Turbulent pressure support and hydrostatic mass-bias in the intracluster medium

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ABSTRACT

The degree of turbulent pressure support by residual gas motions in galaxy clusters is not well known. The X-ray spectrometer on board Hitomi satellite has measured the turbulent velocities in the centre of the Perseus cluster, whereas the mass modelling of combined X-ray and Sunyaev Zel’dovich observations gives an estimate of this support in the outer regions of galaxy clusters. Cosmological simulations may help to quantify the amount of turbulent pressure, but the estimates vary widely. In this work, we test a new filtering technique to disentangle the bulk gas motion from the turbulent one in high-resolution cosmological simulations of galaxy clusters using the cosmological hydro code ENZO. We focused on the ratio of non-thermal pressure to total gas pressure as a function of cluster-centric distance. We find that the radial behavior can be described by a simple polynomial function. The typical non-thermal pressure support in the centre of clusters is $\sim5\%$, increasing to $\sim15\%$ in the outskirts, in line with the pressure excess found in recent X-ray observations. We also find that the total mass recovered under the assumption of hydrostatic equilibrium is affected by a bias with a non-negligible contribution from residual radial accelerations of the gas, often yielding differences compared to the real turbulent energy budget in simulations. Our study highlights the relation between shocks and radial accelerations, and the role of gas dynamical processes in the hydrostatic mass bias.

Key words: galaxy clusters, general – methods: numerical – intergalactic medium – large-scale structure of Universe – turbulence – hydrostatic mass bias

1 INTRODUCTION

Turbulence plays a key role in the assembly of large-scale structure and in controlling the physics of the intracluster medium (ICM) (e.g. Brunetti & Jones 2014). The origin and evolution of turbulence in the ICM have been widely studied using hydrodynamical simulations (e.g. Dolag et al. 2005; Lau et al. 2009; Vazza et al. 2011; Miniati 2014; Gaspari et al. 2014). Various physical processes produce turbulence in galaxy clusters, such as the injection and amplification of vorticity by shock waves (e.g. Ryu et al. 2008; Porter et al. 2015; Vazza et al. 2017) or ram pressure stripping (e.g. Subramanian et al. 2006; Cassano & Brunetti 2005; Roediger & Brüggen 2007). Moreover, winds from star-burst galaxies and outflows from active galactic nuclei stir the ICM, especially in cluster cores (e.g. Brüggen et al. 2005; Gaspari et al. 2011).

Direct observations of turbulent gas motions in the ICM are almost entirely missing. Only the Soft X-ray Spectrometer (SXS) on board Hitomi satellite has directly detected turbulent gas motions in the core of the Perseus cluster, a fairly relaxed cluster. Using the width of atomic lines, the root-mean square velocities were found to be $\sim200\ \text{km/s}$ on $\leq 60\ \text{kpc}$ scales (e.g. Hitomi Collaboration et al. 2016; ZuHone et al. 2018).

Radio observations of Faraday Rotation of polarised sources located behind galaxy clusters hint at a tangled magnetic field in the ICM (with typical coherence scales in the range of $\sim 10 - 50\ \text{kpc}$ (e.g. Murgia et al. 2004; Vogt & Enßlin 2005; Bonafede et al. 2010)), which is naturally explained by volume-filling stretching motions induced by turbulence (e.g. Dolag et al. 2001; Donnert et al. 2017).
In order to explain their observed morphology and strength, other indirect probes of turbulent motions are obtained from highly resolved X-ray surface brightness fluctuations, which are interpreted as indications of moderate density fluctuations induced by turbulence (e.g. Schuecker et al. 2004; Churazov et al. 2012; Gaspari et al. 2014; Zhuravleva et al. 2014). From a comparison between X-ray and radio observations, it has been suggested that the surface brightness fluctuations correlate with the diffuse radio emission (Eckert et al. 2017b; Bonafede et al. 2018). This suggests that turbulence detected in X-ray emission could be linked to the re-acceleration of radio emitting particles (e.g. Brunetti & Lazarian 2011). The observed mass bias may also be caused by turbulent motions in the ICM (Morandi et al. 2011; Parrish et al. 2012; Shi & Komatsu 2014; Shi et al. 2015, 2016; Fusco-Femiano & Lapi 2018; Ota et al. 2018; Fusco-Femiano 2019).

Recently, Eckert et al. (2019) and Ettori et al. (2019) presented results of a systematic study of non-thermal support and hydrostatic mass modeling of several galaxy clusters based on X-ray observations suggests that it is necessary to include a non-thermal pressure support. This hydrostatic mass modeling of several galaxy clusters should vary between 5 to 15%. Such values are a factor of 2 to 3 below what is found in numerical simulations (e.g. Lau et al. 2009; Vazza et al. 2011; Nelson et al. 2014; Biffi et al. 2016; Kay et al. 2004; Faltenbacher et al. 2005; Rasia et al. 2006; Hallman et al. 2006; Nagai et al. 2007). This discrepancy may either be due to missing physics in the simulations such as physical viscosity, magnetic fields, or due to an incorrect separation of turbulent and bulk motions. The results from cosmological simulations may depend on the numerical techniques used to disentangle bulk from laminar motions. As recently discussed in Vazza et al. (2018), different definitions of turbulent motions in numerical simulations could yield non-thermal pressures that differ by factors of 2 to 3, even within the same simulations. Recently, Valdarnini (2019) studied turbulent motions in galaxy clusters simulated with (radiative and non-radiative) N-body/SPH codes, using a multi-scale filtering technique. Their results are consistent with Vazza et al. (2018), despite the difference in the underlying hydro schemes. This suggests that advanced filtering techniques to study the internal dynamics of the simulated ICM are important to assess the mass-bias in galaxy clusters. In this paper, we measure the non-thermal pressure support by gas motions in the simulated ICM. We apply advanced filtering techniques to identify turbulence in a sample of galaxy clusters produced with high-resolution Eulerian simulations. Our results for the turbulent pressures are then compared to the constraints obtained in X-COP sample (Eckert et al. 2019), and in other numerical simulations (Nelson et al. 2014).

The paper is structured as follows: in Sec. 2 we describe our cluster sample and the numerical techniques used in the analysis of turbulent motions in the simulated ICM; in Sec. 3 we give our results from the analysis of our sample and the comparison between our work and recent observational and numerical results. In Sec. 4 we discuss the results and the limitations of our analysis and their implications for future work.
Finally, we have to verify that the mass growth between the snapshots is compatible with the expected growth. In particular, we checked that the corresponding $M_{100}$ mass is below or equal to the predicted mass, within some tolerance ($0 \pm 20\%$), based on the theoretical mass growth for a given $M_{100}$ at $z = 0$ for the given cosmology, as outlined in Giocoli et al. (2012a) and De Boni et al. (2016).

We treat each new selected cluster, along the mass growth, as independent from the previous one when calculating the theoretical mass accretion history (Giocoli et al. 2012b). In Fig. 1, the blue curve displays the mass growth history of one of our clusters from $z = 0$ to $z = 2$. The dashed black line shows the corresponding mass accretion history model starting from the $z = 0$ system. The various data points indicate the selected independent clusters along the growth with different tolerance thresholds. Thus, we obtained a final sample of 68 clusters (with $0\%$ tolerance), yielding the total mass function shown in Fig. 2. For comparison, the Despali et al. (2016) mass function at $z = 0$ for the same cosmology and total volume is shown as a black dashed line, and this suggests that our final sample is sufficiently mass complete for $M > 5 \cdot 10^{13} M_{\odot}$.

