Mode and Torque of Flows around a Rotating Disk in a Cylindrical Casing

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Abstract. The flow around a rotating disk in a cylindrical casing is investigated by the direct numerical simulation. The sizes of the disk and the casing are finite and the flow field has a radial clearance and an axial clearance. The interaction of the flows in these clearances make complex three-dimensional flow structures. In the present study, the patterns and the bifurcation processes of the fully developed flows are classified and the developing process of the flow is identified. The torques acting on the disk surface and the rim of the disk are evaluated and the power laws governing the relations among the torque, the Reynolds number and the thickness ratio of the disk are determined.

1. Introduction

The flows around an axisymmetric bodies can be found in a variety of fluid machinery and bio-reactors and the investigation of the flow is important from the engineering’s point of view. They also show basic three-dimensional and cross-flow phenomena and have an interest in the research area of the fluid dynamics [1]. In the present study, the direct numerical simulation is performed to study the flow around a rotating disk in a cylindrical casing. The disk has a finite thickness. It is supported at the center of the casing by a thin driving shaft. This flow configuration has a radial clearance between the sidewall of the casing and the rim of the disk and an axial clearance between the end wall of the casing and the surface of the disk. The interaction of the flows in these clearances forms a complex three-dimensional flow structures and the bifurcation processes that have not been studies well.

In the radial clearance, Taylor-Couette system is established where the sidewall of the casing and the disk rim correspond to the stationary outer cylinder wall and the rotating inner cylinder wall, respectively [2-4]. Indeed, Taylor vortices have been predicted to exist between the rotating disk rim and the stationary enclosure of the hard disk drive [5]. The axial length is limited by the stationary end walls of the casing and the effect of the Ekman pumping mechanism on the end walls makes secondary flows at the Reynolds number below that for the onset of Taylor vortices and brings the imperfect bifurcation process [6, 7]. This imperfection forms a multiple states and solutions of the flow and the flow structures includes the normal mode flow, the anomalous mode flow and the secondary mode flow in addition to the axisymmetric Taylor vortex flow, wave Taylor vortex flow and modulated flows. The appearance of these flow modes in the accelerated and the decelerated flow was investigated experimentally and numerically [8-10]. Recently, the periodically driven flow [11], the
flow with passive contaminants [12] and the simulation by multiple turbulent model [13] of Taylor-Couette system have been examined.

The flow in the axial clearance can be considered as the flow in the cylindrical cavity flow between rotating and stationary disks and the flow in the annular cavity flow between disks with a hub at their center. Saric et al. [14] and Launder et al. [15] reviewed the stability of these cavity flows. The detail experimental result of the cylindrical cavity flow by Schouveiler et al. [16] showed that the circular vortices and spiral vortices appeared in the wider axial clearance and the turbulent spots and solitary vortices emerged in the narrower axial clearance. Yim et al. [17] numerically represented the similarity of the flow to the von Kármán self-similar solution and examined the stability of the flow between the disk edge and the sidewall of the casing. The experimental study by Wan et al. [18] confirmed the existence of the merged and separated boundary layers on the rotating and stationary disks and estimated the effect of the superhydrophobic surface. In the annular cavity with a hub at its center, the disturbances in the Bödewadt layer on the stationary end wall propagate along the hub and reach the Ekman layer on the rotating disk. The stationary hub suppresses the propagating disturbances [19], while the hub rotating with the disk intensifies the disturbances and makes the flow turbulent [20, 21]. Other than the cylindrical and annular cavity flows, the flow between two co-rotating disks confined by a stationary enclosure has been examined. Huang and Hsieh [22] experimentally showed the polygonal flow structures around the hub and the hysteresis of the flow structures against the change of the Reynolds number. Uenishi et al. [23, 24] carried out experimental and numerical study and showed the symmetric and asymmetric flow structures in the meridional section as well as the polygonal flow structures in the axial clearance.

