THE FIRST HALOS∗

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The size and time of formation of the first gravitationally bound objects in the Universe is set by the microphysical properties of the dark matter. It is argued that observations seem to favour cold and thermal candidates for the main contribution to the dark matter. For that type of dark matter, the size and time of formation of the first halos is determined by the elastic cross sections and mass of the CDM particles. Consequently, the astrophysics of CDM might allow us to measure some of the fundamental parameters of CDM particles. Essential for observations is the survival rate and spatial distribution of the very first objects, which are currently under debate.

Keywords: dark matter; structure formation/

1. Motivation

There are at least four motivations to study the issue of the smallest structures and first objects in the Universe. Firstly, a scale invariant power spectrum, without cut-off at some small scale, would lead to an infinite amount of energy stored in acoustic waves (density fluctuations) in the Universe and would be inconsistent. Thus a cut-off at some scale is necessary and it is interesting to know where it is. Secondly, we lack a fundamental understanding of dark matter. All dark matter candidates give rise to the same structure at the largest scales, but differ in their predictions at smaller scales. E.g. light neutrinos and weakly interacting massive particles (WIMPs) lead to different structures at large subhorizon scales and can be discriminated by present large scale structure observations. Similarly, various WIMP candidates give rise to different smallest scale structure. Thus we hope that the astrophysics of dark matter might allow us one day to exclude some of the many dark matter candidates. A third motivation are

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direct and indirect search experiments. Although experimentalists and observers typically deliver limits on some cross section as a function of dark matter mass, they actually constrain the rate. However, the rates depend on some cross section times the local dark matter density. The local dark matter density might be significantly different from the matter density in the isothermal halo model. It is thus important to understand the small scale distribution of dark matter down to the scale of dark matter (also the velocity distribution is of concern, but not discussed in this contribution). Finally, a complete understanding of hierarchical structure formation must include the study of the very first objects in the hierarchy.

2. Primordial power spectrum

We can only make sensible statements on the power of modes that are generated by quantum fluctuations during cosmological inflation. Larger modes are certainly not observable today and we have no theory for that regime. The smallest primordial modes cross the horizon close to the end of inflation. The energy scale at the end of inflation lies between the Planck scale and nucleosynthesis, which means that the smallest primordial scales are somewhere between 1 mm and 1 pc, measured in comoving length scales. If we argue that the matter-antimatter asymmetry in the Universe cannot be generated below the electroweak scale, the energy scale at the end of inflation should be at least 1 TeV. In that case the maximal mode number is $k_{\text{max}} > 10^6/\text{pc} \approx 1/(0.5\text{au})$. For length scales smaller than the Hubble scale at the end of inflation, no squeezing of quantum fluctuations occurs and thus it seems that those modes will never become "classical" density inhomogeneities. For our discussion it is enough to realise, that the scenario of cosmological inflation, together with a successful mechanism of baryogenesis, allows us to speak about the primordial power spectrum down to at least $10^{12}/\text{Mpc}$ or, expressed in cold matter mass, $10^{-24}M_\odot$.

3. Classification of dark matter candidates

There are basically two important criteria according to which dark matter candidates can be classified. The first one introduces the notion of HOT and COLD dark matter, but has nothing to do with temperature. Hot dark matter obeys a relativistic equation of state ($p \sim \epsilon$) at the time of matter-radiation equality, whereas cold dark matter is non-relativistic at that moment of time ($p \ll \epsilon$). The matter-radiation equality is the relevant moment of time, because growth of structure happens in the matter
Table 1. Classification of dark matter candidates

|           | HOT (relativistic) | COLD (non-relativistic) |
|-----------|--------------------|-------------------------|
| THERMAL   | light $\nu$, ...   | WIMPs (heavy $\nu$, LSP, ...), ... |
| NON-THERMAL | string gas, ...    | misalignment axions, primordial black holes, ... |

dominated epoch only (decoupled dark matter inhomogeneities grow only logarithmically in the radiation dominated epoch).

Whether a dark matter candidate was ever in thermal equilibrium with the hot radiation fluid provides a second criterion. If so, it is called THERMAL, if not NON-THERMAL. Some examples of all four categories are given in table 1.

4. What is the dominant dark matter?

We may ask, given that classification, is any one favoured by observation? A key here is that thermal candidates must have isentropic (often called adiabatic) initial conditions, whereas the non-thermal ones can have more general initial conditions (isocurvature/entropy perturbations). From WMAP and large scale structure data, it follows that the isentropic mode must be the dominant one, a contribution of order 10% of isocurvature modes is not excluded (see e.g.1). This suggests that thermal dark matter candidates are compatible with observations, but non-thermal candidates must provide a mechanism that suppresses isocurvature modes. An especially interesting case in this aspect is the axion, which seems on the first sight to be disfavoured by the data, as it allows isocurvature perturbations. However, as the onset of axion mass is triggered by the temperature of the coloured degrees of freedom in the hot plasma, the axion density perturbations are also largely isentropic, if inflation happens after Peccei-Quinn symmetry breaking. In the case of PQ symmetry breaking after inflation, the axion comes with isocurvature perturbations (see e.g.3). To summarise, the data do not exclude non-thermal candidates in general, but they require that their isocurvature power is suppressed to the few % level.

