I. INTRODUCTION

The knowledge of eta-nucleon coupling constant, \( g_{\eta NN} \), has implications in both hadronic physics as well as nuclear physics. It is required for study of \( \eta \) production off a nucleon target, and also for analysis of NN scattering data. In general, it has been used for construction of realistic nuclear potentials \([1,3]\). In particular, \( \eta \)-exchange along with pion exchange between nucleons can give rise to isospin violation in nuclear potential \([4]\); this has been used for the resolution of the Nolen-Schiffer anomaly \([5]\). \( g_{\eta NN} \) will also be useful for understanding the strange content of the nucleon \( [6] \) via the Sullivan process \([7]\). It may provide important constraints on parton distribution amplitudes (DAs) of \( \eta \). For mesons \( \pi^0, \eta \) and \( \eta' \), the strong interaction induces transitions between quarks of different flavors \((u\bar{u}, d\bar{d}, s\bar{s})\) or gluons \((gg, ...\) \([8]\). The mixing phenomenon is strongly connected with the U(1)\(_A\) anomaly of QCD. Hence, it is expected that a reliable determination of \( g_{\eta NN} \) would shed considerable light on the U(1)\(_A\) dynamics of QCD \([9]\). The measurement of \( g_{\eta NN} \) is a formidable task since the production of \( \eta \) mesons from single nucleons is dominated by the resonance \( S_{11}(1535) \) irrespective of the probe \([10]\). Therefore, a reliable theoretical estimate of \( g_{\eta NN} \) is desirable.

Among the various methods used for calculating hadronic parameters, QCD sum rules are especially useful. This method has already been used for calculating several meson-baryon couplings \([11,20]\) and, in particular, \( g_{\eta NN} \) in SU(3)-flavor symmetry limit \([14,15]\). In this work we wish to apply this method to calculate \( g_{\eta NN} \) with leading SU(3)-flavor violating effect taken into account.

QCD sum rules for calculation of meson-baryon coupling constant was first used by Reinders et al. \([11,12]\) who first considered three-point correlation function for \( g_{\pi NN} \). On finding of the inconsistency with the Goldberger-Treiman (GT) relation they studied two-point correlation function of interpolating fields of nucleons with the pion in the initial state. In the former approach one had to assume an extrapolation from large space-like momentum to zero momentum for the meson whereas no such assumption was needed in the latter. It was pointed out by Birse and Krippa \([13]\) that in the soft pion limit this latter sum rule can be obtained from nucleon mass sum rule by chiral rotation, and hence it does not constitute an independent sum rule from that for the nucleon mass. Therefore, one must take finite meson momentum to arrive at an independent sum rule.

The two-point correlation function with an initial meson state gives rise to several independent sum rules which have been studied in detail \([14,17]\). It has been observed \([14,15]\) that the sum rule with the tensor structure gives the most reliable result. An improvement of this approach has been proposed by Kondo and Morimatsu \([18]\) wherein one takes matrix element of the correlation function with respect to nucleon spinors. The sum rule obtained in this way is free from ambiguity arising from the choice of the effective interaction Lagrangian and the effect of nonzero mass of the meson can also be taken into account.

The evaluation of two-point correlation function of nucleons between vacuum and a one meson state requires knowledge of matrix elements of nonlocal quark and gluon operators between vacuum and one meson state. Such matrix elements have been extensively studied in literature mainly for isospin-non-singlet members of the octet pseudo-scalar family \([21,22]\). There have also been derivations, using various approaches, of Gell-Mann-Okubo type of relations involving two-parton and three-parton light-cone distribution functions of pions, kaons and eta \([24,25]\). These relations can be used to get matrix elements of nonlocal quark and gluon operators between vacuum and one eta-meson state to leading order in SU(3)-breaking corrections. It may be pointed out that constants appearing in DAs, such as meson decay constants and others to be specified below, enter both in the effective Lagrangian relevant.
for low-energy physics and the light-cone wave functions utilized in high energy reactions.

In Sec. II, we derive the sum rule giving necessary details for the projected correlation function. In Sec. III, we analyze the results numerically and discuss them. Finally, in Sec. IV we summarize our work and give conclusion.

II. FORMALISM AND CONSTRUCTION OF SUM RULE

We consider the correlator of the standard nucleon currents between vacuum and one η-state:

$$\Pi(q, p) = i \int d^4 x e^{i q x} < 0 | T \{ J_N(x), \bar{J}_N(0) \} | \eta(p) >,$$

where $J_N$ is the standard proton current

$$J_N = \epsilon^{abc} [ u^a T^\mu C\gamma^\mu u^b ] \gamma^5 \gamma^\mu d^c,$$

where $a, b, c$ are color indices. The most general form of $\Pi(q, p)$ is [18]

$$\Pi(q, p) = i \gamma_5 \hat{p} \Pi^{AV} + i \gamma_5 \hat{q} \Pi^{PS} + \gamma_5 \sigma^{\mu\nu} q_\mu p_\nu \Pi^T + i \gamma_5 \hat{q} \Pi^{AV}. \tag{3}$$

