We investigate statistical properties of daily international market indices of seven countries, and high-frequency S&P500 and KOSDAQ data, by using the detrended fluctuation method and the surrogate test. We have found that the returns of international stock market indices of seven countries follow a universal power-law distribution with an exponent of $\zeta \approx 3$, while the Korean stock market follows an exponential distribution with an exponent of $\beta \approx 0.7$. The Hurst exponent analysis of the original return, and its magnitude and sign series, reveal that the long-term-memory property, which is absent in the returns and sign series, exists in the magnitude time series with $0.7 \leq H \leq 0.8$. The surrogate test shows that the magnitude time series reflects the non-linearity of the return series, which helps to reveal that the KOSDAQ index, one of the emerging markets, shows higher volatility than a mature market such as the S&P 500 index.

In order to test the nonlinearity of the time series, the surrogate test is performed for all time series. We find that the magnitude time series reflects the non-linearity of the return series, which helps to reveal that the KOSDAQ index, one of the emerging markets, shows higher volatility than a mature market such as the S&P 500 index.
II. DATA

We use the return series in eight daily international market indices of seven countries from 1991 to 2005, the S&P 500 index (5 minutes) from 1995 to 2004, and the KOSDAQ index (1 minute) from 1997 to 2004. The seven countries are France (CAC40), Germany (DAX), United Kingdom (FTSE100), Hong Kong (HangSeng), KOREA (KOSPI), America (NASDAQ), Japan (Nikkei225), and America (S&P 500). We make use of the normalized return often used in the financial time series analysis instead of the stock prices. Let \( y_1, y_2, \ldots, y_n \) be the daily stock prices. The normalized return \( R_t \) at a given time \( t \) is defined by

\[
R_t = \ln y_{t+1} - \ln y_t,
\]

(1)

Then, \( R_t \) is multiplied by random phases,

\[
r_k = r_k e^{i \phi_k},
\]

(4)

where \( \phi_k \) is uniformly distributed in \([0, 2\pi]\). The inverse FFT of \( r_k \) gives the surrogate data retaining the linearity in the original time series,

\[
r_n' = \frac{1}{N} \sum_{k=1}^{N} r_k e^{-i 2\pi nk/N}.
\]

(5)

In the third step, non-linear measurements with the entropy, the dimension, and Lyapunov exponents are performed for the original data and the surrogate data, respectively. Finally, the difference in measurements between the original data and the surrogate data is tested for significance. If significant, the hypothesis will be rejected and the original data are regarded as having non-linearity.

B. Detrended Fluctuation Analysis

The typical methods to analyze the long-term-memory property in the time series data are largely classified into three types: the re-scaled range analysis (R/S) method proposed by Mandelbrot and Wallis [19], the modified R/S analysis by Lo et al. [18], and the DFA (detrended fluctuation analysis) method by Peng et al. [20]. In this paper, the DFA method is used due to its effectiveness even for the absence of long-term memory. The Hurst exponent can be calculated by the DFA method through the following process.

Step (1): The time series after the subtraction of the mean are accumulated as follows:

\[
y(i) = \sum_{i=1}^{N} [x(i) - \bar{x}],
\]

(6)

where \( x(i) \) are the i-th time series, and \( \bar{x} \) is the mean of the whole time series. This accumulation process is one that changes the original data into a self-similar process.

Step (2): The accumulated time series are divided into boxes of the same length \( n \). In each box of length \( n \), the trend is estimated through the ordinary least square method, called DFA(m), where \( m \) is the order of fitting. In each box, the ordinary least square line is expressed as \( y_n(i) \). By subtracting \( y_n(i) \) from the accumulated \( y(i) \) in each box, the trend is removed. This process is applied to every box and the fluctuation magnitude is calculated by using

\[
F(n) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} [y(i) - y_n(k)]^2}.
\]

(7)
FIG. 1: Cumulative distribution function (CDF) \( P(R_t > R) \) of normalized returns time series \( R_t \). (a) Normalized return distribution of international market indices of six countries, excluding Korea, from January 1991 to May 2005 in a log-log plot. (b) Linear-log plot for the KOSPI index.

The process of Step (2) is repeated for every scale, from which we obtain a scaling relation

\[
F(n) \approx cn^H, \quad (8)
\]

where \( H \) is the Hurst exponent. The Hurst exponent characterizes the correlation of time series with three different properties. If \( 0 \leq H < 0.5 \), the time series is anti-persistent. If \( 0.5 < H \leq 1 \), it is persistent. In the case of \( H = 0.5 \), the time series correspond to random walks.

**IV. RESULTS**

In this section, we analyze the statistical features of daily international market indices of seven countries from January 1993 to May 31, 2005, the S&P 500 5-minute index from 1995 to 2004, and the KOSDAQ 1-minute index from 1997 to 2004. We present the results of the statistical features such as the cumulated distribution function (CDF) and the time correlation of the various financial time series. Figure 1(a) is a log-log plot of the cumulative distribution function of the market indices of six countries, excluding Korea, from January 1991 to May 2005. Figure 1(b) is a linear-log plot of the distribution function of the KOSPI index.

