QCD Transport Theory

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Nearly Perfect Fluids, 4/7/09
Hot QCD = neutral fluid

- long distance effective theory = hydrodynamics

\[ \partial_\mu T^{\mu\nu} = 0 \]
\[ T^{\mu\nu} = (\varepsilon + p) u^\mu u^\nu + pg^{\mu\nu} + \eta \Pi^{\mu\nu}_{\text{shear}} + \zeta \Pi^{\mu\nu}_{\text{expansion}} \]

- short distance input:

|                      |          |
|----------------------|----------|
| equilibrium thermodynamics | \( \varepsilon \) | energy density  |
| transport coefficients | \( p \)   | pressure        |
|                       | \( \eta \) | shear viscosity |
|                       | \( \zeta \) | bulk viscosity  |
|                       | \( D \)   | flavor diffusion |
Kubo relations

- spectral densities

\[
\rho_{\alpha\beta,\mu\nu}(\omega, k) = \int dt \, d^3x \, e^{i\omega t - ik \cdot x} \langle [T_{\alpha\beta}(t, x), T_{\mu\nu}(0, 0)] \rangle
\]

\[
\rho_{\alpha,\beta}(\omega, k) = \int dt \, d^3x \, e^{i\omega t - ik \cdot x} \langle [J_{\alpha}(t, x), J_{\beta}(0, 0)] \rangle
\]

- transport coefficients

\[
\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \rho_{xy,xy}^{TT}(\omega, 0)
\]

\[
\zeta = \lim_{\omega \to 0} \frac{1}{18\omega} \rho_{ii,jj}^{TT}(\omega, 0)
\]

\[
D = \lim_{\omega \to 0} \frac{1}{6\omega} \rho_{ii}^{JJ}(\omega, 0) \Xi^{-1}
\]

charge susceptibility

transport coefficients \rightleftharpoons \text{zero frequency slope of spectral densities}
Spectral density phenomenology

- $\rho(\omega, k)/\omega = \text{real, positive, even function of } \omega$

$$
\rho(\omega)/\omega
$$

- $\rho(\omega')/\omega' = \text{real, positive, even function of } \omega'$

$$
\rho(\omega')/\omega' = \int \frac{d\omega'}{2\pi} \frac{\rho(\omega', k)}{\omega' + i\omega_n}
$$

**N.B.**

$$
G_E(\omega_n, k) = \int \frac{d\omega'}{2\pi} \frac{\rho(\omega', k)}{\omega' + i\omega_n}
$$

$$
G_E(\omega_n, k) - G_E(\omega_n, k)\bigg|_{T=0}
$$

UV dominated

$$
G_E^\prime(\omega_n, k) - G_E(\omega_n, k)\bigg|_{\text{pert. thy.}}
$$

ill-defined nonsense

$$
G_E(\omega_n, k) - G_E(\omega_n, k)\bigg|_{\text{pert. thy.}}
$$

still UV dominated

N.B. UV dominated still UV dominated ill-defined nonsense
QCD temperature regimes

\[ T \ll T_c \]
- dilute pion gas
- calculable transport coeffs

\[ T \gg T_c \]
- weakly coupled QGP
- calculable transport coeffs

\[ 1.5T_c \lesssim T \lesssim 4T_c \]
- strongly coupled
- quasiconformal?
- similar to N=4 SYM?

\[ T \approx T_c \]
- strongly coupled
- featureless \( \rho \)?
  - Yes: lattice fits useful
  - No: lattice fits useless

“confinement” temperature \( T_c \approx \Lambda_{QCD} \)

= rapid crossover from hadronic gas to quark-gluon plasma
High temperature asymptopia

\[ T \gg T_c \quad \Rightarrow \quad g^2(T) \ll 1 \quad \text{weak coupling} \]

\[ m_D(T)/T \ll 1 \quad \text{small thermal self-energies} \]

\[ \text{lifetimes} \gg 1/T \quad \text{quarks, gluons = good quasiparticles} \]

long distance, long time dynamics
reproduced by effective kinetic theory

Boltzmann equation:
\[ (\partial_t + \mathbf{v}_p \cdot \nabla) f(t, \mathbf{x}, \mathbf{p}) = -C[f] \]

stress energy tensor:
\[ T^{\mu\nu}(x) = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{p^0} f(x, \mathbf{p}) \]

near equilibrium:
\[ C[f_{eq} + \delta f] = C \delta f \]

collision term
linearized collision operator
Calculating transport coefficients

Let: \( f = f_{\text{local-eq}} + \delta f \)

