Yager’s ranking method for solving the trapezoidal fuzzy number linear programming

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Abstract. In the previous research, the authors have studied the fuzzy simplex method for trapezoidal fuzzy number linear programming based on the Maleki’s ranking function. We have found some theories related to the term conditions for the optimum solution of fuzzy simplex method, the fuzzy Big-M method, the fuzzy two-phase method, and the sensitivity analysis. In this research, we study about the fuzzy simplex method based on the other ranking function. It is called Yager’s ranking function. In this case, we investigate the optimum term conditions. Based on the result of research, it is found that Yager’s ranking function is not like Maleki’s ranking function. Using the Yager’s function, the simplex method cannot work as well as when using the Maleki’s function. By using the Yager’s function, the value of the subtraction of two equal fuzzy numbers is not equal to zero. This condition makes the optimum table of the fuzzy simplex table is undetected. As a result, the simplified fuzzy simplex table becomes stopped and does not reach the optimum solution.

1. Introduction

Decision-making issues play an important role in real life. Lots of real-world problems are related to decision-making issues. For examples, problems are related to finding the shortest path, the problem of determining the quantity of a product in order to use the minimum quantity of material, the problem of making the goods such that obtain the maximum profit. One of the problems of decision-making is a linear programming problem.

Analogous to the development of decision-making theory, linear programming theory is developed very rapidly. The concept of decision-making in the fuzzy system was first introduced by Ballmen and Zadeh [1]. In relation to the development of the theory of fuzzy linear programming (FLP), [2] introduces the formulation of a linear fuzzy programming problem. Since then, the theory of fuzzy linear programming has grown enormously. There are several models of fuzzy linear programming problems. The fuzzy parameters can only be as the coefficient variables, on the right side of the problem constraints, or a combination of all them. Ebrahimnejad [3,4] defined a double problem through a parametric linear program and shows that the problem of the primal dual-fuzzy linear program has the same solution.

Fuzzy number ranking function is a mapping from a fuzzy number to a real number [5-10]). There are some ranking functions, and two of them are Maleki’s ranking function and Yager’s ranking function. Karyati \textit{et al.} [6] have conducted research related to FLP. In this case, the FLP model used is the ones where its variables are in the form of a trapezoidal fuzzy number. Some of the research results are the fuzzy simplex method using the Maleki’s ranking function, optimum condition, type of settlement, optimum alternative, fuzzy Big-M method, Two-Phase fuzzy method and the analysis of post optimum. By using Maleki’s ranking method, it is quite successful to derive the properties when applying to linear programming problems based on a crisp number system [5].
Based on the results of the previous research, this research aims to analyse the fuzzy simplex method using Yager’s ranking function and to compare the effectiveness of Maleki’s and Yager’s ranking function.

2. Preliminaries

2.1 Fuzzy Set

We review the fundamental notion of fuzzy set theories based on [6]. A fuzzy set $A$ in $X$ is defined as a set of an ordered pair $\{(x, \mu_A(x)) | x \in X\}$, with $\mu_A(x)$ is a membership function of the fuzzy set [11,12]. The membership function is a map from a set $X$ into the closed interval $[0,1]$ [6,13]. Support of a fuzzy set $A$ is defined as a set of all $x \in X$, with $\mu_A(x) > 0$. The core of a fuzzy set $A$ is a set of all $x \in X$ with $\mu_A(x) = 1$. A fuzzy set $A$ is called normal if the core is not an empty set. A fuzzy set $A$ of $X$ is convex if for any $x, y \in X$ and $\delta \in [0,1]$ then: $\mu_A(\delta x + (1 - \delta)y) \geq \min\{\mu_A(x), \mu_A(y)\}$. A fuzzy set is convex if and only if all of the nonempty $a$-cut is convex ([14]). A fuzzy number $A$ is a normal and convex fuzzy set. There are some kinds of fuzzy numbers. An often used fuzzy number is a trapezoidal fuzzy number. It is defined as follows:

