Seismic modelling of the rotating, slowly pulsating B-type star HD 21071

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ABSTRACT
Interpretation of the oscillation spectrum of the slowly pulsating B-type star HD21071 is presented. We show that non-rotating models cannot account for the two highest amplitude frequencies and taking into account the effects of rotation is necessary. Rotating seismic models are constructed using various chemical compositions, opacity data, core overshooting parameters and rotational velocities. There are prospects for seismic modelling of SPB stars, even if no asymptotic pattern is observed in their oscillation spectra, provided an unambiguous mode identification is doable and the effects of rotation are properly included.

Key words: stars: early-type – stars: oscillations – stars: rotation

1 INTRODUCTION
Main sequence stars of mid to late B spectral types pulsating with periods of the order of days have been dubbed slowly pulsating B-type stars (SPB stars) by Waelkens (1991). Although multiperiodicity and long periods undoubtedly pointed to pulsations in high order gravity modes, the excitation mechanism remained unknown at that time.

Progress has been made possible soon after publication of the revised opacity data OPAL (Iglesias, Rogers & Wilson 1992) with a new maximum around log $T \approx 5.3$ (the so-called metal or $Z$ opacity bump) caused by numerous absorption lines of iron-group elements. Using the new opacity data, Dziembowski, Moskalik & Pamyatnykh (1993) and Gautschy & Saio (1993) showed that high radial order gravity modes in SPB stars are driven by the classical $\kappa$-mechanism operating in the metal opacity bump.

However, studying SPB stars is not an easy task from both observational and theoretical points of view. Observations of SPB stars are challenging because their low frequencies demand long runs of observations. Furthermore, extracting periods of the order of a day is often complicated because of an aliasing effect, especially from ground-based observations. Therefore until the Hipparcos mission only a few SPB stars were known. Thanks to the Hipparcos photometry, Waelkens et al. (1998) increased the number of SPB stars from 11 to about 100. Since the pulsation periods are often of the same order as the rotation periods, the next obstacle is to distinguish them from each other. In many cases this task is impossible without detailed spectroscopic analysis.

Theoretical interpretation of the SPB pulsations is complicated because of three reasons: mode identification, very dense theoretical oscillation spectra and rotation. Firstly, because of the lack of clear structures in the observed oscillation spectra, the only way to determine the spherical harmonic degree, $\ell$, and the azimuthal order, $m$, is the use of the information contained in the light and line profile variations (e.g., Balona & Stobie 1979; Watson 1988; Cugier, Dziembowski & Pamyatnykh 1994; Campos & Smith 1980a,b; Balona 1986a,b; Cugier & Daszyńska 2001; Gies & Kullavanijaya 1988; Kennelly & Walker 1999; Telting & Schrijvers 1997).

An exception are the most recent observational results from the CoRoT and Kepler space missions (e.g., Degroote et al. 2010, Pápics et al. 2012, 2014, 2015). However, we need to emphasize that in very dense oscillation spectra such structures can be accidental. An example is the SPB star HD 50230. Degroote et al. (2010) claim that in the oscillation spectrum of the star there is present a sequence equally-spaced in period associated with asymptotic properties while our studies (Szewczuk, Daszyńska-Daszkiewicz & Dziembowski 2014) contradict this. Secondly, because of very dense theoretical oscillation spectra of SPB stellar models, assignment of the radial order, $n$, to the individual observed peaks is usually very ambiguous. The last problem concerns the effects of rotation, which influence, both, the equilibrium model and pulsational properties. In the case of SPB stars, even at slow rotation, pulsational frequencies can be of the order of the rotational frequency and the effects of the Coriolis force cannot be neglected. On the other hand, the
advantage is that the influence of centrifugal deformation on low frequency gravity modes is small (Ballot et al. 2012). Effects of rotation on low frequency g modes was studied in the framework of the so-called traditional approximation (e.g., Lee & Saio 1997; Townsend 2003, 2005; Daszyńska-Daszkiewicz, Dziembowski & Pamyatnykh 2007) or using the truncated expansion for the eigenfunctions (e.g., Lee & Saio 1989, Lee 2001).

Attempts to match the observed frequencies to the theoretical ones have been undertaken by Walczak, Szewczuk & Daszyńska-Daszkiewicz (2012, 2013), who found seismic models which fit two observed frequencies with well identified degrees, $\ell$, in the two SPB stars HD 74560 and HD 182255. However, they assumed that the observed frequencies are the axisymmetric modes, i.e., $m = 0$, and applied the zero-rotation approximation in pulsational calculations.

