Role of Riemann’s and Goldbach’s hypotheses in the behaviour of complex systems: *Introduction to the concept of “Sciences”*

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**Abstract.** The authors have already established a bi univocal correspondence between Riemann zeta functions and dynamic processes under the control of integro-differential operator of non-integer complex order. We recall that the Riemann zeta function can then be related to hyperbolic geodesics whose angles at the boundary are determined by the real part of the power laws that define the Riemann series. It is suggested that Riemann's conjecture can be reduced to a geometrical phase transition with a reduction of the parameter of order resulting from the combination of a pair symmetries associated with a quasi-self similarity of geodetics.

The well-known relationship with the set of prime numbers must be considered as the result of the local existence of stationary 'state' in the dynamics. The work is focused on the ‘non stationary’ behaviour of Riemann zeta function. It is shown that the main characteristic of the dynamics of complex systems may be associated to a hybridizing between a pair of states and/or processes able to give a geometrical status to the concept of the time and equally to the concept of energy. It is based on the mirror properties of complementary zeta function. It is shown also that the set of prime numbers, which controls the transitions between ‘states’, is the simplest form of the complexity. This analysis suggests the existence of a mathematical relationship between Riemann’s and Goldbach’s hypothesis. Such relationship would be the base of an extension of the principles of the science for the analysis of the complexity. According to previous proposal we name this extension that enlarges the principles of the science: *sciences* with ‘a’.

1. **Introduction**

For any ‘Boeotian’ a Complex System is the system with respect to which the standard reason is impotent [1,2]. Complexity filters the needs of the scientific spirit for clarity within a cloud of confusion, uncertainty and, in its weak form, the randomness. The Greeks invented two approaches [3], the empiricism on the one hand (the human force for testing the beings) and on the other hand the metaphysical world of ideas (for approaching the being of the beings) because the sensitive relationship with the nature is essentially complicated but they did not imagine the complexity. In front of complexity the Greek objective posture has many weaknesses [4,5]. To overcome these weaknesses the Middle Age philosophers (Th. Aquinas, Ockham, Duns Scotus) found the sketching of the future science and the concept of modelization. The closure of our questioning by the existence of God was replaced by the transcendental ontology of the object of science. For a long time religions...
had searched to give meaning to a life without which it makes no sense, but as pointed by Nietzsche ‘the Gods are dead in laughing’ [6]. Despite a high sophistication, the hermeneutics applied to the great myths, including the myth of a pure science, the philosophical analysis did not succeed for reducing the uncertainty imposed to the human slaving, that is why from the Renaissance the thinkers attempts step by step to build the foundations for modern science based on the number and the measurement [7]. Analytic philosophy and conceptual reductionism on the one hand and experimental empiricism on the other help us to formalize in the practice the Science: a new relationship between the thinking subject and the object of its thinking. Without asserting with E. Morin that science is a source of cretinism [1] or with Bergson that science creates no future [8], we can observe the deleterious tendency of science, and especially techno sciences to enter into a dangerous ultra-specialization and the relative failure of the unification initial project for sciences [9]. The worst, in our artificial technological world, is that the conceptual limits we reach cannot be considered as methodological but epistemological [3]. This situation is infinitely deleterious with respect to the efficiency of our relationship with the world [8]. In trying to think the world as source of power, the science, and more specifically the techno-science, attempts to reduce a human to a simple ‘agent of action’ that cannot resist and then destroys the raft of the planet by fitting the ‘quality plans’ of ‘competitive micro societies’ put within concurrency environment. After centuries of assurances about our ability to manage in harmony the thought and the technical power, the postmodernism [10] points out that the techno-scientific approach leads the loss of pragmatism and efficiency of the great tale of the science. Forgetting the specificities of the complexity the engineer-scientist turns the human being into the organ of self-perpetuating complicated machinery, whose teleology is reduced to the chance and/or simple local interactions. The meaning must then be suppressed from such a posture. Would it be possible to think a new scientific spirit, a new science [8], able to unlock the bar that leads to a Maelstrom? Would it be possible to balance the techno-scientific postmodern evolution and the thinking of the creative human being? Probably this issue will has to stay open even if, as it is done in this note, we try to give a scientific answer to this question by using the image of zeta function properties [11-15]. Indeed we think that this mathematical image could be a key of understanding the world of complex systems.

2. Introduction to complex systems

For ten years the Steering Committee of the French National Network on Complex Systems (RNSC) attempts to define what should be the science of the complexity. It tries to organize a scientific community around the question of a scientific access to complexity [2]. A proposal among others is that the object of this science or more precisely the approach of the complexity considered as such an object could be thought through three categories of questions:

(i) the question of the new objects characterized by multi-scale structures (Penrose-tiling, complex fractal geometry, hyperbolic, disordered networks and Gromov groups), topologies of high 'gender', living structures (cells, neurones, cities, companies, etc and their networks.)

