On Discrete Gauge Symmetries in Trinification Model

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Given the important role of discrete gauge symmetries in viable models, we discuss these symmetries in intersecting D6-brane trinification model where the $Z_N$ symmetry is investigated and its identification is shown.

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I. INTRODUCTION

Recently, intersecting D-brane models in string theory have attracted much attention in viable particle physics models [1–5]. Considerable works in this direction have been devoted to Standard-like Models (SM) [6–8]. In these constructions, discrete gauge symmetry seem to play interesting role in realistic model buildings [9, 10]. Indeed, their investigations have important implications, as they guarantee proton stability, in agreement with experimental bounds [11, 12].

Discrete symmetries are strongly constrained by anomaly cancellation conditions [3–10]. In particular abelian $Z_N$ discrete gauge symmetries were imposed to forbid dangerous Lepton and Baryon number violating operators, and can be realized as discrete remnants of continuous $U(1)$ gauge symmetries, with Ramon-Ramon (RR) 2-forms $B_2$ couplings to the $U(1)$ gauge group field strengths $F_a$ (BF couplings), that are broken by scalars with charge $N$ under the respective $U(1)$ acquiring vacuum expectation values (vev’s) [12–14]. One of the well studied models is trinification $U(3)^3$ which can arise in the D-brane constructions, where all fermions representations are charged under three additional anomalous $U(1)$’s [13–15]. One linear combination of these $U(1)$ symmetries is anomaly free and can serve as a hypercharge component, leading to very interesting phenomenological implications [12, 14].

In the present paper we discuss the origin of discrete gauge symmetries $Z_N$ in trinification model $U(3)_C \times U(3)_L \times U(3)_R$. We start with a short description of the basic features of intersecting D6-brane in type IIA constructions trinification model, and we briefly review the basic considerations on the origin of a discrete $Z_N$ symmetries. Then we discuss the constraints and presence of these discrete gauge symmetries imposed by the corresponding winding numbers of the model.

II. INTERSECTING D6-BRANES AND DISCRETE SYMMETRIES

A. Intersecting D6-brane trinification model

In this section, we review the results of the works where the authors describe the gauge symmetry $U(3)_C \times U(3)_L \times U(3)_R$ in the context of intersecting D-branes on Type IIA orientifolds [15–17]. In these works, three-stacks of D6-branes filling up the four-dimensional space-time and wrapping three-cycles $\pi_a$ in the internal Calabi–Yau threefold and moving on orientifold geometry, each stack contains 3 parallel D6-branes almost coincident, and gives rise to the following gauge symmetry,

$$ U(3)_a \times U(3)_b \times U(3)_c. $$

The D-brane analogue of the trinification model construction involves symmetries $U(3) = SU(3) \times U(1)$ and also contains three extra $U(1)$ abelian symmetries, thus, the $U(3)^3$ gauge group can be equivalently written as,

$$ SU(3)_a \times SU(3)_b \times SU(3)_c \times U(1)_a \times U(1)_b \times U(1)_c. $$

The all three abelian $U(1)_{a,b,c}$ factors have mixed anomalies with the non-abelian $SU(3)_a \times SU(3)_b \times SU(3)_c$ gauge parts which are determined by the contributions of three fermion generations are proportional to $A \sim TrQ_i T^a_i$ where $T_I = \{SU(3)_C, SU(3)_L, SU(3)_R\}$ and $Q_I = \{U(1)_C, U(1)_L, U(1)_R\}$ with,

$$ A = \begin{pmatrix} 0 & +1 & -1 \\ -1 & 0 & +1 \\ +1 & -1 & 0 \end{pmatrix}. $$

There is only one anomaly-free $U(1)$ combination, namely,

$$ U(1)_Z = U(1)_C + U(1)_L + U(1)_R. $$

The remaining two orthogonal combinations $U(1)_C - U(1)_R$ and $U(1)_C - 2U(1)_L + U(1)_R$ have anomalies which are canceled by a generalized Green–Schwarz mechanism

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It is very difficult to satisfy the RR tadpole cancellation conditions, because of large RR charges from three $U(3)$ groups. Additional stacks with $U(1)$ groups and filler branes are also used to satisfy the RR tadpole cancellation conditions \cite{15,16,17}.

