Orbital Stability and Precession Effects in the Kepler-89 System

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Abstract
Among the numerous discoveries resulting from the Kepler mission are a plethora of compact planetary systems that provide deep insights into planet formation theories. The architecture of such compact systems also produces unique opportunities to study orbital dynamics in compact environments and the subsequent evolution of orbital parameters. One of the compact Kepler systems is Kepler-89, a system for which the radial velocity follow-up observations place strong upper limits on the masses of the planets and their Keplerian orbital elements. The potential for noncircular orbits in this system make it a compelling system to study dynamical constraints on the measured orbital parameters. I present a dynamical analysis of the system that demonstrates the stability of the circular model and shows that the eccentric model of the system is not stable. The analysis indicates that planets c and d, although close to the 2:1 secular resonance, do not permanently occupy the 2:1 resonance configuration. I explore regions of orbital parameter space to identify the upper bounds of orbital eccentricity for the planets. I further show how the dynamics in the compact system leads to significant periapsis precession of the innermost planets. Finally, I quantify the effect of the periapsis precession on the transit times of the planets compared with the cyclic variations expected from transit timing variations.

Key words: planetary systems – planets and satellites: dynamical evolution and stability – stars: individual (Kepler-89)

1. Introduction
The escalation in exoplanet discoveries has revealed a large diversity in planetary system architectures, many of which are substantially different to the architecture of our solar system (Winn & Fabrycky 2015). The kinds of architectures being unveiled are largely being driven by the observational biases toward relatively small star–planet separation, such as those biases intrinsic to the transit method (Kane & von Braun 2008). For example, the discovery of many compact planetary systems has primarily resulted from the observations by Kepler (Borucki et al. 2010; Borucki 2016) and K2 (Howell et al. 2014). An early example of such a compact system is that of Kepler-11, currently known to have at least six low-density planets orbiting the host star (Lissauer et al. 2011, 2013). Many of these compact planetary systems have also been found to manifest signatures of gravitational interactions in the form of transit timing variations (TTVs; Holman et al. 2010; Holczer et al. 2016). Such compact systems are enticing case studies for dynamical interactions between planets and their long-term stability (Funk et al. 2010; Kane 2015; Quarles et al. 2017; Granados Contreras & Boley 2018) as well as the stability of exomoon companions (Kane 2017).

The Kepler-89 (KOI-94) system is a good example of a compact system and consists of four planets with orbital periods in the range of 3.74–54.32 days (Weiss et al. 2013). The inner planet is a large terrestrial planet with a measured density of $10.1 \pm 5.5\,\text{g/cm}^3$, while the outer planets are gas giants with densities <1 g/cm$^3$. A combined fit of Keck/High Resolution Echelle Spectrometer (HIERES) radial velocity (RV) observations and Kepler photometry by Weiss et al. (2013), combined with a TTV analysis by Masuda et al. (2013), yielded an orbital solution with Keplerian components. The possibility of non-zero eccentricities introduces significantly more dynamical interactions between the planets and the potential for regions of instability within the orbital parameter space. Additionally, the impact of these planetary interactions may result in precession of the orbits that would impact subsequent transit times and durations, along with correct interpretation of TTVs within the system.

In this paper, I present a dynamical analysis of the Kepler-89 system, that includes stability tests for a variety of Keplerian orbital solutions and a study of the effects of periapsis precession. In Section 2 I review the architecture of the Kepler-89 system and calculate the Hill radii and mutual Hill radii separations for each of the planets. Section 3 provides the detailed results of an extended suite of stability simulations for the system, described in terms of the short-term stability (Section 3.1), long-term stability (Section 3.3), and the periapsis precession effects (Section 3.4). The consequences of the precession effects are described in Section 4, particularly the impact on transit times and durations in comparison to TTV amplitudes. I conclude in Section 5 and detail relevance to other compact systems with suggestions for further work and observations.

