Distributed Coverage Maximization via Sketching

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Abstract

In this paper, we present distributed algorithms for coverage optimization problems with almost optimal space complexity and optimal approximation guarantees. These new algorithms also achieve an optimal communication complexity, running in only four rounds of computation, addressing major limitations of prior work. While previous distributed algorithms for submodular maximization rely on ideas of core-sets, our algorithms are based on a new adaptive sampling and sketching technique. We show that the proposed algorithms are implementable in various distributed optimization frameworks such as MapReduce and RAM models. Moreover, our ideas extend to weighted variants of coverage problems, and can solve the related dominating set problems.

Furthermore, we perform an extensive empirical study of our algorithms (implemented in MapReduce) on a variety of datasets. We observe that using sketches 30–600 times smaller than the input, one can solve the coverage maximization problem with quality very close to that of the state-of-the-art single-machine algorithm. Finally, we show an application of our algorithm in large-scale feature selection.

1 Introduction

As important special cases of submodular optimization, maximum $k$-cover and minimum set cover are among the most central problems in optimization with a wide range of applications in machine learning, document summarization, and information retrieval; e.g., see [14, 1, 12, 27]. In order to address the need for handling large datasets, many techniques have been developed for distributed submodular maximization [11, 23, 27, 20, 8, 26, 15]. However, many of these results do not take advantage of the special structure of coverage functions, and consequently achieve suboptimal approximation guarantees and/or poor space complexities in terms of the size of the (coverage) instance. In particular, most previous results on submodular maximization either explicitly or implicitly assume a value oracle access to the submodular function. Such an oracle for coverage functions has the following form: given a subfamily of the (input) family, determine the size of the union of the sets in the subfamily. Implementing this subroutine is costly in the presence of large subsets in the family and/or a large ground set. Indeed communicating entire subsets across machines might be quite impractical. In this paper, we aim to address the above issues, and present almost optimal distributed approximation algorithms for coverage problems with optimal communication and space complexity. Before elaborating on our results, let us describe the problem formulations, and distributed computation models discussed later.

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Problem Formulation Consider a ground set $\mathcal{E}$ of $m$ elements, and a family $\mathcal{F} \subseteq 2^\mathcal{E}$ of $n$ subsets of the elements (i.e., $n = |\mathcal{F}|$ and $m = |\mathcal{E}|$). The coverage function $C$ is defined as $C(S) = |\bigcup_{U \in S} U|$ for any subfamily $S \subseteq \mathcal{F}$ of subsets. Given $k \geq 0$, the goal in $k$-cover is to pick $k$ sets from $\mathcal{F}$ with the largest union size. Set cover asks for the minimum number of sets from $\mathcal{F}$ that together cover $\mathcal{E}$ entirely. In this paper, we also study the following variant of this problem, called set cover with $\lambda$ outliers\footnote{There are two separate series of work in this area. We use the convention of the submodular/welfare maximization formulation \cite{7}, whereas the hypergraph-based formulation \cite{31} typically uses $n,m$ in the opposite way.}, where the goal is to find the minimum number of sets covering at least a $1 - \lambda$ fraction of the elements $\mathcal{E}$.

MapReduce Model The distributed computation model—e.g., MapReduce \cite{17}—assumes that the data is split across multiple machines. In each round of a distributed algorithm, the data is processed in parallel on all machines: Each machine waits to receive messages sent to it in the previous round, performs its own computation, and finally sends messages to the other machines. The total amount of data a machine processes is called its load, which had better be sublinear in the input size. In fact, two important factors determine the performance of a distributed algorithm: (i) the number of rounds of computation, and (ii) the maximum load on any machine. These parameters have been discussed and optimized in previous work \cite{22, 20, 9, 26}.

RAM Model Another model for handling a large amount of data is what we call the RAM model \cite{2}, where the algorithm has random access to any part of the input (say, to the edge lists in the graph) but each lookup takes constant time. For many problems it might be possible to judiciously and adaptively query the data, and solve the problem. In order to implement such a model in practice, distributed hash-tables (such as Bigtable) have been proposed and applied in practice \cite{13}. From a theoretical point of view, this model is closely related to the communication complexity literature.

Our Contributions In this paper, we present distributed algorithms for $k$-cover and set cover addressing several shortcomings of previously studied algorithms and achieving optimal approximation guarantees as well as almost optimal space and communication complexity. To achieve this result, we present an adaptive sampling (or sketching) technique that can be implemented in a distributed manner. We also rule out effectiveness of various simpler sampling techniques by providing lower bound examples. More precisely, our results for coverage problems are as follows: First of all, we develop distributed algorithms for $k$-cover and set cover with $\lambda$ outliers, that are almost optimal from three perspectives: (i) they achieve optimal approximation guarantees of $1 - 1/e$ and $\log \frac{1}{\lambda}$ for the above two problems, respectively; (ii) they have a memory complexity of $O(n)$ and also $O(n)$ communication complexity; and finally (iii) they run in a few (constant) rounds of computation; see Table 1 for brief comparison of our theoretical results with prior work (Sections 2 and 3). We note that the space complexity of our algorithm is independent of the size of the universe of elements and is only a linear function of the number of input sets. This is crucial for tackling coverage instances with very large sets, or large total number of elements.

