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Diffractive Parton Distributions in the Semiclassical Approach

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Abstract

Recently, a semiclassical approach to diffraction has been proposed, which treats the proton as a classical colour field. The present paper demonstrates that this approach is consistent with the concept of diffractive parton distributions. The diffractive quark and gluon distributions are expressed through integrals of non-Abelian eikonal factors in the fundamental and adjoint representation respectively. As a by-product, previously calculated diffractive cross sections for processes with a final state gluon are rederived in a simpler way.
1 Introduction

The measurement of diffractive structure functions in deep inelastic scattering at HERA [1] has triggered renewed theoretical interest in the phenomenon of hard diffraction (which was seen previously in hadron-hadron collisions [2]). A particularly important observation is the absence of a large-$Q^2$ suppression. This suggests a significant leading twist component in diffraction, which is the main subject of the present analysis.

One of the interesting theoretical problems is the description of the colour singlet exchange, responsible for the large rapidity gap in the final state. A large amount of theoretical work has been devoted to two gluon exchange models (see e.g. [3]), which correspond to the simplest possible perturbative description. It is, however, not obvious that such a perturbative approach is valid for the bulk of the events.

A different approach is based on the picture of soft pomeron exchange. The hardness of the process, introduced by the large scale $Q^2$, suggests a partonic interpretation of the pomeron [4]. As pointed out by Berera and Soper [5], instead of applying the parton model to the pomeron, one can also introduce the more general but less predictive concept of diffractive parton distributions, closely related to the fracture functions of [6]. This approach, and the underlying concept of diffractive factorization, has been discussed in more detail in [7].

The semiclassical approach derives leading twist diffraction from soft interactions of fast partons with the proton colour field [8, 9]. Within this model the dominant contribution to diffractive $q\bar{q}$-pair production comes from aligned jet type configurations, which interact with the target in a soft process. Therefore, the semiclassical model is similar in spirit to the non-perturbative models discussed above, although it does not use the concept of a pomeron.

It has been suggested that diffractive factorization can be understood in the semiclassical picture in the proton rest frame [10]. The present paper demonstrates the consistency of the semiclassical calculation with the concept of diffractive structure functions. It is explicitly shown that the amplitude contains two fundamental parts: the usual hard scattering amplitude of a partonic process, and the amplitude for soft eikonal interactions with the external colour field. The latter part is dominated by the scattering of one of the partons from the photon wave function, which has to have small transverse momentum and to carry a relatively small fraction of the photon energy in the proton rest frame. In a frame where the proton is fast this parton can be interpreted as a parton from the pomeron structure function.
Calculating the cross section by standard methods, a result is obtained that can be written as a convolution of a partonic cross section and a diffractive parton distribution. Within the semiclassical model this diffractive parton distribution is explicitly given in terms of integrals of non-Abelian eikonal factors in the background field.

The special rôle played by the soft parton in the photon wave function has also been discussed in [11] in the framework of two gluon exchange. However, the present approach has the two following advantages: firstly, by identifying the hard part as a standard photon-parton scattering cross section the necessity for non-covariant photon wave function calculations is removed. Secondly, once it is established that the main contribution comes from the soft region, non-perturbative effects are expected to become important. The eikonal approximation provides a simple, self-consistent model for this non-perturbative region.

The paper is organized as follows. In Sect. 2 the semiclassical calculation of the diffractive cross section is performed for the simple scalar case. The corresponding diffractive distribution of scalar partons is derived. Sections 3 and 4 introduce the appropriate changes to obtain the diffractive quark and gluon distributions respectively. The application of the developed formalism to the simplest hard diffractive processes of $q\bar{q}$ and $q\bar{q}g$ state production is discussed in Sect. 5. In particular, the agreement with the results of [9] is demonstrated. The conclusions in Sect. 6 are followed by two appendices explaining some technical details relevant to Sections 3 and 4.

2 Diffractive distribution of scalar partons

In this section a simple, yet technically sufficiently interesting, model process will be investigated: the incoming virtual photon with momentum $q \ (q^2 = -Q^2, x \ll 1)$ is assumed to fluctuate into a set of scalar coloured partons. This virtual state is then made real by its interaction with the proton, resulting in a diffractive final state with invariant mass $M$ and an elastically scattered proton.

Following the concept introduced in [8] and further developed in [9], the proton is treated as a soft external colour field. In the high energy limit the only effect of this field is to introduce a non-Abelian phase factor for each of the coloured particles passing the field.

The kinematical situation is shown symbolically in Fig. 1. The process is split into two parts, the hard amplitude for the transition of the photon into a virtual partonic
state and the scattering of this state off the external field.

\[ p(1) \]

\[ p(n) \]

\[ k \]

\[ k' \]

Fig. 1 Hard diffractive process in the proton rest frame. The soft parton with momentum \( k \) is responsible for the leading twist behaviour of the cross section.

To keep the amplitude for the first part (symbolized by a circled “H” in Fig. 1) hard the transverse momenta \( p'_{(j)\perp} (j = 1...n) \) are required to be large, i.e. of order \( Q \). The momentum \( k_{\perp} \) is small, i.e. of the order of some hadronic scale \( \Lambda \), and the corresponding parton carries only a small fraction (\( \sim \Lambda^2/Q^2 \)) of the longitudinal photon momentum in the proton rest frame. While the hardness condition for particles 1 through \( n \) is introduced “by hand”, simply to make the process tractable, the softness of the last particle follows automatically from the requirement of leading twist diffraction. This will become clear from the calculation below (see also [9]).

