Energy Spectra and Energy Correlations
in the Decay $H \rightarrow ZZ \rightarrow \mu^+\mu^-\mu^+\mu^-$

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Abstract

It is shown that in the sequential decay $H \rightarrow ZZ \rightarrow (f_1\bar{f}_1) + (f_2\bar{f}_2)$, the energy distribution of the final state particles provides a simple and powerful test of the $HZZ$ vertex. For a standard Higgs boson, the energy spectrum of any final fermion, in the rest frame of $H$, is predicted to be $d\Gamma/dx \sim 1 + \beta^4 - 2(x-1)^2$, with $\beta = \sqrt{1 - 4m_Z^2/m_H^2}$ and $1 - \beta \leq x = 4E/m_H \leq 1 + \beta$. By contrast, the spectrum for a pseudoscalar Higgs is $d\Gamma/dx \sim \beta^2 + (x-1)^2$.

There are characteristic energy correlations between $f_1$ and $f_2$ and between $f_1$ and $\bar{f}_2$. These considerations are applied to the “gold–plated” reaction $H \rightarrow ZZ \rightarrow \mu^+\mu^-\mu^+\mu^-$, including possible effects of $CP$–violation in the $HZZ$ coupling. Our formalism also yields the energy spectra and correlations of leptons in the decay $H \rightarrow W^+W^- \rightarrow l^+\nu_ll^-\bar{\nu}_l$. 

1


1 Introduction

One of the distinctive signatures of a Higgs particle with mass $m_H > 2m_Z$ is the sequential decay

$$H \rightarrow ZZ \rightarrow (\mu^+ \mu^-) + (\mu^+ \mu^-).$$

(1)

The observation of such a four–muon final state, consisting of two $\mu^+ \mu^-$ pairs with the invariant mass of the $Z$, together resonating at some invariant mass $m_H$, would be unmistakable evidence for a particle with the prima facie characteristics of the Higgs boson [1].

In the standard model, the Higgs boson has the quantum numbers $J^{PC} = 0^{++}$, and a very specific form of coupling to gauge bosons, namely $g_{\mu \nu} \varepsilon_1^\mu \varepsilon_2^\nu$. (We denote the polarization vectors and momenta of the two $Z$ bosons by $(\varepsilon_1, p_1)$ and $(\varepsilon_2, p_2)$.) More generally, a scalar $0^{++}$ particle can couple to a pair of $Z$’s according to $(B g_{\mu \nu} + \frac{C}{m_Z^2} p_{1 \mu} p_{2 \nu}) \varepsilon_1^\mu \varepsilon_2^\nu$. In extended gauge models, there are also pseudoscalar Higgs particles with quantum numbers $0^{-+}$; these can have radiatively induced couplings to two $Z$ bosons of the form $D_{\mu \rho \sigma} \varepsilon_1^\mu \varepsilon_2^\nu \varepsilon_1^\rho \varepsilon_2^\sigma$ [2, 3, 4].

The purpose of this paper is to show that in decays of the type

$$H \rightarrow ZZ \rightarrow (f_1 \bar{f}_1) + (f_2 \bar{f}_2),$$

(2)

of which Eq. (1) is an example, the energy spectrum of the final fermions, in the rest frame of the $H$, is a simple and powerful probe of the $HZZ$ vertex. This is highlighted by the following result (to be obtained in Section 3): For a standard Higgs boson, the energy spectrum of any final fermion (independent of whether it is a lepton, $u$–quark or $d$–quark) has the universal form

$$\frac{1}{\Gamma} \frac{d\Gamma}{dx} = \frac{3/(2\beta)}{3 - 2\beta^2 + 3\beta^4} \left[1 + \beta^4 - 2(x - 1)^2\right]$$

(Scalar Higgs),

(3)
where \( x = 4E/m_H \), \( \beta = \sqrt{1 - 4m_Z^2/m_H^2} \) and the range of \( x \) is \( 1 - \beta \leq x \leq 1 + \beta \).

