Are Superfluid Vortices in Pulsars Violating the Weak Equivalence Principle?

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Abstract

In the present paper we argue that timing irregularities in pulsars, like glitches and timing noise, could be associated with the violation of the weak equivalence principle for vortices in the superfluid core of rotating neutron stars.

1 Introduction

Pulsars are traditionally used to test general relativity [1]. Anomalous inertial mass excess have been found for Cooper pairs in rotating superconductors [2] [3] and for superfluid vortices in rotating superfluids [4]. These anomalies can be understood in terms of a breaking of the weak equivalence principle for Cooper pairs in superconductors [5] and for vortices in superfluids [6]. Can we find phenomenological evidence of similar effects in the context of pulsar physics? In the present paper we argue that the answer to this question is positive. Glitches and timing noise of pulsars [7] might be attributed to a violation of the weak equivalence principle for the superfluid neutron vortices. Although the magnitude of this effect is below the maximum threshold imposed by current tests of the strong equivalence principle in binary systems containing at least one pulsar, it is shown to be too small to be detected by current observational instrumentation.

Section 2 contains a review of the current tests of the equivalence principle in classical and quantum physics (particle physics and superconductors) in the Earth laboratory, and with pulsars. Section 3, discusses the subject of the equivalence principle for superfluids. Section 4 introduces the subject of superfluid physics in the description of the dynamics of pulsars, and reports about the measurement and current physical understanding of glitches and timing noise. In section 5 a possible interpretation of glitches in terms of a breaking of the weak equivalence principle for neutron vortex lines within pulsars is proposed.

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In section 6 this theoretical model is compared with Packard metastable vortex model to account for similar phenomena in Pulsars. In conclusion the parallels which can be established between anomalous mass measurements in superconductors, superfluids and neutron star physics, are critically assessed.

2 Testing the Equivalence Principle on Earth and in Pulsars

The Strong Equivalence Principle (SEP) is completely embodied into general relativity, while alternative theories of gravity predict a violation of some or all aspects of SEP. The SEP is, according to its name, stronger than both the Weak Equivalence Principle (WEP) and the Einstein Equivalence Principle (EEP). The WEP states that all test bodies in an external gravitational field experience the same acceleration regardless of the mass and composition. While the WEP is included in all metric theories of gravity, the EEP goes one step further and also postulates Lorentz-invariance and positional invariance. Lorentz-invariance means that no preferred frame exists, so the outcome of a local non-gravitational experiment is independent from the velocity of the apparatus, while positional invariance renders it unimportant where this experiment is being performed. The SEP includes both the WEP and the EEP, but postulates them also for gravitational experiments.

The violation of WEP means that the ratio of the gravitational mass, \(m_g\), and the inertial mass, \(m_i\), of two bodies \(A\) and \(B\) falling freely, under the single influence of a homogeneous gravitational field, are not equal to each other. This is usually quantified through the Eötvös-factor, \(\eta(A, B)\).

\[
\eta(A, B) = \left(\frac{m_g}{m_i}\right)_A - \left(\frac{m_g}{m_i}\right)_B
\]

The Eötvös-factor is usually obtained from the measurement of the differential acceleration, \(\Delta a\), of two test bodies, \(A\) and \(B\), falling freely in the gravitational field \(g\).

\[
\eta(A, B) = \frac{\Delta a}{g}
\]

The Eötvös-factor, can also be estimated from the measurement of the relative differential rotational frequency, \(\Delta \omega\), of two test bodies, \(A\) and \(B\), freely rotating in a gravitomagnetic field \(B_g\).

\[
\eta(A, B) = \frac{\Delta \omega}{B_g}
\]

The Newtonian gravitational field \(g\), expressed in the SI unit system as an acceleration in \(m/s^2\), and the gravitomagnetic field, with SI units of angular velocity \(Rad/s\), appear both in the weak field linear approximation of Einstein field equations. In this theoretical framework acceleration fields from gravitational and non-gravitational origin cannot be physically distinguished from each other,
the same restriction applies also for angular velocities from gravitomagnetic and non-gravitomagnetic origin. As argued by Anandan [8], any form of the principle of equivalence cannot be demonstrated on a purely theoretical basis. Thus it can only be justified by experiment.

