An Estimate of the Spectral Intensity Expected from the Molecular Bremsstrahlung Radiation in Extensive Air Showers

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Abstract

A detection technique of ultra-high energy cosmic rays, complementary to the fluorescence technique, would be the use of the molecular Bremsstrahlung radiation emitted by low-energy electrons left after the passage of the showers in the atmosphere. The emission mechanism is expected from quasi-elastic collisions of electrons produced in the shower by the ionisation of the molecules in the atmosphere.

In this article, a detailed calculation of the spectral intensity of photons at ground level originating from the transitions between unquantised energy states of free ionisation electrons is presented. In the absence of absorption of the emitted photons in the plasma, the obtained spectral intensity is shown to be \(\approx 5 \times 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}\) at 10 km from the shower core for a vertical shower induced by a proton of \(10^{17.5}\) eV.

1. Introduction

The origin and nature of ultra-high energy cosmic rays still remain to be elucidated despite the recent progresses provided by the data collected at the Pierre Auger Observatory and the Telescope Array\(^1\). This is due to the extremely low intensity of particles at these energies. As of today, the most direct way to infer the nature of the particles at ultra-high energies relies on the observation of the shower longitudinal profile to measure its maximum of development. The use of telescopes detecting the nitrogen fluorescence light emitted after the passage of the electromagnetic cascade is a well-suited technique to achieve such measurements. Moreover, these fluorescence telescopes provide a good calorimetric estimate of the energy of the showers, which is preferable to detectors requiring external information to calibrate the energy estimator of the showers. However, this technique can only be used on moonless nights, resulting in a 10% duty cycle. Together with the low intensity of particles, this makes the study of the cosmic ray composition above few tens of EeV very challenging.
Recently, triggered by microwave emission measurements in laboratory \cite{2}, new telescope techniques based on the detection of the microwave emission in the GHz C-band (3.4-4.2 Hz) have been developed at the Pierre Auger Observatory \cite{3}. These techniques aim at providing measurements of the electromagnetic content of the cascade with quality comparable to the fluorescence detectors but with a 100% duty cycle. Molecular Bremsstrahlung radiation in the GHz band provides an interesting mechanism to detect ultra-high energy cosmic rays due to the expected isotropic and unpolarised radiation. This feature would allow for the possibility of performing shower calorimetry in the same spirit as the fluorescence technique does, by mapping the ionisation content along the showers through the intensity of the microwave signals detected at ground level.

In this paper, we estimate the intensity expected from the molecular Bremsstrahlung radiation in extensive air showers. The approach adopted here does not rely on any free parameter. To start, the production and time evolution of the ionisation electrons along the shower track are detailed in section \cite{2}. In turn, these electrons can produce their own emission, such as Bremsstrahlung emission. The expected spectral intensity at ground level of such an emission is the object of section \cite{3} by making use of the ‘free-free approach’. Possible attenuation or suppression effects are studied in section \cite{4}. Finally, the results obtained in this study are illustrated in section \cite{5} on a toy reference shower. From these results, the perspectives of detection of ultra-high energy cosmic rays by making use of molecular Bremsstrahlung radiation are discussed.

2. Ionisation Electrons along the Shower Track

2.1. A Crude Model of Vertical Air Showers

In this work, an extensive air shower is considered as a thin plane front of high energy charged particles propagating in the atmosphere at the speed of light \( c \). For a given primary type and a given energy \( E \), the longitudinal development of the electromagnetic cascade depends only on the cumulated slant depth \( X \) expressed as the ratio between the vertical thickness of the atmosphere \( X_{\text{vert}} \) (1000 g cm\(^{-2}\) at sea level) and the cosine of the zenith angle of the shower. After the succession of a few initial steps in the cascade, all showers can be described by reproducible macroscopic states. In particular, the shape of the showers is universal except for a translation depending logarithmically on \( E \) and a global factor roughly linear in \( E \). In this way, for any given slant depth \( X \) or equivalently any altitude \( a \), the total number of primary \( e^+ / e^- \) particles, \( N_{e,p} \), can be adequately parameterised by the Gaisser-Hillas function as:

\[
N_{e,p}(a) = N_{\text{max}} \left( \frac{X(a) - X_0}{X_{\text{max}} - X_0} \right)^{X_{\text{max}} - X_0} \frac{X_{\text{max}} - X(a)}{\lambda} \exp \left( \frac{X_{\text{max}} - X(a)}{\lambda} \right),
\]

with \( X(a) \) the depth corresponding to the altitude \( a \), \( X_0 \) the depth of the first interaction, \( X_{\text{max}} \) the depth of shower maximum, \( N_{\text{max}} \) the number of particles observed at \( X_{\text{max}} \), and \( \lambda \) a parameter describing the attenuation of the shower.

