Rare B decays and Tevatron top-pair asymmetry

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Abstract

The recent Tevatron result on the top quark forward-backward asymmetry, which deviates from its standard model prediction by 3.4\,\sigma, has prompted many authors to build new models to account for this anomaly. Among the various proposals, we find that those mechanisms which produce \(t\bar{t}\) via \(t\)- or \(u\)-channel can have a strong correlation to the rare \(B\) decays. We demonstrate this link by studying a model with a new charged gauge boson, \(W'\). In terms of the current measurements on \(B \to \pi K\) decays, we conclude that the branching ratio for \(B^- \to \pi^- K^0\) is affected most by the new effects. Furthermore, using the world average branching ratio for the exclusive \(B\) decays at 2\,\sigma level, we discuss the allowed values for the new parameters. Finally, we point out that the influence of the new physics effects on the direct CP asymmetry in \(B\) decays is insignificant.

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Recently, the forward-backward asymmetry (FBA) in top-pair production is measured by the DØ [1] and CDF [2] Collaborations in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV. With an integrated luminosity of 5.20 fb$^{-1}$, CDF further finds that the large top quark FBA occurs at large $t\bar{t}$ rapidity difference ($\Delta y$) and invariant mass of $t\bar{t}$ ($M_{t\bar{t}}$) [3]:

$$A_{t\bar{t}}(|y| < 1.0) = 0.026 \pm 0.118 [0.039 \pm 0.006],$$

$$A_{t\bar{t}}(|y| \geq 1.0) = 0.611 \pm 0.256 [0.123 \pm 0.008],$$

$$A_{t\bar{t}}(M_{t\bar{t}} < 450 \text{ GeV}) = -0.116 \pm 0.153 [0.040 \pm 0.006],$$

$$A_{t\bar{t}}(M_{t\bar{t}} \geq 450 \text{ GeV}) = 0.475 \pm 0.114 [0.088 \pm 0.013],$$

(1)

where the value in the square brackets denotes the theoretical result calculated by Monte Carlo program (MCFM) [4] at next-to-leading order (NLO). The inconsistency between data and standard model (SM) predictions displayed in Eq. (1) indicates that new physics effects may be at play and hence leading to this anomalous result.

In the wake of this 3.4σ deviation of the observed value of the top quark FBA from the SM predicted one, several possible solutions have been proposed and studied by authors in Refs. [5–30, 32–41]. Among these new mechanisms, we see that a potentially interesting correlation to $B$ meson physics may arise. In particular, the new interaction that contributes to $t\bar{t}$ production by $t$- or $u$-channel will also contribute to rare $B$ decays, e.g. $B \rightarrow \pi(\pi, K)$. This is achieved through box diagrams like those sketched in Fig. 1 where $X$ could be a colored vector [5, 7, 10], $Z'$ [8], $W'$ [9, 26, 30, 32] or colored scalar bosons [11, 12, 14, 35], while $q' = d$ or $u$ depending on the charge of the $X$ particle. It is known that with the enormous $B$ meson production at LHCb, BaBar, Belle and Tevatron, the errors in the measured branching ratios (BRs) for $B \rightarrow \pi K$ decays have now reached percent level. As a result of this high accuracy, it is interesting to investigate how strong is the correlation between rare $B$ decays and top quark FBA. In addition, one can study the influence on the associated CP asymmetry (CPA) in such new physics setups. We present our analyzes on both issues in the following.

In order to illustrate the impact of the new physics (which leads to a large top-pair FBA as measured) on the low energy $B$ sector, in this paper, we shall focus on the case with a new $W'$ interaction, namely, the $t$-$d$-$W'$ coupling. A similar discussion could be applied to other new interactions in the $Z'$, colored vector or colored scalar models. Since we are studying the impact of new physics on $B$ decays, the detailed analysis of top-pair production
FIG. 1: Feynman diagrams for $b \to q'\bar{q}'q$ with $q' = d$ or $u$ and $q = d, s$, where $X$ stands for a generic new particle.

