Star counts in NGC 6397

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ABSTRACT

I-band CCD images of a large area of the nearby globular cluster NGC 6397 have been used to construct a surface density profile and two luminosity and mass functions. The surface density profile extends out to 14′ from the cluster center and shows no sign of a tidal cutoff. The inner profile is a power-law with slope $-0.8$ steepening to $-1.7$ outside of 1′. The mass functions are for fields at 4′ and 11′ from the cluster center and confirm the upturn in the mass function for stars less massive than about 0.4 $M_\odot$. There appears to be an excess of low-mass stars over higher-mass stars in the outer field with respect to the inner, in qualitative agreement with expectations for mass segregation.

Subject headings: Globular clusters: individual: NGC 6397

1. Introduction

NGC 6397, as the nearest globular cluster, is a key observational target. Deep and detailed observations can be made of it with smaller investments of time and requirements for resolution than needed for more distant clusters. Since the central surface brightness profile has a cusp, and the estimated relaxation time is $\sim 10^9$yr, it is probable that NGC 6397 has undergone significant dynamical evolution. Further, its kinematics and low metal abundance ([Fe/H] = -2.0) make this a definite member of the halo population of globular clusters. These factors have not been ignored and several recent papers have turned their attention to this cluster.

Aurière, Ortolani & Lauzeral (1990) followed up on the work of Aurière (1982) in investigating the nature and distribution of the stars in the core of NGC 6397. In addition to seeing the central cusp (also confirmed by Djorgovski & King 1986), they identified many of the bright stars in the center as being blue stragglers. Lauzeral et al. (1992) found that it is these blue stragglers, together with a deficiency of red giants further out, which are responsible for a blue color gradient towards the center of the cluster.

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By removing these stars they were able to remove the color gradient. The resulting surface brightness shows an apparent core radius of 6″. Drukier (1993a) has shown that the low number of stars in the region of interest prevents the exclusion of a model with an unresolved core. Meylan & Mayor (1991) published velocity dispersions which they combined with existing surface density and surface brightness profiles to constrain multi-mass King-Michie models (Gunn & Griffin 1979) of NGC 6397. The high concentration of the cluster makes this a difficult enterprise. Anthony-Twarog, Twarog, & Suntzeff (1992) used CCD photometry in the Strömgren system to explore the distance, age, reddening and metallicity of the cluster.

Da Costa (1982) produced star counts from photographic data to give a surface density profile for the outer part of the cluster as well as a luminosity function. Fahlman et al. (1989; hereafter FRST) continued this work with deep luminosity and mass functions from CCD observations. Their mass function remains as the one probing to the lowest masses on the main sequence and shows no sign of turning over to the limit of the data at 0.12$M_\odot$. The mass function becomes much steeper for stars less massive than $\sim 0.4M_\odot$. In terms of the usual power-law form

$$dN \propto m^{-(x+1)}dm,$$

where $x$ is the mass spectral index (MSI; $x = 1.35$ for a Salpeter mass function), $x$ increases for the stars less massive than $\sim 0.4M_\odot$. Subsequent investigation of five more distant clusters have shown similar upturns (Richer et al. 1990 [M13, confirming Drukier et al. 1988]; [M71], Richer et al. 1991 [ω Cen, M5, and NGC 6752]).

It is to the earlier star count work that this paper serves as a follow up. We have undertaken an extensive set of observations of NGC 6397 in the Cousins $I-$band in order to produce both a surface density profile (SDP) and mass functions at additional radii. The new mass functions allow us to confirm the increase in the mass spectral index for low masses and to search for mass segregation.

Section 2 will discuss the observations and the general reduction procedures used. Section 3 discusses the resulting surface density profile and section 4 the luminosity and mass functions. The final section will briefly summarize the results of these star counts.

### 2. Observations and reduction procedures

The observations of NGC 6397 were obtained during an observing run at the Las Campanas Observatory during May and June 1989. There were two sets of observations, both in the Cousins $I-$band and using the TI #1 800 × 800 pixel CCD. The first data set consists of deep observations of several 4.7 $\square'$ fields using the 2.5 m du Pont Telescope. New observations of three cluster fields will be discussed here. The second set of observations were obtained with the 1 m Swope Telescope and covers four overlapping 33.6 $\square'$ fields, extending outwards from the cluster center, plus two regions further out which overlap the 2.5 m fields. The positions and designations of the fields are listed in Table 1. Figure 2 shows the locations of the program fields with respect to the cluster center. The fields are labeled with their names from Table 1 except that “du Pont” has been shortened to “duP”. All stars in the program frames observed with $I < 13$ have been marked by points. The relative sizes of the points gives an indication of their magnitudes and most of them can also be seen in Fig. 4 of Cannon (1974). In addition to the program fields a field about 1° away from the cluster and at the same galactic latitude was observed with both telescopes. The star counts from this “background field” were used to measure the contribution of non-cluster objects to the counts. Observations
of fields with standard stars were also taken on nights with photometric conditions. The 2.5 m observations were used to construct luminosity and mass functions, while the purpose of the 1 m data was to observe the surface density profile. On the 2.5 m, the plate scale is 0′′162 pixel$^{-1}$. As the seeing was usually around 1″ FWHM, this resulted in heavily over-sampled star images. In order to improve the signal-to-noise ratio and speed up processing, the frames were boxcar averaged 2 × 2 following debiasing and flat fielding, giving an effective plate scale of 0′′324 pixel$^{-1}$. For the 1 m frames the plate scale is 0′′435 pixel$^{-1}$ and the seeing was typically around 1″7 FWHM.

