Halo Intrinsic Alignment: Dependence on Mass, Formation Time, and Environment

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Abstract

In this paper we use high-resolution cosmological simulations to study halo intrinsic alignment and its dependence on mass, formation time, and large-scale environment. In agreement with previous studies using \textit{N}-body simulations, it is found that massive halos have stronger alignment. For the first time, we find that for a given halo mass older halos have stronger alignment and halos in cluster regions also have stronger alignment than those in filaments. To model these dependencies, we extend the linear alignment model with inclusion of halo bias and find that the halo alignment with its mass and formation time dependence can be explained by halo bias. However, the model cannot account for the environment dependence, as it is found that halo bias is lower in clusters and higher in filaments. Our results suggest that halo bias and environment are independent factors in determining halo alignment. We also study the halo alignment correlation function and find that halos are strongly clustered along their major axes and less clustered along the minor axes. The correlated halo alignment can extend to scales as large as 100 h\(^{-1}\) Mpc, where its feature is mainly driven by the baryon acoustic oscillation effect.

\textbf{Key words:} dark matter – galaxies: formation – galaxies: halos – large-scale structure of universe – methods: statistical

1. Introduction

Observational data from large sky surveys have clearly shown that galaxies are aligned with each other and also with the matter distribution on the large-scale structure (LSS). On galactic scales, the satellite galaxies are aligned with the major axis of the central galaxy (e.g., Sales & Lambas 2004; Brainerd 2005; Yang et al. 2005). On scales larger than a few \(h^{-1}\) Mpc, it is found that halo IA is also correlated (e.g., Pen et al. 2000; Lee 2011), as is the shape of galaxies (Brown et al. 2002; Heymans et al. 2004; Hirata et al. 2004; Faltenbacher et al. 2009; Okumura et al. 2009; Joachimi et al. 2011, 2013; Singh et al. 2015). The galaxy shape alignment can be extended to very large scale around 70–100 \(h^{-1}\) Mpc (Smargon et al. 2012; Li et al. 2013). Galaxies are also found to align with the cosmic web (e.g., Lee & Erdogdu 2007; Zhang et al. 2013), though with dependence on galaxy mass and morphology (Tempel & Libeskind 2013). For a recent review on the various kinds of galaxy alignment, we refer the readers to the papers by Joachimi et al. (2015) and Kang et al. (2017) (but in Chinese).

Among the above various galaxy alignments, the shape correlation between galaxies on large scales is of great importance, as it is a major contamination to weak-lensing surveys and precision cosmology requires a good understanding of galaxy alignment (e.g., Kirk et al. 2012). \textit{N}-body simulations have been extensively used to study the intrinsic alignment (IA) of dark matter halos since the beginning of this century (e.g., Croft & Metzler 2000; Heavens et al. 2000; Jing 2002). These studies found that halo IA is strong in massive halos, and a useful fitting formula is given in Jing (2002). To explain the observed galaxy IA at low redshifts, a misalignment between galaxy shape and halo has to be included (e.g., Kang et al. 2007; Faltenbacher et al. 2009; Okumura et al. 2009; Joachimi et al. 2013; Li et al. 2013). However, it is not clear how the misalignment between halo and galaxy shape should depend on galaxy properties. In recent years, cosmological hydrodynamical simulations have also been used to directly measure galaxy IA and its dependence on galaxy mass, luminosity, and redshift (e.g., Dubois et al. 2014; Tenny et al. 2015; Selliscig et al. 2015; Chisari et al. 2016; Hilbert et al. 2017). These studies have found in common that galaxy IA depends on luminosity and morphology. However, due to the difference of the adopted cosmology and the uncertainty of implemented star formation physics, the predicted galaxy IAs are still different in detail from the hydrodynamical simulations. For a review on the progress of measuring galaxy/halo IA using simulations, please refer to Kiessling et al. (2015).

For a better understanding of the physical origin of halo/galaxy IA, we need a theoretical model that can recover the halo/galaxy IA as seen in simulations. The linear alignment model (Hirata & Seljak 2004) is developed based on the tidal theory (e.g., Catelan et al. 2001), with the major assumption that halo/galaxy shape is determined by the local tidal field. Subsequently, the linear model is improved on nonlinear scales (Blazek et al. 2011), and a more comprehensive halo model is also developed (Schneider & Bridle 2010). However, in these analytical models the halo bias is neglected, and galaxy bias is presented only in the second order of power spectrum for galaxy IA. Thus, it is still not clear whether the analytical model can account for the simulated galaxy IA in detail, such as the dependence on galaxy formation time and environment.

As a further step to model the galaxy IA in detail, in this paper we use \textit{N}-body simulation to study halo IA, especially presenting the first measurement of halo IA with dependence on halo formation time and cosmic environment. We also investigate whether the linear model with inclusion of halo bias...
can explain the halo IA with dependence on formation time and cosmic environment.

Our paper is organized as follows. In Section 2 we introduce our simulation data, the method to clarify the LSS, and the modified analytical model to describe the halo IA. In Section 3 we present the simulation results on halo IA with dependence on halo mass, formation time, and large-scale environment, and we also show how our slightly modified linear tidal model can fit the mass and formation time dependence of halo IA. In Section 4 we present the results on the halo alignment correlation functions (ACFs) and investigate the origin behind the oscillation of halo IA on very large scales. In Section 5 we summarize our results and briefly discuss their implications.

2. Methodology

In this section, we introduce the simulation data, calculation of halo IA, and definition of the large-scale environment of the halo. We will also show the slightly modified linear tidal model to describe the halo IA in 3D coordinates measured from our simulations.

