Resonant tunneling of interacting electrons in a one-dimensional wire

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We consider the conductance of a one-dimensional wire interrupted by a double-barrier structure allowing for a resonant level. Using the electron-electron interaction strength as a small parameter, we are able to build a non-perturbative analytical theory of the conductance valid in a broad region of temperatures and for a variety of the barrier parameters. We find that the conductance may have a non-monotonic crossover dependence on temperature, specific for a resonant tunneling in an interacting electron system.

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The phenomenon of resonant tunneling is well-known in the context of electron transport physics [1]. The hybridization of a discrete state localized in the barrier with the extended states outside the barrier may strongly enhance the transmission coefficient for electrons incident on the barrier with energy matching the energy of the localized state. For a single electron, the transmission coefficient at energies close to the resonance is given by the Breit-Wigner formula [1]. However, if the barrier carrying the resonant level separates conductors which in equilibrium have a finite density of mobile electrons, the problem of resonant tunneling becomes more complex due to the electron-electron interaction. Manifestation of resonant tunneling in the conductance of a solid-state device is inevitably sensitive to this interaction.

Some of the effects of electron-electron interaction do not depend on the dimensionality \(d\) of the conductors–leads separated by the barrier. For instance, the on-site repulsion together with the hybridization of the localized state with the states of continua lead to the Kondo effect in the transmission across the barrier [1,2] at any \(d\). The Fermi-edge singularity also strongly affects the resonant tunneling [3,4] in any dimension. The electron-electron interaction within the leads, however, does not have a strong effect if \(d > 1\), and if the leads are not disordered. In contrast, resonant tunneling across a barrier interrupting a one-dimensional (1d) wire is modified drastically by the interaction within the wire. The importance of such a setting is emphasized by the recent transport experiments with nanotubes and nanowires containing a quantum dot [5,6].

The electron-electron interaction enhances the backscattering off the barrier for electrons with energy close to the Fermi level [8]. We find that if the discrete level is not perfectly aligned with the Fermi level in the leads, or the barrier structure has even slight geometrical asymmetry, then the low-temperature linear conductance decreases to zero with the temperature, \(G(T \to 0) \to 0\). At sufficiently high electron energies (measured from the Fermi level) the enhancement of backscattering due to the interaction should get weaker, and the conventional behavior of resonant tunneling prescribed by Breit-Wigner formula may be restored. Consequently, the conductance \(G(T)\) may increase with the temperature being lowered. How to match these two opposite tendencies? We answer this question below by finding the proper non-monotonic crossover function \(G(T)\) for an arbitrary asymmetry of the barrier and arbitrary position of the resonant level with respect to the Fermi level, in the limit of weak interaction. The asymptotes of \(G(T)\) agree with those found in Ref. [7,8] in the context of the Luttinger liquid theory. The universality of latter results was recently doubted in the theoretical part of [6]. We find no ground for such doubts.

We will use an analogue of the renormalization method developed in [9]. Within this method, the complicated picture of many-electron transport is considered within the traditional Landauer-Büttiker elastic scattering formalism. The role of the interaction is to renormalize the elastic scattering amplitudes. The renormalization brings about an extra energy dependence of these amplitudes. It was shown in [9] that in the limit of weak interaction the most divergent terms in perturbation theory indeed correspond to the purely elastic processes, thus justifying the method. The advantage of the method is that it allows one to investigate quantitatively the crossover between the limits of weak tunneling and full transmission across the barrier.

The original formulation of the method [9] disregarded the energy dependence of scattering amplitudes in the absence of interaction. While valid for a generic case of a single scatterer, this assumption obviously fails to describe the resonant tunneling. To circumvent this, we extend the method to arbitrary energy dependence of scattering amplitudes. First step is to derive the first-order interaction correction to scattering amplitudes. This can be readily done along the lines of Ref. [9]. The correction to transmission amplitude reads

\[
\delta t(\epsilon) = \frac{t(\epsilon)}{2} \int_{-\infty}^{0} \frac{d\epsilon'}{\epsilon' - \epsilon} \left[ \alpha_L r_L(\epsilon) r_L^*(\epsilon') + \alpha_R r_R(\epsilon') r_R(\epsilon) \right].
\]
Here the $r_{L(R)}$ are the reflection amplitudes for electrons incoming from the left (right), and the coefficients $\alpha_{L(R)}$ represent the interaction within the left(right) part of the 1d wire; energies $\epsilon$ and $\epsilon'$ are measured from the Fermi level. Transmission and reflection amplitudes $r_{L,R}$ satisfy the unitarity relation: $\frac{\partial t_{\epsilon}}{\partial \epsilon} = -r_{\epsilon}^{*}t$. The coefficients $\alpha$ are related to the Fourier components $V(k)$ of the corresponding electron-electron interaction potential by $\alpha = (V(0) - V(2k_{F}))/2\pi v_{F}$. In the limit of weak interaction, these coefficients determine the exponents in the edge density of states [8] for each part of the channel, $v(\epsilon) \sim \epsilon^{\alpha}$.

