Design of The Thick Wind Turbine Airfoil Based on An Improved MOPSO Algorithm

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Abstract: The design of a thick wind turbine airfoil is a multi-constraint and multi-objective optimal problem. In order to make the thick airfoil with a blunt trailing-edge have better aerodynamic and structural performance simultaneously, a new method for thick airfoil optimal design based on an improved MOPSO algorithm was presented in this paper. To show the performance of the new design technique, the optimization is accomplished on the wind turbine airfoils with a 40\% maximum relative thickness, and the Pareto-optimal set is obtained. The representative airfoils are picked, analyzed, and compared with a traditional wind turbine airfoil (DU00-W2-401) at Re=2x10\textsuperscript{6} for free and fixed transitions, the results show that the new airfoils have better performance than the DU00-W2-401.

1. Introduction

In the design of a wind turbine blade, the shape of the transition section in the first one third of the blade is gotten by interpolating round airfoils with a 40\% relative thickness airfoil. Thus, the 40\% relative thickness airfoil has a significant influence on the aerodynamic and structural performance of the transition section of the blade. For the optimization design of wind turbine airfoils, currently, the single-objective optimization algorithm with achieving the best aerodynamic performance as the optimization objective is the most common one. While with this optimization design method, the authors found that the optimal designed airfoils present small relative thickness at the leading and trailing edges, which exhibiting poor structural performance in the blade structural design.

Inspired by the excellent structural performance of aircraft airfoil FX-77-W-400, K. J. Standish and C. P. van Dam \textit{et al.} [1, 2] designed a flat-back airfoil for the wind turbine by using trailing-edge truncation method. Adopting flat-back airfoils on the transition section of the blade can not only increase the aerodynamic performance of blades, but also reduce the masses of blades. However, it is also found that the flat-back airfoils with a large thick tail will also bring problems such as large drag force, noise and manufacturing cost [3]. Besides, this improved method can only implement on an existing airfoil, which means that this method cannot be utilized to design airfoils fundamentally.

For the universality and fundamentality, this paper only focus on the design of the universal airfoils with the thickness of the airfoil trail not exceed than 3\% of the chord length. In order to make the thick airfoil with a blunt trailing-edge have better aerodynamic and structural performance simultaneously, the method for the generation of blunt tail airfoil was studied firstly, the sharp trailing edge airfoil is parameterized expressed with the airfoil integral theory and the blunt trailing-edge airfoil is created by
enlarging the thickness of trailing edge symmetrically from the location of maximum thickness to the airfoil tail. Then, an improved multi-objective particle swarm optimization (MOPSO) algorithm was proposed, by adopting the improved MOPSO, this paper explored Pareto optimal front sets of the multi-objective optimization for the 40% thickness airfoil, the reliability and superiority of this design and optimization method were validated by comparing the optimization results with the DU00-W2-401 airfoil.

2. Parameter expression of the blunt trailing edge airfoils

Based on conformal transformation theory, a graphic in one plane can be turned into another graphic in another plane, which is shown in Figure 1.

![Figure 1 The transformation of a common airfoil](image)

The corresponding points of an airfoil in the $\zeta$ plane can be expressed as followed:

\[
\begin{align*}
x &= (r + a^2 / r) \cos \theta \\
y &= (r - a^2 / r) \sin \theta
\end{align*}
\]

(1)

Where $x$ and $y$ are the abscissa and the ordinate of an airfoil respectively, $r$ is the radius vector of an airfoil in the $Z$ plane, $\theta$ is the argument.

Based on Theodorsen method, a simple and universal function of the coordinates in the $Z$ plane is presented [4]:

\[
z_c = r = a \rho(\theta) \exp(i\theta)
\]

(2)

Where, $r$, $x$ and $y$ are the radius vector, abscissa and ordinate of the airfoil, respectively, $\theta$ represents the argument, $a$ is 1/4 of the airfoil chord length.

