Performance Analysis of mmWave Ad Hoc Networks

Andrew Thornburg, Tianyang Bai, and Robert W. Heath Jr.

Abstract

Ad hoc networks provide a flexible, infrastructure-free means to communicate between soldiers in war zones, aid workers in disaster areas, or consumers in device-to-device (D2D) applications. Ad hoc networks, however, are still plagued by interference caused by uncoordinated transmissions which leads to poor scaling due to distributed coordination. Communication with millimeter-wave (mmWave) devices offers hope to ad hoc networks through higher bandwidth, reduced interference due to directional antennas, and weaker interference power due to building blockage. This paper uses a stochastic geometry approach to characterize the one-way and two-way signal-to-interference ratio (SINR) distribution of a mmWave ad hoc network with directional antennas, random blockages, and ALOHA channel access. The effect of random receiver location is quantified which shows that random receiver distances do not alter the SINR distribution beyond knowledge of the mean receiver position. A method for computing the distribution of mmWave ad hoc interference-to-noise ratio which shows that mmWave ad hoc networks can still be interference limited. Several reasonable simplifications are used to derive the transmission capacity and area spectral efficiency. The performance of mmWave is then analyzed in terms of rate coverage. The results show that mmWave networks can support higher densities and larger spectral efficiencies, even in the presence of blockage, compared with lower frequency communication for certain link distances. Due to the increased bandwidth, the rate coverage of mmWave can be much greater than lower frequency devices.

A. Thornburg, T. Bai, and R. W. Heath Jr. are with the Wireless Networking and Communications Group at The University of Texas at Austin, Austin, TX, USA (email: {andrew.thornburg,tybai,rheath}@utexas.edu). Parts of this paper were submitted to the IEEE 2015 International Conference on Communications and the IEEE 2015 International Conference on Acoustics, Speech, and Signal Processing. We would like to acknowledge support from Army Research Labs under Grant No. W911NF-12-R-0011 and the National Science Foundation under Grant No. 1218338.
I. INTRODUCTION

Next-generation *ad hoc* networks, such as military battlefield networks, high-fidelity emergency response video, or device-to-device (D2D) entertainment applications, must offer high data rates and high reliability. Typically, *ad hoc* networks are limited by the uncoordinated interference created by proximate transmitters which decreases spectral efficiency. Measurement studies and analysis of indoor, commercial wireless systems (e.g. 802.11ad) have shown that mmWave systems may experience less interference due to directional antennas and building blockage in addition to offering massive bandwidth [1]–[4]. While these results are promising, the potential of outdoor mmWave *ad hoc* networks incorporating key features like directional antennas and building blockage is not yet understood.

The transmission capacity is a popular information theoretic performance metric to assess the viability of network architectures and transmissions strategies [5]–[8]. The transmission capacity is the maximum spatial density of transmitters given an outage constraint [8] and is well studied, e.g. in [8]–[10], and references therein. The transmission capacity of *ad hoc* networks with directional antennas was computed in [6] assuming small-scale Rayleigh fading but neglecting large blockage effects. In mmWave systems, however, small-scale fading is not as severe as lower frequency systems and blockages effects are more significant [1]. In [9], the transmission capacity of MIMO *ad hoc* networks was derived, where it was shown that the receive antennas should be used for interference cancellation not for spatial multiplexing. Compared to results on MIMO, power-efficient hardware architectures limit mmWave devices to simple beamforming [11] rendering the techniques considered in [9] not applicable with present mmWave *ad hoc* network technology. In [12], link blockage results from small-scale fading. In our paper, link blockages due to obstacles like buildings are associated with a distant-dependent blocking probability, which captures the intuition that the longer a link is, the more likely it will be blocked. More recently, D2D overlay networks have been considered in [13] which characterized single-hop *ad hoc* networks, but without including mmWave specific features like directional antennas or blockage. There has been considerable work in mmWave cellular networks [14]–[17]. Cellular networks have a specific exclusion region due to base station association that is not present in *ad hoc* networks. Previous work on mmWave *ad hoc* networks was largely restricted to indoor scenarios with limited range; the new 802.11ad standard which supports multi-hop communication is based on mmWave technology for low mobility, indoor environments [2]. The 802.15.3c and WirelessHD standards also operate in the mmWave bands; they are designed for *ad hoc* WPAN operation but the usage models and design is based on short, indoor, and stationary communication ranges of less than 10m [18].
Beamforming has been considered in *ad hoc* networks under the term *smart antennas*, phased arrays, or adaptive antennas – essentially the use of any antenna array with directional beamforming. Prior work on *ad hoc* networks considered smart antennas and other directional antennas [19]–[22]. While the results are general, they are only generally valid for propagation at UHF frequencies (usually implied by the models assumed in the papers). A directional MAC testbed was benchmarked in [19]. In [20], the analysis and performance of the system assumes Rayleigh fading. It was shown in [1] that mmWave propagation does not exhibit the rich scattering of Rayleigh fading and further has a path-loss characteristic that depends on whether the link is line-of-sight or non-line-of-sight. The optimization of the MAC for directional antennas is discussed in [21], [22].

In this paper, we show that mmWave *ad hoc* networks are viable for outdoor scenarios using the transmission capacity framework. We show that the interference is such that mmWave *ad hoc* networks can support larger network densities than traditional, lower-frequency (e.g. UHF bands like 2.4GHz) *ad hoc* networks. We consider a narrowband mmWave channel model with independent fading and distance-dependent path-loss. The narrowband assumption is justified by the use of OFDM in mmWave systems like IEEE 802.11ad [23]. Each pair of nodes is distributed according to a Poisson point process and either line-of-sight (LOS) or non-line-of-sight (NLOS) which each have a different path-loss exponent as shown in measurements [1], [16], [24], [25]. A sectored directional antenna is considered accounting for main lobe width, main lobe gain, and side lobe gain.

The general mathematical approach is to approximate the mmWave fading power via a Gamma random variable which can be used to compute the Laplace functional of the interference field. We also use a second-order approximation to compute, in closed form, the optimal outage-constrained network density, or transmission capacity. We provide an expression for the SINR distribution of a mmWave *ad hoc* network that incorporates mmWave specific features like channel blockage and directional antennas, validating the theory with simulations. We use the SINR distribution equation to solve for the transmission capacity thus leading to important insights on the area spectral efficiency. We use the probability of achieving or exceeding a given rate to compare mmWave and lower-frequency *ad hoc* networks. Our results show that mmWave is a powerful solution for *ad hoc* networks that can support much higher data rates compared with lower frequency solutions. Additionally, as many wireless communication links require receiver to transmitter communication in the form of control messages, the *two-way* transmission is analyzed. We use the FKG inequality to bound the transmission capacity and SINR distribution. The results indicate that optimal allocation in asymmetric situations can increase performance 50-100% with respect to equal bandwidth allocation or rate based allocation.
Fig. 1: A dipole ad hoc network with direction antennas and blockages. The typical receiver is encountering interference from other nearby transmitters. The link distances are fixed and may be blocked by a building; we allow random link lengths in section II.

II. System Model

Consider an ad hoc network where users act as transmitter or receiver, as shown in Fig. 1. Following the dipole model of [26], each user in the network has a corresponding receiver at distance $r$. The transmitters operate at constant power with no power control. The users within the network are points from a homogeneous Poisson point process (PPP) on the Euclidean plane, $\mathbb{R}^2$, with intensity $\tilde{\lambda}$ which is a standard model for evaluating transmission capacity of ad hoc networks, see [8] and the references therein.

We consider the typical dipole pair at the origin which is valid due to Slivnyak’s Theorem. The channel is accessed using an Aloha-type protocol with parameter $p$; during each block, a user transmits with probability $p$ or remains silent with probability $(1 - p)$. We define the effective density, used throughout the rest of the paper, as

$$\lambda := p\tilde{\lambda},$$

where $\tilde{\lambda}$ is the actual density of users (both silent and transmitting) which is justified through the thinning theorem [26]. We leave the optimization of $p$ to future work, but provide a framework to find the solution in Section 4.

