Dilaton – fixed scalar correlators and $AdS_5 \times S^5$ – SYM correspondence

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Abstract

We address the question of AdS/CFT correspondence in the case of the 3-point function $\langle O_4 O_4 O_8 \rangle$. $O_4$ and $O_8$ are particular primary states represented by $\text{tr} F^2 + ...$ and $\text{tr} F^4 + ...$ operators in $\mathcal{N} = 4$ SYM theory and dilaton $\phi$ and massive “fixed” scalar $\nu$ in $D = 5$ supergravity. While the value of $\langle O_4 O_4 O_8 \rangle$ computed in large $N$ weakly coupled SYM theory is non-vanishing, the $D = 5$ action of type IIB supergravity compactified on $S^5$ does not contain $\phi \phi \nu$ coupling and thus the corresponding correlator seems to vanish on the $AdS_5$ side. This is in obvious contradiction with various arguments suggesting non-renormalization of 2- and 3-point functions of states from short multiplets and implying agreement between the supergravity and SYM expressions for them. We propose a natural resolution of this paradox which emphasizes the 10-dimensional nature of the correspondence. The basic idea is to treat the constant mode of the dilaton as a part of the full $S^5$ Kaluza-Klein family of dilaton modes. This leads to a non-zero result for the $\langle O_4 O_4 O_8 \rangle$ correlator on the supergravity side.

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1. Introduction

Recent studies of 2- and 3-point functions of operators in short multiplets of \( \mathcal{N}=4 \) super Yang-Mills theory in the context of AdS/CFT correspondence \([1,2,3]\) (see \([4]\) for a review) suggest that they are not renormalized in large \( N \) limit \([5,2,6,7,8,9,10,11,12]\). Since the supergravity correlator is interpreted as the large \( N \) strong coupling SYM result, this implies that the exact SYM expressions for the large \( N \) limit of the 2- and 3-point correlators of such operators should not contain non-trivial functions of \('t\) Hooft coupling (the coordinate dependence of the \( n \leq 3 \) correlators is fixed uniquely by conformal invariance). This conclusion is supported by explicit SYM perturbative calculations \([15,16,17,18]\), general arguments based on \( \mathcal{N}=4 \) superspace \([17]\) or \( U(1)_Y \) symmetry \([19]\) (or non-renormalization of anomalies \([20]\)). However, one easily finds an apparent contradiction.

Consider the following \( SU(N) \) \( \mathcal{N}=4 \) SYM operators

\[
O_4 = \text{tr}(F_{\mu\nu} F_{\mu\nu}) + \ldots, \tag{1.1}
\]

\[
O_8 = \frac{3}{2} \text{Str}[F^4 - \frac{1}{4}(F^2)^2] + \ldots
= \text{tr}(F_{\mu\nu} F_{\rho\nu} F_{\mu\lambda} F_{\rho\lambda} + \frac{1}{2} F_{\mu\nu} F_{\rho\nu} F_{\rho\lambda} F_{\mu\lambda} - \frac{1}{4} F_{\mu\nu} F_{\mu\nu} F_{\rho\lambda} F_{\rho\lambda} - \frac{1}{8} F_{\mu\nu} F_{\rho\lambda} F_{\mu\nu} F_{\rho\lambda}) + \ldots, \tag{1.2}
\]

where dots stand for scalar and spinor terms which are required by \( \mathcal{N}=4 \) supersymmetry and \( \text{Str} \) is the symmetrized trace of \( SU(N) \) generators as matrices in the fundamental representation. \( O_4 \) and \( O_8 \) are supersymmetric descendants of chiral primary operators \( \text{tr}(XX) \) and \( \text{tr}(XXXX) \), respectively.\[2\]

\[
O_4 = (Q^4 + \bar{Q}^4) \text{tr} X^2, \quad O_8 = Q^4 \bar{Q}^4 \text{tr} X^4. \tag{1.3}
\]

For all the operators \( \text{tr} X^n \) the precise agreement between the (weighted) 3-point correlators computed in the large \( N \) weakly coupled SYM and the \( AdS_5 \) supergravity was demonstrated in \([10]\), so it seems natural to expect the same for their supersymmetry descendants.

\[1\] For other papers on correlators in the AdS/CFT correspondence see \([13,14]\) and references in \([4]\).

\[2\] The precise definition for these chiral primary operators is \([10]\): \( C_{i_1 \ldots i_k}^I \text{tr}(X^{i_1} \ldots X^{i_k}) \), where \( C^I \) is a totally symmetric traceless rank \( k \) tensor of \( SO(6) \) and \( X^i \) are scalars of \( \mathcal{N}=4 \) SYM theory. The action of supercharges in \( \text{tr} X^2 \) and \( \text{tr} X^4 \) is indicated only schematically. In particular, in \( O_8 \) one needs to specify the ordering of \( Q \)'s and \( \bar{Q} \)'s to resolve the associated total derivative ambiguity.
Using the AdS/CFT correspondence recipe, one finds – either from the supermultiplet considerations \[3,21\] or from the structure of the Born-Infeld action for a D3-brane in a curved background \[22\] – that on the supergravity side \(O_4\) corresponds to the 5-D dilaton \(\phi\) \[23,24\], while \(O_8\) corresponds to the fixed scalar \(\nu\) \[25\]. The latter enters the \(D=10\) Einstein-frame metric as

\[
ds_{10E}^2 = e^{-\frac{10}{3}\nu(x)}g_{5mn}(x)dx^m dx^n + e^{2\nu(x)}d\Omega_5^2.
\]

