Inclusive productions of $\Upsilon(1S, 2S, 3S)$ and $\chi_b(1P, 2P, 3P)$ via the Higgs boson decay

Zhan Sun$^1$ and Yang Ma$^2$

$^1$ Department of Physics, Guizhou Minzu University, Guiyang 550025, People’s Republic of China
$^2$ PITT-PACC, Department of Physics and Astronomy, University of Pittsburgh, PA 15260, USA

(Dated: December 2, 2021)

Abstract

In this paper, we carry out the complete $\mathcal{O}(\alpha_s^2)$-order study on the inclusive productions of $\Upsilon(nS)$ and $\chi_b(nP)$ ($n = 1, 2, 3$) via the Standard Model Higgs boson decay, within the framework of nonrelativistic QCD. The feeddown effects via the higher excited states are found to be substantial. The color-octet $3S_1[^8]$ state related processes consisting of $H^0 \rightarrow b\bar{b}[3S_1[^8]] + g$ and $H^0 \rightarrow b\bar{b}[3S_1[^8]] + Q + \bar{Q}$ ($Q = c, b$) play a vital role in the predictions on the decay widths. Moreover, our newly calculated next-to-leading order QCD corrections to $H^0 \rightarrow b\bar{b}[3S_1[^8]] + g$ can enhance its leading-order result by 3-4 times, subsequently magnifying the total $3S_1[^8]$ contributions by about 40%. Such a remarkable enhancement will to a large extent influence the phenomenological conclusions. For the color-singlet $3P_2[^1]$ state, in addition to $H^0 \rightarrow b\bar{b}[3P_2[^1]] + b + \bar{b}$, the newly introduced light hadrons associated process, $H^0 \rightarrow b\bar{b}[3P_2[^1]] + g + g$, can also provide non-negligible contributions, especially for $3P_2[^1]$. Summing up all the contributions, we have $\mathcal{B}_{H^0 \rightarrow \chi_b(nP)+X} \sim 10^{-6} - 10^{-5}$ and $\mathcal{B}_{H^0 \rightarrow \Upsilon(nS)+X} \sim 10^{-5} - 10^{-4}$, which meets marginally nowadays LHC experimental data and can help in understanding the heavy quarkonium production mechanism as well as the Yukawa couplings.

PACS numbers: 12.38.Bx, 12.39.Jh, 14.40.Pq

$^{*}$Electronic address: zhansun@cqu.edu.cn
$^{†}$Electronic address: mayangluon@pitt.edu
I. INTRODUCTION

Bottomonium, as the heaviest bound state, has its own advantages comparing to the charmonium. Due to the large mass of the constituent heavy quarks, both its typical coupling constant $\alpha_s$ and relative velocity $v$ are smaller than those of charmonium. As a result, the perturbative results over the expansion of $\alpha_s$ and $v^2$ for bottomonium will be more convergent than the charmonium case, which makes $b\bar{b}$ mesons an even better place to apply the nonrelativistic QCD (NRQCD) framework \[1\].

Among the bottomonium family, the $\Upsilon$ and $\chi_b$ are most studied because the two mesons can be easily detected by hunting their decaying into lepton pairs\[1\]. Earlier studies of $\Upsilon$ and $\chi_b$ productions can be found in Refs. \[2–10\] and references therein, where the NRQCD predictions succeeded in explaining almost all the existing experimental measurements. However, considering the fact that the color-octet (CO) long distance matrix elements (LDMEs) that used to well explain the hadroproduction of $J/\psi$ leads to dramatic discrepancies between the theoretical predictions and the measured total cross sections of $e^+e^- \to J/\psi + X_{\text{non-}c\bar{c}}$ from the BABAR and Belle collaborations \[11\], it is indispensable to take investigations on the $\Upsilon(nS)$ and $\chi_b(nP)$ productions in a variety of other processes to further test the validity and universality of the CO LDMEs.

The Higgs boson decay provides a good chance for the studies on $\Upsilon$ and $\chi_b$ because of the large number of $H^0$ events at the high energy colliders, e.g., the HL-LHC and HE-LHC can produce $1.65 \times 10^8$ and $5.78 \times 10^8$ $H^0$ events each year, respectively \[12\]. Although the number of $H^0$ events at the Circular Electron Positron Collider (CEPC) can only reach up to $1.1 \times 10^6$ per year \[12–14\], the “clean” background of CEPC comparing to LHC may help us to more easily hunt the heavy quarkonium related processes. Pioneering studies of inclusive $\Upsilon$ and $\chi_b$ productions via $H^0$ decay can be found in Refs. \[12–14\]. Qiao et al. studied the direct (no feeddown contributions) inclusive production of $\Upsilon(1S)$ via $H^0$ decay, including both color-singlet (CS) and CO contributions \[13\]. Based on the CS mechanism, the investigations on the semi-inclusive productions of $\Upsilon$ and $\chi_b$ in association with a $b\bar{b}$ pair, $H^0 \to b\bar{b}[^3S_1, ^3P_J^1] + b + \bar{b}$, were carried out by Liao et al. \[14\]. Note that, in addition to the processes in \[14\], the other CS process, $H^0 \to b\bar{b}[^3P_J^1] + g + g$, \[1\] The decay of $\chi_b$ into lepton pair is indirect, $\chi_b \to \Upsilon + \gamma \to l^+l^- + \gamma$.
might also have remarkable contributions to $\chi_b$ production. Moreover, we learned from the inclusive productions of heavy quarkonium via the $Z$ boson decay that the $^3S_1^{[8]}$ state played a vital role. As shown in our recent work [15], the lowest order process of the $^3S_1^{[8]}$ state, $Z \to Q\bar{Q}[^3S_1^{[8]}] + g$, could receive a remarkable positive NLO QCD correction, which considerably enhance the NRQCD predictions. It is then natural to wonder whether the NLO QCD corrections to $H^0 \to b\bar{b}[^3S_1^{[8]}] + g$ can bring a similar significant enhancement on the LO results, so as to influence the phenomenological conclusions markedly. Besides the vital sense in the studies on the production mechanism of the heavy quarkonium, the decay of the Higgs boson into heavy quarkonium is also very helpful for understanding the electroweak breaking mechanism, especially the Yukawa couplings. In view of these points, we use NRQCD to have a complete $\mathcal{O}(\alpha_s^2)$-order analysis on the inclusive productions of $\Upsilon(1S, 2S, 3S)$ and $\chi_b(1P, 2P, 3P)$ via $H^0$ decay, where all necessary feeddown effects are included.

