Effect of Strain on Band Engineering in Gapped Graphene

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Abstract

We study the effect of strain on the band engineering in gapped graphene subject to external sources. By applying the Floquet theory, we determine the effective Hamiltonian of electron dressed by a linearly, circularly and an elliptically polarized dressing field in the presence of strain along armchair and zigzag directions. Our results show that the energy spectrum exhibits different symmetries and still isotropic for the strainless case, whereas it is linear as in the case of pristine graphene. It decreases slowly when strain is applied along the armchair direction but increases rapidly for the zigzag case. Moreover, it is found that the renormalized band gap changes along different strain magnitudes and does not change for the polarization phase $\theta$ compared to linear and circular polarizations where its values change oppositely.

PACS numbers: 72.80.Vp, 73.21.-b, 71.10.Pm, 03.65.Pm
Keywords: Graphene, strain, Floquet theory, energy spectrum, band gap.
1 Introduction

The physics of low energy carriers in graphene is governed by a Dirac like-Hamiltonian and carriers are massless fermions having a linear dispersion relation in momentum space [1]. Graphene has many electronic and mechanical properties [2], such as Hall effect [3, 4], Klein tunneling [5], elastic strain engineering [6–9], which would be too many to list. Graphene is considered as a gapless semiconductor and some methods have been used to create band gap. Experimentally, it has been shown one can generate a gap by depositing graphene on substrate hexagonal boron nitride to have a gap of order $\sim 100$ meV [10, 11]. Moreover, the electronic properties of graphene based nanostructures can be adjusted by distorting a deformation on the graphene sample [12–15]. Indeed, since its discovery researchers have conducted extensive research on the influence of elastic strain on mechanical and physical properties of graphene [16, 17]. It showed that graphene has an effective young’s modulus and simultaneously can reversibly support elastic strain up to 25% [18]. It is found that the mechanical strains in graphene can change Dirac points, which causes Dirac fermions to have asymmetrical effective Fermi velocities $v_x \neq v_y$ [8, 19].

On the other hand, controllable quantum systems can be realized using external fields [20, 21] or mechanical deformations [22, 23] allowing to generate novel states of matter. These can be described by effective Hamiltonian based on the Floquet theory of periodically driven quantum systems. Additionally, the interaction between electron and electromagnetic field gives new physics that changes the electronic properties of a driven system. Such coupling actually is known as electron dressed by field or simply dressed electron [24] and has been studied in different occasions. Indeed, the physical properties of dressed electrons were studied in various systems such that quantum wells [25, 26], quantum rings [27, 28] and graphene [29–32] as well as others.

Motivated by the results obtained in [33], we theoretically investigate the electron-field interaction in gapped graphene subject to the tensional strain within the minimal coupling approach. By applying the Floquet theory [34], we end up with an effective Hamiltonian as function of strain for the linearly, circularly and elliptically polarized dressing fields. The solutions of energy spectrum are separately obtained by solving Dirac equation for the three considered dressing fields. Subsequently, we numerically study the effect of strain along armchair and zigzag directions on the energy spectrum as well as the renormalized electronic band gap for different values of the irradiation intensity $I \sim E_0^2$, with $E_0$ being the electric field. Consequently, we show that the effect of strain causes some changes on the energy spectrum and band gap along the armchair direction, but it produces remarkable influence along the zigzag direction. We conclude that, the energy spectrum can be controlled by adjusting the strain amplitude and $I$.

The present paper is organized as follows. In section 2, we present a theoretical model describing a gapped graphene subject to external sources. In section 3, we explicitly determine the solutions of energy spectrum using Schrödinger equation. To do, we apply the Floquet theory of quantum system driven by an oscillating fields to obtain the effective Hamiltonian as function of strain, band gap and spin splitting. Then we calculate the energy spectrum of dressed electron by linearly, circularly and elliptically polarized electromagnetic wave in terms of the physical parameters characterizing our system. To give a better understanding, we numerically analyze and discuss our results under suitable conditions in section 4. Our conclusions are given in the final section.
2 Theoretical model

We study the effect of a tensional strain in gapped graphene illuminated by a continuous wave propagate along the $x$-axis with frequency $\omega$ as shown in Figure 1. The electromagnetic wave can be neither absorbed nor emitted by the electrons and considered as a dressing field for the states around the wave vector $k = 0$. By introducing a vector potential $A = (A_x, A_y)$ of the dressing field, the electronic properties of our system can be described by the two-band Hamiltonian

$$
\mathcal{H}(k) = \frac{|e|}{\hbar} \begin{pmatrix}
0 & \tau \hbar v_x A_x - i \hbar v_y A_y \\
\tau \hbar v_x A_x + i \hbar v_y A_y & 0
\end{pmatrix} + \begin{pmatrix}
\frac{\Delta_g}{2} & \tau \hbar v_x k_x - i \hbar v_y k_y \\
\tau \hbar v_x k_x + i \hbar v_y k_y & -\frac{\Delta_g}{2}
\end{pmatrix}
$$

where $\Delta_g$ is the band gap between the conduction and the valence bands, $k = (k_x, k_y)$ and $\tau = \pm 1$ is the valleys index corresponds to the inequivalent valleys centered at the high-symmetry points $K$ and $K'$. The effective Fermi velocities $v_x$ and $v_y$ are tuned by the tensional strain [19, 35, 36] and in our study we distinguish two cases such that the strain is along either armchair direction

