Effective Field Theory of Heavy Mesons

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Abstract

In this paper we present a detailed formulation for a recently proposed effective field theory to describe the nonperturbative QCD dynamics of heavy mesons. This effective theory incorporates with heavy quark symmetry (HQS) and the heavy quark effective theory (HQET). Heavy mesons in this theory are constructed as composite particles of a heavy quark bounded with the light degrees of freedom. The heavy meson properties in the heavy quark limit and the $1/m_Q$ corrections can then be explicitly evaluated from this effective theory. All the basic parameters of the HQET, namely, the heavy quark mass $m_Q$, the heavy meson residual mass $\Lambda$, and the HQS breaking mass parameters $\lambda_1$ and $\lambda_2$, are consistently determined. $\lambda_1$ is found to be small due to a large cancellation between the heavy quark kinetic energy and the chromo-electric interaction between the heavy quark and light degrees of freedom. We also evaluate the Isgur-Wise function, the decay constant, and the axial-vector coupling constant of heavy mesons.

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I. INTRODUCTION

Within the last decade, the most significant progress in the QCD description of hadronic physics is the discovery of heavy quark symmetry (HQS) [1], which, together with the development of heavy quark effective theory (HQET) from QCD [2], has largely simplified the analysis of heavy hadron physics [3]. However, even in the infinite quark mass limit, the general properties of heavy hadrons, namely, their decay constants, transition form factors and structure functions etc. measured in various exclusive and inclusive decay processes remain unknown in the framework of QCD. The difficulty in understanding the heavy hadronic properties essentially lie in nonperturbative QCD dynamics, but HQET itself does not provide such a description to nonperturbative QCD. Lattice QCD simulations [4] permit a nonperturbative approach to the low-energy QCD problem, but so far a direct lattice calculation with heavy quarks is still not possible due to the difficulty of placing heavy particles on the lattice. An alternative first-principles calculation of nonperturbative QCD to these heavy hadron properties is to solve the heavy meson bound states directly from light-front QCD [5], but to obtain reliable results, further investigations are needed. Therefore, in most recent studies, these heavy hadron properties are usually evaluated using phenomenological models, such as the constituent quark model [6], the MIT bag model [7], the QCD sum rule [8], and the light-front quark model [9].

As is well known, although the constituent quark model (CQM) and the MIT bag model have been widely used in the phenomenological discussion of hadronic structures, applications of these two models are trustworthy only for processes involving small momentum transfers. This is due mainly to the nonrelativistic limitation of the CQM and to the difficulty with boost in the bag model. The light-front quark model (LFQM) which is a relativistic quark model with simple boost kinematics allows us to describe physical processes with large momentum transfers, but it is still not truely Lorentz covariant due to the exclusion of the so-called Z-diagrams [10,11]. As a result, certain theoretical ambiguities arise in LFQM calculation which may lead to inconsistent results as have been shown in [12,13].

To overcome the drawbacks in these phenomenological models, we recently proposed a covariant light-front model [14] which modifies the conventional LFQM in the heavy quark limit by adding a constraint on the light-front wave function. The covariant light-front model rules out some non-covariant light-front wave function often used in the literature, and therefore partially eliminates the ambiguities presented in previous calculations. Meanwhile, it also largely simplifies the light-front formulation, and may further provide a first-principles QCD analysis of the $1/m_Q^2$ corrections within HQET [14].

However, further investigation indicates that although it has overcome some theoretical difficulties encountered in the conventional light-front formulation for heavy hadrons, our covariant quark model [14] still cannot provide fully consistent results for processes involving light quark currents. This is because for those processes involving only heavy quark transitions, the Z-diagram contributions are suppressed in the heavy quark limit; however, when the light quarks (or currents) are involved, the non-covariant light-front treatment of light quarks (due to lack of Z-diagram contributions from the light quark production) will still cause theoretical ambiguities.

In this paper, we will present a detailed formulation of a fully field theoretical description of heavy mesons in terms of an effective theory of composite particles we have proposed very
recently [15]. The composite particle consists of a reduced heavy quark (heavy quark in the heavy quark limit) coupled with the light degrees of freedom, in which a structure function of the composite particle, $\Psi(v \cdot p_q)$, corresponding to the wave function of a heavy meson bound state, is explicitly built in. In this field theory description, the constituents in composite particles are no longer fixed at a given instant point of time, which is a condition imposed in the usual construction of hadronic bound states in various quark models. The field theory structure of the constituents in hadrons allows us to formulate the physical processes in terms of the standard Feynman diagrams in which various time-ordering diagrams are all automatically included. Therefore, the lack of Z-diagrams in the usual quark model descriptions is no longer a problem in the present formulation.

Moreover, combining the effective Lagrangian of the composite heavy mesons with the $1/m_Q$ expansion of the heavy quark QCD Lagrangian, we can systematically evaluate various $1/m_Q$ corrections to heavy meson properties in the standard framework of perturbative field theory. Thus, a self-contained description of heavy mesons (including the bound state structure and $1/m_Q$ corrections) is realized in a field-theoretic framework. This effective field theory allows us to explore the nonperturbative heavy meson dynamics, which is not possible in the light-front quark models.

The rest of the paper is organized as follows: In Sec. II, we will analyze the basic structure of heavy mesons in the heavy quark limit as a composite particle of the reduced heavy quark coupled with the light degrees of freedom. In Sec. III, we construct an effective Lagrangian to describe the composite structure of heavy mesons and combine this effective Lagrangian with HQET to establish a realistic effective field theory which can be used to evaluate the $1/m_Q$ corrections in the standard Feynman diagrammatic approach. As first applications, we evaluate the Isgur-Wise function, the heavy meson decay constant and the axial-vector coupling constant in the heavy quark limit in Sec. IV. In Sec. V, we compute the pseudoscalar and vector heavy meson masses up to $1/m_Q$, which determine the HQET parameters $\lambda_1$ and $\lambda_2$. In Sec. VI, we compare the effective field theory with the covariant light-front model [14], and show how the effective field theory overcomes the lack of relativistic covariance in light-front quark models. In Sec. VII, we present some numerical calculations to check the self-consistency of the theory, and then calculate all the basic parameters in HQET. The summary and perspective are given in Sec. VIII.

II. A COMPOSITE PARTICLE PICTURE OF HEAVY MESONS IN THE HEAVY QUARK LIMIT

We define the general expression for the composite operators of pseudoscalar and vector heavy mesons as follows:

$$
H_c(X) = \int d^4y \overline{\tau}(X - \alpha y) \Gamma_i Q[X + (1 - \alpha)y], \quad i = P, V,
$$

(2.1)

where the subscript $c$ means “composite”, $Q$ and $q$ are the heavy and light quark field operators respectively, $X$ is the center-of-mass coordinate of the heavy meson and $y = x_Q - x_q$ the relative coordinate between the heavy and light quarks, $\alpha = m_Q/(m_Q + m_q)$, and $\Gamma_P = \gamma_5$ and $\Gamma_V = \gamma_\mu$ define the spin structures for the pseudoscalar $(0^-)$ meson and vector $(1^-)$ meson, respectively. Inside the heavy meson, QCD dynamics is nonperturbative, so that
$Q(x)$ and $q(x)$ are strongly coupled, and they are surrounded by infinite number of $qq$ pairs and gluons originating from the nontrivial QCD vacuum. We may phenomenologically rewrite the above composite operator of heavy mesons in terms of the constituent valence heavy and light quark field operators, denoted by $Q_0$ and $q_0$ respectively, coupled through a structure function $F(y)$ which describes the binding effect of infinite number of $qq$ pairs and gluons governed by QCD,

$$H_{ci}(X) = \int d^4y \bar{q}_0(X -\alpha y)F(y)\Gamma_0[X + (1-\alpha)y]. \quad (2.2)$$

Now we take the heavy quark limit, i.e., $m_Q \to \infty$. In the momentum space, the heavy meson carries momentum $P^\mu \equiv m_Qv^\mu + p_H^\mu$, where $v^\mu$ is a four velocity ($v^2 = 1$) of the heavy meson, and $p_H$ the residual momentum. The heavy meson on-mass-shell condition $P^\mu_H = M_Hv^\mu$ corresponds to $p_H^\mu = \overline{\Lambda}v^\mu$, where $M_H$ is the heavy meson mass which approaches to infinity under the heavy quark limit but $\overline{\Lambda} = M_H - m_Q$ is kept finite. Note that $\overline{\Lambda}$ is a basic parameter in HQET known as the residual mass of the heavy meson.

In the heavy quark limit, we define the so-called reduced heavy quark field $h_v$ in the heavy quark expansion [2],

$$Q_0(x) = e^{-im_Qv^\mu x}h_v + O(1/m_Q), \quad \frac{1+\not{\nu}}{2}h_v = h_v. \quad (2.3)$$

We also introduce the reduced heavy meson composite operator $H_{ci}$,

$$H_{ci}(X) = \frac{1}{\sqrt{M_H}}e^{-im_Qv^\mu x}H_{ci}(X). \quad (2.4)$$

Then the Fourier transformation of the composite field operator in the momentum space can be expressed as

$$H_{ci}(v,p_H) = \int d^4X e^{ip_H^\mu X}H_{ci}(X)$$

$$= \int \frac{d^4k}{(2\pi)^4} \frac{d^4p_q}{(2\pi)^4}(2\pi)^4\delta^4(p_H - k - p_q)$$

$$\times \bar{q}_0(p_q)\Psi(v \cdot p_q)\Gamma_ih_v(k), \quad (2.5)$$

where $k^\mu = p_Q^\mu - m_Qv^\mu$ is the residual momentum of the heavy quark, and $p_q$ the momentum of the light antiquark. The function $\Psi(v \cdot p_q)$ is the heavy-quark-limit expression of the Fourier transformation of $F(y)$, which is the analog of heavy meson wave function in the covariant light-front quark model [4].

One may ask why is $\Psi(v \cdot p_q)$ only a function of $v \cdot p_q$? Note that the Fourier transformation of $F(y)$ should be a scalar function of the relative momentum $q = p_Q - p_q = m_Qv + k - p_q$,

$$\tilde{F}(q^2) = \int d^4y e^{iq \cdot y}F(y). \quad (2.6)$$

In the heavy quark limit, the $k^2$- and $p_q^2$-dependences are suppressed by the heavy quark mass:

$$q^2 = m_Q\left(m_Q + 2v \cdot (k - p_q) + O(1/m_Q)\right). \quad (2.7)$$
Also, due to heavy quark symmetry, $\tilde{F}(q^2)$ must be a $m_Q$-independent function in the heavy quark limit. Besides, by momentum conservation [i.e. the $\delta$-function in Eq.(2.3)], $k \sim p_H - p_q$ and hence $q^2 \sim (v \cdot p_H - 2v \cdot p_q)$. Since $\Psi$ describes the composite structure of heavy meson bound states in which $p_H = \Lambda v$, then, without loss of the generality, we have replaced $\tilde{F}(q^2)$ by the $m_Q$-independent function $\Psi(v \cdot p_q)$ in Eq. (2.3).

A motivation of the above analysis is to provide a realistic field theory formulation of the intuitive picture of heavy hadrons; that is, in the heavy quark limit the heavy meson is a composite particle which can be described by a composite field operator of a heavy quark $h_v(k)$ coupled with the light degrees of freedom which is sometimes called the “brown muck” in the literature. From Eq. (2.5), we see that it is natural to define the brown muck by

$$\tilde{q}_v(p_q) = \Psi(v \cdot p_q)\bar{q}_0(p_q).$$ (2.8)

The above definition indicates that the brown muck consists of a light valence antiquark (which contains the tensoral structure of a spin-1/2 Dirac particle) coupled with a brown muck structure function $\Psi(v \cdot p_q)$, which effectively describes the dynamics of infinite number of $q\bar{q}$ pairs and gluons surrounding the light antiquark. Thus, according to Haag, Nishijima and Zimmermann [17], we may rewrite the composite field operator of heavy mesons as a local operator:

$$H_{ci}(x) = \tilde{q}_v(x)\Gamma_i h_v(x),$$ (2.9)

and its momentum representation is given by:

$$H_{ci}(v, p_H) = \int \frac{d^4k}{(2\pi)^4} \frac{d^4p_q}{(2\pi)^4} (2\pi)^4 \delta^4(p_H - k - p_q)\tilde{q}_v(p_q)\Gamma_i h_v(k).$$ (2.10)

From Eq.(2.4), the normalization condition of the heavy meson bound states (with $p_H = \Lambda v$) in the heavy quark limit is given by

$$\langle H_{ci'}(v')|H_{ci}(v)\rangle = 2(2\pi)^3v^e\delta^3(\Lambda v - \Lambda v')\delta_{ii'},$$ (2.11)

where the superscript $e$ denotes the energy component in the momentum space, which can be either the 0 or the + component, depending on whether the light quark on-shell energy is picked on the light-front, $p_q^e = (p_q^2 + m_q^2)/p_q^0$, or on the equal-time form, $p^0 = \sqrt{p_q^2 + m_q^2}$.

