Trajectory tracking sliding mode control for underactuated autonomous underwater vehicles with time delays

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Abstract
Trajectory tracking control of autonomous underwater vehicles in three-dimension always suffers disturbances such as input time delays and model uncertainties. Regarding this problem, an integral time-delay sliding mode control law is proposed in this article with dividing the vehicle’s input time delays model into cascade system consisting of a kinematics subsystem and a dynamics subsystem. Based on the established pose error equation and velocity error equation, a suitable Lyapunov–Krasovskii functional is given to analyze and guarantee the global stability of the whole system under reasonable assumptions. At last, comparative simulations are presented to demonstrate the effectiveness of the proposed method.

Keywords
Motion and tracking, vision systems, robust, adaptive and optimal control, robot manipulation and control, autonomous control, robot manipulation and control, underwater vehicles, field robotics, target tracking and identification, robot sensors and sensor networks

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Introduction
Trajectory tracking control for underactuated autonomous underwater vehicles (AUVs) has strict constraints on time. Considering the practical application of the project, the delays of the AUV’s actuators such as propellers and rudders will cause the delays in output of control force and torque. The most direct effect is system overshoot increases, poor control effect, even causing system divergence instability. At the same time, the input time delays cause the system characteristic equation to have an infinite number of eigenvalues and become an infinite dimensional system. Therefore, the trajectory tracking control strategy based on the traditional Lyapunov stability theory is no longer applicable. Time delays and nonlinear coupling bring challenges to system control research.

Underactuated AUV trajectory tracking control system is a typical time-delay strongly nonlinear system. The stability analysis and robust control of time-delay systems have important theoretical and engineering significance. To make the system stable under bounded uncertainty, the form of free matrix is improved using Leibniz–Newton formula, and a convex optimization algorithm is proposed. Wu et al. considered the influence of the Leibniz–Newton formula when replacing the delay term and introduced the free weighting matrix (FWM) to eliminate the influence. The introduced FWM was determined by linear matrix inequality (LMI), and the criterion with less
conservatism was obtained. Sun et al.\textsuperscript{9} proposed an augmented Lyapunov functional containing triple integral terms. By introducing an FWM,\textsuperscript{10} a new time-domain correlation stability criterion was derived using LMI.

In practical applications, the AUV thrusters and rudders have time-delay characteristics that cannot be ignored, especially for Euler–Lagrangian systems with actuator delays, parameter perturbations, and external bounded disturbances. Sharma et al.\textsuperscript{11} designed tracking controllers for known and unknown inertial forces, introduced prediction terms to deal with the delay in control input, and adopted Lyapunov–Krasovskii function to prove semi-globally consistent ultimately bounded tracking. Mazenc et al.\textsuperscript{12} solved the problem of state feedback stabilization of feedforward system with input delay. Based on the time-varying change of coordinates and the functional of Lyapunov–Krasovskii, the designed controller could still keep the input state stable when parameters were uncertain. This result is applicable to any given constant delay, and the designed controller is applied to the formation tracking control of unmanned aircraft. Han et al.\textsuperscript{13} studied the robust sliding mode control (SMC) problem for discrete singular systems with time-varying delays, parameter uncertainties, and nonlinear disturbances. By constructing the appropriate discrete sliding surface function, the corresponding sliding mode dynamics is obtained and chattering is reduced.

Zhou et al.\textsuperscript{14} address the motion parameter skip problem associated with three-dimensional trajectory tracking of an underactuated AUV using backstepping-based control. The problem of stability analysis for continuous-time/discrete-time systems with time-varying delays has been studied.\textsuperscript{15} A new delay partitioning method is presented, which partitions the delay interval into nonuniform subintervals. An SMC law is proposed for discrete systems with time-varying delays and external disturbances. The delay weight dependence stability of LMI form is obtained using the free weight matrix method.\textsuperscript{16,17} The backstepping control technique is designed as the nominal controller for integral sliding mode control. To enhance the tracking performance of the system, an adaptive technique and a new disturbance observer based on sliding mode technique are developed and integrated into the integrator sliding mode control (ISMC).\textsuperscript{18} Li et al.\textsuperscript{19,20} present a novel swarm control framework for path following of multiple underactuated unmanned marine vehicles with uncertain dynamics and unmeasured velocities. A center-of-swarm guidance scheme is further employed for collision avoidance and obstacle avoidance. In recent years, the globally finite-time control strategy is proposed, which globally stabilizes all trajectory-tracking errors in the finite time.\textsuperscript{21} Although there have been a lot of research results on the stability and control strategy of time-delay systems, there are very few studies on vehicles trajectory tracking control for time-delay nonlinear system.

To reduce the influence of input time delays and disturbances on the trajectory tracking control system, this article presents an integrated SMC strategy based on unknown bounded time delays. Firstly, simplify the vehicle input time delays model. The trajectory tracking pose and velocity error equation are obtained. Secondly, the system is divided into a cascade system consisting of a kinematics subsystem and a dynamics subsystem. Underactuated AUV trajectory tracking control is transformed into tracking error stabilization. The integral time-delay sliding mode control law (TDSMC) is designed for the trajectory tracking position and velocity error. Finally, a suitable Lyapunov–Krasovskii functional is given to prove that the vehicle trajectory tracking error converges to zero. The stability criterion of the system is obtained.

Problem formulation

In this section, firstly, an underactuated AUV five-degree-of-freedom motion model with input delays is established, followed by the transformation between the earth-fixed frame and the body-fixed frame. Secondly, “virtual underactuated AUV” is established. Further, the vehicle three-dimensional trajectory tracking error equation is obtained. The vehicle trajectory tracking control is converted to tracking error stabilization.

The AUV model and frame transformation

To analyze the three-dimensional motion of the vehicle, two coordinate systems are defined, as shown in Figure 1. $E$ and $B$ represent the earth-fixed frame and the body-fixed frame of AUV, respectively.

