The fine tuning problem in pre-big-bang inflation

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We examine the effect of spatial curvature in the pre-big-bang inflationary model suggested by string theory. We study $O(\alpha')$ corrections and we show that, independently of the initial curvature, they lead to a phase of exponential inflation. The amount of inflation in this phase is long enough to solve the horizon and flatness problems if the evolution starts deeply into the weak coupling regime. There is a region of the parameter space of the model where such a long inflationary phase at the string scale is consistent with COBE anisotropies, millisecond pulsar timing and nucleosynthesis constraints. We discuss implications for the spectrum of relic gravitational waves at the frequencies of LIGO and Virgo.

String cosmology \cite{1,2,3} provides a possible implementation of the inflationary paradigm with two major advantages. First, it addresses the problem of the initial singularity and, second, the ‘inflaton’ is identified with a field, the dilaton, whose dynamic is not prescribed \textit{ad hoc}, but rather follows from a fundamental theory.

A principal motivation of any inflationary model is to get rid of the fine tuning of the initial conditions of standard cosmology. In a recent paper \cite{4} the issue of the dependence on the initial conditions in string cosmology has been reanalyzed, with the conclusion that a fine-tuning is still required to obtain enough inflation to solve the horizon/flatness problems. The analysis of ref. \cite{4} focused on the super-inflationary phase of the model (see below). In this Letter we consider the effect of spatial curvature on the ‘string’ phase of the model, and the contribution of this phase to the solution of the fine tuning problem.

The model is based on the low-energy effective action of string theory and depends on the metric $g_{\mu\nu}$ and on the dilaton field $\phi$. The effective action is given by an expansion in powers of the string constant $\alpha'$; including the first order $\alpha'$ correction it can be written (in the so-called ‘string frame’) as

$$S = -\frac{1}{2\lambda_s} \int d^4x \sqrt{-g}e^{-\phi} \left[ R + (\nabla \phi)^2 - \frac{\alpha'}{4} (R_{\text{GB}}^2 - (\nabla \phi)^4) \right], \quad (1)$$

where $\lambda_s$ is the string length, $\alpha' \sim \lambda_s^2$ is the string constant and $R_{\text{GB}}^2$ is the Gauss-Bonnet term. Our sign conventions are $\eta_{\mu\nu} = (+, -, -, -)$ and $R^\mu_{\nu\rho\sigma} = \partial_\rho \Gamma^\mu_{\nu\sigma} + \ldots$. Further terms in the action include an antisymmetric tensor field, higher order corrections in $\alpha'$, corrections $O(e^{\phi})$, a non-perturbative dilaton potential that, as $\phi \to -\infty$, vanishes as a double exponential, matter and gauge fields, and moduli fields.

Out of all these terms, $O(\alpha')$ and $O(e^{\phi})$ corrections are particularly important and play a crucial role in defining a consistent model. This is most easily understood considering the special case of a spatially flat Friedmann-Robertson-Walker (FRW) metric: in this case the evolution starts at $t \to -\infty$ at low curvature and $\phi \to -\infty$, where both $O(\alpha')$ and $O(e^{\phi})$ corrections are negligible. In this regime we can neglect the $\sim \alpha'$ term in eq. (1) and the evolution of the FRW scale factor $a(t)$ is $a(t) \sim 1/(-t)^\gamma$ with $\gamma > 0$. This corresponds to super-inflation, and the solution runs into a singularity as $t \to 0^-$. However, at a value of time $t = t_s < 0$, the curvature becomes of order $1/\lambda_s^2$ and $\alpha'$ corrections become crucial. Let us assume that at this point $e^{\phi}$ is still small, $g_s^2 = e^{\phi_s} \ll 1$. Then $O(e^{\phi})$ corrections can still be neglected. Once we include $\alpha'$ corrections the solution, rather than running into the singularity, approaches asymptotically a stage of exponential inflation with a linearly growing dilaton, $a(t) \sim \exp \{ H_s t \}$, $\phi(t) = \phi_s + c(t - t_s)$, with $H_s, c$ constants of order $1/\lambda_s$. In terms of conformal time $\eta$, this solution reads

$$a(\eta) = \frac{1}{H_s \eta}, \quad \phi(\eta) = \phi_s - 2\beta \ln \frac{\eta}{\eta_s}, \quad (2)$$