Correspondingly, $\delta^3(\Lambda v - \Lambda v')$ becomes $\delta(\Lambda v^+ - \Lambda v'^+)$\delta^2(\Lambda v'_+ - \Lambda v'_-) or $\delta^3(\Lambda \bar{v} - \Lambda \bar{v}')$.

Of course, a first-principles determination of the above composite particle picture lies in the detailed form of $\Psi(v \cdot p_q)$. How to solve $\Psi(v \cdot p_q)$ directly from QCD is one of the most interesting and difficult problems in strong interaction physics. In the present paper, we shall treat $\Psi(v \cdot p_q)$ in a phenomenological manner.

**III. EFFECTIVE FIELD THEORY OF HEAVY MESONS**

Based on the above analysis of the composite particle picture of heavy mesons in the heavy quark limit, we can now build an effective field theory to describe the heavy meson structure in HQET.
A. Effective Lagrangian

In principle, the heavy-meson composite particle structure is determined by the QCD Lagrangian for heavy and light quarks plus gluons,

\[ \mathcal{L} = \mathcal{L}_Q + \mathcal{L}_q + \mathcal{L}_g \]

\[ = \Bar{Q}(i\slashed{D} - m_Q)Q + \Bar{q}(i\slashed{D} - m_q)q - \frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu}. \]  \hspace{1cm} (3.1)

Using the \(1/m_Q\) expansion to the heavy quark QCD Lagrangian \[2\], the above Lagrangian can be rewritten as

\[ \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{m_Q}, \]  \hspace{1cm} (3.2)

where

\[ \mathcal{L}_0 = \Bar{h}iv \cdot Dh_v + \Bar{q}(i\slashed{D} - m_q)q - \frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} \]  \hspace{1cm} (3.3)

governs the structure of the heavy meson composite field operator of Eq. (2.9), and it has the heavy quark \(SU_f(2) \otimes SU_s(2)\) flavor-spin symmetry (or simply the heavy quark symmetry, HQS), while,

\[ \mathcal{L}_{m_Q} = \sum_{n=1}^{\infty} \left( \frac{1}{2m_Q} \right)^n \Bar{h}_v(i\slashed{D}_\perp)(-iv \cdot D)^{n-1}(i\slashed{D}_\perp)h_v \]

\[ = \sum_{n=1}^{\infty} \left( \frac{1}{2m_Q} \right)^n \mathcal{L}_n \]  \hspace{1cm} (3.4)

determines the \(1/m_Q\) corrections to \(\mathcal{L}_0\), which breaks the heavy quark symmetry, where \(D_\perp^\mu \equiv D^\mu - \nu^\mu \nu \cdot D\).

To directly solve the structure of the composite field operator of Eq. (2.9) from \(\mathcal{L}_0\) is not simple and may not even be possible within the known framework of field theory. Instead we shall introduce an effective Lagrangian to phenomenologically describe the heavy meson composite structure,

\[ \mathcal{L}_0 \to \mathcal{L}_{\text{eff}}^M(h_v, q, H_{ci}) = \Bar{h}_viv \cdot \partial h_v + \Bar{q}_0(i\slashed{D} - m_q)q_0 + P_v^\mu(i\nu \cdot \slashed{\partial} - 2\Lambda_0)P_v - V_{v\mu}^\dagger(i\nu \cdot \slashed{\partial} - 2\Lambda_0)V_v^\mu + \text{h.c.} \]

\[ + G_0(\Bar{h}_v i\gamma_5 q_v P_v - \Bar{h}_v \gamma_\mu q_v V_v^\mu + \text{h.c.}), \]  \hspace{1cm} (3.5)

which has the same heavy quark symmetry as \(\mathcal{L}_0\), where \(\slashed{\partial} \equiv \slashed{\partial} - \nu \slashed{\partial} \nu\), \(q_v\) is the brown muck field of the light degrees of freedom inside the heavy mesons, specified by Eq. (2.8) in momentum space. \(P_v\) and \(V_{v\mu}^\mu\) are the reduced pseudoscalar and vector heavy meson fields, respectively:

\[ \Phi(x) = \frac{1}{\sqrt{M_P}} e^{-im_Q v \cdot x} P_v(x), \]  \hspace{1cm} (3.6)

\[ A_\mu(x) = \frac{1}{\sqrt{M_V}} e^{-im_Q v \cdot x} V_{v\mu}(x), \]  \hspace{1cm} (3.7)
with \( v \cdot V_e = 0 \), and \( \Lambda_0 \equiv M_0 - m_{Q_0} \) as the bare residual mass of the heavy mesons, where \( M_0 \) and \( m_{Q_0} \) are defined here as the bare masses of heavy mesons and heavy quarks, respectively. The third and fourth terms in (3.3) are directly obtained from the free Lagrangian of the pseudoscalar and vector fields with the above definition of the reduced fields,

\[
L^M_{\text{free}} = \left( \partial^\mu \Phi^\dagger \partial_\mu \Phi - M_0^2 |\Phi|^2 \right) - \frac{1}{2} F_{\mu\nu}^\rho F^{\rho\mu\nu} + M_0^2 A^{\mu\nu} A_\mu^\nu,
\]

(3.8)

where \( F_{\mu\nu}^\rho = \partial^\rho A_\mu - \partial^\rho A_\mu \).

In principle, the effective Lagrangian \( L_{\text{eff}}^M \) may be derived from \( L_0 \) by integrating out the gluon degrees of freedom in a nonperturbative way, which is unfortunately not practical at present. Therefore, we shall assume that after integrating out the gluon degrees of freedom mediated between heavy and light quarks, we are led to an effective Lagrangian of the form:

\[
L_{\text{eff}}^{Qq} = \bar{q}_v i v \cdot \partial h_v + m_{q_0} q_0 + g_0^2 (\bar{q}_v i \gamma_5 q_0 \gamma_5 h_v - \bar{h}_v q_0 \gamma_5 \gamma_5 h_v + \cdots),
\]

(3.9)

in which the complicated nonperturbative QCD dynamics to the heavy meson structure is effectively described by introducing the brown muck field \( q_v \) through the universal structure function \( \Psi(v \cdot p_{q}) \) of Eq.(2.8). A consistency condition for the composite particle picture of heavy mesons in the heavy quark limit can be determined by demanding the equivalence of Eqs.(3.5) and (3.9).

B. Dynamical description of the composite particle structure

We can now dynamically describe the heavy mesons in the heavy quark limit as composite particles in the above two effective field theories. In terms of Eq. (3.9), the composite particle structure of a heavy-light quark field \( \bar{q}_v \Gamma_i h_v \) can be obtained by considering the heavy-light quark scattering in the chain approximation, as shown in Fig. 1 (18). In the pseudoscalar channel, apart from the factors come from the external quark lines, the scattering amplitude is given by

\[
A_{Qq} = g_0^2 \Psi(v \cdot p_{q}) \Psi^*(v \cdot p_{q}') i \frac{1}{1 - g_0^2 \Pi_H(v \cdot p_H)},
\]

(3.10)

where \( p_H = k + p_q \) is the momentum transfer, \( k \) the residual momentum of the heavy quark, \( p_q \) the momentum carried by the light quark, and \( \Pi_H(v \cdot p_H) \) the “self-energy” correction from the heavy-light quark loop depicted in Fig. 1(a),

\[
- i \Pi_H(v \cdot p_H) = (-i)^2 (-1) \int \frac{d^4 p_q}{(2\pi)^4} |\Psi(v \cdot p_q)|^2 \text{Tr} \left[ i \frac{1 + \gamma^5}{2(v \cdot p_H - v \cdot p_q + i\epsilon)} \right] \times \Gamma_i \frac{-p_q + m_q}{p_q^2 - m_q^2 + i\epsilon},
\]

(3.11)

where \( \Gamma_p = i \gamma_5 \). Thus, we have
The existence of a stable heavy pseudoscalar composite meson with the residual mass $\Lambda$ implies that the above amplitude has a pole at $p_H = \Lambda v$ (which corresponds to $P_H = M_H v$).

This leads to

$$g_0^2 = \frac{1}{\Pi_H(\Lambda)}. \quad (3.13)$$

If we expand $\Pi_H(v \cdot p_H)$ by

$$\Pi_H(v \cdot p_H) = \Pi_H(\Lambda) + (v \cdot p_H - \Lambda)\Pi'_H(\Lambda) + \Pi''_H(v \cdot p_H), \quad (3.14)$$

then, the amplitude can be expressed as

$$A_{Qq} = -i \frac{\Psi(v \cdot p_q)\Psi^*(v' \cdot p'_q)}{(v \cdot p_H - \Lambda) \Pi_H(\Lambda) + \Pi_H(v \cdot p_H)}. \quad (3.15)$$

On the other hand, in Eq. (3.5) the heavy meson field appears as a fundamental field. The physical (i.e., renormalized) meson structure can be determined by considering a similar heavy-light quark scattering process shown in Fig. 1(b). The amplitude is

$$A_M = G_0^2 \Psi(v \cdot p_q)\Psi^*(v \cdot p'_q)\Delta_H(v \cdot p_H), \quad (3.16)$$

where $\Delta(v \cdot p_H)$ is the heavy meson propagator:

$$\Delta_H(v \cdot p_H) = \frac{i}{2(v \cdot p_H - \Lambda) - G_0^2 \Pi_H(v \cdot p_H)}, \quad (3.17)$$

with $\Pi_H(v \cdot p_H)$ being given by Eq. (3.12). Using the expansion (3.14), the above amplitude can be recast into

$$A_M = G^2 \Psi(v \cdot p_q)\Psi^*(v \cdot p'_q)\frac{i}{2(v \cdot p_H - \Lambda) - G^2 \Pi_H(v \cdot p_H)}, \quad (3.18)$$

with

$$G = Z_3^{1/2}G_0, \quad Z_3 = 1 + \frac{G^2}{2}\Pi'_H(\Lambda), \quad (3.19)$$

$$\Lambda = \Lambda_0 + \frac{G_0^2}{2}\Pi_H(\Lambda), \quad (3.20)$$

where $Z_3$ is the wave function renormalization constant of the heavy meson field $P_v$.

In order that the pseudoscalar heavy meson, being a composite particle with the structure $\bar{q}_v \gamma_5 h_v$, can legitimately be represented by the field operator $P_v$, the two scattering amplitudes Eqs. (3.18) and (3.15) must be the same. This results in

$$G^{-2} = -\frac{1}{2}\Pi'_H(\Lambda) = i \int \frac{d^4 p_q}{(2\pi)^4} \frac{|\Psi(v \cdot p_q)|^2}{(\Lambda - v \cdot p + i\epsilon)^2} \frac{v \cdot p_q + m_q}{(p^2 - m_q^2 + i\epsilon)}, \quad (3.21)$$
\[ \Xi = \Xi_0 + iG_0^2 \int \frac{d^4p_q}{(2\pi)^4} \frac{|\Psi(v \cdot p_q)|^2}{(\tilde{\Xi} - v \cdot p_q + i\epsilon)} \frac{v \cdot p_q + m_q}{(p^2 - m^2 + i\epsilon)}. \] (3.22)

We therefore obtain from Eq. (3.19) that
\[ Z_3 = 0. \] (3.23)

The above results have a clear physical interpretation. The fact \( Z_3 = 0 \) implies that the bare fundamental field \( P_v = Z_3^{1/2} P_v^R \) does not exist. In other words, the physical meson field \( P_v^R \) must be a composite particle operator. Such a realization of the composite particle relies upon the introduction of the composite particle structure function \( \Psi(v \cdot p_q) \) in the effective Lagrangians (3.5) and (3.9). One can always choose a structure function \( \Psi(v \cdot p_q) \) to ensure the finiteness of \( \Pi_H(\tilde{\Xi}) \) and \( \Pi'_H(\tilde{\Xi}) \). Without invoking such a structure function in the effective four-fermion point interaction, \( \Pi'_H(0) \) is divergent and hence \( G = 0 \), which leads to obvious inconsistency in the effective theory. This shows the importance of \( \Psi(v \cdot p_q) \) in our construction, which is not surprising since, as we will see shortly, \( \Psi(v \cdot p_q) \) actually corresponds to a heavy meson wave function, while the renormalized coupling constant \( G \) of Eq. (3.21) is just the wave function normalization constant.

