INTRODUCTION
Quantum computing could efficiently solve many essential problems. However, building a quantum computer is not an easy task. One particularly promising approach is to use single-photonics, whose weak interaction with the environment makes them perfectly suitable for encoding and transmitting quantum information. Nonetheless, this weak interaction strength makes the implementation of photon–photon interactions a significant challenge. While this can be overcome at the cost of extra photons,\(^1\) the additional overhead makes purely linear-optical schemes difficult to scale up.\(^2\) Alternatively, single-photon-level nonlinearities can be used to directly create deterministic gates.\(^3\) However, this typically requires complex interactions with atomic systems that cannot readily be miniaturized. Recent work shows that graphene can provide a strong enough nonlinearity without the technical drawbacks of those atomic systems.

Our graphene-based two-qubit logic gate is centered on Franson’s quantum Zeno gate,\(^4\) which is a universal “square-root-of-swap” (SWAP\(^{1/2}\)) gate.\(^5\) If two separable single-qubit states \(| \psi \rangle \) and \(| \phi \rangle \) enter modes 1 and 2, respectively, the gate creates an entangled superposition of these states being swapped and not swapped, i.e.,

$$| \psi \rangle_1 | \psi \rangle_2 - \frac{1}{\sqrt{2}} (| \psi \rangle_1 | \phi \rangle_2 + | \phi \rangle_1 | \psi \rangle_2) . \tag{1}$$

where the subscripts indicate the mode. As illustrated in Fig. 1a, such an operation can be achieved by sending two photons to a 50:50 beamsplitter (BS): The gate succeeds when the two photons exit in different modes, generating the state of Eq. (1), while, half of the time, the gate will fail by allowing both photons to exit the same mode (in reality, the situation is even more complicated because of two-particle interference effects and the logical qubit encoding).

If the SWAP process is made continuous by replacing the 50:50 beamsplitter with coupled waveguides, the quantum Zeno effect\(^6\) (wherein continuous measurement prevents a quantum system from evolving), can boost the success probability of the gate to 100%.\(^7\) In this scenario, however, the quantum Zeno effect requires nonlinear two-photon absorption to occur at the single-photon-level. To date, such a strong optical nonlinearity has only been achieved via complex interactions with atomic systems,\(^8\) which lack scalability.

Plasmon-polaritons, formed when light hybridizes with the collective charge-carrier density oscillations in conducting materials, confine electromagnetic energy to deeply-subwavelength scales, and could potentially enable extremely strong optical nonlinearities in nanoscale photonic circuits—a resonant solution for a scalable quantum logic gate. While plasmons supported by noble metals provide large nonlinear enhancements and are compatible with single-photon-level quantum experiments,\(^9\) they suffer from intrinsically high ohmic losses, severely limiting their application to quantum technologies.

Graphene has recently arisen as a robust material platform for plasmonics, capable of sustaining plasmon resonances with extremely long lifetimes\(^11,12\) that can be tuned actively via electrostatic gating.\(^13\) Furthermore, its low-dimensionality provides unprecedented levels of optical field confinement,\(^14\) boosting optical nonlinearities well above those in noble metals,\(^15-18\) potentially enabling nonlinearities on the single- or few-plasmon level.\(^19,20\) Here we propose that this system can be used to implement a two-qubit quantum logic gate using nanoplasmonic graphene waveguides.

