A Unified Weight Learning and Low-Rank Regression Model for Robust Face Recognition

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Abstract

Regression-based error modelling has been extensively studied for face recognition in recent years. The most important problem in regression-based error model is fitting the complex representation error caused by various corruptions and environment changes. However, existing works are not robust enough to model the complex corrupted errors. In this paper, we address this problem by a unified sparse weight learning and low-rank approximation regression model and applied it to the robust face recognition in the presence of varying types and levels of corruptions, such as random pixel corruptions and block occlusions, or disguise. The proposed model enables the random noise and contiguous occlusions to be addressed simultaneously. For the random noise, we proposed a generalized correntropy (GC) function to match the error distribution. For the structured error caused by occlusion or disguise, we proposed a GC function based rank approximation to measure the rank of error matrix. An effective iterative optimization is developed to solve the optimal weight learning and low-rank approximation. Extensive experimental results on three public face databases show that the proposed model can fit the error distribution and structure very well, thus obtain better recognition accuracy in comparison with the existing methods.

Introduction

The regression-based error model can be roughly classified into two categories: the mean square error (MSE) based ones and the robust function based ones. The most representative approaches of MSE-based approaches is the sparse representation classifier (SRC) (Wright et al. 2009) which takes advantages of the powerful feature selection ability of sparse representation to learn discriminative features for robust face recognition. Then, (Deng, Hu, and Guo 2017) extended the SRC by proposing an auxiliary intraclass variant dictionary to characterise the variation between the training and testing images. (Huang et al. 2013) took advantage of the $l_{2,1}$-norm and took the label information into consideration to obtain more discriminative features. However, performances of these MSE-based methods mentioned above can be significantly deteriorated when the data are corrupted by outliers, which is inevitable in real-world applications. Outliers are typically far away from the centre of the normal data, but MSE-based loss function assigns same weights to all measures without any discriminative constraints on outlier severely or slightly corrupted ones when minimizing the representation error, as a consequence, such an equal weight assignment will result in an incorrect sparse solution. Moreover, the MSE-based loss function assumes that the error follows the Gaussian distribution, which is not only sensitive to non-Gaussian noise but also to outliers. As a consequence, these existing methods fail to approximate the sparse coding if the assumption does not hold (He, Zheng, and Hu 2011). To overcome these drawbacks, (Yang et al. 2012) proposed a regularized robust classifier (RRC) using the local quadratic approximation and a reweighted least squares solution is provided. (Zheng et al. 2017) proposed an iteratively reconstrained group sparse classifier (IRGSC) in which an adaptive weight learning procedure is proposed to give more emphasis on normal image pixels while suppressing the noise and outliers.

However, the above algorithms are based on the vector space and ignore the spatial correlation among error pixels, which cannot preserve the inherent structure of the image, resulting in inferior recognition performance. Recently, many researchers argue that the representation error has a specific structure when there are contiguous errors caused by partial occlusion (Iliadis et al. 2017) [Xie et al. 2017] [Yang et al. 2016]. To make better use of the structure of error, (Qian et al. 2015) proposed to use the low-rank property to approximate the structure of the error image with occlusions. However, the rank minimization is an NP-hard problem and is difficult to be optimized. A convex-relaxation, i.e. nuclear norm, based regression (Yang et al. 2016) was proposed to approximate the rank of the error image. Although the nuclear norm based approximation has improved the performance of face recognition in the presence of occlusions. The nuclear norm based rank approximation treats each singular value equally, i.e. it shrinks each singular value with the same threshold regardless of their contribution to
the image reconstruction, which will lead to biased estimation. In face recognition problem, the larger singular values of the error image represent the error information corresponding to the occlusion, the smaller ones represent normal image pixels. When using the low-rank approximation for the error image, we hope the error image contains the occlusion information as much as possible and face information as less as possible. Thus we should give less punishment to larger singular values and larger punishment to smaller singular values. Then the resulted error image under a low-rank constraint nearly only contains the error information, which means a better approximation. To achieve a better low-rank approximation, some researchers use the non-convex relaxations (Xie et al. 2017; Zheng et al. 2019; Dong, Zheng, and Lian 2019). (Luo et al. 2016) used the Schatten-$p$-norm and obtained a more accurate estimation for error image. (Xie et al. 2017) proposed to use a set of non-convex function to better approximate the low-rank structure of the error image. However, the Schatten-$p$-norm treats all the singular value equally, and the convex relaxations in (Luo et al. 2016) may not an optimal of low-rank approximation.