The integration over $\epsilon'$ in the first-order correction Eq. (1), in general, yields a logarithmic divergence at $\epsilon \to 0$. This indicates that the perturbation series in the interaction potential can be re-summed with the renormalization method. To account for the most divergent term in each order of the perturbation theory in $\alpha$, we proceed with the renormalization in a usual way [10]. On each step of the renormalization, we concentrate on the electron states in a narrow energy strip around $-E$, with $E > 0$ being the running cut-off. We evaluate the interaction correction due to the electrons in these states to the scattering amplitudes at energies $\epsilon$ close to Fermi level, $|\epsilon| < E$. These amplitudes are thus functions of both $\epsilon$ and $E$. We correct those amplitudes according to Eq. 1, reduce the running cut-off by the width of the energy strip, and repeat the procedure. This yields the following renormalization equation:

$$\frac{\partial t(\epsilon, E)}{\partial \ln E} = \frac{i}{2} [\alpha_{L} t_{\epsilon}(\epsilon, E)r_{L}^{*}(-E) + \alpha_{R} r_{R}^{*}(-E)t_{\epsilon}(\epsilon, E)],$$

provided that $|\epsilon| < E$. We abbreviate here $r(\epsilon) \equiv r(\epsilon, |\epsilon|)$ (and similar for $t$) indicating that the renormalization of scattering amplitudes stops when the running cut-off approaches $|\epsilon|$. The initial conditions for this differential equation are set at upper cut-off energy $\Lambda$. If the $\epsilon$-dependence of the transmission amplitude in the absence of interaction, $t(\epsilon, \Lambda)$, can be disregarded, then all the energy dependence of renormalized amplitudes comes about as a result of the renormalization procedure. The corresponding simplification of Eq. (2) then reads

$$\frac{\partial |t(\epsilon)|^{2}}{\partial \ln \epsilon} = (\alpha_{R} + \alpha_{L})|t(\epsilon)|^{2}(1 - |t(\epsilon)|^{2}),$$

and contains the transmission probabilities only. This coincides with the results of Ref. [9]. However, the above simplification is not possible in the more general case we consider here. One can not even deal with a single equation: the equation (2) shall be supplemented with a similar equation for one of the reflection amplitudes,

$$\frac{\partial r_{L}(\epsilon, E)}{\partial \ln E} = \frac{i}{2} [\alpha_{L}|t_{\epsilon}(\epsilon, E)r_{L}^{*}(-E) + \alpha_{R} r_{R}^{*}(-E)t_{\epsilon}(\epsilon, E)].$$

To describe resonant tunneling, we consider a compound scatterer made of two tunnel barriers with tunnel amplitudes $t_{1,2} \ll 1$ separated by a distance $\pi v_{F}/\delta$. This gives rise to a system of equidistant transmission resonances separated by energy $\delta$. We assume that one of the resonances is anomalously close to Fermi energy and concentrate on this one disregarding the others. The scattering amplitudes in the absence of interaction are then given by common Breit-Wigner relations:

$$t(\epsilon, \Lambda) = \frac{i \sqrt{\Gamma_{L} \Gamma_{R}}}{(\Gamma_{L} + \Gamma_{R})/2 - i(\epsilon - \Delta)},$$

$$r_{L}(\epsilon, \Lambda) = \frac{(-\Gamma_{L} + \Gamma_{R})/2 - i(\epsilon - \Delta)}{(\Gamma_{L} + \Gamma_{R})/2 - i(\epsilon - \Delta)},$$

where $\Gamma_{L,R} = |t_{1,2}|^{2}\delta/2\pi$ are the level widths with respect to the electron decay into the left(right) lead and $\Delta$ is the energy shift of the resonance with respect to the Fermi Level; we assume here $\Delta \ll \delta$. We disregard possible energy dependence of $t_{1,2}$ that could be relevant at higher energies, which allows us to take the upper cut-off $\Lambda$ to be of the order of $\delta$. The corresponding transmission probability before the renormalizations,

$$|t(\epsilon, \Lambda)|^{2} = \frac{\Gamma_{L} \Gamma_{R}}{(\epsilon - \Delta)^{2} + (\Gamma_{L} + \Gamma_{R})^{2}/4},$$

is the usual Lorentzian function of energy. The interaction corrections to $\Delta$ and $\Gamma_{L,R}$ which come from higher energy scales, $\delta < E < E_{F}$, are assumed to be included in the definitions of these quantities.

