Polarization-Temperature Correlation from a Primordial Magnetic Field

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Abstract

We propose a new method for constraining a primordial homogeneous magnetic field with the cosmic microwave background. Such a field will induce an observable parity odd cross correlation between the polarization anisotropy and the temperature anisotropy by Faraday rotation. We analyze the necessary experimental features to match, and improve, current constraints of such a field by measuring this correlation.

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In our Galaxy, as well as other spirals, large scale magnetic fields on the order of $10^{-6}$ Gauss have been observed, the origin of which is unknown \[1\]. Schemes that invoke a dynamo mechanism to explain these fields all rely on the presence of an initial seed field \[3\], prompting numerous explanations ranging from nonlinear battery mechanisms, to changes in the nature of the electroweak force \[3\]. Furthermore, the mechanism of dynamo generation itself has yet to be developed in a fully self-consistent manner, and the maximum possible fluxes may have been overestimated \[4\]. Related to the origin of magnetic fields in spiral galaxies is the question of the origin of the $10^{-6}$G fields detected in high redshift galaxies \[5\] and in the damped Ly$\alpha$ clouds \[6\]. At these early times a plausible dynamo explanation has yet to be proposed. Alternatively, large scale galactic and extragalactic fields can be explained by the adiabatic compression of a primordial magnetic field on the order of $10^{-9}$ Gauss today \[7\]. Thus, while it is certainly possible that several of the physical schemes being studied may play a role in the formation of large scale magnetic fields, the considerable debate surrounding this subject can not be resolved until the initial magnetic field configuration is better known. This can only come from further observation.

The cosmic microwave background (CMB) supplies us with the oldest and most extensive probe of the early universe. It has recently been argued that it may provide tight constraints on primordial magnetic fields. In \[8\] it was shown that a random magnetic field with an amplitude of order $10^{-8}$G would lead to distortions in the angular power spectrum of the CMB of around 10%, observable with planned satellite experiments. In \[9\] it was shown that, if one included the full anisotropic effects of the magnetic field one could use the existing large angle measurements of the COBE satellite to set a limit on a homogeneous primordial magnetic field today, $B_0 < 6.8 \times 10^{-9}(\Omega_0 h^2)^{1/2}$ G where $\Omega_0 \leq 1$ is the cosmological density parameter and the Hubble constant is $H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$. Other authors have suggested that Faraday rotation might affect the CMB anisotropy and polarization in a distinct way. In \[10\], the authors derived analytic expressions for its effect on the dipole, quadrupole, and octopole of the CMB anisotropy in a Bianchi I cosmology. More recently in \[11\], the authors found the effect that a homogeneous field with an amplitude of $10^{-8}$G would have in decreasing the polarization of the CMB, while in \[12\] it was proposed that a two frequency measurement of the polarization should be able to set bounds on a random magnetic field on the order of $10^{-9}$G.

In the case of a homogeneous magnetic field a more direct single frequency measurement is possible. The basis of this technique is to exploit the parity and symmetry properties that such a field induces in the CMB. Given a measurement of the Stokes parameters, as a function of position on the sky, this data can be decomposed into a sum of generalized spherical harmonics. Thus

\[
I(\mathbf{n}) = \sum_{lm} a^T_{lm} Y_{lm}(\mathbf{n}); \quad (Q \pm iU)(\mathbf{n}) = \sum_{lm} a^{\pm 2}_{lm \pm 2} Y_{lm}(\mathbf{n})
\]

where $T$ denotes temperature and the spin $\pm 2$ spherical harmonics, $\pm 2 Y_{lm}$, are used to preserve the behavior of $Q$ and $I$ under coordinate rotations. Here we follow the notation discussed in \[13\]; for an alternative, but equivalent approach see \[14\]. Circular polarization can be decomposed as a scalar, but is not expected to occur in the CMB \[17\].
Under parity inversion $s Y_{\ell m} \to (-1)^s Y_{\ell m}$ and thus $2 Y_{\ell m} \pm (-2 Y_{\ell m})$ are parity eigenstates. This motivates us to define