Because of the higher frequencies used in mmWave, the path-loss with omni-directional antennas increases with frequency. The natural approach to combat this is by using a large antenna aperture, which can be achieved using multiple antennas [1], [17], [27]. The resulting array gain overcomes the frequency dependence on the path-loss and allows mmWave systems to provide reasonable link margin.
We assume that adaptive arrays implement directional beamforming. We assume that classic direction-of-arrival based beamforming is implemented at both the transmitter and receiver, where a main lobe is directed towards the dominate propagation path while smaller sidelobes direct energy in other directions. No attempt is made to direct nulls in the pattern to other receivers [28]; this is an interesting problem for future work. To facilitate the analysis, we approximate the actual beam pattern using a sectored model, as in [6]. The beam pattern, \( G_{\theta,M,m} \), is parameterized by three values: main lobe beamwidth (\( \theta \)), main lobe gain (\( M \)), and back lobe gain (\( m \)). The parameters of the gain pattern can be fit to an actual pattern, given an array geometry. The interferers are also equipped with directional antennas. Because the underlying PPP is isotropic in \( \mathbb{R}^2 \), we model the beam-direction of the typical node and each interfering node as a uniform random variable on \([0, 2\pi]\). Thus, the effective antenna gain of the interference seen by the typical node is a discrete random variable described by

\[
M_i = \begin{cases} 
MM & \text{w.p. } p_{MM} = \left( \frac{\theta}{\pi} \right)^2 \\
Mm & \text{w.p. } p_{Mm} = 2 \frac{\theta}{\pi} \frac{\pi - \theta}{\pi} \\
mm & \text{w.p. } p_{mm} = \left( \frac{\pi - \theta}{\pi} \right)^2
\end{cases}
\]  

(2)

We assume that the typical dipole performs perfect beam alignment and thus has an antenna gain of \( MM \). We note that the sectored model is pessimistic with regards to side band power. A typical uniform linear array, for instance, will consist of a main-lobe and many less powerful side-lobes each separated by nulls. The sectored model takes the most powerful side-lobe as the entire side-lobe (i.e. on average, the sectored model provides higher side-lobe power). Other work ignores the side-lobe power [3]. While outdoor, mobile mmWave devices will use directional antennas, due to practical issues like movement and beam-tracking, we do not expect extremely small, pencil beams. As such, the side-lobe power will be non-negligible, and the main-lobe width will not be unreasonably small.

The signal path can be either unobstructed/LOS or blocked/NLOS, each with a different path-loss exponent. This distinction is supported by empirical measurements conducted in Austin and Manhattan [1], [24]. The buildings are modeled as another Poisson point process independent of the communication network. Each point of the building PPP is independently marked with a random width and length. Under such a scenario, it was shown that by using a random shape model of buildings to model blockage [29], the probability that a communication link is LOS is

\[
P[\text{LOS}] = e^{-\beta d},
\]

where \( d \) is the link length and

\[
\beta = \frac{2 \lambda_b (E[W] + E[L])}{\pi}
\]

(3)

with \( \lambda_b \) as the building PPP density, \( E[W] \) and \( E[L] \) are the average width and length, respectively, of the buildings. We ignore correlations between blockages, as in [25]; each blockage is determined

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Fig. 2: An illustration of the sectored antenna model we use. While each antenna is quite directional (90°, 30°, and 9°, respectively), the amount of interference present in the network is not negligible. The network can still be interference limited.

independently. It was shown that the difference in the performance analysis is small when ignoring the correlation [29]. Additionally, blockage models can be derived from building measurements [30]. The path-loss exponent on each interfering link is a discrete random variable described by

$$\alpha_i = \begin{cases} 
\alpha_L & \text{w.p. } p(r) \\
\alpha_N & \text{w.p. } 1 - p(r) 
\end{cases},$$

where $\alpha_L$ and $\alpha_N$ are the LOS and NLOS path-loss exponents.

Nakagami fading is assumed for LOS links. In the measurements of [1], [31], small-scale fading is not very significant. The claim was made that mmWave analysis can, therefore, ignore fading. To help with the analytical tractability, we model the fading as a Nakagami random variable with parameter $N$. Consequently, the received signal power can be modeled as a Gamma random variable. As $N \to \infty$, the fading becomes a deterministic value centered on the mean, whereas $N = 1$ corresponds to Rayleigh fading. This assumption makes the analysis general and flexible.

We assume the typical user is outdoor. In [25], there is a term in the LOS/NLOS link probability to account for the overall area covered by buildings. We assume that the typical communication link is taking place outdoors, and the probability for LOS/NLOS reduces to [3]. We consider this because mmWave is heavily attenuated by many common building materials [1]. For example, brick exhibits losses of 30dB at 28 GHz. While it may be possible to communicate through certain parts of a building (e.g. windows), we ignore this and focus solely on the outdoor setting.
Fig. 3 shows an example realization of the ad hoc network. The blockage is modeled as a Boolean random model [25]. The center of the buildings are distributed as a PPP. The orientation, length, and width of the buildings are sampled from a probability distribution (e.g. uniform). The density and mean building size are modeled to match The University of Texas at Austin [25]. The results in Section III discuss in terms of a dense and sparse network. As Fig. 3 indicates, the sparse network will have a small number of LOS interferers while the dense network will have several dozen LOS interferers. We discuss the implications of this dense network in terms of modeling and system design in Sections III and V. A homogeneous PPP is perhaps overly simplistic, but we leave the investigation of mmWave ad hoc networks modeled with non-homogeneous PPPs to future work.

The SINR, which is the main quantity analyzed in the paper, is defined as

\[
\text{SINR} = \frac{P_t M_0 h_0 A r^{-\alpha_0}}{N_0 + \sum_{i \in \Phi} P_t M_i h_i A d_i^{-\alpha_i}}
\]

where \( P_t \) is the transmit power of each dipole, \( M_0 \) is the antenna gain corresponding to both main beams aligned, \( h_0 \) is the fading power at the dipole of interest, \( A \) is the path-loss intercept, \( r \) is the fixed dipole link length, \( \alpha_i \) is the path-loss exponent, and \( N_0 \) is the noise power. The terms within the sum are for each interfering dipole transmitter; \( d_i \) is used to represent the distance from the interferer to transmitter of interest, \( h_i \) is each interference fading power, and \( M_i \) is the discrete random antenna gain.

When discussing the results and system performance in the subsequent sections, we will explain the network density, \( \lambda \), as the average neighbor distance which allows some intuition on the spacing of nodes. In \( \mathbb{R}^2 \), this is defined as \( d_n := \frac{1}{\sqrt{\lambda}} \). Additionally, we define the expected number of LOS interfering nodes as \( \rho = \mathbb{E}[\#\text{LOS}] = \frac{2 \pi \lambda}{\beta^2} \), which follows as a direct result of Campbell’s Theorem [16].

III. ONE-WAY AD HOC COMMUNICATION

In this section, we will derive the SINR distribution for one-way transmission in the ad hoc network described in Section II. We first characterize the overall instantaneous SINR CCDF by analyzing the network when the desired link is either LOS and NLOS. Next, we present the protocol-gain by limiting communication to LOS links and argue why this is a useful concept. We also investigate the role of a random dipole link length. We argue that, in addition to simplifying analysis, knowing the mean location of the dipole is enough to quantify the SINR distribution. We show that neglecting noise and NLOS interference does not change the SINR distribution, suggesting that mmWave ad hoc networks are LOS interference limited. The transmission capacity and area spectral efficiency are computed using an approximation of the SINR CCDF.
Fig. 3: Example realizations of the random network with blockage. The blue rectangles are random boolean buildings which attenuate the signal. The red triangles are the Poisson point process of interferers. The green star represents the typical node. The user densities are the what we call sparse (a) and dense (b) when discussing the results.

