Roto-translation motion of the stars in close binary systems

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Abstract. This article has to show that the model of p-h which is used to determine the change of the semi major axis of the relative orbit stars is incorrect and leads to large errors in the determination of semi-major axis. The new model, suitable for the elliptical orbits of the stars. To determine relative motion of stars in a close binary system in this paper uses a numerical integration of the equations of motion with the reactive forces, including the rotational component of attraction between the stars and the stream flows into the substance. The calculations of elliptical orbits of close binary stars show that the effect of the reactive force on the evolution of the orbits of stars may be different. The results can be refined by introducing other disturbing factors and making new assumptions based on observations.

1. Introduction

In the mid-20th century the research of the motion of stars in close binary systems was started. This problem is reflected in the works of: A. Kruszewskij [1], D. Hadzhidimetriu [2] S. Piotrowskij [3] etc. That research led to discovery of the conservative mass exchange in a close binary system was obtained a simplified dependence of variation of the semi-major axis $a$ relative stars orbit from the constant rate of increase of the mass of the host star $M_2$ in the form

$$\dot{a} = 2a\dot{M}_2 \left( \frac{1}{M_1} - \frac{1}{M_2} \right). \quad (1)$$

This dependence, which is called Paczynskij-Huang formula, is used in all researches of close binary systems with conservative mass exchange, when $M_1 + M_2 = M = \text{const}$. This formula has been derived using assumptions of the equation of motion of the stars with variable masses permit an integral of the moment of the motion amount.

$$\mathbf{J} = M_1 \mathbf{R}_1 \times \mathbf{V}_1 + M_2 \mathbf{R}_2 \times \mathbf{V}_2 = \text{const}, \quad (2)$$

where $M_i$, $\mathbf{R}_i$, $\mathbf{V}_i$, $(i = 1, 2)$ - the mass, radius-vector and the motion speed of the stars.

The assumption of the existence of the integral of the moment of the motion amount is wrong. To let us prove this suggestion please take a look at, based on the most common form of the equations for the Meshcherskijs motion task of two bodies with variable masses:

$$M_1 \frac{d\mathbf{V}_1}{dt} = G \frac{M_1 M_2}{R^3} \mathbf{R} + \mathbf{Q}_1, \quad M_2 \frac{d\mathbf{V}_2}{dt} = -G \frac{M_1 M_2}{R^3} \mathbf{R} + \mathbf{Q}_2, \quad (3)$$
where \( \mathbf{R} = \mathbf{R}_2 - \mathbf{R}_1 \), \( Q_1 \) \( Q_2 \) — the reaction forces acting on the star \( S_1 \) and \( S_2 \).

The existence of the integral (2) in the task of two bodies with variable masses is possible only in the Meshcherskij-Levi-Civitas problem, the equations of the motion are obtained from (3), if the reaction forces are equal:

\[
Q_1 = -\dot{M}_1 \mathbf{V}_1, \quad Q_2 = -\dot{M}_2 \mathbf{V}_2, \tag{4}
\]

where \( \dot{M}_1 = -\dot{M}_2 = \text{const}, \dot{M}_2 > 0 \).

However, using the Meshcherskij-Levi-Civitas model to research the motion of the stars in a close binary system with a conservative exchange content is unreasonable because reaction forces (4) in close binary system does not exist.

Nowadays, the department of celestial mechanics SAI MSU (L.Lukyanov, S. Hasanov) developed a model, which determines the motion of close binary systems in the case of the elliptical orbit. In this work, we consider a further adjustment of the trajectories of the stars with the influence of the possible distribution of the flowing stream parameters.

2. Restricted elliptical three-body task

To determine the motion of the particles flowing masses we will use a flat elliptical restricted three-body task, the equations of motion, which are located in a rotating and pulsating Shapners barycentric coordinate system

\[
\begin{align*}
\frac{d^2 x}{dv^2} - 2 \frac{dy}{dv} &= \rho \frac{\partial U}{\partial x}, \\
\frac{d^2 y}{dv^2} + 2 \frac{dx}{dv} &= \rho \frac{\partial U}{\partial y},
\end{align*} \tag{5}
\]

where \( x \) axis is always directed at the star-accretor \( S_2 \), \( v \) — true anomaly stars \( \rho = 1/(1+e \cos v) \) — dimensionless radius stardom \( S_2 \) relatively \( S_1 \). \( U \) denotes the Jacobis function in restricted elliptic three-body task

\[
U = \frac{x^2 + y^2}{2} + p^3 \left( \frac{1 - m}{r_1} + \frac{m}{r_2} \right) - \frac{p^2}{2} (3 + m^2 - m), \tag{6}
\]

\( r_1, r_2 \) — the distance from the particle stream flowing respectively to the centres of mass of the first and second stars

\[
r_1 = \sqrt{(x + pm)^2 + y^2}, \quad r_2 = \sqrt{(x + pm - p)^2 + y^2}, \tag{7}
\]

where \( 1 - m = M_1/M \) and \( m = M_2/M \) — relative masses of the stars, \( p = a(1 - e^2) \) — focal parameter of the stars orbit, \( a \) and \( e \) — axis and eccentricity, respectively.

