Resonance and generation of random states in a quenched Ising model

Sunil K. Mishra\textsuperscript{1} and Arul Lakshminarayan\textsuperscript{2}

\textsuperscript{1} Department of Physics, Indian Institute of Technology, Banaras Hindu University - Varanasi-221005, India
\textsuperscript{2} Department of Physics, Indian Institute of Technology Madras - Chennai-600036, India

received 9 October 2013; accepted in final form 2 January 2014
published online 27 January 2014

PACS 03.67.Mn – Entanglement measures, witnesses, and other characterizations
PACS 89.70.Cf – Entropy and other measures of information
PACS 05.45.Mt – Quantum chaos; semiclassical methods

Abstract – The Ising model with a pulsed transverse and a continuous longitudinal field is studied. Starting from a large transverse field and a state that is nearly an eigenstate, the pulsed transverse field is quenched with a simultaneous enhancement of the longitudinal field. The generation of multipartite entanglement is observed along with a phenomenon akin to quantum resonance when the entanglement does not evolve for certain values of the pulse period. Away from the resonance, the longitudinal field can drive the entanglement to near maximum values that is shown to agree well with those of random states. Further evidence is presented that the time-evolved states obtained do have some statistical properties of such random states. For contrast the case when the fields have a steady value is also discussed.

Copyright © EPLA, 2014

Introduction. – Quantum entanglement has been a widely studied feature of quantum mechanics [1]. Of late this study has been substantially enhanced as it finds a central role in various quantum information protocols such as teleportation [1–3]. In the last few years entanglement has also been studied in many condensed-matter contexts, especially for various types of spin chains [4–9]. However, much of the work has focused around integrable models such as the transverse field Ising model, the Heisenberg model etc., with relatively little light being shed on the nonintegrable ones [10–12]. Quantum Ising spins under the application of a magnetic field in the transverse direction present various interesting features while crossing across the quantum critical point [13]. The spin dynamics has been studied by driving the system out of its equilibrium by quantum quenching. In typical quench dynamics studies, the Hamiltonian of the system $\mathcal{H}(\lambda)$ depends on the parameter $\lambda$, that itself is a function of time. The evolution of the state of the system in this case becomes interesting as the system is out of equilibrium. Quench dynamics has been studied theoretically using sudden quench [14] and slow annealing [15]; however, periodic drivings are getting the latest attention in the arena with significant theoretical and experimental advances; see [16,17] and references therein.

Recent work in spin chains singles out the role of entanglement as a method to characterize quantum phase transitions [7–9,18]. These studies show the qualitative change of various entanglement measures near or at the quantum critical point, in turn identifying and characterizing quantum phase transitions. Keeping these studies in mind one can also speculate qualitative changes in various entanglement measures as one crosses the quantum critical point using quench dynamics in the nonintegrable regime.

In the present manuscript we will discuss some results for Ising spin chains under the influence of time-varying and tilted magnetic fields. In a phenomenon reminiscent of quantum resonance in kicked systems [19,20], certain values of the period of the pulses are special and lead to entanglement being absent. In the kicked rotor case, the quantum suppression of momentum diffusion, the so-called dynamical localization, is overcome and in fact a quadratic increase in kinetic energy is seen. In the case studied in this letter, the “resonance” simply leads to a lack of entanglement. However, very close to parameter values that lead to such resonance, the nonintegrable nature of the Hamiltonian renders the time-evolved states to be fairly “complex” and although the interaction is still nearest neighbor, it results in states that have very large multipartite entanglement, in fact as large as that of random or typical states [21].

It is interesting that quenching in nonintegrable spin chains leads to such states. The eigenstates of Hamiltonians such as that of the tilted field Ising model
are also susceptible to such effects; however, they fall short of having properties of random or typical states [11]. Their entanglement entropy is not as large as that of random states. Thus, quenching seems to give rise to this additional randomization. Unitary operators that arise in such quench scenarios may be useful as pseudo-random unitary matrices. Random unitary matrices are used, for example, in encoding and encryption of quantum information [22], process tomography [23], quantum chaos and thermalization studies [24].

