TEMPERATURE FLUCTUATION AND AN EXPECTED LIMIT OF HUBBLE PARAMETER IN THE SELF-CONSISTENT MODEL

A.B. Morcos

Department of Astronomy, National Research Institute
Of Astronomy and Geophysics, Helwan, Cairo, Egypt.
e-mail: morcos@frcu.eun.eg

Abstract

The temperature gradient of microwave background radiation (CMBR) is calculated in the Self Consistent Model. An expected value for Hubble parameter have been presented in two different cases. In the first case the temperature is treated as a function of time only, while in the other one the temperature depends on relaxation of isotropy condition in the self-consistent model and the assumption that the universe expands adiabatically. The COBE’s or WMAP’s fluctuations in temperature of CMBR may be used to predict a value for Hubble parameter.

1 Introduction

The cosmic microwave background radiation (CMBR) temperature is one of the important parameters of any cosmological model. The three characteristics of this radiation are its spectrum, spatial anisotropy and polarization. The COBE Far-Infrared Absolute Spectrophotometer (FIRAS) has determined the black body temperature of (CMBR) to be $2.728 \pm 0.004 \ K$ (Keating et al. (1998)), and the COBE Differential Microwave Radiometer (DMR) experiment has detected spatial anisotropy of the (CMBR) on $10^\circ$ scales of $\Delta T / T \simeq 1.1 \times 10^{-5} \ K$. Ground and balloon-based experiments have detected anisotropy at smaller scales (cf. Silk, and White(1995)). Many authors referred these anisotropies to simple linear or nonlinear processing in the primordial fluctuations (cf. Hu et al. (1997), Challinor and Lasenby(1999), Melek(2002)).

Recently Wilkinson Microwave Anisotropy Probe (WMAP) satellite, which is designed for precision measurement of the CMBR anisotropy on the angular scales ranging from the full sky down to several arc minutes. This ongoing mission has already provided a sharp record of the conditions in the universe from the epoch of last scattering to the present. WMAP results despite the absence of a direct dark energy interaction with our baryonic world (Rebort Caldwell and Michael Doran(2003)). Whenever Joshue et al.(2003) show that the Planck CMBR mission can be significant. In general the observational limit of the temperature fluctuations $\Delta T / T$ becomes lower and lower and it reached almost $10^{-6}$ (Keating et al. (1998)).

In the present work, the COBR or WMAP results for the temperature gradient is used to expect a value for the Hubble constant using the self-consistent model. In the next section a brief review of the self-consistent model will be given. In Section 3 time-temperature relation in the model is calculated. In Section 4 the theoretical technique for calculating the gradient of any scalar cosmic field is described. In Section 5 a lower limit for Hubble parameter is calculated. In Section 6 discussion and concluding remarks are given.
2 The Self-Consistent Model (SCM)

Wanas (1989), has constructed the (SCM), a cosmological model in the framework of Generalized Field Theory (GFT) (Mikhail and Wanas (1977)). This theory is constructed in a 4-dimensional Absolute Parallelism (Ap)-Geometry. In (1986) Wanas suggested a set of conditions to be satisfied by any geometric structure in (AP-Geometry) to be suitable for cosmological applications. This set of conditions, if satisfied, would guarantee that a geometric structure would represent, a homogeneous, isotropic, electrically neutral and non-empty universe. Wanas (1989) has used one of the AP-structures, constructed by Robertson (1932), satisfying the conditions mentioned above, to construct (SCM). The geometric structure used in that model is given in the spherical polar coordinates by,

$$
\chi^\mu_i = \begin{pmatrix}
\sqrt{-1} & 0 & 0 & 0 \\
0 & \frac{L^+ \sin \theta \cos \phi}{4R} & \frac{L^+ \sin \theta \sin \phi}{4R} & \frac{L^+ \cos \theta}{4R} \\
0 & \frac{(L^- \cos \theta \cos \phi - 4K^2 \sin \phi)}{4R} & \frac{(L^- \cos \theta \sin \phi - 4K^2 \cos \phi)}{4R} & \frac{-L^- \sin \phi + 4K^2 \cos \theta \cos \phi}{4R} \\
0 & \frac{-L^- \sin \phi - 4K^2 \cos \theta \sin \phi}{4R} & \frac{(L^- \cos \phi - 4K^2 \cos \theta \sin \phi)}{4R} & \frac{-L^- \cos \phi + 4K^2 \cos \theta \cos \phi}{4R}
\end{pmatrix}.
$$