$$
\hbar v_x = \frac{\sqrt{3}}{2} a (1 - \sigma S) \sqrt{4t_1^2 - t_3^2}, \quad \hbar v_y = \frac{3}{2} a (1 + S) t_3'
$$

or zigzag one

$$
\hbar v_x = \frac{\sqrt{3}}{2} a (1 + S) \sqrt{4t_1^2 - t_3^2}, \quad \hbar v_y = \frac{3}{2} a (1 - \sigma S) t_3'
$$

where the Poisson ratio is $\sigma \approx 0.165$ for graphene, $S$ is the strain and $a$ is the distance between neighboring atoms. The altered hopping integral $t_i'$ can be estimated by a empirical relation [6]

$$
t_i' = t_0 e^{-3.37 (|\delta'_i|/a - 1)}, \quad i = 1, 2, 3
$$

due to stretching or shrinking of the distance vectors between the nearest neighbor carbon atoms and $t_0$ is being the hoping energy. When armchair and zigzag direction is under tension (Figure 1), the vector displacements are changed to

$$
|\delta'_1| (A) = |\delta'_2| (A) = a \left( 1 - \frac{3}{4} \sigma S + \frac{1}{4} S \right), \quad |\delta'_3| (A) = a (1 + S)
$$

$$
|\delta'_1| (Z) = |\delta'_2| (Z) = a \left( 1 + \frac{3}{4} S - \frac{1}{4} \sigma S \right), \quad |\delta'_3| (Z) = a (1 - \sigma S)
$$

![Figure 1](image-url) — Structural deformations of graphene for tensile strain along $\varepsilon_{zz}$ and $\varepsilon_{ac}$ directions. (a): Strain along zigzag direction $\varepsilon_{zz}$. (b) Strain along armchair direction $\varepsilon_{ac}$. 

In the forthcoming analysis, we fix the introduced potential vector by considering three cases of dressing fields. For each case, we will use the Floquet approach to determine the eigenenergies and eigenspinors.

3 Electron dressing field

Our main goal here is to derive the solution of energy spectrum of an electron dressing field by incident light with linear, circular and elliptical polarizations.

3.1 Linearly polarized dressing field

We consider in the case of a linearly polarized electromagnetic wave along the \( x \)-axis, the vector potential \( \mathbf{A} = \frac{E_0}{\omega} (\cos \omega t, 0) \) with \( E_0 \) is the electric field of the electromagnetic wave and \( \omega \) is its frequency. For this, we write (1) as sum

\[ \mathcal{H}(\mathbf{k}) = \mathcal{H}_0 + \mathcal{H}_k \]  

of the Hamiltonian of electron-field interaction (in the unit system \( \hbar = e = 1 \))

\[ \mathcal{H}_0 = \begin{pmatrix} 0 & \Omega \tau \omega/2 \\ \Omega \tau \omega/2 & 0 \end{pmatrix} \cos \omega t \]  

and time-independent Hamiltonian

\[ \mathcal{H}_k = \begin{pmatrix} \frac{\Delta_y}{2} & \tau v_x k_x - iv_y k_y \\ \tau v_x k_x + iv_y k_y & -\frac{\Delta_y}{2} \end{pmatrix} \]  

where the dimensionless parameter describing the strength of electron coupling to the dressing field is given by

\[ \Omega(S) = \frac{2E_0}{\omega^2} v_x = \sqrt{3} \frac{E_0 a}{\omega^2} (1 + \sigma_\pm S) \sqrt{4t_1^2 - t_3^2} \]  

such that \( \sigma_\pm = -\sigma \) for strain along armchair and \( \sigma_\pm = 1 \) for zigzag directions. The corresponding nonstationary Schrödinger equation associated to (8), \( i \frac{\partial \psi_0}{\partial t} = \mathcal{H}_0 \psi_0 \), describes the time evolution of electron states, at \( (\mathbf{k} = 0) \), has the solutions

\[ \psi_0^\pm = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 1 \\ \pm 1 \end{array} \right) \exp \left[ \mp \frac{i\Omega(S)\tau \sin \omega t}{2} \right] \]  

which provide a complete basis for any time of the considered electron system. To seek eigenspinors of the full Hamiltonian (7), we introduce the ansatz

\[ \psi_k = \Xi_1(t) \psi_0^+ + \Xi_2(t) \psi_0^- \]  

that can be injected into the Schrödinger equation \( i \frac{\partial \psi_k}{\partial t} = \mathcal{H}(\mathbf{k}) \psi_k \) to obtain two differential equations describing the quantum dynamics of our system

\[ i \dot{\Xi}_1(t) = \tau v_x k_x \Xi_1(t) + (\Delta_y + iv_y k_y) e^{i\Omega(S)\tau \sin \omega t} \Xi_2(t) \]  

\[ i \dot{\Xi}_2(t) = -\tau v_x k_x \Xi_2(t) + (\Delta_y - iv_y k_y) e^{-i\Omega(S)\tau \sin \omega t} \Xi_1(t) \]
Based on the Floquet theory of periodically driven quantum systems field [37–39], we can rewrite the
eigenspinors (12) as

$$\Psi(r, t) = e^{-i\epsilon(k)t}\phi(r, t)$$  \hspace{1cm} (15)

where $\epsilon(k)$ is the quasi-energy (energy of dressed electron) and $\phi(r, t)$ is periodically function with
respect to $\omega t$ of period $2\pi$, i.e. $\phi(r, t) = \phi(r, t + 2\pi/\omega)$. Taking into account of such periodicity, we
can develop $\Xi_{1,2}(t)$ (12) in Fourier series

$$\Xi_{1,2}(t) = e^{-i\epsilon(k)t}\sum_{n=-\infty}^{+\infty} \Xi_{1,2}^{(n)} e^{in\omega t}.$$  \hspace{1cm} (16)