Current experimental measurements [9][10] indicate that the WEP is verified, for classical macroscopic systems, i.e. systems which do not break gauge invariance, with a fractional precision of the Eötvös-factor

\[ \eta < 5 \times 10^{-13} \]  

Contrasting with classical physics, in quantum mechanics the motion of a particle in the presence of an external gravitational field is mass dependent [11]. Collela, Overhauser and Werner, (COW) [12] measured the phase shift \( \delta \phi \) induced by gravity on a monoenergetic neutron beam propagating with velocity \( v_{n0} \) in the arms of a Fabry-Perot interferometer located in two different gravitational potentials.

\[ \delta \phi = m_{n0}^2 g l_1 l_2 \lambda / \hbar^2 \]  

where \( l_1 \) and \( l_2 \) are the length of the interferometer arms located in two different gravitational potentials, and the height separating the two arms respectively, \( \lambda = \hbar / m_{n0} v_{n0} \) is the de Broglie wavelength of the cold neutron beam, and \( m_{n0} \) is the neutron mass. COW experiments revealed that:

1. The phase shift due to gravity is seen to be verified to well within 1%.
2. The gravitational Newtonian potential enters into the Schrodinger equation as expected.
3. Gravity is not purely geometric at quantum level because the effect depends on \( (m_{n0} / \hbar)^2 \).

Making the distinction between the neutron gravitational mass \( m_{n0g} \) and the neutron inertial mass \( m_{ni} \), \( (m_{n0} / \hbar)^2 \) should be replaced by \( m_{n0i} m_{n0g} / \hbar^2 \). Thus COW experiments indicate that \( m_{n0g} = m_{n0i} \) within 1% for neutrons.

Still in the domain of quantum mechanics, but at macroscopic scales: For superconductors, which do break gauge invariance, the equivalence principle has also been tested to some extent. Cabrera and Tate [2][3], through the measurement of the magnetic trapped flux originated by the London moment, reported an anomalous Cooper pair inertial mass excess in thin rotating Niobium superconductive rings:

\[ \Delta m_i = m_i^* - m_i = 94.147240(21) eV \]  

Here \( m_i^* = 1.000084(21) \times 2m_e = 1.023426(21) MeV \) \( (m_e \) being the standard electron mass) is the experimentally measured Cooper pair inertial mass (with an accuracy of 21 ppm), and \( m_i = 0.999992 \times 2m_e = 1.002331 MeV \) is the theoretically expected Cooper pair inertial mass including relativistic corrections.
This anomalous Cooper pair mass excess has not received, so far, a satisfactory explanation in the framework of superconductor’s physics. If the gravitational mass of the Cooper pairs, $m_g$, remains equal to the expected theoretical Cooper pair inertial mass, $m_i = m_g = 0.999992 \times 2m_e = 1.002331\text{MeV}$, Tate’s experiment would reveal that the Cooper pairs break the WEP with an Eötvös-factor, $\eta(E, T) = 9.19 \times 10^{-5} \gg 5 \times 10^{-13}$, obtained from eq.(1) assuming the Experimental ($E$) and Theoretical ($T$) ratios $m_g/m_i = 0.999908$ and $m_g/m_i = 1$ respectively. The question is thus: Is an excess of mass, similar to the one observed by Tate for the cooper pairs inertial mass, also occurring for the cooper pair’s gravitational mass? In recent work with Christian Beck the author derived a law for the breaking of the WEP for Cooper pairs based on an electromagnetic model of dark energy for the bosonic vacuum fluctuations in superconductors [5].

$$\eta \sim \frac{3 \ln 3}{8 \pi} \frac{k^4 G}{c^4 \hbar^3 \Lambda} T_c^4.$$  (7)

Remarkably, this equation connects the five fundamental constants of nature $k, G, c, \hbar, \Lambda$ with measurable quantities in a superconductor, $\eta$ and $T_c$.

In 1987 Jain et al carried out an experiment to probe the SEP for Cooper pairs [13]. The experiment consisted of two Josephson junctions located in the Earth gravitational field at different heights, connected in opposition by superconducting wires. Jain et al. experiment has shown that using the Cooper pairs as probe masses, we also reach the conclusion that the laboratory is accelerating with respect to a local Minkowski spacetime. This plainly justifies the curved spacetime description, which has been well tested for classical matter, to hold for Cooper pairs as well. This experiment also demonstrated that the inertial and gravitational mass of Cooper pairs are exactly equal to each other within an accuracy of 4%:

$$\frac{m_i}{m_g} = 1 \pm 0.04$$  (8)

Unfortunately the accuracy of Jain’s experiment is not good enough to discard or confirm a difference between the inertial and the gravitational mass of Cooper pairs of 21 ppm streamlined with Tate et al experiments. Tajmar et al. carried out experiments in 2009 to test the WEP for high Tc superconductors using a magnetic suspension balance [14]. although this experiment achieved an higher accuracy than Jain’s experiment, it could only resolve Eötvös-factor, $\eta < 2 \times 10^{-3}$, being 2 orders of magnitude away from the Eötvös-factor $\eta(E, T) = 9.19 \times 10^{-5}$ calculated from Tate’s experiment.

