R-mode oscillations in accreting white dwarfs in cataclysmic variables

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ABSTRACT
Some cataclysmic variables show short-period (∼200 to ∼2000 s) light variations, which are attributable to nonradial pulsations of the accreting primary white dwarf. We regard these periodic variations as r-mode (global Rossby-wave) oscillations which are presumably excited mechanically and confined into hydrogen-rich layers in the white dwarf. Often observed modulations in amplitudes and frequencies even during an observing season may be interpreted as beatings among densely distributed r-mode frequencies. Making the r-mode frequency distribution of a model to be consistent (by adjusting the rotation frequency) with the observed pulsation frequencies, we obtain the best-fit rotation rate for each case. The rotation periods thus obtained for 17 accreting white dwarfs lie between 4 and 10 minutes. Pulsation frequencies observed sometime after a dwarf-nova outburst tend to be higher than the pre-outburst values, so that the best-fit rotation rate for the post-outburst frequencies is also higher. This indicates a spin-up of the hydrogen-rich layers to occur by an enhanced accretion at the outburst. We also discuss the relation between the rotation frequencies of pulsating accreting white dwarfs and the orbital periods.

Key words: binaries:close – stars:dwarf novae – stars:low-mass – stars:oscillations – stars:rotation – white dwarfs

1 INTRODUCTION
Short-period light variations of an accreting white dwarf (WD), which are attributable to nonradial pulsations, were first discovered by Warner & van Zyl (1998) in the cataclysmic variable (dwarf nova) GW Lib at quiescence. Since then, the number of observed accreting WD pulsators has increased to nearly twenty (see e.g. Szkody et al. 2013b, 2015; Mukadam et al. 2017, for reviews). They belong to WZ Sge type cataclysmic variables having very low mass-transfer rates of ∼10^{-11} M☉yr^{-1} at quiescence, and have very large outbursts in very long intervals (a few decades). Because of the very low mass-transfer rate, a large fraction of the flux from a system at quiescence is originated from the WD, so that pulsations of the WD can be detected.

The pulsations of accreting WDs have been generally regarded as g modes excited thermally in the hydrogen and helium ionization zones similarly to the g modes in the ZZ Ceti variables (e.g. Brickhill 1991; Goldreich & Wu 1999; Saio 2013). Although the effective temperature tends to be higher than the blue edge of the ZZ Ceti instability strip (Szkody et al. 2010), it is thought that an enhanced helium abundance would solve the problem (Arras et al. 2006; Van Grootel et al. 2015).

Fig. 1 shows oscillation periods of accreting WDs, which we discuss in the following sections, with respect to the effective temperatures. Although the period range is similar to that of ZZ Ceti variables, there is no trend in the periods of accreting WDs with respect to the effective temperature. This is a stark contrast to the well-established trend of ZZ Ceti variables that cooler ZZ Ceti stars have longer periods (e.g. Mukadam et al. 2006; Hermes et al. 2017). This suggests considerable differences in the oscillation properties between accreting WDs and ZZ Ceti stars.

In addition, there is a significant difference in rotation rates between accreting WDs in cataclysmic variables and ZZ Ceti variables. Rotation periods of ZZ Ceti variables (longer than 5 hr in most cases) are much longer than the g-mode periods (100–1500 s) (Fontaine & Brassard 2008; Kawaler 2015; Hermes et al. 2017), so that rotation is only a small perturbation to g-mode oscillations in ZZ Ceti stars. In contrast, accreting WDs rotate much faster, having projected rotation velocities larger than 100 km s^{-1} for most cases (see e.g. Sion & Godon 2012). Among pulsating accreting WDs, Szkody et al. (2012) obtained a rotation period of 209 s for GW Lib from the spectroscopic projected rotation
velocity $V \sin i = 40 \text{ km s}^{-1}$ combined with an inclination of $11^\circ$ (van Spaandonk et al. 2010). For EZ Lyn Szkody et al. (2013a) obtained $V \sin i = 225\pm 75 \text{ km s}^{-1}$, which corresponds to a rotation period less than 350 s for a typical WD radius. These rotation periods are comparable to or shorter than the oscillation periods of these stars. Therefore, the effects of the Coriolis force should be significant, and g-mode characters should be very different from those in ZZ Ceti variables.

It is known that, among g modes, prograde sectoral modes are predominantly observed in rapidly rotating main-sequence g-mode pulsators (e.g., Van Reeth et al. 2016; Ouazzani et al. 2017). This is explained by the fact that if the pulsation frequency of a star in the co-rotating frame is less than the rotation frequency, a g mode other than the prograde sectoral modes would have amplitude strongly confined into a narrow equatorial zone (e.g., Saio et al. 2018b; Saio 2018), and hence should have low visibility due to geometrical cancellation. Therefore, if pulsations in an accreting WD were g modes, they should be prograde sectoral modes. In the inertial (observer’s) frame, however, a prograde mode should have a period shorter than the rotation period, which contradicts to the fact that most periods observed in GW Lib are longer than the rotation period (Szkody et al. 2012).

Thus, the observed pulsations must be retrograde (in the co-rotating frame) modes. However, as noted above, retrograde g modes should be hardly detected due to the cancellation on the surface. In rapidly rotating $\gamma$ Dor stars, pulsations whose periods longer than the rotation period are identified as r modes (global Rossby waves) (Van Reeth et al. 2016). (The dependence of period-spacing on period clearly distinguishes between g and r modes.) Therefore, it seems more sensible to consider r modes in accreting WDs rather than g modes.

In this paper we consider the pulsations detected in accreting WDs in cataclysmic variables to be r-mode oscillations. The next section (§2) summarises the property of r-mode oscillations in WDs. In §3 (and Appendix) we fit observed pulsation frequencies for each star (in each epoch in some cases) with the frequency distribution of r modes calculated for a WD model with an assumed rotation frequency. In other words, we determine the best-fit rotation frequency for each case. §4 discusses the relation between rotation frequencies of accreting WDs and the orbital periods of the cataclysmic binaries.

## 2 R-MODE OSCILLATIONS IN WHITE DWARFS

The first clear signature of r-mode oscillations was found by Van Reeth et al. (2016) in some rapidly rotating $\gamma$ Dor variables as the period-spacing-period relation having a gradient opposite to that of g modes. Later, from the properties of visibility-frequency relations of r modes, Saio et al. (2018a) identified r modes in various main-sequence stars; r-mode oscillations seem to be commonly present in non-supergiant stars. General properties of r modes are discussed in Saio et al. (2018a); Saio (2018). Here we discuss only the properties to be used in the following section.

In studying low-frequency oscillations of a rotating star, the traditional approximation of rotation (TAR) is useful, in which the horizontal component of the angular velocity of rotation $\Omega \sin \theta$ ( $\theta$ is co-latitude) and the Eulerian perturbation of gravitational potential are neglected. Then, the governing equations are reduced to the same equations for nonradial pulsations of a non-rotating star under the Cowling approximation, except that $\ell (\ell + 1)$ is replaced with $\lambda$, the eigenvalue of the Laplace Tidal equation, and the latitudinal dependence is given by the eigenfunction $\Theta_{\ell m}^n (\cos \theta)$ instead of the Legendre function $P_{\ell m}^n (\cos \theta)$ (see e.g. Lee & Saio 1997). Both $\lambda$ and the functional form of $\Theta_{\ell m}^n$ vary depending on the spin parameter, $s$, defined as

$$s \equiv \frac{2 \Omega}{(2 \pi v^\infty)} = \frac{2 \nu_{\text{rot}}}{v^\infty}. \quad (1)$$

where $v^\infty$ is the cyclic frequency of pulsation in the co-rotating frame, and $\nu_{\text{rot}}$ is the cyclic frequency of rotation. The impact of Coriolis forces on pulsation is significant if $s > 1$.

