Quark gluon plasma as a strongly coupled color-Coulombic plasma

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Abstract

We show that the extensively studied equation of state (EOS) of strongly coupled QED plasma fits the recent lattice EOS data of gluon plasma remarkably well, with appropriate modifications to take account of color degrees of freedom and running coupling constant. Hence we conclude that the quark gluon plasma near the critical temperature is a strongly coupled color-Coulombic plasma.

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Quark gluon plasma (QGP), the deconfined state of quarks and gluons, is a prediction of quantum chromodynamics (QCD). In search of QGP, experiments are on at CERN, BNL etc. where heavy ions are accelerated to relativistic energies and made to collide. At sufficiently large energy (> few GeV), the fireball formed by collision may be QGP which then expands, cools and freezes out into hadrons. These hadrons, photons, leptons etc. are detected and analysed to see whether QGP is formed or not. The expansion of QGP, generally described by relativistic hydrodynamics, affects the experimental observations. In the description of hydrodynamics, the EOS of QGP is needed to complete the set of fluid equations. At present all analysis of experimental results are based on ideal equation of state where quarks and gluons are noninteracting and pressure or energy density are proportional to the 4\textsuperscript{th} power of temperature. In the theoretical calculations of various signatures of QGP, it is assumed that QGP is an ideal plasma.

However, the recent lattice simulation results show that gluon plasma is not ideal and is nonideal even upto five times the critical temperature ($T_c$) \cite{1}. There were lot of attempts \cite{2-4} to explain this numerical experiment based on the properties of QCD, confinement and asymptotic freedom, properties of plasma, etc. All of them, so far, not able to explain the data satisfactorily.

In Ref. \cite{2}, they tried to fit the lattice results assuming Coulombic interaction with running coupling constant, confinement in the form of bag constant and low momentum cut-off in the evaluation of partition function. In Ref. \cite{3}, gluons are assumed as quasi-particles with mass propotional to plasma frequency. Here also a modified, temperature dependent, running coupling constant is used to fit the old lattice results of Ref. \cite{5}. As we will see it does not fit the recent more refined data \cite{1}. Earlier we \cite{4} had assumed Cornel potential interaction \cite{6}, Coulomb + confinement, between quark and antiquark and used Mayer’s cluster expansion method to derive EOS. This EOS was used to fit lattice results on gluon plasma with partial success. Especially near the critical temperature fitting was not good. We suspected that it might be due to the fact that our theory was for weakly interacting system and hence valid only in the high temperature limit. To explain the lattice
results near $T_c$ we need a theory of strongly interacting system. In other words, near $T_c$ QGP
may be a strongly coupled plasma. This is exactly what we find in this letter, namely, QGP
is a strongly coupled color Coulombic plasma.

Generally when we say plasma, it is a quasi neutral system of charged particles which
exhibits collective effects. The so called plasma parameter $\Gamma$, which is the ratio of average
potential energy to average kinetic energy of particles, is assumed to be weak ($<< 1$). Lot of
studies in plasma such as plasma waves, instabilities and other collective effects are in this
range of $\Gamma$. For $\Gamma$ close to 1 and above, it is a strongly coupled plasma (SCP) which modifies
various properties of plasma [7]. We see that QGP is also a strongly coupled plasma near
$T_c$. Partially analytic and partially numerical calculations of SCP do exist in the literature
[7]. In particular, EOS of SCP is extensively studied and parameterized as a function of $\Gamma$.

Let us now discuss about QGP. We take QGP as a deconfined, quasi-color-neutral system
of quarks, antiquarks and gluons with color Coulombic mutual interactions. It is similar to
QED plasma apart from few modifications due to color degrees of freedom. It is charactorized
by plasma parameter,

$$\Gamma \equiv \frac{\langle PE \rangle}{\langle KE \rangle} = \frac{4}{3} \frac{\alpha_s}{r_{av}} T, \quad (1)$$

where we have taken the well known Coulombic interactions used in hadron spectroscopy [8].
The typical value of $\alpha_s \approx 0.5$, $r_{av} \approx 1 fm$ and near the critical temperature, $T_c \approx 200 MeV$,
we estimate $\Gamma \approx 2/3$. Hence QGP is a strongly coupled plasma. Compared to QED, the
fine structure constant, $\alpha$, is replaced by $4 \alpha_s/3$ in Coulomb interaction term. $r_{av}$ may be
estimated as $r_{av} = (3/4\pi n)^{1/3}$ and hence

$$\Gamma = \left(\frac{4\pi n}{3}\right)^{1/3} \frac{4\alpha_s}{3 T}, \quad (2)$$

where '$n$' is the number density. Taking $n \approx a_f T^3$ we get

$$\Gamma \approx \left(\frac{4\pi a_f}{3}\right)^{1/3} \frac{4}{3}\alpha_s, \quad (3)$$