The ηNN coupling constant $g_{\eta NN}$ is defined through the coefficient of the pole as [16]

$$\bar{u}(q r) (\hat{q} - m) \Pi(q, p) (\hat{q} - \hat{p} - m) u(ks) | q^2 = m^2, (q - p)^2 = m^2 = i \lambda^2 g_{\eta NN} \bar{u}(q r) \gamma_5 u(ks), \tag{4}$$

where $k = q - p$ and $u(q r)$ is a Dirac spinor with momentum $q$ and spin $r$ and is normalized as

$$\bar{u}(q r) u(q r) = 2m. \tag{5}$$

Following [18], we define the projected correlation function

$$\Pi^+_+(q^2_0) = \frac{1}{2} [ \Pi(q_0) + \Pi(-q_0) ], \tag{7a}$$

$$\Pi^0_+(q^2_0) = \frac{1}{2q_0} [ \Pi(q_0) - \Pi(-q_0) ]. \tag{7b}$$

The dispersion relations satisfied by these functions are

$$\hat{B} \Pi^E_+(q^2_0) = -\frac{1}{\pi} \int dq'_0 \frac{q'_0}{q'_0 - q_0^2} Im \Pi^+_+(q'_0), \tag{8a}$$

$$\hat{B} \Pi^0_+(q^2_0) = -\frac{1}{\pi} \int dq'_0 \frac{1}{q'_0 - q_0^2} Im \Pi^+_+(q'_0). \tag{8b}$$

We define Borel transform by $\hat{B}$

$$\hat{B} f(q^2_0) = \lim_{n \to \infty, -q^2_0 \to 0, \frac{q^2_0}{M^2} = m} \frac{(-q^2_0)^{n+1}}{n!} \left( \frac{d}{dq^2_0} \right)^n f(q^2_0), \tag{9}$$

so that the dispersion relations become

$$\hat{B} [ \Pi^E_+(q^2_0) ] = \frac{1}{\pi} \int dq'_0 q'_0 \exp \left( -\frac{q^2_0}{M^2} \right) Im \Pi^+_+(q'_0), \tag{10a}$$
The projected correlation function is parameterized as

\[
\hat{B}[\Pi^O_+(q^2)] = \frac{1}{\pi} \int dq_0 \text{exp}\left(-\frac{q_0^2}{M^2}\right) \text{Im}\Pi_+(q_0).
\]  

(10b)

The r.h.s. of Eqs. (10a) and (10b) are expressed in terms of the observed spectral function. The absorptive part of the projected correlation function is parameterized as

\[
\text{Im}\Pi_+(q,p) = \tilde{u}(q) i\gamma_5 u(q') \pi \lambda^2 g(q_0, \mathbf{p}^2) \left[ \frac{\delta(q_0 - m_N)}{q_0 - E_k - \omega_p} + \frac{\delta(q_0 - E_k - \omega_p)}{q_0 - m_N} \right] + [\theta(q_0 - s_\eta) + \theta(-q_0 - s_\eta)] \text{Im}\Pi^{OP}_+(q,p),
\]  

(11)

where \(s_\eta\) is the effective continuum threshold of \(\eta N\) or \(\eta \bar{N}\) channel and \(m_N\) is the mass of the proton and \(\lambda\) is the coupling of the proton current with one-proton state:

\[
\langle 0|J_N(0)|q\rangle = \lambda u(q).
\]  

(12)

We use the following parameterization of the vacuum-to-eta matrix elements of the light-cone operators arising in our calculation [14, 19, 24, 25] \((q = u, d)\):

\[
\langle 0|\bar{q}(0)i\gamma_5 q(x)|\eta(p)\rangle = \frac{f_\eta \mu_\eta}{\sqrt{6}} \left[ 1 - \frac{i}{2} p \cdot x - \frac{1}{6} (p \cdot x)^2 \right],
\]  

(13a)

\[
\langle 0|\bar{q}(0)\gamma_\mu q(x)|\eta(p)\rangle = \frac{f_\eta \mu_\eta}{\sqrt{6}} \left[ i p_\mu + \frac{1}{2} (p \cdot x) p_\mu - \frac{i}{18} \delta^2 p \cdot x x_\mu + \frac{5i}{36} \delta^2 x^2 p_\mu + \frac{5}{72} \delta^2 p \cdot x^2 p_\mu - \frac{1}{36} \delta^2 (p \cdot x)^2 x_\mu \right],
\]  

(13b)

\[
\langle 0|\bar{q}(0)\gamma_5 \sigma^{\mu \nu} q(x)|\eta(p)\rangle = \frac{i}{6\sqrt{6}} \left( p^\mu x^\nu - p^\nu x^\mu \right) f_\eta \mu_\eta (1 - \rho_+^\eta)(1 - \frac{i}{2} p \cdot x),
\]  

(13c)

\[
\langle 0|g_\mu G_{\mu\nu} (\frac{1}{2} x) q_a(x) q_b(0)|\eta(p)\rangle = \frac{if_\eta \mu_\eta}{16\sqrt{6}} \epsilon^{\alpha\beta} a \gamma_5 \lambda^\beta \left[ \sigma_{\lambda\mu} p_\nu - \sigma_{\lambda\nu} p_\mu \right]
\]  

\[
+ \frac{i f_\eta \delta^2}{252 \sqrt{6}} \epsilon^{\alpha\beta} a \gamma_5 (\gamma_\mu p_\nu - \gamma_\nu p_\mu) p x + \gamma_5 \hat{p} (p_\mu x_\nu - p_\nu x_\mu) \right] \frac{f_\eta \delta^2}{16\sqrt{6}} \epsilon^{\alpha\beta} a \gamma_5 \lambda^\beta \left[ \frac{1}{3} - \frac{i}{6} p x \right].
\]  