We will use the so-called single-rail encoding, just as in the original Zeno-gate proposal,\(^4\) where the absence of a particle represents a logical 0, and the presence of a particle a logical 1. In other words, \(| 0 \rangle \langle 1 | \) in the Fock basis represents a logical \(| 0 \rangle \langle 1 | \)
with linear optical elements. One must continuously monitor whether both particles are in the same mode, this holds in general. Even if the particles are made indistinguishable, to circumvent HOM bunching, the two plasmons are found in the same nanoribbon and increases the success probability. For a separation $d_z$ between the ribbons, there is an interaction length $L = L_{\text{SWAP}^{1/2}}$ after which the plasmon has 50-50% probability of remaining in the same mode or having swapped across ribbons. Thus, when a single plasmon is input in each mode, $|1\rangle_1|1\rangle_2$, we find the output state with $a$ one plasmon in each mode, $|1\rangle_1|1\rangle_2$, in which case the gate succeeds, or $b$ both plasmons in one of the modes, $|2\rangle_1|0\rangle_2$ or $|0\rangle_1|2\rangle_2$, in which case the gate fails. When a separable single-qubit is input into each mode ($|\psi\rangle, |\phi\rangle$), an entangled state is created, $|\psi\rangle_1|\phi\rangle_2 \rightarrow \frac{1}{\sqrt{2}} (|\psi\rangle_1|\phi\rangle_2 + |\phi\rangle_1|\psi\rangle_2)$. In the absence of nonlinearity in the waveguide and assuming indistinguishable plasmons, the HOM effect forces the plasmons to exit the gate in the same output mode, meaning that the gate always fails for $|1\rangle_1|1\rangle_2$. However, driven by the Zeno effect, the strong nonlinearity of the graphene waveguides reduces the probability that two plasmons are found in the same nanoribbon and increases the success probability. 

**RESULTS**

As a physical realization of such a graphene-based quantum gate, we envision a system of two graphene nanoribbons that support
propagating single plasmons (see Fig. 2). In this work we will assume that the single plasmons are already excited, which could, in principle, be achieved through the emission of a quantum light source. The two nanoribbons are brought close to each other, so that the plasmons are coupled via Coulomb interaction, forming a graphene plasmon DC, whereby a plasmon starting in one ribbon can couple to the other ribbon. The interaction length, the ribbon width, and the ribbon spacing set the splitting ratio of the DC. At the same time, the ribbon width and the Fermi energy of the nanoribbons determine the two-plasmon absorption rate.

To model this system, we describe each ribbon as a two-level system with energy $\hbar \omega$, where $\omega$ is the resonant plasmon frequency that depends on the nanoribbon width $W$ and doping level (Fermi energy) $E_F$. As shown in Fig. 2c, we consider a maximum of two plasmons, limiting the Hilbert space to six states. States with an equal number of plasmons are coupled via a Coulomb interaction of strength $U$. Decay processes are governed by inelastic scattering rate $\gamma$, and $\gamma'$ denotes the two-plasmon absorption rate.

We quantify the Coulomb interaction by describing plasmons in semi-infinite graphene nanoribbons within the so-called plasmon wave function (PW) formalism, adapted here to include the effect of a non-vanishing optical wave vector $k_0$ in the direction of the ribbon transversal symmetry. Setting the nanoribbons to be aligned horizontally, and separated by a distance $d_z$ in the $z$-direction (see Fig. 2a), the interaction between $N$ plasmons in one ribbon and $N'$ plasmons in the neighboring one, both of them propagating with parallel wave vector $k_0$, is given by

$$U_{k_0,N_0} = \frac{1}{2} \int d^2 R \frac{1}{\sqrt{2\pi \sigma_y}} \left( \rho_{k_0,N_0}^{\text{ind}}(R, \omega) \rho_{k_0,N_0}^{\text{ind}}(R', \omega) \right) \rho_{k_0,N_0}^{\text{ind}}(R', \omega)$$

where the integrals are evaluated over the nanoribbons in a 2D space $R = (x, y)$ and $\rho_{k_0,N_0}^{\text{ind}}(R, \omega)$ is the induced charge associated with $N$ plasmons (see Methods and Fig. 51).

