Exploring the superimposed oscillations parameter space

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Abstract. The space of parameters characterizing an inflationary primordial power spectrum with small superimposed oscillations is explored using Monte Carlo methods. The most interesting region corresponding to high frequency oscillations is included in the analysis. The oscillations originate from some new physics taking place at the beginning of the inflationary phase and characterized by the new energy scale $M_c$. It is found that the standard slow roll model remains the most probable one given the first-year WMAP data. At the same time, the oscillatory models fit the data better on average, which is consistent with previous works on the subject. This is typical of a situation where volume effects in the parameter space play a significant role. Then, we find the amplitude of the oscillations to be less than 22\% of the mean amplitude and the new scale $M_c$ to be such that $H/M_c < 6.6 \times 10^{-4}$ at 1\sigma level, where $H$ is the scale of inflation.

Keywords: trans-Planckian physics, CMBR theory, cosmological perturbation theory, inflation
1. Introduction

The possibility that the cosmic microwave background (CMB) data contain unexpected features which would be signatures of non-standard physics has been recently studied in [1,2]. These features generally consist of superimposed oscillations in the CMB multipole moments $C_\ell$. They could originate either from trans-Planckian effects taking place in the early universe [3]–[32] or from other phenomena, for instance of the type of those investigated in [33]–[38].

In [1,2], it has been demonstrated that the presence of superimposed oscillations can improve the fit to the first-year Wilkinson Microwave Anisotropies Probe (WMAP) data [39]–[43]; see also [44,45]. The corresponding drop in the $\chi^2$ was found to be $\Delta \chi^2 \simeq 10$. Then, based on the so-called $F$-test, it has been argued that this drop is statistically significant. However, it should be clear that the study carried out in [1,2] has only provided a hint as regards the possible presence of non-trivial features in the multipole moments. In order to go further, it is necessary to explore the parameter space in detail, taking into account, of course, the parameters characterizing the oscillations, namely the amplitude, the frequency and the phase. This task is delicate because the numerical computation of the multipole moments in the presence of rapid oscillations in the initial power spectrum necessitates an accurate estimation of the CMB transfer functions and of the line of sight integrals which, in turn, requires one to boost the accuracy with which the $C_\ell s$ are computed (here by means of the \texttt{CAMB} code [46]). This causes an important increase of the computational time and instead of a few seconds, the computation of one model now takes a few minutes. Therefore, the exploration of the full parameter space (i.e. the one including the cosmological parameters characterizing the transfer functions and the primordial parameters encoding the shape of the initial power spectra) remains beyond the capabilities of present day computational facilities, at least in a reasonable time. For this reason, in this paper, we restrict ourselves to the exploration of the so-called fast parameter space only, i.e. the space of parameters characterizing the primordial spectrum, including of course the oscillatory parameters. These parameters are usually called ‘fast’ because restricting the exploration to the corresponding parameter subspace requires the calculation of the transfer functions only once, and hence yields a huge gain in terms of computational time. However, in the present work, the correct computation of the
Exploring the superimposed oscillations parameter space

superimposed oscillations that are transferred from the power spectrum to the multipole moments requires a very accurate evaluation of the line of sight integrals. Therefore, even in the fast parameter subspace, each determination of the $C_\ell$ s remains a heavy time-consuming task [1, 2]. The main new result of the present paper is that, for the first time, the shape of the likelihood function in this space including, and this is a crucial point, the region containing the best fit found in [1, 2] is presented.

The present study also allows us to gain some new insight into the statistical significance of the superimposed oscillations. In particular, we derive the one-dimensional marginalized probabilities and the mean likelihoods for the oscillatory parameters. Our result indicates that the slow roll models are still the most probable ones. However, the marginalized probability distribution of the amplitude possesses a long tail originating from the fact that the mean likelihood is peaked at values corresponding to models with oscillations. From the above-mentioned distributions, we derive constraints on the parameters characterizing the oscillations.

Finally, some comments are in order about previous works on the subject [19, 25, 28]. In those references, the shape of the likelihood function in the parameter space, including cosmological parameters, was studied and its hedgehog shape highlighted. However, the exploration was limited to the low frequency models in order to avoid the above-mentioned numerical difficulties. As a consequence, the region where the fit found in [1, 2] lies was not considered. As mentioned above, the exploration of this region is a crucial ingredient for the present paper and plays a role in constraining the oscillatory parameters.