$$2a^E_{\ell m} \equiv -(a^2_{\ell m} + a^{-2}_{\ell m}) \quad \text{and} \quad 2a^B_{\ell m} \equiv i(a^2_{\ell m} - a^{-2}_{\ell m})$$

so that $a^E_{\ell m} \to (-1)^{\ell} a^E_{\ell m}$ and $a^B_{\ell m} \to (-1)^{\ell} a^B_{\ell m}$ under parity inversion. In an isotropic universe, cross correlations between the B polarization and the temperature and E polarizations are forbidden as this would imply noninvariance under parity. A homogeneous magnetic field, however, is maximally parity violating and therefore one would expect one of its primary signals to be such a cross correlation. Motivated by this expectation, we consider in some detail the effect of a homogeneous magnetic field on CMB polarization.

Polarization of the CMB is generated in the optically thin last scattering surface by quadrupole fluctuations in temperature. Scalar fluctuations result in polarization that is purely of the E sort, while tensor fluctuations result in B and E polarization. The presence of a magnetic field, however, will create B polarization that is correlated with the measured temperature spectra. This happens as the the B and E polarizations are Faraday rotated into each other. Following [12], we can estimate the extent of this coupling by taking the optical depth of the last scattering surface to be approximately 1. For a homogeneous magnetic field we find

$$\phi \approx 2.3^\circ \left(\frac{B_0}{10^{-9} \text{G}}\right) \left(\frac{\nu_0}{30 \text{GHz}}\right)^{-2} \cos \theta$$

where $\theta$ is the angle of the magnetic field with respect to the direction of propagation. This effect can be used to determine the magnetic field by comparing observations at different frequencies [13]. As this B-polarization signal is expected to be small, however, one would like to minimize experimental noise as well as systematic uncertainties by comparing it to quantities at the same frequency and with greater amplitude. Given our discussion above $\langle a^T_{\ell m}, a^B_{\ell m} \rangle$ represents just such a comparison. We would therefore like to examine this quantity in greater detail.

The evolution of radiation in a perturbed Friedman-Robertson-Walker universe can be studied in the linear regime. In this case each plane wave perturbation can be considered separately, with an average over the $\vec{k}$’s being done at the end of the calculation. Defining $\mu = \hat{n} \cdot \hat{k}$ where $\hat{n}$ is the line of sight we can write down the Boltzmann equation in the synchronous gauge as follows [14]:

$$\dot{\Delta}_T + i k \mu \Delta_T = -\frac{1}{6} \dot{h} - \frac{1}{6}(\dot{h} + 6 \dot{\eta}) P_2(\mu)$$

$$+ \dot{\kappa} \left[ -\Delta_T + \Delta T_0 + i \mu \nu_b + \frac{1}{2} P_2(\mu) \Pi \right]$$

$$\dot{\Delta}_Q + i k \mu \Delta_Q = \dot{\kappa} \left[ -\Delta_Q + \frac{1}{2}[1 - Q_2(\mu)] \Pi \right] + 2 \omega_B \Delta_U$$

$$\dot{\Delta}_U + i k \mu \Delta_U = -i \Delta_U - 2 \omega_B \Delta_Q$$

$$\Pi = \Delta T_2 + \Delta Q_2 + \Delta Q_0$$

where $\Delta_T$, $\Delta_Q$, $\Delta_U$ are the brightness functions for $T$, $Q$ and $U$, which can be expanded in multipole moments defined such that $\Delta(\tau, k, \mu) = \sum_{\ell} (2\ell + 1) (-i)^{\ell} \Delta_{\ell}(\tau, k) P_\ell(\mu)$ ($P_\ell(\mu)$ is
the Legendre polynomial of order \(\ell\). Derivatives are taken with respect to the conformal time \(\tau\), \(\kappa\) is the differential optical depth for Thomson scattering \(\kappa = (a/a_0)n_e\sigma_T\) and \(h, \eta, v_b\) are sources of temperature anisotropies from metric perturbations and baryon velocities. We shall restrict ourselves to scalar perturbations. The effect of Faraday rotation comes in through the coupling between polarization components, \(w_B = \kappa/\omega_0\cos \theta\) where

