Mutually Exclusive Modules in Logic Programming

Keehang Kwon
Faculty of Computer Eng., DongA University
khkwon@dau.ac.kr

June 17, 2015

Abstract: Logic programming has traditionally lacked devices for expressing mutually exclusive modules. We address this limitation by adopting choice-conjunctive modules of the form $D_0 \& D_1$ where $D_0, D_1$ are a conjunction of Horn clauses and $\&$ is a linear logic connective. Solving a goal $G$ using $D_0 \& D_1$ – $ex(D_0 \& D_1, G)$ – has the following operational semantics: choose a successful one between $ex(D_0, G)$ and $ex(D_1, G)$. In other words, if $D_0$ is chosen in the course of solving $G$, then $D_1$ will be discarded and vice versa. Hence, the class of choice-conjunctive modules can capture the notion of mutually exclusive modules.

keywords: mutual exclusion, cut, linear logic, choice-conjunction.

1 Introduction

Modern logic programming languages support a notion of modules, \textit{i.e.}, a conjunction of clauses as a unit. Despite their attractiveness, logic programming has traditionally lacked elegant devices for structuring mutually exclusive at the module level. Lacking such devices, structuring mutually exclusive modules in logic programming has been impossible.

This paper proposes a logical, high-level solution to this problem. To be specific, we propose MutexWeb, an extension to LogicWeb with a novel feature called choice-conjunctive modules. This logic extends modules by the choice construct of the form $D_0 \& D_1$ where $D_0, D_1$ are modules and $\&$ is a choice-conjunctive connective of linear logic. Inspired by \cite{3}, this has the following intended semantics: choose a successful one between $D_0$ and $D_1$ in the course of solving a goal. Of course, the unchosen module will be discarded. This expression thus supports the idea of mutual exclusion.

An illustration of this aspect is provided by the following modules $quicksort, heapsort$ which define the usual $qsort, hsort$ relation:

\begin{verbatim}
mod(quicksort). % quicksort
qsort(X, L) :- . . .
\end{verbatim}
mod(heapsort). % heapsort

hsort(X, L) : - . . .

mod(sort). % module sort

mod(quicksort) & mod(heapsort).

Now we want to define a module sort which contains different sorting algorithms. This is show below:

In the above, these two sorting algorithms are defined as mutually exclusive. Hence, only one of these two sorting algorithms can be used.

The remainder of this paper is structured as follows. We describe MutexWeb in the next section. In Section 3 we present some examples of MutexWeb. Section 4 concludes the paper.

2 The Language

The language is an extended version of Horn clauses with choice-conjunctive modules and implication goals. It is described by $G$- and $D$-formulas given by the syntax rules below:

$$G ::= A | G \land G | D \supset G | \exists x \ G$$

$$D ::= A | G \supset D | \forall x \ D | D \& D | D \land D$$

In the rules above, $A$ represents an atomic formula. A $D$-formula is called a module.

In the transition system to be considered, $G$-formulas will function as queries and a set of $D$-formulas will constitute a program.

We will present an operational semantics for this language. The rules of MutexWeb are formalized by means of what it means to execute a goal task $G$ from a program $P$. These rules in fact depend on the top-level constructor in the expression, a property known as uniform provability[7, 8]. Below the notation $\text{bchain}(D, P, A)$ denotes that the $D$ formula is distinguished (marked for backchaining). Note that execution alternates between two phases: the goal-reduction phase (one without a distinguished clause) and the backchaining phase (one with a distinguished clause).

**Definition 1.** Let $G$ be a goal and let $P$ be a program. Then the notion of executing $\langle P, G \rangle$ -- $\text{ex}(P, G)$ -- is defined as follows:
(1) \( bchain(A, P, A). \) % This is a success.

(2) \( bchain(G_1 \supset D, P, A) \) if \( ex(P, G_1) \) and \( bchain(D, P, A) \).

(3) \( bchain(\forall x D, P, A) \) if \( bchain([t/x]D, P, A) \).

(4) \( bchain(D_0 \land D_1, P, A) \) if \( bchain(D_0, P, A) \).

(5) \( bchain(D_0 \land D_1, P, A) \) if \( bchain(D_0, P, A) \).

