The Mathematical Model of Water Flooding in Different Structure Types of Low Permeability Carbonate Rocks

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Abstract. Due to the effect of dissolution and the influence of stratigraphic structure, carbonate rocks’ fracture system is generally developed and the reservoir has a strong heterogeneity. Some fractures exist in some low-permeability carbonate rock matrices, but no fracture system is formed. It is difficult to describe the seepage water flooding characteristics of these rocks with the traditional single medium model and dual medium model. The purpose of this paper is to establish a new carbonate rock seepage model to describe this kind of carbonate rock seepage. By introducing the fracture density to describe the proportion of fractures in the entire infiltration process, the incompletely developed fracture system can be considered as equivalent medium system. The research shows: the apparent permeability of core increases nonlinearly with the increase of fracture density. When the matrix-type carbonate reservoirs are driven by water, the period of non-aqueous oil recovery is long, and the production declines slowly after seeing water. In matrix-fractured carbonate reservoirs, water recovery period is the longest and the output decreased rapidly after seeing water. In fractured carbonate reservoirs, the period of non-aqueous oil recovery is short and the production declines rapidly.

1. Introduction
Carbonate reservoir types can be roughly divided into pore type, cave type, fracture-cave type, pore-fracture type, etc [1, 2]. The most common type is fracture-type dual-media characteristics, such as fracture-porosity, fracture-cavity reservoirs. Although the fracture porosity is low, its permeability is high, so it is often considered as an important channel for communicating bedrock and caverns. Carbonate porous reservoirs have high porosity and are the main reservoir space for oil and gas. In this reservoir, it is both a reservoir space and an oil and gas seepage channel [3]. For this feature, people have proposed a single media model.

For the first time, Barenblatt, Zheltov, and Kochina proposed the concept of dual media for naturally fractured reservoirs, and established a dual medium seepage model [4, 5], also known as the double-hole dual-permeability model. Both the matrix pore system and the fracture system of this model are involved in the seepage, and at the same time there is an exchange of fluid quality. Afterwards, Warren and Root [6] supplemented and defined the fracture system's geometric characteristics and seepage characteristics based on the assumption of Bareblatt model, and proposed a two-hole single-seepage model. The model only considers the seepage in the fracture system, and the pores only serve as the space for fluid storage.

Although there are many conventional seepage models for carbonate rocks, some fractures exist in the carbonate matrix and no fracture system is formed. The dual media model cannot be used to describe the seepage process of this type of media. Therefore, there is an urgent need to establish new
fractured low-permeability carbonate rock seepage model was used to describe this type of carbonate percolation feature.

2. Mathematical model

The carbonates were classified into three types: matrix type, matrix-fractured type and fracture type, as shown in figure 1. The seepage experiment results show that this type of carbonate rock belongs to a low permeability reservoir. In the microscopic experiment of the core, it was found that there are fractures in some carbonate rock matrix, but no fracture system is formed, and the seepage process of this type of medium cannot be described by the dual-media model.

Fig. 1. The classification of low-permeability carbonate rock. The fracture density ($\alpha_f$) is used to define the distribution of fractures in the matrix. (a) Matrix-type carbonate rock, its fracture density is greater than or equal to 0 and less than or equal to 0.1. (b) Matrix-fractured type carbonate rock, its fracture density is greater than 0.1 and less than 0.8. (c) Fractured carbonate rock, its fracture density is greater than or equal to 0.8 and less than or equal to 1.

The fracture permeability is mainly determined by the fracture width, the fracture density ($\alpha_f$) can be described by the ratio of the porosity occupied by the fracture. The fracture density defined by the surface porosity method is:

$$\alpha_f = \phi_f / \phi$$  \hspace{1cm} (1)

Where $\phi_f$ is the porosity of the fracture; $\phi$ is the total porosity of the rock. Both can be measured by microscopic experiments.

