Modeling and adaptive torque computed control of industrial robot based on lie algebra

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Abstract. Computed Torque Control (CTC) is the most direct and effective way to improve the motion control performance of robot. But the computation of the joint torque is quite difficult, and because of the uncertainty of the parameters, an accurate inverse robot dynamic model for torque generation is difficult to obtain. An efficient inverse dynamic model of the industrial robot based on lie algebra is proposed and applied to the computed torque control. In order to overcome the uncertainty of parameters, the inverse robot dynamic model is linearized and an adaptive computed torque control is proposed. In order to validate the adaptive torque computed control method, a multi-domain integrated system model of 6-DOF industrial robot is established and the simulation results show that the adaptive computed torque control system has the function of parameter self-learning, the inaccurate parameters converge to the true value finally. The adaptive control shows better control performance than the traditional computed torque control.

1. Introduction
Industrial robots are abroad used in the field of modern industry manufacture, such as picking and placing, paint spraying, welding, grinding[1], milling [2], drilling[3] and so on. All such applications require the end effector of the robot to achieve precise level of positioning, faster response and robust behavior in the presence of external disturbance and varying load conditions. Therefore, the high-speed and high-precision control of robot is particularly important and it has always been the research area. Many control algorithms have been proposed such as optimal control [4], sliding mode control[5], fraction order control[6], robust neural control[7],computed torque control[8] and so on. Among them, computed torque control is the most effective way to improve the dynamic performance and motion accuracy of robot.

The inner-loop controller of the computed torque control is used to cancel out the nonlinear dynamics and the nonlinear system is linearized by feedback, then various linear control methods can be applied [9]. Compared to the model free control method, the computational torque control has a large variety of advantages, such as high tracking accuracy, small feedback gain, low energy consumption and so on. However, the inverse dynamics calculation of the robot is too complex, and dynamic model parameters are difficult to obtain accurately. These factors limit the development of computational torque control technology.

The traditional modeling method for robot dynamics is based on Recursive Newton-Euler method [10] or Recursive Lagrange method[11]. However, when the two methods are used to establish the
robot model, the calculation and derivation are complex and the amount of calculation is large. Especially, there are essential defects when use the methods to derive the driving torque and its second derivation of the flexible joint robot. In recent years, as an analytical tool of modern geometric mathematics, Lie groups and Lie algebras have attracted more and more attention in the application of robot kinematics and dynamics. Park et al[12] applied the Lie group and Lie algebra theory to robot rigid body dynamics modeling for the first time, and established the robot dynamic model based on Lie group and Lie algebra. In [13], an inverse dynamics modeling method of industrial robot with elastic joints based on Lie group is proposed and applied to estimate the position error of end-effector. However, the related research about applying the dynamic model of robot based on Lie algebra to robot torque computed control is rare.

An accurate inverse robot dynamic model is required for the computed torque control. But There are coupled nonlinearities (friction, backlash due to mechanical transmission) and time-varying dynamics in industrial robot system. Besides, the parameters of robot system are so many and they are difficult to obtain accurately. So the accurate inverse robot dynamic model is hard to obtain. Many researchers are devoted to the study this problem. Two categories of adaptive inverse dynamics controllers [14] have been provided to ensure the robot control performance. In the first category, the parameter adaptations are driven by tracking errors[15]. While in the second category, the driving signals of parameter adaptations are prediction errors of the filtered joint torque[16]. In [17], the inverse robot dynamics model is constructed by using the traditional dynamics modeling method, and an adaptive control method is proposed. The calculation of inverse dynamic model is complex and the model of joint friction is simple.

In this paper, the inverse industrial robot dynamics model with joint friction is established based on Lie group and Lie algebra. For tackling the parameter uncertainty, the inverse robot dynamic model is linearized and an adaptive computed torque control method is developed. A multi-domain integrated system model for the 6-DOF industrial robot is established based on Modelica and the control method is verified by the simulation results.

The rest of the paper is organized as follows. Section 2 introduces the basic background of Lie group. Section 3 introduces the modeling method of industrial robot dynamics based on Lie algebra. Section 4 introduces the adaptive computed torque control of the robot. Section 5 introduces the multi-domain integrated simulation and validation. Section 6 summarized the full text.

2. Basic background of lie group and lie algebra
Based on the traditional D-H parameter method, it is impossible to directly calculate the high-order derivation of the rigid body dynamic equation. So in this paper, based on the theory of Lie algebra, the inverse dynamics model of industrial robot is established. In this section, some basic and essential background of Lie group is introduced. The detail of Lie group can be referenced to the work of Selig [18].

