3D magnetic field vector measurement by magneto-optical imaging

H. Sakaguchi, R. Oya, S. Wada*, T. Matsumura*, H. Saito*, and T. Ishibashi
Department of Materials Science and Technology, Nagaoka University of Technology, 1603-1 Kamitomioka, Nagaoka, Niigata 940-2188, Japan
*Department of Mathematical Science and Electrical-Electronic-Computer Engineering, Akita University, 1-1 Tegata Gakuen-machi, Akita 010-8502, Japan

Magneto-optical (MO) imaging using MO imaging plates is a magnetic imaging technique that enables real-time measurements and is expected to be used for non-destructive testing and for observing the magnetic domains of magnetic materials. In this study, we propose a quantitative measurement method for three dimensional (3D) magnetic field vector measurements. The x- and y-components of the magnetic fields within the measured plane are calculated from the measured z-component by using a signal transformation method based on magnetic field transfer functions. Furthermore, the magnetic field distributions at different heights are also obtained from a one-shot image. In this paper, 3D magnetic field vector measurements are demonstrated for ferrite magnets and an electrical steel sheet.

Key words: magneto-optical imaging, MO imaging plate, polarization camera, magnetic field vector

1. Introduction

Magneto-optical (MO) imaging using an MO imaging plate is a technique to visualize a strayed magnetic field by utilizing the MO effects. By using bismuth-substituted iron garnets, which exhibit a large Faraday effect in the visible light region, as the MO imaging plate, observation of magnetic domains in magnetic materials and in-situ observation of magnetic flux quanta in superconductors have been realized with high spatial resolution and magnetic field sensitivity. In addition, it is attracting attention as a large area magnetic measurement technology that could replace the scanning hall probe method, since the large MO imaging plates using large glass substrates have been developed. In MO imaging, it is necessary to calibrate the magnetic field from the light intensity in order to quantitatively measure the magnetic field distribution, although it is a simple method that can be measured using an optical system like a polarizing microscope. Jooss et al. have proposed a method to calibrate the magnetic field from the light intensity in the crossed Nicol method, in which the large number of parameters to be measured beforehand. We proposed the circular polarization modulation method to quantitatively measure the rotation angle and the ellipticity simultaneously, which can be applied to the MO imaging. However, it was difficult to realize the real-time measurement because three images measured with linearly-, right circularly- and left circularly-polarizations had to be measured.

In this paper, we report on an MO imaging technique that quantitatively measures the magnetic field distribution from the one-shot image measured using the polarization camera that has been widely used in the industry in recent years. We also report on a signal transformation method that calculates the magnetic field vectors from a measured MO image. In this paper, measurements of the magnetic field distributions of ferrite magnets are demonstrated. A measurement of an electrical steel sheet is also demonstrated.

2. Methods

2.1 MO imaging using the polarization camera

The polarization camera is a camera having a structure that integrates an image sensor with a micro-polarizer array consisting of multiple polarizers with different angles of polarization, 0°, 45°, 90° and 135°, formed according to the pixel size. Therefore, four different polarization images with polarization angles of 0°, 45°, 90° and 135° are measured with one-shot measurement.

This means that rotation angles of polarization are quantitatively measured with a one-shot measurement using the polarization camera. Consequently, once four polarization images with intensities \( I_\theta, I_{45\theta}, I_{90\theta}, I_{135\theta} \), we can obtain the Stokes parameter, \( S_0, S_1 \) and \( S_2 \) as described by

\[
\begin{bmatrix}
S_0 \\
S_1 \\
S_2 \\
\end{bmatrix} = \begin{bmatrix}
I_\theta + I_{45\theta} + I_{90\theta} + I_{135\theta} \\\nI_\theta - I_{45\theta} - I_{90\theta} + I_{135\theta} \\\nI_{45\theta} - I_{90\theta} - I_{135\theta} \\\n\end{bmatrix} / 2.
\]