The rest of the paper is organized as follows: In Sec. II, we give a description on the calculation formalism. In Sec. III, the phenomenological results and discussions are presented. Section IV is reserved as a summary.

II. CALCULATION FORMALISM

Within the NRQCD framework, the decay width of $H^0 \to \Upsilon(\chi_b) + X$ can be written as:

$$d\Gamma = \sum_n d\hat{\Gamma}_n(\mathcal{O}^H(n)),$$

where $d\hat{\Gamma}_n$ is the perturbative calculable short distance coefficients (SDCs), representing the production of a configuration of the $Q\bar{Q}$ intermediate state with a quantum number $n^{(2S+1)L_J^{[1,8]}}$. $\langle \mathcal{O}^H(n) \rangle$ is the universal nonperturbative LDME. At LO accuracy in $v$, for the $\Upsilon$ case, four states should be included, i.e. $b\bar{b}[^3S_1^{[1]}], b\bar{b}[^1S_0^{[8]}], b\bar{b}[^3S_1^{[8]}]$, and $b\bar{b}[^3P_J^{[8]}]$. While in the case of $\chi_b$, we only need consider $b\bar{b}[^3S_1^{[8]}]$ and $b\bar{b}[^3P_J^{[1]}]$. All the involved processes are listed below:
FIG. 1: Typical Feynman diagrams for the virtual corrections to the process of $H^0 \rightarrow b\bar{b}[3S^+_1] + g$. The superscript “CT” denotes the counterterms.

FIG. 2: Typical Feynman diagrams for the real corrections to the process of $H^0 \rightarrow b\bar{b}[3S^+_1] + g$.

FIG. 3: Typical Feynman diagrams for the NLO* processes of $3S^+_1$, $H^0 \rightarrow b\bar{b}[3S^+_1] + Q + \bar{Q}$, where $Q = c, b$. For $Q = c$, the first two diagrams are excluded.
• For \( n = 3 S_1^{[8]} \), up to \( \mathcal{O}(\alpha_s^2) \) order, we have

\[
\text{LO : } H^0 \rightarrow b\bar{b}[3S_1^{[8]}] + g,
\]

\[
\text{NLO : } H^0 \rightarrow b\bar{b}[3S_1^{[8]}] + g \text{ (virtual)},
\]

\[
H^0 \rightarrow b\bar{b}[3S_1^{[8]}] + g + g,
\]

\[
H^0 \rightarrow b\bar{b}[3S_1^{[8]}] + u_g + \bar{u}_g \text{ (ghost)},
\]

\[
H^0 \rightarrow b\bar{b}[3S_1^{[8]}] + q + \bar{q},
\]

\[
\text{NLO}^* : H^0 \rightarrow b\bar{b}[3S_1^{[8]}] + b + \bar{b},
\]

\[
H^0 \rightarrow b\bar{b}[3S_1^{[8]}] + c + \bar{c}.
\]

(2)

The label “NLO*” denotes the heavy quark-antiquark pair associated processes, which are free of divergence.

• In the cases of \( n = 3 S_1^{[1]}, 1 S_0^{[8]}, 3 P_0^{[8]} \), and \( 3 P_1^{[1]} \), the involved channels are

\[
H^0 \rightarrow b\bar{b}[3S_1^{[1]}], S_0^{[8]}, 3 P_0^{[8]}, 3 P_1^{[1]}] + b + \bar{b},
\]

\[
H^0 \rightarrow b\bar{b}[1S_0^{[8]}, 3 P_0^{[8]}, 3 P_1^{[1]}] + g + g.
\]

(3)

Typical Feynman diagrams corresponding to Eqs. (2) are presented in Figs. 1, 2, and 3. The diagrams for \( H^0 \rightarrow b\bar{b}[3S_1^{[1]}], 1 S_0^{[8]}, 3 P_0^{[8]}, 3 P_1^{[1]}] + b + \bar{b} \) are the same with the first two diagrams of Fig. 3 and the diagrams for \( H^0 \rightarrow b\bar{b}[1S_0^{[8]}, 3 P_0^{[8]}, 3 P_1^{[1]}] + g + g \) are the same with the ones in the first line of Fig. 2 excluding the 3-gluon vertex diagrams.

In the following, we will briefly present the formalisms for the NLO QCD corrections to \( H^0 \rightarrow b\bar{b}[3S_1^{[8]}] + g \) as well as the calculations for the tree-level process of \( H^0 \rightarrow b\bar{b}[3P_1^{[1]}, 3 P_0^{[8]}] + g + g \). The rest processes in Eq. (3) and the NLO* processes are both free of divergence, thus one can take the calculations directly according to the Feynman rules.

### A. NLO QCD corrections to \( H^0 \rightarrow b\bar{b}[3S_1^{[8]}] + g \)

To the next-to-leading order in \( \alpha_s \), the SDC of the process of \( H^0 \rightarrow b\bar{b}[3S_1^{[8]}] + X_{\text{light-hadrons}} \) reads

\[
\hat{\Gamma} = \hat{\Gamma}_{\text{Born}} + \hat{\Gamma}_{\text{Virtual}} + \hat{\Gamma}_{\text{Real}} + \mathcal{O}(\alpha_s^3),
\]

(4)
\[ \hat{\Gamma}_{\text{Virtual}} = \hat{\Gamma}_{\text{Loop}} + \hat{\Gamma}_{\text{CT}}, \]
\[ \hat{\Gamma}_{\text{Real}} = \hat{\Gamma}_{S} + \hat{\Gamma}_{\text{HC}} + \hat{\Gamma}_{\text{HC}}, \]

(5)

\( \hat{\Gamma}_{\text{Virtual}} \) is the virtual corrections, consisting of the contributions from the one-loop diagrams (\( \hat{\Gamma}_{\text{Loop}} \)) and the counterterms (\( \hat{\Gamma}_{\text{CT}} \)). \( \hat{\Gamma}_{\text{Real}} \) stands for the real corrections, which includes the soft terms (\( \hat{\Gamma}_{S} \)), hard-collinear terms (\( \hat{\Gamma}_{\text{HC}} \)), and hard-noncollinear terms (\( \hat{\Gamma}_{\text{HC}} \)). To isolate the ultraviolet (UV) and infrared (IR) divergences, we adopt the dimensional regularization with \( D = 4 - 2\epsilon \). The on-mass-shell (OS) scheme is employed to set the renormalization constants for the heavy quark mass (\( Z_m \)), heavy quark filed (\( Z_A \)), and gluon filed (\( Z_3 \)). The modified minimal-subtraction (\( \overline{\text{MS}} \)) scheme is used for the QCD gauge coupling (\( Z_3 \)). The renormalization constants are [16],