(ii) the consideration of extreme and singular situations in experimental terms, situation concerning the treatment of huge data base, the coupling Subject-Object, the topology and the metric involves by singularities and burst of singularities, etc.

(iii) the irreducible confrontation between multiple point of views about a same object what we call a 'conceptual anisotropy'.

These proposals are obviously limited, and must be discussed within the community of Complex Systems, but it carries naturally with it many uncomfortable problems raised by the millennium confrontation between Concept and Knowledge in physics and biology. For example in physics we can look over the experimental and financial consequences of the limitations of the standard model [16], the extreme complexification of the attempts for unification of gravitation and quantum mechanics [17], the outstanding issues regarding the relevance of the concept of super-symmetry [18], the always questionable validity of the equivalence principle [17,19], etc. In this perspective we recall
for example that the singularities of the equations of the general relativity theory are far from being completely known. As well, backed onto analytical philosophy, the modern biology (i) carries, far from the concept of complexity, the belief in the ability of genomic analysis of finding answers about the effectiveness of the living (why not also its meaning) and (ii) pushes implicitly a strategic of excellence leading to a dangerous eugenics. The being must then be repaired, fit like a machine and finally has to be created without defect using a perfect design. Obviously these programs are not only based on mercantile research interests on the beings. They aim also to answer to geopolitical requirements using the totalitarian phantasms of numerical controlled of the being and of the society by the mean of techno-sciences. They are also founded upon the idea that a complex system must be reduced to a complicated one, that is to say, to strictly mechanical machinery. Such a point of view ignores that the reliability of a machine is based upon the reliability of its organs and that like a living system the reliability of a Complex Systems overcomes the reliance of its components to stabilize its teleonomy via its homeostasis. The reliability of complex system must be based upon the inverse process of the one of a machine.

In contrast, it is paradoxical that with its insolence freedom, (except in financial artefact(ual) dependence [20,21]), the field of mathematics, has kept a scientific goal and an international organization according to its proximity with the philosophy. Even if the dynamic of the mathematics involves daily progresses there is no more mathematics than modern philosophy. Both are eternal because through a retro process of thinking, the object created by the mind of the thinker is the product of his thinking. This singular position of mathematics may help us to think, upon the specific basis of the singularity of the Complex systems having regard to the standard empiric. Let us also observe that a central object of mathematics is our relationship with the singularity of our position of thinker and the infinity of our environment. That could light the posture that we should have to adopt with respect to the Complex Systems, position that could justified the need for a new definition of a field of science that we suggest to call sci

science [9]. This concept would be able to balance the totalitarian posture focussing on the ‘bit’ and the calculus. Though the generalization of freedom of scientific spirit seems always, as pointed by E. Galois having regard to the solution by radicals, the source of intrinsic ‘ambiguity’, the desire of freedom which is the basis for all sciences, could be to mimic the prominent efficiency of maths using the same ambiguous concerning the relationship between the subject and the object [2].

Coming back to Galois’ remark, we recall that the principle of group theory applied to equation is not to examine each root, but their symmetries. This goal leads our interest to the classes of objects rather than individual objects, and to the structures of equations instead of their solutions. These principles animate the dynamics of Complex Systems. In practice as shown by the attempts of neurobiological understandings of the consciousness, the emergence of Complex System must be the result of a phase transition, a fuzzy situation of emergence that involves divergence of series over infinite number singularities. The phase transition is then led by a gain or lost of symmetries, which concerns most of the time scaling structures! But the paradigms of the standard techno-science roughly assert that the complex is mainly reducible to the mechanical entanglements that could be unravelled. Such hypothesis ignores the existence of complex knots. It also ignores the current failure of the modelization of the phase transition, the complexity of emerging phenomena such as the consciousness as well as the raising of modular structures when passing from integer automorphism to fractional ones (see [22], extension of Mandelbrot set using the automorphism \( z \rightarrow z^\alpha + C \)). It ignores the foundation of the self-organization, of the catastrophes and of the homeostasis that can be spectrally reduced only if we forget the boundary information and the long-range absence of periodicity. The state of our knowledge and the gap between these knowledge and the problems we face with Complex Systems, suggest a great modesty imposed by lake of ontology of the complex object which must be distinguished from a simple mechanical entanglement between basic components [1,3]. Nevertheless, in spite of this criticism, we believe that the major principles of science and rigor stay starting points to handle the complexity and that is why it was suggested to enlighten the complexity starting from practical scientific problems leads by zeta Riemann conjecture.
Complexity confronts out the thinking of our relationship with the nature to an epistemological difficulties leads by the role of the infinitude which is not simply combinatorial, but meta-combinatorial. How to describe this extension problem without using the words given by the standard language [4]? It is in the mathematical field that we must seek this language, not only because mathematicians are always the infinitude for purpose, but more fundamentally (i) because mathematics ignores the distinction between the subject and the object (ii) because the purpose of the mathematical research is precisely the 'brain-food' of the mathematician point of view and (iii) because the concept of forcing in set theory and of limit in categorical theory allow us to think about the extension of the thinking, we mean: the creativity. This paper will focus its proposals on an analogy based on the complexity involved by zeta Riemann function in particular because we have knowledge about its degeneracy in quantum physics [15, 23-28]. According to our proposal concerning a new approach to Riemann conjecture [29-31] we have to start from what it is probably the keystone of the problematic of complexity: the coupling between different scales.