Similar discussion for the vector meson structure can be easily carried out. In the heavy quark limit, the heavy quark symmetry of the effective theory ensures that the vector meson composite particle has the same structure as the pseudoscalar particle. We will therefore not repeat the similar derivation here for vector mesons.

C. The effective field theory of heavy mesons with Feynman rules

After determining the composite particle structure of heavy mesons in the heavy quark limit, we can proceed to evaluate various heavy meson properties by combining the effective Lagrangian \( \mathcal{L}_{\text{eff}}^{MR} \), which obeys heavy quark symmetry, with the \( 1/m_Q \) corrections from \( \mathcal{L}_{m_Q} \):
\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{eff}}^{MR} + \mathcal{L}_{m_Q} ,
\] (3.24)

where
\[
\mathcal{L}_{\text{eff}}^{MR} = \bar{h}_v iv \cdot \partial h_v + \bar{\tau}(i \vec{\sigma} - m_q)q_v
+ P_v^\dagger (iv \cdot \vec{\sigma} - 2\tilde{\Xi}) P_v - V^{\mu\nu} (iv \cdot \vec{\sigma} - 2\tilde{\Xi}) V_{\mu\nu}
+ G(\bar{h}_v i\gamma_5 q_v P_v - \bar{h}_v \gamma_{\mu} q_v V_{\mu} + h.c.),
\] (3.25)

\[
\mathcal{L}_{m_Q} = \sum_{n=1}^{\infty} \left( \frac{1}{2m_Q} \right)^n \bar{h}_v (i \not{D}) (-iv \cdot D)^{n-1} (i \not{D}) h_v ,
\] (3.26)

\( G \) is determined by Eq. (3.21), \( \tilde{\Xi} \) is the physical residual mass of heavy mesons in the heavy quark limit, \( M_H = m_Q + \tilde{\Xi} \), \( q_v = \Psi(v \cdot p_q)q \) and \( \Psi(v \cdot p_q) \) describes the heavy meson structure and is a phenomenological input at this level. The Lagrangian \( \mathcal{L}_{\text{eff}}^{MR} \) gives the
nonperturbative effective coupling of heavy mesons with heavy-light quarks. And \( \mathcal{L}_{mQ} \) is treated as a perturbation to \( \mathcal{L}_{\text{eff}}^{MR} \), which contains all the \( 1/m_Q \) corrections. Then a practical evaluation scheme can be developed in terms of the standard Feynman diagrammatic rules:

(i) The heavy meson bound state \( (p_H = \bar{\Lambda}v) \) in the heavy quark limit gives a vertex as follows:

\[
- - \bullet - - : \quad - i G \Psi^* (v \cdot p_q) \Gamma_H ,
\]

(3.27)

\[
- - \bullet - - : \quad - i G \Psi (v \cdot p_q) \Gamma_H ,
\]

(3.28)

with a momentum conservation factor \( (2\pi)^4 \delta^4(\bar{\Lambda}v - k - p_q) \), where \( \Gamma_P = i \gamma_5 \) for pseudoscalar \( (H = P) \), \( \Gamma_V = - \not\epsilon \) for vector mesons \( (H = V) \), and \( p_q \) is the momentum of the light degrees of freedom.

(ii) The internal line propagators for the heavy quark and the light antiquark are,

\[
\begin{align*}
\underbar{k} : & \quad i \frac{p_H + 1}{2(v \cdot k + i\epsilon)} , \\
\overline{-p_q} : & \quad i \frac{-p_q + m_q}{p_q^2 - m_q^2 + i\epsilon},
\end{align*}
\]

(3.29)

(3.30)

respectively, where \( k \) is the residual momentum of the heavy quark, and \( m_q \) the constituent mass of the light antiquark.

(iii) For internal lines, integrate over the internal four-momenta,

\[
\int \frac{d^4k}{(2\pi)^4} \quad \text{and} \quad \int \frac{d^4p_q}{(2\pi)^4} ,
\]

(3.31)

for heavy and light quarks, respectively. Also there is a factor of \((-1)\) for each fermion loop.

(iv) For all other vertices that do not attach to the bound states, the corresponding diagrammatic rules are standard from the conventional field theory formulation. Most of these vertices mainly come from \( \mathcal{L}_{mQ} \), hence the corresponding \( 1/m_Q \) corrections can be obtained from the standard perturbation field theory (see an explicit example in the next section).

These are the Feynman rules needed for subsequent calculations in this effective field theory.

**IV. HEAVY MESON PROPERTIES IN HEAVY QUARK LIMIT**

In this section, we evaluate the basic heavy meson properties in the heavy quark limit within the present framework. These include the Isgur-Wise function, the decay constants and the axial-vector coupling constants of heavy mesons.
A. Isgur-Wise function

In the heavy quark limit, the transition matrix elements of $B \to D$, and $B \to D^*$ are given by

$$
\langle D(v') | \overline{h}^b \Gamma h^b \rangle_B(v) \quad \text{and} \quad \langle D(v', \epsilon^*) | \overline{h}^b \Gamma h^b \rangle_B(v),
$$

(4.1)

where $\Gamma$ is a Dirac $\gamma$-matrix from the electroweak current of heavy quarks. Using the Feynman rules given in Sec. IIIC, the hadronic matrix elements of $B \to D$ and $B \to D^*$ decays turn out to be (see Fig. 2(a))

$$
\langle D(v') | \overline{h}^b \Gamma h^b \rangle_B(v) = \text{Tr} \left\{ \Gamma_p \left( \frac{1+\not{p}'}{2} \right) \Gamma \left( \frac{1+\not{q}'}{2} \right) \Gamma_p \mathcal{M} \right\},
$$

(4.2)

$$
\langle D(v', \epsilon^*) | \overline{h}^b \Gamma h^b \rangle_B(v) = \text{Tr} \left\{ \Gamma_V \left( \frac{1+\not{p}'}{2} \right) \Gamma \left( \frac{1+\not{q}'}{2} \right) \Gamma_p \mathcal{M} \right\},
$$

(4.3)

or

$$
\langle H'(v') | \overline{h}^b \Gamma h^b \rangle_H(v) = \text{Tr} \left\{ \Gamma_{H'} \left( \frac{1+\not{p}'}{2} \right) \Gamma \left( \frac{1+\not{q}'}{2} \right) \Gamma_H \mathcal{M} \right\},
$$

(4.4)

for the general heavy meson decay process $H \to H'$, where

$$
\mathcal{M} = i G^2 \int \frac{d^4 p_q}{(2\pi)^4} \frac{\Psi^*(v' \cdot p_q) \Psi(v \cdot p_q)}{\left( \Lambda - v \cdot p_q + i\epsilon \right) \left( \Lambda - v' \cdot p_q + i\epsilon \right) p_q^2 - m_q^2 + i\epsilon},
$$

(4.5)

which is actually the transition matrix element of the light antiquark (brown muck).

Since $\mathcal{M}$ is fully covariant, it has the form

$$
\mathcal{M} = A + B \not{q} + C \not{q}',
$$

(4.6)

where the coefficients $A, B, C, D$ can be easily determined to be

$$
A = -i G^2 \int \frac{d^4 p_q}{(2\pi)^4} \frac{\Psi^*(v' \cdot p_q) \Psi(v \cdot p_q)}{\left( \Lambda - v \cdot p_q + i\epsilon \right) \left( \Lambda - v' \cdot p_q + i\epsilon \right) p_q^2 - m_q^2 + i\epsilon},
$$

(4.7)

$$
B = i G^2 \int \frac{d^4 p_q}{(2\pi)^4} \frac{\Psi^*(v' \cdot p_q) \Psi(v \cdot p_q)}{\left( \Lambda - v \cdot p_q + i\epsilon \right) \left( \Lambda - v' \cdot p_q + i\epsilon \right) p_q^2 - m_q^2 + i\epsilon} \times \frac{1}{2} \left\{ \frac{(v + v') \cdot p_q}{1 + v \cdot v'} - \frac{(v - v') \cdot p_q}{1 - v \cdot v'} \right\},
$$

(4.8)

$$
C = i G^2 \int \frac{d^4 p_q}{(2\pi)^4} \frac{\Psi^*(v' \cdot p_q) \Psi(v \cdot p_q)}{\left( \Lambda - v \cdot p_q + i\epsilon \right) \left( \Lambda - v' \cdot p_q + i\epsilon \right) p_q^2 - m_q^2 + i\epsilon} \times \frac{1}{2} \left\{ \frac{(v + v') \cdot p_q}{1 + v \cdot v'} + \frac{(v - v') \cdot p_q}{1 - v \cdot v'} \right\}.
$$

(4.9)

Then Eq. (4.4) can be simplified to

$$
\langle H'(v') | \overline{h}^b \Gamma h^b \rangle_H(v) = -\xi (v \cdot v') \text{Tr} \left\{ \Gamma_{H'} \left( \frac{1+\not{p}'}{2} \right) \Gamma \left( \frac{1+\not{q}'}{2} \right) \Gamma_H \mathcal{M} \right\},
$$

(4.10)
where $\xi(v \cdot v')$ is the Isgur-Wise function

$$\xi(v \cdot v') = -(A - B - C)$$

$$= iG^2 \int \frac{d^4 p_q}{(2\pi)^4} \Psi^*(v' \cdot p_q) \Psi(v \cdot p_q) \frac{m_q + (v + v') \cdot p_q/(1 + v \cdot v')}{(p_q^2 - m_q^2 + i\epsilon)(\Lambda - v \cdot p_q + i\epsilon)(\Lambda - v' \cdot p_q + i\epsilon)}.$$  (4.11)

At zero recoil $v \cdot v' = 1$, i.e., $v' = v$,

$$\xi(1) = iG^2 \int \frac{d^4 p_q}{(2\pi)^4} |\Psi(v \cdot p_q)|^2 \frac{v \cdot p_q + m_q}{(p_q^2 - m_q^2 + i\epsilon)(\Lambda - v \cdot p_q + i\epsilon)^2} = 1,$$  (4.12)

where we have used the result (3.21). This is the well-known normalization of the Isgur-Wise function at the zero-recoil point, as dictated by HQS.

### B. Decay constants in the heavy quark limit

The decay constants of pseudoscalar and vector heavy mesons are defined by $\langle 0|A^\mu|P(p)\rangle = iFP v^\mu$ and $\langle 0|V^\mu|V(p, \epsilon)\rangle = f_V M_V v^\mu$ respectively, where $A^\mu = \overline{q}_5 \gamma_5 \gamma^\mu Q$ and $V^\mu = \overline{q}_5 \gamma^\mu Q$.

In the heavy quark limit, the meson decay constants are redefined as

$$\langle 0|\overline{q}_5 \gamma^\mu h_v|P(v)\rangle = iF_P v^\mu, \quad \langle 0|\overline{q}_5 \gamma^\mu h_v|V(v, \epsilon)\rangle = F_V v^\mu,$$  (4.13)

where in the heavy quark limit, $|P(p)\rangle = \sqrt{M_P} |P(v)\rangle$ and likewise for $|V(p, \epsilon)\rangle$, therefore,

$$F_P = f_P \sqrt{M_P}, \quad F_V = f_V \sqrt{M_V}.$$  (4.14)

HQS demands that $F_V = F_P$.

