Pure spinor superstring in $\text{AdS}_4 \times \mathbb{CP}^3$ with unconstrained ghosts

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ABSTRACT: We construct the action for the pure spinor superstring in the coset description of $\text{AdS}_4 \times \mathbb{CP}^3$ superspace, using the variables which solve the pure spinor condition. As a test of the consistency of the approach, we use the background field method to verify the absence of central charge at the second order in the expansion and to show the one-loop finiteness of the effective action.

KEYWORDS: AdS-CFT Correspondence, Superspaces, Sigma Models

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1 Introduction

The interest in superstring in curved backgrounds has increased considerably during the last fifteen years as a consequence of the AdS/CFT correspondence [1–3]. Attention first focused on type IIB AdS$_5 \times$ S$^5$ superspace, as main object of the correspondence. The general expression of the Green-Schwarz superstring in a generic type IIB supergravity background was known for some time [4]. In particular, for the AdS$_5 \times$ S$^5$ background the explicit form of the metric and the Wess-Zumino term was found in [5], noting that this superspace is homeomorphic to the coset supermanifold PSU(2,2|4)/SO(4,1) $\times$ SO(5) and the superstring action can be written as a sigma model on this coset. This approach is the generalization of the flat space construction, in which the Green-Schwarz superstring is reproduced by a sigma model on the coset SuperPoincaré(D = 10, $\mathcal{N} = 2$)/SO(9,1) [6].

The Wess-Zumino term, typical of the Green-Schwarz action, is given by a 3-form integrated on a three dimensional volume bounded by the world-sheet. The main property of the PSU(2,2|4)/SO(4,1) $\times$ SO(5) coset is to be a semi-symmetric space, i.e. to admit a $\mathbb{Z}_4$-grading, and this fact allows to write the Wess-Zumino term as a world-sheet integral of a 2-form [7].
In general, to quantize the Green-Schwarz action one has to fix the local fermionic kappa-symmetry. Alternatively, one can introduce some ghost fields - specifically bosonic spinors - with their conjugate momenta and provide the action with a BRST symmetry. To assure the on-shell nilpotency of the BRST charge and the BRST invariance of the action, the ghosts have to satisfy a peculiar condition and are called pure spinors. (For a recent review see [9–12].) The pure spinor approach avoids the presence of the kappa-symmetry. In particular, in flat space this formalism provides a quadratic action for the matter fields, hence it does not require to fix the light-cone gauge and preserves the manifest Poincaré covariance. Solving the pure spinor condition and writing the ghost action in terms of free fields, it is possible to show [7] the absence of the conformal anomaly and obtain for the Lorentz currents the same OPE as in the Ramond and Neveu-Schwarz formulation. Nevertheless the constraint solution breaks the SO(10) euclidean Poincaré covariance to U(5).

Pure spinor superstring naturally extends to curved backgrounds in supercoset formulation, especially AdS$^5 \times S^5$ [8, 13]. In this case the global Poincaré covariance typical of flat space becomes a gauge covariance under the little group SO(4, 1) × SO(5). To quantize this model one has to properly take into account, and eventually solve, the pure spinor constraint resulting from the requirement of BRST invariance.

After the conjecture of Aharony, Bergman, Jafferis and Maldacena [14] the attention has been extended to type IIA superstring in the AdS$_4 \times \mathbb{CP}^3$ background as dual of a $\mathcal{N} = 6$ superconformal Chern-Simons-matter theory in three dimensions. The bosonic part of the supercoset OSp(4|6)/SO(3, 1) × U(3) is homeomorphic to AdS$_4 \times \mathbb{CP}^3$, hence it is natural to write the Green-Schwarz superstring as a sigma model on this coset [15–17], like in the AdS$^5 \times S^5$ case. However the supercoset OSp(4|6)/SO(3, 1) × U(3) has 24 fermionic degrees of freedom instead of 32, thus it does not completely describe the IIA AdS$_4 \times \mathbb{CP}^3$ superspace [18]. As discussed in [15], the sigma model action can be thought as the Green-Schwarz action with 8 degrees of freedom gauged away by using half of the kappa-symmetry parameters, indeed the sigma model still presents a local fermionic invariance of rank 8. Nevertheless, for particular configurations, such as a string moving only in the AdS part of the background, the rank of the fermionic symmetry becomes 12 and the coset model does not contain all physical fermionic degrees of freedom.

The supercoset OSp(4|6)/SO(3, 1) × U(3) is a semi-symmetric superspace and admits pure spinor formulation for the superstring [19–21] completely analogous to the PSU(2,2|4)/SO(4, 1) × SO(5) background. The aim of this work is to present a coset formulation of the superstring in AdS$_4 \times \mathbb{CP}^3$ with the ghost action written in terms of the variables which solve the pure spinor condition. After having identified the ghost degrees of freedom, we replace the pure spinor action with a new action for these variables by imposing SO(3, 1) × U(3) gauge covariance. We use the background field method to check the consistency of our model. In particular we show the vanishing of the conformal anomaly up to the second order in the background expansion parameter and the absence of divergent contributions in the one-loop effective action.

The paper is organized as follows. In section 2 we resume the main results for the pure spinor superstring in semi-symmetric spaces and in section 3 we specify the AdS$_4 \times \mathbb{CP}^3$
case as $\text{OSp}(4|6)/\text{SO}(3,1) \times \text{U}(3)$ supercoset. In section 4 we give the solution of the pure spinor constraint in term of independent ghosts and auxiliary variables. In section 5 we present the action term for these fields and quantize the model using the background field method. Finally, in section 6, we make perturbative computations of the central charge and the beta-function. In the appendices we summarize our conventions and give the $\text{OSp}(4|6)$ superalgebra in a form suitable to handle the pure spinor constraint.

2 Pure spinor superstring in semi-symmetric superspaces

Let us consider a superspace described by a supercoset manifold $G/H$, where the bosonic part of the supergroup $G$ - named $\text{Bos}[G]$ - gives the isometries of the space and $H$ is its bosonic stability subgroup. If the Lie superalgebra $G$ of $G$ admits an automorphism $\Omega$ involutive on $\text{Bos}[G]$, the superspace is said semi-symmetric [22]. Thus, defining $\mathcal{H}_k$ the eigenspace of $\Omega$ associated with the eigenvalue $i^k$, $k = 0, 1, 2, 3$, $G$ can be $\mathbb{Z}_4$-graded as

$$G = \bigoplus_{k=0}^{3} \mathcal{H}_k.$$ 

By definition $\Omega([A,B]) = [\Omega(A),\Omega(B)]$ for all $A, B \in G$, hence if $H_{k,l} \in \mathcal{H}_{k,l}$ one has

$$[H_k, H_l] \in \mathcal{H}_{k+l \mod 4}$$

i.e. $\mathcal{H}_0$ and $\mathcal{H}_2$ are the bosonic eigenspaces, while $\mathcal{H}_1$ and $\mathcal{H}_3$ are the fermionic ones. In particular $\mathcal{H}_0$ is a closed subalgebra and generates the subgroup $H$. Semi-symmetric spaces and their automorphisms have been completely classified in [22]. For the types corresponding to $\text{PSU}(n|n)$ and $\text{OSP}(2n|2n + 2)$ symmetries, superstring admits a sigma model description [23]. In these cases the action is written in terms of the canonical form $J = g^{-1}dg$, with $g \in G$, that takes values in $G$, satisfies the Maurer-Cartan equation

$$dJ + J \wedge J = 0$$

and decomposes as

$$J = \sum_{k=0}^{3} J_k, \quad J_k \in \mathcal{H}_k.$$

$J$ is invariant under global left multiplication $g \rightarrow g'g$ with $g' \in G$, while under a local right multiplication $g \rightarrow gh$ with $h \in H$, $J_0$ transforms as a gauge connection

$$J_0 \rightarrow h^{-1} J_0 h + h^{-1} dh$$

and $J_{1,2,3}$ transform according to the adjoint representation of $h$, i.e. like matter fields

$$J_{1,2,3} \rightarrow h^{-1}J_{1,2,3} h.$$ 

In complex worldsheet coordinates, every canonical form $J_k$ has two components $J_k^\alpha, \bar{J}_k^\alpha$ transforming as $(1,0)$ and $(0,1)$ worldsheet tensor respectively.
The pure spinor superstring action \cite{8,13} is given by the sum of a matter and a ghost part
\begin{equation}
S_{PS} = S_{\text{matter}} + S_{\lambda} \tag{2.1}
\end{equation}
with
\begin{equation}
S_{\text{matter}} = \frac{1}{2\pi \alpha'} \int d^2 z \text{STr} \left[ \frac{1}{2} J_2 J_2 + \frac{3}{4} J_3 J_1 + \frac{1}{4} J_1 J_3 \right] \tag{2.2}
\end{equation}
and
\begin{equation}
S_{\lambda} = -\frac{1}{2\pi \alpha'} \int d^2 z \text{STr} \left[ w_3 \nabla \lambda_1 + w_1 \nabla \lambda_3 + \{ w_3, \lambda_1 \} \{ w_1, \lambda_3 \} \right]. \tag{2.3}
\end{equation}
The ghost fields $\lambda_1, \lambda_3$ are worldsheet scalars and take values in the fermionic eigenspaces $\mathcal{H}_1$ and $\mathcal{H}_3$ respectively, while their conjugate momenta $w_3 \in \mathcal{H}_3$ and $w_1 \in \mathcal{H}_1$ are holomorphic and anti-holomorphic one-forms. The gauge field $J_0$ only appears in the covariant derivative
\[ \nabla \lambda \equiv \partial \lambda + [J_0, \lambda] \]
and couples the ghost to the matter sector.

