Finite elements parameterization of optical tomography with the radiative transfer equation in frequency domain

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Abstract. Optical tomography is a technique of probing semi-transparent media with the help of light sources. In this method, the spatial distribution of the optical properties inside the probed medium is reconstructed by minimizing a cost function based on the errors between the measurements and the predictions of a numerical model of light transport (also called forward/direct model) within the medium at the detectors locations. Optical tomography with finite elements methods involves generally continuous formulations where the optical properties are constant per mesh elements. This study proposes a numerical analysis in the parameterization of the finite elements space of the optical properties in order to improve the accuracy and the contrast of the reconstruction. Numerical tests with noised data using the same algorithm show that continuous finite elements spaces give better results than discontinuous ones by allowing a better transfer of the information between the whole computational nodes of the inversion. It is seen that the results are more accurate when the number of degrees of freedom of the finite element space of the optical properties (number of unknowns) is lowered. This shows that reducing the number of unknowns decreases the ill-posed nature of the inverse problem, thus it is a promising way of regularizing the inversion.

1. Introduction

Among the new imaging modalities expected to be available in the future, optical tomography is one of the most promising although numerous difficulties still exist. It is used in flow diagnostics, medical imaging, food processing, etc. This laser-based probing technique may be divided into direct imaging in which the emerging signal is directly used for projection and reconstruction imaging based on inverse problems also called optical tomography. For both of them, recent research tends to show that the use of the long term photons, which have travelled for a long time in the whole sample to be probed, generates more information to the image reconstruction.

Recently, an increasing interest has been devoted to the finite elements formulations when solving the radiative transfer equation. This tendency in favor of the finite elements method follows from its simplicity, flexibility and property of being able to handle complex geometries and advection type equations. From a standard Galerkin formulation [1], a number of studies

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have been done to improve the accuracy of these computational methods such as the Streamline Upwind Petrov Galerkin [2, 3], the Least Square formulation [4, 5] and the Discontinuous Galerkin formulation [6, 7], to name but a few.

Generally, optical tomography with finite elements methods involves continuous formulations [8, 9, 10, 11]. However, continuous finite element formulations suffer from the lack of local conservativity compared to the discontinuous formulation which allows the use of numerical fluxes to achieve local conservativity [12, 13]. In addition, high order accuracy can be achieved with the Discontinuous Galerkin formulation by using higher order polynomial approximation than the finite volume method. The Discontinuous finite elements formulation can be viewed as an all-in-one formulation as with the same formulation, the finite volume method is obtained when constant polynomial elements are used, while using a continuous function yields the standard finite element method [14].

This study presents an analysis of optical tomography where a parameterization of the optical properties is used to improve the accuracy of the reconstruction. Numerical tests of image reconstruction are performed to gauge the reconstruction accuracy in relation with the specific features of the explored finite elements models and the related finite elements space of the optical properties.

2. Physical model
In optical tomography, the forward model aims at computing the prediction of the boundary reading once the optical properties are known. Below, we present the forward model equation based on the frequency domain radiative transfer equation and the used solution method.

2.1. Radiative transfer in frequency domain
In this study, the retained forward model is the Fourier transform of the transient radiative transfer equation which writes in a given direction \( \vec{\Omega} \) and for each spatial position \( r \in D \) by [15]:

\[
\vec{\Omega} \cdot \nabla I(r, \vec{\Omega}, \omega) + \left( \frac{i\omega}{c} + \kappa + \sigma_s \right) I(r, \vec{\Omega}, \omega) = \frac{\sigma_s}{4\pi} \int_{4\pi} I(r, \vec{\Omega}', \omega) \Phi(\vec{\Omega}', \vec{\Omega}) d\Omega'
\]  

(1)

where \( i = \sqrt{-1}, c \) is the light speed in the medium, \( \omega \) is the modulation frequency, \( I(r, \vec{\Omega}, \omega) \) is the radiant power per unit solid angle per unit area at the spatial position \( r \) in direction \( \vec{\Omega} \), \( \kappa = \kappa(r) \) and \( \sigma_s = \sigma_s(r) \) are respectively the absorption and the scattering coefficients, and \( \Phi(\vec{\Omega}', \vec{\Omega}) \) is the scattering phase function. Generally, the scattering phase function in tissues is given by the Henyey-Greenstein phase function [16]:

\[
\Phi(\vec{\Omega}', \vec{\Omega}) = \frac{1 - g^2}{(1 + g^2 - 2g \cos(\Theta))^{3/2}}
\]

(2)

where \( \Theta \) is the scattering angle between directions \( \vec{\Omega} \) and \( \vec{\Omega}' \) and \( g \) is the anisotropic factor.

