Transition of polarized ions through the system of ring permanent magnets

V V Fimushkin, I V Gapienko, Yu A Plis and Yu V Prokofichev
Joint Institute for Nuclear Research, Joliot-Curie 6, Dubna, Moscow region, Russia
E-mail: yuriy@jinr.ru

Abstract. The task of the transportation of the polarized proton and deuteron beam of an energy 10–20 keV in the given magnetic field is considered. Specifically the magnetic field is produced by the system of a solenoid and some ring permanent magnets with contrary directed field. The Lorentz equations together with classic equations for vector and tensor polarizations have been solved. The results of the numerical simulation for a simple two-ring system are presented, which shows there is no essential depolarization in agreement with theoretical estimation.

The problem of transporting a polarized deuteron beam [1] with an energy of 10–20 keV in the magnetic field altering in direction and magnitude is considered. In the particular case of the polarized ion source, we have the system of a solenoid and two ring permanent magnets with counter fields (Fig. 1).

We choose the coordinate system with the origin at the center of the ring system, the axis $x$ is along the direction of motion of the deuteron beam and the axis $z$ is in the vertical direction. For paraxial trajectories the radial magnetic field is calculated as usual:

$$B_r(x) = -0, 5r(dB_x(x)/dx), \quad B_y(x) = -0, 5y(dB_x(x)/dx), \quad B_z(x) = -0, 5z(dB_z(x)/dx).$$

So, the field on the axis of the solenoid is

$$B_x(x) = \frac{\mu_0 I_n}{2(r_2 - r_1)} \left[ (x + L + l) \ln \frac{\sqrt{r_2^2 + (x + L + l)^2 + r_2}}{\sqrt{r_1^2 + (x + L + l)^2 + r_1}} - (x + L) \ln \frac{\sqrt{r_2^2 + (x + L)^2 + r_2}}{\sqrt{r_1^2 + (x + L)^2 + r_1}} \right],$$

where $\mu_0 = 4\pi 10^{-7}$ N/A$^2$, $x_1 = x + L$ - the distance from the point of measuring to the proximal end of the solenoid, $L$ - the distance from the origin to the proximal end of the solenoid, $x_2 = x + L + l$ - the distance from the point of measuring to the distal end of the solenoid, $l$ - the length of the solenoid, $r_1$ - the inner radius of the solenoid, $r_2$ - the outer radius, $n$ - the number of turns per unit of length of the solenoid, $I$ - the solenoid current. The magnetic field on the axis of the annular permanent magnet is calculated as the difference of the field values of two solid cylinders. For a cylinder with magnetization $M = B_0/\mu_0$ ($B_0$ - the residual field) the axial field in the function of the distance $t$ from the center of the magnet can be calculated by:

$$B_z(t) = \frac{1}{2}\mu_0 M [(b - t) \sqrt{a^2 + (b - t)^2 + (b + t) \sqrt{a^2 + (b + t)^2}}],$$

where $a$ and $2b$ - the radius and length of the cylinder.
Figure 1. The magnetic system; 1 - a solenoid, 2 - permanent magnets

Figure 2. Distribution of the magnetic field along the axis, the distance is measured in m, the field is in T

Let the inner and outer radii of the ring be $a_1$ and $a_2$, respectively, and $2d$ - the distance between the rings. Then, shifting the origin to the center of the two rings, we obtain the total field of the two ring system:

$$B_x(x) = \frac{1}{2} \mu_0 M \left[ \left( -x - d \right) \sqrt{a_2^2 + (x + d + 2b)^2} + \left( x + d + 2b \right) \sqrt{a_2^2 + (x + d + 2b)^2} \right]$$

$$- \left[ \left( -x - d \right) \sqrt{a_1^2 + (x - d)^2} + \left( x + d + 2b \right) \sqrt{a_1^2 + (x + d + 2b)^2} \right]$$

$$- \left[ \left( d + 2b - x \right) \sqrt{a_2^2 + (d + 2b - x)^2} + \left( x - d \right) \sqrt{a_2^2 + (x - d)^2} \right]$$

$$+ \left[ \left( d + 2b - x \right) \sqrt{a_1^2 + (d + 2b - x)^2} + \left( x - d \right) \sqrt{a_1^2 + (x - d)^2} \right].$$

The distribution of the magnetic field of the entire system is shown in Fig. 2.