A violation of SEP means that objects with different fractional mass contributions from self-gravitation origin would fall differently in an external gravitational field. This is quantified by the parameter $\zeta$, defined through

$$\left(\frac{m_g}{m_i}\right)_A = 1 + \zeta \left(\frac{E_g}{mc^2}\right)_A$$  (9)

where $m_g, m_i, E_g$ are respectively the gravitational mass, inertial mass, and gravitational self-energy of body $A$. For a binary system composed of two
bodies \( A \) and \( B \), in free fall in a gravitational field \( g \) while orbiting around each other, a non zero value of \( \zeta \) would result in the polarization of the binary orbits in the direction of \( g \) \[15\] \[16\], resulting in a (small) forced eccentricity of the orbit. In this type of systems the parameter to be constrained is \( \Delta \), it is similar to \( \zeta \) but without the requirement of linear dependence on the self-gravitational energy, \( E_g/mc^2 \). It is defined for an individual body \( A \) by

\[
\left( \frac{m_A}{m_i} \right)_A = 1 + \Delta_A
\]  

(10)

Dynamics of a binary orbit depends on the difference

\[
\eta = \Delta_A - \Delta_B
\]  

(11)

between the two objects \( A \) and \( B \) \[17\], where \( \eta \) is given by eq(1). Of course, this effect is most detectable in binary systems composed by bodies with radically different gravitational self energies like, for example, a pulsar orbiting a white dwarf.

Pulsars are rotating neutron stars. The signals received from the pulsars are modulated at a remarkably constant frequency which is resulting from the rotation frequency of the star. The orbital parameters of the binary systems, containing at least one pulsar, are all deduced by fitting the arrival times of pulses. Pulsars are well established test beds for relativity. Observation of the neutron star-neutron star binary PSR B1913+16 have established that its orbit decays at the rate predicted by general relativity within 0.3\% \[1\]. The observation by Wex of an ensemble of long orbit pulsars yields a limit of \(|\eta| < 0.009\) at 95% confidence level \[18\], which represents the most stringent test of SEP violation until the present date. As we will see later, this result is also useful to assess the validity of the WEP in superfluids, since the neutron star core is a superfluid.

In the context of experiments in the Earth laboratory, the WEP is poorly tested for superfluids in general, and superfluid vortices in particular. However the vortex inertial mass in superfluid Helium has been extensively discussed in the literature \[4\] \[19\] \[20\].

### 3 Weak Equivalence Principle in Superfluids

A vortex line in rotating superfluid Helium 4 is a topological singularity, which consists of a normal core region of the size of the coherence length \( \xi \), and an outside region of circulating supercurrent. The coherence length can be estimated from the Heisenberg uncertainty principle.

\[
\xi \sim \frac{\hbar}{mc_s}
\]  

(12)

where \( m \) is the bare atomic mass in \(^4\)He and \( c_s \) is the speed of sound in the superfluid. Taking \( c_s \sim 2 \times 10^2 m/s \[21\] \) we estimate \( \xi \sim 1A \). In the theoretical
framework of the classical fluid model the only obvious contribution to the vortex mass is the core mass\[22\].

$$m_{\text{core}} = L\pi\xi^2 \rho$$  \hspace{2em} (13)

where \(L\) is the length of the vortex line, and \(\rho = Nm\) is the density with \(N\) the bulk number density of \(^4\text{He}\) atoms. This small vortex mass is usually discarded in the equations of motion of vortex dynamics since it contradicts experimental data.

Duan in \[4\] shown that due to spontaneous gauge symmetry breaking in superfluids, the condensate compressibility contributes to a vortex mass which is much larger than the classical core mass. He calculates that the vortex inertial mass turns out to diverge logarithmically with the system size.

$$m_{\text{inertial}} = m_{\text{core}} \ln \left( \frac{L}{\xi} \right)$$  \hspace{2em} (14)

Where \(L\) is the length of the vortex. For a practical superfluid system in the Earth laboratory, \(\ln(L/\xi) \approx 20 - 30\).

The number of vortices \(N_v\) appearing in a cylindrical sample of superfluid \(^4\text{He}\) rotating with angular velocity \(\Omega\) is deduced from the quantization of the vortex canonical momentum.

$$N_v = \frac{2\pi R^2 \Omega}{\hbar/m}$$  \hspace{2em} (15)

where \(R\) is the radius of the superfluid sample. The total increase of the inertial mass of a rotating superfluid sample with respect to the same non-rotating sample, is obtained from eq.(14) and eq.(15).

$$\Delta M_{\text{inertial}} = N_v m_{\text{core}} \left( \ln \left( \frac{L}{\xi} \right) - 1 \right)$$  \hspace{2em} (16)

Assuming that the weak equivalence principle is still valid in superfluids this overall increase of inertial mass should appear together with a similar increase of the gravitational mass of the superfluid sample.

$$\Delta M_{\text{inertial}} = \Delta M_{\text{gravitational}}$$  \hspace{2em} (17)

