Abstract—Distributed energy resources (DERs) such as solar panels have small supply capacities and cannot be directly integrated into wholesale markets. So, the presence of an intermediary is critical. The intermediary could be a profit-seeking entity (called the aggregator) that buys DER supply from prosumers, and then sells them in the wholesale electricity market. Thus, DER integration has an influence on wholesale market prices, demand, and supply. The purpose of this article is to shed light onto the impact of efficient DER aggregation on the market power of conventional generators. Firstly, under efficient DER aggregation, we quantify the social welfare gap between two cases: when conventional generators are truthful, and when they are strategic. We also do the same when DERs are not present. Secondly, we show that the gap due to market power of generators in the presence of DERs is smaller than the one when there is no DER participation. Finally, we provide explicit expressions of the gaps and conduct numerical experiments to gain deeper insights. The main message of this article is that market power of conventional generators can be mitigated by adopting an efficient DER aggregation model.

I. INTRODUCTION

Distributed energy resources (DERs), according to the North American Electric Reliability Corporation (NERC), are defined as “any resource on the distribution system that produces electricity and is not otherwise included in the formal NERC definition of the Bulk Electric System (BES)” [1]. Their increasing integration poses fundamental challenges to the design and operation of electricity markets [2], as they are energy sources for the distribution power system, but at the same time, independent market operators do not have enough visibility over them. This is a concern for market operators because DERs dramatically influence the demand behavior. One common way of integrating DERs into wholesale markets is via an intermediary, called the Aggregator, which could be a profit-seeking entity or a utility with good visibility over the distribution lines.

In [3], we proposed an aggregation model with a profit-seeking aggregator which achieves full market efficiency. Specifically, every prosumer’s behavior (buying/selling amount), along with the market price and social welfare, are exactly the same as if these prosumers participate directly in the wholesale market, which is the ideal but unrealistic case, as aggregators are necessary intermediaries (see Fig. 1 for an illustration). There are two underlying assumptions in the model: all prosumers bid truthfully about their utility of consumption, and all generators bid truthfully about their cost of production. The former assumption is more justifiable as each prosumer represents a small fraction in the energy market; thus, she does not believe her bidding could make a difference in the market price. The later assumption, however, may not hold in general as the generators’ bidding might affect the market price, and they may benefit from non-true (strategic) bidding. Such strategic bidding may result in a higher market price (thus benefiting the generators) and reduced social welfare.

The ability of the generators to influence the market price is referred to as the market power of the generators, which could negatively affect the social welfare [4]. Understanding market power of generators has been, and still is, an active area of research [5]–[9]. While some articles focus on country-specific issues [6], [8], [10]–[15], it is evident that understanding market power and potential price manipulations is of strong interest, especially with DERs being intermittent and uncontrollable [16]–[19]. As DER adoption increases rapidly, it becomes critical to understand the effect of DER integration on the market power of conventional generators, which is, to the best of our knowledge, an issue with plenty of open questions to address. Hence, in this paper, we study the market power of the generators when there is no DER aggregation (no prosumer participation) and when prosumers fully participate (either directly or via efficient aggregation) in the wholesale market. Specifically, we address the following question: When DERs are aggregated, can the market power of conventional power generators be
mitigated? If yes, to what extent? We will show (both qualitatively and quantitatively) that, compared to no prosumer participation, the market power of generators is mitigated, and we will quantify this reduction in terms of social welfare. In particular, we prove that under strategic bidding, the social welfare is higher under efficient aggregation, compared to the case when DERs are not integrated. We also prove that the welfare gap between truthful bidding and strategic bidding is smaller when DERs are aggregated, compared to the case where there are no DERs. Finally, we provide quantifications of such differences.

The paper is organized as follows: Section II discusses the motivation and summarizes the main result of the paper, which we will gradually prove throughout the paper. In Section III, we quantify the equilibrium quantities of the efficient DER aggregation model (or, equivalently, the benchmark case of direct prosumer participation in wholesale markets), and then, in Section IV, we do the same for the for the case in which there is no DER participation. We offer a theoretical discussion along with explicit quantification of market power mitigation from efficient DER aggregation in Section V. Then, we conduct numerical experiments in Section VI, where we provide deeper insights. The paper concludes in Section VII with a summary and suggestions for future research. Proofs are omitted but can be found in an extended version of this paper [20].