The specific powers of \(e^{\nu}\) in the metric are needed to decouple \(\nu\) from the 5-d graviton; this ansatz generalizes to non-linear level the graviton mode decomposition considered in \[27\] where this massive \((m^2 = \frac{32}{R^2})\) singlet scalar \(\nu\) was identified with the zero mode \((S^5\) independent part) of the trace of the perturbation of the metric of \(S^5\). Note that the zero mode \(\nu\) does not mix with the 4-form potential, while higher Kaluza-Klein modes do \[27\]. It is the lowest-mass member of the KK family of scalars with masses \(m^2 = \frac{(k+4)(k+8)}{R^2}, \ k = 0,1,\ldots\) and belongs to a separate massive \(D = 5\) supermultiplet interacting with the \(D = 5, \ N = 8\) supergravity multiplet. Dimensionally reducing the dilaton-graviton sector of type IIB supergravity action to 5 dimensions, one easily finds \[28\] that the action does not contain \(\phi\phi\nu\) coupling (in agreement with the possibility to perform the so-called consistent truncation of the full \(D = 5\) theory to the \(D = 5\) gauged supergravity subsector). This implies that on the AdS supergravity side

\[
<O_4O_4O_8>_{AdS} = 0.
\]

However, the explicit computation of \(<O_4O_4O_8>\) in the SYM theory performed below to the leading order in large \(N\) expansion yields a non-vanishing result for this correlator.\(^4\)

One then faces the following three options:

(i) There may be some subtlety in applying the nonrenormalization theorem for the 3-point functions of chiral primary operators \(\text{tr}X^n\) to their supersymmetry descendants and the correlator \(<O_4O_4O_8>\) may actually depend on gauge coupling in such a way that it vanishes in the limit \(\lambda \equiv g_{YM}^2N \to \infty\).\(^3\)

(ii) The non-renormalization statement for the SYM correlator is correct but there may be a subtlety in the definition of the \(O_8\) operator, e.g., it may contain also \(\frac{1}{N}\text{tr}F^2\text{tr}F^2\)

\(^3\) A relation between fixed scalars and \(F^4\)-type operators was earlier mentioned in \[26\] in the case of the D5+D1 system.

\(^4\) That this correlator of the bosonic gauge-theory parts of \(O_4\) and \(O_8\) computed in the abelian theory limit is non-vanishing was already observed in the first paper in \[25\].

\(^5\) One possibility could be that the \(N = 4\) supersymmetric expression for the 3-point function for the whole multiplet contains several tensor structures. However, this seems to be excluded by the arguments in \[16,17\].
admixtures (with various possible Lorentz contractions of the two $F^2$ factors). That would change the result for the three-point function on the SYM side and may make it zero. However, this suggestion seems to be in contradiction with arguments based on $\mathcal{N} = 4$ supersymmetry \cite{16, 29}.

(iii) There may be subtleties in the supergravity calculation. The supergravity result may turn out to be non-vanishing and in agreement with the non-vanishing large $N$ SYM result.

In this paper we shall propose a natural resolution of this paradox based on the third option. The key observation is the following. Suppose that there was some non-vanishing two dilaton–fixed scalar coupling in the $D = 5$ action. Then the resulting 3-point function computed from supergravity according to the rules of \cite{3} would actually be divergent. This can be seen from the general expression for the 3-point function of the scalar fields in the (Euclidean) $AdS_5$ space found in \cite{6}

$$S = \int d^5x \sqrt{g_5} \left[ \frac{1}{2} \sum_{s=1}^{3} (\partial_m \varphi_s \partial^n \varphi_s + m_s^2 \varphi_s^2) + \lambda_{123} \varphi_1 \varphi_2 \varphi_3 \right],$$

then the corresponding 2- and 3-point correlators of the boundary operators are \cite{3}

$$< O_{\Delta}(x_1)O_{\Delta}(x_2) > = \frac{a_0}{|x_1 - x_2|^{2\Delta}}, \quad a_0 = \frac{2(\Delta - 2)}{\pi^2 \Delta} \frac{\Gamma(\Delta + 1)}{\Gamma(\Delta - 2)},$$

$$< O_{\Delta_1}(x_1)O_{\Delta_2}(x_2)O_{\Delta_3}(x_3) > = \frac{\lambda_{123} a_1}{|x_1 - x_2|^{\Delta_1 + \Delta_2 - \Delta_3}|x_1 - x_3|^{\Delta_1 + \Delta_3 - \Delta_2}|x_3 - x_2|^{\Delta_3 + \Delta_2 - \Delta_1}},$$

where $\Delta_s = 2 + \sqrt{4 + m_s^2 R^2}$ and \cite{3}

$$a_1 = -\frac{\Gamma[\frac{1}{2}(\Delta_1 + \Delta_2 - \Delta_3)]\Gamma[\frac{1}{2}(\Delta_1 + \Delta_3 - \Delta_2)]\Gamma[\frac{1}{2}(\Delta_2 + \Delta_3 - \Delta_1)]\Gamma[\frac{1}{2}(\Delta_1 + \Delta_2 + \Delta_3 - 4)]}{2\pi^4 \Gamma(\Delta_1 - 2)\Gamma(\Delta_2 - 2)\Gamma(\Delta_3 - 2)}$$

It follows from \cite{1.8} that whenever the dimensions of the operators $\Delta_s$ satisfy the relation (or any of its two permutations)

$$\Delta_3 = \Delta_1 + \Delta_2 + n, \quad n = 0, 1, 2, \ldots,$$

\cite{6} We thank I. Klebanov and J. Polchinski for suggesting this possibility.