where $\eta_s = \eta(t_s)$, $\eta_s < \eta < 0$ and $2\beta = c/H_s$. This solution exists at all orders in $\alpha'$ if a set of two algebraic equations that determine the values of $H_s, c$, and that involves all orders in $\alpha'$, has real solutions \cite{5}. We will assume in the following that this is indeed the case, and we will refer to this phase as DeSitter phase, or string phase. Together with
the previous super-inflationary phase (or ‘dilaton dominated’ phase) it defines the so-called ‘pre-big-bang cosmology’. The numerical value of $\beta$ is very difficult to compute since it is sensitive to all higher order corrections in $\alpha'$. It also depends on the dimensionality of space-time, i.e., on whether the compactification of extra dimensions takes place before or after the string phase. We must therefore consider it as a free parameter of the model, although it is in principle fixed by the theory; its numerical value is relevant in the following, and it is also important when considering the phenomenological consequences of the model. In fact pre-big-bang inflation predicts a relic graviton spectrum which is particularly interesting, for the detection in planned gravitational wave experiments, if $|2\beta - 3|$ is very close to 3, i.e., for $\beta \approx 0$ or $\beta \approx 3$. This gravitational wave spectrum also puts important constraints on the model, and we will discuss them in some detail below.

Another very important parameter of the model is the value $\phi(t_a) = \phi_s$ of the dilaton at the transition between the super-inflationary and the DeSitter phases. Note that $g^2 = e^\phi$ plays the role of the gauge coupling ‘constant’ (which actually is only constant in the present era, when the dilaton is frozen at a minimum of the nonperturbative potential). As we have already remarked, the solution (2) is valid provided $g_s^2 < 1$. The string phase is expected to end and to match with the standard radiation dominated era (‘graceful exit’) when $e^\phi \sim O(1)$, although the detailed mechanism is not yet well understood.

The condition $g_s^2 \ll 1$ ensures the existence of a long string phase, whose consequences we will explore below. We also note that a long intermediate string phase might also play an essential role in the generation of density perturbations necessary for the formation of large scale structures, and that the assumption that the Universe starts deeply within the weak coupling region is also necessary in order to relax the hypothesis of homogeneity of the initial conditions.

Let us now turn to the generic case of a spatially closed ($k = 1$) or a spatially open ($k = -1$) Universe. The equations of motion for homogeneous fields derived from the action (1) with a variation with respect to the scale factor $a(t)$ and with respect to the dilaton field read

$$\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} + \frac{\dot{\phi}^2}{2} - \dot{\phi} - 2\ddot{a} \dot{\phi} - \alpha' \left[ \frac{2\ddot{\phi}}{a} + \frac{1}{8} \phi^4 + \left( \ddot{\phi} - \ddot{\phi}^2 \right) \left( \frac{\ddot{a}^2}{a^2} + \frac{k}{a^2} \right) \right] = 0$$

$$-6 \left( \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right) - \phi^2 + 2\ddot{\phi} + 6\frac{\dot{a}^2}{a} \dot{\phi} + 3\alpha' \left[ -2\frac{\ddot{\phi}}{a} \left( \frac{\ddot{a}^2}{a^2} + \frac{k}{a^2} \right) - \frac{1}{4} \phi^4 + \phi' \frac{\dot{\phi}}{a} + \phi'^2 \phi \right] = 0$$

The variation with respect to the lapse function gives a constraint on the initial values,

$$\dot{\phi}^2 + 6 \left( \frac{\ddot{\phi}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right) - 6\ddot{a} \dot{\phi} - 3\alpha' \left[ 2\ddot{\phi} \left( \frac{\ddot{a}^2}{a^2} + \frac{k}{a^2} \right) - \frac{1}{4} \phi^4 \right] = 0$$

that is preserved by the dynamical equations (3,4). Without the $\alpha'$ corrections, these equations have been discussed in [4,11]. For a spatially closed Universe, $k = 1$, the solution cannot be extrapolated back in time to $t \to -\infty$, because it hits a singularity (close to the singularity, of course, $\alpha'$ corrections cannot be neglected). The solution with $k = -1$, instead, can be extrapolated to $t \to -\infty$ but at large negative values of time the scale factor is very large and increases as we go backward in time, and this might be an implausible initial condition.

Following the strategy of ref. [1], we will therefore assign the initial conditions at some finite (negative) value of time $t_0$, not very close to the would-be singularities. Physically, one expects that because of quantum fluctuations, at some value of $t = t_0$ a sufficiently smooth patch emerges, which then starts inflating.