The fracture density describes the proportion of the fracture system in the overall permeability, obviously cannot be determined by the ratio of fracture porosity to total porosity, according to the relation between permeability and porosity Carman-Kozeny equation [7]. It shows that the permeability is proportional to the cubic of porosity and is proportional to the square of the pore size.

$$K = C \cdot \phi^2 r_0^2$$  \hspace{1cm} (2)

Where $C$ is the shape factor; $r_0$ is the average throat radius or fracture width.

Defining the permeability of fractured media:

$$K = \alpha_f^{1/3} \cdot K_f + (1 - \alpha_f^{1/3}) \cdot K_m$$  \hspace{1cm} (3)

From equation (2), the relationship between fracture permeability and matrix pore permeability can be constructed as follows:

$$K_f / K_m = (\bar{r}_f / \bar{r}_m)^2$$  \hspace{1cm} (4)

The fracture permeability can be further obtained:

$$K_f = (\bar{r}_f / \bar{r}_m)^2 K_m$$  \hspace{1cm} (5)
Where $\bar{f}$ is the average width of the fracture; $\bar{r}_m$ is the average radius of the pores. Both can be measured by microscopic experiments or by MRI [8, 9].

Bring equation (5) into equation (3) to get the apparent permeability equation as:

$$K = \left[\alpha_{f}^{1/3} \cdot (\bar{f}/\bar{r}_m) + (1 - \alpha_{f}^{1/3}) \right] \cdot K_m$$  \hspace{1cm} (6)

In practical applications, you need to know $K_f$, $K_m$ and $\alpha_f$. Among them: $\alpha_f$ can be measured by microscopic experiment; $K_f$ and $K_m$ are obtained by equation (5) and equation (6):

$$K_m = K \cdot \left[\alpha_{f}^{1/3} \cdot (\bar{f}/\bar{r}_m)^2 + (1 - \alpha_{f}^{1/3}) \right]^{-1}$$  \hspace{1cm} (7)

Where $K$ is the apparent core permeability measured experimentally.

Therefore, the relationship between apparent permeability and fracture density of three types of carbonate fractures can be determined, as shown in figure 2(a), 2(b), and 2(c). It can be seen that the apparent permeability of the core increases nonlinearly with the increase of the fracture density.

3. Water flooding characteristics

Oil-water two-phase flow theory is the theoretical basis of the oilfield water injection development project. The characteristics of water flooding development of carbonate rocks in this paper are divided into three types: matrix type, matrix-fissure type and fracture type, as shown in figure 3(a), 3(b) and 3(c). It can be seen from the figure that the matrix-type core has the largest common-phase area in water-oil two-phase flow, the single permeability is less than 10mDc, and the flow has nonlinear characteristics, and the efficiency of water flooding is the highest; the matrix-fractured core permeability is highest due to the existence of fracture system communication. It is in the category of Darcy's law of seepage. The pore matrix only provides fluid storage space, and the common phase area of water and oil flow is smaller than matrix core, and the water flooding efficiency is also smaller than the matrix core; in the fracture core, the water and oil common phase area is the smallest, so its oil displacement efficiency is also the lowest.

3.1. The characteristics of water drive recovery factor.

In water-oil two-phase displacement, the water drive recovery factor equation [10] is

$$\eta = 1 - \frac{1 - S_{wi} - S_{or}}{1 - S_{wi}} \cdot \frac{B_{oi}}{B_o}$$  \hspace{1cm} (8)

Where $S_{wi}$ is the irreducible water saturation; $S_{or}$ is the residual oil saturation; $B_{oi}$ is original formation volume factor; $B_o$ is current formation volume factor.
As shown in figure 3(a), for matrix-type reservoirs, $S_{wi} = 0.3$, $S_{or} = 0.2$. It can be calculated that its water drive recovery factor is 71%. Similarly, the water drive recovery factor of matrix-fractured reservoirs was calculated to be 48.5%, and that of fracture reservoirs was 28.6%. The analysis shows that the higher the fracture content was, the lower the recovery efficiency would.

Fig.3. Oil-water two-phase permeability and moisture content curve.

