Neutrino reactions on $^{138}$La and $^{180}$Ta via charged and neutral currents by the Quasi-particle Random Phase Approximation (QRPA)

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Cosmological origins of the two heaviest odd-odd nuclei, $^{138}$La and $^{180}$Ta, are believed to be closely related to the neutrino-process. We investigate in detail neutrino-induced reactions on the nuclei. Charged current (CC) reactions, $^{138}$Ba($\nu_e, e^-$)$^{138}$La and $^{180}$Hf($\nu_e, e^-$)$^{180}$Ta, are calculated by the standard Quasiparticle Random Phase Approximation (QRPA) with neutron-proton pairing as well as neutron-neutron, proton-proton pairing correlations. For neutral current (NC) reactions, $^{139}$La($\nu, \nu'$)$^{139}$La$^*$ and $^{181}$Ta($\nu, \nu'$)$^{181}$Ta$^*$, we generate ground and excited states of odd-even target nuclei, $^{139}$La and $^{181}$Ta, by operating one quasi-particle to even-even nuclei, $^{138}$Ba and $^{180}$Hf, which are assumed as the BCS ground state. Numerical results for CC reactions are shown to be consistent with recent semi-empirical data deduced from the Gamow-Teller strength distributions measured in the ($^3$He, t) reaction. Results for NC reactions are estimated to be smaller by a factor about 4 ~ 5 rather than those by CC reactions. Finally, cross sections weighted by the incident neutrino flux in the core collapsing supernova are presented for further applications to the network calculations for relevant nuclear abundances.

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I. INTRODUCTION

Astrophysical origins of the two heaviest odd-odd nuclei, $^{138}$La and $^{180}$Ta, have been discussed over the last 30 years [1–4]. Destruction rates of the odd-odd nuclei by particle- or photon-induced reactions are generally larger than production rates. These two isotopes are shielded against both $\beta^-$ and $\beta^+$ decays by stable isobars. Thus nucleosynthesis calculations by the slow ($s$), rapid ($r$) and $\gamma$ processes usually underproduced solar abundances of these nuclei.

In a core collapsing supernova, incident neutrino ($\nu$) (antineutrino ($\bar{\nu}$)) energies and flux emitted from a proto-neutron star [4, 5] are presumed to be peaked from a few to tens of MeV energy region by considering the Fermi-Dirac distribution characterized by the temperature in each astrophysical site [6]. The $\nu(\bar{\nu})$ naturally interacts with the nuclei inside the dense matter and usually proceeds via two-step processes, i.e. target nuclei are excited by incident $\nu(\bar{\nu})$ and decay to lower energy states with the emission of some particles [6]. The excitation occurs through various transitions, i.e. super allowed Fermi ($J^\pi = 0^+$), allowed Gamow-Teller (GT) ($J^\pi = 1^+$), spin dipole ($J^\pi = 0^-, 1^-, 2^-$) and other higher multipole transitions [7, 8]. Consequently, the $\nu$-process, neutrino-induced reactions on related nuclei in core collapse supernovae [4, 5, 9–11], are treated as important input data for the network calculations to explain astrophysical origins and abundances of relevant nuclei [7].

Before presenting our results, we briefly summarize recent theoretical and experimental status about the abundances of the two heaviest odd-odd nuclei by following Refs. [9, 12, 13]. Since $^{138}$La lies on the neutron-deficient region, it is bypassed by successive neutron captures, $s$- and $r$-processes [13]. In the 15 solar mass supernova (SN) model [9], the $\gamma$-process by photo-disintegration and the $\nu$-process by neutral current (NC), $^{139}$La($\nu, \nu'$)$^{139}$La*, followed by a neutron emission, are shown to account for 32% of the $^{138}$La abundance [14]. Remainder comes from the charged current (CC) reaction, $^{138}$Ba($\nu_e, e^-$)$^{138}$La. By adding the CC reaction to the SN model at $T_\nu = 4$ MeV, one can reproduce the abundance within about 10% uncertainty. This reproduction could be a reasonable result if we consider the ambiguities inherent in the SN model.