3. Coupling within scales: Dynamics in fractal geometry
There is very interesting link between the fractional differential equations and the fractional dynamics from one side and the fractal geometry from other [31-35]. The significance of the fundamental relationship between these two branches of mathematics must relate the understanding and the mastering of what we call the space-time when the media is heterogeneous. Since the publication of the Mandelbrot's book on fractal geometries [35] the related geometrical issues about such a space-time have been extensively explored in the works done by his students, but many questions stay open and it is very interesting to note that in this matter two schools face each other:

(i) the traditional academics argue that fractal geometries do not disrupt the physical standard rules of physics. Fractal geometries are then considered as simple boundary conditions for standard and simply traditional equations [37,38], for instance, the characteristic length to scale the fractal may be the diffusion length without modification of the diffusion law. Time-space dependence of diffusion coefficient must be written using an anomalous diffusion coefficient depending on time [39];

(ii) the experts in engineering of complex systems a contrario assert that the convolution between scales come from the infinite behaviour, even if the scaling is not infinite and that correlations in scale contributes to the emergence of singularities of behaviour affecting deeply the physical principles themselves for instance the concept of velocity itself disappears [40-42].

Despite many experimental results that link for instance the relationship between the fractal dimension and entropy production of devices challenging nonextensive physical systems over a large range of scale the debate is unresolved. It is already unresolved in spite of many experimental data concerning the diffusion limitation in fractal media. Without going into technical details we can say that the physical standard is based on two paradigms:

(i) a time parameter must be in the set of real numbers; this parameter may disappear when it is merged with the space by using the concept of velocity [43,44] and

(ii) the homogeneity of space-time which gives rise, via Noether's theorems, to invariants such as the energy.

The both invariants of conceptual interest for this paper are the energy and his operational factotum, the velocity. It is precisely this latter that scaling coupling breaks by generalizing the harmonic form of space-time relationship. The main equation is the equation of identity: \( \eta(\omega)^{d_f} (i\omega) = \text{const} \). This equation fits well the Euclidean traditional case \((d_f = 1)\) and the stochastic case \((d_f = 2)\) but it contains much more than both cases. Starting from that generalization, the concept of energy, given by a square of velocity as an invariant must be revisited in heterogeneous media. Related scientific queries lead to discover that the two paradigmatic models of the modern empiricism namely determinism and the stochastic models, are singularities into of a more general
conceptualization of the exchanges of energy whose heart is none other than the class of differential equations of non-integer order [32-35]. However, these equations show, after careful analysis that standard time in the real 1D space is no less than a degenerative form of a general concept of time that must be extended all over the complex numbers [15,45]. The case of both paradigmatic models, deterministic and diffusive, appears in fact as a very specific case (although one should put a reserve for the case of Quantum Mechanics obviously related to the stochastic model [4,46,47]).

The parameterization of geodesic dynamics by complex number due to heterogeneity of the geometry is indicative of the close relationship between fractal and hyperbolic geometry. But precisely this link hence points out the issue of ambiguity in the frame of a rigorous and rational posture, that is to say the competition between many stable solutions. In many complex problems this competition explains the total absence of stable solutions even if the problem may be reduced to a deterministic geodesic on Riemann manifold. We recall that hyperbolic geometry, especially through its folding, involves multiple solutions associated a same state of definition. The usual reductionist intuition related to Euclidean and extensive systems, does not render easy the perception of the general character of such a situation. The need for thinking of linear causality confronted to the hyperbolic character of the reality may be the origin of catastrophic misunderstandings when we face complex systems (4): What is the purpose of our understanding in this case? What is a status of what we call the state of the system? What is the status of the time? To give a simple image of the ambiguity from the dynamics we suggest using the transfer function of a canonical fractional differential equation [35,48].

