Multi-Scale Single-Bit RP-EMS Synthesis for Advanced Propagation Manipulation through System-by-Design

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Abstract

A new method for synthesizing Single-Bit Reconfigurable Passive Electromagnetic Skins (1RP-EMSs) featuring advanced beam shaping capabilities is proposed. By using single-bit unit cells, the multi-scale problem of controlling 1RP-EMSs is formulated as a two-phase process. First, the macro-scale synthesis of the discrete surface current that radiates the electromagnetic (EM) field fitting user-designed requirements is performed by means of an innovative quantized version of the iterative projection method (QIPM). Successively, the meta-atom states of the 1RP-EMS are optimized with a customized implementation of the System-by-Design paradigm to yield a 1RP-EMS that supports such a feasible reference current. A representative set of numerical results is reported to assess the effectiveness of the proposed approach in designing and controlling single-bit meta-atom RP-EMSs that enable complex wave manipulations.

Key words: Reconfigurable EM Skins; EM Holography; Reconfigurable Intelligent Surfaces; Next-Generation Communications; Iterative Projection Method; System-by-Design; Metamaterials.
1 Introduction and Rationale

Electromagnetic Skins (EMSs) are currently the core of a theoretical, methodological, and practical revolution within the academic and industrial communities working on wireless communications [1]-[10]. Several research studies on the foundation, the modeling, the simulation, the design, and the test of EMSs are currently under development with a strong interdisciplinary effort combining chemistry, physics, metamaterial science, electromagnetic (EM) engineering, telecommunications, and signal processing expertises [1]-[3][7][8]. As a matter of fact, starting from their early conceptualization as thin metasurfaces able to manipulate the wave propagation beyond Snell’s laws [11], EMSs are considered as one of the key enabling factors of the revolutionary Smart EM Environment (SEME) paradigm in wireless communications [4]-[6][12][13].

Certainly, a multiplicity of methodological and practical challenges [2]-[4][8][10][11][14] still needs to be addressed to have a full transition from traditional wireless systems to the SEME-enhanced ones. In particular, the complexity associated to the design, the fabrication, the implementation, the control, and the integration within a wireless scenario of EMSs is the main critical issue. More specifically, complexity arises at (i) the EMS design level owing to the multi-scale nature of its layout that features micro/nano-scale descriptors combined with meso/macro-scale reflection and communication properties, (ii) the SEME level due to the interactions between the EMSs and the large-scale propagation scenario, and (iii) the “propagation management” level because of the need to fruitfully integrate the EMSs in a heterogenous wireless infrastructure, which includes the base stations, the integrated access and backhaul (IAB) nodes, and the smart repeaters, as well, to yield measurable performance improvements in the overall wireless network.

Within such a framework, the design of planar artificial materials with advanced propagation management capabilities has been recently demonstrated for static passive EM skins (SP-EMS) by exploiting artificial intelligence (AI) techniques within the System-by-Design (SbD) paradigm [5][6][15]. Such an approach leverages on the decomposition of the problem at hand into a source design phase and a subsequent optimization of the surface descriptors of the SP-EMS within the Generalized Sheet Transition Condition (GSTC) framework [5][6][11]. Thanks to the modularity of such a synthesis tool and its multi-scale-oriented nature, the efficient de-
sign of wide-aperture EMSs that enable advanced pattern shaping properties has been carried out [5]-[6] despite the use of extremely simple unit cells.

Otherwise, reconfigurable passive EMSs (RP-EMSs) have been proposed and widely studied to dynamically control the propagation environment for adaptively improving the communication performance [1]-[3][7][16]. Towards this end, RP-EMS unit cells needs either analog (e.g., varactors/varistor [9][14][17] and mechanically-tuned sub-parts [10]) or digitally-controlled (e.g., p-i-n diodes [18]) components. From an applicative viewpoint, the implementation of a continuous control on each RP-EMS cell can yield to very expensive and complex architectures, thus it is generally avoided [19] and the RP-EMS analog states are often discretized using few bits, \( B \), per cell [9][19] or they are implemented by using binary switches [18]. Therefore, RP-EMSs are usually digitally-controlled systems [18]-[20] with relatively limited per-cell degrees-of-freedom (DoFs) when compared to SP-EMSs [5][6]. A key consequence of such a per-cell constraint, mainly when low-bit \( B \rightarrow 1 \) RP-EMSs are at hand [19], turns out to be the very limited control of the shape of the reflected beam [19]. Thus, the mainstream state-of-the-art literature on RP-EMSs has been concerned with the synthesis of RP-EMSs with “simple” anomalous reflection capabilities and narrow beam focusing (i.e., pencil beam-like) [9][18]-[20]. However, demonstrating more advanced footprint control/shaping with a digital RP-EMS would be of great interest in practice since it would allow one to efficiently concentrate the reflected power in arbitrary desired areas (i.e., roads, squares, streets, buildings) and not just in spots. Unfortunately, the approach derived in [5][6] to design SP-EMSs affording shaped footprint patterns cannot be directly applied to RP-EMSs [5][6]. Indeed, the synthesis of the reference surface current, which is performed in the first step of [5][6] and that exploits the non-uniqueness of the associated inverse source (IS) problem to take advantage of the non-radiating currents (NRCs) [5][21], assumes that the unit cell of the corresponding EMS allows a fine tuning of the reflection phase [5][6]. By definition, this is actually prevented when dealing with digital RP-EMSs [19] making the design process ineffective and potentially unable to fulfill complex coverage requirements.

Dealing with RP-EMSs, the objective of this work is twofold. On the one hand, it is aimed at presenting and validating an innovative method for the synthesis (i.e., the design and the
control) of high-performance holographic 1RP-EMSs. On the other hand, it is devoted to prove that minimum complexity RP-EMSs can be used in SEME scenarios to yield complex wave propagation phenomena despite the coarse tuning of the reflection phase. Starting from the design of a meta-atom of the RP-EMS that features only a single-bit reconfiguration and by generalizing the theoretical concepts on complex large-scale EM wave manipulation systems [5][6][22]-[25], the first step of the proposed method for the synthesis of 1RP-EMSs deals with the computation of a discrete-phase current that radiates a field distribution fitting complex footprint patterns. A digital SbD-based RP-EMS optimization is then carried out to set the 1RP-EMS configuration that supports such a reference discrete-phase current. Towards this end, suitable AI paradigms for building reliable and computationally-efficient “RP-EMS digital twins” [5][6][22]-[25] are exploited to properly address the issues related to the multi-scale complexity of the problem at hand.

The outline of the paper is as follows. First, the 1RP-EMS synthesis problem is formulated (Sect. 2), then Sect. 3 details the proposed two-step (i.e., design and control) synthesis method. Representative results from a wide set of numerical experiments are reported for assessment purposes, while comparisons with state-of-the-art techniques [5][6] are considered (Sect. 4). Finally, some concluding remarks follow (Sect. 5).

2 Mathematical Formulation

Let a single-bit RP-EMS (1RP-EMS) be centered in the origin of the local coordinate system \((x, y, z)\) (Fig. 1). The 1RP-EMS consists of \(M \times N\) reconfigurable binary meta-atoms displaced on a regular grid of cells with sides \(\Delta x\) and \(\Delta y\) on a planar region \(\Psi_{EMS} (\Psi_{EMS} = \{-M \times \frac{\Delta x}{2} \leq x \leq M \times \frac{\Delta x}{2}; -N \times \frac{\Delta y}{2} \leq y \leq N \times \frac{\Delta y}{2}\})\). Each \((m, n)\)-th \((m = 1, ..., M; n = 1, ..., N)\) meta-atom is defined by a set of \(U\) geometrical/material descriptors \(g \triangleq \{g^{(u)}; u = 1, ..., U\}\) and it features, at the \(t\)-th \((t = 1, ..., T)\) time step, a binary state \(s_{mn}(t) \in \{0, 1\}\).