The matrix vector descriptions of the AUV kinematics and dynamics mathematical models are

\begin{equation}
    \dot{\eta} = J(\eta)\nu
\end{equation}

\begin{equation}
    M\dot{\nu} = \tau + C(\nu)\nu - D(\nu)\nu - g(\eta)
\end{equation}

where $\eta = [x, y, z, \theta, \psi]^T \in \mathbb{R}^5$ denotes the position and the heading angle of the AUV in the earth-fixed frame; $J(\eta)$ is the transformation matrix between the earth-fixed frame and the body-fixed frame; $M$ is the inertia matrix, which

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure1.png}
\caption{AUV model. AUV: autonomous underwater vehicle.}
\end{figure}
includes additional quality: \( v = [u, v, w, q, r]^T \in \mathbb{R}^5 \) denotes speed vector in the body-fixed frame; \( C(v) \) denotes the centripetal and Coriolis matrix, including centripetal force and Coriolis force generated by additional mass; \( D(v) \) denotes the hydrodynamic damping matrix; \( g(\eta) \) denotes the vector of buoyancy and gravitational forces and moments; \( \tau = [\tau_u, 0, 0, \tau_q, \tau_r]^T \in \mathbb{R}^5 \) denotes the corresponding control input signal; and \( \tau_v = [\omega_x, \omega_y, \omega_z, \omega_q, \omega_r]^T \in \mathbb{R}^5 \) denotes the external disturbance signal, where

\[
J(\eta) = \begin{bmatrix}
\cos \psi \cos \theta & -\sin \psi & \sin \theta \cos \psi & 0 & 0 \\
\sin \psi \cos \theta & \cos \psi & \sin \theta \sin \psi & 0 & 0 \\
-\sin \theta & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1/\cos \theta
\end{bmatrix} \in \mathbb{R}^{5 \times 5}.
\]

Considering the underactuated characteristics, the AUV here is satisfied: (1) uniform mass distribution, (2) AUV’s center of buoyancy and center of gravity coincide, (3) ignore roll motion and nonlinear hydrodynamic parameters above second order, (4) input delay, that is, \( \tau(t - T') = [\tau_u(t - T'_1), 0, 0, \tau_q(t - T'_2), \tau_r(t - T'_3)]^T \). Establish the following underactuated AUV five-degree-of-freedom mathematical model.\(^{22}\)

**Kinematic model:**

\[
\begin{align*}
\dot{x} &= u \cos \psi \cos \theta - v \sin \psi + w \cos \psi \sin \theta \\
\dot{y} &= u \sin \psi \cos \theta + v \cos \psi + w \sin \psi \sin \theta \\
\dot{z} &= -u \sin \theta + w \cos \theta \\
\dot{\theta} &= q \\
\dot{\psi} &= r / \cos \theta
\end{align*}
\]

**Dynamic model:**

\[
\begin{align*}
\dot{u} &= \frac{m_{22} m_{11} v - m_{33} m_{11} w - X_u u - X_w|u|u}{m_{11}} + \frac{\tau_u(t - T'_1) + \omega_\psi}{m_{11}} \\
\dot{v} &= \frac{-m_{11} m_{22} u - Y_v v - Y_w|v|v}{m_{22}} \\
\dot{w} &= \frac{m_{11} m_{33} u - Z_u u - Z_w|w|w + \omega_w}{m_{33}} \\
\dot{q} &= \frac{(m_{33} - m_{11}) \omega - M_q q - M_{\psi q} q |q| - B G M L_{\psi} \sin \theta}{m_{33}} \\
\dot{r} &= \frac{m_{11} m_{22} \omega - N_r r - N_{\psi r} r |r| + \tau_r (t - T'_3) + \omega_r}{m_{66}}
\end{align*}
\]

where \( m_{11} = m - X_u; m_{22} = m - Y_v; m_{33} = m - N_r; m_{55} = I_y - M_q; m_{66} = I_r - N_{\psi r}; m_{33} = m - N_r; m_{55} = I_y - M_q; m_{66} = I_r - N_{\psi r} \). \( X_u, X_w, X_{\psi u}, Y_v, Y_{\psi v}, Z_u, Z_w, Z_{\psi w}, M_q, M_{\psi q}, N_r, N_{\psi r}, X_u, Y_v, N_r, M_q, N_r, N_{\psi r} \) are hydrodynamic damping coefficient, \( m \) is the total weight of the underactuated, \( B \) is the AUV buoyancy, \( G M_L \) is the metacentric height, \( \omega = [\omega_x, \omega_y, \omega_z, \omega_q, \omega_r]^T \) denotes the external disturbance signal, and \( \tau' = [T'_1, 0, 0, T'_2, T'_3]^T \) is an input time delay caused by actuators in pitch, roll, and yaw in the body-fixed frame. The time delay is an unknown bounded time delay.

Underactuated AUV satisfies the following assumptions:

**Assumption 1.** The speed and control inputs of the underactuated AUV are bounded, that is, \( |\tau_u| \leq \tilde{\tau}_u, |\tau_q| \leq \tilde{\tau}_q, |\tau_r| \leq \tilde{\tau}_r, |u| \leq \bar{u}, |v| \leq \bar{v}, |w| \leq \bar{w}, |q| \leq \bar{q}, |r| \leq \bar{r} \), where \( \tilde{\tau}_u, \tilde{\tau}_q, \tilde{\tau}_r, \bar{u}, \bar{v}, \bar{w}, \bar{q}, \bar{r} \) are known upper bound.

**Assumption 2.** When \( t > 0 \), the variables of the desired trajectory \( u_d, q_d, r_d \) are bounded, and their derivatives \( \dot{u}_d, \dot{q}_d, \dot{r}_d \) are also bounded.

