ARE THERE DYNAMICAL LAWS?

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Abstract

The nature of a physical law is examined, and it is suggested that there may not be any fundamental dynamical laws. This explains the intrinsic indeterminism of quantum theory. The probabilities for transition from a given initial state to a final state then depends on the quantum geometry that is determined by symmetries, which may exist as relations between states in the absence of dynamical laws. This enables the experimentally well confirmed quantum probabilities to be derived from the geometry of Hilbert space, and gives rise to effective probabilistic laws. An arrow of time which is consistent with the one given by the second law of thermodynamics, regarded as an effective law, is obtained. Symmetries are used as the basis for a new proposed paradigm of physics. This gives rise naturally to the gravitational and gauge fields from the symmetry group of the standard model, and a general procedure for obtaining interactions from any symmetry group.

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1 The Paradigm of Laws

At the end of the millennium we are faced with, what may be the most difficult intellectual challenge known to humankind, the problem of constructing a quantum theory of gravity that would unify all the interactions. The great difficulty of this problem is underlined by the fact that it has remained unsolved for more than seven decades. There is another profoundly difficult problem at the heart of quantum theory, which has also remained unsolved for the same period of time, namely the quantum measurement problem. This article will suggest that the present insolubility of both problems may have the same source, namely the paradigm of physics that has been used since the creation of physics about four centuries ago. I shall consider now, briefly, the origin of this paradigm, and argue that it is based on the metaphysical assumption of the existence of dynamical laws, which may be discarded.

Prior to the origin of physics, ‘explanations’ of the world were often attempts to describe the boundary conditions, such as how the world came into being. But some natural philosophers recognized that much more progress could be made in ‘understanding’ the world if we gave up trying to explain boundary conditions and instead described the universal regularities that seem to occur in contingent phenomena. They tried to predict what would happen if certain arbitrary initial conditions were specified, instead of explaining these initial conditions (see, for example, Wigner, 1967, p. 40). The tool for making this prediction is called a dynamical law of physics. More precisely, a dynamical law of physics, which I shall call a law for simplicity in this paper, may be defined as the ability to describe the initial state of a physical system from which the final state can be predicted, deterministically or probabilistically, using the nature of the system and its interaction with its environment.

This profound realization led to the origin of physics. The combination of contingencies, reflected in the boundary conditions or initial conditions, and universal laws that are independent of contingencies has been the paradigm of
physics for four centuries. This will be called the paradigm of laws. It is a curious
fact, however, that the laws we use are always associated with symmetries. This
follows from the fact that the experiments allowed by these laws are reproducible
in different places and times, with different orientations etc. (Wigner, 1967, p. 29).
Moreover, the symmetries of the laws may be thought of as determining
the physical geometry (Anandan, 1980a). Conversely, we may start with the
geometry and conclude that the laws are constrained by the symmetries of the
geometry.

Now, a law of physics is a strange type of necessity that should be confirmable
or refutable (i.e. testable) in order for it to carry information about the world.
So it cannot be a logical necessity, which is tautological and therefore is not
refutable. Belief in such a metaphysical necessity of laws smacks of the belief in
a supernatural agency that is always regulating natural phenomena.

I shall therefore explore the view that there is no such metaphysical necessity,
which implies that there are no fundamental laws of physics. Pierce (1891),
Wheeler (1980, 1985, 1990, 1994), and Smolin (1997) have suggested that the
laws of physics are mutable. But here I shall question even the very existence of
laws, and propose a new paradigm in which ‘dynamical laws’ will be replaced by
symmetries as the fundamental relational structures in the world. Van Fraassen
(1989) has made an empiricist critique of any kind of structural realism such as
laws or symmetries, but regards symmetries as a better clue to theorizing than
laws. While I also do not regard laws as real, I shall however regard symmetries
as real to be associated with the quantum geometry which gives rise to all
the observed interactions, analogous to how space-time geometry gave rise to
gravitation in classical general relativity.

The nonexistence of laws imply that there can be neither deterministic laws
(sections 2,3,4) nor fundamental probabilistic laws (section 5). The probability
rule of quantum theory will be deduced, in section 5, from the quantum geometry
and the associated symmetries, and therefore will not have the status of a
fundamental law. This approach naturally gives an arrow of time (section 3). In
section 6, the modular energy-momentum of Aharonov et al (1969) will be used
to characterize the four fundamental interactions in terms of the symmetry group
of the standard model. This is then generalized to a new principle which gives
the interactions corresponding to the symmetry group of any physical theory. In
section 7, I shall generalize the celebrated Erlanger program of Klein (1872) for
constructing and defining a geometry as the set of properties that are invariant
under a symmetry group acting on a set of points, by giving up the require-
ment that there should be such a set in quantum theory. The advantages of
the present approach over other interpretations of quantum theory are pointed
out in section 8. The relevance of this approach to the construction of quantum
gravity is discussed in the concluding section 9.

2 Processes and Phenomena

Throughout this article, the term ‘state’ refers to a state in which an individual
system has been observed to be. It is not a state that has been constructed
statistically by measurements on an ensemble of identical systems, as it is in
the Copenhagen interpretation of quantum theory. The justification for this
viewpoint and a precise mathematical description of the state, based on possible
observations of an individual system, will be given in section 4.

The absence of deterministic laws implies that there need not be a unique
time evolution for any physical system. This will now be stated as an assump-
tion, called \( A_1 \).

\( A_1: \) Physical systems that start from the same initial state need not end up in
the same final state under the same experimental conditions.

Define therefore

a \textit{phenomenon} to be a sequence of pairs \( \{(\alpha, \beta_1), (\alpha, \beta_2), \ldots (\alpha, \beta_n), \ldots\} \) in an
experiment such that \( \alpha \) is the common initial state of identical systems and \( \beta_i \)
are their final states under the same experimental conditions. The $\beta_i$s need not all be distinct. Each pair $(\alpha, \beta_i)$, regarded as an element of a phenomenon, will be called a trial. Distinct trials will be called *processes*. In the same experiment and for the same system, there can be trials with an initial state $\alpha'$ that is distinct from $\alpha$, but these would then belong to a different phenomenon. (This remark will be relevant to the thought experiment that will be considered in the next section.)

We know that the observation of quantum phenomena is consistent with $A_1$, although no attention was paid to quantum physics in my critique of the metaphysical assumption that there are laws that led to $A_1$. An example is a Stern-Gerlach experiment with neutrons in which the inhomogeneity of the magnetic field is in the $z-$direction, and each incoming neutron is observed as it enters the apparatus to have its spin in the $x-$direction, which corresponds to the initial state $\alpha$. The neutrons then end up in the two possible final states, $\beta_1$ and $\beta_2$, at the two spots on the screen, corresponding to the spin up state and the spin down states with respect to the $z-$direction.

