THERMAL WAVES IN IRRADIATED PROTOPLANETARY DISKS

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ABSTRACT

Protoplanetary disks are mainly heated by radiation from the central star. Since the incident stellar flux at any radius is sensitive to the disk structure near that location, destabilizing feedback may be present. Previous investigations have shown that the disk will be stable to finite-amplitude temperature perturbations if the vertical height of the optical surface is everywhere directly proportional to the gas scale height and if the intercepted fraction of stellar radiation is determined from the local grazing angle. We show that these assumptions may not be generally applicable. Instead, we calculate the quasi-static thermal evolution of irradiated disks by directly integrating the global optical depth to determine the optical surface and the total emitting area filling factor of surface dust. We show that in disks with modest mass accretion rates, thermal waves are spontaneously and continually excited in the outer disk, propagate inward through the planet-forming domains, and dissipate at small radii where viscous dissipation is dominant. This state is quasi-periodic over several thermal timescales, and its pattern does not depend on the details of the opacity law. The viscous dissipation resulting from higher mass accretion stabilizes this instability such that an approximately steady state is realized throughout the disk. In passive protostellar disks, especially transitional disks, these waves induce significant episodic changes in spectral energy distributions, on timescales of years to decades, because the midplane temperatures can vary by a factor of 2 between the exposed and shadowed regions. The transitory peaks and troughs in the potential vorticity distribution may also lead to baroclinic instability and excite turbulence in the planet-forming regions.

Subject headings: accretion, accretion disks — circumstellar matter — instabilities — planetary systems: protoplanetary disks — solar system: formation — stars: pre-main-sequence

1. INTRODUCTION

It has become widely accepted that dusty protoplanetary disks are heated by radiation from the central star, and that this heating mainly determines the physical structure of the outer regions of these disks. Observed infrared spectral energy distributions (SEDs) of T Tauri disks imply that their effective temperatures \( T \) decrease with disk radius \( r \) more slowly than \( T \propto r^{-3/4} \). This temperature distribution is usually explained with a model in which the thermal structure of the disk is assumed to be geometrically flared, that is, the surface height \( z_s \) where stellar radiation is absorbed curves away from the midplane (or equivalently, \( z_s/r \propto r^{\gamma} \) with \( \gamma > 0 \)) (Adams et al. 1987).

The outer regions of these disks are irradiated by the central star (Kenyon & Hartmann 1987), and the flaring enables the disks to absorb more radiation from the star. In a steady state, the flaring index \( \gamma \) of a purely irradiated, optically thick disk can be obtained from the balance between intercepted stellar flux \( F_{\star} \) incident on the surface at a low angle \( \theta \) and emitted blackbody flux from the disk interior, under the assumptions that (1) the surface height \( z_s \) is everywhere proportional to the vertical gas scale height \( h \), (2) the intercepted fraction of stellar radiation is the sine of the local grazing angle \( F_{\star} \propto \sin \theta \), and (3) the central star is a point source.

Given these circumstances, there is a self-consistent power-law solution with \( \gamma \propto \sin \theta \), which corresponds to \( T \propto r^{-3/7} \) (Kusaka et al. 1970; Chiang & Goldreich 1997).

In addition to this particular power-law solution, there exists a one-parameter family of solutions for the disk structure, including solutions with diverging aspect ratio and asymptotically conical solutions (in which \( z_s \propto r \)) (Dullemond 2000). Small variations in the value of \( z_s \) at inner radii where the integration starts can cause large differences in the disk structure at large radii. Dullemond (2000) speculated that such a sensitive nature of the steady solutions suggests an intrinsic instability that must be analyzed with time-dependent governing equations.

More realistic steady solutions can be obtained numerically or semianalytically to take into account the effects of the finite values of stellar radius, the disk optical depth, and the viscous dissipation associated with the mass accretion flow (e.g., Chiang & Goldreich 1997; Chiang et al. 2001; Dullemond et al. 2001; Tanaka et al. 2005; D’Alessio et al. 2006; Garaud & Lin 2007). The most important novel feature of these second-generation models is the assumed presence of superheated surface dust layers above and below the disk midplane (Chiang & Goldreich 1997). Grains in these layers are directly exposed to the stellar flux. Grains much smaller than the peak wavelength of the self-emission are superheated because of their low emissivity. The disk interior is heated by the superheated dust of the layers rather than directly by the central star.

The two-layer disk models clearly explain the silicate and water ice emission bands in the observed SEDs of Herbig Ae/Be and T Tauri stars (Chiang et al. 2001). But despite their triumph in the modeling of the observed SEDs, these models\(^3\) are based on the

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\(^3\) Some authors (e.g., Dullemond et al. 2001; Tanaka et al. 2005; Garaud & Lin 2007) determined \( \gamma \) self-consistently, but these calculations are based on the grazing-angle approximation and the assumption that \( \gamma \) changes slowly with radius.
assumption that \( z_s \) is proportional to the gas scale height \( h \), with a fixed constant of proportionality \( \chi = z_s/h = 4 \) (Chiang et al. 2001). While this assumption has been justified by estimates that suggest changes in \( \chi \) are small throughout the disk, the amount of dust in the superheated layer is a very sensitive function of \( \chi \). This dependence arises because the dust spatial density at \( z_s = \chi h \) is proportional to \( \exp(-\chi^2/2) \). Thus, the assumption of fixed \( \chi \) may cause large discrepancies in the determined values of \( z_s \).

In previous analyses, the magnitude \( F_s \) was directly determined from the grazing angle \( \theta \) (Chiang et al. 2001). This approximation is justified only in the case that the length of the absorption layer (say, where the optical depth to the starlight changes from 0.1 to 1) along the starlight is smaller than the lengths of radial variations of surface density or temperature. This condition would not be satisfied if the disk surface contained fluctuations resulting from the growth of short-wavelength perturbations (see § 2).

In principle, \( z_s \) is determined by the condition that the visual optical depth, which can be obtained by a direct integration along the rays of starlight, is unity and the surface filling factor \( \tau_s \), of the irradiated dust grains can be calculated through a vertical (in the direction normal to the disk plane) integration of the geometric opacity times mass density from \( z_s \) to infinity. This global procedure yields \( z_s \) at any given radius, which depends on not only the local value of \( h \) but also the disk structure interior to that radius. Thus, \( \chi = z_s/h \) varies both in space and in time. Following this procedure, we can check for self-consistency by recalculating \( z_s \) based on the temperature distribution obtained from the steady, constant-\( \chi \) model. With this inductive approach, we demonstrate that there are substantial differences between the iterated values of \( z_s \) and the initial, assumed values of \( \chi h \). We also find that \( A_s \) calculated from the deduced values of \( z_s \) is substantially different from the values of \( \sin \theta \) extrapolated from the constant-\( \chi \) model.

Since the irradiative heating can play such a major role in determining the vertical structure of protoplanetary disks, it is important to investigate the stability of such disks against the excitation of ripples on their surfaces. Under the assumption that the thermal timescale is much longer than the dynamical timescale in protostellar disks, D'Alessio et al. (1999) investigated the thermal stability of irradiation-dominated disks, using a simple cooling equation. They found that vertically isothermal disks are stable against finite-amplitude perturbations. The initial temperature perturbations propagate inward and damp out quickly. However, in their analysis they assumed that \( \chi \) is constant throughout the disk and that \( F_s \) is given by \( \theta \) as in the grazing-angle approximation. The inferred stability in that study may depend on these assumptions. As we show in § 2, changes in \( \chi \) may lead to an instability.