Now inserting (16) into (13-14) and applying the relation involving Bessel function

$$e^{iz\sin \theta} = \sum_{n=-\infty}^{+\infty} J_{n}(z) e^{in\theta}$$  \hspace{1cm} (17)

to obtain the equations of quantum dynamics in time-independent

$$\sum_{n'=-\infty}^{\infty} \sum_{j=1}^{2} \mathcal{H}_{ij}^{(nn')} \Xi_{j}^{(n')} = \epsilon(k) \Xi_{i}^{(n)}$$  \hspace{1cm} (18)

where the matrix elements $\mathcal{H}_{ij}^{(nn')}$ of the stationary Hamiltonian in the Floquet space take the forms

$$\mathcal{H}_{12}^{(nn')} = (\Delta_g + iv_y k_y) J_{n'-n}[\tau\Omega(S)], \quad \mathcal{H}_{21}^{(nn')} = (\Delta_g - iv_y k_y) J_{n'-n}[\tau\Omega(S)]$$  \hspace{1cm} (19)

$$\mathcal{H}_{11}^{(nn')} = (\tau v_x k_x + n\omega)\delta_{nn'}, \quad \mathcal{H}_{22}^{(nn')} = (-\tau v_x k_x + n\omega)\delta_{nn'}.$$  \hspace{1cm} (20)

The explicitly determine the solutions of energy spectrum, we use an approximation based on high
frequency $\omega$ to get the condition [33]

$$\left| \frac{\mathcal{H}_{ij}^{(nn)}}{\mathcal{H}_{ii}^{(00)} - \mathcal{H}_{jj}^{(nn)}} \right| \ll 1, \quad n \neq 0, i \neq j$$  \hspace{1cm} (21)

which allows to treat non-diagonal matrix elements (19) for $n \neq n'$. Therefore, the contribution to the
Schrödinger equation (18) under the condition (21) comes from terms with $n, n' = 0$. By neglecting
the small terms with $n, n' \neq 0$, (18) can be expressed as

$$\sum_{j=1}^{2} \mathcal{H}_{ij}^{(00)} \Xi_{j}^{(0)} = \epsilon(k) \Xi_{i}^{(0)}$$  \hspace{1cm} (22)

where $\mathcal{H}_{ij}^{(00)}$ are matrix elements of the Hamiltonian $\mathcal{H}^{(00)}$ for $n = n'$. By applying the unitary
transformation $U = (\sigma_x + \sigma_z)/\sqrt{2}$

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$  \hspace{1cm} (23)

to $\mathcal{H}^{(00)}$, i.e. $\mathcal{H}_{\text{eff}}(k) = U^{\dagger} \mathcal{H}^{(00)} U$, we end up with the effective stationary Hamiltonian of dressed
electron

$$\mathcal{H}_{\text{eff}}(k) = \begin{pmatrix} \Lambda_g/2 & \tau\alpha_1 k_x - i\beta_1 k_y \\ \tau\alpha_1 k_x + i\beta_1 k_y & -\Lambda_g/2 \end{pmatrix}.$$  \hspace{1cm} (24)
where we have introduced a band gap in terms of the zero Bessel function depending on the strain amplitude
\[ \Lambda_g = \Delta_g J_0[\Omega(S)] \]  
and two effective parameters of electron dispersion relation along the \( x, y \)-axes
\[ \alpha_1 = v_x, \quad \beta_1 = v_y J_0[\Omega(S)] \]  
as function of the effective Fermi velocities tuned by the tensional strain. The eigenenergies of (24) read as
\[ \epsilon(k) = \gamma \left[ \left( \frac{\Lambda_g}{2} \right)^2 + \alpha_1^2 k_x^2 + \beta_1^2 k_y^2 \right]^{\frac{1}{2}} \]
such that \( \gamma = \pm 1 \) is the sign function. Actually, \( \epsilon(k) \) is depending on different parameters including the strain amplitude and effective Fermi velocities. Note that, from (19-20), one can show that (21) is actually equivalent to the condition \( \Delta_g \ll \omega \).

