Abstract—We address a challenge concerning the spectral-domain-based (i.e., plane wave eigenfunction expansion) analysis of electromagnetic fields produced by time-harmonic current sources within planar-layered media, which arises specifically when sources are embedded inside non-birefringent anisotropic medium (NBAM) layers. In NBAM, the highly symmetric permeability and permittivity tensors can induce directionally-dependent, but polarization independent, propagation properties supporting “degenerate” characteristic polarizations. That is to say, the considered NBAM support four linearly independent field polarization eigenvectors associated with only two (rather than four) unique, non-defective eigenvalues. We explain problems that can arise when the source(s) specifically reside within NBAM planar layers when using canonical field expressions as well as obtain alternative expressions, immune to such problems, that form the foundation for a robust eigenfunction expansion-based analysis of electromagnetic radiation and scattering within planar-layered media.

Index Terms—Stratified media; transformation optics.

I. INTRODUCTION

Environments with (locally) planar-layered profile are encountered in diverse applications such as geophysical exploration [1], [2], ground penetrating radar operation [3], atmospheric wave propagation [4], conformal antenna design [5], and so on. To facilitate electromagnetic (EM) radiation analysis in such environments, eigenfunction (plane wave) expansions (PWE) have long been used because of their relative computational efficiency versus brute-force numerical methods such as finite difference and finite element methods [2], [6], [7]. Moreover, PWE can accommodate linear, but otherwise arbitrary anisotropic layers characterized by arbitrary (diagonalizable) $3 \times 3$ material tensors [2]. This proves useful when modeling (for example) the performance of ground-plane-coating “isoimpedance” media and beam-shifting devices [5], [8], [9], designed by means of transformation optics techniques [10], [11]. These named, amongst other, scenarios share in common the potential presence of a particular type of anisotropic media in which the magnetic permeability ($\mu_r$) and electric permittivity ($\epsilon_r$) tensor properties together lead to media supporting four “degenerate” plane wave eigenfunctions that, while possessing four linearly independent field polarization states (eigenvectors) as usual, share only two unique (albeit, critically still, non-defective) eigenvalues [2], [12], [13]. Alternatively stated, propagation characteristics within such media are still (in general) dependent on propagation direction but independent of polarization, eliminating “double refraction” (“birefringence”) effects [12], [14]. Hence our proposed moniker “Non-Birefringent Anisotropic Media” (NBAM), rather than the previously used “pseudo-isotropic” media [14].

From an analytical standpoint, said PWE constitute spectral integrals exactly quantifying the radiated fields [2]. Except for some very simple cases however, these expansions must almost always be evaluated by means of numerical quadratures or cubatures, whose robust computation (with respect to varying source and layer properties) is far from trivial and requires careful choice of appropriate quadrature rules, complex-plane integration contours, etc. to mitigate discretization and truncation errors as well as accelerate convergence [2], [15], [16].

In addition to such considerations of primarily numerical character, a distinct problem occurs, due to said eigenvalue degeneracy, when sources radiate within NBAM layers. This case requires proper analytical “pre-treatment” of the fundamental spectral-domain field expressions to avoid numerically unstable calculations (namely, divisions by zero) during the computation chain as well as to preserve the correct form of the eigenfunctions, viz. $\exp[ik\cdot z]$ instead of $\exp[ik\cdot z]$, the latter resulting from a blanket application of Cauchy’s integral theorem to the canonical field expressions [17], [18].