With a linear perturbation analysis, Dullemond (2000) showed that the flared-disk solution may become unstable to infinitesimal hydrodynamic perturbations when the cooling time of the disk is much shorter than the dynamical time (the opposite limit of D'Alessio et al. 1999). The amplitude of inwardly propagating waves grows exponentially at a rate that is a decreasing function of wavelength. Subsequently, Dullemond & Dominik (2004a) constructed a series of numerical models to examine the two-dimensional structure and evolution of protoplanetary disks around Herbig Ae/Be stars. In these simulations, they studied the radiative transfer process under the assumption that the disk always maintains hydrostatic equilibrium. (This assumption would not be appropriate for the limit in which the timescale of cooling is shorter than that of dynamics.) They found two sets of asymptotically steady state solutions, which include the monotonically flaring solutions and the self-shadowed solutions. For the second set of solutions, the disk has a puffed-up inner rim. They stated that their iteration procedure, in which the hydrostatic equilibrium and the radiative transfer are treated separately in alternate steps, may not, under some circumstances, lead to a set of converged solutions. In their simulations, some wavelike disturbances appear to propagate over the disk from one iterative step to the next, and these transitory features are never damped out completely. Although they limited their presentations to disks around Herbig Ae/Be stars, where the perturbations to the disk structure by these waves are relatively minor, they revealed that this problem appears to be more serious for disks around T Tauri stars. We speculate that this perturbation may be related to the aforementioned instabilities operating in the irradiation-dominated outer regions of protostellar disks.

In this paper, we attempt to address two issues: (1) Are the steady state solutions for irradiated disks constructed from the previous one-dimensional models self-consistent and stable? (2) Do these regions of disks tend to undergo quasi-periodic oscillations rather than attaining an asymptotic steady state? In principle, these questions should be addressed with comprehensive two- or three-dimensional numerical simulations. Such an approach is, however, fairly complicated, time-consuming, and often plagued with problems in the algorithm that implements the radiative transfer processes (e.g., Dullemond & Dominik 2004a). Prior to such detailed simulations, it will be useful to identify the dominant effects that regulate the dynamics of irradiated disks with a set of one-dimensional time-dependent analyses of the thermal evolution of protostellar disks.

Since the thermal timescale is much shorter than the viscous diffusion timescale, most previous studies adopted the assumption of thermal equilibrium during the course of the disks' global evolution. Nevertheless, there have been a few investigations of the thermal evolution of protoplanetary disks. Watanabe et al. (1990) investigated the cooling and quasi-static contraction of protoplanetary disks from an initial high-temperature state. They performed vertical one-dimensional numerical calculations and found that the cooling times can be well estimated with a simple two-temperature (surface and interior temperatures) prescription. In this paper, we utilize this two-layer prescription to examine the stability and thermal evolution of irradiated disks.

The simplest treatments of the thermal evolution of an irradiated disk are radial, one-dimensional models in which the vertical structure of the disk at each radius is analyzed independently. In order to take into account the irradiated surface and the disk interior, we evaluate the surface height directly from the location where the visual optical depth along straight lines from the star is unity. We show that such disks evolve to quasi-periodic states in which thermal waves propagate inward through intermediate disk radii, where planets are formed.

A simple discussion of the nature of thermal instability is given in § 2. The basic assumptions of our model and its governing equations are presented in § 3. The results of our numerical calculations are presented in § 4 for both simple and realistic opacities. Finally, we summarize our findings and discuss some possible evolutionary scenarios.

2. NATURE OF THERMAL INSTABILITY

In this section, we discuss some potential causes for irradiation-dominated regions of disks to become thermally unstable. For convenience of illustration, we adopt the following simplifying assumptions: (1) the star is a point source, (2) the disk's internal heat sources, such as turbulent viscous heating, as well as external heating, other than the stellar radiation, are negligible, (3) the optical depth of the disk is much larger than unity for both stellar radiation and its own emission, and (4) the transport of energy in
the radial direction is much smaller than that in the vertical direction. Note that these assumptions are adopted only in this section for the purpose of pinpointing the physical process that leads to thermal instability. All these idealizations are relaxed in the numerical simulations to be presented below.

Under these assumptions, we consider the heat balance in a geometrically thin disk that is irradiated by a central star. The energy equation reduces to

$$C\Sigma \frac{dT_m}{dt} = 2(F_s - F_m),$$

(1)

where $C$ is the specific heat per unit disk mass, $\Sigma$ is the surface density of the disk, and $T_m$ is the temperature of the disk interior. Under assumption 2, the disk interior has an approximately isothermal structure; $F_m$ is the disk blackbody emission, given by

$$F_m = \sigma T_m^4,$$

(2)

and $F_s$ is the intercepted stellar flux, given by

$$F_s = \frac{1}{2} \frac{L_s}{4\pi r^2} A_s,$$

(3)

where $\sigma$ is the Stefan-Boltzmann constant, $L_s$ is the stellar luminosity, $r$ is the cylindrical radial coordinate, and $A_s$ is the total emitting area filling factor of superheated dust grains. The factor of $\frac{1}{2}$ on the right-hand side of equation (3) comes from the fact that surface irradiated dust reradiates half the absorbed stellar flux toward the disk’s interior (the rest toward infinity). Assuming a homogeneous mixing of gas and dust, we can obtain $A_s$ from

$$A_s = \tau_G \text{erfc} \left( \frac{z_s}{\sqrt{2}h} \right) = \tau_G \text{erfc} \left( \frac{\chi}{\sqrt{2}} \right),$$

(4)

(see Appendix C), where erfc ($x$) is the complementary error function, $\tau_G$ is the geometric optical depth of the disk midplane, and $z_s$ is the surface height where stellar radiation is absorbed. The ratio of $z_s$ to the gas scale height $h$ is denoted by $\chi \equiv z_s/h$. In hydrostatic equilibrium, the gas scale height is given by

$$h = \frac{c_m}{\Omega_K} = \left( \frac{k_B T_m r^3}{\mu m_u G M_*} \right)^{1/2},$$

(5)

where $c_m$ is the disk sound speed, $\Omega_K$ is the Keplerian angular velocity, $k_B$ is the Boltzmann constant, $\mu$ is the molecular weight of disk gas, $m_u$ is the atomic mass unit, $G$ is the gravitational constant, and $M_*$ is the stellar mass.

Figure 1 displays $F_m$ and $F_s$ (at 10 AU) as functions of $T_m$. Assuming that the initial state is in thermal equilibrium (corresponding to the point where the three black lines cross) with $T_m = T_m\text{eq}$, we impose a small positive temperature perturbation. We consider two extreme cases: (1) If $z_s$ is determined by the local disk structure, $\chi = z_s/h$ would also be constant (see eq. [4]) and $F_s$ would not change (dot-dashed line). In this case, the system would be stabilized because $F_m > F_s$ for $T_m > T_m\text{eq}$. (2) If $z_s$ is determined mostly by the attenuation by dust in the inner regions of the disk, $z_s$ would remain constant despite changes in the local disk temperature and $h$ such that $A_s$ would increase rapidly and $F_s$ would increase much faster than $F_m$ (black dashed curve). In this case the system would be unstable, because $F_m < F_s$ for $T_m > T_m\text{eq}$.

