Forms of natural vibrations of a gravitational dam on an elastic foundation, taking into account the attached mass of water

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Abstract. Based on the results of the analysis, we have constructed forms of the dam's vibration forms on different bases, with or without taking into account the reservoir water. The main vibration tone of the 100-meter section of the gravity dam is in the cross direction (along the target) and varies from 0.929 sec to 0.635 sec, depending on the hardness of the base. The main tone of horizontal vibrations varies from 0.687 sec to 0.351 sec. The attached mass of water increases the period of horizontal vibrations of the dam from 0.380 sec to 0.474 sec. Also, we have defined 3 types of vibrations: "pure", "pure shift", "invisible form". We confirmed the need to take into account the water body in solving the contact problem of joint vibrations of the structure with the base and reservoir to obtain real forms of natural vibrations of the structure and improve the accuracy of calculations of seismic resistance of structures in the framework of the linear-spectral theory.

1. Introduction

The water environment and the elastic foundation have a significant impact on the dam’s oscillations during an earthquake. Huge masses of water and soil are involved in the oscillation process. Changing the center of mass of the joint inertial system “dam-foundation-reservoir” leads to a change in the frequencies and forms of the structure’s natural oscillations, and, as a result, to a change in the magnitude of seismic loads affecting the structure, to a change in its stress state [1, 2, 3, 4].

One of the standard methods for calculating all hydraulic structures for seismic loads is the method according to which the seismic stability is assessed considering the conditional quasi-static loads determined according to Rules Set.133330.2018 under the linear spectral theory (LST) under the influence of the effects set by the calculated acceleration of the foundation according to the calculated score at the construction site [5, 6].

In the literature, the calculated loads-seismic forces and seismic moments in the LST are determined on the based on a priori information on forms and frequencies of natural oscillations of the structure, which do not take into account the influence of the water environment and its spatial fluctuations. This significantly distorts the picture of fluctuations in the structure and leads to design errors. Therefore, a reliable determination of the latter ones is a prerequisite for improving the quality and reliability of the study results of the structures’ seismic stability [2, 3, 4, 5, 6, 7].

The purpose of the study was a research of the spatial forms of natural oscillations of a concrete gravity dam in the context of its free oscillations as a free-standing section. This statement of the problem is possible if the dam section being cut by deformation seams has grooved or grouted cross seams, the width of which allows the dam section to make small oscillations.
The established freedom of the dam connections movement (6 degrees of freedom in the connection, including the rotation angles) allows us to study the influence of the reservoir water mass and the foundation soil to the inertial loads that appear not only in case of shear but in case of bending and rotational oscillations of the dam.

During an earthquake, the behavior of a concrete gravity dam section is studied on the assumption of the longitudinal direction of the seismic impact at which the influence of the entrained water mass is the most significant. For this purpose, we consider the problems of free oscillations of a gravity dam section on elastic foundation, which has various stiffness, as well as the problems of free oscillations of the section without and with the consideration of water in the reservoir [8, 9].

2. Methods
Numerical simulation of free oscillations of the dam was performed in SAP2000.v19 software package. When simulating the FEM grid, the size of the calculated area was at least double the height of the dam (2H) in all directions, inclusive of the foundation in depth. We considered a gravity dam with a height of 103 meters, with a reservoir depth of 100 m, the head of 8 m width, and the bottom face of 1:0.8. The section width was 20 meters. The foundation was assumed to be inertia less. The elastic foundation options with different elastic modulus were considered: \( E=5000 \text{ MPa} \) (Option 1), \( E=15000 \text{ MPa} \) (Option 2), \( E=100000 \text{ MPa} \) (Option 3) (Figure.1). The conditions of the flat deformation were set in the foundation structure and the structure itself had no anchors except for the common connections with the foundation.

![Figure 1. Design model](image)

3. Results and Discussion
Forms of natural oscillations with consideration of the entrained water mass. The solution to the problem of the structures’ seismic stability within the LST requires knowledge of the natural oscillation forms (NOF) of the structure [10]. Their calculation is possible based on numerical simulation, considering the physical and mechanical parameters of the calculation model. At present, there are various recommendations for determining the forms and frequencies of natural oscillations of the “reservoir – dam – foundation” complex [11, 12, 13].