### 3.2 Circularly polarized dressing field

Assuming the dressing field to be circularly polarized along the \( x, y \)-axes by choosing the vector potential \( A = \frac{E_0}{\omega} (\cos \xi \omega t, \frac{E_0}{\omega} \sin \xi \omega t) \) and the chirality index \( \xi = \pm 1 \) describes the clockwise/counter-clockwise circular polarizations. We consider the electron states at \( k = 0 \) and write the Hamiltonian \( H_0 \) as
\[ H_0 = \frac{\omega \Omega(S)}{2} \begin{pmatrix} 0 & \tau \cos \omega t - i \frac{v_y}{v_x} \xi \sin \omega t \\ \tau \cos \omega t + i \frac{v_y}{v_x} \xi \sin \omega t & 0 \end{pmatrix} = (H_{01} e^{i\omega t} + H^\dagger_{01} e^{-i\omega t}) \]  
where \( H_{01} \) is the operator of electron interaction with the dressing field
\[ H_{01} = \frac{\omega \Omega(S)}{4} \begin{pmatrix} 0 & \tau - \frac{v_y}{v_x} \xi \\ \tau + \frac{v_y}{v_x} \xi & 0 \end{pmatrix} \]  
giving rise to the total Hamiltonian (7)
\[ H(k) = H_k + (H_{01} e^{i\omega t} + H^\dagger_{01} e^{-i\omega t}). \]

To get the solutions of energy spectrum, we look for a unitary gauge transformation \( e^{iF(t)} \) such that the initial Hamiltonian becomes time-independent in the new gauge. For this, we require that \( F(t) \) should be an anti-Hermitian operator and periodic \( F(t) = F(t + 2\pi/\omega) \) with the same period as for \( H(k) \). Now applying \( e^{iF(t)} \) to both sides of Schrödinger equation and then we can write
\[ i \frac{\partial}{\partial t} \left( e^{iF(t)} \Psi \right) = e^{iF(t)} H(k) \Psi + i \frac{\partial e^{iF(t)}}{\partial t} \Psi. \]  
By setting \( \phi = e^{iF(t)} \Psi \) we get Schrödinger equation \( i \frac{\partial}{\partial t} \phi = H_{\text{eff}}(k) \phi \) such that the effective stationary Hamiltonian is given by
\[ H_{\text{eff}}(k) = e^{iF(t)} H(k) e^{-iF(t)} + i \frac{\partial e^{iF(t)}}{\partial t} e^{-iF(t)}. \]
To go further, let us consider at high frequencies small $F(t)$ of order $1/\omega$ and weak electron coupling to the dressing field, i.e. $\Omega(S) \ll 1$ [34]. In this situation, we can expand both $F(t)$ and $\mathcal{H}_{\text{eff}}(k)$ in power series as

$$\hat{F}(t) = \sum_{n=1}^{\infty} \frac{F^{(n)}(t)}{\omega^n}, \quad \mathcal{H}_{\text{eff}}(k) = \sum_{n=0}^{\infty} \frac{\mathcal{H}_{\text{eff}}^{(n)}(k)}{\omega^n}. \quad (33)$$

By applying the Floquet-Magnus approach [40] to renormalize the time-dependent Hamiltonian (32) and restricting to the second order of $1/\omega^2$, we find

$$\mathcal{H}_{\text{eff}}(k) = \mathcal{H}_k + \frac{\mathcal{H}_{01}, \mathcal{H}_{01}^\dagger}{\omega} + \frac{[\mathcal{H}_{01}, \mathcal{H}_k], \mathcal{H}_{01}^\dagger}{2\omega^2} + \frac{[\mathcal{H}_{01}^\dagger, \mathcal{H}_k], \mathcal{H}_{01}}{2\omega^2}. \quad (34)$$

where different quantities are given by

$$\left[\mathcal{H}_{01}, \mathcal{H}_{01}^\dagger\right] = \begin{pmatrix} \omega^2 \Omega^2(S) \frac{v_y}{v_x} \tau \xi & 0 \\ 0 & \omega^2 \Omega^2(S) \frac{v_y}{v_x} \tau \xi \end{pmatrix},$$

$$\left[\mathcal{H}_{01}, \mathcal{H}_k\right], \mathcal{H}_{01} \right] + \left[\mathcal{H}_{01}^\dagger, \mathcal{H}_k\right], \mathcal{H}_{01} \right] = \begin{pmatrix} A_- & B_+ \\ B_- & A_+ \end{pmatrix}. \quad (35)$$

and the constants $A_\pm, B_\pm$ take the forms

$$A_\pm = \pm \Delta_g \left( \frac{\omega \Omega(S)}{2} \right)^2 \left( 1 + \frac{v_y^2}{v_x^2} \right), \quad (37)$$

$$B_\pm = 2 \left( \frac{\omega \Omega(S)}{2} \right)^2 \left( -\tau v_x k_x \frac{v_y^2}{v_x^2} \pm i v_y k_y \right). \quad (38)$$

Taking into account (9) and (35-36), we can express the effective stationary Hamiltonian of the considered system by a circularly polarized field as

$$\mathcal{H}_{\text{eff}}(k) = \begin{pmatrix} \frac{\Lambda_g}{2} & \tau \alpha_2 k_x - i \beta_2 k_y \\ \tau \alpha_2 k_x + i \beta_2 k_y & -\Lambda_g/2 \end{pmatrix}. \quad (39)$$

and the band gap renormalized by a circularly polarized dressing field is

$$\Lambda_g = \Delta_g \left[ 1 - \frac{\Omega^2(S)}{4} \left( 1 + \frac{v_y^2}{v_x^2} \right) \right] - \tau \xi \Omega^2(S) \frac{v_y}{v_x} \quad (40)$$

and effective Fermi velocities

$$\alpha_2 = v_x \left[ 1 - \frac{\Omega^2(S) \frac{v_y^2}{v_x^2}}{4} \right], \quad \beta_2 = v_y \left[ 1 - \frac{\Omega^2(S)}{4} \right]. \quad (41)$$