By contrast, for a pseudoscalar Higgs boson, the spectrum is

\[
\frac{1}{\Gamma} \frac{d\Gamma}{dx} = \frac{3}{8\beta^3} \left[ \beta^2 + (x - 1)^2 \right] \quad \text{(Pseudoscalar Higgs)}.
\]

This difference between \( 0^{++} \) and \( 0^{-+} \) Higgs decays is exhibited in Fig. 1, for \( m_H = 300 \) GeV.

We will derive in this paper the energy spectrum of the reaction (2) for a general \( HZZ \) coupling. In addition to single particle spectra, we will obtain the correlated two–particle energy distribution of \( f_1 \) and \( f_2 \), and of \( f_1 \) and \( \bar{f}_2 \). In Section 3, we consider Higgs couplings of the scalar and pseudoscalar form, as well as \( CP \)–violating effects when both are present simultaneously. In Section 4, we will relate the energy characteristics of the reactions (1) and (2) to the helicity wave–function of the \( ZZ \) system created in the decay \( H \to ZZ \).

Our work is complementary to other analyses devoted to the sequential decays \( H \to ZZ \to (f_1\bar{f}_1) + (f_2\bar{f}_2) \), in which the structure of the \( HZZ \) coupling is probed by means of the angular distribution of the final particles, particularly the correlation between the \( f_1\bar{f}_1 \) and \( f_2\bar{f}_2 \) planes \([2] - [4]\). Our formalism allows us also to obtain the energy spectrum and correlation of leptons produced in the decay \( H \to W^+W^- \to l^+\nu_ll^-\bar{\nu}_l \), which we have reported earlier \([10]\).

## 2 Differential Decay Rate

Consider the sequential decay

\[
H(P) \to Z(p_1)Z(p_2) \to f_1(q_1)\bar{f}_1(q_2)f_2(q_3)\bar{f}_2(q_4)
\]

induced by a general \( HZZ \) coupling

\[
A[H \to Z(\varepsilon_1,p_1) + Z(\varepsilon_2,p_2)] = 2im_Z^2 \sqrt{G_F} \sqrt{2} \left[ Bg_{\mu\nu} + \frac{C}{m_Z^2} p_{1\mu}p_{2\nu} \right].
\]
The differential decay rate is given by

\[
    d^8 \Gamma = \frac{8\sqrt{2}G^4_F m_Z^4 D_Z}{m_H} (v_1^2 + a_1^2)(v_2^2 + a_2^2) \left[ |B|^2 S + \frac{|C|^2}{m_Z^4} \mathcal{L} + \frac{\text{Re}(B^*C)}{m_Z^2} \mathcal{M} ight.
    + \frac{\text{Im}(B^*C)}{m_Z^2} \mathcal{N} + \frac{|D|^2}{m_Z^2} \mathcal{P} + \frac{\text{Re}(B^*D)}{m_Z^2} \mathcal{Q} + \frac{\text{Im}(B^*D)}{m_Z^2} \mathcal{R} 
    \left.+ \frac{\text{Re}(C^*D)}{m_Z^4} \mathcal{U} + \text{Im}(C^*D) \mathcal{V} \right] \cdot d\text{Lips},
\]

where, neglecting fermion masses,

\[
    S = (q_1 \cdot q_3)(q_2 \cdot q_4) + (q_1 \cdot q_4)(q_2 \cdot q_3) + \xi_1 \xi_2 \left( (q_1 \cdot q_3)(q_2 \cdot q_4) - (q_1 \cdot q_4)(q_2 \cdot q_3) \right),
\]

\[
    \mathcal{L} = 2 \left( \frac{m_Z^2}{4} \right) \left( (p_1 \cdot q_3)(p_1 \cdot q_4) - \frac{m_Z^2}{4} \right),
\]

\[
    \mathcal{M} = (p_2 \cdot q_1) \left( (p_1 \cdot q_3)(q_2 \cdot q_4) + (p_1 \cdot q_4)(q_2 \cdot q_3) \right) 
    + (p_2 \cdot q_2) \left( (p_1 \cdot q_3)(q_1 \cdot q_4) + (p_1 \cdot q_4)(q_1 \cdot q_3) \right) - \frac{m_Z^4}{4} (p_1 \cdot p_2) 
    + \xi_1 \xi_2 \left[ (p_1 \cdot p_2) \left( (q_1 \cdot q_3)(q_2 \cdot q_4) - (q_1 \cdot q_4)(q_2 \cdot q_3) \right) 
    - \frac{m_Z^4}{4} \left( (q_1 - q_2) \cdot (q_3 - q_4) \right) \right],
\]

