Semi-Classical theory of Nonlinear interaction of circularly polarized optical vortex beam with plasma channel

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Abstract. A semiclassical approach of nonlinear interaction of intense circularly polarized optical vortex Laguerre-Gaussian (LG) beam modes with a plasma channel is analyzed theoretically and numerically. We study an exchange of angular momentum between the vortex beam and plasma channel. The transfer of angular momentum and the generated magnetic field are calculated. We have observed that both the generated magnetic field and angular momentum transfer depend on beam mode, intensity, and the polarization state of beam mode.

1. Introduction
It is now well understood that light beam with helical phase front carries orbital angular momentum (OAM) along their direction of propagation in addition to spin angular momentum that describe their polarization. Photons in a light beam have spin \( \sigma_z h \), where \( \sigma_z = \pm 1 \) refers to the left and right circular polarization states respectively. For linear polarized beam, \( \sigma_z = 0 \) in the direction of propagation. In polarized light, spin of photons align in the direction of propagation and contribute to a net spin angular momentum. The helical wavefront exists when the wave vectors spiral around the beam axis and constitute to the OAM [1].

The current understanding of transfer of spin and orbital angular momentum suggest that any beam with an amplitude distribution \( u(r, \phi, z) = u_0(r, \phi, z) \exp \iota \ell \phi \), carries angular momentum about the beam optical axis. These laser beams carry total angular momentum much greater than that associated with the circularly polarized Gaussian laser beam. This problem was addressed recently in plasma physics where its effect on various phenomenon of laser-plasma interaction were taken into account. Cormier-Michel et al [6] and Stupakov et al [7] has recently studied the excitation of quasi-magnetic field due to the interaction of LG higher modes with plasma. Ali et al [4] pointed out that a linearly polarized beam in OAM state could also generate magnetic field in plasma. The propagation of higher laser modes in plasma channel was studied by York et al and has explained direct acceleration of electrons in a corrugated plasma channel [8]. Nesterov et al [9] has analyzed the importance of transfer of OAM by CPVBs to inhomogeneous plasma. Andersen et al [10] has demonstrated the coherent transfer of the orbital angular momentum of a photon to an atom in quantized units of \( h \) using a 2-photon stimulated Raman process with Laguerre-Gaussian beams to generate an atomic vortex state in the Bose-Einstein condensation of sodium atoms. Kyosuke Sakai et al [11] has recently studied the excitation of multipole plasmons by an optical vortex beam and explained the transfer of angular momentum between photons and plasmons.

In the present work, we have proposed a model that allow transfer of OAM of \( LG_\ell^k \) modes and magnetic field generation in plasma channel at relativistic limit. We have theoretically examined
that a normal incident CPVBs with a specific azimuthal mode can excite the magnetic field and couple to the plasmons of spiraling plasma electrons which results into the transfer of angular momentum of CPVBs to plasma electrons. We have demonstrated that the transfer of OAM and magnetic field generation depend on the mode and the polarization states of the vortex beam.

The paper is organized as follows: In section 2, the governing Equations of the angular momentum transferred to plasma electrons is derived using semiclassical approach. The conclusions are presented in section 3.

2. The Model

The OAM of \( \text{LG}_\ell^p \) modes impart helicoidal motion to the plasma electrons and form vortices like structure in a plane perpendicular to the direction of beam propagation. Evolution of these structures result into the excitation of asymmetric quasi-static magnetic fields which depend on the order of beam mode and an axial phase velocity \( v_{ph,z} = c(1 - 1/k(\partial \theta_{lp}/\partial z)) \). Hence, the average torque received by electrons per unit volume also be depend on the beam mode.

An Equation for the average rate of change of angular momentum of the electrons per unit volume is given as

\[
m_e n \frac{d(rv_{\phi})}{dt} = -enr(E_{\phi} + v_z B_r - v_r B_z) - \frac{dM_z}{dt},
\]  

(1)

where \( E_{\phi} \) is the azimuthal electric field, \( B_r \) and \( B_z \) are respectively radial and axial magnetic fields. And \( M_z \) is the axial angular momentum density of photons per unit wavelength. The term \( m_e v_{\phi,r} \) is the angular momentum of electrons in the z direction and \( dM_z/dt \) refers to the pressure like term of quasi-static magnetic field. The relation for \( E_{\phi} \) is given by

\[
E_{\phi} \simeq \frac{-1}{\epsilon \omega L} \frac{\partial I}{\partial r}
\]  

(2)