\[ \delta Z_{m}^{\text{OS}} = -3 C_F \frac{\alpha_s N_c}{4 \pi} \left[ \frac{1}{\epsilon_{\text{UV}}} - \gamma_E + \ln \frac{4\pi \mu_F^2}{m_b^2} + \frac{4}{3} \right], \]
\[ \delta Z_{2}^{\text{OS}} = -C_F \frac{\alpha_s N_c}{4 \pi} \left[ \frac{1}{\epsilon_{\text{UV}}} + \frac{2}{\epsilon_{\text{IR}}} - 3\gamma_E + 3\ln \frac{4\pi \mu_F^2}{m_b^2} + 4 \right], \]
\[ \delta Z_{3}^{\overline{\text{MS}}} = \frac{\alpha_s N_c}{4 \pi} \left[ (\beta_0' - 2 C_A)\left( \frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}} \right) - \frac{4}{3} T_F \left( \frac{1}{\epsilon_{\text{UV}}} - \gamma_E + \ln \frac{4\pi \mu_F^2}{m_b^2} \right) \right], \]
\[ \delta Z_{g}^{\overline{\text{MS}}} = -\frac{\beta_0}{2} \frac{\alpha_s N_c}{4 \pi} \left[ \frac{1}{\epsilon_{\text{UV}}} - \gamma_E + \ln(4\pi) \right], \]

(6)

where \( \gamma_E \) is the Euler’s constant, \( N_c = \Gamma[1 - \epsilon]/(4\pi \mu_F^2/(4m_b^2))^{\epsilon} \), \( \beta_0 = \frac{41}{9} C_A - \frac{4}{3} T_F n_f \) is the one-loop coefficient of the \( \beta \)-function, and \( \beta_0' = \frac{41}{9} C_A - \frac{4}{3} T_F n_F \). \( n_f \) and \( n_F \) are the number of active quark flavors and light quark flavors, respectively. In SU(3), the color factors are given by \( T_F = \frac{1}{2}, C_F = \frac{4}{3}, \) and \( C_A = 3 \). The two-cutoff slicing strategy is utilized to subtract the IR divergences in \( \Gamma_{\text{Real}} \) [17].

B. \( H^0 \to b\bar{b}[^3P_{J}^{[1]};^3P_{J}^{[8]}] + g + g \)

Taking \( ^3P_{J}^{[1]} \) as an example 2, we first divide \( \Gamma_{H^0 \to b\bar{b}[^3P_{J}^{[1]}] + g + g} \) into two ingredients,

\[ d\Gamma_{H^0 \to b\bar{b}[^3P_{J}^{[1]}] + g + g} = d\hat{\Gamma}_{[^3P_{J}^{[1]}]}(O^{\chi_b}(^3P_{J}^{[1]})) + d\hat{\Gamma}_{[^3S_{1}^{[8]}]}(O^{\chi_b}(^3S_{1}^{[8]}))^{NLO}, \]

(7)

2 Since the process of \( H^0 \to b\bar{b}[^3S_{1}^{[1]}] + g \) is forbidden, for the \( H^0 \to b\bar{b}[^3P_{J}^{[8]}] + g + g \) case, the calculation formalism is almost the same except the color factor.
then one can obtain

\[ d\hat{\Gamma}_{3P_{j}^{[1]}}(O^{\chi_b(3P_{j}^{[1]})}) = d\Gamma_{H^0 \rightarrow b\bar{b}[3P_{j}^{[1]}]+g+g} - d\hat{\Gamma}_{3S_{1}^{[8]}(O^{\chi_b(3S_{1}^{[8]})})}^{LO} \]

\[ = d\Gamma_F + d\Gamma_S - d\hat{\Gamma}_{3S_{1}^{[8]}(O^{\chi_b(3S_{1}^{[8]})})}^{LO}. \]  

(8)

\[ d\Gamma_F(= d\hat{\Gamma}_{F}(O^{\chi_b(3P_{j}^{[1]})})) \] is the finite term in \( d\Gamma_{H^0 \rightarrow b\bar{b}[3P_{j}^{[1]}]+g+g} \) and \( d\Gamma_S \) is the soft part which can be written as

\[ d\Gamma_S = -\frac{\alpha_s}{3\pi m_b^2} u^*_c \frac{N_c^2 - 1}{N_c^2} d\hat{\Gamma}_{3S_{1}^{[8]}(O^{\chi_b(3P_{j}^{[1]})})}^{LO}, \]

where

\[ u^*_c = \frac{1}{\epsilon_{IR}} + \frac{E}{|p|} \ln\left(\frac{E + |p|}{E - |p|}\right) + \ln\left(\frac{4\pi \mu^2}{s \delta_s^2}\right) - \gamma_E - \frac{1}{3}. \]

(10)

\( N_c \) is identical to 3 for SU(3) gauge field. \( E \) and \( p \) denote the energy and 3-momentum of \( \chi_b \), respectively. \( \delta_s \) is the usual “soft cut” employed to impose an amputation on the energy of the emitted gluon. Regarding \( \langle O^{\chi_b(3S_{1}^{[8]})} \rangle^{NLO} \), under the dimensional regularization scheme as is adopted in [18], we have

\[ \langle O^{\chi_b(3S_{1}^{[8]})} \rangle^{NLO} = -\frac{\alpha_s}{3\pi m_b^2} u^*_c \frac{N_c^2 - 1}{N_c^2} \langle O^{\chi_b(3P_{j}^{[1]})} \rangle. \]

(11)

Then the third term in Eq. (8) can be written as

\[ d\hat{\Gamma}_{3S_{1}^{[8]}(O^{\chi_b(3S_{1}^{[8]})})}^{LO} = -\frac{\alpha_s}{3\pi m_b^2} u^*_c \frac{N_c^2 - 1}{N_c^2} d\hat{\Gamma}_{3S_{1}^{[8]}(O^{\chi_b(3P_{j}^{[1]})})}^{LO}, \]

where, on the basis of \( \mu_\Lambda \)-cutoff scheme [18], \( u^*_c \) has the following form

\[ u^*_c = \frac{1}{\epsilon_{IR}} - \gamma_E - \frac{1}{3} + \ln\left(\frac{4\pi \mu^2}{\mu^2_\Lambda}\right). \]

(13)