Considering that existing methods mentioned above cannot fit the complex representation error distribution and structure very well, in this paper, we propose a unified sparse weight learning and low-rank approximation regression model based on the generalized correntropy to tackle these problems. By choosing different $\alpha$ values in the generalized correntropy, the proposed model can fit various error distribution and approximate the rank of error very well, which can be seen from Figure 1. The key points of the proposed method are summarized as follows.

- The first attempt to use only one function to address both error distribution and structure estimation.
- The proposed algorithm can better fit the complicated error distribution caused by the variation of illumination, expressions, poses, positions, noise, and occlusions.
- A new and more accurate low-rank approximate estimator is proposed for contiguous error structure estimation.

### Related Works

#### Robust Weight Learning

(He, Zheng, and Hu 2011) proposed a correntropy induced metric (CIM) based loss function for robust face recognition. They adaptively learn a weight for the representation error, by which the larger errors corresponding to the noise and outliers receive smaller weights (larger penalty) while the smaller errors receive larger weights (smaller penalty). Given a query image vector $y \in \mathbb{R}^m$, and the training dataset $D \in \mathbb{R}^{m \times n}$, the CIM-based loss function and weight estimators are defined as follows.

$$
\hat{J} = \max_{x,w} \sum_{j=1}^{m} (w_j (y_j - \sum_{i=1}^{n} d_{ij} x_i)^2 - \phi(w_j)) - \lambda \sum_{i=1}^{n} x_i,
$$

subject to (s.t.) $x_i \geq 0, \ i = 1, \ldots, n.$ \hspace{1cm} (1)

Figure 1: (a) Weight distribution with different parameters; (b) The approximation of different functions for the rank function. Note that the approximated rank of the error by the proposed method (in red) has almost overlapped with the true rank (in green) when $\delta$ is greater than 2.

where the weight is calculated by

$$
w^{t+1}_j = -g(y_j - \sum_{i=1}^{n} d_{ij} x^t_i), \hspace{1cm} (2)
$$

where $g(x)$ is the Gaussian function. Considering that the Gaussian function in (He, Zheng, and Hu 2011) is not robust enough to match the error when there are heavy noise and large occlusions, (Iliadis et al. 2017) and (Yang et al. 2012) proposed to use the logistic function as a weight descriptor to match the error distribution

$$
w^{t+1}_j = \frac{\exp(-\gamma \epsilon_j^2 + \gamma \theta)}{1 + \exp(-\beta \epsilon_j^2 + \beta \theta)}, \hspace{1cm} (3)
$$

Different from learning weight using a specific function as in (He, Zheng, and Hu 2011) (Iliadis et al. 2017), (Zheng et al. 2017) proposed an iterative procedure to adaptively learn the weight by solving a constrained sparse learning problem. Their model is defined as follows:

$$
\arg\min_{w, t=1, \ldots, n, x} \frac{1}{2} \|\sqrt{w}(y - Dx)\|^2_2 + \gamma \|w\|^2_2, \hspace{1cm} (4)
$$

where $w = [w_1, \ldots, w_j, \ldots, w_m]$ with each $w_j$ updated by

$$
w^{t+1}_j = (-\frac{d_i}{2\gamma} + \eta)^+, \hspace{1cm} (5)
$$

where $d_i = \epsilon_i^2$, and $\eta$ is the Lagrangian multiplier. Here $(-\cdot)^+$ is a threshold function.

#### Low-Rank Approximation

Both (Iliadis et al. 2017) and (Yang et al. 2016) introduce the nuclear norm as the low-rank approximation of the error image in the presence of contiguous occlusions. Assume the matrix $E$ is the error image, they calculate the rank-constrained error using the following formulation.

$$
\min_{E} \frac{1}{2} \|\hat{E} - E\|^2_F + \lambda \|E\|_*, \hspace{1cm} (6)
$$

where the nuclear norm $\| \cdot \|_*$ is a rank approximation function. Then the optimal low-rank constrained error image is given by

$$
\hat{E}^* = USV^T, \hspace{1cm} (7)
$$
where \( U, V \) are the left and right singular vectors of \( E, S = \text{sign}(\delta_i)\max(0, |\delta_i| - \lambda) \), and \( \delta_i \) is the singular value of \( E \).