In this letter we will start our discussion by defining some important entanglement measures which will be used to quantify entanglement. Subsequently we will discuss the results of the model extensively. Later we will summarize the results.

Model. – In the present study we start with a kicked Ising model which is a variant of the transverse field Ising model. The usual Ising model Hamiltonian with both a longitudinal and a transverse field term is given by

$$\mathcal{H}_{TL} = J_z \sum_{j=1}^{L} \sigma_j^z \sigma_{j+1}^z + h_x \sum_{j=1}^{L} \sigma_j^x + h_z \sum_{j=1}^{L} \sigma_j^z,$$  

(1)

where $J_z$ is the exchange interaction strength, $h_x$ is the external transverse field and $h_z$ is the external longitudinal field. The presence, simultaneously, of longitudinal and transverse field terms leaves the model nonintegrable. The above model in the absence of the longitudinal field has been widely studied using Jordan-Wigner transformation [13,25] where the spin system is mapped to a system of noninteracting fermions. However, inclusion of the longitudinal field term terminates the liberty to map the problem into noninteracting fermions due to the presence of nonquadratic fermionic operators after the transformation. Hence this study will rely mostly on numerical studies to find various entanglement measures. The inclusion of longitudinal field gives rise to many peculiar phenomena in the quench dynamics, (“quench” here refers to time-dependent fields) and we concentrate on a few of them. The variant of the Ising model which will be studied involves kicks of the transverse magnetic field at regular intervals [10]. Kicking fields lead to products of unitary operators which can hence be interpreted, and implemented, as sequences of two- or single-qubit quantum gates.

The Hamiltonian in this case is

$$\mathcal{H}(t) = J_z \sum_{j=1}^{L} \sigma_j^z \sigma_{j+1}^z + h_x(t) \sum_{j=1}^{L} \sigma_j^x + \sum_{k=-\infty}^{\infty} \delta(k - \frac{t}{\tau}) h_z(t) \sum_{j=1}^{L} \sigma_j^z.$$  

(2)

For the above kicked Hamiltonian, the time evolution between two successive kicks is that of a simple Ising model with longitudinal field. At the kick, the transverse field spike is strong which makes the interaction term unimportant. In the absence of the longitudinal field the model becomes integrable. In all the studies below, we set $J_z = 1$ and most of the time consider periodic boundary conditions i.e., $\sigma_{L+1} = \sigma_1$. The setting of $J_z = 1$ fixes one time scale $1/J_z$ as unity and all other times to be chosen are in these units.

The transverse field is swept from a maximum value $h_{z0}$ with a rate of quenching $\alpha$, $\tau$ is the time between two kicks. The introduction of this additional time scale leads to interesting consequences, but it is also convenient as a numerical scheme as when $\tau \ll 1$ the Hamiltonian $\mathcal{H}$ tends to $\mathcal{H}_{TL}$ in eq. (1), possibly with time-varying fields.

The operator that evolves states of the system from one application of the transverse field to the next is the quantum map [26]. The quantum map, which is the propagator evaluated between $k\tau^+$ and $(k+1)\tau^+$ (where $\tau^+$ indicates a time just after $t$) is given by

$$U_z(k) = \prod_{j=1}^{L} \exp\left(-i \alpha \sigma_j^z \sigma_{j+1}^z - \int_{k\tau}^{(k+1)\tau} h_z(t)d\sigma_j^z\right)$$  

(3)

and

$$U_x(k) = \prod_{j=1}^{L} \exp(-i \alpha h_x(k\tau) \sigma_j^z).$$  

(4)

