Effects of few optical cycles approximation to spectral broadening generated in two-zero dispersion wavelengths fibers

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Abstract. It is both experimentally and numerically confirmed that Supercontinuum can be generated by pumping a few-tens-femtoseconds short pulse in anomalous dispersion region of a two zero-dispersion wavelengths (ZDW) fiber, such as, photonic crystal fiber. In numerical method, a well-known slowly varying envelope approximation (SVEA) has always been used to illustrate how the short pulse expands to a broadband spectrum in two-ZDWs fiber. But, the limit of SVEA is that it can only be applied for the short pulse, which its temporal envelope changes slowly comparing with its optical cycle. However, a few optical cycles short pulse is not fit with the condition of the SVEA approximation. In this report, another model, namely slowly evolving wave approximation (SEWA), is applied for numerical demonstration of supercontinuum generated by pumping the few optical cycles short pulse in two-ZDWs fiber. According to our simulation, under SEWA the final spectral width of the very short input pulse in the two-ZDWs fiber is between about 500 - 2000 nm, even the center wavelength of input pulse is near second-ZDW of fiber. Moreover, other effects of these two approximations since the center wavelength of the very short input pulse is close to both upper and lower ZDWS of the fiber in the same fiber’s length are also compared in our presentation.

1. Introduction

Many studies recently focused on a role of a first-(lower) zero-dispersion-wavelength (ZDW) and a second-(higher) ZDW of two-ZDWs photonic crystal fibers (PCF). [1-3] As the second-ZDW closes to the first-ZDW, some of these researches agreed that the second-ZDW was also one of the factors causing the broadband spectrum of a few tens-femtosecond input pulse. The slowly varying envelope approximation (SVEA) is typically used in those researches. It satisfies to the pulse which its duration is much longer than its optical cycle. Since a few optical cycles pulse was propagated in a bulk silica core fiber, another approximation method for ultra-short pulse is used to investigate the spectrum of the pulse in the fiber. [4] This method is called the slowly evolving wave approximation (SEWA). In order to investigate the effect of the few optical cycles input pulse, propagating in two-ZDWs fiber, this SEWA is applied in this research. To prove SEWA suiting for the ultra-short pulse propagating in the two-ZDWs fiber, its result is compared with the one applied by SVEA. The effect of the pulse center wavelength, which is close to the first- and second-ZDW of fiber, and the pulse width is also reported.
2. Methods
To explore the supercontinuum process of the input pulse in two-ZDWs fiber, a PCF with ZDWs of 751 and 1233 nm is used as our fiber model. In SEWA, propagation and spectral broadening of the few optical cycles short pulse are simulated by solving an extended nonlinear Schrodinger equation (NLS) with a well-known split-step Fourier method (SSFM). The extended NLS is written as: [4, 5]

$$\frac{\partial A(\xi, T)}{\partial \xi} = i(\hat{D}' + \hat{D}_{\text{corr}})A(\xi, T) + i \frac{4g(\omega_0)\omega_0^2}{3c^2\beta_0}(1 + i s \frac{\partial}{\partial T})p_{\text{nl}}(\xi, T)$$  \hspace{1cm} (1)

where $A(\xi, T)$ represent as slowly varying amplitude of pulse envelope, $\omega_0$ is center frequency of pulse, $p_{\text{nl}}$ is nonlinear term, $T = t - \beta_0 z$, $g(\omega) = n(\omega)n'_2(\omega)/A_{\text{eff}}(\omega)$, $s = 2/\omega_0 - \beta_1/\beta_0 + \partial_\omega (\ln g(\omega))|_{\lambda_0}$,

$$\hat{D}' = \sum_{n=0}^{\infty} \frac{n!}{n!} \left[ \frac{\partial}{\partial \omega} \left( \beta(\omega) + \frac{i\alpha(\omega)}{2} \right) \right] \left[ \frac{\partial}{\partial T} - \beta_0 - i\beta_0 \frac{\partial}{\partial t} \right],$$  \hspace{1cm} (2)

and

$$\hat{D}_{\text{corr}} = \left( 1 + \frac{i\beta_1}{\beta_0} \frac{\partial}{\partial T} \right)^{-1} \frac{1}{2\beta_0} \hat{D}'^2,$$  \hspace{1cm} (3)

since $\beta_m$ are $m^{th}$ order dispersion coefficients and $\hat{\beta}_0 = \partial_\omega (\beta(\omega))|_{\lambda_0}$.

3. Results

3.1. Comparison for spectral broadening under SEWA and SVEA
To confirm that SEWA suits for the few optical cycle input pulse, 780-nm center wavelength, 100-kW peak power input Gaussian pulse is simulated to propagate in 10-cm long fiber. Spectral broadenings of 50-fs input pulse width are calculated by SEWA and SVEA as in figure 1(a) and 1(b), respectively.

![Figure 1. Power spectra and temporal pulse under for propagation along 10 cm of a Gaussian pulse with 780-nm center wavelength, 50-fs input pulse width, and 100 kW peak power. These results are calculated from (a) SEWA and (b) SVEA along 10 cm two-ZDWs PCF.](image-url)
From figure 1(a), the spectral broadening of SEWA quickly increases in the first 5-cm propagation distance and its width stays constant from 500- to 2000-nm as the one of SVEA in figure 1(b). Furthermore, temporal evolution of the output pulse remains almost symmetry, but it shifts to the trailing edge due to self-phase modulation (SPM) and second-order group velocity dispersion (GVD). But the spectrum under SVEA is expanded with more flatten and more clean fluctuation as in the previous experimental results. [5] In addition, second- and higher-order GVDs are the cause of loss of symmetry, fluctuation, and noise in time domain as in the previous experimental results.

