On the thermodynamic behaviors and interactions between bubble pairs: A numerical approach

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\textbf{ABSTRACT}

Thermodynamic behaviors and interactions between bubble pairs are important to better understand the cavitation phenomena. In this study, a compressible two-phase model, accounting for thermal effects to investigate the thermodynamic behaviors and interactions between bubble pairs, is developed in OpenFOAM. The volume of fluid (VOF) method is adopted to capture the interface. Validations are performed by comparing the simulation results of a single bubble and bubble pairs with corresponding experimental data. The dynamical behaviors of bubble pairs and their thermodynamic effect at different relative distances \(\gamma\) are investigated and discussed, which help reveal the bubble cloud dynamics. The quantitative analysis of \(\gamma\) effects on the maximum temperature during bubble collapse is performed with three distinct stages identified. For a single bubble collapsing near the rigid surface, the thermodynamic characteristics at different relative distances are similar to that of the bubble pairs, but the maximum temperature is higher since the single bubble can collapse to a smaller volume.

\section{1. Introduction}

Cavitation phenomenon is common in engineering applications of hydrodynamics, such as turbomachinery \cite{1}, pumps \cite{2}, piping systems, and ship propellers \cite{3}. The damage caused by the bubble collapse to nearby surfaces is a major issue to the run life of hydraulic devices and rotating machinery operating under cavitation conditions. Besides, cavitation is also frequently encountered in several important industrial applications, such as water jet drilling and rock cutting \cite{4}, heating systems \cite{5}, and friction reduction in underwater vehicles \cite{6}, where the dynamics of cavitation bubbles play the essential roles. Assuming the incompressible and inviscid liquid phase, Rayleigh \cite{7} first studied the cavitation phenomenon of bubbles. A Rayleigh equation was proposed and later extended by other researchers to consider multiple affecting factors, e.g. surface force, fluid viscosity \cite{8} and compressibility \cite{9}, etc. Thus, the behaviors of spherical cavitation bubbles can then be better analyzed and characterized.

Generally, the collapse of cavitation bubble is vigorous, accompanied by strong local heating and high pressure due to the fast compression of bubbles at the end of collapse process. Since the collapse occurs instantly, the local heating is more rapid than the normal heat transfer \cite{10}, generating an extremely high temperature inside the bubble depending on the experimental setup and operation conditions \cite{11–16}. Such high temperatures may result in melting spots on the special coating used in naval environments to protect the surfaces \cite{17}. Deplancke \cite{18} observed damages on UHMWPE (ultra-high-molecular-weight polyethylene material) under 98-h cavitation flow due to local heating and melting. Hence, the small-scale heat transfer during bubble collapse plays a dominant role in the local melting damage.

Since the cavitation bubble collapses during an infinitesimal time step, the temperature and pressure can jump to extremely high values, causing light emission in sonoluminescence \cite{19} and plasma generation, which further yields an optically opaque area at the center bubble \cite{10}. The accurate measurement of the temperature and pressure in experiments is very difficult. Numerical simulations have therefore been an effective tool to predict and simulate the bubble dynamic behaviors \cite{20}, Müller et al. \cite{21} investigated the laser-generated cavitation bubble in compressible two-phase flows with neglect of the thermodynamic effect between bubbles. Koch et al. \cite{22} observed the shock wave in the cavitation process by implementing an adiabatic model and emphasized that the heat conduction inside bubbles should be considered to better understand the cavitation bubble behaviors. To calculate the instant heat transfer during the bubble collapse, Qin et al. \cite{23} developed a two-phase compressible model. However, the proposed model is inapplicable for larger bubbles with the radii over 1.0 mm. Beig et al. \cite{24} studied the temperature generated by collapsing bubbles near the rigid surfaces and found that the increasing temperature on the wall might depend on the relative distances and
Most existing studies concerning the thermodynamic effect inside cavitation bubbles are based on a single cavitation bubble. However, the cavitation phenomenon usually occurs with swarm bubbles generated, whose interaction effects on the cavitation process should not be neglected. Thus, it is important to investigate bubble interactions to reveal the cavitation mechanism. As the fundamental to study the interaction of multiple bubbles, the interaction between two cavitation bubbles is of great scientific value [25]. Tomita et al. [26] demonstrated that the bubble-pair interaction was significantly affected by relative sizes and the mutual distance. Han et al. [27] numerically investigated the dynamics of two bubbles and observed that the annular jet causes bubble breakup and the axial jet formation. Peng et al. [28] conducted numerical research on the interactions between two neighboring bubbles and analyzed the transition of strong interaction to the weak one. Li et al. [29] studied the nonlinear interaction and coalescence characteristics of the oscillating bubble pairs by defining three different coalescing patterns.

From literature review, although the investigations on cavitation bubble behaviors have been conducted, the thermodynamic effect during the bubble pairs collapse has not been well studied. The temperature change inside cavitation bubbles, as well as in the surrounding liquid phase is not fully understood yet. The accurate assessment of the temperature is important to understand the thermodynamic mechanism of cavitation bubbles.

