Density Fluctuations in Thermal Inflation and Non-Gaussianity

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Abstract

We consider primordial fluctuations in thermal inflation scenario. Since the thermal inflation drives about 10 e-folds after the standard inflation, the time of horizon-exit during inflation corresponding to the present observational scale shifts toward the end of inflation. It generally makes the primordial power spectrum more deviated from a scale-invariant one and hence renders some models inconsistent with observations. We present a mechanism of generating the primordial curvature perturbation at the end of thermal inflation utilizing a fluctuating coupling of a flaton field with the fields in thermal bath. We show that, by adopting the mechanism, some inflation models can be liberated even in the presence of the thermal inflation. We also discuss non-Gaussianity in the mechanism and show that large non-Gaussianity can be generated in this scenario.
1 Introduction

The thermal inflation [1–7], a mini-inflation which occurs long after the standard inflation, has been discussed, in particular, as a possible solution to the cosmological moduli problem [8–10], in which moduli particles would lead to various cosmological difficulties such as destroying light element synthesized by BBN, too much contribution to X(γ)-ray background radiation, overclosure of the universe and so on. The thermal inflation can produce very large entropy to dilute the moduli density sufficiently and thus can be a solution for the moduli problem.

Here we would like to discuss another aspect of the thermal inflation. When one considers the thermal inflation, primordial fluctuations are usually assumed to be generated from the primordial inflation. However notice that the thermal inflation drives the number of e-folds of about 10 due to the large entropy production. This means that fluctuations corresponding to the present observational scales exit the horizon at later time compared with the case of no thermal inflation. Since the time of horizon-exit becomes closer to the end of inflation, the primordial power spectrum is rendered to be more tilted in most cases. Hence even an inflation model consistent with observations in the absence of the thermal inflation can be pushed to outside the allowed range. For example, here let us consider the quadratic chaotic inflation model, where the inflaton potential is given by \( V \propto \phi^2 \). In the usual case, the curvature perturbations probed by cosmological observations are assumed to correspond to the fluctuations which exit the horizon when the number of e-folds is \( N_{\text{inf}} = 50 - 60 \). By adopting this number, the spectral index is \( n_s \sim 0.96 \) and the tensor-to-scalar ratio is \( r \sim 0.13 \), which is consistent with current observations such as WMAP5 [11]. However, when the thermal inflation occurs after the standard primordial inflation, the number of e-folds reduces by about 10, and then the spectral index is more red-tilted and the tensor-to-scalar ratio is more increased. When \( N_{\text{inf}} = 40 \), the tensor-to-scalar ratio is \( r \sim 0.2 \), which is inconsistent with current observations and invalidates the model. Thus in this respect, the thermal inflation can also affect the predictions of primordial fluctuations.

In fact, the primordial fluctuations are not necessarily generated from the inflaton fluctuations. Another light scalar field such as the curvaton [12–14], modulus in the modulated reheating scenario [15, 16] and so on are also known to generate (almost) scale-invariant and adiabatic primordial fluctuations consistent with observations #1. Although by adopting these scenarios, one can alleviate the above mentioned issue, however, here we propose another mechanism which can naturally arise in the framework of the thermal inflation. During the thermal inflation, the effective potential of a flaton, a flat direction in supersymmetric models, is lifted up by a thermal effect due to the coupling between the flaton and particles in thermal bath. The end of the thermal inflation is controlled by the strength of the thermal effect, or the coupling. If the coupling depends on some other scalar field and this scalar field fluctuates, the end of the thermal inflation also differ from place to place in the Universe via the fluctuations of the coupling. If the scalar field is

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#1 Other mechanisms have also been discussed in the literatures [17–20].
light enough during inflation, almost scale-invariant density fluctuations can be achieved similarly to the case of the modulated reheating

Another important issue in the primordial fluctuations is the Gaussian nature of the fluctuations. In fact, although recent observations are consistent with purely Gaussian fluctuations, they may suggest that primordial local type non-Gaussianity is large compared to that of the standard (single-field) inflation model [11, 21]. Since some of the alternative mechanisms such as the curvaton and the modulated reheating scenarios can also produce large non-Gaussianity, they have been the subject of intense study [22, 23]. We show that large non-Gaussianity can be generated in the mechanism as well. Thus the mechanism proposed here would be interesting in this respect too.

