A gap-less mode of the singlet Higgs field

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Abstract

Recent lattice results suggest the existence of a gap-less mode of the singlet Higgs field. We present a description of spontaneous symmetry breaking in $\lambda\Phi^4$ theories showing why one is faced with long-wavelength, collective modes of the scalar condensate whose energy $\tilde{E}(p) \to 0$ in the $p \to 0$ limit.
1. Introduction

Following analogies with condensed matter physics, the idea of a ‘condensed vacuum’ is also playing an increasingly important role in particle physics. Indeed, scalar, gluon and fermion condensates are basic ingredients in the present description of electroweak and strong interactions where one introduces a set of elementary quanta whose perturbative vacuum state $|\phi\rangle$ is not the physical ground state $|v\rangle$ of the interacting theory.

As pointed out in ref.\[1\], the issue of vacuum condensation in Lorentz-invariant theories is somewhat controversial, in view of the apparent contradiction between exact Lorentz-invariance and a non-vanishing energy-density for the vacuum. For this reason, it has been suggested that the origin of vacuum condensation may reflect a ‘principal violation’ \[1\] of Lorentz-invariance, as in the case of the existence of a fundamental length scale associated with an ultimate ultraviolet cutoff $\Lambda$.

This remark is even more relevant if one attempts to model the world as a sequence of effective theories embedded one into the other. In this approach, the problem of exact Lorentz-covariance in an intermediate description is unavoidable since the cutoff will never be really sent to infinity. However, even in a fully continuum, quantum-field theoretical approach \[2\] to $(\lambda\Phi^4)_4$ theory, where the vacuum is found to have exactly zero energy-density, one finds a non-trivial distribution function $f(p^2) \equiv \langle N_p \rangle$ of the elementary quanta in the non-perturbative vacuum. For this reason, in the $p \to 0$ limit, where the $f(p^2)$ function yields significant contributions, the propagation in the physical ground state resembles the one in a condensed medium \[2\].

Here we shall adopt the point of view that Lorentz-covariance has to become exact in the double limit where both the quantization volume and the ultraviolet cutoff are sent to infinity. However, for finite $\Lambda$ one may be faced with deviations from an exact Lorentz-covariant energy spectrum that depend, in general, on the peculiar nature of the ground state. In this sense, the formation of a ‘condensed vacuum’, defined by a macroscopic occupation of the same quantum state, may represent the operative construction of a preferred reference frame suggesting non-Lorentz-covariant effects associated with the long-wavelength excitations of the condensed phase.

To understand the underlying mechanisms, in the case of spontaneously broken scalar theories, we observe preliminarily that for an interacting system the condensed vacuum cannot be a pure Bose condensate. Other components are needed from purely phase-space arguments and take into account all possible processes that, with the same average particle density, can ‘deplete’ the $p = 0$ state.
We shall assume that condensation extends up to some typical momentum $|p| \sim \delta$ and is controlled by an unknown distribution function $f_\delta(p^2)$ of the elementary quanta. It is clear that $\delta \to 0$ in the limit of a vanishingly small interaction where $f_\delta(p^2)$ becomes non zero only for $p = 0$. Therefore, in the ‘trivial’ $\lambda \Phi^4$ theories, we expect the pure Bose condensate to become a better and better approximation to the spontaneously broken vacuum when approaching the continuum limit of quantum field theory.

Let us now consider the excitation spectrum $\tilde{E}_\Lambda(p)$ of the cutoff theory in the limit when $f_\delta(k^2)$ is very strongly peaked at $k = 0$ and vanishes very quickly for $|k| \neq 0$. In this case, we can easily understand the crucial difference with the usual perturbative vacuum. There, the low-lying excitations correspond to single particle states with $\sqrt{p^2 + m^2}$ energy and one has to reach $|p| \sim m$ values before two-particle excitations can become important. Here, in the condensed vacuum, for $|p| < \delta$ it is energetically more convenient to split the momentum among a large number of quanta. This costs very little energy analogously to the small displacements of atoms from their equilibrium positions, at the base of the propagation of elastic waves in continuous media. On the other hand, for momenta larger than $\delta$, where the condensed vacuum looks ‘empty’, a Lorentz-covariant spectrum applies, say $\sqrt{p^2 + M_h^2}$.

