Data-Driven Feedforward Tuning using Non-Causal Rational Basis Functions: with Application to an Industrial Flatbed Printer

Lennart Blanken¹*, Sjirk Koekebakker⁶, Tom Oomen¹

¹Department of Mechanical Engineering, Eindhoven University of Technology, 5600 MB Eindhoven, The Netherlands
⁶Ocê Technologies B.V., St. Urbanusweg 43, 5900 MA, Venlo, The Netherlands

Abstract

Data-driven feedforward tuning enables high performance for control systems that perform varying tasks by using past measurement data. The aim of this paper is to develop an approach for data-driven feedforward tuning that achieves high accuracy and at the same time is computationally inexpensive. A linear parametrization is employed that enables parsimonious modeling of inverse systems for feedforward through the use of non-causal rational orthonormal basis functions in $L_2$. The benefits of the proposed parametrization are experimentally demonstrated on an industrial printer, including pre-actuation and cyclic pole repetition.

Keywords: feedforward control, learning control, non-causal systems, basis functions, mechatronic systems, motion control systems

1. Introduction

Feedforward control is widely used in control systems, since it can effectively compensate for known disturbances before these affect the system. For instance, the main performance improvement for servo tasks in mechatronic systems is typically achieved by compensating for the reference signal, which can vary over different tasks. A relevant example is in printing, see the flatbed printing system in Figure 1. The system has high-order dynamics, with lightly damped mechanical modes. Furthermore, it has to perform varying tasks depending on the to-be-printed surface, see Figure 2. Successful feedforward control approaches include inverse model-based feedforward [1, 2] and iterative learning control (ILC) [3, 4].

On the one hand, inverse model-based feedforward generally results in reasonable performance for a wide class of motion tasks. A parametric model of the inverse system is used, obtained by (i) inverting an identified model of the forward system, see, e.g., [1, 5], or (ii) direct identification of the inverse system, see, e.g., [6, 7]. The achievable performance is highly dependent on the accuracy of the model inversion, and the resulting quality of the inverse model [8]. In particular, it should be capable to accurately compensate for the excited dynamics between reference and error. Especially for lightly damped mechatronic systems, accurate models can be difficult and expensive to obtain, e.g., in terms of computational complexity and required design effort, due to high-order dynamics [9, 10] and numerical issues [11].

On the other hand, ILC enables extremely high performance for a single task that is exactly repeated over and over again. By learning from measured data of previous identical tasks, the

*Corresponding author

Email address: l.l.g.blanken@tue.nl (Lennart Blanken)

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need for an accurate model of the system is avoided. However, extrapolation of the learned control signal to other tasks in general leads to a significant performance deterioration, see, e.g., [12, 13, 14].

Recently, the advantages of model-based feedforward and learning have been combined in several data-driven feedforward algorithms [14, 15]. These data-driven algorithms aim to achieve (i) high performance for varying tasks, (ii) without requiring an accurate system model. This is enabled through a parametrization for the inverse system, whose parameters are iteratively estimated based on data from previous iterations. A key challenge lies in selecting a suitable model structure and complexity. For performance, the parametrization should be parsimonious to avoid variance-induced errors, yet be capable to compensate reference-induced errors.

Model structures that are linear in the parameters are attractive due to the associated convex optimization problem. In [15, 16, 17], a linear combination of polynomial basis functions is used. However, typically a large number of basis functions is required, leading to large variance errors, see [7]. Consequently, the tracking performance is limited due to the polynomial nature of the feedforward controller. Especially for lightly-damped systems, high-order polynomials are required to accurately describe the inverse system, which are susceptible to noise acting on the system.

In [14, 18, 19], data-driven feedforward approaches are developed that employ a rational model structure in combination with polynomial basis functions, which has shown to significantly improve performance using a small number of parameters. However, the associated optimization problem is non-convex. As a result, convergence to the global minimum cannot be guaranteed.

Although data-driven feedforward for varying tasks has been substantially improved towards rational model structures, at present the advantages of improved performance involve a non-convex optimization. The aim of this paper is to develop a suitable inverse-model parametrization for data-driven feedforward control, which (i) is linear in the parameters, (ii) requires a small number of parameters, and (iii) enables infinite pre- and post-action. The developed approach is based on the use of non-causal rational orthonormal basis functions (ROBFs). Whereas the use of causal ROBFs has received significant attention in the field of system identification to model systems in $\mathcal{H}_2$, see, e.g., [20, 21, 22], here the use of non-causal functions is advocated to model inverse systems in $\mathcal{L}_2$. In particular, through stable inversion techniques [23, 24], the functions enable infinite pre-action for high performance feedforward control. The contributions of this paper are twofold:

C1) Development of a parsimonious inverse-model parametrization for data-driven feedforward, that is linear in the parameters and enables non-causal control through ROBFs in $\mathcal{L}_2$ (Section 3);

C2) Experimental validation of the proposed parametrization for feedforward learning on an industrial flatbed printer, demonstrating the benefits of using non-causal rational basis functions, including pre-action (Section 4).

