ABSTRACT

We investigate statistical properties of luminous red galaxies (LRGs) in a sample of X-ray-selected galaxy clusters at intermediate redshift (0.2 \( \leq \) \( z \) \( \leq \) 0.6) of mass range from \( -1 \times 10^{14} M_{\odot} \) to \( -8 \times 10^{14} M_{\odot} \). The LRGs are selected based on carefully designed color criteria, and the cluster membership is assessed via photometric redshifts. As clusters and LRGs are both viewed as promising tracer of the underlying dark matter distribution, understanding the distribution of LRGs within clusters is an important issue. Our main findings include (1) the halo occupation distribution (HOD) of LRGs inside our cluster sample is \( \langle N(M) \rangle = k(M/10^{14} h^{-1} M_{\odot})^a \), where \( a = 0.495 \pm 0.105 \) and \( k = 1.455 \pm 0.285 \) assuming a Poisson distribution for \( N(M) \). If we assume the form of \( \langle N(M) \rangle = 1 + k(M/10^{14} h^{-1} M_{\odot})^a \), where \( a = 0.580 \pm 0.130 \) and \( k = 0.975 \pm 0.240 \) assuming a Poisson distribution for \( N(M) \). (2) The HOD of LRGs \( \langle N(M) \rangle \) and the satellite distribution of LRGs \( \langle N(M) − 1 \rangle \) are both consistent with being Poisson. To be more quantitative, we find \( \text{Var}(N)/\langle N \rangle = 1.43 \pm 0.35 \) and \( \text{Var}(N − 1)/\langle N − 1 \rangle = 1.82 \pm 0.50 \). (3) The radial profile of LRGs within clusters when fitted with a Navarro–Frenk–White profile gives a concentration of \( 17.5^{+3.1}_{−4.3} \) (6.0^{+3.2}_{−1.9}) including (excluding) brightest LRGs (BLRGs). In essence, the BLRGs are more concentrated toward the center of the clusters than the other LRGs in clusters. We also discuss the implications of these observations on the evolution of massive galaxies in clusters.

Key words: cosmology; observations – galaxies: clusters: general – galaxies: formation

1. INTRODUCTION

The recent advent of large-scale galaxy surveys has revolutionized the field of observational cosmology. The enormous amount of data gathered by wide area surveys produce galaxy samples with exquisite statistical precision, which makes it possible to single out the most fundamental properties that govern the physics of galaxy formation from the medley of observables.

Equally impressive has been the progress in the theoretical understanding of the structure formation in the universe. Techniques such as direct numerical simulations and semianalytic models can now reproduce the observed properties of galaxies, such as the luminosity function (LF) and two-point correlation function, color, and mass-to-light ratios over large ranges of environments and cosmic epochs (Kauffmann et al. 1999a, 1999b; Springel et al. 2001; Cole et al. 2000; White & Rees 1978).

Yet another approach, the so-called halo model (Peacock & Smith. 2000; Cooray et al. 2002), which is phenomenological in nature, has enjoyed popularity over the recent years. An essential ingredient of this method is the halo occupation distribution (HOD), which refers to the way galaxies (or substructures of dark matter halos) “populate” dark matter halos. In general, an HOD description includes the probability distribution that a halo of mass \( M \) contains \( N \) galaxies \( P(N|M) \), and the relative distribution (both spatial and velocity) of galaxies and dark matter within halos (Berlind & Weinberg 2002).

The halo model formalism allows fast exploration of a wide range of HODs; an HOD that reproduces the observed clustering properties and LF of galaxies can be further studied to reveal the physical processes that lead to galaxy formation and to measure cosmological parameters. There are ample examples of using halo model formalism to reproduce observables in order to reveal parameters in cosmology, galaxy evolution, and formation (e.g., Abazajian et al. 2005b; White et al. 2007; Yoo et al. 2006; Zheng & Weinberg 2007; Kulkarni et al. 2007).

Despite the success in both observational and theoretical sides, there remains some unsolved problems regarding the formation of the massive, (usually) early type, galaxies. These galaxies appear “red and dead,” with the majority of the stars forming at high redshift (\( z \geq 2 \)) and evolving passively since. Within the cold dark matter (CDM) paradigm, in which massive galaxies are built by smaller galaxies via mergers in the late times, mergers between gas poor systems (“dry” mergers) seem to be a promising route to form giant galaxies. Observationally, however, the overall importance of dry mergers is still under heated debate (Bell et al. 2007; Brown et al. 2007; White et al. 2007).

Luminous red galaxies (LRGs) are massive galaxies composed mainly of old stars, with little or no ongoing star formation (Eisenstein et al. 2001). They demonstrate very consistent spectral energy distribution (SED). Their SEDs mainly consist of old star spectra, most notably for the 4000 Å break. This allows one to photometrically determine their redshifts fairly accurately (see Padmanabhan et al. 2005). With the accurate photometric redshifts of LRGs, one can probe a larger volume of the universe, thus giving better constraints on the formation of massive galaxies. By studying the HOD of the LRGs, we aim to provide a simple quantitative description of these galaxies in massive dark matter halos, which will enable direct comparison with predictions of galaxy formation models.

