Technical Note

Estimation of Settling Velocity and Floc Distribution through Simple Particles Sedimentation Experiments

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Abstract: The tailings that remain after the collection and screening of Mn nodules are directly discharged into the ocean and are anticipated to influence the ocean environment and marine organisms. The primary factors determining the influence (diffusion) range are the ocean currents and the settling velocity of the tailings; the latter is directly correlated with the time that the tailings remain in the water column. Flocculation is affected by the actual tailing discharge conditions. The settling velocity of the tailings is expected to increase as a result of flocculation; therefore, data on the size distribution of flocs are needed to compute the settling velocity of the tailings. In this study, a method for estimating the floc size distribution of the tailings is proposed, and the general flocculation process is analyzed using the apparent settling velocity, which is readily estimated by simple settling experiments conducted with standard tailings at different concentrations. The apparent falling time-curve followed a power function, and the flocculated grain size was 3–4 times larger than that before flocculation. In addition, flocculation and falling were significantly inhibited by the time required for flocculation. The method suggested in this study was validated by using a numerical particle-tracking model based on the autoencoder concept, which estimates the apparent settling velocity using the flocculated grain size distribution. The computed time-velocity curve agreed well with the apparent time curve obtained in the experiment, with an error of approximately 5–10% except in the initial time range (0–30 s), despite the qualitative nature of the assumptions.

Keywords: Mn nodules; apparent settling velocity; floc size distribution; tailings discharge; flocculation; simple settling experiment; particle tracking model

1. Introduction

The mineral resources of the deep ocean such as manganese nodules are collected and screened for their valuable components, and the tailings are directly discharged into the sea. As the discharged tailings are anticipated to affect the environment and organisms of the sea, it is important to estimate the influence (advection-diffusion) range of the tailings. The primary factors determining the influence range are the ocean currents and the settling velocity of the tailings; the latter is directly correlated with the time that the tailings remain in the water column. The tailing flocculation process also should be evaluated for the expected discharge conditions, because the concentration of the discharged tailings is very high. In general, the sediment settling velocity increases with grain size,
and under actual conditions, grain size is not uniform. The flocculation is affected by the tailing discharge conditions. The settling velocity of the tailings is expected to increase as a result of flocculation; therefore, data on the floc size distribution is needed to compute the settling velocity of the tailings. In addition, during the sedimentation of the tailings being discharged, there are currents, salts, water temperature and various external influences, and accordingly, the settling velocity and flocculation phenomenon differ depending on the discharge concentration and particle size. As such, the residue material discharged into the water is influenced by various factors, so it is very complicated and difficult to accurately estimate the settling velocity and floc-distribution. In this study, a method for estimating the tailing floc size distribution is proposed and the general flocculation process is analyzed using the apparent settling velocity, which is readily estimated by simple settling experiments conducted with standard tailings at different concentration conditions.

Typically, sediment settling patterns are classified into three ranges based on (mass) concentration, $C_M$. (1) The free settling range, $C_M < 0.05 \text{ kg/m}^3$, indicates no flocculation. The settling velocity increases as the grain size increases, $w_s = f(d)$. (2) The flocculation settling range, $0.05 < C_M < 0.30$, is the flocculation-dominated range. The settling velocity is expressed as a function of $C_M$ and increases as the sediment concentration increases, $w_s = f(C_M)$. (3) The hindered settling range, $C_M > 1–10 \text{ kg/m}^3$, indicates excessive flocculation. The velocity decreases as the concentration increases because of the self-inhibition process, $w_s = g(C_M)$. The limit (maximum) concentration of the sediment fluid is assumed to be 156 kg/m$^3$ [1]. The settling pattern in this study is between the flocculation and hindered settling regions based on the concentrations $C_M = 2.94$ and $15.88 \text{ kg/m}^3$.

