Relative baryon-dark matter velocities in cosmological zoom simulations

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ABSTRACT

Supersonic relative motion between baryons and dark matter due to the decoupling of baryons from the primordial plasma after recombination affects the growth of the first small-scale structures. Large box sizes (greater than a few hundred Mpc) are required to sample the full range of scales pertinent to the relative velocity, while the effect of the relative velocity is strongest on small scales (less than a few hundred kpc). This separation of scales naturally lends itself to the use of ‘zoom’ simulations, and here we present our methodology to self-consistently incorporate the relative velocity in zoom simulations, including its cumulative effect from recombination through to the start time of the simulation. We apply our methodology to a large-scale cosmological zoom simulation, finding that the inclusion of relative velocities suppresses the halo baryon fraction by 46–23 per cent between \( z = 13.6 \) and 11.2, in qualitative agreement with previous works. In addition, we find that including the relative velocity delays the formation of star particles by \( \sim 20 \) Myr on average (of the order of the lifetime of a \( \sim 9 \) M\textsubscript{☉} Population III star) and suppresses the final stellar mass by as much as 79 per cent at \( z = 11.2 \).

Key words: galaxies: high-redshift – dark ages, reionization, first stars – cosmology: theory

1 INTRODUCTION

The cosmic microwave background (CMB) radiation carries an image of the Universe at the moment of recombination, when the first neutral atoms formed at \( z_{\text{rec}} \approx 1089 \). Prior to recombination, photons and baryons were tightly coupled and oscillated as a single plasma until they decoupled at \( z_{\text{dec}} \approx 1020 \). These oscillations, referred to as the baryon acoustic oscillations (BAO), are observed today as fluctuations of the CMB temperature (e.g. Planck Collaboration et al. 2020). Acoustic oscillations in the baryons’ velocity at the moment of their decoupling lead to clumping in the distribution of baryons at later times, resulting in over and underdense regions. The initially tiny perturbations grew under the effect of gravity and are detected today in the distribution of galaxies on the largest cosmological scales (e.g. Alam et al. 2017).

Not only did the plasma oscillations lead to the BAO features in the post-recombination distribution of baryons, they also impacted the baryons’ velocities (Sunyaev & Zeldovich 1970). Tseliakhovich & Hirata (2010) were the first to point out that the BAO pattern is imprinted in the magnitude of the relative velocity between baryons and dark matter, because the latter was not coupled to the primordial plasma at the time of recombination. At decoupling, the relative velocity had a root-mean-square (RMS) of \( \langle v_{bc}^2 \rangle^{1/2} \approx 30 \text{ km} \text{ s}^{-1} \), or \( \sim 10^{-4} c \) with \( c \) the speed of light. There is a vast separation of scales relevant to the relative velocity. Over scales smaller than a few Mpc (the coherence scale), the relative velocity is roughly constant; however, box sizes greater than a few hundred Mpc are required to properly sample the relative velocity (see Fig. 1 of this work and also fig. 1 in Tseliakhovich & Hirata 2010). At recombination the sound speed of the baryonic fluid dropped from being relativistic, \( \sim c/\sqrt{3} \), to the thermal velocities of hydrogen atoms, \( \sim 2 \times 10^{-3} c \), which is much smaller than the RMS value of \( v_{bc} \). Therefore, on average, at decoupling baryons were travelling with supersonic velocities relative to the underlying potential wells generated by dark matter haloes (Tseliakhovich & Hirata 2010). The relative velocity remained supersonic all the way down to \( z \approx 15 \), sourcing shocks and entropy generation (Tseliakhovich & Hirata 2010; O’Leary & McQuinn 2012). The amplitude of the velocity field decayed with time as \( (1+z) \), and thus the effect weakened as the Universe expanded. For instance, the signature of \( v_{bc} \) in the low-\( z \) power spectrum of BOSS galaxies (Yoo & Seljak 2013; Beutler et al. 2017) and the three-point correlation function of BOSS CMASS galaxies was found to be negligible (Slepian & Eisenstein 2015; Slepian et al. 2018).

In the post-recombination Universe, growth of structure on large cosmological scales is generally described by linear perturbation theory, which follows the evolution of density and velocity fields to the leading order in perturbations. Despite being formally second-order contributions, terms that involve the supersonic relative velocity can actually be as large as the first order terms. Moreover, on scales below the coherence scale, \( v_{bc} \) is position-independent and the second-order terms become effectively linear (Tseliakhovich & Hirata 2010). Using such a ‘quasi-linear’ approach, analytical methods were employed to explore implications of \( v_{bc} \) in the cosmological context.

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Supersonic relative velocities modulate the abundance of minihaloes and their gas content on the BAO scale (e.g. Tseliakhovich & Hirata 2010; Tseliakhovich et al. 2011; Fialkov et al. 2012; Ahn 2016; Ahn & Smith 2018), affecting fluctuations of the 21-cm signal of neutral hydrogen (e.g. Dalal et al. 2010; McQuinn & O’Leary 2012; Visbal et al. 2012; Fialkov et al. 2013; Cohen et al. 2016; Fialkov et al. 2018; Muñoz 2019; Cain et al. 2020; Muñoz et al. 2022; Long et al. 2022).

Numerical simulations were used to explore non-linear effects of $v_{bc}$ on scales well below its coherence scale. Such simulations typically employ boxes of several comoving Mpc or less and assume a position-independent velocity field. These simulations demonstrated that $v_{bc}$ suppresses formation of small dark matter haloes (Naoz et al. 2012, 2013; O’Leary & McQuinn 2012), induces shocks (O’Leary & McQuinn 2012), affects the formation of first stars (e.g. Maio et al. 2011; Stacy et al. 2011; Greif et al. 2011; Schauer et al. 2019, 2021) and black holes (e.g. Hirano et al. 2017; Schauer et al. 2017), and may even influence shaping globular clusters (Naoz & Narayan 2014; Chiu et al. 2018, 2019, 2021; Druschke et al. 2020; Lake et al. 2021).