The structure of this paper is as follows. In the next section, we briefly describe the thermal inflation model which we consider here. Then in Section 3, we summarize the formalism to discuss the curvature perturbation in the model and some observational quantities such as power spectrum, bispectrum and trispectrum or nonlinearity parameters. In Section 4, we present a mechanism of generating the curvature perturbation at the end of thermal inflation and also discuss its non-Gaussianity. In Section 5, we discuss the current observational constraints for several simple inflation models in our scenario. The final section is devoted to summary of this paper.

2 Basic Picture of thermal inflation

In this section, we briefly review the idea of the thermal inflation [5]. The thermal inflation can be realized by utilizing a flat direction which exists in supersymmetric theory. Let us call such a direction as a flaton field $\phi$ and assume that the potential of the flaton field is given by

$$V(\phi) = V_0 - \frac{1}{2}m^2\phi^2 + \frac{\lambda}{6}M_{Pl}^2\phi^6,$$  \hspace{1cm} (1)

where the $\phi^2$ term comes from soft supersymmetry breaking and we have neglected the higher order nonrenormalizable terms ($\phi^{2n+4}, n \geq 2$). The VEV of $\phi$ is given by $\phi_{\text{vev}} \equiv v_{\text{flaton}} = \lambda^{-1/4}m^{1/2}M_{Pl}^{1/2}$ and $V_0 = m^2v_{\text{flaton}}^2/3$ to have $V(v_{\text{flaton}}) = 0$. When the flaton field interacts with some particles in thermal bath, such an interaction gives a thermal contribution to the potential of the flaton field as

$$V_T = \frac{g}{2}T^2\phi^2,$$  \hspace{1cm} (2)

#2 In Ref. [24] the author has presented a mechanism of generating large scale curvature perturbation through the inhomogeneous cosmological phase transitions in the early universe. The mechanism proposed here is similar to this kind.

#3 Two types of non-Gaussianity are often discussed in the literatures, one is the so-called local type and the other is the equilateral type. In this paper, we only focus on the local type non-Gaussianity.
where $g$ is an effective coupling between the flaton and particles in thermal bath and $T$ is the cosmic temperature. Then the total effective potential of the flaton field in thermal bath is given by

$$V_{\text{eff}} = V_0 + \frac{1}{2} \left( gT^2 - m^2 \right) \phi^2 + \frac{\lambda}{6} \frac{1}{M_{\text{Pl}}^2} \phi^6. \quad (3)$$

After the standard primordial inflation ends followed by the reheating, the cosmic temperature decreases as the Universe expands. Then, at some temperature when $V_0$ gets larger than the background radiation energy density, the Universe is dominated by the false vacuum of the flaton’s potential, which drives a mini-inflation. When the temperature of the Universe has dropped down to $T = T_c = m/\sqrt{g}$, the flaton starts rolling down to the VEV and then the mini-inflation ends. The duration of this thermal inflation can be expressed by using the $e$-folding number as

$$N(t_c, t_{\text{in}}) = \int_{t_{\text{in}}}^{t_c} H dt = \ln \left( \frac{a_c}{a_{\text{in}}} \right) = - \ln \left( \frac{T_c}{T_{\text{in}}} \right), \quad (4)$$

where $H$ denotes a Hubble parameter. Here $T_{\text{in}}$ is the temperature at the onset of the thermal inflation, which is defined as $\rho_r(T_{\text{in}}) = V_0 = m^2 v_{\text{flaton}}^2 / 3$, i.e., $T_{\text{in}} \sim m^{1/2} v_{\text{flaton}}^{1/2}$, where $\rho_r$ represents the radiation energy density. Taking some typical values of $m \simeq 10^2$ GeV and $v_{\text{flaton}} \simeq 10^{10}$ GeV, we have $T_{\text{in}} \simeq 10^6$ GeV and $T_c \simeq 10^2$ GeV and hence the size of the $e$-folds during thermal inflation is estimated as $N(t_c, t_{\text{in}}) \simeq 10$, which corresponds to the number $\Delta N_{\text{th}}$ to be subtracted from the standard one. More precisely $\Delta N_{\text{th}}$ is calculated by considering entropy production due to the thermal inflation. The total entropy $S$ of the Universe after thermal inflation increases by a factor $\Delta S = s(T_r) a_r^3 / s(T_c) a_c^3$ where $a_r$ and $a_c$ are the scale factor at the reheating after the thermal inflation and the end of the thermal inflation, respectively. $s$ is the entropy density and $T_r$ is the reheating temperature after thermal inflation. Since the flaton behaves like a matter after the end of the thermal inflation, the energy density of the flaton just before the reheating is given by $\rho_{\text{flaton}}(T_r) = V_0 (a_c/a_r)^3$. This gives the radiation energy density just after the reheating, hence the entropy density can be written as $s(T_r) = 4 \rho_r(T_r)/3 T_r$. Then, $\Delta S$ can be estimated as