Since $\delta \to 0$ when $\Lambda \to \infty$, the form $\sqrt{p^2 + M_h^2}$ would hold everywhere in a strictly continuum theory but, at finite $\Lambda$, one has deviations from exact Lorentz-covariance in an infinitesimal region of momenta $|p| < \delta$ (or equivalently for fluctuations with wavelengths larger than $\delta^{-1}$).

The possibility that the ‘Higgs mass’ $M_h$ differs non-trivially from the energy-gap $\tilde{E}_\Lambda(0)$ of the broken phase has been objectively addressed \[ with lattice simulations. To this end the energy spectrum was measured from the exponential decay of the connected correlator at various values of $|p|$. In this analysis, by approaching the continuum limit, one expects to find a region of 3-momentum where indeed $\tilde{E}_\Lambda(p)$ is well reproduced by the asymptotic form $\sqrt{p^2 + M_h^2}$. The delicate issue is whether this region extends down to $p = 0$ and whether the value of $M_h$ determined in this way agrees with the value of the energy gap $\tilde{E}_\Lambda(0)$. To exclude possible trivial effects, one should also repeat the same analysis in the symmetric phase and compare the corresponding measured energy spectrum $E_\Lambda(p)$ with the form $\sqrt{p^2 + m^2}$. In the symmetric phase the lattice data \[ give $E_\Lambda(0) = m$ to very high accuracy as expected. However, in the broken phase, $\tilde{E}_\Lambda(0) - p^2$ depends on $|p|$ when $p \to 0$ and therefore the attempt to extract $M_h$ from the low-momentum data becomes problematic. If, on the other hand, $M_h$ is extracted from the higher-momentum data where $\tilde{E}_\Lambda(p) - p^2$ does not depend on $|p|$, then one finds $\tilde{E}_\Lambda(0) < M_h$ with a discrepancy between the two values that seems to
increase when taking the continuum limit.

Moreover, new data show that, for the same lattice size, one finds the same $M_h$ at high momenta but smaller and smaller values of $\tilde{E}_\Lambda(0)$. Namely, by using the same lattice parameters that on a $20^4$ lattice give $\tilde{E}_\Lambda(0) = 0.3912 \pm 0.0012$, one finds $\tilde{E}_\Lambda(0) = 0.3791 \pm 0.0035$ on a $24^4$ lattice, $\tilde{E}_\Lambda(0) = 0.344 \pm 0.008$ on a $32^4$ lattice and $\tilde{E}_\Lambda(0) = 0.298 \pm 0.015$ on a $40^4$ lattice. Therefore, differently from $M_h$, which is associated with the higher-momentum part of the energy spectrum, the energy-gap in the broken phase is an infrared-sensitive quantity that, conceivably, vanishes in the infinite-volume limit.

An indication in this sense will be obtained in the next section by studying the $p \to 0$ limit of the propagator $G(p)$. The result is very simple and can be understood by considering the following equation
\begin{equation}
    f^{-1}(x) = 1 + x^2 - g^2 x^2 f(x)
\end{equation}
that bears some analogy to the actual physical situation. In the $x \to 0$ limit there are two distinct limiting behaviours: a) $f(x) \to 1$ and b) $f(x) \sim \frac{1}{g^2 x^2} \to +\infty$ but only the former solution is recovered with a finite number of iterations from
\begin{equation}
    f_0(x) = \frac{1}{1 + x^2}
\end{equation}
for $g^2 = 0$. In the case of $\lambda \Phi^4$ theory, the gapless mode in the broken phase corresponds to the b) type of behaviour.