To check the validity of the paraxial approximation we find an exact solution. For a cylinder with magnetization $M$ the radial field equals

$$B_r = \frac{\mu_0 M}{2\pi} \sqrt{\frac{m}{r}} \left[ \frac{m - 2}{\sqrt{m}} K(m) + \frac{2}{\sqrt{m}} E(m) \right] \zeta_{+},$$

(4)
where $a$ and $L$ are the radius and the length of the cylinder, respectively, $\zeta_\pm = x \pm L/2$,

$$m = \frac{4ar}{(a+r)^2 + \zeta^2}, \quad K(m) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - m \sin^2 \theta}}, \quad E(m) = \int_0^{\pi/2} \sqrt{1 - m \sin^2 \theta} d\theta. \quad (5)$$

Calculation of the radial magnetic field from the exact formula shows that to a radius of 15 mm the difference between the exact value and the approximate one is less than 1%.

Consider the situation with spin rotation in a radial magnetic field in the center of the ring system. The condition of "fast transition" [2] is

$$\gamma B_r \ll \frac{d\varphi}{dt}, \quad (6)$$

where

$$\gamma = \frac{\mu_d}{\hbar}, \quad \cot \varphi = \frac{B_x}{B_r}, \quad \frac{1}{\sin^2 \varphi} \frac{d\varphi}{dt} \approx \frac{1}{B_r} \frac{dB_x}{dt}, \quad \frac{d\varphi}{dt} \approx v \frac{dB_x}{dx} \frac{B_r}{B^2_x + B^2_r}. \quad (7)$$

As $B_x(0) = 0$, $B_r = -(r/2)dB_x/dx$, so

$$\frac{d\varphi}{dt} \approx \frac{v}{B_r} \frac{dB_x}{dx}. \quad (7)$$

In this case, for $E = 20$ keV ($v = 1.38 \times 10^6$ m/s), $dB_x(0)/dx = -0.32$ T/m, $r = 0.015$ m

$$\gamma B_r = \frac{\mu_d B_r}{\hbar} = 10^5 \text{ rad/s}, \quad \frac{d\varphi}{dt} \approx \frac{v}{B_r} \frac{dB_x}{dx} = 1.8 \times 10^8 \text{ rad/s},$$

Thus, the condition of fast passage without changing the polarization is performed.

With the use of the Lorentz equations the trajectories of a deuteron beam in the magnet system were calculated (Fig. 3). Further we consider the system of equations including the motion of deuterons in the magnetic field

$$\frac{d^2 \vec{r}}{dt^2} = \frac{e}{m_d} \left[ \vec{v} \times \vec{B} \right], \quad \frac{d^2 \vec{r}}{dx^2} = \frac{e}{m_d v^2} \left[ \vec{v} \times \vec{B} \right] \quad (8)$$

and the classic equations for vector and tensor polarizations [2]. For the vector polarization we have

$$\frac{d\vec{P}}{dt} = \frac{\mu_d}{h} \left[ \vec{P} \times \vec{B} \right], \quad \frac{d\vec{P}}{dx} = \frac{\mu_d}{hv} \left[ \vec{P} \times \vec{B} \right], \quad (9)$$
Figure 4. Changing the vertical polarization $P_z$ along the way, the energy $E = 20$ keV, the initial polarization value $P_x = 0$, $P_y = -1$, $P_z = 0$, the initial value of $y = 15$ mm, the rest values are zero

where $\mu_d = 0, 433073489 \times 10^{-26}$ JT$^{-1}$ is the magnetic moment of the deuteron. For the tensor polarization

$$\frac{dP_{ik}}{dx} = \frac{-\mu_d}{\hbar v} \left( \varepsilon_{jkl}P_{ij}B_l + \varepsilon_{jkl}P_{kj}B_l \right).$$

(10)

We assume that the residual induction of permanent magnets is $B_0 = 1.124$ T. The length of each magnet is $2b = 0.05$ m, the internal radius is $a_1 = 0.032$ m and the outer radius is $a_2 = 0.064$ m. The distance between the rings $2d = 0.25$ m. The initial polarization of the deuteron beam is horizontal, for injection into the accelerator after the magnetic system it must be vertical.

The parameters of the solenoid are determined by the requirement that the polarization rotates to $90^\circ$. For a deuteron energy of $20$ keV with $L = 0.275$ m, $l = 0.245$ m, $r_1 = 0.028$ m, $r_2 = 0.043$ m, $n = 588$ we need $I = 74$ A.

The calculation results are shown in Fig. 4. With all diversity of initial conditions the loss of polarization is $\Delta P_z \leq 0.02$. The calculation shows also that at the output of the magnetic system there is a nonzero $P_{xy} \simeq 0.1$.

Conclusion

For a polarized deuteron beam of an energy $20$ keV and a normalized emittance of $1.2$ $\pi$ mm rad in the magnetic system consisting of the $90^\circ$ spin-rotating solenoid and $0.4$ T permanent magnet rings the depolarization is expected less then $2\%$. The beam diameter is up to $30$ mm (half of rings aperture).

References

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[2] Schieck Hans Paetz gen. 2012 Nuclear Physics with Polarized Particles (Berlin: Springer)