MULTI-WAVELENGTH EMISSION FROM THE FERMI BUBBLE. III. STOCHASTIC (FERMI) RE-ACCELERATION OF RELATIVISTIC ELECTRONS EMMITED BY SNRS

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ABSTRACT

We analyze the model of stochastic re-acceleration of electrons that are emitted by supernova remnants (SNRs) in the Galactic Disk and then propagate into the Galactic Halo in order to explain the origin of nonthermal (radio and gamma-ray) emission from Fermi bubbles (FB). We assume that the energy for re-acceleration in the Halo is supplied by shocks generated by processes of star accretion onto the central black hole. Numerical simulations show that regions with strong turbulence (places for electron re-acceleration) are located high up in the Galactic Halo several kpc above the disk. The energy of the SNR electrons that reach these regions does not exceed several GeV due to synchrotron and inverse Compton energy losses. At appropriate parameters of re-acceleration these electrons can be re-accelerated up to an energy of \(10^{12}\) eV, which explains in this model the origin of the observed radio and gamma-ray emission from the FB. However, although the model gamma-ray spectrum is consistent with the Fermi results, the model radio spectrum is steeper than that observed by WMAP and Planck. If adiabatic losses due to plasma outflows from the Galactic central regions are taken into account, then the re-acceleration model nicely reproduces the Planck data points.

Key words: acceleration of particles – Galaxy: center – gamma rays: ISM

1. INTRODUCTION

Recently, Fermi discovered two giant gamma-ray bubbles (FBs) that extend nearly 10 kpc in diameter north and south of the Galactic center (GC; see Dobler et al. 2010; Su et al. 2010; for more recent analyses, see also Hooper \\& Slatyer 2013; Ackermann et al. 2014; Yang et al. 2014). These gamma-ray bubbles also correlate with the earlier discovered so-called “microwave haze” observed by the WMAP telescope as described by Finkbeiner (2004) and Dobler \\& Finkbeiner (2008) and with the large scale X-ray emission region first evidenced by analyzing the ROSAT 1.5 keV data, which clearly showed characteristics of a bipolar flow (see, e.g., Snowden et al. 1997; Bland-Hawthorn \\& Cohen 2003). A number of models were suggested to explain the origin of the FBs due to either proton–proton collisions (hadronic model, see, e.g., Crocker \\& Aharonian 2011; Crocker et al. 2014a, 2014b) or inverse Compton scattering of relativistic electrons (leptonic model, see, e.g., Su et al. 2010).

Several requirements for the leptonic models from the observations follow. First, the gamma-ray emission has a cutoff at \(E_\gamma \sim 100\) GeV (see Su et al. 2010; Yang et al. 2014). Then, in the case of an inverse Compton origin of the FB gamma-rays, the electrons have to be accelerated up to the energy

\[
E_\gamma^{\text{max}} \lesssim m_e c^2 \sqrt{\frac{3E_\gamma}{4\varepsilon}} \sim 5 \times 10^{11} \text{eV},
\]

where \(\varepsilon \approx 10^{-3}\) eV is the energy of the microwave photons.

Second, because of the very short lifetime of electrons, they have to be generated in situ in regions of emission. Cheng et al. (2011) assumed that electrons are accelerated by shocks at the FB edge, while in the model of Mertsch \\& Sarkar (2011) it was assumed that this emission is generated by electrons accelerated in the Galactic Halo by an MHD turbulence which is excited by a shock propagating into the Halo. The energy of this shock cascades into turbulence by different processes of plasma instabilities. The interaction of electrons with this turbulence leads to stochastic or second-order Fermi acceleration. Alternatively, particles can be stochastically accelerated in the FBs from interaction with a supersonic turbulence (shocks) which is excited by tidal processes in the GC (Cheng et al. 2012).

These models were investigated in a test particle approximation when the feedback reaction of the accelerated particles on the acceleration mechanism was ignored. Therefore, the number of accelerated particles is usually a free parameter of the models. However, in some cases when the sources of the accelerated electrons are known, this number can be estimated from kinetic equations that provide additional model restrictions. In the case of FBs, there are three evident sources of electrons. The electrons can be supplied by: (a) Coulomb collisions from the FB background plasma, (b) pp collisions in the Halo (secondary electrons), and (c) supernova remnants (SNRs) in the Galactic Disk. We discussed models (a) and (b) in Cheng et al. (2014, Paper I) and Cheng et al. (2015, Paper II), respectively.

In Cheng et al. (2014, Paper I) we analyzed a model of stochastic acceleration of electrons from the background plasma and showed that the problem with the model is the effect of plasma overheating (see Chernyshov et al. 2012). However, for a specified set of acceleration parameters, the in situ stochastic acceleration is able, in principle, to provide high-energy electrons needed for the observed radio and gamma-ray emissions from the FBs.

In Cheng et al. (2015, Paper II) we analyzed the hadronic model of gamma-ray emission from the FBs when gamma-rays are produced by pp collisions while the radio flux is generated by secondary electrons. Owing to the low gas density in the
Halo, the efficiency of gamma-ray production by $pp$ collisions in it is low. In addition, a very long confinement of accelerated protons in the Halo is needed for $\sim 10^{10}$ yr; see Crocker 

Aharonian 2011). We showed that in this model it is problematic to reproduce the gamma-ray and radio fluxes from the FBs, and an additional component of primary electrons is necessary. The magnetic field in this model is strongly restricted. The model reproduces the observed gamma and microwave emission from the FBs if the magnetic field is within the range of 2.5–7 $\mu$G.

