When Did Cosmic Acceleration Start?

Alessandro Melchiorri, Luca Pagano, Stefania Pandolfi
Physics Department and Sezione INFN, University of Rome “La Sapienza”, Ple Aldo Moro 2, 00185, Rome, Italy

A precise determination, and comparison, of the epoch of the onset of cosmic acceleration, at redshift \( z_{acc} \), and of dark energy domination, at \( z_{eq} \), provides an interesting measure with which to parameterize dark energy models. By combining several cosmological datasets we place constraints on the redshift and age of cosmological acceleration. For a ΛCDM model, we find the constraint \( z_{acc} = 0.76 ± 0.10 \) at 95% c.l., occurring 6.7 ± 0.4 Gyrs ago. Allowing a constant equation of state but different from \(-1\) changes the constraint to \( z_{acc} = 0.81 ± 0.12 \) (6.9 ± 0.5 Gyrs ago) and \( z_{eq} = 0.48 ± 0.14 \) (4.9 ± 0.9 Gyrs ago), while dynamical models markedly increase the error on the constraint with \( z_{acc} = 0.81 ± 0.30 \) (6.8 ± 1.4 Gyrs ago) and \( z_{eq} = 0.44 ± 0.20 \) (4.5 ± 1.0 Gyrs ago).

Unified dark energy models as Silent Quartessence yield: \( z_{acc} = 0.80 ± 0.16 \) (6.8 ± 0.6 Gyrs ago).

I. INTRODUCTION

The existence of a dark and unclustered energy component responsible for more than 70% of the overall density of our universe is now suggested at high significance by most of the latest cosmological data (see e.g. [1], [2]).

A cosmological constant provides a possible candidate for the dark energy component, but it needs to have its initial conditions properly ‘tuned’ in order to dominate the universe expansion at precisely the present time. Indeed the energy density in a cosmological constant \( \rho_\Lambda \) does not evolve, while both matter \( (\rho_m) \) and radiation \( (\rho_r) \) energy densities evolve rapidly with the expansion of the universe. The small current value of \( \rho_\Lambda \) implies an extreme fine-tuning of initial conditions with \( \rho_\Lambda/\rho_r \sim 10^{-123} \) at the Planck time (when the temperature of the universe was \( T \sim 10^{19} \) GeV), or \( \rho_\Lambda/\rho_c \sim 10^{-55} \) at the time of the electroweak phase transition (\( T \sim 100 \) GeV). Moreover, \( \rho_\Lambda \) lies in a very small window, since a slightly larger value makes the universe accelerate much before the present epoch, thereby inhibiting structure formation, while a negative value may cause the universe to re-collapse.

The cosmological constant problem has, subsequently, motivated several “dynamical” alternatives (see e.g. [3] for a recent and very complete review) as a slowly-rolling scalar field, “quintessence” [4, 5], or a “k-essence” scalar field with non-canonical kinetic terms in the Lagrangian [6], string-inspired models such as the contribution of nonlinear short distance physics to vacuum energy [7], and modified Friedman equations at late time [8] or large distances [9].

Other possibilities include anthropic arguments [10, 11, 12] and “backreaction” of non linear inhomogeneities (see [13], but see also [14, 15]).

It is plausible that a solution to the dark energy problem could be found by identifying a time correlation between the epoch of appearance of this exotic component and a well understood and physically motivated event such as the time of matter-radiation equality, the origin of non linear structures or, ultimately, life. It is, therefore, clear that a first crucial measurement that has to be made is the determination of the redshift and time of dark energy domination. Evidence for dark energy at very high redshifts \( (z > 1) \), when the cosmological constant is negligible, would indeed favor models based on scalar fields, possibly coupled to dark matter [16, 17]. While the appearance of dark energy at lower redshifts \( (z < 0.2) \) would, on the contrary hint at “phantom” \( (w < -1) \) models. Anthropic principle arguments are definitely less appealing if dark energy dominates well after the epoch of formation of terrestrial planets. At the same time, backreaction models could be perceived as much less convincing if dark energy starts in a time when nonlinear structures are already well present and formed.

