Evaluations and geometrical measurements of the human eye in order to establish the design parameters for the customized contact lens

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Abstract. The paper presents the usual methods and means for geometrical evaluation and measurement for the anterior pole of the eyeball applied in studies conducted on real patients. The results were centralized, evaluated and presented in a summary in the paper. The purpose of the studies is to determine the range of dimensional values of the anterior pole of the human eyeball for the parameterization of the devices and checking tools when manufacturing the contact lenses.

1. Introduction
In this paper are presented several studies, conducted on real patients, in order to determine the range of dimensional values, of shapes of the anterior pole of the human eyeball and the aspects that influence the parameters of the customized contact lens.

The geometrical evaluations and measurements, conducted in order to establish the design parameters for the customized contact lens, contain the following aspects related to the geometry of the anterior pole of the eyeball:

a) Determination of the shape and the dimensions of the corneal contour.
b) Determination of the composed astigmatism (crystalline lens and cornea) of the eye and the one of the central anterior surface of the cornea.
c) Determination of the central curvature radius of the anterior surface of the cornea.
d) Determination of the peripheral curvature radii.
e) Evaluation of the shape of the sclerotic-corneal limbus.
f) Evaluation of the position of the corneal pole.

These measurements and evaluations result from the necessity of the correlation of the corneal geometry to the geometry of the contact lens in general and in particular for the customized ones.

2. Studies and researches regarding the evaluation and measuring of the human eyeball.

2.1. Study for the determination of the shape and the dimensions of the corneal contour
In order to conduct the evaluations and measurements was employed a microscope for ophthalmology known as a bio-microscope with slit lamp. If is designed for the evaluation of the eye tissues using a special light, a high intensity halogen lamp. It is provided with a high performance optical system that determines a very good quality of the images obtained for each individual environment. The slit lamp
provides the option of employing several cornea lighting techniques. This device is composed from two main subassemblies. The subassembly of the actual microscope, composed from 2 identical microscopes, with a common objective and the slit lamp subassembly.

One of the eye glasses of the microscope is provided with a measuring crosshair like the one in figure, which can be rotated at 360° from its own axis.

![Figure 1. Measuring crosshair of the bio-microscope eye glass.](image1)

The lighting is reduced to avoid the tear secretion excitation and to avoid the discomfort for the patient. The zoom of the microscope is adjusted to at least 10x.

For the measuring the crosshair is positioned tangential to the sclerotic-corneal limbus (figure 2), temporal respectively superior, and is read on the graduated scale the value at the part tangentially opposed to the limbus.

During this time the eye is focused on a light spot in order to maintain its position.

![Figure 2. Limbus level.](image2)

The limbus region is located at 1.5 - 2 mm from the cornea-limbus junction to the sclerotic-limbus junction. The depth is about 1 mm.

The measurement diagram is presented in figure 3 a, b.

Was employed an eyepiece with a measuring crosshair with a value of division of the graduated scale of 0.2 mm.
By mapping the shape and dimensions of the cornea can be determined the equation of the frontal corneal ellipsis (figure 4).

\[ X \]
\[ Y \]
\[ O \]
\[ D_H \]
\[ D_V \]

**Figure 4.** Measuring diagram of the cornea.

- \( D_H \) = vertical corneal diameter;
- \( D_V \) = horizontal corneal diameter.

\[ O \] = geometrical center of the corneal projection

Following the measurements results a difference between \( D_H \) and \( D_V \) which justifies our interpretation that at the limbus level the tangential section is an ellipsis, with an equation that we shall determine in the following sections.