The position and attitude of the rigid body can be described by the transformation matrix of the object coordinate system relative to the reference coordinate system, all of which form a set:

\[
g \in \mathbb{SE}(3) = \begin{bmatrix} \mathbb{O} & \mathbb{T} \\
0 & 1 \end{bmatrix} \quad (1)
\]

where \( \mathbb{O} \in \mathbb{SO}(3) \) and \( \mathbb{T} \in \mathbb{R}^3 \). \( \mathbb{SO}(3) \) denotes the group of \( 3 \times 3 \) rotation matrices and \( \mathbb{SE}(3) \) is Special Euclidean Group of rigid-body motions. Both \( \mathbb{SO}(3) \) and \( \mathbb{SE}(3) \) satisfy the properties of Lie groups, so they are subgroups of Lie groups. The corresponding Lie algebras are:

\[
\begin{align*}
\mathfrak{g} &= \frac{\partial g}{\partial \theta} \\
\mathfrak{so}(3) &= \begin{bmatrix} 0 & -\mathbf{\hat{\omega}}_z & \mathbf{\hat{\omega}}_y \\
\mathbf{\hat{\omega}}_z & 0 & -\mathbf{\hat{\omega}}_x \\
-\mathbf{\hat{\omega}}_y & \mathbf{\hat{\omega}}_x & 0 \end{bmatrix} \in \mathbb{SE}(3) \\
\mathfrak{se}(3) &= \begin{bmatrix} 0 & -\mathbf{\hat{\omega}}_z & \mathbf{\hat{\omega}}_y \\
\mathbf{\hat{\omega}}_z & 0 & -\mathbf{\hat{\omega}}_x \\
-\mathbf{\hat{\omega}}_y & \mathbf{\hat{\omega}}_x & 0 \end{bmatrix} \in \mathbb{SE}(3) \\
\end{align*}
\]

(2)
The transformation from Lie algebra to Lie group can be established by using exponential mapping:

$$ T = e^{\hat{\theta}} = I_3 + \frac{1}{\|\omega\|} \omega + \frac{1 - \cos\|\omega\|}{\|\omega\|^2} \omega^2 $$

(4)

$$ g = e^{\hat{\xi}} = T \begin{bmatrix} T & 0 \\ r^T & T \end{bmatrix} \begin{bmatrix} 0^T \\ 1 \end{bmatrix} $$

(5)

An element of a Lie group can also be identified with a linear mapping between its Lie algebra via the adjoint representation. Suppose $g \in SE(3)$, its adjoint map acting on an element $\xi = [\omega, v]^T$ of se(3) is defined as:

$$ Ad^*_g(\xi) = (g^{−1} \xi g)^T = \begin{bmatrix} T & 0 \\ r^T & T \end{bmatrix} \xi $$

(6)

Elements of a Lie algebra can also be identified with a linear mapping between its Lie algebra via the Lie bracket. If $\xi_a$ and $\xi_b$ are elements of se(3), its adjoint representation is:

$$ ad^{*}_\xi (\xi_a) = [\xi_a, \xi_b] = \begin{bmatrix} \hat{\omega} & 0 \\ \hat{v} & \hat{\omega} \end{bmatrix} \xi_b $$

(7)

The dual operator $Ad^*_g$ and $ad^{*}_\xi$ is:

$$ Ad^*_g = (Ad^*_g)^T = \begin{bmatrix} T^T & T^r \\ 0 & T^r \end{bmatrix} $$

(8)

$$ ad^{*}_\xi = (ad^{*}_\xi)^T = \begin{bmatrix} -\hat{\omega} & \hat{v} \\ 0 & -\hat{\omega} \end{bmatrix} $$

(9)

3. Inverse industrial robot dynamic model based on lie algebra

For series industrial robot, the joint between adjacent rigid bodies is revolve joint. According to the properties of Lie algebra, if the revolve joint between adjacent rigid bodies is regarded as a twist $\xi$, then the position and attitude of the link $i$ relative to the base coordinate system can be expressed as:

$$ g_i = e^{\hat{\xi}_0} e^{\hat{\xi}_1} \ldots e^{\hat{\xi}_i} M_{i0} $$

(10)

where $M_{i0}$ is the position and attitude of the link $i$ relative to the base coordinate system when the robot is at zero position.

Through forward recursion (for $i = 1, 2, \ldots, n$), the six dimensional generalized velocity, acceleration, jerk and snap can be calculated as follows:

$$ V_i = g_i^{-1} \frac{d(g_i)}{dt} = Ad_{g_i}^{-1} (V_{i-1}) + \xi_{i1} \dot{\theta}_i $$

(11)

$$ A_i = \frac{d(V_i)}{dt} = Ad_{g_i}^{-1} (A_{i-1}) - ad_{\xi_{i0}}^d(V_i) + \xi_{i1} \ddot{\theta}_i $$

(12)

According to the rigid body dynamics, the resultant force acting on the rigid body in the local coordinate system \{i\} is:

$$ \begin{bmatrix} T_i \\ f_i \end{bmatrix} = \begin{bmatrix} I_i \dot{\omega}_i + \omega_i \times (I_i \omega_i + r_i \times m_i a_i) \\ m_i a_i \end{bmatrix} $$

