Self-similar structure of magnetized ADAFs and CDAFs

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21 May 2008

ABSTRACT

We study the effects of a global magnetic field on viscously-rotating and vertically-integrated accretion disks around compact objects using a self-similar treatment. We extend Akizuki & Fukue’s work (2006) by discussing a general magnetic field with three components \((r, \varphi, z)\) in advection-dominated accretion flows (ADAFs). We also investigate the effects of a global magnetic field on flows with convection. For these purposes, we first adopt a simple form of the kinematic viscosity \(\nu = \alpha c_s^2 / \Omega_K\) to study magnetized ADAFs: a vertical and strong magnetic field, for instance, not only prevents the disk from being accreted but also decreases the isothermal sound speed. Then we consider a more realistic model of the kinematic viscosity \(\nu = \alpha c_s H\), which makes the infall velocity increase but the sound speed and toroidal velocity decrease. We next use two methods to study magnetized flows with convection, i.e., we take the convective coefficient \(\alpha_c\) as a free parameter to discuss the effects of convection for simplicity. We establish the \(\alpha_c - \alpha\) relation for magnetized flows using the mixing-length theory and compare this relation with the non-magnetized case. If \(\alpha_c\) is set as a free parameter, then \(|v_r|\) and \(c_s\) increase for a large toroidal magnetic field, while \(|v_r|\) decreases but \(|v_\varphi|\) increases (or decreases) for a strong and dominated radial (or vertical) magnetic field with increasing \(\alpha_c\). In addition, the magnetic field makes the \(\alpha_c - \alpha\) relation be distinct from that of non-magnetized flows, and allows the \(\rho \propto r^{-1}\) or \(\rho \propto r^{-2}\) structure for magnetized non-accreting convection-dominated accretion flows with \(\alpha + g\alpha_c < 0\) (where \(g\) is the parameter to determine the condition of convective angular momentum transport).

Key words: accretion, accretion disks — black hole physics — MHD
1 INTRODUCTION

Rotating accretion flows with viscosity and angular momentum transfer can be divided into several classes, depending on different structures and energy transfer mechanisms in the flows: advection-dominated accretion flows (ADAFs), advection-dominated inflow-outflows (ADIOs), convection-dominated accretion flows (CDAFs), neutrino-dominated accretion flows (NDAFs) and magnetically-dominated accretion flows (MDAFs).

ADAFs were introduced by Ichimaru (1977) and then have been widely studied over thirty years. The optically-thick ADAFs with super-Eddington accretion rates were discussed by Abramowicz et al. (1988) in details (see also Begelman et al. 1982; Eggum et al. 1988). The optically-thin ADAFs with low, sub-Eddington accretion rates were discussed by Rees et al. (1982) and Narayan & Yi (1994, 1995a, 1995b) (see also Ichimaru 1977; Abramowicz et al. 1995; Gammie & Popham 1998; Popham & Gammie 1998; Wang & Zhou 1999). In particular, Narayan & Yi (1994) introduced self-similar solutions for ADAFs with the fixed ratio of the advective cooling rate to the viscous heating rate in the disk. Wang & Zhou (1999) solved self-similar solutions for optically-thick ADAFs. The effects of general relativity were considered in Gammie & Popham (1998) and Popham & Gammie (1998).

CDAFs were presented in details in Narayan et al. (2000, hereafter NIA). They discussed the effects of convection on angular momentum and energy transport, and presented the relations between the convective coefficient $\alpha_c$ and the classical viscosity parameter $\alpha$. A non-accreting solution can be obtained when convection moves angular momentum inward and the viscosity parameter $\alpha$ is small. Later, a series of works have been published to discuss the disk structure, the MHD instability, the condition of angular momentum transport in CDAFs (e.g., Igumenshchev et al. 2000, 2002, 2003; Quataert & Gruzinov 2000; Narayan et al. 2002; Igumenshchev 2002; Lu et al. 2004; van der Swaluw et al. 2005).

The effects of a magnetic field on the disk were also studied (see Balbus & Hawley 1998; Kaburiki 2000; Shadmehri 2004; Meier 2005; Shadmehri & Khajenabi 2005, 2006; Akizuki & Fukue 2006; Ghanbari et al. 2007). Balbus & Hawley (1998) discussed the MHD turbulence initiated by magnetorotational instability (MRI) and its effects on the angular momentum transportation. Kaburaki (2000) considered an analytic model to describe the ADAFs with a global magnetic field and Meier (2005) considered how a turbulent and
magnetized disk creates a global well-ordered magnetic field, and introduced a magnetically-dominated flow. Shadmehri (2004) and Chanbari et al. (2007) discussed the self-similar structure of the magnetized ADAFs in spherical polar coordinates. Moreover, Shamehri & Khajenabi (2005, 2006, hereafter SK05, SK06) presented self-similar solutions of flows based on the vertically integrated equations. They discussed the relations between magnetic fields components in different directions, and mainly focused on the effects of the magnetic field on the disk structure. Akizuki & Fukue (2006, hereafter AF06), different from SK05 and SK06, emphasized an intermediate case where the magnetic force is comparable to other forces by assuming the physical variables in the disk only as functions of radius. However, they merely discussed a global toroidal magnetic field in the disk.

In this paper, we first extend the work of AF06 by considering a general large-scale magnetic field in all the three components in cylindrical coordinates \((r, \varphi, z)\) and then discuss effects of the global magnetic field on the flows with convection. We adopt the treatment that the flow variables are functions of the disk radius, neglect the different structure in the vertical direction except for the \(z\)-component momentum equation. We also discuss magnetized accretion flows with convection, and compare our results with those in NIA, in which a large-scale magnetic field is neglected.

This paper is organized as follows: basic equations are presented in §2. We obtain self-similar solutions in §3 and discuss the effects of a general large-scale magnetic field on the disk flow. In §4 we investigate the structure and physical variables in magnetized CDAFs, and present the relation of the convective parameter \(\alpha_c\) and the classical viscosity parameter \(\alpha\). We adopt a more realistic form of the kinematic viscosity in §5. Our conclusions are presented in §6.

## 2 BASIC EQUATIONS

In this paper, we use all quantities with their usual meanings: \(r\) is the radius of the disk, \(v_r\) and \(v_\varphi\) are the radial and rotation velocity, \(\Omega = v_\varphi/r\) is the angular velocity of the disk, \(\Omega_K = (GM/r^3)^{1/2}\) is the Keplerian angular velocity, \(\Sigma = 2\rho H\) is the disk surface density with \(\rho\) to be the disk density and \(H\) to be the half-thickness, and \(c_s = (p/\rho)^{1/2}\) is the isothermal sound speed with \(p\) to be the gas pressure in the disk.