The starting point of cosmic acceleration, however, is not a model independent quantity. If the universe is in accelerated expansion today we can identify two crucial epochs. Firstly, the epoch of equality between matter and the dark energy component, at redshift \( z_{eq} \), defined as

\[
\rho_m (z_{eq}) = \rho_\chi (z_{eq})
\]

where \( \rho_m (z) \) and \( \rho_\chi (z) \) are the energy densities of the matter and dark energy components at redshift \( z \) respectively. This epoch is generally different from, and follows, the redshift \( z_{acc} \) when the universe started to accelerate, defined as

\[ q(z_{acc}) = -\frac{1}{H^2} \left( \frac{\dot{a}}{a} \right)^2 (z_{acc}) = 0 \]

where \( H = \dot{a}/a \) is the Hubble parameter at redshift \( z \). The two epochs are model dependent and distinct. In the simple case of a cosmological constant, for example, the two redshifts are not equal and the following simple relation:

\[ z_{acc} = 2^{1/3}(1 + z_{eq}) - 1 \]

holds. The age of the universe at each of those redshifts can then be easily computed from:

\[ t(z) = \int_{z}^{\infty} \frac{dz}{(1+z)H(z)} \]

and compared with the current age of the universe \( t_0 \).
It is clear that constraints on the model-dependent quantities \(z_{acc}, z_{eq}, t(z_{eq})\) and \(t(z_{acc})\) can provide relevant information for several studies. In this paper, we focus on constraining these quantities with current cosmological data with the goal of clarifying the following points: how model independent are the constraints on the epoch of dark energy domination? how does a different choice of cosmological datasets or parameters affect those constraints? Finally, are the constraints, derived in a general dark energy scenario, consistent with the predictions of a cosmological constant?

Our paper is organized as follows, in the next section we introduce our data analysis method, describing the datasets and dark energy parameterizations adopted. In section III we present the results of our analysis and in section IV we derive our conclusions.

II. LIKELIHOOD ANALYSIS

The method we adopt is based on the publicly available Markov Chain Monte Carlo package cosmomc \[18\]. We sample the following dimensional set of cosmological parameters, adopting flat priors on them: the physical baryon and CDM densities, \(\omega_b = \Omega_b h^2\) and \(\omega_c = \Omega_c h^2\), the ratio of the sound horizon to the angular diameter distance at decoupling, \(\theta_s\), the scalar spectral index, \(n_s\), and the optical depth to reionization, \(\tau\). Furthermore, we consider purely adiabatic initial conditions and the possibility of curved universes, \(\Omega_{tot} \neq 1\). We also consider the possibility of having a running of the spectral index \(dn_s/d\ln k\) at \(k = 0.002\ h^{-1}\ Mpc\) (see e.g. \[19\]), an extra-background of relativistic particles (parametrized with an effective number of neutrino species \(N_{eff} \neq 3\), see e.g. \[21\]) and a non-zero, degenerate, neutrino mass of energy density (see e.g. \[21\]):

\[
\Omega_{\nu} h^2 = \frac{\sum m_{\nu}}{92.5 \text{eV}}
\]

in order to establish how robust measurements of \(z_{acc}\) and \(z_{eq}\) are to broader cosmological models.

Finally, we will also investigate the possibility of a dark energy equation of state \(w\) different from \(-1\). Other than a constant equation of state \(w\) we consider the possibility of a varying with redshift equation of state. In particular we consider a linear dependence on scale factor \(a = (1 + z)^{-1}\) as \[22\] and \[23\]:

\[
w(a) = w_0 + w_1(1 - a)
\]

where the equation of state changes from \(w_0\) to \(w_0 + w_1\) at higher redshifts. We refer to this as Chevallier-Polarski-Linder (CPL) parameterization.

We also consider a more sophisticated parametrization that takes in to account the rate and redshift of the transition. We use the model proposed by Hannestad and Mortsell (HM), see \[24\], where:

\[
w(a) = w_0 + w_1 \left( \frac{a^q + a_0^q}{w_1 a^q + w_0 a_0^q} \right)
\]

In this model the equation of state changes from \(w_0\) to \(w_1\) around redshift \(z_s = 1 - 1/\alpha_s\) with a gradient transition given by \(q\). We assume \(w_{0,1} > -3\), \(0.1 < \alpha_s < 1.0\) and \(1 < q < 10\) as external priors for this model.

Finally, we consider the Quartessence (or Chaplygin gas) model (see \[23\]) as unified dark energy-dark matter model. In this scenario cold dark matter and dark energy are the same fluid with equation of state:

\[
w(a) = \frac{w_0}{-w_0 + (1 + w_0)a^{-3(\alpha+1)}}
\]

where \(w_0\) is the current value of the equation of state and \(\alpha\) is a parameter that has to be constrained from observations. From the equation above, it is clear that at early times, when \(a \to 0\), we have \(w \to 0\), and the fluid behaves as non-relativistic matter. At late times, when \(a \gg 1\), we obtain \(w \to -1\). The matter clustering presents strong instabilities and oscillations in this model \[27\] unless one assumes intrinsic non-adiabatic perturbations such that the effective sound speed vanishes \[26\]. In this paper we therefore consider only this “Silent” Quartessence.