This allows us to proceed with a statistical study of the dependence of turbulence on mass, redshift and dynamical state parameters in sub-samples. The limitations connected to this selection procedure are discussed in Sec. 4.

### 2.3 Identifying turbulence in the ICM

To disentangle turbulent from bulk motions, we use a small-scale filtering approach. In this technique, we assume that turbulent velocities are approximated as those parts of the 3D gas velocities that fluctuate on the smallest scales, while bulk motions on the largest scales are approximately laminar. The validity of such approach in cosmological simulations of galaxy clusters is supported by a large body of works on this subject (e.g. Dolag et al. 2005; Lau et al. 2009; Vazza et al. 2011, 2012; Miniati 2014; Vazza et al. 2017). With the use of an appropriate small-scale filter, it is possible to define the velocity of the bulk motions and to calculate the velocity of turbulence motions like the difference between total velocity and bulk ones. In this section we discuss the updated filtering technique which we used to disentangle laminar to bulk motions, and

$$\alpha_{200} = a \cdot \bar{x}^b,$$

where $\bar{x}$ is the value of smoothing scale in physical quantities and $a$ and $b$ are the parameters obtained from Kolmogorov’s theory (Kolmogorov 1941). The expected value for $b$ is close to $\frac{2}{3}$ for stationary and subsonic turbulence. However, the ICM is not such an idealized environment because of density stratification, self-gravity and non-stationary flow patterns, which can lead to deviations from $2/3$. Fig. 3 shows the pressure ratio, $\alpha_{200}$, versus the smoothing scale for our set of clusters at $z = 0$. The trend is very similar across our sample, and can be fitted by a unique power-law. We fit the data
to Eq. 2 and obtain $a \simeq 6 \cdot 10^{-3}$ and $b \simeq 0.77$, with a $\chi^2$ value of 0.02. The value for $b$ is reasonably close to $2/3$ and is consistent with the fact that the power spectra of the velocity field in simulated galaxy clusters are typically steeper than Kolmogorov’s slope because of the stratified cluster atmosphere (Vazza et al. 2011). Only the scales below $\sim 100$ kpc show hints of a steepening, which may partially be ascribed to numerical dissipation in the PPM scheme, which is expected to dampen the velocities on scales close to a few times the spatial resolution (e.g., Porter & Woodward 1994). For scales larger than $\sim 8$ times the numerical resolution ($>200$ kpc) these effects do not occur and the relation between $\alpha$ and the smoothing scale is well fitted by Kolmogorov’s spectrum. Since a number of physical and numerical effects may affect the dynamics of the turbulent flow on $<100$-200 kpc, with these simulations it is hard to tell the different effects apart. In the following, we will mostly focus on the dynamics of turbulence on scales $>100$ kpc, which are also the ones that dominate the non-thermal pressure support. On scales greater than $\sim 1$ Mpc, the spectra show a drop where the peak of Kolmogorov spectrum is reached. The exponent $b$ in Eq. 2 is calculated in the inertial range of Kolmogorov spectrum, from $\sim 200$ to $\sim 800$ kpc, so we can use this value for the multi-scale adaptive filtering which is described below (Vazza et al. 2012).

We will use an adaptive, iterative filtering (Vazza et al. 2018) designed to disentangle turbulent from laminar motions in hydrodynamical grid simulations (Vazza et al. 2012). This algorithm does not assume any a-priori coherence scale and the local mean velocity field around each cell is reconstructed with a multi-scale filtering technique, yielding the maximum scale of turbulent eddies by means of iterations in the smoothing scale length. The key assumption is that the gas flow in these simulations is generally part of a cascade of kinetic energy starting from scales much larger than the cell size. In the original work, the authors applied a fixed tolerance on the increase of the local rms velocity amplitude with the filtering scale to stop the iterations, and find the smoothing scale of each cell (Vazza et al. 2012). For a better removal of spurious contribution from shock waves, the method has been combined with a velocity-based shock finder (Vazza et al. 2017). As a novelty of this work, here we explore a more physical definition for the tolerance needed by our iterative algorithm to stop and converge on the local turbulent velocity field. Based on the dependence of $\alpha$ on the local filtering scale, we modified the multi-scale adaptive filtering by Vazza et al. 2012 to include the scale-dependent expected increase in the local rms velocity. In the original work the authors applied a fixed tolerance of 1% to stop the iterations and find the smoothing scale of each cell. In this work we modified this condition in order to give a more physical condition and, based on Kolmogorov’s theory, we defined a variable tolerance $\epsilon_w$ for each iteration from the following equation:

$$\epsilon_w = \frac{w^f - (w-1)^f}{w^f},$$

where $w$ is the size of the smoothing scale in cell’s unit and $f$ is the exponent of the Kolmogorov’s relations, both the standard value $\frac{2}{3}$ or 0.77 the value which we obtained from our preliminary study. At the lower smoothing scale, this value is too high and the best choice is the minimum value between $\epsilon_w$ and the fixed tolerance used in Vazza et al. 2012. We verified that only for scales smaller than 200 kpc, $\epsilon$ is greater than 1%. As discussed in Vazza et al. 2012, we defined the turbulent velocity in each cell as:

$$\delta v = v - v_{sm},$$

where $v$ is the velocity field obtained from simulations and $v_{sm}$ is the velocity field obtained by a 3D spatial filtering defined as (in the simple 1D case):

$$v_{sm,j} = \frac{1}{w} \Sigma_{j=m-i}^{j=m+i} v_j,$$

where $w$ is the size of the smoothing scale in cell’s unit, which determines the number of cells on which $v_{sm}$ is calculated at each iteration step. We compute the relative variation of the turbulent local velocity $\delta v$ between two successive iterations ‘w-1’ and ‘w’ as:

$$\delta v_w = \frac{\delta v_{w}^2 - \delta v_{w-1}^2}{\delta v_{w}^2},$$

Wherever $\delta_v < \epsilon_v$, we find the value of turbulent velocity and the value of the smoothing scale. We test this procedure with two different exponents for the definition of the tolerance and also with a fixed tolerance as described in Vazza et al. 2012. The distribution of smoothing scales is shown in Fig. 4. The reconstruction of the turbulent velocity before the application of other filtering techniques is shown in Fig. 5 for different configurations of the filtering. Both Fig. 4 and Fig. 5 show that the definitions of tolerance have a minor effect on the distribution of the scales or the reconstruction.
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Figure 5. Maps of central slice of IT92.0 at $z = 0$. From left to right: First panel: Unfiltered velocity field [cm s$^{-1}$]; Second Panel: Filtered velocity field for tolerance determined by Kolmogorov relation with 0.77 exponent [cm s$^{-1}$]; Third panel: Filtered velocity field for tolerance determined by the standard Kolmogorov relation [cm s$^{-1}$]; Forth panel: Filtered velocity field for fixed tolerance equal to 1% [cm s$^{-1}$].

