Ultra-High Energy Cosmic Rays:
Some General Features, and Recent Developments Concerning Air Shower Computations

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We present an introductory lecture on general features of cosmic rays, for non-experts, and some recent developments concerning cascade equations for air shower developments.

1 Discovery

The history of cosmic rays goes back to Coulomb, who realized that a suspended charged metal sphere loses charge. This fact has been unexplained for a long time, until CTR Wilson (1900) found that this is due to the ionization of the surrounding air.

In order to learn about the origin of this ionization, Wulf (1910) climbed the Eifel tower and found that the ionization decreases with increasing altitude, so the ionization seemed to be linked to ground.

To further investigate this question, Victor Hess (1912) made a balloon flight up to an altitude of 5 km. He realized that the ionization increases again above 1 km, which means that the radiation is coming from above. These findings are generally considered to be the discovery of cosmic rays.

Pierre Auger (1932) finally detected coincidences between counters up to 300 m apart, he identified in this way the first extended air showers (EAS).

Today we know: When a high energy proton or nucleus hits an air molecule, it creates many hadrons, which make further collisions, creating more particles, and so on, leading to a so-called hadronic cascade.

Certain hadrons (mainly $\pi^0$) decay into photons, each of them initiating an electromagnetic cascade (of photons and electrons).
2 Characteristics of Cosmic Rays

In fig. 1 we show the spectrum of cosmic rays as measured over the years by many experiments. One observes a surprisingly simple structure, the spectrum is essentially a power law $E^{-2.7}$, with an exponent 2.7 below $3 \times 10^{15}$ eV and 3 above. The position where the slope changes is usually referred to as the knee.

In particular interesting is the high energy end of the spectrum: Protons faster than $10^{20}$ eV interact with the cosmic microwave background (making $\Delta$ resonances), so they cannot come from far away; one expects therefore a galactic source. On the other hand the Larmor radius suggests an extragalactic source...

![Figure 1: The spectrum of cosmic rays](image)

3 Sources

What are the sources of the high energy cosmic rays? Two scenarios are considered [1, 2]:

- Top-Down scenario, which amounts to the decay of some extremely heavy particle
- Bottom-Up scenario, where particles are accelerated by conventional electromagnetic forces. Electric fields are difficult to find, so probably one has to deal with magnetic fields (Fermi)

Fermi proposed that particles may be accelerated by entering and leaving moving magnetic clouds (like tennis balls), which leads to a gain of energy of $\Delta E/E =$
4/3 \beta^2 (second order), or particles are accelerated by entering and leaving moving shock fronts, with \( \Delta E/E = 4/3 \beta \) (first order).

In any case, we have \( \Delta E = \alpha E \). Considering \( n \) iterations, one finds

\[
E = E_0 (1 + \alpha)^n.
\]

Supposing an escape probability \( \varepsilon \), we find for the number of particles

\[
N = N_0 (1 - \varepsilon)^n.
\]

So we get indeed an exponential spectrum,

\[
\frac{N}{N_0} = \left( \frac{E}{E_0} \right)^{\ln(1-\varepsilon)/\ln(1+\alpha)}.
\]

Possible accelerators are

- Supernova shock waves \( \leq 10^{15} \text{eV} \).
- Jets in AGNs: the jets create shocks into the intergalactic medium.
- Rotating neutron stars (ms spin rates, \( B = 10^{13} \text{G} \)): Dynamo effect \( \rightarrow 10^{20} \text{V} \).

## 4 Air Showers

The first step is a primary hadronic interactions of a proton or a nucleus with air producing many hadrons. Then these secondary hadrons interact with air, giving rise again to hadron production, and so on. This is the so-called hadronic cascade. Hadrons may decay before suffering an interaction, for example:

\[
\pi^0 \rightarrow \gamma\gamma \ (98.8\%)
\]
\[
\pi^\pm \rightarrow \mu^\pm + \nu(\bar{\nu}) \ (100\%)
\]
\[
K^\pm \rightarrow \mu^\pm + \nu(\bar{\nu}) \ (63\%).
\]

Most muons reach the ground (at high energies).

Each \( \gamma \) initiates an electromagnetic cascade (photons and electrons). In an EM cascade, photons interact via

- Compton scattering with electrons from an atom (low \( E \)),
- Photoelectric effect: the photon is absorbed by an atom, its energy is used to liberate an electron (low \( E \)).
- Pair production: the photon interacts with the Coulomb field of a nucleus to produce a electron-positron pair (high \( E \)).