The BRST transformation acts on the group element $g$ by right multiplication, $Q(g) = g(\lambda_1 + \lambda_3)$. From $J = g^{-1} dg$ one immediately obtains
\begin{align*}
Q(J_{2n}) &= [J_{2n+3}, \lambda_1] + [J_{2n+1}, \lambda_3], \\
Q(J_{2n+1}) &= \nabla \lambda_{2n+1} + [J_2, \lambda_{2n+3}],
\end{align*}
where $n = 0, 1$ and all indices are modulo 4. For the ghost fields one assumes
\begin{align*}
Q(\lambda_1) &= 0, \quad Q(\lambda_3) = 0, \quad Q(w_3) = J_3, \quad Q(w_1) = J_1,
\end{align*}
according to $\mathbb{Z}_4$-grading and conformal weight. The requirement of BRST invariance for the pure spinor action yields the conditions
\begin{equation}
\{ \lambda_1, \lambda_1 \} = 0, \quad \{ \lambda_3, \lambda_3 \} = 0, \tag{2.4}
\end{equation}
that correspond to the pure spinor constraint in flat space. Because of (2.4), the action (2.1) has an additional local invariance that affects the antighost fields only
\begin{equation}
\delta w_3 = [\lambda_1, \Omega_2], \quad \delta w_1 = [\lambda_3, \Omega_2] \tag{2.5}
\end{equation}
with $\Omega_2 \in \mathcal{H}_2$. Moreover the conditions (2.4) assure $Q^2 = 0$ on shell up to gauge transformations \cite{24,25}.

The explicit form of the pure spinor action (2.1) and pure spinor constraint (2.4) depends on the superspace, i.e. on the supercoset. The AdS$_5 \times S^5$ case was studied in \cite{8,13} using the PSU(2,2|4)/SO(4,1) $\times$ SO(5) coset, while the AdS$_4 \times \mathbb{C}P^3$ case was studied in \cite{19–21} using the OSp(4|6)/SO(3,1) $\times$ U(3) coset.
Let us introduce the \((4 + 6) \times (4 + 6)\) even supermatrix 
\[
M = \begin{pmatrix} A & X \\ Y & B \end{pmatrix}
\]
with Grassmann even entries for \(A, B\) and Grassmann odd entries for \(X, Y\). We define the supertranspose of \(M\)
\[
M^{st} = \begin{pmatrix} A^t & -Y^t \\ X^t & B^t \end{pmatrix}
\]
and the \((4|6)\) metric
\[
K = \begin{pmatrix} C_4 & 0 \\ 0 & 1_6 \end{pmatrix},
\]
where \(C_4\) is a real, antisymmetric matrix with \(C_4^2 = -1_4\) that can be chosen as the 4-dimensional charge conjugation matrix (see appendix A). By definition, \(M\) is in the superalgebra \(osp(4|6)\) of the orthosymplectic supergroup \(OSp(4|6)\) if
\[
M^{st}K + KM = 0,
\]
i.e.
\[
A^tC_4 + C_4A = 0, \quad B^t + B = 0, \quad Y^t - C_4X = 0,
\]
that gives
\[
Bos[OSp(4|6)] \cong Sp(4) \times SO(6) \cong SO(3, 2) \times SU(4).
\]

There exist two real antisymmetric matrices \(K_4, K_6\) of order 4 and 6 respectively, with the properties \([K_4, C_4] = 0, K_4^2 = -1_4, K_6^2 = -1_6\), so that
\[
\Omega(M) = \begin{pmatrix} K_4A^tK_4 & K_4Y^tK_6 \\ -K_6X^tK_4 & K_6B^tK_6 \end{pmatrix}
\]
is an automorphism involutive on \(sp(4) \oplus so(6)\) giving the \(\mathbb{Z}_4\)-grading of \(osp(4|6)\). In particular the \(\Omega\)-invariant subalgebra is \(H_0 = so(3, 1) \oplus u(3)\) and the bosonic part of the supercoset \(OSp(4|6)/SO(3, 1) \times U(3)\) is
\[
Bos \left[ \begin{array}{c} OSp(4|6) \\ SO(3, 1) \times U(3) \end{array} \right] \cong \frac{SO(3, 2) \times SU(4)}{SO(3, 1) \times U(3)} \cong AdS_4 \times \mathbb{C}P^3
\]
as required. It is important to note that
\[
[\Omega(M)]^* = \Omega(M^*),
\]
thus, for all \(H_3 \in H_3, \Omega(H_3^*) = iH_3^*\) i.e. \(H_3^* \in H_1\). Similarly, for all \(H_1 \in H_1, H_1^* \in H_3\)
and one can conclude that there is a one-to-one correspondence between \(H_1\) and \(H_3\).

\footnote{In general this property holds for all OSp semi-symmetric spaces and for most of the PSU ones. In particular it holds for PSU(2, 2|4) [23].}
We choose the SO(3, 1) \times U(3) \times \text{“translations”} basis for the OSp(4|6) superalgebra (see appendix B): The bosonic generators are

\[ H_0 : \{ M^{mn} \in so(3, 1), \quad V_a^b \in u(3) \}, \]
\[ H_2 : \{ P^m \in so(3, 2) \setminus so(3, 1), \quad V_a, V^a \in su(4) \setminus u(3) \} \]

and the fermionic generators are

\[ H_1 : \{ \mathcal{O}_{a\bar{a}}, \mathcal{O}^{\dot{a}a} \}, \quad H_3 : \{ \mathcal{O}^a, \mathcal{O}^{\dot{a}}_a \} \]

with

\[ m, n = 0, 1, 2, 3 \quad a, b = 1, 2, 3 \quad \alpha, \dot{\alpha} = 1, 2. \]

\( M^{mn} \) and \( P^m \) generate the rotations and the translations in AdS\( _4 \) respectively, while \( V_a^b \) and \( V_a, V^a \) play analogous role in \( \mathbb{C}P^3 \). The complete expression for OSp(4|6) superalgebra is given in appendix B. The non-vanishing supertraces are

\[ \text{STr}(M_{ij}M_{km}) = \eta_{[i|m}\eta_{n]}, \quad \text{STr}(P_mP_n) = \eta_{mn}, \]
\[ \text{STr}(V_a^bV_c^d) = -2\delta_a^d\delta_c^b, \quad \text{STr}(V_aV^b) = -\delta_a^b, \]
\[ \text{STr}(\mathcal{O}_{a\bar{a}}\mathcal{O}_\beta^a) = \epsilon_{\alpha\beta}\delta_a^\beta, \quad \text{STr}(\mathcal{O}^{\dot{a}a}\mathcal{O}^{\dot{a}_\beta}_a) = i\epsilon^{\dot{a}\beta}\delta_a^\beta, \]

with \( \eta_{mn} = \text{diag}(+, -, -, -) \). We can define the components of the matter fields

\[ J_0 = J^{mn}M_{mn} + J^a b V_a^b, \quad J_1 = J^{a\alpha}\mathcal{O}_{a\alpha} + J_{a\dot{a}}\mathcal{O}^{\dot{a}a}, \quad J_2 = J^m P_m + J^a V_a + J_a V^a, \quad J_3 = J^a \mathcal{O}^a + J_{\dot{a}}^a \mathcal{O}^{\dot{a}a} \]

and of the ghost/antighost fields

\[ \lambda_1 = \lambda^{a\alpha}\mathcal{O}_{a\alpha} + \lambda_{a\dot{a}}\mathcal{O}^{\dot{a}a}, \quad \lambda_3 = \lambda^{a\alpha}\mathcal{O}^a + \lambda_{\dot{a}}^a \mathcal{O}^{\dot{a}a}, \]
\[ w_3 = w^a\mathcal{O}_a + w_{a\dot{a}}\mathcal{O}^{\dot{a}a}, \quad w_1 = w^{a\alpha}\mathcal{O}_{a\alpha} + w_{a\dot{a}}\mathcal{O}^{\dot{a}a}. \]

