Electrical processes analysis in a wireless power transfer system for electric vehicle battery charging

M V Novolodskiy

1 Novosibirsk State Technical University, 20, Karl Marks Prospect, Novosibirsk, 630073, Russia
E-mail: max_nov94@mail.ru

Abstract. The article considers the electrical processes taking place in a wireless energy transfer system for charging the battery of an electric vehicle. The main way to transfer energy through the airspace between the road and the electric car is the inductive coupling of two coils. It is known that the efficiency of the system increases if electric resonance is established in two independent circuits. The choice of the resonant frequency largely depends on the resonant circuit. The work aims to analyze a series-series resonance circuit and justify the choice of a resonant frequency for a given topology. As a result of the study, the author has obtained the dependences of the active and reactive resistance on the frequency, the possible resonance conditions, and a bicubic equation that allows calculating the resonant frequencies for those two types of resonance. Moreover, a computer model has been created, the parameters of which determined the optimal resonant frequency. This research work helps to choose reasonably resonant frequencies depending on the required efficiency of the system with a series-series resonant circuit.

1. Introduction
At present wired charging stations connected to alternating current or direct current network are used for charging electric vehicles and electric buses. Depending on the power and the type of current, there are at least six basic solutions for connecting an electric vehicle to the power supply system [1]. The absence of a single standardized method for connecting an electric vehicle is, certainly, a drawback, since the equipment of an electric vehicle with many outlets complicates the system as a whole including the charge control system. The complexity of the system in its turn increases with growing transmitted power since additional cooling of conducting parts is required. Therefore, the ideal solution is the creation of wireless charging. The wireless charging system does not require bulky connectors and connectors for them which wear out over time. Obvious advantages are also the elimination of the possibility of electric shock and ease of use. All that the owner needs to do is install the electric car in a special parking place, and after that, the charging process will start automatically.

Currently, in the world, some solutions already exist both from private companies, for example, Plugless Power, WiTricity, Hevo, and from automobile concerns, for example, BMW. Wireless chargers can be located in places of long-term parking of the vehicle, namely: in public parking lots, at intersections, and at bus stops. It is also worth noting that, the electric car has placed several transmitters in succession laid under the asphalt in some sections of the road. The electric car can be charged while very convenient driving.
The transfer of energy from the power supply network to the battery of an electric vehicle is based on the phenomenon of electromagnetic induction. The main components of the system are two or more inductively coupled coils. The presence of mutual inductance allows transferring power from one coil located under the pavement or in some kind of housing to another, attached to the bottom of the electric vehicle. Hence, the first drawback can be called - this is low transmission efficiency (not more than 20-30%) due to the large air gap between the inductors. The solution to the problem is to configure the primary and secondary circuits for electrical resonance [2]. Based on analyzing the work of researchers, it was found that electrical resonance increases the transmission efficiency up to 90% and even more. A wide variety of resonant topologies provides a wide field for research since the most effective of them have not yet been established. The efficiency of energy transfer is also affected by the mutual arrangement of the inductors. Horizontal and vertical displacements change the mutual induction coefficient, and, therefore, the control system must adjust the resonant frequency to the new parameters. In other words, the control system must be adaptive, so it is necessary to study the electrical processes occurring in the system in order to establish the dependence of the transmission efficiency on the resonant frequency and the mutual induction coefficient. In the present work, its author gives a theoretical description of a series-series resonant circuit.

The study aims to analyze a series-series resonant circuit and to establish the dependence of the transmission efficiency on the resonant frequency.

2. Theoretical Basis
Let us consider the main components of a wireless power transfer circuit shown in Figure 1. The system consists of a high-frequency inverter, primary and secondary coil-systems, a bridge rectifier, a filter and a load.

![Figure 1. The electrical circuit of wireless charging system](image)

The output voltage at the contact terminals of the inverter is of periodical and non-sinusoidal and can be expanded in a Fourier series [3]:

$$u_1(t) = \frac{4U_m}{\pi} \sum_{k=1}^{\infty} \frac{\sin k\omega t}{k},$$  \hspace{1cm} (1)

where $U_m$ is the maximum value of the voltage, V; $\omega$ is an angular frequency, s$^{-1}$; $k$ is the number of a harmonic. The number $k$ takes only odd values for the rectangular voltage.

Let us replace the diagram by the equivalent circuit. The instantaneous current and voltage will be replaced by complex values, as well as the bridge rectifier, the filter and the load are replaced by an equivalent resistance based on the power equality:

$$R_{eq} = 0.811R_L,$$  \hspace{1cm} (2)

where $R_L$ is an equivalent resistance of the load.

In addition, the active resistors of the inductors making allowance for active losses in coils are
Figure 2 illustrates the equivalent diagram. Let us compose and solve the system of equations according to the Kirchhoff’s laws with respect to the primary current \( I_{1,k} \).

\[
I_{1,k} = \frac{U_{1,k}}{Z_{\Delta,k} - \frac{Z_{m,k}}{Z_{\Delta,k} + \frac{Z_{m,k}}{2}}} = \frac{U_{1,k}}{Z_{m,k} + \frac{Z_{m,k}}{2}}.
\]

(3)

where \( Z_{in,k} \) is input impedance which can be represented as the sum of two components, namely: the input impedance of the primary side and the reflected impedance of the secondary side to the primary side.

