Charming penguins in $B \to PP$ decays and the extraction of $\gamma$

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Abstract

It is shown that inclusion of charming penguins of the size suggested by short-distance dynamics may shift down by $10^\circ - 15^\circ$ the value of $\gamma$ extracted via the overall fit to the $B \to PP$ branching ratios. A substantial dependence of the fit on their precise values is found, underscoring the need to improve the reliability of data.

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1 Introduction

Various methods of extracting the value of the unitarity-triangle angle $\gamma$ from data have been proposed in the literature. Some of them are based on the analysis of the decays of $B$ mesons into a pair of light pseudoscalar mesons $PP$, and in particular into the $K\pi$ states. With most present data on asymmetries in $B \to PP$ decays still carrying large errors, fits to the branching ratios and asymmetries of $B \to PP$ decays depend mainly on the former.

In the simplest approach [1] to these decays the full $B \to PP$ amplitudes are given in terms of only a few short-distance (SD) amplitudes corresponding to specific quark-line diagrams (tree $T$, colour-suppressed $C$, penguin $P$, singlet penguin $S$) expected to provide the dominant contributions. The penguin amplitude is furthermore assumed to be dominated by the contribution from the internal top quark propagation [2]. The only electroweak penguin that has to be kept is included through an appropriate replacement in colour-suppressed strangeness-changing amplitude. The value of angle $\gamma$ extracted from such analyses depends of course on strong SD phases and on possible modifications of the SD formulae by additional effects. Among the latter effects the issue of the size of rescattering contribution has been addressed by several investigators.

The rescattering (or final state interaction - FSI) contribution is composed of two main parts: the contribution in which the intermediate state contains charmed quarks (so-called charming penguins) [3, 4, 5, 6, 7], and the contribution from elastic and inelastic rescattering through intermediate states involving only light (ie. $u$, $d$, $s$) quarks [8, 9, 10, 11, 12, 13]. In a recent paper [14] the latter contribution was analysed in detail for the SU(3)-symmetry breaking case. The main conclusion of ref [14] was that inclusion of such effects may significantly affect the extracted value of angle $\gamma$. Namely, while for negligible strong SD phases the global fits to the branching ratios of all $B \to PP$ decays yielded the value of $\gamma \approx 100^\circ$, similar fits with rescattering effects included permitted values of $\gamma$ in a broad range of ($50^\circ$, $110^\circ$), and actually even preferred a value of $\gamma$ in qualitative agreement with SM expectations of $\gamma_{SM} \approx 65^\circ$.

Paper [14] left open the issue of the effect induced by charming penguins. Furthermore, the size of the contribution from inelastic rescattering, required for the shift of the extracted value of $\gamma$ down by some $30^\circ$, was a factor of five larger than the estimates of the size of quasi-elastic rescattering in a Regge model [11]. Given low experimental bounds on the size of the observed branching ratios of the $B \to K\bar{K}$ decays, which are thought to provide a bound on the size of rescattering effects, one
might therefore argue that these effects should be much smaller than those resulting from the fits of ref.[14].

In the present paper we address again the question of the size of corrections to the dominant $t$-quark contributions to penguin amplitudes, and show that shifts in the extracted value of $\gamma$ of the order of $10^\circ - 15^\circ$ may result from the inclusion of SD charming penguins. Furthermore, we observe that the use of the updated values of the $B \to PP$ branching ratios shifts the value of $\gamma$ extracted when no rescattering is considered down by $20^\circ$ when compared to the fit of [14]. Although for the recent values of branching ratios the agreement with the data is now worse than in ref.[14], the data do point out to a lower value of $\gamma$.

2 Dominant short-distance amplitudes

In this paper the dominant short-distance amplitudes are parametrized exactly as in [14]. Thus, we assume that all their strong phases are negligible. Although these phases may be non-zero [15, 16], their precise values are not relevant for what we want to discuss here: the aim of this paper is to look at uncertainties not related to these phases (as long as the latter remain small).

