Interference scheme to measure light-induced nonlinearities in Bose-Einstein condensates

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Light-induced nonlinear terms in the Gross-Pitaevskii equation arise from the stimulated coherent exchange of photons between two atoms. For atoms in an optical dipole trap this effect depends on the spatial profile of the trapping laser beam. Two different laser beams can induce the same trapping potential but very different nonlinearities. We propose a scheme to measure light-induced nonlinearities which is based on this observation.

In the framework of the mean-field theory an atomic Bose-Einstein condensate (BEC) represents a nonlinear system where the nonlinearity is caused by atomic collisions. Virtual photon exchange in a BEC of neutral polarizable atoms, caused by off-resonant optical laser radiation with the Rabi frequency and the detuning leads to additional nonlinearities in the condensate’s equation of motion. Since the optically induced nonlinearity (OINL) is strongly suppressed for large detuning, it can be safely neglected for atoms trapped in optical dipole potentials. However, for smaller detunings it must be taken into account. It has been theoretically shown that the OINL in a BEC can lead to nonlinear diffraction of ultracold atomic beams, laser-induced gravity, and one can create Thringing and gap solitons, photonic band gaps and defect states in a periodic BEC.

Inspite of the fact that the OINL can be of the same order of magnitude as collisional nonlinearities, the observation of the OINL is difficult. Due to spontaneous emission an atomic BEC in a largely detuned laser field typically decoheres at a rate of \( \tau_{\text{dec}} = \frac{j}{3} \frac{g^2}{\hbar} \), where \( \hbar \) is the spontaneous emission rate of a single atom in free space (see, for instance, Ref. [4]). Hence, in order to avoid spontaneous emission one has to consider only short interaction times \( \tau \) such that \( \tau_{\text{dec}} \gg \tau \). In this case, the effect of the OINL essentially amounts to a small nonlinear phase imprint on the BEC. The goal of this paper is to develop a scheme to measure this phase imprint.

We would like to note that decoherence of another type can be caused by the fact that there is a distance (the so-called Condon distance) at which a pair of resonant with the light, even if the light is detuned from the atomic resonance for a single atom. However, we neglect this type of decoherence, because in the case of \(^{87}\)Rb at the typical density of \(10^{14} \text{ cm}^{-3} \) and the detuning \( 1 \text{ GHz} \), the Condon distance is almost five times smaller than the mean interatomic distance.

The detailed derivation of the OINL in a BEC was given in Ref. [4]. Its features are determined by the properties of the local electric field, which governs the evolution of the condensate’s wave function in the field of electromagnetic radiation. In the case of low light intensity and in the slowly varying (in time) amplitude approximation, we have the following equation for the local electric-field strength \( \mathbf{E}_\text{loc}(\mathbf{x}, t) \)

\[
\mathbf{E}_\text{loc}(\mathbf{x}, t) = \mathbf{E}_\text{in}(\mathbf{x}) + \sum_k \frac{3 \cos^2 \theta}{R^3} \frac{1}{1 + \frac{3 \cos^2 \theta}{R^2} \left( \frac{k^2 \cos^2 \theta}{R} \right)} \cdot \mathbf{E}_\text{loc}(\mathbf{x})
\]

where \( R = |\mathbf{x}| \), \( \theta = \frac{1}{2} \tan^{-1}(\mathbf{k} \cdot \mathbf{P}) \) is the medium polarization, \( \theta = \frac{1}{2} \tan^{-1}(\mathbf{k} \cdot \mathbf{P}) \) the atomic polarization, and \( \theta \) the angle between the vector \( \mathbf{R} \) and the dipole axis. \( \mathbf{E}_\text{loc}(\mathbf{x}) \) is an incident laser field with the frequency \( \omega = \omega_L \), which is close to the frequency \( \omega_0 \) of the electric-dipole transition. The integral term describes dynamical dipole-dipole interaction and leads to the OINL in the dynamics of the condensate’s wave function. As it was shown in Ref. [4], Eq. (1) describes subsequent pair interactions between the atoms when an off-resonant laser photon enters the medium of ground-state atoms and virtually excites an atom, then this atom goes back to the ground state and emits a photon, which is absorbed by another atom and so on. We would like to stress that the mechanism described above is different from the (quasi)photoassociation, when a pair of atoms first absorbs a photon and undergoes a virtual transition to an electronically excited quasimolecular state and then it reemits the photon and returns to the initial electronic state.