Using the supertraces (3.1) we can write the pure spinor action (2.1) for the AdS\( _4 \times \mathbb{C}P^3 \) background explicitly. The matter term (2.2) becomes

\[ S_{\text{matter}} = \frac{R^2}{2\pi} \int d^2 z \left[ \frac{1}{2} \eta_{mn}J^{mn}J^a - \frac{1}{2} J_a J^a - \frac{1}{2} J^a J_a - \frac{i}{4} \epsilon^{\dot{a}\beta} \left( 3J_a \mathcal{O}_{\beta}^a + J^a \mathcal{O}^a_{\beta} \right) - \frac{i}{4} \epsilon^{\alpha\beta} \left( 3J_{\dot{a}} \mathcal{O}_{\beta}^{\dot{a}} + J^{\dot{a}} \mathcal{O}^{a}_{\beta} \right) \right] \]

and ghost term (2.3) becomes

\[ S_{\chi} = \frac{R^2}{2\pi} \int d^2 z \left[ -i\epsilon^{\dot{a}\beta} \left( w^a_{\dot{a}} \nabla_a \mathcal{O}^{\dot{a}a} + w_{a\dot{a}} \nabla \mathcal{O}^{\dot{a}a} \right) - i\epsilon^{\alpha\beta} \left( w^a_{\alpha} \nabla^a \mathcal{O}_{a\dot{a}} + w_{a\dot{a}} \nabla \mathcal{O}^{\dot{a}a} \right) \right. \]
\[ + \frac{1}{8} \eta_{km} \eta_{ln} \left( w^a_{\alpha} (\sigma^{kl})_{\alpha\beta} \mathcal{O}_{a\beta} w^b_{\dot{a}} (\sigma_{kl})_{\dot{a}\dot{b}} \mathcal{O}^{a\dot{b}} + w^b_{\dot{a}} (\sigma_{mn})_{\dot{a}\dot{b}} \mathcal{O}^{b\dot{b}} + w^b_{\dot{a}} (\sigma^{mn})_{\dot{a}\dot{b}} \mathcal{O}^{a\dot{b}} \right) \]
\[ - \frac{1}{2} \left( \epsilon_{a\dot{a}} w^a_{\alpha} \mathcal{O}_{a\beta} - \epsilon^{a\dot{a}} w_{a\dot{a}} \mathcal{O}^{a\dot{a}} \right) \left( w_{a\dot{a}} \mathcal{O}^{a\dot{a}} \right) \]

where the coupling constant is given naturally by the AdS\( _4 \) radius \( \alpha' = 1/R^2 \). 

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4 Solution of the constrain

By means of the OSp(4|6) superalgebra the ghost constraints (2.4) become

\[ \begin{align*}
\epsilon_{abc} \lambda^{a} \hat{\epsilon}_{a} \lambda^{b} &= 0, \\
\epsilon^{abc} \lambda^{a} \hat{\epsilon}_{a} \lambda^{b} &= 0, \\
\epsilon_{a\hat{a}} \hat{\epsilon}_{a\hat{a}} \lambda^{a} &= 0, \\
\epsilon^{a\hat{a}} \hat{\epsilon}^{a\hat{a}} \lambda^{a} &= 0, \\
\lambda^{a} (\sigma^{m})_{a} \hat{\lambda}^{a} &= 0, \\
\lambda^{a} (\sigma^{m})_{a} \hat{\lambda}^{a} &= 0.
\end{align*} \tag{4.1} \]

The constraint on \( \lambda^{1} \) can be solved setting

\[ \lambda^{1} = \theta \alpha \theta^{a}, \quad \lambda^{1} = \psi \dot{\alpha} \psi_{a} \quad \text{with the condition} \quad |u^{a} v_{a}| = 0. \tag{4.2} \]

Moreover, exploiting the possibility of rescaling the \( \theta \) and \( \psi \) variables, one can normalize \( u \) and \( v \) and set

\[ |u|^{2} \equiv u^{a} u_{a} = 1, \quad |v|^{2} \equiv v^{a} v_{a} = 1. \tag{4.3} \]

In this way the constraint on \( \lambda^{1} \) - i.e. the first column of (4.1) - translates into the conditions (4.3) and (4.4). The ghost fields \( \theta \alpha \) and \( \psi \dot{\alpha} \) are unconstrained spinors which transform under the \((2,1)\) and \((1,2)\) representations of \( \text{SO}(3,1) \) respectively

\[ \delta \theta^{a} = \frac{1}{4} \theta^{b} (\xi_{mn} \sigma^{mn})^{a}_{b}, \quad \delta \psi_{\dot{\alpha}} = \frac{1}{4} \psi_{\dot{\beta}} (\xi_{mn} \bar{\sigma}^{mn})^{\dot{\beta}}_{\dot{\alpha}}, \]

with \( \xi_{mn} = -\xi_{mn} \), and are \( \text{U}(3) \) scalars. On the other hand, \( u^{a} \) and \( v_{a} \) are \( \text{SO}(3,1) \) scalars and transform under the \( 3 \) and \( 3^{*} \) representations of \( \text{U}(3) \)

\[ \delta u^{a} = u^{b} (\xi^{c} \sigma^{d})_{a}^{b} \bar{u}_{b}, \quad \delta v_{a} = (\xi^{c} \sigma^{*} \bar{d})_{a}^{b} \bar{v}_{b}, \quad \text{where} \quad (\sigma^{a} \sigma^{b})_{c}^{d} \equiv -i \delta^{a}_{c} \delta^{b}_{d}. \]

Obviously the hermitian conjugate fields \( u^{a}_{*} \) and \( v^{a*} \) transform under \( 3^{*} \) and \( 3 \) representations of \( \text{U}(3) \).

We can naturally define the covariant derivatives

\[ \nabla \theta^{a} = \partial \theta^{a} - \frac{1}{2} \theta^{b} (J_{mn} \sigma^{mn})^{a}_{b}, \]

\[ \nabla \psi_{\dot{\alpha}} = \partial \psi_{\dot{\alpha}} - \frac{1}{2} \psi_{\dot{\beta}} (J_{mn} \bar{\sigma}^{mn})^{\dot{\beta}}_{\dot{\alpha}} \]

and

\[ \nabla u^{a} = \partial u^{a} + i J^{a}_{b} u^{b}, \]

\[ \nabla v_{a} = \partial v_{a} - i v_{b} J_{a}^{b}. \]
The conditions (4.3) and (4.4) allow to arrange the \((u, v)\) variables in the SU(3) matrix \[ U = \begin{pmatrix} u^a & \epsilon^{abc}v_bu_c^a \end{pmatrix}, \]
furthermore they are invariant under U(1) phase transformations of \(u\) and \(v\) separately. Thus the degrees of freedom of \((u, v)\) are described by the SU(3)/U(1) \(\times U(1)\) coset. The covariant canonical form of the \(su(3)\) Lie algebra, \(U^{-1}\nabla U\), projected onto \(su(3) \backslash [u(1) \oplus u(1)]\), is
\[
j \equiv \begin{pmatrix} 0 & -j_1^* & -j_2^* \\ j_1 & 0 & -j_3^* \\ j_2 & j_3 & 0 \end{pmatrix},
\]
where
\[
j_1 = \epsilon_{abc}v^a u^b \nabla u^c, \quad j_2 = v_a \nabla u^a, \quad j_3 = \epsilon^{abc}u_a v_b \nabla v_c,
\]
and can be used to describe the \((u, v)\) sector.

The constraint on \(\lambda_3\) admits solution identical to \(\lambda_1\) and, recalling the one-to-one correspondence between the eigenspaces \(H_1\) and \(H_3\), we set
\[
\lambda_a^\alpha = \bar{\psi}^\alpha v_a, \quad \lambda^a_\alpha = \bar{\theta}^\alpha u^a
\]
with the SO(3) spinors \(\bar{\psi}\) and \(\bar{\theta}\) given by
\[
\bar{\psi}^\alpha \equiv \psi^\alpha (\bar{\sigma}^2)^{\dot{\alpha}\alpha}, \quad \bar{\theta}^\alpha \equiv \theta^{\alpha*}(\sigma^2)^{\alpha\dot{\alpha}}.
\]

