Kaon Polarizabilities in Chiral Perturbation Theory

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Abstract
We study the kaon polarizabilities in the framework of Chiral Perturbation Theory to order $p^4$. For the neutral kaon we find that them vanish and they have the first non-zero contribution to order $p^6$. We also emphasize the theoretical potential of an eventual measurement of the kaon polarizabilities, in particular of the neutral kaon ones.

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Electric and magnetic polarizabilities are among the fundamental properties of hadrons and provide valuable information on their internal structure. They probe the rigidity of a composite system against the formation of electric (magnetic) dipole moments when an external electric (magnetic) field is switched on. From an experimental point of view, it is possible to determine the polarizabilities of a particle by measuring its Compton scattering. The influence of the polarizabilities on the $\gamma\gamma \rightarrow \pi\pi$ cross-section is small and one cannot measure them very precisely, in general, from these analyses [4].

In this Letter we are interested in studying the polarizabilities of kaons in the framework of Chiral Perturbation Theory (CHPT) [2, 3, 4]. For recent reviews on CHPT see [5]. For the pion polarizabilities, the related process $\gamma\gamma \rightarrow K\bar{K}$ has been studied at $\mathcal{O}(p^4)$ within SU(3)$_L \times$ SU(3)$_R$ CHPT in [6] and in [7] within SU(2)$_L \times$ SU(2)$_R$, and to order $p^6$ within SU(2)$_L \times$ SU(2)$_R$ CHPT in [8] for the neutral pions and in [9] for the charged ones. The charged kaon polarizabilities have been studied within CHPT in [10] to $\mathcal{O}(p^4)$. Though $\gamma\gamma \rightarrow K\bar{K}$ processes are much beyond the applicability of CHPT because of their center of mass energy, one can expect CHPT to give a good description of kaon polarizabilities within the usual 20% because of the SU(3) kaon mass breaking as we shall after see. The experimental situation on kaon polarizabilities is very poor at the moment. One could however expect an improvement in the future either in the projected kaon factories like DAΦNE in Frascati (see [11] for a thorough review of its physics capabilities) or in the high energy beam experiments at Fermilab and CERN (see [12, 13] and references therein). We will see that in particular the neutral kaon polarizabilities can offer a nice way to test hadronic models which predict related form factors in the kaon system. This is particularly interesting since the same models can be used to predict form factors for radiative strangeness-changing processes like the rare decay $K \rightarrow \pi\gamma\gamma$.

Expanding in photon momenta near threshold the Compton amplitude for a pseudoscalar boson $P$ one can write down

$$T (\gamma(q_1)P(p_1) \rightarrow \gamma(q_2)P(p_2)) \equiv 2 \left[ e_1^2 e_2^2 \left( e^2 - 4\pi m \pi \omega_1 \omega_2 \right) - 4\pi m \bar{B} \left( \vec{q}_1 \times \vec{e}_1 \right) \left( \vec{q}_2 \times \vec{e}_2 \right) + \cdots \right]$$

The phase convention we use can be obtained from this amplitude definition. Here $m$ is the pseudo-Goldstone boson mass and $q \equiv (\omega, \vec{q})$ and $\epsilon \equiv (0, \vec{e})$ are photon momentum and polarization vector, respectively. The Compton amplitude above can be decomposed in general as follows

$$T (\gamma(q_1)P(p_1) \rightarrow \gamma(q_2)P(p_2)) \equiv -e^2 A(t, \nu) [(q_1 \cdot q_2)(\epsilon_1 \cdot \epsilon_2^*) - (q_1 \cdot \epsilon_2^*)(q_2 \cdot \epsilon_1)]$$

$$-e^2 B(t, \nu) [ (q_1 \cdot q_2)(\Delta \cdot \epsilon_1)(\Delta \cdot \epsilon_2^*) + (\Delta \cdot q_1)(\Delta \cdot q_2)(\epsilon_1 \cdot \epsilon_2^*)$$

