Supersymmetric Hybrid Inflation with Non-Minimal Kähler potential

M. Bastero-Gil\textsuperscript{1(a)}, S. F. King\textsuperscript{2(b1,b2)}, Q. Shafi\textsuperscript{3(c)},

\textsuperscript{(a)} Departamento de Fisica Teorica y del Cosmos and Centro Andaluz de Fisica de Particulas Elementales (CAFPE), Universidad de Granada, E-19071 Granada, Spain

\textsuperscript{(b1)} TH Division, Physics Department, CERN, 1211, Geneva 23 Switzerland

\textsuperscript{(b2)} School of Physics and Astronomy, University of Southampton, Southampton, SO17 1BJ, U.K.

\textsuperscript{(c)} Bartol Research Institute, University of Delaware, Newark, DE 19716, USA

Abstract

Minimal supersymmetric hybrid inflation based on a minimal Kähler potential predicts a spectral index $n_s \gtrsim 0.98$. On the other hand, WMAP three year data prefers a central value $n_s \approx 0.95$. We propose a class of supersymmetric hybrid inflation models based on the same minimal superpotential but with a non-minimal Kähler potential. Including radiative corrections using the one-loop effective potential, we show that the prediction for the spectral index is sensitive to the small non-minimal corrections, and can lead to a significantly red-tilted spectrum, in agreement with WMAP.

\textsuperscript{1}E-mail: mbg@ugr.es

\textsuperscript{2}E-mail: sfk@hep.phys.soton.ac.uk

\textsuperscript{3}E-mail: shafi@bartol.udel.edu
1 Introduction

Hybrid inflation models [1, 2] are examples of small field inflation models which predict a very small tensor fraction \( r \ll 10^{-2} \). Such models also typically predict an approximately scale invariant spectral index. For such models the WMAP three year central value for the spectral index is about \( n_s \approx 0.95 \) [3], whereas the joint analysis of Ly-\( \alpha \) forest power spectrum from the Sloan Digital Sky Survey, with cosmic microwave background, galaxy clustering and supernovae yields \( n_s = 0.965 \pm 0.012 \) [4]. Consequently hybrid inflation models which predict the spectral index to be too large are now less preferred [5].

Amongst the models that are now less preferred by the WMAP three year measurement of the spectral index are those based on minimal supersymmetric hybrid inflation. Minimal supersymmetric hybrid inflation may be defined by the superpotential \( W \),

\[
W = \kappa \hat{S}(\hat{\phi}\bar{\hat{\phi}} - M^2),
\]

(1)

where \( \hat{S} \) is a gauge singlet and \( \hat{\phi}, \bar{\hat{\phi}} \) are a conjugate pair of superfields transforming as non-trivial representations of some gauge group \( G \), together with a minimal Kähler potential,

\[
K_0 = |S|^2 + |\phi|^2 + |\bar{\phi}|^2,
\]

(2)

with \( S, \phi, \bar{\phi} \) being the bosonic components of the superfields. The gauge singlet \( S \) is a natural candidate for the inflaton in this model. In the true supersymmetric minimum, \( \phi \) and \( \bar{\phi} \) have equal non-zero vevs \( \langle \phi \rangle = \langle \bar{\phi} \rangle = M \) whereas \( \langle S \rangle = 0 \) (or \( \mathcal{O}(m_{3/2}) \) in broken supersymmetry). During inflation, the theory is in a false vacuum where \( \langle \phi \rangle = \langle \bar{\phi} \rangle = 0 \) and \( \langle S \rangle \neq 0 \), driving inflation. Inflation ends when the field value of the inflaton \( S \) falls below some critical value which corresponds to a tachyonic instability for \( \langle \phi \rangle \) and/or \( \langle \bar{\phi} \rangle \). In this minimal model, the vevs \( \langle \phi \rangle \) and \( \langle \bar{\phi} \rangle \) break \( G \) to some subgroup \( H \). If \( \phi, \bar{\phi} \) break e.g. Pati-Salam or SO(10), topological defects are generated after inflation. In order to avoid the monopole problem, one can extend superpotential to so-called shifted [6] or smooth inflation [7, 8], but here we shall restrict ourselves to the minimal \( W \) above.

