Gravitating Stationary Dyons and Rotating Vortex Rings

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We construct dyons, and electrically charged monopole-antimonopole pairs and vortex rings in Yang-Mills-Higgs theory coupled to Einstein gravity. The solutions are stationary, axially symmetric and asymptotically flat. The dyons with magnetic charge \( n \geq 2 \) represent non-static solutions with vanishing angular momentum. The electrically charged monopole-antimonopole pairs and vortex rings, in contrast, possess vanishing magnetic charge, but finite angular momentum, equaling \( n \) times their electric charge.

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I. INTRODUCTION

The non-trivial vacuum structure of SU(2) Yang-Mills-Higgs (YMH) theory gives rise to regular non-perturbative finite energy solutions, such as magnetic monopoles, multimonopoles and monopole-antimonopole systems. While spherically symmetric monopoles carry unit topological charge \([1, 2, 3]\), monopoles with charge \( n > 1 \) \([4, 5, 6, 7]\) and monopole-antimonopole systems \([8, 9, 10, 11]\) are axially symmetric or possess no rotational symmetry at all \([12, 13]\).

To any static solution of the YMH equations there corresponds a family of electrically charged solutions \([2, 3, 14, 15]\). In the Prasad-Sommerfield limit these electrically charged solutions are obtained directly from the electrically neutral solutions via simple scaling relations, by requiring the time component of the gauge field and the Higgs field to be parallel \([14, 15]\).

Bogomol’nyi-Prasad-Sommerfield (BPS) monopoles and Julia-Zee dyons do not admit slowly rotating excitations \([16]\). Indeed, monopoles and dyons cannot rotate in the sense, that they possess finite angular momentum, since they are globally regular solutions which carry magnetic charge \([17]\). Thus monopoles and dyons with higher magnetic charge cannot rotate either. Whereas BPS dyons also have vanishing angular momentum density, non-BPS dyons with higher magnetic charge might possess a finite angular momentum density, yielding a vanishing angular momentum upon integration, though, because of symmetry reasons.

In monopole-antimonopole pairs, on the other hand, the magnetic charge vanishes \([8, 9, 10, 11]\). In these axially symmetric solutions the two nodes of the Higgs field, representing the location of the magnetic charges, are situated symmetrically on the positive and negative \( z \)-axis. When electric charge is added to both the monopole and antimonopole in the pair, they experience a repulsive force and the poles move further apart \([15]\). More importantly, however, the pair begins to rotate about its symmetry axis, yielding an angular momentum equal to its electric charge, \( J = Q \) \([17, 18]\). Thus the presence of electric charge again leads to a finite angular momentum density. But since the magnetic charge of the pair vanishes, it may rotate, and indeed it must rotate with \( J = Q \) \([17, 18]\).

Monopole-antimonopole pairs can also be formed from doubly charged monopoles and antimonopoles \((n = 2)\) \([11, 19]\). For higher values of \( n \), in constrast, a completely different type of solution appears \([11]\). Here the Higgs field vanishes on a ring centered around the symmetry axis. Therefore we refer to these solutions as vortex rings. Adding electric charge to these magnetically neutral solutions then should yield rotating \( n = 2 \) monopole–antimonopole pairs and rotating vortex rings possessing angular momentum \( J = nQ \).

When gravity is coupled to YMH theory, gravitating monopoles \([20, 21]\), gravitating monopole-antimonopole pairs and gravitating vortex rings arise \([22, 23]\). In each case, a branch of gravitating solutions emerges smoothly from the corresponding flat space solution, and extends up to a maximal value of the coupling constant, where, for vanishing Higgs self-coupling constant, it merges with a second branch \([24]\). For monopoles this second branch extends only slightly backwards before it merges with the branch of extremal Reissner-Nordström black holes \([20, 21, 25]\). For monopole-antimonopole pair solutions and vortex rings, in contrast, this second branch extends all the way back to vanishing coupling constant, where the solutions shrink to zero size.

