BPS Saturated Dyonic Black Holes of $N = 8$ Supergravity Vacua\footnote{Based on talks given by M.C. at the SUSY’95 Conference, May 8-12, 1995, Paris, France and the Conference on S-Duality and Mirror Symmetry, May 22-26, 1995, Trieste, Italy.}

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Abstract

We summarize the results for four-dimensional Bogomol’nyi-Prasad-Sommerfield (BPS) saturated dyonic black hole solutions arising in the Kaluza-Klein sector and the three-form field sector of the eleven-dimensional supergravity on a seven-torus. These black hole solutions break $\frac{3}{2}$ of $N = 8$ supersymmetry, fill out the multiplets of $U$-duality group, and the two classes of solutions are related to each other by a discrete symmetry transformation in $E_7$. Using the field redefinitions of the corresponding effective actions, we present these solutions in terms of fields describing classes of dyonic black holes carrying charges of $U(1)$ gauge fields in Neveu-Schwarz-Neveu-Schwarz and/or Ramond-Ramond sector(s) of the type-IIA superstring on a six-torus. We also summarize the dependence of their ADM masses on the asymptotic values of scalar fields, parameterizing the toroidal moduli space and the string coupling constant.
I. INTRODUCTION

Recently, there have been rapid developments in understanding of non-perturbative symmetries in superstring theories. These symmetries, called duality symmetries, relate equivalent vacua in string theories and, especially, S-duality [1,2] and U-duality [3] relate vacua of weakly coupled theories to those of strongly coupled ones and may shed light on non-perturbative structure of string vacua. Duality symmetries are integer valued, i.e., discrete, due to the Dirac charge quantization rules [4] and space-time and world-sheet instanton effects.

Ten-dimensional superstring theories compactified on a torus ($T^n$) possess $T$-duality symmetry [5,6], which relates equivalent points in toroidal moduli space [7] of string vacua. $T$-duality symmetries are exact symmetries of string vacua to all orders in string perturbation. Additionally, superstring theories are conjectured to have strong-weak coupling duality (S-duality) [1,2], which is a generalization of electric-magnetic duality for the case of the system with the dilaton and the axion (pseudo-scalar) fields [2]. S-duality is a non-perturbative symmetry, since it transforms the dilaton, whose vacuum expectation value parameterizes the string coupling constant, non-linearly.

It was recently recognized [3,8] that non-perturbative, BPS saturated states [9] play an important role in gathering evidence for non-perturbative duality symmetries of string vacua. In particular, along with the perturbative string excitations, the non-perturbative BPS saturated states are instrumental in establishing the equivalence of the two dual string vacua at the level on the full spectrum of states. In addition, at the points of moduli space where non-perturbative BPS states become massless [10–13] they play a crucial role in the full, low-energy string dynamics.

The study of BPS saturated states of string vacua is thus of importance. The approach is usually within the effective string action, usually exact in the leading order of $\alpha'$ expansion, only. There, one derives the Bogomol'nyi bound for the ADM mass of a class of non-perturbative configurations, and in some cases obtains the explicit solution for the BPS saturated states, which saturate the corresponding Bogomol'nyi bound, by solving the corresponding Killing spinor equations. In this classical solutions classical charges are replaced with quantized charges, which are consistent with the Dirac quantization condition and the charge constraints of the corresponding string theory. As a next step, it is important to establish the exactness of such semi-classical solutions to all orders in $\alpha'$ expansion [14]. Ultimately, the proper quantization of the corresponding states is needed.

Within four-dimensional string vacua, progress has been made in shedding light on BPS saturated states of $N = 4$ string vacua [1]. In particular, an explicit form of a general class of semi-classical, BPS saturated, spherically symmetric, static dyonic configurations was constructed [15], and parameterized in terms of fields of heterotic string compactified on a

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1 We shall suppress the coefficients, i.e., the set $Z$ of integers, upon which the duality groups are defined. So, for example, $SL(2)$ means the $S$-duality group $SL(2; Z)$, etc.

2 For a review, see Ref. [15] and references therein.
six-torus.

Recently, it was recognized [17,8] that eleven-dimensional supergravity is the strong coupling limit of the type-IIA superstring. Therefore, one can infer the strong coupling behavior of the compactified type-IIA superstring from the compactified eleven-dimensional supergravity. Hull and Townsend [3] conjectured that the type-II superstring compactified on a torus has a larger symmetry than \((T\text{-duality}) \times (S\text{-duality})\), i.e., \(SO(6,6) \times SL(2)\), and the integral version of the \(E_7\) symmetry of the \(N = 8\) supergravity are realized in the full spectrum of the type-IIA superstring on a torus. That is, since the \(S\text{-duality}\) symmetry of the ten-dimensional type-IIA theory does not commute with the \(T\text{-duality}\) symmetry of the type-IIA string on a torus, these two dualities generate a larger symmetry called \(U\text{-duality}\), i.e., \(E_7\), which contains \(SO(6,6) \times SL(2)\) as a maximal subgroup. Since the 16 Ramond-Ramond (RR) \(U(1)\) gauge fields couple to the string through their field strengths only, the fundamental string states of the type-IIA string carry only 12 \(U(1)\) electric charges of the Neveu-Schwarz-Neveu-Schwarz (NS-NS) sector, which mix among themselves under \(SO(6,6) \times SL(2)\). However, the \(U\text{-duality}\) symmetry requires the existence of additional 16 + 16 electric and magnetic charges in the RR sector which are carried by solitons [3], and transform as an irreducible spinor representation under \(SO(6,6)\). In Ref. [3] prescription was given, how to generate four-dimensional singly charged solitons with respect to each of 28 \(U(1)\) fields in the RR and the NS-NS sectors by compactifying p-brane solutions in ten dimensions.

