Evolution of AM CVn binaries with WD donors

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ABSTRACT

The evolution and stability of mass transfer of CO+He WD binaries are not well understood. Observationally they may emerge as AM CVn binaries and are important gravitational wave (GW) emitters. In this work, we have modeled the evolution of double WD binaries with accretor masses of 0.50 – 1.30 M⊙ and donor masses of 0.17 – 0.45 M⊙ using the detailed stellar evolution code MESA. We find that the evolution of binaries with same donor masses but different accretor masses is very similar and binaries with same accretor masses but larger He donor masses have larger maximum mass transfer rates and smaller minimum orbital periods. We also demonstrate that the GW signal from AM CVn binaries can be detected by space-borne GW observatories, such as LISA, TianQin. And there is a linear relation between the donor mass and gravitational wave frequency during mass transfer phase. In our calculation, all binaries can have dynamically stable mass transfer, which is very different from previous studies. The threshold donor mass of Eddington-limited mass transfer for a given accretor WD mass is lower than previous studies. Assuming that a binary may enter common envelope if the mass transfer rate exceeds the maximum stable burning rate of He, we provide a new criterion for double WDs surviving mass transfer, which is below the threshold of Eddington-limit. Finally, we find that some systems with ONe WDs in our calculation may evolve into detached binaries consisting of neutron stars (NSs) and extremely low mass He WDs and further ultra-compact X-ray binaries.

Keywords: : Close binary stars (254); White dwarf stars(1799); AM Canum Venaticorum stars (31); Compact binary stars (283); Gravitational wave sources (677))

1. INTRODUCTION

AM CVn binaries are a kind of interacting binary systems consisting of accreting white dwarfs (WDs) and He-rich donor stars. They are important for studies of binary evolution (e.g. Postnov & Yungelson 2014; Tauris & van den Heuvel 2023), binary population synthesis (e.g. Nelemans et al. 2001a; Han et al. 2020) and common envelope evolution (e.g. Ivanova et al. 2013; Kruckow et al. 2021). Given the short periods (∼ 5 – 66 min) of AM CVn binaries, they are important gravitational wave (GW) sources for space-borne low-frequency GW observatories like LISA (Amaro-Seoane et al. 2017), TianQin (Luo et al. 2016), and Taiji (Ruan et al. 2020). It was also suggested that AM CVn binaries can be the progenitors of type Ia supernovae (Bildsten et al. 2007).

It is known that there are three possible formation channels for AM CVn binaries. In the first channel, a WD accretes material from a semidegenerate He star (e.g. Tutukov & Yungelson 1979; Nather et al. 1981; Iben & Tutukov 1991; Yungelson 2008); In the second channel, a He WD in a double WD system transfers material to the WD accretor (e.g. Nather et al. 1981; Tutukov & Yungelson 1996; Nelemans et al. 2001a; Chen et al. 2022a). In this channel, the accretor is usually a CO WD. In the third channel, an evolved main sequence donor star starts mass transfer around the end of main sequence. After the donor star loses its H-rich envelope, it becomes He-rich and has a remaining mass smaller than 0.10 M⊙, transferring material to the WD (e.g. Podsiadlowski et al. 2003; Liu et al. 2021). In this paper, we mainly focus on the double WD channel.

Regarding the evolution of AM CVn binaries in the double WD channel, it has been widely investigated. Some studies (e.g. Nelemans et al. 2001a; Marsh et al. 2004; Gokhale et al. 2007; Kremer et al. 2017) adopted a semi-analytic method to model the evolution of AM CVn binaries. In these studies, the detailed structure of the He WD was not taken into account and the He WD was assumed to be fully degenerate. This should not be realistic since the He WDs may have small but thick envelopes. Kaplan et al. (2012) have shown that this will have an important impact on the evolution of WD binaries and stability of mass transfer. In addition,
some studies (e.g. Deloye et al. 2007; Wong & Bildsten 2021) have modelled the evolution of AM CVn stars with the He WD structure considered. They found that the initial entropy of the He WD has an impact on the evolution of AM CVn binaries. But it is worth noting that it is widely assumed that the mass transfer in AM CVn binaries is conservative and the evolution of accreted material on the CO WD is not considered in these studies.