2. The propagator for $p \to 0$ in the broken phase

Let us now consider a one-component $\lambda \Phi^4$ theory and the problem of determining the scalar propagator of the fluctuating field $h(x) = \Phi(x) - \varphi$ in the presence of a constant background field $\varphi$. Ultimately, we shall be interested in the $h$–field propagator $G(p)$ in the broken symmetry phase, i.e. at the minima of the effective potential $V_{\text{eff}}(\varphi)$. By defining
\begin{equation}
    \frac{dV_{\text{eff}}}{d\varphi} \equiv J(\varphi) \equiv \varphi T(\varphi^2)
\end{equation}
these are the absolute minima $\varphi = \pm v \neq 0$ where
\begin{equation}
    T(\varphi^2) = 0
\end{equation}
and
\begin{equation}
    \left| \frac{d^2V_{\text{eff}}}{d\varphi^2} \right|_{\varphi = \pm v} > 0
\end{equation}
Usually, in the broken phase, one defines the field propagator from a Dyson sum of 1PI graphs only, say
\[ G(p)_{|1PI} \equiv D(p) \] (2.4)
where
\[ D^{-1}(0) \equiv \frac{d^2V_{eff}}{d\phi^2} \] (2.5)
is evaluated at \( \phi = \pm v \). In this way one neglects the possible role of the one-particle reducible, zero-momentum tadpole graphs. The reason is that their sum is proportional to the 1-point function, i.e. to \( J(\phi) \) in Eq.(2.1) that vanishes by definition at \( \phi = \pm v \). However, the zero-momentum tadpole subgraphs are attached to the other parts of the diagrams through zero-momentum propagators. Therefore, their overall contribution is proportional to \( J(\phi)G(0) \) that vanishes provided \( G(0) \) is non-singular at the minima. In this respect, neglecting the tadpole graphs amounts to assume the regularity of \( G(0) \) at \( \phi = \pm v \) which is certainly true in a finite-order expansion in powers of \( J(\phi) \). However, to check the assumption beyond perturbation theory one has first to control the full propagator in a small region of \( \phi \) values around the minima, by including all zero-momentum tadpole graphs, and finally take the limit \( \phi \to \pm v \). We observe that the problem of tadpole graphs was considered in ref.[7] where the emphasis was mainly to find an efficient way to re-arrange the perturbative expansion. Here, we shall attempt a non-perturbative all-order re-summation of the various effects to check the regularity of \( G(0) \) for \( \phi \to \pm v \).

We shall approach the problem in two steps. In a first step, we shall consider the contributions to the propagator that contain all possible insertions of zero-momentum lines on the internal part of the graphs, i.e. inside 1PI vertices. However, at this stage, the external zero-momentum propagators to the sources maintain their starting value \( D(0) \) at \( J = 0 \). This approximation gives rise to an auxiliary inverse propagator given by
\[ G_{aux}^{-1}(p) = D^{-1}(p) - z\Gamma_3(p,0,-p) + \frac{(z\phi)^2}{2!}\Gamma_4(0,0,0,-p) - \frac{(z\phi)^3}{3!}\Gamma_5(0,0,0,0,-p) + .. \] (2.6)
where
\[ z \equiv T(\phi^2)D(0) \] (2.7)
represents the basic one-tadpole insertion. Eq.(2.6) can be easily checked diagrammatically starting from the tree approximation where
\[ V_{eff} = \frac{1}{2}r\phi^2 + \frac{\lambda}{4!}\phi^4 \] (2.8)
\[ D^{-1}(p) = p^2 + r + \frac{\lambda\phi^2}{2} \] (2.9)
and $\Gamma_3(p, 0, -p) = \lambda \varphi$, $\Gamma_4(0, 0, p, -p) = \lambda$ (with all $\Gamma_n$ vanishing for $n > 4$).

