Some comments about a recent paper on the measurement of the general relativistic Lense-Thirring effect in the gravitational field of the Earth with the laser-ranged LAGEOS and LAGEOS II satellites

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Abstract

In this brief note some comments about the results presented in a recently published paper on the measurement of the general relativistic Lense-Thirring in the gravitational field of the Earth are presented. It turns out that, among other things, the authors might have yielded an optimistic evaluation of the error budget because of an underestimation of the impact of the secular variations of the even zonal harmonics of the geopotential. More tests with real data by varying the observational time spans and the magnitudes of $\dot{J}_4$ and $\dot{J}_6$ in the orbital processor’s force models should have been performed in order to correctly address this important issue. Preliminary analytical evaluations point towards a 15-45% error at 1-3$\sigma$ level, respectively.

1 On the adopted observable

In a recently published paper [1], submitted 2 June 2004 and accepted 10 September 2004, a measurement of the general relativistic Lense-Thirring effect in the gravitational field of the Earth with the laser-ranging satellites LAGEOS and LAGEOS II is reported. The claimed total accuracy is 5 – 10%. The observable used in the analysis is the following linear combination of the residuals of the longitudes of the ascending nodes $\Omega$ of LAGEOS and LAGEOS II

$$\delta \hat{\Omega}^{\text{LAGEOS}} + c_1 \delta \hat{\Omega}^{\text{LAGEOS II}} \sim \mu_{\text{LT}}48.2,$$

(1)

in which $c_1 = 0.546$ and $\mu_{\text{LT}}$ is a scaling parameter which is 1 in the Einstein’s General Theory of Relativity and 0 in the Newtonian mechanics. Eq. [1] allows to cancel out all the static and time-dependent contributions of
the first even zonal harmonic coefficient \( J_2 \) of the multipolar expansion of the Earth gravitational potential which represents one of the major sources of systematic error. The terrestrial gravity model adopted in [1] is the recently released GRACE-only EIGEN-GRACE02S model [2].

The possibility of using only the nodes of the LAGEOS satellites in view of the improvements in the Earth gravity field solutions from the CHAMP and GRACE missions was first presented in [3], although not in an analytic and explicit form. Eq. (1) was explicitly published for the first time in [4]. Then, it has been discussed in a number of other papers [5, 6, 7].

Instead, eq. (1) is presented in [1] as an own result of the authors who, not only miss out to correctly cite the appropriate works [3, 4, 5, 6, 7], but improperly refer to [8] (reference 19, pag. 959 of [1]). In that paper, which is almost twenty years old, there is no mention of eq. (1). It is devoted to the well known LAGEOS III mission in which the launch of a LAGEOS-type satellite in an orbit with supplementary inclination with respect to LAGEOS is presented. The goal of that configuration was to cancel out exactly the contributions of all the even zonal harmonics of the geopotential by using the simple sum of the nodes as observable.

2 Some possible criticisms on the error budget

In this Section we will show that the error analysis presented in [1] could be considered, perhaps, too optimistic, mainly with respect to the impact of the gravitational errors which represent the major source of systematic bias when eq. (1) is adopted.

In Section Error assesment, pag. 960 of [1] the authors correctly assess the systematic error due to the static part of uncancelled even zonal harmonics \( J_2^{(0)} \) of geopotential: indeed, it is \( 3 - 4\% \), according to the EIGEN-GRACE02S model. This result agrees with that obtained in [9]. It should be pointed out that these evaluations are at \( 1 - \sigma \) level; at, say, \( 3 - \sigma \) we would get \( 9 - 12\% \). The first number (3%) comes from a root-sum-square calculation while the second number (4%) is the upper bound obtained by simply summing up the individual error terms. The calibrated standard deviations of the even zonal coefficients of EIGEN-GRACE02S have been used. A possible criticism is that only this Earth gravity model has been used in the presented analysis. Using different GRACE-only Earth gravity models, like, e.g. EIGEN-GRACE01S (available at http://op.gfz-potsdam.de/grace/results) and GGM01S [10], and analyzing the scatter of the so-obtained results would have yielded better results in term of confidence and reliability. Moreover,
also the risk of using that particular model which gives just the expected result would have been avoided.