In Figure 1(a), we find that the tail exponents of the indices of all the countries except Korea follow a universal power-law distribution with an exponent \( \zeta \approx 3 \). However, in Figure 1(b), we find that the Korean stock market follows an exponential distribution with \( \beta \approx 0.7 \), not a power-law distribution. These results indicate that the distribution of returns in the KOSPI index, that belongs to the emerging markets, does not follow a power-law distribution with the exponent \( \zeta \approx 3 \).

Figure 2 shows the Hurst exponents for the returns of each international market index, calculated from the return, magnitude and sign time series. The long-term-memory property is not found for the return and sign series with \( H \approx 0.5 \). However, we find that the magnitude time series has a long-term-memory property with \( H \approx 0.8 \). The surrogate test plots denoted as (surro) in Figure 2 show that the magnitude time series reflects the non-linearity of the original returns, while the sign time series shows the linearity of the original returns.

In order to investigate the scaling in high-frequency data, we use the S&P 500 5-minute index from 1995 to 2004 and the KOSDAQ 1-minute index from 1997 to 2004. Figure 3(a) shows the Hurst exponents of the return, magnitude and sign series for the S&P 500 5-minute index and the KOSDAQ 1-minute index. As for international stock market indices, the sign series corresponds to random walks \( (H \approx 0.5) \), but the magnitude series has a long-term-memory property \( (0.7 \leq H \leq 0.8) \). Figure 3(b) shows that all Hurst exponents of the corresponding surrogate data follow random walks.

In order to find the time evolution of the Hurst exponent, we also investigated the time series by shifting the S&P 500 5-minute index and KOSDAQ 1-minute index by 500 minutes and 100 minutes, respectively. Figure 4 shows the Hurst exponent calculated with 6,000 data points by shifting approximately 500 minutes for the S&P 500 5-minute index, from 1995 to 2004. The average Hurst exponent \( H \approx 0.5 \) for the S&P 500 index sign series of the returns, and \( H \approx 0.7 \) for the magnitude time series. In addition, the surrogate test shows that the non-linearity of the original time series is reflected...
FIG. 3: (a) Hurst exponent of the S&P 500 5-minute index and the KOSDAQ 1-minute index with the time series of the returns divided into magnitude and sign time series. (b) Hurst exponent of the surrogate data of the S&P 500 and KOSDAQ indicies.

by the magnitude time series.

Figure 5 shows the Hurst exponent calculated with 6,000 data points by shifting approximately 100 minutes for the KOSDAQ 1-minute index from 1997 to 2004. Though on average $H \approx 0.5$, the Hurst exponent of the returns changes considerably over time, unlike the S&P 500 index with a more or less uniform Hurst exponent. In particular, in the KOSDAQ index during its bubble period from the second half of 1999 to mid-2000, a large long-term-memory property is observed in the return series. After the market bubble burst, we found that the Hurst exponent of the returns dropped to 0.5. This result indicates that the KOSDAQ index may have improved its market efficiency after the bubble. As in the previous results, the non-linearity of the original time series of the KOSDAQ data is reflected in the magnitude time series, and the linearity in the sign time series.

FIG. 4: Hurst exponent of S&P 500 5-minute index returns divided into magnitude and sign: the black solid line denote the price of S&P 500 from 1995 to 2004. The other lines denotes the Hurst exponents corresponding to the returns, sign and magnitude time series and the Hurst exponents of the returns, sign and magnitude time series of the surrogate data. The notation (surro) denotes the corresponding surrogate data.

V. CONCLUSION

In this paper, we have investigated the statistical features of international stock market indicies of seven countries, high-frequency S&P 500 and KOSDAQ data. For this purpose, the tail index was studied through a linear fitting method by using the Hurst exponent by the DFA method. Also, the non-linearity was measured through the surrogate test method. We find that the absolute value distribution of the returns of international stock market indices follows a universal power-law distribution, having a tail index $\zeta \approx 3$. However, the Korean stock market follows an exponential distribution with $\beta \approx 0.7$, not a power-law distribution.

We also found that in the time series of international market indicies, the S&P 500 index and the KOSDAQ index, the returns and sign series follow random walks ($H \approx 0.5$), but the magnitude series does not. On the other hand, we found that in all the time series, the Hurst exponent of the magnitude time series has a long-term-
FIG. 5: Hurst exponent of the KOSDAQ 1-minute index returns divided into magnitude and sign series. The solid black line shows the KOSDAQ index from 1997 to 2004. The other lines denote the Hurst exponents for the returns, sign and magnitude time series and the corresponding surrogate data. The notation (surro) denotes the corresponding surrogate data.

memory property \(0.7 \leq H \leq 0.8\). Furthermore, we found that in high-frequency data, the KOSDAQ index, one of the emerging markets, shows higher volatility than a mature market such as the S&P 500 index, which is possibly caused by the abnormally generated bubble. We found a long-term-memory property in the magnitude time series of all data, regardless of nation or time scale. Non-linear features of the returns are generally observed in emerging and mature markets. Our results may be useful in analyzing global financial markets, for example, differentiating the mature and emerging markets.

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