NLO and NNLO EWC for PV Møller Scattering

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Motivation:

Low-energy, high-precision, PV electroweak experiments can provide indirect access to physics at multi-TeV scales and play an important complementary role to the LHC research program.

One of such experiments, MOLLER, planned at JLab following the 11 GeV upgrade, will measure the parity-violating asymmetry in the scattering of longitudinally polarized electrons off an unpolarized target and allow a determination of the weak mixing angle with an uncertainty of about 0.1%.

At such precision, any inconsistency with the Standard Model predictions will clearly signal new physics.

However, a comprehensive analysis of radiative corrections is needed before any conclusions can be made.
Polarized Møller Scattering Observable:

The observable is a single-polarization asymmetry corresponding to the scattering of longitudinally polarized electrons on unpolarized target:

$$A_{LR} = \frac{\sigma_{LL} + \sigma_{LR} - \sigma_{RL} - \sigma_{RR}}{\sigma_{LL} + \sigma_{LR} + \sigma_{RL} + \sigma_{RR}} = \frac{\sigma_{LL} - \sigma_{RR}}{\sigma_{LL} + 2\sigma_{LR} + \sigma_{RR}}$$

This asymmetry at low energies and at tree level is given by

$$A^0_{LR} = \frac{s}{2m^2_W} \frac{y(1 - y)}{1 + y^4 + (1 - y)^4} \frac{1 - 4s^2_W}{s^2_W}, \quad y = -t/s$$

which is highly sensitive to small changes in $s_W$. A measurement of $A_{PV}$ to a precision of 0.73 ppb would allow a determination of the weak mixing angle with an uncertainty of $\pm 0.00026$ (stat.) $\pm 0.00013$ (syst.).
Current and Proposed Experiments:

The first observation of Parity Violation in Møller scattering by E-158 at SLAC:

\[ Q^2 = 0.026 \text{GeV}^2, \ A_{LR} = (1.31 \pm 0.14(\text{stat.}) \pm 0.10(\text{syst.})) \times 10^{-7} \]

\[ \sin^2(\hat{\theta}_W) = 0.2403 \pm 0.0013 \text{ in } \overline{MS} \]

The MOLLER experiment (Measurement Of a Lepton Lepton Electroweak Reaction) will allow a determination of the weak mixing angle with an uncertainty of about 0.1%, a factor of five improvement in fractional precision over the measurement by E-158.

J. Benesch et al., MOLLER Proposal to PAC34, 2008.
The MOLLER Experiment:

**MOLLER**

An ultra-precise measurement of the weak mixing angle using Møller scattering

\[ Q_W = 1 - 4 \sin^2 \theta_W \]

\[ Q^c_W G_F + \frac{1}{\Lambda^2} \mathcal{L}_6 \]

\[ \Lambda = 7.5 \text{ TeV} \]

**11 GeV Beam**

\[ A_{PV} = 35.6 \text{ ppb} \]

\[ \delta(Q^c_W) = \pm 2.1 \% \text{ (stat.)} \pm 1.0 \% \text{ (syst.)} \]

**Luminosity:** \( 3 \times 10^{39} \text{ cm}^2/\text{s} \)

**To do better for a 4-lepton contact interaction would require:**

- Giga-Z factory
- Linear collider
- Neutrino factory
- Muon collider

**Compositeness scale:**

\[ \sqrt{g^2_{RR} - g^2_{LL}} = 2\pi \]

\[ \Lambda = 47 \text{ TeV} \]

**Length scale probed:** \( 4 \times 10^{-21} \text{ m} \)
The MOLLER Experiment: Picture by Juliette Mammei, JLab
**Møller Scattering at Tree Level:**

The process of electron–electron scattering (Møller process):
C. Møller, Annalen der Physik 406, 531 (1932)

![Diagram of Møller Scattering](image)

**Figure 1:** Neutral current t-channel (1) and u-channel (2) amplitudes leading to the asymmetry $A_{LR}$ at tree level.
Precision Calculations:

- Make sure that everything is correct for the given level of perturbation (start with one loop)

  For that we choose and compare two approaches: “by hand” and computer-based; with on-shell renormalization and using two different renormalization conditions (RC).

- Determine if higher-order effects (two-loops) are important

  For that we compare results in two renormalization schemes (RS): on-shell and constrained differential renormalization (CDR). Size of the difference between RS will point out importance of higher order effects: W. Hollik and H.-J. Timme, Z. Phys. C. 33, 125 (1986).