Secondly, not requiring value oracle access to the coverage function makes our algorithms and techniques applicable to related problems such as dominating set with applications to influence maximization in social networks (see \cite{27} for application). Indeed we give the first distributed algorithm
Table 1: Comparison of our results to prior work. The first three work for the more general case of submodular maximization.

| Problem                        | Credit | # rounds | Approximation | Load per machine |
|--------------------------------|--------|----------|---------------|-----------------|
| $k$-cover                      | 23     | $O\left(\frac{1}{\varepsilon} \log m\right)$ | $1 - \frac{1}{e} - \varepsilon$ | $O(mk^n)$       |
| $k$-cover                      | 26     | 2        | 0.54          | $\max(mk^2, mn/k)$ |
| $k$-cover                      | 16     | $\frac{1}{\varepsilon}$ | $1 - \frac{1}{e} - \varepsilon$ | $\max(mk^2, mn/k)$ |
| $k$-cover                      | Here   | 4        | $1 - \frac{1}{e} - \varepsilon$ | $\tilde{O}(n)$ |
| set cover with outliers        | Here   | 4        | $(1 + \varepsilon) \log \frac{1}{\lambda}$ | $O(n)$ |
| submodular cover               | 29     | $\Omega(n^{\frac{1}{16}})$ | $\tilde{O}(n)$ | $\tilde{O}(mn)$ |
| submodular cover               | 30     | $O\left(\frac{\log n \log m}{\varepsilon}\right)$ | $(1 + \varepsilon) \log \frac{1}{\lambda}$ | $\tilde{O}(mn)$ |
| dominating set                 | Here   | 4        | $(1 + \varepsilon) \log \frac{1}{\lambda}$ | $O(n)$ |

Table 2: Comparison of results for the RAM model.

| Problem                        | Credit | Approximation | Runtime |
|--------------------------------|--------|---------------|---------|
| $k$-cover                      | 6, 28  | $1 - \frac{1}{e} - \varepsilon$ | $O(nm)$ |
| $k$-cover                      | Here   | $1 - \frac{1}{e} - \varepsilon$ | $\tilde{O}(n)$ |
| set cover with outliers        | Here   | $(1 + \varepsilon) \log \frac{1}{\lambda}$ | $O(n)$ |

for dominating set that does not need to load all edges of a node onto a single machine (Section 4). This is crucial for handling graphs with nodes of very high degree. Thirdly, we show that our algorithm can be implemented in both MapReduce and RAM models, and furthermore, present extensions of our distributed algorithm to a number of variants of weighted coverage problems (Section 5).

Last but not least, we demonstrate the power of our techniques via an extensive empirical study on a variety of applications and publicly available datasets (Section 6). We observe that sketches that are a factor 30–600 smaller than the input suffice for solving $k$-cover with quality (almost) matching that of the state-of-the-art single-machine algorithm; e.g., for a medium-size dataset, we can obtain 99.6% of the quality of the single-machine stochastic-greedy algorithm using only 3% of the input data. Some of the instances we examine in this paper are an order of magnitude larger than the ones studied in prior work [27]. Finally, we show an application of our algorithm in large-scale feature selection by formalizing it as a coverage problem where we aim to choose a subset of features that cover as many pairs of samples as possible. In doing so, we take advantage of the fact that the space complexity of our algorithm is independent of the number of elements in the instance, and we can solve instances of coverage problem with very large sets.

Further Related Work Although maximum $k$-cover may be solved using a distributed algorithm for submodular maximization, all the prior work in this area (have to) assume value oracle access to the submodular function, introducing a dependence on the size of the sets in the running time of each round of the algorithms. In this model, problem, Chierichetti et al. [14] present a $1 - \frac{1}{e}$-approximation algorithm for $k$-cover in polylogarithmic number of rounds of computa-
tion, improvable to $O(\log n)$ rounds \cite{11, 23}. Recently randomized core-sets were used to obtain a constant-approximation 2-round algorithm for this problem \cite{26, 15}, where the best known approximation factor is 0.54. In other recent work, Mirzasoleiman et al. \cite{29, 30} give a distributed algorithm for submodular cover (a generalization of set cover) in the MapReduce framework, however, their algorithm runs in superconstant number of rounds. and compared to the result presented here, they have much larger space complexity when it's applied to set cover.

More Notation Coverage problems may also be described via a bipartite graph $G$, with the two sides corresponding to $F$ and $E$, respectively. The edges of $G$ correspond to pairs $(S, i)$ where $i \in S \in F$. For simplicity, we assume that there is no isolated vertex in $E$. As is customary, we let $\Gamma(G, V')$ denote the set of neighbors of vertices $V'$ in $G$. When applied to a bipartite graph $G$ modeling a coverage instance, we can write the coverage problem as $\mathcal{C}(S) = |\Gamma(G, S)|$ for any $S \subset F$.

2 Distributed Algorithms

In this section we present distributed algorithms for $k$-cover and set cover with $\lambda$ outliers. We aim to develop algorithms that only need $O(n)$ space per machine. As a first attempt, if we want to apply the distributed submodular optimization results to our problems (e.g., DistGreedy \cite{27} or composable core-set algorithm \cite{26, 15}), the underlying algorithms would distribute subsets across machines. The main issue with such an approach is that sending whole subsets does not scale well for large subsets. A natural way to deal with the issue of large subsets is to subsample elements while sending those sets around, and a natural sampling technique would be uniform sampling. We first rule out applicability of such simple sampling schemes for this problem. In particular, we present a hardness example for which the size (i.e., the number of edges) of the instance on each machine has to be $\Omega(nk)$ to obtain a bounded approximation guarantee.

Theorem 1. Pick arbitrary numbers $n, \beta \geq 1$ and $k \leq n/2$. Let $A$ be an algorithm that samples elements uniformly at random and reports an arbitrary optimum solution to $k$-cover on the sampled instance. If the number of edges sampled by $A$ does not exceed $nk/\beta^2$, its approximation factor is at most $\frac{2}{\beta+1}$.

Proof. Consider the following example with $k$ bonus sets and $n-k$ normal sets. Moreover, we have $\beta n$ special elements and $n$ normal ones. Each set has edges to all normal elements, and each bonus set has edges to $\beta n/k$ unique bonus elements. Notice that the optimum $k$-cover solution picks all the $k$ bonus sets, and covers all the $(\beta + 1)n$ elements.

Note that each normal element has $n$ edges. Since $A$ samples at most $nk/\beta^2$ edges, no more than $k/\beta^2$ normal elements in expectation make it to the sample. In other words, each element is sampled with probability at most $\frac{k}{\beta^2 n}$. Therefore, $A$ samples at most $\frac{k}{\beta^2 n} \times \beta n = n/\beta$ bonus elements, in expectation.