(6) \( bchain(D_0 \& D_1, P, A) \) if choose a successful disjunct between \( bchain(D_0, P, A) \) and \( bchain(D_1, P, A) \).

(7) \( ex(P, A) \) if \( D \in P \) and \( bchain(D, P, A) \). % change to backchaining phase.

(8) \( ex(P, G_1 \land G_2) \) if \( ex(P, G_1) \) and \( ex(P, G_2) \).

(9) \( ex(P, \exists x G_1) \) if \( ex(P, [t/x]G_1) \).

(10) \( ex(P, D \supset G_1) \) if \( ex(\{D\} \cup P, G_1) \)

In the rule (6), the symbol \( D_0 \& D_1 \) allows for the mutually exclusive execution of modules. This rule can be implemented as follows: first attempts to solve the goal using \( D_0 \). If it succeeds, then do nothing (and do not leave any choice point for \( D_1 \)). If it fails, then \( D_1 \) is attempted.

Our execution model based on uniform proof is not complete with respect to linear logic. However, it is complete with respect to affine logic (linear logic + weakening). The following theorem connects our language to affine logic. Its proof can be obtained from the fact that the cut rule is admissible in affine logic.

**Theorem 1** Let \( \{D_1, \ldots, D_n\} \) be a program and let \( G \) be a goal. Then, \( ex(\{D_1, \ldots, D_n\}, G) \) terminates with a success if and only if \( G \) follows from \( \{!D_1, \ldots, !D_n\} \) in intuitionistic affine logic.

In the above, \( !D \) represents that \( D \) is a reusable clause.

### 3 MutexWeb

In our context, a web page corresponds simply to a set of \( D \)-formulas with a URL. The module construct \( mod \) allows a URL to be associated to a set of \( D \)-formulas. An example of the use of this construct is provided by the following “lists” module which contains some basic list-handling rules.
mod(lists).
% deterministic version of the member predicate
memb(X, [X|L]) &
memb(X, [Y|L]) :- (neq X Y) \land memb(X, L).
% optimized version of the append predicate
append([], L, L) &
append([X|L_1], L_2, [X|L_3]) :- append(L_1, L_2, L_3).
% the sorting of a list via two mutually exclusive sorting algorithms
mod(quicksort) & mod(heapsort)

Our language makes it possible to use quicksort and heapsort in a mutually exclusive way.
These pages can be made available in specific contexts by explicitly mentioning the module implication. For example, consider a goal mod(lists) \supset qsort([2,60,3,5], L). Solving this goal has the effect of adding the rules in lists to the program before evaluating qsort([2,60,3,5], L), producing the result L = [2,3,5,60].

4 Conclusion

In this paper, we have considered an extension to Prolog with mutually exclusive modules. This extension allows modules of the form $D_0 \& D_1$ where $D_0, D_1$ are modules. These modules are particularly useful for structuring the program space.

We are investigating the connection between MutexWeb and Japaridze’s computability logic [3][4].

References

[1] S.W. Lok and A. Davison, “Logic Programming with the WWW,” Proceedings of the 7th ACM conference on Hypertext, ACM Press, 1996.
[2] J. Davies, D. Fensel, and F.V. Harmelen, Towards the Semantic Web, John Wiley, 2003.
[3] G. Japaridze, “Introduction to computability logic”, Annals of Pure and Applied Logic, vol.123, pp.1–99, 2003.
[4] G. Japaridze, “Sequential operators in computability logic”, Information and Computation, vol.206, No.12, pp.1443-1475, 2008.
[5] J.Y. Girard, “Linear Logic”, Theoretical Computer Science, vol.50, pp.1–102, 1987.
[6] J. Hodas and D. Miller, “Logic Programming in a Fragment of Intuitionistic Linear Logic”, Information and Computation, vol.110, pp.327–365, 1994.
[7] D. Miller, “A logical analysis of modules in logic programming,” Journal of Logic Programming, vol.6, pp.79–108, 1989.

[8] D. Miller, G. Nadathur, F. Pfenning, and A. Scedrov, “Uniform proofs as a foundation for logic programming,” Annals of Pure and Applied Logic, vol.51, pp.125–157, 1991.

[9] A. Porto, “A structured alternative to Prolog with simple compositional semantics”, Theory and Practice of Logic Programming, vol.11, No.4-5, pp.611-627, 2011.