Extracting Work From A Single Heat Bath

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We present here a machine that is capable of extracting work from a single heat bath. Although no significant temperature gradient is involved in the operation of the machine, yet the Carnot efficiency as high as one is achievable. Working of the machine is explained on the basis of a demon suggested by Maxwell. Utilizing the kinetic energy spectrum of the molecules in solution, the demon can send "hotter" molecules to a higher gravitational potential at the expense of their own energies. Difference in chemical potentials due to concentration gradients and use of semi-permeable membranes ensure the continuing cyclic process.
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Perpetual Motion Machine (PMM)$^{1,2}$, although hardly allowed by the laws of thermodynamics, has been a subject of great interest since long. PMM of first kind, in contrast to the law of conservation of energy, is supposed to create energy, thus violating the first law of thermodynamics. On the other hand PMM of second kind should be capable to convert total heat into useful work showing 100% efficiency of the heat engine, which is rather contrary to the second law of thermodynamics.

Another manifestation that forbids PMM of second kind is that no work can be extracted from a single heat bath. This is because according to Carnot, greater efficiency of a heat engine is only possible if temperature of the energy source, $T_h$ is higher and the entropy sink is maintained at a lower temperature $T_c^{2-4}$ whereby the Carnot efficiency, is given as

$$\eta = 1 - \frac{T_c}{T_h}. \quad (1)$$

It is to note further that input of heat energy from a bath is essential for all heat engines but for a PMM of second kind it can take place without requiring any temperature difference and entropy sink.

An additional requirement that prohibits the existence of PMM is pertaining to entropy, which for a spontaneous process must not decrease. Since PMM of second kind is capable of increasing order by separating high and low energy particles without any energy involvement, its existence is not allowed according to the entropy considerations as well. Maxwell’s demon$^{5,6}$, a hypothetical “creature”, can sort hot and cold types of particles without any effort in measurement and consumption of energy, therefore it may also be considered equivalent to a PMM of second kind.

Discovery of quantum non-demolition measurements and computing$^{7-12}$ have given an incentive and led to a refined review of thermodynamics demanding a more careful search for the existence of PMM.

Recently in their excellent paper Scully et al.$^{13}$ working on a new kind of quantum heat engine using “phaseonium”$^{14}$ as fuel and utilizing lasing without inversion$^{15-17}$, showed that
work can be extracted from a single heat bath but they had to consider vanishing quantum coherence to save laws of thermodynamics. In a recent communication Scully\textsuperscript{18} used a Stern-Gerlach apparatus to sort hot and cold spin atoms thus acting as Maxwell’s demon. To achieve a cyclic operation, the atomic center of mass was prepared in a well-defined quantum state, showing that the “cost” of preparing the center of mass wave packet was enough to preserve the second law of thermodynamics.

Distribution of kinetic energy between the particles in a sample of gas or liquid at a given temperature is Gaussian\textsuperscript{19,20}. In an isolated system, one possible way of sorting high and low energy particles can be moving up high-energy particles to a higher gravitational potential at the expense of their own heat energies. The above scheme may be employed for a cyclic process of mass-transfer to achieve useful work without involvement of an initial temperature gradient.

At this point a system of two containers at different gravitational potentials may be considered as shown in Fig. 1. The container $L$ at lower potential is filled with neat water, while the container $H$ at higher potential is initially empty. The system is in thermal equilibrium at an absolute temperature $T$. The demon allows transfer of higher kinetic energy particles from $L$ to $H$ utilizing their own energies. Consequently, mass $m$ of water may reach to $H$ after gaining the gravitational potential energy $mgh$, where $g$ is the (average) gravitational constant and $h$ is the height to which water is raised from the surface of water in $L$. This will result in an overall decrease in temperature of the thermodynamically isolated system. This temperature decrease may be related with potential energy as

$$
\Delta T = \frac{mgh}{M\sigma}, \tag{2}
$$

where $M$ and $\sigma$ are the mass and specific heat capacity of the whole system. The temperature of the system may again be raised back to $T$ if water from $H$ is allowed to drain into container $L$. If the setup is connected to a heat reservoir that can maintain temperature $T$, this demon will continue to pour more and more water into $H$ by increasing the potential energy of water at the expense of heat taken from the bath.
Fig. 1. Maxwell’s demon capable of separating higher energy water molecules to a container at higher potential.

Kinetic energy spectrum of molecules in a solution requires the lighter solvent molecules to be more agile and hence an effective means of energy transfer. A semi-permeable membrane ($SPM$) allows only solvent molecules to pass through it but solute molecules, especially of larger size, are not allowed to cross. Statistically the solvent molecules may move across both sides of $SPM$. However, a net flux of solvent across the $SPM$ may take place under the influence of gravitational and/or osmotic pressure.

