Induced Chern-Simons Like Action by Lorentz Breaking Symmetry in $(3+1)D$ QED

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Abstract

In this paper, we calculate the induced Chern-Simons like action on a system of fermions interacting with a gauge field in $(3+1)$ dimensions. As the articles on this subject in the literature are very difficult to read, we performed the calculations clearly and objectively.

1 Introduction

Lorentz symmetries, together with the CPT theorem [1] (a well known property of the Standard Model (SM) of elementary particle physics) are important for understanding the phenomena that occur within the quantum theory. However the Lorentz symmetry is broken in an attempt to incorporate the theory of General Relativity to the SM. An initial idea for a possible theory of Lorentz symmetry breaking appeared in an work of Kostelecký and Samuel [2]. In this study, the authors argue that such violation can be extended to the SM. Then, in the second half of 1990, there is the Standard Model Extension (SME).

The SME is a theory that has all the usual SM’s properties - such as the structure gauge $SU(3) \times SU(2) \times U(1)$ and renormalizability - and the extension that allows for violations of Lorentz and CPT symmetries. This theory then provides a quantitative description of the violations of Lorentz and CPT symmetries, controlled by coefficients whose values are determined by specific experiments. A striking feature of this theory, as is known from literature, is that the symmetry breaking CPT implies Lorentz breaking symmetry [3]. This fact means that any observable violation of CPT symmetry is described by the SME. In reference [4] a theoretical basis to perform perturbative calculations in this theory, via fermion sector, can be found.

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In the early 80s, S. Deser, R. Jackiw and S. Templeton [7] wrote to Maxwell’s electromagnetic theory in a planar variant in \((2 + 1)D\), which preserves Lorentz invariance and gauge transformations. This model, called the theory of Maxwell-Chern-Simons (MCS), has applicability to planar condensed matter phenomena, with great emphasis in the literature to superconductors and the fractional quantum Hall effect [8]. However, in a 1990 work, Carroll, Field and Jackiw realized in a pioneering work [9], it is possible to formulate a similar theory in \((3 + 1)\) dimensions, by adding the action

\[ S_{CS}^{(3+1)D} = \frac{1}{2} \int d^4 x \varepsilon^{\mu \nu \rho \sigma} \eta_\mu A_\nu \partial_\rho A_\sigma \]  

(1)

to the conventional Maxwell action. This term is known in literature as Carroll-Field-Jackiw term. This term is CPT-odd. Although gauge transformations be preserved, the Lorentz symmetry is violated, because it is necessary to engage the end type Chern-Simons (CS) \(\eta_\mu\) a constant four-vector, which produces an anisotropy of space-time.

In this paper, we calculate the radiative corrections in the approximation of a loop, from the axial coupling of fermions with a gauge field in the presence of Lorentz symmetry breaking. This coupling generates an induction of a term similar to CS [10], as in equation (1), in the action of Quantum Electrodynamics (QED). The induction is ambiguous, since the proportionality between the fields of matter and radiation depends exclusively on the regularization scheme adopted (such schemes can be: dimensional regularization, Pauli-Villars [17] regularization, the method of cut-off [24] and the Schwinger proper time method [21]). Thus, we calculate the term induced by the method of dimensional regularization. The calculations are performed in a clear way, since the articles on the subject in the literature are difficult to understand and the calculations are very tedious. This is the main goal of this work.

2 Fermion propagator expansion and the breaking of Lorentz symmetry

The fermion propagator dependent of Lorentz symmetry breaking [4] is given by

\[ S_b(p) = \frac{i}{\not{p} - m - \not{b} \gamma_5}. \]

(2)

The above inverted propagator is represented by [16]

\[ S_b(p) = \frac{i(\not{p} - \not{b} \gamma_5 + m)(p^2 - b^2 - m^2 + [\not{p}, \not{b}]\gamma_5)}{(p^2 - b^2 - m^2)^2 - 4(p \cdot b)^2 + 4p^2 b^2}. \]

(3)
This propagator has a complicated structure and make it more tedious perturbative calculations (exact perturbative calculations with this propagator can be found in reference [16]). We will not use this propagator, but we will draw on an alternative method. As the four-vector $b_\mu$, which signals the Lorentz breaking symmetry in the theory, is very small compared to the electron mass, the correction he made in the propagator can be treated in perturbative way. So, we apply the following expansion in a sum of terms of an infinite geometric progression

$$S_b(p) = \frac{i}{p - m} + \frac{i}{p - m}(-i\gamma_5)p + \frac{i}{p - m}(-i\gamma_5)p + \cdots$$