A. One-Way Coverage Analysis

In general, the typical link under analysis can be either LOS or NLOS. To analyze the SINR distribution, we compute the conditional SINR distribution in the LOS and NLOS regimes and use total probability to compute the final CCDF. The equation is

\[ P_c = P^L_c P[\text{LOS}] + P^N_c P[\text{NLOS}], \] (6)

where \( P^L_c \) and \( P^N_c \) are the conditional distributions for LOS and NLOS, respectively. From the random building blockage assumption, the probability the link is LOS is \( p(r) \), and conversely, the probability a link is NLOS is \( 1 - p(r) \). What remains is to determine the conditional SINR distribution of the link in both of these regimes. Next, we will explore the probability a link covered given that it is a LOS link.

Using tools from stochastic geometry, we derive the conditional SINR distribution of the typical node of a mmWave ad hoc network located at the origin given that the link is LOS. More formally, the SINR distribution is

\[ P^L_c = P[\text{SINR} > T|\text{LOS}]. \] (7)
Going forward, for brevity, we will drop the conditional notation when using $P_c^L$. Using (5),

$$P_c^L = \mathbb{P} \left[ \frac{P_t M_0 h_0 A r^{-\alpha_L}}{N_0 + \sum_{i \in \Phi} \frac{P_t M_i h_i A d_i^{-\alpha}}{d_i}} > T \right]$$

(8)

$$= \mathbb{P} \left[ h_0 > \frac{T r^{\alpha_L}}{P_t M_0 A} \left( N_0 + \sum_{i \in \Phi} \frac{P_t M_i h_i A}{d_i} \right) \right]$$

(9)

$$= \mathbb{P} \left[ h_0 > \frac{T r^{\alpha_L}}{P_t M_0 A} (N_0 + I) \right]$$

(10)

$$= 1 - \mathbb{P} \left[ h_0 < \frac{T r^{\alpha_L}}{P_t M_0 A} (N_0 + I) \right].$$

(11)

We leverage the tight upper bound of a Gamma random variable of parameter $N$ as $\mathbb{P}[g < \gamma] < (1 - e^{-a \gamma})^N$ with $a = N(N!)^{-1/N}$ [32]. Now we can approximate (11) as

$$P_c^L(T) \approx 1 - \mathbb{E}_\Phi \left[ \left( 1 - e^{-\frac{T r^{\alpha_L}}{P_t M_0 A} (N_0 + I)} \right)^{N_h} \right]$$

(12)

$$= \sum_{n=1}^{N_h} \binom{N_h}{n} (-1)^{n+1} \mathbb{E}_\Phi \left[ e^{-an \frac{T r^{\alpha_L}}{P_t M_0 A} (N_0 + I)} \right],$$

(13)

where $N_h$ is the parameter of the Gamma fading variable $h_0$ and (13) is from the Binomial Theorem [16].

Because the correlation between each random blockage is ignored, which is potential future work, the building blockage is independent which permits the use of the Thinning Theorem from stochastic geometry [33]. Further, because we model the antenna gain between the typical receiver and each interfering user as an independent random variable, we can leverage the notion of mark from stochastic geometry to further split the Poisson point process. Essentially, we can now view the interference as 6 independent PPPs such that

$$I = I_{\Phi_{\text{LOS}}}^{MM} + I_{\Phi_{\text{LOS}}}^{Mm} + I_{\Phi_{\text{LOS}}}^{mm} + I_{\Phi_{\text{NLOS}}}^{MM} + I_{\Phi_{\text{NLOS}}}^{Mm} + I_{\Phi_{\text{NLOS}}}^{mm},$$

(14)

with the superscripts representing the discrete random antenna gain defined in (2) and each interfering node either a LOS transmitter or NLOS transmitter. Because of this, we can distribute the expectation in (13) and re-write (13) as

$$P_c^L \approx \sum_{n=1}^{N_h} \binom{N_h}{n} (-1)^{n+1} \prod_i \prod_j \mathbb{E}_{\Phi_{\text{LOS}}} \left[ e^{-nK_{L,T}T N_0} \prod_{i} \prod_{j} \mathbb{E}_{I_{\Phi_{\text{LOS}}}} \left[ e^{-nK_{L,T}I_{\Phi_{\text{LOS}}}} \right] \right]$$

(15)

with $i \in \{MM, Mm, mm\}$, $j \in \{\text{LOS, NLOS}\}$, and $K_L = \frac{\alpha L}{P_t M_0 A}$. In essence, each expectation is the Laplace transform of the associated sub-PPP, and each of these Laplace transforms are multiplied together.

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Using stochastic geometry theory, we can analytically represent the first Laplace expectation term as

\[ \mathbb{E}\left[e^{-nK_L T N_M M_{\text{LOS}}^m}\right] = e^{-2\pi \lambda \rho_{MM} \int_0^\infty \left(1 - \mathbb{E}_h \left[e^{-\frac{nK_L T N_M M_{\text{MM}} h}{\pi\rho_{MM} h}}\right]\right) p(r) r dr }, \]

(16)

where \( p_{MM} = \left(\frac{\rho}{\pi}\right)^2 \) is the probability of having antenna gain \( MM \) and \( p(r) \) is the probability of being LOS. Notice that \( \mathbb{E}_h[e^{h}] \) corresponds to the moment-generating function (MGF) of the random variable \( h \), which is modeled as a Gamma with parameter \( N_h \) which has a known MGF \([16]\). The final Laplace transform of the PPP is given as

\[ \mathcal{L}_{f_{\text{LOS}}}^{MM} = e^{-2\pi \lambda \rho_{MM} \int_0^\infty \left(1 - \frac{1}{\pi\rho_{MM}} nQ_L T N_0 \right)^N_h p(r) r dr }, \]

(17)

with \( Q_L = K_L P_L M M A = \frac{\alpha\nu_L M M}{M_0} \). Each other Laplace transform is computed similarly, but noting that \( p_{MM}, p(r), \) and \( x^{\alpha_L} \) will change depending on the antenna gain of the sub-PPP and if the sub-PPP is LOS or NLOS. We can summarize the results in the following theorem.

**Theorem 1:** The instantaneous SINR distribution of an outdoor mmWave ad hoc network can be tightly approximated by

\[ P_c(T) \approx \sum_{n=1}^{N_h} \left(\frac{N_h}{n}\right) (-1)^n \left(1 + \frac{nQ_L T N_0}{\pi\rho_{MM} h}\right)^N_h p(x) dx \]

(18)

where

\[ V = \sum_i p_i \int_0^{\infty} \left[1 - 1 \left(1 + \frac{nQ_L T}{x^{\alpha_L} N_h}\right)^N_h \right] p(x) dx \]

(19)

\[ Y = \sum_i p_i \int_0^{\infty} \left[1 - 1 \left(1 + \frac{nQ_N T}{x^{\alpha_N} N_h}\right)^N_h \right] (1 - p(x)) dx \]

(20)

\[ W = \sum_i p_i \int_0^{\infty} \left[1 - 1 \left(1 + \frac{nQ_N T}{x^{\alpha_N} N_h}\right)^N_h \right] p(x) dx \]

(21)

\[ Z = \sum_i p_i \int_0^{\infty} \left[1 - 1 \left(1 + \frac{nQ_L T}{x^{\alpha_L} N_h}\right)^N_h \right] (1 - p(x)) dx \]

(22)

with \( K_L = \frac{\alpha\nu_L}{P_L M_0 A}, K_N = \frac{\alpha\nu_N}{P_L N_0 A}, i \in \{MM, Mm, mm\}, Q_L = \frac{\alpha\nu_L M L}{M_0}, \) and \( Q_N = \frac{\alpha\nu_N M L}{M_0} \).

**Proof:** Substituting each Laplace transform \([17]\) into \([15]\) for the conditional \( P_c^L \) yields the left summation. The same process is done for the Laplace transforms corresponding to \( P_c^N \). These are then put into the form of \([6]\). □

While Theorem 1 may appear unwieldy, the way the terms are decomposed illustrates the insight that can be gained from the Theorem. In the left summation, there are exponential terms that correspond to...
noise, LOS interference (i.e. $V$), and NLOS interference (i.e. $Y$). Further, both $V$ and $Y$ (and similarly $W$ and $Z$) can be decomposed based on each antenna gain. It is possible to compared relative contributions to the total SINR CCDF. For example, by computing $Y$, we were able to see that $Y \approx 0$ for many different system parameters of interest. Therefore, $e^{-2\pi\lambda Y} \approx 1$ which means NLOS interference has relatively no effect on the SINR distribution. We use this insight in Section III-E to conclude that mmWave ad hoc networks are LOS Interference Limited.