As far as the antighosts are concerned, we introduce a pair of spinors \(\omega^\alpha\) and \(\rho^\dot{\alpha}\) with \((1, 0)\) conformal weight that will play the role of the conjugate momenta of \(\theta^\alpha\) and \(\psi^\dot{\alpha}\) respectively. On general grounds, \(w_3\) can be written as
\[
w^a_{\alpha} = \omega^\alpha (u^a_* + A\epsilon_{abc}v^b u^c_* + Bv_a), \quad w^{\dot{a}}_{\dot{\alpha}} = \rho_{\dot{\alpha}} (v^{\dot{a}a} + C\epsilon^{abc}u^a_* v_c + Du^{\dot{a}}),
\]
where \(A, B, C, D\) are arbitrary functions. Exploiting the gauge invariance (2.5), which in our notation reads
\[
\delta w^a_{\alpha} = \frac{i}{2} \Omega_m \psi^{(\bar{\sigma}^m)}_{\dot{\alpha}} v_a + \frac{i}{\sqrt{2}} \epsilon_{abc} \Omega^b u^c \theta^\alpha,
\]
\[
\delta w^{\dot{a}}_{\dot{\alpha}} = \frac{i}{2} \Omega_m \theta^{(\bar{\sigma}^m)}_{\alpha\dot{\alpha}} u^{\dot{a}} - \frac{i}{\sqrt{2}} \epsilon^{abc} \Omega^b v_c \psi^\alpha,
\]
we can finally set
\[
w^a_{\alpha} = \omega^\alpha u^a_* , \quad w^{\dot{a}}_{\dot{\alpha}} = \rho^\dot{\alpha} v^{\dot{a}a}.*
\]
In analogous way, \(w_1\) can be written as
\[
w^{\dot{a}}_{\dot{\alpha}} = \bar{\rho}^{\dot{\alpha} \ast} v^{\dot{a}a} , \quad w_{\alpha a} = \bar{\omega}_{\alpha} u^a_*
\]
with
\[
\bar{\rho}^{\dot{\alpha} \ast} \equiv \rho^{\dot{\alpha} \ast} (\bar{\sigma}^2)^{\dot{\alpha}\dot{\alpha}}, \quad \bar{\omega}_{\alpha} \equiv \omega^{\alpha \ast} (\sigma^2)^{\alpha\dot{\alpha}}.
\]
In conclusion the pure spinors \(\lambda\) and their conjugate momenta \(w\) are given by the unconstrained SO(3, 1) ghost spinors \(\theta, \psi\) and their conjugate momenta \(\omega, \rho\) plus the SU(3)/U(1) \(\times U(1)\) currents \(j_k\). The next step is to construct an action for these variables.
5 Action with unconstrained ghosts

We now propose to replace the ghost action \( S_\lambda \) in (3.3) with an action for the unconstrained ghost and for the currents \( j_k \) and write the AdS\(_4 \times \mathbb{CP}^3 \) superstring action as

\[
S = S_{\text{matter}} + S_{\text{ghost}} + S_j ,
\]

(5.1)

where \( S_{\text{matter}} \) is given in (3.2),

\[
S_{\text{ghost}} = -i \frac{R^2}{2} \int d^2 z \left( \varepsilon_{\alpha\beta} \omega^a \nabla^\alpha \theta^\beta + \varepsilon^{\dot{\alpha}\dot{\beta}} \rho_{\dot{\alpha}} \nabla_{\dot{\beta}} \psi_{\dot{\beta}} + \varepsilon_{\alpha\beta} \bar{\rho}^a \nabla_{\alpha} \bar{\psi}_{\beta} + \varepsilon^{\dot{\alpha}\dot{\beta}} \bar{\omega}_{\dot{\alpha}} \nabla_{\dot{\beta}} \bar{\theta}_{\dot{\beta}} \right) - \frac{1}{8 \pi R^2} \int d^2 z \eta_{mn[k} \eta_{ln]} L^{mn} \bar{L}^{kl} ,
\]

(5.2)

and

\[
S_j = \frac{R^2}{2} \int d^2 z \text{Tr}(\bar{j} \cdot j) = \frac{R^2}{2} \int d^2 z \left[ \sum_{k=1}^3 \bar{j}_k j_k + \text{c.c.} \right] ,
\]

(5.3)

where the normalization in \( S_j \) is chosen for later convenience. The second line of the action (5.2) gives the coupling between the SO(3, 1) ghost currents \( L^{mn}, \bar{L}^{mn} \) via the local AdS\(_4 \) curvature tensor. These currents can be read from the ghost coupling to the gauge fields \( \bar{T}_{mn} \) and \( J_{mn} \) in (5.2) and are

\[
L^{mn} = -i \frac{R^2}{2} \left( \omega^a (\sigma^{mn})_{\alpha\beta} \theta^\beta + \rho_{\dot{\alpha}} (\bar{\sigma}^{mn})_{\dot{\alpha}\dot{\beta}} \psi_{\dot{\beta}} \right) ,
\]

\[
\bar{L}^{mn} = -i \frac{R^2}{2} \left( \bar{\rho}^a (\sigma^{mn})_{\alpha\beta} \bar{\theta}_{\dot{\beta}} + \bar{\omega}_{\dot{\alpha}} (\bar{\sigma}^{mn})_{\dot{\alpha}\dot{\beta}} \bar{\psi}_{\dot{\beta}} \right) .
\]

As we will see, the ghost current coupling is necessary to have one-loop finiteness. Using (A.1)–(A.3), the ghost action (5.2) can be written as

\[
S_{\text{ghost}} = \frac{R^2}{2} \int d^2 z \left[ -i \left( \varepsilon_{\alpha\beta} \omega^a \nabla^\alpha \theta^\beta + \varepsilon^{\dot{\alpha}\dot{\beta}} \rho_{\dot{\alpha}} \nabla_{\dot{\beta}} \psi_{\dot{\beta}} + \varepsilon_{\alpha\beta} \bar{\rho}^a \nabla_{\alpha} \bar{\psi}_{\beta} + \varepsilon^{\dot{\alpha}\dot{\beta}} \bar{\omega}_{\dot{\alpha}} \nabla_{\dot{\beta}} \bar{\theta}_{\dot{\beta}} \right) - \frac{1}{2} \left( \varepsilon_{\alpha(\varepsilon^{\dot{\gamma}\dot{\delta})\beta}} \omega^a \nabla^\alpha \theta^\beta + \varepsilon^{\dot{\alpha}\dot{\beta}} \rho_{\dot{\alpha}} \nabla_{\dot{\beta}} \psi_{\dot{\beta}} + \varepsilon_{\alpha\beta} \bar{\rho}^a \nabla_{\alpha} \bar{\psi}_{\beta} + \varepsilon^{\dot{\alpha}\dot{\beta}} \bar{\omega}_{\dot{\alpha}} \nabla_{\dot{\beta}} \bar{\theta}_{\dot{\beta}} \right) \right] ,
\]

(5.4)

with \( \varepsilon_{\alpha(\varepsilon^{\dot{\gamma}\dot{\delta})\beta}} = -(\varepsilon_{\alpha\gamma} \varepsilon^{\dot{\beta}\dot{\delta}} + \varepsilon_{\alpha\delta} \varepsilon^{\dot{\gamma}\dot{\beta}}) \).

By construction \( S_{\text{matter}} \) and \( S_{\text{ghost}} \) are invariant under SO(3, 1) \times U(3) local transformations. \( S_j \) is obviously invariant under SO(3, 1). As far as the U(3) is concerned, we recall that \( u \) and \( v \) transform as

\[
u^a \rightarrow u^b M_b^a , \quad v_a \rightarrow M^*_{ab} v_b
\]

with \( M \in U(3) \) (see (4.5)). From the definitions (4.6), we see that \( j_2 \) is an U(3) scalar, while \( j_1 \) and \( j_3 \) are U(3) pseudo-scalar, i.e.

\[
J_1 \rightarrow (\det M) j_1 , \quad J_3 \rightarrow (\det M^*) j_3 .
\]

Therefore \( S_j \) is also invariant under local U(3) transformation.
In the following sections we will compute the central charge and the effective action using the background field method \cite{29,30} to treat the action (5.1).

We first discuss the matter sector. In a coset manifold, it is natural to expand around an element of the group $g = \tilde{g} e^X/R$ where $\tilde{g}$ is in OSp(4|6), $X$ is the quantum fluctuation and $R$ is a scale which counts the order of the perturbative expansion and can be identified with the radius of AdS$_4$. The gauge invariance of the action under $g \rightarrow g e^h$ with $h \in so(3,1) \oplus u(3)$ allows to choose $X = \sum_{i=1}^3 X_i \in osp(4|6) \setminus [so(3,1) \oplus u(3)]$. For the Maurer-Cartan form one gets

$$J = \tilde{J} + \frac{1}{R} (dX + [\tilde{J},X]) + \frac{1}{2R^2} [dX + [\tilde{J},X],X] + O\left(\frac{1}{R^3}\right) \tag{5.5}$$

with $\tilde{J} = \tilde{g}^{-1}d\tilde{g}$.