$a_f$ is a constant which depends on degrees of freedom. As we discussed earlier there exists
EOS for strongly coupled Coulombic system as a function of $\Gamma$. Since QGP is also a SCP with
Γ given by Eq. (3), we modify QED nonrelativistic energy density, \( \varepsilon_{QED} = (3/2 + u_{ex}(\Gamma)) n T \) for strongly coupled relativistic QGP (SCQGP) as

\[
\varepsilon = (3 + u_{ex}(\Gamma)) n T ,
\]

where

\[
u_{ex}(\Gamma) = \frac{u_{ex}^{Abe}(\Gamma) + 3 \times 10^3 \Gamma^{5.7} u_{ex}^{OCP}(\Gamma)}{1 + 3 \times 10^3 \Gamma^{5.7}} .
\]

Here we have taken same \( u_{ex}^{Abe} \) and \( u_{ex}^{OCP} \) as used in SCP and are given by

\[
u_{ex}^{Abe}(\Gamma) = -\frac{\sqrt{3}}{2} \Gamma^{3/2} - 3 \Gamma^3 \left[ \frac{3}{8} \ln(3\Gamma) + \frac{\gamma}{2} - \frac{1}{3} \right] ,
\]

\[
u_{ex}^{OCP} = -0.898004 \Gamma + 0.96786 \Gamma^{1/4} + 0.220703 \Gamma^{-1/4} - 0.86097 .
\]

\( u_{ex}^{Abe} \) was derived by Abe [8] exactly in the giant cluster-expansion theory. \( \gamma = 0.57721... \) is Euler’s constant. \( u_{ex}^{OCP} \) was evaluated by computer simulation of one component plasma (OCP), where a single species of charged particles embedded in a uniform background of neutralizing charges. \( u_{ex}(\Gamma) \) is derived for strongly coupled Coulombic plasma and we believe that it should be valid for any Coulombic plasma except a change, \( \alpha \to 4\alpha_s/3 \) in \( \Gamma \), for color Coulombic plasma. Finally, we get,

\[
e(\Gamma) \equiv \frac{\varepsilon}{\varepsilon_s} = 1 + \frac{1}{3} u_{ex}(\Gamma) ,
\]

where \( \varepsilon_s \equiv 3a_f T^4 \), energy density of ideal QGP gas. Using the relation

\[
\varepsilon = T \frac{\partial P}{\partial T} - P ,
\]

we get the pressure

\[
p(T) \equiv \frac{P}{P_s} = \left( \frac{P_c}{a_f T_c} + 3 \int_{T_c}^{T} t^2 e(\tau) \right) / T^3 ,
\]