In this paper, we investigate the flow with a radial clearance and an axial clearance by the direct numerical simulation. For this flow, Schouveiler et al. [16] suggested the important effect of the radial clearance, though they showed no result. We clarified that various flow structure appeared in the radial clearance [25]. Following this work, we study the vortex structures and specify the scenario of their developing processes. The details of the effect of the acceleration rate of the disk from rest on the final flow structures are shown. The torques acting on the disk surface and the disk rim are evaluated and the power lows among the torques, the Reynolds numbers and the thickness ratios of the disks are presented.

2. Formulation
The schematic of the flow field and the cylindrical coordinate system (r, q, ϕ) used in the numerical calculation are shown in figure 1. The origin is at the center of the lower end wall of the casing and the axial q axis is align with the central axis. The length and the inner radius of the stationary casing are r_o and r_0, respectively. The thickness and the radius of the disk are r_0 and r_o, respectively. The disk has a driving shaft of the radius r_o. The disk is supported at the center of the casing. All physical values are made dimensionless by using the disk radius as a reference length and the circumferential velocity at the disk rim as a reference velocity. The Reynolds number, defined by these reference values and the kinematic viscosity of the working fluid. The size of the cylindrical casing is fixed and r_o = 0.3150 and r_0 = 1.118. The radius of the driving shaft r_o = 0.0787 and it is small enough not to have any influence on the flow structures [26]. Eight disks with different thicknesses are used and the width of the axial clearance is settled. The thickness ratios defined by r_0/r_o are 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80 and 0.85.

The equations governing the incompressible fluid flow are the equation of continuity and the three-dimensional unsteady Navier-Stokes equation.
\[ \nabla \cdot \mathbf{u} = 0, \quad (1) \]
\[ \frac{\partial \mathbf{u}}{\partial t} + \nabla (\mathbf{u} \cdot \mathbf{u}) = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u}. \quad (2) \]

Here, \( \Box \) is time and \( \mathbf{u} \) is the velocity vector with its radial, circumferential and axial components of \( \mathbf{u} \), \( \mathbf{u} \) and \( \mathbf{u} \), and \( \mathbf{u} \) is the pressure. The boundary condition is the no-slip condition for the velocity components and the Neumann condition for the pressure. Initially, the fluid is at rest. At the beginning of the calculation, the disk is suddenly accelerated or it is linearly accelerated to the rotation speed defined by the prescribed Reynolds number. The discretization method is the finite difference method proposed by Lygren and Andersson [27]. The fully region of \( \Box \) from 0 to 2\( \Box \) is considered. The verification and the validation of the present code has been confirmed [28].

The numerical results are intended to be compared with the experimental results. The experimental apparatus has a casing with the length of 40 mm and the inner radius of 142 mm, and the disk radius is 127 mm. For the dimensionless time \( \Box \), the dimensional time in second is given by 641.8 \( \Box \) when the kinematic viscosity of the fluid is 4.0 \( \times \) 10\(^{-6} \) [\( \mu^2/\mu \)]. The acceleration rate \( \Box \) is defined by the increment of the Reynolds number per dimensional second. For convenience, we assume that the expression \( \Box - \Box \) means the sudden acceleration. In addition to the sudden acceleration, several values of the acceleration rate are adopted to investigate the effect on the flow development.

