Vortex molecule, fractional flux quanta, and interband phase difference soliton in multi-band superconductivity and multi-component superconductivity

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Abstract. Variety of a vortex and lattice of vortex in two-component superconductivity are
addressed. The balance between an energy gain of the magnetic energy forming the “sub-unit
flux quantum” and cost for inter-component Josephson interaction deforms an axisymmetric
vortex into a non-axisymmetric one, which is a so-called vortex molecule. To deal with this
situation, a crude formalism is introduced, in which London equation for magnetic field and a
sine-Gordon equation for the inter-component Josephson interaction are separated. Giving an
attention to the Josephson interaction, we introduce a “quantum-phase domain”.

1. Introduction
It is widely recognized that multi-component superconductivity, which can be realized in a multi-band
superconductor, have an important internal degree of freedom, namely, the interband phase difference
[1,2].

Leggett has extensively discussed a collective excitation relevant to this degree of freedom, which
resembles a phonon consisting of a quantum phase and a pair density fluctuation. However, other
researchers consider that it is rather unlikely that this excitation plays a role in the superconducting
phenomenon itself. Leggett pointed out himself the difficulty of measuring effects relating to this
mechanism.

However, earlier this decade, topological excitation of the interband phase difference was
theoretically demonstrated [3] and this phenomenon was experimentally demonstrated in 2006 [4]. A
profound consequence of this discovery is the fractional quantization of the magnetic flux generated
by solitons.

These phenomena are strongly related to non-trivial vortices found in ³He triplet superfluidity such
as the spin-mass vortices, coreless vortices and sheet-like vortices [5]. In addition, the analogy of this
phenomenon with fractional charge and confinement of the quark and gluon particles was pointed out \[6\]. One has also observed single winding vortices in Bose-Einstein condensation of alkaline atomic gases having multiple components. It is considered that a similar quantum phase topology has been realized \[7\].

A brand-new physics pertaining to interband (or inter-component) phase-difference solitons (\(i\)-soliton) is very stimulating and has deep significance for the macroscopic quantum state itself. In this report, we discuss some of the peculiar properties of the multi-component superconductivity.

2. Separation of circulating current and \(i\)-soliton current.

For a bright outlook on multi-component superconductivity, separating the circulating current and \(i\)-soliton current gives a good prescription. It is some extension of the London approximation, where it is provided the magnitude of the order of each component is constant outside of a core of a vortex. We can separate London equation \[8\] and sine-Gordon equation for the \(i\)-soliton. \[3\] \[9\]. In this section, we briefly introduce how to separate these two formulae.

Let us start the two-component Ginzburg-Landau equation. In this equation, we introduce the minimal interaction term, which is inter-component Josephson interaction term adding the ordinary Ginzburg-Landau equation \[3\] \[10\]:

\[
f = \sum_{\nu} \left( \alpha_{\nu} |\psi_{\nu}|^2 + \frac{\beta_{\nu}}{2} |\psi_{\nu}|^4 + \frac{1}{2m_{\nu}^*} \left( \frac{\hbar}{i} \nabla - \frac{e^*}{c} A \right) |\psi_{\nu}|^2 \right) + 2\gamma |\psi_{2}| \cos(\theta_{2} - \theta_{1}) + \frac{\hbar^2}{8\pi}.
\]

\(f\) is the free energy density, \( |\psi_{\nu}|^2 \) and \( m_{\nu}^* \) are the density and effective mass of BCS pair for each component indexed by \(\nu\), \(h\) and \(A\) are magnetic field and vector potential, \(\alpha_{\nu}, \beta_{\nu}\) and \(\gamma\) are some constants, \(e^*\) is the charge of BCS pair and \(c\) is the velocity of light.

Introducing the quantum phase for each component, \(\psi_{\nu} = |\psi_{\nu}| \exp(i\theta_{\nu})\) and providing the “London approximation”, where \( |\psi_{\nu}| \) is constant outside of the core of the vortex, eq. (1) can be transformed as following:

\[
f = \sum_{\nu} \left( \alpha_{\nu} |\psi_{\nu}|^2 + \frac{\beta_{\nu}}{2} |\psi_{\nu}|^4 + \frac{|\psi_{\nu}|^2}{2m_{\nu}^*} \left( \frac{\hbar}{i} \nabla \theta_{\nu} - \frac{e^* A}{c} \right)^2 \right) + 2\gamma |\psi_{2}| \cos(\theta_{2} - \theta_{1}) + \frac{\hbar^2}{8\pi}.
\]

The current given by \(J = e^* \left( \frac{|\psi_{1}|^2}{2m_{1}^*} h\nabla \theta_{1} + \frac{|\psi_{2}|^2}{2m_{2}^*} h\nabla \theta_{2} \right) - \frac{e^* A}{c} \left( \frac{|\psi_{1}|^2}{2m_{1}^*} + \frac{|\psi_{2}|^2}{2m_{2}^*} \right) \). We introduce a new (virtual) quantum phase, \(\theta_{\text{circular}}\) and \(\theta_{\text{soliton}}\), contributing to the circulating current, \(J_{\text{circular}}\) and that contributing a “soliton current” \(J_{\text{soliton}}\), which becomes zero by compensation between two components,

\[
\theta_{\nu} = \theta_{\text{circular}} + \theta_{\text{soliton}} \quad ,
\]

\[
\theta_{\text{circular}} = \theta_{\text{circular}} \quad ,
\]

