A Soft-Computing Approach to Fuzzy EOQ Model for Deteriorating Items with Partial Backlogging

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ABSTRACT
Genetic Algorithm (GA) is an optimized method to find a perfect solution which is based on general genetic process of life cycle. In this article we discussed a crisp and a fuzzy inventory model keeping its demand rate constant for the imprecision and uncertainly deteriorating items with special reference to shortage and partially backlogging systems. The objective of this paper is to minimize the total cost of fuzzy inventory environment for which Graded mean representation, Signed distance and Centroid methods are used to defuzzify the total cost of the systems. Consequently, we are comparing the total average cost, obtained through these methods with the help of numerical example, and sensitively analysis is also given to show the effects of the values on these items. Moreover, Genetic Algorithm (GA) is also applied to the optimistic value of the total cost of the crisp model for the effective and fruitful results.

KEYWORDS
Fuzzy inventory model; triangular fuzzy number; de-fuzzification methods; constant demand rate; genetic algorithm

1. Introduction

Genetic algorithm (GA) is a set of tools to get the total inventory cost which is obtained by the fuzzy model. Fuzzification is a technique to transform the crisp set principles into fuzzy set principles, with the help of different sorts of membership functions. Zadeh [1] gives the principle sets of fuzzy, properties and features of membership functions. Economic order quantity prototype, to demonstrate the resonant charge with the help of the fuzzy number, is expressed by Park [2]. Kaufmann and Gupta [3] show the concept and real-time applications of fuzzy logic. Yao and Lee [4] instigate the fuzzy model for inventory control with various constraints such as backlog, fuzzy order amount, crisp principles, trapezoidal, triangular numbers as in fuzzy.

The interval mean value notion helps to resolve the issues of inventory control management, further down the state when the inventory price is less and the fixed order interval system is more explained by Gen et al. [5]. Chang et al. [6] modelled the subjects of backorder fuzzy with the triangular number of fuzzy. To explore a collection of computing...
schematics for the inventory control problems through fuzzy principles with and without backorder, this study is presented by Lee and Yao [7].

In this sequence Yao et al. [8] acknowledged the demand and request frequency is calculated by fuzzy sets without scarcity and accomplished the fuzzy model. Wu and Yao [9] explain a fuzzy model where order quantity is fuzzify. They also used fuzzy cost which is the fuzzy membership function of the fuzzy cost and to defuzzify they used centroid method. Yao and Chiang [10] express a fuzzy model to decrease the total inventory cost without backorders and to equate the result of the model they used signed distance and centroid defuzzification methods.

A fuzzy inventory model with the presence of demand fuzzy random variable and to validate the result of the fuzzy model they used graded the mean integration representation defuzzification method by Dutta et al. [11]. Chang et al. [12] study about the fuzzy model in the presence of backorders, misplaced sales where lead-time belongs to a variable and to validate the model they took help of the centroid method. Wee et al. [13] present a model of inventory for a substandard value of goods in the presence of backorders and scarcity. Lin [14] shows to control inventory for a recurrent study with parameters such as backorder, lead-time of variable and the conformist demand scarcity rate to verify the result they used the signed distance defuzzification method.

Roy and Samanta [15] described a model of fuzzy recurrent study in the absence of backorder and the sequence period and to validate the result they used the signed distance defuzzification method. Gani and Maheswari [16] present a study model of EOQ with the help of different properties of goods such as imperfect value, mandate, inventory costs, shortages and to verify the result of model they used the Graded mean integration method. Ameli et al. [17] shows a modified model of inventory to determine the technique of ordering items or goods for substandard belongings and in the presence of inflation to find the total revenue and validate the model with the signed distance defuzzification process.

Singh et al. [18] expressed a fuzzy two-warehouse model in the existence of interruption in amount is acceptable. Kumar et al. [19] expanded that a model with various parameters of demand rate is dependent on time, backlogged, deteriorating rate and to get the total revenue of inventory and for comparing the results they used defuzzification methods.

In another study of Kumar et al. [19] presented the two-warehouse inventory model with the effect of GA. Agarwal et al. [20] also present a fuzzy model with partially backlog, deteriorating goods and to make an operative and suitable procedure they use linear demand rate. Agarwal et al. [21] inflation should be especially measured for long period investment and forecast such as ice and cold storage industry and weather forecasts, which specifically help in the production industry. Agarwal et al. [22] discussing in which to deal with the actual situation of the ice and cold storage (Agra) and to find the solution to the problem. To get the solution of these problems they used GA.

Here, we present a study of fuzzy model for inventory control with the help of triangular fuzzy number of different inventory cost. To estimate the total inventory cost, we used three methods of defuzzification, i.e. Centroid, Signed distance and Graded Mean Representation methods and to optimised the result we took the help of Genetic Algorithm. To prove the above model, we compared our outputs with the help of mathematical illustration and sensitive examination by MATLAB.
2. Symbolisations and Supposition

This modified model is consequential under the subsequent symbolisations and expectations:

2.1. Symbolisations

- DR(T) the demand rate at any time T.
- Co the ordering cost.
- Φ the deterioration rate, 0 < Φ ≤ 1.
- LC the length of the Cycle.
- OQ the ordering quantity per unit per time.
- Ch the holding cost per unit per time.
- Csh the shortage Cost per unit per time.
- Cu the unit Cost per unit per time.
- TIC the total inventory cost per unit per time.
- FD the fuzzy demand.
- ϕ the deterioration rate.
- C̃h the holding cost per unit per time.
- ˜Csh the shortage Cost per unit per time.
- ˜Cu the unit Cost per unit per time.
- TFIC the total fuzzy inventory cost per unit per time.
- TIC(G) the total inventory cost by applying Graded mean integration method of defuzzification.
- TIC(S) the total inventory cost by applying Signed distance method of defuzzification.
- TIC(C) the total inventory cost by applying Centroid method of defuzzification.