Other problems may arise when the authors show their a priori error analysis for the time-dependent gravitational perturbations (solar and lunar Earth tides, secular trends in the even zonal harmonics of the Earth’s field and other periodic variations in the Earth’s harmonics). Indeed, they claim that, over an observational time span of 11 years, their impact would be 2%. This evaluation is based on reference 30, pag. 960 of [11] which refers to the WEBER-SAT/LARES INFN study; it has nothing to do with the present node-only LAGEOS-LAGEOS II combination. On the contrary, many recent studies [11, 12, 13, 14, 15, 16] mainly focussed on the gravitational part of the error budget in the performed or proposed Lense–Thirring tests with LAGEOS-like satellites are not even included in the attached .doc file which should overcome the unavoidable space limitations posed by the Letter format. Moreover, this estimate may turn out to be optimistic because of the secular variations of the even zonal harmonics \( \dot{J}_\ell \). Indeed, eq. (1) allows to cancel out \( \dot{J}_2 \), but is sensitive to \( \dot{J}_4, \dot{J}_6, \ldots \), as pointed out in [9]. The uncertainties in the \( \dot{J}_\ell \) are still quite large. On the other hand, their impact on the Lense–Thirring measurement grows linearly in time. Indeed, the mismodelled shift, in mas, of eq. (1) due to the secular variations of the uncancelled even zonal harmonics can be written as

\[
\sum_{\ell=2} \left( \dot{\Omega}_{\ell}^{\text{LAGEOS}} + c_1 \dot{\Omega}_{\ell}^{\text{LAGEOS II}} \right) \frac{\delta \dot{J}_\ell}{2} T_{\text{obs}}^2,
\]  

where the coefficients \( \dot{\Omega}_{\ell} \) are \( \partial \dot{\Omega}_{\text{class}} / \partial J_\ell \) and have been explicitly calculated up to degree \( \ell = 20 \) in [13]. It must be divided by the gravitomagnetic shift, in mas, of eq. (1) over the same observational time span

\[
\left( \dot{\Omega}_{\text{LT}}^{\text{LAGEOS}} + 0.546 \dot{\Omega}_{\text{LT}}^{\text{LAGEOS II}} \right) T_{\text{obs}} = 48.2 \text{ mas yr}^{-1} T_{\text{obs}}.
\]

By assuming \( \delta \dot{J}_4 = 0.6 \times 10^{-11} \text{ yr}^{-1} \) and \( \delta \dot{J}_6 = 0.5 \times 10^{-11} \text{ yr}^{-1} \) [18], it turns out that the percent error on the combination eq. (1) grows linearly with \( T_{\text{obs}} \) and would amount to 1% over one year at \( 1 - \sigma \) level. This means

\[1\] On the contrary, a large number of references are devoted to the non-gravitational perturbations which, instead, play a minor role in this case due to the small sensitivity of the LAGEOS nodes to them.

\[2\] The problem of the secular variations of the even zonal harmonics in post-Newtonian tests of gravity with LAGEOS satellites has been quantitatively addressed for the first time in [17]. In regard to the Lense–Thirring measurement with eq. (1), it has been, perhaps, misunderstood in [4].
that, over 11 years, their impact might range from 11% (1-σ) to 33% (3-σ). Alternatively, if we look at the rate, in mas yr\(^{-1}\), these figures must be doubled. Indeed, the mismodelled secular rate due to the \( J_\ell \) is

\[
\sum_{\ell=2} \left( \dot{\Omega}_{\ell}^{\text{LAGEOS}} + c_1 \dot{\Omega}_{\ell}^{\text{LAGEOS II}} \right) \delta \dot{J}_\ell T_{\text{obs}},
\]

which must be divided by the Lense–Thirring secular trend

\[
\dot{\Omega}_{\text{LT}}^{\text{LAGEOS}} + 0.546 \dot{\Omega}_{\text{LT}}^{\text{LAGEOS II}} = 48.2 \text{ mas yr}^{-1}.
\]

This subtle and important point should have been addressed with tests with real data by varying the magnitudes of \( J_2 \) and \( J_4 \) in the force models of the orbital processor over different observational time spans.

Another controversial point is that it is unlikely that the various errors of gravitational origin can be summed in a root-sum-square way because of the unavoidable correlations between the various phenomena of gravitational origin. It would be more conservative to add them. In this case, the \((J^{(0)}_\ell - \dot{J}_\ell)\) error would range from 15% (4%\(\!+\!\!\!11\%\)) at 1\(\!-\!\!\!\sigma\) level to 45% (12%\(\!+\!\!\!33\%\)) at 3\(\!-\!\!\!\sigma\) level over a 11-years long observational time span\(^4\).