Since the nuclear norm based low-rank approximation treats each singular value equally regardless their contributions to the error image, \cite{xie2017generalized} proposed to use the non-convex function to better approximate the low-rank structure of the error image. They use the \( l_p \)-norm, log-sum, atan, and log-exp functions as the non-convex relaxation of the rank function. Their robust low-rank model is

\[
\min_{E} \frac{1}{2} \|E - E^*\|_F^2 + \lambda \|E\|_{\omega,*} \tag{8}
\]

which has the following closed-form

\[
E^* = U S_{\omega,*} V^T, \tag{9}
\]

where \( U \) and \( V \) are the left and right singular vectors of \( E \), and \( S_{\omega,*} \) is the weighted Singular Value Thresholding (SVT) operator,

\[
S_{\omega,*} = \text{diag}(\max(\delta_i - \omega_i \lambda, 0)), \quad i = 1, 2, \cdots, m_2,
\]

where \( \delta_i \) is the singular value of \( E \), and \( \omega_i \) is a weight controlling the shrinkage level of each singular value.

### Proposed Method

In this section, the generalized correntropy metric (GCM) is proposed for face recognition. The GCM introduces a nonconvex penalty to enhance the sparsity in both the intrinsic low-rank structure and sparse corruption. The proposed GCM penalty can overcome the aforementioned drawbacks of the sparse corruption and nuclear norm.

#### The GCM Penalty

Inspired by the remarkable performance of nonconvex regularizers used in image processing \cite{xie2017generalized}, a nonconvex regularizer represented as \( f_{GC}(x) \) is introduced to measure the contribution of each point \( e \in \mathbb{R}^m \). \( f_{GC}(x) \) is defined as the generalized correntropy loss function by

\[
f_{GC}(A, B) = \frac{1}{2} E \left[ \|\varphi_{\alpha,\beta}(A) - \varphi_{\alpha,\beta}(B)\|_H^2 \right]
\]

\[
= \frac{1}{2} E\left[ \langle \varphi_{\alpha,\beta}(A), \varphi_{\alpha,\beta}(A) \rangle - 2 \langle \varphi_{\alpha,\beta}(A), \varphi_{\alpha,\beta}(B) \rangle + \langle \varphi_{\alpha,\beta}(B), \varphi_{\alpha,\beta}(B) \rangle \right] \tag{11}
\]

\[
= E\left[ (G_{\alpha,\beta}(0) - G_{\alpha,\beta}(e)) \right]
\]

where \( e = A - B, E(x) \) is the expectation of \( x, \varphi_{\alpha,\beta}(\cdot) \) denotes a nonlinear mapping which transforms its argument into a high-dimensional Hilbert space \cite{chen2016generalized}, and \( G_{\alpha,\beta} \) is the Generalized Gaussian Density (GGD) function

\[
G_{\alpha,\beta}(e) = \frac{\alpha}{2\beta \Gamma(1/\alpha)} \exp\left( - \frac{|e|^2}{\beta} \right) \tag{12}
\]

Here \( \alpha > 0 \) and \( \beta > 0 \) are the parameters of GGD indicating the peak and width of the probability density function. \( \Gamma(z) = \int_{-\infty}^{\infty} e^{-t^2} t^{z-1} dt, (z > 0) \) is the gamma function. \( \lambda = 1/\beta^\alpha \) and \( \gamma_{\alpha,\beta} = \alpha/(2\beta \Gamma(1/\alpha)) \) are the kernel parameter and the normalization constant, respectively. Obviously, the Gaussian function is just a special case of the generalized Gaussian density function when \( \alpha = 2 \). When \( \alpha = 1, (12) \) becomes the Laplacian distribution. We plot the GGD distributions with several shape parameters in Figure 1(a) which shows that smaller values of \( \alpha \) give heavier tails (sharper distributions) when \( \alpha \to 0 \), the GGD is close to the uniform distribution, while \( \alpha \to \infty \) approaches an impulse function. Thus, owing to the flexibility of shape parameter selection, the GGD function can match the errors of different distributions very well.

