A systematic analysis of transverse momentum spectra of $J/\psi$ mesons in high energy collisions

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Abstract: We aggregate the transverse momentum spectra of $J/\psi$ mesons produced in high energy gold-gold (Au-Au), deuteron-gold ($d$-Au), lead-lead (Pb-Pb), proton-lead ($p$-Pb), and proton-(anti)proton ($p$-$p(\bar{p})$) collisions measured by several collaborations at the Relativistic Heavy Ion collider (RHIC), the Tevatron Proton-Antiproton Collider, and the Large Hadron Collider (LHC). The collision energy (the center-of-mass energy) gets involved in a large range from dozens of GeV to 13 TeV (the top LHC energy). We consider two participant or contributor partons, a charm quark and an anti-charm quark, in the production of $J/\psi$. The probability density of each quark is described by means of the modified Tsallis-Pareto-type function (the TP-like function) while considering that both quarks make suitable contributions to the $J/\psi$ transverse momentum spectrum. Therefore, the convolution of two TP-like functions is applied to represent the $J/\psi$ spectrum. We adopt the mentioned convolution function to fit the experimental data and find out the trends of the power exponent, effective temperature, and of the revised index with changing the centrality, rapidity, and collision energy. Beyond that, we capture the characteristic of $J/\psi$ spectrum, which is of great significance to better understand the production mechanism of $J/\psi$ in high energy collisions.

Keywords: Transverse momentum spectrum, $J/\psi$ meson, TP-like function, convolution

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I. INTRODUCTION

Quantum chromodynamics (QCD) is the standard dynamics theory and an important part of the standard model, which is applicable in the study of heavy quark pair production and correlation $^{[1]}$ such as the transverse momentum spectra, nuclear modification factor, azimuthal correlation, anisotropic flow, and so on. QCD is a kind of non-Abelian gauge field theory $^{[2]}$, which implies that the strong interactions between quarks have three basic characteristics. Firstly, it can explain the asymptotic freedom characteristics proposed in the inelastic electron-proton and electron-deuteron scattering $^{[3]}$. Secondly, it can explain the color confinement which shows quarks and anti-quarks cannot be separated due to very strong interactions. Lastly, it can explain the spontaneous break of the symmetry of the chirality. Understanding these characteristics is of great necessity for researchers to study the interactions among particles and their mechanisms of evolution, structure, and decay $^{[9-13]}$.

The heavy flavor quarkonium is a bound state formed by the heavy flavor quark and anti-quark $^{[14]}$. It plays an important role in the theoretical research of QCD. In hadron induced high energy collisions, the generation of heavy quarkonium can be divided into two processes: One is the appearance of heavy quark pairs and the other one is the evolution of heavy quark pairs into hadrons. The former process can be calculated and analyzed by the perturbative QCD theory $^{[15-17]}$. Particularly, due to the peculiar reason of the heavy quarkonium, in which the relativistic effect can be neglected, some special theories such as the non-relativistic QCD theory $^{[18-20]}$ can be used to calculate and analyze the production process.

As the basic theory of strong interactions of parti-
QCD predicts that the hadronic matter can be heated to a very high temperature when it experiences very strong interactions. Then, the system will go through a phase transition from the hadron matter to quark-gluon plasma (QGP) in the process. The experiment of relativistic heavy ion collisions is the only way of achieving a QCD phase transition in laboratory conditions. Nevertheless, the lifetime of the produced QGP experiencing this phase transition can only reach the order of 10 fm/c (from a few to dozens of fm/c), which cannot be directly observed in experiments. To detect QGP and study its properties, one has to use an indirect method. For example, one may study the spectrum properties of heavy quarkoniums to obtain the excitation degree (temperature) of emission source which is related to the information on QGP.

$J/\psi$ meson is the bound state of charm and anti-charm ($c\bar{c}$) quarks, where the constituent mass of charm quark is about 1.6 GeV/$c^2$. As the first heavy quarkonium discovered experimentally, it has been extensively studied in high energy collisions. In addition, the constraint of $J/\psi$ is considered as an important signal for the generation of QGP. The yield of $J/\psi$ in electron-positron collisions is higher than that in nuclear collisions, so the decay of $J/\psi$ in nuclear collisions is an ideal way and medium for studying the hadron spectrum and finding new particles. $\Upsilon$ meson is a bound state of bottom and anti-bottom ($b\bar{b}$) quarks, where the constituent mass of bottom quark is about 4.6 GeV/$c^2$. The masses of both $J/\psi$ and $\Upsilon$ are very large, which leads to the change scale of energy (momentum) in the collision process to the order of GeV ($c^2$) when we study their structural properties. In addition, $c + \bar{c}$ and $b + \bar{b}$ can form new $J/\psi$ and $\Upsilon$ respectively, which can be researched by non-relativistic approach.

We note that the theories and models based on QCD and related idea are complex in the calculation process. The complex calculation limits the applications of these theories and models in comparison with experimental data. We hope that we could use a simple idea and formalism to describe uniformly the spectra of various particles, in particular the spectra of heavy quarkoniums such as $J/\psi$ due to its abundant early production in the collisions and wide transverse momentum distribution. In the framework of the multi-source thermal model, we have used the idea of quark composition describing tentatively and uniformly the spectra of various particles in our recent work. It is interesting for us to test further the idea systematically, but the idea of two contributor quarks or partons is used due to the fact that some particles such as leptons have no quark composition. Of course, the quark composition and two contributor quarks for heavy quarkoniums are the same.

In this paper, we test systematically the idea of two contributor partons which contribute to the transverse momentum spectrum of $J/\psi$. The experimental data are collected from gold-gold (Au-Au), deuteron-gold ($d$-Au), lead-lead (Pb-Pb), proton-lead (p-Pb), and proton-(anti)proton (p-($\bar{p}$)) collisions over an energy range from dozens of GeV at the Relativistic Heavy Ion Collider (RHIC) to 13 TeV at the LHC. Among the RHIC and LHC, there is the Tevatron Proton-Antiproton Collider from which we cited the data in $p\bar{p}$ collisions at 1.96 TeV. These studies are useful for us to understand one of the three basic characteristics of QCD, the color confinement due to the very strong interactions among quarks and anti-quarks.

II. FORMALISM AND METHOD

According to the multi-source thermal model, we may think that a few emission sources are formed in high energy collisions. For nuclear fragments from the projectile and target in nucleus-nucleus collisions, the sources can be nucleons and nucleon clusters. For produced particles such as pions, kaons, and $J/\psi$, the sources can be participant or contributor quarks or gluons, though the contributors $c + \bar{c}$ may be from gluon fusion at the first. The properties of sources can be described by different statistics such as the Boltzmann-Gibbs, Fermi-Dirac, Bose-Einstein, and Tsallis statistics. There are some relations among these statistics due to the fact that they may result in similar or different distributions while describing the spectra of particles.

The Tsallis distribution describes the transverse momentum ($p_T$) spectra in wider range than the Boltzmann-Gibbs distribution, though the former is derived from the latter. Also, the latter is a special case of the former in which the entropy index $q = 1$. Indeed, the former is widely used in high energy collisions from a few GeV to 13 TeV (the top LHC energy) to parameterize the $p_T$ spectrum of final-state particles, which justifies its usage in the present work. The form of the Tsallis distribution is expressed as

$$E \frac{d^3N}{dp} = \frac{1}{2\pi p_T} \frac{d^2N}{dp_T dy} \frac{d^3N}{dy} = \frac{dN}{dy} \frac{(n-1)(n-2)}{2nT[nT + m_0(n-2)]} \left(1 + \frac{m_T - m_0}{nT}\right)^{-n}.$$  

(1)
Here, $E$, $p$, $N$, $y$, $m_0$, $m_T$, $n$, and $T$ denote the energy, momentum, particle number, rapidity, rest mass, transverse mass, power exponent, and effective temperature, respectively. The transverse mass is given by $m_T = \sqrt{p_T^2 + m_0^2}$. In particular, $n = 1/(q - 1)$, and the entropy index $q$ describes the degree of equilibrium. The closer the parameter $q$ to 1, the more equilibrium.

According to the form of Tsallis distribution Eq. (1), as $p_T \gg m_0$, $m_0$ can be ignored, followed by $Ed^3N/dp^3 \propto p_T^n$. Then it can be observed that the particles are distributed in accordance with the inverse power law. This is the distribution type of particles produced by the hard scattering process in the high energy collision process and in high $p_T$ region. In the non-relativistic limit ($p_T \ll m_0$) condition, there is $m_T = m_0 = p_T^2/2m_0 = E_T^{\text{classical}}$, showing $Ed^3N/dp^3 \propto e^{-E_T^{\text{classical}}}/T$, where $E_T^{\text{classical}}$ is the transverse energy in the non-relativistic limit. We call this distribution the thermodynamic statistical distribution, that is the Boltzmann distribution. Here, we have only discussed the two special cases ($p_T \gg m_0$ and $p_T \ll m_0$), though they are not used by us in the present work.

Usually, the empirical formula, the Tsallis–Pareto-type function, is adopted to outline the $p_T$ spectrum. The general form of the mentioned function is

$$f(p_T) = C \times p_T^{n} \left(1 + \frac{m_T - m_0}{nT}\right)^{-n} \quad (2)$$

which is equivalent to Eq. (1) in the form of probability density function, where $C$ is the parameter dependent normalization constant. As the probability density function, Eq. (2) is normalized as $\int_0^\infty f(p_T)dp_T = 1$. In Eq. (2), $n$ and $T$ reflect the degrees of non-equilibrium and excitation of the source respectively. Larger $n$ corresponds to more equilibrium, and larger $T$ corresponds to higher excitation.

