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Improving fuzzy rule interpolation performance with information gain-guided antecedent weighting

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Published online: 4 September 2017
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Abstract Fuzzy rule interpolation (FRI) makes inference possible when dealing with a sparse and imprecise rule base. However, the rule antecedents are commonly assumed to be of equal significance in most FRI approaches in the implementation of interpolation. This may lead to a poor performance of interpolative reasoning due to inaccurate or incorrect interpolated results. In order to improve the accuracy by minimising the disadvantage of the equal significance assumption, this paper presents a novel inference system where an information gain (IG)-guided fuzzy rule interpolation method is embedded. In particular, the rule antecedents in FRI are weighted using IG to evaluate the relative importance given the consequent for decision making. The computation of antecedent weights is enabled by introducing an innovative reverse engineering process that artificially converts fuzzy rules into training samples. The antecedent weighting scheme is integrated with scale and move transformation-based interpolation (though other FRI techniques may be improved in the same manner). An illustrative example is used to demonstrate the execution of the proposed approach, while systematic comparative experimental studies are reported to demonstrate the potential of the proposed work.

Keywords Fuzzy rule interpolation · Antecedent weighting · Reverse engineering

1 Introduction

Fuzzy set theory (Zadeh 1965) has gained rapid developments in a variety of scientific areas, including mathematics, engineering, and computer science. It has been successfully applied for many real-world problems, such as systems control, fault diagnosis and computer vision, as an effective tool to address the issues of imprecision and vagueness in modelling and reasoning. In particular, fuzzy expert systems have been developed using the idea of linguistic reasoning (also known as approximate reasoning), which reflects the way of cogitation of human beings and leads to new, more human, intelligent systems.

In general, an approximate reasoning system can be formalised as a fuzzy if–then rule-based inference mechanism that derives a conclusion given an input observation. Various techniques have been established to implement generalised modus ponens that facilitates reasoning when provided with imprecise inputs, mostly by following the basic idea of Compositional Rule of Inference (CRI) (Zadeh 1973). However, CRI is unable to draw a conclusion when a rule base is not dense but sparse. Sparse rule bases considered here are not referring to the quantity of rules in a given rule base, but the domain coverage of the antecedents of rules in the universe of discourse. That is, an input observation may have no over-

Communicated by P. Angelov, F. Chao.

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lap with any of the rules available and hence, no rule may be executed to derive the required consequent by applying CRI.

Fuzzy rule interpolation (FRI) (Kóczy and Hirota 1993a, b) plays a significant role in such sparse fuzzy rule-based reasoning systems. It addresses the limitation of conventional fuzzy reasoning that only uses CRI to perform inference, where the antecedents of all the rules within a given rule base cannot cover the whole problem domain. An estimation is able to be made by computing an interpolated consequent for the observation which has no rules matched.

A number of FRI methods have been proposed and improved in the literature (Hsiao et al. 1998; Chang et al. 2008; Huang and Shen 2006; Yang and Shen 2011; Yang et al. 2017; Jin et al. 2014). However, common approaches assume that the rule antecedents involved are of equal significance while searching for rules to implement interpolation. This can lead to inaccurate or incorrect interpolative results. This is because for many application of (fuzzy) decision systems, the decision is typically reached by an aggregation of conditional attributes, with each attribute making a generally different contribution to the decision making process. Weighted FRI methods (Diao et al. 2014) have therefore been introduced to remedy this equal significance assumption. For example, a heuristic method based on Genetic Algorithm is applied to learn the weights of rule antecedents (Chen and Chang 2011), but this leads to a substantial increase in computation overheads. An alternative work is to subjectively redefine the weights on the antecedents of the rules by experts, but this may restrict the adaptivity of the rules and, therefore, the flexibility of the resulting fuzzy system (Li et al. 2005).

In order to assess the relative significance of attributes with regard to the decision variable, information gain has been commonly utilised in data-driven learning algorithms (Mitchell 1997). By observing the property of information gains, this paper presents an innovative approach for rule interpolation. Information gain is integrated within an FRI process to estimate the relative importance of rule antecedents in a given rule base. The required information gains are estimated using an artificially generated decision table through a reverse engineering process which converts a given sparse rule base into a training data set. The proposed work helps minimise the disadvantage of the equal significance assumption made in common FRI techniques, thereby improving the performance of FRI. In particular, the paper presents an information gain-guided FRI method based on the popular scale and move transformation-based FRI (T-FRI) (Huang and Shen 2006). However, alternative FRI techniques may be employed for the same purpose if preferred.

The remainder of this paper is structured as follows. Section 2 outlines the background work that is required for the present development, including T-FRI, the basic concepts of information gain, and a simple iterative rule induction method (for providing the initial rule base). Section 3 describes the proposed information gain-guided fuzzy rule interpolation approach, with a case study illustrating its execution process. Section 4 details the results of comparative experimental evaluations, supported by statistical tests and analysis. Finally, Sect. 5 concludes the paper and points out several further studies.

2 Background work

This section presents an overview of FRI based on scale and move transformations, a description of an iterative rule generation technique, and an outline of the concept of information gain.

2.1 Transformation-based FRI

An FRI system can be defined as a tuple \((R, Y)\), where \(R = \{r^1, r^2, \ldots, r^N\}\) is a non-empty set of finite fuzzy rules (the rule base), and \(Y\) is a non-empty finite set of variables (interchangeably termed attributes). \(Y = A \cup \{z\}\) where \(A = \{a_j\mid j = 1, 2, \ldots, m\}\) is the set of antecedent variables, and \(z\) is the consequent variable appearing in the rules. Without losing generality, a given rule \(r^i \in R\) and an observation \(o^*\) can be expressed in the following format:

\[
\begin{align*}
\text{for } & i = 1 \text{ to } n \\
\text{where } & A_i \text{ represents the value (or fuzzy set) of the antecedent variable } a_j \text{ in the rule } r^i, \text{ and } z^i \text{ denotes the value of the consequent variable } z \text{ in } r^i.
\end{align*}
\]

A key concept used in T-FRI is the representative value \(\text{Rep}(A_j)\) of a fuzzy set \(A_j\), it captures important information such as the overall location in the domain of a fuzzy set and its shape. In general, given an arbitrary polygonal fuzzy set \(A = (a_1, a_2, \ldots, a_n)\) where \(a_i, i = 1, 2, \ldots, n\) denotes the vertex of the polygonal, its representative value \(\text{Rep}(A)\) is defined by (Huang and Shen 2008):

\[
\text{Rep}(A) = \sum_{i=1}^{n} w_i a_i
\]

where \(w_i\) is the weight assigned to the vertex \(a_i\). For simplicity, the weight of each vertex is typically assumed to be equal, i.e., \(w_i = 1/n\).

