Generating functional for the Green’s functions of a two-flavor bosonic model

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Abstract. We examine a simple two-flavor scalar model with a non-diagonal mass matrix. We argue that the conventional definition of the QFT partition function does not allow to evaluate the Green’s functions with respect to the flavor vacuum as it favors only the mass vacuum. By using functional-integral techniques, we derive new generating functional for Green’s functions on the flavor vacuum.

1. Introduction

The particle mixing mechanism is an important topic in the physics of Standard Model and beyond. Quark mixing was discovered theoretically in order to find an explanation for the suppression of some particular mesonic and hadronic processes involving the non conservation of quantum numbers like strangeness and flavor [1, 2]. Later on the concept of mixing was extended [3] so that it could also accommodate the discovery of CP violation in experiments on $K^0$-$\bar{K}^0$ or $B^0$-$\bar{B}^0$ oscillations [4, 5, 6].

A first viable mechanism for flavor oscillations was laid down in Pontecorvo [7, 8] and Bilenky [9] seminal works that allowed to explain quantitatively well the results of the Homestake experiment [10]. Phenomena such as neutrino or $K^0$-$\bar{K}^0$ oscillations are experimentally well accepted [11] nowadays. On the other hand, the theoretical aspects of oscillations still await a deeper understanding. The standard theoretical approach to this subject is based on quantum mechanics (QM) or on perturbative quantum field theory (QFT) [12]. However, in this last case the construction of neutrino’s Fock space is done only in the ultra-relativistic limit.

A non-perturbative QFT approach to mixing was first developed in Ref. [13]. In consequence, phenomenologically relevant corrections to the standard Pontecorvo oscillation formula were found by using Green’s functions evaluated on the flavor Fock vacuum [15]. A general review of these facts can be found in Ref. [14]. Most of developments in this direction were achieved by using operatorial techniques. They include, e.g., a deeper understanding of the mathematical structure of the mixing generator [16] or corrections to the usual Unruh radiation formula due to the presence of flavor mixing [17].

An interesting standpoint is to view the fermion mixing as a dynamical phenomenon, arising as a consequence of the spontaneous symmetry breaking in a some fermionic theory beyond the standard model [18]. This line of research was also recently pursued by the present authors who
used a one-loop effective action for the composite Hubbard-Stratonovich boson field, to find a set of gap equations describing the dynamical generation of both masses and mixing [19]. On the other hand, a richer structure of the gap equations must exist. This was, on a general ground, shown in Ref. [18]. In fact, for each class of unitarily inequivalent Fock spaces one should expect a different class of gap equations [20, 21, 22]. We claim that the reason for this discrepancy can be retraced to the fact that the standard generating functional for the Green’s functions is blind to the vacuum choice (or, perhaps better, to the choice of the particular representation of the Fock space).

Our aim here is to derive the generating functional for the Green’s functions constructed both on mass and on flavor vacuum for a simple two-flavor charged scalar model. To set up the stage, we present in Section 2 some essentials of operatorial approach to scalar-field mixing. In doing so we follow Ref. [23]. In Section 3, we calculate the standard generating functional for the Green’s functions (the mass vacuum one) and show how this can be extended in order to accommodate also the flavor-vacuum states. In order to produce a tractable result, we assume that the mixing angle $\theta$ is small so that the partition functional can be evaluated only to the first order in $\theta$.

2. Boson field mixing

Let us consider the Lagrange density

$$\mathcal{L} = \partial_{\mu} \hat{\varphi}_f^\dagger \partial_{\mu} \hat{\varphi}_f - \hat{\varphi}_m^\dagger M \hat{\varphi}_m,$$

where

$$\hat{\varphi}_f = \begin{bmatrix} \hat{\varphi}_A \\ \hat{\varphi}_B \end{bmatrix}, \quad M = \begin{bmatrix} m_A^2 & m_{A,B} \\ m_{B,A} & m_B^2 \end{bmatrix},$$

which describes the dynamics of two coupled (mixed) scalar fields that we will call flavor fields, in analogy with the terminology used in quark and neutrino physics. However, flavor charge has to be interpreted as a generic quantum number. In fact, in experiments we observe, oscillations of different quantities such as strangeness or isospin. For this reason we consider a complex scalar field doublet, despite the fact that in the experiments are always observed oscillations of neutral mesons.