Next, we compute $\gamma'$ from the nonlinear conductivity $\sigma_{n_0}^{\text{NL}}$, for which an analytical expression in the local and zero-temperature approximation is obtained quantum-mechanically in the Dirac cone approximation and reported in ref. 29. As shown in the Methods, the two-plasmon absorption rate is given by

$$\gamma' = \frac{\hbar \omega \beta_{m_0}^{(2)}(\omega)}{W \beta_{m_0}^{(2)}(\omega)} \left( \frac{\gamma}{\Re \left( \sigma_{m_0}^{(3)}(\omega) \right)} \right)^2 \Re \left( \sigma_{m_0}^{(3)}(\omega) \right),$$

where $\beta_{m_0}^{(2)}(\omega)$ and $\beta_{m_0}^{(3)}(\omega)$ are the momentum-dependent field normalizations, which we consider to be unity for low momentum values. Here $\Delta$ characterizes the spatial extent of the propagating plasmon along the direction of transversal symmetry, which we set to be equal to the ribbon width. We set the single-plasmon lifetime to be $\gamma = 500 \text{ fs}^{-1}$, which is a realistic value, measured at room temperature. Note that this lifetime can be extended by going to cryogenic temperatures; for which lifetimes up to 10 ps have recently been measured.

We can now calculate the density matrix $\rho(t)$ of the system by solving the time-dependent Lindblad master equation, which is the most general type of Markovian and time-homogeneous master equation describing an open-quantum-system evolution that is both trace-preserving and completely positive for any initial condition

$$\dot{\rho} = -\frac{i}{\hbar} [H, \rho] + \sum_{n,m=1}^{N_0} \gamma^{(n)} \left( \gamma^{(n)} a_m^\dagger a_n^\dagger \rho_a - \frac{1}{2} \gamma^{(n)} a_n^\dagger a_m^\dagger \rho_a \right),$$

where $\gamma^{(n)} = \gamma$, $a_m$ denote plasmon creation (annihilation) operators, $n$ is the number of absorbed plasmons and $m$ is the nanoribbon mode. The Hamiltonian of the two-nanoribbon system is

$$H = \hbar \omega \sum_{n,m=1}^{N_0} \gamma^{(n)} (a_m^\dagger a_n^\dagger + U(a_n^\dagger a_0^\dagger + a_m^\dagger a_0))$$

where $U$ is the Coulomb interaction given in Eq. (2), while $\omega$ is the plasmon frequency of each nanoribbon mode.

We numerically solve Eq. (4) using Mathematica, from which we obtain the required time $t_{\text{SWAP1/2}}$ at which a single plasmon incident in either nanoribbon is placed in an equal superposition of both nanoribbon modes at the output. This time is related to the Coulomb interaction $U$ from Eq. (2) (i.e. stronger Coulomb interaction $U$ resulting in shorter $t_{\text{SWAP1/2}}$). To calculate $t_{\text{SWAP1/2}}$ we define our initial state to be $\rho(0) = |\psi_0\rangle \langle \psi_0|$, where $|\psi_0\rangle = |1\rangle_1 |1\rangle_2$, and let it evolve until the probability of the plasmon being in either of the modes is equal: $P_{10}(t_{\text{SWAP1/2}}) = P_{01}(t_{\text{SWAP1/2}})$. We convert this time to a length $t_{\text{SWAP1/2}}$, by computing the plasmon group velocity as shown in Fig. S2. The resulting $t_{\text{SWAP1/2}}$ is plotted in Fig. 3a. For $E_F > 0.1 \text{ eV}$ the required $t_{\text{SWAP1/2}}$ is always less than the single-plasmon decay length, thus showing the potential of long-lived graphene plasmons: novel physical effects can manifest before the plasmon decays.

For all the results presented here, we set the spacing between the two nanoribbons to $d_z = 1 \text{ nm}$. With current technology, such atomically thin spacings can be realized by taking advantage of 2D materials like graphene. This parameter only affects the Coulomb interaction strength, which will determine $t_{\text{SWAP1/2}}$. The PW used in our calculations is applicable for these scales, as discussed in detail in ref. 28. Furthermore, for our parameter regime, the Coulomb interaction does not depend very strongly on $d_z$ (see Fig. S4 of the Supplementary Information).

