1. INTRODUCTION

Milgrom (1983) proposed that a modification of Newton gravitational law at accelerations below a threshold of $10^{-8}\text{ cm s}^{-2}$ could explain the dynamics of galaxies without invoking any dark matter. In the so-called MOdified Newtonian Dynamics (MOND), the rotation velocity of an isolated galaxy is determined by its visible (baryonic) mass. MOND has proven successful in reproducing the shape and amplitude of a significant fraction of spiral galaxies (without any dark matter) with only the mass-to-light ratio of the stellar disk as an adjustable parameter (see Sanders & McGaugh 2002 for a review; Milgrom & Sanders 2007; Sanders & Noordermeer 2007; Swaters et al. 2010).

In this paper, we will consider gas-rich dwarf galaxies to test MOND. This type of galaxy provides a good test for MOND because (1) in most of these galaxies the internal acceleration is below the threshold and (2) their mass is dominated by gas and the predicted rotation curve is not sensitive to the assumed stellar mass-to-light ratio $\Upsilon$. In the MOND framework, the asymptotic velocity, that is, the rotation velocity in the outermost regions where rotation curves tend to be flat, is $(GM\dot{a}_0)^{1/4}$, where $M$ is the total baryonic mass of the system. For a sample of 47 gas-rich galaxies selected by a strict criterion, McGaugh (2012) finds a tight empirical relation between detected baryonic mass and rotation velocity in the outermost measured regions, the baryonic Tully–Fisher (BTF) relation, as that predicted by MOND. This type of galaxy provides a good test for MOND with zero free parameters (but see also Foreman & Scott 2012 and the reply by McGaugh 2011). However, we must note that it was already known at that time that the rotation curves of 22 of the galaxies in McGaugh’s sample (47% of the galaxies) were in agreement with MOND predictions and, hence, they were not really new.

An intrinsic scatter in the BTF relation would be difficult to accommodate in MOND because, in this theory, the BTF relation is a direct consequence of the effective force law. On the contrary, the $\Lambda$CDM paradigm predicts a real scatter albeit small (Desmond 2012). Hence, the BTF relation can in principle be used to distinguish between both scenarios. For now, it is unclear whether the small scatter in the BTF relation is a proof of MOND or is a selection effect. Following this line of arguments, we have carried out a search for gas-rich galaxies that separate from the BTF relation. If MOND is correct, all these galaxies should have an obvious problem with the observed rotation curve like uncertain inclinations and distances, or the presence of non-circular motions. In particular, we are interested in galaxies that rotate more slowly than MOND predicts, so that the amplitude of their rotation curves cannot be fitted by arbitrarily increasing the mass-to-light ratio of the stellar component or by assuming additional undetected matter. The selected galaxies might also provide a good target for studying the issue of a link between the level of stability of galaxies in MOND and the star formation (Sánchez-Salcedo & Hidalgo-Gámez 1999).

The paper is organized as follows. In Section 2, we give a brief description of MOND in galaxies. The selection procedure of galaxies and the outcome of the search are presented in Section 3. Section 4 contains a more exhaustive study focusing on some test cases. Conclusions are given in Section 5.

1. INTRODUCTION

2. MOND: BASIC EQUATIONS

The Lagrangian MOND field equations lead to a modified version of Poisson’s equation given by

$$\nabla \cdot \left[ \mu \left( \frac{|\nabla \Phi|}{\dot{a}_0} \right) \nabla \Phi \right] = 4\pi G \rho,$$

where $\rho$ is the density distribution, $\Phi$ is the gravitational potential, and $\dot{a}_0$ is a universal acceleration of the order of $10^{-8}\text{ cm s}^{-2}$. The interpolating function $\mu(x)$, with $x = |\nabla \Phi|/\dot{a}_0$, has the property that $\mu(x) = x$ for $x \ll 1$ and $\mu(x) = 1$ for $x \gg 1$ (Bekenstein & Milgrom 1984). Brada & Milgrom (1995) showed that, to a good approximation, the real acceleration at the midplane of an isolated, flattened axisymmetrical system, $g$, is related with the Newtonian acceleration, $g_N$, by

$$\mu \left( \frac{|g|}{\dot{a}_0} \right) g = g_N.$$
Binney (2005),
\[ \mu(x) = \frac{x}{1 + x}, \]  
(3)
and the “standard” \( \mu \)-function
\[ \mu(x) = \frac{x}{\sqrt{1 + x^2}}, \]  
(4)
proposed by Milgrom (1983). Unless otherwise specified, we will take \( a_0 = 1.2 \times 10^{-8} \) cm s\(^{-2}\) and use the simple \( \mu \)-function (Famaey et al. 2007; Sanders & Noordermeer 2007; Weijmans et al. 2008). Since we are interested in dwarf galaxies whose dynamics lies in the deep MOND regime (that is, \( x = g/a_0 \approx 1 \)), our results are not sensitive to the exact form of the interpolating function.