The reflected impedance is determined at:

\[
Z_{ref,k} = r_{ref,r} + ix_{ref,k}.
\]

(4)

Based on (3) and (4) equations, the equivalent diagram can be reduced to a single-loop diagram shown in Figure 3.

The reflected active resistance constitutes a resistance in which dissipated energy under the impact of the primary current and is equal to dissipated energy in all resistances under the impact of the secondary current at the secondary side. The reflected reactance constitutes an element taking into consideration a degauss effect of the secondary side to the primary side. Both of them are determined to the following equations:

\[
r_{ref,r} = \frac{k^2\omega^2M^2r_2}{Z_{2,k}^2},
\]

(5)

\[
x_{ref,k} = -\frac{k^2\omega^2M^2x_{2,k}}{Z_{2,k}^2}.
\]

(6)

The electrical resonance can be observed in two cases [4]:

1) If the reactance of the primary side and the reflected reactance are not equal to zero, but their sum is zero, the complex resonance is established in the circuit;

2) If the reactance of both sides equals to zero, the major resonance is established in the system.

Furthermore, if the resistance of the primary side is equal to the resistance of the secondary side, the total resonance is set in the circuit. It is a special case of the major resonance.

While analyzing Figure 2, it is found that the maximum value of the secondary current, and, therefore, the output power in the load, is established only when the active resistance of the primary and secondary side are equal to each other, but not at all with a greater coupling between them.
Let us express the components of the input impedance in terms of two parameters, namely: the coupling parameter and the generalized mismatch which are determined the following equations respectively:

\[
P_{\text{coup}} = \frac{\omega M}{\sqrt{r_1 r_2}}
\]

(7)

\[
\xi = \frac{X_L - X_C}{r}
\]

(8)

Then the input resistance and input reactance are determined as:

\[
r_{\text{in},k} = r_1 + r_{\text{ref},k} = r_1 \left( 1 + \frac{p_{\text{coup},k}^2}{1 + \xi_{2,k}^2} \right)
\]

(9)

\[
x_{\text{in},k} = x_{1,k} + x_{\text{ref},k} = x_{1,k} r_1 \left( 1 - \frac{p_{\text{coup},k}^2}{1 + \xi_{2,k}^2} \right) \xi_{1,k}^2.
\]

(10)

Figures 4 and 5 show the graphs of the dependences of the input active and inductive resistances on frequency. Let us consider the variation of the input reactance: if the frequency \(\omega\) tends to zero, then the input reactance tends to zero too if the frequency \(\omega\) tends to infinity, then the input reactance tends to minus infinity. If the frequency \(\omega\) is equal to the resonant frequency \(\omega_{0,k}\), then \(x_{\text{in},k}\) equals zero.

![Figure 4. Dependence of the input resistance on the frequency](image)

![Figure 5. Dependence of the reactance on the frequency](image)
If the frequency of the primary side equals the frequency of the secondary side, that is, \( \omega_{0,1} = \omega_{0,2} \), both the complex resonance and the major resonance are possible. Otherwise, only the complex resonance is possible. Nevertheless, to define resonance frequencies, it is necessary to solve the following equation:

\[
A_k \omega^6 + B_k \omega^4 + C_k \omega^2 - 1 = 0,
\]

where \( A_k, B_k, C_k \) are coefficients which can be determined as:

\[
A_k = k^6 C_1 C_2 L_2 (L_1 L_2 - M^2),
\]
\[
B_k = K^4 \left[ C_2^2 (C_1 L_1 r_2^2 - L_2^2) + C_1 C_2 (M^2 - 2 L_1 L_2) \right],
\]
\[
C_k = k^2 (2 L_2 C_2 + C_1 L_1 - C_2^2 r_2^2).
\]

The efficiency of the inductive coupling system is defined the following equation for every harmonic:

\[
\eta_k = \frac{P_{2,k}}{P_{1,k}} = \frac{P^2_{\text{coup},k}}{1 + \frac{P^2_{\text{coup},k}}{P^2_{\text{coup},k}}}.
\]

Based on the equation (12), if resonance is established at the secondary side, the efficiency is of the maximum value:

\[
\eta_m(\omega_{0,k}) = \frac{P^2_{\text{coup},k}}{1 + P^2_{\text{coup},k}}.
\]

The frequency diagram of the efficiency for \( L_1 = L_2, C_1 = C_2 \) and \( r_1 \neq r_2 \) is shown in Figure 6. It demonstrates that efficiency takes the maximum value at the major resonance frequency.

