Cellular automaton modelling to predict multi-phase solidification microstructures for Fe-C peritectic alloys

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Abstract. In this study, we developed a cellular automaton (CA) model to simulate the multi-phase microstructural evolution of the solidification of Fe-C binary peritectic alloys. Three phases were considered in the model: δ, γ, and liquid (L). To simulate microstructures formed by the peritectic transformation, moving interfaces at δ-γ, δ-L, and γ-L transformations were introduced for diffusion-controlled growth. The coarsening growth of γ grains after solidification of peritectic alloys was modelled using the CA method. Numerical simulations of multi-phase microstructure evolution for hyper-peritectic and hypo-peritectic alloys were performed in two and three dimensions. In these simulations, the nuclei of γ phases were formed at the δ/L interface below the peritectic temperature. After nucleation of the γ phase in hyper-peritectic alloys, the δ phase in the calculation domain disappeared before γ solidification was completed. Conversely, in hypo-peritectic alloys, the L phase disappeared before the δ-γ transformation was complete. Moreover, the coarsening of γ grains occurred for both alloys after the end of the peritectic transformation. These outcomes agree with the fact that the phase transformation of alloys involves the solidification of peritectic alloys. Thus, we confirmed that the proposed model was a valid approach to simulating multi-phase microstructures formed by the solidification of peritectic alloys.

1. Introduction

Solidification microstructure has a great influence on the quality of metal products. In Fe-C binary alloys, the δ phase solidifies at the liquidus temperature, and the δ-γ transformation, or γ solidification, occurs at the peritectic temperature. It is well known that the phase transformation from the δ phase to the γ phase causes volume shrinkage that is closely related to various defects such as the formation of uneven shell and surface cracks [1, 2]. Thus, it is important to understand the microstructure formation that occurs during the solidification of Fe-C binary alloys. It is well known that the peritectic reaction and transformation occur at the peritectic temperature. In the solidification of peritectic alloys, the nuclei of γ phase are formed on the δ/liquid (L) interface, and the γ phase grows along the δ phase boundary (peritectic reaction). When δ phases are enclosed by the γ phase due to the peritectic reaction, the γ phase grows into the sides of the δ and L phases (peritectic transformation). After the peritectic transformation completes, the coarsening of the γ grains begins. The longer the time required for the peritectic transformation, the larger the size of the γ grains just after the peritectic transformation [3].
Although it has been commonly stated that the coarsening of $\gamma$ grains occurs just after the disappearance of the $\delta$ and L phases [4], new behaviour related to $\delta/\gamma$ transformation and $\gamma$-grain coarsening has recently been reported [5-8]. Tsuchiya et al. reported that the coarsening of $\gamma$ grains does not occur immediately after peritectic transformation under the cooling conditions of continuous casting [5, 6], and Yasuda et al. reported that the massive-like transformation occurs as a new transformation at approximately the peritectic temperature [7, 8]. However, the relationship between the end of peritectic transformation and the start of $\gamma$-grain coarsening is not yet clear. Thus, it is necessary to investigate the relationship between peritectic transformation and the coarsening of $\gamma$ grains in detail.

Numerical simulations have been used as an effective tool for understanding the mechanism of microstructure formation during solidification. Cellular Automaton (CA) models are one type of numerical simulation model describing the microstructure formation of alloys. The majority of CA models for microstructure formations have been proposed or developed for dendritic growth [9-15], which is a single-phase model that simulates the growth of a single solid phase in a liquid phase. We previously developed a single-phase CA model to simulate dendritic growth [11, 13]. However, a multi-phase model to simulate the solidification microstructures of eutectic or peritectic alloys has not been sufficiently developed. Although several multi-phase CA models have been proposed for the solidification of eutectic alloys [14, 15], a model combined with the solidification of peritectic alloys and the coarsening of $\gamma$ grains is still required.