We show that the eigenenergies of (39) take the form

$$\epsilon(k) = \gamma \left[ \left( \frac{\Lambda_g}{2} \right)^2 + \alpha_2^2 k_x^2 + \beta_2^2 k_y^2 \right]^{\frac{1}{2}}. \quad (42)$$

It is interesting to note that $\alpha_2$ and $\beta_2$ are quiet different to $\alpha_1$ and $\beta_1$ given in (26), which show that the obtained energy spectrum is rich than that in (27).
3.3 Elliptically polarized dressing field

For the case of an electromagnetic wave elliptically polarized dressing field oriented along the \( x \)-axis, we consider the vector potential \( \mathbf{A} = \frac{E_0}{\omega} (\cos \omega t, \sin \theta \sin \omega t) \) such that angle \( \theta \) defines its polarization phase. Note that such case involves the two previous potentials because simply by requiring \( \theta = 0 \) and \( \theta = \pm \pi/2 \) we recover the linear and circular polarization cases, respectively. As before, let us write the time-dependent total Hamiltonian (1)

\[
H(k) = H_k + \left( \mathcal{H}_{02} e^{i\omega t} + \mathcal{H}_{02}^* e^{-i\omega t} \right)
\]

in terms of \( H_k \) (9) and the operator of interaction between the electron and dressing field

\[
\mathcal{H}_{02} = \frac{\omega \Omega(S)}{4} \begin{pmatrix} 0 & \tau - \frac{v_y}{v_x} \sin \theta \\ \tau + \frac{v_y}{v_x} \sin \theta & 0 \end{pmatrix}.
\]

To derive the effective time-independent, we use the Floquet-Magnus approach for a periodically driven quantum system in similar way to the previous case of circular polarization. Then, by restricting ourselves to the first three terms in the infinite series, we obtain

\[
H_{\text{eff}}(k) = H_k + \frac{[H_{02}, H_k^*]}{\omega} + \frac{[H_{02}, H_k], H_{02}^* + [H_{02}, H_k], H_{02}]}{2 \omega^2}
\]

and combining all to build (45) as

\[
H_{\text{eff}}(k) = \begin{pmatrix} \Lambda_g/2 & \tau \alpha_3 k_x - i \beta_3 k_y \\ \tau \alpha_3 k_x + i \beta_3 k_y & -\Lambda_g/2 \end{pmatrix}
\]

where the band gap reads as

\[
\Lambda_g = \Delta_g \left[ 1 - \Omega^2(S) \left( 1 + \frac{v_y}{v_x} \sin^2 \theta \right) \right] - \frac{\tau \omega \Omega^2(S) v_y}{2} \frac{v_x}{v_x} \sin \theta
\]

and effective Fermi velocities are given by

\[
\alpha_3 = v_x \left[ 1 - \frac{\Omega^2(S)}{4} \left( \frac{v_y}{v_x} \sin^2 \theta \right) \right], \quad \beta_3 = v_y \left[ 1 - \frac{\Omega^2(S)}{4} \right].
\]

Then, we derive the eigenenergies of (46) as

\[
\epsilon(k) = \gamma \left[ \left( \frac{\Lambda_g}{2} \right)^2 + \alpha_3^2 k_x^2 + \beta_3^2 k_y^2 \right]^{1/2}
\]

which is involving different physical parameters and also generalizes the two former dispersion relations derived for the cases of the linearly and circularly polarized dressing fields. Consequently, we will investigate the behavior of our system based on different configurations of the involved physical parameters. Indeed, the numerical implementation of our theoretical model will be used to study the energy spectrum \( \epsilon(k) \), renormalized band gap \( |\Lambda_g/\Delta_g| \) under suitable conditions of the wave vector components \( (k_x, k_y) \), irradiation intensity \( I \sim E_0^2 \), \( E_0 \) being the electric field, strain amplitude \( S \) and some particular values of polarisation phase \( \theta \).
4 Numerical results

Figure 2 presents the energy spectrum $\epsilon(k)$ of electron dressed by the linearly polarized field versus the wave vector component $k_x$. We choose $k_y = 0, \Delta_y = 2, \omega = 10$ and three values of the irradiation intensities $I = (0, 121, 625)$ with strain amplitudes $S = 0, S = 0.5$ (armchair), $S = 0.5$ (zigzag). For the case without strain ($S = 0.0$), Figure 2(a) reproduces the results obtained in [33] where the up and down bands are symmetrical and the energy spectrum is still isotropic. In Figure 2(b) when strain is applied along the armchair direction with $S = 0.5$, we observe that it causes some changes on $\epsilon(k)$ and we still have the same behavior as seen in Figure 2(a) except that the band gap is reduced. It is interesting to note that $\epsilon(k)$ decreases by increasing the irradiation intensity $I$ and in particular for $I = 625$ we have a linearity behavior of $\epsilon(k)$ in similar way to pristine graphene. Figure 2(c) shows that the strain along zigzag direction produces remarkable influence on $\epsilon(k)$ because it takes elliptical form and increases as long as $I$ increases.