$$\Delta S = \frac{4 V_0}{3 T_r} \frac{45}{2 \pi^2 g_* T_c^3} \simeq 0.01 \frac{v_{\text{flaton}}^2}{m T_r}, \quad (5)$$

where $g_*(\simeq 100)$ counts the effective degrees of radiation. Then $\Delta N_{\text{th}}$ is estimated as

$$\Delta N_{\text{th}} = \frac{1}{3} \ln \Delta S \simeq 12 - \frac{1}{3} \ln \left( \frac{m}{10^2 \text{ GeV}} \right) - \frac{1}{3} \ln \left( \frac{T_r}{\text{GeV}} \right) + \frac{2}{3} \ln \left( \frac{v_{\text{flaton}}}{10^{10} \text{ GeV}} \right). \quad (6)$$

### 3 $\delta N$ formalism and observational quantities

Based on $\delta N$ formalism [25, 26], the curvature perturbation on the uniform energy density hypersurface, $\zeta$, on super-horizon scales is given by the difference of the neighbor background trajectories, which is parametrized by the $e$-folding number, measured between
the initial flat hypersurface and the final uniform energy density hypersurface. That is, we have
\[ \zeta(t_f) \simeq \delta N(t_f, t_*) , \] (7)
where \( N(t_f, t_*) \) denotes the e-folding number measured between those at \( t = t_f \) and \( t = t_* \).

When the curvature perturbation \( \zeta \) originates from fluctuations of a single scalar field \( \sigma \), \( \zeta \) at \( t = t_f \) is given, up to the third order, by
\[ \zeta(t_f) = N_\sigma \delta \sigma_* + \frac{1}{2} N_{\sigma \sigma} (\delta \sigma_*)^2 + \frac{1}{6} N_{\sigma \sigma \sigma} (\delta \sigma_*)^3 , \] (8)
where \( N_\sigma = dN/d\sigma_* \) and so on. The power spectrum \( P_\zeta \), the bispectrum \( B_\zeta \), and the trispectrum \( T_\zeta \) are given by
\[ \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle = (2\pi)^3 P_\zeta(k_1) \delta(\vec{k}_1 + \vec{k}_2) , \] (9)
\[ \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = (2\pi)^3 B_\zeta(k_1, k_2, k_3) \delta(k_1 + \vec{k}_2 + \vec{k}_3) , \] (10)
\[ \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle = (2\pi)^3 T_\zeta(k_1, k_2, k_3, k_4) \delta(k_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4) , \] (11)
where \( B_\zeta \) and \( T_\zeta \) can be expressed with the power spectrum as
\[ B_\zeta(k_1, k_2, k_3) = \frac{6}{5} f_{NL} (P_\zeta(k_1) P_\zeta(k_2) + P_\zeta(k_2) P_\zeta(k_3) + P_\zeta(k_3) P_\zeta(k_1)) , \] (12)
\[ T_\zeta(k_1, k_2, k_3, k_4) = \tau_{NL} (P_\zeta(k_{13}) P_\zeta(k_3) P_\zeta(k_4) + 11 \text{ perms.}) + \frac{54}{25} g_{NL} (P_\zeta(k_{23}) P_\zeta(k_3) P_\zeta(k_4) + 3 \text{ perms.}) . \] (13)

Here \( f_{NL}, \tau_{NL} \) and \( g_{NL} \) are the non-linearity parameters often discussed in the literatures. By adopting \( \delta N \) formalism, these parameters can be calculated as [27]
\[ \frac{6}{5} f_{NL} = \frac{N_{\sigma \sigma}}{N^2_\sigma} , \quad \tau_{NL} = \frac{N^2_{\sigma \sigma}}{N^4_\sigma} , \quad \frac{54}{25} g_{NL} = \frac{N_{\sigma \sigma \sigma}}{N^2_\sigma} . \] (14)

When the curvature perturbation is generated from a single source, the relation \( \tau_{NL} = (36/25) f_{NL}^2 \) holds, which is the case in the following discussion. Thus when we investigate the non-linearity of fluctuations, we do not consider \( \tau_{NL} \), but discuss \( f_{NL} \) and \( g_{NL} \).

