1. Introduction

Development of modern technology requires designing new structures, the work of which is performed under the action of multiple, asymmetrical force and temperature influences. The largest effect of loading in such structures is felt by welded seams, seats of docking of structural elements, technological holes.

In order to prolong the terms of the work of welded seams with pores and inclusions, a method is needed, by which one may identify the main loads that affect crack opening in a chain of pores and inclusions.

2. Analysis of scientific literature and the problem statement

In the paper [1] an approach is proposed based on the use of a method of finite elements for calculation of thermoelastic performance of a structure under symmetrical load, and related formulation of the problem of thermoelasticity is also considered. The results [1] allow considering only a working plane, which decreases due to voids, and defining the assessment of a planar problem, rather than a multi axle one.

The paper [2] proposes performing calculations of external incisions for the calculation of thermoelastic behavior of a structure under symmetrical load and considers associated statement of the problem of thermoelasticity. Calculation of the outer incision [2] does not determine behavior of a material above and below the pore and inclusion and the result passes into a planar solution of the problem. In the work [3] they carried out calculation of external defects to calculate the performance of a structure under symmetrical load and low temperatures. Calculation of external defects [3] does not allow identifying a spatial behavior of a crack. The paper [4] proposed calculation of uniaxial load of external defects under symmetrical load and low temperatures. The decision of a planar problem in [4] does not allow proper discovering full picture of cracking of pores and inclusions along the length of the seam and the behavior of the crack (possible coalescence, not opening). In the paper [5], calculation of multi axle load of external defects under symmetrical load and low temperatures was carried out. The results [5] do not show dependency of temperature and pressure on the spatial

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influence of the pores in a welded seam. The work [6] carried out calculation of the multi axle load of the pores under symmetrical load and high temperatures. The results of the paper [6] give an assessment of the impact of major stresses in the pores and inclusions, but not of the influence of shear and tangential stresses on the development of cracks in the pores. The papers [7, 8] proposed a solution to associated problems of thermoelasticity under symmetrical load and high temperatures. But in these works the issue that is associated with asymmetric thermal force load was not tackled. The work [9] used an approach based on the application of engineering analysis for the calculation of statically loaded structures. However, from the given results it is not clear enough how a welded seam cracks.

The paper [10] introduced an approach based on the results of the tests and use of engineering analysis for the calculation of statically loaded structure. This method [10] allows carrying out a qualitative analysis, rather than numerical, confirming the efficiency of a weld. In the article we perform numerical solution of asymmetrically loaded by pressure and temperature structure according to different mounting schemes. In the above-mentioned works, the object of research is axle symmetric design solutions, which might be obtained with a 5 percent error by analytical methods. This paper presents a welded seam of two plates of different thickness, different geometry, one of the plates is weakened by a hole, load on the plates is caused by not just stretching and compression of the weld, but by shear with torsion. Not only the assessment of the average temperature of the plates on the defects is considered, but also exact distribution of the temperature gradient by thickness and length of the weld. Obtaining a solution to such a problem by simple analytical dependencies is difficult.

3. The purpose and objectives of the study

The performed studies set the goal of developing a method of determining stresses in micro defects (poles and inclusions) of welded seams of plate structures, subject to a simultaneous action of temperature and force loadings.

To achieve the set goal, the following tasks were solved:
- to define geometric parameters of inclusions, with which the results of calculations must be close to those received by engineering methods;
- to determine an interval of the optimal number of elements for prompt obtaining of correct results;
- to create calculation schemes that define the limits of the use of the finite element method.

4. Method of calculation of stress-strain state of a structure

We analyze the stress in the micro defects of welded seams of a plate structure under conditions of non-stationary temperature and force influences.

Let us consider a plate structure in the form of a duct, which consists of plates, locally pivotally-supported on the bottom plate – mounting scheme 1. Pivotally supported on the edges of the plates – mounting scheme 2. Pinching along the edges of the plates – mounting scheme 3.