The slow-roll parameters may be defined as

\[
\epsilon = \frac{m_P^2}{2} \left( \frac{V'}{V} \right)^2,
\]

(3)

\[
\eta = m_P^2 \left( \frac{V''}{V} \right),
\]

(4)

\[
\xi = m_P^4 \left( \frac{V' V'''}{V^2} \right),
\]

(5)

where \( m_P = 2.4 \times 10^{18} \) GeV is the reduced Planck mass. Assuming that the slow-roll approximation is justified (i.e. \( \epsilon \ll 1, \eta \ll 1 \)), the spectral index \( n_s \), the tensor-to-scalar
ratio $r = A_k / A_s$ and the running of the spectral index $dn_s / d \ln k$ are given by

$$n_s \simeq 1 - 6 \epsilon + 2 \eta,$$

$$r \simeq 16 \epsilon,$$

$$\frac{dn_s}{d \ln k} \simeq 16 \epsilon \eta - 24 \epsilon^2 - 2 \xi^2.$$ 

The theory defined above in Eqs. (1) and (2) defines the minimal supersymmetric hybrid inflation model. As we shall see in the next section, it leads to a prediction for the spectral index which is rather close to unity, $n_s \gtrsim 0.98$, which is larger than the central value preferred by WMAP three year data. On the other hand there is no symmetry that protects the minimal form of the Kähler potential. In this paper we study supersymmetric hybrid inflation with non-minimal Kähler potential, including radiative corrections using the one-loop effective potential, and show that the prediction of the spectral index is sensitive to such non-minimal effects, which can lead to a significantly red-tilted spectrum. This is done in Section 3. The summary is presented in the last section, where we also briefly comment on reheating after inflation.

2 Minimal Kähler Potential

In supersymmetric theories based on supergravity, there is a well known problem that $\eta \approx 1$ due to the supergravity corrections, thereby violating one of the slow roll conditions, and leading to the so-called $\eta$ problem [9]. It is an interesting fact that the supergravity potential based on the minimal supersymmetric hybrid inflation theory defined in Eqs. (1), (2) provides a solution to the $\eta$ problem since the mass squared of the inflaton when calculated from the supergravity potential cancels at the tree level.

In general the supergravity potential, including just the F-terms, is:

$$V_F = e^{K/m^2} \left[ K^{-1} D_{z_i} W D_{z_j} W^* - 3 m_p^{-2} |W|^2 \right],$$

with $z_i$ being the bosonic components of the superfields $\hat{z}_i \in \{ \hat{\phi}, \hat{S}, \ldots \}$ and where we have defined

$$D_{z_i} W := \frac{\partial W}{\partial z_i} + m_p^{-2} \frac{\partial K}{\partial z_i} W, \quad K_{ij} := \frac{\partial^2 K}{\partial z_i \partial z_j},$$

and $D_{z_j} W^* := (D_{z_j} W)^*$. For the superpotential in Eq. (1) and the minimal Kähler potential $K_0$ in Eq. (2), the supergravity potential leads to:

$$V_0^{\text{min}} \simeq 2 \kappa^2 |S|^2 |\phi|^2 + \kappa^2 (|\phi|^2 - M^2)^2 \left( 1 + 2 \frac{|\phi|^2}{m_p^2} + \frac{|S|^4}{2m_p^4} + \frac{|\phi|^4}{m_p^4} \right) + \cdots,$$
and we see that to leading order in the supergravity expansion the mass squared term for the inflaton field $S$ has canceled. This is fortunate since, if present, we would expect such a supergravity induced mass squared to have the same form as the $\phi$ mass squared$^1$, namely $\kappa^2M^4/m_p^2 = V_0/m_p^2$ which is of order the Hubble constant squared $H^2 = V_0/3m_p^2$. The fact that the inflaton acquires a mass of order the Hubble constant is a generic feature of supergravity, and gives rise to $\eta \approx 1$, violating the slow roll condition and leading to the so-called $\eta$ problem. However, as already noted, in minimal supersymmetric hybrid inflation above the mass squared term for the inflaton $S$ cancels and there is no $\eta$ problem.