**Assumption 3.** \( \forall t \geq 0 \), underactuated AUV’s trim angle: \( |\theta(t)| < \pi/2 \).

### Establishment of trajectory tracking error equation

The desired motion state of the vehicle under the desired trajectory is described by establishing a “virtual underactuated AUV.”\(^{23}\) The motion state of the virtual AUV is obtained by deriving the position and posture information of the desired trajectory at the current time. Defining the desired position and attitude information of the vehicle is as follows

\[
P_d(t) = [x_d(t), y_d(t), z_d(t), \theta_d(t), \psi_d(t)]^T
\]

where the desired attitude is determined by the desired position

\[
\theta_d = \begin{cases}
-\arctan \frac{\dot{z}_d}{\sqrt{\dot{x}_d^2 + \dot{y}_d^2}}, \dot{x}_d \geq 0 \\
\pi - \arctan \frac{\dot{z}_d}{\sqrt{\dot{x}_d^2 + \dot{y}_d^2}}, \dot{x}_d \leq 0
\end{cases}
\]

\[
\psi_d = \begin{cases}
\pi + \arctan \frac{\dot{y}_d}{\dot{x}_d}, \dot{y}_d \geq 0, \dot{x}_d < 0 \\
-\pi + \arctan \frac{\dot{y}_d}{\dot{x}_d}, \dot{y}_d \leq 0, \dot{x}_d < 0
\end{cases}
\]

The virtual underactuated AUV five-degree-of-freedom model is established as follows
\[
\begin{align*}
\dot{x}_d &= u_d \cos(\psi_d) \cos(\theta_d) - v_d \sin(\psi_d) \\
&\quad + w_d \cos(\psi_d) \sin(\theta_d) \\
\dot{y}_d &= u_d \sin(\psi_d) \cos(\theta_d) + v_d \cos(\psi_d) \\
&\quad + w_d \sin(\psi_d) \sin(\theta_d) \\
\dot{z}_d &= -u_d \sin(\theta_d) + w_d \cos(\theta_d) \\
\hat{\theta}_d &= \dot{q}_d \\
\hat{\psi}_d &= r_d / \cos(\theta_d)
\end{align*}
\]

(8)

Define the trajectory tracking pose and velocity error variables as follows

\[
\begin{bmatrix}
x_e \\
y_e \\
z_e \\
\theta_e \\
\psi_e
\end{bmatrix}
=
\begin{bmatrix}
x - x_d \\
y - y_d \\
z - z_d \\
\theta - \theta_d \\
\psi - \psi_d
\end{bmatrix},
\begin{bmatrix}
u_e \\
v_d \\
w_e \\
q_e \\
r_e
\end{bmatrix}
=
\begin{bmatrix}
u - u_d \\
v - v_d \\
w - w_d \\
q - q_d \\
r - r_d
\end{bmatrix}
\]

(10)

The vehicle trajectory tracking pose error variable is derived along the trajectories equations (3) and (8). Get the tracking pose error equation as follows:

\[
\begin{align*}
\dot{x}_e &= u_e \cos(\psi_e) \cos(\theta_e) - v_e \sin(\psi_e) \\
&\quad + u_d \cos(\psi_e) \cos(\psi_d) \cos(\theta_d) \\
&\quad - v_d \sin(\psi_d) \\
&\quad + w_d \cos(\psi_d) \sin(\theta_d) \\
\dot{y}_e &= u_e \sin(\psi_e) \cos(\theta_e) + v_e \cos(\psi_e) + w_e \sin(\psi_e) \\
&\quad + u_d \sin(\psi_e) \cos(\theta_e) - \sin(\psi_d) \cos(\theta_d) \\
&\quad - v_d \cos(\psi_d) \\
&\quad + w_d \sin(\psi_d) \sin(\theta_d) \\
\dot{z}_e &= -u_e \sin(\theta_e) + v_e \cos(\theta_e) - u_d (\sin(\theta_e) - \sin(\theta_d)) \\
&\quad + w_d (\cos(\theta_e) - \cos(\theta_d))
\end{align*}
\]

(11)

\[
\begin{align*}
\dot{\theta}_e &= q_e \\
\dot{\psi}_e &= r_e / \cos(\theta_d)
\end{align*}
\]

(12)

The vehicle trajectory tracking speed error variable is derived along the trajectories equations (4) and (9). The tracking speed error equation is obtained as follows

\[
\begin{align*}
\dot{u}_e &= \frac{m_{22}}{m_{11}} v_r - \frac{m_{33}}{m_{11}} w_i - \frac{X_u}{m_{11}} - \frac{X_{u|i}}{m_{11}} v_d \\
&\quad + \frac{1}{m_{11}} \tau_{ud} \\
\dot{v}_e &= \frac{m_{11}}{m_{22}} u_d v_d - \frac{Y_v}{m_{22}} v_d - \frac{Y_{v|i}}{m_{22}} v_d v_d \\
\dot{w}_e &= \frac{m_{11}}{m_{33}} u_d w_d - \frac{Z_w}{m_{33}} w_d - \frac{Z_{w|i}}{m_{33}} w_d w_d \\
\dot{q}_e &= \frac{(m_{33} - m_{11})}{m_{55}} u_d w_d - \frac{M_d}{m_{55}} q_d - \frac{M_{d|i}}{m_{55}} q_d v_d \\
&\quad - \frac{BGM \sin(\theta)}{m_{55}} + \frac{1}{m_{55}} \tau_{qd} \\
\dot{r}_e &= \frac{(m_{11} - m_{22})}{m_{66}} u_d v_d - \frac{N_r}{m_{66}} v_d - \frac{N_{r|i}}{m_{66}} v_d v_d \\
&\quad - \frac{BGM \sin(\theta)}{m_{66}} + \frac{1}{m_{66}} \tau_{rd}
\end{align*}
\]

(13)

To facilitate calculation, the trajectory tracking position error equation is written as the following vector form

\[
P_e(t) = P(t) - P_d(t) =
\begin{bmatrix}
x_e \\
y_e \\
z_e
\end{bmatrix}
\]

(15)

Defining the line speed error \(v_e\) of the underactuated AUV is the virtual control input of the position control system, then

\[
v_e(t) = v(t) - v_d(t) =
\begin{bmatrix}
u_e \\
v_e \\
w_e
\end{bmatrix}
\]

(16)

Rewrite equation (11) into the following vector form

\[
\dot{P}_e = \dot{P} - \dot{P}_d = S(\theta, \psi) v_e + \left( S(\theta, \psi) - S(\theta_d, \psi_d) \right) v_d
\]

(17)

where \(S(\theta, \psi) =
\begin{bmatrix}
\cos(\psi) \cos(\theta) & -\sin(\psi) & \cos(\psi) \sin(\theta) \\
\sin(\psi) & \cos(\psi) & \sin(\psi) \sin(\theta) \\
-\sin(\theta) & 0 & \cos(\theta)
\end{bmatrix}\)

is a rotational transformation matrix from body-fixed frame.
to earth-fixed frame, which ignores vehicle’s roll movement and has the following properties

\[ \dot{S}(\theta, \psi) = S(\theta, \psi)S(q, r) \]  
(18)

where \( S(q, r) = \begin{bmatrix} 0 & -r & q \\ r & 0 & r \tan \theta \\ -q & -r \tan \theta & 0 \end{bmatrix} \).