Finally, a hybrid approach was used to incorporate the non-linear effects into the large-scale cosmological picture by tiling regions of fixed $v_{bc}$ together (e.g. Visbal et al. 2012; Fialkov et al. 2013). In such studies, the distribution of $v_{bc}$ on scales larger than the ’pixel’ size was generated from the corresponding density field using the continuity equation, while star formation in each ‘pixel’ was suppressed (Tseliakhovich et al. 2011; Fialkov et al. 2012; Ahn 2016; Ahn & Smith 2018), and black holes (e.g. Hirano et al. 2017; Schauer et al. 2017), and may even influence shaping globular clusters (Naoz & Narayan 2014; Chiu et al. 2018, 2019, 2021; Druschke et al. 2020; Lake et al. 2021).

In the non-linear regime, the evolution of the density and velocity of baryons and dark matter is governed by:

\[
\frac{\partial \delta_c}{\partial t} + \frac{1}{a} \vec{V} \cdot \nabla \delta_c = - \frac{1}{a} (1 + \delta_c) \nabla \cdot \vec{V},
\]

\[
\frac{\partial \delta_b}{\partial t} + \frac{1}{a} \vec{V} \cdot \nabla \delta_b = - \frac{1}{a} (1 + \delta_b) \nabla \cdot \vec{V},
\]

\[
\frac{\partial \vec{V}_b}{\partial t} + \frac{1}{a} \vec{V} \cdot \nabla \vec{V}_b = - \frac{\nabla \Phi}{a} - H \vec{V}_b - \frac{\nabla p}{a \rho_b (1 + \delta_b)},
\]

where $\delta_b$ and $\delta_c$ are dimensionless perturbations in baryonic and dark matter densities respectively, $\vec{V}_b$ and $\vec{V}_c$ are the velocities of baryonic and dark matter respectively, $a$ is the scale factor, $H \equiv \dot{a}/a$ is the Hubble parameter, $\Phi$ is the gravitational potential, $p$ is the baryonic pressure, and $\rho_b$ and $\rho_m$ are the average densities of baryons and total matter, respectively.

Following Tseliakhovich & Hirata (2010), we split the velocities into a coherent bulk motion, $v_{bc}$ (of magnitude $v_{bc}$), and random velocity perturbations, $u_b$ and $u_c$, so that in the cold dark matter frame, the velocities can be written as $v_b = v_{bc} + u_b$ and $v_c = u_c$.

Though they are second-order terms, components involving $v_{bc}$ are large for typical values of $v_{bc}$ at high redshifts. In addition,
these terms become effectively first order on scales where $v_{bc}$ is coherent. In this quasi-linear regime, perturbations in density ($\delta_b$ and $\delta_c$) and velocities ($u_b$ and $u_c$) evolve according to the following set of equations:

$$\frac{\partial \delta_c}{\partial t} = -\theta_c,$$

$$\frac{\partial \delta_b}{\partial t} = -i a v_{bc} \cdot k \delta_b - \theta_b,$$

$$\frac{\partial \theta_b}{\partial t} = -\frac{3H^2}{2}(2\delta_c + \Omega_b \delta_b) - 2H \theta_b,$$

$$\frac{\partial \theta_b}{\partial t} = -\frac{i}{a} v_{bc} \cdot k \delta_b - \frac{3H^2}{2}(2\delta_c + \Omega_b \delta_b) - 2H \theta_b$$

$$+ \frac{k_B T}{\mu m_p a^2} (\delta_b + \delta_T),$$

where $\theta_i = a^{-1} \nabla \cdot u_i$ is the velocity divergence in comoving coordinates (though contrary to Tseliakhovich & Hirata 2010, we work in the rest frame of the dark matter; see also the appendix of O’Leary & McQuinn 2012), $\delta_T$ is the fluctuation in the baryons’ temperature, $k_B$ is the Boltzmann constant, $\mu$ is the mean molecular weight, and $m_p$ is the mass of the proton. Both the baryon and dark matter density parameters $\Omega_b$ and $\Omega_c$ are functions of $t$ (we drop the explicit dependence for clarity).

The importance of baryonic pressure for the growth of density modes was stressed by Naoz & Barkana (2005, 2007), which requires solving an extra equation to track fluctuations in the temperature $\delta_T$. We follow Bovy & Dvorkin (2013) and Ahn (2016) in neglecting tracking fluctuations in photon density and temperature within the evolution equations, since they are subdominant at most of our scales and redshifts of interest. We then add the equation for the temperature fluctuations

$$\frac{\partial \delta_T}{\partial t} = \frac{2}{3} \frac{\partial \delta_b}{\partial t} - \frac{x_e(t)}{a \Gamma_y} T_y \delta_T$$

to equations (2), where

$$T_y^{-1} = 8 \frac{\bar{\sigma}_T \rho_0 c_T}{m_e} = 8.55 \times 10^{-13} \text{ yr}^{-1}$$

and $T_y = 2.726 \text{ K}/a$ is the mean photon temperature, $x_e(t)$ is the electron fraction out of the total number density, of gas particles at time $t$, $\bar{T}$ is the mean gas temperature, $\bar{\rho}_y,0$ is the mean photon energy density at $z = 0$, $\sigma_T$ is the Thomson scattering cross-section for an electron and $m_e$ is the mass of an electron. Both $x_e(t)$ and $\bar{T}$ are calculated using RECFAST++ (Seager et al. 1999; Chluba et al. 2010; Chluba & Thomas 2011). The initial conditions for $\delta_T$ are set as in Naoz & Barkana (2005) by requiring that $\delta(\delta_T - \delta_T^i)/\partial t = 0$ at the initial redshift $z = 1000$, where $\delta_T^i/\partial t$ is calculated from CAMB (Lewis et al. 2000). The above set of linearised equations can be solved using a publicly available code, e.g. cicsass (O’Leary & McQuinn 2012). We reimplement a python version of cicsass, which complements our methodology. In the current implementation, for simplicity, we ignore the directionality of $v_{bc}$ when solving this set of equations, instead taking $v_{bc} \cdot k = v_{bc} k \cos \theta = v_{bc} k$ (i.e. assuming $v_{bc}$ is parallel to $k$). Future implementations will take the directionality of $v_{bc}$ into account.