The equation of an ellipsis with the semi-axis \( a \) and \( b \) is:
If we replace the elements from the figure, we obtain
\[ 4 \frac{x^2}{D_H^2} + 4 \frac{y^2}{D_V^2} = 1 \] (2)

The equation of the frontal ellipsis is
\[ y^2 = \frac{D_H^2}{4} \cdot \left( \frac{D_H}{D_V} \right)^2 \cdot x^2 \] (3)

The frontal ellipsis has an eccentricity
\[ E = \sqrt{1 - \left( \frac{D_H}{D_V} \right)^2} \] (4)

From the study conducted on the corneal frontal contour, results that the numerical eccentricity of the frontal ellipsis is contained between 0.18 and 0.43. The study is presented in part in the table 1.

Table 1. Illustration of the study regarding the eccentricity of the frontal corneal ellipsis for the right eye, study of the corneal contour.

| No. | Corneal diameter | RE | LE | RE | LE |
|-----|------------------|----|----|----|----|
|     |                  | Horizontal | Vertical | Horizontal | Vertical | EN = 1 - \left( \frac{D_H}{D_V} \right)^2 | EN = 1 - \left( \frac{D_V}{D_H} \right)^2 |
| 1   | 9,2              | 8,5          | 8,6          | 7,85          | 0,382603 | 0,408428 |
| 2   | 11,35            | 10,3         | 11,25        | 10,25         | 0,420076 | 0,412161 |
| 3   | 9                | 8,15         | 8,8          | 8,65          | 0,424228 | 0,183849 |
| 4   | 10,85            | 9,65         | 10,95        | 9,95          | 0,457128 | 0,417502 |
| 5   | 9,35             | 8,7          | 8,95         | 8,6           | 0,366339 | 0,276917 |
| 6   | 10,35            | 9,3          | 10,75        | 9,6           | 0,43887  | 0,450011 |
| 7   | 9,6              | 8,5          | 9,75         | 8,45          | 0,464798 | 0,498888 |
| 8   | 11,55            | 10,45        | 11,7         | 10,6          | 0,425918 | 0,423314 |
| 9   | 11,1             | 10           | 11,2         | 10,15         | 0,434025 | 0,422742 |
| 10  | 11,6             | 10,55        | 11,4         | 10,35         | 0,415742 | 0,419198 |

2.2 Determination of the composed astigmatism (crystalline lens and cornea) of the eye and the one of the central anterior surface of the cornea.

The system selected, the auto Ref/Keratometer URK 700-UNICOS, allows several types of measurements like:
- Automated refractometry.
- automated central keratometry.
- Measurement of the corneal diameter.
- Central corneal curvature radii.

The measuring principle, in the case of the refractometry, is the method of the mist of the focus point of the eye. The optical diagram of the objective refractometry is the one presented in figure 5.
After the measuring several situations can occur, determined by the ametropia of the eye, astigmatism or their combinations.

In figure 6 are presented the optical diagrams resulted from the measuring of eyes that are myopic, emmetropic, hypermetropic and astigmatic. Depending on the retinal reflection of the patient, the reflected light is emerging as follows:

- the emmetropia determines a parallel reflection of the ideal light beams.
- the hypermetropia determines the convergence of the ideal light beams.
- the myopia determines the divergence of the ideal light beams.

After the reflection the light beams pass through the optical system of the device and form an image on the CCD camera composed from 6 points (figure 7).

Depending on the coordinates of the 6 points and the center, can be obtained the conventional values SPH (sphere), CYL (cylinder) and AX (axis).
The results of the measurements conducted with the auto Ref/keratometer URK 700 are shown in figure 8.

The values of the eye refraction appear with the conventional formula: $SPH \approx CYL \ AX \ \theta$

In which SPH is the spherical component, CYL is the cylindrical component, and $AX \ \theta$ represents the position of the measuring direction in which the diopter value is the largest.

As measuring premises:
- The value of the vertex corresponding to the position of the glasses lens. In this case this value is 12 mm and is recorded on the measuring chart.
- A negative value for the cylindrical component.
- The value of the refractive index of the cornea is $n = 1.3375$.

The assignment of the values depending on the sign and value of the refraction is made according to table 2 and the graphical representation of the forming of the image in the eye is made according to figure 9.