Since the tree-level mass squared for the inflaton $S$ cancels, in minimal supersymmetric hybrid inflation the curvature of the potential is given by the 1-loop effective potential,

$$V_{1\text{loop}}^{\text{min}} = V_0^{\text{min}} + \Delta V_{1\text{loop}}.$$

with the radiative correction given by

$$\Delta V_{1\text{loop}} = \frac{1}{64\pi^2} \text{Str} \left[ \mathcal{M}^4(\phi) \left( \ln \frac{\mathcal{M}^2(\phi)}{Q^2} - \frac{3}{2} \right) \right],$$

where $\mathcal{M}^2(\phi)$ is the field-dependent mass-squared matrix of the contributing particles, i.e., $\phi$ and $\bar{\phi}$, and $Q$ the renormalization scale$^2$. The leading contributions of the 1-loop effective potential can be expressed analytically as:

$$\Delta V_{1\text{loop}} \simeq \left( \frac{\kappa M}{8\pi^2} \right)^4 N F[x],$$

$$F[x] = \frac{1}{4} \left( (x^4 + 1) \ln \frac{x^4 - 1}{x^4} + 2x^2 \ln \frac{x^2 + 1}{x^2 - 1} + 2 \ln \frac{\kappa^2 M^2 x^2}{Q^2} - 3 \right),$$

$N$ being the dimensionality of the representation of the fields $\phi$, $\bar{\phi}$, and where we have defined $x = |S|/M$.

During the inflationary epoch where $|S| > |S^c| = M$ the waterfall field $\phi$ is held at zero due to its having a large positive mass squared, then when $S$ reaches $S^c$ the waterfall field $\phi$ rolls out towards its global minimum, effectively ending inflation in the usual way

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$^1$The $\phi$ and $\bar{\phi}$ fields also receive field dependent mass squared during inflation given by the $S$ field. Other squark, slepton and Higgs fields are not included explicitly but are necessarily present in any realistic supersymmetric model. Such fields would be expected to get masses of order the Hubble constant during inflation, effectively lifting all flat directions involving these fields, which therefore play no role in inflation.

$^2$None of the derivatives of $\Delta V_{1\text{loop}}$ depend on the renormalization scale $Q$, and therefore it would have no effect on the inflationary predictions. We can always choose for example this scale such that it minimizes the value of $V_{1\text{loop}}$ along the inflationary trajectory.
in hybrid inflation. Writing the potential in terms of the real field \( S_R = \sqrt{2} |S| \), and setting \(|\phi| = 0\), effectively during inflation we are left with the potential\(^3\):

\[
V = V_1^{\text{min}}(\phi = 0) \simeq \kappa^2 M^4 \left( 1 + \frac{S_R^4}{8 m_P^2} + \cdots \right) + \Delta V_{\text{loop}}. \tag{16}
\]

As far as we have inflation for field values well below the Planck scale, and \( \kappa \gtrsim 10^{-3} \), we can neglect the quartic correction induced by the sugra correction\(^4\). The slow-roll parameters are then given:

\[
\epsilon \simeq \frac{\kappa^2}{(4\pi)^2} \left( \frac{\kappa m_p}{4\pi M} \right)^2 \mathcal{N}^2 F'[x]^2, \tag{17}
\]

\[
\eta \simeq -\delta \simeq \left( \frac{\kappa m_p}{4\pi M} \right)^2 \mathcal{N} F''[x], \tag{18}
\]

