Equilibrium Strategies of a Polling Network with Non-zero Switch-over Times

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Abstract. In this paper, we consider a cyclic two-queue polling system with non-zero switch-over times under exhaustive service discipline. Arriving customers decide whether to join queue 1 (Q₁) or queue 2 (Q₂) under two different information levels: fully unobservable case and fully observable case. With the help of the waiting costs, we study the equilibrium strategies for arriving customers in both cases. Furthermore, we analyze the expected waiting cost of an arriving customer in fully unobservable case by numerical examples.

1. Introduction

Equilibrium strategies of customers in queueing systems which can be applied to management in service, communication networks and electronic commerce and so on have been studied for several decades. This work was first introduced by Naor [10]. Since then these problems have been attracted many authors, see for example [1, 2, 5, 9, 12] and reference therein.

Burnetas and Economou [2] first considered the equilibrium strategies of arriving customers in a Markovian queue with exponential vacations (setup times) under four different information levels. Economou [5] studied the equilibrium strategies and socially optimal strategies for arriving customers in general distributed service times and general distributed setup times (vacations) for fully unobservable case and almost observable case. Guo and Hassin [7] investigated equilibrium strategies and socially optimal strategies of arriving customers in a Markovian queue with non-zero setup times under N-policy. Zhang et al. [17] considered the equilibrium strategies of customers in general distributed service times and general distributed setup times for observable and almost observable cases which extended the results in Burnetas and Economou [2] to the non-Markovian case. Therefore, the analysis of equilibrium behavior in queueing systems with setup times is limited to one queue in almost all of the relevant papers.

In this paper, we consider a two-queue polling system with non-zero switch-over times which has applications in various fields such as computers, telecommunications, transportation systems and wireless networks and so on, see for example [4, 8, 15]. The performance evaluations for polling systems with non-zero switch-over times have been studied by many authors. Winands et al. [16] considered a cyclic N -queue (N ≥ 2) polling system with non-zero switch-over times (setup times) for exhaustive and gated service disciplines and obtained the mean queue length and the mean waiting times by using mean value analysis method. Boon et al. [3] considered a cyclic polling system with non-zero switch-over times in which the arrival rates of customers depend on the server's position and received the mean queue length and mean waiting times. However, equilibrium strategy of an arriving customer in a two-queue system is a less studied area. The few literature refers to [1, 11]. Sharafali et
al. [11] studied an optimization problem of production scheduling which can be modeled as a polling system with zero switch-over times in order to minimize the total average cost. Adan et al. [1] analyzed the equilibrium strategies and socially optimal strategies of arriving customers in a two-queue polling system with exhaustive service discipline and zero switch-over times under three different information levels: fully unobservable case, almost unobservable case and fully observable case.

In this paper, we study the equilibrium (Nash) strategies of customers in a two-queue polling system with non-zero switch-over times for exhaustive service discipline. To the best of the authors' knowledge, that problem has not been studied. In this model, we assume that customers arrive at the system according to a Poisson process. The service times and switch-over times are independent and exponentially distributed random variables. Since the waiting cost of an arriving customer depends on the server's position, an arriving customer will join a queue which is not being served to minimize the waiting cost though the total waiting time will increase. We investigate the equilibrium strategies of customers under two different information levels: (1) the fully unobservable case: arriving customers are not informed the number of customers in the system or server's position. (2) the fully observable case: arriving customers are informed the number of customers in the system and server's position. Since the server stops providing service to the arriving customers in switch-over times, the analysis of the equilibrium strategies is more complex than the model with zero switch-over times. Finally, we show some numerical examples to analyze the optimal joining probability such that the expected waiting time of an arriving customer reaches minimum and the effect of several parameters on the expected waiting cost of an arriving customer.

The rest of this paper is organized as follows. In Section 2, we describe precisely the polling model. In Sections 3 and 4, we consider the equilibrium strategies in the fully unobservable case and fully observable case, respectively. Section 5 shows some numerical examples to analyze the expected waiting cost of an arriving customer in fully unobservable case. Section 6 concludes the paper with a summary.

