Development of a detector pair for very high frequency gravitational waves

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Abstract. We are developing a detector with two laser interferometers for gravitational waves at 100 MHz. Each interferometer is a Sagnac interferometer with a 75-cm baseline synchronous recycling (or resonant recycling) cavity. Two such interferometers are constructed to perform cross-correlation analysis. The original signals at around 100 MHz are converted into electrical signals at lower frequencies before we collect the data. The output noise of each interferometer is measured to correspond to less than $1 \times 10^{-18}$ Hz$^{-1/2}$ in strain amplitude at around 100 MHz.

1. Introduction

We are developing a pair of laser interferometers for detecting 100-MHz gravitational waves at the Mitaka campus of the National Astronomical Observatory of Japan. Gravitational waves at 100 MHz are considered to be from astronomical or cosmological sources. Two interferometers have been constructed in order to perform a cross-correlation analysis and improve the signal-to-noise ratio, because the gravitational waves from the sources are expected to be random or stochastic processes.

At above 1 MHz, there have been few experiments for detecting gravitational waves. Today largescale laser-interferometric observatories (e.g. TAMA300, LIGO, VIRGO and GEO600) have been constructed for the detection of gravitational waves from a few Hz to kHz. There are future plans to construct space observatories (e.g. LISA, DECIGO and BBO) for detecting gravitational waves below 1 Hz. However, at 100 MHz, the sole experiment known to us was a detector pair with microwave loops in Birmingham [1].

Our laser interferometer is based on synchronous (or resonant) recycling technique proposed by R W P Drever [2] in 1980s. A synchronous recycling interferometer has a unique response
function such that the signal induced by gravitational waves enhanced at a specific frequency [3, 4, 5]. In this paper, a synchronous recycling interferometer is introduced, then signal extraction schemes are described. Preliminary sensitivities of our interferometers are also shown.

2. Synchronous recycling interferometer

2.1. Optical configuration

A schematic view of a synchronous recycling interferometer is illustrated in Figure 1. The laser interferometer consists of two parts: a four-mirror ring cavity (hereafter recycling cavity) and a Sagnac interferometer (Sagnac part). The recycling cavity is formed by a recycling mirror (RM), a transfer mirror (TM), and two end mirrors (EM1 and EM2). A Sagnac part is formed by a beamsplitter (BS), the recycling cavity, and two steering mirrors. PD at the antisymmetric port represents a photodetector. Coordinate axes (x and y) are also shown.

![Figure 1](image_url)

**Figure 1.** Schematic view of a synchronous recycling interferometer. A recycling cavity is formed by a recycling mirror (RM), a transfer mirror (TM), and two end mirrors (EM1 and EM2). A Sagnac part is formed by a beamsplitter (BS), the recycling cavity, and two steering mirrors. PD at the antisymmetric port represents a photodetector. Coordinate axes (x and y) are also shown.

The laser light is incident on the interferometer from the symmetric port of the Sagnac part. Then the light beam is divided into two directions by the BS before it is incident on the recycling cavity. Those two beams (carrier) are resonant within the cavity; one of the carriers is resonant in the counter-clockwise (CCW) direction, while the other is resonant in the clockwise (CW) direction. The reflected light from the recycling cavity is recombined at the BS. Each light beam experiences the same compound complex amplitude reflectance at the RM, unless a gravitational wave has come. Then the two beams interfere destructively at the antisymmetric port, so this port is called ‘dark port’.

By contrast, when a gravitational wave at a specific frequency has come, a part of the incident light leaks out to the dark port. Therefore we can detect such a gravitational wave by observing the dark port.

2.2. Response of the interferometer to gravitational waves

The synchronous recycling interferometer has the maximum response to a gravitational wave at a specific frequency $f_{GW} = \nu_{FSR}$, where $\nu_{FSR}$ is the free-spectral range of the recycling cavity.