**Definition 2.1.** ([5], [6], [15]) A trapezoidal fuzzy number is a fuzzy number denoted by $\tilde{A} = (a^L, a^U, \alpha_1, \alpha_2)$ with the membership function meets the following mapping:

$$f(x) = \begin{cases} \frac{x - (a^L - \alpha_1)}{\alpha_1}, & a^L - \alpha_1 \leq x \leq a^L \\ 1, & a^L \leq x \leq a^U \\ \frac{(a^U + \alpha_2) - x}{\alpha_2}, & a^U \leq x \leq a^U + \alpha_2 \\ 0, & \text{others} \end{cases}$$

The support of this trapezoidal fuzzy number is $[a^L - \alpha_1, a^U + \alpha_2]$.

2.2 Arithmetical operation of trapezoidal fuzzy number

Based on the [5],[9], [15] we get some basic operations of two trapezoidal fuzzy numbers. Let $\tilde{A} = (a^L, a^U, \alpha_1, \alpha_2)$ and $\tilde{B} = (b^L, b^U, \beta_1, \beta_2)$ be trapezoidal fuzzy numbers, $r \in \mathbb{R}$ then the following arithmetical operations apply:

i. For $r \geq 0$, then $r\tilde{A} = (r a^L, r a^U, r \alpha_1, r \alpha_2)$

ii. For $r < 0$, then $r\tilde{A} = (r a^L, r a^U, -r \alpha_1, -r \alpha_2)$

iii. $\tilde{A} + \tilde{B} = (a^L + b^L, a^U + b^U, \alpha_1 + \beta_1, \alpha_2 + \beta_2)$

iv. $\tilde{A} - \tilde{B} = (a^L - b^L, a^U - b^U, \alpha_1 + \beta_2, \alpha_2 + \beta_1)$

2.3 Fuzzy ranking function

Maleki et al. [4] use a fuzzy ranking method to solve FLP. Karyati et al. [5] also use a fuzzy ranking method to construct fuzzy simplex method. Let $F(\mathbb{R})$ be a set of fuzzy number and $\mathbb{R}$ be a real number. Then an efficient approach for ordering the elements of $F(\mathbb{R})$ is to define a ranking function $R: F(\mathbb{R}) \rightarrow \mathbb{R}$ which map each fuzzy number into a real number (as a line), where a natural order exists. The following relations are the properties of the ranking function [5,9,15]:

i. $\tilde{A}_1 \leq \tilde{A}_2$ if and only if $R(\tilde{A}_1) \leq R(\tilde{A}_2)$.

ii. $\tilde{A}_1 > \tilde{A}_2$ if and only if $R(\tilde{A}_1) > R(\tilde{A}_2)$.

iii. $\tilde{A}_1 = \tilde{A}_2$ if and only if $R(\tilde{A}_1) = R(\tilde{A}_2)$.

A fuzzy ranking function meets $R(k\tilde{A}_1 + l\tilde{A}_2) = kR(\tilde{A}_1) + lR(\tilde{A}_2)$ for every $\tilde{A}_1, \tilde{A}_2 \in F(\mathbb{R})$. For a trapezoidal fuzzy number $\tilde{A} = (a^L, a^U, \alpha_1, \alpha_2)$, then according to Maleki et al. [5], the mapping by $R$ is:

$$R(\tilde{A}) = \frac{1}{2}(a^L + a^U + \frac{4}{\alpha_1 + \alpha_2})$$

The Yager’s fuzzy ranking function is defined as $R(\tilde{A}) = \frac{4}{\alpha_1 + \alpha_2} \left(\frac{a^L}{\alpha_1} + \frac{a^U}{\alpha_2} \right)$.
2.4 Fuzzy simplex method using Maleki’s ranking function
In this case, the general model of FPL is as follow:

Maximized : $\tilde{Z} = \tilde{c}x$

such that : $Ax = b, x \geq 0$  \hspace{1cm} (2.1)

where $b \in (R)^m$, $x \in (R)^n$, $A \in R^{m \times n}$, and $\tilde{c}^T \in (F(R))^n$.