The present paper substantially extends preliminary results in [25] with theoretical details and examples on the use of ROBFs in $\mathcal{L}_2$ for feedforward and new experimental results, and extends [26] with details on the data-driven tuning algorithm.

Notation: All systems are discrete-time, single-input single-output (SISO), and linear time-invariant. The continuous-time case follows analogously. The extension to multi-input multi-output systems is conceptually straightforward, and many of the results in Section 3 directly apply. The complex indeterminate $z \in \mathbb{C}$ is omitted when this does not lead to any confusion. The following standard notation is used, see, e.g., [21]. Let $\mathbb{D}$ denote the open unit disk: $\{z : |z| < 1\}$, $\mathbb{E}$ the complement of the closed unit disk: $\{z : |z| > 1\}$, and $\mathbb{T}$ the unit circle: $\{z : |z| = 1\}$. The space $\mathcal{L}_2(\mathbb{T})$ denotes the set of complex functions that are square integrable on $\mathbb{T}$, and its argument is often omitted for brevity. The real-rational subspace of $\mathcal{L}_2$ is denoted $\mathcal{R}\mathcal{L}_2$. $\mathcal{H}_2$ denotes the set of complex functions that are square integrable on $\mathbb{T}$ and analytic in $\mathbb{E}$. The space $\mathcal{H}_2^+$ denotes all functions in $\mathcal{H}_2$ that are zero at infinity, such as strictly proper systems, and $\mathcal{H}_2^c = \mathcal{L}_2 \setminus \mathcal{H}_2$. Also, $\mathbb{R}[z^{-1}]$ denotes the polynomial ring in indeterminate $z^{-1}$ with coefficients in $\mathbb{R}$. The Laurent polynomial ring in indeterminate $z$ with coefficients in $\mathbb{R}$ is denoted $\mathbb{R}[z, z^{-1}]$, and includes polynomials with both positive and negative exponents of the indeterminate. Signals are often assumed to be of length $N$. A real symmetric matrix $W$ is positive definite if $x^TWx > 0, \forall x \neq 0$. For a vector $x \in \mathbb{R}^N$, the weighted two-norm is given by $\|x\|^2_W = x^TWx$ with $W \in \mathbb{R}^{N \times N}$.

2. Problem Formulation

2.1. Control problem

The class of systems that is addressed in the present paper are mechatronic systems with high-order dynamics. Considering the control configuration in Figure 3, the true and unknown plant $P_0(z)$ is described by the rational representation

$$P_0(z) = \frac{B_0(z)}{A_0(z)},$$

with $A_0(z), B_0(z) \in \mathbb{R}[z^{-1}]$. The control configuration consists of a stabilizing feedback controller $C(z)$, and a feedforward controller $F(z)$. Let $r$ denote the reference task, $y$ the measured output signal, $v$ disturbances, and $f$ the feedforward signal. The tracking error $e = r - y$ is given by

$$e(t) = S_0(q)(I - P_0(q)F(q))r - S_0(q)v(t),$$

with sensitivity function $S_0(q) = (I + P_0(q)C(q))^{-1}$.

The problem considered in this paper is the tuning of feedforward controller $F(z)$ with respect to the following requirements:

1. high control performance, i.e., a small error $e$;
2. for a class of motion tasks $r$.

Optimal reference tracking, i.e., $e(t) = -S_0(q)v(t)$ for all tasks $r \neq 0$, is achieved if an exact model $F = P_0^{-1}$ is used. In practice, only approximate models $P^{-1} \approx P_0^{-1}$ are available, inevitably leading to performance deteriorations [8].
2.2. Data-driven feedforward tuning

The goal of data-driven feedforward is to minimize the error signal in the next iteration, by iteratively optimizing a fixed-structure parametrized controller $F(\theta)$ using batches of measurement data from previous iterations, denoted by index $j = 0, 1, 2, \ldots$. Using measurement $e_j$ and feedforward signal $f_j = F(\theta_j)r_j$ corresponding to the previous motion task $r_j$, a prediction of $e_{j+1}$ for motion task $r_{j+1}$ is given by

$$\hat{e}_{j+1} = e_j + S P (f_j - F(\theta_j+1)r_{j+1}),$$

where $SP$ is a model of $S_0 P_0$. The following performance criterion is considered for tuning of $F(\theta)$.

