Energy potential of dual lead rotors for twin screw compressors

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Abstract. The paper deals with a theoretical investigation into the ways in which fluid properties influence the optimal geometrical parameters of dry running twin screw compressors with uniform and dual lead rotors. Dual lead rotors possess two segments with different rotor leads in order to optimise the progression of chamber volume, clearance size and outlet area. Optimal geometry parameters are determined by means of a multi-chamber model simulation. Fluids are treated as an ideal gas and fluid properties are varied systematically in order to provide guidance on how the machine should be designed for a particular application. The optimal geometry parameters mainly depend on the speed of sound and isentropic exponent of the fluid, because they influence outlet throttling during discharge, and pressure progression due to volumetric compression. In particular, the values for the rotor wrap angle and the internal volume ratio need to be adjusted individually for each compressor application. The results reveal that dual lead compressors possess most potential for fluids with low speed of sound and isentropic exponent.

1. Introduction

Screw machines used for gas compression can be found in numerous engineering applications due to their reliable and low-maintenance operation. Compression is achieved by means of two intermeshing rotors which rotate in a close-fitting casing. The shape of the rotors influences the performance of the compressor and the optimal configuration depends to a large extent on the intended use. In particular, the rotor wrapping characteristics have a decisive impact on the operation of the machine because they influence the size of the clearances and the performance during the discharge phase. Weckes [1] optimised the geometry of a screw air supercharger and estimated the best energy efficiency for a wrap angle of 100 degrees. You et al [2] investigated a screw refrigeration compressor with R22 and identified an optimal wrap angle of 330 degrees for this application. These opposing values are not only a result of the different pressure ratios and rotor profiles under examination, but of the different thermodynamic properties of the fluids.

Newer attempts try to combine the advantages of low and high rotor wrap angles by using variable lead rotors, Fig. 1. So far, an increase in efficiency for variable lead rotors is predicted for high pressure applications [3] and in the field of refrigeration compressors [4]. The improvement is based on the optimised progression of volume curve, clearances and discharge area and is a result of the reduction in the overall wrapping of the rotors. This reduction has positive effects on screw machine performance:
- Decrease in working cycle duration (less time for clearance mass flow results in higher volumetric efficiency)
- Increase in maximum chamber volume leading to higher mass flow for a machine of a given size
- Reduction in throttling during discharge

So far no detailed investigation of possible fields of application where benefits may be expected by using variable rotor leads has been carried out.

![Figure 1. Representative screw machine with dual lead rotors [4]](image)

2. Multi-chamber model simulation

A suitable simulation principle to predict the performance of positive displacement machines is a multi-chamber model simulation which is nowadays used in several applications [5]. The simulation is based on the conservation of mass and energy and calculates the fluid state inside all working chambers simultaneously on the basis of a zero-dimensional chamber model. This chamber model includes the time-dependent progressions of the relevant properties of the machine under examination, e.g. chamber volumes, clearance and port areas. The clearances and port areas maintain an exchange of mass and energy between chambers and ports, thus influencing the fluid pressure inside the working chambers. This interaction necessitates an iterative calculation of the fluid states, which are assumed to be homogeneous in each working chamber. Basically, the change in fluid state inside a working chamber is sectioned in two steps that are calculated consecutively for a very small time step: a change in chamber volume and an exchange of mass and energy via ports and clearances. For the assumption of an ideal gas these two phases will be explained in the following.

A new pressure \( p_2 \), that is the result of a change in chamber volume from \( V_1 \) to \( V_2 \), is calculated assuming an adiabatic change of fluid state with the isentropic exponent \( \kappa \):

\[
p_2 = p_1 \cdot \left( \frac{V_1}{V_2} \right)^{\kappa}
\]  

(1)

The exchange of mass flow between two volumes is calculated by means of the formula introduced by St. Venant and Wantzel [6], which assumes an adiabatic and frictionless flow of an ideal gas from a volume with higher pressure (index hp) to a volume with lower pressure (index lp) through a flow cross-section \( A \) (narrowest cross section with index *):

\[
m = \rho \cdot A \cdot c_s = p_{hp} \cdot A \cdot \sqrt{\frac{2 \cdot \kappa}{(\kappa - 1) \cdot R \cdot T_{hp}}} \cdot \left( \frac{2}{\Pi^k - \Pi^{\frac{k+1}{\kappa}}} \right)
\]  