**Table 2. Assignment of the conventional measurements.**

| Assignment of the conventional measurements - spherical (S), cylinder (C), S+C | S (dpt) | C (dpt) | S+C (dpt) |
|---|---|---|---|
| Emmetropia | -0.50≤S≤0.50 | -0.50≤C≤0.00 | S+C (dpt) |
| Myopia | S<0.50 | -0.50≤C≤0.00 | |
| Simple myopic astigmatism | -0.50≤S≤0.00 | C<0.50 | |
| Composite myopic astigmatism | S<0.50 | C<0.50 | |
| Mixed astigmatism | 0.00≤S | C<0.50 | S+C≤0.00 |
| Hypermetropia | +0.50≤S | -0.50≤C≤0.00 | 0.00≤S+C |
| Simple hypermetropic astigmatism | +0.50≤S | C<0.50 | 0.00≤S+C≤0.50 |
| Composite hypermetropic astigmatism | | | +0.50≤S+C |
Figure 9. Graphical representation of the forming of the image in the eye.

In order to obtain the keratometric measurements, in the optical diagram of the auto Ref/keratometer URK 700, the route of the light beams follows the course from figure 10.

Figure 10. Diagram of keratometric measurements.
The circular light target is reflected on the cornea with a circular or elliptic shape depending on the curvature radius and the corneal astigmatism (figure 11).

![Figure 11. Circular light target.](image)

The analysis of the reflection image determined the value of the corneal astigmatism, the corneal diopter powers on the directions for the minimum and maximum values, the angle of the direction of the long axis of the ellipsis and the curvature radii.

The equation of the reflection ellipsis is:

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = R
\]

(5)

The connection between the incidence angle, the incidence position is presented in figure 12.

![Figure 12. Reflection of the light on the cornea.](image)

The keratometric radius \( R \) has the formula:

\[
R = \frac{h}{\sin \left( \frac{\theta}{2} \right)}
\]

(6)

\( h \) is the incidence height and \( \theta \) is the incidence angle, and the diopter power \( D \) is

\[
D = \frac{1000(n-1)}{R}
\]

(7)

where \( n=1.3375 \) and the value of the radius are entered in millimeters.
The auto refractometric study was conducted on a number of 156 subjects, and the summary of the results is presented in table 3.

**Table 3.** Autorefractometric study.

| Nr. crt | Numele si prenumele | sex | varsta | Refractometrie obiectiva (d+ cil ax) | Keratometrie (raza de curba) | Astigmatismul corneean |
|---------|---------------------|-----|--------|-------------------------------------|----------------------------|-----------------------|
|         |                     |     |        | OD                    | OS                    | OD (+dpt) | OS (+dpt) |
| 1       | RADU AURELIA        | F   | 80     | +2.00SF               | +2.25SF               | r0 7,76   | r0 7,74  |
|         |                     |     |        | OD 43,50              | OS 43,50              | r0 7,76   | r0 7,74  |
| 2       | TRAISTARU ECATERINA| F   | 44     | +1.00SF               | +0.75SF +0.50CIL 176° | r0 8,02   | r0 8,01  |
|         |                     |     |        | OD 42,00              | OS 42,25              | r0 8,02   | r0 8,01  |
| 3       | NEACSU CRISTINA     | F   | 26     | -0.75SF -0.50CIL 98° | +1.25SF -1.0CIL 24°  | r0 7,54   | r0 7,55  |
|         |                     |     |        | OD 41,50              | OS 41,75              | r0 7,54   | r0 7,55  |
| 4       | ZAHARIA VASILE      | M   | 73     | +3.00SF +1.25CIL 172°| +3.50SF +1.25CIL 10° | r0 7,90   | r0 4,25  |
|         |                     |     |        | OD 42,50              | OS 42,50              | r0 7,90   | r0 7,90  |
| 5       | POSTOLACHE VIORICA  | F   | 26     | -0.25SF -0.25CIL 163°| -0.00SF -0.50CIL 90° | r0 7,94   | r0 4,25  |
|         |                     |     |        | OD 42,50              | OS 42,25              | r0 7,94   | r0 7,90  |
| 6       | PETRINGENAR U CLAUD | M   | 46     | +0.50CIL 105°        | -0.50CIL 177°        | r0 8,02   | r0 8,01  |
|         |                     |     |        | OD 42,00              | OS 42,25              | r0 8,02   | r0 8,01  |