Below we analyze an alternative model of stochastic acceleration of electrons in the bubbles. It is known that relativistic electrons are produced by SNRs that are distributed in the Galactic Disk. These electrons fill an extended region (several kpc above the plane) of the Galactic Halo as found from radio and gamma-ray observations (see, e.g., Berezinskii et al. 1990; Strong et al. 2011). However, due to inverse Compton and synchrotron energy losses, only electrons with relatively low energies can penetrate into regions high above the Galactic Plane, while high-energy electrons have shed most of their energy before they reach these regions. The rate of energy loss is described as $dE/dt = -\mu E^2$ where the parameter $\mu$ depends on the density of background photons and the strength of the interstellar magnetic field (for details see, e.g., Berezinskii et al. 1990). For the model of diffusion propagation of cosmic rays (CRs) presented, e.g., in Ackermann et al. (2012), the length scale of the electron mean path length for energies $E = 10^{12}$ eV is less than 1 kpc. Therefore, in order to produce the observed nonthermal emission from the FBs, these electrons should be in situ re-accelerated. An advantage of this model in comparison to the model of stochastic acceleration from the background plasma with a temperature of $\sim 2$ keV (Paper I) is in the case that the initial energy of the accelerated electron is already high, $\sim 1$ GeV, the mechanism of re-acceleration needs to increase the energy of the electrons penetrated into the Galactic Halo by only three orders of magnitude.

Charged particles in the FBs can be accelerated by scattering from MHD waves (Mertsch 

Sarkar 2011) or by interaction with supersonic turbulence (Cheng et al. 2012). These processes can be described as diffusion in the momentum space (for the formal equations of this process see, e.g., Toptygin 1985; Berezinskii et al. 1990; Bykov 

Fleishman 1992; Bykov 

Toptygin 1993). We assume that the power necessary for this turbulence can be supplied by active processes in the GC when stars are captured by the central supermassive black hole. Energy as high as $W = 10^{53} - 10^{54}$ erg can be released from one capture (see Cheng et al. 2006, 2007, 2011). We note that even more energy can be released in the GC, $W \sim 10^{56}$ erg, if a giant molecular cloud is captured by the black hole (see Zubovas 

Nayakshin 2012; Yang et al. 2012). Wardle 

Yusef-Zadeh (2014) may have found traces of the last capture of $10^5 M_\odot$ of gas which occurred $\sim 10^6$–$10^7$ yr ago. A similar conclusion was obtained from UV data by Fox et al. (2015). They found indications of a strong outflow from the GC with velocity $\geq 900$ km s$^{-1}$ that might have been caused by past activity of the GC over the last $\sim 2.5$–5 Myr. This time is comparable with the age of the FBs.

Below, we present our analysis of electron re-acceleration in the Halo. In Section 2 we use the diffusion model to calculate the flux of relativistic electrons emitted by SNRs in the Galactic Disk that reach altitudes of several kpc and the stochastic re-acceleration of these electrons up to energies of about $10^{12}$ eV. In Section 3 we calculate the fluxes of radio and gamma-ray emissions from the region of re-acceleration and compare these results with the data derived from the FB. In Section 4 we analyze the effect of convection transfers on spectra of accelerated particles and radiation. Section 5 provides a conclusion.

2. THE NUMBER AND SPECTRA OF RE-ACCELERATED ELECTRONS IN THE DIFFUSION MODEL

In order to estimate the number of re-accelerated electrons and their spectra in the FBs, the kinetic equation should include processes of particle propagation. This is because the electron sources are in the Galactic Plane while the acceleration processes take place high above the Galactic Plane. The kinetic equation for the distribution function of electrons, $f(r, z, p)$, in this case is

$$
\begin{align*}
-\nabla \cdot \left[ D(r, z, p) \nabla f - u(r, z)f \right] + \frac{1}{p^2} \frac{\partial}{\partial p} p^2 \left[ (\frac{dp}{dt} - \frac{\nabla \cdot u}{3} p) \kappa(r, z, p) \frac{\partial f}{\partial p} \right] &= Q(p, r)\delta(z),
\end{align*}
$$

where $r$ is the galactocentric radius, $z$ is the altitude above the Galactic plane, $p = E/c$ is the momentum of electrons, $u$ is the velocity of the Galactic wind, $D$ and $\kappa$ are the spatial and momentum (stochastic acceleration) diffusion coefficients, $c(dp/dt) = dE/dt = -\mu E^2$ describes the rate of electron energy losses, and $Q$ describes the spatial distribution of CR sources in the Galactic plane ($z = 0$) and their injection spectrum. All parameters of this equation are discussed in Appendix A.

To define regions of stochastic acceleration or re-acceleration in the Halo we used a hydrodynamic code to simulate the propagation of energy released in the GC in an exponential atmosphere of the Halo. We adopted the code PLUTO (Mignone et al. 2007) and ran hydrodynamic simulations with cylindrical symmetry. In the introduction we described that our idea on the formation of FBs is a series of star captures by the SMBH at the GC (e.g., Cheng et al. 2011). We simulate the energy released by the capture as an explosion at the GC. For illustrative purposes a typical result is shown in Figure 1. Kinetic energy distribution of the gas is plotted in the figure to emphasize the turbulent regions since we are going to discuss stochastic acceleration processes in the bubbles. This is the result of 100 captures, where each energy release is $10^{53}$ erg and the interval between two successive captures is $10^7$ yr. The distribution shown is the result at $\sim 10^7$ yr.

From the figure one can see that the shock propagation is mainly in the direction perpendicular to the Galactic Plane and that the morphology resembles the FB. A layer of a highly turbulent region is developed close to the envelope of the bubble. The structure is similar to that excited by the Rayleigh–Taylor instability (RTI). The development of the RTI at the shock front in an exponential atmosphere has been studied analytically by Baumgartner 

Breitschwerdt (2013).