Now, by using the relation of the zero-momentum 1PI vertices with the effective potential at an arbitrary $\varphi$

$$\Gamma_n(0, 0, ...0) = \frac{d^n V_{\text{eff}}}{d\varphi^n}$$

we can express the auxiliary zero-momentum inverse propagator of Eq.(2.6) as

$$G^{-1}_{\text{aux}}(0) = \left. \frac{d^2 V_{\text{eff}}}{d\varphi^2} \right|_{\varphi_{\text{aux}} = \varphi(1-z)}$$

The second step consists in including now all possible tadpole corrections on each external zero-momentum line in (2.6). This is independent of the flowing momentum $p$ and leads to a new infinite hierarchy of Feynman graphs. In fact, in a diagrammatic expansion, a single external zero-momentum leg gives rise to an infinite number of graphs, each producing another infinite number of graphs and so on. Despite of the apparent complexity of the task, the final outcome of this computation can be cast in a rather simple form, at least on a formal ground. In fact, we can re-arrange the infinite expansion for the zero-momentum propagator (all $\Gamma_n$ are evaluated at zero external momenta)

$$G(0) = D(0) + J\Gamma_3D^3(0) + \frac{3J^2\Gamma_3^2}{2}D^5(0) - \frac{\Gamma_4J^2}{2}D^4(0) + \mathcal{O}(J^3)$$

in terms of a modified source

$$\tilde{J} = J - \frac{J^2\Gamma_3}{2}D^2(0) + \mathcal{O}(J^3) \equiv \varphi \tilde{T}(\varphi^2)$$

in such a way that the full power series expansion for the exact inverse zero-momentum propagator can be expressed as

$$G^{-1}(0) = \left. \frac{d^2 V_{\text{eff}}}{d\varphi^2} \right|_{\hat{\varphi} = \varphi(1-\tau)}$$

with

$$\tau \equiv \tilde{T}(\varphi^2)G(0)$$

i.e., as in (2.11) with the replacement $z \to \tau$. In the same way, Eq.(2.6) becomes

$$G^{-1}(p) = D^{-1}(p) - \varphi \tau \Gamma_3(p, 0, -p) + \frac{(\varphi \tau)^2}{2!}\Gamma_4(0, 0, p, -p) - \frac{(\varphi \tau)^3}{3!}\Gamma_5(0, 0, 0, p, -p) + ..$$

Notice that, in the limit $\varphi \to \pm v$ Eq.(2.14) would be generally considered equivalent to Eq.(2.5) that, however, neglects the tadpole graphs altogether. As anticipated, this is true provided $G(0)$ remains non-singular when $J$ and $\tilde{J}$ vanish. Our main point is that, after
resumming the tadpole graphs to all orders, there are now multiple solutions for the zero-momentum propagator that differ from (2.3), even when $\varphi \to \pm v$, and that would not be present otherwise.

To study this problem, we shall analyze Eq.(2.14) in the case of the tree approximation Eq.(2.8). In fact, the simple idea that the broken phase has just massive excitations of mass $M_h$ finds its main motivation in a tree-level analysis. Moreover, we shall assume that when $\varphi \to \pm v$ (and $J \to 0$) also $\tilde{J} \to 0$. Assuming the alternative possibility, i.e. that the full $\tilde{J}$ remains non-zero when $\varphi \to \pm v$, would give even more drastic results. In fact, in this case, an inverse propagator as in Eq.(2.3) would never be recovered from (2.14), even as a particular solution.

By defining the limiting value $\tau \to \bar{\tau}$ for $\varphi^2 \to v^2$, the usual ‘regular’ solution for the inverse propagator (2.14) corresponds to $\bar{\tau} = 0$, namely

$$\lim_{\varphi^2 \to v^2} G^{-1}_{\text{reg}}(0) = \frac{\lambda v^2}{3} \equiv M_h^2$$

so that

$$G^{-1}_{\text{reg}}(p) = p^2 + M_h^2$$

at $\varphi = \pm v$ (2.18)

However, another solution is

$$\lim_{\varphi^2 \to v^2} G^{-1}_{\text{sing}}(0) = \frac{\lambda v^2}{2} [\bar{\tau}^2 - 2\bar{\tau} + \frac{2}{3}] = 0$$

(2.19)

which implies limiting values $\bar{\tau} = 1 \pm \frac{1}{\sqrt{3}}$ for which

$$G^{-1}_{\text{sing}}(p) = p^2$$

at $\varphi = \pm v$ (2.20)

Beyond the tree-approximation finding the singular solution $G^{-1}(0) = 0$ at $\varphi = \pm v$ is equivalent to determine that value of $\varphi^2 \equiv v^2(1 - \bar{\tau})$ where $\frac{d^2V_{\text{eff}}}{d\varphi^2} = 0$. For instance, in the case of the Coleman-Weinberg effective potential

$$V_{\text{eff}}(\varphi) = \frac{\lambda^2 \varphi^4}{256\pi^2}(\ln \frac{\varphi^2}{v^2} - \frac{1}{2})$$

the required values are $\bar{\tau} = 1 \pm e^{-1/3}$. In principle, such solutions should be found in any approximation to the effective potential since their existence depends on the very general assumptions of the broken phase.