Debiasing was done using data from a second read of the CCD. Of this, 32 lines were read, averaged, and stored. For the TI CCD there is no structure to this bias line, so the constant value was taken as the bias level and subtracted. Flat fielding, image registration and averaging, and image trimming were done using procedures within IRAF. The flat fields for most of the 2.5 m data were exposures of clouds from the first night of the run. This flat field did not work very well for the du Pont:bf field so a combination of this flat and dome flats was used. Since the 2.5 m data were acquired under non-photometric conditions, the exposures were weighted by their mean flux levels during the averaging. The sigma clipping algorithm of the IRAF task incombine was used in order to remove cosmic rays. The deepest set of exposures in each field were combined into two independent images for that field and only stars found on both images were counted as detections. While somewhat fainter stars could have been recovered if all the data had been combined into a single frame, the advantage of working with two images is that one can be more certain of the reality of the recovered stars (Stetson 1991). In observing a mass function the additional confidence conferred by this procedure far outweighs the loss in limiting magnitude.

The data in fields Swope:nb, Swope:w and Swope:bf were taken under photometric conditions and a colorless calibration was done using standard stars in fields SA 110, SA 112, SA 113 and in the field of PG 1323-085. The magnitudes used for the standards where preliminary values of the ones in Landolt (1992). The differences between the magnitudes used here and the published values are small and do not affect the photometry. Good results were obtained using the growth curve program DAOGROW (Stetson 1990) and several other photometry calibration programs kindly provided by P. B. Stetson. The transformation to the standard system is good to within about ±0.03 mag. The calibration for the corresponding 2.5 m frames was transferred from the 1 m data. While the remaining four 1 m fields were apparently observed under photometric conditions, it did not prove possible to use the standards observed that night. Fortunately, the Swope:sa and Swope:sb fields overlap that of FRST and a calibration, good to within the ±0.05 quoted by FRST was obtained from magnitudes of stars in that field. This calibration was also transferred onto field du Pont:if.

Aurière et al. (1990) found the center of NGC 6397 to be within 1″ of their star 1. This star, identified with star 335 of Woolley et al. (1961), was also taken as the center by Aurière (1982). Having identified this star in field Swopec, we adopted it as the cluster center. The relative positions of all the other frames within the inner region (ie. Swope:na, Swope:sa, Swope:sb, du Pont:if, and FRST) were found with respect to Swopec. The 2.5 m images were found to be rotated very slightly with respect to the 1 m frames. The positions of the Swope:nb and Swope:w fields with respect to the central four 1 m fields were found using a photograph of NGC 6397. The photograph was a print of a plate obtained at the prime focus of the 4m CTIO telescope in the early seventies. The positions of several stars in the two pairs of regions were measured on the photograph with a measuring engine and identified on the program frames. The required offset between the origins of the Swope:sb and Swope:w fields, and between the origins of the Swope:na and Swope:nb fields, were solved for in a least squares fashion together with a constant scale factor. No rotation was apparent in the transformation relationship and in the final calculation this was assumed to be zero.
The resulting positions for these fields with respect to the cluster accord well with a sketch of the field positions made at the telescope. The positions of the du Pont:n and du Pont:w fields, which are contained within the Swope:nb and Swope:w fields respectively, were calculated with respect to the positions of these 1 m fields.

The data were reduced using versions of DAOPHOT and ALLSTAR (Stetson 1987) in the usual manner. Two finding passes were made on the images with a find threshold of $4\sigma$. Generally, a Moffat function (Moffat 1969) with exponent 1.5 was used for the analytic portion of the point spread function, plus a non-varying residual-look-up-table. Occasionally, a linear variation in the look up table was used but it did not materially affect the photometry. Each of the two independent images for each field was reduced separately. Stars on the two lists were matched and the mean offsets in magnitude and position were calculated. These offsets were then applied and a final star list for each field was produced containing those stars found on both frames with the centroids of the two images closer than 1 pixel.

In order to correct for incompleteness in the star counts, tests were done using the usual procedure of adding artificial star images to the program frames and then re-reducing them. The artificial stars were added with a magnitude distribution based on that estimated from the initial reduction of the frame. Ten percent of the number of stars observed in the magnitude range of interest were added with random magnitudes and positions to each artificial star frame. The same artificial stars, with appropriate magnitude and position offsets, were added to both images in a pair. Ten to thirty such pairs of frames were reduced for each field.

The final star lists were prepared in the following manner. For each field, regions surrounding saturated stars, diffraction spikes, charge overflow columns and other defects, were identified. These areas have a significantly higher rate of spurious detections. All artificial stars added, and objects detected, in these regions were ignored during further analysis. Also ignored were stars found with centers beyond the edges of the frame since no artificial stars were added in these areas. The effects of these restrictions are small. The detection lists for the two images in a pair were then shifted onto common position and magnitude zero points. The stars on the lists were matched and all objects found on both frames with position centroids within 1 pixel were deemed valid detections. The list of valid detections was then compared with the list of positions and magnitudes of the artificial stars added to that pair of frames. All valid detections with positions within 1 pixel and magnitude differences less than 0.7 mag with respect to the artificial star list were counted as recoveries of artificial stars. The remaining, presumably real, stars were put into a separate list. Star counts were performed on the list of real stars from each artificial star test separately and the results averaged. Small variations in the number of stars were seen from test to test due to variations in the local crowding conditions. Errors in the star counts are a combination of the Poisson error and the standard deviation in the number of stars recovered.