**Definition 1** (Performance criterion). The data-driven performance criterion is given by

$$J(\theta_{j+1}) = \left\| \hat{e}_{j+1}(\theta_{j+1}) \right\|_{W_e} + \left\| f_{j+1}(\theta_{j+1}) \right\|_{W_f}$$

$$+ \left\| f_{j+1}(\theta_{j+1}) - f_j \right\|_{W_{\Delta f}},$$

where $W_e > 0$, $W_f, W_{\Delta f} \geq 0$ are user-defined positive-(semi)definite weighting matrices. The optimal parameters $\hat{\theta}_{j+1}$ are defined as

$$\hat{\theta}_{j+1} = \arg\min_{\theta_{j+1}} J(\theta_{j+1}).$$

Criterion (2) resembles the criteria for norm-optimal ILC in [14, 16, 27, 28], and allows for penalizing the input in addition to the error, which provides tuning parameters for balancing tracking performance ($W_e$) with robustness to model uncertainty ($W_f$) and attenuation of iteration-varying disturbances ($W_{\Delta f}$).

**Remark 1.** In conventional norm-optimal ILC [3, 4], the aim is to iteratively learn feedforward signal $f$, i.e., $f_{j+1}$ is the optimization variable in (2) and (3), instead of parameters $\theta_{j+1}$. While ILC is known to achieve extreme performance for repeating identical tasks $r = r_j$, $\forall j$, task variations lead to significant performance deteriorations. This is a key motivation for data-driven tuning of feedforward controllers $F(\theta)$.

The key aspect that is addressed in this paper is the design of a suitable fixed-structure parametrization $F(\theta)$. On the one hand, it should be sufficiently rich to accurately describe general rational systems $P_0$, and allow computation of (3). On the other hand, a restricted model complexity is desirable, since overparametrizations may directly deteriorate control performance due to variance errors.

### 2.3. Problem description and outline

In this paper, a suitable parametrization $F(\theta)$ is presented for general rational systems $P_0$, see (1), with respect to the following requirements:

R1) a convex optimization problem;
R2) inversion of general rational systems $P_0$ with good convergence rate, i.e., a small number of parameters required; and
R3) infinite pre-actuation and post-actuation.

As is outlined in Section 1, pre-existing approaches fail to meet all requirements: the use of polynomial basis functions in [15, 16, 17] does not satisfy R2 and R3; the rational model structure in [14, 18, 19] does not meet R1.

In the next section, the proposed parametrization of $F(\theta)$ is presented that attains R1, R2 and R3 through the use non-causal rational basis functions, constituting contribution C1. The benefits of the approach are demonstrated in Section 4 through an experimental case study on the printer in Figure 1, forming contribution C2. Conclusions are given in Section 5.

### 3. A Compact Feedforward Parametrization Using Non-Causal Rational Basis Functions

In this section, a linear combination of non-causal ROBFs is used to parametrize inverse systems in $\mathcal{L}_2$. This is in contrast with the field of system identification, which generally focuses on modeling stable, causal systems in $\mathcal{H}_2$, see, e.g., [20, 21, 22].

The linear model structure that facilitates convex optimization is defined next. Subsequently, the use of non-causal rational basis functions is investigated.

**Definition 2** (Linear feedforward parametrization). The feedforward controller $F(\theta)$ parameterized in terms of a linear basis $\mathcal{T}_{lin}$ is given by

$$\mathcal{T}_{lin} = \left\{ A(\theta) \mid \theta \in \mathbb{R}^{n_\theta} \right\}$$

with

$$A(z, \theta) = \theta^T \Psi(z) = \sum_{i=1}^{n_\theta} \theta_i \psi_i(z),$$

with basis functions $\Psi(z) = [\psi_1(z), \ldots, \psi_{n_\theta}(z)]^T$.

Since $\mathcal{T}_{lin}$ is linear in parameters $\theta$, criterion $J(\theta)$ in (2) is quadratic in $\theta$. Hence (3) is convex and R1 is directly attained.

**Remark 2.** The pre-existing approaches in [15, 16, 17] employ Laurent polynomials $\psi_i(z) \in \mathbb{R}[z, z^{-1}]$ as basis functions, e.g., $\psi_i(z) = z^i$ with $i \in \{-1, 0, 1, 2, \ldots\}$. The key idea in this paper is that this is an arbitrary choice, and more suitable functions for dynamical systems can be designed by utilizing prior system knowledge.