Electrons interact via
• Ionization energy loss.

• Bremsstrahlung: the electron interacts with the Coulomb field of a nucleus to produce a slowed down electron and plus the radiated photon.

At high energy, the most important processes are pair production and Bremsstrahlung:

\[ e \rightarrow e + \gamma \]

\[ \gamma \rightarrow e + e, \]

so in any case the number of particles doubles for each iteration.

The splitting continues till some critical energy \( E_c \) is reached (after this the particles get absorbed or decay only)

We first discuss a toy model (Heitler 1944):

• At each step the energy is given in equal parts to the two daughters.

• A branching happens always after a collision length \( \lambda \), so the number of branchings at depth \( X \) is \( X/\lambda \).

The number of particles and the energy per particle are

\[ N(X) = 2^{X/\lambda}, \quad E(X) = E_0/N. \]

The shower maximum is at \( E = E_c \), so

\[ N(X_{\text{max}}) = 2^{X_{\text{max}}/\lambda} = \frac{E_0}{E_c} \Rightarrow X_{\text{max}} = \lambda \frac{\ln E_0/E_c}{\ln 2}. \]

So we find the important general results:

\[ N_{\text{max}} \propto E_0, \quad X_{\text{max}} \propto \ln E_0. \]

The standard technique to calculate air shower developments is the Monte Carlo technique [3]: Each particle may decay or interact, whatever happens earlier, at the corresponding depth; new particles from decay or interaction are written to stack and treated subsequently.

This technique is easy to implement, but very slow at high energies, since \( N \propto E_0 \) and even impossible at \( 10^{20}\text{eV} \).

An alternative is the numerical solution of so-called cascade equations (CE) [4–7], which is at high energies considerably faster than the MC approach.

## 5 Characteristics of Air Showers

In the following we show results from the CE calculations (=Conex) for proton induced showers (if not mentioned otherwise), and some comparisons with the CORSIKA Monte Carlo calculations [3]. The cascade equations will be discussed
in the following chapters. The final aim being the solution of the full threedimensional cascade equations, we show here only results concerning the $X$ (depth) dependence of particle numbers, based on the $1D$ version of the approach, which makes use of the small angle approximation. There is no information about the lateral distributions.
6 Hadronic 1D cascade equations

Here we discuss the equations for the energy spectra $h_n(E, X)$ of hadrons of type $n$ at depth $X$, where the depth is defined as

$$X = \int_{P}^{\infty} \rho(x)dx,$$

with $\rho$ being the density of air. The decrease of $h_n(E, X)$ due to collisions is

$$\frac{dh_n}{dX} = -\frac{h_n}{\lambda_n},$$

where the mean inelastic free path $\lambda_n$ is

$$\lambda_n = \frac{m_{\text{air}}}{\sigma_{\text{inel}}^{(n)}}.$$

The decay rate in the particle c.m. system is $dh_n/d\tau = -h_n/\tau_n$, so

$$\frac{dh_n}{dX} = -\frac{m_n \rho^{-1}(X)}{c\tau_n E} h_n.$$
The equation for hadron density, considering collisions and decay (only loss terms) is

$$\frac{\partial h_n(E, X)}{\partial X} = -h_n(E, X) \left( \frac{1}{\lambda_n} + \frac{m_n \rho^{-1}(X)}{c \tau_n E} \right).$$

The full equation, including gain from higher energies, and a source term (generalizing simple initial conditions) is

$$\frac{\partial h_n(E, X)}{\partial X} = -h_n(E, X) \left[ \frac{1}{\lambda_n(E)} + \frac{m_n \rho^{-1}(X)}{c \tau_n E} \right]$$

$$+ \sum_m \int_{E}^{E_{\text{max}}} h_m(E', X)$$

$$\left[ \frac{W_{mn}(E', E)}{\lambda_m(E')} + D_{mn}(E', E) \frac{m_n \rho^{-1}(X)}{c \tau_m E'} \right] dE'$$

$$+ S_n(E, X).$$
where $W_{mn}(E', E)$ and $D_{mn}(E', E)$ are inclusive spectra of secondaries of type $n$ and energy $E$ produced in interactions ($W$) or decays ($D$) of primaries of type $m$ and energy $E'$.