where we have denoted by \( \delta \) the contribution to \( \eta \) from the effective potential. The functions \( F'[x] \) and \( F''[x] \) are the first and second derivative of \( F[x] \) respectively, which for \( x > 1 \) behave like \( F'[x] \simeq 1/x \), \( F''[x] \simeq -1/x^2 \). Therefore, in that regime we have approximately:

\[
\eta \simeq - \left( \frac{\kappa m_p}{4\pi M} \right)^2 \mathcal{N} \frac{x}{x^2}, \tag{19}
\]

\[
\epsilon \simeq \frac{\kappa^2}{(4\pi)^2} \delta \ll |\eta|, \tag{20}
\]

The spectral index is then:

\[
n_s \simeq 1 - 2\delta. \tag{21}
\]

The amplitude of the primordial spectrum is given by:

\[
P_R^{1/2} \simeq \frac{V}{V'} \left( \frac{H}{2\pi m_p^2} \right) \simeq \frac{1}{\sqrt{2\epsilon}} \left( \frac{H}{2\pi m_p} \right) \simeq \sqrt{\frac{2}{3}} \left( \frac{4\pi M}{\kappa m_p} \right)^3 \frac{x_e}{N}, \tag{22}
\]

evaluated for the field value \( x_e = S_R e/\sqrt{2} M \) at \( N_e \) e-folds before the end of inflation,

\[
N_e = \int_{S_{Re}}^{S_R} H dt \simeq \int_{S_{Re}}^{S_R} \frac{3H^2}{V'} dS_R \simeq \left( \frac{4\pi M}{\kappa m_p} \right)^2 \int_1^{x_e} \frac{dx}{F'[x]\mathcal{N}}, \tag{23}
\]

\(^3\)There is also a soft mass term for the inflaton in the potential, \( m_{3/2}^2 |S|^2 \), typically of the order of \( m_{3/2} \simeq O(1 \text{ TeV}) \), but this term is only relevant for values of the coupling \( \kappa < 10^{-5} \), which we do not consider in this letter.

\(^4\)Although this will become relevant for values of the coupling \( \kappa \gtrsim 0.05 \), see later. For values of the coupling \( \kappa \ll 10^{-3} \), the potential is extremely flat and observable inflation takes place quite near the critical value. In the limit \( |S|/M \to 1 \), the sugra term dominates again over the radiative contribution\(^10\)\(^11\).
which again when \( x_e > 1 \) can be approximated by:

\[
S_{Re} \simeq \sqrt{N_e N_e e^{\frac{\kappa}{2\pi} m_P}}. \tag{24}
\]

Finally, using Eq. (24) into Eq. (22), we get the predicted amplitude of the primordial spectrum at \( N_e \):

\[
P_R^{1/2} \simeq 2 \sqrt{\frac{N_e}{3}} \left( \frac{M}{m_P} \right)^2, \tag{25}
\]

The WMAP normalization is \( P_R^{1/2} = 4.86 \times 10^{-5} \), taken at the comoving scale \( k_0 = 0.002 \text{ Mpc}^{-1} \). During inflation, this scale exits the horizon at approximately \( [14] \):

\[
N_e \simeq 53 + \frac{1}{3} \ln(T_R/10^9 \text{GeV}) + \frac{2}{3} \ln(\sqrt{k} M/10^{15} \text{GeV}), \tag{26}
\]

where \( T_R \) is the reheating temperature, which for \( \kappa \geq 10^{-3} \) is expected to be of the order of \( O(10^9 \text{GeV}) \) \([10]\). This sets \( N_e \approx 50 \), and from Eq. (25) this fixes the inflationary scale \( M \approx 6 \times 10^{15} \text{ GeV} \). Thus for the spectral index, using Eqs. (24) and (19), we have the approximated result \([2]\):

\[
n_s \simeq 1 - \frac{1}{N_e} \simeq 0.98, \tag{27}
\]

for \( N_e \approx 50 \). The tensor to scalar ratio is negligible, with \( r \lesssim 10^{-4} \), and also there is no running in the spectral index, with \( dn_s/d \ln k \lesssim 10^{-3} \) \([13]\).

