Determination of $2\beta_s$ in $B_s^0 \to J/\psi K^+K^-$ Decays in the Presence of a $K^+K^-$ S-Wave Contribution

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Abstract

We present the complete differential decay rates for the process $B_s^0 \to J/\psi K^+K^-$ including S-wave and P-wave angular momentum states for the $K^+K^-$ meson pair. We examine the effect of an S-wave component on the determination of the CP violating phase $2\beta_s$. Data from the B-factories indicate that an S-wave component of about $10\%$ may be expected in the $\phi(1020)$ resonance region. We find that if this contribution is ignored in the analysis it could cause a bias in the measured value of $2\beta_s$ towards zero of the order of $10\%$. When including the $K^+K^-$ S-wave component we observe an increase in the statistical error on $2\beta_s$ by less than $15\%$. We also point out the possibility of measuring the sign of $\cos 2\beta_s$ by using the interference between the $K^+K^-$ S-wave and P-wave amplitudes to resolve the strong phase ambiguity. We conclude that the S-wave component can be properly taken into account in the analysis.

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1 Introduction

The decay $B_s^0 \rightarrow J/\psi K^+ K^-$ is a golden channel for the measurement of the $B_s^0$ mixing phase $-2\beta_s$ which is a very sensitive probe of new physics. It has been extensively studied [1, 2, 3, 4, 5, 6, 7, 8]. In the decay $B_s^0 \rightarrow J/\psi \phi$, followed by a two-body decay $\phi(1020) \rightarrow K^+ K^-$, the $K^+ K^-$ meson pair is in an orbital P-wave amplitude. However, in the vicinity of the $\phi(1020)$ mass, the $K^+ K^-$ system can have contributions from other partial waves. The same comment holds for the $K^+ K^-$ system in the decay channels $B^0 \rightarrow K^+ K^- K^0_S$ and $D^0 \rightarrow K^+ K^- \pi^0$. The BaBar experiment showed that in these decays the S-wave and P-wave contributions dominate in the mass range above threshold up to 1.1 GeV/$c^2$ [9, 10]. In both cases there is a dominant resonant $\phi(1020)$ contribution. In addition an S-wave contribution are found to be necessary to describe the data. These results motivated us to investigate the effects of a possible S-wave contribution to $B_s^0 \rightarrow J/\psi K^+ K^-$ in the $\phi(1020)$ mass region.

In the decay $B_s^0 \rightarrow J/\psi K^+ K^-$ the K$^+ K^-$ system can only arise from a $s\bar{s}$ quark pair while in $B^0 \rightarrow K^+ K^- K^0_S$ and $D^0 \rightarrow K^+ K^- \pi^0$ it can have contributions from both $s\bar{s}$ and $d\bar{d}$. This makes it difficult to give a quantitative estimate for the S-wave component. In reference [11] the S-wave $K^+ K^-$ contribution under the $\phi(1020)$ peak is estimated to be $5 - 10\%$ for decay modes in which the $K^+ K^-$ arises from an $s\bar{s}$ quark pair. In this study we consider an S-wave of similar magnitude and assess its impact on the determination of the weak mixing phase $-2\beta_s$.