The dipole field in Eq. (1) contains three terms, which have different dependence on \( R \). The first one (1\( = R^3 \)) is a near field and gives a significant contribution at distances much smaller than the laser radiation wavelength. The last one (1\( = R^2 \)) is a far field and gives a dominant contribution at distances greater than the wavelength. At distances of the order of wavelength all the three terms are of the same order of magnitude. They must be taken into account for condensates with a typical size of several optical wavelengths.

The near and far fields have different dependencies on the angle \( \theta \). The sign of the near-field term can be either negative or positive depending on \( \theta \) and the near field vanishes after the angular integration in the case of a homogeneous density distribution. The sign of the far field does not depend on \( \theta \) and it does not vanish for any atomic distribution. This leads to an attractive effective nonlinear potential in a large system of interacting optically induced dipoles.

The above discussion refers to the microscopic electrodynamics of a BEC. Since usually the condensate density does not change significantly over one wavelength, the nonlinear optical potential can also be described in terms of a macro-
Our scheme to measure the OINL is based on the observation that \( V_{\text{opt}}(\kappa) \) can induce about the same mechanical forces on atoms even for very different laser beam profiles. A doughnut laser beam with Rabi frequency \( \Omega_{\text{do}}(\kappa) = \frac{\hbar}{m} \kappa \frac{\exp(-\frac{r^2}{\kappa^2})}{\exp(-\frac{r^2}{\kappa^2})} \exp(i \kappa z) \exp(i \theta) \) and a Gaussian laser beam with \( V_{\text{Gauss}}(\kappa) = \frac{\hbar}{m} \kappa \frac{\exp(-\frac{r^2}{\kappa^2})}{\exp(-\frac{r^2}{\kappa^2})} \exp(i \kappa z) \) produce the optical potentials

\[
V_{\text{do}} = \frac{\hbar}{m} \kappa \frac{\exp(-\frac{r^2}{\kappa^2})}{\exp(-\frac{r^2}{\kappa^2})} \exp(i \kappa z) \exp(i \theta) ;
\]

\[
V_{\text{Gauss}} = \frac{\hbar}{m} \kappa \frac{\exp(-\frac{r^2}{\kappa^2})}{\exp(-\frac{r^2}{\kappa^2})} \exp(i \kappa z) \exp(i \theta) ;
\]

\[
\frac{M \cdot \Omega_{\text{do}}^2}{2} = \frac{\hbar}{m} \kappa \frac{\exp(-\frac{r^2}{\kappa^2})}{\exp(-\frac{r^2}{\kappa^2})} \exp(i \kappa z) \exp(i \theta) \left( \frac{\exp(-\frac{r^2}{\kappa^2})}{\exp(-\frac{r^2}{\kappa^2})} \exp(i \kappa z) \exp(i \theta) \right) \]

holds. Note that \( \kappa \) must be positive and \( \text{Gauss} \) negative in order to provide a trapping potential.

A very important physical difference between these two potentials is that in the doughnut potential atoms are low-field seekers, whereas in the Gaussian beam they tend to accumulate at positions with high laser intensity. This implies that the atoms experience a strong OINL in a Gaussian beam, but a negligible nonlinearity in a doughnut beam. This effect is the fundamental phenomenon on which our proposal is based on.

We would like to emphasize that in this situation any difference in the evolution of the two BEC components \( j = \pm \) would be produced by the OINL alone. In the harmonic approximation, the trapping potentials only differ by an unimportant spatially homogeneous value \( \kappa \). Furthermore, for \(^{87}\text{Rb} \) the antisymmetric scattering length \( a_{\pm} \) is negligible (see numerical values below), so that the collision-induced nonlinearities are identical for both components.