Because of the similarity of the differential equations, we can formally use an asymptotic expression for g-mode frequencies in a non-rotating star for low-frequency g and r modes in a rotating star by replacing $\ell (\ell + 1)$ with $\lambda$ as

$$v^\infty \approx \frac{\sqrt{\lambda}}{2 \pi n_g} \int N \, d\rho \approx \frac{\sqrt{\lambda}}{n_g} v_0. \quad (2)$$

where $N$ is the Brunt-Väisälä frequency (e.g. Unno et al. 1989; Aerts et al. 2010), $n_g$ the number of radial nodes, and $\int d\rho$ means integration through the propagation layers.

Although the equation is formally similar to the non-rotating case, variation of $\lambda$ as a function of the spin parameter significantly modifies the properties of low-frequency non-radial pulsations if $s > 1$ (in particular for retrograde
The most significant influence of the Coriolis force is the emergence of r modes (global Rossby waves coupled with buoyancy) in a rotating star. In the retrograde side of a s-λ diagram (e.g., Lee & Saio 1997; Saio 2018), a group of lines appear in the positive range of λ for s ≥ 2. These lines represent λ for r modes which are ordered by using negative integer k (= −1, −2, −3…). These r modes are present (i.e., λ > 0) if

\[ 2\pi \nu_0^c (\text{r modes}) < \frac{2m \Omega}{(m + |k|)(m + |k| - 1)} \leq \Omega = 2\pi \nu_{\text{rot}} \]  

with a positive m. (We adopt the convention, as in our previous papers, that a positive m corresponds to retrograde modes.)

While λ for k = −1 increases rapidly as the spin parameter, s, increases, λ for k ≤ −2 stay low as represented by the asymptotic form

\[ \lambda \approx \frac{m^2}{2(k - 1)^2} \]  

if k ≤ −2 & s ≫ 1

derived by Townsend (2003). Fig. 2 shows snapshots of temperature and velocity distributions of m = 1 r modes with k = −1, −2, and −3 on the surface for selected spin parameters. The number of vortices in a (east or west) hemisphere is given by |k|, while an even (odd) |k| corresponds to temperature perturbations symmetric (antisymmetric) to the equator.

Since \( \nu_0^c < \nu_{\text{rot}} \) (eq.3), r-mode frequencies in the inertial frame, \( \nu_{\text{inert}} \), are given as

\[ \nu_{\text{inert}} = m_{\text{rot}} - \nu_0^c \approx m_{\text{rot}} - \frac{\sqrt{\lambda}}{n_g} \]  

for r modes. (5)

Thus, frequencies of r modes with azimuthal order m should be observed between \( (m - 1) \nu_{\text{rot}} \) and \( m_{\text{rot}} \). Fig. 3 shows frequency ranges of m = 1 r modes for k = −1, −2, and −3 with visibilities under the assumption of energy equi-partition (see Saio et al. 2018a, for details) for an inclination of 30°. Generally, k = −2 modes (symmetric) have larger visibilities, while visibilities of odd modes (k = −1, −3) decrease with increasing the inclination.

As the number of radial nodes (n_g) increases, \( \nu_0^c \) decreases (eq.2) so that \( \nu_{\text{inert}} \) increases approaching \( m_{\text{rot}} \) and frequency spacing becomes smaller (see Fig. 3). Each sequence stops, before reaching \( m_{\text{rot}} \), because \( \nu_0^c \) must be larger than the critical frequency, \( \nu_{\text{crit}} \) (effective Lamb frequency at the stellar surface, \( r = R \); see upper panel of Fig. 4); i.e.,

\[ \nu_{\text{inert}} > \nu_{\text{crit}} = \sqrt{\lambda} c_s \]  

\( 2\pi R \)

with \( c_s \) being sound speed at the surface (Townsend 2000). Among the cases of k = −1, −2, −3 the frequency gap to \( m_{\text{rot}} \) (i.e., \( \nu_{\text{crit}} \)) is smallest for k = −3 because the limiting value of \( \lambda \) is smallest (eq. 4), while the gap is larger for hotter stars because of higher sound speed at the surface.

Fig. 3 also shows that visibilities of some modes are very small compared to other ones at similar frequencies. The difference is caused by whether the mode is trapped in the H-rich layers or penetrates into deeper layers. The lower panel of Fig. 4 shows radial displacement \( \xi_r \) for a trapped mode (\( n_g = 45 \); blue line) and two non-trapped modes (\( n_g = 44, 46 \)). Since mass density is very high in deeper layers, non-trapped modes should have higher kinetic energy compared with modes trapped in the H-rich layers for a given surface amplitude. (For all cases \( \xi_r \) is normalized as unity at the surface.) So, if energy is equally given to each mode, trapped modes should have larger amplitude at the surface and hence have larger visibilities. For the cases shown in the lower panel of Fig. 4, the visibility of the trapped mode (\( n_g = 45 \)) is more than 20 times larger than the two non-trapped modes. Thus, only r modes trapped in the H-rich layers should be observable.
Runs of the radial displacement panel: Lower and between the He layers and the C/O degenerate core. The composition at the interfaces between H-rich and He layers is not clearly defined. Sharp peaks in the Väisälä frequency, important parameter for r-mode frequencies is the rotation frequency. Various accreting WDs in cataclysmic variables. The most important parameter for r-mode oscillations to pulsation frequencies observed in white dwarfs. Observed pulsations in accreting (and hence rapidly rotating) WDs are generally thought as g modes similar to those in ZZ Ceti variables in the literature. As noted in § 1, however, g modes in rapidly rotating stars have characters inconsistent with observed pulsations in accreting (and hence rapidly rotating) white dwarfs.

In this section, we fit the theoretical frequency range of r mode oscillations to pulsation frequencies observed in various accreting WDs in cataclysmic variables. The most important parameter for r-mode frequencies is the rotation frequency (f_rot), while details of the interior structure are not very important. To calculate the r-mode frequencies and visibilities for a rotation frequency, we have used a static white dwarf model (with out rotation and atomic diffusion) for each mass was calculated after putting solar-abundance matter of M_H on the He-accreting steady-state model (with 10^{-7} M_⊙ yr^{-1}) taken from Kato et al. (2018) (the accretion was stopped at the start of the cooling calculation). A propagation diagram of a 0.6 M_⊙ model is shown as an example in the upper panel of Fig. 4.

3 R-MODE FITTINGS TO OBSERVED PULSATIONS

While the possibility of r modes was briefly discussed in Mukadam et al. (2013), pulsations in accreting white dwarfs were generally thought as g modes similar to those in ZZ Ceti variables in the literature. As noted in § 1, however, g modes in rapidly rotating stars have characters inconsistent with observed pulsations in accreting (and hence rapidly rotating) white dwarfs.