By combing (11) and (12), the generalized correntropy function in (11) can be rewritten as another form as

\[
f_{GC}(e) = \gamma_{\alpha,\beta}(1 - \exp(-\lambda |e|^\alpha)) \tag{13}
\]

which can be used for the weight learning. Here we use the \( \|e\|_\infty \) to normalize the representation error and ensure the errors to be in the same scale, i.e. \( e = \frac{e}{\|e\|_\infty} \). For the low-rank approximation, the GC-function is defined as follows:

\[
f_{GC}(\delta(E)) = \gamma_{\alpha,\beta}(1 - \exp(-\lambda |\delta(E)|^\alpha)) \tag{14}
\]

where \( E \) is the matrix form of the representation error vector \( e \), and \( \delta(E) \) represents the singular values of \( E \).

We can see that the function \( f_{GC}(\bullet) \) treats each entry adaptively. For the weight learning, it gives a larger weight for a smaller error, while gives a smaller weight for a larger error, which reduces the influence of outliers. For the low-rank constraint, it shrinks the larger singular value less and the smaller singular value more, which can better approximate the rank of the error image. Figure 1 shows the error fitting curve and the rank approximation using different functions. In Figure 1(a), the GC function can fit different levels of error, especially the smaller residual errors. Figure 1(b) shows that the GC-function has a good approximation of the true rank.

#### The Proposed GCM

Motivated by the advantages of the GC-function in learning discriminative weights and in approximating rank for representation errors, we consider to use \( f(e) \) and \( f(\sigma(E)) \) as the vector weight and matrix rank surrogate function to learn more robust features for face recognition in the presence of noise, outliers, and occlusions. The proposed GCM model for robust face recognition is defined as

\[
\min_{w,x} \left[ f_G(e) + \lambda_1 f_G(\sigma(E)) + \lambda_2 v(x) \right],
\]

\[
s.t. \ y - Dx = e, \quad E = TM(e), \tag{15}
\]

where \( \lambda_1 > 0 \) and \( \lambda_2 > 0 \) are regularization parameters used to control the tradeoff between the constraints of sparsity and matrix rank, and \( TM(x) \) means transforming the vector \( x \) to the matrix form \( X \).

### Optimization by Majorization Minimization

In this section, we use a majorization minimization (MM) algorithm to solve problem (15). In MM, instead of solving the complicated nonconvex optimization problem directly, it replaces the original function with its upper bound surrogate function in majorization step and then minimize the resulted function in minimization step.
Majorization Procedure

We first find the majorization function (upper bound) \( f(x|x') \) of \( f(x) \), which satisfies \( f(x|x') \geq f(x) \). Then the optimal solution to the surrogate function can be solved by

\[
x_{t+1} = \arg \min_x f(x|x_t).
\]

As in [Sun, Babu, and Palomar 2016], the first order Taylor expansion of \( f(x) \) is used as a surrogate function as follows:

\[
f(x) \leq f(x_t) + f'(x_t)(x - x_t) = f(x|x_t).
\]

Then \( f(x) \) can be upperbounded as

\[
f(x) \leq f(x_t) + f'(x_t)x + c,
\]

where \( c \) is a constant. Thus, the majorization functions for the weight learning and low-rank approximation are

\[
f_{GC}(e) \leq f_{GC}(e_t) + f'_{GC}(e_t) (e - e_t),
\]

and

\[
f_{GC}(\delta(E)) \leq f_{GC}(\delta_t(E)) + f'_{GC}(\delta_t(E)) (\delta(E) - \delta_t(E)).
\]

Minimization Procedure

Based on the above analysis, minimizing the objective function in (15) can be solved by minimizing the following surrogate function

\[
\arg \min_{e, x, h} f_G(e|e_t) + \lambda_1 f_{GC}(\sigma(E)|\sigma_t(E)) + \lambda_2 v(\mathbf{h}),
\]

s.t. \( y - Dx = e, \quad E = TM(e), \quad x = h. \)

The Lagrangian function of (23) is

\[
L(e, x, h, v_1, v_2) = f_{GC}(e|e_t) + \lambda_1 f_{GC}(\sigma(E)|\sigma_t(E)) + \lambda_2 v(\mathbf{h}) + \nu_1^T(y - Dx - e) + \frac{\rho_1}{2} \|y - Dx - e\|_2^2
\]