- Address higher-order effects

- Add New Physics as necessary
One-Loop Virtual Corrections:

Figure 2: Virtual t-channel one-loop diagrams for $e^-e^- \rightarrow e^-e^-$ scattering process: (1) - boson self-energies (BSE), (2) and (3) - vertex functions (Ver), and (4) and (5) - boxes (Box).
One-Loop Virtual Corrections:

\[ \sigma = \frac{\pi^3}{2s} |M_0 + M_1|^2 = \frac{\pi^3}{2s} \left( M_0 M_0^+ + 2 \text{Re} M_1 M_1^+ + M_1 M_1^+ \right) = \sigma_0 + \sigma_1 + \sigma_Q \]

\[ \sigma_1 = \sigma_{1}^{BSE} + \sigma_{1}^{Ver} + \sigma_{1}^{Box} \]
Photon Emission – $e^-e^- \rightarrow e^-e^-\gamma$:

The bremsstrahlung cross section can be broken down into two parts, soft (IR) and hard (H), by separating the integration domain according to $k_0 < \omega$ or $k_0 > \omega$ where $k_0$ is the emitted photon energy and $\omega$ is a maximum soft photon energy:

$$\sigma^R = \sigma^R_{IR} + \sigma^R_H$$

The analytical dependencies on $\omega$ cancel out when we add $\sigma^R_{IR}$ and $\sigma^R_H$.

**Figure 3:** Bremsstrahlung diagrams for the $e^-e^- \rightarrow e^-e^-\gamma$ process in the $t$-channel. The u-channel diagrams are obtained by interchanging $k_2$ and $p_2$. 

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A. Soft Photons and IR-divergence Cancellation

First, we follow the methods of paper [26] to get a well-known result (see also [11, 12]) for the soft photon cross section:

$$\sigma_{IR} = \alpha \pi \left( 4 \log 2 \omega \lambda \log t^u_{em} - \log 2 s_{em}^2 + 1 - \frac{\pi^2}{3} + \log 2 u^t \right) \sigma_0.$$  \hspace{1cm} (53)

Next, we sum the IR-terms of $V$-and $R$-contributions (formulae (41) and (53)),

$$\sigma_C = \sigma_{V IR} + \sigma_{R IR} = \alpha \pi \left( 2 \log 4 \omega^2 s \log t^u_{em} - \log 2 s_{em}^2 + 1 - \frac{\pi^2}{3} + \log 2 u^t \right) \sigma_0.$$  \hspace{1cm} (54)

and get a result free from IR-divergence which logarithmically depends on $\omega$ and contains $\log 2 (s/m^2)$-terms.

Adding the contribution corresponding to $\Lambda_1$ to $\sigma_C$, we get an expression with the first power of collinear logarithms:

$$\sigma_{Ver} + \sigma_C \sim \alpha \pi \left( \Lambda_1 \left( \lambda \rightarrow s \right) - \log 2 s m^2 \right) + \ldots = \alpha \pi \left( \log 2 - t m^2 - \log 2 s m^2 \right) + \ldots = \alpha \pi \log -t \log -ts m^4 + \ldots$$ \hspace{1cm} (55)

with non-physical dependencies cancelled analytically.

B. Hard Photons. Leading Logarithms Approach

Now we will calculate the hard bremsstrahlung cross section retaining in the result the leading collinear logarithms. This approach allows estimating the EWC very rapidly yet provide a rather accurate result. First
### Treatment of Infrared Singularity:

\[ \sigma_{IR}^{VeR} + \sigma_{IR}^{\gamma\gamma - box} + \sigma_{IR}^{\gamma Z} = -\frac{2\alpha}{\pi} \log \frac{s}{\lambda^2} \log \frac{tu}{em^2s} \sigma_0 \]

\[ \sigma^R = \sigma_{IR}^R + \sigma_H^R \]

\[ \sigma_{IR}^R = \frac{2\alpha}{\pi} \left( \log \frac{4\omega^2}{\lambda^2} \log \frac{tu}{em^2s} - \frac{1}{2} \log^2 \frac{s}{em^2} + \frac{1}{2} - \frac{\pi^2}{6} + \frac{1}{2} \log \frac{u}{t} \right) \sigma_0 \]

\[ \sigma_H^R = \frac{2\alpha}{\pi} \log \frac{\Omega^2}{\omega^2} \log \frac{tu}{em^2s} \sigma_0 + \sigma_{IR}^{R,\Omega} \]
Relative Correction to Asymmetry:

The relative correction to the Born asymmetry $A_{PV}^0$ is defined as follows:

$$\delta^C_A = \frac{A^C_{LR} - A^0_{LR}}{A^0_{LR}}$$

where index C means a specific contribution (C = BSE, Ver, Box, ...), $A_{PV}^0$ is the Born asymmetry, and $A_{PV}^c$ is the total asymmetry including electroweak radiative corrections.

We evaluate these both numerically, using with FeynArts and FormCalc, and independently “by hand”, in a very good approximation.
One-Loop - “By Hand” vs Semi-Automated:

![Graph showing relative weak corrections to the Born asymmetry $A^0_{LR}$ versus $\sqrt{s}$ at $\theta = 90^\circ$.]