Figure 6. Radial profile of non-thermal pressure support for each cluster at $z = 0$ for different definitions of the tolerance, $\epsilon_w$ (Eq.4) used to stop out iterations on the local turbulent velocity field. This behavior is also visible in the radial profile of $\alpha$, as shown in Fig. 6. Here, it is clear that variations in the tolerance lead to small effects on the resulting non-thermal pressure. We also tested if this new definition of tolerance could affect the radial behavior of the smoothing scales. We noticed an increase in the smoothing scale of $\leq 20\%$ from the centre of the cluster to the outskirts, which also results in an average increase of the non-thermal pressure at most by $\leq 30\%$ (e.g. Vazza et al. 2012). However, the radial trend of the turbulent pressure support measured in our data (see following Section) is not an artifact of the filtering procedure: when no filtering is applied, the predicted radial increase of non-thermal support from gas motions in our data (Vazza et al. 2018) as well as in other works (e.g. Nelson et al. 2014) is much steeper.

Figure 7. Radial profile of median value of non-thermal pressure support for a sub-sample of clusters at $z = 0$, obtained by considering the 50%, 75%, or 90% least dense cells at each radial bin from the centre of clusters. In the following, we decided to use the variable tolerance referred to the $f = 0.77$ case. Moreover, in the following we will also combine this with the additional filtering of shocks and gas clumps, to better disentangle turbulent motions from other small-scale hydrodynamical features.

2.3.2 Spurious contributions: shocks and density clumps filtering

Shocks identification

In the study of turbulence, shocks can introduce spurious terms in the estimate of turbulent kinetic energy. In the presence of shocks, it is possible to use the Rankine-Hugoniot conditions and use velocity or temperature jumps to determine the Mach number. The Mach number is used to calculate the flux of kinetic energy that is
dissipated into gas thermal energy. Here we use the shock finding algorithm based on the the velocity jump between neighbouring cells (Vazza et al. 2009, 2017, 2018). Detecting shocks with high Mach numbers is relatively easy task in grid simulations with a uniform resolution (and all clusters in the ITASCA sample were simulated with uniform resolution in the "zoom" region), yet the detection of shocks with small Mach numbers is made uncertain by several factors such as numerical errors due to strong gradients or oblique directions of the shocks. In order to reduce the potential noise in the reconstruction of the local turbulent velocity field due to weak shocks sweeping our volume, we decide to set a lower limit to the Mach number of $M_{\text{thr}} = 1.3$. We refer the reader to Vazza et al. (2017) and to Vazza et al. (2018) for an overview of this shock finding method.

**Clumps excision**

Dense clumps associated with infalling structures can introduce a bias in the estimate of the local velocity field (e.g. Dolag et al. 2005), due to the fact that these structures are correlated with large bulk motions, mostly in the inwards radial direction (e.g. Vazza et al. 2018). These spurious terms could lead to an overestimate of the non-thermal pressure support. Clumps in simulations are routinely identified as peaks with high density contrast in the radial gas density distribution of the host cluster (e.g. Ruszkowski & Oh 2011, Zhuravleva et al. 2011). Therefore, restricting the analysis to a fraction of the gas density distribution at every radius, obtained after excising the highest percentiles in the gas distribution at each radius, is a practical way to limit the bias from the most clumpy structures in the ICM. Hence, we tested three different values for masking the densest cells (considering gas density only) at each radius from the cluster centre: the cells in the top 50%, 25% or 10% of the gas density distribution at every radius. As shown in Fig. 7, the profile of non-thermal pressure support we can derive in our clusters at $z = 0$ is overall quite robust against a more restricting selection of cells in the low density part of the distribution at each radius. Based on our results and previous work (e.g., Zhuravleva et al. 2013, Roncarelli et al. 2013), we decided to use the 90% masking in our analysis. As we will show later, this approach is similar to the techniques applied to X-ray surface brightness maps (e.g., Ghirardini et al. 2017, Eckert et al. 2019).
Based on our tests, the best filtering technique turned out to be one that excludes all cells with $M \geq 1.3$ and/or the cells of the top 10 percentile in density in each shell. In the following, we will refer to the results of our best filtering configuration as the turbulent velocity. The reader is referred to Vazza et al. [2018] for a comparison of this filtering technique to others that have been used in the literature (e.g., Nelson et al. [2014]).

3 RESULTS

To present the sample used in this work we show the maps of a central region for one cluster of our catalog. In Fig. 8 we show the gas density, dark matter density, gas temperature and unfiltered velocity field, as well as the turbulent velocity field and shocks. When presenting radial profiles, we define the center based on the maximum value of the thermal energy of the gas ($E_{th} \propto \rho \cdot T$). This definition of the centre is more stable with respect to the maximum of the total density ($\rho_{gas} + \rho_{DM}$), especially in high perturbed systems. To compute the non-thermal pressure support given by the turbulent motions we define the non-thermal pressure $P_{nt}$ and the thermal pressure $P_{th}$ as:

$$P_{nt} = \frac{1}{3} \cdot \rho \cdot \delta v^2$$

and

$$P_{th} = \frac{k_b}{\mu m_p} \cdot \rho \cdot T,$$

where $\rho$ is the gas density, $T$ is the gas temperature, $k_b$ is the Boltzmann constant, $m_p$ is the proton mass, $\mu$ is the mean molecular mass for electrons gas and its value is 0.59, $\delta v$ is turbulent velocity. To study the ratio of non-thermal pressure versus total one (the sum of the non-thermal and thermal pressure), we used the average radial profile of the pressures, always considering the same selection of cells. We call this ratio $\alpha$ and we defined it as in Eq. 1. In the following, we will study two approaches to estimate $\alpha$ via the kinetic pressure associated with the rms velocity field directly measured in our filtering approach for turbulence ($\alpha_{Turb}$) or by the comparison between the total mass distribution and the mass which can be estimated from the assumption of hydrostatic equilibrium in the cluster atmosphere ($\alpha_{HS}$).

3.1 Parametrising the profile of non-thermal pressure support in galaxy clusters

The radial distribution of non-thermal pressure support we find in our cluster sample is so regular that an analytic formula well reproduces the trend of $\alpha_{Turb}$ with radius:

$$\alpha_{Turb}(r) = a_0 \cdot \left( \frac{r}{R_{200}} \right)^{a_1} + a_2. \quad (10)$$

The physical meaning of our parameters is straightforward: $a_0$ represent the normalization of $\alpha_{Turb}$ at $R_{200}$, $a_1$ gives the slope of the profile and $a_2$ gives the value of non-thermal support in the cluster center. We notice that Shi & Komatsu [2014] developed an analytic model to describe the trend of $\alpha_{Turb}$ with the radius. They use three fundamental time scales to develop their model: turbulence dissipation time-scale, $t_d$; the time elapsed between the initial time and the time of observation, $(t_{obs}, t_0)$, which characterizes the age of the cluster; and a time-scale characterizing the mass growth rate of the cluster defined by $t_{\text{growth}}$. They defined also turbulence injection efficiency $\eta$ and which they constrained to $\eta \approx 0.5 - 1$ based on simulations. However, the turbulence injection efficiency is strongly correlated with the slope of the fitting formula, and compared to Shi & Komatsu [2014], we report a lower injection efficiency, which may also be connected to the role of numerical dissipation of our hydro scheme on small scales. We also notice that in real systems, and especially in the low mass of cluster sampled by our data set, the turbulence in the core may be dominated by the interplay of cooling and feedback (e.g., Brighten et al. [2016]).

