The absolute magnitudes of RR Lyrae stars from HIPPARCOS parallaxes*

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Abstract. Using the method of “reduced parallaxes” for the Halo RR Lyrae stars in the HIPPARCOS catalogue we derive a zero point of $0.77 \pm 0.26$ mag for an assumed slope of 0.18 in the $M_V$-[Fe/H] relation. This is 0.28 magnitude brighter than the value Fernley et al. (1998a) derived by employing the method of statistical parallax for the identical sample and using the same slope.

We point out that a similar difference exists between the “reduced parallaxes” method and the statistical parallax method for the Cepheids in the HIPPARCOS catalogue.

We also determine the zero point for the $M_K$-$\log P_0$ relation, and obtain a value of $-1.16 \pm 0.27$ mag (for a slope of $-2.33$). The distance moduli to the HIPPARCOS RR Lyrae stars derived from the two relations agree well.

The derived distance scale is in good agreement with the results from the Main Sequence fitting distances of Galactic globular clusters and with the results of theoretical Horizontal Branch models, and implies a distance modulus to the LMC of $18.61 \pm 0.28$ mag.

Key words: RR Lyrae - Stars: distances - Magellanic Clouds

1. Introduction

RR Lyrae stars are fundamental standard candles and the accurate determination of their absolute luminosity has a wide range of applications, including the derivation of the Hubble constant and the determination of globular clusters ages. The results of the HIPPARCOS mission allow in principle a calibration of this luminosity, based on the parallaxes and proper motions.

Fernley et al. (1998a, hereafter F98) did that by employing the method of statistical parallax on a sample of 84 RR Lyrae stars (out of the 144 they considered) with [Fe/H] $\leq -1.3$. Combining the statistical parallax result with the absolute magnitude of RR Lyrae itself, computed without applying any Lutz-Kelker (LK) type correction (see Lutz & Kelker 1973, Turon Lacarrue & Crézé 1977, Koen 1992, Oudmaijer et al. 1998), they derived a zero point of $1.05 \pm 0.15$ mag for the $M_V$-[Fe/H] relation, by assuming a slope of $0.18 \pm 0.03$ (Fernley et al. 1998b).

Tsujimoto et al. (1998, hereafter T98) used the statistical parallax method, a maximum likelihood technique and the derived $M_V$ of the star RR Lyrae (with LK correction included) for deriving a combined final value $M_V = 0.6-0.7$ mag at [Fe/H] $= -1.6$. Luri et al. (1998, hereafter L98) applied a maximum-likelihood method that takes all available data into account, including parallaxes, proper motions and radial velocities, considering the sample of 144 RR Lyrae stars given in F98. They derived $M_V = 0.65 \pm 0.23$ at an average metallicity of [Fe/H] = $-1.51$.

The results by F98, T98 and L98 imply dimmer RR Lyrae stars by about 0.3 mag with respect to the results from either the Main Sequence fitting technique using HIPPARCOS subdwarfs (see, e.g., Gratton et al. 1997) or recent theoretical Horizontal Branch models (see, e.g., Salaris & Weiss 1998, Caloi et al. 1997). On the other hand, F98, T98 and L98 agree with the results from Baade-Wesselink analyses, which predict a zero point of about 1.00 mag for the $M_V$-[Fe/H] relation (see, e.g., Clementini et al. 1995).

Turon Lacarrue & Crézé (1977) presented two methods to derive the absolute magnitude of stars from the observed parallaxes, namely using individual LK-corrections and the method of “reduced parallaxes” (hereafter RP) on a sample of stars. The advantages of the RP method are the following: it avoids the biases due to the asymmetry of the errors when transforming the parallaxes into magnitudes, it can be applied to samples which contain negative parallaxes, it is free from LK-type bias if no selection on parallax, or error on the parallax is made (Koen & Laney 1998), and it requires no knowledge about the space distribution of stars. We will apply the RP method to the sample of 144 RR Lyrae stars used by F98, and will derive the zero points of the $M_V$-[Fe/H] and $M_K$-$\log P_0$ relations.