Now, using the Feynman rules of the effective theory, it is very simple to evaluate the above matrix elements (see Fig. 2(b)):

$$\langle 0|\overline{q}_5 \gamma^\mu \gamma_5 h_v|P(v)\rangle = \text{Tr}\{\gamma^\mu \gamma_5 \frac{1 + \not{v}}{2} \Gamma_P \mathcal{M}_1\},$$  (4.15)

$$\langle 0|\overline{q}_5 h_v|V(v, \epsilon)\rangle = \text{Tr}\{\gamma^\mu \frac{1 + \not{v}}{2} \Gamma_V \mathcal{M}_1\},$$  (4.16)

where

$$\mathcal{M}_1 = iG \sqrt{N_c} \int \frac{d^4 p_q}{(2\pi)^4} \Psi(v \cdot p_q) \frac{p_q - m_q}{(p_q^2 - m_q^2 + i\epsilon)(\Lambda - v \cdot p_q + i\epsilon)}.$$  (4.17)

and

$$A_1 = -iG \sqrt{N_c} \int \frac{d^4 p_q}{(2\pi)^4} \Psi(v \cdot p_q) \frac{m_q}{(p_q^2 - m_q^2 + i\epsilon)(\Lambda - v \cdot p_q + i\epsilon)},$$  (4.18)

$$B_1 = iG \sqrt{N_c} \int \frac{d^4 p_q}{(2\pi)^4} \Psi(v \cdot p_q) \frac{v \cdot p_q}{(p_q^2 - m_q^2 + i\epsilon)(\Lambda - v \cdot p_q + i\epsilon)}.$$  (4.19)
Here \( N_c = 3 \) is the number of colors. Thus, it is easily found:

\[
F_P = -2(A_1 - B_1)
\]

\[
= 2iG\sqrt{N_c} \int \frac{d^4p_q}{(2\pi)^4} |\Psi(v \cdot p_q)|^2 \frac{v \cdot p_q + m_q}{(p_q^2 - m_q^2 + i\epsilon)(\Lambda - v \cdot p_q + i\epsilon)}
\]

\[
= F_V ,
\]

(4.20)
as expected from HQS.

C. Axial-vector coupling constants in the heavy quark limit

The axial-vector coupling constants, \( f \) and \( g \), are defined by the transition matrix elements of \( V \to P\pi \) and \( V \to V'\pi \) in the soft pion limit \cite{19,20}:

\[
\langle P(v)|A^a_\mu|V(v, \epsilon) \rangle , \quad \langle V'(v, \epsilon'^*)|A^a_\mu|V(v, \epsilon) \rangle ,
\]

(4.21)

where \( A^a_\mu = \frac{\gamma^a}{2} \gamma^\mu \gamma^5 q \) is the light quark axial-vector current. In the heavy quark limit,

\[
\langle P(v)|q \cdot A^a|V(v, \epsilon) \rangle = i\frac{f}{2} \epsilon \cdot q ,
\]

(4.22)

\[
\langle V'(v, \epsilon'^*)|q \cdot A^a|V(v, \epsilon) \rangle = ig\varepsilon^{\mu\nu\rho\sigma} q_\mu \epsilon'^*_\nu v_\rho \epsilon_\sigma ,
\]

(4.23)

where \( q \approx 0 \) is the momentum carried by the soft-pion, and the SU(3) flavor matrix element \( \chi^\dagger_H \chi \) has been omitted in the above expressions. HQS predicts that \( f = 2g \) \cite{19}.

Diagrammatically, the above two matrix elements are represented by Fig. 2c, from which one can straightforwardly write down these matrix elements:

\[
\langle P(v)|q \cdot A|V(v, \epsilon) \rangle = \frac{1}{2} \mathrm{Tr}\left\{ \Gamma_P \frac{1}{2} \Gamma_V \mathcal{M}_3 \right\} ,
\]

\[
\langle V'(v, \epsilon'^*)|q \cdot A|V(v, \epsilon) \rangle = \frac{1}{2} \mathrm{Tr}\left\{ \Gamma_V^* \frac{1}{2} \Gamma_V \mathcal{M}_3 \right\} ,
\]

(4.24)
or simply

\[
\langle H'(v)|q \cdot A|H(v) \rangle = \frac{1}{2} \mathrm{Tr}\left\{ \Gamma_H \frac{1}{2} \Gamma_H \mathcal{M}_3 \right\} ,
\]

(4.25)

where

\[
\mathcal{M}_3 = -iG^2 \int \frac{d^4p_q}{(2\pi)^4} |\Psi(v \cdot p_q)|^2 \frac{(p_q^2 - m_q^2) q_\mu v_\mu}{(p_q^2 - m_q^2 + i\epsilon)^2(\Lambda - v \cdot p_q + i\epsilon)}
\]

\[
= \left( A_3 v^\dagger q + B_3 q^\dagger + C_3 v \cdot q + D_3 q \cdot q^\dagger \right) \gamma_5 .
\]

(4.26)

and the coefficients are given by
Then, it is easily seen that Eq. (4.25) can be simplified as

\[ A_3 = iG^2 \int \frac{d^4p_q}{(2\pi)^4} |\Psi(v \cdot p_q)|^2 \frac{2m_q v \cdot p_q}{(p_q^2 - m_q^2 + i\epsilon)(\Lambda - v \cdot p_q + i\epsilon)}, \]  

(4.27)

\[ B_3 = -iG^2 \int \frac{d^4p_q}{(2\pi)^4} |\Psi(v \cdot p_q)|^2 \frac{1}{3} \frac{2(v \cdot p_q)^2 + p_q^2 + 3m_q^2}{(p_q^2 - m_q^2 + i\epsilon)^2(\Lambda - v \cdot p_q + i\epsilon)}, \]  

(4.28)

\[ C_3 = iG^2 \int \frac{d^4p_q}{(2\pi)^4} |\Psi(v \cdot p_q)|^2 \frac{2}{3} \frac{4(v \cdot p_q)^2 - p_q^2}{(p_q^2 - m_q^2 + i\epsilon)^2(\Lambda - v \cdot p_q + i\epsilon)}, \]  

(4.29)

\[ D_3 = -A_3. \]  

(4.30)

Then, it is easily seen that Eq. (4.23) can be simplified as

\[ \langle H'(v)|q \cdot A|H(v)\rangle = \frac{g}{2} \text{Tr} \left\{ \Gamma_H \frac{1 + \gamma_5}{2} \Gamma_H \not{q} \gamma_5 \right\}, \]  

(4.31)

with \( f = 2g \) and

\[ g = -(B_3 + D_3) \]

\[ = -iG^2 \int \frac{d^4p_q}{(2\pi)^4} |\Psi(v \cdot p_q)|^2 \frac{2(p_q^2 - m_q^2) + 2(v \cdot p_q + m_q)(v \cdot p_q + 2m_q)}{(p_q^2 - m_q^2 + i\epsilon)^2(\Lambda - v \cdot p_q + i\epsilon)}. \]  

(4.32)

V. THE DETERMINATION OF HEAVY MESON MASSES

In this section, we determine the heavy meson masses up to order \( 1/m_Q \) within the effective theory.

In the heavy quark limit, the heavy meson masses can be written as \( M_H = m_Q + \Lambda \). In this limit, the pseudoscalar and vector mesons are degenerate. The correction to \( M_M \) comes mainly from the leading HQS-breaking \( 1/m_Q \) corrections [see Eq. (3.4)]:

\[ \mathcal{L}_1 = \bar{h}_v (iD_\perp)^2 h_v + \frac{g_s}{2} \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v = \mathcal{O}_1 + \mathcal{O}_2, \]  

(5.1)

where \( \sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu] \) and \( G^{\mu\nu} = \frac{i}{g_s} [D^\mu, D^\nu] \). With these \( 1/m_Q \) corrections included, the heavy meson masses can be expressed as

\[ M_H = m_Q + \Lambda - \frac{1}{2m_Q} (\lambda_1 + d_H \lambda_2), \]  

(5.2)

where \( \lambda_1 \) comes from \( \mathcal{O}_1 \) and \( d_H \lambda_2 \) from \( \mathcal{O}_2 \) (see Fig. 3). \( \Lambda, \lambda_1 \) and \( \lambda_2 \) are the basic parameters in HQET. The parameter \( \Lambda \) is the residual mass of heavy mesons in the heavy quark limit and is associated with the dynamical mass of the brown muck \[3\], \( \lambda_1 \) parametrizes the common mass shift, and \( \lambda_2 \) describes the effect of the hyperfine mass splitting. In the rest frame of the heavy meson, the Clebsch-Gordon coefficient \( d_H \) in Eq. (5.2) is conventionally normalized in such a way that

\[ d_H = -\langle H(v)|4\vec{S}_Q \cdot \vec{S}_\ell|H(v)\rangle \]

\[ = -2[S_{\text{tot}}(S_{\text{tot}} + 1) - S_Q(S_Q + 1) - S_\ell(S_\ell + 1)], \]  

(5.3)
where $\vec S_Q$ ($\vec S_f$) is the spin operator of the heavy quark (light degrees of freedom). Therefore, $d_H = -1$ for vector (1 $^-$) mesons, and $d_H = 3$ for pseudoscalar (0 $^-$) mesons.

Using the Feynman diagrammatic rules of the effective theory, we can immediately write down the expressions for $\lambda_1$ and $d_H \lambda_2$ for heavy mesons as follows:

$$\lambda_1 = \text{Tr}\left\{ \frac{1}{2} \Gamma_H \frac{1 + \not{\gamma}}{2} \Gamma_H \mathcal{M}_2 \right\}, \quad (5.4)$$

$$d_H \lambda_2 = \text{Tr}\left\{ \frac{1}{2} \Gamma_H \frac{1 + \not{\gamma}}{2} \Gamma_H \mathcal{M}_{2}^{\mu\nu} \right\}, \quad (5.5)$$

with

$$\mathcal{M}_2 = iG^2 \int \frac{d^4 \vec{p}_q}{(2\pi)^4} |\Psi(v \cdot \vec{p}_q)|^2 \frac{(p^2_q - (v \cdot \vec{p}_q)^2)(\not{p}_q - m_q)}{(p^2_q - m^2_q + i\epsilon)(\Lambda - v \cdot \vec{p}_q + i\epsilon)^2}$$

$$+ ig_s^2 C_f G^2 \int \frac{d^4 \vec{p}_{1q}}{(2\pi)^4} \frac{d^4 \vec{p}_{2q}}{(2\pi)^4} \Psi(v \cdot \vec{p}_{1q}) \Psi^*(v \cdot \vec{p}_{2q})$$

$$\times \frac{v \cdot (p_{1q} + p_{2q}) u^\mu - (p_{1q} + p_{2q})^\mu}{(\Lambda - v \cdot \vec{p}_{1q} + i\epsilon)(\Lambda - v \cdot \vec{p}_{2q} + i\epsilon)}$$

$$\times D_{\mu\nu} \frac{\not{p}_{1q} - m_q}{p_{1q}^2 - m^2_q + i\epsilon} \frac{\not{p}_{2q} - m_q}{p_{2q}^2 - m^2_q + i\epsilon}, \quad (5.6)$$

$$\mathcal{M}_{2}^{\mu\nu} = g_s^2 C_f G^2 \int \frac{d^4 \vec{p}_{1q}}{(2\pi)^4} \frac{d^4 \vec{p}_{2q}}{(2\pi)^4} \Psi(v \cdot \vec{p}_{1q}) \Psi^*(v \cdot \vec{p}_{2q})$$

$$\times \frac{(p_{1q} - p_{2q})^\mu}{(\Lambda - v \cdot \vec{p}_{1q} + i\epsilon)(\Lambda - v \cdot \vec{p}_{2q} + i\epsilon)}$$

$$\times D^{\nu\mu} \frac{\not{p}_{2q} - m_q}{p_{2q}^2 - m^2_q + i\epsilon} \frac{\not{p}_{1q} - m_q}{p_{1q}^2 - m^2_q + i\epsilon}, \quad (5.7)$$

where $C_f$ is a color factor, $C_f = \frac{N^2 - 1}{2 N_c} = 4/3$ for $N_c = 3$, and $D^{\mu\nu}$ is the gluon propagator given by $D^{\mu\nu} = -\frac{ig^{\mu\nu}}{q^2 + i\epsilon}$ in Feynman gauge ($q = p_{1q} - p_{2q}$).