In this section, we fit the theoretical frequency range of r mode oscillations to pulsation frequencies observed in various accreting WDs in cataclysmic variables. The most important parameter for r-mode frequencies is the rotation frequency (f_rot), while details of the interior structure are not very important. To calculate the r-mode frequencies and visibilities for a rotation frequency, we have used a static white dwarf model (0.6 or 0.8 M_⊙) having an effective temperature roughly consistent with a spectroscopically determined value for each case.

Utilized white dwarf models have masses of helium and H-rich (solar abundance) layers (M_H/M_⊙, M_H/M_⊙) = (8 \times 10^{-3}, 7 \times 10^{-5}) and (2 \times 10^{-3}, 3 \times 10^{-5}) for the 0.6 and 0.8 M_⊙ models, respectively. The cooling evolution (without rotation and atomic diffusion) for each mass was calculated after putting solar-abundance matter of M_H on the He-accreting steady-state model (with 10^{-7} M_⊙ yr^{-1}) taken from Kato et al. (2018) (the accretion was stopped at the start of the cooling calculation). A propagation diagram of a 0.6 M_⊙ model is shown as an example in the upper panel of Fig. 4.

3.1 GW Lib

Short-period light variations indicative of nonradial pulsations of the accreting primary WD in a cataclysmic variable (CV) were first detected in GW Lib by Warner & van Zyl (1998). Since then, many observations have been performed; GW Lib is the most studied case among pulsating accreting WDs. In addition, GW Lib is one of the few cases where pulsations were observed during quiescence before and after an outburst. GW Lib had a very large (9 magnitude) outburst in 2007 after 24 years of the discovery in 1983 (Templeton et al. 2007a). The pulsation signals disappeared at the outburst, but they seemed to return one year after the outburst (Copperwheat et al. 2009) at shifted frequencies.

Fig. 5 compares r-mode frequencies/visibilities with observed frequencies/amplitudes of GW Lib. The upper panel shows frequencies prior to the outburst, taking from van Zyl et al. (2004) (1997–2001), Szkody et al. (2002b) (2002 January), and Copperwheat et al. (2009, 2005 May). The lower panel shows pulsation frequencies/amplitudes after the outburst.
burst obtained by Szkody et al. (2012) (2010 March–2011 August), Chote & Sullivan (2016) (2012 May), Szkody et al. (2015) (2012 June, 2013 March), and Szkody et al. (2016) (2015 April). Each observing run detects only few frequencies which are often similar to but slightly different from the frequencies previously obtained. This is a common property of short-period variations of this type (i.e. accreting WDs). This can be explained as being among densely distributed r-mode frequencies (Fig. 3), and a poor frequency resolution due to a short time baseline. For this reason, all frequencies/amplitudes reported in the literature are included in this figure (and in the following cases).

Gray lines show theoretical visibilities of r-mode oscillations for 0.8-$M_\odot$ WD models. (cf. van Spaandonk et al. (2010) obtained 0.84 $M_\odot$.) A slightly higher effective temperature is adopted for the model after the outburst, taking into account the accretion heating (Szkody et al. 2012), although the property of r modes is not very sensitive to $T_\text{eff}$. To fit r-mode frequency ranges to observed main frequency groups of GW Lib, we have chosen rotation frequencies of 2.85 mHz prior to the 2007 outburst and 3.70 mHz after the outburst (rotation periods of 351 s and 270 s, respectively). These rotation frequencies should represent rotation rates in the H-rich envelope, because visible r modes are confined there. (In other words, the rotation rate of the WD core could be considerably different from that in the outer envelope.) Fig. 5 shows that main frequency groups of GW Lib are roughly consistent with $m = 1$ r modes, while $\sim 4.2$ mHz frequencies in the upper panel can be explained as combination frequencies between $\sim 1.5$ mHz and $\sim 2.6$ mHz (van Zyl et al. 2004). The 3.47 mHz frequency in the upper panel of Fig. 5 cannot be explained. The frequency was only marginally detected in 1998 June data (van Zyl et al. 2004). It may be a high-order combination frequency or a transient feature.

Low frequency features at $\sim 0.43 - 0.2$ mHz should be related with the orbital frequency 0.217 mHz (Thorstensen et al. 2002) and its harmonic and also with superhump frequencies. The strong feature at $\sim 0.9$ mHz (19 min; blue lines in the lower panel) appeared in optical (but not in UV) observations in 2012 May–June (Szkody et al. 2015; Chote & Sullivan 2016), also in 2008 April–June (Copperwheat et al. 2009; Schwieterman et al. 2010; Bullock et al. 2011). Although it is located within the frequency range of $m = 1(k = -1)$ r modes, the 19 min feature is likely originated from the accretion disk because of its transient nature (Bullock et al. 2011).

Another long-period features at 2 and 4 hr (not shown) were detected time to time (Woudt & Warner 2002; Copperwheat et al. 2009; Hilton et al. 2007; Szkody et al. 2016; Toloza et al. 2016). Toloza et al. (2016) found that these light variations (as well as shorter period ones) are caused by considerable temperature variations. They argued a possible explanation for these long periodicities by retrograde low-order g modes despite the expected low visibility due to an amplitude confinement toward a narrow equatorial zone. Long period r-mode pulsations are free from such amplitude confinement. However, the longest period of r model is about 2 hr in the model after the outburst, while it is about 4 hr in the model prior to the outburst. Thus, it does not seem possible to identify 4 hr variability after the outburst as an r mode. The origin of these long periodicities is unclear (see also the discussion in Copperwheat et al. 2009).

Our post-outburst model has an equatorial rotation velocity of 167 km s$^{-1}$. Combining it with the inclination angle $11^\circ$ (van Spaandonk et al. 2010), we have $V\sin i = 32$ km s$^{-1}$. This is roughly consistent with the spectroscopic estimate $V\sin i = 40$ km s$^{-1}$ obtained in 2010 by Szkody et al. (2012).

Our r-mode models for the nonradial pulsations of GW Lib indicate that the H-rich envelope of the white dwarf was considerably spun up by the accretion during the 2007 outburst. Rotation frequencies 2.85 mHz and 3.70 mHz before and after the outburst, respectively, correspond to an increase from $0.033\Omega_K$ to $0.043\Omega_K$ with $\Omega_K$ being the Keplerian angular frequency at the surface of the WD. This indicates that during the outburst an angular momentum of $\sim 0.01\Omega_K (2R^2 M_H/3)$ was accreted, where $M_H$ is the mass of H-rich layers. Letting $M_{acc}$ to be the mass accreted during the outburst, the balance of angular momentum is approximately given as

$$\Omega_K R^2 M_{acc} \approx 0.01\Omega_K \frac{2R^2 M_H}{3} \rightarrow M_{acc} \approx 0.01 \frac{2M_H}{3} \tag{7}$$

Vican et al. (2011) derived a bolometric luminosity of $1.5 \times 10^{34}$ erg s$^{-1}$ during the 26-day outburst of GW Lib; i.e., energy of $\sim 3 \times 10^{40}$ erg was released during the outburst. The energy (= gravitational energy of accreted matter) corresponds to $M_{acc} \approx 2 \times 10^{-10} M_\odot$. Substituting this into equation (7), we obtain $M_H \approx 3 \times 10^{-8} M_\odot$. Combining this $M_H$ with the mean accretion rate $1.3 \times 10^{-11} M_\odot$ yr$^{-1}$ obtained by Vican et al. (2011), we can infer that the last classical nova explosion of GW Lib occurred about $2 \times 10^3$ years ago.

