How to Reach the Correct $\sin^2\theta_W$ and $\alpha_S$ in String Theory*

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Abstract

Effective theories with the matter content of the minimal supersymmetric Standard Model below the string scale $M_{\text{string}}$ predict a wrong value for the weak–mixing angle $\sin^2\theta_W$ and strong coupling constant $\alpha_S$ at the scale $M_Z$. To resolve this problem one needs large threshold corrections. At the same time one would like to avoid introducing new intermediate scales that are small compared to $M_{\text{string}}$. Two requests which seem to be incompatible. We show how both requirements can be satisfied in a class of $(0,2)$ heterotic superstring compactifications with a natural choice of the vevs of the moduli fields entering the moduli dependent string threshold corrections.

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LEP and SLC high precision electroweak data predict for the minimal supersymmetric Standard Model (MSSM) with the lightest Higgs mass in the range $60\text{GeV} < M_H < 150\text{GeV}$

$$\begin{align*}
\sin^2 \hat{\theta}_W(M_Z) &= 0.2316 \pm 0.0003 \\
\alpha_{em}(M_Z)^{-1} &= 127.9 \pm 0.1 \\
\alpha_S(M_Z) &= 0.12 \pm 0.01 \\
m_t &= 160^{+11+6}_{-12-5}\text{GeV}
\end{align*}$$

(1)

for the central value $M_H = M_Z$ in the $\overline{\text{MS}}$ scheme [1]. This is in perfect agreement with the recent CDF/D0 measurements of $m_t$. Taking the first three values as input parameters leads to gauge coupling unification at $M_{\text{GUT}} \sim 2 \cdot 10^{16}\text{GeV}$ with $\alpha_{\text{GUT}} \sim \frac{1}{26}$ and $M_{\text{SUSY}} \sim 1\text{TeV}$ [4, 1]. Slight modifications arise from light SUSY thresholds, i.e. the splitting of the particle mass spectrum, the variation of the mass of the second Higgs doublet and two-loop effects. Whereas these effects are rather mild, huge corrections may arise from heavy thresholds due to mass splittings at the high scale $M_{\text{heavy}} \neq M_{\text{GUT}}$ arising from the infinite many massive string states [3].

In heterotic superstring theories all couplings are related to the universal string coupling constant $g_{\text{string}}$ at the string scale $M_{\text{string}} \sim \alpha'^{-1/2}$, with $\alpha'$ being the inverse string tension. It is a free parameter which is fixed by the dilaton vacuum expectation value $g_{\text{string}}^{-2} = \frac{S+3}{2}$. In general this amounts to string unification, i.e. at the string scale $M_{\text{string}}$ all gauge and Yukawa couplings are proportional to the string coupling and are therefore related to each other. For the gauge couplings (denoted by $g_a$) we have [4]:

$$g_a^2 k_a = g_{\text{string}}^2 = \frac{\kappa^2}{2\alpha'},$$

(2)

Here, $k_a$ is the Kac–Moody level of the group factor labeled by $a$. The string coupling $g_{\text{string}}$ is related to the gravitational coupling constant $\kappa^2$. In particular this means that string theory itself provides gauge coupling and Yukawa coupling unification even in absence of a grand unified gauge group.

To make contact with the observable world one constructs the field–theoretical low–energy limit of a string vacuum. This is achieved by integrating out all the massive string modes corresponding to excited string states as well as states with momentum or winding quantum numbers in the internal dimensions. The resulting theory then describes the physics of the massless string excitations at low energies $\mu \ll M_{\text{string}}$ in field–theoretical terms. If one wants to state anything about higher energy scales one has to take into account threshold corrections $\Delta_a(M_{\text{string}})$ to the bare couplings $g_a(M_{\text{string}})$ due to the infinite tower of massive string modes. They change the relations (2) to:

$$g_a^{-2} = k_a g_{\text{string}}^{-2} + \frac{1}{16\pi^2} \Delta_a,$$

(3)

The corrections in (3) may spoil the string tree–level result (2) and split the one–loop gauge couplings at $M_{\text{string}}$. This splitting could allow for an effective unification at a scale $M_{\text{GUT}} < M_{\text{string}}$ or destroy the unification.