Figure 1. See the text for explanations

The figures 1a left and 1b right reveals two modalities for the conscious representation of an object (related to two types of referential) of the underlying complex phenomenon in fractal media

- The first 1a backed upon internal variables, is directly related to what Poincare called the 'analysis situs' and must be related to a fundamental automorphism domain in space-time which is a punctuated torus characterized by an angle at infinity. The automorphism is based upon a set of (i) a singular universal point at infinity in time \( \omega = 0, \infty \) and a finite distance \( \eta \) in space [35,53]. The relationship between the two limits, \( \omega = 0, \infty \) namely the subject and the object -the sole couple of ontologic reality-, is here represented as dynamic process. This relationship points out the internal correlations of the geometry. The angle at infinity attached to the internal long-range correlations is given via the alpha parameter
(see transfer equation). If the process is deterministic the space-time dimension is two, long-range correlation is given by $\alpha = 1$ and the angle at infinity is $90^\circ$ (subject and object are plunged within two orthogonal space and time: Poincaré automorphism). If the process is stochastic and controlled by chance with 2D long-range interactions $\alpha = 1/2$ the law of the great number emerges, the space dimension versus time is two ($x^2 \propto t$) and the angle at infinity is $45^\circ$. Under the reserve of considering the reversibility of the situation, $\frac{\partial}{\partial t} - D \frac{\partial}{\partial x} = \left( \frac{\partial^{1/2}}{\partial t^{1/2}} - \sqrt{D} \frac{\partial}{\partial x} \right) \left( \frac{\partial^{1/2}}{\partial t^{1/2}} + \sqrt{D} \frac{\partial}{\partial x} \right)$ the situation is the same and an artefact of algebra ($90^\circ = 2 \times 45^\circ$) allows a coming back to deterministic expression $\alpha \cong 1$. Based on these models with $\alpha = 1$ and $\alpha = 1/2$ the time is always immersed in the real space of number.

- The situation changes if we consider the whole field of definition of alpha and especially $1/2 < \alpha < 1$ [29,30,54]. In this case the information carried by the angle at infinity leads the entanglements between object and subject. This is due to the fact that velocity and energy lost their paradigmatic integer dimension, the related extensity of state variables and the invariance that requires the Noether’s axioms [55]. These properties can be easily managed using non integer differential equation because the dimensionality of the main constant is no more a velocity or a diffusion constant (except for $\alpha = 2$) but a length to the power alpha over a standard time $x^d \propto t$. Even if the distance between object and subject stays represented by a density of action by unit of flow (resistance) the quadratic form of the energy is lost. This fundamental observation points out the artificial trick consisting of writing a constant with space time variable properties (anomalous diffusion, spectral physical dimension, etc.) conversely suggests to take into account another referential: 1b. The new referential is obtained by rotation of axis and by trapping the real axis upon the phase angle axe. The so called external representation, based on the conjecture that any quadratic form may be obtained by conjugating two fractional form (see below), restores the legitimacy to an over all ‘energy’. This idea is now also suggested in the frame axiomatic thermodynamics [56]. In the case of the dynamics the time must hence be written using a complex number. The complex part takes in charge the information at infinity. Obviously the angle at the boundary of the first referential and the complex part of the time variable are related together. As the result an automorphism based on the opening of the torus at infinity leads the presence of a singular group associated to the object of the dynamics ($\omega \rightarrow 0$), which may be interpreted as an entanglement between the subject who drive the dynamics and the object passive with respect to it, as well as an entanglement between the past and the future. Time irreversibility (usually pointed out with the dissipation through the resistance) is split between the regression within the frequencies (spectral analysis) and a group of transformation at infinity re-establishing the legacy of the energy. This group being related to negentropic process might be called group of emergence, because it is the source of emerging autonomous structures for any open system exchanging energy with the outside. Let us observe that, in absence of this group at infinity the object in suspension in the space time and reduced to the asymptotic of its spectrum, may be call a ‘tempo-object’ [49]. More generally we suggest the reduction the analytical part of a Complex System to a ‘tempo-object’. To reach its essence within the field of energy exchange with its environment the tempo-object must hence be completed by an additional part taking into account it complex component of the time and therefore of the group of transform at the boundary. The fractional dynamics being used as a model of complex issues, the duality of the representation using the time variable as a complex number to overcome the ambiguity involved at infinity by the tempo object and to trap the excess of energy within
The negentropic effect shows how a complex system may require some huge modification within our paradigmatic analytic posture and why it requires a hybrid point of view. This point of view makes smooth the standard distinction between the subject and the object (see for instance particle and wave as complementary infinite singularities in quantum mechanics [4]). The spectral treatment of the dynamics of complex systems, that is to say the use of a one-dimensional time, is possible but it cannot be exhaustive because it forgets the singular behavior related to singularity at the boundary as well as the need for exchanging external energy with an environment in the absence of invariant parameters able to close the balance of energy. In the frame of non-integer dynamics the series stay diverging (tempo-object) and the renormalisation methods required to take into account the scaling properties of the object [50,57] stay enigmatic. In practice these methods play nevertheless the same role as the group at infinity (figure 1b).