The rest of this section is devoted to the calculation of the above diffractive process using the eikonal model for the scattering off the proton field. The result will be a convolution of a hard photon-parton scattering cross section, as it is typical for deep inelastic scattering seen, e.g., in the Breit frame (brick wall frame), with a diffractive parton distribution calculated from the soft interaction of a high energy parton and the external colour field.

The standard cross section formula for the scattering off a static external field reads

\[
d\sigma = \frac{1}{2q_0} |T|^2 2\pi \delta(q_0 - q'_0) dX^{(n+1)}, \quad \text{where} \quad q' = k' + \sum p'_{(j)}.
\]

All momenta are given in the proton rest frame, \( T \) is the amplitude corresponding to Fig. 1, and \( dX^{(n+1)} \) is the usual phase space element for \( n + 1 \) particles.

Each of the particles scatters off the external field with an effective vertex

\[
V(p', p) = 2\pi \delta(p'_0 - p_0) 2p_0 \tilde{U}(p'_{\perp} - p_{\perp}),
\]

where \( \tilde{U} \) is the Fourier transform of the non-Abelian eikonal factor in impact parameter space,

\[
U(x_{\perp}) = P \exp \left( -\frac{i}{2} \int_{-\infty}^{\infty} A_-(x_+, x_{\perp}) dx_+ \right).
\]
Here \( x_\pm = x_0 \pm x_3 \) are the light-cone components of \( x \), \( A(x_+, x_\perp) \) is the gauge field, and the path ordering \( P \) sets the field at smallest \( x_+ \) to the rightmost position.

This results in the following equation for the combined amplitude \( T' \),

\[
i 2\pi\delta(q_0 - q'_0) T' = \int T_H \prod_j \left( \frac{i}{p^2(j)} 2\pi\delta(p'_j - p_{(j)0}) 2p_{(j)0} \tilde{U}(p'_j - p_{(j)\perp}) \frac{d^4 p_j}{(2\pi)^4} \right) \times \left( \frac{i}{k^2} 2\pi\delta(k'_0 - k_0) 2k_0 \tilde{U}(k'_\perp - k_\perp) \right),
\]

where \( T_H \) stands for the part of the diagram in Fig. [I] that is symbolized by the circled “H”. The prime on the amplitude shows that the above expression is not yet exactly the amplitude required for Eq. (1). The r.h.s. of Eq. (4) includes the unphysical contribution where none of the partons interacts with the field, corresponding to the zeroth order term in an expansion of all \( U \)’s in powers of \( A \). It is convenient to subtract this trivial term later on by hand.

Colour indices have been suppressed in Eq. (4). Notice also, that some of the produced partons are antiparticles. The corresponding matrices \( U \) have to be replaced by \( U^\dagger \). To keep the notation simple, this has not been indicated explicitly.

The integrations over the light-cone components \( p_{(j)\perp} \) can be performed using the appropriate energy \( \delta \)-functions. After that, the \( p_{(j)\perp} \)-integrations are performed by picking up the poles of the propagators \( 1/p^2(j) \). The result is

\[
T = \int T_H \prod_j \left( \tilde{U}(p'_j, p_{(j)\perp}) \frac{d^2 p_{(j)\perp}}{(2\pi)^2} \right) \frac{2k_0}{k^2} \tilde{U}(k'_\perp - k_\perp).
\]

Next, a change of integration variables is performed,

\[
d^2 p_{(n)\perp} \to d^2 k_\perp.
\]

Since the external field is assumed to be soft, it can only transfer transverse momenta of order \( \Lambda \), i.e., \( p_{(j)\perp} \approx p'_{(j)\perp} \) for all \( j \). In general, the amplitude \( T_H \) will be dominated by the hard momenta of order \( Q \). Therefore, it can be assumed that \( T_H \) is constant if the momenta \( p_{(j)\perp} \) vary on a scale \( \Lambda \). In this approximation the integrations over \( p_{(j)\perp} \) \( (j = 1...n - 1) \) can be performed in Eq. (5), resulting in \( \delta \)-functions in impact parameter space. These manipulations give the result

\[
T' = \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{2k_0}{k^2} T_H \int_{x_\perp, y_\perp} \left( \prod_j U(x_\perp) \right) U(x_\perp + y_\perp) e^{-ix_\perp\Delta_\perp - iy_\perp(k'_\perp - k_\perp)},
\]
where $\Delta$ is the total momentum transferred from the proton to the diffractive system and, in particular, $\Delta_\perp = k_\perp' + \sum p_\perp^{(j)}$. It is intuitively clear that the relative proximity of the high-$p_\perp$ partons in impact parameter space leads to the corresponding eikonal factors being evaluated at the same position $x_\perp$.

Now the colour structure of the amplitude will be considered in more detail. Spelling out all the colour indices and introducing explicitly the colour singlet projector $P$ the relevant part of the amplitude reads,

$$T'_{\text{colour}} = T_{H}^{a_1...a_nb} \left( \prod_j U(x_\perp) \right)^{a'_1...a'_n} U(x_\perp + y_\perp)_b \ P_{a_1...a'_b} .$$

(8)

Using the fact that $T_H$ is an invariant tensor in colour space and introducing the function

$$W_{x_\perp}(y_\perp)_b = \left( U(x_\perp) U(x_\perp + y_\perp) - 1 \right)_b ,$$

(9)

the following formula is obtained,

$$T_{\text{colour}} = T'_{\text{colour}} - T_{H}^{a_1...a_nb} P_{a_1...a'_b} = T_{H}^{a_1...a_nb} W_{x_\perp}(y_\perp)_b P_{a_1...a'_b} .$$

(10)

Here the first equality has to be understood as the definition of the corrected amplitude, where the trivial contribution of zeroth order in $A$ has been subtracted. This subtraction has been taken into account in the definition of $W$, Eq. (9).