could be referred to those papers listed in the references. We start by writing the relevant interaction as

$$\mathcal{L} = -i \frac{g'_2}{\sqrt{2}} \bar{t}\gamma^\mu W'^\mu P_R d + h.c,$$

where $g'_2$ is the new gauge coupling which is regarded as a free parameter. By including the contributions of $W$ and the associated Goldstone bosons, the Hamiltonian for $b \to qd\bar{d}$ \hspace{1cm} ($q = d, s$) shown in Fig. 1 is given by

$$\mathcal{H}_{W'} = -\frac{G_F}{\sqrt{2}} V_{tb} V^*_{iq} \left( \frac{g'_2 m_t}{4\pi m_{W'}} \right)^2 I(x_W, x_t) \bar{d}(1 - \gamma_5)b\bar{q}(1 + \gamma_5)d,$$

where $C_{W'}$ stands for the new Wilson coefficient at electroweak scale, $x_W = m_W^2/m_{W'}^2$, $x_t = m_t^2/m_{W'}^2$, and $I(x_W, x_t) = (1 + x_W)I_1(x_W, x_t) + 2(x_W + x_t)I_2(x_W, x_t)$ with

$$I_1(a, b) = \int_0^1 dz_1 \int_0^{z_1} dz_2 \frac{z_2}{1 - (1 - a)z_1 - (a - b)z_2},$$

$$I_2(a, b) = \int_0^1 dz_1 \int_0^{z_1} dz_2 \frac{z_2}{(1 - (1 - a)z_1 - (a - b)z_2)^2}.$$ (4)

We note that there are only two new free parameters in $C_{W'}$. To illustrate the dependence on $g'_2$ and the mass of $W'$, we plot the contours for $C_{W'}$ as a function of $g'_2$ and $m_{W'}$ in Fig. 2. The number on each curve in Fig. 2 denotes the value of $C_{W'}$ in units of $10^{-2}$. For instance, with $g'_2 = 3$ and $m_{W'} = 700$ GeV, we get $C_{W'} = 0.85 \times 10^{-2}$.

Notice that if we perform Fierz transformation on the fields in Eq. (3), the four-Fermi operator could be transformed into $\bar{d}_\alpha \gamma_\mu (1 + \gamma_5) d_\beta \bar{q}_\beta \gamma_\mu (1 - \gamma_5) b_\alpha$, where $\alpha, \beta$ denote the color indices. This operator is the same as the $O_6$ operator in the SM [43] and its corresponding
Wilson coefficient at $m_W$ scale is $C_6 \simeq -0.2 \times 10^{-2}$. According to our brief analysis, we see that the new interaction has the same behavior as $O_6$, and so, its contribution should be similar to that dictated by $C_6$.

![Graph](image)

**FIG. 2:** Contours for $C_{W'}$ as a function of $g'_2$ and $m_{W'}$, where the number on each curve denotes the value of $C_{W'}$ in units of $10^{-2}$.

Since the new four-Fermi interactions in Eq. (3) are induced by loop diagrams, we expect that the influence of the new effects on the tree dominant $B \to \pi\pi$ decays should be small. Therefore, we only concentrate on the $B \to \pi K$ decays. This is because at quark level, they are associated with the $b \to s d \bar{d}$ process which is dominated by gluonic penguins in the SM. In order to comprehend how the new effects contribute to each decay mode in the $B \to \pi K$ processes, we draw the flavor diagrams in Fig. 3, where figures (a) and (b) denote the color-allowed and -suppressed emission diagrams respectively, while figure (c) represents the annihilation diagram. Importantly, we note that the decay $B^- \to \pi^- K^0$, $B^- \to \pi^0 K^-$ and $\bar{B}_d \to \pi^+ K^-$ are influenced by Fig. 3(a), (b) and (c) respectively, whereas $\bar{B}_d \to \pi^0 \bar{K}^0$ involves all three figures.