What remains is the question hot versus cold dark matter. The current limits on light neutrinos can be taken representative for any hot and thermal component. They would lead to a different matter-radiation equality and to a damping of smaller structures due to neutrino free streaming. Fitting WMAP data and large scale structure data indicates that not more than 10% of the dark matter can be hot (see e.g.2). We thus conclude that
observations are most naturally explained by candidates that are classified as cold and thermal dark matter. There are plenty of candidates in that category: the lightest supersymmetric particle (in many scenarios the neutralino), the lightest Kaluza-Klein particle, a heavy fourth generation neutrino (mass of order TeV), any other WIMP, etc.

5. Thermal and non-relativistic dark matter

In the following, we focus our attention on this class of candidates.

5.1. Kinetic decoupling

In contrast to relativistic thermal dark matter, chemical decoupling (freeze-out) and the kinetic decoupling happens at different moments of time.\cite{4-6} For WIMPs with mass \( m \), the freeze out happens at \( T_{cd} \sim m/25 \), whereas kinetic decoupling happens much later; in order to keep thermal equilibrium only elastic scattering is necessary. (A detailed discussion of the relevant elastic rates for neutralinos is given in \cite{5,7}). The decoupling time can be estimated with help of the relaxation time \( \tau_{\text{relax}} \). In each collision the typical change of momentum is

\[
\frac{\Delta p}{p} \sim \sqrt{\frac{T}{m}}.
\]

The CDM particle makes a random walk in momentum space, \( (\Delta p/p)_N \sim \sqrt{N} \Delta p/p \), and thus has to scatter \( N \sim m/T \) times to significantly change to the WIMP’s momentum. When the relaxation time \( \tau_{\text{relax}} \sim N \tau_{\text{coll}} \) reaches the Hubble time \( t_H \) the CDM decouples. For a WIMP with elastic cross sections

\[
\sigma_{\text{el}} \sim \frac{m^2}{m_Z^2} \left( \frac{T}{m} \right)^{1+l},
\]

where \( l = 0 \) or \( 1 \) for domination of s- or p-wave scattering, we find that kinetic decoupling happens at

\[
T_{kd} \sim \left( \frac{m_Z^2 m^4}{m_{\text{Pl}}^4} \right)^{\frac{1}{1+l}},
\]

ranging from a few MeV to a few 10 MeV.\cite{8,9} Profumo et al.\cite{10} performed a more detailed calculation for specific models of supersymmetry and found that the decoupling temperature might be as large as a few GeV. They also investigated models with universal extra dimensions, which are in concordance with our naive estimates.
More recently Bertschinger\textsuperscript{11} improved our estimate of the decoupling temperature by solving the appropriate Fokker-Planck equation. His results agree with our estimates at \( \sim 20\% \).

\subsection*{5.2. Damping and growth of inhomogeneities}

During and after kinetic decoupling the primordial inhomogeneities (from cosmological inflation) are washed out on the smallest scales. Before decoupling the CDM starts to deviate from the prefect fluid behaviour of the radiation fluid and this is very well described by an imperfect fluid initially. Hofmann et al.\textsuperscript{5} calculated the viscosities and used the linearised Navier-Stokes equations to obtain a first estimate of this collisional damping epoch.

After decoupling the CDM particles continue to damp the smallest inhomogeneities due to free streaming.\textsuperscript{5,6,12,13} The comoving free streaming length \( l_{fs} \) is a function of time, but becomes approximately constant as \( z \ll z_{eq} \). It defines the damping scale, which, expressed as the mass of CDM in a homogeneous sphere of radius \( l_{fs}/2 \), is estimated as

\[
M_{fs} \sim 10^{-6} M_\odot \left( \frac{1 + \ln(T_{kd}/30 \text{ MeV})/19.2)^2}{(m/(100 \text{ GeV})(T_{kd}/30 \text{ MeV})} \right)^{3/2}.
\]

Here we assumed a matter density of \( \omega_m = 0.14 \). Collisional damping and free streaming have been incorporated by Green et al.\textsuperscript{8,9} to estimate a typical cut-off scale in the CDM power spectrum. It turns out that for the generic WIMP the free streaming is the dominant contribution to damping and thus sets the scale for the smallest structures.

