Research on Optimal Design of Double Stator Low-speed High-torque Synchronous Motor Based on Surrogate Model

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Abstract. Optimizing double stator low-speed high-torque synchronous motor (DSLHSM) with many parameters is difficult and time-consuming, so an optimization design method suitable for multi-parameter problems was proposed. Through the method, the design optimization of DSLHSM can be more effective, and the optimization time is greatly shortened. Firstly, the optimization variables were selected after the sensitivity of the key parameters of average torque and torque ripple was analysed by using design of experiments (DOE) and analysis of variance (ANOVA). Secondly high-precision dynamic surrogate models of average torque and torque ripple were established based on response surface model (RSM), and then the objective function was established according to the surrogate model and the genetic algorithm (GA) as well as the sequential subspace optimization method (SSOM) was used to solve the objective function step by step. Finally, the optimal solution was substituted into the finite element simulation model. The target performances of the motor before and after optimization were compared and analysed. The results proved the effectiveness and correctness of the optimization design method.

1. Introduction

In the field of low-speed and large-load transmission, the permanent magnet direct drive system has gradually replaced the traditional drive system of asynchronous motor and reducer due to its high efficiency, high power factor, compact structure and other advantages [1]. Low-speed permanent magnet motors are mostly short and thick in theory, that is, a large cavity space inside the motor. In order to make full use of this space, double stator low-speed high-torque synchronous motor (DSLHSM) was proposed. DSLHSM is composed of two parts, which are external surface permanent magnet synchronous motor (SPMSM) and internal synchronous reluctance motor (SyRM). The design variables are numerous and there are strong interactions between the them. Therefore, it is very meaningful to consider multi-variable optimization design. The key issues of optimal design are to improve model accuracy and reduce computational cost [2]. To solve the above-mentioned problems, a research on DSLHSM was carried out and an optimal design method was proposed.

The two-dimensional topology configuration of DSLHSM is shown in Figure 1. The inner and outer rotors have the same rotational speed, and are connected to the same rotating shaft through a magnetic isolation ring. Both inner stator and outer stator have a set of windings, and the inner and outer magnetic circuits are independent of each other, making the control mode more flexible.

There is an offset angle \( \varepsilon \) between the d-axis of the permanent magnet rotor and the d-axis of the reluctance rotor so that both SPMSM and SyRM can run at the maximum torque/current condition. The rated power of the motor is 50kW, the rated speed is 90r/min, and the rated voltage is 1140V.
2. Selection of Key Parameters and Sensitivity Analysis

2.1. Selection of Key Parameters

The key parameters usually choose the independent parameters that have a greater impact on the optimization goal, which can determine other parameters [3]. The DSLHSM proposed in this paper is applied to the field of elevator traction machines. The diameter and length are fixed, and the average torque \( T_{\text{ave}} \) and torque ripple \( T_{\text{rip}} \) are taken as the optimization targets. The PM pole arc coefficient \( a_{p1} \) and thickness \( h_M \) of the PM rotor have a great influence on the PM torque, which are regarded as the key parameters. The salient pole height \( h \) and the width ratio \( m \) between the magnetic and non-magnetic layer affect the reluctance torque, which are taken as key parameters. \( a_{p2} \) is the magnetic barrier pole arc coefficient. The magnetic isolation ring is used as a connecting body of two rotors, and its thickness \( RT \) is taken as a key parameter to determine the outer diameter of the inner rotor. In addition, the air gaps have an important impact on the performance of the motor. Therefore, the outer air gap length \( g_1 \) and the inner air gap length \( g_2 \) are used as key parameters. The influence of slot depth and slot shoulder width on the torque is not obvious, and they are not chosen. The slot width \( b_0 \) affects the air gap flux density distribution and thus the torque, which is regarded as a key parameter. The inner stator is located inside the motor and the number of slots is large. The range of change is limited, so it is not used. In summary, \( a_{p1}, h_M, b_0, g_1, a_{p2}, m, h, g_2 \) and \( RT \) are used as key parameters.