Thus we should observe an increase of the weight of the rotating superfluid sample with respect to the same sample in the stationary state. Taking a cylindrical sample of radius \(R = 1 cm\) and \(\ln(L/\xi) \sim 20 - 30\), rotating at \(\Omega = 1 Rad/s\) in eq.(16), we estimate that the total increase of gravitational mass is of the order of \(\Delta M_{\text{gravitational}} = 10^{-14} - 10^{-9} Kg\). Thus the experimental detection of the associated increase of weight of the overall sample is a challenging task to perform, that has not yet been overcome by experimentalists in the Earth laboratory. In summary Until the present date the weak equivalence principle has not been tested for superfluid vortices.
The breaking of gauge symmetry makes the superfluid sample a preferred frame, this should be associated with a speed of light in the superfluid vacuum different from its classical value \( c_0 \), appearing in Lorentz transformations. As demonstrated by Duan and Popov [4] [20] the vortex inertial mass can be expressed in function of the vortex static energy \( \epsilon_0 \) which is also logarithmically divergent as the sample size.

\[
m_{\text{inertial}} = \frac{\epsilon_0}{c_s^2}
\]

where \( c_s \) is the speed of sound in the superfluid.

Starting from Mach’s principle, which asserts that there is a connection between the local laws of physics and the large scale properties of the universe, Sciama in [23] introduced the relation

\[
c_0^2 = \frac{2GM}{R}
\]

where \( R \) and \( M \) are the radius and the baryonic mass of the universe. Einstein’s relationship linking energy and mass then takes the form

\[
E = mc_0^2 = \frac{2GMm}{R}
\]

which can be interpreted as a statement that the inertial energy that is present in any physical object is due to the gravitational potential energy of all the matter in the universe acting on the object. Therefore the mass \( m \) appearing in eq. (20) should be the gravitational mass of the object.

\[
E = m_{\text{gravitational}} c_0^2
\]

Since the rest mass energy of the vortex \( \epsilon_0 \) must be conserved independently of the effective value of the vacuum speed of light, the gravitational mass will adjust its value to compensate the variation of the speed of light in the superfluid vacuum.

\[
m_{\text{gravitational}} c_0^2 = m_{\text{core}} c_s^2 \ln \left( \frac{L}{\xi} \right)
\]

From eq. (22) we deduce that the gravitational mass of a superfluid vortex \( m_{\text{gravitational}} \) is proportional to the classical vortex core mass and also diverges logarithmically as the size of the vortex.

\[
m_{\text{gravitational}} = \left( \frac{c_s}{c_0} \right)^2 m_{\text{core}} \ln \left( \frac{L}{\xi} \right)
\]

where the proportionality coefficient is equal to the square of the ratio between the speed of sound in the superfluid \( c_s \) and the classical speed of light in vacuum \( c_0 \). Comparing eq. (14) and eq. (23) we conclude that due to the principle of energy conservation and to the breaking of gauge invariance in superfluids the inertial and the gravitational mass of a vortex cannot be equal to each other.
Therefore the weak equivalence principle should break for the case of superfluid vortices.

As we have shown above, measuring the vortices gravitational mass comparing the weight of the superfluid sample in rotating and stationary state is challenging due to the extremely small value of the vortex core mass. However in free fall experiments with rotating superfluid samples it should be possible to measure the differential acceleration $\Delta a$ between the vortex and the bulk superfluid. The Eötvös factor $\eta$ associated with the free fall of a vortex and the superfluid bulk under the single influence of the Earth gravitational field $g_0$ would be obtained from eq. (2):

$$\eta = \frac{\Delta a}{g_0} \quad (24)$$

Let us first assume that the friction force between the vortex and the superfluid bulk is null. On one side, since the superfluid bulk inertial and gravitational mass are equal, the center of mass of the superfluid bulk will fall with and acceleration

$$a_{\text{superfluid}} = g_0 \quad (25)$$

On the other side The vortex will fall according to the equation of motion

$$g_0 m_{\text{gravitational}} = m_{\text{inertial}} a_{\text{vortex}} \quad (26)$$

substituting eq. (23), and eq. (14) in eq. (26) we calculate the vortex falling acceleration

$$a_{\text{vortex}} = g_0 \frac{c_s}{c} \quad (27)$$

Substituting the accelerations $a_{\text{superfluid}},$ eq. (25), and $a_{\text{vortex}},$ eq. (27), in eq. (24) we obtain the Eötvös factor $\eta$ for a superfluid vortex with respect to the superfluid bulk.

$$\eta = 1 - \left( \frac{c_s}{c_0} \right)^2 \quad (28)$$

Taking $c_s \sim 2 \times 10^2 m/s$ we have $\eta \sim 1$ which is much higher than the upper limit measured for classical material systems of $5 \times 10^{-13},$ eq. (4).