Loeb and Zaldarriaga\textsuperscript{13} pointed out correctly, that the acoustic oscillations and the drag of the radiation fluid cannot be neglected at decoupling itself and they showed numerically that this has the effect to give rise to even more damping, however the order of magnitude is correctly estimated by free streaming only. Most recently Bertschinger\textsuperscript{11} showed that the ad hoc treatment of the radiation drag in reference\textsuperscript{13} can be improved with help of the appropriate the Fokker-Planck equation. These studies lead consistently to the conclusion that the order of magnitude of the cut-off in wave number is provided by the free streaming calculation. To fix the order of magnitude of the mass scale a more detailed calculation is needed, as a factor of 2 in wave number is a factor of 8 in mass! Close to the cut-off, the CDM power spectrum inherits the peaks and dips from the acoustic oscillations in the radiation fluid. The typical wavelength of these oscillations is
Fig. 1. CDM power spectrum at $z = 300$ for four different WIMP models (left) and different scenarios of cosmological inflation (right) [from Green et al.]. The dotted lines show the power spectra without damping due to CDM microphysics.

given by the sound horizon at kinetic decoupling, in terms of CDM mass

$$M_{\text{osc}} \sim 10^{-5} M_\odot \left(30 \text{ MeV} \over T_{kd}\right)^3.$$  

The damping of inhomogeneities must be convoluted with the primordial spectrum to provide the cut-off scale and the scale of the maximal primordial power. The CDM becomes fully decoupled only after kinetic decoupling and inhomogeneities in the CDM start to grow logarithmically. This is most easily seen from the evolution equation on subhorizon scales

$$a^2 \frac{d^2 \Delta_{\text{cdm}}}{da^2} + \frac{3}{2} \left(1 - \frac{p}{\epsilon}\right) a \frac{d\Delta_{\text{cdm}}}{da} - \frac{3}{2} \frac{\epsilon_{\text{cdm}}}{\epsilon} \Delta_{\text{cdm}} = 0,$$

where $\Delta_{\text{cdm}}$ is the CDM density contrast, $a$ the scale factor, and $\epsilon$ and $p$ are energy density and pressure, respectively. During the radiation dominated epoch $\epsilon_{\text{cdm}} \ll \epsilon$ and $p \approx \epsilon/3$, which leads to $\Delta_{\text{cdm}} = A + B \ln(a/a_{kd})$, where $A$ and $B$ are constants to be fixed by the initial conditions. It is also possible to take the effects of baryons into account analytically. After CDM kinetic decoupling they are still tightly coupled to the radiation fluid and suppress the growth of structure until they decouple at around $z_b \sim 150$.

Figure 1 shows the influence of the WIMP microphysics, whereas figure 2 shows how the maximum depends on the details of the primordial power spectrum.

5.3. The first objects

According to theory and consistent with present observations, primordial fluctuations are gaussian distributed. The growth of matter perturbations on small scales (compared to the Hubble horizon) is surpressed during the
radiation dominated epoch — they grow logarithmically. After matter-radiation equality all subhorizon fluctuations grow proportional to the scale factor and collapse once the overdensity becomes non-linear. For approximately scale-invariant primordial perturbations, these ingredients lead to the hierarchical scenario of structure formation. The smallest scales are the first that start to grow and are thus the first to reach the nonlinear regime. The redshift when that happens is estimated for the scales $M > M_{fs}$ to be $z_{nl} = 40$ to 80, depending on the details of the WIMP microphysics and the details of the primordial power spectrum.

The very first objects however are formed much earlier, as they are not typical. For gaussian distributed inhomogeneities the $N\sigma$ fluctuations would collapse at $Nz_{nl}$. Let us look at some numbers for $N = 3$ (see\textsuperscript{9}): consider the comoving volume the Milky Way with a mass collecting radius of about 1 Mpc. Let us fix the scale of the very first objects to be $10^{-6} M_\odot$ and let us assume that $z_{nl} = 60$. The density contrast of these CDM clouds would be $\Delta \sim 10^7$ today, which is an order of magnitude above the mean density contrast of the galactic disc. There would be CDM cloud in a pc\textsuperscript{3} within the galaxy. However, the survival of those objects has to be understood before a conclusion for their observational relevance can be drawn. These studies must include numerical simulations and analytic estimates.\textsuperscript{14,16–18}

In a numerical simulation by Diemand et al.,\textsuperscript{14} it was shown that indeed structure on all scales down to the cut-off scale survive the merging of the first halos. The typical fluctuations of a given mass scale merge to give larger ones, however the rare ($N\sigma$, $N > 1$) fluctuations are denser and are less likely to be disrupted in the merger process. Rare fluctuations survive with a certain probability until the formation of the first stars. Once compact baryonic objects (stars and planets) have been formed, encounters of them with the smallest CDM clouds could lead to their disruption.\textsuperscript{15} This process was studied in some detail in references,\textsuperscript{17,18} and the conclusion is that the clouds would loose mass, but their core remains intact, i.e. the cores of the very first objects in the Universe might populate our and other galaxies today. Their distribution in mass, size and space is still a subject of research.

6. Non-thermal and non-relativistic dark matter

The best studied candidate for non-thermal CDM is probably the axion.\textsuperscript{3} Let us just mention that in the case of axions there is a preferred mass scale on which miniclusters of $\sim 10^{-12} M_\odot$ are formed.\textsuperscript{19}
7. Effects from the hot plasma

A sequence of transitions, like the QCD transition, took place in the radiation dominated epoch of the Universe. Especially the QCD transition and the $e^\pm$ annihilation are of interest to the structure formation at the smallest scales. Extra features are imprinted in the CDM power spectra at scales of $10^{-8}M_\odot$ (most relevant in the for non-thermal CDM) and $50M_\odot$ (relevant for any CDM), respectively.

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