where \( P_s \equiv a_f T^4 \) and \( P_c \) is the pressure at critical temperature \( T_c \). Temperature dependence of \( e(\Gamma) \) seen in lattice data can only come from \( \alpha_s \) in our model by using running
coupling constant $\alpha_s(T)$. Let us use the running coupling constant $\alpha_s^L(T)$ used in the lattice calculation \[1\] which may be obtained from the modified second order scaling relation,
\[
\frac{\Lambda_L}{N_\tau T \lambda(\beta)} = (8\pi^2\beta/33)^{51/121} \exp(-4\pi^2\beta/33),
\]
where $\beta \equiv 6/g^2$ and $\Lambda_L, N_\tau$ are lattice parameters which may be replaced in terms of $T_c$ in above equation to get,
\[
\frac{T}{T_c} = (\beta_c/\beta)^{51/121} \exp(-4\pi^2(\beta_c - \beta)/33),
\]
where we have taken $\lambda(\beta) \approx \lambda(\beta_c)$, for simplicity. Also it follows from Ref. \[1\] that $\lambda(\beta)$ varies slowly as a function of $\beta$. After some algebra, we get,
\[
\alpha_s^L(t) = \frac{2\pi}{11 \left( \ln(t/t_0) + \frac{51}{121} \ln(12\pi/33\alpha_s^L(t)) \right)},
\]
where $t \equiv T/T_c$ and $t_0$ is
\[
t_0 = (8\pi^2\beta_c/33)^{51/121} \exp(-4\pi^2\beta_c/33).
\]
$\alpha_s^L(t)$ for given $t$ is obtained by solving Eq. \[13\]. $t_0$ or $\beta_c$ is only one parameter and adjusted to get the best fitting to lattice data \[1\]. In Fig. 1, we plotted $e(t)$, Eq. \[8\] and $p(t)$,
\[
p(t) \equiv \frac{P}{P_s} = \left( p_c + 3 \int_1^t d\tau \tau^2 e(\tau) \right) / t^3,
\]
and obtained a remarkable good fit to lattice data \[1\] for $t_0 = .5763$ or $\beta_c = .57$. We have used $a_f = 16 \pi^2/90$ which is for gluons. However, the value of $\beta_c$ is an order of magnitude smaller than that of lattice results. For small value of $\alpha_s$, $\alpha_s^L(t)$ may be approximated as
\[
\alpha_s^L(t) \to \frac{2\pi}{11 \ln(t/t_0)} \left( 1 - \frac{51}{121} \frac{\ln(2 \ln(t/t_0))}{\ln(t/t_0)} \right),
\]
or
\[
\alpha_s^L(t) \to \frac{2\pi}{11 \ln(t/t_0)}.
\]
The last expression is similar to the running coupling constant used in Ref. \[2\] where $t_0 \equiv \Lambda/T_c$ and also in Ref. \[3\] where $t_0 \equiv \Lambda/\pi T_c$. $\Lambda$ is the QCD scale parameter. For $\Lambda = 200 \text{ MeV}$ we get $T_c = 347 \text{ MeV}$ and $T_c = 110 \text{ MeV}$ respectively. We have also plotted
\[
\Delta = \frac{\varepsilon - 3P}{3a_f T^4},
\]

a measure of nonidealness, and \(c_s^2\). A reasonable good fit to lattice data is obtained, as shown in Fig. 2, except very close to \(T_c\). In Fig. 3, we plotted \(\alpha_s^L(t)\) and \(\Gamma(t)\) as a function of \(t\). We see that even at \(T = 5T_c\), gluon plasma is strongly coupled plasma and running coupling constant is not small.

It is interesting to compare earlier few theories with our present theory. In Fig. 4, the best fits for \(p(t)\) for three models (Ref. [3], Ref. [4] and present model) are plotted. As we see the fitting of lattice data with the theory improves as we go from quasi-particle theory, our earlier result and present result.

In conclusion, we obtained a remarkably good fit to the recent lattice result [1] using EOS of strongly coupled plasma of QED [7] with a modification for color degrees of freedom and with a running coupling constant which was used in lattice calculations. Hence we conclude that QGP is a strongly coupled Coulombic plasma of color charges. Just like in SCP, SCQGP may have a dramatic effects on various signatures, collective effects and other properties. They need to be recalculated before we make confirmation of QGP in relativistic heavy ion collisions. For e.g., screening length decreases rapidly near critical temperature and we may have serious consequences in \(J/\psi\) suppression results.

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FIGURES

FIG. 1. Plots of $p(t) \equiv P/P_s$ (dashed curve) and $e(t) \equiv \varepsilon/\varepsilon_s$ (continuous curve) as a function of $t \equiv T/T_c$ from our model and lattice results (dots), respectively.

FIG. 2. Plots of $\Delta \equiv (\varepsilon - 3P)/\varepsilon_s$ (continuous curve) and $c_s^2$ (dashed curve) as a function of $t \equiv T/T_c$ from our model and lattice results (dots), respectively.

FIG. 3. Plots of $\alpha_s^L(t)$ (dashed curve) and $\Gamma(t)$ (continuous curve) as a function of $t$.

FIG. 4. Plots of the best fits for $p(t)$ from three models, Ref. [3] (dotted curve), Ref. [4] (dashed curve), present model (continuous curve) and lattice results (dots) as a function of $t$. 

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$p(t)$ or $e(t)$