The general expression of $\Delta_a$ for heterotic tachyon–free string vacua is given in [4]. Various contributions to $\Delta_a$ have been determined for several classes of models: First in [3] for two $\mathbb{Z}_3$ orbifold models with a (2,2) world–sheet supersymmetry [3]. This has been
extended to fermionic constructions in [1]. Threshold corrections for (0,2) orbifold models with quantized Wilson lines [8] have been calculated in [9]. Threshold corrections for the quintic threefold and other Calabi–Yau manifolds [10] with gauge group $E_6 \times E_8$ can be found in [11, 12]. In toroidal orbifold compactifications [6] moduli dependent threshold corrections arise only from N=2 supersymmetric sectors. They have been determined for some orbifold compactifications in [13]–[16] and for more general orbifolds in [17]. The full moduli dependence of threshold corrections for (0,2) orbifold compactifications with continuous Wilson lines has been first derived in [18, 19]. These models contain continuous background gauge fields in addition to the usual moduli fields [22]. In most of the cases these models are (0,2) compactifications. In all the above orbifold examples the threshold corrections $\triangle_a$ can be decomposed into three parts:

$$\triangle_a = \hat{\triangle}_a - b_a^{N=2} \triangle + k_a Y .$$

Here the gauge group dependent part is divided into two pieces: The moduli independent part $\hat{\triangle}_a$ containing the contribution of the N=1 supersymmetric sectors as well as scheme dependent parts which are proportional to $b_a$. This prefactor $b_a$ is related to the one–loop $\beta$–function: $\beta_a = b_a g_a^3 / 16 \pi^2$. Furthermore the moduli dependent part $b_a^{N=2} \triangle$ with $b_a^{N=2}$ being related to the anomaly coefficient $b_a$ by $b_a^{N=2} = b'_a - k_a \delta_{GS}$. The gauge group independent part $Y$ contains the gravitational back–reaction to the background gauge fields as well as other universal parts [5, 23, 12, 24]. They are absorbed into the definition of $g_{\text{string}}$: $g_{\text{string}}^2 = \frac{s_{\text{string}}^2}{16 \pi} + \frac{1}{16 \pi} Y$. The scheme dependent parts are the IR–regulators for both field– and string theory as well as the UV–regulator for field theory. The latter is put into the definition of $M_{\text{string}}$ in the DR scheme [3]:

$$M_{\text{string}} = 2 e^{(1-\gamma_E)/2} 3^{3/4} \sqrt{\frac{2\pi}{\alpha'}} = 0.527 \ g_{\text{string}} \times 10^{18} \ \text{GeV} .$$

The constant of the string IR–regulator as well as the universal part due to gravity were recently determined in [24].

The identities (3) are the key to extract any string–implication for low–energy physics. They serve as boundary conditions for our running field–theoretical couplings valid below $M_{\text{string}}$ [24]. Therefore they are the foundation of any discussion about both low–energy predictions and gauge coupling unification. The evolution equations (6) valid below $M_{\text{string}}$

$$\frac{1}{g_a^2(\mu)} = \frac{k_a}{g_{\text{string}}} + \frac{b_a}{16 \pi^2} \ln \frac{M_{\text{string}}^2}{\mu^2} - \frac{1}{16 \pi^2} b_a^{N=2} \triangle ,$$

allow us to determine $\sin^2 \theta_W$ and $\alpha_S$ at $M_Z$. After eliminating $g_{\text{string}}$ in the second and third equations one obtains

$$\sin^2 \theta_W(M_Z) = \frac{k_2}{k_1 + k_2} - \frac{k_1}{k_1 + k_2} \frac{\alpha_{em}(M_Z)}{4 \pi} \left[ A \ln \left( \frac{M_{\text{string}}^2}{M_Z^2} \right) - A' \triangle \right] ,$$

$$\alpha_S^{-1}(M_Z) = \frac{k_3}{k_1 + k_2} \left[ \alpha_{em}^{-1}(M_Z) - \frac{1}{4 \pi} B \ln \left( \frac{M_{\text{string}}^2}{M_Z^2} \right) + \frac{1}{4 \pi} B' \triangle \right] ,$$

1A lowest expansion result in the Wilson line modulus has been obtained in [20, 21].