We can obtain in the same way the value \( n_s \) in general without making use of any approximations: (i) having chosen the no. of e-folds at which the primordial spectrum is normalised, from Eq. (23) one obtains the corresponding value of the field \( S_{Re} \); (ii) knowing the value of the field, the WMAP normalization on the spectrum Eq. (22) fixed the scale of inflation \( M \); (iii) finally, Eqs. (18) and (21) gives the predicted value of the spectral index as a function of \( \kappa \). The predicted value of the spectral index is plotted in Fig. (1), showing the deviations from the approximated value Eq. (27) for small and large values of \( \kappa \). For small values of the coupling \( \kappa \), the approximation \( x_e > 1 \) does not hold. Diminishing the coupling what we have is a flatter potential, with a smaller curvature, so that the last say 50 e-folds of inflation happens to be quite close to the critical value, with \( x_e \approx 1 \), giving rise to a practically scale invariant spectrum. On the other hand, for larger values of the coupling \( \kappa \) although the approximation \( x_e > 1 \) holds, one can see from Eq. (24) that the value gets larger and closer to the Planck scale, so that the quartic term for the inflaton induced by the sugra corrections cannot

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\(^5\)For the numerical results shown in the figures, varying \( \kappa \) and taking \( T_R \simeq 10^9 \text{ GeV} \), we have checked that we always stay in the range \( N_e \approx 50 - 53 \).
be neglected any longer. This tends to give a positive curvature contribution, making the spectrum to turn from red tilted \( n_s < 1 \) to blue tilted \( n_s > 1 \).

The result in Eq. (27) can be viewed as the lower bound on the predicted spectral index, with \( n_s \gtrsim 0.98 \), to be compared to the central WMAP three year central value of \( n_s \approx 0.95 \). This motivates supersymmetric hybrid inflation with a non-minimal Kähler potential, where the spectral index can be lowered.

### 3 Non-Minimal Kähler Potential

We now turn to the non-minimal modification of supersymmetric hybrid inflation. We continue to assume the same minimal superpotential as in Eq. (1). However we now consider a non-minimal Kähler potential, [14, 15, 16].

\[
\mathcal{K} = |S|^2 + |\phi|^2 + |\bar{\phi}|^2 + \kappa_S \frac{|S|^4}{4m_P^2} + \kappa_{S\phi} \frac{|S|^2 |\phi|^2}{m_P^2} + \kappa_{S\bar{\phi}} \frac{|S|^2 |\bar{\phi}|^2}{m_P^2} + \kappa_{SS} \frac{|S|^6}{6m_P^4} + \cdots \tag{28}
\]

Working along the D-flat direction \(|\phi| = |\bar{\phi}|\), and keeping the relevant terms for inflation up to \( O(|S|m_P)^4 \), we get the potential:
\[ V_{\text{non-min}} \simeq V_{0}^{\text{non-min}} + \Delta V_{\text{1loop}}, \]
\[ V_{0}^{\text{non-min}} \simeq 2 \kappa^{2} |S|^{2} |\phi|^{2} + \kappa^{2} (|\phi|^{2} - M^{2})^{2} \left( 1 - \kappa S \frac{|S|^{2}}{m_{P}^{2}} + \kappa_{\phi} \frac{|\phi|^{2}}{m_{P}^{2}} + \gamma S \frac{|S|^{4}}{2 m_{P}^{4}} \right) + \cdots, \tag{29} \]

where \( \Delta V_{\text{1loop}} \) is given in Eq. (14), and we have defined:
\[ \kappa_{\phi} = (1 - \kappa S \phi - \kappa \bar{S} \phi), \tag{30} \]
\[ \gamma_{S} = (1 - \frac{7 \kappa S}{2} + 2 \kappa^{2} S - 3 \kappa S \bar{S}). \tag{31} \]