Pulsation frequencies detected in 2015 (Szkody et al. 2016, green lines in the lower panel of Fig. 5) at $\sim 2.7$ mHz are lower than those observed in 2010–2013, and comparable to the frequencies for even r modes prior to the outburst (upper panel); i.e., consistent with the rotation frequency prior to the outburst. This may indicate accreted angular momentum being re-distributed after the outburst, and rotation frequency is nearing the value prior to the outburst. Further monitoring of pulsations in GW Lib would be very interesting.

### 3.2 EQ Lyn (= SDSS J074531.92+453829.6)

EQ Lyn was first identified as a CV from the SDSS (Sloan Digital Sky Survey) spectrum by Szkody et al. (2006). Mukadam et al. (2007) discovered, in 2005 October, short-period light variations, which are attributed to non-radial pulsations in the primary white dwarf. Similar short-period signals were detected from 2005 November to 2006 January (Mukadam et al. 2011). EQ Lyn underwent an outburst in 2006 October, which was detected by the Catalina Real-Time Transient Survey (CTRS). After the outburst no pulsations had been observed for more than three years. Even after pulsations were detected again in 2010 February–March, pulsations disappeared time to time; pulsations seem to disappear when superhump signatures appear (Mukadam et al. 2013). The superhump is caused by the precession of a fully developed elliptic accretion disk (e.g. Whitehurst 1988; Hirose & Osaki 1990). The cause of the anti-correlation is not clear.
for a frequencies/amplitudes are compared with r-mode visibilities (0 burst in 2006. Obvious superhump (detected before (upper panel) and after (lower panel) the outburst are taken from Mukadam et al. (2011, 2013), and Szkody et al. (2015). Observed years are colour coded as indicated. Gray lines indicate visibilities of r modes. In each panel, the visibility is normalized arbitrarily such that the maximum of even $m = 1$ frequencies ($f_{m=1}$) for each panel is chosen for the predicted visibility distribution of r modes to be consistent with observed frequencies.

Fig. 6 shows observed frequencies and amplitudes detected before (upper panel) and after (lower panel) the outburst in 2006. Obvious superhump (0.19 mHz) and orbital (0.21 mHz) frequencies are not shown. The pulsation frequencies/amplitudes are compared with r-mode visibilities for a 0.6 M$_\odot$ WD model with log $T_{\text{eff}}$ = 4.187. The model parameters are chosen according to the discussion in Mukadam et al. (2013).

In the upper panel, the frequency groups at ~0.8 and ~1.6 mHz are consistent with $m = 1$ modes for a rotation frequency of 1.75 mHz. The frequency group around 2.5 mHz may be identified as $m = 2$ ($k = -1$) odd modes, or combination frequencies between the frequency groups of around 0.8 and 1.6 mHz.

Frequencies in 2010 (red lines in the lower panel of Fig. 6) are very close to frequencies obtained before the outburst, while the frequencies detected in 2012–2014 (blue lines) are slightly higher. So, a slightly faster rotation of 1.90 mHz is needed to be consistent with 2012–2014 frequencies (blue lines), although for the 2010 frequencies the pre-outburst rotation frequency should be good enough. This may indicate that the rotation was spun up slightly by accretion during the quiescent period between 2010 and 2012; during the period, pulsation signals were hardly detected as discussed in Mukadam et al. (2013).

The change in rotation frequency needed to explain pulsation frequency shifts between before and after the 2006 outburst of EQ Lyn is much smaller than the case of GW Lib (3.1). The difference is consistent with the fact that the ~5 mag eruption of EQ Lyn was much less energetic (i.e. much less accretion occurred) compared with the eruption of GW Lib with a brightening of ~9 mag.

3.3 EZ Lyn (= SDSS J080434.20+510349.2)

EZ Lyn, identified as a CV by Szkody et al. (2006), is somewhat different from other systems. EZ Lyn underwent two dwarf nova outbursts in 2006 March (Pavlenko et al. 2007) and 2010 September (Kato et al. 2012), the interval is much shorter than typical recurrence times of decades. In addition, "mini-outbursts" with amplitudes of about 0.5 mag occur at quiescence, indicating the presence of some activities in the accretion disk (the long-term light curve is documented in e.g. Zharikov et al. 2013).

Prior to the 2010 outburst, a periodicity of 12.6 min (1.32 mHz), which can be attributed to nonradial pulsation, was detected from 8 months to 2.5 yr after the 2006 outburst (Pavlenko 2009; Zharikov et al. 2013). But after that no short periodicities were detected before the 2010 outburst.

About 7 months after the 2010 outburst Pavlenko et al. (2012) found a much shorter periodicity at 4.28 min (3.89 mHz). Szkody et al. (2013a) confirmed it 14 months after the 2010 outburst by UV observations with HST (Hubble Space Telescope) as well as ground-based optical obser-
vations. (Pavlenko et al. (2014) reported a higher frequency at 4.25 mHz present for a week in 2012 November.) Furthermore, Sosnovskij et al. (2017) detected similar but slightly lower frequencies in 2015–2016 observations.

Fig. 7 shows pulsation frequencies detected in 2011 (upper panel) and 2015 (lower panel). Visibilities (gray lines) of r modes are shown for a 0.6 M⊙ WD model with $f_{\text{rot}} = 4.05$ mHz ($= 350$ d$^{-1}$) in the upper panel and $f_{\text{rot}} = 3.70$ mHz ($= 320$ d$^{-1}$) in the lower panel. (Model parameters, $M/M_\odot$, $T_{\text{eff}}$ in Fig. 7 are chosen according to the analysis by Szkody et al. (2013a); $T_{\text{eff}} = 13,100$ K, $\log g = 8.0$.) These rotation frequencies are chosen to fit the main frequency groups to be consistent with $m = 1$ even ($k = -2$) modes. Since a relatively high inclination of $70^\circ$ is chosen, visibilities of odd modes (dashed lines) are low.

Lower frequency peaks in both panels cannot be explained by r modes. In the 2011 data (upper panel), the 0.4 mHz peak probably corresponds to a harmonic of the orbital frequency 0.196 mHz of EZ Lyn (Gänsicke et al. 2009), while the 0.8 mHz peak could be the harmonic of 0.4 mHz. More low-frequency features are seen in the 2015 data (lower panel); among them only the 0.8 mHz peaks are common to the 2011 data, while the 0.6 mHz peaks may be explained as combination frequencies between 0.8 mHz and the orbital frequency. Frequencies at $\sim 1.2$ mHz are comparable to the 12.8 min periodicity (1.32 mHz) seen from 8 months to 2.5 yr after the 2006 outburst (Pavlenko et al. 2007), although the cause of these frequencies is not clear. (Frequencies at $\sim 1.4$ mHz are probably combination frequencies between the $\sim 1.2$ mHz frequencies and the orbital frequency.)

