Effects of physical parameters and time-delay coefficients on the amplitude and frequency bandwidth of saturation controller for a nonlinear beam vibration

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Abstract. This paper is focused on the effects of the physical parameters for saturation control bandwidth and the time-delay coefficient for the saturation control stability. The approximate analytical solutions are obtained when the primary external resonance and 1:2 internal resonance occur simultaneously. The effects of amplitude of excitation, nonlinear feedback coefficients and nonlinear coefficients on saturation control are investigated. It is shown that nonlinear feedback coefficients can enlarge the effective saturation control bandwidth and enhance the dynamic vibration absorber’s suppression. Two nonlinear coefficients enhance the system’s nonlinear phenomenon and remain the same saturation control regions. For the time-delay coefficient, the system begins to appear unstable saturation control regions with only a little time-delay. The increasing time-delay coefficients can enlarge the unstable saturation control regions, it should be avoided.

1. Introduction

Saturation as a typical phenomenon in nonlinear systems was first discovered by Nayfeh et al.\cite{1} when analyzing the coupling motion between pitch and roll of a ship. Later, the saturation phenomenon was demonstrated experimentally by Haddow et al.\cite{2}, Nayfeh et al.\cite{3} and Oueini et al.\cite{4,5}. Jerzy et al.\cite{6,7} investigated the active vibration suppression of nonlinear composite beam. The efficiency of the nonlinear saturation control for high level of amplitude and wide range of frequencies of excitation is detailed tested by experiment. Hamed et al.\cite{8} studied an active vibration controller for suppressing the vibration of the non-linear composite beam in the presence of 1:2 internal resonances and the effects of the different parameters of the system.

The technique of delayed feedback control vibration absorber is a new technique of vibration suppression. Delayed position feedback control applied to dynamical structures was first presented by Olgac et al.\cite{9} by introducing a delayed resonator. Sayed et al.\cite{10} introduced time delay feedback control into a non-linear system excited by external and parametric forcing amplitude in the presence of 1:4 internal resonances. Zhao’s research\cite{11} has shown that in the nonlinear system, the vibration suppression performance of delayed feedback control was better than nonlinear dynamical absorber. The construction of this paper is as follows. The equations of motion are shown, and the perturbation analysis and the stability of the equilibrium solutions are given respectively in Section 2. The effects of the physical parameters and the delayed feedback control on saturation control are studied in Section 3. Section 4 gives a brief conclusion and some references are listed in the end.
2. Equations of motion with the saturation control

In the present paper, three time-delay coefficients are introduced into a quadratic nonlinearity controller of the nonlinear beam. The governing equations of the vibrating system can be written as:

$$
\begin{align*}
\ddot{u} + 2\mu \omega_0 \dot{u} + \omega_0^2 u + \beta u^3 - \delta (u^2 + u^2 \dot{u}) &= f \cos(\Omega t) + \gamma v^2 (t - \tau_1) \\
\ddot{v} + 2\zeta \omega_0 \dot{v} + \omega_0^2 v &= \alpha u (t - \tau_2) v (t - \tau_3)
\end{align*}
$$

(1)

2.1. Perturbation Analysis

To find the solution of steady states of Eq. (1) by introduction of a formal small-scale parameter $\varepsilon$:

$$
\begin{align*}
\mu &= \varepsilon \mu, \\
\zeta &= \varepsilon \zeta, \\
\beta &= \varepsilon^{-1} \beta, \\
\delta &= \varepsilon^{-1} \delta, \\
f &= \varepsilon^2 f,
\end{align*}
$$

where $\varepsilon$ is set as $0 < \varepsilon < 1$. Next, the method of multiple scales (MMS) is applied to obtain the second-order approximate solutions of (1), the motion $u$, $v$ and the time-delay motion $1 = u_{\tau_1}, 2 = u_{\tau_2} \ddot{v}, 3 = u_{\tau_3} \ddot{v}$ are written as :

$$
\begin{align*}
u(t, \varepsilon) &= u_0(T_0, T_1) + \varepsilon u_1(T_0, T_1) + \varepsilon^2 u_2(T_0, T_1) + \ldots \\
v(t, \varepsilon) &= v_0(T_0, T_1) + \varepsilon v_1(T_0, T_1) + \varepsilon^2 v_2(T_0, T_1) + \ldots \\
u(t, \varepsilon) &= u_{\tau_1}(T_0, T_1) + \varepsilon u_{\tau_2}(T_0, T_1) + \varepsilon^2 u_{\tau_3}(T_0, T_1) + \ldots \\
v(t, \varepsilon) &= v_{\tau_1}(T_0, T_1) + \varepsilon v_{\tau_2}(T_0, T_1) + \varepsilon^2 v_{\tau_3}(T_0, T_1) + \ldots
\end{align*}
$$

(2)-(6)

where $T_0 = t$, $T_1 = \varepsilon t$, $T_n = \varepsilon^n t^n$, $(n = 0, 1, 2, \ldots)$.