Well assume that the host star has a spherical

\[
(x + pm - p)^2 + y^2 = \frac{p^2}{\rho^2}, \tag{8}
\]

shape radius of \( P \) which in the process of mass transfer changes according to the dependence.

\[
P = P_0 \sqrt[3]{\frac{m}{m_0}}, \tag{9}
\]

Outflow of the matter from the donor star starts when the energy level of the particles is larger than the energy level of the singular point of Euler \( L_1 \). Speed of expiration \( V_0 \) of the mass
particles from the star $S_1$ is always directed toward the star $S_2$, forming the axis $Ox$ an angle $\pm \alpha$ and has a value

$$V_0(v) = V_00V_1(v) \beta, \quad V_1(v) = m\sqrt{\frac{GM}{\rho}}\sqrt{1 + 2e \cos v + e^2}.$$  

(10)

In the initial conditions, the angle $\alpha$ between the jet and the horizontal axis changes within the limits of $-\pi/2 < \alpha < \pi/2$, and the parameter $\beta$ — between $0.5 < \beta < 2$.

Numerical integration is performed on the interval $v_0 \leq v \leq v_0 + \tau$, where the value of the true anomaly $v_0 + \tau$ determined being deposited on the surface of the particles of the second star at $x_2 = x(v_0 + \tau), y_2 = y(v_0 + \tau)$. Without doubts these results can be corrected by introducing other disturbing factors and making new assumptions based on observations.

As the result of numerical integration the components of the speed of expiration $W_1$ of the star $S_1$ and of the speed mass flow $W_2$ of the star $S_2$ are obtained

$$W_{1x} = V_c, \quad W_{1y} = 0, \quad W_{2x} = a_x + b_x(\omega + \Omega), \quad W_{2y} = a_y + b_y(\omega + \Omega),$$  

(11)

where

$$a_x = x'(v_0 + \tau)\dot{v}, \quad b_x = y(v_0 + \tau), \quad a_y = y'(v_0 + \tau)\dot{v}, \quad b_y = -x(v_0 + \tau) - pm + p$$

reaction forces, influencing the stars $S_1$ and $S_2$, are considered to be attached to their center of mass and are given by

$$Q_1 = \dot{m}M\{ -V_c, 0 \}, \quad Q_2 = \dot{m}M\{ x'(v_0 + \tau)\dot{v}, y'(v_0 + \tau)\dot{v} \}. $$  

(12)

A numerical integration is determined as the mass of the steady stream $S_3$ and coordinates $x_3, y_3$ its center of mass

$$M_3 = \frac{\tau}{\dot{v}_c}\dot{m}M, \quad x_3 = \frac{1}{\tau} \int_{v_0}^{v_0 + \tau} x(v)dv, \quad y_3 = \frac{1}{\tau} \int_{v_0}^{v_0 + \tau} y(v)dv,$$  

(13)

where

$$\dot{v}_c = \frac{1}{2\pi} \int_0^{2\pi} \dot{v}dv = \frac{1}{2\pi} \int_0^{2\pi} \sqrt{\frac{GM}{\rho^3} (1 + e \cos v)^2} dv = \sqrt{\frac{GM}{\rho^3}} \left(1 + \frac{e^2}{2}\right)$$

—the mean angular velocity of the orbital motion.

3. Differential equations stars

We explain stars’ motion in the inertial coordinate system on a basis of the model which uses the following equations

$$M_1 \frac{dV_1}{dt} = G \frac{M_1 M_2}{R^3} \mathbf{R} + Q_1 + G \frac{M_1 M_3}{r_{13}^3} \mathbf{r}_{13},$$

$$M_2 \frac{dV_2}{dt} = -G \frac{M_1 M_2}{R^3} \mathbf{R} + Q_2 + G \frac{M_2 M_3}{r_{23}^3} \mathbf{r}_{23},$$

(14)

where $r_{13} = \sqrt{(x_3 + pm)^2 + y_3^2}, \quad r_{23} = \sqrt{(x_3 + pm - p)^2 + y_3^2}$ — the distances between the centres of mass of the stars and the jet.
Differential equations in osculating elements for semi-major axis of the relative orbit \( a \), the eccentricity \( e \) and the angular rotational speed \( \omega \) of the star \( S_2 \) are presented as

\[
\frac{da}{dt} = \frac{2a^2}{\sqrt{GMa(1-e^2)}} [e \sin v \cdot S + (1 + e \cos v) \cdot T],
\]

\[
\frac{de}{dt} = \sqrt{\frac{a(1-e^2)}{GM}} \left[ \sin v \cdot S + \left( \cos v + \frac{\cos v + e}{1 + e \cos v} \right) \cdot T \right],
\]

\[
\frac{d\omega}{dt} = \frac{5}{2} \frac{m}{m'} \sin \gamma \frac{W_{otn}}{P'} = - \frac{5}{2} \frac{m}{P'^2m} [A(\omega + \Omega) + B]
\]