\[
    \mathcal{N} = \varepsilon(q_1, q_2, q_3, q_4) \left[ \xi_1 \left( p_1 \cdot (q_1 - q_3) \right) + \xi_2 \left( p_2 \cdot (q_2 - q_1) \right) \right],
\]

\[
    \mathcal{P} = -\frac{m_Z^8}{8} - 2 \left( (q_1 \cdot q_3)(q_2 \cdot q_4) - (q_1 \cdot q_4)(q_2 \cdot q_3) \right)^2 
    + \frac{m_Z^4}{8} \left[ \left( (q_1 \cdot q_3) + (q_2 \cdot q_4) \right)^2 + \left( (q_1 \cdot q_4) + (q_2 \cdot q_3) \right)^2 \right] 
    + \xi_1 \xi_2 \frac{m_Z^4}{4} \left[ \left( (q_1 - q_3) - (q_2 - q_4) \right)^2 - \left( (q_1 - q_4) - (q_2 - q_3) \right)^2 \right],
\]

\[
    \mathcal{Q} = \varepsilon(q_1, q_2, q_3, q_4) \left[ \left( (q_1 - q_2) \cdot (q_1 - q_3) \right) - \xi_1 \xi_2 (p_1 \cdot p_2) \right],
\]

\[
    \mathcal{R} = (\xi_1 + \xi_2) \left[ \left( (q_1 \cdot q_3) - (q_2 \cdot q_4) \right) \left( \frac{m_Z^4}{4} + (q_1 \cdot q_3)(q_2 \cdot q_4) - (q_1 \cdot q_4)(q_2 \cdot q_3) \right) \right] 
    + (\xi_1 - \xi_2) \left[ \left( (q_1 \cdot q_4) - (q_2 \cdot q_3) \right) \left( \frac{m_Z^4}{4} + (q_1 \cdot q_4)(q_2 \cdot q_3) - (q_1 \cdot q_3)(q_2 \cdot q_4) \right) \right],
\]

\[
    \mathcal{U} = \varepsilon(q_1, q_2, q_3, q_4) \left[ (p_1 \cdot (q_3 - q_4))(p_2 \cdot (q_2 - q_1)) - \xi_1 \xi_2 \left( (p_1 \cdot p_2)^2 - m_Z^4 \right) \right],
\]
\[ \nu = \left[ \xi_1(p_1 \cdot (q_3 - q_4)) + \xi_2(p_2 \cdot (q_1 - q_2)) \right] \left[ \frac{m_Z^4}{4} \left( (q_1 - q_2) \cdot (q_4 - q_3) \right) \right. \\
\left. + (p_1 \cdot p_2) (q_1 \cdot q_3)(q_2 \cdot q_4) - (q_1 \cdot q_4)(q_2 \cdot q_3) \right], \]  
(8)

and \( dLips \) is the Lorentz invariant phase space element

\[ dLips = (2\pi)^4 \delta^{(4)}(P - q_1 - q_2 - q_3 - q_4) \prod_{i=1}^{4} \frac{d^3q_i}{(2\pi)^32q_i^0}. \]  
(9)

The factor \( D_Z \) in Eq. (7) is the product of the two \( Z \) boson propagators,

\[ D_Z = m_Z^4 \prod_{j=1}^{2} \frac{1}{(p_j^2 - m_Z^2)^2 + m_Z^2 \Gamma_Z^2}, \]  
(10)

which, in the narrow–width approximation, may be written as

\[ D_Z \approx \frac{\pi^2 m_Z^2}{\Gamma_Z^2} \delta(p_1^2 - m_Z^2) \delta(p_2^2 - m_Z^2). \]  
(11)

The parameters \( \xi_1 \) and \( \xi_2 \) are given by

\[ \xi_i = \frac{2v_ia_i}{v_i^2 + a_i^2}, \quad i = 1, 2 \]  
(12)

where \( v_i \) and \( a_i \) are the vector and axial vector coupling constants of the fermion pair \( f_i \bar{f}_i \) to \( Z \) \((v_f = 2I_f^3 - 4e_f \sin^2 \Theta_W, a_f = 2I_f^3)\). Eq. (7) is the basis of all the results to be obtained in the following Sections.

