Lack of trinification in $Z_3$ orbifolds of the $SO(32)$ heterotic string

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Abstract

We report results relating to the trinification scenario in some explicit string constructions that contain $SU(3)^3$ as a gauge symmetry. These models are obtained from symmetric $Z_3$ orbifolds of the $SO(32)$ heterotic string with one discrete Wilson line. We highlight the obstacles that were encountered: the absence of the usual Higgs sector that would break $SU(3)^3 \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$; the presence of exotics that would generically befoul gauge coupling unification and lead to fractionally-charged states in the low energy spectrum.

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**Introduction.** Here we continue work begun in [1]. In that letter, all consistent embeddings with one discrete Wilson line were enumerated for the case of symmetric $\mathbb{Z}_3$ orbifolds of the $SO(32)$ heterotic string.

In the present work we study the spectra for a few of the models. Our main focus will be on models that have $SU(3)^3$ gauge symmetry. Given appropriate representations ($\text{reprs}$), this extended gauge symmetry can lead to a **trinification** scenario [2, 3].

Trinification has been suggested as a favorable route for model-building in explicit string constructions [4]. The advantages of this sort of “unified” model have been discussed at length in refs. [3, 4], so we only briefly mention them here. First, the Higgs representations needed to break the $SU(3)^3$ symmetry to $G_{SM} = SU(3) \times SU(2) \times U(1)$ are allowed in affine level 1 constructions, such as the ones we consider here. By contrast, the adjoint Higgses of grand unified theories are not allowed in affine level 1 constructions [5]. Second, because the electroweak hypercharge is embedded into $SU(3)^3$, its normalization is standard, unlike most standard-like constructions. ¹ Third, proton decay can be forbidden by imposing a $Z_2$ discrete symmetry, while still allowing for light fermion masses.

We will encounter difficulties realizing the trinification scenario; the obstruction is due to the absence of the necessary Higgs reprs to break to $G_{SM}$ along a $D$-flat direction. We point out that a similar result occurs in the models ($Z_3$ orbifolds of the $E_8 \times E_8$ heterotic string) constructed in [4], though it was overlooked in that case because $D$-flatness was not checked.

In addition to our explorations of the trinification scenario, we will show that some of the models enumerated in [1] are vector-like and are therefore excluded as extensions of the Standard Model.

**Accomodating trinification.** Here we focus on models from [1] that explicitly contain the gauge symmetry $SU(3)^3$, in addition to other factors. Using the relations discussed in [1], it is straightforward to show the equivalences Model 5.6 ≃ Model 2.12 and Model 5.7 ≃ Model 3.13. The inequivalent models are summarized in Table [1].

In a trinification scenario, such as has been described in detail in [3], we fit the fermion spectrum of the Standard Model into left-handed fermions that fall into the $SU(3)^3$ representation²

$$3[(3, \ar{3}, 1) + (\bar{3}, 1, 3) + (1, 3, \bar{3})] = 3 \text{27s of } E_6$$

(1)

That this is nothing but the $E_6 \supset SU(3)^3$ decomposition of three 27’s has been indicated. Thus [1] contains some extra states beyond the Standard Model, as will be made explicit

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¹By “standard” normalization, we mean that the unification of couplings involves the factor of $\sqrt{5/3}$ in $g_3(\Lambda_U) = g_2(\Lambda_U) = \sqrt{5/3}g_Y(\Lambda_U)$, which is not necessarily the case in heterotic string models with a standard-like gauge group $G_{SM} \times \cdots$. See, for example, Sec. 4 of [3] for an elementary discussion of this generic problem. Exceptions to the normalization problem are the standard-like constructions based on free fermionic models [7].

²Note that there is an irrelevant change of conventions $3 \leftrightarrow 3$ in the second and third $SU(3)$’s relative to those of [3].
below. In what follows, we will use the $E_6$ repr notation where convenient, although the gauge symmetry is never extended to $E_6$.

For the $\mathcal{N} = 1$ supersymmetric constructions we study, these fermions have scalar partners and fall into chiral supermultiplets. We interpret $SU(3)_1 = SU(3)_c$, so that $(3, \bar{3}, 1)$ and $(\bar{3}, 1, 3)$ contain quarks. We decompose

$$SU(3)_2 \supset SU(2)_L \times U(1)_1, \quad SU(3)_3 \supset U(1)_2 \times U(1)_3$$

Then we give a vacuum expectation value (vev) to a scalar in a $(1, 3, \bar{3})$ repr such that $Y \subset U(1)_1 \times U(1)_2 \times U(1)_3$ survives; i.e., the fields contained in $\Pi$ will have the usual hypercharges with respect to the surviving $U(1)$.

