Multifaceted Evaluation of 2-Level Supersaturated Designs

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Abstract:
Experimental design is a statistical technique used in the field of quality control. It is applied to determine the relationships between intended results and the factors considered to influence them. It is a fractional factorial design that is suitable when there are many factors considered to influence the result.

Recently, many supersaturated designs have been studied. These are experimental designs, which can be assigned more factors than the number of experiments. The technique is applied to reduce the number of influencing factors from the total set.

Previous studies focused on structure designs and their respective analyses. Each constitution method is evaluated absolutely particularly. Consequently, prior to this study, the proposed designs had not been systematically evaluated. This paper examines some previously proposed supersaturated designs with the use of several indices. It presents a selection guide that describes recommended constitution methods.

Keywords
Experimental design, Screening designs, Orthogonality

1. Introduction

Statistical quality control helps to ensure that sound products are produced. We base our judgments on data analyses using statistical techniques and on the quantitative results obtained.

One of the statistical methods frequently used in quality control is experimental design, which is used to investigate the relationship between results and the factors that may influence them. When many factors are being considered, the fractional factorial design technique can be applied. Key factors can be identified while reducing the number of experiments by using it. One of the fractional factorial designs is a 2-level supersaturated design, where the number of factors that can be assigned is greater than the number of experiments. Supersaturated design has been recently studied as it is expected to help reduce the total number of factors to identify the key influencing factors.

Satterwaite (1959) presented a basic idea about supersaturated designs, and Booth and Cox (1962) presented how to construct the designs systematically. These studies were actively referenced with respect to the construction of supersaturated designs of Lin (1993), where the number of experiments \( n = N/2 \) and columns \( k = N - 2 \) were made from the \( N \)-th order Hadamard matrix. Wu (1993) proposed a construction method that adds interaction columns to the Hadamard matrix, and Iida (1994) proposed modifying the aforementioned construction method in greater detail. Nyugen (1996) proposed a construction method based on balanced incomplete block design (BIBD). Yamada and Lin (1997) described a supersaturated design method with \( 2n \) number of experiments and \( 2n + 2k - 1 \) columns. This method is made from an orthogonal basis with \( n \) experiments and \( n - 1 \) columns and a matrix with \( n \) experiments and \( k \) columns. Cheng (1997) showed lower bound of \( E(s^2) \), and Butler et al. (2001) and Bulutoglu and Cheng (2004) improved it. Yamada and Lin (1999) and Yamada et al. (1999) expanded designs to 3-level ones, and Fang et al. (2000) and Fang et al. (2003) expanded designs to multi-level ones. Furthermore, previous studies have analyzed supersaturated designs. Cossari (2008) used a Box–Meyer method for screening active factors in supersaturated designs. Phoa et al. (2009) used the Dantzing selector to analyze supersaturated designs. In addition, Georgiou (2014) reviewed supersaturated designs.

The main studies only described the construction method associated with the designs. There has been no
research on the evaluation or comparison of each construction method with the use of some evaluation indicators. Here we consider 2-level supersaturated designs constructed in previous studies using some indicators. In addition, we present a selection guide of 2-level supersaturated designs. In section 2, we explain 2-level supersaturated designs. In section 3, we mention about evaluation indicators of 2-level supersaturated designs. In section 4, we evaluate and consider 2-level supersaturated designs using some indicators. In section 5, we describe the conclusion and future works.

2. 2-level supersaturated designs

2.1 2-level supersaturated designs

Two-level supersaturated designs are designs that can be assigned more factors than orthogonal designs. Because supersaturated designs can treat a lot of potential factors in small experiments, these are effective as screening designs.

Supersaturated designs differ from orthogonal designs in terms of non-orthogonality. For example, an $8 \times 35$ supersaturated design is constructed on the basis of an orthogonal design. In this design, although orthogonality is established among the first 7 columns, it is not established among the remaining columns. Therefore, it is possible that the results of data analysis are different when factors are assigned for the first 7 columns and for the remaining columns.

2.2 Evaluation methods of 2-level supersaturated designs

This paper evaluates 2-level supersaturated designs using 3 evaluation indices: $E(s^2)$, $D_f$, and $A_f$. In addition, this paper describes the application of the design matrix $X$, the number of experiments (the number of rows) $n$, and the number of columns $k$. The column vector $X (x_1, ..., x_k)$ and the elements of $X$ (i.e., 1 or $-1$) are also described. Moreover, the number 1 is the same as $-1$ in the design matrix $X$.