3. Results and discussion

3.1. Flow structures found in the radial clearance and the axial clearance

The modes of Taylor vortex flows between two rotating cylinders with finite lengths are determined by the number of vortices and the rotating direction of the vortices in the meridional section [29]. In the radial clearance around a rotating disk in a casing, the flows similar to Taylor vortex flows emerge. In this study, four types of flows appear as depicted in figure 2. In this result, the acceleration
The radial clearance is the normal 2-cell mode. The flow pattern represents the streamline like lines obtained from the radial and axial velocity components. The colors designate the radial flow direction, and the red and the blue correspond to the radially outward flow and the radially inward flow, respectively. Figure 2(a) shows the flow at $Re = 5000$ and $\varphi = 0.75$. Two large vortices appear in the radial clearance. The radial flow is outward near the axial mid-plane, and the upper and lower vortices rotate in the counterclockwise and clockwise directions, respectively. This indicates that the flows on the upper and lower end walls are radially inward. The flow structure with radial inward flows on the end walls is in the normal mode. This flow has two vortices and the flow mode is the normal 2-cell mode. The flow in figure 2(b) has four vortices. The upper most and lower most vortices rotate in the counterclockwise and clockwise directions, respectively, and the flow mode is the normal 4-cell mode. Figure 2(c) shows the flow that includes two large vortices and the flows near the end walls are radially outward. On the stationary end wall, the flow retards and the centrifugal effect becomes small. This makes our physical intuition say that the flow on the end wall is radially inward. However, the flow in figure 2(c) is outward near the end wall and this contradicts with our insight. Therefore, the vortex with an outward flow on the end wall is anomalous (29), and the flow mode is the anomalous 2-cell mode. The anomalous vortex accompanies an extra vortex. In figure 2(c), the light blue regions at the upper-right and lower-right corners almost correspond to extra vortices attached to the two main anomalous vortices. The flow in figure 2(d) has three vortices and the flow structure is axially asymmetric. The rotations of the upper most and the lower most vortices are counterclockwise and the lower most vortex is anomalous. The flow mode is the anomalous 3-cell mode. The extra vortex form at the lower right corner.

Figure 3 shows the bird’s eye views of the four flow structures in the axial clearance. The disk rotates in the counterclockwise direction when it is seen from above. The vortex structures are extracted by the contour of $Q$ that is the second invariant of the velocity gradient tensor. The color on the contour indicates the value of the circumferential velocity component. Figure 3(a) depicts the circular vortices. The flow mode in the radial clearance is the normal 2-cell mode and two vortices surround the disk. The Reynolds number is 14000 and it is not small, small thin vortices on the sidewall of the casing attach to the two main vortices. These thin vortices have been confirmed in the study of the direct numerical simulation of the turbulent Taylor-Couette system (30). When the Reynolds number is small, no thin vortex appears and the flow structure is axisymmetric. The flow in figure 3(b) has a polygonal vortices of nonagon around the disk. The flow in the radial clearance is the
anomalous 2-cell mode. The anomalous vortex has an extra vortex and the circumferential deformation of the extra vortex results in the polygonal vortices [25]. Following the shape change of the extra vortex, the two main vortices become wavy in the circumferential direction. In figure 3 (c), the disturbances generated in the radial clearance propagate and several tens of spiral vortices extend from polygonal vortices around the disk to the radially inward region over the disk. Because the normal direction of the front of the spiral vortex is opposite to the rotating direction of the disk, the front angle of the spiral vortex is negative. The flow in the radial clearance is the anomalous 2-cell mode and the circumferential frequency of the polygonal structure is higher than that of the flow in figure 3 (b). Figure 3 (d) represents the flow with spiral vortices with a positive front angle. The flow in the radial clearance is in the anomalous 2-cell mode but it is almost unconstricted. The Reynolds number is the high value of 40000 and the spiral vortices with a positive front angle seem to be the ones that spontaneously appear between rotating and stationary disks [31].

3.2. Flow structures and the acceleration rate of the disk
An unconstricted turbulent state in the axial clearance appears in addition to the four types of vortices denoted in the previous subsection. Figure 4 summarizes the flow structures in the axial clearance at the Reynolds number up to 40000. At each thickness ratio \( \theta \), the profile gives the ranges of the Reynolds number in which the circular vortices, polygonal vortices, spiral vortices with a negative front angle, and spiral vortices with a positive front angle appear.
Figure 3. Flow structures in the axial clearance. □ - □. The disk is rotating in the counterclockwise direction when viewed above.