Explicitly, we can decompose the irreducible reprs of $\Pi$ as follows:

$$\Xi_1 = (3, \bar{3}, 1) = \begin{pmatrix} Q \\ D \end{pmatrix}$$

$$\Xi_2 = (\bar{3}, 3, 1) = (u^c, d^c, D^c)$$

$$\Xi_3 = (1, 3, \bar{3}) = \begin{pmatrix} H_u & H_d & L \\ e^c & L & \nu^c & N \end{pmatrix}$$

where the row index is the $SU(3)_2$ index (where applicable, the upper 2 components form an $SU(2)_L$ multiplet) and the column index is the $SU(3)_3$ index. The fields $Q , u^c , d^c , L , e^c , H_u , H_d$ are the usual chiral supermultiplets of the MSSM, except that we have three generations of MSSM Higgses. $D$ are down-like fields and $D^c$ their charge conjugates. $\nu^c$ are a charge conjugates of right-handed neutrinos, and $N$ are singlets. Vevs are given to the scalar components of $\nu^c$ and $N$ in order to break to $G_{SM}$.

The $SU(3)^3$ breaking vev is given at a very high scale, typically $O(10^{16})$ GeV. Since we do not wish to break supersymmetry at this scale, it is necessary that the vev be along a D-flat direction. For this to be true, a vev must also be given to (i) a scalar in the $(1, 3, 3)$ repr, or (ii) to scalars in the $(1, 3, 1)$ and $(1, 1, 3)$ reprs. However, possibility (ii) would not preserve $U(1)_Y$, so we discard it. Since $(1, 3, 3)$ is not contained in the spectrum of chiral supermultiplets $\Pi$, we must add it to the spectrum. Moreover, to have an anomaly free spectrum we must introduce it as part of a vector pair

$$\Xi_3 + \Xi_3^c = (1, 3, \bar{3}) + (1, \bar{3}, 3)$$

These are the minimal Higgs fields of the supersymmetric trinification model. More such pairs could of course be introduced. Indeed, realistic mass spectra for the light fields typically requires that several $SU(3)^3$ Higgses be introduced [3]. However, in the string-based context, we would like to solve the string unification problem by having a uniform running of the gauge couplings above the scale of $SU(3)^3$ breaking. Then it is necessary to have

$$27 + \overline{27} = \sum_{i=1}^{3} [\Xi_i + \Xi_i^c]$$

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3Of course, once D-flatness has been ensured, one must also check F-flatness.
Table 1: Inequivalent models, with an $SU(3)^3$ factor, from [1]. Here, $V$ is the twist embedding and $a_1$ is the nonvanishing Wilson line. “Exponents” indicate the number of times that a particular entry is repeated. $G_{NA}$ denotes the nonabelian factors in the gauge group; $U(1)$ factors are not shown, but should be supplemented as required to have rank 16.

| Model no. | $3V$                | $3a_1$                      | $G_{NA}$        |
|-----------|---------------------|-----------------------------|-----------------|
| 2.12      | $(1^6, 0^{10})$     | $(1^3, 0^3, 1^3, 0^7)$     | $SU(3)^3 \times SO(14)$ |
| 2.21      | $(1^6, 0^{10})$     | $(-2, 1^2, (-1)^3, 1^3, 0^7)$ | $SU(3)^3 \times SO(14)$ |
| 3.13      | $(1^{12}, 0^4)$     | $(1^6, (-1)^3, 0^3, 1^3, 0)$ | $SU(3)^3 \times SU(6)$ |
| 3.24      | $(1^{12}, 0^4)$     | $(-2, 1^2, (-1)^3, 0^6, 1^3, 0)$ | $SU(3)^3 \times SU(6)$ |
| 5.23      | $(-2, 1^8, 0^7)$    | $(-2, 1^2, (-1)^3, 0^3, 1^3, 0^4)$ | $SU(3)^4 \times SO(8)$ |

Instead of [1]. In the models constructed in [3], the full spectrum is $8(27) + 5(\overline{27})$.

Given the number of singlets that occur in the string-derived models considered here, effective Yukawa textures and hierarchies could presumably be manufactured with less Higgses than the $5[27 + \overline{27}]$ employed in [3]. Also, exotics in the spectrum will generically alter the running below the $SU(3)^3$ scale, so we may have to give up simple unification scenarios in any case. Thus we search, minimally, for models that have [1] plus some nonzero number of vector pairs [1]. For the models of Table [1] we will find that even this is not possible.