The Lagrange density (2) can be diagonalized thanks to the following transformation:

$$\hat{\varphi}_f = U \hat{\varphi}_m, \quad U = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}.$$

With this, $\mathcal{L}$ becomes

$$\mathcal{L} = \partial_{\mu} \hat{\varphi}_m^\dagger \partial_{\mu} \hat{\varphi}_m - \hat{\varphi}_m^\dagger M \hat{\varphi}_m,$$

where

$$\hat{\varphi}_m = \begin{bmatrix} \hat{\varphi}_1 \\ \hat{\varphi}_2 \end{bmatrix}, \quad M = \begin{bmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{bmatrix}.$$

Lagrange density (4) thus describes two free scalar particles with masses $m_1$ and $m_2$. The above transformation can be rewritten by introducing the mixing generator:

$$\hat{G}_\theta(t) = \exp \left\{ \theta \left[ \hat{S}_+(t) - \hat{S}_-(t) \right] \right\},$$

where

$$\hat{S}_+(t) = \hat{S}_-^\dagger(t) = -i \int d^3x \left[ \hat{\pi}_1(x) \hat{\varphi}_2(x) - \hat{\varphi}_1^\dagger(x) \hat{\pi}_2^\dagger(x) \right].$$
Here \( \hat{\pi}_i, i = 1, 2 \) are conjugate momenta, so that equal-time canonical commutation relations (ETCR)
\[
[\hat{\varphi}_i(x), \hat{\pi}_i(y)]_{t_x=t_y} = \delta^3(x - y),
\]
are satisfied, with the other commutators equal to zero. We can thus write
\[
\hat{\varphi}_f(x) = \hat{G}_\theta^{-1}(t) \hat{\varphi}_m(x) \hat{G}_\theta(t).
\]
This transformation provides us with the mapping between two representations of ETCR.

Representation spaces (i.e., respective Fock spaces) are connected by the relation
\[
|a(t)\rangle_f = \hat{G}_\theta^{-1}(t)|a\rangle_m.
\]
In particular, the vacuum states in these two representations are related by
\[
|0(t)\rangle_f = \hat{G}_\theta^{-1}(t)|0\rangle_m.
\]