2We neglect the N=1 part of $\hat{\triangle}_a$ which is small compared to $b_a^{N=2} \triangle$ [3, 5, 9].
with $A = \frac{k_2}{k_1} b_1 - b_2$, $B = b_1 + b_2 - \frac{k_1 + k_3}{k_3} b_3$ and $A', B'$ are obtained by exchanging $b_i \rightarrow b'_i$.

For the MSSM one has $A = \frac{28}{5}$, $B = 20$. However to arrive at the predictions of the MSSM (1) one needs huge string threshold corrections $\Delta$ due to the large value of $M_{\text{string}} (3/5k_1 = k_2 = k_3 = 1)$:

$$
\Delta = \frac{A}{A'} \left[ \ln \left( \frac{M_{\text{string}}^2}{M_{\text{GUT}}^2} \right) + \frac{32\pi \delta \sin^2 \theta_W}{5A \alpha_{\text{em}}(M_Z)} \right].
$$

At the same time, the $N=2$ spectrum of the underlying theory encoded in $A', B'$ which enters the threshold corrections has to fulfill the condition

$$
\frac{B'}{A'} = \frac{B}{A} \left[ \ln \left( \frac{M_{\text{string}}^2}{M_{\text{GUT}}^2} \right) + \frac{32\pi \delta \sin^2 \theta_W}{5A \alpha_{\text{em}}(M_Z)} \right],
$$

with the uncertainties $\delta$ appearing in (1). In addition $\delta$ may also contain SUSY thresholds.

For concreteness and as an illustration let us take the $\mathbb{Z}_8$ orbifold example of [26] with $A' = -2$, $B' = -6$ and $b'_1 + b'_2 = -10$. It is one of the few orbifolds left over after imposing the conditions on target–space duality anomaly cancellation [26]. To estimate the size of $\Delta$ one may take in eq. (5) $g_{\text{string}} \sim 0.7$ corresponding to $\alpha_{\text{string}} \sim \frac{1}{26}$, i.e. $M_{\text{string}}/M_{\text{GUT}} \sim 20$. Of course this is a rough estimate since $M_{\text{string}}$ is determined by the first eq. of (6) together with (5). Nevertheless, the qualitative picture does not change. Therefore to predict the correct low–energy parameter (7) eq. (8) tells us that one needs threshold correction of considerable size:

$$
-16.3 \leq \Delta \leq -17.1.
$$

The construction of a realistic unified string model boils down to the question of how to achieve thresholds of that size. To settle the question we need explicit calculations within the given candidate string model. There we can encounter various types of threshold effects. Some depend continuously, others discretely on the values of the moduli fields. For historic reasons we also have to distinguish between thresholds that do or do not depend on Wilson lines. The reason is the fact that the calculations in the latter models are considerably simpler and for some time were the only available results. They were then used to estimate the thresholds in models with gauge group $SU(3) \times SU(2) \times U(1)$ and three families, although as a string model no such orbifold can be constructed without Wilson lines. Therefore, the really relevant thresholds are, of course, the ones found in the $(0,2)$ orbifold models with Wilson lines [18] which may both break the gauge group and reduce its rank. We will discuss the various contributions within the framework of our illustrative model. However the discussion can easily applied for all other orbifolds.

The threshold corrections depend on the $T$ and $U$ modulus describing the size and shape of the internal torus lattice. In addition they may depend on non–trivial gauge background fields encoded in the Wilson line modulus $B$.

Moduli dependent threshold corrections $\Delta$ can be of significant size for an appropriate choice of the vevs of the background fields $T, U, B, \ldots$ which enter these functions. Of course in the decompactification limit $T \rightarrow i\infty$ these corrections become always arbitrarily huge. This is in contrast to fermionic string compactifications or $N=1$ sectors of heterotic superstring compactifications. There one can argue that moduli–independent threshold
corrections cannot become huge at all [27]. This is in precise agreement with the results found earlier in [3, 7]. In field theory threshold corrections can be estimated with the formula [25]

$$\Delta = \sum_{n,m,k} \ln \left( \frac{M_{n,m,k}^2}{M_{\text{string}}^2} \right),$$