For $^{180}$Ta, the $\gamma$-process and the $\nu$-process via NC, $^{181}$Ta($\nu, \nu'$)$^{181}$Ta*, already overproduce the abundance about 15% [14]. Addition of $^{180}$Hf ($\nu_e, e^-$)$^{180}$Ta reaction at $T_\nu = 4$
MeV overestimates the abundance about 3 times \cite{9}. The overestimation is thought to be originated from the unique feature that $^{180}$Ta has a meta-stable isomer $^{9-}$ state whose half-life is larger than $10^{15}$ yr, while the ground $^{1+}$ state is beta unstable with a half life of only 8.15 hr \cite{13}. Linkage transitions between the states could play an important role in reducing the overproduction \cite{15}.

The SN model simulation for the abundances exploited the $\nu$-induced reaction data calculated by the Random Phase Approximation (RPA) model \cite{9}. To reduce the ambiguities persisting on nuclear models, semi-empirical deduction of relevant CC reactions is performed recently by using the GT strength obtained from $^{138}$Ba ($^{3}$He, $t$)$^{138}$La and $^{180}$Hf ($^{3}$He, $t$)$^{180}$Ta reaction data \cite{12}. The $^{138}$La abundance by the data turns out to be about 15 \% higher than that by the theoretical data in the 15 solar mass SN model at $T_\nu = 4$ MeV \cite{9}. Consequently, 25 \% of the population is overproduced by the SN model. The overproduction of $^{180}$Ta becomes larger about 10 \% than the SN model by the theoretical data \cite{9}, i.e. the 3 times overproduction of $^{180}$Ta is still remained to be solved.

However, for more precise estimation of nuclear abundances, one needs more systematic calculations of neutrino reactions by pinning down the ambiguities on nuclear models. For instance, the uncertainty between theoretical and semi-empirical results for CC reactions is within 10 $\sim$ 20 \% at $T_\nu = 4$ MeV. But the discrepancy becomes about 2 times at $T_\nu = 8$ MeV, even if we take it for granted the recipe deducing the CC reaction data from the experimental GT strength \cite{12}. Roles of NC reactions also deserve to be detailed because they do not have any other theoretical and empirical results to be compared with. Moreover the NC reaction is subject to the proper description of ground states of odd-even nuclei.

Here we report more advanced results based on the QRPA calculation, which described very well the available neutrino reaction data on light nuclei \cite{16,17}. Our results are shown to reproduce the recent GT strength data on $^{138}$La and $^{180}$Ta, and their semi-empirical $\nu$-induced reaction data \cite{12}. Results for NC reactions are compared with the calculations used at the SN model \cite{9,12}.

In Sec. II, we summarize our theoretical models for the neutrino reactions on even-even and odd-even nuclei. Numerical results and detailed discussion are presented at Sec. III. Finally summary and conclusion are given at Sec. IV.
II. THEORETICAL FRAMEWORK

Since our QRPA formalism for the the $\nu(\bar{\nu})$-nucleus ($\nu(\bar{\nu}) - A$) reaction is detailed at our previous papers [16, 17], here we summarize two important characteristics compared to other QRPA approaches. First, we include neutron-proton (np) pairing as well as neutron-neutron (nn) and proton-proton (pp) pairing correlations. Consequently, both reactions via CC and NC are described within a framework. In medium or medium-heavy nuclei, the np pairing is usually expected to contribute to some extent for relevant transitions because of small energy gaps between proton and neutron energy spaces [18].

Second, the Brueckner G matrix is employed for two-body interactions inside nuclei by solving the Bethe-Salpeter equation based on the Bonn CD potential for nucleon-nucleon interactions in free space. It may enable us to reduce some ambiguities from nucleon-nucleon interactions inside nuclei. Results by our QRPA have successfully described relevant $\nu-$induced reaction data for $^{12}$C, $^{56}$Fe and $^{56}$Ni [16, 17] as well as $\beta$, $2\nu\beta\beta$ and $0\nu2\beta$ decays [18]. In specific, the double beta decay is well known to be sensitive on the nuclear structure and has more data than the $\nu$-induced reaction data. Therefore, it could be a useful test of nuclear models adopted for the $\nu$-process.