The 1RP-EMS at the \(t\)-th \((t = 1, ..., T)\) instant can be univocally identified by the binary micro-scale state vector \(S(t), S(t) \triangleq \{s_{mn}(t); m = 1, ..., M; n = 1, ..., N\}\), and the time-independent (i.e., it is unrealistic to change the atom layout at each time step) micro-scale descriptor vector \(g\). Otherwise, the RP-EMS can be described from an electromagnetic view-
According to the Generalized Sheet Transition Condition (GSTC) technique \[11\][26][27], the instantaneous far field pattern, \(\overline{E}(r, \theta, \varphi; t)\), reflected by the RP-EMS when illuminated by a time-harmonic plane wave at frequency \(f\) impinging from the incidence direction \((\theta^{inc}, \varphi^{inc})\) and characterized by “perpendicular” and “parallel” complex-valued electric field components \(E_{\perp}^{inc}\) and \(E_{\parallel}^{inc}\) is a function of the surface susceptibility vector \(\mathcal{K}\) through the macro-scale induced surface current \(\overline{J}\) (i.e., \(\overline{E}(r, \theta, \varphi; t) = \mathbb{F}\{\overline{J}(x, y; t)\}\)). More in detail, it turns out to that \[5\][6][11][32]

\[
\overline{E}(r, \theta, \varphi; t) = \frac{ik_0}{4\pi} \exp(-jkr) \int_{\frac{M}{2}}^{\frac{M+1}{2}} \int_{\frac{N}{2}}^{\frac{N+1}{2}} \overline{J}(x', y'; t) \times \exp[jk_0(\nu r' \sin \theta \cos \varphi + r'y' \sin \theta \sin \varphi)] \, dx' \, dy'
\]

where the surface current \(\overline{J}\) is given by

\[
\overline{J}(x, y; t) = \hat{\mathbf{r}} \times \left[ \eta_0 \hat{\mathbf{r}} \times \overline{J}^e(x, y; t) + \overline{J}^h(x, y; t) \right] \quad (x, y) \in \Psi_{EMS}
\]

where \(\overline{J}^o, o \in \{e, h\}\), is the electric/magnetic component of the current induced on the RP-EMS, while \(k_0 = 2\pi f \sqrt{\varepsilon_0\mu_0}\) and \(\eta_0 = \sqrt{\varepsilon_0\mu_0}\) are the free-space wavenumber and the impedance, respectively, which depend on the free-space permeability (permittivity) \(\mu_0 (\varepsilon_0)\).

Subject to the local periodicity condition, the dependence of \(\overline{J}^o, o \in \{e, h\}\), on the entries of the micro-scale electric/magnetic surface susceptibility vector \(\mathcal{K}\) (i.e., \(\overline{J}^o(x, y; t) = \mathbb{G}\{\mathcal{K}^o \{g; s_{mn} (t)\}; \overline{E}_{inc}(x, y, 0; t)\}, o \in \{e, h\}\)) is made explicit in the following form \[5\][6][11][27]

\[
\begin{align*}
\overline{J}^e(x, y; t) &= \sum_{m=1}^{M} \sum_{n=1}^{N} \left\{ j2\pi f \varepsilon_0 \left[ \left[ \overline{K}_{mn}(t) \cdot \overline{E}_{mn}(t) \right]_{\tau} - \hat{\mathbf{r}} \times \nabla_{\tau} \left[ \left[ \overline{K}_{mn}(t) \cdot \overline{H}_{mn}(t) \right]_{\nu} \right] \right] \right\} \Omega_{mn}(x, y)
\end{align*}
\]

\[
\begin{align*}
\overline{J}^h(x, y; t) &= \sum_{m=1}^{M} \sum_{n=1}^{N} \left\{ j2\pi f \mu_0 \left[ \left[ \overline{K}_{mn}(t) \cdot \overline{H}_{mn}(t) \right]_{\tau} + \hat{\mathbf{r}} \times \nabla_{\tau} \left[ \left[ \overline{K}_{mn}(t) \cdot \overline{E}_{mn}(t) \right]_{\nu} \right] \right] \right\} \Omega_{mn}(x, y)
\end{align*}
\]

where \(\hat{\mathbf{r}}\) is the outward normal to \(\Psi_{EMS}\), \([\cdot]_{\tau/\nu}\) stands for the tangential/normal component, and \(\Omega_{mn}(x, y) \equiv \{1 \text{ if } -(m-M-1) \times \frac{\Delta x}{2} \leq x \leq (m-M) \times \frac{\Delta x}{2} \text{ and } -(n-N-1) \times \frac{\Delta y}{2} \leq y \leq (n-N) \times \frac{\Delta y}{2}\}\).
less than or equal to \((n - N) \times \frac{\Delta y}{2}; 0\) otherwise\) is the basis function related to the \((m, n)\)-th \((m = 1, ..., M; n = 1, ..., N)\) cell with support \(\Delta \Psi_{\text{EMS}} (\Delta \Psi_{\text{EMS}} \triangleq \Delta x \times \Delta y)\), while \(E_{mn} (\mathbf{P}_{mn})\) is the surface averaged electric (magnetic) field (see Appendix).

Such a derivation points out that the \(t\)-th \((t = 1, ..., T)\) far-field pattern \(\mathbf{E} (r, \theta, \varphi; t)\) can be controlled by properly adjusting the \(M \times N\) binary entries of \(S (t)\), once the 1RP-EMS is designed (i.e., \(g\) is set - Sect. 2.1). Accordingly, the problem at hand can be mathematically formulated as follows

**1RP-EMS Synthesis Problem** - Find the optimal setting of the micro-scale descriptor vector, \(g^{\text{opt}}\), and the optimal configuration of \(T\) binary micro-scale state vectors, \(\{S^{\text{opt}} (t); t = 1, ..., T\}\), such that

\[
\Phi (g, S (t)) = \int_{\Psi_{\text{obs}}} \Re \left\{ F^{\text{des}} (\tilde{x}, \tilde{y}, \tilde{z}; t) - F (\tilde{x}, \tilde{y}, \tilde{z}; t) \right\} d\tilde{x} d\tilde{y} d\tilde{z}
\]

is minimized at each \(t\)-th \((t = 1, ..., T)\) time instant [i.e., \((g^{\text{opt}}, S^{\text{opt}} (t)) = \arg \min_{g, S(t)} [\Phi (g, S (t))]\), \(t = 1, ..., T\)]

In [4], \(\Re \{ . \}\) is the “ramp” function and \(F^{\text{des}} (\tilde{x}, \tilde{y}, \tilde{z}; t)\) is the user-defined power pattern footprint at the \(t\)-th \((t = 1, ..., T)\) time instant in the observation region \(\Psi_{\text{obs}}\), \((\tilde{x}, \tilde{y}, \tilde{z})\) being the \(RP-EMS\) global coordinate system (Fig. 1). Moreover, the footprint pattern is a function of the reflected far-field pattern \(\mathbf{E} (r, \theta, \varphi; t)\), (i.e., \(F (\tilde{x}, \tilde{y}, \tilde{z}; t) = \Re \{ \mathbf{E} (r, \theta, \varphi; t) \}\)) and it is given by

\[
F (\tilde{x}, \tilde{y}, \tilde{z}; t) = \left| \mathbf{E} \left( \sqrt{\frac{x^2 + y^2 + (z - d)^2}{x}}, \text{arctan} \left( \frac{\sqrt{y^2 + (z - d)^2}}{x} \right); t \right) \right|^2
\]

where \(d\) is the 1RP-EMS height over the ground plane (Fig. 1).

It is worth noticing that, unlike the case of \(SP-EMSs\), the synthesis of a 1RP-EMS cannot be done by minimizing [4] only once since there is a different optimal configuration \(S^{\text{opt}} (t)\) for each \(t\)-th \((t = 1, ..., T)\) user-defined footprint pattern, \(F^{\text{des}} (\tilde{x}, \tilde{y}, \tilde{z}; t)\), as pointed out in the following expression

\[
\Phi (g, S (t)) = \int_{\Psi_{\text{obs}}} \Re \left\{ F^{\text{des}} (\tilde{x}, \tilde{y}, \tilde{z}; t) - \Re \left\{ \mathbf{E} \left( \mathbf{K} \{ g; s_{mn} (t) \}; \mathbf{E}^{\text{inc}} (x, y, 0; t) \right) \right\} \right\} d\tilde{x} d\tilde{y} d\tilde{z}
\]

\[1\]
where the link between \( F^{\text{des}}(\tilde{x}, \tilde{y}, \tilde{z}; t) \) and \( S(t) \triangleq \{ s_{mn}(t); m = 1, ..., M; n = 1, ..., N \} \) is made evident. On the other hand, the \( U \) geometrical/material entries of \( g^{\text{opt}} \) must be set once as the optimal trade-off among all \( T \) propagation scenarios.

Furthermore, the problem at hand is more complex than that of a multi-bit \( RP-EMS \) and (even) much more than of a \( SP-EMS \). Unlike the \( SP \) case, the \( t \)-th \( (t = 1, ..., T) \) micro-scale electric/magnetic surface susceptibility vector \( K(t) \) assumes here only a quantized set of states (i.e., \( 2^{M \times N} \)) instead of a continuity of values \([5][6]\). Thus, the macro-scale (reflection) properties of the arising \( EMS \) turn out to be more severely constrained than those of a \( SP-EMS \) or a multi-bit \( RP-EMS \). Consequently, the fulfilment of complex shaping requirements on the footprint power pattern, as those in \([5][6]\), is certainly more difficult and it may results even physically unfeasible.