A similar effect of the RTI has been seen in SNR shocks (see, e.g., Hester et al. 1996). Numerical calculations near an SNR shock, provided by Yang 

Liu (2013), showed that the stochastic acceleration of electrons by a magnetized turbulence that are near an SNR shock may dominate over the shock acceleration because most of the energy of the magnetic fields
may be generated via the RTI. The total energy density of the accelerated electrons in this case is of the order of the energy density of the magnetic fluctuations. We cannot exclude that a similar mechanism is effective behind the FB shock as Mertsch & Sarkar (2011) assumed.

To facilitate discussion and calculation, we schematically show the region of electron re-acceleration in the FB where \( \kappa \neq 0 \) in dark gray in Figure 2. For calculations we used the model parameters taken from the GALPROP numerical program (Ackermann et al. 2012, for details see Appendix A). The momentum diffusion coefficient is taken in the form

\[
\kappa(p) = a p^2. \tag{3}
\]

The parameter \( a \) can be presented as (see, e.g., Berezinskii et al. 1990)

\[
\alpha \sim \frac{c^2}{\nu} \tag{4}
\]

where \( \nu \), the frequency of the particle scattered by magnetic fluctuations from the wave number \( k \), is, e.g.,

\[
\nu \approx \omega_H \frac{\delta H(k)^2}{H_0^2}. \tag{5}
\]

Here \( \delta H(k) \) is the strength of the magnetic fluctuation with the wave number \( k \), \( H_0 \) is the large-scale magnetic field, and \( \omega_H = eH_0/m_e c \).

In the phenomenological model, which we investigate below, the goal is to estimate the value of \( \alpha \) from observational data.

In the simplest case of pure diffusion propagation of CRs when the convection terms are neglected \( (\mu = 0) \), the number of electrons reaching the re-acceleration region can be calculated from

\[
-\nabla \cdot D(r, z, p) \nabla f + \frac{1}{p^2} \frac{\partial}{\partial p} p^2 f \times \left[ \frac{dp}{dr} - \kappa(r, z, p) \frac{df}{dp} \right] = Q(p, r) \delta(z). \tag{6}
\]

The boundary conditions on the surface of the re-acceleration region are rather questionable. The particle may freely escape from the boundary, as is assumed for the Galactic Halo (see e.g., Berezinskii et al. 1990). On the contrary, Yang et al. (2012) assumed that the particle diffusion across the bubble edge is strongly suppressed, or that there are “magnetic walls” at the edge, as proposed by Jones et al. (2012). Both effects prevent particle propagation through the bubble surface.

Below, we accept the continuity of the particle density and flux across the boundary as the boundary conditions for Equation (6). From Equation (6) we numerically calculated the spectrum of SNR electrons that reach the altitude \( z = 5 \) kpc when the term of re-acceleration is neglected \( (\kappa(p) = 0) \). The spectrum is shown in Figure 3 by the solid line.

The density of high-energy electrons needed for the observed gamma-ray flux is shown by the shaded region in Figure 3. The necessary number of high-energy electrons can be provided by processes of re-acceleration of SNR electrons in the FBs. The spectrum of SNR electrons re-accelerated in the FBs is calculated from Equation (6) when the acceleration term is included \( (\kappa(p) \neq 0) \). However, with acceleration in the form of Equation (3), the spectrum of accelerated particles is too hard \( (f(p) \propto p^{-3}) \). This is shown schematically in Figure 3 by the dash–dotted line, i.e., our numerical calculations show that...
too many high-energy electrons are produced in the re-acceleration region.

The spectrum of energetic particles can be steepened by processes of particle escape from the acceleration region (see, e.g., Cheng et al. 2012). Indeed, the momentum spectrum of accelerated particles is a power law, \( f(p) \propto p^{-\delta} \), with the spectral index \( \delta \) given by

\[
\delta = \frac{3}{2} + \frac{9}{4} + \frac{\tau_{\text{esc}}}{\tau_{\text{acc}}},
\]

where the acceleration time is \( \tau_{\text{acc}} \approx \alpha^{-1} \) and escape time is \( \tau_{\text{esc}} \approx \Delta \tau_{\text{b}}^2/4D_b \). Here, \( \Delta \tau_{\text{b}} \) is the thickness of the re-acceleration region and \( D_b \) is the spatial diffusion coefficient in the re-acceleration region given by

\[
D_b(p) = 4v^2p^2/(6\kappa(p)).
\]

Here, \( v \) is the velocity of turbulent motion. Escape processes make the spectrum steeper and thus decrease the number of emitted electrons.

The spectrum of re-accelerated electrons in the bubbles that was calculated for the model parameters and derived from the observed FB gamma-ray emission (see Section 3) is shown in Figure 3 by the thin dotted line. The ratio \( \tau_{\text{acc}}/\tau_{\text{esc}} \approx 8 \) is shown in next section.

3. GAMMA-RAY AND RADIO EMISSION FROM THE FB

In order to calculate the spectrum of accelerated particles in the bubble, \( f_{\text{b}} \), from Equation (6), we calculated the total distribution function of electrons, \( f(r, z, p) \), with the acceleration term \((\kappa \neq 0)\). We then calculated the distribution function, \( f_{\text{b}}(r, z, p) \), when the acceleration term was neglected \((\kappa = 0)\). We defined the function of the excess of electrons, \( f_{\text{p}} \), due to the acceleration, as \( f_{\text{p}} = f - f_{\text{b}} \). This procedure is similar to the subtraction of the FB gamma-ray flux from the total Galactic emission (see Su et al. 2010; Yang et al. 2014).

Then, the FB gamma-ray intensity in the direction of the galactic coordinates \((\ell, b)\) is

\[
I_{\gamma}(E, \ell, b) = \frac{c}{4\pi} \int s(\ell, b) \, ds \int n(\epsilon, r) \, d\epsilon \times \int_p p^2 f_{\text{b}}(r, p) \left( \frac{d^2\sigma}{d\epsilon \, dp} \right)_{\text{KN}} \, dp,
\]

where \( s(\ell, b) \) is the line of sight in the direction \((\ell, b)\) and \( (d^2\sigma/d\epsilon \, dp)_{\text{KN}} \) is the Klein–Nishina cross-section (see Blumenthal & Gould 1970). We note that the re-accelerated electrons not only fill the region of acceleration, but escape into the surrounding medium of the Halo. Therefore, the total distance of emission is \( l = \Delta r + \sqrt{Dr} \), where \( r \) is the lifetime of emitting electrons.