If the mathematical problematic with regard to the analysis of the dynamics can be almost easily handled, the problematic of managing a complex media remains pending because we have to give physical meaning to the figure 1b. This issue explains the intuitive need of considering the local behavior as Euclidean and, to do so, the need for the reference to quadratic forms even if the system responds to a non-extensive and hyperbolic behavior (tree set of singularities). Let us give the example of batteries - a very complex system of storage of electric energy [51]. This question of engineering was considered for a long time, but only recently the analysis of the efficiency of batteries (generally characterized by fractal electrodes which fractal dimension is trapped all over their life), had understood why the currents required for utilizing the battery, are strongly related to the scaling properties of the electrode and why the negentropic process (the use of energy to produce work) is strongly correlated to the geometry of the heterogeneous structure. The battery use requires the optimization of both the storing of the energy (3D) and the power exchange (2D). One of the fundamental consequence of the characteristics of batteries is the fact that a Maxwell demon on a fractal surface of the electrode, is unable to measure the absolute flux of exchange required for application. This information is below his empirical horizon. This characteristic can be express within a general principle: the local unit of time is controlled via the metric of the space and by the flow of exchange itself. Without any knowledge about this flow we cannot define the numerical absolute value of the time constant of the process. The exchange with the environment must hence be externally normalized. The clock of the Maxwell demon is tuned by the flow to the power the fractal dimension of he geometry. Such a result shows that with respect to a process, the local time may be different of the external time unit. This general property is related to the property of the geometry at infinity and carries the same meaning that the group which must be associated with the tempo-object for reaching the essence of the complex object.

Due to the contact with the outside (open systems), that is to say, the presence of the invariant energy-momentum, the closing of the problematic onto the quadratic form of the energy (Figure 1b) requires the intervention of an additional term which is no more dissipative but organisational (long range correlations). The convolution of the harmonic behaviour with this new negentropic term is associated with the recovering of a pseudo homogeneity of the space-time, considered elsewhere in thermodynamics as a recovering of extensity [56]. Like a mirror effect it is a coupling between local and global properties and a balance between entropy and productive works (see above). The evolution of the energy balance into an auto-organisation appears as an effect associated to open boundary conditions, which has to be taken into account when tempo-object is considered. The absence of distinction between object and subject, which is the paradigm of the so-called objective knowledge, should therefore disappear for Complex Systems. Even if paradigm is questioned mezzo voce in quantum mechanics [4] via a uncertainty principle - to which Feynman’s formulation of quantum mechanics gives precisely a geometrical meaning with 2D fractal path integral - and long distance entanglements in the field theory, this separation stays a pillar of the traditional analytic science [46,47]. The present hypothesis changes the time into a simple avatar of the energy requirement and in the absence of energy, or even of simple irreversibility, the standard time loses both its operational
effectiveness and its psychological meaning. The consequence is that the time may hence be suppressed of the fundamental physics [43].

As the result even if usually it is asserted that the statistical effect is the origin of irreversibility, the statistical correlations -due to the emergence of the law of great number- usually evoked is probably only one reason among others but is clearly not the sole. The irreversibility of time, and especially the auto organization of complex systems should have a relationship with the general scaling effect even if this scaling effect is perfectly deterministic. Within a more rigorous way, we would like to assert that this affirmation could be backed upon the extraordinary mathematical tool, which is the Riemann zeta function [11-13,58-61].

4. The role of Riemann Zeta function and of the prime numbers

We recall that the Riemann zeta-function is given by the series \( \zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \) with \( s = \alpha + i \theta \). It is shown that the zeroes of this function are the same as of \( \chi(s) \) function with \( \chi(s) = (1 - 2^{1-s}) \zeta(s) \). This function assures the validity of the analytic extension to \( \mathbb{C} \). The relationship \( \pi^{-s/2} \Gamma(s/2) \zeta(s) = \pi^{-(1-s)/2} \Gamma((1-s)/2) \zeta(1-s) \) between \( \zeta(s) \) et \( \Gamma(s) \) which was discovered by Riemann in 1859 gave rise to his famous conjecture: in the complex plane of parameters all zeros of zeta function that is to say \( \zeta(s) = 0 \) are located on the line given by \( \alpha = 1/2 \). This relation comes from a mathematical relation between \( \alpha \) and \( \theta \): \( \forall \theta ; \alpha = f(\theta) = \text{const} \) that cannot explained the specific value \( \alpha = 1/2 \). With \( F(s) = \left[ (2\pi)^s / \pi \right]^{1-2s} \Gamma(1-s) \sin(\pi s / 2) \) according to recent works [62] on gamma-function the relationship between \( \zeta(s) \) et \( \zeta(1-s) \) must be also be written by using the \( \chi(s) \) function from the relation \( \chi(s) = F(s) \chi(1-s) \) and \( F(s) F(1-s) = 1 \) but due to the dual expression of \( s \), the conjecture stays open even if the Riemann conjecture should be true if and only if the above functional relation has no other solution than the sole given by this conjecture.