The measurement scheme consists of the following steps.

(i) \( \text{Raman pulse preparing a superposition of the states } j = \pm \text{ and } j = \pm \). We assume that the BEC is initially prepared in the state \( j = \pm \) with a spatial wave function \( \chi_{\pm 0} \) corresponding to the stationary collective ground state in a doughnut beam with \( \pm \) polarization \( (= \text{do}) \). The detuning of the doughnut beam is huge so that both the spontaneous emission and the OINL can be neglected. Half of the atoms are then transferred to the state \( j = \pm \) by a \( \pm \) laser pulse. Since powerful experimental techniques such as stimulated Raman adiabatic passage (\( \text{\ A2} \)) do exist, we can assume that the transfer is nearly perfect and can be described by the unitary operator

\[
U = \exp \left[ \frac{i}{\hbar} \int \frac{v_{\text{opt}}(\kappa)}{2} \right] \exp \left[ \frac{i}{\hbar} \int \frac{v_{\text{opt}}(\kappa)}{2} \right] \exp \left[ \frac{i}{\hbar} \int \frac{v_{\text{opt}}(\kappa)}{2} \right] \]

where \( \psi \) is the phase of the \( \psi \) polarized Stokes laser. It can be freely chosen and will be discussed later. After this transformation with the duration \( \tau = 2 \), the BEC is in a coherent state.
superposition of the states $j \neq i$ with the wave functions

$$\langle \chi; T = 2 \rangle = \frac{\langle \chi; 0 \rangle}{2} e^{i \tau = h} e^{i \nu} ;$$

(9)

(ii) Phase imprint. In the second step, we apply an additional polarized Gaussian laser beam to the BEC so that $+ = \text{Gauss}$. Both detuning and intensity of this beam are much smaller than that of the doughnut beam and are chosen so that relation (7) holds. For small enough detuning the OINL is not negligible for this beam but completely negligible for the doughnut beam. Thus, only the wavefunction for atoms in state $j \neq i$ is changed by the OINL. Since all other parameters are held constant this is the only change the system experiences beside the phase shift caused by the potential difference $\nu$.

Since the interaction time $T$ is small as compared to $\tau_{\text{dec}}$, it has to be of the order of $10^{-3}$ s in a realistic experiment. For such a short interaction time, the spatial distribution of rubidium atoms does not change much and, therefore, it is possible to neglect the center-of-mass motion. The wave functions at time $t = T + T' = 2$ then can be written in the form

$$\langle \chi; T \rangle = \frac{\langle \chi; 0 \rangle}{2} e^{i \tau = h} e^{i \nu} ;$$

(10)

$$+ \langle \chi; T \rangle = \frac{\langle \chi; 0 \rangle}{2} e^{i \tau = h} e^{i (\nu + \nu_{\text{OINL}})} ;$$

where $\nu_{\text{OINL}} = \nu_{\text{opt}}$, $\langle \chi; 0 \rangle$ $j \neq i$ $T = h$ is the phase shift produced by the OINL, i.e., the quantity we are interested in. is the chemical potential and $\nu$ $\tau_{\text{dec}} = \nu T = h$.

(iii) Second $= 2$ Raman pulse. After the phase imprint the Gaussian laser beam is switched off. The interferometer scheme is completed by applying a second $= 2$ Raman pulse that is identical to the first one. After this pulse a number of

$$N = j \langle \chi; 0 \rangle (\nu_{\text{OINL}} + 2 \tau_{\text{s}} + \nu) \frac{2}{2} \mathrm{d}x$$

(11)

atoms are left in the state $j \neq i$, where they are trapped by the doughnut beam. The atoms in the state $j \neq i$ are not trapped anymore and are dispersed in a time that typically is of the order of a few milliseconds. To make $N$ a signal of the OINL alone, one has to eliminate the influence of the potential difference $\nu$ which can be done by choosing $\tau_{\text{s}} = 2 \tau_{\text{dec}}$. In this case, $N$ vanishes in the absence of the OINL.