\+
\nu_2^T(x - h) + \frac{\rho_2}{2} \|x - h\|_2^2,
\]

where \( \rho_1 \) and \( \rho_2 \) are positive penalty parameters, and \( v_1 \) and \( v_2 \) are the dual variables. The optimal parameters can be updated by the following ADMM procedure.

\[
e_{t+1} = \arg \min_e L(e, x_t, h_t, v_1, v_2, v_2), \quad h_{t+1} = \arg \min_h L(e_{t+1}, x_t, h, v_1, v_2, v_2), \quad x_{t+1} = \arg \min_x L(e_{t+1}, x, h_{t+1}, v_1, v_2, v_2),
\]

\[
v_{1,t+1} = v_{1,t} + \rho_1 (y - Dx_{t+1} - e_{t+1}), \quad v_{2,t+1} = v_{2,t} + \rho_2 (x_{t+1} - h_{t+1}).
\]

- **Updating** \( e_{t+1} \): the optimal \( e_{t+1} \) can be updated by the following problem,

\[
e_{t+1} = \arg \min_e f_{GC}(e|e_t) + f_{GC}(\sigma(E)|\sigma_t(E)) + \frac{\rho_1}{2} \|y - Dx - e\|_2^2. \quad (26)
\]

To calculate \( e_{t+1} \), we consider a two-step fast approximation. In Step one, we first solve the following problem.

\[
\hat{e} = \arg \min_e f_{GC}(e|e_t) + \frac{\rho_1}{2} \|y - Dx - e\|_2^2.
\]

where

\[
f_{GC}(e|e_t) = f_{GC}(e|e_t)e = \left[ \gamma_\alpha (1 - \exp(-\lambda|e_i|^\alpha)) \right]' e
\]

\[
= \frac{1}{2} \gamma_\alpha \lambda \exp(-|e_i|^{\frac{\alpha}{\beta}})|e_i|^{\frac{\beta - 1}{\beta}} e^2
\]

\[
\propto \|\sqrt{w} \odot e\|_2^2.
\]

Then (27) can be rewritten as

\[
\hat{e} = \arg \min_e \|\sqrt{w} \odot e\|_2^2 + v_1^T(y - Dx - e) + \frac{\rho_1}{2} \|y - Dx - e\|_2^2.
\]

(29)

Obviously, (29) has a closed-form solution, i.e.,

\[
\hat{e} = \frac{y - Dx + \frac{v_1}{\rho_1}}{I + \frac{v_1}{\rho_1}}.
\]

We then solve the low-rank approximation problem in Step two as follows.

\[
E_{t+1} = \arg \min_e \frac{1}{2} \|E - \hat{E}\|_F^2 + \lambda_1 f_{GC}(\delta(E)|\delta_t(E))
\]

\[
= \arg \min_e \frac{1}{2} \|E - \hat{E}\|_F^2 + \lambda_1 \sum_{i=1}^m f_{GC}(\delta_i(E)|\delta_i(E))
\]

\[
= \arg \min_e \frac{1}{2} \|E - \hat{E}\|_F^2 + \lambda_1 \|E\|_{GC}.
\]

where \( \|E\|_{GC} \) is the proposed robust low-rank approximation. \( E \) can be updated by \( E = U\Sigma V^T \), where \( \Sigma = \text{diag}(a_1, \ldots, a_m) \) with \( a_i = \max(\delta_i - \mu_i \lambda_1, 0) \). Udiag(\( \delta_1, \ldots, \delta_m \)\( V^T \) is the singular value decomposition (SVD) of \( \hat{E} \), and \( w_i = g'(\delta_i(E)) \). Then the optimal \( e_{t+1} \) is obtained by vectorizing \( E \).

- **Updating** \( h_{t+1} \): In this paper, we use the \( l_2 \)-norm to regularize the coefficient \( x \) (or \( h \)). We update \( h_{t+1} \) by solving the following problem.

\[
h_{t+1} = \arg \min_h \lambda_2 \|h\|_2^2 + \frac{\rho_2}{2} \|x_t - h\|_2^2 + v_2^T(x - h)
\]

\[
= \left( x_t + \frac{v_2}{\rho_2} \right). \quad (32)
\]
The UWLLA algorithm is summarized in Algorithm 1.