The MCMC convergence diagnostics are done on 7 chains applying the Gelman and Rubin “variance of chain mean”/“mean of chain variances” \(R\) statistic for each parameter. Our \(1 - D\) and \(2 - D\) constraints are obtained after marginalization over the remaining “nuisance” parameters, again using the programs included in the cosmomc package. Temperature, cross polarization and polarization CMB fluctuations from the WMAP 3 year data \[1\] \[28\] \[29\] \[30\] are considered and we include a top-hat age prior 10 Gyr < \(t_0\) < 20 Gyr. We combine the WMAP data with the real-space power spectrum of galaxies from the Sloan Digital Sky Survey (SDSS) \[31\] and 2dF survey \[32\]. We restrict the analysis to a range of scales over which the fluctuations are assumed to be in the linear regime (technically, \(k < 0.2\ h^{-1}\ Mpc\)) and we marginalize over a bias \(b\) considered as an additional nuisance parameter. We also incorporate the constraints obtained from the supernova (SN-Ia) luminosity measurements by using the so-called GOLD data set from \[32\] and the Supernovae Legacy Survey (SNLS) data from \[34\].

III. RESULTS

A. Cosmological Datasets

Using the analysis method described in the previous section we have constrained the value of \(z_{eq}, t_{eq}, z_{acc}\)
and $t_{\text{acc}}$ in light of the various datasets and cosmological scenarios. This kind of test is extremely useful in order to identify the presence of possible systematics.

The constraints on $z_{\text{eq}}, t_{\text{eq}}, z_{\text{acc}}$ and $t_{\text{acc}}$ for various datasets are reported in Table I.

As we can see, there is a general agreement between the results: namely, in a cosmological constant model, dark energy became the dominant component at redshift $z_{\text{eq}} \sim 0.4$, 4.3 Gyrs ago, and the accelerated expansion of the universe started at $z_{\text{acc}} \sim 0.75$, 6.7 Gyrs ago. Since we are assuming a cosmological constant $z_{\text{eq}}$ and $z_{\text{acc}}$ are not independent but follow Eq. (3). It is interesting, however, to note that the SDSS and GOLD datasets seem to favor a lower redshift for the dark energy’s dominant than that suggested when the 2dF or SNLS datasets are included, respectively.

### B. Theoretical assumptions on the background cosmological model.

Since the results appear stable to the inclusion/exclusion of the experimental datasets, we now consider the full set of cosmological data and study the dependence on some of the theoretical assumptions on the background cosmological model. We consider possible variations from $-1$ in a constant dark energy equation of state, non-flat universes, a running of the spectral index of the primordial inflationary perturbations, massive neutrinos and an extra background of relativistic particles. All those constraints are reported in Table II. As we can see, the results are consistent with those reported in Table I. However, as expected, the constraints are in general weaker. While including a constant dark energy equation of state $w \neq -1$ has little effect, considering a universe with spatial curvature generally doubles the error bars on all the parameters and results in a lower redshift and time of dark energy domination. It is interesting to note that considering an extra background of relativistic particles has a strong effect on the age of the universe and of dark energy. This raises an interesting question of whether the recent discovery of the APM 08279+5255 quasar at $z = 3.91$, whose age of $2 - 3$ Gyr can’t easily be accommodated in the standard scenario, could provide a hint for the presence of an extra background of relativistic particles [33].

### C. Dynamical Dark Energy

We now study the sensitivity of the epochs of dark energy domination and the onset of acceleration to differing dark energy models. We compare models to WMAP+2dF+SNLS datasets, this should be considered a more conservative choice in comparison to the “all” dataset described in the previous section. For the Silent Quartessence, however, we consider only WMAP+SNLS, as a unified dark energy model, include no cold dark matter, we omit the redshift and time of equivalence with the baryonic component. The constraints are reported in Table III.