4 Curvature perturbation generated at the end of thermal inflation

Now we discuss the generation of density fluctuations at the end of thermal inflation. We assume that the coupling \( g \) depends on a light scalar field \( \sigma \), namely, \( g = g(\sigma) \). Then
the coupling $g$ can fluctuate due to the fluctuation of the light field $\sigma$ which originates to quantum fluctuations during inflation and hence the fluctuation of the coupling $g$ gives rise to the inhomogeneous end of thermal inflation. In such a case, from Eq. the curvature perturbation from fluctuations of $\sigma$ at the end of thermal inflation can be given by, up to the third order,

$$
\zeta = \delta N = -\frac{\delta T_c}{T_c} + \frac{1}{2} \left( \frac{\delta T_c}{T_c} \right)^2 - \frac{1}{3} \left( \frac{\delta T_c}{T_c} \right)^3
= \frac{1}{2} \left\{ \frac{g^\prime}{g} \delta \sigma_s + \frac{1}{2} \left[ \frac{g^\prime}{g} \right]^2 \delta \sigma_s^2 + \frac{1}{6} \left[ \frac{g^\prime}{g} - 3 \frac{g^\prime}{g} \frac{g^\prime}{g} + 2 \left( \frac{g^\prime}{g} \right)^3 \right] \delta \sigma_s^3 \right\},
$$

(15)

where a prime denotes the derivative with respect to $\sigma_s$. Here we have used $T_c \propto g^{-1/2}$.

Now the power spectrum is written as

$$
P_\zeta = \frac{1}{4} \left( \frac{g^\prime}{g} \right)^2 \left( \frac{H_s}{2\pi} \right)^2.
$$

(16)

Since $\sigma$ does not evolve much during inflation, the scale dependence of the power spectrum comes from the time variation of the Hubble parameter during inflation. Then the spectral index is given by

$$
n_s - 1 \simeq -2\epsilon,
$$

(17)

where $\epsilon$ is a slow-roll parameter and defined by $\epsilon = (M_{pl}^2/2)(V_{\phi}/V)^2$. Here $V$ is the potential for the inflaton $\phi$ and $V_{\phi} = dV/d\phi_s$. Notice that in the standard inflation scenario, the spectral index is given by $n_s - 1 = -6\epsilon + 2\eta$ with $\eta = M_{pl}^2(V_{\phi\phi}/V)$ being another slow-roll parameter.

To discuss non-Gaussianity of the curvature perturbation in the mechanism, we give the non-linearity parameters defined in the previous section. For the bispectrum, one usually uses the parameter $f_{NL}$ which is given by

$$
\frac{6}{5} f_{NL} = 2 \left[ \frac{g^\prime g}{(g^\prime)^2} - 1 \right].
$$

(18)

For the trispectrum, the non-linearity parameter $g_{NL}$ is

$$
\frac{54}{25} g_{NL} = 4 \left[ \frac{g^\prime g}{(g^\prime)^2} - 3 \frac{g^\prime g}{(g^\prime)^2} + 2 \right].
$$

(19)

To investigate observational quantities in more detail, we assume a simple functional form of $g$ as

$$
g = g_0 \left( 1 + \frac{1}{2} \frac{\sigma^2}{M^2} \right),
$$

(20)
with $g_0$ and $M$ being a coupling constant and some scale, respectively. With this form of $g$, the curvature perturbation generated from the fluctuations of $\sigma$ is given by

$$\zeta_{\sigma} \simeq \left(\frac{\sigma_*}{M}\right)^2 \frac{\delta \sigma_*}{\sigma_*}. \quad (21)$$

In general, however, fluctuations from the inflaton can also be generated. The curvature perturbation $\zeta_{\text{inf}}$ from the inflaton $\phi$ can be expressed as

$$\zeta_{\text{inf}} \simeq \frac{1}{\sqrt{2\epsilon M_{\text{pl}}}} \delta \phi_. \quad (22)$$

Thus the ratio of the amplitude of the power spectra between those from the mechanism proposed here and the inflaton is given by

$$\frac{P_{\zeta_{\sigma}}}{P_{\zeta_{\text{inf}}}} \simeq \frac{1}{2\epsilon} \left(\frac{\sigma_*}{M}\right)^2 \left(\frac{M_{\text{pl}}}{M}\right)^2. \quad (23)$$

