Electron heating and quasiparticle tunneling in superconducting charge qubits

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Abstract.
We have directly measured non-equilibrium quasiparticle tunneling in the time domain as a function of temperature and RF carrier power for a pair of charge qubits based on the single Cooper-pair box, where the readout is performed with a multiplexed quantum capacitance technique. We have extracted an effective electron temperature for each applied RF power, using the data taken at the lowest power as a reference curve. This data has been fit to a standard $T^5$ electron heating model, with a qualitative correspondence with established material parameters.

1. Introduction
In recent years, mesoscopic superconducting devices have attracted considerable interest both as a laboratory for studying fundamental physics and as a building block for advanced technology. Examples of such systems include superconducting qubits [1], probes of nanomechanical oscillators [2], and sensitive radiation detectors [3]. Many of these devices employ microwave or radio-frequency (RF) readout techniques, which are flexible and low in noise [4]. Since the performance of such devices is often sharply enhanced at low temperatures, it is important to understand the impact of RF measurements on the temperature of the sample. These devices are frequently operated at millikelvin temperatures, where the conduction electrons in the metal are only weakly coupled to the lattice phonons. Electron heating effects have been studied extensively at low temperatures, and previous studies have revealed a $T^5$ dependence for the electron energy flow rate in a variety of different materials [5, 6, 7].

In this work, we have studied electron heating effects due to RF measurements in a single Cooper-pair box (SCB) charge qubit with an RF capacitance readout [8, 9]. As a probe, we have used direct measurements of non-equilibrium quasiparticle tunneling rates, which for certain sets of qubit parameters depend very strongly on temperature. Perhaps surprisingly, at very low temperatures the rate at which non-equilibrium quasiparticles tunnel out of the CPB island into the reservoir increases sharply as the electron temperature goes to zero. This effect has recently been observed experimentally and can be well described by a kinetic theory of quasiparticle trapping [10, 11].
In the experiment, we have measured the quasiparticle tunneling rates as a function of temperature for a variety of RF carrier powers, and computed an effective electron temperature using the data taken at the lowest power as a reference curve. We have then fit this effective temperature to a standard $T^5$ model of electron heating, and have found a qualitative correspondence with established material parameters. Such electron heating effects were also observed qualitatively by Ferguson et. al. [12].

2. Experimental Setup

Scanning electron micrographs of the qubit and readout circuitry and a circuit diagram can be found in Fig. 1. A pair of SCB charge qubits were fabricated by double-angle evaporation on a R-plane sapphire substrate. Although the two qubits are weakly coupled with a fixed capacitor, for the purposes of this experiment they can be treated as independent devices. Readout was performed using a quantum capacitance technique, where the center frequency of a lumped-element LC tank circuit is shifted by the state of the qubit. This shift is detected by RF reflectometry using a 556 MHz cw carrier. Further details of the experimental setup and measurement technique can be found in Ref. [10].

For sample temperatures which are low compared to the even-odd free energy difference, equilibrium quasiparticle tunneling events are exponentially suppressed [13]. However, non-equilibrium quasiparticle tunneling has been commonly observed in single-charge devices, even at millikelvin temperatures. The kinetics of quasiparticle tunneling can be directly observed in the time domain [14] using the quantum capacitance readout. As individual quasiparticles tunnel, the capacitance switches stochastically between two values, which are characteristic of odd and even parity in the device. The resulting random telegraph signal can then be digitized directly. By analyzing the statistics of this fluctuation in the time domain, we can independently measure the rates of odd-to-even (island-to-reservoir) and even-to-odd (reservoir-to-island) transitions, corrected for the finite bandwidth of the measurement [15].

![Figure 1](color online) 
A) Scanning electron micrograph (SEM) of multiplexed on-chip LC oscillators. The qubit features are at the center. B) False-color SEM image of qubit structures. Red (1): Left qubit island. Blue (2): Left RF gate. Green (3): Left control gate. Yellow (4): Qubit leads and ground plane. Orange (5): Right qubit island. Purple (6): Right RF gate. Pink (7): Right control gate. C) Schematic diagram of on-chip qubit and quantum capacitance readout circuitry. Nominal component values are given in the text. Not shown explicitly is the fixed interqubit coupling capacitor, which is formed by the shadow of the two qubit islands spanned by a small bridge. Also not shown are the superconducting filters on the RF leads, which can be seen toward the center of figure 1a.
3. Results
To perform the measurements, the sample was mounted on a dilution refrigerator with a base temperature of 18 mK. To measure the effect of the RF carrier power on the electron temperature, we measured the tunneling rates while varying the power from 0.1 to 1 fW, at mixing chamber temperatures of 18 to 200 mK. Below 0.1 fW, the signal-to-noise ratio was too poor to perform measurements. We assume that at the lowest RF excitation power, the electrons are in thermal equilibrium with the lattice and the mixing chamber. As such, the data taken with an 0.1 fW excitation is used as a reference curve to define an effective electron temperature for the data points taken at higher carrier powers. This assumption is admittedly very crude, given that the RF carrier is only one source of power which contributes to electron heating, and that the electron-phonon coupling is extremely weak at 18 mK. Nevertheless, we believe that this analysis gives at least a qualitative picture of the basic physics.

The observed odd-to-even tunneling rates, corresponding to quasiparticles tunneling out of the island into the reservoir, are shown in Fig. 1a. Note that these rates increase dramatically at low temperatures, following the square root singularity in the superconducting density of states [11]. At higher RF powers, this increase is not as steep, since the electron gas is at a higher effective temperature than the lattice. To extract effective temperatures for the data taken at 18 mK, we interpolate between the low-power tunnel rates at 18 and 50 mK, assuming a square root decay. A schematic of this process is shown in figure 2b. The effective temperature as a function of RF excitation power is shown in figure 2c, along with a least-squares fit to a $T^{5}$ power law, as discussed below.

