Chiral Unitary Approach to the $\bar{K}$ Nucleus Interaction and Kaonic Atoms

E. Oset$^1$, S. Hirezaki$^2$, Y. Okumura$^2$, A. Ramos$^3$, H. Toki$^4$ and M.J. Vicente Vacas$^1$

$^1$Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC, Valencia, Spain.
$^2$Physics Department, Nara Women University, Nara, Japan.
$^3$Departament d’Estructura i Constituents de la Materia, Universitat de Barcelona, Spain
$^4$RCNP, Osaka University, Osaka, Japan

Abstract

We review recent work on various topics related to the modification of kaon properties in nuclei. After a brief exposition of the $\bar{K}N$ and $\bar{K}$ nucleus interaction, results from the application to $K^-$ atoms, renormalization of the $f_0$ and $a_0$ scalar resonances in nuclei and $\phi$ decay in the nucleus are shown.

1 Introduction

In this talk I shall be reporting on the properties of antikaons in matter with applications to kaonic atoms, the renormalization of the $f_0(980)$ and $a_0(980)$ scalar meson resonances in nuclei and the $\phi$ decay in nuclei. The starting point is the $\bar{K}N$ interaction which we study using a coupled channel unitary method by means of chiral Lagrangians. The medium modifications are included in the $\bar{K}N$ amplitude and an integration is done over the Fermi sea of occupied nucleons. This selfenergy is then used in the Klein Gordon equation to find out bound states of the kaons in nuclei, which appear in two families, the atomic states and deeply bound nuclear states. The same selfenergy is used in the chiral unitary coupled channel approach which leads to the $f_0(980)$ and $a_0(980)$ resonances in order to investigate the modification of these resonances in nuclei. Finally, the decay modes of the $\phi$ in nuclei and its total width are studied.
2 The properties of the $\bar{K}$ in the nuclear medium

In this section we address the properties of the $\bar{K}$ in the nuclear medium which have been studied in [1]. The work is based on the elementary $\bar{K}N$ interaction which was obtained in [2] using a coupled channel unitary approach with chiral Lagrangians.

The lowest order chiral Lagrangian, coupling the octet of pseudoscalar mesons to the octet of $1/2^+$ baryons, is

$$L_1^{(B)} = \langle \bar{B} i \gamma^\mu \nabla_\mu B \rangle - M_B \langle \bar{B} B \rangle + \frac{1}{2} D \langle \bar{B} \gamma^\mu \gamma_5 \{u_\mu, B\} \rangle + \frac{1}{2} F \langle \bar{B} \gamma^\mu \gamma_5 [u_\mu, B] \rangle,$$

where the symbol $\langle \rangle$ denotes the trace of SU(3) matrices. Expressions for the different magnitudes can be found in [2].

The coupled channel formalism requires to evaluate the transition amplitudes between the different channels that can be built from the meson and baryon octets. For $K^-p$ scattering there are 10 such channels, namely $K^-p$, $\bar{K}^0n$, $\pi^0\Lambda$, $\pi^0\Sigma^0$, $\pi^+\Sigma^-$, $\pi^-\Sigma^+$, $\eta\Lambda$, $\eta\Sigma^0$, $K^+\Xi^-$ and $K^0\Xi^0$. In the case of $K^-n$ scattering the coupled channels are: $K^-n$, $\pi^0\Sigma^-$, $\pi^-\Sigma^0$, $\pi^-\Lambda$, $\eta\Sigma^-$ and $K^0\Xi^-$. At low energies the transition amplitudes can be written as

$$V_{ij} = -C_{ij} \frac{1}{4f_\pi^2} (k^0_j + k^0_i),$$

where $k^0_i,j$ are the energies of the mesons and the explicit values of the coefficients $C_{ij}$ can be found in Ref. [2]. The coupled-channel Bethe Salpeter equation with the kernel (potential) $V_{ij}$ was used in [3] in order to obtain the elastic and transition matrix elements in the $K^-N$ reactions. The diagrammatic expression of this series can be seen in fig. 1. The Bethe Salpeter equations in the center of mass frame read

$$T_{ij} = V_{ij} + V_{il} G_l T_{lj},$$

where the indices $i,l,j$ run over all possible channels and $G_l$ stands for the loop function of a meson and a baryon propagators. Although in the former equation the last term on the right hand side involves in principle the off shell dependence of the amplitudes, it was shown in [3] that the amplitudes factorize on shell in the integral, the off shell part being absorbed into a renormalization of the coupling constant $f_\pi$. 