**Remark 3.** Nonlinear basis functions can directly be included in the employed linear parametrization: due to the linear-in-the-parameter structure, the convexity of the optimization is retained. Relevant examples include gravity effects and nonlinear friction behavior such as Coulomb friction, which can be modeled as $b \text{sgn}(\dot{r})$, with $\text{sgn}(\cdot)$ the sign operator and $\dot{r}$ the first time derivative of $r$. 

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3.1. The role of non-causality for feedforward

If \( P(z) \) has non-minimum phase (NMP) zeros in \( \mathbb{E} \), \( P^{-1}(z) \) has poles in \( \mathbb{E} \). Systems with poles in \( \mathbb{E} \) are often interpreted as causal and unstable operators in \( L_2 \), e.g., in system identification [21, 29]. For feedforward control however, this need not be the case, since filtering operations are performed off-line. This allows to interpret systems with poles in \( \mathbb{E} \) as non-causal and bounded operators on \( L_2 \). In particular, using the bilateral Z-transform, any system \( P^{-1} \in \mathcal{RL}_2 \) can be expressed as

\[
P^{-1}(z) = \mathcal{Z}\{p_k\} = \sum_{k=-\infty}^{\infty} p_k z^{-k}, \tag{5}
\]

where \( \mathcal{Z}\{\cdot\} \) denotes the Z-transform operator, and the sum converges in an annulus that includes \( T \), which inner and outer radii are bounded by the nearest poles of \( P^{-1}(z) \) in \( \mathbb{D} \) and \( \mathbb{E} \), respectively. Indeed, the impulse response, or Markov parameter sequence, \( \{p_k\} \) of \( P^{-1}(z) \) is an element of \( L_2(-\infty, \infty) \) since \( \mathcal{Z}\{\cdot\} \) is an isomorphism between \( \mathcal{RL}_2 \) and \( L_2(-\infty, \infty) \) [30, Theorem 4.5.1], and \( P^{-1} \in \mathcal{RL}_2 \). This response arises from solving the underlying difference equation also for negative time. Crucially, this response is non-causal and infinitely long. The key benefit of this expansion for feedforward control is exemplified next.

Example 1. Consider the non-minimum phase system \( P(z) = \frac{1}{z^2} \), with reference \( r(t) = \left(\frac{1}{2}\right)^t \) for \( t \geq 0 \), and \( r(t) = 0 \) for \( t < 0 \). Applying the Z-transform with region of convergence (ROC) \( |z| > \frac{1}{2} \) to \( r(t) \) gives \( R(z) = \mathcal{Z}\{r(t)\} = \frac{1}{1 - \frac{1}{2}z^{-1}} \). The optimal feedforward is hence described by \( \mathcal{Z}\{f(t)\} = P^{-1}(z)R(z) = \frac{1}{z^2 - \frac{1}{2}z^{-1}} = \frac{1}{z(z-\frac{1}{2})}. \) Now, \( \mathcal{Z}\{f(t)\} \) can be transformed to the time-domain:

- Application of the inverse unilateral Z-transform with ROC \( |z| > 2 \) to \( \mathcal{Z}\{f(t)\} \) gives \( f(t) = \frac{1}{2} 2^t - \left(\frac{1}{2}\right)^t \) for \( t \geq 0 \), and \( f(t) = 0 \) for \( t < 0 \), which is causal and unbounded.
- Application of the inverse bilateral Z-transform with ROC \( \frac{1}{2} < |z| < 2 \) gives \( f(t) = -\frac{1}{2} 2^t \) for \( t < 0 \), and \( f(t) = -\left(\frac{1}{2}\right)^t \) for \( t \geq 0 \), which is non-causal and bounded. Note that \( f(t) \) can directly be computed by solving underlying difference equation \( f(t) = \frac{2}{z} r(t) \) backwards in time from \( f(\infty) = 0 \), where \( q \) is the forward shift operator, i.e., \( qr(t) = r(t+1) \).

The relevant signals are shown in Figure 4. It is observed that although both uses of Z-transforms give perfect tracking, i.e., \( y(t) = P(q)f(t) = r(t) \), the causal feedforward signal grows unbounded, whereas the non-causal signal is bounded.

The example demonstrates the key point of this subsection: since the reference \( r(t) \) is often known a priori in feedforward control applications, non-causal filtering operations are allowed, and this enables exact feedforward control with bounded signals for NMP systems \( P(z) \).

Hence, to accurately approximate (5) using a small number of parameters, a sensible choice is non-causal rational basis functions \( \psi_k(z) \in \mathcal{RL}_2 \), i.e., with non-causal and infinite impulse response. These functions are investigated next.