* Based on data from the ESA Hipparcos astrometry satellite.

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Recently, Koen & Laney (1998) also briefly discussed the application of the RP method to RR Lyrae stars.

2. The “reduced parallax” method

Let us consider a relation of the form:

\[ M_V = \delta [\text{Fe/H}] + \rho, \]

If \( V \) is the intensity-mean visual magnitude and \( V_0 \) its reddening corrected value, then one can write:

\[ 10^{0.2 \rho} = \pi \times 0.01 \ 10^{0.2(V_0 - \delta [\text{Fe/H}])} \equiv \pi \times \text{RHS}, \]

which defines the quantity RHS and where \( \pi \) is the parallax in milli-arcseconds. A weighted-mean of the quantity \( 10^{0.2 \rho} \) is calculated, with the weight (weight = \( \frac{1}{\sigma^2} \)) for the individual stars derived from:

\[ \sigma^2 = (\pi \times \text{RHS})^2 + (0.2 \ln(10) \ \pi \sigma_H \times \text{RHS})^2, \]

with \( \sigma_{\pi} \) the standard error in the parallax. This follows from the propagation-of-errors in Eq. (2). We have adopted the slope \( \delta = 0.18 \) (see the discussion in Fernley et al. 1998b), which is the one used by F98 and which is in agreement with the results from Baade-Wesselink methods (see, e.g., Clementini et al. 1995). Main Sequence fitting (Gratton et al. 1997) and theoretical models (see, e.g., Salaris & Weiss 1998, Cassisi et al. 1999).

The sample we consider is identical to that of F98, that is 144 stars out of a total of 180 stars in the HIPPARCOS catalogue. F98 discuss the reasons for discarding the 36 stars. Arguments include the fact that these stars do not have reddening determinations, are not RR Lyrae variables, or have poor quality HIPPARCOS solutions. Table 1 of F98 (retrievable from the CDS) lists all necessary data to perform the above analysis: periods, intensity-mean \( V \) and \( K \) magnitudes, colour-excesses \( E(B-V) \), and metallicities \( [\text{Fe/H}] \). The extinction is calculated from \( A_V = 3.1E(B-V) \) (as done by F98).

An important requirement when applying this method is that the value of \( \sigma_H \) is small compared to the errors on the parallax. If the dispersion \( \sigma_H \) of the exponent in the factor RHS is large, the distribution of errors on the right-hand term in equation 2 is asymmetrical and a bias towards brighter magnitudes is introduced (Feast & Catchpole 1997, Pont 1999). The adopted value of \( \sigma_H \) has been computed by considering four different contributions: errors on the intensity-mean \( V \) values of the RR Lyrae stars (as given in Table 1 of F98), on the extinction (as derived from the errors on \( E(B-V) \) given in Table 1 of F98), on \( [\text{Fe/H}] \) (again, from Table 1 of F98), and the intrinsic scatter due to evolutionary effects in the instability strip. This last term is the most important one, and we have adopted for it a \( 1 \sigma \) value by 0.12 mag (as in Fernley et al. 1998b), following the results of the exhaustive observational analysis by Sandage (1990). The final value is \( \sigma_H = 0.15 \), a quantity small enough in comparison with the parallax errors so that no substantial bias is introduced on the right-hand term of equation 2, as we have verified by means of numerical simulations. Even a \( \sigma_H \) of 0.20 mag would lead to a bias by at most 0.02 mag.

