Mathematical Description of Dynamic Processes Occurring in Rolling Bearings Used in Oil-And-Gas Sector Rotor Machines as the Basis for Their Vibration-Based Diagnostics

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Abstract. The rolling bearings used in modern rotor machines in the oil-and-gas sector are seen as crucial components. The accuracy of their production complies with the highest operational requirements. However, bearings are also the most probable cause of rotor machine failures. This can be attributed to the friction in support assemblies. This article considers the mathematical dependencies reflecting the dynamic processes occurring in rolling bearing during operation. It will help construct a modern test-range model for vibration-based diagnostics of bearing systems. The experiments with the practical use of this model showed a fault detection accuracy of 82-84%, which is a very good result.

1. Introduction
Modern equipment requires unique reliable evaluation methods for its current, as well as some past and future operations. The developed methods must be applicable in production contexts, sufficiently accurate and reliable, and accessible for production workers. [12, 14].

2. Relevance
Currently, non-destructive inspection methods are widely used to assess the technical condition of machinery in the oil-and-gas sector. The improvement of diagnostic techniques is a very promising area of activity. For instance, it is very important to develop efficient control methods for the engineering parameters of support elements of rotor machines operated in the oil-and-gas sector. Establishing the mathematical bases is basic for the development of vibration-based diagnostic techniques for a bearing system in such rotor machines.

3. Statement of problem
For the correct understanding of the dynamic processes occurring in rolling bearings that impact their operability and cause vibrations, we need to analyze the geometrical and mathematical correlations describing the kinematics and dynamics of the support structures in question. We also need to analyze the vibration ranges of rolling bearings with defects and damages. [8,9,10].

Theory. The graphic representation of dynamic impacts and geometric parameters of a rolling bearing is shown in Figure 1. In that figure, you can see the following circumferential velocities: A - the
circumferential velocity of a point, $B$ - of the running surface of the inner race, $C$ - of the center of the rolling element, which is twice as small as the circumferential velocity of the inner race.

Assume that $d_T$ is the rolling element diameter;

$D_c$ is the diameter of the circumference that passes through all of the rolling element centers;

$D_v$ is the diameter of the circumference that is the geometric location of the contact point of rolling elements and the running surface of the inner race;

$n_c$ is the number of revolutions per minute for the rolling element centers (of the retainer ring) around the bearing axis $O$;

$n_v$ is the number of revolutions per minute for the inner race of the bearing.

\[
\nu_{c} = \frac{\pi D_c n_c}{60}
\]  

(1)

The circumferential velocity \( \nu_{b} \) for the point $B$ on the running surface of the inner race

\[
\nu_{b} = \frac{\pi D_b n_B}{60}
\]

(2)

since

\[
\nu_{b} = 2\nu_c, \quad D_s = D_c - d_T \cos \beta
\]

\[
(D_c - d_T \cos \beta)n_s = 2D_c n_c
\]

then

\[
n_c = \frac{n_s}{2} \cdot \frac{D_c - d_c \cos \beta}{D_c}
\]

(3)

i.e., for one revolution of the inner race, the retaining ring (Figure 1, a) together with the set of rolling elements rotates a little less than half a revolution (for single-row radial bearings $\cos \beta = 1$).
We must note that if the bearing's outer race rotates when the inner ring is unmoved (fixed axis and rotating housing), then for \( \nu_H = 2\nu_c \) and \( D_H = D_c + d_T \cos \beta \) we get (Figure 1, b)

\[
h'_n = \frac{n_H}{2} \cdot \frac{D_c + d_T \cos \beta}{D_c}
\]

where \( n_n \) is the number of revolutions per minute for the rotating outer race;
\( \nu_H \) is the circumferential velocity of the running surface of the outer race;
\( D_H \) is the diameter of the circumference that is the geometric location of the contact point of rolling elements and the running surface of the outer race [1,2,3].

In this case, when the outer ring rotates once, the retaining ring rotates a little more than half a revolution. Thus, the retaining ring rotation rate and the appearance of amplitude modulations in the vibration diagram that signal about retaining ring faults should be within the range of \( f_{ret} = 0.4f_r - 0.6f_r \), where \( f_r \) is the machine rotor speed.

To determine the mathematical dependencies and the locations of the typical frequencies that can be informative for the assessment of the technical condition of the object in question and the description of vibration processes due to the impacts on bearing races, it is necessary to review the application of main loads that occur during rolling bearing operation. [18,19].