2. Orbital Architecture of the Kepler-89 System
The stellar and planetary properties required for the study presented here were extracted from Weiss et al. (2013), including the stellar mass of $M_\star = 1.277 \pm 0.050\, M_\odot$. Since the argument and times of periapsis are not provided by Weiss et al. (2013), I adopt periapsis arguments of $\omega = 90^\circ$ (locations of inferior conjunction) and periapsis times equivalent to the times of mid-transit provided by Weiss et al. (2013). Shown in Table 1 are the orbital period, $P$, eccentricity, $e$, minimum planet mass, $M_p \sin i$, semimajor axis, $a$, and Hill radius, $R_{Hi}$, for each of the known planets. The orbital periods of planets c and d show that they are in near 2:1 resonance, which has implications for their orbital stability. The
The Hill radius is given by

\[ R_H = r \left( \frac{M_p}{3M_*} \right)^{1/3}, \]

where \( r \) is the Keplerian star–planet separation

\[ r = a \left( 1 - e^2 \right) \]

\[ \frac{1}{1 + e \cos f}, \]

where \( f \) is the true anomaly. Since the Hill radius is time-dependent for a Keplerian orbit, the Hill radius shown in Table 1 is the mean Hill radius (where \( r = a \)).

Table 1

| Parameter | b     | c     | d     | e     |
|-----------|-------|-------|-------|-------|
| \( P \) (days) | 3.743208 ± 0.000015 | 10.423648 ± 0.000016 | 22.3429890 ± 0.0000067 | 54.32031 ± 0.00012 |
| \( e \) | 0.25 ± 0.17 | 0.43 ± 0.23 | 0.022 ± 0.038 | 0.019 ± 0.023 |
| \( M_p \sin i \)(M\(_\odot\)) | 10.5 ± 4.6 | 15.6\(^{+5.7}_{-3.6}\) | 106 ± 11 | 35\(^{+18}_{-28}\) |
| \( a \) (au) | 0.05119 ± 0.00067 | 0.1013 ± 0.0013 | 0.1684 ± 0.0022 | 0.3046 ± 0.0040 |
| \( R_H \) (10\(^{-3}\) au) | 1.03 | 2.33 | 7.56 | 9.18 |

Figure 1. Top-down view of the Kepler-89 system, showing the host star (intersection of the dotted crosshairs) and the orbits of the planets (solid lines). The Keplerian orbits are as described by the parameters in Table 1 with \( \omega = 90^\circ \). The scale of the figure is 0.66 au along one edge of the box.

3. Orbital Stability and Precession Effects

The orbital stability simulations described here were carried out using N-body integrations with the Mercury Integrator Package (Chambers 1999). I adopt a similar methodology to that used by Kane & Raymond (2014) and Kane (2016), which systematically explored stability for ranges of orbital eccentricity and inclination. The dynamical simulations made use of the hybrid symplectic/Bulirsch–Stoer integrator with a Jacobi coordinate system, which generally provides more accurate results for multiplanet systems (Wisdom & Holman 1991; Wisdom 2006) except in cases of close encounters (Chambers 1999). I performed a variety of 10\(^6\) and 10\(^7\) yr integrations commencing at the present epoch with the orbital configuration output every 100 simulation years. In accordance with the recommendations of Duncan et al. (1998), I set the time resolution to 0.1 days to meet the minimum required resolution of 1/20 of the shortest orbital period within the system. I randomize the starting positions (mean anomalies) of the planets to create a robust snapshot of the orbital stability for the examined architecture. Stability of simulations may be evaluated via chaos indicators that measure the divergence of orbits, such as the criterion developed by Cincotta & Simó (1999, 2000) and applied to exoplanetary orbits (Goździewski et al. 2001; Goździewski 2002; Satyal et al. 2013, 2014). My criteria for stability is a relatively simple first-order approach that requires all planets survive the duration of the dynamical simulation. This method is based on divergent orbital eccentricities which mean that the planet has either been ejected from the system or succumbed to the host star. The simple stability criteria was compared with chaos indicators by Dvorak et al. (2010) and has been successfully applied to exoplanetary systems in numerous instances (Dvorak et al. 2003; Menou & Tabachnik 2003; Kane 2016). The additional details for the individual simulations are outlined below.
3.1. Short-term Stability