\[
\omega_0 = 0.06 \left(\frac{B_0}{10^{-9} \text{G}}\right) \left(\frac{\nu_0}{30 \text{GHz}}\right)^{-2}
\]

and we have assumed that \(B_0\) lies along the \(z\) axis. From Eq. 2 we see that \(\Delta Q (\Delta U)\) give us the amplitude of the \(E (B)\) component of the polarization.

Before we proceed, we shall now discuss two important approximations. Firstly we will consider an isotropic universe supporting a homogeneous magnetic field. As argued in [9] this is not entirely correct, such a field can only be supported by the existence of a globally anisotropic term. However, as we shall see, most of our statistic will rely on the large \(\ell\) behavior of the cross correlation spectrum and will therefore be insensitive to the large scale behavior of the anisotropy. Clearly a more detailed calculation should include such terms (for a good description see [17]) but for the purpose of this letter we shall not pursue it.

Secondly, we are interested in studying \(B_0\) in the regime where it is competitive with other constraints from the CMB, notably [9], and consequently \(\omega_B\) is sufficiently small that we can drop it from Eq. 5. This means that the only source term for the correlated \(B\) polarization is through a rotation of \(E\) polarized light by \(\sim 2^\circ\); it is expected to be small compared to \(E\) and the coupling of \(B\) into \(E\) even smaller. Thus the effect can be applied only to the evolution of \(B\) polarized light, and there only as a scale factor of \(B_0\cos \theta\) in the source term.

This represents a great simplification in the calculation. As the angle of the magnetic field with respect to the observer appears only as a scale factor, we can sidestep the issue of the relative orientation of the wave vector \(\hat{k}\) and \(\vec{B}_0\), using it only in the angular averages taken at the end of the calculation. With this simplification we find the cross correlation due to scalar perturbations to be

\[
\langle a^{T\ast}_{\ell m} a^B_{\ell' m'} \rangle = C_{T B}^{\ell \ell'}(\hat{z}) \left[ \int d\Omega Y^{\ast}_{\ell' m'}(\hat{n}) Y_{\ell m}(\hat{n}) \cos \theta \right]
\]

where

\[
C_{T B}^{\ell \ell'}(\hat{z}) = \frac{3\omega_0}{4 (4\pi)^2} \left(\frac{(\ell + 2)!}{(\ell - 2)!}\right) \left[ \int k^2 dk P(k) \int_0^{\tau_0} d\tau g(\tau) \int_0^{\tau} d\tau' g(\tau') \frac{j_\ell(x')}{x'^2} \Pi(k, \tau') \int_0^{\tau_0} d\tau'' S_T(k, \tau'') j_{\ell'}(x'') \right]
\]

and

\[
S_T(k, \tau) = g(\Delta \tau_0 + 2\hat{\alpha} + \frac{\nu_b}{k} + \frac{\Pi}{4} + \frac{3\Pi}{4k^2}) + e^{-\kappa(\hat{\theta} + \hat{\alpha})} + g\left(\frac{\nu_b}{k} + \alpha + \frac{3\Pi}{4k^2}\right) + \frac{3\bar{\eta}\Pi}{4k^2}
\]