Before concluding this section, we shall try to provide a possible explanation for the singular zero-momentum behaviour we have pointed out. To this end, let us introduce the
generating functional for connected Green’s functions \( W[J] \) and (for a space-time constant source \( J \)) the associated density \( w(J) \)

\[
W[J] = w(J) \int d^4x
\]  

(2.22)

In this formalism

\[
\phi(J) = \frac{dw}{dJ}
\]  

(2.23)

is just the inverse of

\[
J(\phi) = \frac{dV_{LT}}{d\phi}
\]  

(2.24)

where the effective potential enters as \( V_{\text{eff}}(\phi) = V_{LT}(\phi) \), i.e. the Legendre transform (‘LT’) of \( w(J) \). Notice that in this way \( V_{LT}(\phi) \) is rigorously convex downward. For this reason, it is not the same thing as the usual non-convex (‘NC’) effective potential \( V_{\text{eff}}(\phi) = V_{\text{NC}}(\phi) \) we have considered so far. Moreover, in the presence of spontaneous symmetry breaking, \( V_{LT}(\phi) \) is not an infinitely differentiable function of \( \phi \), differently from \( V_{\text{NC}}(\phi) \). In this language, \( \pm v \) denote the absolute minima of \( V_{\text{NC}}(\phi) \) and Eq.\((2.25)\) becomes

\[
D^{-1}(0) = \left. \frac{d^2V_{\text{NC}}}{d\phi^2} \right|_{\phi=\pm v}
\]  

(2.25)

Now, let us denote \( J = \hat{J}(\varphi) \) the argument of \( w(J) \) that corresponds to determine the full propagator (i.e. including all tadpole graphs) in a given background \( \varphi \). We can look for its inversion in the form

\[
G^{-1}(0) = \left. \frac{d^2V_{LT}}{d\phi^2} \right|_{\phi=f(\varphi)}
\]  

(2.26)

with a suitable \( f(\varphi) \). The possibility to recover the same inverse propagator as in \((2.25)\) requires that, for \( \varphi \to \pm v \), the limiting value of \( f(\varphi) \) has to approach one of the absolute minima of the non-convex effective potential , namely

\[
\lim_{|\varphi| \to v} f(\varphi) = \pm v
\]  

(2.27)

Still, \((2.26)\) is different from \((2.25)\). In fact, the usual identifications \( V_{LT}(\varphi) = V_{\text{NC}}(\pm v) \), in the region \(-v \leq \varphi \leq v \) enclosed by the absolute minima of the non-convex approximation, and \( V_{LT}(\varphi) = V_{\text{NC}}(\varphi) \) for \( \varphi^2 > v^2 \), do not resolve all ambiguities. The identification of the inverse propagators in Eqs. \((2.25)\) and \((2.26)\) amounts to a much stronger assumption: the derivative in Eq.\((2.26)\) has to be a left- (or right-) derivative depending on whether we approach the point \( f = -v \) ( or \( f = +v \)). However, this is just a prescription since derivatives depend on the chosen path (unless one deals with infinitely differentiable functions) and, in general,
Eq. (2.26) leads to multiple solutions for the inverse propagator. Therefore, despite one can define a prescription for which $G_{\text{reg}}^{-1}(0) = D^{-1}(0) = M_h^2$, one is also faced with a $G_{\text{sing}}^{-1}(0) = 0$, as when approaching the points $\pm v$ from the inner region where the Legendre-transformed potential is flat.