It is well-known that the calculations of a nonleptonic exclusive decay include factorizable and nonfactorizable parts. Owing to the color flow, the latter is always color-suppressed and smaller than the former. Hence, to simplify the analysis, we only calculate the factorizable contributions from the $W'$ exchange when displaying the influence of the new physics effects. However, we keep both parts when accounting for the SM sector. In order to study the nonleptonic exclusive $B$ decays, we parametrize the relevant decay constants, transition and
with $P = P_M + P_{M'}$, $q = P_M - P_{M'}$, $Q = P_{M_1} + P_{M_2}$ and $\tilde{q} = P_{M_1} - P_{M_2}$. Applying the equations of motion, we then get

$$
\langle 0|\bar{q}_2\gamma^\mu\gamma_5q_1|\bar{M}\rangle = \langle \bar{M}|\bar{q}_1\gamma^\mu\gamma_5q_2|0\rangle = -if_M\frac{m_M^2}{m_{q_1} + m_{q_2}},
$$

$$
\langle \bar{M}'|\bar{q}_2\gamma^\mu q_1|\bar{M}\rangle = \frac{M^2 - M'^2}{m_{q_1} - m_{q_2}}f_{0}^{MM'}(q^2),
$$

$$
\langle M_1M_2|\bar{q}_1\gamma^\mu q_2|0\rangle = \frac{M_1^2 - M_2^2}{m_{q_1} - m_{q_2}}g_{0}^{MM}(Q^2)\frac{Q\cdot \tilde{q}}{Q^2}.\quad (5)
$$

Using the above form factors, the decay amplitude with $W'$ exchange is formulated by

$$
A^{W'}(\pi K) = \frac{G_FV_{tb}V_{ts}^*}{\sqrt{2}}C_{W'}(\mu_b)\zeta_{\pi K}\quad (7)
$$

for $B^- \rightarrow \pi^-\bar{K}^0$, $\bar{B}_d \rightarrow \pi^+K^-$ and $B^- \rightarrow \pi^0K^-$ processes, while

$$
A^{W'}(\pi^0\bar{K}^0) = -\frac{1}{\sqrt{2}}A^{W'}(\pi^-\bar{K}^0) + A^{W'}(\pi^0\bar{K}^-) - \frac{1}{\sqrt{2}}A^{W'}(\pi^+K^-)\quad (8)
$$
for $B_d \to \pi^0 \bar{K}^0$, where

$$
\begin{align*}
\zeta_{\pi^0 K^0} &= f_K \frac{m_K^2}{m_s + m_d} \frac{m_B^2 - m_d^2}{m_B - m_u} f_0^{B\pi}(m_K^2), \\
\zeta_{\pi^+ K^-} &= f_B \frac{m_B^2}{m_b + m_u} \frac{m_K^2 - m_d^2}{m_s - m_d} g_0^{K\pi}(m_B^2), \\
\zeta_{\pi^0 K^-} &= f_\pi \frac{m_B^2 - m_K^2}{2\sqrt{2}N_c} f_0^{B\pi}(m_\pi^2).
\end{align*}
$$

Clearly, once the first three decays have been determined, the last one will be fixed. Since $m_{K,\pi}^2 \ll m_B^2$, it is a good approximation to take $f_0^{B\pi(K)}(m_K^2(\varepsilon)) \approx f_0^{B\pi(K)}(0)$. Moreover, the transition form factor $f_0^{BM}(0)$ and time-like form factor $g_0^{M_1M_2}(m_B^2)$ are associated with SM QCD, and for calculating them, we employ the perturbative QCD approach (PQCD) \[44\]. The resulting formulae are summarized in Appendix A. We note that in Eq. (7), $C_W(\mu_b)$ is the new Wilson coefficient at $\mu_b = \mathcal{O}(m_b)$ scale.