(2)
The equation assumes the low pressure \( p_{lp} \) to be present in the flow cross section \( A_* \) and is valid for subcritical pressure ratios \( \Pi = p_{in} / p_{hp} \). Mass flow also depends on the isentropic exponent \( \kappa \) and the specific gas constant \( R = R / M \), with the universal gas constant \( R \) and the molar mass \( M \) of the fluid. Assuming a pressure ratio below the critical pressure ratio \( \Pi < \Pi_{crit} \) the fluid reaches the speed of sound in the flow cross-section and the mass flow does not depend on the pressure ratio any more. The choked mass flow can be calculated by:

\[
\dot{m}_{crit} = \rho_* \cdot A_* \cdot a_* = \rho_* \cdot A_* \cdot \sqrt{\kappa \cdot R \cdot T_*} = \frac{p_{hp}}{\sqrt{T_{hp}}} \cdot A_* \cdot \frac{1}{\sqrt{R}} \cdot \sqrt{\frac{2}{\kappa + 1}}
\]  

(3)

The exchanged mass flow is charged to the fluid state inside the working chamber, thereby changing the pressure and energy of the fluid. The estimated mass flow in equation (2) and (3) is usually reduced by a friction coefficient to take friction into account. In this paper friction is ignored and the friction coefficient is set to one.

3. Energy efficiency and delivery rate

As a result of the multi-chamber simulation the required internal compressor power (not including any mechanical power loss) and the displaced mass flow are determined. They are used to calculate the internal isentropic efficiency, which is defined as the ratio between the power in the case of an isentropic change in fluid state, and the internal compressor power

\[
\eta_{is} = \frac{\dot{p}_s}{\dot{P}_l} = \frac{\dot{m}_{real} \cdot \Delta h_s}{\dot{m}_{th} \cdot \rho_{MR} \cdot z_{MR}},
\]  

(4)

and the delivery rate

\[
\lambda_L = \frac{\dot{m}_{real}}{\dot{m}_{th}} = \frac{\dot{m}_{real}}{V_{max} \cdot \rho_{in} \cdot n_{MR} \cdot z_{MR}},
\]  

(5)

which is used to evaluate the amount of clearance flow inside the compressor.

4. Similarity law for screw machines

Geometrical and physical similarity need to be achieved in order to scale a dry-running screw compressor or change the working fluid without a change in internal isentropic efficiency and delivery rate. For example, geometrical similarity implies a constant ratio of clearance heights to rotor diameter. Physical similarity mainly requires the following coefficients to remain constant:

- Isentropic exponent \( \kappa = c_p / c_v \)
- Pressure ratio \( \Pi = p_{in} / p_{out} \)
- Circumferential Mach number \( Ma_u = u_{MR} / a_{in} \)

Here \( u_{MR} \) is the male rotor tip speed and the speed of sound at the suction side is

\[
a_{in} = \sqrt{\kappa \cdot R \cdot T_{in}} = \sqrt{\frac{\kappa \cdot R \cdot T_{in}}{M}}.
\]  

(6)

The minor important characteristic numbers (e.g. the circumferential Reynolds number) are not considered in the following investigation.

5. Relevant fluid properties

It can be seen from equation (1) to (3) that the pressure inside the working chamber depends on the geometry of the machine (volumes and available flow cross-sections) and the fluid properties, isentropic exponent and molar mass as a part of the specific gas constant. Fig. 2 shows an overview of typical values of isentropic exponent and molar mass and how they affect the speed of sound at a given temperature. Although fluids with higher molar mass, such as most refrigerants, are not suitable
for exact simulations employing the ideal gas law, the treatment of all fluids as ideal gas simplifies the examination of the effects of the fluid properties on optimal screw compressor parameters and the analysis of general dependencies.

**Figure 2.** Isentropic exponent and molar mass of vaporous fluids and mixtures from the REFPROP [7] database with lines of constant speed of sound for T = 350K

6. Variation of fluid properties and boundary conditions

Fluid properties are varied systematically in order to show their individual influence on the optimal geometry of screw machines and demonstrate ways in which efficiency can be improved by using dual lead rotors as presented in [4]. Fig. 3 shows the fluids under investigation. The starting point is fluid #1 with a molar mass of 150 g/mol, an isentropic exponent of 1.05 and a temperature of 300 K. At this point the fluid has a speed of sound of 132 m/s. Fluid #2 possesses a higher speed of sound and a lower density due to an increase in inlet temperature. Fluid #3 has a higher molar mass, but the inlet temperature was adjusted so that the speed of sound is equal to the initial fluid #1. Fluid #4 has a higher isentropic exponent, while temperature was also adjusted to guarantee constant speed of sound. The fluid properties under examination are applied to the suction side of the compressors. Although the fluid properties are not based on real operating conditions, the variations presented show the individual influence of the various fluid properties and provide relevant general information for screw compressor design.