Figure 4. Flow patterns found in the axial clearance. □ - □. When the thickness ratio is 0.50 and 0.55, the circular vortices arise at small Reynolds numbers and the polygonal vortices with the anomalous 2-cell mode form in a wide range of the Reynolds number. When the thickness ratio is small, the axial clearance is wide. The flow generated on the Ekman layer on the rotating disk, which has a high value of the circumferential velocity component, easily transfers. This flow tends to bring a radially outward flow on the end wall in the radial clearance and makes the flow mode anomalous.

At □ = 0.60, the spiral vortices with a negative front angle follows the circular vortices. As shown in figure 3 (c), the disturbances originating from the several tens of polygonal vortices travel on the stationary end wall of the casing and spread over the disk. When the thickness ratio is from 0.65 to 0.75, the turbulent state and the spiral vortices with a positive front angle emerge after the appearance of the spiral vortices with a negative front angle. The existence of the turbulent state between the spiral vortices with negative and positive front angles supports the assumption that the spiral vortices with different front angles forms from different fluid dynamics mechanics. Schouveiler et al. [16] showed that the onset Reynolds number of the spiral vortices with a positive angle increases as the axial clearance becomes narrower. This is consistent with the current result that the region of the spiral vortices with a positive front angle found at □ - 0.75 disappears and the region of the turbulent state widens at □ - 0.80. At □ - 0.85, the convection in the axial clearance is weak and no coherent structure other than the circular vortices forms.

The acceleration rate affects the flow structures after the completion of the disk acceleration. The well-developed flow structures obtained at the acceleration rates of □ = □ and □ = 500 are shown in the Table 1 where the Reynolds number is from 3000 to 20000. In each table, the row and the column show the Reynolds number Re and the thickness ratio □, respectively. The colors correspond to the flow modes in the radial clearance and the characters mean the flow structures in the axial clearance. The result at □ = 1000 is in the reference [25]. In the cases with two characters of C and S, the circular vortices and the spiral vortices appear at the inner and outer part of the disk region, respectively. At □
Table 1. Final flow structures at $a = r$ and 500 $a$.

(a) $a = r$.

| $\gamma$ | 0.50 | 0.55 | 0.60 | 0.65 | 0.70 | 0.75 | 0.80 | 0.85 |
|----------|------|------|------|------|------|------|------|------|
| 3000     | C    | C    | C    | C    | C    | C    | C    | C    |
| 4000     | P    | C    | C    | C    | C    | C    | C    | C    |
| 5000     | P    | P    | C    | C    | C    | C    | C    | C    |
| 6000     | P    | P    | C    | C    | C    | S    | C    | C    |
| 7000     | P    | P    | S    | C    | C    | C    | S    | C    |
| 8000     | P    | P    | S    | S    | S    | S    | S    | C    |
| 9000     | P    | P    | S    | S    | S    | S    | S    | C    |
| 10000    | P    | P    | P    | S    | S    | S    | C    | C    |
| 11000    | P    | P    | P    | P    | S    | S    | C    | C    |
| 12000    | P    | P    | P    | P    | P    | S    | C    | C    |
| 13000    | P    | P    | P    | P    | P    | P    | S    | C    |
| 14000    | P    | P    | P    | P    | P    | P    | S    | C    |
| 15000    | P    | P    | P    | P    | P    | P    | C    | C    |
| 16000    | P    | P    | P    | P    | P    | P    | S    | S    |
| 17000    | P    | P    | P    | P    | P    | T    | S    | C    |
| 18000    | P    | P    | P    | P    | T    | S    | S    | C    |
| 19000    | P    | P    | P    | P    | T    | T    | S    | C    |
| 20000    | P    | P    | P    | P    | T    | T    | T    | S    |

(b) $a = 500$.