For colour covariance reasons

$$T_{H}^{a_1...a_nb} P_{a_1...a'_b} = \text{const.} \times \delta_{b'}. $$

(11)

Since the photon is colour neutral, the following equality holds,

$$T_{H}^{a_1...a_nb} T_{H}^{*a_1...a_nb} = | T_{H}^{a_1...a_nb} P_{a_1...a'_b} |^2 = |\text{const.}|^2 N^2 .$$

(12)

Here the partons are assumed to be in the fundamental representation of the colour group $SU(N)$. Combining Eq. (10) with Eqs. (11) and (12) it becomes clear that the colour structure of the hard part decouples from the eikonal factors,

$$|T_{\text{colour}}|^2 = \frac{1}{N} |\text{tr}[W]|^2 |T_H|^2 .$$

(13)

The hard part will be interpreted in terms of an incoming small-$k_\perp$ parton, which collides with the $\gamma^*$ to produce the outgoing partons 1 through $n$. Therefore a factor $1/N$ for initial state colour averaging has been included into the definition of $|T_H|^2$.

In the expression for the cross section the two functions $W$ enter in the combination

$$\left( \text{tr}[W_{x_\perp}(y_\perp)] \right) \left( \text{tr}[W_{x'_\perp}(y'_\perp)] \right)^* e^{-i(x_\perp - x'_\perp)\Delta_\perp} ,$$

(14)
with independent integrations over \(x_\perp, x'_\perp, y_\perp\) and \(y'_\perp\). If the external field is sufficiently smooth, the functions \(W\) vary only slowly with \(x_\perp\) and \(x'_\perp\). Therefore, after integration over \(x_\perp\) and \(x'_\perp\) the expression in Eq. (14) will produce an approximate \(\delta\)-function in \(\Delta_\perp\). Furthermore, it will be assumed that the measurement is sufficiently inclusive, i.e., the hard momenta \(p'_{(j)\perp}\) are not resolved on a soft scale \(\Lambda\). This corresponds to a \(\Delta_\perp\)-integration, which will produce an approximate \(\delta\)-function in \(x_\perp - x'_\perp\). Since the expression in Eq. (14) will always appear under \(x_\perp, x'_\perp\) and \(\Delta_\perp\)-integration, the above considerations justify the substitution

\[
e^{-i(x_\perp - x'_\perp)\Delta} \rightarrow (2\pi)^2 \delta^2(x_\perp - x'_\perp) \delta^2(\Delta_\perp).
\]

Combining Eqs. (1), (7), and (13) the following formula for the cross section results,

\[
d\sigma = \frac{1}{2q_0} \left| T_H \right|^2 \int_{x_\perp} \left| \frac{d^2k_\perp}{(2\pi)^2} \frac{\text{tr}[\hat{W}_{x_\perp}(k'_\perp - k_\perp)]}{\sqrt{Nk^2}} \right|^2 (2k_0)^2 (2\pi)^3 \delta^2(\sum p_{(j)\perp}) \delta(q_0 - q'_0) dX^{(n+1)}.
\]

Note that the soft momentum \(k'_\perp\) has been neglected in the transverse \(\delta\)-function.

To finally establish the parton model interpretation of diffraction the hard partonic cross section based on \(\left| T_H \right|^2\) has to be identified in Eq. (14). Consider the process

\[
\gamma^*(q) + q(yP) \rightarrow q(p'_{(1)}) + \ldots + q(p'_{(n)}),
\]

where the photon collides with a parton carrying a fraction \(y\) of the proton momentum and produces \(n\) high-\(p_\perp\) final state partons. The cross section is approximately given by

\[
d\hat{\sigma}(y) = \frac{1}{2(\hat{s} + Q^2)} \left| T_H \right|^2 (2\pi)^4 \delta^4(q - k - \sum p'_{(j)}) dX^{(n)},
\]

where \(\hat{s} = (\sum p'_{(j)})^2\) and the quantities \(\left| T_H \right|^2\) and \(k\) are the same as in the previous discussion. Eq. (18) is not exact for several reasons. On the one hand, \(\left| T_H \right|^2\) is defined in terms of the unprimed momenta \(p_{(j)}\), which differ slightly from \(p'_{(j)}\). On the other hand, the vector \(k\) is slightly off shell and has, in general, a non-zero transverse component. However, both effects correspond to \(\Lambda/Q\)-corrections, where \(Q\) stands generically for the hard scales that dominate \(T_H\).

Using Eq. (18) the cross section given in Eq. (16) can now be rewritten as

\[
d\sigma = \int dk_- \frac{\hat{s} + Q^2}{2\pi q_0} (2k_0)^2 d\hat{\sigma}(y) \int_{x_\perp} \left| \frac{d^2k_\perp}{(2\pi)^2} \frac{\text{tr}[\hat{W}_{x_\perp}(k'_\perp - k_\perp)]}{\sqrt{Nk^2}} \right|^2 \frac{d^3k'}{(2\pi)^3 2k_0}.
\]

The light-cone component \(k_-\) is given by \(-k_- = yP_- = ym_p\). Note that the minus sign in this formula comes from the interpretation of the parton with momentum \(k\) as an
incoming particle in Eq. (18). This is, in fact, the crucial point of the whole calculation: due to the off-shellness of $k$ the corresponding parton can be interpreted as an incoming particle in both the process Eq. (17) and in the soft scattering process off the external field. The latter process, where an almost on-shell parton with momentum $k$ scatters softly off the external field changing its momentum to the on-shell vector $k'$, is most easily described in the proton rest frame. By contrast, the natural frame for the hard part of the diagram is the Breit frame or a similar frame. In such a frame the $k_-$-component is large and negative, so that the above parton can be interpreted as an almost on-shell particle with momentum $-k$, colliding head-on with the virtual photon.