To obtain a system of differential equations, one proceeds to an energy discretization as

$$E_i = E_{\text{min}} \cdot c^i,$$

which gives

$$\frac{\partial h_{ni}(X)}{\partial X} = -h_{ni}(X) \left[ \frac{1}{\lambda_{ni}} + \frac{m_{n} \rho^{-1}(X)}{c \tau_{n} E_{i}} \right] + \sum_{m} \sum_{j=1}^{j_{\text{max}}} h_{mj}(X) \left[ \frac{W_{ji}}{\lambda_{mj}} + D_{ji} \frac{m_{m} \rho^{-1}(X)}{c \tau_{m} E_{j}} \right] + S_{ni}(X),$$
with \( h_{ni}(X) = h_n(E_i, X) \), \( \lambda_{ni} = \lambda_n(E_i) \), and

\[
W_{mn}^{ji} = \int_{E_{i-1}}^{E_i} f\left(\frac{E}{E_i}\right)W_{mn}(E_j, E)dE + \int_{E_i}^{E_{i+1}} g\left(\frac{E}{E_{i+1}}\right)W_{mn}(E_j, E)dE,
\]

\[
D_{mn}^{ji} = \int_{E_{i-1}}^{E_i} f\left(\frac{E}{E_i}\right)D_{mn}(E_j, E)dE + \int_{E_i}^{E_{i+1}} g\left(\frac{E}{E_{i+1}}\right)D_{mn}(E_j, E)dE,
\]

with

\[
f(x) = \frac{cx - 1}{c - 1}, \quad g(x) = 1 - f(x),
\]

in order to assure energy and particle number conservation.

The solution of the homogeneous equation is

\[
h_{ni}(X) = h_{ni}(X_o) \exp \left\{ -\frac{(1 - W_{nn}^{ii})(X - X_o)}{\lambda_{ni}} - \frac{m_n(L(X) - L(X_o))}{c\tau_n E_i} \right\},
\]

with \( L' = \rho^{-1} \). The solution of the full equation is

\[
h_{ni}(X) = h_{ni}(X_o) \exp \left\{ -\frac{(1 - W_{nn}^{ii})(X - X_o)}{\lambda_{ni}} - \frac{m_n(L(X) - L(X_o))}{c\tau_n E_i} \right\}

+ \int_{X_o}^{X} \left\{ \sum_{m} \sum_{j=i+1}^{j_{max}} h_{mj}(X') \left[ W_{mn}^{ji} \frac{W_{mn}^{ji}}{\lambda_{mj}} + D_{mn}^{ji} \frac{m_n \rho^{-1}(X')}{c\tau_n E_j} \right] + S_{ni}(X') \right\} \exp \left\{ -\frac{(1 - W_{nn}^{ii})(X - X')}{\lambda_{ni}} - \frac{m_n(L(X) - L(X'))}{c\tau_n E_i} \right\} dX'
\]

## 7 Electromagnetic cascade equations

We take the longitudinal coordinate to be \( z \), the lateral coordinates \( x \) and \( y \). The direction of a particle at a given point \( (x, y, z) \) may be written as

\[
\vec{u} = \begin{pmatrix} \sin \theta \cos \phi \\ \sin \theta \sin \phi \\ \cos \theta \end{pmatrix}.
\]

We define

\[
\alpha = \sin \theta \cos \phi,
\]
\[
\beta = \sin \theta \sin \phi.
\]

In the following we work with the coordinates \( \alpha, \beta, x, y, z \) of a particle and its energy \( E \). We consider differential spectra of electrons, positrons, and gammas

\[
l_n(\alpha, \beta, x, y, z, E) = \frac{dN_n}{d\alpha d\beta dx dy dE},
\]

with \( n \in \{e^+, e^-, \gamma\} \), for a given depth \( z \). The energy \( E \) refers to the kinetic energy of electrons and total energy of photons. We suppose a source term
\( S_n = S_n(\alpha, \beta, x, y, z, E) \). In the following we will suppress the arguments of the functions \( l_n, S_n \) etc, unless explicitly needed. The equations of electron-photon cascades may be written in the following form

\[
\vec{u} \nabla l_n = L_n[l] + L_{\text{Coul},n}[l_n] + S_n,
\]

