Advantages of the rain-flow method at the post-processing stage in comparison with the spectral approach

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Abstract. In some massive constructions, like off-shore platforms for example, it is sometimes difficult or impossible to register the loading process during many years of exploitation. Knowing the input random processes characteristics, it sometimes seems promising to estimate the output characteristics using the structural dynamic equation. Following this approach, the spectral densities are employed into longevity estimation. Many methods for estimation of the fatigue damage through the spectral densities were developed. The question arises: is it practical to use the spectral approach in case of post-processing when the process realization already exists? The aim of this paper is to show some limitations of the spectral approach. First, the short review with a brief analysis of the accuracy of the spectral method is presented. It is worth mentioning, that the ultimate goal of the authors of those methods is to achieve the results, similar to those, which were obtained from the rain-flow method. Next, the real loading process is analyzed. To show the fact, that some of the assumptions of spectral methods are doubtful, the method of the digitally modelled process is introduced. Pro and contra of the spectral methods are discussed.

1. Introduction

Nowadays many works are dedicated to machine longevity estimation in the frequency domain, i.e. though operating with spectral densities of the random loading signal. Indeed, there are some situations, when the problem is already described somehow in the frequency domain. That means that the spectral densities of the element loading are known. Spectral densities could be obtained, for example, through the decision of the structural dynamic task. In these situations, spectral methods for estimating longevity represent the natural way to solve the problem [1]. The power spectra of wave loading, for example, can serve as an input in a structural dynamic analysis for estimating of the off-shore longevity by spectral methods at the design stage [2].

On the other hand, during the post-processing stage, when the researchers could register the random loading realization during the exploitation, there is no need to take that controversial way of random loading proceeding. Following the well-known and experimentally proved rain-flow method [3], engineers could estimate longevity for almost every type of the random processes, including non-stationary and non-gaussian types without any difficulty. The rain-flow method is among the best methods of cycle counting – the procedure of substitution of the random amplitudes by the set of
harmonics in a special way. Most of the standards recommend the rain flow method [4-6]. Together with the Miner’s summation rule [7] the rain-flow method gives good agreement with the experiment.

In this paper 1) the short review of the spectral methods is presented; 2) the analysis of some real loading data are shown; 3) special method of process simulation was proposed with the purpose to reveal some contradictions in spectral approach.

2. Short review of the spectral methods
Since the introduction of the famous Rice’s formula [8] in 1944, describing the distribution of peaks of narrowband random processes in a closed form, a lot of effort has been made to find the ways to use the spectral representation of random processes of loading in the evaluation of durability.

2.1. Narrow-band approximation
The Rayleigh approximation, also referred to as the narrowband approximation, assumes that the stress ranges are distributed as the maximums of the stationary Gaussian process. In the narrow-band process for one peak the mean level crossing takes place. In 1965 the formula for accessing the fatigue damage in case on narrow-band process was proposed [9]. The fatigue damage evaluated by the narrow-band Rayleigh approximation (NB) can be defined as $D_{NB}$.

2.2. Wide-band approximation by correction factor
Most processes are not narrow band. The more complex the random process, the more distant the distribution of peaks is from the distributions of amplitudes of the rain flow amplitudes. They might be called as the wide-band process or as the processes with the complex structure.

The damage due to a wideband stress process ($D$) in some cases is proposed to be estimated through the correction of the narrowband estimated damage as:

$$D = \lambda D_{NB}$$

In (1) $\lambda$ is the correction factor. Some authors [10], [11] proposed varied methods for estimation of $\lambda$. In Table 1 and Fig.1 these results will be referred to as WL for [10] and AL for [11].

2.3. Wide-band approximation by composing a few distributions
Some authors tried to compose two or more varied distributions to get the rain-flow amplitudes distributions. For a long time considered as the best method – Dirlik method [12] will be referred to as DK. The other method of this kind is [13] (ZB).