To this end, we first show the key results detailing the degenerate “direct” (i.e., homogeneous medium) radiated fields in the “principal material basis” (PMB) representation with respect to which the material constitutive tensors are assumed simultaneously diagonalized by a real-valued, orthonormal basis (common for NBAM media [10], [11]). Subsequently, for practical incorporation into the PWE algorithm, we transform these PMB expressions to the Cartesian basis. Finally, we propose a robust, numerically-stable NBAM polarization decomposition scheme to obtain the Cartesian-basis direct field polarization amplitudes.$^1$

Section II introduces the problem of PWE-based analysis of EM fields radiated by sources within planar-stratified environments with generally anisotropic layers, and explains major pitfalls when analyzing radiation originating from within NBAM using canonical PWE expressions. Section III exhibits the key derivation steps and results leading to the Cartesian-basis representation for the direct electric field, including the direct field polarization amplitudes, radiated by sources within NBAM. Section IV contains illustrative results demonstrating the high solution accuracy achievable using our algorithm; finally, Section V contains our concluding remarks. We implicitly allow (linear) temporally dispersive NBAM ($\exp[-i\omega t]$)

$^1$Note: The material tensor eigenvectors $\{\hat{\psi}_1, \hat{\psi}_2, \hat{\psi}_3\}$ are not to be confused with the field polarization eigenvectors.
assumed).

II. PROBLEM STATEMENT

Within a homogeneous medium of material properties \( \{ \varepsilon_r, \mu_r \} \), the electric field \( \mathbf{E}(r) \) radiated by electric (\( \mathbf{J} \)) and (equivalent) magnetic (\( \mathbf{M} \)) current sources satisfies the vector wave equation\(^2\)

\[
\mathbf{A}(\cdot) = \nabla \times \mathbf{\hat{\mu}}_r^{-1} \cdot \nabla \times (\cdot) - k_0^2 \varepsilon_r \cdot (\cdot) \quad (\text{II.1})
\]

and can be expressed via a three-dimensional (3-D) Fourier integral over the field’s plane wave constituents \( \{ \mathbf{E}(k) e^{ik \cdot r} \} \)\(^3\)

\[
\mathbf{A}^{-1} = \text{Adj}(\mathbf{A}) / \text{Det}(\mathbf{A}) \quad (\text{II.3})
\]

\[
\mathbf{E}(k) = \mathbf{A}^{-1} \cdot [i k_0 \varepsilon_0 \mathbf{J} - \nabla \times \mathbf{\hat{\mu}}_r^{-1} \cdot \mathbf{M}] \quad (\text{II.4})
\]

\[
\mathbf{E}(r) = \left( \frac{1}{2\pi} \right)^3 \iiint \mathbf{E}(k) e^{ik \cdot r} \, dk_x \, dk_y \, dk_z \quad (\text{II.5})
\]

where, anticipating an environment with planar layering along \( z \) (c.f. Fig. 1 elsewhere \(^2\)), the \( k_z \) spectral integral is “analytically” evaluated for each \( (k_x, k_y) \) doublet manifest in the (typically numerically) evaluated outer 2-D Fourier integral over \( k_x \) and \( k_y \). That is to say, by “analytically” evaluated we mean that the general (symbolic) closed-form solution of the \( k_z \) integral for arbitrary \( (k_x, k_y) \) doublet, obtained by equivalently viewing the \( k_z \) real-axis integral as a contour integral evaluated using Jordan’s Lemma and residue calculus, is well-known and can be numerically evaluated at the \( (k_x, k_y) \) doublets \(^2\), \(^3\). In particular, analytical evaluation of the \( k_z \) integral yields the “direct” field \( \mathbf{E}^d(r) \) \(^2\):

\[
\mathbf{E}^d(r) = \frac{i}{(2\pi)^2} \iiint \left[ u(\Delta z) \sum_{n=1}^{2} a_n^{d} \tilde{e}_n e^{ik_n z} \Delta z + u(-\Delta z) \sum_{n=3}^{4} a_n^{d} \tilde{e}_n e^{ik_n z} \Delta z \right] e^{ik_x x + ik_y y} \, dk_x \, dk_y \quad (\text{II.6})
\]

where \( a_n^d(k_x, k_y) \) is the (source dependent) direct field amplitude of the \( n \)th polarization, while \( \tilde{e}_n \)\(^2\) \( (k_x, k_y) \) and \( \tilde{h}_n \)\(^2\) \( (k_x, k_y) \) are (resp., eigenvalue of the \( n \)th mode \( n = 1, 2, 3, 4 \)) \(^2\). Modes labeled with \( n = 1 \) and \( n = 2 \) correspond to up-going