As we can see from the rolling bearing load diagram in Figure 2, the loads affecting the operating bearing (ball or roller-based) are distributed according to specific laws reviewed in detail below [4].

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**Figure 2.** The distribution diagram for the loads applied to the bearing.

The bearing consists of the outer race I which is rigidly connected to the machine housing, the inner race II connected to the shaft, and the rolling elements (balls or rollers) III. The bearing load \( Q \)
will have an uneven distribution across specific balls or rollers. Those of them that are on the application line of the force $Q$ (according to the antipodal point theory) will experience the highest loads. If we assume that only the balls or rollers located below the line will take the loads, we can claim that the response distribution law $R_n$ will look like the one shown in Figure 2 as the dashed line, i.e. the ball load will reduce the further they are from the line of force $Q$. It will equal zero for the balls whose centers are located on the line.

According to the contact deformation theory for elastic bodies, this problem can be solved assuming that the contacting bodies are smooth and homogeneous (although this is not realistic, some tolerances have to be made to avoid making a multiparameter problem impossible to be solved), only elastic deformations can take place in the contact area, and the pressure forces against the contact surfaces are normal (see Figure 2). At the same time, we can assume that the contact areas of rolling bodies and surfaces are small compared to the sizes of the contacting bodies. Theoretically, the contact area of balls with the rolling surfaces is a point, and the contact area of rollers is a line. Therefore, we can neglect the friction in the contact area occurring when loads are applied when solving this problem [5].

There is a dependency [5] stating that the actual contact area of these surfaces is an ellipse that can transform into a circle or a strip limited by parallel straight lines in extreme cases. These extreme cases take place for two bodies limited by spherical or cylindrical surfaces, which is the case with the rolling bearings in question (ball-based or roller-based with short rollers).

Thus, if some load $Q$ is applied to a bearing (see Figure 2) and rolling elements take forces $P_0$, $P_1$, ..., $P_n$, the balance equation looks as follows:

$$Q = P_0 + 2P_1 \cos \alpha + 2P_2 \cos 2\alpha + ... + 2P_n \cos n\alpha$$

(5)

where $\alpha$ is the angle between the rolling element axes.

If we accept the hypothesis that the pressure distribution law [15] is sinus-shaped and the sinus-shaped pressure distribution law looks the same as harmonic oscillations caused by the contact of rolling elements with the rolling surface (the superposition principle for the specific element within the vibration range of a specific component), i.e. we assume that the normal response $R_n$ for support structure satisfies

$$R_n = R_{n\text{ max}} \cos \alpha,$$

(6)

where $R_{n\text{ max}}$ is the response to the maximum ball load.

If we use the elasticity theory to solve equations (5) and (6) [4], the arithmetic sum of all responses will be as follows:

— for a ball bearing

$$\sum R_n = (1,3...1,35)Q,$$

(7)

— for a roller bearing

$$\sum R_n = (1,4...1,46)Q.$$  

(8)

Thus, the sum $\sum R_n$ for ball and roller bearings may change depending on various hypotheses used between $1.3Q$ and $1.46Q$.

Thus, when discussing the amplitude-frequency parameters, we should assume the following:
1 When the rolling surfaces are not damaged, all of the vibrations in the operating bearing are reflected in the “noise” characteristics, i.e. insignificant and negligible components of the vibration range [13,16].

2 When damages occur, there must be amplitude and frequency modulations for the retaining ring rotation rate of $f_{ret\,r}$ (the centers of rolling elements rotate around the bearing center or the shaft center with this rate). It is logical to assume that the appearance of modulations for a damaged retaining ring will always have typical amplitude surges, which can characterize rolling element damages because the retaining ring contacts them directly. I.e. damaged rolling elements may cause vibrations when they contact the intact retaining ring or when intact rolling elements contact a damaged retaining ring with the rolling element collision rate of $f_{\text{r.b.}}$

$$f_{\text{r.b.}} = f_{\text{ret\,r}} \times z,$$  \hspace{1cm} (9)

where $z$ is the number of rolling bodies.

