Erasing the past of photons

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Abstract. Photons have been proved to hide their past, especially in interferometers. Recently, several surprising results have been shown in nested Mach-Zehnder interferometer (NMZI), proving that the ‘weak trace’ of photons can be hidden. This paper rearranges the three path interference problem which gives the general result of its output and the conditions for eliminating the weak trace. Surprisingly, such conditions strongly depend on the detection method of the output photon, as well as the phase differences and splitting ratios in the NMZI. Further analysis gives a new weak trace elimination phenomenon which is independent of the position sensitive detector. Moreover, we setup an experiment to verify the weak trace elimination conditions and demonstrate the new weak trace elimination phenomenon.

1. Introduction

The past of a quantum particle has been discussed for decades \cite{1,10}. And yet, physicists are still not sure the trajectory a quantum particle took from the source by only measuring it in a detector \cite{11,15}. Such discussions mostly focus on interferometers, such as double slit interferometer \cite{11,16,18} and Mach-Zehnder interferometer (MZI) \cite{3,10,19,21}. For example, Wheler et al. \cite{1,2} tried to infer the past path of photons through the output of photon detector in a double-slit interferometer and found a counter-factual result: we can’t infer the past of photons, because photons can hide their past.

Such phenomenon was further discussed by Vaidman \cite{3,4,10} in a nested Mach-Zehnder interferometer (NMZI). They proposed to use the weak trace as the criterion of whether the light passes through a mirror, then the photon will pass through
discontinuous paths in NMZI. This counter-factual result was explained by the two-state vector formalism [22], which has aroused extensive theoretical discussions [6, 9, 13, 15, 20, 23, 27] and experimental demonstrations [8, 16] in recent years. Danan et al. [8] used a position resolved photon detector in NMZI to find out the weak trace caused by the vibration of mirrors along the trajectory. Alonso et al. [6] found that using dove prism in NMZI can change the weak trace of photons which making the photons’ past change. Yuan et al. [9, 27] interpreted such weak trace problem with three path interference and shows a new condition to hide the past of photons. However, these researches focus on the weak trace of photons under certain conditions and the corresponding photon states.

In this article, we focus on the properties of NMZI which gives a general expression of the output and general conditions to erase the past of photons. A detail discussion shows that the weak trace elimination depends on the detection method of photons, as well as the phase differences and the splitting ratios of beam splitters (BSs) in NMZI. Our results can not only explain the experimental results in Ref. [8], but also predict a new method to eliminate the weak trace. In addition, we experimentally demonstrated such new effects without any position sensitive detector.

2. Theoretical model

![Figure 1. Scheme of NMZI. All the splitting ratios of the beam splitters used in this NMZI are adjustable. The mirrors A, B, C, E, F are vibrating with different frequencies.](image)

Our model is shown in the Fig. 1 where the beam-splitting ratios of the BSs are all arbitrarily adjustable. When a photon is emitted from the light source, the photon state passing through the BS1 is

\[ |\psi_{BS2}\rangle = t_1 |U\rangle + i r_1 |L\rangle, \]  

(1)
where $t_1$ is amplitude transmissivity of the BS1 and $r_1 = \sqrt{1 - t_1^2}$ is the corresponding amplitude reflectivity. Here $|L\rangle$ is the photon state that is reflected by BS1 and goes through the lower arm of the outer MZI, while $|U\rangle$ is the photon state propagates along the upper arm of MZI through BS1.