The coupling constant \( \alpha \), entering the Einstein-Yang-Mills-Higgs (EYMH) equations, is proportional to the Higgs vacuum expectation value \( v \) and the square root of the gravitational constant \( G \). Variation of \( \alpha \) may thus be considered as variation of the gravitational constant \( G \) along the first branch and as variation of the Higgs vacuum expectation value \( v \) along the second branch. Consequently, the Higgs field vanishes in the limit \( \alpha \to 0 \) on the second branch, and, after scaling the coordinates, the mass and the Higgs field with \( \alpha \), solutions of Einstein-Yang-Mills (EYM) theory are obtained \([22, 23]\), which correspond to the lowest mass Bartnik-McKinnon (BM) solution \([26]\) or its generalizations \([27, 28]\).

When dyons \([29]\) or electrically charged monopole-antimonopole pair solutions \([18]\) are coupled to gravity, anal-
ogously a corresponding branch of gravitating dyons or electrically charged monopole-antimonopole pair solutions emerges smoothly from the respective flat space solution \[18, 29\]. Whereas gravitating dyons again merge with the corresponding branch of extremal Reissner-Nordström black holes at a critical value of the coupling constant \[25, 29\], the critical behaviour for gravitating electrically charged monopole-antimonopole pair solutions could not be resolved previously \[18\].

Gravitating spherically symmetric dyons are static in the sense, that their angular momentum density vanishes. In this letter we show, that axially symmetric gravitating dyons \((n \geq 2)\) are stationary, while they carry no angular momentum. We further resolve the critical behaviour of electrically charged monopole-antimonopole pair solutions, and we construct rotating vortex ring solutions. In all calculations we limit ourselves to vanishing Higgs self-coupling.

In section II we present the action, the axially symmetric Ansatz and the boundary conditions. In section III we discuss the properties of stationary dyons, and rotating monopole-antimonopole pairs and vortex rings. We present our conclusions in section IV.

II. EINSTEIN-YANG-MILLS-HIGGS SOLUTIONS

A. Action

We consider the SU(2) EYMH action in the limit of vanishing Higgs potential,

\[
S = \int \left[ \frac{R}{16\pi G} - \frac{1}{2} \text{Tr}(F_{\mu\nu}F^{\mu\nu}) - \frac{1}{4} \text{Tr}(D_\mu \Phi D^\mu \Phi) \right] \sqrt{-g} \, dx^4
\]

with curvature scalar \(R\), SU(2) field strength tensor

\[
F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ie[A_\mu, A_\nu],
\]

gauge potential \(A_\mu = 1/2\tau^n A^a_\mu\), gauge covariant derivative

\[
D_\mu = \nabla_\mu + ie[A_\mu, \cdot]
\]

and Higgs field \(\Phi = \tau^a \Phi^a\); \(G\) is Newton’s constant, and \(e\) is the gauge coupling constant. We impose a Higgs field vacuum expectation value \(v\).

Variation of the action Eq. (1) with respect to the metric \(g_{\mu\nu}\), the gauge potential \(A^a_\mu\), and the Higgs field \(\Phi^a\) leads to the Einstein equations and the matter field equations, respectively.

B. Ansätze

We consider regular stationary, axially symmetric solutions with Killing vector fields \(\xi = \partial_t\) and \(\eta = \partial_\varphi\). We employ the Lewis-Papapetrou form of the metric in isotropic coordinates \[30\]

\[
d s^2 = -f dt^2 + \frac{m}{f} [dr^2 + r^2 d\theta^2] + \frac{l r^2 \sin^2 \theta}{f} \left[ d\varphi - \frac{\omega}{r} dt \right]^2.
\]