If instead of assuming no friction between the vortices and the superfluid bulk, like we did above, we assume an ideal rigid connection between both systems. We deduce from the equation of motion of the freely falling rotating superfluid sample, a falling acceleration $a_z.$

$$a_z = 1 + \left( \frac{c_s}{c} \right)^2 \frac{m_v}{m} g_0 \quad (29)$$

where $m$ is the total classical mass of the superfluid bulk (without the vortices) and $m_v = N_v m_{\text{core}} \ln \left( \frac{L}{\xi} \right)$ is the total inertial mass of vortices in the superfluid sample, with $N_v$ being the effective number of vortices. Comparing this acceleration with the falling acceleration of the same non-rotating sample, $g_0,$
we calculate the Eötvös factor $\eta'$ of the rotating sample with respect to the non-rotating one.

$$\eta' = \frac{g_0 - az}{g_0}$$  \hspace{1cm} (30)

substituting eq. (29) in eq. (30) we obtain [6]

$$\eta' = \frac{m_v \Delta m}{\Delta m \eta}$$  \hspace{1cm} (31)

where $\Delta m = m - m_v$ and $\eta = 1 - \left(\frac{c_s}{c_0}\right)^2$ is the Eötvös factor of one vortex with respect to the superfluid bulk (assuming no friction between both), eq.(28).

Taking a cylindrical sample of radius $R = 1cm$ and $\ln(L/\xi) \sim 20 - 30$, rotating at $\Omega = 1Rad/s$ in eq.(31), we estimate the order of magnitude of $\eta' \sim 10^{-11}$, which is 2 orders of magnitude above the upper limit experimentally determined for normal materials, which do not break gauge invariance, eq.(4).

4 Glitches and Timing Noise in Pulsars

Pulsars are rotating neutron stars, which consist of a solid iron crust and a superfluid neutron core containing also a superconducting proton layer (Ginzburg, 1971). Since the protons represent only a few percent of the total star, we will neglect them in the discussion which follows, and will concentrate on the superfluid neutron part. The temperature of the star is probably $\sim 10^8 K$, and the neutron superfluid phase density is $10^{17} Kg/m^3$, so that the neutrons are highly degenerate. Migdal [24] was the first to propose that the neutrons near the Fermi surface might be paired in such a way as to suffer a BCS type condensation. According to Hoffberg et al. [25] There is a critical neutron density $(1.45 \times 10^{17} Kg/m^3)$ below which s-wave pairing is dominant, as in superconductors, but above which p-wave pairing takes over, as in superfluid $^3He$. Using the standard BCS relations, these authors estimate that the transition temperature for the neutron superfluid is $10^{10} K$, well above the actual star temperature.

Since the data collected from pulsars originates from the stars’ surface, atmosphere or magnetosphere, an important question concerns whether differences in internal structure can be deduced from observations of the pulsar pulses. Having said that, there are phenomena that involve bulk dynamics and which should depend on the internal composition. These phenomena are typically the radio pulsar ”glitches” [7], which are sudden increases in the pulsar rotation rate often accompanied by an increase in slow down rate followed by a period of relaxation (approximately exponential, with time scale of days to years) towards the pre-glitch frequency, and ”timing noise”, which consists of low frequency structures.

During a glitch, the typical fractional increase in pulsar rotation frequency is in the range $\Delta \nu/\nu = 10^{-9} \sim 10^{-6}$, and the relative increment in slow down rate $\Delta \dot{\nu}/\dot{\nu} \sim 10^{-3}$ where $\nu$ and $\dot{\nu}$ are pulsar rotation frequency and frequency derivative respectively. The trigger of the pulsar glitch is presently not well understood. The long relaxation time associated with glitches is seen as indirect
evidence for neutron star superfluidity. On one side, in the superfluid vortex unpinning and re-pinning model, triggering of the glitch is due to coupling of the crust and the superfluid interior as a consequence of a sudden unpinning of vortex lines and the post-glitch relaxation is due to the vortex gradually re-pinning to the crust lattice [26] [27]. On the other side, in the classical starquake model, as a consequence of the long-term spin-down in spin rate, deformation stress in the rigid crust builds up to resist the decreasing oblatness [30]. When the stress exceeds a critical point, the crust cracks suddenly, resulting in a sudden increase in spin rate. Based on the observed typical glitches, both of the models have a sudden increase in rotation frequency and slow down rate (i.e. $\Delta \dot{\nu}/\dot{\nu}$) at the time of the glitch. The post-glitch relaxation represents a return to equilibrium with a linear response of the interior superfluid, while the lack of relaxation represents a non-linear response of the superfluid [27].