Similarly, when photons pass through the mirror E and BS2 of the inner MZI, the state changes to
\[
|\psi_{BS2}\rangle = t_1|U\rangle + i r_1 t_2 e^{i \varphi_c} |u\rangle - r_1 r_2 e^{i \varphi_c} |l\rangle,
\] (2)
where $\varphi_c$ is the complex phase caused by the vibration of mirror E. The real part and the imaginary part represent the change of the optical phase and intensity of light caused by the vibration, respectively. The vibration of mirror causes a tiny change of optical path when photons pass through the mirror, which induces the phase change of the photons. Meanwhile, the reflection angle of light also changes due to the vibration of the mirror, which makes the intensity of the interference change. Therefore, both tiny changes exist in the final output of interferometer, acting as the weak trace of the photon. State $|l\rangle$ is the state that the photon reflected by BS2 with amplitude reflectivity $r_2$ and propagates along the lower arm of the internal MZI. State $|u\rangle$ is transmitted by BS2 with amplitude transmissivity $t_2 = \sqrt{1 - r_2^2}$ and propagates along the upper arm of the inner MZI.

Similar to mirror E, mirrors A, B and C also bring tiny complex phases to photons which are
\[
|\psi_{ABC}\rangle = t_1 e^{i \varphi_a} |U\rangle + i r_1 t_2 e^{i (\varphi_b + \varphi_a)} |u\rangle - r_1 r_2 e^{i (\varphi_c + \varphi_a)} |l\rangle.
\] (3)

When the photons pass through the beam combiner BS3 of the internal MZI, the state of the photons becomes
\[
|\psi_{BS3}\rangle = t_1 e^{i \varphi_a} |U\rangle - (r_1 t_2 r_3 e^{i (\varphi_c + \varphi_b)} e^{i \phi} + r_1 r_2 t_3 e^{i (\varphi_c + \varphi_c)}) |out1\rangle
- i (r_1 r_2 r_3 e^{i (\varphi_c + \varphi_b)} - r_1 r_2 r_3 e^{i (\varphi_c + \varphi_b)} e^{i \phi}) |L'\rangle,
\] (4)
where $t_3$ and $r_3 = \sqrt{1 - t_3^2}$ are the amplitude transmissivity and amplitude reflectivity of BS3, respectively. State $|L'\rangle$ refers to the state that the photons propagate along the lower arm of the external MZI and reach the mirror F after the beam combination. While the state $|out1\rangle$ denotes that the photons leave NMZI without being detected. Here we introduce a real phase difference $\phi$ between the two arms of the internal interferometer $|u\rangle$ and $|l\rangle$ since the output of the interferometer may be different due to the phase.

After BS4, the final photon state from the interferometer is
\[
|\psi_{BS4}\rangle = i \left[ t_1 r_4 e^{i \varphi_a} e^{i \varphi_c} - (r_1 r_2 r_3 e^{i (\varphi_c + \varphi_b)} - r_1 r_2 t_3 e^{i (\varphi_c + \varphi_b)} e^{i \phi}) e^{i \varphi_f r_4} \right] |D\rangle
+ \left[ t_1 t_4 e^{i \varphi_a} e^{i \chi} + (r_1 r_2 r_3 e^{i (\varphi_c + \varphi_b)} - r_1 r_2 t_3 e^{i (\varphi_c + \varphi_b)} e^{i \phi}) e^{i \varphi_f r_4} \right] |out2\rangle
- (r_1 r_2 r_3 e^{i (\varphi_c + \varphi_b)} e^{i \phi} + r_1 r_2 t_3 e^{i (\varphi_c + \varphi_c)}) |out1\rangle,
\] (5)
where $t_4$ and $r_4 = \sqrt{1 - t_4^2}$ are the amplitude transmissivity and reflectivity of BS4. Here we introduce another real phase $\chi$ to represent a phase difference between the two arms of the external interferometer $|L \rightarrow l \rightarrow L'\rangle$ and $|U\rangle$. The state $|D\rangle$ means that the photons are finally received by the detector D, while the other state $|out2\rangle$ means that the photons are not detected after passing through the NMZI.
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Now we can directly get the signal of detector D as
\[ P_D = \left| ae^{i\varphi_a} e^{i\chi} + be^{i(\varphi_e + \varphi_f)} - ce^{i(\varphi_e + \varphi_f)} e^{i\phi} \right|^2, \]  
where \( a = t_1 r_4, b = r_1 t_2 t_3 t_4, c = r_1 r_2 r_3 t_4 \) are the amplitudes of lights inside the NMZI.