2 Time-dependent angular distributions in the decay $B_s^0 \rightarrow J/\psi K^+ K^-$ including S-wave contributions

We consider P- and S-wave amplitudes in the decay $B_s^0 \rightarrow J/\psi K^+ K^-$ where the invariant mass of the $K^+ K^-$ meson pair is in the $\phi(1020)$ mass region and the $J/\psi$ meson decays into a
\(\mu^+\mu^-\) pair. The S-wave contribution can be non-resonant or due to the \(f_0(980)\) resonance\(^1\). We define decay amplitudes for the \(B_s^0 \rightarrow J/\psi K^+K^-\) by \(A = (A_0, A_\parallel, A_\perp, A_S)\). Here \(A_0\), \(A_\parallel\) and \(A_\perp\) are the three P-wave amplitudes consistent with the \(K^+K^-\) system decaying via the \(\phi(1020)\) resonance. \(A_S\) is the amplitude for a possible S-wave contribution in the \(K^+K^-\) system. The amplitudes for the conjugate decay \(\bar{B}_s^0 \rightarrow J/\psi K^+K^-\) are denoted by \(\bar{A} = (\bar{A}_0, \bar{A}_\parallel, \bar{A}_\perp, \bar{A}_S)\), which, in the absence of direct CP violation, are related to \(A\) by \(A_0 = \bar{A}_0, A_\parallel = \bar{A}_\parallel, A_\perp = -\bar{A}_\perp\) and \(A_S = -\bar{A}_S\). Note that \(A_0\) and \(A_\parallel\) are CP-even whereas \(A_\perp\) and \(A_S\) are CP-odd. The amplitudes \((A_0, A_\parallel, A_\perp)\) and the amplitude \(A_S\) may have different dependences on the mass \(m_{K^+K^-}\) of the \(K^+K^-\) system. However, in sufficiently small bins of \(m_{K^+K^-}\), such as the narrow mass region around the \(\phi(1020)\) resonance, the dependences of the amplitudes on \(m_{K^+K^-}\) can be neglected.

We define the total P-wave strength, \(A_P^2 \equiv |A_0|^2 + |A_\parallel|^2 + |A_\perp|^2\), the longitudinal and perpendicular polarisation fractions relative to the P-wave strength \(R_\parallel \equiv |A_\parallel|^2/A_P^2\), and \(R_\perp \equiv |A_\perp|^2/A_P^2\), and the S-wave fraction, \(R_S \equiv |A_S|^2/(A_P^2 + |A_S|^2)\). The phases of these decay amplitudes are defined by \(A_j = |A_j|e^{i\delta}\), where \(j = 0, ||, \perp, S\). As only the relative strong phase differences can be measured we adopt the convention \(\delta_0 = 0\).

An angular analysis is required to disentangle the different CP eigenstates on a statistical basis. The angular observables are denoted as the helicity angles \(\Omega = (\theta_l, \theta_K, \varphi)\). Here \(\theta_l\) is the angle between the \(\mu^+\) momentum and the direction opposite to the \(B_s^0\) momentum in the \(J/\psi\) rest frame; \(\theta_K\) is the angle between the \(K^+\) momentum and the direction opposite to the \(B_s^0\) momentum in the rest frame of the \(K^+K^-\) system; \(\varphi\) is the angle between the decay planes of the \(J/\psi \rightarrow \mu^+\mu^-\) and the \(K^+K^-\) pair, when going from the positive kaon to the positive lepton with a rotation around the opposite direction of the \(B_s^0\) momentum in the \(J/\psi\) rest frame.

The differential decay rate for a \(B_s^0\) meson produced at time \(t = 0\) decaying as \(B_s^0 \rightarrow J/\psi K^+K^-\) at proper time \(t\) is given by

\[
\frac{d^4\Gamma(B_s^0 \rightarrow J/\psi K^+K^-)}{dt \, d\cos\theta \, d\cos\psi \, d\varphi} \propto \sum_{k=1}^{10} h_k(t) f_k(\Omega), \tag{2.1}
\]

whereas the differential decay rate for an initial \(\bar{B}_s^0\) meson is given by

\[
\frac{d^4\Gamma(\bar{B}_s^0 \rightarrow J/\psi K^+K^-)}{dt \, d\cos\theta \, d\cos\psi \, d\varphi} \propto \sum_{k=1}^{10} \bar{h}_k(t) f_k(\Omega). \tag{2.2}
\]

Each of the \(h_k(t), \bar{h}_k(t)\) and \(f_k(\Omega)\) for \(k = 1 - 10\) are defined in Table I. In total there are four amplitude-squared terms for the three polarisations of the P-waves and the S-wave component plus six interference terms.