2.2. Supposition

This Modified Model is Consequential Under the Subsequent Expectations

(i) The period limit is fixed and Replacement is immediate with zero lead-time.
(ii) Shortages and partially backlogs are acceptable. When the cycle occurs no repair is done on deteriorated items.
(iii) The demand is disgruntled backlog and shortages backlogged fraction is 1/(1 + ϕ(LC - T)), where ϕ is a positive constant. And the demand rate is constant which can be presented as DR(T) = a. Where a ≥ 0.

3. Mathematical Formulation

Let us assume IL(T) be the level of inventory on T time with the primary inventory OQ. Moreover, based on the current market scenario and customer consummation, the inventory level regularly moderates on period (0, T1) and afterwards the shortages period (T1, LC). At any random time, IL (T) is administered by the subsequent equations (Figure 1):
3.1. Representation of the Crisp Model

Let the inventory level be $IL(T)$ at any time, the equations are

$$\frac{dIL(T)}{dT} + \phi IL(T) = -a \quad 0 \leq T \leq T_1$$  \hspace{1cm} (1)

$IL(0) = OQ$ and $IL(T_1) = 0$.

$$\frac{dIL(T)}{dT} = -\frac{(a)}{1 + \phi (LC - T)} \quad T_1 \leq T \leq LC$$  \hspace{1cm} (2)

With $IL(T_1) = 0$.

The solution is

$$IL(T) = (1 - \phi T) \left[ OQ - a \left( T + \frac{T^2}{2} \right) \right]$$  \hspace{1cm} (3)

$$IL(T) = \left( \frac{a}{\phi} \right) [\log(1 + \phi (LC - T)) - \log(1 + \phi (LC - T_1))]$$  \hspace{1cm} (4)

Let $IL(T_1) = 0$, so we obtain

$$OQ = a \left( T_1 + \frac{T_1^2}{2} \right)$$  \hspace{1cm} (5)

So, we can write the following equation as follows:

$$IL(T) = \left[ a \left\{ \left( T_1 + \frac{T_1^2}{2} \right) - \left( T + \frac{T^2}{2} \right) \right\} - a\phi T \left\{ \left( T_1 + \frac{T_1^2}{2} \right) - \left( T + \frac{T^2}{2} \right) \right\} \right]$$  \hspace{1cm} (6)

(Ignoring the highest powers of $\phi$).
The entire holding cost is $H_u$ on the period $(0, LC)$ is represented by

$$H_u = \int_0^{T_1} I(L(T)) \, dT = \left[a \left(\frac{T_1^2}{2} + \frac{T_1^3}{3}\right) - a\phi \left(\frac{T_1^4}{8} + \frac{T_1^3}{6}\right)\right]$$  \hspace{1cm} (7)

Deteriorated unit $D_u$ on the period of $(0, LC)$ is specified by

$$D_u = OQ - \text{Total Demand} = OQ - \int_0^{T_1} a \, dT = \frac{aT_1^2}{2}$$  \hspace{1cm} (8)

The shortage unit $S_u$ on the period of $(0, LC)$ is represented by

$$S_u = -\int_{T_1}^{LC} \frac{a}{1 + \varphi(LC - T)} \, dT = \left(\frac{a}{\varphi}\right) \left[\log LC - (LC - T_1) + \frac{1}{\varphi} \log(1 + \varphi(LC - T))\right]$$  \hspace{1cm} (9)

The total cost of inventory, $TIC$, is shown by

$$TIC = \frac{1}{LC} \left[C_0 + C_h \, H_u + C_u \, D_u + C_{sh} \, S_u\right]$$

$$TIC = \frac{1}{LC} \left[C_0 + C_h \left\{a \left(\frac{T_1^2}{2} + \frac{T_1^3}{3}\right) - a\phi \left(\frac{T_1^4}{8} + \frac{T_1^3}{6}\right)\right\}\right]$$

$$+ \frac{1}{LC} \left[C_u \left(\frac{aT_1^2}{2}\right) + C_{sh} \left\{\left(\frac{a}{\varphi}\right) \left[\log LC - (LC - T_1) + \frac{1}{\varphi} \log(1 + \varphi(LC - T))\right]\right\}\right]$$  \hspace{1cm} (10)

### 3.2. Representation of the Fuzzy Model

In the expansion of Economic Order Quantity models, Constant deterioration rate is sup-posed by most authors. But in this model, we let that the entire constraint is static and it can be assumed by certainty; but in the current bazaar state, It is problematic to specify all the parameters because of customer satisfaction, So, here we take the following constraints $\sim b, \sim p, \sim q, \sim \phi, \sim z$ which can be modified with some boundaries.

So, the triangular fuzzy numbers are $\sim b = (b_1, b_2, b_3), \sim p = (p_1, p_2, p_3), \sim q = (q_1, q_2, q_3), \sim \phi = (\phi_1, \phi_2, \phi_3), \sim z = (z_1, z_2, z_3)$.