with

\[
\vec{u} \nabla = \cos \theta \frac{\partial}{\partial z} + \alpha \frac{\partial}{\partial x} + \beta \frac{\partial}{\partial y}.
\]

and

\[
L_n[l] = \sum_m \int_{E_0}^{E} l_m(E') W_{mn}(E', E) \, dE',
\]

with the differential spectra

\[
W_{e^-e^-}(E', E) = W_{\text{brem}}(E', E' - E) + W_{\text{Moeller}}(E', E) \Theta(E - E_{\text{cut}}) - \left( \sigma_{e^-}(E) - \frac{\partial}{\partial E} \beta \right) \delta(E' - E),
\]

\[
W_{e^+e^+}(E', E) = W_{\text{brem}}(E', E' - E) + W_{\text{Bhabha}}(E', E) \Theta(E - E_{\text{cut}}) - \left( \sigma_{e^+}(E) - \frac{\partial}{\partial E} \beta \right) \delta(E' - E),
\]

\[
W_{e^+e^-}(E', E) = W_{\text{Bhabha}}(E', E) \Theta(E - E_{\text{cut}}),
\]

\[
W_{\gamma e^-}(E', E) = W_{\text{pair}}(E', E) + W_{\text{Compton}}(E', E' - E),
\]

\[
W_{\gamma e^+}(E', E) = W_{\text{pair}}(E', E),
\]

\[
W_{e^-\gamma}(E', E) = W_{\text{brem}}(E', E) + W_{\text{annih}}(E', E),
\]

\[
W_{\gamma\gamma}(E', E) = W_{\text{Compton}}(E', E) - \sigma_{\gamma}(E) \delta(E' - E),
\]

and

\[
\sigma_n(E) = \sum_m \int_{E_{\min}}^{E} W_{nm}(E, E') \, dE'.
\]

The ionization energy loss coefficient is obviously zero for photons, \( \beta_{\gamma} = 0 \), the same as the Coulomb term, \( L_{\text{Coul},\gamma} = 0 \). Otherwise

\[
L_{\text{Coul},e}[l_e] = \int d\eta d\zeta \left\{ l_e(\alpha + \eta, \beta + \zeta, ...) - l_e(\alpha, \beta, ...) \right\} \frac{d\sigma_{\text{Coul}}(\eta, \zeta)}{d\eta d\zeta},
\]

where \( l_e \) stands for \( l_{e^-} \) or \( l_{e^+} \). An expansion gives

\[
l_e(\alpha + \eta, \beta + \zeta, ...) - l_e(\alpha, \beta, ...) = \sum_{i, j} \frac{\partial^{i+j} l_e}{\partial \alpha^i \partial \beta^j} \eta^i \zeta^j.
\]

So

\[
L_{\text{Coul},n}[l_n] = \sum_{i, j} C_{ij}^n \cos^2 \theta \frac{\partial^{2i+2j} l_e}{\partial \alpha^{2i} \partial \beta^{2j}}.
\]
with
\[ C_{ij}^0 = 0, \quad C_{ij}^n = \frac{1}{2!^{i+j}} \int d\vartheta^2 d\varphi \frac{\eta^{2_i} \zeta^{2_j}}{\cos^2 \theta} \frac{1}{2\pi} \frac{d\sigma_{\text{Coul}}(\vartheta^2)}{d\vartheta^2}, \] 

So we finally have
\[ (\cos \theta \frac{\partial}{\partial z} + \alpha \frac{\partial}{\partial x} + \beta \frac{\partial}{\partial y})l_n = L_n[l] + \sum_{i+j > 0} C_{ij}^n \cos^2 \theta \frac{\partial^{2i+2j} l_n}{\partial \alpha^{2i} \partial \beta^{2j}} + S_n. \]

In the small angle approximation one may integrate over the variables \( \alpha, \beta, x, y \) and one obtains for
\[ \tilde{l}_n(z, E) = \int l(\alpha, \beta, x, y, z, E) d\alpha d\beta dx dy \]
the equation
\[ \frac{\partial}{\partial z} \tilde{l}_n = L_n[\tilde{l}] + S_n, \]
which can be solved after energy discretisation without major problems.

8 Summary (Cascade Equations)

CONEX 1D works already quite well, it gives practically the same results as the Monte Carlo calculations with Corsika, although the time needed for the computations is enormously reduced. Concerning the 3D version, there is work in progress.

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