\( \mu_\Lambda \) is the upper bound of the integrated gluon energy, rising from the renormalization of the LDME. Substituting Eqs. (9) and (12) into Eq. (8), the soft singularities in \( d\Gamma_S \) and \( d\hat{\Gamma}_{3S_{1}^{[8]}(O^{\chi_b(3S_{1}^{[8]})})}^{LO} \) cancel each other, consequently leading to

\[ d\hat{\Gamma}_{3P_{j}^{[1]}}(O^{\chi_b(3P_{j}^{[1]})}) = \left[d\hat{\Gamma}_F + \frac{\alpha_s}{3\pi m_b^2} (u^*_c - u^*_\epsilon) \frac{N_c^2 - 1}{N_c^2} d\hat{\Gamma}_{3S_{1}^{[8]}(O^{\chi_b(3S_{1}^{[8]})})}^{LO}\right] \langle O^{\chi_b(3P_{j}^{[1]})} \rangle \]

\[ = (d\hat{\Gamma}_F + d\hat{\Gamma}_\epsilon) \langle O^{\chi_b(3P_{j}^{[1]})} \rangle. \]

(14)

The package \texttt{MALT@FDC} that has been adopted in several heavy quarkonium related processes [15, 20–25] is used to deal with \( \hat{\Gamma}_{\text{Virtual}}, \hat{\Gamma}_S \), and \( \hat{\Gamma}_{\text{HC}} \). To calculate the hard-noncollinear
FIG. 4: The verification of the independence on the cutoff parameters of $\delta_{s,c}$ for the SDCs of $^3S_1^{[8]}$ (the upper two diagrams) and $^3P_0^{[1,8]}$ (the lower two diagrams), respectively. The superscript “(0)” denotes the $\epsilon^0$-order terms.

part of the real corrections, $\hat{\Gamma}_{1\text{LC}}$, we employ the FDC \cite{20} package. Both the cancellation of the $\epsilon^{-2(-1)}$-order divergence and the independence on cutoff ($\delta_{s,c}$) have been checked carefully. Taking $^3S_1^{[8]}$ and $^3P_0^{[1,8]} (J = 0)$ as an example, the verification of the independence on the cutoff parameters of $\delta_{s,c}$ is shown in Fig. \ref{fig:4}. The $J = 1, 2$ cases are not presented here since they are quite similar to the $J = 0$ case.

III. PHENOMENOLOGICAL RESULTS

For the numerical calculations, we take $\alpha = 1/128$, $m_c = 1.5$ GeV, $m_b = 4.9$ GeV, $m_t = 173$ GeV, $m_W = 80.4$ GeV, and $m_{H^0} = 125$ GeV. The light quarks $q$ and $\bar{q}$ ($q = u, d, s$) are regarded as massless. For the NLO corrections to $H^0 \to b\bar{b}[^3S_1^{[8]}] + g$ and calculating other $\alpha_s^2$-order processes, we employ the two-loop $\alpha_s$ running. The one-loop $\alpha_s$ running is adopted for the LO cases. The mixed feeddown scheme of $\chi_{bJ}(3P) \to \Upsilon(nS)$ in Ref. \cite{11} is used and the value of $\mu_\Lambda$ is taken as $m_b$, thus the CO LDMEs in Table 4 of Ref. \cite{8} are
chosen to achieve the numerical results. For the CS cases, \( ^3S_1^{[1]} \) and \( ^3P_J^{[1]} \), the LDMEs are related to the radial wave functions at the origin \((n, m = 1, 2, 3)\):

\[
\frac{\langle O^{(nS)(nS)}^{(3S_1^{[1]})} \rangle}{6N_c} = \frac{1}{4\pi}|R_{T(nS)}(0)|^2,
\]

\[
\frac{\langle O^{\chi bJ(mP)(3P_J^{[1]})} \rangle}{2N_c} = (2J+1)\frac{3}{4\pi}|R'_{\chi b(mP)}(0)|^2,
\]

where \(|R_{T(nS)}(0)|^2\) and \(|R'_{\chi b(mP)}(0)|^2\) are taken as [19]

\[
|R_{T(1S)}(0)|^2 = 6.477 \text{ GeV}^3, \quad |R_{T(2S)}(0)|^2 = 3.234 \text{ GeV}^3,
\]

\[
|R_{T(3S)}(0)|^2 = 2.474 \text{ GeV}^3,
\]

\[
|R'_{\chi b(1P)}(0)|^2 = 1.417 \text{ GeV}^5, \quad |R'_{\chi b(2P)}(0)|^2 = 1.653 \text{ GeV}^5,
\]

\[
|R'_{\chi b(3P)}(0)|^2 = 1.794 \text{ GeV}^5.
\]

Branching ratios of \( \chi bJ(mP) \rightarrow \Upsilon(nS), \Upsilon(nS) \rightarrow \chi bJ(mP), \Upsilon(3S) \rightarrow \Upsilon(2S), \Upsilon(3S) \rightarrow \Upsilon(1S), \) and \( \Upsilon(2S) \rightarrow \Upsilon(1S) \) can be found in Refs. [6–8].

**TABLE I**: The SDC of \( ^3S_1^{[8]} \) (in units of keV/GeV\(^3\)).

| \( \mu_r \) | LO | NLO | NLO\(^*\)\(_{bb} \) | NLO\(^*\)\(_{cc} \) | Total |
|---|---|---|---|---|---|
| 2\( m_b \) | 8.79 \times 10^{-2} | 0.340 | 0.568 | 6.44 \times 10^{-2} | 0.97 |
| 2\( m_H \) | 5.42 \times 10^{-2} | 0.170 | 0.223 | 2.53 \times 10^{-2} | 0.42 |

Before presenting the phenomenological results, we first take a look at the effect of the QCD corrections to the process of \( H^0 \rightarrow b \bar{b}[^3S_1^{[8]}] + g \), presented in Table I. We see that the newly calculated NLO terms increase the LO results by about 3-4 times, causing a 40% enhancement on the total \( ^3S_1^{[8]} \) contributions (LO + NLO\(^*\)\(_{bb,cc} \)). This is consistent with the lesson we learn from \( Z^0 \) decay [13]. The \( ^3S_1^{[8]} \) state may provide significant (even dominant) contributions to \( \Gamma_{H^0 \rightarrow \Upsilon, \chi b + X} \), thus the newly introduced NLO ingredient is of great essence in achieving the phenomenological conclusions.

