Modeling of subgrid-scale cloud-clear air turbulent mixing in Large Eddy Simulation of cloud fields

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Abstract. This paper presents computational approach to locally predict the homogeneity of subgrid-scale turbulent mixing between a cloud and its environment in large-eddy simulation of warm (ice-free) shallow convective clouds applying a double-moment bulk microphysics scheme. The term homogeneity of mixing refers to the change of the mean droplet size associated with evaporation of cloud water due to entrainment. The two contrasting limits are the homogeneous mixing, where the radius of all droplets is reduced and the concentration does not change during microscale homogenization, and the extremely inhomogeneous mixing, where the microscale homogenization leads to complete evaporation of some droplets and does not affect the rest. The novel approach is applied to simulations of shallow convective cloud field. The results show that locally the homogeneity of mixing can vary significantly because of the spatial variability of the intensity of turbulence and the mean droplet radius. On average, however, the mixing becomes more homogeneous with height because of higher turbulence intensities and larger droplet sizes aloft.

1. Introduction

Turbulent entrainment and mixing between a cloud and clear air from its immediate environment is an important processes affecting macrophysical (e.g., cloud depth) and microphysical (e.g., spectrum of cloud droplets) properties of boundary layer clouds such as the shallow tropical cumulus and subtropical stratocumulus. There is ample observational evidence that shallow cumuli are strongly diluted by entrainment, with the typical mean liquid water contents around 20% of the adiabatic values (e.g., Warner, 1973; Siebesma, 2003, among many others). Entrainment of clear unsaturated air leads to evaporation of cloud water with associated cooling affecting cloud buoyancy field and leading to buoyancy reversal (e.g., Grabowski, 1993). However, entrainment-related evaporation and cooling do not happen instantaneously because turbulent mixing involves stirring and gradual filamentation of cloudy and cloud-free volumes down to the microscale homogenization scale. In this process, spatial scales of scalar fields (the temperature, moisture, and cloud water) decrease as time progresses due to the development of smaller and smaller eddies, and the microscale homogenization (i.e., evaporation of cloud water) takes place once scales close to the Batchelor and Kolmogorov scales are reached (e.g., Jensen & Baker, 1989; Grabowski, 1993; Malinowski & Zawadzki, 1993; Andrejczuk et al., 2004, 2006). Scales at which microscale homogenization occurs in clouds are much smaller than the typical...
gridlength of a large eddy simulation (LES) cloud model (below 1 cm versus tens or hundreds of meters). It follows that the homogenization is significantly delayed in nature, but there is no representation of this subgrid-scale process in traditional LES models.

As entrainment and mixing leads to the reduction of the liquid water content (LWC), the additional issue is whether the evaporation of cloud droplets results in the reduction of only the droplet size (as in the homogeneous mixing), only the droplet concentration (as in the extremely inhomogeneous mixing), or both the concentration and the size (as in the inhomogeneous mixing). This paper extends an approach for modeling subgrid-scale processes associated with entrainment and mixing proposed in Grabowski (2007) and Jarecka et al. (2009). In Grabowski (2007) and Jarecka et al. (2009), the discussion was limited to the bulk representation of cloud microphysics. Here, the approach is extended to the double-moment bulk microphysics scheme of Morrison & Grabowski (2007, 2008) to locally predict the homogeneity of mixing.

2. Delay of cloud evaporation due to the turbulent mixing

The standard thermodynamic grid-averaged equations for the 2-moment bulk advection-diffusion-condensation problem are as follows (Morrison & Grabowski, 2007):

$$\frac{\partial \theta}{\partial t} + \frac{1}{\rho_o} \nabla \cdot (\rho_o u \theta) = \left( \frac{\partial \theta}{\partial t} \right)_{act} + \frac{L_v \theta_o}{C_p T_e} C + D \theta ,$$  

(1)

$$\frac{\partial q_e}{\partial t} + \frac{1}{\rho_o} \nabla \left[ \rho_o q_e u \right] = \left( \frac{\partial q_e}{\partial t} \right)_{act} - C + D_{q_e} ,$$  

(2)

$$\frac{\partial q_c}{\partial t} + \frac{1}{\rho_o} \nabla \left[ \rho_o (u - V_c) k \right] = \left( \frac{\partial q_c}{\partial t} \right)_{act} + C + D_{q_c} ,$$  

