Top pair production in $e^+e^-$ collisions with virtual and real electroweak radiative corrections.∗

V. Driesen, W. Hollik, A. Kraft

Institut für Theoretische Physik
Universität Karlsruhe, D-76128 Karlsruhe, Germany

March 26, 2022

Abstract

The effect of virtual electroweak corrections to $e^+e^- \rightarrow t\bar{t}$ and the contribution of the radiation processes $e^+e^- \rightarrow t\bar{t}Z, t\bar{t}H$ to the inclusive top pair production cross section and forward-backward asymmetry are discussed in the high energy regime.

∗To appear in the Proceedings of the Workshop ”Physics with $e^+e^-$ Colliders”, Annecy-Gran Sasso-Hamburg 1995, ed. P. Zerwas
Top pair production in $e^+e^-$ collisions with virtual and real electroweak radiative corrections.

V. Driesen, W. Hollik, A. Kraft

Institut für theoretische Physik
Universität Karlsruhe
D-76128 Karlsruhe, Germany

Abstract

The effect of virtual electroweak corrections to $e^+e^- \to t\bar{t}$ and the contribution of the radiation processes $e^+e^- \to t\bar{t}Z, t\bar{t}H$ to the inclusive top pair production cross section and forward-backward asymmetry are discussed in the high energy regime.

For an accurate prediction of top pair production cross sections at a high energy $e^+e^-$ collider, various types of higher-order effects have to be taken into account:

- the QCD corrections, which are treated perturbatively far from the threshold region, but require refinements on threshold and finite width effects close to the production threshold;
- the electromagnetic bremsstrahlung corrections (QED corrections) with real and virtual photons inserted in the Born diagrams. They are complete at the 1-loop level in the virtual and soft photon part as well as in the hard photon contribution;
- the genuine electroweak corrections, which consist of all electroweak 1-loop contributions to $e^+e^- \to t\bar{t}$, without virtual photons in the external charged fermion self energies and vertex corrections. These are also available as a complete set of 1-loop contribution.

An important feature of the electroweak corrections is that they are large and negative at high energies far above the $t\bar{t}$ threshold, and thus lead to a sizeable reduction of the production cross-section. At very high energies, on the other hand, there are also radiation processes besides the conventional photon radiation which contribute to inclusive top pair production $e^+e^- \to t\bar{t}X$: The radiation of $Z$ bosons (Fig. 1) and the radiation of Higgs bosons (Fig. 2).

In previous work these radiation processes were studied with emphasis on searches for Higgs bosons and on investigating the Yukawa interaction. They may, however, also be considered as contributions to the (inclusive) $t\bar{t}$ cross section at 1-loop order. Since they constitute a positive contribution, they increase the production rate of $t\bar{t}$ pairs according to $e^+e^- \to t\bar{t}(X)$ and hence compensate at least partially the negative terms from the virtual corrections.

In this note we present the influence of the virtual electroweak 1-loop corrections on top pair production and the contribution of $e^+e^- \to t\bar{t}Z, t\bar{t}H$ to the inclusive cross section and forward-backward asymmetry. In a first subsection the structure and size of the virtual
contributions are discussed. Subsequently, we include the $Z$ and $H$ radiation processes and show that the large and negative virtual contributions are sizeably compensated by including the $Z,H$ bremsstrahlung processes.

**Virtual contributions**

The integrated cross section and the forward-backward asymmetry for the process $e^+(l_2) + e^-(l_1) \rightarrow t(p_1) + \bar{t}(p_2)$ with purely electroweak virtual corrections can be written in the form\(^1\)

$$
\begin{align*}
\sigma &= \frac{4\pi\alpha(s)^2}{3s} N_C \beta \left\{ \frac{1}{2} (3 - \beta^2) \sigma_1(s) + \beta^2 \sigma_2(s) + \Delta \sigma \right\} \\
A_{FB} &= \frac{3}{4} \frac{\beta}{\frac{1}{2}(3 - \beta^2)} \frac{\sigma_3 + \Delta \sigma'}{\sigma_3}
\end{align*}
$$

where

$$
\begin{align*}
s &= (p_1 + p_2)^2, \\
\beta &= \sqrt{1 - \frac{4m_t^2}{s}}, \quad \alpha(s) = \frac{\alpha}{1 + \Pi^{\gamma}_{\text{ferm}}(s)} \equiv \frac{\alpha}{1 - \Delta \alpha(s)}.
\end{align*}
$$