The mass shift parameter $\lambda_1$ can be further simplified to

$$\lambda_1 = iG^2 \int \frac{d^4 \vec{p}_q}{(2\pi)^4} |\Psi(v \cdot \vec{p}_q)|^2 \frac{2(p^2_q - (v \cdot \vec{p}_q)^2)(v \cdot \vec{p}_q + m_q)}{(p^2_q - m^2_q + i\epsilon)(\Lambda - v \cdot \vec{p}_q + i\epsilon)^2}$$

$$- 2g_s^2 C_f G^2 \int \frac{d^4 \vec{p}_{1q}}{(2\pi)^4} \frac{d^4 \vec{p}_{2q}}{(2\pi)^4} \frac{\Psi(v \cdot \vec{p}_{1q}) \Psi^*(v \cdot \vec{p}_{2q})}{(\Lambda - v \cdot \vec{p}_{1q} + i\epsilon)(\Lambda - v \cdot \vec{p}_{2q} + i\epsilon)}$$

$$\times \frac{1}{((p_{1q} - p_{2q})^2 + i\epsilon)(p_{1q}^2 - m^2_q + i\epsilon)(p_{2q}^2 - m^2_q + i\epsilon)}$$

$$\times \left\{ (v \cdot p_{1q} + m_q)[(p_{1q} + p_{2q}) \cdot p_{2q} - v \cdot (p_{1q} + p_{2q}) v \cdot p_{2q}] + (v \cdot p_{2q} + m_q)[(p_{1q} + p_{2q}) \cdot p_{1q} - v \cdot (p_{1q} + p_{2q}) v \cdot p_{1q}] \right\}. \quad (5.8)$$

The hyperfine mass splitting parameter $\lambda_2$ can also be simplified. By using the identity

$$\frac{1 + \not{\gamma}}{2} \sigma_{\mu\nu} \frac{1 + \not{\gamma}}{2} v^\mu = 0, \quad (5.9)$$
we find that only the antisymmetric component of $\mathcal{M}_{\mu\nu}^\prime$ contributes to $\lambda_2$. Thus we can let

$$\mathcal{M}_{\mu\nu}^\prime \to -\frac{1}{4}\sigma^{\mu\nu}\zeta.$$ (5.10)

With the use of $\Gamma_H \not= -\not{v}\Gamma_H$, it is not difficult to find that $\zeta$ is given by

$$\zeta = \frac{4}{3}g_s^2 C_f G^2 \int \frac{d^4 p_1 q}{(2\pi)^4} \frac{d^4 p_2 q}{(2\pi)^4} \frac{\Psi(v \cdot p_1 q)\Psi^*(v \cdot p_2 q)}{(\Lambda - v \cdot p_1 q + i\epsilon)(\Lambda - v \cdot p_2 q + i\epsilon)} \times \frac{1}{(p_{1q} - p_{2q})^2 + i\epsilon}(p_{1q}^2 - m_q^2 + i\epsilon)(p_{2q}^2 - m_q^2 + i\epsilon) \times \left\{ (v \cdot p_1 q + m_q)[(p_{1q} - p_{2q}) \cdot p_{1q} - v \cdot (p_{1q} - p_{2q})v \cdot p_{2q}] - (v \cdot p_{2q} + m_q)[(p_{1q} - p_{2q}) \cdot p_{1q} - v \cdot (p_{1q} - p_{2q})v \cdot p_{1q}] \right\}.$$ (5.11)

Since

$$\text{Tr}\left\{ \Gamma_H^{1+} + \not{v} \not{\sigma}_{\mu\nu}^{1+} \Gamma_H \sigma^{\mu\nu} \right\} = \begin{cases} -12 & \text{for } \Gamma_H = i\gamma_5, \\ 4 & \text{for } \Gamma_H = -\not{v}, \end{cases}$$ (5.12)

it follows that

$$d_H \lambda_2 = \zeta \times \begin{cases} 3 & \text{for pseudoscalar mesons,} \\ -1 & \text{for vector mesons.} \end{cases}$$ (5.13)

Therefore,

$$\lambda_2 = \zeta.$$ (5.14)

The above results are valid in any Lorentz frame. In other words, we obtain the result of Eq. (5.3) without setting the heavy meson in the rest frame. From Eq. (5.2), we obtain the hyperfine mass splitting,

$$\Delta M_{VP} = M_V - M_P = \frac{2\lambda_2}{m_Q}.$$ (5.15)

As we can see from Eq. (5.8), $\lambda_1$ consists of two contributions: one is the kinetic energy of heavy quarks [the first term in Eq. (5.8)], and the other is the effect of the chromo-electric interaction between the heavy quark and light degrees of freedom. For convenience, we redefine

$$\lambda_1 \equiv -(\bar{\kappa}^2) + \alpha_s \bar{\lambda}_1, \quad \lambda_2 \equiv \alpha_s \bar{\lambda}_2,$$ (5.16)

where $\alpha_s = g_s^2/4\pi$, and $\langle \bar{\kappa}^2 \rangle$, $\bar{\lambda}_1$ and $\bar{\lambda}_2$ are denoted as the kinetic energy, the chromo-electric and chromo-magnetic interaction parameters, respectively. These parameters depend only on the structure function $\Psi(v \cdot p_q)$. 

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VI. COMPARISON OF THE EFFECTIVE THEORY WITH THE COVARIANT LIGHT-FRONT MODEL

In the previous sections we have developed an effective field theory to describe heavy mesons as composite particles, and systematically evaluated the heavy meson properties in the heavy quark limit and the basic HQET parameters. In this section, we will compare the effective theory with the covariant light-front model of heavy mesons that we have constructed recently [14], and show the source of difficulty for developing a fully covariant reformulation of the currently used light-front quark model in the literature.

A. Covariant light-front quark model as a special case

So far we have not specified the form of the heavy meson structure function \( \Psi(v \cdot p_q) \). From the constraint that it forbids on-mass-shell decay of the heavy meson into \( Q\bar{q} \), a possible form is given by

\[
\Psi(v \cdot p_q) = \Lambda - v \cdot p_q \varphi(v \cdot p_q),
\]

where the function \( \varphi(v \cdot p_q) \) is regular at \( v \cdot p_q = \Lambda \). We shall further assume that (1) \( \varphi(v \cdot p_q) \) is analytic except for isolated singularities when continued into the complex plane, and (2) it vanishes “fast enough” as \( |v \cdot p_q| \to \infty \). These two conditions allow us to perform the \( dp^e \) integrations in the expressions derived earlier by Cauchy’s Theorem. Thus by closing the contours in the lower-half complex \( p^e \)-plane, we obtain

\[
\xi(v \cdot v') = iG^2 \int \frac{d^4 p_q}{(2\pi)^4} \varphi^*(v' \cdot p_q) \varphi(v \cdot p_q) \times \frac{m_q + (v + v') \cdot p_q / (1 + v \cdot v')}{(p_q^2 - m_q^2 + i\epsilon)}
= G^2 \int \frac{d^4 p_q}{(2\pi)^4} (2\pi)\delta(p_q^2 - m_q^2)\theta(p_q^e)\varphi^*(v' \cdot p_q)\varphi(v \cdot p_q) \times \left[ m_q + (v + v') \cdot p_q / (1 + v \cdot v') \right],
\]

\[
F_P = 2iG^2\sqrt{N_c} \int \frac{d^4 p_q}{(2\pi)^4} \varphi(v \cdot p_q) \frac{v \cdot p_q + m_q}{p_q^2 - m_q^2 + i\epsilon}
= 2G^2\sqrt{N_c} \int \frac{d^4 p_q}{(2\pi)^4} (2\pi)\delta(p_q^2 - m_q^2)\theta(p_q^e)\varphi(v \cdot p_q)(v \cdot p_q + m_q),
\]

and

\[
G^{-2} = \int \frac{d^4 p_q}{(2\pi)^4} (2\pi)\delta(p_q^2 - m_q^2)\theta(p^e)(v \cdot p_q + m_q)|\varphi(v \cdot p_q)|^2.
\]

Now if we take

\[
\varphi(v \cdot p_q) = \frac{\Phi_{LF}(v \cdot p_q)}{\sqrt{v \cdot p_q + m_q}},
\]

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where $\Phi_{LF}(v \cdot p_q)$ is a normalized light-front wave function, and let $p^e = p^-$, then these results are exactly the same as those obtained in the covariant light-front quark model [14]. Thus we see that, for these quantities, the covariant light-front quark model results can be recovered here with a special choice of $\varphi(v \cdot p_q)$.

### B. Beyond the covariant light-front quark model

Next, we shall consider the mass parameters $\lambda_1$ and $\lambda_2$ (see Sec. V). As before, using Eq. (6.1) and after performing the $p^e$-integrals, we obtain

$$
\lambda_1 = 2G^2 \int \frac{d^4 p_q}{(2\pi)^4} (2\pi)\delta(p_q^2 - m_q^2)\theta(p_q^e)|\varphi(v \cdot p_q)|^2 [m_q^2 - (v \cdot p_q)^2](v \cdot p_q + m_q)
$$

$$
+ 2G^2 g_s^2 C_f \int \frac{d^4 p_{1q}}{(2\pi)^4} (2\pi)^2 \varphi(v \cdot p_{1q})\varphi^*(v \cdot p_2q)
$$

$$
\times \left\{ \frac{(2\pi)\delta(p_{1q}^2 - m_q^2)\theta(p_{1q}^e)(2\pi)\delta(p_{2q}^2 - m_q^2)\theta(p_{2q}^e)}{(p_{1q} - p_{2q})^2 + i\epsilon} \right.
$$

$$
+ \frac{(2\pi)\delta(p_{1q}^2 - m_q^2)\theta(p_{1q}^e)(2\pi)\delta((p_{1q} - p_{2q})^2)\theta(p_{2q}^e - p_{1q}^e)}{p_{2q}^2 - m_q^2}
$$

$$
+ \frac{(2\pi)\delta(p_{2q}^2 - m_q^2)\theta(p_{2q}^e)(2\pi)\delta((p_{1q} - p_{2q})^2)\theta(p_{2q}^e - p_{1q}^e)}{p_{2q}^2 - m_q^2}
\right\}
$$

$$
\times \left\{ (v \cdot p_{1q} + m_q)[(p_{1q} + p_{2q}) \cdot p_{2q} - v \cdot (p_{1q} + p_{2q})v \cdot p_{2q}]
$$

$$
+ (v \cdot p_{2q} + m_q)[(p_{1q} + p_{2q}) \cdot p_{1q} - v \cdot (p_{1q} + p_{2q})v \cdot p_{1q}]ight\},
$$

(6.6)

$$
\lambda_2 = \frac{4}{3} G^2 g_s^2 C_f \int \frac{d^4 p_{1q}}{(2\pi)^4} \frac{d^4 p_{2q}}{(2\pi)^4} \varphi(v \cdot p_{1q})\varphi^*(v \cdot p_{2q})
$$

$$
\times \left\{ \frac{(2\pi)\delta(p_{1q}^2 - m_q^2)\theta(p_{1q}^e)(2\pi)\delta(p_{2q}^2 - m_q^2)\theta(p_{2q}^e)}{(p_{1q} - p_{2q})^2 + i\epsilon} \right.
$$

$$
+ \frac{(2\pi)\delta(p_{1q}^2 - m_q^2)\theta(p_{1q}^e)(2\pi)\delta((p_{1q} - p_{2q})^2)\theta(p_{2q}^e - p_{1q}^e)}{p_{2q}^2 - m_q^2}
$$

$$
+ \frac{(2\pi)\delta(p_{2q}^2 - m_q^2)\theta(p_{2q}^e)(2\pi)\delta((p_{1q} - p_{2q})^2)\theta(p_{2q}^e - p_{1q}^e)}{p_{2q}^2 - m_q^2}
\right\}
$$

$$
\times \left\{ (v \cdot p_{1q} + m_q)[(p_{1q} - p_{2q}) \cdot p_{2q} - v \cdot (p_{1q} - p_{2q})v \cdot p_{2q}]
$$

$$
- (v \cdot (p_{2q} + m_q)[(p_{1q} - p_{2q}) \cdot p_{1q} - v \cdot (p_{1q} - p_{2q})v \cdot p_{1q}]ight\}.
$$

(6.7)

In each of the above expressions, the first delta-function term inside the bracket comes from on-mass-shell light antiquarks. With $\varphi(v \cdot p_q)$ given by Eq. (6.3), one can easily check that this contribution is exactly what one would obtain in the covariant light-front quark model [14] (where the expression for $\lambda_1$ is not explicitly displayed). The rest of the terms are due to the off-mass-shell contributions of the antiquarks, they correspond to the so-called $Z$-diagram contributions which cannot be obtained in any conventional quark model formulations.
As another example, we examine the axial-vector coupling constant $g$, which involves a purely light-quark current. To obtain a covariant result, one has to keep all the time-ordered (including the so-called Z-diagram) contributions, which is obviously beyond the scope of the light-front quark model [11]. However this is not a problem in the present framework, since Feynman diagrams naturally contain all possible time-orderings. As a result, covariance is automatically preserved in deriving all the results in the previous sections.