3.2. Non-causal rational orthonormal basis functions

The key aspect in constructing non-causal rational basis functions is specifying the fixed poles. Indeed, the used Laurent polynomials \( \psi_k(z) \in \mathbb{R}[z, z^{-1}] \) in [15, 16, 17], have all poles at \( z = 0 \), which explains why many parameters are required to accurately approximate rational systems with lightly damped poles, such as the printing system in Figure 1. In system identification, the prespecified poles are often restricted to \( \mathbb{D} \), see, e.g., [20, 21]. The main idea here is to extend this with prespecified poles in \( \mathbb{D} \cup \mathbb{E} \). For feedforward control, this enables the use of non-causal control, as is explained in Subsection 3.1.

Given the sets of poles \( \xi^s = \{\xi^s_k\}_{k=1,2,...} \subset \mathbb{D} \) and \( \xi^u = \left\{ \xi^u_k \right\}_{k=1,2,...} \subset \mathbb{E} \), define the rational basis functions

\[
\psi_k^s(z) = \frac{1 - |\xi^u_k|^2}{z - \xi^u_k} \phi_k^s(z, \xi^s), \tag{6}
\]

\[
\psi_k^u(z) = \frac{1}{1 - |\xi^s_k|^2} \phi_k^u(z, \xi^u), \tag{7}
\]

where the all-pass transfer functions \( \phi_k^s, \phi_k^u \) are defined by

\[
\phi_k^s(z, \xi^s) = \begin{cases} 
1 & \text{if } k = 1, \\
|\prod_{m=1}^{k-1} z - \xi^u_m| & \text{if } k > 1.
\end{cases}
\]

\[
\phi_k^u(z, \xi^u) = \begin{cases} 
1 & \text{if } k = 1, \\
1 - |\xi^s_k|^2 & \text{if } k > 1.
\end{cases}
\]

The set \( \{\psi_k^s\}_{k=0} \in \mathcal{H}_2 \) forms the well-known Takenaka-Malmquist functions, which are standard in system identification [20, 21], and consists of strictly causal functions. In sharp contrast, the set \( \{\psi_k^u\}_{k=0} \in \mathcal{H}_2^u \), see also [31], contains all anti-causal functions and direct feedthrough terms, e.g., select \( \xi^u_1 = 0 \), that are utilized here to enable non-causal control.
Indeed, note their poles are given by $\{1/\xi_k\}_{k=0}^\infty \in \mathbb{E}$, which induce anti-causal responses when using the bilateral Z-transform with ROC as specified in Subsection 3.1.

The basis functions (6), (7) are orthonormal with respect to the standard inner product on $L_2$: $\frac{1}{2\pi} \int_{-\pi}^\pi \phi_k(z)\phi_l(z)dz = \begin{cases} 1 & \text{if } k = l, \\ 0 & \text{if } k \neq l. \end{cases}$

When complex conjugated pole pairs are used, (6) and (7) must be adapted through a unitary transformation to obtain real-valued basis functions, see, e.g., [20, 31]. Together, (6) and (7) form the basis functions to be used:

$$\Psi(z) = [\psi_1^1(z), \psi_2^1(z), \ldots, \psi_p^1(z), \psi_1^2(z), \ldots, \psi_p^2(z)]^T.$$ (8)

Remark 4. The generalization of basis functions (6), (7) to the multi-input multi-output (MIMO) case follows conceptually analogously to the results that are described in [21, Section 10.5] for the causal case, i.e., in $\mathbb{RH}_2$.

3.3. Selection of basis poles

In view of pole sets $\xi^e$ and $\xi^o$, the following observations are made, which can be interpreted as guidelines for pole selection:

- The linear span of the basis functions $\Psi(z)$ is complete in $L_2$, $1 < p < \infty$, if and only if $\sum_{k=1}^\infty 1 - |\xi_k^e| = \infty$ and $\sum_{k=1}^\infty 1 - |\xi_k^o| = \infty$. These mild conditions on the pole locations imply that any such orthonormal basis can arbitrarily well model any system in $L_2$ [20, 31].

- Using a finite number of basis functions, the undermodeling error $|P(z) - F(z)|^2$ can be upper bounded, see, e.g., [32, Theorem VI.1], which can be appropriately extended for systems in $L_2$. Here, $F(z, \theta^e)$ denotes the best approximation of $P(z)$ in $L_2$ sense. Importantly, the bound decreases with the Euclidean distance between $\xi$ and the true poles of $P^{-1}$.