Now that the necessary formulations have been built up, we list the SM theoretical inputs required for our subsequent analysis in Table I, where $V_{ub} = \|V_{ub}\| e^{-i\phi_3}$. Using these values and formulations in the Appendix, the transition and time-like form factor are found to be

$$
\begin{align*}
f_0^{B\pi}(0) &\approx 0.24, \\
f_0^{BK}(0) &\approx 0.35, \\
g_0^{\pi K}(m_B^2) &\approx 0.15e^{i1.1}.
\end{align*}
$$

For estimating $C_{W'}(\mu_b)$, we adopt the relation: $C_{W'}(\mu_b) \approx (\alpha_s(\mu_b)/\alpha_s(m_W))^{\tilde{d}} C_{W'}(m_W)$ with $\tilde{d} = N_c(N_c^2 - 1)/(11N_c - 2n_f)$, $N_c = 3$ and $n_f$ being the number of effective quark flavors \[49\]. In order to combine the SM results with the effects of $W'$ exchange, we write the total decay amplitude for $B \to \pi K$ as

$$
A(\pi K) = A^{SM}(\pi K) + A^{W'}(\pi K).
$$

| | $V_{ub}$ | $\phi_3$ (deg) | $V_{ts}$ | $m_\pi^0$ [GeV] | $m_K^0$ [GeV] |
|-----------------|----------------|----------------|----------------|----------------|----------------|
| $3.9 \times 10^{-3}$ | 65 | -0.041 | 1.34 | 1.70 |
| $m_{d(u)}$ [MeV] | $m_s$ [MeV] | $f_\pi$ [MeV] | $f_K$ [MeV] | $f_B$ [MeV] |
| 4.5 | 100 | 130 | 160 | 190 |
As usual, the SM contributions could be expressed by

\[
A_{SM}^{\pi^-K^0} = -\frac{G_F V_{tb} V_{ts}^*}{\sqrt{2}} P,
\]

\[
A_{SM}^{\pi^+K^-} = -\frac{G_F V_{tb} V_{ts}^*}{\sqrt{2}} P - \frac{G_F V_{ub} V_{us}^*}{\sqrt{2}} T,
\]

\[
\sqrt{2} A_{SM}^{\pi^0K^-} = -\frac{G_F V_{tb} V_{ts}^*}{\sqrt{2}} (P + P_{EW}) - \frac{G_F V_{ub} V_{us}^*}{\sqrt{2}} (T + C),
\]

\[
\sqrt{2} A_{SM}^{\pi^0K^0} = \frac{G_F V_{tb} V_{ts}^*}{\sqrt{2}} (P - P_{EW}) - \frac{G_F V_{ub} V_{us}^*}{\sqrt{2}} C.
\] (12)

In Eq. (12), \(P, P_{EW}, T\) and \(C\) denote, respectively, the contributions from gluonic penguin, electroweak penguin, color-allowed tree and color-suppressed tree. To deal with these hadronic effects, we quote the results calculated by PQCD, where the values in the SM are given by

\[
P = 0.15 e^{-i 0.24}, \quad P_{EW} = 0.018 e^{i 0.44},
\]

\[
T = 1.05 e^{i 0.1}, \quad C = 0.270 e^{-i 1.3}.
\] (13)

Consequently, the resulting SM predicted values of the BRs and CPAs are presented along with the experimental data in Table II.

| Decay | \(B^- \to \pi^-K^0\) | \(\bar{B}_d \to \pi^+K^-\) | \(B^- \to \pi^0K^-\) | \(\bar{B}_d \to \pi^0K^0\) |
|-------|---------------------|---------------------|---------------------|---------------------|
| \(B^{SM}[10^{-6}]\) | 23.5 | 20.46 | 13.25 | 9.16 |
| \(B^{Exp}[10^{-6}]\) | 23.1 ± 1.0 | 19.4 ± 0.6 | 12.9 ± 0.6 | 9.5 ± 0.5 |
| \(A_{SM}^{CP}[\%]\) | 0 | -10.18 | -0.95 | -6.29 |
| \(A_{Exp}^{CP}[\%]\) | 0.9 ± 2.5 | -9.8^{+1.2}_{-1.1} | 5 ± 2.5 | -1 ± 10 |