Much research has adopted triangular membership functions to perform interpolation, which are the most commonly used in fuzzy systems. A triangular membership function is denoted in the form of \(A_j = (a_{j1}, a_{j2}, a_{j3})\), where \(a_{j1}, a_{j3}\) represent the left and right extremities of the support (with membership values 0), and \(a_{j2}\) denotes the normal point (with
Fig. 1 Framework of transformation-based FRI

![Sparse Rule Base](image)

Selection of n Closest Rules → Construction of Intermediate Rule → Scale and Move Transformation → Interpolated Consequent

For such a fuzzy set \( A_j \), the midpoint \( \text{Rep}(A_j) \) is defined as the center of gravity of these three points:

\[
\text{Rep}(A_j) = \frac{a_{j1} + a_{j2} + a_{j3}}{3}  \tag{2}
\]

The definition of representative values for more complex membership functions can be found in (Huang and Shen 2008).

Given a sparse rule base \( R \) and an observation \( o^* \), as illustrated in Fig. 1, the T-FRI works as shown in Algorithm 1.

Without being able to find a rule that directly matches the given observation, the closest rules to the observation are identified and selected instead. The selection criterion is based on the Euclidean distance metric (though other distance metrics may be considered for an alternative), which measures the similarity between the observation \( o^* \) and each rule \( r^p \), \( p = 1, 2, \ldots, N \) in the sparse rule base. In general, the distance between an observation \( o^* \) and a rule \( r^q \), or indeed between any two rules \( r^p, r^q \in R \), is determined by computing the aggregated distances between all the corresponding values of the antecedent variables:

\[
d(v, r^q) = \sqrt{\sum_{j=1}^{m} (A^v_j - A^q_j)^2}  \tag{3}
\]

where \( v \) is \( o^* \) or \( r^p \) (so \( A^v_j \) is \( A^{o^*}_j \) or \( A^{r_p}_j \)), depending on whether the distance is between an observation and a rule or between two rules, and

\[
d(A^v_j, A^q_j) = \frac{|\text{Rep}(A^v_j) - \text{Rep}(A^q_j)|}{\max_{A_j} - \min_{A_j}}  \tag{4}
\]

is the normalised result of the otherwise absolute distance measure, so that distances are compatible with each other over different variable domains. The \( \max_{A_j} \) and \( \min_{A_j} \) in the denominator specify the maximal and minimal value of the antecedent \( A_j \) in its domain, respectively. In general, they will not be identical so that the calculation of the normalised distance between two antecedents [i.e., Eq. (4)] is valid mathematically. In the extreme case, however, the denominator may be zero, which indicates that all the antecedents in the domain of \( a_j \) are the same. In this case, the normalised dis-

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**Algorithm 1** Transformation-based FRI (T-FRI)

**Input:** Sparse rule base \( R \), Observation \( o^* \), Number of closest rules \( n \)

**Output:** Interpolated consequent \( z^* \)

- **Selection of Closest Rules:**
  1. Calculate distance \( d(o^*, r^p) \) between \( o^* \) and each rule \( r^p, p = 1, 2, \ldots, N \) in the sparse rule base \( R \);
  2. Choose \( n \) rules which have minimal distances as the closest \( n \) rules;

- **Construction of Intermediate Rule \( r^* \):**
  3. Assign normalised weights \( w^*_j, i = 1, \ldots, n \) to each the \( j \)th antecedent of the \( i \)th selected closest fuzzy rule;
  4. Aggregate each corresponding weighted antecedents of the \( n \) rules to obtain the antecedent of intermediate rule \( A^*_j, j = 1, 2, \ldots, m \);
  5. Calculate weights to each consequent of the \( i \)th selected closest fuzzy rule \( w^*_j \), which is the mean of the normalised weights associated with the antecedents \( w^*_j \) in each rule:

\[
w^*_i = \frac{1}{m} \sum_{j=1}^{m} w^*_j
\]

- **Computation of Scale and Move Factors:**
  6. The consequent variable of the intermediate rule \( z^* \) is constructed with the weighted aggregation of the consequent variables of \( n \) closest rules \( z^*_i, i = 1, \ldots, n \);

- **Calculation of \( z^* \) via Scale and Move Transformation:**
  7. For Each antecedent do
  8. Scale operation: from \( A^*_j \) to \( A^s_j \) (denoting the scaled intermediate fuzzy set), in an effort to determine the scale rate \( s_{A^*_j} \);
  9. Move operation: from \( A^s_j \) to \( A^m_j \) to obtain a move ratio \( m_{A^*_j} \);
  10. end for

- **Return \( z^* \)**
  11. Compute transformation factors for \( z^* \) by averaging the corresponding values:

\[
s_z = \frac{1}{m} \sum_{j=1}^{m} s_{A^*_j} \quad m_z = \frac{1}{m} \sum_{j=1}^{m} m_{A^*_j}
\]

12. Scale and Move the fuzzy set of the intermediate consequent \( z^* \) with the above calculated factors, resulting in the required interpolated result \( z^* = T(z^*, s_z, m_z) \);

13. Return \( z^* \)
Interpolation via scale and move transformations

The above process of scale and move transformations in an effort to interpolate the consequent variable can be summarised in Fig. 2, which can be collectively and concisely represented by: 

$$z^* = T(z', s_z, m_z),$$

highlighting the importance of the two key transformations required. The detailed computation involved in T-FRI can be referred to the original work (Huang and Shen 2006, 2008).

2.2 Information gain

Information gain has been widely adopted in the development of learning classifier algorithms, to measure how well a given attribute may separate the training examples according to the underlying classes (Mitchell 1997). It is defined via the entropy metric in information theory (Shannon 2001), which is commonly used to characterise the disorder or uncertainty of a system.