In our QRPA, the ground state of a target nucleus, which is assumed as an even-even nucleus, is described by the BCS vacuum for the quasi-particle which comprises nn, pp and np pairing correlations. Excited states, $|m; J^+M\rangle$, in a compound nucleus are generated by operating the following one phonon operator to the initial BCS state

$$Q_{JM}^{+m} = \sum_{k\mu l\nu} [X_{(k\mu l\nu, J)}] C^+(k\mu l\nu J M) - Y_{(k\mu l\nu, J)} C(k\mu l\nu J M)] ,$$

where pair creation and annihilation operators, $C^+$ and $\tilde{C}$, are defined as

$$C^+(k\mu l\nu J M) = \sum_{m_k m_l} C_{j_{kmk,j_{lm}}}^{JM} a_{++}^{+m} a_{++}^{+m} , \quad \tilde{C}(k\mu l\nu J M) = (-)^{J-M} C(k\mu l\nu J M)$$

with a quasi-particle creation operator $a_{++}^{+m}$, and Clebsh-Gordan coefficient $C_{j_{kmk,j_{lm}}}^{JM}$. Here Roman letters indicate single particle states, and Greek letters with a prime mean quasi-particle types 1 or 2.

If the neutron-proton pairing is neglected, quasi-particles become quasi-proton and quasi-neutron, and the phonon operator is easily decoupled to two different phonon operators. One is for the charge changing reaction such as the nuclear $\beta$ decay and CC neutrino reactions. The other is for the charge conserving reaction such as electro-magnetic and
NC neutrino reactions. The amplitudes $X_{\alpha'\beta'}$ and $Y_{\alpha'\beta'}$, which stand for forward and backward going amplitudes from ground states to excited states, are obtained from the QRPA equation, whose detailed derivation was shown at Refs. [16, 18].

Under the second quantization, matrix elements of any transition operator $\hat{O}$ between a ground state and an excited state $|\omega; JM>$ can be factored as follows

$$<\text{QRPA}||\hat{O}_\lambda||\omega; JM> = [\lambda]^{-1} \sum_{ab} \langle a||\hat{O}_\lambda||b > <\text{QRPA}||c_a^+ c_b||\omega; JM>,$$  \hspace{1cm} (3)

where $c_a^+$ is the creation operator of a real particle at state $a$. The first factor $< a||\hat{O}_\lambda||b >$ can be calculated for a given single particle basis independently of the nuclear model [19, 20]. By using the phonon operator $Q^{+m}_{\lambda}$ in Eq.(1), we obtain the following expression for neutrino reactions via CC

$$<\text{QRPA}||\hat{O}_\lambda||\omega; JM> = \sum_{apbn} [N_{apb} < a\alpha'||\hat{O}_\lambda||b\beta'> [u_{pa\alpha'} v_{nb\beta'} X_{\alpha'\beta'} + v_{pa\alpha'} u_{nb\beta'} Y_{\alpha'\beta'}],$$  \hspace{1cm} (4)

where $N_{\alpha'\beta'}(J) = \sqrt{1 - \delta_{ab} \delta_{\alpha'\beta'} (-1)^{J+T}/(1 + \delta_{ab} \delta_{\alpha'\beta'})}$. This form is also easily reduced to the result by the pnQRPA which does not include the np pairing correlations [8].

$$<\text{QRPA}||\hat{O}_\lambda||\omega; JM> = \sum_{apbn} [N_{apb} < a\alpha||\hat{O}_\lambda||b\beta> [u_{pa\alpha} v_{nb\beta} X_{apb} + v_{pa\alpha} u_{nb\beta} Y_{apb}].$$  \hspace{1cm} (5)

Since NC reactions for $^{139}$La and $^{181}$Ta occur at odd-even nuclei, we need to properly describe the ground state of odd-even nuclei. The standard QRPA treats the ground state of the even-even nuclei as the BCS vacuum, so that it is not easily applicable to the reaction on these odd-even nuclei.

Here we present briefly our formalism based on the quasi-particle shell model (QSM) [21] to deal with such NC reactions. First, we generate low energy spectra of odd-even nuclei by operating one quasi-particle to the even-even nuclei constructed by the BCS theory, i.e. $|\Psi_i> = a_{i\mu}^+|BCS>$ and $|\Psi_f> = a_{f\nu'}^+|BCS>$. Then weak transitions by NC are calculated as

$$\sum_{i\mu' f\nu'} < J_f||\hat{O}_\lambda||J_i >$$  \hspace{1cm} (6)

$$= \sum_{\mu' f\nu'} [\lambda]^{-1} \sum_{ab} < a||\hat{O}_\lambda||b > < J_f||[c_{ap}^+ \bar{c}_{bp}]||J_i >$$

$$= \sum_{\mu' f\nu'} < fp||\hat{O}_\lambda||ip > u_{fp\nu'} v_{ip\mu'} + (-)^{j_a + j_b + \lambda} < ip||\hat{O}_\lambda||fp > v_{ip\mu'} u_{fp\nu'} [p \rightarrow n],$$
where we used