Taking into account these considerations, the “\( 1RP-EMS \) Synthesis Problem” \((4)\) is then addressed with a two-step approach where, first, the “\( 1RP-EMS \) Design Problem” (Sect. 2.1) is solved by identifying the \( U \) geometrical/material descriptors of the single-bit meta-atom (i.e., \( g \leftarrow g^{\text{opt}} \)), while the second step is aimed at setting, at each \( t \)-th \( (t = 1, ..., T) \) time-instant, the entries of the micro-scale state vector \( S(t) \) to fulfil the footprint pattern requirements [i.e., \( S(t) \leftarrow S^{\text{opt}}(t) \)] (“\( 1RP-EMS \) Control Problem” - Sect. 2.2).

### 2.1 1RP-EMS Design Problem

As for the \( 1RP-EMS \) unit cell design, a key challenge and preparatory step to enable the footprint pattern control (i.e., \( F \rightarrow F^{\text{des}} \)) is the choice of a meta-atom structure whose reflection properties can be suitably modified when its logical state is changed \([11]\). In principle, an optimal trade-off should be found by minimizing \((5)\) with respect to \( g \) across all \( T \) user-requirements \( \{ F^{\text{des}}(\tilde{x}, \tilde{y}, \tilde{z}; t); t = 1, ..., T \} \), while, in this paper, a “worst case”-strategy is adopted to yield a more general and flexible implementation. The design is then carried out by requiring that the \( 1RP-EMS \) meta-atom supports the widest possible reflection variation to account not only the \( T \) propagation scenarios at hand, but more in general the largest range of admissible conditions. According to \([3]\), such a guideline corresponds to the maximization of the gap between the values of the electric/magnetic local surface susceptibility when switching the status of the
generic \((m, n)\)-th \((m = 1, ..., M; \ n = 1, ..., N)\) meta-atom from \(s_{mn} (t) = 0\) to \(s_{mn} (t) = 1\).

Mathematically, this means to minimize the following cost function

\[
\phi (g) = \frac{1}{\pi} \left[ \left( \angle \Gamma_{mn}^\perp (t) \big|_{f = f_0}^{s_{mn}(t)=1} - \angle \Gamma_{mn}^\perp (t) \big|_{f = f_0}^{s_{mn}(t)=0} \right) - \pi \right]^2 + \\
\left( \left| \Gamma_{mn}^{||} (t) \big|_{f = f_0}^{s_{mn}(t)=1} - \Gamma_{mn}^{||} (t) \big|_{f = f_0}^{s_{mn}(t)=0} \right) - \pi \right]^2 
\]

(7)

to yield the optimal set of the geometrical/material descriptors of the single-bit meta-atom, \(g^{opt}\) [i.e., \(g^{opt} = \arg \left( \min_g [\phi (g)] \right)\)]. In (7), \(f_0\) is the central working frequency, \(\angle \cdot\) stands for the phase of the complex argument, and \(\Gamma_{mn}^\perp (t) / \Gamma_{mn}^{||} (t) \) \(\Gamma_{mn}^\perp (t) = \mathbb{Y}^\perp \{g; s_{mn} (t)\}\) and \(\Gamma_{mn}^{||} (t) = \mathbb{Y}^{||} \{g; s_{mn} (t)\}\) are the TE/TM co-polar components of the reflection tensor in the \((m, n)\)-th \((m = 1, ..., M; \ n = 1, ..., N)\) cell, \(\Gamma_{mn} (t)\), while the logical status of the \((m, n)\)-th cell (i.e., \(s_{mn} (t) \in \{0, 1\}\)) is physically implemented by biasing the diodes in the meta-atom layout (Fig. 2).

2.2 1RP-EMS Control Problem

Once the 1RP-EMS has been designed by setting \(g^{opt}\), the computation of \(S^{opt} (t)\) should be performed by minimizing the constrained \((g \equiv g^{opt})\) version of (4)

\[
\Phi (g^{opt}, S (t)) = \int_{\Psi_{obs}} \mathbb{R} \left\{ \mathcal{F}^{des} (x, y, z; t) - \mathbb{H} \left\{ \mathbb{G} \left\{ \mathbb{K} \{g^{opt}, s_{mn} (t)\}; \mathcal{E}^{inc} (x, y, 0; t) \} \right\} \right\} d\tilde{x} d\tilde{y} d\tilde{z} 
\]

(8)
[i.e., \(S^{opt} (t) = \arg \left( \min_S [\Phi (g^{opt}, S (t))] \right)\)], which directly relates the state vector \(S (t)\) with the footprint target \(F^{des} (\tilde{x}, \tilde{y}, \tilde{z}; t)\). However, when dealing with aperiodic wave manipulation devices [5][28]-[31], such a single-phase solution approach is usually avoided in favour of splitting the problem at hand into two parts. The former phase (“Reference Current Computation”) addresses a macro-scale objective that consists in the computation of an ideal equivalent surface current \(\mathcal{J}^{opt} (x, y; t)\) that affords the desired footprint pattern \(F^{des} (\tilde{x}, \tilde{y}, \tilde{z}; t)\), which is coded into the following macro-scale cost function

\[
\Phi (\mathcal{J} (x, y; t)) = \int_{\Psi_{obs}} \mathbb{R} \left\{ \mathcal{F}^{des} (\tilde{x}, \tilde{y}, \tilde{z}; t) - \mathbb{H} \left\{ \mathcal{F} (\mathcal{J} (x, y; t)) \right\} \right\} d\tilde{x} d\tilde{y} d\tilde{z}, 
\]

(9)
to be minimized

\[ \mathcal{J}^{opt}(x, y; t) = \arg \left( \min_{\mathcal{J}(x, y)} \left[ \Phi (\mathcal{J}(x, y; t)) \right] \right). \]  

(10)

The second (microscale) phase (“1RP-EMS Configuration”) \[5\][28]-[31] is devoted to choose the meta-atoms configuration \( \mathcal{S}^{opt}(t) \) that supports the reference current \( \mathcal{J}^{opt}(x, y; t) \) by solving the following optimization problem

\[ \mathcal{S}^{opt}(t) = \arg \left( \min_{\mathcal{S}} [\psi (\mathcal{S}(t))] \right), \]  

(11)

where

\[ \psi (\mathcal{S}(t)) \triangleq \frac{\| \mathcal{J}^{opt}(x, y; t) - \mathbb{G} \{ \mathbb{K} \{ \mathbb{g}; s_{mn}(t) \} ; \mathcal{E}^{inc}(x, y, 0; t) \} \|}{\| \mathcal{J}^{opt}(x, y; t) \|}. \]  

(12)

This two-phase process exploits the fast Fourier relation between currents and patterns [1], which results in very efficient implementations for large apertures \[5\][28]-[31], as well. Moreover, the arising currents can be re-used to design EMS arrangements with different unit cells [31]. Furthermore, the micro-scale synthesis step does not involve here the optimization of \( \mathbb{K}_{mn} \) to achieve ideal susceptibility distributions (which may yield, even in the SP-EMS case [5][6], to non-feasible anisotropy requirements on the cell), but it is aimed at setting the \((m, n)\)-th \((m = 1, ..., M; n = 1, ..., N)\) atom state \( s_{mn}(t) \) that locally minimizes the mismatch with the target surface current.

On the other hand, it has to be noticed that in principle the problem at hand, whatever the solution approach (direct or two-phases), requires the phase of the wave reflected by the meta-atoms to vary over continuous intervals \[5\][28]-[31]. This is clearly not true when dealing with \( B \)-bits RP-EMSs, since each meta-atom can only assume \( 2^B \) states for each \( t \)-th \((t = 1, ..., T)\) time instant. Such a limitation is even more critical for 1RP-EMSs \((B = 1)\). Moreover, despite the two-phase decomposition, the multi-scale and quantized nature of the 1RP-EMS Control Problem still yields to a solution space with a size (i.e., \( 2^{M \times N} \)) that grows exponentially with the RP-EMS aperture.

To take into account these pros & cons, a dedicated strategy needs to be implemented (Sect. 3).
3 Solution Method

While the “IRP-EMS Design” problem (Sect. 2.1) is a quite standard real-variable optimization problem to be addressed with a standard optimization tool, the “IRP-EMS Control” one (Sect. 2.2) turns out to be a new challenge. As a matter of fact, the most intuitive strategy for solving this latter would be that of exploiting the methodology discussed in [5] by simply replacing the model of the local susceptibility dyadics of the SP-EMS with that of the reconfigurable single-bit meta-atom at hand. However, such an approach has a fundamental drawback when applied to the IRP-EMS control. By ignoring the quantized nature of the IRP-EMS surface currents in the “Reference Current Computation” (10), there may not to be an implementable current distribution, $\mathcal{J}(x,y,t) = \mathcal{G} \left\{ \mathcal{K} \{ g; s_{\text{mn}}(t) \}; \mathcal{E}^{E_{inc}}(x,y,0;t) \right\}$, that approximates the synthesized reference current $\mathcal{J}^{opt}$, regardless of the approach to configure the IRP-EMS (11). Therefore, an innovative method is proposed (Sect. 3.1) to compute a “feasible” ideal equivalent surface current $\mathcal{J}^{opt}$ that affords the desired footprint pattern $F^{des}$, while the approach used in [5] for the design of an SP-EMS is customized here to control the IRP-EMS (3.2).