Formally, let \( O = (O, p) \) be a discrete probability space, where \( O = \{o_1, o_2, \ldots, o_n\} \) is a finite set of domain objects, with each having the probability \( p_i, i = 1, \ldots, n \). Then, the Shannon entropy of \( O \) is defined by

$$\text{Entropy}(O) = - \sum_{i=1}^{n} p_i \log_2 p_i$$

Regarding the task of classification, \( o_i, i = 1, \ldots, n \) represents a certain object, and \( p_i \) is the proportion of \( O \) which is labelled as the class \( j, j = 1, \ldots, m, m \leq n \). Note that the entropy is at its minimum (i.e., \( \text{Entropy}(O) = 0 \)) if all elements of \( O \) belong to the same class (with \( 0 \log_2 0 = 0 \) defined), and the entropy reaches its peak point (i.e., \( \text{Entropy}(O) = \log_2 n \)) if the probability of each category is equal; otherwise, the entropy is between 0 and \( \log_2 n \).
Intuitively, the less the entropy value, the easier the classification problem. It is based on this observation that information gain has been introduced to measure the expected reduction in entropy caused by partitioning the values of an attribute. This leads to the popular decision tree learning methods (Quinlan 1986). Given a collection of examples $U = \{O, A\}$, $o_i \in O$ ($i = 1, \ldots, n$) is an object which is represented with a group of attribute $A = \{a_1, \ldots, a_l\}$ and a class label $m$. Information gain upon a particular attribute $a_k$, $k \in \{1, \ldots, l\}$, is defined as

$$IG(O, a_k) = \text{Entropy}(O) - \sum_{v \in \text{Value}(a_k)} \frac{|O_v|}{|O|} \text{Entropy}(O_v)$$

(6)

where Value($a_k$) is the set of all possible values for the attribute $a_k$, $O_v$ is the subset of $O$ where the value of the attribute $a_k$ is equal to $v$ (i.e., $O_v = \{o \in O | a_k(o) = v\}$), and $|\cdot|$ denotes the cardinality of a set.

From the perspective of entropy evaluation over $U$, the second part of Eq. (6) shows that the entropy is measured via weighted entropies that are calculated over the partition of $O$ using the attribute $a_k$. The bigger the value of information gain $IG(O, a_k)$, the better the partitioning of the given examples with $a_k$. Obtaining a high information gain, therefore, implies achieving a significant reduction of entropy or uncertainty caused by considering the influence of that attribute.

### 2.3 Iterative rule base generation

A data-driven rule base learning mechanism intuitively extracts rules from raw data to generate a rule base, which are in the format of antecedents associated with a corresponding consequent (Wang and Mendel 1992; Hong and Lee 1996). Rule base generation can also follow an iterative procedure (Hoffmann 2004; Galea and Shen 2006) to incrementally add new rules to the rule base. This section outlines an iterative rule base generation procedure, which repeatedly sequentially extracts rules from data into an emerging rule base.

Given a set of instances which consist of $r$ antecedent attributes and a consequent attribute, a rule base is generated in an iterative procedure as illustrated in Algorithm 2. Here, fuzzy rules are considered for generality, which may be readily degenerated into a crisp rule set if preferred. The iteration process is terminated by checking against a pre-set threshold value that determines at least how many data points have been covered by the extracted rules so far.

Before the iterative procedure is executed to generate the rule base, the domains of all $r$ antecedent attributes and the consequent attribute are quantified evenly into $m_1, m_2, \ldots, m_r$ and $m_c$ fuzzy regions, respectively, where $m_c$ denotes the number of regions for the consequent attribute. Each fuzzy region is assigned with a membership function (implemented with triangular membership functions in this work for simplicity). This results in a division of fuzzy region space of the antecedent of an emerging rule in the form of a hypercube, of which each hypergrid stands for those fuzzy values associated with the corresponding hypergrid.

The hypergrid which has the highest hits is larger than the threshold, a rule is extracted from this hypergrid and be added into the rule base $R$.

The iteration process begins with the complete data set of instances $D$. A hypergrid hit by an instance indicates the largest value of membership is obtained for the corresponding combination of fuzzy regions of the $r$ antecedent attributes.

The rule antecedent values returned by this iteration are those fuzzy values associated with the corresponding hypergrid. The rule consequent adopts the fuzzy value which corresponds to one of the $m_c$ values at which the instances have the highest number of hits. After this, those instances hit in this hypergrid are removed from the original data set, and the iterative process repeats by treating the remaining data as the input data set to start the next round for the generation of the rules following the current one. However, if the proportion of hit instances is less than $\delta$, a rule cannot be generated by this hypergrid because those small number of hits may just be due to noise, and the iterative procedure is hence terminated.

### Algorithm 2 Iterative Rule Extraction

**Input:** Data set of instances $D$, threshold value $\delta$

**Output:** Rule base $R$

1. Divide the domain of each antecedent and consequent attribute evenly into a certain number of fuzzy regions, and construct the fuzzy region space (FRS) of the antecedent, which is a hypercube with the dimensionality of $m_1 \times m_2 \times \ldots \times m_r$, where $m_i$, $i = 1, \ldots, r$ stands for the number of regions for the $i$th attribute;
2. while true do
3. Apply the data of instances $D$ into the FRS and match each instance to a corresponding hypergrid in the hypercube in terms of its antecedent attributes;
4. Remove all of the instances which hit this hypergrid from $D$;
5. if $n > \delta$ then
6. Extract a rule from this hypergrid, and add it into the rule base $R$;
7. Remove all of the instances which hit this hypergrid from $D$,
8. else
9. End While Loop;
10. return $R$.
This simple iterative rule generation procedure will be used to learn a rule base to construct the inference system proposed in Sect. 3 (assuming no rules are provided by domain experts). If the generated rule base is dense, any standard fuzzy rule inference technique (e.g., compositional rule of inference (CRI)) can be employed to perform classification once a new input observation is provided. Otherwise, the observation is used as the input to the fuzzy rule interpolation process if it does not match any learned rules. Of course, if it matches a certain rule in the space rule base, CRI will be used as usual.

3 Antecedent weighted T-FRI

This section presents a novel technique for fuzzy rule interpolation which is guided with antecedent weights obtained by information gain. The proposed inference system is illustrated in Fig. 3. The iterative rule learning procedure presented in Sect. 2.3 generates the rule base from data. The scale and move transformation-based fuzzy rule interpolation (T-FRI) is utilised to work with information gain here. Note that the computation on information gain precedes, and its results are used for, all three key stages in T-FRI. The antecedent weighted T-FRI using information gains is described in the following with an illustrative example to show how it works.

3.1 Illustrative case

To illustrate the proposed work, a simple fuzzy classification problem (Yuan and Shaw 1995) is utilised here, involving a small set of training data of 16 instances. The system is set to make a decision on what sports activity to be undertaken (namely, volleyball, swimming and weight lifting) given the status of four conditional attributes regarding the weather, in terms of temperature (hot, mild and cool), outlook (sunny, cloudy and rain), humidity (humid and normal) and wind (windy and not windy).

