Vessel-Bridge Collision Risk Analysis Based on Structural Reliability Theory

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Abstract

With the construction of bridges crossing inland waterways, it becomes vital to analyze the risk of vessel-bridge collision. In this paper, the limitation of the current vessel-bridge collision risk analysis approach recommended by the AASHTO code is addressed, and a new method is proposed based on method of moments of reliability theory. With consideration of the random characteristic of water level, impact angle, impact velocity and deadweight tonnage of vessels even with unknown distribution, the proposed method can reflect the risk level of vessel-bridge collision more accurately. The method can be also applied to other risk analysis of general building structures.

Keywords: vessel-bridge collision; risk analysis; method of moments; structural reliability

1. Introduction

In the world a large number of bridges crossing inland waterways, straits and gulfs are being constructed. The numerous bridges bring generous economic profit while significantly influencing the vessel transportation industry. With the increasing deadweight tonnage and number of vessels as well as the changing environment of bridge area such as scouring, siltation, water current, tidal level etc., collisions between vessels and bridges occur frequently, which may cause serious consequences not only to bridge users but also to vessel owners. According to the statistical data, vessel-bridge collision accidents have occurred more than 100 times in recent decades, which caused enormous casualties, property loss and environmental destruction. Table 1 lists the severe bridge failure accidents due to vessel impact since 1960.

Such catastrophic events force researchers and officials to take a closer look at the frequency of vessel collisions and methods to prevent further accidents from occurring. So it has become a more and more serious problem to carry out rational vessel-bridge collision risk analysis to provide support for management departments to take appropriate measures to reduce risk due to vessel-bridge collisions.

Table 1. Bridge Failure Due to Vessel Collisions with Fatalities

| Bridge name                  | Country | Year | Fatalities |
|------------------------------|---------|------|------------|
| Severn River Railway Bridge  | UK      | 1960 | 5          |
| Lake Ponchartrain Bridge     | USA     | 1964 | 6          |
| Sidney Lanier Bridge         | USA     | 1972 | 10         |
| Lake Ponchartrain Bridge     | USA     | 1974 | 3          |
| Tasman Bridge                | Australia | 1975 | 5          |
| Pass Manchac Bridge          | USA     | 1976 | 1          |
| Tjorn Bridge                 | Sweden  | 1980 | 8          |
| Sunshine Skyway Bridge       | USA     | 1980 | 35         |
| Lorraine Pipeline Bridge     | France  | 1982 | 7          |
| Volga River Railroad Bridge  | Russia  | 1983 | 176        |
| Volga River Railroad Bridge  | Russia  | 1984 | 240        |
| Claiborne Avenue Bridge      | USA     | 1993 | 1          |
| CSX/Amtrak Railroad Bridge   | USA     | 1993 | 47         |
| Port Isabel                  | USA     | 2001 | 8          |
| Webber-Falls                 | USA     | 2002 | 12         |

2. Review of Vessel-bridge Collision Risk Analysis

In view of the severity of vessel-bridge collision, some research work has been developed by a number of researchers all over the world in recent years. These researches are either for specific projects (e.g. the Great Belt Bridge Project in Denmark and the New Sunshine Skyway Bridge in America) or for the establishment of general guidelines for bridge design (e.g. AASHTO 1991, AASHTO 1994 and Eurocode 1.2.7).

With respect to the probability of vessel-bridge collision analysis, some typical models have been
put forward, mainly including the IABSE Model\(^5\), AASHTO Model\(^3\), Eurocode Model\(^4\), Kunz Model\(^9\), Improved Kunz Model\(^1\) etc.

In the AASHTO Guide Specification and Commentary for Vessel Collision Design of Highway Bridges (1991)\(^2\), the probability of vessel-bridge collision is calculated as follows:

\[
F = PA \times PG
\]  
(1)

\(AP\) is the probability of aberrancy of a vessel and is calculated as follows:

\[
PA = BR \times RB \times RC \times XC \times RD
\]  
(2)

Where \(BR\) is aberrancy base rate, \(R_g\) is the factor for bridge location, \(R_c\) is the factor for current acting parallel to vessel transit path, \(R_{XC}\) is the factor for cross-current acting perpendicular to vessel transit path, \(R_D\) is correction factor for vessel traffic density.