Indeed, if $A$ do not pick any bonus elements corresponding to a bonus set $S$, in the sampled graph the set $S$ covers the same elements as any normal set does. Thus $A$ might pick a normal set instead of $S$ in an arbitrary optimum solution on the sampled graph. Notice that $A$ samples at most $n/\beta$ bonus elements in expectation, which corresponds to no more than $n/\beta$ distinct bonus sets, in expectation. Hence there is an optimum solution on the sampled graph with $n/\beta$ bonus sets.
and $n - n/\beta$ normal sets in expectation. The expected total number of elements in this solution is $n + \frac{n}{\beta} \times \beta n = 2n$.

This observation suggests that any distributed algorithm should employ a more nuanced sampling (sketching) technique. To this end, we invoke a recent technique of ours \cite{4} if we can develop a distributed algorithm with $\tilde{O}(n)$ space that outputs a sketch (denoted by $\tilde{H}_{\leq n}(k, \varepsilon, \delta'')$, or simply $H_{\leq n}$) satisfying three special properties, we can prove tight approximation guarantees for the following algorithm: solve the problem by running a greedy algorithm on the sketch.

**Algorithm 1** Distributed algorithm for $k$-cover

**Input:** Input graph $G$ and parameters $k$, $\varepsilon \in (0, 1]$, $\delta''$.

**Output:** Solution to the coverage problem.

Let $h : \mathcal{E} \mapsto [0, 1]$ be a uniform, independent hash function.

**Round 1:** Send the edges of each element to a distinct machine. Let $\tilde{n} = \frac{24n\delta \log(1/\varepsilon) \log n}{(1-\varepsilon)e^4}$. For each element $v$, if $h(v) \leq \frac{2\tilde{n}}{m}$, the machine corresponding to $v$ sends $h(v)$ and its degree to machine one; it does nothing otherwise.

**Round 2:** Machine one iteratively selects elements with the smallest $h$ until the sum of the degrees of the selected vertices reaches $\tilde{n}$. Then it informs the machines corresponding to selected elements.

**Round 3:** For each selected element $v$, if the degree of $v$ is less than $\Delta = \frac{n \log(1/\varepsilon)}{ek}$, machine $v$ sends all its edges to machine one. Otherwise, it sends $\Delta$ arbitrary edges to machine one.

**Round 4:** Machine one receives the sketch $H_{\leq n}$ and solves the coverage problem on it by applying a greedy algorithm.

Here we develop Algorithm 1, a four-round distributed algorithm \cite{4} and prove the main result of this section, by showing that the output of this algorithm satisfies those three properties with high probability in a distributed setting using only $\tilde{O}(n)$ space. More formally, we prove the following.

**Theorem 2.** With probability $1 - \frac{2}{n}$, Algorithm 1 outputs a $(1 - \frac{1}{e} - \varepsilon)$-approximate solution to $k$-cover, and no machine uses more than $\tilde{O}(n)$ space in this algorithm.

The proof has two ingredients. First of all, we show that the sketch $H_{\leq n}$ computed in this algorithm satisfies the following three properties: Given parameters, $\varepsilon$ and $\delta''$, (1) elements are sampled uniformly at random, (2) the degree of each element is upper bounded by $\Delta = \frac{n \log(1/\varepsilon)}{ek}$, and (3) the total number of edges is at least $\frac{24n\delta \log(1/\varepsilon) \log n}{(1-\varepsilon)e^4}$, where $\delta = \delta'' \log \log \frac{1}{1-\varepsilon} m$. Secondly, we need to show that the algorithm uses $\tilde{O}(n)$ space per machine. The following lemma summarizes properties of the algorithm that pave the way for the proof of the theorem.

**Lemma 3.** Given are a graph $G(\mathcal{F} \cup \mathcal{E}, E)$ along with $k$, $\varepsilon \in (0, 1]$ and $\delta'' \in (0, 1]$. Then with probability $1 - 1/n^2$, $H_{\leq n}$

- no element with hash value exceeding $\frac{2\tilde{n}}{m}$, and
- at most $3\tilde{n}$ edges with hash value exactly $\frac{2\tilde{n}}{m}$.

\footnote{In a recent work \cite{5}, we study streaming algorithms for coverage functions and in particular show that an $\alpha$ approximate solution to $k$-cover on a sketch with the above properties is an $\alpha - \varepsilon$ approximate solution on the actual input, with probability $1 - e^{-k}$. This recent work is included as supplementary material and will be made available online.}

\footnote{Number of rounds were not optimized due to readability.}
where \( \tilde{n} = \frac{24n\delta \log (1/\varepsilon) \log n}{(1-\varepsilon)^3} \).

**Proof.** Note that \( H_{\leq n} \) requires to have only \( \tilde{n} \) edges. Clearly to satisfy this it is sufficient to have \( \tilde{n} \) elements. In the rest of the proof we show that, with probability \( 1 - 1/n \), the number of elements with hash value less than \( \frac{2\tilde{n}}{m} \) is within the range \([\tilde{n}, 3\tilde{n}]\). The lower bound together with the fact that \( H \) contains at most \( \tilde{n} \) elements gives us the first part of the theorem. The upper bound directly proves the second part of the theorem.

For every element \( v \in \mathcal{E} \), let \( X_v \) be the binary random variable indicating whether \( h(v) < \frac{2\tilde{n}}{m} \), and let \( X = \sum_{v \in \mathcal{E}} X_v \) denote the number of elements with hash value less than \( \frac{2\tilde{n}}{m} \). The Chernoff bound gives

\[
\Pr\left( |X - \mathbb{E}[X]| \geq \frac{1}{2} \mathbb{E}[X] \right) \leq 2 \exp \left( -\frac{\frac{1}{4} \mathbb{E}[X]}{3} \right) = 2 \exp \left( -\frac{\mathbb{E}[X]}{12} \right).
\]

(1)

Remark that \( h \) is a uniform mapping to \([0, 1]\). Thus, \( \Pr[h(v) \leq \frac{2\tilde{n}}{m}] = \frac{2\tilde{n}}{m} \) for any element \( v \), so we have

\[
\mathbb{E}[X] = \sum_{v \in \mathcal{E}} \mathbb{E}[X_v] = \sum_{v \in \mathcal{E}} \frac{2\tilde{n}}{m} = 2\tilde{n}.
\]

(2)

Putting (1) and (2) together gives us

\[
\Pr\left( |X - 2\tilde{n}| \geq \tilde{n} \right) \leq 2 \exp \left( -\frac{\tilde{n}}{6} \right) = 2 \exp \left( -\frac{4n\delta \log (1/\varepsilon) \log n}{(1-\varepsilon)^3} \right) \leq 2 \exp \left( -2 \log n - 1 \right) < \exp \left( -2 \log n \right) = \frac{1}{n^2}.
\]

Thus with probability \( 1 - \frac{1}{n^2} \) we have \( \tilde{n} \leq X \leq 3\tilde{n} \).