(11)

with \(n, m\) being the winding and momentum, respectively and \(k\) the gauge quantum number of all particles running in the loop. The string mass in the \(N = 2\) sector of the \(\mathbb{Z}_8\) model we consider later with a non–trivial gauge background in the internal directions is determined by [19]:

$$\alpha' M_{n,m,k}^2 = 4 |p_R|^2$$

$$p_R = \frac{1}{\sqrt{Y}} \left[ (\frac{T}{2\alpha'} - B^2) n_2 + \frac{T}{2\alpha'} n_1 - U m_1 + m_2 + B k_2 \right]$$

$$Y = -\frac{1}{2\alpha'} (T - \overline{T})(U - \overline{U}) + (B - \overline{B})^2.$$

(12)

In addition a physical state \(|n, m, k, l\rangle\) has to obey the modular invariance condition

$$m_1 n_1 + m_2 n_2 + k_2^2 - k_1 k_2 + k_3^2 - k_2 k_3 - k_2 k_4 + k_4^2 + k_2^2 = 1 - N_L - \frac{1}{2} L_{E_8}.$$ 

Therefore the sum in (11) should be restricted to these states. This also guarantees its convergence after a proper regularization. In (11) cancellations between the contributions of various string states may arise. E.g. at the critical point \(T = i = U\) where all masses appear in integers of \(M_{\text{string}}\) such cancellations occur. They are the reason for the smallness of the corrections calculated in [3, 9] and in all the fermionic models [7]. Let us investigate this in more detail. The simplest case \((B = 0)\) for moduli dependent threshold corrections to the gauge couplings was derived in [13]:

$$\Delta(T, U) = \ln \left[ \frac{-iT + iT}{2\alpha'} \right] \left| \eta \left( \frac{T}{2\alpha'} \right) \right|^4 + \ln \left[ (-iU + i\overline{U}) |\eta(U)|^4 \right].$$

(13)

Formula (13) can be used for any toroidal orbifold compactifications, where the two–dimensional subplane of the internal lattice which is responsible for the \(N=2\) structure factorize from the remaining part of the lattice. If the latter condition does not hold, (13) is generalized [17].

| \(T/2\alpha'\) | \(U\) | \(M^2\alpha'\) | \(\ln(M^2\alpha')\) | \(\Delta^{II}\) |
|----------------|--------|----------------|----------------|----------------|
| \(Ia\)         | \(i\)  | \(i\)          | 1              | 0              |
| \(Ib\)         | 1.25\(i\) | \(i\)        | \(\frac{4}{5}\) | -0.22          |
| \(Ic\)         | 4.5\(i\) | 4.5\(i\)     | \(\frac{1}{81}\) | -3.01          |
| \(Id\)         | 18.7\(i\) | \(i\)       | \(\frac{10}{187}\) | -2.93          |

Table 1: Lowest mass \(M^2\) of particles charged under \(G_A\) and threshold corrections \(\Delta(T, U)\).
In Table 1 we determine the mass of the lowest massive string state being charged under the considered unbroken gauge group $G_A$ and the threshold corrections $\Delta(T, U)$ for some values of $T$ and $U$.

The influence of moduli dependent threshold corrections to low-energy physics [entailed in eqs. (3)] has until now only been discussed for orbifold compactifications without Wilson lines by using (13). In these cases the corrections only depend on the two moduli $T, U$. However to obtain corrections of the size $\Delta \sim -16.3$ one would need the vevs $\frac{T}{2\alpha'} = 18.7, U = i$ which are unnaturally far away from the self–dual points $[25, 26]$. It remains an open question whether and how such big vevs of $T$ can be obtained in a natural way in string theory.

A generalization of eq. (13) appears when turning on non–vanishing gauge background fields $B \neq 0$. According to (12) the mass of the heavy string states now becomes $B$–dependent and therefore also the threshold corrections change. This kind of corrections were recently determined in [18]. The general expression there is

$$\Delta^{II}(T, U, B) = \frac{1}{12} \ln \left[ \frac{Y_{12}^{12}}{1728^4} |C_{12}(\Omega)|^2 \right],$$

where $B$ is the Wilson line modulus, $\Omega = \left( \begin{array}{cc} \frac{T}{2\alpha'} & B \\ -B & U \end{array} \right)$ and $C_{12}$ is a combination of $g = 2$ elliptic theta functions explained in detail in [19]. It applies to gauge groups $G_A$ which are not affected by the Wilson line mechanism. The case where the gauge group is broken by the Wilson line will be discussed later (those threshold corrections will be singular in the limit of vanishing $B$). Whereas the effect of quantized Wilson lines $B$ on threshold corrections has already been discussed in [4] the function $\Delta^{II}(T, U, B)$ now allows us to study the effect of a continuous variation in $B$.