In the lower $p_T$ range, due to the contribution of light flavor resonance decay, Eq. (2) cannot describe the spectra of light particles very well. For $J/\psi$, the feed-down contribution is more complicated, which is minimal at low $p_T$ and grows with growing $p_T$. This renders that Eq. (2) also fails to describe the $J/\psi$ spectra. As a result, we ought to empirically add a revised index $a_0$ on $p_T$ to modify Eq. (2). Then the revised Eq. (2) becomes

$$f(p_T) = C \times p_T^{a_0} \times \left(1 + \frac{m_T - m_0}{nT}\right)^{-n} \quad (3)$$

Both the normalization constants $C$ in Eqs. (2) and (3) are different, though we have used the same symbol. The two constants are also the parameter dependent. Compared to Eq. (2), Eq. (3) can be used to describe the spectrum in the entire transverse momentum range, having a broader application. For purpose of convenience, as in refs. [40, 41, 87], we also call Eq. (3) the TP-like function in this work. In the TP-like function, the meanings of $n$ and $T$ remain unchanged as what they are in Eq. (2), though their values may be changed.

The discovery of $J/\psi$ provides a direct evidence for the existence of charm quarks, which makes the study of hadron structure theory presenting a new situation. We may think that in the formation of $J/\psi$ there are two participant or contributor (anti-)charm quarks taking part in the collisions. Let $p_{t1}$ and $p_{t2}$ denote the contributions of quarks 1 and 2 to the transverse momentum of $J/\psi$ respectively. The probability density function $f_1(p_{t1})$ ($f_2(p_{t2})$) obeyed by $p_{t1}$ ($p_{t2}$) is assumed to be Eq. (3). We have

$$f_1(p_{t1}) = C_1 p_{t1}^{a_0} \left(1 + \frac{\sqrt{p_{t1}^2 + m_1^2} - m_1}{nT}\right)^{-n} \quad (4)$$

$$f_2(p_{t2}) = C_2 p_{t2}^{a_0} \left(1 + \frac{\sqrt{p_{t2}^2 + m_2^2} - m_2}{nT}\right)^{-n} \quad (5)$$

Here, the two normalization constants $C_1$ and $C_2$ are parameter dependent. $m_1$ and $m_2$ are the constituent masses of quarks 1 and 2 respectively, both are 1.6 GeV/$c^2$ for charm and anti-charm quarks [14] used in this work. Because of the two quarks taking part in the same collisions, the parameters $n$, $T$, and $a_0$ in Eqs. (4) and (5) are separately the same.

The transverse momentum distribution of $J/\psi$ is given by the convolution of two TP-like functions [40, 41, 87]. We have

$$f(p_T) = \int_0^{p_T} f_1(p_{t1}) f_2(p_T - p_{t1}) dp_{t1} = \int_0^{p_T} f_2(p_{t2}) f_1(p_T - p_{t2}) dp_{t2}, \quad (6)$$

where the functions $f_1(x)$ and $f_2(x)$, as well as the parameters $n$, $T$, and $a_0$, are given in Eqs. (4) and (5). It should be noted that we have used two contributors. The convolution of two-parton contributions is applicable even for the spectra of particles with complex quark composition. For example, we may use the convolution of two-parton contributions fitting the spectra of various jets [87]. As the probability density function, Eq. (6) is applicable in high energy collisions with small or large system, no matter what the density of produced particles is. In addition, Eq. (6) results in similar curve as Eqs.
(4) and (5) with different parameters, though the form of Eq. (6) is more complex due to the convolution.

In the above discussions, we assume that $p_T = p_{11} + p_{22}$, which is operated to describe the relationship among $p_T$ of $J/\psi$, $p_{11}$ and $p_{22}$ contributed by quarks 1 and 2, respectively. This treatment assumes the azimuth angle vector we have $p$, which is operated to describe the relationship among more items in the components. For the special case of three or more contributor partons if we add the third or more items in the components. For the special case of $|\phi_1 - \phi_2| = \pi/2$, the analytical expression of Eq. (6) is obtainable, though the special cases of $|\phi_1 - \phi_2| = \pi/2$ and $|\phi_1 - \phi_2| = \pi$ are more easy to fit the spectra due to the calculation itself, though the idea is practicable. In particular, from the point of view of the constituent quarks, the spectra of leptons and jets can be easily fitted from the relationship of $p_T = p_{11} + p_{22}$ and the convolution of two TP-like functions.

This confirms the validity of Eq. (6) based on two contributor sources in the framework of multi-source thermal model.

We would like to emphasize here that we have used two contributor partons as the projectile and target particles/nuclei, no matter what the final-state products are. For $J/\psi$, the two contributor partons (charm and anti-charm quarks) and the constituent quarks are coincidently equal to each other. For baryons, the two contributor partons are not equal to the three constituent quarks. For jets, the two contributor partons are not equal to the sets of two or three constituent quarks, too. For leptons, the two contributor partons have no corresponding constituent quarks. We may consider that two light or slow contributor partons produce leptons and baryons, while two heavy or fast contributor partons produce jets, no matter what the structures of leptons, baryons, and jets are. In fact, the two contributor partons are regarded as two energy resources, but not the constituents.

III. RESULTS AND DISCUSSION

A. Comparison with data

Figure 1 shows the transverse momentum spectra, $d^2N/(2\pi p_T dp_T dy)$, of $J/\psi$ produced in Au-Au collisions at center-of-mass energy per nucleon pair $\sqrt{s_{NN}} = (a) 39$, (b) 62.4, and (c–e) 200 GeV, where $B$ denotes the branching ratio. The symbols in panels (a–c) represent the experimental data of $p_T$ spectra measured by the STAR Collaboration in the rapidity interval of $|y| < 1$ and in the centrality class of 0–60% and its subclasses of 0–20%, 20–40%, and 40–60%. The symbols in panels (d) and (e) represent the experimental data of $p_T$ spectra measured by the PHENIX Collaboration in the rapidity intervals of (d) $|y| < 0.35$ and (e) $y \in [1.2, 2.2]$ and in the centrality classes of 0–20%, 20–40%, 40–60%, and 60–92%. Some data sets are scaled by multiplying or dividing different values marked in the panels for clear indication. The solid curves are our fitting results by using the convolution of two TP-like functions, i.e. Eq. (6) based on Eqs. (4) and (5).
For comparison, the dot-dashed curves are our results refitted by Eq. (6) with \( a_0 = 1 \). The values of fitting parameters \( n, T, \) and \( a_0 \) are listed in Table 1 with the normalization constant \( N_0, \chi^2 \), and the number of degree of freedom (ndof). We use \( \chi^2 \) to characterize the fitting deviation between the experimental data and our fit function and curve. For the given data and fit function, the smaller \( \chi^2 \), the better the fitting result, and the closer to the experimental results. If \( \chi^2 < 1 \), it is rounded to 1 or a decimal fraction; Otherwise, it is rounded to an integer. In the case of ndof is less than or equal to 0, we use “−” to mention in the table. One can see that the mentioned function with changeable \( a_0 \) fits satisfactorily the experimental data in Au-Au collisions measured by the STAR and PHENIX Collaborations at the RHIC, though in many cases the fits with \( a_0 = 1 \) are comparable to those with changeable \( a_0 \).

Similar to Figure 1, Figure 2 shows the transverse momentum spectra, \( d^2\sigma/dp_Tdy \), of \( J/\psi \) produced in \( d-Au \) collisions at 200 GeV. The symbols represent the experimental data measured by the PHENIX Collaboration [44]. Panels (a) and (b) show the spectra of \( J/\psi \to \mu^+\mu^- \) at the backward rapidity of \(-2.2 < y < -1.2\) and the forward rapidity of \(1.2 < y < 2.2\) respectively, with the centrality classes of 0–20%, 20–40%, 40–60%, and 60–88%. Panel (c) shows the spectra of \( J/\psi \to e^+e^- \) at mid-rapidity \(|y| < 0.35\) with the same centrality classes as panels (a) and (b). Panel (d) shows the spectra in minimum-bias collisions with rapidity intervals of \(-2.2 < y < -1.2, 1.2 < y < 2.2, \) and \(|y| < 0.35\). The solid curves are our fitting results by Eq. (6), and the dot-dashed curves are our results refitted by Eq. (6) with \( a_0 = 1 \). The values of fitting parameters are listed in Table 2 with \( N_0, \chi^2 \), and ndof. One can see that the mentioned function with changeable \( a_0 \) fits satisfactorily the experimental data in \( d-Au \) collisions measured by the PHENIX Collaboration at the RHIC. In many cases, the fits with \( a_0 = 1 \) obtain several times larger \( \chi^2 \) than those with changeable \( a_0 \).

The transverse momentum spectra, \( d^2Y/dp_Tdy \), of \( J/\psi \) produced in Pb-Pb collisions at (a) 2.76 TeV and (b) 5.02 TeV in the rapidity interval \( 2.5 < y < 4 \) are displayed in Figure 3, where \( Y \) denotes the yields. The symbols represent the experimental data measured by the ALICE Collaboration [45, 46]. Panel (a) shows the spectra for three centrality classes, 0–20%, 20–40%, and 40–90%. Panel (b) shows the spectra for seven centrality classes, 0–10%, 10–20%, 20–30%, 30–40%, 40–50%, 50–60%, and 60–90%. The sold (dot-dashed) curves are our fitting results by Eq. (6) (with \( a_0 = 1 \)). The values of fitting parameters are listed in Table 3 with other information. One can see that Eq. (6) (with changeable \( a_0 \)) fits satisfactorily the experimental data in \( Pb-Pb \) collisions measured by the ALICE Collaboration at the LHC. In many cases, the fits with \( a_0 = 1 \) obtain several times larger \( \chi^2 \) than those with changeable \( a_0 \).