(13)

Based on the adjoint transformation property of Lie algebra, the equation (13) can be simplified as follows:
\[
\begin{bmatrix}
T_i \\
\mathbf{f}_i
\end{bmatrix} = E_i A_i \mathbf{a} \mathbf{d}^\epsilon_i (E_i \mathbf{V}_i)
\]

where \( E_i \) is the mass matrix of link \( i \)

\[
E_i = \begin{bmatrix}
I_i - m_i \mathbf{r}_i^2 & m_i \mathbf{r}_i \\
m_i \mathbf{r}_i & m_i I_3
\end{bmatrix}
\]

Furthermore, according to the interaction principle of force and the adjoint transformation property of Lie algebra, the external moment and force of rigid body \( i \) are:

\[
\begin{bmatrix}
T_i \\
\mathbf{f}_i
\end{bmatrix} = E_i A_i \mathbf{a} \mathbf{d}^\epsilon_i (E_i \mathbf{V}_i) = F_i - \mathbf{A}^\epsilon_i F_{i+1}
\]

Therefore, by backward recursion (for \( i = n, n-1, \ldots, 1 \)), the generalized forces and their second derivatives of each link can be obtained:

\[
F_i = \mathbf{A}^\epsilon_i (F_{i+1}) + E_i A_i \mathbf{a} \mathbf{d}^\epsilon_i (E_i \mathbf{V}_i)
\]

Then the driving torque of each joint and its derivative can be calculated as follows:

\[
\tau_i = \xi_i^T F_i
\]

### 4. Adaptive control of industrial robot

High calculation efficiency and accuracy of the robot inverse dynamic model are both required for the computed torque control. The accuracy of robot model mainly depends on the accuracy of model parameters. In order to solve the problem that the parameters of robot model are difficult to obtain accurately, the adaptive control is added to the computed torque control. The scheme of adaptive computed torque control is shown in Figure 1:

**Figure 1.** Scheme of adaptive torque computed control.

Firstly, the dynamic model of robot with joint friction torque is linearized. It is generally known that the dynamic model of the robot is a linear function of its inertia parameters when the friction torque of the joint is not considered [19]:

\[
\tau_b = Y_b p^b
\]

where \( Y_b \) is a function of joint motion state only, independent of inertia parameter \( p^b \). Consider the following friction torque model[20]:

\[
\tau_b = \mu_i |\dot{\theta}| + b_i \text{sign} (\dot{\theta}) + c_i |\dot{\theta}|^{\beta / 2} \text{sign} (\dot{\theta})
\]

This friction torque model can be linearized and it is continuous near the zero point of velocity. The friction torque equation can be written as a linear form of coefficient:

\[
\tau_b = \begin{bmatrix} \dot{\theta} , \text{sign} (\dot{\theta}), \text{sign} (\dot{\theta}) \end{bmatrix}^T \begin{bmatrix} \mu_i \\ b_i \\ c_i \end{bmatrix} = Y_b p^b
\]

Then the linear model of friction torque of all joints of robot is:

\[
\tau_f = Y_f p^f
\]

Combined with the rigid body dynamics model of the robot, the required driving torque of the joints can be obtained:
\[ \tau = \tau_b + \tau_f = Y_p \dot{p} + Y_f \dot{p} \]

(23)

In order to design the adaptive control law, the driving torque of the robot joint can be further transformed into the following form:

\[ \tau = [Y_b Y_f] \cdots = Wp \]

(24)

According to the principle of the computed torque control of robot, as shown in Figure 1, the following equation can be obtained:

\[ \dot{M}(0) \ddot{\theta} + K_\theta \dot{\theta} + C(\theta) \dot{\theta} + G(\theta) = \tau_f \]

(25)

By equating equation (21) and equation (22), we can obtain

\[ \dot{M}(0) (\dot{E} + K_\theta \dot{\theta} + K_p E) = \tau - \dot{\tau} = W(p - \dot{p}) = W\Phi \]

(26)

According to [17], if

\[ \Phi = \Gamma W^T M^4 E_\psi \]

then the control system is stable. In equation (24), \( \Gamma = \text{diag}(\gamma_1, \gamma_2, \ldots, \gamma_n) \) with \( \gamma_i > 0 \),

\[ E_\psi = \dot{E} + \Psi E \]

(28)

where \( \Psi = \text{diag}(\psi_1, \psi_2, \ldots, \psi_n) \) with \( \psi_i > 0 \). Combining the equation \( \Phi = p - \dot{p} \), we can obtain the adaptive control law:

\[ \dot{\hat{p}} = \Gamma W^T M^4 E_\psi \]

(29)

5. Simulation and verification

In order to verify the adaptive torque control, a multi-domain integrated system model of industrial robot is required. In this paper, a 6 DOF serial industrial robot system model is established. The mechanical structure dimensions of the robot and the layout of the link coordinates are shown in Figure 2.