3.1 QIPM-Based Reference Current Computation

In order to define a “feasible” reference current, a quantized version of the iterative projection method (QIPM) is derived.

Let $\mathcal{C}$ be the “IRP-EMS Current Space” composed by the whole set of the IRP-EMS admissible surface currents having the following mathematical form

$$\mathcal{J}(x,y,t) = \sum_{m=1}^{M} \sum_{n=1}^{N} \alpha_{mn}(t) \exp[j\chi_{mn}(t)] \Omega_{mn}(x,y) \hat{\iota}$$

where $\hat{\iota}$ denotes the current polarization, while $\alpha_{mn}(t)$ [$\alpha_{mn}(t) = \mathcal{A} \{ s_{\text{mn}}(t) \}$] and $\chi_{mn}(t)$ [$\chi_{mn}(t) = \mathcal{X} \{ s_{\text{mn}}(t) \}$] are the values of the locally-controlled magnitude and phase of the surface current that belong to the discrete (two-elements) alphabets $\mathcal{A}$ and $\mathcal{X}$, respectively. The elements of $\mathcal{A}$ and $\mathcal{X}$ are the magnitude and the phase of the current that each meta-atom can support when configured in one of its binary states, $s_{\text{mn}}(t) \in \{ 0, 1 \}$, ($\mathcal{A} \triangleq \mathcal{A} \{ s_{\text{mn}}(t) = 0 \}$, $\mathcal{A} \{ s_{\text{mn}}(t) = 1 \}$) and $\mathcal{X} \triangleq \mathcal{X} \{ s_{\text{mn}}(t) = 0 \}$, $\mathcal{X} \{ s_{\text{mn}}(t) = 1 \}$).
Starting from a random initialization of the discrete coefficients \( \alpha_{mn}^{(p)}(t) \) and \( \chi_{mn}^{(p)}(t) \) \((m = 1, \ldots, M; n = 1, \ldots, N)\), whose values are randomly drawn from \( \mathcal{A} \) and \( \mathcal{X} \), the QIPM generates a succession of \( P \) trial current distributions, \( \{ \tilde{J}^{(p)}; p = 1, \ldots, P \} \). First, the footprint pattern \( F^{(p)}(\tilde{x}, \tilde{y}, \tilde{z}; t) \) afforded by \( \tilde{J}^{(p)} \) is computed \([1],[21]\). It is then projected into the corresponding feasibility space through the projection operator \( R^{(p)} \) \([12]\).

\[
R^{(p)}(\tilde{x}, \tilde{y}, \tilde{z}; t) = \begin{cases} 
F^{des}(\tilde{x}, \tilde{y}, \tilde{z}; t) & \text{if } F^{(p)}(\tilde{x}, \tilde{y}, \tilde{z}; t) < F^{des}(\tilde{x}, \tilde{y}, \tilde{z}; t) \\
F^{(p)}(\tilde{x}, \tilde{y}, \tilde{z}; t) & \text{otherwise.}
\end{cases}
\]  

(14)

The QIPM convergence is checked and the iterations are stopped if either \( p = P \) or if the index \( \Xi^{(p)}(t) \) \((\Xi^{(p)}(t) = \frac{\int_{\mathcal{X} \mathcal{A}} |R^{(p)}(\tilde{x}, \tilde{y}, \tilde{z}; t) - F^{(p)}(\tilde{x}, \tilde{y}, \tilde{z}; t)| \, d\tilde{x} \, d\tilde{y} \, d\tilde{z}|}{\int_{\mathcal{X} \mathcal{A}} |F^{(p)}(\tilde{x}, \tilde{y}, \tilde{z}; t)| \, d\tilde{x} \, d\tilde{y} \, d\tilde{z}} \) complies with the convergence condition \( \Xi^{(p)}(t) \leq \Xi^{th} \). If this holds true, the reference current is set to the \( p \)-th estimate, \( \tilde{J}^{opt} = \tilde{J}^{(p)} \).

Otherwise, the minimum norm current, \( \tilde{J}^{(p)}_{MN} \), corresponding to \( R^{(p)}(\tilde{x}, \tilde{y}, \tilde{z}; t) \) is retrieved by means of the truncated singular value decomposition \([5],[21]\).

\[
\tilde{J}^{(p)}_{MN} = F^{-1} \{ \mathcal{H}^{-1} \{ R^{(p)}(\tilde{x}, \tilde{y}, \tilde{z}; t) \} \}.
\]  

(15)

The quantization of the minimum norm current is subsequently carried out by approximating it with the closest element of \( \mathcal{C} \) \((\tilde{J}^{(p+1)} \approx \tilde{J}^{(p)}_{MN}, \tilde{J}^{(p+1)} \in \mathcal{C})\). More in detail, the amplitude and the phase coefficients of \( \tilde{J}^{(p+1)} \) are determined by minimizing the mismatch cost function

\[
\rho(\alpha_{mn}(t); \chi_{mn}(t)) = \left\{ \frac{\left\| \sum_{m=1}^{M} \sum_{n=1}^{N} \alpha_{mn}(t) \exp[j \chi_{mn}(t)] \Omega_{mn}(x,y) \hat{t} - \tilde{J}^{(p)}_{MN} \right\|^2}{\left\| \tilde{J}^{(p)}_{MN} \right\|^2} \right\} \right\}.
\]  

(16)

\( || \cdot || \) being the \( \ell_2 \) norm \(\{(\alpha_{mn}^{(p+1)}(t), \chi_{mn}^{(p+1)}(t)) = \arg min_{\alpha_{mn}(t) \in A} \{ \rho(\alpha_{mn}(t), \chi_{mn}(t)) \}\}\), they are then substituted in \([13]\) to yield \( \tilde{J}^{(p+1)} \). The iteration index is then updated \((p \leftarrow p + 1)\) and the entire QIPM process is restarted from the footprint pattern computation.

It is worth pointing out that, unlike state-of-the-art approaches \([5],[6]\), the operation in \([16]\) outputs an estimated current \( \tilde{J}^{(p+1)} \) that fulfills the feasibility condition, thus it is assured that the current distribution determined at the convergence, \( \tilde{J}^{opt} \), can be surely implemented with a IRP-EMS layout.
3.2 1RP-EMS Configuration Method

By following the guidelines in [5], but here customized to a binary control problem, a SbD-based optimization is carried out to identify the 1RP-EMS discrete micro-scale status \( S_{\text{opt}}^t (t) \) of \( M \times N \) binary entries. Towards this end, a set of \( L \) trial 1RP-EMS configurations

\[
\langle S (t) \rangle \triangleq \{ S_l (t) ; l = 1, ..., L \} \tag{17}
\]

is iteratively processed until either the number of SbD iterations reaches the maximum value \( I \) (\( i = I \), \( i \) being the iteration index) or the feasible reference current distribution \( \overline{J}_{\text{opt}} \), computed in Sect. 3.1 is matched \( \psi (S_{\text{opt}}^t (t)) \leq \psi^{th} \), \( S_{\text{opt}}^t (t) = \arg \min_{l,i} \left[ \psi (S_l^t (t)) \right] \), \( \psi^{th} \) being a user-defined convergence threshold.