As more glitches were detected, it became clear that glitch behavior varies in aspects such as glitch rate, amplitude and relaxation. Although, these diverse features suggest glitches are triggered locally in the superfluid interior, no model currently predicts the time between glitches or the size of any given event (for two pulsars with similar rotation parameters, one may glitch frequently while the other may never have been observed to glitch).

As already mentioned above, in addition to glitches, pulsars also suffer another kind of timing irregularity known as timing noise, which is characterized by restless, unpredictable, smaller scale fluctuations in spin rate [31] with time scales from days to years. The timing noise induced fluctuations of pulse frequency are small, with fractional change $\delta \nu/\nu < 10^{-9}$. Timing noise has been explained by random processes [32], unmodelled planetary companions or free precession [33]. However the physical phenomenon underlying most of the timing noise still has not been explained.

Presently the relation between glitches and timing noise is not understood although Janssen and Stappers [34] showed that it is possible to model the timing noise in PSR B1951+32 as multiple small glitches. A better understanding of Pulsar timing irregularities could lead to many important results, explaining the cause of timing noise and glitches could allow us to relate these phenomena and hence provide an insight into the interior structure of neutron stars. However, currently it is still not clear whether the glitch and timing noise phenomena are related.

5 Glitches in Pulsars and Violation of the Weak Equivalence Principle for Superfluid Vortices

Although the critical transition temperature of the neutron superfluid, predicted by current physical models of neutron stars, $10^8 K < T_c < 10^{10} K$, are many orders of magnitude above the critical transition temperatures of ordinary superfluids in the Earth laboratory, typically in the range of $1 K < T_c < 3K$, we will assume the possibility to extrapolate present superfluid physics to the case
of neutron superfluids in pulsars.

Since the major part of the mass of the star consists of neutrons, it appears that most of the rotational energy resides in the superfluid. By analogy with rotating He II, Ginzburg and Kirzhnits [35] concluded that the neutron superfluid would contain an array of quantised vortex lines. In the same way as in He II, one can define critical values of angular velocity, $\Omega_{C1}$, which must be exceeded to form a single vortex, and $\Omega_{C2}$, at which the vortex cores would overlap [36] gave the values $\Omega_{C1} \sim 10^{-14}s^{-1}$ and $\Omega_{C2} \sim 10^{20}s^{-1}$; the periods of all known pulsars correspond to rotational speeds ($\Omega$) ranging from over $1s^{-1}$ to $10^3s^{-1}$.

Thus the neutron superfluid has properties similar to He II undergoing solid-body rotation at the temperature of $\sim 1mK$. Thus we will apply the theoretical model presented in section 3, which predicts that superfluid vortices break the weak equivalence principle with an an E"otv"os factor $\eta'$ given by eq.(31), to the case of pulsars.

Substituting the following typical quantities in pulsars: for the speed of sound in the neutron superfluid $c_s \sim c_010^{-6}m/s$ [28], the length of a superfluid neutron vortex $L \sim 1 \times 10^3m$, the radius of the vortex neutron core $\xi = \hbar/m_n c_s \sim 2.1 \times 10^{-10}m$, the pulsar angular velocity of $\Omega \sim 10^3Rad/s$, the radius of the superfluid neutron shell of $R = 7.5 \times 10^3m$, and the neutron density of $\rho = 10^{17}Kg/m^3$ into eq.(31), one obtains an E"otv"os factor $\eta'$

$$\eta' \sim 1.28 \times 10^{-8}$$ (32)

This quantifies the differential acceleration which would appear between two neutron stars in free fall around another star, one that would be rotating would host vortices, which would break the WEP, and a second one that would not be rotating and would thus be compliant with the WEP. Eq.(32) also predicts the differential angular velocity between two pulsars freely rotating in the gravito-magnetic field generated by a third star, one which do not break the WEP and a second one for which the superfluid vortices break the WEP. Since neutron vortex lines would break the WEP, a change in the effective number of vortex lines would cause the moment of inertia of the fluid core, $I_f$, to change by the fractional amount,

$$\frac{\Delta I_f}{I_f} = \frac{m_v}{\Delta m}$$ (33)

Using eq.31, eq.33 can be expressed in function of the E"otv"os factors $\eta$ and $\eta'$.

$$\frac{\Delta I_f}{I_f} = \frac{\eta'}{\eta}$$ (34)

To conserve angular momentum the Pulsar crust must have a fractional increase in angular speed given by:

$$\frac{\Delta \omega}{\omega} = \frac{\Delta I_f}{I_c}$$ (35)

where $I_c$ is the moment of inertia of the crust. Substituting $\Delta I_f$ from eq.34 in eq.35, we get:

$$\frac{\Delta \omega}{\omega} = \frac{I_f \eta'}{I_c \eta}$$ (36)
Since substituting the numerical values used above in this section, in eq. (28) we obtain $\eta \sim 1$, then eq. (35) simplifies to:

$$\frac{\Delta \omega}{\omega} \sim \frac{I_f}{I_c} \eta'$$

(37)

For any reasonable value of $I_f/I_c (10 - 100)$ [29], and substituting the value of $\eta'$ from eq. (32) in eq. (37), the predicted speedups during glitches in pulsars, $\Delta \omega/\omega (10^{-7} - 10^{-6})$, would be in reasonable agreement with currently observed values, which are in the typical range $\Delta \nu/\nu (10^{-9} - 10^{-6})$. Thus we raise the question of the possibility to apply our crude model for the breaking of WEP for vortices in He II to the case of pulsars, as a possible cause contributing to glitches and eventually other timing noise in this type of stars.