Consider the setup for osmosis as shown in Fig. 2. (A). A tube whose lower end is attached to an $SPM$ contains sugar solution in it. If the $SPM$ is immersed in neat water and the system is left alone to equilibrate, the level of the solution rises in the tube due to osmosis till a height $h$ is finally attained at which the column level in the tube is maintained by a counterbalance of water-flow across the $SPM$. Under this condition the chemical potential of water on the two sides of the $SPM$ becomes equal$^{21}$. The column pressure, $\rho gh$ ($\rho$ being the solution density in appropriate units) makes water to flow out of the tube through $SPM$ while the osmotic pressure tends to maintain the level by an inwards flow. As a result, a state of dynamic equilibrium is reached. The osmotic pressure, $\Pi$ may be related to the mole fraction of water, $X_A$ as

$$\Pi = -(RT/V_m)\ln X_A,$$  \hspace{1cm} (3)

where $R$ is the gas constant, $T$ is absolute temperature and $V_m$ is the molar volume of water. For dilute solutions, eq. (3) simplifies to well-known van’t Hoff’s expression that directly relates osmotic pressure with the solute concentration$^{21,22,23}$. It is evident that the column pressure, $\rho gh$ alone is responsible for the outwards flow of water through $SPM$. Hence in the event where the $SPM$ is not immersed, water will come out of it under the influence of the column pressure. This is depicted in Fig. 2 (B), where shape of the $SPM$ is slightly modified as per requirement of the experiment to be discussed later.
Fig. 2.  (A) Setup for osmosis showing a rise of the solution level due to inwards flow of water through the SPM. (B) Setup for the outwards flow of water under the influence of column pressure, $\rho gl$.

Now we join together the above two setups so that the sugar solution becomes enclosed in a tube whose both ends are attached to $SPM$ as shown in Fig. 3. Since the lower $SPM$ remains immersed in water in $L$, osmosis continues and tends to maintain the column height $h$. The inward pressure at the lower $SPM$ is still governed by eq. (3). On the other hand, the upper $SPM$ has been kept hanging at an average height $k$ above the water level in $L$. An outwards pressure on the upper $SPM$ is an essential requirement for the outflow of water through it; keeping $k < h$ ensures this working condition. A suitable arrangement for the escape of dissolved air or the air that enters into the tube through $SPM$ may be made by providing an “air escape valve” above height $h$ as shown in Fig. 3. Water uptake through the lower $SPM$ and release through the upper $SPM$ makes reaching a dynamic equilibrium at which a steady flow of water from $L$ to $H$ is maintained. The neat water collected in $H$ may be made to drain back into $L$ thus completing the flow cycle and providing an opportunity to extract useful work from the system. We have successfully achieved the working of the setup shown in Fig. 3 at ambient temperature and also repeated the experiment. For this purpose, about 30% by weight sugar solution was used and a natural $SPM$ was employed.

Fig. 3. Setup showing a PMM of second kind, regarded as a Maxwell’s demon, which can totally convert absorbed heat into useful work without any involvement of temperature difference.

Functioning of the above $PMM$ depends upon various factors related to its design and the working conditions. A steady flow rate, besides other factors, depends upon quality & size of the $SPM$, concentration of the sugar solution and temperature of the system. A number of factors tend to hinder the performance of the $PMM$. One important factor is the accumulation of water droplets at the outer surface of the upper $SPM$. This problem may be minimized by properly designing the shape of the $SPM$. Another factor is the concentration gradient developed within the solution due to induction of neat water. Diffusion of sugar
molecules and a slow rate of flow tend to nullify this gradient. The net effect of both the above factors is a reduction of the flow rate than expected.

We conclude that the setup shown in Fig. 3 behaves as a Maxwell’s demon without itself consuming any energy or involving any change of state, but converts heat into potential energy. It is notable that the whole system is at a single temperature, $T$ maintained by the heat bath of the surrounding. Further the system is non-temporal, i.e., all the parameters like concentration and pressure etc. are time independent.

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**Figure Caption**

Fig. 1 Maxwell’s demon capable of separating higher energy water molecules to a container at higher potential.

Fig. 2 (A) Setup for osmosis showing a rise of the solution level due to inwards flow of water through the SPM. (B) Setup for the outwards flow of water under the influence of column pressure, $\rho gl$.

Fig. 3 Setup showing a PMM of second kind, regarded as a Maxwell’s demon, which can totally convert absorbed heat into useful work without any involvement of temperature difference.
Container H

$\beta$

Container L
Sugar solution
Water coming out
(B)(A)
Neat water
Container L
SPM
Sugar solution
Water coming out
Container H
k
(A)
(B)
Neat water
Air escape valve
Container H
Sugar solution
Neat water
Container L