Measuring mathematical literacy of space and shape content as 21st century competency

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Abstract. Mathematical literacy and self-regulated learning is one of the most important cognitive and affective skills to be developed in the class allowing students to participate in the 21st century. This research is qualitatively descriptive that aims to describe students' mathematical literacy of space and shape content reviewed from self-regulated learning. The subject was VII G students of SMP Negeri 1 Purwokerto. The data was collecting by using questionnaire, test, and interview. Students were categorized into high, medium, and low self-regulated learning high. The results showed that high self-regulated learning students were capable to construct a concept based on context. The medium and low self-regulated learning students were not capable to fulfill any indicators.

1. Introduction
Mathematics is an organised, assertive, logical, abstract, and universal mind set to eliminate uncertainty and obscurity [1]. Technological development has changed the concept of mathematics education in terms of policies and practices. Mathematics instructs students to think imaginatively, intuitively, and logically to discover solutions and solve problems [2]. Mathematics helps to understand real world by transforming into a mathematical language [3]. In this case, mathematical literacy plays an important role to understand the real world.

Literacy comes from the English word 'literacy' and the Latin 'littera' which means mastery of writing and language systems [4]. Literacy is defined as ability to read and write from various contexts [5]. However, the meaning of literacy is not limited to those abilities. It requires cognitive skills, language skills and knowledge towards various reading sources [6]. Literacy is a process of identifying, understanding, interpreting, creating, communicating and calculating, using printed and written media in diverse contexts [7]. Literacy relates to a series of learning which simplifies individuals to achieve their goals, develop their knowledge and actively participate.

The essence of literacy is about constructing, understanding, combining what has been understood with new one and expressing it [8]. By combining the meaning of literacy and mathematics, it comes with the term 'mathematical literacy'. It means individual ability to formulate, apply and interpret mathematically [9]. The definition shows that mathematical literacy helps students to understand their roles in real world as a construction and decision-maker [10].

Mathematical literacy is important as one of the mathematical abilities. It is very required for understanding the role of mathematics in real world. Developing mathematical literacy allows students being capable to participate and contribute in the 21st century, which is full of numerations, arguments, and data [11]. It is not only helpful in understanding the content of mathematics concepts,
but also applying knowledge in various situations [12]. Formulating, applying, and evaluating activities of mathematical literacy process can train students' mathematical mind set in facing situations [13].

In the study of PISA 2012, mathematical literacy is defined as an individual's ability to formulate, apply, and interpret mathematical concepts from various contexts [14]. It is related to reasoning, understanding concepts, describing and predicting a situation. In addition, mathematical literacy can be interpreted as knowledge, methods, and processes towards various mathematical problems with reflective insight and steps [15]. In learning process, it means anything as the process of communicating, discussing, and interpreting [16]. Based on those definitions, it can be concluded that mathematical literacy is the ability to formulate problems, apply mathematical procedures, and interpret solutions based on content and context of problems.

Developing mathematical literacy is important thing. Mathematical literacy is not only helping in understanding the content of mathematical concepts, but also applying knowledge in new situations [12]. It is closely related to seven basic mathematical abilities, i.e. communication ability, understanding ability, representation ability, reasoning ability, planning strategies, applying procedures, and mathematical media [10]. Learning the relation of mathematical literacy and the seven basic mathematical abilities, aims to understand real world into mathematics language. Through the mastery of mathematical literacy, students are able to determine and construct decisions critically and logically in their role in real world [17].

The PISA study models three components of mathematical literacy, i.e. content, process, and context components [10]. Content component relates to the understanding of materials for discovering solutions of problems. The content consists of space and shape, change and relationship, uncertainty and data, and quantity. The content of space and shape encompasses a phenomena which is encountered everywhere in our world i.e. patterns, characteristics of objects, representations of objects, and relationship between real shape and its representation. Geometry is the material related to the content of space and shape in mathematical literacy ability [18].

There are several major factors affecting students' mathematical ability, i.e. self-efficacy, self-regulation, learning style, and academic background [19]. Some studies have stated that students need to have self-regulated learning, i.e. organizing their own learning activities, thinking and processing mathematically [20, 21]. Self-regulated learning is an important component supporting mathematical ability, i.e. mathematical literacy, thus it can create more effective and productive mathematics learning [22]. Good self-regulation students can process mathematical literacy well, i.e. analyzing problems, determining strategies, and evaluating [23]. It can help students to achieve their objectives and understand their abilities, strategies and motivations.

Self-regulated learning (SRL) means as activities to acquire knowledge and skill independently [24]. It is a process which students organize their own studies, such as planning, controlling, and evaluating, to achieve their learning objectives cognitively, motivationally, and behaviorally [25]. In addition, SRL has two distinct meanings, i.e. individual and social meaning [26]. As the individual meaning, SRL is an encouragement and students activities of managing their studies. As the social meaning, SRL is a learning strategy to achieve optimal learning objectives. In those cases, students determine their objectives and strategies independently, evaluate, and compare effectiveness of the strategies, thus lessons can be modified and optimized well. Based on those definitions, it can be concluded that self-regulated learning is the ability and encouragement of students in planning, organizing, and evaluating their own learning activities.