Without a doubt, the best example of seismic modelling of the SPB stars so far is that of KIC 10526294. Using the Kepler time-series photometry, Pápics et al. (2014) found 19 frequency peaks quasi-equispaced in period, most of which were split into triplets. These authors interpreted the 19 central peaks according to the asymptotic theory as dipole g modes with consecutive high radial orders, $n$. However, KIC 10526294 is unique among the SPB stars because of its extremely slow rotation. The rotational period, as deduced from the rotationally split modes, is equal to 188 days, which justified neglecting the effects of rotation in seismic modelling.

Furthermore, Pápics et al. (2015) found 36 frequency peaks quasi-equispaced in period being probably a manifestation of the asymptotic properties in the SPB star KIC 776080. The star seems to be even more attractive than KIC 10526294 not only because of the longer equally spaced modes series but also because of the higher rotation velocity of at least 62 km s$^{-1}$. Detailed seismic modelling is still to be done.

The goal of this paper is to present results of seismic modelling of the SPB star HD 21071. The star has a few frequency peaks determined from ground-based photometry with a well identified angular numbers $(\ell, m)$ (Szewczuk & Daszyńska-Daszkiewicz 2015b). The effects of rotation are included via the traditional approximation. In the next section, we present the star and results of earlier studies. In Section 3, the oscillation spectrum of the star is interpreted. Section 4 is devoted to seismic modelling. In the last section we summarize the results and discuss the prospect for future studies.

2 THE SPB STAR HD 21071

HD 21071 (HR 1029, HIP 15988, V576 Per) is a star of brightness $V = 6.1$ mag (Reed 2007) and spectral type B7V (Morgan, Hiltner & Garrison 1971; Mooley et al. 2013). Slightly different spectral classifications can also be found in the literature, e.g., B6V (Abt & Hunter 1963), B5IV (Lash 1968) and B4/B3 (Freire Ferrero et al. 2012).

There are many determinations of the effective temperature from photometric calibrations as well as spectral fitting, e.g., log $T_{\text{eff}} = 4.169$ (Saio & Levato 2014), log $T_{\text{eff}} = 4.149$ (Silaj & Landstreet 2014), log $T_{\text{eff}} = 4.212$ (Freire Ferrero et al. 2012), log $T_{\text{eff}} = 4.157$ (Zorec & Roveri 2012), log $T_{\text{eff}} = 4.130$ (Lefever et al. 2010). In this paper we adopted log $T_{\text{eff}} = 4.164 \pm 0.007$ obtained by Niemczura (2003) from the IUE ultraviolet spectra. This value is approximately equal to the mean of the values obtained by the above mentioned authors. The metallicity, as determined from the IAU spectra, is 0.0082$^{+0.0053}_{-0.0032}$ (Niemczura 2004). To put the star on the HR diagram presented in Fig. 1 we used log $L/L_{\odot} = 2.444 \pm 0.076$ derived by Szewczuk & Daszyńska-Daszkiewicz (2015b).

In Fig. 1 there are also shown the evolutionary tracks calculated with the OP opacity tables (Seaton 2004) and the latest heavy element mixture of Asplund et al. (2003) (hereafter AGSS09). We assumed the initial hydrogen abundance $X_0 = 0.7$, the metallicity $Z = 0.0082$ and the equatorial rotational velocities on the ZAMS equal to the projected value, $V_{\text{rot}} \sin i = 50$ km s$^{-1}$, obtained by Abt, Levato & Grosso (2002). Overshooting from convective core was not taken into account. The Warsaw-New Jersey evolutionary code (e.g. Pamyatnykh et al. 1998) is used in the evolutionary calculations throughout the paper. The estimated evolutionary mass for HD 21071 is $M = 3.69 M_{\odot}$ and the ranges of masses for the 1$\sigma$ and 3$\sigma$ error box are $(3.55, 3.84)$ and $(3.28, 4.16)$, respectively.

3 OSCILLATION FREQUENCIES OF HD 21071

HD 21071 was classified as an SPB star by Waelkens et al. (1998) who found variability with a period of $P = 0.84$ d in the Hipparcos space photometric data. In the seven band of the Geneva photometric system, De Cat et al. (2007) detected four frequencies: $\nu_1 = 1.18843$ d$^{-1}$, which coincides with this given by Waelkens et al. (1998), $\nu_2 = 1.14934$ d$^{-1}$ (also present in the Hipparcos data), $\nu_3 = 1.41968$ d$^{-1}$ and $\nu_4 = 0.95706$ d$^{-1}$. In Tab. 1 we give the values of these frequencies and the corresponding amplitudes.

