Prospects for weak lensing/cosmic shear with VLTs

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ABSTRACT

The present status of weak lensing analyses of clusters of galaxies and of cosmic shear surveys are presented and discussed. We focus on the impact of very large telescopes on present-day and future surveys and compare their potential with HST or wide field 4 meter telescopes.

Keywords: Cosmology, Large Scale Structure, Gravitational Lensing, Weak Lensing, Surveys, Very Large Telescope

1. INTRODUCTION

Over the past decade, gravitational weak lensing offered astronomers an observing tool capable to measure the amount and the spatial distribution of dark matter in the sky. Weak lensing analysis can probe distortion fields of any specific gravitational systems, like groups or clusters of galaxies, in order to provide individual mass reconstruction of those systems. Cosmic shear studies, on the other hand, probe the statistical properties of the gravitational shear, relate them to the geometry and matter/energy content of the universe and attempt to measure some cosmological parameters and the dark matter power spectrum. The results obtained recently are reviewed in (see 1, 2, 3, or 4 in this conference) and some of them will be presented in this proceeding. In this presentation, we only focus on the impact the VLT can have on weak lensing science. In order to answer this question, we first sumarise the present status of weak lensing and cosmic shear surveys, then we point out the main issues and future goals and finally we try to answer how can VLT help in the future.

2. GRAVITATIONAL WEAK LENSING

The distortion of light beams produced by gravitational lensing modifies the image properties of lensed galaxies. In a Friedman-Robertson-Walker metric, and for stationary and weak gravitational fields, the deflection angle writes

\[ \hat{\alpha} = \frac{2}{c^2} \int \nabla \Phi \, dl, \]

where \( c \) is the celerity and \( \Phi \) the 3-dimension gravitational potential associated to the lens. When the deflection angle is small and when lenses can be approximated as thin gravitational systems, the relation between the source (\( S \)) and its image (\( I \)) positions follows the simple geometrical “lens equation”:

\[ \theta^I = \theta^S + \frac{D_{LS}}{D_{OS}} \hat{\alpha}(\theta^I), \]

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where $D_{ij}$ are angular diameter distances.

Equations (1) and (2) express how lens properties depend on (dark) matter distribution and on cosmological models. From an observational point of view, gravitational lenses manifest as image multiplication of galaxies or quasars, strong and weak distortions of galaxy shape or transient micro-lensing effects. These effects, as well as the time delays attached to image multiplications are exploited in order to probe the geometry and matter/energy content of the Universe or to observe high-redshift galaxies (see 1, 2, 3, 5 for reviews).

The image magnification is characterized by the convergence, $\kappa$, and by the shear components $\gamma_1$ and $\gamma_2$:

$$\kappa = \frac{1}{2} (\varphi_{,11} + \varphi_{,22}); \quad \gamma_1(\theta) = \frac{1}{2} (\varphi_{,11} - \varphi_{,22}); \quad \gamma_2(\theta) = \varphi_{,12} = \varphi_{,21}$$

where the $\varphi_{,ij}$ are the second derivatives of $\varphi$ with respect to the $i, j$ coordinates and

$$\varphi(\theta) = \frac{2}{c^2} \frac{D_{LS}}{D_{OS}D_{OL}} \int \Phi(D_{OL}\theta, z) \, dz \ .$$

Note that $\kappa$ is the Laplacian of $\varphi$, so it is the projected mass density.

The shear applied to lensed galaxies increases their ellipticity along a direction perpendicular to the gradient of the projected potential. The lens-induced distortion $\delta$ can then be evaluated from the shape of galaxies as it can be observed from the components of their surface brightness second moment $M_{ij} = \frac{\int I(\theta) \, d^2\theta}{\int I(\theta) \, d^2\theta}$, so it is the projected mass density.

$$\delta = \frac{2\gamma (1 - \kappa)}{(1 - \kappa)^2 + |\gamma|^2} \quad \Rightarrow |\delta| = \frac{a^2 - b^2}{a^2 + b^2} \ ,$$

where $a$ and $b$ are the major and minor axis of galaxies, as derived from their surface brightness second moment. In the weak lensing regime, the relation between the distortion and the gravitational shear simplifies to $\delta \approx 2\gamma$, so in principle, if we assume that unlensed galaxies are randomly oriented (i.e., $S = 0$), the ellipticity of galaxies, as measured from the second moment components, provides a direct estimate of the gravitational shear. The “shear map” can then be used to reconstruct the “mass map” at any galaxy position. However, since each galaxy has its own intrinsic ellipticity, and since the background galaxies only sparsely sample the sky, the final shear-induced ellipticity map is contaminated by shot noise so galaxy ellipticity must be averaged over a small but finite angular-size.

### 3. ANALYSIS OF CLUSTERS OF GALAXIES

#### 3.1. Weak lensing analysis of distant clusters

Because clusters of galaxies are young and the largest relaxed gravitational systems they are surviving witnesses of the cosmic history of structure formation. Their present-day mass function, its evolution with look-back time and their radial mass density profile are remaining footprints of the growth of gravitational perturbations over the past Gigayears as well as physical illustrations of the roles of cosmological parameters and the power spectrum of primordial fluctuations in the cosmic scenario.