2.2. Sensitivity Analysis

The construction process of the surrogate model is complicated and the accuracy of the model is difficult to guarantee due to many parameters. Therefore, sensitivity analysis of key parameters is needed. Compared with other partial factorial designs, the Maximin Latin hypercube design meets the uniformity of the design space and the uniformity of the projection at the same time. The initial values and design space of the key parameters are shown in Table 1. The upper and lower limits of the design space for other key parameters are defined as 1-20% and 1+20% of the initial value.

| Key Parameters | Initial value | Design Space |
|----------------|---------------|--------------|
| \( a_{p1} \)   | 0.80          | [0.64, 0.96] |
| \( h_M \)      | 6.00          | [4.80, 7.20] |
| \( b_0 \)      | 6.00          | [4.80, 7.20] |
| \( g_1 \)      | 2.50          | [2.00, 3.00] |
| \( a_{p2} \)   | 0.75          | [0.60, 0.90] |
| \( m \)        | 2.00          | [1.60, 2.40] |
| \( h \)        | 2.59          | [2.07, 3.11] |
| \( g_2 \)      | 0.60          | [0.48, 0.72] |
| \( RT \)       | 34.5          | [27.6, 34.5] |

Figure 1. Two-dimensional topology configuration
After the design of experiments (DOE) is completed, the sample points extracted are substituted into the finite element model to calculate the response values of the $T_{ave}$ and $T_{rip}$. Then analysis of variance (ANOVA) is used to analyse the sensitivity [4]. The significance level of the test is set to 0.05. Table 2 and Table 3 respectively show the ANOVA results of $T_{ave}$ and $T_{rip}$, where DF is the degree of freedom, AdjSS is the sum of squared deviations, and $F$ is the value of hypothesis testing with $F$ distribution, $P$ value is the probability of hypothesis. The smaller $P$ value, the more significant the influence of the corresponding parameter on the optimization goal. From Table 2 and Table 3, it can be seen that $\alpha_{\theta 1}$, $h_m$, $g_2$ are sensitive to $T_{ave}$ and $T_{rip}$, while $g_1$, $\alpha_{\theta 2}$, $b_0$, and $RT$ are all sensitive to single targets, but $m$ and $h$ are not sensitive to targets. Therefore, $\alpha_{\theta 1}$, $h_m$, $g_2$, $g_1$, $\alpha_{\theta 2}$, $b_0$, $RT$ are selected as the final optimal design variables.

Table 2. ANOVA data of average torque $T_{ave}$

| Parameters | DF | AdjSS  | $F$   | $P$   | Order |
|------------|----|--------|-------|-------|-------|
| $\alpha_{\theta 1}$ | 1  | 3.00331 | 387.99 | 0.000 | 1     |
| $h_m$      | 1  | 1.28628 | 166.17 | 0.000 | 1     |
| $b_0$      | 1  | 0.00022 | 0.0300 | 0.867 | 6     |
| $g_1$      | 1  | 1.50959 | 195.02 | 0.000 | 1     |
| $\alpha_{\theta 2}$ | 1 | 0.00022 | 0.0300 | 0.868 | 7     |
| $m$        | 1  | 0.00490 | 0.6300 | 0.430 | 4     |
| $h$        | 1  | 0.00114 | 0.1500 | 0.703 | 5     |
| $g_2$      | 1  | 0.10342 | 13.360 | 0.001 | 2     |
| $RT$       | 1  | 0.02882 | 3.7200 | 0.060 | 3     |

Table 3. ANOVA data of average torque $T_{rip}$

| Parameters | DF | AdjSS  | $F$   | $P$   | Order |
|------------|----|--------|-------|-------|-------|
| $\alpha_{\theta 1}$ | 1  | 0.00005 | 0.45  | 0.506 | 5     |
| $h_m$      | 1  | 0.00011 | 0.96  | 0.333 | 4     |
| $b_0$      | 1  | 0.00016 | 1.42  | 0.240 | 3     |
| $g_1$      | 1  | 0.00001 | 0.10  | 0.758 | 6     |
| $\alpha_{\theta 2}$ | 1 | 0.00025 | 2.81  | 0.147 | 2     |
| $m$        | 1  | 0.00000 | 0.01  | 0.909 | 9     |
| $h$        | 1  | 0.00000 | 0.03  | 0.868 | 8     |
| $g_2$      | 1  | 0.00077 | 6.69  | 0.013 | 1     |
| $RT$       | 1  | 0.00001 | 0.07  | 0.799 | 7     |