(13d)

The sign of \(\delta^2\) is as per suggestion of Doi et al. [19]. To leading order in SU(3)-flavor symmetry breaking, we use the following Gell-Mann-Okubo-like relations for the parameters of octet pseudo-scalar mesons \(\pi, K\) and \(\eta\) [24, 25]:

\[
3 f_\pi + f_\eta = 4 f_K,
\]  

(14a)

\[
3 f_\eta \mu_\eta = f_\pi \mu_\pi = 4 f_K \mu_K,
\]  

(14b)

\[
3 f_\eta \mu_\eta (1 - \rho_+^\eta) + f_\pi \mu_\pi (1 - \rho_-^\pi) = 4 f_K \mu_K (1 - \rho_+^K).
\]  

(14c)

The constants for pions and kaons are taken from Ref. [23]. As a result, we get the following values for the constants for \(\eta\)-meson:

\[
f_\eta = 0.1695 GeV, \quad \mu_\eta = 1.6380 GeV, \quad f_{3\eta} = 0.0045 GeV^2, \quad \rho_+^\eta = 0.1028.
\]  

(15)

The decay constant \(f_\eta\) obtained is close to the value obtained in a more elaborate scheme of two couplings and two mixing angles [26]. The constant \(\mu_\eta\) is also close to the value directly obtained for \(\eta\) as \(\mu_\eta = 3m_\eta^2/(2m + 4m_\eta)\) [25].
The expression for the correlation function obtained through operator product expansion is:

$$
\Pi(q, p) = i\gamma_5 \hat{\rho} \left[ \frac{f_n}{3\sqrt{6}\pi^2} (q^2 - \frac{1}{2} \hat{q}^2) \ln(-q^2) + \frac{2 f_n \mu_n}{9 \sqrt{6}} < \hat{q} \hat{q} > \frac{1}{q^2} - \frac{1}{36 \sqrt{6}} < \hat{q} \mu \sigma \cdot G q > \frac{1}{q^3} \right.
$$

$$
- \left. - \frac{\alpha_s}{\pi} G^2 > \frac{f_n}{9 \sqrt{6}} \left( \frac{1}{q^2} - \frac{2\hat{q}^2}{3\sqrt{6} q^2} \right) - i\gamma_5 f_n \hat{\mu} \frac{\mu_n}{\sqrt{6}} \frac{q^2}{3\pi^2} \ln(-q^2) + \frac{1}{2q^2} < \alpha_s G^2 > \right] - i\gamma_5 \frac{3 f_n}{4\sqrt{6} \pi^2} p^2 \ln(-q^2)
$$

$$
+ \gamma_5 \sigma^{\mu \nu} \mu \nu \hat{p} \hat{q} \left[ \frac{1}{36 \pi^2} \left( - \frac{5\hat{q}^2}{9\pi^2 q^2} < \hat{q} \mu \sigma \cdot G q > \frac{1}{q^6} - \frac{1}{12} < \alpha_s G^2 > \left( \frac{1}{q^2} - \frac{32\hat{q}^2}{27 q^4} \right) \right] \right)
$$

$$
+ \hat{q} \left( \frac{\mu_n}{18} \left( \hat{q} \mu \sigma \cdot G q > \frac{p^2}{q^6} + \frac{2\mu_n}{9} < \hat{q} \hat{q} > \frac{p^2}{q^4} \right) \right)
$$

$$
+i\gamma_5 f_n \hat{\mu} \frac{\mu_n}{\sqrt{6}} \frac{q^2}{4\pi^2} \ln(-q^2) - \frac{\pi^2}{27 q^4} < \hat{q} \hat{q} > + p \left( \frac{\mu_n}{12 \pi^2 q^2} \frac{1}{3} - \frac{4}{9} \frac{\hat{q} \hat{q} > \frac{1}{q^4} - \frac{40}{27} \frac{\hat{q} \hat{q} > \frac{\delta^2}{q^6} \right]
$$

$$
+ \left. \frac{\mu_n}{18} \left( \frac{\alpha_s}{\pi} G^2 > \frac{1}{q^6} + \frac{4}{9} < \hat{q} \mu \sigma \cdot G q > \frac{1}{q^3} - \frac{\alpha_s}{\pi} G^2 > \frac{1}{q^6} + \frac{4}{9} < \hat{q} \mu \sigma \cdot G q > \frac{1}{q^3} \right) \right].
$$

(16)

In expression above, $q$ in $< \hat{q} \hat{q} >$ and $< \hat{q} \mu \sigma \cdot G q >$ stands for $u$ or $d$. The above expression has some differences from the corresponding expressions obtained previously by other authors \[14\][15]. We have given complete expression for the mixed condensate term, and sign of $\delta^2$, as pointed out earlier, is consistent with Ref. \[19\]. From Eq. (16), we obtain $\Pi^E_\pi(q^2_0)$ and $\Pi^G_\pi(q^2_0)$, as given in Eqs. \[16\], \[17\] and \[18\]. Using energy-momentum conservation, it can be shown that ($k = q - p$)

$$
P^2 = -m^2_n + (m^4_n)/(4m^2_N), \quad \omega_p = (m^2_n)/(2m_N), \quad E_k = m_N - \frac{m^2_n}{2m_N}.
$$