Here $L_0 = L$ or $L - 1$ for periodic or open chains respectively. Note that due to the generally aperiodic time dependence, the quantum map so constructed is itself nonautonomous, and the dependence on $k$, the kick number, is a result of this. As the terms containing $\sigma_x$ and $\sigma_z$ do not commute, the quantum map $U(k)$ is different from $\exp(-i \int_0^T \mathcal{H}_{TL}(t)dt)$, where $\mathcal{H}_{TL}$ is the Hamiltonian in eq. (1), even in the case in which $h_{x,z}(t)$ are independent of time. It has been shown that the kicked transverse Ising model with $h_z = 0$ is integrable [27], and has been solved in [10] using Jordan-Wigner transformation. The kicked Ising model with $h_z = 0$ undergoes a quantum phase transition and belongs to the same universality class as that of the usual transverse Ising model [28]. The state of the system at any time $t = N\tau^+$ is given by $|\psi(t)\rangle = \prod_{k=0}^{N-1} U(k)|\psi(0)\rangle$, where $|\psi(0)\rangle$ is the initial state at time $0^+$, and the product of noncommuting operators $\{U(k)\}$ is time-ordered from right to left. For each $k$, the quantum map $U(k)$ can be considered as a series of two quantum gates $U_z(k)$ and $U_x(k)$ on the pair of nearest-neighbor qubits and on the individual qubits, respectively. In this work we will mostly discuss the nonintegrable model where a time-dependent longitudinal field acts on the system. For the most part we use sinusoidally varying magnetic fields $h_x(t) = h_{x0} \cos \alpha t$ and $h_z(t) = h_{z0} \sin \alpha t$.

The following entanglement measures are studied: the concurrence $C(i,j)$ which is the entanglement of spin $i$ with spin $j$ [29]. This is based on the two-spin reduced density matrix $\rho_{ij}$ (see [10] for details). The block-entropy $S_{L/2}$ is given by [8]

$$S_{L/2} = -\text{Tr}_1,\ldots,L/2[\rho_1,\ldots,L/2 \log_2(\rho_1,\ldots,L/2)],$$  

(5)
Resonance and generation of random states in a quenched Ising model

The time dependence of transverse and longitudinal fields is shown in the figure. At $t = 0$ transverse field is maximum and longitudinal field is set to zero. The kicks of the transverse field are shown as lines separated by the period $\tau$.

where $\rho_{1,\ldots,L/2} = \text{Tr}_{L/2+1,\ldots,L}(|\psi\rangle\langle \psi|)$. It is believed that the entropy $S_{k=1/2}$ is multipartite measure of entanglement and will therefore be complementary to the concurrence, in the sense that we can expect states that maximize concurrence to have low entropy. Generic (random) states have a vanishing probability of a nonzero concurrence, and will therefore be complementary to the concurrence, in particular that of one-body terms and does not create any entanglement, whether multipartite or otherwise, as the case with $h_{z0} = 4$ shows in fig. 2(a), (b) where both concurrence and von Neumann entropy are low.

As the kick interval $\tau = 0.02 \ll 1$ is small the $\delta$-function is almost acting continuously and the evolution is to a good approximation that of the Hamiltonian in eq. (1), with time-dependent fields. The dependence on the time between the kicks will indeed be crucial and this is illustrated in fig. 2(c) where the very large von Neumann entropy (entanglement) when $\tau = \pi/4$ is to be contrasted to the case in which $\tau = 0.02$, although clearly $\tau \ll t_{\text{max}}$ in both cases as $t_{\text{max}} = 20$, the number of kicks for $\tau = 0.02$ ($N = 1000$) are much larger than that ($N = 25$) for the $\tau = \pi/4$ case. Also the case of $\tau = \pi/4$ has a time scale that is of the order of the time scale set by the interaction $J_z$.

The large entanglement produced in this case when the number of kicks is much lower merits a further comment. The initial state is a product state of all the eigenstates of $\sigma^z_i$, while the coupling is along a transverse direction. Thus, entanglement is created that can be very large, in fact without any kicking field a highly entangled state is produced [10]. However, this state also gets equally quickly disentangled and returns to the original state, as the evolution is periodic. Thus, qualitatively it is borne out that the kicking provides dephasing that spoils this periodicity and retains a large, if not maximum, entanglement.