II. MOTIVATION AND MAIN RESULT

Suppose there are \( n \) prosumers, indexed by \( i \), and \( N \) generators, indexed by \( j \). For ease of exposition, we restrict the attention to one node (location). Prosumer \( i \) is equipped with capacity \( C_i \) of electricity generation, and makes a decision \( z_i \), which is the net amount of energy bought, to maximize her total payoff. Generator \( j \) chooses the production amount \( y_j \), which incurs a cost \( c_j(y_j) \), to maximizes its total profit. We also make the following assumptions on prosumers’ utility of consumption and generators’ cost functions.

**Assumption 1.** For each prosumer \( i \), the utility of consumption is quadratic and it is given by

\[
u_i(C_i + z_i) = a_i(C_i + z_i)^2 + b_i(C_i + z_i), \quad \text{where } a_i < 0 \text{ and } b_i \gg |a_i| 
\]

so that the market price always falls into the range \((0, b_i)\). Each generator \( j \)’s (true) cost function is linear in its production, and all generators have the same cost function, i.e., \( c_j(y_j) = \alpha_j y_j \) for some \( \alpha > 0 \), and the optimal total supply always satisfies \( y > 0 \). Furthermore, there exists at least one prosumer \( i \) such that \( 2a_i C_i + b_i > \alpha \).

As we will see in the rest of this paper, Assumption 1 enables us to obtain explicit expressions for the generators’ supply and the social welfare, so that we can characterize the generators’ market power quantitatively. In the following, we denote by \( W^T \) (resp., \( W^S \)) the optimal social welfare of the model with prosumer participation when generators bid truthfully (resp., strategically). When no prosumer participation is allowed, the optimal social welfare under truthful bidding and strategic bidding of the generators are denoted by \( W^{TN} \) and \( W^{SN} \), respectively. Our first main result in this paper is summarized as the following theorem.

**Theorem 1 (Main Result).** Under Assumption 1, the following inequalities hold:

\[
\begin{align*}
W^T &\geq W^{TN}, \quad W^S \geq W^{SN}, \\
W^T &\geq W^S, \quad W^{TN} \geq W^{SN}, \\
W^{TN} - W^{SN} &\geq W^T - W^S
\end{align*}
\]

In Theorem 1, the first two inequalities (1a) state that the optimal social welfare with prosumer participation is always greater than that without prosumer participation, for both truthful bidding and strategic bidding of the generators. The next two inequalities (1b) state that the optimal social welfare under truthful bidding is always greater than that under strategic bidding of generators, for both cases with prosumer participation and without prosumer participation. Finally, the last inequality (1c) imply that the loss of social welfare due to strategic bidding of the generators are reduced when there is prosumer participation, compared to the case when there is no prosumer participation.

For the rest of this paper, we will provide a complete analysis of Theorem 1, along with the explicit analytical expressions for the social welfare and market prices under all four models.

III. FULL PROSUMER PARTICIPATION

We first consider the case when there is prosumer participation. As shown in [3], given a market price, the prosumers’ decisions, as well as the social welfare, under the aggregation model are the same as those as if they participate directly in the wholesale market. Therefore, we will without loss of generality ignore the role of the aggregator and assume the direct participation of prosumers. We analyze the decisions of each party for the cases when generators bid truthfully and strategically.

A. Truthful bidding of the generators

1) **Prosumers:** Let \( \lambda^T \) be the market price when generators bid truthfully. Each prosumer \( i \) solves her payoff maximization problem:

\[
\max_{z_i \geq -C_i} u_i(C_i + z_i) - \lambda^T z_i.
\]

Under Assumption 1, prosumer \( i \)’s optimal response \( z_i^T \) satisfies the first order condition: \( 2a_i(C_i + z_i^T) + b_i = \lambda^T \).