On a graph can be represented the central corneal radius - corneal diopter power in order to obtain using the least squares method an almost linear relation between the two parameters (figure 13).

![Graph showing the relation between central corneal radius and corneal diopter power](image)

**Figure 13.** Relation between central corneal radius - corneal diopter power.

2.3 Determination of the central curvature radius and peripheral curvature radius of the cornea.

For this study was employed a device accessible to all professionals - the Javal keratometer. The measurement diagram to obtain the corneal topography is presented in figure 14.
We assume that the measuring angle, from the center of the entry pupil (P) of the microscope of the ophthalmometer (keratometer) is of $30^\circ$ and that in the peripheral corneal points will be reached the level $z$ for known coordinates.

From a technical perspective it can be observed the arising of the following aspects:
- at the edge of the cornea is lost one of the two images of the target due to the reflection after a very inclined direction. The measuring directions are presented in figure 15.
- the alignment and contact of the two targets is difficult to achieve due to their deformed image.
- for the safety of the measurement is recommended an adjustment of the ophtalmometric center after which, focusing the sight with the target, make an angle of $30^\circ$, will be obtained a first deformed and unaligned image, position in which will be conducted the final adjustment.
There is a point positioning error that is considered to be less than 0.02 mm. The study for determining the central and peripheral curvature radii for the anterior surface of the cornea was conducted on a number of 156 subjects, and the summary of the results is presented in table 4.

Table 4. Example of the study for the determination of the central curvature radius and peripheral curvature radius of the cornea, for the left eye.

