Model of End Milling Force Based on Undeformed Chip Surface with NURBS in Peripheral Milling

Pu Xiongying\textsuperscript{1, 2, a}, Liu Weijun\textsuperscript{1, b} and Zhao Jibin\textsuperscript{1, c}

\textsuperscript{1} Modern Equipment Research & Design Center, Shenyang Institute of Automation, Chinese Academy of Sciences, Shenyang 110016, China
\textsuperscript{2} Graduate School of Chinese Academy of Sciences, Beijing 100049, China
\textsuperscript{a}puxy@sia.cn, \textsuperscript{b}wjliu@sia.cn, \textsuperscript{c}jbzhao@sia.cn

\textbf{Keywords:} End milling; Cutting force; Undeformed chip surface; NURBS

\textbf{Abstract.} A new cutting force model for peripheral milling is presented based-on a developed algorithm for instantaneous undeformed chip surface with NURBS. To decrease the number of the differential element, the contact cutting edges of end-milling cutter with the part and the chip thickness curve are represented by NURBS helix, and the instantaneous undeformed chip is constructed as a ruled surface with the two curves. The cutting force generated by the edge contact length and the uncut chip area. Using the cutting coefficients from Budak\cite{1}, the cutting-force model verified by simulation. The simulation results indicate that new cutting-force model predict the cutting forces in peripheral milling accurately.

\textbf{Introduction}

Peripheral milling is a common manufacturing process in the aerospace, automobile, die and mold. The accuracy of the dimensional and geometric often relate to the product quality and cost. The variable cutting force in peripheral milling cause workpiece deflection, tool deflection, tool wear and chatter vibration which may lead to bad parts and to a considerable waste of time and money.

An accurate dynamic cutting force model is very important for precise prediction of tool and workpiece deflection in peripheral milling. According to Smith and Tlusty\cite{2}, Liu\cite{3} summarized several models to prediction the cutting forces and pointed out the cutting force modes presented by Ismail\cite{4} and Elbestawi\cite{5} that do not consider the size effect of the undeformed chip thickness and the influence of the effective rake angle, and presented an improved model taking account the two factors. Yucesan \cite{6}and Bayoumi\cite{7} developed models from the differential normal and friction forces acting on the rake face that are expressed as a functions of the differential area of the undeformed chip cross-section. In their research, the cutting force coefficients ($K_n, K_f$) were regarded as the variables about the cutter rotation angle and the position angle of a point on the cutting edge. Yun\cite{8} treated the cutting force coefficients as the constant based on the models \cite{6-7} and presented an approach to estimate run-out related parameters.

Since the geometry of tool edge is complex, most of the cutting edges of cylindrical helix are considered as discrete simple elements, as Fig. 1 shown. Engin and Altintas \cite{9} presented a generalized mathematical model of most helical end mills including helical ball, tapered helical ball and bull nose cutters. El-Mounayri and ect.\cite{10} Presented that any shape of cutting edge can be represented by 3D polynomial curves, such as cubic Bezier curves which are general and computationally efficient. These cutting force prediction models normally are the discrete cutting edge in straight segments, the relevant undeformed chip are sliced along the axial direction, too. Each slice with a uniform undeformed chip thickness can be modeled as for single oblique cutting. Divided into the smaller, the more accuracy of the forecasting cutting force is predicted.

A model of end milling force based on undeformed chip surface with NURBS is introduced in this paper. The helical contact cutting edge and thickness curve is represented by NURBS helix and the undeformed chip is expressed as ruled surface in Section 1. In Section 2, a new approach for the...
prediction the cutting force based on the undeformed chip with NURBS in the peripheral milling is detailed. In Section 3, predicted cutting forces for the peripheral milling of the titanium alloy from a series of simulations are presented.