2. Model description

In this paper, we consider a cyclic two-queue polling system with non-zero switch-over time. Now we give a precisely description of the model. The polling model contains a server and two queues, $Q_1, Q_2$, which is served by the order $Q_1 \rightarrow Q_2 \rightarrow Q_1 \rightarrow Q_2 \rightarrow \cdots$. Customers arrive at this system according to a Poisson process with rate $\lambda$. The service times of customers are assumed to be exponentially distributed random variables with rate $\mu$. Indices throughout the paper are modulo 2, then $Q_3$ and $S_3$ can be regarded as $Q_1$ and $S_1$, respectively. Switching from $Q_i$ to $Q_{i+1}$ ($i = 1, 2$), server requires a switch-over time $S_i$. The switch-over times are assumed to be exponentially distributed random variables with parameter $\theta$. We assume that $S_1 + S_2 > 0$. During the switch-over times, customers stop being served and continue arrive. In our model, server will switch from $Q_i$ to $Q_{i+1}$ ($i = 1, 2$) if there are no customers waiting in $Q_i$, that is exhaustive service policy. The inter-arrival times, the service times and the switch-over times are assumed to be mutually independent.

For stability, we assume that $\rho = \lambda/\mu$. Let $I(t)$ be the server's position at time $t$, $I(t) = 1$: working state (a customer is being served); $I(t) = 0$: switching period. When the server is found at the switch-over period (at state 0), a customer incurs a waiting cost of $d$ dollars per unit time. When the server is found at working period (at state 1), a customer will pay a waiting cost of $c$ dollars per unit time if he is waiting for service in a queue which is being served and pay a waiting cost of $d$ dollars per unit time if he is waiting for service in a queue which is not being served. In this paper, the objective is to study how the arriving customers decide whether to join $Q_1$ or $Q_2$ such that their
expected waiting costs reaches minimum in the fully unobservable case and fully observable case, respectively.

3. Fully unobservable case

Now, we consider the fully unobservable case in which arriving customers are not informed the number of customers in $Q_1$ and $Q_2$ or the server's position. The arriving customers join $Q_1$ with probability $q$ and join $Q_2$ with probability $1-q$. For simplicity, let $\rho_1 = q\rho$, $\rho_2 = (1-q)\rho$. $EL_{i,j}$, $i, j = 1, 2$ is the expected queue length of $Q_i$ when the server is serving $Q_j$ or in the switch-over period $S_{j\rightarrow i}$. $EL_{i,j}$, $i = 1, 2$ do not include the customer who is being served. By [16], the equations of $EL_{i,j}$, $i, j = 1, 2$, is given by the following theorem.

**Theorem 3.1** For all $0 \leq q \leq 1$, we have

$$EL_{1,1} = \frac{2\rho_1^2(1-\rho + \rho_2\rho)}{(1-\rho)(1-\rho + 2\rho_2\rho)(1-\rho_2 + \rho_1)} + \frac{2\rho_2(\rho_1^2(1-\rho) + 2(1-\rho + 2\rho_2\rho)(1-\rho_1)(1-\rho_2))}{\theta (1-\rho)(1-\rho + 2\rho_2\rho)(1-\rho_2 + \rho_1)}$$

$$EL_{1,2} = \frac{\rho_1}{1-\rho_2} \left( EL_{2,2} + \frac{2\rho_2}{1+\rho_2 - \rho_1} \right) + \frac{\lambda_2}{\theta} \frac{1-\rho}{1-\rho_2 + \rho_1 - \rho_2};$$

$$EL_{2,1} = \frac{\rho_2}{1-\rho_1} \left( EL_{1,1} + \frac{2\rho_1}{1+\rho_1 - \rho_2} \right) + \frac{\lambda_1}{\theta} \frac{1-\rho}{1-\rho_1 + \rho_1 - \rho_2};$$

$$EL_{2,2} = \frac{2\rho_1^2(1-\rho + \rho_2\rho)}{(1-\rho)(1-\rho + 2\rho_2\rho)(1-\rho_1 + \rho_2)} + \frac{2\rho_2(\rho_1^2(1-\rho) + 2(1-\rho + 2\rho_2\rho)(1-\rho_1)(1-\rho_2))}{\theta (1-\rho)(1-\rho + 2\rho_2\rho)(1-\rho_1 + \rho_2)}$$

$$+ \frac{\lambda_1}{\theta} \frac{\rho_2(\rho_1(1-\rho) + 2(1-\rho + 2\rho_2\rho)(1-\rho_1)(1-\rho_2))}{(1-\rho)(1-\rho + 2\rho_2\rho)(1-\rho_1)}.$$