In this study, to investigate the relationship between the peritectic transformation and coarsening of $\gamma$ grains, we modified the single-phase CA model that we previously proposed, and developed a multi-phase CA model to simulate microstructure formation with peritectic reaction and transformation, which can simulate the growth of the $\delta$, $\gamma$, and L phases of an Fe-C binary alloy. Moreover, we modeled the coarsening of $\gamma$ grains after the peritectic transformation and combined this with the multi-phase CA model of peritectic alloys. The development of three-dimensional (3D) model is important to evaluate quantitatively the multi-phase transition behaviour. However, the computational cost of 3D simulations is very high. Thus, two-dimensional (2D) simulations are also very important to evaluate their behaviour. We performed 2D and 3D numerical simulations of multi-phase microstructural evolution for hyper-peritectic and hypo-peritectic alloys using the proposed model, and the validity of the model was evaluated in terms of modeling the solidification of peritectic alloys and the coarsening of $\gamma$ grains after the peritectic transformation.

2. Model description

2.1. Phase fractions
In the present model, the calculation domain is divided into a regular Cartesian grid referred to as a CA cell, and the five variables of solute concentration, temperature, grain orientation, state of cell, and phase fraction, are provided for each CA cell. The fractions of the L, $\delta$, and $\gamma$ phases are constrained according to the following condition.

$$\sum_{i=L,\delta,\gamma} f_i = 1 \quad (0 \leq f_i \leq 1) \quad (1)$$

where $f_i$ is the fraction of the $i$ phase ($i = L$, $\delta$, and $\gamma$). The state of the CA cell is defined as follows: L phase ($f_L = 1, f_\delta = f_\gamma = 0$), $\delta$ phase ($f_\delta = 1, f_L = f_\gamma = 0$), $\gamma$ phase ($f_\gamma = 1, f_L = f_\delta = 0$), $\delta/L$ interface ($0 \leq f_L$, $f_\delta \leq 1, f_\gamma = 0$), $\gamma/L$ interface ($0 \leq f_L, f_\delta \leq 1, f_\gamma = 0$), and $\gamma/\delta$ interface ($0 \leq f_\delta, f_\gamma \leq 1, f_L = 0$).

2.2. Solute concentration and temperature fields
The solute concentration field is calculated using the expression for solute diffusion and redistribution proposed by equation (2).

$$\frac{\partial C}{\partial t} = \nabla \cdot (D_i \nabla C_i) + \sum_{j \neq i} C_j (1 - k_{i,j}) \frac{\partial f_{ij}}{\partial t} \quad (i = \delta, \gamma, L) \quad (2)$$
where $C_i$ is the solute concentration of the $i$ phase, $D_i$ is the solute diffusion coefficient in the $i$ phase, and $k_{ij}$ is the partition coefficient of the $ij$ interface. The first and second terms on the right-hand side of equation (2) represent the solute diffusion and redistribution, respectively. Local equilibrium at each interface is assumed as $C^*_i = k_{ij} C_j$, where $C^*_i$ is the equilibrium concentration of the $i$ phase at the $ij$ interface. As a condition of solute diffusion between interfaces, only solute diffusion between the $\delta/L$ and $\gamma/L$ interfaces is considered, ignoring solute diffusion between other types of interface. This is to avoid an extreme complexity of the solute partition near the triple junction. The time step ($\Delta t$) is $\Delta x^2/4.5D_i$ in 2D and $\Delta x^2/8.5D_i$ in 3D. Temperature is provided for each cell, and the temperature field is calculated by $T^{\ast +} = T^* - R_i \Delta t$, where $R_i$ is the cooling rate. In this study, the heat flow equation is not solved.

2.3. Change in phase fractions

In the solidification of peritectic alloys, the $\delta/L$, $\gamma/L$, and $\gamma/\delta$ interfaces must be considered to simulate the solidification of the $\delta$ and $\gamma$ phases and the $\gamma/\delta$ transformation. The change in phase fractions of the $ij$ interface cell at every time step ($\Delta t_{ij}$) is calculated using equation (3).

$$\Delta t_{ij} = \frac{C_j - C_i}{C_j (1 - k_{ij})}$$

(3)

$C_j$ is given by

$$C_j = C_0 + \frac{1}{m_{ij}} \left( T_{ij} - T + \Gamma_{ij} \cdot \text{wmc}_{ij} \right)$$

(4)

where $C_0$ is the initial concentration, $m_{ij}$ is the slope of the $j$ phase at the $ij$ interface, $T_{ij}$ is the equilibrium temperature of the $j$ phase at the $ij$ interface for the initial composition, $T$ is the local temperature, $\Gamma_{ij}$ is the Gibbs-Thomson coefficient at the $ij$ interface, and $\text{wmc}_{ij}$ is the weighted mean curvature for 3D simulation at the $ij$ interface [10].