The transverse momentum spectra, (a, b, i, and j) \( d^2\sigma/dp_Tdy \) or (c–h) \( Bd^2\sigma/dp_Tdy \), of (a, c, e, and g) prompt \( J/\psi \), (b) \( J/\psi \) from \( b \), (d, f, and h) nonprompt \( J/\psi \), or (i and j) inclusive \( J/\psi \) produced in \( p-Pb \) collisions at 5.02 TeV are given in Figure 4. As can be seen in the figure, panels (a–h) show the spectra for different rapidity intervals, while panels (i) and (j) show the spectra for given rapidity interval and different centrality classes. The symbols in panels (a and b), (c–f), (g and h), as well as (i and j) represent the experimental data measured by the LHCb [47], CMS [48], ATLAS [49], and ALICE Collaborations [50], respectively. The solid (dot-dashed) curves are our fitting results by Eq. (6) (with \( a_0 = 1 \)). The values of fitting parameters are listed in Table 4. It should be noted here that, in Figures 4(d), 4(f), and 4(h), although nonprompt \( J/\psi \) is produced from the fragmentation of open bottom hadron, it is also regarded as two contributors due to the fact that open bottom hadron has two contributors. This is similar to the view of point of string, in which two contributors form a string. Then, the string is broken to produce a particle, and the particle has two contributors. From Figure 4 one can see that Eq. (6) (with changeable \( a_0 \)) fits satisfactorily the experimental data in \( p-Pb \) collisions measured by several collaborations at the LHC. In many cases, the fits with \( a_0 = 1 \) obtain several times larger \( \chi^2 \) than those with changeable \( a_0 \).

In Figure 5, we show the \( J/\psi \) transverse momentum spectra, (a–c and g–j) \( Bd^2\sigma/(2\pi p_T dp_Tdy) \), (d and e) \( Bds/dp_T \), (f) \( ds/dp_T \), and (k–p) \( d^2\sigma/dp_Tdy \) in \( p-pT \) collisions at center-of-mass energy \( \sqrt{s} = (a) 200, (b) 500, \) and (c) 510 GeV, as well as (d) 1.8, (e) 1.96, (f) 2.76, (g) 5.02, and (h) 5.02 TeV, with different \( y \) or \( \eta \) (pseudorapidity) and other selection conditions marked in the panels. The data symbols in panels (a), (b and c), (d and e), (f), (g–j), and (k–p) are quoted from the PHENIX [51–53], STAR [54], CDF [55–56], LHCb [57], and ALICE [58], CMS [48], and LHCb Collaborations [59, 61], respectively. The solid (dot-dashed) curves are our fitting results by Eq. (6) (with \( a_0 = 1 \)). The values of fitting parameters are listed in Table 5. One can see that Eq. (6) (with changeable \( a_0 \)) fits satisfactorily the experimental data in \( p-p \) collisions measured by several collaborations at the RHIC, Tevatron, and LHC. In many cases, the fits with \( a_0 = 1 \) obtain several times larger \( \chi^2 \) than those with changeable \( a_0 \).
Table 1. Left panel: Values of $n$, $T$, $a_0$, $N_0$, $\chi^2$, and ndof corresponding to the solid curves in Figure 1 for Au-Au collisions. Right panel: Values of $n$, $T$, $N_0$, $\chi^2$, and ndof corresponding to the dot-dashed curves in Figure 1 for Au-Au collisions, in which $a_0 = 1$.

| Figure | Collab. | $\sqrt{s_{NN}}$ (GeV) | Selection | $n$ | $T$ (GeV) | $a_0$ | $N_0$ | $\chi^2$/ndof | $n$ | $T$ (GeV) | $N_0$ | $\chi^2$/ndof |
|--------|--------|----------------------|-----------|-----|---------|-------|-------|----------------|-----|---------|-------|----------------|
| Figure 1(a) | STAR | 39 | 0-60% | 0.70 ± 0.02 0.278 ± 0.001 0.131 ± 0.001 (3.43 ± 0.31) × 10^{-4} | 0.2/− | 1.56 ± 0.01 0.037 ± 0.001 (3.55 ± 0.33) × 10^{-3} | 0.2/− |
|          | Au-Au | | | | | | | | | | | |
|          | $|y| < 1$ | | | 2.37 ± 0.02 0.297 ± 0.001 0.150 ± 0.001 (6.23 ± 0.58) × 10^{-5} | 0.5/− | 1.62 ± 0.01 0.042 ± 0.001 (9.35 ± 0.87) × 10^{-5} | 0.5/− |
|          | 20-40% | | | 2.87 ± 0.02 0.278 ± 0.001 0.112 ± 0.001 (3.19 ± 0.28) × 10^{-5} | 0.1/− | 1.65 ± 0.01 0.039 ± 0.001 (3.19 ± 0.30) × 10^{-5} | 0.3/− |
|          | 40-60% | | | 2.95 ± 0.02 0.252 ± 0.001 0.108 ± 0.001 (8.29 ± 0.79) × 10^{-6} | 0.4/− | 1.67 ± 0.01 0.036 ± 0.001 (8.33 ± 0.81) × 10^{-6} | 0.4/− |
| Figure 1(b) | STAR | 62.4 | 0-60% | 0.86 ± 0.02 0.285 ± 0.001 0.131 ± 0.001 (9.07 ± 0.85) × 10^{-4} | 0.3/− | 1.60 ± 0.01 0.039 ± 0.001 (9.40 ± 0.91) × 10^{-4} | 1/− |
|          | Au-Au | | | | | | | | | | | |
|          | $|y| < 1$ | | | 2.78 ± 0.02 0.275 ± 0.001 0.162 ± 0.001 (2.20 ± 0.20) × 10^{-4} | 0.8/− | 1.66 ± 0.01 0.043 ± 0.001 (2.16 ± 0.19) × 10^{-4} | 1/− |
|          | 20-40% | | | 2.80 ± 0.02 0.271 ± 0.001 0.145 ± 0.001 (9.00 ± 0.86) × 10^{-5} | 0.4/− | 1.69 ± 0.01 0.037 ± 0.001 (8.63 ± 0.84) × 10^{-5} | 0.4/− |
|          | 40-60% | | | 2.94 ± 0.02 0.266 ± 0.001 0.150 ± 0.001 (3.34 ± 0.29) × 10^{-5} | 0.1/− | 1.70 ± 0.01 0.038 ± 0.001 (3.30 ± 0.31) × 10^{-5} | 1/− |
| Figure 1(c) | STAR | 200 | 0-60% | 4.67 ± 0.03 0.169 ± 0.001 0.221 ± 0.001 (7.86 ± 0.76) × 10^{-3} | 4/2 | 3.94 ± 0.03 0.064 ± 0.001 (7.70 ± 0.75) × 10^{-3} | 34/3 |
|          | Au-Au | | | | | | | | | | | |
|          | $|y| < 1$ | | | 4.62 ± 0.03 0.183 ± 0.001 0.225 ± 0.001 (1.07 ± 0.09) × 10^{-3} | 10/2 | 3.99 ± 0.03 0.067 ± 0.001 (1.23 ± 0.10) × 10^{-3} | 29/3 |
|          | 20-40% | | | 4.73 ± 0.03 0.180 ± 0.001 0.221 ± 0.001 (1.02 ± 0.08) × 10^{-4} | 4/2 | 4.08 ± 0.04 0.073 ± 0.001 (1.01 ± 0.08) × 10^{-4} | 14/3 |
|          | 40-60% | | | 4.76 ± 0.03 0.177 ± 0.001 0.215 ± 0.001 (2.16 ± 0.18) × 10^{-5} | 3/2 | 4.12 ± 0.04 0.075 ± 0.001 (2.05 ± 0.19) × 10^{-5} | 10/3 |
| Figure 1(d) | PHENIX | 200 | 0-60% | 2.23 ± 0.02 0.227 ± 0.001 0.208 ± 0.001 (2.76 ± 0.24) × 10^{-4} | 8/1 | 2.26 ± 0.02 0.064 ± 0.001 (2.74 ± 0.25) × 10^{-4} | 9/2 |
|          | Au-Au | | | | | | | | | | | |
|          | $|y| < 0.35$ | | | 2.28 ± 0.02 0.225 ± 0.001 0.204 ± 0.001 (1.61 ± 0.14) × 10^{-4} | 3/1 | 2.20 ± 0.02 0.062 ± 0.001 (1.71 ± 0.15) × 10^{-4} | 4/2 |
|          | 20-40% | | | 2.36 ± 0.02 0.216 ± 0.001 0.195 ± 0.001 (6.17 ± 0.59) × 10^{-5} | 0.7/1 | 2.12 ± 0.02 0.055 ± 0.001 (6.24 ± 0.60) × 10^{-5} | 1/2 |
|          | 60-92% | | | 2.95 ± 0.02 0.179 ± 0.001 0.170 ± 0.001 (1.04 ± 0.10) × 10^{-5} | 1/1 | 2.56 ± 0.02 0.053 ± 0.001 (1.03 ± 0.09) × 10^{-5} | 1/2 |
| Figure 1(e) | PHENIX | 200 | 0-20% | 2.51 ± 0.02 0.199 ± 0.001 0.217 ± 0.001 (1.60 ± 0.13) × 10^{-5} | 3/1 | 2.34 ± 0.02 0.064 ± 0.001 (1.53 ± 0.13) × 10^{-5} | 3/2 |
|          | Au-Au | | | | | | | | | | | |
|          | $|y| \in [1.2, 2.2]$ | | | 2.54 ± 0.02 0.196 ± 0.001 0.215 ± 0.001 (7.51 ± 0.71) × 10^{-5} | 8/1 | 2.30 ± 0.02 0.071 ± 0.001 (7.46 ± 0.73) × 10^{-5} | 8/2 |
|          | 20-40% | | | 2.68 ± 0.02 0.194 ± 0.001 0.213 ± 0.001 (3.70 ± 0.34) × 10^{-5} | 1/1 | 2.48 ± 0.02 0.060 ± 0.001 (3.72 ± 0.35) × 10^{-5} | 2/2 |
|          | 60-92% | | | 3.23 ± 0.02 0.192 ± 0.001 0.211 ± 0.001 (7.16 ± 0.69) × 10^{-6} | 3/1 | 2.59 ± 0.02 0.058 ± 0.001 (7.03 ± 0.68) × 10^{-6} | 6/2 |
Figure 1. Transverse momentum spectra, $Bd^2N/(2\pi p_T dp_T dy)$, of $J/\psi$ produced in high energy Au-Au collisions with various centralities. Different symbols in panels (a–c) show the spectra measured by the STAR Collaboration [42] at $\sqrt{s_{NN}} = (a) 39$, (b) 62.4, and (c) 200 GeV with $|y| < 1$. Panels (d) and (e) are the spectra measured by the PHENIX Collaboration [43] at 200 GeV with $|y| < 0.35$ and $y \in [1.2, 2.2]$ respectively. The solid curves are our fitting results by using Eq. (6) (with changeable $a_0$), and the dot-dashed curves are our results refitted by Eq. (6) with unchangeable $a_0 = 1$. 
Figure 2. Transverse momentum spectra, $Bd^2N/(2\pi p_T dp_T dy)$, of $J/\psi$ produced in $d$-Au collisions at 200 GeV. Panels (a) and (b) show the spectra of $J/\psi \rightarrow \mu^+ \mu^-$ in the backward and forward rapidity regions respectively, and panel (c) shows the spectra of $J/\psi \rightarrow e^+ e^-$ at mid-rapidity $|y| < 0.35$, with different centrality classes. Panel (d) shows the spectra in minimum-bias collisions with different rapidity intervals. The symbols represent the spectra measured by the PHENIX Collaboration [44], the solid curves are our fitting results by Eq. (6), and the dot-dashed curves are our results refitted by Eq. (6) with $a_0 = 1$.