From previous studies of He-accreting WDs, we know that the evolution of He-accreting WDs strongly depends on the WD mass and accretion rate (e.g. Nomoto 1982; Iben & Tutukov 1989; Limongi & Tornambe 1991; Piersanti et al. 2014; Wang et al. 2017; Wu et al. 2017). It is shown that there is a stable burning regime in which the accreted He material can burn stably on the surface of WDs. There is little mass loss in this regime. If the accretion rate is smaller than the minimum stable burning rate, the He-burning is unstable, leading to nova outburst. A fraction of material can be lost during the nova outburst. If the accretion rate is larger than the maximum stable burning rate. The evolution is still under debate. Postnov & Yungelson (2014) and Wang et al. (2017) found that the accreting WDs will evolve into red giants after a small amount of material is accreted. In this case, the binary system is likely to enter common envelope and merge eventually. On the other hand, Hachisu et al. (1999b) found that the optically thick wind (Kato & Hachisu 1994) will occur in this regime. In this scenario, the material on the surface of WDs burns at a rate of the maximum stable burning rate and the excess material is lost in a form of optically thick wind. With these results in mind, we can find that the mass transfer of AM CVn binaries is not likely to be conservative.

This work aims at a comprehensive study of the evolution of AM CVn binaries from double WD channel and the properties of this kind of binaries, in particular, as gravitational wave sources.

The rest of the paper is organized as follows. In section 2, we describe how we make the initial WD models and briefly outline the assumptions underlying the binary evolution. In Section 3, we present the results we obtained. In Section 4, we first discuss the detectability of AM CVn binaries with LISA, TianQin and the properties of AM CVn binaries as GW sources. Then we also discuss the stability of mass transfer of AM CVn binaries from our simulation. In addition, we discuss the uncertainties in our simulation and their influence on our results. Finally, we summarize our conclusion in Section 5.

2. METHOD AND ASSUMPTIONS

2.1. Initial He WD models

In this work, we adopted the He WD models from our previous work about the evolution of NS+He binaries (Chen et al. 2022b). Here we briefly describe how these He WD models are made.

First, the evolution of a grid of low-mass X-ray binaries was computed with the stellar evolution code MESA (Paxton et al. 2011, 2013, 2015, 2018, 2019). The initial NS and donors are assumed to be a point mass and zero-mass main-sequence stars, respectively. Their masses are assumed to be 1.30 and 1.20 $M_\odot$, respectively. The initial orbital periods range from 1.0 to 600 days. In this grid, some donors can evolve into He WDs. Then we extract the WD models when the central temperatures of He WDs are around $10^7$ K. In these models, the He WDs have H envelopes with masses smaller than 0.01 $M_\odot$. The initial He WD masses are 0.17, 0.21, 0.25, 0.30, 0.35, 0.40, 0.45 $M_\odot$. Compared with the He WD models in Li et al. (2019) who modeled the formation of CO+He WDs, our He WD models may have different temperatures and H envelope masses. But from the following discussion, we can find these factors does not influence our results significantly.

2.2. Binary evolutionary models

To model the evolution of AM CVn binaries, we make use of the star plus point mass test suite of MESA code (version 12115). In our simulation, the WD accretor is assumed to be a point mass. The initial masses of WD accretors are assumed to be 0.50, 0.60, 0.70, 0.80, 0.90, 1.00, 1.10, 1.20, 1.30 $M_\odot$. These WDs with masses larger than 1.10 $M_\odot$ are ONe WDs and others are CO WDs. The initial orbital periods are assumed to be 0.05 days.

In our calculation, we consider two kinds of mechanisms of angular momentum loss: GW radiation and angular momentum loss due to mass loss. The angular momentum loss due to GW radiation can be computed with the following formula (Landau & Lifshitz 1971):

$$\frac{dJ_{gw}}{dt} = -\frac{32}{5} \frac{G^{7/2} M_a^2 M_d^2 (M_a + M_d)^{1/2}}{c^{5/2} a^{7/2}},$$

where $G$ is the gravitational constant and $c$ is the speed of light in vacuum; $a$ is the binary separation; $M_a$ and $M_d$ are the masses of the WD accretor and the WD donor, respectively.