Inserting the expansion (5.5) in (2.2) one obtains the kinetic term for matter fluctuations

$$S_{XX} = \frac{1}{2\pi} \int d^2z \text{Str} \left[ \frac{1}{2} \partial X_2 \partial X_2 + \partial X_1 \partial X_3 \right] \tag{5.6}$$

$$= \frac{1}{2\pi} \int d^2z \left[ \frac{1}{2} \partial X^m \partial X_m - \partial X^a \partial X_a - i\varepsilon_{\alpha\beta} \partial X^{a\alpha} \partial X^{\beta a} - i\varepsilon^{\dot{\alpha}\dot{\beta}} \partial X_{\dot{a}\alpha} \partial X_{\dot{\beta} a} \right].$$

Similarly, from the decomposition $U = \tilde{U} e^{x/R}$ where $\tilde{U}$ is a fixed SU(3) matrix and $x \in su(3) \setminus [u(1) \oplus u(1)]$ is the fluctuation, one writes for $j$

$$j = e^{-x/R} \tilde{j} e^{x/R} + e^{-x/R} \partial e^{x/R}$$

$$= \tilde{j} + \frac{1}{R} (\partial x + [\tilde{j},x]) + \cdots, \tag{5.7}$$

where

$$x = \begin{pmatrix} 0 & -x_1^* & -x_2^* \\ x_1 & 0 & -x_3^* \\ x_2 & x_3 & 0 \end{pmatrix}$$

and $\tilde{j}$ is the projection of $\tilde{U}^{-1} \nabla \tilde{U}$ on $su(3) \setminus [u(1) \oplus u(1)]$. The kinetic term for $x$ is

$$S_{xx} = \frac{1}{\pi} \int d^2z \sum_{k=1}^3 \bar{x}_k \partial x_k. \tag{5.8}$$

From the actions (5.6) and (5.8) one computes the OPE for the fluctuations

$$X^m(z)X^n(w) = -\eta^{mn} \ln |z - w|^2, \quad X^a(z)X_b(w) = \delta_a^b \ln |z - w|^2,$$

$$X^{aa}(z)X^{\beta b}(w) = -i\varepsilon^{a\beta} \delta_a^b \ln |z - w|^2, \quad X_{aa}(z)X^b(w) = -i\varepsilon_{a\beta} \delta_a^b \ln |z - w|^2, \tag{5.9}$$

$$x_k^*(z)x_l(w) = -\frac{1}{2} \delta_{lk} \ln |z - w|^2 \tag{5.10}$$

and from the action (5.4) one computes the OPE for the ghost fields

$$\omega^a(z)\theta^\beta(w) = \frac{i}{R^2} \varepsilon^{a\beta} \frac{1}{z - w}, \quad \rho_a(z)\psi_{\beta}(w) = \frac{i}{R^2} \varepsilon_{a\beta} \frac{1}{z - w}. \tag{5.11}$$
6 Central charge

The stress-energy tensor of the action is given by

\[ T = T_{\text{matter}} + T_{\text{ghost}} + T_j, \]

where

\[ T_{\text{matter}} = -R^2 \text{Str} \left( \frac{1}{2} J_2 J_2 + J_1 J_3 \right), \quad (6.1) \]
\[ T_{\text{ghost}} = iR^2 \left( \varepsilon_{\alpha\beta} \omega^\alpha \nabla \theta^\beta + \varepsilon^{\alpha\beta} \rho_\alpha \nabla \psi_\beta \right), \quad (6.2) \]
\[ T_j = -R^2 \text{Tr} \left( j^2 j \right). \quad (6.3) \]

By using the expansions (5.5), (5.7) and the OPE (5.9)–(5.11), one obtains an expansion in power of $1/R$ of the central charge

\[ c = c^{(0)} + \frac{1}{R^2} c^{(2)} + \frac{1}{R^4} c^{(4)} + \cdots. \quad (6.4) \]

We will study separately the matter, ghost and $j$ sectors.

Using (5.5) the matter part of the stress-energy tensor (6.1) without background currents becomes

\[ T_{\text{matter}}(\tilde{J} = 0) = -\text{Str} \left[ \frac{1}{2} \partial X_2 \partial X_2 + \partial X_1 \partial X_3 \right] + O \left( \frac{X^4}{R^2} \right) \]
\[ = -\frac{1}{2} \eta_{mn} \partial X^m \partial X^n + \partial X^a \partial X_a + i\varepsilon \partial X^n a \partial X^a + i\varepsilon \partial X_{\alpha a} \partial X^a \beta + O \left( \frac{X^4}{R^2} \right). \]

Notice that a $O(X^3/R)$ contribute is zero, due to the symmetry properties of the structure constants of the superalgebra. By means of the OPE (5.9) we obtain the matter contribution to the central charge at the zero order

\[ c^{(0)}_{\text{bos. matter}} = 4(\text{AdS}) + 6(\text{CP}) = 10, \quad c^{(0)}_{\text{ferm. matter}} = -12 - 12 = -24. \]

Moreover, the absence of a term $1/R$ in $T_{\text{matter}}$ implies that $c_m^{(2)} = 0$.

The ghost contribution to the central charge can be computed by setting $\tilde{J}_{mn} = 0$ in (6.2) and using the OPE (5.11). At zero order one obtains

\[ c^{(0)}_{\text{ghost}} = 4 + 4 = 8. \]

Moreover corrections to the ghost central charge arising from the expansion of $J_{mn}$ are proportional to $1/R^4$, since $X_0 = 0$.

Finally, using (5.7), the stress-energy tensor of the $j$ sector, becomes

\[ T_j(\tilde{J} = 0) = -2 \sum_{k=1}^{3} \partial x_k^a \partial x_k + O \left( \frac{x^4}{R^2} \right), \]
and gives

\[ c_j^{(0)} = 6. \]

As for the matter sector, \( c_j^{(2)} \) is zero since the \( 1/R \) term of \( T_j \) vanishes.

Collecting the above results one gets

\[ c = c_{\text{bos.matter}} + c_{\text{ferm.matter}} + c_{\text{ghost}} + c_j = 10 - 24 + 8 + 6 = 0, \]

up to the \( 1/R^2 \) order.

7 One-loop effective action

We now discuss the one-loop finiteness of the effective action using the background field method. In general, by a dimensional analysis, the one-particle irreducible diagrams which can diverge are the background field two-point functions, the background field-ghost-ghost vertices and the four-ghost vertices.

To perform the loop integrals one has to go to momentum space and use dimensional regularization. Since we are interested in analysing the UV divergences, we can use the following dictionary \[31,32\] relating the short distance singularities to \( 1/\epsilon \) poles:

\[
\begin{align*}
\ln |0|^2 &\rightarrow -\frac{1}{\epsilon} \\
\delta(z-w) \ln |z-w|^2 &\rightarrow -\frac{1}{\epsilon} \\
\frac{1}{2\pi} \frac{1}{|z-w|^2} &\rightarrow -\frac{1}{\epsilon}.
\end{align*}
\]

7.1 Background field two-point functions

The expansion (5.5) in the action (5.1) gives the interactions between the background fields and the fluctuations. For the one-loop background field two-point functions one has to consider only interactions with two fluctuations and one or two background fields. Indeed one can easily see that the ghosts do not contribute to these two-point functions.

It turns out that the results written in terms of supergroup structure constants are formally analogous in all semi-symmetric spaces and in particular they are identical to the PSU(2,2|4) case \[12,33\]. Specifically, the divergent part of the \( \mathcal{J}_i \mathcal{J}_i \) two-point functions (\( i = 1,2,3 \)) is proportional to the second Casimir operator of the supergroup, that vanishes in OSp(4|6) case. Similarly, the divergent contributes of the one-loop \( \mathcal{J}_0 \mathcal{J}_0 \) two-point function always sums to zero due to general properties of the structure constants of a superalgebra with non-degenerate metric \[12\]. As already mentioned, this result is independent of the ghost sector and therefore does not provide a test for the solutions (4.2) and (4.7) of the pure spinor constraint. On the contrary the one-loop vertices involving ghost fields depend on the chosen parametrization and therefore their finiteness is a non-trivial check of the action (5.1).

\[2\]In the following we will omit the \textit{tilde} on background fields.
7.2 Background field-ghost-ghost vertices

We first write the interaction terms of the action (5.1) required at one-loop. In particular, for the $\mathcal{J}_0 \omega \theta$ vertex, one needs the interaction term

$$S_{\mathcal{J}_0 XX} = \frac{1}{4\pi} \int d^2 z \left\{ \mathcal{J}_{mn} \left[ \partial X^m X^n - \partial X^n X^m \right] 
+ \frac{3}{4} i \left( \partial X^a \partial (\sigma)_{a\beta} X^{\beta a} + \partial X_{\alpha a} (\tilde{\sigma} mn) \dot{\alpha} \beta X^{\alpha a} \right) 
- \frac{1}{4} i \left( \partial X^{\alpha a} (\sigma)_{a\beta} X^{\beta a} + \partial X_{\alpha a} (\tilde{\sigma} mn) \dot{\alpha} \beta X^{\alpha a} \right) \right\},$$

arising from the ghost action (5.4).

