Scaling Law in Cluster Decay

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Abstract

A recently proposed scaling law for the decay time of alpha particles is
generalized for cluster decay. It is shown that for the decay of even-even
parents, \( \log T_{1/2} \) depends linearly on the scaling variable \( S = (Z_c Z_d)^{0.6}/\sqrt{Q_c} \)
and on the square root of the reduced mass of cluster and daughter.

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In the last years alpha decay systematics have been theoretically reinvestigated [1]- [3] partly as a result of the special interest in the studies of superheavy nuclei and cluster decay. In the same time the cluster decay data has been accumulated [7] with a sufficiently high precision so that their systematics can be now studied almost model independently with guidance to the alpha decay. All the predictions or analysis of the cluster decay data given up to now [4]- [16] are based on models. The prescriptions of these models and/or the physical meaning of the parameters used by them have been subjected to criticism. In this letter we present a model independent description of the accumulated data pointing out the most important variables which scale the cluster decay probabilities.

The earliest phenomenological connection between the half lives and the $Q$-values of the alpha decays of radioactive series was proposed by Geiger and Nuttall [17]

$$\log T_{1/2} = a Q_\alpha^{-1/2} + b ,$$

(1)

which proved to be very useful for the prediction of alpha decay half lives. The rule of Geiger and Nuttall has the disadvantage that the $a$ and $b$ parameters are dependent on the isotope chain. A universal scaling law for alpha decay half lives of the even-even parents has recently been proposed [1]

$$\log T_{1/2} = 9.54 \frac{Z_d^{0.6}}{\sqrt{Q_\alpha}} - 51.37 ,$$

(2)

where $Z_d$ is the charge of the daughter. It was shown that the known $[2]$ alpha decay $\log T_{1/2}$ of the even-even nuclei with $Z \geq 76$ stay on this universal line with a rms deviation of 0.33.

It is of interest to know if similar scaling law(s) exist(s) for cluster decay:

$$A_p Z_p \rightarrow A_d Z_d + A_c Z_c$$

(3)

where the subscripts $p$, $d$, $c$ refer to the parent, daughter and cluster, respectively. We look to even-even parents and clusters with the hope that the structure effects [15] are limited and only the collective dynamics dominates the process. Relation (2) has been compared to what one obtains from that obtained from classical Gamow formula for the alpha decay constant.
\[ \lambda = \omega P \circ P \, , \]  

(4)

where \( \omega \) is the frequency with which the alpha particle exists at the barrier, \( P \circ \) is the preformation probability (assumed to be constant in Ref. [1]) and \( P \) is the barrier penetration factor assuming a square well plus a Coulomb potential for the radial dynamics. One obtains

\[ \log T_{1/2} = C_0 + 2 \cdot \log 2 \cdot \frac{Z_d Z_c e^2}{\sqrt{Q_\alpha}} \sqrt{2\mu/\hbar^2} \left[ \arccos(x) - x\sqrt{1 - x^2} \right] \, , \]

(5)

where

\[ x = \sqrt{\frac{Q_c (R_c + R_d)}{Z_c Z_d e^2}} \]

(6)

and where \( Q_c \) is the cluster decay Q-value and the \( R \) are the equivalent hard-sphere charge radii and \( C_0 = \log(\ln 2/\omega P \circ) \).

The scaling law (2) is not obvious from Eq. (5), but for \( x \leq 0.8 \), \( \log P \) behaves approximately linearly as a function of \( Z_d^{0.6}/\sqrt{Q_\alpha} \). This is in fact the region of interest for the alpha decay of heavy nuclei. The above analysis indicates that the scaling variable is \( (Z_c Z_d)^{0.6}/\sqrt{Q_\alpha} \). As a consequence, we have looked for the behavior of the known experimental data on cluster decay (see e.g Table 1 from Ref. [8]) as a function of the cluster scaling variable

\[ S = \frac{(Z_c Z_d)^{0.6}}{\sqrt{Q_c}} \, . \]

(7)

The data are presented in Fig. 1. There are only 3 known “chains” of cluster decay with more than one element: \(^{14}\text{C} (\log T_{1/2}^{\text{exp}}=11.02 \text{ from } \text{\textsuperscript{222}}\text{Ra})\), \(^{24}\text{Ne} (\log T_{1/2}^{\text{exp}}=20.41 \text{ from } \text{\textsuperscript{232}}\text{U})\), \(^{28}\text{Mg} (\log T_{1/2}^{\text{exp}}=21.68 \text{ from } \text{\textsuperscript{236}}\text{Pu})\) and \(^{226}\text{Ra} (\log T_{1/2}^{\text{exp}}=25.24 \text{ from } \text{\textsuperscript{234}}\text{U})\). The experimental data concerning \(^{14}\text{C}\) and \(^{24}\text{Ne}\) cluster decay clearly shows a linear dependence as function of the scaling variable \( S \). This analysis indicates a scaling law for the cluster decay (alpha included) similar to Eq. (2)

\[ \log T_{1/2} = C_1(S - 7) + C_2 \, . \]