The study of sediment settling velocity can be summarized by the diverse formulae in the free settling range. Zhu et al. (1993), Julien (1995), Soulsby (1997), Cheng (1997), Ahrens et al. (2000), Jimenez et al. (2003), Ferguson et al. (2004), Wu et al. (2006), Zhiyao et al. (2008), and Camenen (2008) [2–11] have suggested that the settling velocity formula lies in the specified Re (Reynolds number) range. Zhiyao et al. (2008) suggested the noble formula of the Re and non-dimensional sediment parameters based on Cheng’s (1997) data, and Camenen (2008) suggested the optimal velocity formula under high concentration conditions. In addition, Millares et al. (2018), Park et al. (2018), and Yang et al. (2019) [12–14] determined the settling velocity of fine sediments, and Yang et al. (2019) and Park et al. (2018) experimentally demonstrated the importance of salinity on the settling velocity. Winterwerp (2002) [15] suggested a Eulerian three-dimensional numerical model to estimate the settling velocity of flocculated sediments, and Krishnamoorthy (2010) [16] developed a model applying the Steinour basic equation. Previous research on settling velocity estimation in the flocculation range has focused on formulae using only a reference concentration. In addition, in the study of Je et al. (2004) [17], a simple test of bentonite particles was conducted to estimate the sedimentation rate and floc-distribution as a function of concentration.

Previous research on settling velocity estimation in the flocculation range has focused on formulae using only a reference concentration. One representative settling velocity is computed because only one representative concentration is used. Under actual conditions, however, diverse settling velocities occur in the floc because of the diverse floc sizes. To investigate these practical situations, distribution data on the flocculated sediment are needed. In this study, a simple method for estimating the distribution of flocculated sediment using apparent velocity is proposed and tested using the self-validation process of the apparent settling velocity, which was readily obtained from simple settling experiments. In the flocculation range, the settling velocity increases with the increased sediment grain size caused by flocculation, and the residence time in the water column decreases. It is predicted that the diffusion range decreases and the concentration of the deposited tailings on the bottom increases due to the limited diffusion process.

2. Materials and Methods

2.1. Basic Information on Beneficiation Residue: Particle Size Distribution

The samples used in the experiment were the tailings from crushed manganese nodules collected in the Clarion-Clipperton Fracture Zone (CCFZ) in the Pacific Ocean (Figure 1). The grain size
The distribution of the tailings is shown in Figure 2. Polymetallic nodules (PN) or ferromanganese nodules collected from the Northeast Pacific were crushed with a hammer to sizes less than 2 cm diameter to simulate the process on a pilot-scale miner before feeding the PN into the lifting system that raises the crushed PN from the seabed (5000 m deep) to the sea surface. The crushed PN fragments were then mixed with water in a 1 to 10 ratio and shaken in a ball mill for 20 min to mimic the conditions in the lifting system. The products were wet sieved to separate particles larger than 63 μm. The finer fractions were then further separated into 20–63 μm, 8–20 μm, and less than 8 μm fractions using a standard settling method.

![Figure 1. Manganese nodule sampling station of the Clarion-Clipperton Fracture Zone (CCFZ).](image1)

![Figure 2. Grain-size distribution of manganese nodule tailing samples.](image2)

2.2. Method for Estimating Settling Velocity through Sedimentation Experiment

The experimental design included two main treatments and three sub-treatments (cases) based on grain sizes and concentrations of the tailing discharge (shown in Table 1). The cases were designed...
to evaluate the effect of the expected variation in concentration for the reference discharge concentration conditions.

Table 1. Treatments for the manganese nodule settling velocity experiments.

| Treatments                              | Concentration (%) | Mass (g) |
|-----------------------------------------|-------------------|----------|
| Tailings sample 1. d_r ≤ 8μm            |                   |          |
| (water = 1 L, 19.1 cm)                 |                   |          |
| (ref. water = 1700 t, tailings = 5 t)  |                   |          |
| case 1-1                                | 0.20              | 2.00     |
| case 1-2                                | 0.29              | 2.94     |
| case 1-3                                | 0.40              | 4.00     |
| Tailings sample 2. 8 μm < d_r ≤ 20 μm  |                   |          |
| (water = 0.7 L, 13.5 cm)               |                   |          |
| (ref. water = 1700 t, tailings = 27 t) |                   |          |
| case 2-1                                | 1.00              | 7.00     |
| case 2-2                                | 1.60              | 11.12    |
| case 2-3                                | 1.87              | 13.12    |