The boundary condition for Eq.(1) is a collimated external radiation that penetrates into the medium with the direction \( \vec{\Omega}_c \) at position \( r_0 \). The boundary is taken as non reflecting with the corresponding boundary condition:

\[
I(r_0, \vec{\Omega}) = q_0 \delta(r - r_0) \delta(\vec{\Omega} - \vec{\Omega}_c) \quad \vec{\Omega}_c \cdot \vec{n} < 0 \quad \forall r \in \partial D
\]

(3)

where \( q_0 \) is the total heat flux, \( \vec{n} \) is the outward normal unit vector and \( \delta \) is the Dirac function, \( \partial D \) denotes the boundary of the domain \( D \).

The radiative intensity is separated into two components \( I = I_c + I_s \) where \( I_c \) and \( I_s \) are respectively the collimated and scattered intensities within the medium as treated in [5]. It is
pointed out in [17], that this separation technique removes strong discontinuities due to Dirichlet-type boundary condition (Eq.(3)) and consequently improves the accuracy of the finite element solution. The collimated component obeys the extinction law [16] such that

$$\vec{\Theta}_c \cdot \nabla I_c(r, \omega) + \left(\frac{i\omega}{c} + \kappa + \sigma_s\right) I_c(r, \omega) = 0$$  \hspace{1cm} (4)

where the boundary condition is given by

$$I_c(r_0, \omega) = q_0(r, \omega) \delta(r - r_0) \quad \vec{\Theta}_c \cdot \vec{n} < 0, \quad \forall r \in \partial D$$  \hspace{1cm} (5)

The solution of the collimated part is used as a source term for the scattered component $I_s$ such that

$$\vec{\Theta} \cdot \nabla I_s(r, \vec{\Theta}, \omega) + \left(\frac{i\omega}{c} + \kappa + \sigma_s\right) I_s(r, \vec{\Theta}, \omega) = \frac{\sigma_s}{4\pi} \int_{4\pi} I_s(r, \vec{\Theta}', \omega) \Phi(\vec{\Theta}', \vec{\Theta}) d\Omega' + S_c(r, \vec{\Theta}, \omega)$$  \hspace{1cm} (6)

where $S_c(r, \vec{\Theta}, \omega)$ is the source term induced by the scattering of $I_c$ within the medium. $S_c$ is given by

$$S_c(r, \vec{\Theta}, \omega) = \frac{\sigma_s}{4\pi} \int_{4\pi} I_c(r, \omega) \delta(\vec{\Theta} - \vec{\Theta}_c) \Phi(\vec{\Theta}', \vec{\Theta}) d\Omega' = \frac{\sigma_s}{4\pi} I_c(r, \omega) \Phi(\vec{\Theta}_c, \vec{\Theta})$$  \hspace{1cm} (7)

Let us recall that Eq.(6) uses only a vacuum boundary condition which writes

$$I_s(r, \vec{\Theta}, \omega) = 0, \quad \vec{\Theta} \cdot \vec{n} < 0, \quad \forall r \in \partial D.$$  \hspace{1cm} (8)

For optical tomography applications, the measurable quantity used is the exitance on the boundary obtained by

$$P(r) = \int_{\partial D} I_s(r, \vec{\Theta}, \omega) \vec{n} \cdot d\Omega$$  \hspace{1cm} (9)

Let us point out that here the forward model is a system of two equations (Eq. (4) and Eq.(6)) that makes possible to handle the collimated direction compared to other studies [10, 18, 19, 20]. As shown in [9], the collimated source direction acts in the adjoint equation for the computation of the objective function gradient.

2.2. Solution method
The solution method is based on the Discontinuous Galerkin formulation to solve both the collimated and the scattered Eq. (4) and Eq.(6), whereas the discrete ordinates method is used to handle the angular dependency.