In the weak trace experiment, the complex phases caused by mirror vibrations are usually very small. That is to say, it has little influence on the output of the interference results. Under such condition, we can make a linear approximation \( e^{i\phi_k} \approx 1 + i\phi_k \), where \( k = a, b, c, e, f \). Then the Eq. 6 can be simplified to
\[ P_D \approx \left| a e^{i\chi} + b - c e^{i\phi} \right|^2 - 2Im\left( (a e^{i\chi} + b - c e^{i\phi}) (\varphi_e + \varphi_f) \right) - 2Im\left( (a e^{i\chi} + b - c e^{i\phi}) (a \varphi_a e^{i\chi} + b \varphi_b - c \varphi_c e^{i\phi}) \right). \]  
Here we only keep the linear terms and neglect the high order terms. This is the general output of the NMZI. With different splitting ratios and relative phases \( \phi \) and \( \chi \), we can obtain different results.

3. Analysis and Discussion

According to the general expression in Eq. 7, in principle, we can find and eliminate the weak traces of photons. For example, to eliminate the weak trace caused by mirror A, we can extract the term containing \( \varphi_a \) in Eq. 7 as
\[ -2aIm \left\{ [a + be^{i\chi} - ce^{-i(\phi-\chi)}] \varphi_a \right\} = 0. \]  
Here we neglect the high order terms due to the tiny phase approximation.

As we know, the amplitude \( a \) and the phase \( \varphi_a \) are not always zero. To eliminate the weak trace caused by mirror A, Eq. 8 should be expanded which gives
\[ \begin{cases} a + b\cos(\chi) - c\cos(\phi - \chi) = 0, & \text{eliminating } Re(\varphi_a), \\ b\sin(\chi) - c\sin(\phi - \chi) = 0, & \text{eliminating } Im(\varphi_a). \end{cases} \]

In this case, the output light does not contain any phase terms in first order. However, such conditions make the output of the NMZI becomes zero, i.e. \( P_D \approx 0 \). Obviously, the photon can not reach the detector which makes this condition useless.

But interestingly, the weak trace has been eliminated in Danan et al’s experiment while the photons actually reached the detector. In such experiment, they used a position sensitive quad-cell photodetector with differential detection method to obtain signals from NMZI. Such detection method erases the interference fringes caused by \( Re(\varphi_k), k = a, b, c, e, f \) while the intensity variances caused by \( Im(\varphi_k) \) remain. That is to say, they eliminate the real part of complex phase when doing detection while the imaginary part exist in the outputs. And they tuned the phases of NMZI to cancel the imaginary part.

If a similar position sensitive detection is used in our case, we can keep the term of \( Re(\varphi_a) \) and only eliminate the term of \( Im(\varphi_a) \) as
\[ \begin{cases} a + b\cos(\chi) - c\cos(\phi - \chi) = 0, & \text{eliminating } Re(\varphi_a), \\ b\sin(\chi) - c\sin(\phi - \chi) \neq 0, & \text{remaining } Im(\varphi_a). \end{cases} \]
Then the output of NMZI contains no weak trace of mirror A. This is only true when $\text{Re}(\varphi_a) = 0$ or the detector can not get the information of $\text{Re}(\varphi_a)$. In such case, the weak trace caused by mirror A can be eliminated in the output of NMZI when the beam splitting ratios of BSs and the phases of the interferometer satisfy the relations of Eq. [10].