Total fuzzy inventory cost is

$$TFIC = \frac{1}{LC} \left[C_0 + \tilde{h} \left(\frac{aT_1^2}{2} + \frac{T_1^3}{3}\right) - \tilde{h} a \left(\frac{T_1^4}{8} + \frac{T_1^3}{6}\right)\right]$$

$$+ \tilde{C} \left(\frac{aT_1^2}{2}\right) + \tilde{S} \left(\frac{a}{\varphi}\right) \left[\log LC - (LC - T_1) + \frac{1}{\varphi} \log(1 + \varphi(LC - T))\right]\right]$$  \hspace{1cm} (11)

For defuzzification of the total fuzzy inventory cost, we used three methods.
3.2.1. Graded Mean Representation

The entire inventory price is represented by

\[
TIC(G) = \frac{1}{6}[TIC(G_1), TIC(G_2), TIC(G_3)]
\]

\[
TIC(G_1) = \frac{1}{LC} \left[ C_0 + z_1 b_1 \left( \frac{T_1^2}{3} + \frac{T_1^3}{3} \right) - z_1 b_1 \phi_1 \left( \frac{T_1^4}{8} + \frac{T_1^3}{6} \right) + p_1 \left( \frac{b_1 T_1^2}{2} \right) \right] + q_1 \left( \frac{b}{\psi} \right) \left[ \log LC - (LC - T_1) + \frac{1}{\psi} \log \{1 + \psi (LC - T_1)\} \right]
\]

\[
TIC(G_2) = \frac{1}{LC} \left[ C_0 + z_2 b_2 \left( \frac{T_1^2}{2} + \frac{T_1^3}{3} \right) - z_2 b_2 \phi_2 \left( \frac{T_1^4}{8} + \frac{T_1^3}{6} \right) + p_2 \left( \frac{b_2 T_1^2}{2} \right) \right] + \frac{1}{LC} \left[ + q_2 \left( \frac{b}{\psi} \right) \left[ \log LC - (LC - T_1) + \frac{1}{\psi} \log \{1 + \psi (LC - T_1)\} \right] \right]
\]

\[
TIC(G_3) = \frac{1}{LC} \left[ C_0 + z_3 b_3 \left( \frac{T_1^2}{2} + \frac{T_1^3}{3} \right) - z_3 b_3 \phi_3 \left( \frac{T_1^4}{8} + \frac{T_1^3}{6} \right) + p_3 \left( \frac{b_3 T_1^2}{2} \right) \right] + q_3 \left( \frac{b}{\psi} \right) \left[ \log LC - (LC - T_1) + \frac{1}{\psi} \log \{1 + \psi (LC - T_1)\} \right]
\]

So, we obtain

\[
TIC(G) = \frac{1}{6LC} \left[ \begin{array}{c}
C_0 + b_1 z_1 \left( \frac{T_1^2}{2} + \frac{T_1^3}{3} \right) - z_1 b_1 \phi_1 \left( \frac{T_1^4}{8} + \frac{T_1^3}{6} \right) + p_1 \left( \frac{z_1 T_1^2}{2} \right) \\
+ q_1 \left( \frac{b}{\psi} \right) \left[ \log LC - (LC - T_1) + \frac{1}{\psi} \log \{1 + \psi (LC - T_1)\} \right] + \\
C_0 + z_2 b_2 \left( \frac{T_1^2}{2} + \frac{T_1^3}{3} \right) - z_2 b_2 \phi_2 \left( \frac{T_1^4}{8} + \frac{T_1^3}{6} \right) + p_2 \left( \frac{b_2 T_1^2}{2} \right) \\
+ q_2 \left( \frac{b}{\psi} \right) \left[ \log LC - (LC - T_1) + \frac{1}{\psi} \log \{1 + \psi (LC - T_1)\} \right] + \\
C_0 + z_3 b_3 \left( \frac{T_1^2}{2} + \frac{T_1^3}{3} \right) - z_3 b_3 \phi_3 \left( \frac{T_1^4}{8} + \frac{T_1^3}{6} \right) + p_3 \left( \frac{b_3 T_1^2}{2} \right) \\
+ q_3 \left( \frac{b}{\psi} \right) \left[ \log LC - (LC - T_1) + \frac{1}{\psi} \log \{1 + \psi (LC - T_1)\} \right]
\end{array} \right]
\]

To decrease the entire inventory price per unit per time is \(TIC(G)\) and the optimum rate of \(T_1\) and \(LC\) will be calculated by the following differential equations:

\[
\frac{\partial TIC(G)}{\partial T_1} = 0 \quad \text{and} \quad \frac{\partial TIC(G)}{\partial LC} = 0;
\]

Equation (17) is correspondent to

\[
\frac{1}{6LC} \left[ z_1 b_1 (T_1 + T_1^2) - z_1 b_1 \phi_1 \left( \frac{T_1^3}{2} + \frac{T_1^2}{2} \right) + p_1 b_1 T_1 + z_3 b_3 (T_1 + T_1^2) \\
- z_3 b_3 \phi_3 \left( \frac{T_1^3}{2} + \frac{T_1^2}{2} \right) + p_3 z_3 T_1 \right] + \frac{1}{6LC} \left[ 4 \left[ C_0 + z_2 b_2 (T_1 + T_1^2) - z_2 b_2 \phi_2 \left( \frac{T_1^3}{2} + \frac{T_1^2}{2} \right) + p_2 z_2 T_1 \right] \right] = 0
\]
The entire inventory value is represented by

\[
\frac{1}{6LC} \left[ q_1 b_1 - q_2 b_2 - q_3 b_3 \right] + \frac{1}{6LC^2} \left[ 6C_0 + z_1 b_1 \left( \frac{T_{12}}{2} + \frac{T_{13}}{3} \right) + z_1 b_1 \phi_1 \left( \frac{T_{14}}{8} + \frac{T_{15}}{6} \right) + p_1 \left( \frac{b_1 T_{12}}{2} \right) \right]
\]