According to the Taylor series theory, the generalized shape function $\rho(\theta)$ can be expressed as follows:

\[
\rho(\theta) = c_1 + c_2 \theta + c_3 \theta^2 + c_4 \theta^3 + \cdots c_k \theta^k + k = 1, 2, 3 \cdots n
\]

(3)

Therefore, with the method of airfoil functional integrated parametric expression above, by select various parameters $c_k(k=1,2,3\ldots)$, different airfoils with sharp trailing edge can be generated.

However, in the practical design and manufacture of wind blades, the blunt trailing edge airfoils are generally adopted. Some researchers optimized the blunt airfoils by simply rotating the down airfoil surface or truncating the exiting airfoil’s tail [5]. For the first method, with the leading edge point of airfoils as the center, the down surface is rotated clockwise for a certain angle. With regard to the second method, the trailing edges of airfoils are directly cut off to generate blunt trailing edge airfoils. In the optimization design of large thick airfoils, the authors found that when the trailing edges are required to have large relative thickness, the rotation of the down surface by using the first method is supposed to influence the smoothness and continuity of the leading edges of airfoils, and further aggravates the stall transition at some angles. In contrast, the airfoils generated using trailing-edge truncation method present relatively smooth trailing edges but without favorable maximum lift coefficients.

To avoid the problems above, this paper symmetrically increases the thickness of trailing edge from
the location of airfoil’s maximum thickness to the tail. The expression of generating the blunt airfoil is as follows:

\[
\begin{align*}
    &\begin{cases}
    x \leq ax, & y' = y \\
    x > ax, & y' = y + 0.5p\left(\frac{x - ax}{1 - ax}\right)^n
    \end{cases}
\end{align*}
\]

(4)

Where, \(x\) and \(y\) are the abscissa and ordinate of the upper and lower surfaces of the sharp trailing edge airfoil separately; \(ax\) shows the abscissa of the airfoil at the maximum relative thickness; \(p\) presents the trailing-edge thickness to be increased; \(n\) shows the thickening index of the airfoil and was set as 2 in this paper; \(x'\) and \(y'\) are the abscissa and ordinate of the thickened airfoil separately. The comparison of the blunt trailing edge airfoil and the sharp trailing edge airfoil generated using this method is presented in Figure 2.

![Figure 2 Generation of the blunt airfoil](image)

3. Airfoil optimization design model

3.1. Objective functions and design variables

To make the transition section of the blade have aerodynamic and structural performance simultaneously, this paper takes the maximization of the comprehensive lift-drag ratio and the maximization of the inertia moment of the airfoil to the chord length at the design attack angle under rough and smooth conditions as the design objective:

\[
F(x) = \max(f_1(x), f_2(x))
\]

(5)

Therein to,

\[
f_1(x) = \mu_1 \cdot c_l(\alpha) / c_d(\alpha) + \mu_2 \cdot c_l(\alpha) / c_d(\alpha)
\]

(6)

\[
f_2(x) = I_{xx}
\]

(7)

Where, \(\alpha\) is the maximum lift-drag ratio at a certain attack angle under smooth and rough conditions; \(\mu_1\) and \(\mu_2\) shows the weight coefficients of operation condition under smooth and rough conditions, respectively, \(\mu_1, \mu_2 \in [0,1]\); \(\mu_1 + \mu_2 = 1\); \(c_l\) and \(c_d\) present lift and drag coefficients under smooth airfoil condition, while \(c_l'\) and \(c_d'\) show lift and drag coefficients under rough airfoil condition [6].

According to the cantilever beam theory, the larger rigidity of the blade, the stronger resistance to bending deformation of the blade is. Without loss of generality, this paper takes the inertia moment of the airfoil to chord axis as the index to evaluate the structural characteristics of the airfoils. Based on the practical lamination of the blades, the thickness of blade surface is set as 0.01m.
With the parameterized expression of the wind turbine airfoil as shown in chapter 2, it’s found that the parameter $k$ of the shape function $\rho(\theta)$ should be larger than 11 to express sharp tail airfoils without distortion. For the generation of blunt tail, this paper takes variable $p$ presenting the increased thickness of the upper and lower airfoil tails. Therefore, the design variables are determined as follows:

$$X = \left(c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, p\right)$$

(8)