Once $t_{\text{SWAP1/2}}$ is determined, we proceed to analyze the system when a single plasmon is input in each mode; that is, $\rho(0) = |\psi_0\rangle \langle \psi_0|$, where $|\psi_0\rangle = |1\rangle_1 |1\rangle_2$. For this input, the gate functions correctly if there is still one plasmon in each output mode, which occurs with probability $P_{11}(t_{\text{SWAP1/2}})$.

**DISCUSSION**

In Fig. 3a-c we show the success probability $P_{11}(t_{\text{SWAP1/2}})$, the probability of the plasmons bumping in the same nanoribbon $P_{20}(t_{\text{SWAP1/2}})$, and the probability for both plasmons to decay $P_{00}(t_{\text{SWAP1/2}})$, for a range of nanoribbon widths $W$ and Fermi energies $E_F$. Notice the similarity of the contour features between these figures and the $\gamma'/\gamma$ ratio shown in Fig. 1c. In the upper right corner the two-plasmon absorption is much weaker than the single-plasmon absorption, leading to a very weak Zeno effect, so the HOM effect prevails: that is, $P_{20}(t_{\text{SWAP1/2}}) + P_{02}(t_{\text{SWAP1/2}}) \gg P_{11}(t_{\text{SWAP1/2}})$.

As we decrease both $W$ and $E_F$, $\gamma'/\gamma$ increases, but not enough to drive a noticeable Zeno effect. Instead, both of the plasmons are likely to be absorbed, which is reflected in $P_{00}(t_{\text{SWAP1/2}}) \gg P_{20}(t_{\text{SWAP1/2}})$.

In the region where $\gamma'/\gamma \sim 10^4 - 10^6$, a strong Zeno effect can be realized (light blue area of Fig. 1c). This leads to a large increase in the success probability $P_{11}(t_{\text{SWAP1/2}})$, while $P_{20}(t_{\text{SWAP1/2}})$ becomes negligible, meaning that the Zeno effect completely suppresses the HOM effect. Despite the large $\gamma'/\gamma$, $P_{00}(t_{\text{SWAP1/2}})$ shows a minimum when $\gamma'/\gamma$ is optimal. In this optimal region, we find a maximum success probability of 87.0% for $W = 5 \text{ nm}$ and $E_F = 0.335 \text{ eV}$, which is an increase in the success probability of the SWAP1/2 gate from 0 to 87.0%. This already places us well above the gate success probability rate required to generate universal cluster states for quantum computation. This performance is limited by the single plasmon lifetime. In Fig. 3e we plot the success probability, maximized over the range of $W$ and $E_F$ shown in panels a-d, versus the plasmon lifetime given by $1/\gamma$. For lifetimes longer than 7.5 ps the success probability

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increases above 99%, reaching fault-tolerance regimes for surface codes. Nevertheless, edge imperfections and structural defects would decrease the plasmon lifetime and thus the fidelity of the gate. The predicted nonlinearities, nevertheless, should persist in their presence.

Since single-plasmon decay can also result in logical states changing into other logical states, this process fidelity will be further decreased. To quantify this, we evaluated the process fidelity of our gate by simulating process tomography for the complete range of $W$ and $E_\|$, under consideration (see Methods). The resulting process matrix for $W = 5$ nm and $E_\| = 0.335$ eV with a lifetime of 500 fs is plotted in Fig. 4, and has a fidelity of 93.3%. When the lifetime is increased to 10 ps, the fidelity is 99.6%.

Our proposed gate achieves process fidelities in the fault-tolerance regime for relatively reasonable physical parameters. Doping levels as high as 1–2 eV have been achieved, and nanoribbon widths in the range of 10–40 nm have been constructed using different means, and separation distances of 1 nm are routinely achieved through single-atomic hexagonal boron nitride spacers, which additionally guarantees the preservation of high-quality graphene optical response. Furthermore, by combining ideas from quantum optics with nanoplasmonics, our work opens up an entirely new and promising avenue in the search for single-photon nonlinearities, which could also be used for deterministic optical implementations of quantum teleportation, cluster state generation, and single-photon sources, underlining the applicability of this platform.