Furthermore, the total inventory cost is \( TIC(G) \) to fulfil the result, the following differential equations have to be contended:

\[
\frac{\partial^2 TIC(G)}{\partial T_{12}} > 0, \quad \frac{\partial^2 TIC(G)}{\partial LC^2} > 0; \quad \text{and} \quad \left( \frac{\partial^2 TIC(G)}{\partial T_{12}^2} \right) \left( \frac{\partial^2 TIC(G)}{\partial (LC)^2} \right) - \left( \frac{\partial^2 TIC(G)}{\partial T_{12} \partial (LC)} \right) > 0;
\]

Afterwards the next derivatives of the total inventory cost \( TIC(G) \) are complexes and it is hard to demonstrate mathematically.

### 3.2.2. Signed Distance

The entire inventory value is represented by

\[
TIC(S) = \frac{1}{4} [TIC(S_1), TIC(S_2), TIC(S_3)]
\]

\[
TIC(S_1) = \frac{1}{LC} \left[ C_0 + z_1 b_1 \left( \frac{T_{12}}{2} + \frac{T_{13}}{3} \right) + z_1 b_1 \phi_1 \left( \frac{T_{14}}{8} + \frac{T_{15}}{6} \right) + p_1 \left( \frac{b_1 T_{12}}{2} \right) \right] + \frac{1}{LC} \left[ q_1 \left( \frac{Z}{\psi} \right) \left[ \log LC - (LC - T_1) + \frac{1}{\psi} \log(1 + \psi(LC - T_1)) \right] \right]
\]

\[
TIC(S_2) = \frac{1}{LC} \left[ C_0 + z_2 b_2 \left( \frac{T_{12}}{2} + \frac{T_{13}}{3} \right) + z_2 b_2 \phi_2 \left( \frac{T_{14}}{8} + \frac{T_{15}}{6} \right) + p_2 \left( \frac{b_2 T_{12}}{2} \right) \right] + q_2 \left( \frac{B_2}{\psi} \right) \left[ \log LC - (LC - T_1) + \frac{1}{\psi} \log(1 + \psi(LC - T_1)) \right]
\]

\[
TIC(S_3) = \frac{1}{LC} \left[ C_0 + z_3 b_3 \left( \frac{T_{12}}{2} + \frac{T_{13}}{3} \right) + z_3 b_3 \phi_3 \left( \frac{T_{14}}{8} + \frac{T_{15}}{6} \right) + p_3 \left( \frac{b_3 T_{12}}{2} \right) \right] + q_3 \left( \frac{B_3}{\psi} \right) \left[ \log LC - (LC - T_1) + \frac{1}{\psi} \log(1 + \psi(LC - T_1)) \right]
\]

\[
TIC(S) = \frac{1}{4} [TIC(S_1) + 2TIC(S_2) + TIC(S_3)]
\]
Equation (25) is comparable to hard to demonstrate mathematically. After these next derivatives of the total inventory cost are to be contented:

\[
C_0 + b_3z_3 \left( \frac{T_1^2}{c} + \frac{T_3^2}{3} \right) + z_3b_3\phi_3 \left( \frac{T_4}{b} + \frac{T_6^2}{6} \right) + \frac{p_3}{\varphi} \left( \frac{b_1T_2^2}{2} \right)
\]

To decrease the entire cost of inventory per unit time is \( \text{TIC}(S) \) and the optimum rate of \( T_1 \) and \( LC \) will be calculated by the following differential equations:

\[
\frac{\partial \text{TIC}(S)}{\partial T_1} = 0 \quad \text{and} \quad \frac{\partial \text{TIC}(S)}{\partial (LC)} = 0; \tag{26}
\]

Equation (25) is comparable to

\[
\frac{1}{4LC} \left[ z_1b_1(T_1 + T_2^2) + z_1b_1\phi_1 \left( \frac{T_3}{2} + \frac{T_6^2}{2} \right) + p_1b_1T_1 + + z_3b_3\phi_3 \left( \frac{T_4}{6} + \frac{T_6^2}{2} \right) + p_3b_3T_1 \right]
\]

\[
= 0 \tag{27}
\]

\[
\frac{1}{4LC} \left[ \left[ q_1b_1 + \frac{q_1b_1}{\varphi} - \frac{q_3b_3}{\varphi} \right] + 2 \left[ q_2b_2 + \frac{q_2b_2}{\varphi} - \frac{q_2b_2}{\varphi} \right] + \left[ \frac{q_3b_3}{\varphi} + \frac{q_3b_3}{\varphi} - \frac{q_1b_1}{\varphi} \right] \right] + \frac{1}{4LC^2} \left[ 6C_0 + z_1b_1 \left( \frac{T_3}{2} + \frac{T_6^2}{3} \right) + z_1b_1\phi_1 \left( \frac{T_4}{8} + \frac{T_6^2}{6} \right) + p_1b_1T_2^2 \right] + \frac{1}{4LC} \left[ + z_3b_3 \left( \frac{T_4}{8} + \frac{T_6^2}{3} \right) + z_3b_3\phi_3 \left( \frac{T_4}{6} + \frac{T_6^2}{6} \right) + p_3b_3T_2^2 \right] + \frac{1}{4LC} \left[ + \frac{b_3}{\varphi} \left[ \log \left( \frac{LC}{T_1} + \frac{1}{\varphi} \log \left( 1 + \varphi (LC - T_1) \right) \right] \right] \right] = 0 \tag{28}
\]