The observed luminosity function is a convolution of the true luminosity function with the probability that a star of a given magnitude will be found, and if so, with what observed magnitude. The artificial star tests give an approximation to the probability function and allow for the inversion of the observed luminosity function to give the true one. The luminosity functions were corrected for incompleteness using two techniques. In the simplest approach, all stars added in a given magnitude bin and subsequently recovered were counted as recoveries of stars with that magnitude. The ratio of the number of recovered stars to the number added in that bin gives the recovery rate, and the incompleteness correction factor is the inverse of this. This method tends to underestimate the uncertainties in the corrected counts and does not take into account the effects of stars with true magnitudes lying in one bin being recovered in a neighboring bin. This bin jumping can be corrected for in an approximate fashion using techniques such as
those described by Drukier et al. (1988) and Stetson & Harris (1988).

The second method we used is a simplified version of the matrix inversion method of Drukier et al. (1988). As an intermediate method between using the full matrix, as in Drukier et al., and compressing all the elements onto the diagonal, as in the approach discussed above, a tri-diagonal recovery matrix was constructed. The advantage of a tri-diagonal matrix over the full matrix is that it is straightforward to invert it and to calculate the errors for the inverted matrix. It is also less susceptible to the instability associated with inverting a matrix containing many small elements. The cost paid is that some information on the low-probability tails of the measured-magnitude vs. true-magnitude relations are lost, but not as much as with the diagonal approach. The inverted matrix, together with some assumptions about the recovery rates and magnitudes for stars in the faintest bins, was used to calculate the completeness corrected counts. The results of this method are consistent within the uncertainties with those of the simpler approach discussed above, except that the error estimates and magnitude limit to the star counts are more conservative using the matrix approach. The final counts presented in §4 were corrected using tri-diagonal matrices.

3. Surface Density Profile

The surface density profiles were produced from the 1 m data. Two cutoff magnitudes were adopted and the resulting two surface density profiles combined to give the final profile. The first cutoff, $I = 14.0$, was adopted to obtain the surface density profile into the center of the cluster. Using the artificial star tests for the Swope:c field, the completeness was calculated as a function of magnitude for annuli about the center. In the central region, within a radius of 29 pixels (0′21), the recovery rate was greater than 90% to $I = 14.0$. A similar result was achieved for a pair of images of the same field with exposure times of 5s. The second cutoff magnitude, $I = 15.5$, is discussed below.

For fields Swope:na, Swope:sa, and Swope:sb we had longer exposure images than are discussed here, but on these the brighter stars are saturated and could not be counted in the luminosity function. Therefore, the single short exposure images (averages of the two 5s exposures obtained for each field) were used to produce the surface density profiles. In the Swope:nb, Swope:w, and Swope:bf fields the shallowest images, averages of two 30s exposures, were used. These fields contained no stars bright enough to have poor photometry over this length exposure. For the Swope:c data the detected stars were counted from the output of all 10 artificial star runs and an average taken. For the bright stars, the variations due to crowding between the final star lists on the artificial star frames was quite small, confirming the incompleteness corrections.

The stars were counted in two sets of overlapping annuli to reduce the effects of binning. The outer limits of the annuli were spaced logarithmically with each annulus having a width of 0.2 dex. The radii of the inner disks of the two sets of annuli were 0′21 and 0′17. The areas of the intersections of the annuli with the rectangular fields (we will refer to these as “sections”) were calculated and the star counts were converted to surface densities. The adopted effective radius for each section is the radius which divides the area of that section in half. This radius will vary between sections on the same annulus, but from different fields, depending on the geometry of the section. The field star density was measured on Swope:bf. Twenty-three stars brighter than $I = 14$ were found on that frame giving a surface density of $0.69 \pm 0.14$ stars per square arc minute.
In order to account for areas in which no stars were seen, and to eliminate the apparent anomalies associated with small number statistics, any section of a field which contained fewer than 10 stars was merged with its neighboring section. The area and effective radius for the new, combined section were calculated, and the density and error re-evaluated. The SDP after this merging is shown in Fig. 2 and listed in Table 2. The various symbols indicate the frame of origin for each data point.

A set of surface density profiles were produced in the same way for a set of centers offset by varying amounts from that adopted. Little difference was seen for centers offset by up to ±20 pixels (8.7) in either or both directions. The outer SDP was little affected by all these variations in center position and we conclude that the errors in the determination of the relative positions of the outer frames and the inner ones will have a negligible effect on the surface densities.