Now, as discussed in a very transparent way in ref. [9], the difference between $V_{\text{LT}}(\phi)$ and $V_{\text{NC}}(\phi)$ amounts to include the quantum effects of the zero-momentum mode that cannot be treated as a purely classical background. Namely, one cannot simply use $V_{\text{NC}}(\phi)$ to compute $w(J)$ but has still to perform one more functional integration in field space on the strength of the zero-momentum mode. This is the reason why starting from $w(J)$ one gets $V_{\text{LT}}(\phi)$ as the Legendre transform and not $V_{\text{NC}}(\phi)$. In this respect, our analysis suggests that determining the propagator after this last integration step may finally be equivalent to include all zero-momentum tadpole graphs in the classical background.

We conclude this section with the remark that the singular zero-momentum behaviour we have pointed out does not depend at any stage on the existence of a continuous symmetry of the classical potential. As such, there should be no differences in a spontaneously broken $O(N)$ theory. Beyond the approximation where the ‘Higgs condensate’ is treated as a purely classical background, one has to perform one more integration over the zero-momentum mode of the condensed $\sigma-$field. Therefore, all ambiguities in computing the inverse propagator of the $\sigma-$field through Eq. (2.26) remain. In this sense, the existence of gap-less modes of the singlet Higgs field has nothing to do with the number of field components.

We only note that in the $O(N)$ theory some additional care is needed to re-sum tadpole graphs to all orders in perturbation theory. In fact, the tadpole function of the $\sigma-$field $T_{\sigma}(\varphi) \equiv m_\pi^2$ plays the role of a mass term for the $\pi$-fields. By setting $T_{\sigma} = m_\pi^2 = 0$ in a straightforward diagrammatic expansion around $T_{\sigma} = 0$, the 2-point function of the $\sigma-$field $\Gamma_{\sigma}(p)$ at $p = 0$ (the equivalent of our $D^{-1}(0)$) would become singular due to the massless Goldstone loop. However, beyond perturbation theory, $m_\pi = 0$ can coexist with a non-singular $\Gamma_{\sigma}(0)$ [10], differently from the original perturbative analysis of [8]. In the same approximation [10], the $\pi-$fields decouple from each other and from the $\sigma-$field. This result supports the point of view [11] that, beyond perturbation theory, the dynamics of ‘radial’ and ‘angular’ degrees of freedom may effectively decouple.

### 3. The energy spectrum in the broken phase

By choosing the singular solutions corresponding to $G^{-1}(0) = 0$ at $\varphi = \pm v$, one always ends up with a form of $G^{-1}(p)$ that vanishes when $p_\mu \to 0$ implying the existence of gap-less
modes whose energy also vanishes when $p \to 0$. We can express the required relations for the various modes of the cutoff theory in terms of unknown slope parameters $\eta_\Lambda$

$$\lim_{p \to 0} \tilde{E}_\Lambda^2(p) = \eta_\Lambda p^2$$

at $\varphi = \pm v$ \hspace{1cm} (3.1)

that describe long-wavelength collective excitations of the scalar condensate.

Although determining the $\eta_\Lambda$’s would require the analytical form of the propagator at non-zero momenta, one can draw a certain number of general conclusions. A first observation is that the two distinct solutions for $\tilde{\tau}$ may correspond to different $\eta_\Lambda$’s meaning that the scalar condensate can support different types of oscillations. Think for instance of superfluids where the velocity of second sound $c_2$ approaches $\frac{c_s}{\sqrt{3}}$ in the limit of zero temperature, $c_s$ being the velocity of (first) sound \([12]\).

A second observation is that the possible limiting values for the $\eta_\Lambda$’s can easily be guessed by imposing the requirement of a Lorentz-covariant spectrum in the continuum limit $\Lambda \to \infty$. In this case, there are only two possibilities, namely

$$\eta_\Lambda \to 1$$ \hspace{1cm} (3.2)

and/or

$$\eta_\Lambda = \mathcal{O}\left(\frac{M_h^2}{\delta^2}\right) \to \infty$$ \hspace{1cm} (3.3)

The limit in (3.2) corresponds to ordinary massless fields, quite unrelated to the usual massive Higgs particle. This scenario finds support in the tree-level approximation Eq.(2.20).