(4)

Thus, with each $\times$ representing each insertion $-\gamma_5$ in the propagator, its graph is given by

$$= + + + \cdots$$

3 Calculation of the induced term

The radiative corrections to the action of usual QED will be calculated by considering a system of fermions coupled to a gauge field $A_\mu$ formulated in space-time in $(3+1)$ dimensions. Using the disturbance $\gamma_5$, the Lagrangian of this model is given by

$$L = \bar{\psi}(i\gamma_0 - eA - m)\psi.$$  

(5)

To calculate the induced term, we use the path integrals formalism. The effective action for this model dependent on the term of the Lorentz symmetry breaking $\bar{\psi}\gamma_5\psi$, in an approximation of a one loop, is defined as follows [11]

$$e^{iS_{eff}[\bar{b},m]} = N \int D\bar{\psi}D\psi \exp \left[i \int d^4x \bar{\psi}(i\gamma_0 - eA - \gamma_5 - m)\psi\right],$$

(6)

where $N$ is a normalization constant.

With the use of Grassmann variables, integrating over the fermions fields, we obtain

$$e^{iS_{eff}[\bar{b},m]} = N \det(i\gamma_0 - eA - \gamma_5 - m),$$

(7)

in other words

$$S_{eff}[\bar{b},m] = -iTr \ln[i\gamma_0 - eA - \gamma_5 - m].$$

(8)
Where $A$ and $B$ two matrices do not commute, we obtain the following identity:

$$\ln(B - A) = \ln B - \sum_{n=1}^{\infty} \frac{1}{n} \left[ \frac{1}{B} A \right]^{n}.$$  \hspace{1cm} (9)

Identifying $A = e A$ and $B = i \partial - \gamma_5 - m$ in expression (9), we have

$$S_{\text{eff}}[b, m] = -i Tr \ln[i \partial - \gamma_5 - m] + i Tr \sum_{n=1}^{\infty} \frac{1}{n} \left[ \frac{1}{i \partial - \gamma_5 - m} e A \right]^{n}.$$  \hspace{1cm} (10)

The first term of the above expansion corresponds to a constant term added to the action and therefore does not matter, since it does not depend on the gauge field $A_{\mu}$. The contributions come from terms for $n > 1$. Thus, for $n = 1$ in the expansion above, we have

$$S_{\text{eff}}^{(1)}[b, m] = i e Tr \int d^4x \int \frac{d^4p}{(2\pi)^4} \frac{1}{p - \gamma_5 - m} A.$$  \hspace{1cm} (11)

The contributions of (11) give rise to terms like tadpoles, which are linear in $A_{\mu}$ and are divergent in the ultraviolet. Because these terms do not contribute to the induction of the CS like term, the same will be disregarded in the calculations below. However, we illustrate its graphics to first order in gauge field.

The second order contributions in providing

$$S_{\text{eff}}^{(2)} = -\frac{i e^2}{2} Tr[S_b(p) A S_b(p) A].$$  \hspace{1cm} (12)

The calculation of the trace of the action above is similar to the calculation of an operator $O$ which depends on the Dirac matrices and internal indices of the Lie group. So, your total trace $\text{Tr}$, on $(3+1)$ dimensions, is defined by:
\[
Tr \mathcal{O} \equiv tr_D \int d^4 x \langle x | \mathcal{O} | x' \rangle \bigg|_{x=x'},
\]  

where the symbol \( tr_D \) indicates that the trace will be calculated on the Dirac matrices in standard representation.

The result of this calculation is

\[
S_{\text{eff}}^{(2)} = \frac{ie^2}{2} tr tr_D \int d^4 x \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p - i\theta - \not{\gamma}_5 - m} A(x) \frac{1}{p - i\theta - \not{\gamma}_5 - m} A(x),
\]

Using the change of variable \( p - q \to k \), we obtain

\[
S_{\text{eff}}^{(2)} = \frac{ie^2}{2} tr tr_D \int d^4 x \int \frac{d^4 p}{(2\pi)^4} S_b(p) A S_b(p - i\theta) A.
\]