The time-dependence of the ten functions \(h_k(t)\) for an initial \(B_s^0\) meson state can be written as:

\[
|A_0(t)|^2 = |A_0|^2 e^{-\Gamma_s t} \left[ \cosh \left( \frac{\Delta \Gamma_s t}{2} \right) - \cos \Phi \sinh \left( \frac{\Delta \Gamma_s t}{2} \right) + \sin \Phi \sin(\Delta m_s t) \right], \tag{2.3}
\]

\[
|A_\parallel(t)|^2 = |A_\parallel|^2 e^{-\Gamma_s t} \left[ \cosh \left( \frac{\Delta \Gamma_s t}{2} \right) - \cos \Phi \sinh \left( \frac{\Delta \Gamma_s t}{2} \right) + \sin \Phi \sin(\Delta m_s t) \right], \tag{2.4}
\]

\[\text{\(1\)The mass dependence of the \(f_0(980)\) is distorted as the central value of the resonance is below threshold.}\]
Table 1: Definition of the functions $h_k(t)$, $\tilde{h}_k(t)$ and $f_k(\theta_1, \theta_K, \varphi)$ of Eq. 2.1 and 2.2

\begin{align*}
|A_\perp(t)|^2 &= |A_\perp|^2 e^{-\Gamma_\perp t} \left[ \cosh \left( \frac{\Delta \Gamma_s t}{2} \right) + \cos \Phi \sinh \left( \frac{\Delta \Gamma_s t}{2} \right) - \sin \Phi \sin(\Delta m_s t) \right], \quad (2.5) \\
\Re\{A_\parallel(t)A_\perp(t)\} &= |A_\parallel| |A_\perp| e^{-\Gamma_\perp t} \left[ -\cos(\delta_\perp - \delta_\parallel) \sin \Phi \sinh \left( \frac{\Delta \Gamma_s t}{2} \right) \\
&\quad + \sin(\delta_\perp - \delta_\parallel) \cos(\Delta m_s t) - \cos(\delta_\perp - \delta_\parallel) \cos \Phi \sin(\Delta m_s t) \right], \quad (2.6) \\
\Im\{A_\parallel(t)A_\perp(t)\} &= |A_\parallel| |A_\perp| e^{-\Gamma_\perp t} \left[ -\cos(\delta_\perp - \delta_0) \sin \Phi \sinh \left( \frac{\Delta \Gamma_s t}{2} \right) \\
&\quad + \sin \Phi \sin(\Delta m_s t) \right], \quad (2.7) \\
|A_S(t)|^2 &= |A_S|^2 e^{-\Gamma_s t} \left[ \cosh \left( \frac{\Delta \Gamma_s t}{2} \right) + \cos \Phi \sinh \left( \frac{\Delta \Gamma_s t}{2} \right) - \sin \Phi \sin(\Delta m_s t) \right], \quad (2.9) \\
\Re\{A_S(t)A_\parallel(t)\} &= |A_S| |A_\parallel| e^{-\Gamma_\perp t} \left[ -\sin(\delta_\parallel - \delta_S) \sin \Phi \sinh \left( \frac{\Delta \Gamma_s t}{2} \right) \\
&\quad + \cos(\delta_\parallel - \delta_S) \cos(\Delta m_s t) - \sin(\delta_\parallel - \delta_S) \cos \Phi \sin(\Delta m_s t) \right], \quad (2.10) \\
\Im\{A_S(t)A_\parallel(t)\} &= |A_S| |A_\parallel| e^{-\Gamma_\perp t} \sin(\delta_\parallel - \delta_S) \left[ \cosh \left( \frac{\Delta \Gamma_s t}{2} \right) + \cos \Phi \sinh \left( \frac{\Delta \Gamma_s t}{2} \right) \\
&\quad - \sin \Phi \sin(\Delta m_s t) \right], \quad (2.11) \\
\Re\{A_S(t)A_0(t)\} &= |A_S| |A_0| e^{-\Gamma_s t} \left[ -\sin(\delta_0 - \delta_S) \sin \Phi \sinh \left( \frac{\Delta \Gamma_s t}{2} \right) \\
&\quad + \cos(\delta_0 - \delta_S) \cos(\Delta m_s t) - \sin(\delta_0 - \delta_S) \cos \Phi \sin(\Delta m_s t) \right], \quad (2.12)
\end{align*}