By the *probability* of a process $(\alpha, \beta)$ is meant the relative frequency of this pair in a phenomenon as the number of trials tends to infinity under the same experimental conditions. Since all trials in the phenomenon have the same initial state, this is the conditional probability of observing the system in $\beta$ if it was previously observed in state $\alpha$. These probabilities are well defined in the sense that the probability of a process is the same for phenomena that have the same initial state as the given process under the same experimental conditions. A *symmetry* is a transformation on the set of states with the property that the probabilities of processes are invariant under this transformation. Later I shall associate symmetries *a priori* with the quantum geometry which will then *explain* why processes have well defined probabilities and why these probabilities are invariant under symmetries.

The apparent existence of deterministic laws in classical physics is a mystery,
given $A_1$. But this mystery is removed by the realization that the phenomena that make up classical physics are limiting cases of quantum phenomena that correspond to the probability of some processes being much greater than others. This gives the illusion of deterministic laws in classical physics. E.g. suppose a baseball or cricket ball is thrown and the positions of its center of mass are observed to some uncertainty in a series of measurements. These positions are very likely to be on an approximate parabola. But there is no law compelling them to lie on a parabola as was mistakenly assumed in classical physics.

3 The Arrow of Time

In general, a phenomenon is not time-reversible. This may be illustrated by an example due to Penrose (1989, p. 357). He considered photons emitted from a lamp in a state, denoted here by $\alpha_1$, that is aimed at a detector. Between the lamp and the detector there is a half-reflecting mirror inclined at an angle, say $45^0$, to the line connecting the lamp to the detector. The probability of transmission or reflection of the photon by the glass is $\frac{1}{2}$ for incidence on either side. Then as the number of photons emitted in state $\alpha_1$ tends to infinity, half of them are detected in state $\beta_1$, say, by the detector, while the other half, reflected by the mirror, strike a wall where they are observed in a state $\beta_2$, say. So this experiment gives a phenomenon such as $P \equiv \{(\alpha_1, \beta_1), (\alpha_1, \beta_2), (\alpha_1, \beta_2), (\alpha_1, \beta_1), \ldots \}$. This is consistent with $A_1$ above. Now Penrose points out that while the probability of $\beta_1$ given $\alpha_1$ is $\frac{1}{2}$, the probability of $\alpha_1$ given $\beta_1$ is 1. I.e. the probability of the process $(\alpha_1, \beta_1)$ forwards in time is not the same as the probability of this process backwards in time. More generally, it follows from the above definition of a phenomenon that the probabilities forwards and backwards in time of any process in it are not the same unless the phenomenon is such that all the final states $\beta_i$ are the same.

The time-reversal of the Penrose phenomenon $P$ above is the sequence
\( P^* \equiv \{(\beta_1^*, \alpha_1^*), (\beta_2^*, \alpha_1^*), (\beta_2^*, \alpha_1^*), (\beta_1^*, \alpha_1^*), \ldots\} \), where the * denotes the time-reversal of the unstarred state (in the present case * changes absorption to emission and vice versa). The subsequence of \( P^* \) that consists of pairs with the same first component \( \beta_1^* \) is \( P_1^* \equiv \{(\beta_1^*, \alpha_1^*), (\beta_1^*, \alpha_1^*), (\beta_1^*, \alpha_1^*), \ldots\} \). If \( P_1^* \) were a phenomenon then the probability of the process \((\beta_1^*, \alpha_1^*)\) would be 1. But \( P_1^* \) cannot take place in the forward time direction if we assume rotational symmetry of \( P \). To see this, simply rotate the apparatus about the axis perpendicular to the plane of the apparatus through the middle of the half-reflecting mirror by 180° so that the positions of the lamp and the detector are interchanged. The assumption now that \( P \) is invariant under this rotation implies that we could have the phenomenon \( P_R \) obtained from \( P \) by the above mentioned rotation, i.e.
\[
P_R = \{(\beta_1^*, \alpha_1^*), (\beta_2^*, \alpha_2^*), \ldots\},
\]
with each of the two processes having probability 1/2, where \( \alpha_2^* \) is the state of the photon as it is being absorbed, after reflection by the mirror, in the wall opposite to the wall on which \( \beta_2 \) is absorbed. So \( \beta_1^* \) does not always end up as \( \alpha_2^* \) as required by \( P_1^* \). Therefore, \( P_1^* \) is not allowed in the forward time direction. Hence, by observing \( P \), we can determine the direction of time. Another way of saying this is that if a movie of this experiment is taken, then when the movie is viewed we can say whether the movie is running forwards or backwards, which therefore gives an arrow of time.

It may be argued that a photon may be emitted, albeit with very low probability, in the state \( \alpha_2 \), that is the time reversal of \( \alpha_2^* \), from the corresponding wall, which may then end up as \( \beta_1 \) or \( \beta_2 \) with equal probability. If this occurs then \( P_1^* \) should be modified to include the process \((\beta_1^*, \alpha_2^*)\). But this process has a very low probability, close to 0, in \( P_1^* \), whereas in \( P_R \) it has probability 1/2. So, rotational symmetry again prevents \( P_1^* \) being a phenomenon in the forward time direction. This consideration shows that if all possible initial and final states are included, the violation of time reversal symmetry is due to the non invariance of probabilities under time reversal.

Each phenomenon distributes systems in the same or similar original states
to different states with higher entropy, in general. And the time-reversal of a phenomenon which would lead to decrease in entropy, in general, cannot occur. Hence, the above arrow of time is the same as the arrow of time given by the second law of thermodynamics. Here the second law of thermodynamics is regarded as an effective law and not a fundamental law. By this I mean that the second law of thermodynamics is a statistical statement about the outcomes in a large number of experiments, none of which is governed by a law within the framework of quantum theory. The inherent probabilities in the individual processes, it will be argued later, in section 5, are the outcomes of the quantum geometry that is determined by symmetries. So, even this statistical statement is not a fundamental law.

This appears to resolve at least partially the well known paradox which arises in the paradigm of laws due to the fact that the microscopic laws have time-reversal symmetry (ignoring weak interactions which are not relevant to this problem which arises even in the absence of weak interactions), and yet there is an observable direction of time. In the present approach, because there are no laws, this problem does not arise.

The time-irreversibility mentioned above should be distinguished from the ‘irreversible’ amplification which occurs when a measurement is made on the system, which Penrose (1989) calls the \textit{R}-process (stands for the ‘reduction of the state vector’). The latter irreversibility is associated only with the measurement or observation of each of the initial and final states of an individual system, whereas the former irreversibility is associated with both the initial and final states in a phenomenon, which is given physical meaning by means of an ensemble.