Now substituting Eq. (6.1) into Eq. (4.32), and integrating over $p^-$, we obtain

$$g = \frac{G^2}{3} \int \frac{dp^+_q dp_{q\perp}}{(2\pi)^3 2p^+} \left\{ |\varphi(v \cdot p_q)|^2 \left[ \frac{(v \cdot p_q + 2m_q)(v \cdot p_q + m_q)}{X} - \frac{X - v \cdot p_q}{X} \right] \right\} \| \varphi(v \cdot p_q) \|^2, \tag{6.8}$$

where $X = p_q^+/v^+$ and $p_q^2 = m_q^2$. Obviously, this expression cannot be obtained from any type of light-front quark models because it involves a derivative of the wave function, which is very unusual and cannot be simply related to some matrix elements of hadronic bound states in quark models. However it is interesting to note that the first term in Eq. (6.8) is what one would get for $g$ if we naively calculate it in the light-front quark model [14]. Therefore the second term in (6.8) should be the Z-diagram contribution not present in the light-front quark model formulation.

For completeness, we also display the resulting expression for $g$ by integrating over $p^0$ in Eq. (4.32):

$$g = \frac{G^2}{3} \int \frac{dp^3_q dp^0_q}{(2\pi)^3 2p^0} \left\{ |\varphi(v \cdot p_q)|^2 \left[ \frac{(v \cdot p_q + 2m_q)(v \cdot p_q + m_q)}{X} - \frac{X - v \cdot p_q}{X} \right] \left[ 2v \cdot p_q + 3m_q + X + (v \cdot p_q + 2m_q)(v \cdot p_q + m_q) \left( \frac{1}{p^0_q} + \frac{\partial}{\partial(v \cdot p_q)} \right) \right] \| \varphi(v \cdot p_q) \|^2 \right\}, \tag{6.9}$$

where $X = p_q^0/v^0$ and $p_q^2 = m_q^2$.

Thus we have shown that, by the specific choice of Eq. (6.3), our effective field theory can reproduces the covariant light-front quark model [14] results for those quantities ($\xi(v \cdot v')$, $F_P$, and $F_V$) which do not have Z-diagram contributions. For other quantities ($\lambda_1$, $\lambda_2$, and $g$), the extra Z-diagram contributions missed in the light-front quark model can be explicitly identified here. Hence the effective field theory presented here has the simplicity of a conventional quark model, and is fully covariant at the same time.

C. Choice of $\Psi(v \cdot p_q)$

Now an important question is what kind of $\Psi(v \cdot p_q)$ will allow a proper analytic continuation into the complex plane, so that the $p^\ell$-integrals can be evaluated by Cauchy’s Theorem, and comparisons can be made with results obtained in the quark model. In the light-front formulation, there are three type of mesonic wave functions which have been widely used in the literature. One of them, the so-called BSW wave function (or BSW model) has already been ruled out since it explicitly breaks the relativistic covariance and thereby leads to inconsistent results in HQET, as we have shown in recent publications [12,14]. The other two
wave functions, namely, the Gaussian-type wave function and the invariant light-front mass wave function, have the following forms in the heavy quark limit \[14\]:

\[
\Phi_{LF}^G(v \cdot p_q) = \mathcal{N}_G \sqrt{v \cdot p_q} \exp \left\{ -\frac{1}{2\omega^2} \left[ (v \cdot p_q)^2 - m_q^2 \right] \right\},
\]

(6.10)

\[
\Phi_{LF}^M(v \cdot p_q) = \mathcal{N}_M \exp \left\{ -\frac{v \cdot p_q}{\omega} \right\}.
\]

(6.11)

These two wave functions apparently have a covariant form \[14\]. However, in the light-front quark model formulation, the light antiquark is always on the mass shell: \( p_q^2 = \frac{p_{q^\perp}^2 + m_q^2}{p_q^2} \). It is not obvious how to extend these wave functions to be used in a 4-dimensional covariant framework. If \( p_q \) were allowed to go off-shell in these wave functions, then \( \Phi_{LF}^M(v \cdot p_q) \) would be unbounded when \( |p_{q^\perp}| \to \infty \), whereas \( \Phi_{LF}^G(v \cdot p_q) \) would also be unbounded when analytically continued into the complex plane. To fix these problems, one could instead use \( |v \cdot p_q| \) in the above expressions, but then both wave functions would not be analytic in the complex plane. In any case, we find that it is not possible to modify the exponential form so that the two conditions stated below Eq. (6.1) are satisfied.

Besides the exponential-type functions, we can also consider the so-called Lorentzian-type dependence for \( \Psi(v \cdot p_q) \). In the heavy quark limit, we may write \( \Psi(v \cdot p_q) \) as

\[
\Psi_n(v \cdot p_q) = \frac{\Lambda - v \cdot p_q}{(v \cdot p_q + \omega - i\epsilon)^n},
\]

(6.12)

where an \(-i\epsilon\) has been inserted in the denominator so that \( \Psi_n(v \cdot p_q) \) is analytic in the lower half complex-\( p_{q^\perp}^\epsilon \) plane. Consequently by closing the integration contours in the lower half complex-\( p_{q^\perp}^\epsilon \) plane, we will not pick up contributions from the poles of \( \Psi_n(v \cdot p_q) \). It is easy to check that this choice of \( \Psi(v \cdot p_q) \) corresponds to a covariant light-front wave function of the form

\[
\Phi_{LF}^n(v \cdot p_q) = \mathcal{N}_L \frac{\sqrt{v \cdot p_q + m_q}}{(v \cdot p_q + \omega)^n} \quad n > 2,
\]

(6.13)

where \( \mathcal{N}_L \) is a normalization constant, and \( p_q^2 = m_q^2 \). Here we require that \( n > 2 \) to ensure the finiteness of all the integrals.

In the following, we shall present some numerical analyses to further examine the predictive power of the effective theory with the choice Eq. (6.12) for \( \Psi(v \cdot p_q) \).

**VII. NUMERICAL CALCULATION AND DISCUSSIONS**

A. Short-distance corrections

Before performing the numerical computation, it should be stressed that all the quantities we have evaluated in the effective field theory are mainly governed by the long-distance physics of HQET. That is, a renormalization scale \( \mu (\Lambda_{QCD} << \mu << m_Q) \) has been implicitly employed in the structure function \( \Psi(v \cdot p_q) \). It is necessary to take into account short-distance QCD corrections to match with the full QCD description:
\[ \langle \mathcal{O} \rangle_{\text{QCD}} = C_0(\mu)\langle \mathcal{O}_0(\mu) \rangle_{\text{HQET}} + \frac{C_1(\mu)}{2m_Q(\mu)}\langle \mathcal{O}_1(\mu) \rangle_{\text{HQET}} + \cdots, \]  

(7.1)

where the Wilson coefficients \( C_i(\mu) \), which account for the short-distance corrections, have been computed in the literature [3].

Explicitly, the \( B \)-meson decay constant \( f_B \) is given by

\[ f_B = \frac{1}{\sqrt{M_H}} \left[ \frac{\alpha_s(m_b)}{\alpha_s(\mu)} \right]^{-6/(33-2N_f)} F_P(\mu), \]  

(7.2)

where \( F_P(\mu) \) is the decay constant defined in Sec. IVB, and \( N_f = 4 \) is the number of quark flavors with mass less than \( m_b \).

The Isgur-Wise function discussed in Sec. IV.A is also defined at the renormalization scale \( \mu \). Its relation with the \( \mu \)-independent physical observable, for example, the form factor \( f_+(q^2) \) [or \( F_1(q^2) \)] is given by [21]

\[ f_+(q^2) = \frac{M_B + M_D}{2\sqrt{M_B M_D}} \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{-6/25} \left[ \frac{\alpha_s(m_c)}{\alpha_s(\mu)} \right]^{a_L(v \cdot v', \mu)} \xi(v \cdot v', \mu), \]  

(7.3)

where

\[ a_L(\omega) = \frac{8}{27} \left[ \omega r(\omega) - 1 \right], \]

\[ r(\omega) = \frac{1}{\sqrt{\omega^2 - 1}} \ln(\sqrt{\omega^2 + 1} - 1). \]  

(7.4)

The axial-vector coupling constant \( g \) does not receive short-distance corrections since it involves a light-quark current which is partially conserved.

Although the physical heavy meson mass is scale independent, individual contributions to it are modified by short-distance corrections. Thus Eq. (5.2), written separately for pseudoscalar and vector mesons, becomes

\[ M_H = m_Q(\mu) + \overline{\Lambda}(\mu) - \frac{1}{2m_Q(\mu)} \left[ \lambda_1(\mu) + 3a_2(\mu)\lambda_2(\mu) \right], \]  

(7.5)

\[ M_{H^*} = m_Q(\mu) + \overline{\Lambda}(\mu) - \frac{1}{2m_Q(\mu)} \left[ \lambda_1(\mu) - a_2(\mu)\lambda_2(\mu) \right], \]  

(7.6)

where

\[ a_2(\mu) = \left[ \frac{\alpha_s(m_Q)}{\alpha_s(\mu)} \right]^{-9/(33-2N_f)}. \]  

(7.7)

Note that all the basic nonperturbative parameters in HQET, i.e. \( m_b(\mu), \overline{\Lambda}(\mu), \lambda_1(\mu) \) and \( \lambda_2(\mu) \), are \( \mu \)-dependent; only the physical masses are independent of the renormalization scale \( \mu \).

It should be stressed that in the above expressions, we have distinguished the heavy quark mass in the \( 1/m_Q \) expansion which is the running heavy quark mass defined at the scale \( \mu \), and the mass scale in the strong coupling constant \( \alpha_s(m_Q) \) set at the heavy quark
pole mass: \( \alpha_s(m_Q^{\text{pole}}) \). In the literature, there are many discussions on the ambiguities in the definition of the heavy quark mass due to the presence of the so-called renormalons \[22\]. We believe that the above distinction of heavy quark masses in our formulation is theoretically consistent.

