A strong first order phase transition in the UMSSM

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Abstract. In this work, the electroweak phase transition (EWPT) strength has been investigated within the $U(1)$ extended Minimal Supersymmetric Standard Model (UMSSM) without introducing any exotic fields. We found that the EWPT could be strongly first order for reasonable values of the lightest Higgs and neutralino masses.

1. Introduction
The origin of matter-antimatter asymmetry is one of the main problems of both particle physics and cosmology. The explanation of this asymmetry $n_b/n_\gamma \sim 10^{-10}$ [1], requires the Sakharov criteria, which can be summarized in the existence of such interactions in the early universe that violate the baryon number $B$, the symmetries $C$ and $CP$ and occur out of equilibrium. It appears that the Standard Model (SM) fulfills all these criteria; the baryon number is not conserved at quantum level due to the $B + L$ anomaly [2], a $CP$ violation source does exist in $CKM$ matrix, and a departure from thermal equilibrium could be reached through a strong first order phase transition [3], but its realization was not possible numerically due to smallness of $CP$ violation effect and the weakness of the electroweak phase transition (EWPT) [4]. In the SM, the EWPT is so weak unless the Higgs mass is less than 45 GeV [5], which is in conflict with present data [6], but it could be strong within some SM extensions (for e.g SM with a singlet [7]).

In spite of its success and popularity, the MSSM with R-parity still has two major problems: the $\mu$-problem [8] and the potential proton decay problem due to dimension 5 operators [9]. A natural solution to these problems would probably require that the MSSM be extended by a new mechanism or a new symmetry. The $U(1)'$-extended MSSM (UMSSM) [10] is a straightforward extension of the MSSM with a non-anomalous TeV scale Abelian gauge symmetry. In the minimal supersymmetric standard model (MSSM), the EWPT could be strongly first order if the light stop is lighter than the top quark [11], and in the UMSSM, it is also strongly first order but with the price of introducing 3 new extra singlet scalars [12], or by adding new extra heavy singlet fermions [13]. In this work, we will investigate the possibility of getting a strong first order phase transition within the minimal gauge extension of the MSSM, UMSSM without adding any new field beside the usual singlet.

This paper is organized as follow: in the second section, we give a brief review of the UMSSM model, define the scalar potential and discuss different constraints on the parameters. After that, we discuss the EWPT dynamics and show how to get a first order phase transition, and discuss our numerical results. Finally, we summarize our results.
2. The UMSSM model

In the U(1)MSSM (or UMSSM), the gauge group is extended to \( G = SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)' \) with the couplings \( g_3, g_2, g_1 \) and \( g' \), respectively [10]. The superpotential is given by

\[
W = \lambda S \epsilon_{ij} H_1^+ H_2^+ + Y_U \epsilon_{ij} Q_i^c U^c H_2^+ + Y_D \epsilon_{ij} Q_i^c D^c H_1^+ + Y_L \epsilon_{ij} L^i E^c H_1^+, \tag{1}
\]

where \( \epsilon_{ij} \) is the anti-symmetry \( 2 \times 2 \) tensor, \( Y_U, Y_D \) and \( Y_L \) are Yukawa couplings, and \( \lambda \) is a coupling constant in which \( \lambda \) replaces the \( \mu \)-term in the MSSM. The content here, is the same as the MSSM in addition to a new singlet and gauge boson; and the superpartners.

2.1. The scalar potential

In case where both sneutrinos or/and squarks do not develop vevs, the scalar potential is the combination of the so-called \( D, F \) and soft terms, which are given by

\[
V_D = \frac{g_2^2 + g_1^2}{8} (H_2^+ H_2 - H_1^+ H_1)^2 + \frac{g_2^2}{2} |H_1^+ H_2|^2 + \frac{g^2}{2} |Q_1 H_1^+ H_1 + Q_2 H_2^+ H_2 + Q_S |S|^2|^2, \\
V_F = |\lambda|^2 \left\{ |\epsilon_{ij} H_1^+ H_2^+|^2 + |S|^2 \left[ H_1^+ H_1 + H_2^+ H_2 \right] \right\} \\
V_{soft} = m_{H_1}^2 H_1^+ H_1 + m_{H_2}^2 H_2^+ H_2 + m_S^2 |S|^2 + \left\{ A_\lambda \epsilon_{ij} H_1^+ H_2^+ + h.c. \right\}. \tag{2}
\]

Here \( m_{H_1}^2, m_{H_2}^2, m_S^2 \) and \( A_\lambda \) are usually called soft parameters. The charges \( Q' \)'s should be chosen in a way that ensures the anomaly cancelations. The structure of the tree-level potential seems to allow two explicit CP violating relative phases between the scalar vevs, but these phases can be canceled by such gauge cancelations, and the ground state is independent of any CP violating phases. However, these relative phases could appear at one loop through the superpartner masses (see Appendix A in [14]).