Moreover, we consider a magnetic field in the disk with three components \(B_r, B_\varphi\) and \(B_z\) in the cylindrical coordinates \((r, \varphi, z)\). We define the Alfvén sound speeds \(c_r, c_\varphi\) and \(c_z\) in
three directions of the cylindrical coordinates as $c_{r,\varphi,z}^2 = B_{r,\varphi,z}^2/(4\pi\rho)$. We consider that all flow variables are only functions of radius $r$, and write basic equations, i.e., the continuity equation, the three components $(r, \varphi, z)$ of the momentum equation and the energy equation:

\[
\frac{1}{r} \frac{d}{dr} (r \Sigma v_r) = 2 \dot{\rho} H, \tag{1}
\]

\[
v_r \frac{d}{dr} \left( \frac{v_r^2}{r} - \frac{GM}{r^2} - \frac{1}{\Sigma} \frac{d}{dr} \left( \Sigma c_s^2 \right) - \frac{1}{2\Sigma} \frac{d}{dr} \left( \Sigma c_z^2 + \Sigma c_{\varphi}^2 \right) - \frac{c_{\varphi}^2}{r} \right), \tag{2}
\]

\[
v_r \frac{d}{dr} (rv_{\varphi}) = \frac{1}{\Sigma r^2} \frac{d}{dr} \left( \frac{\Sigma \alpha c_s^2}{\Omega_K} r^3 \frac{d\Omega}{dr} \right) + \frac{c_{\varphi} c_r}{r} + \frac{c_r}{\sqrt{\Sigma}} \frac{d}{dr} \left( \sqrt{\Sigma} c_{\varphi} \right), \tag{3}
\]

\[
\Omega_K^2 H - \frac{1}{\sqrt{\Sigma}} c_r \frac{d}{dr} \left( \sqrt{\Sigma} c_z \right) = \frac{c_s^2 + \frac{1}{2} \left( c_{\varphi}^2 + c_r^2 \right)}{H}, \tag{4}
\]

\[
\frac{v_r}{\gamma - 1} \frac{d}{dr} \left( \frac{c_s^2}{\rho} \right) - v_r \frac{c_s^2}{\rho} \frac{d\rho}{dr} = f \frac{\alpha c_s^2 r^2}{\Omega_K} \left( \frac{d\Omega}{dr} \right)^2. \tag{5}
\]

Here we consider the height-integrated equations using the classical $\alpha$-prescription model with $\alpha$ to be the viscosity parameter, and use the Newtonian gravitational potential. In the mass continuity equation, we also consider the mass loss term $\partial\rho/\partial t$. In the energy equation we take $\gamma$ to be the adiabatic index of the disk gas and $f$ to measure the degree to which the flow is advection-dominated (NY94), and neglect the Joule heating rate.

In AF06, a general case of viscosity $\eta = \rho \nu = \Omega_K^{-1} \alpha p_{gas}^\mu (p_{gas} + p_{mas})^{1-\mu}$ with $\mu$ to be a parameter is mentioned. If the ratio of the magnetic pressure to the gas pressure is constant (as assumed in the self-similar structure), the solution of the basic equations can be obtained with replacing $\alpha$ by $\alpha (1 + \beta)^{1-\mu}$. In our paper, however, we first adopt the classical form $\nu = \alpha c_s^2 / \Omega_K$ for simplicity in §3 and §4, in which we mainly focus on the effects of a magnetic field on the variables $v_r$, $v_{\varphi}$ and $c_s$. A more realistic model requires $\nu = \alpha c_s H$ with both $c_s$ and $H$ as functions of the magnetic field strength. We discuss this model in §5 and compare it with the results in §3 and §4.

Our equations are somewhat different from those in SK05 and SK06, since we only consider the disk variables as functions of radius $r$, while SK05 and SK06 discuss the magnetic field structure in the vertical direction. More details about the basic equations are discussed in Appendix A. When $B_r = 0$ and $B_z = 0$, our equations switch back to the equations in AF06, in which only the toroidal magnetic field is considered and all the variables are taken to depend merely on radius $r$.

In addition, we need the the three-component induction equations to measure the magnetic field escaping rate:

\[
\frac{\partial B_r}{\partial t} + (v_r B_r + v_{\varphi} B_{\varphi}) = 0,
\]

\[
\frac{\partial B_{\varphi}}{\partial t} + (v_r B_{\varphi} + v_{\varphi} B_r) = 0,
\]

\[
\frac{\partial B_z}{\partial t} + (v_r B_z + v_{\varphi} B_{\varphi}) = 0.
\]
\[ \dot{B}_r \approx 0, \quad (6) \]
\[ \dot{B}_\varphi = \frac{d}{dr} (v_\varphi B_r - v_r B_\varphi), \quad (7) \]
\[ \dot{B}_z = -\frac{d}{dr} (v_r B_z) - \frac{v_r B_z}{r}. \quad (8) \]

3 SELF-SIMILAR SOLUTIONS FOR ADAF

If we assume the parameters \( \gamma \) and \( f \) in the energy equation are independent of radius \( r \), then we can adopt a self-similar treatment similar to NY94 and AF06,

\[ v_r (r) = -c_1 \alpha \sqrt{\frac{G M}{r}}, \quad (9) \]
\[ v_\varphi (r) = c_2 \sqrt{\frac{G M}{r}}, \quad (10) \]
\[ c_s^2 (r) = c_3 \frac{G M}{r}, \quad (11) \]
\[ c_{r,\varphi,z}^2 (r) = \frac{B_{r,\varphi,z}^2}{4 \pi \rho} = 2 \beta_{r,\varphi,z} c_3 \frac{G M}{r}, \quad (12) \]

where the coefficients \( c_1, c_2 \) and \( c_3 \) are similar to those in AF06, and \( \beta_r, \beta_\varphi \) and \( \beta_z \) measure the ratio of the magnetic pressure in three directions to the gas pressure, i.e., \( \beta_{r,\varphi,z} = p_{\text{mag},r,\varphi,z}/p_{\text{gas}} \). Following AF06, we also denote the structure of the surface density \( \Sigma \) by

\[ \Sigma (r) = \Sigma_0 r^s. \quad (13) \]

The half-thickness of the disk still satisfies the relation \( H \propto r \) and we obtain

\[ H (r) = H_0 r. \quad (14) \]

Substituting self-similar relations \([9]-[14] \) to equations \([2], [3] \) and \([5] \), we can obtain the algebraic equations of \( c_1, c_2 \) and \( c_3 \):

\[ -\frac{1}{2} c_1^2 \alpha^2 = \frac{c_2^2}{2} - 1 - [(s - 1) + \beta_z (s - 1) + \beta_\varphi (s + 1)]c_3, \quad (15) \]
\[ -\frac{1}{2} c_1 c_2 \alpha = -\frac{3}{2} \alpha (s + 1) c_2 c_3 + c_3 (s + 1) \sqrt{\beta_r \beta_\varphi}, \quad (16) \]
\[ c_2^2 = \frac{4}{9 f} \left( \frac{1}{\gamma - 1} + s - 1 \right) c_1. \quad (17) \]

If in the cylindrical coordinates we assume the three components of magnetic field \( B_{r,\varphi,z} > 0 \), then \( v_\varphi (r) \) can be either positive or negative, depending on the detailed magnetic field structure in the disk. In a particular case where \( B_r = 0 \) or \( B_\varphi = 0 \), we can only obtain
the value of $|c_2|$, but in a general case where $B_r B_\varphi \neq 0$, we are able to determine the value of $c_2$.

