Exact $\beta$-functions in softly-broken $\mathcal{N} = 2$ Chern-Simons matter theories

I. Jack and C. Luckhurst

Dept. of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, UK

Abstract

We present exact results for the $\beta$-functions for the soft-breaking parameters in softly-broken $\mathcal{N} = 2$ Chern-Simons matter theories in terms of the anomalous dimension in the unbroken theory. We check our results explicitly up to the two loop level.
Chern-Simons gauge theories have attracted attention for a considerable time due to their topological nature [1–3] (in the pure gauge case) and their possible relation to the quantum Hall effect and high-$T_c$ superconductivity. More recently there has been substantial interest in $\mathcal{N} = 2$ supersymmetric Chern-Simons matter theories in the context of the AdS/CFT correspondence (see Refs. [4–6] for details and a comprehensive list of references), and it therefore seems timely to consider the softly-broken version of the theory. It is already well-known that the $\beta$-functions for the soft-breaking parameters in softly-broken $\mathcal{N} = 1$ supersymmetric gauge theories in four dimensions may be expressed exactly in terms of the anomalous dimensions and gauge $\beta$-function for the unbroken theory. (See Ref. [7] for a complete description of the most general case.) Moreover this leads [8] to exact renormalisation group invariant solutions for the soft-breaking parameters—the “anomaly-mediated supersymmetry-breaking” (AMSB) solutions [9]–[15]. The purpose of this note is to point out that similar results hold for $\mathcal{N} = 2$ Chern-Simons matter theories in three dimensions; indeed the results are simpler due to the absence of a gauge coupling (which reflects the topological nature of the gauge part of the theory).

Our results are based on a set of rules devised by Yamada [16] for obtaining the $\beta$-functions for the scalar soft-breaking couplings (in four dimensions) starting from the anomalous dimension for the chiral superfields. We shall present here an abridged derivation based on Ref. [18]; see Ref. [7] for the complete version. Yamada’s rules are based on the spurion formalism [19], which enables one to write the softly broken $\mathcal{N} = 2$ theory in terms of superfields. The lagrangian for the theory can be written

$$L = L_{SUSY} + L_{SB} + L_{GF} + L_{FP}$$

(1)

where $L_{SUSY}$ is the usual $\mathcal{N} = 2$ supersymmetric lagrangian [17],

$$L_{SUSY} = \int d^3x \int d^4\theta \left(2k \int_0^1 dt \text{Tr}[\overline{\Psi}(e^{-tV}D_\alpha e^{tV})] + \Phi^j(e^{V A R_A})_{ij} \Phi_i \right) + \left(\int d^3x \int d^2\theta W(\Phi) + \text{h.c.}\right) ,$$

(2)

where $V$ is the vector superfield, $\Phi$ the chiral matter superfield and where the superpotential $W(\Phi)$ is given by

$$W(\Phi) = \frac{1}{4!} Y^{ijkl} \Phi_i \Phi_j \Phi_k \Phi_l + \frac{1}{3!} Z^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2!} \mu^{ij} \Phi_i \Phi_j.$$  

(3)

(We use the convention that $\Phi^i = (\Phi_i)^*$.) We assume a simple gauge group; a gauge group with a $U(1)$ factor could also include a linear term in the superpotential. Gauge invariance requires the gauge coupling $k$ to be quantised, so that $2\pi k$ is an integer. The vector superfield $V$ is in the adjoint representation, $V = V_A T_A$ where $T_A$ are the generators of the fundamental representation, satisfying

$$[T_A,T_B] = if_{ABC}T_C,$$

$$\text{Tr}(T_AT_B) = \frac{1}{2} \delta_{AB}.$$  

(4)

(5)