Starting from a random initial configuration, \( \langle S_i^t (t) \rangle_{i=0} \), each \( i \)-th \( (i = 1, ..., I) \) iteration consists of the following operations:

- **1RP-EMS Surrogate Modeling** - The set of \( L \) micro-scale electric/magnetic surface susceptibility vectors, \( \langle K^{(i)} (t) \rangle \) (\( \langle K (t) \rangle \triangleq \{ K_l (t) ; l = 1, ..., L \} \)), is predicted with an AI-based technique, featuring an Ordinary Kriging implementation, according to the most recent trends in the surrogate modeling of wave manipulating devices [24][33]. For each \( l \)-th entry of \( \langle K^{(i)} (t) \rangle \), the diagonal tensor of the electric/magnetic local surface susceptibility of the \( (m,n) \)-th \( (m = 1, ..., M; n = 1, ..., N) \) meta-atom, \( \overline{K}_{mn} (t) \), is approximated with its digital-twin (DT), \( \overline{K}_{mn} (t) \approx \overline{K}_{mn}^{DT} (t) \) \( \overline{K}_{mn}^{DT} (t) \triangleq \overline{K}_{mn}^{DT} \{ \mathbf{g}; s_{mn} (t) \} \), which is off-line trained starting from \( V \) full-wave evaluations of the meta-atom response \( \{ \mathbf{g}_v, \ K_{mn}^{u} (t); \mathbb{K} \{ \mathbf{u}_v; s_{mn}^{v} (t) \} \} \); \( v = 1, ..., V \) [24][33];

- **Surface Current Computation** - The distribution of the surface current \( \overline{J}_l (x,y; t) \) induced on the \( l \)-th \( (l = 1, ..., L) \) 1RP-EMS, which is modeled with the surrogate susceptibility vector \( K^{DT}_l (t) \), is computed by setting \( \overline{K}_{mn} (t) = \overline{K}_{mn}^{DT} (t) \) in (3);

- **Surface Current Fitness Evaluation** - The mismatch between \( \overline{J}_{l}^{(i)} (l = 1, ..., L) \) and \( \overline{J}_{\text{opt}} \) is quantified by calculating the value of the micro-scale cost function (12), \( \psi (S_l^{(i)} (t)) \);

- **Guess Current Update** - A new set of 1RP-EMS states, \( \langle S_{i+1}^t (t) \rangle \), is generated by ap-
plying the Genetic-Algorithm (GA) operators \[34\] to the previous guesses, \(\langle S^{(i)}(t) \rangle\), according their fitness values, \(\langle \psi^{(i)}(t) \rangle\) \(\triangleq \{\psi(S^{(i)}(t)) ; l = 1, ..., L\). Unlike \[5\][6], a GA-based optimization is performed due to the binary DoFs of the problem at hand.

4 Numerical Results

This section is aimed at illustrating the synthesis process of IRP-EMSs described in Sect. \[3\] as well as at demonstrating its effectiveness and potentialities. Towards this end, the design of the single-bit meta-atom is first presented along with the full-wave validation of its properties (Sect. 4.1). Afterwards, the IRP-EMS control is assessed through a selected set of numerical experiments (Sect. 4.2). For the full-wave modeling of both the meta-atom and the finite IRP-EMS layouts, the Ansys HFSS \[35\] EM simulator has been used.

4.1 Single-Bit Meta-Atom Design and Validation

Since a key objective of this work is to prove that it is possible to achieve advanced beam shaping properties with minimum-complexity RP-EMSs, the design of the single-bit meta-atom has been carried out according to Sect. 2.1 by also taking into account the following constraints: (i) the meta-atom features a single-layer geometry to minimize the fabrication complexity; (ii) the single-bit \((B = 1)\) reconfigurability of the RP-EMS unit cell is obtained by applying a single bias voltage; (iii) the shape of the layout of the printed cell is very regular to keep its EM behavior independent on the accuracy of the fabrication process; (iv) the IRP-EMS structure works whatever the polarization of the incident field.

The unit cell in \[36\] has been then considered as reference model. It consists of a simple square patch (Fig. 2) with two edges connected to the ground plane through two p-i-n diodes [green rectangles - Fig. 2(a)] and two vias [yellow circles - Fig. 2(a)]. By applying a bias voltage at the center of the patch, the diodes can be either both set to the “ON” \([s_{mn}(t) = 1]\) or both to the “OFF” \([s_{mn}(t) = 0]\) states to implement the single-bit-per-atom reconfigurability.

To operate at the central frequency of the sub-6GHz n78 band \[37\] (i.e., \(f_0 = 3.5\) [GHz]),
such a reference model has been tuned by considering a Rogers RO4350 ($\varepsilon_r = 3.66$, $\tan\delta = 4.0 \times 10^{-3}$) substrate with thickness of $1.524 \times 10^{-3}$ [m] that includes $3.5 \times 10^{-5}$ [m]-thick metallizations and the MACOM MADP-000907-14020 diodes. The values of the $g_{opt}$ entries are listed in Tab. I, while the CAD models of the unit cell and of the switching device are shown in Fig. 2(b) and Fig. 2(c), respectively.

The reflection performance of the optimized meta-atom are illustrated in Fig. 3 for the broadside incidence. More in detail, the plots of the phase [Fig. 3(a)] and the magnitude [Fig. 3(b)] of the $TE/TM$ components of the local reflection tensor $\Gamma_{mn}(t)$ indicate that such a meta-atom supports a $\approx 180$ [deg] phase difference between the “ON” [$s_{mn}(t) = 1$] and the “OFF” [$s_{mn}(t) = 0$] states at $f_0$ [Fig. 3(a)]. Thanks to the symmetry of the layout, the arising unit-cell is insensitive to the polarization [Fig. 3(a)]. Moreover, the losses are limited [$< 4$ [dB] - Fig. 3(b)] and the cross-polarization level is low [$< -18$ [dB] - Fig. 3(b)] within the whole frequency band.

It is finally worthwhile to remark that, while the successive control step (Sect. 2.2) has been performed in this paper with the single-bit cell in Fig. 2, the proposed approach for configuring the 1RP-EMS can be adopted regardless of the working frequency, the number of bits per cell, $B$, and the meta-atom complexity [38].

4.2 Single-Bit RP-EMS Control

To assess the features and the potentialities of the 1RP-EMS control method in Sects. 3.1-3.2 different EMS apertures and target radiation performance have been analyzed by considering a SEME scenario where a 1RP-EMS is placed at $d = 5$ [m] over the ground (Fig. 1), it is illuminated by a base station located along the EMS broadside direction [i.e., $(\theta_{\text{inc}}, \phi_{\text{inc}}) = (0, 0)$ [deg] $\rightarrow \hat{e}_\perp = \hat{y}$ and $\hat{e}_\parallel = \hat{x}$], and it is equipped with a slant +45 [deg] linearly polarized antenna (i.e., $E_{\perp}^{\text{inc}} = E_{\parallel}^{\text{inc}} = 1$). As for the calibration setup of the 1RP-EMS control, the following values have been chosen according to the guidelines in [5][24]: $V = 2 \times 10^4$, $P = 10^2$, $L = 20$, $\Xi^{th} = 10^{-4}$, $\psi^{th} = 10^{-3}$, and $I = 10^4$.

The first experiment is aimed at configuring a $M \times N = 10 \times 10$ 1RP-EMS to maximize the reflected power in a square $\Psi_{\text{cov}}$ of size $10 \times 10$ [m$^2$] located in the global coordinate system.
(Fig. 1) at \((\tilde{x}, \tilde{y}, \tilde{z}) = (25, 30, 0) [m]\), which corresponds to set the desired footprint pattern as follows

\[
F_{\text{des}} (\tilde{x}, \tilde{y}, \tilde{z}; t) = \begin{cases} 
-10 [dB] & (\tilde{x}, \tilde{y}, \tilde{z}) \in \Psi_{\text{cov}} \\
-50 [dB] & (\tilde{x}, \tilde{y}, \tilde{z}) \notin \Psi_{\text{cov}} 
\end{cases}
\]  

(18)

with \(t = T = 1\).

Figure 4(a) shows the behaviour of the macro-scale cost function \(\Phi^{(p)}\) during the QIPM-based process (\(p = 1, \ldots, P\)) for the synthesis of the reference surface current in comparison with that of the IPM technique [5]. As expected, the QIPM does not outperform the IPM in terms of footprint pattern matching (i.e., \(\Phi^{(p)}_{\text{QIPM}} > \Phi^{(p)}_{\text{IPM}}, p = 1, \ldots, P\)) since the former is a constrained version of the latter owing to the binary nature of the meta-atoms and the quantization of the arising current distribution. Indeed, unlike the smoothly varying phase distribution of the IPM [Fig. 4(b)] that ignores any limitation to the phase control, the profile of the phase distribution of the QIPM current turns out to be binarized [Fig. 4(c)]. Such an apparent drawback [Fig. 4(a)] is actually a fundamental advantage of the QIPM when dealing with the subsequent SbD-driven micro-scale state optimization (Fig. 5). As a matter of fact, the plot of the local error \(\sigma (m,n) (m = 1, \ldots, M; n = 1, \ldots, N)\)

\[
\sigma (m,n; t) \triangleq \angle J_{\text{opt}} (x_m, y_n; t) - \angle J (x_m, y_n; t)
\]  

(19)

in approximating the phase of the reference current distribution with the 1RP-EMS in Fig. 5(b) points out that it is more difficult to match the IPM-synthesized one, while the mismatch reduces in the QIPM case [Fig. 5(a)] (i.e., \(3 \leq \sigma^{\text{IPM}} (m,n) \leq 121 [\text{deg}]\) vs. \(0.2 \leq \sigma^{\text{QIPM}} (m,n) \leq 0.45 [\text{deg}]\)), as one can visually notice by comparing the phase profiles of the reference and the synthesized currents [\(J_{\text{IPM}}^{\text{opt}} (x,y; t)\) - Fig. 4(b) vs. \(J_{\text{IPM}} (x,y; t)\) - Fig. 5(c); \(J_{\text{QIPM}}^{\text{opt}} (x,y; t)\) - Fig. 4(c) vs. \(J_{\text{QIPM}} (x,y; t)\) - Fig. 5(d)].