With $\hat{\Pi}^{\gamma}_{\text{ferm}} = \hat{\Pi}^{\gamma} - \hat{\Pi}^{\gamma}_{\text{bos}}$ we denote the fermionic part of the renormalized subtracted photon vacuum polarization. $\sigma_1$, $\sigma_2$, and $\sigma_3$ contain the lowest-order contribution and

\(^1\) Up to $O(\alpha^4)$. 

---

**Figure 1:** Diagrams for $e^+e^- \rightarrow t\bar{t}Z$.

**Figure 2:** Diagrams for $e^+e^- \rightarrow t\bar{t}H$. 

---

---
those one-loop corrections which can be incorporated in effective photon-fermion and Z-fermion couplings, i.e. the self-energies and the vertex corrections to the V, A couplings. The remaining terms, the top vertex corrections not of V, A structure and the contribution from the WW and ZZ box diagrams, are collected in $\Delta \sigma$ and $\Delta \sigma'$ for the symmetric and the antisymmetric cross section.

The $\sigma_i$ can be written in the following way:

$$
\sigma_1 = (Q_e^2 + Q_A^2) Q_t^2
+ 2 (Q_e V_e + Q_e A_e) Q_t V_t \frac{s}{s - M_Z^2}
+ (V_e^2 + A_e^2) V_t^2 \left( \frac{s}{s - M_Z^2} \right)^2,
$$

$$
\sigma_2 = Q_e^2 Q_t A^2
+ 2 Q_e V_e Q_t A_s \frac{s}{s - M_Z^2}
+ (V_e^2 + A_e^2) A_t^2 \left( \frac{s}{s - M_Z^2} \right)^2,
$$

$$
\sigma_3 = 2Q_e V_t A_e A_t \frac{s}{s - M_Z^2} + 4V_e V_t A_e A_t \left( \frac{s}{s - M_Z^2} \right)^2.
$$

The effective coupling constants in these formulae are $(f = e, t)$

$$
Q_f^V = Q_f \left[ 1 - \frac{1}{2} \hat{\Pi}_{bos}^V (s) \right] - F_{V}^f (s),
$$

$$
Q_f^A = - F_{A}^f (s),
$$

$$
V_f = \bar{v}_f + F_{V}^Z (s),
$$

$$
A_f = \bar{a}_f + F_{A}^Z (s)
$$

with

$$
\bar{a}_f = \left( \frac{\sqrt{2} G_{\mu} M_Z}{4 \pi \alpha (s)} \right)^{1/2} \bar{\rho}^{1/2} I_3^f
$$

$$
\bar{v}_f = \left( \frac{\sqrt{2} G_{\mu} M_Z}{4 \pi \alpha (s)} \right)^{1/2} \bar{\rho}^{1/2} (I_3^f - 2Q_f s^2).
$$

They contain the propagator corrections

$$
\bar{\rho} = 1 - \Delta r - \hat{\Pi}^{Z} (s),
$$

$$
\bar{s}^2 = s_w^2 - s_w c_w \hat{\Pi}^{Z} (s)
$$

with the self-energies renormalized according to the on-shell scheme

$$
\hat{\Pi}^{Z} = \Re \frac{\hat{\Sigma}^{Z} (s)}{s - M_Z},
$$

$$
\hat{\Pi}^{\gamma Z} = \Re \frac{\hat{\Sigma}^{\gamma Z} (s)}{s}
$$

(3)
together with the $V, A$ form factors $F_{V,A}$ from the vertex corrections (real part). $\Delta r$ is the radiative correction to the Fermi coupling constant in the on-shell scheme. The explicit expressions can be found in [2, 3].

The Born cross section $\sigma^{(0)}$ can be obtained from the formulae above by setting all the self-energies, vertex corrections, $\Delta \sigma$, $\Delta \sigma'$ and $\Delta r$ in Eqs. (4-6) equal to zero.