To facilitate subsequent calculations, the following two lemmas are introduced.

**Lemma 1.** For a given limited time globally stable system\(^{24,25}\)

\[ x = -\eta|x|^\alpha \text{sgn}(x) \]  
(19)

where \( \eta > 0, \alpha \in (0, 1) \). For any initial state \( x(t_0) \), the state will converge to zero at time \( t = t_0 + |x(t_0)|^{1-\alpha}/c(1-\alpha) \). Then the system can be stabilized by the time \( T(x(t_0)) = |x(t_0)|^{1-\alpha}/c(1-\alpha) \).

**Lemma 2 (Barbalat).** Considering function \( f(t) \in L_\infty \), if \( p \in [1, \infty) \), function \( f(t) \) satisfies \( f(t) \in L_p \), then \( \lim_{t \to \infty} f(t) = 0 \).\(^{23}\)

**Controller design**

### The AUV model and frame transformation

**Step 1.** To achieve the target of trajectory tracking, the following first-order nonlinear sliding surface is designed to make the position error \( P_e(t) \) converge to zero

\[ S_0(t) = P_e(t) + \lambda_0 \int_0^t \text{sgn}^a(P_e(t)) \, dt - P_e(0) \]  
(20)

where \( S_0(t) = [S_{01}(t), S_{02}(t), S_{03}(t)]^T \) is a vector consisting of first-order nonlinear sliding modes corresponding to three position errors; \( \lambda_0 = \text{diag}(\lambda_{01}, \lambda_{02}, \lambda_{03}) \) is a diagonal coefficient matrix composed of integral coefficients of three first-order sliding surfaces and satisfied \( \lambda_{01}, \lambda_{02}, \lambda_{03} > 0 \); \( \text{sgn}^a(P_e) = [|x_e|^\alpha \text{sgn}(x_e), |y_e|^\alpha \text{sgn}(y_e), |z_e|^\alpha \text{sgn}(z_e)]^T \), where \( \alpha_1, \alpha_2, \alpha_3 \in (0, 1) \). It has the property \( S_0(0) = 0 \). Therefore, the designed sliding surface is initially on the sliding surface and can directly enter the sliding section.

The derivatives of equation (20) are obtained as

\[ \dot{S}_0(t) = \dot{P}_e(t) + \lambda_0 \text{sgn}^a(P_e(t)) \]  
(21)

While \( \dot{S}_0(t) = 0 \), then

\[ \dot{P}_e(t) = -\lambda_0 \text{sgn}^a(P_e(t)) \]  
(22)

Expand the above formula, that is,

\[ \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{z}_e \end{bmatrix} = \begin{bmatrix} -\lambda_{01} |x_e|^\alpha \text{sgn}(x_e) \\ -\lambda_{02} |y_e|^\alpha \text{sgn}(y_e) \\ -\lambda_{03} |z_e|^\alpha \text{sgn}(z_e) \end{bmatrix} \]  
(23)

Since the position error equation state is completely measurable, designing an equivalent control law eliminates positional errors. Combining equation (3) and \( S_0(t) = 0 \), the expected control law of virtual control input \( v_e \) of the position error system can be obtained as

\[ \alpha_v = -S(\theta, \psi)^T \lambda_0 \text{sgn}^a(P_e(t)) \]
\[ - S(\theta, \psi)^T (S(\theta, \psi) - S(\theta_d, \psi_d)) v_d \]  
(24)

where \( \alpha_v = [\alpha_u, \alpha_v, \alpha_w]^T \). It consists of the desired control law of the corresponding linear velocity \( v_e = [u_e, v_e, w_e]^T \).

From Lemma 1, the designed desired control law \( \alpha_v \) can eliminate the position error, and the time for \( P_e \) to converge to zero in a finite time is

\[ \begin{align*}
& t_{c1} = \frac{|x_e(0)|^{\alpha_1}}{\lambda_{01}(1-\alpha_1)}, \\
& t_{c2} = \frac{|y_e(0)|^{\alpha_2}}{\lambda_{02}(1-\alpha_2)}, \\
& t_{c3} = \frac{|z_e(0)|^{\alpha_3}}{\lambda_{03}(1-\alpha_3)}
\end{align*} \]  
(25)

Considering that \( \alpha_v = [\alpha_u, \alpha_v, \alpha_w]^T \) is not a real control input, the virtual control error is defined as

\[ e = \dot{v}_e - \alpha_v = \begin{bmatrix} e_u \\ e_v \\ e_w \end{bmatrix} = \begin{bmatrix} u_e - \alpha_u \\ v_e - \alpha_v \\ w_e - \alpha_w \end{bmatrix} \]  
(26)

The derivatives of equation (13) are obtained as

\[ \begin{align*}
\dot{e}_u &= m_{22} \frac{m_{33}}{m_{22}} v_r - m_{33} w_q - X_u - X_{uw}|u| - \dot{u}_d \\
& \quad - \dot{\alpha}_u + \frac{1}{m_{11}} \omega_u + \frac{\tau_u(t - T)}{m_{11}} \\
\dot{e}_v &= m_{11} \frac{m_{22}}{m_{33}} u - \frac{m_{33}}{m_{33}} v - Y_v|v| - \dot{v}_d - \dot{\alpha}_v + \frac{\omega_v}{m_{22}} \\
\dot{e}_w &= \frac{m_{11}}{m_{33}} u - \frac{Z_w}{m_{33}}|w| - W_w|w| - \dot{w}_d - \dot{\alpha}_w + \frac{\omega_w}{m_{33}}
\end{align*} \]  
(27)