| Parameter [dimension] | Value |
|-----------------------|-------|
| Cycle parameters:     |       |
| - Compressor inlet pressure [bar] | 1 |
| - Compressor outlet pressure [bar]  | 5 |
| Machine parameters:   |       |
| - Lobe count combination [-] | 4+6 |
| - Male rotor crown circle [mm]  | 72 |
| - Rotor length [mm]  | 130 |
| - Clearance heights [mm] | 0.07 |
| Operating parameters: |       |
| - Circumferential speed [m/s] | 80 |
Other boundary conditions remaining constant throughout all simulations are summarised in Table 1. The rotor profile under investigation is the asymmetric SRM-profile with four male and six female rotor lobes with equal diameters for both rotors. The length-to-diameter ratio is set to 1.8. Analytical functions for the profile can be found in [8]. Dry-running screw machines are simulated by means of the multi-chamber simulation tool KaSim [5], assuming adiabatic flows inside the compressor.

**7. Influence of fluid properties on the pressure-volume diagram**

Idealised and real screw compressor cycles are shown in Fig. 4. The cycle can be sectioned in three characteristic phases: During suction the fluid flows from the low pressure port into the emerging working chamber, until maximum chamber volume $V_{\text{max}}$ is reached. At this point the chamber becomes disconnected from the low pressure port and the compression phase begins. Once the fluid state inside the chamber arrives at discharge conditions the chamber opens to the high pressure port and the fluid is expelled through the outlet area. Maximum chamber volume and the chamber volume when discharge begins are used to calculate the internal volume ratio of the compressor, equation (7).

The change in chamber volume which is necessary for the idealised working cycle to achieve high pressure conditions inside the working chamber as the high pressure port opens, can be estimated by:

$$v_i = \frac{V_{\text{max}}}{V_{\text{comp, end}}} = \left(\frac{P_{\text{out}}}{P_{\text{in}}}\right)^{1/\kappa}$$

(7)

The influence of molar mass, the speed of sound and the isentropic exponent on the screw compressor cycles is discussed in the following for a representative screw compressor geometry.

**7.1. Influence of fluid properties on the pressure-volume diagram**

For the ideal screw compressor working cycle, clearance flow and throttling effects are ignored, so that the load change is isobaric, and the compression is isentropic. Fluids #1, #2 and #3 possess the same isentropic exponent and have the same idealised working cycle so that no isolated influence of molar mass and speed of sound can be observed. Compared to fluid #4 they need a greater change in chamber volume in order to achieve high pressure conditions inside the working chamber as the high pressure port opens, can be estimated by:

The internal volume ratio influences the size of the outlet area which is available for discharge, so that a smaller outlet area is usually available for fluids with a lower isentropic exponent as shown for the
Axial outlet area in Fig. 4. In addition, the discharge temperature and therefore the speed of sound are reduced for these fluids. This in turn influences the discharge flow which is crucial for the real compressor cycle which will be investigated next.

![Diagram](image)

**Figure 4.** Screw compressor cycles for fluids #1 - #4 and corresponding contours of axial discharge area (wrap angle $\varphi = 300^\circ$)

### 7.2. Influence of fluid properties on the real pressure-volume-diagram

In contrast to the ideal cycle the real screw compressor cycle includes throttling and clearance flows. Analogous to equation (3) the maximum exchanged volume flow can be calculated by:

$$ V_{\text{crit}} = A_s \cdot a_s = A_s \cdot \sqrt{\kappa \cdot R \cdot T_s} = A_s \cdot \sqrt{T_{\text{hp}}} \cdot \sqrt{R} \cdot \frac{2\kappa}{\kappa + 1} \quad (8) $$

Critical volume flow for fluid #4 ($\kappa = 1.66$) is 10% higher than for fluid #1 ($\kappa = 1.05$) although the speed of sound of the non-flowing fluid at the inlet side is equal for both fluids. Furthermore, greater volumes can be displaced with a larger outlet area and a higher temperature at the discharge port. For the low isentropic exponent of fluids #1 - #3 the temperature when attaining the high pressure conditions is comparatively low. In practice this is an advantage because less cooling of the compressor is necessary, but the lower temperature results in a lower speed of sound during discharge. Although the speed of sound for fluids #1 and #4 is the same at the inlet port, the speed of sound rises to a value of 137 m/s for fluid #1 and to 182 m/s for fluid #4 on the high pressure side, as calculated via the isentropic equations. In turn, the speed of sound increases clearance flow, pre-heating the fluid even more during compression, thus amplifying the effect referred to. In accordance with the faster compression and the resulting larger outlet area discussed in the last section, fluids with a higher isentropic exponent can discharge greater volumes and are less at risk of throttling at the outlet if the internal volume ratio is well adjusted (Fig. 4). On the other hand, fluid #4 results in lower volumetric efficiency because clearance mass flow is also comparatively high, which can be seen from the difference between the idealised and the real working cycle during the compression phase.