In the following we study the von Neumann entropy at the end of the quench when $h_z(t) = 0$, the state corresponding to the maximum transverse field $h_z(t) = h_{z0}$. During the process, the longitudinal field builds up from zero to reach its maximum value. Shown are cases for vanishing ($h_{z0} = 0$) and $h_{z0} = 4$ peak value of the longitudinal field. For every value of $\tau$ taken, there are an integer number $N$ of kicks, such that $N\tau = t_{\text{max}}$ is fixed. The rate of quench is $\alpha = \pi/2N\tau$. As can be seen in fig. 3, the case in which the longitudinal field is switched on shows peculiar behavior. For an initial state, eigenstate of all $\sigma^z_i$, it can be seen that the entropy increases considerably for moderate kicking periods and interestingly sharply vanishes exactly at $\tau = \pi/2, \pi, 3\pi/2, \ldots$, and recovers almost immediately to very large values. It may be noted that in this plot $t_{\text{max}} = 400$ and has been chosen so as to accommodate a substantial number of kicks even for large values of $\tau$ explored, else this number is arbitrary. In particular the quantities become stationary before this time.

The reason for disappearance of entropy at particular $\tau$ values is not hard to find. Define the interaction part in $U_z(k)$ of eq. (3) as $U_z^\ell(\tau) = \prod_{j=1}^{\ell} \exp\left(-i\tau\sigma^z_j\sigma^z_{j+1}\right)$. Then

\[
\rho_{1,\ldots,L/2} = \text{Tr}_{L/2+1,\ldots,L}(|\psi\rangle\langle \psi|)
\]

where $\rho_{1,\ldots,L/2}$ is multipartite measure of entanglement and will therefore be complementary to the concurrence, in the sense that we can expect states that maximize concurrence to have low entropy. Generic (random) states have a vanishing probability of a nonzero concurrence, and will therefore be complementary to the concurrence, in particular that of one-body terms and does not create any entanglement, whether multipartite or otherwise, as the case with $h_{z0} = 4$ shows in fig. 2(a), (b) where both concurrence and von Neumann entropy are low.

As the kick interval $\tau = 0.02 \ll 1$ is small the $\delta$-function is almost acting continuously and the evolution is to a good approximation that of the Hamiltonian in eq. (1), with time-dependent fields. The dependence on the time between the kicks will indeed be crucial and this is illustrated in fig. 2(c) where the very large von Neumann entropy (entanglement) when $\tau = \pi/4$ is to be contrasted to the case in which $\tau = 0.02$, although clearly $\tau \ll t_{\text{max}}$ in both cases as $t_{\text{max}} = 20$, the number of kicks for $\tau = 0.02$ ($N = 1000$) are much larger than that ($N = 25$) for the $\tau = \pi/4$ case. Also the case of $\tau = \pi/4$ has a time scale that is of the order of the time scale set by the interaction $J_z$.

The large entanglement produced in this case when the number of kicks is much lower merits a further comment. The initial state is a product state of all the eigenstates of $\sigma^z_i$, while the coupling is along a transverse direction. Thus, entanglement is created that can be very large, in fact without any kicking field a highly entangled state is produced [10]. However, this state also gets equally quickly disentangled and returns to the original state, as the evolution is periodic. Thus, qualitatively it is borne out that the kicking provides dephasing that spoils this periodicity and retains a large, if not maximum, entanglement.

In the following we study the von Neumann entropy at the end of the quench when $h_z(t) = 0$, the state corresponding to the maximum transverse field $h_z(t) = h_{z0}$. During the process, the longitudinal field builds up from zero to reach its maximum value. Shown are cases for vanishing ($h_{z0} = 0$) and $h_{z0} = 4$ peak value of the longitudinal field. For every value of $\tau$ taken, there are an integer number $N$ of kicks, such that $N\tau = t_{\text{max}}$ is fixed. The rate of quench is $\alpha = \pi/2N\tau$. As can be seen in fig. 3, the case in which the longitudinal field is switched on shows peculiar behavior. For an initial state, eigenstate of all $\sigma^z_i$, it can be seen that the entropy increases considerably for moderate kicking periods and interestingly sharply vanishes exactly at $\tau = \pi/2, \pi, 3\pi/2, \ldots$, and recovers almost immediately to very large values. It may be noted that in this plot $t_{\text{max}} = 400$ and has been chosen so as to accommodate a substantial number of kicks even for large values of $\tau$ explored, else this number is arbitrary. In particular the quantities become stationary before this time.