\(PG\) is geometric probability which estimates the probability that the vessel will strike the bridge once a vessel has become aberrant. The AASHTO code uses normal distribution to account for geometric probability. The standard deviation is taken as the overall length of the vessel (LOA). This can be seen in Fig.1.

With respect to the probability of collapse analysis, only the AASHTO code and the Eurocode have proposed a method to calculate it so far. The AASHTO code, which addresses the probability of collapse was developed by Cowiconsult (1987) based on studies performed by Fujii and Shiobara (1978) using Japanese historical damage data on vessels colliding at sea\(^7\). The damaged data of bridges is based on observation of the damage situation during vessel-vessel collision while there is little accurate damaged data of bridges during vessel-bridge collision. It proposes a method to calculate the probability of collapse based on the relationship of ultimate lateral resistance of the pier and vessel impact force, as seen in Fig.2., if the ultimate lateral resistance, \(H\), is greater than the vessel impact force, \(P\), the probability of collapse will be zero. As the \(H/P\) ratio decreases, the probability of collapse remains low until the ultimate lateral pier strength becomes less than one-tenth of the vessel impact force. From then on, a small reduction in the \(H/P\) ratio will cause the probability of collapse to increase quite sharply.

While the approach recommended by the AASHTO code for calculation of the probability of collapse can often provide reasonable answers, some of its limitations can be addressed.

Undoubtedly it will be convenient to calculate the probability of collapse by utilizing this curve because there is no need to consider the complicated influence factors. In fact, once the bridge suffers vessel collision, the calculation of probability of collapse is a complicated problem, which is a function of many variables. It depends on the randomness of vessel impact force as well as characteristic of the structure. In order to overcome the limitation of the method in the AASHTO code, a new method of vessel-bridge collision risk analysis is proposed in the present paper.

### 3. Reliability-based Probability of Collapse Analysis

In this paper, reliability theory is introduced to calculate the probability of collapse. The probability models of influence factors which affect the ultimate lateral resistance and vessel impact force are proposed. The bridge safety condition can be expressed by a reliability index through studying the relationship between impact force probability distribution and resistance probability distribution in order to set up the limit state equation of the structure. Then the failure probability is calculated based on reliability theory. Fig.3. shows the process of failure probability analysis.

#### 3.1 The limit state equation of the pier

The limit state equation can be formulated as:

\[
g = R(\Delta, \theta) - P(V, DWT)
\]  
(3)

where \(R(\Delta, \theta)\) is the ultimate lateral resistance of a pier, \(\Delta\) and \(\theta\) represent water level and impact angle respectively. \(P(V,DWT)\) is the vessel impact force,
$V$, $DWT$ represent impact velocity and deadweight tonnage of the vessel respectively.

3.1.1 Probability models of ultimate lateral resistance

Obviously the ultimate lateral resistance varies with water level and impact angle. Supposing that the ultimate lateral resistance conforms to linear variation according to the change of water level and impact angle, the ultimate lateral resistance can be expressed as the function of water level and impact angle. The function expression is as follows:

$$R = k_1\Delta \theta + k_2\Delta + k_3\theta + k_4$$  \hspace{1cm} (4)

Where $k_1$, $k_2$, $k_3$, $k_4$ are deterministic coefficients, $\Delta$ denotes water level, $\theta$ denotes impact angle.

The water level and impact angle are considered as random variables.

The water level is described as normal distribution. The mean takes value as the annual mean water level while the standard deviation is determined by the $3\sigma$ principle. That is to say the water levels of an entire year are in the range of $\mu \pm 3\sigma$.

The impact angle is described as extreme type I distribution. Regarding the parameter value, the mean of impact angle can take the value of 10~15° and standard deviation with a value of 4~6° according to the practical observed data.

3.1.2 Probability models of ultimate lateral resistance

According to the AASHTO code, the vessel impact force based on the vessel impact velocity and the deadweight tonnage of the vessel is computed as follows:

$$P = 0.122V\sqrt{DWT}$$  \hspace{1cm} (5)

The design impact velocity and deadweight tonnage of the vessel are considered as random variables.