To display the influence of the \(W'\) effects on BRs and CPAs in \(B\) decays, we consider the two quantities \(R_B\) and \(R_{CP}\) which represent, respectively, the ratios of BR and CPA in the \(W'\) exchange to those in the SM:

\[
R_B \equiv \frac{B^{W'}(B \to \pi K)}{B^{SM}(B \to \pi K)},
\]

\[
R_{CP} \equiv \frac{A_{CP}^{W'}(B \to \pi K)}{A_{CP}^{SM}(B \to \pi K)}.
\] (14)
Because there is only two new free parameters in the $W'$-mediated effects, we use contour plots as a function of $g'_2$ and $m_{W'}$ to display the deviation from the SM predictions. With the SM inputs discussed earlier, the contours for $R_B$ as a function of $g'_2$ and $m_{W'}$ are shown in Fig. 4, where (a)-(d) correspond to the decays $B^- \to \pi^- \bar{K}^0$, $\bar{B}_d \to \pi^+ K^-$, $B^- \to \pi^0 K^-$ and $\bar{B}_d \to \pi^0 \bar{K}^0$, respectively. From this, we see that $W'$-mediated effects have a significant contribution to the BRs of the $\pi^- \bar{K}^0$, $\pi^0 K^-$ and $\pi^0 \bar{K}^0$ modes, whereas the effects are small for the $\pi^+ K^-$ mode. The main reason for this can be traced to the fact that the associated topology for $\bar{B}_d \to \pi^+ K^-$ decay is of annihilation type, whose corresponding hadronic effects are usually smaller than those arising from the emission diagrams.

In addition, if we compare the three decay modes $\pi^- \bar{K}^0$, $\pi^0 K^-$ and $\pi^0 \bar{K}^0$, one finds that the influence on $\pi^0 K^-$ mode is smaller than the other two because Fig. 3(b) is color-suppressed. From our numerical results, we conclude that $B^- \to \pi^- \bar{K}^0$ decay is the most sensitive to the $W'$-mediated effects. In order to display the constraint from the measured data, we plot the allowed range for $g'_2$ and $m_{W'}$ in Fig. 5, where we have adopted the world average BR for $B^- \to \pi^- \bar{K}^0$ with $2\sigma$ errors, $(2.31 \pm 0.20) \times 10^{-5}$ [31], and the PQCD results with the uncertainties are taken as $|P| = 0.15^{+0.04}_{-0.03}$ and $\text{arg}(P) = -0.24^{+0.1}_{-0.2}$ [47]. From these, we find that only those values of $|P|$ approaching to the central value calculated by PQCD contribute to the range $400 < m_{W'} < 800$ and the allowed $m_{W'}$ from other values of $|P|$ is below 200 GeV, which is disfavored by the analysis in Ref. [30]. Based on the result, we observe that $g'_2 \gtrsim 2.5$ is excluded when $m_{W'} \lesssim 700$ GeV. Such constraint on the parameter space highlights the tension between the need to choose a large enough coupling ($g'_2 \simeq 3$ when $m_{W'} \simeq 550$ GeV [32]) for this type of model to explain the top anomaly and satisfying the limits implied by rare B processes at the same time.\(^1\) As a result, we expect that new models which have the $t\bar{t}$ produced via $t$- or $u$-channel are subject to similar restrictions when fine-tuning for a workable parameter set.

Similarly, using the definition in Eq. (14), the ratio $R_{CP}$ as a function of $g'_2$ and $m_{W'}$ is plotted in Fig. 6, where (a)-(c) correspond to the decays $\bar{B}_d \to \pi^+ K^-$, $B^- \to \pi^0 K^-$ and $\bar{B}_d \to \pi^0 \bar{K}^0$, respectively. Looking at Fig. 6(b), it seems that there is a sizable effect on the CPA for $B^- \to \pi^0 K^-$. However, because the SM result for $A_{CP}(B^- \to \pi^0 K^-)$ is small, the

\(^1\) Note that the choice of the numerical values used here is for convenience and ease of comparison with existing work such as [32]. This selection of parameter space would not in any way bias our conclusion.
FIG. 4: Contours for the ratio $R_B$ as a function of $g'_2$ and $m_{W'}$, where the correspondence of each plot is (a) $B^- \to \pi^- K^0$, (b) $\bar{B}_d \to \pi^+ K^-$, (c) $B^- \to \pi^0 K^-$ and (d) $\bar{B}_d \to \pi^0 \bar{K}^0$.

