Project based pedagogy in the development of physical-mathematical thinking

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Abstract. Physical-mathematical thinking is the visible representation of the relationship between physics and mathematics. It is based on the development of the five types of mathematical thinking and leads to the consolidation of the learning of physics through the construction, interpretation, abstraction and consolidation of meanings for the teacher and the student about physical phenomena. The work allows for the implementation of a pedagogical strategy based on project-based pedagogy, which facilitates the development and application of mathematical algorithms for the analysis, understanding and explanation of physics by students in the context of collaborative field work. The study is framed in the quantitative paradigm, in a quasi-experimental design, working with control and experimental groups. Pre-test is applied to identify the development of physical-mathematical thinking in students and as a support to the design of the intervention proposal. Once the proposal is developed, the results are evaluated by means of a post-test that allows identifying significant improvements in the students' physical-mathematical thinking.

1. Introduction
Human progress has been linked to the need to solve problems [1] supported by the development of thought and its applications. Knowledge management, analyzed in the context of the trends in the evolution of humanity, allows a more complete vision of its social significance based on theories about the functioning of the mind, the stimulation of the intellect and the cognitive phenomena that accompany the mental act [2]. The complexity of thought recognizes the incompleteness and uncertainty; moreover, it supposes and needs diversity because it is the product of the relationship between homogeneity and heterogeneity, and it maintains that the unity of the human being is the unity of diversity [3]. Therefore, the problems associated with mathematics and physics that arise in the context of the student allow, with the support of the teacher

Today's education has a new and greater responsibility: instead of offering information it must train and enable the subject to seek it, select it critically, understand it and apply it effectively to his or her personal needs and those of society. This leads to a number of changes in the way physics teaching is conceived at all educational levels [4], since the way in which different knowledge is learned varies according to the content involving different cognitive processes and involves the exercise of specialized skills. Thus, learning physics involves hard work, the results of which are closely related to the development of students' physical-mathematical thinking [5].

Physical-mathematical thinking is based on the development and existence of the five types of mathematical thinking and refers to the development of skills that allow us to compare, describe, analyze, synthesize, abstract and model physical phenomena; is in itself the representation of the
relationship between physics and mathematics, which places mathematics as the language or the tool that allows the characterization of different physical phenomena through the use of algorithms that characterize the language of mathematics as a strategy that allows the construction, interpretation, abstraction and consolidation of meanings for the teacher and the student [6].

The analysis of physical phenomena and its explanation using a mathematical language offers the possibility of forming the thought for the search of the argumentative coherence, where the knowledge that is acquired is coherent for the thought, they are intelligible and expressible conceptions for the knowledgeable subject in order to have a clear understanding. In this way, the development of physical mathematical thinking involves the student being able to mathematize, which simultaneously involves the development of the different types of mathematical thinking [7].

In this sense, the pedagogical practices developed from classroom projects represent one of the most important aspects of the mediated didactic interventions of teachers, beyond the didactic materials provided to students, since it is the doing of them that will lead them to a significant learning of the concepts of mathematics and physics. Designing projects in real teaching situations does not mean merely stating certain activities. It implies recognizing the students' capacities and cognitive processes in order to select, among a range of possibilities, those that promote the integral development of the student and to identify the didactic methods and resources appropriate to the disciplinary area to be taught [8]. In this order of ideas, we can say that the paradigmatic aspects of the different ways of teaching determine the nature of knowledge [9,10].

Teaching processes in the context of project pedagogy are based on integrated constructivist planning [11,12]. In this sense, this study, from Vygotsky's point of view [13], proposes a teacher as a mediator. Therefore, the student's role is highly participatory, he stops being mechanistic and inserts himself in the world of knowledge based on his own previous experiences.

Classroom teaching projects are an important part of the three-part relationship (teacher, student, content) that exists in every educational act [14] and, as a model of integrated planning, they are based on the contributions of constructivist approaches, which allow for the construction of knowledge, take into account the student's previous knowledge, integrate teaching processes around a generating theme, the teacher as a mediator and dynamic classes with group work, videos, field trips, presentations by experts and plenary sessions, among others [15,16].

Classroom-based pedagogical projects are teaching planning instruments with a comprehensive approach that takes into account the components of the curriculum and is based on the needs and interests of the school and learners in order to provide them with an education that is improved in terms of quality and equity [17]. The methodological process of classroom teaching projects consists of the following phases: conceptual, operational, monitoring and evaluation, and a systematization phase [18].

2. Method

The research is part of a field and quasi-experimental study. It is field-based because the strategy followed by the research was based on methods that allowed data to be collected directly from reality. The field research constitutes a systematic, rigorous and rational process of data collection, treatment, analysis and presentation, based on a strategy of direct collection of the reality of the information necessary for the research. Two groups of students in the last grade of secondary education are guided by the same teacher, and one is randomly selected as a control and the other as an experimental one [19].

