Buoyancy driven flow in counter flow heat exchangers

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Abstract. The temperature distribution, the buoyancy head and the flow rate have been studied in a counter flow heat exchanger having buoyancy driven flow on at least one side. The assumptions made for heat flux distribution are varied and the resulting effects on the flow rate and fluid temperatures are studied. A network model is used to simulate the temperature distribution and oil flow rates in an oil-filled power transformer cooled by radiators. It is found that for operating conditions normally found for mineral oil the counter flow assumptions for heat flux distribution gives approximately the same results as assuming uniform heat flux. When a more viscous oil type is used or the radiators are placed lower than normal relative to the heat generating parts, the counter flow assumptions give more reliable results.

1. Introduction

The cooling circuit of oil-filled power transformers typically consists of a steel tank with the heat generating parts inside and radiators placed at the outside of the tank. The oil is flowing upwards in cooling ducts in the heat generating parts, and at the top of the tank the oil is free to flow into the radiators to be cooled and returned to the tank at a lower position. For sufficiently small transformers, the cooling surface can be an integrated part of the tank wall that could be corrugated to increase the surface area, but for large transformers the radiators can be placed at a distance from the transformer tank due to space limitations and mechanical considerations. There are cooling principles where pumps are used to circulate the oil and fans to blow the air through the heat exchangers, but still the most common cooling principle for power transformers is that the oil and the air should flow only due to buoyancy forces and a cooling capacity up to a specified rating should be achieved without pumps or fans.

The oil circuit is a closed single phase thermosiphon, i.e. the driving pressure differences are created due to density differences without phase change. Oil moves upwards where heating of the oil takes place, and oil moves downwards where cooling of the oil takes place. Due to continuity, the total oil flow is given by the integrated effects of heating and cooling. When there are parallel paths for the oil flow, the flow distribution is dictated by the condition of exactly the same pressure difference for all possible paths connecting two points in the flow system. Using this condition and knowing the geometry and the heat generation inside the transformer it is possible to calculate the flow and temperature distribution in very complex systems of cooling ducts.

A transformer radiator typically consists of steel sheets that are welded together to form flat vertical channels for the oil flowing from top to bottom whereas the opposite sides of the steel sheets face air channels. Headers are used at top and bottom to distribute the oil among a large number of parallel ducts whereas the air ducts are open to ambient at top and bottom as well as at the sides. The radiator assembly is carried by the headers only, so that the radiators are hanging on the sides of the
transformer tank without any other mechanical support. The surface area to volume ratio is typically 200 m²/m³ for this type of radiators.

The flows through the radiator can thus be described as counter flow with oil flowing from top to bottom and air flowing from bottom to top. The air flow is however more complicated since the open boundaries at the sides allow cold air to go into the air channels at positions above the bottom boundary. Therefore the air velocity is higher at the top level than at the bottom level and the average air temperature as function of height is not just given by the heat flux from oil to air but also on the air entrainment from the open sides, see e.g. [1-2]. In order to capture the effects of the air flow distribution, three dimensional flow simulations can be used, see e.g. [3]. When fast network models are used for design purpose, the deviations from pure vertical air flow could be accounted for with correction factors derived from experiments or simulations.

The oil used in power transformers should fulfill high demands on electrical insulation besides serving as coolant. Traditionally, highly purified mineral oils are used, and the viscosity characteristics can be modified through selection of the proportions of aromatic and aliphatic hydrocarbons. For cold climates, an oil type must be used that has sufficiently low viscosity also at the lowest temperatures. Among most types of mineral oils for transformer applications, the viscosity is in the same range at temperatures of 60 – 100°C. More recently, other oils than mineral oils are introduced for transformer applications in order to meet requirements on fire safety and environmental properties. Examples of such oils are natural esters, synthetic esters and silicone oil. The viscosities of these oils are higher than for normal mineral oils. Design rules for cooling that have been established from transformer tests using mineral oil might not be applicable for ester oils, and if mineral oil is replaced by ester oil in an existing transformer, the oil flow rate and the temperature distribution will be different. It is therefore important to use adequate models that take into account the oil properties.