The next step is to solve the renormalization equations (2) and (4). To stay within the accuracy of the method, in the solution we need to retain the terms $\propto \alpha^{n}[\ln(\Lambda/\epsilon)]^{n}$ while same-order terms with a lower exponent of the logarithmic factor should be disregarded. This allows for a substantial simplification. We proceed by solving Eqs. (2) and (4) at higher energy (far from the resonance), where the reflection from the compound scatterer is almost perfect. In this case, we approximate $|r_{L,R}(-E)| \approx 1$. It is possible to see that in this case the renormalization of the tunnel amplitudes $t_{1,2}$ of each constituent of our compound scatterer occurs separate from each other, $d \ln t_{1,2}/d \ln \epsilon = \alpha_{L,R}/2$. This renormalization can be incorporated into the energy dependence of the effective level widths, $\Gamma_{R,L}(\epsilon) = \Gamma_{R,L}(\epsilon/\Lambda)^{\alpha_{R,L}}$. The result for $|t(\epsilon)|^{2}$ thus reads

$$|t(\epsilon)|^{2} = \frac{\Gamma_{L}(\epsilon)\Gamma_{R}(\epsilon)}{(\epsilon - \Delta)^{2} + (\Gamma_{L}(\epsilon) + \Gamma_{R}(\epsilon))^{2}/4}{\epsilon}. $$

The above approximation of the integrand in Eq. (2) becomes invalid at lower energies, where the transmission coefficient may become of the order of unity. The energy scale $\epsilon$ at which this occurs can be evaluated from
Eq. (5), and is given by the solution of equation $2\tilde{\epsilon} = \Gamma_L(\tilde{\epsilon}) + \Gamma_R(\tilde{\epsilon})$. In the simplest case of $\alpha_L = \alpha_R \equiv \alpha \ll 1$, it is $2\tilde{\epsilon} = (\Gamma_L + \Gamma_R)(\Gamma_L + \Gamma_R)/2\Delta^2$. At energies below $\tilde{\epsilon}$, the reflection amplitudes in the integrand can be approximated as $r(\epsilon') \approx r(\epsilon)$. Under this assumption, we immediately recover Eq. (3). At $|\epsilon| < \tilde{\epsilon}$, its solution yields
\[ |t(\epsilon)|^2 = \frac{\tilde{\Gamma}_L(\epsilon)\tilde{\Gamma}_R(\epsilon)}{(\epsilon - \Delta)^2 + \tilde{\Gamma}_L(\epsilon)\tilde{\Gamma}_R(\epsilon) + [\Gamma_L(\tilde{\epsilon}) - \Gamma_R(\tilde{\epsilon})]^2/4}. \] (6)

with
\[ \tilde{\Gamma}_{L,R}(\epsilon) = \Gamma_{L,R}(\tilde{\epsilon}) (|\epsilon|/\tilde{\epsilon})^{-\alpha + \alpha'}. \] (7)

Relations (6) and (7) determine the full crossover function for the resonant tunneling between the interacting 1d electron systems, if $\tilde{\epsilon} \gtrsim |\Delta|$.

In the opposite case of a resonance distant from the Fermi level, $|\Delta| \gtrsim \tilde{\epsilon}$, we shall change the approximation at $\epsilon = |\Delta|$. The answer is thus given by the equations (6), (7) with $\tilde{\epsilon}$ being replaced by $|\Delta|$.

A typical energy dependence of the transmission probability is sketched in the insert of Fig. 1. It combines an overall Lorentz-like shape with a sharp dip at the Fermi level. Note, that the transmission is not suppressed at low energies for a perfectly symmetric barrier with the resonant level tuned to coincide with the Fermi level, in full agreement with Ref. [7].

It may seem that the numerical factors in the definition of the crossover energy $\epsilon$ and in the condition $|\Delta| = \tilde{\epsilon}$ of the crossover between low energy cut-offs are chosen in a rather arbitrary fashion. Indeed, these two definitions could contain any other numerical factors of the order of 1. The point is that fixing the numerical factors with a greater precision would exceed the accuracy of our renormalization method. In other words, the energy dependence of $\Gamma$ in all above relations is assumed to be very slow. It is this slow dependence that, in the limit $\alpha \ll 1$, gives us the luxury of arbitrary choice of those numerical factors.

To present quantitative conclusions, we discuss the linear conductance $G(T)$ in the case of $\alpha_R = \alpha_L \equiv \alpha$. Within the Landauer formalism, the conductance is given by
\[ G(T) = G_Q \int_{-\infty}^{\infty} \frac{d\epsilon}{4T \cosh^2(\epsilon/2T)} |t(\epsilon)|^2, \] (8)

where the conductance quantum unit for one fermion mode is $G_Q = e^2/2\pi h$. The results strongly depend on the ratio of $\Gamma_R$ and $\Gamma_L$. We will characterize this ratio by the asymmetry parameter $\beta \equiv [\Gamma_L - \Gamma_R]/(\Gamma_R + \Gamma_L)$ which ranges from 0 to 1 and does not depend on energy, provided that $\alpha_R = \alpha_L$. To emphasize the effect of interaction, let us recall that in the case of free electrons one finds $G(T) \propto 1/T$ at temperatures $T \gg \Gamma, \Delta$; in the limit $T \to 0$, the conductance saturates at a finite value, which reaches $(1 - \beta^2)G_Q$ if the Fermi level is tuned to the resonance $(\Delta = 0)$. Interaction changes this picture noticeably. Let us start the discussion with the case $\Delta = 0$.

At high temperatures, $T \gtrsim \tilde{\epsilon}$, the conductance can be estimated as
\[ G_{\Delta=0}(T)/G_Q = [\pi(1 - \beta^2)/4](T/\tilde{\epsilon})^{\alpha - 1} \sim \Gamma(T)/T. \] (9)

The temperature dependence can be thus ascribed to interaction-induced power-law temperature dependence of $\Gamma$. Furthermore, the low-temperature behavior differs strikingly for symmetric ($\beta = 0$) and asymmetric ($\beta \neq 0$) resonance. For symmetric resonance, the conductance saturates at the ideal value of $G_Q$. For $\beta \neq 0$ the conductance reaches at $T \approx \tilde{\epsilon}$ its maximum value, which is smaller than $(1 - \beta^2)G_Q$, and drops to zero with the further decrease of temperature,
\[ G_{\Delta=0}(T)/G_Q = (1/\beta^2 - 1)(T/\tilde{\epsilon})^\alpha, \quad T \lesssim \tilde{\epsilon}. \] (10)

The temperature exponents at $T \lesssim \tilde{\epsilon}$ in both cases agree with those obtained in Refs. [7,8]. The exponent at $\beta \neq 0$ is the same as for a single tunnel barrier interrupting the 1d channel. It indicates that at low energies the electrons get over the compound scatterer in a single quantum transition. The high-temperature exponent arises from the separate renormalization of each barrier, which signals the “sequential mechanism” of tunneling: the electron tunnels across one barrier first and waits a while ($\approx \hbar/\Gamma(\epsilon)$) before tunneling across another one. It is important to recognize though that no energy relaxation or decoherence takes place during this waiting time. This is especially clear from our calculation based on the Landauer formula: there are no inelastic processes included which could provide for the relaxation or decoherence.

The increase of $\Delta$ leads to a decrease of the conductance. For non-interacting electrons, the conductance stays at a level of the order of its maximal value, $G_{\Delta=0}$ for $\Delta$ less than $\Gamma_L + \Gamma_R$, which determines the width of the resonance $G(\Delta)$ at $T \lesssim \Gamma_L + \Gamma_R$. At higher temperatures, the effective resonance width is $w \approx T$. Let us discuss now the temperature dependence $w(T)$ and the shape of the resonance $G(\Delta)$ at fixed temperature in the presence of interaction. For $T \gtrsim \tilde{\epsilon}$, the width $w \approx T$ does not reveal any anomalous exponent. The shape of the resonance in this regime is mainly determined by the thermal-activated exponential contribution $G(\Delta) \approx \exp(-|\Delta|/T)\Gamma(T)/T$ in Eq. (8). However, at large $\Delta \gtrsim w$ the power-law "cotunneling" tail
\[ G_{\text{tail}}(\Delta) = G_Q(1 - \beta^2)(T/\tilde{\epsilon})^\alpha e^{\tilde{\epsilon}^2/\Delta^2}, \]