$$\text{wmc}_{ij} = \left(3\varepsilon_{ij} - 1\right) \left(\partial \cdot n_x + \partial \cdot n_y + \partial \cdot n_z\right) - 48\varepsilon_{ij} \left(n_x^2 \partial \cdot n_x + n_y^2 \partial \cdot n_y + n_z^2 \partial \cdot n_z\right)$$

$$+ 12\varepsilon_{ij} \left(\partial \cdot n_x + \partial \cdot n_y + \partial \cdot n_z\right) + 12\varepsilon_{ij} \left(n_x \partial \cdot Q + n_y \partial \cdot Q + n_z \partial \cdot Q\right)$$

(5)

where $Q = n_x^4 + n_y^4 + n_z^4$, $n_c = \partial \cdot n_c/|n_c|$, and $\varepsilon_{ij}$ is the degree of anisotropy of the surface energy of the $ij$ interface. In the present model, the state of the CA cell is varied according to the local rule of Moor’s neighborhood, and updated using the values of the phase fractions of the interface cells. After the change in phase fractions is calculated, the values of each phase fraction and the state of the cell are updated. For example, in the $\delta/L$ interface cell, if the fraction of the $\delta$ phase exceeds one ($f_{\delta} \geq 1$), the state of the cell is updated from the $\delta/L$ interface to the $\delta$ phase. As a result, the cells of the $L$ phase around the updated $\delta$ phase are changed to an $\delta/L$ interface. The same CA algorithm is used for updating the cell states of $\gamma/L$ and $\gamma/\delta$ interfaces. The $\delta/L$ interface cell with all phases in Moor’s neighbourhood is captured as the $\gamma/L$ interface.

2.4. Grain coarsening model

The grain coarsening model used in this study is based on a model proposed by Li et al. [16]. The growth rate of a grain boundary ($v$) is generally described by the expression $v = M \sigma \kappa$, where $M$ is the grain boundary mobility, $\sigma$ is the grain boundary energy, and $\kappa$ is the curvature of the grain boundary. In this model, $\kappa$ is determined using equation (6).

$$\kappa = \frac{A}{\Delta x} \frac{Kink - N_i}{N}$$

(6)

where $A$ is the shape factor, $\Delta x$ is the cell size, $Kink$ is the number of cells required to create a flat grain boundary, $N_i$ is the number of cells belonging to the same grain within Moor’s neighborhood, and $N$ is
the number of all cells within Moor’s neighbourhood. A commonly used value for $A$ is 1.28 in 2D simulations [17]. In 3D simulations, the value of $A$ is unknown (as discussed in Section 3.4). When the cell adjoins a cell belonging to a different grain, if $\kappa \gg 0$, the cell properties (grain orientation) may change to those of the cell belonging to the different grain. The probability ($p_g$) of this event is

$$
p_g = \sum_{(e,m) \in I} \Delta t \frac{\nu}{a_{(e,m)}}
$$

(7)

where $a_{(e,m)}$ is the distance between two neighbouring cells, and $I$ is the set of cells belonging to the different grain within Moor’s neighbourhood. When the random number (0–1) that is generated at each cell is less than $p_g$, the cell changes to the cell match the neighbouring cell that belongs to the other grain. Since $p_g \leq 1$, $\Delta t$ must be carefully chosen. The $\Delta t$ used in the calculation of solute diffusion is sufficient small for both 2D and 3D simulations, as mentioned in Section 2.2. In the grain growth theory of Hillert [18], the grain growth rate $(dr/dt)$ in 2D is calculated with equation (8), while that in 3D is calculated using equation (9), where $r$ is the grain radius, and $\langle r \rangle$ is the average grain radius.

$$\frac{2r}{M\sigma} \frac{dr}{dt} = \left( \frac{r}{\langle r \rangle} - 1 \right)
$$

(8)

$$\frac{r}{M\sigma} \frac{dr}{dt} = \left( \frac{r^2}{\langle r \rangle^2} \right) \left( \frac{r}{\langle r \rangle} - 1 \right)
$$

(9)