The reason for disappearance of entropy at particular $\tau$ values is not hard to find. Define the interaction part in $U_z(k)$ of eq. (3) as $U_z^\ell(\tau) = \prod_{j=1}^{\ell} \exp\left(-i\tau\sigma^z_j\sigma^z_{j+1}\right)$. Then
for Neumann entropy as a function of the quenching transverse field for two very different kick periods, $\tau$. In all the cases $L = 20$, $t_{\text{max}} = 20$ and $\tau = 0.02$. The quenching fields are given by $h_x(t) = h_{\omega 0} \sin(\alpha t)$ and $h_z(t) = h_{\omega 0} \cos(\alpha t)$, with $h_{\omega 0} = 4.0$ and $\alpha = \pi/2t_{\text{max}}$. In (c) the von Neumann entropy as a function of the quenching transverse field for two very different kick periods, $\tau = 0.02$ and $\pi/4$, is shown for $h_{\omega 0} = 4.0$, $h_{\omega 0} = 4.0$ with the above quenching fields. In all cases periodic boundary conditions are used.

Fig. 2: (Colour on-line) (a) Nearest-neighbor concurrence, (b) von Neumann entropy are shown with the quenching in the transverse field for various values of peak longitudinal fields $h_{\omega 0}$. In all the cases $L = 20$, $t_{\text{max}} = 20$ and $\tau = 0.02$. The quenching fields are given by $h_x(t) = h_{\omega 0} \sin(\alpha t)$ and $h_z(t) = h_{\omega 0} \cos(\alpha t)$, with $h_{\omega 0} = 4.0$ and $\alpha = \pi/2t_{\text{max}}$. In (c) the von Neumann entropy as a function of the quenching transverse field for two very different kick periods, $\tau = 0.02$ and $\pi/4$, is shown for $h_{\omega 0} = 4.0$, $h_{\omega 0} = 4.0$ with the above quenching fields. In all cases periodic boundary conditions are used.

Fig. 3: (Colour on-line) The von Neumann entropy at $t_{\text{max}}$, when the quench is complete is plotted against kick interval $\tau$. During the quench $h_x(t)$ and $h_z(t)$ starts from $h_x(t) = 4.0$ and $h_z(t) = 0$ and finishes to $h_x(t) = 0$ and $h_z(t) = h_{\omega 0}$. A comparison between two cases with $h_{\omega 0} = 0$ and $4$ is shown. In all the cases $t_{\text{max}} = 400$ and $L = 12$ with periodic boundary conditions. A reference line corresponding to the random-state entropy is also shown.

with periodic boundary conditions we have that

$$U^I_\tau(\pi/2) = \prod_{j=1}^L \exp \left(-i\frac{\pi}{2} \sigma_j^z \sigma_{j+1}^z \right) = (-i)^L \mathbb{1}_{2^L},$$

where $\mathbb{1}_{2^L}$ is the $2^L$-dimensional identity. This is proved by considering the action of $U^I_\tau(\pi/2)$ on states $|a_1 \cdots a_L \rangle$ where $a_j \in \{0, 1\}$, and $\sigma_j^z |a_j \rangle = (1 - 2a_j) |a_j \rangle$, that is these are the eigenstates of $\sigma_j^z$. Therefore,

$$U^I_\tau(\pi/2) |a_1 \cdots a_L \rangle = e^{-\frac{1}{4} \sum_{j=1}^L \{1 - 2a_j\} (1 - 2a_{j+1})} |a_1 \cdots a_L \rangle$$