We compute the mass transfer rate with the Ritter scheme (Ritter 1988). The mass transfer in our calculation is not conservative and the isotropic re-emission model are adopted (Tauris & van den Heuvel 2006). The mass retention efficiency is computed as follows.

Given the H shell of the He WD, the accreted material is H-rich at the early phase of mass transfer. We simply assume that the retention efficiency is 0 in this phase. We have tested that this assumption has little impact on our results. As for He burning, we adopt the optical thick wind model (Kato & Hachisu 1994; Hachisu et al. 1999a) and the prescription of Kato & Hachisu (2004). If the mass transfer rate ($\dot{M}_{tr}$) is larger than a threshold
value $\dot{M}_{\text{up}}$, we assume that He burns steadily at a rate of $\dot{M}_{\text{up}}$ and the excess material is lost in the form of optically thick wind. The threshold mass transfer rate is

$$
\dot{M}_{\text{up}} = 7.2 \times 10^{-6} \left( \frac{M_a}{M_\odot} - 0.6 \right) M_\odot \text{ yr}^{-1},
$$

(2)

for accretor masses $M_a \geq 0.75 M_\odot$ (Nomoto 1982).

If the mass transfer rate is larger than the minimum rate of stable burning ($\dot{M}_{\text{tr}}$) and smaller than $\dot{M}_{\text{up}}$, we assume that He burns stably on the surface of the WD and there is no mass loss. If the mass transfer rate is smaller than $\dot{M}_{\text{tr}}$ but larger than $\dot{M}_{\text{low}}$, we assume that the He burns unstably, triggering He flashes. In this regime, the retention efficiency is computed following Kato & Hachisu (2004). If the mass transfer rate is smaller than $\dot{M}_{\text{low}}$, we assume that the He flashes is too strong to retain any material. Therefore the retention efficiency for He burning is

$$
\eta_{\text{He}} = \begin{cases} 
\frac{\dot{M}_{\text{up}}}{\dot{M}_{\text{tr}}} & \dot{M}_{\text{tr}} \geq \dot{M}_{\text{up}} \\
1 & \dot{M}_{\text{up}} > \dot{M}_{\text{tr}} \geq \dot{M}_{\text{tr}} \\
\eta_{\text{KH04}} & \dot{M}_{\text{tr}} > \dot{M}_{\text{tr}} \geq \dot{M}_{\text{low}} \\
0 & \dot{M}_{\text{tr}} < \dot{M}_{\text{low}}
\end{cases}
$$

where

$$
\dot{M}_{\text{tr}} = 10^{-5.8} \ M_\odot \text{ yr}^{-1} \\
\dot{M}_{\text{low}} = 10^{-7.4} \ M_\odot \text{ yr}^{-1}.
$$

For WDs with masses $M_a < 0.75 \ M_\odot$, Eq. 2 is not validated any more and we simply assume that the mass transfer is completely non-conservative, i.e., no mass is retained by the accretor. These material not accreted by the accretor leaves the system and takes away the specific angular momentum of the accretor.

In addition, we also take the Eddington limit into consideration. The Eddington limit can be given by (Tauris & van den Heuvel 2023)

$$
\dot{M}_{\text{Edd}} = 4.4 \times 10^{-6} \left( \frac{M_a}{M_\odot} \right) M_\odot \text{ yr}^{-1}.
$$

(3)

Compared with the Eddington limit of Han & Webbink (1999), the Eddington limit in our calculation is lower. This can be understood as follows. In the calculation of Eddington limit, Han & Webbink (1999) only considered the gravitational energy released by the accreted material. However, Tauris & van den Heuvel (2023) suggest that the nuclear burning energy from these accreted material should be also considered, which leads to a lower Eddington accretion rate. Following Han & Webbink (1999), we assume that the binary will merge in a common envelope if the mass transfer rate is larger than the Eddington limit. But we do not stop the calculation in order to know if the binary system can have dynamically stable mass transfer.

The inlist files for our simulations can be made available on request by contacting the corresponding author.