| r_1 | r_2 | r_3 | r_4 | r_5 | r_0 | r_5 | 2r_0 | 2r_5 | 2(r_0/r_5) | E_n |
|-----|-----|-----|-----|-----|-----|-----|-------|-------|------------|-----|
| 1   | 8   | 7.8 | 7.8 | 7.8 | 7.7 | 7.7 | 7.77  | 7.825 | 15.54      | 0.9929712 | 0.1676753 |
| 2   | 8.4 | 8.07| 8.3 | 8.2 | 7.95| 8   | 8.01  | 8.225 | 16.02      | 0.9738602 | 0.3233563 |
| 3   | 7.8 | 7.55| 7.7 | 7.6 | 7.34| 7.7 | 7.44  | 7.7   | 14.89      | 0.9668831 | 0.3639609 |
| 4   | 8.2 | 8   | 8   | 7.9 | 7.92| 7.9 | 7.96  | 8     | 15.92      | 0.995     | 0.1414214 |
| 5   | 7.9 | 7.4 | 7.8 | 7.5 | 7.27| 7.6 | 7.33  | 7.7   | 14.67      | 0.9525974 | 0.4354428 |
| 6   | 8.6 | 8.11| 8.4 | 8.1 | 7.91| 8   | 8.01  | 8.275 | 16.02      | 0.9679758 | 0.357906  |
| 7   | 8.1 | 7.89| 8   | 7.8 | 7.76| 7.8 | 7.82  | 7.925 | 15.65      | 0.9873817 | 0.2246624 |
| 8   | 7.9 | 7.67| 7.8 | 7.5 | 7.62| 7.6 | 7.64  | 7.7   | 15.29      | 0.9928571 | 0.1690309 |
| 9   | 7.9 | 7.71| 7.8 | 7.6 | 7.68| 7.7 | 7.69  | 7.75  | 15.39      | 0.9929032 | 0.1684847 |
| 10  | 8.8 | 8.38| 8.7 | 8.0 | 8.14| 8.3 | 8.26  | 8.465 | 16.52      | 0.9757826 | 0.3112386 |
| 11  | 8.3 | 7.63| 8.2 | 7.8 | 7.63| 7.9 | 7.63  | 8.05  | 15.26      | 0.9478261 | 0.4568322 |
| 12  | 8.5 | 8.22| 8.4 | 8.2 | 7.88| 8.2 | 8.05  | 8.325 | 16.1       | 0.966967  | 0.3634998 |
| 13  | 7.7 | 7.59| 7.6 | 7.6 | 7.48| 7.6 | 7.53  | 7.625 | 15.07      | 0.9881967 | 0.2172858 |
| 14  | 8.5 | 7.4 | 8.4 | 8.3 | 7.27| 8.4 | 7.33  | 8.4   | 14.67      | 0.8732143 | 0.7121396 |
| 15  | 8   | 7.8 | 7.9 | 7.9 | 7.65| 8   | 7.72  | 7.95  | 15.45      | 0.9716981 | 0.3364633 |
| 16  | 8.2 | 7.99| 8.2 | 8.1 | 7.92| 8.1 | 7.95  | 8.15  | 15.91      | 0.9760736 | 0.3093631 |
| 17  | 8.7 | 8.46| 8.6 | 8.4 | 8.33| 8.3 | 8.39  | 8.5   | 16.79      | 0.9876471 | 0.2222876 |
| 18  | 7.5 | 7.33| 7.4 | 7.3 | 7.33| 7.4 | 7.33  | 7.4   | 14.66      | 0.9905405 | 0.1945195 |
| 19  | 8.4 | 7.45| 8.3 | 8.2 | 7.36| 8.3 | 7.40  | 8.3   | 14.81      | 0.8921687 | 0.6567536 |
| 20  | 8.5 | 8.15| 8.4 | 8.4 | 7.93| 8.3 | 8.04  | 8.4   | 16.08      | 0.9571429 | 0.4140393 |

There are obtained 4 peripheral radii which will determine a mean radius. Can be determined a mean of the peripheral radii using the formula:

\[ r_m = \frac{\sum_{i=1}^{4} r_i}{4}. \]  (8)
We can also determine the numerical eccentricity of the anterior pole of the eye, considered a spheroid, with the formula:

\[
E_n = 2 \sqrt{1 - \frac{(r_2 + r_2)^2}{(r_1 + r_2 + r_3 + r_4)^2}}.
\]  

(9)

The range of values of the eccentricity of the human eyeball resulted from the study is presented in figure 17.

**Figure 17.** Range of values for the eccentricity of the human eyeball.

If the measuring of the curvature radii is reported to a reference sphere with the value of 7.8 mm, we can trace the Gauss curves for the scattering of the results of the measurements for all measured radii. We consider the existence of a mean value, the most likely, which is added to the reference value for each measuring direction. The graphical representation of the distribution of errors was made with the aid of a program designed in the Delphi 6 language (figures 18, 19, 20, 21, 22, 23).
Figure 18. Statistical distribution for 50 measurements of the radius R1.

Figure 19. Statistical distribution for 50 measurements of the radius R012.

Figure 20. Statistical distribution for 50 measurements of the radius R2.
Figure 21. Statistical distribution for 50 measurements of the radius R3.

Figure 22. Statistical distribution for 50 measurements of the radius R034.

