Modern Approaches to Mathematical Modeling of River Runoff in the Territory of the European Part of Russia

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Abstract. This article deals with the construction of mathematical models, which describe the dependence of the river runoff modulus in the territory of the European part of Russia on landscape-geographical conditions and anthropogenic load using modern regression-type statistical models. The obtained models reflect about 80% of the variability of the data and provide acceptable forecast accuracy. As well as these models reflect the main dependencies of river runoff on the conditions of its formation in the research scale. Spatial extrapolation of the river runoff values to the hydrologically unexplored areas of the European part of Russia is performed.

1. Introduction
The formation of river runoff is a complex multifactorial process. The study of dependencies between hydrological characteristics and different zonal and azonal factors is often too general, that under conditions of hydrometeorological monitoring network reduction can lead to serious errors in predicting runoff for unexplored river basins. The strengthening of anthropogenic load on basin geosystems and climate change necessitates the development of new approaches to establishing quantitative links between the characteristics of the river runoff and the totality of its controlling factors. One of such approaches can be mathematical modeling of river runoff.

General ideas about the mechanism of formation of river runoff were described in 1940-1950s in the works of such scientists as Alekseev G.A., Befani A.N., Velikanov M.A., Kalinin G.P., Kohler M.A., Linlesley R.K., Horton R.E. As a trend, mathematical modeling took shape in the 1970s, which was associated with the use of computers for processing large amounts of information [1, 2]. This period is characterized by the appearance of closed models that describe the entire hydrological cycle from the arrival of water to the catchment area to the formation of runoff in the main-stream station [3]. An important role in the development of the modeling of hydrological processes on the river basins in the Russian hydrology was played by the Kuchment’s monograph, published in 1972. This work presented the foreign and domestic modeling experience accumulated by that time. At present, scientific groups under the leadership of Kuchment L.S., Burakov D.A., Harzman B.I., Gelfan A.N., Gusev E.M., Kondratiev S.A. and others have significant achievements in runoff modeling [4, 2].

Three main classes of models are distinguished depending on the nature of the description of the runoff formation processes and the extent of using theoretical and empirical information: "black box" type models, conceptual and physical-mathematical models [5]. This classification allows us to identify the most important principles that are the basis of the model. However, it is not unique. So, for example, three types of models are distinguished by the method of specifying a mathematical structure: non-linear...
multifactorial models ("neural networks"), multiple linear regression models and physico-mathematical models [6].

Physico-mathematical models use physical laws for quantitative description of the processes of runoff formation taking into account their spatial heterogeneity [5]. The universal deterministic modeling system "Stock-Erosion-Pollution" and the distributed deterministic hydrological model of the flow formation processes "Hydrograph" can be cited as examples of physico-mathematical models. Both models were developed under the leadership of Vinogradov Yu.B. [1, 7]. Other well-known physico-mathematical models (MIKE, SHE, TOPOG, KINEROS2) are oriented to rivers with a mild climate and, basically, describe the processes of rainwater flow formation [8]. The main direction of practical application of physico-mathematical models is connected with modeling situations in unobserved or in changed conditions, in particular, the pre-calculation of the risk and magnitude of possible hazardous floods, the assessment of the impact of human activities on river runoff [9].

The conceptual models are based on various concepts describing the physical processes of river runoff formation with preliminary spatial averaging of the input effects on the catchment area [10]. This approach to modeling involves the solution of systems of equations, which are based on various concepts describing the physical processes of river runoff formation. Examples of conceptual models are the model of river basin pre-flood cycle (Harzman), the model of spring flood hydrograph generation for plain rivers (Vinogradov), the SWMM model (Huber, Dickens) [1].

Along with models that take into account the spatial heterogeneity of input data and require a large amount of information about the properties of the catchment area, there is a number of models based on empirical relationships between the river runoff and the factors of its formation (the "black box" type model). Empirical relationships are identified on the basis of statistical methods, the theory of random processes or using artificial neural networks [11]. Despite the fact that such models practically do not use a priori information about the processes occurring in the river basin, they are successfully used in forecasting practice. In particular, artificial neural networks are well suited for predicting the structure and trends of data. Using the example of the Bolhovets River (Belgorod, Russia), neural network technologies were successfully applied to predict the dynamics of the water regime of small rivers depending on climatic factors [12].