![Figure 2](image)

Figure 2 – (color online) The energy spectrum $\epsilon(k)$ of electron dressed by the linearly polarized field versus the wave vector component $k_x$ for $k_y = 0, \Delta_y = 2, \omega = 10$ with three values of the irradiation intensities $I = 0$ (blue line), $I = 121$ (green line), $I = 625$ (red line). (a): Without strain ($S = 0.0$). (b): Effect of armchair strain direction $S = 0.5$. (c): Effect of zigzag strain direction $S = 0.5$.

![Figure 3](image)

Figure 3 – (color online) The energy spectrum $\epsilon(k)$ of electron dressed by the linearly polarized field versus the wave vector component $k_y$ for $k_x = 0.01, \Delta_x = 2, \omega = 10$ with three values of the irradiation intensity $I = 0$ (blue line), $I = 121$ (green line), $I = 625$ (red line). (a): Without strain ($S = 0.0$). (b): Effect of armchair strain direction $S = 0.5$. (c): Effect of zigzag strain direction $S = 0.5$.

In Figure 3, we plot the energy spectrum of dressed electron $\epsilon(k)$ versus the wave vector component $k_y$ for $k_x = 0.01, \Delta_x = 2, \omega = 10$ with three values of the irradiation intensities $I = 0$ (blue line), $I = 121$ (green line), $I = 625$ (red line). As far as $S = 0.0$ is concerned, Figure 3(a) tells us
\( \epsilon(\mathbf{k}) \) is isotropic and decreases quickly as long as \( I \) increases. It is shown in Figure 3(b) that the armchair strain direction affects \( \epsilon(\mathbf{k}) \), which becomes anisotropic and decreases up to near zero values for \( I = 625 \). On the other hand, we observe that the zigzag strain direction produces remarkable influence on \( \epsilon(\mathbf{k}) \) as presented in Figure 3(c). More precisely, \( \epsilon(\mathbf{k}) \) is showing different behavior compared to the previous results because it is has hyperbolic form and its bad gap increases as long as the intensity \( I \) increases. Therefore, we conclude that the effects depend on the direction of applied strain.

![Figure 4](image1.png)

**Figure 4** – (color online) The energy spectrum \( \epsilon(\mathbf{k}) \) of electron dressed by the linearly polarized field versus the strain amplitude \( S \) for \( k_y = 0, k_x = 0.05, \Delta g = 2, \omega = 10 \) with three values of the irradiation intensity \( I = 0 \) (blue line), \( I = 121 \) (green line), \( I = 625 \) (red line). (a): Effect of armchair strain direction. (b): Effect of zigzag strain direction.

![Figure 5](image2.png)

**Figure 5** – (color online) Contour plot of the energy spectrum \( \epsilon(\mathbf{k}) \) of electron dressed by the linearly polarized field versus the wave vector components \( (k_x, k_y) \) for \( I = 0, \Delta g = 2, \omega = 10 \). (a-d): Without strain \( (S = 0.0) \). (b-e): Effect of armchair strain direction \( S = 0.5 \). (c-f): Effect of zigzag strain direction with \( S = 0.5 \).
Figure 4 illustrates the energy spectrum of dressed electron $\epsilon(k)$ versus the strain amplitude $S$ for $k_x = 0.01$, $\Delta_g = 2$, $\omega = 10$ and we switch three values of the irradiation intensity $I = 0$ (blue line), $I = 121$ (green line), $I = 625$ (red line). It is clearly seen that in Figure 4(a), $\epsilon(k)$ takes an anisotropy form and decreases slowly when $I$ increases for the strain applied along the armchair direction. However, we observe that there is a remarkable difference by switching the strain to the zigzag direction. Indeed according to Figure 4(b), we notice that $\epsilon(k)$ decreases in the interval $0 < S < 0.23$ but increases rapidly at the numerical value $S = 0.23$. In addition, there is a symmetry separating positive and negative behavior of $\epsilon(k)$. Then, we emphasis that the energy spectrum can be controlled by tuning the strain amplitude $S$ and irradiation intensity $I$.

In Figure 5, we show the energy spectrum $\epsilon(k)$ of dressed electron versus the wave vector components $(k_x, k_y)$ for $I = 0$, $\Delta_g = 2$, $\omega = 10$. Indeed, for a linear polarization we observe that in Figures 5(a), 5(b) for the strainless case $S = 0.0$, $\epsilon(k)$ has a circular form. When the strain follows armchair direction $S = 0.5$, $\epsilon(k)$ grows up vertically between the negative and positive values of $k_x$ and takes an elliptic form. On other hand, by applying the zigzag strain $S = 0.5$, we clearly see that $\epsilon(k)$ is linear and isotropic, also it presents a symmetry at normal incidence $k_y = 0$ as shown in Figures 5(c), 5(f). Moreover, Figures 5(a), 5(b), 5(e) show the same behaviors as those in Figures 5(d), 5(e), 5(f) except that the first Figures correspond to $\gamma = 1$ where $\epsilon(k)$ decreases inside for different values of $k_x$ and $k_y$, but it increases in the second Figures for $\gamma = -1$. The interesting results is that $\epsilon(k)$ in gapped graphene can be exhibited an inter-valley spectrum symmetry and controlled from negative to positive values by changing the sign of $\gamma$.