In this contribution, we summarize the results of our previous work [18] on four-dimensional BPS saturated states of effective, four-dimensional \(N = 8\) string vacua. In this case the results are only partial and pertain to solutions arising from the following two sectors: either the Kaluza-Klein sector or the three-form field sector of the eleven-dimensional supergravity compactified on a seven-torus. These results, through the field redefinition, also provide an explicit construction of classes of dyonic black holes whose charges arise from the RR sector and/or the NS-NS sector(s) of type-IIA string on a torus. By finding solutions of the corresponding Killing spinor equations, we explicitly construct dyonic BPS saturated states, carrying charges of 28 \(U(1)\) fields in the RR and/or the NS-NS sector(s). These solutions fill out the multiplets of the \(U\text{-duality}\) group. Note, however, that some of the configurations are more general, i.e., they preserve only \(\frac{1}{4}\) of supersymmetry, and cannot be obtained by simply compactifying p-brane solutions in ten dimensions.

In Section II, we shall summarize the eleven-dimensional supergravity and discuss its compactification on \(T^7\). In Section III, we give the BPS saturated dyonic black hole solutions of eleven-dimensional supergravity on a seven-torus. In Section IV, field redefinitions relating the fields of eleven-dimensional supergravity on a seven-torus to those of the type-IIA supergravity on a six-torus are given, and the corresponding BPS saturated dyonic black holes carrying \(U(1)\) charges of the NS-NS and/or the RR sector are classified. Conclusions are given in Section V.
II. LAGRANGIAN OF ELEVEN-DIMENSIONAL SUPERGRAVITY ON SEVEN-TORUS

The low energy effective theory of massless states (at generic points of moduli space) of the type-IIA string in ten dimensions can be described by eleven-dimensional supergravity on $S^1$ \cite{19}. Eleven-dimensional supergravity can be thought of as the strong coupling limit of the type-IIA superstring in ten dimensions \cite{3}. Therefore, the strong coupling limit behavior of the lower dimensional type-IIA theory can be deduced from eleven-dimensional supergravity compactified down to lower dimensions.

Eleven-dimensional supergravity \cite{20} contains the Elfbein $E^{(11)A}_M$, gravitino $\psi^{(11)}_M$, and the three-form field $A^{(11)MNP}$ as its field content. The bosonic Lagrangian density is given by

$$\mathcal{L} = -\frac{1}{4}E^{(11)}[\mathcal{R}^{(11)} + \frac{1}{12}F^{(11)MNPQ}F^{(11)MNPQ} - \frac{8}{125}\varepsilon^{M_1\cdots M_{11}}F^{(11)M_1\cdots M_4}F^{(11)M_5\cdots M_8}A^{(11)M_9M_{10}M_{11}}],$$

(1)

where $F^{(11)MNPQ} \equiv 4\partial_{[M}A^{(11)NPQ]}$ is the field strength associated with the three-form field $A^{(11)MNP}$. In the bosonic background, the gravitino $\psi^{(11)}_M$ transforms under supersymmetry as

$$\delta\psi^{(11)}_M = D_M\varepsilon + \frac{i}{144}(\Gamma^{NPQR}_M - 8\Gamma_{NP\varepsilon}^{\varepsilon}_M)F^{(11)}_{NPQR}\varepsilon,$$

(2)

where $D_M\varepsilon = (\partial_M + \frac{1}{4}\Omega^{MAB}\Gamma^{AB})\varepsilon$ is the gravitational covariant derivative on the spinor $\varepsilon$, and $\Omega_{ABC} \equiv -\tilde{\Omega}_{ABC} + \tilde{\Omega}_{BC,A} - \tilde{\Omega}_{CA,B}$ (strike-through) is the spin connection defined in terms of the Elfbein.

The four-dimensional effective action can be obtained by compactifying the extra six spatial coordinates on a seven-torus $T^7$ by using the following Ansatz for the Elfbein:

$$E^{(11)}_M = \left(\begin{array}{ccc}
e^{-\frac{i}{4}\varepsilon^{\mu}} e_{\mu}^a & B_{\mu}^i e_{i}^a \\
0 & e_{i}^a \end{array}\right),$$

(3)

where $\varphi \equiv \ln \det e_{i}^a$ and $B_{\mu}^i$ ($i = 1,\ldots,7$) are Kaluza-Klein Abelian gauge fields. Eleven-dimensional supergravity on $T^7$ becomes $E_7$ on-shell symmetry and 70 scalar fields take values in the moduli space $E_7/[SU(8)/Z_2]$ \cite{21}. However, here we shall write down $SL(7)$ symmetric truncation of the Lagrangian for the purpose of obtaining black hole solutions carrying charges of Kaluza-Klein $U(1)$ or three-form $U(1)$ fields. These $U(1)$ fields will later on be related to various $U(1)$ gauge fields of the RR and the NS-NS sectors of the type-IIA superstring in order to obtain the corresponding dyonic solitons carrying $U(1)$ charges of the RR and/or the NS-NS sectors. The Lagrangian is given by

$$\mathcal{L} = -\frac{1}{4}[\mathcal{R} - \frac{1}{2}\partial_{\mu}\varphi\partial^{\mu}\varphi + \frac{1}{4}\partial_{\mu}g_{ij}\partial^{\mu}g^{ij} - \frac{1}{4}e^{\varphi}g^{ij}G_{\mu
u}^{i}G^{i\mu
u} + \frac{1}{2}e^{\varphi}g^{ik}g^{jl}F_{\mu
u}^{ij}F^{\mu\nu} + \cdots],$$