B. Validation of the Model

Before proceeding, we verify the accuracy of Theorem 1. Table I shows the values used throughout the section. The parameters of (6) are simulated through Monte Carlo, while Theorem 1 is used for the analytical model. For the simulation, a PPP was generated over an area of $4 \text{km}^2$, which we believe to be large enough when compared to the selected dipole lengths. The thermal noise power of 2GHz bandwidth (e.g. the bandwidth of 802.11ad) at room temperature is $-81 \text{dBm}$; this leaves approximately 11dB for other noise and RF losses. We used $N_h = 7$ when computing the analytical expressions. We chose $N_h = 7$ because, while the analytical expression greatly improves computation speed, the number of terms in Theorem 1 increases with $N_h$. We believe $N_h = 7$ provides a good balance between computation speed and accuracy, as Fig. 4 shows. We chose a 30° beamwidth. We believe, that for mobile ad hoc networks, extremely tight beamwidths may not be feasible due to beamtracking and movement. Additionally, 10dB gain corresponds to the theoretical gain of 10 co-phased unit gain antennas with a beamwidth of 30° in a uniform linear array configuration.

Fig. 4a shows the comparison for the analytical SINR distribution with the empirical given a $\lambda = 5 \times 10^{-5} \text{m}^{-2}$ or $d_n \approx 70 \text{m}$. In this network, $\rho \approx 5$, but the performance is quite good. This can be

| Parameter | Value |
|-----------|--------|
| $\lambda$ | $5 \times 10^{-5}, 5 \times 10^{-4}$ |
| $r$ | 25, 50, 75 |
| $\beta, \alpha_{\text{LOS}}, \alpha_{\text{NLOS}}$ | 0.008, 2, 4 |
| $N_0$ | -70 dB |
| $h, N_h$ | Gamma, 7 |
| $\theta, M, m$ | $\frac{\pi}{2}, 10, 0.1$ |
| $P_t$ | 1W (30dBm) |

TABLE I: Parameters of results.
attributed to the directional antennas limiting the interference seen by the typical node. The analytical approximation in Theorem 1 of the mmWave ad hoc network matches extremely well to the simulations. Note that for $r = 25$ m and $r = 50$ m, nearest neighbor is quite far away. For $r = 75$ m, even with a neighbor, on average closer then the receiver, the SINR distribution is still very favorable.

Fig. 4b compares the SINR distribution results for a much denser network, $\lambda = 5 \times 10^{-4}$ which corresponds to $d_n \approx 22$ m, and $\rho \approx 50$. Again, Theorem 1 matches the simulation well. While the performance drop is quite large compared to the network in Fig. 4a for $r = 50$ m which is over $2 \times$ the nearest-neighbor, a user’s SINR > 0 dB 90% of the time. We will show later that with the massive bandwidth of mmWave the achievable rate is still large.

The run-time of the analytical expression was much better compared to the simulation in this denser network. The data for Fig 4b from Theorem 1 was generated in 30 secs whereas the simulation was created in roughly 10 minutes. This difference increases greatly with the number of nodes. For instance, in the dense network, tens of thousands nodes need to be created and simulated in each iteration. This greatly adds to the run-time of the simulation. The analytical expression, however, is invariant to different density within the simulation area.
C. LOS Protocol-Gain

In this section, we discuss what we call the LOS protocol-gain. Figs. 4a & 4b highlight the need for a protocol to enforce, as best as possible, a LOS communication regime. If LOS communication is assumed, the SINR distribution in the LOS regime can be described as $P_c(T) = P_c^L$. The performance gap is evident most clearly in Fig. 4b which is the denser network configuration. Each curve has a plateau at approximately $P_c = 0.8, 0.7, 0.55$, respectively. Perhaps unsurprisingly, these values are the LOS link probabilities for the three distances shown: 25m, 50m, and 75m. Therefore, unless the SINR threshold is very low (e.g. below -20dB), these links will not be able to communicate without LOS communication. This motivates the need for a protocol to ensure LOS communication. The networks in Fig. 4 have $\rho = 5 & 50$. Therefore, on average, the network will have multiple users that could potentially be a LOS receiver. We believe that system designers can and should leverage this when considering network and MAC level system design. Essentially, the performance gap between LOS and NLOS communication is quite large; we believe that, while NLOS communication is possible, the system should prioritize LOS communication. For example, a LOS relay could be used to multi-hop around a building.

At this point, to further motivate this section, we would like to carefully explain what the model we are using does and does not capture. Under the blockage model assumption from Section II, the probability that a link is LOS is $e^{-\beta r}$. This is the probability given a link at distance $r$. It does not capture the probability a LOS link exists at distance $r$. As evident from Fig. 3, any of the nearby LOS interferers in the dense network could be also be a potential receiver in the ad hoc network.

Fig. 5 shows the SINR distribution of a mmWave ad hoc network if the desired link is LOS. The improvement is quite large. The 90% coverage point in Fig. 5a is improved by 10dB for 25m, 20dB for 50m, and 30dB for 75m, compared to the same network in 4a. The improvement in Fig. 5b is even more drastic. For the 25m link, 20dB improvement is seen. We wish to stress what Fig. 5 shows. If, for example, the MAC determines a peer at 50m is LOS, the physical layer may be able to support the transmission, even in extremely dense network of Fig. 3b. This knowledge should influence MAC design, which is why we call it protocol gain.

In the event that no LOS receivers are available, which is still possible in a dense network, using a spread-spectrum technique is advised, as is done in the 802.11ad standard [23]. Because of the large bandwidth (e.g. 2GHz for 802.11ad), bandwidth can be sacrificed in order to increase received SINR which will improve the distribution. For example, if orthogonal codes are used with spreading factor $K$,
the maximum achievable rate with spread spectrum is

\[ R_{ss} = \max_K \frac{B}{K} \log_2(1 + KT)P_c(T/K), \]

where \( B \) is the system bandwidth. We leave the exploration of this trade-off to future work [34].

D. Distributions of \( r \)

One of the limitations of the dipole model is the fixed length of the communication link. In the model, the transmitters that form the PPP all have a receiver at a fixed distance \( r \) away. This model is used for its analytical tractability, yet, a receiver at a fixed distance is not a realistic expectation. In a D2D gaming scenario, for example, the receiver will be located at some random distance from the transmitter as the two users walk around. To quantify this, we can integrate Theorem 1 against a receiver location density function. The SINR distribution accounting for different receiver geometries is

\[ P_c(r) = \int_S P_c(r, T)f_R(r)dr \]

where \( S \) is the support of the location density distribution and \( f_R \) is the density and \( P_c(r, T) \) is Theorem 1, but we allow varying receiver distances. We compare two different distributions against the fixed dipole assumption. Perhaps surprisingly, the results indicate that simply knowing the mean of the distribution captures much of the SINR distribution.
As shown in Fig. 6, we use two receiver geometries to compare against, the uniform and truncated exponential. We use the truncated exponential as the properties of the Poisson point process prohibit another point at the origin. In the case of the Fig. 6a, there is a loss of performance for higher SINR probabilities due to the truncated exponential distribution. This illustrates the degradation of performance when attempting to communicate over a long distance (i.e. the tail of the distribution). From Fig. 4, we know that communication at a longer link length is more difficult because the communication link is more likely operating in the NLOS regime. Fig. 6 shows the effect of the unbounded support as the 90% SINR point is 8dB lower than that of the fixed length result. Conversely, for the uniform distribution with finite support, the distribution matches quite well to the fixed dipole result. At high SINR probabilities, which are of most interest, there is roughly 1dB difference between the uniformly random receiver position and fixed dipole assumption.