Six fuzzy rules have been generated as given below. However, these six rules form a dense rule base where the domains of the antecedent variables are completely covered by the rules. To facilitate the illustration (of interpolation), Rule 6 is purposefully removed to have a sparse rule base.

1. If Temperature is Hot and Outlook is Sunny, then Swimming.
2. If Temperature is Hot and Outlook is Cloudy, then Swimming.
3. If Outlook is Rain, then Weight lifting.
4. If Temperature is Mild and Wind is Windy, then Weight lifting.
5. If Temperature is Mild and Wind is Not Windy, then Volleyball.
6. (If Temperature is Cool, then Weight lifting.)

3.2 Turning rules into training data via reverse engineering

Given a rule base, the proposed information gain-guided T-FRI begins with a reverse engineering procedure which converts the rules into a set of artificial training samples, forming a decision table for the calculation of required information gains. This development is based on the examination of how T-FRI performs its task. Its first key stage is the selection of \(n\) closest fuzzy rules when an observation is presented (which does not match with any existing rule in the sparse rule base and hence, CRI is not applicable).

In conventional T-FRI algorithms, all antecedent attributes of the rules are assumed to be of equal significance while searching for a subset of rules closest to the observation since the original approaches are unable to assess, nor to make use of, the relative importance or ranking of these antecedent attributes. Information gain offers such an intuitively sound and implementation-wise straightforward mechanism for evaluating the relative significance of attributes.

The question is what data are available to act as the learning examples for computing the information gains. T-FRI works with a sparse rule base. When an observation is given, it is expected to produce an interpolated result for the consequent variable. Without losing generality, it is practically presumed that there is no sufficient example data available for use to support the computation of the required information gains due to the sparseness of domain knowledge. However, any T-FRI method does use a given sparse rule base involving a set of antecedent variables \(Y = A \cup \{z\}\) (as shown in Sect. 2.1). This set of rules can be translated into an artificial decision table (i.e., a set of artificially generated training examples), where each row represents a particular rule. In any data-driven learning mechanism, rules are learned from given data samples. Translating rules back to data is therefore a reverse engineering process of data-driven learning.

Generally speaking, a sparse rule-based system may involve rules that use different numbers of antecedent variables and even different variables in the first place. In order to employ the proposed reverse engineering procedure to obtain a training decision table, all rules are reformulated into a common representation by the following two-step procedure:

- Identifying all possible antecedent variables appearing in the rules and all value domains for these variables, and
- Expanding iteratively each existing rule into one which involves all domain variables such that if a certain antecedent variable is not originally involved in a rule,
then that rule is replaced by $q$ rules, with $q$ being the cardinality of the value domain of that variable, so that the variable within each of the expanded rule takes one possible and different value from its domain.

The above procedure makes logical sense. This is because for any rule, if a variable is missing from the rule antecedent, it means that it does not matter what value it takes and the rule will lead to the same consequent value, provided that those variables that do appear in the rule are satisfied.

Given the rule base of Sect. 3.1 which may be reformulated as given in Table 1. Following the two-step procedure, 32 training data are generated as listed in Table 9 in “Appendix A”. The reverse engineering process can be explained using the illustrative case. Without losing generality, assume that the first given rule is used to create the artificial data first. Then, part of the emerging artificial decision table is constructed from this rule first. Note that Humidity and Wind are missing in Rule 1, which means if Temperature is satisfied with the value Hot and Outlook with Sunny, the rule is satisfied and thus, the consequent variable Decision will have the value Swimming no matter which value Humidity and Wind takes. That is, Rule 1 can be expanded by the first four data in Table 9, each having...
the variable *Humidity* and *Wind* taking one of its two possible values. Similarly, more artificial data can be created by translating and expanding the remaining original rules.

Comparing both the antecedent values and the consequent in Table 9, it can be seen that there are several identical samples which are generated from different original rules. Retaining one of them results in a total of 30 training data. Note that in such an artificially constructed decision table, it may appear to include inconsistent data since they may have the same values for the respective antecedent attributes but different consequents (e.g., two inconsistent pairs are italicised in Table 9). This does not matter as the eventual rule-based inference, including rule interpolation does not use these artificially generated rules, but the original sparse rule base. They are created just to help assess the significance of individual variables through the estimation of their respective information gains. It is because there are variables which may lead to potentially inconsistent implications in a given problem that it is possible to distinguish the different abilities of the variables in possessing the power in influencing the consequent. This in turn enables the measuring of the information gains of individual antecedent variables as described below.

### 3.3 Weighting of individual variables

Given an artificial decision table that is derived from a sparse rule base via reverse engineering, the information gain $IG_i$ of a certain antecedent variable $a_i$, $i = 1, \ldots, m$, regarding the consequent variable $z$ is calculated as per Eq. (6):

$$IG_i = \text{Entropy}([z]) - \sum_{v \in \text{Value}(a_i)} \frac{|\{z\}_v|}{|\{z\}|} \text{Entropy}([\{z\}_v])$$

(7)

where $[z]_v$ denotes the subset of rules in the artificial decision table in which the antecedent variable $a_i$ has the value $v$. Repeating the above, the information gains for all antecedent variables $IG_i$, $i = 1, \ldots, m$ can be computed. These values are then normalised into $IG_i$, $i = 1, \ldots, m$ such that

$$IG_i = \frac{IG_i'}{\sum_{i=1,\ldots,m} IG_i'}$$

(8)

| Rules | Variables | Outlook | Humidity | Wind | Decision |
|-------|-----------|---------|----------|------|----------|
| $r^1$ | Hot       | Sunny   | –        | –    | Swimming |
| $r^2$ | Hot       | Cloudy  | –        | –    | Swimming |
| $r^3$ | –         | Rain    | –        | –    | Weight lifting |
| $r^4$ | Mild      | –       | –        | Windy | Weight lifting |
| $r^5$ | Mild      | –       | –        | Not windy | Volleyball |

| Rules | Variables | $a_1$ | $a_2$ | $\cdots$ | $a_m$ | $z$ |
|-------|-----------|-------|-------|-----------|-------|-----|
| $r^1$ | $A_1^1$   | $A_1^2$ | $\cdots$ | $A_m^1$ | $z_1$ |
| $r^2$ | $A_1^2$   | $A_1^2$ | $\cdots$ | $A_m^2$ | $z_2$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ | $\vdots$ |
| $r^N$ | $A_1^N$   | $A_1^N$ | $\cdots$ | $A_m^N$ | $z_N$ |

| Antecedent | Temperature | Outlook | Humidity | Wind |
|------------|-------------|---------|----------|------|
| Normalised IG | 0.5000 | 0.4515 | 0.0000 | 0.0485 |

Given the inherent meaning of information gain, the resulting normalised values can be intuitively interpreted as the relative significance degrees of the individual rule antecedent attributes in the determination of the rule consequent. Therefore, they can be used to act as the weights associated with each individual antecedent variable in the original sparse rule base. In general, through this procedure, an original decision table such as the one shown in Table 1 becomes Table 2 (where $N$ is the number of the distinct rules generated by the procedure), with a weight added to each antecedent variable.