Perturbing acceleration of \( S \) and \( T \) is becoming known after the integration of the equations (5):

\[
S = \frac{\hat{m}}{m} \left( a_y + b_y(\omega + \Omega) + \frac{mV_e}{1-m} + GM\tau \frac{\hat{m}}{\bar{v}_e} \left( \frac{x_3 + pm - p}{r_{23}^3} - \frac{x_3 + pm}{r_{31}^3} \right) \right),
\]

\[
T = \frac{\hat{m}}{m} \left[ a_y + b_y(\omega + \Omega) + GM\tau \frac{\hat{m}}{\bar{v}_e} y_3 r_3, \quad r_3 = \frac{1}{r_{23}^3} - \frac{1}{r_{31}^3} \right],
\]

By choosing the true stars anomaly \( v \) and performing averaging using the independent variable, we obtain

\[
\frac{d\hat{a}}{dv} = \frac{1}{2\pi} \int_0^{2\pi} \frac{2a^2}{\hat{v}} \sqrt{GMa(1-e^2)} [e \sin v \cdot S + (1 + e \cos v) \cdot T] dv,
\]

\[
\frac{d\hat{e}}{dv} = \frac{1}{2\pi} \int_0^{2\pi} \frac{\hat{a}(1-e^2)}{\sqrt{GMa(1-e^2)}} \left[ \sin v \cdot S + \left( \cos v + \frac{\cos v + e}{1 + e \cos v} \right) \cdot T \right] dv
\]

\[
\frac{d\hat{\omega}}{dv} = \frac{5}{2\hat{v}} \frac{m}{m'} \sin \gamma \frac{W_{otn}}{P'} = - \frac{5}{2} \frac{m}{P'^2m} [A(\omega + \Omega) + B]
\]

if, in addition to the independent variable \( m \) we find according to the formula \( \frac{d}{dv} = \frac{\hat{m}}{\bar{v}} \frac{d}{dm} \), the equations of motion for the average of elements \( \hat{a}, \hat{e}, \hat{\omega} \) with regard to the expression \( \hat{v} \) will take the form

\[
\frac{d\hat{a}}{dm} = \frac{2 + \hat{e}^2}{(1-\hat{e}^2)^2} \frac{\tilde{W}_{2y}}{m} + 2\hat{a}^3 \sqrt{1-\hat{e}^2} \tau y_3 r_3 + \frac{\hat{a}^2(2 + \hat{e}^2)}{\sqrt{GM\hat{a}(1-\hat{e}^2)}} \frac{\tilde{b}(\hat{\omega} + \Omega)}{m},
\]

\[
\frac{d\hat{e}}{dm} = \frac{2 + \hat{e}^2}{2\hat{a}(1-\hat{e}^2)} \frac{\hat{e}}{1 + \sqrt{1-\hat{e}^2}} \frac{\tilde{W}_{2y}}{m} - \frac{3}{2} \hat{e} \hat{a}^2 \sqrt{1-\hat{e}^2} \tau y_3 r_3 - \frac{\tilde{b}(\hat{\omega} + \Omega)}{4\sqrt{GM\hat{a}(1-\hat{e}^2)}}
\]

\[
\frac{d\hat{\omega}}{dm} = \frac{5}{4} \frac{2 + \hat{e}^2}{(1-\hat{e}^2)^{3/2} P'^2 m} \left[ \tilde{W}_{2x} \tilde{y} + \tilde{W}_{2y} \tilde{b} + (\tilde{b}^2 + \tilde{y}^2)(\omega + \Omega) \right]
\]

where

\[
\tilde{x} = x (v_0 + \tau), \quad \tilde{y} = y (v_0 + \tau), \quad \tilde{W}_{2x} = x'(v_0 + \tau), \quad \tilde{W}_{2y} = y'(v_0 + \tau), \quad \tilde{b} = p - pm - \tilde{x} = \tilde{a}(1 - \hat{e}^2)(1 - m) - \tilde{x}.
\]
4. Numerical results

Account of the interaction of gravity of stars, star attraction jet and reactive power with the translational and rotational motion

**Figure 1.** On the left picture is dependence of the semi-major axis of the mass ratio, on the right dependence on the mass ratio of eccentricity

**Figure 2.** Dependence of the angular velocity of the mass ratio

Here shows the power modules for dependencies.
5. Conclusions

For half a century the incorrect model of Paczynskij-Huang was used to determine the relative orbit of close binary stars. But to this day in the work associated with the close binary systems, the model still has been used. Because we consider this model is wrong, the purpose of this paper was to show fallacy of this model using other methods. To determine the relative motion of stars in a close binary system in this paper we use a numerical integration of the equations of motion with the reactive forces of attraction and stars flowing stream. The calculations of elliptical orbits of close binary stars show that the effect of the reactive force on the evolution of the orbits of stars may be different. There is no doubt that the results can be corrected by introducing other disturbing factors and making new assumptions based on observations.

6. References

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