To solve the FLP (2.1), Karyati et al. [5] construct an algorithm for solving FLP problem using a fuzzy simplex method. The following are the steps of the fuzzy simplex algorithm for a maximum standard case of FLP problem:

i. Transform the FLP problem into a canonical form (the constraints must be positive, if necessary, change it into ‘=’ relation by adding a slack variable).

ii. Create an initial fuzzy simplex tableau, same as the crisp simplex tableau, only on its coefficient row; the objective function is a trapezoidal fuzzy number.

iii. To test the optimality, it can be seen from the value on the $\tilde{z}_j - \tilde{c}_j$ row. If there is still a $\tilde{Z}_j - \tilde{c}_j < 0$, then is not optimal yet. Use the same way for $\Re(\tilde{z}_j - \tilde{c}_j) < 0$ to determine if $\tilde{Z}_j - \tilde{c}_j < 0$.

iv. To fix the solution, choose an entering variable with the most negative $\Re(\tilde{z}_j - \tilde{c}_j)$, and choose a leaving variable by choosing the most positive smallest $R_i$.

v. On the new tableau, the element pivot must be transformed into 1, and the other elements on the same row must be transformed into 0 using elementary row operations. Repeat step (iii) until an optimal solution is obtained.

3. Fuzzy simplex method using Yager’s ranking function
3.1. Algorithm fuzzy simplex method using Yager’s ranking function

The initial idea of the fuzzy simplex method using a Yager’s ranking function is the algorithm of fuzzy simplex method used by Karyati at al [5] that based on Maleki’s ranking function. Thus the proposed algorithm of the fuzzy simplex method using a Yager’s ranking function is as follows:

i. Transform the FPL problem into a canonical form (all constraints must be positive, if necessary, change them into ‘=’ relation by adding a slack variable to each variable).

ii. Create an initial fuzzy simplex tableau, same as the crisp simplex tableau, only on its coefficient row; the objective function is a trapezoidal fuzzy number.

iii. To test the optimality, it can be seen from the value on the $\tilde{z}_j - \tilde{c}_j$ row. If there is still a $\tilde{Z}_j - \tilde{c}_j < 0$, then is not optimal yet. Use the same way with $\Re(\tilde{z}_j - \tilde{c}_j) < 0$ to determine if $\tilde{Z}_j - \tilde{c}_j < 0$.

iv. Choose an entering variable with the most negative $\Re(\tilde{z}_j - \tilde{c}_j)$, and choose a leaving variable by choosing the most positive smallest $R_i$.

v. On the new tableau, the element pivot must be equal to 1, and the other elements on the same row must be equal to 0 using elementary row operations. Repeat step iii until an optimal solution is obtained.

3.2 Numerical simulation
To describe the previous algorithm, the numerical simulation is given here. Recall the FLP problem used in Karyati et al. [5].

Example 1:

Maximize $\tilde{Z} = (4,6,2,3)x_1 + (5,8,2,4)x_2$

such that $3x_1 + 6x_2 \leq 18$
$5x_1 + 4x_2 \leq 20$
$x_1, x_2 \geq 0$

Based on the standard maximum problem, the canonical form is as follow:

Maximize $\tilde{Z} = (4,6,2,3)x_1 + (5,8,2,4)x_2 + \tilde{0}x_3 + \tilde{0}x_4$

such that $3x_1 + 6x_2 + x_3 = 18$
Using the algorithm fuzzy simplex method based on a Yager’s ranking function, we get the 0’s iteration simplex in Table 1.