Figure 1: Diagrammatic representation of the Bethe-Salpeter equation.
As shown in [2] the results obtained lead to quite good agreement with experimental data for cross sections of the different reactions and the mass distribution of the $\Lambda(1405)$ resonance which is seen in the invariant mass distribution of $\pi\Sigma$ produced in some reactions.

One may wonder why the lowest order Lagrangian gives already good results while in principle there should be important contributions from the next to leading order Lagrangians. This was also the case for the meson meson interaction in the s-wave, as shown in [3]. The reason can be seen in [4] where a more general treatment of the problem was done in all channels of the meson meson interaction. This method is the inverse amplitude method (IAM) in coupled channels which uses the $L^2$ and $L^4$ chiral Lagrangians, imposes unitarity exactly and makes a chiral expansion of the real part of the inverse of the T matrix. The order $O(p^4)$ contribution to the T matrix $T_4$ contains the loop function with two matrices at order $O(p^2)$, $T_2$, and the loop function of two mesons, plus the polynomial contribution $T_{P4}$ which comes from the $L^4$ chiral Lagrangian. The loop function is divergent and must be regularized, with either dimensional regularization, cut off, etc. The total $T_4$ contribution sums these two pieces and is independent of the cut off. This means that we can make a trade off with the cut off and the $T_{P4}$ contribution such as to minimize the contribution of the latter. If then this contribution can be neglected, the IAM method leads to the Bethe Salpeter equation with $T_2$ playing the role of the potential. This is possible for the s-wave, both in meson meson and in the $\bar{K}N$ interaction, but can not be used for p-waves where explicit resonances [5], or alternatively the $L^4$ Lagrangians must be considered [4].

In order to evaluate the $\bar{K}$ selfenergy, one needs first to include the medium modifications in the $\bar{K}N$ amplitude, $T_{\alpha\text{eff}}^\alpha (\alpha = \bar{K}p, \bar{K}n)$, and then perform the integral over the nucleons in the Fermi sea:

\[
\Pi_{\bar{K}}(q^0, \vec{q}, \rho) = 2 \int \frac{d^3p}{(2\pi)^3} n(\vec{p}) \left[ T_{\text{eff}}^{\bar{K}p}(P^0, \vec{P}, \rho) + T_{\text{eff}}^{\bar{K}n}(P^0, \vec{P}, \rho) \right], \quad (4)
\]

The values $(q^0, \vec{q})$ stand now for the energy and momentum of the $\bar{K}$ in the lab frame, $P^0 = q^0 + \varepsilon_N(\vec{p})$, $\vec{P} = \vec{q} + \vec{p}$ and $\rho$ is the nuclear matter density.

We also include a p-wave contribution to the $\bar{K}$ self-energy coming from the coupling of the $\bar{K}$ meson to hyperon-nucleon hole $(YN^{-1})$ excitations, with $Y = \Lambda, \Sigma, \Sigma^*(1385)$. The vertices $MBB'$ are easily derived from the $D$ and $F$ terms of Eq. (1). The explicit expressions can be seen in [1]. At this point it is interesting to recall three different approaches to the question of the $\bar{K}$ selfenergy in the nuclear medium. The first interesting realization was the one in [1], where the Pauli blocking in the intermediate nucleon states in fig.1 induced as a results a shift of the $\Lambda(1405)$ resonance to higher energies and a subsequent attractive $\bar{K}$ selfenergy. The work of [1] introduced a novel an interesting aspect, the selfconsistency. Pauli blocking required a higher energy to produce the resonance, but having a smaller kaon mass led to an opposite effect, and as a consequence the position of the resonance was brought back to
the free position. Yet, a moderate attraction on the kaons still resulted, but weaker than anticipated from the former work. The work of [1] introduces some novelties. It incorporates the selfconsistent treatment of the kaons done in [9] and in addition it also includes the selfenergy of the pions, which are let to excite $\Delta h$ components. It also includes the mean field potentials of the baryons. The obvious consequence of the work of [1] is that the spectral function of the kaons gets much wider than in the two former approaches because one is including new decay channels for the $\bar{K}$ in nuclei. This can be seen in fig. 2. The work of [1] leads to an attractive potential around nuclear matter density and for kaons close to threshold of about 40 MeV and a width of about 100 MeV.

