Ginzburg-Landau theory for the time-dependent phase field in a two-dimensional $d$-wave superconductor

S.G. Sharapov$^a$, H. Beck$^a$, and V.M. Loktev$^b$

$^a$Institut de Physique, Universite de Neuchatel, Neuchatel, Switzerland

$^b$Bogolyubov Institute for Theoretical Physics, 03143 Kiev, Ukraine

We derive a finite temperature time-dependent effective theory for the phase $\theta$ of the pairing field, which is appropriate for a 2D conducting electron system with non-retarded $d$-wave attraction. As for $s$-wave pairing the effective action contains terms with Landau damping, but their structure appears to be different from the $s$-wave case due to the fact that the Landau damping is determined by the quasiparticle group velocity $v_g$, which for the $d$-wave pairing does not have the same direction as the non-interacting Fermi velocity $v_F$. We show that for the $d$-wave pairing the Landau terms have a linear low temperature dependence and in contrast to the $s$-wave case are important for all finite temperatures.

Key words: time-dependent Ginzburg-Landau theory, $d$-wave pairing

1. The microscopic derivation of the effective time-dependent Ginzburg-Landau (GL) theory continues to attract attention since an early paper by Abrahams and Tsuneto [1]. Whereas the static GL potential was derived from the microscopic BCS theory soon after its introduction, the time-dependent GL theory is still a subject of interest [2]. One of the reasons for this is the presence of Landau damping terms in the effective action. For $s$-wave superconductivity these terms are singular at the origin of energy-momentum space, and consequently they cannot be expanded as a Taylor series about the origin. In other words, these terms do not have a well-defined expansion in terms of space and time derivatives of the ordering field and therefore they cannot be represented as a part of a local Lagrangian. We recall that at $T = 0$ and for the static, time-independent case the Landau damping vanishes, so that either at $T = 0$ one still has a local well-defined time-dependent GL theory or for $T \neq 0$ the familiar static GL theory exists. It is known, however, that for $s$-wave superconductivity even though the Landau terms do exist, they appear to be small compared to the main terms of the effective action in the large temperature region $0 < T \lesssim 0.6T_c$, where $T_c$ is the superconducting transition temperature. This is evidently related to the fact that only thermally excited quasiparticles contribute to the Landau damping. The number of such quasiparticles at low temperatures is an exponentially small fraction of the total charge carriers number in the $s$-wave superconductor due to the nonzero superconducting gap $\Delta_s$ which opens over all directions on the Fermi surface.

For a $d$-wave superconductor there are four points (nodes) where the superconducting gap $\Delta_d(k) = 0$ on the Fermi surface. The presence of the nodes increases significantly the number of the thermally excited quasiparticles at given temperature $T$ comparing to the $s$-wave case. Therefore one can expect that the Landau damping is stronger for superconductor with a $d$-wave gap which is commonly accepted to be the case of high-temperature superconductors (HTSC). Moreover, it is believed that at temperatures $T \ll T_c$, these quasiparticles are reasonably well described by the Landau quasiparticles, even though such an approach fails in these materials at higher energies [3]. This is the reason why one...
can hope that a generalization of the BCS-like approach [2] for the 2D $d$-wave superconductivity may be relevant to the description of the low-temperature time-dependent GL theory in HTSC.

We derive such a theory from a microscopical model with $d$-wave pairing extending the approach of [2] developed for $s$-wave superconductivity. As known from [1] the physical origin of the Landau damping is a scattering of the thermally excited quasiparticles (normal fluid) with the group velocity $v_g$ from the phase (or $\theta$-) excitations (quanta). Such scattering occurs only if the $\tilde{\text{C}}$erenkov irradiation (absorption) condition, $\Omega = v_g K$ for the energy $\Omega$ and momentum $K$ of the $\theta$-excitation is satisfied. This phenomenon in superconductivity is also called Landau damping since its equivalent for the plasma theory was originally obtained by Landau.