Tseliakhovich & Hirata (2010) showed that most of the contributions to the variance of $v_{bc}$ come from scales between $0.005 h \text{ Mpc}^{-1}$ and $0.5 h \text{ Mpc}^{-1}$. In a similar fashion to Pontzen et al. (2020), we can compute the RMS $v_{bc}$ inside a box of size $L$ by integrating the power spectrum of $v_{bc}$ fluctuations from the fundamental mode of the box $k_{\text{min}} = 2\pi/L$ to infinity. The mean square $v_{bc}$ in a box of size $L$ is given by

$$\langle v_{bc}^2 \rangle_L = \int_{2\pi/L}^{k_{\text{max}}} \frac{\Delta^2 v_{bc}}{k} dk,$$

where $\Delta^2 v_{bc}$ is the dimensionless power spectrum of the $v_{bc}$, taken from CAMB and, in theory, the upper limit $k_{\text{max}}$ of the integral in equation (5) should be the maximum wavenumber of the box, dictated by the number of simulation elements. In practice, however, any upper limit $k_{\text{max}} > 0.5 h \text{ Mpc}^{-1}$ is sufficient, since the $v_{bc}$ power spectrum drops off rapidly above this value. Fig. 1 shows the RMS $v_{bc}$ calculated as the square root of equation (5), where the oscillatory nature of $\Delta^2 v_{bc}$ at low-$k$ (cf. fig. 1 in Tseliakhovich & Hirata 2010) is clearly visible. From Fig. 1, we can see that even in a box size of $100 h^{-1} \text{ Mpc}$, we do not capture all of the scales relevant to $v_{bc}$. The curve only begins to plateau around $\sim 400 h^{-1} \text{ Mpc}$, so using a box size smaller than this means that we may miss out on some of the effect, for example by not sampling extreme values of $v_{bc}$. Simultaneously simulating this large-scale box and the very high-resolution zoom region needed to observe the effect would be computationally infeasible. In Section 3.2, we discuss our solution to this problem.

3 METHODS

3.1 Simulations

We follow the evolution of dark matter, gas, and stars in the cosmological context using RAMSES, which employs a second-order Godunov method to solve the equations of hydrodynamics. Gas states are

2 The version used here is commit aa56bc01 from the master branch. Note that older versions of RAMSES may not use separate fields for dark matter and baryon velocities by default.

Figure 1. RMS $v_{bc}$ at $z = 200$ as a function of box size, calculated by integrating the $v_{bc}$ power spectrum $\Delta^2 v_{bc}$ (computed for the cosmological parameters listed in Section 1) from $2\pi/L$ to $3365 h^{-1} \text{ Mpc}$. The RMS $v_{bc}$ converges for $L > 400 h^{-1} \text{ Mpc}$, since $\Delta^2 v_{bc}$ drops to zero at small-$k$, and so box sizes of this or larger are needed to capture all the relevant scales.
computed at cell interfaces using the Harten-Lax-van Leer-contact Riemann solver, with a MinMod slope limiter. Dark matter and stars are modelled as a collisionless N-body system, described by the Vlasov-Poisson equations. Grid refinement is performed whenever a cell contains more than eight high-resolution dark matter particles, or has the equivalent amount of baryonic mass scaled by $\Omega_b/\Omega_m$. We allow the AMR grid to refine from the coarsest level $\ell_{\text{min}} = 8$ to the finest level $\ell_{\text{max}} = 23$, but in practice grid hold-back within RAMSES means that the finest level reached is $\ell = 21$, corresponding to a maximum comoving resolution of $47.7 \, h^{-1}$ pc$^3$.

Star formation is allowed whenever the gas density of a cell is greater than $n_e = 1 \, \text{cm}^{-3}$ in units of the number density of hydrogen atoms and when the local overdensity is greater than $200\rho_c$, where the latter condition prevents spurious star formation at extremely high redshift. We impose a polytropic temperature function with index $g = 2$ and $T_0 = 1050$ K, which ensures that the Jeans length is always resolved by at least eight cells. We do not rigorously calibrate the star formation parameters to reproduce any stellar mass-halo mass relation, since we are interested only in the differences between simulations. Star particles, which represent a population of stars, form using periodic boundary conditions with non-periodic ICs, though these errors will be common between runs, so their effect is the default behaviour for older versions of RAMSES, where the dark matter velocity is used to initialise both the dark matter and baryon velocity fields, such as by generating ICs using CAMB. In this case, $v_{bc}$ is included from the start time of the simulation, but the effect of $v_{bc}$ on density and velocity perturbations between recombination and $z_{\text{ini}}$ is missed. In the final, and most realistic, case, $v_{bc}$ is included in the zoom region in the centre of the box. Additionally, these errors will be common between runs, so their effect will wash out when comparing between runs.

Table 1 details the sets of ICs used in this work. We selected a region for zoom-in with $v_{bc,\text{ini}} = 20.09 \, \text{km s}^{-1}$ at $z = 200$, corresponding to $v_{bc,\text{rec}} = 100.07 \, \text{km s}^{-1}$, or $\sim 3.3\sigma_{v_{bc}}$, at recombination. The no $v_{bc}$ case is used in cosmological simulations, for example when using transfer functions that do not have separate amplitudes for the baryon and dark matter velocity fields (in fact, it is the default behaviour for older versions of RAMSES, where the dark matter velocity field is used to initialise both the dark matter and baryon velocities). The $v_{bc,\text{ini}}$ case is where the simulation is initialised using separate transfer functions for the baryon and dark matter velocity fields, such as by generating ICs using MUSIC with transfer functions from CAMB. In this case, $v_{bc}$ is included from the start time of the simulation $z_{\text{ini}}$, but the effect of $v_{bc}$ on density and velocity perturbations between recombination and $z_{\text{ini}}$ is missed. In the final, and most realistic, case, $v_{bc}$ is included across all $z$ by computing a bias factor which is applied to the ICs. The methodology for computing the bias factor is detailed in Section 3.3.