To describe the nearness of external resonance and internal resonance quantitatively and, two detuning parameters $\sigma_s$ and $\sigma_c$ are defined as follows

$$
\begin{align*}
\Omega &= \omega_0 + \varepsilon \sigma_s, \\
2\omega_0 &= \omega_0 + \varepsilon \sigma_c
\end{align*}
$$

(7)-(8)

According to the MMS, the general solutions of Eq. (1) is written in complex form:

$$
\begin{align*}
u(t, \varepsilon) &= A_1(T_1) e^{i\omega_1 T_1} + \bar{A}_1(T_1) e^{-i\omega_1 T_1}, \\
v(t, \varepsilon) &= A_2(T_1) e^{i\omega_1 T_1} + \bar{A}_2(T_1) e^{-i\omega_1 T_1}
\end{align*}
$$

(9)

Substituting Eqs. (2)-(9) into the Eq. (1) we get modulation equations in the following form

$$
\begin{align*}
a_1^2 + \frac{\gamma^2}{4\omega_0^2} \left[ \sin(\phi) \cos(2\tau_1 \omega_1) - \cos(\phi) \sin(2\tau_1 \omega_1) \right] - a_1 \mu \omega_0 + \frac{f \sin(\phi)}{2\omega_0} &= \frac{\alpha_a}{4\omega_0} \\
\phi_a &= \frac{\mu_a}{4\omega_0} \left[ \cos(\phi) \cos(2\tau_1 \omega_1) + \sin(\phi) \sin(2\tau_1 \omega_1) \right] - a_1 \omega_0 \\
\phi_c &= \frac{\mu_c}{4\omega_0} \left[ \cos(\phi) \cos(2\tau_1 \omega_1 - \tau_2 \omega_2) - \cos(\phi) \sin(2\tau_1 \omega_1 - \tau_2 \omega_2) \right] - a_2 \omega_0 \\
\phi_c &= \frac{\alpha_a a_2}{4\omega_0} \left[ \cos(\phi) \cos(2\tau_1 \omega_1 - \tau_2 \omega_2) + \sin(\phi) \sin(2\tau_1 \omega_1 - \tau_2 \omega_2) \right] + \frac{\sigma_c - \sigma_s}{2} a_2
\end{align*}
$$

(10)-(13)

where

$$
\begin{align*}
A_1(T_1) &= \frac{1}{2} a_1(T_1) e^{i\omega_1 T_1}, \\
A_2(T_1) &= \frac{1}{2} a_2(T_1) e^{i\omega_1 T_1}, \\
\sigma_s \tau_1 - \theta_1 = \phi_1 \quad \text{and} \quad 2\theta_2 - \theta_1 + \sigma_c \tau_1 = \phi_2.
\end{align*}
$$

2.2. Equilibrium Solutions and Their Stability.

From Eqs. (10)-(13), get single-mode solution $a_2 = 0$

$$
\begin{align*}
a_1^2 \left[ \sigma_s - \left( \frac{3\beta}{8\omega_0} + \frac{1}{4} \delta \omega_0 \right) a_2^2 \right] + a_1^2 \left( \mu \omega_0 \right)^2 &= \left( \frac{f}{2\omega_0} \right)^2
\end{align*}
$$

(14)

And couple-mode solution $a_2 \neq 0$.\]
To get the steady-state solutions of Eqs. (10)-(13) and to determine their stability, we transform them into Cartesian coordinates as

\[
a_i = \frac{4\omega_s}{\alpha} \sqrt{\left(\xi\omega_s\right)^2 + \left(\frac{\sigma_i - \sigma_s}{2}\right)^2}
\]

(15)

3. Effects of the physical parameters

From Ref[6], the physical parameters are shown as Tab.1, unless otherwise specified.

| \(\alpha\) | \(\beta\) | \(\gamma\) | \(\delta\) | \(\mu\) | \(\zeta\) | \(\omega_s\) | \(\omega_c\) | \(\tau_1\) | \(\tau_2\) | \(\tau_3\) | \(f\) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.7 | 14.41 | 0.01 | 0.93 | 0.01 | 0.0001 | 3.06 | 1.53 | 0 | 0 | 0 | 0.05 |