2. Heat exchanger model

For fast calculations of temperature and flow distribution in complex systems of ducts, network models can be used. This is today the preferred method for cooling calculations of power transformers; see e.g. [4-6]. A pressure network assuring mass continuity and pressure balances is used to calculate the average velocities in all ducts, and a temperature network assuring energy conservation together with heat conduction, convection and – if applicable – radiation is used to calculate the temperature at every solid node and the fluid temperatures in all ducts. In the general case there is a coupling between the pressure network and the temperature network through velocity dependent heat transfer coefficients, temperature dependent physical properties and temperature and velocity dependent buoyancy. The influence of buoyancy might be neglected if there is pure forced flow in all ducts, but in the case of buoyancy driven flow as in a power transformer the buoyancy is important for the fluid motions, see [7].

The equations in the pressure network and temperature network can be solved with many different methods such as electric circuit software, general equation solvers as found in e.g. Matlab, or even the standard version of Microsoft Excel can be used. The formulation and solution of the network equations will not be discussed further here since often individual preferences are behind the selection of method. The advantage of the network approach as compared to CFD is the speed of calculation. Often a network consisting of a few thousand temperature and pressure nodes should be solved within less than a second on a standard computer.

2.1. Pressure drop in radiator

The vertical ducts for the oil flow in a radiator have high aspect ratio and slightly curved walls due to corrugations. The flow is normally laminar and the friction factor can be approximated with the following formula, see [8].

\[
f = \frac{4}{Re} \left[ \frac{3.44}{(x^+)^{1/2}} + \frac{24 + 0.674/(4x^+)^{1/3} - 3.44/(x^+)^{1/2}}{1+0.00029(x^+)^{1/2}} \right] \tag{1}\]
The validity of forced convection friction factors at mixed convection was studied in [9]. A significant part of the total pressure drop in the oil flow circuit takes place in the ducts inside the heat generating parts. The friction factor formula above is often a good approximation also for these ducts with the addition of pressure changes due to area changes and bends. The Reynolds number at the oil side of the radiator in the present investigation is from 2 to 20.

2.2. Air flow in radiator

The air flow rate and average velocities in a radiator might be given through a fan curve or from a CFD calculation; however for the present investigation the air flow rate is determined as function of the average heat flux to the air using the following formula.

\[
Re = \frac{UD_h}{v} \quad (2)
\]

\[
x^+ = \frac{L / D_h}{Re} \quad (3)
\]

\[
D_h = \frac{4A}{P} \quad (4)
\]

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For turbulent flow, \( C = 0.3164 \) and \( n = 0.25 \) as from the Blasius formula [8]. The validity of equation (5) for the air flow rate through a radiator is only verified indirectly through comparison of calculated and measured oil temperatures in radiators. The laminar version of equation (5) was investigated in [10], but for a radiator of approximately 2 m height the laminar formula provide too low flow rate and too high outlet temperature. The air side Reynolds number of radiators often come in the transition regime, and the flow conditions in terms of laminar and turbulent behaviour in actual radiators with buoyancy driven air flow are not well described. In the present study \( Ra'' = 3.9 \times 10^4 \) and \( Re = 5700 \).

2.3. Heat transfer coefficients in radiator

The heat transfer coefficient on the oil side of a radiator can be approximated from the following Nusselt number formula, see [8].

\[
Nu = 8.235 + \frac{0.024 \left( x^+ \right)^{1.14}}{1 + 0.0358 Pr^{0.17} \left( x^+ \right)^{0.64}} \quad (6)
\]

\[
x^+ = \frac{L / D_h}{Re Pr} \quad (7)
\]
\[ Nu = \frac{h_{\text{avg}} D_h}{k_f} \]  

(8)

The heat transfer coefficient on the air side, which represents the dominating thermal resistance, can be approximated from the following Nusselt number formula, see [11].

\[ Nu = 0.023 \text{Re}^{0.8} \text{Pr}^{0.4} \]  

(9)

It should be noted that the choice of equations (5) and (9) plays a large role for the overall accuracy of the calculated oil temperatures in a radiator. A detailed validation would be needed in order to state their applicability, even though the combination of the two formulas has proved to give good agreement with measured oil temperatures for power transformer radiators.

2.4. Vertical temperature distribution

The vertical temperature distribution of the oil in a radiator is needed in order to calculate the buoyancy of the oil. Using the \( \varepsilon \)-NTU nomenclature, the temperature distribution in a counter flow heat exchanger is given by the following expressions; see [11].