The gauge potential is parametrized by \[30\]

\[
A_\mu dx^\mu = \left( B_1 \frac{\tau^r(n,m)}{2e} + B_2 \frac{\tau^\theta(n,m)}{2e} \right) dt - n \sin \theta \left[ H_3 \frac{\tau^r(n,m)}{2e} + (1 - H_4) \frac{\tau^\theta(n,m)}{2e} \right] d\varphi + \left( \frac{H_1}{r} dr + (1 - H_2) d\theta \right) \frac{\tau^r(n,m)}{2e},
\]

and the Higgs field by \[31\]

\[
\Phi = v \left( \Phi_1 \tau^r(n,m) + \Phi_2 \tau^\theta(n,m) \right),
\]

where \(n\) and \(m\) are integers, with \(\pm n\) representing the magnetic charge of single (anti)monopoles in monopole-antimonopole chains and \(m\) the total number of monopoles and antimonopoles in monopole-antimonopole chains \[11\].
Dyons are obtained for \( m = 1 \), and monopole-antimonopole pairs and vortex rings for \( m = 2 \). The \( su(2) \) matrices \( \tau^{(n,m)}_r, \tau^{(n,m)}_\theta \), and \( \tau^{(n)}_\varphi \) are defined as scalar products of the spatial unit vectors

\[
\begin{align*}
\hat{e}^{(n,m)}_r &= (\sin(m\theta) \cos(n\varphi), \sin(m\theta) \sin(n\varphi), \cos(m\theta)), \\
\hat{e}^{(n,m)}_\theta &= (\cos(m\theta) \cos(n\varphi), \cos(m\theta) \sin(n\varphi), -\sin(m\theta)), \\
\hat{e}^{(n)}_\varphi &= (-\sin(n\varphi), \cos(n\varphi), 0),
\end{align*}
\]

with the Pauli matrices \( \tau^a = (\tau_x, \tau_y, \tau_z) \). All functions depend on \( r \) and \( \theta \), only.

The ansatz is form-invariant under Abelian gauge transformations \( U \).

\[
U = \exp\left(\frac{i}{2} \tau^{(n)}_\varphi \Gamma(r, \theta)\right).
\]

With respect to this residual gauge degree of freedom we choose the gauge fixing condition \( r \partial_r H_1 - \partial_\theta H_2 = 0 \).

**C. Boundary Conditions**

Regularity of the solutions at the origin \( (r = 0) \) requires for the metric functions the boundary conditions

\[
\partial_r f(r, \theta)|_{r=0} = \partial_r m(r, \theta)|_{r=0} = \partial_\theta l(r, \theta)|_{r=0} = 0,
\]

whereas the gauge and Higgs field functions \( H_t \) and \( \Phi_t \) satisfy

\[
\begin{align*}
H_1(0, 0, \theta) &= H_3(0, 0, \theta) = 0, & H_2(0, 0, \theta) &= H_4(0, 0, \theta) = 1,
\end{align*}
\]

and for even \( m \)

\[
\begin{align*}
\sin(m\theta) B_1(0, 0, \theta) + \cos(m\theta) B_2(0, 0, \theta) &= 0, \\
\partial_r [\cos(m\theta) B_1(r, \theta) - \sin(m\theta) B_2(r, \theta)]|_{r=0} &= 0, \\
\sin(m\theta) \Phi_1(0, 0, \theta) + \cos(m\theta) \Phi_2(0, 0, \theta) &= 0, \\
\partial_r [\cos(m\theta) \Phi_1(r, \theta) - \sin(m\theta) \Phi_2(r, \theta)]|_{r=0} &= 0,
\end{align*}
\]

whereas for odd \( m \) \( B_t(0, 0, \theta) = \Phi_t(0, 0, \theta) = 0 \).

Asymptotic flatness imposes on the metric functions at infinity \( (r = \infty) \) the boundary conditions

\[
\begin{align*}
f &\to 1, & m &\to 1, & l &\to 1, & \omega &\to 0.
\end{align*}
\]

The boundary conditions for the functions \( H_1 - H_4, B_1, B_2, \Phi_1, \Phi_2 \) read

\[
\begin{align*}
H_1 &\to 0, & H_2 &\to 1 - m, \\
H_3 &\to \frac{\cos \theta - \cos(m\theta)}{\sin \theta} m \text{ odd}, & H_3 &\to \frac{1 - \cos(m\theta)}{\sin \theta} m \text{ even}, \\
H_4 &\to 1 - \frac{\sin(m\theta)}{\sin \theta},
\end{align*}
\]

\[
\begin{align*}
B_1 &\to \gamma, & B_2 &\to 0, \\
\Phi_1 &\to 1, & \Phi_2 &\to 0.
\end{align*}
\]