\cite{7} The leading large $N$ term in $< (\text{tr} F^2)^4 >$ which is of order $O(1)$ comes from disconnected diagram, while the next order term (leading connected contribution) is of order $O(1/N^2)$. One can show (using the results of section 2 below) that $< O_4 O_8 O'_8 > = 0$ if $O'_8 = O_8 - \frac{1}{2N} \text{tr} F^2 \text{tr} F^2$.

\cite{8} Similar expression is found for any dimension $d$ of the boundary: the numbers $-2$ and $-4$ are replaced in general by $-\frac{d}{2}$ and $-d$. 3
the coefficient $a_1$ develops a pole. This is precisely what we encounter in the case of the $\phi\phi\nu$ interaction: since $m_\phi^2 = 0$ and $m_\nu^2 = \frac{32}{R^2}$ one has $\Delta_1 = \Delta_2 = 4$, $\Delta_3 = 8$, i.e. $\Delta_3 = \Delta_1 + \Delta_2$. The final result is thus undefined: a product of a zero and a divergence.

Our proposal is that in such special cases one should use a special regularization prescription in computing the supergravity expressions. One should effectively turn on “a little bit” of $S^5$ KK momentum for the dilaton. Then (i) the dilaton will get an infinitesimal mass $\epsilon$, i.e. the dimension of $O_4$ will be shifted from 4 to $4 + \epsilon$, thus regularizing (1.8), and (ii) the action will get a small coupling term $\epsilon\phi\phi\nu$. As a result, the dependence on $\epsilon$ will cancel out in the final expression in (1.7) ($\epsilon \times \frac{1}{\epsilon} = 1$) and we will obtain a finite result for $<O_4O_4O_8>$. This procedure essentially amounts to an analytic continuation that treats the $S^5$-independent zero mode $\phi$ as an integral part of the whole family of $S^5$ KK modes of the 10-d dilaton field.

Equivalently, we suggest that the AdS/CFT correspondence should be defined for the whole KK families of modes on the supergravity side, i.e. one is to compute, e.g., the supergravity $\phi\phi\nu$ correlator as a limit of the correlator for the two massive KK modes of the dilaton (with equal masses $m_k^2 = \frac{k(k+4)}{R^2}$, $k = 0, 1, 2, ...$) and $\nu$. A non-zero KK quantum number $k$ implies that now $\Delta_k = 4 + k$ (the corresponding SYM operator is $O_{4+k} \sim \text{Str}(F^2X^k+...)$) and thus the $\Gamma$-function factor in (1.7) $\Gamma(\frac{1}{2}(\Delta_1+\Delta_2-\Delta_3))$ is simply $\Gamma(k)$. At the same time, the coupling constant $\lambda_{\phi\phi\nu}$ between $\nu$ and the two KK dilaton modes happens to be non-vanishing for $k \neq 0$ and is proportional to $\frac{k(k+4)}{2^{k-1}(k+1)(k+2)}$. As a result, the product of $\lambda_{123}$ and $a_1$ in (1.7) is well-defined and finite ($\sim k\Gamma(k) = \Gamma(1+k)$) for all $k$, including the case of $k = 0$.

A similar prescription should be applied in all cases where the anomalous dimensions of the three operators in the correlation function satisfy the relation (1.9). With a hindsight, the suggested regularization procedure is actually implicit in the examples discussed in [10], where the 3-point functions $<\text{tr}X^k\text{tr}X^k\text{tr}X^{2k}>$ ($k = 2, 3, ...$) were shown to be finite and equal to their large $N$ (free-field) SYM counterparts. There the poles in the supergravity amplitudes (1.7),(1.8) were similarly cancelled by zeros in the cubic Lagrangian coupling derived from the KK reduction of the 10-d supergravity action.

Close parallel with the case of the $\text{tr}X^n$ 3-point functions and the supersymmetry relations between the corresponding operators on the SYM side (1.3) leave little doubt that the precise numerical AdS/CFT correspondence should then be found also in the present case of the normalized correlator $<O_4O_4O_8>$. To check this explicitly one is to compare the finite numerical coefficient obtained using our prescription on the

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9 If one regularizes the resulting IR divergence (e.g., by shifting the position of the boundary) the final result will be zero because of the vanishing of the bulk interaction constant.

10 The observation about cancellation between poles and zeroes for special combinations of dimensions of the three operators in [10] was also made in ref. [30].
supergravity side with the large \( N \) SYM result. Since the comparison is done for the normalized correlator, one needs to know also the SYM expressions for \( < O_4O_4 > \) and \( < O_8O_8 > \). While the SYM computation of \( < O_4O_4O_8 > \) (discussed below) does not depend on the scalar and fermionic terms in the \( \mathcal{N} = 4 \) supersymmetric completion of the \( F^4 \) operator (1.2), to find \( < O_8O_8 > \) one needs first to determine the full structure of \( O_8 \). This will be done in [31] where we will present the full expression for the \( \mathcal{N} = 4 \) super-invariant \( O_8 \) in terms of \( \mathcal{N} = 1 \) and \( \mathcal{N} = 2 \) superfields and compute \( < O_8O_8 > \) using off-shell superfield formalism.