It is convenient to use the variables $\xi$ (sometimes called $x_{IP}$), defined by $M^2 = (q + \xi P)^2$, and $b$, defined by $b = y/\xi$. Note that the latter one is, in general, different from the conventional observable $\beta = Q^2/(Q^2 + M^2)$. Substituting the variables $y$ and $\xi$ for $k_-$ and $k'_-$ the cross section, Eq. (19), can be given in the form

$$\frac{d\sigma}{d\xi} = \int_x^\xi dy \hat{\sigma}(y) \left( \frac{df(y, \xi)}{d\xi} \right),$$

where the diffractive parton distribution for scalars is

$$\left( \frac{df(y, \xi)}{d\xi} \right)_{\text{scalar}} = \frac{1}{\xi^2} \left( \frac{b}{1 - b} \right) \int \frac{d^2k'_-}{(2\pi)^2 N} \int \frac{d^2k_-}{(2\pi)^2} \frac{\text{tr} [\tilde{W}_{x_\perp}(k'_- - k_-)]}{k'_- b + k_- (1 - b)^2}.$$

This result is in complete agreement with the concepts developed in [7]. However, the parton distributions introduced there are differential in $t = (P' - P)^2$. Since here the proton is modelled by a given external field, the present result is automatically inclusive in $t$.

For an external colour field that is smooth on a soft scale $\Lambda$ and confined to a region of approximate size $1/\Lambda$ the function $\text{tr} [W_{x_\perp}(y_\perp)]$ is also smooth and vanishes at $y_\perp = 0$ together with its first derivative. From this it can be derived that the $k_-$- and $k'_-$-integrations in Eq. (21) are dominated by the soft scale. This justifies, a posteriori, the softness assumption for one of the partons used in the derivation.

The qualitative result is that the eikonal scattering of this soft parton off the proton field determines the diffractive parton distribution. The hard part of the photon evolution can be explicitly separated and expressed in terms of a standard cross section for photon-parton collisions.

Having worked out the kinematics in the simple scalar case, it will be straightforward to extend the calculation to realistic quarks and gluons in the following sections.
3 Diffractive quark distribution

The introduction of spinor or vector partons does not affect the calculations leading to the generic expression in Eq. (20). However, those parts of the calculation responsible for the specific form of Eq. (21) have to be changed if the soft parton is a spinor or vector particle.

The most economic procedure is to first identify the piece of the old squared amplitude $|T|^2$ that depends explicitly on the spin of the soft parton. This piece, which is essentially just the squared scattering amplitude of the soft parton and the external field, is symbolically separated in Fig. 2. The two independent integration variables for the intermediate momentum of the soft parton are denoted by $k$ and $\tilde{k}$ in the amplitude $T$ and its complex conjugate $T^*$ respectively. Note also, that $k_+ = \tilde{k}_+$ and $k_- = \tilde{k}_-$ at leading order.

Fig. 2 Symbolic representation of the square of the amplitude for a hard diffractive process. The box separates the contributions associated with the soft parton and responsible for the differences between diffractive distributions for scalars, spinors and vector particles.

It is straightforward to write down the factor $B_{\text{scalar}}$ that corresponds to the box in Fig. 2 for the scalar case. Since the off-shell denominators $k^2$ and $\tilde{k}^2$ as well as the two eikonal factors and energy $\delta$-functions are present for all spins of the soft parton, they are not included into the definition of $B$. All that remains are the explicit factors $2k_0$ from the effective vertex, Eq. (2). Therefore, the result for the scalar case reads simply

$$B_{\text{scalar}} = (2k_0)^2.$$ \hfill (22)

The next step is the calculation of the corresponding expression $B_{\text{spinor}}$, given by the box in Fig. 2 in the case where the produced soft parton is a quark. Introducing the
factor $B_{\text{spinor}}/B_{\text{scalar}}$ into Eq. (21) will give the required diffractive parton distribution $(df/d\xi)_{\text{spinor}}$.  

Observe first, that the analogue of Eq. (2) for spinors is simply

$$V_q(p', p) = 2\pi\delta(p'_0 - p_0)\frac{\gamma^+}{2} \tilde{U}(p'_\perp - p_\perp) .$$

(23)

The Dirac structure of $V_q$ follows from the fact that in the high energy limit only the light-cone component $A_-$ of the gluon field contributes. To establish the correctness of the normalization in Eq. (23) the vertex $V_q$ is used in a scattering amplitude for an on-shell quark. The result is

$$\bar{u}_{s'}(p')V_q(p', p)u_s(p) = 2\pi\delta(p'_0 - p_0) 2p_0 \delta_{s's} \tilde{U}(p'_\perp - p_\perp) ,$$

(24)

in agreement with the scalar case.