As an efficient and reliable numerical tool, the open-source CFD code OpenFOAM has been widely employed to study the cavitation phenomena and bubble behaviors. The official OpenFOAM package (version 4.1) [30] includes a compressible two-phase solver, named as compressibleInterFoam, to account for the interphase heat transfer. However, the original solver cannot be applied to bubble pairs directly (more details can be referred to in Appendix A). Our previous work [31] already proposed a compressible two-phase model without considering the thermodynamic effect to investigate the single bubble dynamics in a free filed. The shock waves emitted during the bubble collapse can be captured by implementing the Tait equation into the model. In this

![Fig. 1. The computational region and O-grid type mesh for the single bubble in a free field.](image1)

![Fig. 2. Comparison of the bubble equivalent radius histories between the present simulation results and the experiments [21,49].](image2)
paper, the proposed two-phase model is further developed to incorporate the energy equation and Tammann equation of state [32], as described in Section 2.

The objectives of the present work include following aspects. Firstly, a compressible two-phase model accounting for the thermal effects to investigate the interactions between bubble pairs is developed. Secondly, the effects of relative distances $\gamma$ and ambient pressure $p_\infty$ on the maximum temperature inside the bubble pairs of similar sizes

![Fig. 3. The variation of the bubble radius, pressure, and temperature at the bubble center, and the velocity of the bubble wall during collapse. The blue solid line (−) denotes the bubble radius; the red solid line (−) denotes the bubble center temperature; the green solid line (−) denotes the bubble center pressure; the pink solid line (−) denotes the velocity of the bubble wall.](image)

![Fig. 4. Evolutions of the volume fraction, temperature, pressure, and velocity during the bubble first collapse. The size of each frame is 70 $\mu$m × 70 $\mu$m.](image)
during collapse are investigated. Although the movement of each bubble resembles the behaviors of a single bubble collapsing near the rigid boundary \[33\], few studies are available in literature to quantitatively analyze the similarities between them. In this study, the thermodynamic similarities between bubble pairs and the single bubble collapsing near a solid wall are compared in detail.

The rest of the paper is arranged as follows. The theory and physical model are introduced in Section 2. The model is validated by comparing the simulation results with experimental data in Section 3. In Section 4, the dynamical behaviors and thermodynamic effect between bubble pairs with different relative distances \( \gamma \) are analyzed and discussed. The \( \gamma \) effects on the maximum temperature of both bubble pairs and the single bubble collapsing near the solid wall are quantitatively analyzed. Finally, the main conclusions and contributions of this paper are given in Section 5.

2. Physical model and numerical setup

2.1. Physical model

The governing equations comprise a set of non-linear equations based on the conservation laws of mass, momentum, and energy to describe the dynamics of fluids, which can be solved within the whole calculation domain. Due to instant bubble pulsation, the mass transfer between phases is ignored \[22\]. The gas bubble is non-condensable whose saturated vapor pressure can be ignored \[34\].

![Fig. 5. Geometrical parameters in numerical simulations, \( R_{\text{max1}} \) is the maximum radius of bubble 1; \( R_{\text{max2}} \) is the maximum radius of bubble 2; \( d \) is the distance between the center of bubble 1 and of bubble 2.](image1)

![Fig. 6. The computational region (a) and the mapped computational mesh (b).](image2)
The continuity equation for each phase is written as:

\[ \frac{\partial (\rho_i \alpha_i)}{\partial t} + \nabla \cdot (\rho_i \alpha_i \mathbf{U}) = 0 \quad (i = l, g) \]  

(1)

The overall continuity equation is given by:

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0 \]  

(2)

where \( \rho_i \) is the density for phase \( i \) and \( \rho \) is the average density with \( \rho = \rho_l \alpha_l + \rho_g \alpha_g \); \( \mathbf{U} \) is the average velocity; \( \alpha_i \) is the volume fraction with \( \alpha_l = 1 \) denoting the liquid phase and \( \alpha_g = 1 \) denoting the gas phase. \( \alpha_l + \alpha_g = 1 \) and \( 0 < \alpha_l, \alpha_g < 1 \) are the algebraic restrictions for \( \alpha_i \). \( \alpha \) spans from 0 to 1 in the transition zone of the gas–liquid interface, which is captured by the volume of fluid (VOF) method. In VOF method, the following advection equation derived from Eq. (1) for the liquid phase is solved.

\[ \frac{\partial \alpha_l}{\partial t} + \nabla \cdot (\alpha_l \mathbf{U}) = 0 \]  

where \( \frac{\partial \alpha_l}{\partial t} + \nabla \cdot (\alpha_l \mathbf{U}) \) is the artificial term [35], which guarantees the sharpness of the interface; \( \mathbf{U} \) is the relative velocity both phases [36].