We present here observational constraints on the HOD of the LRGs based on a sample of 47 intermediate-redshift clusters from the ROSAT PSPC Galaxy Cluster Survey (hereafter the 400d survey; Burenin et al. 2007), with photometric data from the Sloan Digital Sky Survey (SDSS; Stoughton et al. 2002). Using X-ray properties of these clusters to define the cluster
center and estimate the cluster-binding mass, we determine the mean halo occupation number \( \langle N \rangle \) as a function of mass from \( \sim 1 \times 10^{14} M_\odot \) to \( \sim 8 \times 10^{14} M_\odot \), and investigate the LRG distribution within the clusters.

In Section 2, we briefly describe the X-ray cluster catalog that we utilize and the construction of SDSS LRG sample. In Section 3, we present our method and findings on the LRG distributions within the clusters and the mean halo occupation number. We discuss what is a good mass tracers and evolution of massive galaxies in Section 5. Possible systematics that may affect our results are discussed in Section 4.

Throughout the paper we assume that the cosmological parameters to be the Wilkinson Microwave Anisotropy Probe (WMAP) values (Spergel et al. 2007): \( \Omega_m h^2 = 0.1277 \) and the Hubble parameter \( H_0 = 73 \text{ km s}^{-1} \text{Mpc}^{-1} \) (equivalently \( h = 0.732 \)).

2. DATA
2.1. Cluster Sample

Our cluster sample is drawn from the 400 deg\(^2\) ROSAT 400d survey (Burenin et al. 2007), which is an extension of the 160 deg\(^2\) survey (Vikhlinin et al. 1998). The survey detects extended X-ray sources in archival ROSAT PSPC images down to a flux limit of \( 1.4 \times 10^{-13} \text{ erg s}^{-1} \text{cm}^{-2} \), with extensive optical spectroscopic follow-up. Out of the 266 clusters detected in the survey, 47 lie within the redshift range \( 0.2 \leq z \leq 0.6 \) and are covered by SDSS DR5. The redshift range is chosen to be consistent with the photometric cuts designed to select a homogeneous LRG sample across a wide range in cosmic epochs (see Section 2.2).

The cluster catalog from the 400d survey provides estimates of cluster center, redshift, and X-ray luminosity \( L_X \), which is used to estimate the cluster mass (see Section 3.1). Some of the basic information of the clusters in our sample is given in Table 1.

2.2. LRG Data from Sloan Digital Sky Survey

The SDSS has taken \( ugriz \) CCD images of \( 10^4 \text{ deg}^2 \) of the high-latitude sky. A dedicated 2.5 m telescope at Apache Point Observatory images the sky in five bands between 3200 Å and 11000 Å (Fukugita et al. 1996) using a drift-scanning, mosaic CCD camera (Gunn et al. 1998, 2006), detecting objects to a flux limit of \( r \sim 22.5 \text{ mag} \). The survey selects \( 10^5 \) targets for spectroscopy, most of them galaxies with \( r < 17.77 \text{ mag} \) (Gunn et al. 1998; York et al. 2000; Stoughton et al. 2002). This spectroscopic follow-up uses two digital spectrographs on the same telescope as the imaging camera. Details of the galaxy survey are described in the galaxy target selection papers (Eisenstein et al. 2001; Strauss et al. 2002); other aspects of the survey are mainly described in the Early Data Release paper (Stoughton et al. 2002). All the data processing, including astrometry (Pier et al. 2003), source identification and photometry (Lupton et al. 2001; Hogg et al. 2001; Ivezic et al. 2004), calibration (Fukugita et al. 1996; Smith et al. 2002), spectroscopic target selection (Eisenstein et al. 2001; Strauss et al. 2002; Richards et al. 2002), and spectroscopic fiber placement (Blanton et al. 2003) are done automatically via SDSS software (Tucker et al. 2006). The SDSS is well underway, and has had seven major releases (Stoughton et al. 2002; Abazajian et al. 2003, 2004, 2005a; Finkbeiner et al. 2004; Adelman-McCarthy et al. 2007).

We utilize the photometric LRGs from SDSS constructed as described in Padmanabhan et al. (2005, hereafter P05). Since LRGs are luminous, relatively uniform (in SED), and common, they have been very useful cosmological probe, probing a large volume than other galaxy tracers. They are potentially the most powerful tracer at large-scale structure. On top of this, they also have very regular SEDs and a prominent 4000 Å break, making photometric redshift estimation much easier than the other galaxies. We plot the color magnitude diagram for one of the cluster and show that the LRGs in the cluster are the bright red galaxies that follow nicely along the red sequence of the cluster.

Our selection criteria are based on the spectroscopic selection of LRGs described in Eisenstein et al. (2001), extended to fainter apparent magnitudes (P05). We select LRGs by choosing galaxies that both have colors consistent with an old stellar population, as well as absolute luminosities greater than a chosen threshold. The first criterion is simple to implement since the uniform SEDs of LRGs imply that they lie on an extremely tight locus in the space of galaxy colors; we simply select all galaxies that lie close to this locus. More specifically, we can define three (not independent) colors that describe this locus,

\[
\begin{align*}
\text{Cut I:} & \quad |c_i| < 0.25, \\
\text{Cut II:} & \quad d_i > 0.55, \\
\text{Cut III:} & \quad r - i > 0.18
\end{align*}
\]

where \( g, r, \text{ and } i \) are the SDSS model magnitudes in these bands, respectively. We now make the following color selections:

\[
\text{Cut I:} \quad |c_i| < 0.2, \\
\text{Cut II:} \quad d_i > 0.55, \quad g - r > 1.4
\]

as well as the magnitude cuts designed to give roughly constant comoving density (see P05).
Making two cuts (Cut I and Cut II) is convenient since the LRG color locus changes direction sharply as the 4000 Å break redshifts from the g to the r band; this division divides the sample into low-redshift (Cut I, z < 0.4) and high-redshift (Cut II, z > 0.4) samples. More details of these color selection criteria are thoroughly described in P05.