\[< J_f | [c_{ap}^+ \tilde{c}_{bp}]_\lambda | J_i > = < BCS | a_{f\nu'} [c_{ap}^+ \tilde{c}_{bp}]_\lambda a_{\mu'}^- | BCS > \]

\[= < J_f | [c_{ap}^+ \tilde{c}_{bp}]_\lambda | J_i > (-)^{J_i+M_i} \hat{j}_z^{-1} \delta_{\lambda,\lambda'} \delta_{M_i,\mu'} \]

\[= [u_{ap\alpha'} u_{bp\beta'} \delta_{f\nu',a\alpha'} \delta_{b\beta',\mu'} + (-)^{j_a+j_b+\lambda} u_{ap\alpha'} u_{pp\beta'} \delta_{f\nu',a\alpha'} \delta_{b\beta',\mu'}] .\]

The weak current operator is comprised by longitudinal, Coulomb, electric and magnetic operators, \( \hat{O}_\lambda \), detailed at Ref. [16]. Finally, with the initial and final nuclear states, cross sections for \( \nu(\bar{\nu}) - A \) reactions through the weak transition operator are directly calculated by using the formulas at Ref. [20]. For CC reactions we multiplied Cabbibo angle \( \cos^2 \theta_c \) and took account of the Coulomb distortion of outgoing leptons [7, 8].

Our QRPA includes not only proton-proton and neutron-neutron pairing but also neutron-proton (np) pairing correlations. But the contribution by the np pairing is shown to be only within 1 ∼ 2 % for the weak interaction on \(^{12}\)C, such as \( \beta^\pm \) decay and the \( \nu-^{12}\)C reaction [16, 17]. Such a small effect is easily understood because the energy gap between neutron and proton energy spaces in such a light nucleus is too large to be effective. But in medium-heavy nuclei, such as \(^{56}\)Fe and \(^{56}\)Ni, the np pairing effect accounts for 20 ∼ 30 % of total cross sections [16]. Therefore, in heavy nuclei considered in this work, the np pairing could be one of important ingredients to be considered.

Since \(^{180}\)Ta is a well known deformed nucleus, one needs to explicitly consider the deformation with the Nilsson deformed basis. But, since our QRPA is based on the spherical symmetry, we take a phenomenological approach for the deformation in \(^{180}\)Ta [18].

The np pairing has two isospin contributions, \( T = 1 \) and \( T = 0 \), which correspond to \( J = 0 \) and \( J = 1 \) pairings, respectively. Since the \( J = 0 \) (\( T = 1 \)) pairing takes the coupling of a state and its time reversed state, radial shape is almost spherical, so that the \( J = 0 \) (\( T = 1 \)) np pairing can be included even in the spherical symmetric model. But, most calculations by the RPA, QRPA and Shell models considered only \( T = 1 \) pairing by neutron-neutron (nn) and proton-proton (pp) pairings correlations. Since our framework is based on the Hartree Fock Bogoliubob (HFB) theory, all possible \( T = 1 \) pairings by np, nn and pp pairing correlations can be taken into account properly.

The \( J = 1 \) (\( T = 0 \)) np pairing, which is partly associated with the tensor force, leads to the non-spherical property, \textit{i.e.} the deformation. Therefore, the \( J = 1 \) (\( T = 0 \)) np coupling
cannot be included in the spherical symmetric model, in principle. However, if we use a renormalized strength constant for the np pairing, $g_{np}$, as a parameter to be fitted for the empirical pairing gaps $\delta_{np}^{emp}$, the $J = 1$ ($T = 0$) pairing can be incorporated implicitly even in the spherical symmetric model because the fitted $g_{np}$ may include effectively the deformation in the nucleus.

Empirical np pairing gap $\delta_{np}^{emp}$ is easily extracted from the mass excess data. Theoretical pairing gap $\delta_{np}^{th}$ is calculated as the difference of total energies with and without np pairing correlations \[ \delta_{np}^{th} = -(H_0' + E_1' + E_2' - (H_0 + E_1 + E_2)), \] (8)

where $H_0'(H_0)$ is the Hartree-Fock energy of a ground state with (without) np pairing and $E_1' + E_2'(E_1 + E_2)$ is a sum of the lowest two quasi-particles energies with (without) np pairing correlations. More detailed discussion is given at Ref. [18].