\footnote{To quote Wheeler (1980), “Ask any molecule what it thinks about the second law of thermodynamics and it will laugh at the question. All the same the molecules, collectively, uphold the second law.”}
4 Meaning of States And Processes

The question naturally arises as to what exactly are the initial and final states that were referred to in the previous two sections. An operational meaning may be given to states from the fact that it is possible to observe the extended wave function of a single system by means of protective measurements, which were introduced by Aharonov, Anandan and Vaidman (1993). For example if the system is in a non-degenerate eigenstate of the Hamiltonian, then an adiabatic measurement on it, which does not lead to entanglement between the system and the apparatus, would be a protective measurement. This protective observation therefore shows that an extended wave function is real. But it is not necessary to assume the Hilbert space structure, including wave functions, in doing protective measurements. One may begin with the set of physical states, or simply states, of a physical system as an abstract set without any structure on it, as in the present approach. Then, as shown by Anandan (1993a), from the numbers that an experimentalist could in principle obtain from protective measurements on these states for various observables, the entire Hilbert space structure may be constructed; the states are then rays in this Hilbert space and the numbers obtained by the experimentalist are of the form $<\psi|A|\psi>$, where $|\psi>$ is a normalized vector in the ray representing the state and $A$ is a Hermitian operator now representing the observable that was protectively measured. In the latter approach, due to the absence of the Hilbert space structure at the beginning, it is not possible to determine the state by calculation prior to the protective measurements, as one could do if there were a Hilbert space structure and one knows the protecting Hamiltonian.

Moreover the following purely empirical criterion may be used to ensure and recognize that a given experiment is a protective measurement and is not the usual measurement: If we make repeated measurements on a single system of any pair of arbitrary observables $A, B$ which are alternated indefinitely (i.e. measure $A, B, A, B, ...$), then only if the measurements are protective are the
same values always obtained for $A$ and $B$ (which are $\langle \psi | A | \psi \rangle$ and $\langle \psi | B | \psi \rangle$ respectively). In order for this to be the case for the usual measurements it would be necessary for $A$ and $B$ to commute. This empirical criterion does not require any knowledge of the Hamiltonian. Therefore, this criterion and the observation at the end of the previous paragraph refute the criticism that one could obtain the wave function by calculation if the Hamiltonian is known and therefore no new information is obtained by protective measurements. Hence, protective observations of states have the same epistemological status as the usual measurements, and have the advantage over the latter of establishing the ontology of the states.

I shall therefore take the possible initial or final states of the system (i.e. the $\alpha$s and $\beta$s in the sections 2 and 3) to be any rays in the Hilbert space of the system. This is because the theorem in the first paragraph of this section says that the Hilbert space may be constructed from protective measurements and, conversely, any ray in the Hilbert space may be protectively observed, in principle. For example, suppose an electron in an atom is in an excited state of energy, and decays to the ground state. Then both its initial and final states may, in principle, be protectively observed. So in the present approach there is no ‘preferred basis’.

By two states being orthogonal I mean that any pair of vectors belonging to the rays representing these states is orthogonal. The final states that are not the same (i.e. distinct) are mutually exclusive and are assumed to be orthogonal, which is contained in the second assumption:

$A_2$. Any two distinct possible final states of a phenomenon are orthogonal.

The essential reason for the time irreversibility of a phenomenon that has distinct final states, mentioned in section 2, can now be understood in the usual quantum theory as due to the unitarity of time evolution. This has been called the $U$–process by Penrose (1989). This implies that orthogonal states must necessarily evolve into orthogonal states. Hence it is not possible for identical
systems beginning from different initial states to evolve always to the same final state. Therefore, if measurements are made to determine the final state for systems having two possible initial states we cannot always obtain the same final state. But if a phenomenon such as \( P \) in section 2 is time reversed, then systems in distinct initial states will always go to the same final state, which violates unitarity. So, the time-reversal of a phenomenon cannot be realized, except in the special case when the experimental conditions are such that only one final state is possible. Unitary evolution preserves information. But when there is one initial state and more than one final states for an ensemble of systems, as it is for a phenomenon in general, information is lost. This loss of information is equivalent to an increase in entropy as mentioned in the previous section.

Within the paradigm of laws there is now the following paradox: Suppose a particle's state is protectively observed assigning to it an extended wave function \( \psi_1 \), up to an arbitrary phase factor and gauge transformations if the particle is charged. This direct observation of the wave function implies that it is real. Suppose now that a usual measurement is made on the system and the system is found to have a localized wave function \( \psi_2 \), which is equally real. How do we account for this sudden transition from \( \psi_1 \) to \( \psi_2 \)?

One may try to explain this as being due to a dynamical ‘collapse’, i.e. a transition which is governed by some law (Pearle, 1986; Ghiradi, Rimini and Weber, 1986; Pearle, 1989; Diosi, 1989; Ghiradi, Grassi and Rimini, 1990; Penrose, 1996). But dynamical collapse models generally have the following three problems: a) They violate energy-momentum conservation. Even if this happens so rarely that it is unnoticed, it is unnatural nevertheless from the present point of view which gives great importance to symmetry principles and the associated conserved quantities. b) If the wave function is charged we would expect it to radiate as it ‘collapses’ from \( \psi_1 \) to \( \psi_2 \). c) It has not been possible, as far as I know, to construct a satisfactory Lorentz covariant model of the ‘collapse’. But these problems and the above paradox disappear if we give up the notion
of a law for an individual system.

The above considerations show the problems which arise if we require that each individual system obeys the $U$-process. For the individual system what is observed is the $R$-process and not the $U$-process. (In protective measurement the $U$-process is observed for the individual system but this is in the trivial case where the $U$ process does not change the state.) Hence if there is a conflict between the two processes then we should give up the $U$-process for the individual system. The wave function being associated with a single system by protective measurements and the observed $R$ process that the wave function of this system undergoes suggest that the transition from the initial to the final wave function of the individual system is not governed by any law.

On the other hand, for an ensemble of particles which begin with the same initial state, the $U$-process can be observed. E.g. in the Stern-Gerlach experiment mentioned in section 2, the superposition of the two wave packets that are obtained from the $U$-process can be observed for the entire ensemble of neutrons by the two spots they form on the screen. When an individual neutron undergoes the $R$-process into one of the spots, it is not necessary to ‘collapse’ the wave function if it represents the entire ensemble. Alternatively, we can observe the $U$-process by measuring an Observable, one of whose eigenstates is the state obtained from the initial state by the $U$-process. Then all the systems in the ensemble which had the same initial state will be found in the same final state that is obtained from the $U$-process. But again, to verify this we need to make measurements on an ensemble of a large number of copies of the system. Hence I shall associate the $U$-process with an ensemble of systems with the same initial state and the $R$-process with the individual system.
5 Probabilities of Processes

Assumption $A_1$ implies that there are no deterministic laws. This still leaves open the possibility of there being probabilistic laws. I shall now argue even against probabilistic laws as fundamental laws. To do so, it is necessary to derive the experimentally well confirmed probabilities that were postulated in quantum theory from something more fundamental, which I shall do now. The answer proposed here to Einstein’s famous question as to why God plays dice with the universe is that an individual system is forced to obey only logical necessity and therefore it cannot obey any dynamical law that is not a logical necessity. There can, nevertheless, be well defined probabilities as a consequence of symmetries.

