Solar convection zone dynamics

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Abstract A comprehensive understanding of the solar magnetic cycle requires detailed modeling of the solar interior including the maintenance and variation of large scale flows (differential rotation and meridional flow), the solar dynamo and the flux emergence process connecting the magnetic field in the solar convection zone with magnetic field in the photosphere and above. Due to the vast range of time and length scales encountered, a single model of the entire convection zone is still out of reach. However, a variety of aspects can be modeled through a combined approach of 3D MHD models and simplified descriptions. We will briefly review our current theoretical understanding of these processes based on numerical models of the solar interior.

1 Introduction

The solar convection zone comprises the outer most 30% of the solar radius and contains about 2% of the total solar mass. Due to a density variation of more than 6 orders of magnitude a variety of different physical regimes are encountered. While fluid motions are are highly subsonic ($Ma \approx 10^{-4}$) and strongly influenced by rotation near the base of the convection zone, they turn supersonic in the photosphere and the influence of rotation diminishes. The pressure scale height varies between about 50 Mm at the base of the convection one and about 100 km in the photosphere of the sun. As a consequence a comprehensive model of the entire convection zone is currently out of reach and different aspects have to be modeled independently. The deep convection zone up to about 10-20 Mm beneath the photosphere can be modeled most efficiently using the anelastic approach which is filtering out sound waves, but is fully considering the compressibility in the stratification. Glatzmaier
The uppermost parts of the convection zone require fully compressible MHD (see e.g. Nordlund et al., 2009, for a recent review). While most anelastic models of the solar interior are global models with computational domains covering an entire shell between two radii (or at least a shell segment), MHD models of the outer parts of the convection zone typically focus on details in rectangular computational domains.

Apart from 3D MHD models adapted to the different physical regimes a variety aspects have been modeled based on simplified models, such as the mean field approach. Here the focus is on the large scales, while the effects of unresolved turbulence is parametrized. Non-linear terms in the momentum, energy and induction equations lead to non-vanishing second order correlation terms of small scale quantities that act as drivers for large scale flows or as turbulent induction effects for the large scale magnetic field. The decomposition into large and small scale properties and the arising correlation terms driving large scale flows are the strength and the weakness of this approach at the same time. On one hand the computational expense is decreased by orders of magnitude allowing for simulations covering long time scales as well as exploring wide parameter ranges, on the other hand the results are heavily dependent on parametrization of the second order correlation terms. For a comprehensive description of mean field theory we refer to Rüdiger & Hollerbach (2004).

2 Differential rotation and meridional flow

Differential rotation is the consequence of angular momentum transport in the solar convection zone. Starting with a decomposition of the turbulent velocity field into fluctuating and (axisymmetric) mean flows $v = \langle v \rangle + v'$ leads to the following terms in the angular momentum flux (neglecting magnetic terms for simplicity):

$$\langle F_i \rangle = r \sin \theta \rho \left( \langle v_i' v_\theta' \rangle + \langle v_i \rangle \Omega r \sin \theta \right).$$

Here the Reynolds stress describes the transport due to correlations of fluctuating turbulent velocity components, while the Meridional flow describes the transport due to large scale coherent mean flows in the $r-\theta$ plane. Angular momentum transport through Reynolds stresses requires the presence of rotation and anisotropy and expressions for these transport terms have been derived within the mean field approach by Durney & Spruit (1979), Hathaway (1984) and more recently by Kitchatinov & Rüdiger (1993) using a quasi-linear approach (see also Kitchatinov & Rüdiger, 2005 for an improved representation). In 3D simulations the influence of rotation on convection leads to a preferential north-south alignment of convection cells (Gilman, 1979, Miesch et al., 2000, Brun & Toomre, 2002, Miesch et al., 2008). The consequence is a dominance of east-west motions over north-south motions. By means of the Coriolis force eastward (faster rotating) flows
are deflected equatorward, while westward (slower rotating) flows are deflected poleward, leading on average to an equatorward transport of angular momentum.

Fig. 1 Contour plots of differential rotation (a), entropy perturbation (b) and stream function of meridional flow (c) using the mean field model of Rempel (2005). Panel (d) shows the differential rotation profile obtained using the same parametrization of the Reynolds stress but neglecting the effects of baroclinicity.