$$- (\Delta \cdot q_2)(q_1 \cdot \epsilon_2^*)(\Delta \cdot \epsilon_1) - (\Delta \cdot q_1)(q_2 \cdot \epsilon_1)(\Delta \cdot \epsilon_2^*)]$$

for photons on-shell, where

$$s = (q_1 + p_1)^2; \ t = (q_1 - q_2)^2; \ u = (q_1 - p_2)^2; \ \nu \equiv s - u; \ \Delta \equiv p_1 + p_2.$$  (3)
For \( p_1^2 = p_2^2 \) we have \( 2\Delta \cdot q_1 = 2\Delta \cdot q_2 = s - u \). The above amplitude is manifestly gauge invariant.

The polarizabilities \( \overline{\alpha} \) and \( \overline{\beta} \) in (1) can be obtained from the amplitudes defined in (2) as follows

\[
\overline{\alpha} - \overline{\beta} = \frac{e^2}{4\pi m} \lim_{t \to 0} \left( \overline{A}(t, \nu = t) + 8m^2 \overline{B}(t, \nu = t) \right)
\]

\[
\overline{\alpha} + \overline{\beta} = \frac{e^2}{4\pi m} \lim_{t \to 0} \left( m^2 \overline{B}(t, \nu = t) \right).
\]

The barred amplitudes in (4) are the corresponding amplitudes with the Born contributions using pseudoscalar propagators to the corresponding order in CHPT subtracted. The \( \overline{\alpha} - \overline{\beta} \) combination is pure S wave while the \( \overline{\alpha} + \overline{\beta} \) combination is pure D wave. Here one observes that polarizabilities are properties at \( \nu = t \to 0 \), so the only possible parameters in the chiral expansion are the ones which explicitly break chiral symmetry, i.e. masses of the pseudo-Goldstone bosons. In known examples these corrections are typically of the order of 20 % to 30 % in the strange sector. The point is that these type of corrections find a natural framework within CHPT so that CHPT for kaon polarizabilities is well suited too.

At lowest order in CHPT, the only contribution to the amplitudes \( A(t, \nu) \) and \( B(t, \nu) \) are the Born type diagrams (see Figure 1) with the vertices from the \( \mathcal{O}(p^2) \) Lagrangian

\[
\mathcal{L}^{(2)} = \frac{F_0^2}{4} \left\{ \text{tr} \left( D_\mu UD^\mu U^\dagger \right) + \text{tr} \left( \chi U^\dagger + U\chi^\dagger \right) \right\}
\]
where $U \equiv \exp \left( \frac{i \sqrt{2} \Phi}{F_0} \right)$ is an SU(3) matrix incorporating the octet of pseudoscalar mesons

$$
\Phi(x) = \frac{\bar{\lambda}}{\sqrt{2}} \phi = \begin{pmatrix}
\pi^0 + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\
\pi^- & -\pi^0 + \frac{\eta_8}{\sqrt{6}} & K^0 \\
K^- & K^0 & -2\eta_8/\sqrt{6}
\end{pmatrix}.
$$

(6)

In the absence of the U(1)$_A$ anomaly, the SU(3) singlet $\eta_1$ becomes the ninth Goldstone boson which is incorporated in the $\Phi(x)$ field as

$$
\Phi(x) = \frac{\bar{\lambda}}{\sqrt{2}} \phi + \frac{\eta_1}{\sqrt{3}}
$$

(7)

Light-quark masses are collected in the $3 \times 3$ flavor matrix $M = \text{diag}(m_u, m_d, m_s)$ and $\chi = 2B_0M$. The constant $B_0$ is related to the light-quark vacuum expectation value

$$
\langle 0 | \bar{q} q | 0 \rangle = -F_0^2 B_0 (1 + O(m_q)).
$$

(8)

In this normalization, $F_0$ is the chiral limit value corresponding to the pion decay coupling $F_\pi \simeq 92.4$ MeV. In the presence of electromagnetism the covariant derivative $D_\mu$ is

$$
D_\mu U = \partial_\mu U - i|e|A_\mu [Q, U]
$$

(9)

Here, $A_\mu(x)$ is the photon field and the light-quark electric charges in units of the electron charge $|e|$ are collected in the $3 \times 3$ flavor matrix $Q = \text{diag}(2, -1, -1)/3$.