It is now well known that gravitational lensing studies of clusters of galaxies do not need to assume the geometry of the mass distribution nor its dynamical and thermodynamical properties. In this respect this is an interesting tool to study complex systems like clusters of galaxies. By comnining the previous equations, it is easy to show that the projected mass density can be reconstructed from this integral:

$$\kappa(\theta) = \frac{1}{\pi} \int \Re[D^*(\theta - \theta') \gamma(\theta')] \, d^2\theta' + \kappa_0 \ ,$$

where

$$D(\theta - \theta') = \frac{(\theta_2 - \theta'_2)^2 - (\theta_1 - \theta'_1)^2 - 2i(\theta_1 - \theta'_1)(\theta_2 - \theta'_2)}{|(\theta - \theta')|^4} \ .$$

This inversion produces the mass reconstruction from the shear field.

From a technical and practical point of view clusters of galaxies are attractive systems for weak lensing analysis.
Figure 1. The left panel shows a simulation of a cluster of galaxies at redshift 0.15, lensed by an isothermal sphere with a velocity dispersion of 1300 km\(\text{s}^{-1}\). The lensed population has an average redshift of one. The amplitude of the gravitational shear decreases with radial distance and far from the cluster center, the ellipticity produced by the shear is smaller than the intrinsic ellipticity of the galaxies. The lensing signal must be averaged over a large number of galaxies in order to be measured accurately. The zoom on the right panel shows the images of the galaxies in the weak lensing regime. The contours show their shape as determined from their second moments. The average orientation of these galaxies is given by the solid lines at the top right. The lower line is the true orientation of the shear produced by the cluster at that position and the upper line is the orientation computed from the galaxies of the zoomed area. The difference between the two orientations is random noise due to the intrinsic ellipticity and orientation on distributions of the galaxies (from 2).

Their mass-density contrast is high (> 100) enough to produce significant gravitational distortion, and their angular scale (> 10 arc-minutes) is much larger than the typical angular distances between lensed background galaxies. The details of cluster mass distribution can therefore be well sampled by the grid of lensed galaxy population.

Over the past decade about 50 clusters have been reconstructed using weak gravitational lensing analysis, which all produced mass maps as the one shown in Fig.(2). They are listed in Table 1 (I excluded nearby clusters and reconstruction from depletion by magnification bias). This table reveal some general trends. The averaged mass-to-light ratios from weak lensing \((M/L)_{WL}\) is \((M/L)_{WL} \approx 400 h\) and typical velocity dispersion of those clusters is 1000 km/sec. When scaled with respect to the critical mass-to-light ratio, an estimate of the mass density of the universe can be derived,

\[
\Omega_{m-WL} \approx 0.3 \pm 0.1 ,
\]

which is in very good agreement with values inferred by other methods. In contrast, the mass density profiles are still too noisy to provide robust parameter profiles, like its central value, its typical scale and its slope. Isothermal, power law or NFW “universal” models fit equally well the data (see Fig.(3)), for all clusters analyzed so far (see 6). It is therefore not yet possible to test current numerical CDM predictions using this method. In fact, in view of the present signal-to-noise ratio of mass maps and the large family of possible analytical mass profiles, we are still far from being able to address in details this issue, if one use only weak distortion analysis.

3.2. Dark clusters?

One of the interesting outcomes of weak lensing is the blind discovery of clusters, like in Fig.(4) (see also 15), or the remarkable discovery of dark cluster candidates. Dark clusters are massive objects which are only detected from gravitational lensing effects. The most convincing cases (HST/STIS field (7); Abell 1942 (8); Cl1604+4304, (9); Cl0024+1654, 10) have been detected from a strong shear field spread over more than one arc-minute. For almost all candidates, there is no galaxies associated to the lens*. Their typical mass, estimated from reasonable

*The restriction is for MS1224 reported in Table 1.
Table 1. Results obtained from weak lensing analyses of clusters. The scale is the typical radial distance with respect to the cluster center. The \((M/L)_r\) ratio has been rescaled since most data were obtained with different filters (this is a rough rescaling; it probably increases uncertainties).