Remark 5. Depending on the order in which the poles are included in (6), (7), different bases are obtained. Methods for optimal ordering of the poles are presented in [33].

Remark 6. The orthonormality of the basis functions potentially improves numerical properties of (3) compared to general rational functions [32]. Orthonormality does not directly influence the achievable performance, since any orthonormal basis is equivalent under a linear transformation to any non-orthonormal basis with the same prespecified poles.

Remark 7. The developed parametrization recovers FIR bases as a special case: placing all poles of $\xi^e \in \mathbb{D}$ at $z = 0$ yields basis functions $\phi_k^1(z) = z^{-k}$, and all poles of $\xi^o \in \mathbb{E}$ at $z = \infty$, e.g., setting $\xi_o = 0$, yields functions $\psi_k^o(z) = z^k$. This also means that by placing part of the poles at $z = 0$ or $z = \infty$, and the others anywhere else in $\mathbb{D}$ and $\mathbb{E}$, pure delay or preview can be included in the, otherwise, rational basis.

Summarizing, in practice the pole sets $\{\xi_{l,k}\}_{k=1,2,\ldots}$ and $\{1/\xi_{l,k}\}_{k=1,2,\ldots}$ should be chosen as close as possible to the true poles of $P^{-1}_0$. To further reduce the approximation error and improve feedforward control performance, the orthonormality can be exploited by cyclically repeating these poles.

3.4. Cyclic repetition of poles

In practice, the poles of $P^{-1}_0$ are not exactly known. Interestingly, the same set of basis poles can be repeatedly added to construct a basis that is dense in $L_2$, i.e., covers any LTI dynamical system. This is called cyclic pole repetition. The idea essentially generalizes the case of Laurent polynomials from [15, 16, 17], where the order is increased by adding additional FIR coefficients, i.e., repeatedly adding poles at $z = 0$.

In particular, given a finite set of poles, the span of the basis functions is increased by cyclically repeating the sets of poles as $\xi_{k+1}^e = \xi_k^e$ and $\xi_{k+1}^o = \xi_k^o$. The repeated basis is generated from the original basis by multiplications with all-pass functions $\phi_k^o, \phi_k^o'$, see (6), (7), and hence differs only in phase. The benefits are illustrated next.

Example 2 (Benefits of non-causal ROBFs and cyclic repetition). Consider the continuous-time mechanical system

$$P^{-1}_0(s) = \prod_{i=1}^3 \frac{\omega_i^2}{s^2 + 2\zeta_i \omega_i s + \omega_i^2},$$

where the mode with natural frequency $\omega_1 = 1$ [Hz] and damping ratio $\zeta_1 = 0.1$ yields a complex pair of stable poles, and the mode with negative natural frequency $\omega_2 = -0.5$ [Hz] and $\zeta_2 = 0.2$ gives a complex pair of unstable poles. A discrete-time system $P^{-1}_0(z) \in \mathbb{RL}_2$ is obtained using zero-order-hold with sampling time $0.02$ s.

Three models of $P^{-1}_0(z)$ are estimated: a $100^{th}$ order FIR model with 50 anti-causal and 50 causal terms, and 4th and 12th order rational models with basis functions according to (6), (7). The poles $\{\xi^e\}$ and $\{1/\xi^o\}$ are chosen corresponding to the true poles of $P^{-1}_0(z)$, except the damping ratios are chosen 2 times higher. For the 12th order model, these poles are repeated three times. The parameters are obtained by minimizing

$$\int_0^\pi \left| P^{-1}_0(\omega^e) - \delta \Psi(\omega^e)(\omega^e) \right|^2 d\omega.$$ 

The results are shown in Figure 5, and the following observations are made.

- For the $100^{th}$ order FIR model, a substantial error remains. In time-domain, this can be seen from the truncated response.

- By choosing four poles in $\mathbb{D} \cup \mathbb{E}$, infinitely long impulse responses are obtained. A significant error remains due to the mismatch between the chosen poles and true poles.

- By cyclically repeating the poles three times, the modeling error is significantly reduced using only 12 parameters, and the model closely approximates the true system.