The non-minimal Kähler only introduces a small correction to the \( \phi \) squared mass, so that still for values of the inflaton field \( |S| > |S|^{c} \) this is positive and we can set \( \phi = 0 \) during inflation, with:
\[ V = V_{1}^{\text{non-min}}(\phi = 0) \simeq \kappa^{2} M^{4} \left( 1 - \kappa S \frac{S_{R}^{2}}{2 m_{P}^{2}} + \gamma S \frac{S_{R}^{4}}{8 m_{P}^{4}} + \cdots \right) + \Delta V_{\text{1loop}}. \tag{32} \]

Although it seems that we are introducing an infinite number of arbitrary parameters in the expansion of the Kähler potential, Eq. (28), we remark that in the regime where the inflaton field value is well below the Planck mass, the non-minimal Kähler contributions to the quartic and higher terms for the inflaton have no effect on the inflationary dynamics and therefore only one parameter, \( \kappa_{S} \), will be relevant for the inflationary predictions that follow. Note that \( \kappa_{S} > 0 \) will be required so that the prediction for \( n_{s} \) is in agreement with WMAP.

The non-minimal Kähler induces now a negative correction to both the first and the second derivative of the potential in the inflaton direction:
\[ V' \simeq \frac{\kappa^{2} M^{4}}{m_{P}^{2}} \left( -\kappa S \frac{S_{R}}{m_{P}^{2}} + \gamma S \frac{S_{R}^{3}}{2 m_{P}^{4}} + \frac{\kappa^{2} m_{P}^{2}}{8 \sqrt{2 \pi^{2} M}} N F'[x] \right), \tag{33} \]
\[ V'' \simeq \frac{\kappa^{2} M^{4}}{m_{P}^{4}} \left( -\kappa S + 3 \gamma S \frac{S_{R}^{2}}{2 m_{P}^{2}} + \frac{\kappa^{2} m_{P}^{2}}{16 \pi^{2} M} N F''[x] \right). \tag{34} \]

This correction gives rise to a local minimum and maximum in the potential located at
\[ \frac{S_{R}^{\text{min}}}{m_{P}} \simeq \sqrt{\frac{2 \kappa_{S}}{\gamma_{S}}}, \tag{35} \]
\[ \frac{S_{R}^{\text{max}}}{m_{P}} \simeq \sqrt{\frac{2 N}{\kappa_{S}} \left( \frac{\kappa}{4 \pi} \right)}, \tag{36} \]
which for example for \( \kappa_{S} \approx \kappa \approx 0.01 \) gives \( S_{R}^{\text{min}}/m_{P} \approx 0.14 \), and \( S_{R}^{\text{max}}/m_{P} \approx 0.01 \). After that, for \( S_{R} < S_{R}^{\text{max}} \) we have the standard flat potential with \( V' > 0 \), suitable for
hybrid inflation, with the field rolling towards the critical value. We have demanded then that we can get at least 60-50 e-folds of inflation once the field is in that region of the potential with $V' > 0$, i.e., that $S_{Re} \leq S_{R}^{max}$. We do not address the question of how the field reaches $S_{Re}$ in this letter, that is, the problem of the initial conditions for inflation. Although this problem is also present to some extent in the minimal case, it can be more severe in the non-minimal scenario due to the presence of the local minimum near Planck values. Starting the evolution for the homogeneous inflaton field near or beyond Planck, it may happen that the field gets stuck in this local minimum, and the system may inflate there, but it is not clear how to end inflation. On the other hand depending on the initial field values, the field may overcome the minimum, reach the flat part of the potential, and hybrid inflation may start. We can always find such initial values at least for the inflaton field, but then the question would be how fine-tuned they are. Nevertheless, when studying the evolution of the system prior to inflation, fields such as $\phi$, $\bar{\phi}$, should be taken into account. One should also check that these fields indeed go early enough to their respective local minimum. This is an important issue, but beyond the scope of this letter. Here we just concentrate on the inflationary predictions derived from the potential Eq. (32), assuming that we have suitable initial conditions for hybrid inflation to take place.