Surprisingly the attempts to approach the understanding of the emergence of complex component of the time and therefore the key factor to strengthen the concept of hybrid structure of complex system opens the way to understand deeply the conjecture by the mean of a geometrical phase transition [29]. The main idea is to replace the variable \( \alpha \) by an angle \( \psi \), which will appear as the order parameter transition with respect to Riemann function symmetries. We can summarize this proposal within the following manner. Leaving aside the sum defining \( \zeta(s) \) we can express the Riemann series within the following mathematical form \( \zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \equiv \left\{ \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n}, \ldots \right\} \). The real part of this series suggests associating the Riemann series to the non-integer differential equation of \( \alpha \)-order, or more precisely, to relate the \( \alpha \zeta(s) \) series to Cole and Cole transfer function \( ^{\tau} Z_{\alpha}(\omega) \propto \frac{1}{1+(i\alpha \tau)^{\alpha}} \) [29,30,53] of a canonical function derived from a generalization of a first order equation. It can be shown that the Riemann series may then be considered as a discrete path ‘integral’ of \( \alpha - \text{hyperbolic distance} \) \( 1/\eta(\omega \tau) = \frac{1}{(\omega \tau)^{\alpha}} \) on the geodesic \( ^{\tau} Z_{\alpha}(\omega) \) of the canonical non-integer dynamic in Fourier space.
The homorphism between \( \omega \alpha \tau \rightarrow Z \) and \( s \zeta \) that can be generalized over the all set of Riemann function demonstrates the existence of scaling correlations between each term of the Riemann series. These correlations can be characterized by a phase angle at infinity \( \varphi(\alpha) \) with \( \varphi = \frac{\pi}{2}(1-\alpha) \). Due to \( \zeta(s) \) and \( \zeta(\overline{s}) \) symmetries as well symmetries of \( \zeta(s) \) and \( \zeta(1-s) \) the reference axis must be chosen upon \( \frac{\pi}{4} \) axis and \( \varphi(\alpha) \) can be replaced by \( \psi(\alpha) \) with \( \psi(\alpha) = \frac{\pi}{4}(2\alpha - 1) \). This relation authorizes the replacement of \( \alpha \) by \( \psi \) in the Riemann series expression as well as in the sum; therefore \( 0 \zeta(s) = 0 \zeta(\psi, \theta) \) With the same conventions for the rotation (Figure 2) let us observe (Figure 2) that \( 0 \zeta(s) \) can be associated with \( \tau Z^+(\omega) \) and therefore \( 0 \zeta(s) = 0 \zeta(-\psi, -\theta) \) equality which leads the major following mathematical expression: \( \zeta(\psi, \theta) = \zeta(-\psi, -\theta) \). We can therefore repeat the above arguing to show that \( 0 \zeta(1-s) = 0 \zeta(\psi, -\theta) \) and \( 0 \zeta(1-\overline{s}) = 0 \zeta(-\psi, \theta) \) therefore \( \zeta(\psi, -\theta) = \zeta(-\psi, \theta) \). Let us observe that the use of \( \psi(\alpha) \) as scaling invariant for all set definition of zeta functions leads a dependence between the \( \text{sign}(\psi) \) and the \( \text{sign}(\theta) \). This dependence is an identity for \( s \) and \( \overline{s} \) and is naturally inverse for \( 1-s \) and \( 1-\overline{s} \). We can consider the Riemann Conjecture, now extended to the mathematical relation: \( \forall \theta \zeta(\psi, \theta) = \zeta(-\psi, -\theta) = 0 \) that is to say \( \theta = f(\psi) \). This relation achieves a breaking of the symmetries into the set of zeta functions and creates an implicit relationship between both variable leading the mathematical relation \( \forall \theta \psi = \text{const} \). Nevertheless except if \( \psi = 0 \) this relation cannot fit any more the relations concerning the reciprocal sign relationship such as \( \text{sign}(\psi) = \text{sign}(\theta) \) than \( \psi(\alpha) \) must be equal to zero that is to say \( \alpha = 1/2 \). Obviously the same reasoning may be based from \( \zeta(\psi, -\theta) = \zeta(-\psi, \theta) = 0 \) Even if it is not a mathematical demonstration of the conjecture this analysis valid the Riemann Conjecture as a geometrical phase breaking of symmetries [29,30,53,62] but also unveils the role of the
complementarity between \( \zeta(s) \) and \( \zeta(1-s) \) in the emergence of stable states (related to prime number) in the spectrum of Riemann Zeta function.