Consider now the Dirac propagator with momentum $k$. The hard part $T_H$ requires the interpretation of the quark line as an incoming parton, which collides head on with the photon. Therefore, it is convenient to define a corresponding on-shell momentum $l$, given by $l_- = -k_-$, $l_\perp = -k_\perp$ and $l_+ = l^2_\perp/l_-$. The propagator can now be written as

$$\frac{1}{k} = -\sum_s u_s(l)\bar{u}_s(l) \frac{\gamma^-}{2l_-} .$$

(25)

This is technically similar to [11], where an on-shell momentum has been defined by adjusting the ‘$-$’-component of $k$. In the present treatment, however, the ‘$+$’-component of $l = -k$ has been adjusted, so that a partonic interpretation in the Breit frame becomes possible. It is shown in Appendix A, that the second term on the r.h.s. of Eq. (25) can be neglected, since it is suppressed in the Breit frame by the hard momentum $l_- = yP_-$. This can be intuitively understood by observing that this term represents a correction for the small off-shellness of the quark, which is neither important for $T_H$, nor for the soft high energy scattering off the external field.

When the soft part is separated in Fig. 2 the spinor $\bar{u}_s(l) = -\bar{v}_s(l)$ has to be considered as a part of the hard amplitude $T_H$. Therefore the analogue of Eq. (22) reads

$$B_{\text{spinor}} = \sum_{s,s'} \bar{u}_{s'}(k') \frac{\gamma^+}{2} u_s(l) \bar{u}_s(\tilde{l}) \frac{\gamma^+}{2} u_{s'}(k') ,$$

(26)

where $\tilde{l}$ is defined analogously to $l$, but using the momentum $\tilde{k}$ instead of $k$. The spin summation decouples from the hard part if the measurement is sufficiently inclusive for the hard amplitude square not to depend on the helicity of the incoming parton.
The above expression can be evaluated further to give

\[ B_{\text{spinor}} = k_0 \sum_s \bar{u}_s(\tilde{l}) \gamma_+ u_s(l) = k_0 \frac{4(l_\perp \tilde{l}_\perp)}{\sqrt{l_- l_-}} , \]  

where the last equality is most easily obtained using Table II of [13]. Simple kinematics leads to the result

\[ B_{\text{spinor}} = (2k_0)^2 \frac{2(k_\perp \tilde{k}_\perp)}{k_\perp^2} \left( \frac{\xi - y}{y} \right) . \]  

Comparing this with Eq. (22) the diffractive parton distribution is straightforwardly obtained from the scalar case, Eq. (21),

\[ \left( \frac{df}{d\xi} \right)_{\text{spinor}} = 2 \frac{2(k_\perp \tilde{k}_\perp)}{k_\perp^2} \left( \frac{\xi - y}{y} \right)^2 . \]  

The virtual fermion line corresponds to a right-moving quark with momentum \( k \) in the proton rest frame and to a left-moving antiquark with momentum \( l \) in the Breit frame. Therefore, the above result has in fact to be interpreted as a diffractive antiquark distribution. The diffractive quark distribution is identical.

### 4 Diffractive gluon distribution

To obtain the diffractive gluon distribution the procedure of the last section has to be repeated for the case of an outgoing soft gluon with momentum \( k' \) in Fig. 3. Calculating the contribution separated by the box will give the required quantity \( B_{\text{vector}} \), in analogy to Eqs. (22) and (28).

It will prove convenient to introduce two light-like vectors \( m \) and \( n \), such that the only non-zero component of \( m \) is \( m_- = 2 \) in the proton rest frame, and the only non-zero component of \( n \) is \( n_+ = 2 \) in the Breit frame. Since Breit frame and proton rest frame are connected by a boost along the \( z \)-axis with boost factor \( \gamma = Q/(m_p x) \), the product of these vectors is \( mn = 2\gamma \).

Furthermore, two sets of physical polarization vectors, \( e^{(i)} \) and \( \epsilon^{(i)} \) (with \( i = 1, 2 \)), are defined by the conditions \( e k = \epsilon k = 0 \), \( e^2 = \epsilon^2 = -1 \), and \( em = \epsilon n = 0 \). An explicit choice, written in light-cone co-ordinates, is

\[ e^{(i)} = \left( 0, \frac{2(k_\perp \epsilon^{(i) \perp})}{k_+}, \epsilon^{(i) \perp} \right) \quad \text{and} \quad \epsilon^{(i)} = \left( \frac{2(k_\perp \epsilon^{(i) \perp})}{k_-}, 0, \epsilon^{(i) \perp} \right) . \]
where a transverse basis $\epsilon_{(1)} = (1, 0)$ and $\epsilon_{(2)} = (0, 1)$ has been used. Note, that the above equations hold in the proton rest frame, in the Breit frame, and in any other frame derived by a boost along the z-axis.

These definitions give rise to the two following representations for the metric tensor:

$$g^{\mu\nu} = \left( \sum_i \epsilon^{\mu}_{(i)} \epsilon^{\nu}_{(i)} + \frac{m^\mu k^\nu}{(mk)} + \frac{k^\mu m^\nu}{(mk)^2} k^2 \right)$$

$$= \left( \sum_i \epsilon^{\mu}_{(i)} \epsilon^{\nu}_{(i)} + \frac{n^\mu k^\nu}{(nk)} + \frac{k^\mu n^\nu}{(nk)^2} k^2 \right).$$

The amplitude of the process in Fig. 2, with the lowest parton being a gluon in Feynman gauge, is proportional to

$$A = \epsilon^\mu(k') V_g(k', k) \rho_T H = \epsilon^\mu(k') V_g(k', k) \gamma^{\rho\sigma} \rho_T H,$$

where $V_g^{\mu\nu}$ is the effective vertex for the scattering of the gluon off the external field.

