Crystal, cluster and quantum liquid nuclear states

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Abstract. Nuclear states are often described as quantum liquid states. Using the localisation parameter, it is possible to understand cluster states in nuclei as hybrid states between quantum liquid and crystal states. Recent calculations discussing the effect of the depth of the confining potential, as well as various length scales ratios, are presented in order to provide a complementary view on the cluster phenomenon in nuclei.

1. Introduction: states of matter
It is well known that when temperature decreases and density increases, a system of $A$ constituents interacting with a short range attractive interaction undergoes from a classical gaseous state to a liquid one. When further decreasing the temperature and increasing the density, the system becomes a solid, which is microscopically described as a crystal structure. Microscopically the system went from weakly interacting constituents (gas) towards more interacting ones (liquid) to bound states into crystal having constituents fixed at periodic nodes (solid). What happens if the density further increases, that is adding constituents between the nodes of the crystal? In this case, the constituents at nodes start to overlap, forming a molecule of constituents, and the system becomes clusterised. When the density further increases, the dense system becomes homogeneous and this is the quantum liquid state [1]. If the density increases again the inner structure of the constituents starts to impact, and the system cannot be considered of only $A$ interacting constituents anymore. Figure 1 summarizes the global picture of the above discussed states of matter.

Quantum effects start to have an impact from the solid to the quantum liquid states. This happens when the typical dispersion of the constituents is non-negligible compared to the inter-constituents distance. The wavelength of a constituent can be approximated by [1]:

\[ \lambda = \frac{2\pi}{k} \]

where $k$ is the wave vector.
\[ \alpha = \frac{b}{r_0} ; \Lambda ; \lambda \]

\[ \lambda = \frac{\hbar}{p} \approx \frac{\hbar}{\sqrt{2mE_{kin}}} \]  

where the kinetic energy of the constituent is approximated by \( kT \). One therefore sees that decreasing temperature (or increasing density) generates a larger dispersion of the wavefunction of the constituent, which can lead to quantum effects when \( \lambda \) is non-negligible compared to the inter-constituents distance \( r_0 \). This means that in a dense system (quantum liquid) the filling of the space between the nodes of the crystal is done so to achieve a homogeneous density but with the quantum side-effect that the constituents get also delocalised.

Several concomitant quantities can be further used to analyse this effect:

- i) \( \lambda/r_0 \) as mentioned above. This is also discussed in Ref. [2].
- ii) \( b_0/r_0 \) where \( b_0 \) is a sort of interacting-constituent Compton wavelength:

\[ b_0 \approx \frac{\hbar}{\sqrt{2mV_0}} \]  

where \( V_0 \) is the typical magnitude of the inter-constituent interaction. This ratio is similar to i) where the kinetic temperature has been approximated by the potential magnitude \( V_0 \). This ratio represents the quantality parameter used by Mottelson [3]:

**Figure 1.** Sketch of the states of matter.
\[ \Lambda = 2 \left( \frac{b_0}{r_0} \right)^2 \]  
\[ (3) \]

• iii) \( b/r_0 \) where \( b \) is the typical dispersion of a constituent, taking into account the finite size of the system. This is the localisation parameter defined in Ref. [4]:

\[ \alpha = \frac{b}{r_0} = \frac{\sqrt{\frac{\hbar A^{1/6}}{(2mV_0r_0^2)^{1/4}}}} \]
\[ (4) \]

where \( V_0 \) is the depth of the confining potential. It allows to study the evolution of the states with respect to the number of constituents \( A \) [5] and is well adapted to systems where finite-size effects are relevant \((A \lesssim 10^3)\), such as nuclei. The quantity (Eq. (3)), on the contrary only depends on the interaction and would be the same in nuclei or in infinite nuclear matter. It does not consider finite size effects.

From this picture, cluster states in nuclei are hybrid states between quantum liquid and crystal states. They can be produced in light nuclei [6] and/or in dilute nuclear systems as discussed and noticed both experimentally and theoretically [7, 8]. In Ref. [5] it has been shown that a nuclear crystal cannot be produced in a nucleus, contrarily to in the crust of neutron stars. Another way to produce a nuclear cluster or a nuclear crystal might be by heating the nucleus. Using Eq. (1), the temperature to be reached to produce a cluster \((\lambda \sim r_0)\) or a crystal \((\lambda \sim 0.1r_0)\) are 10 MeV and 1 GeV, respectively. These may be too high in order to keep the finite system bound.

The well-known large mean-free path of nucleons in nuclear matter [9] shows that nuclei usually behave as quantum liquid states. However, as a function of the number of nucleons \( A \), the localisation parameter Eq. (4) shows that the delocalisation induced by the interactions is quenched in light nuclei due to the effect of the confining potential [5]. This effect is depicted on Fig. 1 and is due to the saturation of nuclear interactions, which allows the localisation parameter to depend on \( A \) while the depth of the confining potential remains rather constant.

The depth of the confining potential is therefore a key value and one can wonder about its actual magnitude. The answer is not straightforward since it is not an observable. In other fields of physics, such as in atomic traps, it is possible to experimentally tune the depth of the confining potential [10]. A deeper potential generates more localised structures. What happens in the case of nuclei? How deep is the confining nuclear potential? The question cannot be addressed experimentally, but it can be theoretically by using functionnals predicting different depth of the nuclear confining potential.