**Figure 4:** The relative weak (solid line in DRC (semi-automated)) and dotted line in HRC ("by hand"). The filled circle corresponds to our predictions for the MOLLER experiment.

arXiv:1008.3355
## Comparison Between Two Methods:

| \( \theta,^\circ \)  | 20  | 30  | 40  | 50  | 60  | 70  | 80  | 90  |
|---------------------|-----|-----|-----|-----|-----|-----|-----|-----|
| \( A^0_{LR}, \text{ ppb} \) | 6.63 | 15.19 | 27.45 | 43.05 | 60.69 | 77.68 | 90.28 | 94.97 |
| \( \gamma\gamma\text{-SE, DRC} \) | -0.0043 | -0.0049 | -0.0054 | -0.0058 | -0.0062 | -0.0064 | -0.0066 | -0.0067 |
| \( \gamma\gamma\text{-SE, HRC} \) | -0.0043 | -0.0049 | -0.0054 | -0.0058 | -0.0062 | -0.0064 | -0.0066 | -0.0067 |
| \( \gamma Z\text{-SE, DRC} \) | -0.2919 | -0.2916 | -0.2914 | -0.2912 | -0.2911 | -0.2910 | -0.2909 | -0.2909 |
| \( \gamma Z\text{-SE, HRC} \) | -0.6051 | -0.6043 | -0.6042 | -0.6038 | -0.6034 | -0.6031 | -0.6028 | -0.6028 |
| \( ZZ\text{-SE, DRC} \) | -0.0105 | -0.0105 | -0.0105 | -0.0105 | -0.0105 | -0.0105 | -0.0105 | -0.0105 |
| \( ZZ\text{-SE, HRC} \) | 0.0309 | 0.0309 | 0.0309 | 0.0309 | 0.0309 | 0.0309 | 0.0309 | 0.0309 |
| \( HV, \text{ DRC} \) | -0.2946 | -0.2633 | -0.2727 | -0.2703 | -0.2714 | -0.2712 | -0.2711 | -0.2710 |
| \( HV, \text{ HRC} \) | -0.0015 | -0.0012 | -0.0010 | -0.0009 | -0.0008 | -0.0007 | -0.0007 | -0.0007 |
| \( ZZ\text{-box, exact} \) | -0.0013 | -0.0013 | -0.0013 | -0.0013 | -0.0013 | -0.0013 | -0.0013 | -0.0013 |
| \( ZZ\text{-box, approx.} \) | -0.0013 | -0.0013 | -0.0013 | -0.0013 | -0.0013 | -0.0013 | -0.0013 | -0.0013 |
| \( WW\text{-box, exact} \) | 0.0239 | 0.0238 | 0.0238 | 0.0239 | 0.0239 | 0.0238 | 0.0238 | 0.0238 |
| \( WW\text{-box, approx.} \) | 0.0238 | 0.0238 | 0.0238 | 0.0238 | 0.0238 | 0.0238 | 0.0238 | 0.0238 |
| total \( \text{weak, DRC, exact} \) | -0.5643 | -0.5430 | -0.5508 | -0.5489 | -0.5500 | -0.5495 | -0.5493 | -0.5493 |
| total \( \text{weak, HRC, approx.} \) | -0.5526 | -0.5514 | -0.5511 | -0.5505 | -0.5500 | -0.5496 | -0.5493 | -0.5493 |

**Table 2:** The Born asymmetry \( A^0_{LR} \) and the structure of relative weak corrections to asymmetry at \( E_{lab}=11 \text{ GeV} \) and at different \( \theta \). DRC stands for Denner renormalization conditions and HRC stands for Hollik, respectively.

[arXiv:1010.4185](http://arxiv.org/abs/1010.4185)
One-Loop in On-Shell and CDR – Cross Section:

\[ \delta^{tot} = (\sigma^{tot} - \sigma^0) / \sigma^0 \]

**Figure 5:** The relative total corrections to the unpolarized cross section versus \( \sqrt{s} \) at \( \theta = 90^\circ \). Solid line corresponds to CDR and dotted line to on-shell RS. The filled circle corresponds to our predictions for the MOLLER experiment.

Constrained Differential Renormalization (CDR): F. del Aguila et al., Phys. Lett. B 419 263 (1998)
At small energies, the difference between the two schemes is almost constant and small (~0.01).

However, in the high-energy region the weak correction becomes comparable to QED.

Since the difference between the on-shell and CDR results grows as the weak correction becomes larger, the difference between the on-shell and CDR schemes becomes sizeable at higher $\sqrt{s}$. 
One-Loop in On-Shell and CDR - Asymmetry:

\[ \delta_A^C = \frac{A_{LR}^C - A_{LR}^0}{A_{LR}^0} \]

Figure 6: The relative weak (lower lines) and QED (upper lines) corrections to the Born asymmetry \( A_{LR}^0 \) versus \( \sqrt{s} \) at \( \theta = 90^\circ \). The filled circle corresponds to our predictions to the MOLLER experiment. Solid lines correspond to CDR and dotted lines to on-shell RS.