In their stability analysis, D’Alessio et al. (1999) assumed a constant $\chi$. Based on the above analysis, this assumption naturally leads to stable solutions. In fact, most of the analyses of the structure of irradiated disks are based on the constant-$\chi$ assumption (e.g., Chiang et al. 2001). The usual procedure to determine $z_s$ is based on a geometric consideration, that is,

$$A_s = \sin \theta \approx \frac{z_s}{r} \left( \frac{d \ln z_s}{d \ln r} - 1 \right),$$

(6)

where $\theta$ is the grazing angle (i.e., the angle between the starlight and the disk surface). We refer to this as the grazing-angle approximation. Most previous steady state disk models are constructed with equation (6) under the assumption that $\chi$ is constant throughout the disk.

However, the magnitude of $\chi$ is generally determined by the radial structure of the disk as well as its local properties. The surface height $z_s$ is determined by the optical depth integrated through a ray of the stellar radiation. We derive a ray integral and check the validity of equation (6) in § 3 and Appendix C. In principle, equations (4) and (6) must be resolved simultaneously (Tanaka et al. 2005). However, this set of equations is fairly unstable to solve numerically, because they do not contain contributions that may reduce any steep temperature gradients in the radial direction.

A steep temperature gradient, if present, would invalidate the constant-$\chi$ and the grazing-angle approximations. Physically, the radial transport of heat suppresses the radial temperature gradient, but such a process through the opaque regions of the disk must be analyzed with multidimensional numerical simulations. One of the most efficient processes of radial heat transport is radiative transfer from the superheated dust grains at the surface of any radial location to the disk midplane at adjacent radial regions. Using a simple one-dimensional model, we can take this oblique radiative transfer of heat into account.

3. BASIC EQUATIONS

Following the approaches of Chiang & Goldreich (1997) and Garaud & Lin (2007), we construct numerical models to study
the thermal evolution of a protostellar accretion disk. The surface of the disk is illuminated by the central star. Exposed to the stellar radiation, submillimeter dust grains in the surface layers of the disk are superheated. We consider the case in which the dust mass of the disk is so large that the disk midplane (except for an innermost region where silicates are evaporated) is optically thick to the stellar radiation. In contrast to the previous section, the heat sources for the disk interior in these numerical models include both irradiation from the superheated grains on the disk surface and the viscous dissipation associated with the accretion flow. We adopt a cylindrical coordinate system \((r, \phi, z)\) in which the \(z = 0\) plane represents the disk midplane and the origin is at the location of the central star. Since the star-disk system is symmetric with respect to the midplane, we describe our results for the upper half of the disk only.

In order to simplify the problem, we adopt the two-layer axisymmetric disk model proposed by Chiang & Goldreich (1997). In this model, the disk consists of a superheated surface layer where the dust temperature is \(T_s(r)\) and a disk interior where the dust and gas temperatures are assumed to be uniform at \(T_m(r)\). This model is simple to use and includes all the essential ingredients to analyze the onset, evolution, and stabilization of thermal instability in protostellar disks. However, such a simplification will be invalid if the disk optical depth \(\tau_m(T_m)\) to its intrinsic radiation is much larger than unity and the viscous heating rate is larger than the surface heating rate. However, the dust optical depth may be self-limited by the grains’ rapid growth through coagulation collisions, so that \(\tau_m(T_m) \leq 10\) throughout the disk (see Fig. 6 of Tanaka et al. 2005). Thus, the two-layer model is valid even in the inner disk, where the dust surface density is higher.

The two-layer model is invalid at the inner edge of the disk, where the disk is irradiated not only from the top but also from the radial direction. The disk may have a puffed-up inner rim (e.g., Dullemond & Dominik 2004a), but a set of two-dimensional radiative transfer calculations is needed to determine the structure of the innermost region. In this work, we simply assume that the disk within 0.1 AU is optically thick in the radial direction and has no puffed-up rim that might cast shadows over the outer regions of the disk. We confine our calculations to the regions \(r > 0.1\) AU, where the two-layer model is valid.

The thermal timescale of the disk interior at radius \(r\) is given by

\[
\tau_{th} = \frac{(\gamma_a + 1) c_m^2 \Sigma}{2(\gamma_a - 1) \alpha T_m^4} \approx 53 \left( \frac{\Sigma_0}{\Sigma_{H,0}} \right) \left( \frac{T_{m,0}}{124 \, \text{K}} \right)^{-3} \left( \frac{r}{\text{1 AU}} \right)^{-3/2} \text{yr}
\]

(see eq. [12]), where \(\gamma_a\) is the adiabatic exponent and \(c_m\) is the sound speed of the disk interior. For evaluation, we assume power-law distributions for the total (gas+dust) surface density, \(\Sigma(r) \propto r^{-p}\), and the midplane temperature, \(T_m(r) \propto r^{-q}\). The normalization factors, \(\Sigma_0\) and \(T_{m,0}\), refer to their corresponding values at \(r = 1\) AU. The nominal value of the surface density is given by that in the minimum-mass solar nebula (MMSN) model, in which \(\Sigma_0 = 1.7 \times 10^3 \, \text{g cm}^{-2}\) (Hayashi 1981).

We assume that the thermal timescale \(\tau_{th}\) is much longer than the dynamical time (\(\Omega^{-1}\)) but much shorter than the viscous evolution time \((\nu^2/\nu)\), where \(\nu\) is the turbulent viscosity. In this case we can regard the whole region of the disk as always in hydrostatic equilibrium in the vertical direction and as having time-independent surface densities.

The temperature \(T_s\) of the superheated dust grains is given by

\[
\frac{L_s}{4\pi r^2} = 4\epsilon_s \sigma T_s^4,
\]

(8) where \(\epsilon_s\) is the averaged emissivity of the grains at \(T_s\). Along a ray from the surface of the star, the superheated layer extends outward until the position where the visual optical depth reaches unity. We take into account the attenuation of the stellar photons by defining the height \(z_s\) of the bottom of the superheated layer with the following equation:

\[
\tau_s(T_s; r, z_s) = 1.
\]

(9) Here \(\tau_s(T_s; r, z_s)\) is the optical depth between the central star and the point \((r, z_s)\) to the blackbody radiation peaked at the stellar effective temperature \(T_s\), given by the integration

\[
\tau_s(T_s; r, z) = \int_{R_s}^{r} \kappa_s(T_s) \rho_d(r', z') \left( 1 + \frac{z'^2}{r'^2} \right)^{1/2} dr'.
\]

(10) along a straight path from the star to \((r, z_s)\). Here \(R_s\) is the stellar radius, \(\zeta \equiv z/r\) is the aspect ratio, \(\rho_d(r', z')\) is the spatial mass density of dust at \((r', z')\), and \(\kappa_s(T_{rad})\) is the Planck mean opacity of the grains interacting with the blackbody radiation peaked at \(T_{rad}\). Note that we define here the grain opacity per unit dust mass, not per unit total (gas+dust) mass as in the usual definition, because it is convenient for considering the case in which the dust-to-gas mass ratio may change vertically. The emissivity in equation (8) can be given by

\[
\epsilon_s = \frac{\kappa_s(T_s)}{\kappa_s(T_s)}.
\]

(11) The energy equation includes heating from both stellar irradiation and viscous dissipation, as well as radiative losses from the disk surface, such that

\[
\frac{\gamma_a + 1}{2(\gamma_a - 1)} k_B \Sigma \frac{\partial T_m}{\partial r} = 2(F_s - F_m) + \frac{3}{4\pi} MΩ^2\kappa
\]

(12) (see, e.g., Watanabe et al. 1990), where \(M\) is the mass accretion rate, which we assume to be constant throughout the disk. Note that the steady state assumption is compatible with a power-law surface density distribution for some prescriptions of effective viscosity (Chiang & Goldreich 1997; D’Alessio et al. 2006; Garaud & Lin 2007).