$$= e^{-\frac{1}{4} L} e^{-\pi(a_1 + a_{L+1})} |a_1 \cdots a_L \rangle.$$  

with periodic boundary conditions, $a_{L+1} = a_1$, and hence the phrase becomes independent of the $\{a_j\}$, and as the set $\{a_1 \cdots a_L \}$, $a_j \in \{0, 1\}$ is complete, the identity in eq. (6) follows. As the interaction term in the quantum propagator becomes identity at $\tau = \pi/2$, and indeed any integer multiple of $\pi/2$, at these values of the kick-period no entanglement is created or destroyed. This is reminiscent of the phenomenon of “quantum resonance” that has been extensively studied for kicked systems such as the kicked rotor. In that case at specific values of the effective Planck constant that depends on the time between kicks, the kinetic energy term becomes ineffective, the associated propagator becoming identity, with the consequence that the energy of the kicked rotor increases ballistically in time rather than getting localized [19]. This purely quantum phenomenon has also been observed in cold-atom experiments [20]. In the present instance also the ineffectiveness of the interaction occurs for specific relations between the two time scales, one set by the interaction and the other by the time between the pulses of the transverse magnetic field. However, the effect is most visible in quantities like entanglement, as the lack of interaction leads to no generation of such quantum correlations. An interesting case is for large $\tau$ other than multiples of $\pi/2$. The von Neumann entropy corresponding to these values is equal to the entropy of typical or random states. As conjectured by Page [21], and later proved by others [31], the average entropy of a subsystem of dimension $m \leq n$, for pure random states in the $mn$ dimensional space is given as

$$\sum_{k=m+1}^n \frac{1 - m^{-1}}{2} \approx \ln m - \frac{m}{2}. $$

The approximation is valid for large $m$ and $n$. This shows the somewhat remarkable result that most of the states of pure bipartite systems are nearly maximally entangled, the maximum possible being $\ln m$. In the present case $m = n = 2^6$, hence the value turns out to be $\approx 5.27$. From fig. 3 one finds that for $\tau$ not very small and also except for special values corresponding to $\tau = \pi/2, \pi, 3\pi/2, \ldots$, and their immediate vicinity, the entropy reaches the average random state value. In the absence of longitudinal field, the entropy at $h_x = 0$ never reaches to the random state value. The longitudinal field and the resultant nonintegrability generates the large entanglement.

In order to get more insight into the generation of random states in a simple spin chain problem, we compare the foregoing case with the case of constant field kicking. For
this protocol, we apply constant transverse and longitudinal magnetic fields. In this case, the unitary operators $U_z$ and $U_x$ take the form $U_z = \prod_{j=1}^{L} e^{-i x_j \sigma^z_j}$ and $U_x = \prod_{j=1}^{L} e^{-i y_j \sigma^x_j}$. The quantum map is now $U = U_z U_x$, and time evolution propagator for time $t = n \tau$ is simply given by $U^n$. The operator is independent of the step $k$ and therefore is the stationary Floquet operator. The state at time $n \tau$ is $| \psi_n \rangle = U^n \otimes \mathcal{L} | \rightarrow \rangle$. The fixed-field protocol along with the quench case is shown in fig. 4(a). For the fixed-field situation we consider both periodic ($L_0 = L$) as well as open ($L_0 = L - 1$) boundary conditions. For both these boundary conditions, constant fields $h_z = h_x = 2.0$ are applied during the process in contrast to the quench case where the magnitude of $h_x$ and $h_z$ vary at each time. It should be noted here that in the quench case both, periodic and open, boundary conditions are qualitatively similar, hence only the quench case with periodic boundary conditions is shown. The entropy for fixed-field periodic and open cases at $t_{\text{max}}$ show dips at the new series of times $\tau = \pi/4, 3\pi/4, 5\pi/4, \ldots$ as well as at the already discussed values of $\tau = \pi/2, \pi, 3\pi/2, \ldots$. Interestingly we can see that for a very simple case with constant $h_x$ and $h_z$ and open boundary, except for few special values of $\tau$ like $\alpha t_{\text{max}} = \pi/2$, the entropy at $t_{\text{max}}$ shows the numerical value corresponding to those of random states. However, the same is not true for the case of a periodic chain, where a slightly smaller value is found. Also the autonomous case shows peculiar U-shaped curves at multiples of $\pi/4$. A detailed analysis of these interesting oscillations is warranted but is postponed however for future study. The emergence of random states behavior of $| \psi_n \rangle$ can be further analyzed by calculating the distribution of the intensities defined as $\langle |i| \psi_n \rangle^2$, being the $i$-th component of the state $| \psi_n \rangle$. Here the basis used is the standard product of $\sigma^z$ eigenstates.