The notation used, put in evidence that the unitary generator (6) has the structure of a \( SU(2) \) generalized (Perelomov) coherent state. The \( SU(2) \) algebra is closed by the operators:
\[
[S_+, S_-] = 2\hat{S}_3(t), \quad [\hat{S}_3(t), \hat{S}_\pm(t)] = \pm \hat{S}_\pm(t),
\]
where
\[
\hat{S}_3(t) = -\frac{i}{2} \int d^3x \left[ \hat{\pi}_1(x)\hat{\varphi}_1(x) - \hat{\varphi}_1^\dagger(x)\hat{\pi}_1^\dagger(x) - \hat{\varphi}_2(x)\hat{\varphi}_2(x) + \hat{\varphi}_2^\dagger(x)\hat{\varphi}_2^\dagger(x) \right].
\]
One can prove that these two Fock spaces are orthogonal (i.e., inequivalent) in the large \( V \) limit [13, 14]. In fact, by using the Gaussian decomposition
\[
\hat{G}_\theta(t) = \exp \left[-\tan \theta \hat{S}_+(t)\right] \exp \left[-2 \log \cos \theta \hat{S}_3(t)\right] \exp \left[\tan \theta \hat{S}_-(t)\right],
\]
we can derive the important result:
\[
m(0|0)\rangle_f = \prod_k m(0|G_k^{-1}G_\theta(t)|0\rangle_m = \exp \left[-\frac{V}{(2\pi)^3} \int d^3k \log \left(1 + \sin^2 \theta |V_k|^2 \right)\right],
\]
where
\[
V_k = \frac{1}{2} \left( \frac{\omega_{k1}}{\omega_{k2}} - \frac{\omega_{k2}}{\omega_{k1}} \right) e^{(\omega_{k1} + \omega_{k2})t}, \quad \omega_{ki} = \sqrt{|k|^2 + m_i^2}, \quad i = 1, 2.
\]
In the limit \( V \to \infty \), as said, the RHS of Eq. (15) goes to zero: flavor and mass Fock states are orthogonal and so flavor and mass fields belong to unitarily inequivalent representations of ETCR (8). In other words, in this limit, the unitary generator (6) does not exist as a consequence of the second Schur’s lemma [24]. It is thus important to decide which representation is the one on which we should work. It was argued in Ref. [23] that, in order to have a well defined form of the oscillation formula which is independent from an arbitrary mass parametrization [25], one has to choose the flavor Fock space. The exact oscillation formula reads in this case
\[
Q_{kA}(t) = 1 - \sin^2 2\theta \left[ |U_k|^2 \sin^2 \left( \frac{\omega_{k1} - \omega_{k2}}{2} \right) t - |V_k|^2 \sin^2 \left( \frac{\omega_{k1} + \omega_{k2}}{2} \right) t \right],
\]
\[
Q_{kB}(t) = \sin^2 2\theta \left[ |U_k|^2 \sin^2 \left( \frac{\omega_{k1} - \omega_{k2}}{2} \right) t - |V_k|^2 \sin^2 \left( \frac{\omega_{k1} + \omega_{k2}}{2} \right) t \right],
\]
\[
U_k = \frac{1}{2} \left( \frac{\omega_{k1}}{\omega_{k2}} + \frac{\omega_{k2}}{\omega_{k1}} \right) e^{(\omega_{k1} + \omega_{k2})t}, \quad |U_k|^2 - |V_k|^2 = 1,
\]
where \( Q_{k\sigma}(t) \equiv \langle \nu_\sigma |Q_{kA}(t)|\nu_\sigma \rangle, \ \sigma = A, B \) is the expectation value of the flavor charge, for each family, on the single particle flavor state, in the flavor representation.
3. Mass and Flavor partition function

In this section we evaluate explicitly the generating functional for the Green’s functions (known also as the QFT partition function) both on mass and on flavor vacuum. In doing so, we point out how the standard definition of partition function favors only the first case. The generating functional for the Green’s functions on the flavor vacuum has to be determined in a completely new way.

Let us consider the standard Hamiltonian-based partition function:

$$Z[J_{\varphi}, J_\pi] = \int \mathcal{D}\pi_f \mathcal{D}\varphi_f \mathcal{D}^4 \varphi_f \, e^{i \int d^4x \left( \pi_f^T \partial_\mu \varphi_f + \partial_\mu \varphi_f \pi_f + \mathcal{H}_J \right)},$$

where the Hamilton density is

$$\mathcal{H}_J = \int d^3x \left( \pi_f^T \nabla \varphi_f \cdot \nabla \varphi_f + \varphi_f^T \mathcal{M} \varphi_f - J_f^1 \varphi_f - \varphi_f^T J_{\varphi} - J_{\pi}^1 \pi_f + \pi_f^T J_{\pi} \right).$$

Performing the change of variables

$$\pi_f = \tilde{\pi}_f + \partial_\mu \varphi_f + J_f^1, \quad \pi_f = \tilde{\pi}_f + \partial_\mu \varphi_f + J_{\pi},$$

we have

$$Z[J_{\varphi}, J_\pi] = N e^{i \int d^4x J_{\varphi}^1 J_\pi^1} \int \mathcal{D}\varphi_f \mathcal{D}\pi_f \, e^{i S_J},$$

where the action $S_J$ reads

$$S_J = \int d^4x \left( \partial_\mu \varphi_f \partial_\mu \varphi_f - \varphi_f^T \mathcal{M} \varphi_f + J_f^1 \varphi_f + \varphi_f^T J_{\varphi} + J_{\pi}^1 \partial_\mu \varphi_f + \partial_\mu \varphi_f J_{\pi} \right).$$

