Electron states and the spin density wave phase
diagram in Cr(1 1 0) films

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Abstract. Chromium films offer an excellent system to study the impact of
dimensional confinement on physical properties associated with the spin-density-
wave (SDW) ground state observed in bulk materials. These properties are also of
some technological importance since chromium is a common component of thin
film magnetic structures. We prepared chromium (1 1 0) films of high crystalline
quality on a W(1 1 0) substrate with a wedge-shaped thickness profile so that
the impact of confinement can be systematically studied. We have characterized
these films using a combination of low-energy electron diffraction and microscopy
as well as high-resolution angle-resolved photoemission spectroscopy. We have
probed the Fermi surface and the nesting vectors therein that are relevant to
the SDW ground state. We find these to predict accurately the observed bulk
SDW periodicity. We have also characterized the SDW periodicity in the film
directly by measuring the splitting between backfolded bands, and we find that this
periodicity deviates markedly from the bulk periodicity for thinner films at higher
temperatures. We have systematically mapped the SDW incommensurability
and phase diagram as a function of both film thickness and temperature. We
find commensurate and incommensurate phases that are separated by nearly
continuous transitions. Our results suggest a simple model to explain the delicate
interplay between commensurate and incommensurate phases that involves a
balance between SDW stabilization energy and surface and interface energetics.
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1. Introduction

A key goal of nanoscience is to fabricate artificial structures having targeted properties associated with dimensional confinement. A particularly interesting example of this has been to understand how the rich and fascinating physics of bulk chromium metal [1, 2], the prototypical itinerant antiferromagnet, is modified by the geometrical constraints associated with interfaces and thin film structures [3, 4]. Over the past 10–15 years, a co-ordinated effort involving synthesis, analysis and modelling has produced a good understanding of giant magnetoresistance (GMR) structures that often incorporate chromium layers [3]–[14]. Moreover, the detailed nanoscale understanding developed through this process has helped to spawn a new field that seeks to use electron spin in electronic devices [15, 16].

An important and necessary step to achieve this primary goal of nanoscience is to understand how complexity emerges from rationally designed artificial structures. ‘Complexity’ is conventionally associated with systems exhibiting a high degree of electron correlation or with systems having large unit cells such as soft systems. The strong many-body coupling between low energy, nanometre-scale excitations in these complex materials leads to emergent complexity that is manifested by rich phase behaviours and useful functionality. It is now plainly apparent, however, that nanoscale-dimensional confinement can also lead to emergent complexity even though the underlying materials are ‘simple’. A recent example where chromium films play a key role is a study of Cr(1 0 0): Cr97.5Mn2.5(1 0 0) superlattices that demonstrated an interesting hysteresis in a purely antiferromagnetic structure [17]. This interplay between the spin-density-wave (SDW) structure in chromium and the periodicity of the superlattice [4, 18]–[20] offers an excellent example of how nanoscale-dimensional confinement of a ‘simple’ material can lead to a notably complex and possibly useful macroscopic behaviour.

A hierarchical energy scale in bulk chromium drives many of this metal’s interesting antiferromagnetic properties [1, 2]. Electron–electron exchange energies on the scale of a few
tenths of an eV produce an incommensurate (IC) SDW ground state. This separates into domains with the SDW wave vector, $Q_{SDW}$, oriented along the $\langle 100 \rangle$ axes. The relative populations of these domains can be controlled with external magnetic fields, suggesting weaker underlying energetics. Also, in analogy to the magnetocrystalline anisotropy in ferromagnets, weak spin–orbit interaction determines the spin-polarization direction. This direction is known to undergo a reorientation transition at 123 K. The properties of the SDW state are also modified under hydrostatic pressure, near defects, and by alloying. These weak bulk energetic interactions are enriched by similarly weak surface, interface and quantum-well energetics in chromium thin film structures, and this can lead to interesting complexity as noted above. For this reason, it is not surprising that chromium has become one of the most studied coupling agents in GMR structures.

The vast majority of chromium thin films and multilayers studied to date have been oriented in a $\langle 100 \rangle$ direction. In such structures, $Q_{SDW}$ generally lies normal to the film plane [3, 4]. In addition to the normal electronic quantum well states often observed in metallic thin films [21], this situation also leads to SDW quantization, as shown schematically in figure 1(a). The unusual hysteresis in the experiment on Cr($100$) : Cr$_{97.5}$Mn$_{2.5}$(100) superlattices, for example, resulted from the temperature-dependence of the SDW wavelength [17]. The energetically preferred number of SDW nodes and antinodes in the film is thus also $T$-dependent, yet SDW quantization requires that this number needs to be discrete so that hysteretic transitions result. Many other interesting results have been reported that stem from SDW quantization along the film normal [3, 4].
What would happen if we could eliminate this SDW quantization so that other, weaker interactions govern a film’s phase behaviour? This can be simply accomplished in (1 1 0)-oriented films, as shown schematically in figures 1(b) and (c). In a geometry where \( Q_{SDW} \) does not lie normal to the film, the intersections of the IC-SDW with the interfaces remain incommensurate. There can be no SDW quantization, since all spin densities are equally sampled at the interfaces and varying the phase of the SDW relative to the interface cannot change the total energy. This is a simple idea that does not consider the possibility of other, more exotic interfacial magnetic phases. However, the geometry of the bulk Fermi surface explored below indicates that SDW energetic interactions are only relevant when \( Q_{SDW} \) lies along a [1 0 0] axis. One might expect that the SDW in Cr(1 1 0) films would prefer to orient within the film along the [0 0 1] axis, in a ‘wave-guide’ geometry, as in figure 1(c). A recent STM study of the SDW orientation near the surface of a Cr(1 1 0) shows that the preferred orientation is along the [0 1 0] or [1 0 0] direction [22]. Neutron scattering studies of multilayers including thicker chromium films also observe [0 1 0] or [1 0 0] orientations [5, 23, 24]. The effects involve very subtle energetics.

Measuring the SDW periodicity and orientation in Cr films less than 10 nm thick is difficult. Neutron scattering has been applied only to thicker films and extended multilayers, without considering the layer thickness dependence in general and mostly on (1 0 0)-oriented films [3, 4, 23]. X-ray diffraction offers an attractive alternative, although the low signal available in scattering off the SDW in bulk chromium implies that similar studies of ultrathin films will be difficult [25]. A delicate scanning tunnelling microscopy study [22] measured the parasitic charge density wave associated with the IC-SDW on the (1 1 0) surface of a bulk Cr crystal, and a similar technique could be used to map the C–IC-phase boundaries discussed below. In an earlier paper, we showed that angle-resolved photoemission (ARP) is exquisitely sensitive to SDW periodicity while also offering the ability to probe the Fermi surface nesting that is a key aspect of the underlying energetics [26].