### 3 Energy Spectra and Correlations: Scalar vs. Pseudoscalar Higgs and \( CP \)–Violation

We consider an \( HZZ \) coupling that is a combination of \( B \)– and \( D \)–type terms in Eq. (6), with \( C = 0 \). This will enable us to compare a scalar Higgs \((B \neq 0, D = 0)\) with a pseudoscalar Higgs \((B = 0, D \neq 0)\), as well as discuss certain \( CP \)–violating effects arising from simultaneous presence of both terms.
For the reaction $H \to ZZ \to (f_1\bar{f}_1) + (f_2\bar{f}_2)$, with $f_1 \neq f_2$, we obtain the following energy distributions for the pairs $(f_1, f_2), (f_1, \bar{f}_2), (\bar{f}_1, f_2)$ and $(f_1, \bar{f}_2)$:

\[
\frac{1}{\Gamma} \frac{d\Gamma}{dx(f_1)dx'(f_2)} = \frac{1}{N} \left\{ |B|^2 (F_1 + \xi_1\xi_2F_2) + |D|^2 (F_3 + \xi_1\xi_2F_4) \right. \\
+ \text{Im}(B^*D) \left[ \xi_1(F_5 + F_6) + \xi_2(F_5 - F_6) \right] \right\},
\]

\[
\frac{1}{\Gamma} \frac{d\Gamma}{dx(f_1)dx'(f_2)} = \frac{1}{N} \left\{ |B|^2 (F_1 - \xi_1\xi_2F_2) + |D|^2 (F_3 - \xi_1\xi_2F_4) \right. \\
- \text{Im}(B^*D) \left[ \xi_1(F_5 + F_6) - \xi_2(F_5 - F_6) \right] \right\},
\]

\[
\frac{1}{\Gamma} \frac{d\Gamma}{dx(f_1)dx'(f_2)} = \frac{1}{N} \left\{ |B|^2 (F_1 - \xi_1\xi_2F_2) + |D|^2 (F_3 - \xi_1\xi_2F_4) \right. \\
+ \text{Im}(B^*D) \left[ \xi_1(F_5 + F_6) - \xi_2(F_5 - F_6) \right] \right\},
\]

(13)

with the normalization factor

\[
N = |B|^2 (3 - 2\beta^2 + 3\beta^4) + 8|D|^2\beta^2.
\]

(14)

The functions $F_i$ are defined as follows:

\[
F_1(x, x') = \frac{9}{32\beta^6} \left\{ (1 - \beta^2)^2 \left[ \beta^2 + (x - 1)^2 \right] \left[ \beta^2 + (x' - 1)^2 \right] \\
+ 2(1 + \beta^2)^2 \left[ \beta^2 - (x - 1)^2 \right] \left[ \beta^2 - (x' - 1)^2 \right] \right\},
\]

\[
F_2(x, x') = \frac{9}{8\beta^4} (1 - \beta^2)^2 (x - 1)(x' - 1),
\]

\[
F_3(x, x') = \frac{9}{8\beta^4} \left[ \beta^2 + (x - 1)^2 \right] \left[ \beta^2 + (x' - 1)^2 \right],
\]

\[
F_4(x, x') = 4\beta^2 F_2(x, x')/(1 - \beta^2)^2,
\]

\[
F_5(x, x') = \frac{9(1 - \beta^2)}{8\beta^4} (x + x' - 2) [(x - 1)(x' - 1) + \beta^2],
\]

\[
F_6(x, x') = \frac{9(1 - \beta^2)}{8\beta^4} (x' - x) [(x - 1)(x' - 1) - \beta^2].
\]

(15)