Allowing for an equation of state which is varying with redshift can strongly affect the constraints, with error bars as large as four times those reported in Table I. However, the mean values are generally consistent with the previous results, i.e. there is no indication for deviations from a cosmological constant. In this sense, it is useful to plot the constraints on the $z_{\text{acc}} - z_{\text{eq}}$ plane as we do in Fig.1. A cosmological constant in this plane generates a line described by Eq. 3. As showed by three contour plots, while including a dynamical component leaves the possibility of a different relation between the two redshifts, the case of a cosmological constant is always well inside the $1\sigma$ c.l.

In Fig.2 we plot the $2\sigma$ constraints on the deceleration parameter $q$ in function of the redshift for the four different dark energy parameterizations. While

| Dataset | $z_{\text{eq}}$ | $t_{\text{eq}} - t_{\text{eq}}$ | $z_{\text{acc}}$ | $t_{\text{acc}} - t_{\text{acc}}$ | $t_{\text{eq}}$ |
|---------|----------------|--------------------------|----------------|--------------------------|----------------|
| Alone   | 0.47±0.08     | 0.57±0.07                | 0.86±0.11      | 0.70±0.04                | 13.8±0.3      |
| +SDSS   | 0.40±0.07     | 0.37±0.08                | 0.77±0.10      | 0.63±0.03                | 13.8±0.2      |
| +2dF    | 0.48±0.06     | 0.53±0.07                | 0.87±0.09      | 0.71±0.02                | 13.8±0.2      |
| +GOLD   | 0.38±0.06     | 0.44±0.08                | 0.74±0.06      | 0.64±0.02                | 13.8±0.2      |
| +SNLS   | 0.45±0.07     | 0.43±0.09                | 0.83±0.08      | 0.69±0.03                | 13.8±0.1      |
| +all    | 0.40±0.04     | 0.49±0.07                | 0.76±0.05      | 0.70±0.02                | 13.9±0.1      |

| Model | $z_{\text{eq}}$ | $t_{\text{eq}} - t_{\text{eq}}$ | $z_{\text{acc}}$ | $t_{\text{acc}} - t_{\text{acc}}$ | $t_{\text{eq}}$ |
|-------|----------------|--------------------------|----------------|--------------------------|----------------|
| $w \neq -1$ | 0.43±0.07     | 0.57±0.09                | 0.79±0.07      | 0.68±0.06                | 13.8±0.1      |
| CPL   | 0.44±0.10     | 0.54±0.10                | 0.81±0.13      | 0.68±0.04                | 13.8±0.1      |
| HM    | 0.45±0.08     | 0.56±0.08                | 0.80±0.14      | 0.67±0.04                | 13.9±0.2      |
| SQ    | 0.46±0.08     | 0.57±0.08                | 0.81±0.15      | 0.67±0.04                | 13.8±0.1      |
there is a large spread in the values, especially when more complex parametrizations such as CPL or HM are considered, there is a very good agreement, and all the models point towards the same acceleration redshift value at $z \sim 0.8$.

**IV. CONCLUSIONS**

In this brief paper we have presented several constraints on the epoch and redshift of dark energy domination and of cosmic acceleration. We have derived those constraints using different datasets, different theoretical assumptions and different dark energy parametrizations. We have found that a redshift and epoch of acceleration at $z_{acc} = 0.78$ and $t_0 - t_{acc} = 6.9$ Gys and a redshift and epoch of dark energy domination start at $z_{acc} = 0.43$ and $t_0 - t_{acc} = 4.4$ Gyrs, as expected for a flat universe with $\Omega_\Lambda = 0.7$ is in agreement with all the possible cases considered. Moreover, despite the large set of models and data analyzed, there a very little spread in the best-fit values. Curvature, running of the spectral-index, massive neutrinos and an extra-background of relativistic particles are non-standard cosmological parameters that can strongly enlarge the error bars on $z_{eq}$ and $z_{acc}$ in case of a cosmological constant model. Allowing a constant or dynamical dark energy equation of state different from $-1$ produces similar results, however the best fit values are, again, not significantly altered. A tension in the derived best fit values appears when considering galaxy clustering data from SLOAN and 2dF and supernovae type Ia from Riess et al. and SNLS datasets separately. However the significance of the discrepancy is well below the $2\sigma$ c.l.. As a final remark, we like to stress that the analysis presented here relies nonetheless in the assumption of a class of scenarios. It may be possible to construct more complicated cosmological and/or dark energy models that could evade the constraints presented here. Future data may well falsify those possibilities as well the simple case of a cosmological constant by testing the $z_{acc} - z_{eq}$ relation of Eq. 3.

**Acknowledgments** We are pleased to thank Rachel Bean for help, discussions and useful comments on the manuscript.

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