In order to get concrete estimates, we have considered a BEC of $N = 10^5$ rubidium atoms ($^{87}$Rb, $M = 145$ $10^{-25}$ kg, $a_s=5\text{\,nm}$, $a_{\text{d}}=0.05\text{\,nm}$ $13$, $\omega = 780$ nm, $\omega_{\text{opt}} = 38$ MHz) in a trapping potential of the form $5$ and with the parameters $14$ $\omega_{\text{opt}} = 3\Delta \omega$ $10^5$ Hz (which corresponds to the oscillation frequency $\omega_{\text{opt}} = 576$ Hz), $\omega = 10$ m. Using these parameters, we have performed analytical two-dimensional (2D) calculations along the lines described above by employing the Thomas-Fermi approximation for the calculation of the ground state wave function $\langle \chi; 0 \rangle$. The condensate has been assumed to be homogeneously extended along the $z$ direction with a length of $L_z = 20$ m, which gives the following value of the Thomas-Fermi radius: $R_{TF} = 2 \omega_{\text{opt}} M / \omega_{\text{opt}} \approx 2 (\omega_{\text{opt}} + a) N = L_z$. The number of atoms remaining in the trapped state $N$ can be worked out according to Eq. (11), assuming that the condensate size is much smaller than the laser width $\omega$. The result is given by

$$N = \frac{1}{2} \sin^2 \frac{n}{w} \cos^2 \frac{1}{w_{\text{Gauss}}} ;$$

(12)

and is shown in Fig. 2 (dashed line) for the interaction time $T = 10$ s. As one clearly sees, the number $N$ of trapped atoms increases with the increase of the light-induced nonlinearity which is proportional to $\omega_{\text{Gauss}} / \omega_{\text{opt}}$ and reaches about 0.75$N$ at $\omega_{\text{Gauss}} / \omega_{\text{opt}} = 0.0025$. However, to fulfill $T_{\text{dec}}$ the parameter $\omega_{\text{Gauss}} / \omega_{\text{opt}}$ must not be higher than about 0.001, i.e., values of $N$ up to 0.25$N$ could be observable.

To confirm our analytical results, we have also performed 2D numerical calculations in which the center-of-mass motion, the full form of the potentials $V_{\text{opt}}$, and all nonlinearities are taken into account. The initial ground-state wave function has been obtained by numerical solution of the two-dimensional Gross-Pitaevskii equation. For the time evolution, we have employed the split-step method (see, e.g., Ref. $16$) and combined it with the imaginary time propagation technique (see, e.g., Ref. $17$) to find the ground state.

Starting from state $5$, we have solved the time evolution of the BEC which is governed by Eq. (11). After that the transformation $U$ of the two-component wave function has been performed and the number of atoms in the trapped state $N$ has been worked out. The dependence of $N$ on the parameter $\omega_{\text{Gauss}} / \omega_{\text{opt}}$ is shown in Fig. 2 (dots). The results are in good agreement with the approximate analytical calculations. However, the Thomas-Fermi approximation leads to somewhat higher values of $N$ compared to the exact numerical calculations, because in neglecting the center-of-mass mo-
tion the Thomas-Fermi approximation gives a slightly higher central density. The comparison of our analytical and numerical calculations shows that the center-of-mass motion does not influence significantly $N$, which is mainly determined by the form of the initial atomic distribution and the parameters of the laser radiation in the middle of the trap. After the second Raman pulse those atoms transferred to state $j^+i$ are dispersed within about $1$ ms, because the Gaussian laser is switched off. Since the transfer is not adiabatic the number of atoms in the state $j^-i$ is reduced, but the shape of the wave function remains almost the same. If there would be no atomic collisions this would be again a stationary state of the corresponding Schrödinger equation, but in the presence of collisions this is not the case and the remaining cloud of atoms in state $j^-i$ starts to oscillate radially. They can be detected by taking an absorption image with resonant laser light.

In the present paper, we have developed an interference scheme that allows to measure the OINL caused by dynamical dipole-dipole interaction in the field of an optical laser. The observation of this effect would help to improve our understanding of the interactions between light and atoms.

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