From (7.4) we get the first order diagram in figure 1a which gives the following contribution to the effective action

$$- \frac{i}{2\pi} \int d^2 z \mathcal{J}_{mn}(z) \omega^a(z) (\sigma)_{a\beta} \theta^b(z) \ln |\theta|^2 + \text{finite terms}.$$
From (7.2) and (7.3) we get the second order diagram in figure 1b which gives
\[ -i \frac{1}{4\pi} \int d^2 z \int d^2 w J_{mn}(z) \omega^\alpha(w) (\sigma^{mn})_{\alpha\beta} \theta^\beta(w) \times \left( -\delta(z - w) \ln |z - w|^2 + \frac{1}{2\pi} \frac{1}{|z - w|^2} \right) + \text{finite terms} \]

Summing these contributions and using the dictionary (7.1), we see that the three-point vertex \( J_0 \omega \theta \) is finite at one-loop.

The calculation for the \( J_0 \rho \psi \) vertex is strictly similar, so there are no divergencies once again. Analogously the \( J_0 \bar{\omega} \bar{\theta} \) and \( J_0 \bar{\rho} \bar{\psi} \) functions do not diverge.

### 7.3 Four-ghost vertices

In addition to (7.3), the ghost action (5.4) also gives the interaction
\[ S_{XX\bar{\rho}\bar{\psi}} = \frac{1}{8\pi} \int d^2 z \left[ i\partial X_m X_n (\sigma^{mn})_{\alpha\beta} - \epsilon_{\alpha(\gamma\delta)\beta}(\partial X^{\gamma\delta} X^\alpha_a + \partial X^\gamma_a X^{\delta\alpha}) \right] \bar{\rho}^\alpha \bar{\psi}^\beta. \] (7.5)

The interactions (7.3) and (7.5) contribute to the four-ghost function \( \omega \theta \bar{\rho} \bar{\psi} \) and yield the matter loop in figure 2a, giving
\[ -\frac{1}{4\pi} \int d^2 z \int d^2 w \epsilon_{\alpha(\gamma\delta)\beta} \omega^\alpha(z) \theta^\beta(z) \bar{\rho}^\gamma(w) \bar{\psi}^\delta(w) \times \left( -\delta(z - w) \ln |z - w|^2 + \frac{1}{2\pi} \frac{1}{|z - w|^2} \right) + \text{finite terms}. \] (7.6)

The remaining diagrams in figure 2b and figure 2c originate from the four-ghost terms in the action (5.4) and give
\[ -\frac{1}{(2\pi)^2} \int d^2 z \int d^2 w \left( \epsilon_{\alpha(\gamma\delta)\beta} + \frac{1}{4} \epsilon_{\alpha\beta\gamma\delta} \right) \omega^\alpha(z) \bar{\rho}^\gamma(z) \theta^\beta(w) \bar{\psi}^\delta(w) \frac{1}{|z - w|^2} + \text{finite terms} \]

and
\[ -\frac{1}{(2\pi)^2} \int d^2 z \int d^2 w \left( \epsilon_{\alpha\beta\gamma\delta} - \frac{1}{4} \epsilon_{\alpha\beta\gamma\delta} \right) \omega^\alpha(z) \bar{\psi}^\delta(z) \theta^\beta(w) \bar{\rho}^\gamma(w) \frac{1}{|z - w|^2} + \text{finite terms} \]
respectively. Thus for \( w \to z \) the two diagrams with ghost loop give
\[ \frac{1}{2\pi} \int d^2 z \int d^2 w \epsilon_{\alpha(\gamma\delta)\beta} \omega^\alpha(z) \theta^\beta \bar{\rho}^\gamma \bar{\psi}^\delta \frac{1}{2\pi} \frac{1}{|z - w|^2} + \text{finite terms}. \] (7.7)

Summing up the contributions (7.6) and (7.7) one finds that this four-ghost vertex function is finite. The calculation for the diagrams with external ghosts \( \bar{\omega} \bar{\theta} \rho \bar{\psi} \) is identical.
8 Discussion and outlook

In this work we reformulated the ghost sector of the pure spinor superstring for the AdS$_4 \times \mathbb{CP}^3$ superspace in terms of a new set of unconstrained variables. In the supercoset formulation, the pure spinor ghosts $\lambda_1, \lambda_3$ and their conjugate momenta $w_3, w_1$ take values in the fermionic eigenspaces $\mathcal{H}_1, \mathcal{H}_3$ of the $\text{OSp}(4|6)$ Lie superalgebra and they are subject to the pure spinor condition $\{\lambda_1, \lambda_1\} = \{\lambda_3, \lambda_3\} = 0$. We replaced these variables in terms of the (anti)ghosts $(\omega, \theta)$ and $(\rho, \psi)$, and their complex conjugates, which are free $\text{SO}(3,1)$ Weyl spinors. The remaining degrees of freedom parametrize a SU(3)/U(1) × U(1) coset.

We wrote an action for these new variables, which is $\text{SO}(3,1) \times \text{U}(3)$ gauge invariant. It also contains a coupling between the ghost Lorentz currents $L^{nm}$ and no other four-ghost coupling. The model is constructed to have a tree-level vanishing central charge, since our variables solve the pure spinor constraint. Using the background field method we showed this also holds up to the second order in the expansion parameter. Then we analyzed the possible UV divergences of the effective action and showed its one-loop finiteness. The ghost Lorentz current interaction term included in the action is crucial to prove this fact.

Further checks on our model may be done. In particular, the understanding of the role of the BRST invariance of the original pure spinor action. Obviously our model is not BRST invariant. This also happens in the flat space, where the action written in terms of the $\text{U}(5)$ variables which solve the pure spinor constraint is no longer BRST invariant, although it is equivalent to a BRST invariant gauge-fixed action, modulo a redefinition of the antighosts [34]. A related and crucial problem would be the construction of the vertex operators in terms of the new variables. We plan to address these issues in the nearest future.

Moreover, as already noted, the coset description of the AdS$_4 \times \mathbb{CP}^3$ background cannot describe all possible string configurations, indeed when the string moves entirely in AdS$_4$, the fermionic symmetry of the sigma model removes too many degrees of freedom. This fermionic symmetry is broken in the pure spinor formulation, thus it would be interesting to see if (and how) this issue manifests in the present model.

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A Conventions

For the antisymmetric 2-dimensional tensor we use the following convention:

$$\varepsilon^{12} = -\varepsilon^{21} = 1, \quad \varepsilon^{12} = -\varepsilon^{21} = -1,$$
$$\varepsilon_{\alpha\gamma} \varepsilon^{\gamma\beta} = \delta_\alpha^\beta, \quad \varepsilon^{\dot{\alpha}\dot{\gamma}} \varepsilon_{\dot{\gamma}\dot{\beta}} = \delta^{\dot{\alpha}}_{\dot{\beta}}.$$
To treat the symplectic part of the OSp(4\mid 6) superalgebra we define the 4-dimensional charge conjugation matrix \((\mu, \nu = 1, \ldots, 4)\)

\[
C_{\mu\nu} = \begin{pmatrix}
\varepsilon_{\alpha\beta} & 0 \\
0 & \varepsilon_{\dot{\alpha}\dot{\beta}}
\end{pmatrix} = \begin{pmatrix}
0 & -1 \\
1 & 0 \\
0 & 1 \\
0 & -1
\end{pmatrix}
\]

and its inverse

\[
C^{\mu\nu} = \begin{pmatrix}
\varepsilon^{\alpha\beta} & 0 \\
0 & \varepsilon^{\dot{\alpha}\dot{\beta}}
\end{pmatrix},
\]

so that \(C_{\mu\nu}C^{\nu\rho} = \delta^{\rho}_{\mu}\), where obviously

\[
\delta^{\rho}_{\mu} = \begin{pmatrix}
\delta^{\beta}_{\alpha} & 0 \\
0 & \delta^{\dot{\beta}}_{\dot{\alpha}}
\end{pmatrix}.
\]

By definition the Dirac matrices \(\gamma^m\) in 4 dimensions satisfy the Clifford algebra \((m = 0, \ldots, 3)\)

\[
\{\gamma^m, \gamma^n\} = 2\eta^{mn}
\]

with \(\eta^{mn} = \text{diag}(+,-,-,-)\). In the chiral representation they can be written as

\[
(\gamma^m)_\mu^\nu = \begin{pmatrix}
0 \\
(\sigma^m)_{\alpha\dot{\alpha}} \\
0
\end{pmatrix}
\]

with \(\sigma^m = (1, \sigma^1, \sigma^2, \sigma^3)\), being \(\sigma^i\) the Pauli matrices and

\[
(\bar{\sigma}^m)^{\dot{\alpha}\alpha} = \varepsilon^{\dot{\alpha}\dot{\beta}}\varepsilon^{\alpha\beta}(\sigma^m)^{\beta\dot{\beta}},
\]

i.e. \(\bar{\sigma}^m = (1, -\sigma^1, -\sigma^2, -\sigma^3)\).