Figure 3. Transverse momentum spectra, $d^2Y/dp_T dy$, of $J/\psi$ produced in Pb-Pb collisions at (a) 2.76 TeV and (b) 5.02 TeV, in the rapidity interval $2.5 < y < 4$ and with different centrality classes. The symbols represent the experimental data measured by the ALICE Collaboration [45, 46], the solid curves are our fitting results by Eq. (6), and the dot-dashed curves are our results refitted by Eq. (6) with $a_0 = 1$. 
### Table 2

| Figure 2(a) PHENIX 200 | Selection | $n$ | $T$ (GeV) | $a_0$ | $N_0$ | $\chi^2$/ndof | $n$ | $T$ (GeV) | $a_0$ | $N_0$ | $\chi^2$/ndof |
|------------------------|-----------|-----|-----------|-------|-------|---------------|-----|-----------|-------|-------|---------------|
| $d$-Au $-2.2 < y < -1.2$ | 0–20%     | 5.85 ± 0.05 | 0.310 ± 0.001 | 0.191 ± 0.001 | (1.11 ± 0.11) $\times 10^{-5}$ | 40/23 | 5.35 ± 0.05 | 0.159 ± 0.001 | (1.06 ± 0.09) $\times 10^{-5}$ | 110/24 |
| $d$-Au $-2.2 < y < -1.2$ | 20–40% | 5.88 ± 0.05 | 0.301 ± 0.001 | 0.165 ± 0.001 | (6.90 ± 0.04) $\times 10^{-6}$ | 119/23 | 5.61 ± 0.05 | 0.154 ± 0.001 | (6.89 ± 0.67) $\times 10^{-6}$ | 137/24 |
| $d$-Au $-2.2 < y < -1.2$ | 40–60% | 5.92 ± 0.05 | 0.299 ± 0.001 | 0.161 ± 0.001 | (4.90 ± 0.48) $\times 10^{-6}$ | 31/23 | 5.61 ± 0.05 | 0.163 ± 0.001 | (4.64 ± 0.44) $\times 10^{-6}$ | 136/24 |
| $d$-Au $-2.2 < y < -1.2$ | 60–80% | 5.99 ± 0.05 | 0.296 ± 0.001 | 0.160 ± 0.001 | (2.27 ± 0.21) $\times 10^{-6}$ | 38/22 | 5.69 ± 0.05 | 0.157 ± 0.001 | (2.16 ± 0.20) $\times 10^{-6}$ | 128/23 |

### Table 3

| Figure 3(a) ALICE 2.76 | Selection | $n$ | $T$ (GeV) | $a_0$ | $N_0$ | $\chi^2$/ndof | $n$ | $T$ (GeV) | $a_0$ | $N_0$ | $\chi^2$/ndof |
|------------------------|-----------|-----|-----------|-------|-------|---------------|-----|-----------|-------|-------|---------------|
| Pb-Pb $2.5 < y < 4$ | 0–20%     | 5.35 ± 0.05 | 0.409 ± 0.001 | 0.140 ± 0.001 | (2.15 ± 0.20) $\times 10^{3}$ | 14/9 | 4.89 ± 0.04 | 0.202 ± 0.001 | (2.10 ± 0.18) $\times 10^{3}$ | 45/10 |
| Pb-Pb $2.5 < y < 4$ | 20–40% | 5.37 ± 0.05 | 0.407 ± 0.001 | 0.137 ± 0.001 | (9.70 ± 0.95) $\times 10^{3}$ | 14/9 | 4.91 ± 0.04 | 0.200 ± 0.001 | (8.88 ± 0.86) $\times 10^{3}$ | 70/10 |
| Pb-Pb $2.5 < y < 4$ | 40–90% | 5.39 ± 0.05 | 0.407 ± 0.001 | 0.135 ± 0.001 | (1.61 ± 0.14) $\times 10^{3}$ | 39/9 | 4.93 ± 0.04 | 0.197 ± 0.001 | (1.45 ± 0.12) $\times 10^{3}$ | 64/10 |

| Figure 3(b) ALICE 5.02 | Selection | $n$ | $T$ (GeV) | $a_0$ | $N_0$ | $\chi^2$/ndof | $n$ | $T$ (GeV) | $a_0$ | $N_0$ | $\chi^2$/ndof |
|------------------------|-----------|-----|-----------|-------|-------|---------------|-----|-----------|-------|-------|---------------|
| Pb-Pb $2.5 < y < 4$ | 0–10%     | 5.65 ± 0.05 | 0.431 ± 0.001 | 0.141 ± 0.001 | (5.69 ± 0.55) $\times 10^{3}$ | 47/6 | 5.24 ± 0.05 | 0.215 ± 0.001 | (5.18 ± 0.50) $\times 10^{2}$ | 138/7 |
| Pb-Pb $2.5 < y < 4$ | 10–20% | 5.68 ± 0.05 | 0.430 ± 0.001 | 0.141 ± 0.001 | (3.34 ± 0.32) $\times 10^{3}$ | 23/6 | 4.91 ± 0.04 | 0.212 ± 0.001 | (3.26 ± 0.31) $\times 10^{3}$ | 108/7 |
| Pb-Pb $2.5 < y < 4$ | 20–30% | 5.70 ± 0.05 | 0.429 ± 0.001 | 0.140 ± 0.001 | (2.23 ± 0.20) $\times 10^{3}$ | 40/6 | 4.76 ± 0.04 | 0.211 ± 0.001 | (2.12 ± 0.19) $\times 10^{3}$ | 107/7 |
| Pb-Pb $2.5 < y < 4$ | 30–40% | 4.56 ± 0.04 | 0.428 ± 0.001 | 0.139 ± 0.001 | (1.12 ± 0.10) $\times 10^{3}$ | 40/6 | 4.42 ± 0.04 | 0.210 ± 0.001 | (1.09 ± 0.09) $\times 10^{3}$ | 119/7 |
| Pb-Pb $2.5 < y < 4$ | 40–50% | 4.57 ± 0.04 | 0.427 ± 0.001 | 0.136 ± 0.001 | (6.67 ± 0.65) $\times 10^{3}$ | 34/6 | 4.34 ± 0.04 | 0.207 ± 0.001 | (6.51 ± 0.63) $\times 10^{3}$ | 86/7 |
| Pb-Pb $2.5 < y < 4$ | 50–60% | 4.58 ± 0.04 | 0.426 ± 0.001 | 0.135 ± 0.001 | (3.67 ± 0.35) $\times 10^{3}$ | 57/6 | 4.17 ± 0.04 | 0.204 ± 0.001 | (3.39 ± 0.32) $\times 10^{3}$ | 136/7 |
| Pb-Pb $2.5 < y < 4$ | 60–90% | 4.60 ± 0.04 | 0.425 ± 0.001 | 0.135 ± 0.001 | (1.01 ± 0.08) $\times 10^{3}$ | 160/6 | 4.13 ± 0.06 | 0.202 ± 0.001 | (8.84 ± 0.86) $\times 10^{3}$ | 267/7 |
Figure 4. Transverse momentum spectra, (a, b, i, and j) $d^{2}\sigma/dydy$ or (c–h) $Bd^{2}\sigma/dydy$, of (a, c, e, and g) prompt $J/\psi$, (b) $J/\psi$ from $b$, (d, f, and h) nonprompt $J/\psi$, or (i and j) inclusive $J/\psi$ produced in $p$-$Pb$ collisions at 5.02 TeV. Panels (a–h) show the spectra for different rapidity intervals, while panels (i) and (j) show the spectra for given rapidity interval and different centrality classes. The symbols in panels (a and b), (c–f), (g and h), as well as (i and j) represent the experimental data measured by the LHCb [47], CMS [48], ATLAS [49], and ALICE [50] Collaborations, respectively. The solid curves are our fitting results by Eq. (6), and the dot-dashed curves are our results refitted by Eq. (6) with $a_0 = 1$. 

\[ B = a_0 \times 10^{a_1} \times 10^{a_2} \times 10^{a_3} \times 10^{a_4} \times 10^{a_5} \times 10^{a_6} \]
Table 4. Left panel: Values of $n$, $T$, $\alpha_0$, $\sigma_0$, $\chi^2$, and ndof corresponding to the solid curves in Figure 4 for p-Pb collisions. Right panel: Values of $n$, $T$, $\sigma_0$, $\chi^2$, and ndof corresponding to the dot-dashed curves in Figure 4 for p-Pb collisions, in which $\alpha_0 = 1$.