The plan of this paper is the following. In section 2 we shall present the SYM calculation for the correlator \( < O_4O_4O_8 > \) in the leading large \( N \) (fixed \( \lambda = g_{YM}^2N \)) approximation. In section 3 we shall determine the supergravity expressions for the correlators \( < O_4O_4O_8 > \) using the “analytic continuation in KK momentum” prescription outlined above. Section 4 will contain some concluding remarks. In Appendix A we shall discuss an attempt to determine the normalization of the \( O_8 \) operator from its coupling to the fixed scalar \( \nu \) in the Born-Infeld D3-brane probe action.

2. Gauge theory calculation of \( < O_4O_4O_8 > \)

In this section we shall compute \( < O_4O_4O_8 > \) correlator in large \( N \) SYM theory and demonstrate that it is non-vanishing. As already mentioned above, one is able to find the precise expression for \( < O_4O_4O_8 > \) (at separated points, i.e. modulo contact terms) without knowing the detailed structure of the supersymmetric completion of \( F^4 \) terms in (1.2). It is easy to understand either from the action of supercharges on the chiral primary field (1.3) (see, e.g., [12]) or from the (\( \mathcal{N} \geq 2 \)) superfield expressions [21] that the scalar and fermionic terms in \( O_4 \) are of the “equations of motion” form\(^{11}\)

\[
O_4 = \text{tr}(F^2 + X \partial^2 X + \bar{\psi} \gamma \cdot \partial \psi + ...) . \quad (2.1)
\]

\(^{11}\) Here we are ignoring interaction (commutator \( [X,X], [\psi, X] \) dependent [12]) terms in \( O_4 \) since they produce terms of higher order in \( \lambda \) in the correlators (which altogether should cancel out in the cases where there is a non-renormalization theorem). Since the dilaton should be related to gauge coupling constant, one expects that the dilaton operator should be proportional to the SYM Lagrangian, up to a total derivative term. The \( \mathcal{N} = 2 \) supersymmetry implies that the quadratic scalar term should have the form \( \text{tr}(XD^2X) \), which is different from the standard kinetic term by a total derivative.
Contracted with other fields in the correlators these terms give rise only to contact contributions proportional to delta functions (which are field-redefinition dependent and are omitted in CFT correlation functions).

As a result, the scalar and fermionic terms in $O_4$ and $O_8$ are irrelevant for the computation of $\langle O_4(x)O_4(0) \rangle$ and $\langle O_4(x)O_4(y)O_8(0) \rangle$ which can therefore be done simply in the pure YM theory. The correlator $\langle O_8(x)O_8(0) \rangle$ does, however, receive non-trivial scalar and spinor contributions, i.e. is different from the pure YM result [11].

We shall take the $SU(N)$ Yang-Mills Lagrangian in the form

$$\mathcal{L} = \frac{N}{4\lambda} \text{tr}(F_{\mu\nu}F_{\mu\nu}) \ ,$$

(2.2)

where $\lambda = g_{\text{YM}}^2 N$. We are assuming that the generators are normalized so that $\text{tr}(T_i T_j) = \delta_{ij}$. In the double-line notation, $A_{\mu b}^a = A_i^\mu (T_i)_b^a$, the gauge vector propagator is

$$\langle A_{\mu b}^a(x) A_{\nu d}^c(0) \rangle = \frac{\lambda}{N} (\delta_a^c \delta_b^d - \frac{1}{N} \delta_b^a \delta_d^c) \frac{\delta_{\mu\nu}}{4\pi^2 x^2} \ .$$

(2.3)

In the free-theory limit the field strength propagator is then

$$\langle F_{\mu\nu b}^a(x) F_{\lambda\rho d}^c(0) \rangle = \frac{\lambda}{N} (\delta_a^c \delta_b^d - \frac{1}{N} \delta_b^a \delta_d^c) \frac{2}{\pi^2 x^4} D_{\mu\nu,\lambda\rho}(x) \ ,$$

(2.4)

where

$$D_{\mu\nu,\lambda\rho}(x) = \epsilon_{\mu\nu,\lambda\rho} - \frac{4}{x^2} X_{\mu\nu,\lambda\rho} \ ,$$

(2.5)

$$\epsilon_{\mu\nu,\lambda\rho} \equiv \frac{1}{2}(\delta_{\mu\lambda} \delta_{\nu\rho} - \delta_{\mu\rho} \delta_{\nu\lambda}) \ , \quad X_{\mu\nu,\lambda\rho} \equiv \frac{1}{4}(x_{\mu} x_{\lambda} \delta_{\nu\rho} - x_{\mu} x_{\rho} \delta_{\nu\lambda} + x_{\nu} x_{\rho} \delta_{\mu\lambda} - x_{\nu} x_{\lambda} \delta_{\mu\rho}) \ .$$

All correlators will be found in the free theory limit, i.e. ignoring all contributions which are sub-leading at large $N$ and small $\lambda$. Hence the field strength matrices in (1.1) and (1.2) do not contain the interaction term, i.e. are simply $F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$, and the

12 This explains, in particular, the agreement between the result obtained using the dilatonic $F^2$ operator that follows from the BI action [23,24] (in the Einstein frame the BI action gives $e^{-\phi} F^2 + (\partial X)^2 + ...$, i.e. scalars and fermions do not couple to $\phi$) and the form of the corresponding $\mathcal{N} = 2$ superinvariant that has the structure (2.1). The scalar and fermionic terms are important, however, in most of other relevant cases: for example, the stress tensor operator that couples to the 4-d graviton contains non-trivial scalar and fermionic terms [24] (see also [12]).