Assume that a process of deformation is not adiabatic or isothermal, then there is an increase in the temperature \( \Delta T = T - T_0 \), where \( T = T(\mathbf{x}, t) \) is the body temperature in the point \( \mathbf{x} = (x_1, x_2, x_3) \) in time that is considered, and \( T_0 \) is the body temperature in the same point in the initial non-deformed state in the initial time.

With the change in temperature by the value \( \Delta T = T - T_0 \) in some point \( \mathbf{x} = (x_1, x_2, x_3) \) of the structure, the volume of a small circle that surrounds this point will change in proportion to \( \Delta T \); in this case thermal deformations occur

\[
\varepsilon'_{ij} = \alpha_{ij}(T - T_0), \quad i,j = 1,2,3, \tag{1}
\]

or, in the matrix entry

\[
\{\varepsilon'\} = \{\alpha\}(T - T_0), \tag{2}
\]

where \( \alpha_{ij} \) is the matrix of coefficients of thermal expansion (1/degree).

For the case of isotropic body, expansion in all directions is executed in the same way, therefore, we can write down

\[
\varepsilon = \varepsilon' + \varepsilon''.
\]

Here we determine the value of elastic deformation

\[
\{\varepsilon\} = \{\varepsilon'\} + \{\varepsilon''\}.
\]

Thus, the stress in a linear-strain body can be defined by the Hooke's law (formula (6) from [10])

\[
\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda(\varepsilon_{kk} - \gamma T)\delta_{ij}, \tag{3}
\]

where \( \lambda, \mu \) are the Lamé constants; \( \gamma \) is the thermomechanical constant \( (\gamma = (3\lambda + 2\mu)\alpha_i) \); \( \alpha_i \) is the coefficient of thermal expansion; \( T \) is the temperature.

These are known ratios of Duhamel-Neumann.

Strain tensor components \( \{\varepsilon\} \) are expressed by moving \( u_i \) \( (i = 1, 2, 3) \) of the relevant points of the body according to the Cauchy ratios

\[
\varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right). \tag{4}
\]

It is necessary to determine stress-strain state of elastic plate under conditions of temperature and force influence, that is satisfactory to the equation of motion

\[
(\lambda + 2\mu)\Delta u_i - (\lambda + \mu)\frac{\partial}{\partial x_i} (\gamma u) - \gamma \frac{\partial^2}{\partial x_i^2} - \rho \frac{\partial^2 u}{\partial t^2} - F_i = 0,
\]

the equation of thermal conductivity (2), ratios of Duhamel-Neumann (3), Cauchy ratios (4), the equations of
consolidation of deformations [10], as well as the initial and border-state conditions.

The problem was solved by time. We conduct a comprehensive analysis of the impact of two pores and the inclusion of a range of diameters of 2.5 ÷ 3.5 mm, the inclusion has a diameter of 2 mm. The pores and inclusions are in the welded seam that connects two plates with a 40 mm thickness each.

Consider different distances between the pores and inclusions and conduct an evaluation of the magnitude of the acute angle of the inclusion on the development of cracks in the welded seam. The method allows determining the effect of micro defects on further opening of the cracks. The above mentioned implies an ability to forecast the behavior of micro defects on the overall picture of the strength of the structure.

To solve the associated task of thermoelasticity, we use the finite elements method. Perform approximation of the temperature field (scalar function) within the finite element

\[ T = T_0 \times \{F\}^T \{T\}, \quad k=1,2,...,N, \]  

where \( N \) is the number of elements; \( \{F\} \) is the matrix (vector) of functions of an element form; \( \{T\} = \{T_1, T_2, ..., T_N\}^T \) are the temperature values in an element’s modes.

Here and below we use the rule of summation by indices, which are repeated, that is an expression of the type \( a_{ij} \) should be considered as the sum of \( \sum a_{ij} \).

Then the expression (1) takes the form

\[ \varepsilon_{ij} = \alpha_{ij} \times (T - T_0), \]  

or in the matrix entry

\[ \{\varepsilon\} = \alpha \left\{\{F\}^T \times (\{T\} - \{T_0\})\right\}, \]  

where \( \{T_0\} \) is the vector of nodular values of the initial field of temperature.

