Construction of Superstrings in Wormhole-like Backgrounds

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Abstract

We construct a class of superstring solutions in non-trivial space-time. The existence of an $N = 4$ world-sheet superconformal symmetry stabilizes our solutions under perturbative string loop corrections and implies in target space some unbroken space-time supersymmetries.

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The study of string propagation in non-trivial gravitational backgrounds can provide a better understanding of quantum gravitational phenomena at short distances. Classical string solutions corresponding to such non-trivial backgrounds can be obtained by two different methods. The first makes use of a two-dimensional $\sigma$-model where the space-time backgrounds correspond to field-dependent coupling constants. The vanishing of the relevant $\beta$-functions is identified with the background field equations of motion in target space [1]. The second approach consists in replacing the free space-time coordinates by a non-trivial (super)conformal system whose background interpretation can be seen in the semiclassical limit. The two methods are useful and complementary. The $\sigma$-model approach provides a clear geometric interpretation, but it has the disadvantage that we can only treat it perturbatively in $\alpha'$. Such a treatment is valid only when all curvatures and derivatives of space-time fields are small. In this way, one can easily obtain approximate solutions, but their possible extension to exact ones is in general difficult to prove. The conformal field theory approach takes into account all orders in $\alpha'$ automatically and has the main advantage of describing exact string vacua.

The background interpretation of a given exact string solution is a notion that is ill-defined in general [2],[3]. Indeed, such concepts as space-time dimensionality and topology break down for solutions involving highly curved backgrounds, namely when the metric and/or gauge field curvatures are of the order of the string scale. When the “Kaluza-Klein” excitations (quantized “momentum” modes) are as massive as the string “winding” modes, then the target space interpretation of the string solution is not as clear, since it is possible to describe the same string solution in terms of non-equivalent backgrounds, which have different topology and in some cases different dimensionality as well. This phenomenon is intimately related to the target space duality symmetry among string solutions [3]-[8], which is well known by now. What in my opinion is extremely interesting is the equivalence of regular to singular target spaces via string duality. Once this string phenomenon is well understood, it may shed some light on the initial singularity problem of classical cosmological solutions as well as to paradoxes associated with black holes or other singular objects of classical
Einstein gravity.

In this talk I shall present a special class of exact solutions of superstrings that are based on some $N = 4$ superconformal theories [2]. According to the realization of the underlying superconformal algebra, our solutions are classified into several classes. More explicitly, we arrange the degrees of freedom of the ten supercoordinates in three superconformal systems [2]:

$$\{\hat{c}\} = 10 = \{\hat{c} = 2\} + \{\hat{c} = 4\}_1 + \{\hat{c} = 4\}_2.$$  (1)

The $\hat{c} = 2$ system is saturated by two free superfields. In one variation of our solutions, one of the two free superfields is chosen to be the time-like supercoordinate and the other to be one of the nine space-like ones. In other variations, both supercoordinates are Euclidean or even compactified on a one- or two-dimensional torus. The remaining eight supercoordinates appear in groups of four as $\{\hat{c} = 4\}_1$ and $\{\hat{c} = 4\}_2$. Both $\{\hat{c} = 4\}_A$ systems exhibit $N = 4$ superconformal symmetry of the Ademollo et al. type [9]. The non-triviality of our solutions follows from the fact that there exist realizations of such superconformal theories, based on spaces with non-trivial geometry and topology, other than the $T^4/Z_2$ orbifold and the $K_3$ manifold.

The first class is characterized by two integer parameters $k_1$, $k_2$, which are the levels of two $SU(2)$ group manifolds. For weakly curved backgrounds (large $k_A$) these solutions can be interpreted in terms of a ten-dimensional topologically non-trivial target space of the form $R^4 \times S^3 \times S^3$. In the special limit $k_2 \to \infty$ one obtains the semi-wormhole solution of Callan, Harvey and Strominger [10], based on a six-dimensional flat background combined with a four-dimensional space $W_{k_1}^{(4)} \equiv U(1) \times SU(2)_{k_1}$ that describes the semi-wormhole. The underlying superconformal field theory associated to $W_{k_1}^{(4)}$ includes a supersymmetric $SU(2)_{k_1}$ WZW model describing the three coordinates of $S^3$ as well as a non-compact coordinate with background charge, describing the scale factor of the sphere. Furthermore, it was known that the five-brane background $M^{(6)} \times W_{k_1}^{(4)}$ admits two covariantly constant spinors and, therefore, leaves up to two space-time supersymmetries unbroken, consistently with the $N = 4$ symmetry of the $W_{k_1}^{(4)}$ superconformal system. The explicit realization of the desired $N = 4$
algebra is derived in [11], while the target space interpretation as a four-dimensional semi-wormhole space is given in [10]. In the context of this interpretation, the 10-d backgrounds corresponding to the first class of our solutions are products of topologically non-trivial spaces, $M^2 \times W_{k_1}^{(4)} \times W_{k_2}^{(4)}$ ($M^2$ is the flat $(1+1)$ space-time).