2.5. Material properties and calculation parameters

| Table 1. Physical and thermodynamic properties for Fe–C binary alloy |
|---------------------------|----------------|-------|----------------|
| Name of parameter | Symbol | Values | Unit | Ref. |
| Melting point of pure δ-Fe | $T_{m,\delta}$ | 1811 | K | [19] |
| Melting point of γ-Fe | $T_{m,\gamma}$ | 1801 | K | [19] |
| δ/γ transformation temperature | $T_{m,\delta/\gamma}$ | 1667 | K | [19] |
| Peritectic temperature | $T_p$ | 1768.4 | K | [19] |
| Peritectic composition | $C_p$ | 0.778 | mol\% | [19] |
| Liquidus slope of δ phase | $m_{\delta}$ | -18.28 | K mol\% | [19] |
| Liquidus slope of γ phase | $m_{\gamma}$ | -13.99 | K mol\% | [19] |
| Partition coefficient of δ/L interface | $k_{\delta/L}$ | 0.179 | - | [19] |
| Partition coefficient of γ/L interface | $k_{\gamma/L}$ | 0.334 | - | [19] |
| Partition coefficient of γ/δ interface | $k_{\gamma/\delta}$ | 1.866 | - | [19] |
| Frequency factor of diffusion in L phase | $D_{0,L}$ | $5.2\times10^{-7}$ | m² s⁻¹ | [19] |
| Activation energy of diffusion in L phase | $Q_L$ | $5.0\times10^{4}$ | J mol⁻¹ | [19] |
| Frequency factor of diffusion in δ phase | $D_{0,\delta}$ | $1.27\times10^{-6}$ | m² s⁻¹ | [19] |
| Activation energy of diffusion in δ phase | $Q_\delta$ | $8.3\times10^{4}$ | J mol⁻¹ | [19] |
| Frequency factor of diffusion in γ phase | $D_{0,\gamma}$ | $7.61\times10^{-6}$ | m² s⁻¹ | [19] |
| Activation energy of diffusion in γ phase | $Q_\gamma$ | $13.7\times10^{4}$ | J mol⁻¹ | [19] |
| Gibbs-Thomson coefficient of L/δ interface | $\Gamma_{\gamma/L}$ | $1.9\times10^{-7}$ | K m | [20] |
| Gibbs-Thomson coefficient of L/γ interface | $\Gamma_{\gamma/L}$ | $3.7\times10^{-7}$ | K m | [21] |
| Gibbs-Thomson coefficient of δ/γ interface | $\Gamma_{\delta/\gamma}$ | $5.3\times10^{-7}$ | K m | Estimated |
| Anisotropy of the surface energy | $\varepsilon_{\delta/L, \gamma/L, \gamma/\delta}$ | 0.03 | - | [10] |
| Grain boundary energy of γ grain | $\sigma$ | 0.79 | J m⁻² | [6] |

Table 1 shows the physical and thermodynamic properties used in the simulations performed in this study. The diffusion coefficient is provided in the form $D_i = D_{0,i} \exp(-Q_i/(RT))$. Although the majority of material property values are obtained from references [6, 19, 20, 21], the Gibbs-Thomson coefficient
of the $\gamma/\delta$ interface ($\Gamma_{\gamma/\delta}$) and the anisotropy parameters cannot be obtained from the data in the literature [10]. The $\Gamma_{\gamma/\delta}$ is estimated using the $\gamma/\delta$ interface energy (0.56 J m$^{-2}$ [22]) and the melting entropy of pure Fe (1.07×10$^6$ JK$^{-1}$m$^{-3}$ [20]), while the value for the anisotropy parameter that is often used in simulations of dendritic growth are adopted.

3. Results and discussion

3.1. Model validation for $\gamma$-δ and $\gamma$-L transformations

To evaluate the validity of the proposed model in terms of the $\delta$-$\gamma$ and $\gamma$-$L$ transformations, 1D simulations were performed under isothermal conditions. The calculation domain was set to 100 $\mu$m using a cell of 0.5 $\mu$m, and was divided into two equal parts as an initial setting. The $\delta$ and $L$ phases were set in the left and right halves, respectively, and the $\gamma$ phase was set in one cell between the $\delta$ and $L$ phases. The initial concentrations of the $\delta$, $\gamma$, and $L$ phases were $C_{\delta}^\gamma$, $C_{\gamma}^\delta$, and $C_{L}^\gamma$, respectively. The relationship between the change in positions of the $\gamma/\delta$ and $\gamma/L$ interfaces ($\Delta S_{\gamma/\delta}$ and $\Delta S_{\gamma/L}$) and the transformation time ($t$) was evaluated by comparison with equation (10).