\( \omega = 0.05 \sqrt{s} \) and \( \theta = 90^\circ \)

\( \delta_A = \frac{A_{LR} - A_{LR}^0}{A_{LR}^0} \)

arXiv:1010.4185
**One-Loop in On-Shell and CDR - Asymmetry:**

The difference is growing with increasing $\sqrt{s}$.

The total correction to PV asymmetry is $-69.8\%$ with on-shell and $-58.5\%$ with CDR.

For E-158, the one-loop weak corrections were found to be about $-40\%$ in the MS scheme [Czarnecki &Marciano1996] and about $-50\%$ in the on-shell scheme [Petriello2003].

Higher-order contributions!
The Next-to-Next-to-Leading Order EWC to the Born (\( \sim M_0M_0^+ \)) cross section can be divided into two classes:

- Q-part induced by quadratic one-loop amplitudes \( \sim M_1M_1^+ \), and
- T-part – the interference of Born and two-loop diagrams \( \sim 2\text{Re}M_0M_{2\text{-loop}}^+ \).

\[
\sigma = \frac{\pi^3}{2s} |M_0 + M_1|^2 = \frac{\pi^3}{2s} (M_0M_0^+ + 2\text{Re}M_1M_0^+ + M_1M_1^+) = \sigma_0 + \sigma_1 + \sigma_Q
\]

\[
\sigma_T = \frac{\pi^3}{s} \text{Re}M_2M_0^+ \propto \alpha^4
\]
In order to remove the IR-divergent terms in quadratic cross section we need to consider:

1. Photon emission from one-loop diagrams
2. Two photon photon emission

Next-to-Next-to-Leading Order (NNLO):
The parity-violating asymmetry is defined in a traditional way, $\delta^C_A = (A^C_{LR} - A^0_{LR})/A^0_{LR}$.

For the numerical calculations we used $e$, the electric charge of fermion.

$\Delta_A = (A^{1\text{-loop}+Q}_{LR} - A^{1\text{-loop}}_{LR})/A^0_{LR}$.

Figure 7: The relative corrections to the asymmetry (left) and the absolute correction $\Delta_A$ (right) vs scattering angle $\theta$.

arXiv:1110.1750
NNLO T-Part – Boxes with Lepton Self-Energy:

Figure 7: One-loop crossed box type diagrams with lepton self-energy corrections.

Figure 6: One-loop direct box type diagrams with lepton self-energy corrections.
NNLO T-Part – Decorated-Box Diagrams:

\[ Z_W \]

\[ \nu \]

\[ W \]

\[ Z \]

\[ \gamma \]

\[ \chi \]

\[ \rho \]

\[ S \]

\[ a \]

\[ b \]

\[ c \]

\[ d \]

\[ e \]

\[ f \]

\[ g \]

\[ h \]

\[ i \]

\[ j \]

\[ k \]

\[ l \]

\[ m \]

\[ n \]

\[ o \]

\[ p \]

\[ q \]

\[ r \]

\[ s \]

\[ t \]

\[ u \]

\[ v \]

\[ w \]

\[ x \]

\[ y \]

\[ z \]
NNLO T-Part – Ladder-Box Diagrams:

\[
\begin{align*}
&\text{(a)} \quad \text{Z} \quad \text{Z} \quad \text{Z} \\
&\text{(b)} \quad \text{Z}^* \quad \text{Z} \quad \text{Z} \\
&\text{(c)} \quad \text{Z} \quad \text{Z}^* \quad \text{Z} \\
&\text{(d)} \quad \text{Z} \quad \text{Z} \\
&\text{(e)} \quad \text{Z} \quad \text{Z} \quad \text{Z} \\
&\text{(f)} \quad \text{Z}^* \quad \text{Z} \quad \text{Z} \\
&\text{(g)} \quad \text{Z} \quad \text{Z}^* \quad \text{Z} \\
&\text{(h)} \quad \text{Z} \quad \text{Z} \\
&\text{(i)} \quad \text{Z} \quad \text{Z} \quad \text{Z} \\
&\text{(j)} \quad \text{Z} \quad \text{Z}^* \quad \text{Z} \\
&\text{(k)} \quad \text{Z} \quad \text{Z} \\
&\text{(l)} \quad \text{Z}^* \quad \text{Z} \quad \text{Z} \\
&\text{(m)} \quad \text{Z} \quad \text{Z}^* \quad \text{Z} \\
&\text{(n)} \quad \text{Z} \quad \text{Z} \\
&\text{(o)} \quad \text{Z}^* \quad \text{Z} \quad \text{Z} \\
&\text{(p)} \quad \text{Z} \quad \text{Z}^* \quad \text{Z} \\
&\text{(q)} \quad \text{Z} \quad \text{Z} \\
&\text{(r)} \quad \text{Z}^* \quad \text{Z} \quad \text{Z} \\
&\text{(s)} \quad \text{Z} \quad \text{Z}^* \quad \text{Z} \\
&\text{(t)} \quad \text{Z} \quad \text{Z} \\
&\text{(u)} \quad \text{Z}^* \quad \text{Z} \quad \text{Z} \\
&\text{(v)} \quad \text{Z} \quad \text{Z}^* \quad \text{Z} \\
&\text{(w)} \quad \text{Z} \quad \text{Z} \\
&\text{(x)} \quad \text{Z}^* \quad \text{Z} \quad \text{Z} \\
&\text{(y)} \quad \text{Z} \quad \text{Z}^* \quad \text{Z} \\
&\text{(z)} \quad \text{Z} \quad \text{Z} \\
&\text{Fig. 9(f):} \\
&\text{Fig. 9(g):}
\end{align*}
\]