Our calculations show that in order to reproduce the FB gamma-ray spectrum the following model parameters are required: the spectral index of accelerated electrons \( \delta = 4.8 \), the thickness of the re-acceleration region \( \Delta \tau_{\text{b}} = 3 \) pc, and the characteristic time of acceleration \( \tau = 2 \times 10^{-13} \) s\(^{-1}\).

The calculated FB gamma-ray spectrum and the data from Ackermann et al. (2014) are shown in the top and middle rows of the left panels of Figure 4. As one can see, there is no complete agreement between the calculation and the data.

This model has more serious difficulties in describing the microwave emission from the FBs. The calculated spectrum is shown in Figure 4 (bottom row, left column). As one can see, for the parameters derived from the gamma-ray data, the model gives radio spectrum steeper than \( \nu^{-0.51} \), which follows from the measurements of Planck (see Figure 7 in Ade et al. 2013).

These shortcomings can be alleviated when CR convection propagation is included. Below, we present a model of FBs that takes convection into account.

4. EFFECT OF CONVECTION ON THE RE-ACCELERATION PROCESS

The effect of convective transfer (Galactic wind) may also be essential in the Galaxy, as it follows from theoretical treatments (see Breitschwerdt et al. 1991, 2002) as well as from interpretation of observational data (see Bloemen et al. 1993).

Analysis of the radio emission from the central Galactic region provided by Crocker et al. (2011) and Carretti et al. (2013) might indicate a very strong wind, with a wind velocity estimated up to \( 1100 \) km s\(^{-1}\). Recent three-dimensional hydrodynamic simulations of Mou et al. (2014) showed the existence of strong winds in the FBs caused by the past accretion in Sgr A*. They concluded that the wind is collimated by the Central Molecular Zone toward the Galactic poles, i.e., perpendicular to the Galactic Plane. Thus, we expect that the effect of wind transfer in the GC may not be negligible.

If the wind velocity is spatially non uniform in the Galactic Halo, then CRs lose their energy through adiabatic losses. Indeed, the MHD numerical calculations of Breitschwerdt et al. (1991) showed that the velocity of the Galactic wind increases almost linearly with the altitude \( z \). Bloemen et al. (1993)
Figure 4. Spectra of gamma-ray and microwave emission produced by re-accelerated electrons in the FBs calculated for the diffusion model (left column) and for the case when the effect of convection is included (right column). The top and middle rows are the gamma-ray emission for the latitude range $20^\circ \leq |b| \leq 40^\circ$ and $40^\circ \leq |b| \leq 60^\circ$, respectively. The bottom row presents the spectrum of microwave emission from the bubbles. Data points were taken from Ackermann et al. (2014) for the gamma-ray and from Ade et al. (2013) for the microwave spectrum.
derived the value of the wind gradient from the CR chemical composition. For the wind velocity in the form \( u(z) = 3v_0z \), they estimated the gradient value \( v_0 \approx 10^{-15} s^{-1} \).

Formal solutions for the one-dimensional kinetic equation with the convection term are presented in Appendices \( B \) and \( C \). To demonstrate this effect we solved Equation (2) with the both re-acceleration and convection terms. According to Crocker et al. (2011) we assume that the wind is blowing mainly from the central part of the Galactic disk, i.e., from the bubble region. Therefore, we took the wind velocity in the following simplified form

\[
u_z = 3v_0z/r_0 - r,
\]

where \( \theta(r) \) is the Heavyside function and \( r_0 = 3 \) kpc.

To define the spectrum of accelerated electrons we estimated the number of electrons that can reach an altitude of several kpc (shown in Figure 3 by the thick dash–dotted line). We then calculated the spectrum of electrons re-accelerated in the bubbles. The effect of adiabatic losses shift the spectrum as a whole into the range of smaller energies (see the solution in Appendix \( C \) and an example in Figure 8), and that makes the spectrum of re-accelerated particles flatter than in the diffusion model (see the thin dashed line in Figure 3).

In the case of a leptonic origin of the FB radio and gamma-ray emission, which are generated by synchrotron and inverse Compton energy losses of electrons, respectively, the necessary magnetic field strength in the FBs can be estimated from the simplified equations for these processes presented in Ginzburg (1979). For a power-law spectrum of relativistic electrons \( N(E) = N_0 E^{-\gamma} \), the magnetic field strength is

\[
H \approx \frac{1}{E_T} \left[ \frac{I_e}{I_T} \frac{c \omega_{ph} \sigma_T}{2a(\gamma_e)} \frac{mc^2}{e^3} \right] \frac{\mu G}{r_0^2} (3/4e)^{(\gamma_e-3)/2} \left( \frac{4\pi mc^2 \omega_{ph}}{3e} \right)^{(\gamma_e-1)/2} \frac{E}{E_{\gamma}^{1/2}}.
\]

This value is of the order of one derived below from more accurate numerical calculations.

The procedure of calculating the spectrum of accelerated particles \( f_0 \) is the same as described in Section 2.

The calculated spectra of gamma-ray emission at different latitudes and radio emission from the bubbles are shown in Figure 4 (right column). The best agreement with the data is achieved for \( v_0 = 10^{-15} s^{-1} \). The magnetic field strength is \( H = 3 \mu G \). The parameters of the acceleration are the thickness of the re-acceleration region \( \Delta r_0 = 60 \) pc, and \( a = 2 \times 10^{-14} s^{-1} \).