When the minimum heat capacity rate \( C_{\text{min}} = (\dot{m}c_p)_{\text{min}} \) is the oil flow, the temperature distribution from inlet \((x = 0)\) to outlet \((x = H)\) of the oil is given as

\[
T_{\text{oil}}(x) = T_{\text{oil, in}} - \left( T_{\text{oil, in}} - T_{\text{air, in}} \right) \frac{1 - e^{-\frac{x}{H \text{NTU}(1-R)}}}{1 - \text{Re}^{-\text{NTU}(1-R)}} \]  

(10)

\[
R = \frac{C_{\text{min}}}{C_{\text{max}}} \]  

(11)

\[
\text{NTU} = \frac{UA}{C_{\text{min}}} \]  

(12)

The height averaged temperature is then

\[
\frac{1}{H} \int_0^H T_{\text{oil}}(x) \, dx = T_{\text{oil, in}} - \left( T_{\text{oil, in}} - T_{\text{air, in}} \right) \frac{1 - \left(1 - e^{-\text{NTU}(1-R)}\right)}{1 - \text{Re}^{-\text{NTU}(1-R)}} \]  

(13)

When the maximum heat capacity rate \( C_{\text{max}} = (\dot{m}c_p)_{\text{max}} \) is the oil flow, the temperature distribution of the oil is given as

\[
T_{\text{oil}}(x) = T_{\text{air, in}} + \left( T_{\text{oil, in}} - T_{\text{air, in}} \right) \frac{1 - \text{Re}^{-\frac{1 - e^{-\text{NTU}(1-R)}}{\text{NTU}(1-R)}}}{1 - \text{Re}^{-\text{NTU}(1-R)}} \]  

(14)

The height averaged temperature is then

\[
\frac{1}{H} \int_0^H T_{\text{oil}}(x) \, dx = T_{\text{air, in}} + \left( T_{\text{oil, in}} - T_{\text{air, in}} \right) \frac{1 - R(1 - e^{-\text{NTU}(1-R)})}{1 - \text{Re}^{-\text{NTU}(1-R)}} \]  

(15)
For the case of $C_{\min} = C_{\max}$, i.e. $R = 1$, the oil temperature distribution is given as

$$T_{\text{oil}}(x) = T_{\text{oil,in}} - (T_{\text{oil,in}} - T_{\text{air,in}}) \frac{NTU}{1 + NTU} H x$$

(16)

The height averaged temperature is then

$$\frac{1}{H} \int_0^H T_{\text{oil}}(x) dx = T_{\text{oil,in}} - (T_{\text{oil,in}} - T_{\text{air,in}}) \frac{NTU}{2(1 + NTU)}$$

(17)

2.5. Buoyancy of oil in radiators

The buoyancy in a closed flow circuit is defined as the integral of density variations along a flow path.

$$\Delta p_B = \int \frac{g}{\rho} \delta \rho g_s dx$$

(18)

Using the Boussinesq approximation for the temperature dependence of density, equation (18) can be written as follows.

$$\Delta p_B = \int \frac{g}{\rho} \rho \beta (T_0 - T) g_s dx$$

(19)

The total flow rate in a flow circuit is obtained by finding the velocity and temperature distributions that satisfy the balance between pressure drop and buoyancy.

$$\Delta p_f = \Delta p_B$$

(20)

For the vertical oil ducts in a radiator the important term is the integral of temperature with position, i.e. $\int T_{\text{oil}}(x) dx$. When there is a linear temperature distribution, this integral is simply

$$\frac{1}{H} \int_0^H T_{\text{oil}}(x) dx = \frac{T_{\text{oil,in}} + T_{\text{oil,out}}}{2}$$

(21)

This height averaged temperature is the same as the one given by equation (17). When the oil temperature distribution deviates from linear, equations (13) or (15) should be used instead.

3. Results and discussion

A network model for power transformers implemented in Microsoft Excel has been used to investigate the influence of the heat flux assumption for the radiators, the influence of vertical position of radiators and the influence of oil viscosity. A small power transformer with two disc windings is used as example; however details of the transformer design are not important for the present study. The calculations are made for radiators that are designed to give 80°C oil temperature at the radiator inlet when the ambient air temperature is 20°C at full load.

