Joint 2D DOA and Doppler frequency estimation for L-shaped array using compressive sensing

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Abstract: A joint two-dimensional (2D) direction-of-arrival (DOA) and radial Doppler frequency estimation method for the L-shaped array is proposed in this paper based on the compressive sensing (CS) framework. Revised from the conventional CS-based methods where the joint spatial-temporal parameters are characterized in one large scale matrix, three smaller scale matrices with independent azimuth, elevation and Doppler frequency are introduced adopting a separable observation model. Afterwards, the estimation is achieved by $L_1$-norm minimization and the Bayesian CS algorithm. In addition, under the L-shaped array topology, the azimuth and elevation are separated yet coupled to the same radial Doppler frequency. Hence, the pair matching problem is solved with the aid of the radial Doppler frequency. Finally, numerical simulations corroborate the feasibility and validity of the proposed algorithm.

Keywords: electronic warfare, L-shaped array, joint parameter estimation, $L_1$-norm minimization, Bayesian compressive sensing (CS), pair matching.

DOI: 10.21629/JSEE.2020.01.04

1. Introduction

Joint direction-of-arrival (DOA) and Doppler frequency estimation have been applied in various fields including radar, sonar, communication and so on. Many DOA estimation algorithms have been proposed in the last decades, among which the subspace-based methods are the most popular ones, such as the multiple signal classification method (MUSIC) [1], estimating sign parameters via rotational invariant technique (ESPRIT) [2] and other improved algorithms including root-MUSIC [3], total least squares – ESPRIT (TLS-ESPRIT), QR TLS-ESPRIT algorithm [4] as well as a combined ESPRIT-MUSIC approach [5]. Additionally, in recent years, compressive sensing (CS) based DOA estimations have been reported taking advantages of the spatial sparsity of the signals, which can accurately obtain DOA estimation with only a few samples. Especially, the reconstruction methods are well addressed. For example, this problem was solved through orthogonal matching pursuit (OMP) and compressive sampling matching pursuit (CoSaMP) in [6] and [7]. On the other hand, the $L_1$-norm minimization method was proposed in [8] with stability at a low signal to noise ratio (SNR). Moreover, the Bayesian CS algorithm was proposed in [9,10], showing a promising prospect with its accurate DOA estimation performance.

In this paper, we exploit the L-shaped array to implement the joint two-dimensional (2D) DOA and radial Doppler frequency estimation. Actually, some methods have been proposed to solve this estimation problem. For example, the 2D-MUSIC algorithm [11] and the multi-ESPRIT algorithm [12] were adopted to solve the joint estimation problem for coprime arrays. However, these algorithms need a large amount of sampled data to ensure the accuracy and validity. To solve the limitation, Dang et al. [13] resorted a CS-based method to obtain DOA estimation, then the short-time Fourier transform (STFT) was applied to capture the Doppler frequency using the time window. Nevertheless, this method did not utilize the property that the Doppler frequency is sparse in nature for the moving platform (fighter plane, missile and so on) reconnaissance. In [14], the authors exploited an algorithm based on sparse Bayesian learning (SBL) to obtain the joint DOA and Doppler estimation. Like other conventional CS-based methods, the joint spatial-temporal parameters are characterized in one large scale matrix, which will, in turn, increase the computing burden. Fortunately, Luo et al. [15] proposed a separable observation model, which splited the observation matrix into two small matrices. With the aid of this separable observation framework, we separate the joint spatial-temporal observation matrix into three individual small matrices under the L-shaped array. Further, we extend the joint DOA and Doppler frequency estima-
tion to the three-dimensional (3D) scenario.

The novelty of this paper can be summarized as follows: (i) we achieve a joint estimation under the CS framework; (ii) we solve the DOA pair matching problem under the L-shaped array topology. To this aim, we resort to the $L_1$-norm minimization to estimate the azimuth and the elevation stably at a low SNR. Moreover, we exploit the Bayesian CS theory to estimate the radial Doppler frequency accurately. Although the azimuth and elevation are estimated separately, they are coupled to the same radial Doppler frequency, so the method can also implement pair matching.

The rest of this paper is organized as follows. Section 2 formulates the signal models. In Section 3, the proposed method is described in detail. Section 4 presents our simulation through numerical Monte-Carlo experiments. Section 5 draws some conclusions.

2. Signal models

Suppose an L-shaped array constructed by two orthogonal uniform linear arrays (ULA) in the $x$–$y$ plane, as is illustrated in Fig. 1. Denote $M$ by the amount of antenna in each sub-array. Moreover, in this paper, we follow the definition in [16] that the elevation $\theta$ represents the angle between $ok$ and the $y$-axis, the azimuth $\phi$ represents the angle between $ok$ and the $x$-axis.

![Configuration of an L-shaped array](image)

Suppose that there are $K$ uncorrelated far-field narrow-band signals impinging on the L-shaped array from different directions. Under the geometry shown in Fig. 1, the received signals collected by the ULAs along the $x$-axis and $y$-axis can be respectively written as

$$
\begin{align*}
\mathbf{x}(t) &= \mathbf{A}(\varphi)\mathbf{s}(t) + \mathbf{n}_x(t) \\
\mathbf{y}(t) &= \mathbf{A}(\theta)\mathbf{s}(t) + \mathbf{n}_y(t)
\end{align*}
$$

(1)

where the incident signal vector and the receiving additive white Gaussian noise (AWGN) for ULAs along the $x$-axis and $y$-axis can be expressed as

$$
\mathbf{s}(t) = [s_1(t) \ s_2(t) \ \cdots \ s_K(t)]^T
$$

and the manifold along the $x$-axis and $y$-axis is expressed as

$$
\begin{align*}
\mathbf{A}(\varphi) &= [\mathbf{a}(\varphi_1) \ \mathbf{a}(\varphi_2) \ \cdots \ \mathbf{a}(\varphi_K)] \\
\mathbf{A}(\theta) &= [\mathbf{a}(\theta_1) \ \mathbf{a}(\theta_2) \ \cdots \ \mathbf{a}(\theta_K)] \\
\mathbf{A}(\Phi) &= \mathbf{C}^M \times K
\end{align*}
$$

(2)

whose elements, i.e., the steering vectors, are calculated by

$$
\begin{align*}
\mathbf{a}(\varphi_k) &= \begin{bmatrix} 1 \ e^{-j\beta_k} \ \cdots \ e^{-j(M-1)\beta_k} \end{bmatrix}^T \\
\mathbf{a}(\theta_k) &= \begin{bmatrix} 1 \ e^{-j\alpha_k} \ \cdots \ e^{-j(M-1)\alpha_k} \end{bmatrix}^T
\end{align*}
$$

(3)

where $k = 1, 2, 3, \ldots, K$; $\alpha_k = 2\pi d\cos \theta_k / \lambda$ and $\beta_k = 2\pi d\cos \varphi_k / \lambda$; $d = \lambda / 2$ is the interspace between adjacent antennas; $\lambda = c / f_0$ is the transmitted signal wavelength; $[\ ]^T$ is the transpose operator; $f_0$ is the carrier frequency.

Fortunately, the estimation of this carrier frequency has been addressed in [17]. Therefore, in this paper, we assume the carrier frequency estimation has been accomplished for the sake of simplicity.

Let us first consider the ULA along the $x$-axis (the derivation for the $y$-axis is symmetry), suppose the signal amplitude equals 1, the $k$th signal received by the $m$th antenna can be expressed as

$$
x_{m,k}(t) = \exp[-j2\pi f_0 \left(\frac{d_{r,k} - v_k t}{c}\right) - j(m - 1)\alpha_k] = \\
\gamma_k \exp[j2\pi f_{d,k} t - j(m - 1)\alpha_k]
$$

(4)

where $f_{d,k}$ denotes the radial Doppler frequency of the $k$th target, $d_{r,k}$ and $v_k$ stand for the initial radial distance and the radial velocity of the $k$th target, $c$ is the speed of light, respectively.

Hence, the overall baseband signal received by the $m$th antenna is given by

$$
x_m(t) = \sum_{k=1}^{K} \gamma_k x_{m,k}(t) + n_{x,m}(t)
$$

(5)

where $\gamma_k = \exp(-j2\pi d_{r,k} / \lambda)$ is a complex amplitude. We collect samples simultaneously from the signal with a stable sampling rate $T_s$ satisfying the Nyquist constraint, so as to form a data matrix $\mathbf{X}$. For the derivation simplicity, we ignore the AWGN for time being; and the $(m, l)$th element of this data matrix can be written as

$$
\mathbf{X}(m, l) = \sum_{k=1}^{K} \gamma_k e^{-j(m-1)\alpha_k} e^{j2\pi f_{d,k}(l-1)T_s},
$$

(6)

where $L$ denotes the length of the snapshots.
We divide the azimuth and Doppler space on grids into $N$ and $Q$ grids respectively. Then, we can define the observation matrix $\Phi \in \mathbb{C}^{M \times N}$ and $F \in \mathbb{C}^{Q \times L}$ as

$$\begin{align}
\Phi &= \begin{bmatrix} a(\varphi_1) & a(\varphi_2) & \cdots & a(\varphi_N) \end{bmatrix} \\
F &= \begin{bmatrix} b(f_1) & b(f_2) & \cdots & b(f_Q) \end{bmatrix}^T
\end{align}$$

(7)

where $a(\varphi_k)$ ($k = 1, 2, \ldots, N$) inherits the definition as (3); $b(f_l) = [\text{e}^{2\pi f_l \tau_x} \ \cdots \ \text{e}^{2\pi f_l \tau_{L-1}}]^T \in \mathbb{C}^{L \times 1}$ is the Doppler frequency bin vector. Therefore, we can define an $N \times Q$ index matrix $S_x$ whose $(i,j)$th element can be expressed as

$$S_x(i, j) = \begin{cases} \gamma_k, & \text{if there is a target at } (\varphi_i, f_j) \\ 0, & \text{otherwise} \end{cases},$$

$$i = 1, \ldots, N; j = 1, \ldots, Q.$$ 

(8)

According to the fact that the number of targets is much smaller than the grid size of angles and Doppler frequencies (i.e., $N$), the matrix $S_x$ is indeed a spatial-temporal sparse matrix, wherein the row sparsity of it determines the azimuth of the target and the column sparsity determines the radial Doppler frequencies of the target. Assuming $S_x$ is $K$-sparse ($K < L$), when right-multiplied with an $L$-rank matrix $F$, the sparsity is still guaranteed, i.e., we can introduce a row sparse transition matrix $Z_x = S_x F$, where $Z_x$ shares the same sparsity as the matrix $S_x$. Hence, (9) can be reformulated as

$$X = \Phi Z_x + N_x.$$ 

(9)

Therefore, the index of the non-zero rows of the matrix $Z_x$ determines the azimuth of the target.

Similarly, the elevation space can be expressed as

$$\Theta = [a(\theta_1) \ \cdots \ a(\theta_N)]$$

(11)

where $\Theta \in \mathbb{C}^{M \times N}$. The data matrix received by the ULA along the $y$-axis can be formulated as

$$Y = \Theta S_y F + N_y$$ 

(12)

and the corresponding transition matrix is given by $Z_y = S_y F$.

Therefore, the estimation of the 2D DOA is now equivalent to search for the non-zero rows of transition matrices $Z_x$ and $Z_y$. We can then estimate the Doppler frequency by searching for the non-zero columns of sparse matrices $S_x$ and $S_y$. Also, thanks to the separable observation model, we can divide the azimuth/elevation-Doppler frequency space into $N$ grids separately instead of an $N^2$ grids large scale space matrix.

### 3. Proposed joint estimation algorithm

With analysis above, we can achieve our DOA estimation goal by first reconstructing the transition matrix $Z_x$ and retrieving the sparse matrix $S_x$, thereafter. These two steps can both be categorized as the multiple measurement vector (MMV) problems in the CS theory. Many approaches have shown a superior capability to solve this MMV problem, including the multiple OMP (M-OMP) algorithm [18], multiple focal under-determined system solver (MFOCUSS) algorithm [19], $L_1$-norm minimization and the multitask-Bayesian CS (MT-BCS) [20]. However, most of these methods suffer from a performance digression when the SNR is low. This estimation error will bring failure to our further Doppler frequency estimation. Therefore, we can first resort to $L_1$-norm minimization, which has a superior performance in the noisy environment. The MT-BCS algorithm is then followed by the computational efficiency. This algorithm shows an outstanding accuracy when the first step brings less residual into the radial Doppler frequency estimation.

#### 3.1 Joint 2D DOA and radial Doppler frequency estimation

Since $Z_x$ shares the same row sparsity property as the matrix $S_x$, and the index of the non-zero rows of the matrix $Z_x$ determines the azimuth angles of the target. We resort to the $L_1$-norm minimization, where reconstructing $Z_x$ from (9) can be converted into the following optimization problem [8]:

$$\text{OP1} : \min \|X - \Phi Z_x\|_F^2 + \mu \|Z_x^{(2)}\|_1$$

(13)

where $\|Z_x\|_1 = \|a(\varphi_x[1])\|_1 + \cdots + \|a(\varphi_x[n])\|_1$, $Z_x[n]$ is the $n$th row of $Z_x$; $\| \cdot \|_f$, $\| \cdot \|_1$ and $\| \cdot \|_2$ denote the Frobenius norm, $L_1$ norm and $L_2$ norm respectively. $\mu$ is the regularization parameter, which controls the relative importance applied to the error and the sparseness term, and one can choose $\mu$ such that $\|X - \Phi Z_x(\mu)\|_F^2 \approx E[\|N_x\|_F^2]$ (see Section V in [8] for more details).

Since OP1 in (13) contains a term $\|Z_x^{(2)}\|_1$, which is neither linear nor quadratic, so we turn to the second-order cone (SOC) programming [21], which is a suitable framework for the objective function. Moreover, the main reason for considering SOC programming instead of generic non-linear optimization for our problem is the availability of efficient interior point algorithms for the numerical solution of the former [8].

We define $Z_x[l]$ as the $l$th column of the matrix $Z_x$. 
OP1 is then formulated as an SOC programming problem, i.e., OP2 (see the Appendix in [8] for more details).

\[
\text{OP2} : \min p + \mu q \\
\text{s.t. } \| (b_1, \ldots, b_L) \|_2^2 \leq p, \quad 1^T r \leq q
\]

(14)

where \( b_l = X \{l \} - \Phi Z_x \{l \} \) (\( l = 1, 2, \ldots, L \)), \( \| Z_x[n] \|_2 \leq r_n \) (\( n = 1, 2, \ldots, N \)); \( p, q \) and \( r \) are \( C^{N \times 1} \) are the introduced auxiliary variables; \( X \{l \} \) represents the \( l \)th column of the matrix \( X \). Then we can use the convex optimization (CVX) [22] toolbox in Matlab to reconstruct the matrix \( Z_x \).

Recall that \( Z_x = S_x F \), where \( S_x \) is the sparse matrix, the index of the non-zero elements in \( S_x \) correspond to the estimation of the azimuth angles and the radial Doppler frequencies; \( F \) is the observation matrix. We take the transpose of the sparse matrix \( Z_x \), then we can obtain

\[
Z_x^T = F^T S_x^T.
\]

(15)

Further, we utilize the MT-BCS algorithm to reconstruct the sparse matrix \( S_x \).

Introducing the auxiliary matrices \( \tilde{Z}_x = Z_x^T, \tilde{F} = F^T \) and \( \tilde{S}_x = S_x^T \), (15) can be reformulated as \( \tilde{Z}_x = \tilde{F} \tilde{S} \).

Since the MT-BCS approach cannot process complex data directly, we use the following real data model:

\[
\begin{bmatrix}
\text{Re}(\tilde{Z}_x \{l \}) \\
\text{Im}(\tilde{Z}_x \{l \})
\end{bmatrix} =
\begin{bmatrix}
\text{Re}(\tilde{F}) & -\text{Im}(\tilde{F}) \\
\text{Im}(\tilde{F}) & \text{Re}(\tilde{F})
\end{bmatrix}
\begin{bmatrix}
\text{Re}(\tilde{S}_x \{l \}) \\
\text{Im}(\tilde{S}_x \{l \})
\end{bmatrix} +
\begin{bmatrix}
\text{Re}(\tilde{W} \{l \}) \\
\text{Im}(\tilde{W} \{l \})
\end{bmatrix}, \quad 1 \leq l \leq N
\]

(16)

where \( \text{Re}(\cdot) \) and \( \text{Im}(\cdot) \) denote the real and imaginary operator, respectively; \( \tilde{W} \{l \} \) stands for the \( l \)th column of the noise matrix with unknown variance.

According to the MT-BCS theory, the column vector of the sparse matrix is obtained by the probability density function way by solving the following [23] optimal problem, i.e., OP3, given by

\[
\text{OP3} : \tilde{S}_x \{l \} _{\text{MT-BCS}} =
\frac{1}{N} \sum_{l=1}^{N} \left[ \arg \max_{\tilde{S}_x \{l \}} p(\tilde{S}_x \{l \}, \gamma \tilde{Z}_x \{l \}) \right]
\]

(17)

where \( \tilde{Z}_x \{l \} \) and \( \tilde{S}_x \{l \} \) denote the \( l \)th column of the revised matrix \( \tilde{Z} \) and \( \tilde{S} \), \( \gamma = (\alpha_j)_{j=1}^{N} \) is a hyperparameter vector to be determined which controls the sparseness of the signal vector, respectively. According to [20], the optimal parameter \( \gamma \) is computed by maximizing the following likelihood function:

\[
\gamma_{\text{MT-BCS}} = \arg \max_{\gamma} \left[ \psi(\theta) \right]
\]

\[
\psi(\theta) = -\frac{1}{2} \sum_{l=1}^{L} \left[ (2L + 2a) \ln(\tilde{Z}_x \{l \} \text{T} C_{\text{MT-BCS}} \tilde{Z}_x \{l \}) + 
\right]

2b) + \ln |C_{\text{MT-BCS}}|
\]

(19)

\[
C_{\text{MT-BCS}} = I + \tilde{F} \text{diag}(\alpha^{-1}) \tilde{F}^T
\]

(20)

where \( I \) denotes the \( N \times N \) identity matrix, \( a \) and \( b \) are defined by the users [24]. Finally, the retrieved sparse matrix \( \tilde{S}_x \) turns out to be

\[
\tilde{S}_{\text{MT-BCS}} =
\frac{1}{N} \sum_{l=1}^{N} \left( \tilde{F}^T \tilde{F} + \text{diag}(\alpha_{\text{MT-BCS}})^{-1} \tilde{F}^T \tilde{Z}_x \{l \} \right).
\]

(21)

In addition, we substitute the signal azimuth measurement matrix into (8). The elevation angle observation matrix \( \Theta \) can obtain the baseband discrete signal model received by the ULA along the \( y \)-axis, which is illustrated in (12). Similarly, the row sparse matrix \( S_y \) and the sparse matrix \( S_y \) can be obtained by the way we mentioned above.

### 3.2 2D DOA pair matching based on the radial Doppler frequency

As is stated in Section 3.1, we estimate the azimuth and elevation separately, so pair matching is required after the 2D DOA estimation [16,25]. Fortunately, we can solve this problem by utilizing the evidence that for the same target, the azimuth and elevation angles are coupled with the same radial Doppler frequency.

We define two sets to denote the 2D DOA and radial Doppler frequencies by

\[
\{ \tilde{f}_x = \{ (\tilde{\varphi}_1, \tilde{f}_{x1}), (\tilde{\varphi}_2, \tilde{f}_{x2}), \ldots, (\varphi_K, \tilde{f}_{xK}) \} \}
\]

\[
\{ \tilde{f}_y = \{ (\tilde{\theta}_1, \tilde{f}_{y1}), (\tilde{\theta}_2, \tilde{f}_{y2}), \ldots, (\varphi_K, \tilde{f}_{yK}) \} \}
\]

(22)

The pair matching problem can be illustrated as Fig. 2, where the “distance” between the \( i \)th element in \( \tilde{f}_x \) and the \( j \)th element in \( \tilde{f}_y \) is defined as \( d_{ij} = |f_{xi} - f_{yj}| \). The solid line means the matched azimuth-elevation-Doppler frequency and the dash line means unmatched ones.

We further assume different targets have different Doppler frequencies. Then for a fixed azimuth index \( i = 1, \ldots, K \), the specific pair matching procedure is to repeat the following minimization for \( j = 1, \ldots, K \).

\[
\{i, j\} = \arg \min_{j} \{ |\tilde{f}_{xi} - \tilde{f}_{yj}| \}
\]

(23)
This procedure is addressed as follows. In addition, we notice that some preprocessing needs to be done on matrices \( \hat{Z} \) and \( \tilde{Z} \) considered to belong to the same target. The procedure is addressed as follows.

**Input** The DOA-radial Doppler frequencies pair sets \( \hat{F}_x \) and \( \tilde{F}_y \)

**Loop** for \( i = 1 \) to \( K \)

(i) Calculate the frequency offset using (23);

(ii) Find \( j_{opt} = \arg\min_j \{|\hat{f}_{xi} - \tilde{f}_{yj}\}| \);

(iii) Remove \( \tilde{f}_{yj} \) from set \( \tilde{F}_y \);

**End loop.**

Finally, \( (\varphi_i, \theta_{j_{opt}}) \) is matched where \( i = 1, \ldots, K; j_{opt} = 1, \ldots, K \).

### 3.3 The proposed DOA and Doppler frequency estimator

According to the above process, we can implement the joint 2D DOA and radial Doppler frequency estimation. In addition, we notice that some preprocessing needs to be done on matrices \( Z_x \) and \( Z_y \) to reduce the calculation burden. Delete the rows with small energy and reserve the K rows with strong energy, define new matrices as \( Z_{x, min} \) and \( Z_{y, min} \), then plug \( Z_{x, min} \) and \( Z_{y, min} \) into (13). Note that all the reserved row indices should be recorded as the results of the azimuth and elevation angles estimation.

In summary, with the signal matrix \( X \), observation matrix \( \Phi \) and matrix \( F \), the pseudo code of the proposed algorithm is illustrated as follows.

**Initialization** Construct the observation matrices \( \Phi \), \( \Theta \) and \( F \).

**Estimation of DOA:**

(i) Convert (13) into an SOC programming problem.

(ii) Use the CVX toolbox in Matlab to reconstruct the matrices \( Z_x \) and \( Z_y \).

**Estimation of radial Doppler frequency:**

(iii) Using (16) to convert (15) to the real data model.

(iv) Calculate the likelihood function \( \Psi(\alpha) \) using (19), where the \( C_{MT-BCS} \) is given by (20).

(v) Find \( \alpha_{MT-BCS} = \arg\max_\alpha|\Psi(\alpha)| \).

(vi) Finally, reconstruct the matrix \( S_x \) and \( S_y \) by using (21).

**Pair matching for DOA:**

(vii) Calculate the function (23) repeatedly, until all the azimuth and elevation angles are matched.

### 3.4 Complexity analysis

In this part, we analyze the computational complexity of the proposed approach and the traditional CS-based estimation approach. SALARI et al. [14] solved the one-dimensional (1D) DOA and Doppler frequency estimation, and the scale of the basis matrix is supposed as \( M L \times N Q \), the computational complexity of the traditional CS-based approach is \( O(N^3Q^3) \). If this approach is extended to the 3D scenario, the computational complexity is increased with it. For the proposed approach, the scale of the basis \( F \) is \( M \times N \), of \( F \) is \( Q \times L \), so the complexity for solving the data received by ULA along the x-axis is \( O(N^3 + Q^3) \). Similarly, the scale of the basis \( \Theta \) is \( M \times N \), the complexity for solving the data received by ULA along the y-axis is also \( O(N^3 + Q^3) \), and the overall complexity is \( O(N^3 + Q^3) \).

### 4. Performance evaluation

In this section, we describe some numerical results obtained by Monte-Carlo simulation. To be specific, an L-shaped array illustrated in Fig. 1 is considered, the interval between each antenna is half-wavelength. Moreover, we assume two uncorrelated far-field narrowband signals impinge on the L-shaped array, and their 2D DOA and radial Doppler frequencies are set to be \([\varphi_1, \theta_1, f_{d1}] = [3^\circ, 7^\circ, 260 \text{ Hz}] \) and \([\varphi_2, \theta_2, f_{d2}] = [4^\circ, 7.5^\circ, 320 \text{ Hz}] \). The SNR is defined as \( 10 \cdot \log(\eta/\sigma^2) \), where \( \eta \) is the signal power, \( \sigma^2 \) is the noise power, respectively, \( \theta, \varphi \in (0^\circ, 10^\circ) \) and the direction is divided uniformly into \( N = 101 \) grids with the interval equal to \( 0.1^\circ \). \( f_d \in (250 \text{ Hz}, 350 \text{ Hz}) \) and the radial Doppler frequency is also divided into \( Q = 101 \) grids with the interval equal to \( 1 \text{ Hz} \).

Additionally, in all simulations, 200 independent runs are conducted to calculate the root mean square error (RMSE) of the 2D DOA and Doppler frequency as

\[
\text{RMSE}_{\text{DOA}} = \sqrt{\frac{1}{T} \sum_{i=1}^{T} \sum_{k=1}^{K} \left( (\hat{\theta}_k - \hat{\theta}_{ik})^2 + (\hat{\varphi}_k - \hat{\varphi}_{ik})^2 \right)}
\]

\[
\text{RMSE}_{\text{Doppler}} = \sqrt{\frac{1}{T} \sum_{i=1}^{T} \sum_{k=1}^{K} (f_{d,k} - \tilde{f}_{d,k})^2}
\]

where \( T \) denotes the number of Monte Carlo experiments, \( K \) denotes the total number of targets, \( \hat{\theta}_{ik}, \hat{\varphi}_{ik}, \theta_{ik} \) and \( f_{d,k} \) denote the estimated values of the \( k \)-th target at the \( i \)-th Monte Carlo experiment, respectively.
Simulation 1  Demonstration of the parameter estimation process

In this simulation, we demonstrate the estimation procedure. To this end, we set SNR to be 20 dB, the amount of antennas is 10, and the length of snapshots is 50. Fig. 3(a) and Fig. 4(a) show the spectra of $Z_x$ and $Z_y$ respectively. It can be observed that the energy concentrates on the actual angles. Fig. 3(b) and Fig. 4(b) show the spectra of $S_x$ and $S_y$ respectively, which have the distinct peaks corresponding to the sparse distribution on the spatial-temporal domain.

Simulation 2  2D DOA and radial Doppler frequency estimation performance evaluation

In this simulation, we evaluate the estimation performance with respect to SNR and it can be easily observed from Fig. 5 and Fig. 6 that the accuracy of DOA and radial Doppler frequency estimation improves with the increase of the SNR.
Fig. 6 RMSE of DOA/radial Doppler frequency versus SNR and $L$

Specifically, in Fig. 5 we investigate estimation performance as a function of $M$. To this aim, the length of snapshots is fixed at 50. It is observed that with increasing $M$, the DOA estimation performance is noticeably improved. It is also observed by comparing the two sub-figures that, the number of antennas has a less effect on the Doppler frequency estimation performance than that on the DOA estimation performance. This phenomenon is brought by the fact that, in the proposed method, the Doppler frequency estimation performance is influenced by the length of the snapshots, which is fixed in this simulation; and the row sparse matrix $Z_x$ and $Z_y$ are $M$ related. Meanwhile, the estimation errors of the row sparse matrix have already been small. Hence, $M$ has limited effect on the radial Doppler frequency estimation performance compared with the DOA estimation performance.

Likewise, in Fig. 6 we investigate estimation performance as a function of $L$. To this aim, the amount of antennas is fixed at 12. Fig. 6(b) indicates that the performance of the radial Doppler frequency estimation is significantly improved by increasing the length of snapshots. However, it is also observed by comparing the sub-figures that, the length of snapshots has a less effect on the DOA estimation performance than that on the Doppler frequency estimation performance. Similar to the reasons above, the DOA estimation performance is influenced by the number of the antennas, which is fixed in this simulation. Thus, $L$ has limited effect on the DOA estimation performance compared with the Doppler frequency estimation performance.

Simulation 3 Comparison with existing methods

Some simulations of joint DOA and Doppler estimation are compared with the proposed method in this paper. It is shown in Fig. 7 that the proposed method has better performance than other methods especially in low SNRs.

5. Conclusions

In this paper, a joint 2D DOA and radial Doppler frequency estimation method for the L-shaped array is proposed based on the CS theory. Resorting to the separable observation framework, the joint spatial-temporal observation matrix is separated into three individual small matrices, afterwards the $L_1$-norm minimization method and the MT-BCS algorithm are used to estimate 2D DOA and radial Doppler frequencies respectively. Further, the pair matching problem can be solved since azimuth and elevation angles are coupled to the same radial Doppler fre-
frequency. The simulation results verify the effectiveness and superiority of the proposed method.

On the contrary, the estimation accuracy of the proposed algorithm depends on the grid scale $N$. However, denser grids will increase the computation burden and vice versa. Therefore, refined dynamic grid meshing is to be developed. In addition, for the proposed pair matching algorithm, the optimal solution obtained from each iteration is actually a local optimal solution rather than a global optimal solution. Therefore, in future research, we will investigate a revised matching algorithm. Moreover, as is stated in most DOA estimate studies, since wide-band signals are applied more in practice, utilizing CS based DOA estimation to wide-band models will be another future research.

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