Further, \(F_s\) and \(F_m\) are respectively the thermal radiation fluxes downward from the superheated dust grains high up in the disk atmosphere and upward from dust grains in the disk interior,

\[
F_s(r) = (1 - e^{-2\tau_m(T_m)}) \frac{L_s}{8\pi r^2} \left( \frac{A_s}{r^2} + \frac{4R_s}{3r^3} \right),
\]

(13) \(F_m(r) = (1 - e^{-2\tau_m(T_m)}) \sigma T_m^4,\)

(14) where \(\tau_m(T_s)\) and \(\tau_m(T_m)\) are the optical depths of the disk interior (from \(z = 0\) to \(z = z_s\)) to the radiation from the superheated dust grains and to its own emission, respectively, and \(A_s\) is the total emitting area filling factor of superheated dust grains. We consider the effect of the star’s finite radius in \(F_s\), which is important in the inner part of the disk. We also consider the effects of oblique radiative transfer: the angle brackets on the right-hand
side of equation (13) represent a radial average of radiation emitting from superheated dust within the adjacent regions (see §4). The factors of 2 in the exponential functions in equations (13) and (14) also denote oblique radiative transfer in the disk interior (Tanaka et al. 2005).

Once the dust density distribution $\rho_d(r, z)$ of the disk is specified, $A_s$ and $\tau_m$ can be determined from the following integrations:

$$A_s(r) = 1 - \exp \left[ - \int_0^\infty \kappa_s(T_s) \rho_d(r, z^\prime) dz^\prime \right],$$

(15)

$$\tau_m(T_{\text{rad}}; r) = \int_0^{r/r_0} \kappa_m(T_{\text{rad}}) \rho_d(r, z^\prime) dz^\prime,$$

(16)

where $\kappa_m(T_{\text{rad}})$ is the Planck mean opacity of midplane grains interacting with the blackbody radiation peaked at temperature $T_{\text{rad}}$. Further details of the dust opacity are given in §4 and Appendix A.

We assume that the total (gas+dust) surface density $\Sigma$ of the disk is a simple power-law distribution in the radial direction:

$$\Sigma = \Sigma_0 (r/r_0)^{-p},$$

(17)

which is kept constant during the thermal evolution considered in this work.

Taking the effects of dust settling into account, we can obtain the dust density distribution $\rho_d$ of the disk in the two-temperature model adopted here (see Appendix B). However, a computationally intensive, iterative calculation is needed to determine, self-consistently, the magnitudes of $\rho_d$ and $z_s$ simultaneously. Instead of equations (B4) and (B6), we adopt, in most of the calculations, the following simple density distribution for the dust:

$$\rho_d(r, z) = \frac{\Sigma_d}{\sqrt{2\pi h}} \exp \left( - \frac{z^2}{2h^2} \right),$$

(18)

where $\Sigma_d$ is the surface density of dust. We set $\Sigma_d = f_d \Sigma$, where $f_d$ is the dust fraction in the surface density. We dub $f_d$ the dust-to-gas ratio. In the disk interior, dust sedimentation is not very important unless the dust radii are not so large that we can formally set the dust density distribution to equation (18). In the surface layer, dust settling reduces the dust density while high surface temperatures $T_s$ raise it, so that equation (18) also gives a good estimate. In some cases we compared results with the more realistic dust distribution given in Appendix B, and we found that they are very similar unless the dust sizes are not large enough. We discuss the differences in the results between the two distributions in §5.

We found that if the gradient $d \ln z_s / d \ln r$ changes rapidly in the $r$-direction, the approximation used to derive equation (6) is no longer justified (see Appendix C). For this reason, we use equations (9) and (15) instead of equation (6).

4. NUMERICAL RESULTS

We adopt the following values as fixed parameters for all models: the mass, radius, and luminosity of the central star are set to $M_\star = 1 M_\odot$, $R_\star = 2.085 R_\odot$, and $L_\star = 1 L_\odot$, respectively, so that its effective temperature is $T_\star = 4000$ K. For the disk gas, we specify $\mu = 2.34$ and $\gamma_\alpha = 1.4$.

We adopt the phenomenological MMSN model (Hayashi 1981) for the standard gas and dust surface density distribution. In this model, $\Sigma_0 = \Sigma_{0,0} = 1.7 \times 10^2$ g cm$^{-2}$ with $r_0 = 1$ AU and $p = 1.5$ in equation (17). For comparison, we also calculate a relatively flat $\Sigma$-distribution with $p = 1.0$ and $\Sigma_0 = 3.54 \times 10^2$ g cm$^{-2}$.

These surface densities are kept constant with time. The dust-to-gas ratio $f_d$ and solid-density material $\rho_{\text{mat}}$ are set to 0.14 and 1.4 g cm$^{-3}$, respectively. In §4.1, we neglect any changes of dust surface density due to sublimation and use the value of $f_d$ throughout the disk. In §4.2, we consider the effect of ice sublimation. We also vary the mass accretion rate $\dot{M}$ from zero to $10^{-5} M_\odot$ yr$^{-1}$. These steady state accretion rates are consistent with our specified surface density and temperature distributions provided that the magnitude of $\alpha$ is a function of radius, where $\alpha$ is the non-dimensional turbulent viscosity in the so-called $\alpha$-prescription (Shakura & Sunyaev 1973).

The normalization time unit is the thermal timescale given by equation (7) with $\Sigma = \Sigma_0$ and $T_m = T_m$. We denote the time unit as $t_{\text{th},0}$ and the nondimensional time as $\tau = t/t_{\text{th},0}$. For the standard model, we set $\Sigma_0 = \Sigma_{H,0}$ and $T_0 = 124$ K, so that $t_{\text{th},0} = 53$ yr. Note that the local thermal timescale $t_{\text{th}}$ is nearly constant with $r$ in the standard disk model with $p = 1.5$. The nondimensional time step used in the numerical integration is set to $\delta \tau = 0.005$. In order to verify numerical convergence, we also performed several calculations with a time step of half the standard value and confirmed that the results exhibit no significant changes.

The spatial grid consists of 90 points (the standard case) or 180 points (the high-resolution case), logarithmically distributed, between $r = 0.1$ AU and $r = 100$ AU. Numerical oscillation would be induced if there were no radial exchange of energy at all. At any radius, the disk interior is exposed not only to the superheated surface grains directly overhead, but also obliquely to those at adjacent radial locations. Such a radial exchange of energy tends to suppresses instabilities for short-wavelength perturbations. In our numerical scheme, we assume that the isotropic radiation coming from all radii $r'$ within $|r^\prime - r| < z_s(r)$ contributes to the heating at radius $r$ as expressed in equation (13). This implementation stabilizes the short-wavelength oscillation, and the results are essentially independent of the numerical resolution.