![Figure 6](image)

**Figure 6** — (color online) Contour plot of the energy spectrum $\epsilon(k)$ of electron dressed by the linearly polarized field versus the wave vector components $(k_x, k_y)$ for $I = 625$, $\Delta_g = 2$, $\omega = 10$, $\gamma = 1$, showing the effects of armchair strain direction in (a),(b),(c) and zigzag strain direction in (d),(e),(f).
In Figure 6 we show the energy spectrum of dressed electron $\epsilon(k)$ as function of the wave vector components $(k_x, k_y)$ for $I = 625$, $\Delta_y = 2$, $\gamma = 1$, $\omega = 10$ and different strain magnitudes. It is interesting to note that in Figure 6(a) $\epsilon(k)$ is elliptic, but it develops in a slow manner in Figure 6(b) and 6(c) under the change of values of the strain applied along the armchair direction and it has minima in the center. Now for the zigzag strain direction, we notice that in Figure 6(d) for $S = 0.22$, the energy spectrum is also elliptic as shown in Figure 6(a) but taking another form. Moreover, it is clearly seen in Figure 6(f) when $S = 0.3$, $\epsilon(k)$ is symmetric at $k_y = 0$ and linear along the different values of $k_y$ but remains quadratic along $k_x$ and closes its minimum at $\epsilon(k) = 0$. For large values of $S$ the energy spectrum outside increases rapidly for zigzag strain, which is not the case for the armchair strain. Then, one may notice that the strains along different directions play the opposite roles in the energy spectrum $\epsilon(k)$ behavior.

![Figure 6](image_url)

Figure 7 shows the energy spectrum of electron dressed by the elliptically polarized field versus the wave vector component $k_x$ for $\Delta_y = 2$, $\omega = 10$, $k_y = 0$ with different values of the irradiation intensity $I$. (a)/(b): Effect of zigzag strain direction $S = 0.2$, $I = 625$ (blue line), $\{I = 121, \tau = 0\}$ (green line), $\{I = 121, \tau = 1\}$ (red line) and polarization angle $\theta = \pi/6$, $\pi/2$. (c): Effect of zigzag strain direction $S = 0.5$, $\theta = 0$, $I = 121$ (blue line), $\{I = 625, \tau = 1\}$ (green line), $\{I = 625, \tau = -1\}$ (red line).

Figure 7 shows the energy spectrum $\epsilon(k)$ of electron dressed by the elliptically polarized field versus the wave vector $k_x$ for $\Delta_y = 2$, $\omega = 10$, $k_y = 0$. It follows from (49) that $\theta \in [-\pi/2, \pi/2]$ is the polarization phase: the polarization is linear for $\theta = 0$, circular for $\theta = \pi/2$ and elliptical for other angles. Figure 7(a) presents the effect of zigzag strain direction ($S = 0.2$) with different values of the irradiation intensity $I = 0$ (blue line), $\{I = 121, \tau = 1\}$ (green line), $\{I = 121, \tau = -1\}$ (red line) and $\theta = \pi/6$. We observe that $\epsilon(k)$ is isotropic and has parabolic form for $I = 0$ and $\{I = 121, \tau = -1\}$, while it is linear and similar to that of pristine graphene for $\{I = 121, \tau = 1\}$. We also notice that $\epsilon(k)$ increases for $\tau = 1$ and decreases when $\tau = -1$ by increasing the values of the irradiation intensity $I$. We find that for circular polarization $\theta = \pi/2$, Figure 7(b) presents the same behavior as for Figure 7(a) except that $\epsilon(k)$ is anisotropic for $\tau = \pm 1$ and starts to grow up slowly as long as $I$ increases. On the other hand, one sees that in Figure 7(c) when $S = 0.5$, $\theta = 0$ and for different irradiation intensities $I = 0$ (blue line), $\{I = 625, \tau = 1\}$ (green line), $\{I = 625, \tau = -1\}$ (red line), $\epsilon(k)$ has an elliptic form and is the same for both valley indices $\tau = \pm 1$. We notice that when the zigzag strain direction is considered, the energy spectrum can be adjusted from positive to negative values by changing the irradiation intensity $I$, polarization $\theta$, sign of $\gamma$ and valley index $\tau$.

Figure 8 illustrates the normalized band gap $|\Delta_g/\Delta_y|$ for the linearly (red line) and circularly (green and blue lines) polarized fields, which given in (25) and (40), respectively, versus the irradiation
intensity $I$ corresponds to $\tau \xi = \pm 1$ for $\Delta_g = 2, \omega = 10, \theta = \pi/6$. It is interesting to note that in Figure 8(a) for $S = 0.0$, the $|\Lambda_g/\Delta_g|$ decreases and turns to zero by dressing field, then it increases but becomes also null at $I = 447$, which is due to zero of the Bessel function $J_0[\Omega(S)]$. We also observe that two curves depend of the clockwise/counterclockwise circularly polarization field (polarization indices $\xi = \pm 1$) and different valleys of the Brillouin zone (valley indices $\tau = \pm 1$). Indeed, we find for the case of $\tau \xi = -1$ (blue line), $|\Lambda_g/\Delta_g|$ monotonously increases with irradiation intensity. However, for the case of $\tau \xi = 1$ (green line), $|\Lambda_g/\Delta_g|$ decreases to zero and then starts to grow up. We notice that in Figure 8(b) when strain is applied along the armchair direction, the very slow change concerning the curves of $\tau \xi$ as well as the band gap of the linearly polarized electromagnetic wave presents the same behavior compared to Figure 8(a) except that it is equal zero at $I = 450$. In Figure 8(c), it is clearly shown that the strain along the zigzag direction produces obvious change on $|\Lambda_g/\Delta_g|$, where we observe that by increasing the values of the irradiation intensities $I$, the renormalized gap when the polarization is linear decreases slowly and non null in the interval $0 < I < 800$, while it increases rapidly for circular polarization ($\tau \xi = \pm 1$).