| Table 1. The initial simplex tableau FLP for Example 1 |
|---------------------------------------------|
| Iteration 0  | \( c_j \) | (4,6,2.3) | (5,6,2.4) | (0,0,0.0) | (0,0,0.0) | \( b_j \) | \( R_i \) |
|---------------|------------|------------|------------|------------|------------|----------------|----------------|
| \( x_i \)    | \( x_j \) | \( x_3 \) | \( x_4 \) | \( x_5 \) | \( x_6 \) | \( x_7 \) | \( x_8 \) | \( x_9 \) | \( x_{10} \) |
| (0,0,0,0)     | (0,0,0,0) | 3          | 6          | 1          | 0          | 18             | 6              |
| (0,0,0,0)     | (0,0,0,0) | 5          | 4          | 0          | 1          | 20             | 4              |
| \( x_1 - c_j \) | \( z_j \) | \( (0,0,0,0) \) | \( (0,0,0,0) \) | \( (0,0,0,0) \) | \( (0,0,0,0) \) | \( (0,0,0,0) \) | \( (0,0,0,0) \) | \( (0,0,0,0) \) |
| \( \Re(x_j - c_j) \) | \( -5 \) | \( -31 \) | \( 0 \) | \( 0 \) | \( 0 \) | \( 0 \) | \( 0 \) | \( 0 \) | \( 0 \) |

According to the Table 1, the ‘entering variable’ is \( x_3 \). This variable is going to become a basis variable for the next iteration, as \( x_4 \) is substituted as the ‘leaving variable’.

| Table 2. The first simplex tableau for Example 1 |
|---------------------------------------------|
| Iteration 1  | \( c_j \) | (4,6,2.3) | (5,6,2.4) | (0,0,0.0) | (0,0,0.0) | \( b_j \) | \( R_i \) |
|---------------|------------|------------|------------|------------|------------|----------------|----------------|
| \( x_i \)    | \( x_j \) | \( x_3 \) | \( x_4 \) | \( x_5 \) | \( x_6 \) | \( x_7 \) | \( x_8 \) | \( x_9 \) | \( x_{10} \) |
| (0,0,0,0)     | (0,0,0,0) | 0          | 1          | \( \frac{18}{5} \) | \( \frac{4}{5} \) | \( 0 \) | \( 0 \) | \( 6 \) | \( \frac{5}{3} \) |
| \( 4,6,2,3 \) | \( 4,6,2,3 \) | \( \frac{16}{5} \) | \( \frac{24}{5} \) | \( \frac{8}{5} \) | \( \frac{12}{5} \) | \( 0 \) | \( 0 \) | \( 4 \) | \( \frac{20}{4} \) |
| \( x_1 \)    | \( x_1 \) | \( \frac{1}{5} \) | \( \frac{1}{5} \) | \( \frac{1}{5} \) | \( \frac{28}{5} \) | \( \frac{22}{5} \) | \( 0 \) | \( 0 \) | \( 0 \) |
| \( \Re(x_1 - c_j) \) | \( -1 \) | \( -0 \) | \( 0 \) | \( 0 \) | \( 0 \) | \( 0 \) | \( 0 \) | \( 0 \) | \( 0 \) |

The pivot element is equal to 5, and it must change into 1. So, all elements in this row are divided by 5, and all elements in the same column are changed into zero. Using elementary row operation, the first iteration, we get Table 2. In Table 2, we obtain \( x_2 \) as the ‘entering variable’ and \( x_3 \) as the ‘leaving variable’. The pivot element is equal to \( \frac{18}{5} \), and it must change into 1. So, all elements in this row are divided by \( \frac{18}{5} \), and all elements in the same column must be changed into zero. So, the second iteration tableau is obtained as follow:

| Table 3. The second simplex tableau for Example 1 |
|---------------------------------------------|
| Iteration 2  | \( c_j \) | (4,6,2.3) | (5,6,2.4) | (0,0,0.0) | (0,0,0.0) | \( b_j \) | \( R_i \) |
|---------------|------------|------------|------------|------------|------------|----------------|----------------|
| \( x_i \)    | \( x_j \) | \( x_3 \) | \( x_4 \) | \( x_5 \) | \( x_6 \) | \( x_7 \) | \( x_8 \) | \( x_9 \) | \( x_{10} \) |
| \( 5,6,2,4 \) | \( 5,6,2,4 \) | \( \frac{5}{18} \) | \( 0 \) | \( 0 \) | \( \frac{5}{3} \) | \( \frac{5}{3} \) | \( \frac{5}{3} \) | \( \frac{5}{3} \) | \( \frac{5}{3} \) |
| \( 4,6,2,3 \) | \( 4,6,2,3 \) | \( \frac{7}{18} \) | \( \frac{11}{18} \) | \( \frac{14}{18} \) | \( 0 \) | \( 0 \) | \( \frac{27}{7} \) | \( \frac{26}{7} \) | \( \frac{41}{7} \) |
| \( x_2 \)    | \( x_2 \) | \( 0 \) | \( 1 \) | \( 0 \) | \( 0 \) | \( 0 \) | \( 0 \) | \( 0 \) | \( 0 \) |
| \( x_1 \)    | \( x_1 \) | \( \frac{1}{3} \) | \( \frac{1}{3} \) | \( \frac{1}{3} \) | \( \frac{1}{3} \) | \( \frac{1}{3} \) | \( \frac{1}{3} \) | \( \frac{1}{3} \) | \( \frac{1}{3} \) |
| \( \Re(x_2 - c_j) \) | \( -\frac{1}{3} \) | \( -\frac{2}{3} \) | \( -\frac{3}{3} \) | \( -\frac{4}{3} \) | \( -\frac{5}{3} \) | \( -\frac{6}{3} \) | \( -\frac{7}{3} \) | \( -\frac{8}{3} \) | \( -\frac{9}{3} \) |

Based on Table 3, the simplex tableau is not optimal yet. It is because there are \( \Re(x_j - c_j) \) that have negative values, i.e. \( \Re(x_1 - c_1) = -\frac{1}{3} \) and \( \Re(x_2 - c_2) = -\frac{2}{3} \). Based on the previous algorithm, \( x_2 \) becomes the ‘entering variable’. But if we choose this one, \( x_2 \) also becomes the ‘leaving variable’, which is impossible. Therefore, the optimality condition cannot be satisfied. Unlikely we cannot improve the solution This condition is different when we use a fuzzy simplex
method based on the Maleki’s ranking fuzzy. According to the results of Karyati et al. [5], the problem obtains its optimal solution at the second iteration. Recall its optimum tableau [5] as follow:

**Table 4.** The optimum solution for Example 1 using Maleki’s ranking function

| Iteration 2 | \( \bar{c}_j \) | (4,6,2,3) | (5,8,2,4) | 0 | 0 |
|-------------|----------------|-----------|-----------|---|---|
| \( \bar{c}_j \) | \( x_1 \times x_j \) | \( x_1 \times x_2 \times x_3 \times x_4 \) | \( b_j \) |
| (5,8,2,4) | \( x_2 \) | 0 | 1 | \( \frac{5}{18} \) | \( \frac{1}{6} \) |
| (4,6,2,3) | \( x_1 \) | 1 | 0 | \( -\frac{1}{6} \) | \( \frac{5}{3} \) |
| \( \bar{z}_j \) | (4,6,2,3) | (5,8,2,4) | (0,0,2,4) | (0,0,2,4) |
| \( \bar{z}_j - \bar{c}_j \) | (-2,2,5,5) | (-3,3,6,6) | (0,0,2,4) | (0,0,2,4) |
| \( \Re(\bar{z}_j - \bar{c}_j) \) | 0 | 0 | \( \frac{7}{7} \) | \( \frac{7}{7} \) |

From Table 4, it shows that \( \Re(\bar{z}_j - \bar{c}_j) \geq 0 \), hence the optimal solution is obtained, which is: \( \bar{z} = (57, 88, 3, 5) \) for \( x_1 = \frac{9}{3} \) and \( x_2 = \frac{5}{3} \).