In this paper, we further elucidate our ARP measurements of the SDW phase behaviour of Cr(1 1 0) films grown on a W(1 1 0) substrate. We have measured slices through the bulk Fermi surface and relate these with precision to the observed bulk SDW nesting vector \( Q_{F} \) and SDW periodicity. Moreover, we have also mapped the evolution of the SDW incommensurability, \( \delta_{SDW} \), as a function of film thickness \( (d) \) and temperature \( (T) \) and find a continuous transition between commensurate (C) and IC phases. The behaviour of \( \delta_{SDW} \) cannot be attributed to changes in the Fermi surface nesting since this is found to be independent of \( d \) and \( T \). We propose that the underlying energetics are driven by a balance between SDW stabilization energy (which favours the IC phase) and interfacial energies (which favours the C phase). We believe that this interplay between volume and surface energies is an important contribution to the stability of the C phase in thin Cr(1 1 0) films, and we propose that the surface spin anisotropy play a role in this interplay.

In section 2, we present our experimental procedures. We describe how we prepare high-quality chromium thin films in wedge-shape geometries to facilitate measurements as a function of thickness. We also describe an approach to ARP that makes extensive use of multiparameter datasets. In section 3, we discuss experimental results that probe the nesting of the chromium Fermi surface and also the commensurability of the SDW as a function of \( d \) and \( T \). In section 4, we discuss our results in terms of the underlying energetic interactions, and examine the interplay between surface and volume interactions that leads to our measured phase diagram. In the final section, we speculate on where these results might lead in the future.
2. Experimental procedures

2.1. Chromium film growth

A relatively small number of experiments has been reported on clean chromium surfaces. Aside from being a very reactive material requiring excellent vacuum, bulk chromium nearly always has a notable concentration of nitrogen that migrates to the surface under standard cleaning treatments involving sputtering and annealing. One early goal of our chromium experiments was simply to develop a robust technique for producing clean and well-ordered surfaces by thermal evaporation onto a clean and crystalline substrate. Despite a substantial lattice mismatch with most 3d metals, W(1 1 0) has become a substrate of choice in such efforts, and that is the substrate we chose to use [27]–[29].

One of the most attractive features of studying thin films as a foundation for nanoscience is the systematic way in which synthetic film parameters can be controlled, particularly by using thickness wedges. Thickness wedges, where the microscopic film thickness varies continuously across the macroscopic extent of the film, offered early proof for the thickness-dependent oscillatory magnetic coupling in Fe:Cr GMR structures [30], and since that time have been used to study a variety of thin film magnetic phenomena, including quantum well states [9, 31]–[34], surface magnetic anisotropy [35] and the spin reorientation transition [36]–[41], and the SDW phase diagram in iron thin films [42]. A wedge-shape profile also helped us develop our growth and annealing protocols, since the resulting film morphology depends dramatically on thickness.

Our initial effort to make high quality films involved MBE-like growth at moderate growth temperatures of 400–500°C. Such films produced low-energy electron diffraction (LEED) patterns indicating at all thicknesses a coincidence lattice, with spots corresponding to both the tungsten and chromium lattice constants. Qualitatively similar results were found in 1990 by Shinn [43]. Also, the tungsten 4f core-level photoemission intensity decayed as a function of deposited thickness in a notably non-exponential way, suggesting island growth due to a Stranski–Krastinov growth mode. Later, we examined this growth mode with in situ low-energy electron microscopy (LEEM) at the National Center for Electron Microscopy at Lawrence Berkeley National Laboratory. The images in figure 2(a)–(i) and the movie confirms our conclusion. The growing films are stable until the beginning of the third monolayer, at which point a fairly abrupt onset of island formation occurs in conjunction with a de-wetting mechanism in which material from the second monolayer of the flat film diffuses into three-dimensional (3D) islands, leaving behind a one-monolayer thick wetting-layer between the islands. The growth mechanism we observe on heated substrates is typical for this class of Stranski–Krastinov systems [44].

After some effort to improve the film quality empirically, we determined that the best films were those deposited at or near room temperature and then annealed briefly to 800–1000 K after deposition. As discussed in section 3.3, the SDW phase behaviour depends sensitively on the temperature of this annealing. We used a commercial UHV electron beam evaporator, and deposited the initial film at a rate of ~1 monolayer (ML) minute. The thickness and deposition rate were measured with a quartz microbalance film thickness monitor that was calibrated with a thin film stylus profilometer. The room temperature-deposited film is notably non-crystalline: the sharp W(1 1 0) LEED pattern disappears after the equivalent of 2–3 ML have been deposited. As noted above, the morphology after annealing depends on the deposited thickness. For films less than ~1 nm thick, the Stranski–Krastinov growth mode is observed, where one
Figure 2. LEEM images of chromium films grown at 500 K on W(1 1 0). (a)–(i) Images ranging from the clean W(1 1 0) substrate before deposition to the Stranski–Krastinov island morphology resulting from deposition of ∼10 ML equivalent at 500 K; a movie is included of the first three monolayer growth showing the de-wetting transition which occurs at the onset of deposition of the third ML. Note that nucleation of the chromium islands in the first and second monolayers is preferentially at step edges.

A monolayer of chromium adopts a flat coincidence lattice, and the rest of the chromium resides in islands.

For thicker deposited films, this post-annealing procedure results in high quality crystalline Cr(1 1 0). These results were initially determined empirically, as judged by the quality and character of LEED patterns and ARP spectra and also using core-level spectroscopy. An example of the latter is shown in figure 3, where we plot the W 4f intensity as a function of lateral position on a film with a wedge-shaped profile. At low deposited thickness, we observe a roughly constant W 4f intensity, implying that much of the chromium is tied up in islands and thus is not visible to a surface-sensitive technique like photoemission. Above a deposited thickness of ∼1 nm, the W 4f core-level intensity falls exponentially, indicative of a nominally flat surface.