A higher speed of sound (fluid #2) decreases throttling at the outlet, because more volume can be displaced in the same time (equation (8), Fig. 4). Clearance flow is also increased, so that the pressure progress during the compression phase is slightly increased and volumetric efficiency is reduced.

The pressure progression for fluids #1 and #3 are equal. Compared with fluid #1, fluid #3 possesses a greater molar mass while isentropic exponent and speed of sound remain constant by adjusting the
temperature of the fluid. Due to the equality of speed of sound and isentropic exponent the similarity laws are complied with, so that no individual influence of molar mass, inlet temperature or inlet density on the screw compressor cycle can be observed.

8. Influence of fluid properties on optimal design parameters

Results for an optimisation of the rotor wrap angles for the different fluids are discussed in the following, for constant and dual lead compressors. For constant lead the wrap angle is varied between 150° and 500° to determine the optimal geometry. For dual lead the rotors consist of two segments, each with a constant rotor lead as shown in Fig. 1. For constant overall wrap angles $\varphi_{total}$ the wrappings of the segments $\varphi_{hp}$ and $\varphi_{lp}$ can be varied according to the following equations [4]:

$$L_{total} \cdot \varphi_{total} = L_{hp} \cdot \varphi_{hp} + L_{lp} \cdot \varphi_{lp}$$  \hspace{1cm} (9)

$$L_{total} = L_{hp} + L_{lp}$$  \hspace{1cm} (10)

Each overall wrapping creates hundreds of possible combinations of the two rotor segments which need to be simulated in order to optimise the geometry. The overall wrap angle is varied between 175° and 350° whereas results are only shown for optimised geometries for each overall wrapping. For each compressor (constant or dual lead) the internal volume ratio is optimised individually for steps of 0.25, so that only maximum efficiency values are shown for each rotor wrapping.

8.1. Influence of the speed of sound

The influence of the fluid properties on performance is investigated by comparing their effect on the optimal design parameters of the screw compressors. Simulation results for fluid #1 are shown in Fig. 5 and reveal that maximum efficiency for constant rotor lead is attained at comparatively high wrap angles. Due to the low speed of sound, discharge speed is limited so that a high degree of wrapping is necessary to avoid throttling during discharge. Higher wrap angles reduce the time-dependent change in chamber volume so that less volume flow needs to be displaced during discharge. Furthermore, at the same internal volume ratio, the discharge area increases with higher wrap angles, and outlet throttling is also reduced [9], so that optimal values for the internal volume ratio are increased with higher wrap angles. For low wrap angles the chamber connects to the high pressure port even before the fluid reaches high pressure conditions, but the quasi-isochoric compression under high pressure conditions is less detrimental than throttling at the discharge port caused by a smaller outlet area. On the other hand, high wrap angles extend the duration of the working cycle and clearance areas are larger, so that volumetric efficiency is reduced. Because of the low speed of sound the effect of clearance mass flow is insignificant, so that the maximum efficiency which can be achieved is very high and only slightly reduced if wrap angles are enlarged. With dual lead screw compressors maximal isentropic efficiency can be increased by about 1.5 percentage points and even more for lower rotor wrappings. Delivery rate is somewhat decreased, depending mainly on the blowhole area, which depends entirely on the rotor lead. Optimal energy efficiency is attained with higher internal volume ratios due to the optimised progressions of volume curve and outlet area.