Szkody et al. (2013a) obtained a projected rotation velocity of $V \sin i = 225 \pm 75$ km s$^{-1}$ from the UV spectra of EZ Lyn acquired in November 2011, while our model predicts an equatorial rotation velocity of 228 km s$^{-1}$ for the 2011 pulsation frequencies (upper panel). The presence of a shallow eclipse (Kato et al. 2009; Szkody et al. 2013a) indicates a high inclination. So, the projected velocity is consistent with the equatorial rotation velocity 228 km s$^{-1}$ of our r-mode pulsation model for the year 2011.

Our r-mode models interpret the shift of pulsation frequencies from $\sim 3.9$ mHz (2011) to $\sim 3.5$ mHz (2015) as that the rotation rate of the H-rich envelope of the WD has slowed down slightly during the time interval, due to a re-distribution of angular momentum. Further monitoring the pulsations of EZ Lyn would be very interesting.

### 3.4 V455 And (= HS 2331+3905)

The cataclysmic variable V455 And has shown complex variabilities. It underwent a large dwarf-nova outburst in September 2007 (Templeton et al. 2007b). Prior to the outburst, Araujo-Betancor et al. (2005) and Gänsicke (2007) found short period variations in 2002–2003 and in 2004, respectively. They also found an orbital period of 81.08 min with a shallow eclipse. The short period variations consist of a group at $\sim 300$–400 s and a 67.6 s period with its harmonic. The latter is thought to be caused by a spinning magnetic dipole (intermediate polar), and the former are attributed to pulsations in the primary accreting WD.

After the 2007 outburst, the photometric and spectroscopic variations have been monitored by Silvestri et al. (2012), Szkody et al. (2013c) and Mukadam et al. (2016). While the 67.6 s period is almost constant (Mukadam et al. 2016), the pulsation periods have decreased to $\sim 250$ – 280 s (Silvestri et al. 2012). Furthermore, Szkody et al. (2013c) found that these periods are, in turn, getting longer with time. Fig. 8 shows the pulsation frequencies/amplitudes before (upper panel) and after (lower panel) the 2007 outburst. As mentioned above, the pulsation frequencies got higher after the outburst, while after that they decreased considerably from 2010 (blue lines) to 2014–15 (red lines). Gray lines are frequencies/visibilities of $m = 1$ r modes in WD models of 0.6 M⊙ with rotation frequencies as indicated.

Araujo-Betancor et al. (2005) determined the effective temperature of the accreting WD in V455 And at quiescence prior to the 2007 outburst as $T_{\text{eff}} = 10,500$ K, while Szkody et al. (2013c) obtained effective temperatures of 11,100 ± 250 from 2010 HST UV spectra and 10,850 ± 300 K from 2011 spectra. The effective temperatures of the models in Fig. 8 are chosen taking into account above determinations, although r-mode properties are not sensitive to $T_{\text{eff}}$.

In order to make r-mode frequency ranges to be roughly consistent with the observed pulsation frequencies, we choose rotation frequencies of 3.3 mHz (303 s in period) for the 2003 data (upper panel) and 3.94 mHz (254 s) for the 2010 data (lower panel). Obviously, these rotation rates are much ($\sim 4$ times) slower than the spin rate of the dipole magnetic poles. We can argue that this may indicate the presence of a considerable differential rotation between the

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Figure 8. Upper panel: Short period variations of V455 And before the 2007 outburst. Frequencies/amplitudes obtained by Araujo-Betancor et al. (2005) are compared with r modes of $m = 1$ (gray lines) in a 0.6 $M_\odot$ WD model with a rotation frequency ($f_{\text{rot}}$) of 3.36 mHz. Lower panel: V455 And after the outburst. Frequencies in 2010 (July–September) (blue lines) are taken from Silvestri et al. (2012), while 2014–2015 data (red lines) are taken from Mukadam et al. (2016). These data are compared with $m = 1$ r-mode visibilities for $f_{\text{rot}} = 3.94$ mHz.
H-rich layers and the degenerate core of the WD. Since visible r modes should be trapped in the H-rich layers, the pulsation frequencies reflect the rotation rate there. On the other hand, the dipole magnetic field, which interacts with circum-stellar gas, should be rooted in the central part of the white dwarf and hence rotate at a rate of the core rotation; i.e., the 67.6 s period should be the core-rotation period, which can be different from the rotation period of the envelope. Since a large inertia is expected in the core, it is reasonable that the period 67.6 s was not changed by the outburst (Mukadam et al. 2016).

The gradual decrease of pulsation frequencies mentioned above appears in the lower panel of Fig. 8; the blue-line group for 2010 data is separated from the red-line group for 2014–2015 data. The rotation frequency to fit with the 2014–2015 group should be comparable with the rotation frequency prior to the outburst, which indicates the rotation rate in the outer H-rich layers to be slowing down nearing the pre-outburst rotation rate.

R-mode oscillations of the accreting WD in V455 And should be generated mechanically by the disturbed flows caused by accretion or/and the interaction with the differentially rotating magnetic field. However, it still remains unsolved why the r-mode period group and dipole signal are predominantly detected in emission-line photons as found by Szkody et al. (2013c).

3.5 PQ And

The cataclysmic variable PQ And underwent outbursts in 1938, 1967, and 1988. Short-period light variations at quiescence were discovered in 2004 by Vanlandingham et al. (2005), and confirmed by Patterson et al. (2005b) in 2005. Szkody et al. (2010) detected similar periodicities in 2007–2008. The observed frequencies/amplitudes are shown in Fig. 9 with blue and red solid lines. These frequencies form three groups. Patterson et al. (2005b) found the frequency 0.41 mHz (40 min) to be the harmonic of the orbital frequency.

The other two frequency groups are compared with the visibility distribution (gray lines) of r modes for a 0.6 M⊙ WD model with a rotation frequency of 1.70 mHz (606 s in period). The rotation frequency has been chosen in order for the frequencies around 1.5 mHz to agree with m = 1 (k = −2, −3) r modes. The observed frequencies in the range 0.75 – 0.8 mHz lie in the frequency range of m = 1 (k = −1) r modes. The model parameters are consistent with the spectroscopic analysis by Schwarz et al. (2004), who obtained Teff = 12,000 ± 1,000 K and log g = 7.7 ± 0.3. An inclination angle of 30° is chosen arbitrarily based on no eclipse being reported.

3.6 GY Cet (=SDSS J013132.39−090122.3)

Szkody et al. (2003) found GY Cet to be one of the CV with very low mass-transfer rates. No outburst of GY Cet has been observed. Warner & Woudt (2004) discovered short-period light variations in 2003. Szkody et al. (2007b) obtained an effective temperature of 14500 K for the primary WD in GY Cet. Szkody et al. (2003) obtained an orbital frequency of 0.17 mHz, while Otulakowska-Hypka et al. (2016) gives an orbital inclination of 25° for the system.

Fig. 10 compares the observed pulsation frequencies/amplitudes listed in Szkody et al. (2007b) with r mode visibilities predicted for a 0.6 M⊙ WD model with a rotation frequency of 1.95 mHz. The rotation frequency is chosen in order to include as many observed frequencies as possible into the predicted range of r-mode frequencies, although the frequency group at ~4.7 mHz cannot be explained by r modes. To fit this frequency group with r modes of m = 1 (k = −1, −3), a rotation frequency of ~4.80 mHz would be needed. With this rotation frequency, however, the main frequency group at ~1.7 mHz and the frequency group at 0.9 mHz cannot be consistent with r modes. So, we consider this choice unlikely. The frequency group at 4.7 mHz may correspond to combination frequencies between groups at ~1.7 and ~3.0 mHz (as indicated in table 4 of Szkody et al. 2007b). As another possibility, 4.7 mHz could be the rotation...
rate of the WD core emerged by a magnetic field anchored in the core similarly to the 67 s periodicity of V455 And (§3.4). Needless to say, further observations are desirable; in particular, a spectroscopic determination of $V_{\sin i}$ would be helpful to understand the periodicities of GY Cet.