The experiments were conducted using a 2 L mass cylinder; the volume of the water and height were determined based on the mass and the concentrations of the tailings in each case. The water volume (and the height of the water in the cylinder) of the first and second main treatments were 1.0 (+19.1 cm) and 0.7 L (+13.5 cm), respectively. After repeated shaking to ensure complete mixing, the descent of the tailings in the cylinder was recorded as video image files using a mobile phone until most of the tailings had settled down to the bottom (approximately 180 s). The apparent velocity, defined as the maximum velocity of the tailings at a specific time, was calculated by dividing the distance fallen by the falling time at the specified time using the video image files.

The basic assumptions in the experiment were as follows:

1. The settling velocity in the high concentration range is a function of the diameter of the flocculated tailings, and a suitable formula can be selected from an existing list of formulae.
2. The apparent velocity is regarded as the velocity of the largest tailing size, i.e., the maximum velocity, at the specified time for the different existing velocities.
3. The shape of the flocculated sediment is assumed to be spherical.
4. The falling velocities range from the velocity of the largest particle size to the velocity of the smallest particle size, and the apparent velocities decrease as time increases because the large particles settle more rapidly.
5. There are no additional size changes during the flocculation settling process, i.e., the settling velocity of flocculated tailings is constant.
6. The settling flux is assumed to be proportional to the products of cumulative time and settling velocity.

The detailed steps of the experiment to estimate the time-curve for apparent settling velocity were as follows:

Step 1: The mass cylinder (2 L), sample tailings (sediments), electronic scale (AND HL-100, 0.1 g resolution scale), measuring tape, and camera and chronometer of a smart-phone (Samsung Galaxy Note-9, 1200 pixels) were prepared.

Step 2: The mass of the tailings was weighed to match the target concentration for each treatment, and the cylinder was filled with water (1.0 and 0.7 L for treatments 1 and 2, respectively).

Step 3: The tailings were added to the cylinder and shaken by turning the cylinder end-for-end to create a fully homogeneous mixture of water and tailings.

Step 4: The image of the falling tailings was recorded using a smart phone after fixing the cylinder on the table. Additional lighting was used to generate a clear image for recording.

Step 5: The apparent settling velocity was calculated for the measured time using the falling distance and time data obtained from the recorded image. A measuring tape and timer were used.
2.3. Estimation of Floc Particle Size Distribution Using Settling Velocity

Step 1: The optimal fit for the time curve was estimated using the experimental data set, the measuring time \( t \), and the apparent falling velocity \( w_s \) at that time. Power and hyperbolic functions were used to fit the data.

Step 2: The floc tailing diameter corresponding to the apparent falling velocity at the time of measurement was estimated using the van Rijn formula shown in Equation (1). The Newton Raphson method was used to estimate the grain diameter because the formula becomes implicit in the inverse function shown in Equation (2) [18].

\[
w_s (m/s) = \frac{10v}{d_f} \left[ \left(1 + \frac{0.01(s - 1)gd_f^3}{v^2} \right)^{0.5} - 1 \right]
\]

\[
f(d_f) = \frac{d_f w_s}{10v} - \left[ \left(1 + \frac{0.01(s - 1)gd_f^3}{v^2} \right)^{0.5} - 1 \right] = 0
\]

in which \( s = \) specific gravity (= 2.65), \( v = \) the kinematic viscosity coefficient (= \( 1.011 \times 10^{-6} \) m²/s at 20 °C), \( w_s = \) the settling velocity (m/s), and \( d_f = \) the floc diameter (m).

Step 3: The grain volume and mass were calculated using the density of the tailings \( \rho = 2600 \) kg/m³).

Step 4: The cumulative flux of the tailings was computed for the specific grain size. Hein and Koschinsky (2014) [19] stated numerical values for density because of the porosity properties of manganese mass. However, since the density of the manganese sample used in this study is small enough to ignore the porosity properties, it is judged that there will be no difficulty in using the specific gravity (2.65).