![Image](https://example.com/image.png)
in the $UBV$ Geneva filters (columns from 2 to 5). It should be mentioned that due to a strong aliasing there is a risk that $\nu_3$ and $\nu_4$ can be mistaken with their aliases. Two dominant frequencies, $\nu_1$ and $\nu_2$, were also detected by Andrews et al. (2003) in the APT data. Moreover $\nu_1$ was detected in spectroscopic data [De Cat 2002].

Identification of the degree, $\ell$, for the frequencies of HD 21071 was performed by De Cat et al. [2007] and the results are given in the 7th column of Table 1. Recently, Szewczuk & Daszyńska-Daszkiewicz [2015a,b] included the effects of the Coriolis force in the mode identification. They determined both angular numbers ($\ell, m$) as well as constrained the rotational velocity ($V_{\text{rot}} \in (150, 250)$) [km s$^{-1}$]). The values of ($\ell, m$) are given in the last column of Table 1. For completeness, in the penultimate column we added identification of $\ell$ obtained by Szewczuk & Daszyńska-Daszkiewicz [2015a,b] with the zero-rotation approximation.

As one can see from Fig. 2, three frequencies HD 21071, i.e. $\nu_1$, $\nu_2$, and $\nu_3$, are dipole axisymmetric modes. The frequency $\nu_4$ can be a dipole axisymmetric or a retrograde mode, or a quadrupole retrograde mode. One can also see that $\nu_1$, $\nu_2$, and $\nu_3$ are equally spaced in frequency. If we are not in the asymptotic regime of high order acoustic modes, which is obviously true in our case, a rotational origin of triplets is a most probable explanation for such type of structures. If we are dealing with a rotationally split triplet, the central peak, $\nu_1$, should be an axisymmetric mode, the left side peak, $\nu_4$, should be a retrograde mode and the right side peak, $\nu_3$, a prograde mode. However, this is in contradiction with the identification of $\nu_3$ as an axisymmetric mode.

Another evidence suggesting the non-rotational origin of the triplet is its perfect symmetry. The difference between the central and left side peaks, $\nu_1 - \nu_4 = 0.23137 \pm 0.00009$, is equal within the errors to the difference between the right side and the central peaks, $\nu_3 - \nu_1 = 0.23125 \pm 0.00007$. Linear pulsational theory predicts an asymmetry and the left side peak should be closer to the central one than the right side peak (see also Appendix A for more details, only in the electronic edition of the journal).

Because the study of higher order effects of mode cou-
In this section, we construct seismic models fitting the two \( \nu_1 \) and \( \nu_2 \) and three \( \nu_1, \nu_2, \nu_3 \) observed frequencies of HD 21071. The effects of rotation were included via the traditional approximation in which the effects of the Coriolis force are taken into account whereas centrifugal distortion is neglected. Rigid rotation is assumed.

The method of mode identification developed by Daszyńska-Daszkiewicz et al. (2015) allows, besides determination of \( \ell \) and \( m \), to constrain the rotational velocity. Szewczuk & Daszyńska-Daszkiewicz (2015b) showed that the rotation rate of HD 21071 is in the range \( V_{\text{rot}} \in (150, 250) \text{ km} \text{ s}^{-1} \). We confine our searching of seismic models to this range of the rotational velocity. Due to a high computational cost, it was done in the two steps.

Firstly, we constructed a preliminary grid of models for \( X_0 = 0.7 \), with the OP opacity data, AGSS09 chemical mixture and without overshooting from the convective core \( \alpha_{\text{ov}} = 0.0 \). Models were calculated inside the 3\( \sigma \) error box (see Fig. 1) for different masses, metallicities and rotation velocities with a step \( \Delta M = 0.01 \text{ M}_\odot \), \( \Delta Z = 0.00025 \) and \( \Delta V_{\text{rot}} = 10 \text{ km} \text{ s}^{-1} \). Then, to find the approximate parameters of models which fit the two observed frequencies, the theoretical frequencies were interpolated by means of multi-dimensional linear interpolation.