The initial conditions for eqs. (3,4) are specified by $a_0, \dot{a}_0, \phi_0, \dot{\phi}_0$. However, $\phi_0$ never enters the equations, as can be seen from the fact that, in the absence of external matter fields in the action, a constant shift in $\phi$ gives simply an overall multiplicative factor in the action [4] and therefore does not affect the classical equations of motion; thus we have a family of solutions differing only by a shift in $\phi$. Furthermore, $a_0, \dot{a}_0, \phi_0$ are not independent, because of the constraint, eq. (1). We will take $a_0$ and $H_0 = \dot{a}_0/a_0$ as independent variables in the following. The physical curvature radius of a FRW Universe is $R_{\text{curv}} = a(t)/|k|^{1/2}$ and the spatial curvature is $3R = 6/R_{\text{curv}}^2$. Since we

\*Recently one of us has discussed a different mechanism for the regularization of the singularity, based on the production of massive string modes. In this case the Hubble parameter in the string phase is not constant, but rather bounces back after reaching a maximum value. The detailed evolution in this case is sensitive to both $\alpha'$ and $O(e^\phi)$ corrections, and we hope to report on it in future work. The scenario discussed in the present paper is appropriate if the value of $H_s$, determined by perturbative $\alpha'$ corrections, is smaller than the typical value of $H$ where the production of massive modes sets in.
have performed the usual rescaling so that for a spatially curved space $|k| = 1$, then $a_0$ is the initial curvature radius. The spatially flat limit is recovered as $a_0 \to \infty$.

We have studied eqs. (3,4) for different values of $k, a_0, H_0$. Figs. (1a,1b,1c) show the result of the integration for $a(\eta)$ and for the effective gauge coupling $g^2(\eta) = \exp\{\phi(\eta)\}$, versus conformal time $\eta$, without $\alpha'$ corrections, in the three cases $k = 0, k = \pm 1$ [13]. At a finite value of $\eta$ the solution hits a singularity. Close to the singularity, the solutions for the three cases approach each other, as can be seen analytically.

Figs. (2a,2b,2c) show the result of the numerical integration including $O(\alpha')$ corrections, for $k = 0, k = \pm 1$. We plot $H$ and $\dot{\phi}$ vs. $\eta$, where $H$ is the Hubble parameter, $H = \dot{a}/a$. We see that in all three cases the $\alpha'$ corrections regularize the lowest order solution, and in all three cases we have a phase of exponential inflation with a linearly growing dilaton, $H \simeq H_0, \phi \simeq c$. Furthermore the asymptotic values $H_\pm, c$ are the same in the three cases. This can also be easily seen analytically, since in eqs. (3,4) $k$ always appears in the combination $(\dot{a}/a)^2 + k/a^2$. During superinflation, the term $(\dot{a}/a)^2$ becomes much bigger than $1/a^2$ as $t$ approaches zero, and during exponential inflation $(\dot{a}/a)^2$ continues to increase compared to $1/a^2$, and then the term $k/a^2$ becomes quickly irrelevant.

The total amount of inflation between two generic values of time $t_i$ and $t_f$ can be conveniently measured by the factor

$$ Z = \frac{H(t_f)a(t_f)}{H(t_i)a(t_i)}, \quad (6) $$

The total amount of inflation during the DeSitter phase, for a spatially flat space, is easily found from eq. (3), fixing the end of the inflation from the condition $\phi(t_f) = 0$:

$$ Z_{DS} = \exp\left\{\frac{|\phi_s|}{2\beta}\right\}, \quad (7) $$

and it is very large if at the beginning of the string phase we are in the weak coupling regime, $|\phi_s| \gg 1$, or if $\beta \ll 1$.

Since the scale factor and dilaton field during the string phase are still given by eq. (5) also for $k = \pm 1$, at least within our numerical precision, the amount of inflation in the string phase is still given by eq. (5); furthermore, even the constant $2\beta = c/H_0$ that appears in eq. (5) is the same for $k = 0$ and for $k = \pm 1$.