replaces that exponential dependence [11]. The crossover occurs at $\Delta \approx T \ln(G_Q/G_{\Delta=0})$ and corresponds to the conductance $G_{\text{cross}} \approx G_Q^2/(G_{\Delta=0}/G_Q)$, this being much smaller than $G_{\Delta=0}$.
At $T \ll \bar{\epsilon}$ the apparent width of the non-symmetric resonance saturates at $w \approx \bar{\epsilon}$. The conductance thus drops uniformly at any $\Delta$ following the power law (10). The symmetric resonance presents an exception. In this case, the width shrinks with the decreasing temperature,

$$w(T) \approx (T/\bar{\epsilon})^\alpha \bar{\epsilon},$$

and $G(T, \Delta)$ acquires the scaling form, $G(T, \Delta) = G_{\Omega}/\{1 + [\Delta/w(T)]^2\}$, in agreement with Ref. [7].

We further illustrate our results by a numerical evaluation of Eq. (8), see Figs 1–2. For this calculation, we choose $\alpha = 0.2$. By virtue of our approach, the relative accuracy of the results is expected to be of the order of $\alpha$. The dependence $G(T)$ is not monotonic, and in the limit $T \to 0$ the conductance drops to zero at any $\beta \neq 0$, although for small $\beta$ this is noticeable only at very low temperatures (Fig. 1). The temperature dependence $w(T)$ of the width of the resonance $G(\Delta)$ is shown in the left panel of Fig. 2. If $\beta \neq 0$, this dependence saturates at some value $w(0) \neq 0$.

![FIG. 1. Temperature dependence of resonant ($\Delta = 0$) tunneling conductance. The asymmetry parameter $\beta = 0$ (top curve), 0.2, 0.4, 0.6, and 0.8 (bottom curve). For symmetric resonance ($\beta = 0$), the conductance saturates at $T = 0$. Inset: The typical energy dependence of transmission coefficient.](image)

![FIG. 2. Left: Half width at half maximum $w$ vs. temperature $T$ for the values of asymmetry parameter $\beta = 0, 0.2, 0.4, 0.6, 0.8$ (bottom to top curve). With the decreasing temperature, the half width saturates for a non-symmetric resonance, and continuously decreases for the symmetric one. Right: The conductance dependence on the position of the resonant level with respect to the Fermi level, $G(\Delta)$, for symmetric (top) and non-symmetric with $\beta = 0.5$ (bottom) resonances at three temperatures $T/\bar{\epsilon} = 0.04, 0.2, 1$.](image)

The differences and similarities of symmetric and non-symmetric resonances are further illustrated in the right panels of Fig. 2. The three pairs of line shapes there correspond to "high", "medium", and "low" temperatures, respectively. The two high-temperature curves (the smallest values of $G_{\Delta=0}$) are hardly distinguishable from each other, and correspond to the resonance width $w \approx T$. Both medium-temperature curves show a more narrow resonant peak with increased conductivity $G_{\Delta=0}$, and are still similar to each other, apart from the scale. The real difference becomes visible for the low-temperature curves. In the case of non-symmetric resonance, the low-temperature curve is just reduced in height with no noticeable change of the shape. This is in contrast to the symmetric resonance, where the resonance peak gets taller and thinner. The symmetric resonance does not seem to be very realistic since any randomness in the barriers and/or in nanowire would cause asymmetry. Provided symmetry is achieved, the symmetric resonance can be easily identified by its ideal conductance $G_{\Omega}$.

To conclude, we have investigated the transmission resonances of interacting electrons in 1d wires. For a weak electron-electron interaction the transmission can be considered as an elastic process, which allowed us to build a comprehensive theory of the resonances, valid in a broad range of temperature and parameters of the resonant level. The temperature dependence of the maximum conductance in general is not monotonic, and reveals important differences between symmetric and non-symmetric resonances. The obtained quantitative results present a comprehensive and consistent picture of the effect. It assures us in the qualitative validity of the picture at an arbitrary interaction strength. Although we are not able to come up with an explicit expression for the crossover function $G(T)$ in this case, such function, with known high- and low-temperature asymptotes, does exist by virtue of the renormalizability.

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