Gear Fault Signal Detection based on an Adaptive Fractional Fourier Transform Filter

Xiaojun Zhou\textsuperscript{[1]*}, Yimin Shao\textsuperscript{[1,3]}, Dong Zhen\textsuperscript{[2]}, Fengshou Gu\textsuperscript{[2]}, Andrew. Ball\textsuperscript{[2]}

\textsuperscript{1}State Key Laboratory of Mechanical Transmission, Chongqing University, 400044, PR China;
\textsuperscript{2}School of Computing and Engineering, University of Huddersfield, HD1 3DH, UK;
\textsuperscript{3}College of Automobile and Mechanical Engineering, Changsha University of Science and Technology, Changsha, 410004, PR China.

Abstract: Vibration-based fault diagnosis is widely used for gearbox monitoring. However, it often needs considerable effort to extract effective diagnostic features from noisy vibration signals because of rich signal components contained in a complex gear transmission system. In this paper, an adaptive fractional Fourier transform filter is proposed to suppress noise in gear vibration signals and hence to highlight signal components originated from gear fault dynamic characteristics. The approach relies on the use of adaptive filters in the fractional Fourier transform domain with the optimised fractional transform order and the filter parameters, while the transform orders are selected when the signal have the highest energy gathering and the filter parameters are determined by evolutionary rules. The results from the simulation and experiments have verified the performance of the proposed algorithm in extracting the gear failure signal components from the noisy signals based on a multistage gearbox system.

1. Introduction

Gearbox health monitoring problems have received increased interest for the last two decades due to maximising safety and minimising cost in operations. Vibration signal are sensitive quantities with respect to a fault and it is often used for gearbox fault detection diagnosis. However, gear fault feature signal are always buried in the heavy noise which are from the healthy gear-pair meshing vibration and other machine vibration sources [1-2]. How to extract the gear fault feature signal efficiently or how to improve the signal-noise-ratio (S/N) actively is the main research problems of gearbox monitoring. Many signal detection methods such as wavelet transform (WT) [3], wavelet packet transform (WPT) [4], genetic algorithm (GA) [5], ensemble empirical mode decomposition (EEMD) [6] and artificial neural network (ANN) [7] have been proposed to suppress noise in gear vibration signals and hence to highlight components originated from gear dynamic characteristics.

Adaptive noise cancelling (ANC) is an effective method for estimating signal when it is corrupted by additive noise [8]. The effectiveness of the signal enhancement depends on the correlation degrees between the reference signal and the noise contained in the primary signal. For better effectiveness, many adaptive filter parameters search algorithms have been proposed, e.g. least mean square (LMS)

\*Author correspondence, Phone: (+86 23) 65112520, Email: xiaojun@cqu.edu.cn.
algorithm, recursive least squares (RLS) algorithm, LMS/RLS algorithms in the frequency domain and so on. However, there are little researches on the variation of output signal when the adaptive noise cancelling realized from the time domain to frequency domain.

In the mean time, the fractional Fourier transform (FrFT) belongs to the class of time-frequency representations which have been extensively used in the field of signal processing for extracting useful signal components from complicated noise and interferences data [9-12]. FrFT's response to the ordinary time domain and ordinary Fourier transform when the transform order \( \alpha \) are equal to 0 and \( \pi / 2 \), respectively. An optimal transform fractional order will lead to an optimal energy density and the signal intensity simultaneously in the time and frequency plane. So in this work, a novel algorithm of adaptive fractional Fourier transform filter (AFTF) is proposed for the gear fault signal extraction.

The approach relies on the use of adaptive noise cancelling filters in the fractional Fourier transform domain with the optimised fractional order and optimised filter parameters, while the transform order is selected when the transform signal have the highest energy gathering and the filter parameters are determined by evolutionary rules, because the filters based on the evolutionary rules can converge to a global minimum when deal with the multiple peak problem [13].

The content of this paper is organized into 5 sections. In section 2, the algorithm of adaptive fractional Fourier transform filter is described. The main body of the paper is in section 3 and 4 where both the simulation study and the test evaluations are addressed to show the performance of the method. Finally, section 5 summaries the conclusions.