In the second step, around the approximate model parameters for which theoretical frequencies fit the observed ones, we computed a denser grid of models with steps \( \Delta M = 0.002 \text{ M}_\odot \) and \( \Delta Z = 0.00005 \). A step in effective temperature in both grids was dynamically determined by the evolutionary code and equal approximately to \( \Delta T_{\text{eff}} \approx 0.0005 \).

We will call the grid of models with the input \( (X_0 = 0.7, \text{ OP, AGSS09, } \alpha_{\text{ov}} = 0.0) \) as G1.

In Fig. 4, we put seismic models fitting \( \nu_1 \) and \( \nu_2 \) in the HR diagram for the seven values of \( V_{\text{rot}} \). The colours are assigned to the values of metallicity, \( Z \). In this figure all modes, stable and unstable, were plotted. The use of the instability condition significantly reduces the number of seismic models. Our seismic modelling showed that for \( V_{\text{rot}} \geq 220 \text{ km} \text{ s}^{-1} \) there are very few models with unstable modes which reproduce \( \nu_1 \) and \( \nu_2 \). Moreover, these models have rather high metallicity. For example, in the case of \( V_{\text{rot}} = 250 \text{ km} \text{ s}^{-1} \) unstable modes reproducing \( \nu_1 \) and \( \nu_2 \), were found in models with \( Z > 0.02 \). On the other hand, in the case of the lowest considered rotational velocity, i.e., \( V_{\text{rot}} = 150 \text{ km} \text{ s}^{-1} \), unstable modes reproducing \( \nu_1 \) and \( \nu_2 \) were found in models with metallicity as low as \( Z \approx 0.0054 \).

All our seismic models of HD 21071 can be divided into two families. The first one (hereafter F1), associated with the radial orders from \( n = 13 \) to \( n = 19 \), has on average higher metallicity and is more luminous. The second one (hereafter F2), associated with the radial orders from \( n = 26 \) to \( n = 31 \), has on average lower metallicity and is less luminous. In both cases the models which fit \( \nu_1 \) and \( \nu_2 \) become more luminous with increasing rotation velocity. For the lowest rates of rotation, i.e., \( V_{\text{rot}} = 150 \) and 160 km s\(^{-1}\), we found only models with the F1 solution within 3\( \sigma \) error.
Figure 4. The HR diagrams with the models from the G1 grid which fit $\nu_1$ and $\nu_2$, marked, for seven values of the rotational velocity, $V_{\text{rot}}$. Both, stable and unstable modes were plotted. In each panel, the inner frame corresponds the $1\sigma$ error box and the entire frame to the $3\sigma$ error box. In the bottom right panel there are shown the evolutionary tracks for the three values of metallicity.
Figure 6. The same as in Fig. 4 but models fitting the three frequencies ν₁, ν₂ and ν₃ of HD 21071 are shown.
Table 2. The radial orders, $n$, of dipole modes which fit the three frequencies, $\nu_1$, $\nu_2$ and $\nu_3$, of HD 21071 in the models G1, G2, G3 and G4. In square brackets are stable modes. The first column contains the value of the rotational velocity.

| $V_{\text{rot}}$ (km s$^{-1}$) | G1 | G2 | G3 | G4 |
|-----------------------------|----|----|----|----|
| $X_0 = 0.70$ | $\alpha_{\text{ov}} = 0.0$ | $X_0 = 0.75$ | $\alpha_{\text{ov}} = 0.0$ | $X_0 = 0.70$ | $\alpha_{\text{ov}} = 0.2$ |
| 150 | [g16, g17, g12] | [g16, g17, g12] | [g14, g15, g10] | [g16, g17, g12] | [g16, g17, g12] | [g14, g15, g10] | [g14, g15, g10] |
| 160 | 13, 14, 99 | 13, 14, 99 | 13, 14, 99 | 13, 14, 99 |
| 170 | 14, 15, 99 | 14, 15, 99 | 14, 15, 99 | 14, 15, 99 |
| 180 | 14, 15, 99 | 15, 16, 11 | 14, 15, 99 | [g29, g31, g20] | [g14, g15, g10] |
| 190 | 13, 14, 99 | 13, 14, 99 | 13, 14, 99 | [g14, g15, g10] |
| 200 | 13, 14, 99 | 13, 14, 99 | 13, 14, 99 | [g14, g15, g10] |
| 210 | 13, 14, 99 | 13, 14, 99 | 13, 14, 99 | [g14, g15, g10] |
| 220 | 13, 14, 99 | 13, 14, 99 | 13, 14, 99 | [g14, g15, g10] |
| 230 | 13, 14, 99 | 13, 14, 99 | 13, 14, 99 | [g14, g15, g10] |
| 240 | 13, 14, 99 | 13, 14, 99 | 13, 14, 99 | [g14, g15, g10] |
| 250 | 13, 14, 99 | 13, 14, 99 | 13, 14, 99 | [g14, g15, g10] |