4.1. Constant Opacity

In this subsection, we first illustrate the dominant features using a simple opacity model. We adopt the emissivity and opacity (per unit dust mass) of the grains interacting with blackbody radiation peaked at temperature $T_i$ as

$$\epsilon_s(T_i) = \left( \frac{T_i}{T_s} \right)^\beta, \quad \kappa_s(T_i) = \kappa_{s,0} \left( \frac{T_i}{T_s} \right)^\beta,$$

(19)

where we choose the value $\kappa_{s,0} = 10^2$ cm$^2$ g$^{-1}$, which approximately corresponds to the commonly defined opacity per unit gas mass with the value of 1 cm$^2$ g$^{-1}$. Most of the calculations shown here are for $\beta = 0$, but we also calculate some models with $\beta = 1$ for comparison purposes.

For initial conditions, we adopt the steady state solution obtained from the time integration with fixed $\chi(r)$. This set of initial conditions does not correspond to the asymptotic steady state solutions, because the initial estimate of $\chi$ is not self-consistently compatible with the actual aspect ratio $\zeta$ of the surface. Nevertheless, the numerical calculations relax to nearby steady solutions if they exist. In order to verify that our results are independent of the adopted initial conditions, we calculated the evolution of the disk with several different initial guesses for $\chi$ and found that the system reaches the same asymptotic state.

We first present the results of the calculations with no mass accretion ($\dot{M} = 0$). Figure 2 shows the initial evolution of the midplane temperature $T_m$, as well as the surface temperature $T_s$, which is kept constant with time. The elapsed time is $\hat{t} = 1.6$, that
$t = 1.6 t_{\text{b},0} \approx 85 \text{ yr}$, and each curve corresponds to a time step $\Delta t = 0.2$ ($\Delta t = 10.6 \text{ yr}$). At first, the initial state is almost stable in the innermost and outermost regions. But in the intermediate region ($0.5$–$20 \text{ AU}$), the disk becomes unstable. At a typical instant of time, four local temperature peaks (high $T_m$ and $z_s$) coexist and are amplified. These peaks move inward (toward the star) as they grow. Hence, we refer to these propagating perturbations as waves. During the amplification of the waves, they cast shadows over the outer regions of the peak. The temperature $T_m$ decreases in the shadowed regions. Each fully grown wave has a sharp slope on the inner, “exposed” side of the peak and a gentler decline on the outer, “shadow” side. The waves propagate inward with velocities a few tenths of $r/t_{\text{b}}$. The outermost wave ($\sim 16 \text{ AU}$) begins to grow just outside the shadowed region of the inner adjacent wave when the shadowed region is developed. This tendency shows that the outermost wave may be induced by the wave ahead of it.

Additional time integration shows that as these waves propagate inward, they begin to decay when they reach inside $1 \text{ AU}$. The waves are completely damped out at around $0.25 \text{ AU}$. In contrast, new waves are formed continually in the outermost region ($>20 \text{ AU}$) of the computational domain. The growth region of waves gradually retreats, and the maximum amplitude of each wave during its propagation cycle gradually increases with time. The system reaches a quasi-periodic state when $t \sim 8$. Figure 3 shows the evolution of $T_m$ at this stage. Waves are continually formed and amplified in the outer disk ($>30 \text{ AU}$) and then propagate inward with nearly constant amplitude through the intermediate disk regions, beginning to decay at about $1 \text{ AU}$, and are damped out completely at around $0.25 \text{ AU}$. The temperature in the innermost disk region ($r < 0.25 \text{ AU}$) attains steady values. The propagation speed of the waves is approximately given by $r/t_{\text{b}}$. At intermediate disk radii ($1$–$20 \text{ AU}$), the peak temperature of each wave is $2$–$3$ times higher than the lowest temperature in the inner adjacent shadowed region.

The radial profile of each wave is somewhat skewed. The half-width of an individual wave is about $(0.1$–$0.2)r$ on the inner side and $(0.2$–$0.4)r$ on the outer side. The wavelength is approximately twice as large as $z_s(r)$. Because of the steep radial temperature gradient, the magnitude of $z_s(r)$ is affected by the variation in the thickness $h$ in the disk regions interior to $r$ on this length scale. The ratio of the radii between two adjacent wave peaks is about $20$ if both waves are outside of $1 \text{ AU}$. Near the inner boundary of the propagating-wave zone, the finite size of the star ($R_\star$) becomes comparable to the surface height $z_s$. This time-independent contribution in equation (13) essentially stabilizes the innermost region of the disk.

The changes in other variables at the same epoch as in Figure 3 are shown in Figures 4–8. Figure 4 shows the time evolution of $z_s/r$. Outside $0.25 \text{ AU}$, $z_s$ experiences stepwise changes. The two or three steep jumps correspond to the leading, inner side of the thermal waves. The magnitude of the increase in $z_s$ at...
the leading edge of each step reaches a maximum of 1.5 around 10 AU. The portions with flat $\zeta_s$ correspond to the shadowed regions, where $\zeta_s$ is determined by the stellar ray that passes above the peak of each wave.

Figure 5 shows the time evolution of $\chi = z_s/h = \zeta_s/\zeta_k$. The local minima of $\chi$ are located just ahead of the peaks in $T_m$. The decrease of $\chi$ is essential for the temperature rise. In the shadowed regions, $\chi$ increases because $\zeta_s$ remains almost constant (see Fig. 4) whereas $\zeta_k = h/r$ decreases. Changes in the value of $\chi$ produce the variations in the surface filling factor $A_s$. The time evolution of $A_s$ is shown Figure 6. Sharp peaks, which correspond to the minima of $\chi$ (see eq. [C1]), propagate inward. Note that the variation in the amplitude of $A_s$ is very large, ranging from a few tenths at the peaks to less than $10^{-3}$ in the shadowed regions. Such large changes of $A_s$ induce rapid heating in front of the waves and rapid cooling in the shadowed regions. The unperturbed $z_s$ in the intermediate regions is comparable to the half-width of the propagating waves. Modest variations in $\chi$ can lead to nonlinear dissipation, such that the wave amplitudes are also limited in these regions.

We also plot the evolution of the distribution of the logarithmic pressure gradient $d \ln P/d \ln r$ (Fig. 7), where $P(r)$ is the pressure in the midplane of the disk. This plot indicates that the gas pressure gradient is nearly reversed just in front of the peak of the waves. This inversion occurs because of the steep positive temperature gradient in the exposed, leading, inner face of the waves. Consequently, the velocity of the gas departs significantly from its unperturbed sub-Keplerian value. In a follow-up paper, we will consider the associated gas drag on grains of various sizes. Another interesting quantity is the distribution of the potential vorticity (or vortensity):

$$\frac{\Omega_{ep}}{\Sigma} = \frac{1}{\Sigma \rho^2} \frac{d}{dr} r^4 \Omega^2,$$

where $\Omega_{ep}$ is the epicyclic frequency and $\Omega$ is the angular velocity of gas. Note that in a quasi-Keplerian disk $\Omega_{ep} \simeq \Omega_K \propto r^{-1.5}$, so that the potential vorticity is almost constant for a MMSN with $p = 1.5$. The thermal waves disturb the potential vorticity through the change of pressure gradient. Figure 8 displays the evolution of the distribution of the potential vorticity. There are peaks and troughs around the waves. Local extrema of this quantity can lead to baroclinic instabilities, which may excite turbulence (see, e.g., Klahr & Bodenheimer 2003) in the dead zone where the magnetorotational instability may have a limited influence (Gammie 1996). Further investigation of this possibility will also be considered elsewhere.