In order to analyze the impact of those results on the coverage performance, the plots of the analytically-computed [Figs. 6(c)-6(d)] and the HFSS-simulated [Figs. 6(e)-6(f)] footprint patterns generated by the IPM [Fig. 6(a)] and the QIPM [Fig. 6(b)] 1RP-EMS in an observation region \(\Psi_{\text{obs}}\) of 75 \times 60 [m\(^2\)] located in front of the RP-EMS are reported.
Despite the relatively small EMS aperture and its very limited (binary) reconfiguration capabilities, the QIPM configuration [Fig. 6(b)] of the IRP-EMS focuses the reflected beam in the desired coverage region $\Psi_{cov}$ (i.e., along a non-Snell direction) better than the IPM one [Fig. 6(a)] with a lower number of sidelobes [Fig. 6(c) vs. Fig. 6(d)]. Moreover, the close fitting between analytically-computed [Figs. 6(c)-6(d)] and HFSS-simulated [Figs. 6(e)-6(f)] patterns proves the accuracy of the analytic prediction of the reflection/focusing properties of the IRP-EMS layout despite the finite EMS aperture and the intrinsic approximations of the analytical model. Such an outcome, which is also in line with the conclusions drawn for the SP-EMS case [5][6], further confirms the reliability of the proposed multi-scale design without the need of recurring to expensive full-wave simulations in the on-line synthesis process as well as its effectiveness to control the macro-scale wave manipulation properties of IRP-EMSs.

When increasing the EMS size $\Psi_{EMS} (M = N = 10 \rightarrow M = N = 30)$ by keeping the same target coverage region $\Psi_{obs}$ and footprint requirements [18], similar considerations to those of the first numerical experiment hold true. For the sake of completeness and analogously to the $M \times N = 10 \times 10$ case, Figures 7-9 illustrate the process for configuring the IRP-EMS by also comparing the QIPM-based approach with the IPM one. Figure 7 deals with the current synthesis, while Figure 8 is concerned with the configuration of the IRP-EMS, and Figure 9 gives the radiated footprint patterns. More in detail, Figure 7(a) shows the iterative QIPM/IPM minimization of the macro-scale cost function (9) to define the reference (phase) current profiles in Figs. 7(b)-7(c) that are approximated [Figs. 7(d)-7(e)] by the SbD-optimized setups [Figs. 8(b)-8(c)] of the IRP-EMS in Fig. 8(a) to afford the footprint patterns in Figs. 9(a)-9(b). Once again, the constrained nature of the QIPM solution [Fig. 7(c)] allows one to better configure [Fig. 8(c)] the single-bit RP-EMS [Fig. 8(a)] for more faithfully fulfilling the target coverage [Fig. 9(b)]. The improved focusing performance of the QIPM-based control are quantified by the value of the footprint coverage index $\gamma$,

$$\gamma \triangleq \frac{W_{cov}}{W_{ext}}$$  \hspace{1cm} (20)

where $W_{\Psi} \triangleq \frac{1}{2\pi_0} \int F(\bar{x}, \bar{y}, \bar{z}; t) \, d\bar{x}d\bar{y}d\bar{z}$ is the power reflected in the $\Psi$ region and $\Psi_{ext} = \Psi_{obs} - \Psi_{cov}$, which is equal to $\gamma^{QIPM} \approx 4.3 \times 10^{-1}$, while $\gamma^{IPM} \approx 3.6 \times 10^{-1}$ [Fig. 10(a)].

In order to give the interested readers a more exhaustive picture of the advantages of using the
QIPM approach instead of the IPM one when dealing with discrete RP-EMSs, Figure 10(a) compares the behavior of $\gamma^{IPM}$ and $\gamma^{QIPM}$ versus the size of the IRP-EMS aperture by reporting the relative index $\Delta \gamma$ ($\Delta \gamma \triangleq \frac{\gamma^{QIPM} - \gamma^{IPM}}{\gamma^{IPM}}$), as well. As it can be observed, the proposed method (Sect. 3) always determines a configuration of the same IRP-EMS that better focuses the reflected power towards the coverage region $\Psi_{cov}$ (i.e., $\gamma^{QIPM} > \gamma^{IPM}$) with a non-negligible improvement of the power efficiency [i.e., $8\% \leq \Delta \gamma \leq 30\%$ - Fig. 10(a)] also when wide apertures are at hand.

For illustrative purposes, the synthesized ON/OFF configurations of the IRP-EMS [Figs. 10(b)-10(e)] and the corresponding footprint patterns (Fig. 11) when $M \times N = 50 \times 50$ [Figs. 10(b)-10(c) and Figs. 11(a)-11(b)] and $M \times N = 200 \times 200$ [Figs. 10(d)-10(e) and Figs. 11(c)-11(d)] are reported, as well. Despite the exponentially increasing complexity of the optimization problem at hand (11) owing to the widening of the discrete solution space [i.e., $8.4 \times 10^{270}$ ($M \times N = 30 \times 30$), $3.7 \times 10^{752}$ ($M \times N = 50 \times 50$), and $1.5 \times 10^{12041}$ ($M \times N = 200 \times 200$)] binary configurations, $S(t)$, the control method in Sect. 3.2 turns out to be very effective in finding the optimal IRP-EMS configuration $S^{opt}(t)_{t=1}^{T}$ whatever the size of $\Psi_{EMS}$, thus improving the beam focusing capabilities of the IRP-EMS [Fig. 10(a)] by fully exploiting the aperture enlargement [Fig. 6(d) ($M \times N = 10 \times 10$) vs. Fig. 9(b) ($M \times N = 30 \times 30$) vs. Fig. 11(b) ($M \times N = 50 \times 50$) vs. Fig. 11(d) ($M \times N = 200 \times 200$)].

The next numerical experiment is concerned with the case of a Multi-Static Reconfigurability ($t \in \{t_1, t_2\}$; $t = 1, ..., T$) and it deals with the installation of a IRP-EMS to alternatively target the wireless coverage of Piazza della Signoria or Piazzale degli Uffizi in Florence (Italy) (i.e., one of the most frequented urban areas in Europe) that consist of a L-shaped wider square, $\Psi_{cov}^{(1)}$, and a narrow adjacent site, $\Psi_{cov}^{(2)}$, where the entrance to the Uffizi museum is located [Fig. 12(a)].

The IRP-EMS, which has been assumed to be placed at $d = 15$ [m] on the building in Fig. 12(b), is requested to switch between the “Signoria+Uffizi” coverage (i.e., $F^{des}(\tilde{x}, \tilde{y}, \tilde{z}; t)_{t=t_1}$ as in (18) by setting $\Psi_{cov} = \Psi_{cov}^{(1)}$) and the “Uffizi” coverage (i.e., $F^{des}(\tilde{x}, \tilde{y}, \tilde{z}; t)_{t=t_2}$ as in (18) by setting $\Psi_{cov} = \Psi_{cov}^{(2)}$).

The configurations of the ON/OFF states of a $M \times N = 30 \times 30$ layout (i.e., $\Psi_{EMS} \approx 1.3 \times 1.3$ [$m^2$]), which afford the $T = 2$ footprint patterns in Figs. 13(c)-13(d), are reported in Figs.
13(a)-13(b). As it can be observed, there are few similarities between the two control maps, $S_{\text{opt}}^{t_{1}}(t)$ and $S_{\text{opt}}^{t_{2}}(t)$, even though the coverage regions at hand, $\{\Psi^{(c)}_{\text{cov}}, (c = 1, \ldots, C; C = 2)\}$, partially overlap [Fig. 12(a)]. Such a behavior is not unexpected due to both the strong non-linearity of the control problem at hand and the binary nature of the control DoFs.

Concerning the distribution of the radiated power pattern, Figure 14 confirms the effectiveness of the synthesized layouts in fulfilling the coverage requirements, the footprint patterns faithfully overlapping the area of interest, despite the irregular geometries of the regions-of-interest and the relatively limited number of reconfigurable states [$\leq 1$ [Kbit] - Fig. 8(a)]. On the other hand, it is also worth noticing that, although the wireless coverage of the “Signoria+Uffizi” area, $\Psi^{(1)}_{\text{cov}}$, is a more challenging problem than that of the “Uffizi” site, $\Psi^{(2)}_{\text{cov}}$, since it requires the 1RP-EMS focuses the reflected power also at very low elevation angles with respect to its location [Fig. 13(c)], the amount of power reflected by the 1RP-EMS is kept almost unaltered (i.e., $\frac{W^{(2)}_{\text{cov}}}{W^{(1)}_{\text{cov}}} \approx 0.78$).