2.2.1. Discrete ordinates method  In the Discrete Ordinates Method, integrals over solid angles are replaced by a numerical quadrature [21]. Thus, Eq.(6) is rewritten as a spatial differential equation for each discrete direction $\vec{\Theta}_m$ :

$$\vec{\Theta}_m \cdot \nabla I^m_s(r, \omega) + \left(\frac{i\omega}{c} + \kappa + \sigma_s\right) I^m_s(r, \omega) = \frac{\sigma_s}{4\pi} \sum_{m'=1}^{M} I^m_{s'}(r, \omega) \Phi \left(\vec{\Theta}^m_{m'}, \vec{\Theta}_m\right) w_{m'} + S^m_c(r, \omega)$$  \hspace{1cm} (10)

where $M$ is the number of directions of the quadrature, $w_{m'}$ and $I^m_{s'}$ are respectively the quadrature weight and the radiative intensity in the direction $\vec{\Theta}_{m'}$. The corresponding boundary condition writes

$$I^m_s(r, \omega) = 0 \quad \vec{\Theta}_m \cdot \vec{n} < 0, \quad \forall r \in \partial D.$$

(11)
The phase function in Eq.(10) is re-normalized in order to avoid normalization errors. As in [22, 23], The re-normalized phase function is given by

$$\Phi\left(\vec{\Omega}_m', \vec{\Omega}_m\right) = f_{m'} \Phi\left(\vec{\Omega}_m', \vec{\Omega}_m\right)$$  \hspace{1cm} (12)

where $f_{m'}$ is the renormalization factor defined by

$$f_{m'} = \left(\frac{1}{4\pi} \sum_{i=1}^{M} w_i \Phi\left(\vec{\Omega}_m', \vec{\Omega}_i\right)\right)^{-1}$$  \hspace{1cm} (13)

2.2.2. Discontinuous Galerkin formulation

Equations Eq. (4) and Eq.(6) for each discrete direction $m$, are advection-type equations that are to be solved. There are many methods that can be used to solve these equations, among which the finite element methods are well suited for handling complex geometries. Finite elements formulation of the radiative transfer equation has been used in optical tomography recently [10, 8, 9]. In these works, only standard or continuous finite elements are used. Here we consider the Discontinuous Galerkin formulation [24].

For an easy presentation of the formulation, let us rewrite the equations of the collimated intensity and the scattered intensity for each direction of the quadrature as

$$\beta \nabla u + bu = f \hspace{1cm} \forall x \in D \hspace{1cm} \forall x \in \partial D^-$$  \hspace{1cm} (14)

where $\beta$ is the advection direction ($\beta = \vec{\Omega}_i$ or $\beta = \vec{\Omega}_m$), $u = u(x)$ is the complex value field, $b$ is the complex extinction coefficient, $f$ the complex source term, $\partial D^- = \{x \in \partial D, \beta \cdot \vec{n} < 0\}$ is the inflow boundary according to the advection direction $\beta$ and $h$ is the inflow boundary complex function that is applied to the system.

The variational formulation of (14) with the Discontinuous Galerkin writes [17]:

$$\text{Find } u_h \in V_h \text{ such that } \mathcal{B}(u_h, v_h) = \mathcal{F}(v_h) \hspace{1cm} \forall v_h \in V_h$$  \hspace{1cm} (15)

where $V_h$ is the finite element space where the solution $u_h$ is searched, $\mathcal{B}(u_h, v_h)$ and $\mathcal{F}(v_h)$ are respectively the bilinear and linear forms given by

$$\mathcal{B}(u_h, v_h) = \sum_{k \in D} (-\beta u_h, \nabla v_h)_k + \sum_{k \in D} (\beta \cdot \vec{n} u_h, v_h)_{\partial k^+} + \sum_{k \in D} (\beta \cdot \vec{n} u_h^-, v_h)_{\partial k^- \setminus \partial D^-} + (b u_h, v_h)_D$$  \hspace{1cm} (16)