Note that $F_1, \ldots, F_5$ are symmetric under exchange of $x$ and $x'$, while $F_6$ is antisymmetric.
By integrating the two–particle distributions (13) over the energies of one of the
particles, we obtain the inclusive energy spectrum of any fermion $f$ in the decay
$H \rightarrow ZZ \rightarrow f\bar{f} + \cdots$:

$$\frac{1}{\Gamma} \frac{d\Gamma}{dx(f)} = \frac{3}{2\beta N} \left\{ |B|^2 [1 + \beta^4 - 2(x - 1)^2] + 2|D|^2 [\beta^2 + (x - 1)^2] \right. \quad \left. + 4\text{Im}(B^*D)\xi(1 - \beta^2)(x - 1) \right\}.$$  \hfill (16)

For a pure scalar ($B$–type) or pseudoscalar ($D$–type) Higgs coupling, we obtain the
results already announced in Eqs. (3) and (4).

Although the two–particle distributions given in Eq. (13) were derived for the
reaction $H \rightarrow ZZ \rightarrow (f_1\bar{f}_1) + (f_2\bar{f}_2)$ with $f_1 \neq f_2$, the results are also applicable
to the decay $H \rightarrow ZZ \rightarrow (\mu^+\mu^-) + (\mu^+\mu^-)$. The only requirement is that the two
observed muons belong to different $Z$’s. This is automatically the case for like sign
pairs $\mu^+\mu^+$ or $\mu^-\mu^-$, and is easy to ensure for $\mu^+\mu^-$ by requiring that their invariant
mass be different from $m_Z$. (Note that a $\mu^+\mu^-$ pair from the same $Z$ must fulfil
$x + x' = 2$.) With this proviso the correlated energy distributions for $\mu^+\mu^+$, $\mu^-\mu^-$
and $\mu^+\mu^-$ are

$$\frac{1}{\Gamma} \frac{d\Gamma}{dx(\mu^+)}dx'(\mu^\pm) = \frac{1}{N} \left[ |B|^2 (F_1 + \xi^2 F_2) + |D|^2 (F_3 + \xi^2 F_4) \mp 2\text{Im}(B^*D)\xi F_5 \right],$$

$$\frac{1}{\Gamma} \frac{d\Gamma}{dx(\mu^+)}dx'(\mu^\mp) = \frac{1}{N} \left[ |B|^2 (F_1 - \xi^2 F_2) + |D|^2 (F_3 - \xi^2 F_4) \mp 2\text{Im}(B^*D)\xi F_6 \right].$$ \hfill (17)

Here $\xi = 2va/(v^2 + a^2) \approx 0.16$ is the parameter describing the $Z$ coupling to muons.

From Eqs. (17), we draw the following conclusions.

(i) For a scalar Higgs coupling ($D = 0$), the like sign muon pairs $\mu^+\mu^+$ and $\mu^-\mu^-$
have the energy distribution

$$\frac{1}{\Gamma} \frac{d\Gamma}{dx(\mu^+)}dx'(\mu^\pm) = \frac{1}{3 - 2\beta^2 + 3\beta^4} \left[ F_1(x, x') + \xi^2 F_2(x, x') \right] \quad \text{(Scalar Higgs)}. \hfill (18)$$
This is quite distinct from the pseudoscalar Higgs decay, given by

\[
\frac{1}{\Gamma} \frac{d\Gamma}{dx(\mu^{\pm})dx'(\mu^{\pm})} = \frac{1}{8\beta^2} \left[ F_3(x, x') + \xi^2 F_4(x, x') \right] \quad \text{(Pseudoscalar Higgs).} \tag{19}
\]

These two distributions are contrasted in Fig. 2.

(ii) The unlike sign dimuon \( \mu^+\mu^- \) spectra are likewise quite different for \( 0^{++} \) and \( 0^{-+} \) Higgs decays. Indeed, the \( \mu^+\mu^- \) spectra differ from the \( \mu^\pm\mu^\pm \) given in Eqs. (18) and (19) only in the replacement \( \xi^2 \to -\xi^2 \). Since \( \xi^2 \) is approximately \( 2.56 \times 10^{-2} \), the two–dimensional distributions for unlike sign \( \mu \)’s are very similar to those of the like sign muon pairs, shown in Fig. 2.