3.2. Constraint conditions

The maximum thickness $t_{\text{max}}$ of the airfoil should be:

$$0.375 \leq t_{\text{max}} \leq 0.425$$

(9)

The maximum camber $c_{\text{max}}$ and location of maximum thickness $L_{\text{max}}$ should be constrained as follows:

$$c_{\text{max}} \leq 0.2$$

(10)

$$0.25 \leq L_{\text{max}} \leq 0.35$$

(11)

To obtain the better aerodynamic performance and stall characteristics, this paper limited the maximum lift coefficient and required the lift coefficient of airfoils reducing no more than 0.1 every $3^\circ$ attack angle from the attach angel with maximum lift coefficient $\alpha_{l,\text{max}}$.

$$c_i(\alpha_{l,\text{max}} + 3i) - c_i(\alpha_{l,\text{max}} + 3(i + 1)) \leq 0.1, \quad i = 1, 2, 3, 4$$

(12)

3.3 MOPSO Optimization algorithm

According to the laws of birds cluster activities, American electrical engineer Eberhart and social psychologist Kennedy proposed the particle swarm optimization (PSO) [14]. This algorithm is to change the disordered group activities as ordered ones by sharing individual information, so as to obtain optimal solutions. It has been widely recognized by scholars as it has many advantages such as easy operation, high precision and fast convergence as well as superiority in solving practical problems. Ray et al. presented Pareto decision-based MOPSO algorithm by combining Pareto dominance relation and the PSO [17]. This algorithm constructs non-inferior solutions in the iteration process of the external file through Pareto sorting. Then, the obtained non-inferior solution sets are sorted based on the crowding distance with the global optimal position selected as gbest and individual optimal position as pbest. Afterwards, the positions of particles in the group are updated using Formulas 6 and 7. In addition, Coello et al. [18], Liang et al [19] and M. Janga Reddy et al. [20] put forward adaptive mesh-based MOPSO, comprehensive learning particle swarm optimizer (CLPSO) and elitist-mutated multi-objective particle swarm optimization technique (EM-MOPSO), respectively.

$$V_{i}^{k+1} = wV_{i}^{k} + C_{1}R_{1}(P_{i}^{\text{best}} - X_{i}^{k}) + C_{2}R_{2}(P_{g}^{\text{best}} - X_{i}^{k})$$

$$X_{i}^{k+1} = X_{i}^{k} + V_{i}^{k+1}$$

(14)

(15)

Where, $w$ and $X_{i}^{k}$ are the inertia weight and the position of current particle swarm respectively; $C_1$ and $C_2$ are the learning factors which are set between 0 and 2 according to experience; $R_1$ and $R_2$ are random numbers between 0 and 1, while $P_{i}^{\text{best}}$ and $P_{g}^{\text{best}}$ present the positions of current local optimal solution and global optimal solution respectively.

To solve constraint multi-objective optimization problem, Deb et al. defined a Pareto decision-based method for dealing with multi-constraint and multi-objective optimization problems in non-dominated sorting genetic algorithm (NSGA-II). That is, a solution $X_i$ constrains and dominates another solution $X_j$ if and only if the following conditions are met.
1) The solution $X_i$ is a feasible solution while the solution $X_j$ is not a feasible solution;
2) Neither the solutions $X_i$ nor $X_j$ are feasible solutions, but the general constraint conflict value of the solution $X_i$ is less than that of the solution $X_j$;
3) Both the solutions $X_i$ and $X_j$ are feasible solutions, but $X_i$ dominates $X_j$.

