Signatures of quantum stability in a classically chaotic system

S. Schlunk,1 M.B. d’Arcy,1 S.A. Gardiner,1 D. Cassettari,1 R.M. Godun,1 and G.S. Summy1,2

1Clarendon Laboratory, Department of Physics, University of Oxford, Parks Road, Oxford, OX1 3PU, United Kingdom
2Department of Physics, Oklahoma State University, Stillwater, Oklahoma, 74078-3072

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We experimentally and numerically investigate the quantum accelerator mode dynamics of an atom optical realization of the quantum δ-kicked accelerator, whose classical dynamics are chaotic. Using a Ramsey-type experiment, we observe interference, demonstrating that quantum accelerator modes are formed coherently. We construct a link between the behavior of the evolution’s fidelity and the phase space structure of a recently proposed pseudoclassical map, and thus account for the observed interference visibilities.

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The relationship between the behaviour of classical and quantum systems, and how macroscopic classical phenomena originate in the quantum regime, remain subjects of dispute [1]. The issues involved are particularly marked for quantum versions of classically chaotic systems [2]. Experimental investigations of such systems began with studies of microwave-driven hydrogen [3]; subsequent work has also centred on microwave cavities [4], mesoscopic solid-state systems [5], and atom optics [6], the approach we adopt. In this Letter we consider the quantum δ-kicked accelerator [7, 8, 9], a δ-kicked rotor with an additional static linear potential. The δ-kicked rotor is one of the most extensively investigated systems in chaotic dynamics [10], and is equivalent to a free particle subjected periodically to instantaneous momentum kicks from a sinusoidal potential. Quantum mechanically, the effect of these kicks is to diffract the particles’ constituent de Broglie waves into a series of discrete momentum states. In the δ-kicked accelerator, the linear potential modifies the chaotic classical dynamics only slightly, yet can radically change the quantum behaviour. The phases accumulated between consecutive kicks by the momentum states are altered, leading to the creation of quantum accelerator modes [7, 8, 9]. We realize quantum δ-kicked accelerator dynamics in laser-cooled cesium atoms by the application of short pulses of a vertical standing wave of off-resonant laser light, which constitutes a sinusoidal potential; gravity provides the linear potential.

In our interference experiment, the atoms undergo δ-kicked accelerator dynamics, between the application of two π/2 microwave pulses that couple two atomic hyperfine levels. In the absence of coherence-destroying spontaneous emission, the contrast of any interference fringes is related to the overlap of two initially identical motional states evolved under the influence of slightly different Hamiltonians [15], i.e., the fidelity. It can therefore yield information on the sensitivity of the atoms’ evolution to variations in the kicking strength. Strong sensitivity can be considered a quantum signature of chaos, particularly in the semiclassical limit (ℏ → 0), hence the use by Peres [16] of f as a measure of quantum stability.

After magneto-optic trapping and molasses cooling to 5μK, we prepare around 10⁶ freely falling cesium atoms in the F = 3, m_F = 0 hyperfine level (denoted |a⟩) of the ⁶²S₁/₂ ground state [7]. The first π/2 microwave pulse creates an equal superposition of the atoms’ internal states, i.e., |a⟩ → (√2|a⟩ − i|b⟩)/√2, where |b⟩ denotes the F = 4, m_F = 0 level. The phase θ of this pulse can be changed with respect to the phase space structure of the δ-kicked accelerator in a pseudoclassical limit recently proposed by Fishman et al. [14]. Finally we explain differences in the observed fringe visibilities by examining the effect of the experimental range of kicking strengths.

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FIG. 1: Experimentally measured momentum distributions as the microwave phase difference θ is varied in a π/2 − 20 kick − π/2 sequence where δ_L^0 ≈ 35 GHz, with (a) T = 60.5μs (accelerator mode at −17ℏG), and (b) T = 74.5μs (accelerator mode at 20ℏG). The corresponding numerically generated momentum distributions, where δ_L^0 = 35 GHz, are shown in (c) and (d). Population has been arbitrarily normalized to maximum value = 1.
to that of the second $\pi/2$ pulse, applied following 20 equally spaced 500-ns pulses from a standing wave of light. This is formed by retroreflection of a Ti:sapphire laser beam; its maximum intensity is $\sim 1 \times 10^8$ mW/cm$^2$ [3], and the light is red-detuned by 45 and 35 GHz from the D1 transition for atoms in states $|a\rangle$ and $|b\rangle$, respectively. After the second $\pi/2$ microwave pulse, we measure the momentum distribution in state $|b\rangle$ by a time-of-flight method. For more details of our experimental setup see Refs. [3, 4]. Measurement of a periodic variation with $\theta$ in the accelerator mode population in state $|b\rangle$, i.e. interference, directly implies coherent evolution.