We introduce a \((1 + 4)\)-index \(m = (0', m)\) and define the matrices \(\gamma^{mn} = -\gamma^{nm}\)

\[
(\gamma^{0'm})_\mu^\nu \equiv i\gamma^m,
\]

\[
(\gamma^{mn})_\mu^\nu \equiv \frac{1}{2}[\gamma^m, \gamma^n] = \begin{pmatrix}
(\sigma^{mn})^{\alpha\beta} & 0 \\
0 & (\bar{\sigma}^{mn})_{\dot{\alpha}\dot{\beta}}
\end{pmatrix}
\]

with

\[
(\sigma^{mn})^{\alpha\beta} = \frac{1}{2} \left( (\sigma^m)_{\alpha\dot{\alpha}}(\bar{\sigma}^n)^{\dot{\beta}\beta} - (\sigma^n)_{\alpha\dot{\alpha}}(\bar{\sigma}^m)^{\dot{\beta}\beta} \right),
\]

\[
(\bar{\sigma}^{mn})^{\dot{\alpha}\dot{\beta}} = \frac{1}{2} \left( (\bar{\sigma}^m)^{\dot{\alpha}\alpha}(\sigma^n)_{\alpha\beta} - (\bar{\sigma}^n)^{\dot{\alpha}\alpha}(\sigma^m)_{\alpha\beta} \right).
\]

Useful identities are

\[
(\sigma^{mn})_{\alpha\beta}(\sigma^{mn})_{\gamma\delta} = 4(\varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta} + \varepsilon_{\alpha\delta}\varepsilon_{\beta\gamma}) \equiv -4\varepsilon_{\alpha}(\varepsilon_{\gamma}\delta_{\beta}), \tag{A.1}
\]

\[
(\bar{\sigma}^{mn})^{\dot{\alpha}\dot{\beta}}(\bar{\sigma}^{mn})^{\dot{\gamma}\dot{\delta}} = 4(\varepsilon^{\dot{\alpha}\dot{\gamma}}\varepsilon^{\dot{\beta}\dot{\delta}} + \varepsilon^{\dot{\alpha}\dot{\delta}}\varepsilon^{\dot{\beta}\dot{\gamma}}) \equiv -4\varepsilon^{\dot{\alpha}}(\varepsilon^{\dot{\gamma}}\delta_{\dot{\beta}}). \tag{A.2}
\]
\[ (\sigma_{mn})_{\alpha\beta} (\bar{\sigma}^{mn})_{\dot{\alpha}\dot{\beta}} = 0. \]  \hspace{1cm} (A.3)

The indices of \( \gamma^m \) can be raised and lowered by the charge conjugation matrix

\[ (\gamma^m)^{\mu\nu} \equiv C^{\mu\rho} (\gamma^m)^{\rho\nu}, \quad (\gamma^m)_{\mu\nu} \equiv (\gamma^m)_{\mu}^{\rho} C_{\rho\nu}, \]  \hspace{1cm} (A.4)

and the Clifford algebra can be written as

\[ (\gamma^m)_{\mu\rho} (\gamma^n)_{\rho\nu} + (\gamma^n)_{\mu\rho} (\gamma^m)_{\rho\nu} = 2\eta^{mn} \delta_{\mu\nu}. \]

Notice that \( \gamma^{mn} \) can be also written in terms of the (A.4) matrices

\[ (\gamma^{mn})_{\mu\nu} \equiv C_{\mu\rho} (\gamma^{mn})_{\rho\nu}, \quad (\gamma^{mn})_{\mu\nu} \equiv C_{\rho\nu} (\gamma^{mn})_{\rho\mu}, \]  \hspace{1cm} (A.5)

The matrix \( C \) also raises and lowers the indices of \( \gamma^{mn} \)

\[ (\gamma^{mn})_{\mu\rho} (\gamma^{nm})_{\rho\nu} = 2 \delta_{\mu\nu}. \]

Notice that by definition \( C^{-1} \gamma^m C = -(\gamma^m)^T \), so \( C^{-1} \gamma^{mn} C = -(\gamma^{mn})^T \) i.e. \( \gamma^{mn} C = (\gamma^{mn} C)^T \), using \( C^T = -C \). Thus \( (\gamma^{mn})_{\mu\nu} \) and \( (\gamma^{mn})_{\mu\nu} \) are symmetric in \( \mu, \nu \) indices.

To treat the orthogonal part of the superalgebra, we define the 4 x 4 antisymmetric chiral matrices \( \rho^M \) \( (M = 1, \ldots, 6) \) satisfying the Clifford algebra

\[ [\rho^M_{\mu\nu}, \rho^N_{\rho\sigma}] = C_{\mu\rho} \rho^N_{\nu\sigma} + C_{\mu\sigma} \rho^N_{\nu\rho} + C_{\nu\rho} \rho^M_{\mu\sigma} + C_{\nu\sigma} \rho^M_{\mu\rho} \]

with \( a = 1, \ldots, 4 \) and

\[ (\rho^M)^a_b = \frac{1}{2} \epsilon^{abcd} (\rho^M)^c_d \]  \quad i.e. \quad \[ (\rho^M)^a_b = \frac{1}{2} \epsilon^{abcd} (\rho^M)^c_d, \]  \hspace{1cm} (A.6)

where \( \epsilon^{abcd} \) is the completely antisymmetric tensor \( (\epsilon^{1234} = 1) \). As in (A.5), we define the matrices

\[ (\rho^{MN})^b_a \equiv \frac{1}{2} \left( (\rho^M)^a_b (\rho^N)^c_d - (\rho^N)^a_b (\rho^M)^c_d \right). \]

B \hspace{0.5cm} \textbf{OSp}(4|6) superalgebra

The natural form of the \( \text{OSp}(4|6) \) superalgebra is in \( \text{Sp}(4) \times \text{SO}(6) \) basis. Denoting by \( O_{\mu\nu} = O_{\nu\mu} \) and \( O_{MN} = -O_{NM} \) the bosonic generators of \( \text{Sp}(4) \) and \( \text{SO}(6) \) respectively and by \( O_{\mu M} \) the fermionic ones, the algebra writes

\[ [O_{\mu\nu}, O_{\rho\sigma}] = C_{\mu\rho} O_{\nu\sigma} + C_{\mu\sigma} O_{\nu\rho} + C_{\nu\rho} O_{\mu\sigma} + C_{\nu\sigma} O_{\mu\rho} \]

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\[ -17 - \]
\[ [O_{MN}, O_{KL}] = \delta_{MK}O_{NL} - \delta_{ML}O_{NK} - \delta_{NK}O_{ML} + \delta_{NL}O_{MK} \]
\[ \{O_{\mu M}, O_{\rho L}\} = i(-\delta_{ML}O_{\rho \mu} + C_{\mu \rho}O_{ML}) \]
\[ [O_{\mu \nu}, O_{\rho L}] = C_{\mu \rho}O_{\nu L} + C_{\nu \rho}O_{\mu L} \]
\[ [O_{MN}, O_{\rho L}] = \delta_{ML}O_{\rho N} - \delta_{NL}O_{\rho M} \]

with \( \mu, \nu = 1, \ldots, 4 \) and \( M, N = 1, \ldots, 6 \).

Due to the homomorphisms \( \text{Sp}(4) \cong \text{SO}(3,2) \) and \( \text{SO}(6) \cong \text{SU}(4) \), the \( \text{OSp}(4|6) \) superalgebra can be written in \( \text{SO}(3,2) \times \text{SU}(4) \) basis. The generators of \( \text{SO}(3,2) \) and \( \text{SU}(4) \) are obtained by the change of basis

\[
M_{mn}^{\mu \nu} = \frac{1}{4}(\gamma_{mn})^{\mu \nu}O_{\mu \nu}, \quad O_{\mu \nu} = -\frac{1}{2}(\gamma_{mn})^{\mu \nu}M_{mn},
\]
\[
U_{ab}^b = -\frac{i}{4}(\rho^{MN})^b_a O_{MN}, \quad O_{MN} = -\frac{i}{2}(\rho^{MN})^b_a U_{ab}^b.
\]

For the fermionic generators, we define
\[
O_{\mu \nu a b} = \frac{1}{2}O_{\mu a}(\rho^M)^{ab}, \quad O_{\mu a b} = -\frac{i}{2}(\rho^M)^{ab}O_{\mu a b}.
\]