2. Adaptive fractional Fourier transform filter

2.1. Review of fractional Fourier transform

The Fractional Fourier transform (FrFT) is a time-frequency distribution extended from the classical Fourier transform (FT). There are several applications of FrFT in the areas of signal processing, especially in noise reduction. The FrFT of signal \( x(t) \) with order \( \alpha \) denoted by \( X_{\alpha}(u) \) is defined as [9]

\[
X_{\alpha}(u) = \int_{-\infty}^{\infty} x(t) K_{\alpha}(t, u) dt
\]

Where the kernel function \( K_{\alpha}(t, u) \) is shown as

\[
K_{\alpha}(t, u) = \begin{cases} 
\delta(t - u) & \text{if } \alpha \text{ is a multiple of } 2\pi \\
\delta(t + u) & \text{if } \alpha + \pi \text{ is a multiple of } 2\pi \\
\frac{1 - j \cot \alpha}{2\pi} e^{-j\frac{\alpha^2 + 1}{2t} + j\alpha \cot \alpha} & \text{if } \alpha \text{ is not a multiple of } \pi 
\end{cases}
\]

It is found that the FrFT of the signal \( x(t) \) exists under the same conditions in which its Fourier transform exists. And the FrFT of order \( \alpha=0 \) is the input signal itself, while the FrFT of order \( \alpha=\pi/2 \) corresponds to the Fourier transform of the input signal.

if a signal \( x(t) \) is a periodic signal and can be expressed as [10]

\[
x(t) = \sum_{n=-\infty}^{\infty} X_n \exp(j2\pi nt)
\]

where \( X_n = \int_{0}^{1} x(t) \exp(-j2\pi nt) dt \), the FrFT of the signal can be shown as
where \( \phi(u, \alpha) = \sum_{n=\pm\infty}^n X_n \exp\left(\frac{j2\pi nu}{\cos \alpha}\right) \exp(-j\pi n^2 \tan \alpha). \)

Figure 1 gives the chirp signal and its time-frequency distribution with the transform order in the fractional Fourier domain. Figure 1(b) shows the chirp signal can convergence to an appropriate transform order in the fractional Fourier transform domain.

2.2. Algorithm adaptive fractional Fourier transform filter (AFTF)

Figure 2 shows the diagram of the classic adaptive noise cancelling algorithm. Based on the reference signal \( N_2 \), which is related to the noise signal \( N_1 \) and unrelated to the feature signal \( S_1 \), using the adaptive algorithms, we can extract the feature signal \( S_1 \) from sensor signal \( S \).

The diagram of the adaptive fractional Fourier transform filter (AFTF) is shown in Figure 3. It contains the fractional Fourier transform algorithm and the adaptive noise cancelling algorithms which is based on the evolutionary algorithm. And the proposed adaptive fractional Fourier transform filter contains the following steps:

1) The primary input signal and the reference signal are transformed into the fractional Fourier domain by FrFT with a fractional order;
2) The primary signal in the fractional transform domain is filtered by a series of filter with different parameters, while the filter parameters are controlled and refreshed based on the strategy of the evolutionary rules;
3) The error signals among the reference signal and the outputs from the filters are calculated;

4) The rules are used to evaluated the performance of the filters and for selection of the optimum filter parameters and the fractional orders, and go to step 2 if the output signal don’t satisfy the requirements;

5) The optimum output signal is transformed to the time domain by the inverse fractional Fourier transform with the inverse of the optimum fractional order, and the gear fault diagnosis based on the noise cancelled signal can be much more accuracy.

\[ y(k) = \sum_{n=1}^{N} a_n(k)y(k-n) + \sum_{m=0}^{M} b_m(k)x(k-m) \]  \hspace{1cm} (5)

Where, \( M \) and \( N \) are the orders of the moving averaging part and the autoregressive part, respectively.

