RED SHIFT FROM GRAVITATIONAL BACK REACTION

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Abstract

Deviations from geodesic motion caused by gravitational radiation have been discussed in the last decades to describe the motion of particles or photons in strong fields around collapsed objects. On cosmological scale this effect, which in the first order is caused by the finite speed of gravitational interaction, is important also in the weak field limit. In this paper the energy loss by transfer to the gravitational potential is determined in a quasi-Newtonian approximation for the examples of a static Einstein universe and for an expanding universe with flat metric. In both cases the resulting red shift is a considerable fraction of the total red shift and requires an adjustment of the age and the matter composition in our models of the universe.

1 Introduction

Today the theory of general relativity (GRT) is accepted as the correct description of gravitation, but due to its non-linear character and the complicated mathematical formalism by now the application to practical problems is restricted to approximate solutions in most cases. Especially in the weak field limit most of the work, beginning with the ‘classical’ problems discussed by Einstein, like perihelion advance or gravitational aberration, is restricted to the ‘geodesic approximation’. That means that the trajectories of test masses or photons are determined as geodesic motions in non-Minkowskian space-time with the metric set up by the surrounding matter fields. A comprehensive description of this method can be found in every textbook on GRT (see e.g. R.M.Wald 1984).

But of course, distinction between field masses and test masses is somewhat artificial, as according to GRT every matter particle or energy quantum contributes to the metric. Thus by principle any motion of a test particle or photon changes the metric in its neighbourhood, which then leads to a back reaction on the particle itself. In the last decades with the advent of growing observational

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information on compact sources of gravitation like neutron stars and black holes
the need has increased for a better description of deviation from geodesic motion in curved space, caused by back reaction processes. Not only the loss of angular momentum by emission of gravitational radiation, observed in the rotational motion of double pulsars (Weisberg and Tailor 1984), but also the inspiralling of matter in the accretion disk of black holes have focussed increasing attention to the problem of back reaction. A review of the state of activities in this field has been given by Poisson (Poisson 2004).

Due to the non-linear form of the basic equations of GRT analytical solutions require an enormous effort of mathematical calculations, even if deviations from geodesic motion is treated as small perturbations. But common to all the papers in the field is the result that the deviation from geodesic motion in curved space the leading order of the perturbation results in an energy loss, which is emitted as gravitational radiation. It is this gravitational radiation, which attracts the interest of many researchers, as they hope to prove the existence of this radiation experimentally on earth. Of course, these people are looking for events which lead to strong radiation pulses, as they are expected from situations near the surface of black holes.

These problems may require higher order perturbation models, but the effect of energy loss by gravitational radiation is not limited to the strong field regime, but affects all motions in mass filled space. By principle every moving particle or photon will suffer such an energy loss, even if the effect is so tiny that we never can expect to observe it experimentally. But we cannot exclude that by accumulation over cosmological times the energy loss may be important. If it plays a role, this would lead to corrections of our cosmological models.

2 Energy loss in curved space

To estimate the order of magnitude of the expected effects we must not go into the full treatment by GRT. The more exact calculations mentioned above have shown that in the leading order the deviations from geodesic motion are caused by the fact that gravitational interaction is limited to the speed of light. The most important difference is, compared to Newtonian gravity, that we have to use retarded forces to solve the equation of motion. It is this retarded interaction which gives rise to the loss of energy by transfer to the gravitational potential. Thus, to calculate this effect in first order, we will study the behaviour of a test particle, moving with respect to a reference frame, given by a homogeneous mass-filled universe. We will study the effects in a quasi-Newtonian approximation, but with the constraints imposed by general relativity:

1. Gravitational interaction is limited by the speed of light.

2. Mass or energy cause an intrinsic curvature of space. A spatially homogeneous universe can thus be regarded as a three-dimensional surface of constant curvature. By restriction to closed space we avoid the infinity problems of Newtonian physics.
3. Lines of force between masses follow the geodesic lines connecting them.

This mixture of the geometrical description of GRT with the force field description on Newtonian theory appears justified, as we restrict our calculations to the weak field limit.