### 3.7 MT Com (= RE J1255+266)

MT Com was discovered as a bright Extreme Ultraviolet (EUV) transient in 1994 with ROSAT (Dahlem et al. 1995), which is regarded as a normal dwarf nova outburst (Wheatley et al. 2000). From optical time-series photometry in 2000–2004, Patterson et al. (2005a) detected many short period signals mainly around 1344, 1236, and 668 s (0.744, 2000–2004, Patterson et al. 2005a) detected many short period signals mainly around 1344, 1236, and 668 s (0.744, 8.29 and 8.37 mHz detected by Patterson et al. (2005a) are not shown.) Observational years are colour-coded as indicated. (The vertical axis in the upper panel is arbitrarily normalized.)

\[ f_m = 1.75 \text{ mHz} \]
\[ v_a = 77.3 \text{ km/s} \]
\[ \text{Inc} = 29^\circ \]

\[ m = 1 \]
\[ m = 2 \]

\[ MT \text{ Com} (2000–2001, 2002, 2004, 2008) \]

**Figure 11.** Frequencies (crosses; lower panel) of short period variations detected in MT Com by Patterson et al. (2005a) (in 2000–2004) and Szkody et al. (2010) (in 2008) are compared with r-mode frequency ranges (gray lines) for a 0.8 $M_\odot$ WD model with a rotation frequency of 1.75 mHz (upper panel). (Two very high frequencies of 8.29 and 8.37 mHz detected by Patterson et al. (2005a) are not shown.) Observational years are colour-coded as indicated. (The vertical axis in the upper panel is arbitrarily normalized.)

Patterson et al. (2005a) estimated $T_{\text{eff}} = 13,000 \pm 2000$ K for the primary WD in MT Com, while Szkody et al. (2010) obtained 12,000±1000 K. Patterson et al. (2005a) also argued the primary white dwarf in MT Com should be fairly massive and the inclination should be $i \sim 20^\circ$ if the mass ratio is as low as $\sim 0.03$. Our model parameters are chosen by taking into account these results.

The upper panel of Fig. 11 presents visibility of r modes predicted for a WD model of 0.8 $M_\odot$ with a rotation frequency of 1.75 mHz (9.5 min). The rotation frequency is chosen such that r-mode frequencies should be consistent with as many observed frequencies as possible. Fig. 11 shows that r modes are roughly successful in explaining most short periodicities observed in MT Com.

\[ 0.60 M_\odot \text{ WD, log } T_{\text{eff}} = 4.187, \log g = 7.99, \log L = -2.073 \]

**BW Scl (1999, 2001, 2009)**

\[ f_m = 1.86 \text{ mHz} \]
\[ v_a = 105.0 \text{ km/s} \]
\[ \text{Inc} = 30^\circ \]

**Figure 12.** Frequencies of short period variations of BW Scl obtained by Uthas et al. (2012) in 1999 (red), 2001 (green), and 2009 (blue) are shown, while combination frequencies with the orbital frequency and its harmonic are removed. (Amplitudes are taken from table 2 and Fourier spectra for each observing campaign of Uthas et al. (2012)). Gray lines show visibilities of $m = 1$ r modes in a 0.6 $M_\odot$ WD with a rotation frequency of 1.85 mHz. Since no information on the inclination of BW Scl is available, a relatively low inclination, 30°, is chosen arbitrarily.

### 3.8 BW Scl (=2MASS J23530086–3851465)

Abbott et al. (1997) identified BW Scl as a CV. Uthas et al. (2012) discovered short period light variations in BW Scl at quiescence. The short-period (10 to 20 min) variations are attributed to nonradial pulsations of the accreting primary WD. Uthas et al. (2012) reported modulations in the pulsation frequencies and amplitudes, suggesting the presence of more complex frequency structure than detected. Fig. 12 compares the detected pulsation frequencies with r-mode visibilities calculated for a 0.6 $M_\odot$ WD model assuming a rotation frequency of 1.85 mHz. Model parameters are chosen according to the spectroscopic analysis by Gansicke et al. (2005), who obtained $T_{\text{eff}} = 14,800 \pm 900$ (with assumed log $g = 8.0 \pm 0.5$).

A group of observed frequencies between 1.4 – 1.7 mHz (12 – 9.8 min in periods) is fitted to $m = 1$ even ($k = -2$) or odd ($k = -3$) r-modes, while frequencies around 0.8 mHz (20 min in period) to odd ($k = -1$) r-modes. Predicted equatorial rotation velocity, 105 km s$^{-1}$, is consistent with the upper bound $V_{\sin i} \leq 300$ km s$^{-1}$ obtained by Gansicke et al. (2005) for the primary WD in BW Scl. The first outburst of BW Scl was detected by M. Linnolt in 2011 October as mentioned in Kato et al. (2013). So far, no information on short-period variations after the outburst is available in the literature.

### 3.9 V386 Ser (= SDSS J161033.64–010223.3)

V386 Ser is one of the CVs selected from SDSS spectra by Szkody et al. (2002a). No outburst of V386 Ser has been detected. Short period oscillations of the accreting WD in V386 Ser were discovered, next to GW Lib, by Woudt & Warner (2004) in 2003. The short periodicities were also observed by Copperwheat et al. (2009) in 2005, and Mukadam et al. (2010) in 2007. The observed frequencies are similar but not the same among the three seasons of observations.
These pulsation frequencies are shown in Fig. 13, and are compared with the visibility distribution of r modes for a 0.6 $M_\odot$ WD model with a rotation frequency of 1.75 mHz. The predicted visibilities of $m = 3$ r modes are multiplied by a factor 5, to make the frequency range more apparent.

Figure 13. Observed frequencies of V386 Ser in 2003 by Woudt & Warner (2004) (magenta), in 2005 by Copperwheat et al. (2009); Szkody et al. (2007b) (blue), and in 2007 by Mukadam et al. (2010) (red) are compared with visibility distribution of r modes consistent with r modes of 1 frequency is chosen for the main frequencies at 1.75 mHz. The rotation frequency is chosen according to Szkody et al. (2010). The rotation (a) is a combination between the harmonic of the main frequency at 1 (b) is caused by narrow frequency spacings among adjacent r-mode frequencies; the triplet splitting (c) is an equally spaced triplet. Assuming the triplet (d) as rotational splitting, they derived a rotation period of 4.8 d (0.002 mHz), which is much slower than our rotation rate assumed (1.75 mHz) for the model (the latter is more typical as an accreting white dwarf). The discrepancy would be resolved if the triplet splitting $\sim$0.001 mHz is caused by narrow frequency spacings among adjacent r-mode frequencies; the frequency spacing is in fact comparable to $\sim$0.001 mHz at $\sim$1.64 mHz for the $m = 1$ r modes shown in Fig. 13. In addition, beatings among dense r-mode frequencies may explain the small frequency differences in different observing runs as shown in Figs. 7 and 8 of Mukadam et al. (2010). The dense r-mode frequency spectrum is also consistent with the amplitude variations found and inferred as unresolved multiplets by Woudt & Warner (2004).