The system of equilibrium equations of a finite element of elastic body in the presence of temperature action takes the form

\[ \{K\} \times \{\lambda\} = \{f\}, \]  

where, as in [5], \( \{K\} \) is the matrix of body rigidity; \( \{\lambda\} \) is the vector of nodular displacement; \( \{f\} \) is the vector of efforts in the nodes of elements, which are calculated by the formula (in the matrix record)

\[ \{f\} = \int q \times \{\bar{F}\} dV + \int P \times \{\bar{F}\} dS + \int (B) \times \{\bar{D}\} \times \{e\} dV. \]  

The last term in (9) is the load in the nodes of elements, caused by temperatures’ field. The equations of equilibrium for the whole finite element model of the structure will look the same.

Thus, the ratios (7)–(9) provide a solution to the problem of thermal strength at the known distribution of current and initial temperature of the body. Compared with the usual calculation (excluding thermal action), in our case it is necessary to assign additionally the matrices of coefficients of thermal expansion that are used in the model of materials, and nodular values of specified temperatures.

5. Source data for carrying out calculation by strength

Consider rectangular plates of the dimensions: plate 1 \( a=400 \text{ cm}; b=2100 \text{ cm}; \text{ thickness } h=4 \text{ cm} \), plate 2 \( a=400 \text{ cm}; b=1100 \text{ cm}; \text{ thickness } h=3 \text{ cm} \). The material of the plates is steel 10HSND.

Modulus of elasticity and Poisson’s ratio, accordingly, equal \( E=2.1 \times 10^6 \text{ kgs/cm}^2 \); \( \nu=0.3 \). The density of the material of the plate equals \( \rho_p=0.0079 \text{ kgs/cm}^3 \). The density of the material of inclusions (tungsten) equals \( \rho_p=0.01925 \text{ kgs/cm}^3 \).

A basic finite element model of the duct is designed using three-dimensional elements of a "solids" type.

For the calculation of SSS with the action of internal pressure and heating, we use a 4-layer model on the plates with the thickness of 4 cm, a 3-layer – on the plates with the thickness of 3 cm. On the welded seams we use 3-layer models, in the location of pores and inclusions we held thickening of the elements, which consist of 200000–300000 elements.

For defining a stable solution, we used 4 models: model number one – 200000 elements, model number two – 240000 elements, model number three – 280000 elements, model number four – 300000 elements. A finite-element grid is depicted in Fig. 1.

![Fig. 1. Finite-element body grid](image)

Calculation is performed by the temperature load and pressure on the lateral surfaces.

Consider the range of temperatures from 135–170 °C in the welded seam. The pressure on the lateral surfaces is adopted equal to \( P_1=1.6 \text{ kgs/cm}^2 \) and \( P_2=–0.5 \text{ kgs/cm}^2 \).

Fig. 2 shows features of the mounting scheme 1.

![Fig. 2. Mounting scheme 1](image)
Fig. 3 shows features of the mounting scheme 2.

Fig. 4 shows features of the mounting scheme 3.

Location of the pores and inclusions in the welded seam is shown in Fig. 5.

6. Results of calculations of maximal stress in the welded seam

The initial data and the results of the calculation are shown in Table 1 for the pores with diameter of 2.5 mm, \( \alpha \)-acute angle of inclusion, which equals 45° and the mounting scheme 1.

| L, mm | T, °C | \( \sigma_{0,2t} \), kgs/cm² | \( \sigma_{\text{eqv}}^1 \), kgs/cm² | \( \sigma_{\text{eqv}}^2 \), kgs/cm² |
|-------|-------|----------------|----------------|----------------|
| 0.875 | 159   | 3280           | 3350           | 3500           |
| 1.75  | 152   | 3300           | 3320           | 3400           |
| 2.625 | 148   | 3300           | 3300           | 3350           |
| 3.5   | 146   | 3300           | 3300           | 3300           |

Note: \( \sigma_{\text{eqv}}^1 \) is the maximal tension in the node of a pore with maximal temperature; \( \sigma_{\text{eqv}}^2 \) is the maximal tension in the node of inclusion with maximal temperature; \( \sigma_{0,2t} \) is the minimal acceptable tension in the welded seam (on the border of proportionality); L is the distance between a pore and inclusion.