At next-to-leading order there are contributions from the Born-type diagrams in Figure 4 but with vertices from the order $p^4$ Lagrangian (see [4] to find the explicit form) and one-loop diagrams in Figure 5 and Figure 6 with vertices from the $O(p^2)$ Lagrangian in (5). The terms of the $O(p^4)$ Lagrangian in [4] are the needed counterterms (the so-called $L_s$ couplings) to make the one-loop diagrams with $O(p^2)$ vertices UV finite. For neutral pseudo-Goldstone bosons, the first not vanishing contribution to the Compton scattering amplitude come only from diagrams in Figure 3 and 4. In fact, using the gauge invariance structure of the Compton amplitude in (2), one can reduce the calculation of the $O(p^4)$ contributions for neutral pseudo-Goldstone bosons to just diagram (d) in Figure 3. This is because the coefficients of the non $(\epsilon_1 \cdot \epsilon_2^* )$ terms plus gauge invariance determine uniquely the amplitudes $A(t, \nu)$ and $B(t, \nu)$ and at this order only diagram (d) can generate non $(\epsilon_1 \cdot \epsilon_2^* )$ terms. Of course the result is finite. In addition there are no counterterms to this order so that the result we get for the $K^0$ Compton scattering amplitudes up to order $p^4$ in CHPT is

$$
A(t, \nu) = -\frac{1}{16\pi^2 F_0^2} \left[ 2 - \frac{4m_\pi^2}{t} \arctan^2 \left( \sqrt{\frac{t}{4m_\pi^2}} \right) \right]
$$
Figure 2: Pseudoscalar electromagnetic form factor diagrams contributing to $\gamma P \to \gamma P$ at order $p^4$. S-channel and symmetric diagrams have to be included too. Lines like in Figure 1.

Figure 3: Diagrams contributing to $\gamma P \to \gamma P$ at order $p^4$. Crossed diagrams have to be included too. Lines like in Figure 1.
\[-\frac{4m_K^2}{t} \arctan^2 \left( \sqrt{\frac{t}{4m_K^2 - t}} \right) ;
\]

\[B(t, \nu) = 0.\quad (10)\]

The complete neutral kaon polarizabilities to order \(p^4\) is

\[\overline{\alpha}_{K^0} = \overline{\beta}_{K^0} = 0.\quad (11)\]

Remember that the polarizabilities for the \(\pi^0\) at this same order obtained from the results in [6, 7] are

\[\overline{\alpha}_{\pi^0} = -\overline{\beta}_{\pi^0} = -\frac{e^2}{4\pi m_\pi} \frac{1}{96\pi^2 F_\pi^2} = -0.54 \times 10^{-4}\text{ fm}^3 \quad (12)\]

where we have resummed the higher order corrections that change \(F_0\) into \(F_\pi\).

For charged pseudo-Goldstone bosons, there are order \(p^4\) contributions from the diagrams in Figures [1], [2], and [3]. In addition there are the diagrams which give wave function and mass renormalization. The complete result up to \(O(p^4)\) for the charged kaon is the following

\[A(t, \nu) = \frac{2}{t - \nu} + \frac{2}{t + \nu} + \frac{8}{F_0^2} (L_9 + L_{10}) - \frac{1}{16\pi^2 F_0^2} \left[ \frac{3}{2} - \frac{2m_\pi^2}{t} \arctan^2 \left( \sqrt{\frac{t}{4m_\pi^2 - t}} \right) \right] - \frac{4m_K^2}{t} \arctan^2 \left( \sqrt{\frac{t}{4m_K^2 - t}} \right) ;
\]

\[B(t, \nu) = \frac{1}{t} \left[ \frac{1}{t - \nu} + \frac{1}{t + \nu} \right].\quad (13)\]