On the other hand, if we assume that $(\sigma_*/M)^2 \ll 1$, the non-linearity parameter $f_{\text{NL}}$ can be written as

$$f_{\text{NL}} \simeq \left(\frac{\sigma_*}{M}\right)^{-2}. \quad (24)$$

The constraint on the non-linearity parameter from the WMAP5 [11, 21] is $f_{\text{NL}} < O(100)$, which indicates that $\sigma_*/M \gtrsim O(10^{-1})$. By using above equations, we can rewrite the ratio in Eq. (23) as

$$\frac{P_{\zeta_{\sigma}}}{P_{\zeta_{\text{inf}}}} \simeq 3 \times 10^{14} \epsilon \left(\frac{100}{f_{\text{NL}}}\right) \left(\frac{10^{10} \text{GeV}}{M}\right)^2. \quad (25)$$

Hence as far as $\epsilon$ is not extremely small, the fluctuations from $\sigma$ in this mechanism can be responsible for cosmic density perturbations today.

When the fluctuations from $\sigma$ are fully responsible for the scalar primordial fluctuations, the tensor-to-scalar ratio is

$$r = 8 \left(\frac{\sigma_*}{M}\right)^{-4} \sigma_*^2 \simeq 10^{-15} \times \left(\frac{f_{\text{NL}}}{100}\right) \left(\frac{M}{10^{10} \text{GeV}}\right)^2. \quad (26)$$

Thus, as other mechanisms of generating primordial fluctuations such as the curvaton and modulated reheating models, the tensor-to-scalar ratio is generally very small in this mechanism.

Here we make some comments on $g_{\text{NL}}$. With the form of $g$ assumed in Eq. (20) and when $g'''$ is negligible, we have the relation between $f_{\text{NL}}$ and $g_{\text{NL}}$:

$$g_{\text{NL}} = -\frac{10}{3} f_{\text{NL}} - \frac{50}{27}. \quad (27)$$
Thus in this model, the sizes of $f_{NL}$ and $g_{NL}$ are almost the same. In the above discussion, we have neglected the evolution of the light field $\sigma$ after the horizon crossing until the end of thermal inflation. This assumption is valid when $m_\sigma \ll H_{th}$ with $H_{th}$ being the Hubble parameter during the thermal inflation. As we discussed in Section 2, $H_{th}$ can be estimated as $H_{th} = V_0^{1/2}/M_{Pl} \simeq 10^3 \text{eV}$ and hence $\sigma$ has to be light as $m_\sigma \ll 10^3 \text{eV}$. Moreover, $\sigma$ gets the effective mass from the coupling $\sim g(\sigma) \phi^2 T^2$. Thus such an effective mass should be also smaller than the Hubble parameter during the thermal inflation. In our scenario, however, this effective mass coming from the coupling can be negligible until the end of thermal inflation because the flaton field is trapped at $\phi = 0$ during thermal inflation phase.

5 Primordial fluctuations in scenarios with thermal inflation and models of inflation

Now in this section, we discuss what inflation models can be compatible with the thermal inflation in the light of current cosmological observations. As mentioned in the introduction, when a mini-inflation occurs after the primordial inflation, the time when the fluctuations of the reference scale in the present observations exit the horizon during inflation is shifted toward the end of inflation. This, in most cases, means that the spectral index is more tilted. Thus even if an inflation model consistent with current observations in the standard case can be excluded when the thermal inflation occurs. However, this situation can be relaxed by adopting the inhomogeneous end of thermal inflation discussed in Section 4 since fluctuations from the mechanism are generally more scale-invariant as seen from Eq. (17). We investigate this issue by working out explicitly some inflation models and discuss in what cases they are consistent with current observations.

5.1 The number of $e$-folds

To predict the quantities such as the spectral index and tensor-to-scalar ratio, we need to specify the number of $e$-folds when the reference scale exits the horizon, at which above mentioned quantities are measured. The number of $e$-folds at the time when fluctuations of the scale $k$ exit the horizon can be given by

$$N_{\inf} = 50 - \ln \left( \frac{k}{a_0 H_0} \right) - \frac{2}{3} \ln \left( \frac{10^{15} \text{GeV}}{V_{\inf}^{1/4}} \right) - \frac{1}{3} \ln \left( \frac{10^6 \text{GeV}}{\rho_{\text{reh}}^{1/4}} \right),$$  (28)