Next, the first and second metric tensor appearing in this expression for $A$ are rewritten according to Eq. (31) and Eq. (32) respectively. It is shown in Appendix B that only the first terms from Eqs. (31) and (32) contribute at leading order in $x$ and $\Lambda/Q$. The intuitive reason for this is the relatively small virtuality of $k$, which ensures that for both the hard amplitude $T_H$ and the soft scattering vertex $V$ only the appropriately defined transverse polarizations are important.

The leading contribution to $|A|^2$, with appropriate polarization summation understood, now reads

$$|A|^2 = \sum_{i,j,i',j'} \left[ (\epsilon_{(i)}(k') V_g(\epsilon_{(i)}) (\epsilon_{(i)}(j) (\epsilon_{(j)} T_H) ) \left[ (\epsilon_{(i)}(k') V_g(\epsilon_{(i)}) (\epsilon_{(i)}(j) (\epsilon_{(j)} T_H) ) \right]^*,

$$

where the arguments $k$ and $\tilde{k}$ of the polarization vectors in the first and second square bracket respectively have been suppressed.

In the high energy limit the scattering of a transverse gluon off an external field is completely analogous to the scattering of a scalar or a spinor,

$$\epsilon_{\nu}(p') V_g(p', p) \epsilon_{\nu}(p) = 2\pi \delta(p'_0 - p_0) 2p_0 \delta_{\nu\nu} U_A(p_{\perp} - p_{\perp}).$$

The only difference comes with the non-Abelian eikonal factor, which is now in the adjoint representation,

$$U_A(x_{\perp}) = A(U(x_{\perp})).$$
In analogy to the spinor case, the polarization sum decouples from the hard part for sufficiently inclusive measurements, so that the squared amplitude is proportional to

$$|A|^2 = |T_H|^2 \left( \epsilon_i(k') V_g e_i(k) \right) \left( \epsilon_i(k') V_g e_i(k) \right) \sum_{i,j} \left( e_{(i)}(k) e_{(j)}(k) \right) \left( e_{(i)}(k) e_{(j)}(k) \right). \quad (37)$$

Note, that there is no summation over the index $l$. Recall the definition of $B$, the soft part of the amplitude square, given at the beginning of the last section and illustrated in Fig. 2. The corresponding expression in the case of a soft gluon can now be read off from Eqs. (35) and (37):

$$B_{\text{vector}} = (2k_0)^2 \sum_{i,j} \left( e_{(i)}(k) e_{(j)}(k) \right) \left( e_{(i)}(k) e_{(j)}(k) \right). \quad (38)$$

This is further evaluated using the explicit formulae in Eq. (30) and the identity

$$\sum_i e_{(i)}^a e_{(i)}^b = \delta^{ab} \quad (a, b \in \{1, 2\}). \quad (39)$$

The resulting expression,

$$B_{\text{vector}} = (2k_0)^2 \left( \delta^{ij} + \frac{2k_i^j k_i^j}{k^2} \left( 1 - \frac{b}{b} \right) \right) \left( \delta^{ij} + \frac{2k_i^j k_i^j}{k^2} \left( 1 - \frac{b}{b} \right) \right), \quad (40)$$

is now compared to Eq. (22), which gives the diffractive gluon distribution

$$\left( \frac{df(y, \xi)}{d\xi} \right)_{\text{vector}} = \frac{1}{\xi^2} \left( \frac{b}{1 - b} \right) \int \frac{d^2k_\perp}{(2\pi)^2} \frac{(k^2)^2}{(N^2 - 1)} \int_{x_\perp} \left| \int \frac{d^2k_\perp}{(2\pi)^2} \frac{\text{tr}[\hat{W}_x A (k' - k)] t^{ij}}{k^2 b + k^2_\perp (1 - b)} \right|^2,$$

with

$$t^{ij} = \delta^{ij} + \frac{2k_i^j k_i^j}{k^2} \left( 1 - \frac{b}{b} \right). \quad (42)$$

Note, that the factor $N$ appearing in the denominator of Eq. (21) has been replaced by the dimension of the adjoint representation, $N^2 - 1$.

## 5 Simple applications

In this section some results for specific diffractive processes, which can be easily obtained within the developed framework, will be discussed.

The lowest order process in leading twist diffraction is the production of a colour neutral $q\bar{q}$-pair. This corresponds to Bjorken’s aligned jet model [14], where the relative softness of one of the produced quarks gives rise to a large elastic amplitude. It has
been shown \[8, 9\] that within the eikonal model the aligned jet configuration is indeed responsible for the formally leading contribution to diffraction in deep inelastic scattering.

In the framework of the present paper the above process is described by taking the hard part \(T_H\) to be the amplitude for virtual photon-quark scattering, \(\gamma^* q \rightarrow q\). Obviously, in this simplest case neither of the quarks has large \(p_{\perp}\). The hard parton is only hard in the sense that it carries most of the photon’s longitudinal momentum. It can be easily checked that this does not invalidate the discussion of Sect. 2.

In the case of one generation of quarks with one unit of electric charge the well-known partonic cross section reads

\[
\hat{\sigma}_T(y)_{\gamma^* q \rightarrow q} = \frac{\pi e^2}{Q^2} \delta(1 - y/x). \tag{43}
\]

The corresponding cross section for longitudinal photons vanishes.

