Connecting cosmological accretion to strong Lyα absorbers

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ABSTRACT

We present an analytical model for the cosmological accretion of gas onto dark matter halos, based on a similarity solution applicable to spherical systems. Performing simplified radiative transfer, we compute how the accreting gas turns increasingly neutral as it self-shields from the ionising background, and obtain the column density, NHI, as a function of impact parameter. The resulting column-density distribution function (CDDF) is in excellent agreement with observations. The analytical expression elucidates (1) why halos over a large range in mass contribute about equally to the CDDF as well as (2) why the CDDF evolves so little with redshift in the range z = 2 → 5. We show that the model also predicts reasonable DLA line-widths (σ0), bias and molecular fractions. Integrating over the CDDF yields the mass density in neutral gas, ΩHI, which agrees well with observations. ΩHI(z) is nearly constant even though the accretion rate onto halos evolves. We show that this occurs because the fraction of time that the inflowing gas is neutral depends on the dynamical time of the halo, which is inversely proportional to the accretion rate. Encapsulating results from cosmological simulations, the simple model shows that most Lyman-limit system and damped Lyman-α absorbers are associated with the cosmological accretion of gas onto halos.

Key words: galaxies: high-redshift, intergalactic medium, quasars: absorption lines

1 INTRODUCTION

UV-photons emitted by a cosmologically-distant source, such as for example a quasar, may be absorbed en route by scattering off neutral Hydrogen atoms in the Lyman (n = 1 → n’) or Lyman-limit (n = 1 → ∞) transitions (n and n’ are the Hydrogen atom’s principle quantum number). This generates a series of absorption lines due to intervening ‘Lyman-α’ clouds. The spectral signature when many lines are close together and overlap is that of a ‘forest’ of lines, hence the name ‘Lyman-α’ forest (Weymann et al. 1981). The taxonomy of these lines depends on how column-density, NHI, is determined. When NHI is sufficiently small, absorption lines have a nearly Gaussian shape which can be fit accurately with a Voigt profile. Lines become increasingly square in shape and insensitive to NHI with increasing NHI, but once NHI ≥ 10^{17.2} cm^{-2}, the Lyman-limit optical depth is so high that the presence of the absorber results in a strong depression of the flux below (1 + z) × 912Å, where z is the absorber’s redshift. Finally at even higher values of NHI ≥ 10^{20.3} cm^{-2}, absorption is so strong that the line-shape can be measured far from the line centre where it is dominated by the natural line-width of the Lyman-α transition. The Lorenzian shape of this natural line-profile leads to such high NHI absorbers to be called ‘damped Lyman-α absorbers’ (DLAs), in analogy with the Lorenzian shape of the resonance of a damped harmonic oscillator (Beaver et al. 1972; Wolfe et al. 1986). Lines are therefore classified based on column-density as Lyman-α forest, Lyman-limit system (LLSs), and DLAs for NHI < 10^{17.2}, < 10^{20.3}, and ≥ 10^{20.3} cm^{-2}, respectively (see e.g. Rauch 1998; Péroux & Howk 2020, for reviews). This paper focuses on DLAs, foraying briefly into the realm of LLSs.

The original motivation for studying DLAs by Wolfe et al. (1986) was to identify high-redshift galaxies. Indeed, a sight-line through the Milky Way’s disk is likely to have NHI ≥ 10^{20.3} cm^{-2}, therefore a sight-line through a high-z Milky Way-like galaxy would likely result in a DLA in the spectrum of a background quasar - and, of course, high-redshift DLAs were already known and detectable back then. This realisation inadvertently initiated decades of relatively unsuccessful attempts to identify optical counterpart(s) to DLAs (with some exceptions, e.g. Møller et al. 2004; Fynbo et al. 2010; Péroux et al. 2012; Fumagalli et al. 2015). The low success rate of such searches was attributed to the DLA galaxies being generally too faint to be detectable (see e.g. the discussion by Krogager et al. 2017). This endeavour has been revolutionised by the advent of integral field units (ifu’s) which allow for the identification of faint galaxies at a range of distances from the sight-line to the quasar in a single telescope pointing (e.g. Péroux et al. 2016 using SINFONI, Fumagalli et al. 2017; Mackenzie et al. 2019 using MUSE). Such studies suffer from the opposite problem: there
may be be several galaxies detected close to the sight-line at similar redshift as the DLA, in which case it becomes unclear which particular galaxy - if any - to associate with the DLA.

The prevailing view that DLAs are gas closely associated with a galaxy was also the motivation for several theoretical models (e.g. Maller et al. 2001; Fynbo et al. 2008; Di Gioia et al. 2020; Krogager et al. 2020). Typically, these struggle to reproduce the observed number density of DLAs for a realistic value of size of the galaxy. In the relatively few cases where a galaxy is identified with a particular DLA, it is not unusual for the impact parameter to be \( b \gtrsim 10 \) kpc (and sometimes much larger than that, e.g. Fumagalli et al. 2017), and that is of course (much) larger than the sizes of (stellar components of) high-z galaxies, which are closer to \( \sim 3 \) kpc even for massive galaxies (e.g. van der Wel et al. 2014).

DLAs are relatively rare, with of order \( \sim 0.3 \) DLA per decade in column-density per redshift interval \( \Delta z = 1 \) at \( z = 3 \). Statistically robust studies of DLAs, especially at high \( N_{\text{HI}} \), have been revolutionised by the advent of very large samples of quasar spectra (see for example DLAs identified by Noterdaeme et al. 2012 in the Sloan Digital Sky Survey (SDSS) data release 9, Ahn et al. 2012, or by Ho et al. 2020 in SDSS DR12, Alam et al. 2015). Where earlier numerical simulations (e.g. Nagamine et al. 2004; Tescari et al. 2009) and cosmological zoom simulations (e.g. Pontzen et al. 2008) reproduced the earlier data by Péroux et al. (2001): Prochaska et al. (2005) relatively well, the leap in data brought about by the SDSS made it harder for simulations to reproduce the data.

Nevertheless, several relatively recent simulation can reproduce the vastly improved DLA statistics reasonably well (e.g. Krogager et al. 2011; Cen 2012; Bird et al. 2014). The simulations differ in numerical technique (particle based, grid based, moving mesh, respectively) and vary the prescriptions associated with the formation of the galaxy (both how stars form and importantly in the way that feedback is implemented.) What these simulations have in common is that the majority of the DLA cross section is not associated with the stellar component of the galaxy, but rather the gas that gives rise to the DLA is distributed throughout and even outside the galaxy’s halo, (‘every galaxy is a DLA, but not every DLA is a galaxy’), typically in the form of several filaments of high density gas, as also seen in the high-resolution zoom simulations of Fumagalli et al. (2011): Faucher-Giguère & Kereš (2011). In the simulations presented by van de Voort et al. (2012), the majority of the gas associated with LLS and DLAs at redshift \( z \sim 3 \) is falling rapidly towards the central galaxy of a dark matter halo while remaining cool (temperature \( T \lesssim 10^5 \text{K} \)). As it accretes onto the galaxy, the majority of gas is ejected in the form of a galactic outflow powered by supernovae. Gas inside the galaxy that is neither inflowing nor outflowing and the outflow itself adds a small contribution to the total DLA cross section.

Two aspects of these simulations form the motivation for the analytical model described here. Firstly, the DLA gas is mostly accreting towards the centre of dark matter halos (van de Voort et al. 2012), therefore our model is based on cosmological accretion (rather than assuming DLA gas is in centrifugal or hydrostatic equilibrium in a halo as in the model by Padmanabhan et al. 2016, or is associated directly with galaxies, as in the recent models by Krogager et al. 2020; Di Gioia et al. 2020). We will assume that the accretion is mostly smooth (as opposed to accretion via satellites, say), an idea supported by simulations (e.g. Crain et al. 2017; Wright et al. 2020), at least at higher \( z \gtrsim 1 \), say. Both assumptions are consistent with the ‘cold accretion’ paradigm of Keres et al. (2005): Dekel et al. (2009). Secondly, although the importance of feedback from the galaxy on DLAs is somewhat contested (e.g. Sommer-Larsen & Fynbo 2017; Rhodin et al. 2019), the simulations by Altay et al. (2013) that are based on the OWLS project described by Schaye et al. (2010), include a very wide variety of feedback implementations and show convincingly that outflows driven by feedback affect DLA statistics mainly at the high column-density end, \( N_{\text{HI}} \lesssim 10^{18} \text{cm}^{-2} \). This encourages us to simply neglect feedback, either from outflows, or indeed from local ionising radiation which may play at role at the high column-density end (e.g. Rahmati et al. 2013). Obviously, this is only an approximation. As stressed by e.g. Krogager et al. 2017, correlations between DLAs and galaxies depend on metallicity: since we neglect feedback, we cannot study this interesting observational finding. A third motivation for developing an analytical model is to elucidate why the DLA column-density distribution (CDDF) evolves so weakly with redshift (as observed and reproduced by simulations) and why halos with a range of masses contribute about equally to the CDDF. Having an analytical expression for the CDDF makes this straightforward.

This paper is organised as follow: the model is described in section 2, which starts by discussing the radial distribution of gas accreting onto halos and goes on to derive the resulting CDDF. Section 3 discusses in more detail the validity of some of the assumptions and where improvements could be made. Finally, section 4 summarizes. We will use the Planck Collaboration et al. (2014) values of the cosmological parameters were needed.

2 THE MODEL

Our model for connecting DLAs to dark matter halos has several ingredients and makes many simplifying assumptions. It is described in §2.1, which starts by relating the radial and column-density distribution of neutral gas to a halo of a given mass at a given \( z \), and explores the differential contribution of halos of a given mass to the CDDF. We explain there why a relatively extended range in halos masses contributes significantly to the CDDF. Next, we calculate the CDDF in §2.2 and use the analytical expression to explore its redshift evolution. The dynamics of the gas is examined in terms of DLA bias in §2.3 and in terms of line-widths in §2.4. We explore the molecular contents of DLAs in §2.5, and the cosmological fraction of gas in DLAs in §2.6.

2.1 Cosmological accretion onto halos

The equations describing cosmological accretion onto a spherically symmetric over-density associated with a dark matter halo admits a similarity solution in the case of an Einstein-de Sitter (EdS) universe\(^1\) (Bertschinger 1985). In

\(^1\) For studying structure formation at redshift \( z \gtrsim 1 \), an EdS model should be sufficiently accurate.
The cosmological growth of a dark matter halo is described by

\[ M_h(z) = M_{h,0} m_h(z); \quad m_h(z) = (1 + z)^a \exp(-bz), \]

where \( M_h \) is the virial mass of the halo at redshift \( z \), \( M_{h,0} \) is the mass of the halo at \( z = 0 \), and \( a \approx 0.24 \) and \( b \approx 0.75 \) are fits by Correa et al. (2015a,b) to simulations.