| $\gamma$ | 0.50 | 0.55 | 0.60 | 0.65 | 0.70 | 0.75 | 0.80 | 0.85 |
|----------|------|------|------|------|------|------|------|------|
| 3000     | C    | C    | C    | C    | C    | C    | C    | C    |
| 4000     | P    | C    | C    | C    | C    | C    | C    | C    |
| 5000     | P    | P    | C    | C    | C    | C    | C    | C    |
| 6000     | P    | P    | C    | C    | C    | C    | S    | C    |
| 7000     | P    | P    | S    | C    | C    | S    | S    | C    |
| 8000     | P    | P    | S    | S    | S    | S    | S    | C    |
| 9000     | P    | P    | S    | S    | S    | S    | S    | C    |
| 10000    | P    | P    | P    | S    | S    | S    | C    | C    |
| 11000    | P    | P    | P    | P    | S    | S    | C    | C    |
| 12000    | P    | P    | P    | P    | P    | S    | C    | C    |
| 13000    | P    | P    | P    | P    | P    | P    | S    | C    |
| 14000    | P    | P    | P    | P    | P    | P    | S    | C    |
| 15000    | P    | P    | P    | P    | P    | P    | C    | C    |
| 16000    | P    | P    | P    | P    | P    | P    | S    | S    |
| 17000    | P    | P    | P    | P    | P    | T    | S    | C    |
| 18000    | P    | P    | P    | P    | T    | T    | S    | C    |
| 19000    | P    | P    | P    | P    | T    | T    | T    | S    |
| 20000    | P    | P    | P    | P    | T    | T    | T    | S    |

- $a$ and $-500$ and $a$ up to 0.55, the flow has the anomalous 2-cell mode in all cases and the polygonal vortices in cases except those at the lowest Reynolds number. When the thickness ratio is from 0.60 to 0.70, the spiral vortices tend to emerge instead of the polygonal vortices. At $a - 0.75$ and 0.8, the number of cases of the circular vortices in the normal 2-cell mode increases. When $a$ is 500, the region of the normal 4-cell mode expands at $a$ from 0.70 to 0.80 and the incoherent turbulent state appear at $a - 0.65$ and $Re \geq 17000$. At both acceleration rates, all cases have the normal 2-cell mode at $a - 0.85$.

Table 2 represents the flow structures obtained at lower acceleration rates of $r - 333, 250, 220$ and 200. The Reynolds number is from 3000 to 7000 and the thickness ratio is from 0.50 to 0.70. The flow at $r - 0.70$ has the circular vortices in the normal 2-cell mode at the Reynolds number up to 7000. The cases of this flow structure occupy wider region as the decrease of the acceleration rate and the all cases are the circular vortices in the normal 2-cell mode at $r - 200$. As shown above, the flow on the rotating disk makes the radially outward flow on the end wall in the radial clearance. At higher acceleration rate, the strong radially outward flow appears just after the beginning of the rotation and its effect is large. When the acceleration rate is low, the radially outward flow is not strong enough to suppress the radially inward flow generated by the Ekman pumping that is consistent with the physical intuition, and the normal mode tends to appear frequently.

Table 2. Final flow structures at lower acceleration rates.

| $r$ | 333. | 250. |
|-----|------|------|
| 0.50 | C    | C    |
| 0.55 | C    | C    |
| 0.60 | C    | C    |
| 0.65 | C    | C    |
| 0.70 | C    | C    |
| 0.75 | C    | C    |
| 0.80 | C    | C    |
| 0.85 | C    | C    |
3.3. Process of the flow development

The final flow modes appear some developing processes from the rest. Figure 5 shows one of the processes. The acceleration is sudden and the final flow includes the circular vortices in the normal 2-cell mode. First, small vortices appear at the tips of the rotating disk ($t = 2.0, 0.257$ (s) in the dimensional time). Then, as these vortices grow, a pair of another vortices form on the rim of the disk ($t = 4.0, 0.513$ (s) in the dimensional time), and the flow have four vortices in the radial clearance. This 4-cell mode is transient. The central two vortices collapse ($t = 60, 7.70$ (s) in the dimensional time), and then the flow stabilizes in the final normal 2-cell mode state. The appearance and disappearance of vortices and the mode competition as shown in figure 5 have been studied in Taylor-Couette system [32, 33].