The profile of differential rotation cannot be determined on the basis of angular momentum transport processes alone. As stationary state requires beside vanishing divergence of the total angular momentum flux also a force balance in the meridional plane between Coriolis, centrifugal, buoyancy and pressure forces. The latter is most conveniently expressed by (follows from \(\phi\)-component of vorticity equation):

\[
r \sin \theta \frac{d \Omega^2}{dz} = \frac{g}{c_p r} \frac{\partial s}{\partial \theta}
\]  

(2)
Helioseismic observations by Thompson et al. (1996) show clearly a differential rotation profile with contours of constant $\Omega$ inclined by about 25 deg with respect to the rotation axis (deviation from Taylor-Proudman state). It turns out that avoiding the Taylor-Proudman state is a key problem for a theoretical understanding of solar differential rotation. While early models attempted to achieve this by assuming large viscosities ('Taylor-number puzzle' after Brandenburg et al. (1990)), Kitchatinov & Rüdiger (1995) showed that an alternative solution of this problem can be given if the anisotropic convective energy transport is considered, leading to a pole-equator temperature difference of about 10 K. Anisotropic convective energy transport is automatically considered in global 3D simulations, but in many cases it turns out to be insufficient for obtaining solar-like differential rotation.

Recently Rempel (2005) showed that coupling between the tachocline and convection zone can also provide the latitudinal entropy variation needed to explain the observed profile of solar differential rotation. A typical solution from that model is shown in Fig. (1), displaying differential rotation (a), corresponding entropy perturbation (b) and the stream function of the meridional flow (c). Panel (d) shows for comparison the profile of differential rotation obtained if the effects of the entropy perturbation displayed in (b) are neglected. Inclusion of this effect through the bottom boundary condition in 3D models allows also for more solar-like differential rotation in 3D convection models (Miesch et al., 2006).

While there is a general agreement that thermal effects are essential for solar-like differential rotation, it is still unclear whether the required latitudinal entropy variation is a consequence of anisotropic convective energy transport, imposed by the tachocline, or a combination of both.

How does the meridional flow come into play here? A stationary solution requires that the divergence of the angular momentum flux Eq. (1) vanishes. While in very special situations the Reynolds stress can be divergence free on its own, in general a contribution from the meridional flow is required to close the system. It turns out that primarily the component of the Reynolds stress that transports angular momentum parallel to the axis of rotation influences most strongly the direction of the meridional flow. If the transport of angular momentum is inward directed, the resulting meridional flow is poleward at the surface and equatorward near the bottom of the convection zone. While this is found in most mean field models such as Kitchatinov & Rüdiger (1995), 3D simulations present a more complicated situation. Early models with lower resolution (Brun & Toomre, 2002) typically show multi-cellular flow pattern, while a recent high resolution run (Miesch et al., 2008) shows a single flow cell (poleward at top, equatorward at bottom of CZ) in the bulk of the convection zone. To which degree these results are converged with respect to numerical resolutions remains to be seen in the future.

Differential rotation shows also cyclic variations known as torsional oscillations, which point toward a close relation to the solar magnetic cycle. We refer here to Howe (2009) and Brun & Rempel (2009) for reviews of observations as well as theoretical models for the time varying component of solar differential rotation.
3 Solar Dynamo

Similar to models of differential rotation and meridional flow we discussed in section 2 also the solar dynamo is modeled through a combination of mean field models and 3D simulations. Currently mean field models of the solar dynamo are the only models that provide dynamo solutions that are compatible with basic cycle features and can be evolved over time scales much longer than a cycle. However, as already stated above, these models are heavily dependent on parametrization of turbulent induction processes and cannot provide an explanation from first principles. On the other hand, 3D MHD simulations describe currently only aspects of the dynamo process, a comprehensive model of a solar dynamo with features compatible with the basic dynamo constraints derived from the solar butterfly diagram is still an open challenge.

