The Three-Box “Paradox” and other Reasons to Reject
the Counterfactual Usage of the ABL Rule

R.E. Kastner*

Version of February 8, 1999

Department of Philosophy
University of Maryland
College Park, MD 20742 USA.

Abstract

An apparent paradox proposed by Aharonov and Vaidman in which a single particle
can be found with certainty in two (or more) boxes is analyzed by way of a simple
thought experiment. It is found that the apparent paradox arises from an invalid
counterfactual usage of the Aharonov-Bergmann-Lebowitz (ABL) rule, and effectively
attributes conflicting properties not to the same particle but to different particles. A
connection is made between the present analysis and the consistent histories formulation
of Griffiths. Finally, a critique is given of some recent counterarguments by Vaidman
against the rejection of the counterfactual usage of the ABL rule.

1. Background.

The Aharonov-Bergmann-Lebowitz (ABL) rule is a well-known formula for
calculating the probabilities of the possible outcomes of observables measured
at an intermediate time \( t_1 \) between pre- and post-selection measurements at

*rkastner@wam.umd.edu
times \( t_0 \) and \( t_f \), respectively [Aharonov, Bergmann, and Lebowitz, 1964]. If an intermediate measurement of (possibly degenerate) observable \( C \) with eigenvalues \( \{c_i\} \) is performed at time \( t_1 \), the ABL rule states that the probability of outcome \( c_j \) in the case of a preselection for the state \( |\psi_0\rangle \) and a post-selection for the state \( |\psi_f\rangle \) is given by:

\[
P_{ABL}(c_j|\psi_0,\psi_f) = \frac{|\langle \psi_f|P_{c_j}|\psi_0\rangle|^2}{\sum_i |\langle \psi_f|P_{c_i}|\psi_0\rangle|^2} \tag{1}
\]

where \( P_{c_j} \) is the projection operator on the eigenspace corresponding to outcome \( c_j \).

In the formulation of quantum theory known as “Time Symmetrized Quantum Theory,” (cf. Aharonov and Vaidman, 1991), a system thus pre- and post-selected is labelled by a time-symmetric “two-state vector” \( \Psi = \langle \psi_f||\psi_0 \rangle \).

It has recently been shown\(^2\) that the ABL rule cannot be used in a counterfactual sense: i.e., in general, it cannot be used to calculate the probabilities of possible outcomes of observables that have not actually been measured at time \( t_1 \). This result has been implicit all along in the consistent histories formulation of Griffiths (1984, 1996, 1998), which gives precise conditions under which meaningful, “classical” probabilities can be assigned to the outcomes of real or hypothetical measurements. (Indeed, the ABL rule can be seen as one simple instance of the consistent histories approach; this is shown in the Appendix.)

A quantitative distinction is made in Kastner (1998) between the valid, non-counterfactual usage and the generally invalid, counterfactual usage. This in-

\(^1\)In this case, the Hamiltonian \( H = 0 \); the ABL rule also applies to the more general case of nonzero Hamiltonian, with a time-dependence of the pre- and post-selection states in the usual way.

\(^2\)Sharp and Shanks (1993), Cohen (1995), Miller (1996), Kastner (1998).
volves augmenting the ABL probability $P_{ABL}$ in (1) with an additional parameter specifying which observable has actually been measured in the selection of any particular system. Thus if system X has been pre- and post-selected in the state $\Psi = \langle \psi_f | \psi_0 \rangle$ via an intervening measurement of observable $C$ at time $t_1$, the augmented ABL probability is written as:

$$P_{ABL}(x_j | \psi_0, \psi_f; C) = \frac{|\langle \psi_f | P_{x_j} | \psi_0 \rangle|^2}{\sum_i |\langle \psi_f | P_{x_i} | \psi_0 \rangle|^2}, \quad (1')$$

where the outcome whose probability is being calculated is denoted by the general parameter $x_j$. The correct, non-counterfactual usage restricts $x_j$ to the set of eigenvalues $\{c_i\}$ of $C$; the (generally) incorrect, counterfactual usage consists of allowing $x_j$ to vary over values not in the range of $C$.

2. The three-box example.

In his ‘Weak-Measurement Elements of Reality’ (1996), Vaidman discusses an example of what he terms an “ideal-measurement element of reality of the pre- and post-selected system” (‘ideal’ in the sense of ideal measurements rather than weak measurements; cf. Vaidman (1996, p. 899)). This consists of a single particle and three boxes labeled A, B, and C. The particle is pre-selected at time $t_0$ in the state

$$|\psi_0\rangle = \frac{1}{\sqrt{3}} \left( |a\rangle + |b\rangle + |c\rangle \right) \quad (2a)$$

and post-selected at time $t_f > t_0$ in the state

$$|\psi_f\rangle = \frac{1}{\sqrt{3}} \left( |a\rangle + |b\rangle - |c\rangle \right), \quad (2b)$$

where the states $|a\rangle$, $|b\rangle$, and $|c\rangle$ correspond to the particle being found in box A, B, or C, respectively. The pre- and post-selected states $|\psi_0\rangle$ and $|\psi_f\rangle$ are not
orthogonal and can be viewed as eigenvectors of two different observables $Q_0$ and $Q_f$. Let the eigenbasis of observable $Q_0$ be labelled as $\{\ket{\psi_0}, \ket{\psi'_0}, \ket{\psi''_0}\}$, and similarly for $Q_f$.

The two possible intermediate measurements at time $t_1$ are of two observables $A$ and $B$, corresponding to opening box A or opening box B. They are defined as follows:

$$A = a\ket{a}\bra{a} + a'[\ket{b}\bra{b} + \ket{c}\bra{c}], \quad (3a)$$

and

$$B = b\ket{b}\bra{b} + b'[\ket{a}\bra{a} + \ket{c}\bra{c}], \quad (3b)$$

where $a'$ and $b'$ are the eigenvalues corresponding to finding the particle to be not in box A or B, respectively. Their associated eigenspaces are the planes $bc$ and $ac$ (See Figure 1). We also define the states

$$\ket{a'} = \frac{1}{\sqrt{2}}(\ket{b} + \ket{c}) \quad (4a)$$

and

$$\ket{b'} = \frac{1}{\sqrt{2}}(\ket{a} + \ket{c}) \quad (4b)$$

obtained by projecting the initial state $\ket{\psi_0}$ onto the degenerate eigenspaces corresponding to the planes $bc$ and $ac$, respectively. These states describe the system after a minimally disturbing measurement of $A$ or $B$ yielding outcome $a'$ or $b'$, i.e., the outcome in which the particle is found to be not in the box which was opened.
Figure 1

The pre- and post-selected states $|\psi_0\rangle$ and $|\psi_f\rangle$ in the three-box basis.

$|b'\rangle = \frac{1}{\sqrt{2}}(|a\rangle + |c\rangle)$

$|a'\rangle = \frac{1}{\sqrt{2}}(|b\rangle + |c\rangle)$
With the above pre- and post-selection, the ABL rule (1) gives probability one for an outcome of either $a$ or $b$ at time $t_1$ ($t_0 < t_1 < t_f$) upon measurement of observable $A$ or $B$ corresponding to opening box A or box B.

3. Analysis and resolution.

Vaidman interprets the above results as indicating that there are two ‘elements of reality’ for this system, corresponding to both the particle being in box A (if we look for it there) and the particle being in box B (if we look for it there). These ‘elements of reality’ are, indeed, highly peculiar and counterintuitive. But need we really accept them as ‘elements of reality’? I will argue in the negative: these results cannot be interpreted as applying to an individual system such as the particle in the above example.

Consider first an experiment in which we start with a large number of systems pre-selected in state $|\psi_0\rangle$ and we choose to open box A at time $t_1$, thus measuring observable $A$ (see Figure 2(a)).
(a): A measurement of observable $A$ is performed at time $t_1$. The numbers in parentheses indicate the fraction of particles selected in the given state at each measurement.

(b): A measurement of observable $B$ is performed at time $t_1$. 
Of those systems, roughly 1/3 will be found in box A and 2/3 will be found to be not in box A. Subsequently, when we perform the post-selection measurement of \( Q_f \), the distribution will be as follows: of those systems found at time \( t_1 \) in box A, 1/3 will be post-selected in the state \( |\psi_f\rangle \). However, none of the particles that were found to be not in A can be post-selected in state \( |\psi_f\rangle \), since the state \( |a'\rangle \) corresponding to “not in box A” is orthogonal to \( |\psi_f\rangle \). Thus the actual process occurring in this experiment is one in which roughly 1/9 of the pre-selected particles will be post-selected; and all the particles that are post-selected will be guaranteed to be ones that were found in box A at time \( t_1 \).

If we consider an experiment in which the observable \( B \) is measured at time \( t_1 \), we observe exactly the same statistics but with the roles of observables \( A \) and \( B \) interchanged (Figure 2b). In each case, any particles which end up post-selected are ones which could not have been in any box except the one which was opened (be it A or B). Thus, we see that a necessary (but not sufficient) condition for post-selection of a particular particle \( X \) via a measurement of (for instance) \( A \) is that particle \( X \) was found in box A at time \( t_1 \). Since the same particle \( X \)'s being found in box A and being found in box B at the same instant of time are mutually exclusive states of affairs, for purely physical reasons it is clearly incorrect to say of any such particle \( X \) that it had a probability 1 of being found in box B at time \( t_1 \).

To make a claim about the elements of reality of an individual system, we have to consider the physical situation involved in an individual run of the experiment. But in each run, we have to make a choice as to measure \( A \) or \( B \). In the cases that we choose to measure \( A \), all successfully post-selected particles had to be found in box A at \( t_1 \), and mutatis mutandis for a measurement of
B. This means that it is not valid to say of any *individual particle*, “If in the intermediate time it was searched for in box A, it has to be found there with probability one, and if, instead, it was searched for in box B, it has to be found there too with probability one...” (Vaidman, 1996).

The same argument applies to the generalized example of a particle in \( N + 1 \) boxes as discussed by Aharonov and Vaidman (1991). Aharonov and Vaidman say of this example, “in spite of the fact that we have only one particle in the above situation, we find this particle with probability one in any one of the first \( N \) boxes” (1991, p. 2318). However, as shown above, this statement is inaccurate in the sense that the property of being with certainty in any one of \( N \) boxes (depending on which one is opened) *cannot apply to the same individual particle* in any given run of the experiment. Thus these ‘elements of reality,’ as defined by Vaidman, are not really properties of individual systems but apply only to ensembles.

The fallacy of attributing these “peculiar” properties to a single particle can be seen as arising directly from the counterfactual reading of the ABL rule. As noted above, in any given run of the experiment in which a given particle \( X \) is post-selected, we can measure only one of the two observables \( A \) and \( B \).

Once we have chosen one of these, say \( A \), it is erroneous to apply a counterfactual reading of the ABL rule to particle \( X \) with respect to a measurement of observable \( B \) which has not actually occurred in the process of post-selecting that particle. (As noted in section 1, this point is further elaborated upon in the Appendix, which discusses conditions for validity of the counterfactual ABL usage in terms of Griffiths’ consistent histories formalism.)

4. Critique of Vaidman’s counterarguments

In his most recent paper, Vaidman (1998) presents counterarguments to
some recent refutations of the counterfactual usage of the ABL rule. Vaidman claims that the refutations are based on confusion about the evaluation of counterfactual statements in quantum theory, especially with respect to its indeterminism; and about the role of time symmetry in counterfactual statements. However, none of these counterarguments succeed in identifying any flaw in the refutations, as will be shown below.

Vaidman starts by noting that “there is a general philosophical trend to consider counterfactuals to be asymmetric in time” (1998, p.2). He then quotes Lewis: “I believe that indeterminism is neither necessary nor sufficient for the asymmetries I am discussing. Therefore I shall ignore the possibility of indeterminism in the rest of this paper, and see how the asymmetries might arise even under strict determinism.” (Lewis 1986, p.37) Vaidman states that, “in contrast to this opinion”, he believes that “indeterminism is crucial for allowing non-trivial time-symmetric counterfactuals, and that Lewis’s and other general philosophical analyses are irrelevant for the issue of counterfactuals in quantum theory.”

This opening statement is a very curious one for several reasons. It starts by making an observation that time asymmetry is usually assumed in theories of counterfactuals, while making no attempt to show that such theories depend in any way on time asymmetry—the latter being the crucial consideration. The mere fact that time asymmetry may be a metaphysical presupposition or prejudice falls far short of showing that classical theories of counterfactuals are inimical or irrelevant to time-symmetric counterfactuals. So the observation about time-asymmetry preferences in no way casts doubt on the soundness or appropriateness of theories such as Lewis’ or Stalnaker’s (1968) for analyzing time-symmetric counterfactuals.
Now to the question of indeterminism. The sketch of an argument by Vaidman, given above, is incomplete at best. It appears to take Lewis’ above-quoted statement in favor of the sufficiency of determinism for time asymmetric counterfactuals as being contrary to Vaidman’s view in favor of the necessity of indeterminism for time symmetric counterfactuals, which it is not. Vaidman then summarily takes the same argument by Lewis as an indication against the applicability of Lewis’ theory to indeterministic or time-symmetric situations.

However, none of these negative assertions about Lewis’ theory has been supported by sound, or even anything approaching complete, arguments. In particular, it has not been shown that Lewis’ theory is intrinsically inapplicable either to indeterminism or to time symmetry. Thus, it is specious to suggest that any theory of counterfactuals that considers deterministic and/or time asymmetric situations is automatically disqualified for use with indeterministic and/or time-symmetric situations. Vaidman’s conclusion that “Lewis’s and other general philosophical analyses are irrelevant for the issue of counterfactuals in quantum theory” would appear to be a completely groundless dismissal of a perfectly general and sound counterfactual theory.

Furthermore, Vaidman then goes on to claim that “the key questions in [Lewis’ and similar theories] are related to [the antecedent] A....Do we need a ‘miracle’ (i.e., breaking the laws of physics) for A?...”, and asserts that “miracles” are “the main topic of discussion on counterfactuals in general philosophy.” (1998, p.2) Again, this is simply incorrect. In his seminal book *Counterfactuals*, Lewis spends three pages, out of a total of 142, discussing miracles (1973, pp. 75-77); this is in connection with a discussion of the importance of laws of nature in determining the closeness of worlds under the

---

4A good indication that Lewis in no way presupposes determinism in his theory is the following statement, (Lewis 1973, p. 75): “Suppose that the laws prevailing at a world i are deterministic as we used to think the laws of our own world were.”
relevant similarity relation. Thus, despite Vaidman’s suggestions to the contrary, Lewis’ theory is perfectly applicable in an indeterministic universe where miracles are irrelevant.

Vaidman goes on to propose a definition for time-symmetric counterfactuals in quantum theory. His definition is based on a proposed similarity relation which “fixes” the pre- and post-selection outcomes. A detailed critique of this definition has already been given in Kastner (1998); these arguments will not be repeated here except to note that the “fixing” requirement is highly problematic and amounts to proposing an unphysical similarity relation.

In this paper, we discuss an additional problem with Vaidman’s proposal for a time-symmetrized counterfactual: a problem with the syntax of the definition which reflects a confusion between the non-counterfactual and counterfactual usage of the ABL rule.

Vaidman’s proposed definition is:

“If it were that a measurement of an observable $A$ has been performed at time $t_1$, $t_0 < t_1 < t_f$, then the probability for $A = a_i$ would be equal to $p_i$, provided that the results of measurements performed on the system at times $t_0$ and $t_f$ are fixed.”

The above definition incorporates a strange and awkward mixing of tenses: “If it were that...” (the subjunctive tense) juxtaposed with “a measurement of an observable $A$ has been performed...” (the past perfect tense). This muddling of tenses suggests that the definition attempts to strike a ‘middle ground’ between two distinct usages: (a) “If observable $A$ was measured, then the probability for $A = a_i$ (at time $t_1$) was equal to $p_i$.” This is the non-counterfactual (i.e., material conditional) usage of the ABL rule, and it is

---

*I have slightly changed the notation for consistency with that used in this discussion and in the Appendix. In the original quote, Vaidman uses $t_1$ instead of $t_0$, $t$ instead of $t_1$, and $t_2$ instead of $t_f$. 

12
(b) “If observable \( A \) had been measured (instead of some other observable which was actually measured), then the probability for \( A = a_i \) (at time \( t_1 \)) would have been \( p_i \).” This is the \textit{bona fide} subjunctive conditional, or counterfactual, usage, and it is generally incorrect, as discussed in Kastner (1998).

Consider the two usages (a) and (b) in the context of an actual experimental situation. If observable \( A \) was \textit{actually} measured at time \( t_1 \), then usage (a) applies nontrivially and usage (b) reduces to the case of a counterfactual with true antecedent, or a material conditional (see Lewis 1973, p. 26); in other words it becomes logically equivalent to (a). If, on the other hand, observable \( A \) was \textit{not} actually measured, but instead some other (noncommuting) observable \( B \), then usage (a) is still correct but now applies only vacuously (i.e., proposition (a) is vacuously true). Meanwhile, usage (b) becomes a \textit{bona fide} counterfactual with false antecedent; this usage is now incorrect, and proposition (b) is false.

Once it is admitted that some definite observable (perhaps the trivial observable \( I \) corresponding to no measurement) was \textit{actually} measured in the selection of any given system—regardless of whether or not we are privy to that information—we are forced to choose between the two situations described above. If observable \( A \) was actually measured at time \( t_1 \), then (a) is equivalent to (b) and they both apply; if observable \( A \) was \textit{not} actually measured at time \( t_1 \), then (a) is still (vacuously) correct but (b) is not (except in certain special cases, as discussed in Kastner (1998), Cohen (1995)). Thus Definition (*), as it stands, is grammatically incorrect in a way that reflects its lack of clarity and rigor with respect to the physically crucial point concerning which measurement has \textit{actually} taken place.
5. Conclusion

It has been argued that an apparent paradox proposed by Aharonov and Vaidman (1991), and further amplified by Vaidman (1996), to illustrate peculiarities of time-symmetric quantum systems is not a true paradox, but arises from an invalid counterfactual usage of the ABL rule. The paradox consists in the apparent assignment of mutually exclusive properties to a system; however it is resolved by noting that these properties can never be simultaneously attributed to the same individual system. A connection is made between this problem and the consistent histories approach of Griffiths. In addition, some counterarguments by Vaidman against refutations of the counterfactual usage of the ABL rule are analyzed and shown to be ineffective.

6. Appendix

The consistent histories (henceforth ‘CH’) approach pioneered by Griffiths (1984, 1996, 1998) has been widely discussed in connection with the problem of assigning properties to quantum systems independent of measurement. In particular, Cohen (1995, pp. 4376-7) gives a concise summary of the basic features of the formulation in that context. In this Appendix we will very briefly review the fundamental features of CH in order to relate it to the ABL rule. Readers desiring a more complete exposition of CH may wish to refer to the Griffiths and/or Cohen references noted above.

A “history” as defined by Griffiths is a series of events

\[ D \to E_1 \to E_2 \to \ldots \to F, \]  

occurring at times \( t_0, t_1, \ldots t_f \), with the subscripts on the events denoting the time of their occurrence; events D and F represent the initial and final events occurring at \( t_0 \) and \( t_f \), respectively.
A particular history is considered as a member of a family of histories associated with the “events sets” \([E^\alpha_i]\), where \([E^\alpha_i]\) is a set of orthogonal projections, i.e.,

\[
E^\alpha_k E^\beta_k = \delta_{\alpha\beta} E^\alpha_k
\]

comprising a decomposition of the identity:

\[
1 = \sum_\alpha E^\alpha_k.
\]

For the special case in which we are interested, that is, three time indices \((t_0 < t_1 < t_f)\) and zero Hamiltonian, a given history is considered to be a consistent history if and only if, for all \(\alpha \neq \beta\) the histories in its associated event sets satisfy the condition:

\[
ReTr((D E^\alpha F)^\dagger D E^\beta F) = 0.
\]

This condition ensures that the probabilities of disjoint individual histories comprising the family are additive, thus disallowing quantum mechanical ‘interference’ between mutually exclusive histories. Families of histories satisfying condition (8) are called “consistent families”, or “frameworks”.

The probability of a given consistent history \(Y\),

\[
Y = D \land E^\alpha \land F
\]

is then given by

\[
P(Y) = Tr((D E^\alpha F)^\dagger D E^\alpha F) = Tr(E^\alpha D E^\alpha F),
\]
using the fact that projection operators are idempotent and self-adjoint, and
that the trace is invariant under cyclic permutations. Since the above now
behaves exactly as an ordinary classical probability, this result can be extended
in the usual way to conditional probabilities via Bayes’ rule, i.e.:

\[ P(E^\alpha|D \land F) = \frac{P(D \land E^\alpha \land F)}{P(D \land F)} \]  \hspace{1cm} (11)

\[ = Tr(E^\alpha DE^\alpha F)/Tr(DF). \]  \hspace{1cm} (12)

The consistent histories formulation applies to a closed system usually taken
to be a composite of quantum system S and measuring apparatus M, with
associated Hilbert space \( H = S \otimes M \). In order to relate the conditional
probability in (11) to the ABL rule, consider a typical experiment in which sys-
tem S is preselected in state \( |D\rangle \) and post-selected in state \( |F\rangle \). In accordance
with the notation of Griffiths, we denote the projection operators associated
with quantum states simply by the letters labeling the state. Apparatus M,
which measures the post-selection observable, starts out at time \( t_0 \) in a ready
state \( M_0 \). At time \( t_1 \), the apparatus remains untriggered but we consider a
framework in which S has some value \( C_k \) associated with an arbitrary observ-
able \( C \) defined over the system Hilbert space \( S \). At time \( t_f \), apparatus M has
been triggered in state \( M_F \) corresponding to finding the system in state \( F \).

Thus the history analogous to (5) is in this case:

\[ D \otimes M_0 \rightarrow C_k \otimes M_0 \rightarrow F \otimes M_F, \]  \hspace{1cm} (13)

We can now make the connection with the ABL rule, which (in its counter-
factual form) essentially asks: What is the probability that the system is in
state $C_k$ at time $t_1$, given that it was preselected in state $D$ and post-selected in state $F$?  

In the Griffiths formalism, this probability is given by:

\[
P(C_k|(D \otimes M_0) \wedge (F \otimes M_F)) = \frac{Tr[(D \otimes M_0)(C_k \otimes I)(F \otimes M_F)(C_k \otimes I)]}{Tr[(D \otimes M_0)(F \otimes M_F)]} \tag{14}
\]

\[
= \frac{Tr(DC_kFC_k)Tr(M_0M_F)}{Tr(DF)Tr(M_0M_F)} = \frac{\sum_i \langle C_i|DC_kFC_k|C_i \rangle}{\sum_i \langle C_i|DF|C_i \rangle} = \frac{\langle C_k|DC_kFC_k|C_k \rangle}{\sum_{i,j} \langle C_i|D|C_j \rangle \langle C_j|F|C_i \rangle} \tag{15}
\]

Note that (15) is only equivalent to the ABL rule if we further assume that the history $Y$ is consistent, i.e., that for $i \neq j$,

\[
Re(\langle C_i|D|C_j \rangle \langle C_j|F|C_i \rangle) = 0. \tag{16}
\]

Applying that condition, we then can say:

\[
P_{CH}(C_k|D \otimes M_0 \wedge F \otimes M_F) = \frac{\langle C_k|DC_kFC_k \rangle}{\sum_i \langle C_i|D|C_i \rangle \langle C_i|F|C_i \rangle} = \frac{|\langle D|C_k|F \rangle|^2}{\sum_i |\langle D|C_i|F \rangle|^2}, \tag{17}
\]

which is the ABL rule. Thus, the ABL rule can be obtained as a special case of the consistent histories approach.

The counterfactual usage of the ABL rule is equivalent to asserting that

\footnote{The non-counterfactual usage corresponds to the question: Given that a measurement of observable $C$ is performed at $t_1$ on the pre- and post-selected system, what is the probability that the measurement outcome is $C_k$?}

\footnote{This discussion assumes we are considering a counterfactual measurement, i.e., probabilities associated with outcomes of an observable $C$ that has not actually been measured. If $C$ is actually measured at time $t_1$, then (according to the orthodox interpretation) the interference terms corresponding to $i \neq j$ vanish upon measurement.}
histories associated with an observable that was not actually measured at time 
$t_1$ may be meaningfully added to the event set \{D, [E^\alpha], F\}. In the three-box 
example, the two observables that could be measured at $t_1$ are $A$ and $B$. The 
counterfactual usage thus corresponds to the set of histories

$$
\psi_0 \rightarrow [A^\alpha, B^\beta] \rightarrow \psi_f
$$

where

- $A^1 = a$,
- $A^2 = b + c$,
- $B^1 = b$,
- and $B^2 = a + c$.

However, (18) is not a consistent family, as can be seen by applying the con-
 sistency condition (8) or (16). For $E^\alpha = a$ and $E^\beta = b$ we find the nonvanishing 
result:

$$
Re Tr \left[ (\psi_0 a \psi_f)^\dagger \psi_0 b \psi_f \right] = Re \left[ \langle a | \psi_0 | b \rangle \langle b | \psi_f | a \rangle \right] = \frac{1}{9} \neq 0.
$$

Since condition (16) fails, we cannot use the ABL rule to calculate the prob-
ability of any particular value of either $A$ or $B$ at time $t_1$, but instead must use 
(15). As Griffiths (1984, 1996, 1998) has shown, only if the family of histories 
is consistent can we combine probabilities in the classical way so as to make 
inferences such as “If I had measured $B$ instead of $A$ at $t_1$, particle X would 
have been in box B”.

According to Griffiths, it is “not meaningful” to consider together the prob-
abilities of histories that are not consistent, i.e., which belong to different frameworks. While terms such as “not meaningful” might be criticized on the same basis as has been classical positivism (i.e., it has long been recognized that dismissals by positivists of certain statements as “meaningless” have been untenable), Griffiths’ admonition not to combine probabilities of histories belonging to inconsistent families has important physical content, as can be seen in reference to the three-box example. Specifically, in this example, the proscription against combining those probabilities obtained by applying the ABL rule first to a measurement of observable \( A \) and then again to a measurement of observable \( B \) corresponds physically to the fact that the associated properties (being in box A or being in box B) cannot be possessed by the same individual particle.

**Acknowledgements**

The author gratefully acknowledges valuable correspondence and/or discussions with J. Bub, R. B. Griffiths, and J. Malley.

**References**

Aharonov, Y, Bergmann, P.G., and Lebowitz, J.L. (1964), ‘Time Symmetry in the Quantum Process of Measurement,’ *Physical Review B* 134, 1410-16.

Aharonov, Y. and Vaidman, L. (1991), ‘Complete Description of a Quantum System at a Given Time,’ *Journal of Physics A* 24, 2315-28.

Albert, D.Z., Aharonov, Y., and D’Amato, S. (1986), “Comment on ‘Curious Properties of Quantum Systems Which Have Been Both Preselected and Post-Selected,” *Physical Review Letters* 56, 2427.

Cohen, O. (1995), ‘Pre- and postselected quantum systems, counterfactual measurements, and consistent histories,’ *Physical Review A* 51, 4373-4380.

---

*\(^8\)Cf. Griffiths (1998, p. 1607).*

*\(^9\)Cf. Popper (1959, pp. 35-6).*
Griffiths, R.B. (1984), ‘Consistent Histories and the Interpretation of Quantum Mechanics,’ *Journal of Statistical Physics* 36, 4373.

Griffiths, R.B. (1996), ‘Consistent Histories and Quantum Reasoning,’ *Physical Review A* 54, 2759-2774.

Griffiths, R.B. (1998), ‘Choice of Consistent Family, and Quantum Incompatibility,’ *Physical Review A* 57, 1604-1618.

Kastner, R.E. (1998), ‘Time-Symmetrized Quantum Theory, Counterfactuals, and “Advanced Action,”’ to be published, *Studies in History and Philosophy of Modern Physics; quant-ph/9806002*.

Miller, D.J. (1996), ‘Realism and Time Symmetry in Quantum Mechanics,’ *Physics Letters A* 222, 31.

Popper, Karl R. (1959), *The Logic of Scientific Discovery*, London: Routledge.

Sharp, W. D. and Shanks, N. (1993), ‘The Rise and Fall of Time-Symmetrized Quantum Mechanics,’ *Philosophy of Science* 60, 488-499.

Stalnaker, R. (1968), ‘A Theory of Conditionals,’ in *Studies in Logical Theory*, edited by N. Rescher. Oxford: Blackwell.

Vaidman, L. (1996), ‘Weak-Measurement Elements of Reality,’ *Foundations of Physics* 26, 895-906.

Vaidman, L. (1998), ‘Time-Symmetrized Counterfactuals in Quantum Theory,’ preprint, *quant-ph/9802042*.