Although the violation of the WEP in Pulsars within a predicted Eötvös factor $\eta' \sim 10^{-8}$, is not ruled out by current observational tests of SEP in binary systems containing at least one pulsar, (which, as referred above in section 2, set an upper limit on SEP violation with $|\eta| < 0.009$ at 95% confidence level); the experimental detection of this phenomena in these systems is a challenging task for current observational capabilities, being 5 orders of magnitude away.

6 Discussion

In the Packard model [29], Glitches are accounted for by various metastable states of a vortex array, which are possible for a given angular velocity of the pulsar. In the neutron star, a transition between two such states would involve a decrease in the angular momentum of the superfluid, with a compensating increase of of the angular momentum of the pulsar’s crust. If the angular momentum in the fluid core changes by the fractional amount $\Delta L_f/L_f$, then the crust must have a fractional increase in speed of:

$$\frac{\Delta \omega}{\omega} = \frac{I_f}{I_c} \frac{\Delta L_f}{L_f}$$

(38)

comparing eq. (38) with eq. (37), one concludes that the Packard model is coherent with the breaking of WEP in pulsars proposed in the present paper only if:

$$\eta' \sim \frac{\Delta L_f}{L_f}$$

(39)

which means that both theoretical models depend on the effective number of vortex lines in the neutron superfluid. However the transfer of angular momentum from the rotating superfluid to the pulsar crust, which is taking place in Packard model through pinning and unpinning of vortex lines, is taking place in the present model, through the change of the moment of inertia of the superfluid core associated with a breaking of the WEP for neutron vortex lines, according to eq. (34).

We wish to emphasize that the physical interpretation of glitches as signs of a breaking of WEP in the pulsar neutron superfluid, cannot provide by itself a
physical mechanism capable to induce transition from a pulsar state in which the WEP is broken to the ground state which is compliant with the WEP.

The proposed model for the breaking of the WEP for vortex lines in superfluid is also leaving us with a puzzle: Since the individual atoms making the superfluid vortices (presumably) do satisfy the weak equivalence principle, it is hard to see how vortices, as a whole, can do other than obey weak equivalence. Therefore attribution of large vortex mass excess to spontaneous gauge symmetry breaking by Popov, Duan and others ought to imply either that this symmetry breaking creates gravitational mass, or that it breaks weak equivalence. In the present paper arguments have been presented to support the latter physical possibility.

7 Conclusions

Since gauge invariance is broken in superconductors and superfluids, it seems pertinent to investigate if the WEP is broken for cooper pairs in superconductors and for vortices in superfluids. Although, as pointed out by Cosimo Bambi in 2007 [37], the interpretation of the anomalous Cooper pair inertial mass excess in terms of a gravitomagnetic-type London moment in rotating superconductors [35], is not tenable with respect to the experimental observation of orbital parameters of pulsars in binary systems, as well as with respect to recent experiments carried out by Tajmar et al. [39]. It seems that the physical interpretation of the anomalous mass of Cooper pairs in superconductors in terms of a violation of the WEP for this particles is more promising, since it is quite well accounted for by an electromagnetic model of dark energy in the superconductor [5] [40], and it leads to the physical interpretation of the logarithmically diverging inertial mass of vortices in rotating superfluids as a breaking of the WEP for vortex lines [6]. In the present paper we demonstrated that the breaking of WEP for superfluid vortices, predicted by our crude theoretical model, seems suitable to account for glitches and timing noise in pulsars, at an E"otvös level $\eta' \sim 10^{-8}$. Although this prediction is not ruled out by present tests of SEP in binary systems containing at least one pulsar $|\eta| < 0.009$, it is unfortunately too small, by 5 orders of magnitude, to be detected by current astronomical observational capabilities.

References

[1] J. H. Taylor, J. M. Weisberg, Astrophys. J., 345, 434, (1989)

[2] J. Tate, B. Cabrera, S.B. Felch, J.T. Anderson, ”Precise determination of the Cooper-pair mass”, Phys. Rev. Lett. 62 (8) 845848 (1989)

[3] J. Tate, B. Cabrera, S. B. Felch, J.T. Anderson, ”Determination of the Cooper-pair mass in niobium”, Phys. Rev. B 42 (13) 78857893 (1990).