From Eq. (35), the condition of having enough inflation, $S_{Re} \leq S_{R}^{max}$, might be expressed as an upper bound on the possible value of $\kappa_S$, with

$$\kappa_S \lesssim 2N \left( \frac{\kappa}{4\pi} \right)^2 \left( \frac{m_P}{S_{Re}} \right)^2. \quad (37)$$

However, the value of the field at $N_e$ e-folds given by Eq. (33), $S_{Re}$, itself depends on the value of $\kappa_S$ through $V'$. The contribution from $\kappa_S$ tends to decrease $V'$ and makes the potential flatter, so that the corresponding value of $S_{Re}$ decreases and it will stay below $S_{R}^{max}$. This is shown in Fig. (2), where we have plotted the ratio $S_{Re}/S_{R}^{max}$ depending on $\kappa_S$, for different values of $\kappa$. As $\kappa_S$ increases the ratio also increases but remains below one. On the other hand, the minimum/maximum in the potential disappears whenever $\kappa_S \lesssim \sqrt{\gamma_S N_e} \kappa/(2\pi)$, this being the lower value of $\kappa_S$ shown for each curve in Fig. (2).

In addition, we have now in Eq. (32) a mass term for the inflaton field proportional to $\kappa_S$, and therefore this parameter has to be small enough in order to satisfy the slow-roll conditions. The slow-roll parameters are given by:

$$\epsilon \simeq \left( -\kappa_S \frac{S_R}{m_P} + \left( \frac{\kappa^2}{4\pi} \right) \frac{m_P}{M} \mathcal{N} F' [x] \right)^2, \quad (38)$$

$$\eta \simeq -\kappa_S - \delta, \quad (39)$$

where we have assumed $S_R \ll m_P$ so that we can neglect the quartic term in the
analytical expression\(^6\), \(\delta\) is the contribution from the 1-loop effective potential, Eq. (18), and again \(\epsilon \ll |\eta|\). Therefore, for slow-roll inflation, \(|\eta| < 1\), we only require \(\kappa_S < 1\).

The spectral index is given by:

\[
ns \simeq 1 - 2\kappa_S - 2\delta.
\]  

(40)

From the previous analysis with minimal Kähler potential, we could think naively that the one-loop contribution \(\delta \leq 0.01\), and then we would need for example \(\kappa_S \geq 0.01\) if we want the spectral index around or below \(n_s \approx 0.96\). However, as previously noted, the non-minimal Kähler contribution will decrease \(V'\) which, from Eq. (22), tends to increase the amplitude of the curvature perturbation. Thus, in order to keep the WMAP normalization, the scale of inflation \(M\) (i.e. \(V\)) has to decrease accordingly, see Fig. (3). Also, a decrease in \(V'\) means a smaller value of the field at 50 e-folds. Therefore, for a given value of the coupling \(\kappa\), both the scale of inflation \(M\) and the value of the inflaton field \(S_{Re}\) decrease, but in such a way that their ratio \(x_e = S_R / (\sqrt{2}M)\) remains practically constant\(^7\). Comparing the prediction for \(\delta\) (Eq. (18)) in the minimal \((\kappa_S = 0)\)

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\(^6\)The quartic term for the inflaton is taken into account in all the numerical calculations, and therefore in the results presented in the plots.

\(^7\)We have checked this numerically.
and non-minimal case ($\kappa_S \neq 0$), the latter gets enhanced with respect to the minimal case due to the reduction in $M$. Therefore, when taking into account the effects of the non-minimal Kähler potential we have also that the 1-loop contribution can be well above the previous upper limit of 0.01. Note that there is no regime where the 1-loop contribution could be neglected with respect to the non-minimal Kähler one, as far as the approximation $S_R < m_P$ is fulfilled. This can clearly be seen in Fig. (4) where we show the 1-loop contribution to the spectral index, $\delta$, for different values of $\kappa_S$. The general trend is that the 1-loop effective potential contribution always remains non-negligible, and besides $\delta > \kappa_S$.