![Figure 2](image)

**Figure 2.** The mechanical structure of 6-DOF industrial robot and the layout of link coordinates.

The multi-domain integrated system model of the industrial robot is established based on Modelica for its excellent characteristics. Modelica is a multi-domain system modeling language, which supports the modeling of component models based on equations and the modeling of complex systems based on component non-causal connection. In Modelica, the interface of the component model is called the connector, and the coupling relationship established on the component connector is called the connection. If the connection expresses a causal coupling relationship, it is called the causal connection. If the connection expresses a non-causal coupling relationship, it is called the non-causal
connection. In non-causal connection, the interface in each domain usually contains two types of variables: flow variable and potential variable, while in causal connection, the connector includes input variables and output variables. The non-causal connection between the interfaces satisfies the generalized Kirchhoff theorem, that is, the sum of flow variables is zero and the potential variables are equal.

The multi-domain integrated robot system model mainly include mechanical subsystem, control subsystem and electrical subsystem and its block diagram is shown in figure 3. Based on the multi-domain integrated system model of the robot, multi-domain integrated simulation in different working conditions can be performed, and the adaptive torque computed control can be validated. In this paper, simulation in two conditions are carried out and the simulation results are compared. One is the simulation of the computed torque control when the parameters are not accurate, the other is the simulation of the adaptive computed torque control. In general, it is difficult to obtain the parameters of friction torque accurately.

Figure 3. The block diagram of multi-domain integrated model of industrial robot system.

So in this paper, it is assumed that the dynamic inertia parameters of rigid links are accurately known, while the parameters of friction torque are not accurate. Suppose that there is a 30% deviation between the estimated friction torque parameters of the first three joints and the true value, the estimated value and the true value are shown in table 1:

Table 1. The assumption of the true value and the estimated value of the friction torque parameters.

| Joint | Name | Unit                        | True value | Estimated value |
|-------|------|-----------------------------|------------|-----------------|
| 1     | \(\mu_1\) | N.m/(rad/s)                 | 0.2        | 0.2*(1+0.3)     |
|       | \(b_1\)  | N.m                        | 0.5        | 0.5*(1+0.3)     |
|       | \(c_1\)  | N.m/(rad/s)                 | 0.3        | 0.3*(1+0.3)     |
| 2     | \(\mu_2\) | N.m/(rad/s)                 | 0.15       | 0.15*(1+0.3)    |
|       | \(b_2\)  | N.m                        | 0.4        | 0.4*(1+0.3)     |
|       | \(c_2\)  | N.m/(rad/s)                 | 0.14       | 0.14*(1+0.3)    |
| 3     | \(\mu_3\) | N.m/(rad/s)                 | 0.3        | 0.3*(1+0.3)     |
|       | \(b_3\)  | N.m                        | 0.6        | 0.6*(1+0.3)     |
|       | \(c_3\)  | N.m/(rad/s)                 | 0.1        | 0.1*(1+0.3)     |

Both the torque computed control with inaccurate friction torque parameters and the adaptive torque computed control are simulated and the simulation results are compared. The simulation results of each joint control deviation are shown in the figure 4. It can be seen from the figure 4 that the control error of the adaptive torque computed control is more smaller than the control error of the traditional torque computed control with inaccurate parameters.

In fact, the adaptive computed torque control system can learn the dynamic characteristics of the robot itself and adjust the parameters in the dynamic model until the control error reaches the minimum. Therefor, this adaptive control method has good control effect. The learning process of friction torque parameters is show in figure 5. From the figure 5, it is can be seen that the friction torque parameters of each joint gradually converge to the true value with the advance of simulation time.
6. Conclusion

The inverse robot dynamic model is complex to calculate and the parameters of the model are difficult to obtain accurately, which are the main obstacle for the development of computed torque control. In this paper, based on the theory of Lie algebra, the inverse robot dynamics model is established to simplify the calculation of inverse dynamics. In order to solve the problem that the parameters in the model are difficult to obtain accurately, the inverse robot dynamic model including the friction torque is linearized in this paper, and the self-adaptive control law is obtained. The adaptive control strategy is added into the traditional computed torque control, so as to avoid the influence of the inaccurate parameters on the control effect. The multi-domain integrated system model of industrial robot is established to validate the adaptive torque computed control. The simulation results show that the inaccurate friction torque model parameters converge to the true value finally. That is to say the adaptive computed torque control system has the function of parameter self-learning. From the simulation results, it easily can be seen than the control effect is better than that of traditional calculation torque control. The method proposed in this paper has theoretical significance for improving the control accuracy of industrial robots.

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