In fig. 4(b) is shown the distribution of intensities for $L = 12$ spins with $\tau = 1.14$ and $t_{\text{max}} = 400$. In order to reach the maximum time $t_{\text{max}}$ we need approximately 350 kicks. During the time evolution we collect all the states $| \psi_n \rangle$ between kicks 330 to 340. The intensity is calculated for the ensemble of these 40960 values. Typical or random states have real and imaginary parts that are to an excellent approximation independently Gaussian distributed, hence the intensity follows the exponential distribution. We see that the quench case (open and periodic) and the fixed-field open chain follows such an exponential distribution; however the same is not true for the fixed-field periodic case, which shows deviations. The origins of these deviations must be a combination of translational symmetry and the autonomous nature of the system.

The eigenvalue spectrum of the reduced density matrix is of evident interest and is closely connected to the so-called entanglement spectrum (if $\lambda_i$ are the eigenvalues, the entanglement spectrum is $-\log \lambda_i$ [32]). The distribution of the eigenvalues of the reduced density matrices of pure random states is known to follow the Marchenko-Pastur distribution [33] for large system dimensionality. If $\lambda_1, \lambda_2, \ldots, \lambda_{2L/2}$ are the eigenvalues of the reduced density matrix for a state of $L$ qubits, the distribution for a rescaled variable $Y_i = 2L/2 \lambda_i$ is given by

$$P(Y) = \frac{1}{2\pi} \sqrt{\frac{4-Y}{Y}}. \quad (8)$$

The cumulative density is $P_n(Y) = \int_0^Y P(x)dx$.

In fig. 4(c) this cumulative distribution is plotted for the quench case (periodic), the fixed-field open-chain case and the fixed-field periodic chain. In all cases $L = 12$ and $t_{\text{max}} = 400$. The ensemble of states used is the same as one for the data in fig. 4(b). For each state of the ensemble, consisting of pure states of 12 spins, we calculate all the $2^6$ eigenvalues of the reduced density matrix of $L/2$ contiguous spins. There are 10 such states calculated at consecutive times, and hence an ensemble of $2^6 \times 10$ eigenvalues is used for the cumulative density. A reference curve for Marchenko-Pastur distribution is also shown. We find that the distribution of eigenvalues in quench and fixed-field open-chain cases follow Marchenko-Pastur law while...
the fixed-field periodic chain case shows a deviation. This is again in agreement with the intensity analysis. Thus, it is interesting that quenching encourages the production of states that follow properties of random states more closely.

The above studies are generic in Nature as we find similar phenomena for the linear quenches also. The quantum states that follow properties of random states more closely.

is interesting that quenching encourages the production of states that follow properties of random states more closely. This is again in agreement with the intensity analysis. Thus, it is interesting that quenching encourages the production of states that follow properties of random states more closely.

Conclusion. – We have studied various entanglement properties for the nonintegrable kicked Ising model with both transverse and longitudinal fields. The concurrence and von Neumann entropy are calculated numerically for varying longitudinal field. Oscillations occur in the concurrence, and entropy as the transverse field quenches to zero, with an overall decrease in the concurrence and an increase in the other measures for small longitudinal fields, signaling the creation of multipartite entanglement. This multipartite entanglement is not produced if the longitudinal fields are large enough so that the disordering effects of the transverse field are not felt. This paper has also shown the effect of the number of kicks (or kicking interval) on entanglement measures such as von Neumann entropy. In one protocol we applied sinusoidal transverse and longitudinal time-dependent fields and ensured that the rate at which transverse field vanishes is the same as that at which the longitudinal field reaches its maximum value. We find discrete values of the kicking period where entanglement exactly vanishes while for almost all other values of the period it can be identified with that of a random state. The eigenvalues and eigenstates distribution confirms the random-state nature of generic quenched states in the presence of both transverse and longitudinal fields. For a qualitative comparison, we have also presented the case of fixed-field kicking using open as well as periodic chains. Further work to elucidate the interesting structure of entanglement in these states is underway.

***

SKM acknowledges the support of DST project “Quantum chaos and quantum information in condensed matter systems”, SR/S2/HEP-12/2009, during his stay as a research fellow at IIT Madras where this work was initiated.