To understand the dependence of $Z_{DS}$ on the initial conditions we therefore study $\phi_s(a_0, H_0, \phi_0, k)$. Since the equations of motion depend on $\dot{\phi}$ and not on $\phi$, $\phi_s$ has the general form

$$ \phi_s(a_0, H_0, \phi_0, k) = \phi_0 + f(a_0, H_0, k). \quad (8) $$

The function $f(a_0, H_0, k)$ is shown in fig. 3 for the three values of $k$ vs. $a_0$ for fixed $H_0$ (actually, for $k = 0$, $f$ does not depend on $a_0$, so we can introduce $f(H_0) = f(a_0, H_0, 0)$). Each point in the graph is the result of a numerical integration with the given values of $a_0, H_0$ as initial condition; $\phi_s$ is defined, operatively, as the value of $\phi$ when both $H$ and $\dot{\phi}$ are constant within an accuracy of one per cent. For large values of $a_0$, $f(a_0, H_0, k = \pm 1)$ approaches smoothly its flat space limit, $f(\infty, H_0, k = \pm 1) = f(H_0)$. In fig. 3, where we have fixed $H_0 = 0.5/\sqrt{\alpha'}$, the value of $|\phi_s|$ for $k = 1$ is larger than the value for $k = -1$, starting with the same $\phi_0$. For smaller values of $H_0$ we have found that the situation is reversed and $|\phi_s|$ is larger for $k = -1$.

We now ask when the amount of inflation during the string phase alone, given by eq. (6), is sufficient to solve the cosmological problems. The solution of the horizon problem requires [13] in $Z \gg O(60)$, and therefore in our case we must have

$$ |\phi_s| \gtrsim 120 \beta. \quad (9) $$

In general, we can expect that a long inflationary phase at the string scale will produce a large density of stochastic gravitational waves, and we should ask whether the condition (9) is consistent with the experimental bounds on the gravitational wave spectrum. The spectrum is conveniently characterized by the quantity

$$ \Omega_{GW}(f) = \frac{1}{\rho_c} \frac{d\rho_{GW}}{d\ln f}, \quad (10) $$

where $f = \omega/(2\pi)$ is the frequency, $\rho_{GW}$ is the energy density in gravitational waves and $\rho_c$ is the critical density of the Universe. The quantity to be compared with the experimental results is actually $h_0^2\Omega_{GW}$, where $h_0$ is the uncertainty on the Hubble constant, $H_0 = h_0100\text{km}/(\text{sec} \cdot \text{Mpc})$, since $h_0^2\Omega_{GW}$ is independent of the uncertainty in the quantity $\rho_c$ that we use to normalize $\rho_{GW}$. The main observational bounds for the spectrum are [13,15,14]:  

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• the COBE bound
\[ h_0^2 \Omega_{GW}(f) < 7 \times 10^{-11} \left( \frac{H_0}{f} \right)^2 \quad \text{for} \quad H_0 < f < 30H_0, \] (11)

• the millisecond pulsar timing constraint
\[ h_0^2 \Omega_{GW}(f = 10^{-8} \text{Hz}) < 10^{-8}, \] (12)

• the compatibility with the standard nucleosynthesis scenario
\[ \int \Omega_{GW}(f) d(\ln f) \leq 6.3 \times 10^{-6} \] (13)

(in the last equation we have used the value \( N_e < 3.9 \) for the equivalent number of neutrino species, see [17]). The spectrum of gravitational waves predicted by string cosmology [8] is characterized by two parameters \( f_1, f_s \) with dimension of frequency, and by the dimensionless constant \( \beta \); the spectrum is an increasing function of \( f \) up to the cutoff frequency \( f_1 \) which is of the order of 10 GHz. The parameter \( f_s \) is related to \( \phi_s, \beta \) by \( Z_{DS} = f_1 / f_s \), where \( Z_{DS} = \exp|\phi_s / (2\beta)| \), see eq. (7).

In the range \( f_s < f < f_1 \) the spectrum is approximately [8] given by
\[ h_0^2 \Omega_{GW}(f) \simeq 3 \times 10^{-7} \left( \frac{f}{f_1} \right)^{3-3|2\beta|}. \] (14)