The investigation starts from the diagnosis and recognition of the initial situation. The process began with the identification of the problem area based on preliminary ideas to arrive at the subject matter. Subsequently, the instruments were developed, the pilot test was applied, the pretest was applied, and the data were ordered, grouped, arranged and related, in accordance with the objectives of the research; in other words, the information was prepared in order to proceed with its analysis and interpretation in both the problem statement component and the pretest. Based on these results, an intervention plan is drawn up. When we know what is happening, i.e. a situation has been diagnosed, we have to decide what to do. During this moment an action plan is elaborated, to improve what is already happening. In the action plan, the pedagogical priorities are studied and established, and options for possible
alternatives are proposed. Implementation involves acting to put the plan into practice and observing its effects in the context in which it takes place. Finally, a post-test is applied in order to evaluate the result of the intervention.

The project methodology used in this work is framed in the cascade or traditional projects that are quite common in the educational and industrial fields, and consists of dividing the project in different processes that are executed in a sequential way until the project objectives are achieved. These processes are: Initialization: initial definition of the project, together with the approval of its start by the teacher. Planning: development of the different management plans with the active participation of teachers and students. Execution: carrying out of the tasks planned in the previous phase in order to complete the deliverables, carried out by the students with the collaboration of the teacher. Follow-up and control: supervision of the executed tasks, carried out by the teacher. Closing: end of the project, it implies the delivery of the final report, socialization of results by the students and feedback by the teacher and students.

3. Results
A test is designed to evaluate mathematical skills in communicating, reasoning and problem solving in numerical, spatial, metric, random and variational thinking. The test is developed in two phases, one consisting of a ten-item test that evaluates reasoning and problem solving in mathematics and basic physics, and another application test that evaluates creativity in the proposal of problem solving and physical and mathematical argumentation of results. The item response theory [20,21] is used in the evaluation of the test, assigning at the end a value, on a scale of 1 to 10, for the development of mathematical and physical thinking. The practical work is also assessed on the same scale, evaluating the two types of thinking separately. The final result corresponds to the weighted average of the evaluations of each component. Finally, adjustments are made to the scale to achieve normality. The results are illustrated in Figure 1.

![Figure 1. Pre-test results for the groups in the study.](image)

The results in the pretest, schematized in Figure 1, are presented below indicating for each group the mean and standard deviation, mathematical thinking in control group (5.1; 1.1), physical thinking in control group (4.5; 0.8), mathematical thinking in experimental group (5.5; 1.3), physical thinking in experimental group (4.7; 1.1). The results allow to infer homogeneity between averages and standard deviations.
The intervention is carried out in the experimental group through the realization of classroom projects that involve physics and the use of mathematical models in the realization and explanation of the physical phenomena involved in the project. In the control group, the classes are developed in the traditional way of working in the educational institution.

A project is developed with the analysis of car suspension; in them the waves generated by the springs that support the vehicle are analyzed. The student analyzes the deformation of the springs and the waves generated when they are pressed and released freely. He makes measurements using metric and numerical thinking; he analyzes differences between them, random thinking; he observes displacements and deformations, spatial and variational thinking. The analysis leads to the use of the shock absorber to avoid continuous vibrations in the vehicle; the spring suspends the vehicle, but the shock absorber prevents it from continuing to vibrate, the laws of physics applied are explained and a lecture is given on results, development of physical thinking.

A project related to Newton’s first law. The student observes objects and their location in space, spatial thinking. Discussion of the forces that act on him both at rest and in motion, metric and spatial thinking; the student explains the movements and tries to present mathematical relationships, variational thinking; comes to characterize the force as an action that produces changes in the state of movement of the body and performs vectorial schematics, numerical, metric, spatial, variational and physical thinking.

One project, which can be linked to ecology, is related to the analysis of a riverbed, spatial thinking; explaining why water moves, physical, variational, metric thinking; analyzing why water flows at different speeds, random, physical, metric and variational thinking; describing the type of waves that are formed, numerical, metric and spatial thinking; schematic presentation of observed waves and description of the underlying mathematical models, spatial thinking, metric and physical.

After the pedagogical intervention of classroom projects, a post-test designed in a similar way to the pre-test is applied to the two groups of students, but with different items, which evaluate the same thoughts and a field practice similar, in terms of design, to the previous one. The same procedure is followed for the evaluation of the results, which are shown in Figure 2.

![Figure 2](image_url)

**Figure 2.** Post-test results for the groups in the study.

The results in the post-test, schematized in Figure 2, are presented below indicating for each group the mean and standard deviation, mathematical thinking in control group \((5.5; 0.8)\), physical thinking in control group \((5.7; 1.5)\), mathematical thinking in experimental group \((7.3; 0.9)\), physical thinking in
experimental group (6.8; 1.3). The results allow to infer homogeneity between averages of the two types of thinking in each group, but a significant increase is appreciated in both mathematical thinking and physical thinking in the experimental group with respect to the control group.