3. Construction Method of High-precision Surrogate Model

Response surface method (RSM) model has the advantages of simplicity, short modelling time, high transparency, and ability to remove digital noise. RSM uses a reasonable DOE to generate sample points and fits the function relationship between the response value and the design parameters through the response value of the corresponding sample point [5]. In this paper, the response surface function with a quadratic polynomial is selected to construct the surrogate models of $T_{ave}$ and $T_{rip}$. The basic expression of the model is

$$y = \beta_0 + \sum_{j=1}^{n} \beta_j x_j + \sum_{j=1}^{n} \beta_j x_j^2 + \sum_{j=1}^{n} \sum_{j=1}^{n} \beta_{ij} x_i x_j \tag{1}$$

Where $\beta$ is the regression coefficient, $x$ is the design variable, $y$ is the response value, and $n$ is the number of input variables. The accuracy of the surrogate model in engineering is judged by the coefficient of determination $R^2$. When $R^2 \geq 0.9$, it can basically meet the engineering needs.
For the poor accuracy of RSM model in dealing with highly dimensional and multivariate problems, a dynamic surrogate model is proposed to solve this problem. The model establishment flow chart is shown in Figure 2. $k$ is the number of iterations. The proposed dynamic surrogate model construction method can use fewer sample points, greatly reduce the number of finite element calculation calls, and the accuracy of the model is higher. $R^2>0.95$ is set in this paper, from a statistical point of view, the model accuracy is higher. For a general model, when sample points are used more, the constructed model is close to the real model. However, the number of finite element calculations increases. The proposed construction method adds the optimal sample points of the current model to the initial design space, purposefully increases the samples and avoids sample exhaustion. The method not only increases the accuracy of the entire model, but also increases the accuracy of the model in the area near the optimal solution.

![Figure 2. Construction flowchart of high-precision RSM model](image)

According to the above method, RSM surrogate models of optimized targets $T_{ave}$ and $T_{ip}$ are established. Theoretically, there are 36 terms in the two-dimensional RSM with seven variables.

For $T_{ave}$ surrogate model, the model accuracy is very high when the initial sample points are used for model, and $R^2$ is 99.78%, which provides convenience for the simplification of the surrogate model. In order to simplify the model, that is, to reduce the number of terms, the ANOVA method is again used to analyse the sensitivity of every term of the $T_{ave}$ surrogate model. After multiple calculations and model reconstruction, it can be seen that when the P value is set to 0.5, the accuracy of the model can still be guaranteed. At this time, the coefficient of determination $R^2$ is 98.89%. On this basis, a simplified $T_{ave}$ surrogate model that contains 18 terms is finally obtained. For $T_{ip}$ surrogate model, when the initial sample points are used for modelling, $R^2$ is 89.14%, which does not meet the accuracy requirements $R^2>0.95$ of the surrogate model in this article, so it needs to follow the construction process of the high-precision RSM dynamic surrogate model to add sample points and reconstruct the model. After many repeated calculations, it can be seen that when the $k=11$, the accuracy of the corresponding surrogate model meets the requirements. At the same time, the $R^2$ is 95.23%, and the number of surrogate model terms is 36.

In the process of motor optimization design, in addition to establishing the surrogate models of the optimization targets, it is also necessary to establish the surrogate models of the performance constraints. The effective value of the air gap flux density $B_1$ and $B_2$ of the external and internal motors are used as performance constraints. Since the internal and external magnetic circuits of the motor are independent of each other, $a_{P1}$, $h_M$, $g_1$, $b_0$ are selected to construct the surrogate model of $B_1$, and the $a_{P2}$, $g_2$, $RT$ are selected for $B_2$. 


After calculation, the $R^2$ of the surrogate model of $B_1$ is 99.92%, and $R^2$ of the surrogate model of $B_2$ is 99.87%. The accuracy of the model is high. The number of model terms is small, which are 15 and 10, so there is no need to simplify.