Figure 1. Evolution of mass transfer rate (upper panel), WD masses (middle panel) and orbital period (lower panel) as a function of time. The initial binary parameters in this example are $M_a = 0.90 \ M_\odot$, $M_d = 0.21 \ M_\odot$ and $P_{\text{orb}} = 0.05$ days. In the upper panel, the three dashed lines from up to bottom indicate $\dot{M}_{\text{up}}$, $\dot{M}_{\text{tr}}$ and $\dot{M}_{\text{low}}$, respectively. In the middle panel, the red and blue lines are for the donor and accretor masses, respectively.

3. RESULTS

3.1. Examples of binary evolution

In Fig. 1, we present an example of binary evolution of AM CVn binaries. The initial masses of the accretor
and donor are $0.90 \, M_\odot$ and $0.21 \, M_\odot$, respectively. The initial orbital period is 0.05 days. From this plot, we can find that the mass transfer rate in this example is always below the stable burning regime of He (indicated by the green and red dashed lines in the upper panel). The mass of WD accretor increases when the mass transfer rate is between $M_{\text{low}}$ (see the purple dashed line) and $M_{\text{cr}}$ (see the red dashed line). At the early phase of evolution, the orbital period decreases because of GW radiation. When the H envelope is stripped, the mass transfer rate is around its maximum value and the orbital period reaches its minimum. Afterwards, the mass transfer leads to the increase of orbital period.

In Fig. 2, we show the evolution of mass transfer rate, orbital period and accretor mass for binaries with different WD accretor masses and a same donor mass. From the upper and middle panels, we can find the evolution of mass transfer rate and orbital period are very similar for these binaries. The minimum orbital periods during the evolution are almost the same for these binaries. This is because the evolution of orbital period during the mass transfer mainly depends on the donor mass (Chen et al. 2022b). In the binary system with an accretor mass of $1.30 \, M_\odot$, the accretor mass reaches $1.40 \, M_\odot$ during its evolution. The WD accretor can collapse into a NS and we do not stop the calculation at that point. We have a further discussion on this kind of systems in Sec. 4.3. For the systems with initial accretor masses of 0.50 and 0.70 $M_\odot$, the accretor masses do not change during their evolution. This is because we assume that the mass transfer is completely non-conservative for these systems with accretor masses $M_a \leq 0.75 \, M_\odot$.

Fig. 3 presents the evolution of mass transfer rate as a function of orbital period and He WD masses for binaries with different He WD masses and a same accretor mass. From this plot, we can also find that these tracks converge to a single branch after the peaks of mass transfer rate. Compared with these systems with smaller He WD masses, the systems with massive He WDs have larger maximum mass transfer rate and smaller minimum orbital period. This is mainly because massive He WDs have smaller radius.

4. DISCUSSION

4.1. Properties as GW sources

Given the short orbital periods of AM CVn binaries, they are expected to be important sources of low GW frequency. In Fig. 4, we present an example of the evolution of GW frequency, chirp mass and signal-to-noise ratio (SNR) as a function of time for a binary system. In addition, the evolution of donor mass as a function of GW frequency is also shown. Here the SNR for LISA and TianQin is computed with the Python package LEGWORK (Wagg et al. 2022). The initial binary parameters are $M_a = 0.90 \, M_\odot$, $M_d = 0.21 \, M_\odot$ and $P_{\text{orb}} = 0.05$ days. From these plots, we can find that the AM CVn binaries have a strong GW emission in the mHz regime. The SNR can be up to $\sim 800$ ($\sim 50$) if the source is located at 1 kpc (15 kpc). If we adopt the critical SNR = 7, above which the source becomes detectable, then we can find that this source can be detected by LISA and TianQin. The chirp mass for the detectable source is between 0.11 $M_\odot$ and 0.36 $M_\odot$. 

![Figure 2. Evolution of mass transfer rate (upper panel), orbital period (middle panel) and accretor mass (bottom panel) for AM CV binaries with different accretor masses. In the upper panel, the arrows indicate the evolutionary direction. In these binaries, the initial donor masses and the initial orbital periods are the same, i.e. 0.21 $M_\odot$ and 0.05 days, respectively.](image-url)
Modeling the evolution of CO+He WD binaries

4.2. Stability of mass transfer

The stability of mass transfer of double WDs is still under debate. This problem could strongly influence the number of AM CVn and GW sources predicted by binary population synthesis model.