We note, however, that as follows from Equation (11), it is problematic to reproduce the gamma-ray spectrum in the leptonic model if we accept the magnetic field strength in the bubbles to be 15 \( \mu G \), as derived by Carretti et al. (2013). The density of relativistic electrons estimated from the radio data is too low to generate enough gamma-ray photons by inverse Compton in the FBs.

At low Galactic latitudes the contribution of FBs should decrease, especially in the high-energy range, because of the energy losses from electrons. On the other hand, at these latitudes the contribution from electrons emitted by SNRs in the Disk increases. This effect is shown in Figure 5 where we present the gamma-ray spectrum in the direction of low \( (10^\circ < |b| < 20^\circ) \) and high \( (40^\circ < |b| < 60^\circ) \) Galactic latitudes. In this figure we show the IC component of the gamma-ray emission. Here, \( a(\gamma_e) = 0.1 \), \( \sigma_T \) is the Thomson cross-section, \( I_e \) and \( I_T \) are intensities of radio and gamma-ray emission from the FBs, and \( c \) and \( \omega_{ph} \) are the energy and the energy density of background photons in the FBs.

For the FB radio intensity \( I_e = 0.52 \) kJy sr\(^{-1}\) at the frequency \( \nu = 23\) GHz and gamma-ray intensity \( I_T = 4 \times 10^{-9} \) ph cm\(^{-2}\) s\(^{-1}\) GeV\(^{-1}\) sr\(^{-1}\) at \( E_R = 10 \) GeV, which is produced by scattering of optical or IR photons (see Cheng et al. 2011) whose energy density in the Halo is about \( w = 0.2 \) eV cm\(^{-3}\), for the electron spectral index \( \gamma_e = 2 \) we obtain a magnetic field strength of

\[
H \approx 5 \mu G.
\]
Nevertheless, the model qualitatively depicted in Figure 2 is quite schematic, we do not expect gamma-ray intensity. As the geometry of the acceleration energy, respectively, the kinetic energy, distribution function, and (et al. p where 90° ≤ b ≤ 50° latitude range Figure 6. The Astrophysical Journal, 2014)

Figure 6. Longitudinal distribution of the FB gamma-ray intensity for the latitude range 40° ≤ b ≤ 50°. Data points were taken from Ackermann et al. (2014).

ray emission (solid lines), produced only by the FBs, and the total IC emission (FB+SNR, dashed lines), produced by both FBs and SNR electrons. As one can see, in the framework of the model the contribution of the FBs to the total gamma-ray flux is significant at high enough latitudes. At low latitudes, the IC emission from SNR electrons is dominant and, if we also take into account the gamma-ray component from proton–proton collisions, which is very intensive at low latitudes, we conclude that it is almost impossible to subtract the FB component from the total gamma-ray flux in these directions.

The total power supplied by external sources (Fermi acceleration) needed to produce high-energy electrons in the FBs can be estimated from (see Chernyshov et al. 2012)

\[ W = -\int_0^\infty E \frac{\partial \kappa}{\partial\epsilon} \left( \frac{p^2 \kappa(p)}{\partial p} \right) dp, \]  

where \( p \) and \( \epsilon \) are the particle momentum and particle kinetic energy, respectively, the kinetic energy, \( f(p) \) is the particle distribution function, and \( \kappa \) is the diffusion coefficient of the Fermi acceleration.

We estimate \( W \) from the observed FB gamma-ray and microwave fluxes. We take into account all processes of electron energy losses as well as their escape from the Galaxy (see Appendix A for detail). The power estimated numerically is about \( W \sim 2 \times 10^{38} \text{ erg s}^{-1} \).

This estimate is a lower limit for \( W \) because part of the energy released in the GC is transformed into accelerated protons and the plasma that is heating in the Halo. As shown in Cheng et al. (2011), processes of star accretion onto the central black hole can provide, on average, about \( 10^{41} - 10^{42} \text{ erg s}^{-1} \). Thus tidal accretion supplies enough energy for particle acceleration in the GC.

Finally, we address the question of whether or not the model is able to reproduce sharp edges of the bubbles. As an example in Figure 6 we presented the longitudinal distribution of gamma-ray intensity. As the geometry of the acceleration depicted in Figure 2 is quite schematic, we do not expect complete coincidence between the calculations and the data. Nevertheless, the model qualitatively reproduces the effect of the sharp edges and neither one-dimensional diffusion nor magnetic walls at the bubbles edges are required (in contrast to Jones et al. 2012; Yang et al. 2012).

5. CONCLUSION

In summary, we studied a leptonic model for the gamma-ray and microwave emissions from the FB. The source of electrons is the SNR in the Galactic Disk. The electrons propagate into the Galactic Halo and are re-accelerated in the high turbulence regions in the FB which are located several kpc about the Galactic Disk. We have two goals to achieve; the first is to obtain the density needed for the observed gamma-ray flux from the FB, and the second is to obtain the Planck microwave spectrum from the FB. We presented models of different levels of sophistication.

To meet the first goal, the high-energy electron density should be within the shaded area in Figure 3. We introduced the pure diffusion propagation model where we calculated the spectrum of electrons (from SNRs in the disk) penetrating into the region of re-acceleration and calculated the spectrum of re-accelerated electrons. The re-acceleration process generates very hard spectra \( (E^{-1}) \). Therefore the electron density we obtained was too high (the thin dash–dotted line above the shaded area in Figure 3). We then introduced a fast particle escape from the re-acceleration region. For small enough escape times, the spectrum (thin dotted line in Figure 3) passes through the shaded area in Figure 3.