(17)

Upon Borel transform, $\Pi^E_\pi$ gives the following sum rule:

$$
2m^2_N \lambda e^{-\frac{m^2_n}{2M^2}} (g_{NN} - M^2) =
$$

$$
- \frac{f_n \mu_n}{\sqrt{6}} \frac{1}{\pi^2} M^2 E_1 \left( \frac{\pi^2}{M^2} \left( \frac{E_k + m_N - \omega_p}{3 \mu_n} - \frac{1}{4} \right) + \frac{M^4}{\pi^2} E_0 \left( \frac{\pi^2}{M^2} \left( \frac{5\delta^2}{2} \left( E_k + m_N - \omega_p \right) - \frac{f_n \mu_n}{4 \sqrt{6}} \frac{m_n^2}{\mu_n^2} \right) \right) \right.
$$

$$
+ \left( E_k + m_N - \omega_p \right) \left( \frac{2}{9} < \hat{q} \hat{q} > \frac{1}{9} \frac{\mu_n}{\pi} G^2 > \frac{1}{2} \frac{\delta^2}{M^2} \frac{M^2}{\mu_n} \right) - \frac{f_n \mu_n}{\sqrt{6}} \left[ \frac{\alpha_s}{\pi} G^2 > \right]
$$

$$
- \left. \frac{2}{81 M^2 \mu_n} \frac{\delta^2}{2} < \frac{\alpha_s}{\pi} G^2 > \right] + \frac{f_n \mu_n}{\sqrt{6}} \left[ \frac{\alpha_s}{\pi} m_n^2 \left( \frac{M^4}{\pi^2} \frac{\mu_n^2}{2} \frac{E_0}{M^2} \left( \frac{\pi^2}{M^2} \left( \frac{5\delta^2}{2} \left( E_k + m_N - \omega_p \right) - \frac{f_n \mu_n}{4 \sqrt{6}} \frac{m_n^2}{\mu_n^2} \right) \right) \right) \right.
$$

$$
+ \left( E_k + m_N - \omega_p \right) \left( \frac{2}{9} < \hat{q} \hat{q} > \frac{1}{9} \frac{\mu_n}{\pi} G^2 > \frac{1}{2} \frac{\delta^2}{M^2} \frac{M^2}{\mu_n} \right) - \frac{f_n \mu_n}{\sqrt{6}} \left[ \frac{\alpha_s}{\pi} G^2 > \right]
$$

$$
+ \left. \frac{1}{18 M^2} \frac{\delta^2}{\mu_n} \frac{\alpha_s}{\pi} G^2 > \right] + \frac{m_n^2}{12 \pi^2} E_0 \left( \frac{\pi^2}{M^2} \left( \frac{5\delta^2}{2} \left( E_k + m_N - \omega_p \right) - \frac{f_n \mu_n}{4 \sqrt{6}} \frac{m_n^2}{\mu_n^2} \right) \right)
$$

$$
+ \left( E_k + m_N - \omega_p \right) \left( \frac{2}{9} < \hat{q} \hat{q} > \frac{1}{9} \frac{\mu_n}{\pi} G^2 > \frac{1}{2} \frac{\delta^2}{M^2} \frac{M^2}{\mu_n} \right) - \frac{f_n \mu_n}{\sqrt{6}} \left[ \frac{\alpha_s}{\pi} G^2 > \right]
$$

$$
- \left. \frac{1}{108} < \frac{\alpha_s}{\pi} G^2 > \right].
$$

(18)

On the l.h.s. $l$ is a constant independent of $M^2$. On the r.h.s., $E_0(x) = 1 - e^{-x}$ and $E_1(x) = 1 - (1 + x)e^{-x}$ are used to model contributions of excited states. The expression in the square bracket on the r.h.s. in Eq. (18)
gives purely SU(3) symmetry breaking contribution. After Borel transform $\Pi^B_\chi$ gives the following sum rule

$$2m_N\lambda^2 e^{-\frac{m_N^2}{2\pi^2}}(g_{\eta NN} - hM^2)$$

$$= \frac{f_0\mu_H}{\sqrt{6}}\left[\frac{M^4}{12\pi^2}(E_k + m_N)E_0(\frac{s_0}{M^2}) - \frac{4}{3}\frac{q\tilde{q}}{\mu_\eta}(E_k + m_N)M^2 + \left(\frac{26}{27}\frac{q\tilde{q}}{\mu_\eta}\delta^2 - \frac{1}{2}\frac{m_N^2}{\mu_\eta}\right) + \frac{2}{3}\frac{\omega_p}{\mu_\eta}(E_k + m_N - \omega_p) + 3\omega_p + \frac{\omega_p}{\mu_\eta}(E_k + m_N)\right] \frac{M^4}{12\pi^2}E_0(\frac{s_0}{M^2})$$

$$- \frac{1}{216}\left\{ \frac{\alpha_\pi}{\pi} G^2 > \right\}\sum_{E_k} \left\{ \frac{-2m_N^2}{\mu_\eta} - \frac{\alpha_\pi}{\pi} M^2 \right\} \frac{M^4}{12\pi^2}E_0(\frac{s_0}{M^2})$$

$$+ \left\{ \frac{7}{18\pi^2}\delta^2 \frac{m_N^2}{\mu_\eta} - \frac{5}{9\pi^2}(E_k + m_N - \omega_p)\delta^2 \frac{\omega_p}{\mu_\eta} \right\} \sum_{E_k} \frac{M^4}{12\pi^2}E_0(\frac{s_0}{M^2})$$

$$+ \left\{ \frac{7}{18\pi^2}\delta^2 \frac{m_N^2}{\mu_\eta} - \frac{5}{9\pi^2}(E_k + m_N - \omega_p)\delta^2 \frac{\omega_p}{\mu_\eta} \right\} \sum_{E_k} \frac{M^4}{12\pi^2}E_0(\frac{s_0}{M^2})$$