where $V_{\inf}^{1/4}$ denotes the potential energy of the inflaton during the inflation and $\rho_{\text{reh}}$ denotes the energy density of radiation at the reheating after the primordial inflation. Here we have assumed that the inflaton oscillates around a minimum of a quadratic potential for some time after the inflation ends.
When the thermal inflation occurs, $N_{\text{inf}}$ given above would be reduced by $\Delta N_{\text{th}}$ of Eq. (5). Thus, in this case, the number of the $e$-folds is modified as

$$N_{\text{inf}} \simeq 38 - \ln \left( \frac{k}{a_0 H_0} \right) - \frac{2}{3} \ln \left( \frac{10^{15}}{V_{\text{inf}}^{1/4}} \right) - \frac{1}{3} \ln \left( \frac{10^6}{\rho_{\text{reh}}^{1/4}} \right) + \frac{1}{3} \ln \left( \frac{m}{10^2 \text{GeV}} \right) + \frac{1}{3} \ln \left( \frac{T_r}{\text{GeV}} \right) - \frac{2}{3} \ln \left( \frac{v_{\text{flaton}}}{10^{10} \text{GeV}} \right).$$

(29)

Since the spectral index and the tensor-to-scalar ratio depend on the number of the $e$-folds, some models with the reduced $N_{\text{inf}}$ due to the thermal inflation may be excluded by current cosmological observations.

In the following, we consider some representative models of inflation such as chaotic inflation, new inflation and hybrid inflation models. First we focus on how the spectral index and tensor-to-scalar ratio are modified in the presence of the thermal inflation when primordial fluctuations are generated from the inflaton and discuss the compatibility with observational constraints. Then we investigate the case where primordial fluctuations are produced by the inhomogeneous end of thermal inflation, which is discussed in Section 4, and see that some excluded inflation models can be relaxed by using this mechanism.

### 5.2 Chaotic inflation

Chaotic inflation models have the potential of the following form:

$$V(\phi) = \lambda M^4_{\text{pl}} \left( \frac{\phi}{M_{\text{pl}}} \right)^n,$$

(30)

where $\phi$ is the inflaton field and $n$ is assumed to be even integer to have a minimum at the origin. The spectral index and the tensor-to-scalar ratio in chaotic inflation models are respectively given by, in terms of $N_{\text{inf}},$

$$n_s - 1 = -\frac{n + 2}{2N_{\text{inf}}}, \quad r = \frac{4n}{N_{\text{inf}}}.$$  

(31)

From the above expressions we can find that as $N_{\text{inf}}$ becomes smaller, $n_s$ is more red-tilted and $r$ becomes larger. In Fig. 1 contours are shown for 68% (green dashed line) and 95% (red solid line) C.L. allowed region derived from WMAP+BAO+SN. The blue dotted line corresponds to the predictions of chaotic inflation models on the $n_s$–$r$ plane for $n = 2$ (left) and $n = 4$ (right) cases, respectively. The values of $n_s$ and $r$ in the standard inflation case are also shown with small circles for $N_{\text{inf}} = 30, 40, 50$ and $60$. When $N_{\text{inf}} \lesssim 45$, we find that the quadratic model is excluded at 95% C.L. For the case of $n = 4$, as is well-known, the model is ruled out even if $N_{\text{inf}} = 60$. However, if fluctuations come from the inhomogeneous end of thermal inflation, the situation drastically alters. As discussed in the previous section, the spectral index is given by $n_s - 1 = -2\epsilon$ and the tensor-to-scalar...
ratio would be negligibly small. In Fig. 1 we also plot the predictions of \( n_s \) and \( r \) from the inhomogeneous end of thermal inflation with small squares. Although the quadratic chaotic inflation model with \( N_{\text{inf}} \lesssim 45 \) are excluded by current observations when the inflaton generates primordial fluctuations, such a model becomes viable in the present mechanism even if \( N_{\text{inf}} \) is small as 30. More interestingly, the quartic inflation model, which is excluded with its original form, can be viable in the present mechanism as shown in the right panel of Fig. 1.

\[ V = V_0 \left[ 1 - 2 \left( \frac{\phi}{v} \right)^n + \left( \frac{\phi}{v} \right)^{2n} \right] , \quad (32) \]

where \( v \) denotes the VEV of the inflaton. We assume that \( \phi/v \ll 1 \) during inflation. Then, the slow-roll parameters are given by

\[ \epsilon = 2n^2 \left( \frac{\phi_s}{v} \right)^{2(n-1)} \left( \frac{M_{\text{pl}}}{v} \right)^2 , \quad \eta = -2n(n-1) \left( \frac{\phi_s}{v} \right)^{n-2} \left( \frac{M_{\text{pl}}}{v} \right)^2 . \quad (33) \]