Consider, as an example, the tossing of a coin. The equal probabilities of heads and tails are due to the symmetry between the two sides, and are independent of the particular law which governs the falling of the coin. This therefore suggests (but does not prove) that the equal probabilities may exist as a consequence of the symmetry alone and may be independent of whether there is a law. A classical coin obeys (apparent) dynamical laws and the indeterminacy of outcomes is due to our ignorance of the initial conditions while the equal probabilities are due to the symmetry of the coin and of the possible initial conditions with respect to the possible final outcomes. But for a quantum system, due to the absence of law in the present approach, there is an intrinsic indeterminacy. Nevertheless, there are well defined probabilities which may be regarded as due to the symmetries of the initial state with respect to the possible final states, as I shall show now.

Consider first the Stern-Gerlach experiment, mentioned in section 2. The initial wave function $\psi$ of the neutron is an equal superposition of the two normalized wave packets $\psi_\uparrow$ and $\psi_\downarrow$ that end up in the two spots with probability $1/2$, i.e. $\psi = \frac{1}{\sqrt{2}}\psi_\uparrow + \frac{1}{\sqrt{2}}\psi_\downarrow$. Consider the two dimensional Hilbert space spanned by $\psi_\uparrow$ and $\psi_\downarrow$. The set of rays or the projective space of this Hilbert space is a
sphere with the Fubini-Study metric (Kobayashi and Nomizu, 1969; Anandan and Aharonov 1990; Anandan, 1991) being the usual metric on this sphere. Also, $\psi_\uparrow$ and $\psi_\downarrow$ correspond to opposite points of this sphere and $\psi$ is half-way between them on a geodesic connecting them. This is somewhat analogous to the symmetry of the coin discussed above. It is reasonable therefore to suppose that $\psi$ has equal probabilities of going into $\psi_\uparrow$ and $\psi_\downarrow$.

Given an arbitrary quantum system, there is a group of unitary and anti-unitary transformations which act on its Hilbert space. Physically, this group may be thought of as relating states that belong to the same physical system and preserving mutual exclusivity (represented by orthogonality) of the distinct states among the final states of a phenomenon as required by $A_2$. The properties that are invariant under this group, which include the Fubini-Study metric (Anandan and Aharonov, 1990) may therefore be regarded as a geometry in the sense of Klein’s Erlanger program (Klein, 1872). These properties are of course also invariant under the universal symmetry group of the fundamental interactions which is a subgroup of this meta-symmetry group of unitary and anti-unitary transformations.

Define the distance between two states to be the Fubini-Study length of the shortest geodesic joining them. The distance $s$ between two states that are represented by normalized state vectors $|\psi\rangle$ and $|\psi'\rangle$ is then given by

$$\cos \frac{s}{2} = |\langle \psi | \psi' \rangle|,$$

where $s \in [0, \pi]$. Here I have scaled the Fubini-Study metric so that the sphere that is the projective space of the Hilbert subspace spanned by $\psi$ and $\psi'$ has unit radius. Then the distance between two orthogonal states (opposite points on the sphere) is $\pi$. In the following only the definition of distance given by (1) is used, and no other knowledge of the Fubini-Study metric is needed.

The present work, to use the usual formalism of quantum theory, corresponds to the Heisenberg picture in which the $U$-process governs the observables, while the state is changed only during an $R$-process. Since these time dependent
observables are common to all systems which have the given Hamiltonian, this is consistent with the conclusion above that the $U$-process should be associated with an ensemble of systems while the $R$-process may be associated with the individual system. I now generalize the equiprobabilities in the above Stern-Gerlach experiment to the following assumption:

$A_3$. Suppose a system is in an initial state $\alpha$, which has equal distances to possible final states $\beta_1$ and $\beta_2$. Then the probabilities of transition from $\alpha$ to $\beta_1$ and $\beta_2$ are equal.

In the Heisenberg picture the initial and final states are on the same footing because they are all time independent. Also, in $A_2$, the mutual exclusivity of the final states of a phenomenon was expressed by their orthogonality. This suggests that a state that is orthogonal to the initial state cannot be a possible final state. Similarly, we would expect that a state that is orthogonal to all possible final states must be orthogonal to the initial state. The last two statements are equivalent to the following assumption:

$A_4$. The state vector representing the initial state in a phenomenon can be expressed as a linear combination of the state vectors that represent all possible final states with non zero coefficients.

Probabilities of transition between an arbitrary pair of states will now be obtained using the ‘classical’ or ‘standard’ definition of probability (Gnedenko, 1967; Zurek, 1998). The basic idea here is to derive an arbitrary probability by expressing it in terms of equiprobabilities of equally likely events, which is a primitive concept. In the present case, transitions to states that are equidistant from the initial state have equal probabilities by $A_3$. Consider a phenomenon in which the initial state is represented by $|\psi>$ and all possible distinct final states by $|\psi_i>$, $i = 1, 2, ..., N$. Then from $A_2$ and $A_4$, we can write

$$|\psi> = \sum_{i=1}^{N} c_i |\psi_i>$$

(2)
where without loss of generality $c_i$ are real and positive, and

\[ <\psi_i|\psi_j> = \delta_{ij}, \quad <\psi|\psi> = 1. \]  \tag{3}

First consider the special case

\[ |\psi> = \sum_{i=1}^{N} \cos \frac{\theta}{2} |\psi_i>. \]  \tag{4}

Then (3) implies

\[ N \cos^2 \frac{\theta}{2} = 1. \]  \tag{5}

It also follows from (1) that the state represented by $|\psi>$ has equal distance $\theta$ from all the states represented by $|\psi_i>$. Replacing $|\psi_i>$ by $e^{i\theta_i} |\psi_i>$ in (4) or (2) does not change the distances between states. This is why $A_3$ was stated in terms of distances in the state space. It follows from $A_3$ that the probabilities of transition from $\psi$ to all $\psi_i$ are the same. Since these probabilities must add up to 1, the probability of transition from $\psi$ to each $\psi_i$ is $\frac{1}{N}$. But from (3) and (4)

\[ \frac{1}{N} = \cos \frac{\theta}{2} = |<\psi|\psi_i>|^2 \]  \tag{6}

which is therefore the probability of transition from $\psi$ to $\psi_i$.

Consider now the more general case of (2) in which the $c_i$s need not be equal. Introduce now an auxiliary system whose possible states belong to an infinite dimensional Hilbert space. The state vector of the combined system is the direct product $|\psi>|\phi>$, where the state $|\phi>$ of the auxiliary system does not interact with $|\psi>$ and both state vectors are normalized. The purpose of introducing the auxiliary system is to use the assumption $A_3$ on $|\psi> |\phi>$ in order to derive the the probabilities of transition from $\psi$ to all $\psi_i$. For each positive integer $n_i$, there exist orthonormal state vectors \{ $|\phi_{ij}>, j = 1, 2, ..., n_i$ \} such that

\[ |\phi> = \sum_{j=1}^{n_i} \frac{1}{\sqrt{n_i}} |\phi_{ij}> \]  \tag{7}
Using (7), the state vector of the combined system may be written as

$$|\psi\rangle |\phi\rangle = \sum_{i=1}^{N} \sum_{j=1}^{n_i} \frac{c_i}{\sqrt{n_i}} |\psi_i\rangle |\phi_{ij}\rangle.$$  

Choose the positive integers $n_i$ so that for all $i,j$,

$$\frac{c_i}{\sqrt{n_i}} \approx \frac{c_j}{\sqrt{n_j}}.$$  

If the ratios $c_i^2/n_i$ are rational for all $i$ and a fixed $k$, and $N$ is finite, then there exist positive integers $n_i$s such that $c_i^2 = n_i/M$, where $M$ is a common real denominator. Then (8) is an exact equality. If at least one of these ratios is irrational then, owing to the rational numbers being dense in the real line, the positive integers $n_i$ may be made large enough to make (8) as close as possible to equalities.