**METHODS**

Classical electrostatic description of plasmons in graphene nanoribbons

We consider a single graphene nanoribbon occupying the $R = (x, y)$ plane that has a finite width $W$ in the $x$-direction and is infinitely-extended along the $y$-direction. In the linear approximation, following refs. 19,23 the self-consistent electric field within the ribbon $E_\|$, produced by an impinging field $E^{ext}(R, \omega) = e^{ext}e^{i(\omega t - ky)} + c.c.$, i.e., having frequency $\omega$ and momentum $k_y \equiv q/W$ along $y$, is given by

$$E_\| = E^{ext}_\| - \frac{1}{\varepsilon_0} \nabla_R \int \frac{d^2R'}{R-R'} E^{ind}_\| (R', \omega),$$

where $E^{ext}_\| = (e^{ext}_x + e^{ext}_y)/2$ is the average of the dielectric functions describing media above ($e^{ind}_x$) and below ($e^{ind}_y$) the 2D layer and $E^{ind}_\| (R, \omega)$ is the induced charge. From the continuity equation, we express $E^{ind}_\|$ in terms of the local, linear 2D graphene conductivity $\sigma^{ind}_\|$ as

$$\rho^{ind}_\| (R, \omega) = -\frac{1}{\omega C^{ind}_\|} \nabla_R \left[ i E_\| (R, \omega) \right],$$

where we have introduced the occupation factor $f_\omega$, which is equal to one when $-W/2 < x < W/2$ and is vanishingly small everywhere else. In practice, we employ the optical conductivity obtained for zero temperature in the local limit (i.e., for vanishing in-plane optical momentum) of the random-
can be expressed as
\[ \eta \text{ is a coefficient form an orthonormal set} \]

Incidentally, we have neglected inelastic damping in the interband transitions. In terms of normalized coordinates \( \bar{\theta} \equiv R \theta / W \) and the normalized electric field \( \bar{E}_{q}(\bar{\theta}, \omega) \equiv W \sqrt{\bar{F}_{q}(\bar{\theta}, \omega)} \), Eq. (6) can be expressed as
\[ \bar{E}_{q}(\bar{\theta}, \omega) = \bar{E}_{q}^{\text{est}}(\bar{\theta}, \omega) + \eta_{q}^{(1)} \int d^{2} \bar{\theta} \bar{M}(\bar{\theta}, \bar{\theta}') \cdot \bar{E}_{q}(\bar{\theta}', \omega), \]

where \( \eta_{q}^{(1)} \equiv \text{io}^{(1)} / \epsilon_{q}^{0} W W \) is a dimensionless parameter characterizing the intrinsic linear optical response of graphene, and
\[ \bar{M}(\bar{\theta}, \bar{\theta}') = \sqrt{\bar{F}_{q}^{\text{est}}(\bar{\theta}, \bar{\theta}') \cdot \bar{F}_{q}^{\text{est}}(\bar{\theta}', \omega)}, \]

which we identify as a real, symmetric operator that admits a complete set of real eigenvalues. The electric field of Eq. (9) is expanded in eigenmodes of the matrix \( \bar{M}(\bar{\theta}, \bar{\theta}') \) as
\[ \bar{E}_{q}(\bar{\theta}, \omega) = \sum_{m} a_{q,m} \bar{E}_{q,m}(\bar{\theta}) e^{i \theta}, \]

where the modes \( \bar{E}_{q,m}(\bar{\theta}) e^{i \theta} \) and their corresponding eigenvalues \( \eta_{q,m} \) satisfy
\[ \bar{E}_{q,m}(\bar{\theta}) e^{i \theta} = \eta_{q,m} \int d^{2} \bar{\theta} \bar{M}(\bar{\theta}, \bar{\theta}') \cdot \bar{E}_{q,m}(\bar{\theta}') e^{i \theta'}, \]

and form an orthonormal set
\[ \int d \theta \bar{E}_{q,m}(\theta) \cdot \bar{E}_{q,m}(\theta) = \delta_{nm}. \]