On the one hand, it is possible to determine the forms of natural oscillations within the solution of the contact problem of the water environment interaction with the dam body and the foundation, on the assumptions of fluid compressibility or incompressibility, consideration or neglecting of wave generation on the free surface, the shape of the reservoir bed [14, 15, 16]. Such solutions are highly
complex and carried out in large software systems (ANSYS) with the use of special acoustic elements describing the fluid behavior.

On the other hand, it is possible to solve the problem of natural oscillations of a dam on the elastic foundation without considering its interaction with the water environment, neglecting the water environment influence to forms of the dam oscillations, assuming that they do not differ a lot with and without consideration of water in the reservoir [17, 18]. The values of natural oscillation periods when considering the reservoir water are defined more precisely on the bases of the available recommendations, for example, for the $i$ – form of natural oscillations:

$$T_i^B = T_i \sqrt{1 + \frac{M_i^B}{M}}$$

where $T_i$ is the period of natural oscillations of a structure by $i$-tone without consideration of water; $M$ is the mass of the structures per 1 running meter of length; $M_i^B$ is the entrained mass of water corresponding to the motion by $i$-tone of the structure oscillation; it is determined by an epure of the entrained masses:

$$M_i^B = \int_0^H m_i^B(x)dx$$

where the $x$-axis is directed along the dam height ($H$).

The nature of the hydrodynamic water pressure epure depends on many factors related to the problem statement, design of the structure, and limiting conditions; for example, the hydrodynamic water pressure epures for structures located in a narrow site differ depending on the coefficient of the dam site and its shape. Thus, in calculations under LST, especially for spatial conditions, it is necessary to distinguish the nature of the structure’s oscillations in different forms, and the calculation formulas for determining the coefficient of the entrained mass should correspond to the nature of this movement.

The hydrodynamic water pressure to the structure during the translational oscillations of the structure is simulated as a mass entrained for elements of the upstream side of the structure according to Westergaard hypothesis that is determined according to a provision in SP by the following expression:

$$m_k^B = \rho_o H \mu_k \psi \quad [\text{kg/m}^2]$$

where $\rho_o$ is the water density (kg/m$^3$); $H$ is the water depth at the foot of the structure (m); $\psi$ is a coefficient that takes into account the limited length of the reservoir, $\mu_k$ is the coefficient of the entrained water mass in the $k$-element of the upstream face.

The hydrodynamic effect of water to the dam is twice considered in the calculations of seismic stability of structures under LST, first time - in determining the forms of natural oscillations, and second time – in calculating of seismic forces, where the hydrodynamic effect is simulated by the entrained mass. Since the hydrodynamic pressure epure depends on the nature of the structure’s oscillations, the first calculation is based on the condition of the translational movement of the dam-reservoir system as an initial approximation. At the second stage, when the oscillation forms are determined, the calculation of seismic forces and moments is based on the actual nature of the structure interaction with the fluid according to recommendations of Rules Set and All-Union Scientific Research Institute of Hydraulic Engineering.

In the studies performed, the solution of the problem of natural oscillations of the “dam – foundation” system was carried out in two statements: in the first statement - without consideration of water in the reservoir, and the second statement – with consideration of water in the reservoir, which was simulated by the entrained mass according to the solution of Westergard [19, 20]. The epures of the entrained mass for the translational nature of the joint dam-reservoir oscillations per 1 running meter of the dam section width are shown in Figure 1.

The NOF analysis in the spatial problem was carried out by the value of the element nodes’ displacements normalized to the center of mass of the element. Table 1 contains the periods of the
dam’s natural oscillations for these problems and the “layout” of displacements along the first 12 NOFs for the problem with the elastic module of the foundation of 15000 MPa (Option 2).