The relation between the Stokes parameter and the polarization state is defined by

\[
\begin{bmatrix}
S_0 \\
S_1 \\
S_2 \\
\end{bmatrix} = \begin{bmatrix}
1 \\
\cos 2\eta \cos 2\theta \\
\sin 2\theta \\
\end{bmatrix},
\]

where intensity is normalized as \( S_0 = 1 \), \( \theta \) is the rotation angle of the polarization plane and \( \eta \) is the ellipticity. Consequently, the rotation angle \( \theta \) can be written by...
\[ \theta = \frac{1}{2} \tan^{-1}\left(\frac{S_z}{S_y}\right). \]  (3)

In MO imaging, since the rotation angle due to the MO effect is measured, it is necessary to adjust the origin of the polarization, \( \theta = 0^\circ \), corresponding to the absence of an applied magnetic field, which could be obtained by subtracting a background image from a measured image. In addition, the rotation angle measurement using the polarization camera has a disadvantage that it analyzes one polarization from four adjacent pixels, which may result in deterioration of the spatial resolution. For this problem, the pixel interpolation method using Fourier analysis has been proposed.

### 2.2 Transformation of magnetic field

Fig. 1 shows a schematic drawing of calculation model for the MO imaging. The magnetic field \( H(r) \) at the measurement point \( r = (x, y, z) \) in the MO imaging plate is given by the convolution integral of the magnetic pole \( \rho(r') \) at the certain place \( r' = (x', y', z') \) on the sample surface and the transfer function \( G_H \), which is given by

\[ H(r) = \int r \cdot \rho(r') \cdot G_H(r-r') dr', \]  (4)

where \( G_H(r) \) is given from Coulomb's law for magnetism as

\[ G_H(r) = \frac{1}{4\pi\mu_0 |r|^3}. \]  (5)

In the frequency domain, the convolution integral in Eq. (5) is given in the form of multiplication according to the convolution theorem as

\[ H(k_x, k_y, z) = \rho(k_x, k_y, z') \cdot G_H(k_x, k_y, z - z'). \]  (6)

where \( H(k_x, k_y, z) \) and \( G_H(k_x, k_y, z) \) are the Fourier transforms of the magnetic field distribution and the transfer function. Consequently, the Fourier transforms of \( x \)-component of the magnetic field distribution \( H_x(r) \) and \( y \)-component of the magnetic field distribution \( H_y(r) \) can be deduced from \( z \)-component of the magnetic field distribution \( H_z(r) \) and the transfer functions as

\[ H_x(k_x, k_y, z) = \frac{G_{Hx}(k_x, k_y, z - z')}{G_{Hz}(k_x, k_y, z - z')} \cdot H_z(k_x, k_y, z) \]
\[ = T[H_z \rightarrow H_x]H_z(k_x, k_y, z) \]  (7)

and

\[ H_y(k_x, k_y, z) = \frac{G_{Hy}(k_x, k_y, z - z')}{G_{Hz}(k_x, k_y, z - z')} \cdot H_z(k_x, k_y, z) \]
\[ = T[H_z \rightarrow H_y]H_z(k_x, k_y, z). \]  (8)

where components of \( G_H(k_x, k_y, z) \) are given by

\[ G_{Hz}(k_x, k_y, z) = \frac{-ik_x}{2\mu_0 \sqrt{k_x^2 + k_y^2}} \exp\left(-\frac{k_x^2 + k_y^2}{2\mu_0}z\right), \]  (9)

\[ G_{Hy}(k_x, k_y, z) = \frac{-ik_y}{2\mu_0 \sqrt{k_x^2 + k_y^2}} \exp\left(-\frac{k_x^2 + k_y^2}{2\mu_0}z\right), \]  (10)

\[ G_{Hz}(k_x, k_y, z) = \frac{1}{2\mu_0} \exp\left(-\frac{k_x^2 + k_y^2}{2\mu_0}z\right). \]  (11)

Consequently, the transformation filters, \( T[H_z \rightarrow H_x] \) and \( T[H_z \rightarrow H_y] \), are given by

\[ T[H_z \rightarrow H_x] = \frac{-ik_x}{\sqrt{k_x^2 + k_y^2}}, \]  (12)

\[ T[H_z \rightarrow H_y] = \frac{-ik_y}{\sqrt{k_x^2 + k_y^2}}. \]  (13)

Finally, \( H_x(r) \) and \( H_y(r) \) are obtained by as the inverse Fourier transforms of \( H_x(k_x, k_y, z) \) and \( H_y(k_x, k_y, z) \), respectively.