In Fig. (5) we plot the prediction for $n_s$ as function of $\kappa$ for different values of $\kappa_S$. We can see that even for small values of $\kappa$, already for $\kappa_S \simeq 5 \times 10^{-3}$ we obtain a spectral index smaller than what we would have expected only from the non-minimal Kähler contribution, due to the increase in $\delta$. As the value of $\kappa_S$ increases, the effect gets larger and the spectrum more and more red-tilted. However, for a given value of $\kappa_S$, the prediction for the spectral index is practically independent of the value of $\kappa$, for values of the coupling in the range $[0.001, 0.05]$.

On the other hand, as we increase the coupling and $\kappa \approx 0.1$, the field value at 50 e-folds also increases and approaches the Planck scale, as in the minimal case. The

Figure 3: Value of $M$ depending on $\kappa$, for different values of $\kappa_S$; from top to bottom $\kappa_S = 0, 0.005, 0.01, 0.015, 0.02$. ($\mathcal{N} = 1$)
quartic term in the potential then takes over and gives rise to a blue-tilted spectrum, just as with the minimal Kähler potential. At which value of $\kappa$ this effect dominates depends on the value of the quartic coefficient $\gamma_S$, which in turn may depend now also on the next parameter in the expansion of the non-minimal Kähler potential, i.e. $\kappa_{SS}$. Nevertheless, for values of $\kappa_{SS} < 1/3$, this parameter has no effect on the spectral index.

4 Reheating and Baryon Asymmetry

To proceed further, an inflationary model should specify the transition to radiation domination, and also explain the origin of the observed baryon asymmetry. For hybrid inflation models this has been extensively studied (see [8] for a review and additional references). For the non-minimal models under discussion, let us consider two well motivated examples. For the first one we identify $G$ with the local $U(1)_{B-L}$ symmetry, and introduce three MSSM singlet right-handed neutrinos $N_i$ ($i=1,2,3$). These acquire masses via the non-renormalizable couplings

$$W_2 = \frac{1}{m_p} \hat{N} \hat{N} \hat{\phi} \hat{\psi},$$

(41)
where, for simplicity, we will ignore family indices. The couplings in Eqs (I) and (II) ensure that the inflaton fields $\phi, \bar{\phi}$ and $S$ decay into right handed neutrinos and sneutrinos. The reheat temperature is roughly given by \[ T_R \simeq (10^{-1} - 10^{-2}) M_N, \] where $M_N$ denotes the mass of the heaviest singlet neutrino which satisfies $2M_N \leq m_{\text{inf}} = \sqrt{2}\kappa M$. Here $m_{\text{inf}}$ denotes the inflaton mass in the global minimum after inflation. The gravitino constraint usually requires that $T_R \leq 10^6 - 10^9$ GeV, and so non-thermal leptogenesis \[17\] is the most plausible scenario in these models for generating the observed baryon asymmetry in the universe.

Our second example is based on the flipped $SU(5)$ model. The reason one avoids GUTs such as $SU(5)$ and $SO(10)$ has largely to do with the primordial monopoles which appear at the end of inflation and create a serious cosmological problem. The well known doublet-triplet problem in $SU(5)$ is also nicely resolved in flipped $SU(5)$. Hybrid inflation in flipped $SU(5)$ was recently discussed and shown to yield a spectral index $n_s = 0.99 \pm 0.01$ \[18\]. By using a non-minimal Kähler problem we can obtain $n_s$ close to 0.95, in much better agreement with the three year WMAP results. Note that baryogenesis via leptogenesis is also automatic in flipped $SU(5)$.
In summary, we have argued that a relatively modest extension of minimal supersymmetric hybrid inflation preserves many of its successful features and also yields a scalar spectral index which appears to be more consistent with the most recent data.

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