In Fig. 6, we present the stability limits for mass transfer of AM CVn binaries and compare our results with previous works. Following Chen & Han (2008), if a binary system has a runaway mass transfer, we assume that the binary system will have dynamically unstable mass transfer. With this criteria, we can find that all binaries in our calculation have dynamically stable mass transfer. In addition, following Marsh et al. (2004) and Kremer et al. (2017), if we adopt a critical mass transfer rate of $\dot{M} = 0.01 M_\odot/yr$ as the limit for dynamically stable mass transfer, we can also find that all binaries in our calculation have dynamically stable mass transfer. This is very different from Nelemans et al. (2001a) and Marsh et al. (2004) (see the solid lines in Fig. 6).

The mass transfer of these binaries with smaller accretor and larger donor masses are dynamically stable in our calculation and unstable in Nelemans et al. (2001a) and Marsh et al. (2004). This is partially due to that we have non-conservative mass transfer in our calculation, while they assumed conservative mass transfer in their calculation (see sec. 4.4 for detail discussion). In addition, a zero temperature is assumed for the WD donors in Nelemans et al. (2001a) and Marsh et al. (2004), which may lead to higher mass transfer rate (see sec. 4.5 for more discussion).

Han & Webbink (1999) suggested that a common envelope may form if the mass transfer rate in double WD binaries is larger than the Eddington limit. With this restriction, we can find the threshold for Eddington-limited mass transfer and show these binaries below the threshold with crosses in Fig. 6. From the plot, we can find that the threshold in our calculation is slightly below that found by Han & Webbink (1999) and Nelemans et al. (2001b). This is because the Eddington limit we adopted is lower than that in Han & Webbink (1999) and Nelemans et al. (2001b).

In our calculation, we assume that the optically thick wind occurs if the mass transfer rate in double WD binaries is larger than the maximum stable burning rate of He. But there is another possibility in this regime. For example, Piersanti et al. (2014) found that the accreting WD may evolve into a red giant. In this scenario, the binary may enter

![Figure 3. Evolution of mass transfer as a function of orbital period (upper panel) and He WD mass (lower panel). In the plot, the arrows indicate the evolutionary direction of binaries. In these binaries, the initial masses of WD accretors and the initial orbital periods are the same, i.e. 1.30 $M_\odot$ and 0.05 days, respectively. In these plots, we do not show these evolutionary tracks for these binaries with donor masses $M_d \geq 0.30 M_\odot$. This is because their mass transfer rates during their evolution can exceed the Eddington limit and we assume these binaries will merge.](image-url)
common envelope and merge eventually. With this scenario in mind, we can find all binaries with maximum mass transfer rates smaller than the maximum stable burning rates of He\textsuperscript{1}, which are shown as empty squares in Fig. 6. This may provide a new criterion for double WDs surviving mass transfer, which is slightly below the threshold for Eddington-limit.

\textsuperscript{1}Since Eq. 2 is only validated for WDs with masses $M_a \geq 0.75 \, M_\odot$, we adopt the maximum stable burning rate of He from Piersanti et al. (2014) for WDs with masses smaller than $0.75 \, M_\odot$.}

Carter et al. (2013) calculated the AM CVn space density to be $(5 \pm 3) \times 10^{-7} \, \text{pc}^{-3}$, which is 50 times smaller than the predicted value by the optimistic population synthesis model from Nelemans et al. (2001a). Recently, van Roestel et al. (2022) found a space density of $6^{+6}_{-2} \times 10^{-7} \, \text{pc}^{-3}$ and confirmed this discrepancy. Our new stability limits may have an important implication for the formation of AM CVn binaries in binary population synthesis study. Since the stability limits in our calculation are lower than that in Nelemans et al. (2001a), we expect that less binaries will have stable mass transfer and less AM CVn binaries will be produced in the population synthesis models. This will be helpful to mitigate this discrepancy between the theoretical model and observation.

