Evolvable Image Filter based on Rotation Invariant Pixel Correlations

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Abstract. As one of the most typical nonlinear filters, weighted median filter has considerable filter behaviour by assigning weights. But each pixel has only one weight in the traditional weighted median filter. Correlations between pixel and other pixels under the filtering template are not considered. A novel rotation-invariant image filter model is proposed here which has a stable filtering performance when filtering on rotated and reversed images, symmetry of the correlations is utilized to reduce the complexity. Cartesian Genetic Programming is adopted to evolve image filter operators. Experiments about rotated images show that this rotation invariant model exhibits robust characteristics. Image denoising of different images show that it has better properties of edge or details preservation, efficient noise attenuation.

1. Introduction

In case of traditional image processing, the image filters are designed for gaussian noise, salt and pepper noise, such as Weighted Median Filter (WMF) [1]. Linear function and Sigmoidal function are used for the weights (coefficients) optimization under the Mean Absolute Error criterion [2]. Evolutionary computation has a growth in both theoretical analyses and industrial applications [3,4]. Evolvable hardware (EHW) is a field of evolutionary computation that focuses on the hardware design [5], Cartesian Genetic Programming (CGP) is adopted to evolve image filters which are composed of simple functional operators (CGPF) [6].

In this paper, the correlations between pixel and other pixels under the filtering template are considered. A novel rotation-invariant image filter based on CGP algorithm (RI-CGPF) is proposed here, the image filter has a stable filtering performance when filtering on rotated and reversed images, symmetry of the correlations is utilized to reduce the complexity. Experiments about rotated images show that RI-CGPF exhibits better stability than the other image filters. Image denoising of different gray images show that RI-CGPF has better properties of edge or details preservation, efficient noise attenuation.
2. Cartesian Genetic Programming
Cartesian Genetic Programming (CGP) was originally proposed by Miller and Thomson [7,8] for the purpose of evolving digital circuits. It represents a program as a directed graph which allows the implicit reuse of nodes. A Cartesian program is defined as a set \( \langle G, n_i, n_o, n_{in}, F, n_f, n_e, n_c, l \rangle \) where \( G \) represents the genotype and is itself a set of integers representing the indexed \( n_i \) program inputs, the \( n_e \) node input connections and functions, and the \( n_o \) program output connections. The set \( F \) represents the \( n_f \) functions of the nodes. The number of nodes in a row and column are given by \( n_r \) and \( n_c \) respectively. The program inter-connectivity is defined by the levels back parameter \( l \), which determine how many previous columns of cells may have their outputs connected to a node in the current column. The algorithm was as follows [7]:
   a) Generate initial population at random
   b) Evaluate fitness of genotypes in population
   c) Promote fittest genotype to new population
   d) Fill remaining places in the population with mutated versions of the fittest
   e) Return to step 2 until stopping criterion reached

3. Architecture of System
Image filtering is mostly operated in the spatial domain where the input image convolves with the filter function. The kernel is shifted over image and multiplies its values with the corresponding pixel values of the image, each pixel value in an image is replaced with the median or the average value of its neighbors.

In this paper, the correlations between pixel and other pixels under the filtering template are considered. The kernel is considered as \( 3 \times 3 \) in this paper showed in Figure 1, take the input pixels \( I_1, I_2, I_5 \) for example, the correlation between \( I_1 \) and \( I_5 \) is different from the correlation between \( I_2 \) and \( I_5 \), then the correlations among different pixels must be considered. The correlations are represented as functions \( f(x, y) \) listed in Table 1. The input x and y and the outputs operate over 8 bits and the following functions are used: \( \lor \) means binary OR, \( \land \) means binary AND, \( \oplus \) means binary exclusive-OR, \( + \) means 8bit adder, \( +^\text{sat} \) means 8bit adder with saturation.

Table 1. The set of functions implemented in RI-CGPF

| \( x \lor y \) | \( x \land y \) | \( x \oplus y \) | \( x +^\text{sat} y \) |
|--------------|---------------|-------------|-----------------|
| 0            | \( 4 \)       | \( 5 \)     | \( 6 \)         |
| 1            | \( x \land y \) | mean(x,y)  | \( x +^\text{sat} y \) |
| 2            | \( x \oplus y \) | \( x +^\text{sat} y \) | min(x,y) |
| 3            | \( x +^\text{sat} y \) | min(x,y)  |                 |

In order to have a stable denoising performance when filtering on rotated and reversed images, symmetry of the correlations is discussed here. In Figure 1.1, the symmetry is discussed as follows:

According to the diagonal symmetry, the correlation between \( I_1 \) and \( I_4 \) is equivalent to the correlation between \( I_3 \) and \( I_6 \), the other correlations are concluded as follows:

\[
\begin{align*}
  f(I_1, I_2) &= f(I_1, I_4), \quad f(I_3, I_2) = f(I_3, I_6) \\
  f(I_1, I_5) &= f(I_9, I_8), \quad f(I_1, I_6) = f(I_9, I_7) \\
  f(I_7, I_8) &= f(I_7, I_4) \\
  f(I_1, I_2) &= f(I_3, I_6), \quad f(I_1, I_5) = f(I_9, I_8) \\
  f(I_7, I_8) &= f(I_7, I_4)
\end{align*}
\]