Inserting Eq. (11) into Eq. (9), we make use of Eqs. (12) and (13) to write
\[ b_{q,m} = a_{q,m} / \left( 1 - \eta_{q,m}^{0}/\eta_{q,m} \right)^{-1}, \]

where
\[ b_{q,m}(\theta') = \int d \theta \bar{E}_{q,m}^{\text{est}}(\theta') \bar{E}_{q,m}(\theta), \]

is a coefficient that depends on the form of the external field. In what follows we take \( \bar{E}_{q,m}^{\text{est}} \) to be independent of \( \chi \); so we may write
\[ b_{q,m}(\theta') = -W \bar{E}_{q,m}^{\text{est}}(\theta') \bar{E}_{q,m}(\theta), \]

where \( \bar{E}_{q,m}^{\text{est}}(\theta') \) contains the \( \chi \)-dependence of the external field and \( \bar{E}_{q,m} \equiv -\int d \theta \bar{E}_{q,m}(\theta) \), so that the normalized electric field in Eq. (11) becomes
\[ \bar{E}_{q}(\bar{\theta}, \omega) = \sum_{m} \frac{W \bar{E}_{q,m}^{\text{est}}(\bar{\theta}, \omega)}{1 - \eta_{q,m}^{0}/\eta_{q,m}} \left( \frac{\bar{E}_{q,m}(\bar{\theta}) e^{i \theta}}{\eta_{q,m}} \right) \bar{E}_{q,m}(\bar{\theta}, \omega). \]

Electrostatic energy in nanoribbons
The electrostatic energy for identical, parallel ribbons separated by a distance \( d_{z} \) in the \( z \)-direction is given by
\[ U_{q,v}(d_{z}) = \frac{W^{2}}{2} \int d^{2} \bar{\theta} \int d^{2} \bar{\theta}' \ \frac{\left[ \rho_{q,v}(\bar{\theta}, \omega) \right]^{*} \rho_{q,v}(\bar{\theta}', \omega)}{\left( \theta - \theta' \right)^{2} + \left( \theta - \theta' \right)^{2} + (d_{z}/W)^{2}}, \]

where, from Eq. (7) (taking \( ab = 1 \) for simplicity), we can express the induced charge in ribbon \( l \) as
\[ \rho_{q,v}^{(i)}(\bar{\theta}, \omega) = \frac{\eta_{q}^{(1)}}{W} \sqrt{\bar{F}_{q}(\bar{\theta}, \omega)}, \]

Inserting the above expression into Eq. (16) and making use of Eq. (15), the electrostatic energy becomes
\[ U_{q,v}(d_{z}) = \frac{W^{2}}{2} \sum_{nm} \left( \frac{\bar{E}_{q,n}^{\text{est}}(\bar{\theta}) e^{i \theta}}{\eta_{q,m}^{0}} \right)^{*} \left( \frac{\bar{E}_{q,m}(\bar{\theta}) e^{i \theta}}{\eta_{q,m}^{0}} \right) \eta_{nm}(d_{z}), \]

where
\[ \eta_{nm}(d_{z}) = \int_{-1/2}^{1/2} d \theta_{x} \int_{-1/2}^{1/2} d \theta_{y} \left[ \sqrt{\left( \theta_{x} - \theta_{x}' \right)^{2} + \left( \theta_{y} - \theta_{y}' \right)^{2} + (d_{z}/W)^{2}} \right] \]

and \( L \to \infty \) is the nanoribbon length.