Combining this with the Eqs. (21) and (29) and specifying to \(N = 3\) the corresponding contribution to the diffractive structure function is easily obtained,

\[
F_2^D(x, Q^2, \xi)_{\gamma^* q \rightarrow q} = 2\beta \xi \int d^2k_{\perp} \int \frac{d^2k_{\perp}}{(2\pi)^4} \left( \frac{k_{\perp}^2 \text{tr}[\tilde{W}_{x\perp}(k_{\perp}' - k_{\perp})]}{k_{\perp}^2 \beta + k_{\perp}^2 (1 - \beta)} \right)^2. \tag{44}
\]

Note that in this specific case \(y = x\) and \(b = \beta = x/\xi\). Adding the antiquark contribution and replacing the integration variable \(k_{\perp}'\) by \(\alpha = k_{\perp}'/q_0 = k_{\perp}/M^2\) the result of \[9\] is exactly reproduced.

A simple model for colour neutral quark pair production, based on soft colour exchange, had been suggested earlier in \[15\]. The present more precise model shows the dominance of the aligned jet configuration and replaces the probabilistic colour rotation of \[15\] with the colour trace condition in Eq. (44).

Consider next the situation where the diffractive final state contains three partons, a \(q\bar{q}\)-pair and a gluon, two of which have high \(p_{\perp}\). The soft parton responsible for leading twist diffraction can be either the gluon or one of the quarks (The cross section is the same for the \(q\)- and the \(q\bar{q}\)-case.). Considering the transverse and longitudinal photon cross sections separately, four different diffractive contributions arise, all of which have been calculated and discussed in some detail in \[9\]. In the present framework these contributions can be easily obtained using the diffractive quark and gluon distributions calculated above together with the partonic cross sections for the processes \(\gamma^* q \rightarrow qg\) and \(\gamma^* g \rightarrow q\bar{q}\) (see, e.g., \[10\]).

As an example, the longitudinal photon contribution in the soft quark case will be
discussed in more detail: the required partonic cross section reads

\[
\frac{d\hat{\sigma}_L(y)\gamma^*q\rightarrow qg}{dt} = \frac{8\varepsilon^2\alpha_S}{3} \frac{Q^2(Q^2 + \hat{s} + \hat{t})}{(Q^2 + \hat{s})^4},
\]

(45)

where \(\hat{s}\) and \(\hat{t}\) are the usual partonic Mandelstam variables and \(\hat{s} = (y/x - 1)Q^2\).

The corresponding contribution to the longitudinal structure function can be written as

\[
F_D^L(x, Q^2, \xi)\gamma^*q\rightarrow qg = \frac{\alpha_S}{9\pi^5} \int db \int d\hat{t} \frac{\beta^3(bQ^2 + \beta\hat{t})}{b^4Q^4} \times \int d^2k'_\perp(k'^2_\perp) \int \left| \int \frac{d^2k_{\perp}}{(2\pi)^2} \frac{k_{\perp} \text{tr}[\tilde{W}_x(\hat{k}'_\perp - k_{\perp})]}{k_{\perp}^2 b + k_{\perp}^2(1 - b)} \right|^2.
\]

(46)

Adding the antiquark contribution and performing an appropriate change of integration variables the corresponding result of [9] is again exactly reproduced.

The longitudinal photon contribution in the soft gluon case is straightforwardly calculated using the diffractive gluon distribution of Sect. 4. The result is in agreement with the formula presented in [9]. Similarly, the cross sections for the remaining two transverse photon contributions with a \(q\bar{q}g\)-final state are found to agree with [9].

Notice finally, that the diffractive parton distributions calculated in the present model have to be interpreted as parton distributions at some low scale \(\sim \Lambda\), i.e., as the input for the QCD evolution equations. The simple applications to \(q\bar{q}g\)-final states discussed in the present section are a part of this evolution.

6 Conclusions

In this paper the calculation of hard diffractive processes in the semiclassical model has been organized in a way that shows its consistency with the concept of diffractive factorization.

The essentials of the method can already be understood in a model theory of scalar partons. Working in the proton rest frame it has been shown that high-\(p_\perp\) partons from the photon wave function are not sensitive kinematically to the colour rotation introduced by the external field. Leading twist diffraction appears if a relatively slow parton with small \(p_\perp\) is found in the photon wave function. The cross section is only sensitive to the momentum transfer associated with the impact parameter dependence of the eikonal factor of this soft parton. Since the soft parton has small negative virtuality, its interpretation as an incoming or outgoing particle is ambiguous. Therefore, working e.g. in
the Breit frame, the hard part of the amplitude can be viewed as describing a head-on photon-parton collision.

As a result, the cross section can be written as a convolution of the hard partonic cross section with a function of the non-Abelian eikonal factors, that does not involve the hard scale. This function is the diffractive parton distribution, calculated in the eikonal model.

The above factorization of soft and hard parts makes it easy to generalize the calculation to a realistic theory with gluons and quarks. In the case where the soft parton is a spinor or a vector particle, only the soft part of the squared amplitude has to be recalculated. The resulting new functions of the eikonal factors represent the diffractive quark and gluon distributions.

In the proton rest frame the above soft parton from the photon wave function is still a very fast particle. This parton undergoes a soft scattering off the colour field and appears in the diffractive final state together with the products of the hard partonic process. Its kinematical rôle is similar to the pomeron remnant appearing in partonic pomeron models (compare the discussion in [17]). Notice however, that no pomeron has been introduced in the present calculation. Instead, correlated colour singlet parton pairs are found to be a natural consequence of the fast proton colour field.

It appears likely that the concept of diffractive factorization will be established as a rigorous consequence of QCD. The present semiclassical calculation supplies a QCD-based model for the required diffractive parton distributions. The main problem of this model is that once a soft parton is found in the photon wave function, further calculations involving this parton are not perturbatively justified. In particular, it is not clear how large corrections due to additional soft partons in the photon wave function and their eikonal scattering will be.