Tests for the short-term stability of the system were conducted for 10^6 yr with both the eccentric model (Table 1) and circular model (\(e = 0.0\)) parameters as input starting conditions. For the eccentric model, all simulations were found to be dramatically unstable, generally resulting in the inner (b) planet being removed from the system within the first several hundred years. The circular model remained stable for the full 10^6 yr simulation in all cases and the eccentricities of the planets remained below 0.001 for planets b, d, and e, and below 0.005 for planet c.

It is worth noting that the removal of planets does not necessarily mean that they are ejected from the system. Particularly for compact systems such as Kepler-89, it is nontrivial for inner planets to escape the gravitational potential well of the host star, resulting in the planets being consumed by the host star rather than ejected.

3.2. Near Resonance of Planets c and d

It was noted by Weiss et al. (2013) that planets c and d are in near 2:1 resonance. I investigated the extent to which the resonance scenario is true by executing a short-term (10^3 yr) simulation that assumes circular orbits for all planets, and using a high-resolution (1 day) time output. To test for sustained resonance between the planets, I calculated the resonance angle using the methodology of Ketchum et al. (2013) and investigated both the short-term and long-term behavior of the c and d planetary orbits.

The results of the resonance analysis are encapsulated in Figure 2. The top panel shows the full 10^3 yr variation of the resonance angle between planets c and d assuming a 2:1 resonance. The plot demonstrates that, although the planets are indeed close to the 2:1 secular resonance, they regularly diverge from resonance and so do not permanently occupy the 2:1 resonance configuration in a stable fashion. The bottom panel is a zoomed-in portion of the top panel during one of the periods of pseudo-resonance, during which the resonance angle of the planets oscillates with a peak-to-peak amplitude of \(\sim 100^\circ\) for a period of \(\sim 80\) yr before the planets separate from resonance. I additionally investigated the resonance angle of the other planet pairs in the system but did not locate any resonant pairs.

3.3. Long-term Stability

To investigate the long-term dynamical stability of the Kepler-89 system, I extended the duration of the N-body integrations with a variety of initial orbital architectures. The chosen duration of 10^7 yr represents \(\sim 10^5\) orbital periods of the inner planet and \(\sim 6.7 \times 10^7\) orbital periods of the outer planet. For context, 10^7 yr represents \(\sim 4.2 \times 10^7\) orbital periods of Mercury and \(\sim 6.1 \times 10^6\) orbital periods of Neptune. Since it is the relative number of orbital periods that is used as a metric for time durations relative to orbital stability, 10^7 yr is sufficient for a compact system such as Kepler-89.

Figure 2. Resonance angle for planets c and d assuming circular orbits, as described in Section 3.2. Top panel: the results for the full 10^3 yr simulation, demonstrating that the planets do not remain in a stable long-term 2:1 secular resonant configuration. Bottom panel: a zoomed version of the top panel for a period when the planets are in a short-term 2:1 pseudo-resonance.
An important aspect of the system architecture to note is that planet c is only \( \sim 50\% \) more massive than planet b. However, the eccentricity of planet c is the primary cause of the system instability for the eccentric model of the system. For example, even if planet b begins in a circular orbit, while the other planets begin with the eccentricities shown in Table 1, then planet b is very quickly removed from the system due to the perturbing effects of planet c. The outcome of this simulation is depicted in the top panel of Figure 3, where the three remaining planets maintain long-term stability. The angular momentum loss from the removal of planet b results in a dampened eccentricity of planet c which is then periodically transferred to the two remaining outer planets. On the other hand, if planet c begins in a circular orbit while all the other planets start with their measured eccentricities, then the system is stable as is shown in the bottom panel of Figure 3. Therefore I explored the eccentricities of planet c that allow system stability for at least \( 10^7 \) yr. Hereafter, I refer to the eccentricities of planets b and c as \( e_b \) and \( e_c \), respectively.