From the above simulation, we indicate that the fuzzy simplex method based on Yager’s ranking function is not suitable for this kind of problem, not like when Maleki’s ranking function is used. To get a clearer sight, the next FLP example is given as follow:

**Example 2:**

Maximize \( \bar{z} = (2,6,5,6)x_1 + (1,5,10,3)x_2 \)

Such that

\[
4x_1 + 2x_2 \leq 8
\]

\[
x_1 + x_2 \leq 8
\]

\[
x_1, x_2 \geq 0
\]

Then, its canonical form is:

Maximize \( \bar{z} = (2,6,5,6)x_1 + (1,5,10,3)x_2 + \bar{o}x_3 + \bar{o}x_4 \)

Such that

\[
4x_1 + 2x_2 + x_3 = 8
\]

\[
x_1 + x_2 + x_4 = 8
\]

\[
x_1, x_2, x_3, x_4 \geq 0
\]

\[
\bar{o} = (0,0,0,0)
\]

According to the 2\textsuperscript{nd} iteration on Table 5, the table is not optimized yet. It can be seen that all \( \Re(\bar{z}_j - \bar{c}_j) \) is less than zero. So, the solution can be improved by choosing \( x_2 \) as the ‘entering variable’, but it causes \( x_2 \) as the ‘leaving variable’. On the other hand, if \( x_4 \) is chosen as the ‘entering variable’, it will cause \( x_1 \) as the ‘leaving variable’. Again, it ends up with the same situation like in our first example (Example 1). Both in Example 1 and Example 2, the same case happens, i.e. the iteration cannot be continued because the entering variable is the same as the leaving variable.

According to the canonical form, the initial solution is obtained in the following fuzzy simplex:
We have tried to substitute the Maleki’s ranking function by a formula as follows:

\[
\text{optimal solution and the iterations never stopped. Why did this case happen? Here is our further analysis.}
\]

\[
\begin{array}{cccccc}
\text{Iteration 0} & \mathcal{z}_j & (2,6,6,5) & (2,8,3,10) & (0,0,0,0) & (0,0,0,0) \\
(0,0,0,0) & x_1 & x_1 & x_2 & x_3 & x_4 & b_i & R_i \\
(0,0,0,0) & x_3 & 4 & 2 & 1 & 0 & 8 & 4 \\
\end{array}
\]

\[
\begin{array}{cccc}
\text{Iteration 1} & \mathcal{z}_j & (2,6,6,5) & (2,8,3,10) \\
(0,0,0,0) & x_2 & 1 & 1 & 0 \\
(2,8,3,10) & x_1 & 2 & 0 & 1 & -2 & 2 & 1 \\
\end{array}
\]

\[
\begin{array}{cccc}
\text{Iteration 2} & \mathcal{z}_j & (2,6,6,5) & (2,8,3,10) \\
(2,6,6,5) & x_2 & 0 & 1 & -1 & 1 & -1 \\
(2,8,3,10) & x_1 & 1 & 0 & -1 & 2 & 2 & 2 \\
\end{array}
\]

### 3.3 Analysis of fuzzy simplex method using Yager’s ranking function

Based on the algorithm of the fuzzy simplex method using Maleki’s ranking function, we get the same condition when the optimum solution is obtained for the crisp simplex method. The fuzzy big-M method and fuzzy two-phase method can be found. Moreover, the analysis of post optimum is analogue with the crisp simplex method. We have tried to substitute the Maleki’s ranking function with Yager’s ranking function. It can be concluded that the fuzzy simplex method never meets the optimal solution and the iterations never stopped. Why did this case happen? Here is our further analysis.