The expected waiting cost of an arriving customer who joins $Q_1$ or $Q_2$ is $EW_1$ or $EW_2$, respectively. Therefore, we get

$$EW_1 = \frac{c}{\mu} \left( \frac{1+\rho_1-\rho_2}{2} EL_{1,1} + \frac{1+\rho_2-\rho_1}{2} EL_{1,2} + \rho_1 \right)$$

$$+ d \left( \frac{1-\rho}{2\theta} + \frac{1+\rho_2-\rho_1}{2} \left( \frac{1}{1-\rho_2} \left( EL_{2,2} + \frac{2\rho_2}{1+\rho_2 - \rho_1} + \frac{1-\rho}{\theta (1+\rho_2 - \rho_1)} \right) \right) \right);$$

$$EW_2 = \frac{c}{\mu} \left( \frac{1+\rho_1-\rho_2}{2} EL_{2,1} + \frac{1+\rho_2-\rho_1}{2} EL_{2,2} + \rho_2 \right)$$

$$+ d \left( \frac{1-\rho}{2\theta} + \frac{1-\rho_2 + \rho_1}{2} \left( \frac{1}{1-\rho_1} \left( EL_{1,1} + \frac{2\rho_1}{1+\rho_1 - \rho_2} + \frac{1-\rho}{\theta (1+\rho_1 - \rho_2)} \right) \right) \right);$$

Hence, the expected waiting cost of an arriving customer in fully unobservable case is

$$S = qEW_1 + (1-q)EW_2.$$
Since the first and second order derivatives of the expected waiting cost of an arriving customer in fully unobservable are too complicated, so we analyze the social benefit in fully unobservable case by numerical experiments.

The equilibrium strategies of an arriving customer are specified by the joining probability, that is an arriving customer will join $Q_1$ with probability $q_e$ and join $Q_2$ with probability $1-q_e$. The value of $q_e$ is described in the following theorem.

**Theorem 3.2** In fully unobservable case of the polling system with non-zero switch-over times, Nash equilibrium strategies are given by:

**Case A:** If $(1 - \rho) - \frac{d}{(1 - \rho)} \leq 0$, then $q_e = 0$, $q_e = 1/2$, $q_e = 1$.

**Case B:** If $(1 - \rho) \geq d + \frac{2(1 + \rho^2)}{4(1-\rho)}$, then $q_e = 1/2$.

**Proof** By the definition of $EW_1$ and $EW_2$, we obtain

$$
EW_1 - EW_2 = \frac{1}{\mu} \left( (c(1-\rho)-d) \left( \frac{1+\rho_1-\rho_2}{2(1-\rho)} EL_{1,1} + (d-c(1-\rho)) \frac{1+\rho_2-\rho_1}{2(1-\rho)} EL_{2,2} \right) \right)
+ \frac{c}{\mu} \left( \frac{\rho_1 \rho_2 - \rho_1^2 - \rho_2^2}{1 - \rho_1} \right) + \frac{c}{\theta} \left( \frac{2(1-\rho)}{2(1-\rho)}(1-\rho) \right) + \frac{d}{\mu} \left( \frac{\rho_2 - \rho_1}{1 - \rho_1} \right) + \frac{d}{\mu} \left( \frac{1 - \rho_1}{2(1-\rho)} \right) + \frac{d}{\theta} \left( \frac{\rho_1 - \rho_2 - \rho_1^2}{2} \right) \right)
+ \frac{1}{\mu} \left( (c(1-\rho)-d) \left( \frac{1+\rho_2-\rho_1}{2(1-\rho)} EL_{1,1} - \frac{1+\rho_2-\rho_1}{2(1-\rho)} EL_{2,2} \right) \right)
+ \frac{1}{\mu} \left( (c(1-\rho)-d) \left( \frac{(\rho_1 - \rho_2)}{2(1-\rho)} \right) \right) + \frac{1}{\theta} \left( (c(1-\rho)-d) \left( \frac{(\rho_1 - \rho_2)}{2(1-\rho)} \right) \right) + \frac{d}{\theta} \left( \frac{\rho_1 - \rho_2}{2(1-\rho)} \right).