Figures 1 and 2 show the self-similar coefficients $c_1$, $|c_2|$ and $c_3$ as functions of the advection parameter $f$ with different $(\beta_r, \beta_\varphi, \beta_z)$. We consider the disk to be radiation dominated with $\gamma = 4/3$, and take $s = -1/2$ (i.e., $\rho \propto r^{-3/2}$ as the common case) and the viscosity parameter $\alpha = 0.1$, which is the widely used value.

Figure 1 shows changes of the coefficients $c_1$, $|c_2|$, $c_3$ with $\beta_z$ and $\beta_\varphi$. We neglect the radial magnetic fields $B_r$, and take the parameters $(\beta_r, \beta_\varphi, \beta_z)$ in the left three panels in Figure 1 for $(0, 1, 0)$, $(0, 1, 1)$, $(0, 1, 3)$ and $(0, 1, 10)$, and then take the value of $\beta_\varphi$ to be 10 in the right three panels. As $\beta_r = 0$, we can only obtain $|c_2|$ without needing to determine the direction of $v_\varphi$. The coefficients $c_1$ and $c_3$ increase with increasing the advection parameter $f$, but $|c_2|$ decreases monotonously as a function of $f$ except for a strong toroidal magnetic field. Moreover, with the fixed ratio $\beta_z$, an increase of $\beta_\varphi$ makes all the coefficients $|c_i|$ become larger. Oppositely, $|c_i|$ decreases with increasing $\beta_z$. In fact, with a small radial magnetic field $\beta_r \approx 0$, we can obtain an analytical solution of $c_i$ from equations (15)-(17), which are similar to expressions (27)-(29) in AF06, but we should replace $(1 - s)/(1 + s)$ by $(1 - s)(1 + \beta_z)/(1 + s)$ and $\beta$ by $\beta_\varphi$ in those expressions instead. Also, we have $c_2^2 \propto f^{-1} c_1$, $c_3 \propto c_1$, and $c_1 \propto \beta_\varphi$ for large $\beta_\varphi$ and $c_1 \propto \beta_z^{-1}$ for large $\beta_z$, all of which are consistent with the results in Figure 1.

As a result, from Figure 1, we first find that a strong toroidal magnetic field leads to an increase of the infall velocity $|v_r|$, rotation velocity $|v_\varphi|$ and isothermal sound speed $c_s$, and $|v_r|$ and $c_s$ are large in the case where the disk flow is mainly advection-dominated, but the rotation velocity $|v_\varphi|$ increases with increasing $f$ only in the case where the toroidal magnetic field is large enough. This conclusion is consistent with the case 1 in AF06. Second, the high ratio $\beta_z$ decreases the value of $|v_r|$, $|v_\varphi|$ and $c_s$, which means that a strong magnetic pressure in the vertical direction prevents the disk matter from being accreted, and decreases the effect of gas pressure as accretion proceeds.

Figure 2 shows how $c_1$, $|c_2|$ and $c_3$ change with $\beta_r$ and $\beta_\varphi$, where we neglect the vertical magnetic $B_z$. For a small value of $\beta_r$, the coefficients $c_1$, $|c_2|$ and $c_3$ also increase with increasing $\beta_\varphi$. However, a change of $c_i$ is not obvious for a large value of $\beta_r$. We are able to calculate the limiting value of $c_i$ in the extreme case where $\beta_r$ is large enough and $\beta_r \beta_\varphi \neq 0$ using an analytical method. From equations (15)-(17), we can obtain $c_1 = 2/[\epsilon'' + \sqrt{(\epsilon'')^2 + 2\alpha^2}]$ and
\[ c_2 = \epsilon'' c_1 \text{ for large } \beta_r, \text{ where } \epsilon'' = \frac{4}{9} (\gamma - 1)^{-1} + s - 1. \] If \( \epsilon'' \gg \alpha \), we have \( c_1 \sim (\epsilon'')^{-1} \propto f \) and \( |c_2| \sim 1 \), which means that the infall velocity \( |v_r| \) increases with advection parameter \( f \) linearly, and the radial velocity \( |v_\phi| \) is nearly the Keplerian velocity, no matter whether the disk is efficiently cooled or not. Also, for a large value of \( \beta_r \), equation (16) becomes

\[ -(c_1 c_2 \alpha)/2 \sim c_3 (s + 1) \sqrt{\beta_r \beta_\phi}. \] Since \( c_1, c_3 > 0 \) in the accretion disk, we obtain \( c_2 < 0 \), which means that the direction of rotation in the disk is opposite to the toroidal magnetic field \( B_\phi \). Actually, in the case where \( \beta_r \) is sufficient large and \( \beta_r \beta_\phi \neq 0 \), the angular momentum transported due to the magnetic field stress is dominated over that due to the viscosity, and balances with the advection angular momentum. As we take \( B_{r,\phi,z} > 0 \), from equation (3), we obtain that the large angular momentum due to the magnetic field stress makes the value of the advection angular momentum \( (\dot{M} v_r r, \text{ where } \dot{M} \text{ is the mass accretion rate}) \) increase in the disk, which requires \( v_\phi < 0 \) in the self-similar structure.