4. Optimal design of DSLHSM based on GA and SSOM

Genetic algorithm (GA) can directly operate on structural objects such as motor optimization variable arrays, matrices. Therefore, GA has been widely used in motor parameter identification, motor control [6]. However, there are the problems of slow convergence speed, large calculation amount, and low solution efficiency in the later stage of GA. Especially when there are many optimization variables, the time cost is greater. In order to solve this problem, a method combining GA and sequential subspace optimization method (SSOM) is proposed. SSOM divides optimization variables into multiple subspaces with fewer optimization variables according to their sensitivity to the optimization goal, so that the optimization algorithm can be solved directionally along the path of different subspaces [7].

4.1. Subspace Division of Optimization Variables

The optimized variables have been given in the previous section, namely $X = (\alpha_p, h_M, g_2, g_1, \alpha_R, b_0, RT)$. From the sensitivity analysis of the optimization variables, $\alpha_p, h_M, g_2$ are all sensitive to $T_{ave}$ and $T_{rip}$, so they are divided into the first subspace $X_1$: $g_1, \alpha_R, b_0, RT$ are sensitive to a single target, so they are divided into the second subspace $X_2$. Table 4 shows the subspace division and optimization sequence of optimization variables.

| Subspace | Variables | Sensitivity | Sequence |
|----------|-----------|-------------|----------|
| X1       | $\alpha_p, h_M, g_2$ | very strong | first optimize |
| X2       | $g_1, \alpha_R, b_0, RT$ | strong | then optimize |

4.2. Objective Function and Constraint Conditions

In order to increase $T_{ave}$ and minimize $T_{rip}$, a weighted method is used to establish the objective function, which makes the optimization design become a problem to find the minimum of the objective function. The objective function $F(X)$ is defined as follows:

$$F(X) = \omega_1 \frac{T_{ave_{initial}}}{T_{ave}} + \omega_2 \frac{T_{rip}}{T_{rip_{initial}}}$$  \hspace{1cm} (2)

Where $\omega_1$ and $\omega_2$ are the weight coefficients. Since $T_{ave}$ and $T_{rip}$ are equally important, $\omega_1$ and $\omega_2$ set to 0.5. $T_{ave_{initial}}$ is the average torque value of the original scheme before optimization, which is 5.45kNm, and $T_{rip_{initial}}$ is the torque ripple value of the original scheme before optimization, which is 6.79%.

Regarding the processing of constraint conditions, the performance constraints in the algorithm are as follows:

$$G_i(X) = B_i - 0.8 \leq 0$$

\hspace{1cm} (3)

The air gap flux density is constrained within the range of not more than 0.8T, so that the optimal solution can meet the requirements of the electromagnetic design of the motor. In addition, the one-pole permanent magnet volume $Cost$ is selected as the constraint condition for selecting the optimal solution $X$ from the Pareto solution set $X^*$ obtained from the above algorithm. The constraint in the algorithm is

$$G_t(X) = \min Cost(X) , \ X \in X^*$$

4.3. Process and Results of Motor Optimization
Figure 3 shows the flow chart of the optimization algorithm based on SSOM and GA. The population size M is set to 60, the number of termination iterations G is set to 20, the crossover probability $P_c$ is set to 0.9, the mutation probability $P_m$ is set to 0.4, and floating-point coding method is adopted. Convergence criterion is $\delta$ does not exceed 0.1%.

![Flow chart of the proposed optimization algorithm](image)

**Figure 3.** Flow chart of the proposed optimization algorithm

The calculation process data of the proposed optimization algorithm is shown in Table 5. It can be concluded that when the $k=2$, that is, after the optimization process is performed twice, the objective function $F(X)$ meets the convergence criterion. The output optimal solution $X= [0.885 5.358 0.509 2.216 0.608 6.939 32.42]$ while $F(X)$ is 0.7777 at this time.