On the other hand, high energy particles constituting the core of the shower are collimated along the initial shower axis. The lateral extension of the core depends on the
mean free path and can be expressed in terms of the Moliere radius $R_M$ such that 90% of the energy is contained within a distance $r$ from the axis such as $r < R_M$. Motivated by general arguments to describe the electromagnetic cascade of showers, the NKG lateral distribution function denoted hereafter by $l d f (r, a)$ is known to reproduce reasonably well the observations:

$$l d f (r, a) = C(s(a)) R_M^{2} \left( \frac{r}{R_M} \right)^{s(a)-2} \left( 1 + \frac{r}{R_M} \right)^{s(a)-4.5}.$$  \hspace{1cm} (2)

Here, $s(a)$ stands for the age parameter at altitude $a$ defined as $s(a) = 3X(a)/(X(a) + 2X_{\text{max}})$, and $C(s)$ is a normalisation factor.

The number of primary $e^+/e^-$ per unit surface, $n_{e,p}(r, a)$, is then simply obtained by folding the longitudinal profile to the normalised lateral one. For a vertical shower whose geometry is depicted in figure 1, $n_{e,p}(r, a)$ reads as:

$$n_{e,p}(r, a) = N_{e,p}(a) \frac{l d f (r, a)}{2\pi \int dr \, r \, l d f (r, a)}.$$  \hspace{1cm} (3)

Noticeably, this description is only a crude model of an extensive air shower. This shall allow us, however, to derive in the following a realistic number of ionisation electrons left along the shower track and thus to estimate relevant orders of magnitude for...
the spectral intensities (in W m\(^{-2}\) Hz\(^{-1}\)) that can be expected from molecular Bremsstrahlung radiation by these ionisation electrons.

To facilitate comparisons of the results obtained in this study with the values reported in [2] and [4], the parameters of both the Gaisser/Hillas and the NKG functions are tuned to apply to vertical proton showers with primary energy \(E = 10^{17.5}\) eV.

2.2. Production of Ionisation Electrons along the Shower Track

Through the passage of charged particles in the atmosphere, the energy of an extensive air shower is deposited mainly through the ionisation process. The resulting numerous ionisation electrons can, in turn, produce their own emission such as continuum Bremsstrahlung emission through quasi-elastic scattering with molecular nitrogen and, to smaller extent, oxygen. To evaluate the spectral intensity of this radiation, it is necessary to derive the flux of secondary ionisation electrons created by the development of the shower.

For one single primary electron travelling over an infinitesimal distance \(da\), and for a mass density \(\rho_m(a)\) of molecular nitrogen or oxygen, the average number of ionisation electrons per unit length and per kinetic energy band reads as:

\[
\frac{d^2 N_{e,i}}{da dT_e}(a, T_e) = \rho_m(a) f_0(T_e) \frac{dE}{dX} \frac{1}{(I_0 + T_e)}. \tag{4}
\]

with \(I_0\) the ionisation potential to create an electron-ion pair in air. The bracketed expression \(\langle dE/dX \rangle\) stands for the mean energy loss of primary electrons per grammage unit. This energy loss is due quasi-exclusively to ionisation and is almost independent of the electron energy over a range of few tens of MeV, typical of the primary electrons energy in the cascade. The distribution in kinetic energy of the resulting ionisation electrons is described here by the normalised function \(f_0(T_e)\) parameterised as [5]:

\[
f_0(T_e) = \frac{K}{1 + (T_e/\bar{T})^{2.1}}, \tag{5}
\]

where the constant \(K\) is tuned to guarantee \(\int dT_e f_0(T_e) = 1\), and \(\bar{T} = 13.0\) (17.4) eV for nitrogen (oxygen). Note that there is a recent work to account for relativistic effects as well as indistinguishability between primary and secondary electrons [6], but this does not affect the results presented in this study. The instantaneous number of ionisation electrons per unit volume and per kinetic energy band is then obtained by coupling equation (4) to the number of primary charged particles per surface unit:

\[
n_{e,i}^0(r, a, T_e) = f_0(T_e) \langle dE/dX \rangle \rho_m(a) n_{e,p}(r, a). \tag{6}
\]