The effect of the virtual electroweak corrections on the cross section for $e^+e^- \to t\bar{t}$ is displayed in Fig. 3, where the relative deviation from the "Born" cross section is shown. Note that "Born" already includes the QED running of the effective charge in the $\gamma$-exchange amplitude. The results in Fig. 3 are thus the residual corrections which are specific for the electroweak standard model.

![Figure 3: Relative electroweak corrections to $e^+e^- \to t\bar{t}$ for $M_H = 100$ GeV (solid line) and $M_H = 1000$ GeV (dashed line).](image)

**Real contributions**

Next we consider the inclusive cross section for $t\bar{t}$ production in association with Higgs and $Z$ bremsstrahlung. We thereby restrict our discussion to Higgs masses below $2m_t$, such that real Higgs production with subsequent decay into $t\bar{t}$ cannot occur. The latter case would be of specific interest for investigating the Higgs Yukawa interaction, and was explicitly studied in ref. [7].

The bremsstrahlung processes are of higher order in the coupling constants than the $2 \to 2$ process. The cross section at energies sufficiently above the threshold reach 10%
and more of the lowest order cross section. Their contribution to the $t\bar{t}$ final state hence becomes of the same order as the virtual electroweak corrections.

For the computation of the cross sections corresponding to the amplitudes in Figs. 1 and 2 the following set of couplings has been chosen:

\begin{align*}
\gamma ff & : - \sqrt{4\pi \alpha(s)} \gamma^\mu Q_f \\
Z ff & : - \sqrt{2} G_\mu M_Z^2 \gamma^\mu [(I_3 - 2Q_f s_w^2) - I_3 \gamma_5] \\
H ff & : - \sqrt{2} G_\mu m_f \\
H ZZ & : \sqrt{4\sqrt{2}} G_\mu M_Z M_Z
\end{align*}

For $s_w^2$ the approximate expression

$$s_w^2 = \frac{1}{2} - \frac{1}{4} \frac{\pi \alpha(M_Z)}{\sqrt{2} G_\mu M_Z^2} = 0.2311$$  \hspace{1cm} (8)

is used.

In Fig. 4 we put together the integrated cross section for $e^+e^- \rightarrow t\bar{t}Z$ and $e^+e^- \rightarrow t\bar{t}H$ as function of the energy $\sqrt{s}$. The dominating contribution to the inclusive final state for $\sqrt{s} \geq 1$ TeV is from the "Z-strahlung", as can be seen from Fig. 4.

![Figure 4: Integrated cross section for $e^+e^- \rightarrow t\bar{t}Z$ and $e^+e^- \rightarrow t\bar{t}H$.](image)

We now can define an inclusive cross section for $t\bar{t}$ production in the following way:

$$\frac{d\sigma}{d\Omega}(t\bar{t}) = \frac{d\sigma_V}{d\Omega}(t\bar{t}) + \int d^3p_3 \frac{d\sigma(t\bar{t}H)}{d\Omega d^3p_3} + \int d^3p_3 \frac{d\sigma(t\bar{t}Z)}{d\Omega d^3p_3}$$  \hspace{1cm} (9)

where $d\Omega = d\cos\theta_t$ $d\phi$ is the solid angle of the outgoing top, and $\theta_t$ the scattering angle between $e^-$ and $t$. $d\sigma_V$ denotes the 2-particle final states including the virtual contributions, and $d^3p_3$ is the phase space element for $H$ and $Z$, respectively.
The integrated cross section is obtained as

\[ \sigma_{tt} = \sigma_V(tt) + \sigma(e^+e^- \rightarrow t\bar{t}H) + \sigma(e^+e^- \rightarrow t\bar{t}Z). \]  

(10)

The forward-backward asymmetry \( A_{FB} \) is given by

\[ A_{FB} = \frac{1}{\sigma_{tt}} \left( \int_0^1 d\cos\theta_t \frac{d\sigma(tt)}{d\cos\theta_t} - \int_{-1}^0 d\cos\theta_t \frac{d\sigma(tt)}{d\cos\theta_t} \right) \]  