Feynman diagrams in one loop approximation used to calculate the action (15).
The radiative corrections will be obtained by introducing the expanded fermion propagator (4). Thus, we have

\[ S_b(p) = \frac{i(p + m)}{p^2 - m^2} + \frac{i}{(p^2 - m^2)^2} \gamma_5(p + m) \]

\[ + \frac{1}{(p^2 - m^2)^3} \gamma_5(p + m) \gamma_5(p + m) + \cdots \]  

(16)

Using the notations

\[ P = p + m \]  

(17)

\[ B_5 = \bar{\psi} \gamma_5 \]  

(18)

\[ \mathcal{D}^2 = p^2 - m^2 \]  

(19)

the propagator is written in compact form to linear terms in \( B_5 \):

\[ S_b(p) = \frac{i}{\mathcal{D}^2} P + \frac{i}{\mathcal{D}^4} P B_5 P + \cdots \]  

(20)

Now, expanding the propagator \( S_b(p - i\partial) \) to linear terms in \( \bar{B}_5 = \bar{\psi} \gamma_5 + i\partial \), we obtain

\[ S_b(p - i\partial) = \frac{i}{\mathcal{D}^2} P + \frac{i}{\mathcal{D}^4} \bar{B}_5 P + \cdots \]  

(21)

and

\[ S_b(p) A S_b(p - i\partial) A = -\frac{1}{\mathcal{D}^4} P A P A - \frac{1}{\mathcal{D}^6} P A \bar{B}_5 P A \]

\[ - \frac{1}{\mathcal{D}^6} P B_5 P A P A - \frac{1}{\mathcal{D}^8} P B_5 P A \bar{B}_5 P A. \]  

(22)

The terms that give rise to the CS induction are those that depend only on a derivative of the field \( A_\mu : \bar{\psi} A \partial A \gamma_5 \) and that will give the term structure of CS like term. This contribution comes from the last term of the above expression. Thus, using the explicit forms (17-18), it follows that

\[ PB_5 P A \bar{B}_5 P A = i \bar{\psi} A \partial A \gamma_5 + \text{im}^2 \bar{\psi} A \partial A \gamma_5 + \text{im}^2 \bar{\psi} A \partial A \gamma_5 \]

\[ + \text{im}^2 \bar{\psi} A \partial A \gamma_5 - \text{im}^2 \bar{\psi} A \partial A \gamma_5 - \text{im}^2 \bar{\psi} A \partial A \gamma_5 \]

\[ - \text{im}^2 \bar{\psi} A \partial A \gamma_5 + \cdots \]  

(23)
In the above expression, we have omitted terms that do not contain derivatives and those that are in odd number of Dirac matrices, because the result is zero. Now, we reduce the terms of the above expression of eight and six to four $\gamma$ matrices using the properties $\not{c} \not{d} = - \not{d} \not{c} + 2(c \cdot d) e$ $\not{c}^2 = c^2$ for any four-vector $c^\mu$:

\[ i p^4 \not{b} \not{A} \not{\partial} \not{A} \gamma_5 - 2 i p^2 \not{b} \not{A} (p \cdot \not{\partial}) \not{A} \gamma_5 - 2 i p^2 (b \cdot p) \not{A} \not{\partial} \not{A} \gamma_5 \]

\[ + 4i (b \cdot p) \not{A} (p \cdot \not{\partial}) \not{A} \gamma_5 + 2 i m^2 (b \cdot p) \not{A} \not{\partial} \not{A} \gamma_5 + 2 i m^2 \not{b} \not{A} (p \cdot \not{\partial}) \not{A} \gamma_5 \]

\[ - 2i m^2 (p \cdot A) \not{b} \not{A} \not{\partial} \not{A} \gamma_5 - i m^4 \not{b} \not{A} \not{\partial} \not{A} \gamma_5 \]

(24)

The next step is to insert the above result in action (15) to calculate the integral in moments. It should be noted that, by counting the powers, the integrals proportional to $p^4$ are logarithmic divergence, while proportional to $p^2$ are finite:

\[ \int_{inf} d^4 p (2\pi)^4 \frac{p^4}{(p^2 - m^2)^4} ; \int_{fin} d^4 p (2\pi)^4 \frac{p^2}{(p^2 - m^2)^4} \]

In an arbitrary dimension $D$, these finite dimensionally regularized integrals [12] are given by

\[ \int \frac{d^D p}{(2\pi)^D (p^2 - m^2)^\alpha} = \frac{(-1)^\alpha i}{4\pi^{D/2} m^{2\alpha - D}} \frac{\Gamma(\alpha - D/2)}{\Gamma(\alpha)} \]