Using Eq. (5.16), Eqs. (7.5-7.6) can be rewritten as

\[
M_H = m_Q(\mu) + \overline{\lambda}(\mu) + \frac{1}{2m_Q(\mu)} \left\{ \langle \vec{k}^2 \rangle - \alpha_s(\mu) \left[ \overline{\lambda}_1 + 3a_2(\mu) \overline{\lambda}_2 \right] \right\}, \tag{7.8}
\]

\[
M_{H^*} = m_Q(\mu) + \overline{\lambda}(\mu) + \frac{1}{2m_Q(\mu)} \left\{ \langle \vec{k}^2 \rangle - \alpha_s(\mu) \left[ \overline{\lambda}_1 - a_2(\mu) \overline{\lambda}_2 \right] \right\}. \tag{7.9}
\]

Note that \( \langle \vec{k}^2 \rangle \), \( \overline{\lambda}_1 \) and \( \overline{\lambda}_2 \) depend only on the structure function \( \Psi(v \cdot p_q) \), and therefore also on the scale \( \mu \). The heavy meson mass splitting between pseudoscalar and vector mesons is simply given by

\[
M_{H^*} - M_H = 2 \frac{\alpha_s(\mu)a_2(\mu)}{m_Q(\mu)} \overline{\lambda}_2. \tag{7.10}
\]

**B. Numerical analysis**

For the sake of demonstration, we first set \( \mu = m_b^{\text{pole}} \approx 4.89 \text{ GeV} \) (the pole mass) \[27\], and see if a consistent set of parameters can be found. At this scale, short-distance corrections are not present. The light quark mass at this scale is given by the current quark mass which is about a few MeV for up and down quarks. We take \( m_{u,d}(m_b^{\text{pole}}) \approx 5 \text{ MeV} \). Then the parameter \( \omega \) in the structure functions can be fixed by the decay constant \( f_B \) via Eq. (4.20). Taking

\[ f_B = 0.180 \text{ GeV}, \tag{7.11} \]

the values of \( \omega \) for different structure functions \( \Psi_n(v \cdot p_q) \) are listed in Table I. Since the strong coupling constant \( \alpha_s \) is also known at \( \mu = m_b^{\text{pole}} \), knowing \( \omega \), we can predict various heavy meson properties obtained in the effective theory at this scale.

We first calculate the vector-pseudoscalar \( B \) meson mass difference which is determined by \( \lambda_2 \). As \( \mu = m_b^{\text{pole}} \gg \Lambda_{\text{QCD}} \), \( \alpha_s(m_b^{\text{pole}}) \) can be determined from \( \alpha_s(M_Z) \) by the perturbative evolution equation (at the one-loop level in the \( \overline{\text{MS}} \) scheme),

\[
\overline{\alpha}_s(m_b^{\text{pole}}) = \frac{\alpha_s(M_Z)}{1 + \alpha_s(M_Z) \beta_0 \ln((m_b^{\text{pole}})^2/M_Z^2)/(4\pi)} \approx 0.22, \tag{7.12}
\]

where \( \beta_0 = 11 - \frac{2}{3}N_f = 11 - 8/3 \) for \( N_f = 4 \), \( M_Z = 91.19 \text{ GeV} \), and \( \alpha_s(M_Z) = 0.119 \) from experimental fits. Thus, the \( B^* - B \) mass splitting at the scale \( \mu = m_b^{\text{pole}} \) in our calculation is given by

\[
\Delta M_{B^*B} = M_{B^*} - M_B = 2 \frac{\overline{\alpha}_s(m_b^{\text{pole}})}{m_b(m_b^{\text{pole}})} \overline{\lambda}_2, \tag{7.13}
\]
with the \( \overline{\text{MS}} \) \( b \)-quark mass at this scale: \( \overline{m}_b(m_b^{\text{pole}}) = 4.339 \ \text{GeV} \) \[27\]. The result is also listed in Table I, from which it is evident our results do not fit the experimental value of \( \Delta M_{B^+B} = 0.046 \ \text{GeV} \) \[23\]. This numerical analysis indicates that the real scale \( \mu \) implicitly used in the effective theory should be lower than \( m_b^{\text{pole}} \).

Table I. The parameter \( \omega \) in the structure functions \( \Psi_n(v \cdot p_q) \) \[\(6.12\)] with different \( n \) values fitted to \( f_B = 180 \ \text{GeV} \) at the scale \( \mu = m_b^{\text{pole}} \) and the resulting \( \Delta M_{B^+B} \) and \( \langle \vec{k}^2 \rangle \). With \( n \) larger than 12, the change of \( \Delta M_{B^+B} \) and \( \langle \vec{k}^2 \rangle \) becomes insignificant.

| \( n \) | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \( \omega \) (GeV) | 0.100 | 0.374 | 0.689 | 1.011 | 1.335 | 1.660 | 1.986 | 2.312 | 2.638 | 2.965 |
| \( \Delta M_{B^+B} \) (GeV) | 0.011 | 0.023 | 0.028 | 0.031 | 0.032 | 0.034 | 0.034 | 0.035 | 0.035 | 0.035 |
| \( \langle \vec{k}^2 \rangle \) (GeV²) | 0.111 | 0.274 | 0.374 | 0.432 | 0.470 | 0.496 | 0.515 | 0.529 | 0.541 | 0.550 |

Of course, as is well known, the HQET works well only at the scale \( \Lambda_{QCD} << \mu << m_b \). The results shown in Table I indicate that the implicit scale \( \mu \) of our effective theory is not close to \( m_b \). On the other hand, for perturbation expansion to work, \( \mu \) should not be too close \( \Lambda_{QCD} \) either. Hence, we must choose a value of \( \mu \) such that the corresponding \( \alpha_s \Lambda_{QCD}/m_Q \) is small enough to ensure the validity of the HQET. In Table I, we have also calculated \( \langle \vec{k}^2 \rangle \), the mean square momentum carried by the quarks in the meson. We find that \( \langle \vec{k}^2 \rangle \) increases with increasing \( n \), which indicates that in fact the scale \( \mu \) also implicitly increases with \( n \). As a result, the corresponding \( \alpha_s \) would be larger for smaller \( n \). Thus, in order that higher order \( 1/m_Q \) corrections can be ignored, we should choose a structure function \( \Psi_n(v \cdot p_q) \) with a suitably larger \( n \).

In the following numerical analysis, we shall consider the scale with

\[
\alpha_s(\mu) < 0.5 \quad \text{and} \quad m_q(\mu) = 0.22 \sim 0.25 \ \text{GeV} \quad (7.14)
\]

and try to find a consistent set of parameters for each \( \Psi_n(v \cdot p_q) \). The condition \( \alpha_s(\mu) < 0.5 \) guarantees that \( \alpha_s(\mu) \Lambda_{QCD}/m_b(\mu) \) cannot be too large. Also, at this scale, we hope that the short-distance corrections given in Sec. VII.A are still applicable. The range of the light quark mass we have assumed is close to the constituent quark mass often used in relativistic quark models. Now we proceed as follows. For each \( \Psi_n(v \cdot p_q) \), \( \omega \) is fixed by \( f_B = 180 \ \text{GeV} \) through Eq. \( (7.2) \). We can then predict the HQET parameters \( \lambda_1 \) and \( \lambda_2 \). Subsequently, \( m_b(\mu), \overline{\Lambda}(\mu) \), and \( \alpha_s(\mu) \) are determined by fitting the experimental \( \Delta M_{B^+B} = 0.046 \ \text{GeV} \) \[23\] and the experimental \( B \)-meson mass \( M_B = 5.279 \ \text{GeV} \) \[23\] via Eq. \( (7.10) \) and Eq. \( (7.8) \), respectively. We found that for \( n \leq 3 \), \( \Psi_n(v \cdot p_q) \) allows no consistent fit to the experimental data. Only with \( n > 4 \), can we have good descriptions for various \( B \) meson properties with small \( 1/m_b \) corrections. The results are summarized in Table II.

Table II. The parameter \( \omega \) in the structure functions \( \Psi_n(v \cdot p_q) \) \[\(6.12\)] and the bottom quark mass \( m_b \) fitted to \( f_B = 180 \ \text{GeV} \) and \( \Delta M_{B^+B} = 0.046 \ \text{GeV} \) with a suitable giving scale by \( \alpha_s(\mu) \) and \( m_q(\mu) \) for each giving \( n \) in Eq. \( (6.12) \), and all the predicted nonperturbative HQET parameters in the effective theory, where \( m_q, \omega, m_b \) and \( \overline{\Lambda} \) are in unit GeV, and \( \langle \vec{k}^2 \rangle, \alpha_s \lambda_1, \lambda_1 \) and \( \lambda_2 \) in unit GeV².
| \( \Psi_n(v \cdot p_q) \) | \( \alpha_s(\mu) \) | \( m_q(\mu) \) | \( \omega(\mu) \) | \( m_b(\mu) \) | \( \langle k^2 \rangle \) | \( \alpha_s \Lambda \) | \( -\lambda_1 \) | \( \lambda_2 \) | \( \Lambda \) | \( m_c(\mu) \) | \( g \) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| \( n = 5 \) | 0.457 | 0.251 | 0.405 | 4.907 | 0.292 | 0.254 | 0.038 | 0.087 | 0.333 | 1.626 | 0.380 |
| \( n = 6 \) | 0.431 | 0.248 | 0.671 | 4.901 | 0.329 | 0.265 | 0.064 | 0.089 | 0.337 | 1.614 | 0.329 |
| \( n = 7 \) | 0.417 | 0.245 | 0.937 | 4.894 | 0.352 | 0.271 | 0.081 | 0.090 | 0.342 | 1.604 | 0.328 |
| \( n = 8 \) | 0.408 | 0.242 | 1.205 | 4.888 | 0.368 | 0.275 | 0.093 | 0.090 | 0.347 | 1.595 | 0.337 |
| \( n = 9 \) | 0.401 | 0.238 | 1.477 | 4.880 | 0.381 | 0.278 | 0.103 | 0.091 | 0.354 | 1.584 | 0.372 |
| \( n = 10 \) | 0.395 | 0.234 | 1.752 | 4.869 | 0.391 | 0.280 | 0.111 | 0.091 | 0.364 | 1.572 | 0.428 |
| \( n = 11 \) | 0.390 | 0.230 | 2.029 | 4.859 | 0.399 | 0.282 | 0.117 | 0.091 | 0.373 | 1.560 | 0.479 |
| \( n = 12 \) | 0.385 | 0.226 | 2.310 | 4.844 | 0.406 | 0.283 | 0.123 | 0.091 | 0.389 | 1.542 | 0.556 |

*In the literature, the usual experimental value of \( \lambda_2 \approx 0.12 \) GeV\(^2\) is obtained by taking \( m_b = M_B \) approximately. Here the experimental \( \lambda_2 \approx 0.090 \) GeV\(^2\) is obtained by using \( m_b^{\text{pole}} = 4.89 \) GeV, the \( b \)-quark pole mass, in order to make a consistent comparison with the theoretical calculation used by Eq. (5.15) with the short-distance correction.

** There are no experimental data for the constituent quark masses \( m_b \) and \( m_c \). Here we list the bottom and charm quark pole masses [27].

From Table II, we see that \( \langle k^2 \rangle \) increases with increasing \( n \). As noted earlier, this indicates that the scale \( \mu \) in the phenomenological structure function \( \Psi_n(v \cdot p_q) \) also goes up with \( n \). Then to be consistent, the running strong coupling constant \( \alpha_s(\mu) \) and the running quark masses \( m_q(\mu) \) and \( m_b(\mu) \) should decrease with increasing \( n \). This expected behavior is clearly shown in Table II. From table II, it is also interesting to see that the resulting \( \text{b}-\text{quark mass} \) is in between the constituent mass \( \sim 4.8 \) GeV used in various model calculations and the pole mass \( (4.89 \pm 0.05) \) GeV. Furthermore, we also find that the hyperfine mass splitting parameter \( \lambda_2 \) is quite stable and insensitive to the function \( \Psi(v \cdot p_q) \). This is not unexpected since \( \lambda_2 \) is directly related to the heavy meson mass splitting which is well measured. The mass shift parameter \( \lambda_1 \) is very small in our calculation. In this work, \( \lambda_1 \) receives two contributions: one is the heavy quark kinetic energy \( (\langle k^2 \rangle \approx 0.3 \sim 0.4 \text{ GeV}^2 \) for \( n \geq 5 \)) which is a negative contribution to \( \lambda_1 \) [see Eq. (5.16)], and the other is the chromo-electric interaction between the heavy quark and light degrees of freedom characterized by the parameter \( \Lambda \) which is close to 0.3 GeV\(^2\) and is a positive contribution. Therefore, the kinetic energy is largely compensated by the chromo-electric interaction energy, leading to a small and negative \( \lambda_1 \): \( \lambda_1 = 0.04 \sim 0.11 \) GeV\(^2\) for \( n = 5 \sim 10 \). Then a fit to the physical masses of the \( B \) mesons yield the nonperturbative HQET parameter \( \Lambda \): \( \Lambda \approx 0.33 \sim 0.36 \) GeV, which is consistent with results from HQET analyses of the semi-inclusive \( B \) decay data [20]. These numerical results provide a vote of confidence to the reliability of the effective theory proposed in this work.