### Table 1. The sets of ICs used for the main zoom simulation

| Case        | $v_h$ | Modified? | $v_{bc,\text{rec}}$ | $v_{bc,\text{ini}}$ (km s$^{-1}$) |
|-------------|-------|-----------|---------------------|-----------------------------------|
| no $v_{bc}$ | $v_c$ | no        | 0.0                 | 0.0                               |
| $v_{bc,\text{ini}}$ | $v_h$ | no        | 0.0                 | 20.09                             |
| $v_{bc,\text{rec}}$ | $v_h$ | yes       | 100.07              | 20.09                             |

3.2 Initial conditions

As described in Section 2, large box sizes of $\geq 400 \, h^{-1}$ Mpc are required in order to capture all of the scales pertaining to $v_{bc}$. By performing calibration runs, we found that very high resolution (a cell size of $\Delta x \lesssim 2 \, h^{-1}$ kpc) is needed in the ICs in order to properly resolve the effect. To this end, we employ ‘zoom’ initial conditions (ICs), generating density and velocity fields at $z_{\text{ini}} = 200$ first in a $400 \, h^{-1}$ Mpc box using MUSIC (Hahn & Abel 2011). The ICs are refined from the base level $\ell_{\text{min}} = 10$ ($1.024^3$) up to $\ell = 18$ ($262,144^3$ effective) in a cube of side length 543 $h^{-1}$ kpc at the finest level. Such extremely high resolution is required because $v_{bc}$ suppresses structure formation on very small scales. We found that the resolution we used in this work is the minimum necessary to observe the effect of $v_{bc}$, and using lower resolution largely misses the effect. Since the zoom region is very small compared to the box size, we use extra padding between zoom levels, increasing the number of padding cells on each side for each dimension from the typical value of 4 to 32. We use transfer functions from CAMB (Lewis et al. 2000), which gives distinct density and velocity fields for the baryons and dark matter.

In order to make the simulation tractable, we extract a $100 \, h^{-1}$ Mpc ($\ell_{\text{min}} = 8, 256^3$) base grid from the 400 $h^{-1}$ Mpc ($\ell_{\text{min}} = 10, 1,024^3$) box and use this as our coarsest level, meaning that the maximum refinement level in the zoom region also drops two levels from $\ell = 18$ to $\ell = 16$ (65,536$^3$ effective). In principle, this methodological choice could introduce some error around the edges of the box due to using periodic boundary conditions with non-periodic ICs, though in practice we expect the impact of this to be negligible since we are concerned with a sub-$h^{-1}$ Mpc region in the centre of the box. Additionally, these errors will be common between runs, so their effect will wash out when comparing between runs.

Table 1 details the sets of ICs used in this work. We selected a region for zoom-in with $v_{bc,\text{ini}} = 20.09 \, \text{km s}^{-1}$ at $z = 200$, corresponding to $v_{bc,\text{rec}} = 100.07 \, \text{km s}^{-1}$, or $\sim 3.3\sigma_{v_{bc}}$, at recombination. The no $v_{bc}$ case is often used in cosmological simulations, for example when using transfer functions that do not have separate amplitudes for the baryon and dark matter velocity fields (in fact, it is the default behaviour for older versions of RAMSES, where the dark matter velocity field is used to initialise both the dark matter and baryon velocities). The $v_{bc,\text{ini}}$ case is where the simulation is initialised using separate transfer functions for the baryon and dark matter velocity fields, such as by generating ICs using MUSIC with transfer functions from CAMB. In this case, $v_{bc}$ is included from the start time of the simulation $z_{\text{ini}}$, but the effect of $v_{bc}$ on density and velocity perturbations between recombination and $z_{\text{ini}}$ is missed. In the final, and most realistic, case, $v_{bc}$ is included across all $z$ by computing a bias factor which is applied to the ICs. The methodology for computing the bias factor is detailed in Section 3.3.

3.3 Bias factor

Using transfer functions that have distinct amplitudes for the baryon and dark matter velocity fluctuations naturally yields the $v_{bc}$ field at the start time of the simulation $z_{\text{ini}}$. First, we interpolate the dark matter particle velocities onto the same grid as the baryons, then take the difference of these two fields to calculate the magnitude as $v_{bc} = |v_h - v_c|$. A 0.39 $h^{-1}$ Mpc thick slice through the resultant $v_{bc}$ field is shown in Fig. 2.

With the $v_{bc}$ field in hand, we split our ICs into cubic patches, aiming for a patch extent of 0.5 $h^{-1}$ Mpc, though the actual extent depends upon how many patches can be fit in each level of the GRAFIC files. The size of these patches is chosen to be smaller than the scale over which $v_{bc}$ is coherent (Tseliakhovich & Hirata 2010). Within each patch, the average value of $v_{bc}$ is calculated and used as $v_{bc}$ in equations (2). The initial values for equations (2) are set using the transfer functions from CAMB at $z = 1000$, and the equations are integrated from $z = 1000$ to $z = 200$ using the LSODA ordinary differential equation solver. Equations (2) are solved for the average patch value of $v_{bc}$ and also for $v_{bc} = 0 \, \text{km s}^{-1}$, which yields power...
spectra for the baryon perturbations both with and without $v_{bc}$. We use these power spectra to calculate a ‘bias’ factor at $z_{ini} = 200$ that depends both upon scale $k$ and the magnitude of the relative velocity $v_{bc}$