$$\Delta S_{ij} = B_{ij} t^{0.5}$$

where $\Delta S_{ij}$ is the change in position of the $i/j$ interface, $B_{ij}$ is the material parameter of the $i/j$ interface, and is temperature dependent. The values of $B_{ij}$ were determined by comparing the simulation results and equation (10), as shown in figure 1(a). Figure 1(b) shows the $B_{ij}$ obtained from the simulation results at different temperatures. For comparison, the experimental results reported by Matsura et al. [23, 24] are plotted on the figure. The simulation results were in good agreement with the experimental results. From these results, we confirmed that the present model could appropriately simulate the solidification of peritectic alloys ($\gamma$-δ and $\gamma$-$L$ transformations).

![Figure 1](image_url) (a) Change in position of $\gamma/\delta$ and $\gamma/L$ interfaces at 1760 K, and (b) the relationship between temperature and the $B_{ij}$ of the $\gamma/\delta$ and $\gamma/L$ interfaces.

3.2. 2D simulations of peritectic reaction, peritectic transformation, and grain coarsening

We performed 2D simulations of microstructure formation in the hypo-peritectic and the hyper-peritectic alloys. The calculation domain was set to 250 $\mu$m × 250 $\mu$m using a cell size of 0.5 $\mu$m. The Neumann boundary condition was used for all variables. As an initial condition, the calculation domain was cooled at $R_c = 10$ K/s from an initial temperature of an undercooling of 5 K, and the forty small solids of the $\delta$ phase were randomly set in the calculation domain. The nuclei of the $\gamma$ phase were provided at the $\delta/L$ interface, below the peritectic temperature. Grain boundary mobility ($M$) was determined using the formula $M = 2.53 \times 10^3 \exp(-177 \times 10^3/(RT))$ m$^2$/J s$^{-1}$ [4]. Figures 2 and 3 show snapshots of simulated microstructures and the change in fractions of each phase in the Fe-0.7mol%C hypo-peritectic and the Fe-1.0mol%C hyper-peritectic alloys, respectively. In the hypo-peritectic alloy, the $\delta$ dendrites grew in the domain at 1793.0 K (figure 2(a)). Below the peritectic temperature, the $\gamma$ phases were nucleated at the $\delta/L$ interface and the grains of $\delta$ phase were immediately enclosed by the $\gamma$ phases (figures 2(b), (c)). Although the amount of $\delta$ phase decreased due to $\delta/\gamma$ transformation, the $L$ phase disappeared before the $\delta/\gamma$ transformation finished (figure 2(e)). Moreover, the coarsening of $\gamma$
grains occurred after peritectic transformation had ended (figure 2(f)). Conversely, in the hyper-peritectic alloy, although the microstructure formation behaviour before the peritectic reaction was similar to that of the hypo-peritectic alloy (figures 3(a)-(d)), the variations in each phase fraction differed. Although the L phase decreased due to the γ solidification, the δ phase disappeared before γ solidification finished (figure 3(e)). The coarsening of γ grains occurred after γ solidification ended (figure 3(f)). These simulation results provided reasonable description of the phase transformations of peritectic alloys. Thus, it is found that the proposed 2D model effectively simulated the multi-phase microstructures formed by the peritectic reaction, peritectic transformation, and grain coarsening.

**Figure 2.** (a)-(f) Microstructure formation of the Fe-0.7mol%C hypo-peritectic alloy at different temperatures, simulated using the proposed 2D model.

**Figure 3.** (a)-(f) Microstructure formation of the Fe-1.0mol%C hyper-peritectic alloy at different temperatures, simulated using the proposed 2D model.

**Figure 4.** (a)-(f) 2D simulated grain structures at different times, and (g) the relationship between normalized grain radius and grain growth rate obtained from the 2D simulations.