As shown in figure 2, the homomorphism and the symmetries within rotation are based on the exchange of the space and time axis. This inversion leads the shift from \( s \rightarrow \bar{s} \). Hence the long range geometrical correlations would be replaced by long-range correlations in time (history and anticipation). But taking \( \psi(\alpha) \) as reference of geometrical phase to express the set of zeta functions, the constraint \( \alpha = 1/2 \), appears related to standard ergodic properties of the fractional dynamics. Due to spatial mixing properties that may be expressed locally through an integer differential equation

\[
\frac{\partial}{\partial t} - D \frac{\partial}{\partial x} = \left( \frac{\partial^{1/2}}{\partial t^{1/2}} - \sqrt{D} \frac{\partial}{\partial x} \right) \frac{\partial^{1/2}}{\partial t^{1/2}} + \sqrt{D} \frac{\partial}{\partial x} \]

the process is irreversible. Conversely for \( \alpha \neq 1/2 \) a phase \( \psi(\alpha) \) exists that cancels the commutativity of the dynamics and of the underlying \( \alpha \) - Riemannian geometry. These properties associate our analysis to non-commutative geometry theory [44].

The fractional differential equation contains a long-range memory that prohibits time-space extensive properties and a local description able to be later involved within integration. These constraints are strongly related to the local irreversibility of the dynamics and its measurement [63] but nevertheless according to the fact that

\[
\frac{\partial^\alpha}{\partial t^\alpha} * \frac{\partial^{1-\alpha}}{\partial t^{1-\alpha}} = \frac{d}{dt}
\]

we may observe that, contingent to the right phase angle, the merging of transfer functions (Figure 2) may be useful for re-establishing pseudo extensive behaviour. Indeed above analysis leads the following assertion: All quadratic form can be decomposed into to complementary fractional forms one of them being causal and the other a-causal. As well, the merging of \( \zeta(\psi,\theta) \cup \zeta(-\psi,\theta) \) and \( \zeta(-\psi,-\theta) \cup \zeta(-\psi,\theta) \) (figure 2) should contribute to the definition of pseudo stable states associated with the ‘teleonomic’ like behaviour Complex Systems Issues from this analysis this assertion is the main conjecture concerning the specificity of complex systems. A certain form of causality may then be re established by considering the couple of the complementary component of the overall description (i) the objective spectral part for one side, and (ii) what it has called the ‘group at infinity’ that contains the long range correlations. Let us observe that the zeroes of zeta function, we means the stable states, then disappears to be replaced by a goal (group at infinity) located below a local horizon of the physical action.

In summary, the analysis shows the existence of two type of irreversibility. The first with \( \alpha = 1/2 \) or \( d_f = 2 \) (related to the underlying 2D fractal geometry in the dynamics, with the existence of a velocity and the relevance of standard energy) is due to the loss of information within a process that may be locally time reversible and deterministic but which is globally dispersive. The second with \( \alpha \neq 1/2 \) or \( d_f \leq 2 \) is characterized by a local irreversibility and unveiled by the existence of phase angle at infinity. This irreversibility leads negentropic effects able to lead emergence, auto-organisation and homeostasis. In both case the dynamics may be expressed with the same parameter \( t \), the time, but if \( \alpha \neq 1/2 \) the time lost a part of its symmetries and may be expressed using a complex number. We conjecture that the specific associated irreversibility is then due to the exchange with the environment. In the case of \( \alpha = 1/2 \) the environment is no more than the object itself and the total dissipation of negentropic effect is therefore dissipated within entropic effect through balance energy exchanges. It is at this stage that the zeros of the Riemann function have to play a role in the differentiation between complicated systems (singular stable states) with respect to Complex Systems (dual stationary states). This balance is not in equilibrium if \( d_f \neq 2 \) and all specificity of complex systems [2] may appears through internal and external energy exchange process.

In spite of the limits of paradigmatic support [4] the aim of the scientific research looking for the spectrum of stable state looks like the research of the whole set of zeros of the zeta function [15, 55-
We know that these zeros are related to the prime numbers. The ontological existence of these states is an indirect epistemological meaning of Riemann hypothesis. Our proposal to approach the conjecture gives the limits of this reductionist point of view. The analysis of the role of the phase upon the raising of a sign of rotation in the complex plan of parameters, that is to say the nature of the irreversibility of time, unveils the key role of hybridizing and coupling for systems strongly influenced by a memory. The image of such a system is enlightened by a general form of non-integer equation when \( \alpha \neq 1/2 \) and by the meaning of the merging symmetric Riemann functions. By analogy, and taking into account the role of stable state as minimum of energy - that is zero value for the derivative of the energy - the properties of the Riemann zeta function suggest a radical breakthrough within the scientific paradigms able to defined a Complex Systems. Clearly to address the analytical problems, the scientist needs to deal with converging series to stable states. All means are good to go through divergences and to reduce the description of the reality through his stable states, but this point of view can not be exclusive especially when interaction takes place with environment. In this situation zeta function suggests an analogy. Certain forms of stationary - we mean for living in an open system with a management of inside and outside energy - requires additional ingredient for managing the out stable state flow of energy. The being of this flow must keep the divergences as diverging for its survival, and the state related to the minimum of energy must create a specific dynamics on time. Complex Systems behaves like a tempo object. To solve its relationship with an instable environment it has to use transparency fences taking into account the correlations characterizing obviously the bulk, the properties of the boundary and the external environment. The analysis of the properties of the set of zeta function enlightens the methods to do so using the symmetries. Obviously these methods may take into account the existence of stable states somewhere but at the same time they must overcome the reductionism that could be associated to them. This method leads us to embrace the total fields of the problematic and not solely the state given by the zeros of Riemann function. The scientific meaning of the zeros appears as a step.