V355 UMa was identified as a CV from the SDSS spectrum by Szkody et al. (2005). G¨ansicke et al. (2006) and Nilsson et al. (2006) independently discovered, in 2005, rapid periodic signals at 600–660 s attributable to non-radial pulsations of the primary WD in V355 UMa. In 2008 Szkody et al. (2010) obtained a somewhat longer period at 1539 s. As shown in Fig. 14, these periods are consistent with r modes in a 0.6 $M_\odot$ WD model with a rotation frequency of 1.75 mHz (571 s). The effective temperature of the model is consistent with the spectroscopic analysis by G¨ansicke et al. (2006); they obtained $T_{\text{eff}} = 12500$ K for a fixed log g = 8.

The outburst of V355 UMa was detected for the first time in 2011 February by J. Shears (Kato et al. 2012). But so far, no information on short period variations after the outburst is available in the literature.

Figure 14. R-mode visibilities calculated for a 0.6 $M_\odot$ WD models with a rotation frequency of 1.75 mHz are compared with observed pulsation frequencies in V355 UMa. G¨ansicke et al. (2006) obtained a frequency of 1.558 mHz in 2005 April, Nilsson et al. (2006) obtained frequencies of 1.517 and 1.678 mHz in 2005 August, while Szkody et al. (2010) obtained a frequency of 0.650 mHz in 2008 January.

3.10 V355 UMa ( = SDSS J133941.11+484727.5)

V355 UMa was identified as a CV from the SDSS spectrum by Szkody et al. (2005). G¨ansicke et al. (2006) and Nilsson et al. (2006) independently discovered, in 2005, rapid periodic signals at 600–660 s attributable to non-radial pulsations of the primary WD in V355 UMa. In 2008 Szkody et al. (2010) obtained a somewhat longer period at 1539 s. As shown in Fig. 14, these periods are consistent with r modes in a 0.6 $M_\odot$ WD model with a rotation frequency of 1.75 mHz (571 s). The effective temperature of the model is consistent with the spectroscopic analysis by G¨ansicke et al. (2006); they obtained $T_{\text{eff}} = 12500$ K for a fixed log g = 8.

3.11 SDSS J1457 (= SDSS J145758.21+514807.9)

SDSS J1457 was identified as a CV from the SDSS spectrum by Szkody et al. (2005). Uthas et al. (2012) discovered short periodicities in SDSS J1457 and a orbital frequency of 0.214 mHz (77.9 min). Fig. 15 compares these pulsation frequencies/amplitudes with r-mode visibilities calculated for a 0.6 $M_\odot$ WD model with a rotation frequency of 1.80 mHz (556 s). Since no estimates of $T_{\text{eff}}$ and log g are available in the literature, the same equilibrium model as that for BW Scii (which has similar pulsation frequencies) is adopted. Fig. 15 shows the observed short periodicities in SDSS J1457 consistent with r modes of $m = 1$.

3.12 SDSS J2205 (= SDSS J220553.98+115553.7)

SDSS J2205 was identified as a CV from the SDSS spectrum by Szkody et al. (2003). Warner & Woudt (2004) discovered 330, 475, 575 s periodicities in 2003, while Szkody et al. (2007b) confirmed the 575 s periodicity from UV observations with HST. These periodicities are consistent with
$m = 1 \, r$ modes predicted for a rotation frequency of 3.30 mHz (303 s) as shown in Fig. 16. The effective temperature of the model is chosen to be consistent to the value 15,000 K obtained by Szkody et al. (2007b).

However, Southworth et al. (2008) found in August 2007 that SDSS J2205 had stopped pulsating. Since, $r$ modes in an accreting white dwarf are probably generated by flow disturbances on the stellar surface due to accretion, diminishing $r$ mode oscillations might mean that accretion on the white dwarf had stopped or had been very weak for a long time in SDSS J2205.

An dwarf-nova outburst of SDSS J2205 was, for the first time, detected in 2011 May by CRTS as noted in Kato et al. (2013). So far, no information on short periodicities in SDSS J2205 after the outburst are available in the literature.
primary WD in a CV is expected because it has accreted matter as well as angular momentum from the accretion disk.

The rotation velocities given in Table 1, however, tend to be smaller than other primary WDs in CVs (in which no pulsations have been detected); see Table 2 of Sion & Godon (2012). The difference is probably related to the fact that pulsations in the primary WDs are detected only in CV systems with extremely small mass-transfer rates.

The lower panel of Fig. 17 shows rotation frequencies of accreting WDs with respect to the orbital periods \(P_{\text{orb}}\). (Rotation rates of the same star at different epochs are connected by a vertical line.) This figure shows the presence of a sequence, runs from upper-right towards shorter orbital period, bounced at \(P_{\text{orb}} < 1.1\) hr, and goes toward lower right. For comparison, the upper panel of Fig. 17 shows a theoretical evolutionary track of the mass transfer rate \(M\) computed by Howell et al. (2001) for a cataclysmic binary model. The evolution of \(M\) starts from upper-right, comes down to the minimum orbital period at \(P_{\text{orb}} = 1.1\) hr, and decreases to longer orbital period (lower-right). The evolution is caused by losing the orbital angular momentum due to gravitational radiation, and by decreasing mass of the secondary star. The shape of the sequence is similar to our sequence of rotation frequencies in the lower panel.

The similarity of the \(f_{\text{rot}}-P_{\text{orb}}\) sequence to the \(M-P_{\text{orb}}\) evolution track seems to indicate that the rotation of an accreting primary WD slows down with decreasing mass-transfer rate, rather than monotonically being spun up by accretion. This means that some mechanism of redistributing angular momentum in the WD works effectively. This is consistent with the fact we found in the previous section that the rotation frequency needed to fit observed pulsation frequencies with \(r\) modes sometimes decreases after a sharp rise at an outburst as in the cases of GW Lib (Fig. 5), EZ Lyn (Fig. 7), and V455 And (Fig. 8).

The binary evolution (upper panel of Fig. 17; Howell et al. 2001) predicts CVs on the lower branch (sometimes called ‘period bouncers’) to have smaller (secondary to primary) mass ratios \(q\) compared to the cases on the upper branch at a same orbital period. According to the mass ratios from Kato (2015) (listed in Table 1), systems with lower mass ratios tend to fall on the slower rotation branch in Fig. 17. In addition, Patterson et al. (2005a) estimated \(q < 0.06\) for MT Com, which supports the tendency of lower \(q\) for the slower rotation branch. (Patterson et al. (2005a) also estimated a mass-transfer rate of \(4 \times 10^{-12} M_\odot\) yr\(^{-1}\) for MT Com, which agrees with the evolutionary model.)

Thus, our \(f_{\text{rot}}-P_{\text{orb}}\) sequence is likely related to the evolution of cataclysmic binaries, and the property \(f_{\text{rot}} < 2\) mHz may be considered as an additional character for the period bouncers. Among the CVs discussed in this paper, Patterson (2011) lists MT Com, V455 And, GW Lib, PQ And, BW ScI, EZ Lyn, and PP Boo as period-bouncer candidates. Based on our rotation rates, we confirm MT Com, PQ And, BW ScI, and PP Boo as period bouncers.