3.1. Phenomenon of Saturation Control and Bandwidth of Saturation Control

The equilibrium solutions can be obtained by setting \(p_{1}^* = q_{1}^* = p_{2}^* = q_{2}^* = 0\) in Eqs. (16)-(19) and using \(a_i = \sqrt{p_i^2 + q_i^2} (i = 1, 2)\). When \(\sigma_s - \sigma_i = d_0 = 0\), Fig. 1 shows that the amplitude of the external excitation increases from 0.05 to 0.15, the response amplitude of the beam remains a constant value and the response amplitude of the controller increases during the saturation control regions, where the solid line and dot with blank represent stable and unstable solutions, respectively. We can observe that the system is in the couple-mode motion when the amplitude of the controller is not equal to zero. This means that the controller does work as an absorber. The results imply that the extra energy added to the system has been absorbed by the controller so the response amplitude of the beam can remain constant value.

![Figure 1. Amplitude-frequency response curves of: (a) beam; (b) controller.](image-url)
3.2. Effects of the nonlinear coefficient $\beta$ and $\delta$ when $\sigma_s-\sigma_c=d_0=0$

In the following part, the effects of the nonlinear coefficients $\beta$ and $\delta$ on amplitude-frequency response curve are investigated when $d_0 = 0$. And, the other physical parameters of the system are shown as Tab.1. From Fig. 2 and Fig. 3, it can be observed that the nonlinear coefficients $\beta$ and $\delta$ have the similar effects. The controller maximum amplitude of saturation control $a^2$ remains a constant for different values. While, the non-linear phenomena of beam will be enhanced with the increasing coefficients $\beta$ and $\delta$, but the bandwidth of saturation control can’t be changed.

The results show that the nonlinear coefficients $\beta$ and $\delta$ don’t change the bandwidth of saturation control, but they enhance the non-linear phenomena.

![Figure 2. Amplitude-frequency response curves (a) beam; (b) controller.](image)

![Figure 3. Amplitude-frequency response curves (a) beam; (b) controller.](image)

3.3. Effects delayed feedback control on saturation control when $\sigma_s-\sigma_c=d_0=0$

From the couple-mode solution of Eq.(15), we observe that the amplitude of the beam $a_i$ is dependent on the feedback gain coefficient $\alpha$ and independent on the feedback gain coefficient $\gamma$ when $d_0 = 0$. And, the other physical parameters of the system are not changed as Tab.1.

Fig. 4 shows that as the increase of the feedback gain coefficient $\alpha$, the beam amplitude of saturation control decreases and the bandwidth of saturation control increases. Whereas, the controller maximum amplitude of saturation control stays on a constant and the bandwidth of saturation control increases.
Fig. 5 shows that as the increase of the feedback gain coefficient $\gamma$, the beam amplitude of saturation control and the bandwidth of saturation control all stay on a constant. Whereas, the controller maximum amplitude of saturation control decreases and the bandwidth of saturation control stays on a constant.

The results show that when the system is in the saturation control state, the amplitude of the beam could be suppressed while the bandwidth of saturation control could be broadened by increasing the feedback gain coefficient $\alpha$.

If there are some delay time in the control system, the stability analysis must be discussed. In the following section, the effects of delayed feedback on vibrating system are investigated when the system is in the saturation state. Fig. 6 shows the amplitude-frequency response curves of the beam and the controller for different values of delay time. For convenience, we set the delay time same. As $\tau_1 = \tau_2 = \tau_3$ increases, the unstable amplitudes of the beam and the controller on the saturation control region increase.

It has been predicted from the above analysis and the Fig. 6 that the delay time can affect the stability of the beam and control when the system is in the saturation control.

![Figure 4. Amplitude-frequency response curves (a) beam; (b) controller.](image)

![Figure 5. Amplitude-frequency response curves (a) beam; (b) controller.](image)
4. Conclusion

In the present paper, the effects of physical parameters and controlling parameters on saturation control are studied. The method of multiple scales is employed to obtain the approximate solutions of the vibrating system when the primary resonance and 1:2 internal resonance all occur simultaneously. From the above analysis, the main conclusions are as follows.

(1) The bandwidth of saturation control could be enlarged by increasing amplitude of external excitation.

(2) Properly increasing the feedback gain coefficient \(\alpha\) can enhance the performance of the improved saturation control, but increasing the feedback gain coefficient \(\gamma\) doesn't enhance the performance.

(3) The increasing nonlinear coefficients \(\beta\) and \(\delta\) enhance the non-linear phenomena.

(4) The increasing time-delay coefficients \(\tau_1\), \(\tau_2\) and \(\tau_3\) enlarge the unstable saturation control regions.

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