2.2.1 $E(s^2)$

The index $E(s^2)$ is the average of squares of the inner product and a measure of orthogonality between 2 columns of designs. It is defined by equation (1):

$$E(s^2) = \frac{\sum_{i < j} s_{ij}^2}{\binom{k}{2}},$$

where $s_{ij}$ represents the inner product of $x_i$ and $x_j$. The index $E(s^2)$ is most frequently used as an evaluation index of 2-level supersaturated designs. When this value approaches 0, it indicates that the design is closer to orthogonality.

This paper will compare $E(s^2)$ of designs constructed from various methods for each experiment. It is assumed that factors are assigned from the left column in order. In addition, the study considers whether $E(s^2)$ of each supersaturated designs will change if the number of factors that are assigned increases.

2.2.2 $D$-optimality

The index $D_f$ is a measure that is focused on the orthogonality between 2 columns in designs. Wu (1993) used $D_f$ and $A_f$ as scales to simultaneously measure the relationship among 3 or more columns. Here $f$ represents the number of columns we want to use to measure the relationship. $D_f$ is a measure of the $D$-optimality and indicates the accuracy of the estimation. Wu (1993) defined it using the following equation:

$$D_f = \frac{\sum_{i=1}^{f} \left| X_i^T X_i \right|^{1/f}}{\binom{k}{f}},$$

where $X_i$ is the $n \times f$ partial matrix of the design of interest. $D_f$ shows the effect when $f$-number of factors are randomly assigned to the design.

2.2.3 $A$-optimality

The index $A_f$ is defined using the following equation:

$$A_f = \frac{\sum_{i=1}^{f} \left( \frac{1}{n} X_i^T X_i \right)^{-1}}{\binom{k}{f}}.$$ 

The index $A_f$ is a measure of the $A$-optimality and indicates the accuracy of the forecast. $A_f$, as with $D_f$, shows the effect when $f$-number of factors are randomly assigned to the design.
3. Construction methods of 2-level supersaturated designs

This paper constructs and evaluates 2-level supersaturated designs using the construction methods proposed in previous studies. This chapter describes how the construction methods were applied and the resultant designs that were constructed. This paper compares 2-level supersaturated designs constructed by Lin (1993), Wu (1993), Iida (1994), Nguyen (1996) and Yamada and Lin (1997) because they are constructed uniquely.

3.1 Construction methods by Lin (1993)

Lin (1993) constructed 2-level supersaturated designs with \( n = N/2 \) and \( k = N - 2 \), from an \( N \)-order Hadamard matrix.

First, the first column of the \( N \)-order Haramard matrix is eliminated. Because the level of the first column is only 1, the columns are not necessary in the designs. Following this, the branching column is arbitrarily selected, and the \( N \)-order Haramard matrix without the first column is split into two half fractions according to the branching column whose elements equal 1 or \(-1\). Specifically, rows with level 1 branching column are defined as group 1, and rows with \(-1\) level branching column are defined as group 2. When the branching column from group 1 is removed, group 1 without the branching column becomes a 2-level supersaturated design with \( n = N/2 \) and \( k = N - 2 \). Similarly, if group 2 is used, an equivalent one will be obtained.

3.2 Construction methods by Wu (1993)

Wu (1993) constructed 2-level supersaturated designs by adding interaction columns to 12- and 24-order orthogonal matrices. In this study, 2-level supersaturated designs with \( n = 12 \) and \( k = 66 \) are included as examples. The first 11 columns of the design are the orthogonal matrix and the remaining columns are the interaction columns of the orthogonal matrix.

The 2-level supersaturated design of Wu (1993) has many columns, and many factors can be assigned. However, when there are interactions between 2 factors in this design, there are cases when these factors completely confound the main effect. To address this problem, there are some designs with a different number of columns.

3.3 Construction methods by Iida (1994)

Iida (1994) constructed 2-level supersaturated designs from interaction columns of the basic orthogonal design. To prevent the main effect from being completely confounded by the interaction of factors, he divided 12 columns, i.e., 11 columns of the orthogonal design and the mean, into 2 groups. One column from each group was then chosen and a resultant interaction column was made from this. The division was as follows: the first 2 columns and the later 10 columns, the first 3 columns and the later 9 columns, and so on.

3.4 Construction methods by Nguyen (1996)

Nguyen (1996) proposed how to construct 2-level supersaturated designs from balanced incomplete block design (BIBD). The design is constructed by circulating 2 column vectors.