2.4. Methods, which employ spectral moments
The geometrical characteristics of spectral density $W(f)$, which represents the frequency spectrum of the random stress, or the moments $m_j$, for longevity estimation are calculated by [1]:

$$m_j = \int_0^\infty f^j W(f)df$$

2.5. Brief comparison
Detailed review with the thorough description of the methods is presented in [11]. Some extracts from [11] in chronological order are shown in Tabl.1.
Table 1. List of some popular spectral methods (from [17] with the permission of the author).

| Definition | Method                  | Year | Reference |
|------------|-------------------------|------|-----------|
| NB         | narrow-band method      | 1965 | 9         |
| WL         | Wirsching-Light method  | 1980 | 10        |
| DK         | Dirlik method           | 1985 | 12        |
| ZB         | Zhao-Baker method       | 1992 | 13        |
| AL         | the α0.75 method        | 2004 | 11        |
| PZ         | Petrucci-Zuccarello method | 2004 | 15        |
| TB         | Tovo-Benasciutti method | 2006 | 14        |

In [17&18] the accuracy of fatigue-life estimates of the frequency-domain methods was compared to the life estimate in the time domain using a combination of the rain-flow count [4,5] and the Palmgren-Miner [7] hypothesis. This value was denoted as TRF and was assumed to be an exact reference value in the studies. The relative error is calculated by:

$$ERR = \frac{T^{XX} - TRF}{TRF}$$  \hspace{1cm} (3)

where $T^{XX}$ – the estimations based on the methods shown in Table 1.

Several random processes were modelled and the longevity estimations for varied fatigue exponent $m$ values were performed. Here, among many other processes, the process MM (multi-modal, [18]) has been chosen, as the most interesting example, because of its wide-band nature. In Figure 1 according to works [17] and [18] the accuracy of some spectral methods is shown.

![Figure 1. The comparison of varied methods accuracy in relation to rain-flow.](image)

Note, that the results in Fig.1 were taken just for example. It is impossible to judge about the accuracy of the methods looking at this Figure. More thorough investigations usually conclude about advantages of DK and TB methods above the other methods.
The fact, that we have learned from this analysis, is the following. The accuracy of the estimations by the spectral approach methods is satisfactory, no matter which of the methods was used.

3. Material

As an example of the random loading, the real loading processes in the torsion shaft of the cantilever machine during exploitation were investigated [19]. Figure 2 shows one realization registered during 1,5 min. move. It represents a typical loading of the machine part.

![Figure 2. The realization #1 of random loading in the part of cantilever machine.](image)

The main statistical characteristics of the process are:

- Mean value = 348.9 MPa;
- Std = 60.84 MPa.

Two studies of the random processes were performed.

3.1. Stationarity test

To calculate moments \( m_j \) (2) the spectral densities \( W(f) \) should be evaluated. By the definition they exist only for stationary processes. In this study Augmented Dickey-Fuller Test was employed. The analysis was performed in R statistical package [20]. Since the p-values for the sample processes were small, then with the significance level of 0.01 those series could be considered stationary (the hypothesis of the presence of a single root is rejected). It allows estimating the spectral densities (Fig.5, the solid line).

3.2. Analysis the of irregularity factor

The important characteristic, which shows the degree of complexity of the process is irregularity factor, IF. Knowing \( W(f) \) and the moments \( m_j \) the irregularity factor might be estimated [8]:

\[
IF = m_2 \sqrt{\frac{m_0 m_4}{\mu_4}}
\]

In other way it might be defined as in [4]:
where \( N_0 \) and \( N_E \) are the numbers of mean level crossing and the number of extremums accordingly.

The study of dependability of the \( IF \) indicator was performed by the investigation of several realizations of stresses in the torsion shaft, like what one which is shown in the Fig. 2. The initial process was divided by the classes by the ordinate [4]. Classes number in this study was taken in the interval \( M = 12 \ldots 100 \). This procedure of dividing by classes might be used as a first step of filtration. The dependence of \( IF \) on \( M \) for some realizations is shown in the Fig. 3.

![Figure 3. Dependence the estimated \( IF \) on the number of classes](image)

It can be seen, that with the increase in the number of classes, \( IF \) decreases by up to 35%. Hence the conclusion: the value \( IF \) cannot be used as an objective characteristic of the random process. If the value \( M \) is taken \( M = 36 \), (following the recommendations [4]), the estimated for realization #1 by equation (4) is \( IF = 0.525 \), that means, that the process is complex enough.

**4. Method**

As a method to confirm the validity of the transition from a continuous process to a sequence of extrema, the “saw”, the numerical simulation was carried out. That question is of great importance while speaking of spectral densities because during this transition the information of time is lost. Fig. 4 schematically shows the three stages of the process transformation. As an example, the initial part of the random process #1 was taken (Fig.2). The first transition from "initial" (I) to the "saw" (II) is recommended by the majority of researchers [15, 16] and is included in all standards [4, 5]. This transition is a necessary stage for all the cycle counting methods, including rain-flow. What happens with the spectral density during this transformation? This question is important while speculating about the spectral approach.