This class of models includes the models of multiple linear regression, the mathematical structure of which is determined by the choice of the approximating function. The main goal of multiple regression is to construct a model with several factors and determine the impact of each factor separately, as well as their combined effect on the studying indicator under investigation. The advantages of this approach are clear interpretation and simplicity of the definition of parameters; limitations are the problem of choosing the essential factors and their independence, i.e. the absence of a linear connection between them. An example is the use of a linear regression model for the formation of the annual runoff of rivers in the Volga basin in a variety of meteorological factors [13].

The purpose of this research is to study the statistical dependencies of river runoff characteristics on landscape-geographical conditions and anthropogenic load using modern regression-type statistical models. The territory of the European part of Russia, where the main part of the population, industrial and agricultural potential of the country is concentrated, was selected as the study territory. The modulated element is the river runoff modulus. The use of river runoff modulus as a characteristic of the water runoff from a catchment area unit will allow, in the presence of a model describing the dependence of this indicator on landscape-geographical and anthropogenic conditions, extrapolate its value to the hydrologically unexplored areas of the territory of the European part of Russia.

2. Material and methods
Two multidimensional samples for the statistical study of the regularities of the river runoff formation were formed. The catchments referred to hydrological posts, which provided with river runoff data, as elements of the first sample, were made. The second sample is small river basins and their intervening areas (over 60.000), planar covering the entire territory of the European part of Russia. Both samples include a list of quantitative and categorical variables: the catchment area, 16 climate indicators,
morphometric relief characteristics in the basin, interest forest coverage, ploughness, grassing, wetlands in the basin, the characteristics of the soil cover (the predominant type of soil and type of parent rock), and the predominant class of pre-Quaternary sediments. These variables \{X\} are regarded as explaining, i.e. describing the conditions of formation of river runoff. The first sample also includes a dependent variable Y - the river runoff modulus of catchments allocated relatively hydrological posts.

The basins of small rivers and their intervening areas of the European part of Russia were obtained by constructing in automatic mode with GIS technologies using the corrected model of the relief GMTED2010 and the raster model of the hydrographic network [14]. The boundaries of the basins, related to the hydrological posts, were constructed on the basis of the obtained relief model and a point layer representing hydrological posts.

Published materials of long-term regime observations at hydrological posts (Surface Water Resources of the USSR), data from open sources (http://caspi.ru/), as well as data from the «All Russian Research Institute of Hydrometeorological Information» were used as sources of information on river runoff. The source of the data on climate factors was the data of the Federal Service for Hydrometeorology and Environmental Monitoring weather stations (open data of the RIHMI-WDC). Source of data on soil cover characteristics: The Unified State Register of Soil Resources of Russia, presented as a vector layer, the objects of which are digitized contours of a soil map of the RSFSR with scale 1: 2,500,000. The model of the relief GMTED2010 with a spatial resolution of 250 m was used as a source of information on the relief of the study area. The map of land cover types, developed at the Space Research Institute for the Earth (Moscow-2015), served as a source of information on the types of earth cover and the degree of anthropogenic load. The geological conditions were recorded using the State geological map of pre-Quaternary deposits of scale 1: 1,000,000, represented as the sheets of maps of three generations in vector form and the form of scanned paper maps.

The construction of models was preceded by an analysis of the structure of explanatory variables, the study of their mutual correlation with the subsequent choice of an independent subset \{X\}.

The modeling of the dependence of the river runoff modulus on the conditions of its formation was carried out from the data of the first sample using methods such as the generalized linear model (GLM) and the generalized additive model (GAM), separately for subsamples of basins located in plain and mountain areas. Calculations were carried out using programs written in the statistical environment R. The best subset of regressors in constructing models was selected by analyzing the statistical significance of the contribution of each regressor to the model and analyzing the VIF factor. In addition, the value of the information criterion AIC, which allows comparing models for a given dependent variable, was monitored. Standardized values of independent variables, which were used in the construction of models, allow considering the absolute values of the model coefficients as an estimate of the contributions of the relevant factors to the value of Y.

3. Results and discussion
Two models reflecting the main regularities of river runoff for mountainous and plain areas were obtained as a result of the calculations. The criterion for assessing the quality of the obtained models was the adjusted coefficient of determination. It should be noted that the complication of modeling methods in comparison with linear methods does not significantly improve the quality of the model.

Consider the model of the river runoff modulus constructed by linear methods for plain areas.

$$
\log (Y) = -5.15 - 0.51 \cdot T + 0.21 \cdot \log (R) + 0.08 \cdot \log (S) + 0.10 \cdot F
$$

The presented model explains more than 80% of the variability of the data and well reflects the main regularities of river runoff in a given research scale.