3.5 Construction methods by Yamada and Lin (1997)

Yamada and Lin (1997) proposed the construction of novel 2-level supersaturated designs from previously proposed designs with an orthogonal base.

The \( i \)-th column of the original design is represented by \( c_i \). In addition, \( 1 \) represents \( n \times 1 \) column vector whose elements are all defined as 1 and \( -1 \) represents \( n \times 1 \) column vector whose elements are all defined as \(-1\). Among the original design matrix, the part of the matrix whose columns are orthogonal to each other is represented by \( C_o \), while the rest of the matrix is represented by \( C_p \).

\[
C_o^n = [c_1 \ldots c_{n-1}]. \quad (4)
\]

\[
C_p^n = [c_n c_{n+1} \ldots c_{n+k-1}]. \quad (5)
\]

Yamada and Lin (1997) constructed new 2-level supersaturated designs and used them as follows:

\[
X = \begin{bmatrix}
1 & C_o^n & C_o^n & C_p^n & C_p^n \\
-1 & -C_o^n & -C_o^n & C_p^n & -C_p^n
\end{bmatrix}. \quad (6)
\]
4. Evaluation results

4.1 Evaluation by $E(s^2)$

Table 1 shows $E(s^2)$ of each of the supersaturated designs.

| $n$ | $k$ | Lin (1993) | Wu (1993) | Iida (1994) | Nguyen (1996) | Yamada and Lin (1997) |
|-----|-----|-------------|------------|-------------|---------------|-----------------------|
| 6   | 10  | 4.00        | 4.00       |             |               |                       |
| 10  | 18  | 5.88        |            |             |               |                       |
| 12  | 16  | 7.53        | 7.47       | 5.88        |               |                       |
| 18  | 22  | 6.86        |            | 6.86        |               |                       |
| 20  | 24  | 9.74        |            | 9.85        |               |                       |
| 27  | 32  | 10.84       |            |             |               |                       |
| 32  | 35  | 11.29       |            |             |               |                       |
| 36  |     |             |            |             |               |                       |
| 66  |     |             |            |             | 11.08         |                       |
| 14  | 26  | 7.84        | 7.84       |             |               |                       |
| 16  | 30  | 8.83        |            |             |               |                       |
| 18  | 34  | 9.82        |             | 9.82        |               |                       |
| 20  | 38  | 10.81       |            |             |               |                       |
| 54  | 58  | 17.02       |            |             |               |                       |
| 22  | 42  | 11.80       |             | 11.80       |               |                       |
| 24  | 46  | 12.80       |             | 12.80       |               |                       |
| 133 |     |             |            |             | 21.65         |                       |
| 276 |     |             |            |             | 23.04         |                       |

A comparison of each of the designs is based on the value of $E(s^2)$. In many cases, the designs of Lin (1993) and Nguyen (1996) were better. Compared with the designs of Lin (1993) and Nguyen (1996), the values associated with the designs of Wu (1993) are slightly worse, although close in value. It seems that the values of $E(s^2)$ of the designs of Iida (1994) are greater than those of the other designs; this is because it was specifically constructed to minimize the influence of confounding main effects and interactions with the main effects. While designs of Yamada and Lin (1997) have advantages (i.e., the assignment of more factors to the matrix than that possible with other designs), the associated values of $E(s^2)$ are lower.

Next, this paper analyzes the transition of $E(s^2)$ when the number of columns increases from the number of experiments to the greatest number of columns.

4.1.1 An example with parameter $n = 10, 14, 18,$ and $22$

When $n = 10, 14, 18,$ or $22$, only 2 designs constructed by Lin (1993) and Nguyen (1996) are suitable. The study compares these 2 designs further. Figure 1 shows the transition of each $E(s^2)$ when $n = 10$.

![Figure 1: Transition of $E(s^2)$ when $n = 10$](image-url)
The index $E(s^2)$ of the designs of Lin (1993) decreases each time the number of columns increases. On the other hand, $E(s^2)$ of the designs of Nguyen (1996) increases each time the number of columns increases. In other words, when the number of factors we want to assign to the designs is close to the maximum number of factors that can be assigned, there is no difference between designs of Lin (1993) and Nguyen (1996). However, when the number of factors we want to assign to the designs is less than the maximum number of factors can be assigned, the designs of Nguyen (1996) are recommended.