1
The chiral superfield can be in a general representation, with gauge matrices denoted \( R_A \) satisfying
\[
[R_A, R_B] = i f_{ABC} R_C, \tag{6}
\]
\[
\text{Tr}(R_A R_B) = T(R) \delta_{AB}. \tag{7}
\]
In three dimensions the Yukawa couplings \( Y^{ijkl} \) are dimensionless and the theory is renormalisable. The soft breaking part \( L_{SB} \) may be written \(^{20}\)
\[
L_{SB} = \int d^2 \theta \eta \left( \frac{1}{4!} h^{ijkl} \Phi_i \Phi_j \Phi_k \Phi_l + \frac{1}{3!} g^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2!} b^{ij} \Phi_i \Phi_j + \text{h.c.} \right) - \int d^4 \theta \eta^* \eta \Phi^j_i (m^2)_{ik} Y^{k}_{AB} \Phi_{B}, \tag{8}
\]
where \( \eta = \theta^2 \) is the spurion external field. Note that in three dimensions there is no soft term corresponding to the four-dimensional gaugino mass term. The gauge-fixing and Fadeev-Popov terms are contained in \( L_{GF} \) and \( L_{FP} \) respectively. It is convenient to introduce a generalised form \( \gamma_\eta \) of the anomalous dimension \( \gamma \) of the chiral supermultiplet, given by:
\[
\gamma_\eta = \gamma + \gamma_1 \eta + \gamma_2 \eta^*, \tag{9}
\]
It was shown by Yamada \(^{16}\) that \( (\gamma_\eta)^{ij} \) could be obtained from \( (\gamma)^{ij} \) by the following rules (simpler in three than in four dimensions due to the absence of a running gauge coupling):

1. Replace \( Y^{lmno} \) by \( Y^{lmno} - h^{lmno} \eta. \)

2. Insert \( \delta'_{l} + (m^2)_{l'} \eta^* \eta \) between contracted indices \( l \) and \( l' \) in \( Y \) and \( Y^* \), respectively:
\[
Y^{lmno} Y^{l'n'o'} \rightarrow Y^{lmno} Y^{l'n'o'} + Y^{lmno} (m^2)_{l'} Y^{l'n'o'} \eta^* \eta \tag{where, here and subsequently, \( Y^{lmno} = (Y^{lmno})^*).}
\]

3. Replace a term \( T^i_j \) in \( \gamma^i_j \) with no Yukawa couplings by \( T^i_j - (m^2)_{ik} T^k_j \eta^* \eta. \)
\( \gamma_1 \) and \( \gamma_2 \) may then be obtained by extracting the coefficients of \( \eta \) and \( \eta^* \eta \) respectively. In the case of \( \gamma_1 \), the above rules can be subsumed by the simple relation
\[
(\gamma_1)^{ij} = \mathcal{O} \gamma^{ij}, \tag{10}
\]
where
\[
\mathcal{O} = -h^{lmno} \frac{\partial}{\partial Y^{lmno}}. \tag{11}
\]
It is straightforward to show that
\[
\beta^{ijkl}_h = \gamma^{ij} h^{jkl} m - 2 \gamma_1^{ij} Y^{jkl} m. \tag{12}
\]
This result is similar in form to the standard result for \( \beta_Y \) which follows from the non-renormalisation theorem (which is valid for \( \mathcal{N} = 2 \) supersymmetric theories in three dimensions), namely
\[
\beta^{ijkl}_Y = \gamma^{ij} Y^{jkl} m. \tag{13}
\]
We also have the analogous results for the soft couplings corresponding to dimensionful supersymmetric couplings,

\[
\beta_{ij} = \gamma(i_m g^{jk})_m - 2\gamma(i_m Z^{jk})_m,
\]
\[
\beta_i^j = \gamma(i_m b^{jm})_m - 2\gamma(i_m H^j)_m.
\]