Recall the example case. The normalised information gains calculated for each antecedent variable using those 30 training samples are shown in Table 3. The information gain of the antecedent attribute *Temperature* is relatively higher than the other three, which indicates *Temperature* plays a much important role in the decision on the sports activity. This can be verified from the five fuzzy rules where the antecedent variable *Temperature* appears in 4 rules. On the other hand, *Humidity* and *Wind* are assigned a very small amount of weight. In particular, the normalised IG of *Humidity* is 0, signifying its irrelevance on the decision in this rule base.
3.4 Weighted T-FRI

Given the weights associated with the rule antecedent attributes T-FRI can be modified. Such modification will involve three key stages as detailed below.

3.4.1 Weight-guided selection of n closest rules

First of all, when an observation is present which does not entail a direct match with any rule in the sparse rule base, \( n \) \(( n \geq 2)\) closest rules to it are required to be chosen to perform rule interpolation. The original selection is based on the Euclidean distance measured by aggregating the distances between individual antecedent variables of a certain rule and the corresponding variable values in the observation [as per Eq. (3)]. Considering the weights assessed by information gain, the distance between a given rule \( r^p \) and the observation \( o^* \) can now be calculated by

\[
\tilde{d}(r^p, o^*) = \frac{1}{\sum_{j=1}^{m} IG_j} \sqrt{\sum_{j=1}^{m} (IG_j d(A^p_j, A^*_j))^2} \tag{9}
\]

where \( d(A^p_j, A^*_j) \) is computed according to Eq. (4).

Choosing the \( n \) closest rules this way allows those rules which involve certain antecedent variables that are regarded more significant to be selected with priority. Note that the normalisation term \( \frac{1}{\sum_{j=1}^{m} IG_j} \) is a constant and, therefore, can be omitted in computation since the purpose of calculating the distance \( \tilde{d}(r^p, o^*) \) is in order to rank the rules and only information on the relative distance measures is required.

To continue illustration with the case study, suppose that the membership functions used to describe the antecedent and consequent variables are defined as given in Fig. 5 of “Appendix B”. Also, suppose that the observation of Table 4 (involving only singleton fuzzy sets) is presented, resulting in the membership values for the observation as shown in the bottom of row of Table 4. This does not match with any of the rules in the sparse rule base. Thus, no rule in the sparse rule base can be fired directly and FRI is applied to derive a conclusion. Both the information gain-guided T-FRI (IG-T-FRI) and the original T-FRI are employed here for comparison. Given the rule base and the observation, the 2 closest rules selected by T-FRI and those by IG-T-FRI are different, with Rules 4 and 5 and Rules 3 and 5 are selected by T-FRI and IG-T-FRI, respectively.

3.4.2 Weighted parameters for intermediate-rule construction

Unlike the conventional T-FRI, the significance of individual antecedent variables is captured and reflected in their contribution towards the derivation of the (interpolated) consequent, by the use of their associated weights. To emphasise this, weights are integrated in all calculations during the transformation process, including the initial construction of the intermediate rule. In particular, the weighting on the consequent \( w^*_c \) is now computed as follows:

\[
\tilde{w}^* = \sum_{j=1}^{m} IG_j w^*_j \tag{10}
\]

This is a direct extension to the original construction process of the intermediate rule as shown in step 5 of Algorithm 1 where all variables are equally regarded in terms of their significance. Referring to Eq. (8), it is clear that if antecedent attributes are of equal significance, \( \tilde{w}^*_c \) degenerates to \( w^*_c \).

3.4.3 Weighted transformation

In performing the scale and move transformations, the previous computation of the required scale and move factors, namely those equations in step 11 of Algorithm 1, is now modified to:

\[
\tilde{z} = \sum_{j=1}^{m} IG_j s_{A_j}, \quad \tilde{m} = \sum_{j=1}^{m} IG_j m_{A_j} \tag{11}
\]

From these modifications, given an observation (that does not match with any rule in the sparse rule base), an interpolated consequent variable \( \tilde{z}^* \) can be obtained by performing the transformation \( T(\tilde{z}', \tilde{z}, \tilde{m}) \). Note that when all weights are equal, i.e., when all antecedent variables are assumed to be of equal significance, the above modified version degenerates to the original T-FRI. Mathematical proof for this is straightforward and, hence, omitted here.

**Table 4** Observation in illustrative example

| Antecedent attribute | Temperature | Outlook | Humidity | Wind |
|----------------------|-------------|---------|----------|------|
| Observed value       |             |         |          |      |
| Membership value     | Hot         | Mild    | Cool     | Rain | Humid | Normal | Windy | Not windy |
|                      | 0.0         | 0.0     | 0.775    | 0.0  | 0.733 | 0.5    | 0.49  | 0.51      |

3.4 Weighted T-FRI

Given the weights associated with the rule antecedent attributes T-FRI can be modified. Such modification will involve three key stages as detailed below.

### 3.4.1 Weight-guided selection of n closest rules

First of all, when an observation is present which does not entail a direct match with any rule in the sparse rule base, \( n \) \(( n \geq 2)\) closest rules to it are required to be chosen to perform rule interpolation. The original selection is based on the Euclidean distance measured by aggregating the distances between individual antecedent variables of a certain rule and the corresponding variable values in the observation [as per Eq. (3)]. Considering the weights assessed by information gain, the distance between a given rule \( r^p \) and the observation \( o^* \) can now be calculated by

\[
\tilde{d}(r^p, o^*) = \frac{1}{\sum_{j=1}^{m} IG_j} \sqrt{\sum_{j=1}^{m} (IG_j d(A^p_j, A^*_j))^2} \tag{9}
\]

where \( d(A^p_j, A^*_j) \) is computed according to Eq. (4).