In Eq. (19), also, the expression in the second square bracket on the r.h.s. gives purely SU(3) symmetry breaking contribution. On the l.h.s., $h$ is a constant independent of $M^2$. We also write the nucleon mass sum rules [26, 27] for our use. The chiral-even mass sum rule is

$$m_N\lambda^2 e^{-\frac{m_N^2}{2\pi^2}} = \frac{1}{4\pi^2}\left[ -\frac{M^4}{8}E_1(\frac{s_0}{M^2}) < q\tilde{q} > + \frac{5\pi^2}{18} < q\tilde{q} > < \frac{\alpha_{s\pi}}{\pi} G^2 > \right].$$

In above equation, $s_0$ is the continuum threshold for the mass sum rule. The coefficient of dimension 7 operator has been taken from Ref. [27]. The chiral-odd mass sum rule is

$$\lambda^2 e^{-\frac{m_N^2}{2\pi^2}} = \frac{1}{4\pi^2}\left[ -\frac{M^6}{8}E_2(\frac{s_0}{M^2}) + \frac{M^2}{8}E_0(\frac{s_0}{M^2})^2 < \frac{\alpha_\pi}{\pi} G^2 > + \frac{8\pi^4}{3} < q\tilde{q} > - \frac{2\pi^4}{3} < q\tilde{q}\cdot G\cdot q > < q\tilde{q} > \right].$$

In Eq. (21), $E_2(x) = 1 - (1 + x + x^2/2)e^{-x}$. By dividing the two coupling sum rules with the two mass sum rules we get a total of four equations, each of which can be used independently for the determination of the coupling $g_{\eta NN}$. We will study full sum rules as well as SU(3) symmetric part of sum rules in the next section.

III. ANALYSIS OF SUM RULES AND RESULTS

We have used the following numerical constants in our analysis

$$< q\tilde{q} > = -(0.23 \text{ GeV})^3, \quad < \frac{\alpha_{s\pi}}{\pi} G^2 > = (0.33 \text{ GeV})^4, \quad < q\tilde{q}\cdot G\cdot q > = m_N^2 < q\tilde{q} >$$

with $m_N^2 = 0.8 \text{ GeV}^2$, $f_{\eta NN} = 0.0045$, $m_N = 0.939 \text{ GeV}$, $m_\eta = 0.547 \text{ GeV}$, $\delta^2 = 0.2 \text{ GeV}^2$. (22)

For continuum threshold $s_\eta$ in the coupling sum rules we have uniformly used $s_\eta = 2.57 \text{ GeV}^2$ in both coupling constant sum rules while $s_0 = 2.00 \text{ GeV}^2$ in both mass sum rules. Later on we shall study the effect of changes in the continuum thresholds on results obtained for $g_{\eta NN}$. We use a notation in which $f_{\text{total}}^{\text{EO}}$ is the function obtained by taking the ratio of full OPE side of even sum rule in coupling to odd sum rule in mass while $f_{\text{sym}}^{\text{EO}}$ has a similar meaning except that in coupling sum rule only SU(3) symmetric terms are taken, and similarly for others. We have shown the plots of all the functions obtained by taking the ratio of a coupling sum rule to a mass sum rule. Furthermore, for coupling sum rules both the full expressions as given in Eqs. (18) and (19) as well as SU(3) symmetric parts have been considered. We have found straight line fits for all the curves over a range of Borel mass $1.0 \text{ GeV}^2 \leq M^2 \leq 1.8 \text{ GeV}^2$. To get a quantitative idea of the goodness of fit, we define a $\chi^2$ by

$$\chi^2 = \sum_{i=0}^{n}\frac{[f(x_i) - f_{fit}(x_i)]^2}{[f(x_i) + f_{fit}(x_i)]^2}/(1 + n).$$

(23)

All the curves obtained from ratios and their straight line fits are shown in Figs. [11] to [12]. The Borel windows for both coupling sum rules are common, since, in principle, couplings for all the Lorentz structures are related under SU(3) rotations [13]. We have listed the results of fits along with $\chi^2$’s in Table[1]. It is observed that ratios containing even mass sum rule, namely, $f_{\text{total}}^{\text{EO}}$ and $f_{\text{total}}^{\text{KE}}$, exhibit poor fits. Hence, we have also shown their fits over a shorter Borel
window (1.2 GeV^2 \leq M^2 \leq 1.8 GeV^2) in Figs. 5 and 10. This gives better \( \chi^2 \), but still not so good as that for \( f_{\text{total}}^{\text{OO}} \) and \( f_{\text{total}}^{\text{EO}} \). Hence, hereafter we analyze these two latter sum rules.