We observed that the total angular momentum is non-zero for the plane polarized light \( (\sigma_z = 0) \). Following [?] and Faraday’s law, the time derivative of the generated axial magnetic field is given as

\[
\frac{\partial B_z}{\partial t} = \frac{1}{enr_{ch}^3 L \epsilon_0 c} \frac{d}{dt} \left( \frac{1}{2\omega c} \frac{\partial}{\partial r} \left( r \frac{\partial I(r,z,\phi)}{\partial r} \right) \right) \times (\ell_{pz} + p \pm \sigma_z) \hbar \cos \theta_{lpz}.
\]  

(3)

Integrating Eq.(6) within the time limit \( t=0 \) and \( t \) and assuming that the damping rate of the laser energy over this period is almost negligible, the generated magnetic field in plasma channel turns out to be of the following form

\[
B_z = \frac{\eta(r)}{enr_{ch}^3 \omega L \epsilon_0 c} \left( \frac{\partial}{\partial r} \left( r \frac{\partial I}{\partial r} \right) \right) \times (\ell_{pz} + p \pm \sigma_z) \hbar \cos \theta_{lpz},
\]  

(4)

It is important to note here that for linearly polarized light \( B_z \) is nonzero.

To estimate \( B_z \) in Mega Gauss (MG), we assume \( \partial/\partial r = 1/r_{ch} \). Thus Eq.(7) can be written as

\[
B_z = \eta(r) \left( \frac{\lambda^2}{r_{ch}^2} \right) \left( \frac{n_c}{n} \right) [(\ell_{pz} + p \pm \sigma_z) \hbar \cos \theta_{lpz} ] \left( \frac{I(r,\phi) \lambda^2}{7.3 \times 10^{22} W m^{-2} (\mu m)^2} \right),
\]  

(5)
Figure 1. The generated axial magnetic field $B_z$ of mode $LG_0^1$ at different values of the phase shifts (a) $\theta_{lpz} = 0$ (b) $\theta_{lpz} = \pi/3$ (c) $\theta_{lpz} = \pi/4$ and (d) $\theta_{lpz} = \pi/6$. The continuous and dashed lines respectively refers to the variation right circular and left circular polarized beam.

where $\lambda$ is wavelength of the laser beam, $n_c$ is critical density (which is approximately $1.1 \times 10^{15} m^{-3}$).

The intensity profile of LG beam modes in the focal plane ($z=0$) is given as

$$I(r, \phi) = I_0 \frac{(-1)^{2\ell} \rho!}{(\ell + p)!} \exp(-\rho^2) \times (\sqrt{2}\rho)^{2\ell} \times L_\rho^\ell(\rho^2) \cos^2(\theta_{lpz}), \quad (6)$$

where $I_0 = a_0^2 (4\omega^2/k^2\pi^2)(1/(w_0^2(1 + \delta_{0}^2)))$ is the maximum intensity of LG beam. It is clear from Eqs.(8-9) that the generated axial magnetic field depends on the beam mode, plasma density, channel width, the phase shift $\theta_{lpz}$, and polarization state of the beam. The generated axial magnetic field for the mode $LG_0^1(\ell = 0, p = 0)$ can be obtained via Eq.(8) and Eq.(9) and read as

$$B_z \approx \frac{I_0 \eta(r) \cos \theta_{pl}}{enr_3^3 \omega c} \left[ \sigma_z \hbar \cos \theta_{lpz} \exp(-\rho^2) \right], \quad (7)$$

for $LG_0^1(\ell_{pz} \approx 1$ and $p = 0$), we have

$$B_z \approx \frac{I_0 \eta(r)}{enr_3^3 \omega c} \left[ (1 \pm \sigma_z)(1 - \rho^2)\hbar \cos \theta_{lpz} \right] \exp(-\rho^2), \quad (8)$$

Figure 1 shows the variation of the generated magnetic field as a function of $r$ for different values of beam phase transfer and polarization states for right and circularly polarized beams of $LG_0^1$ mode. The continuous and broken curves show the variation for right circular and left circular polarized beams respectively. We have observed that the magnitude of the generated magnetic field for right circular polarized beam mode is higher than the left circularly polarized beam.

Figure 2 shows the variation of an angular momentum density $M_z$ of the plasma electrons as a function of $r$ for the different beam modes $LG_0^0$, $LG_0^1$, and $LG_0^2$. It is observed that the angular momentum density transfer increases with increasing mode order and also depends on the polarization state of the beam modes.

3. Conclusion

In the present work, we have examined theoretically and numerically the effect of polarization states on the transfer of angular momentum and the generated magnetic field of CPVBs in the
plasma channel using a semi-classical approach. We note that both the generated magnetic field and the angular momentum transfer depends on the $LG^p_\ell$ mode order, polarization state of beam and the intensity of beam. The present study may be useful in various context of laser plasma interaction such as in angular dependent wake fields, various laser acceleration schemes, laser fusion research, generation of electron vortex beams and in photonic science.

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