DEEP-INELASTIC DIFFRACTION AND THE POMERON AS A SINGLE GLUON

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Abstract

Deep-inelastic diffractive scaling provides fundamental insight into the QCD pomeron. It is argued that single gluon domination of the structure function, together with the well-known Regge pole property, determines that the pomeron carries color-charge parity $C_c = -1$ and, at short distances, is in a super-critical phase of Reggeon Field Theory. The main purpose of the talk is to describe the relationship of the super-critical pomeron to QCD.

Presented at the International Conference (VIIth Blois Workshop) On Elastic and Diffractive Scattering - Recent Advances in Hadron Physics.
Seoul, Korea. June 10-14, 1997.

*Work supported by the U.S. Department of Energy, Division of High Energy Physics, Contracts W-31-109-ENG-38 and DEFG05-86-ER-40272
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1 Introduction

Understanding the pomeron in QCD is equivalent to solving the theory at high-energy. In this talk I want to focus on two, apparently very different, experimental properties of the pomeron that I believe provide important insight into the problem.

(i) 35 years of experiment/phenomenology/theory have shown that the pomeron is, approximately, a Regge pole.

(ii) Recent DIS diffractive scaling violation results\(^1\) show that (at \(Q^2 \sim 5 \text{ GeV}^2\)) the pomeron is, approximately, a single gluon.

It has been known for a long time that (i) is very difficult to reconcile with perturbative QCD and the 2 gluon BFKL pomeron. (ii) is a relatively new result that, as I discuss further below, is similarly in conflict with leading-twist perturbative QCD (and even, at first sight, with gauge invariance). In this talk I will argue that (i) and (ii) are closely related and reflect a subtle mixture of perturbative and non-perturbative physics in the Regge limit. We will see that both (i) and (ii) are satisfied if the pomeron carries color-charge parity \(C_c = -1\) and, at short distances, is in a super-critical phase of Reggeon Field Theory. The focus of the talk will be on the relationship of the super-critical pomeron to QCD.

It was more than sixteen years ago that I first suggested\(^2\) the pomeron could appear, in a super-critical phase, as a single (reggeized) gluon in a soft gluon background. Subsequently\(^3\),\(^4\), I laid out what I hoped might be the basis for a full dynamical understanding of the QCD pomeron. Although my arguments were very incomplete, they implied a fundamentally different picture of the pomeron to what might be called the conventional BFKL perturbative picture. It was clear that in my picture the nature of the pomeron is intimately related to confinement and chiral symmetry breaking. The scaling violation results\(^1\) from H1 have encouraged me to return to this work and attempt to put it on firmer ground. I briefly outline below the central elements of a new paper in preparation. Although the global picture presented in\(^3\) re-emerges, the details are different in important ways.

2 Outline

I begin, in Section 3, by reviewing the DIS diffractive results and the H1 analysis of the scaling violations. In Section 4, I go on to discuss why the large logarithmic violations can not be reproduced by conventional leading-twist perturbative QCD calculations. I then list, very briefly, in Section 5 the elements of multi-Regge theory\(^5\) that provide the underlying basis for my analysis. The main body of the talk, Sections 6, 7 and 8, outlines how multi-regge theory can be used to simultaneously derive both the dynamical pomeron and hadron reggeons via super-critical RFT. Finally, in Section 9, I briefly discuss how the single gluon approximation appears in DIS diffraction.
The essential points of the talk are the following.

- Existing Regge limit QCD calculations together with new calculations of further “helicity-flip” reggeon vertices allow multi-Regge theory to be used to construct the asymptotic behavior of very complicated scattering processes in terms of $J$-plane reggeon diagrams ($k_\perp$ integrals with reggeon propagators $[J - 1 + \sum \Delta(k_\perp^2)]^{-1}$ and interactions). This description is unitary (and complete) in the small $k_\perp$ region when gluons are massive.

- In such processes bound states and their scattering amplitudes can be simultaneously studied and the infra-red (massless gluon) limit taken.

- In helicity-flip multi-Regge kinematics, perturbative massless quark contributions to vertices contain the infra-red triangle anomaly and produce the chirality violation normally associated with non-perturbative instanton interactions.

- An effective multi-regge reggeon theory of (massless) quarks and gluons that implicitly includes effects of instanton interactions can be constructed.

- In the infra-red limit giving SU(2) gauge symmetry (recall that an instanton interaction is associated with an SU(2) subgroup), infra-red divergent multi-regge diagrams appear which contain “hadron” reggeon states scattering via “pomeron” exchange.

- The pomeron appears as a Regge pole which is, in first approximation, a reggeized gluon in a “reggeon condensate”. All the features of super-critical RFT are present.