3. GAS-RICH DWARF GALAXIES WITH “LOW ROTATION”

Define \( \Gamma(R) \) as the ratio between the real acceleration \( g \) and the Newtonian acceleration \( g_N \) at galactocentric radius \( R \), that is,
\[ \Gamma(R) = g/g_N = v_c^2/(v_c^2 + v_g^2), \]  
where \( v_c \) is the observed circular and \( v_c, v_g \) are the Newtonian contributions of the stars and the gas to the rotation curve, respectively. At large enough galactocentric radii, \( \Gamma \) is a measure of the ratio between the dynamical and the baryonic mass. According to Equation (2), if we know \( v_c(R) \), MOND predicts \( \Gamma(R) \) as \( \Gamma_M = 1/(\mu g/a_0) \), where \( g = v_c^2/R \). The predicted value \( \Gamma_M \) can be confronted with the observed value \( \Gamma_{obs} \). If MOND is correct, \( \Gamma_M = \Gamma_{obs} \).

In the outer parts of dwarf galaxies, MOND predicts high \( \Gamma \)-values for galaxies with low-rotation velocities. For instance, for a galaxy with \( v_c \approx 40 \) km s\(^{-1}\) at a radius of 7 kpc, MOND predicts \( \Gamma_M \approx 15 \). In the present study, we will evaluate \( \Gamma_{obs} \) in the outer parts of gas-rich galaxies and select those having the smallest values of \( \Gamma_{obs} \). Doing so, we will be able to identify potential galaxies for which MOND could fail to reproduce the observed amplitude of their rotation curves. The selected galaxies will deserve a detailed analysis of all the available rotation curve data.

To be completely successful, MOND should account for not only the amplitude of the rotation curves at the outskirt of galactic disks but also the detailed shape of the rotation curves. However, a test of MOND based on the capability of reproducing the fine structure of the rotation curves is a more delicate issue. Indeed, a handful of galaxies exists for which MOND does not provide a good fit to the shape of the rotation curve (e.g., Lake & Skillman 1989; Bottema et al. 2002; Sánchez-Salcedo & Lora 2005; Corbelli & Salucci 2007; Swaters et al. 2010). However, this fact does not necessarily rule out MOND because of uncertainties in distance and inclination of the galaxies, beam smearing, non-circular motions, morphological asymmetries, corrections for asymmetric drift, warps and uncertainties in the photometric calibration (e.g., Swaters et al. 2010). Moreover, the simple relation between \( g \) and \( g_N \) given in Equation (2) has only been tested with the underlying assumption of axisymmetry and may induce some error when estimating the MOND-predicted rotation curves in non-axisymmetric galaxies.

Another source of uncertainty in the exact shape of the predicted rotation curve is the specific form adopted for \( \mu \), which is important at intermediate galactic radii, where the transition between the Newtonian and MOND regimes takes place. In our selection procedure, we will evaluate \( \Gamma_{obs} \) and \( \Gamma_M \) at the outer parts, where the predicted \( \Gamma_M \)-value is not sensitive to the exact form of the interpolating function \( \mu \).

3.1. Selection Procedure and Computation of \( \Gamma_{obs} \)

In order to check if all galaxies with “slow rotation” present large values of \( \Gamma \), we have estimated \( \Gamma_{obs} \) for more than 80 galaxies with published rotation curves whose rotation velocities, at the last measured point, are between 35 and \( 80 \) km s\(^{-1}\). Galaxies with rising rotation curves in the outermost measured regions were also included. We have discarded galaxies with circular velocities \(<35 \) km s\(^{-1}\) at the last measured point to avoid the inclusion of rotation curves with large uncertainties due to asymmetric drift corrections (e.g., IC 1613 and UGC 7577). On the other hand, we restrict to galaxies with circular velocities \(<80 \) km s\(^{-1}\) because they are more likely to be gas-rich galaxies (e.g., McGaugh 2012) and their dynamics lie in the deep MOND regime. We selected those galaxies with low \( \Gamma \)-values, i.e., galaxies with \( \Gamma_{obs} < 5.5 \).