while for \( f < f_s \) it varies as \( f^3 \ln^2 f \). This form of the spectrum has been computed for \( k = 0 \). We have repeated the computation for \( k = \pm 1 \) [16] and we find that, with very high accuracy, it is unchanged. For wavelengths much smaller than the present Hubble radius of the Universe, i.e. \( f \gg H_0 \), this is obvious, since the gravitational wave does not feel the structure of the Universe on a length scale much bigger than its wavelength. For sufficiently large wavelengths one should expect a modification of the spectrum, especially for a closed Universe, because in this case the modes of the gravitational field are discrete [8]; however the spacing is \( \Delta f_{com} = 1/(2\pi) \), where \( f_{com} \) is the comoving frequency. This corresponds to a spacing in the physical frequencies today \( \Delta f = 1/(2\pi a(t_{pres})) \), where \( a(t_{pres}) \) is the present value of the scale factor, in units \( k = -1 \). In general, also for an open Universe, this is the frequency scale where the spectrum is modified compared to the case of flat space. However, because of the inflationary evolution, \( 1/a(t_{pres}) \) is much smaller than \( H_0 \), and therefore at COBE frequencies, \( f \sim H_0 \), the spectrum is still indistinguishable from the flat space case, \( k = 0 \).

Both the frequency \( f_1 \) and the peak value \( h_0^2 \Omega_{GW}(f_1) \) are fixed, either using the so-called one-graviton level [7], or using the expected value for the string mass scale and therefore for \( H_1 \) [6], in the range \( f_1 = 6-20 \text{ GHz} \), \( h_0^2 \Omega_{GW}(f_1) = (0.1-8) \times 10^{-7} \). In the following, for definiteness, we use \( f_1 = 10 \text{ GHz} \) and \( h_0^2 \Omega_{GW}(f_1) = 3 \cdot 10^{-7} \), as in [8]. Our results can be easily rescaled using different values for these quantities.

Since \( f_1 \) is fixed, the spectrum depends only on two free parameters \( \beta, f_s \), or equivalently \( \beta, \ln Z_{DS} \). We now study the range of values of these parameter allowed by the three observational constraint discussed above.

We insert the explicit form of the spectrum in order to compute the integral in the bound [13], combining it with the COBE and pulsar bounds we obtain the results presented in fig. 4. The shaded area is the region of parameter space forbidden by these observational constraints. We observe that if we require \( \ln Z_{DS} > 60 \) we must have \( \beta > 0.12 \) in order to evade the COBE bound. This is due to the fact that with \( \ln Z_{DS} > 60 \) the frequency \( f_s = f_1 / Z_{DS} \) becomes smaller than the maximum frequency explored by COBE \( f \simeq 10^{-16} \text{ Hz} \), and we cannot take advantage of the \( f \) behavior of \( \Omega_{GW} \) for \( f < f_s \) in order to lower the value at COBE frequencies in comparison to the value at \( f = f_1 \). Rather, from \( 10^{-16} \text{ Hz} \) up to \( f = f_1 \sim 10 \text{ GHz} \) the spectrum varies as \( f^{3-3|2\beta|} \) (i.e. as \( f^{2\beta} \) for \( \beta \leq 3/2 \), and, because of this, \( \beta \) cannot be too close to zero.

We can also consider a situation in which the amount of inflation \( \ln Z > 60 \) is given partly by the DeSitter phase and partly by the super-inflationary phase. This reduces the requirement on \( Z_{DS} \) alone and therefore on \( \beta \). A value

\[^{1}\text{We neglect here the finer details of the spectrum, which are discussed in ref. [8].}\]
of $\beta$ as close as possible to zero is the most favorable situation for the observation of the gravitational wave spectrum at LIGO/Virgo frequencies, $f = 6\text{Hz}–1\text{kHz}$. Therefore, if we ask that the required amount of inflation is provided uniquely by the string phase, the maximum value of the spectrum at, say, 1kHz, is lowered. We present in the figure the lines in the parameter space that correspond to a value of $h_0^2\Omega_{GW}(1\text{kHz})$ equal to $10^{-9}$, $10^{-8}$ and $10^{-7}$, respectively.

In the range $40 < \ln Z < 56$ the stronger limit on $\beta$ is given by the pulsar-timing constraint, and is $\beta > 0.04$. Finally for $21 < \ln Z < 40$ the primordial nucleosynthesis constraint is the strongest one and we find the final smooth branch of the curve. For $\ln Z < 21$ we have no more restrictions on the value of $\beta$.

To understand the issue of fine tuning, it is also useful to discuss our results in terms of the original parameters of the model $\phi_s$ and $\beta$, rather than $\ln Z_{DS}$ and $\beta$. A large value of $Z_{DS} = -\phi_s/(2\beta)$ can be obtained as a combination of two limiting cases: (i) if $\beta$ is very close to zero, so that for any reasonable value of the initial curvature eq. (1) is satisfied, even without requiring an especially large value of $|\phi_0|$ and hence of $|\phi_s|$; or (ii) if $\phi_0$ is very large and negative; in this case the dependence of $\phi_s$ on $a_0$ is negligible for natural values of $a_0$, and again the amount of inflation is sufficiently large.