\[\begin{align*}
&\text{(a)} \quad \gamma^2_{123} \\
&\text{(b)} \quad \gamma^2_{13} \\
&\text{(c)} \quad \gamma^2_{231} \\
&\text{(d)} \quad \gamma^2_{132} \\
&\text{(e)} \quad \gamma^2_{123} \\
&\text{(f)} \quad \gamma^2_{13} \\
&\text{(g)} \quad \gamma^2_{231} \\
&\text{(h)} \quad \gamma^2_{132} \\
&\text{(i)} \quad \gamma^2_{123} \\
&\text{(j)} \quad \gamma^2_{13} \\
&\text{(k)} \quad \gamma^2_{231} \\
&\text{(l)} \quad \gamma^2_{132} \\
&\text{(m)} \quad \gamma^2_{123} \\
&\text{(n)} \quad \gamma^2_{13} \\
&\text{(o)} \quad \gamma^2_{231} \\
&\text{(p)} \quad \gamma^2_{132} \\
&\text{(q)} \quad \gamma^2_{123} \\
&\text{(r)} \quad \gamma^2_{13} \\
&\text{(s)} \quad \gamma^2_{231} \\
&\text{(t)} \quad \gamma^2_{132} \\
&\text{(u)} \quad \gamma^2_{123} \\
&\text{(v)} \quad \gamma^2_{13} \\
&\text{(w)} \quad \gamma^2_{231} \\
&\text{(x)} \quad \gamma^2_{132} \\
&\text{(y)} \quad \gamma^2_{123} \\
&\text{(z)} \quad \gamma^2_{13} \\
&\text{Fig. 9(g):}
\end{align*}\]
Numerical Estimations (Preliminary!):

At the MOLLER kinematic conditions, the part of the quadratic EWC we considered here can increase the asymmetry up to ~ 4%.

For the high-energy region $\sqrt{s} \sim 2000$ GeV the contribution of the quadratic EWC can reach +30%.

For the two-loop relative corrections, we can say that the general effect of all the box contributions gives $\delta_A = -0.93\%$.

Combining this value with one-loop electroweak corrections, we obtain $\delta_A = (-0.93\%) \times 1.4514 = -1.35\%$ for the central point of MOLLER kinematics. Since the combined statistical and systematic uncertainty of MOLLER is $\delta_{\text{exp}} \sim \pm 2\%$, one can clearly see that it is essential to include the two-loop radiative corrections.

arXiv:1110.1750, arXiv:1202.0378
New Physics:

Once all the SM corrections are under control, it is worth considering corrections including new-physics particles, starting with the Minimal Supersymmetric Standard Model (MSSM).

For e−e− scattering, MSSM contributions will arise at the one-loop order; the large suppression of the SM weak charge can make the weak charge sensitive to the effects of new physics.

\[
A_{LR}^0 = \frac{s}{2m_W^2} \frac{y(1 - y)}{1 + y^4 + (1 - y)^4} \frac{1 - 4s_W^2}{s_W^2}, \quad y = -t/s
\]

According to [Kurylov et al., 2003], the loop corrections in the MSSM can be as large as ~ 4% for the weak charge of the proton and ~ 8% for the weak charge of the electron, which is close to the current level of experimental and theoretical precision available for the low-energy studies.
Let's consider a simple example of the $ZZ'$-box (it is gauge-invariant and is thus not affected by the choice of renormalization), to investigate if the two complimentary methods we used before, "by-hand" and semi-automated, can be applied in the NP domain.

For now, we assume that there is just one additional neutral boson (ANB), or $Z'$-boson, with the usual $V - A$ structure of interaction with fermions, vector(axial) coupling constants $v^{Z'}(a^{Z'})$ and $m_{Z'}$.