A second class of solutions is based on a different realization of the $N = 4$ superconformal system with $\hat{c} = 4$. Here one replaces the $W_{k}^{(4)}$ space by a new $N = 4$ system, $\Delta_{k}^{(4)} \equiv \left\{ \left( \frac{SU(2)}{U(1)} \right)_k \times \left( \frac{SU(2,1)}{U(1)} \right)_{k+4} \right\}_{\text{SUSY}}$, i.e. a gauged supersymmetric WZW model, with $\hat{c}[\Delta_{k}^{(4)}] = 4$ for any value of $k$. The choice of the levels $k$ and $k+4$ is necessary for the existence of an $N = 4$ symmetry with $\hat{c} = 4$. Using $\Delta_{k}^{(4)}$ or $W_{k}^{(4)}$ as four-dimensional subspaces, we can construct non-trivial 10-d solutions, which admit $N = 2$ target space supersymmetries in the heterotic string, or even $N = 2 + 2$ target space supersymmetries in type-II strings.

Another class of solutions is obtained using the dual space of $W_{k}^{(4)}$, $C_{k}^{(4)} [2],[12],[13]$. It turns out that the $C_{k}^{(4)}$ conformal system with $\hat{c} = 4$ shares with $\Delta_{k}^{(4)}$ and $W_{k}^{(4)}$ the same $N = 4$ superconformal properties. The explicit realization of the $C_{k}^{(4)}$ space is given in [2]. From the conformal theory viewpoint $C_{k}^{(4)}$ is based on the supersymmetric gauged WZW model $C_{k}^{(4)} \equiv \left( \frac{SU(2)}{U(1)} \right)_k \otimes U(1)_R \otimes U(1)_Q$ with a background charge $Q = \sqrt{\frac{2}{k+2}}$ in one of the two coordinate currents ($U(1)_Q$). The other free coordinate ($U(1)_R$) is compactified on a torus with radius $R = \sqrt{2k}$.

Having at our disposal non-trivial $N = 4$, $\hat{c} = 4$ superconformal systems, we can use them as building blocks in order to obtain new classes of exact and stable string solutions in both type II and heterotic superstrings. Some typical 10-d target spaces obtained via the above-mentioned conformal construction are:

A) $i) F^{(2)} \otimes W_{k_1}^{(4)} \otimes W_{k_2}^{(4)}$

$ii) F^{(2)} \otimes F^{(4)} \otimes W_{k}^{(4)}$

B) $i) F^{(2)} \otimes C_{k_1}^{(4)} \otimes C_{k_2}^{(4)}$

$ii) F^{(2)} \otimes F^{(4)} \otimes C_{k}^{(4)}$

C) $i) F^{(2)} \otimes C_{k_1}^{(4)} \otimes W_{k_2}^{(4)}$
In the above expressions, $F^{(4)}$ stands for a 4-d flat space, compact or non-compact, as well as for the $T^4/Z_2$ orbifold; $F^{(2)}$ denotes a two-dimensional flat space, compact or non-compact, with Lorentzian or Euclidean signature. Note that the Euclidean version of subclasses A)ii), B)ii), D)ii) (i.e. when $F^{(2)} \otimes F^{(4)}$ is a compact six-dimensional flat space) can be identified with three different kinds of 4-d gravitational and/or dilatonic instanton solutions. In this interpretation, the subspace described by the last factor denotes the Euclidean version of our (4-d) space-time.

The type A) constructions based on $W^{(4)}$ conformal theories describe, from the target space point of view, stable solutions of 4-d gauged supergravities [14], which leave some of the space-time supersymmetries unbroken. In fact, consider the 10-d heterotic or type-II superstring compactified on a product of two three-dimensional spheres. The corresponding superconformal field theory is then given by a supersymmetric WZW model based on a $K^{(6)} \equiv SU(2)_{k_1} \otimes SU(2)_{k_2}$ group manifold, where the affine levels $k_A$ define the radii of the spheres $r_A$, $k_A = r_A^2$. In contrast to the toroidal compactification ($T^{(6)} = U(1)^6$) where the six graviphotons are Abelian, in $K^{(6)}$ compactification they become non-Abelian. As expected from field theory Kaluza-Klein arguments, in the large radius limit the resulting effective theory is an $SU(2)_{k_1} \otimes SU(2)_{k_2}$ gauged supergravity [14]. This can be easily shown in the 2d $\sigma$-model approach by means of the $\alpha'$-expansion.