$$b(k, v_{bc}) = \left[ \frac{P(k, v_{bc})}{P(k, v_{bc} = 0)} \right]^{1/2},$$

where the square root arises from $P \propto \langle \delta^2 \rangle$. In Fig. 3, we show the bias factor for the baryon and dark matter densities ($\delta_b$ and $\delta_c$) and velocities ($v_b$ and $v_c$), computed for the average $v_{bc}$ in our zoom region. The strongest suppression is seen in the baryons and in particular the baryon density, while the dark matter is hardly affected. We do not expect the oscillatory features in $b(k, v_{bc})$ at the very small scales to have much, if any, impact since the power spectrum of fluctuations in the baryon density contrast begins to fall rapidly for $k \gtrsim 300$ h Mpc$^{-1}$, while for the velocity most of the power is at much larger scales. Ali-Haimoud et al. (2014) also found oscillatory features in the small-scale baryon perturbations, and we have checked that we find similar oscillations for typical values of $v_{bc}$ and also find that increasing the magnitude of $v_{bc}$ increases the frequency of oscillations for the larger-scale ($\gtrsim 100$ h Mpc$^{-1}$) modes too. For a detailed study into the origin of these small-scale oscillations, we defer the reader to Ali-Haimoud et al. (2014). This factor is then convolved with the Fourier transform of the corresponding patch of baryon overdensity

$$\delta_b(k, v_{bc}) = b(k, v_{bc}) \cdot \delta_b(k)$$

(7)

to give individual patches of biased overdensity $\delta_b$, which are then stitched together to generate the $v_{bc}$–rec set of ICs. In this way, the bias factor compensates for the suppression of baryon perturbations between $z = 1000$ and $z_{ini}$ that is missing if $v_{bc}$ is included only from $z_{ini}$. We only modify the baryons, since as discussed earlier, they are much more strongly affected than the dark matter, as can be seen from Fig. 3.

We deal with the peculiar velocity field for the baryons in a similar way, by first converting the velocity divergence to peculiar velocities as $v_{p}(k) = -iak \theta_b(k)/k^2$. Note again that we do not include directionality when solving the evolution equations, and therefore, the bias factor is applied to each direction of $v_b$ equally. In reality, there would be preferential directions for the bias factor, depending on the direction of $v_{bc}$, but we defer that implementation to future work.

Fig. 4 shows a 1.53 h$^{-1}$ kpc thick slice through the highest resolution level of the zoom ICs directly from MUSIC (‘unmodified’, left column) and after the bias factor $b(k, v_{bc})$ has been applied (‘modified’, right column). For $\delta_b$ (top row), the unmodified ICs contain a lot of small-scale structure, which is almost totally washed out after applying $b(k, v_{bc})$. Most of what remains is in the form of lower amplitude, larger scale fluctuations. For $v_{bc,i}$ (bottom rows) there is less small-scale structure to begin with, since the peculiar velocity fields are dominated by large scales. The effect of $b(k, v_{bc})$ on $v_{bc,i}$ is therefore much less striking than on the $\delta_b$, with the main effect being smoothing and a slight reduction in amplitude.

### 3.4 Haloes

After the ICs have been correctly initialised with $v_{bc}$, we can characterise the effect of $v_{bc}$ on structure formation, principally by exploring

Note that we show the $v_b$ for each direction for completeness, but the effect is independent of direction in our methodology.
Figure 4. Slices of the unmodified (left column) and modified (through convolution with the bias factor, as in equation (7), right column) baryon overdensity (top row), peculiar velocity in the $x$-direction (second row), $y$-direction (third row) and $z$-direction (bottom row) in the high-resolution zoom region, of side length $543 \, h^{-1} \, \text{kpc}$. Each pixel corresponds to a cell width of $1.53 \, h^{-1} \, \text{kpc}$ and the slice has a thickness of one cell width. The effect of applying $b(k, v_{bc})$, defined in equation (6), can be clearly seen in the baryon overdensity, in that it washes out the small-scale fluctuations. The effect is less pronounced in the peculiar velocities, which are dominated by large-scale modes.
4 RESULTS

4.1 Halo abundances

Throughout this section, we calculate the cumulative number of haloes \(N(> M)\) as the number of haloes with a total mass greater than \(M\) and estimate the Poisson uncertainty on \(N(> M)\) for each case as \(\sqrt{N(> M)}\). To begin with, we ignore the conditions described in Section 3.4, instead considering the cumulative number of all haloes formed in the simulation \(N_{\text{all}}(> M)\), shown in Fig. 5. Aside from a slight suppression in \(N_{\text{all}}(> M)\) for the \(v_{\text{bc}}-\text{rec}\) case below \(10^8 \ h^{-1} M_\odot\) at \(z = 14.2\), the \(v_{\text{bc}}-\text{ini}\) and \(v_{\text{bc}}-\text{rec}\) cases are consistent with each other and with the no \(v_{\text{bc}}\) case at the 1\(\sigma\) level. Analysing the haloes in this fashion (i.e. by performing no cleaning on the catalogue) gives a picture of the global impact of \(v_{\text{bc}}\) on halo abundance, however this picture is inaccurate because a significant fraction of the haloes are contaminated by, or formed entirely of, lower-resolution particles, affecting the accuracy of their properties (see Section 3.4 for a discussion of contamination).

If we now apply the conditions described in Section 3.4, namely that we do not include in our analysis any haloes that are contaminated at any point in the simulation, we are left with a reduced catalogue. In Fig. 6, we show the cumulative number of haloes in this cleaned catalogue \(N_{\text{clean}}(> M)\), which shows a much more striking suppression of \(N_{\text{clean}}(> M)\) for the \(v_{\text{bc}}-\text{rec}\) case and an apparent increase in the abundance of low-mass \((M < 10^8 \ h^{-1} M_\odot)\) haloes for the \(v_{\text{bc}}-\text{ini}\) case at \(z = 11.2\) (though still consistent with no difference to the no \(v_{\text{bc}}\) case at the 1\(\sigma\) level).