2) **System operator:** The system operator solves the economic dispatch problem to maximize the social welfare \( W^T \): 

\[
W^T = \max_{z_i \geq -C_i, y_j \geq 0} \sum_i u_i(C_i + z_i) - \sum_j c_j(y_j) \\
\text{s.t., } \sum_j y_j = \sum_i z_i.
\]

Since the generators are identical, we may without loss of generality restrict attention to \( y_j = y_j^*, \forall j \neq j', \) i.e., each generator supplies the same amount of energy \( y_j \). Let \( y := \sum_j y_j \) and \( C := \sum_i C_i \). Considering the prosumers’ and the system operator’s problems, we have the following result.
Proposition 1. Under Assumption 1, an optimal solution to (3) is given by
\[ z_i^T = -C_i + \frac{\alpha - b_i}{2a_i}, \quad \forall i, \quad y_j^T = \frac{-C_j + \alpha - b_j}{N} \]
Furthermore, the equilibrium market price (the optimal Lagrange multiplier of the constraint) is given by \( \lambda^T = \alpha \).

B. Strategic bidding of the generators

1) Prosumers: Let \( \lambda^S \) be the market price when generators bid strategically. Then, each prosumer’s optimal response \( z_i^S \) (under Assumption 1) now becomes \( 2a_i(C_i + z_i^S) + b_i = \lambda^S \).

2) System operator: The system operator solves the economic dispatch problem to maximize the apparent social welfare, which is the ‘social welfare’ when the system operator assumes the generators’ bids are true, but are actually based on the (nontruthful) bidding \( \tilde{c}_j \) of the generators:
\[
\max_{z_i > -C_i, y_j \geq 0} \sum_i u_i(C_i + z_i) - \sum_j \tilde{c}_j(y_j) \quad \text{s.t.} \quad \sum_j y_j = \sum_i z_i.
\]

For each possible total supply \( y \), we define the overall utility of consumption as
\[
u(C + y) = \left\{ \max_{z_i > -C_i} \sum_i u_i(C_i + z_i) \right\} \quad \text{s.t.} \quad \sum_i z_i = y.
\]

In other words, \( u(C + y) \) computes the total utility of consumption of all prosumers when the total energy supply from the generators is \( y \). We can therefore rewrite the system operator’s problem equivalently as \( \max_y u(C + y) - \sum_j \tilde{c}_j(y_j/N) \).

3) Generators: Each generator sets an optimal \( y_j \) to supply, and bids a tilted cost function \( \tilde{c}_j \) instead of the true cost function \( c_j \). The generator \( j \) considers the market price \( \lambda^S \) as a function of the total supply \( y \), and aims to solve the profit maximization problem:
\[
\max_{y_j} \lambda^S \left( y_j + \sum_{j' \neq j} y_{j'} \right) - c_j(y_j).
\]

To this end, the generator needs to compute the market price as a function of total supply. Since the total supply and the total (net) demand are matched by the system operator, we have that \( \sum_i z_i^S = y \). This, together with prosumer’s optimal condition \( 2a_i(C_i + z_i^S) + b_i = \lambda^S \), implies that \( \lambda^S(y) = \frac{y + C + \sum_i b_i}{\sum_i \frac{b_i}{2a_i}} \). Therefore, the generator’s profit maximization problem becomes:
\[
\max_{y_j \geq 0} \frac{y_j + \sum_{j' \neq j} y_{j'} + C + \sum_i b_i}{\sum_i \frac{b_i}{2a_i}} - \alpha y_j.
\]

Each generator solves (8), and since they are all identical, we only look at the equilibrium where \( y_j = y_{j'}, \forall j, j' \in [N] \). Thus, from the first-order condition of (8), we obtain the optimal supply amount for generator \( j \):
\[
y_j^S = \frac{-C_j + \alpha - b_j}{N + 1} \quad \forall j.
\]
In order to sell \( y_j^S \) as given in (9), generator \( j \) will bid \( \tilde{c}_j \) such that the system operator will allocate \( y_j^S \) amount of energy to \( j \). The generator thus considers the economic dispatch problem that the system operator solves. Therefore, the generator will bid \( \tilde{c}_j \) such that \( y_j^S = y_j \) solves the system operator’s problem optimally, i.e.,
\[
\frac{\partial u(C + y_j)}{\partial y_j} \bigg|_{y_j = y_j^S} = \frac{\partial \tilde{c}_j(y_j)}{\partial y_j} \bigg|_{y_j = y_j^S}.
\]

While there are many possible choices of \( \tilde{c}_j \) that satisfies (10), one of the optimal bids for the generator is to bid a linear cost function.