Nelson et al. [2014] presented the following analytical fit to the radial distribution of the non-thermal pressure in dataset of 65 simulated galaxy clusters in a similar mass range of our dataset:

$$F_{rad}/F_{total}(r) = 1 - A \left\{ 1 + \exp \left[ - \left( \frac{r}{B} \right)^\gamma \right] \right\}, \quad (11)$$

with best-fit values $A = 0.452 \pm 0.001, B = 0.841 \pm 0.008$ and $\gamma = 1.628 \pm 0.019$ (Nelson et al. [2014]). This fit formula is based on three-dimensional gas velocity fields with a less aggressive filtering of bulk motions, as discussed in Vazza et al. [2018]. The same function also fits our data after filtering, albeit with a slightly higher $\chi^2$ value (see Tab. 1). In Fig. 9 we show the median radial profiles of our sample along with the fits.

From the comparison of the $\chi^2$, it appears that our model yields a better fit to the data than, or as good as, the model in Nelson et al. [2014]. The fit suggested by Nelson et al. [2014] can also fit our data, albeit with different parameters. However, the advantage of our best-fit form is that the fit parameters have a simple physical meaning.

![Figure 9. Radial profile of the median value (blue solid line) of the non-thermal pressure support (shadow regions represent the $1\sigma$, $2\sigma$ and $3\sigma$ variance) and the fits obtained with our model (dash-dotted red line) and the one proposed by Nelson et al. [2014] (dashed green line). We also show the model proposed in Nelson et al. [2014] with the values of the parameters which they found for their sample (dotted gray line).](image)
Figure 10. Radial profile of median value (blue solid line) of non-thermal pressure support for sub-samples of mass (top row), redshift (central row) and mass sparsity (bottom row). The shadow regions represent the $1\sigma$, $2\sigma$ and $3\sigma$ variance. The fit obtained with our model is shown as dash-dotted red line in each panel.
They compared the hydrostatic mass recovered up to $R_{500}$ to a sample of 12 clusters observed in X-rays using XMM-Newton. Recently, Eckert et al. (2019) studied turbulence in the ICM using XMM-Newton. Their mass estimates are based on the assumptions that hydrodynamical simulations provide the correct baryon fraction distribution in clusters, that the gas mass is correctly inferred from X-ray measurements and that the contribution from the stellar mass fraction can be evaluated statistically from published work. From the mismatch between the two estimates of the total mass, it is thus possible to infer the hydrostatic bias, which turns out to be, on average, consistent with the results obtained by other methods (see Ettori et al. 2019). If one attributes the origin of this hydrostatic bias to the contribution from any non-thermal pressure component, then, following Eckert et al. (2019), one can write:

$$\frac{d}{dr}(P_{th}(r) + P_{nt}(r)) = -\rho \frac{GM_H(<r)}{r^2},$$

where $P_{th}$ and $P_{nt}$ are the thermal and non-thermal pressure components, respectively, and $M_H$ is the total mass. By defining $\alpha(r) = P_{nt}(r)/P_{tot}(r) = P_{nt}(r)/(P_{th}(r) + P_{nt}(r))$, the equation above can be rewritten as:

$$M_H(<r) = \alpha(r)M_T(<r) - \frac{P_{th}r^2}{(1 - \alpha)\rho G} \frac{d\alpha}{dr},$$

where $M_H$ is the hydrostatic mass:

$$M_H(i) = -\left(\frac{dP_{th}}{dr}\right) \frac{r^2}{G\rho_{i-1}}.$$

From the equations above and using our radial profiles of total mass and hydrostatic mass, we can then define $\alpha_{HS}$ at each radius $r$ as:

$$\alpha_{HS} = 1 - \frac{M_H + \sqrt{(M_H)^2 - 4M_TP_{th}^2/\rho G}}{2M_T},$$

and link it to the parameter $b$, which is usually used in literature to identify the hydrostatic mass bias (e.g., Salvati et al. 2019; Pratt et al. 2019) and defined as:

$$M_H = (1 - b)M_T,$$

to obtain

$$b = \frac{\alpha + A}{1 + A},$$

where $A$ encloses the pressure’s contributions

$$A = (P_{th} + P_{nt}) \frac{d\alpha}{dr} \frac{dP_{th}}{dr}.$$

We notice that if $\alpha$ is radially constant, then $b = \alpha$; however, in general $\alpha$ is not really constant with radius in our sample (Sec. 3.1), hence the $d\alpha/dr$ term plays a small but non-negligible role here.

As we already discussed in Vazza et al. (2018) the differences between our results and Nelson et al. (2014) stem from the different choices in filtering velocities, and the two methods yield formally the same result if no filtering is applied to the 3-dimensional velocity field in our simulations. However, our work suggests that this filtering yields the isotropic part of the turbulent pressure, while filtering out the spurious contribution to the non-thermal pressure support by inward radial motions. Finally, we investigated the possible correlations between the non-thermal pressure and mass, redshift and sparsity of each clusters in our sample. For all of these quantities, we divided our samples into three sub-samples which contain the same number of objects. The results are shown in Fig. 10. Then, we applied the same fitting formula used above at each sub-sample, and the results are shown in Tab. 2. Our best-fit model gives similar results when applied to sub-samples in mass, redshift or mass sparsity.

Recently, Eckert et al. (2019) studied turbulence in the ICM using a sample of 12 clusters observed in X-rays using XMM-Newton. They compared the hydrostatic mass recovered up to $R_{200}$ by using a combination of SZ data from the Planck satellite and X-ray spectral and spatial constraints on the ICM derived from XMM-Newton. Their mass estimates are based on the assumptions that hydrodynamical simulations provide the correct baryon fraction distribution in clusters, that the gas mass is correctly inferred from X-ray measurements and that the contribution from the stellar mass fraction can be evaluated statistically from published work. From the mismatch between the two estimates of the total mass, it is thus possible to infer the hydrostatic bias, which turns out to be, on average, consistent with the results obtained by other methods (see Ettori et al. 2019). If one attributes the origin of this hydrostatic bias to the contribution from any non-thermal pressure component, then, following Eckert et al. (2019), one can write:

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From the equations above and using our radial profiles of total mass and hydrostatic mass, we can then define $\alpha_{HS}$ at each radius $r$ as:

$$\alpha_{HS} = 1 - \frac{M_H + \sqrt{(M_H)^2 - 4M_TP_{th}^2/\rho G}}{2M_T},$$

and link it to the parameter $b$, which is usually used in literature to identify the hydrostatic mass bias (e.g., Salvati et al. 2019; Pratt et al. 2019) and defined as:

$$M_H = (1 - b)M_T,$$

to obtain

$$b = \frac{\alpha + A}{1 + A},$$

where $A$ encloses the pressure’s contributions

$$A = (P_{th} + P_{nt}) \frac{d\alpha}{dr} \frac{dP_{th}}{dr}.$$

We notice that if $\alpha$ is radially constant, then $b = \alpha$; however, in general $\alpha$ is not really constant with radius in our sample (Sec. 3.1), hence the $d\alpha/dr$ term plays a small but non-negligible role here.