### 3 Kaonic atoms

In the work of [10] the kaon selfenergy discussed above has been used for the case of kaonic atoms, where there are abundant data to test the theoretical predictions. One uses the Klein Gordon equation and obtains two families of states. One family corresponds to the atomic states, some of which are those already measured, and which have energies around or below 1 MeV and widths of about a few hundred KeV or smaller. The other family corresponds to states which are nuclear deeply bound states, with energies of 10 or more MeV and

![Kaon spectral function at several densities.](image-url)
widths around 100 MeV. In fig. 3 we can see the results obtained for shifts and widths for a large set of nuclei around the periodic table. The agreement with data is sufficiently good to endorse the fairness of the theoretical potential. A best fit with a strength of the potential slightly modified around the theoretical values can lead to even better agreement as shown in [11] and serves to quantify the level of accuracy of the theoretical potential, which is set there at the level of 20-30 per cent as an average. The curious thing is that there are good fits to the data using potentials with a strength at $\rho = \rho_0$ of the order of 200 MeV [12]. As shown in [10], the results obtained there and those obtained using the potential of [12] are in excellent agreement for the atomic states. The differences in the two potentials appear in the deeply bound nuclear states. The deep potential provides extra states bound by about 200 MeV, while the potential of [1] binds states at most by 40 MeV. This remarkable finding can be interpreted as saying that the extra bound states, forcing the atomic states to be orthogonal to them, introduces extra nodes in the wave function and pushes the atomic states more to the surface of the nucleus, acting effectively as a repulsion which counterbalances the extra attraction of the potential. This observation also tells that pure fits to the $K^-$ atoms are not sufficient to determine the strength of the $K^-$ nucleus potential. Other solutions with even more attraction at $\rho = \rho_0$ are in principle possible, provided they introduce new states of the deeply bound nuclear family. On the other hand, the work of [11] also tells us that at least an attraction as the one provided by the theoretical potential is needed.
4 Scalar mesons

Having the selfenergy of the kaons under control one can tackle new problems where the kaon interaction with the medium is a necessary ingredient. One of such cases is the modification of the $f_0(980)$ and $a_0(980)$ resonances in the nuclear medium. As shown in [3] and [4] one can obtain a good description of the scalar resonances within the context of the chiral unitary approach. In particular, in [3] it is possible to generate them using the lowest order Lagrangian and the Bethe Salpeter equation, as also done for the $\bar{K}N$ interaction in section 2. Here the coupled channels are $K\bar{K}$ and $\pi\pi$ for the $I=0$ channel, where the $f_0$ resonance appears, and $K\bar{K}$ and $\pi\eta$ in the $I=1$ channel where the $a_0$ resonance appears. In this case we must introduce in the Bethe Salpeter equation the proper selfenergies of pions, kaons and eta, and in addition one has to include also vertex corrections which cancel the off shell part of the vertices [13, 14]. The explicit calculations are done in [15] and we show the results in figs. 4 and 5 for different densities. For the case of $I=1$ we have shown the modulus squared of the $\eta\pi$ amplitude, since this is what would appear in the reaction where one looks at the invariant mass of the $\eta\pi$ system in the final state. What we observe in the figures is that for the case of the $a_0$ resonance the shape becomes gradually wider as the density increases and at densities of around $\rho_0$ the resonance is already washed away in the medium. The case of the $f_0$ resonance, which is already narrower to begin with, is more hopeful, since even at $\rho_0$ one still can see the resonance shape, however, the width passes from a free value around