Lemma 1. The following linear cost function is an optimal bidding strategy for generator \( j \):
\[
\tilde{c}_j(y_j) = \frac{Na \sum_i \frac{b_i}{2a_i} + C + \sum_i b_i}{(N + 1) \sum_i \frac{b_i}{2a_i}} \cdot y_j.
\]

Considering the prosumers’ and the system operator’s problems, we have the following result.

Proposition 2 (Competitive Equilibrium). Under Assumption 1, if all generators bid as in (11), then, an optimal solution to (5) is given by
\[
z_i^S = -C_i + \frac{\lambda^S - b_i}{2a_i}, \quad \forall i, \quad y_j^S = \frac{-C_j + \alpha - b_j}{N + 1}, \quad \forall j.
\]

where the equilibrium market price (the optimal Lagrange multiplier of the equality constraint) is \( \lambda^S = \frac{N\alpha}{N+1} \). Furthermore, the solutions in (12) optimally solve the prosumer’s/generator’s problem.

IV. NO PROSUMER PARTICIPATION

We next consider the case when the prosumers cannot sell back to the grid, i.e., each prosumer \( i \) can only purchase some \( z_i \geq 0 \) amount of energy. We first look at the prosumers’ problem.

Prosumers: Let \( \lambda \) be the market price. Each prosumer \( i \) solves her payoff maximizing problem: \( \max_{z_i \geq 0} u_i(C_i + z_i) - \lambda z_i = \max_{z_i \geq 0} a_i(C_i + z_i)^2 + b_i(C_i + z_i) - \lambda z_i \). The optimal decision of prosumer \( i \) is thus
\[
z_i = \left[ \frac{\lambda - b_i}{2a_i} - C_i \right] +.
\]

Therefore, given any market price \( \lambda \), the set of prosumers who make a strictly positive amount of purchase is \( S(\lambda) := \{ i \mid 2a_iC_i + b_i > \lambda \} \). We will without loss of generality sort the prosumers in decreasing order of \( 2a_iC_i + b_i \), i.e., for any two prosumers \( i \) and \( i' \), we have that \( 2a_iC_i + b_i \geq 2a_{i'}C_{i'} + b_{i'} \) if \( i \leq i' \). Under such ordering, for any \( i \leq i' \), if prosumer \( i' \) makes a positive purchase of electricity, then prosumer \( i \) must also make a positive purchase. This sorting of prosumers will be helpful when we write the social welfare.

A. Truthful bidding of the generators

1) System operator: The system operator solves the economic dispatch problem that maximizes the social welfare:
\[
W^{TN} = \max_{z_i \geq 0, y_j \geq 0} \sum_i u_i(C_i + z_i) - \sum_j c_j(y_j)
\]
\[\text{s.t.} \quad \sum_j y_j = \sum_i z_i.\]
Since the generators are identical, we again restrict solutions to $y_j = y_{j'}$, $\forall j \neq j'$, i.e., each generator supplies the same amount of energy $y_j$. Considering the prosumers’ and the system operator’s problems, we have the following result.

**Proposition 3.** Under Assumption 1, an optimal solution to (14) is given by

$$z_i^{TN} = \left[ -C_i + \frac{\alpha - b_i}{2a_i} \right]^+, \quad y_j^{TN} = \frac{\sum_i \left[ -C_i + \frac{\alpha - b_i}{2a_i} \right]^+}{N}.$$  

Furthermore, the equilibrium market price (the optimal Lagrange multiplier of the constraint) is given by $\lambda^{TN} = \alpha$.

**B. Strategic bidding of the generators**

1) System operator: The system operator solves the economic dispatch problem to maximize the apparent social welfare, which is the “social welfare” when the system operator assumes the generators’ bids are true, but are again actually based on the (nontruthful) bidding $\tilde{c}_j$ of the generators:

$$\max_{z_i \geq 0, y_j \geq 0} \sum_i u_i(C_i + z_i) - \sum_j \tilde{c}_j(y_j)$$

s.t. $\sum_j y_j = \sum_i z_i$  

(15)

For each possible total supply $y$, the overall utility of consumption now becomes

$$u(C + y) = \left\{ \max_{z_i \geq 0} \sum_i u_i(C_i + z_i) \text{ s.t. } \sum_i z_i = y \right\}.$$  

(16)

As $y$ increases, the number of prosumers with $z_i > 0$ will change in a discrete manner. However, we have the following useful result.