The deadweight tonnage of vessels is difficult to describe based on common distribution because of large discreteness and variability according to navigation conditions. As the probabilistic characteristics of deadweight tonnage of vessels can be expressed using statistical moments, the distribution is approximated with its known first four moments. The distribution proposed by Zhao is defined on the basis of the following polynomial normal transformation.

$$\frac{x-\mu}{\sigma} = a_0 + a_1u + a_2u^2 + a_3u^3$$  \hspace{1cm} (6)

The CDF and PDF corresponding to Eq(6) are expressed as:

$$F(x) = \Phi(u)$$

$$f(x) = \frac{\phi(u)}{\sigma(a_0 + 2a_1u + 3a_2u^2)}$$  \hspace{1cm} (7)

Where $\mu$ and $\sigma$ are the mean value and standard deviation of random variables, $a_0$, $a_1$, $a_2$, $a_3$ are deterministic coefficients which can be obtained by the first four moments of the random variable.

With respect to the design impact velocity, by now only the AASHTO code proposes a means for determination of the vessel velocity. A linear interpolation is used to represent the variation in velocity from the centerline of the waterway to the edges of the channel. According to the AASHTO code, the design impact velocity can be formulated as:

$$V = \begin{cases} 
\frac{V_T}{x_c} & x \leq x_c \\
\frac{V_T}{x_c} \frac{V_{min}}{x_c} - \frac{x(V_T - V_{min})}{x_c} & x_c < x \leq x_e \\
V_{min} & x > x_e 
\end{cases}$$  \hspace{1cm} (8)

Where $V$ denotes design impact velocity, $V_T$ denotes typical vessel transit velocity, $V_{min}$ denotes minimum
design impact velocity (not less than the yearly mean current velocity), \( x \) denotes distance to face of pier from centerline of channel, \( x_c \) denotes distance to edge of channel, \( x_L \) denotes distance equal to three times the overall length of the vessel.

In the formula stated above, \( V_T \) and \( V_{\text{min}} \) are considered as random variables. They are both described as normal distribution. The mean and standard deviation of \( V_T \) are obtained based on the observed data while those of \( V_{\text{min}} \) are determined by the characteristics of the current near the area of the bridge. While \( x \leq x_c \) or \( x > x_L \), the mean and standard deviation of \( V \) are the same with \( V_T \)'s and \( V_{\text{min}} \)'s respectively. While \( x_c < x \leq x_L \), the mean of \( V \) can be calculated by Eq(8), the derivation formula for calculating standard deviation of \( V \) is as follows:

\[
\sigma_V = \frac{1}{x_L - x_c} \sqrt{\sigma^2_{V_T} (x - x_c)^2 + \sigma^2_{V_{\text{min}}} (x_L - x)^2}
\]  

(9)

3.2 Method of moments for structural system reliability

With respect to the bridge system reliability assessment due to vessel impact, the failure mode among piers is obviously correlative. The failure probability of the bridge system can be determined using bounding techniques\(^9\), however, the bounds would be wide even though these bounds can be improved by second-order bounds\(^{10,11}\). The failure probability of the system may also be estimated approximately with the probabilistic network evaluation technique (PENT) developed by Ang and Ma\(^{12}\) in which mutual correlations among the failure modes have to be computed. The direct or smart Monte Carlo simulation can also be applied, but is time-consuming.

The method of moments\(^{13,14}\), being very simple, has no shortcomings with respect to design points, and requires neither iteration nor computation of derivatives and computation of mutual correlations among failure modes, and thus is convenient for application to structural system reliability analysis.

A structural system will invariably have multiple modes of potential failure. In the case of a series system, occurrence of one or more of these failure modes will constitute failure of the system. Supposing each of the failure modes corresponds to a performance function \( g_i \), the failure probability of the system is as follows:

\[
P_F = \text{Prob}[g_1 \leq 0 \cup g_2 \leq 0 \cup \cdots \cup g_k \leq 0]
\]  

(10)

Conversely, the safety of a system is the event in which none of the \( k \) potential failure modes occur, this means:

\[
P_F = \text{Prob}[g_1 > 0 \cap g_2 > 0 \cap \cdots \cap g_k > 0]
\]  

\[
P_F = \text{Prob}[\text{min}[g_1, g_2, \cdots, g_k] > 0]
\]  

(11)

Thus the performance function of a series system, \( G \), can be expressed as the minimum of the performance functions corresponding to all the potential failure modes, that is:

\[
G(X) = \min[g_1, g_2, \cdots, g_k]
\]  

(12)

Where \( g_i \) denotes the performance function of the \( i \)th failure mode.

