. The MILC collaboration used the standard Wilson and clover actions to simulate the charm quark in their simulations of heavy exotic mesons, as previous work has shown this to be reliable [12].

The first results for heavy exotic [17] hybrids were done in the adiabatic surfaces approach, where the effect of the excited glue is subsumed in a potential measured on the lattice (see [18, 19] for a review). Juge, Kuti and Morningstar [14] have completed a systematic study of these potentials. The NRQCD approach is a more accurate approximation to QCD than the adiabatic potential technique; however Juge, Kuti and Morningstar [14] found that the adiabatic potential approach
Table 4: Mass splitting between the $\psi c$ $1^{-+}$ hybrid and the $\psi c$ $1S$ state.

| Group       | comments                              | mass MeV               |
|-------------|---------------------------------------|------------------------|
| MILC 4      | Wilson action                         | $1340^{+60}_{-150} + \text{sys}$ |
| MILC 5      | clover action                          | $1220^{+110}_{-190} + \text{sys}$ |
| CP-PACS 13  | NRQCD $O(M_v^4)$ errors, Asymmetric   | 1323(13)               |

reproduced the level splittings from NRQCD up to 10%. A preliminary result for the calculation of the adiabatic surfaces with the effects of dynamical fermions included has been reported by Bali [18] and collaborators. No dramatic differences between the quenched theory result were observed.

5 Conclusions

All the lattice simulations agree that the light $1^{-+}$ state has a mass of 2.0(2)GeV. The first simulation that included the effects of dynamical fermions has not changed the result.

The experimental results for light exotics are reviewed by S.U. Chung [20], so I just briefly compare the lattice results to experiment. There is an experimental signal for a $1^{-+}$ state at 1.4 GeV with a decay into $\eta\pi$ [20] from E852, Crystal barrel, VES, and KEK. It is surprising that this state is only seen in the $\eta\pi$ channel, as this decay is theoretically suppressed relative to other decays [21]. The E852 collaboration have also reported [22] a signal for $1^{-+}$ state decaying into $\rho\pi$ with a mass of around 1.6 GeV. The decay width is in reasonable agreement with theoretical calculations [23].

Clearly the agreement between the possible experimental signals for the $1^{-+}$ states and the lattice results is very poor. The errors on the lattice results for $1^{-+}$ states are large relative to the errors on $Q\bar{Q}$ states. To quantify the disagreement between experiment and lattice results the systematic errors on the lattice simulation results should be reduced. In particular the masses of the quarks used in the lattice simulation should be reduced. It is possible that the states seen experimentally are really $Q\bar{Q}QQ$ states, in which case the operators used in the lattice simulations would not couple strongly to them.

Although the adiabatic lattice potential approach is not expected to be a good description of the physics of light hybrids, we note that the results of Juge, Kuti and Morningstar [14], show that the splitting between the ground and first excited state is about 200 MeV, in broad agreement with the experimental results.
of the E852 collaboration. Although no insight is gained about the different decay widths.

To definitely identify an exotic hybrid meson requires both the calculation of the mass as well as the decay widths. There has been very little work on hadronic decays on the lattice. The most obvious hadronic process to study using lattice gauge theory is the $\rho \to \pi \pi$ decay, however there have only been a few attempts to calculate the $g_{\rho \pi \pi}$ coupling. The GF11 lattice group has recently computed the decay widths for the decay of the $0^{++}$ glueball to two pseudoscalars.

The MILC collaboration have started to investigate the mixing between the operator in eq. 6 and the operator $(\pi \otimes a_1)$ eq. 8.

$$\psi^a \gamma_5 \gamma_i \psi^b$$

which has the quantum numbers $1^{-+}$. This type of correlator would naively be expected to yield the decay width of the $1^{-+}$ state to $\rho$, and $a_1$. Unfortunately the analysis of Maiani and Testa shows that the matrix element required in the computation of the decay width is hidden beneath an unphysical term that increases exponentially with time. The origin of the unphysical term comes from the requirement that both final mesons should be onshell and is deeply related to theory being defined in Euclidean space (required for us to have a well defined theory to simulate). Some information may be extracted for onshell processes, using the methods proposed by Michael.

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References

1. T. H. Burnett and S. R. Sharpe, Ann. Rev. Nucl. Part. Sci. 40, 327 (1990).
2. I. Montvay and G. Munster, Quantum fields on a lattice, Cambridge, UK, (1994) (Cambridge monographs on mathematical physics).
3. P. Lacock and K. Schilling, hep-lat/9809022, 1998.
4. C. Bernard et al., Phys. Rev. D56, 7039 (1997).
5. C. Bernard et al., hep-lat/9809087, 1998.
6. T. D. Cohen, Phys. Lett. B427, 348 (1998).
7. S. Aoki et al., [hep-lat/9904012], 1998.
8. R. D. Kenway, [hep-lat/9810054], 1998.
9. Y. Uehara, N. Konno, H. Nakamura, and H. Noya, *Nucl. Phys.* **A606**, 357 (1996). D. B. Lichtenberg, R. Roncaglia, and E. Predazzi, *J. Phys.* **G23**, 865 (1997). D. B. Lichtenberg, R. Roncaglia, and E. Predazzi, [hep-ph/9611428], 1996.
10. M. Hess, F. Karsch, E. Laermann, and I. Wetzorke, *Phys. Rev.* **D58**, 111502 (1998).
11. P. Lacock, C. Michael, P. Boyle, and P. Rowland, *Phys. Lett.* **B401**, 308 (1997).
12. C. Davies, [hep-ph/9710394], 1997.
13. T. Manke et al., [hep-lat/9812017], 1998.
14. K. J. Juge, J. Kuti, and C. J. Morningstar, [hep-ph/9902336], 1999.
15. C. J. Morningstar and M. Peardon, *Phys. Rev.* **D56**, 4043 (1997).
16. T. Manke, I. T. Drummond, R. R. Horgan, and H. P. Shanahan, *Phys. Rev.* **D57**, 3829 (1998).
17. S. Perantonis and C. Michael, *Nucl. Phys.* **B347**, 854 (1990).
18. J. Kuti, [hep-lat/9811021], 1998.
19. C. Michael, [hep-ph/9809211], 1998.
20. S. U. Chung, these proceedings.
21. P. R. Page, *Phys. Lett.* **B402**, 183 (1997).
22. G. S. Adams et al., *Phys. Rev. Lett.* **81**, 5760 (1998).
23. P. R. Page, *Phys. Lett.* **B415**, 205 (1997).
24. S. Gottlieb, P. B. Mackenzie, H. B. Thacker, and D. Weingarten, *Phys. Lett.* **134B**, 346 (1984). R. L. Altmeyer et al., *Z. Phys.* **C68**, 443 (1995).
25. J. Sexton, A. Vaccarino, and D. Weingarten, *Phys. Rev. Lett.* **75**, 4563 (1995).
26. L. Maiani and M. Testa, *Phys. Lett.* **B245**, 585 (1990).
27. C. Michael, *Nucl. Phys.* **B327**, 515 (1989).