This result agrees with the one found in [10]. The corresponding complete charged kaon polarizabilities to order \(p^4\) are

\[\overline{\alpha}_{K^+} = -\overline{\beta}_{K^+} = \frac{e^2}{4\pi m_{K^+}} \frac{4}{F_{K^+}^2} (L_9 + L_{10}) = (0.64 \pm 0.10) \times 10^{-4}\text{ fm}^3 \quad (14)\]

where we have included the higher corrections that change \(F_0\) into \(F_\pi\) and used the recent result in [14].

\[L_9 + L_{10} = (1.6 \pm 0.2) \times 10^{-3}.\quad (15)\]

Let us now study the results we have obtained in [11] and [14]. Could we have gotten them from some symmetry relation? In the \(\gamma P \rightarrow \gamma P\) process, the pseudoscalar boson \(P\) has to be in one of the three SU(2) subgroups of SU(3). Then, unless the center of mass energy in some channel is enough to produce any of the rest of the octet multiplet particles in [3], they can be integrated out.
of the effective theory and make the calculation within the corresponding SU(2) CHPT. Of course in each one of these SU(2) the coupling constants depend on the ratio of the masses of the particles not integrated over the ones of the integrated particles and of $F_0^2$ over the masses of the integrated particles. It has then little sense for numerical purposes to integrate out the pion in the case of calculations involving kaons just because in nature we have obviously not access to the effective couplings of the U-spin SU(2) and the V-spin SU(2) CHPT. It can nevertheless be useful for understanding some results and/or getting new results. The effective couplings corresponding to the SU(2) isospin are known to $O(p^4)$, they are the so-called $l_i$s couplings defined in [3]. For $m_p \neq 0$ polarizabilities are in the situation described above.

Therefore, formally the calculation of pion polarizabilities can be done in the isospin SU(2) CHPT, of charged kaon within the V-spin SU(2) CHPT and of neutral kaons within the U-spin SU(2) CHPT. Of course, the couplings in each case will be different. Let us see what are the things we can obtain from this observation. Since the quarks in the u-d isospin system and in the u-s V-spin system only differ for QCD in the presence of electromagnetism by the mass of the quarks the calculations of charged kaon polarizabilities can be obtained from the ones of charged pions by exchanging pion masses by kaon masses. If we know the relation between the order $p^4$ isospin SU(2) couplings $l_i$s and the SU(3) couplings $L_i$s, we can have the result for the charged kaons too just by translating the $l_i$s couplings in the complete isospin SU(2) result to SU(3) couplings and changing pion by kaon masses. The relation between $l_i$s and $L_i$s can be found in [4, 15]. We find the result in (14) without any SU(3) calculation. Of course, some SU(3) calculation was needed in order to make the relation between the SU(2) and the SU(3) couplings, but these could be easier (for instance two-point or three-point functions calculations) and they are universal.

We cannot apply the same trick for the neutral kaon polarizabilities because this SU(2) subgroup differs from the other SU(2) subgroups also in the electric charge of the components (s-d in this case). So the result in (14) cannot be obtained from the neutral, charged pion or charged kaon polarizabilities. This system has the peculiarity in turn that we can only form electrically neutral bosons. From this, we can easily obtain the result in (11) as follows. Making the calculation of the neutral kaon polarizabilities to $O(p^4)$ in the U-spin means that there are no loop contributions since there are no photon–pseudo-Goldstone boson order $p^2$ vertices. Noticing that there are no counterterms in the U-spin CHPT calculation either leads to the result in (11). This is because we get zero at $O(p^4)$ in the complete (counterterm plus loops) SU(2) calculation which can only give zero when making the trick above explained to go to SU(3). Therefore the vanishing result for the neutral kaon polarizabilities at order $p^4$ is a result of chiral symmetry plus the fact that the $K^0$ belongs to an SU(2) subgroup where there are only neutral pseudo-Goldstone bosons.