This expression can be simplified by integrating by parts:

$$S_J = \int d^4x \left( \partial_\mu \varphi_f \partial_\mu \varphi_f - \varphi_f^T \mathcal{M} \varphi_f + J_f^1 \varphi_f + \varphi_f^T J_f \right),$$

where $J_f(x) = J_{\varphi}(x) - \partial_0 J_\pi(x)$ and $J_f^1(x) = J_{\varphi}^1(x) - \partial_0 J_{\pi}^1(x)$. Performing now the transformation

$$\varphi_f = U \varphi_m, \quad U = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix},$$

we obtain

$$J_m = U^T J_f, \quad J_m^1 = J_f^1 U.$$  

The solution can be written as:

$$Z[J_m, J_\pi] = e^{i \int d^4x J_m^1 J_\pi} e^{-i \int d^4x f \int d^4y J_m^1(x) \Delta_F(x - y) J_m(y)},$$

where

$$\Delta_F(x - y) = \begin{bmatrix} \Delta_F^1(x - y) & 0 \\ 0 & \Delta_F^2(x - y) \end{bmatrix},$$

and $\Delta_F^i(x - y)$ is the Feynman propagator of the $i$-th mass field

$$\Delta_F^i(x - y) = \int_{C_F} d^4p \frac{e^{-ip(x - y)}}{p^2 - m_i^2 + i\varepsilon}.$$
If \( m_1 = m_2 \), it follows, from Eq. (27) that the partition function (28) is the same in terms of both mass and flavor currents.

First we consider the case \( J_n = 0 \). In this case Eq. (28) becomes

\[
\mathcal{Z}[J_m] = e^{-i \int d^4x \int d^4y J_m^i(x) \Delta_F(x-y) J_m^j(y)}.
\]

This is the generating functional of mass Green’s functions on the mass vacuum:

\[
m_m \langle \varphi_i(y) \varphi_j^\dagger(x) |0\rangle_m = (-i)^2 \left\{ \frac{\delta \mathcal{Z}[J_m]}{\delta J_m^i(x) \delta J_m^j(y)} \right\}_{J_m=0}, \quad i, j = 1, 2.
\]

In this case, rewriting the partition function in terms of flavor currents, we get:

\[
\mathcal{Z}[J_{\varphi}] = e^{-i \int d^4x \int d^4y J_{\varphi}^i(x) \Delta_{F}^i(x-y) J_{\varphi}^j(y)}.
\]

Now we can define the \textit{flavor propagator} as:

\[
\Delta_{F}^i(x-y) = U \Delta_F(x-y) U^T
\]

\[
= \begin{bmatrix}
\Delta_{F}^1(x-y) \cos^2 \theta + \Delta_{F}^2(x-y) \sin^2 \theta & (\Delta_{F}^2(x-y) - \Delta_{F}^1(x-y)) \cos \theta \sin \theta \\
(\Delta_{F}^2(x-y) - \Delta_{F}^1(x-y)) \cos \theta \sin \theta & \Delta_{F}^1(x-y) \cos^2 \theta + \Delta_{F}^2(x-y) \sin^2 \theta
\end{bmatrix}.
\]

Therefore

\[
\mathcal{Z}[J_{\varphi}] = e^{-i \int d^4x \int d^4y J_{\varphi}^i(x) \Delta_{F}^i(x-y) J_{\varphi}^j(y)}.
\]

This is indeed the generating functional for the flavor Green’s functions on the mass vacuum:

\[
m_m \langle \varphi_\alpha(y) \varphi_\beta^\dagger(x) |0\rangle_m = (-i)^2 \left\{ \frac{\delta \mathcal{Z}[J_{\varphi}]}{\delta J_{\varphi}^\alpha(x) \delta J_{\varphi}^\beta(y)} \right\}_{J_{\varphi}=0}, \quad \alpha, \beta = A, B.
\]