- Hadron reggeons have a confinement and chiral symmetry breaking spectrum and (although details remain to be worked out it appears that) chiral symmetry breaking is essential for the self-consistency of the pomeron.

- When there is a $k_\perp$ cut-off, “complementarity” implies the full SU(3) gauge symmetry can be smoothly restored. Critical behavior of the Pomeron is involved.

- In DIS diffraction the reggeized gluon appears as a single gluon.
3 DIS Diffractive Scaling Violations

The definition of the diffractive structure function $F_2^D(x_F, \beta, Q^2)$ is illustrated in Fig. 3.1. We use the usual kinematic variables

$$W^2 = (P+Q)^2 \quad \beta = \frac{Q^2}{Q^2 + M_X^2} \quad x_F = \frac{Q^2 + M_X^2}{Q^2 + W^2}$$

Fig. 3.2 shows the H1 results for the $Q^2$ and $\beta$ dependence of $F_2^D(x_F, \beta, Q^2)$ at small-$x_F$. The presence of positive $\ln(Q^2)$ scaling violations over a large range of $\beta$ is clear.

By fitting to DGLAP evolution, H1 have extracted the low $Q^2$ pomeron structure function shown in Fig. 3.3. To reproduce (at medium $\beta$) the logarithmic rise at large $Q^2$, a single gluon must carry nearly all the pomeron momentum.

As we discuss next, for perturbative two gluon exchange, a single gluon can not carry nearly all the momentum. This is closely related to the scale-invariance property of the BFKL pomeron which produces a fixed singularity in the $J$-plane rather than a Regge pole.
4 Perturbative Scaling Violations

As illustrated in Fig. 4.1, in lowest order there are two diagrams for \((t\text{-}channel)\) two-gluon exchange. The short-distance region \(k_\perp^2 \sim Q^2 \gg t\) gives non-leading twist behavior \((\int \frac{d^2 k_\perp}{k_\perp^2} \sim \frac{1}{Q^2})\). Consequently leading-twist necessarily involves \(k_\perp \ll Q^2\) for one gluon. If we consider \(k_\perp^2 > \Lambda_{IR}^2 (\equiv \Lambda_{QCD}^2)\), the \(\Lambda_{IR}\) dependence cancels between the two diagrams. This cancellation is the well-known infra-red finiteness property of the BFKL pomeron (which follows directly from gauge invariance). The absence of \(\Lambda_{IR}\) dependence implies there is no scale for any potential \(\ln [Q^2]\) contributions, which indeed are absent. In effect, gauge invariance implies that the \(k_\perp\) regions for the two gluons can not be separated sufficiently to produce a leading-twist, leading-order, \(\ln [Q^2]\).

It is straightforward to extend the above argument to any number of perturbative gluons coupling directly to the quark loop and to “soft Pomeron models” defined via two (or more) “non-perturbative” gluon propagators. \(\ln [Q^2]\) dependence will arise, of course, when gluon exchanges between the gluons of Fig. 4.1 are taken into account, but this will be down by \(O(\alpha_s)\) and will be too small to describe the scaling violations of Fig. 3.2.

As illustrated in Fig. 4.2, to obtain a large \(\ln [Q^2]\) dependence in “leading-order” and reproduce the \(H1\) analysis, requires one hard gluon, with cut-off \(\Lambda_{IR}\), plus a color compensating interaction with \(\langle k_\perp \rangle \ll \Lambda_{IR}\). The above argument implies that gauge invariance will be violated unless the soft interaction is distinguished from single gluon exchange - via a quantum number. Even so, it is clearly non-trivial to suppose that gauge invariance can be maintained when colored exchanges at different scales are combined.

5 Multi-Regge Theory - the Key Ingredients

\textit{i) Angular Variables} - For the \(N\)-point amplitude we write

\[ M_N(P_1, \ldots, P_N) \equiv M_N(t_1, \ldots, t_{N-3}, g_1, \ldots, g_{N-3}) \]

where, in the notation of Fig. 5.1, \(t_j = Q_j^2\) and \(g_j \in \text{the little group of } Q_j\), i.e. \(g_j \in \text{SO}(3)\) or \(g_j \in \text{SO}(2,1)\) for \(t_j \geq 0\). There are \(N-3\) \(t_i\) variables, \(N-3\) \(z_j (\equiv \cos \theta_j)\) variables and \(N-4\) \(u_{jk} (\equiv e^{i(\mu_j-\nu_k)})\) variables.
ii) A Multi-Regge Limit is defined by \( z_j \to \infty \forall j \). In a Helicity-Pole Limit, some \( u_{jk} \to \infty \) and some \( z_j \to \infty \).