(3.1)

Now we compute

$$
1 + \frac{1}{\mu} \left( \left( \frac{1+\rho_1-\rho_2}{2(1-\rho)} EL_{1,1} - \frac{1+\rho_2-\rho_1}{2(1-\rho)} EL_{2,2} \right) \right)
= S_1 + S_2,
$$

where

$$
S_1 = \frac{1}{\mu} \left( \frac{\rho_1 - \rho_2}{2(1-\rho)} EL_{1,1} - \frac{\rho_2 - \rho_1}{2(1-\rho)} EL_{2,2} \right)
$$

and

$$
S_2 = \frac{1}{\mu} \left( \frac{1+\rho_1-\rho_2}{2(1-\rho)} EL_{1,1} - \frac{1+\rho_2-\rho_1}{2(1-\rho)} EL_{2,2} \right).
$$

Therefore, by Theorem 3.1, we obtain

$$
S_1 = (\rho_1 - \rho_2) \left( \frac{S_{1,1}}{\mu S_{1,0}(1-\rho_1)(1-\rho_2)} + \frac{S_{1,2}}{S_{1,0}(1-\rho_1)(1-\rho_2)} + \frac{S_{1,3}}{S_{1,0}} \right);
$$

$$
S_2 = (\rho_1 - \rho_2) \left( \frac{S_{2,1}}{\mu S_{2,0}(1-\rho_1)(1-\rho_2)} + \frac{S_{2,2}}{S_{2,0}(1-\rho_1)(1-\rho_2)} + \frac{S_{2,3}}{S_{2,0}} \right),
$$

where

$$
S_{1,0} = (1-\rho)(1-\rho + 2\rho_1 \rho_2)(1+\rho_1 - \rho_2)(1+\rho_2 - \rho_1);
$$

$$
S_{1,1} = (1-\rho)(1-\rho)\rho_1^2 + \rho_1 \rho_2 + (1-\rho)\rho_2^2 + \rho^2 \rho_2
$$

$$
+(1+\rho+\rho^2)\rho_1^2 \rho_2 + \rho_1 \rho_2 (2\rho + 1)(\rho_1^2 + \rho_1 \rho_2 + \rho_2^2);\n$$

$$
S_{1,2} = (1-\rho)(1-\rho)\rho_1 + \rho_1 \rho_2 + (1-\rho)\rho_2
$$

$$
+(1+\rho+\rho^2)\rho_1^2 \rho_2 + \rho_1 \rho_2 (2\rho + 1)(\rho_1^2 + \rho_1 \rho_2 + \rho_2^2);\n$$

$$
S_{2,0} = (1-\rho)(1-\rho + 2\rho_1 \rho_2)(1+\rho_1 - \rho_2)(1+\rho_2 - \rho_1);
$$

$$
S_{2,1} = (1-\rho)(1-\rho)\rho_1^2 + \rho_1 \rho_2 + (1-\rho)\rho_2^2 + \rho^2 \rho_2
$$

$$
+(1+\rho+\rho^2)\rho_1^2 \rho_2 + \rho_1 \rho_2 (2\rho + 1)(\rho_1^2 + \rho_1 \rho_2 + \rho_2^2);\n$$

$$
S_{2,2} = (1-\rho)(1-\rho)\rho_1 + \rho_1 \rho_2 + (1-\rho)\rho_2
$$

$$
+(1+\rho+\rho^2)\rho_1^2 \rho_2 + \rho_1 \rho_2 (2\rho + 1)(\rho_1^2 + \rho_1 \rho_2 + \rho_2^2);\n$$

$$
S_{2,3} = (1-\rho)(1-\rho)\rho_1 + \rho_1 \rho_2 + (1-\rho)\rho_2
$$

$$
+(1+\rho+\rho^2)\rho_1^2 \rho_2 + \rho_1 \rho_2 (2\rho + 1)(\rho_1^2 + \rho_1 \rho_2 + \rho_2^2).\n$$
\[ S_{1,2} = \frac{\rho \rho_2}{2 \theta} (\rho_1 \rho_2 (1 - \rho)(1 - \rho + 2 \rho_1 \rho_2) + 2(1 - \rho + \rho_1 \rho_2)(\rho_1^2 + \rho \rho_2 + \rho_2^2 + \rho_2 \rho_2)) ; \]

\[ S_{1,3} = \frac{1 - \rho}{2 \theta} (\rho_1^2 + \rho_1 \rho_2 + \rho_2^2) - \rho_2 \rho_1^2 - \rho_2 \rho_1^2 + \rho_1 (1 - \rho) + \rho_2 (1 - \rho) + \rho_2 \rho_1) ; \]

\[ S_{2,1} = \frac{(1 - \rho_1)(1 - \rho_2)}{\theta} (\rho_1 - \rho_2) (\rho_1 - \rho) + \rho_1 \rho_2 (\rho_1 - \rho_2) + 2 \rho_1 \rho_2 (\rho_1^2 + \rho_2^2 + \rho_2 \rho_2) ; \]