We do however apply slightly different cuts than those adopted in P05: we limit our samples to sky regions where

\[ E(B-V) \leq 0.08 \quad (4) \]

and data taken under seeing condition of

\[ \text{FWHM} < 2''0. \quad (5) \]

These cuts in extinction and seeing are applied simply by excluding areas at which the galaxy overdensity drops significantly. Furthermore, there are a few regions in the SDSS that have 60% more red objects and less blue objects; we decide to throw away these regions.

We also regularize our redshift distribution as described in P05. For our sample, we have 855,534 galaxies, covering
2,025,731 resolution 10 HEALpix pixels (Górski et al. 2005), each with area of 11.8 arcmin$^2$, giving 0.422 gal pixel$^{-1}$. The spatial density of the LRGs is $1.326 \times 10^{-4}$ Mpc$^{-3}$.

We then estimate the photometric redshift of these LRGs with the algorithm developed by P05. The typical uncertainty of the photo-$z$’s is $\delta_z = 0.03$ (see P05).

3. ANALYSIS

3.1. Method

We estimate the cluster virial mass $M_{200} \equiv (4\pi/3) r_{200}^3 200 \rho_c$ from the X-ray luminosity using the mass–luminosity relation given by Reiprich & Böhringer (2002):

$$\log \left[ \frac{1.46^2 L_X(0.1–2.4 \text{ keV})}{h_7^2 10^{40} \text{ erg s}^{-1}} \right] = A + \alpha \log \left( \frac{1.46 M_{200}}{h_7 M_\odot} \right),$$

where $A = -20.055$ and $\alpha = 1.652$. The radius $r_{200}$ is defined such that the enclosed mean overdensity is 200 times the critical density $\rho_c$. The corresponding angular extent is $\theta_{200}$. The mass–luminosity scaling relation provides a mass estimate accurate to <50% (Reiprich & Böhringer 2002) and a virial radius $r_{200}$ estimate accurate to 15%.

As we now have the redshifts and positions of these clusters, we locate the LRGs as described in Section 2.2 in each of these clusters. We look for LRGs that are within a cylinder of radius $\theta_{200}$ and length of $\Delta_z = 0.06$ from the cluster center in both position and redshift space (i.e., $z_{LRG} = z_c \pm 0.03$, $z_c$ is the cluster redshift). We choose $\delta_z = 0.03$ since that is the typical 1$\sigma$ error on the LRG photometric redshift (P05). More discussion on the choice of cluster radius and $\delta_z$ will be given in Section 4.

Since we are relying on the photometric redshifts of the LRGs to find out whether an LRG sits in certain cluster or not, we take into account the effects of the following mechanisms that may lead to over(or under)estimate of the number of LRGs in each cluster:

1. **LRG identification failure**: this is the rate of which an LRG (photometrically chosen) is actually a star or a quasar after we get the spectra of the object. There is an identification failure rate of ~1% (Padmanabhan et al. 2007).

2. **Interlopers**: there is a finite probability of finding LRGs inside the cluster purely by chance (i.e., interlopers). We access the expected number of interlopers in each cluster by looking at the average number of LRGs in sky (twodimensional projected) in the solid angle of radius $\theta_{200}$ of the cluster and the average probability of finding an LRG in redshift range of $z_c \pm \delta_z$ where $\delta_z = 0.03$ (as defined above). We can write down the expected number of interlopers ($\langle N_{\text{int}} \rangle$) as

$$\langle N_{\text{int}} \rangle = \bar{n} \pi \theta^2 \int_{z_c - \delta_z}^{z_c + \delta_z} P(z_p) dz_p,$$

where $P(z_p)$ is the normalized (photometric) redshift distribution of LRGs and $\bar{n}$ is the two-dimensional average LRG density.

3. **Missing galaxies due to errors in photometric redshift**: as an LRG can be scattered out of the cluster (due to photo-$z$ error), we need to account for this process by looking at the probability of an LRG having been photometrically determined to be outside of the cluster, but in fact has spectroscopic redshift that falls within the range of the cluster:

$$P(|z_p - z_c| > \delta_z, |z_s - z_c| < \delta_s) = \int_{z_{\text{max}}}^{z_{\text{min}}} \left[ E(z_s) + B(z_s) \right] dz_s,$$

$$z_{\text{min}} = \max(0.05, z_c - \delta_{z, in}),$$

$$z_{\text{max}} = \min(0.7, z_c + \delta_{z, in}),$$

$$E(z) = \int_{z_c - \delta_z - z}^{z_c - \delta_z} P(\delta, z) d\delta,$$

$$B(z) = \int_{z_c + \delta_z - z}^{z_c + \delta_z} P(\delta, z) d\delta,$$

where $P(\delta, z)$ is the probability of finding $\delta (= z_s - z_p)$ at $z_c$, given by Padmanabhan et al. (2005) and these are only characterized within the spectroscopic redshift range from $z = 0.05$ to $z = 0.7$. $\delta_{z, in}$ is the redshift range, we allow an LRG to be a cluster member when we have its spectroscopic redshift, and this is set to be 0.01.

