Optimising the feedback loop of a millimetre-wave absolute photoacoustic power meter using $H$-infinity control synthesis

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Abstract. An H-infinity control strategy has been developed for the design of controllers used in feedback controlled electrical substitution measurements (FCESM). The methodology has the potential to provide substantial improvements in both response time and resolution of a millimetre-wave absolute photoacoustic power meter.

1. Description of the millimetre-wave photoacoustic power meter

An area of current interest at Reading is that of free-space absolute power detection of millimeter and sub-millimetre wave radiation, using a photoacoustic power meter (PAPM) which has replaced the previously used UK standard, a modified from the visible region thermopile radiometer. The PAPM comprises a nichrome film on a thin Mylar substrate enclosed within a gas cell formed by the space between a transparent window and an adjustable back reflector [1].

Figure 1a) mm-wave PAPM marketed by TK Ltd, b) typical closed-loop response assuming c) short and d) long averaging times.
Square wave modulated near-mm wave radiation causes temperature and pressure modulation in the gas cell to be picked-up by a microphone and the effect of absorbed dissipation is nulled using ohmic dissipation. In this mode, electrical heating is applied continuously and a control system is used to vary this heating in such a way that the output of the sensor is kept constant; the radiant power is then equated to the feedback-induced change in electrical heating power.

The problem that we treat here is the choice of control parameters to minimize the measurement time \( t_m \) which is the sum of a response time \( t_r \) and averaging time \( t_a \). Speed of measurement is important because it reduces errors caused by drift in the radiometer. The response time is defined as the time following a step change in input until the heating power is settled at its new level to within some fraction \( \varepsilon \) of the change in electrical power as shown in Figure 1 (b to d). This is a radiometer bandwidth selection problem which was firstly identified by Clare and White [2] to which any solution is a compromise between noise and resolution on the one hand and response time on the other.

2. Casting the FCESM problem as an \( H\)-infinity control problem

A general description of a feedback control electrical substitution system is shown in Figures 2 and 3 below.

![Figure 2. Archetypal FCESM scheme as described by Clare and White in [2].](image)

![Figure 3. Block diagram representation of the above FCESM scheme.](image)

From figures 2 and 3, using transfer function notation the controller response is:

\[
W_f(s) = \left( \frac{G(s)K(s)}{1 + G(s)K(s)} \right) W_e(s) + \left( \frac{G(s)}{1 + G(s)K(s)} \right) V_n(s) \tag{1}
\]

where \( G(s) = R(1 + s \tau_{th})^{-1} \) is the PAPM transfer function composed of the transducer responsivity \( R \) and thermal time constant \( \tau_{th} \), \( K(s) \) is the controller transfer function and \( W_e(s) \) is the external influence to be measured. \( V_n(s) \) is the equivalent voltage noise of power spectral density \( V_n^2(s) \) entering the system at the controller input. In conventional control systems, the goal is to attain the
highest possible gain–bandwidth product and ensure that the output of the controller is a direct measurement of the effect of an external influence on the plant, i.e. \( G(s)\{W_f(s)+W_e(s)\} = 0 \). This is different in FCESM, where the controller is designed to maintain the plant in a constant state, i.e. \( \{W_f\} = -\{W_e\} \). From (1), we want:

\[
G(s)K(s)(1+G(s)K(s))^{-1}W_e(s) = -1
\]

\[
G(s)(1+G(s)K(s))^{-1}V_n(s) = 0
\]

across all frequencies. A controller design strategy is required that takes into consideration these two conflicting requirements.

Figure 4 is an equivalent block diagram representation of the FCESM scheme, \( P \) represents the generalised plant, \( K \) the controller, \( w = [r \ d \ n]^T \) the reference, disturbance and noise exogenous inputs respectively, \( z = [z_1 \ z_2]^T \) the exogenous outputs, \( u \) the control signals and \( \nu \) the sensed outputs.

Figure 4. Equivalent block diagram representation of the FCESM scheme appropriate for \( H\)-infinity problem formulation.

The overall control objective is to minimise the \( H_\infty \) norm: \( \|F_j(P,K)\|_{\infty} = \sup_{\omega} \sigma F_j(P,K)(j\omega) \) of the transfer function between \( w \) and \( z \): \( z = F_j(P,K)w \), where \( F_j(P,K) \) is the linear fractional transformation \( F_j(P,K) = P_{11} + P_{12}K(I-P_{22}K)^{-1}P_{21} \) and \( \sigma \) is the maximum signal value of matrix \( F_j(P,K) \) at each frequency. The controller design problem then becomes finding a controller \( K \) which based on the information \( \nu \), generates a control signal \( u \), which counteracts the influence \( w \) on \( z \), thereby minimising the closed loop norm from \( w \) to \( z \). The transfer function of \( P \) can be expressed as:

\[
\begin{bmatrix}
  z_1 \\
  z_2 \\
  \nu
\end{bmatrix} =
\begin{bmatrix}
  P_{11} & P_{12} \\
  P_{21} & P_{22}
\end{bmatrix}
\begin{bmatrix}
  w \\
  u
\end{bmatrix} \quad \Rightarrow \quad
\begin{bmatrix}
  z_1 \\
  z_2 \\
  \nu
\end{bmatrix} =
\begin{bmatrix}
  W_p & -GW_p & -W_p & -GW_p \\
  0 & 0 & W_T & 0 \\
  I & -G & -I & -G
\end{bmatrix}
\begin{bmatrix}
  r \\
  d \\
  n \\
  u
\end{bmatrix}
\]

with \( W_p \) and \( W_T \) frequency dependent designer weights [3].
3. Results and discussion

Figure 5 shows simulated generic results from designing controllers for first order transducers such as the PAPM using $H$-infinity control synthesis. It can be observed that a range of sub-optimal controllers offering superior characteristics than the ones calculated using the Clare and White methodology can be implemented.

![Figure 5. Clare and White's figure of merit plotted against response time for controllers designed using the Clare and White tuning method (dashed line) and the $H$-infinity method (solid line).](image)

4. Conclusion

An $H$-infinity control strategy has been developed for the design of controllers that maximise the absolute precision of FCESM systems while at the same time minimising the response time. For single measurements, the correct choice of control system variables has the potential to provide substantial improvements whereas for repeated measurements, the methodology also answers the question whether it is better to set the system up for a relatively fast but noisy response and average a sequence of observations of the response after it is settled, or to make a single observation of a slower but less noisy response. The currently developed controller design procedure exemplified using a mm-wave photoacoustic power meter is applicable to all terahertz radiometric detectors governed by first order dynamics and may be extended to 2nd order transducers such as kinetic inductance detectors or higher order transducers e.g. transition-edge superconducting bolometers which are not currently employed for absolute measurements. Furthermore the methodology is generic and may be adopted for use in different parts of the electromagnetic spectrum e.g., in true rms ac/dc transfer systems, RF power meters and calorimeters.

References

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[3] S. Skogestad, I. Postlethwaite, Multivariable Feedback Control, Wiley, Chichester, 2005.