Furthermore, the total inventory cost is \( \text{TIC}(S) \), to fulfill the result, the following differential equations have to be contented:

\[
\frac{\partial^2 \text{TIC}(S)}{\partial T_1^2} > 0, \quad \frac{\partial^2 \text{TIC}(S)}{\partial LC^2} > 0; \quad \text{and} \quad \left( \frac{\partial^2 \text{TIC}(S)}{\partial T_1^2} \right) \left( \frac{\partial^2 \text{TIC}(S)}{\partial (LC)^2} \right) - \left( \frac{\partial^2 \text{TIC}(S)}{\partial T_1 \partial LC} \right) > 0; \tag{29}
\]

Afterwards the next derivatives of the total inventory cost \( \text{TIC}(S) \) are complexes and it is hard to demonstrate mathematically.

3.2.2.1. Centroid Method. Entire Cost is Given by

\[
\text{TIC}(C) = \frac{1}{3} [\text{TIC}(C_1), \text{TIC}(C_2), \text{TIC}(C_3)]
\]
where the values of $TIC(C_1)$, $TIC(C_2)$, $TIC(C_3)$ are

$$
TIC(C_1) = \frac{1}{LC} \left[ C_0 + z_1 b_1 \left( \frac{T_1^2}{2} + \frac{T_1^3}{3} \right) + z_1 b_1 \phi_1 \left( \frac{T_1^4}{8} + \frac{T_1^3}{6} \right) + p_1 \frac{b_1 T_1^2}{2} \right] + q_1 \frac{b_1}{\psi} \left[ \log LC - (LC - T_1) + \frac{1}{\psi} \log(1 + \psi(LC - T_1)) \right] 
$$

$$
TIC(C_2) = \frac{1}{LC} \left[ C_0 + z_2 b_2 \left( \frac{T_2^2}{2} + \frac{T_2^3}{3} \right) + z_2 b_2 \phi_2 \left( \frac{T_2^4}{8} + \frac{T_2^3}{6} \right) + p_2 \frac{b_2 T_2^2}{2} \right] + q_2 \frac{b_2}{\psi} \left[ \log LC - (LC - T_1) + \frac{1}{\psi} \log(1 + \psi(LC - T_1)) \right] 
$$

$$
TIC(C_3) = \frac{1}{LC} \left[ C_0 + z_3 b_3 \left( \frac{T_3^2}{2} + \frac{T_3^3}{3} \right) + z_3 b_3 \phi_3 \left( \frac{T_3^4}{8} + \frac{T_3^3}{6} \right) + p_3 \frac{b_3 T_3^2}{2} \right] + q_3 \frac{b_3}{\psi} \left[ \log LC - (LC - T_1) + \frac{1}{\psi} \log(1 + \psi(LC - T_1)) \right] 
$$

$$
TIC(C) = \frac{1}{3} \left[ TIC(C_1) + TIC(C_2) + TIC(C_3) \right] 
$$

$$
TIC(C) = \frac{1}{3LC} \left[ C_0 + z_1 b_1 \left( \frac{T_1^2}{2} + \frac{T_1^3}{3} \right) + z_1 b_1 \phi_1 \left( \frac{T_1^4}{8} + \frac{T_1^3}{6} \right) + p_1 \frac{b_1 T_1^2}{2} \right] + q_1 \frac{b_1}{\psi} \left[ \log LC - (LC - T_1) + \frac{1}{\psi} \log(1 + \psi(LC - T_1)) \right] 
$$

$$
+ \left[ C_0 + z_2 b_2 \left( \frac{T_2^2}{2} + \frac{T_2^3}{3} \right) + z_2 b_2 \phi_2 \left( \frac{T_2^4}{8} + \frac{T_2^3}{6} \right) + p_2 \frac{b_2 T_2^2}{2} \right] + q_2 \frac{b_2}{\psi} \left[ \log LC - (LC - T_1) + \frac{1}{\psi} \log(1 + \psi(LC - T_1)) \right] 
$$

$$
+ \left[ C_0 + z_3 b_3 \left( \frac{T_3^2}{2} + \frac{T_3^3}{3} \right) + z_3 b_3 \phi_3 \left( \frac{T_3^4}{8} + \frac{T_3^3}{6} \right) + p_3 \frac{b_3 T_3^2}{2} \right] + q_3 \frac{b_3}{\psi} \left[ \log LC - (LC - T_1) + \frac{1}{\psi} \log(1 + \psi(LC - T_1)) \right] 
$$

To decrease the entire inventory price per unit per time is $TIC(C)$ and the optimum rate of $T_1$ and LC will be calculated by the following differential equations:

$$
\frac{\partial TIC(C)}{\partial T_1} = 0 \quad \text{and} \quad \frac{\partial TIC(C)}{\partial LC} = 0; 
$$