Although we found the density of electrons needed for the FB gamma-ray flux, the spectrum was too steep to produce the Planck microwave spectrum from the FB \((\nu^{0.51})\), and we fail to complete the second goal. To remedy this situation, we then included the wind transport and adiabatic losses in the diffusion with escape model. The adiabatic losses shift the spectrum to a lower energy range. Adjusting the parameters of particle escape and the wind in the Halo, we can obtain a flatter spectrum that satisfies both goals (gamma-ray flux and Planck microwave spectrum).

The conclusions of the Paper can be itemized as follows.

1. Numerical calculations showed that the energy of SNR electrons penetrating into the upper Halo region is not high enough to generate the FB gamma-ray emission using inverse Comptonization. Therefore, further re-acceleration up to energies of about \( 10^{12} \text{ eV} \) is needed to generate the gamma-ray flux.

2. Re-acceleration (without convection) generates electron spectra that are too steep. Therefore, this model is unable to correctly reproduce the gamma-ray and microwave emissions from the FBs.

3. There are indications of an intensive outflow of plasma from the GC. The effect of the wind leads to adiabatic losses of CRs. We expect that the adiabatic losses make the spectrum of electrons in the acceleration region harder than for the case without convection. Our calculations show that the gamma-ray and radio emissions of the re-accelerated electrons nicely reproduce the Fermi-LAT and Planck data points in this case.

4. In the re-acceleration model with convection, the gamma-ray flux produced by the FBs is more significant at high Galactic latitudes than at low latitudes; see Figure 5. At low latitudes, the IC emission from the FB’s electrons is dominated by that from the Galactic disk SNR electrons.
Additionally, gamma-rays can be produced by YY collisions (hadronic model), but this process is not effective in the Galactic Halo where the gas and CR densities are low. As a whole, at low latitudes the contribution of FBs to gamma-ray flux is subordinate to other processes. We should point out that gamma-ray production by the hadronic model (e.g., YY collisions) could be distinguished from the leptonic model (e.g., inverse Compton process of electrons) because the YY collisions not only produce gamma-rays, but also neutrinos (see e.g., Crocker & Aharonian 2011; Lunardini & Razzaque 2012; Taylor et al. 2014).

5. An advantage of the re-acceleration model in comparison with that of acceleration from the background plasma is that in the first case the energy of the SNR electrons should be increased in the FBs by only three orders of magnitude, while in the second case the electrons are accelerated from the thermal plasma with a temperature of about 2 keV, i.e., an increase of nine orders of magnitude is needed.

6. We compared the efficiency of electron acceleration for the two models: acceleration from the background plasma presented in Paper I and the re-acceleration presented in this work. To do this, we included both processes into the kinetic equation for electrons, Equation (6). Our numerical calculations showed that for the same momentum diffusion (acceleration term) and time of escape, the re-acceleration mechanism is more effective for the production of high-energy electrons in the FBs (see Figure 7).

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APPENDIX A

PARAMETERS OF THE CR KINETIC EQUATION

The distribution function of electrons, \( f(r,z,p) \), is derived from the kinetic equation (see also Equation (2))

\[
\begin{align*}
-\nabla \cdot [D(r,z,p)\nabla f - u(r,z)f] \\
&+ \frac{1}{\beta^2} \frac{\partial}{\partial \beta} \left[ \left( \frac{dp}{dt} - \frac{\nabla \cdot u}{3} \right) f - \kappa(r,z,p) \frac{\partial f}{\partial \beta} \right] \\
&= Q(p,r)\delta(z).
\end{align*}
\]

The relation between momentum and energy loss is

\[
\frac{dp}{dt} = \frac{1}{\beta} \frac{dE}{dt},
\]

where \( \beta = \nu/c \) is the dimensionless velocity of the electron and the rate of electron energy loss can be presented as (see e.g., Ginzburg 1979):

\[
\frac{dE}{dt} = \left( \frac{dE}{dt} \right)_{cn} + \left( \frac{dE}{dt} \right)_{ci} + \left( \frac{dE}{dt} \right)_{br} + \left( \frac{dE}{dt} \right)_{sc},
\]

where the rate of Coulomb losses in a neutral medium and in a fully ionized plasma are, respectively,

\[
\left( \frac{dE}{dt} \right)_{cn} = 7.62 \times 10^{-18} \text{ GeV s}^{-1} \times n_{H}\beta^{-1}\left[ \log(\gamma - 1)(\gamma^2 - 1) + 20.5 \right],
\]

\[
\left( \frac{dE}{dt} \right)_{ci} = 7.62 \times 10^{-18} \text{ GeV s}^{-1} \times n_{H}\beta^{-1}\left[ \log(\gamma/n_{H}) + 73.6 \right].
\]

Here \( \gamma = E/(mc^2) = 1/\sqrt{1 - \beta^2} \) is the Lorentz factor and \( n_{H} \) and \( n_{H}\) are the densities of neutral and ionized hydrogen in the disk and in the halo.

The bremsstrahlung losses in the interstellar medium are

\[
\left( \frac{dE}{dt} \right)_{br} = 5.1 \times 10^{-19} \text{ GeV s}^{-1} \times (n_{H} + n_{H})\gamma,
\]

and the synchrotron and inverse Compton losses (in the Compton limit) can be presented as

\[
\left( \frac{dE}{dt} \right)_{sc} = 1.6 \times 10^{-11} \text{ GeV s}^{-1} \times (H^2/8\pi + w_0)\gamma^2\beta^2.
\]
The distribution of the magnetic field in the Galaxy was taken in the form

$$H = 6 \mu G \times \exp \left[ -\frac{z}{5 \text{kpc}} - \frac{r}{20 \text{kpc}} \right]$$  \hspace{1cm} (21)

while the densities of neutral $n_{\text{HI}}$, ionized $n_{\text{HII}}$, and the density of the interstellar radiation field $w_{\text{sf}}$ were taken from the GALPROP code of Ackermann et al. (2012).