(4)

where $G_{\mu
u}^{i} \equiv \partial_{\mu}B_{\nu}^{i} - \partial_{\nu}B_{\mu}^{i}$, $F_{\mu
u}^{i} \equiv F_{\mu
u}^{i} + G_{\mu
u}^{i}A_{ij}$, and the dots ($\cdots$) denote the terms involving the pseudo-scalars $A_{ij}$ and the two-form fields $A_{\mu i}$. Here, $F_{\mu
u}^{ij}$ is the field strength of $A_{\mu ij} \equiv A_{\mu ij} - D_{\mu}A_{ij}$, which are canonical four-dimensional $U(1)$ fields that are defined to be scalars under the internal coordinate transformations $x^i \rightarrow x'^i = x^i + \xi^i$ and transform as a $U(1)$ field under $\delta A^{(11)MNP}_{\mu ij} = \partial_{\mu}\varsigma_{NP} + \partial_{\nu}\varsigma_{PM} + \partial_{\rho}\varsigma_{MN}$. The four-dimensional effective action \cite{3} has symmetry under the following $SL(7)$ target space transformations:

$$g_{ij} \rightarrow U_{ik}g_{kl}U_{jl}, \quad G_{\mu
u}^{i} \rightarrow (U^{-1})_{ik}G_{\mu
u}^{k}, \quad F_{\mu
u}^{ij} \rightarrow (U^{-1})_{ik}(U^{-1})_{jl}F_{\mu\nu}^{ik},$$

(5)

and the dilaton $\varphi$ and the four-dimensional metric $g_{\mu\nu}$ remain intact, where $U \in SL(7)$. 


Given the \( SL(7) \) symmetric effective four-dimensional theory, we would like to solve static, spherically symmetric solutions carrying either Kaluza-Klein or three-form \( U(1) \) gauge field. The spherically symmetric Ansatz for the four-dimensional space-time metric is chosen to be

\[
g_{\mu\nu}dx^\mu dx^\nu = \lambda(r)dt^2 - \lambda^{-1}(r)dr^2 - R(r)(d\theta^2 + \sin^2\theta d\phi^2)
\]

and the scalar fields depend on the radial coordinate \( r \) only. By solving the Maxwell’s equations with the above spherically symmetric Ansätze, one obtains the following non-zero components of \( U(1) \) field strengths:

\[
G_{\mu\nu}^i = \frac{\tilde{Q}_j}{Q^i}g_{\mu\nu}^j, \quad G_{\theta\phi}^i = P^i \sin\theta;
F_{\mu\nu}^{ij} = \frac{g_{ik}g_{jl}^{\mu} \tilde{Q}^{kl}}{R^2}, \quad F_{\theta\phi}^{ij} = P_{ij} \sin\theta,
\]

where the physical electric charges are given by \( Q^i = e^{-\phi}g_{\infty}^{ij} \tilde{Q}_j \) and \( Q_{ij} = e^{-\phi}g_{\infty}^{ij} \tilde{Q}^{kl} \).

### A. BPS States in the Kaluza-Klein Sector

The first class of solutions is black holes carrying charges of Kaluza-Klein \( U(1) \) gauge fields. With the other fields turned off except the dilaton, the internal metric and Kaluza-Klein \( U(1) \) fields, the four-dimensional effective action reduces to that of eleven-dimensional Kaluza-Klein theory compactified on a seven-torus. It was shown [22] that with a diagonal internal metric, the most general spherically symmetric configuration corresponds to \( U(1)_M \times U(1)_E \) charged dyonic black hole. By implementing this solution with \( SO(7)/SO(5) \) transformations [23], one obtains the most general supersymmetric spherically symmetric Kaluza-Klein black holes with one constraint \( \vec{P} \cdot \vec{Q} = 0 \), where \( \vec{P} \equiv (P_1, ..., P_7) \) and \( \vec{Q} \equiv (Q_1, ..., Q_7) \) are magnetic and electric charge vectors of Kaluza-Klein \( U(1) \) gauge fields.

Explicit supersymmetric \( U(1)_M \times U(1)_E \) solutions [24] of eleven-dimensional Kaluza-Klein theory with the \( j \)-th gauge field magnetic and the \( k \)-th gauge field electric are given by

\[
\lambda = \frac{r - |P_j\infty| - |Q_k\infty|}{(r - |P_j\infty|)^2}, \quad R\lambda = r^2, \quad e^{2(\phi - \varphi_\infty)} = \frac{r - |P_j\infty|}{r - |Q_k\infty|},
\]

\[
\frac{g_{KK}^{i\infty}}{g_{KK}^{ii\infty}} = 1 \quad (i \neq j), \quad \frac{g_{KK}^{jj\infty}}{g_{KK}^{jj\infty}} = \frac{r - |P_j\infty| - |Q_k\infty|}{r - |Q_k\infty|}, \quad \frac{g_{KK}^{kk\infty}}{g_{KK}^{kk\infty}} = \frac{r - |P_j\infty| - |Q_k\infty|}{r - |Q_k\infty|},
\]

where \( P_j\infty \equiv e^{\frac{1}{2}\phi_\infty}g_{KK}^{jj\infty}P_j \) and \( Q_k\infty \equiv e^{\frac{1}{2}\phi_\infty}g_{KK}^{kk\infty}Q_k \) are the “screened” magnetic and electric charges. Here, the subscript \( \infty \) denotes the asymptotic \( (r \to \infty) \) value of the corresponding field and the ADM mass of the configuration is given by \( M = |P_j\infty| + |Q_k\infty| \).
B. BPS States Carrying the Three-Form $U(1)$ Charges

The second class of solutions corresponds to black holes carrying charges of the three-form $U(1)$ gauge fields, i.e., the $U(1)$ gauge fields obtained from the dimensional reduction of the three-form field $A^{(1)}_{MNP}$. With all the other fields, except the dilaton, the internal metric and the three-form $U(1)$ gauge fields, turned off, the four-dimensional Lagrangian density reduces to

$$\mathcal{L} = -\frac{1}{4} e^{-\phi} \left[ R - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{1}{4} \partial_\mu g_{ij} \partial^\mu g^{ij} + \frac{1}{2} \delta^{[k} g^{i]l} F_{\mu \nu ij} F_{\mu \nu kl} \right],$$

where $F_{\mu \nu ij} \equiv \partial_\mu A_{\nu ij} - \partial_\nu A_{\mu ij}$. This Lagrangian has a manifest invariance under $SO(7) \subset SL(7)$ with $g_{ij}$ and $A_{\mu ij}$ transforming as 27 symmetric and 21 antisymmetric representations of $SO(7)$, respectively.