Regardless of the adopted modeling approach the primary uncertainties regarding the underlying dynamo process are similar. Many of these uncertainties result directly from our limited ability to model processes from first principles and the rather sparse observational constraints on the solar interior. The best known ingredient is differential rotation ($\Omega$-effect) due to observational constraints from helioseismology on the mean profile and variation of differential rotation (Howe, 2009). But even the exact knowledge of the differential rotation profile is not sufficient to determine whether radial velocity gradients at the base of the convection zone (tachocline) or latitudinal gradients in the bulk of the convection zone play the major role in the generation of toroidal magnetic fields, since this would require knowledge of the detailed distribution of poloidal field in the convection zone. Even less known are the processes related to the regeneration of poloidal field ($\alpha$-effect). A third unknown are the transport processes of magnetic flux in the convection zone. Since in general the locations where the $\alpha$-effect and $\Omega$-effect operate do not coincide, transport of magnetic flux in between these regions is crucial for a coherent operation of the large dynamo in the convection zone.

It is beyond the scope of this paper to review all the possible dynamo scenarios which have been considered and we refer to Charbonneau (2005) for further reading. In the following three subsections we will briefly discuss some of the key uncertainties and open questions.

3.1 Role of tachocline

Soon after helioseismology revealed the detailed structure of differential rotation in the solar interior (Thompson et al., 1996) it was suggested that the base of the convection zone with its strong radial shear layer (tachocline) is a likely location for the solar dynamo (production of strong toroidal field by shear). In addition, the stable stratification found in the solar overshoot region at the base of the convection zone allows for storage of magnetic field over time scales comparable to the solar cycle. Both aspects are crucial since simulations of rising flux in the convection zone
as well as studies of magnetic stability in the solar overshoot regions (see section 4 for further detail) point toward a rather strong toroidal magnetic field of $10^5$ Gauss at the base of the convection zone. More recently the role of the tachocline for the organization and amplification of large scale toroidal field has been also seen in global 3D MHD simulations of the solar dynamo (Browning et al., 2006). However, Brown et al. (2009) presented simulations of solar like stars at faster rotation rates, which point toward the possibility that substantial magnetic field can be produced and maintained within the convection zone in near equatorial regions. It is currently not clear to which degree this result can be also relevant for the solar rotation rate.

While most models of flux emergence point toward a field strength of $10^5$ Gauss at the base of the convection zone, it is far from trivial to amplify field to this strength solely through differential rotation. Dynamo models that include non-linear feedbacks consistently (Rempel, 2006) lead to an upper limit more around $10^4$ Gauss, similar values are also found in most 3D simulations such as (Browning et al., 2006). Whether this discrepancy can be bridged through an alternative field amplification mechanism (e.g. harvesting potential energy of the stratification as proposed by Rempel & Schüssler (2001)) or the possibility that also initially weaker magnetic field from the bulk of the convection zone can lead to active region formation is currently an open question.

### 3.2 Regeneration process of poloidal field

The details of the processes rebuilding the poloidal field from toroidal field are still very uncertain. In the meanfield language these processes are formally described as $\alpha$-effect and in the context of solar dynamo models the following 3 classes of $\alpha$-effects are typically considered: 1. Helical turbulence, 2. MHD shear flow instabilities in the tachocline, 3. Rising flux tubes in the convection zone (Babcock-Leighton). While all these processes are likely to contribute, their amplitude and spacial distribution is not known well enough to clearly quantify their individual role.

Furthermore recent research also points toward highly non-linear and also time dependent $\alpha$-effects resulting from additional constraints due to conservation of magnetic helicity (Brandenburg & Sandin, 2004). Indirect constraints on the operation of the $\alpha$-effect might be gained from helicity fluxes observable in the photosphere and above.

The only $\alpha$-effect contribution that is directly constrainable through observations is the Babcock-Leighton $\alpha$-effect (Babcock, 1961; Leighton, 1969), which has been used in most of the recent flux-transport dynamo models (Dikpati & Charbonneau, 1999; Dikpati et al., 2004; Rempel, 2006). The Babcock-Leighton $\alpha$-effect is based on the flux emergence process leading to the formation of active regions, the key ingredient is the systematic tilt resulting from the action of the Coriolis force twisting the rising flux tube. While it is possible to construct dynamo models entirely based on the Babcock-Leighton $\alpha$-effect, these models lead in general to rather strong po-
lar fields at the surface in contradiction with observations, unless a strong magnetic diffusivity gradient and additional contributions from $\alpha$-effects at the base of the convection zone are considered (Dikpati et al., 2004).