Table 1 lists the values of the zero point with error we obtain with different sample selections for the \( M_V-[\text{Fe/H}] \) relation. Solution 1 corresponds to the case of the whole sample; the zero point of 0.67 ± 0.24 mag is about 0.4 mag brighter than the value derived by F98, and consistent with the value listed in Koen & Laney (1998) using the same method with slightly different values for \( \sigma_H \). The sample with \( [\text{Fe/H}] \leq -1.3 \) (Solution 2) corresponds to a sample constituted entirely (according to the discussion in F98) by Halo RR Lyrae stars, with a negligible contamination from the Disk population. In this case the zero point is equal to 0.77 ± 0.26 mag; it is slightly fainter than Solution 1, but well in agreement within the statistical errors. We also re-derived the zero point for Solution 2 in the case of \( \sigma_H = 0.0 \), and we found a change by only 0.04 mag. A systematic change in the metallicity scale (Solution 4) by 0.15 dex does not affect appreciably the zero point determination, while the result is more sensitive to a systematic variation of the adopted reddening (Solution 5).

The RP method has also been used to derive the zero point of the \( M_K-logP_0 \) relation. This relation appears to be insensitive to the metallicity (Fernley et al. 1987, Carney et al. 1995) and is also very weakly affected by reddening uncertainties, since \( A_K = 0.112 A_V \) (Rieke & Lebofsky 1985). Moreover, the intrinsic scatter around this relation is smaller than in the case of the \( M_V-[\text{Fe/H}] \) relation (Fernley et al. 1987). In the sample considered here there are 108 RR Lyrae stars with an observed intensity-mean \( K \) magnitude. The procedure is the same as described before, the only difference is that now, instead of Eq. 1, we use the expression \( M_K = \delta logP_0 + \rho \) where \( P_0 \) is the fundamental pulsation period. For the first-overtone RRc variables we have derived the fundamental periods using the relation \( logP_0/P_1 = +0.120 \) (Carney et al. 1995). We adopt a slope \( \delta = -2.33 \) following Carney et al. (1995); for the value of \( \sigma_H \) we have considered the same contributions previously described (with the exception, of course, of the contribution due to the error on \( [\text{Fe/H}] \)). In this case the observational estimate of the intrinsic scatter due to the width of the instability strip comes from Carney et al. (1995), and the final value results to be \( \sigma_H = 0.10 \).

In Tab. 1 the values of the zero point for the \( M_K-logP_0 \) relation are listed. When considering the entire sample we obtain a zero point of \(-1.28 \pm 0.25 \) mag, \( \approx 0.4 \) mag brighter than the value from the Baade-Wesselink method (see, e.g., Carney et al. 1995). In the case of a pure Halo RR Lyrae sample \(( [\text{Fe/H}] \leq -1.3 \) we obtain \(-1.16 \pm 0.27 \) mag, slightly dimmer but again in agreement with the value derived for the whole sample. The influence of \( \sigma_H \) is even less than for the \( M_V-[\text{Fe/H}] \) relation.

As the sample of the RR Lyrae stars is not volume complete it may be subject to Malmquist type bias. If the space distribution of RR Lyrae is spherical it
is done over 10

et al. (1999) showed empirically that when the averaging applies when average absolute magnitudes of a volume

\[ M \] stars with \([\text{Fe/H}] \leq -0.01 \text{ mag} \), respectively, for the adopted values of

implies that the true zero points of the

\[ M_{\text{K}} - \log P_0 \]

relations may be fainter by up to 0.03 and 0.01 mag, respectively, for the adopted values of \( \sigma_H \). This applies when average absolute magnitudes of a volume and brightness limited sample are compared. Oudmaijer et al. (1999) showed empirically that when the averaging is done over \( 10^{M_{\text{V}} - 0.3} \) the effect of Malmquist bias is less.

In Fig. 1 we compare, for the 62 HIPPARCOS RR Lyrae stars with \([\text{Fe/H}] \leq -1.3\) and both observed K and V magnitudes, the true distance moduli derived from the \( M_{\text{V}} - [\text{Fe/H}] \) and \( M_{\text{K}} - \log P_0 \) relations, using zero points of 0.77 and \(-1.16 \) mag, respectively. Each data point has an error bar of 0.26 mag in \( x \)- and 0.27 mag in the \( y \)-direction. The comparison of the two photometric distances can in principle give us an independent indication for possible biases in the determination of the zero points of the two relations with the RP method. As it is evident from the figure, the distance moduli from both relations agree very well. A linear fit to the data is consistent with a slope of unity, and the dispersion around the 1:1 relation is equal to 0.098 mag. A dispersion of this order is what is expected from the dispersions in the observed \( \log P - [\text{Fe/H}] \) and \( (V - K)_0 - \log P \) relations for the RR Lyrae sample.