Equation (34) is correspondent to

$$
\frac{1}{3LC} \left[ z_1 b_1 (T_1 + T_1^2) + z_1 b_1 \phi_1 \left( \frac{T_1^3}{2} + \frac{T_1^2}{2} \right) + p_1 b_1 T_1 + z_2 b_2 (T_1 + T_1^2) \right. 
\left. + z_2 b_2 \phi_2 \left( \frac{T_2^3}{2} + \frac{T_2^2}{2} \right) + p_2 b_2 T_1 + z_3 b_3 (T_1 + T_1^2) + z_3 b_3 \phi_3 \left( \frac{T_3^3}{2} + \frac{T_3^2}{2} \right) + p_3 b_3 T_1 \right] = 0 
$$

$$
\frac{1}{3LC} \left\{ \left( \frac{q_1 b_1}{\psi LC} + \frac{q_1 b_1}{\psi} - \frac{q_3 b_3}{\psi} \right) + \left( \frac{q_2 b_2}{\psi LC} + \frac{q_2 b_2}{\psi} - \frac{q_3 b_3}{\psi} \right) + \left( \frac{q_1 b_1}{\psi LC} + \frac{q_1 b_1}{\psi} - \frac{q_3 b_3}{\psi} \right) \right\} 
$$

$$
\left\{ \left( \frac{3C_0 + z_1 b_1 \left( \frac{T_1^2}{2} + \frac{T_1^3}{3} \right) + z_1 b_1 \phi_1 \left( \frac{T_1^4}{8} + \frac{T_1^3}{6} \right) + p_1 \frac{b_1 T_1^2}{2} \right)}{3LC^2} \right. 
\left. + q_1 \frac{b_1}{\psi} \left[ \log LC - (LC - T_1) + \frac{1}{\psi} \log(1 + \psi(LC - T_1)) \right] \right\} 
$$

$$
\left. + \left( \frac{z_2 b_2 \left( \frac{T_2^2}{2} + \frac{T_2^3}{3} \right) + z_2 b_2 \phi_2 \left( \frac{T_2^4}{8} + \frac{T_2^3}{6} \right) + p_2 \frac{b_2 T_2^2}{2} \right)}{3LC^2} \right. 
\left. + q_2 \frac{b_2}{\psi} \left[ \log LC - (LC - T_1) + \frac{1}{\psi} \log(1 + \psi(LC - T_1)) \right] \right\} 
$$

$$
\left. + \left( \frac{z_3 b_3 \left( \frac{T_3^2}{2} + \frac{T_3^3}{3} \right) + z_3 b_3 \phi_3 \left( \frac{T_3^4}{8} + \frac{T_3^3}{6} \right) + p_3 \frac{b_3 T_3^2}{2} \right)}{3LC^2} \right. 
\left. + q_3 \frac{b_3}{\psi} \left[ \log LC - (LC - T_1) + \frac{1}{\psi} \log(1 + \psi(LC - T_1)) \right] \right\} = 0 
$$

$$
\left( \frac{1}{3LC^2} \right) \left\{ \left( \frac{3C_0 + z_1 b_1 \left( \frac{T_1^2}{2} + \frac{T_1^3}{3} \right) + z_1 b_1 \phi_1 \left( \frac{T_1^4}{8} + \frac{T_1^3}{6} \right) + p_1 \frac{b_1 T_1^2}{2} \right)}{3LC^2} \right. 
\left. + q_1 \frac{b_1}{\psi} \left[ \log LC - (LC - T_1) + \frac{1}{\psi} \log(1 + \psi(LC - T_1)) \right] \right\} 
$$

$$
\left. + \left( \frac{z_2 b_2 \left( \frac{T_2^2}{2} + \frac{T_2^3}{3} \right) + z_2 b_2 \phi_2 \left( \frac{T_2^4}{8} + \frac{T_2^3}{6} \right) + p_2 \frac{b_2 T_2^2}{2} \right)}{3LC^2} \right. 
\left. + q_2 \frac{b_2}{\psi} \left[ \log LC - (LC - T_1) + \frac{1}{\psi} \log(1 + \psi(LC - T_1)) \right] \right\} 
$$

$$
\left. + \left( \frac{z_3 b_3 \left( \frac{T_3^2}{2} + \frac{T_3^3}{3} \right) + z_3 b_3 \phi_3 \left( \frac{T_3^4}{8} + \frac{T_3^3}{6} \right) + p_3 \frac{b_3 T_3^2}{2} \right)}{3LC^2} \right. 
\left. + q_3 \frac{b_3}{\psi} \left[ \log LC - (LC - T_1) + \frac{1}{\psi} \log(1 + \psi(LC - T_1)) \right] \right\} = 0 
$$
Furthermore, the total inventory cost is $TIC(C)$, to fulfil the result, the following differential equations have to be contented:

$$\frac{\partial^2 TIC(C)}{\partial T_1^2} > 0, \frac{\partial^2 TIC(C)}{\partial LC^2} > 0; \text{ and } \left( \frac{\partial^2 TIC(C)}{\partial T_1^2} \right) \left( \frac{\partial^2 TIC(C)}{\partial (LC)^2} \right) - \left( \frac{\partial^2 TIC(C)}{\partial T_1 \partial LC} \right) > 0; \quad (38)$$

Afterwards the next derivatives of the total inventory cost $TIC(C)$ are complexes and it is hard to demonstrate mathematically.