We also probed the quality of these films using high-resolution LEED experiments to measure the evolution of the lattice constant and LEEM to probe film morphology. Figures 4(a)–(c), for example, show LEED patterns of the clean, unreconstructed W(1 1 0) substrate, the Cr(1 1 0)/W(1 1 0) coincidence lattice showing diffraction beams corresponding to both the tungsten and chromium lattice constants, and the thick Cr(1 1 0) film, respectively. These were collected from different positions on the same wedge. We now understand that the coincidence lattice is associated with an inhomogeneous surface, with some areas covered by single monolayer (1 × 1)-Cr/W(1 1 0) and others covered by chromium islands with a surface lattice close to that
Figure 3. W 4f core level study of the growth of the chromium wedge on W(1 1 0). The as-deposited layer exhibits an exponential decay of the 4f core level as a function of thickness, indicating a smooth, flat film. The absence of a LEED pattern indicates that the film is essentially amorphous. After annealing, the intensity of the core level exhibits a ‘plateau region’ for \( d \sim 1.3 \) nm. At small deposited thickness, the film has broken up into islands on top of a monolayer thick film, and most of the chromium is invisible to a surface-sensitive technique. For \( d > 1.3 \) nm, an exponential dependence of the W 4f core level intensity on film thickness indicates that a flat film has been produced by this post-annealing procedure. LEED measurements in figure 4 show that this film is of high crystalline quality.

of an ideal Cr(1 1 0) truncation. We present a movie of a series of such patterns which indicates the fairly abrupt transitions between these structures along the wedge. Figure 4(d) shows an image plot of the intensity along a line connecting (0 1) and (0 1 1) LEED beams as a function of film thickness, extending from bare W(1 1 0) through the flat Cr(1 1 0) positions on the wedge. The separations of the inner and outer vertical streaks are determined by the W(1 1 0) and Cr(1 1 0) lattice constants, respectively. We fitted slices through these images to measure the evolution of the chromium thin film lattice constant with high precision and determined that this varies <0.2% over the entire thickness range of the wedge. Moreover, as is clear in panels (a)–(c), we find that the widths of the Cr(1 1 0) LEED spots are not much larger than those of the W(1 1 0) substrate, suggesting similar morphologies for these two. This suggestion is confirmed by the LEEM results shown in figure 5. Figure 5(a) shows the typical step morphology of the W(1 1 0) substrate. Some of the same structures are observed on a nominally flat Cr(1 1 0) film in figure 5(b). A film grown with this post-annealing procedure is therefore largely conformal with the tungsten substrate. We present a movie taken during chromium growth at \( T = 470 \) K.
Figure 4. LEED patterns collected as a function of position/thickness from the Cr/W(110) wedge. (a) Clean W(110) substrate; (b) coincidence lattice formed by an ultrathin film of Cr islands on a single monolayer (1×1)-Cr/W(110), and (c) thick, well-ordered Cr(110) film produced by post-annealing a film deposited at room temperature as described in the text. We present a movie here of images like these, showing the evolution from clean W(110) and (1×1)-Cr/W(110) spots, W(110)/Cr(110) coincidence spots, and the sharp spots associated with the Cr(110) film for thickness greater than ∼1.3 nm. The slight variation of the positions of the LEED spots is an experimental artifact associated with a slightly non-planar crystal surface over the 5 mm wedge. Panel (d) presents an image portraying the intensity of the (0 1) and (0 1 ̅) LEED beams as a function of thickness. The slight tilting of these streaks is caused by the experimental artifact noted above. Fitting horizontal slices through these streaks allow us to determine that the Cr(110) lattice spacing is constant to within ∼0.2%.
Figure 5. LEEM images of (a) clean W(1 1 0) and (b) the flat Cr(1 1 0) film produced by post-annealing following room-temperature-deposition, collected from the same region of the crystal. Some of the same step structures are clearly visible, indicating essentially conformal film growth. A Movie is included of LEEM images of the Cr(1 1 0) film during growth at 470 K. Step flow, homoepitaxial growth is clearly visible.

on top of the flat chromium film produced by post-annealing. We observe step-flow growth in this circumstance, similar to what would typically be observed in homoepitaxy on very good crystals.

2.2. Photoemission experiments

ARP experiments were performed at the Electronic Structure Factory on beamline 7.0.1 at the Advanced Light Source (ALS) at Lawrence Berkeley National Laboratory. Films were deposited in a wedge-shaped profile with $0 < d < 15$ nm over a 5 mm distance—much greater than the 50 $\mu$m focus of the photon beam. In this circumstance, in any given photoemission experiment the film thickness is effectively constant, although of course we average over microscopic inhomogeneities associated with steps and other defects. After preparation, the films were transferred in vacuo onto a liquid He-cooled sample goniometer with temperature control between 20 and 300 K. Various ARP datasets were then produced that measure the $(d, T)$-dependence of the electronic structure and SDW periodicity, as discussed below.

The Gammadata SES-100 electron energy analyser employed in these experiments was fixed in space in the horizontal plane defined by the incident photon beam and polarization directions at an angle of 30° to the latter. The angular flexibility of our experiments was provided primarily in the rotations facilitated with the sample goniometer. Three precision angular degrees of freedom were available, as shown in figure 6(a): the polar angle of the surface normal relative to the energy analyser ($\theta$), azimuthal angle of rotation about the normal ($\phi$) and tilt of the normal out of the horizontal plane ($\beta$). All of these angles are continuously driven with stepper-motors and are under computer control. The precision is typically 0.1°. Figure 6(b) shows a picture of the goniometer, and we present a movie of the goniometer operating in real time. In our geometry, the SES-100 analyser naturally multiplexes over a range of $\sim 8^\circ$ in $\theta$. A particularly important degree of freedom in many of our measurements is the tilt angle, $\beta$. To build up a desired dataset, we typically do many $\beta$-scans as a function of $\theta$ and possibly also as a function of the photon energy, $h\nu$. A sufficiently broad $(\beta, \theta, h\nu)$ dataset probes more than one entire bulk Brillouin zone, and full a 3D construction of the band structure results. More typically, we interpolate these datasets to produce truly planar slices through $k$-space, an example of which is shown.

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in figure 7. When electron binding energy, film thickness, and temperature are included, in principle a 6D dataset is possible.

3. Experimental results

3.1. Fermi surface nesting

The calculated bulk chromium Fermi surface is displayed in figure 8(a), along with the body-centred cubic Brillouin zone [45, 46]. The primary segments are the electron jack centred at the $\Gamma$ point, and the hole octahedron and ellipsoid centred at the H and N points, respectively. The electron jack is formed by the combination of octahedral and knob-shaped segments along the $\Gamma$–H lines, and a small lens-shaped feature results at this intersection. Cubic symmetry requires that the faces of the electron and hole octahedra have parallel tangents, and their similar curvature implies very good nesting along the $\langle 001 \rangle$ directions. This nesting, which is responsible for the SDW ground state [1, 2, 47], is illuminated in figure 8(b), which shows sections of the two relevant surfaces.