Then (8) reduces to the special case considered before in which the coefficients are equal. Therefore, the probability of transition to $|\psi_i\rangle |\phi_{ij}\rangle$ is $c_i^2/n_i (j = 1, 2, ... n_i)$, using (8). Since there are $n_i$ such mutually exclusive states for a given $i$, the probability of transition to the state $|\psi_i\rangle$ is $n_i \times c_i^2/n_i = c_i^2 = |<\psi|\psi_i>|^2$, using (2) and (3). So the quantum probability rule has been derived from the assumptions $A_1$ to $A_4$. A somewhat similar argument has been given by Zurek (1998) using density matrices in the context of decoherence.

It follows that the probability of transition between Heisenberg states $|\alpha\rangle$ and $|\beta\rangle$ is $|<\alpha|\beta>|^2$ irrespective of the time elapsed between the observations of $|\alpha\rangle$ and $|\beta\rangle$. Hence, the converse of $A_3$, above, is also valid.

6 Symmetries and Interactions

If there are no laws then how do we explain the regularities we observe in the world? I shall argue here that the answer lies in symmetries. To make this argument, I shall first use the presently used laws of physics in order to obtain
a characterization of all fundamental interactions in terms of symmetries. This statement will be so elegant and simple that it will suggest that we may discard the laws and keep the symmetries.

First consider the realization of quantum probabilities obtained in Section 5 in a simple experiment, namely diffraction through a narrow slit. In this phenomenon, particles that were initially prepared in the same state, after they go through the slit, end up at various points on a screen. It is well known that, the density of distribution of particles on the screen (the diffraction pattern), which gives the probability density for the possible final states in the limit of large number of particles, has an oscillatory behavior: It has a central maximum where the particles are most likely to strike, and the probability density decreases, as would be expected, as we move away from this point. But then after reaching a minimum, strangely enough, the probability density increases, then decreases again to a minimum and increases again, and so on. This is very different from what would be expected classically, namely a monotonic decrease in probability density as we move away from the central point.

I believe that nature is telling us something very important about quantum geometry even in this simple experiment. It is that given two points $A$ and $B$ separated by a distance $\ell$, the translation operator

$$\exp(i\mathbf{p}\ell),$$

where $\mathbf{p}$ is the momentum (generator of translations) in the direction from $A$ to $B$, is more fundamental than the distance $\ell$ between $A$ and $B$, because the eigenvalues of $\exp(i\mathbf{p}\ell)$ oscillate with $\ell$. It is $\exp(i\mathbf{p}\ell)$ which relates appropriate pairs of quantum states (represented by vectors in Hilbert space), whereas $\ell$ may be regarded as a derived concept. Indeed, $\ell$ regarded as the distance between $A$ and $B$ is an approximate concept because there is inevitably a fuzziness in the determination of points owing to the uncertainty principle, as shown for example by Wigner (1967, p. 62-69). But $\exp(i\mathbf{p}\ell)$, regarded as a translation of any quantum state, has no such fuzziness.
Now (10) is equivalent to the ‘modular momentum’ introduced by Aharonov et al (1969) as a more fundamental variable than the momentum $p$. They showed that this variable captures an essential non local aspect of quantum theory, which makes quantum physics fundamentally different from classical physics. To see this consider the wave packet

$$\psi_\alpha(x, y, z) = \frac{1}{\sqrt{2}} f(x, y, z) + \frac{1}{\sqrt{2}} \exp(i\alpha) f(x - \ell, y, z)$$

(11)

where $f(x, y, z)$ is a normalized wave function such that $f(x, y, z)$ and $f(x - \ell, y, z)$ are non overlapping functions. For example, $\psi_\alpha$ may be the superposition of two wave packets in a double slit interference experiment at the time when they are emerging from the two slits that are separated by a distance $\ell$ in the $x-$direction. Then

$$\langle \psi_\alpha | \exp(i\mathbf{p}\ell) | \psi_\alpha \rangle = \frac{1}{2} \exp(i\alpha).$$

(12)

So the expectation value of (10) gives the phase difference between the two superposed wave packets. What makes this non local is that no experiment performed on the wave packets at the given time could detect this phase difference between the non overlapping wave packets. But $\alpha$ may be observed when they interfere subsequently. This is unlike in classical physics where all experiments performed locally on the two wave packets can predict what will happen subsequently. In classical physics also the translational operator generated by the momentum may be defined on the phase space, which is the classical analog of the Hilbert space, but it contains no more information than the momentum. But in quantum physics, (10) contains more information than $p$, which is non local and observable.

The result (12) may be applied to the momentum distribution in the Aharonov-Bohm (AB) effect on the electron beams interfering around a solenoid due to the magnetic field confined inside the solenoid. Aharonov et al (1969) showed that although there is no exchange of momentum between each electron and the solenoid in the AB effect, for the obvious reason that there is no force on
the electron due to the magnetic field, there is nevertheless a non local exchange of modular momentum between them. The magnetic field causes a shift in the interference fringes due to the exchange of \( \exp(i p \ell) \). But the envelope of the interference pattern does not shift because there is no exchange of any of the moments of momentum \( p^n \) \((n \text{ is any positive integer)}\). Hence, (10) is a more fundamental observable than \( p \). Combining this with the earlier considerations in this section, I conclude that the group element \( \exp(i p \ell) \) is more fundamental than either \( p \) or \( \ell \).

In the presence of the electromagnetic field, \( \exp(i p \ell) \) is gauge dependent. The gauge invariant re-statement of the above is that in the AB effect the modular kinetic momentum \( \exp i(p\ell - e \int_0^\ell A_0 dx) \) is exchanged but not the kinetic momentum \( p - eA \). The last two operators are gauge invariant. Aharonov et al (1969) also showed that when there is a forceless interaction, there could be an exchange of modular energy in the absence of an exchange of energy. Also, the above considerations on the AB effect will also apply to the generalization of the AB effect to arbitrary gauge fields (see Wisnivesky and Aharonov (1967), Anandan(1970)). I therefore generalize the modular kinetic momentum covariantly, in an arbitrary gauge field, to the modular kinetic energy-momentum

\[
\hat{f}_\gamma = P \exp\{i \int_0^\ell (p_\nu - A_\nu^k T_k) dx^\nu \}
\]

where \( T_k \) generate the gauge group, \( A_\nu^k \) is the gauge potential and \( P \) denotes path ordering.