The momentum equation reads:

\[ \frac{\partial \rho \mathbf{U}}{\partial t} + \nabla \cdot (\rho \mathbf{U} \mathbf{U}) = -\nabla p + \nabla \cdot \tau + \int_{S_0} \sigma_{\text{int}} \delta(x - x')ds \]  

(4)

where \( \rho \) is the pressure field; the last term in the right hand of Eq. (4) represents the effect of surface tension force acting on the two-phase interface; More detailed descriptions of the momentum equation are given by Yin et al. [31]. \( \tau \) satisfies the relationship of

\[ \tau = \mu \left( \nabla \mathbf{U} + (\nabla \mathbf{U})^T - \frac{2}{3} (\nabla \cdot \mathbf{U}) \mathbf{I} \right) \]  

(5)

where \( \mathbf{I} \) is the unit tensor; \( \mu \) is the average dynamic viscosity with

\[ \mu = \mu_l \alpha_l + \mu_g \alpha_g \]  

The evaluation of the surface tension between the liquid and gas phases is achieved by Brackbill et al. [37] approach:

\[ \int_{S_0} \sigma_{\text{int}} \delta(x - x')ds \approx \sigma \nabla \alpha_l \]  

(6)

where \( \sigma \) is the interface curvature:

\[ \sigma = -\nabla \cdot \left( \frac{\nabla \alpha}{|\nabla \alpha|} \right) \]  

(7)

where \( \alpha \) is obtained from the volume fraction \( \alpha_l \) by smoothing it over a finite region around the interface using Lafaurie filter [38]. It is an effective and simple way to reduce the non-physical spurious currents. However, this filter tends to level out high curvature regions, where is very effective to suppress parasitic current [39]. Hoang et al. [40] found that the magnitude of parasitic current significantly decreases by using \( \alpha_l \) instead of \( \alpha_i \).

The energy equation expressed in terms of temperature \( T \) can be written as:

\[ \frac{\partial (\rho C_l U T)}{\partial t} + \nabla \cdot (\rho C_l U U T) = -\nabla \cdot \lambda_l \nabla T + \rho C_p \frac{\partial T}{\partial t} \]  

(8)

where \( C_{pl} \) and \( C_{pg} \) are the constant pressure heat capacity of the two phases; \( \lambda = \frac{U^2}{2} \) is the kinetic energy; \( \frac{\partial \rho}{\partial T} \) and \( \nabla \cdot \tau \) are the heat effects of pressure variation and shear stress on the flow [41]; \( \lambda_l \) and \( \lambda_g \) is the thermal conductivity of the two phases.

For compressible flow, the relationship between density, pressure, and temperature can be given by the equation of state (EOS) for the gas and liquid phases. The EOS for an ideal gas is used as follows [42,43]:

\[ \frac{p}{\rho g} = \frac{p}{R_g T} \]  

(9)

where \( R_g \) is the gas constant with \( R_g = 284.75 \text{ J/(kg·K)} \) [44].

The compressibility for gas \( \psi_g \) is related to the speed of sound by

\[ \psi_g = \frac{\alpha_l}{\alpha_l} \]  

Fig. 7. The convergence of \( R_{\text{min}} \) for different numbers of cells per initial bubble diameter.
\( \psi = \frac{1}{c_s^2} \) [22] as is defined by:

\[
\psi = \frac{d\rho}{dp} = \frac{1}{R_g T}
\] (10)

The EOS for the pure liquid is proposed by Tammann [32]. The so-called Tammann EOS can be applied to model a wide range of fluids even with the presence of strong shock waves [45], which is usually given as a relationship among pressure, density, and specific internal energy [46]. A similar relationship proposed by Shin et al. [42] is used in this paper:

\[
\rho_l = \frac{p + p_c}{K_c(T + T_c)}
\] (11)

where \( p_c, T_c \) and \( K_c \) are the pressure, temperature, and liquid constants for the liquid with \( p_c = 1944.61 \text{ MPa}, T_c = 3837 \text{ K} \) and \( K_c = 472.27 \text{ J/(kg·K)} \).

The compressibility for liquid \( \psi_l \) reads:

\[
\psi_l = \frac{d\rho}{dp} = \frac{1}{K_c(T + T_c)}
\] (12)

2.2. Numerical setup

Based on the finite volume method (FVM) [47], the governing equations are discretized to generate a system of linear algebraic equations. The preconditioned bi-conjugate gradient (PBiCGStab) solver with a simplified diagonal-based incomplete LU (DILU) preconditioner is used to solve the matrix system [30]. The first-order implicit Euler scheme is adopted to discretize the transient term. To reduce the numerical diffusion, the second-order TVD scheme with van Leer limiter is applied for the spatial discretization. The adjustable time-step is employed to improve computing efficiency. The interface distortion is avoided by setting the maximum Courant number to < 0.4 [48]. The tolerance to satisfy the convergence criterion is \( 10^{-14} \) for solving \( U, p, T, \) and \( \alpha \). All simulations in this study are set to be 2D axisymmetric to reduce the computing cost. The computational zone and grid strategies are shown in Fig. 1 for the single bubble and Fig. 6 for the bubble pairs.