\( f_{\text{local-eq}} = \left[ \exp\left(-\beta(x) u_\nu(x)p^\nu\right) \mp 1 \right]^{-1} \)

\( \delta f \ll f_{\text{local-eq}} \)

Choose slowly varying \( \beta(x), u_\nu(x) \)

Insert into Boltzmann equation, solve for \( \delta f \)

Evaluate \( T^{\mu\nu} \) and extract transport coefficients

\[
(\partial_t + \mathbf{v}_p \cdot \nabla) f_{\text{local-eq}} = S
\]

\( \delta f = C^{-1} S \)

\( (S, C^{-1} S) \)
**Fate of quasiparticles**

Typical momenta = $O(T)$

Thermal self-energies $\sim E^2 - p^2 = O(g^2 T^2)$

$\Rightarrow$ Quasiparticles ultrarelativistic

| Diagram | Process | $\Delta \theta$ | Rate |
|---------|---------|-----------------|------|
| ![Small Angle Scattering](image1) | Small angle scattering, $\Delta \theta = O(g)$ | $\Delta \theta = O(g)$ | Rate = $O(g^2 T \ln 1/g)$ |
| ![Large Angle Scattering](image2) | Large angle scattering, $\Delta \theta = O(1)$ | $\Delta \theta = O(1)$ | Rate = $O(g^4 T \ln 1/g)$ |
| ![Gluon Bremsstrahlung](image3) | Gluon bremsstrahlung, $\Delta \theta = O(g)$ | $\Delta \theta = O(g)$ | Rate = $O(g^4 T)$ |
| ![Pair Creation/Aannihilation](image4) | Pair creation/annihilation, $\Delta \theta = O(g)$ | $\Delta \theta = O(g)$ | Rate = $O(g^4 T)$ |
Effective kinetic theory

\[(\partial_t + \hat{p} \cdot \nabla_x) f_s(x, p, t) = -C_s^{2\rightarrow 2}[f] - C_s^{1\rightarrow 2}[f] \]

\[C_s^{2\rightarrow 2}[f] = \frac{1}{4|\mathbf{p}| \nu_a} \sum_{b,c,d} \int_{k_{p'}k'} |\mathcal{M}^{ab}_{cd}(p, k; p', k')|^2 (2\pi)^4 \delta^{(4)}(P + K - P' - K') \]
\[\times \left\{ f_a(p) f_b(k) [1 \pm f_c(p')] [1 \pm f_d(k')] - f_c(p') f_d(k') [1 \pm f_a(p)] [1 \pm f_b(k)] \right\} \]

\[C_s^{1\rightarrow 2}[f] = \frac{(2\pi)^3}{2|\mathbf{p}|^2 \nu_a} \sum_{b,c} \int_0^\infty dp' dk' \delta(|\mathbf{p}| - p' - k') \gamma^{a}_{bc}(p; p' \hat{p}, k' \hat{p}) \]
\[\times \left\{ f_a(p)[1 \pm f_b(p' \hat{p})][1 \pm f_c(k' \hat{p})] - f_b(p' \hat{p}) f_c(k' \hat{p})[1 \pm f_a(p)] \right\} \]
Results

• Next-to-leading log \((N_c = 3, N_f = 3)\):

\[
\eta = \frac{T^2}{\alpha_s^2} \left[ \frac{1.35}{\ln(\frac{0.46}{\alpha_s})} + O\left(\frac{1}{\ln^3(\alpha_s^{-1})}\right) \right] \\
D = \frac{1}{\alpha_s^2 T} \left[ \frac{0.15}{\ln(\frac{0.46}{\alpha_s})} + O\left(\frac{1}{\ln^3(\alpha_s^{-1})}\right) \right]
\]

• Full leading-order:

\[
\eta \approx 34.4 T^3, \quad \eta/s \approx 1.6
\]
Domain of validity

- Differing 'reasonable' treatments, equally valid at leading order, produce range of results with 100% uncertainty when \( m_D/T > 1.7 \).

- \( m_D^2 = 6\pi \alpha_s T^2 \) \[\alpha_s > 0.21 \Rightarrow m_D/T > 2!\]
Conclusions

- weak coupling analysis: beautiful theory, but not reliable in experimentally relevant temperature range. (“Reliable” ≡ less than 100% uncertainty.)

- weak coupling results: equally unreliable at RHIC & LHC.

- estimates of transport coefficients via lattice fits to models of spectral density: utterly dependent on unverifiable assumption of negligible structure.

- results from gravitational duals of SUSY plasmas: similarity to QCD depends on observable, may be reasonable at 30-50% level in quasiconformal regime.