| Figure 4(a) | LHCb | 5.02 | $1.5 < y < 2.0$ | $4.55 \pm 0.03$ | $0.598 \pm 0.002$ | $0.172 \pm 0.001$ | $(5.97 \pm 0.57) \times 10^2$ | 2/4 | $4.27 \pm 0.04$ | $0.278 \pm 0.001$ | $(5.38 \pm 0.58) \times 10^2$ | 41/5 |
| Figure 4(b) | LHCb | 5.02 | $1.5 < y < 2.0$ | $4.01 \pm 0.03$ | $0.679 \pm 0.002$ | $0.226 \pm 0.001$ | $(9.00 \pm 0.88) \times 10^1$ | 3/4 | $3.64 \pm 0.03$ | $0.299 \pm 0.001$ | $(8.47 \pm 0.83) \times 10^2$ | 9/5 |
| Figure 4(c) | CMS | 5.02 | $-0.9 < y < 0$ | $4.96 \pm 0.04$ | $0.566 \pm 0.002$ | $0.254 \pm 0.001$ | $(4.74 \pm 0.45) \times 10^1$ | 25/1 | $4.86 \pm 0.04$ | $0.291 \pm 0.001$ | $(5.32 \pm 0.51) \times 10^2$ | 59/2 |
| Figure 4(d) | CMS | 5.02 | $-0.9 < y < 0$ | $4.49 \pm 0.03$ | $0.583 \pm 0.002$ | $0.278 \pm 0.001$ | $(1.24 \pm 0.10) \times 10^1$ | 30/1 | $4.06 \pm 0.04$ | $0.286 \pm 0.001$ | $(1.16 \pm 0.10) \times 10^2$ | 31/2 |
| Figure 4(e) | CMS | 5.02 | $0 < y < 0.9$ | $4.96 \pm 0.03$ | $0.562 \pm 0.002$ | $0.254 \pm 0.001$ | $(4.64 \pm 0.43) \times 10^1$ | 23/1 | $4.90 \pm 0.04$ | $0.294 \pm 0.001$ | $(5.49 \pm 0.53) \times 10^2$ | 39/2 |
| Figure 4(f) | CMS | 5.02 | $0 < y < 0.9$ | $4.43 \pm 0.03$ | $0.586 \pm 0.002$ | $0.281 \pm 0.001$ | $(1.13 \pm 0.10) \times 10^1$ | 43/1 | $4.08 \pm 0.04$ | $0.287 \pm 0.001$ | $(1.21 \pm 0.10) \times 10^2$ | 47/2 |
| Figure 4(g) | ATLAS | 5.02 | $-1.94 < y < 0$ | $5.76 \pm 0.04$ | $0.583 \pm 0.002$ | $0.248 \pm 0.001$ | $(5.23 \pm 0.50) \times 10^1$ | 2/1 | $5.67 \pm 0.05$ | $0.329 \pm 0.001$ | $(5.63 \pm 0.54) \times 10^2$ | 16/2 |
| Figure 4(h) | ALICE | 5.02 | $0 < y < 1.94$ | $5.85 \pm 0.04$ | $0.583 \pm 0.002$ | $0.241 \pm 0.001$ | $(5.23 \pm 0.50) \times 10^1$ | 4/1 | $5.79 \pm 0.05$ | $0.337 \pm 0.001$ | $(5.16 \pm 0.50) \times 10^2$ | 21/2 |
| Figure 4(i) | ALICE | 5.02 | $2-10\%$ | $5.08 \pm 0.04$ | $0.521 \pm 0.002$ | $0.184 \pm 0.001$ | $(1.18 \pm 0.16) \times 10^3$ | 3/4 | $4.87 \pm 0.04$ | $0.263 \pm 0.001$ | $(1.15 \pm 0.10) \times 10^3$ | 60/5 |
| Figure 4(j) | ALICE | 5.02 | $2-10\%$ | $5.50 \pm 0.04$ | $0.561 \pm 0.002$ | $0.205 \pm 0.001$ | $(9.61 \pm 0.96) \times 10^2$ | 10/4 | $4.40 \pm 0.04$ | $0.280 \pm 0.001$ | $(9.06 \pm 0.89) \times 10^2$ | 86/5 |
| Figure 4(k) | ALICE | 5.02 | $2-10\%$ | $5.50 \pm 0.04$ | $0.560 \pm 0.002$ | $0.198 \pm 0.001$ | $(9.86 \pm 0.89) \times 10^2$ | 6/4 | $4.46 \pm 0.04$ | $0.278 \pm 0.001$ | $(8.61 \pm 0.84) \times 10^2$ | 102/5 |
| Figure 4(l) | ALICE | 5.02 | $2-10\%$ | $5.50 \pm 0.04$ | $0.559 \pm 0.002$ | $0.192 \pm 0.001$ | $(8.12 \pm 0.78) \times 10^2$ | 5/4 | $4.49 \pm 0.04$ | $0.274 \pm 0.001$ | $(8.00 \pm 0.78) \times 10^2$ | 126/5 |
| Figure 4(m) | ALICE | 5.02 | $2-10\%$ | $5.50 \pm 0.04$ | $0.558 \pm 0.002$ | $0.183 \pm 0.001$ | $(6.15 \pm 0.55) \times 10^2$ | 4/4 | $4.52 \pm 0.04$ | $0.271 \pm 0.001$ | $(5.91 \pm 0.57) \times 10^2$ | 112/5 |
| Figure 4(n) | ALICE | 5.02 | $2-10\%$ | $5.50 \pm 0.04$ | $0.555 \pm 0.002$ | $0.171 \pm 0.001$ | $(3.96 \pm 0.39) \times 10^2$ | 9/4 | $5.55 \pm 0.04$ | $0.268 \pm 0.001$ | $(3.71 \pm 0.35) \times 10^2$ | 111/5 |

Figure 4. Left panel: Values of $n$, $T$, $\alpha_0$, $\sigma_0$, $\chi^2$, and ndof corresponding to the solid curves in Figure 4 for p-Pb collisions. Right panel: Values of $n$, $T$, $\sigma_0$, $\chi^2$, and ndof corresponding to the dot-dashed curves in Figure 4 for p-Pb collisions, in which $\alpha_0 = 1$. 
Figure 5. Transverse momentum spectra, (a–c and g–j) $B d^2 \sigma / (2 \pi p_T d\eta dy)$, (d and e) $d\sigma / dp_T$, (f) $d\sigma / dp_T$, and (k–p) $d^2 \sigma / dp_T dy$, of $J/\psi$ produced in $p$-$p(\bar{p})$ collisions at different energies. The data symbols in panels (a), (b and c), (d and e), (f), (g–j), and (k–p) are quoted from the PHENIX [51–53], STAR [54], CDF [55, 56], LHCb [57] and ALICE [58], CMS [48], and LHCb Collaborations [59–61], respectively. The solid curves are our fitting results by Eq. (6), and the dot-dashed curves are our results refitted by Eq. (6) with $a_0 = 1$. 
Figure 5. Continued. Panels (i–p) are presented.
Table 5. Left panel: Values of \( n \), \( T \), \( a_0 \), \( \sigma_0 \), \( \chi^2 \), and \( \text{ndof} \) corresponding to the solid curves in Figure 5 for \( p-p \) collisions. Right panel: Values of \( n \), \( T \), \( \sigma_0 \), \( \chi^2 \), and \( \text{ndof} \) corresponding to the dot-dashed curves in Figure 5 for \( p-p \) collisions, in which \( a_0 = 1 \).

