Coding Schemes for a Class of Receiver Message Side Information in AWGN Broadcast Channels

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Abstract—This paper considers the three-receiver AWGN broadcast channel where the receivers (i) have private-message requests and (ii) know some of the messages requested by other receivers as side information. For this setup, all possible side information configurations have been recently classified into eight groups and the capacity of the channel has been established for six groups (Asadi et al., ISIT 2014). We propose inner and outer bounds for the two remaining groups, groups 4 and 7. A distinguishing feature of these two groups is that the weakest receiver knows the requested message of the strongest receiver as side information while the in-between receiver does not. For group 4, the inner and outer bounds coincide at certain regions. For group 7, the inner and outer bounds coincide, thereby establishing the capacity, for four members out of all eight members of the group; for the remaining four members, the proposed bounds reduce the gap between the best known inner and outer bounds.

I. INTRODUCTION

We consider private-message broadcasting over the three-receiver additive white Gaussian noise broadcast channel (AWGN BC) where the receivers may know some of the source messages a priori. We investigate the capacity of the channel for a class of side information where the weakest receiver knows the requested message of the strongest receiver as side information while the in-between receiver does not.

A. Background

The capacity of BCs [1] with receiver message side information, where each receiver may know some of the messages requested by other receivers as side information, is of interest due to applications such as multimedia broadcasting with packet loss, and multi-way relay channels [2]. The capacity of these channels is known when each receiver needs to decode all the source messages (or equivalently, all the messages not known a priori) [3], [4]. Otherwise, the capacity of BCs with receiver message side information is not known in general.

The capacity of the two-receiver discrete-memoryless BC when one of the receivers need not decode all the source messages has been established by Kramer et al. [5]. The capacity of the two-receiver AWGN BC is known for all message request and side information configurations [6]. Oechtering et al. [7] established the capacity of the three-receiver less-noisy and more-capable broadcast channels for some message request and side information configurations where (i) only two receivers possess side information and (ii) the request of the third receiver is only restricted to a common message.

B. Existing Results and Contributions

Considering private-message broadcasting over the three-receiver AWGN BC, Yoo et al. [8] proposed a separate index and channel coding scheme that achieves within a constant gap of the channel capacity for all side information configurations. For this setup, all side information configurations have been recently classified into eight groups and the capacity of the channel has been established for six groups [9].

In this paper, we propose new inner and outer bounds for the two remaining groups, groups 4 and 7. For group 4, the proposed inner and outer bounds coincide at certain regions. The inner bound is achieved by two schemes employing dirty paper coding [10] with different order of encoding. The outer bound employs the notion of an enhanced channel [11], and is shown to be tighter than the best existing one [8]. For group 7, the proposed inner and outer bounds coincide, thereby establishing the capacity, for four members out of all eight members of the group; for the remaining four members, we improve both the best existing inner bound [9] and outer bound [8].

II. SYSTEM MODEL

In the channel model under consideration, as depicted in Fig. 1, the signals received by receiver $i$, $Y_i^{(n)} = (Y_{i1}, Y_{i2}, \ldots, Y_{in})$ $i = 1, 2, 3$, is the sum of the codeword transmitted by the sender, $X^{(n)}$, and an i.i.d. noise sequence, $Z_i^{(n)} i = 1, 2, 3$, with normal distribution, $Z_i \sim \mathcal{N}(0, N_i)$. This channel is stochastically degraded, and without loss of generality, we can assume that receiver 1 is the strongest and receiver 3 is the weakest in the sense that $N_1 \leq N_2 \leq N_3$.

The transmitted codeword has a power constraint of $\sum_{i=1}^{n} E(X_i^2) \leq nP$ and is a function of source messages $\mathcal{M} = \{M_1, M_2, M_3\}$. The messages are independent, and $M_i$ is intended for receiver $i$ at rate $R_i$ bits per channel use, i.e., $m_i \in \{1, 2, \ldots, 2^{nR_i}\}$. The capacity of the channel is the closure of the set of all rate triples $(R_1, R_2, R_3)$ that are achievable in the Shannon sense [1].

We define the knows set $\mathcal{K}_i$ as the set of messages known to receiver $i$. The side information configuration of the channel is modeled by a side information graph, $\mathcal{G} = (\mathcal{V}_G, \mathcal{A}_G)$, where $\mathcal{V}_G$ is the set of vertices and $\mathcal{A}_G$ is the set of arcs. Vertex $i$ represents both $M_i$ and receiver $i$ requesting it. An arc from

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vertex $i$ to vertex $j$, denoted by $(i \to j)$, exists if and only if receiver $i$ knows $M_j$. The set of out-neighbors of vertex $i$ is then $O_i \triangleq \{ j | (i \to j) \in A_i \} = \{ j | M_j \in K_i \}$. For instance, in the following side information graph

receiver 1 knows $M_3$, and receiver 3 knows $M_2$.

All possible side information graphs for the three-receiver case, i.e., $\mathcal{V}_s = \{1, 2, 3\}$ have been classified into eight groups [9]. Any side information graph is the union of $G_{ij}$ (depicted in Fig. 2) and $G_{jk}$ (depicted in Fig. 3) for some unique $j$ and $k$ where $j, k \in \{1, 2, \ldots, 8\}$. According to this classification, the side information graphs $\{G_{ij}\}_{j=1}^8$ are considered as the group leaders, and group $j$ consists of the side information graphs formed by the union of $G_{ij}$ with each of the $\{G_{jk}\}_{k=1}^8$.

In this work, we investigate the capacity of the channel for graphs formed by the union of $G_j$, i.e.,

\[ G_j \triangleq \bigcup_{k=1}^8 G_{jk} \]

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receiver 2, \(m_2\) is decoded without being affected by \(x_1^{(n)}\) due to dirty paper coding. At receiver 3, \(x_1^{(n)}\) is just decoded while \(x_2^{(n)}\) is treated as noise.

The achievability of \(\mathscr{R}_m\) is verified by using the following decoding methods. At receiver 3, \(m_3\) is decoded without being affected by \(x_2^{(n)}\) due to dirty paper coding. At receiver 2, \(x_2^{(n)}\) is decoded while \(x_1^{(n)}\) is treated as noise. At receiver 1, \(x_1^{(n)}\) is first decoded while \(x_2^{(n)}\) is treated as noise, and then \(x_1^{(n)}\) is decoded.

Note that the receivers utilize their side information during channel coding. Also, for \(G_{14} \cup G_{22}\) and \(G_{14} \cup G_{25}\), Fourier-Motzkin elimination is used subsequent to channel decoding to obtain \(\mathscr{R}_m\), and \(\mathscr{F}_m\) in terms of \((R_1, R_2, R_3)\).

In this paper, all rate bound derivations use standard techniques, and are omitted due to space limitations.

**B. Proposed Outer Bound**

The proposed outer bound for group 4, stated as Theorem 2, is formed from the intersection of two outer bounds.

**Theorem 2:** If the rate triple \((R_1, R_2, R_3)\) is achievable for a member of group 4, then it must lie in \(\mathcal{R}_{out1} \cap \mathcal{R}_{out2}\), where \(\mathcal{R}_{out1}\) is the set of all rate triples, each satisfying

\[
R_1 \leq C \left(\frac{P}{N_1}\right),
\]

\[
R_2 \leq C \left(\frac{\alpha P}{N_2}\right),
\]

\[
R_3 \leq C \left(\frac{(1-\alpha)P}{\alpha P + N_3}\right),
\]

for some \(0 \leq \alpha \leq 1\), and \(\mathcal{R}_{out2}\) is the capacity of the enhanced channel for the member obtained by decreasing the received noise variance of receiver 3 from \(N_3\) to \(N_2\).

**Proof:** Condition [1] in \(\mathcal{R}_{out1}\) is due to the point-to-point channel capacity between the transmitter and receiver 1. Conditions [2] and [3] follow from the capacity of the two-receiver AWGN BC (from the transmitter to receivers 2 and 3) where only the stronger receiver (receiver 2) may know the requested message of the weaker receiver (receiver 3) as side information. In this group, the side information of receivers 2 and 3 about each other’s requested messages has this property.

Outer bound 2, \(\mathcal{R}_{out2}\), is developed using the idea of enhanced channel [11]. The capacity of the enhanced channel is an outer bound to the capacity of the original channel. Since the received noise variance of the two weakest receivers in the defined enhanced channel are equal, this channel can also be considered as a member of group 5 or 3 depending on whether receiver 2 in the original channel knows \(M_3\) or not, respectively. The capacity of the channel with a side information configuration in group 3 or 5 is known [9]. For example, the enhanced channel for \(G_{14} \cup G_{21}\) can be considered as \(G_{13} \cup G_{21}\), and the one for \(G_{14} \cup G_{22}\) as \(G_{15} \cup G_{21}\).

**C. Evaluation of the Proposed Inner and Outer Bounds**

In this subsection, we first show that the proposed outer bound is tighter than the best existing one. We next characterize the regions where the proposed inner and outer bounds coincide.

In order to prove that our proposed outer bound is tighter than the best existing one, we show that, for any condition that must be met in the best existing outer bound, the proposed outer bound includes some more restrictive conditions. We present the proof for \(G_{14} \cup G_{21}\) in the following; the proof for the other members is similar. Our proposed outer bound is the intersection of the bound given in (1–3) and the capacity of the enhanced channel for \(G_{14} \cup G_{21}\). According to the enhanced channel, if the rate triple \((R_1, R_2, R_3)\) is achievable for \(G_{14} \cup G_{21}\), it must satisfy

\[
R_1 + R_3 \leq C \left(\frac{\gamma P}{N_1}\right),
\]

\[
R_2 \leq C \left(\frac{(1-\gamma)P}{\gamma P + N_2}\right),
\]

\[
R_3 \leq C \left(\frac{P}{N_3}\right),
\]

for some \(0 \leq \gamma \leq 1\). Based on the best existing outer bound [8] if the rate triple \((R_1, R_2, R_3)\) is achievable, it must satisfy

\[
\sum_{i \in V_S} R_i \leq \max_{i \in V_S} C \left(\frac{P}{N_1}\right),
\]

for all induced acyclic subgraphs, \(S\), of the side information graph. Then, for \(G_{14} \cup G_{21}\), if the rate triple \((R_1, R_2, R_3)\) is achievable, it must satisfy \(R_3 \leq C \left(\frac{P}{N_3}\right)\), \(R_2 + R_3 \leq C \left(\frac{P}{N_2}\right)\), and \(R_1 + R_2 + R_3 \leq C \left(\frac{P}{N_1}\right)\). Concerning \(R_3 \leq C \left(\frac{P}{N_3}\right)\), if condition [3] in \(\mathcal{R}_{out}\), is satisfied, this condition is also satisfied. Conditions [2] and [3] in \(\mathcal{R}_{out1}\) are more restrictive than \(R_2 + R_3 \leq C \left(\frac{P}{N_2}\right)\), and conditions [4] and [5] in \(\mathcal{R}_{out2}\) are more restrictive than \(R_1 + R_2 + R_3 \leq C \left(\frac{P}{N_1}\right)\). This completes the proof for \(G_{14} \cup G_{21}\).

Here, we characterize the certain regions where the proposed inner and outer bounds coincide. For any fixed \(R_1\) where \(0 \leq R_1 \leq C \left(\frac{P}{N_1}\right)\), the proposed bounds are tight when \(R_3 \leq R_{thr3}\) or \(R_3 \geq R'_{thr3}\) where \(R_{thr3} \leq R'_{thr3}\) or similarly, when \(R_2 \leq R_{thr2}\) or \(R_2 \geq R'_{thr2}\) where \(R_{thr2} \leq R'_{thr2}\). The thresholds are functions of \(R_1\). The same behavior can be observed for any

**TABLE I**

| Transmission Scheme 1 & Inner Bound 1 (\(\mathcal{A}_{thr1}\)) | Transmission Scheme 2 & Inner Bound 2 (\(\mathcal{A}_{thr2}\)) |
|---------------------------------------------------------------|---------------------------------------------------------------|
| \(x_1^{(0)}([m_1, m_3]) + x_2^{(0)}([m_2, m_3])\) for all \(\alpha \in \mathcal{Q}\) | \(x_1^{(0)}([m_1, m_3]) + x_2^{(0)}([m_2, m_3])\) for all \(\alpha \in \mathcal{Q}\) |
| \(\sum_{i \in \{3\}} R_i < C \left(\frac{1-\alpha P}{N_1}\right), R_2 < C \left(\frac{\alpha P}{N_2}\right), R_3 < C \left(\frac{(1-\alpha)P}{\alpha P + N_3}\right)\) | \(\sum_{i \in \{3\}} R_i < C \left(\frac{1-\alpha P}{N_1}\right), R_2 < C \left(\frac{\alpha P}{N_2}\right), R_3 < C \left(\frac{(1-\alpha)P}{\alpha P + N_3}\right)\) |

Transmission Scheme 1 & Inner Bound 1 (\(\mathcal{A}_{thr1}\))

Transmission Scheme 2 & Inner Bound 2 (\(\mathcal{A}_{thr2}\))
TABLE II
GROUP 7: PROPOSED TRANSMISSION SCHEMES AND INNER BOUNDS

| Member | Graph | Transmission Scheme | Inner Bound ($\mathcal{R}_i^0$) |
|--------|-------|---------------------|----------------------------------|
| $\mathcal{G}_{17} \cup \mathcal{G}_{21}$ | $\mathcal{G}_{17} \cup \mathcal{G}_{23}$ | $x_1^{(n)}([m_{10}, m_{30}]) + x_2^{(n)}([m_{21}, m_{31}])$ | $R_2 + \sum_{\ell \in \{1,3\}\setminus i} R_{\ell}^{(a)} < C \left( \frac{(1-\alpha)P}{\alpha N_1} \right) + C \left( \frac{\alpha P}{N_1} \right), R_2 < C \left( \frac{(1-\alpha)P}{\alpha N_1 + N_2} \right); R_2 + R_3 < C \left( \frac{(1-\alpha)P}{\alpha P + N_2} \right) + C \left( \frac{\alpha P}{N_1} \right); R_3 < C \left( \frac{P}{N_3} \right)$. |
| $\mathcal{G}_{17} \cup \mathcal{G}_{23}$ | $\mathcal{G}_{17} \cup \mathcal{G}_{24}$ | $x_1^{(n)}([m_{10}, m_{30}]) + x_2^{(n)}([m_{21}, m_{13} \oplus m_{31}])$ | |
| $\mathcal{G}_{17} \cup \mathcal{G}_{26}$ | $\mathcal{G}_{17} \cup \mathcal{G}_{22}$ | $x_1^{(n)}([m_{10}, m_{30}]) + x_2^{(n)}([m_{21}, m_{31}])$ | |
| $\mathcal{G}_{17} \cup \mathcal{G}_{25}$ | $\mathcal{G}_{17} \cup \mathcal{G}_{27}$ | $x_1^{(n)}([m_1, m_2 \oplus m_3]) + x_2^{(n)}(m_2 \oplus m_3)$ | |

The proposed inner bound is the convex hull of the union of inner bounds 1 and 2, and the proposed outer bound is the intersection of outer bounds 1 and 2.
C. Evaluation of the Proposed Inner and Outer Bounds

In this subsection, we show that the proposed inner and outer bounds for group 7 coincide for four members and reduce the gap between the best known inner and outer bounds for the remaining four members.

For $G_{17} \cup G_{22}$ and $G_{17} \cup G_{25}$, the proposed outer bound, $R_{out}^{\prime}$, coincides with $R_{in}$ which consequently establishes the capacity. This is while $R_{out}$ (the best existing outer bound) alone is not tight for these members.

For $G_{17} \cup G_{27}$ and $G_{17} \cup G_{28}$, $R_{out}^{\prime}$, given in (8)–(10), coincides with $R_{in}^{\prime}$. This establishes the capacity for these members and shows that $R_{out}^{\prime}$ is strictly tighter than $R_{out}$ for these members (this is because $R_{out}$ has some curved surfaces while $R_{out}^{\prime}$ is a polyhedron).

For the remaining four members, we show that the inner and outer bounds are both tighter than the best existing ones.

The best existing inner bound $[9]$ for the remaining four members with unknown capacity is the set of all rate triples $(R_1, R_2, R_3)$, each satisfying

\begin{align}
& R_2 + \sum_{i \in \{1, 2\} \setminus C_1} R_i < B_1 + B_2 + B_3, \quad (11) \\
& R_2 + R_3 < B_2 + B_3, \quad (12) \\
& R_3 < \min\{C\left(\frac{\alpha B}{N_3}, B_3\right), B_3\}, \quad (13)
\end{align}

where $B_1 = C(\alpha B/N_1)$, $B_2 = C(\alpha B/(\alpha_1 P + N_2))$, and $B_3 = C(\alpha B/(\alpha B + \alpha P + N_2))$ for some $\alpha_k \geq 0$ for $k = 1, 2, 3$ such that $\sum_{k=1}^{3} \alpha_k = 1$. This inner bound for $G_{17} \cup G_{21}$ and $G_{17} \cup G_{23}$ is achieved by using the following scheme

\begin{align*}
& x_1^{(n)}(m_{10}) + x_2^{(n)}(m_{20}) + x_3^{(n)}(m_{31}), \\
& x_1^{(n)}(m_{10}) + x_2^{(n)}(m_{20}) + x_3^{(n)}(m_{21}),
\end{align*}

and for $G_{17} \cup G_{24}$ and $G_{17} \cup G_{26}$ by using the following scheme

\begin{align*}
& x_1^{(n)}(m_{10}) + x_2^{(n)}(m_{10}) + x_3^{(n)}(m_{32}),
\end{align*}

where the three subcodebooks are constructed independently using i.i.d. codewords generated according to $X_k \sim \mathcal{N}(0, \alpha_k P)$ for $k = 1, 2, 3$ for some $\alpha_k \geq 0$ such that $\sum_{k=1}^{3} \alpha_k = 1$. Also, the receivers employ a joint decoding approach $[9]$ which utilizes side information during successive decoding.

For these members, we now show that for any chosen set of $\{\alpha_k\}_{k=1}^{3}$, the region in $[11]$–[13] is smaller than $S_{in}^{\prime}$ for $\alpha = \alpha_1$. Noting that $B_2 + B_3 = C\left(\frac{(1-\alpha_1)P}{\alpha_1 P + N_2}\right)$, then condition (a) is the same as $[11]$, conditions (b) and (c) are more relaxed than $[12]$, and condition (d) is more relaxed than $[13]$. This proves that the proposed inner bound is larger than the best existing inner bound for these members.

Concerning the outer bound, since the proposed outer bound is the intersection of the best existing outer bound and a new outer bound, $S_{out}^{\prime}$, the proposed outer bound is tighter than the best existing outer bound. As an example, for $G_{17} \cup G_{24}$, Fig. 6 depicts that the proposed inner bound is strictly larger than the best existing one, and the proposed outer bound is strictly tighter than the best existing one.

V. CONCLUSION

We considered the problem of private-message broadcasting over the three-receiver AWGN BC with receiver message side information. Following the recent classification of all possible side information configurations into eight groups and the establishment of the capacity for six groups, we investigated the capacity of the channel for the two remaining groups with unknown capacity, groups 4 and 7. We proposed inner and outer bounds for these two groups. For group 4, the proposed inner and outer bounds coincide at certain regions. For group 7, the proposed inner and outer bounds coincide for four members, and for the remaining four members, the proposed inner and outer bounds are both tighter than the best existing ones.

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