According to the symmetry of horizontal axis, the correlation between \( I_1 \) and \( I_4 \) is equivalent to the correlation between \( I_7 \) and \( I_4 \), the other correlations are concluded as follows:

\[
\begin{align*}
  f(I_1, I_4) &= f(I_7, I_4), \quad f(I_3, I_6) = f(I_9, I_8) \\
  f(I_1, I_5) &= f(I_9, I_8), \quad f(I_1, I_6) = f(I_9, I_7) \\
  f(I_7, I_8) &= f(I_9, I_7), \quad f(I_7, I_2) = f(I_9, I_6) \\
  f(I_1, I_2) &= f(I_3, I_2) \\
  f(I_7, I_8) &= f(I_9, I_7)
\end{align*}
\]

According to the symmetry of vertical axis, the correlation between \( I_1 \) and \( I_4 \) is equivalent to the correlation between \( I_3 \) and \( I_6 \), the other correlations are concluded as follows:

\[
\begin{align*}
  f(I_1, I_4) &= f(I_3, I_6), \quad f(I_1, I_5) = f(I_9, I_8) \\
  f(I_1, I_6) &= f(I_9, I_7), \quad f(I_1, I_2) = f(I_9, I_6) \\
  f(I_7, I_8) &= f(I_9, I_7)
\end{align*}
\]
From the symmetry discussion in Figure 1.1, the correlations in Figure 1.1 have the same effect on the image filtering, so the correlations showed in Figure 1.1 can be simplified as the same function \( f_1 \). Furthermore, the symmetry of diagonal, horizontal axis and vertical axis of the other correlations among Figure 1.2 to Figure 1.8 are discussed, then the correlations can be simplified as \( f_2 \) in Figure 1.2 and so on. All the correlations among the input pixels are simplified as a function set of \( \{ f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8 \} \).

Then the evolvable RI-CGPF is modeled as an array of \( n_f \) and \( n_c \) programmable elements based on CGP algorithm showed in Figure 2, the input pixels under the filter template are processed by the functions set \( \{ f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8 \} \) listed in Table 1, the outputs of functions set are used as input of the reconfigurable directed graph. Also, an output function \( f_{out} \) is needed to get the final value of the image filtering.

**Figure 1.** Symmetry of correlations under filtering template

**Figure 2.** An example of RI-CGPF. Only utilized programmable elements are marked.

**4. Experimental Details**

The following parameters of CGP were set up as default: \( n_f = 9, n_o = 1, n_c = 7, n_p = 4, l = 2, n_r = 8 \). The population size is 10, the evolution was typically stopped after 6000 generations.
Mutation of two randomly selected genes is applied for the evolution. The MSE criterion between the filtered image and the original image is used for the fitness function:

\[ MSE = \frac{1}{K_1K_2} \sum_{i=1}^{K_1K_2} (o_i - y_i)^2 \]

Lena image is used for the evolution training. In order to evaluate the stability of RI-CGPF, eight rotated and reversed Lena images in Figure 3 are tested. The kernels of LWMF and SWMF are defined as [2]:

\[
LWMF = \begin{pmatrix}
0.06 & 0.10 & 0.05 \\
0.11 & 0.41 & 0.11 \\
0.08 & 0.12 & 0.08 \\
\end{pmatrix}
\]

\[
SWMF = \begin{pmatrix}
0.45 & 0.71 & 0.44 \\
0.84 & 2.33 & 0.85 \\
0.42 & 0.68 & 0.45 \\
\end{pmatrix}
\]

From the results showed in Figure 4, we can see that RI-CGPF has the same MSE values among the eight rotated and reversed Lena images, but the values tested by the others are not, which means RI-CGPF has better stability than the others.
that RI-CGPF produces the noise-free image with better preserved image details. The results of PSNR also show that RI-CGPF exhibits acceptable noise attenuation capabilities.

| Image    | MAE       | MSE       | PSNR   | MAE       | MSE       | PSNR   |
|----------|-----------|-----------|--------|-----------|-----------|--------|
| Lena     | 6.502     | 920.768   | 18.490 | 6.345     | 845.624   | 18.859 |
| Peppers  | 6.320     | 52.449    | 30.933 | 6.214     | 48.122    | 28.552 |
| Baboon   | 1.184     | 47.631    | 31.352 | 2.334     | 47.279    | 31.307 |
| Camera   | 1.408     | 44.126    | 33.040 | 2.767     | 52.662    | 31.384 |
| Goldhill | 0.871     | 12.643    | 37.112 | 2.380     | 15.275    | 30.915 |

5. Conclusions
In this paper, a novel rotation-invariant image filter based on CGP algorithm is proposed. Symmetry of the correlations in the kernel is utilized to reduce the complexity. Experiments of rotated images show that RI-CGPF exhibits better robust characteristics than the other traditional image filters. Results of different images show that RI-CGPF has better properties of edge or details preservation, efficient noise attenuation. Furthermore, RI-CGPF may have good performance in the complex environment with unknown noises by learning and evolving.

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