In the models that we study, the untwisted sector has a degeneracy factor of 3, while the three twisted sectors, denoted by the fixed point label of the first complex plane, $n_1$, have a degeneracy factor of 9, since we have only 1 discrete Wilson line. Thus we attempt to accommodate trinification by obtaining, say, [1] in the untwisted sector, and 9, or 18, etc., copies of [1] from twisted sectors. In addition, we anticipate exotic reps such as (underlining here and below indicates that all possible permutations should be considered)

$$(1, 1, 3), \quad (1, 1, \overline{3})$$

(6)

to appear in some of the models. Ultimately, it would be important to find flat directions which remove these exotics at a high scale while preserving $G_{SM}$, since these states lead to particles with fractional electric charge. For example, the representation $(3, 1, 1)$ is an electrically neutral quark, which would lead to the observation of hadrons with fractional electric charge unless it is supermassive. 4

**Spectral analysis.** The methods for calculating the spectrum in heterotic orbifolds are well-known [9]. Here we only review the aspects most pertinent to our considerations. See, for example, [10] and refs. therein for a more detailed discussion.

The representations of the massless spectrum are characterized in terms of the $spin(32)/Z_2$

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4 The mass of an exotic is fixed by the requirement that its relic abundance be made essentially zero in a typical inflation scenario. See, for example, [8] or the end of Sec. 5 of [6] for further details.
lattice, which we will denote by \( \Lambda \). This lattice consists of all 16-vectors of the form
\[
(n_1, \ldots, n_{16}), \quad (n_1 + \frac{1}{2}, \ldots, n_{16} + \frac{1}{2}),
\]
subject to the constraints \( n_I \in \mathbb{Z} \) and \( \sum_I n_I = 0 \mod 2 \). We remind the reader that spin(32) is the covering group for \( SO(32) \). The \( SO(32) \) roots are
\[
(\pm 1, \pm 1, 0^{14}).
\]
Here, signs are not correlated. The “exponent” indicates that the entry is repeated 14 times. Analogous notations will be used below.

In the untwisted sector we have for massless states with nontrivial spin(32)/\( Z_2 \) weights:
\[
K \in \Lambda, \quad K^2 = 2.
\]
The Wilson lines \( a_i \) enforce a projection on these states. Only those that satisfy
\[
a_i \cdot K \in \mathbb{Z} \quad \forall i = 1, 3, 5
\]
survive. Those that do survive fall into three categories, depending on their inner product with the twist embedding \( V \):
\[
3V \cdot K = \begin{cases} 
0 \mod 3 & \text{gauge} \\
1 \mod 3 & \text{matter} \\
-1 \mod 3 & \text{antimatter}
\end{cases}
\]
In truth this is a further projection onto states with differing right-moving quantum numbers.

For the massless twisted states, corresponding to string states with nontrivial monodromy, we have shifted weights \( \tilde{K} \) which satisfy
\[
\tilde{K}^2 = \frac{4}{3} - 2N_L, \quad \tilde{K} = K + V + \sum_{i=1,3,5} n_i a_i, \quad K \in \Lambda.
\]
If left-moving oscillators are excited in the 6-dimensional compact space, we can have \( N_L = 1/3 \) or \( 2/3 \). The integers \( n_i = 0, \pm 1 \) label fixed point locations in each of the 3 complex planes. Each twisted state is labeled by a triple \((n_1, n_3, n_5)\). Note that \( 3\tilde{K} \in \Lambda \).

We have applied these formulae to the calculation of the spectra of the models in Table 1. The results are presented in Table 2. It is obvious from inspection of this table that none of the models work. What is missing are the necessary Higgs representations to break \( SU(3)^3 \rightarrow G_{SM} \). Bifundamentals are always strictly chiral. It can also be seen that there are many exotics. Even if we had the Higgses to break to \( G_{SM} \) in the usual way, many fractionally charged particles would generically occur in the massless spectrum.
seen from (11). In the solutions to (12) exist that have nontrivial weight \( \tilde{\nu} \) to the so-called critical orbit.

As a matter of fact, a breaking of SU(3) \( ^c \) is not possible to cancel this D-term. Similar arguments hold for the other broken generators.

Comparison to \( E_8 \times E_8 \) case. Indeed, this result is similar to what occurs in the models constructed in [1]. However, there it was overlooked that the set of vevs suggested—in our notation, two vevs in the \((1,3,\bar{3})\) repr—is not a D-flat direction. For instance, for the \( U(1)_1 \) that appears in (2), we would have

\[
D_{U(1)_1} \propto \langle |\tilde{\nu}_1|^2 + |\tilde{\nu}_2|^2 + |\tilde{N}_1|^2 + |\tilde{N}_2|^2 \rangle
\]  

(13)

Here the notation indicates the scalar components of the first and second \((1,3,\bar{3})\) that are supposed to get vevs [cf. Eq. (3)]. Without a conjugate repr \((1,3,3)\)—which carries opposite charges with respect to \( U(1)_1 \), and hence comes in with opposite sign of the terms in (13)—it is not possible to cancel this D-term. Similar arguments hold for the other broken generators.