13 A more conventional choice of normalization of the generators in the fundamental representation is $\text{tr}(T_i T_j) = \frac{1}{2} \delta_{ij}$, $\mathcal{L} = \frac{N}{2\lambda} \text{tr}(F_{\mu\nu}F_{\mu\nu})$. 
computation of the correlators reduced to a straightforward application of \((2.4)\). After lengthy calculations one finds\(^\text{14}\)

\[
< O_4(x)O_4(0) > = \frac{48\lambda^2}{\pi^4} \frac{1}{x^8} (1 - \frac{1}{N^2}) ,
\]

\[
< O_4(x)O_4(y)O_8(0) > = \frac{128 \times 9\lambda^4}{N \pi^8} \frac{1}{x^8y^8} + O(\frac{1}{N^3}) .
\]

Similarly, one finds that the gauge-field part \(\frac{3}{2}\)Str\([F^4 - \frac{1}{4}(F^2)^2]\) of the operator \(O_8\) contribution to the correlator \(< O_8(x)O_8(0) >\) is

\[
< O_8(x)O_8(0) >_{g.f.} = \frac{8 \times 27\lambda^4}{\pi^8} \frac{1}{x^{16}} + O(\frac{1}{N^2}) .
\]

The contributions of the fermionic and scalar terms in \(O_8\) increase the numerical coefficient in \((2.8)\) \([31]\).

### 3. \(D = 5\) supergravity results for the correlators

The type IIB supergravity action for the (Einstein-frame) metric, dilaton, RR scalar and 4-form is \((2\kappa_{10}^2 = (2\pi)^7 g_s^2 \alpha'^4)\)

\[
I_{10} = -\frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{g} \left[ R - \frac{1}{2} (\partial \phi)^2 - \frac{1}{4} e^{2\phi} (\partial C)^2 - \frac{1}{4 \cdot 5!} (F_5)^2 \right] .
\]

In what follows we shall concentrate on the dilaton – fixed scalar correlators, but the case where the dilaton is replaced by the RR scalar \(C\) (and the SYM operator tr\(F^2\) is replaced by tr\(FF^*\)) is very similar. The background \(AdS_5 \times S^5\) metric (which is an extremum of \((3.1)\) \([32]\) ) is

\[
ds_{10}^2 = R^2 \left[ \frac{1}{z^2} (dz^2 + dx_\mu dx_\mu) + d\Omega_5^2 \right] , \quad R^4 = 4\pi g_s N \alpha'^2 .
\]

In what follows we shall set the radius \(R\) to be 1. To dimensionally reduce to \(D = 5\), we use the ansatz \((1.4)\) for the 10-d Einstein frame metric. The relevant part of the type IIB supergravity action reduced to 5 dimensions for the zero modes of the dilaton and the metric is

\[
I_{5} = -\frac{1}{2\kappa_{5}^2} \int d^5x \sqrt{g_5} \left[ \mathcal{R}_5 - \frac{1}{2} (\partial \phi)^2 - \frac{40}{3} (\partial \nu)^2 - V(\nu) + ... \right] ,
\]

\(^\text{14}\) In the case of abelian gauge theory these correlators were computed in \([25]\). The numerical coefficients in the correlators found in the abelian theory and the large \(N\) non-abelian theory are different.
where

\[ V(\nu) = 8e^{-\frac{4\nu}{3}} - 20e^{-\frac{16\nu}{3}} = -12 + \frac{40}{3} \times 32\nu^2 + O(\nu^3) \, . \] (3.4)

This action does not contain \( \phi - \nu \) coupling (in particular, \( \phi\phi\nu \) term) – this is a consequence of the fact that the dilaton does not couple to both \( R \) and \( (F_5)^2 \) in (3.1).

As was already explained in the Introduction, our main suggestion is that to establish a correspondence between the SYM and the supergravity expressions for the \( <O_4O_4O_8> \) correlator one should “regularize” the dimensional reduction procedure. One should first assume that \( \phi \) has some “small” dependence on \( S^5 \) coordinates, leading to an infinitesimal 5-d dilaton mass term and an infinitesimal \( \phi\phi\nu \) coupling, and take the corresponding small parameter to zero only after the computation of the \( \phi\phi\nu \) correlator. The meaning of this prescription is that treating the zero mode of the dilaton on the same footing as higher KK modes resolves the ambiguity in the definition of the \( \phi\phi\nu \) correlator and leads to a finite value for its overall numerical coefficient.

Let us start with decomposing \( \phi \) in terms of spherical harmonics on \( S^5 \)

\[ \phi(y, x) = \sum_I Y^I(y) \phi^I(x) \, . \] (3.5)

We shall follow the notation of [10] and consider \( S^5 \) as embedded in a Euclidean space \( R^6 \). The spherical harmonics \( Y^I \) are defined by \( Y^I = C^I_{i_1...i_k} y^{i_1} \cdots y^{i_k} \), where \( y^i \) \((i = 1, \ldots, 6)\) are the Cartesian coordinates of the embedding \( R^6 \) space \((y^i y^i = 1)\) and \( C^I_{i_1...i_k} \) are totally symmetric traceless tensors of rank \( k \).