From equations (24), (26), and (17), we have

\[ \dot{P}_e = -\lambda_0 \text{sgn}^a(P_e) + S(\theta, \psi)e = f_1(t, P_e) + g(t, P_e, e) \]  
(28)

From equation (27), we obtain

\[ \dot{e} = f_2(t, e) \]  
(29)
Therefore, the underactuated AUV trajectory tracking time-delay control system can be represented by two subsystems as follows

\[
\begin{align*}
\sum_1: \dot{P}_e &= f_1(t, P_e) + g(t, P_e, e) \\
\sum_2: \dot{e} &= f_2(t, e)
\end{align*}
\]  
(30)

It is known from the stability theorem of cascade system\(^2\): If the two subsystems are globally uniformly ultimately bounded, then the system equation (30) is globally uniformly ultimately bounded.

**Design of virtual controller**

In step 1, the stabilization problem of trajectory tracking error has been transformed into the stabilization problem of virtual control error. Now controllers need to be designed to stabilize the subsystem \(\sum_2\), and the virtual control error \(e = [e_u, e_r, e_w]^T\) converges to zero. Considering the input time delays of the system, the vehicle five-degree-of-freedom motion mechanics time-delay model is

\[
\dot{v} = M^{-1}[\tau(t - T') - C(v)v - D(v)v - g(\eta) - \tau_d]
\]  
(31)

Considering that the input time delay has an essential effect on the system is the hysteresis output of the control force and the control torque, further delaying the change in motion state. Therefore, the input time delay can be equivalent to the state time delay of the system. From equation (31)

\[
\dot{\tau}(t - T) = M^{-1}\begin{bmatrix} \tau - C(v)v(t - T) \\ -D(v)v(t - T) - g(\eta) - \tau_d \end{bmatrix}
\]  
(32)

where \(T\) is an unknown bounded state time delay equivalent to the input time delay, and the upper bound is \(T_{\text{max}}\). The following formula is established

\[
\dot{\tau}(t - T) = \frac{\partial v}{\partial(t - T)} \frac{\partial(t - T)}{\partial t} = \dot{v}(t)
\]  
(33)

From equations (33) and (32), we obtain

\[
\dot{\tau}(t) = M^{-1}\begin{bmatrix} \tau - C(v)v(t - T) \\ -D(v)v(t - T) - g(\eta) - \tau_d \end{bmatrix}
\]  
(34)

**Step 2.** For the roll virtual control error, the following first-order nonlinear sliding surface is defined as

\[
S_1 = e_u + \lambda_1 \int_0^t e_u(l) dl, \quad \lambda_1 > 0
\]  
(35)

The derivatives of equation (35) are obtained as

\[
\dot{S}_1 = \dot{e}_u + \lambda_1 e_u
\]  
(36)

\[
\dot{S}_1 = \frac{m_{22}}{m_{11}}v(t - T)r(t - T) - \frac{m_{33}}{m_{11}}w(t - T)q(t - T)
\]
\[
- \frac{X_u}{m_{11}}u(t - T) - \frac{X_{u|u|}}{m_{11}}u(t - T)|u(t - T)|
\]
\[
- \dot{u}_d - \dot{\tau}_u + \frac{\tau_u}{m_{11}} + \lambda_1 e_u + \omega_1
\]  
(37)

While

\[
\dot{r}_{eq} = \frac{\ddot{m}_{22}}{m_{11}}v(t - T)r(t - T) - \frac{\ddot{m}_{33}}{m_{11}}w(t - T)q(t - T)
\]
\[
- \frac{\ddot{X}_u}{m_{11}}u(t - T) - \frac{\ddot{X}_{u|u|}}{m_{11}}u(t - T)|u(t - T)| - \dot{u}_d
\]
\[
- \dot{\tau}_u + \lambda_1 u_e
\]  
(38)

where \(\ddot{\ }\) is the estimated value of the model uncertainty parameter. \(\ddot{\ }\) is the upper bound of the model uncertainty parameter, and satisfies the following boundary conditions

\[
|m_{11} - \ddot{m}_{11}| \leq \dddot{m}_{11}, |m_{22} - \dddot{m}_{22}| \leq \dddot{m}_{22}, |m_{33} - \dddot{m}_{33}| \leq \dddot{m}_{33},
\]
\[
|X_u - \dddot{X}_u| \leq \dddot{X}_u, |X_{u|u|} - \dddot{X}_{u|u|}| \leq \dddot{X}_{u|u|},
\]

From equations (38) and (24), we have

\[
\dot{S}_1 = \frac{1}{m_{11}}\tau_u + \omega_1 + \dot{r}_{eq}
\]  
(39)

where \(\omega_1\) is the bounded environmental interference. Choose the constant upper bound as \(\xi_1\).

Choose following saturation function to weaken the buffet caused by the sign function in the sliding mode

\[
sat(S_1/\Delta_i) = \begin{cases} 
1, & S_1 > \Delta_i \\
S_1/\Delta_i, & |S_1/\Delta_i| \leq 1 \\
-1, & S_1 < \Delta_i
\end{cases}
\]  
(40)

where \(\Delta_i\) is usually chosen as a small positive value. Define a boundary layer with an arbitrarily small thickness around \(S_1\).