The distribution of SNRs in the disk was taken in the form

$$Q(p, r) = Q(p) \times \left( \frac{r}{8 \text{kpc}} \right)^{1.2} \exp \left( -3.22 \frac{r}{8 \text{kpc}} \right)$$  \hspace{1cm} (22)

As follows from the analyses of radio and gamma-ray emissions, the spectra of CR electrons and protons have depletion at energies below 1 GeV (see e.g., Strong et al. 2011; Neronov et al. 2012; Dermer et al. 2013). Here, we take the source spectrum of electrons as a broken power law from Ackermann et al. (2012)

$$Q(p) \propto \begin{cases} p^{-3.6}, & \text{if } pc \leq 2.2 \text{ GeV} \\ p^{-4.4}, & \text{if } 2.2 \text{ GeV} < pc \leq 2.1 \text{ TeV} \\ p^{-6}, & \text{if } pc > 2.1 \text{ TeV} \end{cases}$$  \hspace{1cm} (23)

The spatial diffusion coefficients in the Galaxy are assumed to be a function only of the momentum,

$$D(p) = D_0 \times \beta \left( \frac{pc}{4 \text{GeV}} \right)^{0.33},$$  \hspace{1cm} (24)

where $D_0 = 9 \times 10^{28} \text{ cm}^2 \text{s}^{-1}$ for an 8 kpc Halo.

Parameters inside the acceleration region were derived from the FB gamma-ray and radio emission. The momentum diffusion coefficient has the form

$$\kappa_b(p) = \frac{\alpha_b}{\beta} p^2,$$  \hspace{1cm} (25)

where the acceleration rate $\alpha_b$ is estimated from calculations and the spatial diffusion coefficient is

$$D_p(p) = \frac{4 \nu^2 p^2}{6 \kappa_b(p)},$$  \hspace{1cm} (26)

where $\nu$ is the velocity of the turbulent motions that provide stochastic acceleration. In Ackermann et al. (2012) this acceleration is due to particle interaction with Alfvénic waves when $\nu = \nu_A = H/\sqrt{3 \pi \rho}$.

The boundary conditions were taken in the form

$$\begin{align*}
\frac{df}{dr} &= 0, \text{ at } r = 0 \\
\frac{df}{dz} &= 0, \text{ at } z = 0 \\
f &= 0, \text{ at the boundary of the Galactic halo. } \hspace{1cm} (27)
\end{align*}$$

At the boundary of the acceleration region, continuity of the particle function and flux are enforced.

APPENDIX B

RE-ACCELERATION IN A DIVERGENT WIND FLOW

In this appendix we present a solution to a one-dimensional re-acceleration problem in a divergent flow. The governing equation is

$$\frac{df}{dt} + u \frac{df}{dz} - \frac{\partial}{\partial z} \left( D \frac{df}{dz} \right) - \left( \frac{\partial u}{\partial z} \right) \frac{1}{3} \frac{df}{f} - \frac{1}{2} \frac{\partial}{\partial z} \left( \frac{\rho^2 \kappa f}{\partial z} \right) = Q.$$  \hspace{1cm} (28)

We consider a specific case where $u = u_0 \frac{z}{H}, D = D_0, \kappa = \kappa_0 p^2 = \sigma^2 \frac{p^2}{\beta D_0},$ and $Q = -f/\tau$, where $u_0, D_0, \kappa_0, \sigma,$ and $\tau$ are constants. $H$ is the characteristic length of the system or flow. We seek a steady state solution with boundary conditions $f = s(p)$ at $z = 0$ and $f = 0$ as $z \to \infty$. Introducing the dimensionless quantities

$$\tilde{u}_0 = \frac{H u_0}{D_0}, \quad \bar{\sigma} = \frac{H \sigma}{D_0}, \quad \bar{\tau} = \frac{D_0 \tau}{H^2}, \quad \bar{\xi} = \frac{z}{H},$$

$$\tilde{p} = \frac{p}{p_0}, \quad \eta = \int \frac{3 \, d\tilde{p}}{\bar{\sigma} \bar{\rho}} = \frac{3}{\bar{\sigma}} \log \frac{\tilde{p}}{p_0},$$  \hspace{1cm} (29)

and

$$f(\xi, \eta) = g(\xi, \eta) \exp \left[ -\frac{3}{2 \bar{\sigma}^2} \left( \bar{\sigma}^2 + \tilde{u}_0 \right) \right] = g(\xi, \eta) \exp(-\bar{\sigma}^2),$$  \hspace{1cm} (30)

the steady state of Equation (28) becomes

$$\frac{\partial^2 f}{\partial \xi^2} - \tilde{u}_0 \bar{\sigma} \frac{\partial f}{\partial \xi} + \frac{\partial^2 f}{\partial \eta^2} + 2 \nu \frac{\partial f}{\partial \eta} = \frac{f}{\bar{\tau}},$$  \hspace{1cm} (31)

and the boundary conditions for $g$ become $g = s(\tilde{p}) \tilde{p}^{\nu_0/\beta} = S(\eta) e^{\nu_0 \eta}$ at $\xi = 0$ and $g = 0$ as $\xi \to \infty$. We can solve Equation (31) using a Fourier transform with respect to $\eta$. We obtain an ODE

$$\frac{\partial^2 \tilde{g}}{\partial \xi^2} - \tilde{u}_0 \bar{\sigma} \frac{\partial \tilde{g}}{\partial \xi} - \left( \nu^2 + \frac{1}{\bar{\tau}} \right) \tilde{g} = 0,$$  \hspace{1cm} (32)

subject to boundary conditions $\tilde{g} = S(\omega + i \nu)$ at $\xi = 0$ and $\tilde{g} = 0$ as $\xi \to \infty$. Here the Fourier transform pairs are

$$g(\xi, \eta) = \int_{-\infty}^{\infty} \tilde{g}(\xi, \omega) e^{i \omega \eta} d\omega,$$  \hspace{1cm} (33)

and

$$S(\eta) = \int_{-\infty}^{\infty} S(\omega) e^{i \omega \eta} d\omega.$$  \hspace{1cm} (34)

