Testing the Copernican Principle with Hubble Parameter

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By way of expressing the Hubble expansion rate for the general Lemaître-Tolman-Bondi (LTB) metric as a function of cosmic time, we test the scale on which the Copernican Principle holds in the context of a void model. By performing parameter estimation on the CGBH void model, we show the Hubble parameter data favors a void with characteristic radius of 2 ~ 3 Gpc. This brings the void model closer, but not yet enough, to harmony with observational indications given by the background kinetic Sunyaev-Zeldovich effect and the normalization of near-infrared galaxy luminosity function. However, the test of such void models may ultimately lie in the future detection of the discrepancy between longitudinal and transverse expansion rates, a touchstone of inhomogeneous models. With the proliferation of observational Hubble parameter data and future large-scale structure observation, a definitive test could be performed on the question of cosmic homogeneity.

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Introduction — The Copernican Principle (CP) is the hypothesis that we do not occupy a privileged position in the Universe. It leads to the Friedmann-Robertson-Walker (FRW) metric as the metric of the homogeneous and isotropic background spacetime [1]. However, one may not expect the CP to hold on all scales of cosmological interest, for both theory and observation shows that large-scale structure can emerge even if a homogeneous and isotropic initial background is assumed. Recently, the observed near-infrared luminosity function from a complete sample of galaxies indicates that the data cannot rule out the possibility of our vicinity being described by a void model [2]. In addition, the void model may also serve as a possible explanation to the emergence of accelerated expansion of the Universe without employing an exotic component dubbed ‘dark energy’. To further investigate the ramifications of such a non-CP scenario and ascertain the possible existence of a local void, we shall consider a variety of other cosmological tests, as laid out in the following Letter. We mainly make use of the observational Hubble parameter data (OHD) which is independent of CMB and galaxy distribution measurement, and their observational properties have not been elucidated well enough in the inhomogeneous void model.

LTB dynamics and the void model — The Lemaître-Tolman-Bondi (LTB) line element reads

\[
\frac{ds^2}{ds^2} = -dt^2 + \frac{A'(r,t)^2}{1-k(r)} dr^2 + A^2(r,t) d\Omega^2, \tag{1}
\]

where \( \frac{d}{dr} \) denotes \( \partial / \partial r \), and \( k(r) \) is associated with the spatial curvature. The Friedmann-Robertson-Walker (FRW) metric can be recovered by imposing \( A(r,t) = a(t)r \) and \( k(r) = kr^2 \).

From the LTB metric one can go on writing down and solving the dynamical equations for LTB void models.

One notices along the way that the spherical symmetric configuration gives rise to two expansion rates

\[
H_\perp \equiv \frac{\dot{A}}{A}, \quad H_\parallel \equiv \frac{\dot{A}}{A} \tag{2}
\]

After choosing a gauge \( A_0(r) = r \), and a homogeneous ‘bang time’, one needs only the boundary conditions to finally obtain the evolution history (see Ref. [3, 4] for more detailed treatments). Expressed as two functions, \( \Omega_m \) and \( H_{\perp,0} \), these boundary conditions define an LTB void model. Throughout this work, we employ the Constrained GBH (CGBH) model [5], in which

\[
\Omega_m(r) = 1 + (\Omega_0 - 1) \left( \frac{1 - \tanh[(r-r_0)/2\Delta r]}{1 + \tanh(r_0/2\Delta r)} \right), \tag{3}
\]

where \( \Omega_0 \) describes the density at the symmetric center, \( r_0 \) is the characteristic size of the void, and \( \Delta r \) describes the steepness of the void near the edge.

To illustrate how the universe and its evolution in CGBH model look like, we choose \( \Omega_0 = 0.05 \), \( H_0 = 73 \text{ km s}^{-1} \text{ Mpc}^{-1} \), \( r_0 = 6 \text{ Gpc} \), \( \Delta r = 0.1 \text{ r}_0 \), and plot in Fig. 1 and Fig. 2 the density profile on different cosmic time (in Gyr) slices and on the light cone, respectively, and in Fig. 3 and Fig. 4 the profiles of the two expansion parameters on time slices and the light cone, respectively.