Almost all forms of oscillations, except the main tone, have components of movement in all directions. For the convenience of their analysis, the relative displacements (hereinafter, the \( \text{displacements} \)) in each oscillation form were normalized by the maximum value inside the form, which allows us to compare the obtained displacements by a magnitude and evaluate the direction of the resulting displacement vector. Empty cells in Table 1 mean that there is no displacement in one of the directions, i.e. their small value compared to other components with a simultaneous zero value of the displacement in the center of mass of the element.

**Analysis of the received forms of natural oscillations**

Graphical representation of oscillation epures was performed for the vertical section of the dam near the upstream face. Analysis of the forms allows us to note that all oscillations can be divided into 3 types:

- **The first type of oscillation** in which there is a dominant direction of displacement by one of the axes (\(0X, 0Y, 0Z\)), possibly by two axes. This type includes oscillations in the main tone with “pure” (only in one direction) oscillations along the dam’s site, the period of which is 0.738 s and it is equal for problems with water and without water due to the lack of an upstream face for contact with water in the area of the deformation seams.

For forms 2, 6, 7, 11, the dam oscillations are translational in nature and have components by axes \(0X\) and \(0Z\), but have no spatial components by \(0Y\).

**Table 1. Forms of oscillations with and without the entrained mass of water**

| Elasticity module of the foundation of 15,000 MPa | without the entrained mass of water | with the entrained mass of water |
|--------------------------------------------------|------------------------------------|----------------------------------|
| \(T, \text{sec}\)                             | \(X\) \(Y\) \(Z\)          | \(X\) \(Y\) \(Z\)          |
| 0.738                                           | (1) \(1\) \(1\)          | (1) \(1\) \(1\)          |
| 0.380                                           | (1) \(1\) Tor Tor        | (2) \(2\) Tor Tor        |
| 0.208                                           | Tor (2) Tor \(1\)* \(2\) | Tor \(1\) Tor Tor        |
| 0.203                                           | (1) \(1\)* \(1\) Tor Tor | 0.211 \(2\) \(2\) Tor Tor |
| 0.180                                           | Tor \(2\) Tor \(1\)* Tor | 0.196 \(2\) \(2\) Tor Tor |
| 0.172                                           | (2) Tor \(2\) Tor \(2\)  | 0.191 Tor \(2\) Tor       |
| 0.106                                           | (3) Tor \(2\) Tor \(2\)  | 0.121 \(2\) \(2\) Tor Tor |
| 0.105                                           | Tor \(2\) Tor \(2\) Tor | 0.120 Tor \(3\) Tor Tor   |
| 0.093                                           | Tor \(3\) Tor \(3\) Tor | 0.096 Tor \(3\) Tor Tor   |
| 0.077                                           | Tor \(3\) Tor \(3\) Tor | 0.088 Tor \(3\) Tor Tor   |
| 0.068                                           | Tor \(3\) Tor \(3\) Tor | 0.083 Tor \(3\) Tor Tor   |
| 0.066                                           | Tor \(3\) Tor \(3\) Tor | 0.080 Tor \(3\) Tor Tor   |

**Note:** - the number in parentheses corresponds to the form number in the console diagram of the displacement wedge method; - tor: oscillations-torsion; \((1)*\) - oscillations with an invisible shape

- **The second type of oscillation** in which one or two directions are present, in which the displacements of the nodes of the opposite faces of the elements are oppositely directed and have equal absolute values, which gives a zero displacement in the center of mass of the element. Such oscillations can be described as “net displacement” oscillations within an element. The multidirectional swaying of opposite faces and the smallness of deformations allow us to consider such oscillations as torsion relative to the axis passing through the center of gravity of the element, with an angular acceleration \(\varepsilon\), which is defined by the expression: \(\frac{d^2\varphi}{dt^2}\), where \(\varphi(t)\) is the angle of
rotation or relative displacement. The relative displacement value can be determined based on the existing displacements of the opposite faces of the element. Taking into account the smallness of deformations under free oscillations, one can, instead of the angle tangent, take the value of the angle of displacement itself (angular displacement), expressed in radians as: \( \theta = \frac{U}{B/2} \), where \( U \) is the displacement of faces, \( B \) is the distance between the faces.