| Cluster        | \(z\) | \(\sigma_{obs}\) \((\text{kms}^{-1})\) | \(\sigma_{wl}\) \((\text{kms}^{-1})\) | \((M/L)_{100}\) | Scale \((h_{100}\text{ Mpc})\) | Tel. | Ref.                      |
|----------------|-------|----------------------------------------|----------------------------------------|-----------------|-------------------------------------|------|---------------------------|
| A2218          | 0.17  | 1370                                   | 1000-1400                              | ≈300            | 0.5                                 | CFHT | Squires et al (19)         |
| A1689          | 0.18  | 2400                                   | 1200-1500                              | -               | 0.5                                 | CTIO | Tyson et al (21)           |
|                |       |                                        | -                                      | 400             | 1.0                                 | CTIO | Tyson & Fischer (22)       |
|                |       |                                        | 1030                                   | -               | 1.0                                 | ESO/2.2 | Clove & Schneider (23)    |
| A2163          | 0.20  | 1680                                   | 740-1000                               | 300             | 0.5                                 | CFHT | Squires et al (24)         |
| A2390          | 0.23  | 1090                                   | ≈1000                                  | 320             | 0.5                                 | CFHT | Squires et al (24)         |
| < 8 clusters > | < 0.2 >| -                                      | -                                      | < 295 >         | 1.0                                 | CTIO | Wittman et al (26)         |
| MS1455+22      | 0.26  | 1133                                   | -                                      | 1080            | 0.4                                 | WHT  | Smail et al (27)           |
| AC118          | 0.31  | 1950                                   | -                                      | 370             | 0.15                                | HST  | Smail et al (20)           |
| MS1008-1224    | 0.31  | 1054                                   | 940                                    | 340             | 0.5                                 | VLT  | Athreya et al (28)         |
|                |       |                                        | 850                                    | ≈320            | 0.5                                 | VLT  | Lombardi et al (29)        |
| MS2137-23      | 0.31  | -                                      | 950                                    | 300             | 0.5                                 | VLT  | Gavazzi et al (in prep.)   |
| MS1358+62      | 0.33  | 940                                    | 780                                    | 180             | 0.75                                | HST  | Hoekstra et al (30)        |
| MS1224+20      | 0.33  | 802                                    | -                                      | ≈800            | 1.0                                 | CFHT | Fahlman et al (31)         |
|                |       |                                        | 1300                                   | 890             | 1.0                                 | MDM2.2 | Fischer et al (32)        |
| Q0957+56       | 0.36  | 715                                    | -                                      | -               | 0.5                                 | CFHT | Fischer et al (33)         |
| Cl0024+17      | 0.39  | 1250                                   | -                                      | 150             | 0.15                                | HST  | Smail et al (20)           |
| Cl0939+47      | 0.41  | 1080                                   | -                                      | 120             | 0.2                                 | HST  | Smail et al (20)           |
| Cl0302+17      | 0.42  | 1080                                   | 80                                     | 250             | 0.2                                 | HST  | Seitz et al (34)           |
| RXJ1347-11     | 0.45  | -                                      | 1500                                   | 400             | 1.0                                 | CTIO | Fischer & Tyson (35)       |
| 3C295          | 0.46  | 1670                                   | 1100-1500                              | -               | 0.5                                 | CFHT | Tyson et al (21)           |
| Cl0412-65      | 0.51  | -                                      | -                                      | 330             | 0.2                                 | HST  | Smail et al (20)           |
| Cl1601+43      | 0.54  | 1170                                   | -                                      | 190             | 0.2                                 | HST  | Smail et al (20)           |
| MS0016+16      | 0.55  | 1230                                   | -                                      | 180             | 0.2                                 | HST  | Smail et al (20)           |
|                |       |                                        | 740                                    | 740             | 0.6                                 | WHT  | Smail et al (36)           |
|                 |       |                                        | 800                                    | -               | 0.6                                 | Keck | Clowe et al (6)            |
| MS0451         | 0.55  | 1371                                   | 980                                    | -               | 0.6                                 | Keck | Clowe et al (6)            |
| Cl0054+27      | 0.56  | -                                      | -                                      | 400             | 0.2                                 | HST  | Smail et al (20)           |
| MS2053         | 0.59  | 820                                    | 730                                    | -               | 0.5                                 | Keck | Clowe et al (6)            |
|                |       |                                        | 886                                    | 360             | 0.5                                 | HST  | Hoekstra et al (40)        |
| MS1137+60      | 0.78  | 884                                    | 1190                                   | 270             | 0.5                                 | Keck | Clowe et al (37,6)         |
| RXJ1716+67     | 0.81  | 1522                                   | -                                      | 190             | 0.5                                 | Keck | Clowe et al (37)           |
|                |       |                                        | 1030                                   | -               | 0.5                                 | Keck | Clowe et al (6)            |
| MS1054-03      | 0.83  | 1360                                   | 1100-2200                              | 350-1600        | 0.5                                 | UH2.2 | Luppino & Kaiser (38)      |
|                |       |                                        | 1310                                   | 250-500         | 0.5                                 | HST  | Hoekstra et al (39)        |
|                |       |                                        | 1080                                   | -               | 0.5                                 | Keck | Clowe et al (6)            |
| < 10 clusters >| < 0.5 >| -                                      | -                                      | 1.0             | VLT                                 | White et al (2002) and Clowe et al (2002) |
assumptions on their redshift, corresponds to $M/L > 500$.

There is not yet any clear understanding about "dark clusters". Abell 1942, has also been observed in the infrared in order to detect a very high-redshift cluster of galaxies (11) which could be missed on visible data. But nothing has been detected so far. It is premature to claim that these systems are totally dark until X-ray observations confirm they do not contain hot intra-cluster gas. But if those objects were confirmed to be dark gravitational systems that will be a serious theoretical issue in order to understand how gravitational collapse would accrete only non-baryonic dark matter inside clusters. If the dark cluster population is numerous, then X-ray or optically selected samples underestimate their mass fraction and the cluster abundance in the universe. Whether they contribute significantly to $\Omega_m$ has to be clarified. However, since only few dark clusters have been observed in wide field surveys, it is likely that they do not represent a significant fraction of mass. This is also confirmed by the fact that the $\Omega_m - \sigma_8$ relations inferred from cosmic shear (see next section) and from cluster abundance are not in contradiction, as it should be if a significant fraction of clusters were dark.