3. Discussion

For their preferred sample of 84 stars with \([\text{Fe/H}] \leq -1.3\) F98 obtain a zero point of \( 1.05 \pm 0.15 \) mag for the \( M_{\text{V}} - [\text{Fe/H}] \) relation (assuming a slope of 0.18), in agreement with results from Baade-Wesselink methods. When applying the RP method to the same sample of stars, we find a zero point 0.28 mag brighter. An analogous result, which means a zero point \( \approx 0.30 \) mag brighter than the Baade-Wesselink one, is derived for the \( M_{\text{K}} - \log P_0 \) relation.

Even if within the error bar the results derived with the different methods formally agree, there appears to exist a systematic difference between zero points obtained using the parallaxes directly and zero points obtained by employing methods which are sensitive to proper motions and radial velocities (F98, T98, L98), especially if one also takes into account the results for the HIPPARCOS Cepheids. Also with the Cepheids one finds that methods where the results are mostly sensitive to the proper motions and radial velocities find dimmer zero points for the Cepheids PL-relation compared to methods which directly use the parallax. In particular, using the RP method Feast & Catchpole (1997) derived a zero point of \(-1.43 \pm 0.10 \) mag, and Lanoix et al. (1999) using a slightly bigger sample find \(-1.44 \pm 0.05 \) mag. Oudmaijer et al. (1998), using only the positive parallaxes but then correcting for the LK-bias, find \(-1.29 \pm 0.08 \) mag. On the other hand, L98 find a zero point of \(-1.05 \pm 0.17 \) mag using a maximum likelihood method that takes into account parallaxes, proper motions and velocity informations. As discussed by Pont (1999), in this technique the parallaxes do not influence the result to first order, and the method is similar to a statistical parallax analysis. A careful check of all assumptions implicit in the kinematical methods could be the key to understanding the nature of this puzzling disagreement. In the case of the RP method, as discussed extensively in the previous section, the condition for deriving the zero point without introducing a bias is to have \( \sigma_H \) small with respect to

Table 1. Values for the zero point of the \( M_{\text{V}} - [\text{Fe/H}] \) and \( M_{\text{K}} - \log P_0 \) relations from the RP method

| Solution | \( M_{\text{V}} - [\text{Fe/H}] \) | Total | Slope | \( M_{\text{K}} - \log P_0 \) | Total | Slope | Remarks |
|----------|-----------------|-------|-------|-----------------|-------|-------|---------|
| 1        | 144             | 0.67 ± 0.24 | 45.5  | 0.18            | 108  | -1.28 ± 0.25 | 241.3 | -2.33 | whole sample |
| 2        | 84              | 0.77 ± 0.26 | 35.3  | 0.18            | 62   | -1.16 ± 0.27 | 188.4 | -2.33 | \([\text{Fe/H}] \leq -1.3\) |
| 3        | 84              | 0.81 ± 0.24 | 40.0  | 0.18            | 62   | -1.14 ± 0.26 | 201.4 | -2.33 | \([\text{Fe/H}] \leq -1.3, \sigma_H = 0\) |
| 4        | 144             | 0.64 ± 0.24 | 46.7  | 0.18            |       | -1.28 ± 0.25 | 242.8 | -2.33 | as 1, all \([\text{Fe/H}]\) larger by 0.15 dex |
| 5        | 144             | 0.60 ± 0.24 | 48.2  | 0.18            | 108  | -1.28 ± 0.25 |       |       | as 1, all \( E(B - V) \) larger by 0.02 |

Fig. 1. A comparison of the true distance moduli to the 62 metal-poor RR Lyrae with \([\text{Fe/H}] \leq -1.3\) from the \( M - [\text{Fe/H}] \) and \( M_{\text{K}} - \log P \) relations. Each point has an error bar of about 0.26 mag in both \( x \)- and \( y \)-direction. The solid line is the 1:1 relation. The dispersion is less than 0.10 mag.
the errors on the parallaxes; this condition appears to be fulfilled in the sample considered.