\( k_1 = \omega \sqrt{\mu_0 \varepsilon_0} \), \( \mu_0, \mu_r, \varepsilon_0 = \sqrt{\mu_0 / \varepsilon_0} \), \( \varepsilon_r \), and \( \mathbf{\hat{\mu}}_r \) are the vacuum wave number, vacuum permeability, vacuum permeability, vacuum plane wave impedance, NBAM relative permittivity tensor, and NBAM relative permeability tensor, respectively. An infinitesimal point/Hermit dipole current resides at \( r' = (x', y', z') \), the observation point resides at \( r = (x, y, z) \), \( \Delta r = r - r' = (\Delta x, \Delta y, \Delta z), u(\cdot) \) denotes the Heaviside step function, and \( k = (k_x, k_y, k_z) \) denotes the wave vector. Furthermore, \( \mathbf{\hat{\mu}}_r = \mu_r^{-1} \) and \( k_0^2 \) of \( k_{1,2} (x, y, z) \), \( k_{3,4} (x, y, z) \), where \( \gamma = \mathbf{\hat{\gamma}} = t \gamma_t \gamma_s \mathbf{\hat{\gamma}} \gamma_t \); \( \mathbf{\hat{\gamma}} = i \mathbf{\hat{e}}_x \mathbf{\hat{e}}_y \mathbf{\hat{e}}_z \) and \( t, s = x, y, z \). All derivations are performed for the electric field, but duality in Maxwell’s Equations makes immediate the magnetic field solution. Finally, a tilde over variables denotes they are Fourier/wave-number domain quantities.

\(^2\) \( \text{Adj(\cdot)} \) and \( \text{Det(\cdot)} \) denote the adjugate \(^1\) and determinant of said argument, respectively. \( \text{Det(\mathbf{A}) = Det(\mathbf{A}) = d(\mathbf{k}_2 - \mathbf{k}_1)(\mathbf{k}_3 - \mathbf{k}_2)(\mathbf{k}_4 - \mathbf{k}_3)(\mathbf{k}_4 - \mathbf{k}_4), \) \( \{ \mathbf{k}_n \} \) are the eigenvalues (i.e., longitudinal \( z \) propagation constants).

The problem with the canonical numerical implementation of this residue calculus approach lies in its tacit assumption of non-degeneracy (distinctness) in the eigenvalues \( \{ k_{1,2}, k_{3,4}, k_{5,6} \} \), which does not hold for NBAM media. As an illustration of the polarization-independent dispersion behavior of NBAM, consider the dispersion relations of a uniaxial-anisotropic medium slab \( \{ \varepsilon_r = \text{Diag}[a, a, b], \mu_r = \text{Diag}[c, c, d] \} \) \(^2\), \(^3\): \n
\[
\tilde{k}_{12} = \left[ k_0^2 ac - (c/d) k_p^2 \right]^{1/2}, \quad \tilde{k}_{34} = - \left[ k_0^2 ac - (c/d) k_p^2 \right]^{1/2}
\]

\( \tilde{k}_{22} = \left[ k_0^2 ac - (a/b) k_p^2 \right]^{1/2}, \quad \tilde{k}_{42} = - \left[ k_0^2 ac - (a/b) k_p^2 \right]^{1/2}
\]