(iii) The simultaneous presence of \( B^-\) and \( D^-\)–type couplings produces a \( CP\)–violating asymmetry between \( \mu^+\mu^+ \) and \( \mu^-\mu^- \). There is also an asymmetry between the single particle \( \mu^+ \) and \( \mu^- \) spectra, that can be read off Eq. (16):

\[
A = \frac{d\Gamma/dx(\mu^-) - d\Gamma/dx(\mu^+)}{d\Gamma/dx(\mu^-) + d\Gamma/dx(\mu^+)}
= \frac{4\xi(x-1)(1-\beta^2)}{|B|^2 \left(1 + \beta^4 - 2(x-1)^2\right) + 2|D|^2 \left[\beta^2 + (x-1)^2\right]}.
\tag{20}
\]

It should be noted that all of these asymmetries are odd under \( CP \) but even under \( T \). Thus they require a non–zero phase difference between the amplitudes \( B \) and \( D \), induced by final state interactions. This is evident from the appearance of the factor \( \text{Im}(B^*D) \) in the asymmetric terms in Eq. (17), and in Eq. (20). The asymmetries are also proportional to the parameter \( \xi \), which is approximately 0.16 for muon pairs.
4 Energy Spectra and Correlations: Relation to Helicity Structure of $H \rightarrow ZZ$ Amplitude

Quite generally, the decay $H \rightarrow ZZ$ produces a system of two $Z$ bosons in the helicity state

$$|ZZ\rangle = c_+|++\rangle + c_-|--\rangle + c_0|00\rangle. \quad (21)$$

Couplings of the form $B$, $C$ and $D$ (Eq. (6)) give rise to the following characteristic helicity wave–functions:

$$B g_{\mu\nu} \varepsilon_1^\mu \varepsilon_2^\nu : |++\rangle + |--\rangle + \frac{1 + \beta^2}{1 - \beta^2} |00\rangle$$

$$C \frac{p_2 p_1 \varepsilon_1^\mu \varepsilon_2^\nu}{m_Z^2} : |00\rangle \quad (22)$$

$$D \frac{p_1^\rho p_2^\sigma \varepsilon_1^\mu \varepsilon_2^\nu}{m_Z^2} : |++\rangle - |--\rangle.$$

Notice that the standard Higgs boson coupling generates a transversely polarized state $|++\rangle + |--\rangle$ mixed with a specific amount of longitudinal polarization $|00\rangle$. In the high energy limit $\beta \rightarrow 1$, the longitudinal component dominates. In comparison, a pseudoscalar Higgs decays into a transversely polarized state $|++\rangle - |--\rangle$. A scalar coupling of the form $C \frac{p_1 p_2 \varepsilon_1^\mu \varepsilon_2^\nu}{m_Z^2}$ generates a state of longitudinal polarization $|00\rangle$ only.

It is possible to relate the energy distributions derived in the preceding Section to the helicity structure of the $H \rightarrow ZZ$ amplitude. To this end, it is important to note (i) that, as far as the energy distributions are concerned, the transverse and longitudinal components of the $ZZ$ wave–function add incoherently; (ii) that the energy spectra do not distinguish between the states $|++\rangle + |--\rangle$ and $|++\rangle - |--\rangle$. In the $CP$–invariant limit, therefore, the energy distribution of the final fermion system can be written as follows:
(a) One–dimensional distribution

\[
\frac{1}{\Gamma} \frac{d\Gamma}{dx} = P_T f_T(x) + P_L f_L(x),
\]

where \( P_T \) and \( P_L \) are the probabilities for transverse and longitudinal polarization in the ZZ wave–function \((P_T + P_L = 1)\), and \( x \) is the scaled energy \((x = 4E/m_H)\) of any of the final fermions in \( H \rightarrow ZZ \rightarrow (f_1\bar{f}_1) + (f_2\bar{f}_2)\). The functions \( f_T \) and \( f_L \) are given by

\[
\begin{align*}
  f_T(x) &= \frac{3}{8\beta^3} \left[ \beta^2 + (x - 1)^2 \right], \\
  f_L(x) &= \frac{3}{4\beta^3} \left[ \beta^2 - (x - 1)^2 \right].
\end{align*}
\]