Our zero point for the $M_V$-[Fe/H] relation is in agreement with results from the Main Sequence fitting technique (Gratton et al. 1997), and from theoretical Horizontal Branch models. In particular, the Horizontal Branch models by Salaris & Weiss (1998) and Cassisi et al. (1999) give a zero point for the Zero Age Horizontal Branch (ZAHB) at the RR Lyrae instability strip in the range 0.74-0.77 mag. To compare the results for the ZAHB with the $M_V$-[Fe/H] relations mentioned in this paper which consider the mean absolute brightness of the RR Lyrae stars population at a certain metallicity, one has to apply a correction by $\approx -0.1$ mag (see, e.g., Caloi et al. 1997 and references therein) to the ZAHB result; this takes into account the evolution off the ZAHB of the observed RR Lyrae stars. Even after applying this correction the theoretical results are in good agreement with the results from the RP method. Moreover, the zero point derived with the RP method is also in agreement with the recent results by Kovacs & Walker (1999), who derive, by employing linear pulsation models, RR Lyrae luminosities that are brighter by 0.2-0.3 mag with respect to Baade-Wesselink results.

Finally, we want to derive the LMC distance implied by our zero point of the RR Lyrae distance scale. Table 2 collects the available data on RR Lyrae stars in LMC clusters: the name of the cluster, the observed mean $V$-magnitude, reddening, metallicity and the difference in distance modulus ($\Delta$) between the cluster and the main body of the LMC. All these data are taken from the references listed. From them the dereddened magnitude at the centre of the LMC (Col. 7), and this value minus the quantity $0.18 \times$ [Fe/H] (Col. 8) have been calculated for those clusters with $\Delta <0.1$ mag. At this point we have taken into account the difference in metallicity between the clusters before deriving the LMC distance. More in detail, we have derived the weighted mean of the values in Col. 8 to find an average of $19.38$ with a rms dispersion of $0.10$ mag, which can be compared directly to the zero point of the $M_V$-[Fe/H] relation to find a distance modulus of $18.61 \pm 0.28$. This result turns out to be consistent with the Cepheids distance to the LMC as derived by Feast & Catchpole (1997) or Oudmaijer et al. (1998).

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Table 2. Data on RR Lyrae in LMC clusters

| Name         | $<V>$ | $A_V$ | [Fe/H] | $\Delta$ | Reference       | $V_0 + \Delta$ | $V_0 + \Delta - 0.18$ [Fe/H] |
|--------------|-------|-------|--------|---------|----------------|----------------|--------------------------------|
| NGC 1466     | 19.33 | 0.28  | -1.85  | 0.0     | Walker 1992b   | 19.05          | 19.38                          |
| NGC 1786     | 19.27 | 0.23  | -2.3   | 0.0     | Walker & Mack  | 19.04          | 19.45                          |
| NGC 1835     | 19.38 | 0.40  | -1.8   | -0.03   | Walker 1993    | 19.84          | 19.26                          |
| NGC 2210     | 19.12 | 0.19  | -1.9   | +0.09   | Reid & Freedman| 19.02          | 19.36                          |
| Reticulum    | 19.07 | 0.09  | -1.7   | -0.08   | Walker 1992a   | 18.91          | 19.22                          |
| NGC 1841     | 19.31 | 0.56  | -2.2   | (~ 0.2) | Walker 1990    |                |                                |
| NGC 2257     | 19.03 | 0.12  | -1.8   | +0.18   | Walker 1989    |                |                                |