Transition Radiation by Neutrinos at an Edge of Magnetic Field

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We calculate the transition radiation process $\nu \rightarrow \nu \gamma$ at an interface of magnetic field and vacuum. The neutrinos are taken to be with only standard-model couplings. The magnetic field fulfills the dual purpose of inducing an effective neutrino-photon vertex and of modifying the photon dispersion relation. The transition radiation occurs when at least one of those quantities have different values in different media. The neutrino mass is ignored due to its negligible contribution. We present a result for the probability of the transition radiation which is both accurate and analytic.

**Introduction.**

In many astrophysical environments the absorption, emission, or scattering of neutrinos occurs in media, in the presence of magnetic fields \cite{1} or at the interface of two media. Of particular conceptual interest are those reactions which have no counterpart in vacuum, notably the plasmon decay $\gamma \rightarrow \bar{\nu}\nu$, the Cherenkov and transition radiation processes $\nu \rightarrow \nu \gamma$. These reactions do not occur in vacuum because they are kinematically forbidden and because neutrinos do not couple to photons. In the presence of a media (or a magnetic field), neutrinos acquire an effective coupling to photons by virtue of intermediate charged particles. In uniform media (or external field) the dispersion relations are modified of all particles so that phase space is opened for neutrino-photon reactions of the type $\nu \rightarrow \nu \gamma$. In \cite{2}, the plasma process was studied in \cite{5}. Transition radiation by neutrinos with large magnetic/electric dipole moment was studied in \cite{10}.

The magnetic field causes an effective $\nu-\gamma$-vertex in the standard-model. Also the magnetic field changes photon’s dispersion relation. We neglect neutrino masses and medium-induced modifications of their dispersion relation due to their negligible role. Therefore, we study the transition radiation entirely within the particle-physics standard model.

We proceed by deriving a general expression for the transition radiation rate (assuming a general $\nu-\gamma$-vertex) in quantum field theory. We derive the standard-model effective vertex in a magnetic field, then we calculate the transition radiation rate by performing semi-analytical integrations and summarize our findings.

**Transition Radiation.**

Let us consider a neutrino crossing the interface of two media with refraction indexes $n_1$ and $n_2$ (see Fig. 1). In terms of the matrix element $M$ the transition radiation probability of the process $\nu \rightarrow \nu \gamma$ is \cite{2}

$$W = \frac{1}{(2\pi)^3} \frac{1}{2E\beta z} \frac{d^3p'}{2E'\gamma} \frac{d^3k}{2\omega} \sum_{\text{pol} s} \int_{-\infty}^{\infty} dz e^{i(p_z-k_z)z} |M|^2 \times \delta(E-E' - \omega) \delta(p'_x + k_x) \delta(p'_y + k_y).$$

(1)

Here, $p = (E, \mathbf{p})$, $p' = (E', \mathbf{p}')$, and $k = (\omega, \mathbf{k})$ are the four momenta of the incoming neutrino, outgoing neutrino, and photon, respectively. The matrix element $M$ is related to the four-momentum transfer $q = (\omega, \mathbf{k})$ by

$$M(p, p', k) = \int dp'' \int d^3k'' \rho(p'') \rho(k'') \rho(p'') \rho(k'') \mathcal{M}(p, p'', k, k').$$

\section{Introduction}

\section{Transition Radiation}

\section{Conclusion}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{transition_radiation}
\caption{Transition radiation by neutrino at an interface of two media with refractive indexes $n_1$ and $n_2$.}
\end{figure}
trino, and photon, respectively and \( \beta_z = p_z/E \). The sum is over the photon polarizations.

We shall neglect the neutrino masses and the deformation of its dispersion relations due to the forward scattering. Thus we assume that the neutrino dispersion relation is precisely light-like so that \( p^2 = 0 \) and \( E = |p| \).

The transition radiation formation zone length of the medium is

\[
|p_z - p_z' - k_z|^{-1}.
\]

The integral over \( z \) in Eq. (1) oscillates beyond the length of the formation zone. Therefore, the contributions to the process from the depths over the formation zone length may be neglected. The neutrino transfers the \( z \) momentum \( (p_z - p_z' - k_z) \) to the medium. Since photon propagation in the medium suffers from the attenuation (absorption) the formation zone length must be limited by the attenuation length of the photons in the medium when the latter is shorter than the formation zone length.