Of relevant importance for the following is the flux \(\phi_{e,i}^0(r, a, T_e)\) of secondary electrons per kinetic energy band. For any surface element \(dS\), and considering a coordinate system with the zenith angle defined along the axis perpendicular to the surface \(dS\), the total number of electrons \(N_{e,i}(r, a, T_e, \chi, \psi)\) (per kinetic energy band) crossing \(dS\) during a short time interval \(dt\) under zenith and azimuth incidence angles \(\chi\) and \(\psi\) is:

\[
N_{e,i}^0(r, a, T_e, \chi, \psi) = c\beta(T_e)dt \int d\psi \sin \chi d\chi dS |\cos \chi| n_{e,i}^0(r, a, T_e, \chi, \psi), \tag{7}
\]
with $\beta(T_e)$ the relativistic factor. Since ionisation electrons can be assumed to be emitted isotropically, the quantity per solid angle unit $n_e^0_i(r, a, T_e, \chi, \psi)$ is in fact independent of the incidence angles and reduces to $n_e^0_i(r, a, T_e)/4\pi$. This yields to the expression of the instantaneous flux per kinetic energy band, which is the relevant quantity for the following:

$$
\phi_{e,i}^0(r, a, T_e) = \frac{c\beta(T_e)}{2(\langle \rho \rangle + T_e)} \int \frac{dE}{d\chi} \rho_m(a) n_{e,p}(r, a).
$$

(8)

2.3. Time Evolution of the Flux of Ionisation Electrons

The instantaneous flux per kinetic energy band obtained through equation (8) corresponds to the number of secondary electrons per surface, time and kinetic energy units left just after the passage of the primary high-energy electrons of the shower. The flux of secondary low-energy electrons $\phi_{e,i}(r, a, T_e, t)$, still per kinetic energy band, available at any time $t$ after the passage of the shower is governed by the interactions that these electrons undergo in the atmosphere. In turn, the evolution in time of the function $\phi_{e,i}(r, a, T_e, t)$ can be fully encompassed in the time dependence of the distribution in kinetic energy $f(T_e, t)$ of the ionisation electrons. This evolution is determined by a Boltzmann equation accounting for all the interactions of interest at work, Boltzmann equation that is now detailed and solved numerically.

Ionisation electrons can be considered as static in space to a good approximation given their low energy and given that their rate of disappearance is governed by attachment processes which occur on a time scale of at most few hundreds of nanoseconds in the atmospheric layers of interest. It is consequently comfortable to neglect the space diffusion term in the Boltzmann equation. The time evolution of the distribution function $f$ is then exclusively governed by a collision term in the following Boltzmann equation:

$$
\frac{\partial f}{\partial t}(T_e, t) = -n_m(a)c\beta(T_e)\left[\sigma_{att}(T_e) + \sigma_{exc}(T_e) + \sigma_{ion}(T_e)\right]f(T_e, t)
$$

$$
+ n_m(a)c \int_{T_e}^{T_{e,max}} dT_e' \beta(T_e') \left(\frac{d\sigma_{ion}}{dT_e}(T_e', T_e) + \frac{d\sigma_{ion}}{dT_e}(T_e', T_e' - T_e)\right) f(T_e', t)
$$

$$
+ n_m(a)c \int_{T_e}^{T_{e,max}} dT_e' \beta(T_e') \left(\frac{d\sigma_{exc}}{dT_e}(T_e', T_e)\right) f(T_e', t).
$$

(9)

where $\sigma_i$ denotes the cross sections of interest (ionisation, excitation of electronic levels and attachment processes), and $n_m(a)$ is the density of molecules at an altitude $a$ obtained by multiplying the corresponding mass density by the Avogadro number $\mathcal{N}_A$ and by dividing by the corresponding molar mass $\mathcal{A}$. The first term in the right hand side stands for the disappearance of electrons with kinetic energy $T_e$, while the second and third terms stand for the appearance of electrons with kinetic energy $T_e$ due to ionisation and excitation reactions initiated by electrons with higher kinetic energy $T_e'$. Note that in the case of ionisation, a second electron emerges from the collision with kinetic energy $T_{e'} - T_e$.