When star counting is done on photographic plates, a reseau consisting of sectored annuli is placed on the plate and the stars are counted by sector. The density and uncertainty for a given annulus is calculated from the counts for all the sectors in the annulus. Here, with the exception of the central region, only a limited area of each annulus was surveyed. In order to check the consistency of the star counts and the assigned errors we split the lists of recovered stars for Swope:bf into quadrants about the cluster center and repeated the counting procedure for each quadrant separately. Figure 3 shows the results of this test. The symbols without error bars are the densities for the four quarters of field Swope:bf. The line connects the mean of the four measurements in each annulus and the error bars are the standard deviations of the samples. The solid circles offset +0.025 dex in radius are the overall densities from the full field together with the uncertainties computed from Poisson statistics and frame-to-frame variations in the counts. With the exception of the innermost point, the quarter-to-quarter errors are within a factor of two of the Poisson errors. There is excellent agreement in the errors for mean points between 0.25 and 1′ but a larger quadrant-to-quadrant variation is seen for the annuli beyond 1′. The large scatter in the innermost point is an artifact of the linear arrangement of the bright stars in the core of NGC 6397 (q.v. Aurière et al. 1990).

The second surface density profile, containing all stars brighter than $I = 15.5$ and more than 20″ from the center, was produced in the same manner as the $I \leq 14.0$ SDP. The inner radial limit is due to incompleteness. Beyond this point the counts from Swope:bf are more than 90% complete between $I = 15.0$ and $I = 15.5$, and greater than 95% complete for all stars brighter that $I = 15.5$. These incompletenesses would change the final surface density profile by less than 0.05 dex, well within the estimated errors. The magnitude cutoff of $I = 15.5$ was chosen since this is the magnitude of the turnoff on a roughly calibrated $I - (V - I)$ color-magnitude diagram produced with data taken at the same time as that of FRST. A field density of $2.06 \pm 0.25$ stars per square arc minute was derived from the Swope:bf data. This SDP is listed in Table 3. The two surface density profiles are shown in Fig. 4.

The SDP of NGC 6397 seems to be best characterized by two power-laws with a break at 1′. Both single and multiple power-law models were fit to the $I \leq 14.0$ SDP and for the multiple power-laws both the cases of continuity and non-continuity across the break point were tried. Quite consistently, the double power-law fits had significantly lower values of reduced-$\chi^2$ (by a factor of two or more) than the single power-laws. The preferred position for the break point in these fits was at 1′ to within 10%. The introduction of a third power-law does not significantly improve the reduced-$\chi^2$. For the $I \leq 14.0$ SDP, the region with $r \leq 1′'26$ has a slope of $-0.9 \pm 0.1$ and the region $1′ \leq r \leq 6′3$ has a slope of $-1.7 \pm 0.1$. The slopes are the mean of those calculated by a weighted linear regression for the two sets of annuli independently. The cutoff at $r = 6′3$ was chosen given that this is the radius where the field and cluster densities are about equal and the observational uncertainties in the SDP become large. Over the region $1′ \leq r \leq 6′3$ the $I \leq 15.5$ SDP has a slope of $-1.73 \pm 0.07$. The intercepts are better determined and give an offset of $0.49 \pm 0.04$ dex.
Since all the stars on both the $I \leq 14.0$ and $I \leq 15.5$ SDPs lie at or above the turnoff they should have about the same mass and so should have the same radial density distribution. A comparison between the two SDPs is shown in Fig. 4 where the offset of 0.49 dex has been applied to the $I \leq 14.0$ SDP. The background level for the $I \leq 14.0$ SDP has been shifted by the same amount. The linear fits to the profile are also shown in Fig. 4.

Given the low galactic latitude of NGC 6397 ($b = -12^\circ$), determination of a limiting radius for this cluster is a difficult proposition. For example, Da Costa (1979), from a single-mass King model fit, finds a limiting radius of 38$'$ and Meylan & Mayor (1991) using multi-mass King models find a mean limiting radius of 95$'$. These values are much larger than the limit of the new SDP (14$'$.)

In view of the large uncertainties in the density and the preponderance of field stars at these radii, much larger areas will need to be observed in order to say anything about the limiting radius of NGC 6397.

4. Luminosity and Mass Functions

The luminosity functions (LFs) for fields du Pont:if, du Pont:n, du Pont:w, and du Pont:bf were produced in accordance with the procedures outlined in §2. The stars were counted in two sets of overlapping 0.5 mag bins offset by 0.25 magnitudes in order to reduce the distortions due to binning. For each field the true observed area, exclusive of the ignored regions, was computed and the star counts adjusted to be the number of stars which would be observed in a 4.67 $'$ field (800 $\times$ 800 pixels at a plate scale of 0.$''$162 pixel$^{-1}$)

At first glance the completeness corrected counts in Tables 4–7 may appear to present some anomalies. When the incompleteness corrections do not take into account bin jumping, the corrected counts cannot be less than the raw counts. The inclusion of the effects of bin jumping can lead to the corrected number of stars in a bin being smaller than the original number. To see that this can be the case consider the following. First, for a given bin, bin $i$, the next fainter bin, bin $i + 1$, generally has more stars for a rising luminosity function as is usually the case. Second, since the errors in magnitude increases with magnitude, a higher proportion of the stars with true magnitudes in bin $i + 1$ will be observed with magnitudes in bin $i$ than the other way around. This is so even if the distribution of magnitude differences are symmetric about zero, but there is a tendency at the faintest magnitudes for stars to be found too bright, since those sitting on positive noise spikes have a better chance of being found than those sitting on negative background fluctuations. The result of a rising luminosity function and the increase of errors with magnitude is a net flux of stars being found with magnitudes brighter than their true ones. When the correction is made for bin jumping, bin $i$ could then end up with fewer stars than it started with before the corrections. Since some fraction of observed stars are not recovered at all, the incompleteness corrections (ie. the diagonal elements in the recovery matrix) will increase the counts in bin $i$. However, if the fraction of the stars observed in bin $i$ which belong in bin $i + 1$ is greater than the incompleteness correction, the number of stars in bin $i$ will decrease after being corrected.