In this setup, the scalar potential (2) may admit another minimum \((0, 0, x)\), This wrong vacuum could play a very important role in the EWPT dynamics.

2.2. The parameters

In this model, we have many parameters, some of them are free like: \( g', \lambda, v_2, \tan \beta = v_1/v_2 \), and the soft terms: \( m_Q, m_U, A_t, A_\lambda, M_2, M_1 \) and \( M_1' \); and others are fixed by such measured physical quantities: \( g_1, g_2, v \) and \( y_t \); or such conditions like the elimination \( m_1^2, m_2^2 \) and \( m_S^2 \) by imposing \((v_1, v_2, v_2)\) to be the absolute minimum of the effective potential at zero temperature. In addition to the constrains that are coming from the extra U(1)' gauge interactions, like mixing between the neutral gauge boson \( Z \) and the new one \( Z' \) [15]

\[
2 M_{ZZ'}^2 /(M_{Z'Z'}^2 - M_{ZZ}^2) < 10^{-3}, \tag{3}
\]

and the bound on the heavy \( Z' \) mass [16]

\[
M_{Z'} > (500 - 800) \text{ GeV}. \tag{4}
\]

Keeping the new charges as free parameters beside the condition \( Q_1 + Q_2 + Q_s = 0 \), we take random values for the free parameters, taking into account such conditions like the elimination of \( m^2 \)'s parameters in (2), the perturbativity of the quartic couplings in (2), and the vacuum stability. Another constrains could be derived from the upper bound on the mixing between the gauge boson \( Z \) and the new one \( Z' \) (3), and the lower bound on the new gauge boson \( Z' \) mass (4). The condition (4), could be achieved by considering relatively large \( v_2 \), or large \( g'Q' \)'s. The condition (3) could be fulfilled by vanishing the mixing term \( M_{ZZ'}^2 \), i.e., if

\[
Q_1 = Q_2 \tan^2 \beta, \tag{5}
\]
or if $M_{Z'}^2 > M_{Z'}^2, M_{ZZ},$ which roughly means

$$g' |Q_S| v_x \gtrsim (500 - 800) \text{ GeV},$$

(6)

or a serious tuning in the values of $Q_{1,2}$ and $tan\beta$.

In our search for the parameter’s space that fulfills the strong first-order phase transition criterion: $v (T_c)/T_c > 1$, we will focus our search on two regions:

a) Moderate values for the parameters $Q_{1,2}$ and $tan\beta$, where (3) is nearly satisfied. In this case, the singlet vev $v_x$, could be the order $v$ or even relatively smaller.

b) The two terms $M_{Z'}^2$ and $M_{ZZ}$, in the mass-squared matrix of $(Z, Z')$, should be suppressed with respect to the mass term $M_{Z'}^2$. In this case, the values of $U'(1)$ charge and the vev of the singlet, $Q_x$ and $v_x$, should be large enough (6).

The mass parameters: $m_Q, m_U, A_t, M_2, M_1$ and $M_1^1$ appear at one-loop level in the effective potential, therefore we expect that their role is less important in the EWPT dynamics. But each of these parameters: $g', \lambda, v_x$ and $tan \beta$, as well the charges $Q's$ that appear multiplied by $g'$, seems to be very important, therefore we focus on these parameters while fixing the mass parameters: $g' = g_1, m_Q = m_U = 1 \text{ TeV}$, and giving different values for the rest $A_t, M_2, M_1$ and $M_1^1$.

3. The electroweak phase transition

3.1. The phase dynamics

Due the condition (3) and (4), there could exist a hierarchy between the vev of the singlet and those of the doublets, i.e., $v_x > v_{1,2}$. In this case, the gauge symmetry could be broken in two steps: $(0, 0, 0) \rightarrow (0, x) \rightarrow (0, x, v_x)$, or just in once $(0, 0, 0) \rightarrow (v_1, v_2, v_x)$. Since the singlet dynamics does not affect the $SU(2)$ sphaleron processes, we will not be interested in distinguishing between the one- and two-steps symmetry breaking. We will treat our field dynamics using the effective potential where the singlet is replaced by it thermal vev. At the critical temperature $T_c$, the two minima get degenerate

$$V_{eff} (v_1^c, v_2^c, v_x^c, T_c) = V_{eff} (0, 0, x_c, T_c).$$

(7)

Below this temperature, the new minimum becomes the absolute one, and the system has to move from the old (false) vacuum to the new (true) one. In the case where a barrier does exist between the two minima, this transition has to occur via tunneling trough bubbles nucleation at certain points, which expand and fill the whole space by the new vacuum $v_{1,2} (T) \neq 0$, i.e, the symmetry is broken.

The $B + L$ anomalous interactions [2], that violate the baryon number have not the same rate in the symmetric and broken phases (i.e., at both sides of the bubble wall). In the symmetric phase, this rate behaves like $\sim T^4$ [17], and suppressed as $exp(-E_{Sp}/T)$ [18], in the broken phase, where $E_{Sp}$ is the system static energy within such field configuration called the sphaleron [20]. Therefore any generated baryon number at the symmetric phase will erase at the broken phase, unless these interactions are switched off at the broken phase, this is translated to the famous criterion [19].

Here, we are interested in the in the value of the EW vev at the transition temperature, and since the singlet field does not play an important role in the sphaleron processes [7], the criterion of a strong first order phase transition in our case is given by [19]

$$v (T_c)/T_c \equiv \sqrt{v_1^2 (T_c) + v_2^2 (T_c)}/T_c > 1.$$  

(8)

In the general case where the relative phases $\theta_{1,2} \neq 0$, the field ground state should be written as $\{v_i\}_{i=1,5} = (v_1, v_2 \cos \theta_1, v_2 \sin \theta_1, v_x \cos \theta_2, v_x \sin \theta_2)$ instead of $(v_1, v_2, v_x)$ and the
two relative phases. These 5 variables should be treated independently when looking for \(v_{1,2,x}\) and \(\theta_{1,2}\) at any temperature \(T\). Then the phase transition could be defined through the equations

\[
\frac{\partial}{\partial v_i} V_{\text{eff}} (v_i, T_c) = 0, \quad V_{\text{eff}} (v_1, v_2, v_3, v_4, v_5, T_c) = V_{\text{eff}} (0, 0, 0, x_1, x_2, T_c),
\]

where \(x_{1,2}\) are the real and imaginary parts of the singlet vev in the wrong vacuum.

### 3.2. Numerical results

In the following figures, we show the dependence of the quantity \(\nu(T_c)/T_c\) in (8) on the lightest Higgs mass (in Fig. 1 left), and on the lightest neutralino mass (in Fig. 1 right), for a random choice of about 2000 cases in both regions (a) and (b), where different conditions are fulfilled.

![Figure 1](image)

**Figure 1.** The dependence of the quantity \(\nu_c/T_c\) on the lightest Higgs (right) and the lightest neutralino (left) masses. The green points refer to cases in region (a), and the red ones to cases in region (b).

As it is clear from Fig. 1, the EWPT could be strongly first order in the two regions (a) and (b), for different values of the lightest Higgs mass. The fact that the lightest Higgs mass is around 90\(\sim\)100 GeV might be consistent with experiment because the doublet couplings will be modified due to the mixing with singlet. The EWPT could be also, strongly first order for different the lightest neutralino masses. This leads to the conclusion that the EWPT strength is a factor beyond the fields contributions to the effective potential at zero temperature.

The critical temperature is, in general, higher when comparing with minimal SM (\(\sim\)100 GeV), the generic value is larger than 500 GeV, this is a consequence of the the interaction of the doublets with the singlet that has, in general, a very large vev. However for the benchmarks, giving a strong first order EWPT, it is relatively smaller than the generic values. As it is small, the EWPT is stronger. In order to understand this point, we take a benchmark from Fig. 1, and study the dependance of the scalar vevs on the temperature \(T\). We will see also how could this behavior be changed with respect to the charges \(Q's\), and some parameters like \(A_t\) and \(\theta^{(v)}_{1,2}\), that appear in the effective potential at one-loop level. Therefore we consider the following modifications:

From this table, it is clear that the EWPT strength is sensitive to the relative phases \(\theta^{(v)}_{1,2}\), that appear in the effective potential at one-loop through the superpartners masses. For this parameters set, these relative phase shift the critical temperature by some units. The effect of the parameter \(A_t\), that appears in the effective potential through the one-loop corrections of the stops, is more important. The critical temperature, in this case (4) is shifted by 200 GeV, which makes the EWPT so weak. From case (3), the effect of the charges \(Q's\) is extremely important.
Table 1. Values of the parameters used to study the scalar vevs dependance with respect to the temperature. We used common values for the parameters: $\lambda = 0.01$, $\tan\beta = 1$, $A_\lambda = 1210$ GeV, $\nu_x = 1046.4$ GeV, $M_1 = M_2 = 100$ GeV, $M'_1 = 300$ GeV. The mass-dimension parameters are given in GeV.

In Fig. 2, we show the dependance of the ground state on the temperature below the critical temperature for the benchmark (1) in Table-1, and its modifications (2)-(4). From Fig. 2, one remarks that the common feature between all these cases is that the dependance of the singlet vev on the temperature is very weak around and below the critical temperature.

Another important remark, is that the Higgs vevs for this benchmark (Fig. 2-1), are increasing when the Universe gets cooled unlike the SM, or MSSM. In case (3) (Fig. 2-3), the Higgs vevs decrease, but slower than the SM or MSSM. This can be seen clearly in Fig. 1-3, for the benchmarks whose critical temperature is between $250 \sim 400$ GeV, where the corresponding value of $\nu(T_c)/T_c$ is larger when comparing with the SM. The fact that the doublets vev values at high temperatures are larger than their zero temperature values has been mentioned in a similar work [13]. This behavior: increasing Higgs vevs w.r.t the temperature or decreasing slower than the SM case, is a consequence of the interaction of the singlet with the doublet. These interactions lead to relax the shape of the potential in the direction of the doublets, and therefore enhances the ratio in (8), and strengthen the EWPT. This is a common feature for models with singlets [7].

Unlike the SM and MSSM, the wrong vacuum $(0,0,x(T))$, is evolving w.r.t the temperature, therefore its evolution could be a very important factor that strengthens the EWPT. It could delay the transition, and then enhances the ratio (8). The EWPT dynamics seems to be less sensitive to some input parameters like $m_Q$, $m_U$, $A_t$, $M_2$, $M_1$, $M'_1$ and especially the relative...
phases $\theta^{(v)}_{1,2}$, since they appear in the effective potential at one loop, however, we have shown in Table-1 that the EWPT dynamics is sensitive to these parameters more than expected. This sensitivity is a consequence of the dependance of the two vacua (instead one) on these input parameters.

The possible one-loop spontaneous $CP$ violation due to the relative phases $\theta^{(v)}_{1,2}$, could have an important phenomenological impact. An important feature in this model is that the dark matter candidate (lightest neutralino) could be dominated by the new ingredient $\tilde{B}'$. This possibility will be investigated in further work [21].

4. Conclusion

In this work, the electroweak phase transition nature within the minimal $U(1)$ extension of the MSSM (UMSSM) has been investigated. We found that the EWPT could be strongly first order for reasonable masses of scalar Higgs bosons and the lightest neutralino, without adding extra singlet scalars or fermions. We evaluated the effective potential at one-loop taking into account the whole particle spectrum, and its temperature dependant corrections were estimated exactly using the known techniques. We found that EWPT strength could be enhanced due to two factors: first, the interactions of the singlet scalars with the doublets that relax the shape of the effective potential in the doublets directions, which lead to a large value for the ratio $\upsilon(T_c)/T_c$, at the critical temperature. The second factor is that the temperature-dependant local minimum $(0,0,x(T))$, could play an important role during the EWPT dynamics. It can delay the phase transition until relatively low temperatures (even below 100 $GeV$), which favor the ratio $\upsilon(T_c)/T_c$ to be large enough.

We mention also that the reliability of the parameter choice, as well as the EWPT strength are more sensitive to the input parameters that appear in the effective potential at one-loop.

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