From the mass-continuity equation (1) and the induction equations (6)-(8) as well as the solved coefficients \( c_i \), we can solve the self-similar structure of the mass loss and magnetic field escaping rate with forms of \( \dot{\rho} = \dot{\rho}_0 r^{s-5/2} \) and \( \dot{B}_{r,\phi,z} = \dot{B}_{r_0,\phi_0,z_0} r^{(s-5)/2} \) where \( \dot{\rho}_0 \) satisfies

\[ \dot{\rho}_0 = - \left( s + \frac{1}{2} \right) \frac{c_1 \alpha \Sigma_0 \sqrt{GM}}{2H_0}. \] (18)

As mentioned in AF06, when \( s = -1/2 \), i.e., \( \Sigma \propto r^{-1/2} \) or \( \rho \propto r^{-3/2} \), there is no wind in the disk, and thus we can use the formula \(-2\pi r v_r \Sigma = \dot{M}\) to determine the surface density \( \Sigma \). In addition, in the region of the disk where is adiabatic with \( \rho \propto r^{-1/(\gamma-1)}, \ p \propto r^{-\gamma/(\gamma-1)}, \ v_r \propto r^{(3-2\gamma)/(\gamma-1)} \) (i.e., \( s = (\gamma - 2)/(\gamma - 1) \)), we obtain the self-similar solution of \( c_1 \alpha = \sqrt{2}, c_2 = 0 \) and \( c_3 = 0 \), which describe the Bondi accretion. However, if the disk region satisfies the entropy-conservation condition with \( f = 0 \), we can still obtain an accretion-disk solution beyond the self-similar treatment. For a CDAF with \( \rho \propto r^{-1/2} \) (NIA), a steady disk without wind requires \( c_1 = 0 \) or \( v_r = 0 \).

\[ \dot{B}_{r_0,\phi_0,z_0} \text{ satisfy } \dot{B}_{r_0} \approx 0, \] (19)

1 In some previous works (e.g., Wang 95, Lai 98, SK05 and SK06), the rotation velocity is taken to be positive and the toroidal magnetic field \( B_\phi \) to be negative. The advection transports angular momentum inward, while the magnetic stress transports angular momentum outward instead. This previous result is consistent with our result here if we change the cylindrical coordinate used above from \((r, \phi, z)\) to \((r, -\phi, z)\). In this paper it is convenient for us to take \( B_{r,\phi,z} > 0 \) and to obtain a series of self-similar solutions about magnetized flows in many different cases.
\[ \dot{B}_{\phi_0} = \left( \frac{s - 3}{2} \right) GM \left\{ c_2 \sqrt{\frac{4\pi \beta r c_3 \Sigma_0}{H_0}} + c_1 \alpha \sqrt{\frac{4\pi \beta \phi c_3 \Sigma_0}{H_0}} \right\}, \quad (20) \]

\[ \dot{B}_{z_0} = \left( \frac{s - 1}{2} \right) c_1 \alpha (GM) \sqrt{\frac{4\pi \beta z \Sigma_0 c_3}{H_0}}, \quad (21) \]

where \( H_0 \) in the expression (14) can be obtained from the hydrostatic equilibrium equation (4), that is,

\[ H_0 = \frac{1}{2} \left[ (s - 1)c_3 \sqrt{\beta r \beta_z} + \sqrt{c_3^2 (s - 1)^2 \beta r \beta_z} + 4(1 + \beta \phi + \beta z)c_3 \right], \quad (22) \]

and thus we obtain the half-thickness of the disk \( H = H_0 c_s / (\sqrt{c_3^3 \Omega_K}) \). We will discuss the effects of the magnetic field on \( H \) later in §5.

4 SELF-SIMILAR SOLUTIONS FOR CDAF

In CADFs, both advection and convection play contributions to the angular momentum and energy transportation. We propose a CDAF model in a global magnetic field in order to compare it with non-globally-magnetized CDAFs. We follow the idea of NIA in this section and consider the effect of a magnetic field. The viscosity angular momentum flux is

\[ \dot{J}_v = -\alpha c_s^2 \frac{\Omega_K \rho r^3 d\Omega}{dr}, \quad (23) \]

and the convection angular momentum flux can be written as

\[ \dot{J}_c = -\alpha c_s^2 \frac{\rho_r^3 (1 + g) / 2}{r} \frac{d}{dr} \left( \Omega r^{3(1 - g) / 2} \right), \quad (24) \]

where \( \alpha_c \) is the dimensionless coefficient to measure the strength of convective diffusion, \( g \) is the parameter to determine the condition of convective angular momentum transport. Convection transports angular momentum inward (or outward) for \( g < 0 \) (or \( > 0 \)).

The energy equation of a CADF is

\[ \rho v_r T \frac{ds}{dr} + \frac{1}{r^2} \frac{d}{dr} \left( r^2 F_c \right) = Q^+ = \int \left( \alpha + g \alpha_c \right) \rho c_s^2 r^2 \left( \frac{d\Omega}{dr} \right)^2, \quad (25) \]

where the convective energy flux \( F_c \) is

\[ F_c = -\alpha_c c_s^2 \frac{\rho T}{\Omega_K} \frac{ds}{dr}, \quad (26) \]

where we still consider the general energy equation without vertical integration as in NIA, and still neglect the Joule heating rate.

\[ ^2 \text{We adopt the (} \alpha, \alpha_c \text{)-prescription following NIA. The MHD simulations beyond this prescription can be seen in Igumenshchev et al. (2002, 2003), Hawley & Balbus (2002) and so on. Moreover, Quataert & Gruzinov (2000) also develop an analytical model for CDAFs.} \]
Using the angular momentum equation, the energy equation of the CDAF and the self-similar structure (9)-(14), we can obtain the relevant algebraic equations of self-similar structure of the CDAF,

\[
-\frac{1}{2}c_1c_2\alpha = -\frac{3}{2}(\alpha + g\alpha_c)(s + 1)c_2c_3 + c_3(s + 1)\sqrt{\beta_r\beta_\varphi},
\]

\[
\left(s - \frac{1}{2}\right)\left(\frac{1}{\gamma - 1} + s - 1\right)c_3\alpha_c + \left(\frac{1}{\gamma - 1} + s - 1\right)c_1\alpha = (\alpha + g\alpha_c)\frac{9f}{4}c_2^2.
\]

The radial momentum equation is still the same as that in ADAF. Combining equation (15), (27) and (28), we can finally solve the coefficients \(c_1, c_2,\) and \(c_3\) in the case of CDAF and compare them with those in ADAF. The dimensionless coefficient \(\alpha_c\) can be calculated using the mixing length theory, and we adopt equation (15) in NIA, who describes the relation of \(\alpha_c\) with \(s, c_3\) and \(\gamma\) (in NIA, they used the symbols \(a\) and \(c_0\), where \(a = 1 - s\), and \(c_0^2 = c_3\) in our paper).

To simplify the problem, we first prefer using a simpler treatment with a fixed \(\alpha_c\) to discussing the solutions of (15), (27) and (28), i.e., we take \(\alpha_c\) as a free parameter rather than a calculated variable, since \(\alpha_c\) does not dramatically change in many cases. Then we can adopt a similar treatment as in §3 to solve equations (15), (27) and (28). Similarly as in §3, we first use the analytical method to discuss some particular cases.