This paper is organized as follows. In the next section we briefly recall the theoretical model used to generate primordial oscillations, together with the numerical implementations. In section 3, marginalized probabilities and constraints on the fast parameters are discussed. We give our conclusions in section 4.

2. The fast parameter space

We assume that inflation of the spatially flat Friedman–Lemaître–Robertson–Walker (FLRW) space–time is driven by a single scalar field $\phi(t)$. Then, the scalar fluctuations can be characterized by the curvature perturbations $\zeta$ and the tensor perturbations by a transverse and traceless tensor $h^{(T)}_{ij}$ [47]–[50]. If the trans-Planckian effects are taken into account, the corresponding power spectra take the form of a standard inflationary part plus some oscillatory corrections. The standard part is parametrized by the Hubble parameter during inflation $H$, the slow roll parameters $\epsilon_1$ and $\epsilon_2$, and the pivot scale $k_*$ [47]–[50]. The two slow roll parameters that we use are defined by $\epsilon_1 \equiv -\dot{H}/H^2$ and $\epsilon_2 = 2(\epsilon_1 - \delta)$ with $\delta \equiv -\ddot{\phi}/(H\dot{\phi})$. The oscillatory part is determined by three new parameters. Two of them are the modulus $|x|$ and the phase $\varphi$ of a complex number $x$ which characterizes the initial conditions and the other is $\sigma_0 = H/M_c$, $M_c$ being a new energy scale. Note that $\sigma_0$ is evaluated at a time $\eta_0$ during inflation which is a priori arbitrary but, and this is the important point, does not depend on $k$. In the following we will choose this time such that $k_*/a_0 = M_c$ where $a_0$ is the scale factor at time $\eta_0$.

Explicitly, for density perturbations, the power spectrum reads [1, 2]

$$k^3P_\zeta = k^3P_{\text{sr}} \left\{ 1 - 2|x|\sigma_0 \cos \left( \frac{2\epsilon_1}{\sigma_0} \ln \left( \frac{k}{k_*} \right) + \psi \right) + O(|x|\sigma_0 \epsilon_2) \right\},$$

(1)
Exploring the superimposed oscillations parameter space

...and there is a similar expression for the gravitational waves. Typically, the amplitude of the oscillations is given by $|x| \sigma_0$, while the wavelength is such that $\Delta k/k = \pi \sigma_0/\epsilon_1$. As discussed in [1], the factor $x$ parametrizing the initial conditions plays an important role in our analysis. The quantum state in which the cosmological fluctuations are ‘created’, when their wavelength equals the new characteristic scale $M_c$, is a priori not known. Usually, it is assumed that this quantum state is characterized by the choice $x = 1$. In the present paper, following [20, 22, 24, 27], we generalize these considerations and describe our ignorance of the initial state by the parameter $x$ which is therefore considered as a free parameter. From the point of view of the statistical study presented here, this turns out to be very important. Indeed, as mentioned in [1] (see the discussion after equation (8)), this allows us to decouple the amplitude of the oscillations from the value of the frequency which, therefore, can now be considered as independent parameters. As a consequence, high frequency waves no longer necessarily have a small amplitude as is the case if one chooses $x = 1$. Let us also note that, despite the fact that the parametrization of the initial conditions used is more general, we have nevertheless restricted our considerations to the case where $x$ is scale independent over the scales of interest.

In order to compute the resulting CMB anisotropies, we use a modified version of the CAMB code [46] whose characteristics and settings are detailed in [1, 2]. The exploration of the parameter space is performed by using the Markov chain Monte Carlo methods implemented in the COSMOMC code [51] together with our modified CAMB version. As already mentioned, in order to avoid prohibitive computational time, only the ‘fast parameter space’ is explored. Notice again that, even in this case, the required accuracy (see [1, 2]) renders the numerical computation much longer than the one necessary to the exploration of the ‘full parameter space’ in the standard inflationary case.