**Proof of Theorem 3.** We first show that Algorithm 1 uses \( O(n) \) space per machine. Next we prove that the algorithm constructs by Round 4 a sketch satisfying the desirable three properties mentioned above. As a result, Lemma 13 guarantees that invoking the greedy algorithm in Round 4 produces the promised solution.

The degree of each element is at most \( n \), the number of sets; thus, the space consumption of each machine in the first and third rounds is \( O(n) \). In the second round machine number 1 receives \( O(1) \) bits from each machine independently with probability \( \frac{2\tilde{n}}{m} \). Using the second condition of Lemma 3, the number of messages that this machine receives is at most \( 3\tilde{n} \). Therefore, this machine uses \( O(n) \) space. The number of edges machine one receives in the fourth round is at most \( \tilde{n} + n = \tilde{O}(n) \).

By the first condition of Lemma 3, no element in \( H_{\leq n} \) has hash value more than \( \frac{2\tilde{n}}{m} \). Thus the machines with no output in the first round do not miss any elements of \( H_{\leq n} \). Then the set of elements selected by machine one in round two is the same as in \( H_{\leq n} \). Therefore, what machine one receives in the fourth round is \( H_{\leq n} \). Discussion at the beginning of the proof finishes the argument. \( \square \)
While our algorithm for $k$-cover runs a greedy algorithm on the sketch, our algorithm for set-cover with $\lambda$ outliers makes logarithmically many guesses on the number of sets in the solution, constructs $H_{\leq n}$ sketches for each (simultaneously), and solves the problem on each resulting sketch. The proof of the following theorem is deferred to the full version.

**Theorem 4.** There exists a four-round distributed algorithm that reports a $(1+\varepsilon) \log \frac{1}{\lambda}$-approximate solution to set cover with $\lambda$ outliers, with probability $1 - \frac{2}{n}$. Moreover, each machine uses $O(n)$ space in the algorithm.

### 3 Algorithms for RAM Model

In this section we explain how our results apply to the RAM model, as well. Recall that in this model, we have random access to the edge lists, however, each access takes $O(1)$ time.

**Algorithm 2** Abstract construction of the sketch

**Input:** Graph $G(F \cup E, E)$ and numbers $k, \varepsilon, \delta' \in (0, 1/\lambda]$.  

**Output:** Sketch $H(V_H, E_H) = H_{\leq n}(k, \varepsilon, \delta')$.

1. $\delta \leftarrow \delta' \log \frac{1}{1-\varepsilon} m$  
2. $h : E \mapsto [0, 1]$ uniform, independent hash function  
3. $V_H \leftarrow F$ and $E_H \leftarrow \emptyset$  
4. **while** $|E_H| < \frac{24n \log(1/\varepsilon) \log n}{(1-\varepsilon) \varepsilon^2}$ **do**  
5. $v \leftarrow \arg \min_{v \in E \setminus V_H} h(v)$  
6. $V_H \leftarrow V_H \cup \{v\}$  
7. Add min($\frac{n \log(1/\varepsilon)}{\varepsilon k}, |\Gamma_G(v)|$) edges of $v$ to $E_H$

**Theorem 5.** There exists an algorithm that, given random access to the edge lists of coverage instance $G(F \cup E, E)$, computes the sketch $H = H_{\leq n}$ in time $\tilde{O}(n)$.

**Proof.** We show how Algorithm 2 can run in the RAM model. Since $|E_H| = \tilde{O}(n)$ at the end, total work done in Line 7 is $\tilde{O}(n)$. In Line 2 we do not need to define the hash function explicitly. When Line 5 seeks the next vertex, it is equivalent to pick a random new vertex. We only need to keep a list of already selected vertices to avoid repetition.

Once the sketch is constructed we can run a sequential algorithm on the sketch (or sketches) to solve $k$-cover and set cover with outliers. The proof is almost identical to those of Theorems 2 and 4 and is omitted.

**Theorem 6.** There is an $\tilde{O}(n)$-time, $1 - \frac{1}{\varepsilon} - \varepsilon$-approximation algorithm in the RAM model for $k$-cover.

**Theorem 7.** There is an $\tilde{O}(n)$-time algorithm in the RAM model that finds a $(1 + \varepsilon) \log \frac{1}{\lambda}$-approximate solution for set cover with $\lambda$ outliers.

### 4 Dominating Set

In dominating set problem, we are given a graph $G(V, E)$ and we aim to find the minimum number of vertices $S \subseteq V$ such that $S \cup \Gamma_G(S) = V$. We say a set of vertices $S \subseteq V$ is a dominating set
with $\lambda$ outliers if $|S \cup \Gamma_G(S)| \geq (1 - \lambda)|V|$. A set of vertices $S$ is an $\alpha$-approximate solution to dominating set with $\lambda$ outliers if (1) it is a dominating set with $\lambda$ outliers, and (2) $|S|$ is at most $\alpha$ times the size of the smallest dominating set.

The following theorem provides the first distributed algorithm for dominating set with $\lambda$ outliers.

**Theorem 8.** There exists a four-round distributed algorithm that reports a $(1+\varepsilon) \log \frac{1}{\lambda}$-approximate solution to dominating set with $\lambda$ outliers, with probability $1 - \frac{2}{n}$, while each machine use only $\tilde{O}(n)$ space.