But the renormalization constant $g_{np}$ may deviate largely from $g_{np} = 1.0$, while the deformed effects are reasonably included by this approach. In the model including the deformation explicitly, for example, the deformed QRPA model [26], the value may be only scattered slightly from the $g_{np} = 1.0$.

**III. RESULTS**

In Fig.1, we show results for NC reactions on odd-even nuclei, $^{139}$La($\nu_e, \nu_e'$)$^{139}$La* and $^{181}$Ta($\nu_e, \nu_e'$)$^{181}$Ta*. Since both nuclei have $7/2^+$ ground states and have $3/2 \sim 9/2$ states for $^{139}$La and $7/2 \sim 15/2$ states for $^{181}$Ta as excited states, higher multipole transitions between ground and excited states are possible. But the GT ($1^+$) transition dominates the cross sections for $^{139}$La. For $^{181}$Ta, $2^-$ transition emerges to be dominant with the GT ($1^+$) transition.

In the case of even-even nuclei, such as $^{12}$C and $^{56}$Ni, NC reactions are dominated by the GT transition [16, 17]. In specific, below 40 MeV region, neutrino cross sections are fully ascribed to the GT transition. But with the higher energy, other contributions such as isospin analogue state (IAS) and spin dipole resonance (SDR) transitions account for $30 \sim 40\%$ of total cross sections, as shown at figures 2, 4 and 5 in Ref. [16]. Therefore, results for $^{139}$La show a tendency similar to those of even-even nuclei. But, for $^{181}$Ta, one
could see a significant contribution by the $2^{-}$ transition, which becomes larger than the contribution by the GT transition, with the higher incident energy.

Ground and excited states generated by $|\Psi_f> = a^+_f \nu |BCS : A(e-e) >$ reproduce experimental spectra of the lower excited states. For both nuclei, $^{40}$Ca is used as a core. One more point to be noticed is that we did not find any discernible differences between $\nu_e$ and $\bar{\nu}_e$ reactions. It means that main contributions for both reactions stem from the Coulomb and longitudinal parts because the difference originates from the magnetic and electric interference term [20].

Fig.2 shows results for CC reactions on even-even nuclei, $^{138}$Ba($\nu_e, e^-)^{138}$La* and $^{180}$Hf($\nu_e, e^-)^{180}$Ta*, respectively. Main contribution for CC reactions on both nuclei is the GT transition below 40 MeV region, and remained contributions, $20 \sim 30 \%$ of total cross sections, are ascribed to other transitions, such as IAS and SDR $(1^-, 0^\pm, 3^\pm$ and $2^+)$. With the increase of incident energy, those contributions are increased to $60 \sim 70 \%$, which are almost 2 times of those in NC reactions. These roles of the GT and other transitions are also typical of CC neutrino reactions on even-even nuclei, for example, $^{12}$C and $^{56}$Fe as shown in figure 1 and 6 at Ref. [16].

The Fermi function for the Coulomb distortion is used on the whole energy region in Fig.2. Strength constants of the pairing correlations, $g_{nn} = 1.1667(1.441)$, $g_{pp} = 0.950(0.899)$ and $g_{np} = 2.3435(2.9915)$, are adjusted to the empirical pairing gaps $\Delta_{nn} = 0.883(0.712)$ MeV, $\Delta_{pp} = 1.087(1.065)$ MeV and $\delta_{np} = 0.262(0.192)$ MeV for $^{138}$Ba ($^{180}$Hf), respectively [18]. These strength parameters should be understood as renormalized constants introduced to consider finite Hilbert particle model spaces used here. Therefore, they deviated a little bit from 1.0. But, the $g_{np}$ value for $^{180}$Hf is quite larger than the $g_{np}$ for $^{138}$Ba because of the deformation as explained above.

In experimental sides, we do not have any data for neutrino reactions on these nuclei. But a recent experiment [12] for the GT transition by the $(^3$He, t) reaction could help us to constrain theoretical estimations of neutrino reactions via CC. Moreover, Ref. [12] deduced neutrino reaction data from the measured GT strength distribution by using the recipe on Ref. [27] and compared to the theoretical calculations by the RPA.