After integration of (1) over \( p' \) and \( z \) we find

\[
W = \frac{1}{(2\pi)^3} \frac{1}{8E\beta_z} \int |k|'^2|d|k| \sin \theta \, d\theta \, d\varphi \times \sum_{\text{pols}} \left| \frac{\mathcal{M}^{(1)} - \mathcal{M}^{(2)}}{p_z' - }\right|^2,
\]

where \( \beta_z' = p_z'/E' \), \( \theta \) is the angle between the emitted photon and incoming neutrino. \( \mathcal{M}^{(1,2)} \) are matrix elements of the \( \nu \rightarrow \nu \gamma \) in each medium. \( k_z^{(1)} \) and \( p_z^{(1)} \) are the incoming neutrino and the boundary surface of two media (if that angle is not close to zero). The integration over \( \varphi \) drops out and we may replace \( d\varphi \rightarrow 2\pi \). \( k_z^{(1)} \) and \( p_z^{(1)} \) have the forms

\[
k_z^{(1)} = n(1)\omega \cos \theta, \quad p_z^{(1)} = \sqrt{(E - \omega)^2 - n(1)^2\omega^2\sin^2 \theta}, \quad (4)
\]

here we used \( n_{1,2} = |k|^{(1,2)}/\omega \).

The Neutrino-Photon-Vertex

In a magnetic field, photons couple to neutrinos by the amplitudes shown in Figs. 2(a) and (b). The electron propagator, represented by a double line, is modified by the field to allow for a nonvanishing coupling. It has been speculated that superstrong magnetic fields may exist in the early universe, but we limit our discussion to field strengths not very much larger than \( B_{\text{crit}} = m_e^2/e \) which is the range thought to occur in pulsars. Therefore, while in principle similar graphs exist for \( \mu \) and \( \tau \) leptons, we may neglect their contribution. For the same reason we may ignore field-induced modifications of the gauge-boson propagators. Moreover, we are interested in neutrino energies very much smaller than the \( W^- \) and \( Z^- \)-boson masses, allowing us to use the limit of infinitely heavy gauge bosons.

The \( \nu - \gamma \)-vertex in a magnetic field has been investigated in [2]. According to that result the matrix element for the \( \nu - \gamma \) vertex can be written in the form

\[
\mathcal{M} = - \frac{G_F}{\sqrt{2}} e Z \varepsilon_{\mu} \bar{\nu}_\gamma \gamma_\mu (1 - \gamma_5) \nu (g_V \Pi^{\mu\nu} - g_A \Pi_5^{\mu\nu}) \quad (6)
\]

here \( g_V = 2 \sin^2 \theta_W + \frac{1}{2} \) and \( g_A = \frac{1}{2} \) for \( \nu_e \), and \( g_V = 2 \sin^2 \theta_W - \frac{1}{2} \) and \( g_A = -\frac{1}{2} \) for \( \nu_\mu, \nu_\tau \). \( \Pi \) is the photon polarization tensor or vector-vector (VV) response function in the magnetic field, while \( \Pi_5 \) is the vector-axial vector (VA) response function. \( \varepsilon \) is the photon polarization vector and \( Z \) its wave-function renormalization.
FIG. 3: Neutrino-photon coupling in an external magnetic field. The double line represents the electron propagator in the presence of a $B$-field. (a) $Z$-$A$-mixing. (b) Penguin diagram (only for $\nu_e$). (c) Effective coupling in the limit of infinite gauge-boson masses.

factor. For the physical circumstances of interest to us, the photon refractive index will be very close to unity so that we will be able to use the vacuum approximation $Z = 1$.

The VA response function is

$$
\Pi^\mu_5(k) = \frac{e^3}{(4\pi)^2me} \left\{ -C_\parallel k_\nu^\mu(\vec{F}k)^\mu + C_\perp \left[ k_\nu^\mu(\vec{F}k)^\mu + k_\nu^\mu(\vec{F}k)^\mu - k_1^2 \vec{F}^\mu\nu \right] \right\},
$$

(7)

where

$$
C_\parallel = im_e^2 \int_0^\infty ds \int_{-1}^{+1} dv e^{-is\phi_0}(1-v^2)
$$

$$
C_\perp = im_e^2 \int_0^\infty ds \int_{-1}^{+1} dv e^{-is\phi_0} R
$$

(8)

are dimensionless coefficients which are real for $\omega < 2m_e$, i.e. below the pair-production threshold. In Eq. (8)

$$
\phi_0 = m_e^2 + \frac{1-v^2}{4}k_\parallel + \frac{\cos zv - \cos z}{2z\sin z} k_1^\perp,
$$

(9)

$$
R = \frac{1 - v \sin zv \sin z - \cos z \cos z}{\sin^2 z},
$$

(10)

$z = eBs$. $\bar{F}^\mu\nu = \frac{1}{4}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ with $\epsilon^{0123} = 1$ is the dual of the field-strength tensor.