Next, to evaluate the validity of the γ-grain coarsening model in 2D, we performed 2D simulation of polycrystalline structure evolution under isothermal conditions of 1768.4 K for an Fe-2.1mol%C peritectic alloy. The calculation domain of the Neumann boundary condition was set to 1500 μm × 1500 μm using a cell size of 0.5 μm. In the initial γ-grain distribution, 40000 grains with various orientations were randomly set in the calculation domain. Solute diffusion was not considered in this simulation. Figure 4 shows the relationship between the normalized grain radius and the normalized grain growth
rate for each grain. The solid line on the figure shows the results that were calculated using equation (8). Although dispersions in the data were observed, there was good agreement between the tendencies of the simulated and analytical (obtained from equation (8)) results. Thus, we confirmed that the proposed model produced a reasonable description of the coarsening of $\gamma$ grains in 2D.

3.3. 3D simulations of peritectic reaction, peritectic transformation, and grain coarsening

We performed 3D simulations of microstructure formation for unidirectional solidification in the Fe-2.1mol%C hyper-peritectic alloy. The calculation domain was set to $625 \mu m \times 125 \mu m \times 500 \mu m$ using a cell size of 1.0 $\mu m$, and was cooled at $R_c = 5$ K/s under a temperature gradient of 1.0 K/mm. The Neumann boundary condition was used for all variables. Calculation of $\gamma$-grain coarsening after peritectic transformation was not performed in 3D because of a huge computational time requirement in 3D simulation for many grains as performed in the 2D simulation. At an undercooling of 3 K, five small solids of $\delta$ phase were initially added to the bottom of the calculation domain. When the temperature on the bottom of the calculation domain was below the peritectic temperature, the $\gamma$ phase was set up around the $\delta$ dendrites on the bottom of the domain. Figure 5 shows snapshots of the simulated microstructure. After the $\delta$ columnar dendrites grew from the bottom, the $\gamma$ phase grew and enclosed the $\delta$ phase from the bottom (figures 5(a)-(c)). The $\delta$ phase then disappeared due to the end of the $\delta/\gamma$ transformation (figure 5(d)). From this result, we confirmed that the proposed 3D model was able to simulate the peritectic reaction and transformation.

![Figure 5](imageurl)
Figure 5. Microstructures formation of the Fe-2.1mol%C hyper-peritectic alloy simulated using the proposed 3D model.

![Figure 6](imageurl)
Figure 6. (a)-(f) The 3D simulated grain structures at different times, and (g) the relationship between normalized grain radius and grain growth rate that was obtained from the 3D simulations.

Finally, to evaluate the validity of the $\gamma$-grain coarsening model in 3D, we performed a 3D simulation of polycrystalline structure evolution under isothermal conditions of 1768.4 K for the Fe-2.1mol%C peritectic alloy. The calculation domain of the Neumann boundary condition was set to $225 \mu m \times 225 \mu m \times 225 \mu m$ using a cell size of 0.5 $\mu m$. In the initial $\gamma$-grain distribution, 0.12 million grains with
various orientations were randomly set in the calculation domain. As before, solute diffusion was not considered in this simulation. Figure 6 shows the relationship between the normalized grain radius and the normalized grain growth rate for each grain. The solid line in the figure shows the result that was calculated using equation (9). Since the value of $A$ in equation (6) for the 3D case was unknown, this value was obtained by comparing the simulated results with the analytical line calculated using equation (10). From this comparison, a value of $A = 0.394$ was estimated. The simulated results were in good agreement with the result of phase-field simulations performed by Kim et al. [25]. Thus, we confirmed that the present model could reasonably simulate the coarsening of $\gamma$ grains in 3D.

4. Conclusion
In this study, we developed a multi-phase CA model to simulate the microstructural evolution of the peritectic solidification of an Fe-C binary peritectic alloy, combining a multi-phase model for solidification with a model for $\gamma$-grain coarsening after the peritectic reaction and transformation. First, 2D and 3D simulations of microstructure formations appearing with the solidification of peritectic alloys were performed using the proposed model. Reasonable 2D and 3D simulation results were obtained that describe the solidification of peritectic alloys. Next, 2D and 3D simulations of isothermal grain growth were performed, and simulation results were compared with the analytical results obtained using Hillert’s equation. Both 2D and 3D simulation results were in good agreement with those obtained with Hillert’s equation. From those results, we confirmed that the proposed model can accurately simulate both the microstructures involved in the solidification of peritectic alloys and the grain coarsening that occurs after peritectic transformation. In the near future, we plan to modify and develop our model to simulate microstructure formations such as the massive-like transformation, which has been recently reported by Yasuda’s group as a new transformation model for steel [7, 8].

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