\[ S_{2,2} = \frac{1}{\theta} (\rho_1 \rho_2 (1 - \rho_1)(1 - \rho_2) (2(1 - \rho + \rho_1 \rho_2) + \rho_1 \rho_2^2 (1 - \rho) (3 + \rho_1^2 + \rho_2^2 + \rho_2 \rho_2) + 2 \rho_1 \rho_2 (2 - \rho)) ; \]

\[ S_{2,3} = \frac{1}{\theta} (\rho_1 \rho_2 (1 - \rho_1)(1 - \rho_2)(\rho_1 - \rho_2) + \rho_2 (1 - \rho_1)(1 - \rho_2) + 2(1 - \rho + \rho_1 \rho_2) . \]

Since \( \rho < 1 \), \( \rho_1 < \rho \) and \( \rho_2 < \rho \), we have \( S_{1,0} > 0 \), \( S_{1,1} > 0 \), \( S_{1,2} > 0 \), \( S_{1,3} > 0 \), \( S_{2,1} > 0 \), \( S_{2,2} > 0 \) and \( S_{2,3} > 0 \).

Therefore, by equation (3.1), we obtain

\[ EW_1 - EW_2 = \frac{1}{\mu} (c(1 - \rho) - d)(\rho_1 - \rho_2) \left( \frac{S_{1,1}}{\mu S_{1,0}(1 - \rho_1)(1 - \rho_2)} + \frac{S_{1,2}}{S_{1,0}(1 - \rho_1)(1 - \rho_2)} + \frac{S_{1,3}}{S_{1,0}} \right) \]

\[ + \left( \frac{S_{2,2}}{S_{2,2}} + \frac{S_{2,3}}{S_{2,2}} \right) + \frac{1}{\mu} (c(1 - \rho) - d) \left( \frac{\rho_1 - \rho_2}{(1 - \rho_1)(1 - \rho_2)} \right) \]

\[ + \frac{1}{\theta} (c(1 - \rho) - d - \frac{2(1 - \rho + \rho_1 \rho_2)}{1 - \rho} \frac{\rho_2 - \rho_1}{2(1 - \rho_1)(1 - \rho_2)} . \]

Case A: We study the case \( c(1 - \rho) > d + 2 (1 + \rho_1^2 / (4(1 - \rho))) d \).

1) If all arriving customers join \( Q_1 \) with probability \( q > 1/2 \), then \( EW_1 > EW_2 \), so that the arriving customers will join \( Q_2 \), that is \( q = 0 \).

2) If all arriving customers join \( Q_1 \) with probability \( q < 1/2 \), then \( EW_1 < EW_2 \), so that the arriving customers will join \( Q_1 \), that is \( q = 1 \).

Therefore, from case (1) and case (2), we know that there is no equilibrium strategy.

3) If all arriving customers join \( Q_1 \) with probability \( q = 1/2 \), then there is no different for the arriving customer to join \( Q_1 \) or \( Q_2 \), so that \( q_e = 1/2 \).

Case B: We study the case \( c(1 - \rho) < d \), with the same method of Case A, it is easy to obtain three equilibrium strategies \( q_e = 1 \), \( q_e = 1/2 \) and \( q_e = 0 \).

4. Fully observable case
We now proceed to the fully observable case where arriving customers are informed the number of customers in \( Q_1 \) and \( Q_2 \) and the server’s position. When the server is found at state 1 (a customer in \( Q_i \), \( i = 1, 2 \) is being served), then \( Q_r \) is called busy queue and the other queue is called idle queue.

When the server is found in switch-over period \( S_i \) for \( i = 1, 2 \) (the server is found at state 0), \( Q_{ri+1} \) is called busy queue and \( Q_i \) is called idle queue. In the fully observable case, the probability of joining
busy queue for an arriving customer is \( q \) and the probability of joining idle queue is \( 1 - q \). 
\((N_1(t), N_2(t), I(t))\) represents that there are \( N_1(t) \) customers in the busy queue, \( N_2(t) \) customers in the idle queue and server is in state \( I(t) \) at time \( t \).