(11)

with

\[ \frac{d\sigma(tt)}{d\cos\theta_t} = \int_0^{2\pi} d\phi \frac{d\sigma}{d\Omega}(tt) \]  

(12)

The results are displayed in Figs. 5, 6. As one can see from Fig. 5, the virtual and real contributions to the integrated cross section cancel each other to a large extent. For \( A_{FB} \), however, the situation is different. Fig. 5 contains \( A_{FB} \) in Born order, with virtual corrections, and after including the real contributions. For \( A_{FB} \), the totally inclusive \( t\bar{t} \) asymmetry deviates more from the Born result than the one with only virtual corrections to \( e^+e^- \rightarrow t\bar{t} \).

Figure 5: Integrated cross section (times \( s \)). The dotted line shows the Born cross section \( \sigma^{(0)} \), the dashed line contains all electroweak 1-loop virtual corrections, the dash-dotted line depicts the real contributions from \( e^+e^- \rightarrow t\bar{t}H \) and \( e^+e^- \rightarrow t\bar{t}Z \), and the solid line shows the inclusive cross section \( \sigma \).
Figure 6: Forward-backward asymmetry. The dotted line shows the Born prediction, the dashed line contains also all electroweak 1-loop virtual corrections, and the solid line depicts the result for the inclusive production.

Thereby, $A_{FB}$ in Born approximation is obtained from Eq. (1) with expressions (3-6), where all self energies, vertex corrections, $\Delta r$ and $\Delta \sigma, \Delta \sigma'$ are put to zero.
References

[1] J. Jersak, E. Laerman, P.M. Zerwas, Phys. Rev. D25 (1980) 1218;
S. Güsken, J.H. Kühn, P.M. Zerwas, Phys. Lett. B155 (1985) 185;
J.H. Kühn, P.M. Zerwas, Phys. Rep. 167 (1988) 321;
V.S. Fadin, V.A. Khoze, Pi’sma v Zh. Eksp. Teor. Fiz. 46 (1987) 417; Yad. Fiz. 48
(1988) 487;
V.S. Fadin, V.A. Khoze, T. Sjöstrand, Z. Phys. C48 (1990) 613;
V.S. Fadin, O.I. Yakovlev, Novosibirsk preprint IYF 90-138 (1990);
W. Kwong, Phys. Rev. D43 (1991) 1488;
H. Inazawa,, T. Morii, J. Morishita, Phys. Lett. B203 (1988) 279; Z. Phys. C42 (1989)
569;
K. Hagiwara et al., Nucl. Phys. B344 (1990) 1;
J. Feigenbaum, Phys. Rev. D43 (1991) 264;
M.J. Strassler, M.E. Peskin, Phys. Rev. D43 (1991) 1500;
J.H. Kühn, Proceedings of the Workshop on on Physics and Experiments with Linear
e+e− Colliders, Waikoloa, Hawaii 1993, eds.: F.A. Harris, S.L. Olsen, S. Pakvasa, X.
Tata

[2] W. Beenakker, W. Hollik and S.C. van der Marck, Nucl. Phys. B365 (1991) 24.

[3] A.A. Akhundov, D.Y. Bardin and A. Leike, Phys. Lett. B261 (1991) 321;
A. Arbuzov, D. Bardin, A. Leike, Mod. Phys. Lett. A7 (1992) 2029; E: A9 (1994)
1515.

[4] W. Beenakker and W. Hollik, Phys. Lett. B269 (1991) 425.

[5] W. Beenakker, A. Denner and A. Kraft, Nucl. Phys. B410 (1993) 219.

[6] W. Hollik, Fortschr. Phys. 38 (1990) 165;
M. Consoli, W. Hollik, F. Jegerlehner, in: Z Physics at LEP1, CERN 89-08(1989),
eds. G. Altarelli, R. Kleiss, C. Verzegnassi.

[7] K. Hagiwara, H. Murayama, I. Watanabe, Nucl. Phys. B367 (1991) 257.

[8] A. Djouadi, J. Kalinowski, P.M. Zerwas, Z. Phys. C54 (1992) 255

8