In intersecting D-brane models the cubic non-abelian anomalies are cancelled automatically when the RR tadpole cancellation conditions are satisfied. The mixed anomalies $SU(N)^2 \times U(1)$ appears when the gauge group structure of the D-brane constructions involves symmetries $U(N) = SU(N) \times U(1)$ and does not vanish even after imposing the tadpole conditions presented above. These anomalies are cancelled by a generalized GSM
which involves some set of $BF$ couplings to a set of RR 2-forms. In particular, for one stack of $N_a$ D-brane with wrapping numbers $(n_{ai}, m_{ai})$, the four-dimensional couplings of the $B_i^2$ to the $U(1)$ field strength $F_a$ read,

$$N_a m_{a1} m_{a2} m_{a3} \int_{M_4} B_2^0 \wedge F_a, \quad N_a n_{aj} n_{ak} m_{ai} \int_{M_4} B_2^0 \wedge F_a,$$

which are of the form $\sum \epsilon^a_i B_2^i \wedge F_a$ and $\epsilon^0_a = N_a m_{a1} m_{a2} m_{a3}$, $\epsilon_a = N_a n_{aj} n_{ak} m_{ai}$ depend on the winding numbers.

B. Discrete gauge symmetries

Discrete symmetries are well motivated in supersymmetric field theories from a phenomenological point of view, as they guarantee proton stability in agreement with experimental bounds. These symmetries, that are less constrained by anomalies than their continuous counterparts, arise naturally in string theory, as it will be shown in the next section. $Z_N$ symmetries thus offer excellent ingredients for viable models [10, 11].

Generically, in field theory, discrete $Z_N$ symmetry acts on the chiral superfields $\Psi_j$ by a global phase rotation,

$$\Psi_j \rightarrow e^{i \alpha_j 2\pi/N} \Psi_j,$$

where the integer charge $\alpha_j$ is defined modulo an integer shift $N$,

$$\alpha_j \sim \alpha_j + m_j N \quad \alpha_j, m_j \in \mathbb{Z}.$$

More specifically, discrete symmetries offer excellent extensions to the Supersymmetric SM in order to prohibit the lepton-and/or baryon-number violating operators. The possible $Z_N$ generation independent discrete symmetries of the Supersymmetric SM with generator $g_N$ can be expressed in terms of products of powers of three mutually discrete generators $A_N = e^{i 2\pi N_0 / N}$, $L_N = e^{i 2\pi N_1 / N}$ and $R_N = e^{i 2\pi N_2 / N}$,

$$g_N = A_N^m \times L_N^n \times R_N^p,$$

where the exponents run over $m, n, p = 0, 1, \ldots N - 1$. The most general $Z_N$ symmetry allowing for the presence of the charges of the chiral Supersymmetric SM matter fields are given in table 2,

| Generator | $Q_L$ | $\overline{U}_R$ | $\overline{D}_R$ | $L$ | $\overline{E}_R$ | $\overline{N}_R$ | $H_u$ | $H_d$ |
|-----------|-------|------------------|------------------|-----|------------------|------------------|-------|-------|
| $R$       | 0     | $n - 1$          | 1                | 0   | $n - 1$          | 1                | $n - 1$ | $n - 1$ |
| $L$       | 0     | 0                | $n - 1$          | 1   | 1                | 0                | 0     | 0     |
| $A$       | 0     | 0                | $n - 1$          | 1   | 0                | 1                | 0     | 1     |

TABLE II: Chiral fields and their discrete $R$, $L$, $A$ charges assignment.

| $q_{QL}$ | $q_{Q_R}$ | $q_{U_R}$ | $q_{D_R}$ | $q_{E_R}$ | $q_{N_R}$ | $q_{g_{DL}}$ | $q_{g_{DR}}$ |
|-----------|-----------|-----------|-----------|-----------|-----------|-------------|-------------|
| 0         | $-m$      | $-m - n$  | $m + n$   | $m - n$   | $m + n$   | $-m - n$    | $m - n$    |

TABLE III: Chiral fields and their discrete $g_N$ charges.

$SU(3) - SU(3) - Z_N : \quad 3n = 0 \mod N,$

$SU(2) - SU(2) - Z_N : \quad 2n + 3p = 0 \mod N,$

$G - G - Z_N : \quad -13n - 3p + 3m = 0 \mod N + \eta N / 2,$

where $\eta = 0$ for $N$ being odd and $\eta = 1$ for $N$ being even.

III. DISCRETE GAUGE SYMMETRIES AND TRINIFICATION MODEL

A. $Z_N$ in intersecting D6-branes

In type II orientifold constructions a stack of $N$ identical D6-branes gives rise to a $U(N)$, that splits into $U(N) = SU(N) \times U(1)$ where the abelian part is generally broken to $Z_N$ discrete gauge symmetries by the presence of $N B \wedge F_a$ couplings, with $B$ being RR 2-form fields. Abelian discrete symmetries $Z_N$ are remnants of continuous $U(1)$ symmetries that are broken by scalars with charge $N$ under the respective $U(1)$ acquiring vev’s.