So in the fractional Fourier domain with fractional order \( \alpha \), the output of the ith sub-filter can be given by

\[ Y_i(k,\alpha) = \sum_{n=1}^{N} a_{n,i}(k)Y_i(k-n,\alpha) + \sum_{m=0}^{M} b_{m,i}(k)X_j(k-m,\alpha) \]  \hspace{1cm} (6)

where \( Y(k,\alpha) \) and \( X(k,\alpha) \) stand for the fractional Fourier transform of \( y(k) \) and \( x(k) \) with the order \( \alpha \). And the coefficients \( a_{n,i} \) and \( b_{m,i} \) are correspond to the feature of an individual in the evolutionary algorithm. In this paper, the filter coefficients of each sub-filter at time \( k \) are given as a column vector

\[ W_i(k) = [a_{n,i}(k), b_{m,i}(k)] \]  \hspace{1cm} (7)

The evolutionary algorithm is constructed to refresh the coefficients of the AFTF. The cloning and mating method are used according to the parents’ objective function fitness value, while cloning is for the higher fitness, and the rules are described as follows:

\[ W_{i,sc,j}(k) = W_{i,cp}(k) + \beta \cdot n_{i,j} \]  \hspace{1cm} (8)

\[ W_{i,mc,j}(k) = (W_{i,mpa}(k) + W_{i,mpb}(k)) / 2 + \gamma \cdot n_{i,j} \]  \hspace{1cm} (9)
where $W_{i,ec,j}(k) W_{i,mc,j}(k)$ are the children from the cloning method and mating method for the $i$th filter, respectively. $W_{i,cpp}(k)$ and $W_{i,cmp}(k)$ are the parents for the cloning method and mating method, respectively. $\beta, \gamma$ are fluctuation coefficients. $n_{i,j}$ is a Gaussian random variable vector with zero mean and unit variance.

The choice of the objective function is crucial for the algorithm definition since it can affect several characteristics of the overall algorithm including computational complexity, speed of convergence, robustness, and most importantly for the IIR adaptive filtering case, the occurrence of biased and multiple solutions. In this paper, the objective function for the $i$th filter is given by mean square of the error signal:

$$\{\hat{\alpha}, \hat{\beta}\} = \min_{\alpha, \beta} E[|e_i(k)|^2] = \min_{\alpha, \beta} E[|Y_i(k) - D(k)|^2]$$

(10)

And the optimum FrFT order is estimated according to the following rule:

$$\{\hat{\alpha}\} = \max_{\alpha} (\text{kurtosis}(e_i(k)))$$

(11)

where $e_i(k)$ is the residual signal from the $i$th sub-filter; \text{kurtosis} stands for the kurtosis calculation, and it is sensitive to the shape of the signal, hence is well suited to the impulsive nature of the stimulating forces generated by component damage.

3. Simulation Validations

According to the references [1,6], there will be periodic impulse signal which is induced by the gear failure, when the gearbox works under the constant speed condition. Supposed the noisy signal, $x(t)$, contains the feature signal, $s(t)$, the additive random noise, $n(t)$, and the period noise signal, $r(t)$. It expresses as

$$x(t) = s(t) + n(t) + r(t)$$

(12)

where the feature signal $s(t)$ is a periodic impulse signal, the noise $n(t)$ is the random signal, and the noise signal $r(t)$ is a periodic signal which has a different period with the feature signal.

To validate the proposed algorithm under this condition, the periodic pulse signal with the noise is simulated. The feature signal is simulated using a periodic Gaussian pulse, with 50Hz signal frequency and 10Hz repetitive frequency, as shown in Figure 4(a), and the noise signal combines the 15 Hz sinusoidal signal and the random noise with zero mean is shown in Figure 4(b), while the feature signal and the noise signal is formed into the noisy signal (the primary signal) which have the -5dB signal-noise-ratio, as shown in Figure 4(d). The reference signal is shown as in Figure 4(c), which is relative to the noise contained in the primary signal but with different amplitude and phase, and the sampling frequency is 2 kHz.
According to the Figure 5(a), the optimum transform order is \( \pi/2 \) for this simulation, and in the evolutionary algorithm, the number of parents and children are 16 and 4, and the generation is 60. From Figure 5(b), we can find the indicator calculated from the output signal converged quickly, that is to say, the proposed algorithm can converge quickly and not time-consuming. And the extracted signal with the proposed algorithm and frequency domain adaptive filter (FDAF) using the LMS algorithm [14] are shown in Figure 6, compared to the Figure 4(a), the noise signal is eliminated and the feature signal is highlighted by the proposed algorithm, while the output signal from the FDAF is still heavy noised. The simulation results verified the performance of the proposed algorithm is much better than the classic frequency domain adaptive filter algorithm.