![Figure 5. Formation process of the normal 2-cell mode. □ - σ, Re - 5000, □ - 0.75.](image)

3.4. Torque of the disk and its power law

The evaluation of the torque acting on the disk is important to improve the efficiency of fluid machinery. In this subsection, we estimate the effects of the Reynolds number and the thickness ratio on the torques on the surface and the rim of the rotating disk in the anomalous 2-cell mode.

Figure 6 shows the relations among the torque on one side of the disk $τ_σ$, the Reynolds number and the thickness ratio. The profiles in figure 6 (a) give the variations of the torque with respect to the Reynolds number for the thickness ratios from 0.50 to 0.85. In the logarithmic scales, the torque $τ_σ$ is almost linearly proportional to the Reynolds number. Thought the effect of the thickness ratio on the torque is not large, the torque decreases as the thickness ratio increases and the axial clearance...
becomes narrower. In order to figure out the effect of the thickness ratio, we assume the torque $\tau_a$ has a power law with the Reynolds number $Re$ and the thickness ratio $\gamma$ and drive the following equation

$$T_a = 0.05 Re^{1.58} \gamma^{-0.26}.$$  \hspace{1cm} (3)$$

The profile of the normalized torque is in figure 6 (b). The normalization properly reduces the effect of the thickness ratio and the all plots are almost on the power law given by the yellow line. The figure includes the empirical result by Daily and Nece [34], which evaluates the torque of a rotating plane disk enclosed by a cylindrical chamber. The empirical profile gives lower values when compared with the law of equation (3). This may be because that the existence of the radial clearance in the present study gives an increase of the torque on the disk.

![Figure 6. Torque on a disk surface in the anomalous 2-cell mode.](image)

Figure 7 presents the torque acting on the disk rim $\tau_r$. Though the area causing the torque $\tau_r$ is smaller than that of the torque $\tau_a$, the radius is maximum at the disk rim and the value of $\tau_r$ is comparable with that of $\tau_a$. The thickness ratio directly affects the area of the rim, and each thickness ratio gives its own straight profile of the torque as shown in figure 7 (a). The power law among the torque, the Reynolds number and the thickness ratio gives the equation

$$T_r = 0.32 Re^{1.46} \gamma^{1.65}.$$  \hspace{1cm} (4)$$

The normalized torque in figure 7 (b) gives the unique profile regardless the thickness ratio. We may reasonably consider that the torque $\tau_r$ is linearly proportional to the thickness ratio. However, the exponent of $\gamma$ in equation (4) is not unity. This nonlinearity may depends on the interaction of the flows in the radial and axial clearances, and the large interaction at the lower thickness ratio decreases the torque on the disk rim.
4. Conclusion

The flow around a rotating disk in a cylindrical casing is numerically investigated. The flow field has a radial clearance and an axial clearance and characteristic flows develop in these clearances. The Reynolds number based on the disk radius and the circumferential velocity at the disk rim is from 3000 to 20000 or 40000. The ratio of disk thickness to the length of the cylinder is from 0.50 to 0.85. In the radial clearance, the modes of the normal 2-cell, normal 4-cell, anomalous 2-cell and the anomalous 3-cell appear. In the axial clearance, the circular vortices, polygonal vortices, spiral vortices with a negative and a positive front angle and the turbulent state exist. The diagram of these flow structures in the space of the Reynolds number and the thickness ratio is clarified for the case of the sudden acceleration. The anomalous mode and the polygonal vortices tend to appear at the smaller thickness ratios where the interaction of the flows between the radial and the axial clearances is rather strong. When the thickness ratio is large, the normal 2-cell mode usually found in Taylor-Couette system frequently forms. In case that the acceleration rate is small and the flow development is quasi-steady, the circular vortices in the normal 2-cell mode appear. It is shown that the flow develops to the final state via several structures. The torques acting on the disk surface and the disk rim are evaluated and the torque laws are introduced, which represent the relations with the Reynolds number and the thickness ratio.

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