Algorithm 1 The UWLLA Algorithm

Input: Given a test image \( y \in \mathbb{R}^m \), and a set of training images \( D = \{d_1, d_2, \ldots, d_n\} \in \mathbb{R}^{m \times n} \) with each \( d_i \in \mathbb{R}^m \) being a training sample, \( \alpha, \beta, \lambda_1, \lambda_2, \rho_1 \) and \( \rho_2 \), Initializing \( e \) and \( y \).

Output: \( x^* \), \( w^* \).

1: while \( t = 1, \ldots, T \) do
2: \hspace{1em} Updating weights by \( w = \exp(-\beta_2 \|e_t\|^2) / \|e_t\|^2 - 1 \).
3: \hspace{1em} Updating \( e \) by (27)-(30).
4: \hspace{1em} Updating \( E_{t+1} \) by (31).
5: \hspace{1em} Updating \( h_{t+1} \) by (32).
6: \hspace{1em} Updating \( x_{t+1} \) by (35).
7: \hspace{1em} Updating \( v_{1,t+1} \) by \( v_{1,t+1} = v_{1,t} + \rho_1(y - Dx - e) \).
8: \hspace{1em} Updating \( v_{2,t+1} \) by \( v_{2,t+1} = v_{2,t} + \rho_2(x_{t+1} - h_{t+1}) \).
9: \hspace{1em} If \( \epsilon > 1e-5 \) then
10: \hspace{2em} repeat
11: \hspace{2em} else
12: \hspace{2em} \hspace{1em} \( t \leftarrow t + 1 \); Break;
13: \hspace{1em} end if
14: end while
15: \hspace{1em} Identity \( y = \arg\min_{x} \|\sqrt{w}^{-1}(y - Dk_i(x^*))\|^2_2 \), where \( k_i(x^*) \) is a subvector of \( x^* \) corresponding to the coefficients of training samples from the \( i \)-th class.

Computational Complexity and Convergence

Suppose \( y \in \mathbb{R}^m \) and \( Y \in \mathbb{R}^{m_1 \times m_2} \) \((m_1 \leq m_2)\) are a testing image vector and its matrix form, and the training set is \( D \in \mathbb{R}^{m \times n} \). The computational complexity for Step 3 in Algorithm 1 is \( O(n) \), for Step 4 is \( O(mn) \) which is determined by the matrix multiplication \( D x \), and for Step 5 is \( O(m_1 m_2^2) \) which is determined by the SVD of matrix \( E \). Step 7 requires \( mn \) multiplications for \( D^T y \). Thus, the total computational complexity for Algorithm 1 is \( O(T(n + mn + m_1 m_2^2 + mn)) \), where \( T \) is the number of iterations.

The convergence analysis of MM has been well studied in (Hunter and Lange 2004), thus according to the theory of MM, we have \( f_{GC}(e_{t+1}) \leq f_{GC}(e_t), \) and \( f_{GC}(\delta(E)_{t+1}) \leq f_{GC}(\delta(E)_{t}) \), which indicates that the objective function in (15) will monotonically decrease. Moreover, the ADMM optimization problem in the UWLLA algorithm can be divided into three main subproblems (refer to (25)), each of which is convex with respect to one variable. The convergence analysis of the ADMM algorithm has been well studied in (Boyd et al. 2011) [Yang et al. 2016] [Shang et al. 2017], and has been further verified in different applications, such as (Hu and Chen 2018) [Zhu et al. 2017] [Piao et al. 2019]. Thus, we can find a locally optimal solution for each subproblem.

Experimental results

Databases and Parameter Settings

To verify the effectiveness of the proposed algorithm, we carry out experiments on three public face databases, including Extended Yale B (ExYaleB) (AS, Georgiades and Kriegman 2001), AR (Martinez and Benavente 1998), and aligned Labeled Face in the wild (LFW-a) (Wolf, Hassner, and Taigman 2009). The proposed algorithm is tested and compared with recently published face recognition approaches, including CESR (He, Zheng, and Hu 2011), RRC-L1 and RRC-L2 (Yang et al. 2012), HQ-A and HQ-M (He et al. 2013), F-LR-IRNNLS (Iliadis et al. 2017), IRGSC (Zheng et al. 2017), NMR (Yang et al. 2016).