(3)

$$\frac{\partial N_c}{\partial t} + \frac{1}{\rho_o} \nabla \left[ \rho_o (u - V_N) k \right] = \left( \frac{\partial N_c}{\partial t} \right)_{act} + \left( \frac{\partial N_c}{\partial t} \right)_{cond/evap} + D_{N_c} ,$$  

(4)

where $\theta$, $q_e$, $q_c$ and $N_c$ are the potential temperature, the water vapor mixing ratio, the cloud water mixing ratio, and the cloud droplets number concentration, respectively; $\rho_o(z)$ is the base state density profile; $V_{N/e}$ is the number-/mass-weighted mean particle fall speed; $\theta_e(z)$ and $T_e(z)$ are the environmental potential temperature and temperature profiles; $u$ is the wind velocity vector; $L_v$ and $C_p$ denote the latent heat of condensation and specific heat at constant pressure, respectively; $C$ is the condensation rate; $(\partial N_c/\partial t)_{cond/evap}$ is the source term that describes changes of the cloud droplets number concentration due to the condensation/evaporation; $(\partial * / \partial t)_{act}$ are the source terms representing activation/deactivation of cloud condensation nuclei (CCN); and $D$ terms represent subgrid-scale turbulent transport terms.

To simulate the entrainment-related delay of cloud water evaporation, Grabowski (2007) and Jarecka et al. (2009) suggested a relatively simple approach by including two additional variables: the scale (or width) of a filament $\lambda$, which characterizes the progress of turbulent stirring (Broadwell & Breidenthal, 1982; Jensen & Baker, 1989) and the fraction of the gridbox containing cloudy air $\beta$. The evolution of $\lambda$ is supposed to represent the progress of subgrid-scale turbulent mixing toward the microscale homogenization (e.g., Broadwell & Breidenthal, 1982; Jensen & Baker, 1989). Local values of the cloudy-air fraction $\beta$ are affected by resolved advection and subgrid-scale diffusion, and by the subgrid-scale homogenization. When extended into the multidimensional framework and written in the conservative (flux) form, the equation for $\lambda$ and $\beta$ are:

$$\frac{\partial \lambda}{\partial t} + \frac{1}{\rho_o} \nabla \cdot (\rho_o u \lambda) = -\gamma (\epsilon \lambda)^{1/3} + S_\lambda + D_\lambda ,$$  

(5)

$$\frac{\partial \beta}{\partial t} + \frac{1}{\rho_o} \nabla \cdot (\rho_o u \beta) = S_\beta + D_\beta ,$$  

(6)
where the first term on the right-hand side of eq. 5 describes the decrease of $\lambda$ as the turbulent mixing progresses [$\epsilon$ is the local dissipation rate of the turbulent kinetic energy (TKE) and $\gamma \sim 1$ is a nondimensional parameter taken as $\gamma = 1.8$; (see Grabowski, 2007; Jarecka et al., 2009)], $S\lambda$, $S\beta$ are the source/sink terms, and $D\lambda$, $D\beta$ are the subgrid transport terms. The source/sink terms $S\lambda$ and $S\beta$ consider three processes that affect the scale $\lambda$ and the cloudy-air fraction $\beta$: (a) initial formation of a cloudy volume due to grid-scale condensation, (b) removal of a cloudy volume due to complete evaporation of cloud water, and (c) homogenization of a cloudy volume.

A uniform cloudy gridbox is characterized by $\lambda = \Lambda$ and $\beta = 1$, where $\Lambda \equiv (\Delta x \Delta y \Delta z)^{1/3}$ ($\Delta x$, $\Delta y$, $\Delta z$ are model gridlength in $x$, $y$, and $z$ direction, respectively). A cloud-free gridbox has $\lambda = 0$ and $\beta = 0$. It follows that the source/sink term $S\lambda$ resets the current value of $\lambda$ to $\Lambda$ in cases (a) and (c), or resets $\lambda$ to 0 in the case (b). Similarly, the source/sink term $S\beta$ resets the current value of $\beta$ to 1 in cases (a) and (c), or resets $\beta$ to 0 in the case (b). Microscale homogenization of a cloudy gridbox is assumed once the scale predicted by eq. 5 falls below the threshold value $\lambda_0$ taken as 1 mm (note that $\lambda_0 = 1$ cm was used in Grabowski (2007) and Jarecka et al. (2009)).