In the limit of large detuning, the Hamiltonian is

$$\hat{H} = \hat{H}_a |a\rangle \langle a| + \hat{H}_b |b\rangle \langle b| + \frac{\hbar \omega_{ab}}{2} (|b\rangle \langle a| - |a\rangle \langle b|),$$

where $\hbar \omega_{ab}$ is the energy gap between $|a\rangle$ and $|b\rangle$, and

$$\hat{H}_a = \frac{\hbar^2 \rho}{2m} + mg\ddot{z} - \frac{\hbar^2 G^2 L}{8\delta E_L^2} [1 + \cos(\hat{G} \ddot{z})] \sum_n \delta(t - nT)$$

is the quantum $\delta$-kicked accelerator Hamiltonian, acting on atoms in internal state $|\sigma\rangle \in \{|a\rangle, |b\rangle\}$. Here $\ddot{z}$ is the vertical position, $\rho$ the $z$-direction momentum, $m$ the particle mass, $g$ the gravitational acceleration, $t$ the time, $\delta$ the pulsing period, $\Omega$ the Rabi frequency, $t_p$ the pulse duration, $\delta E_L$ the detuning from the D1 transition for the state $|\sigma\rangle$, and $G = 4\pi/\lambda$ with $\lambda = 894$ nm is the laser wavelength, and $hG$ is a grating recoil (the momentum separation of adjacent diffracted states). We denote the amplitude of the phase modulation to atoms in state $|b\rangle$ that results from application of the standing wave as $\phi_d$. The experimental mean value of $\phi_d$ is $0.8\pi$, and, due to the different detuning, that of the corresponding quantity for atoms in state $|a\rangle$ is $\phi^0_d = \phi_d \delta E_L / \delta E_L = 0.6\pi$. We thus have effectively two different Hamiltonians, applied to the same initial motional state. The pulse train leads to the creation of a quantum accelerator mode, the momentum of which is the same for the two internal states [3]. We consider pulse periods $T = 60.5\mu s$ and $74.5\mu s$, close to $T_{1/2} = 2\pi m / hG^2 = 66.7\mu s$, which corresponds to the lowest second-order quantum resonance in the $\delta$-kicked rotor [3, 8]. For these $T$, well-populated accelerator modes involving substantial momentum transfer are created [7, 8, 9].

Figure 1(a) shows the measured final momentum distributions of $|b\rangle$ atoms, for $T = 60.5\mu s$. We see a period-2 $\pi$ variation with $\theta$ in the accelerator mode population (at around $-17hG$), the visibility $V$ of which is $(21 \pm 2)\%$ [20]. We observe similar fringes for a range of detunings ($\delta E_L = 20$–40 GHz) and total number of kicks $N = 10$–30. However, depending on their exact values, $V$ can vary between 10% and 40%. This periodic variation of the population demonstrates interference, and hence that the accelerator mode transfers momentum coherently. At $T = 74.5\mu s$ [Fig. 1(b)], however, fringes in the accelerator mode (at around $20hG$) are practically invisible, despite the expected coherent nature of the momentum transfer. In Figs. 1(c) and 1(d) diffraction-based numerical simulations [7, 9, 13], incorporating the experimental range of $\phi_d$ ($0.3\pi$ to $1.2\pi$), also show this difference in the fringe visibility for the two values of $T$. The range of $\phi_d$ is due to the Gaussian profile of the standing wave intensity (FWHM $\sim 1$ mm) and the spatial extent of the atomic cloud (Gaussian density distribution, FWHM $\sim 1$ mm) [3]. As we optimized the overlap of the laser beams with the atomic cloud, the intensity and density maxima can be assumed to be coincident. The calculated visibility is then $25\%$ for $T = 60.5\mu s$ [Fig. 1(c)] but only $8\%$ for $T = 74.5\mu s$ [Fig. 1(d)].

In order to explain these surprising observations, we introduce the Floquet operator $F_b(\phi_d)$. This describes the effect of one kick and the subsequent free evolution on the motional state of atoms in state $|b\rangle$, where

$$\tilde{F}_b(\phi_d) = \exp(-i[\gamma \hat{\chi} + \hat{P}^2/2]/\hbar) \exp(i\phi_d [1 + \cos(\hat{\chi})]).$$