Since we restrict ourselves to the ‘fast parameter space’, we have to fix several degrees of freedom. Firstly, the cosmological parameters determining the CMB transfer functions should be chosen. Since the oscillatory features in the multipole moments are small corrections to the standard inflationary predictions, a reasonable choice is to fix the (‘non-primordial’) cosmological parameters to their best fit values obtained from the vanilla slow roll power spectrum, i.e. from a pure slow roll model without any additional feature, in a flat $\Lambda$CDM universe [1, 2, 52]. Indeed, it has been shown in [28] that the oscillatory parameters and cosmological parameters are not very degenerate. This amounts to taking $h = 0.734$, $\Omega_b h^2 = 0.024$, $\Omega_{\text{dm}} h^2 = 0.116$, $\Omega_\Lambda = 0.740$, $\tau = 0.129$, $\tau_{\text{re}} = 14.3$ and $10^2 \theta = 1.049$, $\theta$ being approximately the ratio of the sound horizon to the angular diameter distance; see [51].

Secondly, priors on the remaining fast parameters have to be imposed. For the standard inflationary parameters, $\epsilon_1$, $\epsilon_2$ and the logarithm of the initial amplitude of the scalar power spectrum at the pivot scale, $\ln P_{\text{scalar}}$, wide flat priors around their standard values have been chosen. Let us now discuss the three oscillatory parameters. For the phase, the sampling has been performed on the quantity $\psi$ assuming a flat prior in the interval $[0, 2\pi]$ because there is also a sine function in the term that we have not written explicitly in equation (1). For the amplitude of the oscillations, i.e. $|x| \sigma_0$, we have also imposed a flat prior and have required this parameter to vary in the range $[0, 0.45]$, the upper limit being chosen in order to avoid negative primordial power spectra.

The case of the last oscillatory parameter, i.e. the frequency, should be treated with great care. The frequency of the superimposed oscillations is given by $2\epsilon_1/\sigma_0$ and, hence,
Exploring the superimposed oscillations parameter space

Figure 1. 1D marginalized probabilities (solid curve) and normalized mean likelihoods (dashed curve) for the slow roll parameters $\epsilon_1$, $\epsilon_2$, and the scalar amplitude $P_{\text{scalar}}$, in the case of the reference model. The three-dimensional plots show the normalized mean likelihoods and the corresponding 1$\sigma$ and 2$\sigma$ contours of the 2D marginalized probabilities.

we have chosen to sample the quantity $\log (1/\sigma_p)$. At this point, it is necessary to clarify exactly what we mean by ‘superimposed oscillations’. Indeed, if the frequency is very low and is such that less than half of a period covers the whole observable range of multipoles, then the resulting effect is just a modification of the overall amplitude of the power spectra and we do not really have oscillations in the multipole moments any longer. In this regime, there is a strong degeneracy between the amplitude and $P_{\text{scalar}}$. Clearly, it is not desirable to reach this extreme case. Another regime corresponds to the case where
the frequency of the superimposed oscillations is of the same order of magnitude as the
frequency of the acoustic oscillations. In this situation, new peaks would appear in the $C_\ell$
is and this could be problematic since the WMAP data are well described by the standard
inflationary model. As a matter of fact, the amplitude of such oscillations is strongly
constrained; see e.g. [28]. Finally, there is the case where high frequency oscillations are
present. This is the case that we are mostly interested in since it contains the improved
fits found in [1, 2]. Nevertheless, very high values of the frequency lead to oscillations
from multipole to multipole and any dependence on the frequency is in fact lost. It is
clear that we would also like to avoid this extreme regime. All of the above considerations
have led us to restrict our consideration to $\log(1/\sigma_0) \in [2.3, 3.8]$ with a flat prior. This is
equivalent to imposing a Jeffreys-like prior on $\sigma_0$.

Our next step was to generate Markov chains with COSMOMC for a standard slow roll
inflationary model without oscillation (i.e. for our reference, or vanilla, model) as well as
for an oscillatory model characterized by $\psi$, $|x|\sigma_0$ and $\log(1/\sigma_0)$ with the above priors, both
cases being investigated with the non-primordial cosmological parameters chosen before.
The standard inflationary model can be used as a reference for the restricted framework
of the ‘fast parameter space’. For the two models, the Markov chains were started
from widespread points in the initial parameter space and stopped when the posterior
distributions no longer evolved significantly, which corresponds to about 250,000 elements.
The generalized Gelman and Rubin $R$-statistics implemented in COSMOMC [51, 53] is found
to be less than 5% for the slow roll parameters and equal to approximately 9% for the
oscillatory parameters. However, the phase $\psi$ remains unconstrained. Let us recall that
the $R$-statistics gives, for each parameter, the ratio of the variance of the chain means to
the mean of the chain variances. In other words, it quantifies the errors in the parameter
distributions obtained from the Markov chains exploration.