To highlight the effect of the cleaning procedure we also show the raw cumulative number of haloes at \(z = 14.2\) and \(z = 11.2\) in Fig. 7. The impact of the cleaning process described in Section 3.4 can be clearly seen in the bottom panel of Fig. 7, which shows the ratio of the cleaned catalogue to the full catalogue \(N_{\text{clean}}(> M)/N_{\text{all}}(> M)\). Comparing the different runs, it can be clearly seen that a different fraction of haloes is removed between each run at each of the times shown.

Figure 5. Ratio of the cumulative number of haloes \(N_{\text{all}}(> M)\) in the \(v_{\text{bc}}-\text{ini}\) (pink short-dashed) and the \(v_{\text{bc}}-\text{rec}\) (blue dotted) cases to the no \(v_{\text{bc}}\) case at \(z = 14.2\) (top), 12.6 (middle), and 11.2 (bottom). All the haloes found by \(\text{AHF}\) are included in \(N_{\text{all}}(> M)\). The shaded regions indicate the 1\(\sigma\) Poisson uncertainty on the ratio. \(N_{\text{all}}(> M)\) is broadly the same between all three sets of simulations, except for a suppression in the number of haloes with \(M < 10^6 \ h^{-1} M_\odot\) at \(z = 14.2\) in the \(v_{\text{bc}}-\text{rec}\) case compared to the no \(v_{\text{bc}}\) case.
Figure 6. As in Fig. 5, but this time $N_{\text{clean}}(> M)$ includes only haloes that have been selected by the process detailed in Section 3.4. Selecting never-contaminated haloes in this fashion leads to the removal of an unequal and inconsistent number of haloes between each run—this leads to the odd behaviour of the $v_{bc}$–ini curve, which jumps from a suppression to a boost in the number of haloes with $M < 2 \times 10^6 \, h^{-1} M_\odot$ between $z = 12.6$ and 11.2.

4.2 Baryon fraction

We allow star formation in these runs, so the total baryon fraction of a given halo is defined as
\[
    f_b = \frac{M_g + M_*}{M_g + M_e + M_*},
\]
where $M_g$ is the gas, $M_*$ the stellar, and $M_e$ the dark matter mass in each halo. We upweight the best resolved (i.e. most massive) haloes by calculating the mass-weighted average baryon fraction as
\[
    \langle f_b \rangle_M = \frac{\sum_i f_{b,i} M_i}{\sum_i M_i},
\]
and the associated mass-weighted standard deviation as
\[
    \sigma_M = \left( \frac{\sum_i f_{b,i}^2 M_i}{\sum_i M_i} - \langle f_b \rangle_M^2 \right)^{\frac{1}{2}},
\]
where the sum is over all haloes that satisfy the conditions in Section 3.4. Fig. 8 shows $\langle f_b \rangle_M$ and associated 1σ errorbars as function of $z$. We show each $z$ where $\geq 30$ haloes have formed that satisfy the criteria in Section 3.4, starting from $z = 13.6$ where we are able to match 38 haloes between the three cases. The gas fraction is suppressed at all $z$ for the $v_{bc}$–ini and $v_{bc}$–rec cases compared to the no $v_{bc}$ case. At earlier $z$, the suppression is stronger, though even by the final snapshot at $z = 11.2$, $\langle f_b \rangle_M$ for both the $v_{bc}$–rec and $v_{bc}$–ini cases are not within 1σ of the no $v_{bc}$ case. Notably, at all $z$, $\langle f_b \rangle_M$ in $v_{bc}$–ini and $v_{bc}$–rec cases are almost indistinguishable from, and certainly consistent with, one another.

4.3 Star formation

In Fig. 9, we show the cumulative $M_*$ formed in the simulation, not accounting for mass loss due to supernovae, and the corresponding number of stellar particles $N_*$, which each have a mass of $108.0 \, h^{-1} M_\odot$. In each case, all of the star particles in the simulation formed inside a single halo. In total, 29 star particles formed by $z = 11.2$ in the no $v_{bc}$ case, 10 in the $v_{bc}$–ini case, and 7 in
5 DISCUSSION

To understand the perceived uptick in \(v_{bc}\)–ini haloes in Fig. 6, we need to explore the raw cumulative number of haloes, shown in Fig. 7 at \(z = 14.2\) and \(z = 11.2\). The key point is that while only keeping haloes that are never contaminated ensures that haloes do not disappear between timesteps of the same run, it also means that different numbers of haloes will be removed between different runs at any given timestep. This discrepancy in the number of haloes removed is responsible for the apparent low-mass boost in the \(v_{bc}\)–ini case, because over the mass range \(3 \times 10^3 < M / h^{-1} M_{\odot} < 10^6\) fewer haloes are removed in the \(v_{bc}\)–ini case than in the no \(v_{bc}\) case. Since the cumulative numbers of haloes are already very similar, removing a smaller number of haloes in the \(v_{bc}\)–ini case manifests itself as a boost relative to the no \(v_{bc}\) case, when in fact it is simply an artefact of the cleaning procedure.

Given the difficulties in extracting a sample of haloes that is both free from contamination and comparable between runs, we will not draw any quantitative conclusions on the impact of \(v_{bc}\) on global properties like the number of haloes formed. We retain this discussion of the well-known impact of contamination and, crucially, the importance of verifying any mitigation techniques as it may prove helpful for other works employing zoom simulations.

Including \(v_{bc}\) also significantly affects the baryon fraction \(f_b\), where we see that the mass-weighted baryon fraction \(\langle f_b \rangle_M\) is suppressed at all redshifts in both cases, with the suppression stronger at higher redshift. Even by \(z = 11.2\), the \(v_{bc}\)–ini and \(v_{bc}\)–rec cases are still not in agreement with the no \(v_{bc}\) case, though the difference between the two populations has decreased. Again, this is likely due to the decay in the magnitude of \(v_{bc}\), which allows the haloes to accrete more gas. Interestingly, \(\langle f_b \rangle_M\) is almost indistinguishable between the \(v_{bc}\)–ini and \(v_{bc}\)–rec cases, suggesting that including \(v_{bc}\) from \(z_{ini}\) is sufficient to observe its impact on halo baryon fraction, though a larger sample is required to confirm this. The suppression in \(\langle f_b \rangle_M\) when \(v_{bc}\) is included is in qualitative agreement with previous studies.