How can we find the Green’s functions on the flavor vacuum? Here we follow the approach presented in Ref. [26] for the evaluation of the Green’s functions on exotic vacua. Flavor fields correlation functions on the flavor vacuum will be given by:

\[
f(0(t_+)) \varphi_\alpha(y) \varphi_\beta^\dagger(x) |0(t_-)\rangle_f = (-i)^2 \left\{ \frac{\delta \mathcal{Z}[J_{\varphi}]}{\delta J_{\varphi}^\alpha(x) \delta J_{\varphi}^\beta(y)} \right\}_{J_{\varphi}=0}, \quad \alpha, \beta = A, B,
\]

and mass fields correlation functions on the flavor vacua are:

\[
f(0(t_+)) \varphi_i(y) \varphi_j^\dagger(x) |0(t_-)\rangle_f = (-i)^2 \left\{ \frac{\delta \mathcal{Z}[J_m]}{\delta J_m^i(x) \delta J_m^j(y)} \right\}_{J_m=0}, \quad i, j = 1, 2,
\]

where, taking into account the form of mixing generator (6) and using the Gaussian decomposition (14):

\[
\mathcal{Z}[J_{\varphi}] = \frac{-i \tan \theta S_+ \left[ \frac{\delta}{\delta J_{\varphi}^\alpha} \frac{\delta}{\delta J_{\varphi}^\beta} \frac{\delta}{\delta J_{\varphi}^\gamma} \right]_{(\tau_+)}}{e} \left. \frac{\delta \mathcal{Z}[J_{\varphi}]}{\delta J_{\varphi}^\alpha(x) \delta J_{\varphi}^\beta(y)} \right\}_{J_{\varphi}=0}
\]

\[
\times \frac{i \tan \theta S_+ \left[ \frac{\delta}{\delta J_{\varphi}^\alpha} \frac{\delta}{\delta J_{\varphi}^\beta} \frac{\delta}{\delta J_{\varphi}^\gamma} \right]_{(\tau_+)} \left. \frac{\delta \mathcal{Z}[J_{\varphi}]}{\delta J_{\varphi}^\gamma(x) \delta J_{\varphi}^\delta(y)} \right\}_{J_{\varphi}=0}}{e}
\]

\[
\times \frac{-i \tan \theta S_+ \left[ \frac{\delta}{\delta J_{\varphi}^\alpha} \frac{\delta}{\delta J_{\varphi}^\beta} \frac{\delta}{\delta J_{\varphi}^\gamma} \right]_{(\tau_+)} \left. \frac{\delta \mathcal{Z}[J_{\varphi}]}{\delta J_{\varphi}^\gamma(x) \delta J_{\varphi}^\delta(y)} \right\}_{J_{\varphi}=0}}{e}
\]

1 This fact can be proved (cf. Ref [26]) by noticing that in the standard derivation of the generating functional for the Green’s functions (see e.g. [27], one makes use of the resolution of unity \( \sum_n \langle n |n \rangle = 1 \) in terms of Hamiltonian eigenstates, which, in this case, can be constructed only on \( |0\rangle_m \).
Here $f(\theta)$ has to be determined by fixing a particular operator ordering and $J_\pi = 0$ is a shorthand notation for $J_\pi = J_\pi^\dagger = 0$. This definition can be understood by means of the definition (11) and by regarding the generator $\hat{G}_\theta(t)$ as an extra interaction term. Mass and flavor field Green’s functions evaluated on the flavor vacuum at time $t$ are thus given, by

$$ f(0(t)|\varphi_\alpha(y)\varphi_\beta^\dagger(x)|0(t))_f = \lim_{t_\pm \to t} (-i)^2 \left\{ \frac{\delta Z_f[J_\varphi]}{\delta J^*_\beta(x)\delta J_\alpha(y)} \right\}_{J_\varphi=0}, \quad \alpha, \beta = A, B; \quad (40) $$

$$ f\langle 0(t)|\varphi_i(y)\varphi_j^\dagger(x)|0(t)\rangle_f = \lim_{t_\pm \to t} (-i)^2 \left\{ \frac{\delta Z_f[J_\varphi]}{\delta J^*_j(x)\delta J_i(y)} \right\}_{J_m=0}, \quad i, j = 1, 2. \quad (41) $$