In order to estimate \( v_c, g \), the gas mass was taken as 1.4 times the H \( i \) content to correct for the presence of He and metals. The typical uncertainty in the H \( i \) mass is less than 5% (except for E364-G029 which is of 25%). Hence, most of the uncertainties in \( v_c, g \) comes from the content of molecular gas and other forms of gas. In order to estimate \( v_{c,s} \), the stellar disk contribution to the rotation curve, we convert luminosity to mass using the models of Bell & de Jong (2001). The \( (B − R) \) color was determined from the magnitudes of the galaxies as given in NED. When available, the 25th isophote magnitude was preferred. In Column 7 of Table 1, we give a range of the stellar mass-to-light ratios in the blue band \( \Upsilon_B \). The range is so ample that it includes values previously reported by other authors, as well as the bursting model, which is the model that best describes most of the properties of these galaxies. Note that the stellar mass-to-light ratios are very dependent on the color and on the model. However, since the gas mass dominates in these galaxies, uncertainties in the stellar mass-to-light ratio do not have a strong impact on the estimates of the total baryonic mass or in \( \Gamma \) (see Columns 8 and 10 in Table 1).

3.2. Search Outcome: Galaxies with Low \( \Gamma_{obs} \)

3.2.1. General Comments

The search turned out only seven gas-rich galaxies with \( \Gamma_{obs} < 5.5 \): NGC 3077, NGC 2366, UGC 4173, NGC 4861, ESO 245-G05, ESO 364-G029, and HoII. The main properties of these galaxies are compiled in Table 1; the rotational velocities at the outer parts, \( V_{rot} \), are given in Column 9, the empirical values of \( \Gamma \) for the seven selected galaxies are given in Column 10 (denoted by \( \Gamma_{obs} \)), whereas the values predicted by MOND, \( \Gamma_M \), are provided in Column 11. The uncertainty in \( \Upsilon_B \) was also treated as an additional source of error. However, the most important error source in the quoted values of \( \Gamma \) is due to uncertainties in the H \( i \) rotation curve of these galaxies. Figure 1 shows the predicted MOND \( \Gamma \)-value and the observed one for these galaxies.

For NGC 3077 and NGC 2366, the values of \( \Gamma_{obs} \) and \( \Gamma_M \) are consistent within the 1σ uncertainty. The H \( i \) kinematics of NGC 3077 is highly perturbed by the tidal interaction with M81 and M82 and, therefore, is not a good candidate to test MOND. On the other hand, whereas the inclination of NGC 2366 \( (i = 63°) \) is adequate to accurately estimate the rotation curve, the uncertainties in the rotation curve are very large. Therefore, these two galaxies should be discarded until more precise data are available. For the remainder five galaxies, MOND predicts too large a value of \( \Gamma \) and, hence, it appears to fail for those galaxies with small \( \Gamma_{obs} \).
the uncertainties in this parameter. Warren et al. (2004) report McGaugh’s consistency criterion.

The Astronomical Journal, 145:61 (7pp), 2013 March

Sánchez-Salcedo, Hidalgo-Gámez, & Martínez-García

Note that the derived values of $\Gamma_{\text{obs}}$ and $\Gamma_{\text{M}}$ depend on the distance to the galaxy and on the adopted inclination. The error bars quoted in Table 1 do not include uncertainties in the inclination angle or distance to the galaxy.

Consider first uncertainties in the inclination angle. Beyond a few disk scale radii, where the mass distribution has essentially converged, $v_{c,*}$ and $v_{c,g}$ do not depend on the adopted inclination. The amplitude of the rotation curve at the outer parts $V_{\text{rot}}$ depends on the inclination $i$ of galaxy and changes as $V_{\text{rot}} \propto (1/\sin i)$. For modest changes in the inclination so that the outer dynamics lie in the deep MOND regime, it holds that $\Gamma_{\text{M}} \propto V_{\text{rot}}^{2} \propto \sin^{2} i$, whereas $\Gamma_{\text{obs}} \propto V_{\text{rot}} \propto (1/\sin^{2} i)$.

In order to avoid strong effects on the determination of the rotation curve, Begeman et al. (1991) required that the galaxies should have H I inclinations between $50^\circ$ and $80^\circ$. On the other hand, McGaugh (2012) demands consistency between optical and H I inclinations. Whereas ESO 245-G05, NGC 4861, and ESO 364-G029 have comfortable H I inclination angles between $50^\circ$ and $80^\circ$, none of the selected galaxies comply with McGaugh’s consistency criterion.