The following parameters are used in this paper: density, \( \rho_l = 998.2 \text{ kg/m}^3 \) and \( \rho_g = 1.0 \text{ kg/m}^3 \); dynamic viscosity, \( \mu_l = 9.982 \times 10^{-4} \text{ Pa·s} \) and \( \mu_g = 1.589 \times 10^{-5} \text{ Pa·s} \); the constant
pressure heat capacity, $C_{pl} = 4220 \text{ J/(kg·K)}$ and $C_{pg} = 1000 \text{ J/(kg·K)}$; the thermal conductivity, $\lambda_l = 0.677 \text{ W/(m·K)}$ and $\lambda_g = 0.026 \text{ W/(m·K)}$; surface tension between the two phases, $\sigma = 0.07 \text{ N/m}$.

3. Model validation

3.1. Validation via the single laser-generated cavitation bubble experiment

To validate the proposed numerical model, the comparison was conducted first using the single bubble experiment results. As shown in Fig. 1, the computational domain is implemented by ICEM (ANSYS), with the local refinement for the cavitation bubble (green area). The size of the blue area is $27 \mu\text{m} \times 27 \mu\text{m}$. The uniform grids are used here with the minimum length of $0.5 \mu\text{m}$ for the cell edges. The green area is meshed with radial grids with the sizes ranging from $30 \mu\text{m} \times 30 \mu\text{m}$ to $2 \text{mm} \times 2 \text{mm}$. The red and gray areas are also meshed with the radial grids, corresponding to the transition region and the surrounding water domain, respectively. The total amount of the grids is 200,880. The no-slip boundary condition (BC) is applied to the far-field boundary. The gravity is ignored in this case to guarantee the spherical shape of the bubble. The initial conditions in terms of pressure, temperature, and radius are 10 Pa, 293.15 K, and 747 $\mu\text{m}$. The ambient pressure and temperature for water are 101,325 Pa and 293.15 K.

Fig. 2 shows the comparison of the bubble equivalent radius between the simulation results and the experimental measurements. As seen, the simulation results agree well with the experimental data in Kröninger et al. [21,49], including the first and second collapse/rebound. At the final stage of the first bubble collapse, an upper bound for the minimum radius, $R_{\text{min}} \leq 12 \mu\text{m}$, was observed in experiments due to the limited optical resolution and exposure time of the camera [21]. In this study, the simulated minimum radius is about 3 $\mu\text{m}$.

Fig. 3 presents the time-dependent profiles of bubble radius, temperature, and pressure in the bubble center, as well as the bubble wall velocity during collapse, which is driven by the pressure difference between bubble and the surrounding liquid. Due to gas compressibility, the central pressure and wall velocity of the bubble increase before collapsing. However, the temperature changes subtly. The initial potential energy $E_0 (E_0 = \Delta p V_0$, where $\Delta p$ is the pressure difference and $V_0$ is the initial bubble volume) is converted to the liquid kinetic energy since the liquid auto-fills the space created by the collapsed bubble. Besides, the bubble internal energy increases as its radius decreases. When the bubble shrinks to the minimum volume, both pressure and temperature at the bubble center become extremely high. Also, the wall speed reaches a very high value. As can be seen in Fig. 3, the maximum temperature and pressure during the bubble first collapse are 22,190.5 K and 13.3 GPa, respectively.

Through adjusting the initial gas pressure to make the Keller-Miksis model [50] comparable with the numerically-observed radial evolution of bubble, Supponen et al. [51] found that the peak pressure could reach 12 GPa. Beig at al. [24] reported that the temperature around 40,000 K could be achieved during the spherical bubble collapse. The maximum velocity of bubble wall can reach 2060.4 m/s. Such high pressure and velocity indicate that the gas compressibility is of great importance to cavitation bubble behaviors. As the first collapse finishes, the high pressure inside the bubble is released in the form of shock waves [24], which propagate outwards with a circular format (Fig. 4(c3)), leading to the bubble energy loss. Thus, the bubble maximum radius and collapse time reduce significantly from the first collapse to the second one, corresponding to the bubble internal pressure and temperature of 6.3 MPa and 887.6 K, respectively.

In Fig. 4, the detailed descriptions of the bubble, temperature, pressure, and velocity before and after the first collapse are demonstrated. Before the first collapse ($t = 67.0 \mu\text{s}$), a high-pressure region appears around the bubble (frame a3). The velocity of the bubble wall and the internal temperature are very high, and the liquid surrounded the bubble moves radially towards the bubble (frame a4). At $t = 67.86 \mu\text{s}$, the bubble collapses to the minimum volume (frame b1) with a radius around 3.0 $\mu\text{m}$. Such a tiny volume is very difficult to capture in experiments. After the first collapse ($t = 69.0 \mu\text{s}$), the shock wave emitted and propagated radially (frame c3) with the intensity decreasing over time.