REFERENCES

[1] Nielsen M. A. and Chuang I. L., Quantum Computation and Quantum Information (Cambridge University Press) 2000.
[2] Peres A., Quantum Theory: Concepts and Methods (Kluwer Academic Publishers, Dordrecht) 1993.
[3] Bennett C. H., Brassard G., Crepeau C., Jozsa R., Peres A. and Wootters W. K., Phys. Rev. Lett., 70 (1993) 1895.

[4] Bose S., Phys. Rev. Lett., 91 (2003) 207901.
[5] Subrahmanyan V., Phys. Rev. A, 69 (2004) 034304.
[6] Latorre J. I. and Riera A., J. Phys. A: Math. Theor., 42 (2009) 540402.
[7] Osborne T. J. and Nielsen M. A., Phys. Rev. A, 66 (2002) 032110.
[8] Amico L., Fazio R., Osterloh A. and Vedral V., Rev. Mod. Phys., 80 (2008) 517.
[9] Sengupta K. and Sen D., Phys. Rev. A, 80 (2009) 032304.
[10] Lakshmimamay A. and Subrahmanyan V., Phys. Rev. A, 71 (2005) 062334.
[11] Kartik J., Sharma A. and Lakshmimamay A., Phys. Rev. A, 75 (2007) 022304.
[12] Mejia-Monasterio C., Benenti G., Carlo G. G. and Casati G., Phys. Rev. A, 66 (2002) 032110.
[13] Sachdev S., Quantum Phase Transitions (Cambridge University Press) 1999.
[14] Polkovnikov A., Sengupta K., Silva A. and Vengalattore M., Rev. Mod. Phys., 83 (2011) 883.
[15] Santoro G., Martošák R., Tosatti E. and Car R., Science, 295 (2002) 2427.
[16] Russomanno A., Silva A. and Santoro G., Phys. Rev. Lett., 109 (2012) 257201.
[17] Russomanno A., Silva A. and Santoro G., J. Stat. Mech. (2013) P09012.
[18] Tribedi A. and Bose I., Phys. Rev. A, 79 (2009) 012331.
[19] Izrailev F. M. and Shepelyansky D. L., Theor. Math. Phys., 43 (1980) 553; Izrailef F. M., Phys. Rep., 196 (1990) 299.
[20] Dana I., Ramareddy V., Talukdar I. and Summy G. S., Phys. Rev. Lett., 100 (2008) 024103; Oskay W. H., Steck D., Milner V., Klappauf B. G. and Raizen M. G., Opt. Commun., 179 (2000) 137.
[21] Page D. N., Phys. Rev. Lett., 71 (1993) 1291.
[22] Abeyesinghe A., Devetak I., Hayden P. and Winter A., Proc. R. Soc. A, 465 (2009) 2537.
[23] Emerson J., Alicki R. and Zyczkowski K., J. Opt. B: Quantum Semiclass. Opt., 7 (2005) S347.
[24] Vinayak and Znidaric M., J. Phys. A: Math. Theor., 45 (2012) 125204; Masanes L., Roncaglia A. J. and Acín A., Phys. Rev. E, 87 (2013) 032137.
[25] Jordan P. and Wigner E., Z. Phys., 47 (1928) 631; Lieb E., Schultz T. and Mattis D., Ann. Phys. (N.Y.), 16 (1961) 406.
[26] Berry M. V., Balazs N. L., Tabor M. and Voros A., Ann. Phys. (N.Y.), 122 (1979) 26.
[27] Prosen T., Prog. Theor. Phys. Suppl., 139 (2000) 191; Phys. Rev. E, 65 (2002) 036208; Physica D, 187 (2004) 244.
[28] Baraktarevic J. P., Milburn G. J. and McKenzie R. H., Phys. Rev. A, 71 (2005) 012335.
[29] Wootters W. K., Phys. Rev. Lett., 80 (1998) 2245.
[30] Scott A. J. and Caves C. M., J. Phys. A: Math. Gen., 36 (2003) 9553.
[31] Sen Siddhartha, Phys. Rev. Lett., 77 (1996) 1.
[32] Li H. and Haldane F. D. M., Phys. Rev. Lett., 101 (2008) 010504.
[33] Marchenko V. A. and Pastur L. A., Mat. Sb. USSR, 72 (1967) 507.