To avoid confusion with effects stemming from time-dependence of the metric, we first confine the discussion to the time-independent metric of a static Ein-
stein universe. To describe the motion on a surface of constant curvature, it is convenient to use 4-dimensional spherical coordinates defined by

\[ \begin{align*}
x &= r \cos \gamma \cos \theta \cos \phi, \\
y &= r \cos \gamma \cos \theta \sin \phi, \\
z &= r \cos \gamma \sin \theta, \\
w &= r \sin \gamma
\end{align*} \]  

(1)

where \( x, y, z \) and \( w \) is a set of four Cartesian coordinates. In this system we determine the force exerted on a test mass at \( P = (R, 0, 0, 0) \) by the mass contained in a volume element \( dV \) of the surface \( r = R \), the volume element being given by

\[ dV = R^3 \cos^2 \gamma \cos \theta \, d\gamma \, d\theta \, d\phi \]  

(2)

The distance between the element and \( P \) as measured along the geodesic of the surface \( r = R \) is given by

\[ s = 2R \arcsin \frac{r_P}{2R} = 2R \arcsin \sqrt{1 - \cos^2 \gamma \cos^2 \theta \cos^2 \phi} \]  

(3)

The component of gravitational force at point \( P \) in some direction, defined by the unit vector \( \vec{e} \), is given by

\[ dF = \frac{G \rho m dV}{s^2} (\vec{e} \cdot \vec{e}) \]  

(4)

where \( \rho \) is the mass density, \( G \) the gravitational constant and \( \vec{e}_s \) the unit vector in the direction of the geodesic at point \( P \). As all directions are equivalent, we can choose \( \vec{e} \) in the direction of the y-coordinate. In this case the projection onto the direction of the force component is

\[ (\vec{e} \cdot \vec{e}) = \cos \gamma \cos \theta \sin \phi \sqrt{1 - \cos^2 \gamma \cos^2 \theta \cos^2 \phi} \]  

(5)

For a universe with constant mass density we obtain the total force in the y-

direction by integrating equation (4) over all distances and directions:

\[ F_y = G \rho m R^3 \int_{-\infty}^{\infty} \int_{-\pi/2}^{\pi/2} \int_{-\pi/2}^{\pi/2} \frac{\cos^3 \gamma \cos^2 \theta \sin \phi}{s^2 \sqrt{1 - \cos^2 \gamma \cos^2 \theta \cos^2 \phi}} \, d\gamma \, d\theta \, d\phi. \]  

(6)

The limits of integration over the angle \( \phi \) are set to \( \pm \infty \), as the lines of force may extent beyond the reciprocal pole \((-R, 0, 0, 0)\).

For any mass at rest with respect to the frame of reference it is obvious that this integral is zero due to symmetry. In the Newtonian limit this holds also for
a moving mass, as the gravitational force from matter in all volume elements is assumed to act instantaneously. If, however, the velocity of interaction is limited to the speed of light, we have to use retarded coordinates. Thus to determine the force at some instant \( t \) we have to replace the distance \( s(t) \) by \( s'(t) = s(t-\tau) \), with \( \tau \) being the running time of the signal \( \tau = s'/c \). That means that the integral does not remain symmetrical. All distances in the direction of motion are enhanced and all distances in the direction opposite to the motion are reduced. As a result we find that there exists a force that tends to reduce the momentum of every mass or energy quantum moving relative to the rest frame of the universe. Due to the finite velocity of interaction gravity acts like a kind of viscosity of mass-filled space, that slows down every motion in the universe and transfers energy to the gravitational potential. It is this transfer to the gravitational potential, which is commonly labeled as gravitational radiation.

For the numerical evaluation of the force integral it is convenient to change the coordinate system. Instead of replacing the distance \( s \) and its projection onto the direction of motion in equation (6), we keep these quantities unchanged and change the integration variable instead, introducing the co-moving angular coordinate

\[
\phi' = \phi(t') = \phi(t) + \frac{d\phi}{dt} \tau = \phi(t) + \frac{vs'}{Rc}
\]

where \( v \) is the velocity of the moving mass. This can be done, as \( \phi \) is the only coordinate which is affected by the motion. Expressing the distance \( s' \) according to equation (3), we find the differential

\[
d\phi = d\phi' \left(1 - \frac{v}{c} \frac{\cos \gamma \cos \theta \sin \phi'}{\sqrt{1 - \cos^2 \gamma \cos^2 \theta \cos^2 \phi'}}\right)
\]

The limits of the integral are not changed by the transformation. So finally, omitting the primes for convenience, for the total force we obtain the expression

\[
F_y = -2G\rho m R^3 \frac{v}{c} \int_{-\infty}^{\infty} \int_{-\pi/2}^{\pi/2} \int_{-\pi/2}^{\pi/2} \times \frac{\cos^4 \gamma \cos^3 \theta \sin^2 \phi d\gamma d\theta d\phi}{(1 - \cos^2 \gamma \cos^2 \theta \cos^2 \phi) \left(\arcsin \sqrt{(1 - \cos \gamma \cos \theta \cos \phi)/2}\right)^2}
\]

The integral has been solved numerically. Its value is \( Y = 3.0695 \).