Remember that Maleki’s ranking function for a trapezoidal fuzzy number \( \tilde{A} = (a^L, a^U, \alpha_1, \alpha_2) \) has a formula as follow:

\[
\mathcal{R}(\tilde{A}) = \frac{1}{2}(a^L + a^U + \frac{1}{2}(\alpha_2 - \alpha_1)).
\]

If the trapezoidal fuzzy number has \( \tilde{A} = (a^L, a^U, \alpha_3, \alpha_2) \), then:

\[
\mathcal{R}(\tilde{A} - \tilde{A}) = \mathcal{R}((a^L, a^U, \alpha_1, \alpha_2) - (a^L, a^U, \alpha_1, \alpha_2))
\]

\[
= \mathcal{R}((a^L, a^U, \alpha_1, \alpha_2))
\]

\[
= \mathcal{R}((a^L - a^L, a^U - a^U, \alpha_1 + \alpha_1, \alpha_1 + \alpha_1))
\]

\[
= \frac{1}{4}(0, 0, 0, 0)
\]

The Yager’s fuzzy ranking function is defined as \( \mathcal{R}(\tilde{A}) = \frac{1}{2}(a^L + a^U - \frac{1}{2}a_1 + \frac{1}{2}a_2) \). Then for \( \tilde{A} = (a^L, a^U, \alpha_1, \alpha_2) \), we have:

\[
\mathcal{R}(\tilde{A} - \tilde{A}) = \mathcal{R}((a^L, a^U, \alpha_1, \alpha_2) - (a^L, a^U, \alpha_1, \alpha_2))
\]

\[
= \mathcal{R}((a^L, a^U, \alpha_1, \alpha_2))
\]

\[
= \mathcal{R}((a^L - a^L, a^U - a^U, \alpha_1 + \alpha_1, \alpha_1 + \alpha_1))
\]

\[
= \frac{1}{4}(0, 0, 0, 0)
\]

\[
\mathcal{R}(\tilde{A} - \tilde{A}) = \frac{1}{2}(a^L + a^U - \frac{1}{2}a_1 + \frac{1}{2}a_2).
\]
Based on Figure 1, we know the values of $\alpha_1$ and $\alpha_2$ are always positive. Thus the Yager's ranking function of $\mathcal{R}(\bar{A} - \bar{A})$ is always negative. According to the Maleki’s ranking function, the value of $\mathcal{R}(\bar{A} - \bar{A})$ is always zero. When we use a fuzzy simplex method based on Maleki’s ranking function, the method is consistent like when a crisp simplex method is used. This has happened since the value $\mathcal{R}(\bar{A} - \bar{A})$ is zero. See the Table 4. Since $\mathcal{R}(\bar{A} - \bar{A})$ is zero, so the optimum condition is held. On the other hand, see Table 3. This table is the fuzzy simplex method based on Yager’s ranking function, which has a negative value for its ranking function. In the maximum case, this is suspected as the cause of the fuzzy simplex method cannot get the optimal solution.

The alternative to solve the problem, we can define the $\mathcal{R}(\bar{A} - \bar{A}) = 0$, for Yager’s ranking function. It means that $\mathcal{R}(\bar{A} - \bar{A}) = -\frac{1}{10}(\alpha_1 + \alpha_2) = 0$ or $\alpha_1 = -\alpha_2$. It is impossible since the values of $\alpha_1$ and $\alpha_2$ are always positive and $\alpha_1, \alpha_2 \neq 0$. It is impossible if $\alpha_1$ and $\alpha_2$ are equal to zero. Based on these conditions, we conclude that the algorithm for fuzzy simplex method is not effective if it is based on Yager’s ranking function. If ones want to use it, they must redefine the value of $\mathcal{R}(\bar{A} - \bar{A}) = 0$, but its consistency with the other operations must be re-check.

4. Conclusion

Based on the results of the previous discussion, it can be concluded that Yager’s ranking function is not as well as Maleki’s ranking function to construct a fuzzy simplex method. Using the Yager’s function, the simplex method cannot work as well as when using the Maleki’s function. By using the Yager’s function, the value of the subtraction of two equal fuzzy numbers is not equal to zero. This condition makes the optimum table of the fuzzy simplex table is undetected. As a result, the simplified fuzzy simplex table becomes stopped and does not reach the optimum. If ones want to use it, they must redefine the value of $\mathcal{R}(\bar{A} - \bar{A}) = 0$, but its consistency with the other operations must be re-check.

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