It is useful to introduce also the generators
\[
O^{ab}_{\mu} \equiv \frac{1}{2} \varepsilon^{abcd}O_{\mu cd} = \frac{1}{2}O_{\mu a}(\rho^M)^{ab}, \quad (B.1)
\]
see (A.6). In this basis the \( \text{OSp}(4|6) \) superalgebra becomes

\[
[M_{mn}^{\mu \nu}, M_{kl}^{\rho \sigma}] = \eta^{\mu k}M_{ml}^{\rho \nu} - \eta^{\mu l}M_{mk}^{\rho \nu} - \eta^{\nu k}M_{ml}^{\rho \mu} + \eta^{\nu l}M_{mk}^{\rho \mu}
\]
\[
[U_{ab}^b, U_{cd}^c] = i\left(\delta_{ab}^c U_{cd}^d - \delta_{cd}^a U_{ab}^b\right)
\]
\[
\{O_{\mu a b}, O_{\nu c d}\} = i\left(\delta_{ab}^d \delta_{cd}^\nu - \delta_{cd}^a \delta_{ab}^\nu\right)(\gamma_{mn})^{\mu \nu}M_{mn}
\]
\[
+ \frac{i}{2} C_{\mu \nu} \left(\delta_{ab}^c U_{cd}^d - \delta_{cd}^a U_{ab}^b - \delta_{ab}^d U_{cd}^c + \delta_{cd}^b U_{ab}^a\right)
\]
\[
[M_{mn}^{\mu \nu}, O_{\rho c d}] = -\frac{1}{2}(\gamma_{mn})^{\mu \nu}O_{\rho c d},
\]
\[
[M_{mn}^{\mu \nu}, O_{\rho a b}] = -\frac{1}{2}(\gamma_{mn})^{\mu \nu}O_{\rho a b},
\]
\[
[U_{ab}^b, O_{\rho c d}] = i\left(\delta_{ab}^b O_{\rho c d} - \delta_{cd}^b O_{\rho a b} - \frac{1}{2} \delta_{ab}^d O_{\rho c d}\right)
\]
\[
[U_{ab}^b, O_{\rho a b}] = -i\left(\delta_{ab}^c O_{\rho c d} - \delta_{cd}^c O_{\rho a b} - \frac{1}{2} \delta_{ab}^d O_{\rho c d}\right)
\]

with \( m, n, k, l = 0', 0, \ldots, 3 \) and \( a, b, c, d = 1, \ldots, 4 \).

We split the bosonic generators as
\[
M_{mn} = (M^{0'm} m, M^{mn}), \quad U_{ab}^b = (U_{a b}^a, U_{a 4}^a, U_{4 a}^a),
\]
with \( a = 1, 2, 3 \). \( M^{mn} \) and \( U_{a b}^b \) generate the subgroups \( \text{SO}(3,1) \subset \text{SO}(3,2) \) and \( \text{U}(3) \subset \text{SU}(4) \) respectively and we call them generators of “rotations”. On the other hand, \( M^{0'm} \)
and $U_a^4$, $U_4^a$ lie in the cosets $\text{SO}(3, 2)/\text{SO}(3, 1)$ and $\text{SU}(4)/\text{U}(3)$ respectively and we call them generators of “translations”. Thus we define

$$P^m \equiv M^{0' m}, \quad V_a \equiv \frac{1}{\sqrt{2}} U_4^a, \quad V^a \equiv \frac{1}{\sqrt{2}} U_a^4.$$  

We split the fermionic generators $\mathcal{O}_{\mu \dot{b} \dot{b}}$ into $(\mathcal{O}_{\mu 4 a}, \mathcal{O}_{\mu b c})$ and then substitute $\mathcal{O}_{\mu b c}$ with

$$\mathcal{O}_{\mu 4 a} = -\frac{1}{2} \epsilon^{a b c} \mathcal{O}_{\mu b c},$$

using (B.1) and $\epsilon^{a b c} = -\epsilon^{a c b} = -\epsilon^{b a c}$. Finally we split the $\mu$ index in the $(\alpha, \dot{\alpha})$ indices and write

$$\mathcal{O}_{\alpha a} \equiv \mathcal{O}_{\mu 4 a}, \quad \mathcal{O}_{\dot{\alpha} a} \equiv \mathcal{O}_{\mu 4 a}, \quad \mathcal{O}_{\alpha \dot{\alpha}} \equiv \mathcal{O}_{\mu 4 a}$$

when $\mu = 1, 2$ and $\mathcal{O}_{\alpha \dot{\alpha}} \equiv \mathcal{O}_{\mu 4 a}$ when $\mu = 3, 4$ with $\alpha, \dot{\alpha} = 1, 2$. By means of these definitions and

$$V_a^b \equiv U_a^b - \delta_a^b U_c^c,$$

one writes the OSP(4|6) superalgebra in $\text{SO}(3, 1) \times \text{U}(3) \times “translations”$ basis

$$[M^{m n}, M^{k l}] = \eta^{n k} M^{m l} - \eta^{m k} M^{n l} - \eta^{m l} M^{n k} + \eta^{k l} M^{m n},$$

$$[P^m, P^n] = -M^{m n},$$

$$[V_a^b, V_c^d] = i \left( \delta_c^b V_a^d - \delta_d^b V_c^a \right),$$

$$[V_a^b, V_c] = i \left( \delta_c^b V_a - \delta_a^b V_c \right),$$

$$[V_a^b, V^b] = \frac{i}{2} \left( V_a^b - \delta_a^b V_c^c \right),$$

$$\{ \mathcal{O}_{\alpha a}, \mathcal{O}_{\beta b} \} = -\frac{1}{\sqrt{2}} \epsilon_{\alpha \beta} \epsilon^{a b c} V_c^c,$$

$$\{ \mathcal{O}_{\alpha a}^\dot{\alpha}, \mathcal{O}_{\beta b}^\dot{\beta} \} = -\frac{1}{\sqrt{2}} \epsilon_{\alpha \beta} \epsilon^{a b c} V_c^c,$$

$$\{ \mathcal{O}_{\alpha a}, \mathcal{O}_{\beta b}^\dot{\beta} \} = \frac{1}{2} \delta_a^b (\sigma^m)_\alpha^\beta P_m,$$

$$\{ \mathcal{O}_{\alpha a}^\dot{\alpha}, \mathcal{O}_{\beta b}^\dot{\beta} \} = \frac{1}{2} \delta_a^b (\sigma^m)_\alpha^\beta P_m,$$

$$\{ \mathcal{O}_{\alpha a}, \mathcal{O}_{\beta}^\dot{\beta} \} = -\frac{i}{4} \delta_a^b (\sigma^{m n})_{\alpha \beta} M_{m n} + \frac{i}{2} \epsilon_{\alpha \beta \delta} V_a^b,$$

$$\{ \mathcal{O}_{\alpha a}^\dot{\alpha}, \mathcal{O}_{\beta}^\dot{\beta} \} = -\frac{i}{4} \delta_a^b (\sigma^{m n})_{\alpha \beta} M_{m n} + \frac{i}{2} \epsilon_{\alpha \beta \delta} V_a^b,$$

$$\{ \mathcal{O}_{\alpha a}, \mathcal{O}_{\beta}^\dot{\beta} \} = 0,$$

$$\{ \mathcal{O}_{\alpha a}^\dot{\alpha}, \mathcal{O}_{\beta}^\dot{\beta} \} = 0,$$

$$[M^{m n}, \mathcal{O}_{\alpha a}] = -\frac{1}{2} (\sigma^{m n})_{\alpha \beta} \mathcal{O}_{\beta a},$$

$$[M^{m n}, \mathcal{O}_{\alpha a}^\dot{\alpha}] = -\frac{1}{2} (\sigma^{m n})_{\alpha \beta} \mathcal{O}_{\beta a}^\dot{\beta},$$

$$[P^m, \mathcal{O}_{\alpha a}] = -\frac{i}{2} (\sigma^m)_{\alpha}^\beta \mathcal{O}_{\beta a},$$

$$[P^m, \mathcal{O}_{\alpha a}^\dot{\alpha}] = -\frac{i}{2} (\sigma^m)_{\alpha}^\beta \mathcal{O}_{\beta a}^\dot{\beta}.$$
\[
\begin{align*}
[P^m, O^{\dot{a}}a] &= -\frac{i}{2} (\bar{\sigma}^m)^{\dot{a}\dot{b}} O^a_{\beta} \\
[V^a_{\dot{b}}, O_{\alpha a}] &= i \delta^b_c O_{\alpha a} \\
[V^a_{\dot{a}}, O^{\dot{a}c}] &= -i \delta^c_a O_{\dot{a}b} \\
[V^a_{\dot{b}}, O_{\alpha b}] &= 0 \\
[V^a_{\dot{a}}, O^{\dot{b}c}] &= 0 \\
[V^a_{\dot{b}}, O_{\alpha c}] &= i \sqrt{2} \epsilon^{abc} O_{\dot{b}c} \\
[V^a_{\dot{b}}, O_{\alpha b}] &= 0 \\
[V^a_{\dot{a}}, O^{\dot{b}c}] &= 0.
\end{align*}
\]

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