In the case of the few optical cycles input pulse, 10-fs input pulses with the same parameters are also calculated by SEWA and SVEA as in figure 2(a) and 2(b), respectively.

![Figure 2](image-url)

**Figure 2.** Power spectra and temporal pulse under for propagation along 10 cm of a Gaussian pulse with 780-nm center wavelength, 10-fs input pulse width, and 100 kW peak power. These results are calculated from (a) SEWA and (b) SVEA along 10 cm of PCF.

From figure 2(a), the spectral broadening under SEWA is about 500- to 2000-nm, which is wider and more flatten than the one of SVEA in figure 2(b), which extends only about 1500-nm. That is because of GVD and nonlinearity of PCF. In both cases, soliton fission can be observed, however, solitons in SVEA clearly shift to leading edge. In contrast, the main soliton carry almost energy in SEWA. It is the same as the behaviour of soliton, approximated as the ultra-short pulse in the experiment. [4, 5] It can be concluded that SEWA suits for the few optical cycles pulses.

3.2. *Effect of center wavelength pumped in anomalous dispersion region*

In case of the input pulse pumped close to the first-ZDW, an input Gaussian pulse with 780-nm center wavelength is simulated to propagate in 10-cm long fiber by SEWA. The width and the peak power of the pulse are 10-fs and 100-kW peak power, respectively. The spectral broadening in this case shows as in figure 3(a). For the one pumped close to the second-ZDW, the same input Gaussian pulse but with 1200-nm center wavelength is also simulated in the same condition. Its result shows in figure 3(b).

According to our results, the spectral width becomes broaden about 500- to 2000-nm, in both two cases. For 780-nm center wavelength input pulse, the energy of the pulse almost transfers to the longer wavelength regime because of self-phase modulation (SPM) and Raman scattering. On the other hands, some energy from longer wavelength region of the input pulse with 1200-nm center wavelength transfers to the shorter one due to the four-wave mixing (FWM) effect. These cause fluctuation of pulse envelope. For both cases, GVD also leads the energy transformation to the around 500-nm wavelength. [4, 6] The soliton fission to leading edge is also observed. Furthermore, in both cases, the speed of pulse in all range of spectrum is almost the same as shown in the spectrogram of figure 4.
Figure 3. Power spectra and temporal pulse for propagation along 10 cm of a Gaussian pulse with 10-fs input pulse width, 100 kW peak power. Their center wavelengths are (a) 780 nm and (b) 1200 nm.

Figure 4. Spectrogram for propagation along 10 cm of a Gaussian pulse with 10-fs input pulse width, 100 kW peak power. Their center wavelengths are (a) 780 nm and (b) 1200 nm.

3.3. Effect of width of peak power of input pulse pumped in anomalous dispersion region
To study the effect of input pulse in the first-ZDW region, 780-nm center wavelength, 10-fs input Gaussian pulses with 50-kW, 100-kW and 200-kW peak power are separately simulated to propagated in 10-cm long two-ZDW fiber by SWEA. The results show in figure 5.

According to our simulation, the broadening of spectrum in every case is about 500 nm to more than 2000 nm. But the greater peak power produces more spectrum fluctuation. Also, the shape of temporal spectral of output pulses trends to separate due to SPM and higher order GVDs. [4, 6] For 200-kW peak power input pulse, the soliton insignificant moves to the longer wavelength.
Figure 5. Power spectra and temporal pulse of output pulse under SEWA for propagation along 10 cm of a Gaussian pulse with 10-fs input pulse width, 780 nm center wavelength. Their peak powers are 50 kW, 100 kW and 200 kW.

Figure 6. Power spectra and temporal pulse of output pulse under SEWA for propagation along 10 cm of a Gaussian pulse with 10-fs input pulse width, 1200 nm center wavelength. Their peak powers are 50 kW, 100 kW and 200 kW.

Furthermore, the effect of peak power of the input pulse in the second-ZDW region is investigated by applied SEWA to simulate the propagation of 10-fs Gaussian input pulses with 50-kW, 100-kW and 200-kW peak power in 10-cm long fiber. Center wavelength of the input pulse is set to 1200-nm. From our results in figure 6, the range of output spectral in all cases is around 500- to more than 2000-nm. Spectral broadening in this case is also owing to SPM, and FWM inducing the components of the spectrum in 500-nm short wavelength region. Moreover, the greater peak power produces more
spectrum fluctuation as in 780-nm center wavelength input pulse. Also, the shape of temporal spectral of all pulses trend to separate as solitons. But they move to the longer wavelength comparing with the one of 200-kW, 780-nm pulse. This confirms that the second-ZDW causes the spectrum extension to near-infrared region. [1-3]

4. Discussion and Conclusion
In conclusion, SEWA suits for simulating the propagation of the few optical cycles pulses in two-ZDWs PCF. That is because the main soliton carries almost energy, which is the same as the behaviour of solitons in the experiment. Input pulse with high energy gives spectral broadening from 500- to more than 2000-nm whether the center wavelength of input pulse is near first-ZDW or second-ZDW. Moreover, peak power of input pulse is also affected and contributes energy to the pulse propagation in the short- and long-wavelength range. The greater peak power produces more spectrum fluctuation. For the input pulse near the first-ZDW fiber, more peak power produces more broaden output spectrum. However, the fission soliton moves to longer wavelength, since the center wavelength of the pulse is closer to the second-ZDW. This confirms that the second-ZDW causes the spectrum extension to near-infrared region.

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