3.3. Issues and prospects for VLTs

The investigation of a large sample of clusters of galaxies using weak gravitational lensing is motivated by the potential of this method to directly probe matter, regardless its nature nor its dynamical stage. In principle, any mass-scale can be observed, so that a detailed mass function of gravitational systems could be derived from this kind of survey. However, there are still technical issues have to be addressed. Some of these key questions are listed below:

- What is the lower mass limit weak lensing can probe?
- Is it possible to build a complete mass-limited sample of clusters of galaxies with a weak lensing survey?
Figure 3. The total mass profile of the lensing cluster MS1007-1224 as it is derived from a mass reconstruction of VLT FORS/ISAAC and NTT/SOFI data. The solid lines on the left show the best fit of a singular isothermal profile and an isothermal model with core radius. The solid line on the right in an NFW mass profile. Both isothermal and NFW profiles fit these data equally well (from 28, see also 29.).

Figure 4. A blind mass reconstruction in a randomly selected VLT field. The right panel shows the $\kappa$ mass automatically produced by the cosmic shear pipeline used by (53) to process their 50 VLT fields. After this strong detection, this field was checked by eye and a galaxy concentration corresponding to a cluster of galaxies was detected. This shows the strength of weak lensing analysis: in principle, this technique allows astronomers to make mass-selected cluster samples without using the light distribution of galaxies (from 41).

- Are dark clusters real?
- Could projection effects contaminate weak lensing reconstruction? How projection effect could spoil the individual mass maps and the definition of a mass-limited sample?
- What is the cluster mass profiles of clusters of galaxies and groups? Down to what accuracy can we measure it with weak lensing?
- What is the redshift distribution of sources? Do we have a good absolute mass calibrations of clusters?
- Can we use weak lensing to probe substructures in cluster?
- How can we suppress the mass-sheet degeneracy?
Figure 5. Redshift distribution of sources. The left panel shows the sensitivity of the $D_{LS}/D_{OS}$ ratio to the source and lens redshifts. For a fixed lens redshift, the ratio monotonically increases with the source redshifts. Conversely, for a fixed source redshift, the ratio decreases with the lens distance, making mass reconstruction of very distant clusters more and more challenging. On the right panel, the histogram shows the photometric redshift distribution of the van Waerbeke et al (42) cosmic shear shear survey. This photometric distribution was obtained using 1000 galaxies observed with VLT FORS1, ISAAC and NTT/SOFI. The thick line is the best fit.

- What is the mass of the most massive high-$z$ clusters of galaxies?

The VLT can certainly help to answer this questions. Although no ground based telescopes can compete with the outstanding image quality of HST, Keck, VLT, Gemini and SUBARU have much larger fields of view which better match cluster angular sizes.

In order to emphasize the strength of VLT, the relevant quantity is the signal-to-noise of the weak lensing analysis of a gravitational system. For a typical mass reconstruction it expresses as follows:

$$
\frac{S}{N} \approx 10 \left( \frac{n}{20 \ \text{arcmin}^{-2}} \right)^{\frac{1}{2}} \times \left[ \frac{\sigma_{\text{gal}}}{0.4} \right]^{-1} \times \left[ \frac{\sigma_{v}}{800 \ \text{km/sec}^{-1}} \right]^{2} \times \langle \frac{D_{LS}}{D_{OS}} \rangle,
$$

where $n$ is the galaxy number density, $\sigma_{\text{gal}}$ the intrinsic ellipticity dispersion of galaxies and $\sigma_{v}$ the cluster velocity dispersion. Equation (9) shows that we only have two ways to improve weak lensing analysis of a specific target: either increasing the depth of observation in order to increase $n$, or increasing $D_{LS}/D_{OS}$ by increasing the source redshifts and therefore the amplitude of the shear. In addition, since the “Universal mass profile” is only an averaged view of cluster radial mass distribution, we expect strong mass profile fluctuations from cluster to cluster, so it is important to observe a large sample.

$n$ and $D_{LS}/D_{OS}$ are correlated, since the redshift of sources increases with the depth of observations. However, both work in several and somewhat different ways: $n$ also provides a better sampling of each field, making the analysis of substructures in clusters easier. On the other hand, as it is shown on Fig.(5), changing $D_{LS}/D_{OS}$ is more complicated since it both depends on the lensed sources and the distance of the lensing cluster. Nevertheless, for distant clusters of galaxies the need for very high redshift sources is obvious. Furthermore, it is worth noticing that the absolute mass of clusters cannot be derived if the redshift of the lensed sources is unknown.