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Box. The seismic models from the F2 solution appear close to the lowest edge of the 3σ error box ($\log L/L_\odot \approx 2.22$) from $V_{\text{rot}} = 170$ km s$^{-1}$. With increasing $V_{\text{rot}}$, we obtained more and more models from F2 and less from F1 in the space of parameters we considered. For the highest rotational velocity, $V_{\text{rot}} = 250$ km s$^{-1}$, we found only seismic models from F2. In both cases, there is a clear trend: the lower metallicity the higher luminosity.

The seismic models of HD 21071 lie along sloped lines in the HR diagram. For one pair of the radial orders we usually have a few nearly parallel sequences of such models differing in metallicity. A similar structure occurs in the diagram $M$ vs. $\log T_{\text{eff}}$. This is because the frequencies of the g modes in the range of parameters and frequency we consider, in general, decrease with decreasing effective temperature and increasing mass. The dependence on Z is varied, for the higher radial orders, $n > 10$, the pulsational frequency is an increasing function of Z, whereas for $n < 10$ there is an oscillatory character of $\nu(Z)$. In Fig. 5 for clarity, we show the diagram $M$ vs. $\log T_{\text{eff}}$ for only one pair of the radial orders, $g_{14}$ and $g_{15}$, and one value of the rotational velocity.

Gaps which sometimes appear along the lines of seismic models result from the adopted steps in $M$, $Z$, $T_{\text{eff}}$ and $V_{\text{rot}}$, which, although small, can cause the omission of some models.

It is worth to notice that in the case of F1 we have always consecutive radial orders whereas in the F2 solution we have every second radial order. This is because with increasing $V_{\text{rot}}$ the eigenvalue $\lambda$ of the $\ell = 1, m = 0$ mode increases and as a result of denser oscillation spectrum of the high radial order modes is shifted towards higher frequencies. This explains why we have more models from F1 than F2 for lower rotation rates and vice versa. On the other hand coexistence solutions from F1 and F2 for fixed rotational velocity (e.g., $V_{\text{rot}} = 200$ km s$^{-1}$) is associated with considerably different parameters of the models from both families of solutions (mainly metallicity and mass) as described in previous paragraph.

An interesting fact is that most unstable modes come from the F1 seismic models. The only exception is for $V_{\text{rot}} = 250$ km s$^{-1}$ where some modes from the F2 solution are unstable. With increasing $V_{\text{rot}}$ there are less and less seismic models with unstable modes fitting the two frequencies $\nu_1$ and $\nu_2$. On the one hand this is due to lower metallicity of the F2 models which dominate for the higher rotational velocities, on the other, for higher $V_{\text{rot}}$ the instability domain is shifted towards higher frequencies than the observed ones.

In the next step we selected models which fit the third observational frequency, $\nu_3$, which also corresponds to a dipole axisymmetric mode. Models fitting $\nu_1$, $\nu_2$ and $\nu_3$ are shown in Fig. 6. We can see a further reduction in the number of models. Moreover, we did not find a solution for each value of $V_{\text{rot}}$. The radial orders of modes which fit $\nu_1$, $\nu_2$ and $\nu_3$ from the G1 grid are given in the second column of Table 2. Unstable solutions exist only for $V_{\text{rot}} \leq 180$ km s$^{-1}$.

4.3 Effects of hydrogen abundance, core overshooting and opacities

There are many input parameters, both, from a model and microphysics, that may affect pulsational frequencies. Here, we examine the influence of the initial hydrogen abundance, $X_0$, overshooting from the convective core (in terms of the
parameter $\alpha_{\text{ov}}$) and the opacities. The effect of the core overshooting was included according to the formulation of Dziewonski & Paputynski (2003) which takes into account both, the distance of the overshooting and partial mixing in the overshoot layer.

To this end we constructed three additional grids of models in the same way as explained in Section 4.2. In the second grid, G2, we used $X_0 = 0.75$ (comparing to $X_0 = 0.70$ in G1), in the third grid, G3, we added the core overshooting, $\alpha_{\text{ov}} = 0.2$, and in the fourth one, G4, we used OPAL (Iglesias & Rogers 1996) opacity tables (instead of OP used in G1).