| Figure | Collab. | \( \sqrt{s_{NN}} \) (TeV) | Selection | \( n \) | \( T \) (GeV) | \( a_0 \) | \( \sigma_0 \) (\( \mu b \)) | \( \chi^2/\text{ndof} \) |
|--------|---------|-----------------|----------|-----|--------|-----|--------|-------------|
| Figure 5(a) | PHENIX | 0.2 | | \( |y| < 0.35 \) | 4.42 \pm 0.03 | 0.240 \pm 0.001 | 0.363 \pm 0.001 | (4.15 \pm 0.38) \times 10^{-2} | 19/17 |
| p-p | | \( y \in [1.2, 2.2] \) | 5.26 \pm 0.04 | 0.239 \pm 0.001 | 0.306 \pm 0.001 | (2.76 \pm 0.26) \times 10^{-2} | 26/15 |
| Figure 5(b) | STAR | 0.5 | full cross section | 5.99 \pm 0.04 | 0.350 \pm 0.001 | 0.149 \pm 0.001 | (1.19 \pm 0.11) \times 10^{-2} | 11/15 |
| p-p | fiducial cross section | 4.67 \pm 0.03 | 0.389 \pm 0.001 | 0.135 \pm 0.001 | (1.79 \pm 0.17) \times 10^{-2} | 143/15 |
| Figure 5(c) | STAR | 0.51 | full cross section | 4.00 \pm 0.03 | 0.350 \pm 0.001 | 0.149 \pm 0.001 | (6.58 \pm 0.65) \times 10^{-1} | 1/1 |
| p-p | fiducial cross section | 3.33 \pm 0.03 | 0.368 \pm 0.001 | 0.146 \pm 0.001 | (1.11 \pm 0.11) \times 10^{-1} | 1/1 |
| Figure 5(d) | CDF | 1.8 | prompt \( J/\psi \) from b | 6.47 \pm 0.04 | 0.535 \pm 0.002 | 0.174 \pm 0.001 | (1.96 \pm 0.18) \times 10^{-1} | 4/7 |
| p-\( \bar{p} \) | \( J/\psi \) from b | 5.03 \pm 0.03 | 0.529 \pm 0.002 | 0.191 \pm 0.001 | (3.54 \pm 0.34) \times 10^{-1} | 11/7 |
| Figure 5(e) | CDF | 1.96 | prompt \( J/\psi \) from b | 5.13 \pm 0.04 | 0.570 \pm 0.002 | 0.285 \pm 0.001 | (2.30 \pm 0.23) \times 10^{-1} | 4/22 |
| p-\( \bar{p} \) | \( J/\psi \) from b | 6.67 \pm 0.04 | 0.495 \pm 0.002 | 0.322 \pm 0.001 | (2.05 \pm 0.20) \times 10^{-1} | 6/22 |
| Figure 5(f) | LHCb | 2.76 | \( 2.0 < y < 4.5 \) | 5.03 \pm 0.04 | 0.452 \pm 0.002 | 0.128 \pm 0.001 | (2.24 \pm 0.22) \times 10^{0} | 6/4 |
| ALICE | 2.75 | \( 0 < y < 4.0 \) | 5.76 \pm 0.04 | 0.505 \pm 0.002 | 0.148 \pm 0.001 | (2.23 \pm 0.22) \times 10^{0} | 3/3 |
| Figure 5(g) | CMS | 5.02 | \( -0.9 < y < 0 \) | 4.85 \pm 0.03 | 0.540 \pm 0.002 | 0.244 \pm 0.001 | (2.47 \pm 0.24) \times 10^{-1} | 30/1 |
| p-p | \( -1.5 < y < -0.9 \) | 4.97 \pm 0.04 | 0.553 \pm 0.002 | 0.239 \pm 0.001 | (2.21 \pm 0.20) \times 10^{-1} | 24/1 |
| Figure 5(h) | CMS | 5.02 | \( -0.9 < y < 0 \) | 4.39 \pm 0.03 | 0.583 \pm 0.002 | 0.267 \pm 0.001 | (5.60 \pm 0.50) \times 10^{-2} | 67/1 |
| p-p | \( -1.5 < y < -0.9 \) | 4.45 \pm 0.03 | 0.591 \pm 0.002 | 0.264 \pm 0.001 | (4.71 \pm 0.45) \times 10^{-2} | 52/1 |
| Figure 5(i) | CMS | 5.02 | \( 0 < y < 0.9 \) | 4.89 \pm 0.03 | 0.542 \pm 0.002 | 0.243 \pm 0.001 | (2.38 \pm 0.23) \times 10^{-1} | 25/1 |
| p-p | \( 0.9 < y < 1.5 \) | 5.07 \pm 0.04 | 0.546 \pm 0.002 | 0.239 \pm 0.001 | (2.22 \pm 0.22) \times 10^{-1} | 27/1 |
| Figure 5(j) | CMS | 5.02 | \( 0 < y < 0.9 \) | 4.38 \pm 0.03 | 0.593 \pm 0.002 | 0.269 \pm 0.001 | (4.99 \pm 0.40) \times 10^{-2} | 78/1 |
| p-p | \( 0.9 < y < 1.5 \) | 4.48 \pm 0.03 | 0.597 \pm 0.002 | 0.266 \pm 0.001 | (4.57 \pm 0.42) \times 10^{-2} | 57/1 |
| Figure 5(k) | LHCb | 7 | \( 2.0 < y < 2.5 \) | 5.46 \pm 0.04 | 0.544 \pm 0.002 | 0.149 \pm 0.001 | (5.69 \pm 0.56) \times 10^{0} | 5/10 |
| p-p | \( 2.5 < y < 3.0 \) | 5.75 \pm 0.04 | 0.563 \pm 0.002 | 0.149 \pm 0.001 | (4.97 \pm 0.48) \times 10^{0} | 3/10 |
| Figure 5(l) | LHCb | 7 | \( 2.0 < y < 2.5 \) | 5.06 \pm 0.04 | 0.666 \pm 0.002 | 0.226 \pm 0.001 | (6.62 \pm 0.66) \times 10^{-1} | 3/10 |
| p-p | \( 2.5 < y < 3.0 \) | 5.37 \pm 0.04 | 0.668 \pm 0.002 | 0.226 \pm 0.001 | (5.86 \pm 0.58) \times 10^{-1} | 3/10 |
| Figure 5(m) | LHCb | 7 | \( 3.0 < y < 3.5 \) | 5.90 \pm 0.04 | 0.668 \pm 0.002 | 0.226 \pm 0.001 | (4.76 \pm 0.50) \times 10^{-1} | 5/10 |
| p-p | \( 3.5 < y < 4.0 \) | 6.14 \pm 0.04 | 0.677 \pm 0.002 | 0.226 \pm 0.001 | (3.01 \pm 0.29) \times 10^{-1} | 6/9 |
| Figure 5(n) | LHCb | 7 | \( 4.0 < y < 4.5 \) | 6.88 \pm 0.04 | 0.681 \pm 0.002 | 0.226 \pm 0.001 | (1.78 \pm 0.18) \times 10^{-1} | 3/7 |

\( \chi^2/\text{ndof} \)
Table 5. Continued. The parameters for the curves in Figures 5(m), 5(n), 5(o), and 5(p) are listed.

| Figure | Collab. | \(\sqrt{s_{NN}}\) (TeV) | Selection | \(n\) | \(T\) (GeV) | \(a_0\) | \(\sigma_0\) (\(\mu\)b) | \(\chi^2/\text{ndof}\) | \(n\) | \(T\) (GeV) | \(\sigma_0\) (\(\mu\)b) | \(\chi^2/\text{ndof}\) |
|--------|---------|--------------------------|-----------|------|----------|--------|-----------------|-------------|------|----------|-----------------|-------------|
| Figure 5(m) LHCb | 8 | 2.0 < \(y\) < 2.5 | 5.79 ± 0.04 0.602 ± 0.002 0.149 ± 0.001 | (5.25 ± 0.52) × 10^9 | 4/10 | 4.72 ± 0.04 0.260 ± 0.001 | (5.15 ± 0.50) × 10^9 | 133/11 |
| | | | 2.5 < \(y\) < 3.0 | 6.11 ± 0.04 0.607 ± 0.002 0.146 ± 0.001 | (5.11 ± 0.48) × 10^9 | 6/10 | 4.78 ± 0.04 0.257 ± 0.001 | (4.95 ± 0.48) × 10^9 | 153/11 |
| | | | 3.0 < \(y\) < 3.5 | 6.51 ± 0.04 0.608 ± 0.002 0.141 ± 0.001 | (4.56 ± 0.42) × 10^9 | 6/10 | 4.97 ± 0.05 0.256 ± 0.001 | (4.41 ± 0.42) × 10^9 | 168/11 |
| | | | 3.5 < \(y\) < 4.0 | 6.86 ± 0.04 0.607 ± 0.002 0.133 ± 0.001 | (3.84 ± 0.38) × 10^9 | 8/10 | 5.31 ± 0.05 0.270 ± 0.001 | (3.78 ± 0.36) × 10^9 | 157/11 |
| | | | 4.0 < \(y\) < 4.5 | 7.98 ± 0.05 0.613 ± 0.002 0.133 ± 0.001 | (3.15 ± 0.30) × 10^9 | 17/10 | 5.91 ± 0.06 0.276 ± 0.001 | (3.01 ± 0.28) × 10^9 | 181/11 |
| Figure 5(n) LHCb | 8 | 2.0 < \(y\) < 2.5 | 4.88 ± 0.03 0.660 ± 0.002 0.247 ± 0.001 | (7.10 ± 0.66) × 10^-1 | 4/10 | 4.04 ± 0.04 0.296 ± 0.001 | (7.05 ± 0.69) × 10^-1 | 42/11 |
| | | | 2.5 < \(y\) < 3.0 | 5.03 ± 0.04 0.673 ± 0.002 0.242 ± 0.001 | (6.55 ± 0.65) × 10^-1 | 7/10 | 4.16 ± 0.04 0.298 ± 0.001 | (6.34 ± 0.62) × 10^-1 | 62/11 |
| | | | 3.0 < \(y\) < 3.5 | 5.48 ± 0.04 0.673 ± 0.002 0.242 ± 0.001 | (5.20 ± 0.47) × 10^-1 | 8/10 | 4.39 ± 0.04 0.300 ± 0.001 | (5.11 ± 0.49) × 10^-1 | 58/11 |
| | | | 3.5 < \(y\) < 4.0 | 5.99 ± 0.04 0.681 ± 0.002 0.214 ± 0.001 | (3.92 ± 0.39) × 10^-1 | 12/10 | 4.67 ± 0.04 0.302 ± 0.001 | (3.73 ± 0.35) × 10^-1 | 120/11 |
| | | | 4.0 < \(y\) < 4.5 | 6.69 ± 0.04 0.682 ± 0.002 0.214 ± 0.001 | (2.43 ± 0.29) × 10^-1 | 8/10 | 4.98 ± 0.05 0.304 ± 0.001 | (2.22 ± 0.29) × 10^-1 | 94/11 |
| Figure 5(o) LHCb | 13 | 2.0 < \(y\) < 2.5 | 5.57 ± 0.04 0.615 ± 0.002 0.145 ± 0.001 | (6.91 ± 0.66) × 10^9 | 21/10 | 4.87 ± 0.05 0.288 ± 0.001 | (6.70 ± 0.65) × 10^9 | 362/11 |
| | | | 2.5 < \(y\) < 3.0 | 5.61 ± 0.04 0.619 ± 0.002 0.138 ± 0.001 | (6.91 ± 0.68) × 10^9 | 19/10 | 4.92 ± 0.05 0.292 ± 0.001 | (6.72 ± 0.65) × 10^9 | 360/11 |
| | | | 3.0 < \(y\) < 3.5 | 5.81 ± 0.04 0.623 ± 0.002 0.130 ± 0.001 | (6.30 ± 0.60) × 10^9 | 12/10 | 5.03 ± 0.05 0.294 ± 0.001 | (6.24 ± 0.60) × 10^9 | 320/11 |
| | | | 3.5 < \(y\) < 4.0 | 6.89 ± 0.04 0.658 ± 0.002 0.116 ± 0.001 | (5.31 ± 0.50) × 10^9 | 16/10 | 5.31 ± 0.05 0.296 ± 0.001 | (5.21 ± 0.50) × 10^9 | 243/11 |
| | | | 4.0 < \(y\) < 4.5 | 7.61 ± 0.05 0.661 ± 0.002 0.116 ± 0.001 | (4.27 ± 0.41) × 10^9 | 19/10 | 5.45 ± 0.05 0.297 ± 0.001 | (4.04 ± 0.38) × 10^9 | 153/11 |
| Figure 5(p) LHCb | 13 | 2.0 < \(y\) < 2.5 | 4.30 ± 0.03 0.668 ± 0.002 0.247 ± 0.001 | (1.22 ± 0.11) × 10^9 | 5/10 | 3.73 ± 0.03 0.282 ± 0.001 | (1.22 ± 0.10) × 10^9 | 46/11 |
| | | | 2.5 < \(y\) < 3.0 | 4.44 ± 0.03 0.669 ± 0.002 0.239 ± 0.001 | (1.09 ± 0.11) × 10^9 | 9/10 | 3.78 ± 0.03 0.287 ± 0.001 | (1.05 ± 0.09) × 10^9 | 134/11 |
| | | | 3.0 < \(y\) < 3.5 | 4.72 ± 0.03 0.665 ± 0.002 0.229 ± 0.001 | (9.41 ± 0.90) × 10^-1 | 11/10 | 3.97 ± 0.04 0.289 ± 0.001 | (9.27 ± 0.91) × 10^-1 | 146/11 |
| | | | 3.5 < \(y\) < 4.0 | 4.92 ± 0.04 0.669 ± 0.002 0.192 ± 0.001 | (7.60 ± 0.73) × 10^-1 | 10/10 | 4.17 ± 0.04 0.291 ± 0.001 | (7.50 ± 0.73) × 10^-1 | 117/11 |
| | | | 4.0 < \(y\) < 4.5 | 5.51 ± 0.04 0.673 ± 0.002 0.169 ± 0.001 | (5.35 ± 0.52) × 10^-1 | 13/10 | 4.25 ± 0.04 0.292 ± 0.001 | (5.09 ± 0.49) × 10^-1 | 82/11 |
B. Tendencies of parameters

Due to the worse results obtained from the fit function with fixed $a_0 = 1$, we give up to analyze the changing laws of the parameters listed in the right panel in the above tables and extracted from the dot-dashed curves in the above figures. To better analyze the changing laws of the parameters used in the fit function with changeable $a_0$ for different centralities, system sizes, and energies, Figure 6 shows the dependences of the parameters (a) $n$, (b) $T$, and (c) $a_0$ on centrality percentage $C$ in Au-Au, d-Au, Pb-Pb, and $p$-Pb collisions at the RHIC and LHC. The symbols represent the parameter values listed in the left panel in Tables 1–4 and extracted from the solid curves in Figures 1–4. One can see that with the increase of $C$ (with the decrease of centrality from central to peripheral collisions), the power exponent $n$ increases slightly, the effective temperature $T$ does not change obviously, and the revised index $a_0$ decreases slightly in most cases. At the given energy (the top RHIC or LHC energy), the values of $n$, $T$, and $a_0$ for d-Au and $p$-Pb collisions are larger than those for Au-Au and Pb-Pb collisions.

For given collision system, the tendencies in Figure 6 indicate that the spectra of $J/\psi$ are affected slightly by the centrality or impact parameter. The size of participant region and the effect of cold spectator nucleons have weak influences on the production of $J/\psi$. The $J/\psi$'s produced in the early stage of collisions have less interactions with the hot and dense QGP and the cold spectator nucleons. For different collision systems, the system size dependence of parameters is caused by the secondary cascade collisions of produced $J/\psi$ with the subsequent nucleons in the incident path way of $d(p)$ in target Au(Pb), where the subsequent nucleons are the remainder nucleons after the binary nucleon-nucleon collisions in the incident path way. The subsequent nucleons are also the cold spectator nucleons in d-Au and $p$-Pb collisions. It is believed that the subsequent nucleons have larger influence on $J/\psi$ than other cold spectator nucleons outside the incident path way. In Au-Au (Pb-Pb) collisions, there is no subsequent nucleon after the binary nucleon-nucleon collisions. In most cases, the values of $n$ are large enough, which implies that the entropy index $q$ is close to 1 due to the fact that $n = 1/(q - 1)$ in the TP-Like function. This means that the emission source of $J/\psi$ and the collision system stay approximately at equilibrium.

Figure 7 shows the dependences of (a) $n$, (b) $T$, and (c) $a_0$ on rapidity $y$ in the forward region, which are extracted from the spectra of prompt $J/\psi$ and $J/\psi$ from $b$ in $p$-$p$ collisions at the LHC (5.02, 7, 8, and 13 TeV). The symbols represent the parameter values listed in the left panel in Table 5 and obtained from the solid curves in Figure 5. One can see that with the increase of $y$, the parameters $n$ and $T$ increase and the parameter $a_0$ decreases. The three parameters depend on the type of $J/\psi$. For prompt $J/\psi$, $n$ is slightly larger and $T$ and $a_0$ are significantly less than those for $J/\psi$ from $b$.

Being the parameter which describes the excitation degree of emission source, the effective temperature $T$ of emission source for prompt $J/\psi$ is lower than that for $J/\psi$ from $b$. This implies the more energy deposition, lower threshold energy, and larger particle yields in the production process of prompt $J/\psi$. The $J/\psi$ from $b$ is produced from the fragmentation of open bottom hadron. Its emission source has larger $T$ and smaller $n$ due to larger mass of bottom, which has a harder spectrum than charm. Not only for prompt $J/\psi$ but also for $J/\psi$ from $b$, we have used the convolution of two TP-like functions to fit the spectra.

The situation of emission source for $J/\psi$ is different from light particles which are produced in the later stage of collisions. As early produced particle, $J/\psi$ at forward rapidity means less influence from the hot and dense QGP region, then higher degree of equilibrium and higher temperature of the emission source in the early stage are obtained. Indeed, in the forward rapidity region, the larger power exponent $n$ implies the entropy index $q$ being closer to 1 and higher degree of equilibrium of the emission source. Compared with that for $J/\psi$ with small rapidity, the larger $T$ implies that $J/\psi$ with large rapidity loses less energy when it goes through the hot and dense QGP region due to large flying velocity and less interaction time. Compared to the light particles produced in the later stage, $J/\psi$ produced in the early stage has different properties of source.

The dependences of (a) $n$, (b) $T$, and (c) $a_0$ on $y$ in the forward and backward regions, which are extracted from the spectra of prompt $J/\psi$ and nonprompt $J/\psi$ in $p$-Pb and $p$-$p$ collisions at 5.02 TeV, are displayed in Figure 8. The symbols represent the parameter values listed in the left panel in Tables 4 and 5 and obtained from the solid curves in Figures 4 and 5. One can see that with the increase of $|y|$, the parameters $n$ and $T$ increase and the parameter $a_0$ decreases. The three parameters depend on the types of $J/\psi$ which are produced from different sources. For prompt $J/\psi$, $n$ is slightly larger and $T$ and $a_0$ are significantly less than those for nonprompt $J/\psi$. These conclusions are consistent with Figure 7.

The discussions on the tendencies of parameters for Figure 7 are suitable to those for Figure 8, if the term “$J/\psi$ from $b$” is replaced by “nonprompt $J/\psi$”. The conclusions from Figures 7 and 8 confirm the current knowl-
edge of $J/\psi$ from different sources.

The dependences of (a) $n$, (b) $T$, and (c) $a_0$ on energy $\sqrt{s_{NN}}$ or $\sqrt{s}$ in the forward rapidity region, which are extracted from the spectra of prompt $J/\psi$ and $J/\psi$ from $b$ in $p$-Pb and $p$-$p$ collisions at the LHC (5.02, 7, 8, and 13 TeV), are displayed in Figure 9. The symbols represent the parameter values listed in the left panel in Tables 4 and 5 and obtained from the solid curves in Figures 4 and 5. One can see that with the increase of $\sqrt{s_{NN}}$ or $\sqrt{s}$ from 5.02 to 13 TeV, the three parameters do not show an obvious change. The relative sizes of the parameters for emissions of prompt $J/\psi$ and $J/\psi$ from $b$ are consistent with those from Figures 7 and 8.

The reason for the less obvious change is that the energy range considered in Figure 9 is very narrow, or the parameters saturate indeed in the considered energy range. Even if there are changes in the parameters with an increase in energy, it is hard to detect them in the narrow energy range. The production of prompt $J/\psi$ is from approximate equilibrium due to the fact that $n$ is large enough (and hence $q$ is close to 1). The production of $J/\psi$ from $b$ is also from approximate equilibrium, though its production source is very different from prompt $J/\psi$. Comparing with the production of $J/\psi$ from $b$, the production of prompt $J/\psi$ corresponds to lower excitation degree (smaller $T$) of emission source, which results in steeper spectrum and smaller $a_0$.

Figure 10 gives the dependences of (a) $n$, (b) $T$, and (c) $a_0$ on $\sqrt{s_{NN}}$ for different centrality classes in Au-Au, d-Au, Pb-Pb, and p-Pb collisions at the RHIC and LHC. The symbols represent the parameter values listed in the left panel in Tables 1–4 and obtained from the solid curves in Figures 1–4. The curves are the results fitted by empirical formulas which will be discussed later. One can see that $n$ and $T$ increases and $a_0$ does not change obviously with the increase of $\sqrt{s_{NN}}$ when it varies from...
the RHIC to LHC energies.

The parameter tendencies in Figure 10 indicate that, at the LHC, the higher collision energy results in more energy deposition in the collisions and higher excitation degree of the emission source. Then, more yields are produced, and quicker equilibrium is approached, from the statistical point of view. The almost invariant $a_0$ reveals that the spectrum shapes and hence the production mechanisms of $J/\psi$ at the RHIC and LHC are similar. Or, in the two energy ranges, the early produced $J/\psi$ traverse through similar participant regions, in which the matters formed in both the cases are hot and dense QGP. The initial energy of QGP formation is below 39 GeV [90].

Based on the above discussions we see that Eq. (3) is a generalization of Eq. (2) with one more parameter, and surely the TP-like function of Eq. (3) can fit experimental data better. As showed in Figures 1–5 by the curves and listed in Tables 1–5 for the $\chi^2$/ndof, we have obtained several times larger $\chi^2$ with fixed $a_0 = 1$ than those with changeable $a_0$ in most cases. That is, if we do not introduce the parameter $a_0$ in Eq. (3) and utilize directly Eq. (2) or the convolution Eq. (6) with $a_0 = 1$ to fit the data, worse fits will be obtained, though in some cases the two fits are comparable each other. So, we summarize that the introduction of $a_0$ is indeed necessary.

It seems that Tables 1–5 show that for each different set of data we need a different set of four parameters (three free parameters and one normalization constant), and there are no obvious relations between 4 parameters of different sets. In fact, there are tendencies of free parameters on centrality, rapidity, and collision energy. Then, we may predict some results in central collisions at other energies.

For example, the fitted curves in Figure 10 can be
Figure 8. Dependences of (a) \( n \), (b) \( T \), and (c) \( a_0 \) on \( y \) in the forward and backward regions, which are extracted from the spectra of prompt \( J/\psi \) and nonprompt \( J/\psi \) in \( p\text{-}Pb \) and \( p\text{-}p \) collisions at 5.02 TeV. The symbols represent the parameter values listed in Tables 4 and 5 and obtained from Figures 4 and 5.

approximately parameterized by (a) \( n = a_1 + b_1/[1 + c_1(\sqrt{s_{NN}})^{d_1}] \) (with \( a_1 = 5.279, b_1 = -5.361, c_1 = 0.018, \) and \( d_1 = 1.082 \)), (b) \( T = a_2 + b_2 \exp\{-2[(\sqrt{s_{NN}} - c_2)/d_2]^2\} \) (with \( a_2 = 0.517, b_2 = -1.832, c_2 = -6901.400, \) and \( d_2 = 7924.553 \)), and (c) \( a_0 = \sqrt{s_{NN}}/[a_3 + b_3\sqrt{s_{NN}} + c_3(\sqrt{s_{NN}})^{1/2}] \) (with \( a_3 = 210.596, b_3 = 6.214, \) and \( c_3 = -25.252 \)), respectively, where \( \sqrt{s_{NN}} \) is in the units of GeV. For 0–20\% Pb-Pb collisions at 8.8 TeV, we may obtain approximately \( n = 5.26, T = 0.516 \) GeV, and \( a_0 = 0.168 \) which are less centrality dependent according to Figure 6. Because of different types of spectra being used in experiments, the normalization constants are not uniform. We have not given the normalization constant for 0–20\% Pb-Pb collisions.

C. Further discussions

Before summary and conclusions, we would like to point out that the effective temperature \( T \) discussed above is not the real temperature. Because the effect of transverse flow is not excluded, we call \( T \) the effective temperature. If we exclude the effect of transverse flow, a smaller \( T \) will be obtained. However, being an early produced particle, \( J/\psi \) has less influence from the transverse flow which appears in the later stage of collisions. So, we think that the values of \( T \) extracted from \( J/\psi \) spectra in this work can be approximately regarded as the initial temperature \( T_i \) of the collision system. Because prompt \( J/\psi \) and nonprompt \( J/\psi \) from \( b \) are from different sources, the initial temperatures extracted from the two \( J/\psi \) spectra may be different. This is a reflection of the multi-source picture discussed by us.

Generally, the initial temperature \( T_i \) is larger than the
chemical freeze-out temperature $T_{ch}$ ($\sim 160$ MeV) which is extracted from the ratios of different particle yields. $T_{ch}$ is larger than the kinetic freeze-out temperature $T_{kin}$ or $T_0$ which is extracted from the transverse momentum spectra in the last stage of collisions. The results of the present work support this statement. Although the transverse flow affects slightly $T_i$, even if it does not affect $T_{ch}$, it affects obviously $T_0$. Because the transverse flow affects the transverse momentum spectra in the last stage of collisions, it also affects the quantities extracted from the mentioned spectra.

The revised index $a_0$ is a dimensionless quantity. However, its introduction causes the dimension or unit of $p_T^{a_0}$ to be (GeV/c)$^{a_0}$. This unit should be combined with the unit of normalization constant $C$ in Eq. (3) to result in the unit of $f(p_T)$ to be $(\text{GeV/c})^{-1}$. That is to say, the unit of $C$ in Eq. (3) is $(\text{GeV/c})^{-a_0-1}$. The values of $a_0$ are changeable. Thus, the units of $p_T^{a_0}$ and $C$ are also changeable. As a result, the unit of $f(p_T)$ is always $(\text{GeV/c})^{-1}$ which is independent of $a_0$.

In the above discussions, we have used the concept of equilibrium. This means that lots of particles should be considered. However, in many cases, the yields of $J/\psi$ are not too large. To consider lots of particles, we may include lots of events which are from the same collisions. Although the particles from different events are not correlated, their production proceeds under similar conditions, which allows increasing their statistics. From the statistical point of view, the particle productions in high energy collisions are a statistical process, rather than a thermodynamical process, though some quantities such as temperature can be used to connect the two processes.

In low statistics with single or few events, the concept of temperature can be also used, if the temperature reflects the average kinetic energies of the considered particles, even the kinetic energy of a given particle. Anyhow,
the temperature reflects the excitation degree of emission source. A low excitation degree of emission source corresponds to a low kinetic energy of the considered particle. This is similar to the case in single atomic and molecular physics, in which a cold atom or molecule has a low kinetic energy and then corresponds to a low temperature of emission source.

At the level of partons, we have an alternative discussion on the temperature of emission source in the collision system. Generally, we may think that lots of gluons and sea quarks taking part in the collisions, though only two partons are considered in Eq. (6) to contribute to the transverse momentum of particle. These partons stay at equilibrium or local equilibrium state. The concept of temperature is suitable and the transverse momentum or kinetic energy of particle is a reflection of the temperature.

Although the tendencies of parameters have been obtained in the above discussions, the main significance of the present work is in the field of methodology. According to the probability density functions (the TP-like functions) contributed by the two contributor partons, the transverse momentum distribution of particles is described by the convolution of two TP-like functions. The same idea can be used in the description of the spectra of different particles and jets.

The TP-like function is generalized from Eq. (2) which is one form of the Tsallis distribution \([65, 71, 82, 86]\). In those studies, although the spectra of \(J/\psi\) are approximated described by the Tsallis distribution, the present fits are better in most cases due to smaller \(\chi^2/\text{ndof}\). In addition, the present fits are suitable for the spectra of different particles which include leptons, mesons, and baryons \([40, 11]\), as well as various jets \([87]\) due to the introduction of \(a_0\). Not only for other studies but also for this work, \(q\) is close to 1 which means the system or
parton sources being approximately in equilibrium when different particles emit. However, the partons are not in equilibrium in some cases when jets emit.

Except the widely applications of the Tsallis distribution with different forms [68, 71, 82, 83], there are some studies on the Tsallis distribution itself. The Tsallis distribution with \( q > 1 \) may cover two or three Boltzmann-Gibbs, Fermi-Dirac, or Bose-Einstein distributions [91]. If one associates the Tsallis statistics with the Tsallis-2 statistics [94–97], the Tsallis distribution belongs to the Tsallis-2 statistics [94, 95]. In the case when one associates the Tsallis distribution [69, 92, 93] with the Gibbs, Fermi-Dirac, or Bose-Einstein distributions [91].

IV. SUMMARY AND CONCLUSIONS

We have aggregated the transverse momentum spectra of \( J/\psi \) produced in high energy collisions measured by several collaborations at the RHIC, Tevatron, and LHC. Two contributor partons, a charm quark and an anti-charm quark, are considered in the production of \( J/\psi \). The contribution of each quark to \( J/\psi \) transverse momentum is described by the TP-like function. The convolution of two TP-like functions is successful in fitting the \( J/\psi \) spectrum. Three free parameters, the power exponent \( n \), effective temperature \( T \), and revised index \( a_0 \) are extracted from the spectra with different centrality classes, rapidity intervals, and energy ranges.

With the decrease in collision centrality from central to peripheral collisions, \( n \) increases slightly, \( T \) does not change noticeably, and \( a_0 \) decreases a little bit in most of the cases. The spectra of \( J/\psi \) are not affected largely by the centrality, or by the size of participant region and the effect of cold spectator nucleons. The \( J/\psi \) produced in the early stage of collisions has less interactions with the hot and dense QGP and the cold spectator nucleons. The emission source of \( J/\psi \) and the collision system stay approximately at equilibrium.

With the increase of energy, \( n \) and \( T \) increases and \( a_0 \) does not change obviously. The higher collision energy results in more energy deposition, higher excitation degree, more particle yields, and quicker equilibrium. At the RHIC and LHC, the spectrum shapes and the production mechanisms of \( J/\psi \) are similar. The early produced \( J/\psi \) undergo through similar hot and dense QGP regions with less influence from the transverse flow. The values of \( T \) extracted from \( J/\psi \) spectra can be approximately regarded as the initial temperature of the system.

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Data availability statement

This manuscript has no associated data or the data will not be deposited. [Authors’ comment: The data used to support the findings of this study are included within the article and are cited at relevant places within the text as references.]

Compliance with ethical standards

Ethical approval

The authors declare that they are in compliance with ethical standards regarding the content of this paper.

Disclosure

The funding agencies have no role in the design of the study; in the collection, analysis, or interpretation of the data; in the writing of the manuscript; or in the decision to publish the results.

Conflicts of interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.
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