On the symmetry axis, we impose \( \partial_\theta f = \partial_\theta m = \partial_\theta l = \partial_\theta \omega = 0 \), \( H_1 = H_3 = 0 \), \( \partial_\theta H_2 = \partial_\theta H_4 = 0 \), \( \partial_\theta B_1 = 0 \), \( B_2 = 0 \), \( \partial_\theta \Phi_1 = 0 \), \( \Phi_2 = 0 \). Regularity further requires \( m(r, \theta) = l(r, \theta) \) and \( H_2(r, \theta) = H_4(r, \theta) \) on the symmetry axis.
III. RESULTS

We here discuss our numerical results for stationary dyons, and rotating monopole-antimonopole pairs and vortex rings, and determine the dependence of these solutions on the coupling constant $\alpha$.

A. Numerical procedure

To construct solutions subject to the above boundary conditions, we map the infinite interval of the variable $r$ onto the unit interval of the compactified radial variable $\tilde{r} \in [0 : 1],

$$\tilde{r} = \frac{r}{1 + r},$$

i.e., the partial derivative with respect to the radial coordinate changes according to $\partial_r \to (1 - \tilde{r})^2 \partial_{\tilde{r}}$. The numerical calculations are then performed with the help of the FIDISOL package based on the Newton-Raphson iterative procedure\[32\].

B. Charges

Let us introduce dimensionless quantities,

$$x = e v r , \quad \hat{B}_i = \frac{1}{e v} B_i , \quad \hat{\gamma} = \frac{1}{e v} \gamma .$$

(21)

For fixed $n$ and $m$, the equations then depend only on the dimensionless coupling constant $\alpha$\[20\]

$$\alpha = \sqrt{4 \pi G v} ,$$

(22)

since we restrict to vanishing Higgs potential.

The dimensionless mass $M$ and angular momentum $J$ of the solutions are obtained from the asymptotic expansion of the metric functions

$$M = \frac{1}{2 \alpha^2} \lim_{x \to \infty} x^2 \partial_x f , \quad J = \frac{1}{2 \alpha^2} \lim_{x \to \infty} x^2 \omega ,$$

(23)

the dimensionless electric charge $Q$\[31\] and magnetic charge $P$\[11\] are obtained from

$$Q = - \lim_{x \to \infty} x \left( \hat{B}_1 - \hat{\gamma} \right) , \quad P = \frac{n}{2} (1 - (-1)^m) .$$

(24)

Magnetically charged solutions have vanishing angular momentum, $J = 0$\[17\], and vanishing magnetic dipole moment, $\mu_{\text{mag}} = 0$\[11\]. Magnetically neutral solutions possess a finite dipole moment\[11\], which can be obtained from the asymptotic form of the non-Abelian gauge field, after transforming to a gauge where the Higgs field is constant at infinity, $\Phi = \tau_z$,

$$A_\mu dx^\mu = -\mu_{\text{mag}} \frac{\sin^2 \theta}{x} \frac{\tau_z}{2} d\varphi ,$$

(25)

and they satisfy the relation

$$J = \frac{n}{2} (1 + (-1)^m) Q ,$$

(26)

generalizing the previous relations\[17\]. The full asymptotic expansion will be given elsewhere\[31\].

C. Stationary dyons: $n > 1$

Gravitating dyons with magnetic charge $n = 1$ are spherically symmetric and static\[29\]. Their $\alpha$-dependence is completely analogous to the $\alpha$-dependence of gravitating monopoles. Thus a branch of gravitating dyons emerges from
the flat space dyon solution and extends up to a maximal value \( \alpha_{\text{max}} \). There it merges with a short second branch, which bends backwards and then merges at a critical value \( \alpha_{\text{cr}} \) with the branch of extremal Reissner-Nordström solutions \[25, 29\].