The ability to afford more elaborated/over-constrained footprints by controlling a single-bit RP-EMS has been the assessed with the synthesis of a $M \times N = 100 \times 100$ 1RP-EMS devoted to manipulate the reflected power for matching the “ELEDIA” logo pattern [Fig. 15(b)]. The plot of the full-wave simulated footprint pattern, $F^{\text{opt}}(\bar{x}, \bar{y}, \bar{z}; t)$, in an observation region $\Psi^{(c)}_{\text{obs}}$ of extension $80 \times 40$ [m$^2$] [Fig. 15(b)] proves the reliability of the EMS configuration $S_{\text{opt}}^{t_{1}}(t)$ in Fig. 15(a) to match the coverage requirements on a complex region $\Psi_{\text{cov}}$. The readers are suggested to notice that this test case has been already successfully addressed in [5] with SP-EMSs, but here the DoFs are far less than those available in the SP-EMS case [5].

Finally, the numerical assessment ends with a test on the performance of the proposed 1RP-EMS synthesis method in a scenario that needs a Dynamic Multi-Beam ($C = 3$) Reconfigurability. More in detail, the problem at hand is that of $C = 3$ users, each occupying a coverage region $\Psi^{(c)}_{\text{cov}} (c = 1, \ldots, C)$ of size $10 \times 10$ [m$^2$], that move at different speeds in different directions as sketched in Fig. 16. By still considering the $M \times N = 100 \times 100$ 1RP-EMS aperture, the plots of the footprint pattern [Figs. 17(e)-17(h)] radiated by the corresponding ON/OFF configuration of the EMS [Figs. 17(a)-17(d)] in $T = 4$ subsequent time-instants confirm that
such a technological solution fits the users’ needs without installing multiple \textit{RP-EMS}s or using multi-bit-per-atom reconfiguration schemes.

From a computational perspective and to give some insights on the burden for dynamically managing an \textit{EMS}-driven wireless planning, let us consider that a $t$-th ($t = 1, \ldots, T$) reconfiguration of the $M \times N = 100 \times 100$ \textit{RP-EMS} for the last scenario (Fig. 16) required less than 0.2 [s] to a non-optimized MATLAB implementation of the control algorithm (Sects. 3.1, 3.2) running on a standard laptop equipped with a single-core 1.6 GHz CPU. Such a quite impressive result has been obtained thanks to the profitable integration of the \textit{QIPM} strategy (Sect. 3.1) and the \textit{SbD}-based binary optimization (Sect. 3.2). Moreover, to the best of our knowledge on the state-of-the-art literature on \textit{EMS}s, it turns out that the proposed \textit{EMS} implementation/control can be properly considered as a suitable candidate/tool for the real-time coverage of time-varying wireless scenarios.

\section{Conclusions}

An innovative method for the synthesis of \textit{RP-EMS}s, based on single-bit meta-atoms able to support advanced propagation manipulation features in \textit{SEME} scenarios, has been proposed. The arising multi-scale optimization problem has been addressed by means of a two-step approach starting from the design of a meta-atom that features only a single-bit reconfiguration. First, a discrete-phase current, which radiates a field distribution fitting complex user-defined requirements on the footprint pattern, has been computed. Then, a digital \textit{SbD}-based optimization has been carried out to set the binary configuration of the \textit{RP-EMS} atoms that supports such a reference discrete-phase current.

To the best of the authors’ knowledge on the state-of-the-art literature, the main theoretical and methodological advancements of this work lie in:

\begin{itemize}
  \item the assessment that \textit{RP-EMS} architectures featuring single-bit meta-atoms can allow complex wave manipulations without the need of continuous phase variations \cite{5, 6};
  \item the derivation of an approach for the control of the \textit{RP-EMS} to afford complex footprint shapes and not only pencil beams \cite{9};
\end{itemize}
the non-trivial extension of the synthesis paradigm, adopted so far to synthesize static reflectarrays and \textit{SP-EMS}s \cite{5} \cite{6} \cite{21}, to minimum-complexity \textit{RP-EMS}s by deriving a computationally effective reconfiguration method.

From the numerical validation, the following outcomes can be drawn:

- the \textit{QIPM}-based approach for the definition of the reference currents significantly improves the coverage efficiency with respect to state-of-the-art techniques \cite{5} regardless of the \textit{1RP-EMS} aperture at hand [Fig. 10(a)];

- despite the minimum complexity of the meta-atoms ($B = 1$), the synthesized \textit{RP-EMS}s feature advanced wave manipulation properties in realistic scenarios (Fig. 14) as well as in very complex “demonstrative” cases (e.g., Fig. 15);

- the proposed method turns out to be an enabling tool for multi-beam reconfiguration and/or independent user-tracking through \textit{1RP-EMS} layouts (Fig. 16).

Future works, beyond the scope of this manuscript, will be aimed at assessing the performance of the proposed method when using multi-bit meta-atoms and/or different meta-atom geometries.

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Expression of $E_{mn}(t)$ and $H_{mn}(t)$

The surface averaged fields $E_{mn}(t)$ and $H_{mn}(t)$ can be expressed as \[5\][6][11]

\[
E_{mn}(t) = \frac{\int_{M/2}^{M/2} \int_{N/2}^{N/2} \left\{ \mathbb{T} + \mathbb{P}_{mn}(t) \right\} \cdot \mathcal{E}^{inc}(x, y, 0) \Omega_{mn}(x, y) \, dx \, dy}{2 \times \Delta x \times \Delta y},
\]

and

\[
H_{mn}(t) = \frac{\int_{M/2}^{M/2} \int_{N/2}^{N/2} \left\{ \mathbf{k}^{inc} \times \mathcal{E}^{inc}(x, y, 0) + \mathbf{k}^{ref} \times \mathbb{P}_{mn}(t) \cdot \mathcal{E}^{inc}(x, y, 0) \right\} \Omega_{mn}(x, y) \, dx \, dy}{2 \times \Delta x \times \Delta y \times \eta_0 \times k_0},
\]

respectively, where $\mathbb{P}_{mn}(t)$ is the local reflection tensor in the $(m, n)$-th cell \[5\][6][11] \( \mathbb{P}_{mn}(t) = \mathbb{Y} \{ g; s_{mn}(t) \} \), $\mathcal{E}^{inc}$ is the incident electric field \[32\],

\[
\mathcal{E}^{inc}(x, y, z) \triangleq (E^{inc}_\perp \hat{e}_\perp + E^{inc}_\parallel \hat{e}_\parallel) \exp \left[ -j \mathbf{k}^{inc} \cdot (x \hat{x} + y \hat{y} + z \hat{z}) \right],
\]

where $\mathbf{k}^{inc}$ is the incident wave vector ($\mathbf{k}^{inc} \triangleq -k_0 [\sin (\varphi^{inc}) \cos (\varphi^{inc}) \hat{x} + \sin (\varphi^{inc}) \sin (\varphi^{inc}) \hat{y} + \cos (\varphi^{inc}) \hat{z}]$), $\mathbf{k}^{ref}$ is the corresponding reflected wave vector according to standard plane wave theory \[11\], and $\hat{e}_\perp = \frac{\mathbf{e}_\perp \times \mathbf{k}^{inc}}{|\mathbf{e}_\perp \times \mathbf{k}^{inc}|}$ and $\hat{e}_\parallel = \frac{\mathbf{e}_\parallel \times \mathbf{k}^{inc}}{|\mathbf{e}_\parallel \times \mathbf{k}^{inc}|}$ are the “perpendicular” and “parallel” unit vectors (i.e., TE and TM modes) \[5\][6][11].
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**FIGURE CAPTIONS**

- **Figure 1.** *Problem geometry*. Sketch of the smart EM environment scenario.

- **Figure 2.** *1RP-EMS Design* - Unit cell geometry: (a) top view and 3D CAD model of (b) the meta-atom and zoom on (c) the switching device.

- **Figure 3.** *1RP-EMS Design* - Plots of (a) the phase and (b) the magnitude of the TE/TM components of the local reflection tensor $\Gamma_{mn}$ versus the frequency in correspondence with the “ON” ($s_{mn} = 1$) and the “OFF” ($s_{mn} = 0$) states.

- **Figure 4.** *1RP-EMS Control* (“Square” Footprint, $M = N = 10$, $d = 5$ [m]; $P = 10^2$) - Behaviour of (a) the macro-scale cost $\Phi^{(p)}$ versus the iteration index ($p = 1, ..., P$) and plot of (b)(c) the phase of the reference current, $\{J_{opt}^{pt}(x_m, y_n; \tau); (m = 1, ..., M; n = 1, ..., N)\}$, synthesized with (b) the IPM, $\overline{J}_{IPM}^{opt}$, and (c) the QIPM, $\overline{J}_{QIPM}^{opt}$.