The \( I = 0 \) term in (3.5) \( \phi^0(x) \equiv \phi(x) \) is the 5-dimensional dilaton while \( \phi^I(x) \) are higher KK modes of the \( D = 10 \) dilaton. The dimensionally reduced form of the dilaton part of the action (3.1) is found to be

\[ \frac{1}{2\kappa_{10}^2} \int d^{10}x \frac{1}{2\sqrt{g}} g^{MN} \partial_M \phi \partial_N \phi = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{g_5}(L_1 + L_2) \, , \] (3.6)

where \((\omega_5 = \pi^3)\)

\[ \frac{1}{2\kappa_5^2} \frac{R^8\omega_5}{2\kappa_{10}^2} = \frac{N^2}{8\pi^2} \, . \] (3.7)

The two parts of the dilaton Lagrangian are

\[ L_1 = \frac{1}{2} g_5^{mn} \sum_I A(k) \partial_m \phi^I \partial_n \phi^I \, , \] (3.8)

and

\[ L_2 = \frac{1}{2} e^{-\frac{16\nu}{3}} \sum_I B(k) \phi^I \phi^I \, , \] (3.9)

\[ ^{15} \text{The two terms in the potential originate from the curvature of } S^5 \text{ and the } (F_5)^2 \text{ term.} \]
with the normalization constants $A(k)$ and $B(k)$ given by ($k$ is the rank of $C_{i_1 \ldots i_k}$)

$$A(k) = \frac{1}{2^{k-1}(k+1)(k+2)}, \quad B(k) = k(k+4)A(k). \quad (3.10)$$

The factors of $e^{\nu}$ in (1.4) where chosen so that the Einstein $R$-term and hence $L_1$ have no $\nu$-dependence. The form of the $\nu$-dependent factor in $L_2$ (3.9) follows from the fact that $L_2 = e^{-\frac{2\mu(x)}{\nu}} \int g^{ss'} \partial_s \phi \partial_{s'} \phi$, where $g_{ss'} \sim e^{2\nu}$ is the $S^5$ part of the metric in (1.4).

Since for $I = 0$ one has $k = 0$ and thus vanishing $B(k)$, we conclude once again that there is no coupling between $\nu$ and the zero mode of the dilaton. Instead of taking $k = 0$ directly, one may formally set $k = \varepsilon$ and take the limit $\varepsilon \to 0$ only at the end of the calculation of correlation functions. As follows from (3.10), this prescription generates both a small mass for the 5-d dilaton and a coupling between the dilaton and $\nu$,

$$m^2 = 4\varepsilon, \quad \lambda_{\phi \phi \nu} = -\frac{32}{3} \varepsilon = -\frac{8}{3} m^2. \quad (3.11)$$

Introducing the couplings between the $AdS_5$ boundary values of the 5-d fields $\phi$ and $\nu$ and the corresponding $\mathcal{N} = 4$ SYM $F^2$ and $F^4$ operators (1.1),(1.2) as in [2,3] we get

$$\int d^4x \left[ a_4 O_4 \phi(x) + a_8 O_8 \nu(x) \right], \quad (3.12)$$

where $a_4$ and $a_8$ are some normalization constants.

We learn from (3.3) and (3.12) that the operators $O_4$ and $O_8$ have the following dimensions ($m^2 \to 0$)

$$\Delta_4 = 2 + \sqrt{4 + m^2} = 4 + \frac{m^2}{4} + O(m^4), \quad \Delta_8 = 2 + \sqrt{4 + 32} = 8. \quad (3.13)$$

Following [2,3] and using the expressions for the 2- and 3-point functions in [4] we find from (3.3) (3.12) the following supergravity predictions for the correlators involving $O_4$ and $O_8$ (see eqs. (1.6)–(1.8))

$$<O_4(x)O_4(0)> = \frac{1}{2\kappa_5^2} \frac{24}{\pi^2 a_4^2} \frac{1}{x_8}, \quad (3.14)$$

$$<O_8(x)O_8(0)> = \frac{1}{2\kappa_5^2} \frac{42 \times 24 \times 40}{3\pi^2 a_8^2} \frac{1}{x_16}, \quad (3.15)$$

$$<O_4(x)O_4(y)O_8(0)> = \frac{1}{2\kappa_5^2} \frac{8 \times 36 m^2}{3\pi^4 a_4^4 a_8} \Gamma(\frac{m^2}{4}) \left( \frac{1}{x^8 y^8} \right) = \frac{1}{2\kappa_5^2} \frac{32 \times 36}{3\pi^4 a_4^2 a_8} \frac{1}{x^8 y^8} + O(m^2). \quad (3.16)$$

There is an extra factor of 2 in (3.16) coming from the fact that there are two dilaton field legs in the $\phi \phi \nu$ coupling (3.11). In (3.16) we have used that for $m^2 \to 0$, $\Gamma(\frac{m^2}{4}) \to \frac{4}{m^2}$. The resulting expression for the $\phi \phi \nu$ correlation function is thus finite: the zero in the $\phi \phi \nu$
bulk interaction vertex cancels against the pole in the $\Gamma$-function coming from the integral over $AdS_5$.

Let us note in passing that there is also no $\phi \nu \nu$ coupling in (3.3), i.e.

$$< O_4(x)O_8(y)O_8(0)> = 0 .$$

(3.17)

Since the dimensions of the three operators here do not satisfy (1.9), in contrast to the case of $< O_4(x)O_4(y)O_8(0)>$, the expression (1.8) for this correlator is well-defined and there is no ambiguity. The vanishing of $< O_4(x)O_8(y)O_8(0)>$ in (3.17) can be deduced from the argument based on $U(1)_Y$ charge conservation, see [19,17]. From the supergravity point of view, this $U(1)$ is part of $SL(2,R)$ which is a classical symmetry of type IIB supergravity. The dilaton transforms non-trivially under this group, while $\nu$ is inert being part of the Einstein-frame metric. Thus the coupling like $\phi \nu \nu$ is forbidden in the classical supergravity Lagrangian.