### 4.1.2 An example with parameter $n = 12$

When $n = 12$, the designs of Lin (1993), Wu (1993), Iida (1994), and Nguyen (1996) are considered to be suitable. There are several designs that have been proposed by Iida (1994). Figure 2 shows the transition of each $E(s^2)$ when $n = 12$.

![Figure 2: Transition of $E(s^2)$ when $n = 12$](image)

Compared with the other designs, $E(s^2)$ of the designs of Iida (1994) is always large. As the study evaluates the suitability of designs with $E(s^2)$ based on the value of Iida, the designs of Iida (1994) are not considered to be suitable. When the number of factors we want to assign is small, the designs of Wu (1993) are recommended because the first 11 columns are orthogonal to each other. When the number of factors we want to assign is from 17 to 21, there is no significant difference among the designs proposed by Lin (1993), Wu (1993), and Nguyen (1996). We recommend designs of Lin (1993) or Nguyen (1996) when the number of factors to be assigned is 22. We recommend the designs of Wu (1993) when the number of factors to be assigned is greater than 22.

### 4.1.3 An example with parameter $n = 16$ and 20

When $n = 16$ or 20, designs of Nguyen (1996) and Yamada and Lin (1997) can be considered. Figure 3 shows the transition of each $E(s^2)$ when $n = 16$. 

![Figure 3: Transition of $E(s^2)$ when $n = 16$](image)
In both cases, the designs of Yamada and Lin (1997) have a greater number of columns. If the number of factors we want to assign is small, $E(s^2)$ associated with the designs of Yamada and Lin (1997) is smaller. However, as the number of columns increases, $E(s^2)$ associated with the designs of Yamada and Lin (1997) increases. Consequently, the designs of Nguyen (1996) are recommended when the number of factors to be assigned is greater than 21 when $n = 16$ and 30 when $n = 20$.

### 4.1.4 An example with parameter $n = 24$

When $n = 24$, the designs of Lin (1993), Wu (1993), Nguyen (1996), and Yamada and Lin (1997) can be considered. Figure 4 shows the transition of each $E(s^2)$ when $n = 24$.

The designs of Yamada and Lin (1997) have a greater number of columns. If the number of factors we want to assign is small, the designs of Wu (1993) or Yamada and Lin (1997) are recommended. When the number of factors we want to assign is greater than 34, $E(s^2)$ associated with the designs of Yamada and Lin (1997) is very large. Therefore, other designs are recommended. The designs with the smallest $E(s^2)$ are associated with the designs of Wu (1993). Therefore, it is recommended that these designs are used, regardless of the number of factors that will be assigned.
### 4.2 Evaluation by D-optimality

This paper evaluates the effectiveness of the aforementioned designs based on the values of $D_f$. The value of $D_f$ is 1 when the designs are orthogonal. The value decreases with increasing distance from the orthogonal. The number of columns we want to measure the relationship $f$ is 3 or 4. In the graph, the solid line represents $D_f$ when $f = 3$ and the dotted line represents $D_f$ when $f = 4$.

#### 4.2.1 An example with parameter $n = 10, 14, 18,$ and $22$

When $n = 10, 14, 18, \text{ or } 22$, there are only 2 designs of Lin (1993) and Nguyen (1996) that are considered to be suitable. This study compares the 2 aforementioned designs. Figure 5 shows the transition of each $D_f$ when $n = 10$.

![Figure 5: Transition of $D_f$ when $n = 10$](image)

When $n = 10, 14, 18, \text{ or } 22$, for any number of experiments, as the number of columns is reduced, the value of $D_f$ associated with the designs of Lin (1993) decreases and the value of $D_f$ associated with the designs of Nguyen (1996) increases. When the number of columns is most common, $D_f$ has the same value in both designs. Therefore, we recommend using the designs of Nguyen (1996) when the initial designs are used and using the designs of Lin (1993) when almost all the columns of the designs are considered.