Even more interesting is that there is another way to eliminate the weak traces of photons. When $\text{Im}(\varphi_a) = 0$ or the variance of intensity can not be detected, the weak trace caused by mirror A can also be eliminated when

$$
\begin{align*}
\begin{cases}
a + b \cos(\chi) - c \cos(\phi - \chi) & \neq 0, \\
bsin(\chi) - c \sin(\phi - \chi) & = 0,
\end{cases}
\end{align*}
$$

remaining $\text{Re}(\varphi_a)$, 

$$
\begin{align*}
\begin{cases}
a \cos \chi + b - c \cos \phi & = 0, \\
a \sin \chi - b - c \sin \phi & = 0,
\end{cases}
\end{align*}
$$

eliminating $\text{Im}(\varphi_a)$. (11)

Through these two cases, it is easy to find that the condition of eliminating weak trace of photons completely depends on the way of detection. The weak traces of photons vary with different detection methods, as well as the conditions of eliminating weak traces. Moreover, due to the simultaneous interference of three paths in NMZI, the weak trace elimination effect is much easier to achieve than the cases mentioned in previous literatures [3, 8, 27]. Actually, by adjusting the beam splitting ratios of BSs and the phases of interferometer, eliminating the weak traces of photons has become a completely controllable effect.

According to the discussion above, we can further obtain the conditions for eliminating weak traces caused by mirrors B, C, E and F from Eq. [8] which shows in table [1].

| Weak trace to be eliminated | Condition |
|-----------------------------|-----------|
| $\text{Im}(\varphi_a)$      | $a + b \cos \chi - c \cos(\phi - \chi) = 0$ |
| $\text{Re}(\varphi_a)$      | $b \sin \chi - c \sin(\phi - \chi) = 0$ |
| $\text{Im}(\varphi_b)$      | $a \cos \chi + b - c \cos \phi = 0$ |
| $\text{Re}(\varphi_b)$      | $a \sin \chi - b - c \sin \phi = 0$ |
| $\text{Im}(\varphi_c)$      | $a \cos(\phi - \chi) + b \cos \phi - c = 0$ |
| $\text{Re}(\varphi_c)$      | $a \sin(\phi - \chi) + b \sin \phi - c = 0$ |
| $\text{Im}(\varphi_e)$ & $\text{Im}(\varphi_f)$ | $b^2 + c^2 - 2bc \cos \phi + ab \cos \chi - ac \cos(\phi - \chi) = 0$ |
| $\text{Re}(\varphi_e)$ & $\text{Re}(\varphi_f)$ | $ab \sin \chi + ac \sin(\phi - \chi) = 0$ |

It can be seen from the table that the weak traces caused by any mirrors can be eliminated under certain conditions, but all weak traces cannot be eliminated at the same time. In the previous work [3, 4, 27], the weak trace caused by mirror E and mirror F can be surprisingly eliminated together. However, in the table we can see that the weak traces caused by any two mirrors can be eliminated at the same time, not only by mirror E and mirror F. What we need to do is carefully adjusted the phase differences and the beam splitting ratios of BSs. We can even eliminate the weak traces caused by three mirrors which is hardly seen in the previous work.
4. Experimental Results

To verify the above results, we set up an experiment as shown in Fig. 2. We use a half wave plate and a PBS to realize an adjustable BS. A normal detector is used to detect the output of the interferometer directly, so as to avoid detecting the signal caused by the change of light intensity and obtain the interference signal caused by phase. As shown in Fig. 2, each mirror vibrates with different frequencies. The final output signal of the NMZI is detected by D1 and analyzed by the power spectrum analyzer to obtain the weak traces of photons.

\[
\text{Figure 2.} \quad \text{Experimental setup of NMZI. The mirror A, B, C, E, F are vibrating with frequency } f_A = 1.1kHz, f_B = 1.2kHz, f_C = 1.3kHz, f_E = 1.4kHz, f_F = 1.5kHz, \text{ respectively.}
\]