The background counts, converted to the number expected on a 4.67 $'$ field, are tabulated in Table 4.

In the bin centered at $I = 22.5$, 50% of the artificial stars were recovered with magnitudes within 0.7 mag of their true values. For the next independent 0.5 mag bin, centered at $I = 23.0$, only 8% of the artificial stars were recovered. Since it is impossible, because of the poor statistics, to estimate the number of stars in the
I = 22.5 bin which originated in the \(I = 23.0\) bin, the \(I = 22.5\) bin could not be used in the luminosity function. Similarly, for the overlapping bins centered on the quarter magnitude, the last useful bin is at \(I = 21.75\). Although the next bin has a recovery rate of 78\%, the one following it, centered at \(I = 22.75\), has a recovery rate of only 26\%, and of these, 44\% were found in the next higher bin, 12\% in the next fainter bin, and the remaining 44\% were found in the same magnitude bin they were added in. The recovery rate drops quite quickly, so the last usable bin usually has a rather high recovery fraction. As a result of these considerations, the cutoff magnitude for the du Pont:bf luminosity function is \(I = 22.25\); the last bin used being the one centered at \(I = 22.0\).

Tables 5–7 contain the raw and incompleteness corrected star counts for the three program fields. The du Pont:if star counts were found to have a 50\% recovery rate at \(I = 21.7\) and when the bin jumping effects are taken into account the limiting magnitude of the final bin is \(I = 21.5\). The 50\% recovery rate magnitude for the du Pont:n data is \(I = 22.8\), and for the du Pont:w data it is \(I = 22.1\). The magnitude limits of the incompleteness corrected star counts are 22.5 and 21.75 respectively. The du Pont:n LF goes 0.75 mag deeper than the du Pont:w LF because of better seeing and a longer total exposure. Since the du Pont:w and du Pont:n fields are at about the same distance from the cluster center, and the LFs for them have low signal-to-noise, the LFs for these two fields have been averaged where they overlap and a final, outer LF produced. The bins with centers with \(I \geq 21.75\) are from the du Pont:n data only. The magnitude limit of the background LF cuts off this combined LF, which will be referred to as the du Pont:out LF, at \(I = 22.25\).

Figure 5 shows a comparison of the star counts for the background fields observed in this work and in FRST. The agreement is good despite the two fields being on opposite sides of the cluster. This agrees with the observation by Da Costa (1982) that the field stars are evenly distributed in the vicinity of the cluster. The final, background subtracted, cluster luminosity-functions are listed in Table 8 and are plotted in Figs. 6 and 7. These two figures show the two new LFs and compares them with the background star counts. At the distance of the two outer fields the number of field stars is equal to the number of cluster stars, so deriving a more precise luminosity function will require observing a much larger area.

Combined with the luminosity function of FRST, the new LFs give three deep LFs for NGC 6397. Figure 8 brings together all three of the observed deep LFs for NGC 6397. The two types of filled symbols are the new LFs from this study, while the open symbols are the deeper LF of FRST. No further normalizations have been applied; the offsets between the LFs are a reflection of the overall decrease in density with radius. For the magnitude range shown, all the stars should be on the main sequence, since the turn-off lies at about \(I = 15.5\). The FRST LF goes deeper because of the better seeing for that data, and because of the more conservative approach to incompleteness corrections used here. The agreement between the three luminosity functions over their common range indicates that the faint stars counted by FRST were real. As discussed in §2, the conservative, tri-diagonal method for incompleteness corrections gives substantially the same results as the method used by FRST. Therefore, were the FRST star counts to be recalculated with the more conservative method, the previous results would stand. The origin of the dip around \(I = 19\) in the du Pont:out LF is unclear, but is seen, at about the same magnitude, in both fields. Comparison with Fig. 5 shows that it cannot be explained by structure in the background star counts. The general depression in the counts between \(I \sim 18.5\) and \(I \sim 20\) suggests that whatever is causing the dip is not restricted to just the two low bins.

In order to convert the luminosity functions into mass functions a distance modulus and a mass-luminosity law are required. We adopt an apparent \(I\)-band distance modulus of 12.0 as discussed in Appendix A. The \(I\)-band mass-luminosity law of FRST, with an extension to more massive stars taken from VandenBerg & Bell (1985), was used. This mass-luminosity relationship is for \(Y = 0.2\), \(Z = 10^{-4}\), and
an age of 16 Gyr. Table 9 lists the two new mass functions. These are shown, together with the MF of FRST, in Fig. 9.