In contrast, the higher speed of sound of fluid #2 reduces the optimal value of the wrap angle from 400° to 300°, Fig. 6. The higher speed of sound increases maximum possible volume flow, so that outlet throttling is reduced, resulting in higher values for the optimal internal volume ratio. On the other hand, clearance mass flow is also higher, resulting in a lower delivery rate, so that maximum overall efficiency is reduced by almost seven percentage points. Delivery rate drops sharply for higher wrap angles. With dual lead rotors no significant benefit in maximal isentropic efficiency is possible. An enhancement in maximal isentropic efficiency in case of dual lead rotors is therefore more likely for fluids with a low speed of sound.
8.2. Influence of inlet density and temperature
Comparison of fluids #1 and #2 reveals that the density at the compressors inlet varies from 6 to 1.2 kg/m³. Density could be held constant without varying the speed of sound, molar mass and isentropic exponent by simply changing the inlet pressure while the pressure ratio of the compressor remains constant. However, the exchanged volume (equation (8)) does not depend on the absolute pressure of the fluid, and in the case of subcritical flow it depends entirely on the pressure ratio. Furthermore, the change in temperature due to the volumetric compression would remain the same because it depends entirely on the volume ratio. Thus, a change in density at the inlet - e.g. due to the unrealistically high inlet temperature of fluid #2 - would change absolute values such as mass flow and internal power of the compressor, but delivery rate and isentropic efficiency would remain exactly the same. A comparison of fluids with different densities, e.g. by using different inlet temperatures, is therefore reasonable, because density is not a crucial parameter for the design parameters of screw compressors.
8.3. Influence of the isentropic exponent

Fig. 7 shows the results for fluid #4, which obtains a higher isentropic exponent of 1.66 while the speed of sound is the same as for fluid #1, due to an adjustment in the temperature. Results for the optimal internal volume ratio differ considerably from the other fluids. Less change in volume is necessary to compress the fluid so that high pressure conditions are attained at lower internal volume ratios.

As discussed in section 7, the results are comparable with an increase in the speed of sound (Fig. 6) and maximum possible efficiency, delivery rate and optimal wrap angle are reduced. As for fluid #2, no significant benefit concerning maximum isentropic efficiency or delivery rate is achievable with dual lead rotors. Besides speed of sound, the isentropic exponent is the second major parameter that influences the optimal geometry parameters.

Figure 7. Internal isentropic efficiency, corresponding delivery rate and optimised internal volume ratio for different rotor overall wrappings for fluid #4

9. Conclusion

The present study indicates the ways in which the relevant fluid properties influence the screw compressor working cycle and the optimal geometry parameters of twin screw machines for uniform and dual lead rotors. Speed of sound and the isentropic exponent are the major parameters influencing the design of screw machines, while no isolated influence of molar mass, density and inlet temperature was observed. Potential for dual lead compressors exists for fluids with a low speed of sound and a low isentropic exponent. In this case the optimal wrap angle for constant lead needs to be very large in order to avoid throttling during discharge. An optimised progression of volume curve and outlet area enables the screw machine to operate with a lower overall wrap angle and results in higher efficiency due to decreased throttling at the outlet. Besides, the reduced overall wrapping of dual lead rotors may increase maximum chamber volume and therefore significantly increase the absolute mass flow for a given compressor size.

For wet-running screw machines, which are not investigated in this paper, the injected fluid reduces the temperature of the working fluid as well as the effective size of the available flow cross section and is therefore comparable to a reduction in speed of sound, so that an enhancement in efficiency for this application can be expected too.

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**Nomenclature**

| Symbol | Description |
|--------|-------------|
| A      | Area (m²)   |
| a      | Speed of sound (m/s) |
| c      | Flow velocity (m/s) |
| cp, c_v | Isobaric heat capacity (J/kg/K), Isochoric heat capacity (J/kg/K) |
| h      | Specific enthalpy (J/kg) |
| L      | Length (m) |
| M      | Molar mass (kg/mol) |
| Ma_u  | Molar Mach number (-) |
| m      | Mass flow (kg/s) |
| n      | Rotational speed (1/s) |
| P      | Power (W) |
| p      | Pressure (Pa) |
| R      | Specific gas constant (J/kg/K) |
| R_d    | Universal gas constant (J/mol/K) |
| T      | Temperature (K) |
| u      | Circumferential speed (m/s) |
| V      | Volume (m³) |
| V_th   | Theoretical flow (m³/s) |
| V_r    | Internal volume ratio (-) |
| z      | Number of lobes (-) |
| Δ      | Difference (-) |

**Subscript**

- 1, 2 Fluid states
- compr,end End of compression
- crit Critical
- i Internal
- in Suction / inlet side
- hp High pressure
- lp Low pressure
- MR Male rotor
- max Maximum
- out Discharge / outlet side
- real Real flow
- th Theoretical flow
- total Total wrapping
- s Constant entropy
- * Critical cross section

**Acknowledgement**
The work leading to these results has received funding from the European Community’s Horizon 2020 Programme (2014-2020) under grant agreement no 678727. The opinions expressed in the document are of the authors only and no way reflect the European Commission’s opinions. The European Union is not liable for any use that may be made of the information.