However, in order to compare the levels of physical-mathematical thinking of the students participating in the study, the value of the quartiles in the results of each of the tests was taken as a reference and the high category was assigned to the upper quartile, the medium category to students located between the first and third quartiles and the low category to students with results below the first quartile. The resulting information is used to develop a multiple correspondence diagram, Figure 3. The location of the two groups is then projected, based on the total results.

The location of the modalities of each variable is relative and is given by the test scores of all students on each of the variables. Figure 3 then allows us to establish relationships of association between the levels of physical-mathematical thinking.

![Figure 3. Students' levels of physical-mathematical thinking; thinking: variational (VT), metric (MT), numerical (NT), random (RT), spatial (ST), physical (FT).](image)

4. Discussion

The results show that the project methodology is a very good alternative for the development of physical-mathematical thinking. These results coincide with those obtained by [4,22].

In relation to physical-mathematical thinking, it is evident that the experimental group increased the scores significantly, while the control group did so, but very slightly. Therefore, it is possible to think that the result of the pedagogical intervention plan was positive in terms of content associated with the development of physical-mathematical thinking. Part of the success achieved in the work developed in the framework of the research was due to the motivation, preparation and participation of the students to address the process; this makes the resulting proposal particularly different from other proposals developed in the same direction. One of the most important effects of using the project methodology was the degree of satisfaction of the students compared to their peers attending traditional classes.

The fact that the students showed a good level of development in mathematical and physical thinking through the pretest also constitutes a pillar for the success of the work. It is important that other studies
involve the students' capacity for expression and their capacity for creativity and innovation. Mathematical thinking allows the continuous generation of new procedures and algorithms; it is not possible, therefore, to terminate the domain of any concept in a short period of time, nor to pretend that a significant connection between a new knowledge and those previously established knowledge is automatically achieved; however, the application of mathematics to the understanding and modelling of physics, allows to favorably promote mathematical physical thinking.

In this regard, it can be mentioned that the class, particularly the one related to physics and mathematics, is influenced by factors that depend on the students, the school context and the actions of the teacher. As for the students, they are influenced by their conceptions and attitudes related to mathematics and physics, their knowledge and previous experiences of academic work and, in general, their way of facing the school. The influence of the context is related to the school and social work environment, the organization and functioning, the existing resources and the teacher's own knowledge and professional competence, and very especially the way in which he or she introduces the different projects and supports the students in their realization.

Figure 3 identifies four groups of categories of the variables analyzed. It can be seen that the two groups, control and experimental, are located in the central part of the graph, which represents a certain homogeneity between them, so they are not determining the level of physical-mathematical thinking of the students.

Students with high levels of physical thinking are associated with high levels of metric and numerical thinking. Students with medium physical thinking levels are associated with high levels of variability and low levels of numerical thinking. Students with low physical thinking levels are associated with medium levels of vocational and numerical thinking. Thus, relationships are found that identify levels of physical-mathematical thinking, since relevant associations are found between the level of physical thinking and the students' metric, numerical and variational thinking. There is no evidence of a relationship between physical thinking and random and spatial thinking.

5. Conclusions
The results of the research allow us to infer that the project methodology is an efficient alternative for the development of physical-mathematical thinking, since it allows the student to integrate the knowledge into his or her context, visualize the usefulness of the learning and apply knowledge to the solution of problems in his or her environment, in other words, it allows the student to achieve significant learning. The methodology not only facilitates the knowledge of physics, but also stimulates autonomous learning, since the student must discover by himself the theoretical foundations and apply them to the development of projects, at the same time that it stimulates the collaborative learning, since, most of the projects must be approached in work teams to find their solution, but it is responsibility of all to reach with success the goal of the realized work.

Through this research it can be concluded that the teaching of Physics in high school students using the methodology of problem-based learning, leads to better educational performance than with traditional teaching while generating greater motivation and interest in their learning and the development of physical-mathematical thinking. The didactic proposal became an enriching experience. The activities developed allowed an active participation of the students. Thus, with an appropriate pedagogical approach, it allowed the students to find new ways of learning, since they felt motivated, interested and attracted by the activities. The integration in the learning of physics explained from the mathematical model was quite attractive for the students, since they not only develop physical-mathematical thinking, but also find in physics an experimental model where mathematics becomes very relevant.

The association between levels of physical-mathematical thinking identified in Figure 3 constitutes a fundamental pillar for the design of class projects that strengthen the relationship between physical, numerical, spatial and variational thinking, but which in turn stimulate relationships between physical thinking and geometric and random thinking.
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