**A.\( \chi b(3P, 2P, 1P) \)**

The NRQCD predictions on the decay width of \( H^0 \rightarrow \chi bJ(3P, 2P, 1P) + X \) are listed in Tables. [II, III, and IV] In order to show the relative importance of different production
TABLE II: The decay widths of $H^0 \rightarrow \chi_{bJ}(3P) + X$ (in units of ev). The superscripts “DR” and “FD” denote the direct production processes and feeddown effects, respectively.

| $\chi_{bJ}$ | $\mu_r$ | $^3S_1^{[8]}$ | $^3P_0^{[1]}_{|gg|}$ | $^3P_0^{[1]}_{|bb|}$ | $\Gamma_{DR}$ | $\Gamma_{FD}$ | $\Gamma_{Total}$ | $\text{Br} \times 10^{-6}$ |
|-------------|---------|----------------|-----------------|----------------|-------------|-------------|----------------|-----------------|
| $J=0$       | $2m_b$  | 6.22           | 0.69            | 12.9           | 19.8        | –           | 19.8          | 4.71            |
| $m_{H^0}$   | 2.67    | 0.27           | 5.06            | 8.00           | –           | 8.00        | 1.90          |
| $J=1$       | $2m_b$  | 18.7           | 0.91            | 14.0           | 33.6        | –           | 33.6          | 8.00            |
| $m_{H^0}$   | 8.03    | 0.36           | 5.49            | 13.9           | –           | 13.9        | 3.31          |
| $J=2$       | $2m_b$  | 31.1           | 4.09            | 5.06           | 40.3        | –           | 40.3          | 9.60            |
| $m_{H^0}$   | 13.4    | 1.60           | 1.99            | 17.0           | –           | 17.0        | 4.05          |

TABLE III: The decay widths of $H^0 \rightarrow \chi_{bJ}(2P) + X$ (in units of ev). The superscripts “DR” and “FD” denote the direct production processes and feeddown effects, respectively.

| $\chi_{bJ}$ | $\mu_r$ | $^3S_1^{[8]}$ | $^3P_0^{[1]}_{|gg|}$ | $^3P_0^{[1]}_{|bb|}$ | $\Gamma_{DR}$ | $\Gamma_{FD}$ | $\Gamma_{Total}$ | $\text{Br} \times 10^{-6}$ |
|-------------|---------|----------------|-----------------|----------------|-------------|-------------|----------------|-----------------|
| $J=0$       | $2m_b$  | 4.86           | 0.64            | 11.9           | 17.4        | 5.83        | 23.2          | 5.52            |
| $m_{H^0}$   | 2.09    | 0.25           | 4.66            | 7.00           | 2.33        | 9.33        | 2.22          |
| $J=1$       | $2m_b$  | 14.6           | 0.84            | 12.9           | 28.3        | 12.5        | 40.8          | 9.71            |
| $m_{H^0}$   | 6.27    | 0.33           | 5.06            | 11.7           | 4.98        | 16.7        | 3.98          |
| $J=2$       | $2m_b$  | 24.3           | 3.76            | 4.66           | 32.7        | 12.9        | 45.6          | 10.9            |
| $m_{H^0}$   | 10.5    | 1.48           | 1.83            | 13.8           | 5.18        | 19.0        | 4.52          |

channels in a wide range of $\mu_r$, we provide the predictions at $\mu_r = 2m_b$ and $\mu_r = m_{H^0}$ simultaneously. It is noticed that the branching ratios for $H^0 \rightarrow \chi_{bJ}(3P, 2P, 1P) + X$ are calculated to be on the order of $10^{-6} - 10^{-5}$, indicating the probability of these processes to be observed at the HE-LHC, HL-LHC, and other colliders in near future. In addition to the direct production processes that are dominant, the feeddown effects via the higher excited states, e.g., $\Upsilon(2S)$ and $\Upsilon(1S)$, are also significant, accounting for about 30% of the total decay width, as is shown in Table III and Table IV. The direct productions consist of two parts, i.e. the CS state $^3P_j^{[1]}$ and the CO state $^3S_1^{[8]}$.

- For the CS cases, the processes of $H^0 \rightarrow b\bar{b}[^3P_j^{[1]}] + b + \bar{b}$ (“$b\bar{b}$”) serve as the leading role in the total CS prediction due to the $b$-quark fragmentation mechanism. However, the light hadrons associated process $H^0 \rightarrow b\bar{b}[^3P_j^{[1]}] + g + g$ (“$gg$”) can also provide non-
TABLE IV: The decay widths of $H^0 \to \chi_{bJ}(1P) + X$ (in units of ev). The superscripts “DR” and “FD” denote the direct production processes and feeddown effects, respectively.

| $\chi_{bJ}$ | $\mu_r$ | $3^3S_1^{[8]}$ | $3P_0^{[1]}$ | $gg$ | $3P_0^{[1]}$ | $bb$ | $\Gamma_{DR}$ | $\Gamma_{FD}^{(2S)}$ | $\Gamma_{Total}$ | $\text{Br} \times 10^{-6}$ |
|-------------|---------|----------------|-------------|------|-------------|------|--------------|---------------|----------------|----------------|
| $J = 0$     | $2m_b$  | 3.76           | 0.54        | 10.2 | 14.5        | 5.94 | 20.4         | 8.22          | 4.86           |
| $m_{H^0}$   |         | 1.62           | 0.21        | 4.00 | 5.83        | 2.39 | 8.22         |               | 1.96           |
| $J = 1$     | $2m_b$  | 11.3           | 0.72        | 11.1 | 23.1        | 10.8 | 33.9         | 13.8          | 3.29           |
| $m_{H^0}$   |         | 4.85           | 0.28        | 4.34 | 9.47        | 4.33 | 13.8         |               | 3.29           |
| $J = 2$     | $2m_b$  | 18.8           | 3.23        | 4.00 | 26.0        | 11.2 | 37.2         | 15.4          | 3.67           |
| $m_{H^0}$   |         | 8.08           | 1.27        | 1.57 | 10.9        | 4.49 | 15.4         |               | 3.67           |

negligible contributions. To be specific, for $3P_0^{[1]}$ and $3P_1^{[1]}$ states, the contribution of the “$gg$” channel enhance the “$b\bar{b}$” cases by about 5% and 7%, respectively. Moreover, for the $3P_2^{[1]}$ case, the “$gg$” contribution can surprisingly reach up to about 81% of the “$b\bar{b}$” contribution. Therefore, to achieve a sound estimate, besides $H^0 \to b\bar{b}[3P_1^{[1]}] + b + \bar{b}$, the contributions of $H^0 \to b\bar{b}[3P_2^{[1]}] + g + g$ must be also taken into consideration.