3.3 Transport of magnetic flux in convection zone

Traditionally most models considered only turbulent transport in the convection zone, which can be decomposed (in the meanfield language) into diffusive transport (turbulent diffusion) but also advection like transport in form of turbulent pumping. The latter has been also studied extensively through 3D MHD simulations (Tobias et al., 1998, 2001). If magnetic field becomes sufficiently strong magnetic buoyancy drives additional transport in terms of rising flux bundles that can lead to the formation of active regions on the visible surface (see section 4 for more detail). Additional to these processes magnetic flux can be transported by the large scale meridional flow in the convection zone. Dynamo models based primarily on the latter are called flux transport dynamos and were first introduced by Choudhuri et al. (1995) and Dikpati & Charbonneau (1999) and have been developed further by several groups since then.

The attraction of flux transport dynamos comes primarily from the fact that a meridional flow which is poleward at the top and equatorward at the bottom of the convection zone gives a very robust explanation for the equatorward propagation of the activity in the course of the solar cycle. In addition the poleward flow in the near surface levels in combination with the systematic tilt angle of sunspot groups leads automatically to the correct phase relation between toroidal and poloidal field. However, as pointed out by Schüssler (2005), the phase relation is primarily a consequence of the tilt angle of active regions and in that sense only a weak constraint on dynamo processes in the solar interior. For meridional flow speeds consistent with surface observations and an extrapolation based on mass conservation for the deeper layers, these models also yield dynamo periods in agreement with the solar cycle.

Overall the flux transport picture is currently one of the most successful scenarios for the large scale solar dynamo, but (as many other models) it is based on two strong assumptions which cannot be proven from first principles: 1. The meridional flow is dominated by one flow cell with poleward flow close to the surface layers and equatorward flow at the base of the convection zone. 2. Turbulent transport processes are sufficiently weak to allow advection effects to dominate. While meanfield models of differential rotation and meridional flow typically lead to the required flow patterns (see e.g. Küker & Rüdiger, 2005), the situation is more complicated in 3D simulations. Most of the earlier models at moderate resolution lead to multi-cellular flows (Brun & Toomre, 2002), in contrast a more recent model at higher resolution shows a flow pattern dominated by a single cell in the convection (Miesch et al., 2008). Overall the situation cannot be considered as converged yet. The amplitude of turbulent transport estimated from simple mixing length ar-
arguments is typically 1 - 2 orders of magnitude larger than the values required for the flux transport picture. Since turbulent transport is in general more complicated than a simple diffusive transport this aspect needs to be studied in more detail through 3D simulations taking into account the presence of large scale flows and the full non-linearity of the problem.

4 Flux emergence process

It is generally accepted that sunspots form from magnetic field rising from the base of the convection zone to the surface (see reviews by Moreno-Insertis 1997; Fisher et al. 2000; Fan 2004, and further references therein). Solar dynamo models as presented in section 3 focus on the large scale evolution of magnetic field and cannot address detailed processes such as the flux emergence process leading to the formation of sunspots on the visible surface of the sun. The latter is primarily a consequence of limited numerical resolution. Nevertheless, studying flux emergence is integral to our understanding of solar magnetism, since it allows us to connect the solar dynamo to observational constraints on the magnetic field structure in the solar photosphere. To date the flux emergence process has been studied decoupled from dynamo models using a variety of different approaches.

4.1 Flux emergence in lower convection zone

Early work was based on the thin flux tube approximation (Choudhuri & Gilman 1987; Fan et al. 1993, 1994; Moreno-Insertis et al. 1994; Schüssler et al. 1994; Caligari et al. 1995). These studies concluded that the overall properties of active regions, such as the low latitude of emergence, latitudinal trend in tilt angles as well as asymmetries between leading and following spots can be understood on the basis of rising flux tubes, provided the initial field strength at the base of the convection zone is around 100 kG. This conclusion was also consistent with stability considerations of flux in a subadiabatic overshoot region (Ferriz-Mas & Schüssler 1993, 1995).

Based on two-dimensional MHD simulations it was early realized by Schüssler (1979) that untwisted magnetic flux tubes cannot rise coherently and fragment. It was shown later by Moreno-Insertis & Emonet (1996); Emonet & Moreno-Insertis (1998) that this fragmentation can be alleviated provided that flux tubes have enough initial twist.