4. Mathematical Illustration

Consider the following parameters in the above model.

**Crisp Model values are**

- $C_O = \text{Rs}179/\text{order}$, $C_u = \text{Rs}17/\text{unit}$, $C_h = \text{Rs. 3 unit per year}$, $b = 99 \text{ units per year}$, $\phi = 0.01 \text{ year}$, $C_{sh} = \text{Rs 12 unit per year}$.

The crisp model solution is $TIC = \text{Rs 392.2823}$, $T_1 = 0.5346 \text{ year}$, $LC = 0.7585 \text{ year}$.

| Table 1. The results of defuzzication methods. | 'Graded Mean Representation Method' | 'Signed Distance Method' | 'Centroid Method' |
|-----------------------------------------------|-----------------------------------|--------------------------|------------------|
| $T_1$  | $LC$     | $TIC(G)$ | $T_1$  | $LC$     | $TIC(S)$ | $T_1$  | $LC$     | $TIC(C)$ |
| 0.5406 | 0.9027   | 391.1083 | 0.6407 | 0.8394   | 411.5095 | 0.6264 | 0.8886   | 416.6585 |
| 0.7134 | 0.9367   | 383.2551 | 0.6734 | 0.8361   | 406.7851 | 0.7054 | 0.9262   | 408.9852 |
| 0.6914 | 0.8393   | 382.5273 | 0.6914 | 0.8495   | 397.6273 | 0.6714 | 0.8491   | 404.8283 |
| 0.6094 | 0.8512   | 379.6577 | 0.6109 | 0.8507   | 396.5263 | 0.6717 | 0.8502   | 403.2690 |

**Figure 2.** Graphical representation of the fuzzy model by MATLAB.
**Fuzzy Model Values are**
The triangular fuzzy numbers are represented by
\[ \tilde{b} = (47, 79, 111), \tilde{p} = (14, 19, 221), \tilde{q} = (11, 13, 15), \tilde{\phi} = (0.005, 0.009, 0.011), \tilde{z} = (2, 3, 5) \]

The solutions of this inventory model can be represented as (Table 1 and Figure 2):

**Sensitivity Examination**

We can compare the values of parameters with changes in effects. And the outcomes are presented in tables.

**Observations:**
The observations from the above table are mentioned as follows.

1. It is clearly visible if the parameter value of \( b \) will be increasing and also parametric values of \( T_1 \) and \( LC \) will be decreasing, so the total fuzzy inventory cost \( TIC(G) \) will also be increasing, as shown in Table 2 and Figure 3.

| \( B \) | \( T_1 \) | \( LC \) | \( TIC(G) \) |
|---|---|---|---|
| 49 | 0.8411 | 1.1641 | 315.6461 |
| 69 | 0.7325 | 1.0248 | 356.1480 |
| 89 | 0.6401 | 0.9181 | 415.7494 |
| 109 | 0.6145 | 0.8513 | 454.7471 |
| 129 | 0.5497 | 0.7645 | 488.3596 |

**Table 2.** Effect of changes on \( b \) in the graded mean representation method.

![Graphical representation of changes effect in \( b \) by MATLAB.](image-url)
Table 3. Effect of changes on $\emptyset$ in the graded mean representation method.

| $\emptyset$ | $T_1$ (year) | LC (year) | TIC(G) (Rs) |
|-------------|--------------|-----------|-------------|
| 0.004       | 0.6872       | 0.9341    | 410.2483    |
| 0.006       | 0.6845       | 0.9346    | 411.4161    |
| 0.008       | 0.6704       | 0.9183    | 413.6272    |
| 0.010       | 0.6643       | 0.9153    | 414.6861    |
| 0.012       | 0.6695       | 0.9120    | 415.9681    |

2. Similarly, if the parameter value of $\emptyset$ will be increasing and at the time parametric values of $T_1$ and LC will be decreasing, so the total fuzzy inventory cost $TIC(G)$ will also be increasing, as shown in Table 3 and Figure 4.

5. Execution of Genetic Algorithm

Here we are implementing (GA) Genetic algorithm in Table 2. So, we get the optimal values. Again, GA is applying in Table 3. So, changes have effects on parameter $\emptyset$

Observations:

Here it is clearly visible that we are getting optimal result by using mutation and the crossover method of GA.

1. It is clearly visible if the parameter value of $b$ will be increasing and parametric values of $T_1$ and LC will be decreasing, so the total fuzzy inventory cost $TIC(G)$ will also be increasing, as shown in Table 4 and Figure 5.
Table 4. Sensitivity analysis after applying GA on parameter $b$.

| $b$ | $T_1$ | LC | $TIC(G)$ |
|-----|-------|----|----------|
| 49  | 0.7325| 1.0250 | 316.6530 |
| 69  | 0.8411| 1.1640 | 348.1470 |
| 89  | 0.6145| 0.8520 | 401.7570 |
| 109 | 0.6401| 0.9180 | 449.8775 |
| 129 | 0.5497| 0.7644 | 475.3750 |

Figure 5. MATLAB results in $b$ by after applying GA.