- Regarding the CO cases, including the $3S_1^{[8]}$ state contributions can significantly enlarge the predicted decay width. Taking $\chi_{bJ}(3P)$ as an example, when $\mu_r = 2m_b$, the $3S_1^{[8]}$ contributions account for about 31%, 56%, and 77% of $\Gamma_{DR}$, corresponding to $J = 0, 1,$ and 2, respectively. As for the $\chi_b(2P)$ and $\chi_b(1P)$ cases, the proportions are about 28%, 52%, 74% and 26%, 49%, 72%, respectively.

TABLE V: The ratios of $\Gamma_{\chi_{b2}}/\Gamma_{\chi_{b0}}$ and $\Gamma_{\chi_{b2}}/\Gamma_{\chi_{b1}}$. “CS” denotes the sum of the CS direct ($3P_J^{[1]}$) and feeddown ($3S_1^{[1]}$) contributions, while “NR” means the NRQCD results including both CS and CO contributions. $\mu_r$ is varied in $[2m_b, m_{H^0}]$.

|                  | $\Gamma_{\chi_{b2}}/\Gamma_{\chi_{b0}}$ | $\Gamma_{\chi_{b2}}/\Gamma_{\chi_{b1}}$ | $\Gamma_{\chi_{b2}}/\Gamma_{\chi_{b0}}$ | $\Gamma_{\chi_{b2}}/\Gamma_{\chi_{b1}}$ | $\Gamma_{\chi_{b2}}/\Gamma_{\chi_{b0}}$ | $\Gamma_{\chi_{b2}}/\Gamma_{\chi_{b1}}$ |
|-----------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
|                  | $3P$                                      | $3P$                                      | $2P$                                      | $2P$                                      | $1P$                                      | $1P$                                      |
| CS              | 0.674                                     | 0.613                                     | 1.097                                     | 0.794                                     | 1.029                                     | 0.786                                     |
| NR              | 2.035                                     | 2.125                                     | 1.199                                     | 1.223                                     | 1.966                                     | 2.036                                     |

In addition to the large contributions to the total decay width, the $3S_1^{[8]}$ state also has crucial effect on the ratios of $\Gamma_{\chi_{b2}}/\Gamma_{\chi_{b0}}$ and $\Gamma_{\chi_{b2}}/\Gamma_{\chi_{b1}}$, as shown in Table V, where the
feeddown effects have been incorporated. Since the dependence of the CS channels, \textit{“gg”} and \textit{“bb”}, on \( \mu_r \) is only in the strong coupling constants \( \alpha_s \), varying \( \mu_r \) of course does not affect the ratios. However, for the CO cases, due to the NLO corrections to \( H^0 \rightarrow \bar{b}b[3S^1_1] + g \), the form of the dependence on \( \mu_r \) is not only \( \alpha_s \). Although varying \( \mu_r \) in \([2m_b, m_{H^0}]\) greatly influence the total decay widths, the ratios of \( \Gamma_{\chi_{b2}}/\Gamma_{\chi_{b0}} \) and \( \Gamma_{\chi_{b2}}/\Gamma_{\chi_{b1}} \) are quite insensitive to the choice of \( \mu_r \). Taking \( \Gamma_{\chi_{b2}}/\Gamma_{\chi_{b0}} |_{3P} \) for example, when \( \mu_r \) is varied from \( 2m_b \) (9.8 GeV) to \( m_{H^0} \) (125 GeV), the ratios just increase by about 4%. In addition, the differences between the CS and NRQCD results are rather conspicuous, which can be regarded as an outstanding probe to distinguish between the two heavy quarkonium production mechanism.

B. \( \Upsilon(3S,2S,1S) \)

### TABLE VI: The decay widths of \( H^0 \rightarrow \Upsilon(3S) + X \) (in units of ev). The superscripts “DR” and “FD” denote the direct production processes and feeddown effects, respectively.

| \( \mu_r \) | \( 3S^8_{1} \) | \( 1S^8_{0} \) | \( 3P^8_{J} \) | \( 3S^1_{1} \) | \( \Gamma_{DR} \) | \( \Gamma_{FD}^{3P} \) | \( \Gamma_{FD}^{3S} \) | \( \Gamma_{Total} \) | \( \text{Br}( \times 10^{-5}) \) |
|---|---|---|---|---|---|---|---|---|---|
| \( 2m_b \) | 14.8 | -3.29 \times 10^{-2} | -1.69 \times 10^{-2} | 77.1 | 91.9 | 6.96 | - | 98.9 | 2.35 |
| \( m_{H^0} \) | 6.35 | -1.29 \times 10^{-2} | -6.62 \times 10^{-3} | 30.3 | 36.6 | 2.89 | - | 39.5 | 0.94 |

### TABLE VII: The decay widths of \( H^0 \rightarrow \Upsilon(2S) + X \) (in units of ev). The superscripts “DR” and “FD” denote the direct production processes and feeddown effects, respectively.

| \( \mu_r \) | \( 3S^8_{1} \) | \( 1S^8_{0} \) | \( 3P^8_{J} \) | \( 3S^1_{1} \) | \( \Gamma_{DR} \) | \( \Gamma_{FD}^{2,3P} \) | \( \Gamma_{FD}^{3S} \) | \( \Gamma_{Total} \) | \( \text{Br}( \times 10^{-5}) \) |
|---|---|---|---|---|---|---|---|---|---|
| \( 2m_b \) | 28.6 | -0.11 | 0.47 | 101 | 130 | 16.1 | 10.5 | 157 | 3.74 |
| \( m_{H^0} \) | 12.3 | -4.23 \times 10^{-2} | 0.19 | 39.6 | 52.1 | 6.60 | 4.19 | 62.9 | 1.50 |

### TABLE VIII: The decay widths of \( H^0 \rightarrow \Upsilon(1S) + X \) (in units of ev). The superscripts “DR” and “FD” denote the direct production processes and feeddown effects, respectively.

| \( \mu_r \) | \( 3S^8_{1} \) | \( 1S^8_{0} \) | \( 3P^8_{J} \) | \( 3S^1_{1} \) | \( \Gamma_{DR} \) | \( \Gamma_{FD}^{1,2,3P} \) | \( \Gamma_{FD}^{2,3S} \) | \( \Gamma_{Total} \) | \( \text{Br}( \times 10^{-5}) \) |
|---|---|---|---|---|---|---|---|---|---|
| \( 2m_b \) | 4.57 | 2.12 | -0.83 | 202 | 208 | 27.3 | 48.0 | 283 | 6.74 |
| \( m_{H^0} \) | 1.96 | 0.83 | -0.32 | 79.3 | 81.8 | 12.1 | 19.3 | 113 | 2.69 |
The NRQCD predictions on the decay width of $H^0 \to \Upsilon(3S, 2S, 1S) + X$ are presented in Tables VI, VII, and VIII, respectively. In these tables, one would see that the branching ratios of the inclusive productions of $\Upsilon(3S, 2S, 1S)$ via $H^0$ decay are about $10^{-5} - 10^{-4}$, indicating the potential to be detected at the high energy collider. For $H^0 \to \Upsilon(3S, 2S, 1S) + X$, the feeddown contributions from the higher excited states are remarkable, accounting for about 7%, 17%, and 27% of the total decay widths of $\Upsilon(3S)$, $\Upsilon(2S)$, and $\Upsilon(1S)$, respectively.