Modeling of Instantaneous Undeformed Chip with NURBS

**Contact Helical Edge with NURBS.** When the bottom point of the first flute of the end mill with helical angle $\beta$ is at angle $\phi$, the contact helix will have a circular angle $\theta$. According to the algorithm presented by Pu and Liu[11], a helix curve

$$h(\phi) = R(\cos\phi, \sin\phi, \phi \cot \beta) \quad \phi \in [0, \theta]$$

(1)

Can be represented by initial NURBS helix and it can be approximated using subdivision algorithm for a given tolerance. The contact helical edge can be presented

$$C(u) = \frac{\sum_{j=1}^{n} N_{i,p}(u)w_{j}B_{i}}{\sum_{j=1}^{n} N_{i,p}(u)w_{j}}$$

(2)

where the degree $p = 2$, the $\{B_{i}\}$ are the control points, the $\{w_{j}\}$ are the weights, and $\{N_{i,p}\}$ are the $p$th-degree B-spline basis functions defined on the nonuniform knot vector $E = \{u_{1}, u_{2}, \ldots, u_{n+p+1}\}$. As shown in Fig. 2, using rotation and translation to $C(u)$, the helix contact edge is the curve $C_{1}$ with the circular angle $\theta_{c}$, the center is at $O_{c}$. Assuming that the rotation angle of the first flute at an elevation $z = 0$ is $\phi$, the rotation angle of the flute $j$ at the bottom of the flute is expressed as $\phi_{j} = \phi + (j - 1)\phi_{p}$, $j = 1, 2, \ldots, (N - 1), \phi_{p} = 2\pi / N$. The NURBS helix of the flute $j$ can be expressed using the same way.

**Undefomed Chip Surface with NURBS.** At any moment during the tool-workpiece engagement, the undeformed chip thickness for a tooth engaged in cutting is determined through finding the intersection point of the surface left by the preceding tooth and the line passing through the current tooth tip and cutter axis $O_{c}$ as shown Fig. 2, and the feed rate is $f$. Assume the surface left by the preceding tooth to be a cylinder with the axis $O_{c}$, the intersection points make of a thickness helical curve $C_{2}$ with a circular angle $\theta_{t}$ and the helical angle $\beta'$ is determined by

$$\tan \beta' = \frac{\theta_{t}}{\theta_{c}} \tan \beta$$

(3)
The angle at full axial depth of cut \( z = a_p \) is \( \varphi_a = a_p / (R \cot \beta) \), the cutter entry angle \( \phi_{st} = 0 \) and the cutter exit angle \( \phi_{ex} = \arccos(1 - a_e / R) \). The following computer algorithm is used in determining the circular angle of the contact helix \( \theta_c \) and the circular angle of the thickness helix \( \theta_t \).

If \( \phi_{st} < \phi_j < \phi_{ex} \),

- Case 0: \( \phi_j - \varphi_a \geq 0 \), then \( \theta_t = \varphi_a, \theta_c = \varphi_a + \alpha - \alpha_c \),
- Case 1: \( \phi_j - \varphi_a < 0 \), then \( \theta_c = \varphi + \delta, \theta_t = \varphi + \alpha - \delta \),

If \( \phi_j > \phi_{ex} \),

- Case 2: \( 0 < \phi_j - \varphi_a < \phi_{ex} \), it is same with case 0.
- Case 3: \( \phi_j - \varphi_a < 0 \), it is same with case 1.
- Case 4: \( \phi_j - \varphi_a > \phi_{ex} \), the flute is out of cutting.

Application of sine theorem, \( \alpha_c = \sin^{-1} \left( \frac{f_t \cos \phi}{R} \right) \), \( \alpha_t = \sin^{-1} \left( \frac{f_t \cos(\phi - \theta_c)}{R} \right) \), and \( \delta = \sin^{-1} \left( \frac{f_t}{R} \right) \). In case 2 and case 3, a part of undeformed chip area is out of cutting, the lower bound of the integrating \( \xi_{low} \) needs to be determined by finding the close point at \( z_{low} = (\phi_j - \phi_{ex})R \cos \beta \). Using the knots insertion algorithm insert the knot \( \xi_{low} \), the number of elements of the curve is increased.

Once the angels have been determined, the undeformed chip thickness curve \( C_2(u) \) will be represented by NURBS helix, too. The undeformed chip surface \( S(u, v) \) is viewed as a ruled surface since it is a straight line segment in the thickness direction. Let \( C_2(u) \) have the same knot vector with \( C_1(u) \). The undeformed chip form is

\[
S(u, v) = \frac{\sum_{i=1}^{\mu} \sum_{j=1}^{\nu} N_{i,2}(u)M_{j,0}(v)w_jB_{ij}}{\sum_{i=1}^{\mu} \sum_{j=1}^{\nu} N_{i,2}(u)M_{j,0}(v)w_j} \quad \text{(4)}
\]

where the knot vectors are \( E = \{u_1, \cdots, u_{n+1}\} \) and \( F = \{0, 0, 1, 1\} \) and \( S(u, 0) = C_1(u) \). The algorithm for rule surface with NURBS come form Piegl[12].

**Modeling of End Milling Forces**

**Mechanics of End Milling Forces.** The immersion angle for flute \( j \) at axial depth of cut \( z \) therefore is \( \phi_j = \phi + (j-1)\phi_p - z \tan \beta / R \). In mechanistic model, an elemental cutting forces model is applied to each differential cutting edge discrete element. The element tangential \( dF_{t,j} \), radial \( dF_{r,j} \), and axial \( dF_{a,j} \) cutting forces acting on a differential flute element with height \( dz \) are given by

\[
\begin{align*}
  dF_{t,j}(\phi, z) &= \left[ K_{t} + K_{t,c}f_j \right] \cos(\phi - \theta_c) dz \\
  dF_{r,j}(\phi, z) &= \left[ K_{r} + K_{r,c}f_j \right] \cos \phi dz \\
  dF_{a,j}(\phi, z) &= \left[ K_{a} + K_{a,c}f_j \right] dz
\end{align*} \quad \text{(5)}
\]

where \( f_j(\phi, z) \) is the chip thickness, \( K_{t,c}, K_{r,c}, K_{a,c} \) can be expressed in terms of the modified mechanics of cutting analysis variables as[1, 13]:
\[ K_c = \frac{r_s \cos(\beta_n - \alpha_n) + \tan \beta \tan \eta \sin \beta_n}{\sin \phi_n} \]
\[ K_{rc} = \frac{r_s \sin(\beta_n - \alpha_n)}{\sin \phi_n \cos \beta} \]
\[ K_{ac} = \frac{r_s \cos(\beta_n - \alpha_n) \tan \beta - \tan \eta \sin \beta_n}{\sin \phi_n} \]  

where \( c = \sqrt{\cos^2(\phi_n + \beta_n - \alpha_n) + \tan^2 \eta \sin^2 \beta_n} \), \( \phi_n, \beta_n, \alpha_n \) are the normal shear angle, average friction angle and the normal rake angle; the chip flow angle \( \eta \) is equal to the oblique angle \( (\eta = \beta) \) by adopting Stabler’s chip flow rule[14].

**Model of End Milling Force Based on NURBS Chip Surface.** The new proposed approach is based on the expressions presented by Altintas[15] for the prediction the cutting force in the peripheral milling. Instantaneous tangential \( F_t \), radial \( F_r \), and axial \( F_a \) cutting forces are expressed as a function of varying undeformed chip area (A) and edge contact length (S)

\[
F_{t,j} = K_c A + K_{rc} S \\
F_{r,j} = K_{rc} A + K_{ac} S \\
F_{a,j} = K_{ac} A + K_{ac} S
\]  

The cutting coefficients (6) are modified

\[ K_p = K_p \cos \beta, \quad p = tc, rc, ac \]  

The undeformed chip surface \( S(u,v) \) is constructed a rule surface by the contact cutting edge curve \( C_1(u) \) and the undeformed chip thickness curve \( C_2(u) \) as shown Fig. 2. So the undeformed chip area and the edge contact length are respectively

\[
A = \int_\Omega dA = \int_\Omega J(u,v)dudv = \sum_{i=1}^{n_p} \sum_{j=1}^{2} \int_{u_i}^{u_{i+1}} \int_{v_j}^{v_{j+1}} J(u,v)dudv
\]  

where \( J(u,v) = |S_u(u,v) \times S_v(u,v)| \),

\[
S = \int_\Gamma ds = \int_\Gamma |S_u(u,0)|du \sum_{i=1}^{n_p} \int_{u_i}^{u_{i+1}} |S_u(u,0)|du
\]

The integrals of the differential cutting forces can be calculated numerically using Gaussian quadrature over the NURBS domain. The Gaussian integration parameters are chosen according to the degree of the NURBS curve or surface. Cutting force distribute on the part as shown in Fig. 2. There are three Gaussian points for quadratic NURBS curve element and there are six Gaussian points for the ruled surface element to evaluate the integration according to observations of Hughes et al.[16]. The integral limits are first changed to -1 and 1 by using the following transformation:

\[
u = \frac{1}{2} ((1 - \xi)u_{j} + (1 + \xi)u_{j+1})
\]

For example:
\[
\int_{u_i}^{u_{i+1}} \int_{v_j}^{v_{j+1}} J(u,v)dudv = \frac{1}{4} (u_{i+1} - u_i)(v_{j+1} - v_j) \int_{-1}^{1} \int_{-1}^{1} H(\xi, \eta) d\xi d\eta \\
= \frac{1}{4} (u_{i+1} - u_i)(v_{j+1} - v_j) \sum_{k=1}^{3} \sum_{l=1}^{3} W_k W_l H(\xi_k, \eta_l)
\]
\[
\int_{u_i}^{u_{i+1}} |C'(u)| du = \frac{1}{2} (u_{i+1} - u_i) \int_{-1}^{1} D(\xi) d\xi = \frac{1}{2} (u_{i+1} - u_i) \sum_{k=1}^{3} W_k D(\xi_k)
\]

\(W_k, W_l\) are the Gaussian weights and \(\xi_k, \eta_l\) are the Gaussian points.

**Resolve Force.** The different tangential, radial and axial force acting on the edge of the flute can be expressed

\[
dF_{t,j} = K_{nc} dA + K_{nc} dS \\
dF_{r,j} = K_{rc} dA + K_{rc} dS \\
dF_{a,j} = K_{ac} dA + K_{ac} dS
\]  

(11)

where \(dA = J(u,v)dudv, dS = |S_u(u,0)| du\). Thus

\[
dF_{t,j} = \left( K_{nc} \int_{0}^{1} J(u,v)dv + K_{nc} |S_u(u,0)| \right) du \\
dF_{r,j} = \left( K_{rc} \int_{0}^{1} J(u,v)dv + K_{rc} |S_u(u,0)| \right) du \\
dF_{a,j} = \left( K_{ac} \int_{0}^{1} J(u,v)dv + K_{ac} |S_u(u,0)| \right) du
\]  

(12)

The normal vector of the curve \(S(u,0)\) is

\[
n(u) = \frac{S_u(u,0)}{|S_u(u,0)|} = \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix}
\]  

(13)

Actually, the value of \(n_z\) of the normal of the helix curve is zero. The different tangential, radial and axial force can be resolved in the feed (x), normal (y) and axial (z) directions acting on the edge of the flute.

The total cutting force can be obtained by integrating the differential cutting forces along the contact cutting edge.
\[
\begin{pmatrix}
\frac{dF_{x,j}}{u}
\frac{dF_{y,j}}{u}
\frac{dF_{z,j}}{u}
\end{pmatrix}
= \begin{pmatrix}
n_x & n_y & 0 \\
n_y & n_z & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\frac{dF_{x,j}}{a}
\frac{dF_{y,j}}{a}
\frac{dF_{z,j}}{a}
\end{pmatrix}
\] (14)

\[
F_{q,j}(u) = \int_{u_{j-1}}^{u_j} \frac{dF_{q,j}}{u}(u), \quad q = x, y, z
\] (11)

The cutting forces contributed by all flutes are calculated and summed to obtain the total instantaneous forces on the cutter at rotation angle \( \phi \).

\[
F_x(\phi) = \sum_{j=0}^{N} F_{x,j}; \quad F_y(\phi) = \sum_{j=0}^{N} F_{y,j}; \quad F_z(\phi) = \sum_{j=0}^{N} F_{z,j}.
\] (12)

**Milling Force Simulation**

Fig. 4 shows the undeformed chip surfaces in the milling process. The predicted milling forces for a half immersion up milling cutting using a 19.05 mm diameter, four flute end milling with 30 degree helix angle. The feed rate is \( f_t = 0.25 \) mm/tooth, and the axial depth of cut is 5.08 mm. From the knots vector of the curve \( C_i(u) \), the number of the integral elements is only two, and the undeformed chip surface is only two elements. The red start points are the Gaussian points.

Predicted cutting forces for the peripheral milling of the titanium alloy, the cutting force coefficients \( (K_{xc}, K_{rc}, K_{nc}, K_{xc}, K_{rc}, K_{nc}, K_{ac}) \) can be found experimentally using cutting forces per tooth averaged for a specific type of tool and material[1]. The constants used in the simulation were \( K_{xc} = 1619 N / mm^{2}, K_{rc} = 253 N / mm^{2}, K_{nc} = 604 N / mm^{2}, K_{xc} = 24 N / mm^{2}, K_{rc} = 43 N / mm^{2}, K_{ac} = 3 N / mm^{2} \). The modified coefficients were \( K_{xc}' = 1402 N / mm^{2}, K_{rc}' = 219 N / mm^{2}, K_{ac}' = 604 N / mm^{2} \). The simulation results for the predicted milling force shows in Fig. 5 which the feed rate \( f_t = 0.05 \) mm/tooth. Compare the predicted milling forces using new method with the cutting forces using the traditional method, the different is very small.

**Conclusions**

![Fig. 4 The undeformed chip in the milling process](image)

![Fig. 5 Predicted milling force comparison](image)

In order to decrease the number of the differential elements, a new approach to predict the cutting force based on undeformed chip surface with NURBS has been presented. The method involves the establishment of undeformed chip surface with NURBS, the resolve cutting forces and the integrals of the differential cutting forces. The predicted cutting forces can be calculated by using a few
differential elements. The simulation shows a good agreement between the new method and the traditional method.

References

[1] E. Budak, Y. Altintas, and E.J.A: Journal of Manufacturing Science and Engineering(1996), No. 118, p.216-224.
[2] S. Smith, J. Tlusty: Journal of Engineering for Industry(1991), No.113, p.169-175.
[3] X.W. Liu: The International Journal of Advanced Manufacturing Technology Vol.20(2002), No. 9, p. 631-638.
[4] F. Ismail: Journal of Engineering for Industry(1993), No.115, p.245-252.
[5] M.A. Elbestawi: Journal of Engineering for Industry(1994), No.116, p.435-439.
[6] G.B. Yucesan, Y. Altintas: International Journal of Machine Tools and Manufacture Vol.34(1994), No.4, p.473-487.
[7] A.E. Bayoumi, G.Yucesan, and L.A. Kendall: Journal of Engineering for Industry(1994), No. 116, p.324-339.
[8] W.S. Yun, D.W. Cho: The International Journal of Advanced Manufacturing Technology Vol.16( 2000), No.12, p.851-858.
[9] S. Engin, Y. Altintas: International Journal of Machine Tools and Manufacture Vol.41(2001) , No.15, p.2195-2212.
[10]H. El-Mounayri: The International Journal of Advanced Manufacturing Technology Vol.13(1997), No.4, p.237-247.
[11]X. PU, W. Liu: IEEE 10th International Conference on Computer-Aided Industrial Design & Conceptual Design(2009), p.620-624.
[12]L. Piegl, W. Tiller: The NURBS book.: Springer Verlag (1997).
[13]E.J.A. Armarego, R.H. Brown: The machining of metals. New Jersey : Prentice-Hall(1969).
[14]G.V. Stabler: Advances in Machine Tool Design and Research(1964), p.243-251.
[15]Y. Altintas, Manufacturing automation: Metal Cutting Mechanics, Machine Tool Vibrations, and CNC Design, ed. Cambridge. Cambridge Universit Press(2000).
[16]T.J.R. Hughes, J.A. Cottrell, and Y. Bazilevs, Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. Comput. Methods Appl. Mech. Engrg., 2005. 194: p. 4135-4195.