When communication is limited to LOS, the effect of the unbounded exponential distribution largely disappears. At the 90% SINR point, the difference in the SINR distributions is negligible. For higher SINR thresholds, the varying geometry begins to help, as the chance for a shorter link length becomes increasing crucial to link SINR.
E. LOS Interference Limited Networks

Intuition might suggest that mmWave devices would be noise limited because the high bandwidth (e.g. 1GHz versus 20MHz for lower frequency UHF devices) \cite{30}, \cite{35}. We have found this is not the case in all circumstances. From our analytical equations and simulations, we have found that mmWave networks can still be interference limited. Additionally, because of the differing NLOS and LOS path loss, for network densities and building densities such that the average number of LOS interferers is large (e.g. $\rho := \mathbb{E}[\#\text{LOS}] \geq 1$), the network is considered \textit{LOS Interference Limited} which we illustrate by characterizing the interference-to-noise ration (INR). Specifically, the INR distribution can quantify if the network is noise limited

$$P_{\text{NL}}(T) = \mathbb{P}[\text{INR} < T].$$

We leave the threshold value up to system designers to determine what value of $T$ is appropriate for defining noise limited. A natural choice may be 1 (0dB) or 10 (10dB). The INR CDF can be written as

$$P_{\text{NL}}(T) = \mathbb{P}\left[ \sum_{i \in \Phi} \frac{P_t M_i h_i A r_i^{-\alpha_i}}{N_0} < T \right]$$

$$= \mathbb{P}\left[ C > \frac{\sum_{i \in \Phi} P_t M_i h_i A r_i^{-\alpha_i}}{TN_0} \right]$$

$$= \mathbb{P}\left[ C > \frac{I_{\Phi}}{TN_0} \right]$$

$$= 1 - \mathbb{P}\left[ C < \frac{I_{\Phi}}{TN_0} \right]$$

where $C = 1$ is a constant and the total interference field power given by $I_{\Phi}$. We approximate the constant $C$ as a Gamma random variable with large parameter $N_C$ as $\lim_{N \to \infty} \frac{N^{N-1} e^{-N}}{\Gamma(N)} = \delta(x)$, which is the PDF of a Gamma random variable. Further, we leverage the tight upper bound of a Gamma random as before with the SINR distribution. The INR distribution can then be approximated as

$$P_{\text{NL}} \approx 1 - \mathbb{E}_{\Phi}\left[ (1 - e^{-\frac{I_{\Phi}}{TN_0}})^N \right]$$

$$= \sum_{n=1}^{N_C} \binom{N_C}{n} (-1)^{n+1} \mathbb{E}_{\Phi}\left[ e^{-\frac{an I_{\Phi}}{TN_0}} \right],$$

where (31) is from the Binomial Theorem. The total interference field $I_{\Phi}$ is $I_{\Phi,\text{LOS}} + I_{\Phi,\text{NLOS}}$ by the Thinning Theorem, and are thus independent. We again can view the transmitters as six independent PPPs as explained in (14). Because each sub-process is independent, we can re-write (31) as a product of expectations. The form of each expectation in is the Laplace transform of the Poisson point process.
We can analytically represent the first Laplace expectation term as
\[
E\left[e^{-\frac{an}{N_0 T I_{\text{LOS}}}}\right] = e^{-2\pi \lambda_{\text{PM}} \int_0^\infty \left(1 - \mathbb{E}_h\left[e^{-\frac{anP_AMMh}{\frac{\alpha N_0 T N_h}}}ight]\right)p(r)rdr},
\] (32)
where \((\theta^2)\) is the probability of having antenna gain \(MM\) and \(p(r)\) is the probability of being LOS. Notice that \(\mathbb{E}_h[e^{\eta h}]\) corresponds to the moment-generating function (MGF) of the random variable \(h\), which is modeled as a Gamma with parameter \(N_h\) which has a known MGF. The final Laplace transform of the PPP is given as
\[
\mathcal{L}_{I_{\text{LOS}}}^{MM} = e^{-2\pi \lambda_{\text{PM}} \int_0^\infty \left(1 - 1/(1 + \frac{anP_tAM_i}{N_0 T N_h})^{N_h}\right)p(r)rdr}.
\] (33)
Each other Laplace transform is computed similarly but \(p_{MM}\) will the correspond to the probability of the antenna gain \(\{MM, Mm, mm\}\) and the NLOS probability is \(1 - p(r)\). We can summarize our results in the following theorem.

**Theorem 2:** The instantaneous INR distribution of a mmWave ad hoc network can be tightly approximated by
\[
P_{\text{NL}}(T) \approx \sum_{n=1}^N \binom{N}{n} (-1)^{n+1} e^{-2\pi \lambda (R+U)}
\] (34)
where
\[
R = \sum_i p_i \int_0^\infty \left(1 - 1/(1 + \frac{anP_tAM_i}{\frac{\alpha N_0 T N_h}})^{N_h}\right)p(x)x dx,
\] (35)
\[
U = \sum_i p_i \int_0^\infty \left(1 - 1/(1 + \frac{anP_tAM_i}{\frac{\alpha N_0 T N_h}})^{N_h}\right)(1 - p(x))x dx
\] (36)
with \(i \in \{MM, Mm, mm\}\).

**Proof:** Substituting the Laplace transform (33) into (31) yields the result. \(\square\)

The result allows system designers to determine the statistics of the interference as a function of antenna pattern, transmitter density, and building blockage. By understanding the statistics of the interference, designers can determine if more complicated interference reduction schemes are needed. In Section V, we show numerical results and argue that mmWave ad hoc networks can still be interference limited.

**F. One-Way Capacity Analysis**

Next, we wish to characterize the transmission capacity, \(\lambda_c\). This is the largest \(\lambda\) the network can support given an SINR threshold, \(T\) and outage \(\epsilon\). More simply, \(1 - \epsilon = P_c\) of users will have an SINR larger than \(T\). It can also be defined as the number of successful transmissions per unit area, which is
directly connected to the number of users supported by the network. To do this, we can approximate the exponential terms of Theorem 1 as

\[ P_c^L \approx \sum_{n=1}^{N_h} (-1)^{n+1} \binom{N_h}{n} e^{-n K L T N_0} \left( 1 - 2\pi \lambda \Theta + 2\pi \lambda^2 \Theta^2 \right) \]  

(37)

where \( \Theta = V + Y \). We leverage the approximation, \( e^x \approx 1 + x + x^2/2 \) for small \( x \), for the Laplace functional term. We rationalize this because we are interested in analyzing the optimal \( \lambda \) for \( P_c \) near 1. As a result, the Laplace functional will be close to 1; the argument will be close to 0. A similar approximation is done for the NLOS term in Theorem 1. We combine (37) and the NLOS approximation to form

\[ P_c(T) \approx \sum_{n=1}^{N_h} (-1)^{n+1} \binom{N_h}{n} e^{-n K L T N_0} \left( 1 - 2\pi \lambda \Theta + 2\pi \lambda^2 \Theta^2 \right) 
+ \sum_{n=1}^{N_h} (-1)^{n+1} \binom{N_h}{n} e^{-n K N T N_0} \left( 1 - 2\pi \lambda \Psi + 2\pi \lambda^2 \Psi^2 \right) \]  

(38)

with \( \Psi = W + Z \). Because of this approximation, \( P_c \) is now a quadratic equation in \( \lambda \) which can be solved in closed-form. The exact solution depends on \( N_h \). Symbolic tools, such as Mathematica, can factor and solve (38) such that

\[ \lambda = f(T, \epsilon). \]  

(39)

Area spectral efficiency is a useful metric because it can characterize the network performance, rather than just a single link, as SINR does. We define area spectral efficiency as

\[ ASE := \lambda \epsilon \frac{\log_2(1+T)}{\text{getting rate } R} \frac{(1-\epsilon)}{\% \text{ of the time}}. \]  

(40)

Substituting (39) into (40) yields a function of just \( T \) and \( \epsilon \). Rather than numerically finding solutions for \( \lambda_\epsilon \), (40) yields the result directly.