We can now analyze the contribution of $Z'$-Born and $ZZ'$-box diagrams to the observable scattering asymmetry for the MOLLER experiment.

arXiv:1010.4185
New Physics:

The relative correction to the Born asymmetry coming from $Z'$-boson is additive, and is given by

$$
\delta^{Z'}_A = \frac{v^Z a^Z}{v^Z a^Z m_Z^2} m_{Z'}^2
$$

The relative correction to the observable asymmetry from the ANB contribution (i.e. from ZZ'-box) $\delta_{ANB}$ is:

$$
\delta_{ANB} = \frac{6\alpha m_{Z'}^2}{\pi} \frac{v^{B'} a^{B'}}{v^Z a^Z} L
$$

$$
L = \frac{1}{m_Z^2 - m_{Z'}^2} \log \frac{m_Z}{m_{Z'}}
$$

If we take the MOLLER kinematics and assume that $v^{Z'} = v^Z$, $a^{Z'} = a^Z$, then for $r_m \equiv m_{Z'}/m_Z = 1$ the correction is twice the contribution from ZZ-box is $\delta_{ANB} \approx -0.0025465$. As $r_m$ grows, the correction decreases: at $r_m = 2$ the correction is $\delta_{ANB} \approx -0.0011768$, and for $r_m = 10$ the correction is $\delta_{ANB} \approx -0.0001185$. Since MOLLER will measure $A_{PV}$ to 2%, it should be possible to detect ANB with a mass up to $m_{Z'} \approx 7m_Z$.

arXiv:1010.4185
Conclusions:

At certain kinematic conditions, NLO EWC can reduce the asymmetry up to 70%, and they strongly depend on the experimental cuts. Obviously, before we can interpret the high-precision scattering experiments in terms of possible new physics, it is crucial to have the SM electroweak radiative corrections under a very firm control.

For the 11 GeV relevant for the JLab experiment, the NLO results obtained by two different methods with two different renormalization conditions are identical. This gives us assurance that our NLO EWC calculations are error-free.

To estimate the importance of higher orders, we provide a tuned comparison between the one-loop results obtained in two different renormalization schemes: on-shell and constrained differential renormalization.

The analysis of a set of two-loop corrections shows that the next-to-next order contributions may be larger than previously thought.

Once NNLO is done, the new physics particles can be incorporated into NLO theoretical predictions for the Møller asymmetry using the same well-tested computational model employed before.
Thank You!

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Additional Slides
Observing New Physics:

Many options: heavy Z’s and neutrinos, technicolor, compositeness, extra dimensions, SUSY…

Electromagnetic amplitude interferes with Z-exchange as well as any new physics:

\[
\left| A_\gamma + A_Z + A_{\text{new}} \right|^2 \rightarrow A_\gamma^2 \left[ 1 + 2 \left( \frac{A_Z}{A_\gamma} \right) + 2 \left( \frac{A_{\text{new}}}{A_\gamma} \right) \right]
\]
Electroweak Radiative Corrections:

Although PV asymmetry (\( A_{LR} \sim 10^{-7} \)) is very small, the accuracy of modern experiments exceeds the accuracy of the theoretical result in Born approximation.

We know that the one–loop contributions are rather big:

A. Czarnecki, W. J. Marciano, Phys. Rev. D53, 1066 (1996);
A. Denner, S. Pozzorini, Eur. Phys. J. C7, 185 (1999);
A. Aleksejevs, S. Barkanova, A. Ilyichev, V. Zykunov, Phys. Rev. D82, 093013 (2010).

At certain kinematic conditions, EWC can reduce the asymmetry up to 70%, and they depend quite strongly on the experimental cuts.
**Relative Correction to Asymmetry:**

The relative correction to the Born asymmetry $A^0_{PV}$ is defined as follows:

$$\delta^C_A = \frac{A^C_{LR} - A^0_{LR}}{A^0_{LR}}$$

where index $C$ means a specific contribution ($C = \text{BSE}, \text{Ver}, \text{Box}, \ldots$), $A^0_{PV}$ is the Born asymmetry, and $A^C_{PV}$ is the total asymmetry including electroweak radiative corrections.

We evaluate these both numerically, using with FeynArts and FormCalc, and independently “by hand”, in a very good approximation.

For example, here are analytic expressions for boxes:

$$\delta_A^{ZZ} \approx -\frac{3\alpha}{2\pi} \nu_B, \quad \delta_A^{WW} \approx \frac{\alpha}{4\pi s^2_W (1 - 4s^2_W)}$$
One-Loop - Comparison with the Literature:

It is essential to compare the corrected parity-violating asymmetry, which is sensitive to input parameters and calculation scheme, with well-known existing results.