Since the subsystem \(\sum_2\) model is constrained by parameter uncertainty, the following constant velocity approach law is designed to eliminate this negative effect

\[
\eta_1 = \frac{\dddot{m}_{22}}{m_{11}}|v(t - T)r(t - T)| + \frac{\dddot{m}_{33}}{m_{11}}|w(t - T)q(t - T)|
\]
\[
+ \frac{\dddot{X}_u}{m_{11}}|u(t - T)| + \frac{\dddot{X}_{u|u|}}{m_{11}}u^2(t - T)
\]
\[
+ |\dot{u}_d| + |\dot{\tau}_u| + \lambda_1 |u_e| + \rho_1
\]  
(41)

where \(\rho_1\) is the positive constant to be determined.
To eliminate the effects of time delays, uncertainty parameter, and external disturbances, the longitudinal control law is designed as
\[
\tau_u = m_{11}[-\hat{\tau}_{eq} - \eta_1 \text{sat}(S_1/\Delta_1) - K_1 S_1 - H_1 S_1(t - T_m)]
\]
(42)
where \( \eta_1 \geq \xi_1, K_1 \) is a positive constant, \( H_1 \) is the gain coefficient matrix to be determined.

**Step 3.** For the pitch virtual control error, the following second-order nonlinear sliding surface is designed as
\[
S_2 = \dot{e}_v + 2\lambda_2 e_v + \lambda_2^2 \int_0^t e_v(l) \, dl, \; \lambda_2 > 0
\]
(43)
The derivatives of equation (43) are obtained as
\[
\dot{S}_2 = \ddot{e}_v + 2\lambda_2 \ddot{e}_v + \lambda_2^2 e_v
\]
(44)
Taking the derivatives of equation (27)
\[
\dot{e}_v = -\frac{m_{11}}{m_{22}} (\alpha u + \alpha_\mu \tan \theta) - \frac{Y_v}{m_{22}} \dot{v} - \frac{2 Y_v}{m_{22}} \dot{v} |v(t)| - \dot{v}_d - \alpha \dot{v} + \frac{1}{m_{22}} \omega_v
\]
\[
= \frac{m_{22} (\alpha u + \alpha_\mu \tan \theta) - m_{11} u}{m_{22} m_{66}} \left( \frac{(m_{11} - m_{22}) u (t - T)v(t - T)}{m_{22} m_{66}} \right) - N_v r(t - T) + \tau_r + \omega_v
\]
\[
-\frac{m_{11}}{m_{22}} \dot{u} (t - T) - \frac{Y_v}{m_{22}} \dot{v} |v(t)| - \dot{v}_d + \frac{1}{m_{22}} \omega_v - \Theta_r
\]
(45)
Submitting equation (45) into equation (44) produces
\[
\dot{S}_2 = \frac{\beta_r}{m_{22} m_{66}} \tau_r - \frac{\kappa_r}{m_{22} m_{66}} + \omega_2
\]
(46)
where \( \omega_2 \) is a bounded environmental interference. Select the constant upper bound as \( \xi_2, \beta_r = m_{22} (\alpha u + \alpha_\mu \tan \theta) - m_{11} (e_u + \alpha u + u_d), \) and
\[
\kappa_r = \left( m_{22} (\alpha u + \alpha_\mu \tan \theta) - m_{11} (e_u + \alpha u + u_d) \right)
\]
\[
\left( (m_{11} - m_{22}) u (t - T)v(t - T) \right)
\]
\[
- N_v r(t - T) + \Theta_r - \lambda_2^2 e_v
\]
Design the following constant velocity approach law to eliminate the negative effects of parameter uncertainty of subsystem \( \sum_2 \)
\[
\eta_2 = \frac{\kappa_r}{m_{22} m_{66}} + \rho_2
\]
(47)
where \( \rho_2 \) is a positive constant to be determined. The uncertain model parameters involved satisfy the following boundary conditions
\[
|m_{66} - m_{56}| \leq \tilde{m}_{66}, \; |Y_v - \tilde{Y}_v| \leq \tilde{Y}_v, \; |Y_{vi} - \tilde{Y}_{vi}| \leq \tilde{Y}_{vi}
\]
\[
|N_v - \tilde{N}_v| \leq \tilde{N}_v, \; |N_{vi} - \tilde{N}_{vi}| \leq \tilde{N}_{vi}
\]
\[
\tilde{\kappa}_r = \left( \tilde{m}_{22} (|\alpha u| + |\alpha_\mu \tan \theta| + \tilde{m}_{11} (|e_u| + |\alpha u| + |u_d|) \right)
\]
\[
\left( (\tilde{m}_{11} + \tilde{m}_{22}) u (t - T)v(t - T) \right)
\]
\[
+ \tilde{N}_v r(t - T) + \tilde{N}_{vi} \dot{r}(t - T)
\]
\[
+ \tilde{m}_{11} \tilde{m}_{66} \dot{u} (t - T) + \tilde{m}_{66} |v(t - T)| + \tilde{m}_{22} \tilde{m}_{66} (|\dot{v}_d| + |\Theta_r| + 2\lambda_2 |\dot{e}_v| + \lambda_2^2 |e_v|)
\]
To eliminate the effects of time delays, uncertainty parameter, and external disturbances, the pitch control law is designed as
\[
\tau_r = \frac{\rho_r}{\beta_r}
\]
(48)
where \( \eta_2 \geq \xi_2, K_2 \) is a positive constant, \( H_2 \) is the gain coefficient matrix to be determined.

**Step 4.** For the yaw virtual control error, the following second-order nonlinear sliding surface is designed as
\[
S_3 = \dot{e}_w + 2\lambda_3 e_w + \lambda_3^2 \int_0^t e_w(l) \, dl, \; \lambda_3 > 0
\]
(49)
The derivatives of equation (49) are obtained as
\[
\dot{S}_3 = \ddot{e}_w + 2\lambda_3 \ddot{e}_w + \lambda_3^2 e_w
\]
(50)
Similar to step 3, we can obtain
\[
\dot{S}_3 = \frac{\beta_q}{m_{33} m_{55}} \tau_q - \frac{\kappa_q}{m_{33} m_{55}} + \omega_3
\]
(51)
where \( \omega_3 \) is a bounded environmental interference. Select the constant upper bound as \( \xi_3, \beta_q = m_{33} \alpha_u - m_{11} (e_u + \alpha u + u_d), \) and
\[
\kappa_q = \left( m_{11} (e_u + \alpha u + u_d) + m_{33} \alpha_u \right)
\]
\[
\left( (m_{33} + m_{11}) u v - M_0 q - M_{q|q} |q| - B G M_{\sin \theta} \right)
\]
\[
+ m_{11} m_{55} u q - m_{55} Z_{3z} \ddot{w} - 2 Z_{3w} |w| |\dot{w}| \]
\[
- m_{33} m_{55} (\dot{w}_d + \Theta_q - 2\lambda_3 \dot{e}_w - \lambda_3^2 e_w)
\]
Design the following constant velocity approach law to eliminate the negative effects of parameter uncertainty of subsystem \( \sum_2 \)
\[
\eta_3 = \frac{\kappa_q}{m_{33} m_{55}} + \rho_3
\]
(52)
where \( \rho_3 \) is a positive constant to be determined. The uncertain model parameters involved satisfy the following boundary conditions
Define the sliding surface vector as $\zeta$ globally uniformly ultimately bounded stable. The time-delay sliding mode surface converges and the system is globally uniformly ultimately bounded, the gain matrix of designed control law satisfies Assumptions 1 to 3, then the sliding surface designed as