In the case of the eccentric model and with planet b starting a circular orbit, the conducted simulations result in system stability for \( 10^7 \) yr when \( e_c \leq 0.26 \). An example of this is shown in the top panel of Figure 4 where \( e_b = 0.0 \) and \( e_c = 0.26 \). As can be seen, planet b cannot remain in a circular orbit due to perturbation from the other planets. However, planets b and c maintain a stable configuration while transferring angular momentum to each other, except for a brief period at \( \sim 1.5–3 \) Myr where planet e gains some angular momentum resulting in a slight increase in eccentricity.

As demonstrated in Section 3.1, the fully eccentric model of the system is highly unstable. I gradually decreased the eccentricity of planet c until long-term dynamical stability was achieved. I found that the eccentric model of the system is generally stable for \( 10^7 \) yr when \( e_c \leq 0.22 \). An example of one of these simulations is represented in the bottom panel of Figure 4. It can be seen in the figure that a relatively chaotic exchange of angular momentum between planets b, c, and e commences after a simulation duration of \( \sim 5.5 \) Myr. Planet d remains relatively unaffected by this sudden onset of momentum exchange due to its mass being substantially larger than the other three planets (see Table 1). A further aspect worth noting is that, even though the eccentric model of the system is unstable for \( e_c > 0.22 \), not all such simulations result in the loss of planet b. In some cases planet b survives while planet c is ejected. An example of this is when \( e_c = 0.24 \), in which case the planetary orbital dynamics result in the loss of planet c after \( \sim 1.5 \) Myr. Recall that planet c is in a near 2:1 secular resonance with planet d (see Section 3.2), and the rare occasions when planet c loses its orbital integrity are because it has been pushed out of the near 2:1 resonant island that provides temporary orbital stability (Agnew et al. 2019).
for which each planet survived. Planets d and e are not shown in the plot since they survived in all of the cases encapsulated by the plot.

I further investigated the dynamical behavior of planets b and c for the eccentric model with a starting eccentricity of $e_c = 0.22$ (Figure 4, bottom panel) by calculating the trajectory of the apsidal modes. Detailed descriptions of apsidal motion in the context of interacting exoplanetary systems are provided by Barnes & Greenberg (2006a, 2006b), where the two basic types of apsidal behavior, libration and circulation, are separated by a boundary called a secular separatrix (Barnes & Greenberg 2006a; Kane & Raymond 2014). The apsidal trajectories for planets b and c in the Kepler-89 system are represented graphically in polar form in Figure 6. The time span over which these are shown are the first 2 Myr of the simulation before the relatively chaotic transfer of angular momentum between the system planets occurs at $\sim$5.5 Myr. During this stable period, Figure 6 shows that the planets are circulating since the polar trajectories consistently encompass the origin.

It was described in Section 2 that I adopted periastron argument values of $\omega = 90^\circ$ for all planets since these are not provided by the published orbital solutions and there is a vast number of possible combinations. The periastron arguments were assigned the same value to ensure that apastron for the planets occurs on the same side of the star, thus minimizing close encounters between the planets and optimizing the potential system stability. The previous paragraph described how the eccentric model is only stable where $e_c \leq 0.22$. I performed an additional test of stability by changing the argument of periastron for the inner planet to $\omega_b = 270^\circ$. As expected, this places more stringent constraints on the stable Keplerian orbital elements for the other planets. For this scenario using the eccentric model described above, system stability is only achieved for the full $10^7$ yr when $e_c \leq 0.19$.