Choosing the \( n \) closest rules this way allows those rules which involve certain antecedent variables that are regarded more significant to be selected with priority. Note that the normalisation term \( \frac{1}{\sum_{j=1}^{m} IG_j} \) is a constant and, therefore, can be omitted in computation since the purpose of calculating the distance \( \tilde{d}(r^p, o^*) \) is in order to rank the rules and only information on the relative distance measures is required.

To continue illustration with the case study, suppose that the membership functions used to describe the antecedent and consequent variables are defined as given in Fig. 5 of “Appendix B”. Also, suppose that the observation of Table 4 (involving only singleton fuzzy sets) is presented, resulting in the membership values for the observation as shown in the bottom of row of Table 4. This does not match with any of the rules in the sparse rule base. Thus, no rule in the sparse rule base can be fired directly and FRI is applied to derive a conclusion. Both the information gain-guided T-FRI (IG-T-FRI) and the original T-FRI are employed here for comparison. Given the rule base and the observation, the 2 closest rules selected by T-FRI and those by IG-T-FRI are different, with Rules 4 and 5 and Rules 3 and 5 are selected by T-FRI and IG-T-FRI, respectively.

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Unlike the conventional T-FRI, the significance of individual antecedent variables is captured and reflected in their contribution towards the derivation of the (interpolated) consequent, by the use of their associated weights. To emphasise this, weights are integrated in all calculations during the transformation process, including the initial construction of the intermediate rule. In particular, the weighting on the consequent \( w^*_c \) is now computed as follows:

\[
\tilde{w}^* = \sum_{j=1}^{m} IG_j w^*_j \tag{10}
\]

This is a direct extension to the original construction process of the intermediate rule as shown in step 5 of Algorithm 1 where all variables are equally regarded in terms of their significance. Referring to Eq. (8), it is clear that if antecedent attributes are of equal significance, \( \tilde{w}^*_c \) degenerates to \( w^*_c \).

### 3.4.3 Weighted transformation

In performing the scale and move transformations, the previous computation of the required scale and move factors, namely those equations in step 11 of Algorithm 1, is now modified to:

\[
\tilde{z} = \sum_{j=1}^{m} IG_j s_{A_j}, \quad \tilde{m} = \sum_{j=1}^{m} IG_j m_{A_j} \tag{11}
\]

From these modifications, given an observation (that does not match with any rule in the sparse rule base), an interpolated consequent variable \( \tilde{z}^* \) can be obtained by performing the transformation \( T(\tilde{z}', \tilde{z}, \tilde{m}) \). Note that when all weights are equal, i.e., when all antecedent variables are assumed to be of equal significance, the above modified version degenerates to the original T-FRI. Mathematical proof for this is straightforward and, hence, omitted here.

**Table 4** Observation in illustrative example

| Antecedent attribute | Temperature | Outlook | Humidity | Wind |
|----------------------|-------------|---------|----------|------|
| Observed value       |             |         |          |      |
| Membership value     | Hot         | Mild    | Cool     | Rain | Humid | Normal | Windy | Not windy |
|                      | 0.0         | 0.0     | 0.775    | 0.0  | 0.733 | 0.5    | 0.49  | 0.51      |
Differently, the intermediate rule created by the two closest rules, Rules 4 and 5 using T-FRI is:

If Temperature is (0.61,0.91,1.21) and Outlook is (0.42, 0.42,0.42) and Humidity is (0.50,0.50,0.50) and Wind is (0.01,0.51,1.01), then Decision is (2.51,2.51,2.51)

Given the simplified case where observations are all singleton fuzzy sets, the above intermediate results imply that the final interpolated result with IG-T-FRI is \(\tilde{z}^* = (2.49,2.49,2.49)\), using the IG-guided transformation \(T(\tilde{c'} = (2.49,2.49,2.49), \tilde{s}_c = 0, m_{\tilde{s}} = 0)\), and that the result with the standard T-FRI is \(z^* = (2.51,2.51,2.51)\), using a transformation of \(T(\tilde{c'} = (2.51,2.51,2.51), \tilde{s}_c = 0, m_{\tilde{s}} = 0)\). From this, through defuzzification (to obtain a classification result), the conclusions drawn by the use of these two different methods are Weight lifting and playing Volleyball, respectively. Clearly, the outcome of applying IG-T-FRI has a better intuitive appeal given the particular observation. Indeed, recall the original rule base for this illustrative case given in (Yuan and Shaw 1995), the observation used for illustration actually matches Rule 6 (i.e., the one purposefully removed to form a sparse rule base). This results in the same decision if fired as the interpolated consequent derived by the proposed IG-T-FRI method.

The workflow of the construction of the intermediate rule and of the computation of the interpolative results for both methods is outlined in Fig. 6 in “Appendix C”.

This illustrative case is very simple, involving only a small number of instances and a rather specific rule base. It is therefore not surprising that similar interpolated values may result by the use of either the original T-FRI or the proposed IG-T-FRI. Even though the above still demonstrates the strength of the proposed approach, the following section will systematically evaluate such strength using more complicated datasets.

### 4 Experimental evaluation

This section presents a systematic experimental evaluation of the proposed inference system, where the information gain-guided T-FRI approach is embedded. The work is assessed for the task of performing pattern classification over nine benchmark datasets. Classification results are compared with those obtained by the original T-FRI method and also, with the standard Mamdani inference (Mamdani and Assilian 1999) without involving rule interpolation but directly firing those (possibly partially) matched rules. In addition, a statistical analysis is utilised to further evaluate the performance of the proposed approach over the original T-FRI.

#### 4.1 Experimental set-up

##### 4.1.1 Datasets

The nine benchmark datasets are taken from the UCI machine learning (A Asuncion 2007) and KEEL (Knowledge Extraction based on Evolutionary Learning) (Alcalá et al. 2010) dataset repositories, with their details summarised in Table 5.

| Dataset   | Attributes # | Classes # | Instances # |
|-----------|--------------|-----------|-------------|
| Iris      | 4            | 3         | 150         |
| Diabetes  | 8            | 2         | 768         |
| Phoneme   | 5            | 2         | 5404        |
| Appendicitis | 7       | 2         | 106         |
| Magic     | 10           | 2         | 1902        |
| NewThyroid| 5            | 3         | 215         |
| Banana    | 2            | 2         | 5300        |
| Haberman  | 3            | 2         | 306         |
| Monk-2    | 6            | 2         | 432         |

#### 4.1.2 Experimental methodology

Triangular membership functions are used to represent the fuzzy sets of the antecedent variables due to their popularity and simplicity. Given that the problems are all for classification, the consequent variable always adopts a singleton fuzzy set (i.e., a crisp value) as its value. In general, different variables have their own underlying domains. However, to simplify knowledge representation, these domains are normalised to take a value from the common range of 0 to 1, as illustrated in Fig. 4. Note that such a simple fuzzification is used in this work; no optimisation of the value domain is carried out. This is used for all methods under comparison. A fine-tuned definition of the membership functions will no doubt improve the performance of the classification results.