It also follows from Eqs. (8) and (9) that

\[
(\beta_{m^2})_j = \frac{1}{2}\gamma^i (m^2)^i_j + \frac{1}{2}(m^2)^i_k \gamma^k_j + \gamma^j_i,
\]

which we may write using Yamada’s rules as

\[
(\beta_{m^2})_j = \left[2OO^* + \tilde{Y}_{lmn} \frac{\partial}{\partial Y_{lmn}} + \tilde{Y}_{lmn} \frac{\partial}{\partial Y_{lmn}}\right] \gamma^j_i,
\]

where

\[
\tilde{Y}_{ijkl} = (m^2)^i_m Y_{mjkl} + (m^2)^i_m Y_{imkl} + (m^2)^i_m Y_{ijml} + (m^2)^i_m Y_{ijkm}.
\]

The exact results Eqs. (12) and (17) for the \(\beta\)-functions lead to exact renormalisation group invariant solutions for the soft-breaking couplings, namely

\[
h_{ijkl} = -M_0 \beta_Y^{ijkl},
\]
\[
g_{ij} = -M_0 \beta_Z^i + \kappa_1 Z_{ijk},
\]
\[
b_{ij} = -M_0 \beta_\mu^i + \kappa_2 \mu^{ij},
\]
\[
(m^2)_j = \frac{1}{2} |M_0|^2 \mu \frac{d^2 \gamma^i_j}{d\mu},
\]

where \(M_0, \kappa_1, \kappa_2\) are constant masses. These results can be proved following the four-dimensional discussion in Ref. [8] (though the terms with \(\kappa_{1,2}\) were given for the first time in Ref. [21]); but once more the details are simpler due to the non-running of the gauge coupling. We note that in the case of a gauge group with a \(U(1)\) factor and a linear term in the superpotential, additional terms are expected [7] in the expressions for \(\beta_g\) and \(\beta_b\) in Eq. (15), and hence corresponding extra terms in Eqs. (22); there should also be an exact expression for the \(\beta\)-function corresponding to the linear soft coupling, and an exact RG-invariant solution for this coupling. There is also potentially an additional term [22] in the solution for \(m^2\) corresponding to the possible Fayet-Iliopoulos term.

We now turn to our check of the results Eqs. (12) and (17) up to two loops using the component formulation of the theory (there are no divergences at odd loop orders for a theory in odd dimensions, so this is the simplest non-trivial check). The first ingredient is the anomalous dimension of the chiral superfield, which is given at two loops by

\[
64\pi^2 \gamma^{(2)} = \frac{1}{3} Y_2 - 2k^{-2} C_2(R) C_2(R) - k^{-2} T(R) C_2(R) + k^{-2} C_2(G) C_2(R)
\]
where

\[
(Y_2)_i^j = Y^{iklm} Y_{jklm} \tag{24}
\]

\[C_2(R) = R_A R_A, \tag{25}\]

\[C_2(G) \delta_{AB} = f_{ACD} f_{BCD} \tag{26}\]

\[\beta_{ijkl} (2) h = \left[ \frac{1}{3} Y_{ijkl} + \frac{1}{3} (m^2)_{k} Y_{jklm} + 2 Y^{iklm} (m^2)_{k} Y_{jk'lm} \right] \tag{29}\]

\[64 \pi^2 (Y_2)_i^j = -\frac{1}{3} h_{ilmn} Y_{jlmn} \tag{28}\]

and \(T(R)\) is defined in Eq. (7). This result may readily be obtained by \(\mathcal{N} = 2\) superfield methods \([5,6,20,23]\); see the appendix for the \(\mathcal{N} = 2\) superfield conventions.

An expression for the two-loop anomalous dimension for an \(\mathcal{N} = 1\) theory in three dimensions (with no Yukawa coupling) is given in Ref. [24]. This does not agree with the \(k^{-2}\) terms in Eq. (23) when specialised to the \(\mathcal{N} = 2\) case. Presumably this is because the result is in general gauge-dependent and the \(\mathcal{N} = 1\) and \(\mathcal{N} = 2\) Feynman gauges are not equivalent. Since \(\mathcal{N} = 2\) supersymmetry is not manifest in the \(\mathcal{N} = 1\) formalism, one would not expect Eq. (13) to be valid using the anomalous dimension computed using the \(\mathcal{N} = 1\) formalism. We have however checked explicitly via a component calculation that the \(\beta\) function for the Yukawa coupling is indeed given by Eq. (13) with the anomalous dimension of Eq. (23).