(25)

\[ \int \frac{d^D p}{(2\pi)^D (p^2 - m^2)^\alpha} \frac{p_\mu p_\nu}{2} = g_{\mu\nu} \frac{(-1)^{\alpha - 1} i}{4\pi^{D/2} m^{2\alpha - D - 2}} \frac{\Gamma(\alpha - D/2 - 1)}{\Gamma(\alpha)} \]

(26)

Explicitly compute the fifth term (finite) of the above expression using the trace of the Dirac matrices, $tr_D (\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma)$, and the integral (26) for $D = 4$:

\[ - \frac{ie^2}{2} tr_D \int d^4 x \int \frac{d^4 p}{(2\pi)^4} \frac{2 i m^2 (b \cdot p) \not{A} \not{\partial} \not{A} \gamma_5}{(p^2 - m^2)^4} = \frac{e^2}{48\pi^2} \int d^4 x \varepsilon^{\mu\rho\sigma\lambda} b_\mu A_\nu \partial_\rho A_\sigma \]

(27)

The first four terms of expression (24) generate infinite terms in effective action. To perform the calculations, we make the regularization $D = 4 - 2\epsilon$ for the calculation of divergent integrals. While we keep $\epsilon \neq 0$, these originally divergent integrals are kept finite and thus we can add them and remove them.
This infinity integral is represented by \[12,13\]

\[
\int \frac{d^D p}{(2\pi)^D} \frac{p_\mu p_\nu p_\rho p_\sigma}{(p^2 - m^2)^4} = \frac{i}{384\pi^2} \left[ \frac{1}{\epsilon} + \log \frac{4\pi}{m^2} - \gamma + O(\epsilon) \right] (g_{\mu\nu} g_{\rho\sigma} + g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\rho})
\]

(28)

The sum of infinity parts vanishes

\[
2e^2 \int d^4x \varepsilon^{\mu\nu\rho\sigma} \left\{ i \frac{i}{16\pi^2} \left[ \frac{1}{\epsilon} + \log \frac{4\pi}{m^2} - \gamma + O(\epsilon) \right] b_\mu A_\nu \partial_\rho A_\sigma \\
-2i \frac{i}{64\pi^2} \left[ \frac{1}{\epsilon} + \log \frac{4\pi}{m^2} - \gamma + O(\epsilon) \right] b_\mu A_\nu \partial_\rho A_\sigma \\
-2i \frac{i}{64\pi^2} \left[ \frac{1}{\epsilon} + \log \frac{4\pi}{m^2} - \gamma + O(\epsilon) \right] b_\mu A_\nu \partial_\rho A_\sigma \\
+4i \frac{i}{384\pi^2} \left[ \frac{1}{\epsilon} + \log \frac{4\pi}{m^2} - \gamma + O(\epsilon) \right] (b_\mu A_\nu \partial_\rho A_\sigma - b_\mu A_\nu \partial_\rho A_\sigma) \right\} = 0
\]

So, the induced term depends only on the finity part of action and is given by

\[
\delta_{CS}^{(3+1)D} = \frac{e^2}{12\pi^2} \int d^4x \varepsilon^{\mu\nu\rho\sigma} b_\mu A_\nu \partial_\rho A_\sigma.
\]

(29)

And we obtain the relation

\[
\eta_\mu = \frac{e^2}{6\pi^2} b_\mu.
\]

(30)

Therefore, we conclude that the addition of a term with a background field that breaks Lorentz symmetry of the Lagrangian gauge theory leads to the usual Chern-Simons type action in four-dimensional spacetime. As is well noted in the literature, this term is finite and obtained by various regularization methods. The only difference is the constant of proportionality, which depends exclusively on the type of regularization method used.

4 Conclusions

We calculate the induced Chern-Simons like action in a quadrimensional space. We note that this is only possible if we introduce the term of the Lorentz symmetry breaking, since the term $-\gamma_5$ induces the calculation of $tr_D(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma)$, which generates the necessary structure $\varepsilon^{\mu\nu\rho\sigma}$ relevant to that induced term. The procedure for obtaining such action had as its starting point the expansion
of the modified fermion propagator given by the new theory. Our result is in agreement with those obtained in the literature, which can differentiate between them in relation to a constant, if different methods of regularization are used. In our calculation, using the method of dimensional regularization, we note that the induced action of the different terms cancel each other, and the limit $m \to 0$ is not necessary and the induced action is finite.

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