**Table 5.** Calculation process data

| Parameters | Initial value | $X_1$ ($k=1$) | $X_2$ ($k=1$) | $X_1$ ($k=2$) | $X_2$ ($k=2$) |
|------------|---------------|---------------|---------------|---------------|---------------|
| $\alpha_{p_1}$ | 0.80 | 0.842 | 0.842 | 0.885 | 0.885 |
| $h_M$ (mm) | 6.00 | 5.700 | 5.700 | 5.358 | 5.358 |
| $g_2$ (mm) | 0.60 | 0.506 | 0.506 | 0.509 | 0.509 |
| $g_1$ (mm) | 2.50 | 2.500 | 2.220 | 2.220 | 2.216 |
| $\alpha_{p_2}$ | 0.75 | 0.750 | 0.602 | 0.602 | 0.608 |
| $b_0$ (mm) | 6.00 | 6.000 | 7.050 | 7.050 | 6.939 |
| $RT$ (mm) | 34.5 | 34.50 | 32.51 | 32.51 | 32.42 |
| $F(X)$ | 1.00 | 0.866 | 0.7867 | 0.7778 | 0.7777 |
| $T_{ave}$ (kNm) | 5.45 | 5.577 | 5.640 | 5.645 | 5.654 |
| $T_{trip}$ (%) | 6.79 | 5.400 | 4.000 | 4.008 | 4.016 |

Figure 4 shows the convergence process. It can be obtained intuitively that when $k=1$, the variables in the first subspace $X_1$ are optimized, and the value of the objective function quickly approaches the optimal solution. When optimizing the variables in the second subspace $X_2$, the objective function also moves closer to the optimal solution at a faster speed. Then the objective function value gradually moves closer to the optimal solution until convergence.
4.4. Comparison with optimization results by only using GA

In order to verify the feasibility and effectiveness of the proposed optimization algorithm, GA is directly used to solve the objective function. The two results obtained by only using GA and proposed method combining GA and SSOM are compared and analyzed. The optimal solution by only using GA is \( X = (a_{p1} h_m g_2 g_1 a_{p2} b_0 RT) = (0.870 \ 5.61 \ 0.481 \ 2.26 \ 0.600 \ 7.00 \ 32.82) \).

Table 6 shows the optimization results by only using GA and using the proposed optimization method combining GA and SSOM. The deviation of the two values is only 2.03%. And the total calculation time of the algorithm is reduced by 36%. The solution efficiency of the algorithm is greatly improved.

|                | Only GA | GA and SSOM |
|----------------|---------|-------------|
| \( F(X) \)    | 0.7623  | 0.7777      |
| Number of iterations | 15 | 12 |
| Calculation time(s) | 50 | 32 |

5. Analysis and Verification by FEM

The optimal solution obtained above is substituted into the finite element model for calculation, and the optimization goals of the motor before and after optimization are compared and analysed. Figure 5 shows the torque curve before and after optimization. Table 7 shows the results of the optimal parameters and optimization goals before and after optimization.

It can be seen that the average torque \( T_{ave} \) after optimization is 5.68kNm, which is 4.22% higher than the initial scheme, and the torque ripple \( T_{ripp} \) after optimization is 4.26%, which is reduced by 37.3%.
compared with the initial scheme, which meet the expected optimization requirements. The feasibility of the proposed optimization method is proved.

Table 7. Comparison of before and after optimization

| Parameters | Before optimization | After optimization |
|------------|---------------------|--------------------|
| $a_{p1}$   | 0.800               | 0.885              |
| $h_M$ (mm) | 6.00                | 5.36               |
| $b_0$ (mm) | 6.00                | 6.94               |
| $g_1$ (mm) | 2.50                | 2.22               |
| $a_{p2}$   | 0.750               | 0.608              |
| $g_2$ (mm) | 0.50                | 0.51               |
| $RT$ (mm)  | 34.5                | 32.42              |
| $T_{ave}$ (kNm) | 5.45      | 5.68               |
| $T_{tip}$ (%) | 6.79         | 4.26               |

6. Conclusion
Aiming at the problem that optimal design of DSLHSM with numerous parameters is difficult and time-consuming, an optimization design method is proposed. The sensitivity analysis of optimization parameters is mainly based on the Latin hypercube test design and the analysis of variance method. The response surface dynamic surrogate model established can use the least sample points and call the least finite element calculation to build a high-precision model, rather than blindly enumerating. The optimal solution of the model uses a method combining of GA and SSOM to make the algorithm proceed along a certain path, which improve the efficiency of the solution. Finally, the finite element model is established and the optimization goals after optimization are compared with them before optimization. $T_{ave}$ after optimization is 4.22% higher than the initial scheme, and $T_{tip}$ after optimization is reduced by 37.3%, which verified the correctness and effectiveness of the method. The multivariable optimization method proposed in this paper can be extended to other complex electromagnetic equipment with multivariable nonlinearity.

7. References
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