We then find from Eq. (10) that

\[64 \pi^2 (\gamma_1^{(2)})_i^j = -\frac{1}{3} h_{ilmn} Y_{jlmn} \tag{28}\]

and that therefore (using Eq. (12))

\[64 \pi^2 \beta_h^{ijkl(2)} = \left[ \frac{1}{3} Y_{2} - 2 k^{-2} C_2(R) C_2(R) - k^{-2} T(R) C_2(R) + k^{-2} C_2(G) C_2(R) \right] m h_{mjkl} + \frac{2}{3} h_{ilmn} Y_{plmn} Y_{njkl} + \text{cyclic perms.} \tag{29}\]

We also find from Eq. (17) that

\[64 \pi^2 (\beta_{m^2})_i^j = \frac{2}{3} h_{iklm} h_{jklm} + \frac{1}{3} (m^2)_k Y_{2}^j + \frac{1}{3} (Y_2)_k (m^2)_j + 2 Y^{iklm} (m^2)_{k'} Y_{jk'lm}. \tag{29}\]

It is straightforward to verify these results by a component calculation. The supersymmetric Lagrangian is given in components by [25]

\[L_{\text{SUSY}} = L_{\text{CS}} + L_m \tag{30}\]

\[L_{\text{CS}} = 2k \text{Tr} [\epsilon^{\mu\nu\rho} (A_\mu \partial_\nu A_\rho + \frac{2}{3} A_\mu A_\nu A_\rho) - \overline{\lambda} \lambda + 2 D \sigma] \tag{31}\]

\[L_m = D_\mu \phi^i D^\mu \phi_i + i \overline{\psi} \gamma^\mu D_\mu \psi_i + F_i^i F_i - i \phi^i \sigma^2 \phi_i + \phi^i D_\mu \phi_i + i \phi^i \overline{\lambda} \phi + i \overline{\psi} \lambda \phi \]

\[+ \left( \frac{1}{3} Y^{ijkl} \phi_i \phi_j \phi_k F_i + \frac{1}{2} Y^{ijkl} \phi_i \phi_j \phi_k \overline{\psi} \psi_l + \text{h.c.} \right), \tag{32}\]

where \(\lambda\) and \(\psi\) are two-component Dirac spinors, \(\overline{\lambda} = \lambda^\dagger \gamma_0\), \(D_\mu = \partial_\mu + i A_\mu\) and we have set \(\mu^{ij} = Z^{ijk} = 0\) for simplicity, in order to focus on the dimensionless couplings. After
eliminating the auxiliary fields $D$, $\sigma$ we obtain

$$L_{CS} = \text{Tr} [\epsilon^{\mu \nu \rho \sigma} (A_\mu \partial_\nu A_\rho + \frac{2i}{3} A_\mu A_\nu A_\rho)],$$

$$L_m = D_\mu \phi^i D^\mu \phi_i + i \bar{\psi} \gamma^\mu D_\mu \psi_i$$

$$- (\phi^i R_A \phi)(\phi^* R_B \phi)(\phi^* R_A R_B \phi) + (\phi^i R_A \phi)(\bar{\psi} R_A \psi) + 2(\bar{\psi} R_A \phi)(\phi^* R_A \psi)$$

$$- \frac{1}{(3!)^2} Y_{ijkm} Y_{ij'k'n} \phi_i \phi_j \phi_{k'} \phi_{l'} + (\frac{1}{4} Y_{ijkl} \phi_i \phi_j \bar{\psi}_k \psi_l + \text{h.c.})$$

The soft-breaking lagrangian is given by

$$L_{SB} = - \left( \frac{1}{4!} h_{ijkl} \phi_i \phi_j \phi_k \phi_l + \text{h.c.} \right) - (m^2)^{ij} \phi_i \phi^j,$$

where we have set $b^{ij} = g^{ijk} = 0$.

The diagrams contributing to the anomalous dimension of the scalar component field $\phi$ at two loops are depicted in Fig. 1, with scalar, fermion, gauge and ghost propagators denoted by dashed, unbroken, wavy and dotted lines respectively. We work in a standard Feynman gauge in components (see the Appendix). The divergent contributions from the diagrams in Fig. 1 to $\partial_\mu \phi^i \partial^\mu \phi$ are given by (using dimensional regularisation and working in $d = 3 - \epsilon$ dimensions)

$$L^{(2)}_{\phi(a)} = \frac{1}{3} Y_2 + \frac{1}{6} k^{-2} [4C_2(R) - 2C_2(G) + 5T(R)] C_2(R),$$

$$L^{(2)}_{\phi(b)} = \frac{1}{12} k^{-2} [-4C_2(R) + C_2(G)] C_2(R),$$

$$L^{(2)}_{\phi(c)} = -\frac{2}{3} k^{-2} T(R) C_2(R),$$

$$L^{(2)}_{\phi(d)} = -\frac{2}{3} k^{-2} T(R) C_2(R),$$

$$L^{(2)}_{\phi(e)} = \frac{1}{6} k^{-2} C_2(G) C_2(R),$$

$$L^{(2)}_{\phi(f)} = -\frac{1}{6} k^{-2} C_2(G) C_2(R),$$

$$L^{(2)}_{\phi(g)} = \frac{2}{3} k^{-2} [-2C_2(R) + C_2(G)] C_2(R),$$

$$L^{(2)}_{\phi(h)} = \frac{1}{3} k^{-2} C_2(G) C_2(R),$$

where $L = 64\pi^2 \epsilon$, leading to

$$\gamma^{(2)}_{\phi} = \frac{1}{3} Y_2 - k^{-2} C_2(R) C_2(R) - \frac{1}{2} k^{-2} T(R) C_2(R) + \frac{3}{4} k^{-2} C_2(G) C_2(R)$$

which agrees (up to an overall factor of 4, whose origin we have not been able to identify) with the component-field calculation in Ref. [24], when the relevant result is specialised to the case of $\mathcal{N} = 2$ supersymmetry. Note that since there are no simple poles at one loop, there are no double poles at two loops and no need to consider diagrams with counterterm insertions at this order. The list of diagrams contributing to $\beta_\lambda$ and $\beta_{m^2}$ can be shortened by noting that any logarithmically divergent diagram where an external scalar emerges
from a $\phi^* A \phi$ vertex is zero by symmetry, due to the form of the gauge propagator (see the Appendix). The diagrams contributing to the two-loop $\beta$ functions for $m^2$ and $h$ are shown in Figs. 2 and 3 respectively. They yield divergent contributions to the effective quantum properties of this theory were discussed in Ref. [27] based on the identities using Eqs. (30), (29), (52), (53), (44).

The maximal supersymmetry for a Chern-Simons theory with a single gauge group is $\mathcal{N} = 3$. The component formulation of this theory was presented in Ref. [26]. The soft-breaking $\beta$-functions will satisfy

$$2\Gamma^{(2)}_h = \frac{1}{4} \left( \beta^{ijkl(2)}_h - 4 (\gamma^{(2)}_h)_{ij} h^{ijkl} \right) \phi_i \phi_j \phi_k \phi_l,$$

$$2\Gamma^{(2)}_{m^2} = (\beta^{(2)}_{m^2})_{ij} \phi^i \phi_j - (\gamma^{(2)}_{m^2})_{ij} \phi^i \phi_j - (m^2 \gamma^{(2)}_h)_{ij} \phi^i \phi_j,$$

writing the results in this form to avoid cumbersome symmetrisations. We easily verify these identities using Eqs. [30], [29], [52], [53], [11].