Furthermore, the signal transformation based on the transfer function can be used to calculate \( H_x(x, y, z + \Delta z) \) at a position that is increased by a certain distance \( \Delta z \) from the measured \( H_x(x, y, z) \). In this case, the transformation filter \( T[H_z, z \rightarrow z + \Delta z] \) becomes

\[ T[H_z, z \rightarrow z + \Delta z] = \exp\left(-\frac{k_x^2 + k_y^2}{2\mu_0}\Delta z\right). \]  (14)

By combining the distance transformation method with the \( x \) and \( y \)-components calculation method, it is possible to obtain the three dimensional (3D) magnetic field distribution from the one-shot measurement.

### 3. Experimental

#### 3.1 MO imaging system using the polarization camera

Fig. 2 shows a schematic drawing of the MO imaging system used in this study. A red LED panel (MISUMI, LEDXR120) with a central wavelength of 630 nm was used as a light source. The linearly-polarized light obtained by a polarizer put on the LED panel vertically illuminated the MO imaging plate. The reflected light
was reflected by a half mirror and captured by a polarization camera (Baumer, VCXU-50MP) having a CMOS with \(2448 \times 2048\) pixels. Image data were imported to a computer via USB connection and were processed by a program written in Python. For the MO imaging plate, Bi-substituted iron garnet films \(\text{Nd}_{0.5}\text{Bi}_{2.5}\text{Fe}_5\text{O}_{12}\), with a thickness of 600 nm were prepared by a metal-organic decomposition (MOD) method on a glass substrate (Eagle XG, CORNING). Details of the garnet films were described in Ref.14,15,21,22. A 150 nm-thick silver layer was deposited by the sputtering method on the garnet films as a reflective film. The Kerr rotation hysteresis measured by applying a magnetic field in the perpendicular direction to the plane of the MO imaging plate is shown in Fig. 3. Here, the Kerr rotation is equivalent to the Faraday rotation of the garnet film, since the light goes through the garnet film and come back after being reflected by the reflective film. The shape of the hysteresis shows that the rotation angle varies continuously with the perpendicular magnetic field, indicating the easy axis of magnetization is in the in-plane direction. The calibration from the Kerr rotation angle to the magnetic field was done using a linear approximation formula obtained within the range of magnetic saturation. Measurement errors caused by coercivity could solve by using \(\text{Gd}_3\text{Ga}_5\text{O}_{12}\) (GGG) substrates, which exhibit small coercivity \(^{23}\). Four rectangular-shaped ferrite magnets with a size of 10 mm \(\times 30\) mm \(\times 5\) mm were used as a sample. The distance \(\Delta z\) from the MO imaging plate was varied from 0 mm to 10 mm.

### 3.2 MO imaging of ferrite magnets

Fig. 4 shows a photo of ferrite magnets, \(H_\perp(x,y,0)\) measured by the one-shot measurement with an exposure time of 10 msec, and \(H_x(x,y,0)\) and \(H_y(x,y,0)\) calculated by the process described in Sec. 2. The 3D magnetic field vectors plotted using \(H_\perp(x,y,\Delta z)\) measured at \(\Delta z = 0, 3, 6\) mm and calculated \(H_x(x,y,\Delta z)\) and \(H_y(x,y,\Delta z)\) are shown in Fig. 5. The magnetic poles of the magnets were clearly observed in \(H_\perp(x,y,0)\), and it was confirmed that sufficient contrast could be obtained even with the one-shot measurement. The values of \(H_x(x,y,0)\) consistent with that measured on the sample surface using a Gauss meter, \(H_x(x,y,0)\) showed a strong contrast at the boundaries and left and right edges of the magnetic poles, and a weak contrast at the center of each.
magnet. This result is consistent that the magnetic flux travels from the N-pole to the S-pole. $H_z(x,y,\Delta z)$ showed a strong contrast at the top and bottom edges of the magnet and a weak contrast at the center, which is consistent with the behavior of the magnetic flux. These features are consistent with a result of a simulation using Femtet (Murata Software Co., Ltd.), which it is not shown here.