The quantities $\tau_i^{-1} = n_m(a)c\beta(T_e)\sigma_i(T_e)$ characterise the rates of collisions. The electron attachment processes with oxygen have been studied in detail in reference [7]. The
corresponding collision rate can be adequately parameterised as:

$$\tau_{\text{att}}^{-1}(a) = k_{\text{att}1} n_{O_2}^2(a) + k_{\text{att}2} n_{O_2}(a)n_{N_2}(a),$$  \hspace{1em} (10)

with $k_{\text{att}1} = 2 \times 10^{-30} \text{ cm}^6\text{s}^{-1}$ and $k_{\text{att}2} = 8 \times 10^{-32} \text{ cm}^6\text{s}^{-1}$. This leads to a characteristic time scale $\tau_{\text{att}} = 15 \text{ ns}$ at sea level and $\tau_{\text{att}} = 50 \text{ ns}$ at an altitude of 6 km. The ionisation cross-section is taken from reference [8], it varies between $\simeq 1$ and $\simeq 10$ atomic units in the energy range of interest, leading to a characteristic time scale between two ionisations below $\sim 1$ picosecond. Finally, collisional rates describing the excitation of the different electronic levels of interest of $O_2$ and $N_2$ molecules are taken from [9]. Overall, they lead to characteristic time scales around $\sim 1$ picosecond. The different collision rates are shown in figure 2 at sea level. For energies above the ionisation potential $I_0$, ionisation is the dominant energy loss process, while for energies between $\simeq 4.5$ eV (corresponding to the threshold to excite one of the electronic levels of oxygen) and $I_0$, excitation is the dominant process. Below $\simeq 4.5$ eV, only the attachment process is at work, causing a real disappearance of electrons.

The second and third terms in equation 9 account for the migration of electrons with kinetic energies $T_e'$ to lower energies through ionisation and excitation processes. From the ionisation collision rate plotted in figure 2, the population of electrons produced with an energy larger than the ionisation potential $I_0$ is expected to be confined in the energy range below $I_0$ on a time scale much smaller than a nanosecond. New ionisation electrons created through this process undergo the same mechanism. On almost the same time scale, the excitation collision rate is expected to confine all electrons in a small energy window below $\simeq 4.5$ eV - corresponding to the smallest threshold of possible excitations. Then, on a much larger time scale (a hundred of nanoseconds), the electrons disappear through the attachment process to oxygen. To quantify precisely this picture, the Boltzmann equation can be solved numerically in different ways. We
show in figure 3 the evolution with time of the function $f$ as obtained by Monte-Carlo for electrons produced at sea level. The high collision rate of ionisation drives the energy loss of electrons with kinetic energies larger than the ionisation potential $I_0$. On a time scale of $\approx 10$ ps, all electron energies pass below this threshold and the excitation becomes the dominant process. After $\approx 1$ ns, all electron energies get below the lowest excitation threshold. This results in a sharp cutoff in the $f$ function, which confines all electron energies below $\approx 4.5$ eV on a short time scale. On longer time scales, free electrons disappear through the attachment process. Since the number of electrons is not conserved, the normalisation of $f$ against the kinetic energy varies with time; it starts by quickly increasing due to the new electrons produced by ionisation, and then exponentially decreases on a longer time scale driven by the attachment process.

Given that the time scale of interest for the microwave detection is the nanosecond, the detailed behaviour of the function $f$ below this time scale is quite irrelevant. To make easier the calculations in the following sections, and from the result shown in figure 3, the approximate analytical expression will be used:

$$f(T_e, t) \approx \tilde{f}_0(T_e) \exp\left(-\frac{t}{\tau_{\text{att}}} \right),$$

with $\tilde{f}_0$ tabulated from the Monte-Carlo results obtained for $t = 1$ ns.

### 3. Microwave Emission from Molecular Bremsstrahlung: the Free-Free Approach

As long as they remain free, ionisation electrons can produce photons through the process of quasi-elastic collisions with neutral molecules in the atmosphere:

$$e^\pm + M \rightarrow e^\pm + M + \gamma.$$
In this approach, the production of photons with energies $h\nu$ corresponds to transitions between unquantised energy states of the free electrons ('free-free' transitions). The spectral intensity at ground level can be deduced in a straightforward way from the collision rate of ionisation electrons with neutral molecules in air. This approach has been shown to be successful for describing, for instance, the production of free-free radiations in collisions of low energy electrons with neutral atoms in measurements using drift-tube techniques [10].

By considering the production rate $r_\gamma$ of photons with energy $h\nu$ per volume unit proportional to the target density only, it is then governed by the electron flux and by the free-free cross section $\sigma_{ff}(T_e, h\nu)$, leading to the following expression:

$$r_\gamma(r, a, t, \nu) = n_m(a) \int_0^{T_e^{\max}} dT_e \phi_{e,i}(r, a, T_e, t) \sigma_{ff}(T_e, h\nu).$$

(13)

Possible effects of absorption or suppression of the emission due to destructive interferences of the photons within the interaction zone are neglected at this step. Such effects will be further discussed in section 4.