![Fig.1 – Dependence of the threshold corrections $\Delta^{II}$ on the Wilson line modulus $B = B_1 + iB_2$ for $\frac{T}{2\alpha'} = i = U$.](image)

We see in Fig.1 that the threshold corrections change very little with the Wilson line.
modulus $B$. They are comparable with $\Delta = -0.72$ corresponding to the case of $B = 0$. In this case eq. (14) becomes eq. (13) for $\frac{T}{2\alpha'} = i = U$.

So far all these calculations have been done within models where the considered gauge group $G_A$ is not broken by the Wilson line and its matter representations are not projected out. To arrive at SM like gauge groups with the matter content of the MSSM one has to break the considered gauge group with a Wilson line.

From the phenomenological point of view [29], the most promising class of string vacua is provided by (0,2) compactifications equipped with a non–trivial gauge background in the internal space which breaks the $E_6$ gauge group down to a SM–like gauge group [30, 31, 8, 22, 32]. Since the internal space is not simply connected these gauge fields cannot be gauged away and may break the gauge group. Some of the problems present in (2,2) compactifications with $E_6$ as a grand unified group like e.g. the doublet–triplet splitting problem, the fine–tuning problem and Yukawa coupling unification may be absent in (0,2) compactifications. It is important that these properties can be studied in the full string theory, not just in the field theoretic limit [30]. The background gauge fields give rise to a new class of massless moduli fields again denoted by $B$ which have quite different low–energy implications than the usual moduli arising from the geometry of the internal manifold itself. In this framework the question of string unification can now be discussed for realistic string models. The threshold corrections for our illustrative model take the form [18]

$$
\Delta^I(T, U, B) = \frac{1}{10} \ln \left[ Y^{10} \left| \frac{1}{128} \prod_{k=1}^{10} \vartheta_k(\Omega) \right|^4 \right],
$$

where $\vartheta_k$ are the ten even $g = 2$ theta–functions [19]. Equipped with this result we can now investigate the influence of the B–modulus on the thresholds and see how the conclusions of ref. [28, 29] might be modified. The results for a representative set of background vevs is displayed in Fig.2.

Fig.2 – Dependence of the threshold corrections $\Delta^I$ on the Wilson line modulus $B = B_1 + iB_2$ for $\frac{T}{2\alpha'} = 4.5i = U$. 
From this picture we see that threshold corrections of $\Delta \sim -16.3$ can be obtained for the choice of $\frac{T}{2\alpha'} \sim 4.5i \sim U$ and $B = \frac{1}{2}$. This has to be compared to the model in ref. [26] where such a value was achieved with $T = 18.7i$ and $B = 0$. This turns out to be a general property of the models under consideration. With more moduli, sizeable threshold effects are achieved even with moderate values of the vevs of the background fields.

The modulus plays the rôle of an adjoint Higgs field which breaks e.g. the $G_A = E_6$ down to a SM like gauge group $G_a$. According to eq. (12) the vev of this field gives some particles masses between zero and $M_{\text{string}}$. This is known as the stringy Higgs effect. Such additional intermediate fields may be very important to generate high scale thresholds. Sizeable threshold corrections $\Delta$ can only appear if some particles have masses different from the string scale $M_{\text{string}}$ and where not a cancellation between different states as mentioned above takes place. In particular some gauge bosons of $G_A$ become massive receiving the mass:

$$\alpha' M_I^2 = \frac{4}{Y} |B|^2. \quad (16)$$

As before let us investigate the masses of the lightest massive particles charged under the gauge group $G_a$. For our concrete model we have $M_{\text{string}} = 3.6 \cdot 10^{17}\text{GeV}$.