Since the supergravity expressions (3.14)–(3.16) depend on the normalization constants appearing in (3.12), the direct comparison with SYM result (2.7) is not possible. The weighted three-point function does not depend on the absolute normalization,

$$< O_4(x)O_4(y)O_8(0)> = \frac{1}{N} \sqrt{\frac{16}{105}} \frac{1}{x^8 y^8} ,$$

(3.18)

where $O_4$ and $O_8$ are rescaled $O_4$ and $O_8$ satisfying

$$< O_4(x)O_4(0)> = \frac{1}{x^8} , \quad < O_8(x)O_8(0)> = \frac{1}{x^{16}} .$$

(3.19)

To compare (3.18) with the SYM result we need the expression for $< O_8O_8 >$ in SYM theory which requires a detailed separate computation and will be presented in [31]. Here we note only that the value of $< O_8O_8 >$ in SYM theory which would lead to the agreement with (3.18) is

$$< O_8(x)O_8(0)> = \frac{105 \times 36 \lambda^4}{\pi^8 x^{16}} .$$

(3.20)

Not surprisingly, this expression is different from the pure gauge theory part (2.8) of the full SYM correlator.

One heuristic way to fix the values of the normalization constants $a_4$ and $a_8$ in (3.12) is to derive (3.12) from the BI action for a D3-brane propagating in a curved background. As shown in Appendix this leads to

$$a_4 = \frac{N}{4\lambda} , \quad a_8 = \frac{10N\pi^2}{9\lambda^2} ,$$

(3.21)

where $\lambda = g_{YM}^2 N = 4\pi g_s N$. The leading large $N$ term in (2.6) is in perfect agreement with the supergravity expression (3.14) with $a_4$ given by the BI normalization in (3.21) (this
agreement is essentially the one originally found in \[23,24,2\]). If we use \(a_8\) from \(3.24\) in \(3.13\) we find that the supergravity expression for \(<O_8(x)O_8(0)>\) is

\[
<O_8(x)O_8(0)>_{BI} = \frac{84 \times 81 \lambda^4}{5 \pi^8} \frac{1}{x^{16}},
\]  

(3.22)

and from (3.16) we get

\[
<O_4(x)O_4(y)O_8(0)>_{BI} = \frac{1}{N} \frac{96 \times 36 \lambda^4}{5 \pi^8} \frac{1}{x^8y^8}.
\]  

(3.23)

The numerical coefficient in (3.23) is different from the one in (2.7) by factor of \(\frac{3}{5}\). This disagreement may be related to ambiguities in using the BI action for fixing the normalization of operators beyond the leading second-derivative terms (see Appendix).

4. Concluding remarks

What follows from our proposal is that though some of the couplings in the \(S^5\) compactified 5-d (supergravity + KK multiplets) action may appear as missing they are actually ‘hidden’ and must be taken into account in establishing the AdS/CFT correspondence. This may carry an important message about how the AdS/CFT correspondence is actually working. The 10-d picture with non-zero KK momentum seems to be more fundamental than the 5-d one. This is after all what the original precise conjecture \[1\] about the duality of ten-dimensional string on \(AdS_5 \times S^5\) and \(\mathcal{N} = 4\) SYM theory is implying.

On the supergravity/string side the 5-d fields appear as KK modes of \(D = 10\) fields upon compactification on \(S^5\). The families of 5-d fields originating from a single type IIB supergravity field should definitely ‘remember’ their common 10-dimensional origin. At the same time, this 10-dimensional origin of the corresponding composite operators on the SYM side is quite obscure. The \(AdS_5 \times S^5\) string – SYM correspondence suggests that there may exist a ‘10-dimensional’ reformulation of \(\mathcal{N} = 4\) SYM theory where these operators are somehow unified in ‘KK families’.

One of our motivations for trying to get a detailed understanding of how the AdS/CFT correspondence works at the level of 3-point functions is to shed some light on the factorization properties of much more complicated 4-point correlators. In the previous discussions of the supergravity expression for the 4-point function of the ‘dilatonic’ \(O_4\) operator \[23,33,34,35\], the contribution of the scalar \(\nu\) exchange to the four-dilaton amplitude was not included. This may be justified by noting that although the 3-point amplitudes \(1.7\) are divergent for the operators satisfying \(1.9\), the exchange amplitude contributing to the 4-point function \(<O_{\Delta_1}O_{\Delta_1}O_{\Delta_2}O_{\Delta_2}>>\) with the exchanged state corresponding to the operator \(O_{\Delta_3}\) is actually finite \[36,50\]. Naively, one may conclude that the \(\nu\)-exchange contribution to the four-dilaton amplitude is indeed zero. This seems to provide an example when
the existence of a non-vanishing three-point function ($< O_4 O_4 O_8 >$) does not imply the presence of the corresponding factorization contribution to the 4-point ($< O_4 O_4 O_4 O_4 >$) correlator.

The story may be more intricate. In line with our proposal for the 3-point function $< O_4 O_4 O_8 >$ we suggest that the supergravity expression for the 4-dilaton amplitude should be defined as a $k \rightarrow 0$ limit of the amplitude $< O_{4+k} O_{4+k} O_{4+k} O_{4+k} >$ for four massive \( m_k^2 = \frac{k(k+4)}{R^2} \) dilaton modes. This amplitude receives contributions from both graviton and fixed scalar exchanges. It may happen that when the correlator $< O_4 O_4 O_4 O_4 >$ is defined in this way, its factorization will become more transparent (cf. \cite{33, 30, 35}).