From the more massive, upper main-sequence stars it is difficult to see any sign of mass segregation, although there is some indication that the outer field is more deficient in higher mass stars than the two fields closer to the center. In relation to this, a series of papers (Capaccioli, Ortolani, & Piotto 1990, Piotto 1991, Capaccioli, Piotto, & Stiavelli 1993, Djorgovski, Piotto, & Capaccioli 1993) has been investigating correlations between globular cluster mass functions and other cluster parameters such as galactic position and metallicity. The mass function data are quantified in terms of the mass spectral index over the range $0.5 \leq M/M_\odot \leq 0.8$. Rather than the observed mass spectral index at the particular location in the cluster of the observation, global values are desired. In the above series of papers, the conversion from apparent to global MSI has been accomplished following the prescription of Pryor, Smith & McClure (1986). In this technique multi-mass, King-Michie models are used to calculate the apparent mass spectral index, $x_a$, as a function of radius for a given global mass spectral index, $x_g$, and concentration. There are several things to be concerned about with this procedure. First, the mass spectrum used is very simple, containing only five mass species, and will not necessarily be a good match to the cluster mass function. Second, it only extends to $0.2 M_\odot$ whereas FRST have counted stars to $0.12 M_\odot$ with no indication that the mass spectrum ends. Third, different forms for the global mass function will lead to different local-to-global corrections and, while a power law may be a good approximation over a small mass range, the observed mass functions show much more structure. Fourth, the mass spectral index is computed over a very small mass range. Fifth, in the case of clusters having central cusps, this technique is not really practical since King models were never meant to represent clusters undergoing core-collapse (King 1966).

For the cusp clusters Djorgovski et al. (1993) use models with concentration parameter $c = 2.50$. With the caveats discussed above in mind we show in Fig. 11 a Pryor et al. (1986) style diagram for a model with this concentration together with the mass spectral indices over $0.5 \leq M/M_\odot \leq 0.8$ from the three mass functions in Fig. 9. The core radius has been taken to be $6''$ based on the profile of Lauzeral et al. (1992). While a global value of $x_g \sim 0$ is consistent with all three local measurements, the agreement is very unsatisfactory. There is a disturbing trend for the “global” value derived from this diagram to increase with radius. Table 10 lists the apparent mass spectral indices for the three fields and their inferred global values from Fig. 11 together with their errors. The mean is $x_g = 0.2 \pm 0.6$. The difficulty in finding a single, global, mass spectral index from the three local values, indicates that the Pryor et al. method is indeed not useful in doing these conversions for, at least, post-core-collapse clusters.

For the less massive stars, there is certainly an impression that the outer field contains a relatively higher proportion of these than do the two inner fields. For these stars there is a somewhat clearer signal for mass segregation. Table 11 gives the slopes of weighted, least-square, power-law fits to the mass bins with masses lower than $0.4$, $0.32$, and $0.28 M_\odot$, together with the numbers of points fit and error estimates. The final point in the du Pont:out MF has been excluded from this fitting due to its large uncertainty. The errors for the slopes of the new MFs are somewhat underestimated since the points going into the fits are not independent. For the du Pont:if and FRST MFs the three slopes are quite consistent with one another, and are also consistent with there being no mass segregation between the two fields. As the high-mass cutoff is moved to lower masses the MF from the outer field gets steeper. The evidence for mass segregation is strongest for the stars with $M < 0.32M_\odot$ but is not highly significant. There is also an indication in the three MFs that the mass of the break in the MSI decreases with distance from the cluster center. For the du Pont:if MF the upturn in the MF takes place at around $0.4M_\odot$, for the FRST MF this occurs between $0.4M_\odot$ and $0.3M_\odot$, and for the du Pont:out MF the data suggests an upturn nearer to $0.3M_\odot$. To some
extent this is borne out by the power laws fit to the MFs. If this variation of the break with radius is real, then this indicates that the break is not an artifact of the knee in the mass-luminosity relation, as has been suggested (Capaccioli et al. 1993).

Drukier, Fahlman, & Richer (1992) introduced the concept of a segregation measure, the ratio of two mass functions, in order to quantify the differences between mass functions independent of the overall shape of the cluster’s mass function. If the segregation measure is computed between mass functions measured for two fields at different radii, then the differences associated with mass segregation will show up more clearly. If \( N^a(m) \) is the number per unit mass of stars with mass \( m \) in field ‘a’, then the radial segregation measure for mass \( m \) and field ‘a’ with respect to field ‘b’ is given by

\[
S_r(m; a, b) = \log\left[ \frac{N^a(m)}{N^b(m)} \right].
\] (2)

If \( S_r(m_1; a, b) \) is larger than \( S_r(m_2; a, b) \) then, with respect to field ‘b’, field ‘a’ has an excess of stars of mass \( m_1 \) over stars with mass \( m_2 \). If, for example, field ‘a’ lies further from the cluster center than does field ‘b’, then the segregation measure for the low mass stars should be larger than that for the high mass stars if mass segregation has occurred in the cluster. The segregation measures between the three pairs of fields are shown in Fig. 11. One difficulty with these diagrams are the large errors associated with taking the ratio of two, already somewhat poorly determined, quantities. The segregation measure between the inner two fields, du Pont:if and FRST, is flat except for the most massive stars, indicating that there is little mass segregation between the two fields. The segregation measure between du Pont:if and du Pont:out increases by 0.3 over a factor of 3.7 in mass. We can define a mass-normalized segregation measure over an observed mass range by

\[
S_{r,m}(m_1, m_2; a, b) = S_r(m_1; a, b) - S_r(m_2; a, b),
\] (3)

where \( S_{r,m}(m_1, m_2; a, b) \) is the segregation measure of mass \( m_1 \) with respect to mass \( m_2 \) for field ‘a’ with respect to field ‘b’. In the event that a power law is a good fit to the mass function then, if \( x_a \) and \( x_b \) are the MSIs in field ‘a’ and ‘b’ respectively,

\[
x_a - x_b = -S_{r,m}(m_1, m_2; a, b)/\log(m_1/m_2).
\] (4)

If the full NGC 6397 mass functions could be characterized by single power laws, then the mass-normalized segregation measure would imply that the du Pont:out mass function has a MSI 0.5 larger than does the du Pont:if mass function. The more complex structure observed in the mass functions precludes such an inference, but the segregation measure does support the conclusion that there is mass segregation in NGC 6397.