(1)

where $L^\pm = 4 \pm kr^2$, k is the curvature of the space and $R(t)$ is an unknown function of (t) only. It is to be considered that the Riemannian space, associated with (1), is given by

$$
dS^2 = \hat{g}_{\mu \nu} \ d\chi^\mu \ d\chi^\nu.
$$

(2)

with the metric tensor given by,

$$
\hat{g}_{\mu \nu} = \sum_i e_i^i \chi^\mu_i \chi^\nu_i,
$$

(3)

where $e_i (= 1, -1, -1, -1)$ is Levi-Civita’s indicator. Wanas (1989) got the following set of the differential equations:

$$
\frac{\ddot{R}^2}{R^2} + \frac{4k}{R^2} = 0,
$$

(4)

$$
\frac{2\dot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{4k}{R^2} = 0,
$$

(5)

where the dots represents differentiation with respect to time (t). Integration of (4), gives immediately

$$
R = \tilde{R} \pm 2(-k)^{\frac{1}{2}} \ t,
$$

(6)

where $\tilde{R}$ is a constant of integration, giving the value of the scale factor at $t=0$. If $k$ takes the value zero , the SCM model will be a static empty one, and when $k=+1$ it will give an imaginary scale factor. So we must take $k=-1$ for non-static, non-empty model, and the solution (6) will take the form,

$$
R = \tilde{R} + 2t,
$$

(7)
with $k = -1$. It is worth of mention that the SCM is a cosmological model fixing the the curvature constant to be -1. It is also satisfying the weak and strong energy conditions, and it is free of particle horizon (for more details see Wanas(1989), (2003)). The model is consistent with recent Supernovae observations Riess et al. (2004). The negative curvature, uniquely fixed by the model, is among recent discussed reasons of the WMAP low multi-pole anomaly (cf. Gurzadyan et al. (2003)). So, this model deserves further examination.

3 The Time-Temperature Relation in SCM

In what follows we are going to find the relation between time and temperature in SCM, we are going to assume that in the early stages of the universe, the radiations behaves as if it is coming from a black body with temperature $T$ given by the will known relation (cf. Narlikar (1983))

$$B_0^0 = a T^4,$$

(8)

where $B_{\mu \nu}$ is the phenomenological energy-momentum tensor, and $a$ is the radiation constant. But the energy momentum tensor in the SCM is a geometric one, say $S_{\mu \nu}$, that has the non-vanishing values:

$$S_0^0 = \frac{9k}{R^2}, S_1^1 = S_2^2 = S_3^3 = \frac{3k}{R^2}.$$

(9)

We can assume that the geometric energy momentum tensor is related to the phenomenological one via the relation,

$$S_{\mu \nu} = \mathcal{H} B_{\mu \nu},$$

(10)

where $\mathcal{H}$ is a conversion constant equal to $\frac{8 \Pi c^2}{G}$, $G$ is the gravitational constant and $c$ is the speed of light. If we use (7), (8), and (9), we get

$$T = \left(\frac{9}{4a\mathcal{H}}\right)^{1/4} (\frac{2}{R + 2t})^{1/2}.$$ 

(11)

But it is well known that the relation between temperature and time depends on the type of particles filling the model and the kind of interaction between them at a certain temperature range. Thus it is more convenient to rewrite the relation (11) in the form,

$$T = \left(\frac{9}{a\mathcal{H} \gamma (R + 2t)^2}\right)^{1/4},$$

(12)

where $\gamma$ is a parameter depending on types of particles and their interactions. The relation (12) may be used to determine the parameter $\tilde{R}$, if all other constants are known.