![Fig. 9. The maximum temperature inside the bubble corresponding to different $T_0$.](image-url)
3.2. Validation via the laser-induced bubble pairs experiment

In this section, the experimental results of the laser-induced bubble pairs from Han et al. [52] are selected as the reference for model validation. Several parameters dominating bubble behaviors and jet formation of bubble pairs are shown in Fig. 5. To exclude the influences of the initial bubble radius and pressure on the dynamic properties [53], the normalized variables are defined below:

The relative bubble size, $\xi$:

$$\xi = \frac{R_{\text{max}2}}{R_{\text{max}1}}$$

(13)

Fig. 10. Pressure and velocity fields for bubble pairs with $\gamma = 2.5$, $\zeta = 1.0$. (a) $t = 40$ μs, (b) $t = 107$ μs, (c) $t = 210$ μs, (d) $t = 225$ μs, (e) $t = 229$ μs, and (f) $t = 260$ μs.
The relative initiation bubble distance, $\gamma$:  

$$\gamma = \frac{d}{(R_{\max 1} + R_{\max 2})} \tag{14}$$

As shown in Fig. 6, the computational domain (50 mm $\times$ 100 mm) is meshed with mapped-type structured grids with the local refinement of 0–2 mm and 45–55 mm in $x$ and $y$ directions, respectively. The mesh independence is checked using six different grid numbers. According to Ref. [54], the mesh size is characterized by the numbers of cells per initial bubble diameter (CPBD). Thus, six CPBDs, i.e. 14 (20 μm $\times$ 20 μm), 28 (10 μm $\times$ 10 μm), 56 (5 μm $\times$ 5 μm), 70 (4 μm $\times$ 4 μm), 85 (3.3 μm $\times$ 3.3 μm) and 93 (3 μm $\times$ 3 μm) are used according to Ref. [54], the mesh size is characterized by the numbers of cells per initial bubble diameter (CPBD). Thus, six CPBDs, i.e. 14 (20 μm $\times$ 20 μm), 28 (10 μm $\times$ 10 μm), 56 (5 μm $\times$ 5 μm), 70 (4 μm $\times$ 4 μm), 85 (3.3 μm $\times$ 3.3 μm) and 93 (3 μm $\times$ 3 μm) are used to perform the mesh independence check in this study.

The convergence of minimum radius ($R_{\min}$) for different CPBDs is shown in Fig. 7. As seen, the maximum deviation of $R_{\min}$ is < 0.1% when CPBD is equal to 85, which is employed in this paper to conduct following numerical simulations. The cell edge length in the refinement area around the bubble pairs (the red area in Fig. 6(a)) is about 3.3 μm. The entire computational region comprises 2,232,006 cells. The no-slip wall boundary condition is used for the right side and the bottom of the region. The total pressure boundary condition is placed on the top of the domain. The gravity is considered as an external force in this case.

The initial pressure and radius inside the bubble are determined based on the initial conditions in Ref [52]. Therefore, $p_0 = 1.0 \times 10^7$ Pa and $R_0 = 0.14$ mm are specified in the bubble pairs. Since the thermodynamic effect was not accounted for in Ref [52], an initial temperature $T_0 = 593.15$ K inside the bubble is predetermined in this paper. The initial pressure and temperature of water are set to be 101,325 Pa and 293.15 K, respectively. The simulation results are compared with Ref [52] in Fig. 8. The corresponding pressure and velocity fields, as well as the temperature variation inside the bubble, are illustrated in Figs. 10 and 11.

As can be seen in Fig. 8, each bubble slowly grows with identical spherical shapes during expansion. After reaching the maximum volume, the bubble pairs start to collapse. Each bubble develops a liquid jet towards the other. Finally, the jets penetrate the bubble pairs and move to the symmetry plane. During the bubble rebound, the protrusion generated by the liquid jet in experiments was noticeable, which can be well represented by the simulation results in this study. However, the simulated bubble shape is slightly different from the visual observations since the experimental bubble pairs were not identically the same, resulting in the asynchronous jet formations. Overall, the simulated shape evolutions of bubble pairs agree well with the corresponding experimental observations.

Fig. 9 demonstrates the maximum temperature inside the bubble versus time corresponding to different values of $T_0$. As $T_0$ increases, no distinguished difference is observed except the initial expansion, since it takes longer to achieve the equilibrium status for higher $T_0$. In addition, the maximum temperature during the collapse is almost unaffected by $T_0$. Therefore, the initial bubble temperature $T_0 = 593.15$ K is used in our following simulations.

4. Result and discussion

4.1. Analysis of the variation of the maximum temperature inside the bubble pairs at different relative distances

In this section, the dynamical behaviors of bubble pairs and their thermodynamic effects at different relative distances $\gamma$ are analyzed and discussed. Fig. 10 shows the pressure and velocity fields at $\gamma = 2.5$, where the solid black circle represents the gas–liquid interface (iso-contour $\alpha = 0.5$). As shown in Fig. 10(a), the spherical bubble pairs expand outward due to high internal pressure. Fig. 10(b) presents that the bubble pairs reach the maximum volume with the internal pressure lower than that of the surrounding liquid. During the collapse of bubble pairs, the high-pressure regions emerge over the distal sides of the bubble pairs (Fig. 10(c)), driving the high-speed liquid jets into the bubble pairs (Fig. 10(d)). Fig. 10(e) shows the minimum volume of the bubble pairs during collapse. At this moment, the high-speed liquid jets have not pierced through the bubble pairs. Then, the bubbles start to rebound due to high internal pressure, during which the liquid jets pierced the bubbles with the direction towards the center of the bubble pairs as shown in Fig. 10(f).