Experiments are validated by tenfold cross-validation which is repeated for 10 times per dataset. The rule base
is generated from training data after the fuzzification by the presented iterative rule induction method. In particular, the domain interval of antecedent variables is divided into three fuzzy regions, and the threshold δ is empirically set to 2 in order to determine whether to promote a rule into the emerging rule base. 10% of the learned rules are deliberately removed randomly, ensuring that the resultant rule base is sparse. The information gains for weighting are computed using an artificial decision table which is translated from the learned rule base. The number of the closest rules to perform rule interpolation is set to 2, which is commonly used in the existing literature. The classification performance is assessed in terms of accuracy over the testing data. A statistical t test (p = 0.05) is utilised to determine the statistical significance of the improvement of the information gain-guided T-FRI over the original T-FRI for each of the nine data set.

4.2 Results and discussion

4.2.1 Comparison on overall classification accuracy

Table 6 shows the classification performance over the nine datasets, measured with the average accuracy and the standard deviation (SD) through a process of 10 × 10 cross-validation. In particular, the column of CRI presents the results obtained using the compositional rule of inference directly by firing those matched rules only; the T-FRI column shows the results obtained by the use of the original T-FRI; and the IG-T-FRI column summaries the results obtained using the information gain-guided T-FRI approach. A pairwise t test (p = 0.05) validates the experimental evaluation furthermore. Note that the asterisk (*) after a result in the column T-FRI indicates that the improvement made by the original T-FRI over CRI is statistically significant, and similarly the asterisk (*) in the IG-T-FRI column shows that the improvement made by IG-T-FRI is in turn, statistically significant over T-FRI.

The accuracies achieved by CRI indicate the sparseness of the rule base, which is expected to have a relatively poorer performance in classification. Both the interpolative reasoning approaches clearly show their significant advantage in dealing with the sparse rule base. Importantly, the information gain-guided T-FRI method has consistently achieved better classification accuracies over all nine datasets, with an overall 5.9% higher accuracy than that reachable by the original T-FRI and, a 23.95% improvement over the use of CRI which only fires matched rules without any rule interpolation method. The SD values compared between the three methods indicate that more robust classification performance is achieved by IG-T-FRI also. Together, these results clearly demonstrate the potential of the proposed work.

4.2.2 Comparison on false negatives and false positives

Apart from the classification accuracies, in many real-world applications, it is worth to examine the statistical rates on true positives (TP), true negatives (TN), false positives (FP) and false negatives (FN). Without overly complicating the experimental investigation while having a focused discussion, the Diabetes dataset, as a binary classification problem, is selected for this comparison. Tables 7 and 8 show the confusion matrices computed by the use of the original T-FRI and that of IG-T-FRI, respectively. ‘Positive’ in both tables is interpreted as an instance in which a person is diagnosed to have diabetes. The numbers shown in both tables are computed by averaging the results obtained in 10 × 10 cross-validation.

First of all, recall the results shown in Table 6, the classification accuracy of T-FRI is 62.5%, which is improved to 68.49% using IG-T-FRI. As can be seen from comparing Tables 7 and 8, as the classification precision increases with

| Dataset     | CRI (± SD)  | T-FRI (± SD) | IG-T-FRI (± SD) |
|-------------|-------------|-------------|-----------------|
| Iris        | 66.66 ± 0.25| 76.99 ± 0.16*| 82.53 ± 0.13*   |
| Diabetes    | 32.10 ± 0.08| 62.50 ± 0.06*| 68.49 ± 0.05*   |
| Phone     | 38.40 ± 0.09| 60.53 ± 0.05*| 66.18 ± 0.07*   |
| Appendicitis| 32.27 ± 0.10| 57.72 ± 0.12*| 69.69 ± 0.13*   |
| Magic      | 49.15 ± 0.05| 58.40 ± 0.09*| 64.67 ± 0.05*   |
| NewThyroid | 43.33 ± 0.28| 47.43 ± 0.24*| 53.28 ± 0.22*   |
| Banana      | 44.83 ± 0.08| 60.49 ± 0.05*| 63.27 ± 0.04*   |
| Haberman   | 54.00 ± 0.09| 71.73 ± 0.08*| 77.47 ± 0.07*   |
| Monk-2     | 32.63 ± 0.05| 60.01 ± 0.11*| 63.31 ± 0.06*   |
| Average    | 43.70 ± 0.12| 61.75 ± 0.11 | 67.65 ± 0.09    |

Table 6 Average classification accuracy (%) and standard deviation with 10 × 10 fold cross-validation

| Actual | Classified |   |   |
|--------|------------|---|---|
|        | Positive   | 9.5| 17.6|
|        | Negative   | 11.2| 38.5|

Table 7 Confusion matrix of T-FRI on diabetes dataset by averaging 10 × 10 cross-validation

| Actual | Classified |   |   |
|--------|------------|---|---|
|        | Positive   | 17.2| 9.9|
|        | Negative   | 14.3| 35.4|

Table 8 Confusion matrix of IG-T-FRI on Diabetes dataset by averaging 10 × 10 cross-validation
the use of IG-T-FRI, the rate of FN reduces significantly from 64.94 to 36.53% [where the false negative rate is calculated by FN/(TP + FN)]. This makes a great sense in performing medical diagnosis since the rate of missing disease detection (i.e., the proportion of the disease tested as not present when it is really present) is reduced. Although the number of FP is slightly increased, the diagnostic sensitivity (true positive rate) has raised significantly also, with 28.41% in average. This promising result clearly indicates considerable improvement on the decisions made by the use of IG-T-FRI.

5 Conclusion

This paper has presented a novel fuzzy rule-based inference system to address the situation when the rule base is sparse. The proposed information gain-guided fuzzy rule interpolation approach is embedded in this system, where the rule antecedent variables are weighted via computing the information gains. In particular, the computation is enabled through an innovative reverse engineering procedure which converts fuzzy rules into training samples. The proposed method is illustrated by a case study with a small data set and is systematically evaluated by solving benchmark classification problems over nine datasets. The experimental results have confirmed that the relative significance of the individual rule antecedent variables can indeed be captured by the information gains, forming the weights on the variables to guide FRI. This remarkably improves the performance of the interpolative reasoning, thanks to the exploitation of the information gains in differentiating the significances of different antecedent variables.