If it is assumed that at time $t=0$ the temperature of the universe is $10^{12} \ oK$, as it is usually used in the thermal literature (cf. Narlikar(1983)), the value of the parameter $\tilde{R}$ is obtained from the relation (12) to be $3.7 \times 10^{-4} sec$. Relation (12) then takes the form

$$T = \left(\frac{9}{a\mathcal{H} \gamma (3.7 \times 10^{-4} + 2t)^2}\right)^{1/4},$$

(13)

where $\gamma = 1.45$ (cf. Narlikar (1983)).


4 The Gradient of any Cosmic Scalar Field

Melek(1992) generalized a procedure, used in meteorology, in studying the temperature gradient in the Earth’s atmosphere, to study the matter density and temperature gradients in the universe. For any cosmic measurable scalar field \( S \) which can be related to the energy momentum tensor \( B_{\mu \nu} \), he defined the function \( F_g \), in a curved space-time with metric \( g_{\mu \nu} \), as :

\[
F_g \overset{\text{def.}}{=} \frac{dG}{d\tau},
\]

where \( G \overset{\text{def.}}{=} (g_{\mu \nu} S_\mu S_\nu)^{1/2}, \)

and \( S_\mu \overset{\text{def.}}{=} \frac{\partial S}{\partial x^\mu}, \)

where \( S_\mu \) is a time-like covariant vector, \( \tau \) is the cosmic time and \( \mu = 0, 1, 2, 3 \). Melek has shown that the function \( F_g \) has the form:

\[
F_g = \frac{1}{G} g^{\mu \nu} S_{\mu ; \sigma} S_\nu u^{\sigma},
\]

where \( S_{\mu ; \sigma} \) is the usual covariant derivative with respect to \( x^\sigma \).

The second derivative of the absolute value of the gradient of any cosmic scalar field \( S \), with respect to the cosmic time \( \tau \), is given by:

\[
\frac{d^2 G}{d\tau^2} = \frac{dF_g}{d\tau} = \frac{1}{G} \{ g^{\mu \nu} u^\alpha [ S_{\mu ; \sigma \alpha} S_\nu + S_{\mu ; \sigma} S_{\nu ; \alpha} ] - F_g^2 \}.
\]

Melek(1995) applied this procedure to suggest an expression for the function \( F_g \) in a spatially perturbed Friedmann-Robertson-Walker cosmological model (FRW). He put a lower limit on the Hubble parameter. Melek (2000) used the same technique for (FRW) to study limits on cosmic time scale variations of gravitational and cosmological constants. Melek(2002) used the same procedure to find the primordial angular gradients in the temperature of the microwave background radiation and the density functions in the same cosmological model.

In what follows we are going to use the same technique to find the gradient of microwave back ground radiation’s temperature in SCM. Also, we are going to get a relation between this gradient and the value of Hubble parameter.

5 CMBR Temperature Gradient in the SCM and Expected Limit of Hubble Parameter

The metric of the Riemannian space, associated with the AP-space (1), can be written using, equations (2) and (3), as

\[
dS^2 = dt^2 - \frac{16}{L+4} \frac{R^2}{L^2} \big[ dr^2 + r^2 d\theta^2 + r^2 \sin \theta \, d\phi \big],
\]
where \( L^+ = 4 + r^2k \).

Now if we follow the coordinate transformation,

\[
dt = R(t) d\tau,
\]

in the metric (19), then we can write

\[
dS^2 = R^2(t) \{ d\tau^2 - \frac{16}{L^+} \{ dr^2 + r^2 d\theta^2 + r^2 \sin\theta \, d\phi \} \},
\]

where \( R(t) \) is the scale factor and \( \tau \) is the cosmic time. If we assume that the microwave background radiation temperature (\( T(t) \)), is our scalar field and this field varies with time only, then following (16), we can write

\[
T_{\mu}^{\text{def.}} = \frac{d}{dx^\mu} T,
\]

\[
T_0 = \frac{d}{dt} \frac{d}{d\tau} = R \, \dot{T},
\]

where \( \dot{T} = \frac{dT}{d\tau} \). Then,

\[
T_{\mu ; \nu} = T_{\mu , \nu} - \left\{ \rho \right\}_{\mu \nu} T_\rho.
\]