|   | $T/2\alpha'$ | $U$ | $B$ | $M_I$ [GeV] | $\ln(M_I^2\alpha')$ | $\Delta^I$ |
|---|-------------|----|----|-------------|---------------------|----------|
|IIa| $i$         | $i$| $\frac{1}{10^6}$| $8.4 \cdot 10^{12}$|$-23.0$               | $-10.03$ |
|IIb| $i$         | $i$| $\frac{1}{2}$  | $4.2 \cdot 10^{17}$|$-1.39$              | $-1.72$  |
|IIc| $1.25i$     | $i$| $\frac{1}{2}$  | $3.7 \cdot 10^{17}$|$-1.61$              | $-2.12$  |
|IId| $4i$        | $i$| $\frac{1}{2}$  | $2.1 \cdot 10^{17}$|$-2.78$              | $-7.86$  |
|IIe| $4.5i$      | $4.5i$| $\frac{1}{2}$  | $9.3 \cdot 10^{16}$|$-4.39$              | $-16.3$  |
|IIf| $18.7i$     | $i$| $\frac{1}{2}$  | $1.1 \cdot 10^{16}$|$-4.31$              | $-43.3$  |

Table 2: Lowest mass $M_I$ of particles charged under $G_a$ and threshold corrections $\Delta^I$ for $B \neq 0$.

Whereas $\Delta^{II}$ describes threshold corrections w.r.t. to a gauge group which is not broken when turning on a vev of $B$, now the gauge group is broken for $B \neq 0$ and in particular this means that the threshold $\Delta^I$ shows a logarithmic singularity for $B \to 0$ when the full gauge symmetry is restored. This behaviour is known from field theory and the effect from the heavy string states can be decoupled from the former: Then the part of $\Delta^I_a$ in (3) which is only due to the massive particles becomes [18, 21]

$$\frac{b_A - b_a}{16\pi^2} \ln \frac{M_{\text{string}}^2}{|B|^2} - \frac{b'_A}{16\pi^2} \ln \left| \eta \left( \frac{T}{2\alpha'} \right) \eta(U) \right|^4, \quad (17)$$
where the first part accounts for the new particles appearing at the intermediate scale of \( M_I \) and the other part takes into account the contributions of the heavy string states. One of the questions of string unification concerns the size of this intermediate scale \( M_I \). In a standard grand unified model one would be tempted to identify \( M_I \) with \( M_{GUT} \). While this would also be a possibility for string unification, we have in string theory in addition the possibility to consider \( M_I > M_{GUT} \). The question remains whether the thresholds in that case can be big enough, as we shall discuss in a moment. Let us first discuss the general consequences of our results for the idea of string unification without a grand unified gauge group. Due to the specific form of the threshold corrections in eq. (6) unification always takes place if the condition \( A B' = A' B \) is met within the errors arising from the uncertainties in (1). It guarantees that all three gauge couplings meet at a single point \( M_X \) [26]:

\[
M_X = m_{\text{string}} e^{\frac{1}{2} A' \triangle}.
\] (18)

For our concrete model this leads to \( M_X \sim 2 \cdot 10^{16} \text{GeV} \). Given these results we can now study the relation between \( M_I \) and \( M_X \), which plays the role of the GUT–scale in string unified models. As a concrete example, consider the model IIe in Table 2. It leads to an intermediate scale \( M_I \) which is a factor 3.9 smaller than the string scale, thus \( \sim 10^{17} \text{GeV} \), although the apparent unification scale is as low as \( 2 \times 10^{16} \text{GeV} \). We thus have an explicit example of a string model where all the non–MSSM particles are above \( 9.3 \cdot 10^{16} \text{GeV} \), but still a correct prediction of the low energy parameters emerges. Thus string unification can be achieved without the introduction of a small intermediate scale.