It is to be noted that \( \Pi_E^Q \) contains combination of SU(3) symmetric axial vector and pseudo-scalar terms whereas \( \Pi_E^Q \) contains tensor terms in SU(3) symmetry limit. The use of tensor structure has been advocated on account of its nice features. It has been concluded that for the later the physical parameter, in SU(3) symmetry limit, is independent of the form of the nucleon current for currents with no derivative [13], the sum rule does not depend on the form of the coupling in the effective Lagrangian [10]. Borel curves are rather insensitive to the continuum threshold [15]. In the present case, we see from Table I that the odd-odd sum rule has the lowest \( \chi^2 \). We have found that the contribution of excited states to \( \Pi_E^Q \) is 6.5\% at \( M^2 = 1.8\% \text{GeV}^2 \) while the operators of highest dimension contribute 6.7\% at \( M^2 = 1.0 \text{GeV}^2 \). Hence, the Borel window is quite appropriate for this case. The change of continuum threshold \( s_0 \) from 2.57 GeV^2 to 2.07 GeV^2 (see Fig. 11) increases the value of \( g_{\eta NN} \) by 2.3\% while the change of the continuum threshold \( s_0 \) from 2.0 GeV^2 to 2.3 GeV^2 increases it by 3.7\%. Change of \( <qq> \) in both the coupling and mass sum rules of \( f_{\text{total}}^{\text{OO}} \) as \((0.23 \pm 0.025 \text{GeV}^3) \) changes \( g_{\eta NN} \) by 9.6\%. Uncertainties of gluon condensate, \( m_0^2 \) and \( \delta^2 \) may change the value of \( g_{\eta NN} \) by up to 5\%. The uncertainties in \( f_{\eta}, \mu_{\eta}, f_{\rho \eta} \) and \( \rho_{\eta}^2 \) (which are small); higher powers of \( m_0 \) not considered and extrapolation of the coupling constant to the physical point may result in another couple of percent of change in \( g_{\eta NN} \). In all, we expect an uncertainty of up to 25\% in the value of \( g_{\eta NN} \) obtained from analysis of \( f_{\text{total}} \) due to uncertainties in various constants used in the sum rule and extrapolation.

The even-odd sum rule has \( \chi^2 \) marginally higher than the odd-odd case. Also, the value of \( g_{\eta NN} \) obtained from even-odd sum rule coincides with the one obtained from odd-even sum rule, albeit with a lower Borel window (1.2 –1.8 GeV^2). Moreover, this sum rule has a minimal dependence on the continuum threshold \( s_0 \), as is evident from Fig. 12. However, contribution of excited states as per the model used here is 23\% at \( M^2 = 1.8 \text{ GeV}^2 \). Highest dimensional operators contribute 24\% to \( \Pi_E^Q \) in the entire range of Borel window. This means that the omitted higher order terms in OPE may contribute significantly to \( \Pi_E^Q \). Changing \( <qq> \) to \(-0.255 \text{ GeV}^3 \) in both the coupling and the mass sum rule of \( f_{\text{total}}^{\text{EO}} \) changes \( g_{\eta NN} \) by 23\%. This means that the result obtained from even-odd sum rule has a larger uncertainty. Hence, we consider the result obtained from odd-odd sum rule as the most reliable and as our main result we have:

\[
g_{\eta NN} = 4.20 \pm 1.05
\]  

A few comments on the size of SU(3)-breaking effects in the four sum rules. For even-odd and even-even sum rules, the coupling \( g_{\eta NN} \) increases on introducing symmetry breaking effects and the increments (with respect to the SU(3) symmetric results) are almost uniform (by 56 to 60\%) in these cases. On the other hand, the coupling decreases for odd-odd and odd-even cases on introducing symmetry breaking effects. Furthermore, the change is minimum (about 21\%) for odd-odd sum rule and moderate (about 38\%) for odd-even case.

In Table I we compare our result with the values of \( g_{\eta NN} \) obtained by other authors using various methods in the recent literature. In Refs. 8, 9, 29, Goldberger-Treiman relation has been used to calculate \( g_{\eta NN} \) and \( g_{\eta' NN} \) simultaneously. Authors of Refs. 8, 9 use the measured values of singlet and octet axial charges of nucleon. Nasrallah [29], on the other hand, has used dispersion relation for the nucleon matrix element of the divergence of axial current saturating it with \( \eta \) and \( \eta' \) states and accounting the continuum contribution through a model. Feldmann [8] has neglected the contribution of higher excited pseudo-scalar states. In the absence of knowledge of gluon topological susceptibility and gluonic coupling to nucleon \( (g_{GNN}) \), Shore [9] ends up getting a plot of \( g_{\eta NN} \) and \( g_{GNN} \) as a function of \( g_{\eta NN} \). Kim \textit{et. al.} [14] have performed calculation of \( g_{\eta NN} \) to leading order in \( p_\eta \) by considering the two separate Dirac structures \( (\gamma_\mu \gamma_5 p^\mu) \) and \( (\gamma_5 \sigma_{\mu \nu} p^\mu p^\nu) \) separately. They have introduced SU(3)-breaking only through the decay constant \( f_\eta \). In Ref. 8, while fitting the data in a potential model that accounts for various meson exchanges, the coupling \( g_{\eta NN} \) was not searched independently; rather it was constrained via SU(3) symmetry. In Ref. 31 the authors have worked with effective Lagrangian approach to calculate \( \eta \) photoproduction in the N* (1535) region. They find that the observed differential cross section is not very sensitive to either the nature of the \( \eta \)-nucleon coupling or to the precise value of the coupling constant. They extract a broad range of values for the \( \eta NN \) pseudo-scalar...
coupling constant: $0.2 \leq g_{\eta NN} \leq 6.2$ from analysis of all available data.