Next, we describe the results of $H_z(x,y,\Delta z)$ calculated from $H_z(x,y,0)$ shown in Fig. 4(b). $H_z(x,y,\Delta z)$ calculated at $\Delta z = 1.5, 10\, \text{mm}$ are shown in the lower part of Fig. 6(a), and those of actually measured images at $\Delta z = 1.5, 10\, \text{mm}$ are shown in the upper part for comparison. The line profiles were measured along the center of the magnets as shown with a dotted line in Fig. 6(a). In the actual measurement, the intensity decreases and becomes gradually blurred as the height is increased. The same trend was confirmed in the results of the signal transformation. From the results of the line profile comparison, it was found that the calculated values of the magnetic fields were in good agreement with the measured values except in the vicinity of the peaks. For the height of 1 mm, the calculated data was 18% lower than the measured data, and for the height of 10 mm, the calculated data was 11% higher than the measured data. The difference between the measured and calculated data may be due to the fact that the image contains unnecessary information at the edge of the plate and outside of the MO imaging plate, that is not related to the magnetic field distributions, or that the sample size is too large as compared to the size of the image, resulting in lack of information in the Fourier transforms.

### 3.3 MO imaging of the electrical steel sheet

We have reported MO imaging over a wide area of a few centimeters, on the other hand, the spatial resolution of MO imaging can reach 0.3 $\mu\text{m}$, indicating the microscopic scale observations are also possible. The MO imaging plate was placed on a 0.2 mm-thick unpolished electrical steel sheet, and MO imaging was performed with a magnetic field of 465 Oe applied along in-plane direction. Fig. 7 shows an optical image of the sample surface, measured $H_z(x,y,0)$ and calculated $H_z(x,y,0)$, $H_y(x,y,0)$ and azimuth angle $\phi(x,y,0)$ of magnetic field vector. From $H_z(x,y,0)$, strayed magnetic fields from grain boundaries were clearly observed, and stripe magnetic domains were also confirmed. The strayed magnetic field from the grain boundary and the stripe magnetic domain were also observed in the calculated in-plane components. Coarse surface observed in the entire image is thought to be caused by the grain structure of the garnet. It should be noted that longitudinal Kerr microscopy, which is commonly used to observe magnetic domains, requires mirror like surfaces obtained by surface treatments such as polishing, on the other hand, the MO imaging using MO imaging plate has capabilities for visualizing magnetic domain structures without surface treatment.

![Fig. 6](image-url) (a) Measured $H_z(x,y,\Delta z)$ at $\Delta z = 1.5, 10\, \text{mm}$ and calculated $H_z(x,y,\Delta z)$, (b) line profile along dotted line.

![Fig. 7](image-url) (a) Optical image of untreated electrical steel sheet and magnetic field images, (b) measured $H_z(x,y,0)$ and calculated (c) $H_y(x,y,0)$, (d) $H_y(x,y,0)$ and (e) azimuth angle $\phi(x,y,0)$.

### 4. Conclusion

The 3D magnetic field vectors measurements in a 75 mm diameter area with the one-shot measurement by MO imaging using the polarization camera was demonstrated.

The in-plane components, $H_x(r)$ and $H_y(r)$, were calculated using the signal transformation based on the
transfer function from measured $H_z(r)$ by the MO imaging. It was also shown that $H_z(x,y,\Delta z)$ were obtained from measured $H_z(x,y,0)$. We conclude that this MO imaging technique combined with the signal transformation technique could be a powerful tool for measuring 3D magnetic field vectors in a short time with high accuracy.

Acknowledgements The author thanks Mr. Takashi Yamaguchi of YAMAGUCHI MFG Co., Ltd. for providing the electrical steel sheet. This research was supported in part by JSPS KAKENHI (JP18H03776), the Nanotechnology Platform Program (Molecule and Material Synthesis) of the Ministry of Education, Culture, Sports, Science and Technology (MEXT), Japan, and a joint research program with the Institute of Materials and Systems for Sustainability, Nagoya University.