5. Summary

We have observed 168.4 \( \square' \) of NGC 6397 in the Cousins \( I \)–band with deep photometry on a 13.7 \( \square' \) sub-region. The shallower, large-area observations have been used to construct a surface brightness profile for the giants, subgiants and turn-off stars. The limiting magnitude for these observations is \( I = 14 \) for \( r < 0\,'42 \) and \( I = 15.5 \) for \( 0\,'42 < r < 14\,'1 \). The field star surface density is equal to that of the cluster at about \( 7\,' \) in this magnitude range.

The core of the cluster is unresolved in these data. This is not surprising in that the innermost point is at \( 9\,' \) and surface brightness data suggest a core radius of order \( 6\,' \) (Lauzeral et al. 1992). The surface
density profile can be characterized by two power laws with a break at about 1′. Within this radius the power law slope is \(-0.9\). This slope is consistent with those observed for other globular clusters with central cusps (Djorgovski & King 1986, Lugger et al. 1991). Beyond 1′ it is \(-1.7\). There is no sign of a tidal cut off to the limit of the observations at 14′.

The deeper observations were used to produce local mass functions at 4′ and 11′ from the cluster center. Although these do not extend to as low a mass as that of FRST—which was for a field at 6.5′ from the cluster center—they do confirm the change in slope of the mass function which occurs at about 0.4\(M_\odot\). No mass segregation is seen between the 4′ and 6.5′ fields. There is some indication of mass segregation between the 11′ field and the others for the stars with \(M < 0.32M_\odot\), but the difference in the mass spectral indices is not significant. More stars will need to be counted in order to reduce the statistical uncertainties and confirm the existence of any mass segregation in this cluster. For the stars with masses in the range 0.5 \(\leq M/\odot \leq 0.8\), the procedure suggested by Pryor et al. (1986) for converting apparent MSIs to their global values does not give a consistent result. There is a trend for the inferred global values to increase with radius. This indicates that this procedure should not be used for at least those clusters with unresolved cores. Since, in the case of NGC 6397, neither the core nor tidal radii are known, it is impossible to say what concentration model to use. In any case, King models were never designed for clusters with such extreme concentrations (King 1966).

The prospects for further star count studies of NGC 6397 are encouraging. As the discussion of the mass functions highlights, the difficulty of CCD star counting has been the limited areas which can be observed with telescopes large enough to get to faint magnitudes in reasonable times. With the advent of large area CCDs and mosaics of CCDs this can be overcome and it will be possible to count enough stars to make firm statements on the variation of NGC 6397’s mass function with radius and its surface density profile with magnitude.

Drukier et al. (1992) examined a similar set of observations, in that case for M71, in terms of a series of Fokker-Planck models. They found that they could not fit the M71 observations with any of the models they considered since M71 showed too much mass segregation for a cluster with such a large core radius. NGC 6397 is in many ways a useful foil to M71. Both lie at about the same galactocentric distance—7.4 kpc for M71, 6.9 kpc for NGC 6397 (Webbink 1985)—but M71 has the kinematics of a disk cluster while NGC 6397 belongs to the galactic halo (Cudworth 1992). Further, NGC 6397 is the more massive and more concentrated cluster of the two. A future paper (Drukier 1993b) will compare the observations discussed here with Fokker-Planck models and comment further on both the current dynamical state of NGC 6397 and the differences and similarities between it and M71.

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A. *I*-band distance modulus to NGC 6397

There are two sources for the distance modulus to NGC 6397. The first is a venerable measurement of the magnitude of the horizontal branch by Cannon (1974). Depending on the choice of horizontal branch calibration, a distance modulus of \((m - M)_V = 12.3 \pm 0.3\) is obtained (q.v. Harris 1980, Zinn 1985). More recently, Anthony-Twarog, Twarog, & Suntzeff (1992) derived an independent distance using Strömgren photometry. Using main sequence fitting and the \(M_V, (b - y)\) relationship method of Laird, Carney, & Latham (1988) they found \((m - M)_V = 12.1 \pm 0.3\). With this distance modulus, the color-magnitude diagram does not give a good fit to the theoretical isochrones of VandenBerg & Bell (1985) as modified for enhanced oxygen abundance by McClure *et al.* (1986). A better fit is achieved if \((m - M)_V = 12.4\) is used. Anthony-Twarog *et al.* attribute some of the discrepancy to the bolometric corrections used to transfer the luminosities onto the observable plane. In any event, their distance moduli are consistent with \((m - M)_V = 12.3\) adopted here.