iii) Partial-wave Expansions. Using \( f(g) = \sum_{j=0}^{\infty} \sum_{|n|,|n'|<J} D_{n,n'}^j(g) a_{Jn',n} \), for a function \( f(g) \) defined on SO(3), leads to \( M_N(t, g) = \sum_{J} \sum_{n} \Pi_{i} D_{n,n_i}'(g_i) a_{J,n_i,n_i}'(t) \)

iv) Asymptotic Dispersion Relations. We can write \( M_N = \sum_{C} M_N^C + M^0 \) where
\[
M_N^C = \frac{1}{(2\pi)^N} \int \frac{d\tau_1'...d\tau_N'}{(\tau_1' - \tau_1)(\tau_2' - \tau_2)...(\tau_N' - \tau_N)}
\]
and \( \sum_{C} \) is over all sets of (N-3) Regge limit asymptotic cuts. \( (M^0 \) is non-leading in the multi-regge limit.) The resulting separation into (hexagraph) spectral components is crucial for the development of multiparticle complex angular theory.

v) Sommerfeld-Watson Representations of Spectral Components e.g.
\[
M_4^C = \frac{1}{8} \sum_{N_1,N_2} \int \frac{dn_1 dn_2 d\lambda_1 u_2^0 a_{1,1}^0}{\sin \pi n_2 \sin \pi (n_1 - n_2) \sin \pi (J_1 - n_1)} a_{N_2 N_3}^{N_1, N_2} (J_1, n_1, n_2, t)
\]
from which the form of multi-Regge behaviour in any limit can be extracted.

vi) t-channel Unitarity in the J-plane Multiparticle unitarity in every t-channel can be partial-wave projected, diagonalized, and continued to complex \( J \) in the form
\[
a_+^J a_-^J = i \int d\rho \sum_{\Delta N} \int \frac{dn_2 dn_2'}{\sin \pi (J_1 - n_2)} \int \frac{dn_2'}{\sin \pi (n_1 - n_2 - 3)} \cdots a_+^J \Delta N a_-^J \Delta N
\]
Regge poles at \( n_i = \alpha_i \), together with the phase-space integral and the “nonsense poles” at \( J = n_1 + n_2 - 1, n_1 = n_3 + n_4 - 1, \ldots \) generate Regge cuts.

vii) Reggeon Unitarity. In ANY partial-wave amplitude, the J-plane discontinuity due to \( M \) Regge poles \( \alpha = (\alpha_1, \alpha_2, \ldots \alpha_M) \) is given by the reggeon unitarity equation
\[
\text{disc}_{J=\alpha_M(t)} a_+^J a_-^J = \xi_M \int d\rho \ a_+^J a_-^J \frac{\delta(J-\sum_{k=1}^{M} (\alpha_k - 1))}{\sin \frac{\pi}{\alpha_1 - \tau_1} \cdots \sin \frac{\pi}{\alpha_M - \tau_M}}
\]
Writing \( t_i = k_i^2 / \lambda \) (with \( \int dt_1 dt_2 \lambda^{-1/2}(t_1, t_2) = 2 \int d^2 k_1 d^2 k_2 \delta^2(k - k_1 - k_2) \)), \( \int d\rho \) can be written in terms of two dimensional “ k-1” integrations, anticipating the results of direct (s-channel) high-energy calculations (leading-log, next-to-leading log etc.).Because the gluon “reggeizes” in perturbation theory, i.e. is a Regge pole in the J-plane, reggeon unitarity is a powerful constraint on the higher-order contributions of multigluon exchange.

6 Reggeon Diagrams for Multi-Regge Limits

Reggeon unitarity is particularly powerful when applied to multiparticle “helicity-pole” limits. If the SW representation shows only one partial-wave amplitude is involved, reggeon unitarity implies that the limit is fully described by two-dimensional “ k-1” diagrams. (In fact the “physical” transverse planes actually contains lightlike momenta.)
E.g. for an 8-point amplitude introduce angular variables as in Fig. 6.1 and consider the “helicity-flip” helicity-pole limit in which 
\[ z, u_1, u_2, u_3, u_4 \to \infty \]. The behavior of invariants is 
\[ P_1.P_2 \sim u_1 u_2, \quad P_1.P_3 \sim u_1 z u_3, \quad P_2.P_4 \sim u_2 u_4, \]
\[ P_1.Q_3 \sim u_1 z, \quad Q_1.Q_3 \sim z, \quad P_3.Q_1 \sim z u_4 \]
\[ \cdots \]
\[ P_1.Q, \quad P_2.Q, \quad P_3.Q, \quad P_4.Q \quad \text{finite} \]

Since the S-W representation shows that only one partial-wave amplitude is involved, a simple reggeon unitarity equation holds in all channels. As a result the complete multi-Regge behavior can be described by \( k_\perp \) integrals of the form illustrated in Fig. 6.2, where \( T^L, T^R \) contain connected and disconnected interactions that involve both elastic scattering (s-channel “helicity non-flip”) reggeon vertices and also new “helicity-flip” vertices.