Nevertheless, it would certainly be interesting to obtain numerical predictions for diffractive structure functions from the equations derived above and non-perturbative models for the proton colour field.

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Appendix A

The claim that the $\gamma_-$-term in Eq. (25) can be neglected requires some additional considerations.

The relevant part of the amplitude has the form

$$\bar{u}(k') \gamma_+ \frac{1}{k} T_H = -\bar{u}(k') \gamma_+ \left( \sum_s u_s(l) \bar{u}_s(l) \frac{\gamma_-}{2l_-} \right) T_H \equiv -(C_1 + C_2). \quad (47)$$

Working in the Breit frame, it will be shown that the $\gamma_-$-term $C_2$ is suppressed with respect to $C_1$.

Using the techniques of Sect. 3 and assuming $k^2 \sim k_{\perp}^2 \sim \Lambda^2$ the first term can be estimated to be

$$C_1 \sim \sqrt{\frac{\alpha}{\Lambda}} \left[ \bar{u}(l) T_H \right], \quad (48)$$

where factors $O(1)$ have been suppressed.

Defining a vector $a$ by the requirements $a_+ = 1, a_- = 0, a_{\perp} = 0$ in the Breit frame, so that $\gamma_-/2 = \phi$, the $\gamma_-$-term can be written as

$$C_2 = \sum_s \left[ \bar{u}(k') \gamma_+ u_s(a) \right] \frac{1}{l_-} \left[ \bar{u}_s(a) T_H \right]. \quad (49)$$

In the proton rest frame, where both $a_+ = q_+/Q$ and $k'_+ = \alpha q_+$ are large, it is easy to see that $[\bar{u}(k') \gamma_+ u(a)] = 2 \sqrt{a_+ k'_+} = 2 q_+ \sqrt{\alpha/Q}$ for appropriate helicities. This gives, in the Breit frame, $[\bar{u}(k') \gamma_+ u(a)] = 2 Q \sqrt{\alpha/Q}$. Since in this frame $l_-$ is a hard momentum $\sim Q$, the following estimate is obtained,

$$\frac{C_2}{C_1} \sim \frac{\Lambda}{Q} \frac{[\bar{u}(a) T_H]}{[\bar{u}(l) T_H]} \sim \frac{\Lambda}{Q}, \quad (50)$$

where it has been assumed that no specific cancellation makes $\bar{u}(l) T_H$ small, i.e., $[\bar{u}(l) T_H]/[\bar{u}(a) T_H] \sim \sqrt{Q}$. Eq. (50) establishes the required suppression of the $\gamma_-$-term.

Appendix B

It has been claimed in Sect. 4 that only the first terms of Eqs. (31) and (32), i.e., the transverse polarizations, contribute in Eq. (33) if the metric tensors are rewritten according to these formulae. To see this explicitly, consider the expression

$$A = e V_g \left[ \sum ee + \frac{mk}{(mk)} + \frac{km}{(mk)} - \frac{mm}{(mk)^2} (k^2) \right] \left[ \sum e\epsilon + \frac{nk}{(nk)} + \frac{kn}{(nk)} - \frac{nn}{(nk)^2} (k^2) \right] T_H, \quad (51)$$
where the appropriate contractions of vector indices are understood.

Several estimates involving products of $V_g$ and $T_H$ with specific polarization vectors will be required.

Note that both $\epsilon$ and $n$ are $\mathcal{O}(1)$ in the Breit frame. For appropriate polarization the amplitude ($\epsilon T_H$) involves no particular cancellation, i.e., it has its leading (formal) power behaviour in the dominant scale $Q$. Therefore, $(n T_H)$ is not enhanced with respect to $(\epsilon T_H)$,

$$\frac{(n T_H)}{(\epsilon T_H)} \sim \mathcal{O}(1).$$

(52)

By analogy, it can be argued that $(\epsilon V_g m)$ is not enhanced with respect to $(\epsilon V_g e)$: since both $\epsilon$ and $m$ are $\mathcal{O}(1)$ in the proton rest frame and $(\epsilon V_g e)$ has the leading power behaviour for appropriate polarizations $\epsilon$ and $e$, the following estimate holds,

$$\frac{(\epsilon V_g m)}{(\epsilon V_g e)} \sim \mathcal{O}(1).$$

(53)

Gauge invariance requires $(k T_H)$ to vanish if $k^2 = 0$. Since the amplitude $T_H$ is dominated by hard momenta $\mathcal{O}(Q)$, and $k^2 \sim \Lambda^2 \ll Q^2$, this leads to the estimate

$$\frac{(k T_H)}{(\epsilon T_H)} \sim \frac{k^2}{Q}.$$  

(54)

Analogously, from $(\epsilon V_g k) = 0$ at $k^2 = 0$, the suppression of this quantity at small virtualities $k^2$ can be derived,

$$\frac{(\epsilon V_g k)}{(\epsilon V_g e)} \sim \frac{k^2}{k_+}.$$  

(55)

For this estimate it is also important that none of the soft scales involved in $V_g$, like $k^2_\perp$ or the gauge field $A$, can appear in the denominator to compensate for the dimension of $k^2$.

All the vector products $nk$, $mk$, $ne$, $me$, $mn$, and $ee$ can be calculated explicitly. Using the relations in Eqs. (52), (53), (54), and (55) it is now straightforward to show that $(\epsilon V_g e)(ee)(\epsilon T_H)$ is indeed the leading term in Eq.(51). The other terms are suppressed by powers of $Q$ or $k_+$.

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