Like monopoles with higher magnetic charge \[21\] also dyons with higher magnetic charge show a similar \( \alpha \)-dependence: they merge with the corresponding branch of extremal Reissner-Nordström solutions \[25\]. However, numerical accuracy does not allow to definitely conclude, whether a short second branch is present, i.e., whether \( \alpha_{\text{max}} \neq \alpha_{\text{cr}} \) \[21\]. We exhibit the scaled mass \( \alpha M \) and the electric charge \( Q \) for dyons with magnetic charge \( n = 2 \) and \( n = 3 \) in Figs. 1.

**Fig. 1** The scaled mass \( \alpha M \) (a) and the charge \( Q \) (b) are shown as functions of the coupling constant \( \alpha \) for dyon solutions with \( n = 2, 3 \) and for monopole-antimonopole resp. vortex ring solutions with \( n = 1, 2, 3 \) at \( \hat{\gamma} = 0.32 \).

Gravitating dyons with magnetic charge \( n = 1 \) are static, since they have vanishing angular momentum density \( T^i_\phi \). We here demonstrate that gravitating dyons with magnetic charge \( n > 1 \) are stationary but not static. They possess a finite angular momentum density. But since their angular momentum density is antisymmetric with respect to reflection, \( z \to -z \), their angular momentum vanishes. In Fig. 2 we exhibit the energy density \( T^t_t \) and the angular momentum density \( T^t_\phi \) for a typical gravitating dyon with \( n = 2 \) for \( \alpha = 1.4 \). The energy density is toruslike \[12, 21\], but becomes spherical in the limit \( \alpha \to \alpha_{\text{cr}} \). The angular momentum density vanishes both for \( \alpha = 0 \) and \( \alpha = \alpha_{\text{cr}} \). With increasing \( \alpha \) it increases in magnitude, but is localized in a decreasing region of space.

**Fig. 2** The energy density \( T^t_t \) (a) and the angular momentum density \( T^t_\phi \) (b) are shown for a dyon solution with \( n = 2, \alpha = 1.4, \hat{\gamma} = 0.32 \).

D. Rotating monopole-antimonopole pairs: \( n = 1 \)

Let us now consider rotating monopole-antimonopole pairs composed of poles of charge \( \pm 1 \). For these one expects an analogous coupling constant dependence as for static monopole-antimonopole pairs, i.e., when gravity is coupled, a branch of gravitating monopole-antimonopole pair solutions emerges from the flat space solution, and merges with
a second branch of monopole-antimonopole pair solutions at a maximal value of the coupling constant $\alpha_{\text{max}}$. The second branch then extends all the way back to $\alpha = 0$. Along the second branch, with decreasing $\alpha$, the solutions shrink to zero size \[22\]. Scaling the coordinates and the Higgs field with $\alpha$ however, leads to a limiting solution with finite size and finite scaled mass $\hat{M} \[22\]$, representing the lowest BM solution \[26\] of EYM theory.

The dependence of the rotating monopole-antimonopole pair solutions on the coupling constant $\alpha$ has in part been studied before \[19\]. There indeed two branches of solutions were found, however, these were not smoothly connected: at a value $\alpha_{\text{cr}}$ the mass of the solutions on both branches agreed, but their angular momenta did not. Thus the existence of further branches was hypothesized.

Repeating the numerical study reveals, however, that both branches can be extended beyond $\alpha_{\text{cr}}$, up to a maximal value $\alpha_{\text{max}}$, where they merge. Thus rotating monopole-antimonopole pair solutions indeed show the expected coupling constant dependence, except that the two branches of solutions cross before they merge. This is illustrated in Figs.1.

Along the two branches, the two nodes of the Higgs field, which represent the locations of the magnetic poles, move continuously closer together, until they merge at the origin in the EYM limit on the second branch. Thus the nodes of the rotating monopole-antimonopole pair solutions also exhibit the same $\alpha$ dependence as the nodes of static solutions \[22, 23\].

![Fig. 3](image-url) The magnetic moment $\mu_{\text{mag}}$ for monopole-antimonopole resp. vortex ring solutions with $n = 1, 2, 3$ at $\hat{\gamma} = 0.32$ (a) and the location of the nodes for monopole-antimonopole resp. vortex ring solutions with $n = 2$ at $\hat{\gamma} = 0.32$ and $\hat{\gamma} = 0$ (b) are shown as functions of the coupling constant $\alpha$.