Given the total hydrogen profile, we proceed to compute the neutral hydrogen density profile, \( n_{H}(r) \). The ionizing radiation from the UV-background will photo-ionize the intergalactic medium (IGM) as well as gas in the outskirts of the halo. As the gas gets denser, the optical depth to the ionizing radiation increases, and eventually hydrogen starts to self-shield. To describe this, we assume that (1) hydrogen is in photo-ionization equilibrium at a constant temperature of \( T = 10^4 K \), (2) collisional ionization can be neglected, (3) the presence of helium and of all other elements can be neglected, (4) the frequency dependence of the photo-ionization cross-section can be neglected, and (5) the optical depth can be approximated by its value in a slab (rather than spherical) geometry. Under these assumptions, of 2.2. The steeper profile leads to an apparent divergence of the enclosed mass at the centre. However, in reality, the power-law profile ceases to hold inside the accretion shock. Such a normalization should be approximately consistent with the similarity solution.

\[ 200 \times \rho_c \]

This has a minor effect which can just be noticed outside the virial radius of the \( M_h = 10^{13} M_\odot \) halo in Fig. 1.

\[ 3 \]

See Zheng & Miralda-Escudé 2002; Sykes et al. 2019 for calculations in spherical geometry.

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2 We will use a subscript \( h \) to denote properties of a halo, and subscript \( H \) to denote properties that refer to hydrogen.

3 The factor 200/3 that relates \( \rho_h \) to the value of the critical density, \( \rho_c \), assumes that \( \alpha = 2 \) rather than our preferred value of 2.2. The steeper profile leads to an apparent divergence of the enclosed mass at the centre. However, in reality, the power-law profile ceases to hold inside the accretion shock. Such a normalization should be approximately consistent with the similarity solution.

4 This has a minor effect which can just be noticed outside the virial radius of the \( M_h = 10^{13} M_\odot \) halo in Fig. 1.

5 See Zheng & Miralda-Escudé 2002; Sykes et al. 2019 for calculations in spherical geometry.
the neutral hydrogen fraction, \( x \), at radius \( r \), is given by

\[
x(r) \equiv \frac{n_{\text{HI}}(r)}{n_{\text{H}}(r)} = \frac{\alpha_B}{\Gamma_0} n_{\text{HI}}(r)
\]

\[
\Gamma(r) = \Gamma_0 \exp(-\tau(r))
\]

\[
\tau(r) = \sigma_{n_{\text{HI}}} \int r(r') n_{\text{HI}}(r') \, dr', \tag{3}
\]

where \( \alpha_B \) is the ‘case-B’ recombination coefficient evaluated at a temperature of \( T = 10^4 \text{K} \), \( \sigma_{n_{\text{HI}}} \), the photo-ionization cross-section at the ionization threshold, and \( \Gamma_0 \) the value of the photo-ionization rate in the IGM, for which we will take the values as a function of redshift computed by Haardt & Madau (2012). Rather than performing the integral for the optical depth to infinity, we integrate towards \( r = 10 \times R_h \), setting \( \Gamma = \Gamma_0 \) at \( r = 10 \times R_h \). Since the gas is highly ionised outside \( R_h \), the exact value taken for this integration limit is mostly irrelevant, provided it is larger than \( R_h \). Obtaining the run of \( x \) versus \( r \) numerically is straightforward, it is also easy to understand the result: in the outskirts of the halo the gas is highly ionized so that \( \tau \approx 0 \). Once \( \tau \approx 1 \), the reduction in ionizing flux \( \exp(-\tau) \) increases very rapidly with decreasing \( r \), hence the neutral fraction increases rapidly from very small to \( x \approx 1 \) over the narrow extent of the ionization front. Inside the front, \( x \approx 0 \). Therefore we expect that

\[
\begin{cases}
 x(r) \approx (\alpha_B/\Gamma_0) n_{\text{HI}}(r) \to n_{\text{HI}}(r) \approx (\alpha_B/\Gamma_0) n_{\text{HI}}(r)^2 \text{ for } r > r_I \\
 x(r) \approx 1 \to n_{\text{HI}}(r) \approx n_{\text{HI}}(r), \text{ for } r < r_I,
\end{cases}
\tag{4}
\]

where \( r_I \) is the location of the ionization where \( \tau \approx 1 \). The value of \( r_I \) can be estimated in the slab geometry by using the first expression for the neutral hydrogen density, and solving \( \sigma_{n_{\text{HI}}} \int \frac{R_h}{r} n_{\text{HI}}(r') \, dr' = \tau_I = 1 \) for \( r_I \), as we discuss in more detail below.

The run of total density and neutral hydrogen density obtained by numerically integrating Eq. (1) together with Eq. (3) for halos with a range of mass is shown at redshift \( z = 3 \) in panel (a) of Fig.1, where the solid lines show \( n_{\text{HI}}(r) \) and the dotted lines \( n_{\text{H}}(r) \); different colours correspond to different halo masses. As expected, \( n_{\text{HI}} \approx n_{\text{HI}} \) close to the centre, and \( n_{\text{HI}} \approx n_{\text{HI}}^\beta \) in the outskirts: the inner most red-hydrogen density (Bahcall & Peebles 1969 or see for example Fumagalli et al. (2011), §5.1). Therefore the number of times a sight-line intersects an absorber with a given column-density, \( n(z) \), to a halo with a given mass per dex in halo mass, is given

\[
r^{-\beta} \text{ (where } \beta = \alpha \text{ for } r < r_I \text{ and } \beta = 2 \text{ for } r > r_I),
\]

we expect that approximately

\[
N_{\text{HI}}(b) \approx 2 n_0 b^{1-\beta} R_h^2 \int_0^\infty \frac{1}{(1+y^2)^{3/2}} \, dy \approx b^{1-\beta} R_h^2,
\tag{6}
\]

that is, also a power-law in \( b \), with the dependence on halo mass a power-law as well, since \( R_h \propto M_h^{1/3} \). Here, \( n_0 = n_{\text{HI}} \) with \( x \) the neutral fraction at the virial radius for \( r > r_I \), and \( x = 1 \) for \( r < r_I \), as in Eq. (4). \( N_{\text{HI}}(b) \), integrated numerically, is shown in panel (b) of Fig. 1 for the halos shown in panel (a); the power-law relation captures the shape of the \( N_{\text{HI}} \) profile well, as expected. Notice also that \( N_{\text{HI}} \) increases rapidly by a factor of several hundred at the location of the ionization front, \( r = r_I \), where \( x \) changes rapidly from \( x \ll 1 \) to \( x = 1 \).

We estimate the approximate location of the ionization front, \( r_I \), by assuming that the gas outside \( r_I \) is highly ionised as in Eq. (4). Integrating from \( r_I \) outwards, the optical depth to infinity is

\[
\tau_I = \sigma_{n_{\text{HI}}} \int_{r_I}^{\infty} n_{\text{HI}}(r) \, dr = \frac{\sigma_{n_{\text{HI}}} \alpha_B}{\Gamma_0} \int_{r_I}^{\infty} n_{\text{HI}}(r)^2 \, dr
\]

\[
= \frac{\sigma_{n_{\text{HI}}} \alpha_B}{\Gamma_0} \int_{r_I}^{\infty} n_{\text{HI}}(r)^2 \frac{R_h}{r} \, \frac{R_h}{r_I} \, R_h \sim 1,
\tag{7}
\]

so that

\[
r_I = \left( \frac{\sigma_{n_{\text{HI}}} \alpha_B}{\Gamma_0} \frac{R_h}{(2a-1)\Gamma_0} \right)^{1/(2a-1)} R_h.
\tag{8}
\]

The locations corresponding to this value of \( r_I \) are indicated in Fig. 1 using filled diamonds. Defined in this way, the column-density measured outwards from \( r_I \) to infinity is \( N_{\text{HI}} = \tau/\sigma_{n_{\text{HI}}} \) with \( \tau = 1 \), corresponding to \( \log N_{\text{HI}}(\text{cm}^{-2}) \approx 17.2 \). The characteristic column-density of Lyman-limit systems. However, the column-density of the halo for \( b = r_I \) is of course a bit higher because of the different geometry of the two sight lines: radially outwards for the definition of \( r_I \) versus hitting the absorber at a given impact parameter for an absorber.

We can use the \( N_{\text{HI}} - b \) relation to compute the cross-section, \( \sigma(N_{\text{HI}}) \), within which the column density is larger than some value, \( \sigma = \pi b^2 \propto N_{\text{HI}}^{2(1-\beta)} \). Its derivative, \( d\sigma/dN_{\text{HI}} \), is closely related to the shape of the CDFD, \( f(N_{\text{HI}}) \).

To see this, from the average number of times, \( dN \), that a sight-line with proper length \( dl \) intersects absorbers with proper cross-section \( \sigma \) and co-moving number-density \( n(z) \),

\[
dN = n(z) (1+z)^3 \sigma \, dl = n(z) (1+z)^3 \sigma \, e \, dz
\]

\[
= \frac{c}{H_0} n(z) \sigma \, dX
\]

\[
dX = \frac{H_0 (1+z)^2}{H(z)} \, dz,
\tag{9}
\]

where \( H(z) \) is the Hubble constant at redshift \( z \), \( H_0 = H(z = 0) \) and the dimensionless quantity \( dX \) is called the ‘absorption distance’; the factor \( (1+z)^3 \) converts the co-moving density of absorbers, \( n(z) \), to a proper number density (Bahcall & Peebles 1969) or see for example Fumagalli et al. (2011), §5.1). Therefore the number of times a sight-line intersects an absorber with a given column-density, due to a halo with a given mass per dex in halo mass, is given

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by
\[ g(M_h, N_{HI}) \equiv \frac{d^3N}{dM_{HI}d\log M_h dX} = \frac{c}{H_0} \frac{dn}{d\log M_h} \frac{d\sigma(M_h, N_{HI})}{dN_{HI}}. \] (10)

In this paper we use the \textsc{colossus python} package described by Diemer (2018) to compute the number density of halos per dex in halo mass (the halo mass function) \( dn/d\log M_h \), stipulating a cosmology with the cosmological parameters from Planck Collaboration et al. (2014). We selected the Reed et al. (2007) fitting function to the mass function as implemented in \textsc{colossus}.