The maximum temperature inside the bubble pairs and the equivalent radius of single bubble versus time are given in Fig. 11, which also shows the temperature inside the bubbles at typical collapse stages in Fig. 10. At the initial expansion stage, heat transfer between bubble pairs and the surrounding liquid occurs due to temperature difference. Then, the temperature becomes stable as shown by points (a) and (b) in Fig. 11, corresponding to frames (a) and (b) in Fig. 10. The maximum temperature inside the bubble pairs increases during collapse (points (c) and (d) in Fig. 11). Eventually, it reaches around 1000 K when the bubbles collapse to the minimum volume as depicted by point (e). Compared with the collapse of a single bubble in a free

![Fig. 11.](image-url)
field, the maximum temperature inside the bubble pairs is much smaller. The existence of multiple bubbles breaks the collapse symmetry, casting a retardation effect on the collapse of the other bubble. Thus, the bubble internal energy is reduced while the kinetic energy of surrounding liquid increases evidently, featured by the strong liquid jet formation. Moreover, the retardation effect is also noticeable by a larger minimum volume, e.g., $R_{\text{min}}$ is 3 μm and 27 μm for the collapse of single bubble and bubble pairs, respectively. During the bubble pairs rebound and expansion again, the temperature inside the bubbles gradually decreases as depicted by point (f) in Fig. 11.

When $\gamma$ reduces, the dynamic behaviors of one bubble is more influenced by the other. The pressure and velocity fields with $\gamma = 0.9$ are shown in Fig. 12. The bubbles expand outward (Fig. 12(a)) and reach the maximum volume. The liquid film between bubbles gets thinned and squeezed inducing a larger escaping velocity (Fig. 12(b)). During the collapse of bubble pairs, the distal sides of the bubble pairs become more curved with the high-pressure regions emerging as can be seen in Fig. 12(c). Fig. 12(d) shows the liquid jets impact on the bubble pairs when the liquid film still exists. Then, the jets pierced the bubble pairs with two toroidal bubbles generated in Fig. 12(e). Fig. 12(f) presents

![Fig. 12. Pressure and velocity fields for bubble pairs with $\gamma = 0.9$, $\zeta = 1.0$, (a) $t = 40 \mu s$, (b) $t = 118 \mu s$, (c) $t = 220 \mu s$, (d) $t = 240 \mu s$, (e) $t = 242 \mu s$, (f) $t = 260 \mu s$, (g) $t = 262 \mu s$, (h) $t = 264 \mu s$, and (i) $t = 290 \mu s$.](image-url)
that two toroidal bubbles continue falling apart and breaking up into ones of different sizes. Then, the tiny bubbles collapse to the minimum volume in Fig. 12(g). Fig. 12(h) shows that the bubbles start to rebound and coalesce into a new larger toroidal bubble in Fig. 12(i). Unlike the previous simulation case, the bubbles collapse to the minimum volume after the liquid jets piercing the bubble pairs. This phenomenon was also investigated by Supponen et al. [55] when studying the single bubble dynamics near different boundaries.

Fig. 13 shows the maximum temperature inside the bubble pairs and the equivalent radius for single bubble at $\gamma = 0.9$. The temperature

![Graph showing radius and temperature versus time](image)

**Fig. 13.** The equivalent radius (black solid line) of single bubble and the maximum temperature inside the bubbles (red solid line) versus time at $\gamma = 0.9$.

![Pressure and velocity fields for bubble pairs](image)

**Fig. 14.** Pressure and velocity fields for bubble pairs with $\gamma = 0.4, \zeta = 1.0$, (a) $t = 40 \mu s$, (b) $t = 125 \mu s$, (c) $t = 230 \mu s$, (d) $t = 240 \mu s$, (e) $t = 250 \mu s$, and (f) $t = 271 \mu s$. 
inside the bubbles at typical stages in Fig. 12 is also denoted. During bubble expansion, the bubble temperature is stable, as shown by points (a) and (b) in Fig. 13. When bubble pairs collapse, the maximum temperature gradually increases (points (c)–(g) in Fig. 13). Eventually, the maximum temperature of 830 K (point (h)) is reached, which is inconsistent with point (g) when the bubble pairs collapse to the minimum volume. This is because only a portion of small bubbles achieve the minimum volume when they collapse together. Thus, some bubbles continue collapsing while the others begin to rebound. The temperature of small bubbles increases further until the minimum volume is reached. Then, the temperature decreases as the bubbles rebound.