One can see from the results in [6, 7] that, at order $p^4$, the kaon loops do
not contribute to the pion polarizabilities. This is just an accidental symmetry
and it has not to happen at higher orders. In general, there can be terms in the
pion polarizabilities that go to a constant when $m_K \to \infty$ in the kaon loops of
$SU(3)_L \times SU(3)_R$ CHPT. These terms are included in the values of the $l_i$s $SU(2)$
couplings.

The order $O(p^6)$ calculation of the charged pion polarizabilities has been done
in [9]. We could use the trick above again to calculate the $O(p^6)$ chiral log con-
tributions to the charged kaon from the charged pion calculation. Here we also
need the relation between the corresponding order $p^6$ couplings for which one
needs $SU(3)$ calculations. Notice that these relations mix the $SU(3)$ chiral loop
contributions with the $SU(3)$ counterterms. The needed $SU(3)$ calculations could
be easier however than $\gamma K^+ \to \gamma K^+$ itself and as said before universal. For
instance from two-point and three-point function calculations to order $p^6$. For
the neutral kaon polarizabilities, the order $p^6$ relative size cannot be guessed be-
cause its a complete new contribution so both logs and counterterms to order
$p^6$ have to be computed. If calculated in the U-spin CHPT, the $O(p^6)$ chiral logs
for the neutral kaons polarizabilities are again zero because there neither
photon–pseudo-Goldstone boson order $p^4$ vertices, there are however non-zero
counterterm contributions of order $p^6$ this time. To get the corresponding $SU(3)$
result we need again the relation between the corresponding order $p^6$ which how-
ever could be obtained from easier $SU(3)$ calculations as said before.

Let us finally analyze the possibilities that offer kaon polarizabilities for study-
ing the low-energy hadronic interactions between pseudo-Goldstone bosons.

For pions and the charged kaon, the combination of polarizabilities $\alpha - \beta$
is not zero already at order $p^4$ while the combination $\alpha + \beta$ starts at order $p^6$.
Remember that many experimental fits to pion polarizabilities are made with the
order $p^4$ constraint $\alpha + \beta = 0$; this has no sense for the neutral kaon. For the
neutral kaon we have obtained that both combinations are first non-zero at order
$p^6$, so they are naturally expected to be of the same order of magnitude. This
does not happen in any of the other systems.

The study of both the charged and the neutral kaon polarizabilities is very
interesting in relation with the information they can give on the explicit breaking
of chiral symmetry through kaon masses. As said before CHPT is the natural
framework to study such effects. In particular the neutral kaon polarizabilities are
proportional to $m_K$, so the proportionality factor between the neutral kaon polar-
izabilities and $m_K$ is a direct measure of such explicit chiral symmetry breaking
effects.

The fact that the neutral kaon polarizabilities start at order $p^6$ make them
also very interesting for checking different hadronic models for counterterms. In
particular in checking the way they incorporate the explicit chiral symmetry
breaking effects. In the case of the neutral kaon, notice that $SU(3)$ chiral sym-
metry together with electromagnetism forces the amplitudes $A(t, \nu)$ and $B(t, \nu)$
in (2) to go to zero when both $m_K$ and $t$ go to zero. Any hadronic model has
to satisfy this constraint. For the neutral pions this is only true in the large $N_c$ limit and $m_\pi$ and $t$ going to zero.

Calculations of the counterterms appearing in charged and neutral kaon polarizabilities can be found for instance in [16] using a Nambu–Jona-Lasinio model with no vector-like interactions, in [17] using an extended Nambu–Jona-Lasinio model with also spin-one interactions, in [18] using the so-called quark confinement model. The measurement of kaon polarizabilities and in particular the neutral kaon ones can then test the predictability for the order $p^6$ counterterms of these models and others as vector meson dominance ones Here we want to emphasize that these checks are only meaningful if a full CHPT calculation (i.e. chiral logs and counterterms) at order $p^6$ is made since we have no way of estimating their relative weight. The analysis of neutral kaon polarizabilities can also provide very useful information that could be used in predicting some rare kaon decays form factors for instance. We find then the neutral kaon polarizability very interesting theoretically and deserving further experimental effort at the planned kaon facilities and experiments.

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