\(P(k)\) is the initial power spectrum of scalar perturbations, \(x \equiv k(\tau_0 - \tau)\), \(\alpha = (\dot{h} - 6\dot{\eta})/2k^2\), \(j_\ell(x)\) is the spherical Bessel function of order \(\ell\), and \(g(\tau) = \kappa e^\kappa\). Physically, \(g(\tau)\) is a
leads to a slight increase in amplitude of the \( C_{\ell m} \) error in determining axial directions. We shall consider a full sky survey such as would be performed by a satellite. To estimate the coefficients in Eq. 10 we find that suppressing power in the various cross correlations. However, the broader support of the \( R \) could be taken to be infinitely thin, then \( g(\tau) = \delta(\tau - \tau_s) \) and \( C_{\ell m}^{TB} \) would be identical up to the difference introduced by integrating over two spherical harmonics whose orders differ by one. As polarization is generated in the optically thin last scattering surface, the visibility function is quite sensitive to reionization. Thus we have also calculated \( C_{\ell m}^{TB} \) for a CDM universe with an optical depth of \( \kappa = 1 \) and we present the results in the bottom panel of Figure 4. Note that reionization will suppress power in the various cross correlations. However, the broader support of \( g(\tau) \) leads to a slight increase in amplitude of \( C_{\ell m}^{TB} \) relative to \( C_{\ell m}^{TE} \).

Let us now turn to the observability of such a signal. For the purpose of this paper we shall consider a full sky survey such as would be performed by a satellite. To estimate the error in determining \( C_{\ell m}^{TB} \) we follow the analysis given in [14]. Being careful to include the \( m \) coefficients in Eq. 11 we find

\[
\Delta C_{\ell m}^{TB} \simeq \frac{1}{2\ell^2} \left[ 2 \ln(2\ell) \left( C_{\ell+1}^{TT} + w_T^{-1} e^{\ell^2 \sigma^2} \right) \left( C_{\ell}^{BB} + w_P^{-1} e^{\ell^2 \sigma^2} \right) + (C_{\ell m}^{TB})^2 \right]
\]

(11)

where \( \sigma^2 \) is the full width at half maximum of the beam, \( w_T^{-1} \) and \( w_P^{-1} \) are measures of the experimental sensitivity, independent of pixel size and we have assumed full sky coverage. We have also approximated...
FIG. 1. The top panel shows a comparison of $\ell(\ell + 1)C_{\ell,\ell}^{TE}/(2\pi)$ (solid line), $\ell(\ell + 1)C_{\ell-1,\ell}^{TB}/(2\pi\omega_0)$ (dotted line) and $\ell(\ell + 1)C_{\ell+1,\ell}^{TB}/(2\pi\omega_0)$ (dashed line) in a standard CDM cosmology, while the bottom panel shows such a comparison in a reionized universe with $\kappa = 1$. 
FIG. 2. The sensitivity of \( \frac{B_0}{10^{-9}G} \) (\( \nu_{30\text{GHz}} \))^{-2} given a full sky measurement of the CMB temperature and anisotropy with noise characteristic \( w_T^{-1} \) and beamsize at full width half maximum \( \sigma \).
\[
\sum_{m=-\ell}^{\ell} \frac{1}{2\ell+1} \left[ \frac{4(\ell+1)^2 - 1}{(\ell+1)^2 - m^2} \right] \equiv (2\ell + 3)2 \int_{0}^{\ell} \frac{dm}{(\ell + 1)^2 - m^2}
\]

which is valid for large \(\ell\)’s. Due to the high cosmic variance and low signal at small \(\ell\) we will want to base our measurement of the magnetic field on correlations with \(\ell > 50\) where this approximation is justified.

Following [20] we can define a function for the goodness of fit of a theory given an observation:

\[
\chi^2(\omega_0) = \sum_{\ell} \left[ \frac{C_{\ell+1,\ell}^{TB \text{exp}} - C_{\ell+1,\ell}^{TB \text{th}}(\omega_0)}{(\Delta C_{\ell+1,\ell}^{TB})^2} + \frac{C_{\ell-1,\ell}^{TB \text{exp}} - C_{\ell-1,\ell}^{TB \text{th}}(\omega_0)}{(\Delta C_{\ell-1,\ell}^{TB})^2} \right] 
\]

where the sum is taken from \(\ell > 50\) and we assume that we know all other cosmological parameters. We hope the true value of the magnetic field will minimize the \(\chi^2\) and its sensitivity to \(B_0\) will tell us how well we can constrain it. We are interested in determining the minimum value of \(B_0\) to which we are sensitive. In this limit we expect noise to dominate over \(C^{TB}\) and \(C^{BB}\) as they scale as \(B_0\) and \(B_0^2\) respectively, so we can neglect these terms in Eq. 11.