The \textit{CDFD} - the number of absorbers with a given column-density per absorption distance \( dX \) - is the integral of the function \( g \) over halo mass,
\[ f(N_{HI}) \equiv \frac{d^2N}{dX dN_{HI}} = \int_{-\infty}^{\infty} g(M_h, N_{HI}) d\log M_h. \] (11)

The contribution of halos of given mass to the \textit{CDFD}, \( g(M_h, N_{HI}) \), is plotted in panel (c) of Fig. 1. Its dependence on column-density is due to the factor \( d\sigma(M_h, N_{HI})/dN_{HI} \propto N_{HI}^{-2/(\beta-1)}-1 \), with the power-law approximation shown by red-dashed lines. The power-law shape of \( g \) is therefore a consequence of the power-law profile of the density in the halos, shown in panel (a), with the change in slope a consequence of the transition from fully neutral in the centre of the halos, so that \( n_{HI}(r) \propto n_{HI}^0(r) \), to the highly ionized regime in the outskirts, where \( n_{HI}(r) \propto n_{HI}^2(r) \). The inflection around \( N_{HI} \sim 10^{20}\text{cm}^{-2} \) occurs when the value of the impact parameter is (approximately) equal to \( r_I. \)

It is striking how at higher values of \( N_{HI} \), \( N_{HI} \sim 10^{21}\text{cm}^{-2} \) say, the contribution of halos is almost independent of halo mass over 2 orders of magnitude in \( M_h \). The underlying reason for this is as follows: firstly, we will neglect the contribution of the ionised halo gas to the column-density - this is an excellent approximation at sufficiently high \( N_{HI} \). Therefore, at impact parameter \( b \ll r_I \), we find that
\[ N_{HI}(M_h, b, z) \approx 2 n_{HI,0}(z) R_h^2(M_h, z) b^{1-\alpha} I \]
\[ I \equiv \int_0^{(r_I(z)/b)^{2-1/2}} \frac{dx}{(1 + x^2)\alpha/2} \] (12)

where we have explicitly written-out the redshift dependence for later use. We can solve this equation for the value of the impact parameter \( b \) at which a sight-line through a halo of mass \( M_h \) has column-density \( N_{HI} \) or greater, and calculate the corresponding cross section \( \sigma \equiv \pi b^2 \),
\[ \sigma(N_{HI}, M_h, z) = \pi (2 n_{HI,0}(z))^{2/(\alpha-1)} R_h^{2/(\alpha-1)} \times \frac{N_{HI}^{-2/\alpha} b^{2/(\alpha-1)}}{M_h^{2/(\alpha-1)}}. \] (13)

To gain insight and obtain a simpler analytical scaling relation, we further neglect the somewhat awkward dependence of \( \sigma \) on the integral \( I \), in which case \( d\sigma/dN_{HI} \propto R_h^{2/(\alpha-1)} \propto M_h^{2/(\alpha-1)} \), where we have used the fact that \( n_{HI,0} \) depends on redshift but not on \( M_h \). Finally, we find that the contribution of halos of mass \( M_h \) to the \textit{CDFD} depends on halo mass as
\[ g(M_h, N_{HI} \sim 10^{21}\text{cm}^{-2}) \propto \frac{dn}{d\log M_h} \frac{d\sigma}{dN_{HI}} \]
\[ \approx M_h^{-1.1} M_h^{2\alpha/(3(\alpha-1))} \approx M_h^{0.13}, \] (14)

where we have taken a value of \( -1.1 \) for the slope of the halo mass function at low masses at \( z = 3 \), and our default value of \( \alpha = 2.2 \) for the slope of the density distribution within a given halo. The contribution of halos to the \textit{CDFD} is thus a slowly increasing function of \( M_h \) at high \( N_{HI} \), increasing by a factor of \( \sim 2 \) for \( M_h \) increasing by a factor of 100 from \( M_h = 10^{10}M_\odot \) to \( 10^{12}M_\odot \). At even higher mass, the contribution eventually drops due to the exponential factor in the halo mass function. The scaling exhibited by Eq. (14) explains the weak dependence of the \textit{CDFD} on halo mass. We will verify the scaling in more detail numerically below, including the impact of the integral \( I \).

We could now integrate Eq. (10) over halo mass to obtain the \textit{CDFD}, as in Eq. (11). Before doing so, we should be more careful in considering for which halos our model assumptions hold. The ionizing background will photo-evaporate gas from halos below some critical mass, \( M_{crit}(z) \), and such halos will not contribute to the \textit{CDFD}. Here, we will use the values of \( M_{crit}(z) \) obtained using numerical simulations by Okamoto et al. (2008). At high halo masses, \( M_h \gg 10^{12}M_\odot \), on the other hand, a lot of the halo gas is presumably shock heated rather than cold, in which case we should not neglect collisional ionization and additionally the radial density profile is likely to be affected. Fortunately above \( z \sim 2 \), such halos are relatively rare and their contribution to the integral in Eq. (11) is already exponentially suppressed due to the shape of the halo mass function, meaning we may not need to further suppress their contribution.

We will therefore compute the \textit{CDFD} as
\[ f(N_{HI}) \equiv \frac{d^2N}{dX dN_{HI}} = \int_{\log M_{crit}(z)}^{\infty} g(M_h, N_{HI}) d\log M_h, \] (15)

where in practice we integrate up to halos of mass \( 10^{13}M_\odot \). There remains one further subtlety regarding the definition of \( M_h \). To compute the number density of halos, \( dn/d\log M_h \), we use the \textsc{colossus} routines of Diemer (2018), which are fits to dark matter only (\textsc{dmo}) simulations. In such simulations, the mass of a halo, \( M_h \), is the sum of the dark matter plus baryon mass. Therefore when comparing to hydrodynamical simulations, we should not compare the dark matter halo mass to \( M_h \) but rather \( \Omega_m/\Omega_{dm} \times \) the dark matter mass (with, of course, \( \Omega_m \) and \( \Omega_{dm} \) the mean cosmological mass and dark matter density in units of the critical density). Of course in reality, outflows of baryons due to feedback from star formation slow the growth in mass of a halo (e.g. Sawala et al. 2015), a process we will neglect in this analytical investigation. We are now in a position to compute the \textit{CDFD} and study its evolution.

### 2.2 The \textit{CDFD} and its redshift evolution

Numerically integrating Eq. (15) yields the \textit{CDFD} with the result plotted in panel (a) of Fig. 2 at four redshifts. The model (coloured curves for different redshifts) is compared to the results from a hydrodynamical simulation at \( z = 3 \)
Figure 2: Panel (a): The model CDFD function \( f(N_{\text{HI}}, z) \), at various redshifts plotted in solid lines. Over plotted is \( f(z = 3) \) for the owl simulations (Schaye et al. 2010) presented by Altay et al. (2011) (yellow line connecting diamonds) and \( f(z = 2.5) \) for DLAs measured by Noterdaeme et al. (2012) (yellow squares) and sub-DLAs from Zafar et al. (2013) (yellow circles), which both fall approximately on top of the yellow simulations line. The inset shows the logarithm of the ratio of model over simulation at \( z = 3 \). Panel (b): \( f(N_{\text{HI}}, z) \) at \( z = 2 \) and \( z = 5 \) for the reference model (solid lines). A variation on the reference model in which the critical mass, \( M_{\text{crit}} \) (below which halo gas is photo-evaporated and no longer contributes to \( f \)) does not evolve, \( M_{\text{crit}}(z) = M_{\text{crit}}(z = 2) \), is shown as dotted lines connecting diamonds (this variation overlaps with the reference model at \( z = 2 \)). Another variation in which the amplitude of the ionising background, \( \Gamma \), is multiplied by a factor of 3 at all \( z \) is shown as dot-dashed lines connecting crosses.

Figure 3: Dependence of the contribution of halos with given mass, \( \log(M_h/M_\odot) = 9, 10, 11 \) and 12 shown in red, green, yellow and blue, respectively) at a given column-density \( \log(N_{\text{HI}}/\text{cm}^{-2}) = 19.3, 20.3 \) and 21.3 as dotted, thick solid, and dashed, respectively) to the CDFD as a function of redshift, from Eq. (10). Panel (a): cross-section, \( d\sigma/dN_{\text{HI}} \), as a function of \( z \). Panel (b): dark matter halo number density, \( dN/d\log M_h \), as a function of redshift. Panel (c): net contribution \( g(N_{\text{HI}}, M_h, z) \) of halos of mass \( M_h \) to the CDFD as column density \( N_{\text{HI}} \), as a function of redshift. Cyan dash-dotted lines in each panel indicates the redshift scaling as annotated. Panels (a) and (b) show that the evolution of cross section and number density is relatively weak, as expected (see text). The trends are also in the opposite direction, leading to even weaker evolution of \( g \), as shown in panel (c).

(yellow line connecting diamonds) from the owls project (Schaye et al. 2010), post-processed with radiative transfer using the urchcl ray-tracing code described by Altay & Theuns (2013), as presented by Altay et al. (2011). Also shown as a yellow diamonds with error bars is the observed CDFD of DLAs at a mean redshift of \( \langle z \rangle \approx 2.5 \) as measured by Noterdaeme et al. (2012) from the sdss DR9 (Ahn et al. 2012), and the ‘sub-DLAs’ measured by Zafar et al. (2013) at \( z \approx 3 \) (yellow circles). Simulation and model are in excellent agreement with the data (better than 50%) above the DLA threshold, \( N_{\text{HI}} > 10^{20.3} \text{cm}^{-2} \). The model also agrees with the simulation at lower columns although not as well, with in particular the transition from mostly neutral absorbers (i.e. DLAs) to absorbers that are highly ionized in the model (sub-DLAs, \( N_{\text{HI}} \sim 10^{19–20.3} \text{cm}^{-2} \) and Lyman-limit systems, \( N_{\text{HI}} \geq 10^{17.2} \text{cm}^{-2} \) more abrupt than in the simulation or in the data. However the overall agreement at higher column densities between the simple model and the simulations, as plotted in the inset of Fig. 2a, is encouraging. We note that the latter include molecule formation but the model does not (but see §2.5).

What determines the shape of the ‘knee’ feature at \( N_{\text{HI}} \sim 10^{20} \text{cm}^{-2} \)? As illustrated in Fig. 1, when the impact parameter \( b \) becomes smaller than \( r_r \), the radius of the ionization front, the column density rapidly increases by \( \sim 2 \) orders of magnitude for a small change in \( b \). For a given halo,
We therefore examine the predicted evolution of the function $g(N_{\text{HI}}, M_h) \propto (d\sigma/dN_{\text{HI}}) (dn/d\log M_h)$ of Eq. (14) in more detail, starting from Eq. (13) for the evolution of the cross section, $d\sigma/dN_{\text{HI}}$. Using $\sigma = \pi b^2$, we solve Eq. (13) for the value of the impact parameter $b$ for a sightline through a halo of mass $M_h$ to have a given column density $N_{\text{HI}}$. Keeping $M_h$ and $N_{\text{HI}}$ both constant, and neglecting the contribution from the integral $I$, the redshift dependence is due to the evolution of $n_{\text{HI},0}$ - the (total) hydrogen density at the edge of the halo, and $R_h$ - the virial radius of a halo of given mass $M_h$. This yields for the net redshift dependence of $\sigma$ and its derivative with respect to $N_{\text{HI}}$

$$
\frac{d\sigma}{dN_{\text{HI}}} \propto n_{\text{HI},0}^{2/(\alpha - 1)} \pi b^2 \propto (1 + z)^{6/(\alpha - 1)} (1 + z)^{-2\alpha/(\alpha - 1)} \propto (1 + z)^{2(3-\alpha)/(\alpha - 1)}.
$$

This scaling is a result of halos becoming larger at later times, $R_h \propto (1 + z)^{-1}$, but less dense, $n_{\text{HI},0} \propto (1 + z)^{3}$. The density dependence is stronger and the cross section (at a given column density and given mass) decreases with time.