For the new inflation model, we consider the following potential:

\[ \text{Figure 1: Contours indicate 68} \% \text{ (green dashed line) and 95} \% \text{ (red solid line) C.L. allowed regions derived from WMAP+BAO+SN [11]. The blue dotted line corresponds to the predictions for the quadratic (left) and the quartic (right) chaotic models. The small purple circles and red squares represent the values of } n_s \text{ and } r \text{ from the standard inflation and inhomogeneous end of thermal inflation, respectively, for } N_{\text{inf}} = 30, 40, 50 \text{ and } 60. \]

5.3 new inflation

For the new inflation model, we consider the following potential:
From these expressions, we find that $\epsilon/\eta = (\phi_e/v)^n \ll 1$. Hence the tensor-to-scalar ratio is very small and the spectral index is given by

$$n_s - 1 \simeq 2\eta.$$  \hfill (34)

The e-folding number and the value of the inflaton field at the time of horizon crossing are related by

$$N_{\text{inf}} = \begin{cases} \frac{1}{4} \left( \frac{v}{M_{\text{pl}}} \right)^2 \ln \left( \frac{\phi_e}{\phi_*} \right) \simeq \frac{1}{\eta} \left( \frac{\phi_e}{\phi_*} \right) & (n = 2) \\ \frac{1}{2n(n-2)} \left( \frac{v}{M_{\text{pl}}} \right)^2 \left( \frac{\phi_*}{v} \right)^{-n+2} = -\left( \frac{n-1}{n-2} \right) \frac{1}{\eta} & (n \geq 3) \end{cases}$$

From these equations, the spectral index is given by, in terms of $N_{\text{inf}},$

$$n_s - 1 = \begin{cases} -\frac{2}{N_{\text{inf}}} \left( \frac{\phi_e}{\phi_*} \right) & (n = 2) \\ -\left( \frac{n-1}{n-2} \right) \frac{2}{N_{\text{inf}}} & (n \geq 3) \end{cases}$$

From the above expression, we find that when the curvature perturbation is generated from the inflaton, $|n_s - 1|$ becomes larger as $N_{\text{inf}}$ becomes smaller. For examples, when $n = 3$, $n_s = 0.933, 0.92$ and 0.9 for the cases with $N_{\text{inf}} = 60, 50$ and 40, respectively. From WMAP 5yr data, the limit on $n_s$ is obtained as $0.925 < n_s < 0.997$ (95\% C.L.) for negligible tensor-to-scalar ratio [11]. Hence, the models with $N_{\text{inf}} = 50$ and 40 lie out of the 95\% C.L. allowed region.

Now we consider the primordial fluctuations from the inhomogeneous end of thermal inflation. As discussed in the previous section, the condition $P_{\sigma}/P_{\zeta_{\text{inf}}} \gg 1$ should be satisfied when the fluctuations from the mechanism dominate over those from the inflaton. However, this requires $\epsilon(\sigma_*/M)^2(M_{\text{pl}}/M)^2 \gg 1$, thus when $\epsilon$ is very small as in the case of the new inflation, the above condition is difficult to be satisfied. Even if the fluctuations from the inflaton give a negligible contribution and those from the inhomogeneous end of thermal inflation dominate, its power spectrum becomes fairly scale-invariant since the spectral index is given by $n_s - 1 = -2\epsilon \ll O(0.01)$. Since the tensor-to-scalar ratio is also very small, such fluctuations are also excluded. Thus, the new inflation may not be well suited to the framework of the thermal inflation.

### 5.4 Hybrid inflation

Next we consider hybrid inflation. Although there are some variations for the potential for the hybrid inflation, we discuss some representative ones here. First we consider the
potential of the following form:

\[ V(\phi) = V_0 + \frac{1}{2}m_\phi^2\phi^2. \]  

(35)

When \( V_0 \gg m_\phi^2\phi^2 \), the slow-roll parameter can be written as

\[ \epsilon \simeq \frac{M_{\text{pl}}^2 m_\phi^4\phi^2}{2V_0^2}, \quad \eta \simeq \frac{m_\phi^2 M_{\text{pl}}^2}{V_0}. \]  

(36)

