A GEVREY CLASS SEMIGROUP FOR A THERMOELASTIC PLATE MODEL WITH A FRACTIONAL LAPLACIAN: BETWEEN THE EULER-BERNOULLI AND KIRCHHOFF MODELS

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Abstract. In a bounded domain, we consider a thermoelastic plate with rotational forces. The rotational forces involve the spectral fractional Laplacian, with power parameter $0 \leq \theta \leq 1$. The model includes both the Euler-Bernoulli ($\theta = 0$) and Kirchhoff ($\theta = 1$) models for thermoelastic plate as special cases. First, we show that the underlying semigroup is of Gevrey class $\delta$ for every $\delta > (2 - \theta)/(2 - 4\theta)$ for both the clamped and hinged boundary conditions when the parameter $\theta$ lies in the interval $(0, 1/2)$. Then, we show that the semigroup is exponentially stable for hinged boundary conditions, for all values of $\theta$ in $[0, 1]$. Finally, we prove, by constructing a counterexample, that, under hinged boundary conditions, the semigroup is not analytic, for all $\theta$ in the interval $(0, 1]$. The main features of our Gevrey class proof are: the frequency domain method, appropriate decompositions of the components of the system and the use of Lions’ interpolation inequalities.

1. Problem formulation and statement of the main results. Let $\Omega$ be a bounded smooth (e.g. with boundary of class $C^4$) open connected subset of $\mathbb{R}^N$, $N \geq 1$. Let $\alpha, \beta, \kappa$ be positive constants. Let $\nu$ denote the unit outward normal vector to the boundary $\Gamma$ of $\Omega$. Let $\theta \in [0, 1]$, and consider the thermoelastic plate equation involving the spectral fractional Laplacian (that is, the fractional powers $2010$ Mathematics Subject Classification. 35Q74, 35B65, 47D03, 74K20, 93D20.

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of the realization of $-\Delta$ in $L^2(\Omega)$ with the zero Dirichlet boundary condition. See e.g. [1] and the references therein for the precise definition):

$$\begin{aligned}
B_\theta y_{tt} + \Delta^2 y + \alpha \Delta z &= 0 \quad \text{in } \Omega \times (0, \infty), \\
z_t - \kappa \Delta z - \beta \Delta y_t &= 0 \quad \text{in } \Omega \times (0, \infty), \\
y(0) = y^0 \in V, \quad y_t(0) = y^1 \in H_\theta, \quad z(0) = z^0 \in H,
\end{aligned}$$

(1.1)

where $B_\theta u = u + (-\Delta)^\theta u$, with the boundary conditions:

- **(Hinged plate/Dirichlet temperature):**
  $$z = 0, \quad y = 0, \quad \Delta y = 0 \text{ on } \Sigma = \partial \Omega \times (0, \infty),$$

or, else

- **(Clamped plate/Dirichlet temperature):**
  $$z = 0, \quad y = 0, \quad \partial_\nu y = 0 \text{ on } \Sigma = \partial \Omega \times (0, \infty).$$

System (1.1) represents the Euler-Bernoulli model for a thermoelastic plate when $\theta = 0$, while it stands for the Kirchhoff model for a thermoelastic plate when $\theta = 1$. Physically relevant models of thin plates correspond to the space dimension $N = 2$. For the Euler-Bernoulli model, it is well known that the underlying semigroup is both analytic and exponentially stable. However, for the Kirchhoff model, only the exponential stability of the semigroup is true; Lasiecka and Triggiani showed that, not only is the semigroup not analytic, it is not even differentiable [13, 15]. As far as exponential stability is concerned, the first result was proved by Kim in [8], where, for the first time, in the case of an Euler-Bernoulli clamped plate, it was shown that the exponential stability of the semigroup could be achieved without the need of additional mechanical damping. Earlier, Lagnese, [10] in his seminal book on the boundary stabilization of thin plate, had added mechanical boundary damping in the case of a Kirchhoff model with free boundary conditions in order to prove the exponential stability of the semigroup. Later on, many other works followed, establishing the exponential stability of thermoelastic plates (both models) with various boundary conditions e.g. [2, 3, 20, 23, 27]. As for the analyticity of the semigroup for the Euler-Bernoulli model, the first result was established by Liu and Renardy, [19] in the case of hinged and clamped boundary conditions. Subsequently, Liu and Liu, [18], and Lasiecka and Triggiani [11, 12, 13, 14] proved other analyticity results under various boundary conditions. More recently, Tébou [27] proved uniform analyticity of the semigroup for the Euler-Bernoulli thermoelastic plate with perturbed boundary conditions, and Dell’Oro et al analyzed an abstract Kirchhoff/Euler-Bernoulli thermoelastic model where the coupling involves the spectral fractional Laplacian; they proved exponential and polynomial stability results, but no regularity result.

Thus, on the one hand, the Euler-Bernoulli thermoelastic plate, ($\theta = 0$), is analytic, and on the other hand, it is known that when $\theta = 1$, the semigroup associated with the Kirchhoff thermoelastic plate, though exponentially stable, is not even differentiable [13, 15]. Inspired by the earlier works of Triggiani and Chen [4] for elastic systems, it then makes sense for us to wonder whether the semigroup associated with (1.1) would be analytic, or else, have an intermediate behavior when $0 < \theta < 1$. The purpose of this paper is to tackle that outstanding question.

To this end, we are going to reformulate System (1.1) as an abstract evolution equation. Set

$$V_1 = H^2(\Omega) \cap H_0^1(\Omega), \quad H_\theta = H^\theta(\Omega) = D(B_\theta^{\frac{1}{2}}),$$
\[ W = H_0^1(\Omega), V_2 = H_0^2(\Omega), H = L^2(\Omega). \]

For \( j = 1, 2 \), we introduce the Hilbert space \( \mathcal{H}_{	heta,j} = V_j \times H_0 \times H \) over the field of complex numbers, and we endow it with the norm given by (notice that the norm is the same in both cases):

\[
||(u, v, w)||_\theta^2 = \int_\Omega \left\{ |\Delta u|^2 + |B_\theta^{-1}v|^2 + \frac{\alpha}{\beta}|w|^2 \right\} \, dx. \tag{1.2}
\]

Setting \( Z = \left( \begin{array}{c} y \\ y_t \\ z \end{array} \right) \) and \( Z^0 = \left( \begin{array}{c} y_0 \\ y_1 \\ z_0 \end{array} \right) \), then System (1.1) may be rewritten as:

\[
\begin{align*}
\dot{Z} - A_\theta Z &= 0 \quad \text{in } (0, \infty), \\
Z(0) &= Z^0,
\end{align*}
\tag{1.3}
\]

where the dot denotes differentiation with respect to time, and the unbounded operator matrix \( A_\theta \) is given by

\[
A_\theta = \begin{pmatrix}
0 & I & 0 \\
-B_\theta^{-1}\Delta^2 & 0 & -\alpha B_\theta^{-1}\Delta \\
0 & \beta\Delta & \kappa\Delta
\end{pmatrix},
\tag{1.4}
\]

with domain

\( D(A_\theta) = \left\{ (u, v, w) \in V_1 \times V_1 \times W : B_\theta^{-1}(\Delta^2 u + \alpha \Delta w) \in H_0, \: \beta \Delta v + \kappa \Delta w \in H \right\} \]

or, else

\( D(A_\theta) = \left\{ (u, v, w) \in (H^{4-\theta}(\Omega) \cap W) \times V_1 \times V_1 \right\}, \)

by elliptic regularity

Before stating our results, we find it useful to recall the following definition, and result (adapted from [25, Theorem 4, p. 153]).

**Definition 1.1.** Let \( t_0 \geq 0 \) be a real number. A strongly continuous semigroup \( T = (T(t))_{t \geq 0} \), defined on a Banach space \( X \), is of Gevrey class \( s \geq 1 \) for \( t > t_0 \), if \( T(t) \) is infinitely differentiable for \( t > t_0 \), and for every compact set \( K \subset (t_0, \infty) \) and each \( \mu > 0 \), there exists a constant \( C = C(\mu, K) > 0 \) such that

\[
||T^{(n)}(t)||_{L(X)} \leq C\mu^n(n!)^s, \quad \text{for all } t \in K, \: n = 0, 1, 2, ...
\]

**Theorem 1.2.** ([25]) Let \( T = (T(t))_{t \geq 0} \) be a strongly continuous and bounded semigroup on a Hilbert space \( X \). Suppose that the infinitesimal generator \( A \) of the semigroup \( T \) satisfies the following estimate, for some \( 0 < \alpha < 1 \):

\[
\limsup_{\lambda \to \infty} |\lambda|^\alpha ||(i\lambda I - A)^{-1}||_{L(X)} < \infty. \tag{1.5}
\]

Then \( T = (T(t))_{t \geq 0} \) is of Gevrey class \( \delta \), for \( t > 0 \), for every \( \delta > \frac{1}{\alpha} \).

Our main results read as follows:
Theorem 1.3. For every $\theta \in [0, 1]$, the linear operator $A_\theta$, given in (1.4), generates a strongly continuous-semigroup of contractions $(S_{\theta,j}(t))_{t \geq 0}$ on the Hilbert space $H_{\theta,j}$, $j = 1, 2$.

Furthermore, for $j = 1, 2$, and for every $\theta$ in $(0, 1/2)$ the semigroup $(S_{\theta,j}(t))_{t \geq 0}$ is of Gevrey class $\delta$ for every $\delta > \frac{2-\theta}{\theta}$, as there exists a positive constant $C$ such that we have the resolvent estimate:

$$|\lambda|^{\frac{2-\theta}{\theta} - \frac{\delta}{2}} \| (i \lambda I - A_\theta)^{-1} \|_{\mathcal{L}(H_{\theta,j})} \leq C, \quad \forall \lambda \in \mathbb{R}. \tag{1.6}$$

Thus, the semigroup $(S_{\theta,j}(t))_{t \geq 0}$ is infinitely differentiable on $H_{\theta,j}$ for all $t > 0$.

Theorem 1.4. Assume that the hinged boundary conditions hold. Then for every $\theta \in [0, 1]$, the strongly continuous semigroup $(S_{\theta,1}(t))_{t \geq 0}$ is exponentially stable on the Hilbert space $H_{\theta,1}$. More precisely, there exist positive constants $M$ and $\omega$ such that:

$$\| S_{\theta,1}(t) \|_{\mathcal{L}(H_{\theta,1})} \leq M \exp(-\omega t), \quad \forall t \geq 0. \tag{1.7}$$

However, for every $\theta$ in $(0, 1)$, and every $r$ in $((2 - 2\theta)/(2 - \theta), 1]$, we have:

$$\limsup_{|\lambda| \to \infty} |\lambda|^r \| (i \lambda I - A_\theta)^{-1} \|_{\mathcal{L}(H_{\theta,1})} = \infty.$$ 

Remark 1.5. Thanks to [7, Theorem 3] or [24, Corollary 4], one derives from the resolvent estimate in Theorem 1.1 that the semigroup $S_{\theta,j}(t)$ is exponentially stable for $j = 1, 2$ and for all $\theta$ in $(0, 1/2)$; in fact our proof shows that the exponential stability of the semigroup would hold for $\theta \in [0, 1/2]$ in both cases considered here. In particular, choosing $\theta = 0$, leads to the known aforementioned analyticity results in the case of the Euler-Bernoulli model with clamped or hinged boundary conditions. We think it worthwhile to call the reader’s attention to the fact that the exponential stability in Theorem 1.4 is valid for all values of $\theta$ in $[0, 1]$, and as such, cannot be derived from Theorem 1.3 for the values of $\theta$ greater than $1/2$. The last statement in Theorem 1.4 shows in particular that analyticity fails for all values of $\theta$ in $(0, 1]$, thereby generalizing an earlier result of Lasiecka and Triggiani [13, 15], while simultaneously simplifying its proof, in the case of hinged boundary conditions.

The rest of the paper is organized as follows. Section 2 is devoted to the proof of Theorem 1.3, while Section 3 focuses on the proof of Theorem 1.4, and some final comments.

2. Proof of Theorem 1.3. Throughout this section and the rest of the paper, the quantity $|u|_2$ denotes the $L^2(\Omega)$-norm of $u$. In addition, throughout the proof, for a complex number $z$, we shall denote by $\text{Re}(z)$, its real part.

The fact that the operator $A_\theta$ generates a $C_0$-semigroup of contractions is pretty straightforward, thanks to the Lumer-Phillips Theorem [22], as $D(A_\theta)$ is dense in $H_\theta$, $A_\theta$ is dissipative, and one easily checks, via Lax-Milgram Lemma, that $I - A_\theta$ is onto. We will focus our attention on the Gevrey regularity of the semigroup. Since $D(A_\theta)$ is dense in $H_\theta$, one checks that $A_\theta$ has a compact resolvent; therefore its spectrum is discrete. Furthermore, $\sigma(A_\theta) \cap i\mathbb{R} = \emptyset$, which means $i\mathbb{R} \subset \rho(A_\theta)$, where $\sigma(A_\theta)$ and $\rho(A_\theta)$ denote respectively the spectrum and the resolvent set of
\( A_\theta \). Now, thanks to Theorem 1.2, we shall prove that there exists a positive constant \( C_0 \) such that, for each \( j = 1, 2 \):

\[
\sup \left\{ \left| b \right| \frac{2 - 4\theta}{\beta} \| (ibI - A_\theta)^{-1} \|_\theta \mid \mathcal{Z}(H_{\theta,j}) \right. ; \ b \in \mathbb{R} \right\} \leq C_0. \tag{2.1}
\]

The constant \( C_0 \) may vary from line to line and depends on the parameters of the system, but not on the frequency variable \( b \).

Invoking the continuity of the resolvent, it is enough to prove the inequality (2.1) for all \( b \) with \( \left| b \right| > 1 \). To prove (2.1), we will show that there exists \( C_0 > 0 \) such that for every \( U \in H_{\theta,j} \), one has:

\[
\left| b \right| \frac{2 - 4\theta}{\beta} \| (ibI - A_\theta)^{-1}U \|_\theta \leq C_0 \| U \|_\theta \quad \forall \ b \in \mathbb{R}, \ \text{with} \ \left| b \right| > 1. \tag{2.2}
\]

Thus, let \( b \in \mathbb{R} \) with \( \left| b \right| > 1 \), \( U = (f, g, h) \in H_{\theta,j} \), and let \( Z = (u, v, w) \in D(A_\theta) \) such that

\[
(ib - A_\theta)Z = U. \tag{2.3}
\]

Multiply both sides of (2.3) by \( Z \), then take the real part of the inner product in \( H_{\theta,j} \) to derive:

\[
\frac{\alpha \kappa}{\beta} \int_\Omega |\nabla w|^2 \, dx = \text{Re}(U, Z) \leq \| U \|_\theta \| Z \|_\theta. \tag{2.4}
\]

Equation (2.3) may be rewritten as:

\[
\begin{cases}
ibu - v = f, \\
ibB_\theta v + \Delta^2 u + \alpha \Delta w = B_\theta g, \\
ibw - \kappa \Delta w - \beta \Delta v = h.
\end{cases} \tag{2.5}
\]

Therefore, (2.2) will be established if we show the following estimate:

\[
\left| b \right| \frac{2 - 4\theta}{\beta} \| Z \|_\theta \leq C_0 \| U \|_\theta, \quad \forall b \in \mathbb{R} \text{ with } \left| b \right| > 1. \tag{2.6}
\]

Several steps will be needed in the proof of (2.6). Henceforth, we assume \( b \in \mathbb{R} \) with \( \left| b \right| > 1 \).

**Step 1.** In this step, we are going to show that for every \( \varepsilon > 0 \), there exists a positive constant \( C_\varepsilon \), independent of \( b \) such that:

\[
\left| b \right| \frac{2 - 4\theta}{\beta} \| \{ w \}_{2} + |B_{\varepsilon}^{\frac{1}{2}} v \|_{2} \| \varepsilon \mid \frac{2 - 4\theta}{\beta} \| Z \|_\theta + C_\varepsilon \| U \|_\theta. \tag{2.7}
\]

To this end, we shall borrow some ideas from [19]. Set \( w = w_1 + w_2 \), where \( w_1 \in W \) and \( w_2 \in H \), with

\[
ibw_1 - \Delta w_1 = h, \quad ibw_2 = \kappa \Delta w + \beta \Delta v - \Delta w_1. \tag{2.8}
\]

Firstly, multiplying the first equation in (2.8) by \( \bar{w}_1 \), then by \( \Delta \bar{w}_1 \) and using Green’s formula, one easily derives

\[
|b|\|w_1\|_2 + |b|^{\frac{1}{2}}\|w_1\|_W + |\Delta w_1\|_2 \leq C_0 \|U\|_\theta. \tag{2.9}
\]

On the other hand, it follows from the second equation in (2.8) that

\[
|b|\|w_2\|_{H^{-2}(\Omega)} \leq C_0(\|w_2\|_2 + |v|_2 + |w_1|_2) \leq C_0(\|Z\|_\theta + |b|^{-1}\|U\|_\theta). \tag{2.10}
\]

Now, by Lions’ interpolation inequality as well as (2.9), (2.10), the fact that \( \|w_2\|_W \leq \|w\|_W + \|w_1\|_W \), and (2.4), we derive

\[
|b|\|w_2\|_2 \leq C_0|b|\|w_2\|_{H^{-2}(\Omega)}\|w_2\|_{H^1(\Omega)}^{\frac{1}{2}}.\tag{2.11}
\]
On the other hand, we have

\[
\|v\|_{H^2} \leq C_0 b^{\frac{3}{2}} \left( \|Z\|_{\partial} + |b|^{-1} \|U\|_{\partial} \right)^{\frac{3}{4}} \left( \|U\|_{\partial}^\frac{3}{2} \|Z\|_{\partial}^\frac{1}{2} + |b|^{-\frac{1}{2}} \|U\|_{\partial} \right)^{\frac{3}{2}}
\]  

(2.11)

Now, set

\[
\Delta v \equiv \nabla \cdot (\nabla v) - \frac{\partial^2}{\partial t^2} v + \frac{\partial^2}{\partial y^2} v + \frac{\partial^2}{\partial z^2} v.
\]

In the sequel, we also need to estimate \( |b| \|v_2\|_{H^{-1}(\Omega)} \). Applying Lions' interpolation inequality once more and proceeding as above, one gets

\[
|b| \|v_2\|_{H^{-1}(\Omega)} \leq C_0 \left( |b| \|v_2\|_{H^{-2}(\Omega)} \right)^\frac{3}{4} \left( |b| \|v_2\|_{H^{-1}(\Omega)} \right)^\frac{1}{4} + \frac{3}{2} \left( |b| \|v_2\|_{H^{-1}(\Omega)} \right)^\frac{3}{4} \left( |b| \|v_2\|_{H^{-1}(\Omega)} \right)^\frac{1}{4}.
\]  

(2.12)

The combination of (2.9) and (2.12) yields:

\[
|b| \left( \|v_2\|_{H^{-1}(\Omega)} \right) \leq C_0 \left( |b| \|v_2\|_{H^{-2}(\Omega)} \right)^\frac{3}{4} \left( |b| \|v_2\|_{H^{-1}(\Omega)} \right)^\frac{1}{4} + \frac{3}{2} \left( |b| \|v_2\|_{H^{-1}(\Omega)} \right)^\frac{3}{4} \left( |b| \|v_2\|_{H^{-1}(\Omega)} \right)^\frac{1}{4}.
\]  

(2.13)

In the sequel, we also need to estimate \( |b| \|v_2\|_{H^{-1}(\Omega)} \). Applying Lions' interpolation inequality once more and proceeding as above, one gets

\[
|b| \|v_2\|_{H^{-1}(\Omega)} \leq C_0 |b| \left( \|v_2\|_{H^{-2}(\Omega)} \right)^\frac{3}{4} \left( |b| \|v_2\|_{H^{-1}(\Omega)} \right)^\frac{1}{4} + \frac{3}{2} |b| \left( \|v_2\|_{H^{-1}(\Omega)} \right)^\frac{3}{4} \left( |b| \|v_2\|_{H^{-1}(\Omega)} \right)^\frac{1}{4}.
\]  

(2.14)

Proceeding as above, one gets the following estimate:

\[
|b| \|v_2\|_{H^{-1}(\Omega)} \leq C_0 |b| \left( \|v_2\|_{H^{-2}(\Omega)} \right)^\frac{3}{4} \left( |b| \|v_2\|_{H^{-1}(\Omega)} \right)^\frac{1}{4} + \frac{3}{2} |b| \left( \|v_2\|_{H^{-1}(\Omega)} \right)^\frac{3}{4} \left( |b| \|v_2\|_{H^{-1}(\Omega)} \right)^\frac{1}{4}.
\]  

(2.15)

We should now estimate \( |b| \|v_2\|_{H^{-2}(\Omega)} \sim |b| \|v_2\|_{H^{-2}(\Omega)} \). Note that with that norm equivalence, we can deal both with the cases of hinged and clamped boundary conditions simultaneously.

On the one hand, we have

\[
|b| \|v_2\|_{H^{-2}(\Omega)} \leq C_0 (|b| \|v_2\|_{H^{-2}(\Omega)} + |b| \|v_2\|_{H^{-2}(\Omega)}).
\]

(2.16)

On the other hand, we have

\[
|b| \|v_2\|_{H^{-2}(\Omega)} \leq C_0 (|b| \|v_2\|_{H^{-2}(\Omega)} + |b| \|v_2\|_{H^{-2}(\Omega)})
\]

(2.17)

Consequently,

\[
|b| \|v_2\|_{H^{-2}(\Omega)} \leq C_0 (|b| \|v_2\|_{H^{-2}(\Omega)} + |b| \|v_2\|_{H^{-2}(\Omega)}).
\]  

(2.18)

Now, multiply the third equation in (2.5) by \( \tilde{v} \), and apply Green’s formula first, then Cauchy-Schwarz and Poincaré inequalities to derive (using the decomposition \( w = w_1 + w_2 \)),

\[
|\nabla v|^2 \leq \text{Re} \int_{\Omega} \{ h\tilde{v} - \kappa w\tilde{v} - i b w_1 \tilde{v} - i b w_2 \tilde{v} \} \, dx
\]  

(2.20)
where $s$ is in some sense optimal, provided $v \leq b\|w_1\|v_2 \leq b\|w_2\|_{H^{-1}(\Omega)}$. Hence,

$$|v|_{W} \leq C_0(|U|_{\theta}^2 \|Z\|_{\theta}^3 \theta + C_0|b|\|w_2\|_{H^{-1}(\Omega)}).$$

Substituting (2.14) into (2.22), we obtain

$$|v|_{W} \leq C_0|U|_{\theta}^2 \|Z\|_{\theta}^{3 \theta} \theta$$

Applying Lions’ interpolation inequality once more, and using the fact that $|v_2|_W \leq |v_1|_W + |v|_W$, as well as (2.16) and (2.19), we find

$$|b|B^2_\theta v_2 |_{2} \leq C_0|b|\|v_2\|^{\frac{1-\theta}{2\theta}}_{2}(\Omega) \|v_2\|^{\frac{2-\theta}{2\theta}}_{H^1(\Omega)}$$

Using (2.23) and (2.24), we get

$$|b|B^2_\theta v_2 |_{2} \leq C_0|b|\|v_2\|^{\frac{1-\theta}{2\theta}}_{2}(\Omega) \|v_2\|^{\frac{2-\theta}{2\theta}}_{H^1(\Omega)}$$

From which, we derive

$$|b|^s B^s_\theta v_2 |_{2} \leq C_0|b|\|v_2\|^{\frac{1-\theta}{2\theta}}_{2}(\Omega) \|v_2\|^{\frac{2-\theta}{2\theta}}_{H^1(\Omega)}$$

where $s$ is a positive constant to be determined. We will show that $s = (2 - 4\theta)/(2 - \theta)$ is in some sense optimal, provided $\theta < 1/2$; this is precisely where the restriction on $\theta$ appears.

Now, we shall estimate each of the three terms in the right hand side of (2.26). For the first term, we have

$$C_0|b|^{\frac{1-\theta}{2\theta}}_{2} \cdot \frac{2-\theta}{2\theta}$$

(2.27)
One readily checks that the exponents of $2^s$ in (2.27)-(2.29) only for $s \in [0,(2-4\theta)/(2-\theta)]$; so the best value for $s$ is then given by $s = (2-4\theta)/(2-\theta)$ as claimed above. To simplify the notations, we use $s$ to denote $(2-4\theta)/(2-\theta)$ in the rest of this proof.

Combining (2.26)-(2.29), we arrive at

$$\sum_{i=1}^{2} B_i^\frac{1}{2} v_2 \leq C_0 \left( \|b^s Z\|_{\theta} \frac{1-\theta}{2-\theta} \|\nabla U\|_{\theta}^{\frac{2-\theta}{2}} + C_0 \left[ \frac{1}{2} \|\nabla U\|_{\theta} + \|b^s Z\|_{\theta} \frac{4-3\theta}{2-\theta} \|\nabla U\|_{\theta} \right] \right).$$

As for the second term, we have

$$\sum_{i=1}^{2} B_i^\frac{1}{2} v_2 \leq C_0 \left( \|b^s Z\|_{\theta} \frac{1-\theta}{2-\theta} \|\nabla U\|_{\theta}^{\frac{2-\theta}{2}} + C_0 \left[ \frac{1}{2} \|\nabla U\|_{\theta} + \|b^s Z\|_{\theta} \frac{4-3\theta}{2-\theta} \|\nabla U\|_{\theta} \right] \right).$$

Proceeding similarly for the third term, we find

$$\sum_{i=1}^{2} B_i^\frac{1}{2} v_2 \leq C_0 \left( \|b^s Z\|_{\theta} \frac{1-\theta}{2-\theta} \|\nabla U\|_{\theta}^{\frac{2-\theta}{2}} + C_0 \left[ \frac{1}{2} \|\nabla U\|_{\theta} + \|b^s Z\|_{\theta} \frac{4-3\theta}{2-\theta} \|\nabla U\|_{\theta} \right] \right).$$

One readily checks that the exponents of $|b|$ are all nonpositive in (2.27)-(2.29) only for $s \in [0,(2-4\theta)/(2-\theta)]$; so the best value for $s$ is then given by $s = (2-4\theta)/(2-\theta)$ as claimed above. To simplify the notations, we use $s$ to denote $(2-4\theta)/(2-\theta)$ in the rest of this proof.

Combining (2.26)-(2.29), we arrive at

$$\sum_{i=1}^{2} B_i^\frac{1}{2} v_2 \leq C_0 \left( \|b^s Z\|_{\theta} \frac{1-\theta}{2-\theta} \|\nabla U\|_{\theta}^{\frac{2-\theta}{2}} + \|\nabla U\|_{\theta} + \|b^s Z\|_{\theta} \frac{4-3\theta}{2-\theta} \|\nabla U\|_{\theta} \right).$$

(2.28)
Consequently, (2.35) becomes (keeping in mind that for this purpose, set $u = u_1 + u_2$ with

$$|b|^2 |B_{\theta}^2 v|^2 \leq C_0 b^{2s} (|B_{\theta}^2 B_{\theta} B_{\theta}^2 v|^2 + |w|^2) + C_0 |U|^2,$$

(2.32)

Next, combining (2.16) and (2.30), we find

$$|b|^2 |\Delta u|^2 \leq C_0 b^{2s} (|B_{\theta}^2 v|^2 + |w|^2) + C_0 |U|^2,$$

(2.33)

Multiplying equation (2.34) by $\bar{u}$ and using Green’s formula, one gets

$$b^{2s} |\Delta u|^2 = b^{2s+2} |B_{\theta}^2 u_2|^2 + \text{Re} \int_{\Omega} \left\{ b^{2s+2} b_{\theta}^2 u_2 B_{\theta}^2 \bar{u}_1 - b^{2s+2} \alpha w \Delta \bar{u} - b^{2s+2} v_1 \Delta \bar{u} \right\} dx.$$

(2.35)

The application of Young’s inequality readily shows

$$\left| \text{Re} \int_{\Omega} \{-b^{2s} \alpha w \Delta \bar{u} - b^{2s} v_1 \Delta \bar{u} \} dx \right| \leq \frac{b^{2s}}{2} |\Delta u|^2 + C_0 b^{2s} (|w|^2 + |v_1|^2).$$

(2.36)

Consequently, (2.35) becomes (keeping in mind that $ibu_2 = v_2$):

$$b^{2s} |\Delta u|^2 \leq 2b^{2s} |B_{\theta}^2 u_2|^2 + 2 \text{Re} \int_{\Omega} b^{2s+2} B_{\theta}^2 u_2 B_{\theta}^2 \bar{u}_1 dx + C_0 b^{2s} (|w|^2 + |v_1|^2).$$

(2.37)

Now, one checks, denoting by $\langle \cdot, \cdot \rangle$ the duality product between $H_{\theta}$ and its topological dual:

$$b^{2s} \int_{\Omega} B_{\theta}^2 u_2 B_{\theta}^2 \bar{u}_1 dx = b^{2s} \int_{\Omega} B_{\theta}^2 v_1 B_{\theta}^2 v_2 dx + b^{2s} (B_{\theta} v_2, \bar{f})$$

$$= b^{2s} \int_{\Omega} B_{\theta}^2 \bar{v}_1 B_{\theta}^2 v_2 dx + b^{2s} (ib)^{-1} (\Delta^2 u, \bar{f}) + (ib)^{-1} b^{2s} \int_{\Omega} \bar{f} (\alpha \Delta w + \Delta v_1) dx$$

$$= b^{2s} \int_{\Omega} B_{\theta}^2 \bar{v}_1 B_{\theta}^2 v_2 dx + (ib)^{-1} b^{2s} \int_{\Omega} (\Delta u + \alpha w + v_1) \Delta \bar{f} dx.$$
Thanks to the Hölder and Young inequalities, one derives from (2.38):
\[ 2b^{2s} + 2s \int_{\Omega} B_{\theta}^2 u_2 B_{\theta} \bar{u}_1 \, dx \leq \frac{1}{b} \left| B_{\theta}^2 v_1 \right|_2 \left| b^s B_{\theta}^2 v_2 \right|_2 + C_0 b^{2s} (|w|^2_2 + |v|^2_2) + C_0 |b^{2s} w|^2 + \frac{1}{2} b^{2s} |\Delta u|^2_2. \] (2.39)

Substituting (2.39) in (2.37), we derive
\[ b^{2s} |\Delta u|^2_2 \leq C_0 b^{2s} (|B_{\theta}^4 v_1|^2_2 + |B_{\theta}^4 v_2|^2 + |w|^2_2) + C_0 |U|^2_\theta, \] (2.40)
from which, (2.32) readily follows.

Thanks to Young’s inequality, Steps 2 and 3 yield
\[ |b|^s |Z|_\theta \leq C_0 \varepsilon |b|^s |Z|_\theta + C_\varepsilon |U|_\theta, \quad \forall \varepsilon > 0. \] (2.41)
Choosing \( \varepsilon = 1/2C_0 \) in (2.41), one gets (2.6), which completes the proof of Theorem 1.3.

3. Proof of Theorem 1.4. First, we are going to prove the exponential decay of the energy for all \( \theta \in [0, 1] \). To this end, it suffices, thanks to [7, Theorem 3] or [24, Corollary 4], to prove the following two assertions:
\[ i\mathbb{R} \subset \rho(A_\theta), \quad \text{and} \quad \sup \left\{ ||(ib - A_\theta)^{-1}||_{L(H_{\theta,1})}; b \in \mathbb{R} \right\} < \infty. \] (3.1)

First, we are going to prove that \( i\mathbb{R} \subset \rho(A_\theta) \). Given that \( A_\theta \) has a compact resolvent, it is enough to show that \( A_\theta \) has no imaginary eigenvalue. Notice that it is easy to check that \( 0 \in \rho(A_\theta) \). Now, let \( b \) be a nonzero real number, and let \( Z = (u, v, w) \in D(A_\theta) \) be such that
\[ (ib - A_\theta)Z = 0. \] (3.2)
We shall prove that \( Z = (0, 0, 0) \). Taking the inner product of both sides of (3.2) and \( Z \), one derives at once
\[ \Re (A_\theta Z, Z) = -\frac{\alpha}{\beta} |\nabla w|^2_2 = 0, \]
from which it follows that \( w = 0 \), thanks to Poincaré’s inequality. Now, (3.2) can be rewritten as
\[ \begin{cases} v = bu, \\
ibB_{\theta}v + \Delta^2 u + \alpha \Delta w = 0, \\
ibw - \kappa \Delta w - \beta \Delta v = 0. \end{cases} \] (3.3)
Since \( w = 0 \), the third equation in that system reduces to \( \Delta v = 0 \). Since \( v \in H_0^1(\Omega) \), the application of Poincaré’s inequality shows that \( v = 0 \); replacing this in the first equation in (3.3), one deduces that \( u = 0 \), as \( b \neq 0 \). Hence \( i\mathbb{R} \subset \rho(A_\theta) \).

We shall now prove the resolvent estimate in (3.1). To this end, we will show that there exists a constant \( C_0 > 0 \) such that for every \( U \in H_{\theta,1} \), one has:
\[ ||(ibI - A_\theta)^{-1}U||_\theta \leq C_0 ||U||_\theta, \quad \forall b \in \mathbb{R}, \] (3.4)
then invoke the continuity of the resolvent for all real numbers \( b \) with \( |b| \leq 1 \).

Thus, let \( b \in \mathbb{R} \) with \( |b| > 1 \), \( U = (f, g, h) \in H_{\theta,1} \), and let \( Z = (u, v, w) \in D(A_\theta) \) be such that
\[ (ib - A_\theta)Z = U. \] (3.5)
Multiply both sides of (3.5) by $Z$, then take the real part of the inner product in $H^{1,0}$ to derive:

$$\alpha \kappa \beta \int_{\Omega} |\nabla w|^2 \, dx = \text{Re}(U, Z) \leq ||U||_\theta ||Z||_\theta.$$ (3.6)

Equation (3.5) may be rewritten as:

$$\begin{cases}
ib u - v = f, \\
ib B_\theta v + \Delta^2 u + \alpha \Delta w = B_\theta g, \\
ib w - \kappa \Delta w - \beta \Delta v = h.
\end{cases}$$ (3.7)

Multiplying the second equation in (3.7) by $\bar{u}$, using Green’s formula and taking real parts, we find

$$\text{Re} \int_{\Omega} B_\theta g B_\theta ^{1/2} \bar{u} \, dx + |\Delta u|^2 \leq |B_\theta ^{1/2} v|^2 + \text{Re} \int_{\Omega} B_\theta ^{1/2} g B_\theta ^{1/2} \bar{u} \, dx + \alpha \text{Re} \int_{\Omega} \nabla w \cdot \nabla \bar{u} \, dx.$$ (3.8)

Using the first equation from (3.7) in (3.8), it follows that

$$|\Delta u|^2 \leq |B_\theta ^{1/2} v|^2 + \text{Re} \int_{\Omega} B_\theta ^{1/2} g B_\theta ^{1/2} \bar{u} \, dx + \alpha \text{Re} \int_{\Omega} \nabla w \cdot \nabla \bar{u} \, dx$$

$$+ \text{Re} \int_{\Omega} B_\theta ^{1/2} v B_\theta ^{1/2} \bar{f} \, dx.$$ (3.9)

Thanks to the Cauchy-Schwarz inequality and Sobolev embedding theorems, it follows that

$$\left| \text{Re} \int_{\Omega} B_\theta ^{1/2} g B_\theta ^{1/2} \bar{u} \, dx \right| \leq |B_\theta ^{1/2} v|^2 + C_0 |B_\theta ^{1/2} g|^2 |\Delta u|^2 \leq C_0 ||U||_\theta ||Z||_\theta.$$ (3.10)

Similarly, one derives

$$\left| \text{Re} \int_{\Omega} \nabla w \cdot \nabla \bar{u} \, dx \right| \leq C_0 ||U||^{1/2}_\theta ||Z||^{1/2}_\theta ,$$ (3.11)

and

$$\left| \text{Re} \int_{\Omega} B_\theta ^{1/2} v B_\theta ^{1/2} \bar{f} \, dx \right| \leq C_0 ||U||_\theta ||Z||_\theta .$$ (3.12)

Replacing (3.10)-(3.12) in (3.9), we find

$$|\Delta u|^2 \leq |B_\theta ^{1/2} v|^2 + C_0 \left(||U||^{1/2}_\theta ||Z||^{1/2}_\theta + ||U||_\theta ||Z||_\theta \right).$$ (3.13)

It remains to estimate the first term in the right hand side of (3.13). For this purpose, multiply the last equation in (3.7) by $(-\Delta)^{-1} B_\theta \bar{v}$, apply Green’s formula, and take real parts to get

$$|B_\theta ^{1/2} v|^2 = \text{Re} \frac{1}{\beta} \int_{\Omega} \{ B_\theta ^{1/2} (-\Delta)^{-1} h - \kappa B_\theta ^{1/2} w \} B_\theta ^{1/2} \bar{v} \, dx - \text{Re} \frac{ib}{\beta} \int_{\Omega} (-\Delta)^{-1} w B_\theta \bar{v} \, dx.$$ (3.14)

The application of the Cauchy-Schwarz inequality and Sobolev embedding theorems yields

$$\left| \text{Re} \frac{1}{\beta} \int_{\Omega} \{ B_\theta ^{1/2} (-\Delta)^{-1} h - \kappa B_\theta ^{1/2} w \} B_\theta ^{1/2} \bar{v} \, dx \right| \leq C_0 \left(||U||_\theta ||Z||_\theta + ||U||^{1/2}_\theta ||Z||^{1/2}_\theta \right).$$ (3.15)
Indeed, if we have sequences \( Z \), there exist a sequence of positive real numbers \( \beta \), using Young’s inequality.

Applying the Cauchy-Schwarz inequality, Poincaré’s inequality, and Sobolev embedding theorems once more, yields

\[
|B_\beta^2 v|^2 \leq C_0 \left( |U||\theta||Z||\theta + |U||\theta|^2 \right) - \text{Re} \frac{ib}{\beta} \int_\Omega (-\Delta)^{-1} \omega B_\beta \bar{v} \ dx. \tag{3.16}
\]

To estimate the integral term in (3.16), we are going to rewrite it in a different form. To this end, take the conjugate of the second equation in (3.7), multiply that new equation by \((-\Delta)^{-1} \omega\), and apply Green’s formula to get

\[
-ib \int_\Omega (B_\beta \bar{v})(-\Delta)^{-1} \omega \ dx - \int_\Omega (\Delta \bar{u}) \omega \ dx - |w|^2 = \int_\Omega B_\beta^2 \bar{g} B_\beta^- \bar{g} (-\Delta)^{-1} \omega \ dx. \tag{3.17}
\]

Dividing each term by \( \beta \), and taking real parts, we find

\[
- \text{Re} \frac{ib}{\beta} \int_\Omega (-\Delta)^{-1} \omega B_\beta \bar{v} \ dx = \text{Re} \frac{1}{\beta} \int_\Omega \left\{ \omega \Delta \bar{u} + B_\beta^2 \bar{g} B_\beta^- \bar{g} (-\Delta)^{-1} \omega \right\} \ dx + \frac{\alpha}{\beta} |w|^2. \tag{3.18}
\]

Applying the Cauchy-Schwarz inequality, Poincaré’s inequality, and Sobolev embedding theorems once more, yields

\[
\left| \text{Re} \frac{1}{\beta} \int_\Omega \left\{ \omega \Delta \bar{u} + B_\beta^2 \bar{g} B_\beta^- \bar{g} (-\Delta)^{-1} \omega \right\} \ dx \right| + \frac{\alpha}{\beta} |w|^2 \leq C_0 \left( |U||\theta|^2 \|Z\|_{\theta}^2 + |U||\theta||Z||_{\theta} \right). \tag{3.19}
\]

Hence

\[
- \text{Re} \frac{ib}{\beta} \int_\Omega (-\Delta)^{-1} \omega B_\beta \bar{v} \ dx \leq C_0 \left( |U||\theta|^2 \|Z\|_{\theta}^2 + |U||\theta||Z||_{\theta} \right). \tag{3.20}
\]

Combining (3.20) and (3.16), we obtain

\[
|B_\beta^2 v|^2 \leq C_0 \left( |U||\theta||Z||\theta + |U||\theta|^2 \right). \tag{3.21}
\]

The combination of (3.6), (3.13) and (3.21) produces

\[
\|Z\|_{\theta} \leq C_0 \left( |U||\theta||Z||\theta + |U||\theta|^2 \right), \tag{3.22}
\]

from which, one easily derives the desired resolvent estimate for all \( b \) with \( |b| > 1 \), by using Young’s inequality.

Using the continuity of the resolvent, one derives a similar estimate for all \( b \) in \([-1, 1]\), which completes the proof of the claimed exponential decay.

Let us now turn to the proof of the boundedness of the weighted resolvent. To this end, let \( \theta \in (0, 1) \). Let \( r \in ((2 - 2\theta)/(2 - \theta), 1) \). We are going to show that there exist a sequence of positive real numbers \( \{b_n\}_{n \geq 1} \), and for each \( n \), an element \( Z_n \in \mathcal{D}(\mathcal{A}_\theta) \) such that:

\[
\lim_{n \to \infty} b_n = \infty, \quad \|Z_n\|_{\theta} = 1, \quad \lim_{n \to \infty} b_n^{-r} \|((ib_n - \mathcal{A}_\theta)Z_n)\|_{\theta} = 0. \tag{3.23}
\]

Indeed, if we have sequences \( b_n \) and \( Z_n \) satisfying (3.23), then we set

\[
V_n = b_n^{-r} (ib_n - \mathcal{A}_\theta)Z_n, \quad U_n = \frac{V_n}{\|V_n\|_{\theta}}. \tag{3.24}
\]

Therefore \( \|U_n\|_{\theta} = 1 \) and

\[
\lim_{n \to \infty} b_n^{-r} \|((ib_n - \mathcal{A}_\theta)^{-1}U_n\|_{\theta} = \lim_{n \to \infty} \frac{1}{\|V_n\|_{\theta}} = \infty, \tag{3.25}
\]

which would establish the claimed result, thereby completing the proof of Theorem 1.4.
Thus, it remains to prove the existence of such sequences. We shall borrow some ideas from [26]. For each \( n \geq 1 \), we introduce the eigenfunction \( e_n \), with \( |e_n|_2 = 1 \) and:

\[
\begin{cases}
-\Delta e_n = \omega_n e_n & \text{in } \Omega \\
e_n = 0 & \text{on } \partial \Omega.
\end{cases}
\] (3.26)

It is a well-known fact that \( (\omega_n) \) is an increasing sequence of positive real numbers with \( \lim_{n \to \infty} \omega_n = \infty \). We seek \( Z_n \) in the form \( Z_n = (a_n e_n, ib_n a_n e_n, c_n e_n) \), with \( b_n \) and the complex numbers \( a_n \) and \( c_n \) chosen such that \( Z_n \) fulfills the desired conditions. Let \( \gamma \) and \( \mu \) be two nonzero real numbers satisfying:

\[ 2(\gamma^2 + \mu^2) = 1. \] (3.27)

For some large enough \( N_\theta \), to be specified later, and for \( n \geq N_\theta \), set:

\[ b_n = \frac{\omega_n}{\sqrt{1 + \omega_n^\theta}}, \quad \omega_n a_n = \mu_n + i\gamma_n, \quad c_n = -\frac{i\beta \omega_n a_n}{i + \kappa \sqrt{1 + \omega_n^\theta}}. \] (3.28)

Now, we shall specify what \( \mu_n \) and \( \gamma_n \) are. To this end, introduce the sequence of real numbers

\[ r_n = -\frac{\alpha \beta}{4(2(1 + \kappa^2)1 + \omega_n^\theta) + \alpha \beta}. \] (3.29)

Since \( r_n \) goes to zero as \( n \) goes to infinity, for any positive \( \theta \), and \( \gamma \) and \( \mu \) are nonzero, there exists a positive integer \( M_\theta \) such that both numbers \( \gamma^2 + r_n \) and \( \mu^2 + r_n \) are positive for each \( n \geq M_\theta \); we choose \( N_\theta = M_\theta \), and set

\[ \mu_n = \pm \sqrt{\mu^2 + r_n}, \quad \gamma_n = \pm \sqrt{\gamma^2 + r_n}. \] (3.30)

For \( n < N_\theta \), one may just set \( a_n = 0 \) and \( c_n = \sqrt{\beta/\alpha} \); however, we are mostly interested in what would happen when \( n \) goes to infinity.

Next, we show that \( ||Z_n||_\theta = 1 \). For \( n \geq N_\theta \), the use of (3.27)-(3.30) yields:

\[
||Z_n||_\theta^2 = \omega_n^2 |a_n|^2 + b_n^2 (1 + \omega_n^\theta) |a_n|^2 + \frac{\alpha}{\beta} |c_n|^2
\]

\[
= 2(\mu_n^2 + \gamma_n^2) + \frac{\alpha \beta (\mu_n^2 + 2 \gamma_n^2)}{1 + \kappa^2 (1 + \omega_n^\theta)}
\]

\[
= 2(\mu_n^2 + \gamma_n^2) + 4r_n + \frac{\alpha \beta (\mu_n^2 + \gamma_n^2 + 2r_n)}{1 + \kappa^2 (1 + \omega_n^\theta)}
\]

\[
= 1,
\] (3.31)

as \( 4r_n + \frac{\alpha \beta (\mu_n^2 + 2r_n)}{1 + \kappa^2 (1 + \omega_n^\theta)} = 0 \), thanks to (3.27) and (3.29).

It remains to show that the second limit in (3.23) holds. We have:

\[
(i b_n - A_\theta) Z_n = \begin{pmatrix}
0 \\
((\omega_n^2 - b_n^2 (1 + \omega_n^\theta)) a_n - \alpha \omega_n c_n) B_\theta^{-1} e_n \\
i \beta b_n \omega_n a_n e_n + (i b_n + \kappa \omega_n) c_n e_n \\
0
\end{pmatrix}
\]

\[
= \begin{pmatrix}
-\alpha \omega_n c_n B_\theta^{-1} e_n \\
0
\end{pmatrix}, \] (3.32)

by the definitions of \( b_n \) and \( c_n \). Consequently, we have (keeping in mind that \( \lim_{n \to \infty} (\mu_n^2 + \gamma_n^2) = 1/2 \)):

\[
\lim_{n \to \infty} b_n^{-2r} ||(i b_n - A_\theta) Z_n||_\theta^2
\]
\[
\lim_{n \to \infty} \alpha^2 b_n^{-2} \omega_n^2 |c_n|^2 |B_n^{-\frac{1}{2}} e_n|_2^2 = \lim_{n \to \infty} \frac{\alpha^2 b_n^{-2} \omega_n^2 |c_n|^2}{1 + \omega_n^2} = \lim_{n \to \infty} \frac{\alpha^2 \beta^2 b_n^{-2} \omega_n^2 (\mu_n^2 + \gamma_n^2)}{(1 + \kappa^2 (1 + \omega_n^2))(1 + \omega_n^2)} = \lim_{n \to \infty} \frac{\alpha^2 \beta^2 \omega_n^{-2r + \theta r} \omega_n^{-2\theta}}{2\kappa^2} = 0,
\]
provided that
\[
2 - 2\theta - r(2 - \theta) < 0, \quad \text{or} \quad r > \frac{2 - 2\theta}{2 - \theta},
\]
which proves the claimed result, and completes the proof of Theorem 1.4.

**Final Remark 3.1.** In this work, we are able to prove that the semigroup corresponding to the thermoelastic plate model with rotational forces involving the spectral fractional Laplacian is of Gevrey class \( \delta > (2 - \theta)/(2 - 4\theta) \) for all \( \theta \) in \((0, 1/2)\); this is also valid for the hinged or clamped plate. The case \( \theta \) in \([1/2, 1)\) remains open, though the second part of Theorem 1.4 shows that the semigroup fails to be analytic for all \( \theta \) in \((0, 1)\) in the case of a hinged plate. Our work shows that analyticity occurs only for the Euler-Bernoulli model, corresponding to \( \theta = 0 \), at least for hinged boundary conditions.

This work also falls within the framework of stability of thermoelastic systems whose study was pioneered by Dafermos [5] in the late sixties; it was shown in his work that the standard thermoelasticity system fails to be strongly stable except for special domains. The work of Dafermos did not discuss the decay rate for such systems when strong stability was possible; that question was tackled by Lebeau and Zuazua [16], where the authors used microlocal analysis tools to characterize such domains as well as establishing polynomial and exponential decay rates.

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