The so obtained global gravitational error can be added in quadrature to the non-gravitational error. Even by assuming the 2% authors’ estimate of the time-dependent part of the gravitational error, the upper bound errors would be \(\sqrt{(4 + 2)^2 + 22\%} = 6\%\) at 1-\(\sigma\), \(\sqrt{(8 + 4)^2 + 42\%} = 13\%\) at 2-\(\sigma\) and \(\sqrt{(12 + 6)^2 + 62\%} = 19\%\) at 3-\(\sigma\). Instead, at the end of the Section Total uncertainty, pag. 960 of [1] and in their Supplementary Information .doc file the authors add in quadrature the doubled error due to the static part of the geopotential (the 2\(\times\)4\% value obtained from the sum of the individual error terms), their perhaps optimistic evaluation of the error due to the time dependent part of the geopotential and the non-gravitational error getting \(\sqrt{8^2 + 4^2 + 4^2\%} = 10\%\) at 2-\(\sigma\). On the other hand, in the Supplementary Information .doc file it seems that they triple the 3\% error due to the static part of the geopotential obtained with a root-sum-square calculation and add it in quadrature to the other (not tripled) errors getting \(\sqrt{9^2 + 2^2 + 22\%} \leq 10\%\) at 3-\(\sigma\). These calculations look like tricks to get just a desired value, i.e. 10%.

Finally, it is hard to understand why the authors very often refer to the LAGEOS-LARES proposed experiment and to the related simulations

\[\text{---}
^3\text{Indeed, the normalized slope of the time series is measured.}
\]

\[\text{---}
^4\text{The evaluations of eq. 2 have been used; if eq. 4 is used the upper bounds become 4\% + 22\% = 26\% (1-\(\sigma\)) and 12\% + 66\% = 78\% (3-\(\sigma\)).}
\]
Table 1: Orbital parameters of the existing LAGEOS and LAGEOS II and of the proposed LARES.

| Satellite | \( a \) semimajor axis (km) | eccentricity \( e \) | inclination \( i \) (deg) |
|-----------|-----------------------------|------------------|---------------------|
| LAGEOS    | 12270                       | 0.0045           | 110                 |
| LAGEOS II | 12163                       | 0.0135           | 52.64               |
| LARES     | 12270                       | 0.04             | 70.0                |

and error budgets. It is rather confusing and misleading. The LAGEOS-LAGEOS II combination of eq. (1) is, by construction, designed in order to exactly cancel out the \( J_2 \) term with an approach which can be applied to any orbital configuration given a pair of satellites in different orbits or a pair of different Keplerian orbital elements of the same satellite. On the contrary, the observable originally proposed for the LAGEOS-LARES mission is the simple sum of their nodes. If the orbital parameters of LARES, quoted in Table 1, were exactly equal to their nominal values, all the even zonal harmonics would be exactly cancelled out. Instead, the sum of the nodes would be affected, to a certain extent, by the whole range of the even zonal harmonics of the geopotential due to unavoidable departures from the LARES nominal configuration because of the orbital injection errors and mission design (the eccentricity of LARES would be one order of magnitude larger than that of LAGEOS), i.e. the coefficients of the classical nodal precessions would not be exactly equal and opposite \( \dot{\Omega}_{\ell}^{\text{LAGEOS}} \neq -\dot{\Omega}_{\ell}^{\text{LARES}} \) for \( \ell = 2, 4, 6, 8, \ldots \). In the Supplementary Information .doc file the combination of the nodes of LAGEOS and LAGEOS II of eq. (1) is presented as if it is only slightly different with respect to the sum of the nodes of the originally proposed LAGEOS-LARES configuration, apart from a 18 deg offset in the inclination of LAGEOS II with respect to LARES. The differences in the eccentricities and the semimajor axes, which do play a role [12], have been neglected.

## 3 Conclusions

The main objections to the results presented in [1] can be summarized as follows

- The authors attribute to themselves the combination of eq. (1) used in their analysis and consistently ignore almost all the works of other scientists on the gravitational part of the error budget which, in this case, represents the major source of systematic bias because of the
low sensitivity of the nodes of the LAGEOS satellites to the non–
gravitational perturbations.

- No different GRACE-only Earth gravity models have been used to
  support the error assessment.

- A major point is represented by the impact of the secular variations
  of the uncancelled even zonal harmonics $\dot{J}_4$, $\dot{J}_6$. Their
  mismodelling might induce errors as large as tens percent over an
  observational time span of 11 years, according to preliminary analytic
  analyses. Tests with real data by varying the observational time spans and the
  magnitudes of $\dot{J}_4$, $\dot{J}_6$ in the orbital processor’s force models should
  have been extensively conducted. The authors present a 2% estimate of the
  global time-dependent part of the gravitational error and support it
  with an improper reference to the preparatory study for the WEBER-
  SAT/LARES mission submitted to the Italian National Institute of
  Nuclear Physics (INFN).

- The static and time-dependent gravitational errors should be simply
  added together because of the correlations among the various gravi-
  tational phenomena. Instead, in [1] they are summed in quadrature
  and presented in a way which seems a ad hoc trick just to get a 10%.
  Even by assuming the authors’ 2% estimate of the time-dependent part
  of the gravitational error, the 3-$\sigma$ upper bound error would amount
  to 19%, contrary to the 10% value presented in the Supplementary
  Information .doc file.

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