However the (co-moving) number density of halos with mass $M_h$, $dn/d\log M_h$, increases with time. In the Press-Schechter approximation, the increase is proportional to the linear growth rate, $D(z)$, which to a good approximation is $D(z) \approx 1/(1 + z)$ in the redshift range $z = 2 \rightarrow 5$. However, in the mass-range of interest, $9 \leq \log M_h/M_\odot \leq 13$, the evolution is slightly stronger, more like $dn/d\log M_h \propto (1 + z)^{-1.7}$.

The opposite evolution of the cross section and the number density of halos yields for the net evolution for the contribution of halos with given $M_h$ to the CDDF at a given (self-shielded) column,

$$
g(N_{\text{HI}}, z) \propto \frac{dn}{d\log M_h} \frac{d\sigma}{dN_{\text{HI}}} \propto (1 + z)^{-1.7} (1 + z)^{2(3-\alpha)/(\alpha - 1)} \approx (1 + z)^{-0.4}.
$$

Using the value at $z = 3$ as a pivot point, $g(z = 2)/g(z = 3) \approx 1.1$, and $g(z = 5)/g(z = 3) \approx 0.86$. Therefore as time progresses, more halos of a given mass $M_h$ appear at lower $z$ but the cross section of individual halos decreases. These two factors nearly compensate each other, resulting in weak evolution of the function $g$ and explaining the weak evolution in the column-density distribution of DLA, $f(N_{\text{HI}})$.

These analytical estimates are tested in more detail in Fig. 3. In panel (a) of that figure, we show the evolution of the cross section, $d\sigma/dN_{\text{HI}}$ for halos with various masses (coloured lines), and for column densities of $\log N_{\text{HI}}[\text{cm}^{-2}] = 19.3$ (dotted), 20.3 (solid) and 21.3 (dashed lines). Evolution is relatively weak, following approximately the evolution $d\sigma/dN_{\text{HI}} \propto (1 + z)^{2(3-\alpha)/(\alpha - 1)}$ shown as the dot-dashed cyan line, for all $M_h$ and values of $N_{\text{HI}}$ shown.

In panel (b) we show the evolution of the co-moving number density, $dn/d\log M_h$, computed using COLOSSUS (Diemer 2018). To guide the eye, we over plot as the dot-dashed cyan line, the scaling $\propto (1 + z)^{-1.7}$, which captures at least approximately the evolution of the lower mass halos, underestimating the evolution for $\log M_h/M_\odot = 12$. Finally, in panel (c) we plot the evolution of the contribution of halos of given mass and given column density, $g(N_{\text{HI}}, M_h)$. Given that the
in the opposite direction, the evolution in trends in cross section and number density are weak and in the opposite direction, the evolution in $g$ is even weaker, following reasonable well the trend $\propto (1+z)^{-0.4}$ predicted earlier for self-shielded column-density systems, and shown as the dot-dashed cyan line.

It is also worth exploring the evolution of the covering factor, the ratio $\sigma/(\pi R^2_h)$, which follows from Eq. (13) and is plotted in Fig. 4. The DLA cross section (in proper kpc) increases slowly with redshift at constant halo mass $M_h$, and is approximately $\propto M_h$, as can be seen in the upper panel. Because the virial radius decreases with $z$, the covering factor increases faster with $z$ than the cross section (lower panel). The values of the covering factor and their evolution are in very good agreement with those computed by Rahmati et al. (2015) for galaxies in the Eagle simulation (Schaye et al. 2015) with $M_h > 10^{12} M_\odot$ (filled red diamonds). Another comparison is with the high resolution simulations of Faucher-Giguère & Kereš (2011) which yield a covering factor of $\sigma_{\rm DLA}/(\pi R^2_h)$ of a $\sim 3$ - 10 percent for $M_h \sim 2 - 3 \times 10^{11} M_\odot$ at $z = 2$ - values comparable to what we find here.

According to Eq.(13), $\sigma \propto R^{2\alpha/(\alpha-1)} \propto M^{2\alpha/(3\alpha-1)} \approx M_h^{1.22}$. We have multiplied the values of $\sigma$ for each halo by $(10^{12} M_\odot/M_h)^{1.22}$ and over plotted them on Fig.4 as filled diamonds. The fact that these symbols fall nearly on top of the $M_h = 10^{12} M_\odot$ values demonstrates that this scaling works very well. The value of the exponent $\beta \sim 1.22$ in $\sigma \propto M_h^{1.22}$ is a bit larger than found in the numerical simulations presented by Bird et al. (2014) (who finds that $\beta < 1$ depending on the feedback strength in their simulations) and more comparable to that inferred from the observed DLA bias by Font-Ribera et al. (2012); Pérez-Rafols et al. (2018), who prefer $\beta \sim 1.1$. With $\beta = 1.22$, the DLA covering factor scales as $\sigma/R^2_h \propto M_h^{0.56}$ in our model.

Two other trends are worth noting in panel (c) of Fig. 3. Firstly, at high column densities ($\log N_{\rm HI} [\text{cm}^{-2}] = 21.3$), halos of mass $M_h/M_\odot = 10 - 12$ contribute about equally to $g$ (see the dashed lines). As discussed before, the dependence of $g$ on $N_{\rm HI}$ is weak at high column, see Eq. (14). Secondly, the dependence becomes stronger at lower columns, with lower mass halos contributing significantly more particularly at higher $z$ (contrast the dotted and the solid lines).

The second observation brings us back to the shape of the knee in $f(N_{\rm HI})$. At higher $z$, lower mass halos contribute relatively more to $f(N_{\rm HI})$ per decade in halo mass. In addition, the value of $M_{\rm crit}$ - the halo mass below which gas is evaporated from halos by photo-heating by the ionising background - is lower at higher $z$, meaning lower mass halos contribute even more to $f$ at $z = 5$ compared to $z = 2$. The result is a noticeable increase in the number density of absorbers slightly below the DLA threshold moving the location of the knee to lower column densities: this increase is due to an increase in the contribution of lower mass halos. The consequence of this is best appreciated by comparing $f(N_{\rm HI}, z)$ at $z = 5$ (red lines) versus at $z = 2$ (blue line) in panel (a) of Fig. 2: there are considerably more sub-DLAs (with $N_{\rm HI}$ slightly below $10^{20.3}\text{cm}^{-2}$ at $z = 5$ compared to $z = 2$. Such a change in shape is also apparent in simulations (see e.g. Fig. 1 in Rahmati et al. (2015), see also Sykes et al. in prep.). The data also suggest an increased abundance of sub-DLAs with increasing $z$ (Zafar et al. 2013).

To illustrate the impact of $M_{\rm crit}$ on the shape of the knee, we plot in panel (b) of Fig. 2 the cddf, $f(N_{\rm HI}, z)$, for $z = 2$ and $z = 5$, comparing the default model in which $M_{\rm crit}$ evolves (solid lines) with a variation in which the critical mass is kept fixed at its value at $z = 2$ (dotted lines connecting diamonds). These models obviously fall on top of each other at $z = 2$, but the shape of the knee for the $z = 5$ curves changes significantly - resulting in a large decrease in the number density of sub-DLAs. The panel also shows another model variation, in which $M_{\rm crit}$ evolves as before, but the amplitude $\Gamma$ of the ionising background is multiplied by a factor of 3 at all $z$ above its value in the default model (that of Haardt & Madau (2012); this variation is shown

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8 The simulations by Faucher-Giguère et al. 2015 that include feedback are claimed to give a far higher covering factor of 0.2-0.4 for such halos.
biasing factors. It was possible to constrain \( g \) robustly by observations at lower columns, for example in the Lyman-\( \alpha \) forest (e.g. Becker & Bolton 2013), then evolution of the shape of the knee could be used to test models for the evolution of \( M_{\text{crit}} \). In fact, it is likely easier to constrain robustly the evolution of \( g \) rather than its amplitude at any \( z \) - this might be enough to test whether \( M_{\text{crit}} \) evolves as expected. Other effects may also play a role in shaping the knee, for example the temperature of the absorbing gas (McQuinn et al. 2011) or the hardness of the ionizing radiation which will affect the width of the ionization front. Finally, we note the relatively substantial effect of the value of \( M_{\text{crit}} \) on the number density of Lyman limit systems (see also Fumagalli et al. 2013).

Armed with an estimate of how much halos of given mass contribute to the CDFD, it is now straightforward to compute the bias of DLAs, to which we turn next.

2.3 DLA bias

The term bias was famously coined by Davis et al. (1985) to describe the fact that observed galaxies are more strongly clustered than the mass seen in numerical simulations, with Kaiser (1984) showing that such biasing naturally follows from the assumption that galaxies form at local maxima in a Gaussian density field. Mathematically, the bias factor \( b_{\text{bias}} \) of a population of objects - for example halos with a given mass - can be defined as the ratio of their power-spectrum, \( P(M_h, z) \), to that of the mass, \( P_m \) : 

\[
b_{\text{bias}}(M_h, z) = \frac{P(M_h, z) / \delta^2}{P_m}.
\]

The COLOSSUS tool of Diemer (2018) implements several methods for estimating \( b_{\text{bias}} \) for halos of a given mass at a given redshift. Below we use Diemer (2018)’s implementation of the model by Tinker et al. (2010). Given the function \( g(N_{\text{HI}}, M_h, z) \) and this bias function \( b_{\text{bias}}(M_h, z) \), we compute the bias of lines with a given value of the column density as (see also Padmanabhan et al. 2016)

\[
b_{\text{bias}}(N_{\text{HI}}, z) = \frac{\int_{\log M_{\text{crit}}(z)}^{\infty} d \log M_h \left\{ b(M_h, z) \times g(N_{\text{HI}}, M_h, z) \right\}}{\int_{\log M_{\text{crit}}(z)}^{\infty} d \log M_h \left\{ g(N_{\text{HI}}, M_h, z) \right\}}.
\] (18)

and the bias of DLAs as

\[
b_{\text{bias}}(\text{DLA}, z) = \frac{\int_{20.3}^{\infty} d \log N_{\text{HI}} \int_{\log M_{\text{crit}}(z)}^{\infty} d \log M_h F_1(M_h, N_{\text{HI}}, z) d \log M_h F_2(M_h, N_{\text{HI}}, z)}{\int_{20.3}^{\infty} d \log N_{\text{HI}} \int_{\log M_{\text{crit}}(z)}^{\infty} d \log M_h F_1(M_h, N_{\text{HI}}, z) d \log M_h F_2(M_h, N_{\text{HI}}, z)}.
\]

The results are illustrated in Fig. 5. In panel (a) we plot the bias function \( b_{\text{bias}}(M_h, z) \) computed using COLOSSUS (Diemer 2018). Superposed as filled diamonds are the values of the critical mass, \( M_{\text{crit}} \), computed from Okamoto et al. (2008) for reference. In panel (b) we plot the function \( g(N_{\text{HI}}, M_h, z) \) for three values of \( N_{\text{HI}} \) (colours). As we noted and explained before, the function \( g \) depends relatively weakly on \( M_h \) over a large range in halo mass. In our model, it falls abruptly to zero for halos with mass \( M_h < M_{\text{crit}} \), by construction, and at high mass decreases exponentially due to the exponential cut-off in the halo mass function. A comparison between the solid and dotted lines, which correspond to redshifts \( z = 3 \) and \( z = 5 \), respectively, shows how the function \( g \) evolves. \( M_{\text{crit}} \) decreases with increasing \( z \) - so that lower mass halos contribute more to \( g \) at higher \( z \). Additionally, the location of the exponential cut-off in the mass function also decreases with increasing \( z \) and consequently the relative contribution of lower mass halos to the CDFD increases with \( z \).