The total current is given by \(J = J_{\text{circular}} + J_{\text{soliton}}\), where \(J_{\text{circular}}\) and \(J_{\text{soliton}}\) are given as follows:

\[
J_{\text{circular}} = -e^* \left( \frac{|\psi_{1}|^2}{2m_{1}^*} h\nabla \theta_{\text{circular}} - \frac{e^* A}{c} \right), \quad J_{\text{soliton}} = -e^* \left( \frac{|\psi_{2}|^2}{2m_{2}^*} h\nabla \theta_{\text{circular}} - \frac{e^* A}{c} \right).
\]

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\[ J_{\text{soliton}} = e^* \left( \frac{\psi_1^2}{2m_1^*} \hbar \nabla \theta_{\text{soliton1}} + \frac{\psi_2^2}{2m_2^*} \hbar \nabla \theta_{\text{soliton2}} \right) = 0. \] (4-2)

We can consider eq. (3) is a transformation of variables under the condition of eq. (4-2). Introducing the inter-component phase difference, \( \phi_{\text{soliton}} = \theta_1 - \theta_2 = \theta_{\text{soliton1}} - \theta_{\text{soliton2}} \), eq. (1) can be considered as follows:

\[
\begin{align*}
 f &= \left( \sum_{\nu} e^* |\psi_{\nu}|^2 + \frac{B}{2} |\psi_{\nu}|^4 \right) + \left( \frac{\psi_1^2}{2m_1^*} + \frac{\psi_2^2}{2m_2^*} \right) \hbar \nabla \theta_{\text{circular}} - \frac{e^* A}{c} \right)^2 + \frac{\hbar^2}{8\pi} \\
 &+ \frac{|\psi_1|^2 |\psi_2|^2}{m_1^* + m_2^*} \hbar^2 (\nabla \phi_{\text{soliton}})^2 + 2\gamma |\psi_1| |\psi_2| \cos(\phi_{\text{soliton}}).
\end{align*}
\] (5)

The first three terms give the formula for the circulating current and the magnetic field. Two remained terms give that for the \( i \)-soliton. Of course we can not deal with the variation of \( |\psi_{\nu}| \) with this formula. Instead of that we apply the London equation for the first three terms and the sine-Gordon equation for the remained terms. Under the London approximation, these equations can be independently solved. However it requires the topological condition, which is a single-valued condition for an arbitrary closed path, \( \int \nabla \theta_{\nu} ds = 2\pi n_{\nu} \) (\( n_{\nu} \) is an integer). When \( n_1 \neq n_2 \), the path surrounds fractional vortex [11], which has fractional flux quanta, and cross an \( i \)-soliton wall somewhere. (The \( i \)-soliton wall is a two-dimensional extension of soliton, which is similar to the Maki planar soliton seen in superfluid \( ^3 \text{He} \). [12-14]) To achieve \( J=0 \) at an infinite faraway location from the vortex, the \( i \)-soliton wall should be terminated by two fractional vortices. This is a vortex molecule. The fractional vortex has sub-unit flux quanta and the total flux carried by two fractional vortices becomes a unit flux quantum. The magnetic energy of two fractional vortices is less than the conventional vortex having a unit flux quanta. It means the magnetic energy tends to separate one conventional vortex into two fractional vortices. When energy gain balances with the cost of the formation of the \( i \)-soliton wall, the vortex molecule is stabilized [9]. Because of this non-axisymmetric structure, the lattice of vortex molecule has a unique phase diagram like an amphiphile (or Langmuir) monolayers [15,16]. Experimental indications relevant to the dynamics of the vortex molecule have been experimentally detected [17].

3. Phase domain
We can consider a domain surrounded by the \( i \)-soliton wall. It is not energetically stable but is topologically possible. The schematic sketch is shown in Fig.1. Because the phase-slip accompanies with the \( i \)-soliton, the inside and outside of the domain have different phases. It results in the formation of the fractional vortex and vortex molecule, when the domain bursts. The vortex molecule can be considered as the

![Fig. 1. A schematic picture of a phase domain. The red and blue arrows indicate the quantum phase for each component. The area, where the red and blue arrows do not aligned, corresponds to the \( i \)-soliton domain wall.](image-url)
fragment of the phase domain. The formation of the phase domain by the external field will becomes one of basic procedures, considering the superconducting electronics using the $i$-soliton [10].

Though we need cost to form the $i$-soliton, we can imagine several shapes (or configuration) of the phase domain having the same energy stored on the domain wall. It leads increment of the entropy. At a certain temperature, this entropy term may assist the thermal excitation of the phase domain and the superconducting quantum phase no longer settles [6]. Above such a temperature, the situation can not be considered as the superconducting state. (It can not be considered as the conventional normal state either.)

The properties and dynamics of the phase domain have not been explored in detailed yet. It will be a crucial subject for the academic study like the quantum statistics of the fractional vortex and the design of the $i$-soliton device.

4. Summary

We discuss some interesting and peculiar phenomena on two-component superconductivity emerged in the multi-band superconductor. A crude formalism separating the conventional London equation and sine-Gordon equation helps the basic understanding of the topology of this system. Experimental indication also starts to be reported.

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