Just as in the case of Kaluza-Klein BH’s, one can obtain constraints on charges by solving the Killing spinor equations. These are given by

$$\sum P_{ab} P_{cd} \gamma^{abcd} = 0 = \sum Q_{ab} Q_{cd} \gamma^{abcd}, \quad \sum P_{ij} Q^{ij} = 0 = \sum P_{ab} Q_{cd} \gamma^{abcd},$$

where $P_{ab} \equiv e^a e^b P_{ij}$, $Q_{ab} \equiv e^a e^b Q_{ij}$, and $\gamma^a$’s are gamma matrices satisfying $SO(3, 1)$ Clifford algebra. Analyzing the asymptotic form of the above constraints and applying subsets of global $SL(7)$ transformations, one can see that static, spherically symmetric solutions with the most general three-form $U(1)$ charge configurations can be obtained by applying $SO(7)/SO(3)$ transformation on the configurations with non-zero charges given by $P_{ij} \neq 0$ and $Q^{ik} \neq 0 (j \neq k)$, where $i$ is a fixed index. Therefore, the static, spherically symmetric configurations carrying three-form $U(1)$ charges have $(3n - 6) + 2 = 17$ independent charge degrees of freedom.

Once again, this generating solution with non-zero charges $P_{ij}$ and $Q^{ik}$ is configuration with a diagonal internal metric, and is given by

$$\lambda = \frac{r - 2 |P_{ij}| - 2 |Q_{ik}|}{(r - 2 |P_{ij}|)^\frac{2}{3} (r - 2 |Q_{ik}|)^\frac{2}{3}}, \quad R\lambda = r^2, \quad e^{3(\phi - \varphi_\infty)} = \left( \frac{r - 2 |P_{ij}|}{r - 2 |Q_{ik}|} \right)^\frac{2}{3},$$

$$g_{\infty ii}/g_{\infty ii} = \left( \frac{r - 2 |Q_{ik}|}{r - 2 |P_{ij}|} \right)^\frac{2}{3}, \quad g_{\infty jj}/g_{\infty jj} = \left( \frac{r - 2 |P_{ij}|}{r - 2 |Q_{ik}|} \right)^\frac{2}{3},$$

$$g_{\infty kk}/g_{\infty kk} = \left( \frac{r - 2 |Q_{ik}|}{r - 2 |P_{ij}|} \right)^\frac{2}{3}, \quad g_{\infty \ell \ell}/g_{\infty \ell \ell} = \left( \frac{r - 2 |P_{ij}|}{r - 2 |Q_{ik}|} \right)^\frac{2}{3} \left( \ell \neq i, j, k \right),$$

where $P_{ij} \equiv e^{\phi_{\infty}/2} g_{\infty ii} g_{\infty jj} P_{ij}$ and $Q_{ik} \equiv e^{\varphi_{\infty}/2} g_{\infty ii} g_{\infty kk} Q_{ik}$.

C. Relation between Kaluza-Klein and Three-Form Black Hole Solutions

The above two classes of solutions have the same global space-time structure. That is, for non-zero magnetic and electric charges, the temperature $T_H \propto 1/\sqrt{PQ}$ is finite,
the singularity is null and the entropy is zero. If either of $P$ or $Q$ is zero, the singularity becomes naked and the temperature diverges. This can be traced back to the fact that eleven-dimensional supergravity compactified on a seven-torus has $E_7$ global symmetry [21], which puts all the 28 $U(1)$ gauge fields and 28 duals on the same putting, and leaves the four-dimensional space-time metric intact. The discrete subset of $E_7$ which relates the above two classes of solutions is given by

$$P_{ij} = \frac{P_{i}}{2}, \quad Q^{ik} = \frac{Q^{k}}{2}; \quad \mathcal{g}_{H j j} = \frac{\tilde{g}_{KK j j} \tilde{g}_{KK k k}}{\mathcal{g}_{H k k}},$$

$$\mathcal{g}_{H j j} \mathcal{g}_{H k k} = (\tilde{g}_{KK k k} \tilde{g}_{KK j j})^{-1/3}, \quad \mathcal{g}_{H i i} = (\tilde{g}_{KK j j} \tilde{g}_{KK k k})^{-2/3}, \quad \prod_{j \neq (ijk)} \mathcal{g}_{H \ell \ell} = (\tilde{g}_{KK j j} \tilde{g}_{KK k k})^{4/3}, \quad (12)$$

where $\tilde{g}_{ij} \equiv g_{ij}/g_{i j \infty}$.

By explicitly solving the Killing spinor equations we have shown that the above two classes of solutions admit Killing spinors, i.e., preserve some of supersymmetries. The spinor $\epsilon$ is constrained by one constraint for each type (electric or magnetic) of non-zero charges. So, purely electric [or magnetic] solutions preserve $\frac{1}{2}$ of the original supersymmetry, while dyonic solutions preserve only $\frac{1}{4}$ of the original supersymmetry.

Note, however, that a set of configurations obtained in the manner discussed above constitutes only a subset of the most general BPS saturated configuration. One can obtain a more general class of solutions carrying charges of Kaluza-Klein $U(1)$ and the three-form $U(1)$ gauge fields, which will turn out to be the “generating” solution that yields the most general supersymmetric black hole solution of eleven-dimensional supergravity on a seven-torus after imposing a subset of the $E_7$ transformations [1]. Such a generating solution would in turn allow us to study the full spectrum of the U-duality symmetry of the $N = 8$ superstring vacua.

