Analysis of FSS Using NWCS-FDTD Method
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Abstract. In this paper, the frequency selective surfaces (FSS) are analyzed by using a novel weakly conditional Stable (NWCS) finite-difference time-domain (FDTD) method, which is extremely useful for periodic problems with thin slots in two directions. Compared with the ADI-FDTD method, the presented method has higher accuracy and efficiency. Two examples are provided.

Introduction
The research of frequency selective surface (FSS) starts from 1950, and then developed quickly. It has been applied to antenna and radar. Frequency selective surfaces (FSS) are often analyzed using the frequency domain integral equation formulations and Method of Moments (MoM) solutions [1]. Because they need to analyze repeatedly to get the multiple frequencies, these approaches can become computationally intensive. In this paper we employ the Novel Weakly Conditionally Stable Finite Difference Time Domain (NWCS-FDTD) method[2] to analyze FSS structures. The NWCS-FDTD method is not only capable of dealing with arbitrary and inhomogeneous FSS elements, but also has the advantage that it yields a wideband response in a single run.

The main difficulty to implement the FDTD method for FSS structures is the handling of the periodic boundary condition (PBC), especially when the angle of the excitation is oblique. To solve this problem, a number of methods have been proposed, such as sine-cosine method [3], multiple unit cells method [4] and split-field method [5]. Recently, a spectral FDTD method for periodic structures was introduced, which is simple and efficient for analyzing periodic structures with oblique incidence. To overcome the restriction of the Courant Friedrich Levy (CFL) stability condition, Mao and Chen [2] have combined the spectral FDTD method with the NWCS method, namely, the NWCS-SFDTD method. In this paper, the the NWCS-SFDTD method is applied to analyze FSS structures. The advantage of the method is demonstrated through numerical examples.

Method of Solution
Periodic Boundary Condition in FDTD
For a periodic structure with a periodicity of $N_x$ along the $x$ direction, the periodic boundary condition (PBC) in the frequency domain is expressed as

$$ \tilde{E}(x=0, y, z) = \tilde{E}(x=Nx, y, z) \exp(jk_xNx) $$

where $j = \sqrt{-1}$, $k_x$ represents the transverse wave-number, which is assumed to be constant (independent of frequency). So Eq. (1) can be transformed into time domain as

$$ E(x=0, y, z, t) = E(x=Nx, y, z, t) \exp(jk_xNx) $$

We find that $\exp(jk_xNx)$ is a constant number and no future time data are needed in Eq. (2). As a result, the PBC can be implemented similarly as that in normal incidence.

NWCS-FDTD
Combined the NWCS technique with the FDTD method, relations between field components of the novel weakly conditionally stable FDTD scheme can be represented as
\[(I+[A])U^{n+\frac{1}{2}} = (I+[B])U^n\]  
\[(I+[C])U^{n+1} = (I+[D])U^{n+\frac{1}{2}}\]  
where \(U^n = [E_x^n, E_y^n, E_z^n, H_x^n, H_y^n, H_z^n]\) and \([I]\) denotes the unit matrix.

For the stability analysis of the optimized NWCS-FDTD, we adopted the same procedure in [7], so the limitation of the time step size can be calculated as follows:

\[\Delta t \leq \Delta y / c \beta\]  

**The Analysis of FSS Structures**

In this section, two different FSS examples are given.

A periodic array of metallic patches with thin slots along \(x\) axis plotted in Figure 1 is presented. The sizes of the slots are 0.1 mm. The unit cell is chosen as \(\Delta y = \Delta z = 5 \Delta x = \Delta = 0.5\) mm. And the computational domain is truncated by 8-layer PML in the \(z\) direction. So it contains \(100 \times 20 \times 86\) cells. The CTW wave is introduced into the computational domain, with the transverse wave-number \(k_y = 20 m^{-1}\) and \(\varphi = \pi / 6\) (\(k_x = k_y / \tan(\varphi)\)). To satisfy the stability condition of the FDTD algorithm, the time step size for conventional SFDTD \(\Delta / 6c\) is chosen, for the proposed method and ADI-SFDTD \(\Delta / 3c, \Delta / 1.5c\) are chosen. Figs. 2-4 show the electric component \(E_y\) at the observation point \(Q(50, 10, 43)\), which are calculated using conventional SFDTD, ADI-SFDTD and proposed method with different time step sizes. It can be seen from these figures that when the time step size is \(\Delta / 6c\) the proposed method and ADI-SFDTD method agree well with conventional SFDTD method, which shows that when the time step is small both proposed method and ADI-SFDTD have high accuracy. When the time step is \(\Delta / 3c\), the results of proposed method and ADI-SFDTD still shows good agreement with conventional method, however, proposed method has higher accuracy. When the time step increases to \(\Delta / 1.5c\), the results of ADI-SFDTD begin to deviate from the conventional results, however, a good agreement between proposed method and the conventional one is still achieved. It is apparent that the proposed method has higher accuracy than the ADI-SFDTD method.

The CPU time for the proposed method can be reduced to about 45% of the ADI-FDTD method. So it can be concluded that the proposed method has higher efficiency than ADI-FDTD method.

Figure 1. Periodic array of metallic patches with thin slots.
The second example is FSS structure with aperture, the geometry is shown in Figure 5.
The dimensions of the structure are provided in Figure 5. The same incident wave as it is in the first example is used. The scattering results are calculated with the incident angle 0° and 30°. The reflection coefficient results are plotted in Figure 6 and Figure 7.

Figure 6 shows that the resonance frequency is about 13.8GHz, we can find the reflection coefficient almost equals to zero. So we can conclude this FSS structure has the trait of full
transmission at the resonance frequency. Figure 7 shows that the resonance frequency reaches to 14.3GHz when the incident angle is 300, and the scattering trait becomes complicated.

Summary

In this paper, the frequency selective surfaces are analyzed by using the NWCS-FDTD method. Two examples show that when the FSS arrangement changes or there are apertures in the structure, the scattering traits are totally different.

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