**Lemma 2.** $u(C + y)$ is continuous and differentiable in $y$.

We can thus rewrite the system operator’s problem equivalently as $\max_{z_i \geq 0} u(C + y) - \sum_j \tilde{c}_j(y_j/N)$.

2) Generators: Each generator sets an optimal $y_j$ to supply, and bids a tilted cost function $\tilde{c}_j$ instead of the true cost function $c_j$. The generator $j$ considers the market price $\lambda^{SN}$ as a function of the total supply $y$, and aims to solve the profit maximization problem:

$$\max_{y_j} \lambda^{SN}(y_j + \sum_{j' \neq j} y_{j'}) \cdot y_j - c_j(y_j).$$  

(17)

To this end, the generator needs to compute the market price as a function of total supply. Since the total supply and the total (net) demand are matched by the system operator, we have that $\sum_j y_j^{SN} = y$. At any given $y$, we need to consider which prosumers have $z_i > 0$ and which prosumers have $z_i = 0$. For all $i = \{1, 2, \ldots, n\}$, define

$$y_i^i := (2a_iC_i + b_i)\sum_{i'=1}^{i-1} \frac{1}{2a_i} = \sum_{i'=1}^{i-1} \left( C_{i'} + \frac{b_i}{2a_i} \right).$$  

(18)

Then, we have the following lemma.

**Lemma 3.** Prosumer $i$’s optimal decision $z_i > 0$ if and only if the total supply satisfies $y > y_i^i$.

In other words, the set of prosumers with $z_i > 0$ may be written as $S(y) = \{ i \mid y > y_i^i \}$.

Combining (13) with $\lambda = \lambda^{SN}$, Lemma 3, and the supply-demand balance, we obtain that

$$\lambda^{SN}(y) = \frac{y + \sum_i \left( C_i + \frac{b_i}{2a_i} \right) \cdot 1_{\{y>y_i^i\}}}{\sum_i \frac{1}{2a_i} \cdot 1_{\{y>y_i^i\}}}.$$  

(19)

Therefore, each generator essentially solves (17) with the market price function given by (19). Since they are all identical, we only look at the equilibrium where $y_j = y_{j'}$, $\forall j, j' \in [N]$. Thus, from the first-order condition of (17), we obtain the condition on optimal supply amount for generator $j$:

$$y_j^{SN} = \sum_i \left( \frac{\alpha - b_i}{2a_i} - C_i \right) \cdot 1_{\{N y_j^{SN} > y_i^i\}} \text{ s.t. } \sum_j y_j^{SN} = \sum_i z_i^{TN} - y_i^i.$$  

(20)

In order to sell $y_j^{SN}$, each generator $j$ will bid $\tilde{c}_j$ such that the system operator will allocate $y_j^{SN}$ amount of energy to $j$. The generator thus considers the economic dispatch problem that the system operator solves. Therefore, the generator will bid $\tilde{c}_j$ such that $y_j^{SN} := \sum_j y_j^{SN} = y_j^{SN}$ solves the system operator’s problem optimally, i.e.,

$$\frac{\partial u(C + y)}{\partial y} \bigg|_{y = N y_j^{SN}} = \frac{\partial \tilde{c}_j(y_j)}{\partial y} \bigg|_{y_j = y_j^{SN}}.$$  

(21)

While there are many possible choices of $\tilde{c}_j$ that satisfies (21), one of the optimal bidding strategies for the generator is again to bid a linear cost function.

**Lemma 4.** The following linear cost function is an optimal bidding strategy for generator $j$: $\tilde{c}_j(y_j) = \frac{N \alpha \sum_i \frac{1}{2a_i} 1_{\{N y_j^{SN} > y_i^i\}} + \sum_i \left( C_i + \frac{b_i}{2a_i} \right) 1_{\{N y_j^{SN} > y_i^i\}}}{(N + 1) \sum_i \frac{1}{2a_i} 1_{\{N y_j^{SN} > y_i^i\}}}, \forall y_j.$

(22a)

Considering the prosumers’ and the system operator’s problems, we have the following result.

**Proposition 4 (Competitive Equilibrium).** Under Assumption 1, if all generators bid as in Lemma 4, an optimal solution to (15) satisfies the following:

$$z_i^{SN} = \left[ -C_i + \frac{\lambda^{SN} - b_i}{2a_i} \right]^+, \quad y_j^{SN} = \frac{\sum_i \left( \frac{\alpha - b_i}{2a_i} - C_i \right) \cdot 1_{\{N y_j^{SN} > y_i^i\}}}{N + 1}, \quad \forall j,$$

(22b)

where the equilibrium market price (the optimal Lagrange multiplier of the constraint) is $\lambda^{SN} = \frac{N \alpha}{N + 1} + \sum_i \left( C_i + \frac{b_i}{2a_i} \right) 1_{\{y_j^{SN} > y_i^i\}}$. Furthermore, $z_i^{SN}$ and $y_j^{SN}$ optimally solve the prosumer’s/generator’s problem.

V. DISCUSSION

In previous sections, we have analyzed and derived the equilibrium quantities for all four models, depending on whether we allow prosumer participation and whether generators can bid strategically. For each model, Fig. 2 provides an explicit characterization of the equilibrium social welfare.

While we have obtained explicit expressions (in terms of parameters of the prosumers and generators) for all four social welfare models, it remains to compare them to finish the proof of Theorem 1, which is not obvious in their current form. In this section, we provide alternative expressions for
these social welfare. To proceed, we first define the social welfare when there is no generator and no electricity market, i.e., each prosumer consumes the amount of energy that her capacity allows: $W_0 := \sum_i u_i(C_i)$. We also define the amount of market price rise due to strategic bidding of the generators as $\delta := \lambda_S - \lambda_T$, and $\delta^N := \lambda^N_S - \lambda^N_T$. With these definitions, we have the following expressions for the social welfare under different models, which use $W_0$ as a reference.

**Proposition 5.** We have the following expressions for the four social welfare.

$$W^T = W_0 - \sum_i a_i (z_i^T)^2,$$

$$W^S = W_0 + \sum_i \left( -\alpha (z_i^S)^2 + \delta z_i^S \right),$$

$$W^{TN} = W_0 - \sum_{\{|i|S^T > 0\}} a_i (z_i^T)^2,$$

$$W^{SN} = W_0 - \sum_i a_i (z_i^{SN})^2 + \delta^N \sum_i z_i^{SN}.$$

We next obtain the expressions for $\delta$ and $\delta^N$. Under full prosumer participation and strategic bidding of the generators, we have that $u'(C_i + z_i^S) = \lambda_S - \lambda_T + \delta = \alpha + \delta$. From the truthful bidding case, we also have that $\alpha = u'(C_i + z_i^T)$. Thus, we can write $\delta = u'(C_i + z_i^T) - u'(C_i + z_i^S) = 2a_i(z_i^T - z_i^S)$ for all $i$, or equivalently, by noting $\sum_i z_i^T = y^T \cdot \frac{N+1}{N+1} \sum_i z_i^T$, we have that

$$\delta = \sum_i (z_i^T - z_i^S) = \left( \frac{1}{N+1} \right) \sum_i z_i^T.$$

(27)

Under the models that prosumers cannot sell, with generators bidding strategically, we have that $u'(C_i + z_i^{SN}) = \lambda^SN_S - \lambda^T N + \delta^N = \alpha + \delta^N$ for those prosumers with $z_i^{SN} > 0$. From the truthful bidding case, we also have that $\alpha = u'(C_i + z_i^{TN})$ for those prosumers with $z_i^{TN} > 0$. Thus, we can write $\delta^N$ as

$$\delta^N = u'(C_i + z_i^{SN}) - u'(C_i + z_i^{TN}) = 2a_i(z_i^{SN} - z_i^{TN}) = 2a_i(z_i^{SN} - z_i^{TN}), \forall i, s.t. z_i^{SN} > 0.$$  