Combining panels (a) and (b) yields the bias of absorbers, plotted in panel (c). In addition to the bias of the absorbers shown in panels (a) and (b), we also plot the bias of DLAs (red line connecting solid diamonds). The bias increases with \( z \) for all column-densities shown in the panel. So, even though there is a tendency for lower mass halos to increase their contribution to the cross section with increasing redshift (which would decrease \( b_{\text{bias}} \)), the bias of such halos increases even more rapidly with \( z \), leading to a net increase in \( b_{\text{bias}} \). We also note that the bias increases with column density, which is not surprising, but at \( z \approx 2 \), the bias is approximately the same for \( \log N_{\text{HI}} [\text{cm}^{-2}] = 19.3 \) and 20.3 (blue and orange lines, respectively). Since bias increases with column density, the bias of DLAs, \( b_{\text{bias}}(\text{DLA}) > b_{\text{bias}}(\log N_{\text{HI}} [\text{cm}^{-2}] = 20.3) \). For reference we over plot the bias measurements of DLAs from Pérez-Rafols et al. (2018) as black symbols with error bars\(^\text{10}\). Although the agreement is not perfect, our DLA model yields reasonably high values of \( b_{\text{bias}} \), even though low-mass halos contribute significantly.

2.4 DLA line widths

To measure the velocity extent of DLAs expected in the model, we consider sight lines that intersect neutral gas. In the current model, those have impact parameter \( b \leq r_f \), the radius of the ionization front. We further assume that the gas is flowing radially with hydrogen accretion rate

\[
M_h = \omega_X X M_*.
\] (20)

where we use Eq. (1) for the profile and Eq. (2) for the halo accretion rate. Using the continuity equation then yields for the radial inflow velocity of the hydrogen gas

\[
v_r = \frac{M_h}{4 \pi r^2 \rho_h n_h(r)} = \frac{1 + z}{10 H} \left( b - \frac{a}{1 + z} \right) \frac{(r/R_h)^{a-2}}{v_b}
\]

\[
\approx 210 \text{km} \text{s}^{-1} \left( \frac{M_h}{10^{12} M_*} \right)^{1/3} \left( \frac{1 + z}{4} \right)^{3/2} \left( \frac{r}{R_h} \right)^{a-2},
\] (21)

where the numerical values use thePlanck Collaboration et al. (2014) values of the cosmological parameters. This infall velocity depends little on radius in the cases we are interested, i.e. when the slope of the density profile \( \alpha \approx \)

\(^9\) We use the symbol \( b_{\text{bias}} \) to denote the bias factor to avoid confusion with the impact parameter, \( b \).

\(^{10}\) These correspond to the red filled circles in Fig. 4 of that paper.
2. The component of the velocity along the line-of-sight is \( v_\perp = \cos(\theta) v_z \), where \( \cos(\theta) = z/r \) depends on the distance \( r \) to the centre and on \( z = (r^2 - b^2)^{1/2} \). Given that \( v_\perp \) only depends weakly on \( r \) means that absorption in neutral gas occurs over a velocity range of approximately \([-v_\perp, v_\perp]\), where \( v_\perp = (1 - b^2/r^2)^{1/2} v_z (r = r_I) \), where \( v_z (r = r_I) \) is the value of the infall velocity at the location \( r_I \) of the ionization front.

Observationally, the line width associated with a DLA is quantified by a velocity called \( v_{90} \), defined such that 90 per cent of the optical depth of a line associated with a metal transition occurring in neutral gas is enclosed by this velocity interval. To connect this to our velocity profile, we will identify \( v_{90} \) with \( 2v_\perp \),

\[
v_{90} = 2v_\perp (r_I) (1 - b^2/r_I^2)^{1/2}, \tag{22}\]

which is thus the velocity extent\(^{11} \) of neutral gas at impact parameter \( b \) for a halo of given mass, \( M_h \). As we will show below, there is a (relatively weak) dependence of \( v_{90} \) on column-density for any given halo, since at low columns the sight line grazes the ionization front at \( r = r_I \), gas flows mostly perpendicular to the sight line, yielding a low value of \( v_{90} \). At low \( b \), the column-density is near maximum, gas flows along the sight line, and hence \( v_{90} \) is maximal. This geometric dependence does not translate directly into a correlation between \( N_\text{HI} \) and \( v_{90} \) for a population of DLAs, as we show below.

We obtain the joint probability distribution function (PDF) for lines to have a given value of column-density and \( v_{90} \) starting from Eq. (22), by changing variables from \( b \) to \( N_\text{HI} \) using Eq. (12), eliminating \( r_I \) using Eq (8) and finally using Eq. (10):

\[
g(N_\text{HI}, v_{90}) \equiv \frac{\partial^2 N}{\partial X_\text{HI} \partial v_{90}} = g(M_h, N_\text{HI}) \left( \frac{dv_{90}}{d \log M_h} \right)^{-1}. \tag{23}\]

Integrating this quantity over \( N_\text{HI} \), for \( \log N_\text{HI} \) \( > 20.3 \), yields the line width distribution for DLAs. It turns out that not surprisingly most of the dependence of \( v_{90} \) on halo mass arises from its dependence on the halo’s virial velocity, so that approximately \( d \log v_{90}/d \log M_h \approx d \log v_z/d \log M_h = 1/3 \) and hence \( dv_{90}/d \log M_h \propto v_{90} \). Results are shown in Fig. 6, where model results are shown at \( z = 3 \).

In panel (a) of Fig. 6 we plot as coloured lines the relation between \( v_{90} \) and the column density for various values of the halo mass; \( M_h \), in units of \( M_\odot \), can be read from the legend. The more massive the halo, the higher \( v_{90} \) at a given value of \( N_\text{HI} \), reflecting the \( v_{90} \propto v_z \) dependence. The impact parameter decreases with increasing \( N_\text{HI} \) for a given value of \( M_h \), as explained this increases \( v_{90} \) as gas flows increasingly along the line of sight. For reference we show as red symbols the data of Neeleman et al. (2013). Panel (b) shows the fraction of DLAs with a given value of \( v_{90} \) as a black thick line, as well as the line width distribution for two values of the column density, \( \log N_\text{HI} \) \( > 20.3 \) (dark blue) and 21.3 (cyan). This reveals as weak trend for lower column-density DLAs to have lower values of \( v_{90} \). Shown as red symbols are the histogrammed data of Neeleman et al. (2013), showing that the model gives a reasonable distribution of velocity widths. In panel (c) we plot the number of lines with a given value of \( v_{90} \) from Eq. (23), for the same cuts in column-density as in panel (b). Over plotted as red symbols is the observed distribution for DLAs from Prochaska et al. (2003), which should be compared to the black line.

The model is in reasonable agreement with the data, producing a small number of quite broad lines with \( v_{90} > 400 \text{ km s}^{-1} \), and a large number of narrow lines with a rather abrupt cut-off at \( v_{90} \lesssim 25 \text{ km s}^{-1} \), as observed. Comparing panels (b) and (c), it seems that the model slightly under-shoots the number of narrow-lined DLAs when compared to the Prochaska et al. (2003) data, whereas it overshoots the number of narrow-lined DLAs when compared to the Neeleman et al. (2013) data. Either this means that the two data sets are not quite consistent, possibly just due to the small number statistics, or it may point to redshift evolution in the data.

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\(^{11}\) With the factor 2 accounting for the gas falling away from as well as towards the observer.
2.5 Molecules

Since at least some fraction of DLAs correspond to a sight line puncturing a galaxy, some DLA sight line should also intersect molecular gas. Here we try to estimate how often this occurs. To do so we use the simple empirical pressure-\(H_2/\text{HI}\) relation from Blitz & Rosolowsky (2006).

Measuring the surface density of local spiral galaxies in both neutral and molecular gas, Blitz & Rosolowsky (2006) found that the ratio of these surface densities scaled with central gas pressure \(p\) approximately as (their equation (12))

\[
R_{\text{mol}} = \frac{\Sigma(H_2)}{\Sigma(\text{HI})} = \left(\frac{p/k_B}{3.5 \times 10^4 \text{Kcm}^{-3}}\right)^{0.92},
\]

where \(k_B\) is Boltzmann’s constant. In their paper, Blitz & Rosolowsky (2006) verify this relation in the range \(3 \times 10^3 \leq (p/k_B)/(\text{K cm}^{-3}) \leq 2 \times 10^9\). We will use this fit to relate the volume densities of molecular and atomic hydrogen inside the ionization front, setting

\[
\frac{2n_{H_2}}{n_{\text{HI}}} \approx \left(1 + \frac{1}{2R_{\text{mol}}}\right)^{-1}.
\]

Given the assumed hydrogen density profile in halos, \(n_{\text{HI}} \propto r^{-\alpha}\), and setting the gas temperature to \(T \approx 10^4\text{K}\), we compute the gas pressure \(p\), and substitute this value in the previous two relations to compute the molecular hydrogen density, \(n_{H_2}\). Combining this with the number density of halos with a given mass allows us to compute the \(H_2\) column-density distribution function. The results are illustrated in Fig. 7.