According to table 1, the two relative phase \( \phi \) and \( \chi \) plays a central role of eliminating the weak traces. Therefore, we need to simultaneous detect the two phases of the NMZI accurately. As shown in Fig. 2, D2 is used to measure the photon state \( |\text{out1}\rangle \), the interference fringes of the internal interferometer. And D3 is used to measure the photon state \( |\text{out2}\rangle \), another output of NMZI. Such two detectors give the interference fringes which can be used to obtain the two phases in real time. According to Eq. 5 we can get the two phase as

\[
\phi = \cos^{-1} \left[ \frac{P_{D2} - r_1^2 (t_2^2 r_3^2 + r_2^2 t_3^2)}{2r_1^2 t_2 r_2 t_3 r_3} \right],
\]

\[
\chi = \cos^{-1} \left[ \frac{P_{D3} - (t_1^2 t_4^2 + r_1^2 r_2^2 r_3^2 + t_2^2 t_3^2 r_4^2 + r_2^2 t_1^2 t_2^2 r_3^2 - 2r_1^2 r_2^2 t_2 t_3 r_3 r_4^2 \cos \phi)}{2r_1 t_1 r_4 t_4 \sqrt{r_3^2 r_4^2 + t_2^2 t_3^2} - 2r_2 r_3 t_2 t_3 \cos \phi} \right] - \beta,
\]

where \( P_{D2} \) and \( P_{D3} \) are the optical power detected by D2 and D3, and

\[
\beta = \cos^{-1} \left( \frac{r_2 r_3 - t_2 t_3 \cos \phi}{\sqrt{r_2^2 r_3^2 + t_2^2 t_3^2} - 2r_2 r_3 t_2 t_3 \cos \phi} \right).
\]
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Figure 3. The power spectrum of the photon current from D1 shows the elimination of weak trace from (a) mirror A, (b) mirror B, (c) mirror E and F.

When we set $r_1 = r_4 = \sqrt{2/3}, r_2 = r_3 = \sqrt{1/2}$, the intensity of the light fields in the three paths of NMZI are the same. Then the output of NMZI varies with the two phases $\phi$ and $\chi$ as shown in Fig. 3. Since the detector D1 is a normal detector which can only detect the phase sensitive interference fringes, the weak traces we can detect is only the real part of complex phases. As we can see in Fig. 3 (a), the weak trace of mirror A is eliminated when $\phi = 0.223 \text{rad}, \chi = 0.11 \text{rad}$ which is consistent with Eq. 11.

When the phase of the internal and external interferometer becomes $\phi = -0.33 \text{rad}$ and $\chi = 0.74 \text{rad}$, the weak trace caused by mirror B is eliminated as shown in Fig. 3 (b). Moreover, the weak traces of mirror E and mirror F are eliminated as shown in Fig. 3 (c) if we set the phases to $\phi = -0.45 \text{rad}$ and $\chi = 0.44 \text{rad}$. Such results are all consistent with the predictions of table 1.

Figure 4. The experimental conditions and the corresponding fittings of eliminating the weak trace of mirror A. As we can see, the relation between the two phases $\phi$ and $\chi$ varies with the splitting ratio of BS2 $r_2$. 
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Furthermore, by tuning the half wave plates before each PBS we change the beam splitting ratios of BSs. Then we can still eliminate the weak trace caused by mirrors through adjusting the phases $\phi$ and $\chi$. Fig. 4 shows the relationship between the phases and $r_2$ when eliminating the weak trace of mirror A. As the phase difference $\chi$ changing, the phase $\phi$ varies and fits well with the theoretical simulation. Also, due to the variance of the splitting ratio $r_2$, the relation between the two phases changes which is also consistent with the theoretical simulation.

5. Conclusion

We rearrange the three path interference problem which gives the general conditions for eliminating weak traces. This general result can not only explain the previous experimental results but also predicted a new weak trace elimination effect. Further discussion shows that the elimination of photon’s weak trace depends on both photon detection method and the output photon state from the interferometer while the photon state depends on the phase differences and beam splitting ratio of BSs. Since the photon detection method determines the manifestation of the weak trace, we discover a new method to eliminate the weak trace. In summarize, we can change the way of photon detection, control the beam splitting ratio and phase to eliminate any weak trace. Moreover, these results have potential applications in quantum information and quantum precision measurement.

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