The galaxy ESO 215-G009 exemplifies the importance of having galaxies with high inclinations in order to ameliorate the uncertainties in this parameter. Warren et al. (2004) report a circular velocity of $51 \pm 8$ km s$^{-1}$ at 7 kpc for an inclination $i = 36^\circ \pm 10^\circ$. However, van Eymeren et al. (2009b), using a smaller inclination of $28^\circ$, derive a remarkably larger amplitude of the rotation curve of $75$ km s$^{-1}$ at the same angular distance.

Given the low inclination and the uncertainties in the distance to this galaxy, the amplitude of the rotation curve may be well in accordance to MOND predictions. Hence, ESO 215-G009 must be excluded.

Uncertainties in the distance D may also play a role. Consider now how $\Gamma_{\text{obs}}$ depends on the adopted distance to the galaxy. Assuming that the H I disk is infinitely thin, $v_{c,g}^{2} \propto D$. The stellar contribution $v_{c,*}^{2}$ is also proportional to $D$ if the stellar mass-to-light ratio is kept fixed. Therefore, $\Gamma_{\text{obs}} \propto (v_{c,g}^{2} + v_{c,*}^{2})^{-1} \propto D^{-1}$. The variation of $\Gamma_{\text{M}}$ to changes in the adopted distance is also simple to derive. Suppose a galaxy that rotates at velocity $V_{\text{rot}}$ at a radius $R_{\text{m}}$. In the deep MOND regime, we have that $\mu \simeq g/\alpha_{0} = V_{\text{rot}}^{2}/(\mu R_{\text{m}}) = r_{m}$. Since $r_{m}$ scales as $r_{m} \propto D$, we obtain $\Gamma_{\text{M}} = 1/\mu \propto r_{m} \propto D$ (provided that the distance change is small enough that the outer galaxy is still in the deep MOND regime).

Can uncertainties inherent to such a sample of galaxies like uncertainties in the inclinations and distances explain the discrepancy between $\Gamma_{\text{obs}}$ and $\Gamma_{\text{M}}$? In the following we will try to answer this question. Before making any further interpretation, it is convenient to briefly comment on individual galaxies.

### 3.2.2. Comments on Individual Galaxies

The MOND rotation curve fitting of UGC 4173 was already studied by Swaters et al. (2010). Using the standard $\mu$-function, MOND overestimates the circular velocity in all the points

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**Table 1**

| Name            | $D$  | $M_{B}$ | $L_{B}$ | $(i)$ | $M_{\text{gas}}$ | $M_{\text{rot}}^{\theta}$ | $M_{\text{total}}$ | $V_{\text{rot}}$ | $\Gamma_{\text{obs}}$ | $\Gamma_{\text{M}}$ | Source |
|-----------------|------|---------|---------|-------|-----------------|--------------------------|------------------|---------------|----------------------|-------------------|--------|
| E215-G?009      | 4.2  | $-12.9$ | 0.23    | 36°   | 1.12            | 7.2–7.6                  | $51 \pm 5$       | 7 kpc         | 5.6$^{+1.2}_{-1}$   | 10.7$^{+0.1}_{-0.1}$ | 1      |
|                 | 5.25 | $-13.4$ | 0.36    | 28°   | 1.11            | 11–12                    | 75$^{+0.1}_{-0.1}$ | 8 kpc         | 9.0$^{+0.1}_{-0.1}$   | 6.1$^{+0.1}_{-0.1}$ | 2      |
| NGC 3077        | 3.8  | $-17.5$ | 19.6    | 46°   | 12.3            | 0.2–0.6                  | 16–24            | 6 kpc         | 3.0$^{+0.1}_{-0.1}$   | 6.5$^{+0.1}_{-0.1}$ | 3.4    |
| NGC 2366        | 3.4  | $-17.1$ | 11.5    | 63°   | 9.1             | 10.8–12.5                | 60 ± 16          | 7 kpc         | 5.4$^{+0.1}_{-0.1}$   | 8.0$^{+0.1}_{-0.1}$ | 2.3,5   |
| UGC 4173        | 18.0 | $-16.5$ | 6.2     | 40°   | 34.0            | 37–40                    | 48 ± 5           | 9 kpc         | 2.8 ± 0.1           | 15$^{+0.5}_{-0.5}$ | 6      |
| NGC 4861        | 7.5  | $-16.7$ | 7.9     | 65°   | 6.7             | 8.0–9.1                  | 46$^{+0.5}_{-0.5}$ | 7 kpc         | 4.7$^{+0.1}_{-0.1}$   | 12.9$^{+0.1}_{-0.1}$ | 2.7    |
| E245-G05        | 2.5  | $-15.0$ | 1.6     | 54°   | 2.8             | 3.9–5.2                  | 43 ± 1           | 3.5 kpc       | 3.0 ± 0.1           | 7.8 ± 0.3         | 8      |
| E364-G029       | 10.8 | $-16.6$ | 6.6     | 70°   | 9.0             | 10–16                    | 42 ± 3           | 7 kpc         | 2.0$^{+0.1}_{-0.1}$   | 15.0$^{+0.5}_{-0.5}$ | 9      |
| HoII            | 3.4  | $-16.8$ | 8.7     | 49°   | 9.0             | 0.17                     | 10.5            | 4 ± 7 kpc     | 2.0 ± 0.4           | 19.5$^{+0.5}_{-0.5}$ | 3,5    |