|                | Our model |         |         |         |         |
|----------------|-----------|---------|---------|---------|---------|
| $a_0$          | $(6.04\pm0.02) \cdot 10^{-2}$ |         |         |         |         |
| $a_1$          | $1.323 \pm 0.005$ |         |         |         |         |
| $a_2$          | $(5.88\pm0.02) \cdot 10^{-2}$ |         |         |         | 0.006   |

|                | Nelson’s model |         |         |         |         |
|----------------|----------------|---------|---------|---------|---------|
| $A$            | $4.690\pm0.001 \cdot 10^{-1}$ |         |         |         |         |
| $R$            | 0.45            | 0.84        | 1.63           |         |         |
| $\gamma$       | $1.526\pm0.006$ |         |         |         | 0.007   |

Table 1. Parameters and values of $\chi^2$ statistical test for the different formula used to fit the radial behavior of $\alpha_{Purb}$ of our sample. We show also the values of the parameters presented in Nelson et al. (2014). The errors on the parameters are the values at 3$\sigma$ confidence.

Figure 11. Radial profile of median value (blue solid line) of non-thermal pressure support for our data (shadow regions represent the 1$\sigma$, 2$\sigma$ and 3$\sigma$ variance) and the fit obtained with our model (dash-dotted red line). We also show the values of non-thermal pressure support for the clusters presented in Eckert et al. (2019) at $R_{500}$ (green points) and $R_{200}$ (gold points).
cluster radius correlate with the large-scale mass distribution, as in order to see whether the non-thermal pressure support at a large
Table 2.

Parameters and values of $\alpha$ that we want to study. In Fig. 13 we used the following colour leg-
port

3.2 Properties of the non-thermal pressure support in the
cluster sample

In Fig. 12 we show values of mass and non-thermal pressure support $\alpha_{\text{Turb}}$ at radius $R_{200}$ in function of redshift. From Fig. 12 we notice that there is a strong relation between mergers and an increase of $\alpha_{\text{Turb}}$. Instead, when the cluster is not affected by mergers the value of $\alpha_{\text{Turb}}$ decrease. The red points in Fig. 12 are the selected snapshots obtained by the selection described in Sec. 2.2. We studied the relations between $\alpha_{\text{Turb}}$ computed at radii $R_{200}$ or $R_{500}$ and mass (at the same radii), redshift and mass sparsity, $s$. The latter is defined as the ratio between the total mass within $R_{100}$ and $R_{200}$:

$$s = \frac{M_{100}}{M_{200}}$$

We seek here a relation between the mass sparsity and $\alpha_{\text{Turb}}$, in order to see whether the non-thermal pressure support at a large cluster radius correlate with the large-scale mass distribution, as in [Vazza et al. 2018] we already reported very little correlation between the turbulent pressure support at $R_{200}$ and other X-ray morphological parameters (the emission centroid shift, $w$, and concentration parameter, $c$, e.g. Cassano et al. 2010), which are typically biased towards the innermost cluster regions. We divided our sample into three different sub-samples for all the physical quantities that we want to study. In Fig. 13 we used the following colour leg-
port

- Mass: $M_{100}/M_{200} < 4.86 \times 10^{13}$ (red), $4.86 \times 10^{13} < M_{100}/M_{200} < 8.15 \times 10^{13}$ (grey), $M_{100}/M_{200} > 8.15 \times 10^{13}$ (green);
- Redshift: $z < 0.21$ (red), $0.21 < z < 0.54$ (grey), $z > 0.54$ (green);
- Sparsity: $s < 1.23$ (red), $1.23 < s < 1.30$ (grey), $s > 1.30$ (green).

To evaluate a possible relation between the value of $\alpha_{\text{Turb}}$ with mass, redshift and mass sparsity, we compute the median value in a single bin of each quantities and we show these values in the "Best" column of the Tab. 3 or the values at $R_{200}$, and Tab. 4 for the values at $R_{500}$.

In summary, from Fig. 13, Tab. 3 and Tab. 4 it can be noticed that no strong correlations are found between $\alpha_{\text{Turb}}$ and the host cluster mass, redshift or mass sparsity. The lack of correlation in the first two cases can be interpreted as a-posteriori validation of our cluster selection procedure (Sec 2.2), in the sense that the resulting turbulence budget is overall self-similar, in line with previous numerical simulations (e.g. Vazza et al. 2006; Nelson et al. 2014). On the other hand, the lack of correlation with the mass sparsity indicates that also this observational proxy is not a robust indicator of the mass accretion rate, which is instead found to correlate well with the turbulent budget (e.g. Vazza et al. 2011; Nelson et al. 2014; Vazza et al. 2017).

Concerning the comparison with real data, we find that the median $\alpha_{\text{Turb}}$ is in line with the values inferred from X-ray/SZ observations by Eckert et al. (2019) (blue shadow regions in Fig. 13), if we restrict to a subsample of clusters within the same mass and redshift range of observations.

We explored some possible variations on the filtering techniques which we applied at our data and the results are show in Tab. 3 and Tab. 4. In particular, we remove the clumpiness filter or the shocks filter and to change the exponent of the Kolmogorov’s relation (as we tested in Sec. 2.3.1). As for the case calling "Best", also for any possible variation of filtering technique, we do not find any strong correlations between $\alpha_{\text{Turb}}$ and physical parameters of the clusters. We also notice that from our "Best" configuration and the other ones there are not any strong variations in the values of the median, both at $R_{200}$ and $R_{500}$.

3.3 A closer look at the hydrostatic mass bias in the simulated ICM

To test the possible relation between the non-thermal pressure and turbulence we identify in our data and the X-ray derived proxy (e.g. Eckert et al. 2019), we compute for each cluster the values of $\alpha_{\text{HS}}$ at radii $R_{200}$ and $R_{500}$. To this end, we defined the cluster’s center as the cell with the maximum thermal pressure, and compute

| Sample | $a_0$ | $a_1$ | $a_2$ | $\chi^2$ |
|--------|------|------|------|--------|
| $M_{100}/M_{200} < 4.86 \times 10^{13}$ | (8.58 ± 0.05) × 10^{-2} | 1.153 ± 0.007 | (3.94 ± 0.05) × 10^{-2} | 0.008 |
| $4.86 \times 10^{13} < M_{100}/M_{200} < 8.15 \times 10^{13}$ | (5.90 ± 0.01) × 10^{-2} | 1.384 ± 0.002 | (5.96 ± 0.01) × 10^{-2} | 0.02 |
| $M_{100}/M_{200} > 8.15 \times 10^{13}$ | (3.44 ± 0.01) × 10^{-2} | 1.601 ± 0.005 | (8.09 ± 0.01) × 10^{-2} | 0.03 |
| Redshift | $z < 0.21$ | (6.67 ± 0.01) × 10^{-2} | 1.017 ± 0.002 | (3.82 ± 0.01) × 10^{-2} | 0.005 |
| | $0.21 < z < 0.54$ | (3.36 ± 0.01) × 10^{-2} | 1.940 ± 0.002 | (9.80 ± 0.01) × 10^{-2} | 0.03 |
| | $z > 0.54$ | (8.60 ± 0.05) × 10^{-2} | 1.143 ± 0.007 | (4.18 ± 0.05) × 10^{-2} | 0.01 |
| Sparsity | $s < 1.23$ | (6.05 ± 0.02) × 10^{-2} | 1.284 ± 0.004 | (5.74 ± 0.02) × 10^{-2} | 0.02 |
| | $1.23 < s < 1.30$ | (7.63 ± 0.01) × 10^{-2} | 0.987 ± 0.001 | (4.18 ± 0.01) × 10^{-2} | 0.01 |
| | $s > 1.30$ | (5.65 ± 0.03) × 10^{-2} | 1.571 ± 0.008 | (7.35 ± 0.03) × 10^{-2} | 0.008 |