We define $\tilde{F}_a(\phi_d)$ analogously for state $|a\rangle$, with $\phi_d$ replaced by $\phi^0_d$ [3]. As in Ref. [3], we use scaled position and momentum variables $\chi = Gz$ and $p = G T_p / m$, while $\gamma = g/2T^2$ describes the effect of gravity, and $\hat{P} = \hbar G^2 T / m = -i[\chi, \hat{\rho}]$ is an effective scaled Planck constant. After $N$ pulses an initial plane wave $|q\rangle$ of wavenumber $q$ evolves to $F_b(\phi_d)^N |q\rangle = e^{-i N |\psi^q_b(\phi_d)\rangle}$, where $\phi = \phi_d$ or $\phi^0_d$, as appropriate. Regarding the initial motional state as an incoherent superposition of $|q\rangle$, the momentum distribution in state $|b\rangle$ for a given $\phi_d$ after the $\pi/2 - N$ kick — $\pi/2$ sequence, is

$$P_b(\phi_d, p) = \frac{1}{4} \left[ \int dq C(q) \left[ |\psi^q_b(\phi_d, p)\rangle \langle \psi^q_b(\phi_d, p)| \right. \right]$$

$$+ \frac{1}{2} \left\{ \int dq C(q) \left[ \psi^q_b(\phi_d, p) \right. \langle \psi^q_b(\phi_d, p) | \right.$$}

$$\times \cos(\phi_T(\phi_d, p) + N \delta \phi_d + \theta),$$

where $\phi_T = \phi_d - \phi^0_d$, $\psi^q_b(\phi_d, p) = \langle p | \psi^q_b(\phi_d) \rangle$, and $\phi_T$ is the phase of the interference term, i.e., $\int dq C(q) \psi^q_a(\phi_d, p) \langle \psi^q_b(\phi_d, p) | \langle p | \psi^q_b(\phi_d) \rangle$. The weighting $C(q)$ describes the initial Gaussian momentum distribution (FWHM = 6 grating recoils $hG$). The third (interference) term in Eq. (4) is responsible for the appearance of fringes in the accelerated $|b\rangle$ population. We denote the amplitude of the modulation in $P_b$ by $A(\phi_d, p) / 2 = \left| \int dq C(q) \psi^q_a(\phi_d, p) \langle \psi^q_b(\phi_d, p) | \langle p | \psi^q_b(\phi_d) \rangle \right| / 2$, where $\int dp A(\phi_d, p)^2 = f(\phi_d)$ is the fidelity for a given $\phi_d$.

We have calculated the individual terms of $P_b$ numerically for a wide range of $\phi_d$, obtaining as a consequence an important result linking the pseudoclassical analysis of Fishman et al. [14] with the quantum stability measure of Peres [16]. Comparison of Figs. 2(a) and 2(c) with Fig. 2(b) shows that the region in momentum space corresponding to an accelerator mode is also a region of high $A$. This remains high up to large values of $\phi_d$ [3], continuing beyond the point at which it has decayed to nearly zero in other regions of momentum space. As $f = \int dp A^2$, its large value when determined by integrating over the momenta populated by atoms in the quantum accelerator mode implies that these atoms inhabit a stable region of quantum state space. Note that small $A$ does not necessarily imply low atomic population, as can be seen in the plots of the non-interfering population $\int dq C(q) (|\psi^q_a|^2 + |\psi^q_b|^2) / 4$ in Figs. 3(b) and 3(d). Contrasting Fig. 3(a) with Fig. 3(c), we
see that this large value of $A$ extends over a significantly wider range of $\phi_d$ for $T = 60.5\mu s$ than for $T = 74.5\mu s$. Hence, we can interpret the accelerator mode at $T = 60.5\mu s$ as being more robust to variations in $\phi_d$, i.e. more stable, compared with that at $T = 74.5\mu s$. However given our comparatively narrow experimental range of $\phi_d$, this does not explain the difference in fringe visibilities seen in Fig. 1.

The appearance of quantum accelerator modes in the $\delta$-kicked accelerator is explained in the analysis of Fishman et al. [14] in terms of islands of stability centred on stable fixed points in the phase space generated by the map [22]:

$$\begin{align*}
\tilde{\rho}_{n+1} &= \tilde{\rho}_n - \hat{k} \sin(\chi_n) - \text{sign}(\epsilon)\gamma, \\
\chi_{n+1} &= \chi_n + \text{sign}(\epsilon)\epsilon|n+1,
\end{align*}$$

(5)