| Ref. no. | 29          | 8          | 14         | 3         | 30         | 9          | 31         | This work |
|----------|-------------|------------|------------|-----------|------------|------------|------------|-----------|
| $g_{\eta NN}$ | 5.20±0.25   | 3.4±0.5    | 5.76       | 7.95      | 6.8        | 3.96 ± 0.16 | 3.2±3.0   | 4.20±1.05 |
| [SU(3) sym ] | (g'$_{\eta NN}$ = 2.0) | 7.34      | 3.59±0.15  |           |            |            |            |           |
| [Brok. SU(3)] | (g'$_{\eta NN}$ = 1.0) |           |            |           |            |            |            |           |

### IV. SUMMARY AND CONCLUSION

Eta-nucleon coupling constant has a definite role in analyzing experiments involving NN scattering in general and $\eta$ production in particular. However, due to the dominant role of $N^*(1535)$ in $\eta$ production and lack of precise knowledge of $g_{\eta NN}$, the role of $\eta NN$ vertex is usually ignored in analysis [1, 32]. Hence a credible determination of $g_{\eta NN}$ is desirable. Separating the even and odd parts in the projected correlation function, we obtained two independent sum rules for the coupling $g_{\eta NN}$. These sum rules incorporate the effect of SU(3)-flavor symmetry breaking in leading order. The symmetry breaking parameters themselves come from matrix elements of quark and gluon operators between vacuum and one-eta state. Taking the ratios of the two coupling sum rules with the two mass sum rules for the nucleon, we obtained a total of four sum rules. We have analyzed the full expression for the four cases as well as their SU(3) symmetric forms. The results obtained from three of these full sum rules are quite close to each other and this enhances the credibility of our result. The result from odd-odd sum rule is quite robust and is the main result of this work.

We have ignored the mixing of octet state with singlet and glueball states and the consequent role of instantons in our analysis. Nevertheless, we have quantified the SU(3) breaking corrections to eta-nucleon coupling constant to leading order in SU(3) symmetry breaking, and found the effects to be significant. We conclude that inclusion of SU(3) symmetry breaking effects in a realistic way in this work is a step forward in realizing a correct estimate of this poorly determined quantity.

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[1] R. Machleidt, Phys. Rev. C 63, 024001 (2001).
[2] S. Ceci, A. Svarc and B. Zauner, arXiv:0904.2430.
[3] V. G. J. Stoks and T. A. Rijken, Phys. Rev. C 59, 3009 (1999); T.A. Rijken, Phys. Rev. C 73, 044007 (2006).
[4] I. Halperin, Phys. Rev. D 50,4602 (1994).
[5] J. A. Nolen and J. P. Schiffer, Ann. Rev. Nucl. Sci. 19, 471 (1969); G. A. Miller, B. M. K. Nefkens and I. Slaus, Phys. Rep. 194, 1 (1990).
[6] J. Ellis, hep-ph/0411369.
[7] J. D. Sullivan, Phys. Rev. D 5, 1732.
[8] T. Feldmann, Int. J. Mod. Phys. A15, 159 (2000).
[9] G. M. Shore, Nucl. Phys. B744, 34 (2006).
[10] C. Hanhart, arXiv: nucl-th/0511045 and references therein.
[11] L. J. Reinders, H. R. Rubinstein and S. Yazaki, Phys. Rep. 127, 1 (1985).
[12] L. J. Reinders, Acta Phys. Pol. B15, 329 (1984).
[13] M. C. Birse and B. Krippa, Phys. Lett. B373, 9 (1996).
[14] H. Kim, T. Doi, M. Oka and S. H. Lee, Nucl. Phys. A678, 295 (2000).
[15] T. Doi, H. Kim and M. Oka, Phys. Rev. C 62, 055202 (2000).
[16] H. Kim, S. H. Lee and M. Oka, Phys. Rev. D 60, 034007 (1999).
[17] Y. Kondo and O. Morimatsu, Phys. Rev. C 66, 028201 (2002).
[18] Y. Kondo and O. Morimatsu, Nucl. Phys. A717, 55 (2003).
[19] T. Doi, Y. Kondo and M. Oka, Phys. Rep. 398, 253 (2004).
[20] T. M. Aliev et al., Phys. Rev. D 74, 116001 (2006).
[21] V. M. Belyaev, V. M. Braun, A. Khodjamirian and R. Ruckel, Phys. Rev. D 51, 6177 (1995).
[22] P. Ball, JHEP 9901, 010 (1999).
[23] P. Ball, V. M. Braun and A. Lenz, arXiv: hep-ph/0603063 and references therein.
[24] J-W Chen, H-M Tsai and K-C Weng, Phys. Rev. D 73, 054010 (2006).
[25] C. Kim and A. K. Leibovich, Phys. Rev. D 78, 054026 (2008).
[26] Janardan P. Singh and J. Pasupathy, Phys. Rev. D 79, 116005 (2009).
[27] B. L. Ioffe, Acta Phys. Polon. B16, 543 (1985).
[28] D. B. Leinweber, Ann. Phys. 254, 328 (1997); ibid 198, 203 (1990).
[29] N. F. Nasrallah, Phys. Lett. B645, 335 (2007).
[30] O. Dumbrajs et al., Nucl. Phys. B216, 277 (1983) and references therein.
[31] M. Benmerrouche, N. C. Mukhopadhyay and J. F. Zhang, Phys. Rev. D 51, 3237 (1995).
[32] R. Shyam, Phys. Rev. C 75, 055201 (2007).