Table 2. Parameters and values of $\chi^2$ statistical test for our model applied to mass, redshift and mass sparsity sub-samples of our data. The errors on the parameters are the values at 3$\sigma$ confidence.
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Figure 12. $M_{100}$ growth (blue solid line) and non-thermal pressure support at $R_{100}$ time behavior (green solid line) for IT92.0 and IT90.4. The red points are the selected snapshots as explain in Sec. 2.2.

Table 3. Summary table of values of $\alpha_{Turb}$ at radius $R_{200}$ for different configurations of the filtering techniques. The errors are computed as 1σ variance.

| Sample         | Best   | No Clump | No Shocks | toll: k066 | toll: 1% |
|----------------|--------|----------|-----------|------------|----------|
| All            | 0.12±0.04 | 0.12±0.04 | 0.12±0.04 | 0.12±0.04 | 0.12±0.04 |
| Mass $M_{100}/M_\odot < 4.86 \cdot 10^{13}$ | 0.12±0.04 | 0.12±0.04 | 0.12±0.04 | 0.12±0.04 | 0.12±0.04 |
| $M_{100}/M_\odot > 8.15 \cdot 10^{13}$ | 0.11±0.03 | 0.12±0.03 | 0.11±0.03 | 0.11±0.03 | 0.11±0.03 |
| Redshift       | $z < 0.21$ | 0.1±0.05 | 0.1±0.05 | 0.1±0.05 | 0.1±0.05 |
| $0.21 < z < 0.54$ | 0.13±0.03 | 0.13±0.02 | 0.13±0.03 | 0.13±0.03 | 0.13±0.02 |
| $z > 0.54$     | 0.13±0.05 | 0.13±0.04 | 0.13±0.04 | 0.13±0.05 | 0.13±0.05 |
| Sparsity       | $s < 1.23$ | 0.12±0.05 | 0.12±0.05 | 0.12±0.05 | 0.12±0.05 |
| $1.21 < s < 1.30$ | 0.11±0.02 | 0.12±0.02 | 0.11±0.02 | 0.11±0.02 | 0.11±0.02 |
| $s > 1.30$     | 0.13±0.05 | 0.13±0.04 | 0.13±0.05 | 0.13±0.05 | 0.13±0.05 |

Table 4. Summary table of values of $\alpha_{Turb}$ at radius $R_{500}$ for different configurations of the filtering techniques. The errors are computed as 1σ variance.

| Sample         | Best   | No Clump | No Shocks | toll: k066 | toll: 1% |
|----------------|--------|----------|-----------|------------|----------|
| All            | 0.09±0.04 | 0.09±0.04 | 0.09±0.04 | 0.09±0.04 | 0.09±0.04 |
| Mass $M_{100}/M_\odot < 4.86 \cdot 10^{13}$ | 0.1±0.06 | 0.1±0.05 | 0.1±0.05 | 0.1±0.06 | 0.1±0.06 |
| $4.86 \cdot 10^{13} < M_{100}/M_\odot < 8.15 \cdot 10^{13}$ | 0.09±0.03 | 0.09±0.03 | 0.09±0.03 | 0.09±0.03 | 0.09±0.03 |
| $M_{100}/M_\odot > 8.15 \cdot 10^{13}$ | 0.11±0.04 | 0.11±0.04 | 0.11±0.04 | 0.11±0.04 | 0.11±0.04 |
| Redshift       | $z < 0.21$ | 0.08±0.03 | 0.08±0.03 | 0.08±0.03 | 0.08±0.03 |
| $0.21 < z < 0.54$ | 0.11±0.03 | 0.11±0.03 | 0.11±0.03 | 0.11±0.03 | 0.11±0.03 |
| $z > 0.54$     | 0.1±0.06 | 0.1±0.05 | 0.1±0.05 | 0.1±0.06 | 0.1±0.06 |
| Sparsity       | $s < 1.23$ | 0.09±0.04 | 0.09±0.04 | 0.09±0.04 | 0.09±0.04 |
| $1.21 < s < 1.30$ | 0.09±0.03 | 0.09±0.03 | 0.09±0.03 | 0.09±0.03 | 0.09±0.03 |
| $s > 1.30$     | 0.1±0.06 | 0.1±0.05 | 0.1±0.05 | 0.1±0.06 | 0.1±0.06 |
the hydrostatic mass $M_H$ through the radial derivative of thermal pressure, computed as follows:

$$
\left( \frac{dP_{th}}{dr} \right)_i = \frac{P_{th,i+2} - 6 P_{th,i-1} + 3 P_{th,i} + 2 P_{th,i+1}}{6r_i} 
$$

where $P_{th}$ is the thermal pressure defined as in Eq. 9 and $i$ represents each radial shell. To limit the contribution from dense, self-gravitating clumps, we use the same masking procedure of Sec. 2.3.1, in order to consider only the thermal pressure exerted by

the gas within the cluster. For each radius, we computed the value of $\alpha_{HS}$ applying the Eq. 15. In the Fig. 14 we show the relation between $\alpha_{Turb}$ and $\alpha_{HS}$ computed at radii $R_{200}$ and $R_{500}$. At first glance, there is almost no correlation between the two proxies for the non-thermal pressure support, even if the two distributions span a similar range of values, and also are in the same ballpark of the XCOP data by Eckert et al. (2019). We also notice that while $\alpha_{Turb}$ is defined as a positive quantity by construction, $\alpha_{HS}$ scatters from positive to negative values, with a very significant presence of negative points, meaning that in several system one would measure an hydrostatic mass larger than the total mass, at odds with the general expectation on the role of non-thermal pressure in the ICM.