In the presence of matter the $\nu$-$\gamma$-vertex is the sum of the vacuum one (as discussed above) and the matter contribution. It is well known that the last one is proportional to the small parameter $(n-1)^2 \ll 1$. We'll ignore matter contribution to the $\nu$-$\gamma$-vertex in the present work.

**Transition Radiation Probability**

Armed with these results we may now turn to an evaluation of the rate for $\nu \to \nu\gamma$ at an interface of magnetic field and vacuum. It is easy to see that for both photon eigenmodes the parity-conserving part of the effective vertex ($\Pi^\mu\nu$) is proportional to the small parameter $(n||L - 1)^2 \approx (\alpha/2\pi)\eta_{||L} \sin^2 \beta$. It is important to note that the parity-violating part ($\Pi^\mu_5$) is not proportional to this small parameter for the $||$ photon mode, while it is proportional to it for the $\perp$ mode.

For neutrinos which propagate perpendicular to the magnetic field the transition radiation probability of $||$ photons is

$$
W = \frac{\alpha G_F^2}{9 \cdot 2\pi^5} \left( \frac{B}{B_{\text{crit}}} \right)^2 \int h(B) \frac{\omega^5 \sin \theta d\omega d\theta}{(p_z - p_z')^2}.
$$

(11)

where

$$
h(B) = \frac{9}{16}(C_\parallel - 2C_\perp)^2.
$$

(12)

It turns out that in the range $0 < \omega < 2m_e$ the expression $C_\parallel/2 - C_\perp$ depends only weakly on $\omega$ so that it is well approximated by its value at $\omega = 0$. Explicitly, this is found to be

$$
h(B) = \begin{cases} 
(4/25) (B/B_{\text{crit}})^4 & \text{for } B \ll B_{\text{crit}} \\
1 & \text{for } B \gg B_{\text{crit}}.
\end{cases}
$$

(13)

$p_z'$ in Eq. (11) is

$$
p_z' = \sqrt{(E - \omega)^2 - n^2\omega^2 \sin^2 \theta}.
$$

(14)

The maximal allowed angle, $\theta_{\text{max}}$, for the photon emission is $\pi/2$ when $\omega < \frac{E_c}{m_e}$ and $\sin \theta_{\text{max}} = \frac{E_c - \omega}{E_c}$ when $\omega > \frac{E_c}{2}$. Now we expand the integrand (11) into the series in small angle, since only in that case the denominator is small (and the formation zone length is large). Thus we write the transition probability in the form

$$
W \approx \frac{\alpha G_F^2}{9 \cdot 2\pi^5} \left( \frac{B}{B_{\text{crit}}} \right)^2 h(B) \int \frac{(1 - \omega^2)^2 \omega^3 \theta d\omega d\theta}{(\theta^2 + (1 - n^2)(1 - \omega^2))^2}.
$$

(15)

When $n < 1$ eq. (15) can be written in the form

$$
W \approx \frac{\alpha G_F^2}{9 \cdot 2\pi^5} \left( \frac{B}{B_{\text{crit}}} \right)^2 h(B) \int \frac{(1 - \omega^2)^2 \omega^3 d\omega}{(1 - n^2)^2}.
$$

(16)

Eqs. (15) and (16) present the main results of the present work.

The index of refraction of photon is greater than 1, $n > 1$, for photon energies $\omega < 2m_e$ and in pure magnetic field. The pole appears in the denominator under the integral of eq. (15). Therefore outgoing radiation will be a combination of Cherenkov [7] and transition (15) ones.
When \(0 < (1 - n) \ll 1\) and we limited the thickness of magnetic field to \(d\), the resulting radiation probability will be one from eq (15 with replacement of \(1 - n\) under the integral to \(2\pi\omega\). This probability will exactly coincide with the probability of the Cherenkov radiation, \(d\Gamma_{Cher}\), where \(\Gamma_{Cher}\) is Cherenkov radiation rate from [7]. Thus, in that case both results gave the same answer.

When the process is taking place in presence of some matter (plasma, air), matter contribution to the index of refraction may overcome the magnetic field contribution, and the index of refraction will be less than 1, \(n < 1\). In that case, only transition radiation will be responsible for outgoing photon according to eq. (16).

Our result is very similar to the electron transition radiation. Our previous work on the subject, transition radiation by neutrino at an interface of matter and vacua [9], has an additional suppression by the square of the small angle, \(\theta^2\).

When the magnetic field has an opposite direction on each side from the interface, both sides give the same sign contribution to the transition radiation amplitude (unlike usual electron transition radiation when they subtract from each other and exactly cancel each other when \(n_1 = n_2\)) and the resulting probability will be 4 times larger than [16].

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\]