Consider a linear combination of the form $\sum_a q_a U(1) \alpha$ of brane $a$, its $BF$ couplings are,

$$\sum \left[ \frac{1}{2} \sum a q_a N_a (\pi_a - \pi'_a) \right] B_2 \wedge F_a,$$

where $\pi'_a$ denotes the orientifold image cycle of $\pi_a$. In [14], the authors give the criteria for the remaining massless combination,

$$\frac{1}{2} \sum a q_a N_a (\pi_a - \pi'_a) = 0.$$

In practice, it is convenient to work with a basis of three-cycles $\{ \alpha_i \}, \{ \beta_i \}$ which are even and odd under the orientifold action, with $i = 0, \ldots, h_{21}$, such that $\alpha_k \wedge \beta_i = \delta_{ki}$. The wrapped cycles are expanded in terms of this basis as,
\[ \pi_a = \sum_i \left( r_i^a \alpha_i + s_i^a \beta_i \right), \quad \pi_a' = \sum_i \left( r_i^a \alpha_i - s_i^a \beta_i \right), \]

where the coefficients \( r_i^a \) and \( s_i^a \) are integers and are usually referred to as wrapping numbers, with,

\[ s_a^0 = m_a^1 n_a^2 n_a^3, \quad s_a^1 = m_a^2 n_a^2 n_a^3, \quad s_a^2 = n_a^1 n_a^2 n_a^3, \quad s_a^3 = n_a^1 n_a^2 m_a^3. \]

The structure of the \( BF \) coupling shows that a discrete \( Z_N \) gauge symmetry appears when the quantities \( \sum_a q_a N_a s_a^i \) are multiples of \( n_i \), for all \( i \). Under a \( U(1) \) gauge transformation, the constraint \( (23) \) takes the form,

\[ \sum_a \sum_i q_a N_a s_a^i \beta_i = 0. \]

We will assume that the three-cycles are orthogonal to each other \( (25) \) reads,

\[ \sum_a q_a N_a s_a^i = 0 \forall i, \text{ with } q_a \in \mathbb{Q}. \]

This condition can be generalised for a discrete gauge symmetry \( Z_N \) arising from a linear combination \( Z_N = \sum_i k_i U(1) \) as,

\[ \sum_a k_a N_a s_a^i = 0 \mod N \forall i, \text{ with } k_a \in \mathbb{Z}. \]

### B. Trinification model with discrete symmetries

We will now discuss the origin of discrete symmetries in trinification model. To show how these symmetries appear we proceed further by imposing the necessary condition \( (24) \). The hypercharge remains massless as long as,

\[ m_{c_1} n_{c_2} n_{c_3} + m_{l_1} n_{l_2} n_{l_3} + m_{r_1} n_{r_2} n_{r_3} = 0, \]

\[ n_{c_1} n_{c_2} m_{c_3} + n_{l_1} n_{l_2} m_{l_3} + n_{r_1} n_{r_2} m_{r_3} = 0, \]

\[ m_{c_1} m_{c_2} m_{c_3} + m_{l_1} m_{l_2} m_{l_3} + m_{r_1} m_{r_2} m_{r_3} = 0, \]

\[ n_{c_1} n_{c_2} n_{c_3} + n_{l_1} n_{l_2} n_{l_3} + n_{r_1} n_{r_2} n_{r_3} = 0, \]

where we have accounted for a factor of \( N_a = 3 \). In our model building, we need to consider the constraints imposed on the winding numbers. We may use the condition \( I_{c^*} = m_{c_1} m_{c_2} m_{c_3} = 0 \) to eliminate the existence of the symmetric and antisymmetric representations originated from open strings with both end-points

on \( SU(3)_c \), for example, this condition can be satisfied by setting \( m_{r_2} = 0 \), which imply,

\[ m_{c_1} n_{c_2} n_{c_3} + m_{l_1} n_{l_2} n_{l_3} + m_{r_1} n_{r_2} n_{r_3} = 0, \]

\[ n_{c_1} n_{c_2} m_{c_3} + n_{l_1} n_{l_2} m_{l_3} + n_{r_1} n_{r_2} m_{r_3} = 0, \]

\[ m_{l_1} m_{l_2} m_{l_3} + m_{r_1} m_{r_2} m_{r_3} = 0, \]

\[ n_{l_1} m_{l_2} n_{l_3} + n_{r_1} m_{r_2} n_{r_3} = 0. \]

