Push Recovery of a Humanoid Robot Based on Model Predictive Control and Capture Point

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Abstract—The three bio-inspired strategies that have been used for balance recovery of biped robots are the ankle, hip and stepping Strategies. However, there are several cases for a biped robot where stepping is not possible, e.g. when the available contact surfaces are limited. In this situation, the balance recovery by modulating the angular momentum of the upper body (Hip-strategy) or the Zero Moment Point (ZMP) (Ankle strategy) is essential. In this paper, a single Model Predictive Control (MPC) scheme is employed for controlling the Capture Point (CP) to a desired position by modulating both the ZMP and the Centroidal Moment Pivot (CMP). The goal of the proposed controller is to control the CP, employing the CMP when the CP is out of the support polygon, and/or the ZMP when the CP is inside the support polygon. The proposed algorithm is implemented on an abstract model of the SURENA III humanoid robot. Obtained results show the effectiveness of the proposed approach in the presence of severe pushes, even when the support polygon is shrunken to a point or a line.

I. INTRODUCTION

The main destination of humanoid robots research is realizing a robot that is able to work in real environments. Because of unstable nature of the biped robots, the ability of recovering from unexpected external disturbances is essential. In recent years, several attempts have been made by researchers to generate robust locomotion of biped robots (1–7). A common criterion for ensuring dynamic balance during walking is to maintain the Zero Moment Point (ZMP) or the Center of Pressure (CoP) within the support polygon of the contact points. The main approaches that have been used for balancing and walking of humanoid robots in the presence of disturbances are based on the Model Predictive Control (MPC) or controlling the Capture Point (CP) [1]–[10].

Kajita et al. [1] introduced preview control of ZMP and paved a way for robust walking pattern generation. This method was expressed more generally as an MPC problem by Wieber et al. [2]. To increase the robustness of the gaits, the MPC formulation in [2] has been modified to adapt the step locations [4], [5]. However, the upper-body angular momentum has not been employed in these works. As a result, Aftab et. al [6] proposed a single MPC that uses all the ankle, hip, and stepping strategies for balance recovery of humanoid robots. In all of these works, the CoM has been considered as the state of the system. However, relating the problem in this way constrains both divergent and convergent components of motion [8].

Pratt et al. [3], [7] introduced the CP by splitting the Center of Mass (CoM) dynamics into stable and unstable components. The state variable related to the unstable part of the CoM dynamics has been named the Capture Point (CP). The CP specifies when and where a humanoid must step in order to maintain balance, however it requires a controller for stabilizing unstable nature of dynamic of the CP. To this end, Englsberger et al. [8], [9] developed a controller for CP tracking without using the effect of upper-body angular momentum (CMP modulating) and by guiding the CP only by CoP modulation. The effect of upper-body angular momentum plays a key role for balance recovery especially in the situation that stepping is not possible or contact surface is small [10]–[12].

In this paper, in order to utilize the usefulness of the two mentioned approaches, the CP concept is used in an MPC. To do so, an effective MPC scheme is developed for push recovery by manipulating the CoP when the CP is within the support polygon, and employing the CMP modulation when the CP is out of the support polygon. The main goal of this controller is to maintain the CP, CMP and CoP on the center of support polygon. The proposed algorithm is capable of dealing with severe pushes while the contact surface is a line or a point. The remainder of this paper is organized as follows. The CoM dynamics, and the CP formulations are reviewed in Sec II. The proposed push recovery controller is
presented in Sec III. In IV, the obtained simulations results are presented and discussed. Finally, Section V concludes the findings.

II. CENTER OF MASS DYNAMICS

A. Linear Inverted Pendulum

Using the full nonlinear dynamics of a humanoid robot for gait planning makes the corresponding optimization problem non-convex [13]. However, the dynamics of a biped robot can be approximated by the Linear Inverted Pendulum Model (LIPM) [14]. This model is a good dynamic approximation of a biped robot, particularly for the standing posture. The LIPM uses the following assumptions [14]:

- The rate of change of angular momentum is zero,
- The CoM height remains constant

Based on the mentioned assumptions and Fig.1, the equation of Motion of the LIPM can be expressed as follows:

\[
\ddot{x}_c = \omega_n^2 (x_c - p_x) \tag{1}
\]

where \( m \) is the robot mass, the CoM position is given by \( P_c = [x_c, y_c, z_c]^T \), \( P_{cmp} = [p_x, p_y, 0]^T \) is the position of the ZMP and \( \omega_n = \sqrt{(g/z_c)} \) is the natural frequency of the LIPM. The Ground Reaction Force (GRF) intersects with the CoM because the base joint of the pendulum is torque-free and the rate of change of angular momentum is zero. As shown in Fig.1, \( F_z \) is the vertical component of the GRF. It compensates the gravitational force \( F_z \) acting on the CoM. The inertial force \( F_i = mx\dot{x} \) completes the equilibrium of forces in \( P_c \). The equation of motion in frontal plane and sagittal plane is independent. By adding the external force (Disturbance) to the dynamics of the LIPM, the equations of motion can be modified and written as:

\[
\begin{align*}
\ddot{x}_c &= \omega_n^2 (x_c - p_x) + \frac{F_{ext,x}}{m} \\
\ddot{y}_c &= \omega_n^2 (y_c - p_y) + \frac{F_{ext,y}}{m}
\end{align*} \tag{2}
\]

The effect of angular momentum of the upper-body, especially the torso and arms, can play an important role in push recovery. These joints can be used to apply a torque about the CoM. The CMP is equal to the COP in the case of zero torque about the CoM such as the LIPM. For a non-zero moment about the COM, however, the CMP can be out of the support polygon, while the COP still remains inside the support polygon. This effect can be embedded by considering the upper body as a flywheel that can be actuated directly as shown by Pratt [3]. In other words, the CMP is the point where a line parallel to the ground reaction force and passing through the COM intersects the ground. Therefore, by adding this effect to the LIPM dynamics, the equations of motion can be written as:

\[
\begin{align*}
\ddot{x}_c &= \omega_n^2 (x_c - p_x) - \frac{H_y}{mz} + \frac{F_{ext,x}}{m} \\
\ddot{y}_c &= \omega_n^2 (y_c - p_y) + \frac{H_x}{mz} + \frac{F_{ext,y}}{m}
\end{align*} \tag{3}
\]

B. Capture Point Dynamics

The unstable part of the LIPM dynamics has been called the CP and can be defined as follows [3], [7], [8]:

\[
\begin{align*}
\dot{\xi}_x &= x_c + \frac{\dot{x}_c}{\omega_n} \\
\dot{\xi}_y &= y_c + \frac{\dot{y}_c}{\omega_n}
\end{align*} \tag{6}
\]

From (6), the CoM dynamics is given by:

\[
\begin{align*}
\dot{x}_c &= \omega_n (\xi - x_c) \\
\dot{y}_c &= \omega_n (\xi - y_c)
\end{align*} \tag{7}
\]

By differentiating (7) and substituting (5) the CP dynamics is given by:

\[
\begin{align*}
\dot{\xi}_x &= \omega_n (\xi - x_c) + \frac{F_{ext,x}}{m\omega_n} \\
\dot{\xi}_y &= \omega_n (\xi - y_c) + \frac{F_{ext,y}}{m\omega_n}
\end{align*} \tag{8}
\]

As it is obvious in (8), the CMP can push the CP. In order to recover the balance of a humanoid robot, the CP should
be controlled. When the CP is located within the support polygon, it can be controlled by the CoP [8], and when it is located out of the support polygon it can be controlled by the CMP or stepping.

Using the concept of CP we can determine when and where to take a step to recover from a push [3]. If the CP is located within the support polygon, the robot is able to recover from the push without having to take a step. In order to stop in one step, the support polygon must have an intersection with the capture region as it shown on Fig. 2.(b). The robot will fail to recover from a severe push in one step, if the capture region does not intersect with the kinematic workspace of the swing foot. In the next sections we will discuss how to use the CP in Push recovery controller based on the MPC scheme.

C. Human-Inspired Balancing Strategies

The response of a human to progressively increasing disturbances can be categorized into three basic strategy: (1) ankle strategy, (2) hip strategy (3) and stepping strategy. Humans tend to use the ankle strategy in case of small pushes to bring back the CP to its desired position as depicted in Fig.3(a). However, the contact between the foot and floor is a unilateral constraint and if the ankle torque becomes too large, the CoP locates on the edge of the support polygon and the foot starts to rotate. Angular momentum of the upper body can be generated in the direction of the disturbance by applying a torque on the hip joint or arm joint as shown in Fig.3(b). This strategy also called CMP Balancing. With increasing the disturbance the useful strategy will be stepping Fig.3(c). However, there are several situations might occur where stepping is not possible as shown in Fig.4. In this situation the balance recovery by Hip-Ankle strategy is necessary [11].

Moreover in the situations that contact surface is small such as right side of Fig.4, generating upper body angular momentum for balance recovery is unavoidable. In this paper, the Hip-Ankle strategy is used in a single MPC scheme that will be presented in the following section.

III. PUSH RECOVERY CONTROLLER

A. discrete state-space form of LIPM+flywheel dynamics

We discretize the LIPM dynamics in the sagittal plane, while the procedure for the other direction is similar:

\[ x_{c,t+1} = (1 - \omega_n T)x_{c,t} + \omega_n T \bar{\xi}_{x,t} \]
\[ \xi_{x,t+1} = (1 + \omega_n T)\xi_{x,t} - \omega_n T(p_{x,t} + \frac{H_{x,t}}{mg}) + \frac{F_{ext,x}}{m\omega_n} \quad (9) \]
\[ p_{x,t+1} = p_{x,t} + \dot{p}_{x,t}T \]
\[ H_{x,t+1} = H_{x,t} + \dot{H}_{x,t}T \]

This system can be re-written in discrete state-space form:

\[ X_{t+1} = A_t X_t + B U_t \quad (10) \]

where \( X_t = [x_{c,t}, \xi_{x,t}, p_{x,t}, H_{x,t}, F_{ext,x}] \) is the vector of state variables and \( U_t = [\dot{p}_{x,t}, \dot{H}_{x,t}] \) specifies the control inputs. The last state variable \( F_{ext} \) is activated in the step time that a push is exerted by defining \( \mu \). Therefore, when a push is exerted we have \( \mu = 1 \) and in the other step times \( \mu \) is equal to zero:

\[ A_t = \begin{bmatrix} (1 - \omega_n T) & \omega_n T & 0 & 0 & 0 \\ 0 & (1 + \omega_n T) & -\omega_n T & -\frac{\omega_n T}{mg} & \frac{1}{m\omega_n} \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \mu \end{bmatrix} \]

\[ B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & T \\ 0 & 0 \end{bmatrix} \]

Given a sequence of control inputs \( \hat{U} \), the linear model in (10) can be converted into a sequence of states \( \hat{X} \), for the whole prediction horizon:

\[ \dot{X} = \hat{A} X + \hat{B} \hat{U} \]
\[ \hat{X} = [X^T_t, X^T_{t+1}, \ldots, X^T_{t+N}] \]
\[ \hat{U} = [U^T_t, U^T_{t+1}, \ldots, U^T_{t+N-1}] \quad (11) \]
where \( \hat{A} \) and \( \hat{B} \) are defined recursively from (10). The control inputs are the rate of change of ZMP position and the rate of upper-body angular momentum. As a result, the core of the Proposed MPC is based on combined hip and ankle strategies.

B. Model Predictive Control (MPC)

We present an MPC Controller that uses the concept of hip and ankle strategies in its core by modulating the ZMP and CMP as control inputs, considering future constraints on the CP. Using the LIPM+Flywheel, the trajectory optimization is simplified to a Quadratic Programming (QP) problem. The LIPM+Flywheel has a linear dynamics and the corresponding optimization problem is linear and can be solved in real-time. The push recovery control objective is simplified to an optimization problem that is realizable on our considered robot. The trajectory of upper-body angular momentum is generated by the MPC to move the CP. The forth term is used for minimizing the rate of change of angular momentum. The second and third terms are considered for modulating the ZMP and CMP in order to coincide the CP with the ZMP in the support polygon center at the end of motion, while the rate of change of upper-body angular momentum is zero. This means we have the following constraints:

\[
\begin{align*}
\dot{\xi}_{c,N} &= \dot{\xi}_{ref,s} \\
x_{c,N} &= x_{ref,s} \\
p_{x,i} &= p_{x,i} \\
\dot{H}_{x,N} &= 0
\end{align*}
\]

(14)

where \( \dot{\xi}_{ref,s} \) is the reference CP that is located on the center of support polygon. The first four constraints are equality constraints for the last step time of motion. The last equation is an inequality constraint that enforces the ZMP to remain inside the support polygon. Similar equations can be derived for the lateral direction. Using the objective function of (12) and adding the constraints of (14), control inputs can be optimized during push recovery by the QP.

IV. SIMULATION AND DISCUSSION

To verify the performance of the push recovery controller, we performed simulations using MATLAB. The proposed controller is implemented on an abstract model of the SURENA III humanoid robot. Parameters that have been used in the simulation is shown in Table.I. The time of balance recovery is considered 1.5 s. The allowable rate of upper-body angular momentum that can be applied is 190 N.m during 1.5 s according to [6].

A. Simulation Results

In the first scenario, a push with the magnitude of 360 N in sagittal direction, and another one with the magnitude of 140 N in frontal plane are exerted on the CoM of the robot. As we expected, the large push throws the CP out of the support polygon, and the ZMP cannot navigate it. Therefore, the angular momentum is generated by the MPC to move the CMP outside the support polygon for controlling the CP. The maximum flywheel torque for push recovery is about 50 N.m that is realizable on our considered robot. The trajectory of

| Variable | Symbol | Value |
|----------|--------|-------|
| Height   | -      | 190 cm |
| CoM Height | \( \zeta_c \) | 75 cm |
| Mass     | m      | 98 kg |
| Foot Length |       | 25 cm |
| Foot Width |       | 15 cm |
| Trunk Inertia |     | 8 Kg.m² |
| Arms Inertia |     | 3 Kg.m² |
| Step-time | T      | 0.05 s |
| MPC gain | \( \alpha_1 \) | 1 m⁻¹ |
| MPC gain | \( \alpha_2 \) | 3 s.(N.m)⁻¹ |
| MPC gain | \( \alpha_3 \) | 10⁻⁶ N.m⁻¹ |
| MPC gain | \( \alpha_4 \) | 10⁻³ (N.m)⁻¹ |
| MPC gain | \( \alpha_5 \) | 1 m⁻¹ |
| MPC gain | \( \alpha_6 \) | 1 s.(N.m)⁻¹ |
| MPC gain | \( \alpha_7 \) | 10⁻⁶ s.m⁻¹ |
| MPC gain | \( \alpha_8 \) | 10⁻³ (N.m)⁻¹ |

TABLE I: Variable used in the simulation (Based on SURENA III)
CP, CoP, CMP and CoM during balance recovery is shown in Fig.5.

In the second scenario, the robot stands on one leg, while the contact surface is shrunken to a line or a point. Two examples for this situation are standing on lumber and standing on rock. In this case, the CMP modulation recovers the robot from the disturbance, because the support polygon is too small and the ankle strategy is not helpful anymore. In this situation, the CP leaves the support polygon and the CoP remains on the bounds of the support polygon, while the CMP pushes the CP to the desired position. As shown in Fig.6, in the first case, pushes with magnitude of 350 N in sagittal and 100 N in lateral direction are exerted on the CoM, while the surface contact is a line. In the second case, pushes with magnitude of 140 N in sagittal and 100 N in frontal plane are exerted on the CoM, while the surface contact is a point.

As shown in Fig.5, 6, in all simulations the angle of the hip pitch joint is smaller than 1.5 rad that is allowable [6].

Based on the simulation results, the regulation of angular momentum is so beneficial during push recovery, especially in the standing on small contact surfaces or in the situations where stepping is not possible. Based on presented results the proposed method has the following features:

- The presented MPC scheme is capable of generating human-like response to external disturbances; for example, when the exerted force is small, it uses the ankle strategy for balance recovery. Furthermore, in the presence of large disturbances, it generates angular momentum and uses hip-ankle strategy simultaneously.
- The proposed push recovery controller can compensate the severe pushes, when the robot stands on small contacts such as a line or a point and also is capable of saving the robot from falling in the situations that stepping is not possible.

V. CONCLUSION AND FUTURE WORK

In this paper, a push recovery controller based on the CP concept and through an MPC framework is developed. The core of the proposed MPC is based on a combined hip and ankle strategies by modulating the CMP and ZMP to control the CP. The results showed that this controller is capable of rejecting severe pushes, even in the case where the support polygon is limited to a line or a point, and stepping is not allowed. The effectiveness of the proposed MPC scheme was demonstrated by simulating an abstract model of the SURENA III humanoid robot.

Despite all above advantages, this controller is implemented only in simulation. Implementing on the experimental setup has more practical challenges [12]. For example, accurate state estimation to obtain the CP position, the saturation of actuators especially in the case where the support polygon is a point or a line, foot slipping and bringing the upper-body back into an upright position are some of the main challenges of experimental implementation that will be discussed in the future works.

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A summary of the simulation scenarios is available on https://youtu.be/bDPafm-6CLk
The push (140,100) N is exerted during 0.05 s, while the contact surface is a point.

The push (350,100) N is exerted during 0.05 s, while the contact surface is a line.

Fig. 6: Simulation results of push recovery (The robot stands on one leg)

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