3. Results

In figure 1, we have plotted the one-dimensional marginalized probabilities and the
normalized mean likelihoods for the slow roll parameters and the scalar amplitude in the
case of the reference model. Three-dimensional plots of the mean likelihoods, obtained
by averaging with respect to one parameter, are also represented with the $1\sigma$ and $2\sigma$
contours of the two-dimensional marginalized probabilities which appear through the
surface. These plots are obviously consistent with the previously derived constraints [52],
extcept that fixing the cosmological parameters has made the likelihood isocontours slightly
tighter.

In figure 2, we have plotted the 1D marginalized probabilities and the normalized
mean likelihoods for the parameters of the oscillatory model. In figures 3 and 4, the $1\sigma$
and $2\sigma$ contours of the 2D marginalized probabilities and the normalized mean
likelihoods are displayed. They have been plotted in various planes of parameters, the parameter pairs
being chosen to be the most correlated ones.

Let us now discuss the constraints that can be put on the amplitude of the oscillations,$|x|\sigma_0$. As can be seen in figure 2, the vanilla slow roll model, corresponding to $|x|\sigma_0 = 0$,
remains the most probable one with the currently available WMAP data. From the
probability distribution, one finds that the $2\sigma$ marginalized upper limit is given by

$$|x|\sigma_0 < 0.11.$$  (2)
Figure 2. 1D marginalized probabilities (solid curves) and normalized mean likelihoods (dashed curves) for the oscillatory model.

However, the most striking feature of figure 2 is that the corresponding mean likelihood does not behave similarly to the marginalized probability, as is the case for the other parameters. Indeed, it exhibits a maximum around $|x| \sigma_0 \simeq 0.1$ which is consistent with the best fits found in [1, 2]. This unusual behaviour is expected if volume effects are present (more precisely, since we are dealing with quantities defined as integrals over...
Exploring the superimposed oscillations parameter space

Figure 3. $1\sigma$ and $2\sigma$ contours of the 2D marginalized probabilities and normalized mean likelihoods (see also figure 4).

As can be seen in figure 4, strong correlations appear between the amplitude and the frequency of the oscillations. On the three-dimensional plot $[|x|\sigma_0, \log(1/\sigma_0)]$, one remarks that the mean likelihood is non-vanishing either for large values of $\log(1/\sigma_0)$, when $|x|\sigma_0 \neq 0$, or in a ‘long thin tube’ centred around $|x|\sigma_0 = 0$, whose amplitude...
Exploring the superimposed oscillations parameter space

Figure 4. 1σ and 2σ contours of the 2D marginalized probabilities and normalized mean likelihoods (see also figure 3). Strong correlations between the amplitude and the frequency of the oscillations appear (on the left).

is much smaller (recall that $\Delta \chi^2 \simeq 10$ between these two models). This behaviour is expected since any frequency is possible provided that its amplitude is very small; hence the 'long thin tube' around $|x|\sigma_0 = 0$. On the other hand, for large values of $\log(1/\sigma_0)$, one recovers a peak which corresponds to the oscillations fitting the outliers at relatively small scales [1, 2]. The marginalized probability is peaked at $|x|\sigma_0 = 0$ because the tube has a bigger statistical weight, i.e. occupies a larger volume in the parameter space. Note also that this is the very existence of the likelihood peak at large $\log(1/\sigma_0)$ which explains the long tail of the 1D marginalized probabilities for $|x|\sigma_0$.