When \(\beta_r\beta_\varphi \sim 0\) (which implies that the radial or toroidal magnetic fields are weak), we can obtain an analytical solution similar to that in §3 (see Appendix B for more details about the calculation). We discuss two cases. One is that the toroidal magnetic field \(B_\varphi\) is dominated and the radial magnetic field is weak \((B_r \approx 0)\). Then we can have an approximate solution,

\[
c_1\alpha \sim \frac{2\beta_\varphi}{3(\alpha + g\alpha_c)},
\]

\[
c_2^2 = \frac{c_1\alpha}{\alpha + g\alpha_c} \left[ e'' - |\xi''| \frac{\alpha_c}{3(\alpha + g\alpha_c)(s + 1)} \right],
\]

\[
c_3 \sim \frac{2\beta_\varphi}{9(\alpha + g\alpha_c)^2(s + 1)},
\]

where \(\xi'' = \frac{4}{9f}(s - \frac{1}{2})(\frac{1}{\gamma - 1} + s - 1)\). From these equations, we know the coefficients \(c_1\) and \(c_3\) increase with increasing the convective parameter \(\alpha_c\) for \(g < 0\) (i.e., convection transports angular momentum inward), but we cannot obtain the relation between \(\alpha_c\) and \(|c_2|\) unless the values of \(s, g\) and \(\gamma\) are given in detail. The other case is that the vertical magnetic field \(B_z\) is dominated. In this case, we obtain
We find that the coefficients $c_1$ and $|c_2|$ decrease with increasing $\alpha_c$ for $g < 0$ while the value of $c_3$ is more or less the same for a fixed $\beta_z$.

Another analytical solution can be obtained when $\beta_r$ is large and $\beta_\phi \neq 0$, and we have the relations

$$c_1 \propto \left[ \frac{\alpha}{\alpha + g\alpha_c} e'' + \sqrt{\left( \frac{\alpha}{\alpha + g\alpha_c} \right)^2 e''^2 + 2\alpha^2} \right]^{-1},$$

$$|c_2| \propto \left[ e'' + \sqrt{e''^2 + 2(\alpha + g\alpha_c)^2} \right]^{-1/2},$$

$$c_3 \propto -c_1 c_2. \tag{37}$$

From formulae (35) and (36), we again find changes of $c_1$ and $|c_2|$ with $\alpha_c$, which is similar to the former case. From equation (37) and $c_{1,3} > 0$, we get $c_2 < 0$, which has also been obtained in §3.

Figure 3 shows some examples of the effect of the convection parameter $\alpha_c$ on the three coefficients $c_i$. In order to see the results clearly, we take $\alpha = 1$ and change the value of $\alpha_c$ from 0 to 0.9 with several sets of magnetic field parameters $(\beta_r, \beta_\phi, \beta_z) = (0, 3, 0), (3, 3, 0)$ and $(0, 0, 3)$. Also we set $\gamma = 4/3$, $s = -1/2$ and $g = -1/3$. The basic results in Figure 3 are consistent with the above discussion using the analytical method. In particular, we notice that the three coefficients do not change dramatically in the case of $\beta_r \sim \beta_\phi$, since the magnetic field gives a contribution to the angular momentum rather than the viscosity, and reduces the effect of convection on the disk.

Next we want to obtain the relation between $\alpha_c$ and $\alpha$ following NIA, i.e., we consider $\alpha_c$ to be the variable as a function of $s$, $\gamma$ and $c_3$. Using the treatment in NIA based on the mixing length theory and equations (15), (27) and (28), we can establish the $\alpha_c-\alpha$ relation. NIA discussed such a relation with $g = 1$ and $g = -1/3$, and find that the solution with $s = -1/2$ is available only for $\alpha$ greater than a certain critical $\alpha_{crit}$ when the isothermal sound speed reaches its maximum value, and the value of $\alpha_c$ decreases monotonously as $\alpha$ increases. However, our results are quite different from those in NIA for two reasons. First, we keep the term $v_r dv_r/dr$ in the radial momentum equation, while in NIA this term is
neglected. As a result, in many cases, the sound speed to determine the actual critical $\alpha$ for available solutions does not reach exactly its maximum value. Second and more importantly, we consider the effect of the large-scale magnetic field on the disk.

From equation (15), we obtain

$$\frac{1}{2} c^2 \alpha^2 + [(1 - s)(1 + \beta_z) - \beta_\varphi(1 + s)]c_3 - 1 < 0.$$ (38)

If the radial magnetic field is weak, then we have $c_3 < [(1 - s)\beta_z]^{-1}$ for a large vertical magnetic field and $c_3 < 2\beta_\varphi/[9(\alpha + g\alpha_e)^2(1 + s)]$ for a large toroidal magnetic field. On the other hand, from NIA, we have $c_3 > \gamma/[(2 - s)(2 + s\gamma - s)]$ for the convective process to be available. Therefore, the structure of flows with convection cannot be maintained for a large vertical magnetic field. Moreover, if the term $\beta_r \beta_\varphi$ is large, we still obtain a small value of $c_3 \sim c_1|c_2|\alpha/(2\sqrt{\beta_r \beta_\varphi})$ with a small value of $\alpha_e$, which is almost independent of the variation of $\alpha$.

Figure 4 shows examples of the $\alpha_c - \alpha$ relation with different magnetic field structures. We take $\gamma = 1.4$ and $s = -1/2$. The left panel shows the $\alpha_c - \alpha$ relation with different values of $\beta_\varphi$. When the magnetic field is small ($\beta_\varphi = 0$ and 1 in this panel), $\alpha_c$ decreases with increasing $\alpha$, and $\alpha$ has its critical (minimum) value for the solution to be available. These results are basically consistent with those in NIA. However, when $\beta_\varphi$ becomes large, $\alpha_c$ increases as the viscosity parameter $\alpha$ increases, and the critical value of $\alpha$ becomes extremely small or even disappears. The right panel of Figure 4 shows the $\alpha_c - \alpha$ relation with different values of $\beta_z$. When $\beta_z$ becomes large, the critical value of $\alpha$ also disappears, but $\alpha$ has its maximum value. This result is quite different from NIA, who found that only the minimum value of $\alpha$ exists and becomes important.

From the above discussion, we conclude that a strong vertical magnetic field or large $\beta_r \beta_\varphi$ prevents the convective process in flows, while a moderate vertical magnetic field is available for small $\alpha$. A strong toroidal magnetic field with weak radial field makes the convective process become important even for large $\alpha$ in flows.