We see an excellent agreement with the result for weak correction by A. Denner and S. Pozzorini (if we use their values for standard model parameters):

| √s, GeV | Result of Denner and Pozzorini | Our result |
|--------|-------------------------------|------------|
| 100    | −0.2787                       | −0.2790    |
| 500    | −0.3407                       | −0.3406    |
| 2000   | −0.9056                       | −0.9066    |

Table 1: Comparison of our result for the weak correction to asymmetry with the result of arXiv:hep-ph/9807446.
One-Loop Renormalization Conditions:

- For a gauge invariant set, physical results should be invariant under different renormalization conditions.
- Renormalization constants are fixed by the renormalization conditions.
- Consider two classes:
  1. The first determines the renormalization of the parameters and is related to physical observables at a given order of perturbation theory. These conditions are identical in both Hollik RC (HRC) and Denner RC (DRC).

\[
\text{Re} \hat{\Sigma}_T^W(m_W^2) = \text{Re} \hat{\Sigma}_T^Z(m_Z^2) = \text{Re} \hat{\Sigma}_f^f(m_f^2) = 0, \\
\hat{\Gamma}^{ee\gamma}_\mu(k^2 = 0, p^2 = m^2) = i e \gamma_\mu.
\]

2. The second class fixes the renormalization of fields and is related to the Green’s functions and has no effect on calculations of S-matrix elements.

\[
\hat{\Sigma}_T^Z(0) = 0, \quad \frac{\partial}{\partial k^2} \hat{\Sigma}_T^Z(0) = 0, \\
\text{Re} \hat{\Sigma}_T^Z(m_Z^2) = 0, \quad \text{Re} \frac{\partial}{\partial k^2} \hat{\Sigma}_T^Z(m_Z^2) = 0, \\
\text{Re} \frac{\partial}{\partial k^2} \hat{\Sigma}_T^W(m_W^2) = 0.
\]

W. Hollik, Fortschr. Phys. 38, 165 (1990).

A. Denner, Fortschr. Phys. 41, 307 (1993).
### APPENDIX C: NUMERICAL ANALYSIS

**TABLE III.** The unpolarized Born cross section and the relative weak and total corrections to it at $E_{\text{lab}} = 11$ GeV at different $\gamma_1$ ($\gamma_1 = 0.005, 0.01, 0.05$) and $\theta$.

| $\theta,^\circ$ | $\sigma^0, \text{mb}$ | Weak | S, 0.005 | S + H, 0.005 | S, 0.01 | S + H, 0.01 | S, 0.05 | S + H, 0.05 |
|-----------------|----------------------|------|-----------|--------------|-------|------------|-------|------------|
| 20              | $0.1277 \times 10^2$ | 0.0087 | -0.2149 | -0.2148 | -0.1754 | -0.1758 | … | … |
| 30              | $0.2607 \times 10^1$ | 0.0101 | -0.2417 | -0.2415 | -0.1972 | -0.1978 | … | … |
| 40              | 0.8734               | 0.0111 | -0.2595 | -0.2591 | -0.2118 | -0.2124 | -0.1012 | -0.1067 |
| 50              | 0.3920               | 0.0119 | -0.2721 | -0.2716 | -0.2222 | -0.2227 | -0.1063 | -0.1136 |
| 60              | 0.2176               | 0.0126 | -0.2810 | -0.2805 | -0.2295 | -0.2303 | -0.1099 | -0.1183 |
| 70              | 0.1444               | 0.0131 | -0.2870 | -0.2867 | -0.2344 | -0.2356 | -0.1124 | -0.1219 |
| 80              | 0.1131               | 0.0135 | -0.2905 | -0.2904 | -0.2373 | -0.2389 | -0.1139 | -0.1241 |
| 90              | 0.1043               | 0.0136 | -0.2916 | -0.2916 | -0.2383 | -0.2400 | -0.1144 | -0.1249 |

**TABLE V.** The Born asymmetry and the QED corrections to it at $E_{\text{lab}} = 11$ GeV at different $\gamma_1$ ($\gamma_1 = 0.005, 0.01, 0.05$) and $\theta$.

| $\theta,^\circ$ | $A_1^0, \text{ppb}$ | S, 0.005 | S + H, 0.005 | S, 0.01 | S + H, 0.01 | S, 0.05 | S + H, 0.05 |
|-----------------|---------------------|-----------|--------------|-------|------------|-------|------------|
| 20              | 6.63                | -0.0710   | -0.0649      | -0.0676 | -0.0566 | … | … |
| 30              | 15.19               | -0.0758   | -0.0736      | -0.0716 | -0.0686 | … | … |
| 40              | 27.45               | -0.0792   | -0.0790      | -0.0744 | -0.0744 | -0.0651 | -0.0567 |
| 50              | 43.05               | -0.0817   | -0.0826      | -0.0763 | -0.0778 | -0.0663 | -0.0660 |
| 60              | 60.69               | -0.0833   | -0.0843      | -0.0777 | -0.0797 | -0.0671 | -0.0691 |
| 70              | 77.68               | -0.0844   | -0.0859      | -0.0785 | -0.0805 | -0.0675 | -0.0701 |
| 80              | 90.28               | -0.0849   | -0.0863      | -0.0789 | -0.0806 | -0.0677 | -0.0702 |
| 90              | 94.97               | -0.0850   | -0.0863      | -0.0790 | -0.0806 | -0.0678 | -0.0702 |

arXiv:1008.3355
Although the two-loop corrections are suppressed relative to the one-loop corrections, they can not be dismissed.