![Graph](image.png)

Figure 4: Modulus squared of the $\pi\eta$ scattering amplitude. Solid line, free amplitude; long dashed line, $\rho = \rho_0/8$; short dashed line, $\rho = \rho_0/2$; dotted line, $\rho = \rho_0$; dashed dotted line, $\rho = 1.5\rho_0$. 
40 MeV to about more than 150 MeV. How to observe these changes might not be so easy. In the talk of [16] it was shown that using pions or protons as probes it was rather difficult. Using photoproduction of two pions, and πη photoproduction, might give better chances [17]. Other possibilities would come from radiative decay of the φ in nuclei where recent experiments show a clean $f_0$ and $a_0$ peak [18, 19], which incidentally can be very well described within the approach followed here to generate the resonances [20] and a similar one where a separable meson meson potential is used rather than the amplitudes provided by the chiral Lagrangians [21].

5 φ decay in nuclei

Finally let us say a few words about the φ decay in nuclei. The work reported here [22] follows closely the lines of [23, 24], however, it uses the updated $\bar{K}$ selfenergies of [1]. In the present case the φ decays primarily in $K\bar{K}$, but these kaons can now interact with the medium as discussed previously. For the self-energy of the $K$, since the $KN$ interaction is not too strong and there are no resonances, the $t\rho$ approximation is sufficient. In fig. 6 we show the results for the φ width at $\rho = \rho_0$ as a function of the mass of the φ, separating the contribution from the different channels. What we observe is that the consideration of the $s$-wave $\bar{K}$-selfenergy is responsible for a sizeable increase of the width in the medium, but the $p$-wave is also relevant, particularly the Λ$h$ excitation and the Σ$^*h$ excitation. It is also interesting to note that the vertex corrections [25].
(Yh loops attached to the $\phi$ decay vertex) are now present and do not cancel off shell contributions like in the case of the scalar mesons. Their contribution is also shown in the figure and has about the same strength as the other p-wave contributions. The total width of the $\phi$ that we obtain is about 22 MeV at $\rho = \rho_0$, about a factor two smaller than the one obtained in [23, 24], yet, the important message is the nearly one order of magnitude increase of the width with respect to the free one. We are hopeful that in the near future one can measure the width of the $\phi$ in the medium, from heavy ion reactions or particle nucleus interactions, although it will require careful analyses as shown in [26] for the case of $K\bar{K}$ production in heavy ion collisions, where consideration of the possibility that the observed kaons come from $\phi$ decay outside the nucleus leads to nuclear $\phi$ widths considerably larger than the directly observed ones.

6 Summary

In summary we have reported here on recent work which involves the propagation of kaons in the nuclear medium. All them together provide a test of consistency of the theoretical ideas and results previously developed and reported here. If we gain confidence in those theoretical methods one can proceed to higher densities and investigate the possibility of kaon condensates in neutron stars [27]. The weak strength of our $\bar{K}$ potential would make however the phenomenon highly unlikely. On the other hand, we can also extract some conclusions concerning the general chiral framework: 1) The chiral Lagrangians have much information in store. 2) Chiral perturbation theory allows one to extract some of this information. 3) The chiral unitary approach allows one to extract much more informa-
tion. 4) These unitary methods combined with the use of standard many body
techniques are opening the door to the investigation of new nuclear problems
in a more accurate and systematic way, giving rise to a new field which could
be rightly called "Chiral Nuclear Physics". As chiral theory becomes gradually
a more accepted tool to deal with strong interactions at intermediate energies,
chiral nuclear physics is bound to follow analogously in the interpretation of old
and new phenomena in nuclei.

Acknowledgments

This work is partially supported by DGICYT contract numbers PB98-1247 and
PB96-0753, and by the EEC-TMR Program under contract No. CT98-0169.

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