Thus, for the tree amplitudes we use

$$T' = \frac{V_{us} f_K}{V_{ud} f_\pi} T \approx 0.276 T$$

with (un)/primed amplitudes denoting strangeness (preserving)/changing processes. Both tree amplitudes have the same weak phase: $T'/|T'| = T'/|T'| = e^{i\gamma}$.

Assuming that the penguin SD amplitudes are dominated by the $t$ quark, the weak phase factor is $e^{-i\beta}$ for $P$ and $-1$ for $P'$ (ie. $P' = -|P'|$). We use the estimate [17]

$$P = -e^{-i\beta} |\frac{V_{td}}{V_{ts}}| P' \approx -0.176 e^{-i\beta} P'. \quad (2)$$

In the following we use $\beta \approx 24^\circ$, which is in agreement with the world average [18] $\sin 2\beta = 0.734 \pm 0.054$.

We accept the relations between the tree and the colour-suppressed amplitudes given by the SD estimates:

$$C = \xi T \quad (3)$$

and

$$C' = T' (\xi - (1 + \xi) \delta_{EW} e^{-i\gamma}) \quad (4)$$
where we take $\xi = C_1 + \zeta C_2 \approx 0.17$, assuming $\zeta \approx 0.42$, i.e. midway between $1/N_c$ and the value of 0.5 suggested by experiment, and using $C_1 \approx -0.31$ and $C_2 \approx 1.14$ [19]. The contribution from the electroweak penguin $P_{EW}'$ has been included in Eq.(4), with $\delta_{EW} \approx +0.65$ [20] (other electroweak penguins are neglected).

Finally, since data suggests that the singlet penguin amplitude $S'$ is sizable (cf. [21, 17]) we include it in our calculations as well, with weak and strong phases as for $P'$. The remaining SD amplitudes (exchange $E$ and $E'$, singlet penguin $S$, penguin annihilation $PA$ etc.) are neglected. Thus, the dominant SD amplitudes depend on four SD parameters: $|T|$, $P'$, $S'$, and the weak phase $\gamma$.

Because rescattering effects induced by Pomeron exchange are fully calculable, we correct for them following ref.[14] (the relevant theoretical formulae for all $B \rightarrow PP$ amplitudes in question are given in Table 1 therein). Actually, it is only when SU(3) is broken that these corrections are different for different decay channels, and the resulting deviations from the standard SD form could be observed.

As in ref.[14] we minimize the $\chi^2$ function defined as:

$$\chi^2 = \sum_i \frac{(B_i^{\text{the}} - B_i^{\text{exp}})^2}{(\Delta B_i)^2},$$

(5)

where $B_i^{\text{the(exp)}}$ denote theoretical (experimental) CP-averaged branching ratio for the $i$-th decay channel. We consider the same 16 decay channels as in ref.[14] (see Table 1). Their experimental branching ratios and errors taken from [22] are given in the second column of Table 1. These numbers differ from the ones used in [14] in a couple of entries, the most significant ones (i.e. where the new average is more than one old standard deviation away from the old average) being for $\pi^+\eta$, $\pi^+K^0$, and $\pi^0K^0$. In the calculations themselves, the branching ratios were corrected for the deviation of the ratio of the $\tau_{B^+}$ and $\tau_{B^0}$ lifetimes from unity (using $\tau_{B^+}/\tau_{B^0} = 1.086$). For a given value of $\gamma$ the $\chi^2$ function was minimized with respect to $|T|$, $P'$, and $S'$.