**Proof.** We give a reduction from dominating set with $\lambda$ outliers to set cover with $\lambda$ outliers. Let graph $H(V, E)$ be an instance of the former. We construct an instance $G(\mathcal{F} \cup \mathcal{E}, E')$ of the latter problem. The $n$ sets in $\mathcal{F}$ correspond to the vertices of $H$. Similarly, the elements in $\mathcal{E}$ correspond to the vertices in $H$. An edge $(a, b) : a \in \mathcal{F}, b \in \mathcal{E}$ appears in $G$ if $a = b$ or $(a, b) \in E$.

Any solution $S \subseteq \mathcal{F}$ to the set-cover instance $G$ corresponds to a subset of $V$ that dominate the vertices the corresponding sets cover, hence a one-to-one correspondence between the solutions of the two problems.

The input for $k$-dominating set includes a number $k$ in addition to the graph $G$. The goal is to select $k$ vertices that maximize the number of dominated vertices. Then a subset of vertices $S$ is an $\alpha$-approximate solution to $k$-dominating set if it covers $\alpha$ times that of the optimum.

The following theorem provides the first distributed algorithm for $k$-dominating set.

**Theorem 9.** There exists a four-round distributed algorithm that reports a $1 - \frac{1}{e} - \varepsilon$-approximate solution to $k$-dominating set, with probability $1 - \frac{2}{n}$. Moreover, each machine uses only $\tilde{O}(n)$ space in the algorithm.

**Proof.** Similarly to the proof of Theorem 8, we give a reduction from $k$-dominating set to $k$-cover. From an instance $H$ of the former problem, we construct an instance $G(\mathcal{F} \cup \mathcal{E}, E')$ of the latter. Once again, each set in $\mathcal{F}$ corresponds to a vertex in $H$, as is each element in $\mathcal{E}$. We place an edge between $a \in \mathcal{F}$ and $b \in \mathcal{E}$ if and only if $a = b$ or there is an edge between $a$ and $b$ in $H$.

For any set $v \in \mathcal{F}$, the set of elements covered by $v$ is exactly the union of $v$ and all its neighbors in $H$. Similarly, for any subset $S \subseteq \mathcal{F}$, the elements that $S$ covers is the union of vertices in $S$ and all their neighbors. Therefore any $k$-cover solution on $G$ corresponds to a $k$-dominating set solution with the same coverage, and vice versa.

## 5 Weighted Variants

In this section we extend our results to three variants of the coverage problem. In element-weighted $k$-cover, a weight $w_v$ is associated with each element $v \in \mathcal{E}$, and the objective is to maximize the total weight of covered elements. A fractional $k$-cover instance has quantity $\alpha_{u,v} \in [0, 1]$ for each $S \in \mathcal{F}, v \in \mathcal{E}$, denoting that set $S$ covers $\alpha_{S,v}$ fraction of element $v$. A solution $S \subseteq \mathcal{F}$ covers $\max_{S \subseteq \mathcal{F}} \max_{S \subseteq \mathcal{S}} \alpha_{S,v}$ fraction of element $v$. Here the objective is to find a solution $S \subseteq \mathcal{F}$ of size $k$ that maximizes $\sum_{v \in \mathcal{E}} \max_{S \subseteq \mathcal{F}} \alpha_{S,v}$. Finally in probabilistic $k$-cover, quantity $\alpha_{S,v} \in [0, 1]$ is provided for each pair of $S \in \mathcal{F}$ and $v \in \mathcal{E}$: set $S$ covers element $v$ with probability $\alpha_{S,v}$. A solution $S \subseteq \mathcal{F}$ covers $1 - \prod_{S \subseteq \mathcal{F}} (1 - \alpha_{S,v})$ fraction of element $v$. The objective then is to find a solution $S \subseteq \mathcal{F}$ of cardinality $k$ that maximizes $\sum_{v \in \mathcal{E}} \left(1 - \prod_{S \subseteq \mathcal{F}} (1 - \alpha_{S,v})\right)$. 

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In the first problem, for simplicity we assume that all weights are integers upper-bounded by a number $U$. Similarly, in the second and the third problems, we assume that $\alpha_{S,v}$ is a factor of $1/U$ for any $v \in S \subseteq F$.

**Theorem 10.** There exists a four-round distributed algorithm that finds a $(1 - \frac{1}{e} - \epsilon)$-approximate solution to element-weighted $k$-cover, with probability $1 - \frac{2}{n}$. Moreover, each machine uses $\tilde{O}(n)$ space in this algorithm.

**Proof.** Let’s replace an element $v \in E$ of weight $w_v$ with $w_v$ copies of $v$ of weight one. This does not change the coverage of any solution, however, it may significantly increase the size of the problem—we can end up with $Um$ elements. The sketch size, though, is logarithmic in terms of the number of elements, though, and we can also sample the new elements implicitly: simply pick each element with probability proportional to its weight. \qed

**Theorem 11.** There exists a four-round distributed algorithm that reports a $(1 - \frac{1}{e} - \epsilon)$-approximate solution to fractional $k$-cover, with probability $1 - \frac{2}{n}$. Moreover, no machine uses more than $\tilde{O}(n)$ space in this algorithm.

**Proof.** Once again we reduce to unweighted $k$-cover and show how to perform the sampling implicitly. Replace each element $v$ with $U$ copies and connect the first $\alpha_{S,v}U$ copies of $v$ to $S$. We observe that the coverage of any solution grows by exactly a factor $U$.

The sketch size is logarithmic in terms of the number of elements, which grows by a factor $U$. To sample an element form the unweighted $k$-cover instance uniformly at random, we equivalently first sample an element from the original fractional $k$-cover instance uniformly at random, and then pick an index from $[1, U]$ to decide how many of its copies should appear in the sketch. \qed

**Theorem 12.** There exists a four-round distributed algorithm, using $\tilde{O}(n)$ space per machine, that finds a $(1 - \frac{1}{e} - 2\epsilon)$-approximate solution to probabilistic $k$-cover with probability $1 - \frac{3}{n}$.