By noting that $\sum_{\{|i|S^N > 0\}} z_i^{SN} = \sum_{\{|i|S^N > 0\}} z_i^{SN} = \sum_{\{|i|S^N > 0\}} z_i^{SN}$, we have that

$$\delta^N = \sum_{\{|i|S^N > 0\}} (z_i^{SN} - z_i^{TN}) = \left( \frac{1}{N+1} \right) \sum_{\{|i|S^N > 0\}} z_i^{SN}.$$

(28)

With Proposition 5, (27), and (28), we are ready to show the relations in Theorem 1. After some algebra, we restate Theorem 1 as follows.

$$W^T - W^{TN} = -\sum_{\{|i|T^S \leq 0\}} a_i (z_i^T)^2 \geq 0,$$

(29a)

$$W^S - W^{SN} = \sum_i -\alpha (z_i^S)^2 + \delta z_i^S + \sum_i a_i (z_i^{SN})^2 + \delta^N \sum_i z_i^{SN} \geq 0,$$

(29b)

$$W^{TN} - W^S = \left( \frac{1}{N+1} \right) \sum_{\{|i|S^N = 0, T^S > 0\}} a_i z_i^2,$$

(29c)

$$W^{TN} - W^{SN} \geq W^T - W^S.$$

(29d)

This finishes the proof of Theorem 1, and verifies the benefit of aggregating distributed energy resources (DERs), i.e., by allowing the aggregation of DERs, the optimal social welfare is improved with either truthful bidding generators or strategic bidding generators, and the loss of social welfare due to strategic bidding of the generators is reduced in the full participation model compared to that in the no participation model. Thus, we can state the main takeaway point of the paper, which is, efficient DER aggregation mitigates market power of generators, and such mitigation is quantified explicitly as in (29).

VI. ILLUSTRATIVE EXAMPLE

Next, we consider an example with $n = 2$ and provide illustrations of our results. We let the true cost of each generator be $c(y) = \alpha y = 5y$. For each prosumer $i$, let $a_i = -0.1$ and $b_i = 10$, and we distinguish between them via the capacities, with $C_1 = 10$ and $C_2 = 30$. The parameters are picked such that $2a_1C_1 + b_1 > \alpha$ and $2a_2C_2 + b_2 < \alpha$, i.e., prosumer $i = 1$ will always have a positive demand. To make our example more interesting, we vary the number of generators from $N = 1$ to $N = 20$ (outcomes saturate for $N > 20$). In Fig. 3, we plot the social welfare for each market setup, efficiency loss due to strategic behavior of the
Fig. 3. **Left:** Social welfare for each market setup vs. $N$. DER participation improves the social welfare. As $N$ increases, strategic bidding converges to truthful bidding, and all inequalities provided in Theorem 1 hold. **Middle:** Quantification of market power in terms of social welfare loss. When DERs are integrated, market power is mitigated. **Right:** Price for each market setup vs. $N$. Highest price corresponds to strategic bidding without DER participation, but when DERs are present, the price becomes lower. All prices converge to $\alpha = 5$.

generators, and prices. The key outcomes can be summarized as follows:

1) $W_{SN} \rightarrow W_{TN}$ and $W^S \rightarrow W^T$ as $N \rightarrow \infty$.
2) $\lambda^{SN} \rightarrow \alpha$ and $\lambda^S \rightarrow \alpha$ as $N \rightarrow \infty$.
3) For $N < \infty$, $\alpha < \lambda^{SN} < \lambda^S$.
4) Efficient DER aggregation mitigates the market power of generators.

VII. CONCLUSIONS AND FUTURE WORKS

There are many open questions related to DER aggregation, as they are intermittent, uncontrollable, and rapidly increasing. While the work of [3] provides an efficient DER aggregation model, this work serves as a stepping stone towards understanding the impact of DER aggregation onto market power of generators. Particularly, we have proven the inequalities stated in Theorem 1 and provided explicit expressions later in (29). The main results of this paper state that the optimal social welfare under efficient DER integration is always greater than that without prosumer participation, the optimal social welfare under truthful bidding is always greater than that under strategic bidding of generators, and the loss of social welfare due to strategic bidding of the generators is mitigated under efficient DER aggregation.