The left panel repeats the (total) hydrogen profile, \(n_{\text{HI}} \propto r^{-\alpha}\), with \(\alpha = 2.2\), from Eq. (1), replacing the atomic hydrogen density \(n_{\text{HI}} \rightarrow n_{\text{HI}} - 2n_{H_2}\) to account for molecular gas. The profiles are shown for two halo masses and apply to redshift \(z = 3\). The inclusion of molecules flattens the atomic profile towards the centre. When \(R_{\text{mol}} \ll 1\) in self-shielded gas, \(n_{H_2} \approx R_{\text{mol}} n_{\text{HI}} \approx n_{\text{HI}}^{1.92}\), since the gas pressure \(p \propto n_{\text{HI}}\) when gas is isothermal. Therefore \(n_{H_2} \propto r^{-1.92}\) in the outskirts of the halo, shown in the panel as the red-dashed line. At smaller radii, gas becomes mostly molecular so that \(2n_{H_2} \rightarrow n_{H_2}\). The central diamond corresponds to the location where \(R_{\text{mol}} = 1\), with the outer and inner diamonds enclosing the region to which Blitz & Rosolowsky (2006) fitted Eq. (24). As could have been anticipated, gas in the central regions is mostly molecular, in a shell around that it is mostly neutral, and in the outskirts it is highly ionized. Another point to take away from this is that there is a large range at low values of \(n_{H_2}\) in the neutral shell of gas where relation Eq. (24) is extrapolated: we should therefore treat the predicted values of the molecular gas fraction at these lower densities with caution.

The middle panel of Fig. 7 shows the corresponding column-density as a function of impact parameter, \(b\), computed as in Eq. (5). As explained when discussing Eq. (6), if the radial density profile is a power-law in radius, \(\propto r^{-\beta}\), the column-density is approximately a power-law in impact parameter, \(\propto b^{1-\beta}\). Using the values of \(\beta\) from the left panel, we show the predicted power-law slopes as dashed red lines, for both the molecular and atomic column-densities. The power-law approximations fit the curves well over a large range in impact parameter.

Comparing the \(N_{\text{HI}}\) profile with that in Fig. 1 shows that, not surprisingly, including molecules decreases the \(\text{HI}\) column-density at small impact parameter, leading to a plateau in \(N_{\text{HI}}\) value at low \(b\). The height of the plateau increases with halo mass, \(M_h\). We can estimate the approximate value of this maximum column density at \(b = 0\) by assuming that gas in the spherical shell \(r_I \leq r \leq r_I\) is neutral, gas at \(r > r_I\) is fully ionised, and gas at \(r < r_I\) is fully molecular. Here, \(r_I\) is the radius of the ionization front from Eq. (8), and \(r_I\) is the radius within which the gas is mostly molecular. For the latter we use the value of the radius where \(R_{\text{mol}} = 1\), as in the left panel. The maximum column-density is then

\[
N_{\text{HI, max}} \approx 2 \int_{r_I}^{r_f} n_I(r) \, dr
\]

\[
= \frac{n_{I, 0} R_{b}}{\alpha - 1} \left[\frac{n_{I, 0}}{n_{I, 0}^{(\alpha - 1)/\alpha} - \frac{R_b}{r_I} - 1}\right],
\]

which indeed increases with increasing \(R_b\) and hence \(M_h\). The corresponding values are plotted in the middle of Fig. 7 as the horizontal dashed lines, for both values of \(M_h\); they capture the numerical result quite accurately. The factor of 2 in Eq. (26) accounts for the sight-line crossing the neutral shell twice.

We also plot the values of \((b = r_{H_2}, N_{\text{HI, max}})\) for halo masses log \(M_h[M_\odot] = 10\) and 13 in black, and for log \(M_h[M_\odot] = 11\) and 12 in blue and orange, respectively. For the last two values of \(M_h\), we also plot the point \((b = r_I, N_{\text{HI, max}})\) as left pointing triangles. Comparing the triangles and diamonds, it is clear that for decreasing \(b\), when \(b \leq r_I\), the HI column density increases rapidly as gas becomes mostly neutral, and once \(b \leq r_I\), there is a kink in the HI column-density-b relation, because the gas interior to \(r_I\) is mostly molecular.

Given the run of the molecular column density as a function of \(b\) and \(M_h\), we now sum over all halos to compute the molecular hydrogen \(cddf\), shown in the right panel of Fig. 7 at \(z = 3\). The blue curve sums the contribution of all halos with \(M_h > M_h, \text{cddf}\) - the critical halo mass below which halos lose gas due to reionization - as we did for the HI cddf. The slope of the \(cddf\) is once more set by the dependence of the \(H_2\) cross-section on impact parameter, just as we found for the HI cddf. Given the power-laws from the middle panel, this predicts approximately scalings of \(f(N_{H_2}) \propto N_{H_2}^{(-2/1.92) - 1} \approx N_{H_2}^{-4.47}\) at low columns, \(N_{H_2} \lesssim 10^{22}\text{cm}^{-2}\), and \(N_{H_2} \lesssim N_{H_2}^{-2.66}\) above that. The steepening at high column-density is caused by the fact that these absorbers are mostly molecular, so that \(2n_{H_2} \rightarrow n_{H_2}\). For comparison we overplot the observed \(cddf\) at \(z = 3\) from Balashev & Noterdaeme (2018), for which the slope at the lower column-densities is slightly flatter at \(-1.29\). We also overplot the \(cddf\) at \(z = 0\) from Zwaan & Prochaska (2006).

There is no obvious reason for the \(z = 0\) data to smoothly fit onto the \(z = 3\) data - although, as Balashev & Noterdaeme (2018) point out, in fact they do. At high columns, \(N_{H_2} \lesssim 10^{22}\text{cm}^{-2}\), our default model (blue curve) reproduces the \(z = 3\) data well, and fits smoothly onto the \(z = 0\) data. However below \(N_{H_2} \lesssim 10^{20}\text{cm}^{-2}\), the model increasingly overestimates the \(cddf\), although the observed and simulated slopes of the \(cddf\)s are quite similar. Recall-
Figure 7: Molecular hydrogen in halos at redshift $z = 3$. Panel (a): Assumed radial distribution of the total hydrogen density ($n_{HI}$, solid line), the atomic hydrogen density ($n_{HI}$, dotted line) and the molecular hydrogen density ($n_{H2}$, dashed line) in halos with mass $M_h = 10^{11}$ and $10^{12}\, M_\odot$ (blue and orange lines, respectively); the power-law exponent is $\alpha = 2.2$. The $n_{HI}$ and $n_{H2}$ distributions are repeated from Fig.1, except that now we subtract $2 \times n_{HI}$, which causes the down-turn in the $n_{HI}$ density at small radii. When the molecular fraction is low, $n_{H2}$ is approximately proportional to $n_{HI}^{1.92}$, with the factor 0.92 the exponent appearing in Eq. (24). This scaling is indicated by the red dashed line. At higher densities, the molecular fraction tends to unity so that $2n_{H2} \approx n_{HI}$. The left and right most filled diamonds correspond to the maximum and minimum density for which Blitz & Rosolowsky (2006) measured the fitting relation of Eq. (24); the middle filled diamond corresponds to the density where $R_{mol} = 1$. Panel (b): Corresponding column-densities of atomic gas (solid line) and molecular gas (dotted lines) as a function of impact parameter, $b$. The slopes for pure power-law profiles are plotted as red dashed lines. When $N_{HI} \lesssim 10^{22}\, cm^{-2}$, gas in the model becomes increasingly molecular, with the transition column dependent on halo mass. The thick horizontal dashed lines show the maximum HI column-density as computed from Eq. (26), with the diamonds at the location where $b = r_{H2}$, the radius within which $R_{mol} > 1$. The black line shows the relation $r_{H2} - N_{HI,max}$, with further black diamonds indicating halo masses of $\log M_h [M_\odot] = 10$ and $13$. The left pointing triangles show the location of the ionization front, plotted at $b = R_I$. Panel (c): Corresponding molecular hydrogen column density distribution function. The blue and orange lines are the model prediction when summing over all halos more massive than $10^{10.99}$ and $10^{11.01}\, M_\odot$, respectively. At column $N_{HI} \gtrsim 10^{22}\, cm^{-2}$, gas is mostly atomic and the $H_2$ CDDF is a power-law with slope $f(N_{H2}) \propto N_{H2}^{-12/(1.92\alpha)}$, indicated by the upper red-dashed line. At higher column-densities, gas becomes mostly molecular, and the CDDF steepens to $f(N_{HI}) \propto N_{HI}^{-2(\alpha - 1)}$ - the same slope as the $N_{HI}$ CDDF at high column-density - indicated by the lower red-dashed line. These (approximate) power-law dependences result from the dependence of cross-section on column-density (see text). The purple diamonds depict the observed CDDF for molecular hydrogen at $z \approx 3$ reported by Balashev & Noterdaeme (2018), the cyan crosses are the $z = 0$ data from Zuwaan & Prochaska (2006). The orange dotted line assumes that the molecular abundances is 5 times lower than predicted by Eq. (24).

2.6 $\Omega_{HI}$, $\Omega_{H2}$ and their evolution

Given the CDDF and the molecular fraction in halos, we can integrate over column density to compute the fraction of the cosmological mass that is in neutral and molecular gas in units of the critical density,

$$\Omega_{HI} = \frac{H_0 \mu m_H}{c \rho_c} \int_0^{\infty} N_{HI} f(N_{HI}) \, dN_{HI}$$

$$\Omega_{H2} = \frac{H_0 \mu m_H}{c \rho_c} \int_0^{\infty} (2N_{H2}) f(N_{H2}) \, dN_{H2}. \quad (27)$$

Here, $\mu = 1.3$ is the mean molecular weight per particle in units of the proton mass, $m_H$, to account for Helium and other elements, the factor of two in the expression for $\Omega_{H2}$ counts the two hydrogen atoms per $H_2$ molecule. The predictions for these two quantities are illustrated in Fig. 8.

Panel (a) of Fig. 8 repeats the HI CDDF from Fig. 2 at two redshifts (dotted lines), but also shows the CDDF corrected for molecules as discussed in the previous section (solid lines). The onset of $H_2$ introduces a cut-off in the CDDF which is particularly sharp at $z = 2$. Over most of

12 Notice that this is the traditional definition of $\Omega_{H2}$: the cosmological mean density in gas that is mostly neutral - as opposed to the mass density of HI - in units of $\rho_c$. 

the observed range (yellow line connecting error bars) is the observed CDDF from Noterdaeme et al. (2012). Panel (b): as in panel (a), but plotting the cumulative fraction of gas in absorbers with column density $N_{\text{HI}} \leq N_{\text{HI}}$. The dashed lines correspond to the cumulative fraction of mass in molecular gas in absorbers with $N_{\text{HI}} \leq 2N_{\text{HI}}$. Panel (c): mass density in neutral gas in units of the critical density, Eq. (33), for the model when molecules are included (red squares connected with a solid line) and when molecules are not included (black diamonds connected with a dotted line). The red line connecting error bars are the observed values from Noterdaeme et al. (2012). Circles connected by a dashed line refers to $\Omega_{\text{HI}}$, defined in Eq. (33); the filled blue squares are $\Omega_{\text{HI}}$ from the COLDZ survey, Riechers et al. (2019).