$$
\frac{\hat{\zeta}_q}{m_{33}} = \left( \begin{array}{c}
\hat{m}_{11}[(e_q + |o_q| + |u_q|) + m_{33}|o_q|] \\
(\hat{m}_{33} - \hat{m}_{11})|o_q| + \hat{M}_{q}q + \hat{M}_{q}q^2 + \hat{B}Gd_{q}^{\sin} \theta \\
m_{33} - \hat{m}_{11}u_q + m_{33}|u_q| - 2\hat{Z}_w|u_q|w_u
\end{array} \right)$$

where $\eta_3 \geq \xi_3$, $K_3$ is a positive constant, $H_3$ is the gain coefficient matrix to be determined.

### System stability analysis

**Theorem 1.** For the underactuated unmanned underwater vehicle (UUV) three-dimensional trajectory tracking system, if the UUV kinematic model (3) and the dynamic model (4) satisfy Assumptions 1 to 3, then the sliding surface designed by equations (35), (43), and (49) and the control law shown in equations (42), (48), and (53) when the control gain matrix $\lambda_0 = \text{diag}(3.94, 3.1475, 1.888)$, adjustable gain matrix as $\hat{H} = \text{diag}(H_1 H_2 H_3)$, bounded interference as $\omega = [\omega_1 \omega_2 \omega_3]^T$, and satisfied $|\omega_i| < \xi_i$, $i = 1, 2, 3$; $\eta = \text{diag}(\eta_1 \eta_2 \eta_3)$, $\eta_i < \xi_i$, $i = 1, 2, 3$.

The Lyapunov–Krasovskii functional of the integral term with time delay is chosen as follows

$$
V = \frac{1}{2} \dot{S}^T S + \int_{t-T_m}^t \dot{S}^T P S \, dt
$$

where $P \in R^3$ is a positive diagonal matrix. Obviously $V \geq 0$ is established. Combined with the designed controller, the equation (54) is derived along the sliding surface, which can be obtained as

$$
\dot{V} = \dot{\zeta}^T \dot{\zeta} + \dot{S}^T P S - \dot{S}^T(t - T_m) P S(t - T_m)
$$

For the underactuated AUV time-delay control algorithm designed in this article, and robustness to model parameters uncertainty and disturbances, the yaw control law is designed as

$$
\tau_y = \frac{\hat{m}_{33}m_{55}}{\beta_q} \left[ \frac{\hat{\zeta}_q}{m_{33}m_{55}} - \eta_3 \text{sat}(S(T_3) - K_3S_3 - H_3S_3(t - T_m)) \right]
$$

To eliminate the effects of time delays, uncertainty parameter, and external disturbances, the yaw control law is designed as

$$
\tau_y = \frac{\hat{m}_{33}m_{55}}{\beta_q} \left[ \frac{\hat{\zeta}_q}{m_{33}m_{55}} - \eta_3 \text{sat}(S(T_3) - K_3S_3 - H_3S_3(t - T_m)) \right]
$$

where $\eta_3 \geq \xi_3$, $K_3$ is a positive constant, $H_3$ is the gain coefficient matrix to be determined.

### Simulation results and analysis

To verify the performance of the underactuated AUV time-delay control algorithm designed in this article, and robustness to model parameters uncertainty and disturbances, the spiral descent trajectory was selected, and the following two comparative simulation tasks were designed:

1. The true value of time delays is 0.5 s. Considering that it may face a worse situation, the upper bound
of the time delay is 0.52 s. Select TDSMC given in equations (42), (48), and (53).

(2) The true value of time delays is 0.5 s. The upper bound of the time delay is 0.52 s. Select SMC as

$$
\tau'_u = m_1 \left( -\tilde{\tau}_{eq} - \eta_1 \text{sat}(S_1/\Delta_1) \right)
$$

$$
\tau' q = \frac{m_{33} \overline{m}_{55}}{\beta_{\dot{q}}} \left[ \frac{\bar{\dot{q}}}{m_{33} \overline{m}_{55}} - \eta_3 \text{sat}(S_3/\Delta_3) \right]
$$

$$
\tau'_r = \frac{m_{22} \overline{m}_{66}}{\beta_{\dot{r}}} \left[ \frac{\bar{\dot{r}}}{m_{22} \overline{m}_{66}} - \eta_2 \text{sat}(S_2/\Delta_2) \right]
$$

Figure 2. Three-dimensional trajectory tracking curve with time delays: (a) time-delay sliding mode control and (b) sliding mode control.

AUV: autonomous underwater vehicle.