The free-free cross-section as obtained in reference [11] can be related to the electron momentum transfer cross-section through:

$$\sigma_{ff}(T_e, h\nu) = \frac{4}{3\pi R_y} \alpha^3 \left(1 - \frac{h\nu}{2T_e}\right) \sqrt{1 - \frac{h\nu}{T_e}} T_e \sigma_m(T_e),$$

(14)

with $\alpha$ the fine-structure constant and $R_y$ the Rydberg constant. For electrons with kinetic energies in the range of few tens of eV and photons in the GHz energy range, this expression is very accurately independent of $h\nu$ and can be reduced to $\sigma_{ff}(T_e) = 1.211 \times 10^{-8} T_e^{3/2} T_e \sigma_m(T_e)$. The electron momentum transfer cross-section, on the other hand, has been well measured on various targets. Compiled tables provided in reference [12] were used for the following.

As already stressed, the space volume that these electrons can probe during their relatively small lifetime is negligible compared to the volume in which an extensive air shower develops. In this way, it is comfortable to consider each electron as a point-like source of photons during its whole lifetime. Hence, from the knowledge of the collision rate per volume unit $r_\gamma$, the emitted power per volume unit at each point $(r, a)$ can be simply obtained by coupling this rate to the energy of the emitted photons, so that the emitted spectral power per volume unit can be written as:

$$\frac{d^2P}{d\nu dV}(r, a, t) = \frac{d}{d\nu}(h\nu r_\gamma(r, a, t))$$

$$= \frac{h\nu}{2A(\langle I_0 + T_e\rangle)} \left\langle \frac{dE}{dX} \right\rangle \bar{\sigma} n_{e,p}(r, a) \exp(-t/\tau_{att}(a)),$$

(15)

where $\bar{\sigma} \approx 2.5 \times 10^{-30}$ m$^2$ (for nitrogen targets) is an effective cross-section defined as:

$$\bar{\sigma} = \int_0^{T_e^{\max}} dT_e \tilde{f}_0(T_e) \beta(T_e) \sigma_{ff}(T_e).$$

(17)
The transparency to photons of the electrons-neutral molecules will be justified in the next section, so that the radiation produced by individual electrons-nitrogen/oxygen encounters can be considered here to pass out of the interaction volume without absorption or reflection. At any distance $R$, the spectral intensity received from sources contained in any infinitesimal volume $dV$ is proportional to $d^2P/d\nu dV$ times $dV$ and weighted by $4\pi R^2$ given that photons are emitted isotropically \footnote{The assumption on isotropy is justified in the regime where the energy of the photons is low compared to the energy of the electrons \cite{13}.} from each source. In this way, the observable spectral intensity at any ground position $x_g$, $\Phi_g$, is simply the sum of the uncorrelated contributions of the individual encounters:

$$
\Phi_g(x_g, t) = \int_0^\infty r \, dr \int_0^{2\pi} d\phi \int_0^\infty da \frac{1}{4\pi R^2(r, \phi, a)} \frac{d^2P}{d\nu dV}(r, a, t_d(t, r, \phi, a)).
$$

(18)

Here, $R$ is the distance between the position at ground $x_g$ and the position of the current source in the integration, and $t_d$ is the delayed time at which the emission occurred. Fixing the reference time $t_0$ to the time at which the shower front crosses the ground, each source at altitude $a$ started emitting radiation at the time the shower front passed at that altitude (i.e. at $t_0 - a/c$). Each photon crossing the ground at time $t$ (with the condition that $t \geq t_0$) coming from a source at altitude $a$ and located at a distance $r$ from the shower axis was emitted at time $t - Rn(a, \nu)/c$ - with $n(a, \nu)$ the refractive index of the atmosphere. The delayed time $t_d(t)$ is thus expressed as:

$$
t_d(t, r, \phi, a) = t - t_0 - \left(\frac{R(r, \phi, a)n(a, \nu)}{c} - \frac{a}{c}\right).
$$

(19)

With the evident condition that emissions occur only at $t_d \geq 0$, a Heaviside function denoted $\Theta$ is introduced leading to the following semi-analytical expression for the observable spectral intensity:

$$
\Phi_g(x_g, t) = \frac{hcN_A\bar{\sigma}}{8\pi A(I_0 + T_e)} \frac{dE}{dX} \int_0^\infty r \, dr \int_0^{2\pi} d\phi \int_0^\infty da \frac{\rho_{m}(a)n_{e,p}(r, a)}{R^2(r, \phi, a)} e^{-\frac{t_d(t, r, \phi, a)}{\tau(a)}} \Theta(t_d(t, r, \phi, a)).
$$

(20)

Besides the free-free approach presented here, an independent estimation of the radiated power using the Classical Field Theory formalism is presented, resulting in the same predictions. Detailed expressions in this frame can be found in \cite{5}.

**4. Possible Attenuation Effects in Molecular Bremsstrahlung Radiation**

**4.1. Absorption effects**

In addition to free-free emissions, ionisation electrons can also experience inverse Bremsstrahlung and stimulated Bremsstrahlung within the electrons-neutral molecules plasma. A convenient way to quantify the size of these effects is to calculate the absorption coefficient, $\alpha_\nu$, defined as the relative attenuation per unit length of the emitted photons. The absorption coefficient is defined as the net balance between the number
of absorbed photons per unit length subtracted to the number of stimulated emitted photons (due to a photon that causes an electron in the potential of a neutral molecule to emit another photon of the same frequency) per unit length.

Re-writing equation 15 in a way such that $\eta_{\nu,V}(r,a,T_e)$ is introduced, the emitted spectral power per volume unit at a fixed energy $T_e$ is:

$$\frac{d^2 P}{d\nu dV}(r,a,t) = \int dT_e \tilde{f}(T_e, t) \eta_{\nu,V}(T_e), \quad (21)$$

the absorption coefficient $\alpha_\nu$ is then known to be related to $\eta_{\nu,V}(r,a,T_e)$ through [14]:

$$\alpha_\nu = \frac{c^2}{h\nu^3} \int dT_e' \left[ \tilde{f}_0(T_e) - \tilde{f}_0(T_e') \right] \eta_{\nu,V}(T_e'). \quad (22)$$

This expression accounts for both the absorption of a photon of energy $h\nu$ by an electron with initial energy $T_e$ and a final one $T_e'$ and the stimulated emission due to a photon that causes a neighboring electron to emit another photon of the same energy within the electrons-neutral molecules plasma. Given the low frequencies of the photons considered here compared to the mean energy of the electrons, expanding $\tilde{f}_0(T_e)$ to first order leads to:

$$\tilde{f}_0(T_e') = \tilde{f}_0(T_e) + h\nu \frac{\partial \tilde{f}_0}{\partial T_e}, \quad (23)$$

so that the absorption coefficient $\alpha_\nu$ reads as:

$$\alpha_\nu = -\frac{c^2}{\nu^2} \int dT_e \frac{\partial \tilde{f}_0(T_e)}{\partial T_e} \eta_{\nu,V}(T_e). \quad (24)$$

Injecting explicitly the expression of $\eta_{\nu,V}(T_e)$ into this expression, $\alpha_\nu$ turns out to read:

$$\alpha_\nu(r,a) = -\frac{hc^3 \rho^2(a) N_A}{2A(L_0 + T_e)} \frac{1}{\nu^2} \left\langle \frac{dE}{dX} \right\rangle n_{e,p}(r,a) \int dT_e \beta(T_e) \sigma_{\Pi}(T_e) \frac{\partial \tilde{f}_0(T_e)}{\partial T_e}. \quad (25)$$

Close to the shower core and to the maximum of shower development (that is, within the denser plasma region), this leads to $\nu^2 \alpha_\nu \approx 10^{-4}$ m$^{-1}$ Hz$^2$. At GHz frequencies, the absorption is thus negligible.

### 4.2. Suppression Effects

The spectral intensity predicted by equation 20 is based on the assumption that the emitted radiation passes out of the interaction volume without undergoing any dispersive properties of the plasma caused by the successive interactions of the electrons.

Dispersive properties are commonly described on a macroscopic basis by a dielectric coefficient $K$. This coefficient allows a derivation of the absorption coefficient which, in contrast to the previous derivation, accounts for successive collisions within the radiation formation zone of each electron-neutral collision. The effect of the successive collisions, known as the plasma dispersion effect [14], can lead to destructive interferences of the radiated fields.

Accounting for the coupling of electrons to the emitted radiation turns out to be a difficult task here, since it consists in considering an additional term in equation 20.
proportional to $\partial f / \partial T_e$ times the radiated electric fields. To get an order of magnitude of the effects which might be at work, we adopt the commonly used method consisting in linearising a simplified Boltzmann equation pertaining only to the case of a distribution function $f(T_e)$ stationary in time in the absence of any emitted radiation. In this case, it can be shown that the plasma dispersion effects result in a suppression factor in the integrand of $\tilde{\sigma}$ defined in equation 17, reading as:

$$\frac{1}{1 + (\nu_c(T_e, t)/\nu)^2}, \quad (26)$$

where $\nu_c(T_e, t)$ is the time-dependent rate of inelastic collisions of the electrons of kinetic energy $T_e$. From the analysis of section 2.3, this collision rate amounts to several THz within the first nanosecond for highly energetic electrons and then decreases to the level of a few tens of MHz. Consequently, for frequencies $\nu$ around the GHz, the suppression factor can be important only during the first nanosecond, as long as the collision rate is much larger than the frequency considered for the radiation field.

It is noteworthy that the suppression factor aforementioned is derived by means of a series of hypotheses which are not really relevant in the case considered here. However, it is clear that such plasma dispersion effects can be important only when $\nu_c(T_e, t)$ is larger than $\nu$. As a kind of proxy to probe these effects in the most pessimistic way, the impact can be evaluated by suppressing the emission as long as $\nu_c(T_e, t) > \nu$.

5. Discussion

Although semi-analytical integrations of equation 20 are possible without random number generators, a Monte-Carlo sampling of the integrand function in $r$ and $\phi$ allows a much faster integration in terms of CPU time. Results presented here have thus been obtained using random number generators to carry out these particular integrations.

A vertical proton shower of $10^{17.5}$ eV is used as a proxy to illustrate the estimation of the spectral intensity expected from molecular Bremsstrahlung radiation presented in section 3. Experimental setups using regularly spaced antennas oriented vertically or nearly vertically detect showers crossing the field-of-view and impacting the ground. The spectral intensity expected at different distances from the shower core at ground level as derived from equation 20 is shown in figure 4. This figure is relevant for the case of the currently running EASIER installation at the Pierre Auger Observatory [3] or of the recent CROME experiment [16]. It is seen that the spectral intensity is rapidly decreasing in amplitude for increasing distances to the shower core. An alternative detection method of microwave radiation is the use of large aperture receivers pointing just above the horizon to observe the longitudinal profiles of the showers at large distances, such as MIDAS and AMBER installations at the Pierre Auger Observatory [3]. The received power as a function of time at a distance of 10 km from the shower axis is shown in figure 5. It turns out to be $\approx 5 \times 10^{-26}$ W m$^{-2}$ Hz$^{-1}$, smaller than the one reported in reference [2] by a factor $\approx 50$ when scaling the beam measurements to air showers.

To probe the maximal impact of the plasma dispersion effects as discussed in the previous section, the same simulation is repeated with the condition of suppressing the
emission as long as $v_c(T_e, t) > v$. At 10 km from the shower axis, the signal is found to be $\approx 4.8 \times 10^{-26}$ W m$^{-2}$ Hz$^{-1}$, not significantly different from the spectral intensity value ob-
tained without accounting for any suppression effect. This gives an idea about the small systematic uncertainty which affects the estimate due to possible collective suppression within the plasma.

Close to the shower core, the measured signal amplitude from geomagnetic radiation [16] is greater than or of the same order as the one expected from molecular Bremsstrahlung radiation found in this study. However, the expected signal duration from molecular Bremsstrahlung radiation appears significantly larger (about a factor 100) comparing to the measured signal from geomagnetic radiation.

The estimated intensity is found to be much lower than expected from laboratory measurements interpreted exclusively in terms of molecular Bremsstrahlung [2]. We note that these measurements are still to be confirmed by independent experiments [17, 18]. Based on this study, significant increases in sensitivity should be achieved from an experimental point of view to be able to detect showers induced by ultra-high energy cosmic rays by means of the molecular Bremsstrahlung emission mechanism.

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Classical Field theory approach in Molecular Bremsstrahlung Radiation estimation

From the point of view of the classical field theory, the radiated power by the ionisation electrons is associated to the deviations caused by the collisions with the neutral molecules. In this framework, although the formal expression of the spectral intensity expected at ground level is unchanged with respect to equation[18] the expression of the emitted spectral power per volume unit has to be revised to remove any reference to free-free transitions.

$$\Delta \nu = 2v \sin (\chi/2).$$

Figure 6: Geometry of a classical binary collision. The vector change in velocity is $|\Delta \nu| = 2v \sin (\chi/2)$.

In a classical way, when an electron approaches a neutral molecule, the electric field of the electron polarises the neutral molecule. This polarisation gives rise to a dipole moment which induces an attractive interaction potential at a short distance range: $V(d) \propto d^{-4}$ - with $d$ the distance between the electron and the molecule. The time-dependent radiated power during the interaction is known to obey the Larmor formula: $p^e(t) = e^2 |\dot{\nu}(t)|^2 / 6\pi \varepsilon_0 c^3$, with $e$ the elementary charge and $\varepsilon_0$ the vacuum permittivity.

Then, by making use of the Parseval identity, one can derive directly the frequency spectrum of the radiated energy for one collision as:

$$\frac{dE}{d\nu}(v, \nu, \chi) = \frac{e^2}{3\pi \varepsilon_0 c^3} \left| \int_{-\infty}^{\infty} dt \, \dot{\nu}(t) \exp(-i2\pi \nu t) \right|^2. \quad (27)$$

Since the interaction potential acts at a short distance range only, the average time during which the interaction is taking place can be estimated as $\Delta t \approx b / \langle v \rangle$, with $b$ the impact parameter of the collision. For typical electron velocities of the order of a few percents of the speed of light, a realistic order of magnitude for $\Delta t$ is $\approx 10^{-16}$ s so that for GHz frequencies, $2\pi \nu \Delta t \ll 1$ for any $\tau$ within $\Delta t$. Hence, the argument in the exponential of the integrand is very small during the time where the electron is accelerated, so that for an electron undergoing a total deviation by an angle $\chi$, the corresponding frequency spectrum of the radiated energy can be accurately estimated as:

$$\frac{dE}{d\nu}(v, \chi) = \frac{e^2}{3\pi \varepsilon_0 c^3} |\Delta \nu|^2 \quad (28)$$

$$\approx \frac{2e^2}{3\pi \varepsilon_0 c^3} \nu^2 (1 - \cos \chi), \quad (29)$$
where the expression of $|\Delta v|$ has been easily derived from the geometry of the collision depicted in figure 6.

The collision rate per volume unit, per electron velocity band and per solid angle unit (with here $d\Omega = \sin \chi d\chi d\psi$) is governed by the same ingredients as in the previous section, except that the free-free cross-section is now replaced by the classical differential cross-section:

$$\frac{d^2 r^c}{dvd\Omega}(r, a, t, v, \chi) = n_m(a) \phi_{e,i}(r, a, v, t) \frac{d\sigma^c}{d\Omega}(v, \chi), \quad (30)$$

where the flux of ionisation electrons is now expressed, for convenience, per velocity band instead of kinetic energy band. This is achieved by expressing the kinetic energy in terms of the velocity and by substituting the normalised $f$ function by the one obtained through the relevant Jacobian transformation:

$$\tilde{f}(v, t) = \frac{m_e v}{(1 - (v/c)^2)^{3/2}} f(T_e(v), t). \quad (31)$$

Since the collision rate is independent of the frequency, the emitted spectral power per volume unit can be obtained by integrating this collision rate directly coupled to the frequency spectrum of the energy radiated per collision over solid angle and the velocity $v$. This leads to:

$$\frac{d^2 P^c}{dvdV}(r, a, t) = \frac{e^2 p^2_m(a) N_A}{3\pi\epsilon_0 A I_0} \left\langle \frac{dE}{dX} \right\rangle \tilde{\sigma}^{cl} n_{e,p}(r, a) \exp(-t/\tau_{att}(a)), \quad (32)$$

with the classical effective cross-section defined as:

$$\tilde{\sigma}^{cl} = \int dv \tilde{f}_0(v) \left(\frac{v}{c}\right)^3 \frac{I_0}{I_0 + T_e(v)} \int d\Omega (1 - \cos \chi) \frac{d\sigma^c}{d\Omega}(v, \chi). \quad (33)$$

Note that the result of the solid angle integration is, by definition, the momentum transfer cross-section $\sigma_m(T_e(v))$.

Hence, it is clear by identification that equation 32 can be equivalent to equation 15 only if the following correspondence holds:

$$h\tilde{c}^{cl} \rightarrow \frac{2e^2}{3\pi\epsilon_0} \tilde{\sigma}^{cl}. \quad (34)$$

And, it turns out that both energy volumes equal, for nitrogen targets for instance, to $\approx 3.1 \times 10^{-36}$ eV m$^3$. Given the low energy of the photons considered here compared to the electron energies, the classical approach results in the same prediction as the free-free approach.
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