IV. BPS STATES OF TYPE-II STRING COMPACTIFIED ON SIX-TORUS

By using the fact that the type-IIA superstring on a six-torus is dual to eleven-dimensional supergravity on a seven-torus, one can transform the above two classes of solutions in eleven-dimensional supergravity on a seven-torus into those in the type-IIA superstring on a six-torus. The zero slope limit of the ten-dimensional type-IIA superstring can be described by the eleven-dimensional supergravity compactified on a circle $S^1$. The dimensional reduction is accomplished by the following choice of the Kaluza-Klein Ansatz for the Elfbein $E_{M}^{(11)A}$:

3The corresponding generating solution in the heterotic string on a six-torus has been obtained [16]: it carries two electric [and two magnetic] charges of the Kaluza-Klein $U(1)$ field and the two-form $U(1)$ field having common indices. Each type (electric or magnetic) of charge break $\frac{1}{2}$ of the original supersymmetry. Note that such dyonic solutions cannot be obtained from the singly charged solutions through the $S$-duality transformations.
\[ E^{(11)}_\mu A = \begin{pmatrix} e^{-\frac{\Phi}{2}} e^{(10)\tilde{\alpha}} & e^{\frac{3\Phi}{2}} B_{\mu} \\ 0 & e^{\frac{5\Phi}{2}} \end{pmatrix}, \] (13)

where \( \Phi \) corresponds to the ten-dimensional dilaton field in the NS-NS sector, \( e^{(10)\tilde{\alpha}} \) is the Zehnbein in the NS-NS sector, and \( B_{\mu} \) corresponds to a one-form in the RR sector of superstring. Here, the breve denotes the ten-dimensional space-time vector index. And the three-form \( A^{(11)}_{MNP} \) is decomposed into \( A^{(11)}_{MNP} = (A_{\mu\tilde{\alpha}\tilde{\beta}}, A_{\mu\tilde{\alpha}11} \equiv A_{\mu\tilde{\alpha}}) \), where \( A_{\mu\tilde{\alpha}\tilde{\beta}} \) is identified as the three-form in the RR sector and \( A_{\mu\tilde{\alpha}} \) is the antisymmetric tensor in the NS-NS sector.

Then, the eleven-dimensional bosonic action reduces to the following effective action for the massless bosonic fields in the type-IIA superstring:

\[ \mathcal{L} = \mathcal{L}_{NS} + \mathcal{L}_R, \] (14)

with

\[ \mathcal{L}_{NS} = -\frac{1}{4} e^{(10)} e^{-2\Phi} [\mathcal{R} + 4 \partial_\mu \Phi \partial^\mu \Phi - \frac{1}{3} F_{\mu\nu\rho} F^{\mu\nu\rho}], \]

\[ \mathcal{L}_R = -\frac{1}{4} e^{(10)} \left[ \frac{1}{4} G_{\mu\tilde{\alpha}} G^{\mu\tilde{\alpha}} + \frac{1}{12} F'_{\mu\tilde{\alpha}\tilde{\beta}} F'^{\mu\tilde{\alpha}\tilde{\beta}} - \frac{6}{(12)^3} \varepsilon^{\mu_1...\mu_{11}} \varepsilon_{\mu_1...\mu_{11}} F_{\mu_1...\mu_4} F_{\mu_5...\mu_4} A_{\mu_0\mu_1\mu_2} \right], \] (15)

where \( F_{\mu\nu\rho} \equiv 3 \partial_\mu A_{\nu\rho} \), \( G_{\mu\tilde{\alpha}} \equiv 2 \partial_\mu B_{\tilde{\alpha}} \), \( F'_{\mu\tilde{\alpha}\tilde{\beta}} \equiv 4 \partial_\mu A_{\tilde{\alpha}\tilde{\beta}} - 4 F_{[\tilde{\alpha}\tilde{\beta}]\mu} B_\mu \), and \( \varepsilon^{\mu_1...\mu_{11}} \equiv \varepsilon^{\mu_1...\mu_{11}} \). The eleven-dimensional gravitino \( \psi^{(11)}_M \) is decomposed into the ten-dimensional gravitino \( \psi_\mu \) and a fermion \( \psi_{11} \) as \( \psi^{(11)}_M = (\psi_\mu, \psi_{11}) \). These spinors can be split into two Majorana-Weyl spinors of left- and right-helicities, thereby describing \( N = 2 \) supergravity.

To compactify the above ten-dimensional effective action on a six-torus down to four dimensions, one chooses the following Kaluza-Klein Ansatz for the Zehnbein:

\[ e^{(10)\tilde{\alpha}} = \begin{pmatrix} e^{\frac{\tilde{\alpha}}{2}} & e^{\frac{\tilde{\alpha}}{2}} \\ 0 & e^{\frac{\tilde{\alpha}}{2}} \end{pmatrix}, \] (16)

where \( \tilde{B}_m^m \) (\( m = 1, ..., 6 \)) are Abelian Kaluza-Klein gauge fields (in the NS-NS sector), \( e^{\frac{\tilde{\alpha}}{2}} \) is the string frame Vierbein and \( e^{\frac{\tilde{\alpha}}{2}} \) is the Sechsbein. Setting all the scalars except those associated with the Sechsbein \( e^a_m \) and the ten-dimensional dilaton \( \Phi \) to zero, one has the following string-frame four-dimensional Lagrangian for the type-IIA superstring on a six-torus:

\[ \mathcal{L}_{II} = -\frac{1}{4} e^{2\Phi} (\mathcal{R} + 4 \partial_\mu \tilde{\phi} \partial^\mu \tilde{\phi} + \frac{1}{4} \partial mn \tilde{g}^mn \tilde{g}^mn - \frac{1}{4} \tilde{g}_{mn} \tilde{G}_{\mu\nu} \tilde{G}^{\mu\nu} - \tilde{g}^{mn} \tilde{F}_{\mu\nu} m n \tilde{F}^{\mu\nu} m n) \]