The sum of the active temperatures (T), the annual precipitation (R), the average steepness of the slopes (S), and the percentage of forest coverage in the catchment area (F) are the most significant predictors of the model.

The obtained model is well interpreted in terms of the water balance equation, which is confirmed by the positive contribution of the sum of atmospheric precipitation and the negative contribution of the
sum of the active temperatures. The positive effect of the average steepness of the slopes is due to its reverse effect on the amount of evaporation, in other words, with increasing steepness of the slopes the speed of water movement increases and evaporation losses decrease. The positive contribution of the percentage of forest coverage in the catchments is manifested in the following. The soil within forest areas is characterized by increased filtration capacity due to the presence of loose forest litter and less depth of soil freezing. This contributes to the replenishment of groundwater resources, which are drained by rivers during the low streamflow period, thereby increasing the annual runoff modulus.

The model built for mountain areas explains 72% of the variability of the data. The height of the catchment area (H) and the percentage of ploughness in the catchment area (A) were added to the model as the main predictors.

\[ \log(Y) = -5.01 - 0.19 \cdot T + 0.29 \cdot \log(R) + 0.72 \cdot \log(S) - 0.20 \cdot A - 0.43 \cdot H \] (2)

The negative impact of the ploughness percentage in catchment rate on the river runoff modulus is associated with the depletion of ground waters, which are drained by rivers during low streamflow period. This, in turn, is related to an increase in the agricultural development of catchments and, as a consequence, deterioration in the filtration properties of soils. The negative contribution of the average height of the catchments is due to the fact that the thickness of the aeration zone and the depth of occurrence of groundwater increase with increasing height. Deterioration of drainage conditions leads to a reduction in runoff.

Figure 1 shows diagrams illustrating the qualities of the constructed models - the observed values of the river runoff modulus are displayed against the model values.

![Figure 1](image)

\textbf{Figure 1.} Diagrams illustrating the quality of the constructed models for the river runoff modulus: a). for plain territories, b). for mountain territories

The constructed models provide an acceptable forecast accuracy, which allows using the data of the second sample to calculate the predicted (model) values of the river runoff for river basins and intervening areas planar covering the study area. In this way, spatial extrapolation of the river runoff values (river runoff modulus and the annual runoff layer) to the hydrologically unexplored river basins of the European part of Russia was conducted. The adequacy of the obtained spatial models was verified by comparing them with previously constructed small-scale maps (National Atlas of Russia, etc.).

4. Acknowledgements

The work is performed according to the Russian Science Foundation (15–17–10008).
References

[1] Antokhina E 2012 Water regime of the rivers of the European territory of Russia and its study on the basis of the flow formation model (Moscow: Moscow State University Press) p 219

[2] Kuchment L Mathematical modeling of river flow (Gidrometeoizdat Press) p 191

[3] Koren' V Mathematical models in river flow forecasts (Gidrometeoizdat Press) p 200

[4] Gorshkova A, Urbanova O and Karimova A 2015 International Scientific and Research Journal 8 (39) 66-72

[5] Motovilov Yu 2013 Int. Conf. "The First Vinogradov Readings. The Future of Hydrology " (St. Petersburg) pp 25-27

[6] Ignatov A 2011 Geography and natural resources 4 114–118

[7] Vinogradov Yu and Vinogradova T 2010 Mathematical modeling in hydrology (Moscow: Publishing Center "Academy" Press) p 304

[8] Guide to hydrological practice 2012 (Geneva: World Meteorological Organization) p 324

[9] Kuchment L, Gelfan A, Kondratiev S and Lavrov S 2013 VII All-Russian Congress

[10] Geyman T 2015 South-Siberian Scientific Herald 4(12) 47-49

[11] Belyakova P 2015 Flood discharge of the Russian rivers of the Black Sea coast of the Caucasus (Moscow: Moscow State University Press) p 200

[12] Kuzmenko Ya, Lisetskiy F and Pichura V 2012 Modern problems of science and education 6

[13] Frolova N 1984 Geographical regularities of the spatio-temporal variability of the annual runoff of the Volga River basin (Moscow: Moscow State University Press) p 201

[14] Ermolaev O, Maltsev K, Muharamova S, Kharchenko S and Vedeneeva E 2017 Geography and natural resources 2 27-36