To generalize this further to include the gravitational field, I shall use the principle of equivalence according to which at each space-time point there exists a local Minkowski coordinate system \((x^a, a = 0, 1, 2, 3)\). The kinetic energy-momentum operator in this coordinate system is

\[
\Pi_a \equiv p_a - A_\nu^k T_k.
\]

This may be written in an arbitrary coordinate system as

\[
\theta_\mu^a \Pi_a \equiv \theta_\mu^a p_a - A_\nu^k T_k,
\]

20
where $\theta^a_\mu$ is the canonical 1-form (Kobayashi and Nomizu (1963) or the solder form (Trautman, 1973, Anandan ,1993b), so called because it solders the 1-forms in the local inertial frame to the cotangent space of space-time at each point. The generalization of (13) in the presence of gravity and gauge fields is then

$$g_\gamma = P e^{\int \Gamma_\mu dx^\mu},$$

where

$$\Gamma_\mu = \theta^a_\mu P_a + \frac{1}{2} \omega^a_\mu b M^b_a - A^k_\mu T_k.$$  (15)

Here, $P_a$ generate space-time translations, $M^b_a$ generate Lorentz transformations and $T_k$ generate the gauge group $G$. The $\theta$, $\omega$ and $A$ are defined along the curve $\gamma$. ($\theta$ and $\omega$ should not be thought of as functions of space in the definition of $g_\gamma$ because this would make $g_\gamma$ non unitary.) Hence, $g_\gamma \in$ Poincare group $\times G$.

$\gamma$ may be any open or closed curve. When $\gamma$ is an infinitesimal closed curve spanning an area element $d\sigma^{\mu\nu}$, using (14) and the Lie algebra relations of the Poincare group and the gauge group, we obtain (Anandan, 1980, 1996)

$$g_\gamma = 1 + i(Q^{\mu\nu} a P_a + \frac{1}{2} R^{\mu\nu} b M^b_a + F^{k} M^k T_k) \frac{d\sigma^{\mu\nu}}{2}$$

(16)

where the 2-forms

$$Q^{\mu} = d\theta^a + \omega^a_b \wedge \theta^b, R^{\mu} = d\omega^a_b + \omega^a_c \wedge \omega^c_b, F = dA^k + C_{mn}^k A^m \wedge A^n$$

are called the torsion, linear curvature, and the Yang-Mills field strength, respectively, and $C_{mn}^k$ are the structure constants of the gauge Lie algebra. Whenever two quantum systems interact via gravity and gauge fields, it can be shown that there exist curves $\gamma$ so that $g_\gamma$ is exchanged between the two systems.

This result was obtained from laws which had the above symmetries, at least locally. But since $g_\gamma$ is what is observed $\gamma$, it may be possible to keep

$\text{2}$In practice, we observe Hermitian operators such as $g_\gamma + g_\gamma^\dagger$ and $i(g_\gamma - g_\gamma^\dagger)$ from which of
it and discard the laws. Then symmetries need not be regarded as originating as invariances of laws but instead regarded as relations between states. These relations, and hence the symmetries, are universal in that they have representations in every Hilbert space. (The gauge group part of the symmetries would be trivial in representations in which the gauge charges are zero.) They may therefore be regarded as defining the geometry of quantum theory.

The above considerations suggest the following principle which characterizes all interactions: *Two quantum systems interact if and only if there is an exchange of a symmetry group element between them.* In terms of the usual quantum theory, this means that the expectation value of the symmetry group element for each of the states of the quantum system changes, but not for the state of the combined system. If the symmetry group $S$ is chosen to be that of the standard model, i.e. $S = P \times G$, where $P$ is the Fermionic covering group of the Poincare group (semi-direct product of $SL(2, C)$ with the space-time translational group) and $G = U(1) \times SU(2) \times SU(3)$, then I conjecture that in an appropriate limit the usual gravity and gauge fields are obtained. The appropriate limit means that the systems which produce the gravity and gauge fields may be treated classically. In this limit the group element which is exchanged may be written in the form (14) so that classical gravity and gauge fields may be obtained from (15). But we may need to change $S$ when the standard model is superseded by a deeper theory. The above principle would then still hold and give us a new set of fields corresponding to the new symmetry group. I shall therefore call these fields, corresponding to arbitrary symmetry groups, symmetry fields.

In the above treatment, gravitational and gauge fields were treated classically. When the fields are quantized, the probability amplitude for a process is at present obtained by adding integrals represented by Feynman diagrams. At course $g_*$ may be determined. See Anandan (1986a,b) for a discussion of how these Hermitian operators may be observed in interference experiments.
each vertex of a Feynman diagram the energy-momentum is exchanged. But it may be useful to reformulate quantum field theory in terms of exchange of modular energy-momentum. At distance scales of the order of Planck length, space-time geometry breaks down, and the usual Feynman diagrams may not be meaningful. Also, in (13), \( \gamma \) cannot then be meaningfully defined. However, we may then be able to replace (13) by an element of the symmetry group that does not require a space-time curve \( \gamma \) for its definition. Such a theory would be a quantum theory of gravity. Since quantum gravity is expected to unify all the interactions, the unified treatment of all the fundamental interactions in (13) suggests that it may be useful in constructing quantum gravity.

An advantage of the present approach is that non local interactions, such as the non local exchange of modular momentum between the solenoid and the charge in the AB effect and its generalization to gauge fields and gravitation, is easily treated. But in the AB effect for example, the change in the modular momentum undergone by the charged wave function gives the AB phase shift around a closed curve because of the \( U \)-process undergone by the interfering beams. Also, the \( U \)-process due to Schrödinger evolution undergone by the state vector in the Schrödinger picture or the observables in the Heisenberg picture was associated earlier with ensembles of identical quantum systems because this determines the probabilities of processes of individual systems. The Hamiltonian that generates the \( U \) process comes from a Lagrangian. It may appear that the specification of the Lagrangian constitutes a law. But in fact, the general form of the Lagrangian for a system of non relativistic interacting particles may be obtained from Galilean symmetries (Landau and Lifshitz, 1976; Lawrie, 1994). In relativistic quantum field theory, the fields in the Lagrangian must provide representations of the symmetry group and the Lagrangian is invariant under this group. This is a major constraint on the fields and the Lagrangian.

But even after the fields are specified, there are still an infinite number of Lagrangians that are invariant under the symmetry group. However, the
requirement of renormalizability places a severe restriction on the possible Lagrangians. In the case of the symmetry group $S$ of the standard model, after specifying the fields and the coupling constants (which are the contingencies of this model), the requirement of renormalizability pretty much uniquely determines the Lagrangian of all the fields, excluding gravity, as mentioned by Weinberg (1980). A quantum theoretic description of the gravitational field cannot be obtained this way at present, which I believe to be due to an insufficient understanding at present of the connection between symmetries and the gravitational field. One of the purposes of the present article is to elucidate this connection, which hopefully would help in constructing a quantum theory of gravity. In a future theory, renormalizability may be realized as being due to logical self-consistency of the theory.