![Figure 6. The dependence of the magnetically coupled system efficiency on the frequency](image)

The dependence of the maximum efficiency on the coupling parameter at the major resonance is shown in Figure 7. It illustrates that the efficiency of the system equals 50 per cent, and the transmitted power is of the highest value if the coupling equals to the critical coupling \( P_{\text{coup},k} = 1 \). The following increase of the coupling parameter leads to the decline of both the transferred power and the consumed power, but the efficiency rises and theoretically can attain 99 per cent. If the coupling parameter is duplicated, the efficiency achieves 80 per cent. If it is tripled, the efficiency achieves 90 per cent.
Figure 7. Dependencies of the maximum value of the efficiency of the system and the relative consumed and transmitted powers on the communication parameter in the individual resonance mode.

3. Computer Modeling

Figure 8 shows a model of magnetically coupled coils created in the COMSOL Multiphysics software environment. The parameters of the model are presented in Table 1.

![Figure 8. The model of the magnetically coupled system](image)

| Parameter                          | Symbol | Unit of measurement | Value  |
|------------------------------------|--------|---------------------|--------|
| Input voltage frequency            | \( f \) | kHz                 | 5…100  |
| Angular frequency                  | \( \omega \) | s\(^{-1}\)          | \( 2\pi f \) |
| Magnitude of input voltage         | \( U_m \) | V                   | 100    |
| Number of turns in the primary     | \( N_1 \) | -                   | 5      |
| inductance coil                    |        |                     |        |
| Number of turns in the secondary   | \( N_2 \) | -                   | 5      |
| inductance coil                    |        |                     |        |
| Inner radius                       | \( R_{in} \) | mm                 | 50     |
| Diameter of the conductor          | \( D_{cond} \) | mm                | 5      |
| Air gap                            | \( H_{coil} \) | mm                | 100    |
The selected range of frequencies requires taking into account the skin effect. The higher a current frequency is, the more pronounced the skin effect would be. It leads to an uneven distribution of the current density within a conductor. The concentration of current density near the surface of the conductor leads to the increase of its active resistance. The skin depth is determined the following equation [5]:

\[
\delta = \sqrt{\frac{2}{\sigma \mu_r \mu_0 \omega}},
\]

there\(\sigma\) is electrical conductivity, \(S/m\); \(\mu_r\) – relative magnetic permeability of the conductor; \(\mu_0\) – the permeability of free space, \(\mu_0 = 4\pi \times 10^{-7} \text{ H/m}\); \(\omega\) – angular frequency of current, \(s^{-1}\).

4. Results and Discussion
Results of the modelling illustrating dependences of the active resistance and the inductance of the spiral coil on the frequency are presented in Figure 9. The coupling coefficient \(k_{coup}\) equals 0.16.

Dependences of the relative active power of the primary side, the relative transferred active power and the highest value of the efficiency on the frequency are presented in Figure 10. The coupling parameter is calculated for a Nissan Leaf battery (0.389Ω).

![Figure 9](image.png)

**Figure 9.** Dependences of the coil resistance and inductance on the frequency

![Figure 10](image.png)

**Figure 10.** Dependences of the relative, active power of the primary side, the relative transferred active power and the highest value of the efficiency on the frequency
5. Discussion

The uneven distribution of current within the conductor occurs at high or very high frequencies results appearing not only in increasing of conductor resistance but also in the decreasing of the inductance (Fig 9). The distribution of current depends on electrical conductivity of a conductor, magnetic permeability and the current frequency, therefore, both the mutual inductance and the inductance of a conductor indirectly depend on the above variables.

It is known that inductance is the ratio of two variables, namely: self-inductance flux and the current flowing through the conductor. At a high-frequency current, the magnetic linkage can be decomposed into two components, in particular: an external stream whose lines cover the entire wire, and an internal stream whose lines are fully or partially closed inside the wire. The external flow is irrespective of the distribution of the current flowing through the conductor. Therefore, it is the same for all frequencies. The internal magnetic linkage is decreased with the growth of frequency, and it tends to zero at extra high frequencies. Hence, the total magnetic linkage leads to decreasing of the total impedance [6].

According to Figure 10, the resonance frequency should be chosen under the condition of the coupling parameter is no less than six because part of the power is dissipated in the inductors in the form of heat. The further growth of the coupling parameter does not change the efficiency of the system. According to the theoretical calculations, the efficiency of the system will be over 96 per cent, while the efficiency of the charge will be over 90 per cent. The resonance frequency has been taken 90 kHz.

6. Conclusion

The study allows the following conclusions:

1) It has been found that both the complex resonance and the major resonance may set at the magnetically coupled system. The major resonance is the preferable type of resonance from the highest efficiency.

2) It has been shown that the transferred power is the highest value for the critical coupling, but losses can get 50 per cent. If the coupling parameter is increased at two times concerning the critical coupling parameter, the efficiency of the system grows to 80 per cent, four times - to 95 per cent.

The transmitted power is reduced with the increase of the coupling parameter; for example, if the coupling parameter is 2, the transmitted power is reduced by 50 per cent compared to maximal possible value. However, the transferred power is decreased with the growth of the coupling parameter, for example, if the coupling parameter equals 2, the transferred power is decreased by 50 per cent compared to the highest possible value.

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