IV. TWO-WAY AD HOC COMMUNICATION

The derivations from the Section III are for one-way communication. There is no consideration for the reverse link (i.e. receiver to transmitter). In real systems, however, successful transmission usually relies on a two-way communication stack. On multiple layers of a communication link, control packets are generally used. TCP and 802.11 use acknowledgment packets for instance. Upon successful reception of a message, the receiver will acknowledge the success with a short ACK message. If the transmitter does not receive the ACK, it will attempt to re-send the message. A successful exchange, therefore, involves
the receiver successfully decoding the large data packet and the transmitter successfully decoding the control message. This example illustrates two-way asymmetric communication. There are, of course, examples of symmetric two-way communication (i.e. two D2D users playing a game). With asymmetry, however, we show that significant gains can be realized by intelligently allocating bandwidth. The two-way transmission capacity quantifies the maximum density of users a network can support while both the forward and reverse link are subject to outage constraint, $\epsilon$ [10].

The same network from Fig. 1 is considered. The forward link is defined as the transmitter to receiver link (i.e. what was discussed in Section III), while the reverse link is the receiver to transmitter control link. Frequency division duplexing is used between the forward and reverse links. Consider the bandwidth from Section III split among the forward and reverse links. Hence, $B_{\text{total}}$ is the bandwidth available to the system. The forward link is allocated $B_F$, while the reverse link is allocated $B_R := B_{\text{total}} - B_F$. The SINR is similarly defined as $\text{SINR}_F$ and $\text{SINR}_R$. Correspondingly, from Shannon’s equation, the links achieve rates, $R_F$ and $R_R$.

A. Two-way Coverage Analysis

The two-way SINR probability is the probability that the forward link and reverse link exceed an SINR threshold. More precisely,

$$P^\text{tw}_{\text{c}} = P[\text{SINR}_F > T_F, \text{SINR}_R > T_R].$$

We assume that the forward and reverse link do not have the same SINR threshold because the reverse control link is generally low-rate compared to the forward link. To analyze this probability, we leverage the following definitions and lemma.

Definition IV.1 [10]: A random variable $X$ defined on $(\Omega, \mathcal{F}, \mathbb{P})$ is increasing if $X(\omega) \leq X(\omega')$ for a partial ordering on $\omega, \omega'$. $X$ is decreasing if $-X$ is increasing.

The SINR is a random variable defined on the probability space which is determined by how the interferers are placed on the plane. Let $\omega$ be a set of active interferers from the PPP. Then, $\omega' \geq \omega$ if $\omega'$ is a superset of $\omega$. The SINR (5) decreases if another interferer is added: $\text{SINR}(\omega) \geq \text{SINR}(\omega')$. Therefore, SINR is a decreasing random variable.

Definition IV.2 [10]: An event $A$ from $\mathcal{F}$ is increasing if $\mathbb{I}_A(\omega) \leq \mathbb{I}_A(\omega')$ when $\omega \leq \omega'$. The event is decreasing if $A^c$ is increasing.
The SINR probability event, \( \{ \text{SINR} > T \} \) is a decreasing event. If another interfering user is added to \( \omega \), the probability of successful transmission decreases. Now, we can leverage the Fortuin, Kastelyn, Ginibre (FKG) inequality \[36\].

Lemma IV.1 \[36\]: If both \( A, B \in F \) are increasing or decreasing events then \( P(AB) \geq P(A)P(B) \).

The FKG inequality can give a lower bound on the two-way SINR probability. In \[10\], this was shown to be a very tight lower bound. Using FKG, we can define the two-way SINR probability as

\[
P_{\text{tw}}^c \geq \mathbb{P}[\text{SINR}_F > T_F] \mathbb{P}[\text{SINR}_R > T_R].
\]

Therefore, the two-way SINR probability can be lower-bounded as

\[
P_{\text{tw}}^c \geq \left[ \sum_{n=1}^{N_h} (-1)^{n+1} \left( \frac{N_h}{n} \right) e^{-2\pi\lambda \left( V(T_F) + Y(T_F) \right)} \right] \left[ \sum_{n=1}^{N_h} (-1)^{n+1} \left( \frac{N_h}{n} \right) e^{-2\pi\lambda \left( V(T_R) + Y(T_R) \right)} \right].
\]

B. Two-Way Capacity Analysis

Using a similar approach as with the one-way, we use a Taylor expansion of the exponential function to yield

\[
P_{\text{tw}}^c \approx \left[ \sum_{n=1}^{N_h} (-1)^{n+1} \left( \frac{N_h}{n} \right) \left( 1 - 2\pi\lambda \Theta(T_F) + 2\pi^2\lambda^2 \Theta^2(T_F) \right) \right] \times \left[ \sum_{n=1}^{N_h} (-1)^{n+1} \left( \frac{N_h}{n} \right) \left( 1 - 2\pi\lambda \Theta(T_R) + 2\pi^2\lambda^2 \Theta^2(T_R) \right) \right].
\]

This yields a quartic equation in \( \lambda \) which has an analytic expression. The general solution, however, is quite messy, and the equation is a page long, so it is omitted here. An analytical solver, such as Mathematica, can factor the coefficients of \(44\) which can be input into a polynomial root solver to yield the transmission capacity \( \lambda_c \). The two-way area spectral efficiency can be defined as

\[
\text{ASE}_{\text{tw}} := \lambda_c \left( \frac{R_F + R_R}{B_{\text{total}}} \right) (1 - \epsilon).
\]

The interest then becomes, given rate requirements \( R_F \) and \( R_R \), what is the allocation of bandwidth that maximizes \(45\)? We explore this trade-off in Section \[V\]. For example, we show that simply splitting the resource according to the different rate requirements is sub-optimal.
Fig. 7: The largest $\lambda$ for a 10% outage at various SINR thresholds and dipole distances for NLOS/LOS communication (a) and LOS-only communication (b). Note the different y-axis scales.

V. COVERAGE AND CAPACITY RESULTS

We present the results for the transmission capacity, $\lambda_c$. Further, we compute the area spectral efficiency to define the best $\lambda$, given by $\lambda^\star$. We compare the achievable rates for mmWave networks with classic results for lower frequency ad hoc networks. The section is concluded with an investigation into two-way communication. The results indicate that simply performing equal bandwidth allocation or rate-proportional allocation is sub-optimal in asymmetric two-way communication.

A. Transmission Capacity

Mathematica is used to solve the quadratic form of (37) to obtain an analytical solution $\lambda$. The same parameters from Table I are used. Additionally, we compare the mmWave results to UHF ad hoc networks (e.g. 2.4 GHz). For the UHF network, we use the classic results from [26] which use omni-directional antennas, assume Rayleigh fading, and SISO flat-fading model. Fig. 7 shows the transmission capacity for mmWave and lower frequency networks with a 10% outage. It shows the potential for very dense mmWave networks. If users require SINR $> 0$ dB 90% of the time, 25m link length can support networks with $d_n = 15$m ($\lambda = 10^{-3}$m$^{-2}$) whereas lower frequency networks require a neighbor distance of 50m ($\lambda = 10^{-4}$m$^{-2}$). This increase in density is reflected in the massive increase in area spectral efficiency.

Fig. 7 shows the relationship between providing a higher SINR (and thus rate) to users while maintaining a constant outage constraint. As expected, the shortest dipole length can support the highest density
B. Area Spectral Efficiency

The improvement in NLOS and LOS networks of Fig. 7a leads to nearly a $2 \times$ increase in the area spectral efficiency as shown in Fig. 8a. Thus, mmWave networks provide more bandwidth and are more efficient with the bandwidth. Fig. 8b shows the area spectral efficiency of an mmWave network if only LOS communication is used. For link lengths of 25m and 50m, $10 \times$ gains are realized when compared to lower-frequency networks.

The shape of the curves suggests an optimal density with respect to ASE. This leads to the optimization
Fig. 9: Optimal network density for various dipole lengths, subject to 10% outage.