We then calculate the corrected LRG counts in each cluster via the following:

$$\langle N_{\text{corr}} \rangle = \left( \langle N_{\text{obs}} \rangle - \langle N_{\text{int}} \rangle \right)/f(z_p, z_c, z_s),$$

$$f(z_p, z_c, z_s) = [1 - P(|z_p - z_c| > \delta_z, |z_s - z_c| < \delta_s)] \times (1 + F),$$

and $F$ is the LRG identification failure rate.

We list these corrected LRG counts in Table 1.

To convert the observed magnitudes of the LRGs into the rest-frame luminosity at $z = 0$, we follow the evolution of a simple stellar population formed in a burst at $z = 5$, with solar metallicity and Salpeter initial mass function, using the model of Bruzual & Charlot (2003). The LRGs are selected so that their present-day magnitude lies in the range $-23.5 \leq M_g \leq -21$ (roughly corresponding to $1–7 L_\odot$, where $L_\odot$ is the characteristic luminosity).

For each cluster, we visually inspect the spatial and color distributions of LRGs with respect to all objects detected by SDSS. An example is shown for cluster 142. Perhaps not surprisingly, the spatial distribution of the LRGs seems concentrated toward the cluster center (Figure 2).

A general scenario that has been painted about LRGs and clusters is that there is a massive red galaxy sitting right in the middle of the cluster. Other process may bring in other massive red galaxies, but they will sink into the center over several dynamical times Tremaine & Ostriker (1999). As we have the number and positions of LRGs inside the clusters, we can test if the scenario described above is true. We present and discuss the spatial distribution of LRGs in clusters (Section 3.2), the halo occupation number (Section 3.3), and the LRG multiplicity function (Section 3.5).

3.2. Spatial Distribution of LRGs within Clusters

We show the spatial distribution of LRGs within the clusters in Figure 3. There is a significant concentration of LRGs within 10% of $r_{200}$ of the cluster; however, there are LRGs distributed throughout the whole clusters with 28% of the LRGs lying outside 0.5$r_{200}$. Previous studies (e.g., Jones & Forman 1984; Lin & Mohr 2004) have shown that brightest galaxies tend to be red.
Figure 2. Spatial distribution of LRGs in Cluster 142. The points represent all objects detected in the SDSS photometric survey. Those satisfying Cuts I and II are shown as crosses and squares, respectively. The LRGs that have photo-z consistent with the cluster redshift ($z = 0.353$) are represented as circles. The cross denotes the centroid position of the intracluster medium. The large circle is the region encircled by the virial radius of the cluster.

Figure 3. Distribution of LRGs in the clusters. The number of LRGs in each bin are normalized by dividing the number of LRGs in each bin by the total number of LRGs in all bins. The solid (long dashed; short dashed) line denotes all LRGs in cluster (LRGs that are not the brightest; LRGs that are not CLRG).

Figure 4. Top: the distribution of BLRGs (solid) and the non-BLRGs (dashed) in the clusters. As shown above, the B-LRGs tend to lie at centers of the clusters, while those that are not B-LRGs have a shallower radial distribution. We fit the B-LRGs (N-BLRGs) to an NFW profile and find the concentration to be $105^{+316}_{-66}$ ($5.97^{+3.13}_{-1.80}$). Bottom: the distribution of most CLRGs (solid) and the noncentral LRGs (N-CLRGs) (dashed) in the clusters. The distribution of the CLRGs are very similar to the B-LRGs. We also fit the CLRGs (N-CLRGs) to the NFW profile and find the concentration to be $225^{+711}_{-143} (4.69^{+2.58}_{-1.69})$.

The question of whether the centers of intracluster gas coincide with the central LRGs (CLRGs; defined as the LRG closest to the centroid of the X-ray emitting gas) is also very important to the understanding of the formation of galaxies. We investigate the distribution of the CLRGs inside the cluster (see Figure 4). There are $\sim20\%$ of the “central” LRGs which are not central at all. This may suggest the following scenario: the cluster is not relaxed enough for the CLRG to sit at the center of the gravitational potential (which is supposedly traced by the centers of the X-ray emission). The centroiding of the clusters in X-ray is called into question, and we will address this in Section 4.

The profile of galaxies in clusters is a key ingredient to the halo model formalism. One would like to understand how statistically LRGs populate the clusters they are residing. We try to fit the NFW profile to LRG surface density of stacked clusters in our sample and find that the concentration of the surface density to be $17.5^{+4.7}_{-4.3}$ with $\chi^2 = 4.29$ and dof = 7 (dof = degrees of freedom). The fitted profile is shown in Figure 5. We also fit the NFW profile to LRG surface density of stacked clusters without the B-LRGs, and this gives a concentration of $6.0^{+3.2}_{-1.9}$ with $\chi^2 = 6.6$ and dof = 7. Both profiles have very similar concentration as the K-band galaxy profile discussed in Lin & Mohr (2007; see Figure 6). Errors in $r_{200}$ determination do not affect the fit in any significant fashion as demonstrated in the appendix of Lin & Mohr (2007).