FIG. 5: Allowed range (gray) for $g'_2$ and $m_{W'}$, where the world average BR for $B^- \to \pi^- K^0$ with $2\sigma$ errors has been used.

resulting shift in magnitude of CPA due to the presence of new physics is only around 1%. Furthermore, if we combine the result in Fig. 6(a) with the allowed range depicted in Fig. 5 we see that the modification from the $W'$ effects will be about 10\% of $A_{CP}^{SM}(\bar{B}_d \to \pi^+ K^-)$ only. Therefore, the change in the absolute value of CPA for $\bar{B}_d \to \pi^+ K^-$ is at about the 1\% level. Finally, we note from Fig. 6(c) that the CPA for $\bar{B}_d \to \pi^0 \bar{K}^0$ is more or less insensitive to the $W'$ effects.
FIG. 6: Ratio of CPA in the $W'$ exchange to that in the SM for (a) $\bar{B}_d \to \pi^+ K^-$, (b) $B^- \to \pi^0 K^-$ and (c) $\bar{B}_d \to \pi^0 \bar{K}^0$.

In summary, the fascinating discovery of an unexpectedly large FBA in the top quark pair production at the Tevatron strongly suggests that some new physics may be at play. While the exact nature of this is not clear, we observe that any new interactions introduced to solve the anomaly which produce $t\bar{t}$ via the $t$- or $u$-channel are naturally correlated to rare $B$ decays. In this work, we illustrate this connection by studying a model with a new charged gauge boson $W'$ added to the SM. Using the current precision measurements on $B \to \pi K$ decays, we estimate the constraints on the parameters of the theory. In particular, we find that strong conditions are imposed on the new gauge coupling and the mass of the associated gauge boson. From the data for $B^- \to \pi^- \bar{K}^0$, which is the most stringent among the set of $B \to \pi K$ decays considered, our study shows that the size of the coupling $g'_2$ is strongly constrained from above for the relevant $W'$ mass range of several hundreds GeV. On the other hand, these $W'$-mediated effects are not expected to play a significant role on the direct CPA of $B$ decays.
Acknowledgments

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Appendix A: formulations of $f_0^BM(0)$ and $g_0^M_{1,2}(m_B^2)$ in PQCD approach

For calculating the transition and time-like form factors in PQCD approach, the necessary distribution amplitude of $B$ meson is defined by

$$\langle 0|\bar{q}_{\beta}(z)\gamma_{\alpha}(0)|B(p_B)\rangle = \frac{i}{\sqrt{2N_c}} \int d^4k e^{ikz} \left\{ (\not{p}_B + m_B)\gamma_\gamma \left[ \not{\phi}_B(k) - \frac{H_+ - H_-}{\sqrt{2}} \bar{\phi}_B(k) \right] \right\}_{\alpha\beta},$$

$$\langle \bar{M}(p)\bar{q}_{2\beta}(z)q_{1\alpha}(0)|0\rangle = -\frac{i}{\sqrt{2N_c}} \int^1_0 dx e^{ixp} \left\{ \gamma_\gamma \not{\phi}_M(x) + \gamma_\gamma m_0 \not{\phi}_M^p(x) + m_\gamma \gamma_\gamma (H_+ - H_- - 1)\not{\phi}_M^p \right\}_{\alpha\beta}.$$ (A1)

with $p_B = \frac{m_B}{\sqrt{2}}(1,1,0,0)$, $p = (p^+,0,0,0)$, $n_+ = (1,0,0,0)$, $n_- = (0,1,0,0)$. Since the effects of $\bar{\phi}_B$ are small \[50\], in the calculations we only focus on the contributions from $\phi_B$. By hard gluon exchange, the $B \to \bar{M}$ transition form factor is expressed by

$$f_0^{BM}(0) = 8\pi C_F m_B^2 \int^1_0 dx_1 dx_2 \int^\infty_0 b_1 b_2 b_3 d\Phi_B(x_1, b_1)$$