Setting \( a = \hat{y}^2 c, b = \hat{y}^2 d \) (\( \hat{y} \) is an arbitrary, non-zero multiplicative constant) renders \( \tilde{k}_{12} = \tilde{k}_{34} = \tilde{k}_{42}, \) and \( \tilde{k}_{22} = \hat{y}^2 k_p^2 \), demonstrating the plane wave propagation direction dependent, but polarization independent, dispersion characteristics of uniaxial NBAM \(^2\). This conclusion applies also for more general uniaxial NBAM material tensors possessing PMB rotated with respect to the Cartesian basis \(^2\). Similarly, for biaxial-anisotropic NBAM with PMB-expressed material tensors \( \{ \mu_{\text{pmb}} = \text{Diag}[a, b, c], \varepsilon_{\text{pmb}} = \hat{y}^2 \mu_{\text{pmb}} \} \), the polarization-independent dispersion relations are:

\[
\tilde{k}_{12} = \tilde{k}_{34} = \left[ (\hat{y} k_0)^2 ab - (a/c) k_x^2 - (b/c) k_y^2 \right]^{1/2}
\]

\( \tilde{k}_{34} = \tilde{k}_{42} = - \left[ (\hat{y} k_0)^2 ab - (a/c) k_x^2 - (b/c) k_y^2 \right]^{1/2}
\]

Moreover, the uniaxial and biaxial NBAM polarization vectors can be derived (e.g., see \(^2\), \(^3\), \(^4\)).

Now, the two-fold degenerate eigenvalue \( \tilde{k}_z \) has associated with it two linearly independent field polarizations describing up-going plane waves \(^4\); this holds likewise for the two down-going polarizations with common eigenvalue \( \tilde{k}_z \). Mathematically speaking, the eigenvalues \( \{ \tilde{k}_z, \tilde{k}_z \} \) are each twice-repeating (i.e., algebraic multiplicity of two) but have associated with each of them two linearly independent eigenvectors (i.e., geometric multiplicity of two), making them non-defective and rendering the four NBAM polarization states suitable as a local EM field basis within NBAM layers \(^3\). Despite the existence of four linearly independent eigenvectors, it is worthwhile to further exhibit the key results of the systematic analytical treatment of the two fictitious double-poles of \( \mathbf{A}^{-1} \) to render numerical PWE-based EM field evaluation robust to the two said sources of “breakdown”; this treatment is performed in the next section.

Let us first make two preliminary remarks, however. First, assume that the source-containing layer is a biaxial NBAM with \( \mu_{\text{pmb}} = \text{Diag}[a, b, c] \) and \( \varepsilon_{\text{pmb}} = \hat{y}^2 \mu_{\text{pmb}} \). Second, the orthogonal matrix \( \mathbf{U} = [\hat{v}_1 \hat{v}_2 \hat{v}_3] \) transforms vectors between the PMB and Cartesian basis. For example, the

\( \hat{v}_1 \) Please see background references for other expressions relevant to layered-medium calculations \(^2\), \(^3\).
relationship between the $n$th PMB eigenmode wave vector $\mathbf{k}_{n} = (k_{nx}^{\text{pmb}}, k_{ny}^{\text{pmb}}, k_{nz}^{\text{pmb}})$ and the (assumed available) $n$th Cartesian-basis wave vector $\mathbf{k}_{n} = (k_{x}, k_{y}, k_{z})$ writes as $k_{n}^{\text{pmb}} = \mathbf{U}^{-1} \cdot \mathbf{k}_{n}$.

### III. Direct Electric Field Radiated within NBAM

The (Cartesian basis) Fourier domain representation of the electric field, radiated in a homogeneous NBAM, writes as $\tilde{\mathbf{E}} = -\mathbf{A}^{-1} \cdot \nabla \times \tilde{\mathbf{A}} r$. $\tilde{\mathbf{M}}$ for a (equivalent) magnetic current source or $\tilde{\mathbf{E}} = ik_{0}\mathbf{A}^{-1} \cdot \mathbf{J}$ for an electric current source. These two equations, moreover, hold equally when re-represented in the NBAM’s PMB (i.e., adding “pmb” superscript to all quantities), which is what we will employ. Indeed, the components $\{A_{m,w}\} = \{A_{m,w}\}^{\text{pmb}}$ write as $(A_{m,w} = A_{m,w}^{\text{pmb}})$ and $\mathbf{K} = \mathbf{k}_{n}^{\text{pmb}} / k_{0}$:

$$\begin{align*}
\tilde{\mathbf{E}} &= -\mathbf{A}^{-1} \cdot \nabla \times \tilde{\mathbf{A}} r, \quad \tilde{\mathbf{M}} = \frac{1}{\mu} \tilde{\mathbf{E}} r, \\
A_{11} &= \left(\tilde{k}_{x} - b\tilde{g}_{y}\right) / \tilde{B}, \quad A_{12} = \tilde{k}_{y} / \tilde{B}, \\
A_{13} &= \tilde{k}_{x} / \tilde{B}, \quad A_{22} = \tilde{k}_{y} - a\tilde{g}_{y} / \tilde{B}, \\
A_{23} &= \tilde{k}_{y} / \tilde{B}, \quad A_{33} = \tilde{k}_{x} - b\tilde{g}_{y} / \tilde{B}
\end{align*}$$

(III.1) - (III.4)

while the components of $\tilde{\mathbf{A}}^{-1} \cdot \mathbf{V}^{\text{pmb}} \times \mathbf{K}^{-1} \cdot \mathbf{A}$ write as $(A_{m,w} = A_{m,w})$:

$$\begin{align*}
\tilde{B} &= \tilde{B} / \tilde{g}_{y}, \quad A_{12} = -ik_{z} / \tilde{B} r, \\
A_{13} &= ik_{y} / \tilde{B} r, \quad A_{23} = -ik_{x} / \tilde{B} r
\end{align*}$$

(III.5) - (III.6)

The expressions within Eqns. (III.1) - (III.4) describe the electric field from an electric current source while the expressions within Eqn. (III.6) describe the electric field from an (equivalent) magnetic current source. Duality in Maxwell’s Equations makes immediate the magnetic field results.

Next the PMB electric field $\mathbf{E}^{\text{pmb}}(k_{x}, k_{y}, z, z')$, after re-expressing Eqns. (III.1) - (III.6) in terms of $\{k_{x}, k_{y}, k_{z}\}$ to identify the $k_{z}$ (rather than $k_{n}^{\text{pmb}}$) eigenvalues $\{k_{nz}\}$ (using the relation $k = \mathbf{U} \cdot k_{n}^{\text{pmb}}$) as well as “analytically” performing the $k_{z}$ contour integral, can be decomposed into a linear combination of the degenerate up-going modes $\{\mathbf{e}_{1}^{\text{pmb}}, \mathbf{e}_{2}^{\text{pmb}}\}$ (for $z > z'$) or down-going modes $\{\mathbf{e}_{3}^{\text{pmb}}, \mathbf{e}_{4}^{\text{pmb}}\}$ (for $z < z'$) $\tilde{\mathbf{A}}^{-1} \cdot \mathbf{V}^{\text{pmb}} \times \mathbf{K}^{-1} \cdot \mathbf{A}$.

For an electric source, we have for $\mathbf{e}_{1}^{\text{pmb}}, \mathbf{e}_{2}^{\text{pmb}}$:

$$\begin{align*}
\pm 2\pi i \left[ ik_{0} \mathbf{A}^{-1} \cdot \tilde{\mathbf{E}} r \right] k_{z} = k_{z}
\end{align*}$$

and similarly for a (equivalent) magnetic source upon replacing $\mathbf{A}^{-1} \cdot \mathbf{V}^{\text{pmb}} \times \mathbf{K}^{-1} \cdot \mathbf{A}$ in Eqn. (III.7). Next, the degenerate PMB modal electric fields are re-expressed in the Cartesian basis:

$$\mathbf{e} = \begin{bmatrix} \mathbf{e}_{x} & \mathbf{e}_{y} & \mathbf{e}_{z} \end{bmatrix}^{T} = \mathbf{U} \cdot \mathbf{e}^{\pm}$$

(III.8)

The Cartesian basis wave vectors and polarization eigenvectors are assumed available (e.g., via the state matrix method [13]). Indeed, recall that the operations discussed herein are performed within the backdrop of numerical 2-D Fourier integral evaluations [12].