Concerning condition (i), we see from fig. 4 that we cannot choose $\beta$ arbitrarily small because of the various observational constraints; still, we can reach moderately small values $\beta \approx 0.15$. Condition (i) is analogous to the slow roll conditions in standard implementations of the inflationary scenario. It is a requirement on the dynamics of the theory, not on the initial conditions, and it ensures that $\phi$ is sufficiently small so that the mechanism that terminates inflation, and that presumably takes place when $e^{\phi} = O(1)$, is sufficiently delayed. A solution of the fine tuning problem based on the option (i) is therefore very similar, conceptually, to what is done in other inflationary models.

String cosmology, however, also has the option (ii). In this case the inflationary phase is long not because the field $\phi$ obeys a slow-roll condition, but rather because its initial value $\phi_0$ is such that $e^{\phi_0} \ll 1$ is very far from the point where inflation terminates, $e^{\phi} \sim 1$. We can compare this with what happens in chaotic inflation [20]. In this case the ‘natural’ initial value of the inflaton field $\varphi_0$ is fixed by the condition $V(\varphi_0) \sim M_{\text{Planck}}^4$, where $V$ is the potential that triggers inflation, and this fixes the dimensionful field $\varphi_0$ in terms of the Planck mass and of the dimensionless parameters of the potential.

In our case, instead, $\phi$ is a dimensionless field and $g_0^2 = e^{\phi_0}$ is the initial value of the gauge coupling. The initial condition $g_0^2 \ll 1$ means that the evolution starts deeply into the perturbative regime. As such, we do not regard $g_0^2 \ll 1$ as a fine-tuned initial condition; rather, it is possibly the most natural initial condition in this context. (Furthermore, we should note that the requirement that some coupling constant is small is quite common even in standard implementations of the inflationary scenario. For instance in chaotic inflation with $\lambda \phi^4$ theory we must have $\lambda \sim 10^{-15}$ in order to obtain a correct value for the density fluctuations.)

In conclusion, there is a region of the parameter space of the model that gives a long inflationary phase at the string scale, while at the same time the existing observational bounds on the production of relic gravitational waves are respected. This DeSitter inflationary phase can be long enough to solve the horizon/flatness problems, or it can be combined with the superinflationary phase to provide the required amount of inflation. In the former case, the value of the intensity of the relic gravitational wave spectrum to be expected at ground based interferometers is of order $\Omega_{GW} \sim 10^{-8}$ while in the latter case it can reach a maximum value $\Omega_{GW} \sim 10^{-7}$.

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FIG. 1. Evolution of the cosmic scale factor $a$ (solid line) and of the gauge coupling $g^2 \equiv e^\phi$ (dashed line) from eqs. (3, 4) with $\alpha' \equiv 0$ for the three possible cases of spatial curvature. The scale of the vertical axes are arbitrary, depending on the values of $a_0$ and $\phi_0$. 
FIG. 2. Evolution of the Hubble parameter $H$ (solid line) and of the derivative of the dilaton with respect to cosmic time $\dot{\phi}$ (dashed line) from eqs. (3,4), including $\alpha'$ corrections, for the three possible cases of spatial curvature. The vertical axes are in units of $1/\sqrt{\alpha'}$. The initial conditions $H_0$ and $\dot{\phi}_0$ are chosen so that they lie on the pre-big-bang solutions with $\alpha' = 0$ exhibited in figs. (1a,1b,1c).
FIG. 3. $\phi_s$ (defined as the value $\phi$ when both $H$ and $\dot{\phi}$ are constant within an accuracy of one per cent) as a function of $a_0$ (with $H_0 = 0.5$ and $\phi_0 = -20$) for $k = 0$ (solid line), $k = 1$ (dashed line) and $k = -1$ (dotted line).

FIG. 4. The forbidden region in the parameter space is the shaded area. Along the dot-dashed line $\Omega = h_0^2 \Omega_{\text{SW}}(1 \text{kHz}) = 10^{-7}$, along the dotted one $\Omega = 10^{-8}$ and along the dashed line $\Omega = 10^{-9}$. 