Of course, there are also other possibilities which lead to the correct low–energy predictions. Instead of large threshold corrections one could consider a non–standard hypercharge normalization, i.e. a \( k_1 \neq 5/3 \) [33]. This would maintain gauge coupling unification at the string scale with the correct values of \( \sin^2 \theta_W (M_Z) \) and \( \alpha_S (M_Z) \). However, it is very hard to construct such models. A further possibility would be to give up the idea of gauge coupling unification within the MSSM by introducing extra massless particles such as \((3, 2)\) w.r.t. \( SU(3) \times SU(2) \) in addition to those of the SM [34, 27]. A careful choice of these matter fields may lead to sizable additional intermediate threshold corrections in (7) thus allowing for the correct low–energy data [4]. Unfortunately the price for that is exactly an introduction of a new intermediate scale of \( M_I \sim 10^{12-14} \text{GeV} \). It seems to be hard to explain such a small scale naturally in the framework of string theory. In some sense such a model can be compared to the model IIa in table 2. Other possible corrections to (7) may arise from an extended gauge structure between \( M_X \) and \( M_{\text{string}} \). However this might even enhance the disagreement with the experiment [27]. Finally a modification to (7) appears from the scheme conversion from the string– or SUSY–based \( \overline{\text{DR}} \) scheme to the \( \overline{\text{MS}} \) scheme relevant for the low–energy physics data [4]. However these effects are shown to be small [27].

Therefore we conclude with stating again the new result that string unification is easily achieved with moduli dependent threshold corrections within (0,2) superstring compactification. The Wilson line dependence of these functions is comparable to that on the \( T \) and \( U \) fields thus offering the interesting possibility of large thresholds with background configurations of moderate size. All non–MSSM like states can be heavier than 1/4 of the string scale, still leading to an apparent unification scale of \( M_X = \frac{1}{20} M_{\text{string}} \). We do not need vevs of the moduli fields that are of the order 20 away from the natural scale, neither
do we need to introduce particles at a new intermediate scale that is small compared to $M_{\text{string}}$. The situation could be even more improved with a higher number of moduli fields entering the threshold corrections: They may come from other orbifold planes giving rise to $N=2$ sectors or from additional Wilson lines. We think that the actual moderate vevs of the underlying moduli fields can be fixed by non-perturbative effects as e.g. gaugino condensation.

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**References**

[1] For a recent review see: P. Langacker, *Grand Unification and the Standard Model*, hep-ph/9411247

[2] J. Ellis, S. Kelley and D.V. Nanopoulos, *Phys. Lett. B* 249 (1990) 441;
U. Amaldi, W. de Boer and H. Fürstenau, *Phys. Lett. B* 260 (1991) 447;
P. Langacker and M.X. Luo, *Phys. Rev. D* 44 (1991) 817

[3] P. Langacker and N. Polonsky, *Phys. Rev. D* 47 (1993) 4028 and references therein

[4] P. Ginsparg, *Phys. Lett. B* 197 (1987) 139

[5] V. Kaplunovsky, *Nucl. Phys. B* 307 (1988) 145, Erratum: *Nucl. Phys. B* 382 (1992) 436

[6] L. Dixon, J. Harvey, C. Vafa and E. Witten, *Nucl. Phys. B* 261 (1985) 678; *B* 274 (1986) 285;
L.E. Ibáñez, J. Mas, H.P. Nilles and F. Quevedo, *Nucl. Phys. B* 301 (1988) 157

[7] I. Antoniadis, J. Ellis, R. Lacaze and D. V. Nanopoulos, *Phys. Lett. B* 268 (1991) 188;
L. Dolan and J.T. Liu, *Nucl. Phys. B* 387 (1992) 86;
M. Chemtob, *Threshold corrections in orbifold models and superstring unification of gauge interac-
tions*, Saclay T95/086 [hep-th/9506178]

[8] L.E. Ibáñez, H.P. Nilles and F. Quevedo, *Phys. Lett. B* 187 (1987) 25;
L.E. Ibáñez, J.E. Kim, H.P. Nilles and F. Quevedo, *Phys. Lett. B* 191 (1987) 283

[9] P. Mayr, H.P. Nilles and S. Stieberger, *Phys. Lett. B* 317 (1993) 53

[10] P. Candelas, G. Horowitz, A. Strominger and E. Witten, *Nucl. Phys. B* 258 (1985) 46

[11] M. Bershadsky, S. Cecotti, H. Ooguri and C. Vafa, *Nucl. Phys. B* 405 (1993) 279; *Comm. Math.
Phys. 165* (1994) 311;
S. Hosono, A. Klemm, S. Theisen and S.T. Yau, *Nucl. Phys. B* 433 (1995) 501