One of the main physical differences between the $s$- and $d$-wave cases is related to the fact that for $d$-wave superconductivity the direction of the quasiparticle group velocity $v_g(k) \equiv \partial E(k)/\partial k$ ($E(k)$ is the quasiparticle dispersion law) does not coincide with the Fermi velocity $v_F$ [3,4] and a gap velocity $v_\Delta \equiv \partial \Delta(k)/\partial k$ also enters into the $\tilde{\text{C}}$erenkov condition along with $v_F$. We also show that the intensity of the Landau damping is proportional to $T$ at low temperatures.

2. We consider the Hamiltonian $H$ for fermions on the square lattice with the lattice constant $a$, the dispersion law $\xi(k)$ and the attractive potential $V(r)$ the momentum representation of which contains only $d$-wave pairing

$$H = \sum_\sigma \int dt \left\{ \int d^2r \psi_\sigma^\dagger(\tau, r) \xi(-i\nabla) \psi_\sigma(\tau, r) \right.$$

$$- \frac{1}{2} \int d^2r_1 \int d^2r_2 \psi_\sigma^\dagger(\tau, r_1) \psi_\sigma^\dagger(\tau, r_2) \psi_\sigma^\dagger(\tau, r_2) \psi_\sigma(\tau, r_1) \right\} .$$

(1)

Here $\psi_\sigma^\dagger(\tau, r)$ is a fermion field with the spin $\sigma = \uparrow$, $\downarrow$, $\bar{\sigma} \equiv -\sigma$ and $\tau$ is the imaginary time. The final results will be formulated in terms of the Fermi velocity $v_F \equiv \partial \xi(k)/\partial k|_{k_F}$ and the gap $v_\Delta$ velocities which proved to be very convenient both in the theory of $d$-wave superconductors [34] and for the analysis of various experiments [5].

The Hubbard-Stratonovich method is employed to derive the effective “phase-only” action (see the review [3] and Refs. therein). The present derivation has some specific features related to the non-local character of the interaction in coordinate space, so that a bilocal Hubbard-Stratonovich field has to be used [6] and an additional Born-Oppenheimer approximation is necessary to separate the terms describing a relevant phase dynamics from the rest of the effective action. The detail of this rather lengthy calculation will be presented elsewhere [6], so that here we present only the final results.

3. If the Landau terms are neglected it is possible to express the thermodynamical potential $\beta \Omega_{\text{kin}} = -i \int dt \int d^2r \mathcal{L}_0^\alpha(t, r) \beta \equiv 1/T$ in terms of a local effective Lagrangian

$$\mathcal{L}_0 = -\frac{n_f}{2} \dot{\theta}(t, r) + \frac{K}{2} |\theta(t, r)|^2 - \frac{J}{2} |\nabla \theta(t, r)|^2 ,$$

(2)

which is valid for $T \ll \Delta_d$, where $\Delta_d$ is the amplitude of the superconducting gap and $n_f$ is the carrier density. In [2]

$$J = \left( \frac{\sqrt{\pi} v_F v_\Delta}{2a} - \frac{\ln 2}{2\pi} \frac{v_F}{v_\Delta} T \right) , K = \frac{1}{4a\sqrt{\pi} v_F v_\Delta}$$

(3)

are the phase stiffness and compressibility, respectively. The Lagrangian [2] describes the collective phase excitations (Berezinskii-Kostelitz-Thouless mode) which is the 2D analog of the well-known 3D Bogolyubov-Anderson mode. The Landau terms which will be considered in what follows are, in fact, the corrections to [2] non-local in coordinate, which result in the damping of $\theta$-excitations.

4. The Landau terms originate from the terms

$$\sim \frac{(v_F k)^\alpha}{v_g k - \Omega - i0} \frac{d n_F(E)}{dE} v_g k \quad (\alpha = 0, 1, 2)$$

(4)

which are present in the momentum - real frequency representation of the effective action [3] ($n_F$ is the Fermi distribution). The proper treatment of these denominators leads to the following
result:
\[ \beta \Omega F_{\mathrm{lin}}^d = \frac{i}{2} \int d\Omega \int K dK \frac{K dK}{2\pi} \int_0^{2\pi} d\phi \frac{d\phi}{2\pi} \theta(\Omega, K) F_{v,d}^d(\Omega, K, \phi) \theta(-\Omega, -K). \]

For s-wave pairing [3] all three terms with \( \alpha = 0, 1, 2 \) result in \( F_s^d \sim \Omega^3/v_F K \) because \( v_g \parallel v_F \). In this case the intensity of the Landau damping does not depend on the direction of the vector \( K = (K \cos(\phi-\pi/4), K \sin(\phi-\pi/4)) \) in the plane.

(The angle \( \phi \) is chosen in such a way that \( \phi = \pi/4 \) corresponds to the first node of the \( d \)-wave gap when \( d \)-wave pairing is considered.) Furthermore, the damping process is exponentially suppressed by the factor \( \exp(-\Delta_s/T) \) (\( \Delta_s \) is the \( s \)-wave superconducting gap) reflecting a small number of thermally excited quasiparticles at \( T \ll \Delta_s \) which contribute into the Landau damping.

Since for \( d \)-wave pairing \( v_g \parallel v_F \), the terms with different \( \alpha \) do not produce the same analytical structure with all terms \( \sim \Omega/v_F K \) as for \( s \)-wave pairing. As the result we arrive at the following more complex expression:

\[
F_d^d(\Omega, K, \phi) = -\frac{T \ln 2}{v_\Delta 4\pi} \left( \frac{\Omega^3}{K v_F^2} f_1(\phi) + \frac{\Omega K f_2(\phi) - \Omega^2 \frac{2}{v_F} f_3(\phi)}{v_F} \right), \quad \frac{\Omega}{v_F K} \ll 1,
\]

where the functions \( f_1, f_2 \) and \( f_3 \) obtained in [3] describe the directional dependence of the corresponding damping term. The analytical expression for these functions are simple, but rather lengthy, so that here we show only the graphic for one them in Fig. 1. It appears that the calculation of ultrasonic attenuation for \( d \)-wave superconductivity [4] gives a result similar to the first term of [3], and indeed the angular dependence described by \( f_1 \) coincides with the dependence obtained in [3]. The presence of angular dependence in [3] demonstrates explicitly that the intensity of damping depends on the direction of \( \theta \)-particle motion with respect to the nodes on the Fermi surface. One can see from [3] that the Landau damping is linear in \( T \) and thus it is much stronger than for the \( s \)-wave case.

5. It is very important to recall that the collective phase excitations described here can and have been studied experimentally. Indeed the measurements of the order parameter dynamical structure factor in the dirty Al films allowed to extract the dispersion relation of the corresponding Carlson-Goldman mode and to investigate its temperature dependence [1]. The model considered here shows that it would be interesting to address experimentally the physics of the phase excitations in \( d \)-wave superconductors [10] which as we have demonstrated has many specific features.

This work was supported by the research project 2000-061901.00/1 and SCOPES-project 7UKPJ062150.00/1 of the Swiss National Science Foundation. The work of V.M.L. is partially supported by NATO grant CP/UN/19/C/2000/PO.

REFERENCES
1. E. Abrahams, T. Tsumeto, Phys. Rev. 152 (1966) 416.
2. I.J.R. Aitchison, G. Metikas, D.J. Lee, Phys. Rev. B 62 (2000) 6638.
3. A.C. Durst, P.A. Lee, Phys. Rev. B 62 (2000) 1270.
4. I. Vekhter, E.J. Nikol, J.P. Carbotte, Phys. Rev. B 59 (1999) 7129.
5. M. Chiao, R.W. Hill, C. Lupien, et al. Phys. Rev. B 62 (2000) 3554.
6. V.M. Loktev, R.M. Quick and S.G. Sharapov, Physics Reports, in press; preprint cond-mat/0012082.
7. H. Kleinert, Forts. der Physik 26 (1978) 565.
8. S.G. Sharapov, H. Beck, V.M. Loktev, preprint cond-mat/0012511.
9. R.V. Carlson, A.M. Goldman, Phys. Rev. Lett. 34 (1975) 11.
10. Y. Ohashi, S. Takada, Phys. Rev. B 62 (2000) 5971.