More recently also 3D MHD simulations of rising flux tubes based on the anelastic approximation have become possible (Fan 2008) and give support for results from earlier simulations based on the thin flux tube approximation. It was however found by Fan (2008) that there is a very delicate balance between the amount of twist required for a coherent rise and the amount of twist allowed to be in agree-
ment with observations of sunspot tilt angles (twist with the observed sign produces a tilt opposite to the effect of Coriolis forces on rising tubes).

The simulations presented above consider the flux emergence process decoupled from convection. First attempts to address flux emergence in global simulations of the convection zone were made recently by Jouve & Brun (2007, 2009). Understanding the interaction of emerging flux with the ambient convective motions in the convective envelope is a crucial step toward more realism; however, currently the focus on the global scale limits the resolution required to resolve this interaction in detail. Substantial progress will likely happen in the next decade with increase in computing power.

### 4.2 Flux emergence in upper convection zone

Another challenge is understanding the last stages of the flux emergence process in the near surface layers (last 10 - 20 Mm). All of the models presented above exclude the upper most 10 - 20 Mm since the adopted approximations (thin flux tube, anelastic) lose their validity and also the steep decrease of pressure scale height and increase in convective velocities would lead to very stringent resolution and time step constraints. The upper most layers of the convection zone require fully compressible MHD and also the inclusion of radiative processes is necessary if a detailed comparison with the available high resolution observations is desired (Cheung et al., 2007, 2008, 2010). While the primary modeling focus in the deep convection zone lies on large scale properties of active regions, near surface simulations focus on the detailed interaction of emerging flux with convection on the scale of granulation and below. One of the major open questions concerns the re-amplification process of magnetic flux into coherent sunspots from flux that has risen through a convection zone with a density stratification of six orders of magnitude.

Recently MHD simulations with radiative transfer also provided a breakthrough in our understanding of sunspot fine structure in the photosphere such as umbral dots, penumbral filaments, light bridges and the Evershed flow in terms of a common magneto-convection process modulated by inclination angle and field strength (Schüssler & Vögler, 2006; Heinemann et al., 2007; Rempel et al., 2009b,c).

### 4.3 Open questions, connection to dynamo models

While we have seen strong progress in modeling the flux emergence process over the past decades, we do not have at this point a fully consistent model. The latter is a consequence of the fact that different aspects are modeled independently due to computational constraints. As a consequence there are some ‘missing links’ between different models, which have to be addressed in the future through a more coherent coupling of models. Here we mention just a few of the open questions:
1. As pointed out before most models of emerging magnetic flux require an initial field strength of about $10^5$ Gauss at the base of the convection zone to be consistent with observational constraints. On the other hand the majority of dynamo models falls short of such values, more typical are $10^4$ Gauss. 2. Due to the strong density stratification in the convection zone even magnetic field with initially $10^5$ is weakened to sub kG field strength in the upper most layers of the convection during the emergence process. It is currently an open question if such weak field can get re-amplified to sunspot field strength. Near surface simulations start very often from 10 kG field about 5 Mm beneath the photosphere (Cheung et al., 2008) to overcome the influence of convective motions. 3. Rising $\Omega$-shaped flux tubes in the deep convection zone form typically as low wave number instability ($m = 1$ and $m = 2$ modes are preferred). In the near surface layers such low wave numbers should lead to much faster diverging motions in bipolar sunspot groups as observed (due to magnetic tension) if sunspots stay connected to their deep roots. A possible dynamical ‘disconnection’ mechanism has been proposed by Schüssler & Rempel (2005), but it is also unclear if sunspots are sufficiently stable if they are rather shallow.

5 Summary

We presented here a brief summary of the state of the art of modeling of dynamical processes in the solar convection zone with focus on differential rotation/meridional flow, the large scale solar dynamo and the flux emergence process. We see currently in this field a dramatic change from more simplified models toward large 3D MHD simulations, primarily driven by the strong increase in computing resources. At the same time the field suffers from very limited observational constraints on processes in the solar interior. Progress in the future will heavily rely on improving and exploiting helioseismic constraints and also on coupling models to allow for a check of consistency. In the near terms the latter is likely to be most successful for models of the flux emergence process.

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