References
1) S. Gotoh, N. Koshizuka, M. Yoshida, M. Murakami, and S. Tanaka: J. Appl. Phys., 29 1083 (1990).
2) M. V. Indenbom, N. N. Kolesnikov, M. P. Kulakov, I. G. Naumenko, V. I. Nikitenko, A. A. Polyanskii, N. F. Vershinin, and V. K. Vlasko-Vlasov: Physica C, 166 486 (1990).
3) T. H. Johansen, M. Baziljevich, H. Bratsberg, Y. Galperin, P. E. Lindelof, Y. Shen, and P. Vase: Phys. Rev. B, 54, 16264 (1996).
4) Ch. Jooss, A. Forkl, R. Wrthmann, H.-U. Hbermeier, B. Leibold, and H. Kronmüller: Physica C, 266, 235 (1996).
5) Ch. Jooss, J. Albrecht, H. Kuhn, S. Leonhardt, and H. Kronmüller: Rep. Prog. Phys., 65, 651 (2002).
6) T. Ishibashi, Z. Kuang, S. Yufune, T. Kawata, M. Oda, T. Tani, Y. Imura, and K. Sato: J. Appl. Phys., 100, 093903 (2006).
7) H. Lee, T. Kim, S. Kim, Y. Yoon, S. Kim, A. Babajanyan, T. Ishibashi, B. Friedman, and K. Lee: J. Magn. Magn. Mater., 322, 2722 (2010).
8) T. Ishibashi, G. Lou, A. Meguro, T. Hashinaka, M. Sasaki, and T. Nishi: Sensors and Materials, 27, 965 (2015).
9) W. C. Patterson, N. Garraud, E. E. Shorman, and D. P. Arnold: Rev. Sci. Instrum., 86, 097404 (2015).
10) T. Ishibashi, T. Yoshida, K. Baba, T. Liu, G. Lou, and T. Ishibashi, J. Magn. Soc. Jpn., 41, 29 (2017).
11) T. Ishibashi: J. Magn. Soc. Jpn., 44, 108 (2020).
12) P. E. Goa, H. Hauglin, À. A. F. Olsen, M. Baziljevich and T. H. Johansen: Rev. Sci. Instr., 74, 141 (2003).
13) S. W. Clark and D. Stevens: IEEE Trans. Ind. Appl., 52, 1469 (2016).
14) T. Yoshida, K. Oishi, T. Nishi, and T. Ishibashi: European Phys. J. Web of Conf., 75, 05009 (2014).
15) G. Lou, T. Yoshida, and T. Ishibashi: J. Appl. Phys., 117, 17A749 (2015).
16) G. Lou, T. Kato, S. Iwata, and T. Ishibashi: Opt. Mater. Express, 7, 2248 (2017).
17) Y. Nagakubo, M. Sasaki, S. Meguro, M. Nishikawa, and T. Ishibashi: J. Appl. Phys., 57, 09TC02 (2018).
18) Y. Maruyama, T. Terada, Y. Yamazaki, Y. Uesaka, Y. Nakamura, Y. Matoba, K. Komori, Y. Ohba, S. Arakawa, Y. Hirasawa, Y. Kondo, N. Murayama, K. Akiiya, Y. Oike, S. Sato, and T. Ezaki: IEEE Trans. Elect. Devices, 65, 6 (2018).
19) J. S. Tyo, C. F. LaCasce, and B. M. Ratliff: Opt. Lett., 34, 29 (2009).
20) H. Saito, J. Chen, S. Ishio: J. Magn. Magn. Mater., 191, 153 (1999).
21) T. Ishibashi, T. Kawata, T. H. Johansen, J. He, N. Harada, and K. Sato: J. Magn. Soc. Jpn., 32, 150 (2008).
22) T. Ishibashi, A. Mizusawa, M. Nagai, S. Shimizu, and K. Sato: J. Appl. Phys., 97, 013516 (2005).
23) M. Sasaki, G. Lou, Q. Lin, M. Ninomiya, T. Kato, S. Iwata, and T. Ishibashi: J. Appl. Phys., 55, 055501 (2016).

Received Nov. 4, 2021: Revised Dec. 27, 2021: Accepted Jan. 15, 2022