$$\mathcal{F}(v_h) = (f, v_h)_D - (\beta \cdot \vec{n} h, v_h)_{\partial D^-}$$

and where $u_h^+$ and $u_h^-$ are respectively the values of $u_h$ in cell $k$ and in the neighboring cells. In the following, the space of piecewise linear discontinuous elements is chosen for the solutions of the collimated and the scattered component of the light intensity. The solution method of the forward model equation is done by solving first the collimated component solution. The resulting source term (Eq. (7)) is used for the solution of the scattered part through an iterative procedure where the initial scattered field is null. The iterative algorithm stops when the maximum absolute relative error between the current iteration and the previous one on the scattered field is lower than a user-defined value, i.e. when

$$\max_{i,m} \left| \frac{I_{s,i}^{m,k-1} - I_{s,i}^{m,k}}{I_{s,i}^{m,k}} \right| \leq 10^{-6}$$  \hspace{1cm} (17)
where \( \mathbf{r}_{m,k}^{i} \) is the discretized scattered component solution in direction \( m \) at iteration \( k \) and \( i \) is a degree of freedom of the finite element solution. In all computations, linear equations are solved with the SuperLu solver [25]. The criterion Eq. (17) is chosen in order to make sure that both the global convergence and the local convergence are achieved. The accuracy of the method is given in [17].

3. Inversion
In the inversion procedure, the aim is to recover the optical properties of the medium through the minimization of an objective function. Here, a gradient-type optimization is used where the objective function gradient is deduced through an additional adjoint state formulation.

3.1. Objective function and gradient computation
The objective function to be minimized in the inversion procedure is based on the errors between the measurements and the predictions at the detectors positions with a numerical forward model. Here, our model is based on a Discontinuous Galerkin formulation which leads us to use an integral form of objective function in order to be consistent with discontinuous fields as the values of the solution is not well defined at boundaries due to the discontinuity of the finite element space but their integral can be computed. Hence, the objective function writes

\[
J(\theta) = \frac{1}{2} \sum_{n=1}^{N_s} \sum_{d} \int_{\partial D_d} \| P(r, \omega_n, I_s) - M(r_d, \omega_n) \|^2 ds
\]

where \( \partial D_d \) is the surface of the \( d \)-th detector such that \( \partial D_d \subset \partial D \), \( \partial D \) being the boundary of the domain whose properties are to be recovered. \( \theta = (\kappa, \sigma) \) is the vector of parameters, i.e. the absorption and the scattering coefficients. The complex values \( P(r, \omega_n, I_s) \) and \( M(r_d, \omega_n) \) are the predictions and the measurements whom differences are integrated over all the detectors for all collimated sources. Next, \( N_s \) is the number of collimated sources, and \( N_d \) is the number of detectors, and \( \| z \|^2 = zz^* \forall z \in \mathbb{C} \) where \( z^* \) is the conjugate of \( z \). We suppose that \( \kappa \) and \( \sigma \) belong to the same finite element space such that \( \theta = (\kappa, \sigma) \in \mathbb{R}^{2Nc} \) where \( N_c \) is the number of degrees of freedom of the chosen finite element space. In the following, the space of piecewise polynomial functions of degree 1 is chosen for \( \kappa \) and \( \sigma \) fields.

The gradient of the objective function is computed with the adjoint method with the following expressions [8]:

\[
\nabla J(\theta) \cdot \delta \kappa = (\lambda_s|I_s \delta \kappa)_s + (\lambda_c|I_c \delta \kappa)_c
\]

\[
\nabla J(\theta) \cdot \delta \sigma = (\lambda_s|I_s \delta \sigma)_s + (\lambda_c|I_c \delta \sigma)_c - \left( \lambda_s| \left( \frac{1}{4\pi} \int_{4\pi} I_s(r, \Omega', \omega) \Phi(\Omega', \Omega)|d\Omega \right) \delta \sigma \right)_s
\]

where \((.|)_s\) and \((.|)_c\) are inner products associated to the solution space respectively of \( I_s \) and \( I_c \) and \( \lambda_s = \lambda_s(r, \Omega, \omega) \), \( \lambda_c = \lambda_c(r, \omega) \) are the corresponding complex vectors which represent the solution of the following adjoint equation system:

\[
\left[ -\vec{\Omega} \cdot \nabla - \frac{i\omega}{c} + \kappa_a + \sigma_s \right] \lambda_s(r, \vec{\Omega}, \omega) - \frac{\sigma_s}{4\pi} \int_{4\pi} \lambda_s(r, \vec{\Omega}', \omega) \Phi(\vec{\Omega}', \vec{\Omega}) d\Omega' = 0
\]

and

\[
\left[ -\vec{\Omega} \cdot \nabla - \frac{i\omega}{c} + \kappa_a + \sigma_s \right] \lambda_c(r, \omega) - \frac{\sigma_s}{4\pi} \int_{4\pi} \lambda_s(r, \vec{\Omega}', \omega) d\Omega' = 0
\]
where $\lambda_c = 0$ and $\bar{\Omega} \cdot \vec{n}_\lambda s + \frac{\partial J(\theta)}{\partial I_s} = 0$ for $\bar{\Omega} \cdot \vec{n} > 0$, $\forall r \in \partial D$. We refer to references [8, 9, 26, 27, 28] for more details on how to deduce the gradient of the objective function through a Lagrangian formulation.

3.2. Reconstruction algorithm

Gradient-based algorithms have shown to be efficient for large scale optimization problem in optical tomography [18, 29]. Here, our reconstruction scheme is based on the limited-memory version of the BFGS method [30] where a scaling of both the objective function and its gradient are used to handle round-off errors and the low level of the boundary measurements due to the high extinction in the medium [9].

4. Tests and results

4.1. Test description

A 2 cm $\times$ 2 cm domain which contains two inclusions is chosen where the heterogeneity in the optical properties of the medium is represented by two inclusions. The optical properties of the medium are given in Table 1. It is assumed that the medium is forward-scattering where the phase function is given by the Henyey-Greenstein function with an anisotropic factor $g=0.9$. Four collimated source with zero-phased, modulated at the frequency of 600 MHz are placed at mid-centres of each side of the square and the measurements are done with eight detectors of 0.8 cm of extension regularly placed around the boundaries beyond the source positions.

The experiments measurements data is computed numerically. The above forward model is used with a regular fine triangular mesh of 8142 elements and 24 angular discrete ordinates ($S_6$ quadrature). The inverse problem is based on a coarser mesh of 3786 elements coupled to 24 discrete directions. In this study, the parameters ($\kappa$ and $\sigma_s$) and the state variables ($I_c$ and $I_s$) are computed on the same mesh, but they belong to different finite elements spaces for parameterization purpose. The obtained complex-valued intensities with the original properties (Table 1) are noised with a Gaussian distribution such that the level of the noise in dB unit is given by

$$SNR = 10 \log_{10} \left( \frac{M_d}{\sigma_{M_d}} \right)$$

where $\sigma_{M_d}$ is the standard deviation. The inversion computations are performed with the limited memory quasi-Newton method of Brodyen-Fletcher-Goldfarb-Shanno [30, 31]. The quality of the reconstruction is measured with errors $\epsilon_1$ and $\epsilon_2$ defined by:

$$\epsilon_1 = \frac{1}{N_c} \sum_{i=1}^{N_c} \left( \frac{\theta^r_i - \theta^o_i}{\theta^o_i} \right)^2, \quad \epsilon_2 = \left( \int_D (\theta^r_i - \theta^o_i)^2 \, dx / \int_D (\theta^o_i)^2 \, dx \right)^{1/2}$$

where $N_c$ is the number of degrees of freedom related to the parameters finite element space, and superscripts $r$ and $o$ refer to the reconstructed and original images, respectively. $\epsilon_1$ represents the mean quadratic errors per degree of freedom and $\epsilon_2$ is the relative error of the reconstruction with respect to the original distribution.

Table 1: Optical properties of the test medium

|           | Background | Bottom inclusion | Top inclusion |
|-----------|------------|-----------------|--------------|
| $\kappa$  | 0.25 cm$^{-1}$ | 0.35 cm$^{-1}$  | 0.15 cm$^{-1}$ |
| $\sigma_s$ | 20 cm$^{-1}$  | 30 cm$^{-1}$    | 10 cm$^{-1}$   |
Figure 1: Original distribution of the optical properties. (a) is the absorption coefficient, (b) is the reduced scattering coefficient.