In panel (b) of Fig. 8 we plot the cumulative mass-density in neutral in units of $\rho_\ast$, $\Omega_{\text{HI}}(< N_{\text{HI}})$, at two redshifts (different colours) and with or without including H$_2$ formation (solid and dotted lines, respectively). This panel demonstrates that $\Omega_{\text{HI}}$ is dominated by gas in sub-DLAs and DLAs, as is well known. The panel also shows $\Omega_{\text{HI}}$ at the same two redshifts as dashed lines. There is little evolution in $\Omega_{\text{HI}}$ and $\Omega_{\text{HI}} \ll \Omega_{\ast}$ - therefore including or excluding molecules makes less than $\sim 20$ per cent difference to $\Omega_{\text{HI}}$.

In panel (c) of Fig. 8 we plot the evolution of $\Omega_{\text{HI}}$ (solid and dotted lines show the model including or excluding molecules, respectively), compared to the data of Noterdaeme et al. (2012). The model prediction is within 20 per cent of the observed value. Within the observational error bars, there is little evidence for evolution in the data as has been stressed by e.g. Prochaska & Wolfe (2009). That panel also compares the predicted value of $\Omega_{\ast}$ (blue circles connected by a dashed line) to the value reported by Riechers et al. (2019) from the COLDZ survey (the blue filled rectangles, which correspond to the green error boxes in Figure 5 of Riechers et al. 2019). Given the relative simplicity of the model, the prediction is in relatively good agreement with the data. There is a hint that the evolution in the data is stronger than in the model.

In models in which HI is mostly associated with the ISM of galaxies, the non-evolution of $\Omega_{\text{HI}}$ over the redshift range $z = 5 \rightarrow 2$ is surprising because the galaxy stellar mass function builds-up significantly over this interval. If HI is the gas reservoir associated with galaxies, then we might expect $\Omega_{\text{HI}}$ and $\Omega_{\ast}$ (the cosmological mass density in stars divided by $\rho_\ast$) to evolve in tandem - but they do not, see also Péroux et al. (2012). However, in the current model, most of the HI mass is actively accreting onto halos rather than a reservoir that is being build-up. As gas accretes onto a halo, it is initially highly ionised and hence does not contribute to $\Omega_{\text{HI}}$. Once gas flows past the ionization front, $r < r_I$ from Eq. (8), it becomes neutral. Even further in, $r < r_{\text{H}_2}$, the radius where $R_{\text{mol}}$ from Eq. (24) becomes $\sim 1$, gas becomes molecular. This means that the cosmological mean density of HI, $\rho_{\text{HI}} = \Omega_{\text{HI}}\rho_\ast/\mu$, is due to gas in halos at a distance from the centre $r_{\text{H}_2} < r < r_I$. We can compute the evolution of the cosmological density of such gas as follows.

We first notice from panel (b) in Fig. 8 that $\Omega_{\text{HI}}$ is dominated by gas with neutral fraction $x \approx 1$. This means that its numerical value does not depend (strongly) on the radial distribution of gas in halos. Indeed, if instead $\Omega_{\text{HI}}$ were to be dominated by the small neutral fraction in highly ionised gas for which $n_{\text{HI}} \propto n_{\ast}^2$ according to Eq. (4), then $\Omega_{\text{HI}}$ would depend strongly on the radial density profile, $n_{\ast}(r)$. Since we concluded that $\Omega_{\text{HI}}$ does not depend on the radial distribution (as long as molecules can be neglected as well), $\rho_{\text{HI}}$ should be related to the rate at which mass collapses into halos that contain DLAs, i.e. those with halo mass $M_h > M_{h,\text{crit}}$. This means that we should be able to compute $\Omega_{\ast}$ directly from the rate of collapse into halos without computing the CDDF first. We do so in the following.

We compute the collapsed fraction in halos that host DLAs as the integral

$$\Omega_{\ast} \equiv \frac{1}{\rho_c} \int_{\log M_{h,\text{crit}}}^{\infty} M_h \frac{dn}{d \log M_h} d \log M_h ,$$

where in practise we integrate to an upper limit of $M_h = 10^{13} M_\odot$. This does not affect the results here, since the halo mass function is already decreasing exponentially well before that. We evaluate the integral in Eq. (28) numerically, computing $dn/d\log M_h$ with the COLOSSUS tool of Diemer (2018). The result is plotted in the top panel of Fig. 9, together with the Taylor-series fit

$$\ln \left( \frac{\Omega_{\ast}(z)}{0.065} \right) = -0.22(z-2) - 0.05(z-2)^2 + 0.0008(z-2)^3 ,$$

from which we compute that

$$\frac{d\Omega_{\ast}}{dz} \approx -0.015(1 + 0.23(z-2) - 0.18(z-2)^2 + \cdots) ,$$
The fit from Eq. (29) (red diamonds) computed numerically (blue solid line) using the dynamical time (\(\tau_{\text{dyn}}\)), Eq. (33), respectively. Given this reasoning, we compute \(\Omega_{\text{HI}}\) as

\[
\Omega_{\text{HI}} = \left( \frac{\omega_{z}}{\Omega_{t}} \right) \times (\eta(t)), \tag{33}
\]

where \(\eta = \tau_{\text{dyn}}\) or \(\tau = \tau_{\text{acc}}\).

This equation forecasts that \(\Omega_{\text{HI}}\) depends relatively weakly on redshift. Indeed, taking \(\tau = \tau_{\text{acc}}\) we find that \(\Omega_{\text{HI}}(z) \approx \eta(z)(-d\Omega_{\text{HI}}/dz)\). Given that \(d\Omega_{\text{HI}}/dz\) is approximately constant, the redshift dependence of \(\Omega_{\text{HI}}\) is mostly due to that of \(\eta(z)\), which we discuss in more detail below. The reason for the lack of evolution is illuminating: it occurs because the rate of collapse into halos is approximately inversely proportional to the accretion time of those halos. Therefore the product, \((d\Omega_{\text{HI}}/dt)\) \(\tau_{\text{acc}}\), is almost constant. This is the main reason for the lack of evolution in \(\Omega_{\text{HI}}\) in the current model.

We examine the evolution of \(\eta\) starting from Eq. (8),

\[
\eta(z) \approx 0.25 \left( \frac{\Omega_{\text{HI}}(z)/\Omega_{\text{HI}}(2)}{\Gamma(2)/\Gamma(2)} \right)^{1/(2\alpha - 1)}
\]

We used \(z = 2\) as the reference point, reading the value of \(\eta(z = 2) \approx 0.25\) for a halo with mass \(M_{h} = 10^{13}\,M_{\odot}\) from Fig. 1. The virial radius of a halo with given mass decreases with \(z\) as \(R_{h} \propto 1/(1 + z)\), whereas the density of the halo increases \(\propto (1 + z)^{3}\). As a result \(\tau_{\text{I}}\) increases whereas \(R_{h}\) decreases with \(z\). This evolution, combined with the evolution of \(\Gamma\), causes the ratio \(\tau_{\text{I}}/R_{h}\) to increase with \(z\).

The numerical values we obtain are

\[
\Omega_{\text{HI}}(z = 2) = \begin{cases} 
0.74 \times 10^{-3} & \text{for } \tau = \tau_{\text{acc}}, \\
0.34 \times 10^{-3} & \text{for } \tau = \tau_{\text{dyn}}.
\end{cases}
\]

13 For an \(1/r^{2}\) density distribution - which is close to the radial profile that we assume when taking \(\alpha = 2.2\) - the enclosed mass in a sphere of radius \(r\) is proportional to \(r\). However, here we need the mass enclosed in a circular aperture. Since we take a single value of \(\eta\), whereas in reality we should take the average, weighing each halo mass with its cross-section, we think this approximation is good enough to understand the scaling of \(\Omega_{\text{HI}}\).
The predicted evolution for both models is plotted in the lower panel of Fig. 9 (solid line: accretion model which uses $\tau = \tau_{ac}$, dash-dot line: dynamical model which uses $\tau = \tau_{dyn}$). The value obtained for the accretion model ($\Omega_{HI}(z = 2) = 0.75 \times 10^{-3}$) agrees very well with that obtained by integrating the $z = 2$ CDDF ($\Omega_{HI}(z = 2) = 0.77 \times 10^{-3}$) for the full model (solid squares connected by a red line) as well as the value inferred observationally by Noterdaeme et al. (2012) (solid red line connecting error bars, $\Omega_{HI}(z = 2.2) = (0.99 \pm 0.05) \times 10^{-3}$) repeated from Fig. 8 in the lower panel of Fig. 9. The dynamical model under estimates $\Omega_{HI}$ by a factor of $\sim 2$.

We stress that these models are meant to reproduce the full model, shown as red squares connected with a solid line. The stronger evolution in the accretion model compared to the full model is due to the stronger evolution in $\eta(z)$ in the former. This is demonstrated by the blue dotted line, which is $\propto \eta(z)$ - it captures the evolution of the accretion model well. What the accretion model does not capture is that, at higher $z$, the CDDF and hence $\Omega_{HI}$, is dominated by lower-mass halos. Such lower-mass halos have lower values of $\eta$. The accretion model does not account for this evolution of the DLA halo mass function.

In summary: the value of $\Omega_{HI}$ obtained by integrating the model CDDF agrees well with the value observed by Noterdaeme et al. (2012). Neither model nor data evolve strongly with redshift. In the model, HI gas is accreting onto halos. We discussed how such an accretion model can be derived directly without making reference to the CDDF, Eq. (33). In this accretion model, the relative constancy of $\Omega_{HI}$ results mostly from the fact that the accretion rate scales $\propto H(z)$ whereas the dynamical time of a halo scales $\propto 1/H(z)$ so that $\Omega_{HI}$, which is proportional to their product, depends weakly on $z$.

3 DISCUSSION

We have made several simplifications in developing the simple model for predicting the absorption properties of gas as it accretes onto halos and feeds a central galaxy. We briefly discuss their impact and how they could be improved.

**Spherical accretion.** Numerical simulations show convincingly that cold gas accretes onto halos in the form of filaments rather than spherically symmetric (e.g. Fumagalli et al. 2011; Cen 2012; Bird et al. 2014; Rahmati et al. 2015; van de Voort et al. 2019). Observational evidence for such filamentary accretion is mounting (e.g Fumagalli et al. 2017). At the very least, the accreting gas also has angular momentum which will affect how it accretes. In addition, galactic outflows driven by energy injected in the interstellar medium are a major ingredient in simulations of galaxy formation (see e.g Somerville & Davé 2015 for a review). Such outflows must, at some level, impact the accretion of gas. Finally, as gas accretes onto the galaxy, it eventually must encounter an accretion shock. Assuming that gas accretion is spherical, as we have done in this paper, is clearly a major simplification. One reason we suggest that this may not be as unreasonable as it looks at first sight, is that a major fraction of the DLA cross section of gas in a halo occurs relative far out. That accreting gas may not be affected so much by angular momentum or bipolar galactic outflows. In addition, provided that the halo mass is low, the location of any accretion shock may be relatively close to the galaxy, in which case it does not strongly affect the DLA cross section. All these processes are likely to affect the CDDF at higher values of the column-density. There is support for this view from the simulations by Altay et al. (2011), who show that the CDDF is little affected by feedback for columns $N_{HI} \leq 10^{15.5}$ cm$^{-2}$, but there is increasingly significant impact at higher column-densities.