- **Figure 5.** *1RP-EMS Control* (“Square” Footprint, $M = N = 10$, $d = 5$ [m]) - Plots of (a) the distribution of the local error $\sigma_\ell$ [$\ell = n + N \times (m - 1); (m = 1, ..., M; n = 1, ..., N)$]
and of (c)(d) the phase of the current, \( \mathbf{J} (x_m, y_n; t); (m = 1, ..., M; n = 1, ..., N) \), generated by the 1RP-EMS (b) configured with (c) the IPM-based approach, \( \mathbf{J}^\text{IPM} \), or (d) the QIPM one, \( \mathbf{J}^\text{QIPM} \).

- **Figure 6.** 1RP-EMS Control ("Square" Footprint, \( M = N = 10, d = 5 \) [m]) - Plots of (a)(b) the ON/OFF states and the corresponding (c)(d) analytically-computed and (e)(f) HFSS-simulated footprint patterns of (a) the IPM and (b) the QIPM 1RP-EMS.

- **Figure 7.** 1RP-EMS Control ("Square" Footprint, \( M = N = 30, P = 10^2 \)) - Behaviour of (a) the macro-scale cost \( \Phi^{(p)} \) versus the iteration index \( (p = 1, ..., P) \) and plots of the phase of (b)(c) the reference current, \( \{ \mathbf{J}^{\text{opt}} (x_m, y_n; t); (m = 1, ..., M; n = 1, ..., N) \} \), and of (d)(e) the current, \( \{ \mathbf{J} (x_m, y_n; t); (m = 1, ..., M; n = 1, ..., N) \} \), generated by the configured 1RP-EMS in [Fig. 8(a)] when applying (b)(d) the IPM-based approach or (c)(e) the QIPM one.

- **Figure 8.** 1RP-EMS Control ("Square" Footprint, \( M = N = 30, d = 5 \) [m]) - Plot of (a) the 3D model of the RP-EMS and map of (b)(c) the ON/OFF states of the 1RP-EMS yielded with (b) the IPM, \( S^\text{IPM} \), and (d) the QIPM, \( S^\text{QIPM} \), approaches.

- **Figure 9.** 1RP-EMS Control ("Square" Footprint, \( M = N = 30, d = 5 \) [m]) - Plots of the HFSS-simulated footprint pattern radiated by (a) the IPM and (b) the QIPM 1RP-EMS.

- **Figure 10.** 1RP-EMS Control ("Square" Footprint, \( d = 5 \) [m]) - Behaviours of (a) the total, \( \gamma \), and the relative, \( \Delta \gamma \), coverage indexes versus the 1RP-EMS size and maps of (b)-(e) the ON/OFF configurations of the 1RP-EMS synthesized with (b)(d) the IPM and (c)(e) the QIPM when (b)(c) \( M \times N = 50 \times 50 \) and (d)(e) \( M \times N = 200 \times 200 \).

- **Figure 11.** 1RP-EMS Control ("Square" Footprint, \( d = 5 \) [m]) - Plots of the HFSS-simulated footprint pattern radiated by (a)(c) the IPM and (b)(d) the QIPM 1RP-EMS when (a)(b) \( M \times N = 50 \times 50 \) and (c)(d) \( M \times N = 200 \times 200 \).

- **Figure 12.** 1RP-EMS Control ("Signoria+Uffizi" and "Uffizi" Footprints, \( M = N = 30, d = 15 \) [m]) - View of (a) the scenario and of (b) the location of the 1RP-EMS.
• **Figure 13.** *1RP-EMS Control (“Signoria+Uffizi” and “Uffizi” Footprints, M = N = 30, d = 15 [m]; QIPM)* - Plots of (a)(b) the ON/OFF states of the *1RP-EMS* and (c)(d) the corresponding HFSS-simulated footprint patterns when focusing on (a)(c) the “Signoria+Uffizi” area and (b)(d) the “Uffizi” area.

• **Figure 14.** *1RP-EMS Control (“Signoria+Uffizi” and “Uffizi” Footprints, M = N = 30, d = 15 [m]; QIPM)* - Coverage check when dealing with (a) the “Signoria+Uffizi” and (b) the “Uffizi” coverage scenarios.

• **Figure 15.** *1RP-EMS Control (“ELEDIA” Footprint, M = N = 100, d = 15 [m]; QIPM)* - Plots of (a) the ON/OFF states of the *1RP-EMS* and (b) the corresponding HFSS-simulated footprint pattern.

• **Figure 16.** *1RP-EMS Control (Multi-Beam Footprint, M = N = 100, d = 5 [m]; QIPM)* - Users’ trajectories.

• **Figure 17.** *1RP-EMS Control (Multi-Beam Footprint, M = N = 100, d = 5 [m]; QIPM)* - Plots of (a)-(d) the ON/OFF states of the *1RP-EMS* and (e)-(h) the corresponding HFSS-simulated footprint patterns radiated at (a)(e) \( t = 1 \), (b)(f) \( t = 2 \), (c)(g) \( t = 3 \), and (d)(h) \( t = T \ (T = 4) \).

**TABLE CAPTIONS**

• **Table 1.** *IRP-EMS Design - Geometrical descriptors.*
Fig. 1 - G. Oliveri et al., “Multi-Scale Single-Bit RP-EMS Synthesis for ...”
Fig. 2 - G. Oliveri et al., “Multi-Scale Single-Bit RP-EMS Synthesis for ...”
Fig. 3 - G. Oliveri et al., “Multi-Scale Single-Bit RP-EMS Synthesis for ...”
M=N=10, ‘Square’ Footprint

Fig. 4 - G. Oliveri et al., “Multi-Scale Single-Bit RP-EMS Synthesis for ...”
Fig. 5 - G. Oliveri et al., “Multi-Scale Single-Bit RP-EMS Synthesis for ...”
Fig. 6 - G. Oliveri et al., “Multi-Scale Single-Bit RP-EMS Synthesis for ...”
Fig. 7 - G. Oliveri et al., “Multi-Scale Single-Bit RP-EMS Synthesis for ...”
Fig. 8 - G. Oliveri et al., “Multi-Scale Single-Bit RP-EMS Synthesis for ...”
Fig. 9 - G. Oliveri et al., “Multi-Scale Single-Bit RP-EMS Synthesis for ...”
Fig. 10 - G. Oliveri et al., “Multi-Scale Single-Bit RP-EMS Synthesis for ...”
Fig. 11 - G. Oliveri et al., “Multi-Scale Single-Bit RP-EMS Synthesis for ...”
Fig. 12 - G. Oliveri et al., “Multi-Scale Single-Bit RP-EMS Synthesis for ...”
Fig. 13 - G. Oliveri et al., “Multi-Scale Single-Bit RP-EMS Synthesis for ...”
Fig. 14 - G. Oliveri et al., “Multi-Scale Single-Bit RP-EMS Synthesis for ...”
Fig. 15 - G. Oliveri et al., “Multi-Scale Single-Bit RP-EMS Synthesis for ...”
$M=N=100, \bar{z}=0$ [m], Footprint Multibeam

Fig. 16 - G. Oliveri et al., “Multi-Scale Single-Bit RP-EMS Synthesis for ...”
Fig. 17 - G. Oliveri et al., “Multi-Scale Single-Bit RP-EMS Synthesis for ...”
| Parameter | Value [m] |
|-----------|-----------|
| $g_{1}^{\text{opt}} = g_{2}^{\text{opt}}$ | $3.854 \times 10^{-2}$ |
| $g_{3}^{\text{opt}} = g_{4}^{\text{opt}}$ | $2.191 \times 10^{-2}$ |
| $g_{5}^{\text{opt}} = g_{11}^{\text{opt}}$ | $1.616 \times 10^{-4}$ |
| $g_{6}^{\text{opt}} = g_{12}^{\text{opt}}$ | $2.488 \times 10^{-3}$ |
| $g_{7}^{\text{opt}} = g_{13}^{\text{opt}}$ | $3.300 \times 10^{-4}$ |
| $g_{8}^{\text{opt}} = g_{14}^{\text{opt}}$ | $1.777 \times 10^{-3}$ |
| $g_{9}^{\text{opt}} = g_{15}^{\text{opt}}$ | $2.000 \times 10^{-4}$ |
| $g_{10}^{\text{opt}} = g_{16}^{\text{opt}}$ | $6.000 \times 10^{-4}$ |

Table I - G. Oliveri et al., “Multi-Scale Single-Bit RP-EMS Synthesis for ...”