We will comment on the issue of large negative values of $\alpha_{HS}$ below. To test for relations between $\alpha_{HS}$ and physical quantities such as mass, redshift and mass sparsity of clusters, we used the same analysis presented in Sec. 3.2. The results are shown in Fig. 15. Here, we notice that the variance of the data (the shadow regions on the Fig. 15) is larger with respect Fig. 13. As for $\alpha_{Turb}$, also

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure13}
\caption{$\alpha_{Turb}$ against redshift, mass and mass sparsity of the clusters. The shadow regions identify the variance for the three different bins of $z$, mass and mass sparsity (see Sec. 3.2 for details) while the blue shadow is the range of value identified by Eckert et al. (2019). On top the values computed at $R_{200}$, bottom panel at $R_{500}$.}
\end{figure}
for $\alpha_{\text{HS}}$ we could not identify any strong correlation with physical property of clusters. However, from the comparison between the values presented in Eckert et al. (2019) and the values of median and variance of our sample are roughly in the same ballpark. In Tab. 5, we summarize the median values and the variance of each and variance of our sample are roughly in the same ballpark. In

In order to follow the same convention by Biffi et al. (2016), we

This is understood in the hierarchy cosmological scenario, in which equal mass systems can have significantly more substructures than lower redshift ones, even if they have equal mass. This introduces crucial problems for the $\alpha_{\text{HS}}$ estimate. First, the spherical symmetry and the coincidence between the centre of the gas pressure and of the gravitational mass (and between gas and dark matter densities) are often violated for systems which had only a little time to relax. Gas substructures are also more prominent, as they are often found in their first crossing of the ICM. This also leads to an ICM with a multiphase structure, also correlated with the crossing of shocks. In summary, most of the assumptions on which the hydrostatic equilibrium analysis is based are violated at high-$z$, while on the other hand the above factors little affect our estimate of $\alpha_{\text{Turb}}$, because through our filtering procedure the measure of turbulence is local and do not rely in assumptions of symmetry or isothermality. Interestingly, the above problems should also play an important role for the mass modelling of high-$z$ galaxy clusters in real observations (e.g. Maughan et al. 2006; Jee et al. 2011; Schrabback et al. 2018). To further investigate the physical origin of the difference between $\alpha_{\text{Turb}}$ and $\alpha_{\text{HS}}$, we added a few important dynamical proxies to characterize the dynamics of our systems, and computed their radial profile in order to compare to the radial quantities listed above.

First, we computed the radial profile of the kinetic flux weighted Mach Number of the shocks in each cluster, $M_w$, based on the three-dimensional distribution of shocks. The presence of shocks in the ICM is important for their dynamical equilibrium, as an effect of the passage of the shocks is that a portion of the cluster volume experiences a thrust, usually in the outward direction. This effect generates a radial acceleration of the gas that could affect the compute of the hydrostatic mass, miming an excess of thermal pressure if the hydrostatic equilibrium is (wrongly) imposed on the structure (e.g. Nelson et al. 2014). We additionally computed the radial profile of gas acceleration, and derived the residual acceleration from gas motions which are out of equilibrium in the presence of mergers, following Biffi et al. (2016). While in their work they could directly access the acceleration values of single smoothed-particle-hydrodynamics (SPH) particles from the hydrodynamical solver, in our approach we rely on the post-processing of Eulerian data, taking the derivative of two close timesteps. We defined the gravitational acceleration in each radial shell as:

$$g(r) = -\frac{GM_r}{r^2},$$

(21)

while the residual gas acceleration is computed by first taking the radial velocity in each cell, and then reconstructing the radial profile of this quantity for every selected snapshot. To define the residual gas acceleration in the radial direction, we take the difference in each radial shell, between two snapshots, $\delta(r)$ as:

$$\delta(r) = \frac{V_r(t_2) - V_r(t_1)}{(t_2 - t_1)}.$$

(22)

In order to follow the same convention by Biffi et al. (2016), we

Figure 14. Comparison between $\alpha_{\text{Turb}}$ and $\alpha_{\text{HS}}$ at $R_{200}$ (left panel) and $R_{500}$ (right panel). The shadow regions represented the range values presented by Eckert et al. 2019.
Figure 15. $\alpha_{\text{HS}}$ against redshift, mass and mass sparsity of the clusters. The shadow regions identify the variance for the three different bins of $z$, mass and mass sparsity (see Sec. 3.2 for details) while the blue shadow is the range of value identified by Eckert et al. (2019). On top the values computed at $R_{200}$, bottom panel at $R_{500}$.

Table 5. Summary table for $\alpha_{\text{Turb}}$ and $\alpha_{\text{HS}}$. We list the median of each bin and the variance computed at $1\sigma$. 

| Sample          | $R_{200}$ | $R_{500}$ | $R_{200}$ | $R_{500}$ |
|-----------------|-----------|-----------|-----------|-----------|
| All             | 0.12±0.04 | 0.09±0.04 | 0.02±0.24 | -0.15±0.28 |
| Mass            |           |           |           |           |
| $M_{100}/M_\odot < 4.86 \cdot 10^{13}$ | 0.12±0.04 | 0.1±0.06  | -0.1±0.21 | -0.29±0.3  |
| $4.86 \cdot 10^{13} < M_{100}/M_\odot < 8.15 \cdot 10^{13}$ | 0.12±0.04 | 0.09±0.03 | 0.05±0.3  | -0.11±0.21 |
| $M_{100}/M_\odot > 8.15 \cdot 10^{13}$ | 0.11±0.03 | 0.11±0.04 | 0.07±0.17 | -0.05±0.24 |
| Redshift        |           |           |           |           |
| $z < 0.21$      | 0.1±0.05  | 0.08±0.03 | 0.15±0.27 | -0.06±0.26 |
| $0.21 < z < 0.54$ | 0.13±0.03 | 0.11±0.03 | 0.02±0.22 | -0.06±0.22 |
| $z > 0.54$      | 0.13±0.05 | 0.1±0.06  | -0.06±0.22| -0.26±0.3  |
| Sparsity        |           |           |           |           |
| $s < 1.23$      | 0.12±0.05 | 0.09±0.04 | -0.07±0.3 | -0.15±0.28 |
| $1.23 < s < 1.30$ | 0.11±0.02 | 0.09±0.03 | 0.09±0.19 | -0.05±0.19 |
| $s > 1.30$      | 0.13±0.05 | 0.1±0.06  | -0.002±0.2| -0.23±0.32 |
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defined an acceleration term consistent with the one extracted from their SPH simulations:

\[ H(r) = g(r) + \delta(r). \]  

From the above we can thus introduce a factor, \( \delta_{HE} \), which compensates for the residual gas radial acceleration by motions which are not in equilibrium with the gravitational pull of the cluster:

\[ \delta_{HE}(r) = \frac{g(r)}{H(r)} - 1. \]  

From this definition we notice that when the gas is in hydrostatic equilibrium \( \delta_{HE} \) is equal to 0. Finally, as in [Biffi et al.] we define \( \xi : \)

\[ \xi = | \alpha_{HS}(r) - \delta_{HE}(r) |, \]  

which allows us to consider at the same time the contributions given at the hydrostatic bias from the acceleration terms and the term obtained from \( \alpha_{HS} \). We quantify the amount of departure from the hydrostatic equilibrium in each shell through the median value of \( \xi \), within the shell, \( \xi \). As an example, in Fig. 16 we show the central slice of Mach Number, which allows us to identify shocks sweeping the clusters volume at a given epoch. In left panel we can see a wide \( M \approx 3 \) shocks in the inner part of cluster IT90.0 at the epoch of \( z \approx 0.15 \), while in right slight through cluster IT92.1 at the same epoch there are no relevant shocks inside \( r_{200} \). We can therefore expect in the first case a stronger departure from equilibrium, following the gasdynamical acceleration downstream of the shock wave. These trends are well captured in Fig. 17 which gives the radial behavior of total mass (blue solid line), hydrostatic mass (red solid line) and \( M_{\text{trace}} \) (green solid line). In the bottom panels of Fig. 17 we also show the radial profiles of \( \alpha_{HS} \) and of the radial acceleration term.