(8)
The constant "seven" is subtracted from $S$ in this equation simply so that the parameter $C_2$ has a numerical value which is close to the actual experimental values shown in Fig. 1. The coefficients $C_1$ and $C_2$ can be extracted from the fit of the experimental data. The values of the $C_1$ parameters are 6.3 for $^4\text{He}$, 17.4 for $^{14}\text{C}$, 20.7 for $^{24}\text{Ne}$ and 27.1 for $^{28}\text{Mg}$. The corresponding values of $C_2$ are: -7.3, 8.0, 19.1 and 21.7.

It is interesting to examine if there are some correlations between the $C_1$ and $C_2$ coefficients corresponding to different cluster decays. Eq. (5) suggests an additional dependence on the masses. In the alpha decay case the dependence on $\sqrt{\mu}$ (the reduced mass) is very small: 0.5 % for the mass of the daughter ($A_d$) in the range 150 to 250. For clusters heavier than alpha the dependence on $\sqrt{\mu}$ is very important. The analysis of the heavy cluster decay case is complicated by the fact that the preformation probability (prescission probability) plays a more important role in the majority of the theoretical models. There are also models which assure the prescission probability to be unity.

Guided by Eq. (5) and by the fact that some models indicate a $\sqrt{\mu}$ dependence of the prescission part also, we have plotted in Fig. 2 the $C_1$ and $C_2$ coefficients as a function of

$$\sqrt{\mu} = \sqrt{\frac{A_c \cdot A_d}{A_c + A_d}}.$$  \hfill (9)

For the plotting purpose only we have used $A_d = 208$ neglecting the small variation due to the different daughter masses. One can clearly see a linear dependence on $\sqrt{\mu}$ of these coefficients (the fitted lines are given by $C_1 = 6.3\sqrt{\mu} - 6.2$, $C_2 = 9.8\sqrt{\mu} - 26.9$).

An alternative fit of the cluster decay data, in the spirit of those models which consider this preformation probability equal to unity, can be performed with the help of Eq. (5). We have taken $R_c = 0.0354A_c + 2.008$ (fm) for the cluster radius which empirically reproduces the charge radii of the light clusters, and $R_d = r_o A_d^{1/3}$ for the daughter radius. The experimental data were fitted by using the two parameters $C_o$ and $r_o$. A 0.64 rms deviation from the experimental values has been obtained and the following values for the fitted parameters: $C_o = -23.1$, $r_o = 0.976$ fm. The $r_o$ value is 0.25 fm smaller than the
typical values for heavy nuclei, and this make the touching radius 1 - 1.5 fm smaller. This is an unreasonable reduction, but the ”extra-penetrability” could simulate the preformation probability (the -23.1 value is consistent with a preformation probability $P_o$ of about unity contributing to $C_o$).

Assuming that the preformation probability is different from unity, our analysis indicates that not only the postscission but the prescission dynamics also is dominated by the square root of the reduced mass. It is interesting to compare this conclusion with the prescriptions presented by different models. The present analysis is in accord with the prescription of Ref. [13] and [4]. The work of Blendowske and Walliser [5] indicates a linear dependence on $A_c$ (the cluster mass) of the prescission probability (the spectroscopic factor in Ref. [5]), differing from the present conclusion. Barranco, Broglia and Bertsch [6] have obtained in their superfluid tunneling model a dependence of the prescission probability on the number of steps to the scission (an extra dependence of the gap parameter entering their formula on the mass of the parent nucleus does not affect this analysis). This number is very close to the reduced mass $\mu$ and not to $\sqrt{\mu}$, again differing from the our findings.