Finally, I note that the eccentricity constraints presented in this section for the eccentric model are well within the eccentricity uncertainties for the system, shown in Table 1. The final example described in the previous paragraph retains the original eccentricities for planets b, d, and e and reduces the initial eccentricity of planet c to $e_c = 0.22$, compared with the value of $e_c = 0.43 \pm 0.23$ shown in Table 1.

### 3.4. Argument of Periastron Precession

The dynamical nature of compact planetary systems results in constant adjustments of the Keplerian orbits for each of the planets. One of the ways in which these adjustments are observed is through the precession of the orbital periastron arguments. As an example of the rate at which periastron precession can occur for the inner planet to $\omega_b = 270^\circ$. As expected, this places more stringent constraints on the stable Keplerian orbital elements for the other planets. For this scenario using the eccentric model described above, system stability is only achieved for the full $10^7$ yr when $e_c \leq 0.19$.

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shown in the bottom panel of Figure 4. The precession described here includes the effects of both general relativity (GR) and perturbations of other planets (Bolmont et al. 2015).

The resulting rate of change in periapsis over the first 10^7 simulation years are shown in Figure 7 for planet b (top panel) and planet c (bottom panel). The orbits of both planets are subjected to significant periapsis precession, with planet c having a much more rapid precession due to the combined influences of the surrounding planets. As can be seen in Figure 7, the rate of periapsis precession is nonuniform, but the mean rate of precession over the 10^7 yr simulation are 0°.51 yr^-1 and 1°.76 yr^-1 for planets b and c, respectively. For comparison, the perihelion of Mercury precesses at a rate of 0°.0119/century due to GR effects, and 0°.148/century due to perturbations from other solar system planets (Clemence 1947; Iorio 2005). Thus the precession rates of the planets are generally accounted for in TTV calculations and the observability of this effect may be utilized as an additional constraint upon eccentricity if it is not observed in the transit data.

The Kepler-89 system has been subjected to several independent TTV analyses (Masuda et al. 2013; Xie et al. 2014; Holczer et al. 2016; Hadden & Lithwick 2017), including that of the discovery paper, Weiss et al. (2013). For example, Masuda et al. (2013) found that the TTVs for Kepler-89 c have a semi-amplitude ~7 minutes and a period of ~155 days, caused mostly by the aforementioned near 2:1 resonance with planet d. The periapsis precession rate of planet c described in Section 3.4 is sufficiently large that it can impact the calculation of the predicted transit observables. The precession rate of planet c in the eccentric model is 1°.76 yr^-1, or 0°.05/ orbit. The net effect of this precession is to bring the predicted transit time forward by ~2 minutes/orbit. This effect is generally accounted for in TTV calculations and the observability of this effect may be utilized as an additional constraint upon eccentricity if it is not observed in the transit data.

Keplerian orbital elements can also have a significant effect on the duration of a planetary transit (Barnes 2007; Burke 2008). From Burke (2008), the duration of a transit scales according to the relation

\[
\tau = \frac{\sqrt{1 - e^2}}{1 + e \cos(\omega - 90^\circ)}.
\]

Combining this scaling relationship with periapsis precession results in a time-dependent transit duration. For Kepler-89 c, the rate of precession results in a ~180° change in the argument of periapsis over the course of 100 yr. This is represented in Figure 8, where the fractional change in the transit duration is plotted against the time (in years) since the periapsis argument was located at 90°. For the eccentricity of \( e_c = 0.22 \), the transit duration has a fractional change of ~0.4 between inferior and superior conjunction, or a fractional change of 0.004 yr^-1 1 part in 10,000 per orbit. Thus, although the effect of periapsis precession on transit duration is unlikely due to planet–planet perturbations. The effect of precession on transit times and duration has been studied by various authors (Miralda-Escudé 2002; Heyl & Gladman 2007; Jordán & Bakos 2008; Pál & Kocsis 2008; Ragozzine & Wolf 2009; Carter & Winn 2010; Damiani & Lanza 2011; Herman et al. 2018), and the additional impact on transit probabilities has been similarly quantified (Kane et al. 2012). Depending upon the system architecture, the effects of the periapsis precession may be observable over a timescale less than a few years.