The impact of VLTs are therefore clear: in order to improve the spatial resolution of mass reconstruction, to produce mass maps of very distant clusters of galaxies and to uncover their absolute total mass, one need to go very deep and to get redshifts of galaxies. The need for redshift is a critical point and indeed a very strong case for VLT observations rather than 4-meter class telescopes: beyond $I \approx 24$, even VLT spectroscopic capabilities are not sufficient. Photometric redshifts are the only technique to probe high-$z$ galaxies. Moreover, has it is demonstrated by several works done in the past on photometric redshift (see for example 43), in order to precisely sample the redshift domain between $z = 0$ and $z = 2$, both optical and near infrared photometric data are necessary, with S/N of 10 in each band. Otherwise, part of the redshift range of the distorted sources could
be systematically missed by the photo-z machinery, which would bias the mass estimate of distant clusters. In practice, only VLTs can go fast enough to probe these galaxies in all $UBVRIJHK$ bands in a reasonable amount of time (see the results obtained with VLT/FORS+ISAAC on Fig. (5)). In this respect it is hard to HST and 4 meter class telescopes to be competitive. This is even more obvious if one wants to carry out a cluster weak lensing survey over a large cluster sample (see the EDICS cluster survey for instance).

Regarding dark clusters, the first priority is to test alternative solutions, like very distant (almost invisible) clusters of galaxies or projection effects. VLTs are certainly very useful, in particular if dark clusters results from projection effects along the line of sights. Deep optical and near infrared images of these fields can provide a detailed description of source clustering which accumulate along each line of sight. This contamination has been addressed recently by 44) and it seems to be a serious concern. Ultra deep multicolor observations of each known dark cluster candidate can quickly test this possibility with VLTs.

Finally, it is worth noticing that even for Poisson noise argument the depth is also a strong requirement for very high-z clusters observations. Beyond $z \approx 1$, the galaxy number density of background sources significantly decreases, making the mass reconstruction more and more noisy (and undersampled). Going to extremely deep imaging is the only way to compensate the decreasing fraction of lensed population. This critical point is also valid for photo-z, which will be more and more demanding in observing time in order to keep the signal-to-noise of photometric redshifts high enough. This is clearly beyond the capabilities of 4 meter class telescopes.

There is indeed an alternative to depth in order to sample accurately clusters and to investigate the detailed substructures in clusters of galaxies. It consists in using weak lensing in nearby clusters of galaxies (see 45). In that case, for a given depth, one can probe small physical scales than for high-z clusters. In order to probe clusters on megaparsec scales, this alternative would favor wide field observations rather than ultra deep VLT imaging. However (46) pointed out that cosmic variance produced by all structure along the line of sight may be a severe limitation of this approach for the nearest clusters.

A general limitation of VLTs is the rather small field of view most faint imaging-spectrographs have. Even the VLT/FORS instruments with of field of view of 7 arc-minutes cannot probe the periphery of $z \approx 0.5$ is one single shot. This then hampers to assume that $\kappa = 0$ at the border of the field. The mass-sheet degeneracy is still an issue for VLT and Keck telescopes, but likely not for SUBARU/SUPRIME.

In conclusion, the VLTs are certainly the most precise telescope for a thorough investigations of clusters of galaxies, in particular those at high redshift. The two weak points are the image quality which cannot compete with HST and the small field of views they have. On the first point, since galaxy angular size are significantly larger than the PSF and the pixel size, the capability of ground based telescopes to measure weak lensing down to a limiting shear of 1% is indeed excellent, even for VLT. Regarding the other point (which is even more dramatic for HST), clearly we should recommend to use SUPRIME at SUBARU or VMOS at the VLT in the future. It would be extremely valuable to have similar wide field instrument for the near infrared. Even $J$ and $H$ bands, as it is proposed in the NIRMOS instrument design, would be sufficient.

4. COSMIC SHEAR

Light propagation over Gigaparsec distances and across an inhomogeneous universe produces weak lensing effects which accumulates along the line of sight of each ray bundle. Assuming structures formed from gravitational growth of Gaussian fluctuations, the shape and amplitude of cosmological weak lensing as function of angular scale can be predicted from Perturbation Theory. To first order, the convergence $\kappa(\theta)$ at angular position $\theta$ is given by the line-of-sight integral

$$
\kappa(\theta) = \frac{3}{2} \Omega_0 \int_0^{z_s} n(z_s) dz_s \int_0^{\chi(z)} \left[ \frac{D(z)}{D(z_s)} \right] \delta(\chi, \theta) [1 + z \chi] d\chi
$$

(10)

where $\chi(z)$ is the radial distance out to redshift $z$, $D$ the angular diameter distances, $n(z_s)$ is the redshift distribution of the sources and $\delta$ is the mass density contrast responsible for the deflection at redshift $z$. Its amplitude at a given redshift depends on the properties of the (dark) matter power spectrum and its evolution with look-back-time.