![Figure 2](image-url)

**Figure 2.** (color online) **A)** Odd-to-even (island-to-reservoir) quasiparticle tunnel rates as a function of mixing chamber temperature for a variety of RF power levels. Dashed lines are interpolations between data points. **B)** Illustration of the extraction of an effective temperature for each RF carrier power when the mixing chamber is at 18 mK. **C)** Plot of the applied RF carrier power as a function of the effective temperature. The solid line is a fit to Eq. (1) with $\alpha \gamma / 5 = 4.4$ nW $\cdot$ K$^{-5}$

4. Discussion
The elevation of the electron temperature above the phonon temperature will follow $dT = R_{ep} d\dot{Q} = d\dot{Q} \tau_{ep} / C$, where $dT$ is the increment in the electron temperature, $1 / R_{ep}$ is the thermal conductance between the electrons and the phonons, $d\dot{Q}$ is the rate at which the energy is injected onto the electrons, $\tau_{ep}$ is the electron-phonon energy-relaxation time, and $C = \gamma T$ is the electron heat capacity. The electron-phonon energy-relaxation time can be defined by $dT_{e}/dt = -(T_{e} - T_{ph}) / \tau_{ep}$, where $T_{e}$ and $T_{ph}$ are the electron and phonon temperatures, respectively. The functional form of the electron-phonon relaxation rate depends on the
dimensionality of the phonons, and whether or not one is operating in the limit of a clean or dirty material. Assuming that $1/\tau_{ep} = \alpha T^3$, it is found that [5]

$$\dot{Q} = \frac{\alpha \gamma}{5}(T_e^5 - T_{ph}^5).$$

(1)

We may take the value of $\alpha$ to be of the order $10^7 \text{s}^{-1}\text{K}^{-3}$, based on its value in normal metals [5, 7]. The value of $\gamma$ can be estimated from the electron heat capacity,

$$\gamma = \frac{C}{T} = \frac{\pi^2}{3}D(\epsilon_F)k_B^2V_L = \frac{\pi^2}{3}N_Fk_BV_L$$

(2)

where $N_F$ is the electron density, $D(\epsilon_F)$ is the density of states at the Fermi level, and $V_L$ is the volume of the lead. The effective lead volume $V_L$ can be estimated by considering the physical volume of the reservoir which is within one diffusion length of the SCB tunnel junctions. For the aluminum films produced in our evaporation system, we have extracted the diffusion constant $D = 35 \text{cm}^2/\text{s}$ from magnetoeresistance experiments. To compute the diffusion length $L = \sqrt{D\tau_{ep}}$, we have taken the quasiparticle recombination time to be $\tau_{ep} = 100 \mu\text{s}$ [3]. This gives an effective lead volume of $V_L = 10^{-16}\text{m}^3$. By comparison, the fit to Eq. (1) for $\alpha\gamma/5 = 4.4 \text{nW}\cdot\text{K}^{-5}$ shown in fig. 2c is in qualitative correspondence with the theoretical value, $\alpha\gamma/5 = 27 \text{nW}\cdot\text{K}^{-5}$, given the considerable error in the quasiparticle tunneling rates, the extraction technique for the effective temperature, and the uncertainty in the effective lead volume, the quasiparticle recombination time, and the electron-phonon coupling constant $\alpha$.

In summary, we have shown that the RF carrier power applied in a time-domain measurement of quasiparticle tunneling rates in a single Cooper-pair box contributes to electron heating at low temperatures. This was done by assigning effective temperatures for each value of the applied power, using the lowest-excitation data set as a calibration curve. While coarse, this analysis agrees reasonably well with a standard electron heating model. In order to minimize electron heating in systems probed with RF reflectometry, it is imperative to use very low excitation power and/or pulsed readout techniques.

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[1] Nakamura Y, Pashkin Y A and Tsai J S 1999 Nature 398 786
[2] LaHaye M D, Bun O, Camarota B and Schwab K C 2004 Science 304 74
[3] Zmuidzinas J, Vayonakis A, Day P K, LeDuc H G and Mazin B A 2003 Nature 425 817
[4] Schoelkopf R J, Wahlgren P, Kozhevnikov A A, Delsing P and Prober D E 1998 Science 280 1238
[5] Roukes M L, Freeman M R, Germain R S, Richardson R C and Ketchen M B 1985, Phys. Rev. Lett. 55 422
[6] Wellstood F C and Clarke J 1989, Bull. Am. Phys. Soc. Series II 34 1019
[7] Echternach P M, Thoman M R, Gould C M and Bozler H M 1992 Phys. Rev. B 46 3839
[8] Duty T, Johansson G, Bladh K, Gunnarsson D, Wilson C and Delsing P 2005 Phys. Rev. Lett. 95 206807
[9] Sillanpää M A, Lehtinen T, Paila A, Makhlin Yu, Roschier L and Hakonen P J 2005 Phys. Rev. Lett. 95, 206806.
[10] Shaw M D, Schneiderman J, Lutchyn R M, Delsing P and Echternach P M 2008 Phys. Rev. B, in press
[11] Lutchyn R M and Glazman L I 2007 Phys. Rev. B 75 184520
[12] Ferguson A J, Court N A, Hudson F E and Clark R G 2006 Phys. Rev. Lett. 97 106603
[13] Tuominen M T, Hergenrother J M, Tighe T S and Tinkham M 1992 Phys. Rev. Lett. 69 1997
[14] Naaman O and Aumentado J 2006 Phys. Rev. B 73 172504
[15] Naaman O and Aumentado J 2006 Phys. Rev. Lett. 96 100201