Self-regulated learning is an important affective aspect to be developed. SRL can predict the success of students' academic process [27]. It assists students in preparing long-term learning plan (knowledge and skills). Successful students are those who have self-regulating, analyze their needs, discover, and organize their learning strategies to achieve their success [28]. The SRL process of managing and controlling cognition, motivation, and behavior is directed and encouraged by the purpose of prioritizing environmental context [29]. Students who have good self-regulation, are able to define problems, correct mistakes, and evaluate their planning if some obstacles occur [30].
Developing cognitive and affective aspects is important in the learning process. One of the affective aspects supporting mathematical literacy is self-regulated learning. Self-regulated learning (SRL) is not limited to have meaning as self-study, but it means as managing and being responsible towards learning independently [26]. SRL helps to perform learning activities encouraged by motivation or specific skills to solve the problems [30]. This article will describe students' mathematical literacy of space and shapes content reviewed from self-regulated learning.

2. Method
This research is a qualitative which aims to describe the mathematical literacy of space and shape content reviewed by self-regulated learning. The subjects were students of class VII G in SMP Negeri 1 Purwokerto. Data was collecting by questionnaire, test, and interview. The questionnaire contains 25 statements in Likert Scale to classify self-regulated learning categories. The test contains 3 essay items of space and shape contents to know students' mathematical literacy ability. The interview is used to obtain deeper information from students. The subjects were categorized into three groups of self-regulated learning and chosen three students each category by using purposive sampling.

Data was analyzed by descriptive analysis using three stages i.e. data reduction, data display and drawing conclusion. The data analysis is described by elaborating and triangulating mathematical literacy result documents and interview based on mathematical literacy indicators. The indicators are constructing a concept based on context, generalizing, and create a representation picture.

3. Results and Discussion
The results of SRL questionnaire of VII G students showed that there were 6 students of low SRL category, 20 medium SRL category, and 3 high SRL category. Based on the subject selection criteria using purposive sampling, it was selected three students from each SRL category to be the subject of research.

3.1. Constructing a concept based on context
The first indicator aims to determine mathematical literacy in constructing a concept based on context of square and rhombus characteristics. Concept is an abstract intellectual property against a particular situation or event. Concept can be formed from analysis process of some concepts which have existed before [31].

Figure 1. Sample of High SRL category number 1

English Translation:
a. No, it is not because square has four angles and diagonals in equal size. Whereas, rhombus has equal size facing angles and unequal diagonals.

b. No, it is not. Rhombus and square are different. Every square’s angle is 90°, while rhombus’ is not.
Based on Figure 1, high SRL student knew differences characteristics of square and rhombus. The facing angles of rhombus have same size and each square angle is 90°. In other words, the angle size of those two quadrilaterals is different. Therefore, students concluded that square was not rhombus and vice versa.

**Figure 2.** Sample of Medium SRL category number 1

It can be seen on Figure 2, medium SRL student explained that square had a wide meaning and another quadrilateral became a part of it. It means square encompasses rectangle, parallelogram, trapezoid, rhombus, and kite. Therefore, the student concluded that square was not a rhombus, but rhombus was square. In this case, student was mistaken in understanding the concept of square, it caused student being mistaken in constructing a concept based on those contexts.

**Figure 3.** Sample of Low SRL category number 1

English Translation:

a. Square is rhombus, if it is rotated 45°.

b. Rhombus is square

Low SRL student explained the reason for answering both context issues based on visual reasons on Figure 3. If square is rotated, it turns into rhombus and vice versa. Students did not give reason based on characteristics of square and rhombus. Trouble in understanding the concept of those characteristics, caused students being difficult to construct a concept based on context.

In the context of 'whether square is a rhombus', the characteristics of square needs to be seen from the characteristics rule of rhombus, whether fulfilling or not. It is apparent that the facing angles of rhombus have same size and each square angle is 90°, have no similarity. In deeper understanding, the facing angles of square have same size. It is 90° and suited with angle characteristics of rhombus. Therefore, square is rhombus. In this case, it had not been thought by the students.

3.2. Generalizing

The second indicator aims to determine mathematical literacy in generalizing patterns. Generalization is a process of identifying a common thing that has been known in a particular context, then applying it to find another certain thing [32].
Figure 4. Sample of High SRL category number 2

English Translation:

Known:

- 1st pillar = 8 m
- 2nd pillar = 14 m

Asked: 3rd pillar?

If the two remains combine, they become 1st pillar = 8 m.
Thus, the conclusion is 8 x 2 = 16 m for 3rd pillar.

Figure 4 showed that high SRL student is mistaken about the problem question. Student thought that it was height of third pillar. However, students had idea to find the composition of fifth pillar. The fifth pillar is composed of two first pillars in the middle, then added a rectangle and a trapezoid on the edges. Therefore, it is composed of six rectangles and six trapezoids. The same way in defining pattern of third pillar, student defined the composition of the fifth pillar. It showed that student could discover specific pattern to determine fifth pillar.