Table 5. Sensitivity analysis after applying GA on parameter $\vartheta$.

| $\vartheta$ | $T_1$ (year) | LC (year) | $TIC(G)$ (Rs) |
|-----------|-------------|----------|--------------|
| 0.004     | 0.6850      | 0.9345   | 408.2450     |
| 0.006     | 0.6870      | 0.9350   | 410.2590     |
| 0.008     | 0.6646      | 0.9155   | 411.2460     |
| 0.010     | 0.6704      | 0.9183   | 412.4564     |
| 0.012     | 0.6680      | 0.9120   | 414.5780     |

2. Similarly, if the parameter value of $\vartheta$ will be increasing and similarly parametric values of $T_1$ and LC will be decreasing, so the total fuzzy inventory cost $TIC(G)$ will also be increasing, as shown in Table 5 and Figure 6.
Figure 6. MATLAB result after applying GA on demand parameter $\emptyset$ on total fuzzy cost.

6. Conclusion

The presenting model for worsening objects in the situations of where shortage, partially backlog is acceptable. The main objective of this study is to abate the inventory value and increase the revenue. So, we used three defuzzification methods, i.e. Centroid, Signed Distance and Graded Mean Representation Method. And we can see that if parameter values of $b$ and $\emptyset$ will be increasing and also parametric values of $T_1$ and $LC$ will be decreasing, so the total fuzzy inventory cost $TIC(G)$ will also be increasing.

Furthermore, we used GA to get optimistic values of different cost factors and to conclude our result with the help of arithmetical illustrations and changes in the effects in parametric values of $b$ and $\emptyset$. To show the graphical representation and to analyse the values of different cost factors MATLAB and Java are used.

This model can be modified with various factors such as stock-dependent demand rate, time-based demand and many more.

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References

[1] Zadeh LA. Fuzzy sets. Inf Control. 1965;8(3):338–353.
[2] Park K. Fuzzy-set theoretic interpretation of economic order quantity. IEEE Trans Syst Man Cybernet SMC-17. 1987;17(6):1082–1084.
[3] Kaufmann A, Gupta M. Introduction to fuzzy arithmetic: theory and applications. New York: Van Nostrand Reinhold; 1991.
[4] Yao JS, Lee HM. Fuzzy inventory with backorder for fuzzy order quantity. Inf Sci (Ny). 1996;93:283–319.
[5] Gen M, Tsujimura Y, Zheng D. An application of fuzzy set theory to inventory control models. Comput Ind Eng. 1997;33:553–556.
[6] Chang SC, Yao JS, Lee HM. Economic reorder point for fuzzy backorder quantity. Eur J Oper Res. 1998;109:183–202.
[7] Lee HM, Yao JS. Economic order quantity in fuzzy sense for inventory without backorder model. Fuzzy Sets Syst. 1999;105:13–31.
[8] Yao JS, Chang SC, Su JS. Fuzzy inventory without backorder for fuzzy order quantity and fuzzy total demand quantity. Comput Oper Res. 2000;27:935–962.
[9] Wu K, Yao JS. Fuzzy inventory with backorder for fuzzy order quantity and fuzzy shortage quantity. Eur J Oper Res. 2003;150(2):320–352.
[10] Yao JS, Chiang J. Inventory without backorder with fuzzy total cost and fuzzy storing cost defuzzified by centroid and signed distance. Eur J Oper Res. 2003;148:401–409.
[11] Dutta P, Chakraborty D, Roy A. A single-period inventory model with fuzzy random variable demand. Math Comput Model. 2005;41(8–9):915–922.
[12] Chang HC, Yao JS, Ouyang LY. Fuzzy mixture inventory model involving fuzzy random variable lead-time demand and fuzzy total demand. Eur J Oper Res. 2006;169(1):65–80.
[13] Wee HM, Yu J, Chen MC. Optimal inventory model for items with imperfect quality and shortage backordering. Int J Manage Sci. 2007;35:7–11.
[14] Lin YJ. A periodic review inventory model involving fuzzy expected demand short and fuzzy backorder rate. Comput Ind Eng. 2008;54(3):666–676.
[15] Roy A, Samanta G. Fuzzy continuous review inventory model without backorder for deteriorating items. Electron J Appl Stat Anal. 2009;2(1):58–66.
[16] Gani N, Maheshwari S. Economic order quantity for items with imperfect quality where shortages are backordered in fuzzy environment. Adv Fuzzy Math. 2010;5(2):91–100.
[17] Ameli M, Mirzazadeh A, Shirazi M. Economic order quantity model with imperfect items under fuzzy inflationary conditions. Trends Appl Sci Res. 2011;6(3):294–303.
[18] Singh S, Kumari R, Kumar N. Two-warehouse fuzzy inventory model under the conditions of permissible delay in payments. Int J Oper Res. 2011;11(1):78–99.
[19] Kumar S, Agarwal P, Kumar N. An inventory model for deteriorating items under inflation and permissible delay in payments by genetic algorithm. Belgium: Uses of Sampling Techniques & Inventory Control with Capacity Constraints. Pons publishing Brussels; 2016. 53–68
[20] Agarwal P, Sharma A, Kumar N. Fuzzy production inventory model with stock dependent demand using genetic algorithm (GA) under inflationary environment. J Sci Technol. 2018a;26(4):1637–1658.
[21] Agarwal P, Kumar S, Kumar N, et al. An inventory model for deteriorating items with stock-dependent demand rate and partial backlogging under permissible delay in payments. Int J Pure Appl Math. 2018b;118(22):1339–1352.
[22] Agarwal P, Sharma A, Kumar N. Inventory control in ice and cold storage management using GA. Int J Comput Math Sci. 2017;6(6):37–41.