2.4. Verification of Test Results on Settling Velocity

2.4.1. Autoencoder Method

The suggested method for estimating floc size cannot be directly validated because there are no data on the floc size distribution. Thus, indirect validation methods using an apparent velocity were adopted based on the autoencoder concept, which is widely used in neural network model validation.

The concept is described as follows. The first transformation, i.e., an encoder, \( y = f(Wx + b) \), estimates the floc grain size using input data such as apparent velocity, and the second transformation, i.e., a decoder, \( \hat{x} = f(Wy + b) \), estimates the input (velocity) using the numerical particle-tracking model, which uses another input, the floc grain size distribution. This process, called a neural network autoencoder, finds and reproduces the input data; the concept is expressed by the following mathematical equations [20]. In this study, the cross-validation was conducted for the apparent velocity by generating input data in the experiment, and the output data (with the same input variable) was simulated by the numerical model:

\[
d_f = f(W \cdot w_s + b) \text{ at the floc size estimation stage.}
\]

\[
\hat{w}_s = f(W \cdot \hat{d}_f + b) \text{ at the settling velocity estimation stage}
\]

in which, \( w_s \) and \( \hat{w}_s \) are the (input) settling velocity measured in the experiment and the estimated settling velocity for the calibration, respectively; \( d_f \) and \( \hat{d}_f \) are the estimated floc size and the (input) floc size, respectively; and \( W \) and \( b \) are the model control parameters corresponding to the weighting factors and bias of the neural network model, respectively.

2.4.2. Validation Method and Results

The numerical model computes the time curve of the apparent velocity by using the input grain size distribution from the considerable grain numbers generated at the different grain sizes. This method is regarded as a self-validation process. This process is the inverse of the estimation of the
apparent velocity using grain size if the basic step is regarded as the estimation of the grain size using the apparent velocity. The adoption of this validation method is inevitable under the limiting condition where there are no data for direct validation or efficient validation. This method can be expressed in mathematical form in Equation (3):

$$d_t = f(w_t) \rightarrow \tilde{w}_t = g(d_t) = g[f(w_t)]$$  \hspace{1cm} (3)

From a theoretical point of view, if the g function is the inverse of the f function, the estimated falling velocity should be the same as the input apparent falling velocity. However, it is impossible to have a perfect match between the velocity values because the input conditions of the numerical model cannot encompass all of the true input conditions, and the conditions should be assumed to be practical approximations. This method is acceptable as the validation method in terms of the autoencoder and decoder concept. The detailed steps are as follows.

Step 1: The particle numbers were input based on the regular grain size intervals. The number of particles is the same as the particle mass ratio of the distribution.

Step 2: The particles were placed in a random pattern in the (numerical) cylinder (computational domain) as the initial location of each particle. The apparent falling velocity was computed for each particle.

Step 3: The new locations of the particles were computed at each regular time step by considering the falling velocity. The particles reaching the bottom were considered deposited particles (unmoving particles, velocity = 0). At each time step, the maximum falling velocity was selected, and the velocity was assumed to be the apparent velocity at that time.

Step 4: The estimation step was repeated for each step (at 1-s intervals). The maximum falling velocity should diminish because the particles with higher velocities disappear more rapidly than particles with lower velocities.

Step 5: After completion of the process, the time-velocity curve was constructed and compared with the apparent settling velocity of the experiment.

3. Results

3.1. Apparent Settling Velocity and Curve Fitting

The optimal falling time curves were estimated using the time and apparent velocity data sets (shown in Figure 3) for all experimental cases. The power and hyperbolic functions were tested and the coefficients of the determination showed values greater than 0.95. However, the power function exhibited a more suitable pattern compared with the hyperbolic function, as shown in Figures 4 and 5. Thus, the power function was selected as the reference function to estimate the floc grain size because of the overall R^2 values.