[12] V. Kaplunovsky and J. Louis, *Nucl. Phys. B* 444 (1995) 191

[13] L. Dixon, V. Kaplunovsky and J. Louis *Nucl. Phys. B* 355 (1991) 649

[14] S. Ferrara, C. Kounnas, D. Lüst and F. Zwirner, *Nucl. Phys. B* 365 (1991) 431

[15] I. Antoniadis, K.S. Narain and T.R. Taylor, *Phys. Lett. B* 267 (1991) 37;
I. Antoniadis, E. Gava and K. S. Narain, *Nucl. Phys. B* 383 (1992) 93; *Phys. Lett. B* 283 (1992) 209;
P. Mayr and S. Stieberger, *Nucl. Phys. B* 412 (1994) 502

[16] M. Cvetić, A. Font, L. E. Ibáñez, D. Lüst and F. Quevedo, *Nucl. Phys. B* 361 (1991) 194

[17] P. Mayr and S. Stieberger, *Nucl. Phys. B* 407 (1993) 725;
D. Bailin, A. Love, W.A. Sabra and S. Thomas, *Mod. Phys. Lett. A* 9 (1994) 67; *A* 10 (1995) 337

[18] P. Mayr and S. Stieberger, *Phys. Lett. B* 355 (1995) 107
[19] P. Mayr and S. Stieberger, TUM–HEP–212/95 to appear; 
S. Stieberger, One–loop corrections and gauge coupling unification in superstring theory, Ph.D. thesis, 
TUM–HEP–220/95

[20] I. Antoniadis, E. Gava, K.S. Narain and T.R. Taylor, Nucl. Phys. B 432 (1994) 187

[21] G. Lopes Cardoso, D. Lüst and T. Mohaupt, Nucl. Phys. B 450 (1995) 115

[22] L.E. Ibáñez, H.P. Nilles and F. Quevedo, Phys. Lett. B 192 (1987) 332

[23] J.P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, Nucl. Phys. B 372 (1992) 145; 
I. Antoniadis, E. Gava, K.S. Narain and T.R. Taylor, Nucl. Phys. B 407 (1993) 706

[24] E. Kiritsis and C. Kounnas, Nucl. Phys. B 41 [Proc. Sup.] (1995) 331; Nucl. Phys. B 442 (1995) 
472; Infrared–regulated string theory and loop corrections to coupling constants, hep–th/9507051

[25] S. Weinberg, Phys. Lett. B 91 (1980) 51

[26] L.E. Ibáñez and D. Lüst, Nucl. Phys. B 382 (1992) 305

[27] K.R. Dienes and A.E. Faraggi, Making ends meet: string unification and low–energy data, Princeton 
IASSNS–HEP–95/24 (hep-th/9505018); Gauge coupling unification in realistic free–fermionic string 
models, Princeton IASSNS–HEP–94/113 (hep–th/9505046)

[28] L.E. Ibáñez, D. Lüst and G. G. Ross, Phys. Lett. B 272 (1991) 251

[29] For a recent review see: P. Mayr and S. Stieberger, Proceedings 28th International Symposium on 
Particle Theory, p. 72–79, Wendisch–Rietz (1994) (hep–th/9412196, DESY 95–027)

[30] E. Witten, Nucl. Phys. B 258 (1985) 75

[31] E. Witten, Nucl. Phys. B 269 (1986) 79

[32] G. Aldazabal, A. Font, L.E. Ibáñez and A.M. Uranga, String GUTs, Madrid FTUAM–94/28 (1994); 
hep-th/9410206

[33] L.E. Ibáñez, Phys. Lett. B 318 (1993) 73

[34] U. Amaldi, W. de Boer, P.H. Frampton, H. Fürstenau and J. Liu, Phys. Lett. B 281 (1992) 374; 
I. Antoniadis, J. Ellis, S. Kelley and D.V. Nanopoulos, Phys. Lett. B 272 (1991) 31; 
A.E. Faraggi, Phys. Lett. B 302 (1993) 202; 
M.K. Gaillard and R. Xiu, Phys. Lett. B 296 (1992) 71