This is the case of relativistic functionnals compared to non-relativistic ones. In the relativistic approach, the depth of the potential is related to the so-called scalar and vector potentials generated by the mesons exchange. It has a deep connection with the spin-orbit potential, also related to these scalar and vector ones [4]. In the non-relativistic approach, the depth of the confining potential depends on the magnitude of the attraction of the nucleon-nucleon interaction, which remains a free parameter. As a result the depth of the relativistic potential is significantly larger than the non-relativistic one. This induces more localisation and it may be possible that it has sounder roots than the non-relativistic one. Fig. 3 shows the comparison between relativistic and non-relativistic potentials in \( ^{20}\text{Ne} \).

It should be noted that studying the depth of the potential highlights this important quantity on clusterisation features and allows to connect the nuclear cluster phenomenology to various analogous previous approaches in other fields of physics such as in condensed matter [10, 11, 12], where the depth of the confining potential is known to impact clusterisation.
2. EDF approaches and cluster states

As a result, light nuclei densities are more localised in the relativistic approaches, as can be seen on numerous Figs. of Refs. [13, 14]. Therefore relativistic approaches predict clusterisation in light nuclear systems in agreement with cluster phenomenology. Moreover this means that relativistic energy density functionals (EDFs) are able to describe both quantum liquid and clusters aspects of nuclei and hence encompasses interesting features about the origin of clusterisation in nuclei. Let us therefore underline the specificities of EDF approaches with respect to clusters, since this way of studying clusters is maybe not well known by the community because at the interface between two domains:

- EDF approaches allow for the most general antisymmetrized product of nucleonic wave functions to describe a nuclear state: there is no a priori assumption on the nucleon’s wavefunction
- Correlations beyond the mean-field are effectively included by the EDF itself, its parameter being determined from nuclear observables including full correlations, allowing for saturation and \( \alpha \) particle description
- Results are obtained in the intrinsic frame of the nucleus, allowing for cluster shapes
- EDFs are well suited to investigate nuclear structure on the whole nuclear chart including the quantum liquid vs cluster state transition

The contribution of J.-P. Ebran in the present proceedings [14] shows the large variety of clusters states which can be predicted with a microscopic relativistic EDF.

3. Towards a global picture

The fact that EDF approaches are able to describe clusterisation effects in nuclei may help to move towards a global picture of nuclear states. It should be stressed again that the present
Figure 3. Comparison between the relativistic and non-relativistic potentials and single-neutrons energies in the mean-field framework.

approach is the regular relativistic one, used to describe usual quantum liquid states in nuclei without any further assumption [15]: clusters states naturally come out.

What is already known from decades of study on cluster states in nuclei? i) Deformation and single particle levels degeneracy play an important role [6], ii) excited states are easier to clusterise, as we know from the Ikeda picture [16] and iii) neutron excess can favorize clusterisation, the excess neutron playing a role analogous to molecular bonding [6].

All these 3 features are included, predicted and understood by the present fully microscopic EDF approaches: i) level degeneracy and deformation studies shows that both large deformation and single particle level degeneracy raising favor clusterisation [13, 14], ii) excited states are more likely to clusterise [17, 18], and iii) molecular bonding are naturally predicted in neutron-rich nuclei using relativistic EDF in $^{10}$Be for instance [13, 14]. This shows that EDFs are able to recover what is known from the cluster states phenomenology.

It should be also noted that this provides a global framework to understand haloes and cluster states [5]: in both cases it is the energy of the system which is related to the spatial localisation in nuclei: large delocalisation outside of the potential for weakly bound systems in the case of haloes, and localisation inside a deep potential in the case of clusters.

4. The origin of clusterisation
One can go one step further and try to investigate about the origin of clusterisation in the view of the present EDF tool. It is known that when diluting a nuclear system, at about 1/3 of the saturation density, it reaches a transition from a homogeneous to a cluster system [7, 8]:
at low density, the system gains energy to have separated clusters at saturation density rather than having a homogeneous diluted density. This points out saturation as a relevant quantity to understand the origin of clusterisation.

Fig. 4 shows the competition between the localisation parameter and the quantality one [18]. In this framework cluster states are hybrid states where quantality (related to the nuclear interaction) is large enough to provide delocalised nucleons, but in light systems, the radius of the confining potential attenuates this delocalisation and provides some clusterisation effect. This is possible because the depth of the confining potential remains rather constant over the nuclear chart, due to saturation properties.

![Diagram showing the competition between localisation parameter and quantality](image)

**Figure 4.** Effects of both quantality and localisation parameter on fermionic systems of $A$ constituents [18].

5. Conclusion
The role of the depth of the confining potential and of the localisation parameter in nuclear clusterisation has been analysed, allowing for a connection with similar cluster studies in other fields of physics. Saturation plays a key role in the nuclear clusterisation. Relativistic EDF are able to describe clusterisation phenomena and to provide microscopic light on its phenomenology. These EDF approaches do not contain any further a priori assumption aimed to describe clusterisation. The present approach allows to better connect the EDF approach with previous nuclear cluster studies, and also the nuclear cluster field with clusters studies in other areas of physics.

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