In a homogeneous universe one always has \( H_\parallel = H_\perp \). Observation of a difference, like that shown in Fig. 4 at any redshift, would imply spatial inhomogeneity.

Observational Hubble parameter data — There are three main methods to measure the Hubble parameter \( H(z) \): by measuring the differential age of passively evolving galaxies (differential age method) [6, 11], by measuring the baryon acoustic oscillation (BAO) along the line-of-sight direction from the spectroscopic galaxy sample [12], and the dipole of the luminosity distance \( d_L \) of gravitational wave sources (luminosity dipole method of standard sirens) [13, 14]. The radial BAO size method depends on the detailed evolution of perturbations not.
well understood in the LTB cosmology, although progresses have been made [15,16]. The luminosity dipole method of standard sirens by now has produced no observational data yet. Therefore, the OHD used in this work refer exclusively to that by the differential age method.

The Hubble parameter for FRW models with scale factor $a$ reads

$$H \equiv \frac{\dot{a}}{a} = -\frac{1}{1+z} \frac{dz}{dt_{ct}}, \quad (4)$$

where $dt_{ct}$ is the variation of the cosmic time due to a small change in the redshift $dz$. For any galaxy one has $T_{CA}(z) = T_F + T_{GA}(z)$, which simply states that the cosmic age $T_{CA}$ at redshift $z$ equates the summation of the formation time of the galaxy, $T_F$, and the age of this galaxy, $T_{GA}$ that can be determined spectroscopically. If we could find a group of galaxies that share a uniform formation time, i.e. $T_F = \text{const.}$, we would then get a handle of $dt_{ct}$ by simply measuring the age difference of those galaxies: $dt_{ct}(z) = dT_{CA} = dT_F + dT_{GA}(z) = dT_{GA}(z)$. The passively evolving galaxies can be identified by figuring out at every redshift the oldest galaxies, which together define the 'red envelop'. One assumes in this process the oldest galaxies formed at the same time (standard cosmic chronometers), which is a natural assumption in an FRW universe (one may call it the galaxy-formation version of the cosmic Copernican Principle).
Of the two expansion rates defined in Eq. (2), the longitudinal expansion rate $H_\parallel$ turns out to have the same form as Eq. (1), $H_\parallel = -[1/(1+z)](dz/dt_\parallel)$, hence corresponds to the observed $H(z)$ [4].

The problem of using OHD in LTB models is that the basic assumption that the oldest galaxies share a same formation time might not hold any more, as discussed recently in Ref. [4], because the background in LTB models has considerable inhomogeneities. However, we argue that OHD is still valid in our context (see Discussion). In the following we will use the latest 23 data entries as listed in Refs. [9, 11].

Constraints on the void model — We perform the constraints on the void model by both OHD and the background inhomogeneity-induced kSZ (BIkSZ) effect [17, 18]. The direction dependent, therefore observable, temperature shift reads

$$\Delta T_{\text{BI}} = T_{\text{CMB}} \times \int_0^{z_e} \delta_c(\hat{n}, z) \frac{\ddot{v}_H(\hat{n}, z) \cdot \hat{n}}{c} d\tau_e,$$

(5)

where $T_{\text{CMB}} = 2.73$ K, $z_e = 100$ (the result essentially does not change as long as $z(\tau_0) \ll z_e$), and

$$v_H(z) \approx [H_\parallel(r(z), t(z)) − H_\parallel(r(z_e), t(z))]A(r(z), t(z)).$$

(6)

We calculate BIkSZ power spectrum $\Delta T_{\text{BI}}^2$ at $l \approx 3000$ and its constraints on the $(r_0, \Omega_0)$ parameter plane, with $\Delta r/r_0 = 0.21$, $H_0 = 74$ km s$^{-1}$ Mpc$^{-1}$ fixed at their respective best-fit values, which are in turn obtained from the Hubble parameter dataset. The resulting contours, as well as confidence regions from the OHD and the supernovae Union2 dataset, are plotted in Fig. 5. The OHD data favor a smaller (and more tightly constrained) void than what the supernovae Union2 data do. Indeed, there is a clear discrepancy between those two datasets, as found in Ref. [4]. This is a sign of inadequacy for this specific LTB model. Also, as pointed out in Ref. [17], one can see from Fig. 5 that the Gpc-sized voids, as those favored by the supernovae data, are incompatible with the BIkSZ measurement, hence are largely excluded. Now we can tell from the figure that the OHD dataset give slightly weaker, though basically the same, conclusion.