\( \alpha \) and \( \beta \) are two important parameters in the proposed regression model where the former models the shape of the error distribution, and the latter is the kernel width. Denoted by \( \alpha_1 \) and \( \beta_1 \) the parameters for the weight learning, and \( \alpha_2 \) and \( \beta_2 \) for the rank approximation. In this paper, \( 1 < \alpha_1 < 2 \) is used for weight learning for all the experiments. We fixed \( \alpha_2 = 1 \) and \( \beta_2 = 0.7 \) for a better low-rank approximation for all the experiments. For the proposed ADMM optimization algorithm, we use \( \rho_1 = 1, \rho_2 = 0.1, \lambda_1 = 0.01, \lambda_2 = 1 \).

Experiments on the ExYaleB database

In this experiment, the images from the ExYaleB face database are resized to \( 96 \times 84 \) pixels. We adopt two experimental settings, one is the testing data with different percentages of occlusions, another is the testing data with different percentages of occlusion-pixel mixed corruptions. For both experiments, we choose all the images in subsets 1 and 2 for training, and subset 3 for testing. Thus, the total number of images for training and testing are 719 and 455, respectively.

In the first experiment, we evaluate the proposed algorithm on the dataset with occlusion percentage varying from 20% to 70%. To simulate occlusions, we randomly selected local region in each testing image and replace this area with an unrelated image. In this experiment, we use the baboon image, as used in (Iliadis et al. 2017) [Xie et al. 2017], for occlusion. To simulate a specific percentage of occlusion for a testing image \( Y \in \mathbb{R}^{m_1 \times m_2} \), we resize the baboon image to \( z \times z \), where \( z = \sqrt{m_1 \times m_2 \times \alpha} \) and replace the local region in the testing image. One example of the occluded test-
Table 1: The recognition accuracy of all the algorithms on the ExYaleB face database with 60% occlusion (60% Occ.) and 60% mixed corruption (60% Mix.).

| Corruptions | CESR  | RRC-L1 | RRC-L2 | HQ-A  | HQ-M  | FLR-IRNLS | IRGSC | NMR  | Proposed |
|-------------|-------|--------|--------|-------|-------|------------|-------|------|----------|
| 60% Occ.    | 41.85 | 69.67  | 70.54  | 48.02 | 68.13 | 95.82      | 66.15 | 79.12 | 98.46    |
| 60% Mix.    | 27.27 | 34.72  | 35.60  | 17.92 | 32.15 | 49.01      | 27.91 | 8.64 | 64.75    |

The recognition accuracy of all algorithms on the data with 60% occlusion are shown in Table 1 where the proposed algorithm obtained the highest accuracy 98.46%. The recognition rates from the proposed method and all the benchmarks under different percent of occlusions are shown in Figure 3(a), which show that our method nearly 100% recognize testing images when occlusion percent is no larger than 50%, and still maintain the highest accuracy when occlusion percent is larger than 50%. Especially, the accuracy of the proposed method are nearly 3% and 30% higher than that of the second largest one under case of 60% and 70% occlusion, respectively.

Experiments on the AR database

In this experiment, we also test the proposed algorithm on the dataset with real sunglasses and scarf occlusions, and mixed corruptions. In AR database, there are two sessions of facial images from 100 subjects (50 male and 50 female). In each session, there are 2 natural unoccluded face images, 3 face images with scarf disguise and 3 with sunglasses. Some examples of the testing image are shown in Figure 4.
Table 2: The recognition accuracy of all the algorithms on the AR face database with different occlusions.

| Evaluation Types | CESR | RRC-L1 | RRC-L2 | HQ-A | HQ-M | FLR-IRNNLS | IRGSC | NMR | Proposed |
|------------------|------|--------|--------|------|------|------------|-------|-----|----------|
| Session 1        |      |        |        |      |      |            |       |     |          |
| sunglass scarf   | 60.54| 75.33  | 77.00  | 68.00| 72.67| 83.33      | 73.33 | 75.67| 93.00    |
|                  | 19.46| 61.66  | 64.33  | 30.54| 35.23| 55.67      | 54.33 | 60.74| 66.33    |
| Session 2        |      |        |        |      |      |            |       |     |          |
| sunglass scarf   | 67.11| 82.66  | 83.66  | 70.13| 72.48| 86.24      | 77.67 | 80.47| 93.67    |
|                  | 13.67| 60.66  | 61.66  | 25.33| 30.00| 48.00      | 54.33 | 54.00| 63.00    |
| Both sessions    |      |        |        |      |      |            |       |     |          |
| sunglass scarf   | 66.72| 83.00  | 83.16  | 70.23| 72.58| 85.79      | 77.33 | 72.53| 94.67    |
|                  | 17.33| 67.33  | 69.33  | 29.67| 34.17| 57.33      | 62.83 | 64.00| 71.33    |