The evaluation of Eq. (39) is long and cumbersome. However, it is possible to perform calculations relatively easily under the assumption that $\theta \ll 1$. With this we keep only the first order terms in $\theta$. Since

$$ \log \cos \theta \approx -\theta^2/2, \quad (42) $$

we can neglect the terms containing $S_\beta(t)$ in the leading order in $\theta$. We thus have:

$$ Z_f[J_\varphi] \approx Z[J_\varphi] - i\theta [ (S_+(\tau_+) - S_-(\tau_+) + S_-(\tau_-) - S_+(\tau_-)) Z[J_\varphi, J_\pi] ]_{J_\pi=0}. \quad (43) $$

To explicitly evaluate Eq. (43) we consider each term in the sum on the RHS separately, namely

$$ S_+(\tau_+)Z[J_\varphi, J_\pi] $$

$$ = i \int d^3x \left[ \frac{\delta}{\delta J^{\pi+A}(\tau_+, x)} \frac{\delta}{\delta J^{\phi B}_A(\tau_+, x)} - \frac{\delta}{\delta J^{\phi A}(\tau_+, x)} \frac{\delta}{\delta J^{\pi B}_A(\tau_+, x)} \right] Z[J_\varphi, J_\pi]. \quad (44) $$

$$ S_+(\tau_-)Z[J_\varphi, J_\pi] $$

$$ = -i \int d^3x \left[ \frac{\delta}{\delta J^{\pi+A}(\tau_-, x)} \frac{\delta}{\delta J^{\phi B}_A(\tau_-, x)} - \frac{\delta}{\delta J^{\phi A}(\tau_-, x)} \frac{\delta}{\delta J^{\pi B}_A(\tau_-, x)} \right] Z[J_\varphi, J_\pi]. \quad (45) $$

$$ S_-(\tau_+)Z[J_\varphi, J_\pi] $$

$$ = -i \int d^3x \left[ \frac{\delta}{\delta J^{\pi+A}_A(\tau_+, x)} \frac{\delta}{\delta J^{\phi B}_A(\tau_+, x)} - \frac{\delta}{\delta J^{\phi A}_A(\tau_+, x)} \frac{\delta}{\delta J^{\pi B}_A(\tau_+, x)} \right] Z[J_\varphi, J_\pi]. \quad (46) $$

$$ S_-(\tau_-)Z[J_\varphi, J_\pi] $$

$$ = -i \int d^3x \left[ \frac{\delta}{\delta J^{\pi+A}_A(\tau_-, x)} \frac{\delta}{\delta J^{\phi B}_A(\tau_-, x)} - \frac{\delta}{\delta J^{\phi A}_A(\tau_-, x)} \frac{\delta}{\delta J^{\pi B}_A(\tau_-, x)} \right] Z[J_\varphi, J_\pi]. \quad (47) $$

First we consider the expression (44). This explicitly reads as

$$ \{ S_+(\tau_+)Z[J_\varphi, J_\pi] \}_{J_\pi=0} = i \int d^3x_+ \int d^4x \int d^4y \left[ J^{\alpha_\pi}_\phi(x) \partial_\tau \Delta^{\alpha A}(x-x_+) \Delta^{\beta B}(x+y) J^{\beta_\phi}_\pi(y) - J^{\alpha_\pi}_\phi(x) \Delta^{\alpha A}(x-x_+) \partial_\tau \Delta^{\beta B}(x+y) J^{\beta_\phi}_\pi(y) \right], \quad (48) $$