**Proof.** Similarly we transform an instance of probabilistic $k$-cover to one of $k$-cover: substitute each element $v \in E$ with $\zeta = \frac{12(n+1+\log n)U}{\epsilon^2}$ copies of $v$, and for each set $S$ that contains $v$ we connect $S$ to each copy of $v$ with probability $\alpha_{S,v}$. We show that with probability $1 - \frac{1}{n}$, for all solutions $S \subseteq F$, the coverage of $S$ in the $k$-cover instance is within a factor $\zeta(1 \pm \epsilon/2)$ of that in the original probabilistic $k$-cover instance. Fix a solution $S$, and let $\beta_v = 1 - \prod_{S \in S}(1 - \alpha_{S,v})$. The Chernoff bounds gives for $X$, the number of copies of $v$ covered by $S$, as follows.

$$
    \Pr\left(|X - \zeta \beta_v| \geq \zeta \beta_v \epsilon/2\right) \leq 2 \exp\left(- \frac{\zeta \beta_v \epsilon^2}{12}\right)
$$

$$
= 2 \exp\left(- \frac{12(n+1+\log n)U}{\epsilon^2} \beta_v \epsilon^2}\right)
$$

$$
\leq 2 \exp\left(- \frac{12(n+1+\log n)}{\epsilon^2} \epsilon^2\right) \quad \text{since } \beta_v \geq 1/U,
$$

$$
\leq 2 \exp\left(- n + 1 + \log n\right) \leq 2^{-n}/n.
$$

There are $2^n$ choices for $S$, hence for all solutions $S \subseteq F$, the coverage of $S$ on the $k$-cover instance is within the promised interval with probability $1 - \frac{1}{n}$. \hfill \Box
Since the number of elements in the \( k \)-cover instance is at most \( \zeta m \), the size of the sketch grows logarithmically in \( U \). To sample an element from the \( k \)-cover instance uniformly at random, we sample an element from the original fractional \( k \)-cover instance uniformly at random and connect it to each set \( S \in \mathcal{F} \) with probability \( \alpha_{S,v} \).

6 Empirical Study and Results

We begin this section by a brief overview of the datasets and corresponding applications used in our empirical study (see Table 3), and then move to the methodology as well as the experiment results. Detailed information on the datasets is given in Appendix B.

| Name        | Type           | \(|\mathcal{F}|\) | \(|\mathcal{E}|\) | \(|E|\) |
|-------------|----------------|------------------|------------------|--------|
| livej-3     | dominating set | 4M               | 4M               | 73B    |
| livej-2     | dominating set | 4M               | 4M               | 3.4B   |
| dblp-3      | dominating set | 320K             | 320K             | 330M   |
| dblp-2      | dominating set | 320K             | 320K             | 27M    |
| gutenberg   | bag of words   | 42K              | 100M             | 1B     |
| s-gutenberg | bag of words   | 925              | 11M              | 27M    |
| reuters     | bag of words   | 200K             | 140K             | 15M    |
| planted-A   | planted coverage | 10K           | 10K              | 1.2M   |
| planted-B   | planted coverage | 100K             | 1M               | 1.2B   |
| planted-C   | planted coverage | 100K             | 10M              | 2.4B   |
| planted-D   | planted coverage | 101K             | 10M              | 1.2B   |
| wiki-main   | contribution graph | 2.9M          | 11M              | 75M    |
| wiki-talk   | contribution graph | 1.7M            | 1M               | 7.3M   |
| news20      | feature selection | 1.4M            | 200M             | 4.3B   |

Table 3: General information about our datasets.

Our empirical study is based on five types of instances covering a variety of applications:

1. **Dominating-set** instances are formed by considering vertices of a graph as *sets* and their two- or three-hop neighborhoods as *elements* they dominate. The Dominating-set problem is motivated by sensor placement and influence maximization applications [27].
2. The “bag of words” instances correspond to documents and bigrams they contain. The goal is pick a few documents that cover many bigrams together. This instance highlights the application of coverage maximization in document summarization, or finding representative entities in a corpus [27].
3. We have synthetic “planted set cover” instances that are synthetically generated, and known to be hard for greedy algorithms.
4. “Contribution graphs” model interaction between users on a set of documents. We seek a small subset of users that collectively have contributed to a majority of documents. This, in turn, has application in team formation [5].
5. A **feature-selection** instance proposes a column subset selection problem on a matrix of news articles and their features. This application is described in Section 6.2.

We remark that, to the best of our knowledge, some of these datasets are an order of magnitude larger than what has been considered in prior work.
6.1 Approach

Recall that the sketch construction is based on two types of prunings for edges and vertices of the input graph:

- subsampling the elements, and
- removing edges from large-degree elements.

The theoretical definition of the sketch provides (i) the probability of sampling an element, and (ii) the upper bound on the degree of the elements. Though these two parameters are almost tight in theory, in practice one can use smaller values to get desirable solutions. Here we parameterize our algorithm by $\rho$ and $\sigma$, where $\rho$ is the probability of sampling elements, and $\sigma$ is the upper bound on element degrees. We investigate this in our experiments.

The StochasticGreedy algorithm [28] achieves $1 - \frac{1}{e} - \varepsilon$ approximation to maximizing monotone submodular functions (hence coverage functions) with $O(n \log(1/\varepsilon))$ calls to the submodular function. This is theoretically the fastest known $1 - \frac{1}{e} - \varepsilon$ approximation algorithm for coverage maximization, and is also the most efficient in practice for maximizing monotone submodular functions, when the input is very large. We plug it into our MapReduce algorithm, which then runs much faster, while losing very little in terms of quality. For smaller instances we compare our algorithm to StochasticGreedy, but for larger ones we provide convergence numbers to argue that the two should get very similar coverage results.