In the left panels of Fig.3, we present our results for the GT(−) strength distribution for $^{138}$La and $^{180}$Ta, which are calculated as

$$B(GT_{\pm}) = \frac{1}{2J_i + 1} |< f||\sum_k \sigma_k \tau_k||i >|^2 .$$

(9)
Table I: CC cross sections in a unit of \(10^{-42} \text{cm}^2\) for \(^{138}\text{Ba}(\nu_e, e^-)^{138}\text{La}^*\) and \(^{180}\text{Hf}(\nu_e, e^-)^{180}\text{Ta}^*\) reactions. They are averaged by the neutrino flux in a core collapsing SN for a given temperature. Experimental data and RPA results are cited from Ref. [12].

| T(MeV) | \(^{138}\text{La}\) | \(^{180}\text{Ta}\) |
|--------|-----------------|-----------------|
|        | Exp. | RPA | Ours | Exp. | RPA | Ours |
| 4      | 74   | 61  | 68   | 151  | 115  | 76   |
| 6      | 226  | 156 | 254  | 399  | 272  | 316  |
| 8      | 435  | 281 | 554  | 752  | 485  | 672  |

Various peak positions in lower energy states are confirmed to be well suited to the experimental data [12] performed below 10 MeV region. Contributions above nucleon thresholds, in particular, around 20 MeV region, are found to play significant roles in relevant neutrino reactions.

Running sums of the GT(–) strength distribution up to 10 MeV are shown in the right panels. Our results for the running sum up to 8 MeV are 5.5 and 3.8 for \(^{138}\text{La}\) and \(^{180}\text{Ta}\), if we take the universal quenching factor \(f_q = 0.74\) for the axial coupling constant \(g_A\) into account. They reproduce well the experimental data, 5.8 ± 1.6 and 4.4 ± 0.9, respectively, reported at Ref. [12]. For other multipole transitions, we did not use the quenching factor by following the discussions at Ref. [10]. More data on the higher energy region for the GT and other spin-isospin excitations by the \(^{3}\text{He},t\) or \((p,n)\) reactions could give more reliable information on the nuclear structure for the relevant weak transitions.

In table 1, our full calculations for CC cross sections weighted by the assumed neutrino spectra are tabulated with the semi-empirical data deduced from the \(^{3}\text{He},t\) reaction and the RPA calculations. To compare with the experimental data, we exploited the Fermi function below 40 MeV and the effective momentum approach (EMA) above 40 MeV for the Coulomb correction [8, 17]. Results for \(^{138}\text{La}\) are more or less consistent with the semi-experimental data, if we consider the inherent error bars in the data, which arise from the ambiguity on the GT strength obtained from the experiment and the uncertainty in the theoretical deduction of the corresponding cross sections from the data.

Results for \(^{180}\text{Ta}\) seem to underestimate the empirical data about 10 ~ 20 % for T = 6 ~ 8 MeV. But it leads to about 2 times difference at T = 4 MeV. Cross sections
at low temperature need to be further studied. Since our calculations are carried out in the spherical basis, the $T = 0 \ (J = 1)$ np pairing is included implicitly by increasing the strength parameter $g_{np}$ at the $T = 1 \ (J = 0)$ np pairing contribution. But, as well known, single particle energy states may be changed by the deformation, which could affect relevant transitions at low temperature. Nevertheless, our results are more advanced to the semi-empirical data compared with previous theoretical calculations [12].

For the supernovae application, in Fig. 4, we show the cross sections averaged by the presumed neutrino flux in a core collapsing SN. The heavier nuclei we go to, the larger cross sections are obtained. For the light and medium-heavy nuclei, magnitudes of CC reactions are about 5 times larger than those by NC. But, on the heavier nuclei, $^{138}$La and $^{180}$Ta, CC reactions are larger about 10 times than those by NC reactions. Applications to the $\nu$-process for the relevant nuclei are in progress.

IV. SUMMARIES AND CONCLUSION

We calculated neutrino reactions via neutral and charged currents related to the two heaviest odd-odd nuclei, $^{138}$La and $^{180}$Ta, by including multipole transitions up to $J^\pi = 4^\pm$ with explicit momentum dependence. Our QRPA includes neutron-proton (np) pairing as well as neutron-neutron and proton-proton pairing correlations. Since energy gaps between proton and neutron energy spaces in heavy nuclei are adjacent to each other, the np pairing may affect significantly the nuclear weak interaction. We included explicitly the $T = 1 \ (J = 0)$ np pairing. The $T = 0 \ (J = 1)$ contribution in the np pairing, which is believed to cause the deformation of $^{180}$Ta, is included implicitly by increasing the strength parameter $g_{np}$ at the $T = 1 \ (J = 0)$ pairing matrix element to reproduce the empirical np pairing gap.