Furthermore, the observed normalization of the near-IR galaxy luminosity function indicates that a void, if exists, amounts to a few hundred Mpc [2]. This could in principle be consistent with the BIkSZ measurement.

Future BAO constraint — As is shown above, unlike the homogeneous cosmological models, $H_\parallel$ can differ from $H_\perp$ in LTB models. Therefore one straightforward way is to define the ratio between these two expansion rates $E = H_\parallel/H_\perp$, which always equals to 1 for the homogeneous cosmological models, but deviates from 1 in LTB models. Specifically, we can further write $E$ as

$$E = H_\parallel/H_\perp = 1 + \frac{\Delta}{\Delta_{\parallel}} H_\perp,'$$

(7)

since $H_\parallel = \frac{\Delta_{\parallel}'}{\Delta_{\parallel}} H_\perp + \frac{A'}{A} H_\perp'$.

Therefore, the violation of $E = 1$ could also be an indicator of LTB-type models. To adopt this criteria however, one needs independent measurements of $H_\parallel$ and $H_\perp$ at the same redshift or just the variation of $H_\perp$ on the light cone. The BAO feature imprinted in the non-relativistic matter such as galaxies distribution yields a further geometric test of homogeneity, and future large-volume BAO surveys will also allow us detect the BAO scale in both radial and transverse directions. Thus, we expect future BAO measurement would supply the information of the criteria $E$ by radial Hubble parameter $H_\parallel$ and transverse Hubble parameter $H_\perp$, and improve greatly the testing of violation of homogeneity.

Discussion — In our calculation of OHD constraints on the void model, we assume that OHD could be used in LTB models. Actually, the same formation time of the oldest galaxies is a basic assumption in obtaining OHD. However, in LTB models where the universe has a considerable background inhomogeneity, this assumption becomes unreasonable and some arguments are also recently given in Ref. [23] where galaxy ages are used. First, despite the overall uniform-formation-age assumption, the validity of an $H(z)$ data point requires the same formation time only inside the redshift bin where OHD is locally defined and obtained (Eq. [1]), even if the global density – hence the formation time of the oldest galaxies...
at different redshifts varies much. Secondly, a standard viewpoint (referred to as the onion approximation) is to treat the LTB void universe as a group of thin shells structured together, and inside each of these spherical shells the matter is homogeneously distributed [24]. For the OHD used in this paper, the size of each redshift bin is between 0.1 and 0.15, where the first limit is so chosen that the age evolution between the two bins is larger than the error in the age determination [7]. As the precision of the age determination improves, we expect an even smaller bin size. To be sure about the validity of OHD used in LTB models, one needs the exact knowledge about the thickness of the shell given the size of a redshift bin, as well as the steepness of the density profile at the time the oldest galaxies formed. We discussed this issue in Ref. [4].

On the other hand, future observation is expected to yield $\sim 2000$ measurements for passively evolving galaxies in the redshift range $0 < z < 1.5$ in the future [7]. It has been estimated that about 1000 OHD entries at a 15% accuracy level will be determined with 10% error of the galaxies ages. In Ref. [25] the power of OHD in the context of $\Lambda$CDM model has been assessed. With the increase of high quantity OHD, the power of OHD constraining void model should also be greatly improved for constraining the void models [4].

Although the large void model appear to be ruled out by some cosmological observations, future OHD measurement in both radial and transverse directions, as an alternative and complementary cosmological test, could give a tight constraint on LTB model with a small void. If the transverse BAO information can be realized from future large-scale structure observations, we should be able to arrive at a definite test of spatial homogeneity of the Universe. In this context, the role played by the transverse BAO is complementary to the radial BAO discussed in Ref. [26].

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[1] R. M. Wald, General Relativity (The University of Chicago Press, Chicago, Illinois, 1984) Chap. 5.