Table 1. Underactuated AUV desired trajectory and related parameters.

| Spiral desired reference trajectory | xₜ = 100sin(0.01t) |
|-------------------------------------|---------------------|
|                                     | yₜ = 100cos(0.01t)  |
|                                     | zₜ = 0.1t           |

Underactuated AUV initial state:

| x₀ | y₀ | z₀ |
|----|----|----|
| 10 | 90 | 10 |

Controller parameter:

| η | = 10⁻² × diag(4 5 0.5) |
|---|------------------------|
| P | = 10⁻⁴ × diag(1.25 1 0.6) |
| H | = 10⁻⁴ × diag(1.5 1.5 0.4) |
| K | = 10⁻² × diag(3.94 3.1475 1.888) |
| λ₀ | = diag(3.94 3.1475 1.888) |
| λ₁ | = 1.5 |
| λ₂ | = 1.5 |
| λ₃ | = 1.5 |
| α₁ | = 0.9 |
| α₂ | = 0.9 |
| α₃ | = 0.9 |

Figure 3. (a to c) Position tracking error with time delays.

Table 1. Underactuated AUV desired trajectory and related parameters.

| Spiral desired reference trajectory | xₜ = 100sin(0.01t) |
|-------------------------------------|---------------------|
|                                     | yₜ = 100cos(0.01t)  |
|                                     | zₜ = 0.1t           |

Underactuated AUV initial state:

| x₀ | y₀ | z₀ |
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| η | = 10⁻² × diag(4 5 0.5) |
|---|------------------------|
| P | = 10⁻⁴ × diag(1.25 1 0.6) |
| H | = 10⁻⁴ × diag(1.5 1.5 0.4) |
| K | = 10⁻² × diag(3.94 3.1475 1.888) |
| λ₀ | = diag(3.94 3.1475 1.888) |
| λ₁ | = 1.5 |
| λ₂ | = 1.5 |
| λ₃ | = 1.5 |
| α₁ | = 0.9 |
| α₂ | = 0.9 |
| α₃ | = 0.9 |

The model parameters used in the simulations are:

- $m = 185$ kg, $I_z = 50$ kg m², $X_{\dot{u}} = -30$ kg, $Y_{\dot{v}} = -80$ kg, $N_{\dot{r}} = -30$ kg, $X_{\dot{u}} = 70$ kg/s, $X_{\dot{u}|\dot{u}|} = 100$ kg/s, $Y_{\dot{v}} = 100$ kg/m, $Y_{\dot{v}|\dot{v}|} = 200$ kg/m, $Z_{\dot{w}} = 100$ kg/s, $Z_{\dot{w}|\dot{w}|} = 200$ kg/m²/s, $N_{\dot{r}|\dot{r}|} = 100$ kg m², $M_{\dot{q}} = 50$ kg m²/s, $M_{\dot{q}|\dot{q}|} = 100$ kg m². To verify the robustness of the trajectory tracking controller to parameter uncertainties, the perturbation of ±10% is added to the model parameters of the vehicle. As to external disturbances, the external disturbance values in the simulation are $\omega_u = 0.3$ N, $\omega_v = 0.1$ N, $\omega_w = 0.1$ N, $\omega_q = 0.1$ N and $\omega_r = 0.3$ N. The expected trajectory and related parameters in the simulation task are presented in Table 1.

Simulation results are shown in Figures 2 to 9.

In the simulations, the control effects between the TDSMC and the existing conventional SMC are compared. Among them (a) is TDSMC and (b) is SMC.

Figure 2 compares the dive spiral trajectory tracking of the time-delay sliding mode controller and the sliding mode controller in the same time delays. It can be revealed that under the control of the designed TDSMC, the vehicle achieves smooth tracking of the spiral dive reference trajectory. And it has novel robustness to model parameter perturbation and disturbances. The vehicle also tracks the reference trajectory under the control of the SMC. However, under the control of SMC, there is a large oscillation in the entire tracking process. The above analysis shows that the control parameters of the SMC are greatly affected by the input time delays.
Figure 4. (a to c) Trajectory tracking translation speed response curve with time delays.

Figure 5. (a to c) Velocity tracking error with time delays.

Figure 6. (a and b) Attitude angle tracking error with time delays.

Figure 7. (a and b) Trajectory tracking angular velocity response curve with time delay.

Figure 8. (a and b) Angular velocity tracking error with time delay.

Figure 9. (a to c) Control input force and moment with time delays.
Figures 3, 4, and 5 are position tracking error with time delays, trajectory tracking translation speed response curve with time delays, and velocity tracking error with time delays, respectively. From the simulation results of TDSMC, it can be revealed that within 100 s after the start of the task, due to the large position tracking error, the longitudinal and transverse velocities respond quickly to track the desired position. After 100 s, the underactuated AUV tracks the desired reference trajectory. The linear velocities also converge to the expected value. $u_e, v_e, w_e$ converge to a small area of origin. Comparing the simulation results of SMC, it can be revealed that although the position error of the vehicle also converges to zero, the linear velocity response and the speed tracking error also oscillate and cannot converge smoothly to the origin.

Figures 6, 7, and 8 are attitude angle tracking error, trajectory tracking angular velocity response curve, and angular velocity tracking error with time delays, respectively. From the simulation results of TDSMC, it can be revealed that within 100 s after the start of the task, after the task starts 100 s, AUV tracks the position and starts spiral dive. The pitch angular velocity $q$ and the yaw angular velocity $r$ have a larger response than before. The final angular velocity error also converges to zero. Comparing the simulation results of SMC, it can be revealed that there is a very serious oscillation on the attitude angle tracking. This is consistent with the severe oscillations that occur on the actual trajectory in Figure 2. SMC cannot control the vehicle to smoothly track the desired trajectory.

Figure 9 is the control input of the underactuated AUV under the two controllers. In the trajectory tracking control of the vehicle, although the actual actuator delay is small, if the controller design is simplified by directly ignoring the time delays, it will cause severe control oscillations. This oscillation may cause significant wear, shorten the service life, and even not under command on the actuator hardware.

**Conclusion**

This article presents a trajectory tracking SMC for underactuated AUVs with time delays, model parameter uncertainty, and external disturbances. The integral TDSMC law is designed for the trajectory tracking pose and velocity error of the cascade subsystem. Based on the Lyapunov–Krasovskii functional, the stability conditions of the LMI form are obtained, which guarantees that the designed controller is globally stable. The simulation results reveal that under the designed controller, the vehicle realizes the trajectory tracking control target under the constraint condition. The control accuracy has reached the expected target.

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