5 CONCLUSIONS

Short-period light variations detected in accreting WDs in cataclysmic variables are identified as \(r\)-mode oscillations (global Rossby waves) that are trapped in the H-rich layers of the WD. Since the \(r\)-mode frequencies are mainly determined by rotation frequency, we can determine a rotation frequency by making predicted \(r\)-mode frequency range to be consistent with observed pulsation frequencies for each case. The rotation frequency should be regarded as representing the rotation speed of the outermost layers. Thus determined rotation speeds are found to be much faster than those of non-interacting pulsating WDs (ZZ Ceti variables), but tend to be somewhat slower than the non-pulsating accreting WDs in other cataclysmic variables.

We have found that on the \(P_{\text{orb}}-f_{\text{rot}}\) plane, accreting WDs in cataclysmic variables lie on a sequence, which is likely related with the evolutionary sequence of mass-transfer rate as a function of orbital period for a cataclysmic binary model computed by Howell et al. (2001).

The pulsation frequencies of the primary white dwarfs in GW Lib, EQ Lyn, and V455 And indicate that rotation in their outermost layers spun up during a dwarf-nova outburst, while in some cases rotation gradually slows down after the spin up. Such shifts of rotation rates and the presence of a sequence of accreting WDs in the \(P_{\text{orb}}-f_{\text{rot}}\) plane suggest that the rotation rate of the outermost layers of an accreting WDs does not increase monotonically with accretion; rather, it seems to settle at the ‘equilibrium’ rate corresponding to each accretion rate. This suggests the presence of an efficient mechanism to transport the angular momentum in accreting WDs.

Monitoring pulsation frequencies and spectroscopic \(V\sin i\) of accreting WDs in cataclysmic variables would be
very useful to understand the angular momentum accretion onto the surface and its re-distribution.

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APPENDIX A: R-MODE FITTINGS TO OBSERVED PULSATIONS; LESS CERTAIN CASES

This appendix, as in §3, fit r-mode oscillations to pulsations observed in accreting WDs in cataclysmic variables, but for the cases where the fittings are less certain.

A1 LV Cnc (= SDSS J091945.10+085710.0)
LV Cnc was identified as a CV from the SDSS spectrum by Szkody et al. (2005). Mukadam et al. (2007) discovered, in 2005 December, pulsation periods of ~260 s. Woudt et al. (2012) confirmed the presence of similar periodicities. Although these short period variations had been observed until 2007 March, they were not present in 2007 November and 2008 December (Szkody et al. 2010); i.e., the star seems to stop pulsating.

Fig. A1 shows pulsation frequencies obtained by Mukadam et al. (2007) and Woudt et al. (2012), including low frequencies related to the orbital frequency $0.204 \text{ mHz}$ ($1.36 \text{ hr}$) (Dillon et al. 2008) and its first and second harmonics. Observed ~260 s pulsations ($3.8 \text{ mHz}$) are fitted with

Figure A1. Frequencies detected in LV Cnc are compared with visibilities of r modes for a $0.6 M_\odot$ WD model with a rotation frequency of $4.0 \text{ mHz}$. The effective temperature is adopted from Szkody et al. (2010). Low frequencies less than ~0.6 mHz correspond to the orbital frequency and its first and second harmonics.

A2 PP Boo (= SDSS J151413.72+454911.9)
PP Boo was identified as a CV from the SDSS spectrum by Szkody et al. (2005). Nilsson et al. (2006) discovered, in 2005 August, short period variations. Fig. A2 shows a comparison of these frequencies with r-mode visibilities (gray lines) for a WD model for a rotation frequency of 2.0 mHz.

m = 1 r modes for a rotation frequency of 4.0 mHz (250 s), while frequencies at 4.7 and 6.9 mHz cannot be explained by r modes of the model.

A3 SDSS J0755 (= SDSS J075507.70+143547.6)
SDSS J0755 was identified as a CV from the SDSS spectrum by Szkody et al. (2005). Mukadam et al. (2017) discovered
R modes in accreting white dwarfs

Figure A3. Frequencies of SDSS J0755 obtained by Mukadam et al. (2017) (amplitudes are assumed as 10 mmag) are compared with r-mode ($m = 1$) visibilities (gray lines) for a 0.6 $M_\odot$ WD model with a rotation frequency of 4.10 mHz.

Figure A4. Frequencies/amplitudes of DY CMi obtained in 2009 December by Woudt & Warner (2011) are compared with r-mode ($m = 1$) visibilities for a 0.6 $M_\odot$ WD model with a rotation frequency of 4 mHz. The peak at 0.2 mHz corresponds to the orbital frequency. Two periods of 265.5 and 252 s. These periodicities are compared with the visibility distribution of $m = 1$ r modes in a 0.6 $M_\odot$ WD model with a rotation frequency of 4.15 mHz. The adopted model has an effective temperature within the range 15,862 ± 716 K obtained by Pala et al. (2017). The orbital frequency of SDSS J0755 is 84.76 min (Gänsicke et al. 2009).

Figure A5. Frequencies of rapid variations of OV Boo (blue solid lines) obtained by Patterson et al. (2008) are compared with r-mode visibilities (gray lines) for a 0.8 $M_\odot$ WD model with a rotation frequency of 2.2 mHz. These rapid variations of OV Boo (Szkołdy et al. 2010) are adopted as indicated in this figure.

A5 OV Boo (= SDSS J150722.30+523039.8)

Szkołdy et al. (2005) identifies OV Boo as a CV from the SDSS spectrum and found it to be an eclipsing binary by photometry. The orbital period is 1.11 hr (0.25 mHz in frequency) (Szkołdy et al. 2005; Littlefair et al. 2007; Patterson et al. 2008) and inclination angle is 83° (Littlefair et al. 2007; Patterson et al. 2008; Uthas et al. 2011). The orbital period is shortest among known CV binaries. Uthas et al. (2011) analysed HST UV spectra for OV Boo to obtain $T_{\text{eff}} = 14200 \pm 500$ K, $\log g = 8.2 \pm 0.3$, and a $V \sin i = 180 \pm 20$ km s$^{-1}$.

Patterson et al. (2008) found short-period pulsations of 77–75 d$^{-1}$ (0.89–0.87 mHz), 133–128 d$^{-1}$ (1.53–1.48 mHz), and 176–171 d$^{-1}$ (2.04–1.98 mHz) in 2005 and 2006 observations. Patterson et al. (2008) noted the frequency difference between the higher two frequency groups to be equal to twice of the orbital frequency.

Fig. A5 compares these frequencies with r-mode visibility distribution for a 0.8 $M_\odot$ model with a rotation frequency of 2.2 mHz. This model fits the group at $\sim 2$ mHz to $m = 1$ even ($k = -1$) r modes (Szkołdy et al. 2005). The latter group were r modes instead, the rotation frequency to fit would be reduced to $\sim 1.7$ mHz. In this case, however, the model at $-0.9$ mHz would get outside of the frequency range of $m = 1$ ($k = -1$) r modes. So, the former model with faster rotation fits the data. This paper has been typeset from a TeX/LaTeX file prepared by the author.