2 natural images are selected as training images, and 6 images with sunglasses and 6 images with scarf for testing. The recognition rates from the proposed algorithm and all the benchmarks are shown in Table 2, which shows that the proposed algorithm outperforms all the benchmarks in terms of single session and both session testing. Figure 5 displays the performance of all the algorithms with different levels of sunglasses-random pixel mixed and scarf-random pixel mixed corruptions. The accuracy curves show that the proposed algorithm significantly outperforms the benchmarks.

Experiments on the LFW database

To evaluate the robustness of the proposed algorithm for face recognition under an unconstrained environment, we then carry out experiments on the LFW database. We use the aligned version LFW-a database for all the experiments. We select 158 subjects with each subject no less than 10 samples. For each subject, we randomly select 5 samples for training and 5 samples for testing. Thus, the number of training samples and testing samples are both 790.

First, we test all the algorithms on the clean images to verify the effectiveness of the proposed algorithm in fitting the representation error caused by illumination, pose, and expression changes. Two types of corruptions, i.e., block occlusion (baboon image) and occlusion-pixel mixed corruption, are adopted to further evaluate the robustness of the proposed algorithm. All the algorithms are tested on the data with 20% block occlusion (20% Occ.), 20% mixed corruptions (20% Mix.), 40% Occ., and 40% Mix. Table 3 lists the recognition accuracy of different algorithms on the clean and corrupted datasets, which shows that the proposed algorithm is superior to other benchmarks in a complex environment.

Discussion on the selection of \(\alpha\) and \(\beta\)

According to above experimental results, we empirically choose values of \(\alpha_1\) and \(\beta_1\) from \(\alpha_1 \in [1.2]\) and \(\beta_1 \in [0.05, 0.15]\) for the robust weight learning. When the errors are disturbed by larger outliers which will cause heavy-tailed noise, then a lower-order statistical measure (smaller \(\alpha_1\)) for the error is usually more robust. In this paper, we use \(\alpha_1 = 1.7\) for all the experiments to handle different types of corruptions. Choosing the value of \(\beta_1\) is also important for the proposed model. A smaller \(\beta_1\) leads to a thinner distribution, while larger \(\beta_1\) leads to a fatter distribution. The error with a thiner distribution is usually caused by a simple corruption, e.g., corruption from different occlusions. A fatter distribution is caused by more complicated corruptions, e.g., occlusion-pixel mixed corruptions. Thus, for all the experiments, we use a smaller \(\beta_1 = 0.07\) for the experiments with occlusions, and a larger \(\beta_1 = 0.11\) for the experiments with occlusion-pixel mixed corruptions. Different from matching the error distribution in weight learning, the GC function for low-rank approximation tries to give more emphasis on the larger singular values and thus can maintain the low-rank structure of the error image. In all the experiments, \(\alpha_1 = 1\) and \(\beta_2 = 0.7\) can provide a good low-rank approximation.

Table 3: The recognition accuracy of all the algorithms on the clean LFW database and data with various corruptions.

| Methods     | Clean | 20% Occ. | 20% Mix. | 40% Occ. | 40% Mix. |
|-------------|-------|----------|----------|----------|----------|
| CESR        | 59.95 | 54.75    | 50.19    | 35.28    | 23.35    |
| RRC-L1      | 68.98 | 65.06    | 63.29    | 47.34    | 25.56    |
| RRC-L2      | 69.36 | 60.12    | 57.08    | 40.63    | 19.62    |
| HQ-A        | 51.27 | 51.33    | 57.09    | 35.28    | 30.33    |
| HQ-M        | 58.30 | 58.63    | 58.94    | 43.91    | 32.99    |
| FLR-IRNNLS  | 71.65 | 63.37    | 62.74    | 51.02    | 37.56    |
| IRGSC       | 73.42 | 65.33    | 50.51    | 37.09    | 13.04    |
| NMR         | 72.62 | 54.63    | 41.83    | 32.23    | 14.45    |
| Proposed    | 74.18 | 66.78    | 65.19    | 52.03    | 46.70    |
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