Next we examine the dependence of wave excitation and propagation on the mass accretion rate ($\dot{M}$). Other than the value of $\dot{M}$ in the energy equation, we adopt the same model parameters as for the no-accretion case shown in Figures 2–8. Thus, in the present context the primary physical effect associated with the accretion flow is the viscous dissipation. This internal energy source (viscous dissipation) in equation (12) has a greater fractional contribution to the energy budget in the inner regions than in the outer regions of the disk (Garaud & Lin 2007).
Although quasi-periodic oscillations are excited in all cases, the radial extent to which they propagate depends on the magnitude of $\dot{M}$. Figure 9 shows the time evolution of $T_m$ in a quasi-periodic state for the case of $\dot{M} = \frac{10^{-8}}{C_{0.1}} M_{\odot} \text{yr}^{-1}$. The result is similar to that obtained when neglecting viscous dissipation. In this case, waves are excited in the outer region, propagate inward, and are damped out in the innermost region. Compared with the no-accretion model, the innermost region in this case is hotter [$T_m(0.1 \text{ AU}) \sim 600 \text{ K}$] and thicker. The time-independent contribution of viscous heating to the energy equation provides a stabilizing effect to a slightly larger radial extent ($r < 0.5 \text{ AU}$) than in the no-accretion case [$T_m(0.1 \text{ AU}) \sim 500 \text{ K}$ and $r < 0.25 \text{ AU}$].

The quasi-periodic state, however, is drastically changed in the $\dot{M} = \frac{10^{-7}}{C_{0.1}} M_{\odot} \text{ yr}^{-1}$ model (Fig. 10). The disk becomes stable interior to about 6 AU. In the outer region there are two high-temperature peaks, both oscillating quasi-periodically. The positions of the two peaks do not coherently propagate inward as in the $\dot{M} = \frac{10^{-8}}{C_{0.1}} M_{\odot} \text{ yr}^{-1}$ case but fluctuate to-and-fro between 12 and 40 AU. The range of temporal temperature changes is less than 2 K over the disk, so that it can be regarded to be in an approximately steady state. This result shows that the disk stabilizes as $\dot{M}$ increases.

Finally, we show the dependence on the disk surface density $\Sigma$. Note that $\Sigma$ affects the evolution not only through the optical depth of the surface layer but also through the thermal timescale. We performed several calculations for $\Sigma_0 = 3.54 \times 10^2 \text{ g cm}^{-2}$ and $\rho = 1.0$. The time unit for this case is $t_{\text{th},0} = 11.0 \text{ yr}$. We calculated the disk evolution for this surface density distribution with several values of $\dot{M}$ and found that the results are quite similar to the case with the standard surface density distribution.
The quasi-periodic wave solutions are obtained for small ˙\(M\), whereas the disk becomes nearly steady for large ˙\(M\).

Figure 11 shows the temporal variation of ˙\(T\) in the quasi-periodic state for this prescribed ˙\(S(\rho)\) distribution with ˙\(M = 10^{-8}\) \(M_\odot\) yr\(^{-1}\). As compared with the case with ˙\(p = 1.5\) (Fig. 9), the propagating-wave region is relatively narrow, confined to between 1 and 20 AU, and the maximum wave amplitude is also smaller. In the outermost region, where \(r > 20\) AU, the amplitudes of the waves are limited and do not exceed about 2 K. The amplitudes grow as waves propagate from 20 to 8 AU, and then they maintain a nearly constant value from 8 to 1.5 AU. The waves are finally damped at 1.5–1 AU. For an analogous inviscid model [i.e., with an identically prescribed ˙\(S(\rho)\) distribution but without viscous dissipation], the result is quite similar to that in Figure 11, except that the wave propagation region extends slightly closer to the star. The results for the ˙\(M = 10^{-7}\) \(M_\odot\) yr\(^{-1}\) model show that the disk attains an approximately steady state, with only very small fluctuations (amplitude less than a few tenths of a kelvin) remaining in the outermost region.

All the above results are for \(\beta = 0\). We also calculated cases with \(\beta = 1\). Other than a modification in the distribution of ˙\(T\) and ˙\(T\), the time-dependent nature of the wave excitation and propagation is essentially independent of the value of \(\beta\). In all cases, we find that the quasi-periodic nature of the inwardly propagating thermal waves is realized for a disk with ˙\(M \leq 10^{-8}\) \(M_\odot\) yr\(^{-1}\). The basic features of this state do not strongly depend on other parameters such as ˙\(p\) and \(\beta\). In order to verify the universality of these results, we perform further calculations with more realistic opacities.

4.2. Realistic Opacities

In this subsection, we present results based on models with a more realistic grain opacity prescription, given by Tanaka et al. (2005). In this prescription, grains are assumed to consist of a uniform mixture of H\(_2\)O ice, organics, olivine, pyroxene, metallic iron, and troilite, the abundances of which are taken from Pollack et al. (1994). For relatively high temperatures (\(T > 160\) K), we use the dust opacity of grains without ice or organics. For the low-temperature (\(T < 160\) K) state, the opacity includes the contribution from ice and organic grains. Based on a single-sized monochromatic opacity table generated by H. Tanaka (see Tanaka et al. 2005), we calculate the Planck mean of the size-averaged monochromatic opacity table generated by H. Tanaka (see Tanaka et al. 2005), we calculate the Planck mean of the size-averaged monochromatic opacity table generated by H. Tanaka (see Tanaka et al. 2005), we calculate the Planck mean of the size-averaged monochromatic opacity table generated by H. Tanaka (see Tanaka et al. 2005), we calculate the Planck mean of the size-averaged monochromatic opacity table generated by H. Tanaka (see Tanaka et al. 2005), we calculate the Planck mean of the size-averaged monochromatic opacity table generated by H. Tanaka (see Tanaka et al. 2005), we calculate the Planck mean of the size-averaged monochromatic opacity table generated by H. Tanaka. For relatively high temperatures (\(T > 160\) K), we use the dust opacity of grains without ice or organics. For the low-temperature (\(T < 160\) K) state, the opacity includes the contribution from ice and organic grains. Based on a single-sized monochromatic opacity table generated by H. Tanaka (see Tanaka et al. 2005), we calculate the Planck mean of the size-averaged monochromatic opacity table generated by H. Tanaka. For the low-temperature (\(T < 160\) K) state, the opacity includes the contribution from ice and organic grains. Based on a single-sized monochromatic opacity table generated by H. Tanaka (see Tanaka et al. 2005), we calculate the Planck mean of the size-averaged monochromatic opacity table generated by H. Tanaka.

Note that even in the constant-opacity case in which \(\beta = 0\) (Fig. 9), \(T_s\) is everywhere larger for the realistic grain opacity, because of the effect of superheating. There is a small jump at the radius where the opacity law changes (\(T_s \approx 160\) K). For the same value of \(M\), the structure of the wave propagation region is very similar to that of the constant-opacity case.