(b) Two–dimensional distribution

\[
\frac{1}{\Gamma} \frac{d\Gamma}{dx \, dx'} = P_T \left[ f_T(x) f_T(x') \pm \xi_1 \xi_2 g_T(x) g_T(x') \right] + P_L \left[ f_L(x) f_L(x') \right],
\]

with

\[
g_T(x) = \frac{3}{4\beta^2} (x - 1).
\]

In Eq. (25), the + sign applies to a pair \((f_1, f_2)\) or \((\bar{f}_1, \bar{f}_2)\), while the – sign applies to a pair \((f_1, \bar{f}_2)\). It is easy to see that Eqs. (23) to (26) reproduce the results (13) and (16), in the limit of \( CP \)–conservation.

It is clear from the structure of Eq. (25) that, for \( P_T \neq 0 \), the two–dimensional distribution is not simply a product of one–dimensional spectra. The term proportional to \( g_T(x) g_T(x') \) is indicative of the fact that the transverse helicities of the two \( Z \)'s are \emph{correlated}. On the other hand, for \( P_T = 0 \), the two–particle distribution factorises into a product of one–particle spectra. The three types of coupling in Eq.
are obviously characterised by:

\[
\begin{align*}
B - \text{type} : & \quad P_T = \frac{2(1 - \beta^2)^2}{3 - 2\beta^2 + 3\beta^4}, \quad P_L = \frac{(1 + \beta^2)^2}{3 - 2\beta^2 + 3\beta^4} \\
C - \text{type} : & \quad P_T = 0, \quad P_L = 1 \\
D - \text{type} : & \quad P_T = 1, \quad P_L = 0.
\end{align*}
\]

(27)

5 Remarks on the Decay \(H \to W^+W^- \to \mu^+\nu\mu^-\bar{\nu}_\mu\)

Our results for the sequential decay \(H \to ZZ \to \mu^+\mu^-\mu^+\mu^-\) are immediately adaptable to the reaction

\[
H \to W^+W^- \to \mu^+\nu\mu^-\bar{\nu}_\mu.
\]

(28)

One has only to put \(\xi_{1,2} = 1\) in Eqs. (13) or (16), and interpret \(\beta\) as \(\sqrt{1 - 4m_W^2/m_H^2}\). If the \(W^+W^-\) state is characterised by probabilities \(P_T\) and \(P_L\) for transverse and longitudinal polarization, the one–particle spectrum of \(\mu^+\) or \(\mu^-\) (in the \(CP\)-conserving limit) is identical to that produced by \(H \to ZZ\), namely the spectrum given by Eq. (23). On the other hand the correlated \(\mu^+\mu^-\) distribution in reaction (28) differs significantly from that in \(H \to ZZ \to \mu^+\mu^-\mu^+\mu^-\), since it is obtained from Eq. (25) with \(\xi_1\xi_2 = 1\) instead of \(\xi_1\xi_2 \approx 2.56 \times 10^{-2}\). This gives rise to a marked difference between the two reactions as illustrated in Fig. 3, for the case of a standard Higgs boson, and in Fig. 4 for the case of a pseudoscalar Higgs. Our results concerning the energy distribution of the secondary leptons in \(H \to W^+W^- \to l^+\nu_l l^-\bar{\nu}_l\) coincide with those that we have reported in an earlier paper [10].
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References

[1] J.F. Gunion, H.E. Haber, G. Kane and S. Dawson, *The Higgs Hunter’s Guide* (Addison–Wesley Publishing Company, 1990)

[2] C.A. Nelson, Phys. Rev. D37 (1988) 1220

[3] A. Soni and R.M. Xu, Phys. Rev. D48 (1993) 5259

[4] D. Chang and W.–Y. Keung, Phys. Lett. B305 (1993) 261

[5] D. Chang, W.–Y. Keung and I. Phillips, Phys. Rev. D48 (1993) 3225

[6] A. Skjold and P. Osland, Phys. Lett. B311 (1993) 261; Phys. Lett. B329 (1994) 305