Assuming a plane wave field profile along the \( y \)-direction corresponding to \( \bar{E}_{q,n}(\bar{\theta}) = e^{i \theta_{y}}, \) in a single ribbon (i.e., taking \( l = \infty \) and \( d_{z} = 0 \)), the use of Eqs. (10) and (11) yields \( \lambda_{nm}(d_{z}) = -L_{nm} \eta_{nm}^{0} / \eta_{nm} \), and so the electrostatic energy per unit length in ribbon \( l \) is
\[ U_{q,l} = \frac{W^{2}}{2} \sum_{nm} \frac{-1}{\eta_{nm}^{0}} \left( \frac{\bar{E}_{q,n}^{\text{est}}(\bar{\theta}) e^{i \theta}}{\eta_{nm}^{0}} \right)^{2} \]

In practice, we restrict our study to the lowest-order \( m = 1 \) mode, and fix the number of plasmon quanta in this mode using the condition \( i = 2 \Delta U_{q,l} \), where \( \Delta \) is an effective length for the plasmon mode along the ribbon (i.e., the characteristic spatial width of a pulse), leading to
\[ \left| \frac{\bar{E}_{q,n}^{\text{est}}(\bar{\theta}) e^{i \theta}}{\eta_{nm}^{0}} \right|^{2} = \frac{i \lambda_{nm}^{0}}{W^{2} \Delta} \left( \frac{1}{\eta_{nm}^{0}} - \frac{1}{\eta_{nm}^{0}} \right)^{2}, \]

where it is now understood that the indices \( l \) and \( l' \) denote the number of plasmons in the first and second ribbon, respectively. Using the above
condition, the coupling energy between ribbons containing l and l' plasmons is obtained directly from Eq. (18), again considering only the m = m' = 1 contribution.

Plasmon normalization
We normalize the electric field amplitude of the plasmon mode by equating the absorbed and dissipated power at linear order, i.e.,

$$\text{ih}_\omega y = \left\langle d^2 \mathbf{R}_{\mathbf{j}(1)} (\mathbf{R}, t) \cdot \mathbf{E}_\omega (\mathbf{R}, t) \right\rangle,$$

where l is the number of plasmon quanta, \(\mathbf{j}(1) (\mathbf{R}, \omega) = \alpha_{\mathbf{j}(1)} \mathbf{E}_\omega (\mathbf{R}, \omega), \) and <..> denote the time-averaging. Using the result of Eq. (15) with only the m = 1 mode, we obtain

$$\text{ih}_\omega y = \frac{2W^2}{L} \text{Re}\left(\alpha_{\mathbf{j}(1)}^* \alpha_{\mathbf{j}(1)} \right) \frac{E_{\text{ext}}^2 - E_{\text{eq}}^2}{1 - \eta_{\text{th}}^{(1)}/\eta_{\text{th}}},$$

where \(\beta_{\mathbf{j}(1)}^{(0)} = \int L_1 d_\mathbf{q} c_\mathbf{q}(\theta) \phi_{\mathbf{j}(1)}^{(0)}(\mathbf{q}). \) For a mode defined as a plane-wave along the ribbon, such that \(c_\mathbf{q}(\theta) = e^{i\mathbf{q} \cdot \mathbf{R}}\), within an effective length \(\Delta\), we write the normalization condition for N plasmons as

$$\left(\frac{E_{\text{ext}}^2 - E_{\text{eq}}^2}{1 - \eta_{\text{th}}^{(1)}/\eta_{\text{th}}} \right)^2 = \frac{\text{ih}_\omega y}{2W \text{Re}\left(\alpha_{\mathbf{j}(1)}^* \alpha_{\mathbf{j}(1)} \right) \Gamma_{\text{eq}}^2 / \Delta},$$

Two-plasmon absorption rate
Power absorption in a nanoribbon via two-plasmon absorption arises from the nonlinear current \(j_2^{(3)} (\mathbf{R}, \omega) = j_2^{(3)} (\mathbf{R}, \omega) \mathbf{e}^{i \omega_2 t} + \text{c.c., where} \)

$$j_2^{(3)} (\mathbf{R}, \omega) = \alpha_{j_2^{(3)}}^* \mathbf{E}_\omega (\mathbf{R}, \omega) \mathbf{E}_\omega (\mathbf{R}, \omega)$$