The cumulative weak lensing effects of structures induce a shear field which is primarily related to the power
spectrum of the projected mass density, $P_\kappa$. Its statistical properties can be analyzed using several statistics on galaxy ellipticities, like the shear top-hat variance (18, 47, 48),

$$\langle \gamma^2 \rangle = \frac{2}{\pi \theta_c^2} \int_0^\infty \frac{dk}{k} P_\kappa(k) [J_1(k\theta_c)]^2, \quad (11)$$

or others described in Mellier et al (4, this conference) where $J_n$ is the Bessel function of the first kind. $P_\kappa(k)$ is directly related to the 3-dimension power spectrum of the dark matter along the line of sight, $P_{3D}$.

$$P_\kappa(k) = \frac{9}{4} \Omega_m^2 \int_0^\infty P_{3D} \left( \frac{k}{D_L(z)}, z \right) W(z, z_s) \, dz, \quad (12)$$

where $W(z, z_s)$ is an efficiency function which depends on the redshift distribution of sources and lenses. Therefore, in principle an inversion permit to reconstruct the 3-dimension power spectrum of the dark matter from the weak distortion field.

4.1. Analysis and interpretation of the shear variance

The amplitude of cosmic shear signal and its sensitivity to cosmology can be illustrated in the fiducial case of a power law mass power spectrum with no cosmological constant and a background population at a single redshift $z$. In that case, the variance of the convergence, $\langle \kappa(\theta)^2 \rangle$, writes:

$$\langle \kappa(\theta)^2 \rangle >^{1/2} = < \gamma(\theta)^2 >^{1/2} \approx 1% \sigma_8 \Omega_m^{0.75} z_s^{0.8} \left( \frac{\theta}{1'} \right)^{-\left( \frac{z_s}{z} \right)}, \quad (13)$$

where $z_s$ is the source redshift, $n$ is the spectral index of the power spectrum of density fluctuation and $\sigma_8$ the power spectrum normalization. Several teams succeeded to get a significant signal (see Table 2). Since each group used different telescopes, adopted different observing strategies and used different data analysis techniques, one can figure out their reliability by comparing these surveys. Figure 6 show that they are all in very good agreement†. This is a convincing demonstration that the correlation of ellipticities do not only results from unexpected systematic effects. The cosmological origin of the coherent distortion signal detected

Table 2. Present status of cosmic shear surveys with published results.

| Telescope     | Pointings | Total Area | Lim. Mag. | Ref.      |
|---------------|-----------|------------|-----------|-----------|
| CFHT          | $5 \times 30' \times 30'$ | 1.7 deg$^2$ | $I=24.17$ | [vWME+]   |
| CTIO          | $3 \times 40' \times 40'$ | 1.5 deg$^2$ | $R=26.$  | [WTK+]    |
| WHT           | $14 \times 8' \times 15'$ | 0.5 deg$^2$ | $R=24.$  | [BRE]     |
| CFHT          | $6 \times 30' \times 30'$ | 1.0 deg$^2$ | $I=24.$  | [KWL]     |
| VLT/UT1       | $50 \times 7' \times 7'$ | 0.6 deg$^2$ | $I=24.$  | [MvWM+]   |
| HST/WFPC2     | $1 \times 4' \times 42'$ | 0.05 deg$^2$ | $R=27.$  | [WTK+]    |
| CFHT          | $4 \times 120' \times 120'$ | 6.5 deg$^2$ | $I=24.$  | [vWMR+]   |
| HST/STIS      | $121 \times 1' \times 1'$ | 0.05 deg$^2$ | $V\approx26$ | 60       |
| CFHT          | $5 \times 126' \times 140'$ | 24. deg$^2$ | $R=23.5$ | 61        |
| CFHT          | $10 \times 126' \times 140'$ | 53. deg$^2$ | $R=23.5$ | 62        |
| CFHT          | $4 \times 120' \times 120'$ | 8.5 deg$^2$ | $I=24.$  | 63        |
| HST/WFPC2     | $271 \times 2.1 \times 2.1$ | 0.06 deg$^2$ | $I=23.5$ | 59        |
| Keck+WHT      | $173 \times 2' \times 8'$ | 1.6 deg$^2$ | $R=25$   | 56        |

†The Hoekstra et al (61) data are missing because depth is different so the sources are at lower redshift and the amplitude of the shear is not directly comparable to other data plotted.
by all these surveys has been confirmed by decomposing the distortion field into E- and B- modes. The E-mode is a gradient term which contains signal produced by gravity-induced distortion. The B-mode is a pure curl-component, so it only contains intrinsic ellipticity correlation or systematics residuals. Both modes have been extracted using the aperture mass statistics by van Waerbeke et al (55, 42) and Pen et al (63), in the VIRMOS-DESCART‡§ survey, as well as by Hoekstra et al (62) in the Red Cluster Sequence survey. In both samples, the E-mode dominates the signal, which strongly supports the gravitational origin of the distortion.

The comparison of the top-hat shear variance with some realistic cosmological models has been done for these surveys. Although some cosmological models are ruled by these data, as expected from Eq.(13) the degeneracy between $\Omega_m$ and $\sigma_8$ hampers a strong discrimination among most popular cosmological models. The present-day constraints resulting from these studies, including the VLT and Keck samples (see Figure(7)), can be summarized as follows (90% confidence level):

$$0.05 \leq \Omega_m \leq 0.8 \quad \text{and} \quad 0.5 \leq \sigma_8 \leq 1.2,$$

and, in the case of a flat-universe with $\Omega_m = 0.3$, we infer $\sigma_8 = 0.9 \pm 0.1$ (68% c.l.).