As the resulting force is always in line with the momentum vector \( \vec{p} = m\vec{v} \), we can rewrite equation (9) in vectorial form, replacing \( F \) by the time derivative of momentum

\[
\frac{d\vec{p}}{dt} = -\frac{2Y \rho GR}{c} \vec{p}.
\]

Accordingly the loss of kinetic energy is given by

\[
\frac{dE}{dt} = \frac{d}{dt} \frac{p^2}{2m} = -\frac{4Y \rho GR}{c} E.
\]
As these equations have been derived without any restriction of the particle velocity, they should be valid also for relativistic motion and thus also for photons. As in this case the velocity is fixed to the speed of light, the energy loss will show up as a change of frequency or wavelength:

\[ \lambda = \lambda_0 \exp \left( \frac{4Y \rho GR}{c} t \right), \]  

(12)

implying, as a first approximation, a red shift increasing linearly with the distance of the source. In an expanding universe this red shift would be superimposed to the red shift resulting from the change of metric. The quantity

\[ H_R = 4Y \rho GR/c \]  

(13)

must be regarded as an additional part of the observed Hubble constant $H$. To estimate the contribution of $H_R$ on $H$, we consider a universe at the critical closing density $\rho_{cr} = 3H^2/(8\pi G)$. The radius of the corresponding Einstein universe is given by $R = \sqrt{c^2/(4\pi G \rho)}$. Introducing these values into Eq.(13) we find the relation

\[ H_R = \sqrt{\frac{3}{2}} \frac{Y}{\pi} \times H = 1.197H. \]  

(14)

With other words: the red shift caused by energy loss due to gravitational radiation could account for the complete observed red shift, leaving no room for expansion at all. Exact agreement would be obtained at $\rho = 0.7\rho_{cr}$. Of course, a quantitative comparison of the effect in a static Einstein universe with that in an expanding universe in this way is not correct. But it is obvious that an analogous energy loss must be present in this case, too.

### 3 Energy loss in expanding flat space

While the Einstein universe is curved in a definite way, the curvature of an expanding universe may be much lower or even zero. But also in this case particles and photons move through mass filled space and thus lose energy and feel the "gravitational viscosity".

The energy loss rate can be easily estimated for the limiting case of an expanding universe with flat geometry. In this case space is unlimited, but gravitational interaction is limited to the region which is causally connected to the moving particle. Denoting the fraction of the Hubble constant due to expansion by $H_E$, interaction is limited to a sphere of radius $R = c/H_E$.

To determine the radiative energy loss rate, we consider the motion of a particle in the centre of a mass filled sphere of Radius $R$, moving in x-direction of a Cartesian coordinate system. The contribution of a volume element at distance $\vec{s}$ to the x-component of the gravitational force is

\[ dF = \frac{G\rho m dV}{s^2} (\vec{c_s} \cdot \vec{c_x}). \]  

(15)
Analogous to the case of the Einstein universe, to include retardation of interaction, we have to introduce retarded distances or, changing to comoving coordinates, instead of \( x' \) we have to use the comoving coordinate \( x = x' - \frac{vs'}{c} \) and \( dx = dx' \left( 1 - \frac{vx's'}{s'c} \right) \) (16).

Using spherical coordinates, integration of the force over a sphere of radius \( R \) leads to

\[
F_x = -G\rho m \frac{v}{c} \int_0^R dr \int_{-\pi/2}^{\pi/2} \cos^3 \theta d\theta \int_0^{2\pi} \cos^2 \phi d\phi.
\] (17)

From this equation we find \( H_R \) in a similar way as for the Einstein universe. Using \( R = c/H_E \) and the value for the density \( \rho = 3H_E^2/(8\pi G) \), which follows from the GRT field equations in the case of a flat metric, we get

\[
H_R = \frac{8\pi}{3} \frac{\rho G}{c} R = \frac{8\pi}{3} \frac{3H_E^2}{8\pi G} \frac{G}{c} \frac{c}{H_E} = H_E
\] (18)

That means that only half of the observed red shift can be attributed to expansion, the other half is caused by energy loss to the gravitational potential. Thus, compared to the presently favoured standard model, the age of the universe is doubled. This may explain the fact that we see fully developed galaxies with metallicities comparable to nearby ones at red shifts up to \( z=6 \). Also the number of absorption lines, the Lyman forest, in the spectra of distant quasars implies a much higher age than discussed in the ‘concordance model’ (see Liebscher, Priester, Hoell 1992). As the mean density according to the GRT field equation scales with the square of \( H_E \), the total density is reduced to 25% of the presently discussed values, making the adoption of a dark energy component unnecessary, at least from the viewpoint of red shift.

Though the results presented here may be inaccurate due to unjustified simplifications, the fact remains that, if we trust GRT, the energy loss of moving particles and energy quanta exists, and it is important not only in the range of strong gravitational fields, but also for the global processes which determine the history of the universe.

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