When the arriving customer knows information \((n, m, i)\), the expected waiting cost of an arriving customer for joining the busy queue and idle queue \( EW_b(i) \) and \( EW_i(i) \), \( i = 0, 1 \), are, respectively

\[
EW_b(i) = \frac{c(n+1)}{\mu} + \frac{d(1-i)}{\theta}; \quad EW_i(i) = \frac{c(m+1)}{\mu} + d\left(\frac{n}{\mu - \lambda q} + \frac{2-i}{\theta}\right).
\]

The arriving customer joins the busy queue if

\[
\frac{c(n+1)}{\mu} + \frac{d(1-i)}{\theta} < \frac{c(m+1)}{\mu} + d\left(\frac{n}{\mu - \lambda q} + \frac{2-i}{\theta}\right),
\]

that is

\[
\frac{cn}{\mu} < \frac{cm}{\mu} + \frac{dn}{\mu - \lambda q} + \frac{d}{\theta}.
\]

The arriving customer joins the idle queue if

\[
\frac{c(n+1)}{\mu} + \frac{d(1-i)}{\theta} > \frac{c(m+1)}{\mu} + d\left(\frac{n}{\mu - \lambda q} + \frac{2-i}{\theta}\right),
\]

that is

\[
\frac{cn}{\mu} < \frac{cm}{\mu} + \frac{dn}{\mu - \lambda q} + \frac{d}{\theta}.
\]

When \( \frac{cn}{\mu} = \frac{cm}{\mu} + \frac{dn}{\mu - \lambda q} + \frac{d}{\theta} \), there is no different to join busy queue or idle queue. If \( d = 0 \), he decides to join a queue in which the number of customers is least.

5. Numerical experiments
In this section, we present some numerical experiments to analyze the optimal joining probability such that the expected waiting cost of an arriving customer reaches minimum and the effect of some parameters (the service rate \( \mu \), the arrival rate of customers \( \lambda \), the switch-over rate \( \theta \), the costs of waiting per unit time \( c \) and \( d \) ) on the expected waiting cost of an arriving customer in the fully unobservable case.

We first study the optimal joining probability to minimize the expected waiting cost of an arriving customer in the fully unobservable case. From the left of Figure 1, we can see that the expected waiting cost is a concave function, so that there exists a unique optimal joining probability, which is \( q = 0.65 \).

We then consider the effect of several parameters on expected waiting cost of an arriving customer. Obviously, we know that the expected waiting cost decreases as the service time or the switch-over time increase, this phenomenon can be observed in the right of Figure 1 and 2. These are because that when service rate \( \mu \) and switch-over rate \( \theta \) increase, the service time and switch-over time become smaller, so that the expected waiting cost of an arriving customers reduces. When the service rate and switch-over rate are greater than a certain point, the expected waiting cost of an arriving customer reaches a stable value. The left of Figure 2 shows that the relationship between the arrival rate of customers \( \lambda \) and the expected waiting cost. The expected waiting cost increases as the arrival rate of customers increases, which is intuitive. From Figure 3, we can observe that the expected waiting cost of an arriving customer increases with the costs of waiting per unit time \( c \) and \( d \) increase.
Figure 1 Left: Optimal joining probability in fully unobservable case for $\lambda = 0.6$, $\mu = 1$, $c = 20$, $d = 1$; Right: Expected waiting cost of an arriving customer in fully unobservable case with respect to $\mu$ for $\lambda = 10$, $\theta = 10$. $c = 20$, $d = 1$.

Figure 2 Left: Expected waiting cost of an arriving customer in fully unobservable case with respect to $\lambda$ for $\mu = 20$, $\theta = 10$, $c = 20$, $d = 1$; Right: Expected waiting cost of an arriving customer in fully unobservable case with respect to $\theta$ for $\lambda = 50$, $\mu = 20$, $c = 20$, $d = 1$.

Figure 3 Left: Expected waiting cost of an arriving customer in fully unobservable case with respect to $c$ for $\lambda = 0.6$, $\mu = 1$, $\theta = 10$, $d = 10$; Right: Expected waiting cost of an arriving customer in fully unobservable case with respect to $d$ for $\lambda = 0.6$, $\mu = 1$, $\theta = 1$, $c = 20$. 
6. Conclusions
In this paper, we studied a two-queue polling system with non-zero switch-over times under exhaustive service discipline. The arriving customers decide whether to join $Q_1$ or $Q_2$ under two different information levels: fully unobservable case and fully observable case. The equilibrium strategy of an arriving customer is specified by joining probability, that is an arriving customer will join $Q_1$ with probability $q$ and join $Q_2$ with probability $1-q$. We studied the equilibrium strategies of arriving customers in both cases. We studied the optimal joining probability to minimize the expected waiting cost of an arriving customer and the effect of several parameters on the expected waiting cost of an arriving customer by numerical experiments.

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