The existing Pareto mechanism-based multi-objective optimization algorithm merely depends on the penalty degree of the design object to the constraints to evaluate the non-inferior solutions. However, in some practical problems, though with favorable fitness, certain solutions are supposed to be considered as poor non-feasible solutions once they slightly violate the constraint conditions relating the essential characteristics of the design object. While for some solutions, they are still superior infeasible solutions than the former though they are with poorer fitness than the former and largely violate some non-critical constraint conditions. For example, in the optimization design of airfoils, if the airfoils do not satisfy the constraint conditions for the relative thickness, this solution is regarded as a poorer infeasible solution; if the airfoil cannot satisfy the non-critical constraint conditions including the maximum bending and stall characteristics, this solution is believed as a better infeasible solution than the former. However, the existing multi-objective optimization algorithm applied in the multi-constraint and multi-objective optimization cannot precisely evaluate the infeasible solutions.

To solve these problems, this paper proposed an adaptive penalty function-based method for dealing with multi-constraint conditions. This method calls the key constraint conditions relating the essential characteristics of objects as the primary constraint conditions, and the non-critical constraints to the performance indexes as secondary one. In Formula (5), a larger penalty function coefficient is applied to the primary constraint conditions, while a smaller penalty function coefficient is adapted to the secondary ones. Therefore, the multi-constraint optimization is transferred into the optimization without constraints, as shown in the following formula.

$$\max (F(x) = f(x)^{\ast}P$$

$$p_i = \exp(g_i(x))^{-p_i}$$

$$p_j = \exp(h_j(x))^{-q_j}$$

Where, $P$ is the product of all penalty factors, showing the general violation of the current objects to the constraint conditions; $p_i$ and $q_j$ are the penalty factors of the $i^{th}$ inequality constraint and $j^{th}$ equality constraint separately.

Based on the improved MPSO, the flow chart of the optimization algorithm is shown in Fig.3.
4. Optimization results

Figure 4 demonstrates the Pareto optimal front obtained by optimizing the airfoil with 40% relative thickness using the MOPSO algorithm. As shown in the figure, the Pareto optimal sets had favorable design university and diversity.

Figure 5 shows the leading edge shapes and trailing edge shapes of three optimized airfoils selected from the Pareto optimal set. From the perspective of the geometric features, the relative thickness of the leading edge part of the airfoils B and C (0–0.2 c, where c is the chord length of the airfoil) were larger than that of the airfoil A. The thickness of the airfoil A at 0.1 relative chord length was 0.27, while those of the airfoils B and C were 0.291 and 0.282 separately. In fact, the reduced thickness of the leading edge of airfoils was beneficial to improve the aerodynamic performance of airfoils. For the trailing edge parts (0.4 c–1 c) of airfoils, the trailing edge of the airfoil C was thicker than those of the airfoils A and B, which was helpful to enhance the section rigidity of blades. In addition, similar to the DU00-W2-401, the optimized airfoils presented favorable symmetry in the leading edge. Therefore, they are more easily to be compatible with round airfoils and middle thick airfoils in blades.

By using the software RFOLL for aerodynamic analysis, the aerodynamic performances of three
types of airfoils under the same operation condition ($Re=2\times 10^6$ and $Ma=0.15$) were calculated and compared with that of the DU00-W2-401, as shown in Figures 6 to 8. Under smooth condition, the lift curves of these airfoils did not immediately reduce after reaching the maximum values. While under rough condition, the lift curves presented transitions at different degrees at an attack angle of $2^\circ$ where the lift coefficients rapidly reduced and then increased. These transitions of the lift curves under rough condition were mainly attributed to flow separation when the attack angle of large thick airfoils changed from a negative one to a positive one, which induced the rapid reduction of the lift coefficients. The transitions of the airfoils DU00-W2-401, A, B and C occurred at the attack angles of $4^\circ$, $2^\circ$, $2^\circ$ and $1^\circ$, respectively. The earlier transitions of the optimized airfoils widened the stable range of the lift curves of the airfoils. In addition, when the lift curves of airfoils A and B turned under rough condition, the reducing amplitudes of the curves were less than that of the DU00-W2-401 to a certain degree, which was beneficial to reduce the load fluctuation of blades.
Table 1 presents the comparison of the lift and lift/drag ratio of the three types of airfoils with the DUoo-W2-402 airfoil. Under smooth condition, compared with the DUoo-W2-402 airfoil, the maximum lift coefficient and lift/drag ratio of the airfoil A increased by 34.33% and 38.62%, respectively; while those of the airfoil B raised by 37.2% and 23.51% separately; those of the airfoil C grew up by 38.12% and 12.56%, respectively.