3.5. Data-driven learning using non-causal ROBFs

To summarize this section, a feedforward parametrization is developed that enables parsimonious modeling of general rational systems, and facilitates convex optimizations in data-driven
1. Choose a finite set of poles in $\mathbb{D} \cup \mathbb{E}$ approximating those of $P_z^{-1}(z)$, e.g., based on FRF measurements or prior knowledge.
2. Construct a set of non-causal basis functions $\psi_k(z) \in \mathcal{L}_2$ for the feedforward parametrization (4).
3. Select weights $W_r, W_f, W_{\Delta f}$ to define criterion $J(\theta_{j+1})$ in (2).
4. Set iteration index $j = 0$, initialize $\theta_0$, and perform the next steps.
   a. Implement $F(\theta_j)$, execute task $r_j$, and measure $e_j$.
   b. Construct solution (9) to convex optimization problem (3).
   c. Set $j \to j + 1$, and return to step 4a.
5. If the achieved performance is unsatisfactory, return to
   - step 1, and add or update fixed pole locations, or
   - step 2, and add set of basis functions through cyclic repetition of same poles, see Subsection 3.4, or
   - step 3, and retune weights $W_r, W_f, W_{\Delta f}$.

In the next section, Procedure 1 is experimentally applied to an industrial flatbed printer.

4. Experimental Application on Industrial Flatbed Printer

In this section, the proposed use of non-causal rational basis functions for data-driven feedforward tuning is experimentally validated on the industrial flatbed printer, shown in Figure 1. The experimental contributions are:

- The control performance upon task variations is compared to standard norm-optimal ILC, see, e.g., [3, 4] and Remark 1.
- The tracking performance using ROBFs is compared to pre-existing Laurent polynomials, including non-causal control actions.
- The performance benefits of cyclically repeating the poles of the basis functions are demonstrated.

4.1. Experimental setup

The considered motion system is an Océ Arizona 550 GT, see Figure 1. In contrast to standard consumer printers, the medium is fixed on the printing surface. The carriage, which contains the printheads, translates in $y$-direction along the gantry, which translates in $x$ and rotates in $R_z$. The system is actuated by three current-driven brushless electrical motors. The measurement system consists of three optical encoders, collocated with the actuators, yielding position measurements $x_L$, $x_R$ and $y$. A stabilizing multivariable feedback controller is implemented. The system is operated with sampling time 1 ms.

To clearly illustrate the benefits of the developed approach, the input and output are selected such that the system has pronounced NMP dynamics: the input is current $u_L$ [A] of the actuator on the left side of the gantry, and the output is position $x_R$ [m] on the right side of the gantry, i.e., the input and output are non-collocated. Note that in industrial practice, it is typically aimed to avoid NMP dynamics through mechanical design, e.g., by collocated actuator placement or additional actuators [34]. By closing all feedback loops, an equivalent system $P$...
is obtained, i.e. $P : u_k \rightarrow x_R$. Since all performed translations and rotations are small, the system is assumed linear. An identified frequency response function of $P$ is shown in Figure 6. The NMP system dynamics can be observed from Figure 6 by the Bode gain-phase relationship: around the anti-resonance at 7 Hz, the phase lag increases by 180 degrees. An approximate 10th order parametric model $\hat{P}$ is identified, containing three NMP zeros in $\mathbb{E}$, two of which can clearly be observed in Figure 6 at 7 Hz.

4.2. Design of basis functions

The basis functions are constructed according to Procedure 1. In step 1, the poles of the rational basis functions are chosen equal to the zeros of model $\hat{P}$, see Figure 6, hence approximating true poles of $P_{\infty}^{\infty}$, and are given by

$$\xi^e = \{0.9039, 0.9853, 0.9854 \pm 0.0457i, 0.9988\}, \quad \xi^u = \left\{0, \frac{1}{1.008 \pm 0.0477i}, 1 \right\}.$$  \hspace{1cm} (10)

$$\xi^u = \left\{0, \frac{1}{1.008 \pm 0.0477i}, 1 \right\}.$$  \hspace{1cm} (11)

Based on these, in the second step basis functions (6), (7) are constructed. Note that $\xi^u_1 = 0$ generates a feedthrough term, i.e., $\psi^u_1 = 1$. To compare the developed rational basis functions with pre-existing polynomial basis functions, the same number of causal and anti-causal Laurent polynomials is generated, i.e., $\psi_k = z^{-k}$, with $k \in [-3, 6]$. The weights in step 3 are selected $W_c = 50I$, $W_f = 10^{-7}I$, and $W_{af} = 0$. In step 4, all parameters are initialized as $\theta = 0$, i.e., only feedback is active in iteration $j = 0$.

4.3. Case I - control performance upon task variations

The performance upon task variations to non-repeating tasks is demonstrated, and compared to standard norm-optimal ILC (NO-ILC) see, e.g., [3, 4]. In particular, reference $r_1$ is performed in iterations $j = 0, 1, \ldots, 9$, which is changed to $r_2$ at $j = 10$, see Figure 7. Both signals have length $N = 2501$ samples, where $r_2$ requires less aggressive motion than $r_1$. The results are presented in Figures 8, 9 and 10, and the following observations are made.