**Notes.** Column 1: name of the galaxy; Column 2: adopted distance; Column 3: absolute B magnitude; Column 4: total blue-band luminosity; Column 5: average value of the inclination; Column 6: total mass in gas; Column 7: stellar mass-to-light ratio in the blue band; Column 8: total baryonic mass; Column 9: rotation velocity; Columns 10 and 11: $\Gamma_{\text{obs}}$ and $\Gamma_{\text{M}} = 1/(\mu g/\alpha_{0})$; Column 12: references—(1) Warren et al. 2004; (2) van Eymeren et al. 2009b; (3) Walter et al. 2008; (4) Martin 1998; (5) Oh et al. 2011; (6) Swaters et al. 2010; (7) van Eymeren et al. 2009a; (8) Côté et al. 2000; (9) Kouwenhoven et al. 2007.
beyond 7 kpc. If the inclination is taken as a free parameter in the fits, the MOND curve and the data agree well for an inclination of 24° ± 5°. Swaters et al. (2010) argue that an inclination of 25°–30° is consistent with the H I morphology because this galaxy has an optical bar with a faint surrounding disk.

NGC 4861 is an irregular dwarf galaxy with a luminosity $L_B = 7.9 \times 10^8 L_\odot$ and shows no evidence for spiral structure (van Eymeren et al. 2009a). This galaxy was studied in H I by Wilcots et al. (1996), Thuan et al. (2004), and van Eymeren et al. (2009a, 2009b). For $\Upsilon H_\text{I}^R = 0.3 M_\odot / L_\odot$, the total baryonic mass is $9.1 \times 10^8 M_\odot$. It is one of the most inclined galaxies in our selected sample. The optical inclination from HYPERLEDA is 90° (Paturel et al. 2003). Thuan et al. (2004) derived an inclination of 82° for the outermost H I tilted ring, whereas van Eymeren et al. (2009a, 2009b) estimated a mean H I inclination of 65° ± 5° within 7 kpc. Using the latter inclination, we infer $\Gamma_{\text{obs}} = 4.7$ at 7 kpc.

Tilted-ring fits for ESO 245-G05 give an inclination of 54° ± 10° (Côté et al. 2000). Its H I mass is $2 \times 10^8 M_\odot$. Adopting $\Upsilon H_\text{I}^R = 1$, the total baryonic mass is $4.4 \times 10^8 M_\odot$. At the last measured radius of 3.5 kpc, we find $\Gamma_{\text{obs}} = 3.0 \pm 0.6$, which is a factor of two smaller than the predicted value by MOND. Whereas this is a potential galaxy to test MOND, it is still premature to make any conclusion given the large errors quoted on the inclination and its uncertain kinematics due to the presence of a strong bar.

HoII is an irregular galaxy in the M81 group. Careful analyses of H I observations of HoII have been carried out by three independent groups: Puche et al. (1992), Bureau & Carignan (2002), and Oh et al. (2011). The H I rotation curves derived by all these authors are very similar. Oh et al. (2011) derived the stellar mass-to-light ratio in the K band, $\Upsilon K^*$, for HoII. With such a $\Upsilon K^*$-value (which we will refer to as the nominal value), the stellar mass in the disk of HoII is of $1.5 \times 10^8 M_\odot$. Given that the total mass in gas is $9 \times 10^8 M_\odot$ (Bureau & Carignan 2002), the total (gas plus stars) baryonic mass in HoII is about $10.5 \times 10^8 M_\odot$. With these empirical values, we infer $\Gamma_{\text{obs}} = 2 \pm 0.4$ at a radial distance of 7 kpc. This is much smaller than the value predicted by MOND, which is about 19.