Adding to the model new variables $\lambda$ and $\beta$ allows representing the chain of events characterizing turbulent mixing and to include a corresponding delay of evaporation in the model. In the bulk $\lambda - \beta$ model discussed in Jarecka et al. (2009) and in Grabowski (2007), the evaporation of cloud water due to the turbulent mixing was delayed until the predicted filament scale $\lambda$ reached the scale of molecular homogenization $\lambda_0$. However, one might anticipate a gradual increase of the evaporation as the scale of $\lambda_0$ is approached instead of an abrupt transition from zero to finite evaporation. This is supported by simulations using the DNS approach (Andrejczuk et al., 2004, 2006, 2009) and simulations using the linear eddy model (e.g., Krueger, 1993; Krueger et al., 1997, S. Krueger, personal communication). This is also consistent with a heuristic argument that, during the turbulent stirring, complete evaporation of cloud droplets is anticipated near the edges of the filament, while droplets away from the interface should not experience any evaporation at all (except due to resolved vertical motions). To include a gradual increase on the evaporation due to the turbulent mixing, we postulate that the amount of cloud water $\Delta q_c^*$ that evaporates at the filament edges is a fraction of the cloud water mixing ratio $\Delta q_c$ that would evaporate during model time step in the traditional model, that is, when the microphysical adjustment is applied without any subgrid-scale considerations (i.e., applying model-predicted values of $\theta$, $q_c$, $N$, and $q_v$). Heuristic arguments following ideas discussed in Sreenivasan et al. (1989); Malinowski & Zawadzki (1993) and considering the increase of the surface area of the cloud-clear air interface during turbulent stirring suggest that:

$$\Delta q_c^* = \frac{\lambda_0}{\lambda} \Delta q_c. \quad (7)$$

As expected, eq. 7 implies almost no evaporation when $\lambda >> \lambda_0$ and the correct evaporation $\Delta q_c^* \rightarrow \Delta q_c$ when $\lambda \rightarrow \lambda_0$. Note that the above considerations apply only for gridpoints affected by the entrainment and mixing, that is, when $\Delta q_c < 0$ and $\lambda < \Lambda$.

The key differences between traditional cloud models and the model with delayed entrainment-related evaporation of cloud droplets are illustrated in Figure 1.

### 3. Droplet spectrum changes due to the turbulent mixing

Mixing of cloud air with dry environmental air changes also the spectrum of cloud droplets. In general, microscale homogenization may result in the reduction of only the mean droplet size without changing the number of droplets. This is typically referred to as the homogeneous mixing. In contrast, the size may remain unchanged and only the number of droplets may be reduced. This is the extremely inhomogeneous mixing. In practice, both the size and the number of droplets may change as a result of the microscale homogenization.
Figure 1. Evaporation of cloud water as a result of turbulent mixing between cloudy and cloud-free gridboxes. The horizontal axis represents time. The two gridboxes are shown on the left hand side of the figure, at time $t_0$. During a model time step, from $t_0$ to $t_1$, parametrized turbulent mixing creates a gridbox containing both cloudy and cloud-free air. The traditional model immediately homogenizes the gridbox, resulting in either a saturated and cloudy or subsaturated and cloud-free gridbox at time $t_1$. In the modified model, homogenization is only possible once turbulent stirring reduces the filament width $\lambda$ from the initial value $\sim \Lambda$ to the value corresponding to the microscale homogenization $\lambda_0$. This process may take several model time steps. Before homogenization, the condensation rate is calculated using eq. 7.

On theoretical grounds, homogeneity of mixing depends on the relative magnitude of the time scales for droplet evaporation and turbulent homogenization. If the turbulent homogenization time scale is much shorter than the droplet evaporation time scale, the subsaturated air dilute the cloud fast enough that all droplets experience the same subsaturated conditions. In that situation sizes of all droplets decrease, so the homogeneous mixing is thought to take place. In the opposite limit, when the droplet evaporation time scale is much shorter than the turbulent homogenization time scale, droplets which are in the vicinity of the subsaturated air evaporate so fast that there is no time to uniformly dilute the cloudy air. In that case only some droplets evaporate completely and the rest does not experience any change, so the extremely inhomogeneous mixing is thought to take place.

In the Morrison & Grabowski (2008) double-moment scheme, the mixing scenario is determined by a single parameter $\alpha$. This parameter is used to calculate the final droplet concentration after entrainment and turbulent mixing according to:

$$N_f = N_i \left( \frac{q_{fc}^f}{q_{fc}^i} \right)^\alpha,$$

where $N_f$ is the final droplet concentration after microphysical adjustment due to the evaporation, $N_i$ is the droplet concentration after advection and turbulent mixing (i.e., the initial value for the microphysical adjustment), and $q_{fc}^i$ and $q_{fc}^f$ are the initial and final cloud water mixing ratios (i.e., before and after the microphysical adjustment). Note that, in the Morrison & Grabowski (2008) scheme, the microphysical adjustment of the cloud water mixing ratio $q_c$ takes place before adjusting $N$, and it is dictated by the predicted supersaturation.
and characteristics of the cloud droplet population (i.e., the droplet concentration and size). Thus, $q_c^\ell$ and $q_c^f$ in eq. 8 are already known, and eq. 8 predicts the corresponding microphysical adjustment of the droplet concentration $N$ once $\alpha$ is known. The parameter $\alpha$ varies from 0 for the case of the homogeneous mixing (i.e., no change to $N$) to 1 for the extremely inhomogeneous mixing (i.e., when $N$ changes in the same proportion as $q_c$ and thus the mean volume radius remains unchanged). In simulations presented in Morrison & Grabowski (2008), $\alpha$ could only be assumed constant in space and time during the simulation. The same applies to simulations discussed in Slawinska et al. (2011).

To predict the local value of $\alpha$, we take advantage of the direct numerical simulations (DNS) results reported in Andrejczuk et al. (2009). Andrejczuk et al. (2009) performed 72 simulations of decaying moist turbulence mimicking turbulent mixing and microscale homogenization of cloudy and clear air using detailed (bin) microphysics. They analyzed DNS results in terms of the instantaneous change of microphysical characteristics versus the ratio between the turbulent mixing and droplet evaporation time scales, $\tau_{mix}$ and $\tau_{evap}$, respectively. The change in the microphysical characteristics was measured by the slope $\delta$ of the line depicting the evolution of the total number of droplets plotted against the mean volume radius cubed, both normalized by the initial values, the $r - N$ diagram, applied in Andrejczuk et al. (2004, 2006). In this diagram, the homogeneous mixing corresponds to the horizontal line (i.e., changing droplet size without changing the number of droplets; $\delta = 0$). The vertical line (reduction of the number of droplets without changing the size; $\delta \rightarrow \infty$) implies extremely inhomogeneous mixing. The simulations suggested approximately one-to-one relationship between the ratio of the two time scales and the slope of the mixing line, that is, $\delta \sim \tau_{mix}/\tau_{evap}$ (see Fig. 2 in Andrejczuk et al. (2009)).

The slope $\delta$ is related to the parameter $\alpha$ in eq. 8. Since $q_c \sim N r^3$, eq. 8 implies that $N \sim (r^3)^{\alpha/(1-\alpha)}$. It follows that the slope $\delta \equiv dN/d(r^3)$ equals $\alpha/(1 - \alpha)$ which leads to

$$\alpha = \frac{\delta}{1 + \delta}.$$  

(9)

The turbulent homogenization time scale is calculated following Andrejczuk et al. (2009) as

$$\tau_{mix} = \lambda/u(\lambda),$$  

(10)

where $u(\lambda)$ is the characteristic velocity at spatial scale equal to the filament scale $\lambda$. It can be related to the model-predicted TKE as

$$u(\lambda) = (TKE^{1/2}((\lambda/\Lambda)^{1/3}).$$  

(11)

This relationship assumes inertial range scaling for subgrid-scale turbulence and considers TKE to be dominated by the eddies of scale $\Lambda$ [i.e., $u(\Lambda) \sim (TKE)^{1/2}$]. The droplet evaporation time scale is estimated as

$$\tau_{evap} = \frac{r^2}{A(1 - RH_d)},$$  

(12)

where $r$ is the mean volume radius of cloud droplets (predicted by the double-moment microphysics scheme), $RH_d$ is the relative humidity of the cloud-free portion of the gridbox, and $A \approx 10^{-10}$ m$^2$s$^{-1}$ is the constant in the droplet diffusional growth equation (i.e., $dr/dt = AS/r$, where $S = RH - 1$ is the supersaturation). $RH_d$ can be estimated using the mean (model-predicted) relative humidity of a gridbox $RH$, and assuming that $RH = 1$ for the cloudy part of the gridbox. These lead to

$$RH_d = \frac{RH - \beta}{1 - \beta}.$$  

(13)

Once the values of the two time scales are derived, their ratio provides a prediction of the slope $\delta$ using the relationship suggested in Andrejczuk et al. (2009), and the parameter $\alpha$ can be calculated from eq. 9 and subsequently applied in eq. 8.
4. Application of the subgrid-scale model to BOMEX shallow convection

The subgrid-scale microphysics model described above was included in the anelastic semi-Lagrangian/Eulerian cloud model EULAG documented in Smolarkiewicz & Margolin (1997, model dynamics), Grabowski & Smolarkiewicz (1996, model thermodynamics), and Margolin et al. (1999, subgrid-scale turbulent mixing). The double-moment microphysics of Morrison & Grabowski (2007, 2008) was recently added to the model; simulations using the scheme and prescribed values of the parameter \( \alpha \) are reported in Sławinska et al. (2011). The model simulates quasi-steady-state trade-wind shallow nonprecipitating convection observed during the Barbados Oceanographic and Meteorological Experiment (BOMEX; (Holland & Rasmusson, 1973)) and used in the model intercomparison study described in Siebesma (2003). In BOMEX observations, a 1.5-km-deep trade-wind convection layer overlays a 0.5-km-deep mixed layer near the ocean surface and lies under a 0.5-km-deep trade-wind inversion layer. The cloud cover is about 10% and quasi-steady conditions are maintained by the prescribed large-scale subsidence, large-scale moisture advection, surface heat fluxes, and radiative cooling. Simulations with either prescribed \( \alpha \) (0 or 1) or \( \alpha \) calculated locally as described in the previous section are run for 6 hrs and data from last 3 hrs are used in the analysis. CCN characteristics are assumed as in the pristine case in Morrison & Grabowski (2008).

Figure 2. Scatter plot and the mean \( \alpha \) as a function of height for the simulation with the new subgrid-scale model.

Figure 2 shows the scatter plot and the mean \( \alpha \) as a function of height from the simulation where \( \alpha \) is locally predicted. Only points with \( q_c \) exceeding 0.1 g kg\(^{-1}\) are included in the analysis. As the figure illustrates, a relatively wide range of mixing scenarios exists at any given height. The average mixing scenario shifts with height towards the homogeneous mixing. The decrease of \( \alpha \) can be understood considering scatter plots of TKE and the mean droplet radius. As illustrated in Figs. 3 and 4, both TKE and the mean droplet radius increase with altitude. These make the turbulence mixing more effective compared to the droplet evaporation (i.e., a reduction of \( \tau_{mix} \) and an increase \( \tau_{evap} \)), so the mixing becomes more homogeneous.

Finally, Fig. 5 compares profiles of the mean droplet number concentration between the simulation with \( \alpha \) predicted and the simulations where \( \alpha \) was fixed at 0 or 1 for the entire simulation. The mean concentrations are calculated as averages including only gridpoints with \( q_c \) exceeding 0.01 g kg\(^{-1}\). Overall, the differences in conditionally-sampled droplet concentrations...
are rather small, perhaps even insignificant. This echoes results presented in Slawinska et al. (2011). Slawinska et al. (2011) argued that the small difference between various mixing scenarios may result from compensating effects of entrainment and in-cloud activation (i.e., complete evaporation of some droplets in the extremely inhomogeneous mixing is subsequently compensated by additional CCN activation above the entrainment level) and from the fact that entrained air is relatively close to saturation (in which case little evaporation is needed and only small differences between various mixing scenarios exist). Current results seem to support such conjectures.
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