NGC 6397 lies at \(b = -11^\circ 959\). Reddening measurements give consistent values: Cannon (1974) found \(E(B - V) = 0.18 \pm 0.01\), van den Bergh (1988) \(E(B - V) = 0.19 \pm 0.02\), and VandenBerg, Bolte & Stetson (1990) \(E(B - V) = 0.19\). VandenBerg *et al.* also find evidence for a small amount of variable reddening. From the data in their paper this appears to be at the 0.013 magnitude level, and should not affect the results here where a constant reddening of \(E(B - V) = 0.19\) is assumed. These values for the distance modulus and reddening gives a heliocentric distance of 2.2 kpc to NGC 6397. At this distance 1 pc subtends 1.6 ± 0.2′, given the uncertainty in the distance modulus.

Since the present observations are in the \(I\)–band, an apparent distance modulus in that color is required. The formula in Dean, Warren, & Cousins (1978), extrapolated from O and B stars, implies \(E(V - I)/E(B - V) = 1.35\) for the \((B - V) \sim 1.3\) typical for the lower main sequence of NGC 6397 (Alcaino *et al.* 1987). Consideration of the entry for K3 III stars in Table 3 of Taylor (1986) gives a value of 1.38 for this ratio. On the other hand, Grieve’s (1983) calibration of the color excess ratio for F and G supergiants gives a higher value, more like 1.6. This difference will only affect \((m - M)_I\) by 0.05 mag, well within the uncertainties in the distance modulus itself. Given these values for the ratio of the reddenings, we adopt an apparent \(I\) distance modulus \((m - M)_I = 12.0\). This is the same as the distance modulus used by FRST.
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Fig. 1.— Positions of the program fields observed for this study. The coordinates are with respect to the adopted center marked by a ‘+’. The dots mark the positions of all the stars observed with $I < 13$ in the program fields and have sizes related to their magnitudes. Most of these also appear in the plate of Cannon (1974) and can be used to locate these fields. The abbreviation “duP” indicates the du Pont fields listed in Table I. These were selected to avoid bright stars. The field marked “FRST” is that observed by Fahlman et al. (1989).

Fig. 2.— The surface density profile for all stars with $I < 14$. The field star density has been subtracted and is shown by the horizontal line. All sections with fewer than ten stars (including those with no stars) have been merged with their neighbors in the same field. As the relatively empty sections are always at the edge of a field there is no ambiguity in the direction of merging. Each density point is shown at the radius which bisects the area of each (possibly composite) section and with a symbol indicating the field of origin.

Fig. 3.— A comparison of the central surface density profile for $I < 14$ derived from the four quadrants of the field Swope:c with that derived from the entire field. The various symbols without error bars are the surface densities from star counts in the four quadrants. The line connects their mean at each radius with error bars indicating the standard deviation in the four measurements. The solid points, offset outward by 0.025 dex for clarity, are the surface densities from the whole field with errors based on a combination of Poisson errors and frame to frame errors as described in §2.

Fig. 4.— The combined surface density profiles for all stars with $I \leq 14.0$ (open circles) and $I \leq 15.5$ (filled circles). The former has been shifted upward by 0.49 dex on the basis of the least-squares fits to the slopes and intercepts over the region $0.0 \leq \log r \leq 0.8$. The field star surface densities to the same limiting magnitudes are also shown. The $I \leq 14.0$ field star density (horizontal dotted line) has also been shifted by 0.49 dex. The overall fits to various radial ranges of the surface density profile are shown: solid line $\log r \leq 0.1$, dot-dash line $0.0 \leq \log r \leq 0.8$. The positions are given in arc minutes.

Fig. 5.— Comparison of the field star counts from FRST (histogram) and this study (field du Pont:bf; points). The two fields are at a distance of 1° on either side of, and at the same galactic latitude as the cluster. Like the new luminosity functions, these star counts are shown for two sets of overlapping half-magnitude bins.

Fig. 6.— Luminosity function for the the inner field du Pont:if. The field star counts are shown as a histogram for comparison. Two sets of overlapping half-magnitude bins are shown.

Fig. 7.— Mean luminosity function for the outer fields du Pont:n and du Pont:w. The field star counts are also shown as a histogram. These fields are about as far from the center of the cluster as is practical without surveying very large areas. Two sets of overlapping half-magnitude bins are shown.

Fig. 8.— All the NGC 6397 luminosity functions are compared. The three LFs are from the inner field of this study, du Pont:if (filled circles); that of FRST at an intermediate radius (open circles); and the mean of the outer fields of this study (squares). For the two LFs from this study two overlapping half-magnitude bins are shown. The offsets between the luminosity functions are real and reflect the drop in stellar density with radius.

Fig. 9.— All the NGC 6397 mass functions are shown. The symbols and binning are as in Fig. 8.
Fig. 10.— Apparent mass spectral index vs. radius diagram following Pryor et al. (1986). The five curves are labeled with the global values of the mass spectral index for each model. A concentration of $c = 2.5$ and core radius of $6''$ have been used. The points are the observed values of the MSI over the mass range $0.5 \leq M/M_\odot \leq 0.8$. The symbols for each mass function are as in Fig. 9. While $x_g \sim 0.$ is in marginal agreement with the observations, the inferred global value for each point increases with radius. This inconsistency indicates that this is not a useful approach for doing this conversion.

Fig. 11.— The segregation measures between the three pairs of fields are shown. The radial segregation measure, $S_r(m; a, b)$, is defined by eq. (2). (a) $S_r(m; \text{du Pont:out, du Pont:if})$. (b) $S_r(m; \text{du Pont:out, FRST})$. (c) $S_r(m; \text{FRST, du Pont:if})$. 
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