Can, Mahir Bilen; Houser, Hayden; Wolfe, Corey
On the Borel submonoid of a symplectic monoid. (English) Zbl 07242720
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Summary: In this article, we study the Bruhat-Chevalley-Renner order on the complex symplectic monoid $\text{MSp}_n$. After showing that this order is completely determined by the Bruhat-Chevalley-Renner order on the linear algebraic monoid of $n \times n$ matrices $M_n$, we focus on the Borel submonoid of $\text{MSp}_n$. By using this submonoid, we introduce a new set of type B set partitions. We determine their count by using the “folding” and “unfolding” operators that we introduce. We show that the Borel submonoid of a rationally smooth reductive monoid with zero is rationally smooth. Finally, we analyze the nilpotent subsemigroups of the Borel semigroups of $M_n$ and $\text{MSp}_n$. We show that, contrary to the case of $\text{MSp}_n$, the nilpotent subsemigroup of the Borel submonoid of $M_n$ is irreducible.

MSC:
20M32 Algebraic monoids
20G99 Linear algebraic groups and related topics
06A99 Ordered sets

Keywords:
symplectic monoid; Renner monoid; Borel submonoid; rationally smooth; set partitions; (un)folding

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