[7] V. Barger, K. Cheung, A. Djouadi, B.A. Kniehl and P.M. Zerwas, Phys. Rev. D49 (1994) 79; M. Krämer, J. Kühn, M.L. Stong and P.M. Zerwas, DESY preprint 93-174

[8] T. Matsuura and J.J. van der Bij, Z. Phys. C51 (1991) 259

[9] A. Djouadi and B.A. Kniehl, in *e^+e^- Collisions at 500 GeV: The Physics Potential*, Proceedings of the Workshop – Munich, Annecy, Hamburg, edited by P.M. Zerwas (DESY report 93-123C)

[10] T. Arens, U.D.J. Gieseler and L.M. Sehgal, “Energy correlation and asymmetry of secondary leptons originating in $H \rightarrow t\bar{t}$ and $H \rightarrow W^+W^-$”, Aachen preprint PITHA 94/25, to appear in Phys. Lett. B
Figure Captions

Fig. 1. Single particle energy spectra of a fermion \( f \) in the decay \( H \to ZZ \to f + \cdots \).

The full curve represents the scalar case and the dashed curve the pseudoscalar case, for \( m_H = 300 \) GeV.

Fig. 2. Normalized energy distribution of like sign muon pairs \( \mu^+\mu^+ \) or \( \mu^-\mu^- \) in the decay \( H \to ZZ \to \mu^+\mu^-\mu^+\mu^- \). Fig. 2(a) shows the spectrum of a standard model Higgs boson, and Fig. 2(b) that of a pseudoscalar Higgs, with \( m_H = 300 \) GeV.

Fig. 3. Normalized energy distribution of unlike sign muon pairs \( \mu^+\mu^- \) in the decay of a standard model Higgs boson with mass \( m_H = 200 \) GeV. (a) \( \mu^+\mu^- \) distribution in \( H \to ZZ \to \mu^+\mu^-\mu^+\mu^- \), with \( \mu^+ \) and \( \mu^- \) chosen from different Z’s; (b) \( \mu^+\mu^- \) distribution in \( H \to W^+W^- \to \mu^+\nu_\mu\mu^-\bar{\nu}_\mu \).

Fig. 4. Normalized energy distribution of unlike sign \( \mu^+\mu^- \) pairs in the decay of a pseudoscalar Higgs boson (\( m_H = 200 \) GeV). (a) \( \mu^+\mu^- \) distribution in \( H \to ZZ \to \mu^+\mu^-\mu^+\mu^- \), with \( \mu^+ \) and \( \mu^- \) chosen from different Z’s; (b) \( \mu^+\mu^- \) distribution in \( H \to W^+W^- \to \mu^+\nu_\mu\mu^-\bar{\nu}_\mu \).
\( \Gamma^{-1} \frac{d\Gamma}{dx} (H \rightarrow ZZ \rightarrow f + ...) \)

Fig. 1.

\( m_H = 300 \text{ GeV} \)
H → ZZ → μ⁺μ⁻μ⁺μ⁻

(scalar)

$m_H = 300 \text{ GeV}$

Fig. 2(a).
$H \rightarrow ZZ \rightarrow \mu^+ \mu^- \mu^+ \mu^-$ (pseudoscalar)

$m_H = 300 \text{ GeV}$

Fig. 2(b).
$H \rightarrow ZZ \rightarrow \mu^+ \mu^- \mu^+ \mu^-$

$(\text{scalar})$

$m_H = 200 \text{ GeV}$

Fig. 3(a).
$H \rightarrow W^+ W^- \rightarrow \mu^+ \nu \bar{\mu} \bar{\nu}$

$\text{(scalar)}$

$m_H = 200 \text{ GeV}$

Fig. 3(b).
$H \rightarrow ZZ \rightarrow \mu^+\mu^-\mu^+\mu^-$

$\text{(pseudoscalar)}$

$m_H = 200 \text{ GeV}$

Fig. 4(a).
\[ H \rightarrow W^+ W^- \rightarrow \mu^+ \nu_{\mu} \mu^- \bar{\nu}_{\mu} \]

(\text{pseudoscalar})

\[ m_H = 200 \text{ GeV} \]

Fig. 4(b).