We leave the exploration of analytical solutions to this problem for future work. Fig. 9 shows the numerically obtained $\lambda^*$ from Fig. 8b. Notice that if the dipole length is 50, the optimal density is about what was shown in Fig. 5b while a dipole length of 150 optimal density is roughly what was shown in Fig. 5a. Furthermore, the optimal density is exponentially decreasing in $r$. The optimal density, $\lambda^*$, corresponds to a neighbor distance 1/2 the link distance in the LOS-only (protocol gain) case (i.e. for $r = 25m$, $d_n \approx 12.5m$). **MmWave ad hoc networks can not only support high density, but this density is best for overall network efficiency.** This is due to both the directional antennas and blockage. The blockage thins the interference PPP as shown in Section III-E. The remaining LOS interferers are effectively pushed away. The interference power from a close neighbor into the side-lobe (i.e. the power is heavily attenuated) is the same as that interferer being further away but using omni-directional antennas. Of course, if an interferer is in the main-lobe of the antenna, this phenomenon works against the receiver, but more often, it helps.
Fig. 10: mmWave *ad hoc* networks provide significant increase in rate coverage over lower frequency networks.

**C. Rate Analysis**

We wish to analyze the rate achieved by mmWave *ad hoc* networks. As alluded to earlier, the potential for extremely high data rates is real with mmWave networks. Fig. 10 shows the rate coverage probability, where \( R = W \log_2(1 + T) \), and \( W \) is the system bandwidth. From Theorem 1, a user will achieve SINR \( > T \) with some probability as shown in Fig. 4a and Fig. 4b which leads to an achievable rate probability. For example, according to Fig. 5a, a LOS mmWave communication link will have an SINR of at least 10dB 90% of the time. Assuming Gaussian signaling, the link can achieve \( R = W \log_2(1 + 10) \) 90% of the time.

The system bandwidth used in Fig. 10 is 100MHz for the mmWave and 20MHz for the lower frequency system. While the bandwidth is only a 5× increase, we see orders-of-magnitude increase in the rate coverage for mmWave (note the densities: the higher density lower frequency network barely appeared on the plot and was omitted). Because of the propagation properties of the higher frequency wave, this reduces or eliminates much of the interference.

**D. INR Distribution**

Figs. 11, 12, and 13 show the instantaneous INR CDF for three values of \( \lambda \) for each of the beam patterns in Fig. 2. Indeed, in all antenna patterns, the sparsest network exhibits noise limited behavior. For example, the \( \mathbb{P}[\text{INR} < 0\text{dB}] = 0.5 \) for 30° antennas in the sparsest network. On the other hand, these
results show compelling evidence that a mmWave *ad hoc* network can still be considered interference limited. In dense networks (22m and 70m spacing), in all but the very narrow beam case, the network exhibits strong interference. Because of this, we urge caution when considering mmWave networks to be noise limited.

Fig 14 shows the instantaneous INR distribution if we ignore NLOS interference for when $\theta = 30^\circ$. It shows that for many mmWave networks the interference is largely driven by the LOS interference in the two denser networks. The CDF of the two denser networks in Fig. 14 is nearly identical to Fig. 12 which indicates that NLOS interference plays no role at those densities. We believe this shows compelling evidence that interference reduction schemes will be useful, even at mmWave frequencies. In particular, eliminating LOS interference is most important.

**E. Two-Way Communication Results**

The results presented in this section consider a system under rate constraints. We show that, in asymmetric traffic, the transmission capacity of a two-way network is can be vastly improved compared to equal bandwidth allocation or rate-proportional allocation. The two-way area spectral efficiency is compared to one-way area spectral efficiency. We show that 75% of the one-way efficiency can be
Fig. 12: The INR CDF for $\theta = 30^\circ$. In the sparsest network, the interference power is more dominant than the noise power (i.e. $P[\text{INR} < 0\text{dB}] = 0.4$ for the green circle network), but the red triangle curve shows that the network is always interference limited.

Fig. 13: The INR CDF for $\theta = 90^\circ$. In all networks, the interference power is nearly always more dominant than the noise power (i.e. $P[\text{INR} < 0\text{dB}] = 0.05$ for the green circle network).
Fig. 14: The INR CDF for $\lambda = 5 \times 10^{-5}$ and $\theta = 30^\circ$ with only LOS interference. Compare to Fig. 12 we find that the shape of INR distributions is largely determined by the LOS interference when the network is dense.

achieved for outage of 10% which is a 100% increase over the baseline equal allocation. In all the results, the dipole link length is 50m.

We consider asymmetric traffic. For example, in TCP assuming 1000 byte data packets, the receiver must reply with 40 byte ACK packets [37]. Hence, the rate asymmetry in TCP is $1/25$. The following results consider a system bandwidth of 100MHz, a forward rate requirement of 200Mbps, and a reverse link rate requirement of 8Mbps.

Fig. 15 shows the transmission capacity as a function of forward bandwidth allocation. As more bandwidth is added to the forward link, the required SINR$_F$ decreases to meet the rate requirement. Because the reverse link rate requirement is quite small, the increase in SINR$_R$ does not change the SINR probability much (i.e. we are operating at very low SINR which is where the SINR probability plateaus to 1). Fig. 15 shows the naivet of simply splitting the bandwidth in half. A nearly 2x improvement in transmission capacity is achieved by going from 50% to the optimal allocation of 90%. What is somewhat more surprising is that a $1/25$ split (i.e. splitting according to the rate requirement) results in nearly the same performance as a naive 50/50 allocation. Lastly, Fig. 15 shows that this allocation is invariant to outage constraint.
Fig. 15: The transmission capacity of a two-way network can be improved by allocating bandwidth in an optimal way.

Fig 16 shows the performance gains in terms of area spectral efficiency that can be achieved by various bandwidth allocations. In all curves, the sum rate of the system is 208Mbps. As expected from Fig. 15, the area spectral efficiency is the worst in the naive 50/50 bandwidth allocation. The rate based (96%/4%) allocation performs better, but additional gains can be made by further optimizing the allocation. With the optimal allocation, the two-way system can achieve 75% the area spectral efficiency of the one-way system.

VI. CONCLUSIONS

We presented an analysis that characterized the performance of mmWave ad hoc networks for both one-way and two-way communication. We showed that mmWave networks can improve on the performance and efficiency of UHF networks when considering both LOS and NLOS communication. Massive improvements (e.g. 100×) are possible when limiting the communication to LOS links which motivates LOS aware protocols. Further, we showed the NLOS interference is negligible and LOS interference can still be the limiting factor for a mmWave ad hoc network. We believe this motivates the need for LOS interference mitigation strategies. Lastly, by, understanding the asymmetries in the mmWave network for two way traffic, 75% of the one-way capacity can be achieved.
Fig. 16: Significant gains can be achieved by intelligently allocating bandwidth.

VII. ACKNOWLEDGMENTS

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REFERENCES

[1] T. Rappaport, S. Sun, R. Mayzus, H. Zhao, Y. Azar, K. Wang, G. Wong, J. Schulz, M. Samimi, and F. Gutierrez, “Millimeter Wave Mobile Communications for 5G Cellular: It Will Work!” IEEE Access, vol. 1, pp. 335–349, 2013.

[2] X. Zhu, A. Doufexi, and T. Kocak, “Throughput and coverage performance for IEEE 802.11ad millimeter-wave WPANs,” in Proc. of 2011 IEEE 73rd Vehicular Technology Conference (VTC Spring), 2011, pp. 1–5.

[3] S. Singh, R. Mudumbai, and U. Madhow, “Interference Analysis for Highly Directional 60-GHz Mesh Networks: The Case for Rethinking Medium Access Control,” vol. 19, no. 5, pp. 1513–1527, 2011.

[4] T. Rappaport, R. W. Heath Jr., R. C. Daniels, and J. Murdock, Millimeter Wave Wireless Communications. Prentice-Hall, September 2014.