If a gravitational wave propagates in the vertical direction (the normal direction to the cavity’s plane), and in plus polarization (the coordinate axes are shown in Figure 1), the gravitational wave modulates the phases of the both carriers being resonant in the recycling cavity in CW and CCW, respectively. In other words, the gravitational wave produces signal sidebands (at $\nu_0 \pm f_{GW}$) on each carrier (at $\nu_0$). Assume that the phase of the gravitational wave is zero at a certain initial time $t = 0$. At $t = 1/(2\nu_{FSR})$, both carriers arrive at the TM; the CCW carrier has experienced a positive phase shift, while the CW carrier has experienced a negative phase shift.
shift, because a gravitational wave is a quadrupole radiation. Then the TM reflects each carrier, at the same time, the phase of the gravitational wave is reversed. At $t = 1/(\nu_{\text{FSR}})$, both carriers come back to the RM; the CCW carrier has experienced again a positive phase shift, while the CW carrier has experienced again a negative phase shift. As a result, the signal sidebands on both carriers are opposite in sign to each other, and resonantly enhanced inside the cavity. At the dark port, such the signal sidebands interfere constructively, and thus appear.

The light at the dark port is: $\propto \int_{-\infty}^{\infty} \hat{h}(f) G_{\text{sync}}(f) e^{2\pi i f t} \, df$, where $\hat{h}(f)$ is a Fourier component of the gravitational wave, and $G_{\text{sync}}(f)$ is the response function of the interferometer. The magnitude of the response function is written as [5]

$$|G_{\text{sync}}(f)| = \frac{t_r^2 r_c \nu_0}{(1 - r_r r_c)^2} \frac{2 \sin^2(\pi f/(2\nu_{\text{FSR}}))}{\sqrt{1 + N^2 \sin^2(\pi f/\nu_{\text{FSR}})}}$$

where $r_r$ and $t_r$ are amplitude reflectance and transmittance of TM, EM1, and EM2; $N$ is the folding number for the cavity defined as $N = 2\sqrt{r_r r_c}/(1 - r_r r_c) \simeq 2\mathcal{F}/\pi$, where $\mathcal{F}$ is known as the finesse. The response function is maximized at $f = \nu_{\text{FSR}}$.

### 2.3. Shot noise limited sensitivity

The synchronous recycling interferometer has the shot noise limited sensitivity written as

$$h_{\text{shot}}(f) = \frac{2\hbar \Omega_0}{\eta P_0 2|G_{\text{sync}}(f)|}$$

for plus-polarized gravitational waves in the vertical direction, where $P_0$ and $\Omega_0 = 2\pi \nu_0$ are the power and angular frequency of the laser light source, respectively; $\hbar$ is the reduced Planck constant; $\eta$ is the quantum efficiency of the photo diode at the dark port. The highest sensitivity is achieved at $f = \nu_{\text{FSR}}$.

Now our purpose is to detect 100 MHz gravitational waves, so the free-spectral range of the cavity $\nu_{\text{FSR}}$ is tuned at 100 MHz. In other words, we construct a recycling cavity with 75-cm baseline (or 3-m round-trip length) for each interferometer. Figure 2 shows the calculated sensitivity using the parameters as follows: $P_0 = 0.5\, \text{W}$, laser wavelength $\lambda_0 = 1064\, \text{nm}$, $\eta = 1$, $r_r^2 = 99.999\%$, $t_r^2 = 1 - r_r^2$, and $r_c^2 = 99.994\%$. Then the finesse $\mathcal{F}$ and the center sensitivity $h_{\text{shot}}(f = \nu_{\text{FSR}})$ are calculated as $4.5 \times 10^4$ and $4.7 \times 10^{-21} \text{Hz}^{-1/2}$, respectively. Note that we now construct the interferometers in air as described later, so the actual finesse of each cavity is designed to be around 100. The sensitivity for an arbitrary direction is described in our previous paper [5].

**Figure 2.** The shot noise limited sensitivity of a synchronous recycling interferometer (tuned at 100 MHz) is calculated. Power and wavelength of the source are assumed to be 0.5 W and 1064 nm, respectively. The sensitivity at 100 MHz is calculated to be $4.7 \times 10^{-21}\, \text{Hz}^{-1/2}$. 
3. Signal extraction schemes

We need to extract two kinds of signals. First, it is required to extract the signal sideband induced by a gravitational wave and convert it into an electrical signal. Moreover, we have to suppress the fluctuation of the laser frequency relative to the recycling cavity length with feedback control. In other words, it is required to extract an error signal for the control. Note that the dark port is kept in a dark fringe without any controls, unlike a Michelson interferometer. Thus there is only one degree of freedom to be controlled.