When $\gamma$ further reduces to 0.4, the dynamic behaviors of bubble pairs are different from those discussed above. The bubble pairs get closer during expansion and the liquid film is much thinner, as shown in Fig. 14(a). Bremond et al. [56] explained that the film thinning was dominated by the inertia force and the film rupture would lead to the bubble coalescence. Fig. 14(b) shows the coalescence of two bubbles with a larger bubble generated. An obvious horizontal velocity is also observed. During the collapse of bubbles, the high-pressure regions appear near the upper and lower sides of the coalesced bubble and the collapse velocity is relatively high as shown in Fig. 14(c). Fig. 14(d) presents two liquid jets driven by high-pressure regions impact on the bubble. The jets pierce the bubble and turn it into a toroidal shape (Fig. 14(e)). Then, the bubble continues collapsing to the minimum volume (Fig. 14(f)).

Fig. 15 shows the maximum temperature inside the bubble pairs and the equivalent radius for single bubble at $\gamma = 0.4$. It also depicts the temperature inside the bubbles at the typical stages in Fig. 14. As aforementioned, the maximum temperature inside the bubbles increases during the collapse of bubble pairs, which eventually achieves 900 K when the bubbles collapse to the minimum volume as shown by point (f) in Fig. 15.

Fig. 16. Maximum temperature inside the bubble pairs (red dot), the temperature at the minimum radius (black cross in the black circle), and the minimum radius (black square) of a single bubble under different relative distances ($\gamma$).
4.2. Comparison of the bubble pairs’ dynamics with a single bubble collapsing near a rigid boundary

The effect of $\gamma$ on the maximum temperature inside the bubble pairs during collapse is shown in Fig. 16. Three distinct stages, namely stages I, II and III, can be obtained. For stage I ($\gamma < 0.6$), the liquid film between bubbles ruptures during bubble expansion. Two bubbles coalesce to form a larger one, generating two liquid jets in the opposite directions. After the jets piercing the bubble, it turns into a toroidal shape and continues collapsing to the minimum volume, at which the temperature inside the bubble also reaches the maximum value. In addition, the maximum temperature increases as $\gamma$ decreases, which results in two bubbles coalescing earlier and the retardation effect weakened. The coalesced bubble collapses more fiercely and the energy focusing inside the bubble is greater, corresponding to a smaller minimum radius. Thus, the maximum temperature inside the bubbles

![Fig. 17. The maximum temperature inside the single bubble near a rigid boundary (red dot), the temperature at the minimum radius (black cross in the black circle) and the bubble minimum radius (black square) under different relative distances ($\gamma_1 = L_1/R_{\text{max}}$, where $L_1$ is the distance from the center of the initial bubble to the solid wall and $R_{\text{max}}$ is the maximum bubble radius.). Initial parameters are $p_0 = 1.0\times10^7$ Pa, $T_0 = 593.15$ K, and $R_0 = 0.14$ mm.]

![Fig. 18. The typical bubble cases of $\gamma_1 = 0.3$, $\gamma_1 = 0.8$, and $\gamma_1 = 1.5$.]

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during the collapse of bubble pairs is higher.

For stage II (0.6 ≤ γ ≤ 0.9), the bubbles become toroidal and break into many smaller ones of different sizes once two liquid jets in the opposite directions interact. The occurrence of maximum temperature inside the bubbles is inconsistent with that the small bubbles collapse to the minimum volume.

For stage III (γ > 0.9), the coalescence does not occur when the bubble pairs collapse to the minimum volume, meaning the bubble behavior resembles that of a single bubble adjacent to a solid surface. The bubble pairs coalesce during the rebound stage. The occurrence of maximum temperature inside the bubble coincides with that the minimum radius is achieved. Furthermore, the maximum temperature inside the bubbles increases as γ increases, since the retardation effect weakens and the energy focusing inside the bubble is stronger, which is also noticeable by a smaller minimum radius.

To further investigate the dynamics similarities between bubble pairs and a single bubble collapsing near the rigid wall quantitatively, the γ1 effects on the maximum temperature inside a single bubble are shown in Fig. 17. Same as the thermodynamic characteristics of bubble pairs, three distinct stages are obtained. For stage I (γ1 < 0.6), a

![Diagram](https://example.com/diagram1.png)

**Fig. 19.** (a) Maximum temperature inside the bubble pairs (red dot) and the minimum radius (black square) of a single bubble under different ambient pressure (p_∞). (b) Radius change histories of bubble collapse of different p_. Initial parameter γ = 3.
typical simulation case is shown in Fig. 18(a1–a3). The bubble bottom clings to the rigid wall during its collapse. It is very similar to the coalescence of bubble pairs if the solid wall is regarded as the symmetry plane. The occurrence of maximum temperature inside the bubble coincides with that the minimum radius is achieved. In addition, the decrease of $\gamma_1$ increases the maximum temperature inside the bubble.

For stage II ($0.6 \leq \gamma_1 \leq 0.9$), another typical simulation case is given in Fig. 18(b1–b3). As seen, a thin liquid layer exists between the bubble bottom and the solid wall, resulting in an outward radial flow along the solid boundary. Thus, the bubble breaks into smaller ones due to the strong annular splashing. Due to small bubbles of different sizes collapsing, the occurrence of maximum temperature inside the bubble is inconsistent with that the small bubbles collapse together to the minimum volume.