**HI in the central galaxy.** At the centre of the halo, some fraction of the accreted gas will remain in the form of HI in the galaxy, with star formation occurring in molecular gas. We have not accounted for such a ‘reservoir’ of gas, but did try to account for $H_2$ formation. For the model to be viable, the majority of the accreted gas must therefore be ejected again in the form of a galactic wind, rather than be contained in the galaxy, and moreover this wind should not itself contribute significantly to HI in absorption. The galaxy formation model by Sharma & Theuns (2020) makes it plausible that the galactic outflow rate traces the accretion rate, rather than building a gas reservoir in the galaxy. Most models of galaxy formation appeal to strong outflows, see e.g. Somerville & Davé (2015) for a recent review.

**Cosmological accretion** We assumed that the matter that accretes onto halos does so purely in the form of cosmological accretion with the cosmic baryon fraction. In reality, some fraction of baryons will have collapsed into smaller galaxies before, and some may have been ejected by feedback. Conversely, a fraction of the gas accreting onto the halo may have been ejected by the halo’s galaxy previously and is now ‘recycled’. The extent to which this affects accretion onto halos likely depends on mass and redshift, Garratt-Smithson et al. (2020); Wright et al. (2020) examined some of these effects in the EAGLE simulations at $z \lesssim 2$.

**Recombinations.** We used the ‘case-B’ recombination rate for hydrogen, appropriate in situations where a recombination directly to the ground state releases a photon that ionizes a neutral hydrogen in the immediate vicinity - also called the ‘on-the-spot approximation’. This might be a good approximation inside and close to the ionization front, but will underestimate recombinations in the highly-ionized outskirts of the gas profile. At $T = 10^8$ K, the case-A (total) recombination rate is $\approx 1.6$ times higher than the case-B value. Improving this aspect might increase the number of LLS and sub-DLAs by a small fraction, and could also affect the shape of the knee in the CDDF. In a similar vein we also neglected spectral hardening, which could also impact the shape of the knee in the CDDF. Both effects were included in the urCHIN radiative transfer calculations of Altay et al. (2011) to which we compared our model in Fig. 3. In particular, urCHIN switches between case-A and case-B recombination rates when the shielding optical depth $\tau > 1$ (Altay & Theuns 2013). It also uses 100 frequency bins to account for spectral hardening. We also assumed that the accreting gas remains isothermal at $T = 10^4$ K. The temperature-density relation seen in cosmological simulations shows thatigm gas instead heats as gas accretes from $T \sim 10^8$ K to $\approx 10^{4.4}$ K before cooling back to $10^4$ K (see for example Fig. 2 in Theuns et al. (1998)). This increase in $T$ decreases the recombination rate by a factor $\approx 2.2$. To get this temperature evolution right requires radiation-hydrodynamical simulations, which can simultaneously account for shielding from the UV-background and its impact on the cooling and
heating rate of the accreting gas. We also neglected any impact of radiation from the central galaxies on the DLA (see e.g. Rahmati et al. 2013).

- **DLA line-widths.** A characteristic feature of the line-shape of low-ionization metal absorption associated with DLAs is that the strongest absorption tends to occur at the edges of the absorption systems (‘edge-leading spectra’, Neeleman et al. 2013). Simulations seem to be able to reproduce this (e.g. Bird et al. 2015). We have not examined whether our model is consistent with this.

- **Molecules.** At face-value, the agreement between the model prediction and observations of the molecular CDDF is not very good, with the model over predicting \( N_{\text{HI}} \) by a significant amount. It would be interesting to examine how well simulations do. It seems reasonable to expect that a model that accounts for the impact of metals on the \( \text{H}_2 \) abundance would improve the agreement with the data. In any case, we expect that some model features would still emerge, such as the flattening of the \( N_{\text{HI}} \) column at small impact parameter parameter and the associated bend in the HI CDDF at high \( N_{\text{HI}} \).

## 4 SUMMARY AND CONCLUSIONS

The high star formation rate measured in galaxies at \( z \gtrsim 2 \) is mostly fuelled by cosmological accretion, however observing this accreting gas directly has proved to be challenging (e.g. Fumagalli et al. 2011). In this paper we have examined a simple model for cosmological accretion onto dark matter halos, based on the similarity solution of Bertschinger (1985), and including approximate radiative transfer to account for self-shielding from an ambient ionising background taken from Haardt & Madau (2012). In this model, the accreting gas gives rise to Lyman-limit systems (LLS) in the surroundings and outskirts of the halo where it is highly ionised, and to damped Lyman-\( \alpha \) systems (DLAs) in the inner halo where the gas is mostly neutral. The resulting column-density distribution function (CDDF) is in excellent agreement with the data (Fig.2). To the extent that the model captures accretion of gas onto halos, it seems we have been observing the accreting gas that fuels star formation all along.

The shape of the CDDF reflects that of the power-law radial distribution of the gas, \( n_{\text{HI}}(r) \propto r^{-\alpha} \), with \( \alpha \approx 2.2 \). The reason is that the density profile is self-similar and hence the same for all halos. In the LLS regime, the slope of the CDDF is \( f(N_{\text{HI}}) \propto N_{\text{HI}}^{2/(1-2\alpha)-1} \propto N_{\text{HI}}^{-1.6} \), whereas in the DALA regime it is \( f(N_{\text{HI}}) \propto N_{\text{HI}}^{2/(1-\alpha)-1} \propto N_{\text{HI}}^{-2.7} \). Our analytical expression explain why halos contribute about equally to the CDDF over a large range of halo masses, \( M_7 = 10^{10} - 10^{12} M_\odot \) (lower-mass halos are more numerous but their cross-section is smaller, see Eq. 14) as well as why the amplitude of the CDDF evolves so little with redshift (although halos become more abundant at lower \( z \), their cross-section decreases, see Eq. 17). We also explain the origin of a lower-mass cut-off (reionization introduces a redshift-depend critical mass, \( M_{\text{em}}(z) \), below which halos lose their gas, Okamoto et al. (2008)) and why high-mass halos do not contribute more to DLAs (the exponential cut-off in the halo mass function).

The relative contribution of halos of a given mass to the CDDF does evolve in the sub-DLA regime (\( N_{\text{HI}} \lesssim 10^{20} \text{cm}^{-2} \)), where lower-mass halo contribute more at higher \( z \). This leads to a change in the location of the ‘knee’ in the CDDF - the transition from LLS to DALAs- with the knee shifting to lower values of \( N_{\text{HI}} \) at higher \( z \) (see Fig. 2). This shift is caused by the evolution of the critical mass \( M_{\text{crit}}(z) \), which becomes smaller at higher \( z \). Detecting such an evolution in the data might constrain the evolution of this critical mass.

Computing the differential contribution of halos of a given mass to the CDDF allows to evaluate the DLA bias (§2.3), which agrees well with the observed value, as well as the distribution of DLA line-widths, \( v_{90} \) (§2.4). We assumed that accreting gas falls in radially, at a rate consistent with the cosmological accretion rate, and weight the infall velocity with the cosine of the angle with the line of sight. For a given halo, this introduces a relation between column-density and \( v_{90} \), because at a smaller impact parameter, the DLA column density is higher and the gas flows increasingly parallel to the line of sight so that \( v_{90} \) is larger. Integrating over all halos yields the distribution of \( v_{90} \), which has a relatively sharp cut-off at \( \sim 30 \text{ km s}^{-1} \) and a long tail towards high values of \( v_{90} \) (Fig.6). The sharp cut-off is related to the value of \( M_{\text{em}}(z) \). The extended tail results from DLA sight-lines that intersect rare massive halos at an unusually small impact parameter.

We used the model of Blitz & Rosolowsky (2006) to attempt to compute where the DLA gas turns molecular (§2.5). This over predicts the \( \text{H}_2 \) CDDF, especially at low \( N_{\text{HI}} \), but does reasonably well at higher column-densities. The model predicts a maximum value of \( N_{\text{HI}} \) caused by high column-density gas turning increasingly molecular (see also Schaye 2001; Erkal et al. 2012). The value of this maximum column-density depends on halo mass. The presence of such a maximum leads to a down-turn in the HI CDDF. Notwithstanding the superb statistics of the observed CDDF (Noterdaeme et al. 2012), such a down-turn is not yet clearly detected.

Integrating over the CDDF yields the fraction of the mass in the Universe that is in neutral gas, \( \Omega_{\text{HI}} \), or in molecular gas, \( \Omega_{\text{H}_2} \) (§2.6). The model prediction agrees well with observations (Fig.8). We presented a model that captures the accretion origin of the HI gas, and which reproduces well the results of the full model. This simpler model elucidates why \( \Omega_{\text{HI}} \) evolves so slowly, as observed. The underlying reason is that the accretion rate onto halos increases with redshift at the same rate as the flow time scale of the gas within halos decreases. Since \( \Omega_{\text{HI}} \) is proportional to the product of these (Eq. 33), it evolves slowly.

It has long been posited that the HI seen in absorption fuels star formation in galaxies (e.g Wolfe et al. 2005). Within this interpretation, the fact that the mean HI density, \( \Omega_{\text{HI}} \), evolves weakly with \( z \) whereas the star formation rate density evolves strongly, is puzzling. The solution to the puzzle as discussed in this paper is twofold: firstly, the observed HI is dominated by gas accreting onto halos rather than a reservoir of gas in the galaxy’s interstellar medium (ISM). Although a sight-line through a galaxy likely produces a DLA, the reverse is not true: every galaxy is a DLA, but not every DLA is a galaxy. The accreting gas is in the form of HI for a fraction of time as it accretes onto a halo. This explains why the accretion rate (and hence also the star formation rate) can vary with redshift while \( \Omega_{\text{HI}} \) remains approximately constant. Secondly, as the HI gas gets close to the centre and enters the galaxy’s ISM, it becomes molecular, some fraction forms stars, but the majority is ejected.
in the form of a galaxy-wide outflow. This implies that any ‘reservoir’ of HI in the ISM of the galaxy remains small.

In short we claim that Lyman-limit systems (LLS) and DLAs are both related to the accretion of gas onto halos, and only a small fraction of this accreted gas fuels star formation in galaxies. This simple picture is consistent with the observed properties of LLS and DLAs.

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DATA AVAILABILITY

No new data were generated or analysed in support of this research.

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