Helicity-flip vertices play a crucial dynamical role. They do not appear in either elastic scattering or multi-Regge production processes. Such vertices are most simply isolated in a “non-planar” triple-regge limit involving three light-cone momenta. In the notation of Fig. 6.3 we can define a non-planar triple-regge limit by
\[ P_1 \to (p_1, p_1, 0, 0), \quad p_1 \to \infty, \quad Q_1 \to (0, 0, q_2, -q_3) \]
\[ P_2 \to (p_2, 0, p_2, 0), \quad p_2 \to \infty, \quad Q_2 \to (0, -q_1, 0, q_3) \]
\[ P_3 \to (p_3, 0, 0, p_3), \quad p_3 \to \infty, \quad Q_3 \to (0, q_1, -q_2, 0) \]

7 Reggeized Gluon Helicity-Flip Vertices Involving Massless Quarks

Consider three quarks scattering via gluon exchange with a quark loop triple-gluon coupling as illustrated in Fig. 7.1. The non-planar triple-regge limit discussed above gives
\[ g^6 \frac{p_1.p_2.p_3}{t_1.t_2.t_3} \Gamma_{1+2+3+}(q_1, q_2, q_3) \] where \( t_i = Q_i^2, \gamma_i = \gamma_0 + \gamma_i \) and \( \Gamma_{\mu_1 \mu_2 \mu_3} \) is given by the quark triangle diagram, i.e.
\[ \Gamma_{\mu_1 \mu_2 \mu_3} = i \int d^4k \frac{Tr\{\gamma_{\mu_1}(\gamma^\ast_2 + k + m)\gamma_{\mu_2}(\gamma^\ast_3 + k + m)\gamma_{\mu_3}(\gamma^\ast_4 + k + m)\}} {[q_1 + k - m]^2 - m^2} \]

It is a crucial property of the “\( O(m^2) \)” part of \( \Gamma_{1+2+3+} \) that the limits \( q_1, q_2, q_3 \sim Q \to 0, m \to 0 \) do not commute. (An “infra-red anomaly” due to the triangle Landau singularity).
\[ \Gamma_{1+2+3+,m^2} \sim Q \sim 0 \quad \text{m}^2 \int \frac{d^4k}{[k^2 - m^2]} \quad m \to 0 \]
After adding color factors and summing diagrams, $\Gamma_{1+2+3+,m^2}$ survives only in triple-regge vertices that couple reggeon states which all have “anomalous color parity”, i.e. $C_c = -\tau$. ($\tau$ = signature and color parity is defined via $A_{ab} \rightarrow -A_{ba}$ for gluon color matrices.) An important example is shown in Fig. 7.2. In this case each three reggeon state has odd signature but even color parity, e.g. $f_{ijk}d_{jrs}A^{k\gamma}A^{\gamma}_{\delta}$. In these circumstances, the survival of $O(m^2)$ processes as $m \rightarrow 0$ reproduces the chirality violation of instanton interactions.

Note that $\Gamma_{1+2+3+,m^2}$ has $[k_\perp]$-dimension 1 !! Normal reggeon interactions have $[k_\perp]$-dimension - 2. When combined with transverse momentum conservation i.e. $\delta^2(k_\perp)$, the normal interactions naturally produce a scale-invariant (and even conformal invariant) massless theory. Because of it’s anomalous dimension, $\Gamma_{1+2+3+,m^2}$ can only appear in special multi-Regge limits where the large invariants contribute an an extra momentum factor. The non-planar triple-regge limit discussed above is an example. In the notation of Fig. 7.3

$$p_1p_2p_3Q_i \leftrightarrow (p_1p_3)^{1/2}(p_2p_3)^{1/2}(p_1p_2)^{1/2}Q_i \equiv (s_{31})^{1/2}(s_{23})^{1/2}(s_{12})^{1/2}Q_i$$

Fig. 7.3

8 Infra-red Divergences

The anomalous dimension $\Gamma_{1+2+3+,m^2}$ also implies a “violation” of conventional reggeon Ward identities, i.e. when $m \rightarrow 0$, the vertex does not vanish sufficiently fast when all $Q_i \rightarrow 0$. (When $m \neq 0$, other diagrams combine to produce the normal reggeon Ward identities.) As a consequence, when the gluon mass is zero, an infra-red divergence appears as $m \rightarrow 0$ in a large class of non-planar multi-regge diagrams where $Q_1 \sim Q_2 \sim Q_3 \sim 0$ is part of the integration region. A candidate diagram is shown in Fig. 8.1. Both $\quad$ and $\quad$ are reggeon states.