For a further consistency check with HQET and HQS, we use the above results to calculate the \( D \) meson observables. The \( D \) meson mass is given by

\[
M_D = m_c(\mu) + \Lambda(\mu) - \frac{\lambda_1}{2m_c(\mu)} - \frac{3}{4} \Delta M_{D^*D},
\]

where \( \Lambda \) and \( \lambda_{1,2} \) are the same for both the \( B \) and \( D \) mesons. Then using the experimental \( D \) mass data [23]
we can uniquely determine the charmed quark mass. We get (see Table II)

\[ m_c(\mu) = 1.57 \sim 1.62 \text{ GeV} \ (n = 5 \sim 10), \]  

\[ \rho^2 = -\xi'(1), \]  

which is close to the \( m_c \) used in various quark model calculations, and also the pole mass \( m_c^\text{pole} = 1.59 \pm 0.02 \text{ GeV} \ [27]. \)

Next we proceed to calculate the Isgur-Wise function \( \xi(v \cdot v') \) [cf. Eq. (6.2)], its slope parameter at the zero-recoil point

\[ \rho^2 = -\xi'(1), \]  

and the axial-vector coupling constant \( g \) from Eq. (6.8) or Eq. (6.9). The Isgur-Wise function depends on the parameter \( \omega \) and the light quark mass \( m_q \), and the axial-vector coupling constant also depends on \( \Lambda \). Hence, once \( \omega \) and \( \Lambda \) are determined for a given \( m_q \), \( \xi(v \cdot v') \), \( \rho^2 \), and \( g \) can all be predicted. In Fig. 4, we plot the Isgur-Wise function as a function of \( v \cdot v' \). It is remarkable to see that the structure functions \( \Psi_n(v \cdot p_q) \) (6.12) with different \( n \) give very similar results. In particular, the Isgur-Wise function obtained from \( \Psi_{n=10} \) is almost identical to that form the light-front wave functions (6.10) and (6.11). The slope parameter of the Isgur-Wise function at the zero-recoil point is found to be:

\[ \rho^2 = 1.1 \sim 1.2 \ (n = 5 \sim 10), \]  

which is also consistent with other theoretical analyses [4].

Having determined \( \Lambda \) from the heavy meson mass calculations, we can now predict the axial-vector coupling constant \( g \). As listed in Table II, we have

\[ g = 0.33 \sim 0.43 \ (n = 5 \sim 10), \]  

which is consistent with the experimental constraint \( g < 0.7 \ [20,25] \) derived from \( D^* \rightarrow D + \pi \) decays. Our result is also close to the QCD sum rule results, which tend to concentrate within the range \( 0.2 < g < 0.4 \).

\section*{VIII. CONCLUSIONS AND PERSPECTIVE}

In this paper, we have given a detail account of a recently proposed field theoretical description of heavy mesons [15]. The effective theory incorporates heavy quark symmetry and the heavy quark effective theory, from which a natural realization of heavy mesons in the heavy quark limit as a composite particle of the reduced heavy quark coupled with a brown muck of light degrees of freedom is provided. This theory is fully covariant, so that Feynman diagrammatic techniques can be use to carry out perturbative calculations. Moreover the effective theory preserves the simplicity of a conventional quark model, in fact, for those quantities which do not have the so-called \( Z \)—diagram contributions, light-front quark model results can be reproduced as a special case in the present theory. Thus this theory provides a link between the fundamental QCD and the phenomenologically successful quark model,
so that the difficult subject of hadronic bound state physics can be quantitatively studied in a covariant framework.

The effective theory provides a quasi-first-principles description of the heavy meson dynamics. Although at present the description of the heavy mesons structure function $\Psi(v \cdot p_q)$ is still phenomenological in nature, it offers a systematic approach to evaluate the $1/m_Q$ corrections based on the first-principles $1/m_Q$ expansion of QCD. This resembles very much the situation of the QCD analysis of deep inelastic scatterings, in which the low energy dynamics (described by parton distribution functions) is determined phenomenologically and perturbative corrections are given in a fully first-principles way. Here the situation may even be better since the phenomenological part is constrained by HQS and HQET, and the nonperturbative QCD dynamics should be much simpler in the heavy quark limit.

As we have seen, the introduction of the structure function $\Psi(v \cdot p_q)$ is crucial for a field theoretic realization of the heavy mesons as composite particles. In fact, this structure function is related to the covariant wave functions of heavy meson bound states, and it essentially describes the brown muck structure of the light degrees of freedom inside the heavy mesons in the heavy quark limit. Our results show that it is not possible to extend the popular Gaussian-type wave functions to be used in a fully covariant formulation. However, we find that a Lorentzian-type function can be readily adopted in a covariant formulation for heavy meson structures, and numerically it gives a very good description of the physical properties of heavy mesons. The predictions for all the HQET parameters are consistent with experiment. $\Lambda$ is about $0.33 \sim 0.39$ GeV [for $\Psi_n(v \cdot p_q)$ with $n = 5 \sim 12$] which is consistent with results from HQET analyses of the experimental data [26]. The heavy quark masses $m_b$ and $m_c$, determined by fitting to the $B^* - B$ and $D^* - D$ mass differences, are about $4.84 \sim 4.91$ GeV and $1.54 \sim 1.63$ GeV respectively, which agree with those used in various relativistic quark model calculations, and also their respective pole masses. We find that the mass shift parameter $\lambda_1$ in HQET is negative and very small (about $-0.04 \sim -0.12$) because of a large cancellation between the heavy quark kinetic energy and the chromo-electric interaction between the heavy quark and the light degrees of freedom. The Isgur-Wise function we obtained is also consistent with other calculations. Thus, we have self-consistently determined and predicted all the HQET parameters within the effective theory.

We expect that the effective theory presented in this paper can now be applied to describe various heavy meson processes, such as the inclusive $B$ decays and various exclusive $B$ decays without involving light mesons. For processes involving light mesons, we must first extend our framework so that light mesons are also included. Finally, to solve the structure function $\Psi(v \cdot p_q)$ from the fundamental theory in the heavy quark limit is an important and interesting challenge. We shall leave these topics to future investigations.

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REFERENCES

[1] N. Isgur and M. B. Wise, Phys. Lett. B232, 113 (1989); B237, 527 (1990).
[2] H. Georgi, Phys. Lett. B240, 447 (1990); E. Eichten and B. Hill, Phys. Lett. B234, 511 (1990); B243, 427 (1990).
[3] M. Neubert, Phys. Rep. 245, 261 (1994); Also see hep-ph/9801269.
[4] C. Bernard, Y. Shen, and A. Soni, Phys. Lett. B317, 164 (1993); A. Abada et al. (ELC Collaboration), Nucl. Phys. B416, 675 (1994); C. R. Allton et al. (APE Collaboration), Phys. Lett. B345, 513 (1995); K. C. Bowler et al. (UKQCD Collaboration), Phys. Rev. D52, 5067 (1995); Nucl. Phys. B461, 327 (1996).
[5] W. M. Zhang, Phys. Rev. D56, 1528 (1997).
[6] N. Isgur, D. Scora, B. Grinstein, and M. B. Wise, Phys. Rev. D 39, 799 (1989); D. Scora and N. Isgur, Phys. Rev. D52, 2783 (1995); H. Y. Cheng and B. Tseng, Phys. Rev. D53, 1457 (1996).
[7] M. Sadzikowski and K. Zalewski, Z. Phys. C59, 677 (1993).
[8] P. Ball, V. M. Braun, and H. Dosch, Phys. Rev. D44, 3567 (1991); P. Ball, Phys. Rev. D48, 3190 (1993); V. M. Belyaev, A. Khodjamirian, and R. Ruckl, Z. Phys. C60, 349 (1993); T. Huang and C.-W. Luo, Phys. Rev. D50, 5775 (1994); A. Ali, V. M. Braun, and H. Simma, Z. Phys. C83, 437 (1994); P. Colangelo, F. De Fazio, P. Santorelli, and E. Scrinieri, Phys. Rev. D53, 3672 (1996).
[9] W. Jaus, Phys. Rev. D 41, 3394 (1990); 44, 2851 (1991); 53, 1349 (1996); P. J. O’Donnell and Q. P. Xu, Phys. Lett. B325, 219 (1994); 336, 113 (1994); P. J. O’Donnell, Q. P. Xu, and H. K. K. Tung, Phys. Rev. D52, 3966 (1995).
For a review on light-front dynamics, see W. M. Zhang, Chin. J. Phys. 32, 717 (1994).
[10] A. Dulib and A. Kaidalov, Yad. Fiz. 56, 164 (1993), [Phys. At. Nucl. 56, 237 (1993)].
[11] C. Y. Cheung, C. W. Hwang, and W. M. Zhang, Z. Phys. C75, 657 (1997).
[12] H. Y. Cheng, C. Y. Cheung, and C. W. Hwang, Phys. Rev. D55, 1559 (1997).
[13] N. B. Demchuk, P. Yu. Kulikov, I. M. Narodetskii and P. J. O’Donnell, Phys. At. Nucl. 60, 1292 (1997).
[14] H. Y. Cheng, C. Y. Cheung, C. W. Hwang, and W. M. Zhang, hep-ph/9709412, to appear in Phys. Rev. D57, (1998).
[15] C. Y. Cheung and W. M. Zhang, submitted to Phys. Lett. , hep-ph/9712258.
[16] M. Wirbel, S. Stech, and M. Bauer, Z. Phys. C29,
[17] R. Hagg, Phys. Rev. 112, 669 (1958); K. Nishijima, ibid, 111, 995 (1958); 122, 298 (1961); R. L. Zimmermann, ibid, 141, 1124 (1964).
[18] D. Lurie, Particles and Fields (Interscience, New York, 1969).
[19] T. M. Yan, H. Y. Cheng, C. Y. Cheung, G. L. Lin, Y. C. Lin, and H. L. Yu, Phys. Rev. D 46, 1148 (1992); M. B. Wise, Phys. Rev. D 45, R2188 (1992); G. Burdman and J. Donoghue, Phys. Lett. B 280, 287 (1992).
[20] H. Y. Cheng, C. Y. Cheung, G. L. Lin, Y. C. Lin, T. M. Yan, and H. L. Yu, Phys. Rev. D 46, 5060 (1992); ibid. D 47, 1030 (1993);
[21] A. Falk et al., Nucl. Phys. B343, 1 (1990).
[22] I.I. Bigi, M.A. Shifman, N.G. Uraltsev, and A.I. Vainshtein, Phys. Rev. D50 2234 (1994).
[23] Particle Data Group, Phys. Rev. D54, 1 (1996).
[24] M. Neubert, Int. J. Mod. Phys. A11, 4173 (1996).
[25] ACCMOR Collaboration, S. Barlag et al. Phys. Lett. B278, 480 (1992).
[26] M. Gremm and I. Stewart, Phys. Rev. D55, 1226 (1997); A. F. Falk, M. Luke and M. J. Savage, Phys. Rev. D53, 6316 (1996).
[27] H. Fusaoka and Y. Koide, hep-ph/9712201.
Figure Captions

Fig. 1  Heavy-light quark scattering in (a) quark-quark coupling picture, and (b) meson-quark coupling picture, which determine the composite particle structure of heavy mesons.

Fig. 2  Feynman diagrams for (a) the Isgur-Wise function, (b) decay constant, and (c) strong axial-coupling constant of heavy mesons.

Fig. 3  Feynman diagrams for $1/m_Q$ corrections to heavy meson masses.

Fig. 4  The Isgur-Wise functions as a function of $v \cdot v'$ that obtained from the wave functions $\Psi_n(v \cdot p_q), (n = 6, 8, 10)$ [(6.12)] and compare with the Isgur-Wise functions obtained from the light-front wave functions $\Psi^G$ [(6.10)] and $\Psi^M$ [(6.11)] [14].
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