In NIA, a self-similar convection-driven non-accreting solution with $s = 1/2$ (i.e. $\Sigma \propto r^{1/2}$) was given for $\alpha + g\alpha_e = 0$ when $\alpha$ is smaller than the critical value $\alpha_{\text{crit}}$. However, the relation $\alpha + g\alpha_e = 0$ cannot be satisfied if $\beta_r \beta_\varphi \neq 0$ for magnetized CDAFs. In fact, we are still able to get a self-similar structure for magnetized CDAFs when the $\alpha_e - \alpha$ relation mentioned is no longer satisfied (i.e., inequality (38) is not satisfied). For $\beta_r \beta_\varphi \neq 0$, the zero
infall velocity (i.e. \( c_1 = 0 \)) requires \( s = -1 \) \((\rho \propto r^{-2})\) from equation (27), and \( \alpha \) as a function of \( c_3 \),

\[
\alpha = \alpha_c \left| g - \frac{\xi'' c_3}{c_2^2} \right| < \alpha_c |g|,
\]

with the maximum value of \( \alpha_c \) to be

\[
\alpha_{c,\text{crit}2} = \frac{(1 + \beta_z)}{4 \sqrt{2}} \left| \frac{2 - (1 + \beta_z) \gamma}{\gamma(1 + \beta_z)} \right|.
\]

(40)

Furthermore, if we turn the radial momentum equation from its vertical integration to its general form, we can still have the relation (39) but \( s = 0 \) \((\rho \propto r^{-1})\), and the maximum value of \( \alpha_c \) to be

\[
\alpha_{c,\text{crit}2} = \frac{(1 + \beta_z)}{9 \sqrt{2}} \left| \frac{9 - (2 \beta_z + 5) \gamma}{2 \gamma(1 + \beta_z)} \right|.
\]

(41)

Such a structure of \( \rho \propto r^{-1} \) was also obtained by Igumenshchev et al. (2003), who explained the structure as a result of vertical leakage of convective energy flux from the disk. In our model, however, we show that this structure is due to the inefficient angular momentum transfer by viscosity and the zero Lorentz force in the \( \phi \)-direction.

As a result, we obtained a self-similar solution for magnetized CDAFs with \( c_1 = 0 \), \( s = -1 \) or \( s = 0 \) (for the general form) and \( \alpha + g \alpha_c < 0 \). This solution is adopted when the normal self-similar solutions mentioned above for convective flows cannot be satisfied.

5 A MORE REALISTIC FORM OF KINEMATIC VISCOSITY

In the above sections §3 and §4, we assume the kinematic viscosity \( \nu = \alpha c_s^2 / \Omega_K \) and take the viscosity parameter \( \alpha \) as a constant in our discussion for simplicity. A more realistic model based on the physical meaning of the viscosity parameter is \( \nu = \alpha c_s H \) with \( H \neq c_s / \Omega_K \) in the magnetized disk. In this section we consider the effect of different forms of kinematic viscosity \( \nu \). In order to compare with the results in §3 and §4, we replace \( \alpha \) in the last two sections by \( \alpha' \) and take \( \nu = \alpha c_s H = \alpha' c_s^2 / \Omega_K \) in this section. Also, we still adopt the definition of \( c_1 \) using equation (9). From formula (22), we are able to obtain

\[
\alpha' = \alpha \left\{ \left( \frac{1 - s}{2} \right)^2 \beta_r \beta_z c_3 + (1 + \beta_r + \beta_\varphi) \right\}^{1/2} - \left( \frac{1 - s}{2} \right) (\beta_r \beta_z c_3)^{1/2}. \]

(42)

When \( \beta_{r,z} = 0 \), we have \( \alpha' = \alpha \sqrt{1 + \beta_\varphi} \) and equation (42) switches back to the case of \( \mu = 1/2 \) in AF06. For large \( \beta_r \) or \( \beta_\varphi \) and small \( \beta_z \), we have \( \alpha' \sim \alpha \sqrt{1 + \beta_r + \beta_\varphi} \) and \( \alpha' \gg \alpha \).

For large \( \beta_z \), we obtain \( \alpha' \sim \alpha (1 + \beta_r + \beta_\varphi) / (1 - s) \sqrt{\beta_r \beta_z c_3} \) and \( \alpha' \ll \alpha \). This result can be
explained as being due to the fact that a large toroidal or radial magnetic field makes the disk half-thickness $H$ become large and increase the kinematic viscosity (since $\nu \propto H$), but a large vertical field reduces the height $H$ and decreases the kinematic viscosity.

Figure 5 shows the effect of a modified kinematic viscosity on the three coefficients $c_1$, $|c_2|$ and $c_3$ in ADAFs. A more realistic expression of $\nu$ increases the infall velocity, but decreases the radial velocity and the isothermal sound speed. However, a difference between these two cases of kinematic viscosity is obvious for a large toroidal magnetic field rather than a large vertical field. In fact, if the toroidal magnetic field $B_\phi$ is strong and dominated in $(B_r, B_\phi, B_z)$, we can adopt a similar solution of (49)-(51) in AF06 for $\mu = 1/2$, and find that $c_1$ increases but $|c_2|$ and $c_3$ reaches their limiting values with increasing $\beta_\phi$. If the radial magnetic field $B_r$ is strong and dominated, we can obtain $c_1 \sim \text{const}$, $|c_2| \propto \beta_r^{-1/4}$ and $c_3 \propto \beta_r^{-1/2}$, which are different from §3 in which $|c_2| \sim 1$ for large $\beta_r$. Furthermore, if $\beta_z$ is large enough, we have the limiting value $c_1 \sim 3(1 + \beta_r + \beta_\phi)(1 - s)^{-3/2}\beta_z^{-1}\beta_r^{-1/2}$, $c_3 \sim (1 - s)^{-1}\beta_z$ and $c_2^2 = 3\epsilon''(s + 1)c_3$, and the values of $|c_2|$ and $c_3$ are more or less the same, no matter what the form of kinematic viscosity is.

For flows with convection, it is convenient for us to adopt the general definition of $\alpha_c$ from NIA, which measures a degree of convection in the flows. We find that the conclusions in §4 are not basically changed if we replace $\alpha$ in §4 by $\alpha'$. The $\alpha' - \alpha_c$ relation can be turned back to the $\alpha - \alpha_c$ relation using equation (42). However, there is no dramatic change between these two relations except for extremely strong magnetic fields.