where $\Phi = -2\beta_s$, $\Delta m_s$, $\Delta \Gamma_s$ and $\Gamma_s$ denote the weak mixing phase, mass difference, decay width difference and average decay width of the $B_s^0$-$\bar{B}_s^0$ system, respectively. Here we have assumed that each of the decay amplitudes in $\mathbf{A}$ is dominated by a single weak phase, therefore a common effective $2\beta_s$ can be used for all CP eigenstates. The time evolution functions $\tilde{h}_k(t)$ for an initial $\bar{B}_s^0$ meson can be obtained by reversing the sign of each term proportional to $\sin(\Delta m_s t)$ or $\cos(\Delta m_s t)$ in $h_k(t)$. 

3
3 Measuring $2\beta_s$ in the presence of a $K^+K^-$ S-wave

In this section we investigate how the measurement of $2\beta_s$ is affected by the presence of a possible $K^+K^-$ S-wave contribution. We use Monte Carlo simulated toy data based on the differential decay rate expressions of Section 2. We generate signal decays only and ignore backgrounds underneath the $B^0_s$ mass peak as well as all detector effects. The inclusion of these effects does not alter the qualitative results of this study.

We assume a tagging efficiency $\epsilon_{\text{tag}} = 56\%$ and a wrong tag probability $\omega_{\text{tag}} = 33\%$, which correspond approximately to the expected flavour tagging performance for this channel at the LHCb experiment [12]. In Table 2 we summarize the values of the physical parameters used to generate the toy data sets.

We generate 500 data sets for different scenarios where we vary the values of the S-wave fraction $R_S$ and its phase $\delta_S$ and the weak phase $-2\beta_s$. Each data set contains 30000 signal events corresponding to approximately one quarter of a nominal LHCb year of 2 fb$^{-1}$.

| $\Delta m_s$ | $\Gamma_s$ | $\Delta \Gamma_s$ | $\delta_0$ | $\delta_\parallel$ | $\delta_\perp$ | $R_\parallel$ | $R_\perp$ | $R_S$ | $\delta_S$ | $2\beta_s$ |
|--------------|-----------|------------------|----------|-------------------|--------------|------------|---------|-------|----------|---------|
| Input        | 17.8 ps$^{-1}$ | 0.68 ps$^{-1}$ | 0.05 ps$^{-1}$ | 0 | -2.93 | 2.91 | 0.207 | 0.233 | vary | vary | vary |
| Fit          | fix       | fix              | fix      | fix               | float       | float     | float   | float* | float | float |

$^*$ $R_S$ is fixed to 0 when the S-wave component is neglected.

Table 2: Values of the physical parameters used in the generation of signal decays and how these parameters are treated in the fit.

We perform fits to each data set where $2\beta_s$, $R_S$, $\delta_S$, $R_\parallel$, $\delta_\parallel$, $R_\perp$, $\delta_\perp$ are free parameters and all other parameters are kept fixed. We also perform fits where the S-wave component is present in the generated toy data, but ignored in the fit ($R_S$ is set to 0) in order to investigate the bias in the determination of $2\beta_s$.

The results of these fits for the statistical error and mean value of the weak phase $-2\beta_s$ are summarized in Table 3, 4 and 5 for several different scenarios with $-2\beta_s = -0.0368$, $-2\beta_s = -0.2$ and $-2\beta_s = -0.5$, respectively. As an example, in Figure 1 we show the distributions of the fitted values of $-2\beta_s$ for $R_S = 0.1$, $\delta_S = \pi/2$ and $-2\beta_s = -0.5$ for both the S-wave fraction $R_S$ fixed to zero and $R_S$ left free in the fits. In Figure 2 we show the distributions of the fitted values of $R_S$ and the strong phase of the S-wave component $\delta_S$ for the same case with $R_S$ left free in the fit. It can be seen that when all parameters are fitted the results are unbiased, but when it is wrongly assumed that $R_S = 0$, the result for $-2\beta_s$ acquires a bias with regard to the true input value.