ESO 364-G029 has a luminosity of $L_B = 6.6 \times 10^8 L_\odot$ for a distance of $D = 10.8$ Mpc (Kouwenhoven et al. 2007). The models of Bell & de Jong (2001) predict a stellar mass-to-light ratio of $\sim 0.2$–0.6 in the blue band, implying a stellar mass of $(1.5-4) \times 10^8 M_\odot$. The H I distribution is mildly asymmetric and roughly follows the stellar brightness distribution. The H I mass is $6.4 \times 10^8 M_\odot$ and the rotation velocity, assuming $i = 70°$, reaches a value of 42 km s$^{-1}$ at a distance of 7 kpc. Despite the asymmetric appearance of the H I, the rotation curve is fairly symmetric. The corresponding $\Gamma_{\text{obs}}$-value at $R = 7$ kpc is $2 \pm 0.5$.

We conclude that, at present, the galaxies NGC 4861, HoII, and ESO 364-G029 are promising targets to test MOND. For the remaining four galaxies, further decreasing systematic uncertainties in these galaxies could provide a strong test to the MOND framework. In the next section, we will consider the whole available rotation curve of the galaxies NGC 4861, HoII, and ESO 364-G029 to quantify the difference between predicted circular velocities and measured velocities. The galaxies HoII and ESO 364-G029 are expected to be the most problematic because they present the lowest $\Gamma$-values ($\Gamma_{\text{obs}} \simeq 2$).

4. ROTATION CURVES: NGC 4861, HOII, AND ESO 364-G029

4.1. The Galaxy NGC 4861

The left panel of Figure 2 shows the observed rotation curve of NGC 4861 from van Eymeren et al. (2009a) together with the predicted MOND rotation curve (solid lines), for the simple and the standard interpolating functions. The adopted distance was $D = 7.5$ Mpc. We have modeled only the inner 7.5 kpc from the dynamic center because beyond this radius the observed rotation curve is affected by uncertainties caused by the sparsely filled tilted rings. For the stellar disk, we have assumed $\Upsilon K^* = 0.3 M_\odot / L_\odot$. We see that MOND overestimates the observed rotation velocities at any radius.

If the distance and $\Upsilon K^*$ are left as free parameters, an acceptable fit is obtained for $D = 4.2$ Mpc and $\Upsilon K^* = 0.12$ (see Figure 2). The distance to this galaxy has been determined with different methods and all gives $D > 7$ Mpc. Using the Virgocentric infall model of Schechter (1980) with parameters $\gamma = 2$, $\nu_{\text{virgo}} = 976$ km s$^{-1}$, $w_\odot = 220$ km s$^{-1}$, and a Virgo distance of 15.9 Mpc, a distance of 10.7 Mpc is derived for NGC 4861 with an uncertainty of $\sim 20$% (Heckman et al. 1998; Thuan et al. 2004). Thus, it is unlikely that changes in distance alone can explain the discrepancy between the predicted and the observed rotation curves.
The Astronomical Journal, 145:61 (7pp), 2013 March

Sánchez-Salcedo, Hidalgo-Gámez, & Martínez-García

Figure 3. Rotation curve of HoII from Oh et al. (2011) together with the contributions of the stellar disk (dotted lines) and gas (dashed lines) for the nominal $\Upsilon_\star$-value (left panel), minimum disk plus gas (central panel) and twice the nominal $\Upsilon_\star$-value. The solid lines represent the rotation curve according to MOND prescription using the simple $\mu$-function (upper solid curves) or the standard $\mu$-function (lower solid curves). MOND is unable to provide the amplitude of the rotation curve. Here we take $D = 3.4$ Mpc and $a_0 = 1.2 \times 10^{-8}$ cm s$^{-2}$.

The MOND fit to the rotation curve can be improved if the inclination of the galaxy and $\Upsilon_\star$ are left free in the fit. If, instead of adopting the nominal inclination $i = 65^\circ$ derived by van Eymeren et al. (2009a), we use an inclination of $i = 43^\circ$, the MOND curve and the derived rotation curve are consistent. However, the change required in inclination is much larger than the associated uncertainty $\sim 5^\circ$, indicating that this inclination is unlikely.