At this step, however, the reasoning points out an additional opportunity of progress given by the Goldbach hypothesis applied upon the quadratic forms unveiled via the set of \( \{ Z^{\omega \rightarrow} \} \). We recall that the simplest Goldbach conjecture asserts that every even number is the sum of two prime numbers \( p \). Today this hypothesis has not received any mathematical demonstration. However, it may be illuminated by the above approach about the degeneracy of the quadratic forms for \( \alpha = 1/2 \) and the associated stable states. Let us recall that the set of all primes numbers and Riemann zeta function are associated within the relation \( \zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p} (1/1-(1/p^s)) \). The homomorphism with the set \( \{ Z^{\omega \rightarrow} \} \) stays pending the role of the time constant. This time constant have to assure the incommensurability and the independence of the couple of terms involved in the procedure of merging \( \zeta(s) \cup \zeta(1-s) \) Due to the fact that we consider only the case \( \alpha = 1/2 \) and as noticed in chapter 5 stable states \( \omega \rightarrow 0 \) are associated to an underlying geometry characterized by a space-time relationship based on diffusive equation that is to say \( \eta^2 \sim \tau \). Combined with Pythagoras theorem, see figure 2, this relation leads the Goldbach relation between prime number: \( \tau^{-1} + \tau^1 = 2 \tau \). This is therefore a heuristic approach of Goldbach hypothesis, which appears strongly related to the spectrum of zeros of Riemann function.

5. Conclusion
The results obtained during last 5 years about the relationship between Zeta function, fractional differential equations, \( \alpha \sim \) Riemann Manifold, irreversibility and complex systems can be summarized as follows. The Riemann's conjecture that we consider under the angle associated to the topology and to the differential equations of non-integer complex order, must be regarded as a geometrical phase.
transition leads by the order parameter which is the phase angle $\psi(\alpha)$ of the dynamics at infinity. This transition contributes to the stabilisation of all possible stable solutions of dynamic problem controlled by local Euclidean geometry of the space-time. That why some scientists look for the solution of the conjecture analysing the spectrum of stochastic matrix [27,28] or of fractal string structures [15]. However, due to the limited information contains at the boundary when $\alpha = 1/2$ (open systems with simple mirror effect at the boundary) these solutions only gain a physical Noetherian meaning (recovery of fundamental invariants such as energy relevance of the concept of standard velocity as a ratio between space and time etc.). The system may be complicated but never complex; a spectral analysis is sufficient to unveil the ontology of the stable states and associated objects. Due to the role of the major information at the boundary Complex Systems requires more advanced methods of approach. The simplest one requires the closing within a quadratic form using a couple of complementary operator to impose an asymmetry of time and to plunge this parameter in the Complex space. This time asymmetry is surprisingly related to the fact that the stable solutions of the zeta function require to take into account whether the angle theta is run positively or negatively (fundamental anisotropy of the complex plane is introduced by the group of tempo object). Here we find the properties of a non-commutative underlying $\alpha \sim$ geometry. The specific role of transcendence number $\sqrt{2}$ and intervention Goldbach’s conjecture when $\alpha = 1/2$ are pointed out. These properties have probably very strong correlation with non-commutative geometry [44] but also with $\alpha \sim$ Gromov group [65, 68].

Under this perspective, we suggest to look the Complex Systems as a system that is able to survive only as hybrid open structure represented by a merging of a couple of system associated to $\zeta(s) \cup \zeta(1-s)$, the internal pseudo stable states then emerging being stable if and only if the boundary properties, that contain information and energy, are balance by a internal flow of energy related to auto organisation homeostasis or negentropic effects, or in other word by a complimentarily local and holistic joined attitude. Obviously in this case the standard paradigms of the science and especially the strict separation between the object and the subject being cancelled an objective posture may again be designed. We had called this new strategy with respect to Complex System a Science [9]. By the way Michel Bitbol in his book [9] about the blinding near real: anti realism and quasi realism, analyses in detail these problems already emerging in the quantum mechanics. But quantum mechanics stays inside the science because the Noether axioms keep their meanings. It is no more the case for instance for living structure and a cell for instance or a brain cannot be reduced to their chemical components. What is need in addition is still unknown but it is sure that it is related to their relationships with its boundary. We though that above analysis and especially the spectral concept of tempo object and group at infinity may help us to unveil a part of the mysteries still hidden by complexity of the Complex Systems

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