We found that independently of the adopted grid, all trends noted in G1 are also present in G2, G3 and G4. Firstly, there are always the two families of solutions. The trends noted in G1 are also present in G2, G3 and G4. (Iglesias & Rogers 1996) opacity tables (instead of OP used in G1).

With an increased abundance of hydrogen (the G2 grid), we found less seismic models compared to G1, in particular for the F2 solution.

Seismic models with and without overshooting from the convective core (G3 vs. G1) lie approximately in the same places of the HR diagram but the G3 models are confined to slightly narrower bands. A similar picture emerges when we considered seismic models only with unstable modes.

Changing the opacity tables (G4 vs. G1) has very little effect on the position of seismic models in the HR diagram. The G4 models are only slightly more luminous. The same is true if only unstable modes are considered. The exception is for seismic models with $V_{\text{rot}} = 240$ km $s^{-1}$. In this case, unstable modes occurs only in the F2 solution. This is opposite to seismic models calculated with the OP tables. There are fewer seismic models with unstable modes obtained with the use of the OPAL opacities. This result is not surprising as it is well known that for SPB models the computations with the OP tables give more unstable modes than those with the OPAL ones (e.g. Panutynski 1999).

Models which fit the three frequencies $\nu_1$, $\nu_2$ and $\nu_3$ in the grids G2, G3 and G4 are presented in Fig. 1. In Table 2 we list the allowed combinations of the radial orders.

As in the case of G1, only for the lower rotation velocity, $V_{\text{rot}} \leq 180$ km $s^{-1}$, we were able to find models which fit the three frequencies with the instability condition fulfilled. For $V_{\text{rot}} \geq 190$ km $s^{-1}$, the number of these seismic models is significantly lower and the modes are stable (cf. Table 2).

To summarize our results of seismic modelling of the star HD 21071, in Table 3 we give the ranges of the stellar parameters of models with unstable modes which fit the three frequencies, $\nu_1$, $\nu_2$ and $\nu_3$, for the four grids. The parameters are provided for each separate group of models with similar properties, shown in Figs 3a and 3b by ovals. The groups are marked as 'grp' in Table 3. In addition, we give also the parameters of representative models for each group (marked as 'rep' in Table 3). In the last column we give, for the representative seismic models, the goodness of the fit defined as

$$d = \frac{1}{3} \sum_{n=1}^{3} \frac{(\nu_n^o - \nu_n^t)^2}{\sigma_n^2}.$$

where $\nu_n^o$ and $\nu_n^t$ are observed and theoretical frequency, respectively, and $\sigma_n$ are the observational errors of the frequencies.

It should be mentioned that masses of seismic models from Table 3 are in the range $M \in (3.909, 4.661)$ $M_\odot$ whereas masses of evolutionary models which fall into the $3\sigma$ error box are in a slightly wider range $M \in (3.04, 4.99)$ $M_\odot$. The range of masses of evolutionary models given above is wider than the one given in Section 2 because here we take into account models with all considered metallicities ($Z \in (0.005, 0.025)$) and from all grids.

In Fig 3 we compare the oscillation spectrum of HD 21071 with the theoretical spectra corresponding to the representative seismic models from Table 3. In the bottom panel of Fig 3 we show a zoom-in on the frequencies around the observed values. As one can see, the model G2g reproduces also the frequency $\nu_4$ within the observational errors if it is the mode $\ell = 1$, $m = 0$, $g_{22}$, a possibility consistent with our mode identifications. Similarly, the quadrupole retrograde mode $\ell = 2$, $m = -2$, $g_{34}$ in the model G3i fits the frequency $\nu_4$. This mode is also allowed by our mode identifications.

We can also see that the frequency of the mode $\ell = 1$, $m = 0$, $g_{22}$ is close to $\nu_4$ in the models G1d and G4m. In a similar distance is also the mode, $\ell = 2$, $m = -2$, $g_{34}$ in the model G3k. Given the numerical accuracy of theoretical frequencies, which is about $\Delta \nu = 0.0001$ d$^{-1}$, we have to accept also these solutions. The important result is that these identifications are valid for all seismic models constructed in this paper.