3.1. Recycling cavity error signal

The laser frequency is locked to the recycling cavity length with the Pound-Drever-Hall technique [6]. For this purpose, the carrier is phase modulated at radio frequency (RF) to produce RF sidebands at \( \nu_0 \pm f_{RF} \) before it is incident on the interferometer. The error signal is obtained from in-phase signal of the PD at the symmetric port (see Figure 3). This is because the round-trip period \( 1/\nu_{FSR} \) is much shorter than the timescale in which the cavity mirrors (i.e. RM, TM, and two EMs) change their longitudinal positions. Therefore both of the two carriers experience the same amount of optical phase shift, although two carriers (CCW and CW) are resonant within the identical recycling cavity. Thus the sidebands produced by these mirror motions appear in the symmetric port, like a Fabry-Perot cavity.

3.2. Signal sideband downconversion

We convert the original signals at around \( \nu_{FSR} \) (\( \approx 100\) MHz) to electrical signals in the audio frequency range (see Figure 4). The original signals are too fast to be sampled with an available data acquisition system. On the other hand, the signal bandwidth \( \Delta \nu \) of the interferometer is rather narrower than 100 MHz if the recycling cavity has a high finesse. Note that the signal bandwidth is determined by the finesse as follows: \( \Delta \nu = \nu_{FSR}/F \). Thus the relation \( \Delta \nu \ll \nu_{FSR} \) is derived for a high finesse cavity.

Assume that a gravitational wave at the frequency of \( f_{GW} \) (\( \approx \nu_{FSR} \)) has come. Then it produces signal sidebands at \( \nu_0 \pm f_{GW} \) in the interferometer. Meanwhile, the optical local oscillator (1st LO) at \( \nu_0 \pm f_{RF} \), which is also required for the cavity lock, can appear at the dark port if the BS has unequal splitting ratio. The beat between the 1st LO and signal sidebands produces a signal at \( f_{GW} - f_{RF} \), since the dark port PD has a resonant circuit at \( \nu_{FSR} - f_{RF} \).

The signal is mixed with an electrical LO (2nd LO) at \( \nu_{FSR} - f_{RF} - \Delta \nu/2 \) and then low-pass filtered. As a whole, a gravitational-wave signal at \( f_{GW} \) is downconverted to an electrical signal at \( \nu_{FSR} - f_{GW} + \Delta \nu/2 \). Note that the original signals at \( f_{GW} = \nu_{FSR} - \Delta \nu/2 \) and \( \nu_{FSR} + \Delta \nu/2 \) are converted to the electrical signals at 0 Hz and \( \Delta \nu \) Hz, respectively.

4. Current status of the experiment

4.1. Experimental setup

The current experimental setup of one of the interferometers is illustrated in Figure 3. Two such interferometers have been constructed to perform a cross-correlation analysis. They are co-aligned, and located about 12 cm apart. Hereafter they are referred to as Detector-1 and -2, respectively.

Each light source is a Nd:YAG laser, which has a wavelength of 1064 nm and an output power of 0.5 W. In Figure 3, QWP, HWP, EOM, and FI indicate a quarter-wave plate, a half-wave plate, an electro-optic modulator, and a Faraday isolator, respectively. The EOM between the laser source and the Sagnac part is used to introduce RF sideband (\( f_{RF} = 85.4 \) MHz). The output from a RF signal generator is split and supplied to Detector-1 and -2. The length \( L \) between RM and EM1 is about 75 cm, and the round-trip length of the recycling cavity is about 3 m. The laser power at each dark port was measured to be about 15 mW, that is, about 3% of the incident light.
Both of the two interferometers were constructed in air. The amplitude reflectance of the RM is designed to be 98.5% (nominal) such that the cavity finesse is around 100. In the future, we will construct both of the two interferometers in vacuo to improve their finesse.