For a performance function \( Z = G(X) \), using inverse Rosenblatt transformation, the \( k \)th moments about zero, of \( Z \) can be defined as:\(^{15}\)

\[
\mu_{G} = \mu_1
\]

\[
\sigma^2_{G} = \sqrt{\mu_2 - \mu_1^2}
\]

\[
\alpha_{4G} = \left( \mu_4 - 4\mu_2\mu_1 - 3\mu_1^2 + 12\mu_2\mu_1^2 - 6\mu_1^4 \right) / \sigma^4_{G}
\]

\[
\alpha_{5G} = \left( \mu_5 - 3\mu_2\mu_1 + 2\mu_1^2 \right) / \sigma^3_{G}
\]  

(14)

The \( \mu_l \) denotes the \( l \)th moments about zero which can be calculated by the point estimate method with estimating points and the corresponding weights\(^{15}\).

With the first few moments of the performance function obtained by DRI, the moment-based reliability index can be evaluated as follows\(^{15}\):

\[
\beta_{2M} = \frac{\mu_2}{\sigma_2}
\]

\[
\beta_{3M} = \frac{3\sqrt{9 + \alpha_{4G} - 6\alpha_{5G}\beta_{2M}^2}}{\alpha_{5G}}
\]

\[
\beta_{4M} = -\Phi^{-1}\left[ \int_{-\infty}^{Z_u} f(Z_u)dZ_u \right]
\]

\[
Z_u = (Z - \mu_2) / \sigma_2
\]  

(15)

Then the failure probability can be obtained.

3.3 Calculation of system reliability under conditional probability

The probability of collision should be considered while calculating structural system reliability due to vessel collision. As the probability of collision corresponding to each pier is often different, it is a problem of calculation of system reliability under conditional probability.

In the present paper, a method based on the method...
of moments for calculating system reliability under conditional probability is proposed which does not require the calculation of mutual correlations among the failure mode of each pier. The failure probability of each pier, which can be readily calculated by the method of moments can be expressed as:

\[ P_{fi} = \text{Prob}[g_i \leq 0] = \int_{g_i \leq 0} f(X) dX \]  

(16)

The probability of collision with respect to each pier can be calculated as \( P_i \), the failure probability of a pier with consideration of probability of collision can be expressed as:

\[ P_{fi} \times P_i = \text{Prob}[g_i + a_i \leq 0] = \int_{g_i + a_i \leq 0} f(X) dX \]  

(17)

Where \( g_i \) is the performance function of one pier, \( a_i \) is a deterministic coefficient. If \( a_i \) can be obtained, the performance function of each pier considering the probability of collision can be written as:

\[ g'_i = g_i + a_i \]  

(18)

One can see that the first four moments of \( g_i \) and \( g_i + a_i \) are the same except mean with \( \mu_{g_i} \) and \( \mu_{g_i + a_i} \) respectively. As the reliability index corresponding to failure probability can be computed by moments of performance function, thus, \( a_i \) can be obtained. Then, performance function of the system considering the probability of collision is expressed as:

\[ G(X) = \min[g'_1, g'_2, \ldots, g'_k] \]  

(19)

Obviously the failure mode of each pier is correlative, thus, the failure probability of a system can be calculated by the method of moments without calculating the mutual correlations among the failure mode of each pier.

### 3.4 Application in practical engineering

A detailed example of vessel-bridge collision risk analysis is presented in this paper. Fig.4 shows the general view of a suspension bridge. The span arrangement of this suspension bridge is 154m+452m+154m. The navigation width is 211.52m and navigation clearance is 46m. The annual highest water level is 2.89m, annual lowest water level is -1.22m and annual mean water level is 1.32m. According to observed data, the taken values of parameters for risk analysis are listed in Table 2.