\[ + \frac{1}{4} e^\tilde{\sigma} \tilde{G}_{\mu\nu} \tilde{G}^{\mu\nu} + \frac{1}{2} e^\tilde{\sigma} \tilde{g}^{mn} \tilde{g}^{pq} \tilde{F}_{\mu\nu} m p \tilde{F}^{\mu\nu} n q, \] (17)

where \( \tilde{\phi} \equiv \tilde{\Phi} - \frac{1}{2} \ln \det \tilde{e}^a_m \) is the four-dimensional dilaton and \( \tilde{\sigma} \equiv \ln \det \tilde{e}^a_m \) parameterizes the volume of a six-torus, \( \tilde{g}_{mn} \equiv \eta_{ab} \tilde{e}^a_m \tilde{e}^b_n \), and \( \tilde{G}_{\mu\nu} \equiv \partial_\mu \tilde{B}_\nu - \partial_\nu \tilde{B}_\mu \). Here, the field strengths \( \tilde{F}_{\mu\nu} m n \), \( \tilde{G}_{\mu\nu} \) and \( \tilde{F}_{\mu\nu} m n \) are defined in terms of the Abelian gauge fields decomposed from the ten-dimensional two-form \( A_{\mu\tilde{\alpha}} \), the one-form \( B_\mu \) and the three-form \( A_{\mu\tilde{\alpha}\tilde{\beta}} \) fields, respectively. The following Einstein-frame action can be obtained by the Weyl rescaling \( g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = e^{-2\tilde{\phi}} g_{\mu\nu} \):

8
\[
\mathcal{L}_{11} = -\frac{1}{4}e^E [\mathcal{R}^E - 2\partial_\mu \phi \partial^\mu \phi + \frac{1}{4} \partial_\mu \bar{g}_{mn} \partial^\mu \bar{g}^{mn} - \frac{1}{4} e^{-2\phi} \bar{g}_{mn} \bar{G}^m \bar{G}^n - \frac{1}{2} e^\phi \bar{g}^{mn} \bar{F}_{\mu \nu} \bar{F}^{\mu \nu} 
+ \frac{1}{4} e^\sigma \bar{G}_{\mu \nu} \bar{G}^{\mu \nu} + \frac{1}{2} e^\sigma \bar{g}^{mn} \bar{g}^{pq} \bar{F}_{\mu \nu \rho \sigma} \bar{F}^{\mu \nu \rho \sigma}].
\] (18)

One has to notice here that when we relate the fields of eleven-dimensional supergravity to those of type-IIA supergravity on a torus, we kept only scalar fields that are associated only with the Elfbein \(E^{(11)A}_M\) (thereby, breaking \(E_7\) to \(SL(7)\)), and turned off \(g^{(11)\text{m}}\) or, equivalently, \(B_m\) (thereby, breaking \(SL(7)\) to \(SL(6)\)). Therefore, the symmetry transformations of the four-dimensional Lagrangian (18) do not mix RR \(B\) with the Elfbein those of type-IIA supergravity on a torus, we kept only scalar fields that are associated only

\[\bar{m},\bar{n}\]

where

\[\bar{m},\bar{n} \in \{1,\ldots, 6\}\]

and \(\bar{\rho}_{mn}\) is the unimodular part of the internal metric \(\bar{g}_{mn}\) (\(\bar{g}_{mn} = -e^{\sigma/3} \bar{\rho}_{mn}\)).

By using the relations (19), one can obtain the dyonic BH solutions carrying charges of each \(U(1)\) fields in the RR and/or the NS-NS sector(s) of the type-IIA superstring. In the following, we shall classify the type-IIA superstring BH solutions according to the type of the four-dimensional field, \(E_M^{(11)A}\) and \(A_M^{(11)A}\), from which the four-dimensional \(U(1)\) gauge fields are originated, and summarize the dependence of the ADM masses of these solutions on the asymptotic values of four-dimensional scalars. Note, the string frame ADM mass is related to the Einstein frame ADM mass as

\[M_s = e^{-\phi_\infty} M_E.\]

The first class of solutions corresponds to black holes carrying charges of \(U(1)\) gauge fields associated with the off-diagonal components of the eleven-dimensional space-time metric:

- **Type-KNR solutions** Magnetic charge \(P\) associated with \(\bar{B}_\mu\), \(i.e.,\) the one-form \(U(1)\) field in the RR sector, and electric charge \(Q_m\) associated with \(\bar{B}_m\), \(i.e.,\) one of six Kaluza-Klein gauge fields in the NS-NS sector:

\[
e^{(\phi - \phi_\infty)} = \left(\frac{r - P_\infty - Q_{m\infty}}{r - P_\infty}\right)^{\frac{1}{2}}, \quad e^{2(\sigma - \sigma_\infty)} = \left(\frac{(r - P_\infty - Q_{m\infty})^2}{(r - P_\infty)^{1/2}(r - Q_{m\infty})^{3/2}}\right),
\]

\[
\bar{\rho}_{mn}/\bar{\rho}_{mn\infty} = \left(\frac{r - P_\infty}{r - P_\infty - Q_{m\infty}}\right)^{\frac{1}{2}}, \quad \bar{\rho}_{kk}/\bar{\rho}_{kk\infty} = \left(\frac{r - P_\infty - Q_{m\infty}}{r - P_\infty}\right)^{\frac{1}{2}} (k \neq m),
\]

\[M_E = |P_\infty| + |Q_{m\infty}| = e^{\sigma_\infty/2} |P| + e^{-\phi_\infty} \bar{g}_{mn\infty} |Q_m|.\] (20)
The $SO(6)/SO(5)$ rotations on this solution induce $\frac{6 \cdot 5}{2} - \frac{6 \cdot 4}{2} = 5$ new magnetic charge degrees of freedom in the gauge fields $\tilde{B}_m$. For the case where electric charge $Q$ and magnetic charge $P_m$ are associated with $B_\mu$ and $\tilde{B}_\mu^m$, respectively, one can obtain the corresponding solutions by imposing the electric-magnetic duality transformations.