6 CONCLUSIONS

In this paper we have studied the effects of a global magnetic field on viscously-rotating and vertically-integrated accretion disks around compact objects using a self-similar treatment. Our conclusions are listed as follows:

(1) We have extended Akizuki and Fukue’s self-similar solutions (2006) by considering a three-component magnetic field $B_r, B_\phi,$ and $B_z$ in ADAFs. If we set the kinematic viscosity $\nu = \alpha c_s^2/\Omega_K$ as its classical form, then with the flow to be advection-dominated, the infall velocity $|v_r|$ and the isothermal sound speed $c_s$ increase, and even the radial velocity $|v_\phi|$ can exceed the Keplerian velocity with a strong toroidal magnetic field. The strong magnetic field in the vertical direction prevents the disk from being accreted, and decreases the effect of the gas pressure. For a large radial magnetic field, $v_r, v_\phi$ and $c_s$ can reach their limiting
values, and the direction of radial velocity is actually negative, since the angular momentum transfer due to the magnetic field stress in this case is dominated over that due to the viscosity in the disk, and makes the value of advection angular momentum increase inward.

(2) If the convective coefficient $\alpha_c$ in flows is set as a free parameter, $|v_r|$ and $c_s$ increase with increasing $\alpha_c$ for large $B_\psi$ and weak $B_r$. Also, $|v_r|$ becomes smaller and $|v_\phi|$ becomes larger (or smaller) with increasing $\alpha_c$ for a strong and dominated radial (or vertical) magnetic field.

(3) The $\alpha_c - \alpha$ relation in the magnetized disk is different from that in the non-magnetized disk. For large $B_\psi$ and weak $B_r$, $\alpha_c$ increases with increasing $\alpha$, the critical value $\alpha_{\text{crit}}$ to determine different cases of the $\alpha_c - \alpha$ relation disappears, and $\Sigma \propto r^{-1/2}$ can be satisfied for any value of $\alpha$. A moderate vertical magnetic field is available for small $\alpha$. The large $B_z$ or $B_r B_\psi$, on the other hand, prevents the convective process in flows.

(4) The self-similar convection envelope solution in NIA should be replaced by $c_1 = 0$, $\alpha + g\alpha_c < 0$ and $s = -1$ ($\rho \propto r^{-2}$) for the vertical integration form of angular equations and $s = 0$ ($\rho \propto r^{-1}$) for the general form in magnetized CDAFs. This solution can be adopted in the region that does not satisfy the normal self-similar solutions for flows with convection and $\alpha_c < \alpha_{c,\text{crit}2}$.

(5) The magnetic field increases the disk height $H$ for large $B_r$ and $B_\psi$, but decreases it for large $B_z$ in the magnetized disk. A more realistic model of the kinematic viscosity $\nu = \alpha c_s H$ makes the infall velocity in ADAFs increase and the sound speed and toroidal velocity decrease compared with the simple case when the form $\nu = \alpha c_s^2 / \Omega_K$ is assumed.

ACKNOWLEDGEMENTS

We would like to thank the referee, Jun Fukue, for useful comments. We also thank X. D. Li and Y. W. Yu for their helpful discussions. This work is supported by the National Natural Science Foundation of China (grants 10221001 and 10640420144) and the National Basic Research Program of China (973 program) No. 2007CB815404.

APPENDIX A:

The momentum equation of accretion flows can be written as (Frank et al. 2002)

$$ (v \cdot \nabla)v = -\frac{1}{\rho} \nabla P - \nabla \Phi + \Omega^2 r + (\nabla \cdot \sigma) + \frac{1}{\rho c} j \times B, $$

(A1)
where $\sigma$ is the viscosity stress tensor, $j \times B/(\rho c)$ is the density Lorentz force. Also, the Ampère’s law and the induction equation (Faraday’s law) are

$$j = \frac{c}{4\pi} (\nabla \times B). \quad (A2)$$

$$\frac{\partial B}{\partial t} = \nabla \times (v \times B) + \eta_m \nabla^2 B. \quad (A3)$$

where $\eta_m = c^2/(4\pi \sigma_e)$ is the magnetic diffusivity and $\sigma_e$ is the electrical conductivity. For simplicity, we consider the extreme case that $\sigma_e \to \infty$ and $\eta_m \approx 0$, and then neglect the second term in the right side of the induction equation (A3). Combining equations (A1) and (A2), we can obtain the three components of the momentum equation. In particular, the three components of the Lorentz force in the cylindrical coordinates are

$$\frac{4\pi}{c} (j \times B)_r = -\frac{1}{2} \frac{\partial}{\partial r} (B^2_r + B^2_\varphi) + B_r \frac{\partial B_r}{\partial z} - \frac{B^2_\varphi}{r}, \quad (A4)$$

$$\frac{4\pi}{c} (j \times B)_\varphi = \frac{1}{r} B_\varphi B_r + B_r \frac{\partial B_\varphi}{\partial r} + B_z \frac{\partial B_z}{\partial z}, \quad (A5)$$

$$\frac{4\pi}{c} (j \times B)_z = -\frac{1}{2} \frac{\partial}{\partial z} (B^2_r + B^2_\varphi) + B_r \frac{\partial B_z}{\partial r}. \quad (A6)$$

Based on the consideration that all flow variables including the magnetic field are mainly functions of radius $r$, we can conclude $v_z = 0$ and $\partial/\partial z = 0$. Or a more realistic consideration requires $\partial/\partial z \sim (H/r)\partial/\partial r \ll \partial/\partial r$. Also, we take $\partial/\partial \varphi = 0$ for the axisymmetric disk. We rewrite the Lorentz force using the Alfven sound speed as

$$\frac{1}{\rho c} (j \times B)_r = -\frac{1}{2\rho \partial r} [\rho (c^2_z + c^2_\varphi)] - \frac{c^2_\varphi}{r}, \quad (A7)$$

$$\frac{1}{\rho c} (j \times B)_\varphi = \frac{1}{r} c_\varphi c_r + \frac{c_r}{\sqrt{\rho}} \frac{\partial}{\partial r} (\sqrt{\rho} c_\varphi), \quad (A8)$$

$$\frac{1}{\rho c} (j \times B)_z = \frac{c_r}{\sqrt{\rho}} \frac{\partial}{\partial r} (\sqrt{\rho} c_z). \quad (A9)$$

These expressions are different from SK05 and SK06, who considered the magnetic field structure as a function of both radius $r$ and height $z$: $B_r(r, z) = z(B_r)_H/H$, $B_\varphi(r, z) = z(B_\varphi)_H/H$ with $H$ to be the half-thickness of the disk, and $B_z(r, z) = B_z(r)$. However, in this paper we take the magnetic field to be homogeneous in the vertical direction and neglect the term of $\partial/\partial z$ as mentioned above except for the $z$-component equation, in which we take the total pressure as $P_{\text{tot}} = P_{\text{gas}} + (B^2_\varphi + B^2_r)/8\pi$ in the vertical direction, and adopt $\partial P_{\text{tot}}/\partial z \sim -P_{\text{tot}}/H$ to estimate the value of $H$. In §2, we use the height-integration equations.
APPENDIX B:

Expressions (29)-(37) in §4 can be derived as follows:

When $\beta_r = 0$ or $\beta_\varphi = 0$, we obtain equations for the three coefficients $c_i$ in CDAFs as

$$-\frac{1}{2}c_1^2\alpha^2 = c_2^2 - 1 - [(s - 1)(1 + \beta_z) + (1 + s)\beta_\varphi]c_3,$$

(B1)

$$c_1\alpha = 3(\alpha + g\alpha_c)(s + 1)c_3,$$

(B2)

$$c_2^2 = \epsilon''\frac{\alpha c_1}{\alpha + g\alpha_c} + \xi''\frac{\alpha c_3}{\alpha + g\alpha_c},$$

(B3)

with $\epsilon'' = \frac{4}{9}(\frac{1}{\gamma - 1} + s - 1)$ and $\xi'' = \frac{4}{9}(s - \frac{1}{2})(\frac{1}{\gamma - 1} + s - 1)$. Then we can write the equation for $c_1$ as

$$\frac{1}{2}c_1^2\alpha^2 + c_1\alpha\left\{\epsilon'' + \xi''\frac{\alpha c_3}{3(s + 1)(\alpha + g\alpha_c)} + \frac{1}{3}\left[(1 - s)(1 + \beta_z) - \beta_\varphi\right]\right\} - 1 = 0.$$

(B4)

When $\beta_z$ is large, the above equation can be simplified as

$$\frac{1}{2}c_1^2\alpha^2 + \frac{c_1\alpha}{3}\left(\frac{1 - s}{1 + s}\right)\beta_z - 1 = 0,$$

(B5)

and we obtain

$$c_1\alpha \sim \frac{3(\alpha + g\alpha_c)(1 + s)}{(1 - s)\beta_z}.$$  

(B6)

Similarly, we can get the solution for large $\beta_\varphi$ and small $\beta_r$.

On the other hand, if the radial magnetic field is strong and dominated and $\beta_\varphi \neq 0$, then equation (B2) should be replaced by

$$-\frac{1}{2}c_1c_2\alpha = (s + 1)c_3\sqrt{\beta_r}\beta_\varphi,$$

(B7)

and we obtain an equation in the extreme case,

$$\frac{1}{2}c_1^2\alpha^2 + \epsilon''\frac{\alpha c_1}{\alpha + g\alpha_c} - 1 = 0,$$

(B8)

and get

$$\alpha c_1 = 2(\alpha + g\alpha_c) \left[\epsilon'' + \sqrt{\epsilon''^2 + 2(\alpha + g\alpha_c)^2}\right]^{-1}.$$  

(B9)

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Figure 1. The self-similar coefficients $c_1$, $|c_2|$, and $c_3$ as functions of the advection parameter $f$ for different sets of parameters $\beta_r$, $\beta_\phi$, and $\beta_z$. We take $\alpha=0.1$, $\gamma=4/3$ and $s=-1/2$. The left three panels correspond to $(\beta_r, \beta_\phi, \beta_z) = (0, 1, 0)$ (solid lines), (0, 1, 1) (dashed lines), (0, 1, 3) (dotted lines), and (0,1,10) (dash-dotted lines). The right three panels correspond to $(\beta_r, \beta_\phi, \beta_z) = (0, 10, 0)$ (solid lines), (0, 10, 1) (dashed lines), (0, 10, 3) (dotted lines), and (0, 10, 10) (dash-dotted lines).
Figure 2. The self-similar coefficients $c_1$, $|c_2|$, and $c_3$ as functions of the advection parameter $f$ for different sets of parameters $\beta_r$, $\beta_\phi$, and $\beta_z$ with $\alpha=0.1$, $\gamma = 4/3$ and $s = -1/2$. The left three panels correspond to $(\beta_r, \beta_\phi, \beta_z) = (0, 1, 0)$ (solid lines), (0.1, 1, 0) (dashed lines), (1, 1, 0) (dotted lines), and (10, 1, 0) (dash-dotted lines). The right three panels correspond to $(\beta_r, \beta_\phi, \beta_z) = (0, 2.5, 0)$ (solid lines), (0.1, 2.5, 0) (dashed lines), (1, 2.5, 0) (dotted lines), and (10, 2.5, 0) (dash-dotted lines).
Figure 3. The coefficients $c_1$, $|c_2|$, and $c_3$ as functions of $f$ with different sets of parameters $\alpha_c$ and $(\beta_r, \beta_\phi, \beta_z)$. We take $\alpha=1$, $\gamma = 4/3$, $s = -1/2$ and $g = -1/3$. The left three panels correspond to $(\beta_r, \beta_\phi, \beta_z) = (0, 3, 0)$, the middle three panels to $(\beta_r, \beta_\phi, \beta_z) = (3, 3, 0)$, and the right panels to $(\beta_r, \beta_\phi, \beta_z) = (0, 0, 3)$. Different lines refer to $\alpha_c = 0$ (solid lines), $\alpha_c = 0.3$ (dashed lines) and $\alpha_c = 0.9$ (dotted lines).
Figure 4. The convective coefficient $\alpha_c$ as a function of viscosity parameter $\alpha$ with $s = -1/2, \gamma = 1.4, f = 1$ and different sets of parameters $(\beta_r, \beta_\varphi, \beta_z)$. (a) Left panel: $(\beta_r, \beta_\varphi, \beta_z) = (0, 0, 0)$ (solid line), $(0, 1, 0)$ (dashed line), $(0, 3, 0)$ (dotted line) and $(0, 5, 0)$ (dash-dotted line); (b) Right panel: $(\beta_r, \beta_\varphi, \beta_z) = (0, 0, 0.1)$ (solid line), $(0, 0, 0.5)$ (dashed line), $(0, 0, 0.7)$ (dotted line) and $(0, 0, 0.8)$ (dash-dotted line)
Figure 5. Comparison between two forms of the kinematic viscosity $\nu$, while the results of $\nu = \alpha c_s^2 / \Omega K$ are shown by thin lines, and those of $\nu = \alpha c_s H$ are shown by thick lines. We adopt $s = -1/2$, $\gamma = 4/3$ and $\alpha = 1$. (a) Left panels: $(\beta_r, \beta_\phi, \beta_z) = (2, 0, 0)$ (solid lines) and $(2, 2.5, 0)$ (dashed lines); (b) Right panels: $(\beta_r, \beta_\phi, \beta_z) = (2, 0, 1)$ (solid lines) and $(2, 0, 5)$ (dashed lines).