This decrement in baryon fraction for the \(v_{bc}\)–ini and \(v_{bc}\)–rec cases is reflected in the cumulative stellar mass formed, as fewer star particles formed in both cases than in the no \(v_{bc}\) case. Not only do they form fewer star particles, they also start forming star particles later since the effect of \(v_{bc}\) is to wash out the peaks (and troughs) in the baryon density contrast, meaning that it takes longer for gas to reach the densities required for star formation. The mean delay for forming...
star particles is 19.4 Myr for the \(v_{bc\text{-ini}}\) case and 34.9 Myr for the \(v_{bc\text{-rec}}\) case. From Schauer (2002), we find that these delays are all of the order of the lifetime of a 9 M\(_\odot\) first-generation Population (Pop) III star, which has a lifetime of 20.02 Myr (table 3 in Schauer 2002). More massive Pop III stars have even shorter lifetimes, for example a 120 M\(_\odot\) Pop III star lives for only 2.52 Myr. Pop III stars form from initially pristine gas, and their death pollutes their immediate surroundings with metals, introducing new cooling channels into the high-redshift Universe. Any delay in this introduction of metals will delay the transition between Pop III to Pop II (i.e. from metal-enriched gas), which can, for example, affect the 21 cm signal (Magg et al. 2022). In our case, though we do not form Pop III stars, chemical enrichment is still vitally important for star formation to get properly underway, particularly as all of the star particles form in the same halo.

Despite there being almost no difference in \(\langle f_b\rangle_M\) between the \(v_{bc\text{-ini}}\) and \(v_{bc\text{-rec}}\) cases at most redshifts, there is a clear hierarchy in the amount of stars formed—no \(v_{bc}\) forms the most, \(v_{bc\text{-ini}}\) forms fewer, and \(v_{bc\text{-rec}}\) forms the least—albeit on the order of a few star particles. This effect is expected, since the bias factor washes out baryonic density peaks, and there are slightly more haloes (i.e. star formation locations) present in the \(v_{bc\text{-ini}}\) case than in the \(v_{bc\text{-rec}}\) case.

6 CONCLUSIONS

We have performed the first cosmological zoom simulations that self-consistently sample the relative baryon-dark matter velocity \(v_{bc}\) from a large 400 h\(^{-1}\) Mpc box. This relative velocity naturally arises when simulations are initialised using transfer functions that have separate amplitudes for the baryon and dark matter velocities, and we have shown that a box roughly as large as this is required to properly sample all of the scales associated with the relative velocity. However, solely initialising simulations in this manner misses out on the effect of the relative velocities from \(z = 1000\) to the start time of the simulation, \(z_{ini}\). We developed a methodology that compensates for the effect of \(v_{bc}\) on baryonic density and velocity perturbations by computing a ‘bias’ factor \(b(k, v_{bc})\), which is convolved with the ICs. We verified that our methodology performs as expected by comparing to previous works (see Appendix A).

As a first demonstration of our methodology, we applied it to an extremely high-resolution zoom region in a 100 h\(^{-1}\) Mpc subbox, extracted from the main 400 h\(^{-1}\) Mpc box. The zoom region is centred on the region with the largest relative velocity in the 400 h\(^{-1}\) Mpc box, which has an RMS value of \(v_{bc} = 100.07\) km s\(^{-1}\) at \(z = 1000\), corresponding to \(3.3\sigma_{v_{bc}}\). We find qualitative agreement with previous works, namely a reduction in the halo baryon fraction, a delay in the onset of star formation at high redshift, and a suppression of the final stellar mass. The strength of the effect decreases with redshift, but the two simulations still exhibit some differences by \(z = 11.2\). We find that the delay in the onset of star formation is of the order of the lifetime of a ~ 9 M\(_\odot\) Pop III star. We also test the effect of incorporating the bias factor by running a simulation that includes the relative velocity from the start time of the simulation only. In this case, we find that more stars are formed when compared to the simulation that includes the bias factor, but there is almost no change in the average baryon fraction, except at the earliest redshift. Due to the small size of our zoom region, the vast majority of haloes in our simulation are contaminated with low-resolution particles, and we are thus unable to draw any robust conclusions regarding the halo mass function.

Our code for producing these compensated ICs is publicly available\(^5\), and we hope will be of use for studying this effect in the full cosmological context.

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DATA AVAILABILITY

The code for computing and applying the bias factor \(b(k, v_{bc})\) is available at https://github.com/lconaboy/drft. The data underlying this article will be shared on reasonable request to the corresponding author.

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\(^5\) https://github.com/lconaboy/drft
We run a series of test simulations set up as in Naoz et al. (2012, 2013). The specific case shown here has a base resolution $\ell_{\text{min}} = 9 h^{-1}$ kpc periodic box. The simulations in Naoz et al. (2012, 2013) were performed using the SPH code {	exttt{arepo}} (Springel 2005), while we use the AMR code {	exttt{ramses}}. We allow the AMR grid to refine freely up to $\ell_{\text{max}} = 14$, corresponding to a maximum comoving resolution of $28.9 h^{-1}$ pc, comparable to the gravitational softening length of $45.8 h^{-1}$ pc comoving used in Naoz et al. (2012). We use our fiducial cosmology (Section 3.1), whereas Naoz et al. (2012) used a later study performed using the moving-mesh code {	exttt{arepo}} found better agreement with the analytical gas fraction relation (Popa et al. 2016; Chiou et al. 2018). For the comparison in this section, Naoz et al. (2012, 2013) are sufficient.
Indeed this is borne out by the simulations (cyan dashed), which are working in the period of matter domination so the difference will be small. While the final halo masses in Naoz et al. (2012) are defined using a spherical overdensity method, the initial halo finding is done using a friend-of-friends method, which could be susceptible to ‘overlinking’ (e.g. Davis et al. 1985), whereby disparate groups of particles are spuriously connected by a diffuse particle bridge. Their redefinition of the halo mass using the spherical overdensity method will go some way to alleviating the impact (if any) of overlinking.