### 4.2. Analysis of non-Gaussian features in cosmic shear signal

Higher order statistics, like the skewness of the convergence, $s_3(\theta)$, can also be computed. These statistics are sensitive to non-Gaussian features in the projected mass density field produced by non-linear systems like massive clusters or compact groups of galaxies (12, 13). As for the simple fiducial models discussed in the

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1Data of the VIRMOS-DESCART survey were processed and analyzed at the TERAPIX data center: [http://terapix.iap.fr](http://terapix.iap.fr)
previous section, one can express also $s_3(\theta)$:

$$s_3(\theta) = \frac{\langle \kappa^3 \rangle}{\langle \kappa^2 \rangle^2} \approx 40 \Omega_m^{-0.8} z^{-1.35}.$$  (15)

Therefore, in principle the degeneracy between $\Omega_m$ and $\sigma_8$ can be broken when both the variance and the skewness of the convergence are measured. Unfortunately, the measurements of $s_3(\theta)$ is presently hampered by a number of practical difficulties which are not yet fixed. In particular, the masking process which masks bright stars and defects on CCD images degrades the quality of the mass reconstruction. The reliability of its skewness as well as its cosmological interpretation are therefore seriously questioned. Bernardeau, van Waerbeke & Mellier (65) have proposed an alternative method using some specific patterns in the shear three-point function. Their detection strategy based on their method has been tested on ray tracing simulations and turns out to be robust, usable in patchy catalogs, and quite insensitive to the topology of the survey. They have recently used the analysis of the 3-point correlations function on the VIRMOS-DESCART data. Their results show a 2.4σ signal over four independent angular bins corresponding to angular scales between 2 and 4 arc-minutes. The amplitude and the shape of the signal are consistent with theoretical expectations obtained from ray-tracing simulations. This result supports the idea that the measure corresponds to a cosmological signal due to the gravitational instability dynamics. Although the errors are still large to permit secure conclusions, one clearly sees that the amplitude and the shape of the 3-point correlations function match the most likely cosmological models. Remarkably, the ΛCDM scenario perfectly fit the data points.

The Bernardeau et al (66) result is the first detection of non-Gaussian features in a cosmic shear survey and it opens the route to break the $\Omega_m - \sigma_8$ degeneracy. However, there are still some caveats which may be considered seriously. One difficulty is the source-lens clustering which could significantly perturb high-order statistics (Hamana et al 2002 (67)). Fig.(8) shows that source clustering may change the amplitude of the skewness of the convergence by 20%. If so, multi-lens plane cosmic shear analysis will be necessary which implies a good knowledge of the redshift distribution. According to Hamana et al (Fig. 8), the source clustering contamination decreases with source redshift, so that this issue is less critical when lensed galaxies are a very high-z. For sources at redshift one, the effect of source clustering can drop down to less than 5% if the sources are within a narrow range in redshift. It implies that source redshift must be measured with an accuracy better than 0.2 at redshift one, and better than 0.05 for source at redshift 0.5. When looking at the Bolzonnela et al plot on photo-z (see 43), this accuracy demands both optical and near infrared photometric data, with a photometric precision of about 10% in all filters.
Figure 8. Sensitivity of the convergence skewness to source-lens clustering. The top-left panel shows how the amplitude of the skewness varies with source-lens clustering for three cosmological models. The lower plot gives the relative variation. In all cases, the skewness decreases by a factor larger than 10%, a serious limitation to precision cosmology with cosmic shear. To solve this issue, one needs to get the redshift of source-lenses. On the right panel, the width of the source distribution is plotted as function of the averaged source-redshift. In order to minimize the clustering effect, the source distances must be large as possible and spread over a narrow redshift range. Clearly, the accuracy of photometric redshift must be better than 10%. This goal can be achieved if UBVRIJ and $K$ band data are obtained for most lensed galaxies, with a photometric accuracy of about 10% in each filter. (From 67)

4.3. Issues and prospects for VLTs

Although there are still a lot of technical issues that must be addressed prior to go toward high precision cosmology with cosmic shear, there are some immediate points which turn out to be important for the near future and where VLTs can help. As for clusters, let us present them by the following questions:

- What is the redshift distribution of sources?
- Can we have an idea of source clustering for skewness?
- How can we minimize cosmic variance?
- How high order statistics are contaminated by $E$ and $B$ modes?
- Are VLT competitive with respect to wide field 4-meter telescopes for cosmic shear?