Regarding the direct productions, the main contributions come from the CS state, $^3S_1^{[1]}$, via the heavy-quark pair associated process. The CO states can also provide considerable contributions, which account for about 16%, 22%, and 3% on $\Gamma_{DR}$ of $\Upsilon(3S)$, $\Upsilon(2S)$, and $\Upsilon(1S)$, respectively.

In addition to the total decay width, we also calculate the ratios of $\Gamma_{\Upsilon(2S)}/\Gamma_{\Upsilon(3S)}$ and $\Gamma_{\Upsilon(1S)}/\Gamma_{\Upsilon(3S)}$. By varying $\mu_r$ in $[2m_b, m_{H^0}]$, we have

\begin{align}
\text{CS} & : \quad \Gamma_{\Upsilon(2S)}/\Gamma_{\Upsilon(3S)} = 1.471, \\
& \quad \Gamma_{\Upsilon(1S)}/\Gamma_{\Upsilon(3S)} = 3.170, \\
\text{NR} & : \quad \Gamma_{\Upsilon(2S)}/\Gamma_{\Upsilon(3S)} = 1.587 \sim 1.592, \\
& \quad \Gamma_{\Upsilon(1S)}/\Gamma_{\Upsilon(3S)} = 2.861 \sim 2.862, \tag{17}
\end{align}

where “CS” denotes the sum of the CS direct ($^3P_J^{[1]}$) and feeddown ($^3S_1^{[1]}$) contributions, while “NR” is the total results including both CS and CO contributions. The difference between the CS and NRQCD predictions reflects that the CO influence on $\Gamma_{\Upsilon(2S)}/\Gamma_{\Upsilon(3S)}$ and $\Gamma_{\Upsilon(1S)}/\Gamma_{\Upsilon(3S)}$ is moderate.

Finally, to serve as a useful reference, we analyze the uncertainties of the predictions due to the choices of the renormalization scale $\mu_r$, Higgs mass $m_{H^0}$, the bottom quark mass $m_b$, and the CO LDMEs.
where the four columns are the uncertainties caused by $\mu$, $m_{H^0}$, $m_b$, and the CO LDMEs, respectively. The center values in Eqs. (18) and (19) are calculated at $m_{H^0} = 125$ GeV, $m_b = 4.9$ GeV, and $\mu_r = m_{H^0}/2$, with the LDMEs taken as the center values in Table 4 of Ref. [8]. To estimate the uncertainty, we vary $m_{H^0}$ in [123, 127] GeV, $m_b$ in [4.7, 5.1] GeV, $\mu_r$ in [$m_{H^0}/4$, $m_{H^0}$] with $m_{H^0} = 125$ GeV, and the LDMEs from the upper limit to the lower limit, respectively. The numerical results show that the ambiguities of $\mu_r$, $m_b$, and the LDMEs are responsible for the main uncertainties, while varying $m_{H^0}$ only slightly influence the predictions on the total decay widths.

\section*{IV. SUMMARY}

In this paper, we used NRQCD factorization to investigate the inclusive productions of the $\Upsilon(1S, 2S, 3S)$ and $\chi_b(1P, 2P, 3P)$ via the Standard Model Higgs boson decay up to $\mathcal{O}(a_s^2)$ order. It is found that the CO states, especially $3^1S_1$\cite{8}, provide remarkable contributions,
leading to vital effect on the predictions on the total decay widths. The newly calculated NLO QCD corrections to the lowest order process of $^{3}S_{1}^{[8]} \rightarrow b \bar{b}^{[3}S_{1}^{[8]之外] + g}$, can significantly (3-4 times) enhance the LO results, subsequently enlarging the total $^{3}S_{1}^{[8]}$ contributions by about 40%. In addition to the crucial effect on the total decay widths of $\Upsilon(nS)$ and $\chi_b(nP)$, including the CO states also influence the ratios of $\frac{\Gamma_{\Upsilon(nS)}}{\Gamma_{\chi_b(nP)}}$ a lot. Regarding the $^{3}P_{J}^{[1]}$ state, besides the dominant $H^0 \rightarrow b \bar{b}^{[3}P_{J}^{[1]之外] + b + \bar{b}$ process, the newly introduced light hadrons associated process, $H^0 \rightarrow b \bar{b}^{[3}P_{J}^{[1]之外] + g + g$, can also provide non-negligible contributions, especially for $J = 2$. The feeddown contributions via the decay of the higher excited states are found to be substantial, significantly influencing the NRQCD predictions. In the end, the branching ratios of $H^0 \rightarrow \Upsilon(nS) + X$ and $H^0 \rightarrow \chi_b(nP) + X$ are predicted to be on the order of $10^{-5} - 10^{-4}$ and $10^{-6} - 10^{-5}$, reflecting the great potential of these processes to be detected at high energy colliders. As a conclusion, the decay of Higgs boson into $\Upsilon(nS)$ and $\chi_b(nP)$ can be considered as an ideal laboratory not only to study the heavy quarkonium production mechanism, but also to understand the electroweak breaking mechanism especially the Yukawa couplings.

V. ACKNOWLEDGMENTS

Acknowledgments: Z. Sun is supported in part by the Natural Science Foundation of China under the Grant No. 11647113. and No. 11705034., by the Project for Young Talents Growth of Guizhou Provincial Department of Education under Grant No.KY[2017]135, and by the Project of Guizhou Provincial Department of Science and Technology under Grant No. QKHJC[2019]1160. Y. Ma is supported by PITT PACC.

[1] G. T. Bodwin, E. Braaten and G. P. Lepage, “Rigorous QCD analysis of inclusive annihilation and production of heavy quarkonium, Phys. Rev. D 51 (1995) 1125 Erratum: [Phys. Rev. D 55 (1997) 5853] doi:10.1103/PhysRevD.55.5853, 10.1103/PhysRevD.51.1125.