#### 4.2.2 An example with parameter $n = 12$

When $n = 12$, designs of Lin (1993), Wu (1993), Iida (1994), and Nguyen (1996) are considered to be suitable. Iida (1994) proposed several designs. Figure 6 shows the transition of each $D_f$. For clarity, the case of $f = 3$ is isolated. In this graph specifically, the solid line represents $D_f$ associated with the designs of Lin (1993), Wu (1993), and Nguyen (1993) and the dotted line represents $D_f$ associated with the designs of Iida (1994).
Compared with other designs mentioned above, $D_F$ associated with the designs of Iida (1994) is always smaller. Based on these evaluation criteria, these designs are not considered to be suitable. When the number of factors we want to assign is 15 or less, the designs of Wu (1993) are recommended because the first 11 columns of the matrix are orthogonal to each other. When the number of factors we want to assign is 16, there is no significant difference among the designs of Lin (1993), Wu (1993), and Nguyen (1996). Consequently, the designs of Lin (1993) or Nguyen (1996) are recommended when the number of factors we want to assign is 22, and the designs of Wu (1993) are recommended when the number of factors we want to assign is greater than 22.

4.2.3 An example with parameter $n = 16$ and 20

When $n = 16$ or 20, the designs of Nguyen (1996) and Yamada and Lin (1997) can be considered. Figure 7 shows the transition of each $D_F$ when $n = 16$.

In both cases, the designs of Yamada and Lin (1997) have more columns. If the number of factors we want to assign is small, $D_F$ associated with the designs of Yamada and Lin (1997) are larger. However, as the number of columns increases, $D_F$ decreases. The designs of Nguyen (1996) are recommended when the number of factors we want to assign is greater than 20 when $n = 16$ and 29 when $n = 20$. 
4.3 Evaluation by A-optimality

This paper evaluates the suitability of designs based on the evaluation of $A_f$. The value of $A_f$ is 1 when the designs are orthogonal. The value becomes larger with increasing distance from the orthogonal. Therefore, if this paper uses $A_f - 1$, the value of orthogonal designs is 0. The number of columns we want to measure the relationship $f$ is 3 or 4. In Figures 8–11, the solid line represents $A_f$ when $f = 3$ and the dotted line represents $A_f$ when $f = 4$.

4.3.1 An example with parameter $n = 10, 14, 18, \text{ and } 22$

When $n = 10, 14, 18, \text{ or } 22$, the designs of Lin (1993) and Nguyen (1996) are considered to be suitable for evaluation. Figures 8 and 9 show the transition of each $A_f$ when $n = 10$ and 14.

![Figure 8: Transition of $A_f$ when $n = 10$](image)

![Figure 9: Transition of $A_f$ when $n = 10$](image)

When $n = 10$ and 22, if the number of columns is the same, $A_f$ has the same value in both designs. However, when the number of columns is small, $A_f$ associated with the designs of Nguyen (1996) is smaller. When $n = 14$ and 18, because $A_f$ associated with the designs of Nguyen (1996) is always small, these designs are recommended.

4.3.2 An example with parameter $n = 12$

When $n = 12$, there are the designs of Lin (1993), Wu (1993), Iida (1994), and Nguyen (1996). There are several designs of Iida (1994). Figure 10 shows transition of each $A_f$. For clarity, it shows the case of $f = 3$ only. In only this graph, the solid line represents $A_f$ of designs of Lin (1993), Wu (1993), and Nguyen (1996) and the dotted line represents $A_f$ of designs of Iida (1994).
Compared with the other designs, $A_f$ associated with the designs of Iida (1994) is always large. Based on these evaluation criteria, these designs is not considered to be suitable. When the number of factors we want to assign is 16 or less, we recommend the designs of Wu (1993) because the first 11 columns of the matrix are orthogonal to each other. When the number of factors we want to assign is from 17 to 21, there is no noticeable difference among the designs of Lin (1993), Wu (1993), and Nguyen (1996). We recommend the designs of Nguyen (1996) when the number of factors to be assigned is 22. Otherwise, the designs of Wu (1993) are recommended when the number of factors to be assigned is greater than 22.

4.3.3 An example with parameter $n = 16$ and $20$

When $n = 16$ or 20, designs of Nguyen (1996) and Yamada and Lin (1997) are considered to be suitable for evaluation. Figure 11 shows the transition of each $A_f$ when $n = 16$. However, because there are cases where $X_i$ associated with the designs of Yamada and Lin (1997) does not have an inverse matrix when $f = 4$, $A_f$ could not be evaluated in this study. Therefore, $A_f$ associated with this particular design was omitted in the comparison.
In both cases, the designs of Yamada and Lin (1997) have more columns. If the number of factors we want to assign is small, $A_f$ associated with the designs of Yamada and Lin (1997) is smaller than that of Nguyen (1996). The designs of Nguyen (1996) are recommended if the number of factors to be assigned is greater than 19 when $n = 16$ and 29 when $n = 20$.