We use two cosmological N-body simulations in this paper. One is the Pangu simulation, carried out by the Computational Cosmology Consortium of China (Li et al. 2012; hereafter this simulation is abbreviated as C4), which simulated the evolution of the universe in a cubic box with each side of 1000 h⁻¹ Mpc. The other simulation is a part of the ELUCID project (Wang et al. 2014, 2016; Li et al. 2016) with box size 500 h⁻¹ Mpc (hereafter L500). Both simulations are run by the GADGET-2 code (Springel 2005) with 3072³ particles; the cosmological parameters can be found in Table 1. Note that the cosmological parameters in the two simulations are very close and we do not expect significant differences in our results, and we will also show comparison between them.

Using the friends-of-friends (FOF) algorithm with a linking length b = 0.2 of the mean particle separation, we identified dark matter halos in both simulations. For each halo, we project its dark matter particles along one axis of the simulation box and calculate the halo’s reduced 2D moment of inertia tensor $I_{ij}$ (Bailin & Steinmetz 2005) as

$$I_{ij} = \sum_k \frac{m_k x_k^i x_k^j}{s_k^2}, \text{ with } i, j \in \{1, 2\},$$

where $x_k$ is the distance of the $k$th particle to the halo center, which is set as the position of the most bound particle. The eigenvalues $\lambda_1, \lambda_2 (\lambda_1 > \lambda_2)$ of the inertia tensor then define the axis ratio $q = \lambda_2/\lambda_1$. Then, following Heavens et al. (2000) and Jing (2002), we define the halo shape by the ellipticity vector,

$$\epsilon = \frac{1 - q^2}{1 + q^2} \cos 2\alpha, \sin2\alpha^T = (\epsilon_1, \epsilon_2)^T,$$

which in the complex form reads

$$\epsilon = \epsilon_1 + i\epsilon_2 = \frac{1 - q^2}{1 + q^2} \exp(i2\alpha),$$

where $\alpha$ is the position angle of the halo measured anticlockwise from the x-axis.

As pointed out by earlier studies (Jing 2002; Bailin & Steinmetz 2005), a lower limit on particle number (∼300 particles) in the halo has to be used to get a converged shape measurement. To ensure a safer measurement, in this work we select halos with more than 500 particles. In addition, it is found that the halo shape is also dependent on the linking length to find halos in the FOF algorithm. Croft & Metzler (2000) had shown that this can lead to the discrepancy of halo IA at the low-mass end. We will later compare our results with those of Jing (2002).

To define the local large-scale environment of a halo, we use the Hessian matrix of the density field at the position of the halo (e.g., Aragón-Calvo et al. 2007). Adopting the cloud-in-cell (CIC) algorithm, the density field is smoothed via a length of 2 h⁻¹ Mpc, and the Hessian matrix of the smoothed density field $\rho$ at halo position is described as

$$H_{ij} = \frac{\partial^2 \rho}{\partial x_i \partial x_j}. \quad (4)$$

The eigenvalues of the Hessian matrix define the large-scale environment of the halo, and it can be classified into cluster, filament, sheet, and void, depending on the sign of the eigenvalues. By ordering the eigenvalue as $\lambda_1 > \lambda_2 > \lambda_3$, the environment is linked as

$$(-,-,-) \rightarrow \text{cluster}$$
$$(+,-,-) \rightarrow \text{filament}$$
$$(+,+,-) \rightarrow \text{sheet}$$
$$(+,+,:) \rightarrow \text{void}.$$

We note that this way of defining the LSS environment is different from the tidal environment defined in Hahn et al. (2007), in which the Hessian of the potential (i.e., tidal field) is used and classification is done using an opposite signature of eigenvalues. In Figure 1, we plot the halo mass function in

| Table 1: Simulation Parameters |
|--------------------------------|
| Name | $\Omega_m$ | $\Omega_k$ | $\Omega_b$ | $h$ | $L$ (h⁻¹ Mpc) | $m_p$ (h⁻¹ Mpc) |
|--------------------------------|
| L500 | 0.28 | 0.72 | 0.045 | 0.7 | 500 | $3.4 \times 10^8$ |
| C4 | 0.26 | 0.74 | 0.045 | 0.7 | 1000 | $2.5 \times 10^9$ |
Table 2

| LSS Environment Ratios (%) | Cluster | Filament | Sheet | Void |
|---------------------------|---------|---------|-------|------|
| Density Hessian           | 20.0359 | 61.8152 | 17.7563 | 0.3926 |
| Tidal field               | 19.3583 | 58.0268 | 21.1922 | 1.4227 |
| In total                  | Same    | Different | …… | …… |
|                           | 76.9173 | 23.082  | …… | …… |

cluster and filament environments from the two different classification schemes, and in Table 2 we list the ratio of each LSS environment under the two classification methods. It is seen from Figure 1 that the halo mass functions agree well in the two environments. By checking the environment of each halo, we find that 77% of all halos keep their environment unchanged and 23% of them changed when changing the classification method. For this paper, we will adopt the density-Hessian classification scheme. We will still be using the density Hessian as a “tidal field” in our context, but the reader should be aware of the mentioned difference when comparing with previous studies that adopt the tidal tensor. We also direct the reader to Libeskind et al. (2017) for detailed comparison and discussions on different LSS classification schemes.

Basically, the environment of the halo is related to its mass: usually massive halos live in cluster regions and low-mass halos live in sheets and voids. The halo shape is also closely related to the eigenvector of the environment; for a short summary of how halo shape is related to the environment and its physical origins, we refer the reader to Kang & Wang (2015) and Wang & Kang (2017) and references therein.

Now we describe how the halo IA is usually defined in the weak-lensing theory. Following the conventional description (Heavens et al. 2000; Jing 2002), for a pair of halos (halo 1 and halo 2) let \( w_1 = x_1 + i y_1 \) and \( w_2 = x_2 + i y_2 \) be projected positions of two halos in the complex plane, and call the direction defined by \( \varphi = \arg(w_2 - w_1) \) (anticlockwise from the positive real axis) the separation direction. Then, for this pair of halos we can define the tangential shear \( \gamma_t \) and the cross shear \( \gamma_x \),

\[
\gamma_t = -\text{Re} (\epsilon \exp(-2i\varphi)), \quad \gamma_x = -\text{Im} (\epsilon \exp(-2i\varphi)),
\]

where \( \epsilon \) is the complex shear of a given halo defined in Equation (3). The halo IA is usually defined as \( c_{11}(r) = \langle \gamma_t(0) \gamma_t(r) \rangle \), sometimes also labeled as \( \eta_t(r) \), and \( \langle \gamma_x(0) \gamma_x(r) \rangle \) is labeled as \( c_{22}(r) \).

By assuming that galaxy shapes are determined by their local tidal shear, the linear alignment model (Hirata & Seljak 2004) suggests that galaxy ellipticity is

\[
\gamma_{(a,x)} = -\frac{C_1}{4\pi G} (\nabla_x^2 - \nabla_y^2, 2\nabla_x \nabla_y) S[\Psi_p],
\]

where \( C_1 \) is a free parameter that can be determined by fitting the observed galaxy IA. Up to now, most studies have determined the value of \( C_1 \) by fitting the IA correlation of low-\( z \) massive galaxies (luminous red galaxies) as a whole. Hence, it is not clear how the normalization will vary as a function of galaxy luminosity. Also, as the fitting is for galaxy IA, it is not clear whether the tidal model (Equation (6)) can describe the IA of dark matter halo and its mass dependence.

To investigate the impact of galaxy IA on probing cosmology, it is important to measure the IA signal in the shear correlation function, which is defined as

\[
\xi_{\pm}(\theta) = \langle \gamma_{+}(\theta) \rangle \pm \langle \gamma_{\times}(\theta) \rangle,
\]

where \( \theta \) is the angular separation on the sky. Blazek et al. (2011), their Equation (3.5), showed that under the linear alignment model the projected tangential and cross components of the shear correlation function to the linear order are

\[
\langle \gamma_{+}(\theta) \rangle = \frac{1}{2\pi^2} \int \frac{C_{\kappa\rho,0}}{D(z)} \frac{\kappa^5}{k^5} \tilde{P}_B(k, z) \times \sin(k_z \theta_{\max}) \left[ J_0(k_r \sigma_p) \pm J_0(k_r \sigma_p) \right].
\]

However, for \( c_{11} \) in Jing (2002), the correlation of projected shapes is calculated at each 3D distance between halo pairs. We explicitly calculate our model prediction for \( c_{11} \) by first performing the integral

\[
\xi_{\pm}(r) = \int_0^\pi \xi_{\pm}(r_p = r \sin \phi, \Pi = r \cos \phi) \, d\phi.
\]

Then, after analytically integrating over azimuthal angles, we obtain the final formulae,

\[
\xi_{\pm}(r) = \left( \frac{C_{\kappa\rho,0}}{D(z)} \right) \frac{1}{4\pi} \int_0^\infty \frac{d\tau}{k} \frac{2\pi}{k^2} \frac{2\pi}{k^2} \left( \frac{3}{8} J_2^2(k r) \right)
\]

and

\[
\xi_{\times}(r) = \left( \frac{C_{\kappa\rho,0}}{D(z)} \right) \frac{1}{4\pi} \int_0^\infty \frac{d\tau}{k} \frac{2\pi}{k^2} \frac{2\pi}{k^2} \left( \frac{9}{280} J_2^2(k r) + \frac{4}{21} J_2^2(k r) + \frac{56}{105} J_2^2(k r) \right),
\]

where \( J_{\pm} \) is the Bessel function of half-integer order. Hence, by relation \( c_{11}(r) = \langle \epsilon_k^a \epsilon_{a'}^k(r) \rangle = \frac{1}{2} (\xi_{\pm}(r) + \xi_{\times}(r)) \), we now have the prediction of \( c_{11}(r) \) from the linear alignment theory.

Since we are calculating the shear correlation functions at halo positions, we will be using \( P_{B,\text{halo}} \) instead of \( P_B \) in the above equations. For halos at fixed mass, the power spectrum at halo position is \( P_{B,\text{halo}}(M) = b_h^2(M) P_B \), where \( b_h \) is the halo bias. The fact that we will be calculating the ellipticity correlation of halos within different mass ranges suggests that we should extend to \( P_{B,\text{halo}}(M_0 < M) \), which by conditioning on mass with \( M_0 > M \) and to linear order gives

\[
P_{B,\text{halo}}(M_0 > M) = b_h^2(M_0 > M) P_B.
\]

Therefore, we get the mass-dependent \( c_{11} \) for halos,

\[
c_{11}(r_0 > M) = \left( \int_M^{\infty} b_h^2(M') \Phi(M') dM' \right)^2 \left( \int_M^{\infty} \Phi(M') dM' \right) \langle \epsilon_{k_1}^a \epsilon_{a'}^k(r) \rangle,
\]

where \( \Phi(M) \) is the halo mass function that can be obtained for the given cosmology.

### 3. Dependencies of Halo IAS

#### 3.1. Mass Dependence of Halo IA

In Section 2 we have outlined the theoretical model for halo IA, and in this section we compare the simulation results to model predictions to see whether the linear model can fit the mass dependence of halo IA.

First we note that, as pointed out by Jing (2002), when calculating \( c_{11} \) from Equation (5), not only is the direction of...
the halo shape correlated, but the ellipticity of the halo is also taken into account (the term \((1 - q^2)/(1 + q^2)\)), as is the theoretical description given by Equation (6). So basically that is the ellipticity weighted \(c_{11}\). In fact, we can also only consider the orientation correlation by assuming that \(q = 0\) (unweighted by ellipticity) in the simulations. In Figure 2 we show the measured weighted and unweighted \(c_{11}\) for halos larger than the given mass in the left and right panels. The circles are from L500 simulation, and diamonds are from C4. For clarity, the results for different halo mass bins have been shifted by different factors. It is found that the results from the two simulations agree quite well for both weighted and unweighted correlations. For the following we will only show results from the L500 simulation unless otherwise stated.

Jing (2002) measured the weighted \(c_{11}\) from their simulation and found that it can be well fitted by a simple formula with mass dependence as

\[
c_{11}^{YPI}(\geq M_h; r) = 3.6 \times 10^{-2} \left(\frac{M_h}{10^{10} h^{-1} M_{\odot}}\right)^{0.5} r^{0.4}(7.5^{1.7} + r^{1.7}).
\]

We plot their fitting as dashed lines in Figure 2. It is worth noting that their fitting formula was intended for FOF halos with a linking length with \(b = 0.1\). As pointed out in Jing (2002), the mass of FOF halos using \(b = 0.1\) is about half of the FOF halo mass with \(b = 0.2\). Therefore, the dashed lines here are plotted using the fitting formula, but with half of the halo mass in our simulation (with \(b = 0.2\)). It is found that the fitting formula can also well fit our results, except at the low-mass end, where our \(c_{11}\) is lower than that of \(c_{11}^{YPI}\). This indicates that \(c_{11}\) in our simulation has a slightly stronger mass dependence. As shown by Croft & Metzler (2000), such a slight difference is expected, as the measured halo shape is slightly different with different link lengths, especially for low-mass halos. In addition, given the additional difference in cosmological parameter, simulation box, and resolution, we believe that such a slight difference is acceptable. The right panel shows that, without considering the halo ellipticity, our \(c_{11}\) agrees with the fitting formula much better. We also note that for the unweighted \(c_{11}\) the fitting formula is multiplied by a factor of \(\sim 6\) to produce a good match with the data, similar to the finding of Okumura et al. (2009). The fact that the fitting formula at \(q = 0\) works well also confirms that the difference seen in the left panel for low-mass halos mainly comes from randomness of halo ellipticities at low mass.

In Figure 3, we compare our simulation results to the theoretical model. The left panel is the same as that of Figure 2, but now the dashed lines are the model predictions (Equation (12)). For the theoretical model we adopted the halo bias from Pujol & Gaztañaga (2014), and the halo mass function \(\Phi(M)\) is taken from our simulation. For the power spectrum we used the nonlinear power spectrum from halofit (Peacock & Smith 2000), adhering to the L500 cosmological parameters. In addition, to better match the real power spectrum in simulation, we added the CIC window function (Cui et al. 2008) with a \(2 h^{-1}\) Mpc smoothing window in the computation. We tune the free model parameters for weighted and unweighted \(c_{11}\), respectively, to better match the simulation results for the highest-mass bin (upper line). It is seen that the simulation results can be well matched by the theoretical predictions, indicating that it is mainly the halo bias accounting for the mass dependence of halo IA, \(c_{11}\). Again, we find that the mass dependence in the simulation is slightly stronger than in the linear tidal model. The right panel shows a better agreement on the unweighted \(c_{11}\) with the model. The better agreement on the unweighted \(c_{11}\) indicates that the correlation between direction of the halo (major axis) is better described than the ellipticity by the linear tidal model.

In the right panel of Figure 3 we plot the \(c_{11}\) at \(z = 1\) as an additional test of the linear tidal model. Amazingly, the agreement at higher redshift is also good, and even better than the \(z = 0\) results. This is also expected, as it is known that the tidal field at earlier time can be better described by the linear theory. Following Equation (B.5) in Joachimi et al. (2011), we also note that the ratio between the free parameter in our unweighted model \(\frac{C(z = 1)_{\nu_{IA}}}{D(z = 1)}\) and the ratio \(\frac{C(z = 0)_{\nu_{IA}}}{D(z = 0)}\) is about 1.40 is slightly lower than the ratio from the linear growth factor \(\frac{D(z = 0)}{D(z = 1)} \approx 1.61\). This indicates that the redshift evolution can be roughly captured by the power spectrum at different redshifts, yet there are still other factors that are absorbed in the normalization factor \(C_{11}\), which should slightly depend on redshift.

### 3.2. Formation Time and Environment Dependence

It is well known that halo properties, such as concentration and bias, are closely related to halo formation time (e.g., Navarro et al. 1997; Gao et al. 2004). In this section we investigate whether the halo IA is also dependent on halo formation time. Here we use the most common definition of formation time, \(z_f\), at which the mass assembled in the main progenitor of a halo is half of its present \((z = 0)\) mass, i.e.,...
As there is a strong correlation between halo formation time and its mass (e.g., Navarro et al. 1997), we select halos in a few narrow mass bins and divide the 20% oldest and youngest halos into the old and young samples. In addition, as we will soon see that halo IA is also dependent on their environment, we refine the old and young halo samples to make sure they have the same distributions of cosmic environment as identified in Section 2.

In Figure 4, we plot the unweighted $c_{11}$ signal for old and young halos in a few mass bins. It is seen that in low-mass bins ($\log M < 12.5$) old halos have stronger alignment than young halos, and at intermediate scales $\sim 10 \text{ Mpc} h^{-1}$, the IA of old halos is around 2 times that of the young halos. For massive halos (bottom right panel) the difference between the two samples is small. This trend can be quantitatively explained by our tidal alignment model (Equation (12)) with inclusion of halo bias. These differences of halo IA in old and young halos are in good agreement with the dependence of halo bias on formation time as found by Gao et al. (2004).

Previously, to explain the mass dependence of halo IA, Smith & Watts (2005) argued that massive halos are more strongly aligned as they are formed later, so they have less time to virialize and still keep the memory of the tidal field that they collapsed within. Our results above show that halo IA is dependent on both halo mass and the formation time, and it can be understood using our linear model in terms of halo bias with its dependence on mass and formation time separately.

Many studies have shown that halo spins are correlated with the large-scale environment (e.g., Hahn et al. 2007; Kang & Wang 2015, and references therein). There is also a strong possibility that halo IA will also depend on the large-scale environment. In this part we compare the halo IA in different environments, mainly in clusters and filaments. We select question halos with mass $>10^{12} h^{-1} M_{\odot}$ and label those in cluster and filament environments as the pure $Q_{\text{clus}}$ and pure $Q_{\text{fil}}$ sample, respectively. As halo mass is strongly correlated with its environment (e.g., Hahn et al. 2007), halos in the cluster environment are usually larger; therefore, direct comparison of $c_{11}$ between these two samples will suffer from a significant mass-dependence effect as seen in the previous section. Hence, we construct two control samples, $Q_{\text{clus}}^{\text{control}}$ and $Q_{\text{fil}}^{\text{control}}$, ensuring that they have the same halo mass distribution as the pure cluster/filament samples, but regardless of their environment.

After eliminating the halo mass dependence, to see whether there is additional difference in the formation time between the

Figure 3. Ellipticity correlation function $c_{11}$ of halos at two redshifts. Left: correlation at $z = 0$. Similar to Figure 2, but the dashed lines are the theoretical model predictions. Right panel: correlation at $z = 1$.

Figure 4. Unweighted halo ellipticity correlation function $c_{11}$ ($q = 0$) at $z = 0$ for old and young halos in a few mass bins. The difference between their correlation is decreasing with halo mass, consistent with the age dependence of halo bias as a function as halo mass.
question and control samples, we plot their formation time distributions in Figure 5. By comparing the solid and dashed lines, it is seen that under the same mass distribution the formation time is identical between the question and control samples. Previous work (e.g., Hahn et al. 2007) has found that for low-mass halos \(< 10^{12} \, h^{-1} M_{\odot}\) the formation time is dependent on the environment, in which cluster halos form earlier than those in filaments and voids, but for massive halos there is no environmental dependence. As we are looking at halos larger than \(10^{12} \, h^{-1} M_{\odot}\), our results agree well with their results. The formation time of halos in the cluster environment is slightly lower than for halos in filaments; this is because the average halo mass in clusters is larger, so the formation time is lower.

In Figure 6, we plot \(c_{11}\) for the pure cluster/filament samples in solid lines and the control samples in dashed lines. The error bars are taken from 10 realizations of the control samples. We find that under the same mass distribution \(c_{11}\) from halos in the cluster environment (red solid line) is much stronger than the control sample, which has a mixture of different environments (red dashed line). On the other hand, the \(c_{11}\) for halos in filaments (blue solid line) is much weaker than that in the control samples (blue dashed line). The difference between solid lines includes both mass and environment dependence. The fact that \(c_{11}\) for \(Q_{\text{clu}}\) is higher than that of \(Q_{\text{fil}}\) is mainly due to the mass dependence.

In order to understand the origin of environment dependence of halo IA and to see whether it can be explained by halo bias as introduced in our theoretical model in Section 2, here we investigate the halo bias in different environments. We calculate the cross-correlation function \(\xi_{\text{hm}}\) between the halos and the dark matter particles. For reference we also calculate the autocorrelation function \(\xi_{\text{mm}}\) of the background particles.

In Figure 7, we show the cross-correlations of halos in different environments with the background particles. First, we note that, below the scale \(\sim 2 \, h^{-1} \text{Mpc}\), the halo–matter cross-correlation function mainly describes the clustering of dark matter particles inside halos. Therefore, this part of the correlation function mainly depends on the average halo mass in the studied sample. Since the question and control samples have the same mass distribution, the clusterings on small scales are very similar between them.

On scales beyond \(2 \, h^{-1} \text{Mpc}\), the cross-correlation function then reflects the level of clustering between particles in...
different halos. It is found that halos in cluster environments (black dashed line) are less clustered than halos in filament environments (black dotted line). The control samples have similar clustering to the corresponding question samples on small scales, but on large scales they are similar to the background dark matter particles, and the control cluster sample (red dashed line) has slightly higher correlations than the control filament sample (blue dashed line), due to their higher average halo mass.

The higher clustering on large scales for halos in filaments than for halos in clusters is unexpected. We note that it is not due to the formation time difference between the two samples, as Figure 5 shows that their formation time distributions are very similar. Interestingly, in a recent paper by Borzyszkowski et al. (2017) from the ZOMG project, the authors found that stalled (filament) halos have a larger halo bias $b_h > 1$ than the accreting (cluster) halos with $b_h < 1$. The environment dependence is also recently confirmed by Yang et al. (2017), who study the halo bias in much more detail using the ELUCID simulations, and they also find that cluster halos are less clustered on larger scales. More recently, Paranjape et al. (2017) have found some very interesting results. Although they used a different method to define the large-scale environment (the anisotropy factor $\alpha_R$), they found that halo bias evolves smoothly with $\alpha_R$, and in particular, halos in isotropic (node) environments have a negative bias while those in anisotropic (filament) environments have a positive bias (see their Figure 12). This is qualitatively in agreement with our results in Figure 7. In general, our results agree with the recent findings.

The fact that halo bias is higher in filaments and lower in clusters is opposite to the trend of environment dependence seen in Figure 6, and therefore we cannot use the halo bias term in our theoretical model to explain the environment dependence in halo IA. These two opposite trends for large-scale environment dependence also suggest that additional factors other than halo bias must be taken into account to understand this difference. In particular, these factors need to produce a significant difference in halo IA between cluster and filament to overcome any halo bias effect and result in the final $c_{11}$ that we see in Figure 6.

One of the potential factors is the level of alignment between halo orientation and the large-scale environment. The basic assumption in the linear alignment model (Hirata & Seljak 2004) is that halo shape is perfectly aligned with the tidal field. As reported in Zhang et al. (2015) and Kang & Wang (2015), although the halo shape is strongly correlated with the local tidal field, there is still a misalignment with dependence on halo mass and environment. Here we calculate the alignment between the major axis ($v_1$) of the halo shape and the direction of the tidal field, i.e., $v_3$ from the Hessian matrix, which is the direction of the slowest collapse direction (for detail, see Kang & Wang 2015). We calculate their alignment as

$$v_3 \cdot v_1 = \cos \theta,$$

where larger $|\cos \theta|$ means that the halo major axis is better aligned with the tidal field. In Figure 8 we plot the probability distribution of $|\cos \theta|$ for halos in three mass bins $M_{\text{halo}} > 10^{12}, 10^{13}, 10^{13.5} \, M_{\odot}$ and in both filament and cluster environments. It is found that halos in clusters have a stronger alignment with the local tidal field in all mass ranges. This result, given that large-scale tidal field correlation underlies the large-scale IA (Catelan & Porciani 2001), provides a plausible reason why the halo IA is stronger in clusters and weaker in filaments. We believe that this effect should be taken into account to fully explain the environment dependence of halo IA. In this paper we do not study this effect in detail.

4. The ACF

In Section 3, we have shown the dependence of halo IA and comparison between simulation results and theoretical model predictions. There the halo IA is defined in a way similar to that often used in weak-lensing analyses. In fact, the halo alignment has been measured in another way, called ACF, in which the halo clustering is a function of the direction along one halo axis (e.g., Faltenbacher et al. 2009). Although the ACF and halo IA can be connected (especially the GI term; Blazek et al. 2011), the ACF provides more direct insight on how halos/galaxies are correlated along their major or minor axes. An illustration of the ACF statistic is shown in Figure 9, where $\theta_p$ is the angle between the major axis of a halo and the separation vector to another halo.

Given the sample of halos in question (sample $Q$), which is a subset of the reference sample $G$, the ACF $\xi(\theta_p, r)$ is defined as (Faltenbacher et al. 2009)

$$\xi(\theta_p, r) = \frac{N_{QG}(\theta_p, r)}{N_{QG}N_{QR}(\theta_p, r)} - 1,$$

where $R$ is a random sample.

The halo and galaxy ACFs have been measured in a few studies (Faltenbacher et al. 2009; Smargon et al. 2012; Chisari et al. 2016, 2015). Li et al. (2013) showed the direct measurement of the ACF using the CMASS galaxy catalog and in the meantime calculated the ACF for dark matter halos using cosmological simulations. They found evidence that galaxies and halos are more clustered than the average within a spanned region $\theta_p = (0^\circ, 30^\circ)$ around the major axis and less clustered within $\theta_p = (60^\circ, 90^\circ)$ at almost all scales. Furthermore, they found that the correlation can be seen up to very large scales at around $70 \, h^{-1} \, \text{Mpc}$. The results of Li et al. (2013) motivate us to study halo ACFs with their dependence on environment and investigate the origin of the correlation on very large scales.

Here we present an investigation on the environment dependence of halo ACFs. In order to look for signals on very large scales, we use the full halo sample in the C4 simulation with a box size of $1 \, h^{-1} \, \text{Gpc}$ at $z = 0.6$ (to match the Li et al. 2013 data from observations) as the reference sample $G$ ($N_G = 12,640,839$), and we use the set of halos with mass $M_h > 10^{12} \, h^{-1} \, M_\odot$ as sample $Q$. The random sample $R$ is generated with $N_R/N_Q = 10$. We divide halos in sample $Q$ into two samples, $Q_{\text{clus}}$ and $Q_{\text{str}}$, based on their environment identified as cluster and filament. In the top panel of Figure 10 we plot the ACFs for these samples.

Figure 10 shows a few interesting features. First, it is found that without limitation on the alignment angle $\theta_p$, the halo ACF in clusters (red solid line) is lower than that in filaments (blue solid line). This is consistent with the results in Figure 7, although there we plot the cross-correlation between halo and dark matter and here we show the halo–halo cross-correlation. The halo ACF also displays a peak at around $110 \, h^{-1} \, \text{Mpc}$, which is exactly the signal from the baryon acoustic oscillations (BAOs). Second, it is found that, in agreement with Li et al. (2013), the halo ACF with $0^\circ < \theta_p < 30^\circ$ (dashed
lines) is higher in both filament and cluster environments than the average, indicating that halo clustering is enhanced along their major axes. Accordingly, the clustering is decreased with \( 60^\circ < \theta_p < 90^\circ \). An illustration of this effect is seen in the right panel of Figure 9.

In the bottom panel of Figure 10 we show the ratio between halo ACFs along different alignment angles with the average ACFs in the cluster and filament environments, respectively. The dashed lines are for halos clustering along the major axes (\( 0^\circ < \theta < 30^\circ \)), and the dotted lines are for results along the minor axes of the halos (\( 60^\circ < \theta_p < 90^\circ \)). It is seen that the halo ACF is lower in clusters, but the correlation along the major axis is significantly more enhanced in clusters than in filaments. It is also interestingly found that the clustering along the minor axes will become negative at some scales with dependence on halo environment. For example, for halos in clusters the ACF is negative at \( r > 35 \ h^{-1} \) Mpc and reaches the lowest dip at around \( 90 \ h^{-1} \) Mpc, after which it increases to the peak at around \( 110 \ h^{-1} \) Mpc. For halos in filaments, the clustering along minor axes becomes negative at around \( 70 \ h^{-1} \) Mpc and reaches a dip at around \( 90 \ h^{-1} \) Mpc. These results indicate that halos in clustering environments have a strong dependence on the alignment angle.

In addition to the halo ACF, we also calculate the alignment signal \( \cos(2\theta_p) \) over correlated pairs, which is defined in Faltenbacher et al. (2009) as

\[
\langle \cos(2\theta_p) \rangle_{\text{cor}}(r) = \frac{\int_0^{\pi/2} \cos(2\theta_p) \xi(\theta_p, r)d\theta_p}{\int_0^{\pi/2} \xi(\theta_p, r)d\theta_p}
\]

(16)

and is estimated by

\[
\langle \cos(2\theta_p) \rangle_{\text{cor}}(r) = \frac{QG_{\delta_h}(r)}{QG(r) - (N_G/N_{\delta_h}) \times QR(r)}.
\]

(17)

In the left panel of Figure 11, we show the calculated \( \langle \cos(2\theta_p) \rangle_{\text{cor}} \) for the above samples, together with the CMASS measurement from Li et al. (2013). It is seen that for all halo samples (black line), the ACF is larger than the observed galaxy ACF (data points). This is not surprising, as Li et al. (2013) have obtained similar results and they also showed that a misalignment between galaxy orientation and halo orientation has to be included so as to dilute the halo ACF to agree with the data. As here we are focusing on the environmental dependence, we do not show such a test.

What is interesting in the left panel of Figure 11 is that the ACF is stronger in cluster environments than in filament environments and both have oscillations with a peak around \( 90 \ h^{-1} \) Mpc. This environment dependence seems to be inconsistent with the above result that halo ACF is stronger in filament environments. In order to understand the physical origin of the oscillation of the \( \langle \cos(2\theta_p) \rangle_{\text{cor}} \) on large scales and the environment dependence in more detail, we take a further look at the \( \langle \cos(2\theta_p) \rangle_{\text{cor}} \) statistic and find that the estimator in Equation (16) can be decomposed into two parts. By recognizing the two-point cross-correlation function as

\[
\xi = \frac{N_G QG - N_{\delta_h} QG_{\delta_h}}{N_{\delta_h} QG_{\delta_h}} - 1,
\]

the estimator of the \( \langle \cos(2\theta_p) \rangle_{\text{cor}} \) statistic is equivalent to

\[
\langle \cos(2\theta_p) \rangle_{\text{cor}}(r) = \frac{QG_{\delta_h} \xi + 1}{QG} = \langle \cos(2\theta_p) \rangle_{\text{avg}} \left( 1 + \frac{1}{\xi} \right),
\]

(18)

where \( \langle \cos(2\theta_p) \rangle_{\text{avg}} \) is the mean value of \( \cos(2\theta_p) \) over all halo pairs instead.

The above equation shows that the correlated halo alignment \( \cos(2\theta_p)_{\text{cor}} \) is determined by two components: the halo correlation and the average halo alignment angle in all pairs. In regions where the halo correlation \( \xi \) is very small (\( \xi \ll 1 \)), the correlated alignment signal \( \cos(2\theta_p)_{\text{cor}} \) is the average alignment \( \cos(2\theta_p)_{\text{avg}} \) divided by the halo correlation. We have shown one component, the clustering for halos \( \xi(r) \), in filaments and clusters in Figure 10, and we plot the other component, average halo alignment angle, for halos in different environments in the right panel of Figure 11.

The right panel shows that the average alignment angles of halos are very similar between filament and cluster environments, and they decrease with halo separation. Since the clustering of halos in cluster environments is much lower than that of filaments (red and blue solid lines in Figure 10) and \( \cos(2\theta_p)_{\text{cor}} \) is proportional to \( 1/\xi(r) \), this leads to a higher correlated halo alignment angle in clusters. The oscillation in the clustering at \( 90 \ h^{-1} \) Mpc \( < r < 120 \ h^{-1} \) Mpc due to the BAO effect also explains the peak and dip of halo correlation.
alignment seen in the left panel of Figure 11. It is also interesting to note that the correlated halo alignment \( \cos(2\theta_p)_{\text{cor}} \) in clusters is higher than 1 at \( r \sim 90 \, h^{-1} \) Mpc. This seems to be impossible from the definition in Equation (16), which implies that the statistic should range between -1 and 1. However, it is true only when the halo ACF (\( \xi(\theta_p, r) \)) in Equation (16) is always positive. We have seen from Figure 10 that the halo ACF is negative for \( 60^\circ < \theta_p < 90^\circ \) at some distances, so the product of two negative components, \( \xi(\theta_p, r) \) and \( \cos(2\theta_p) \), leads to a positive value and results in a final \( \cos(2\theta_p)_{\text{cor}} \) being larger than 1 at some distances.

Finally, we note the seeming inconsistency between the environmental dependence of \( c_{11} \) (higher in clusters) and the ACF (higher in filaments). The ACF is the clustering along the halo major axis, so the ACF is mainly dominated by halo clustering. In Figure 7 we have shown that halo clustering is higher in filaments, so the ACF is also higher in filaments than in clusters. On the other hand, \( c_{11} \) (unweighted by halo ellipticity) measures the average of the product of \( \cos(2\theta_p) \) between halo pairs, and it can be written as

\[
c_{11} = \langle \cos(2\theta_p) \cos(2\beta) \rangle
\]

\[
= \int_0^{\sigma/2} \int_0^{\sigma/2} \cos(2\theta_p) \cos(2\beta) \frac{\xi(\theta_p, \beta, r)}{\xi(r)} d\theta_p d\beta
\]

\[
= \int_0^{\sigma/2} \int_0^{\sigma/2} \cos(2\theta_p) \cos(2\beta) \frac{1 + \xi(\theta_p, \beta, r)}{\xi(r)} d\theta_p d\beta,
\]

where the definition of \( \theta_p, \beta \) can be found from the halo configuration in the left panel of Figure 9. It is seen that \( c_{11} \) arises from two terms. The term \( (1 + \xi(\theta_p, \beta, r))/(1 + \xi(r)) \) is the enhancement of clustering along halo major axis. We have shown in Figure 8 and the bottom panel of Figure 10 that in cluster environments halo major axis is better aligned with the tidal field and the clustering is significantly more enhanced along the halo major axis than in filaments. We also found that the term \( \cos(2\theta_p) \cos(2\beta) \) is higher for halo pairs in clusters than for those in filaments. Therefore, the environmental dependence of \( c_{11} \) and ACF enhancement are consistent with each other, as both can be explained by the alignment between halo major axis and tidal field.

5. Summary and Conclusion

IA of galaxies is a result of complicated physics of galaxy formation and the tidal interaction of dark matter halos with the LSS. Galaxy IA has demonstrated its importance as a challenge to weak lensing in the era of precision cosmology. Understanding the dependence of galaxy IA on galaxy properties is a key step to modeling its effect on the cosmic shear and other useful statistics in the weak-lensing survey. As a first step, it is important to understand the halo IA with its dependence on mass, formation time, and large-scale environment. In this paper we make use of two \( N \)-body simulations to investigate these dependences, and the main results are summarized as follows.

1. IA of dark matter halos has long been found to be higher for massive halos in \( N \)-body simulations (Jing 2002). However, the halo alignment and its mass dependence are often neglected in the linear model for galaxy alignment, and it is not clear whether the linear tidal model (e.g., Hirata & Seljak 2004) can describe the halo IA and its...
mass dependence. By modifying the linear alignment to its 3D form and including the halo bias term, our slightly modified model is able to predict the mass dependence for halo IA in simulations. The redshift dependence can also be closely followed using the nonlinear power spectrum at different redshifts.

2. We find that for a given mass older halos have stronger alignment than younger halos. The trend in halo formation time can be well captured by our modified linear alignment model, and the difference in formation is consistent with that of halo bias found by Gao et al. (2004). Our results confirm that halo formation time is an independent factor in determining the IA.

3. We have measured the halo ACF from the C4 simulation. In agreement with previous results (e.g., Faltenbacher et al. 2009; Li et al. 2013), we find that the halo clustering is significantly enhanced along the halo major axes and is less clustered along the minor axes.

4. For halos in different large-scale environments, the $c_{11}$ is found to be higher in clusters and lower in filaments. A further look into the dark matter halo cross-correlation function reveals an opposite trend that halo bias is stronger in filaments than in clusters. Such a bias with dependence on environment is consistent with recent findings (e.g., Paranjape et al. 2017; Yang et al. 2017). This opposite trend in halo bias can explain the environment dependence in ACF, but not the $c_{11}$. We find that the enhancement of ACF in clusters is much stronger than in filaments, which can explain the environmental dependence in $c_{11}$. The enhancement of ACF along the halo major axes is due to the alignment between halo shape and the tidal field, and this effect is stronger in clusters. Our results suggest that, besides the halo bias, the linear tidal model for halo alignment should also include a factor to describe the correspondence between halo shape and the local tidal field, as well as its environmental dependence.

5. Furthermore, we have found that the alignment angle between correlated halo pairs has a peak at around $90 \ h^{-1} \text{Mpc}$ and a dip at around $110 \ h^{-1} \text{Mpc}$. We pointed out that such an oscillation is caused by the BAO.

Finally, we note that the study of halo IA still needs to be extended in future work to model galaxy IA in order to shed more light on weak-lensing observations. In the language of the halo model, this paper mainly focused on the two-halo term and therefore does not include any recipe for the one-halo alignment for satellite galaxies. For a detailed discussion of the effect of satellite alignment on observed lensing signal, we refer readers to C. Wei et al. (2017, in preparation). On the other hand, the formation time dependence on halo IA can be an important feature to look for in galaxy surveys. Observations have found the ACF signal using CMASS galaxies, and the environment dependence on ACF can be tested using galaxy catalogs with LSS classification, e.g., GAMA. In general, the above dependencies in alignment statistics can be extended to galaxy IA statistics, which would be useful for future development in galaxy IA both theoretically and observationally.

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