While very promising, much can be done to further improve this proposed work. The present implementation assumes the use of a data-driven rule learning mechanism that converts a given dataset into rules, with a simple fuzzification procedure. The size of the rule base may be very large due to a large dataset. Any other rule induction techniques (e.g., those reported in (Janikow 1998; Afify 2016)) that may be used as an alternative to generate a more compact rule base would be helpful, improving the performance of the interpolation method further. With the introduction of information gain in support of weighted rule interpolation, there may be an additional computation overhead overall as compared to the use of the original T-FRI algorithm. An experimental analysis of the runtime expense, in comparison with T-FRI, forms another piece of interesting further work. Finally, the current approach assumes a fixed (sparse) rule base. However, having run the process of rule interpolation, intermediate fuzzy rules are generated. These can be collected and refined to form additional rules to support subsequent inference, thereby enriching the rule base and avoiding unnecessary interpolation afterwards (Naik et al. 2017).

Acknowledgements The first author is grateful to the China Scholarship Council and Aberystwyth University for their support in this research. The authors would like to thank the reviewers of the original version of this paper that was presented at the 16th UK Workshop on Computational Intelligence, 2016; their constructive comments have helped improve this work significantly, leading to it receiving one of the two best paper awards at the Workshop.

Funding This study was partly funded by the National Key Research and Development Program of China (Grant No. 2016YFB0502502).

Compliance with ethical standards

Conflict of interest All authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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Appendix A: Reverse engineering in illustrative example

The artificial decision table converted from the five fuzzy rules by reverse engineering in the illustrative case is shown in Table 9, where two pairs of inconsistent data are italicised.
Table 9  32 Training data after reverse engineering given five rules

| Temperature | Outlook | Humidity | Wind   | Decision   |
|-------------|---------|----------|--------|------------|
| Hot         | Sunny   | Humid    | Windy  | Swimming   |
| Hot         | Sunny   | Humid    | Not Windy | Swimming  |
| Hot         | Sunny   | Normal   | Windy  | Swimming   |
| Hot         | Cloudy  | Humid    | Windy  | Swimming   |
| Hot         | Cloudy  | Humid    | Not Windy | Swimming  |
| Hot         | Cloudy  | Normal   | Windy  | Swimming   |
| Hot         | Cloudy  | Normal   | Not Windy | Swimming  |
| Hot         | Rain    | Humid    | Windy  | Swimming   |
| Hot         | Rain    | Humid    | Not Windy | Weight lifting |
| Hot         | Rain    | Normal   | Windy  | Weight lifting |
| Hot         | Rain    | Normal   | Not Windy | Weight lifting |
| Mild        | Rain    | Humid    | Windy  | Weight lifting |
| Mild        | Rain    | Humid    | Not Windy | Weight lifting |
| Mild        | Rain    | Normal   | Windy  | Weight lifting |
| Mild        | Rain    | Normal   | Not Windy | Weight lifting |
| Mild        | Sunny   | Humid    | Windy  | Weight lifting |
| Mild        | Sunny   | Humid    | Not Windy | Weight lifting |
| Mild        | Cloudy  | Humid    | Windy  | Weight lifting |
| Mild        | Cloudy  | Humid    | Not Windy | Weight lifting |
| Mild        | Cloudy  | Normal   | Windy  | Weight lifting |
| Mild        | Cloudy  | Normal   | Not Windy | Weight lifting |
| Mild        | Cloudy  | Normal   | Not Windy | Weight lifting |
| Mild        | Rain    | Humid    | Not Windy | Volleyball |
| Mild        | Rain    | Normal   | Not Windy | Volleyball |
| Mild        | Rain    | Humid    | Not Windy | Volleyball |
| Mild        | Rain    | Normal   | Not Windy | Volleyball |
| Mild        | Rain    | Humid    | Not Windy | Volleyball |
| Mild        | Rain    | Normal   | Not Windy | Volleyball |

Appendix B: Membership functions used in illustrative example

The triangular membership functions adopted in the illustrative case are shown in Fig. 5, which are used to represent all the antecedent variables for the original data set given in (Yuan and Shaw 1995). In particular, the variables Outlook and Temperature adopt the membership functions defined on the left of the figure, where A, B, C stand for Sunny, Cloudy, Rain, or Hot, Mild, Cool, respectively, in relation to the two variables. Humidity and Wind adopt the membership functions on the right, with D, E standing for Humid, Normal or Windy, Not Windy, respectively. Since this is a classification problem, the consequent generally adopts the singleton fuzzy set in the description of the decision.

Appendix C: Workflow of illustrative example

The workflow of the construction of the intermediate rule and of the computation of the interpolative results using the original T-FRI or the information gain-guided T-FRI is shown in Fig. 6. The left hand side illustrates the workflow of the T-FRI and the right does that of IG-T-FRI. In particular, the fuzzy sets of the antecedent variables taken by the selected closest rules, observation and the intermediate rule are shown in the first row, while the consequent variable of those are displayed in the last. The observation of each antecedent variable and the consequent of the selected rules are both illustrated using singleton fuzzy sets for simplicity.

For T-FRI on the left of this figure, the fuzzy sets in dashed lines represent the variable values in Rule 4, while those in dash-dotted lines represent the sets in Rule 5. For IG-T-FRI on the right, the fuzzy sets in dashed lines represent the variable values in Rule 3, and those in dash-dotted lines represent the sets in Rule 5. The dotted lines represent the (fuzzy) values of the computed intermediate antecedent and consequent variables.

Note that in general and also, as in this simple case, a certain antecedent variable may not be present in both selected rules (e.g., Temperature and Wind are involved in Rule 5 but missing in Rule 3). In such situations, the corresponding value of the given observation is employed to replace the missing one in the closest rules, facilitating the interpolation. This makes logical sense as the missing value of an antecedent variable in a rule indicates that any value in its domain may be matched, so the observation naturally provides the best replacement guided by the representative value. Note also that since the transformation factors in this very simple illustrative case are zero, the interpolated consequents are also shown in dotted lines in the last row of the figure.

Fig. 5  Definition of linguistic terms for domain variables
Fig. 6 Workflow of illustrative example
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