Now by taking into consideration that the temperature is a function of time only and using (24), we can write

\[
T_{0 ; 0} = R(R - \dot{R} \, \dot{T}).
\]

Using equations (14), (15), (16), (17), (18) and (25), taking into consideration that the CMBR is independent of the radial coordinate at any fixed cosmic time and the motion in the Universe is only due to its expansion, then we get after some straightforward calculations,

\[
F = (\ddot{T} - \frac{\dot{R}}{R} \, \dot{T}).
\]

Since the SCM has been assumed to be homogenous and isotropic then \( F = 0 \) i.e,

\[
\ddot{T} - \frac{\dot{R}}{R} \, \dot{T} = 0.
\]

Equation (27) leads directly to the following result

\[
\frac{\dot{T}}{T} = \frac{\dot{R}}{R}.
\]

Noting that \( \frac{\dot{R}}{R} = H \), as usually done, then we get

\[
\frac{\dot{T}}{T} = H.
\]
It is clear from the last equation that all the quantities on its left hand side are unmeasurable quantities till now, so if these quantities are measured by COBE, WMAP or any other satellite, the Hubble parameter is determined completely and at that moment can be fixed.

As it is mentioned above, the most recently detected value of the anisotropy in the temperature of the CMBR is determined by COBE and WMAP for each $10^3\,\text{o}$. This means that it is more suitable to relax the condition of isotropy in the cosmological model used. To satisfy this aim we are going to use the spatially perturbed form of the metric of the SCM in the spherical polar coordinates. The metric (21) will take the form:

$$dS^2 = R^2(t)\{d\tau^2 - \frac{16(1 + h_1)}{L^2}dr^2 - h_2 \, r \, dr \, d\Omega - r^2 \,(1 + h_3) \, d\Omega^2\}, \quad (30)$$

where $\Omega$ is the solid angle defined in terms of $\theta$ and $\phi$ as $d\Omega^2 = d\theta^2 + \sin^2 \theta \, d\phi^2$ and $h_1, h_2$ and $h_3$ are small spatial perturbations. If we use the metric (30) taking into our consideration that the homogeneity is valid (i.e $\frac{\partial T}{\partial r} = 0$, and $\frac{\partial^2 T}{\partial t \, \partial r} = 0$), and assuming that the expansion is the only motion in the universe, then this expansion affects the temperature. If we assume now that the temperature of the CMBR is a function of the cosmic time and direction i.e $T(t, \Omega)$ and one follows the same procedure as before, then equation (17) takes the form

$$F = \left(\frac{1}{G \, R(t)}\right)(\dot{T})(\ddot{T} - \frac{\dot{R}}{R} \, \dot{T}) - \left(\frac{1 - h_2}{r^2}\right)(T')(\frac{\partial T'}{\partial t} - \frac{\dot{R}}{R} \, T'), \quad (31)$$

where $T' \overset{\text{def.}}{=} \frac{\partial T}{\partial \Omega}$. If we write now the metric of the SCM, which is homogenous and isotropic, in the form

$$dS^2 = R^2(t)\{d\tau^2 - \frac{16}{L^2}dr^2 - r^2 \, d\Omega^2\}, \quad (32)$$

then, after some straightforward calculations, we can find the temporal variation of the magnitude of the gradient of $T$ as

$$F_{SCM} = \left(\frac{1}{G \, R(t)}\right)(\dot{T})(\ddot{T} - \frac{\dot{R}}{R} \, \dot{T}) - \left(\frac{1}{r^2}\right)(T')(\frac{\partial T'}{\partial t} - \frac{\dot{R}}{R} \, T'). \quad (33)$$

This gradient will be zero if the model is homogeneous and isotropic. From equations (31) and (33), assuming that the universe expands adiabatically, we get

$$\left(\frac{\partial T'}{\partial t} - \frac{\dot{R}}{R} \, T'\right) = 0. \quad (34)$$