In the calculations performed, torsional oscillations were obtained for forms 3, 5, 8, 9, 10, 12, and all of them have horizontal and vertical components of displacements, which in total determine the direction of twisting of elements relative to the inclined axis passing through their center of mass. The visual representation of oscillations in this form is similar to the periodic twisting of a dam along an inclined axis in different directions.

For such oscillations, the total dynamic moment of the entire structure is defined as the sum of the moments of rotation of all elements of the computational space, each of which in its turn is determined by a known expression:

\[
M = I \cdot \varepsilon
\]

\( I \) is the moment of inertia of the element when rotating relative to one of the axes passing through the center of mass of the element, \( \varepsilon \) is the angular acceleration.

The presence of a significant number of torsional oscillation forms suggests that these forms do not contribute to the longitudinal component of the seismic force, but form a seismic moment that must be considered when calculating the stress state of the dam. When calculating the seismic moment, the coefficient of the entrained water mass is determined by the formulas given for the rotational movement of structures with a vertical upstream face in the reference literature [14].

- **The third type** of oscillation is not shown in all calculations. In some problems, under the “cover” of the dominant direction of oscillations (along the 0Z axis), small horizontal displacements are obtained, which in normalized form have values several times smaller than in the dominant direction, while the shape of the epure of horizontal displacements by the dam height has a triangular or rectangular shape, with small (relative to one) values of one sign, which significantly increases the role of this form in the magnitude of the seismic force. Despite the fact that the horizontal components in the “invisible form” are much smaller than the vertical ones, their unidirectionality gives a significant inertial contribution to the longitudinal oscillations of the dam.

The resulting type of oscillation occurs at periods from 0.115 sec to 0.228 sec, which corresponds to the maximum values of the dynamic coefficients according to LST. The selected horizontal component has a high coefficient of oscillation form, almost twice increasing the same value for the first NOF. The noted features of the “invisible form” indicate its significant contribution to the resulting seismic force, both for the horizontal and vertical direction of the seismic impact. This type of oscillation occurs at these frequencies in the problem without consideration of the entrained water mass at NOF 4 (Table 1), and in the problem with a consideration of water for the foundation of 100000 MPa (Option 3) - at NOF 7 (Table 2).

**Forms of natural oscillations on various foundations**

The layout of the oscillation forms along the oscillation directions for problems with different foundation properties when considering the entrained water mass is shown in Table 2.
Table 2. Forms of oscillation of dams located on different foundations considering the entrained water mass

| Elasticity module of the foundation | 5000 MPa | 15000 MPa | 100000 MPa |
|------------------------------------|----------|-----------|------------|
| T, sec                            |          |           |            |
| Horizontal oscillations           |          |           |            |
| Vertical oscillations              |          |           |            |
| T, sec                            |          |           |            |
| Horizontal oscillations           |          |           |            |
| Vertical oscillations              |          |           |            |
| T, sec                            |          |           |            |
| Horizontal oscillations           |          |           |            |
| Vertical oscillations              |          |           |            |
| T, sec                            |          |           |            |
| Horizontal oscillations           |          |           |            |
| Vertical oscillations              |          |           |            |
| T, sec                            |          |           |            |