Since the condition \( V_0 \gg m_\phi^2\phi^2 \) implies that \( \epsilon \ll \eta \), the spectral index of this model can be written as \( n_s - 1 = 2\eta \), which is blue-tilted since \( \eta \) is positive here. The smallness of \( \epsilon \) also indicates that the tensor-to-scalar ratio is also negligible in this model, and in such a case the blue-tilted spectrum conflicts with the observations (see Fig. 1). Thus, this type of hybrid inflation is excluded by current observations in its original form. As in the case of the new inflation, since \( \epsilon \) is very small in this model as well, it could be difficult to have the case where fluctuations from the inhomogeneous end of thermal inflation dominate over those from the inflaton. Furthermore even if the fluctuations from the mechanism dominates over those from the inflaton, the spectral index is \( n_s \simeq 1 \), thus this again contradicts with current observations.

Another possible realization of the hybrid inflation is so-called mutated hybrid inflation [29], in which the potential can be written as

\[ V(\phi) = V_0 - \mu M_{\text{pl}}^4 \left( \frac{M_{\text{pl}}}{\phi} \right)^n \]  

(37)

where \( \mu \) is a constant. The potential of this kind can appear, for example, smooth hybrid inflation [30] for \( n = 4 \). With this potential, the slow-roll parameters are

\[ \epsilon \simeq \frac{\mu^2 n^2}{2} \left( \frac{M_{\text{pl}}^4}{V_0} \right)^2 \left( \frac{M_{\text{pl}}}{\phi} \right)^{2(n+1)}, \quad \eta \simeq -\mu n(n+1) \left( \frac{M_{\text{pl}}^4}{V_0} \right) \left( \frac{M_{\text{pl}}}{\phi} \right)^{n+2}, \]  

(38)

where we assumed that \( V_0 \gg \mu M_{\text{pl}}^4 \left( \frac{M_{\text{pl}}}{\phi} \right)^n \). This condition also implies that \( \epsilon \ll |\eta| \), thus the spectral index can be given by

\[ n_s - 1 = -\frac{2(n+1)}{(n+2)N_{\text{inf}}}. \]  

(39)

When \( n = 2 \), the spectral index is \( n_s = 0.95, 0.962, 0.97 \) and \( 0.975 \) for \( N_{\text{inf}} = 30, 40, 50 \) and \( 60 \), respectively. Thus even if the number of e-folds is reduced due to the thermal inflation, this model is not excluded. In fact, the central values of the allowed range for the spectral index from WMAP5 analysis is \( n_s \sim 0.96 \), thus a slight reduction of \( N_{\text{inf}} \) may be favored. It should be noticed that the slow-roll parameter \( \epsilon \) in this model could also be very small, the fluctuations from the inhomogeneous end of thermal inflation may not
give a dominant contribution for the curvature fluctuations. Even if fluctuations from the inhomogeneous end of thermal inflation are the main component of $\zeta$, the spectral index is again too scale-invariant to be consistent with observations, which was also the case for the new inflation and hybrid inflation with the potential of Eq. (35). The mutated hybrid inflation discussed here can be compatible to cosmological observations in the framework of thermal inflation even in its original form.

6 Summary

In this paper, we have presented a mechanism of generating primordial curvature perturbation at the end of thermal inflation by considering the case where the coupling of a flaton field with the fields in thermal bath can fluctuate. We also show that there is a possibility of generating large non-Gaussianity in this scenario.

We have also investigated the constraint on inflation models in the case where the thermal inflation is realized after the primordial inflation. When such a mini-inflation occurs, the time when the fluctuations of the reference scale in the present observations exit the horizon during inflation is shifted toward the end of inflation. This, in most cases, means that the spectral index is more tilted. Thus even if an inflation model is consistent with current observations in the standard case, some models can be excluded when the thermal inflation occurs. We have discussed that this situation can be relaxed by adopting the inhomogeneous end of thermal inflation since fluctuations from the mechanism are generally more scale-invariant as seen from Eq. (17). We have explicitly worked on some simple inflation models, such as chaotic inflation, new inflation and hybrid inflation. Then we find that some excluded models can be relaxed by using this mechanism proposed in this paper.

As a final comment, we mention baryogenesis in a scenario with the thermal inflation. Since the thermal inflation dilutes the preexisting baryon asymmetry, one needs to generate the baryon number after thermal inflation. Some authors have discussed Affleck-Dine baryogenesis after thermal inflation [31]. It may also be interesting to investigate observational signature in such model with the inhomogeneous end of thermal inflation scenario, for example, baryon isocurvature fluctuations. This would be a subject of the future study.

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