[5] J. Andrews, S. Shakkottai, R. Heath, N. Jindal, M. Haenggi, R. Berry, D. Guo, M. Neely, S. Weber, S. Jafar, and A. Yener, “Rethinking information theory for mobile ad hoc networks,” vol. 46, no. 12, pp. 94–101, 2008.

[6] A. Hunter, J. Andrews, and S. Weber, “Transmission capacity of ad hoc networks with spatial diversity,” IEEE Trans. Wireless Commun., vol. 7, no. 12, pp. 5058–5071, 2008.

[7] K. Huang, J. Andrews, D. Guo, R. Heath, and R. Berry, “Spatial interference cancellation for multiantenna mobile ad hoc networks,” vol. 58, no. 3, pp. 1660–1676, 2012.
[8] S. Weber, J. Andrews, and N. Jindal, “An overview of the transmission capacity of wireless networks,” vol. 58, no. 12, pp. 3593–3604, 2010.

[9] R. Vaze and R. Heath, “Transmission capacity of ad-hoc networks with multiple antennas using transmit stream adaptation and interference cancellation,” vol. 58, no. 2, pp. 780–792, 2012.

[10] R. Vaze, K. Truong, S. Weber, and R. Heath, “Two-way transmission capacity of wireless ad-hoc networks,” vol. 10, no. 6, pp. 1966–1975, 2011.

[11] A. Alkhateeb, O. El Ayach, G. Leus, and R. Heath, “Hybrid precoding for millimeter wave cellular systems with partial channel knowledge,” in Information Theory and Applications Workshop (ITA), 2013, pp. 1–5.

[12] R. Gowaikar, B. Hochwald, and B. Hassibi, “Communication over a wireless network with random connections,” IEEE Transactions on Information Theory, vol. 52, no. 7, pp. 2857–2871, Jul. 2006.

[13] N. Lee, X. Lin, J. G. Andrews, and R. W. Heath, “Power Control for D2D Underlaid Cellular Networks: Modeling, Algorithms and Analysis,” p. 12, May 2013. [Online]. Available: http://arxiv.org/abs/1305.6161

[14] T. Bai, A. Alkhateeb, and R. W. Heath Jr, “Coverage and capacity in millimeter wave cellular networks,” To appear in IEEE Commun. Mag., Sep. 2014.

[15] T. Bai and R. W. Heath Jr., “Coverage in dense millimeter wave cellular networks,” in Signals, Systems and Computers (ASILOMAR), 2013 Conference Record of the Forty Seventh Asilomar Conference on, 2013.

[16] ———, “Coverage and rate analysis for millimeter wave cellular networks,” submitted to IEEE Trans. Wireless Commun., March 2014. Available Online: http://arxiv.org/abs/1402.6430.

[17] S. Akoum, O. El Ayach, and R. Heath, “Coverage and capacity in mmwave cellular systems,” in Signals, Systems and Computers (ASILOMAR), 2012 Conference Record of the Forty Sixth Asilomar Conference on, 2012, pp. 688–692.

[18] T. Baykas, C.-S. Sum, Z. Lan, J. Wang, M. Rahman, H. Harada, and S. Kato, “IEEE 802.15.3c: the first IEEE wireless standard for data rates over 1 Gb/s,” IEEE Communications Magazine, vol. 49, no. 7, pp. 114–121, Jul. 2011. [Online]. Available: http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=5936164

[19] R. Ramanathan, J. Redi, C. Santivanez, D. Wiggins, and S. Polit, “Ad hoc networking with directional antennas: a complete system solution,” vol. 23, no. 3, pp. 496–506, 2005.

[20] S. Bellofiore, J. Foutz, R. Govindarajula, I. Bahceci, C. Balanis, A. Spanias, J. Capone, and T. Duman, “Smart antenna system analysis, integration and performance for mobile ad-hoc networks (manets),” vol. 50, no. 5, pp. 571–581, 2002.

[21] J. Winters, “Smart antenna techniques and their application to wireless ad hoc networks,” vol. 13, no. 4, pp. 77–83, 2006.

[22] R. Choudhury, X. Yang, R. Ramanathan, and N. Vaidya, “On designing mac protocols for wireless networks using directional antennas,” Mobile Computing, IEEE Transactions on, vol. 5, no. 5, pp. 477 – 491, may 2006.

[23] “IEEE Standard for Information technology–Telecommunications and information exchange between systems–Local and metropolitan area networks–Specific requirements-Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications Am,” pp. 1–628, 2012.

[24] T. Rappaport, E. Ben-Dor, J. Murdock, and Y. Qiao, “38 GHz and 60 GHz angle-dependent propagation for cellular amp; peer-to-peer wireless communications,” in Proc. of 2012 IEEE International Conference on Communications (ICC), 2012, pp. 4568–4573.

[25] T. Bai, R. Vaze, and R. W. Heath Jr., “Analysis of Blockage Effects on Urban Cellular Networks,” IEEE Transactions on Wireless Communications, vol. 13, no. 9, pp. 5070–5083, Sept 2014.

[26] F. Baccelli and B. Blaszczyszyn, Stochastic Geometry and Wireless Networks, Volume II - Applications. NoW Publishers, 2009, vol. 2. [Online]. Available: http://hal.inria.fr/inria-00403040
[27] Z. Pi and F. Khan, “An introduction to millimeter-wave mobile broadband systems,” IEEE Communications Magazine, vol. 49, no. 6, pp. 101–107, Jun. 2011. [Online]. Available: http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=5783993

[28] S. Akoum, M. Kountouris, M. Debbah, and R. Heath, “Spatial interference mitigation for multiple input multiple output ad hoc networks: MISO gains,” in 2011 Conference Record of the Forty Fifth Asilomar Conference on Signals, Systems and Computers (ASILOMAR), 2011, pp. 708–712.

[29] T. Bai, R. Vaze, and R. W. Heath Jr., “Using random shape theory to model blockage in random cellular networks,” in Proc. of Int. Conf. on Signal Processing and Communications (SPCOM), Jul. 2012, pp. 1–5.

[30] S. Singh, M. N. Kulkarni, A. Ghosh, and J. G. Andrews, “Tractable model for rate in self-backhauled millimeter wave cellular networks,” CoRR, vol. abs/1407.5537, 2014. [Online]. Available: http://arxiv.org/abs/1407.5537

[31] M. Samimi, K. Wang, Y. Azar, G. N. Wong, R. Mayzus, H. Zhao, J. K. Schulz, S. Sun, F. Gutierrez, and T. S. Rappaport, “28 GHz Angle of Arrival and Angle of Departure Analysis for Outdoor Cellular Communications Using Steerable Beam Antennas in New York City,” in 2013 IEEE 77th Vehicular Technology Conference (VTC Spring). IEEE, Jun. 2013, pp. 1–6. [Online]. Available: http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=6691812

[32] H. Alzer, “On Some Inequalities for the Incomplete Gamma Function,” Mathematics of Computation, vol. 66, 1977. [Online]. Available: http://www.jstor.org/stable/2153894

[33] F. Baccelli and B. Blaszczyszyn, Stochastic Geometry and Wireless Networks, Volume I - Theory. NoW Publishers, 2009, vol. 1. [Online]. Available: http://hal.inria.fr/inria-00403039

[34] S. Weber, X. Yang, J. Andrews, and G. DeVeciana, “Transmission Capacity of Wireless Ad Hoc Networks With Outage Constraints,” IEEE Transactions on Information Theory, vol. 51, no. 12, pp. 4091–4102, Dec. 2005. [Online]. Available: http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=1542405

[35] J. G. Andrews, S. Buzzi, W. Choi, S. V. Hanly, A. Lozano, A. C. K. Soong, and J. C. Zhang, “What Will 5G Be?” IEEE Journal on Selected Areas in Communications, vol. 32, no. 6, pp. 1065–1082, Jun. 2014.

[36] G. Grimmett, Percolation. Springer Verlag, 1989.

[37] H. Balakrishnan and V. Padmanabhan, “TCP Performance Implications of Network Asymmetry.” [Online]. Available: http://www.ietf.org/proceedings/48/I-D/pilc-asym-01.txt