5 CONCLUSIONS

This paper focused on the challenges and prospects for seismic modelling of slowly pulsating B-type stars with the effects of rotation taken into account. We used as an example the star HD 21071 which pulsates in four frequencies of which the three have a unique mode identification. Firstly, we showed that non-rotating models cannot account for the two highest amplitude frequencies, $\nu_1 = 1.18843$ and $\nu_2 = 1.14934$ d$^{-1}$, and including the effects of rotation on high-order g modes is indispensable. Then, having unambiguous determination of the two angular numbers ($\ell$, $m$) for the three observed frequencies of HD 21071 ($\nu_1$, $\nu_2$ and $\nu_3 = 1.41968$ d$^{-1}$), and constraints on the range of the rotational velocity, $V_{\text{rot}} \in (150, 250)$ km $s^{-1}$, we constructed rotating seismic models which reproduce these three frequencies. We examined the effects of the initial abundance of hydrogen, an amount of the core overshooting and the opacity data, considering four grids of parameters. Due to the high density of the theoretical oscillation spectra, a large number of solutions have been obtained. Despite of that, only two combinations of the radial orders, $n$, were allowed: one set is around $n = 15$ and the second one around $n = 30$. Moreover, the instability condition reduced the number of seismic models significantly. Therefore, accurate calculations of the opacity data are of the utmost importance.

Among seismic models fitting the three frequencies, we found some that reproduce also the forth frequency, $\nu_4 = 0.95706$ d$^{-1}$. In all grids of models only the dipole axisymmetric modes $g_{22}$ or quadrupole modes $\ell = 2$, $m = -2$, $g_{34}$, $g_{31}$ have frequencies close to $\nu_4$ (given the observa-
Figure 7. The same as in Fig. 6 but selected models from the grids G2, G3 and G4, are shown.
Figure 8. The frequencies of dipole axisymmetric modes in the representative models (see Table 3) which reproduce three well identified frequencies, $\nu_1$, $\nu_2$ and $\nu_3$, of HD 21071. A zoom-in on the frequencies around the observed values is shown in the bottom panel for clarity. In this panel the frequency of the quadrupole retrograde mode ($\ell = 2$, $m = -2$) is added as an inverted triangle. The vertical solid lines indicate the observed frequencies and the vertical dashed lines, the observational errors of the frequencies.
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Table 3. Ranges of the astrophysical parameters for the groups of seismic models (rows indicated by ‘grp’ in the sixth column) and the parameters of the selected models for each group (rows indicated by ‘rep’ in the sixth column). All models reproduce the three observed frequencies with the unstable dipole axisymmetric modes. Groups of models are defined as models with similar parameters forming

| Grid | opacity | $X_0$ | $\alpha_{ov}$ | $V_{rot}$ (km s$^{-1}$) | ID | Z | $M$ ($M_\odot$) | $\log T_{\text{eff}}$ | $\log L/L_\odot$ | d |
|------|---------|-------|---------------|-----------------|----|---|-------------|----------------|----------------|----|
| G1   | OP      | 0.70  | 0.0           | 150             | rep a | 0.0157 – 0.0160 | 4.031 – 4.039 | 4.1542 – 4.1559 | 2.452 – 2.458 | – |
| G1   | OP      | 0.70  | 0.0           | 160             | rep b | 0.0231 – 0.0236 | 4.372 – 4.388 | 4.1568 – 4.1597 | 2.479 – 2.489 | – |
| G1   | OP      | 0.70  | 0.0           | 160             | rep c | 0.0163 – 0.0164 | 4.218 – 4.232 | 4.1587 – 4.1611 | 2.541 – 2.549 | – |
| G1   | OP      | 0.70  | 0.0           | 180             | rep d | 0.0216 – 0.0222 | 4.351 – 4.364 | 4.1547 – 4.1574 | 2.503 – 2.513 | – |
| G1   | OP      | 0.70  | 0.0           | 180             | rep e | 0.0184 – 0.0188 | 4.647 – 4.661 | 4.1686 – 4.1712 | 2.524 – 2.534 | – |
| G1   | OP      | 0.70  | 0.0           | 160             | rep h | 0.0164 – 0.0165 | 3.909 – 3.919 | 4.1448 – 4.1467 | 2.409 – 2.415 | – |
| G1   | OP      | 0.70  | 0.0           | 160             | rep i | 0.0132 – 0.0135 | 4.035 – 4.045 | 4.1619 – 4.1639 | 2.523 – 2.531 | – |
| G1   | OP      | 0.70  | 0.0           | 160             | rep j | 0.0134 – 0.0135 | 4.0365         | 4.16247        | 2.5255 – 0.497 | – |
| G1   | OP      | 0.70  | 0.0           | 160             | rep k | 0.0203 – 0.0213 | 4.382 – 4.412 | 4.1675 – 4.1732 | 2.504 – 2.524 | – |
| G1   | OP      | 0.70  | 0.0           | 160             | rep l | 0.0204 – 0.0209 | 4.058 – 4.071 | 4.1496 – 4.1523 | 2.386 – 2.395 | – |
| G1   | OP      | 0.70  | 0.0           | 180             | rep m | 0.02174          | 4.3860         | 4.16833        | 2.5072 – 0.326 | – |