3.1. Influence of radiator position

The calculated temperature distribution of the oil and the air in the radiators is shown in Figure 1. The inlet temperature of the oil is 80.1°C and the outlet temperature is 57.3°C. The air temperature is increased from 20°C at the inlet to 40.4°C at the outlet. The temperature distributions are thus close to linear, and there is no significant difference whether equation (13) or equation (21) is used for the buoyancy calculation of the oil in the radiators. The calculated oil flow rate is 2.05 kg/s and the air flow rate is 4.83 kg/s.
If the radiators are placed 0.25 m lower than in the reference design, the oil flow rate is decreased but the air flow rate is maintained since the average heat flux – that is unchanged – is used in the Rayleigh number of equation (5). The inlet temperature of the oil is now 86.6°C and the outlet temperature is 51.7°C. The oil flow rate is calculated to be 1.34 kg/s, i.e. significantly smaller than in the reference case. The reason for the decreased oil flow rate is mainly that the buoyancy generated between the top of the heat generating parts and the inlet of the radiators is decreased. However, also the nonlinear oil temperature distribution in the radiators has some influence on the oil flow rate. If the same calculation is made but calculating the buoyancy of the oil in the radiators with equation (21) instead of equation (13), the oil flow rate is 1.03 kg/s. The inlet temperature of the oil is 92.7°C and the outlet temperature is 47.3°C. The nonlinear temperature distribution thus gives an oil inlet temperature that is 6.1 K lower and an outlet oil temperature that is 4.4 K higher as compared to the linear temperature distribution.

If the radiators are placed 0.25 m higher than in the reference design, the inlet temperature of the oil is 77.6°C and the outlet temperature is 59.7°C. The oil flow rate is increased to 2.61 kg/s that is almost twice as large as the oil flow rate with the low radiator position. If the radiators are placed higher up, the flow rate will increase further and the difference between inlet and outlet temperature of the oil will be smaller. The vertical temperature distribution at the air side is also slightly influenced by the radiator position, but the outlet air temperature is unchanged since the average heat flux has been used in equation (5).

**Figure 1.** Temperature distribution of oil and air in radiators with three different vertical positions of the radiators.
3.2. Influence of oil viscosity

If oil with higher viscosity is used in a power transformer, the oil flow rate will be decreased. Calculations have been made with two different oils – mineral oil and ester oil. The viscosities as function of temperature are shown in Figure 2. At 80°C the viscosity of the ester oil is 3.2 times higher than the viscosity of the mineral oil whereas at -40°C the viscosity ratio is as high as 15.

![Figure 2. Kinematic viscosity of mineral oil and ester oil.](image)

The calculated temperature distributions in the radiators with mineral oil and ester oil are shown in Figure 3. The inlet oil temperature is 87.8°C for the ester oil as compared to 80.1°C for the mineral oil. The outlet oil temperatures are 49.0°C and 57.3°C for the ester oil and mineral oil, respectively. The oil flow rate is 1.20 kg/s and 2.05 kg/s for the ester oil and mineral oil, respectively. If the buoyancy of the oil in the radiators is calculated with equation (21) instead of equation (13), the oil flow rate of the ester oil is 1.08 kg/s, the inlet oil temperature is 90.4°C, and the outlet oil temperature is 47.2°C.

![Figure 3. Temperature distribution of oil and air in radiators with mineral oil and ester oil.](image)
When mineral oil is replaced with ester oil in a power transformer it can be expected that the oil flow rate is decreased due to the higher viscosity of the ester oil. For the calculation example above, it can be found that the radiators need to be placed 0.8 m higher with the ester oil in order to get the same oil flow rate as with the mineral oil. In order to get the same inlet oil temperature the elevation should be slightly lower since the ester oil has slightly higher specific heat and thermal conductivity, but these effects are smaller. When the radiators are place 0.63 m higher than in the reference case, the inlet oil temperature is 80.1°C with the ester oil, but then the outlet oil temperature is 55.3°C as compared to 57.3°C for the mineral oil. When changing from mineral oil to ester oil it is not possible to get identical temperature distributions due to the differences in physical properties of the oils. In order to check that all temperatures are within acceptable limits, a calculation of the actual power transformer with the actual oil is needed.

4. Concluding remarks
A model for the buoyancy driven flows of oil and air in radiators used for cooling of power transformers has been presented. The model is a part of a more complex network model of the entire oil flow circuit of a power transformer. The radiators are treated as counter flow heat exchangers and analytical expressions are used for the temperature distributions in oil and air.

The calculation of the buoyancy of the oil in the radiators takes into account the nonlinear temperature distribution that can be significant at relatively low oil flow rates. One example of low oil flow rate is when the vertical position of the radiators is low as compared to the vertical position of the heat generating parts. Another example is when the viscosity of the oil is high such as when mineral oil is replaced with ester oil.

5. References
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