4. Effect of Orbital Dynamics on Transits

As described in Section 3.4, the inner planets of the Kepler-89 system undergo significant periapsis precession, largely
to be detected between orbits, the effect could be reasonably observed with further transit observations in subsequent years.

5. Conclusions

The orbital dynamics of compact planetary systems is a topic of crucial importance at the present time, since current detection methods are biased toward the detection of such systems and the extraction of reliable masses via TTVs is dependent upon the mutual interactions of the planets. Orbital stability in compact systems is exceptionally sensitive to the eccentricities of the individual planets, which in turn are difficult to measure for the majority of Kepler systems due to the relative faintness of the host star. The Kepler-89 system is an example of this, where the Keplerian orbital parameters are largely determined from the Kepler photometry rather than the RV data, whose utility is mostly to provide a mass estimate for the giant planet (planet d).

In this work, I have used the Kepler-89 system as a template from which it is demonstrated how the orbital eccentricity may be constrained through the use of dynamical simulations. The results presented here demonstrate that dynamical simulations may be used as powerful tools to explore numerous possible dynamical architectures that result from systems. For example, I show that there is a limited range of eccentricities for planets b and c (e.g., $e_c \lesssim 0.22$ for the fully eccentric model) that ensure long-term dynamical stability, and I have shown how the eccentricities vary with time for several of the stable orbital architectures. I have further demonstrated the dramatic periastron precession that may be occurring within the Kepler-89 system, a result of both the compact architecture combined with eccentricities that lie at the upper boundaries of stability. The periastron precession for Kepler-89 c is significant enough that it should result in observable effects, or else rule out the eccentricity that would produce such precession. In either case, the precession effects should be carefully considered when performing accurate transit timing and duration measurements.

The near 2:1 secular resonance of planets c and d is an important aspect of the overall system architecture, and particularly important since planet d is the dominant planetary mass in the system. As shown in Section 3.2, planets c and d are not quite in resonance but occasionally exhibit resonant behavior. A 2:1 secular resonance would raise the question of whether the system could dynamically harbor a three-body Laplace resonance with a 4:2:1 orbital period ratio. There are several known examples of Laplace resonances among exoplanetary systems, such as GJ 876 (Rivera et al. 2010), Kepler-60 (Goździewski et al. 2016), and TRAPPIST-1 (Luger et al. 2017). It was also proposed by Libert & Renner (2013) that Laplace resonances could be detected via long-term observations of TTV effects. In order for Kepler-89 to contain such a resonance that extends the existing near 2:1 secular resonance of planets c and d to a Laplace configuration, an additional planet would need to be located close to either a 5 day or 44 day orbital period. Given the relatively tenuous stability of the b and c planets and the high mass of the d and e planets, such a Laplace resonance may be difficult to achieve, but nonetheless important for future observers of the system to keep in mind.

Many uncertainties remain in the Keplerian orbital elements of the Kepler-89 system that will require further RV observations and/or precise transit times to resolve. However, the compact nature of the system combined with the possibility of non-zero eccentricities make the system a good case study
for exploring the dynamical effects of the orbits on each other and observable effects. The dynamical effects include gravitational perturbations, precession of orbits, mean motion resonances, and secular resonances. Strategies for the refinement of planetary orbits are being applied to numerous known exoplanetary systems (Kane et al. 2009), and could be further applied to systems that are predicted to produce similar observable signatures. It is possible that the time baseline of Kepler observations is too short to observe the discussed precession effects, but observations of the Kepler field by the Transiting Exoplanet Survey Satellite may sufficiently extend the baseline to reveal eccentricities hidden in the photometry.

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Software: Mercury (Chambers 1999).

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