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APPENDIX A: ASYMMETRY OF THE ROTATIONALLY SPLIT MODES IN THE FRAMEWORK OF THE TRADITIONAL APPROXIMATION

Linear theory of pulsation predicts asymmetry between rotationally split components of the given mode, ie., retrograde modes should be closer to the axisymmetric ones than prograde modes. In Table A1 we list frequencies of dipole modes and the differences between their components for the model at the centre of the error box ($M = 3.69 M_\odot$, $\log T_{\text{eff}} = 4.164$, $\log L/L_\odot = 2.444$, $X_0 = 0.7$, $Z = 0.0082$, $R = 2.62 R_\odot$). The rotation splitting was computed with the traditional approximation and the rotational velocities were chosen to approximately reproduce the value of the observed splitting in HD 21071. As one can see, in all cases retrograde modes are closer to axisymmetric modes than prograde ones and asymmetry is of the order of 0.01 d$^{-1}$. This property is also clearly seen in Fig. A1, where the observed triplet is compared with the theoretical spectrum calculated for the model at the centre of error box of HD 21071. The observed frequencies were shifted in order to align the central peak, $\nu_1$, with the $g_{12}$ ($m = 0$) mode.
Table A1. Theoretical frequencies of dipole modes for several radial orders, \( n \), and the differences between their rotationally split components, for the central model of HD 21071 for the three values of the rotational velocity.

| \( V_{\text{rot}} \) (\( \text{km s}^{-1} \)) | \( n \) | \( \nu (m = -1) \) (\( \text{d}^{-1} \)) | \( \nu (m = 0) \) (\( \text{d}^{-1} \)) | \( \nu (m = 1) \) (\( \text{d}^{-1} \)) | \( \nu (m = 0) - \nu (m = -1) \) (\( \text{d}^{-1} \)) | \( \nu (m = 1) - \nu (m = 0) \) (\( \text{d}^{-1} \)) |
|---|---|---|---|---|---|---|
| 60 | 13 | 0.86855 | 1.08271 | 1.30635 | 0.21416 | 0.22304 |
|   | 12 | 0.93075 | 1.14612 | 1.37041 | 0.21537 | 0.22429 |
|   | 11 | 1.01204 | 1.22871 | 1.45373 | 0.21667 | 0.22502 |
|   | 10 | 1.11148 | 1.32946 | 1.55497 | 0.21798 | 0.22551 |
| 62 | 13 | 0.86067 | 1.08742 | 1.31828 | 0.22075 | 0.23086 |
|   | 12 | 0.92855 | 1.15066 | 1.38221 | 0.22211 | 0.23155 |
|   | 11 | 1.00942 | 1.23300 | 1.46530 | 0.22358 | 0.23230 |
|   | 10 | 1.10854 | 1.33333 | 1.56619 | 0.22479 | 0.23286 |
| 65 | 13 | 0.86408 | 1.09459 | 1.33623 | 0.23051 | 0.24164 |
|   | 12 | 0.92547 | 1.15755 | 1.39986 | 0.23208 | 0.24231 |
|   | 11 | 1.00584 | 1.23941 | 1.48263 | 0.23357 | 0.24322 |
|   | 10 | 1.10431 | 1.33940 | 1.58344 | 0.23509 | 0.24404 |

Figure A1. The theoretical frequencies (dashed lines) of dipole modes calculated with the traditional approximation for the model at the centre of the error box of HD 21071 with \( V_{\text{rot}} = 63 \) km s\(^{-1}\). The mode \( g_{12} \) is highlighted with a thin solid line. The observed triplet (thick solid lines) is shifted by \( -0.036 \text{ d}^{-1} \) to align its central frequency with the \( g_{12} (m = 0) \) mode.