Figure 3. Typical time-specific settling velocity curves.
The parameters of the power functions in each experimental case are shown in Table 2. The power function shows a better fit than the hyperbolic function in cases 1-1, 1-2, 2-2, and 2-3, and the hyperbolic function shows a slightly better fit in cases 1-3 and 2-1. As shown in Figure 4, the larger particles rapidly disappeared from the water column (within 10 s) and the smaller particles settled at a relatively lower speed (<1 cm/s) after approximately 30 s.

| Case No. | Power Function, $w_s = \alpha t^\beta_f$ | $\alpha$ | $\beta$ | $R^2$ |
|----------|------------------------------------------|---------|---------|-------|
| 1-1      |                                          | 4.931   | −0.451  | 0.9583|
| 1-2      |                                          | 6.206   | −0.475  | 0.9726|
| 1-3      |                                          | 7.354   | −0.463  | 0.9739|
| 2-1      |                                          | 5.486   | −0.473  | 0.9549|
| 2-2      |                                          | 8.687   | −0.572  | 0.9812|
| 2-3      |                                          | 8.429   | −0.552  | 0.9797|

Figure 4. Comparison of observed data using a power function.

3.2. Flocculation-Detention

The assumption of instantaneous flocculation was shown not to be practical. In the experiment, it was shown the flocculation time depends on the grain size. If the falling velocity was 0.01 m/s, the time needed to reach the bottom is less than 20 s. However, after 20 s, the apparent speed was greater than 0.01 m/s. These situations demonstrated that the flocculation of small particles requires a certain amount of time; it is not instantaneous. In this study, these effects represented the flocculation-inhibition time ratio as defined by the following equation, and the time curves of the ratio are shown in Figure 5.

Based on the flocculation detention and the flocculation time, the flocculation-detention ratio $F_R$ (%) was computed using Equation (4):

$$F_R = \frac{S_f - z_f}{S_f} = 1 - \frac{z_f}{S_f}$$

where $S_f$ is the expected distance (cm) computed by the formula $S_f = w_s \times t_f$, $w_s$ is the settling velocity (cm/s), $t_f$ is the elapsed time (s), and $z_f$ is the length of water column (cm).
Figure 5. Results of flocculation-detention ratio (%).

3.3. Particle Size Distribution of Final Floc Particles (Time-Specific Effects in the Flocculation Process)

Figure 6 shows the ratio of the number of particles included in the size of the floc particle, and Figure 7 shows the comparison with the initial sieve analysis results by calculating the volume ratio in each particle size range.

As shown in Figure 7, the floc size increased by approximately 10 times after flocculation. The settling velocity also increased after flocculation. In the tailing discharge conditions, the flocculant or precipitation accelerator effects were not tested in this study because there was no injection of an additional precipitation agent.
3.4. Numerical Simulation

Verification of the estimated apparent settling velocity and the floc grain size distribution is necessary. Direct verification is very difficult in laboratory-scale experiments because it requires an instrument based on laser-diffraction theory to detect the grain size distribution and advanced particle image velocimetry. Practically, these are time- and effort-intensive tasks. However, verification is a basic step in the new proposed method, so self-validation of the apparent settling velocity was conducted. This process was based on the autoencoder method, which is widely used in neural network model verification. Comparison plots of the computed velocity and measured velocity are shown in Figure 8. The computed apparent velocity agreed well with the input velocity time curve even though the conditions of the numerical model are different from the experimental conditions. Also, detailed coding for the numerical simulation is presented in Appendix A.

4. Discussion

A particle settling velocity experiment using the tailing material from Mn nodule particles was performed. Based on the results of the settling velocity shown in Figure 4, the power function fit the settling velocity time curve well for all experimental cases.

The limitations of the experiment are shown in the initial time range, because the rapid flocculation and settling processes were dominated by the large floc tailings. The different concentrations did not show clear differences. The settling process was dominated by the large particles in the initial stage. After an initial rapid settling period, the falling velocity decreased as the
time increased. Additionally, flocculation occurred between floating particles with a low settling velocity during sedimentation, and thus flocculation errors occurred over time; i.e., it was confirmed that the flocculation error increased as the particles remained in the water column for a longer time.