Figure 3 shows the bias in $-2\beta_s$ from neglecting an S-wave component with $R_S = 0.1$ and $\delta_S = \pi/2$ versus the value of $-2\beta_s$ used to generate the data sets. A linear dependence is observed, which demonstrates that the bias in $-2\beta_s$ is proportional to the true value of $-2\beta_s$. From Tables 3, 4 and 5 we observe biases for these scenarios which range from 7 – 17% in the measurement of $2\beta_s$ if an S-wave component is present, but left unaccounted for in the fits. The bias moves the measured value of $2\beta_s$ towards zero. This implies that the neglected CP-odd S-wave contribution has a bigger probability to be mis-identified as the CP-even longitudinal or parallel components than as the CP-odd perpendicular component. Therefore, although the bias from neglecting an S-wave contribution is unlikely to lead to false signal of new physics, it will cause a loss of...
sensitivity to new physics. On the other hand, including the S-wave in the fit removes the bias in the central value of $2\beta_s$ at a cost of an increase of less than 15% in the statistical error.

| $R_S$ | Float $R_S$ in fit | Fix $R_S$ to 0 in fit |
|-------|-------------------|---------------------|
| $R_S = 0$ | $\sigma(2\beta_s) = 0.048$, Mean$(2\beta_s) = 0.035$ | $\sigma(2\beta_s) = 0.045$, Mean$(2\beta_s) = 0.038$ |
| $R_S = 0.1, \delta_S = \pi/2$ | $\sigma(2\beta_s) = 0.048$, Mean$(2\beta_s) = 0.035$ | $\sigma(2\beta_s) = 0.045$, Mean$(2\beta_s) = 0.032$ |
| $R_S = 0.1, \delta_S = 0$ | $\sigma(2\beta_s) = 0.054$, Mean$(2\beta_s) = 0.040$ | $\sigma(2\beta_s) = 0.048$, Mean$(2\beta_s) = 0.036$ |
| $R_S = 0.05, \delta_S = \pi/2$ | $\sigma(2\beta_s) = 0.048$, Mean$(2\beta_s) = 0.040$ | $\sigma(2\beta_s) = 0.045$, Mean$(2\beta_s) = 0.036$ |
| $R_S = 0.05, \delta_S = 0$ | $\sigma(2\beta_s) = 0.055$, Mean$(2\beta_s) = 0.038$ | $\sigma(2\beta_s) = 0.047$, Mean$(2\beta_s) = 0.032$ |

Table 3: Statistical errors and mean values of $2\beta_s$ from 500 fits for different scenarios with $2\beta_s = 0.0368$. The errors on $\sigma(2\beta_s)$ and mean$(2\beta_s)$ are approximately 0.003 and 0.002, respectively. The same data sets are used to obtain the results in the second and third columns.

| $R_S$ | Float $R_S$ in fit | Fix $R_S$ to 0 in fit |
|-------|-------------------|---------------------|
| $R_S = 0$ | $\sigma(2\beta_s) = 0.052$, Mean$(2\beta_s) = 0.199$ | $\sigma(2\beta_s) = 0.044$, Mean$(2\beta_s) = 0.198$ |
| $R_S = 0.1, \delta_S = \pi/2$ | $\sigma(2\beta_s) = 0.056$, Mean$(2\beta_s) = 0.202$ | $\sigma(2\beta_s) = 0.049$, Mean$(2\beta_s) = 0.170$ |
| $R_S = 0.1, \delta_S = 0$ | $\sigma(2\beta_s) = 0.049$, Mean$(2\beta_s) = 0.197$ | $\sigma(2\beta_s) = 0.048$, Mean$(2\beta_s) = 0.182$ |
| $R_S = 0.05, \delta_S = \pi/2$ | $\sigma(2\beta_s) = 0.053$, Mean$(2\beta_s) = 0.198$ | $\sigma(2\beta_s) = 0.048$, Mean$(2\beta_s) = 0.180$ |