### 4.4 Summary of 3 indicators

This chapter presents the summary of evaluation by 3 indices. Table 2 shows the best 2-level supersaturated designs by each indicator and summary when $n = 12$. Upper rows indicate $k$ of designs, and lower rows indicate the best designs. When each indicator results in a different value of certain $k$, the results of $E(s^2)$ are preferential because $E(s^2)$ is most frequently used as the indicator of 2-level supersaturated designs. Table 3 shows the best designs when $n = 10, 12, 14, 16, 18, 20, 22$. This table is a selection guide of 2-level supersaturated designs.

| $E(s^2)$ | $12 \leq k \leq 16$ Wu (1993) | $17 \leq k \leq 21$ Lin (1993), Wu (1993), Nguyen (1996) | $k = 22$ Lin (1993), Nguyen (1996) | $23 \leq k \leq 66$ Wu (1993) |
|----------|-------------------------------|---------------------------------|----------------------------------|---------------------------------|
| $D_f$    | $12 \leq k \leq 15$ Wu (1993) | $k = 16$ Wu (1993), Nguyen (1996) | $17 \leq k \leq 22$ Lin (1993), Nguyen (1996) | $23 \leq k \leq 66$ Wu (1993) |
| $A_f$    | $12 \leq k \leq 16$ Wu (1993) | $17 \leq k \leq 21$ Lin (1993), Nguyen (1996) | $k = 22$ Nguyen (1996) | $23 \leq k \leq 66$ Wu (1993) |
| Summary  | $12 \leq k \leq 16$ Wu (1993) | $17 \leq k \leq 21$ Lin (1993), Nguyen (1996) | $k = 22$ Nguyen (1996) | $23 \leq k \leq 66$ Wu (1993) |

Table 2: Best 2-level supersaturated designs by each indicator and summary when $n = 12$

| $n$ | Best designs |
|-----|--------------|
| 10  | $10 \leq k \leq 16$ Nguyen (1996) |
|     | $17 \leq k \leq 18$ Lin (1993), Nguyen (1996) |
| 12  | $12 \leq k \leq 16$ Wu (1993) |
|     | $17 \leq k \leq 21$ Lin (1993), Nguyen (1996) |
|     | $k = 22$ Nguyen (1996) |
|     | $23 \leq k \leq 66$ Wu (1993) |
| 14  | $14 \leq k \leq 26$ Nguyen (1996) |
| 16  | $16 \leq k \leq 20$ Yamada and Lin (1997) |
|     | $21 \leq k \leq 30$ Nguyen (1996) |
|     | $31 \leq k \leq 71$ Yamada and Lin (1997) |
| 18  | $18 \leq k \leq 34$ Nguyen (1996) |
| 20  | $20 \leq k \leq 29$ Yamada and Lin (1997) |
|     | $30 \leq k \leq 38$ Nguyen (1996) |
|     | $39 \leq k \leq 54$ Yamada and Lin (1997) |
| 22  | $22 \leq k \leq 38$ Nguyen (1996) |
|     | $39 \leq k \leq 42$ Lin (1993), Nguyen (1996) |

Table 3: Selection guide of 2-level supersaturated designs

### 5. Conclusion

This paper evaluated and compared designs that were constructed using 5 construction methods. Although these methods are proposed in previous studies, which design is better is not known. From the results, this paper aimed to determine which 2-level supersaturated designs should be used with respect to the number of associated experiments.

The orthogonality of the designs of Lin (1993) was lowered in this study. This reduction is related to the number of columns reduced from the right column in the designs. On the other hand, the orthogonality of the other 4 designs was increased. In particular, the orthogonality of the designs of Wu (1993) and Yamada and Lin (1997) was rapidly increased because the initial associated columns were orthogonal. For the designs of Iida (1994), the value of the evaluation index was worse because the designs primarily focused on the main effects that were confounding and on the effects of their interactions.

Two-level supersaturated designs are applied when we want to assign more factors than columns in the orthogonal designs. If the goal is to assign 1 or 2 more factors than the orthogonal designs, the designs of Wu (1993) and Yamada and Lin (1997) should be used. If the number of factors is more than 4 and less than the
number of factors that can be assigned, the designs of Lin (1993) and Nguyen (1996) should be used. This classification is based on 3 evaluation indices. The designs of Wu (1993) or Yamada and Lin (1997), which are characterized by a large number of columns, should be used when we want to assign more factors than columns in the designs of Lin (1993) and Nguyen (1996).

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