- Using the employed non-causal rational basis functions, the performance is significantly improved compared to pre-existing Laurent polynomials. Note that the only difference are the pole locations: poles at $z = 0$ for polynomials $z^{-k}$, in contrast to to poles (10) and (11) in $\mathbb{D}$ and $\mathbb{E}$.

- Standard NO-ILC achieves the best tracking performance in iterations 1 to 9. However, performance is severely deteriorated upon the task change at $j = 10$, see Figure 9.

- Through the developed framework for data-driven learning with basis functions, the performance upon task variations is significantly improved.

- The three anti-causal basis functions (7) with poles in $\mathbb{E}$ generate infinitely long pre-actuation to effectively compensate the non-minimum phase dynamics of $P$. This can directly be seen in Figure 10: a control action is applied before the start of motion task $r$. The Laurent polynomials generate only a finite amount of preview, i.e., 3 discrete time steps.

4.4. Case II - cyclic pole repetition

Next, the benefits on tracking performance of cyclically repeating the basis poles are demonstrated. The sequences (10) and (11) are repeated once, hence 10 strictly causal and 8 non-causal basis functions are generated. $W_c$ is increased to $W_c = 10^5I$. The experimental results are presented in Figure 11, and the following observation is made.

- By repeating the poles once, the tracking performance $\|e_0\|_2$ is decreased by 17%: the residual cumulative power at the Nyquist frequency (this is the RMS value $\|e_0\|_2^2$) is $(1 - 0.17)^2 \times 100\% = 69\%$. The main performance improvement is achieved below 20 Hz.

In conclusion, the developed parametrization of $F(\theta)$ using non-causal rational orthonormal basis functions meets the requirements posed in Section 2.3. The rational basis functions enable a convex optimization problem, parsimonious modeling of inverse systems $P_0^{-1} \in \mathcal{RL}_2$, and their orthonormality can be exploited for further enhancing performance through cyclic pole repetitions.
Figure 8: Performance criterion values $J(\theta_j)$ (lower values indicate better control performance). Through data-driven tuning of feedforward controllers, enhanced performance at task variations ($j = 10$) is obtained compared to norm-optimal ILC ($\bigcirc$). Through the proposed use of non-causal rational basis functions ($\bigtriangledown$), increased tracking performance is achieved compared to pre-existing Laurent polynomial functions ($\times$).

Figure 9: Tracking error signals: $e_9$ (dashed) and $e_9$ (solid) with $r^1$ (left column); $e_{10}$ (dashed) and $e_{10}$ (solid) with $r_2$ (right column). Non-causal rational basis functions (top row); Laurent polynomial basis functions (middle row); norm-optimal ILC (bottom row). With basis functions, performance is maintained upon reference changes. The proposed rational basis functions achieve the smallest errors. Standard norm-optimal ILC leads to a performance deterioration at the reference change, i.e., an increase from $e_9$ to $e_{10}$.

Figure 10: Feedforward signals $f_9$. The non-causal rational basis functions ($\bigtriangledown$) with poles in $\mathbb{D}$ and $\mathbb{E}$ and norm-optimal ILC ($\bigcirc$) enable pre-actuation and post-actuation. This improves performance compared to the (non-causal) Laurent polynomial functions ($\bigtriangleup$), which enable only a small and finite amount of preview. The vertical dotted lines indicate the start and end of motion task $r^1$.

Figure 11: Results of cyclic pole repetition. Top: time-domain tracking errors $e_9$; bottom: sample normalized cumulative power spectrum (CPS) of $e_9$. By repeating the poles once ($\cdot$) compared to the initial pole sets (10) and (11) ($\cdot$), the power of $e_9$ is reduced at frequencies where $r$ has dominant power.

5. Conclusions

The data-driven feedforward tuning approach in this paper enables high tracking accuracy for varying tasks in combination with an analytical solution, both of which are foreseen to facilitate adoption in control applications. Experimental demonstrations on an industrial flatbed printer show very promising results, including task flexibility and high accuracy through non-causal control and cyclic pole repetitions. The main underlying idea in this paper is the use of rational orthonormal basis functions.
functions in $L^2$, which enables i) convex optimization, ii) parsimonious modeling of general rational systems, and iii) infinite pre-actuation and post-actuation. The approach is in sharp contrast with the use of rational orthonormal basis functions in system identification, which is typically focused on parametrizing models in $H_2$. Ongoing research is aimed at including nonlinear behavior in the basis functions, such as friction characteristics and position-dependent dynamics.

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