Using the above results and the conditions to obtain three fermion generations, we solve for \( m_{c_1} \),

\[ m_{c_1} = -\frac{m_{l_1} n_{c_3}}{n_{l_1}}, \]

which imply the relations,

\[ I_{c^*} = 2 m_{l_2} n_{c_2} m_{l_1} n_{c_1} (m_{c_3} n_{l_3} - m_{l_3} n_{c_3}) = 3, \]

and substitute to the conditions \( I_{c^*} = 0 \), this requires,

\[ (m_{c_1} n_{r_1} + m_{r_1} n_{c_1}) (m_{c_3} n_{r_3} + m_{r_3} n_{c_3}) = 0, \]

and,

\[ \frac{n_{c_1}}{n_{l_1}} (m_{l_1} n_{r_1} - m_{r_1} n_{l_1}) (m_{c_3} n_{r_3} + m_{r_3} n_{c_3}) = 0, \]

we hence deduce,

\[ m_{c_3} n_{r_3} = -m_{r_3} n_{c_3}. \]

Then with the conditions \( I_{r^*} = 0 \),

\[ (m_{l_1} n_{r_1} + m_{r_1} n_{l_1}) (m_{l_2} n_{r_2} + m_{r_2} n_{l_2}) (m_{l_3} n_{r_3} + m_{r_3} n_{l_3}) \]

\[ = 0, \]

and,

\[ \frac{n_{c_1} m_{r_3}}{n_{l_1} m_{c_3}} (m_{l_2} n_{r_2} + m_{r_2} n_{l_2}) (m_{c_1} n_{r_1} - m_{r_1} n_{c_1}) (m_{c_3} n_{l_3} - m_{l_3} n_{c_3}) \]

\[ = 0, \]

we therefore deduce,

\[ m_{l_2} n_{r_2} = -m_{r_2} n_{l_2}. \]
\[ m_r n_{c1} n_{c3} n_{l3} = m_{l3} n_{c3} n_{c1} n_{r1} , \]  
\[ F_c \land 3 \left( m_{c1} n_{c2} n_{c3} B_2^3 + n_{c1} n_{c2} m_{c3} B_3^3 \right), \]  
\[ F_l \land 3 \left( m_{l1} n_{l2} n_{l3} B_2^1 + n_{l1} m_{l2} n_{l3} B_3^1 \right), \]  
\[ F_r \land 3 \left( n_{r1} m_{r2} n_{r3} B_2^1 + n_{r1} n_{r2} m_{r3} B_3^1 \right). \]

Using the results (42), we end up with,

\[ F_c \land 3 \left( m_{c1} n_{c2} n_{c3} B_2^1 + n_{c1} n_{c2} m_{c3} B_3^1 \right), \]  
\[ F_l \land 3 \left( -m_{c1} n_{c2} n_{c3} B_2^1 - \frac{3}{m_{c1} n_{c2} m_{c3}} B_3^1 \right), \]  
\[ F_r \land 3 \left( \frac{3}{m_{c1} n_{c2} m_{c3}} B_2^1 - n_{c1} n_{c2} m_{c3} B_3^1 \right). \]

It is easy to see that this structure naturally contains the discrete gauge symmetries:

the discrete symmetry \( Z_3 \) appears whenever \( m_{c1} n_{c3} n_{c2} = 3 \) and \( n_{c1} n_{c2} m_{c3} = 1 \). Note that this \( Z_3 \) symmetry is associated with the \( U(1)_r \) brane.

the discrete symmetry \( Z_N \) appears whenever \( m_{c1} n_{c2} n_{c3} = N \) and \( n_{c1} n_{c2} m_{c3} \) is a multiple of \( N \).

\[ IV. \ \text{CONCLUSIONS} \]

In this work, we have interested in the discrete gauge symmetries in viable models. We have considered a trinification model in the context of type IIA orientifolds with intersecting D6-branes, and discussed the phenomenological importance of \( Z_N \) symmetries.

We have investigated the origin of \( Z_N \) gauge symmetries from \( B \land F \) couplings in trinification model \( U(3)_C \times U(3)_L \times U(3)_R \). More precisely, we have considered the imposing conditions on the winding numbers for the particular solution required for the presence of a massless \( U(1) \) in the low energy effective theory where these discrete gauge symmetries appear.

We hope the discussion of discrete gauge symmetries in D-brane model buildings as done in this work is sufficient to show and motivate that they still deserve more attention and more systematic understanding for future works.

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