On the other hand, the alternative (3.3) corresponds to the point of view expressed in the Introduction. This can also be understood by noticing that in the presence of several solutions for the propagator, the physical energy spectrum is dominated by the lowest excitations for each region of momenta. Therefore, if there is a massive mode, the form $\sqrt{p^2 + M_h^2}$ has to evolve somehow into Eq.(3.1) for $p \to 0$. By denoting $|p| \sim \delta \ll M_h$ the typical range of momenta where the transition takes place, this leads to $\eta_\Lambda \delta^2 \sim M_h^2$. However, exact Lorentz-covariance in the local limit $\frac{\Lambda}{M_h} \to \infty$ requires that any possible deviation from $\sqrt{p^2 + M_h^2}$ can only reduce to the zero-measure set $p_\mu = 0$, the only value of the 4-momentum that is left invariant by the transformations of the Poincaré group. Therefore, in the continuum limit of the broken phase $\eta_\Lambda \to \infty$ meaning that the form $\sqrt{p^2 + M_h^2}$ holds ‘almost’ everywhere, i.e. with the exception of the point $p = 0$ where one has $\tilde{E}(0) = 0$. One such example is represented by the toy-model

$$\tilde{E}_\Lambda(p) = (1 - e^{-\frac{|p|}{\delta}})\sqrt{p^2 + M_h^2}$$ \hspace{1cm} (3.4)
with
\[ \frac{\Lambda}{M_h} = \frac{M_h}{\delta} \equiv \sqrt{\eta \Lambda} \] (3.5)
that leads to
\[ \lim_{\Lambda M_h \to \infty} \frac{|\tilde{E}_\Lambda(p) - \sqrt{p^2 + M_h^2}|}{\sqrt{p^2 + M_h^2}} = \lim_{\Lambda M_h \to \infty} e^{-\frac{|p|}{\sqrt{\eta \Lambda}}} = 0 \] (3.6)
provided \[ \frac{|p|}{M_h} > 0 \]. Nevertheless, for any \( \Lambda \) there is a far infrared region of momenta near \( p = 0 \) where \( M_h \) and \( \tilde{E}_\Lambda(p) \) differ non trivially.

This scenario is supported by the experimental example of superfluid He\(^4\). In this case the same spectrum has two different branches: a phonon branch \( \omega_{\text{ph}}(p) = c_s |p| \) describing the spectrum for \( p \to 0 \) and a roton branch \( \omega_{\text{rot}}(p) = \Delta + \frac{|p|^2}{2\mu} \) starting at momenta \( |p| > p_0 \).

With this analogy, (3.3) would represent the matching at \( |p| \sim \delta \ll M_h \) between a ‘phonon’ with sound velocity \( c_s = \sqrt{\eta \Lambda} \) and a ‘Lorentz-covariant roton’ with \( \Delta = \mu \equiv M_h \).

4. Summary and outlook

In this Letter, we have presented several arguments that, quite independently of the Goldstone phenomenon, suggest the existence of gap-less modes of the scalar condensate in the broken phase of \( \lambda \Phi^4 \) theories. Although the argument can be purely diagrammatic, this result will be better understood by representing spontaneous symmetry breaking along the lines presented in the Introduction, i.e. as a real condensation process of physical quanta \[ [13] \]. In this way, one will also understand the puzzling lattice data of ref.[5]. These show that the energy-gap in the broken phase of a one-component \( \lambda \Phi^4 \) theory is not the ‘Higgs mass’ \( M_h \) but an infrared-sensitive quantity that becomes smaller and smaller by increasing the lattice size.

Now, it is well known that condensed-matter systems can support long-range forces even with elementary constituents that only have short-range 2-body interactions. For this reason, it should not be surprising that a gap-less collective mode in the condensed phase of \( \lambda \Phi^4 \) theories will give rise to a long-range potential. The phenomenological aspects of this analysis and a possible wider theoretical framework will be presented elsewhere.

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