The connection with gauged supergravities is very important, because it allows us to derive the 4-d effective supergravity action, up to two space-time derivatives, which is induced by the $K^{(6)}$ compactification using 4-d supergravity arguments.

The type A), B) and C) constructions based on $W^{(4)}_{k_A}$, and $C^{(4)}_{k_A}$ superconformal systems are strongly connected to the non-critical superstrings in the so-called strong coupling regime ($1 \leq \hat{c}_{\text{matter}} \leq 9$). In fact, the Liouville superfield of non-critical
strings can be identified with the supercoordinate of the above spaces, which has a non-zero background charge. The central charge of the Liouville supercoordinate can be easily determined \[2, 14, 16\],

\[
\hat{c}_L = 1 + 2(Q_1^2 + Q_2^2) = 1 + 4\left(\frac{1}{k_1 + 2} + \frac{1}{k_2 + 2}\right),
\]

where we have used the relation among the levels \(k_A\) and the background charges \(Q_A\), \(Q_A^2 = 2/(k_A + 2)\). This relation follows from the \(N = 4\) superconformal symmetry in both \(W\) and \(C\) systems. The remaining matter part consists of tensor products of unitary \(N = 1\) superconformal theories based on \(SU(2)_{k_A}\) WZW, \([SU(2)/U(1)]_{k_A}\) KS cosets, with \(U(1)\) factors. The matter central charge is always given by

\[
\hat{c}_M = 9 - 4\left(\frac{1}{k_1 + 2} + \frac{1}{k_2 + 2}\right),
\]

and it varies in the region \(5 \leq \hat{c}_M \leq 9\). Thus, our explicit constructions show the existence of super-Liouville theories coupled to \(N = 1\) superconformal unitary matter systems in the strong coupling regime. The problematic complex conformal weights, usually present in this regime, are projected out by the \(N = 4\) induced generalized GSO projection. This projection phenomenon is similar to the one observed in ref. \[15\] in the case of the \(N = 2\) globally defined superconformal symmetry.

In ref. \[2\] the reader can find more details about the \(N = 4\) Realizations for the \(\hat{c} = 4\) Superconformal Building Blocks. Here I will present the target-space metric together with the dilaton \((\Phi)\) and torsion field strength \((H_{ijk})\) corresponding to the \(W_k^{(4)}, C_k^{(4)}, \Delta_k^{(4)}\) 4-d subspaces in the large \(k\) limit:

\[
ds^2 \left[ W_k^{(4)} \right] = k \frac{dzd\bar{z} + dwd\bar{w}}{z\bar{z} + w\bar{w}} \]
\[
-2\Phi = \log(z\bar{z} + w\bar{w}) + \text{const.}
\]
\[
H_{ijk} = e^{2\Phi} \epsilon_{ijk} \partial_\ell \Phi
\]

\[
ds^2 \left[ C_k^{(4)} \right] = k \frac{dzd\bar{z}}{1 - z\bar{z}} + k dwd\bar{w}
\]
\[
-2\Phi = w + \bar{w} + \log(1 - z\bar{z}) + \text{const.}
\]

\[5\]
It is interesting to note that the $C^{(4)}_k$ space can be obtained by performing a (supersymmetric) duality transformation on $W^{(4)}_k$ [12],[13]. In $C^{(4)}_k$ the torsion is zero, while it is non-trivial in $W^{(4)}_k$. The metric of $C^{(4)}_k$ is singular, while that of $W^{(4)}_k$ is regular. The question about the relevance of the singularity at the stringy level is still an open question.

There are two non-equivalent 4-d spaces associated to the $\Delta^{(4)}_k$ superconformal system that correspond to gauging either the axial or vector $U(1)$ in the $SL(2, \mathbb{R})/U(1)$ gauged WZW model.

\[
d s^2 \left[ \Delta^{(4)}_k \right]_\epsilon = \kappa \frac{d z d \bar{z}}{1 - z \bar{z}} + \kappa' \frac{d w d \bar{w}}{w \bar{w} + \epsilon} - 2 \Phi = \log(1 - z \bar{z}) + \log(w \bar{w} + \epsilon) + \text{const.}
\]  

Here $\epsilon = 1$ corresponds to the axial gauging and $\epsilon = -1$ to the vector one. In the axial case the metric is regular while in the vector case it is singular. The two versions of $\Delta^{(4)}_k$ spaces ($\epsilon = \pm 1$) are dual to each other. Here also the relevance of the singularity is not obvious, but an intriguing issue to understand.

We hope that our explicit construction of a family of consistent and stable solutions will give a better understanding of some fundamental string properties, especially in the case of strongly curved backgrounds ($\text{small } k_A$), where the notion of space-time dimensionality and topology breaks down.

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