From there we can quantify how shocks in the inner parts of the cluster influence the hydrostatic mass. There is a strong correlation between the maximum values of the Mach Number and a negative hydrostatic mass bias, meaning that the total mass that would be inferred through a standard hydrostatic equilibrium analysis would be larger than the total (true) mass, as shown by the radial trend of \( \alpha_{HS} \). These behaviors are not observed for the more relaxed cluster in the right panel, which does not show strong shocks. Therefore, shocks introduce an additional term which one must consider when inferring non-thermal pressure from the hydrostatic mass bias. We remark that such behaviours in the radial profile therefore expect in the first case a stronger departure from equilibrium.

\[ \tilde{\alpha}_{\text{Turb}} = a + b(\tilde{\alpha}_{\text{HS}} + \xi) \]  

for the normalization and \( b = 0.29 \pm 0.09 \) for the slope of the \( \tilde{\alpha}_{\text{Turb}} = a + b(\tilde{\alpha}_{\text{HS}} + \xi) \) relation, and with the BCES-bisector method [Akritas & Bershady 1996], which treats the two variables symmetrically and accounts for the intrinsic scatter in the data via bootstrapping (e.g. Cassano et al. 2013). In this second case, the best fit gives \( a = 0.05 \pm 0.01 \) and \( b = 0.39 \pm 0.09 \). Compared with Fig. 14 this test shows indeed that the acceleration term is important to reconcile the hydrostatic estimate of the non-thermal pressure support by turbulence, bringing them closer to a one to one correlation. However, the high variance suggests that there still is (at least in our simulated data) an irreducible level of discrepancy between \( \alpha_{HS} \) and \( \tilde{\alpha}_{\text{Turb}} \), which does not yield a perfectly linear correlation. Several effects in simulations can lead to this: the volume filling of bulk motions producing gas acceleration, which only affects a fraction of the cluster volume; the fact that such bulk motions can propagate with an oblique angle with respect to the radial direction considered in the simplistic derivation of \( \alpha_{HS} \); the fact that in perturbed systems the estimate of the cluster centre may become uncertain due to multiple substructures, and in general the fact that in several cases the assumptions needed to perform a standard analysis of the hydrostatic equilibrium are severely violated in our objects.

We further divided our dataset in three different sub-samples of mass and redshift (as in Tab. 4) as well as in three bins of \( \xi \) (\( \xi < 0.20 \) in red, \( 0.20 \leq \xi < 0.26 \) in grey, \( \xi \geq 0.26 \) in green), to verify whether the correlation improves for specific subsets. However, we do not find very significant differences in the best-fit relations (see values in Fig. 19).

We shall notice that the use of \( \xi \) significantly mitigates the problem of the negative \( \alpha_{HS} \) for clusters at high \( z \), further confirming that significant radial acceleration terms in clusters undergoing strong merger activity is key to model out-of-equilibrium condition and recover an estimate of the total mass which is closer to the real one. Although the fitting parameters from Fig. 19 are quite different, we could not conclude that any sub-samples presented a strong correlation between \( \tilde{\alpha}_{\text{Turb}} \) and \( (\hat{\alpha}_{\text{HS}} + \xi) \). This behavior confirms that correcting for the acceleration term is important to minimize the hydrostatic bias. However, additional analysis is necessary to investigate whether a closer one-to-one relation between \( \alpha_{HS} \) and \( \hat{\alpha}_{\text{Turb}} \) can be derived in the realistic case. We notice that similar problems were reported in the literature by [Nelson et al. 2014] and [Biffi et al. 2016], employing entirely different numerical codes and methods to estimate the non-thermal pressure, which suggests that this is a physical rather than a numerical problem. Whether it can be minimised for observational applications, such as precision cosmology with eRosita or other future X-ray surveys, is a topic that deserves further investigation and will be the subject of future work.

4 DISCUSSION AND CONCLUSIONS

We have analyzed a high-resolution sample of galaxy clusters, simulated with the cosmological code ENZO, specifically designed to study turbulent motions in the ICM [Vazza et al. 2017; Wittor et al. 2017; Vazza et al. 2018]. We developed and optimized algorithms to disentangle laminar from bulk motions in the simulated ICM, and we applied few tools to limit the spurious contribution from gas clumps and shocks. This work improves on our previous works on the subject in various respects. Firstly, we designed a procedure to extract a larger sample of objects and conduct larger statistical studies, by extracting multiple time snapshots of the same objects...
Figure 16. Maps of shock Mach number in a slice through the center or cluster IT90,0 at $z \simeq 0.15$ (left panel), and through the center of IT92,1 at $z \simeq 0.15$ (right panel). The inner circle shows the location of $R_{500}$, while the outer one shows $R_{200}$.

Figure 17. Total mass profile (blue solid line), hydrostatic mass profile (red solid line) and median Mach Number profile (green solid line) in the top panel, and radial acceleration (blue solid line) and $\alpha_{HS}$ profiles (red solid line) in the bottom panel, for IT90,0 at $z \simeq 0.15$ (left panel) and IT92,1 at $z \simeq 0.15$ (right panel).
Figure 18. Median value of corrected hydrostatic bias ($\alpha_{\text{HS}} + \xi$) against median turbulent pressure support within the radius ($\alpha_{\text{Turb}}$), in both cases computed within $R_{200}$. The solid blue and green lines give the fits obtained with $\chi^2$ method (blue) or BCES one (green), while the shadowed areas give the 1σ errors around the fits (see Sec. 3.3 for details).

We have developed a new fitting formula for the radial profile of $\alpha_{\text{Turb}}$ in the form presented in Eq. 10. This formula produces a good fit of the data of our simulations, both for the complete sample and for the different sub-samples which we studied. We found that the three parameters can be easily related to the physics of the ICM (see Sec. 3.1 for a detailed explanation and the necessary bibliographic references). Our fitting formula differs from Nelson et al. (2014), the main difference coming in the definitions of turbulent velocity.

The average non-thermal pressure support in our sample is in agreement with the recent X-ray observational campaign by Eckert et al. (2019), both at $R_{500}$ and $R_{200}$, albeit with a large scatter due to a larger variety of dynamical states compared to the X-COP sample used by Eckert et al. (2019).

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Figure 19. Median value of corrected hydrostatic bias ($\tilde{\alpha}_{HS} + \xi$) against median value of $\alpha_{Turb}$ ($\tilde{\alpha}_{Turb}$), computed within $R_{200}$ for different samples ($\xi$, mass and redshift sub-samples as described in Sec. 3.3). The solid blue line is the fit computed on complete sample (as shown in Fig. 18), while the colored solid line are the fit computed in single sub-sample. Left panel: $\xi$ sub-samples; Center panel: Mass sub-samples; Right panel: Redshift sub-samples. The color legend used is explained in Sec. 3.3.

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