With a sufficiently large accretion rate (\(M = 1 \times 10^{-7}\) \(M_\odot\) yr\(^{-1}\)), the disk with realistic opacity is also stabilized by the effect of viscous dissipation (Fig. 13). There are three local maxima in the ˙\(T\)-distribution. The innermost peak at about 6 AU corresponds to an opacity transition in the disk’s surface layer (\(T_s \sim 160\) K). These local peaks do not propagate but do fluctuate quasi-periodically with small amplitude.

Finally, we show the effect of ice sublimation. Figure 14 displays the evolution of ˙\(T\) for a model in which the dust surface density is modified by the sublimation of ice. Taking into account the effect of dust’s size sorting on the opacity, we assume that surface dust grains have a smaller maximum size \(s_{\text{min}}\) than those in disk interior, \(s_{\text{min}} = 1\) mm. We choose \(s_{\text{min}} = 1\) mm. Owing to the increase in \(A_r\) associated with the condensation in the surface layer, the ˙\(T\)-distribution attains a local maximum near 5 AU. This peak does not propagate over time. Interior to this snow line, a wave propagates to about 0.5 AU. Outside this line, there is another local maximum near 20 AU, which may be induced by the emergence of the first peak. Compared with the model in which ice sublimation is neglected (Fig. 12), the ice condensation induces the formation of a local thermal maximum, which prevents waves from emerging at large radii and propagating inward.

5. SUMMARY AND DISCUSSION

We have performed a set of one-dimensional radial calculations to examine the thermal evolution of hydrostatic disks, using the direct integration of optical depths \(\tau(T_s)\) to determine the optical surface \(z_s\) and total emitting area filling factor \(A_r\) of a superheated layer. Our results suggest that in regions both with
modest and steep radial temperature gradients, the assumption of a constant $\chi = z_s/h$ is incompatible with the computed height of the surface, where $\tau(T, r, z_s) = 1$. The initial state obtained by a fixed-$\chi$ iteration evolves spontaneously to a state in which thermal waves grow. The disks evolve to a quasi-periodic state where thermal waves continuously propagate toward the star through intermediate radii.

The mechanism driving this thermal instability is the intense stellar irradiation high in the disk’s atmosphere. It is a consequence of a “shadowing effect” in which the surface where most of the stellar photons are intercepted at any given radial location may be affected by the vertical structure in the disk regions interior to that radius. This quasi-periodic state is stabilized by viscous dissipation associated with the mass accretion flow through the disk. For the cases of $M = 10^{-7} M_\odot\ yr^{-1}$, wave excitation and propagation are suppressed and the disks reach approximately steady states. In order to eliminate the possibility of an artificial dependence on the initial conditions, we performed the following numerical experiments: By setting the initial condition for an approximately steady state with $M = 10^{-7} M_\odot\ yr^{-1}$ and decreasing the mass accretion rate to $M = 10^{-8} M_\odot\ yr^{-1}$, we calculate the time evolution of the disk. We find that the system evolves to a quasi-periodic state within about 10 thermal timescales. The asymptotic quasi-periodic state is identical to that obtained with the standard calculation for $M = 10^{-7} M_\odot\ yr^{-1}$, which is shown in Figure 9. Inversely, we also start a calculation from a quasi-periodic state with $M = 10^{-7} M_\odot\ yr^{-1}$ and increase the mass accretion rate to $M = 10^{-8} M_\odot\ yr^{-1}$. The disk evolves into an approximately steady state, which is almost identical to that obtained with the standard calculation for $M = 10^{-7} M_\odot\ yr^{-1}$, which is shown in Figure 10. These results show that the state of the disks is determined by the instantaneous mass accretion rate (note that the viscous evolution time is much longer than the thermal timescale).

The result that quasi-periodic wave-propagating states exist in disks with modest disk accretion rates is robust to variations in the disks’ structural parameters, such as their surface density profile, opacity law, and vertical dust distribution. Whereas these parameters weakly affect the positions of the inner and outer boundaries of the wave-propagating domains, they do not affect the basic features of the waves such as their amplitude and shape.

We also performed some calculations for the constant-opacity case using a general surface density profile (eq. [B6] instead of eq. [18]). The results are quite similar to those for the simple density profile. The difference is even less noticeable than that due to a change in opacity law.

Using the calculated thermal structure of the disk, we obtain the disk SED. We assume that the disk is oriented face-on and has an inner truncation at 0.1 AU and an outer truncation at 100 AU. Figure 15 shows the evolution of the SEDs associated with the model illustrated in Figure 14. The contribution from the central star to the SEDs is included in this figure. The SEDs are expected...
to oscillate periodically within mid-infrared wavelengths. This variation corresponds to the periodic propagation of the thermal waves. In this case, the period of oscillation is about \( t_{th} \approx 53 \) yr. In general, this timescale varies depending on the surface density distribution and accretion rate in the disk, as well as the luminosity of the central star. The change of disk SED is due not only to emission from the disk interior where \( T_m \) changes, but also to emission from the superheated surface layer, where \( A_\lambda \) would modulate even if \( T_s \) were kept constant. The surface emission also contributes to the water ice and silicate emission bands. We expect the relative SED variations due to propagation of the thermal waves. In this case, the period of oscillation is about 53 yr.

Effects can suppress the thermal waves in the outer disk, the waves in the outer regions (say, beyond 20 AU; see eq. [7]). But our calculations show that the wave-propagating region extends to the inner and outer edges. Thus, a direct comparison from the superheated surface layer, where \( T_s \) if \( h > 1 \) mm. The grain compositions and optical constants are adopted from Tanaka et al. (2005 and references therein). We use the Planck mean opacity is given by

\[
\tilde{\kappa}_\nu(T, r) = \frac{\int_0^\infty \tilde{\kappa}_\nu(T, r) B_\nu(T_{\text{rad}}) d\nu}{\int_0^\infty B_\nu(T_{\text{rad}}) d\nu},
\]

where subscript \( j \) represents either “s” (surface) or “m” (disk interior), \( \nu \) is the frequency, \( B_\nu(T_{\text{rad}}) \) is the Planck function, and

\[
\tilde{\kappa}_\nu(T_j) = \frac{\int_{s_{\text{min}}}^{s_{\text{max}}} \kappa_j(T_j, s) s^3 n(s) ds}{\int_{s_{\text{min}}}^{s_{\text{max}}} s^3 n(s) ds}.
\]

Here \( n(s) ds \) is the number density of grains with radii between \( s \) and \( s + ds \), and the size distribution has a lower cutoff \( s_{\text{min}} \) and upper cutoff \( s_{\text{max}} \). In addition, \( \kappa_j(T, s) \) is the single-sized (s) monochromatic (\( \nu \)) dust opacity of grains with temperature \( T \). We assume a power-law size distribution \( n(s) \propto s^{-3.5} \), which is truncated with a minimum and maximum of \( s_{\text{min}} = 0.1 \) μm and \( s_{\text{max}} = 1 \) mm. The grain compositions and optical constants are adopted from Tanaka et al. (2005) and references therein. We use the resultant opacity table provided by H. Tanaka. The table gives two single-sized monochromatic dust opacities as functions of \( \nu \) and \( s \): one for grains without ice or organics at high temperatures, \( T > 160 \) K, and the other for grains including ice or organics at \( T < 160 \) K.