Following [21], an approximate 1−σ error is given by

\[
\sigma^{-2}(\omega_0) = \left[ \frac{1}{2} \frac{\partial^2 \chi^2}{\partial^2 \omega_0} \right]_{\omega_0=0} \approx \sum_{\ell} \ln 2\ell \frac{(C_{\ell+1,\ell}^{TB}/\omega_0)^2 + (C_{\ell-1,\ell}^{TB}/\omega_0)^2}{(C_{\ell}^{TT} + w_{T}^{-1})e^{2\ell^2\sigma^2 w_{T}^{-1}}} 
\]

In Figure 2 we show two contour plots for the expected precision in \((\frac{B_0}{10^{-9} \text{G}})(\frac{\nu_0}{\text{GHz}})^{-2}\) for a range of experimental parameters. Here we take \(w_P = 2w_T\) [14]. For low frequency detectors \((\nu_0 < 50\text{GHz})\), the top plot shows that we can get comparable limits to that of [9] for \(w_T < 10^{-15}\). Such frequencies are currently only accessible through HEMT detectors where the sensitivity has yet to reach such high levels. With low frequencies there is the additional problem of having to consider large beamwidths. For example an optimistic, space based 30GHz receiver can achieve at most 15′ resolution. The planned MAP mission [22] will have a 30GHz with \(\sigma = 42′\) and \(w_T^{-1} \approx 4.7 \times 10^{-15}\); the Planck mission [23] will have a 31GHz detector with \(\sigma = 30′\) and slightly smaller \(w_T\). This means we will get a constraint of \(\sigma(B_0) \approx \text{few} \times 10^{-8}\text{Gauss}\) from both of them.

For high frequency measurements one is faced with the \(\nu_0^2\) term, i.e. the higher the frequency the smaller the effect. In the bottom plot of Fig 2 we have consider a lower range of \(w_T\) appropriate for these measurements, and labeled isocontours in steps of 0.1. The important thing to note is that current bolometer technology is reaching these levels of sensitivity. If we consider the example of the 150GHz detector on the proposed Planck mission, we find that it may be possible to get a constraint of \(\sigma(B_0) \approx 10^{-9}\), an improvement over the limits set in [9]. With the rapid advances in both high frequency HEMT and bolometer technology it is conceivable that one may do even better (for a realistic assessment of future prospects see [24]).
In this letter we have presented a new method for constraining a homogeneous, primordial, magnetic field. In describing the technique and its potential sensitivity, we have restricted ourselves to a full sky measurement of the CMB anisotropy and polarization. Clearly this is a simplification. A full analysis should include different experiments with varying degrees of sky coverage [25]. We have also focused on scalar perturbations in a CDM universe. The inclusion of gravity waves and vorticity will generate a non-negligible $C^{BB}$ which may increase the cosmic variance of our statistic and necessarily reduce its sensitivity. Naturally the inclusion of global anisotropy, the effects of foreground sources and different thermal histories may also change our results. For example, for the reionized universe considered in Figure 1 the constraint is worse by a factor of two as compared to the sCDM case. Again a full analysis should include a wide range of cosmological parameters and scenarios.

This method has two appealing features which we restate. Firstly all current scenarios of structure formation assume statistical isotropy which necessarily leads to a zero cross correlation between B type polarization and temperature anisotropy. It is only through the existence of magnetic field (or some form of global anisotropy) that such a cross correlation can be induced. Secondly the constraint derived in [4] was strongly dominated by the large angle anisotropies. The authors pointed out that a good measurement of the temperature anisotropies already exists on these scales and future measurements will do little to improve it. This is not the case of our method. Clearly, the better the measurements become the better the constraint of $B_0$ will be.

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