That a D-flat direction does not exist for vevs in two \((1,3,\bar{3})\) reprs can be seen another way, by appealing to the more powerful method of invariants [11]. Since an \( SU(3)^3 \) basic invariant cannot be constructed with just two \((1,3,\bar{3})\) reprs, a D-flat direction does not exist.

As a matter of fact, a breaking of \( SU(3)_2 \to SU(2) \) using just 3’s but no \( \bar{3} \)’s corresponds to the so-called critical orbit. This very case has been discussed previously, and has been shown not to be a D-flat direction [12].

Models with \( V = 0 \). These are Models 1.1-1.6 from [1]. Model 1.1 is just the \( SO(32) \) heterotic string—it has no embedding; the model possesses no matter. Models 1.2-1.6 are 9 generation, non-chiral models. Because \( V = 0 \), there is no untwisted matter, as can be seen from [11]. In the \( n_1 = 0 \) twisted sector there is no gauge-charged matter because no solutions to [12] exist that have nontrivial weight \( \tilde{K} \). (It is interesting to note, however,

| Model no. | \( G_{NA} \) reprs |
|-----------|------------------|
| 2.12      | \( 3[(3,1,\bar{3},1) + 3(\bar{3},\bar{3},1,1) + 3(1,3,\bar{3},1) + (1,3,1,14) + 6(3,1,1,1) \right.|
|           | \( + 7(1,3,1,1) + 3(1,\bar{3},1,1) + 6(1,1,\bar{3},1) + 18(1,1,1,1)] \). |
| 2.21      | \( 3[4(\bar{3},\bar{3},1,1) + 4(3,1,\bar{3},1) + 4(1,3,\bar{3},1) + 3(3,1,1,1) + 3(\bar{3},1,1,1) \right.|
|           | \( + 3(1,3,1,1) + 3(\bar{3},1,1,1) + 3(1,1,\bar{3},1) + 3(1,1,3,1) + 27(1,1,1,1)] \). |
| 3.13      | \( 3[(3,1,3,1) + (\bar{3},1,1,6) + (1,1,\bar{3},6) + 6(3,1,1,1) + 3(\bar{3},1,1,1) \right.|
|           | \( + 3(1,3,1,1) + 3(1,\bar{3},1,1) + 6(1,1,3,1) + 3(1,1,\bar{3},1) + 18(1,1,1,1)] \). |
| 3.24      | \( 3[(\bar{3},\bar{3},1,1) + (3,1,\bar{3},1) + (1,3,3,1) + 3(3,1,1,1) + 3(\bar{3},1,1,1) \right.|
|           | \( + 3(1,3,1,1) + 3(1,3,1,1) + 3(1,1,3,1) + 2(1,1,1,6) \right.|
|           | \( + (1,1,1,\mathbb{T}_0) + 27(1,1,1,1)] \). |
| 5.23      | \( 3[(\bar{3},\bar{3},1,1,1) + (3,1,1,\bar{3},1) + (1,3,1,3,1) + (1,1,3,1,8) + (1,1,3,1,1) \right.|
|           | \( + 3(3,1,1,1,1) + 3(\bar{3},1,1,1,1) + 3(1,3,1,1,1) + 3(1,3,1,1,1) \right.|
|           | \( + 9(1,1,3,1,1) + 3(1,1,1,3,1) + 3(1,1,1,3,1)] \). |

Table 2: Matter content of the models.
that there are gauge-neutral double-oscillator, \( N_L = 2/3 \), states in this sector.) From (12) it can be seen that \( n_1 = \pm 1 \) are related by a conjugation of the weights, \( \tilde{K} \leftrightarrow -\tilde{K} \). Thus the gauge charges in the \( n_1 = 1 \) sector are opposite of those in the \( n_1 = -1 \) sector; there is no way to accommodate a chiral gauge theory such as the MSSM.

Conclusions. It is encouraging that we have been able to find models with the reprs (1). However, the Higgs reprs (4) are absent for the class of constructions considered here. We have explained how this result is similar to what occurs in the models constructed in [4].

The trinification group \( SU(3)^3 \) may be obtained as a decomposition of the gauge group of many of the other models listed in [1]. Thus, it may be possible to embed the trinification scenario into a theory with extended gauge symmetry. However, we have not yet investigated matter representations of these other models and whether or not they will provide for both (1) and (4) simultaneously, or for the Higgs reprs of the extended gauge symmetry that would break it to \( SU(3)^3 \) along a flat direction.

Another alternative is to use some of the extra \( U(1) \)'s, that are present in the five models studied here, in the construction of \( Y \). This generalization of the hypercharge embedding might allow for the reprs (6) to be given vevs. However, the MSSM hypercharge has the feel of a contrivance in such a scheme; we would also expect to have the usual problems with nonstandard hypercharge normalization.

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