4.2. Calibration
We need to perform a calibration, that is, relating an output voltage of an interferometer and an incident gravitational wave. First, a broadband EOM is put within the recycling cavity; the gravitational wave is simulated by phase modulation with this EOM at around 100 MHz. The output voltage from a RF signal generator is split and supplied to these EOMs in detector 1 and 2, using cables with the same length. Then we measure the response function from the phase-modulation index to the output of the interferometer. As a result, it is possible to estimate a calibration factor which relates the output of the interferometer and the gravitational wave.

At the dark port, the relation between the phase-modulation index $\Gamma$ with this EOM and the amplitude $h$ of a gravitational wave incident in the vertical direction is written as

$$\frac{h}{\Gamma(f)} = \frac{t_{BS}^2 - r_{BS}^2 \exp(i\pi f/\nu_{FSR} \cdot l_1/L) \exp(-i2\pi f/\nu_{FSR})}{2\Gamma \sin(\pi f/(2\nu_{FSR})) (r_{BS}^2 + t_{BS}^2) \exp(-i\pi f/\nu_{FSR})},$$

where $r_{BS}$ and $t_{BS}$ are amplitude reflectance and transmittance, respectively; $l_1 = 25$ cm is a distance from RM to the center of this EOM. When $f = \nu_{FSR}$, the response $h/\Gamma$ is maximized if $l_1 = L$, while it is minimized if $l_1 = 0$ or $2L$.

4.3. Current sensitivity to gravitational waves
The current sensitivities of the interferometers to the gravitational waves propagating in the vertical direction were measured and shown in Figure 5. For Detector-1, the sensitivity was about $6.4 \times 10^{-17}$ Hz$^{-1/2}$ at 100.1 MHz with finesse of 199. For Detector-2, the sensitivity was about $8.5 \times 10^{-17}$ Hz$^{-1/2}$ at 100.2 MHz with finesse of 142. In particular, Detector-1 now has the highest sensitivity for detecting gravitational waves at around 100 MHz.

The current sensitivity of each interferometer is limited by shot noise. However the sensitivity is worse than a theoretical limit. This is due to little amount of 1st LO at the dark port. Now we are using a BS designed to be $r_{BS} = t_{BS} = 1/\sqrt{2}$ (nominal), thus there should be no 1st LO

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**Figure 4.** Downconversion scheme. The PD with a resonant circuit is put at the dark port, see Figure 3. Signal sideband at $\nu_0 \pm f_{GW}$ and 1st LO at $\nu_0 \pm f_{RF}$ are incident on the PD. PD output is an electrical signal at $f_{GW} - f_{RF}$. This signal is mixed with 2nd LO at $\nu_{FSR} - f_{RF} - \Delta/2$ then low-pass filtered to be the output signal at $\nu_{FSR} - f_{GW} + \Delta/2$.

**Figure 3.** Schematic view of the 100 MHz gravitational-wave detector.
for the signal downconversion scheme in principle. In fact, the alignment of the Sagnac part is slightly tuned so that small amounts of 1st LO appear at the dark port. This BS will be replaced by a new one having an asymmetric splitting ratio so that the sufficient amounts of 1st LO appear at the dark port. With the current experimental configuration, the theoretical limit on the strain sensitivity would be about one order of magnitude lower. We expect that the sensitivity would be improved by using a new BS.

We are now performing a cross-correlation analysis for the output data of the interferometers. The sensitivity of our detector will be improved due to this signal processing.

5. Conclusions
We are developing a detector for 100-MHz gravitational waves. The detector consists of two synchronous recycling interferometers. Now the two interferometers are constructed in air. The output noise of each interferometer is estimated to correspond to less than $1 \times 10^{-16}$ Hz$^{-1/2}$, which is the highest sensitivity for detecting gravitational waves at around 100 MHz. We are now performing a cross-correlation analysis to improve the sensitivity of our detector.

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Figure 5. Strain sensitivities of Detector-1 and -2 to gravitational waves were measured at around 100 MHz. Red (solid) and green (dashed) curve are for Detector-1 and -2, respectively.