**Type-KNN solutions** Magnetic charge $P_m$ associated with $\tilde{B}_\mu^m$ and electric charge $Q_n$ associated with $B_\mu^n$, i.e., both charges correspond to Kaluza-Klein $U(1)$ fields of the NS-NS sector:

\[
e^{(\phi - \phi_\infty)} = \left(\frac{r - Q_n}{r - P_m}\right)^{\frac{1}{6}}, \quad e^{2(\sigma - \sigma_\infty)} = \frac{r - P_m}{r - Q_n}, \quad \tilde{\rho}_{mn}/\tilde{\rho}_{nn} = \frac{(r - P_m)\overline{Q}_n (r - Q_n)^{\frac{1}{6}}}{(r - P_m - Q_n)^{\frac{1}{6}}}, \quad \tilde{\rho}_{\ell\ell}/\tilde{\rho}_{\ell n} = \left(\frac{r - Q_n}{r - P_m}\right)^{\frac{1}{6}} (\ell \neq m, n),
\]

\[
M_E = |P_m| + |Q_n| = e^{-\phi_\infty} \overline{g}_{nm\infty}|P_m| + e^{-\phi_\infty} \overline{g}_{nn\infty}|Q_n|,
\]

Upon imposing the $SO(6)/SO(4)$ rotations, one has the most general supersymmetric eleven-dimensional Kaluza-Klein BH’s with the constraint $\sum P_l Q_l = 0$.

Secondly, we have the following classes of dyonic solutions that correspond to $U(1)$ gauge fields associated with the eleven-dimensional three-form field $A_{MNP}^{(11)}$:

**Type-HNR solutions** Magnetic charge $P_m$ associated with $\tilde{A}_\mu^m$, i.e., one of six 2-form $U(1)$ fields in the NS-NS sector, and the electric charge $Q_{mn}$ associated with $\tilde{A}_{\mu mn}$, i.e., one of fifteen three-form $U(1)$ fields in the RR sector:

\[
e^{(\phi - \phi_\infty)} = \left(\frac{r - 2Q_{mn}}{r - 2P_m - 2Q_{mn}}\right)^{\frac{1}{6}}, \quad e^{2(\sigma - \sigma_\infty)} = \frac{(r - 2P_m)(r - 2Q_{mn})}{(r - 2P_m - 2Q_{mn})^2},
\]

\[
\tilde{\rho}_{mn}/\tilde{\rho}_{mm} = \frac{(r - 2P_m - 2Q_{mn})^{\frac{1}{6}} (r - 2Q_{mn})^{\frac{1}{6}}}{(r - 2P_m - 2Q_{mn})^{\frac{1}{6}}}, \quad \tilde{\rho}_{nn}/\tilde{\rho}_{mn} = \left(\frac{r - 2P_m}{r - 2P_m - 2Q_{mn}}\right)^{\frac{1}{6}} (r - 2Q_{mn})^{\frac{1}{6}},
\]

\[
\tilde{\rho}_{\ell\ell}/\tilde{\rho}_{\ell n} = \left(\frac{r - 2P_m}{r - 2P_m - 2Q_{mn}}\right)^{\frac{1}{6}} (r - 2Q_{mn})^{\frac{1}{6}} (\ell \neq m, n),
\]

\[
M_E = 2|P_m| + 2|Q_{mn}| = 2e^{-\phi_\infty} \overline{g}_{mn\infty}|P_m| + 2e^{\phi_\infty/2} \overline{g}_{mn\infty} \overline{g}_{nn\infty} |Q_{mn}|.
\]

The $SO(6)/SO(4)$ rotations induce $\frac{6 \cdot 5}{2} - \frac{6 \cdot 3}{2} = 9$ new charge degrees of freedom. For the case of electric charge $Q_m$ coming from $\tilde{A}_\mu^m$ and magnetic charge $P_m$ coming from $A_{\mu mn}$, the corresponding solutions can be obtained by imposing the electric-magnetic duality transformations.

**Type-HRR solutions** Magnetic charge $P_{mn}$ coming from $A_{\mu mn}$ and electric charge $Q_{mp}$ coming from $A_{\mu mp}$, both of which are the charges of 3-form $U(1)$ fields in the R-R sector:
e^{(\phi-\phi_\infty)} = 1, \quad e^{2(\sigma-\sigma_\infty)} = \frac{r - 2P_{mn\infty}}{r - 2Q_{mp\infty}},
\]
\[
\tilde{\rho}_{mn}/\tilde{\rho}_{mn\infty} = \left(\frac{r - 2P_{mn\infty}}{r - 2Q_{mp\infty}}\right)^{\frac{1}{2}}, \quad \tilde{\rho}_{mn}/\tilde{\rho}_{mn\infty} = \left(\frac{r - 2P_{mn\infty}}{r - 2Q_{mp\infty}}\right)^{\frac{1}{2} + \frac{1}{6}}.
\]

Upon imposing the $SO(6)/SO(2)$ transformations on this solution introduce $\frac{65}{2} - \frac{21}{2} = 14$ charge degrees of freedom in the gauge fields $A_{\mu ij}$.

- **Type-HNN solutions** Magnetic charge $P_n$ associated with $\tilde{A}_{\mu n}$ and electric charge $Q_n$ associated with $\tilde{A}_{\mu n}$, i.e., both charges arise from 2-form $U(1)$ fields in the NS-NS sector.

\[
M_E = 2|P_{mn\infty}| + 2|Q_{mp\infty}| = 2e^{\sigma_\infty/2}g_{mn\infty}|P_{mn}| + 2e^{\phi_\infty/2}g_{mn\infty}|Q_{mp}|.
\]

In the above, the expressions for the four-dimensional metric components are given by $\lambda(r)$ and $R(r)$ of either ($8$) or ($11$) with the corresponding charges in the NS-NS or the RR sector replaced.