| C | X   | Y   | Z   | X   | Y   | Z   | X   | Y   | Z   |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1 | 0.929 | (1) |     | 0.738 | (1) |     | 0.635 | (1) |     |
| 2 | 0.687 | (1) |     | 0.474 | (1) |     | 0.351 | (1) |     |
| 3 | 0.334 | (2) |     | 0.229 | (1) |     | 0.215 | (1) |     |
| 4 | 0.254 | (2) |     | 0.211 | (2) |     | 0.178 | Tor | (2) |
| 5 | 0.247 | Tor | (1) | 0.196 | (2) |     | 0.161 | (2) |     |
| 6 | 0.201 | Tor | Tor | 0.191 | Tor | Tor | 0.116 | Tor | Tor |
| 7 | 0.140 | (3) |     | 0.121 | (2) |     | 0.115 | Tor | Tor |
| 8 | 0.124 | Tor | (2) | 0.120 | Tor | (3) | 0.099 | (3) |     |
| 9 | 0.100 | Tor | (3) | 0.096 | Tor | (3) | 0.090 | (3) | Tor |
| 1 | 0.089 | Tor | (3) | 0.088 | Tor | (3) | 0.086 | Tor | (3) |
| 1 | 0.087 | (4) | (3) | 0.083 | Tor | (3) | 0.079 | Tor | (3) |
| 1 | 0.086 | (3) | Tor | 0.080 | (4) | (3) | 0.073 | Tor | (4) |

Note: - the number in parentheses corresponds to the form number in the console diagram of the displacement wedge method; - tor: oscillations-torsion; (1)* - oscillations with an invisible shape

Table 2 shows that among all forms of oscillation, the “pure” form, as in the problem without water consideration, is only the oscillation along the main tone in the direction along the site.

For all foundations, the second form of oscillation is dominated by horizontal components, a third of which are vertical displacements. Vertical oscillations dominate in the third form of oscillations only for a “weak” foundation of 5.000 MPa, the form with vertical oscillations are displaced to the region of high oscillation frequencies for a more rigid foundation. So, for a “weak” foundation, the period of dominant vertical oscillations is 0.334 sec, for a “medium” foundation – 0.211 sec, for a “hard” foundation – 0.115 sec.

Translational oscillations for all forms, except the first one, are characterized by the presence of displacement components in several directions. For all foundations, the maximum amplitudes of longitudinal horizontal displacements in the direction of the seismic wave propagation are obtained on the ridge of the dam for all forms of oscillations. In “weak foundations”, the displacement at the bottom of the dam can be up to 20-30% of the maximum one on the ridge, with an increase in the elastic module of the foundation, the displacements at the foundation decreases to zero except for the third type of oscillations.

Forms with the rotational character of oscillations appear at oscillation frequencies of 4 Hz and higher, and all oscillations are characterized by the rotation of the nodes of the opposite faces of the element in the XOZ plane and a zero displacement of the center of mass of the elements.

Changes in natural oscillation periods for the considered problems with different foundations, with or without water in the reservoir are shown in Figure 2.
All tasks with different foundations were calculated considering the entrained water mass. When changing the properties of the foundation, the pitch periods of transverse oscillations naturally decrease from 0.929 sec. for the weakest foundation (Option 1) to 0.635 sec. for the strongest foundation (Option 3). The highest frequency for the 12th NOF in all tasks is about 13.7 Hz.

As you can see, the difference in periods is observed up to and including Form 6, higher forms of oscillations, starting from the seventh, practically do not react to changes in the properties of the foundation and for all three foundations are almost the same. In all problems, the main oscillation tone is associated with the transverse oscillations of the section along the side.

The obtained values allow us to conclude that the full range of periods for the first 12 forms of natural oscillations without water is longer – from 0.738 sec to 0.066 sec (NOF 12); when considering water, the range is from 0.738 sec to 0.080 sec (for NOF 12), which gives a more dense frequency spectrum at the same frequency range.

_Epures of oscillation forms for a foundation with the elastic module E = 5.000 MPa_

Figure 3 shows the forms of natural oscillations considering the entrained water on the upstream side of the section of gravity dam resting on elastic foundation with elasticity module $E = 5.000 \text{ MPa}$, which corresponds to the average strength of the rock foundation typical to sandstones, siltstones of average jointing. The epures clearly show that the main oscillation tone is $0Y$. The main tone in longitudinal oscillations is NOF 2 with a period of 0.687 sec. The main tone in vertical oscillations is NOF 3 with a period of 0.334 sec. Higher forms of oscillation are mainly represented by forms with spatial displacement components.