On the three-dimensional plot $[\epsilon_1, \log(1/\sigma_0)]$, the wavelets in the mean likelihood originate from the degeneracy existing between $\sigma_0$ and $\epsilon_1$, for a fixed value of the frequency $2\epsilon_1/\sigma_0$. Furthermore, the correlation between $\epsilon_1$ and $\epsilon_2$ [52] is still present and, as a consequence, a correlation between $\epsilon_2$ and $\log(1/\sigma_0)$ also exists. We also remark that the correlations appearing when the oscillatory parameters are taken into account in the
Another interesting property concerns the parameter \( \log \left( \frac{1}{\sigma_0} \right) \). As can be seen in figures 2–4, both the mean likelihood and the marginalized probability are peaked at a high value of \( \log \left( \frac{1}{\sigma_0} \right) \). Quantitatively, a 1σ constraint on the energy scale \( M_c \) at which the new physics shows up can be derived:

\[
\sigma_0 \equiv \frac{H}{M_c} < 6.6 \times 10^{-4}.
\] (3)

Let us stress that this constraint is valid only if the amplitude and the frequency of the superimposed oscillations are considered as independent parameters. Indeed, it mainly comes from the region where the likelihood is strongly peaked and, as already noted, this region does not exist if the parameter \( x \) is chosen such that \( x = 1 \). In this case, the likelihood is rather flat and, as a consequence, one would obtain a weaker constraint on the ratio \( H/M_c \).

Finally, we have found that the phase \( \psi \) is not constrained, as is clear from figure 2.

To end this section, one can ask how well the oscillatory model fits the data on average, compared to the standard slow roll one. This can be estimated by comparing the mean likelihood over all the parameters in each of these models [51]. For the reference model one gets \( \ln \bar{L}_{\text{sr}} \simeq -715.1 \) while the oscillatory model has \( \ln \bar{L}_{\text{wig}} \simeq -712.9 \); hence there is a ratio of \( \exp(715.1 - 712.9) \simeq 9 \) in favour of the oscillatory model. This shows that including the oscillatory parameters permits one to improve the goodness of fit on average [51].

4. Discussion and conclusion

In this paper, we have explored, by means of Monte Carlo methods, the fast parameter space of an inflationary model with superimposed oscillations. This restricted framework is, at the time of writing, the only way to derive constraints in a reasonable computational time if the region where high frequency oscillations that better fit the CMB anisotropy outliers at relatively small scales [1, 2] is included. Among the main results derived in the present paper are two constraints, one on the amplitude, \( |x|\sigma_0 < 0.11 \), and the other of the energy scale \( M_c \), \( H/M_c < 6.6 \times 10^{-4} \).

The overall situation is quite interesting: on one hand, the fact that the marginalized probability is peaked at a value corresponding to a vanishing amplitude shows that the most probable model remains the standard slow roll one for which \( |x|\sigma_0 = 0 \). On the other hand, this distribution exhibits a long tail in the regions corresponding to non-vanishing amplitudes of the oscillations, precisely where the mean likelihood function is peaked, i.e. around \( |x|\sigma_0 \simeq 0.1 \). Moreover, the ratio of the vanilla slow roll model to the oscillatory model total mean likelihood is about 9, in favour of the oscillatory model. This shows that, on average, the oscillatory model fits the first-year WMAP data better.

The interpretation of the situation described above is as follows. The marginalized probabilities are quantities which are especially sensitive to ‘volume effects’, i.e. to the shape of the likelihood in regions of the parameter space where this one is significant. On the other hand, the mean likelihood is sensitive to the absolute value of the likelihood regardless of its occupied ‘volume’. Therefore, if in the parameter space there are regions

Journal of Cosmology and Astroparticle Physics 01 (2005) 007 (stacks.iop.org/JCAP/2005/i=01/a=007) 10
which, at the same time, correspond to good fits and occupy a quite confined volume, then these regions will appear particularly significant from the mean likelihood point of view but will be ‘diluted’ and, hence, will appear less significant from the marginalized probabilities point of view. Ultimately, the marginalized probability is the relevant quantity; that is to say, it is the quantity which should be used in order to reach a conclusion as regards the statistical meaning of a given model [54]. It will be interesting to study how the situation evolves when better data become available; in particular, it will be interesting to see whether the models with oscillations can occupy a volume which makes them probable from the marginalized probabilities point of view. This is one of the reasons that the next WMAP data release will be of great interest.

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Exploring the superimposed oscillations parameter space

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