Figure 23. Statistical distribution for 50 measurements of the radius R4.
3. Conclusions

Invariably the measurements showed that the shape of the cornea is an ellipsis with the long axis horizontal and the short axis vertical. The difference is due to the flattening on the vertical axis of the eyeball due to the fatty tissues from the eye socket. This deformation can also determine a corneal astigmatism, called physiological. The corneal astigmatism is determined by the power difference between the minimum and maximum power directions. The corneal curvatures influence the selection of the posterior radius of the contact lens, respectively the weight and tear circulation. The type of astigmatism determines the stretching of the contact cornea-lens and the movement direction of the lens on the cornea due to the blinking. The selection of the diameter of the contact lens is determined by the vertical diameter and the position of the lower eyelid, respectively the centering of the lens. The diameter of the lens is influenced by the corneal diameter. There is also a correlation between the corneal diameter, the central corneal radius and the degree of flattening of the corneal periphery. The position of the corneal pole is determined essentially by the flattening of the corneal periphery in various meridians. The apex is slightly off center from the geometrical center of the cornea. The corneal apex defines the corneal optical center. In the case of using a rigid contact lens, the cornea-sclerotic transition influences the shape of the profile of the lens edge. A lens that tends to be carried by the eyelid can crush the nerves and vessels found under the upper eyelid. The form of the lower limbus matters when using rigid contact lenses. A fused lower limbus, as for the case of a very high regular astigmatism, can generate centering problems. A convex fused limbus allows a posterior geometry of the contact lens of the single curve type, eventually aspheric. A conical fused limbus requires an aspheric shape, eventually single curved with small diameter. The strongly convex limbus requires a bi-curved or tri-curved surface with high displacement to avoid the resting on the conjunctive tissue. The strongly conical limbus requires a bi-curved or tri-curved surface, eventually with small diameter. Frequently the nasal cornea-sclerotic limbus is concave, which explains the temporal misalignment. The study aimed to identify ranges of curvature radii and numerical eccentricities in order to establish their reference values. In a radius - diopter power graph can be shown the range of curvature radii, and also a linear relation between these two dimensions. The curvature radii, for the studied directions, can be found in an inequality formula: \( r_{012} < r_{034} \); \( r_4 < r_3 < r_2 < r_1 \); \( r_{012} < r_2 < r_1 \); \( r_{034} < r_4 < r_3 \). The numerical eccentricities of the shape of the anterior eyeball pole can be assigned to 5 values intervals, with the highest probability between 0.3 - 0.7. From this probability results the requirement of base surfaces for the customized contact lenses with 5 possible eccentricities: 0.3, 0.4, 0.5, 0.6, 0.7. From the study conducted on the corneal frontal contour, results that the numerical eccentricity of the frontal ellipsis is contained between 0.18 and 0.43.

4. References

[1] Burek H, Conics 1987 Cornea and keratometry Optician pp 18-33
[2] Brancato R and Carones F 1994 Topografia corneale computerizata
[3] Bennett A G and Rabbetts R B 1984 Clinical Visual Optics (London: Butterworths)
[4] Charman W N 1989 Diffraction and the precision of measurement of corneal and other small radii Am J Opt Arch Am Acad Optom 49 pp 672-679
[5] Clark B 1974 Mean topography of normal corneas Aust.J. Optom pp 107-114
[6] Cohen K L, Tripoli N K, Holmgren D E and Coggins J.M 1995 Assessment of the power and height of radial aspheres reported by a computer-assisted keratoscope Am.J.Ophthalmol pp 723-732
[7] Chaston J and Irving F 1982 Optical measurement of front and back radius of soft contact lenses in saline ICLC No.9 pp 11-18
[8] Conte M 1996 Mesures, travaux pratique Lab. LMC INSA Lyon France
[9] Doss J D, Hutson R L, Rowswy J J and Brown R 1981 Method for calculation of corneal profile and power distribution Arch.Ophthalmal 99 pp 1261-1265
[10] IACLE 2000 Contact lens course- Modoules 1 to 6 (Sydney: First Edition)
[11] Pascu A T 2000 Rolul, proiectarea și fabricarea lentilelor de contact (București: Edit. Atkins)
[12] Pascu A T 2005 *Metode si mijloace de evaluare a topografiei corneene* (Edit. Man-Dely)
[13] Manual de utilizare – *Autokeratorefractometru URK700 UNICOS*