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Appendix A. Normalizations of $O_4$ and $O_8$ operators from Born-Infeld action

Here we describe an attempt to fix normalizations in (3.12) by starting with a Born-Infeld action.\[^{16}\] The Born-Infeld Lagrangian for (a collection of) D3-branes in curved background has the form:

$$\mathcal{L} = T_3 \text{Str} \sqrt{\det(\hat{g}_{\mu\nu} + e^{-\frac{2}{3}F_{\mu\nu}})} + \text{WZ - terms},$$

(A.1)

where

$$F_{\mu\nu} = 2\pi\alpha' F_{\mu\nu}, \quad T_3 = \frac{1}{(2\pi)^3 g_s \alpha'^2}. \quad (A.2)$$

We take the background metric Einstein-frame metric as a combination of (3.2) and (1.4):

$$ds^2 = R^2 [e^{-\frac{16}{3}\nu} dz^2 + dx_{\mu} dx_{\mu} + e^{2\nu} d\Omega_5^2], \quad (A.3)$$

$$R^4 = \lambda \alpha'^2, \quad \lambda = 4\pi g_s N = g_{YM}^2 N. \quad (A.4)$$

\[^{16}\] Similar approach of fixing normalizations from BI action was used in \cite{2, 37}.
Then the relevant part of the Lagrangian in (A.1) becomes

\[ \mathcal{L} = T_3 \left( \frac{R^2}{z^2} e^{-\frac{2\phi z^2}{R^2}} e^{\frac{10\nu}{3}} \right)^2 \text{Str} \left( \left[ \det(\delta_{\mu\nu} + e^{-2\phi} \frac{z^2}{R^2} e^{\frac{10\nu}{3}} \mathcal{F}_{\mu\nu}) \right]^{\frac{1}{2}} - 1 \right) \]

\[ = T_3 \left( \frac{R^2}{z^2} e^{-\frac{2\phi}{3}} \right)^2 \text{Str} \left[ \frac{1}{4} e^{-2\phi} \frac{R^2}{z^2} e^{\frac{10\nu}{3}} \mathcal{F}^2 - \frac{1}{8} e^{-2\phi} \left( \frac{z^2}{R^2} e^{\frac{10\nu}{3}} \right)^4 \left( F^4 - \frac{1}{4} (F^2)^2 \right) + ... \right] \]

\[ = \frac{1}{4} T_3 e^{-\phi} (2\pi \alpha')^2 O_4 - \frac{1}{8} T_3 e^{-2\phi} \left( \frac{z^2}{R^2} e^{\frac{10\nu}{3}} \right)^2 (2\pi \alpha')^4 \frac{2}{3} O_8 + ... \]

\[ = ... - \left( \frac{N}{4\alpha} \phi O_4 + \frac{10N\pi^2}{9\lambda^2 - \nu z^4} O_8 \right) + \frac{N\pi^2}{3\lambda^2} \phi z^4 O_8 + ... . \]  

(A.5)

Here we used the definitions of \( O_4 \) and \( O_8 \) in (1.1), (1.2). Comparison with (3.12) then implies\( ^{17} \)

\[ a_4 = \frac{N}{4\lambda} , \quad a_8 = \frac{10N\pi^2}{9\lambda^2} . \]  

(A.6)

The normalization of \( O_8 \) derived in this way is only heuristic and should be treated with caution.

Indeed, this argument may look suspicious for several reasons. First, \( \nu \) already couples to scalars \( X \) at the second derivative level but \( (\partial X)^2 \) cannot be part of the \( O_8 \) operator. Also, \( \nu \) interacts differently with \( F^4 \) and \( (\partial X)^4 \) terms, in contradiction with the fact that they must be parts of the same super-invariant. The scalar partners of the gauge field couple to \( \nu \) at quartic order as \( e^{4\nu}(\partial X)^4 \), which leads to the normalization coefficient different from the one in (A.6) – since the coefficient of \( \nu \frac{10}{3} \) is replaced by 2, we get

\[ a_8' = \frac{2N\pi^2}{3\lambda^2} . \]  

(A.7)

Remarkably, this is precisely the coefficient one needs for the correspondence between the supergravity and SYM expressions for the 3-point functions (see eqs. (3.16) and (2.7)). However, due to the ambiguities mentioned above it seems hard at the moment to make this argument convincing.

In general, the BI action \([38,39,40]\) summarizes the low-energy limit of scattering amplitudes of massless open string modes in flat space. Its direct curved-space generalization correctly accounts for certain zero-momentum limits of the disc amplitudes with few closed string vertex operator insertions (see, e.g., \([11]\)). One should keep in mind, however, that the standard BI action does not include terms with derivatives of the vector field strength, higher than the first derivative of the scalar (coordinate) fields and terms with derivatives...
of the closed string fields. As a result, there may be a clash with manifest (linearly realized) $\mathcal{N} \leq 4$ supersymmetry: the manifestly supersymmetric extensions of the structures appearing in the expansion of the bosonic BI action may involve higher derivatives; elimination of such higher-derivative terms via integration by parts may produce terms with derivatives of the background supergravity fields, i.e. terms which are not included in the standard BI action. In particular, one should not be surprised if one does not find the correct bosonic (scalar) parts of the corresponding superinvariants from the expansion of the BI action.
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