Figure 2: Estimated distributions where the optical properties ($\kappa$ and $\sigma$) belong to the piecewise constant finite elements space ($P0$). (a) is the estimated absorption coefficient, (b) is the estimated reduced scattering coefficient.

Table 2: Comparative accuracy of the reconstruction. $\epsilon_1$ and $\epsilon_2$ are the corresponding errors, $ndof$ is the number of degrees of freedom of the finite element space.

|     | $P0$ | $P1dc$ | $P1$ | $P2$ |
|-----|------|--------|------|------|
| $ndof$ | -    | 3790   | 11370| 1976 | 7741 |
| $\epsilon_1$ | $\epsilon_{1,\kappa}$ | 0.0015 | 0.0008 | 0.0018 | 0.0009 |
|      | $\epsilon_{1,\sigma}$ | 0.0018 | 0.0010 | 0.0021 | 0.0010 |
| $\epsilon_2$ | $\epsilon_{2,\kappa}$ | 0.0932 | 0.0895 | 0.0651 | 0.0720 |
|      | $\epsilon_{2,\sigma}$ | 0.1005 | 0.0964 | 0.0715 | 0.0770 |
4.2. Results and analysis
A comparative test is carried out with different finite element spaces for the optical properties where the intensity fields are supposed to be in the space of linear discontinuous elements ($P_{1dc}$). The original distribution of the optical properties are presented in Fig. 1 where the reduced scattering coefficient $\sigma_s^g = (1 - g)\sigma_s$ is used. A comparative estimation is carried out with the computed noised data. The reconstruction is done by taking the parameters respectively in the space of piecewise constant ($P_0$), piecewise linear discontinuous ($P_{1dc}$), piecewise linear continuous ($P_1$) and piecewise quadratic ($P_2$) elements. The recovered distribution are reported
Figure 5: Estimated distributions where the optical properties ($\kappa$ and $\sigma$) belong to piecewise quadratic finite elements space ($P2$). (a) is the estimated absorption coefficient, (b) is the estimated reduced scattering coefficient.

for piecewise constant (Fig.2), piecewise linear discontinuous (Fig.3), piecewise linear continuous (Fig.4) and piecewise quadratic (Fig.5) elements.

It is seen that continuous finite elements spaces give better results than the discontinuous one. The results show that continuous finite elements approximations of the optical properties introduce some implicit regularization on the inversion by smoothing the results. For each type of finite element space (continuous or discontinuous), it is seen that increasing the number of degrees of freedom do not improve the quality of the reconstruction. The smallest errors defining the quality of the reconstruction is given by the $P1$ finite element space whose number of degrees is the lowest (see $\epsilon_2$ in Table 2). The errors given by $\epsilon_1$ take into account the number of degrees of freedom and show that increasing the number of degrees of freedom lower the errors $\epsilon_1$ as expected. However, taking into account the total number of degrees of freedom, the piecewise linear elements ($P1$) solution remains the more accurate.

From the inverse analysis point of view, high order parameterization may increase the ill-posed nature of the inverse problem such as the existence of correlations, the number of possible solutions and the sensitivity to round errors and noise. And one may end-up with less accurate solutions. Then, reducing the number of unknowns is a way of reducing the number of possible solutions i.e a way of regularizing the inversion. Physically, with continuous parameterization, information is well conducted among the whole computational nodes which improves the reconstruction as seen in the results.

5. Conclusion
In this study, numerical tests of reconstruction of optical properties of scattering and absorbing medium are done. The forward model is a finite element model based on the discrete ordinates method and the Discontinuous Galerkin method. A comparative study of the parameterization of the finite element space of the optical properties is done with piecewise constant, linear (continuous and discontinuous), and quadratic elements. The results show that better results are obtained when the finite element space of the unknown coefficients have a low number of degrees of freedom through a decrease of the ill-posed nature of the inverse problem as a regularization technique. The analysis shows that continuous finite element parameterization
gives better results because the information is well conducted among the whole computational nodes which improves the reconstruction as seen in the results.

In a future step, we hope to speed up the forward and the adjoint model by using multiprocessing solvers in order to use high order quadrature. Also, new optimization schemes based on the finite elements flexibilities will be worked to improve the reconstruction.

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