where

$$ \alpha, \beta = A, B, \quad \bar{x}_+ \equiv (\tau_+, x_+), \quad \bar{x}_- \equiv (\tau_-, x_+), \quad (49) $$

$$ \bar{x}_+ \equiv (\tau_+, x_+), \quad \bar{x}_- \equiv (\tau_-, x_+), \quad (49) $$
where we explicitly wrote components of the flavor matrix (34), and a sum over repeated index is understood. In the same way we have

\[ \{ S_+ (\tau_-) \} | J_x, J_\rho \} | J_\sigma = 0 = i \int d^3 \mathbf{x}_+ \int d^4 x \int d^4 y \left[ J_{\rho}^\alpha (x) \partial_{\tau_+} \Delta_{x} A(x - x_-) \Delta_{x_+} B^\beta (x_- - y) J_{\beta}^\gamma (y) - J_{\beta}^\gamma (y) \partial_{\tau_+} \Delta_{x_+} A^\alpha (x_- - y) \right] Z[J], \]

(50)

\[ \{ S_- (\tau_+) \} | J_x, J_\rho \} | J_\sigma = 0 = - i \int d^3 \mathbf{x}_+ \int d^4 x \int d^4 y \left[ J_{\rho}^\alpha (x) \partial_{\tau_+} \Delta_{x_+} B(x_- - x_+) \partial_{\tau_+} \Delta_{x} A^\alpha (x_- - y) J_{\beta}^\gamma (y) - J_{\beta}^\gamma (y) \partial_{\tau_+} \Delta_{x} A^\alpha (x_- - y) \right] Z[J], \]

(51)

\[ \{ S_- (\tau_-) \} | J_x, J_\rho \} | J_\sigma = 0 = - i \int d^3 \mathbf{x}_+ \int d^4 x \int d^4 y \left[ J_{\rho}^\alpha (x) \partial_{\tau_-} \Delta_{x_-} B(x_- - x_+) \partial_{\tau_-} \Delta_{x} A^\alpha (x_- - y) J_{\beta}^\gamma (y) - J_{\beta}^\gamma (y) \partial_{\tau_-} \Delta_{x} A^\alpha (x_- - y) \right] Z[J]. \]

(52)

The result can be written in a shorthand notation as

\[ Z_f [J_x] \approx Z[J_x] \left\{ 1 - \theta \int d^3 \mathbf{x}_+ \int d^4 x \int d^4 y \left[ - J_{\rho}^\alpha (x) \partial_{\tau_+} \Delta_{x} A(x - x_-) \Delta_{x_+} B^\beta (x_- - y) J_{\beta}^\gamma (y) + J_{\rho}^\alpha (x) \partial_{\tau_+} \Delta_{x_+} A(x_- - y) \right] Z[J], \]

\[ + J_{\rho}^\alpha (x) \partial_{\tau_-} \Delta_{x_-} B(x_- - x_+) \partial_{\tau_-} \Delta_{x} A^\alpha (x_- - y) J_{\beta}^\gamma (y) - J_{\beta}^\gamma (y) \partial_{\tau_-} \Delta_{x} A^\alpha (x_- - y) \right] Z[J], \]

(53)

If \( \theta = 0 \) this reduces, correctly, to Eq. (33), i.e. the standard definition\(^2\).

4. Conclusions

In this paper we have shown how the conventional definition of generating functional of Green’s functions has to be modified in order to take properly into account the rich vacuum structure associated to mixing transformation in QFT. Although we performed our calculations in the approximation of a small mixing angle, the partition function obtained (53) is evidently structurally richer than its standard counterpart given by Eq. (35). This extra information is encoded in the presence of two-point mass-vacuum correlations and their temporal changes.

It should be stressed that we have not touched other subtle issues that are relevant in relativistic QFT. Clearly, we have not discussed a renormalization issues which would need to be included in more realistic models. Second, our Hamiltonian-based computations were not manifestly Lorentz invariant. This should be no problem, say in the condensed matter framework, but is a clearly disadvantageous for particle physics applications. So, in the context of mixing, a Lagrangian formulation till needs to be done. For some related discussion see, e.g. [28, 29, 30].