I therefore make the following conjecture: Regularities in physical phenomena are due to symmetries, logical or mathematical consistency, and contingencies. The contingencies here include algorithmically undecidable propositions which exist, according to Gödel’s theorem, in any axiomatic system that is at least as rich as the natural numbers. In view of Gödel’s theorem, it may not be possible to construct a physical theory that is due entirely to logical or mathematical necessity. Also, at present it seems possible for the world to have, in principle, symmetries that are different from the ones we observe, or have no symmetries at all, without violating logical or mathematical necessity. So, if we choose a particular symmetry group, in the new paradigm of symmetries proposed here, it would restrict the physical theory to a particular, but it seems contingent, class of mathematical models.

7 Quantum Geometry

There are parallels in physical geometry to the two views of having dynamical laws, or not having dynamical laws but having symmetries. In the former
case, it is natural to adopt the Riemannian geometry of space-time because the
dynamical laws give evolutions which are best described in space-time. E.g.
geodesics are world-lines of free particles in classical general relativity. But it
was pointed out that a different conception of geometry is better suited for
quantum theory (Anandan, 1980a). This is based on Klein’s Erlanger program,
according to which a geometry is a set of properties invariant under a group
of transformations acting on a set of points. It is difficult to take this set to
be space-time, because space-time points are not observable owing to the un-
certainty principle in quantum theory, as already mentioned. Also, it does not
seem plausible to talk of an electron as being immersed in space-time, because
its states belong to a Hilbert space, which has a very different geometry and
interpretation.

It therefore seems reasonable to generalize Klein’s Erlanger program so that
the symmetry group does not act on a universal set of points. Instead, I shall
let the same symmetry group $S$ to simply act on each Hilbert space, which is
the set of possible states of a system. Each $geS$ is universal in the sense that
the relation it determines between states $\alpha$ and $\beta \equiv g\alpha$ is independent of $\alpha$,
including which Hilbert space $\alpha$ belongs to. Also the relation defined by the
condition that a pair of states are related by arbitrary $geS$ is an equivalence
relation. The last two properties justify associating a geometry directly with
$S$. The available evidence at present is consistent with $S \equiv$ Poincare group
$\times G$, where $G = U(1) \times SU(2) \times SU(3)$. This universal symmetry group $S$,
as shown in the previous section, gives rise to all the fundamental interactions.
Replacing dynamical laws by symmetries as the fundamental concept in physics
corresponds to changing the physical geometry to the above concept of geometry
in which symmetries directly relate equivalent states without being required to
act on a universal set of points, such as the Riemannian space-time.

The Riemannian space-time enabled the natural inclusion of the classical
gravitational field as the metrical relations between space-time points. Similarly,
the generalized Klein geometry described above enables the natural inclusion of all fundamental interactions as symmetry relations between quantum states. This is provided by (14) in an approximate manner because $\gamma$ is a space-time curve, which has no operational meaning in quantum physics. A more precise treatment may give the quantized fields instead of the classical fields contained in (14).

The acceptance of the primary role of symmetries in physics would remove the mystery of why there are complex numbers in quantum mechanics. This is because the representations of groups act more naturally on complex vector spaces than on real vector spaces. For example, the $U(1)$ group has a real faithful representation $SO(2, \mathbb{R})$, where $\mathbb{R}$ is the real line. But the matrices of this representation and its Lie algebra do not have eigenvalues or eigenvectors, if we restrict to the field of real numbers. The elements of $SO(2, \mathbb{R})$ may be written in the form $\exp(\phi X)$, where $\phi \in \mathbb{R}$ and $X^2 = -I$. Hence, $X$ defines a complex structure on the vector space $\mathbb{R}^2$ on which $SO(2, \mathbb{R})$ acts. Since $X$ commutes with $SO(2, \mathbb{R})$, this complex structure is invariant under $SO(2, \mathbb{R})$. It is then natural to treat this vector space as a one-dimensional complex vector space $\mathbb{C}$, with $X$ replaced by $i$ and $SO(2, \mathbb{R})$ correspondingly replaced by $U(1)$. The different representations of the $U(1)$ group then correspond physically to different charges. The compactness of the $U(1)$ group then implies that charges are integer multiples of a fundamental charge (see for example Yang, 1969), which is consistent with observation.

In the case of the Lorentz group, the existence of Fermions implies that it should be $SL(2, \mathbb{C})$ and not its $(2 - 1)$ isomorphic group $SO(3, 1, \mathbb{R})$. The fundamental representation of $SL(2, \mathbb{C})$ acts on $\mathbb{C}^2$, which is the space of spinors. Here also we may regard this action as the action of a real linear group on $\mathbb{R}^4$, which however contains a complex structure $Y$ ($Y^2 = -I$) that commutes with this group. So, it is natural to regard this $\mathbb{R}^4$ as $\mathbb{C}^2$. The action of $Y$ on this $\mathbb{C}^2$ is simply $iI$, where $I$ is the identity transformation. But a vector in
any tangent space $V$ of space-time is a quadratic combination of the spinors and their complex conjugates (or the vectors in the above $\mathbb{R}^4$) at that point. Therefore, the action of $Y$ corresponds to the identity operation on $V$. This explains why the complex structure $Y$ was ‘hidden’ prior to the discovery of Fermions. It also explains why general relativity, which was formulated in terms of tensors that take their values in the tensor products of $V$ with itself and its dual, does not need complex numbers. But it should be noted that even prior to the discovery of Fermions, complex numbers were used in quantum mechanics because of unitary representations of symmetry groups. As remarked above, these representations are more natural than real representations because of the existence of eigenvalues and eigenvectors over the field $\mathbb{C}$.

The view that there is no law corresponds to our experience of a ‘flow of time’, which is a mystery in the paradigm of laws. As mentioned above, in the latter paradigm space-time geometry is the appropriate geometry. But if everything is laid out on space-time, there does not seem to be any room for time to ‘flow’. Moreover, the time of our consciousness has an arrow due to our remembering our past, which is unique, but we do not know of our uncertain future. But this is the same as the arrow of time determined by phenomenena, obtained in section 3, because the final states of a phenomenon, as expressed by $A_1$ above, is uncertain in general. So, the present approach has the advantage that it is in accordance with our conscious experience of time, which may be the most immediate experience we have, and which therefore science should take into account.

8 Comparison With Other Approaches To Quantum Theory

The present approach will now be compared with some other interpretations of quantum theory. Two consistent interpretations of quantum theory within the
paradigm of laws are the Bohm (1952) (see also Holland, 1993), and the Everett (1957) interpretations. Both of them take Schrödinger’s equation seriously as the deterministic law governing quantum evolution. The Bohm interpretation avoids the ‘many worlds’ of Everett by postulating a dual ontology that requires the existence of particles as well as the wave. This also has the advantage that it is possible to introduce probabilities which are consistent with experiment by associating them with the particles while the wave undergoes deterministic evolution. Whereas in the Everett interpretation, since there is only a wave that undergoes deterministic evolution, it is not possible to explain the probabilities observed in quantum experiments by simply postulating a measure on the relative states (Everett, 1957). In the Bohm interpretation, on the other hand, it is not clear why the wave function which deterministically guides the particle should also give the probability density for finding it.