APPENDIX B

EXACT SOLUTION OF THE DUST DENSITY DISTRIBUTION

The gas density distribution \( \rho_g \) in the two-temperature model is given by

\[
\rho_g(r, z) = \begin{cases} 
\rho_g(r, 0) \exp\left(-\frac{z^2}{2h^2}\right), & \text{if } |z| \leq z_s, \\
\rho_g(r, z_s) \exp\left(-\frac{z^2 - z_s^2}{2H^2}\right), & \text{if } |z| \geq z_s,
\end{cases}
\]

where \( h = c_s \Omega_K^{-1} \) (see eq. [5]) and \( H = c_s \Omega_K^{-1} \) (\( c_s \) is the sound speed in the surface layer) are the gas scale height in disk interior and in the surface layer, respectively.
In a steady state, the sedimentation flux of the dust grains with their terminal velocity balances the diffusive mass flux due to gas turbulence (see eq. [30] of Takeuchi & Lin 2002) such that

\[-\rho_d \Omega_K \dot{z}_{stop}^s = \frac{\rho_d \nu}{\text{Sc}} \frac{\partial (\rho_d/\rho_g)}{\partial z},\]  

(B2)

where \(\dot{z}_{stop}\) is the stopping time normalized by \(\Omega_K^{-1}\), \(\nu\) is the turbulent viscosity, and Sc is the Schmidt number, representing the coupling strength between grains and gas. The nondimensional stopping time \(\dot{z}_{stop}\) is given by

\[\dot{z}_{stop} = \frac{\rho_{mat} \Omega_K}{\rho_g c_s} ,\]  

(B3)

where \(\rho_{mat}\) is the material mass density, \(s\) is the dust radius, and \(c_s\) is the mean thermal velocity.

Solving equation (B2) with equations (B1) (in the case of \(|z| \leq z_s\)) and (B3), we obtain the dust density distribution of the disk interior as

\[\rho_{dm}(r, z) = \rho_{dm}(r, 0) \exp \left\{ -\frac{z^2}{2h^2} - \frac{\dot{z}_{stop}}{\alpha} \left[ \exp \left( \frac{z^2}{2h^2} \right) - 1 \right] \right\} \]  

(B4)

(see eq. [31] of Takeuchi & Lin 2002), where \(\dot{z}_{stop}\) is the stopping time in the midplane and \(\alpha = \nu/(c_g h)\) is the nondimensional turbulent viscosity in the so-called \(\alpha\)-prescription (Shakura & Sunyaev 1973). The dimensionless stopping time in the midplane is given by

\[\dot{z}_{stop,m} = \frac{\pi}{2} \frac{\rho_{mat}}{\Sigma}.\]  

(B5)

The midplane dust density \(\rho_{dm}(r, 0)\) is determined by the vertical integration of equation (B4) to be the surface density of dust \(\Sigma_d = f_d \Sigma\), where \(f_d\) is the dust fraction in the surface density.

Solving equation (B2) with equations (B1) (in the case of \(|z| \geq z_s\)) and (B3), we obtain the dust density distribution in the surface layer:

\[\rho_{ds}(r, z) = \rho_{dm}(r, z_s) \exp \left\{ -\frac{z^2 - z_s^2}{2h^2} - \frac{\dot{z}_{stop,m} h}{\alpha H} \left[ \exp \left( \frac{z^2 - z_s^2}{2h^2} \right) - 1 \right] \right\}, \]  

(B6)

where we set \(\rho_{ds}(r, z_s) = \rho_{dm}(r, z_s)\) and assume that Sc and \(\alpha\) in the surface layer have the same values as in the disk interior.

In the disk interior, dust sedimentation is not so important unless the dust radii are not so large that we can formally set \(\rho_{dm} = \rho_d\). In the surface layer, dust settling reduces the dust density while high surface temperatures \(T_s\) raise it, so that \(\rho_d\) also gives a good estimate. Thus, instead of equations (B4) and (B6), we adopt, in most of the calculations, the simple density distribution given by equation (18).

**APPENDIX C**

**VALIDITY OF THE GRAZING-ANGLE APPROXIMATION**

Here we derive the grazing-angle approximation (eq. [6]) and discuss its validity. Substituting \(\rho_d(r, z)\) in equation (15) with that in equation (18) and assuming \(A_s \ll 1\), we obtain

\[A_s = \tau_s \text{erfc} \left( \frac{z_s}{\sqrt{2h}} \right) = \tau_s \text{erfc} \left( \frac{\chi}{\sqrt{2}} \right),\]  

(C1)

where \(\tau_s = \tilde{\tau}_s(T_s) \Sigma_d/2\) and \(\chi = z_s/h\).

We derive the expression for \(A_s\) with a path integration from the star to the point \((r, z)\). Using equations (10) and (18), equation (9) can be written as

\[\tau_s(T_s; r, z_s(r)) = \tau_s(1 + \zeta_s^2)^{1/2} \sqrt{\frac{2}{\pi}} \int_{R_s}^{\tilde{r}} \frac{\tilde{\Sigma}_d'}{h(r')} e^{-\chi'^2/2} dr' = 1\]  

(C2)

with \(\zeta_s = z_s/r\), \(\tilde{\Sigma}_d = \Sigma_d(r')\Sigma_d(r)\), and \(\chi' = \zeta_s r'/h(r')\). Here we assume that \(\tilde{\tau}_s(T_s)\) is independent of position on the path. Changing the variable of integration from \(r'\) to \(\chi'\) and noting that \(\chi' \gg 1\) for \(r' = R_s\), we obtain

\[\tau_s(1 + \zeta_s^2)^{1/2} \zeta_s^{-1} \sqrt{\frac{2}{\pi}} \int_{\chi}^{\infty} \frac{d \ln \zeta_{sh}}{d \ln r'} (\frac{d}{d \ln r'})^{-1} e^{-\chi'^2/2} d\chi' = 1,\]  

(C3)
where we define $\zeta_h = h/r$ and $\zeta_h' = h(r')/r'$. Here we assume that $\chi'$ increases monotonically with $r'$. From equations (C1) and (C3), we obtain

$$A_s = (1 + \zeta_h^2)^{-1/2} \zeta_h \left\langle \int d\chi' \left( \frac{d \ln \zeta_h'}{d \ln r'} \right)^{-1} \right\rangle_{\chi' , \chi}$$

(C4)

with

$$g(\chi') = \left[ \text{erfc} \left( \frac{\chi}{\sqrt{2}} \right) \right]^{-1} \sqrt{\frac{2}{\pi}} \exp \left( -\chi'^2 / 2 \right),$$

(C5)

where $\langle X(\chi') \rangle = \int_{-\infty}^{\infty} X(\chi') f(\chi') d\chi'$ represents the weighted average, with a weight function $f(\chi')$.

If we replace the Gaussian weight $g(\chi')$ with a delta function $\delta(\chi' - \chi)$ and assume $\zeta_s \ll 1$, we obtain

$$A_s = \zeta_s \left( \frac{d \ln \zeta_h}{d \ln r} \right).$$

(C6)

Except for a small difference, this equation corresponds to equation (6). However, if the gradient $d \ln \zeta_h / d \ln r$ changes rapidly in the $r$-direction, the approximation used to derive equation (C6) is no longer justified. In this case $A_s$ is determined not only by the local gradient of $\zeta_h$ but by the gradients in the inner regions, because the weight function $g(\chi')$ extends to the inner radii. For this reason, we use equations (9) and (15) instead of equation (C6).

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