### 3.3. Halo Occupation Number

In halo model formalism, the halo occupation number of the clusters provides us with a recipe in distributing the galaxies...
within the clusters. As the HOD assumes that the distribution of galaxies depend only on the masses of the halos, we investigate the number of LRGs in these clusters as a function of their masses. As the size of our sample is not large and the mass estimate of the clusters are accurate to 30\%–50\% only, one will have to be extra cautious in finding a fit for the average number of LRGs in the mass range of these clusters. We take the following approach, assuming first the simplest form to fit the data:

\[ N(M_t) = a \times M_t + k, \tag{12} \]

and also the commonly assumed power-law form

\[ N(M_t) = k \times M_t^\nu, \tag{13} \]

where \( M_t \) is the true value of cluster virial mass in \( 10^{14} h_{73}^{-1} M_\odot \). We assume a Poisson distribution for \( N(M_t) \) and two distributions for the probability finding \( M_t \), given \( M_t \), where \( M_t \) is the inferred mass of the \( i \)th cluster (in same units as in \( M_t \)): log-normal and Gaussian. The distribution is reported to be log-normal in (Burein et al. 2007) and we test how important it is to use the correct distribution by assuming a Gaussian distribution too.

In short, we have the following:

\[
\mathcal{L}_\text{tot} = \prod_i \int P(N_i, M_{t,i}|a, k) P(M_{t,i}|M_t)dM_{t,i}
\]

\[
\log P(N, M|a, k) = N \times \log(v) + \text{const} - v
\]

\[
\log P_g(M_t|M_i) = -(M_t - M_i)^2/(2\sigma_t^2) + \text{const [Gaussian]}
\]

\[
\log P_n(M_t|M_i) = -(\mu_t - \mu_i)^2/(2\sigma_t^2) + \text{const [log-normal]},
\tag{14}\]

where \( v = a \times M + k \) or \( v = k \times M^n \). \( M_{t,i} \) stands for the \( M_t \) for the \( i \)th cluster and \( \mu_t \) stands for \( \log_{10}(M_t) \). We test a variety of ranges for both \( a \) and \( k \) (such as \( 0 < a < 20 \), \( -10 < a < 10 \), and \( 0 < k < 3 \), and \( 0 < k < 20 \), \( -20 < k < 20 \), and \( 0 < k < 5 \)), and make sure answers converge. We maximize the total likelihood within a grid of resolution 100, 1000, 10,000 for both \( a \) and \( k \) and we also vary the size of \( dM_{t,i} \) to ensure that our results are robust with respect to varying grid size. The linear fit with \( P_g(M_t|M_i) \) gives \( a = 0.330 \pm 0.180 \) and \( k = 1.530 \pm 0.550 \) for 68.3\% confidence intervals. The power-law fit with \( P_g(M_t|M_i) \) gives \( a = 0.385 \pm 0.225 \) and \( k = 1.695 \pm 0.450 \) for 68.3\% confidence intervals. The data and the fit using \( P_n(M_t|M_i) \) are shown in Figure 7.

3.4. \( N(M) \) Distribution: Poisson or Not?

Since we assume a Poisson distribution for \( N(M_t) \) (number of LRGs given the true measure of cluster mass, hereafter \( N \) for simplicity in this section), we test if this is a good assumption.
by looking at \( \gamma_N \equiv ((N^2) - \langle N^2 \rangle)/\langle N \rangle \), \( \gamma \) would be 1 if the distribution is completely Poisson. Since, we only have \( N(M_i) \) (number of LRGs given the inferred cluster mass) but not \( N_i \) we have to consider the contribution of scatter from the various systematic effects we mentioned in Section 3.1. First consider

\[
N_i = N_{\text{int}} + \text{Bin}(N, f)
\]  

where \( N_i \) is the measured number of LRG in the cluster, \( N \) is the true measure of the number of LRG in the cluster, and \( N_{\text{int}} \) is the number of interloper as discussed in Section 3.1. Bin\((N, f)\) is the combination of the true distribution of number of LRGs \( (N) \) in each cluster with the \( f = f(z_p, z_c, z_s) \) (as defined in Section 3.1) which we model as a binomial distribution. The event of finding \( N \) LRGs in one cluster is independent of finding \( N \) LRGs in another cluster, and the errors of finding \( N \) LRGs in each cluster is independent of each other too. We therefore consider the true distribution of the number of LRGs in each cluster to be a binomial distribution, hence Bin\((N, f)\).

We then take the following estimate of \( N^2 \):

\[
\langle N_i^2 \rangle_{(M_i)} = \langle N_{\text{int}}^2 \rangle + \langle \text{Bin}(N, f)^2 \rangle + D,
\]  

where \( D = 2\langle N_{\text{int}} \rangle \langle \text{Bin}(N, f) \rangle \). We then simplify the equation by assuming Poisson distribution for the interlopers and also expanding \( \langle \text{Bin}(N, f)^2 \rangle \):

\[
\langle N_i^2 \rangle_{(M_i)} = \langle N_{\text{int}}^2 \rangle + \langle N_{\text{int}} \rangle + \langle N^2 f^2 + N f(1 - f) \rangle + D.
\]  

After some more algebraic manipulation and expanding \( \langle N^2 f^2 + N f(1 - f) \rangle \) using \( \gamma \), we get the following:

\[
\langle N_i^2 \rangle_{(M_i)} = \langle N_{\text{int}}^2 \rangle + \langle N_{\text{int}} \rangle + Y + Z + W
\]

\[
Y = 2 f \langle N_{\text{int}} \rangle \langle N \rangle_{(M_i)}
\]

\[
Z = \int dM \int P(M_i|M)\langle N \rangle^2_{(M_i)} + V
\]

\[
V = \gamma \langle N \rangle_{(M_i)}
\]

\[
W = f(1 - f) \langle N \rangle_{(M_i)}
\]

where \( X_{(M_i)} (X_{(M)}) \) means the quantity \( X \) given \( M_i (M) \).

Subtracting \( \langle N(M_i)^2 \rangle_{(M_i)} \) from the equation will reduce to

\[
\langle N_i^2 \rangle_{(M_i)} - \langle N(M_i)^2 \rangle_{(M_i)} = P Q + R + S + T
\]

\[
P = f \gamma + f(1 - f)
\]

\[
Q = \langle N \rangle_{(M_i)}
\]

\[
R = \langle N_{\text{int}} \rangle
\]

\[
S = f^2 \int dM_i P(M_i|M) \langle N_i^2 \rangle_{(M_i)}
\]

\[
T = - f^2 \langle N \rangle^2_{(M_i)}
\]

Note that \( \langle N \rangle_{(M_i)} = \int dM_i P(M_i|M) \langle N \rangle_{(M_i)} \).

We calculate \( \gamma \) from the combined sample of 400d and \( Y_X \) (refer to Section 4 for a description of \( Y_X \) sample) sample using \( \langle N \rangle \) from the fit of \( N(M) = k(M/10^{14} f)^\alpha \). We bin the cluster such that there are equal number of clusters in each mass bin (see Figure 8). We find that \( \gamma = 1.428 \pm 0.351 \) and thus the \( N(M_i) \) distribution is consistent with being Poisson.

Furthermore, one important ingredient of HOD is the assumption of Poisson distribution of the satellite galaxies (Zheng et al. 2005). We test the assumption here by computing

\[
\frac{(N - 1)^2}{N}
\]

for the combined sample of 400d + \( Y_X \). It is consistent with being Poisson. However, one should note that it is mathematically impossible for both \( N \) and \( N - 1 \) to be both exactly Poisson for the same distribution.

### 3.5. LRG Multiplicity Function

Finally, we study the multiplicity function of LRGs in clusters (Figure 10). The multiplicity function of LRGs is defined as the density of cluster as a function of number of LRGs.

We calculate the multiplicity function by counting the \( 1/V_{\text{max}} \) weighted number of cluster in each bin LRG number. We compute \( V_{\text{max}} \) (the comoving search volume of the cluster) by the following procedure:
1. We find the flux of the cluster following Burenin et al. (2007):

\[ f = \frac{L}{4\pi d_L(z)^2} K(z), \]  

where \( L \) is the luminosity of the cluster, \( d_L(z) \) is the cosmological distance, and \( K(z) \) is the \( k \)-correction factor for X-ray clusters (for more details see Burenin et al. 2007).

2. We find the comoving search volume that each cluster with luminosity \( L \) can be detected via

\[ V_{\text{max}}(L) = \int_{z=0}^{z_c} P_{\text{sel}}(f, z) \frac{dV}{dz} dz, \]

where \( P_{\text{sel}}(f, z) \) is the selection efficiency of the 400d survey kindly provided by A. Vikhlinin and R. Burenin and \( dV/dz \) is the cosmological comoving volume per redshift interval (see Burenin et al. 2007 for more details).

As 400d survey is a flux-limited survey, we impose an X-ray luminosity cut so that we are not losing the low-luminosity clusters when we calculate the volume-weighted multiplicity function. This offers an additional check for HOD as one would need to reproduce the multiplicity function in their simulations.

4. SYSTEMATICS

4.1. Uncertainties in the Choice of Cluster Radius

We choose to use \( \theta_{200} \) since it is closest to the virial radius of the clusters (Evrard et al. 1996). We also look at how the uncertainties of \( r_{200} \) affect our results. \( r_{200} \) is accurate up to \( \sim 10\% \) (Reiprich & B"ohringer 2002). We calculate the following to determine the effective number of LRGs, we would miss due to uncertainties in \( r_{200} \):

\[ \int_0^{r_{200}} \rho(r) dr / \int_0^{1.1r_{200}} \rho(r) 4\pi r^2 dr = 0.95, \]  

and

\[ \int_0^{r_{200}} \rho(r) dr / \int_0^{0.9r_{200}} \rho(r) 4\pi r^2 dr = 1.06. \]

We set the density profile \( \rho(r) \) as an NFW profile with concentration of 8 (which is approximately what we get when we fit the surface density of the cluster when we exclude the BLRG). This shows that the uncertainties in \( \theta_{200} \), and thus \( r_{200} \), only affect our estimation of \( N(M) \) at the level of \( \sim 5\% \).

4.2. Mass Estimation and Sample Selection

Cluster mass estimation is crucial in our analysis, as it defines the cluster virial region to search for member LRGs, and provides a fundamental radius to scale the distance of LRGs to cluster center. We infer cluster mass through the X-ray luminosity–mass scaling relation (Reiprich & B"ohringer 2002), which has been shown as an unbiased estimator (Reiprich 2006). Compared with other X-ray-based cluster proxies such as temperature and \( Y_X \) (the product of gas mass and temperature, which is proportional to the thermal energy of the cluster; Kravtsov et al. 2006), \( L_X - M \) correlation shows higher degree of scatter. We therefore seek for another cluster sample with better measured mass (despite without well-defined selection criteria).

Recently, Maughan et al. (2006) have presented a large cluster sample selected from the Chandra archive, for which the cluster mass is inferred from \( Y_X \) and the cluster center is inferred from the Chandra images. Twenty-six of these clusters lie within our 400d survey clusters cover a larger range in mass than clusters of other redshifts. It is also clear that the halo occupation numbers of clusters of lower masses in Figure 7 would be biased to those at \( z \lesssim 0.3 \). In Figure 11, we also show the distribution of the \( Y_X \) sample. Very curiously, the distribution of this sample on the mass–redshift space seems to be roughly orthogonal to that of our 400d sample. Since our results derived from SDSS DR5 masks and the redshift range 0.2 \( \leq z \leq 0.9 \). Sixteen of these 26 clusters do not overlap with our 400d sample and we use them to examine and confirm the results presented in Sections 3.2 and 3.3 (hereafter the \( Y_X \) sample).

Because of the flux-limited nature of the 400d survey, low-mass \( \sim 10^{14} M_\odot \) clusters will be only detected at lower redshifts. In Figure 11, we show the mass distribution of the whole 400d sample (open squares) and the subsample used in our analysis (solid points) within 0.2 \( \leq z \leq 0.9 \). It shows that our sample is a random subsample of the whole 400d sample. Interestingly, at \( z \sim 0.3 \) the 400d survey clusters cover a larger range in mass than at other redshifts. It is also clear that the halo occupation numbers of clusters of lower masses in Figure 7 would be biased to those at \( z \lesssim 0.3 \). In Figure 11, we also show the distribution of the \( Y_X \) sample. Very curiously, the distribution of this sample on the mass–redshift space seems to be roughly orthogonal to that of our 400d sample. Since our results derived...
from the $Y_X$ sample is consistent with those based on the 400d sample, we combine the two samples to expand the mass coverage (especially for clusters at $z \geq 0.4$) and the statistical signal. We calculated the $N(M)$ for the combined sample and assuming power-law model, we have $N(M) = k(M/10^{14})^a$, where $a = 0.620 \pm 0.105$ and $k = 1.425 \pm 0.285$ (see Figure 12). Results are recorded at Table 2.

5. DISCUSSION AND SUMMARY

5.1. What is a Good Mass Tracer?

With the ongoing experiments such as Red Cluster Sequence, Actacama Cosmology Telescope, and South Pole Telescope, clusters are becoming more important as cosmological tools (Gladders et al. 2007; Fowler et al. 2007; Ruhl et al. 2004). An important ingredient of extracting cosmological parameters from cluster statistics is the mass estimate of the clusters.

“What is a good mass tracer?” has been a very well-motivated question. We try to investigate a few options which have been suggested before as possible solutions. First, as we see earlier in Section 3.3, the mean number of LRGs does not trace the masses accurately.

We quantify this by looking at the scatter of the $N_{\text{LRG}}-M$ relation by the following quantities in a three mass bins:

$$\sigma(\ln(N_{\text{LRG}})) = \frac{\sqrt{\sigma^2}}{\sqrt{N_{\text{LRG}}}}$$

$$\sigma(\ln(M)) = \frac{1}{a} \sigma(\ln(N_{\text{LRG}})),$$

where $a$ is as defined in $N(M) = k \times (M/10^{14})^a$. We found that the scatter in $\ln(N_{\text{LRG}})$ ($\ln(M)$) in low, middle, and high mass bins are $0.332 \pm 0.535$ dex (at $M = 2.22 \times 10^{14}$ $h^{-1} M_\odot$), $0.281 \pm 0.452$ dex (at $M = 3.81 \times 10^{14}$ $h^{-1} M_\odot$), and $0.21 \pm 0.340$ dex (at $M = 9.56 \times 10^{14}$ $h^{-1} M_\odot$), respectively. According to Reiprich & Böhringer (2002), the scatter in $\ln(M)$ is $0.21$ dex, which is lower than any of the scatters in $\ln(M(N_{\text{LRG}}))$. Therefore, we conclude that the X-ray luminosity is a better mass tracer than the number of LRGs.

Second, we look at the luminosities of the CLRG. As previous studies suggested, the brightest cluster galaxies traces the mass of the cluster (Lin & Mohr 2004) and that the brightest cluster galaxies tend to be the central galaxies of the cluster, we look at the relation between the luminosities of the CLRG in clusters and their X-ray masses. However, the correlation in our sample does not look promising (see Figure 13).

We then examine the correlation between the luminosities of the BLRG and their cluster X-ray masses. However, it does not seem to be promising either (see Figure 13). Lin & Mohr (2004) agree with our observation when considering the same
mass range and when one looks at the correlation between the BLRGs and the richness of the maxBCG catalog (Koester et al. 2007), there is not a strong correlation for 14,000 optically selected clusters (R. Reyes 2007, private communication). However, there are several caveats that would require further investigations, such as the possibility of photo-z failure for the CLRG or BLRG in the clusters and possible photometric problem that could destroy the correlation. We look into the available spectroscopic data in the SDSS and found no extra LRGs that are targeted by the SDSS spectroscopy. This rules out the possible missing LRGs that have $M_z$ of range $\sim 20.8$ (at $z = 0.2$) and $\sim 22.5$ (at $z = 0.6$). Furthermore, as we investigate earlier, only four clusters do not have LRGs, and we find $\sim 70\%$ of BLRGs lie in the central $\sim 20\%$ of the virial radius, and therefore, most clusters do have an LRG at their centers. If we are missing BLRGs in centers of clusters, we need to expect the scenario of having more than one LRG at the central $\sim 20\%$ of cluster virial radius to be prevalent. This scenario is not supported by the distribution of LRGs as shown in Figure 4. Given the caveats and findings here, we conclude that further work will be needed to make this more quantitative, especially to quantify the effect of photometry errors on the correlation.

5.2. Evolution of Massive Galaxies

What do our results imply for the evolution of massive galaxies in clusters?

We first examine the spatial distribution of massive galaxies within clusters. Lin & Mohr (2007), with a large sample of clusters at $z < 0.2$, find that luminous cluster galaxies ($M_K \leq -25$) follow an NFW profile with concentration of 18.2 (5.8), when the brightest cluster galaxy is included (excluded). This result is in very good agreement with our finding in Section 3.2.

The second comparison is made with the halo occupation number. We construct the occupation number for $M_K \leq -25.6$ with the $z < 0.1$ cluster sample presented in Lin et al. (2004). The magnitude limit is chosen so that the number density of the $K$-band-selected luminous galaxies agrees with that of the LRGs (based on LF from Kochanek et al. 2001). For our luminous magnitude range the contamination of blue galaxies should be minimal, thus no color selection analogous to that presented in Section 2.2 is used. Nevertheless, the mean occupation number $\langle N \rangle$ is an upper limit. We derive $\langle N \rangle$ of the local sample using similar method as discussed in Section 3.3. We find $\langle N(M) \rangle = k(M/10^{14} h^{-1} M_\odot)^a$ where $a = 0.4 \pm 0.1$ and $k = 1.32 \pm 0.165$ when assuming a Poisson distribution. The normalization of the $\langle N \rangle - M$ relation is quite similar to that we derive in Section 3.3, but the slope is consistent within 1σ.

Taken at face value, these comparisons suggest that there is not much evolution in the massive cluster galaxy populations between $z \sim 0.5$ and $z \sim 0$. The occupation number comparison implies that the shape of the LF is similar in clusters at these two epochs, after the passive evolution has been taken into account.

In the LCDM model, formation of massive objects through mergers of less massive ones is a generic feature. Indeed, evidence for mergers that produce massive galaxies has been found (Tran et al. 2005; Rines et al. 2007). However, the frequency and importance of the mergers in shaping the present-day LF is still under debate (van Dokkum 2005; Bell et al. 2006; Brown et al. 2007; Wake et al. 2006). Their spatial distribution seems to be similar out to $z \sim 0.5$. How can we reconcile the lack of evolution implied by our data with the merger hypothesis? The Gao et al. (2004) “attractor” hypothesis offers one solution: self-similar evolution of the spatial distribution of LRG. This would be seen more clearly through (Monte Carlo) simulations where the merger history of the halos is fully followed. In a companion paper, we offer another solution: we simulate the merger history of halos and compare the simulations to the observations reported here (Conroy et al. 2007).

5.3. Summary

We investigate statistical properties of LRGs in a sample of X-ray-selected galaxy clusters at intermediate redshift (0.2 $\leq z \leq$ 0.6). The LRGs are selected based on carefully designed color criteria, and the cluster membership is assessed via photometric redshift. We put constraints on spatial distributions of LRG within clusters. We find that the distribution of BLRGs in cluster to be concentrated as discussed in previous studies (Jones & Forman 1984; Lin & Mohr 2004). We also find that the radial distribution can be fitted by an NFW profile with a concentration of 17.5$^{+2.1}_{-4.3}$ when we include the B LRG. When we do not include the B LRG, we find concentration of 6.0$^{+3.2}_{-1.8}$. Considering the sample size and mass errors on our sample, we use the maximum likelihood method to find the best-fit parameters for HOD ($N(M)$). The result depends slightly on what kind of models we adopt, and they are shown in Table 2.

Uncertainties in photometric redshifts are taken into account by including different possible effects such as interlopers and missing LRGs due to errors in photometric redshifts (see Section 3). We estimate that the errors in cluster radius can only contribute to our uncertainty in $N(M)$ at the level of $\sim 5\%$. Errors in mass estimation are fully taken into account throughout the analysis. We also employ an independent sample of better measured masses ($Y_X$ sample) to test the mass estimation of our sample. However, we do implicitly assume that the scatter of $M - L_X$ relation does not correlate with $N(M)$ during the analysis. The result we derive from a combined analysis of both sample on $N(M)$ is consistent with using our sample alone (see Table 2). We also find that there are no obvious good mass tracer as we look at different correlations between various quantities of clusters and their galaxies. Last, we discuss the evolution of massive galaxies from different perspectives. We conclude that it would be important to study low-$z$ LRG population to better constrain the evolution of the population.

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