The absence of some displacement components in the forms is due either to the absence of this displacement or to the presence of rotational motion (torsional oscillations), which is obtained for NOFs 5, 6, 8, 9, 10, 12 (Table 2, Figure 3).
Figure 3. NOF on the foundation of $E = 5000$ MPa with entrained water mass

*Ejures of oscillation forms for a foundation with an elastic module $E = 15000$ MPa*

Figure 4 shows the forms of natural oscillations, considering the entrained water mass on an elastic foundation with an elastic module $E = 15000$ MPa, which corresponds to a strong rock foundation with weak jointing. The main oscillation tone is $0Y$. The main tone in longitudinal oscillations is NOF 2 with a period of 0.474 sec. In contrast to the less solid foundation, torsional oscillations appear on the third NOF with a period of 0.29 sec, similar to the torsional motion of the main axis of inertia of the dam located in the $X0Z$ plane with simultaneous translational oscillations in the transverse direction.

The main tone in vertical oscillations is the oscillations along FSK 4 with a period of 0.211 s, with a small proportion of horizontal components. At NOF 5, the proportion of horizontal displacements is greater than vertical ones, but the pattern of oscillations is similar to the previous form. NOF 6 is similar to NOF 3, a rotation is also available, and oscillations from the plane occur with greater frequency. NOF 7, as in the previous problem, characterizes flexural oscillations in the $X0Z$ plane.

There are torsional oscillations for NOF 3, 6, 8, 9, 10, 11 at higher frequencies (Table 2, Figure 4).

*Epures of oscillation forms for a foundation with an elastic module $E = 100000$ MPa*

Figure 5 shows the forms of natural oscillations, considering the entrained water mass, on an elastic foundation with an elastic module $E = 100,000$ MPa, which corresponds to an absolutely rigid foundation. It is typical for all forms that the ordinates are equal to zero at the dam foundation.

As before, the main tone of the longitudinal oscillations of the dam was obtained for NOF 2, the period is 0.351 sec. The main tone of vertical oscillations in the problem on a rigid foundation displaced to the bending NOF 7, which began to show “abnormal” properties of increasing the proportion of horizontal displacements on the dam ridge in the absence of oscillations on the rest of the dam, due to which its contribution to the seismic force on the dam ridge became highly significant.
As in the previous problems, there is a large number of torsional waveforms corresponding to the frequencies for NOFs 3, 4, 6, 9, 10, 11, 12 (Table 2, Figure 5).

**Figure 4.** NOF on the foundation of $E = 15000$ MPa with entrained water mass.
4. Conclusions
- Calculations of forms in the spatial statement showed that the main oscillation tone of the 100-meter section of the gravity dam goes in the transverse direction (along the site), with the longest period for the “weak” foundation equal to 0.929 sec, for the average one equal to 0.738 sec, and for the rigid one equal to 0.635 sec.
- The main tone of the longitudinal oscillations of the dam, considering the entrained water, is 0.687 sec for the “weak” foundation, 0.474 sec for the average one, and 0.351 sec for the rigid one.
- Considering the entrained water mass leads to an increase of the period of the main tone of longitudinal oscillations from 0.38 sec to 0.474 sec for the average foundation ($E = 15,000 \text{ MPa}$), which is slightly higher than the period of 0.443 sec obtained on the basis of the recommendations of B. E. Vedeneev All-Union Scientific Research Institute of Hydraulic Engineering;
- Considering the entrained water mass in solving the NOF problem changes not only the oscillation frequencies but also the forms themselves, which clarifies the traditional assumption about the identity of the dam’s oscillation forms with water and without water.
- Reducing the rigidity of the foundation leads to an increase in the periods of longitudinal oscillations and changes in the forms themselves. For a “soft” foundation, the oscillation forms have “residual” displacements at the level of the structure’s foot, the value of which is up to 30% of the maximum displacements at the dam ridge.
- All horizontal NOFs have a maximum displacement at the dam ridge;
- All forms with oscillations along the site are “clean” with different frequency and order of approximation;
- Among the considered oscillation forms, more than half belong to forms with torsional oscillations, most of which occur in the $XOY$ plane.

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