The initial data and the results of the calculation are shown in Table 2 for the pores with diameter of 2.5 mm, \( \alpha \)-acute angle of inclusion, which equals 45° and the mounting scheme 2.

| L, mm | T, °C | \( \sigma_{0,2t} \), kgs/cm² | \( \sigma_{\text{eqv}}^1 \), kgs/cm² | \( \sigma_{\text{eqv}}^2 \), kgs/cm² |
|-------|-------|----------------|----------------|----------------|
| 0.875 | 159   | 3280           | 3350           | 3560           |
| 1.75  | 152   | 3300           | 3320           | 3440           |
| 2.625 | 148   | 3300           | 3300           | 3390           |
| 3.5   | 146   | 3300           | 3300           | 3330           |

Note: \( \sigma_{\text{eqv}}^1 \) is the maximal tension in the node of a pore with maximal temperature; \( \sigma_{\text{eqv}}^2 \) is the maximal tension in the node of inclusion with maximal temperature; \( \sigma_{0,2t} \) is the minimal acceptable tension in the welded seam (on the border of proportionality); L is the distance between a pore and inclusion.

The initial data and the results of the calculation are shown in Table 3 for the pores with diameter of 2.5 mm, \( \alpha \)-acute angle of inclusion, which equals 45° and the mounting scheme 3.

| L, mm | T, °C | \( \sigma_{0,2t} \), kgs/cm² | \( \sigma_{\text{eqv}}^1 \), kgs/cm² | \( \sigma_{\text{eqv}}^2 \), kgs/cm² |
|-------|-------|----------------|----------------|----------------|
| 0.875 | 159   | 3280           | 3370           | 3550           |
| 1.75  | 152   | 3300           | 3330           | 3470           |
| 2.625 | 148   | 3300           | 3310           | 3390           |

Note: \( \sigma_{\text{eqv}}^1 \) is the maximal tension in the node of a pore with maximal temperature; \( \sigma_{\text{eqv}}^2 \) is the maximal tension in the node of inclusion with maximal temperature; \( \sigma_{0,2t} \) is the minimal acceptable tension in the welded seam (on the border of proportionality); L is the distance between a pore and inclusion.

The initial data and the results of the calculation are shown in Table 4 for the pores with diameter of 3.0 mm, \( \alpha \)-acute angle of inclusion, which equals 30° and the mounting scheme 1.
The initial data and the results of the calculation are shown in Table 5 for the pores with diameter of 3.0 mm, \( \alpha \)-acute angle of inclusion, which equals 30\(^\circ\) and the mounting scheme 2.

### Table 5

| L, mm | T, °C | \( \sigma_{0,2r} \), kgs/cm\(^2\) | \( \sigma_{\text{eqv}}^1 \), kgs/cm\(^2\) | \( \sigma_{\text{eqv}}^2 \), kgs/cm\(^2\) |
|-------|-------|-------------------------------|-------------------------------|-------------------------------|
| 0.875 | 159   | 3280                          | 3380                          | 3580                          |
| 1.75  | 152   | 3300                          | 3340                          | 3450                          |
| 2.625 | 148   | 3300                          | 3310                          | 3400                          |
| 3.5   | 146   | 3300                          | 3300                          | 3300                          |

Note: \( \sigma_{\text{eqv}}^1 \) is the maximal tension in the node of a pore with maximal temperature; \( \sigma_{\text{eqv}}^2 \) is the maximal tension in the node of inclusion with maximal temperature; \( \sigma_{0,2r} \) is the minimal acceptable tension in the welded seam (on the border of proportionality); L is the distance between a pore and inclusion.