The above analysis indicates a model independent law for the whole body of cluster decay data of the following form:

$$\log T_{1/2} = (a_1 \mu^x + b_1) \left[ \frac{(Z_cZ_d)^y}{\sqrt{Q - 7}} \right] + (a_2 \mu^x + b_2).$$

A fit of the 119 alpha decays [1] and 11 cluster decays [8] from even-even parents has been done. Besides the 8 ”in chain” cluster data considered in Fig. 1, 3 ”single” cluster data have been taken into the fit: $^{20}$O from $^{228}$Th ($\log T_{1/2}^{exp}=20.9$ [21]), $^{32}$Si from $^{238}$Pu ($\log T_{1/2}^{exp}=25.3$ [21]) and $^{34}$Si from $^{242}$Cm ($\log T_{1/2}^{exp}=23.2$ [22]). The fit result gives $a_1 = 9.1$, $b_1 = -10.2$, $a_2 = 7.39$, $b_2 = -23.2$, $x = 0.416$ and $y = 0.613$ with a 0.34 rms deviation of $\log T_{1/2}$.

Considering the important parameter $x$, a range of values from 0.4 to 0.6 can be obtained depending upon the various subsets of data used in the fit. A 0.58 rms is extracted for the heavy clusters only, which represents a fairly good description of the data if one has in mind that the largest deviation comes from $^{34}$Si ($\log T_{1/2}^{exp}=24.45$ as compared with the
23.2 experimental value). This may indicate that the extrapolation of Eq. (10) to heavier clusters must be taken with caution. One would also like to see independent confirmation of the $^{34}$Si experimental investigations [22]. The apparent breakdown of this scaling law when going from cluster decay to fission could be understood by the fact that in the latter case the dynamics is not dominated by the Coulomb potential but by the collective potential up to the scission point. We compared our formula with model dependent results for heavier cluster decay like $^{48}$Ca from $^{256}$No. Our result is $logT_{1/2} = 27.9$ while the result from Ref. [19] is significantly smaller ($logT_{1/2} = 18.9$). Experimental information in this mass range are crucial.

The scaling law, Eq.(10), can be straightforwardly used to produce tables with cluster decay half live predictions similar with those in Ref. [19]. Input parameters are the mass and charge numbers of cluster and daughter and the Q-value of the reaction. A detailed search through all the possible decays of the parents with $82 \leq Z_p \leq 106$ and clusters with $2 \leq Z_c \leq 20$ shows the possibility to obtain experimental data for the decay of new clusters in this region: e.g $^{12}$C from $^{220}$Ra ($logT_{1/2} = 10.4$) and from $^{222}$Th ($logT_{1/2} = 10.08$), $^{18}$O from $^{226}$Th ($logT_{1/2} = 17.75$), etc. To select these cases we have used similar constraints as in Ref. [19], namely $logT_{1/2} \leq 28$ and $logT_{1/2} - logT_{1/2}(\alpha) \leq 18$. Only those nuclides for which the experimental masses are known [23] have been used.

One can try to test the $\sqrt{\mu}$ behavior in decays for which the daughter is different from the $^{208}$Pb region. The neutron deficient A $\approx$ 120 region is particularly interesting. For example, the decay of $^{118}$Ba into a $^{12}$C cluster and $^{106}$Sn. Eqs. (10) together with an experimentally extracted Q-value, $Q_c$=15.10 MeV [19], gives $logT_{1/2} = 18.0$. This extrapolation of the scaling law (8) to light parents gives significantly lower half lives as compared with other model dependent treatments (e.g. $logT_{1/2}$=21.3 in Ref. [19]). Further experimental tests are required to validate one of these approaches.

Eq. (10) represents the first model independent description of all known cluster decay data. The parameters $a_1$, $b_1$, $a_2$, $b_2$, $x$ and $y$ contain information on the dynamics of the decay. The actual theoretical models describing the cluster decay data are rather crude.
Often their parameters lose their physical meaning, as for the unphysically small $r_0$ discussed above or e.g. the use of a zero-point motion energy even in the asymptotic region [19]. In our approach we have emphasized the most important variables $(S, \sqrt{\mu})$ scaling the experimental data. We expect this new approach to be an important step toward a theoretical description of the cluster decay.

In conclusion, we have obtained a new scaling law for the alpha and cluster decay of the even-even heavy nuclei. The scope of these scaling laws is to describe the regularities of the data, to put in evidence the peculiar behavior with respect to these regularities, to reveal the most important parameters entering the theoretical models and to guide new prediction for the cluster decay. New experimental data are necessary to further support the present analysis.

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Figure captions

**Figure 1** Experimental data for $\log T_{1/2}$ (sec) of the cluster decay of even-even parents as functions of the scaling variable $S$ (Eq. (7)). The line noted by $^4$He is given by Eq. (2) which represents the best linear fit to the experimental data. Other lines are drawn to guide the eye.

**Figure 2** $C_1$ and $C_2$ coefficients entering Eq. (8) as function of $\sqrt{\mu}$ defined in Eq. (9). Lines represent the best linear fit.
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Figure 1.
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Figure 2.