Since $H = \frac{\dot{R}}{R}$, then the Hubble parameter can be written as

$$H = \left(\frac{\partial T'}{\partial t}\right) / T'. \quad (35)$$

Since COBE, WMAP and other space and ground based measurements have detected and confirmed anisotropy in the temperature of the CMBR, this means that the right hand side quantities of the equation (35), can be measured easily fixing the value of the Hubble parameter.
6 Discussion and Concluding Remarks

Using the AP-structure (1), Wanas (1989) has got a unique pure geometric world model. This model is non-empty and has no particle horizons. This model fixes a value for $k(= -1)$ i.e. it has no flatness problem, and as it is clear from equation (7), it has no singularity at $t=0$. A further advantage of using pure geometric theories is that one did not need to impose any condition from outside the geometry used (e.g. equation of state) in order to solve the field equations (Wanas (1986), (1989)).

In the present work the generalized procedure for studying gradients, which has been used by Melek (1992), is used to find the temperature gradient in the SCM. It is shown that when it is assumed that the CMBR temperature is a function of time only, the Hubble parameter $(H)$ is given by (22). But all the quantities on the right hand side are nonmeasurable, so this relation cannot determine the numerical value of $H$ except for a satellite or a ground based observation arises the gradient of temperature and its rate of change with respect to time.

When the isotropy condition in the self-consistent model is relaxed and the universe is assumed to expand adiabatically, the Hubble parameter is given by the relation (35). The quantity in the denominator of the right hand side of (35) may be determined by COBE or WMAP while the quantity in the numerator can not be determined at time being. It can be calculated after the accumulation of further data, and then the Hubble parameter can be determined.

It is clear also from the relation (34) that the value of the Hubble parameter decreases as the temperature gradient decreases. This result is in agreement with Bellini (2001) results.

It is worth of mention that the gradient relation my give the same form for many of cosmological models but each result depends essentially on the value of the scale factor fixed by the model under consideration, i.e this procedure is model dependent.

7 Acknowledgement

The author would like to express his deep thankfulness to Professor M.I.Wanas for his useful discussions. Also he is indebted to the late Dr. M.Melek for his previous guiding points. Part of this work has been accomplished during the author’s visit to the High Energy Section of the Abdos Sallam ICTP. So, the author would like to thank Professor S. Randjbar-Daemi the Head of High Energy Section, ICTP, Italy, for inviting him to visit and use the facilities of the ICTP, during the period from 25 July to 3 August 2004.
8 References

Bellini, M. (2001) GRG, 33, 2081.
Challinor, A. and Lasenby, A. (1999) Ap. J., 512, 1.
Gurzadyan V.G. et al. (2003), astro-ph/0312305.
Hu, W., Sugiyama, N. and Silk, J. (1997) Nature, 386.
Joshue, A. F., Dragan H., Eric, V. L., Michael, S. T. (2003)
astro-ph/0208100v2.
Keating, B., Timbie, P., Polnarev, A. and Steinberger, J. (1998)
Ap. J., 495, 580.
Melek, M. (1992) ICTP print no. IC/92/95.
Melek, M. (1995) Astrophys. Space. Sci., 228, 327.
Melek, M. (2000) Astrophys. Space. Sci., 272, 417.
Melek, M. (2002) Astrophys. Space. Sci., 281, 743.
Mikhail, F.I. and Wanas, M.I. (1977) Proc. Roy. Soc. Lond. A, 356, 471.
Narlikar (1983), Introduction to Cosmology, Jones and Bartlett
Publishers, Inc.
Rebort, R. C. and Doran, M. (2003), astro-ph/0305334v1.
Robertson, H. P. (1932) Ann. Math. Princeton(2), 33, 496.
Riess, A. G., et al. (2004) Ap. J., 607, 665.
Wanas, M. I. (1986) Astrophys. Space. Sci., 127, 21.
Wanas, M. I. (1989) Astro. Space Sci., 154, 165.
Wanas, M. I. (2003) Chaos, Solitons and Fractals, 16, 621.