In the expressions for the Einstein frame ADM mass $M_E$ [string frame ADM mass $M_s = e^{-\phi_\infty}M_E$], the screened charges from the RR sector do not scale [scale as $e^{-\phi_\infty}$] with respect to the asymptotic string coupling $e^{\phi_\infty}$, while the screened charges from the NS-NS sector scale as $e^{-\phi_\infty}$ [scale as $e^{-2\phi_\infty}$], in agreement with a general analysis in Ref. ($8$). Note also the following scaling dependence of screened charges on the asymptotic volume of the six-torus, parameterized by $e^{\phi_\infty}$: the screened charges, associated with the $U(1)$ fields in the RR and the NS-NS sectors, scale as $e^{\phi_\infty/2}$ and do not scale, respectively.

\[\text{Note, that for a special case with either } P_m = 0 \text{ or } Q_n = 0, \text{ the result reduces to a solution first found in Ref. [24], and it is related to } H\text{-monopoles of the heterotic string [25,26].} \]
These classes of solutions do not become massless at any point of moduli space or the choice of the charge lattice except at the boundary of the moduli space, i.e., $g_{mm\infty} = \pm\infty$, or in the strong coupling limit, i.e., $e^{\phi_{\infty}} = \infty$, of the string theory.

The massless solutions arise for $N = 4$ string vacua \cite{13} when BPS saturated states carry charges of both, the Kaluza-Klein $U(1)$ field and the two-form $U(1)$ field in the NS-NS sector, which is the common sector of the type-IIA and the heterotic string theory. In Ref. \cite{12}, it was argued that in general $N = 8$ theory in four dimensions, i.e., the eleven-dimensional supergravity on a seven-torus or the type-I IA superstring on a six-torus, does not give rise to massless BPS saturated states for non-trivial charge configurations. However, the latter observation is not in contradiction with the fact that the BPS saturated states carrying charges of the NS-NS sector, which is a part of the type-IIA as well as the heterotic superstring, can become massless for particular charge configurations at special points of moduli space. Namely, in order to turn off the $U(1)$ charges of the RR sector of the type-IIA superstring, one has to discard one chirality of the spinor in order to ensure the consistency of the supersymmetry transformations for the corresponding bosonic fields, thus ensuring that in this case the theory possesses only $N = 4$ supersymmetry in four dimensions.

V. CONCLUSION

We have discussed two classes of dyonic black hole solutions in the eleven-dimensional supergravity on a seven-torus, i.e., one associated with the Kaluza-Klein $U(1)$ fields coming from the off-diagonal components of the Elbein $E_{M}^{(11)A}$ and the other one, associated with the three-form $U(1)$ fields arising from the three-form field $A_{MNP}^{(11)}$. These two classes of solutions have the same four-dimensional space-time property, since all of the $U(1)$ gauge fields and their duals are related through the global $E_7$ symmetry, which leaves the Einstein frame four-dimensional metric intact \footnote{Note that the $E_7$ symmetry puts all the scalars on the same footing. So, the dilaton mixes with other scalars under the $E_7$ transformation.}. It is a subject of future investigation to explore the full $E_7$ symmetry of the BPS saturated states in the four-dimensional $N = 8$ supergravity theory and its relationship to the full BPS soliton spectrum of the type-IIA superstring on a six-torus.

Since the eleven-dimensional supergravity on a seven-torus and the type-IIA supergravity on a six-torus are described by the four-dimensional $N = 8$ supergravity, which is unique, the effective Lagrangian of these two theories are related through the field redefinition. In fact, the former is the strong coupling limit of the latter. In this paper, we obtained the field redefinitions for the special case when the original $E_7$ symmetry of the Lagrangian is broken down to $SL(6)$, i.e., all the other scalar fields except those associated with $e_{m}^{(10)\alpha}$ and $\Phi$ are turned off. By using the field redefinitions we have obtained dyonic black hole solutions carrying charges of the RR and/or the NS-NS sector(s) of the type-IIA superstring from the dyonic solutions carrying either KK or the 3-form $U(1)$ charges of the eleven-dimensional
supergravity on a seven-torus. The ADM masses of these configurations scale with the asymptotic string couplings in accordance with the general analysis in Ref. [8]. In addition, the scaling of the ADM mass with the asymptotic value of the volume of the six-torus is discussed.

The classes of solutions we have discussed in this work are a special case of more general “generating” solution, which would induce all the BPS saturated solutions after imposing a subset of $E_7$ group. This “generating” solution will carry electric charges and magnetic charges, each of which coming from Kaluza-Klein $U(1)$ and three-form $U(1)$ fields (of the eleven-dimensional supergravity on a seven-torus), respectively. More general classes of solutions, including non-supersymmetric as well as BPS saturated solutions, can be obtained by introducing another parameter (into the generating solutions of the most general BPS states) of the $E_8$ group, which is the symmetry of the three-dimensional effective action for stationary solutions of the eleven-dimensional supergravity.

By explicitly solving the Killing spinor equations, we have shown that classes of dyonic extreme black hole solutions carrying the $U(1)$ charges of the NS-NS and/or the RR sectors of the type-IIA superstring preserve part of supersymmetries in the four-dimensional vacua of the effective supergravity theory. Purely electrically (or purely magnetically) charged solutions preserve $\frac{1}{2}$ of the original supersymmetries, thereby forming short multiplets, and dyonic solutions preserve $\frac{1}{4}$ of the original supersymmetries, thereby forming intermediate multiplets. The ultimate goal is to obtain the most general BPS solutions in the $N = 8$ vacua of the superstring theories.

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