The maximal supersymmetry for a Chern-Simons theory with a single gauge group is $\mathcal{N} = 3$. The component formulation of this theory was presented in Ref. [26]. The quantum properties of this theory were discussed in Ref. [27] based on the $d = 3 \mathcal{N} = 3$
harmonic superspace formalism developed in Ref. [28], and it was shown that this theory is all-orders finite. It would be interesting to investigate whether the softly-broken version of this theory is also finite. Theories with higher degrees of supersymmetry (up to $\mathcal{N}=8$) [29] may be obtained in the case of direct product groups and matter in the bi-fundamental representation. A rich variety of these theories [30]-[42] are expected to be superconformal by virtue of the AdS$_4$/CFT$_3$ correspondence, originally stated in Ref [43]. These theories can be expressed in terms of $\mathcal{N}=2$ superfields and are obtained by a judicious choice of field content and also a particular choice of Yukawa couplings (as a function of the gauge couplings). The conformal properties of a range of these models was checked explicitly at the two-loop level in Refs. [30]-[42]. It would be quite straightforward to extend our results to the case of direct product gauge groups and thereby derive exact results for the softly broken versions of these theories. One could then ask whether there were a choice of soft couplings which would maintain finiteness. In the case of $\beta_h$ this would entail arranging for $\gamma_1$ to vanish; this is not guaranteed by the vanishing of $\gamma$, since the derivative in Eq. (11) would be taken before specialising to the special form for the Yukawa couplings which guarantees the extended supersymmetry. Nevertheless it was shown in the four-dimensional case [8] that there was a choice of soft couplings which would guarantee $\gamma_1 = 0$. However this relied on the existence of the gaugino mass as a soft coupling and a similar choice is not possible here; there is therefore no obvious way to guarantee the vanishing of $\beta_h$. The same argument applied to Eq. (16) would imply that we could not render $\beta_{m^2}$ zero. The softly-broken versions of these superconformal theories would therefore not be finite.

Finally, it would be interesting to address the question of gauge groups with a $U(1)$ factor, where, as we have noted, there are additional technical subtleties.

**Acknowledgements**

One of us (CL) was supported by a University of Liverpool studentship. IJ is grateful for useful discussions with Tim Jones.

**Appendix**

In this appendix we list our superspace and supersymmetry conventions. We use a metric signature $(+--)$ so that a possible choice of $\gamma$ matrices is $\gamma^0 = \sigma_2$, $\gamma^1 = i\sigma_3$, $\gamma^2 = i\sigma_1$ with

$$ (\gamma^\mu)_{\alpha}^{\beta} = (\sigma_2)_{\alpha}^{\beta}, $$

etc. We then have

$$ \gamma^\mu \gamma^\nu = \eta^{\mu\nu} - i\varepsilon^{\mu
u\rho\sigma}\gamma_\sigma. $$

We have [6] two complex two-spinors $\theta^\alpha$ and $\theta^\alpha$ with indices raised and lowered according to

$$ \theta^\alpha = C^{\alpha\beta} \theta_\beta, \quad \theta_\alpha = \theta^\beta C_{\beta\alpha}, $$

with $C^{12} = -C_{12} = i$. We then have

$$ \theta_\alpha \theta_\beta = C_{\beta\alpha} \theta^2, \quad \theta^\alpha \theta^\beta = C^{\beta\alpha} \theta^2, $$
where
\[ \theta^2 = \frac{1}{2} \theta^\alpha \theta^\alpha. \]  \hspace{1cm} (59)

The supercovariant derivatives are defined by
\[ D_\alpha = \partial_\alpha + i \frac{1}{2} \theta^\beta \partial_{\alpha \beta}, \]  \hspace{1cm} (60)
\[ \overline{D}_\alpha = \overline{\partial}_\alpha + i \frac{1}{2} \theta^\beta \partial_{\alpha \beta}, \]  \hspace{1cm} (61)

where
\[ \partial_{\alpha \beta} = \partial_\mu (\gamma_\mu)_{\alpha \beta}, \]  \hspace{1cm} (62)

satisfying
\[ \{D_\alpha, \overline{D}_\beta\} = i \partial_{\alpha \beta}. \]  \hspace{1cm} (63)

(We have used the notation \( \theta^* \) rather than the usual \( \bar{\theta} \) to avoid confusion with \( \bar{\lambda} \) defined earlier in the component formulation.) We also define
\[ d^2 \theta = \frac{1}{2} d\theta^\alpha d\theta_\alpha, \quad d^2 \theta^* = \frac{1}{2} d\theta^{*\alpha} d\theta^*_{\alpha}, \quad d^4 \theta = d^2 \theta d^2 \theta^*, \]  \hspace{1cm} (64)
so that
\[ \int d^2 \theta d^2 \theta^* = \int d^2 \theta^* d^2 \theta^* = -1. \]  \hspace{1cm} (65)

The vector superfield \( V(x, \theta, \theta^*) \) is expanded in Wess-Zumino gauge as
\[ V = i \theta^\alpha \theta^*_{\alpha \sigma} + \theta^\alpha \theta^*_{\alpha \beta} A_{\alpha \beta} - \theta^2 \theta^* \lambda^*_{\alpha} - \theta^*^2 \theta^\alpha \lambda^\alpha + \theta^2 \theta^*^2 D, \]  \hspace{1cm} (66)

and the chiral field is expanded as
\[ \Phi = \phi(y) + \theta^\alpha \psi^\alpha(y) - \theta^2 F(y), \]  \hspace{1cm} (67)

where
\[ y^\mu = x^\mu + i \theta \gamma^\mu \theta^*. \]  \hspace{1cm} (68)

The scalar, fermion and gauge propagators \( \Delta_S, \Delta_F \) and \( \Delta_V \) are given by (using a standard Feynman-type gauge)
\[ \Delta_S = \frac{1}{k^2}, \quad \Delta_F = \frac{k_\mu \gamma^\mu}{k^2}, \quad (\Delta_V)^{\mu \nu} = \frac{i \epsilon^{\mu \nu \rho \sigma} k_\rho}{k^2}. \]  \hspace{1cm} (69)

**References**

[1] A.S. Schwarz, *Lett. Math. Phys.* 2 (1978) 247; *Commun. Math. Phys.* 67 (1979) 1

[2] E. Witten, *Commun. Math. Phys.* 121 (1989) 351

[3] S. Deser, R. Jackiw and S. Templeton, *Ann. Phys.* 140 (1982) 372

8
Figure 1: Diagrams contributing to $\gamma_\phi^{(2)}$
Figure 2: Diagrams contributing to $\beta_{m^2}^{(2)}$

Figure 3: Diagrams contributing to $\beta_h^{(2)}$
[4] M. S. Bianchi, S. Penati and M. Siani, *JHEP* 1001 (2010) 080
[5] N. Akerblom, C. Saemann and M. Wolf, *Nucl. Phys.* B826 (2010) 456
[6] M.S. Bianchi, S. Penati and M. Siani, *JHEP* 1005 (2010) 106
[7] I. Jack, D. R. T. Jones and R. Wild, *Phys. Lett.* B509 (2001) 131
[8] I. Jack and D. R. T. Jones, *Phys. Lett.* B465 (1999) 148
[9] L. Randall and R. Sundrum, *Nucl. Phys.* B557 (1999) 79
[10] G.F. Giudice et al, *JHEP* 9812 (1998) 27
[11] A. Pomarol and R. Rattazzi, *JHEP* 9905 (1999) 013
[12] T. Gherghetta, G.F. Giudice and J.D. Wells, *Nucl. Phys.* B559 (1999) 27
[13] M.A. Luty and R. Rattazzi, *JHEP* 9911 (1999) 001
[14] Z. Chacko, M.A. Luty, I. Maksymyk and E. Ponton, *JHEP* 0004 (2000) 001
[15] E. Katz, Y. Shadmi and Y. Shirman, *JHEP* 9908 (1999) 015
[16] Y. Yamada, *Phys. Rev.* D50 (1994) 3530
[17] E.A. Ivanov, *Phys. Lett.* B268 (1991) 203
[18] I. Jack and D. R. T. Jones, *Phys. Lett.* B415 (1997) 383
[19] L. Girardello and M.T. Grisaru, *Nucl. Phys.* B194 (1982) 65; J.A. Helayël-Neto, *Phys. Lett.* B135 (1984) 78; F. Feruglio, J.A. Helayël-Neto and F. Legovini, *Nucl. Phys.* B249 (1985) 533; M. Scholl, *Z. Phys.* C28 (1985) 545
[20] S.J. Gates and H. Nishino, *Phys. Lett.* B72 (1992) 72
[21] R. Hodgson, I. Jack and D. R. T. Jones, *Nucl. Phys.* B728 (2005) 192
[22] I. Jack and D. R. T. Jones, *Phys. Lett.* B482 (2000) 167
[23] L.V. Avdeev, G.V. Grigoryev and D.I. Kazakov, *Nucl. Phys.* B382 (1992) 561
[24] L.V. Avdeev, D.I. Kazakov and I.N. Kondrashuk, *Nucl. Phys.* B391 (1993) 333
[25] J.H. Schwarz, *JHEP* 0411 (2004) 078
[26] H.C. Kao and K.M. Lee, *Phys. Rev.* D46 (1992) 4691
[27] I.L. Buchbinder, E.A. Ivanov, O. Lechtenfeld, N.G. Pletnev, I.B. Samsonov and B.M. Zupnik, *JHEP* 0910 (2009) 075
[28] B.M. Zupnik and D.V. Khetselius, *Sov. J. Nucl. Phys.* 47 (1988) 730

[29] J. Bagger and N. Lambert, *Phys. Rev.* D75 (2007) 045020; *Phys. Rev.* D77 (2008) 065008

[30] M. Benna, I. Klebanov, T. Klose and M. Smedback, *JHEP* 0809 (2008) 072

[31] O. Aharony, O. Bergman and D.L. Jafferis, *JHEP* 0811 (2008) 043

[32] M. Schnabl and Y. Tachikawa, *JHEP* 1009 (2010) 103

[33] D. Martelli and J. Sparks, *Phys. Rev.* D78 (2008) 126005

[34] A. Hanany and A. Zaffaroni, *JHEP* 0810 (2008) 111

[35] S. Franco, A. Hanany, J. Park and D. Rodriguez-Gomez, *JHEP* 0812 (2008) 110

[36] A. Hanany and Y.H. He, [arXiv:0811.4044](https://arxiv.org/abs/0811.4044)[hep-th]

[37] E. Imeroni, *JHEP* 0810 (2008) 026

[38] D.L. Jafferis and A. Tomasiello, *JHEP* 0810 (2008) 101

[39] D. Gaiotto and A. Tomasiello, *JHEP* 1001 (2010) 015; *J. Phys* A 42 (2009) 465205

[40] S. Hohenegger and I. Kirsch, *JHEP* 0904 (2009) 129

[41] D. Gaiotto and D.L. Jafferis, [arXiv:0903.2175](https://arxiv.org/abs/0903.2175)[hep-th]

[42] Y. Hikida, W. Li and T. Takayanagi, *JHEP* 0907 (2009) 065

[43] O. Aharony, O. Bergman, D.L. Jafferis and J. Maldacena, *JHEP* 0810 (2008) 091