At the MOLLER kinematic conditions, the part of the quadratic EWC we considered here can increase the asymmetry up to $\sim 4\%$.

For the high-energy region $\sqrt{s} \sim 2000$ GeV the contribution of the quadratic EWC can reach $+30\%$.

The large size of the Q-part demands detailed and consistent consideration of two-loop corrections, which is the current task of our group.

It is impossible to say at this time if the Q-part will be enhanced partially or completely cancelled by other two-loop radiative corrections, although it seems probable that the two-loop EWC may be larger than previously thought.
Numerical Estimations (Preliminary!):

\[
A = -\tilde{A}_0 \frac{s}{M_Z^2} \left\{ a_V \left[ R_B^Z + \frac{\alpha}{\pi} R_{(1)}^Z + \left( \frac{\alpha}{\pi} \right)^2 R_{(2)}^Z + \cdots \right] + \frac{\alpha}{\pi} R_{(1)}^W + \left( \frac{\alpha}{\pi} \right)^2 R_{(2)}^W + \cdots \right\}
\]

\[
R_B^Z = \frac{1}{2 c_W^2 s_W^2} \approx \frac{8}{3} = 2.66,
\]

\[
R_{(1)}^Z = 0.93775 \ln \frac{M_Z^4}{t u} - 0.622, \quad R_{(1)}^W = -33.4253,
\]

\[
R_{(2)}^Z = -0.834656 L_Z + 40.9947, \quad R_{(2)}^W = 20.3961 L_W - 2.16291 L_Z + 99.5243.
\]

Translating this result to the relative corrections, we can say that the general effect of all the box contributions gives \( \delta_A = -0.93\% \).

Combining this value with one-loop electroweak corrections, we obtain \( \delta_A = (-0.93\%) \times 1.4514 = -1.35\% \) for the central point of MOLLER kinematics. Since the combined statistical and systematic uncertainty of MOLLER is \( \delta_{\text{exp}} \sim \pm 2\% \), one can clearly see that it is essential to include the two-loop radiative corrections.

\[\text{arXiv:1202.0378}\]
New Physics Beyond LHC

\[ \sqrt{2} G_F \delta(Q^e_W) = \frac{1}{(7.5 \text{ TeV})^2} \]

**Doubly-charged Scalars**

\[ \mathcal{M}^{PV} \sim \frac{|h^{ee}_{L,R}|^2}{2M^{2}_{\delta L}} \bar{e}_L \gamma_{\mu} e_L \bar{e}_L \gamma_{\mu} e_L \]

\[ \frac{M_{\delta L}}{|h^{ee}|} \sim 5.3 \text{ TeV} \]

improves reach significantly beyond LEP-200

**Heavy Photons: The Dark Sector**

\[ \Delta L = 2 \]

Hypothesis could explain \((g-2)_\mu\) discrepancy as well as several intriguing astrophysical anomalies related to dark matter

Beyond kinetic mixing:
introduce mass mixing with Z

\[ \epsilon Z = \frac{m_{Zd}}{M_Z} \delta \]

**Complementary to direct heavy photon searches:**

Lifetime/branching ratio model dependence vs mass mixing assumption

**Monday, 11 June, 12**

Slide by Krishna Kumar, JLab
MOLLER Status
Director’s Review chaired by C. Prescott: positive endorsement

Technical Challenges

- **~ 150 GHz scattered electron rate**
  - Design to flip Pockels cell ~ 2 kHz
  - 80 ppm pulse-to-pulse statistical fluctuations

- **1 nm control of beam centroid on target**
  - Improved methods of “slow helicity reversal”

- **> 10 gm/cm² liquid hydrogen target**
  - 1.5 m: ~ 5 kW @ 85 µA

- **Full Azimuthal acceptance with θ_{lab} ~ 5 mrad**
  - novel two-toroid spectrometer
  - radiation hard, highly segmented integrating detectors

- **Robust and Redundant 0.4% beam polarimetry**
  - Pursue both Compton and Atomic Hydrogen techniques

- **MOLLER Collaboration**
  - ~ 100 authors, ~ 30 institutions
  - Expertise from SAMPLE A4, HAPPEX, G0, PREX, Qweak, E158
  - 4th generation JLab parity experiment

- **~ 20M$ project funding sought**
- **3-4 years construction**
- **2-3 years running**

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Slide by Krishna Kumar, JLab