In addition, there are obvious limitations in applying the relational expression of \( w_s \sim f(d) \) proposed by van Rijn to this study to calculate floc size through settling velocity, but there are limitations on experimental results through simulation performance tests within the macro range. Verification was performed.

The large floc grains disappeared from the water column due to the deposition process. The small quantity of sediment that remained in the water column gradually settled, but the settling velocity was much greater than the velocity of the no-flocculation condition. This finding indicates that the flocculation process should be considered and the falling velocity should be estimated based on floc grain diameter. In general, the grain size diameter was several orders of magnitude larger after flocculation. The settling velocity also increased significantly in proportion to the increase in particle size.

Although the other cases did not show it clearly, the case 1-3 results showed that as the concentration increased, larger flocs accumulated and the settling velocity increased.

In addition, since the amount of the sample used in this study is an extremely limited amount, a small scale experiment was performed, and it is considered that it is necessary to perform additional experiments (using sea water) in consideration of the actual situation in which the particles settle later.

On the other hand, as shown in Figure 8, it is judged that accurate analysis is necessary only when a high-speed camera is used when the speed changes rapidly before 10 s. This is one of the problems that our paper has not solved. However, what we intend to present in this paper is that it is not important in this paper because it is key to present a method for estimating the aggregation distribution of particles using apparent settling velocity in all sections except for the initial rapidly changing point.

5. Conclusions

A new and simple method for estimating grain size distribution is proposed using the apparent falling velocity under high tailing concentration conditions, i.e., within the range of expected flocculation. Verification of the method was conducted using apparent velocity based on the numerical particle-tracking model and the autoencoder method from the neural network model. The primary conclusions are summarized as follows:

1. Flocculation of the tailings was verified under high discharge concentration conditions.
   It was shown that the required time for flocculation increases as grain size decreases.
2. The reproducibility of the apparent velocity was acceptable based on the autoencoder concept, even though some unverified assumptions were introduced. This means that the assumptions can be accepted for qualitative evaluations. For a more accurate estimation and strict validation, the settling velocity formulae for the flocculated grain size and shape are required and a grain size detector such as LISST-100 should be used.
3. A limitation of the existing falling velocity formula is shown under the high concentration conditions because the only value computed is for the one concentration. The suggested method in this study reflects the actual conditions more accurately, because the falling velocities were computed using the floc grain size distribution and showed the different velocities present in one high-concentration condition.

The initial rapid flocculation that occurred during the mixing time process in this study cannot be fully understood using this simple estimation method; it is beyond the scope of this study. However, this new and simple method for the floc grain size distribution can be regarded as an acceptable method even though the complexity of the flocculation process is not fully explained. This method allows a rough estimation of the floc grain size distribution because the overall pattern matches the flocculation and settling processes of the sediments.
**Appendix A**

Description of the variables used in the numerical model and the source code (in R).

```r
vis <- 1.011*10^-6
gra <- 9.806
sgr <- 2.65
sgr1 <- 1.65
rho <- 2600 ## (kg/m^3)
n_case <-  1 ## Case No. 1, 2, 3, 4, 5, 6 are 1-1, 1-2, 1-3, 2-1, 2-2, and 2-3 cases, respectively.
alphaT <- c(4.9312, 6.2063, 7.3541, 5.4861, 8.6872, 8.4289)
betaT <- c(-0.451, -0.475, -0.463, -0.473, -0.572, -0.552)
smass <- c(2.00, 2.94, 4.00, 7.00, 11.12, 13.12)  ## Unit : gram
cdist <- c(19.1, 19.1, 19.1, 13.5, 13.5, 13.5)   ## Unit: cm
alpha <- alphaT[n_case]
beta <- betaT[n_case]
kc <- smass[n_case]
clen <- cdist[n_case]

# Code description: Estimation of the floc size distribution and their settling velocity.