[4] J. M. Duan, Phys. Rev. B 49, 12381 (1994)
[5] C. J. de Matos, "Physical Vacuum in Superconductors", arXiv: 0908.4495 (2009), presented at ICSM 2010

[6] C. J. de Matos, "Are Vortices in Rotating Superfluids Breaking the Weak Equivalence Principle?" arXiv:0909.2819 (2009), Presented at ICSM 2010

[7] A. G. Lyne, et al. MNRAS, 315, 534, (2000)

[8] J. Anandan, "Relativistic Gravitation and Superconductors", Class. Quant. Grav. 11 A23-A37 (1994)

[9] S. Baessler et al., Phys. Rev. Lett. 83 3585 (1999)

[10] G. L. Smith et al., Phys. Rev. D61 022001 (2000)

[11] S. Huerfano, et al., "Quantum Mechanics and the Weak Equivalence Principle", arXiv:quant-ph/0606172, (2006)

[12] R. Collela, A. Overhauser, S. A. Werner, Phys. Rev. Lett. 34, 1472, (1975)

[13] A. K. Jain, J. Lukens, and J. S. Tsai, Phys. Rev. Lett. 58 1165 (1987)

[14] M. Tajmar, F Plesescu and B Seifert "Measuring the dependence of weight on temperature in the low temperature regime using a magnetic suspension balance" Meas. Sci. Technol. 21, 015111, (2010)

[15] K. Nordtvedt, Phys. Rev. 169, 1014, (1968)

[16] K. Nordtvedt, Phys. Rev. 170, 1186, (1968)

[17] T. Damour, G. Schafer, Phys. Rev. Lett. 66, 2549, (1991)

[18] N. Wex, in IAU Colloq. 177, Pulsar Astronomy: 2000 and Beyond, ed. M. Kramer, N. Wex, & R. Wielebinski (ASP conf. Ser. 202; San Francisco; ASP), 113, (2000)

[19] D. J. Thouless, J. R. Anglin, Phys. Rev. Lett. 99, 105301 (2007)

[20] V. N. Popov, Sov. Phys. JETP 37, 341 (1973)

[21] P. Nozieres, D. Pines, "The theory of Quantum Liquids" (Addison-Wesley, New York, 1990), Vol. II.

[22] G. Baym, E. Chandler, J. Low Temp. Phys. 50, 57 (1983)

[23] D. W. Sciama, Mon. Not. Roy. Astr. Soc. 113, 34

[24] A. B. Migdal, Nucl. Phys. 13, 655, (1959)

[25] M. Hoffberg, et al., Phys. Rev. Lett., 24, 775, (1970)

[26] P. W. Anderson, N. Itoh, Nature, 256, 25, (1975)
[27] M.A. Alpar et al., *ApJ*, **346**, 823, 1989

[28] M. E. Gusakov, N. Andersson, *Mon. Not. R. Astron. Soc.*, **372**, 1776-1790, (2006)

[29] R. E. Packard, "Pulsar Speedups Related to Metastability of the Superfluid Neutron-Star Core", *Phys. Rev. Lett.*, **28**, 1080, (1972)

[30] G. Baym, D. Pines, "Neutron starquakes and pulsar speedup", *Annals of Physics* **66**, 2, pp 816-835, (1971)

[31] J. M. Corder, G. S. Downs, *ApJS*, **59**, 343, (1985)

[32] J. M. Cordes, D. J. Helfand, *ApJ*, **239**, 640, (1980)

[33] I.H. Stairs, et al., *Nature*, **406**, 484, (2000)

[34] G. Janssen, B. Stappers, *A & A*, **457**, 611, (2006)

[35] V. L. Ginzburg, D. A. Kirzhnits, Soviet Phys. JETP **20**, 1946, (1964)

[36] G. Baym, et al. *Nature*, **224**, 673, 674, (1969)

[37] C. Bambi, "Gravitomagnetism in Superconductors and Compact Stars", *Int.J.Mod.Phys.D* **17** pp 327-336,2008, [arXiv:0710.2042](http://arxiv.org/abs/0710.2042) (2007)

[38] M. Tajmar, C. J. de Matos, "Extended analysis of gravitomegnetic fields in rotating superconductors and superfluids" *Physica C* **420**, 56 (2005).

[39] M. Tajmar, et al., "Anomalous Fiber Optic Gyroscope Signals Observed above Spinning Rings at Low Temperature", *Jour. Phys.: Conf. Series*, **150** (2009) 032101

[40] C. J. de Matos, "Electromagnetic dark energy and gravitoelectrodynamics of superconductors", *Physica C* **468** 210-213 (2008)