FIG. 1: Plot of leading order terms in the odd-odd sum rule as a function of $M^2$ (solid curve). A straight line fit of the form $5.20M^2 + 5.31$ (broken) over the range $1.0 \text{ GeV}^2 \leq M^2 \leq 1.8 \text{ GeV}^2$ gives $g_{\eta NN} = 5.31$.

FIG. 2: Plot of full expression obtained in the odd-odd sum rule as a function of $M^2$ (solid curve). A straight line fit of the form $1.96M^2 + 4.20$ (broken) over the range $1.0 \text{ GeV}^2 \leq M^2 \leq 1.8 \text{ GeV}^2$ gives $g_{\eta NN} = 4.20$. $\chi^2$ (defined in the text) for this fit is $2.0 \times 10^{-3}$.
FIG. 3: Plot of leading order terms in the odd-even sum rule as a function of $M^2$ (solid curve). A straight line fit of the form $-1.86M^2 + 9.98$ (broken) over the range $1.0 \text{ GeV}^2 < M^2 < 1.8 \text{ GeV}^2$ gives $g_{\eta NN} = 9.98$.

FIG. 4: Plot of full expression obtained in the odd-even sum rule as a function of $M^2$ (solid curve). A straight line fit of the form $-1.58M^2 + 6.29$ (broken curve) over the range $1.0 \text{ GeV}^2 \leq M^2 \leq 1.8 \text{ GeV}^2$ gives $g_{\eta NN} = 6.29$. $\chi^2$ (defined in the text) for this fit is $9.2 \times 10^{-3}$. A better fit is shown in Fig. 5.
FIG. 5: Plot of full expression obtained in the odd-even sum rule as a function of $M^2$ (solid curve). A straight line fit of the form $-1.27M^2 + 5.82$ (broken curve) over the range $1.2 \text{ GeV}^2 \leq M^2 \leq 1.8 \text{ GeV}^2$ gives $g_{\eta NN} = 5.82$. $\chi^2$ for this fit is $4.0 \times 10^{-3}$.

FIG. 6: Plot of leading order terms in the even-odd sum rule as a function of $M^2$ (solid curve). A straight line fit of the form $1.21M^2 + 3.73$ (broken curve) over the range $1.0 \text{ GeV}^2 \leq M^2 \leq 1.8 \text{ GeV}^2$ gives $g_{\eta NN} = 3.73$. 
FIG. 7: Plot of full expression obtained in the even-odd sum rule as a function of $M^2$ (solid curve). A straight line fit of the form $2.13M^2 + 5.82$ (broken curve) over the range $1.0 \text{ GeV}^2 \leq M^2 \leq 1.8 \text{ GeV}^2$ gives $g_{\eta NN} = 5.82$. $\chi^2$ for this fit is $2.61 \times 10^{-3}$.

FIG. 8: Plot of leading order terms in the even-even sum rule as a function of $M^2$ (solid curve). A straight line fit of the form $-1.42M^2 + 5.19$ (broken curve) over the range $1.0 \text{ GeV}^2 \leq M^2 \leq 1.8 \text{ GeV}^2$ gives $g_{\eta NN} = 5.19$. 
FIG. 9: Plot of full expression obtained in the even-even sum rule as a function of $M^2$ (solid curve). A straight line fit of the form $-2.22M^2 + 8.30$ (broken curve) over the range $1.0 \text{ GeV}^2 \leq M^2 \leq 1.8 \text{ GeV}^2$ gives $g_{\eta NN} = 8.30$. $\chi^2$ for this fit is $10.6 \times 10^{-3}$. A better fit is shown in Fig. 10.

FIG. 10: Plot of full expression obtained in the even-even sum rule as a function of $M^2$ (solid curve). A straight line fit of the form $-1.78M^2 + 7.60$ (broken curve) over the range $1.2 \text{ GeV}^2 \leq M^2 \leq 1.8 \text{ GeV}^2$ gives $g_{\eta NN} = 7.60$. $\chi^2$ for this fit is $4.5 \times 10^{-3}$. 
FIG. 11: Plot of full expression for odd-odd sum rule for $s_\eta = 2.57$ GeV$^2$ (solid curve) and $s_\eta = 2.07$ GeV$^2$ (broken curve) as a function of $M^2$. This changes $g_{\eta NN}$ from 4.20 to 4.29.

FIG. 12: Plot of full expression for even-odd sum rule for $s_\eta = 2.57$ GeV$^2$ (solid curve) and $s_\eta = 2.07$ GeV$^2$ (broken curve) as a function of $M^2$. This does not change $g_{\eta NN}$ to the accuracy we are working.