The initial data and the results of the calculation are shown in Table 6 for the pores with diameter of 3.0 mm, \( \alpha \)-acute angle of inclusion, which equals 30\(^\circ\) and the mounting scheme 3.

### Table 6

| L, mm | T, °C | \( \sigma_{0,2r} \), kgs/cm\(^2\) | \( \sigma_{\text{eqv}}^1 \), kgs/cm\(^2\) | \( \sigma_{\text{eqv}}^2 \), kgs/cm\(^2\) |
|-------|-------|-------------------------------|-------------------------------|-------------------------------|
| 0.875 | 159   | 3280                          | 3450                          | 3660                          |
| 1.75  | 152   | 3300                          | 3400                          | 3580                          |
| 2.625 | 148   | 3300                          | 3370                          | 3490                          |
| 3.5   | 146   | 3300                          | 3320                          | 3400                          |

Note: \( \sigma_{\text{eqv}}^1 \) is the maximal tension in the node of a pore with maximal temperature; \( \sigma_{\text{eqv}}^2 \) is the maximal tension in the node of inclusion with maximal temperature; \( \sigma_{0,2r} \) is the minimal acceptable tension in the welded seam (on the border of proportionality); L is the distance between a pore and inclusion.

Analysis of the results of the calculation by Tables 1–6 displayed:
- a local mounting of the structure will not lead to rapid cracking of a welded seam;
- hinging mounting of the structure will reduce the operation life of the structure.

7. Discussion of the results of the research

A finite elements method allows assessing not only the compression and extension of a construction, but also the shift and torsion at asymmetric thermo force loading. We have determined that with the initial data that are examined, the shift provides no great influence on the development of cracks. The cracks develop under the influence of loads of extension, which is a positive quality of the method.

To construct a model that describes the physics of the research process, one needs a lot of time for manual building of the elements of a “solids” type. The realization of correct weld takes a lot of time and is a disadvantage of the specified method.

Maximal loads in four models with different number of elements in the welded seam vary from 3600 to 3602 kgs/cm\(^2\).

The calculation of the model 1 takes two hours. The calculation of the model 2 lasts 3 hours, and the calculation of model 3 lasts 3 hours and 30 minutes, the calculation of model 4 takes 4 hours.

Presented research is a continuation of the development of a method for determining stresses and deformations in welded seams under the influence of asymmetrical thermal force loading from [12].

8. Conclusions

Based on the obtained results when performing the proposed calculations, we can make the following conclusions:

1. In the distance between the pore and the inclusion of L=0.875 mm at a pore diameter of 3.0 mm and the inclusion that has a diameter of 2 mm, there are preconditions for the opening of the crack from the inclusion to the pore for the mounting scheme 3.

2. Based on the fact that the difference in loads is less than 0.1 per cent, then the optimal number of elements for the calculation of the loaded state of the body is 200000 elements.

3. The most appropriate to the analytical calculations is the calculation scheme number three, the distinguishing feature of which is the absence of displacements and angles of turning underlying the structure. The results of numerical method in comparison with analytical method do not extend beyond a 3 percent error.

4. In the pore with diameter of 3.0 mm, which is exposed to the maximal temperature, the tension exceeds the adopted \( \sigma_{\text{eqv}}^1 = 3600 \) kgs/cm\(^2\). In the nodes around the inclusion, the process of opening of the cracks will occur.

5. It should be noted that of all considered calculation schemes, the most dangerous for a weld is the mounting scheme 3. The mounting scheme 3 leads to greater stresses under the action of thermo force loading in welded seams. With the mounting schemes 1 and 2, the set-up has more degrees of freedom due to which the tension is less than in the estimated scheme 3.

Special features of the research include identifying of different stresses by time, by the depth of the plates, by the length of the plates and welded seams.

In future we will consider the assessment of fatigue of a weld taking into account changes in physical-mechanical properties of the material around the inclusion and of the material of the inclusion itself.
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