4.2. The Galaxies HoII and ESO 364-G029

4.2.1. A Comparative Study

The galaxies HoII and ESO 346-G029 have similar baryonic mass but the amplitudes beyond galactocentric distances of $R = 5$ kpc are rather similar. In the case of HoII, the method of Oh et al. (2011) minimizes the effect of non-circular motions. The rotation curve of HoII was corrected for asymmetric drift whereas this correction was not made for ESO 364-G029. Asymmetric drift corrections would cause a boost of a few km s$^{-1}$ in the circular velocities of these galaxies.

Remind that the asymptotic velocity is defined as $(GM_{\text{bar}}a_0)^{1/4}$, with $M_{\text{bar}}$ the baryonic mass. Using the estimates of the baryonic masses given in Table 1 and $a_0 = 1.2 \times 10^{-8}$ cm s$^{-2}$, MOND predicts an asymptotic velocity of 63 km s$^{-1}$ for HoII and 66 km s$^{-1}$ for ESO 364-G029. The predicted rotational speed is in excess by 25 km s$^{-1}$ in HoII and ESO 364-G029. In the following, we will concentrate on the case of HoII because the rotation curve has a much better spatial resolution than in ESO 364-G029. In addition, the irregular and lopsided morphology of ESO 364-G029 could induce systematic biases if one assumes axisymmetry.

4.2.2. MOND in HoII

Since HoII is embedded in the external gravitational field of M81 group, one has to quantify the external field effect (EFE) described in Bekenstein & Milgrom (1984), by comparing the internal and external accelerations. Assuming that the M81 group is bound, the external acceleration is $0.7 \times 10^{-10}$ cm s$^{-2}$ (Karachentsev et al. 2002), which is much smaller than its internal accelerations ($\sim 6 \times 10^{-10}$ cm s$^{-2}$ and $2 \times 10^{-10}$ cm s$^{-2}$ at $R = 7$ kpc and $R = 20$ kpc, respectively). Thus, EFE should be small at $R < 10$ kpc.

Figure 3 shows the predicted HoII rotation curve in MOND under various $\Upsilon_\star$ assumptions (the nominal value as derived by Oh et al. 2011, “minimum disk plus gas,” and twice the nominal value). The “minimum disk plus gas” includes the gas component and uses the minimum value of $\Upsilon_\star$ compatible with the requirement that the theoretical circular velocity must be positive and reasonably smooth. The discrepancy between the observed and the predicted rotation curves is apparent. The effect of varying $\Upsilon_\star$ or the interpolating function on the MOND circular velocity at the outer disk is small, less than 10 km s$^{-1}$ at $R = 7$ kpc. For illustration, Figure 4 shows the combined rotation curve from Bureau & Carignan (2002) and Oh et al. (2011) for the updated HoII distance of 3.4 Mpc. Note that at
closer than the nominal distance. The key to lines is the same as in Figure 3. It turns out that if the inclination is taken as a free parameter, a mean inclination of 25° (Carignan 2002). It is required to reconcile MOND with observations. This value is far lower than the one derived by Bureau & Carignan (2002) because MOND severely overpredicts by 50% the observed rotation curve in HoII. An amplitude of the rotation curve lower than expected could be caused by an overestimate of either the inclination or the distance to the galaxy. In order to quantify these effects, we have focused on two galaxies: NGC 4861 and HoII. For these galaxies, we find that the discrepancy between the observed and predicted rotation curves in MOND cannot be a consequence of adopting an incorrect distance because it is unlikely that the distances are uncertain by that much. An inclination of 25°, instead of 40°, is required to make the rotation curve of HoII compatible with MOND. It is clear that the main source of systematic uncertainties in the determination of the amplitude of the rotation curve in HoII is its low inclination. The strategy is to look for a galaxy as similar as possible to HoII but with a higher inclination to reduce geometrical uncertainties. We found that NGC 4861 and ESO 364-G029 rotate at a similar velocity than HoII, have similar baryonic mass but their inclinations are significantly larger. In the case of NGC 4861, the change required in inclination to fit the observed rotation curve in MOND is much larger than the associated uncertainty. The galaxy ESO 364-G029 is very interesting because MOND severely overpredicts by 50% the observed circular velocity at the outer parts of the disk. Contrary to HoII, it has a comfortable inclination of ∼70°. We conclude that deeper photometric and H i observations of ESO 364-G029, together with further decreasing systematic uncertainties, may provide a strong test to MOND.