```
d_f0 <- d_f1

micro_df = df0 * 10^6
d_floc[kk] <- micro_df

setdf <- d_floc

#==================================================================================
Ftt = function(t) {alpha*t^beta}
value = integrate(Ftt, lower=0.5, upper=180)
cfactor <- kc/(value$value)
Ft <- cfactor*alpha*tt^beta
dfc1 <- as.matrix(setdf)
dfc2 <- sort(dfc1, decreasing=T)

## Floc-limitation process
dis <- tt*aa  ## aa = (computed) settling velocity
dif <- dis - clen/100
pp <- (dif/dis)*100  ## percentage(%)  
nidx <- which(pp <=0)
pp[nidx] <- 0.0
plot(pp, xlim = c(0,180), ylim = c(0,100))

##=======================================
Mflux <- alpha*tt^beta*(4/3)*pi*(dfc2/2/1000000)^3*rho*(1-pp/100)*tt

## Cumulative time, pp(0-1, probability unit)
Mass_in <- kc/1000  ## Conversion to the kg unit
MCF <- Mass_in/(sum(Mflux))
c_mass <- Mflux*MCF

##umass <- (4/3)*pi*(dfc2/2/1000000)^3*rho
#nparticle <- c_mass/umass

dfLL <- c(32, 64, 128, 256, 512)
dfUU <- c(64,128,256,612, 1024)
dfLL <- seq(64,256,4)
dfUU <- c(dfLL[-1],256+4)

idx <- rep(0, length(dfLL))
iMass <- rep(0,length(dfLL))
iDF <- (dfLL+dfUU)/2

for (ii in 1:length(dfLL))
{
```r
tidx <- which(setdf >= dfLL[ii] & setdf < dfUU[ii])
idx[ii] <- length(tidx)
iMass[ii] <- sum(Mflux[tidx])
}

plot(iDF,iMass)
abline(h=0, col="red", lwd=2)

umass <- (4/3)*pi*(iDF/2/1000000)^3*rho
nparticle <- iMass/umass

plot(dfc2,Mflux)
plot(iDF, nparticle)

#===============================================

nz_idx <- which(round(nparticle) < 1)
nzDF <- iDF[-nz_idx]
NP <- round(nparticle[-nz_idx])
nfloc <- rep(nzDF[1],NP[1])
for (ii in 2:length(NP))
{
  nfloc <- c(nfloc, rep(nzDF[ii],NP[ii]))
}

xx <- nfloc/1000/1000 ## Conversion micro-m to m
yy <- runif(length(xx), -1, 1)
zz <- runif(length(xx), 0, 2*clen/100)

plot(yy,zz, xlab="Width", ylab="Height", type="p", pch=1, cex=xx*10000)
abline(h=c(0,10), col="black", lwd=2)

ws <- (10*vis/xx)*(sqrt(1+ 0.01*sgr1*gra*xx^3/(vis*vis)) -1.0)  ## Unit : m/s
hist(ws, xlab="Settling velocity (m/s)", ylab="Frequency")

stime <- seq(0.5,179.5,1) # unit: second
vapp <- matrix(0,nrow=length(stime), ncol=1)

par(mfrow=c(3,3))    # < x
hobs <- 1.0   ## Hindered ratio 0 - 1
for (ii in 1:length(stime))
{
  hobs <- 1-pp[ii]/100
  zzn <- zz - ws*hobs ##stime[ii]  ## change the function of the time(ii)
  yy <- yy + 10*rnorm(length(xx),0.0,0.1)
  idx <- which(zz > 0.0 & zz < 0.5)
  vapp[ii] <- max(ws[idx])
  for (jj in 1:length(xx))
  {
    if(zzn[jj] < 0.0) zzn[jj] <- 0.0
  }
}
```

zz <- zzn ##+ 0.1*rnorm(length(xx),0.0,0.1)

plot(yy,zz, xlim=c(-10,10),ylim=c(0,0.5),xlab="Width", ylab="Height", type="p", pch=1, cex=xx) 
}
dev.new()
plot(stime,vapp, xlab="Time (s)", ylab="App. settling velocity (m/s)", ylim=c(0,0.05), cex=1.5)
lines(tt,aa, col="red", lwd=2)
result <- cbind(time,vapp)

References
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