When $z = z'$, assuming the source does not lie exactly at a planar material interface, one can write the direct fields as a linear combination of either the up-going or down-going modes since both combinations lead to identical field results (save at $r'$) on the plane $z = z'$ [2, 13].

### IV. Results

Now we exhibit some illustrative results demonstrating the developed algorithm’s performance. We investigate both the electric field $\tilde{\mathbf{E}}$ radiated by a vertical (i.e., z-directed) Hertzian electric current dipole (VED), as well as the magnetic field $\tilde{\mathbf{H}}$ radiated by a z-directed Hertzian (equivalent) magnetic current dipole (VMD). In both scenarios, the source resides at depth $z = 0$m within an NBAM, occupying the region $-1 \leq z \leq 1$ [m], of material properties $\varepsilon_{r} = \mu_{r} = \text{Diag}[5, 5, 1/\mu]$. The top layer ($z \geq 1$m) is vacuum ($\varepsilon_{r} = \mu_{r} = 1$) while the bottom layer is a perfect electric conductor (PEC); note that this layered-medium configuration was specifically chosen to facilitate comparison with closed-form solutions through invocation of Transformation Optics theory and EM Image theory [11, 21, 22]. Indeed the EM field solution within the region $z > 1$m, for our three-layered configuration involving a VED source, can be shown identical to two VED’s located at depths $z = -4$m and $z = 1$m of identical orientation to the original VED and radiating in homogeneous, unbounded vacuum. Similarly, the VMD problem can be shown identical to two VED’s (located at depths $z = -4$m and $z = -1$m) radiating in homogeneous, unbounded vacuum; in this scenario however, image theory prescribes that the $z = 4$m VMD possess identical orientation to the original VMD, but that the $z = -1$m VMD possess opposite orientation [21].

Observing Figs. 2a-2d we note the relative errors in both the electric field ($\delta_{e}$) and magnetic field ($\delta_{h}$) are very low, approaching in most of the observation plane near the limits of floating point double precision-related numerical noise (approximately -150 in [dB] scale). This is consistent with our having set an adaptive relative integration error tolerance of $1.2 \times 10^{-14}$. For reference, Figs. 2a [23] are the computed field distributions themselves from our algorithm. Due to space limitations, a similar set of results showing the radiation pattern modification versus the “reference scenario” (i.e., replacing the NBAM with vacuum) could not be shown.

Let $\mathbf{E}_{c}$ be the electric field computed using our algorithm, and let $\mathbf{E}_{o}$ be the closed-form reference solution. Then $\delta_{e} = \|\mathbf{E}_{c} - \mathbf{E}_{o}\| / \|\mathbf{E}_{o}\|$ (likewise for $\delta_{h}$).
Finally, we emphasize that given the design of our particular implementation, which always first computes the direct electric field and then (if need be) computes the magnetic field using ancillary relations [13][Ch. 2], we have in fact tested both the soundness of Eqns. (III.1)-(III.4) (VED scenario) and of Eqns. (III.5)-(III.6) (VMD scenario).

V. CONCLUSION

We addressed a fundamental origination of instability in the spectral-domain-based evaluation of EM fields due to sources embedded within NBAM planar slabs. This instability arises due to eigenvalues that, while non-defective, have an algebraic multiplicity equal to two rather than one. The remedy is to apply a proper (analytical) “pre-treatment” of the spectral-domain tensor operators prior to polarization amplitude extraction, resulting in robust analysis of EM fields in arbitrary anisotropic planar-layered media. Numerical results validated the high-accuracy of numerical computations based on this analytical pre-treatment.

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