To calculate the effect of the \( v_{bc} \), we compare to simulations without \( v_{bc} \), where the velocity field of the baryons is equal to that of the dark matter. In order to quantify this effect, we calculate the fractional difference of a quantity \( A \) as

\[
\Delta_A = \frac{A_{\text{bc}} - A_{\text{no bc}}}{A_{\text{bc}}}. \tag{A2}
\]

First, we look at the effect on the cumulative halo mass function \( N(> M) \), as in Naoz et al. (2012). Fig. A1 shows the decrement in \( N(> M) \) for the case with \( v_{bc} \) compared to the case without, both for our simulations and for the Naoz et al. (2012) run. We see qualitatively similar behaviour, observing a decrement between ~0 per cent and 50 per cent at all redshifts shown and for almost all masses.

However, the overall shape of our \( \Delta_N \) is slightly different to Naoz et al. (2012); we match well below \( 3 \times 10^5 \) \( h^{-1} M_\odot \) but show more relative suppression above this mass. This discrepancy is due, at least in part, to the different simulation codes used and the different white noise fields in the initial conditions. One further significant source of difference is the cosmologies used. Fig. A2 shows the difference expected at \( z = 15 \) by comparing the analytic Watson et al. (2013) \( N(> M) \) mass functions (magenta solid). From this, we would expect the Naoz et al. (2012) simulation to have ~9 per cent more haloes with \( M > 3 \times 10^5 \) \( h^{-1} M_\odot \). This increase in the number of haloes increases with mass and for \( M > 1 \times 10^7 \) \( h^{-1} M_\odot \), we would expect ~11 per cent more haloes in the Naoz et al. (2012) simulation. Indeed this is borne out by the simulations (cyan dashed), which show that the Naoz et al. (2012) simulations do produce more haloes at all masses. At higher masses, \( \Delta_N \) diverges as the absolute number of haloes becomes small. As discussed earlier, Naoz et al. (2012) use the friends-of-friends groups as a starting point, which could be a source of the high-mass discrepancy in Fig. A2.

Next, we turn our attention to the gas fraction of haloes, as studied in Naoz et al. (2013). Since we do not include star formation in these runs, the baryon fraction is simply the halo gas mass divided by the total halo mass

\[
f_b = \frac{M_g}{M_g + M_d}. \tag{A3}
\]

Fig. A3 shows the binned gas fractions (top panel) for our and the Naoz et al. (2013) simulations, each normalised to the cosmic mean \( \Omega_b/\Omega_m \) for the appropriate cosmology, and the decrement (bottom panel) as defined in equation (A2). We take the midpoint of the mass bin to be the mean of all the mass values in that bin. The binned gas fractions for Naoz et al. (2013) are slightly higher than in this work, though they exhibit roughly the same mass dependence.
Figure A2. Comparison of the fractional difference $\Delta N$ in halo mass function $N(> M)$ between our simulations and the simulations from Naoz et al. (2012) (cyan short-dashed) and between the Watson et al. (2013) curve, which is fit to $N$-body simulations, for the cosmology used in our work and the one used in Naoz et al. (2012) (magenta solid). The fractional difference is calculated as $\Delta N = (N_1 - N_0) / N_0$, where $N_0$ are the data corresponding to our work and $N_1$ to Naoz et al. (2012), so $\Delta N > 0$ means there are more haloes in Naoz et al. (2012). Neither run includes $v_{bc}$. The shaded cyan region indicates the combined 1σ Poisson uncertainty on $N(> M)$ from the simulations. For $M \lesssim 4 \times 10^6 \, h^{-1} M_\odot$ the difference in $N(> M)$ between the simulations is mostly consistent with the expected difference due to cosmology and halo mass definitions (i.e. the difference between the Watson et al. 2013 curves). Potential sources of the high-mass ($M > 4 \times 10^6 \, h^{-1} M_\odot$) discrepancy are discussed in the text.

The agreement between the two simulations for the decrement is striking – they have an extremely similar mass dependence. There is some difference in the binned baryon fractions, in particular we find slightly more suppression at lower masses. This is likely due to differences in code used since, as mentioned previously, Naoz et al. (2013) used gadget2 (Springel 2005), where we use ramses. There are well documented differences between Lagrangian (e.g. SPH) and Eulerian (e.g. AMR) codes (e.g. Agertz et al. 2007), and indeed it has been shown that numerical diffusion due to baryon-grid relative velocities can artificially smooth densities in Eulerian codes (Pontzen et al. 2020). In any case, we are not interested in comparing the merits of different codes, so by calculating the difference between the runs with and without $v_{bc}$, we can remove artefacts due to the choice of code.

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Figure A3. Binned baryon fraction $f_b$ (top panels) and relative fractional difference $\Delta f_b$ between each run with $v_{bc}$ to the run without, calculated with equation (A2) (bottom panels). We show data from our simulations (red triangles, points and plusses) and from the Naoz et al. (2013) work (grey squares, stars and crosses). The panels show $z = 25$ (left), 19 (centre) and 15 (right). The errorbars indicate the 1$\sigma$ standard deviation in each mass bin (top panels) and the combined 1$\sigma$ uncertainty (bottom panels). We find excellent agreement in the relative difference between our work and Naoz et al. (2013), and broad agreement in the absolute value of $f_b$. Some slight disagreement in the absolute value could be due to the different choice of simulation methodology, since we employ an Eulerian code where Naoz et al. (2013) use a Lagrangian code.