For stage III ($\gamma_1 > 0.9$) in Fig. 18(c1–c3), the occurrence of maximum temperature inside the bubbles coincides with that the minimum radius is reached. As $\gamma_1$ increases, the maximum temperature increases. Besides, the maximum temperature of the single bubble collapsing near a rigid wall is higher than that of the bubble pairs, since the single bubble could collapse to a smaller radius, as shown in Figs. 16 and 17.

4.3. Analysis of the varying maximum temperature inside the bubble pairs at different ambient pressure

To further discuss the simulated temperature within bubble pairs, the effects of ambient pressure $p_\infty$ on the maximum collapse temperature are shown in Fig. 19(a). Five different $p_\infty$ is chosen and the corresponding maximum temperatures within the bubble pairs are obtained. As seen, the maximum temperature inside the bubbles increases as $p_\infty$ increases, since the faster shrinkage of the bubbles for greater ambient pressure corresponds to a smaller minimum radius. Meanwhile, as $p_\infty$ increases, the collapse time also decreases, as shown in Fig. 19(b).

5. Conclusion

In this study, a compressible two-phase model accounting for the thermal effects is developed in OpenFOAM to investigate the thermodynamic effect during the collapses of bubble pairs and a single bubble near a rigid boundary. Validations are performed by comparing the simulation results with the data collected from the single bubble and the bubble pairs’ experiments. Through parametric studies, the effects of relative distances $\gamma$ and ambient pressure $p_\infty$ on the thermodynamic behaviors of bubbles are investigated and discussed. The conclusions can be drawn as follows:

Appendix A

In this study, the heat transfer across the interface of two phases is included in the numerical model by incorporating the equation of state. The perfect fluid equation of state [30] is used in official OpenFOAM as below:

$$
\rho_i = \frac{1}{R_l T} \rho_0 + \rho_i
$$

(A-1)

where $\rho_0$ is the initial density of water, $\gamma = 1/R_l T$ is the compressibility coefficient with $R_l = 3000 \, \text{J/(kg·K)}$ [57].

Fig. A1 shows the comparison of the bubble equivalent radius and the maximum temperature inside the bubbles from the official OpenFOAM and the present work. As seen, the rebound bubble’s maximum radius predicted by the official OpenFOAM is almost the same as the initial bubble’s maximum radius, which is unreasonable since the bubble radius should decrease due to the energy loss during the first collapse (energy dissipation through shock wave emission, jet, and heat transfer [58]). The perfect fluid equation of state is not suitable for investigating the heat transfer between bubble pairs. Therefore, the official OpenFOAM is modified by incorporating Tammann equation of state for the liquid phase in this paper.

When the bubble reaches its maximum radius, the velocity of the surrounding liquid is small enough to be ignored with the kinetic energy equal to zero. The temperature difference between bubble and liquid is stable. Thus, the initial bubble energy can be computed from the potential energy at its maximum size, i.e., $E_0 = 4\pi/3R_{\text{max}}^3 \Delta p$, which means the reduction of maximum bubble radius corresponds to the energy dissipation. As shown in Fig. A1, the maximum radius in the second cycle is around 80% of the first one, indicating that the dissipated energy occupies 50% of the total energy in the system.

1. For a single bubble collapsing in a free field, the bubble can shrink to a small volume and the temperature inside the bubble can reach an extremely high value since it is not affected by external interference.

2. For the bubble pairs, the presence of an additional bubble breaks the collapse symmetry, generating the retardation effect on the other one during collapse. Thus, the energy focusing inside the bubbles is reduced. The quantitative analyses of the $\gamma$ effect on the maximum temperature inside collapsing bubbles is performed with three distinct stages identified. For stage I, the maximum temperature inside the bubble increases as $\gamma$ decreases. For stages II and III, the increase of $\gamma$ results in the maximum temperature increasing as well. Besides, as $p_\infty$ increases, the maximum temperature increases, and the bubble collapse time decreases.

3. For a single bubble collapsing near a rigid boundary, the thermodynamic characteristics at different relative distances are similar to that of the bubble pairs, but the maximum temperature is higher due to it can collapse to a much smaller volume.

CRediT authorship contribution statement

Jianyong Yin: Conceptualization, Methodology, Software, Data curation, Validation, Writing - original draft. Yongxue Zhang: Conceptualization, Funding acquisition, Project administration, Supervision, Writing - review & editing. Jianjun Zhu: Data curation, Validation, Funding acquisition, Project administration, Writing - review & editing. Yuning Zhang: Data curation, Validation, Resources, Supervision, Writing - review & editing. Shida Li: Investigation, Validation, Resources, Visualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Fig. A1. Results of the single bubble equivalent radius (black line) and the maximum temperature inside the bubbles (red line) calculated from the official OpenFOAM (dot line) and the present model (solid wall). The initial radius and pressure are 0.14 mm and 1.0 $\times e^2$ Pa, $\gamma = 2.5$; $R_{\text{max}}$ is the maximum bubble radius; $t_{\text{exp}}$ is the expansion time of the bubble from the inception to the maximum radius.

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