![Figure 8](image)

**Figure 8** — (color online) The band gap $|\Lambda_g/\Delta_g|$ versus the irradiation intensity $I$ for $\tau \xi = \pm 1, \Delta_g = 2, \omega = 10, \theta = \pi/6$. (a): Without strain ($S = 0.0$). (b): Effect of armchair strain direction $S = 0.2$. (c): Effect of zigzag strain direction $S = 0.2$.

Figure 9 presents the band gap $|\Lambda_g/\Delta_g|$ versus the irradiation intensity $I$ and the polarization $\theta$ for $\tau = \pm 1, \Delta_g = 2, \omega = 10$, correspond to elliptically polarized dressing field. The different colors from purple to red correspond to different values of $|\Lambda_g/\Delta_g|$ from 0 to a maximum value which varies along the propagation direction induced by the zigzag and armchair strains. Indeed, we observe that a symmetry at normal incidence angle $\theta = 0$, separating the absolute values of the band gap for different valley indices $\tau = \pm 1$. Note that in Figure 9(a) for $S = 0.0$, the band gap increases rapidly by increasing the irradiation intensities. Also, there is another type of gap which contains different behavior and increases quickly in the interval $1 < |\Lambda_g/\Delta_g| < 3.5$. We show in Figure 9(b) when we introduce a armchair strain with $S = 0.5$, the band gap drops dramatically at $I > 20$, until it becomes equal zero (purple) but we observe an increase of the gap that exists between these low band gaps. Applying the strain along zigzag direction we clearly see in Figure 9(c), that $|\Lambda_g/\Delta_g|$ increases rapidly for large irradiation intensities and takes the value $|\Lambda_g/\Delta_g| = 4$, as a maximum value, which is not the case for the previous Figures. We conclude that the band gap can be changed along the armchair and zigzag directions and not changed for the polarization phases $\theta$ compared to linearly and circularly polarized fields which change the gap values oppositely like Figure 8.

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In Figure 10, we investigate the changing gap versus the irradiation intensity $I$ correspond to circularly and elliptically polarized field for $\Delta_g = 2$, $\omega = 10$, $\theta = \pi/6$ with $\tau \xi = -1$ (red line), $\tau = -1$ (green line). It is clearly shown that for $S = 0.0$, the values of $\Lambda_g - \Delta_g$ increase as long as $I$ increases. In Figure 10(b) when the strain is applied along the armchair direction we observe that the two curves are shifted and for the elliptically polarized electromagnetic wave $\Lambda_g - \Delta_g$ decreases rapidly by increasing the values of the $I$ where it becomes equal $\Lambda_g - \Delta_g = 6.5$ for the value $I = 500$. In Figure 10(c), there is significant change because $\Lambda_g - \Delta_g$ starts increasing for both polarizations when $I$ increases, but decreases quickly compared to previous Figures.

5 Conclusion

We have studied the electron-field interaction in gapped graphene subjected to the tensional strain applied along armchair and zigzag directions. By applying the Floquet theory, we have analytically determined the effective Hamiltonian in terms of strain, band gap and valley index for linearly, circularly and elliptically polarized dressing field. Solving Dirac equation, we have obtained the solutions of energy spectrum as function of the physical parameters characterizing our system.
Subsequently, we have discussed our results numerically for various choices of the physical parameters. Indeed, we have investigated the energy spectrum for two directions of strain including zigzag and armchair as function of the wave vectors and the irradiation intensity. It is observed that for the linear polarization there is a symmetry separating positive and negative behavior of $\epsilon(k)$ as in the case of the pristine graphene. Also the energy spectrum is still isotropic for the strainless case but linear and decreases slowly when strain is along the armchair direction. On the other hand, for the zigzag strain direction the energy spectrum takes different forms and increases rapidly with increasing the values of irradiation intensity. The contour plot of the energy spectrum was illustrated and we also have noticed through it that $\epsilon(k)$ decreases for $\gamma = 1$ but increases for $\gamma = -1$. As a result, we have found that the armchair strain direction causes some changes on the energy spectrum of dressed electron while the zigzag strain direction produces remarkable influence.

Interesting numerical results concerning the renormalized band gap have been reported. It was shown that the band gap decreases and turns to zero by dressing field, then it increases slowly but becomes null at certain values of the irradiation intensity. Whereas, for the circular polarization we have observed that in the case $\tau \xi = -1$, the band gap monotonously increases but decreases to zero and starts to grow up for $\tau \xi = 1$. By applying the zigzag strain direction, we have showed that as long as $I$ increases the renormalized band gap for the both polarizations decreases and equal zero. Furthermore, for the elliptically polarized dressing field the band gap drops dramatically by altering the strain magnitude and did not change for the polarization phase $\theta$.

Acknowledgment

The generous support provided by the Saudi Center for Theoretical Physics (SCTP) is highly appreciated by all authors.

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