Small x divergences in a heavy quark-antiquark state.

Martina Brisudová*

*a Institute of Physics, Slovak Acad. Sciences, Dúbravská cesta 9, Bratislava 842 28, Slovakia

With the current state of similarity renormalisation group approach to light-front QCD, it is possible to address with a degree of generality the issue of light-cone zero modes. We find, contrary to earlier results in a less general framework, that infrared divergences associated with the zero modes do not cancel out in a color singlet heavy quark-antiquark states, except for the lowest order in the nonrelativistic expansion.

1. Introduction

1.1. Light-cone zero modes

Light-cone zero modes are surely the most controversial subject in light-cone field theories. Practically everybody in this field worked on this at some point, and I apologize that I cannot mention everyone for the lack of space. Nevertheless, however extensive the literature on light-cone zero modes, there are basically two attitudes to this issue. One is to try to explicitly solve for the zero modes. This is typically done for various, often lower dimensional, field theories (for review and references see [1], for extensive list of references see also [2]) in the context of discreet light cone quantization (DLCQ) (for a recent review of the method as well as excessive list of references see [3]).

I would like to mention a somewhat untypical paper that falls into this category, a recent work by Tomaras, Tsamis and Woodard [4] on back reaction in light-cone QED. Though motivated by the back-reaction in quantum gravity occurring on an inflating background, their work addresses some issues of the light-front vacuum without having to evoke DLCQ. They consider a free QED coupled to a constant external electric field in continuum (3+1) dimensions, and a full operator solution to the model is constructed. In this set up, all modes are forced to go through the zero mode at which point particle pairs are created. The zero mode of the constraint components of the fermionic field is shown to be crucial for unitarity.

The other approach to the problem of zero mode is more pragmatic. Instead of trying to solve for the zero mode, it is simply cutoff, be it with DLCQ [5] or an explicit infrared cutoff in a continuous formulation [2]. Physics associated with this mode can then be put in form of counterterms, if needed, for example, to restore symmetries or account for phenomena associated with the vacuum. Traditionally, spontaneous symmetry breaking was viewed as an example of such a phenomenon. However, Rozowsky and Thorn [5] have argued recently that, while conceding that the inclusion of a fundamental zero mode is a valid theoretical option, it is not necessary to describe spontaneous symmetry breaking where its presence seems to be most needed. Indeed, in scalar quantum field theory in (1+1) dimensions DLCQ the physics of spontaneous symmetry breaking is completely and accurately described without the zero modes [5].

I hope that the two examples I mentioned explicitly are sufficient to remind you how confusing and controversial the zero modes are.

1.2. Similarity renormalisation group approach to light-cone QCD

The similarity renormalisation group approach has been presented by various authors at the light-cone conferences many times since its introduction [6]. The basic assumption upon which the approach is based is that it is possible to derive...
a constituent picture for hadrons from QCD If this is possible, nonperturbative bound state problems in QCD can be approximated as coupled, few-body Schrödinger equations.

The starting point is a regulated canonical light-front Hamiltonian. The apparent difficulties with renormalization of light-front Hamiltonians (compared to Lagrangians) are turned into an advantage by using similarity renormalization which allows to transform the standard perturbative QCD Hamiltonian at high energy scales into an effective Hamiltonian at hadron scales. Unfortunately, renormalization group as we presently know it can systematically remove dependence only on one regulator. That leaves the infrared, or small $k^+$, regulator in the game.

The need to put in new counterterms associated with the infrared (IR) regulator in our approach was anticipated but was not encountered yet in the applications to hadronic physics so far. Perry has shown that even though the one body and two-body effective operators are each separately divergent as $k^+$ goes to zero, the divergences exactly cancel in any color singlet state. The cancelation does not occur for nonsinglet states, leaving them with an infinite mass. This together with a naturally generated confining potential (imprecisely referred to as “logarithmic”) is a plausible feature of the approach. Note that both the effective confining potential and the infrared regulator. We wish to study the issue of small $k$ (or infrared) divergences in color singlet states consisting of a heavy quark and antiquark of the same flavor, for simplicity. Does the cancellation found by Perry occur also in the boost invariant formulation?

2. Cancelation of infrared divergences?

The short answer to this question is: No. Details of this calculation can be found in [10].

Here we just show the resultant bound state equation for the binding $E$ and wavefunction

$$\Phi_{12} \equiv \Phi(x_1, x_2 = 1 - x_1, \kappa_{12}^\perp),$$

(all other symbols will be explained below),

$$\Phi_{12} = \frac{(4m^2 + 4mE)}{x_1 x_2} \kappa_{12}^\perp \left( \frac{g^2}{4\pi^2} C_F \times \right.$$

$$\left. \int \frac{dx_3 d^3k_{12}^\perp}{\pi} \left[ \mathcal{V}_1 + \mathcal{V}_2 + \mathcal{V}_{inst} \right] f_\Lambda (M_{12}^2 - M_{31}^2) \Phi_{34} 
- \frac{\sqrt{\pi} \lambda^2}{\sqrt{x_1^2 + x_2^2}} \int_0^1 \frac{dy}{y} r_\delta (x_1 y) r_\delta (x_2 y) \Phi_{12} \right)$$

$$+ \frac{g^2}{4\pi^2} C_F \frac{\sqrt{\pi}}{2} \lambda^2 \mathcal{I}_\delta (x_1, x_2) \Phi_{12},$$

with

$$\mathcal{I}_\delta (x_1) \equiv$$

$$+ \frac{1}{\sqrt{2} x_1 x_2} \int_0^1 \frac{dy}{y} (r_\delta (y))^2$$

$$- \frac{2}{\sqrt{x_1^2 + x_2^2}} \int_0^1 \frac{dy}{y} r_\delta (x_1 y) r_\delta (x_2 y).$$
Here \( m \) is current (heavy) quark mass, \( m(\lambda) \) is quark mass including finite contribution from effective one-body operators at finite \( \lambda \), similarity scale. \( \kappa_{ij}^\perp \) is the standard relative transverse momentum between particles \( i \) and \( j \). \( \mathcal{M}_{ij}^2 = (p_i + p_j)^2 \) is the invariant mass of the state consisting of particles \( i, j \). \( f_\lambda \) is the similarity form-factor, \( V_1, V_2 \) are the coefficients of the effective two-body operator at order \( g^2 \) and \( \nu_{\text{inn}} \) is the instantaneous interaction (for details see \[1\]). Finally, \( r_\delta(y) \) is a (general) infrared regulator satisfying \( \lim_{\delta \to 0} r_\delta(y) = 1 \), and \( \delta \) is the IR cutoff.

The apparently \( \delta \)-dependent term in curly brackets in (1) is in fact ensuring that the integral part of the bound state equation is independent of \( \delta \). All \( \delta \)-dependence is contained in the last line of the bound state equation, i.e. in the \( x_1 \) dependent \( I_\delta(x_1) \) given in \[2\]. If \( I_\delta(x_1) \) vanished identically for all values of \( x_1 \), the bound state equation \[1\] would be independent of \( \delta \), just as in Hamiltonian matrix element formulation of the similarity renormalization \[3\].

It is obvious that this does not happen here, except for the leading order in the nonrelativistic expansion, i.e. the point \( x_1 = 1/2 \). However, an arbitrarily small deviation from \( x = 1/2 \) introduces a positive divergent constant into the bound state equation in the limit \( \delta \to 0 \). The bound state equation therefore is not defined in this limit.

Is there a solution to this problem within the framework of the (IR) cutoff theory? Do we need to solve for the zero modes first? These questions are open at present. Hopefully, by the next light-cone meeting we will have answers.

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