The present approach does not require a dual ontology as in the Bohm interpretation. It resembles most the Copenhagen interpretation, but differs from it in not making an arbitrary division between the classical and quantum worlds. By including protective measurements, which were unknown to the founders of the Copenhagen interpretation, the present approach enables an extended wave function to be treated as real, instead of treating only the localized ‘events’ associated with the ‘classical’ measuring apparatus as real as in the Copenhagen interpretation. The indeterminacy of quantum theory was introduced here by the denial of deterministic evolutionary laws governing the state, and not by denying the reality of the wave function and treating it as containing a catalog of probabilities as done by the Copenhagen interpretation. The present interpretation shares, however, with the Copenhagen and other interpretations the mystery of what a measurement really is.

There are two advantages to the present approach over the other interpretations of quantum theory: 1) The indeterminacy of quantum mechanics is built in right from the beginning by denying the existence of deterministic laws. In
the other interpretations, which work within the paradigm of laws, it is not clear why the dynamical laws which are believed to exist are not deterministic; this intrinsic indeterminacy is a mystery and needs to be postulated in an arbitrary manner\(^3\).

2) If there are also no probabilistic laws but only symmetries at a fundamental level, then the probabilities are constrained only by the symmetries. Hence, the probabilities must depend on the invariants under these symmetries which include the geometry of the Hilbert space. So it is not surprising that in the derivation of probabilities, in section 5, the geometry of the Hilbert space played a role. This throws light on another long standing mystery of quantum mechanics, namely why the probabilities come from the geometry of the Hilbert space\(^4\), unlike in general relativity where geometry plays no stochastic role.

9 Further Discussion and Conclusion

In the foregoing considerations, the paradigm of laws was rejected because it is based on the metaphysical assumption that physical systems should in some mysterious way be ‘railroaded’ into obeying dynamical laws. Such a ‘railroad’ cannot be observed. For example, the evolution of the state of an electron between successive observations, say at the source and the detector, cannot be operationally defined. In the Bohm interpretation, considered in the previous section, the electron may be assigned a trajectory between the source and the

\(^3\)The Copenhagen interpretation tries to rationalize this indeterminacy within the paradigm of laws by saying that the observation of a microscopic system disturbs it, as in the well known Heisenberg microscope experiment. The use of the wave function to describe the state of the microscopic system is then argued as being necessary in order to represent this indeterminacy. But this argument cannot be made in the case of protective measurements of the quantum system which do not disturb the state of the system represented by its wave function, which is directly observed.

\(^4\) For a different geometric approach to the stochasticity of quantum mechanics see Hughston (1996).
detector, but this trajectory is not observable. To describe the evolution of the wave function of the electron as it interacts with other macroscopic systems requires the introduction of the ‘many worlds’ of Everett, which are also not observable. On the other hand, the generators of the symmetry group are conserved quantities, such as charge or total momentum, which are observable. The existence of these conserved quantities is also shown from the fact that they generate gauge fields and gravity which have observable effects.

The history of physics has shown the usefulness of rejecting metaphysical assumptions, e.g. Einstein’s rejection of absolute simultaneity in developing special relativity and the notion of a global inertial frame in developing general relativity. The rejection of the metaphysical assumption of laws led immediately to the intrinsic indeterminacy of quantum theory, in section 2, which in turn gave an arrow of time, in section 3. In section 5, the observed probabilities of quantum theory were also obtained in a natural manner. The postulate that symmetries should play the fundamental role in the new proposed paradigm naturally gave, in section 6, the observed gravitational and gauge fields for the symmetries of the standard model.

The last result may also be used to argue for the reality of symmetries. (14), in section 6, may be regarded as an alteration of the symmetries experienced by a quantum state due to gravity and gauge fields. This changes the probabilities of processes undergone by the quantum system that experiences these fields. The quantum system may then be expected to react back to modify the fields contained in (14). We know this to be the case experimentally: all systems modify the gravitational fields, charged systems modify the electromagnetic field etc. So, if the system is affected by the fields, which depends on the particular representation of the symmetry group to which it belongs to, then it reacts back on the fields. A criterion for reality has been proposed by Anandan and Brown(1995) according to which an object may be regarded as real if it satisfies the action-reaction principle. According to this criterion, the symmetry
group element (14), which replaces the usual gravitational and gauge fields, may be regarded as real. I believe that it is these symmetry group elements (14), instead of the usual fields, which should be used to construct a quantum theory of gravity.

We may draw encouragement for the last remark from the following historical fact. As pointed out by Yang (1987), the importance of the electromagnetic phase factor,

$$\exp\left(\frac{ie}{\hbar} \int \gamma A_\mu dx^\mu\right),$$

(18)

was recognized by Schrödinger (1922), in his study of Weyl’s gauge theory, four years before he introduced the wave function. I wish to emphasize that (18) is an element of the $U(1)$ symmetry group, and is part of (14), which belongs to the entire symmetry group. From the present point of view, (18) is not only historically but also logically prior to the Schrödinger equation introduced in 1926. Similarly, (14) may be a precursor to a fully developed quantum theory of gravity which is yet to come.

If the last conjecture is realized then there may still be the ultimate question of why (14) belongs to a particular symmetry group $S$ and not some other symmetry group. The $U(1)$, $SU(2)$ and $SU(3)$ symmetries, which are contained in $S$ according to the standard model, are the simplest unitary symmetries and may be justified on the grounds of simplicity. But why should $S$ contain the Poincare group of symmetries instead of some other symmetries? The anthropic principle may provide the answer to this question. According to this principle, as interpreted here, there are parallel universes in which there could be different symmetries, and hence different effective laws that are obtained from these symmetries, logical self-consistency, and contingencies. The universe in which we live must be one in which the symmetries are such that they allow life to evolve. This places a restriction on the symmetries in our universe. However, we should also be open to the possibility that quantum gravity may modify $S$.

\footnote{I thank Dennis Sciama for a discussion concerning this point.}
to some other symmetry group.

The purpose of physics is to simplify our understanding of nature. Simplification is often accompanied by immense progress, not only in physics but also in all of science. Giving up trying to explain the initial conditions in favor of laws as our basis for understanding nature, mentioned at the beginning of this article, led to an enormous simplification. This also resulted in tremendous progress in physics. Similarly, foregoing laws and accepting symmetries as our basis for understanding nature would lead to another enormous simplification, and perhaps also much more progress. This is particularly relevant to the construction of quantum gravity. Instead of trying to discover new laws, we should perhaps focus on understanding how symmetries give rise to all the fundamental interactions in nature, including gravity. The quantum mechanical nature of these interactions, as discussed above, is then a natural consequence of the denial of the existence of fundamental laws, the central role of symmetries, and the nature of the states of the systems that undergo these interactions.

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