The resulting dependence on $\gamma$ is shown in Fig.1 as solid line. The fitted values of the branching ratios together with their deviations from the experimental numbers are given in columns 3 and 4 of Table 1. When comparing with the fits of ref.[14] one observes a strong shift of the minimum (from just above 100° in [14] to 82° here), and a significant increase in the size of $\chi^2$ (from 14.3 to 26.0). The size of both shifts underscores the need to improve the reliability of data. One observes that the updated fit has problems with the description of not only $\pi^0\pi^0$ and $\pi^0K^0$ as in ref.[14], but also, though to a lesser extent, with $\pi^+\pi^0$ and $\pi^+\eta$. 
3 Rescattering effects and short-distance charm-ing penguins

The fits of the preceding section assumed SD penguin amplitudes to be totally dominated by top quark contribution $P_t$. Various kinds of rescattering effects generate additional contributions due to intermediate $u$ and $c$ quarks, and may modify $P_t$ so that the full penguin contributions (denoted by $\tilde{P}$) may be written as:

\[ \tilde{P} = \lambda_u^{(d)} \tilde{P}_u + \lambda_c^{(d)} \tilde{P}_c + \lambda_t^{(d)} \tilde{P}_t \]  
\[ \tilde{P}' = \lambda_u^{(s)} \tilde{P}_u + \lambda_c^{(s)} \tilde{P}_c + \lambda_t^{(s)} \tilde{P}_t \]  

where

\[ \lambda_q^{(k)} = V_{qk}V_{qb}^* \]  

with $V$ being the CKM matrix.

Ref.[14] was concerned with contributions of $\tilde{P}_u$ type. In the SU(3)-symmetry breaking case studied in [14] this contribution varied from channel to channel. Its SU(3)-symmetric part was parametrized by a single complex parameter $d$ (one of three effective FSI parameters discussed in [14]), so that for SU(3)-symmetric FSIs all formulas for individual $B \to PP$ strangeness-changing amplitudes in [14] depended on a single FSI-corrected penguin amplitude:

\[ \tilde{P}' = P'_{SD}(1 + i3d) + idT'_{SD} \]  

where $P'_{SD} = \lambda_t^{(s)} P_t$, and $T'_{SD} = T' \propto \lambda_u^{(s)}$. The expression for $\tilde{P}$ was, of course, completely analogous. It was the $idT'$ term above which generated the $\lambda_u^{(s)} \tilde{P}_u$-type term of Eq.(7) in [14]. Thus, in the case of SU(3)-symmetric FSIs all rescattering effects not involving intermediate charmed quarks can be hidden into the $\lambda_u^{(k)} \tilde{P}_u$ term in Eqs.(6,7). (However, this cannot be done in a decay-channel-independent manner if FSI break SU(3), the case considered in [14].)

As discussed in [14], FSI effects may depend on two further effective parameters ($c$ and $u$). The first of them ($c$) takes care of ”crossed” quark-line diagrams and modifies the effective ”tree” and ”color-suppressed” diagrams. In refs.[13, 14] it was shown that nonzero value of $c$ leads to effective $\tilde{T}^{(s)}$ ($\tilde{C}^{(s)}$) amplitudes being mixtures of SD tree and colour-suppressed amplitudes with different strong phases. The penguin and singlet penguin get similarly mixed. Since in the fits of [14] small values of $c$ were obtained, we shall not be interested here in these corrections. Nonzero value of the other parameter ($u$) leads to effective annihilation $A$, exchange $E$ and penguin annihilation $PA$ amplitudes. Parameters $u$ and $d$ describe the contributions from
quasi-two-body intermediate states in which the two intermediate mesons belong to
multiplets classified by the same or different charge conjugation parities \( C \) [13]. If
only states composed of two pseudoscalar mesons contributed to the FSI effects, the
parameters \( u \) and \( d \) would be proportional to each other (\( u = d/2 \) in the normalization
of [13, 14]). Then, from the size of \( A, E, PA \) amplitudes from \( B^0_d \to K^+K^- \)
one could determine \( u \) and evaluate the size of rescattering contribution to penguin
amplitudes. However, intermediate states of \( C \) parity opposite to that of the PP
state may also contribute. The relation between \( u \) and \( d \) is then relaxed, and one
may have small \( B^0_d \to K^+K^- \) branching ratio and substantial FSI contribution to
penguin amplitudes. Ref.[14] was concerned with this possibility. The fits performed
in [14] suggest that the \( \tilde{P}_u \) term could be substantial. Since a large size of this term
may be questioned it would be worthwhile to find other arguments that could sup-
port one of the claims of ref.[14], namely that keeping only the \( \tilde{P}_t \) term may lead
to a significant error in the extracted value of \( \gamma \). We shall do that below on the
example of the SD charming penguin.