3.2. The characteristics of water content change and production decline.

The water content equation without consideration of capillary force and gravity [11] is:

$$f_w = \left( 1 + \frac{K_{or} \mu_w}{K_{wr} \mu_o} \right)^{-1}$$  \hspace{1cm} (9)

Theoretically, the JBN method can be used to calculate the oil-water phase percolation curve [12, 13].

$$K_w = K_{or} (s_{wi}) (1 - s_{wi})^b$$

$$K_{wr} = K_{or} (s_{or}) (s_{wn})^b$$  \hspace{1cm} (10)

Where $K_{or}$ is water phase permeability; $K_{wr}$ is oil phase permeability; $S_{wi}$ is water saturation, and $S_{wn}$ is standardized saturation.

Substituting equation (10) and equation (9) into equation (11), a new moisture content equation can be derived

$$f_w (S_{wi}) = \left( 1 + \frac{K_{or} (S_{wi}) (1 - S_{wi})^b \mu_w}{K_{wr} (S_{or}) (S_{wn})^b \mu_o} \right)^{-1}$$  \hspace{1cm} (11)
Where $\mu_o$ is oil viscosity, $\mu_w$ is water viscosity, and both of them can be determined by experiment.

The isosaturation surface displacement equation [14] is:

$$L - x_0 = f_w(S_w)\int_0^t Q_o dt$$

(12)

Where $L$ is injector producer distance; $Q_m$ is constant water injection rate; $A$ is flowing area.

Let $x_0 = 0$, $L = 500m$, $Q_m = 120m^3/d$, $\phi = 0.19$, $A = 9000m^2$. When the oil well outlet just saw water $f_{ol} = f_{wa}$, utilize the equation of isosaturation surface displacement (equation (13)) to get the time of seeing the water at 1155d.

$$\frac{(L-x_0)\phi A}{f_{ol}Q_m} = t$$

(13)

After the deformation of equation (13), the integral of both sides can be obtained equation (14):

$$f_w(S_w) = \frac{(L-x_0)\phi A}{Q}\int_{f_{ol}}^{t} dt + f_{ol}(S_{ul})$$

(14)

The moisture content at the outlet can be calculated as shown in figure 4(a).

Fig. 4. The curve of moisture content in the outlet.

Oil well production equation is:

$$Q_o = (1-f_{ol})Q_m$$

(15)

From the oil well production equation, the characteristic curve of oil production decline can be obtained. Figure 5(a) shows: when the matrix-type carbonate reservoirs are flooded with water, the period of non-aqueous oil recovery is long, and the decline in production after water seepage is also slow.

Fig. 5. Oil production decreasing characteristic curve

Similarly, it can be found that the matrix-fractured, fractured carbonate rock oil reserves change and yield decline characteristics are shown in Figure 4(b), 4(c) and Figure 5(b), 5(c), respectively. When the matrix-fractured carbonate reservoirs are driven by water, the water-free period is long, and
the yield decreases rapidly after seeing water. When the fractured carbonate reservoirs are driven by water, the period of non-aqueous oil recovery is short and the production declines rapidly.

4. Conclusions
A mathematical model of matrix-fracture carbonate rock was established and the concept of fracture density was introduced. The single-medium model well describes the carbonate flow seepage characteristics of the unformed fracture system. It lays the foundation for further study of the seepage law of low permeability carbonate rocks.

The apparent permeability of the core can be expressed by the fracture density and the matrix permeability, and the apparent permeability of the core increases nonlinearly with the increase of the fracture density.

When the matrix-type carbonate reservoirs are flooded with water, the period of non-aqueous oil recovery is long, and the decline in production is slower after seeing water. When the matrix-fractured carbonate reservoirs were flooded with water, the period of non-aqueous oil recovery is the longest and the output decreased rapidly after seeing water. When the fractured carbonate reservoirs are driven by water, the period of non-aqueous oil recovery is short and the production declines rapidly.

Acknowledgments
This work was supported by the National Science Foundation of China under grant No.11472246 and National Major Projects of China under grant No. 2017ZX05072005.

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