Table 1 Performance of the airfoils

| Airfoils | The maximum lift coefficient | Lift/drag ratio at an attack angle of 9° under smooth condition | Lift/drag ratio at an attack angle of 9° under rough condition | Inertia moment (m^4) |
|----------|-----------------------------|-------------------------------------------------|-------------------------------------------------|------------------|
| A        | 1.17(10°)                  | 69.494(5°)                                      | 3.510                                          | 3.965e-4         |
| B        | 1.195(10°)                 | 61.915(5°)                                      | 3.638                                          | 4.134e-4         |
| C        | 1.203(10°)                 | 56.428(5°)                                      | 1.959                                          | 4.240e-4         |
| DU       | 0.871(9°)                  | 50.131(5°)                                      | 1.546                                          | 4.110e-4         |

Note: the data in the brackets present the attack angle positions at maximum lift coefficient or lift/drag ratio.

Table 1 also shows the comparison of inertia moments of the chord axes of these airfoils. Therein, the airfoil A showed the minimal inertia moment, which was 3.64% smaller than that of the DU00-W2-402 airfoil; the inertia moment of the airfoil B increased by 4.26% and 0.58% compared with those of the airfoil A and the DU00-W2-402 airfoil, respectively. Compared with the inertia moment of the airfoil A, that of the airfoil C increased by 7%. Thus it can be seen that the airfoils B and C exhibited superior aerodynamic and structural performance to that of the DU00-W2-402 airfoil.

Compared with the airfoils A and B, the airfoil C showed increased inertia moment. However, its aerodynamic performance reduced. As shown in Figure 7, the drag coefficient of the airfoil C in an attack angle range of 4°~15° was larger than those of the airfoils A, B and DU00-W2-401. Moreover,
by analyzing other airfoils in the optimal sets, it can be found that with the increase of the inertia moment of airfoils, the thicknesses of the leading and trailing edges of airfoils increased. However, in the meanwhile, their aerodynamic performances reduced, as demonstrated by the reduced lift coefficient under rough condition and the increased drag coefficient under smooth condition.

By comprehensively comparing the aerodynamic and structural performances of the four airfoils, it can be found that these two factors were contradictory to each other for airfoils. When much attention is paid to the aerodynamic performance of airfoils, the structural characteristics are supposed to be weakened. On the contrary, if we focus on the structural characteristics of airfoils, the drag coefficient of airfoils increases and the whole aerodynamic performance of airfoils declines.

## 5 Conclusions
This paper proposed a parametric expression method for blunt trailing edge airfoils. In addition, this paper studied the design features of large thick wind turbine airfoils with blunt trailing edges, and then proposed a multi-objective optimization design which comprehensively considered the aerodynamic and structural performance of airfoils based on the MCMOPSO algorithm. By using the method, wind turbine airfoils with a 40% relative thickness was optimized to achieve multiple objectives. In this way, Pareto optimal sets were obtained. Then, this paper compared the airfoils in the optimal set with the existing DU00-W2-401 airfoil. The results indicated that the new airfoils designed using the optimization method proposed in this paper presented better aerodynamic and structural performances than that of the airfoil DU-W2-401. Moreover, by using the multi-objective optimization for airfoils in this paper, there is no need to determine the weight factors in the general single-constraint optimization as designers can select airfoils from the optimal sets according to the operating requirement and decision preference. All these provide a practical and effective method to design large thick wind turbine airfoils. Besides, the airfoils designed by using this method contribute to increase the aerodynamic and structural performances of the transition sections at the root of wind turbine blade of low wind speed.

## Acknowledgments
This research is one of the phased achievements of the general project of the National Natural Science Foundation of China "The collaborative optimization method of wind turbine blade aeroelastic cutting and pre-bending shape design" (No: 51705545).

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