**Figure 4.** Waveform of the signals

**Figure 5.** Kurtosis value vs. transform order and Indicators’ evolutions vs. generations

**Figure 6.** Waveform of the output signal using different adaptive algorithms.

### 4. Experiment Validations

#### 4.1. Experiment Set-up

To confirm the performance of the proposed algorithm, an experimental study was carried out based on a gearbox transmission system. The gear system under experiment is designed by Gear fault diagnosis group, University of Huddersfield (UK). As shown in Figure 7(a), the test rig comprises a 3-
phase electrical induction motor coupled with a DC generator that applies load to the motor, and the
motor is connected to a two stage tested helical gearbox, which contains a gear with fault, and the
schematic of the gearbox is shown as Figure 7(b). The accelerometer sensors were set on the gearbox
to collect the vibration on the vertical and horizontal direction, respectively, and the sampling
frequency of the acquisition system is 84kHz. The input motor speed is about 1500rpm, because it
varies with the current fluctuation. The experiment will take when load condition of the system is 78%
of the full loaded. The numbers of the teeth and the feature frequencies are listed in the table 1.

Table 1. The parameters of the gear system

| Items                  | 1st pinion | 2nd gear | 3rd pinion | 4th gear |
|------------------------|------------|----------|------------|----------|
| No. of teeth           | 58         | 47       | 13         | 59       |
| Feature frequency (Hz) | 25         | 30.9     | 30.9       | 6.8      |
| Mesh frequency (Hz)    | 1450       |          |            | 401.1    |

4.2. Experiment Results

The experiment was conducted in two steps. Vibration measurements were first captured for brand
new gears and then with a fault which was created on one tooth of the first gear wheels with 58 teeth.
The longer the data sets, the time-consuming the program running, so the original signal collected
from the sensors are decimated to a lower sampling rate of 28kHz firstly, another reason is that
frequency higher than 14kHz carry little information. Figure 8(a) and 8(b) show the collected signals
of the vertical vibration in time domain for normal gear pairs and gear pairs with failure teeth
respectively. Compared between them it is seen that the amplitude of vibration signal increases when
there is a fault and there are some impulses buried in the noised signal. However, the interval period is
not very clear and it is hard to diagnose which part of the gearbox is abnormal.

Figure 7. Gearbox transmission system test rig.
Applying the proposed method has found that the optimal transform order is also 1 and the number of parents and children are 16 and 4 while the generation is 60. The extracted two signals, shown in Figure 9, are from the proposed algorithm and the frequency domain LMS adaptive filter algorithm, respectively. Compared to the signals in Figure 8(b), the noise signal is eliminated significantly and the feature signal is much more clearly by the proposed algorithm. The interval for the impulse can be seen is about 0.04s (0.283-0.2436=0.0364s), which is the rotation frequency of the first shaft (25Hz). Based on this the gear is diagnosed to have is a local fault on the first pinion. On the other hand there are no such clear impulses in the output signal from the FDAF. The experimental case results verified the better performance of the proposed algorithm in extracting the gear fault feature signal.

5. Conclusions
To detect the gear fault signal from the heavy noise signal, the adaptive fractional Fourier transform filter (AFFT) is proposed in the paper, which is realized the adaptive noise cancelling algorithm in the fractional transform domain. It also provides the rules for the selection of the optimal fractional transform order and the evolutionary rules for the refreshments of the filter parameters. The results from simulation confirm the algorithm’s quickly convergence and results from the experiment verify that the proposed algorithms are effective in cancelling the heavy noise and highlighting the gear fault feature signal.

Future work: apply the algorithm to the speed variation condition (especially for the speed-up condition) and loose relationship between the reference signal and the noise contained in the primary signal.

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