LiveJournal social network  We try different values for $\rho, \sigma, k$ when running our algorithm on livej-3; see Figure 1. For small $k$, the result improves as $\sigma$ grows, but increasing $\rho$ has no significant effect. On the other hand, the improvement for larger $k$ comes from increasing $\rho$ while $\sigma$ is not as important. This observation matches the definition of our sketch, in which the degree bound is decreasing in $k$ and the sampling rate is increasing in $k$.

DBLP coauthorship network  Figure 2 compares results of our algorithm on dblp-3 (with a range of parameters) to that of StochasticGreedy. Each point in these plots represents the mean of three independent runs. Interestingly, a sketch with merely 3% of the memory footprint of the input graph attains %99.6 of the quality of StochasticGreedy.

We run our algorithm on induced subgraphs of dblp-3 of varying sizes; see Figure 2. Interestingly, the performance of our algorithm improves the larger the sampled graph becomes. In other words, if one finds parameters $\rho$ and $\sigma$ on a subgraph of the input and applies it to the whole graph, one does not lose much in the performance.

Project Gutenberg dataset  We run our algorithm on gutenberg with different values for $\rho$ and $\sigma$. As shown in Figure 3, the outcome of the algorithm converges quickly. In other words, for $\rho = 0.003$ and $\sigma = 100$, the outcome of StochasticGreedy on our sketch and on the input graph are quite similar, while our sketch is 600 times smaller.

Other datasets  Due to space constraints we cannot report detailed results for the other datasets. However, Table 4 shows that for these datasets, a small sketch suffices to get close to the single-machine greedy solution. In fact, these are small enough for the greedy algorithm to run on one machine.
Figure 1: For the dominating-set instance livej-3, these plots show the number of covered nodes against the relative size of the sketch with $\rho \in [10^{-3}, 3 \cdot 10^{-2}]$, $\sigma \in [100, 5000]$, and $k \in [10^2, 10^4]$. Curves in one plot correspond to different choices for $\sigma$. With large $\sigma$, the results of some runs are indistinguishable from the one next to it in the plot, hence invisible.

Figure 2: The results for dblp-3 are shown for $\rho \in [2 \cdot 10^{-3}, 5 \cdot 10^{-2}]$, $\sigma = 100$. We plot our performance relative to STOCHASTICGREEDY against the fraction of edges from the input graph retained in our sketch.

Table 4: Results for other datasets.

| Instance   | Sketch Quality | Instance   | Sketch Quality |
|------------|----------------|------------|----------------|
| wiki-main  | 0.06% 94.4%    | dblp-2     | 1.7% 92%       |
| wiki-main  | 2.4% 99.5%     | dblp-2     | 3.1% 96%       |
| wiki-main  | 7.7% 99.9%     | reuters    | 1.2% 87%       |
| wiki-talk  | 1.5% 99.2%     | reuters    | 3.6% 92%       |
| planted-A  | 8.2% 96%       | reuters    | 10% 96%        |
Figure 3: The above are results of running the algorithm on the sampled version of dblp-3 with $\rho = 0.02$, $\sigma = 100$. The $x$ axis denotes the size of the sampled graph relative to the whole. The $y$ axis shows the quality relative to STOCHASTICGREEDY.

Figure 4: Here we plot the number of covered bigrams against $\rho$ for gutenberg with $\rho \in [10^{-5}, 3 \cdot 10^{-2}]$, $\sigma \in [10^2, 10^4]$, and $k \in [10^2, 1000^3]$. The curves corresponding to different values of $\sigma$ are practically indistinguishable. The livej-2 instance is too big for the single-machine greedy algorithm. Still we can compare our result to what is achievable for a 10% sample of the instance (with about 340 million edges). With a 0.8% footprint we obtain a solution of essentially the same quality. With footprints 0.3%, 0.2%, 0.1% and 0.75%, we lose no more than 1%, 3%, 4% and 9%, respectively.

Except for the smallest, the planted instances are also too big for the greedy algorithm. Nonetheless, looking at the numbers, e.g., for planted-B, we notice that the quality of the greedy solution is almost the same for sketches of relative sizes 0.3% and 42%—the latter has about 500 million edges. In particular, sketches of relative size 0.3%, 1% and 10% produce 3%, 2% and 1% error, respectively, compared to the sketch of size 42%. The results are similar for the other two planted instances.
6.2 Feature-selection Problem

Our algorithm is applicable to the feature-selection problem, which is a first step in many learning-based applications \cite{19}. It is often too expensive to carry out a learning task on the entire matrix or there might be overfitting concerns. Typically a small subset of “representative” features are picked carefully, so as not to affect the overall quality of the learning task. In practice, we gauge the performance of feature selection by reconstruction error or prediction accuracy; see \cite{3} for details of evaluation criteria.

In order to compare our preliminary results to previous work \cite{3}, we model the problem as a maximum \( k \)-cover instance by treating columns (i.e., features) as sets and pairs of rows (i.e., pairs of sample points) as elements. We say a row covers a pair of rows, if that column (feature) is active for both of those rows (sample points), and seek to pick \( k \) columns that cover as many pairs the rows as possible.\footnote{We also studied covering rows as opposed to covering pairs of rows, but that approach was not effective.}

Table 5 compares our results to prior work. Numbers show prediction accuracy in percentage. For description of the data set and the first four algorithms, see \cite{3}. We note that these algorithms may only run on a 8\% sample of the dataset, hence poorer performance compared to the latter two. The fifth column exhibits a distributed version of 2-P (the two-phase optimization): Features are carefully partitioned across many machines via taking into account some cut-based objective, and then the two-phase optimization handles each part separately. It is noteworthy that the (distributed) partitioning phase itself takes significant amount of time to run. The last column corresponds to our distributed \( k \)-cover algorithm, which is more efficient than the algorithm of the fifth column. The results are similar to that of PART.

| \( k \)  | RND | 2-P | DG | PCA | Part | Cover |
|--------|-----|-----|----|-----|------|-------|
| 500    | 54.9| 81.8| 80.2| 85.8| 84.5 | 86.2  |
| 1000   | 59.2| 84.4| 82.9| 88.6| 88.4 | 89.4  |
| 2500   | 67.6| 87.9| 85.5| 90.6| 92.3 | 91.2  |

We emphasize that we can run our algorithm on much larger datasets; the evidence of this was provided above where we reported results for livej-3, for instance.