To describe the NC reactions on odd-even nuclei, $^{139}$La and $^{181}$Ta, we exploited the quasi-particle shell model (QSM). Their results are consistent with the trend of NC reactions on even-even nuclei, which are fully dominated by the GT transition below 40 MeV with the increase of other transitions above 40 MeV region. But, for $^{181}$Ta, the GT dominance is relatively weakened compared to results for other even-even nuclei because of the deformation peculiar to the nucleus.

In the CC reactions on even-even nuclei, such as $^{12}$C, $^{56}$Fe and $^{56}$Ni, about 60 %
of cross sections is ascribed to the GT transition in the energy region below 40 MeV. Contributions by the IAS and SDR transitions become much larger than those in NC reactions. Namely, the GT transition in CC reactions is not so dominant as that of NC reactions. Our results for CC reactions, $^{138}$Ba($\nu_e, e^-$)$^{138}$La and $^{180}$Hf($\nu_e, e^-$)$^{180}$Ta, also show such a tendency typical of the CC reactions on even-even nuclei.

Recent experimental data carried out at RCNP [12], the Gamow-Teller (GT) strength distributions and their total strength, are well reproduced with the universal quenching factor and a proper choice of the Coulomb correction. Neutrino cross sections averaged by the neutrino flux emitted from core collapsing supernovae are also compared with the semi-empirical results deduced from the GT strength data. Our results are found to be more consistent with the semi-empirical data than previous RPA calculations. But, for $^{180}$Ta, which is a well known deformed nucleus, more careful refinement is necessary. Since the exotic nuclei of astrophysical importance are subtly deformed, we need to develop the Deformed QRPA (DQRPA) which explicitly includes the deformation in the Nilsson basis under the axial symmetry [28].

Since our theoretical data for relevant neutrino reactions are shown to be consistent with the recent empirical data, nuclear abundances of $^{138}$La and $^{180}$Ta would not be changed so much enough to affect previous predictions. Abundance of $^{138}$La is more or less reproduced by considering CC reactions $T_{\nu_e} = 4$ MeV neutrino as claimed previously, but $^{180}$Ta is still overproduced. Linkage transitions between ground and isomer states in $^{180}$Ta may be one of the reasonable solutions to explain the overproduction properly [15].

The QRPA is a very efficient method to consider multi-particle and multi-hole interactions and their configuration mixing, and successfully described nuclear reactions sensitive on the nuclear structure, such as $2\nu 2\beta$ and $0\nu 2\beta$ decays. Possible ambiguities on the neutrino-induced reaction caused by the nuclear weak structure can be reduced by reproducing the available data related to the nuclear $\beta$ decay as well as the forthcoming GT transition data by ($^3$He, t) or (p,n) reactions [29].

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Fig. 1: (Color online) Cross sections by neutral current, $^{139}\text{La}(\nu_e, \nu'_e)^{139}\text{La}^*$ and $^{181}\text{Ta}(\nu_e, \nu'_e)^{181}\text{Ta}^*$ reactions for $J_\pi = 0^\pm \sim 4^\pm$ states. Transition matrix elements are calculated by the QSM, Eq.(6).
Fig. 2: (Color online) Cross sections by charged current, $^{138}$Ba($\nu_e, e^-$)$^{138}$La* and $^{180}$Hf($\nu_e, e^-$)$^{180}$Ta*, for $J_\pi = 0^\pm \sim 4^\pm$ states. Transition matrix elements are calculated by the QRPA, Eq.(4).
Fig. 3: The Gamow-Teller strength distribution $B(GT_-)$ by Eq. (9) and their running sums for $^{138}\text{Ba} \rightarrow ^{138}\text{La}$ (upper panels) and $^{180}\text{Hf} \rightarrow ^{180}\text{Ta}$ (lower panels). The universal quenching factor $f_q = 0.74$ is not used for the results in these figures.
Fig. 4: (Color online) Temperature dependence of the energy weighted cross section for $\nu - A$ reactions, whose neutrino spectra for the supernovae are assumed as the Fermi-Dirac distribution. Results for $^{12}$C, $^{56}$Fe and $^{56}$Ni are referred from our previous calculations [16, 17].