Using the unitarity property of the CKM matrix one may rewrite expressions
(6,7) as [23]:

\[
\tilde{P} = \lambda^{(d)}_t (\tilde{P}_c - \tilde{P}_u) + \lambda^{(d)}_t (\tilde{P}_t - \tilde{P}_u) \\
\tilde{P}' = \lambda^{(s)}_c (\tilde{P}_c - \tilde{P}_u) + \lambda^{(s)}_t (\tilde{P}_t - \tilde{P}_u)
\]

Since

\[
\lambda^{(d)}_t = -\lambda^{(s)}_t \left| \frac{V_{td}}{V_{ts}} \right| e^{-i\beta}
\]

\[
\lambda^{(d)}_c \approx \lambda^{(s)}_t \lambda
\]

\[
\lambda^{(s)}_c \approx -\lambda^{(s)}_t,
\]

where \( \lambda \approx 0.22 \) is the Wolfenstein parameter, for negligible \( \tilde{P}_u \) the above formulae
may be rewritten as

\[
\tilde{P} = -\lambda^{(s)}_t \tilde{P}_t \left( \left| \frac{V_{td}}{V_{ts}} \right| e^{-i\beta} - \lambda \zeta \right)
\]

\[
\tilde{P}' = \lambda^{(s)}_t \tilde{P}_t (1 - \zeta)
\]

with

\[
\zeta = \frac{\tilde{P}_c}{\tilde{P}_t}.
\]

For nonnegligible \( \zeta \) the simple connection (2) between \( P \) and \( P' \) gets modified to:

\[
\tilde{P} = -\tilde{P}' \frac{1}{1 - \zeta} \left( \left| \frac{V_{td}}{V_{ts}} \right| e^{-i\beta} - \lambda \zeta \right).
\]
Estimates of $\zeta$ using the perturbative approach of ref.[24] have been performed in ref.[23] with the result that

$$0.2 \lesssim \left| \frac{\tilde{P}_c - \tilde{P}_u}{\tilde{P}_t - \tilde{P}_u} \right| \lesssim 0.5 \quad (19)$$

and

$$70^\circ \lesssim \arg \frac{\tilde{P}_c - \tilde{P}_u}{\tilde{P}_t - \tilde{P}_u} \lesssim 130^\circ. \quad (20)$$

Although the above numbers are certainly very uncertain it is interesting to see how the inclusion of a charmed penguin of this size will affect the results of the fits of Section 2. In the fit discussed here we assume that $S'$ gets modified in a way completely analogous to that for $P'$ (cf. Eq.(16)). As Fig. 1 shows (for which we have selected the limiting cases of $\arg \zeta = 0$ and $180^\circ$), including penguin contributions from the charmed-quark loops (and assuming negligible $u$-quark terms) may shift down the extracted value of $\gamma$ significantly. Specifically, for $\zeta = 0.4$ the shift is of the order of $10^\circ$. However, the value of $\chi^2$ is not meaningfully smaller (Table 1). Furthermore, problems persist with the description of $B \to \pi^0 K^0$, $\pi^0\pi^0$, and $\pi^+\eta$ decays (Table 1). Slightly larger values of $\zeta$ may shift $\gamma$ much more (see Fig.1). In fact, some calculations suggest that the contributions from the charmed penguins could be much larger than the upper limit of Eq.(19). For comparison, the calculations in the second reference of [5] correspond to $|\zeta| \approx 2$, i.e. to charming penguins being dominant.

4 Conclusions

From the considerations of this paper it follows that:

1. shifts in the extracted value of $\gamma$, obtained in the fits with nonzero $\tilde{P}_c$ (and negligible $\tilde{P}_u$) of the size suggested by SD dynamics, are quite similar to those found in ref.[14] for nonzero $\tilde{P}_u$ (and vanishing $\tilde{P}_c$), and.