Figures 7(a) and (b) display Cr-ARP intensity maps collected at the Fermi level ($E_F$) and 260 meV below $E_F$, respectively, in the $\Gamma$–H–P–N plane of the bulk BCC Brillouin zone. The data display 2D planar slices, produced by interpolating a 3D dataset as described above, through the electron and hole octahedra and the electron knob centred on $\Gamma$–H, as in figure 8. In such an ARP experimental slice, a maximum in intensity indicates the presence of a band at that particular (energy, momentum) combination. The data in figure 7(a), therefore, provide a visual image of a slice through the chromium Fermi surface. To accumulate these data, a photon energy range from 90 to $\sim$200 eV was required, with angles set appropriately to produce the desired slice at constant perpendicular momentum. The electron and hole characters of the two pockets are readily apparent: the electron and hole segments decrease and increase in size as the binding
Figure 7. Constant binding energy slices through the chromium bands that produce the octahedral segments of Fermi surface, in the $\Gamma$–H–P–N plane of the bulk BCC Brillouin zone. These were collected at (a) and (c) the Fermi energy and (b) 0.3 eV below the Fermi energy. The flat faces in panel (a) are strongly nested and lead to the SDW ground state, with nesting vectors indicated. Note that the pockets near $\Gamma$ and H in panel (b) shrink and expand indicating electron and hole behaviours, respectively. Backfolded replicas of the hole pocket are visible in the enhanced contrast insets in panel (b). Calculated sections [49] of the Fermi surface of paramagnetic chromium are also plotted in panel (c), indicating generally a very good match between theory and experiment.

The excellent nesting between the faces of the electron and hole octahedra is clearly seen in figure 7(a), where curves representing the face of the hole octahedron have been mapped so that they overlap the electron octahedron as closely as possible. The Fermi surface topology in Cr stabilizes an IC-SDW because the associated nesting vectors, also illustrated in figure 7(a), are not commensurate with the lattice [1, 2, 47]. This topology results from the fact that the hole octahedron is slightly larger than the electron octahedron, as is clearly evident in figure 7(a). These magnitudes of the two nesting vectors are given by $Q_F \equiv 2\pi/a (1 \pm \delta_F)$, where $\delta_F$ is the incommensurability defined by the Fermi surface dimensions. Within experimental uncertainty,
Figure 8. (a) Calculated chromium Fermi surface, showing the $\Gamma$-centred electron jack, the H-centred hole octahedron, and the N-centred hole ellipsoid. A lens-shaped electron pocket is produced at the intersection of the $\Gamma$-centred electron octahedron and the $\Delta$-centred knobs which form the electron jack. (b) A section of the nested electron and hole octahedra responsible for the SDW ground state in chromium. The flat faces are well-nested with a slightly incommensurate wave vector since the hole octahedron is larger than the electron octahedron.

we determine $\delta_F = 0.05 \pm 0.005$, in agreement with the value of the SDW incommensurability measured with high-resolution x-ray scattering in bulk Cr [25]. We believe this measurement provides the most precise test of the connection between Fermi surface nesting and the chromium SDW phase. Within this stated precision, $\delta_F$ does not change measurably over the thickness and temperature ranges discussed below. Therefore, the nesting condition is not a function of film thickness or temperature.

The SDW’s actual incommensurability, $\delta_{SDW}$, can be determined using a slightly different procedure. Close examination of the constant energy contours below $E_F$, as in figure 7(b), indicates two faint replicas of the hole octahedron that are backfolded from H towards $\Gamma$. These can be similarly described by a nesting vector $Q_{SDW} \equiv 2\pi/a (1 \pm \delta_{SDW})$, in terms of the SDW incommensurability $\delta_{SDW}$. For the $(d, T)$ combination used to accumulate the data in figure 7, we find that $\delta_F = \delta_{SDW}$. As we will see below, however, $\delta_{SDW}$ can depart significantly from $\delta_F$ for thinner films.

Modern band-structure calculations generally predict the observed nesting vector with good accuracy as well [1, 48]–[50]. Such calculations have formed the basis for further calculations of the generalized spin susceptibility that also predict an instability at the observed SDW wave vector [51, 52]. A direct comparison between experimental and calculated Fermi surface orbits provides a more stringent test of theory. An example is provided in figure 7(c), where we plot the calculated section of the chromium Fermi surface [49] on top of our experimental data from figure 7(a). For the octahedral segments, we see that the match is quite good in overall size and position. The electron knob appears to be less well-reproduced by the calculation, although this region displays a rich diversity of electron quantum-well states and these might lead to confusion. A more careful analysis of this region will be published elsewhere [26].

Finally, it is interesting to note that the observed $k$-space widths of the nested Fermi surface segments in figure 7 are quite narrow. This is surprising, given that these states are intrinsically 3D and thus are subject to broadening due to the final state lifetime of the photoelectron [53]–[56].
and photohole states, it can be shown that the \( k \)-space Lorentzian width of an ARP direct transition peak is given by [57]

\[
\gamma_k = \frac{v_{\perp e} \Gamma_h + v_{\perp h} \Gamma_e}{v_{\perp e} v_{\parallel h} - v_{\parallel e} v_{\perp h}},
\]

where the band velocities (\( v \)) and inverse lifetimes (\( \Gamma \)) are indexed for the photohole (h) and photoelectron (e) states and the velocities are also broken into components parallel (\( \parallel \)) and perpendicular (\( \perp \)) to the surface plane. The geometry of our experiment was chosen to overcome these lifetime limitations partially and thereby to provide narrow widths. Firstly, the contribution of the photohole inverse lifetime (\( \Gamma_h \)) to the observed peak width is small close to the Fermi level. Ignoring coupling to disorder, phonons and other low-energy excitations, the width due to electronic damping goes to zero at \( E_F \). Moreover, the contribution of the photoelectron’s inverse lifetime (\( \Gamma_e \)) to the momentum width (as in equation 1), and also to the energy width [53]–[56], is weighted by the photohole band velocity normal to the surface (\( v_{\perp h} \)) [53]–[56]. A careful examination of figure 8(a) indicates that the nominally flat faces of the octahedral Fermi surface segments are normal to the (1 1 0) surface plane, which means that \( v_{\perp h} \) is very close to zero. These states actually act like 1D states in this limited context. This geometrical narrowing was characterized first long ago in photoemission studies of the sp-band of bulk copper [55] and was rediscovered more recently in a study of the sp-band of silver [58]. The reduced damping in low-dimensional solids and for surface states has enabled ARP to have a profound impact, for example, in studies of high-temperature superconductors. Using this geometry significantly enhances the precision of our measurements. An unfortunate associated outcome of this analysis is that we have not been able to probe the entire electron and hole octahedra with similar sensitivity. Away from the \( \Gamma–H–P–N \) plane of the BCC Brillouin zone \( v_{\perp h} \) deviates increasingly from zero and the definition of the slices of Fermi surface degrades significantly. Possibly, working at lower or (much) higher photon energy would allow this limitation to be overcome.

### 3.2. Band maps and the C–IC-phase transition

The constant energy cuts shown in figure 7 are very useful for probing Fermi surface nesting, but less so to probe gap formation and band backfolding associated with SDW formation. Band maps, i.e., slices through the band structure along a given line in \( k \)-space, are more illustrative. Figure 9 presents such ARP bandmaps, acquired along the dashed line in figure 7(a), which corresponds to the \( \Gamma–\breve{S} \) line in the surface Brillouin zone. This line intersects the face of the electron octahedron near its middle, where the nesting with the hole octahedron is excellent. Above the Néel temperature (not probed in these experiments), a single band is observed crossing the Fermi level [27]. This band is not modified away from the Fermi level by SDW formation and is therefore essentially the same as the intense feature that approaches \( E_F \) (zero binding energy) in figure 9. The locus of points where this band crosses \( E_F \) forms the electron octahedron shown in figure 7. Panels (a) and (b) show such band maps for film thickness \( d = 3 \) and 12 nm, respectively, at \( T = 50 \) K. The most intense features of these band maps do not change as a function of thickness, meaning that the underlying band structures and Fermi surfaces are not thickness-dependent. However, panel (a) exhibits a single backfolded band characteristic of a C phase (\( \delta_{SDW} = 0 \)), while panel (b) exhibits two backfolded bands characteristic of an IC phase (\( \delta_{SDW} > 0 \)). These assignments will be explained further below. Such backfolded bands generally have a low photoemission cross section and those in figure 9, while among the strongest ever...
Figure 9. ARP bandmaps acquired along the dashed line in figure 7(a), which corresponds to the $\Gamma-\bar{\Sigma}$ line in the surface Brillouin zone. Panels (a) and (b) show such band maps for film thickness $d = 3$ and 12 nm where the SDW is in the C and IC phases, respectively, at $T = 50$ K. The most intense features reflect very nearly the underlying undistorted chromium band structure and are not modified by SDW formation except close to $E_F$. The weak features that disperse away from $E_F$ are the single- and double-backfolded bands characteristic of the C and IC-phases, respectively. We present a movie of such band maps as a function of thickness. The abrupt and continuous onset of the splitting is clearly evident. Panel (c) shows a band map from a lower quality film at a thickness where coexistence of the C and IC phases is observed.

The splitting of the backfolded band in figure 9 is equivalent to the splitting of Bragg diffraction peaks commonly observed in C–IC transitions, including bulk chromium [1, 2, 25]. Figure 10 presents a simple 1D schematic that clarifies the associations drawn in figure 9 between band structure and SDW commensurability and periodicity. Reflecting the situation in Cr, we place a hole pocket at the zone boundary (labelled H) that is slightly larger than an electron pocket at zone centre (labelled $\Gamma$). Above the Néel temperature, ungapped bands cross $E_F$, as shown in figure 10(a). A period doubling C phase in this system renders the $\Gamma$ and H points equivalent and the hole band is mapped directly onto the electron band as shown in figure 10(b). The two bands mix and form an SDW band gap centred above $E_F$. A careful analysis of band maps like those in figure 9(a) reveals that the bottom of the C-phase SDW band gap is only $\sim 10$ meV below $E_F$. The location of the lower-band edge so close to $E_F$ means that SDW gap formation does not significantly stabilize the C phase.

The case of perfect nesting, shown in figure 10(c), couples the Fermi crossing of the central electron pocket with that of the neighbouring zone boundary hole pockets. Unlike the situation
Figure 10. Schematic 1D representation of the backfolding in C and IC SDW phases. (a) Paramagnetic phase with an electron pocket at zone centre (Γ) and a slightly larger hole pocket at the zone boundary (H). (b) C SDW phase, where the Γ and H points become equivalent. The two bands will mix to form a gap centred slightly above $E_F$ and a single backfolded band. (c) IC SDW phase where backfolded hole pockets on the two sides of Γ do not coincide. A more complicated band structure with two-backfolded bands results.

In the commensurate coupling, perfectly nested hole pockets from opposite sides of Γ do not precisely overlap and a more complicated band diagram with two backfolded bands results, as observed in figure 9(b). Unlike the C phase, there is a notable lowering of the total electronic energy since the lower-band edge lies well below $E_F$, measured to be $\sim 90$ meV in the present case. Aside from the appearance of band gaps and backfolding, we find that the bands that produce the flat faces of the octahedra, and therefore the nesting conditions, do not change measurably as a function of $d$ or $T$. This argument can be extended to higher-order Umklapp processes from other Brillouin zones. Indeed, for an IC structure, one expects fundamentally a countably infinite dense band diagram when all folding is considered. Photoemission tends to select only the lower-order folded bands [59]–[61]. In the present case, we measure only the lowest-order bands. We have searched unsuccessfully, for example, for bands nested from second and third Brillouin zones which would be split by two or three times as much as those observed in figure 9.

A cut through parameter space that graphically illustrates the C–IC transition is essentially a set of momentum distribution curves (MDCs) as a function of film thickness. An MDC plots photoemission intensity as a function of momentum and is beneficially used to probe $k$-space band positions and splittings. A line plot of such a slice is shown in figure 11(a) for the same film...
3.3. C–IC-phase diagram and incommensurability surface

The images in figure 11(b) demonstrate that the thickness at which the C–IC transition occurs depends on temperature—higher $T$ implies larger transition thickness. A C–IC-phase diagram can thus be extracted from these data. We fitted curves like those in figure 11(a) to two Lorentzians on a smooth background to extract the isothermal $d$-dependence of the incommensurability. Note that the measured separation of these MDC peaks needs to be corrected geometrically to determine the $k$-space splitting along the [0 0 1] direction, which is directly related to $\delta_{SDW}$. This correction is required both because the tangent to the contour is not perpendicular to the [0 0 1] axis, and also because our scan line (dashed line in figure 7(a)) does not intersect the contour’s Fermi surface in an orthogonal orientation. A sampling of the values of $\delta_{SDW}$ extracted from this analysis is shown in figure 12. As suggested in figure 11, $\delta_{SDW}$ varies smoothly and continuously with film thickness. The two peaks become too closely separated to fit with accuracy for $\delta_{SDW} < 0.01$. The
SDW wavelength, given by \( a/\delta_{SDW} \) where \( a = 0.288 \) nm is the bulk chromium lattice constant, varies from the bulk value of \( \sim 6 \) nm to nearly 30 nm—much larger than the film thickness. This observation suggests that, in these thin films, the SDW orients in-plane along the [0 0 1] direction. We return to this issue shortly.