[2] E. Braaten and J. Lee, “Polarization of $\nu_{nS}$ at the Tevatron,” Phys. Rev. D 63 (2001) 071501 doi:10.1103/PhysRevD.63.071501
[3] P. Artoisenet, J. M. Campbell, J. P. Lansberg, F. Maltoni and F. Tramontano, “ϒ Production at Fermilab Tevatron and LHC Energies,” Phys. Rev. Lett. 101 (2008) 152001 doi:10.1103/PhysRevLett.101.152001

[4] K. Wang, Y. Q. Ma and K. T. Chao, “ϒ(1S) prompt production at the Tevatron and LHC in nonrelativistic QCD,” Phys. Rev. D 85 (2012) 114003 doi:10.1103/PhysRevD.85.114003

[5] A. K. Likhoded, A. V. Luchinsky and S. V. Poslavsky, “Production of χb-mesons at LHC,” Phys. Rev. D 86 (2012) 074027 doi:10.1103/PhysRevD.86.074027

[6] B. Gong, L. P. Wan, J. X. Wang and H. F. Zhang, “Complete next-to-leading-order study on the yield and polarization of ϒ(1S, 2S, 3S) at the Tevatron and LHC,” Phys. Rev. Lett. 112 (2014) no.3, 032001 doi:10.1103/PhysRevLett.112.032001

[7] H. Han, Y. Q. Ma, C. Meng, H. S. Shao, Y. J. Zhang and K. T. Chao, “ϒ(nS) and χb(nP) production at hadron colliders in nonrelativistic QCD,” Phys. Rev. D 94 (2016) no.1, 014028 doi:10.1103/PhysRevD.94.014028

[8] Y. Feng, B. Gong, L. P. Wan and J. X. Wang, “An updated study of ϒ production and polarization at the Tevatron and LHC,” Chin. Phys. C 39 (2015) no.12, 123102 doi:10.1088/1674-1137/39/12/123102

[9] Z. Sun, X. G. Wu, G. Chen, J. Jiang and Z. Yang, “Heavy quarkonium production through the semi-exclusive e+e− annihilation channels round the Z0 peak,” Phys. Rev. D 87 (2013) no.11, 114008 doi:10.1103/PhysRevD.87.114008

[10] Z. Sun, X. G. Wu, G. Chen, Y. Ma, H. H. Ma and H. Y. Bi, “Bottomonium production associated with a photon at a high luminosity e+e− collider with the one-loop QCD correction,” Phys. Rev. D 89 (2014) no.7, 074035 doi:10.1103/PhysRevD.89.074035

[11] Y. J. Zhang, Y. Q. Ma, K. Wang and K. T. Chao, “QCD radiative correction to color-octet J/ψ inclusive production at B Factories,” Phys. Rev. D 81 (2010) 034015 doi:10.1103/PhysRevD.81.034015

[12] J. Jiang and C. F. Qiao, “Bc Production in Higgs Boson Decays,” Phys. Rev. D 93 (2016) no.5, 054031 doi:10.1103/PhysRevD.93.054031

[13] C. F. Qiao, F. Yuan and K. T. Chao, “Quarkonium production in SM Higgs decays,” J. Phys. G 24 (1998) 1219 doi:10.1088/0954-3899/24/7/004

[14] Q. L. Liao, Y. Deng, Y. Yu, G. C. Wang and G. Y. Xie, “Heavy P-wave quarkonium production via Higgs decays,” Phys. Rev. D 98 (2018) no.3, 036014 doi:10.1103/PhysRevD.98.036014
[15] Z. Sun and H. F. Zhang, “Next-to-leading-order QCD corrections to the decay of Z boson into $\chi_c(\chi_b)$,” Phys. Rev. D 99 (2019) no.9, 094009 doi:10.1103/PhysRevD.99.094009

[16] B. Gong and J. X. Wang, “QCD corrections to $J/\psi$ plus $\eta_c$ production in $e^+e^-$ annihilation at $S^{(1/2)} = 10.6$-GeV,” Phys. Rev. D 77 (2008) 054028 doi:10.1103/PhysRevD.77.054028

[17] B. W. Harris and J. F. Owens, “The Two cutoff phase space slicing method,” Phys. Rev. D 65 (2002) 094032 doi:10.1103/PhysRevD.65.094032

[18] H. F. Zhang, L. Yu, S. X. Zhang and L. Jia, “Global analysis of the experimental data on $\chi_c$ meson hadroproduction,” Phys. Rev. D 93 (2016) no.5, 054033 Addendum: [Phys. Rev. D 93 (2016) no.7, 079901] doi:10.1103/PhysRevD.93.054033, 10.1103/PhysRevD.93.079901

[19] E. J. Eichten and C. Quigg, “Quarkonium wave functions at the origin,” Phys. Rev. D 52 (1995) 1726

[20] Q. R. Gong, Z. Sun, H. F. Zhang and X. M. Mo, “$\eta_c$ production associated with light hadrons at the B-factories and the future Super B-factories,” Eur. Phys. J. C 76 (2016) no.9, 518 doi:10.1140/epjc/s10052-016-4360-x

[21] Y. Feng, Z. Sun and H. F. Zhang, “Is the color-octet mechanism consistent with the double $J/\psi$ production measurement at B-factories?,” Eur. Phys. J. C 77 (2017) no.4, 221 doi:10.1140/epjc/s10052-017-4770-4

[22] H. F. Zhang and Z. Sun, “Leptonic current structure and azimuthal asymmetry in deeply inelastic scattering,” Phys. Rev. D 96 (2017) no.3, 034002 doi:10.1103/PhysRevD.96.034002

[23] Z. Sun and H. F. Zhang, “QCD leading order study of the $J/\psi$ leptoproduction at HERA within the nonrelativistic QCD framework,” Eur. Phys. J. C 77 (2017) no.11, 744 doi:10.1140/epjc/s10052-017-5323-6

[24] Z. Sun and H. F. Zhang, “QCD corrections to the color-singlet $J/\psi$ production in deeply inelastic scattering at HERA,” Phys. Rev. D 96 (2017) no.9, 091502 doi:10.1103/PhysRevD.96.091502

[25] Y. Jiang and Z. Sun, “Further studies on the exclusive productions of $J/\psi + \chi_{cJ}$ ( $J = 0,1,2$ ) via $e^+e^-$ annihilation at the $B$ factories,” Eur. Phys. J. C 78 (2018) no.11, 892 doi:10.1140/epjc/s10052-018-6392-x

[26] J. X. Wang, “Progress in FDC project,” Nucl. Instrum. Meth. A 534 (2004) 241 doi:10.1016/j.nima.2004.07.094