$$\times \left\{ [1 + x_2] \Phi_M(x_2) + r_M (1 - 2x_2) (\Phi_M^p(x_2) + \Phi_M^\sigma(x_2)) \right\} E_e(t_e^{(1)})$$

$$\times h_e(x_1, x_2, b_1, b_2) + 2r_M \Phi_M^p(x_2) E_e(t_e^{(2)}) h_e(x_2, x_1, b_1, b_1) \}$$ (A2)

with $C_F = 4/3$ and $r_M = m_0^0/m_B$, and the $0 \to \bar{M}_1M_2$ time-like form factor is written as

$$g_0^{M_1M_2}(m_B^2) = -\frac{(m_b + m_a)(m_s - m_d)}{m_K^2 - m_\pi^2} \times 8\pi C_F m_B^2 \int^1_0 dx_2 dx_3 \int^\infty_0 b_2 b_3 d\Phi_B$$

$$\times \left\{ r_M x_2 \Phi_M(x_2) \left( \Phi_M^p(x_1 - x_3) + \Phi_M^\sigma(x_1 - x_3) \right) + 2r_M \Phi_M^p(x_2) \Phi_M(x_1 - x_3) \right\}$$

$$\times E_a(t_a^1) h_a(x_2, x_3, b_2, b_3) + [x_3 r_M (\Phi_M^p(x_2) - \Phi_M^\sigma(x_2)) \Phi_M(x_1 - x_3)$$

$$+ 2r_M \Phi_M(x_2) \Phi_M^\sigma(x_1 - x_3)] E_a(t_a^2) h_a(x_3, x_2, b_3, b_2) \}.$$ (A3)
The hard functions $h_{e,a}$ are given by

$$
\begin{align*}
    h_e(x_1, x_2, b_1, b_2) &= S_t(x_2)K_0(\sqrt{x_1x_2m_Bb_1}) \\
    &\quad \times [\theta(b_1 - b_2)K_0(\sqrt{x_1m_Bb_1})I_0(\sqrt{x_2m_Bb_2}) \\
    &\quad + \theta(b_2 - b_1)K_0(\sqrt{x_2m_Bb_2})I_0(\sqrt{x_1m_Bb_1})],
\end{align*}
$$

(A4)

$$
\begin{align*}
    h_a(x_2, x_3, b_2, b_3) &= \left(\frac{\pi}{2}\right)^2 S_t(x_2)H_0^{(1)}(\sqrt{x_2x_3m_B^2b_2}) \\
    &\quad \times \left[ \theta(b_2 - b_3)H_0^{(1)}(\sqrt{x_3m_B^2b_2})J_0(\sqrt{x_3m_B^2b_3}) \\
    &\quad + \theta(b_3 - b_2)H_0^{(1)}(\sqrt{x_3m_B^2b_3})J_0(\sqrt{x_3m_B^2b_2}) \right],
\end{align*}
$$

(A5)

and the evolution factor $E_{e,a}$ are defined by

$$
\begin{align*}
    E_e(t) &= \alpha_s(t) S_B(t) S_P(t), \\
    E_a(t) &= \alpha_s(t) S_{P_1}(t) S_{P_2}(t),
\end{align*}
$$

(A6)

where $S_M(t)$ denotes the Sudakov factor of $M$-meson and $S_t$ is the threshold resummation effect. Their explicit expressions could be found in Ref. [45] and the references therein. The hard scales for emission and annihilation are chosen to be

$$
\begin{align*}
    t^1_e &= \max(\sqrt{x_2m_B^2}, 1/b_1, 1/b_2), \\
    t^2_e &= \max(\sqrt{x_1m_B^2}, 1/b_1, 1/b_2), \\
    t^1_a &= \max(\sqrt{x_3m_B^2}, 1/b_2, 1/b_3), \\
    t^2_a &= \max(\sqrt{x_2m_B^2}, 1/b_2, 1/b_3).
\end{align*}
$$

(A7)

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