It is also worth noting that we do not consider the rotation of the accretors in our calculation. During the mass transfer, the accretor may be spun up due to accretion. The coupling between the accretor’s spin and orbit may lead to extra orbital angular momentum loss during binary evolution. Marsh et al. (2004) showed that the coupling may have an important effect on the stability of mass transfer depending on the synchronization timescale. On the other hand, Kupfer et al. (2016) have shown that the accretor velocities of two AM CVn systems, GP Com and V396 Hya, are much slower than the critical.
4.3. Accretion induced collapse of ONe WDs

As we show in Fig. 2, some ONe WDs in our calculation can increase their masses to the Chandrasekhar mass limit (~1.40 $M_\odot$) and will collapse into NSs. At this point, the He WD masses are around 0.09−0.15 $M_\odot$. After the WD collapses into a NS, the binary system will become eccentric because of the mass loss during collapse and the natal kick of NS (e.g. Tauris et al. 2013). Because of mass loss during the collapse, the Roche lobe radius of the donors increase. Then these systems may evolve into detached binaries consisting of NSs and extreme low mass He WDs. Due to the GW radiation, the He WDs in these systems may fill their Roche lobe again at some point. Then these binaries will evolve into ultra-compact X-ray binaries.

4.4. Influence of accretion efficiency

In order to understand the influence of accretion efficiency, we compute the evolution of a binary system assuming the mass transfer is conservative, i.e. all the material lost by the donor is accreted by the accretor. In Fig. 7, we show the comparison of the evolution of mass transfer rate between models with two different prescriptions of accretion efficiency. The blue solid (green dashed) line is for the model assuming conservative (completely non-conservative) mass transfer. The initial binary parameters in this example are $M_a = 0.50 M_\odot$, $M_d = 0.25 M_\odot$, and $P_{\text{orb}} = 0.05$ days.

4.5. Influence of initial effective temperature and orbital period
In our calculation, the initial temperatures of He WDs and orbital periods are assumed to be the same for all binaries. This may be not realistic, given that the double WD systems can be produced from stable mass transfer and common envelope ejection channels (Li et al. 2019). The He WDs in these binaries with long (short) orbital periods have long (short) time to cool down, leading to low (high) temperatures at the onset of mass transfer. Therefore, the influence of initial effective temperature and orbital period should be similar.

In order to understand their influence, we make a new 0.25 $M_\odot$ He WD model with a high temperature following the method described in Sec. 2.1. The central and effective temperatures in this model are $T_c = 3.24 \times 10^7$ K and $T_{\text{eff}} = 2.74 \times 10^3$ K, respectively. With this model, we compare the evolution of mass transfer rate for binaries with different initial orbital periods, which is shown in Fig. 8. From this plot, we can find that the mass transfer rate is slightly higher for binaries with larger orbital periods at early phase of mass transfer. At later times, the difference is very small.

5. CONCLUSIONS

In this work, we have comprehensively studied the evolution of AM CVn binaries with WD donors using the stellar evolution code MESA. Instead of simply assuming conservative mass transfer, we have considered the dependence of retention efficiency on accretor mass and accretion rate. In our calculation, the accretor mass ranges from 0.50 to 1.30 $M_\odot$ and the donor mass ranges from 0.17 to 0.45 $M_\odot$. The main results are as follows:

- These binaries with same He WD masses but different accretor masses have very similar evolution and similar minimum orbital periods. These binaries with same accretor masses but larger donor masses have larger maximum mass transfer rate and smaller minimum orbital periods.
- We demonstrate that the GW signal from AM CVn binaries can be detected by LISA and Tian-Qin. Moreover, there is a linear relation between the WD donor mass and GW frequency during the mass transfer phase.
- In our calculation, all binaries have dynamically stable mass transfer, which is very different from previous studies. The threshold donor mass for Eddington-limited mass transfer for a given accretor mass is lower than previous studies. Assuming that the binary may enter common envelope if the mass transfer rate is larger than the maximum stable burning rate of He, we provide a new criterion for double WDs surviving mass transfer, which is slightly below the threshold of Eddington limit (see Fig. 6).
- These binaries with ONe WD accretors may evolve into binaries consisting of NS and extremely low mass WDs and further ultra-compact X-ray binaries.

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**Software:** MESA (Paxton et al. 2011, 2013, 2015, 2018, 2019), LEGWORK (Wagg et al. 2022), matplotlib (Hunter 2007), numpy (Harris et al. 2020), astropy (Astropy Collaboration et al. 2013, 2018), Python from python.org.

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