A divergence will occur for $Q, Q_1, Q_1' \sim 0$ as $m \rightarrow 0$ if the vertex $V$ contains $\Gamma_{1+2+3+,m^2}$. This requires that $\quad$ be an anomalous color parity reggeon state. To discuss whether the divergence is cancelled by other diagrams requires a systematic analysis which begins with the SU(3) gauge symmetry initially restored only to SU(2) (c.f. an instanton is associated with an SU(2) subgroup). In this case the divergence occurs when

$\quad$ is an SU(2) singlet state containing one or more massive reggeized gluons (or quarks)

$\quad$ is an SU(2) singlet combination of massless gluons with $C_c = -\tau = +1 \equiv (\text{“anomalous odderon”})$.

Fig. 8.1
Higher-order diagrams containing a divergence include those of the form shown in Fig. 8.2. There is always an overall logarithmic divergence from that part of the integration region where all the $Q_i$ entering each $V$ vertex vanish. “Physical amplitudes” are obtained by extracting the coefficient of the divergence. In this coefficient all anomalous odderon reggeon states carry zero $k_\perp$ and so, effectively, the definition of reggeon states includes “an anomalous odderon reggeon condensate”. There is also, as illustrated in Fig. 8.3, an SU(2) singlet “parton process” which carries the kinematic properties of the dynamical interaction. Since the condensate background results from the quark triangle anomaly it (qualitatively) can be thought of as originating from instanton interactions.

The “pomeron” appears as an SU(2) singlet reggeized gluon (with $C_c = \tau = -1$) in the $C_c = -\tau = +1$ condensate, i.e. an even signature Regge pole with $C_c = -\tau = -1$. “Hadrons” are similarly (constituent) quark reggeon states in the condensate background. (It appears that “stability” within the condensate requires chiral symmetry breaking, but I will not discuss this here. Note, however, that $C_c = -1$ for the pomeron implies that hadrons cannot be eigenstates of $C_c$. This does not imply there is no charge conjugation symmetry.) All features of super-critical pomeron RFT are present, but I will not discuss the details.

Restoration of SU(3) gauge symmetry (i.e. the decoupling of a “Higgs” scalar) is straightforward provided there is a cut-off $|k_\perp| < \Lambda_\perp$. The most important point is that the pomeron is described by RFT and carries a crucial remnant of the construction i.e. odd SU(3) color charge parity. An immediate implication is that the BFKL Pomeron does not appear (since it has $C_c = +1$). To remove $\Lambda_\perp$ the pomeron should be Critical. (This is related to the quark content of the theory and again we will not discuss it here.) A-priori, at a fixed value of $\Lambda_\perp$, the theory may be above, or below, the critical point.

9 Deep-Inelastic Scattering

For this discussion we consider the SU(3) gauge symmetry to be restored and assume the critical cut-off $\Lambda_{\perp c}$ is above the physical cut-off so that the full theory is sub-critical. DIS diffractive scattering will expose the simplest perturbative contribution to the pomeron. $C_c = -1$ determines this to be four gluon exchange. A “non-perturbative” coupling $I_c$ to chirality-violating processes must be involved. We assume the chirality-violating scale is smaller than the “perturbative” $Q^2$ scale.
As illustrated in Fig. 9.1, there are two distinct reggeon/gluon diagram contributions, distinguished by whether the anomalous color parity state coupling via $I_c$ involves three or two reggeized gluons. In the Regge limit (finite $Q^2$) the $s_8$ two gluon state gives an even signature reggeon with a “pointlike” coupling to the quark loop. In higher-orders, the $s_A$ single gluon and the $s_8$ two gluon configurations reggeize with identical trajectories. Hence regge-limit infra-red divergences (due to the anomalous reggeon states) from the two contributions can directly cancel.

At large $Q^2$ the first diagram, with the single hard gluon, dominates. The same effect is achieved by breaking the SU(3) gauge symmetry to SU(2). This is why we can say that the pomeron is “in the super-critical phase” at the deep-inelastic scale. The result is the hard gluon behavior seen by H1!

Acknowledgements

I am particularly grateful to Mark Wüsthoff for extensive discussions.

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