There is a desire to compare spectral densities of those processes. However, it is impossible to judge of the spectral density of the “saw”: there is no time factor, the process is not differentiable, because there are singularities. To show the fact, that the spectra do not influence much upon the longevity, a method of the peaks approximating of the process II by the half-waves was developed. It
was proposed to connect the peaks of the “saw” by the half-cosines. The equation of the smoothing process corresponds for each half-cosine wave would be:

\[ X = A \cos (wt + \varphi) \]  

(5)

where \( x \) is the resulting extrapolating process. For each half-wave (5) the parameters \( A, w \) and \( \varphi \) are unique. The amplitude \( A \) is defined as the module of the half difference of successive extremums \( e[I-1] \) and \( e[i] \):

\[ A_i = \frac{\text{abs}(e[i] - e[i-1])}{2} \]  

(6)

The frequency \( w \) is chosen in a such way, that the half-period would have enough points \((k = 5 \ldots 40)\), depending on the \( A_i \) value. The initial cosine phase \( \varphi \) depends on the type of half-wave. If the downward sweep is modelled, \( \varphi = 0 \). For the rising half-wave \( \varphi = \pi/2 \). Of course, the choice of \( k \) will influence the frequency characteristics of the modelled process (Fig. 4, III).

This modelling was performed in R language [20].

The resulting process, obtained as outcome of this approximation, is shown in Fig. 4, III. After this transformation it becomes possible to estimate the its spectral density, since process III represents the discretization of a continuous function. Spectral densities of the processes I and III are shown in Fig. 5. The dashed line, laying below in the high-frequency domain, corresponds to the process III.

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**Figure 4.** Three processes
Figure 5. Comparing the spectral densities of two similar by damage level processes

Figures 4 and 5 show that the processes I and III are similar at some extent, the frequency characteristics are close. On the other hand, the natures of the spectra are different. In the region of high frequencies (above 1.5 Hz), process III has significantly lower level than I: the simulated process has less "high-frequency" noise for the power spectrum by two orders of magnitude. This difference cannot but affect the calculated moments of $m_j$ according to the formula (2), and hence the durability estimates, produced by the processes I and III using moments (3) will be different. Research practice shows, on the contrary, the legitimacy of the transition from I to III and the similarity of these processes by the damaging impact.

5. Discussion
In this paper, only the stage of post-processing is for uniaxial loading was considered. The question of multiaxial loading was not covered. Multiaxial loading is addressed in [1, 21].

Table 2. Arguments and counterarguments concerning rain-flow and spectral approach.

| Person | Argument | Counterargument |
|--------|----------|-----------------|
| D. Benashuti, private exchange of views in researchgate | “If you have, say, only one sample value from a Gaussian distribution, what can you say about the parameters (true 'mean' and 'std') of that distribution? Very little... …nothing can be said about the true mean value of the damage statistical distribution. In addition, nothing can be said about cycles not directly observed in that time-history. By contrast, if you have a PSD, you can estimate the amplitude PSD and the expected damage from PSD data. The main difference between time- and frequency-domain approach basically relies in the reliability of statistical estimates” | Estimating the spectral density by only ONE realization also restricts the horizons. Some extra steps concerning the loading variability should be taken in the both approaches |
| F. Cianetti [16] | Rain-flow has a heavy computational cost… | While choosing the proper classes number [4] it is possible to perform the rain-flow treating in a real-time mode |
Currently, many researchers are working to expand spectral methods towards non-Gaussian and non-stationary processes [1, 22]. Using rain-flow method the researchers do not experience special difficulties because of these peculiarities.

As for the discussion about the use of spectral methods in the post-processing phase, table 2 attempts to collect some arguments and respond to them.

6. Conclusions

In some situations (especially at the design stage) methods for estimating longevity by the spectral density might be helpful.

Many methods for estimating the longevity by the spectral methods exist and it is tough to choose the best one for all the situations concerning the varied processes types and material properties.

The irregularity factor seems not to be the robust indicator of the random processes.

All types of random loading processes, including non-Gaussian and non-stationary, could be easily treated by rain-flow method.

Developed intendedly for the analysis of possibility of transmission from continuous process to the extremum discontinuous sequence method proved the fact, that there is no strong correlation between the spectral density and longevity.

Following the recommendation [23], which was given many years ago, the spectral methods would be recommended at the design stage only.

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