Table 4: Statistical errors and mean values of $2\beta_s$ from 500 fits for different scenarios with $2\beta_s = 0.2$. The errors on $\sigma(2\beta_s)$ and mean$(2\beta_s)$ are approximately 0.003 and 0.002, respectively. The same data sets are used to obtain the results in the second and third columns.

| $R_S$ | Float $R_S$ in fit | Fix $R_S$ to 0 in fit |
|-------|-------------------|---------------------|
| $R_S = 0$ | $\sigma(2\beta_s) = 0.059$, Mean$(2\beta_s) = 0.501$ | $\sigma(2\beta_s) = 0.051$, Mean$(2\beta_s) = 0.501$ |
| $R_S = 0.1, \delta_S = \pi/2$ | $\sigma(2\beta_s) = 0.061$, Mean$(2\beta_s) = 0.501$ | $\sigma(2\beta_s) = 0.053$, Mean$(2\beta_s) = 0.415$ |
| $R_S = 0.1, \delta_S = 0$ | $\sigma(2\beta_s) = 0.051$, Mean$(2\beta_s) = 0.506$ | $\sigma(2\beta_s) = 0.052$, Mean$(2\beta_s) = 0.417$ |
| $R_S = 0.05, \delta_S = \pi/2$ | $\sigma(2\beta_s) = 0.053$, Mean$(2\beta_s) = 0.501$ | $\sigma(2\beta_s) = 0.048$, Mean$(2\beta_s) = 0.463$ |
| $R_S = 0.05, \delta_S = 0$ | $\sigma(2\beta_s) = 0.053$, Mean$(2\beta_s) = 0.501$ | $\sigma(2\beta_s) = 0.049$, Mean$(2\beta_s) = 0.461$ |

Table 5: Statistical errors and mean values of $2\beta_s$ from 500 fits for different scenarios with $2\beta_s = 0.5$. The errors on $\sigma(2\beta_s)$ and mean$(2\beta_s)$ are approximately 0.003 and 0.002, respectively. The same data sets are used to obtain the results in the second and third columns.
Figure 1: Distributions of the fitted values of $-2\beta_s$ for the scenario $R_S = 0.1, \delta_S = \pi/2, 2\beta_s = 0.5$. The left and right plots are obtained with or without fixing $R_S$ to 0 in fitting the data, respectively.

Figure 2: Distributions of the fitted values of $R_S$ and $\delta_S$ for the scenario $R_S = 0.1, \delta_S = \pi/2, 2\beta_s = 0.5$ without fixing $R_S$ to 0 in fitting the data.

Figure 3: The bias in $-2\beta_s$ from neglecting an S-wave component with $R_S = 0.1$ and $\delta_S = \pi/2$ versus the value of $-2\beta_s$ used to generate the data sets. The bias is the difference of the mean of the fitted to the generated $-2\beta_s$ values. A linear fit is superimposed on the graph.
4 Measuring $\cos 2\beta_s$

In Eqs. 2.1 and 2.2 one observes that the differential decay rates are invariant under the transformation

$$\left(\delta_\parallel - \delta_0, \delta_\perp - \delta_0, \delta_S - \delta_0, -2\beta_s, \Delta \Gamma_s\right) \leftrightarrow \left(\delta_0 - \delta_\parallel, \pi + \delta_0 - \delta_\perp, \delta_0 - \delta_S, \pi - (-2\beta_s), -\Delta \Gamma_s\right).$$

(4.1)

As a consequence the measurement of $2\beta_s$ is subject to a two-fold ambiguity, which is equivalent to $\cos 2\beta_s$ transforming into $-\cos 2\beta_s$. A measurement of $\cos 2\beta_s$ including its sign would allow us to resolve this ambiguity.