DERs are naturally stochastic, so, incorporating uncertainty and its impact on market power mitigation would be an interesting question to address. Moreover, we have assumed identical generators, so it would also be interesting to explore generators of different kinds, each with generator-specific considerations, while taking into account the competition among them and network considerations.

REFERENCES

[1] “Distributed Energy Resources: Connection Modeling and Reliability Considerations,” North American Electric Reliability Corporation, Tech. Rep., 2017.
[2] I. Ritzenhofen, J. R. Birge, and S. Spinler, “The structural impact of renewable portfolio standards and feed-in tariffs on electricity markets,” European Journal of Operational Research, vol. 255, no. 1, pp. 224–242, 2016.
[3] Z. Gao, K. Alshehri, and J. R. Birge, “On efficient aggregation of distributed energy resources,” in 2021 60th IEEE Conference on Decision and Control (CDC), 2021, pp. 7064–7069.
[4] M. Al-Gwaiz, X. Chao, and O. Q. Wu, “Understanding how generation flexibility and renewable energy affect power market competition,” Manufacturing & Service Operations Management, vol. 19, no. 1, pp. 114–131, 2017.
[5] P. Twomey, R. J. Green, K. Neuhoff, and D. Newbery, “A review of the monitoring of market power the possible roles of tsos in monitoring for market power issues in congested transmission systems,” Tech. Rep., 2006.
[6] S. Borenstein, J. Bushnell, E. Kahn, and S. Stoft, “Market power in california electricity markets,” Utilities Policy, vol. 5, no. 3–4, pp. 219–236, 1995.
[7] A. Sheffrin and J. Chen, “Predicting market power in wholesale electricity markets,” Proc. Western Conf. Adv. Regul. Competition, South Lake Tahoe, CA, USA, 2002.
[8] Market Power and Competitiveness, California Independent Syst. Operator, Folsom, CA, USA, 1998.
[9] W. W. Hogan, “A market power model with strategic interaction in electricity networks,” The Energy Journal, vol. 18, no. 4, 1997.
[10] H. Weigt and C. von Hirschhausen, “Price formation and market power in the german wholesale electricity market in 2006,” Energy Policy, vol. 36, no. 11, pp. 4227–4234, 2008.
[11] D. Brennan and J. Melanie, “Market power in the australian power market,” Energy Economics, vol. 20, no. 2, pp. 121–133, 1998.
[12] O. J. Olsen and K. Skytte, “Competition and Market Power in Northern Europe,” in Competition in European Electricity Markets, ser. Chapters, J.-M. Glachant and D. Finon, Eds. Edward Elgar Publishing, 2003, ch. 7.
[13] V. Salas and J. Saurina, “Deregulation, market power and risk behaviour in spanish banks,” European Economic Review, vol. 47, no. 6, pp. 1061–1075, 2003.
[14] A. Garcia and L. E. Arbeláez, “Market power analysis for the colombian electricity market,” Energy Economics, vol. 24, no. 3, pp. 217–229, 2002.
[15] N.-H. M. von der Fehr and S. Ropenus, “Renewable energy policy instruments and market power,” The Scandinavian Journal of Economics, vol. 119, no. 2, pp. 312–345, 2016.
[16] N. A. Ruhi, K. Djivotham, N. Chen, and A. Wierman, “Opportunities for price manipulation by aggregators in electricity markets,” IEEE Trans. Smart Grid, vol. 9, no. 6, pp. 5687–5698, 2018.
[17] C. B. Martinez-Anido, G. Brinkman, and B.-M. Hodge, “The impact of wind power on electricity prices,” Renewable Energy, vol. 94, pp. 234–247, 2016.
[18] K. Alshehri, S. Bose, and T. Başar, “Centralized volatility reduction for electricity markets,” International Journal of Electrical Power & Energy Systems, vol. 133, 2021.
[19] D. Cai, A. Agarwal, and A. Wierman, “On the inefficiency of forward markets in leader–follower competition,” Operations Research, vol. 68, no. 1, pp. 35–52, 2020.
[20] Z. Gao, K. Alshehri, and J. R. Birge, “Aggregating distributed energy resources: efficiency and market power,” Available at SSRN 3931052, 2021.