Figure 5. HoII rotation curve in MOND adopting the standard μ-function with $a_0 = 0.9 \times 10^{-8}$ cm s$^{-2}$ and a distance to the galaxy of 1.5 Mpc, a factor 2.3 closer than the nominal distance. The key to lines is the same as in Figure 3.

$R > 10$ kpc, velocities were only measured in the approaching side.

The value of $a_0$ is universal but needs to be fixed from observations. Even adopting a value of $a_0$ at the lower end of the best-fit interval derived by Begeman et al. (1991) and Gentile et al. (2011), $a_0 = 0.9 \times 10^{-8}$ cm s$^{-2}$, the MOND circular rotation speed is only a few km s$^{-1}$ slower.

In the MOND prescription, the amplitude of the rotation curve can be accounted for by adopting a distance to HoII of 1.5 Mpc and $a_0 = 0.9 \times 10^{-8}$ cm s$^{-2}$ (see Figure 5). Given that the uncertainty in the distance is of 0.4 Mpc (Karachentsev et al. 2002), this likely indicates that MOND cannot be made compatible with that rotation curve by a reasonable adjustment of galaxy’s distance.

A more delicate issue is the error resulting from the uncertainty in the inclination of the galaxy. The optical inclination from LEDA is of 45° (Paturel et al. 2003). Here, we have used a global inclination of the H i disk of 40° in the inner parts (Oh et al. 2011), and 84° for tilted rings at $R > 12$ kpc (Bureau & Carignan 2002). It turns out that if the inclination is taken as a free parameter, a mean inclination of 25° would yield a circular velocity of ∼60 km s$^{-1}$ at $R = 7$ kpc. In a recent posting during the course of submitting this paper and motivated by our preprint Sánchez-Salcedo & Hidalgo-Gámez (2011), Gentile et al. (2012) re-analyze the rotation curve in HoII by modeling its H i data cube and find that the inclination is much closer to face-on than previously derived. At this lower inclination, the rotation velocity becomes commensurate with what is expected from MOND.

In the very outer disk, at $R > 12$ kpc, an inclination of 45° is required to reconcile MOND with observations. This value is far lower than the one derived by Bureau & Carignan (2002) fitting tilted ring models (84°). We conclude that more accurate determinations of HoII inclination in the outer parts will provide a more definitive test to MOND.

5. FINAL REMARKS AND CONCLUSIONS

MOND predicts a tight correlation between the asymptotic circular velocity and the total baryonic mass of the galaxy. Gas-rich dwarf galaxies are an interesting and unique test of modified theories of gravity. Their internal accelerations are below $a_0$ and the stellar mass in these galaxies is not important in the budget of the total mass.

For a large sample of gas-rich dwarf galaxies with rotation velocities between 35 and 80 km s$^{-1}$, we have computed the parameter $\Gamma_{\text{obs}}$, defined as the ratio of real to Newtonian accelerations, and the predicted value in MOND $\Gamma_M$. We found that five galaxies (UGC 4173, HoII, ESO 245-G05, NGC 4861, and ESO 364-G029) have $\Gamma_{\text{obs}} < \Gamma_M$. For these galaxies, MOND overestimates the presently available rotation speeds, and hence, they cannot be fitted by arbitrarily increasing the mass-to-light ratio of the stellar component or by assuming additional undetected matter.

An amplitude of the rotation curve lower than expected could be caused by an overestimate of either the inclination or the distance to the galaxy. In order to quantify these effects, we have focused on two galaxies: NGC 4861 and HoII. For these galaxies, we find that the discrepancy between the observed and the predicted rotation curves in MOND cannot be a consequence of adopting an incorrect distance because it is unlikely that the distances are uncertain by that much. An inclination of 25°, instead of 40°, is required to make the rotation curve of HoII compatible with MOND. It is clear that the main source of systematic uncertainties in the determination of the amplitude of the rotation curve in HoII is its low inclination. The strategy is to look for a galaxy as similar as possible to HoII but with a higher inclination to reduce geometrical uncertainties. We found that NGC 4861 and ESO 364-G029 rotate at a similar velocity than HoII, have similar baryonic mass but their inclinations are significantly larger. In the case of NGC 4861, the change required in inclination to fit the observed rotation curve in MOND is much larger than the associated uncertainty. The galaxy ESO 364-G029 is very interesting because MOND severely overpredicts by 50% the observed circular velocity at the outer parts of the disk. Contrary to HoII, it has a comfortable inclination of ∼70°. We conclude that deeper photometric and H i observations of ESO 364-G029, together with further decreasing systematic uncertainties, may provide a strong test to MOND.

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