3 If the rolling surfaces (bearing races) are damaged, there appear subharmonic modulations at a rate of $f_{\text{r.h.}} = f_{\text{r.b.}} \cdot (k \cdot n)$

where $k=1.3 \ldots 1.35$ for ball bearings and $k=1.4 \ldots 1.46$ for roller bearings,

$n=1,2,3$ is the order of harmonics. These subharmonic frequencies for inner rolling surfaces are located above the $f_{\text{r.h.}}$, and below $f_{\text{r.b.}}$ For outer races due to the different circumferential velocities on the surfaces of the rolling elements when they contact the inner or the outer race [6,7].

4. Research findings

Based on the mathematics provided above, we constructed a mathematical graph model (testing range) reflecting the correlations between the dynamic processes, as well as the amplitude and frequency parameters, and the faults occurring in the support structures of rotor machines [11,15,17].

![Mathematical Graph Model](image)

**Figure 3.** Integrated mathematical graph model (testing range) for the correlations of the amplitude and frequency parameters and the rolling bearing damages.

A – modulation amplitude, $f_{\text{rev}}$ – rotational modulation frequency of the machine rotor, $f_{\text{ret\,r}}$ – modulation frequency of the bearing retaining ring, $f_{\text{r.h.}}$ – modulation frequency of the rolling elements, $k$ – modulation harmonics of rolling surfaces
Based on the above, as well as the field tests conducted on the compressor plants of gas transmission pipelines, we can conclude the following:

1. When there are no damages on rolling surfaces, all of the vibration processes in the operating bearing stay within the vibration limits (the fault-free vibration according to the ISO 2373-74 standard is 0.2-0.3 mm/s). There is only the modulation of rotational frequency \( f_{\text{rev}} \) that does or does not exceed the standard value depending on the technical condition of the machine in question.

2. According to the statistics, one of the most widespread defects is the retaining ring wear or damage responsible for up to 28.3% of the total bearing failure count. According to the mathematics provided above, these problems result in frequency modulation of \( 0.45f_{\text{rev}} - 0.55f_{\text{rev}} \). (Figure 3).

3. The percentage of bearing failures caused by the rolling elements is comparable to those caused by the retaining ring (27.9% of the total failure count). Field experiments showed that this amplitude surge occurs at a rate of \( f_{\text{r.b.}} = f_{\text{sep}} \times z \), where \( z \) is the number of rolling elements (Figure 3).

4. Outer and inner race defects commonly occur after the rolling elements and the retaining ring are damaged. They also help obtain informative frequencies for the diagnostics of the outer and inner rolling surfaces. Following the dependencies shown above, race defects cause subharmonic modulations with frequencies of

\[
\begin{align*}
    f_{r,b.}^n &= f_{r,b.}(k \cdot n)
\end{align*}
\]

where \( k=1.3...1.35 \) for ball bearings and \( k=1.4...1.46 \) for roller bearings, and \( n=1,2,3 \) is the order of harmonics. The frequency modulations caused by the damages of the outer race are located below \( f_{r,b.} \) (Figure 3), and the inner race defects (Figure 3) are above the \( f_{r,b.} \) frequency. This can be attributed to the higher circumferential velocity on the surfaces of the rolling elements when they contact the inner race compared to when they contact the outer race.

5. To calculate the amplitude value for Order 1, 2, and 3 modulation harmonics of the damaged rolling surfaces in bearing supports, as well as the amplitude values for the rotational frequency modulation of the damaged retaining ring and rolling elements, we had to perform long-term field studies and measurements and develop a mathematical model reflecting their distribution pattern [thesis]. We observed the highest amplitude with the rotational frequency modulation \( A(f_{\text{rev}}) \approx 0.6 - 0.8A(f_{\text{rev}}) \). The modulation amplitude for damaged rolling bodies of the first harmonic in absolute values is at least \( 0.72A_{r,b.} \), while the rolling element amplitude of the second harmonic is \( 0.216A_{r,b.}, 0.072A_{r,b.} \) for the third harmonic. The development of harmonics depends on the degree of rolling surface damages. One or two harmonics can manifest themselves like this, and they do not have to be in both races at the same time. We noted that damages may occur on the inner race first and then on the outer race, or vice versa [6,7].

5. Conclusions
When the developed testing range was used at industrial facilities, the accuracy of bearing support fault detection was 82-84%. Thus, we can claim that the suggested mathematical techniques and the developed diagnostic procedures for the bearing supports of rotor machines used in the oil-and-gas sector based on vibration parameters are highly efficient.

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