(6)

where the population of a mode is proportional to the size of the corresponding island. This is a pseudoclassical [ε = ($\hat{k} - 2\pi$) → 0] rather than semiclassical ($\hat{k} → 0$) limit of the quantum dynamics characterized by the Floquet operator of Eq. (3). We have introduced $\tilde{\rho} = \rho \epsilon/\pi$ (in an accelerating frame [14]) and $k = \phi_d/\epsilon$. Classically, the system is globally chaotic for our parameter regime. Figure 3 shows the pseudoclassical phase spaces generated by iteration of Eqs. (5) and (6) for the experimentally investigated values of $\epsilon = 2\pi(T/11/2)$, and a range of $\phi_d$. When $\phi_d = 0.3\pi$ [Figs. 3(a) and 3(d)], the island (based around a stable fixed point in phase space) is substantially smaller for $T = 74.5\mu s$ than for $T = 60.5\mu s$. For the average experimental value of $\phi_d$ (0.8$\pi$) [Figs. 3(b) and 3(e)], the islands have both grown to be about the same size. For $\phi_d = 1.5\pi$ [Figs. 3(c) and 3(f)], the island has shrunk dramatically in the case of $T = 74.5\mu s$, while at $T = 60.5\mu s$ the island has also shrunk, but not to the same extent. We therefore conclude that the stable island representing the accelerator mode is much more robust to perturbations in the kicking strength for $T = 60.5\mu s$ than for $T = 74.5\mu s$. The fact that $A$ (and therefore $f$) remains large at the accelerator mode momentum for a significantly broader range of $\phi_d$ than for $T = 60.5\mu s$ than for $T = 74.5\mu s$, as shown in Fig. 3, exactly the observed greater stability of the island in the pseudoclassical phase space for $T = 60.5\mu s$. This is consistent with Peres’s identification of the behaviour of fidelity as reflecting stability properties of the phase space in the semiclassical limit [14], even though our experiment (and numerics) operate in a pseudoclassical regime which is far from semiclassical.

The position of the islands in pseudoclassical phase space in Fig. 3 indicates the region of the quantum accelerator modes spatial localization. For $T = 60.5\mu s$ this is where there is zero standing wave amplitude, whereas when $T = 74.5\mu s$, it is where the amplitude, and hence phase shift, are maximal [23]. We thus expect the modulation of the $P_0$ interference term in Eq. (4), $\cos(\phi_1 + N\delta\phi + \theta)$, to have a strong dependence on $\phi_1$ for the momenta at which accelerator modes are found when $T = 74.5\mu s$, but not when $T = 60.5\mu s$. This is confirmed by Figs. 4(a) and 4(b), where $\cos(\phi_1 + N\delta\phi + \theta)$
FIG. 4: Plots of \( \cos(\phi T + N\delta\phi_d) \) against \( \phi_d \) with \( N = 20 \), for (a) \( T = 60.5 \mu s \) and (b) \( T = 74.5 \mu s \). Crosses mark the boundaries of the experimental range.

is plotted as a function of \( p \) and \( \phi_d \) for constant \( \theta \) (set to 0 for convenience) for \( T = 60.5 \mu s \) and \( T = 74.5 \mu s \), respectively. At the accelerator mode momentum, the value of \( \cos(\phi T + N\delta\phi_d) \) at \( T = 60.5 \mu s \) is almost independent of \( \phi_d \), whereas at \( T = 74.5 \mu s \) there is an approximate frequency doubling, relative to other moments. We can now explain the presence or absence of interference fringes in Fig. 1 in terms of visibility at \( \phi \). Note, however, that as \( \phi \) increases, so does the difference between \( \cos(\phi T + N\delta\phi_d) \) for which the corresponding value of \( A \) is close to zero.

In summary, we have performed a Ramsey-type interference experiment and thus demonstrated the coherence of the production of quantum accelerator modes, and hence their suitability for applications in atom interferometry. Numerically, we have found the accelerator modes to correspond to regions of greater quantum stability, as quantified by the visibility of the fringes at both \( \phi \). Crosses mark the boundaries of the experimental range of kicking strengths. Our investigation of coherence in quantum accelerator modes has allowed observation of their quantum-stable dynamics in this classically chaotic system.

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[17] Here \( F \) is the total electronic plus nuclear angular momentum, and \( m_F \) is the projection on the quantization axis.
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[19] As \( \phi_d^0 \) is determined by \( \phi_d, F_n \) is written as a function of \( \phi_d \).
[20] We define the visibility by \( V = (S_{\text{max}} - S_{\text{min}})/(S_{\text{max}} + S_{\text{min}}) \), where \( S \) is the accelerator mode population in state \( |b \rangle \).
[21] Note that \( \delta\phi_d = \phi_d(1 - \delta\phi_d^0/\delta\phi_d^0) \) grows linearly with \( \phi_d \). Thus, as \( \phi_d \) increases, so does the difference between \( H_a \) and \( H_b \).
[22] Here \( \varphi_n \) and \( \rho_n \) specify \( n \) and \( r \) just prior to kick \( n + 1 \), rather than just after kick \( n \), as in Ref. [14].
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