and is given by

$$\text{P}_{\text{PPA}} = \frac{2W^2}{L} \text{Re}\left(\alpha_{j_2^{(3)}}^* \alpha_{j_2^{(3)}} \right) \frac{E_{\text{ext}}^2 - E_{\text{eq}}^2}{1 - \eta_{\text{th}}^{(1)}/\eta_{\text{th}}} \int L_1 d_\mathbf{q} c_\mathbf{q}(\theta) \phi_{j_2^{(3)}}^{(0)}(\mathbf{q}),$$

where \(j_2^{(3)} (\mathbf{R}, \omega) = \alpha_{j_2^{(3)}}^* \mathbf{E}_\omega (\mathbf{R}, \omega) \mathbf{E}_\omega (\mathbf{R}, \omega)\) and \(\alpha_{j_2^{(3)}}^{(0)}\) is the local third-order conductivity of extended graphene, for which we adopt the analytical result obtained quantum-mechanically at zero temperature in the Dirac cone approximation, as reported in ref. \(^2\). Using Eq. (15) we write the time-average of the absorbed power per unit length as

$$\left\langle \text{P}_{\text{PPA}} \right\rangle = \frac{2W^2}{L} \text{Re}\left(\alpha_{j_2^{(3)}}^* \alpha_{j_2^{(3)}} \right) \frac{E_{\text{ext}}^2 - E_{\text{eq}}^2}{1 - \eta_{\text{th}}^{(1)}/\eta_{\text{th}}} \int L_1 d_\mathbf{q} c_\mathbf{q}(\theta) \phi_{j_2^{(3)}}^{(0)}(\mathbf{q}).$$

Equating \(\text{P}_{\text{PPA}}\) with the power dissipated by two-plasmon absorption, \(2\lambda_\omega y^{(2)}\), we make use of the field normalization condition in Eq. (19) to write the two-plasmon absorption rate for a ribbon containing \(l = 2\) plasmons in the \(m = 1\) mode as

$$\lambda_\omega y^{(2)} = \frac{\text{ih}_\omega y^{(2)}}{W \phi_{j_2^{(3)}}(\mathbf{q}) \phi_{j_2^{(3)}}(\mathbf{q}) \Delta} \left( \frac{\mathbf{Y}}{\text{Re}(\alpha_{j_2^{(3)}}^{(0)})} \right)^2.$$

In obtaining the above expression, we have again chosen the field along the ribbon to have the form of a plane-wave (i.e., \(c_\mathbf{q}(\theta) = e^{i \mathbf{q} \cdot \mathbf{R}}\)), and an effective length \(\Delta\).

Process tomography
We send a complete set of 16-qubit states through our simulation and compute the output states at \(t = t_{\text{swap}}\). To deal with failure events, when \(|2\rangle, |0\rangle_2\) and \(|1\rangle, |1\rangle_2\) terms arise in the output states, we truncate the output density matrix and renormalize the result. Such events only occur when states involving two plasmons are input. We also numerically correct for local single-qubit phases which arise in the output of the simulation. We feed these output states in a least-squares process tomography routine, generating a process matrix \(\chi_{\text{sim}}\). This process matrix is defined as,

$$\rho_{\text{out}} = \sum_{m,n} \chi_{mn} E_{mn} \rho_{\text{in}},$$

where \(\rho_{\text{in}}\) is the input (output) density matrix, and \(E_i\) are the basis operators constructed from the Kronner product of the Pauli matrices (labels of Fig. 4). We calculate the process fidelity between these, and the ideal process (given by Eq. (10) of ref. \(^2\) as \(\text{Tr}(\chi_{\text{sim}})^{\text{ideal}}\).
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**AUTHOR CONTRIBUTIONS**
I.A.C., L.A.R., and P.W. proposed the design of the coupled nanoribbons and performed the numerical calculations. I.A.C., and M.R. derived the master equation for our system. J.D.C., J.R.M.S., and F.J.G.A. derived analytical expressions for the graphene parameters. I.A.C., J.D.C., M.R., and J.R.M.S. wrote the simulation code. I.A.C., J.D.C., and L.A.R. wrote the manuscript. All authors read and commented on the manuscript.

**ADDITIONAL INFORMATION**
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