If the interference between the P-wave and S-wave amplitudes were to be significant in the $\phi(1020)$ mass region, we could use this effect to measure $\cos 2\beta_s$, in the same way as BaBar measured $\cos 2\beta$ in $B^0 \to J/\psi K^0\pi^0$ [13]. This requires measuring $\delta_S - \delta_0$, the strong phase difference between the S-wave and the longitudinal P-wave, as a function of the $K^+K^-$ mass in the $\phi(1020)$ mass region. When plotting this function, two branches are expected with each corresponding to a different solution for the weak phase (see Figure 4 left). It is straightforward to choose the physical solution since the phase of the P-wave Breit-Wigner amplitude is expected to rise rapidly through the $\phi(1020)$ mass region (dashed red curve in Figure 4 right), while the phase of the S-wave amplitude, which can be described either by a coupled channel Breit-Wigner function in case of an $f_0$ contribution or by a constant term in case of a non-resonant contribution, is expected to vary relatively slowly (dotted green curve in Figure 4 right), resulting in $\delta_S - \delta_0$ rapidly falling with increasing $K^+K^-$ mass (solid blue curves in Figure 4).

Figure 4: An example to illustrate the dependence of the strong phase of the S-wave $\delta_S$, of the strong phase of the longitudinal P-wave $\delta_0$, and of their difference $\delta_S - \delta_0$, on the $K^+K^-$ mass. Left: the solid blue curve is the physical solution for $\delta_S - \delta_0$ and the dashed black curve shows the mirror solution. Right: the dashed red, dotted green and solid blue curves are for $\delta_0$, $\delta_S$, and $\delta_S - \delta_0$, respectively.

Below we use a Monte Carlo simulated toy data set to demonstrate the feasibility of this method in measuring the sign of $\cos 2\beta_s$. We generate 30000 $B^0 \to J/\psi K^+K^-$ events in the $K^+K^-$ mass region between 1 and 1.05 GeV/$c^2$, roughly corresponding to 0.5 fb$^{-1}$ of integrated luminosity. The P-wave and $f_0$ contributions are included coherently. The values of the parameters used to generate the toy data set are the same as in Table 2 except that we set $-2\beta_s = -0.0368$, and that the values of both $R_S$ and $\delta_S$ depend on...
the $K^+K^-$ mass. The $f_0$ contribution accounts for about 10% of the total decay rate in the given mass region, as is shown in Figure 5.

![Figure 5: The data points correspond to the $K^+K^-$ mass distribution of a generated sample of $B^0_s \rightarrow J/\psi K^+K^-$ events including 10% $f_0$ contribution in the mass region. The dotted red curve indicates the $f_0$ contribution.](image)

5 Conclusions

In the decay $B^0_s \rightarrow J/\psi K^+K^-$ we expect that a $K^+K^-$ S-wave contribution in the narrow $\phi(1020)$ mass region could be as large as 10%. The full differential decay rates for this decay including the S-wave contribution have been presented. We have considered a range
Figure 6: The fitted values of $\delta_S - \delta_0$ versus $K^+K^-$ mass are shown in red and black data points, corresponding to opposite values of $\cos 2\beta_s$. The blue curve shows the dependence of $\delta_S - \delta_0$ on $K^+K^-$ mass implemented in simulation.

of scenarios which include S-wave components of 5% and 10%. We have shown that within these scenarios, if an S-wave component is ignored in the analysis, the measurement of the weak phase $-2\beta_s$ would be biased by between 7% and 17% towards zero. We have demonstrated that by properly allowing for this S-wave component in the fit, an unbiased measurement of $2\beta_s$ may be obtained with a slightly increased statistical error. Finally, we have shown that the interference between the $K^+K^-$ S-wave and P-wave amplitudes can be used to resolve the two-fold ambiguity in the measurement of the weak phase $-2\beta_s$.

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