Figure 5. Sample of Medium SRL category number 2

Translation

Known:

- Height of 1st pillar = 8 m
- 1 + 3 + 3 + 1 = 8 m

- Height of 2nd pillar = 14 m
- 2 + 3 + 2 + 3 + 2 = 14 m

Asked: height of 2nd pillar?

(2 x 7) + (2 x 3) = 14 + 6 = 20 m
Based on Figure 5, medium SRL student explained that number of 3 was height of rectangle and number of 1 was height of trapezoid. However, student fell confused about number of 2. Those calculations were obtained by equating size and using formulas but student forget the formula. \((2\times7)\) and \((2\times3)\) is multiplications which is suited to number of 14 and 6. The number of 14 was height of second pillar and the number 6 come from height subtraction of second pillar and first pillar. In this case, student is mistaken the problem question. In addition, student was unable to give any ideas about the fifth pillar.

In low SRL category, student did not understand the problem well, thus it was not able to answer it. Student did not have any ideas how to find height of fifth pillar because student did not understand the picture. In this case, students fell difficult to understand visual information from picture. Finally, student was instructed to know there was a pattern in those pillars. By giving gradual direction, student knew that there was a pattern of adding two rectangles and two trapezoid repeatedly and continually. However, it has not been thought.

### 3.3. Create a representation picture

The third indicator aims to determine mathematical literacy in creating a representation picture. This can be interpreted as understanding mathematics problem and representing it into an image form.

![Figure 6. Sample of High SRL category number 3](image)

**English Translation:**

wooden ornament’s perimeter : not colour part’ perimeter

\[
2\times20 + 2\times26 : 2\times20 + 2\times14
\]

\[
40 + 52 : 40 + 28
\]

\[
92 : 34
\]

\[
46 : 34
\]

\[
23 : 17
\]

It can be seen on Figure 6, high SRL student created a representation picture of a wooden ornament, but it is not exact. In the picture, student did not give descriptions of cm unit and coloured part. The size of the outer parallelogram is 26×20 cm and the size of the inner parallelogram is 20×14 cm. In that case, student made mistakes in determining the sizes of wooden ornament. Although the step of calculation was correct, but the error caused the result becoming incorrect.
Figure 7. Sample of Medium SRL category number 3

Perimeter of outer parallelogram:
2(20+14) = 68 m

Perimeter of inner parallelogram:
2(17+11) = 56 m

Figure 8. Sample of Medium SRL category number 2

Based on Figure 7 and Figure 8, medium SRL student did not answer problem completely and exactly. Student was not thorough in determining the size of wooden ornament. In the picture, student did not give description of colored part. The size of the outer parallelogram is 20×14 cm and the size of the inner parallelogram is 17×11 cm. In addition, students performed calculations to determine perimeter of the outer and the inner parallelogram. However, students did not answer the question about perimeter comparison of wooden ornament and the uncoloured parts.

Figure 9. Sample of Low SRL category number 3

Figure 9 showed that low SRL student created a representation picture of wooden ornament, but it is not exact. In the picture, student just created a simple picture of a parallelogram without giving description of cm unit and coloured parts. Student explained that number of 68 was perimeter of wooden ornament and number of 56 was perimeter of uncoloured part. The perimeter of wood ornament used the sides of 14 cm and 20 cm while the perimeter of uncoloured used the sides of 17 cm and 11 cm.

In this case, students misunderstood about information of wood ornaments which was known in the problem. Students was mistaken in interpreting the meaning of 'wooden ornament edges is coloured 3
cm wide. Some students meant that the parallelogram of $20 \times 14 \text{ cm}$ was added 3 cm out of it. Therefore, students created representation picture of wood ornament with the outer parallelogram size of $26 \times 20 \text{ cm}$ and the size of the inner parallelogram of $20 \times 14 \text{ cm}$. Other students meant by subtracting 3 cm only one of the sides of the wooden ornament. Therefore, students created representation picture of wood ornament with the outer parallelogram size of $20 \times 14 \text{ cm}$ and the inner parallelogram size of $17 \times 11 \text{ cm}$. It caused the comparison results becoming incorrect.

Based on the results and discussion, self-regulated learning can support and assist students' mathematical literacy process. Good self-regulation students are able to define problems, correct mistakes, and evaluate planning [29]. It is according with the opinion that SRL can give good success in planning, organizing, and evaluating students' academic process [33].

4. Conclusion
Based on the research result and discussion, it showed that the high SRL students were capable to construct a concept based on the context of square and rhombus characteristics, not capable to generalize the pattern of pillar structures and create representation picture of wooden ornament. The medium and low SRL students were not capable to fulfill any indicators. They are not capable to construct a concept based on the context of square and rhombus characteristics, generalize the pattern of pillar structures and create representation picture of wooden ornament.

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