### E. Rotating pairs and vortex rings: $n \geq 2$

For static monopole-antimonopole pairs composed of monopoles and antimonopoles of charge $\pm 2$ \[11, 19\], the $\alpha$ dependence is completely analogous as for pairs with $n = 1$. Two branches of solutions exist, which merge at a maximal value $\alpha_{\text{max}}$. The $\alpha$-dependence of the two nodes of the solutions was not considered before \[23\].

For rotating monopole-antimonopole pairs we also obtain two branches of solutions. Again the two branches cross before they merge at a maximal value $\alpha_{\text{max}}$. However, when the two nodes of the Higgs field are inspected, one finds a surprise. The nodes merge at the origin at a value $\alpha_{\text{cr}}$. There the solutions change their character, turning into vortex rings beyond $\alpha_{\text{cr}}$, since their Higgs field then vanishes on a ring in the $xy$-plane. The nodal ring first increases in size, reaches a maximum at $\alpha_{\text{max}}$, and then decreases to zero size in the EYM limit on the second branch. The locations of the nodes and the nodal rings are exhibited in Fig.3. As illustrated, the nodes of the static solutions possess an analogous $\alpha$-dependence.

For $\alpha < \alpha_{\text{cr}}$, the energy density of the solutions on the first branch consists of two tori, located symmetrically with respect to the $xy$-plane, whereas for $\alpha > \alpha_{\text{cr}}$, the energy density is a single torus, as illustrated in Fig. 4. The figure also shows that the angular momentum density of the respective solutions is always symmetrical with respect to the $xy$-plane.
The energy density $T^t_1$ (a,c) and the angular momentum density $T^t_\varphi$ (b,d) are shown for a monopole-antimonopole pair solution with $n = 2$, $\alpha = 0.4$, $\hat{\gamma} = 0.32$ (a,b) and a vortex ring solution with $n = 2$, $\alpha = 0.67$, $\hat{\gamma} = 0.32$ (c,d).

When $n = 3$, already the static solutions correspond to vortex ring solutions [11, 23]. Like these, the rotating vortex ring solutions possess two branches, merging at $\alpha_{\text{max}}$. As seen in Figs. 1 and 3, the mass, charge and magnetic moment exhibit an analogous $\alpha$ dependence for these rotating vortex ring solutions as for the rotating $n = 1$ and $n = 2$ solutions. The nodal ring continuously decreases in size along both branches, shrinking to zero size in the EYM limit.

**IV. CONCLUSIONS**

We have constructed stationary dyons and rotating monopole-antimonopole pairs and vortex rings in EYMH theory. Gravitating dyons cannot rotate, since they carry magnetic charge [17]. We have shown, that gravitating dyons with magnetic charge $n \geq 2$, however, are not static but stationary, possessing finite angular momentum density and a non-diagonal metric (see also [33]). Electrically charged monopole-antimonopole pairs as well as (magnetically neutral) vortex rings, on the other hand, must rotate with angular momentum $J = nQ$. For monopole-antimonopole pairs, consisting of singly charged magnetic poles, we have resolved the critical behaviour, by showing that the two branches of solutions cross before they merge. The same feature is present for rotating monopole-antimonopole pairs with $n \geq 2$ and rotating vortex rings. Interestingly, for monopole-antimonopole pairs, consisting of doubly charged magnetic poles, we have observed a transition to vortex rings at a value $\alpha_{\text{tr}}$ on the first branch. The transition value depends on the amount of electric charge present and thus on the rotation.

For the dyons, monopole-antimonopole pair solutions and vortex rings presented here, we have restricted the integers $m$ and $n$ in the general ansatz for the matter fields to $m = 1 - 2$ and $n = 1 - 3$. The solutions thus represent only the simplest types of EYMH solutions. For larger values of $m$ and $n$ new types of static solutions appear [11, 23, 28].
representing e.g. monopole-antimonopole chains and multi-vortex solutions. Study of their stationary or rotating generalizations may lead to further surprises.

Acknowledgments

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