This work should be a starting point in an exact (non-perturbative) FI-based study of meson oscillations. Moreover, the extension to the fermionic case is required to the study of flavor oscillations and to a complete description of the dynamical generation of fermionic mixing. An exact calculation will reveal if the flavor Green’s functions derived in this way are in agreement with those derived in Ref. [15].

\(^2\) This choice affects the vacuum only. Here is understood that we choose currents independently.
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References
[1] Cabibbo N 1963 Phys. Rev. Lett. 10 531
[2] Glashow S L, Iliopoulos J and Maiani L 1970 Phys. Rev. D 2 1285
[3] Kobayashi M and Maskawa T 1973 Prog. Theor. Phys. 49 652
[4] Gell Mann M and Pais A 1955 Phys. Rev. 95 1387
[5] Christenson J H, Cronin J W, Fitch W R and Turlay R 1964 Phys. Rev. Lett. 13 138
[6] KTeV Collaboration 1999; Phys. Rev. Lett. 83 22
[7] Pontecorvo B 1957 Sov. Phys. JETP 6 429
[8] Pontecorvo B 1957 Sov. Phys. JETP 7 172-02
[9] Bilenky S and Pontecorvo B 1957 Phys. Rep. 41 225
[10] Davis R 1994 Prog. Part. Nucl. Phys. 32 13
[11] Smy M B (Super-Kamiokande) 2003 Nucl. Phys. Proc. Suppl. 118 25 (2003) Hallin A L et al. 2003 Nucl. Phys. Proc. Suppl. 118 3; Eguchi K et al. (KamLAND) 2003 Phys. Rev. Lett. 90 021802; Hampel W et al. (GALLEX) 1999 Phys. Lett. B 447 127; Altmann M et al. (GNO) 2000 Phys. Lett. B 490 16
[12] Giunti C 2004 Preprint arXiv:hep-ph/0401244
[13] Blasone M and Vitiello G 1995 Ann. Phys. 244 283
[14] Blasone M, Jizba P and Vitiello G 2011 Quantum Field Theory and its Macroscopic Manifestations (London: World Scientific & ICP)
[15] Blasone M, Henning P A and Vitiello G 1999 Phys. Lett. B 451 140
[16] Blasone M, Gargiulo M V and Vitiello G 2016 Phys. Lett. B 716 104
[17] Blasone M, Lambiase G and Luciano G G 2015 J. Phys. Conf. Ser. 631 no.1 012053
[18] Blasone M, Jizba P, Lambiase G and Mavromatos N E 2014 J. Phys. 538 012003
[19] Blasone M, Jizba P and Smaldone L 2015 Il Nuovo Cimento 38 C 201
[20] Umezawa H, Takahashi Y and Kamefuchi S 1964 Ann. Phys. 26 336
[21] Umezawa H, Matsumoto H and Tachiki M 1982 Thermo Field Dynamics And Condensed States (Amsterdam: North-Holland)
[22] Haag R 1996 Local Quantum Physics: Fields, Particles, Algebras (Berlin: Springer-Verlag)
[23] Blasone M, Capolupo A, Romei O and Vitiello G 2001 Phys. Rev. D 63 125015
[24] Tung W K, Group Theory in Physics 1980 (Singapore: World Scientific)
[25] Fuji K, Habe C and Yabuki T 1999 Phys. Rev. D 59 113003
[26] Blasone M, Jizba P and Smaldone L 2017 in preparation
[27] Greiner W and Reinhardt J 1996 Field Quantization (Berlin Heidelberg: Springer-Verlag)
[28] Giunti C 2004 Am. J. Phys. 72 699
[29] Blasone M, Magueijo J and Pires Pacheco P 2005 Braz. J. Phys. 35 447
[30] Blasone M, Di Mauro M and Lambiase G 2005 Acta Phys. Polon. B 36 3255