Despite the difficulty of measuring the \( \delta_{SDW} \) as it approaches zero, the data can be fitted to a simple power law with a variable threshold. These extrapolated thresholds then correspond to C–IC transition thickness at a given temperature, i.e., to a point on the C–IC-phase line. The resulting phase diagram, \( T_c(d) \), is shown by open circles in figure 13(a) as a function of \( d \) and in figure 13(b) as a function of \( 1/d \). These phase diagrams were collected from a high quality film annealed at the lower end of the temperature range discussed in section 2.1, that is, about 800 K. We have recently observed that higher annealing temperature (\( \sim 1000 \) K) shifts the phase line to lower thickness while not otherwise measurably changing the film quality. This suggests destabilization of the C phase; we will return to this point shortly.

4. Discussion

4.1. BCS model for C–IC-phase diagram

Figure 13(b) indicates that \( T_c(d) \) varies linearly with \( 1/d \), an observation bearing similarity to thin-film superconductors. This similarity, along with the general applicability of BCS theory to charge-density-wave and SDW transitions [62], as motivation to adapt the strong coupling BCS theory to model our results in terms of a small number of parameters. We certainly acknowledge that our results would benefit from a first-principles analysis, where microscopic interactions discussed in the following sections are included at the outset. We assume that the IC-SDW energy gap \( \Delta \), as a function of \( d \) and \( T \) can be described using formulas from the BCS theory [62]. Furthermore, a simple two-band Hamiltonian (a slightly more quantitative version
of the pedagogical analysis in figure 10) suggests that the incommensurability \( \delta_{\text{SDW}}(T, d) \) is proportional to \( \Delta(T, d) \). Then the strong-coupling BCS equation for \( \Delta(T, d) \) gives \[63\]

\[
\frac{\delta_{\text{SDW}}(T, d)}{\delta_{\text{SDW}}(0, d)} = \tanh \left( \frac{T_c(d)}{T} \frac{\delta_{\text{SDW}}(T, d)}{\delta_{\text{SDW}}(0, d)} \right),
\]

where \( T_c(d) \) describes the transition temperature for the onset of incommensurability as a function of film thickness and was determined as discussed above. The other unknown function is \( \delta_{\text{SDW}}(0, d) \), which describes the SDW-IC splitting at zero temperature. Empirically, for the samples annealed at a lower temperature which give the phase diagram in figure 13, this must exhibit a threshold at some thickness \( d_o \), and it must increase smoothly above the threshold to approach the bulk value of \( \delta_{\text{SDW}} = \delta_F \) for thick films. Several functions satisfy these requirements and provide an adequate description of our results. One with two simple parameters is

\[
\delta_{\text{SDW}}(0, d) = \delta_F B_{\frac{1}{2}} \left( \frac{d - d_o}{d_s} \right),
\]

where \( B_{\frac{1}{2}} \) is the spin-\( \frac{1}{2} \) Brillouin function, \( d_o \) is the threshold thickness, and \( d_s \) is a scale thickness over which the function approaches its asymptotic value. These two equations provide a model function for the \( (d, T) \)-dependence of \( \delta_{\text{SDW}} \). This function is plotted as both a contour map and

Figure 13. C–IC-phase diagram for Cr-thin films determined by analysis of temperature- and thickness-dependent measurements of \( \delta_{\text{SDW}} \), plotted as a function of (a) \( d \) and (b) \( 1/d \). The colour contours represent the incommensurability parameter \( \delta_{\text{SDW}} \) calculated with equations (1) and (2).
colour coded in figures 12(a) and (b). The predicted incommensurability is also plotted on top of the grey-scale plot of our measurements in figure 11(c). A very good representation of our data is achieved despite the use of just two parameters—\(d_o = 23.6 \text{ Å}\) and \(d_s = 25.7 \text{ Å}\)—combined with the presumed functional dependences in equations (2) and (3).

The value of \(d_o\) quoted above can be deduced with reasonable accuracy simply from the extrapolated \(d\)-intercept in the phase diagram in figure 13: this value does not depend on any of the above modelling. \(d_s\) can be similarly estimated, although with less precision. Both numbers are clearly less than the bulk SDW wavelength, an observation that attests the novelty of our results and also suggests that the SDW is oriented in the plane along the [0 0 1] axis.

Another curious feature of our results is offered by the extrapolated \(T\)-intercept \(T_c(\infty) = 187 \text{ K}\) in the \(1/d\) plot of the phase diagram. This suggests that there is a hidden C–IC-phase transition in bulk chromium, although this has never been observed to our knowledge. In bulk chromium, the value of \(\delta_{SDW}\) evolves smoothly and over a small range as a function of \(T\). At the Néel temperature, \(\delta_{SDW}\) remains greater than 0.04. We do not understand why our results predict otherwise; perhaps the extrapolation to \(d = \infty\) in figure 13(b) is not valid.

A final interesting facet of these data is the apparent plateaux observed near \(\delta_{SDW} \sim 0.025\) in figure 13. If true, these plateaux suggests that there might be preferred periodicities, possibly of rational commensurability. This would be moderately surprising, since a high-resolution x-ray diffraction study of bulk Cr as a function of temperature saw no evidence for preferred periodicities [25]. Rather, a smooth evolution of incommensurability was observed. The range of commensurability probed in that study was necessarily small, since the commensurability varies only by several per cent between \(T = 0 \text{ K}\) and the Néel temperature. In figure 13, the commensurability varies by a much larger range and possibly a preferred periodicity has emerged. These results suggest a more complete study.

4.2. Energetics of the C–IC-phase diagram

According to figure 13, the C phase is favoured at small \(d\) and high \(T\). What determines the stability of the C phase in our Cr(1 1 0) films? While the stabilization of the IC phase at low \(T\) is induced by a gap opening, the stability is low (<1 meV per atom [64]) and is sensitive to external factors such as strain, defects and surface/interface magnetic anisotropy and electronic states. Furthermore, the quantization of the bulk bands due to confinement in films of finite thickness might alter the band structure itself, and thus alter the SDW phase diagram. Therefore, IC stabilization (a volume effect) can be easily balanced by one or more surface, interface, or thin-film effects which favour the C-phase. This conclusion is also supported by the linearization of the phase diagram in figure 13(b), since related confinement effects in traditional and high-temperature superconductors are understood in terms interfacial energetics [65, 66].

Although tensile strain can partially or completely stabilize the C phase [23, 67], strain appears not to explain our results completely, despite the 9% Cr-W lattice mismatch. Our LEED studies indicate that the strain in our films is completely relaxed (within 0.2% accuracy) in the first few layers (see figure 4(e)). It is, of course, plausible that residual strain smaller than what we can measure using LEED helps determine SDW commensurability. Furthermore, the dimensions of the Fermi surface show no appreciable change with thickness at small \(d\) that might be associated with increased strain. Strain might indirectly favour the C-phase SDW, but apparently its impact is not through direct modification of the nesting vectors and the underlying band structure. This observation also rules out a change in electron doping with film thickness, which might
stabilize the C phase by equalizing the dimensions of the electron and hole octahedra, or other strain effects, such as the alteration of the shape of the nested segments induced by hydrostatic pressure [68]. While quantum-well states exist in this system [26], as noted above, the nested sections of the Fermi surface have practically no dispersion perpendicular to the (1 1 0) direction and therefore do not support electronic quantum well states that might change the nesting condition with thickness to stabilize the C phase.

We found that defects can alter the phase diagram from that in figure 13. In analogy with the C-phase stabilization in defective bulk Cr [1], a lower-quality film favours the C phase as well, reducing $T_c(d)$ and inducing wide thermal hysteresis and coexistence between the C and IC phases (see figure 11(c)). We do not believe, however, that residual defects stabilize the C phase at low $d$, since (i) the phase line saturates as a function of sample annealing at a given temperature presumably reflecting the annealing of defects, (ii) the presence of sharp surface states and LEED beams indicates a low-defect density for our typical films, (iii) it would be difficult to understand the phase diagram based on defect energetics and (iv) the coexistence is not observed in our best films.

While we cannot rule out an impact from residual tensile strain on our observed phase diagram, it seems likely that some other energetic mechanism must also contribute. We propose that the C phase is at least partially stabilized by interfacial electronic effects, either at the free Cr surface or at the Cr/W interface. An interesting interaction to consider is the interfacial spin anisotropy. This plays an important role in the energetics of thin ferromagnetic films and can lead to the preferred magnetization direction being normal to the film plane [35]–[41], [69]–[82]. For thicker films, the volume anisotropies dominate and the magnetization normally changes to an in-plane preferred magnetization direction. This leads to a $(d, T)$ phase diagram vaguely similar to that in figure 13. In an antiferromagnetic Cr(1 1 0) film, the situation is more complicated since phases exist with different magnetic periodicities and SDW orientations and spin polarizations relative to the plane of the film. Also, the shape anisotropy, often dominant in ferromagnetic films, is irrelevant. The microscopic interactions at ferromagnetic and antiferromagnetic interfaces should be of similar magnitude, and it is certainly plausible that the spin anisotropy is of large enough magnitude to help determine the phase diagram in figure 13.

The non-uniform magnetization density in an antiferromagnet makes the impact of the interfacial spin anisotropy potentially much richer than in ferromagnetic films. As indicated in figure 14(c), the C-phase SDW produces a $c(2 \times 2)$ magnetic superstructure at a (1 1 0) interface. In this case, the spin density is of maximum magnitude at each site and flips alternately between neighbouring interface atoms. By contrast, the (1 1 0) interface continuously samples all spin densities (down to zero) in the IC phase. Given that the interfacial anisotropy does not distinguish between directly opposing spin directions, the C phase can naturally minimize the surface magnetic anisotropy energy at low $d$, while the IC phase cannot. Note that this is true regardless of the preferred interfacial spin polarization: this interaction will always favour the C-phase (1 1 0) antiferromagnetic surface. This effect is absent on the (0 0 1) surface (figure 14(a)), where, ignoring the significant impact of monatomic steps, a given surface terrace will have uniform spin density regardless of whether the SDW is C or IC. Other low index surfaces, for example the (2 1 0) surface shown in figure 14(b), will be intermediate between these extremes.

We propose this interfacial spin interaction as a plausible extension from ferromagnetic films and surfaces. It is not possible for us to estimate the precise magnitude of this interaction. Other influences of the interfacial electronic structure, such as interface states, enhanced spin–orbit
Figure 14. Spin structures for the C phase near (a) (100), (b) (210), and (c) (110).

coupling, etc [83, 84] cannot be ruled out, and we also cannot comment on the possible role of the spin-flip transition observed at $T = 123$ K in bulk Cr. Regardless of the detailed mechanism, the proposed interplay between volume and surface energies in (110) films appears to be a new and important mechanism for producing C-phase Cr, with several features that distinguish it from the C phase observed in bulk Cr and [100]-oriented films.

Why does a higher annealing temperature destabilize the C phase, thereby shifting the C–IC-phase line to lower thickness? For higher annealing temperature, the phase line actually intersects the $T$-axis rather than the $d$-axis: at low enough temperature the film remains in the IC phase for all thickness. We have only recently observed this behaviour, and while it is reproducible and apparently robust, we have not done many experiments to try to understand it. A plausible explanation is that higher annealing temperature leads to increased interdiffusion at the Cr:W interface. This could presumably weaken the stabilization of the C phase through the magnetic anisotropy effect discussed above, thereby destabilizing the C phase relative to the IC phase.

4.3. SDW orientation

The C–IC transition discussed in this paper raises interesting issues about the orientation of $Q_{\text{SDW}}$ and the spin polarization in the SDW [1, 2]. In bulk C-phase chromium, a longitudinally spin-polarized SDW with $Q_{\text{SDW}}$ along [001] is equivalent to a transversely spin-polarized SDW with $Q_{\text{SDW}}$ along either [100] or [010]. The issue of longitudinal versus transverse spin and $Q_{\text{SDW}}$ orientation is, therefore, irrelevant in the C phase. This means that there are no $Q_{\text{SDW}}$ domains in the C phase. It also rules out a spin-reorientation transition, which is observed at 123 K in the bulk IC phase, since that transition pertains to longitudinal versus transverse spin orientations and these are not distinguishable in the C phase. The C phase looks qualitatively like a local-moment antiferromagnet, and some authors choose not to call it a SDW at all. The C phase will support spin-orientation domains, since domains of commensurate spins will orient along any coordinate axis (or any arbitrary direction, for that matter), and these domains are
physically different. Differently oriented domains would, for example, have rotated susceptibility tensors. They would also present different interfacial spin anisotropies in the thin film energetics discussed above.

We have little sensitivity to spin orientation using our technique, but one might hope to obtain useful information about the SDW wave vector in the IC phase. The IC phase breaks the cubic symmetry and the orientation of both the spin and the SDW become relevant and important issues, as mentioned a few times in the previous pages. We have suggested evidence that $Q_{SDW}$ lies along the in-plane [0 0 1] crystalline axis. Most of this evidence stems from the rather long SDW wavelengths over much of the phase diagram. It seems to make good physical sense, with $\Lambda_{SDW} \gg d$, that a planar orientation should be adopted. Part of this prejudice stems from thinking of the Cr film as a ‘planar SDW waveguide’. This concept is almost certainly wrong, since the SDW is static, with no dispersion relation, group and phase velocity, etc, that are key features of the physics of waveguides. Figures 1(b) and (c) also indicate the error of this thinking. In the same way that these figures indicate no IC-SDW quantization, they also indicate that an IC-SDW will not be preferentially oriented by the presence of an interface. Stated differently, any bulk IC-SDW truncated at any surface, other than one normal to its wave vector, will produce a truncation that is IC. On Cr(1 1 0), a [0 0 1]-oriented SDW will produce a truncated surface SDW wavelength equal to that in the bulk, while a [1 0 0]- or [0 1 0]-oriented SDW will produce a surface wavelength that differs by the factor $\sqrt{2}$ from that of the bulk. Such an effect was observed in the STM measurement mentioned earlier [22]. In either case, the spin density at the surface is incommensurate, so that all surface spin densities are produced and there can be no obvious preferred SDW orientation at the surface. The energetics of the preferred SDW orientation, in all Cr films other than those of $\langle 1 0 0 \rangle$ orientation, are apparently fairly subtle.

One might hope to determine the orientation of $Q_{SDW}$ from the geometrical correction, discussed at the beginning of section 3.3, needed to get from a $k$-space splitting of peaks in an MDC to the incommensurability parameter, $\delta_{SDW}$. It turns out that the geometric transformation is not dependent on SDW orientation, at least near the flat faces of the octahedra. Simply stated, the flat faces of the electron and hole octahedra are nested with neighbouring octahedra, either along the [0 0 1], [0 1 0], or [1 0 0] directions, and the flat faces map into the same places in all cases. There are small differences near the corners of the octahedra that can, in principle, provide the desired orientation information, but our sensitivity in these regions is not high. A careful analysis of these regions and also of quantum well states [26] associated with the hole octahedron suggests that $Q_{SDW}$ does in fact align with the [0 0 1] axis, as one might have expected in the first place. Further work will be required to refine this result.

This result is contrary to that of the recent STM study of $Q_{SDW}$ near the surface of a bulk Cr(1 1 0) crystal, which found that the SDW lies preferentially along the out-of-plane [1 0 0] or [0 1 0] axes [22]. Moreover, a similar result was obtained in neutron scattering studies of thicker Fe : Cr multilayers [5, 23, 24]. If our preliminary result is correct, there must therefore be an SDW-reorientation transition at some intermediate thickness.

5. Future directions in chromium thin films

Magnetic phase diagrams of bulk materials are typically mapped with neutron scattering. That technique has also been applied to magnetic multilayers and, indeed, the existence of the chromium C phase has been detected in several such systems using neutron scattering,
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[3]–[8], [10]–[12] even in (1 1 0)-oriented thin films and multilayers [23, 24]. While our results represent an unusual application of ARP, they could probably not be obtained with neutron scattering, partly because of the very small sample volume involved in a single film. Also, the use of thickness wedges, enabled by the tightly focused soft x-ray beam employed, allows very systematic studies as a function of film thickness simply by measuring at different points on the sample. It would be extremely tedious to make the systematic studies as a function of thickness using a broad neutron beam. Finally, an obvious and important benefit of mapping such phase diagrams with ARP is that we can simultaneously probe the SDW band gap, the Fermi surface, and, in principle, the many-body couplings that are directly related to the underlying energetics [28, 85]–[90]. These combined capabilities suggest a very productive future for such studies.

What other physical phenomena might we hope to probe?

Aside from allowing a systematic study of the phase diagram, using a thickness wedge is important in the context of quantum critical points (QCPs), since thickness is a non-thermal parameter [91, 92]. Thickness is fundamentally discrete, although lateral roughness renders it essentially continuous in the context of a QCP. The phase diagrams in figure 13 suggest that there are QCPs in Cr films on W(1 1 0) where the phase line extrapolates to zero temperature. The situation is roughly analogous to dilute Cr$_{1-x}$V$_x$ alloys that are of much current interest for the same reason [13, 14, 93, 94]. Predictions concerning non-Fermi liquid or pseudogap behaviour can be checked using ARP to probe the electron states near the Fermi level and determine how these couple to low-energy spin excitations [27]–[29], [83]–[89], [95]–[105]. Even in a conventional Fermi liquid system, such couplings remain of much fundamental interest, and they also play an important role in high-speed magnetic switching and recording [106]–[117]. Recent results on iron thin films, for example, indicate that ARP can directly probe electron–magnon coupling [118]. It would be very interesting to see how such couplings change through the C–IC transition and near the QCP. Also, the change of the phase line with annealing temperature suggests that the QCP can be ‘quenched’, allowing incisive comparative studies.

It is interesting to compare our phase diagram to the spin-reorientation transition observed in some ferromagnetic films [35]–[41], [69]–[82]. The surface magnetic anisotropy dominates for thin ferromagnetic films and the preferred magnetization direction can be normal to the film plane. For thicker films, the volume anisotropies dominate and the magnetization changes to in-plane, leading to a (d, T) phase diagram. In an antiferromagnetic Cr(1 1 0) film, the situation is more complicated since phases exist with different magnetic periodicity and SDW orientations relative to the plane of the film. Also, the shape anisotropy, often dominant in ferromagnetic films, is irrelevant. Despite this complexity, we can surmise some features of the underlying energetics that lead to the observed phase diagram. The results in figure 13 represent one of the first systematic studies of the interplay between surface and volume effects in an itinerant antiferromagnetic film. The resulting phase diagrams can be more complex than in comparable thin-film ferromagnetic systems.

Finally, we note that film thickness is just one of several non-thermal parameters that might be of interest in chromium films. A wealth of interesting phenomena are exhibited, for example, by chromium alloys [2]. The possibility of observing NFL behaviour near the QCP in dilute Cr$_{1-x}$V$_x$ alloys has rejuvenated interest in this subject. In that case, the interest is in the paramagnetic–antiferromagnetic Néel transition rather than the C–IC transition. In principle, one could study these alloys simply and systematically using a composition wedge. One could also grow a 2D thickness/composition wedge to probe the interplay between confinement and band filling.

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