These issues express the sensitivity of the variance and the skewness to source redshift, as shown in Eq.(13,15) and to the source-lens clustering, as pointed out by (67). On the other hand, the strength of VLT with respect to other telescopes can also be discussed by looking at the signal-to-noise ratio of cosmic shear surveys. Adopting the null hypothesis that no gravitational weak lensing signal is present, we can predict the expected limiting shear if only shot noise and sampling are taken into account (in particular, we assume there is no systematic residual errors associated to image processing or PSF corrections as those discussed later). For a $3\sigma$
shear variance limiting detection it writes:

\[ < \gamma(\theta)^2 >_{\text{limit}} = 1.2\% \left( \frac{A_T}{1 \text{ deg}^2} \right)^{-\frac{1}{4}} \times \left( \frac{\sigma_{\text{gal}}}{0.4} \right)^{-\frac{1}{2}} \times \left( \frac{n}{20 \text{ gal/arcmin}^2} \right)^{-\frac{1}{2}} \times \left( \frac{\theta}{10''} \right)^{-\frac{1}{2}}, \]

where \( A_T \) is the total area covered by the survey, \( \sigma_{\text{gal}} \) is the intrinsic ellipticity dispersion of galaxy and \( n \) the galaxy number density of the survey.

The sensitivity to \( A_T \) favors CCD panoramic cameras on 4 meter telescopes against VLTs with small field of view. However, the sensitivity to \( n \) compensates this weak point. Furthermore, since VLT are faster photon collectors than other ground based telescopes, they can observe much more uncorrelated fields, which can be spread over a very large sky area, which then lower the cosmic variance significantly. Therefore, exactly as for clusters of galaxies, the need for VLT on cosmic shear projects is primarily the need for very accurate photometric redshifts and for very deep exposure to increase the galaxy number density. All conclusions relevant for clusters are therefore valid for cosmic shear. In addition, it is important to benefit from the speed of VLT to define an observing strategy based on a sparse sampling using a large number of fields rather than a very large total and compact field of view.

5. SUMMARY AND CONCLUSIONS

The very large aperture of VLTs make these telescopes the most competitive for all scientific goals which demand very deep or very fast capabilities. By increasing limiting magnitudes, VLTs permit to increase the galaxy number density of lensed galaxies, which in return drops the Poisson noise, improves the spatial resolution of mass maps and increases the shear signal by increasing the redshift of sources. As we discussed in previous sections, the knowledge of redshift of sources as well as their clustering will be more and more critical in the future for high precision cosmology. Both optical and near infrared capabilities will therefore be necessary. This requirement is hard to reconcile with high depth unless unrealistic amount of telescope time is spent on weak lensing and cosmic shear studies. Indeed VLTs are in this respect the best instruments, and a join FORS1+ISAAC-like configuration looks promising for future surveys, provided the near infrared field of view is large enough.

The speed capabilities of VLTs is almost of equal importance. As (53) and (56) showed, 10 meter class telescopes can spend only 20 mn per field instead of one hour for 4 meter class telescopes. This advantage can be used to sample in a clever way the sky and make a sparse coverage of a huge field. Maoli et al (53) spread 50 6'\times6' FORS1 field overs 2000 deg^2 and Bacon et al (56) spread 170 fields in the same way. Clearly, one could envision to observe at least 1000 fields with VLT. Figure (9) show the gain when a survey is based on 100, 150 or 300 fields... One can see the potential to constrain cosmological models, with in addition accurate photometric redshift and therefore clustering informations for each field. This strategy can be also used to probe in a clever way the power spectrum on the projected mass density. However, there is not yet a clear quantitative evaluation of the optimum efficiency of such a survey as function of its topology and the sampling strategy. This point has to be clarified.

The weakness of VLTs is definitely the rather small field of view. SUBARU/SUPRIME with of field of view of 24\' turns out to be the most competitive configuration (Miyazaki 2002, (68)). For weak lensing studies of distant clusters of galaxies, the field of view of is not a serious issue since 1 Megaparsec roughly corresponds to 5\'. Therefore, VLT are definitely superior for weak lensing analysis of distant clusters of galaxies. In contrast, as Eq.(16) shows, this may be a limitation for cosmic shear surveys. The incoming VMOS camera on the VLT, which will have a 15' field of view will be a very competitive instrument for cosmic shear, provided it is demonstrated that is image quality is not degraded at the boundary of the field. The need for a wide field near infrared instrument with similar field of view as optical camera is clear. The K-band is not really necessary since even J and H bands are sufficient to get precise photometric redshifts. With such an optical+NIR configuration, a cosmic shear survey covering at least 300 hundred uncorrelated fields with photo-z for all galaxies will produce the most precise data set for high precision cosmology using gravitational lensing tools.
Figure 9. Illustration of the signal-to-noise of the variance of the shear for 100, 150 and 300 VLT/FORS1 fields. One can see that for 300 hundred fields, the two models will be disentangled with a $3\sigma$ confidence level.

Acknowledgements

We thank D. Bacon, M. Bartelmann, D. Clowe, R. Ellis, B. Fort, H. Hoekstra, L. King, M. Lombardi, A. Renzini and A. Réfrégier for useful discussions on the potential of very large telescopes for weak lensing. We thank S. White and the EDICS team and in particular D. Clowe for providing the mass map of cl1232-1250 prior to publication and D. Bacon for providing authorization to publish a figure of his paper. This work was supported by the TMR Network “Gravitational Lensing: New Constraints on Cosmology and the Distribution of Dark Matter” of the EC under contract No. ERBFMRX-CT97-0172.

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