The solution of $g(\xi, \omega)$ can be written in terms of a parabolic cylinder function

$$g(\xi, \omega) = \frac{2^{-n/2}}{\sqrt{\pi}} \Gamma \left( \frac{1 - n}{2} \right) S(\omega + i \nu) \times \exp \left( \frac{\tilde{u}_0 \xi^2}{4} \right) D_n \left( \sqrt{\tilde{u}_0} \xi \right),$$  \hspace{1cm} (35)
where $\Gamma(x)$ is the gamma function, $D_0(x)$ is the parabolic cylinder function, and

$$n = -\frac{1}{\bar{u}_0} \left( \bar{u}_0 \frac{\bar{u}_0 \bar{x}^2}{4} + \frac{\bar{u}_0^2}{4 \bar{x}^2} + \frac{1}{\bar{x}} + \alpha^2 \right).$$

(36)

Defining

$$\bar{G}(\xi, \omega) = \frac{2^{-n/2}}{\sqrt{\pi}} \Gamma \left( \frac{1 - n}{2} \right) \exp \left( \frac{\bar{u}_0 \bar{x}^2}{4} \right) D_0 \left( \sqrt{\bar{u}_0 \bar{x}} \right),$$

(37)

and the Fourier transform pair

$$\bar{G}(\xi, \eta) = \int_{-\infty}^{\infty} \bar{G}(\xi, \omega) e^{i\mu \omega} d\omega,$$

$$\bar{G}(\xi, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{G}(\xi, \eta) e^{-i\omega \eta} d\eta.$$

(38)

The solution can then be written as

$$g(\xi, \eta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\eta') e^{i\mu(\eta - \eta')} d\eta',$$

(39)

and

$$f(\xi, \eta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\eta') e^{-i\eta(\eta - \eta')} G(\xi, \eta - \eta') d\eta'.$$

(40)

APPENDIX C

ANALYTICAL SOLUTION OF THE ONE-DIMENSIONAL WIND EQUATION

The one-dimensional equation for relativistic electrons can be presented in the form

$$-D_0 E \frac{\partial^2 N}{\partial z^2} + 3v_0 \frac{\partial N}{\partial z} = \frac{\partial}{\partial E} \left( \mu E^2 + v_0 E \right) N = Q_0 E^{-\gamma} \delta(z).$$

(41)

Here $D_0 E^\mu$ is the coefficient of electron diffusion in the $z$ direction, $V(z) = 3v_0 z$ is the wind velocity in the $z$ direction, $dE/dt = -\mu E^2$ is the rate of synchrotron and inverse Compton energy losses, and $Q_0 E^{-\gamma} \delta(z)$ is the source function of electrons in the Galactic disk.

Introducing variables

$$\tau = \frac{\mu}{v_0 E}, \quad \tilde{z} = \frac{z}{z_d}, \quad z_d = \sqrt{\frac{D_0}{v_0} \left( \frac{v_0}{\mu} \right)^\mu},$$

$$t = \int \frac{dt}{\tau^\alpha (1 + \tau)^\gamma}, \quad \eta = \frac{\tilde{z}}{(1 + \tau)^\gamma},$$

(42)

and the function

$$K = \frac{z_d v_0}{Q_0} \left( \frac{v_0}{\mu} \right)^\gamma \frac{(1 + \tau)^\delta}{\tau^2} N,$$

(43)

we obtain the standard one-dimensional diffusion equation

$$\frac{\partial}{\partial t} K - \frac{\partial^2}{\partial \eta^2} K = F(t) \delta(\eta),$$

(44)

where $F(t) = \tau^\mu (1 + \tau)^7$ with $\tau$ expressed as a function of $t$ by inverting $t = \int dt [\tau^\mu (1 + \tau)^7]$. The Green function of Equation (44) can be found in Morse & Feshbach (1953).

The solution of this equation can be obtained in the analytical form

$$N = \frac{Q_0 h}{\sqrt{D_0 v_0}} \frac{E^{-\left[\frac{\gamma + \alpha}{2}\right]}}{(1 + \mu E/v_0)^{\frac{\gamma}{2}}} \int_0^1 \frac{dx}{x^\alpha (1 + \mu E/v_0)^{\frac{\gamma}{2}}} \exp \left[ \frac{z}{\Sigma} \left( \frac{v_0}{\mu} \right) \frac{1}{(1 + \mu E/v_0)^{\frac{\gamma}{2}}} \right].$$

(45)

where

$$\Sigma = \int_0^1 \frac{dx}{x^\alpha (\mu E/v_0 + x)^{\frac{\gamma}{2}}}.$$

(46)

The effect of adiabatic losses is a shift of the spectrum into the region of low energies. In Figure 8 we presented the spectrum of electrons at the altitude $z = 6.6$ kpc for the velocity gradients $v_0 = 2 \times 10^{-15}$ s$^{-1}$ (upper line) and $v_0 = 5 \times 10^{-14}$ s$^{-1}$ (bottom line) for the velocity gradients. The production spectrum of electrons by SNRs was taken from Strong et al. (2011) as

$$Q(E) = \left( \frac{4 \text{ GeV}}{E} \right)^{-1.6} \theta(4 \text{ GeV} - E) + \left( \frac{4 \text{ GeV}}{E} \right)^{-2.5} \theta(E - 4 \text{ GeV}).$$

(47)

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Figure 8. Illustration of adiabatic loss in divergent flows. Spectra of electrons at the altitude $z = 6.6$ kpc for the velocity gradients $v_0 = 2 \times 10^{-15}$ s$^{-1}$ (upper line) and $v_0 = 5 \times 10^{-14}$ s$^{-1}$ (bottom line).