7 Conclusions

In this paper, we present almost optimal distributed algorithms for coverage problems. Our algorithms beat the previous ones in several fronts: e.g., (i) they provably achieve the optimal approximation factors for these problems, (ii) they run in only four rounds of computation (as opposed to logarithmic number of rounds), and (iii) their space complexity is independent of the number of elements in the ground set. Moreover, our algorithms can handle coverage problems with huge subsets (in which even one subset of the input may not fit on a single machine). Our empirical study shows practical superiority of our algorithms. Finally, we identified a new application of our algorithms in feature selection, and presented preliminary results for this application. It would be nice to explore this application in more details in the future.
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A Omitted Proofs

We use the following lemma to prove Theorems 2 and 4.

**Lemma 13** (From [4]). For any $\varepsilon \in (0, 1]$ and any set-cover graph $G$, there exist sketch-based algorithms that succeed with probability $1 - \frac{1}{n}$ in finding the following.

1. One finds a $(1 - \frac{1}{e} - \varepsilon)$-approximate solution to $k$-cover on $G$, working on a sketch with $\tilde{O}(n)$ edges.

2. The other finds a $(1 + \varepsilon)\log \frac{1}{\lambda}$ approximate solution to set cover with $\lambda$ outliers on $G$. The sketches used altogether have $\tilde{O}(n/\lambda^3) = \tilde{O}(\lambda(n))$ edges.

**Proof of Theorem 4.** We run $\log_{1+\varepsilon/3} n$ copies of the first three stages of Algorithm 1 (simultaneously) to construct the $\log_{1+\varepsilon/3} n$ different sketches required by Lemma 13. The discussion in Theorem 2 implies that each copy of the sketch is constructed correctly, with probability $1 - \frac{1}{n^2}$. Together with Lemma 13 this proves that our algorithm gives a $(1 + \varepsilon)\log \frac{1}{\lambda}$-approximate solution to set cover with $\lambda$ outliers, with probability $1 - \frac{1}{n} - \log_{1+\varepsilon/3} n \frac{1}{n^2} > 1 - \frac{2}{n}$. 

B Description of Datasets

We run our empirical study on five types of instances. A summary is presented in Table 3. Dominating set instances include livej-3, livej-2, dblp-3 and dblp-2, where the objective is to cover the nodes via multi-hop neighborhoods. We have two sets of bag-of-words instances, where the goal is to cover as many words/bigrams via selecting a few documents: gutenberg and s-gutenberg for books and reuters for news articles. Instances wiki-main and wiki-talk are our contribution graphs where we want to find a set of users who revised many articles. Finally we have some planted set-cover instances that are known to fool the greedy algorithm: planted-A, etc.

**Dominating-set instances** We build set-cover instances based on graphs for LiveJournal (social network) [33] and DBLP (database of coauthorship) [5]. The vertices of these graphs form both $\mathcal{F}$ and $\mathcal{E}$ in the new instances livej-3 and dblp-3, and a set $S$ covers an element $v$ if and only if $v$ is within the three-hop neighborhood of $S$ in the original graph—reachable by a path of length at most three. Similarly, we build two smaller instances dblp-2 and livej-2 that are based on 2-hop neighborhoods. To show scalability further, we also use a sampled version of livej-3: each vertex is picked with a fixed probability.

**“Bag of words” instances** We build gutenberg based on documents and bigrams inside them. The starting point is the set of 50,284 books on Project Gutenberg with IDs less than 53,000 [18], downloaded via [32]. We then remove the English stopwords [10], and throw away 8,568 books we think are not in English, leaving us with 41,716 books. This was done heuristically: any book with more than half its distinct words missing from an English word list [21] was deemed non-English. (Non-English books in the collection make the set-cover instances much simpler, since picking books from different languages allows us to cover a lot of new words/bigrams. This process reduces the number of distinct words by about 63%.) Natural Language Toolkit [10] was then used to turn words into their stems, before generating the list of bigrams in each book. A smaller version of this

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6The upper bound was picked because the project claims to host 52,031 books [18].
Table 6: Parameters used to generated “planted” datasets.

| Name      | $k$   | $m$         | $k'$       | $\epsilon$ |
|-----------|-------|-------------|------------|-------------|
| planted-A | 100   | 10,000      | 10,000     | 0.2         |
| planted-B | 100   | 1,000,000   | 100,000    | 0.2         |
| planted-C | 500   | 10,000,000  | 100,000    | 0.2         |
| planted-D | 1,000 | 10,000,000  | 100,000    | 0.2         |

dataset, called s-gutenberg, was also generated using books with IDs less than 1,000. We also consider another bag-of-words instance, reuters, which is a collection of Reuters news articles [25] written 1996–1997. The words in each article have already been changed to their stems. There are four medium-size subdatasets in the collection. We only report results on the p0 part. The others behave similarly.

**Contribution graphs** The Wikipedia edit history (until 2008) is available [24]. We build two datasets wiki-main and wiki-talk based on this. Users have made revisions in either namespace (main article texts or talk pages). We place edges between users and pages they have revised. A set-cover solution then consists of a group of users who have revised all (or many) pages.

**Planted set-cover instances** We also generate instances where a small set cover is planted in an otherwise random graph. The advantage is that we know the optimum solution even for large instances. We build such graphs with parameters $k$, $m$, $k'$ and $\epsilon$. Such instances have $m$ elements and $k + k'$ sets, $k$ of which have the same size and perfectly cover the entire ground set; the other $k'$ have random elements but are a factor $1 + \epsilon$ larger than the planted sets. We use four such graphs in our experiments with parameters mentioned in Table 6.

**Feature-selection instances** Built from a column subset selection instance, sets correspond to columns and elements corresopnd to rows. We aim to pick a small number of columns that collectively appear in a majority of rows. We focus on news20 dataset that is discussed in detail in [3].