The statistical moments of deadweight tonnage of vessels are obtained by statistical data. The mean, standard deviation, skewness, kurtosis of deadweight tonnage of vessels are 3067, 2968, 1.8, 6.8. The traffic density is 12168 trips per year.

As for calculation of ultimate lateral resistance, the finite element model of the main tower and foundation is set up with an entity model which includes 107479 nodes, 88784 units altogether. Calculation shows that the pile can be easily destroyed under horizontal force. According to the failure mode mentioned above, the ultimate lateral resistance is the horizontal force while the main compressive stress of the piles progressively exceeds the allowable value with an increase in the horizontal force. Table 3. shows the results of calculation.

The limit state equation of piers can be obtained as follows:

South pier:

\[ g_1 = -0.19\Delta \theta - 0.61\Delta - 5.01\theta + 31.76 - 0.122V_c \sqrt{DWT} \]

North pier:

\[ g_2 = 0.58\Delta \theta - 1.22\Delta - 11.23\theta + 28.52 - 0.122V_c \sqrt{DWT} \]

According to the method mentioned above, the first four moments of performance functions obtained from

![Fig.4. General View of Suspension Bridge](image)

| Table 2. The Value of Analysis Parameters |
|-----------------------------------------|
| Parameter                               | Distribution type   | Mean    | Variation coefficient |
|-----------------------------------------|---------------------|---------|----------------------|
| Typical vessel transit velocity         | Normal distribution | 3m/s    | 0.2                  |
| Minimum design impact velocity          | Normal distribution | 1m/s    | 0.2                  |
| Impact angle                            | Extreme type I distribution | 10°    | 0.4                  |
| Water level                             | Normal distribution | 1.32m   | 0.2                  |
DRI with seven estimating points are listed in Table 4. The results of reliability index and failure probability of each pier calculated by method of moments (MM) are listed in Table 5, with comparison with Monte Carlo simulation (MCS).

From Table 5, one can see that the results calculated by method of moments are in good agreement with results obtained by MCS.

The probability of collision calculated by the method in the AASHTO code and failure probability considering probability of collision for each pier are listed in Table 6.

Assuming that the occurrence of one pier failure will constitute failure of the entire-bridge, the bridge system can be defined as a series system. The limit state equation of each pier with consideration of conditional probability can be obtained as follows:

South pier:
\[ g_1' = -0.19\Delta \theta - 0.61\Delta - 5.01\theta + 31.76 - 0.122\sqrt{DWT} + 53.6 \]

North pier:
\[ g_2' = 0.58\Delta \theta - 1.22\Delta - 11.23\theta + 28.52 - 0.122\sqrt{DWT} + 54.2 \]

The limit state equation of a bridge system can be expressed as:

\[ G(X) = \min(g_1', g_2') \]

Using DRI with seven estimating points, the first four moments are obtained as \( \mu_G = 65.71, \sigma_G = 6.95, \alpha_{3G} = -1.15, \alpha_{4G} = 4.02. \) The reliability index and corresponding failure probability of the system are given as \( \beta = 4.98, P_F = 3.22 \times 10^{-7}. \)

With consideration of traffic density (12168 trips per year), the vessel-bridge collision risk of the entire-bridge is \( 3.92 \times 10^{-3}. \)

### Conclusions
1. A new method of vessel-bridge collision risk analysis is proposed based on the reliability theory. Compared with the AASHTO method, this method can reflect the risk level of vessel-bridge collision accurately with consideration of the random characteristic of water level, impact angle and impact velocity deadweight tonnage of vessels.
2. The probability models of influencing factors are presented for vessel-bridge collision risk analysis.
As the deadweight tonnage of vessels is difficult to describe by common distribution for the large discreteness and variability according to navigation conditions, the distribution is approximated with its known first four moments. The limit state equations of each pier and bridge system are set up through studying the relationship between impact force probability distribution and resistance probability distribution.

3. With respect to the reliability of bridge system due to vessel collision, the probability of collision should be considered, a method based on method of moments for calculating the system reliability under conditional probability is proposed which does not require calculation of mutual correlations among the failure mode of each pier.

4. The accuracy of results of failure probability has been thoroughly examined by comparison with large sample Monte Carlo simulation.

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