2. given the uncertainty in the size of both $u-$ and $c-$type penguins (as well as in the strong phases of all amplitudes), a reliable extraction of $\gamma$ requires using additional information (data on asymmetries), possibly combined with a judicious choice of data either insensitive or least sensitive to such uncertainties. This may be achieved by restricting the considerations to the analysis of the branching ratios and asymmetries of the $B \to \pi K$ decays [25]. Clearly, all information provided by the $B \to \pi K$ sector will be included in the fits to all $B \to PP$ decays, if these fits take into account not only the branching ratios but also the asymmetries. At present,
such fits based on the branching ratios only seem to depend quite strongly on the precise values of the latter.

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FIGURE CAPTIONS

Fig. . Dependence of $\chi^2$ on $\gamma$: a) $\tilde{P}_t$ only - solid line ($\zeta = 0$); b) $\tilde{P}_t$ with corrections: long-dashed line - $\zeta = 0.4$; short-dashed line - $\zeta = 0.6$; dotted line - $\zeta = -0.6$. 
Table 1: Fits to branching ratios of $B \rightarrow PP$ decays (in units of $10^{-6}$)

| decay   | expt  | $B$ deviation (in stand. dev's) | $B$ deviation (in stand. dev's) |
|---------|-------|---------------------------------|---------------------------------|
|         |       | $B_{\text{SD}}$ only           | $B_{\text{SD, with } \zeta = +0.4}$ |
| $B^+ \rightarrow \pi^+\pi^0$ | 5.3 ± 0.8 | 4.00 ± 1.0 | 4.53 ± 1.0 |
| $K^+\bar{K}^0$       | 0.0 ± 2.4 | 0.58 ± 0.2 | 0.56 ± 0.2 |
| $\pi^+\eta$        | 4.2 ± 0.9 | 2.66 ± 1.7 | 2.34 ± 2.1 |
| $\pi^+\eta'$       | 0.0 ± 4.5 | 1.29 ± 0.3 | 1.13 ± 0.3 |
| $B_d^0 \rightarrow \pi^+\pi^-$ | 4.6 ± 0.4 | 5.00 ± 1.0 | 4.93 ± 0.8 |
| $\pi^0\pi^0$       | 1.9 ± 0.5 | 0.47 ± 2.9 | 0.54 ± 2.7 |
| $K^+K^-$           | 0.0 ± 0.6 | 0.0 ± 0   | 0.0 ± 0 |
| $K^0\bar{K}^0$     | 0.0 ± 1.8 | 0.54 ± 0.3 | 0.52 ± 0.3 |
| $B^+ \rightarrow \pi^+K^0$ | 21.8 ± 1.4 | 21.04 ± 0.5 | 21.79 ± 0.0 |
| $\pi^0K^+$         | 12.8 ± 1.1 | 12.68 ± 0.1 | 12.61 ± 0.2 |
| $\eta K^+$         | 3.2 ± 0.7 | 2.53 ± 1.0 | 2.32 ± 1.3 |
| $\eta' K^+$        | 77.6 ± 4.6 | 76.44 ± 0.3 | 76.60 ± 0.2 |
| $B_d^0 \rightarrow \pi^-K^+$ | 18.2 ± 0.8 | 19.00 ± 1.0 | 18.76 ± 0.7 |
| $\pi^0K^0$         | 11.9 ± 1.5 | 7.76 ± 2.8 | 8.02 ± 2.6 |
| $\eta K^0$         | 0.0 ± 4.6 | 2.31 ± 0.5 | 2.28 ± 0.5 |
| $\eta' K^0$        | 65.2 ± 6.0 | 70.86 ± 0.9 | 71.68 ± 1.1 |
| $\chi^2$          | 26.0 | 23.7 |
| $|T|$              | 2.32 | 2.47 |
| $\tilde{P}^u$      | -4.48 | -4.56 |
| $\tilde{S}^u$      | -2.29 | -2.25 |
| $\gamma_{\text{fit}}$ | 82°  | 73°  |
Figure 1: