Broad-band electron spectroscopy: a novel concept based on Thomson scattering

P. Tomassini, M. Galimberti, A. Giulietti, D. Giulietti, L.A. Gizzi, and L. Labate

Intense Laser Irradiation Laboratory - IPCF,
Area della Ricerca CNR, Via Moruzzi 1, 56124 Pisa, Italy

Abstract

The spectrum of relativistic electron bunches with large energy dispersion is hardly obtainable with conventional magnetic spectrometers. We present a novel spectroscopic concept, based on the analysis of the photons generated by Thomson Scattering of a probe laser pulse impinging with arbitrary incidence angle onto the electron bunch. The feasibility of a single-pulse spectrometer, using an energy-calibrated CCD device as detector, is investigated. Numerical simulations performed in conditions typical of a real experiment show the effectiveness and accuracy of the new method.

PACS numbers: 13.60.Fz, 29.30.Dn, 41.75.Ht, 41.75.Jv
Thomson Scattering of intense laser beams from charged particles has attracted a great interest for a variety of research areas, including particle acceleration [1], laser-plasma interactions [2] [3] [4], plasma fusion [5] [6], X-rays generation [7] and medical diagnostics [8]. In this letter we investigate the possibility of using linear Thomson scattering of a laser beam impinging at arbitrary incidence angle onto a relativistic electron bunch to design a novel electron energy spectrometer. The basic idea is to measure the spectral and angular distribution of the scattered laser pulse and use it to infer the energy spectrum of the electrons. The concept can be in principle applied to a wide class of electron bunches, but it seems particularly attractive for the study of laser accelerated electrons.

Electron spectroscopy is currently performed with magnetic spectrometers [9], whose use is practically limited to rather well collimated electron beams, with a moderate energy spread around a roughly known mean value. The magnetic electron spectroscopy is much more difficult to apply to electron bunches having a broad band energy spectrum and whose mean energy and degree of collimation are not well predictable. Such conditions are typical, for example, of electron bunches produced by laser acceleration of plasma electrons [10] [11]. In this latter case, when magnetic spectrometers are used, the electron spectrum is usually obtained with several laser shots, each one devoted to produce a portion of the whole spectrum, which results in a poor accuracy. As the research on laser acceleration of electrons is rapidly growing, there is a need of a new class of spectrometers.

The computation of the Thomson scattered photon yield by a bunch of charged particles is based on a classical electrodynamic approach [12] and, in the case of incoherent scattering, it can be expressed as an integration of the single electrons spectra [13]:

$$\frac{d^2 N}{d\omega dO}(\omega, \theta) = \int dx dp f(x, p, t) \frac{d^2 N}{d\omega dO}\big|_{\text{single}}$$  \hspace{1cm} (1)

where the general form of the single electron scattered photon yield $d^2 N/d\omega dO|_{\text{single}}$ can be found in [13] and [14], $\omega$ is the pulsation of the scattered radiation, $\theta$ is the scattering angle in a spherical coordinate system, $n(\theta, \phi)$ is the scattering direction, $dO \equiv d(\cos \theta)d\phi$ is the solid angle and $f(x, p, t)$ is the phase-space statistical distribution of the bunch.

Let us consider a relativistic electron bunch moving in the 'Laboratory' frame along the $z$ direction, whose statistical distribution function is parametrized as $f(x, p, t) \equiv F(\gamma)\Theta_\gamma(\theta)n_\gamma(r - \beta ct)$, being $F(\gamma)$ the energy distribution function and $\Theta_\gamma$, $n_\gamma$ describe the angular and spatial statistics of the population of electrons having relativistic factor $\gamma$. 
A laser pulse of Gaussian longitudinal and transverse envelopes of duration $T$ and waist $w$, intensity $I_0$ and reduced vector potential of nonrelativistic amplitude $a_0 = 8.5 \times 10^{-10} \lambda [\mu m] \sqrt{I_0} [W/cm^2] \ll 1$, propagates in the $x-z$ plane with angle $\alpha^L$ with respect to $z$ and intersects the electrons trajectories, thus making them oscillating and irradiating.

In order to obtain a close expression of the photon yield generated by Thomson Scattering of the laser pulse by the electron bunch, we will assume very general assumptions. First, the number of cycles $N_0$ of the scattered light emission by a single electron is very large, so that the spectrum of the radiation scattered at a certain angle by the electron is monocromatic. Second, the angular distribution of the bunch with collimation angle $\Delta \theta_e$ is known (a collimator is introduced or the bunch divergence is monitored e.g. with the method reported in Ref. [1]). Finally, each population of electrons having the same energy has a Gaussian density distribution with $\sigma_L$, $\sigma_T$ the (energy dependent) longitudinal and transversal bunch sizes, respectively, and position of the center of the spatial distribution at $t = 0$ denoted with $\zeta$.

The evaluation of Eq. 1 with the assumptions reported above, yields the following expression for the emittance

$$
\frac{d^2 N}{d\omega d\cos \theta} (\omega, \theta) = \frac{\alpha \sqrt{2\pi} a_0^2}{8} \int d\gamma d\theta_e \Theta(\theta_e) N_0(\gamma) F(\gamma) \left( H(\gamma) e^{-K(\gamma)(\Delta^T - \zeta/\beta(\gamma))^2} \right)
$$

where the 'resonant' scattered radiation pulsation $\tilde{\omega}(\theta, \gamma)$ is given by

$$
\tilde{\omega}(\theta, \gamma) = \frac{\Omega (1 - \beta(\gamma) \cos \alpha^L)}{(1 - \beta(\gamma) \cos \theta)}
$$

being $\Omega$ the pulsation of laser pulse. The envelope functions $H, K$ take into account the spatial integration of the single electron photon yield and are computed as

$$
H^{-2} = \left(1 + 2 \frac{\sigma_T^2}{w^2}\right) \left(1 + 2 \frac{T^2}{T^2} \frac{\sigma_L^2 \gamma^2 \cos^2 \delta}{w^2}\right) \times
$$

$$
\left(1 + 2 \frac{T^2}{T^2} \frac{\sigma_T^2 \sin^2 \delta}{w^2} + 2 \gamma^2 \sigma_T^2 T^2 \cos^2 \delta\right)
$$

$$
K = \frac{2 \tilde{T}^2}{T^2 w^2 + 2(\tilde{T}^2/T^2) \left(\gamma^2 \sigma_L^2 \cos^2 \delta + \sigma_T^2 \cos^2 \delta\right)}
$$

being $\cos \delta = \sin \alpha^L/\gamma(1 - \beta \cos \alpha^L)$ and $\tilde{T} \equiv N_0/\Omega = T \cdot w/(w^2 + (\gamma \beta \cos \delta cT)^2)^{1/2}$. Expressing the Dirac function in Eq. 2 in terms of the relativistic $\gamma$ factor of the scattering electron(s),

3
we obtain a close expression which links the scattered photon yield spectrum to the energy distribution function of the electron bunch $F(\gamma)$:

$$\frac{d^2N}{d\omega d\cos\theta}(\omega, \theta) = \frac{\alpha}{8} \int d\theta_e \Theta_\gamma(\theta_e) N_0(\tilde{\gamma}) F(\tilde{\gamma}) \times$$

$$\times \frac{\gamma H(\gamma) e^{-K(\gamma)(\Delta T - \zeta/\beta(\gamma)c)^2}}{\Omega(\cos(\theta - \theta_e) - \cos\alpha L)} ,$$

being

$$\tilde{\gamma} \equiv \frac{(\omega \cos(\theta - \theta_e) - \Omega \cos\alpha L)}{\sqrt{\omega \cos(\theta - \theta_e) - \Omega \cos\alpha L)^2 - (\omega - \Omega)^2}}$$

the relativistic factor of the electron(s) which generates the scattered photon(s).

We are now able to build up the formula to be used by the energy spectrometer. Let $\text{Eff}(E)$ be the experimental revelation efficiency of a photon of energy $E$ and

$$S(E, \theta) \equiv \frac{d\text{N}}{dE d\cos\theta}|_{\text{Exper}}$$

the detected spectrum of the photon yield, being $E$ and $\theta$ the energy and scattering angle. The energy spectrum of the electron bunch $F(\gamma)$ can be retrieved by $S(E, \theta)$ and by the prior knowledge of the size and divergence of the electron bunch as

$$F(\gamma) = \frac{8}{\sqrt{2\pi\alpha a_0^2 N_0(\gamma)}} E^{\text{Laser}}(1 - \cos\alpha_L)$$

$$\times \frac{e^{K(\gamma)(\Delta T - \zeta/\beta(\gamma)c)^2}}{\gamma H(\gamma) D\left(\frac{S}{\text{Eff}}, \Theta, \gamma\right)} ,$$

where $D\left(\frac{S}{\text{Eff}}, \Theta, \gamma\right)$ means the deconvolution of $S(E, \theta)/\text{Eff}(E)$ with the known angular distribution function $\Theta_\gamma(\theta_e)$, $E = E^{\text{Laser}}(1 - \beta(\gamma) \cos\alpha)/(1 - \beta(\gamma) \cos(\theta - \theta_e))$ and $E^{\text{Laser}} = h\Omega$ is the energy of the incoming photons.

Formulae (3-6) strongly simplify in the case of very large waist size and probe pulse incidence angle $\alpha_L$ approaching $\pi$. In the limit case $w/\sigma_T \rightarrow \infty$, $\alpha_L \rightarrow \pi$, the laser pulse contains all the electron bunch provided that the length of the bunch $\sigma_L$ does not exceeds the probe Reyleigh length $Z_R \approx \pi w^2/\lambda$, so the envelope functions $H, K$ reduce to

$$H \rightarrow 1, \ K \rightarrow 0 .$$

In this case the energy spectrum of the electron bunch can be very accurately estimated as

$$F(\gamma) = 2.4 \cdot 10^{-2} \frac{w^2}{E\lambda} E^{\text{Laser}}\frac{1}{(1 - \beta(\gamma) \cos(\theta - \theta_e))} D\left(\frac{S}{\text{Eff}}, \Theta, \gamma\right)$$

$$E = 2E^{\text{Laser}}(1 - \beta(\gamma) \cos\alpha)/(1 - \beta(\gamma) \cos(\theta - \theta_e)) .$$
where $E$ is the energy delivered by the probe pulse (in Joules), $w$ and $\lambda$ are in $\mu m$, $E^{Laser}$ is in $eV$ and we have assumed $\theta << 1$ and $\gamma >> 1$.

In order to present a full simulation of a possible experimental setup, we will focus on the measure of the spectrum of a relativistic electron bunch produced e.g. by Laser-Plasma acceleration (say Laser Wake Field Acceleration, LWFA, or Self-Modulated-LWFA) [11]. In this framework, an ultraintense laser pulse impinges onto a plasma and the strong electric fields of the wake accelerate a large number of electrons at energies exceeding tens of MeV’s [10]. Usually the energy spectra of the electron bunches are broad and, in the cases of acceleration with uncontrolled trapping, an exponentially decreasing distribution is obtained [1]. Attempts to control the trapping of the accelerated electrons, thus producing electron bunches with low spectral dispersion, are in progress [1] [16].

To face with a realistic electron bunch, we simulated $N_e = 6 \times 10^8$ electrons with Gaussian angular distribution $\Theta_N(\theta_e)$ having divergence ranging from $20 mrad$ at low energies to $10 mrad$ at large energies. The spatial distribution has transverse and longitudinal size $\sigma_T = 20 \mu m$, $\sigma_L = 50 \mu m$, respectively. Finally, the energy distribution $F(\gamma)$ is composed both by an exponentially decreasing background (whose parameters have been estimated by experimental data [1]) and a few MeV’s thick Gaussian peak, taking into account the portion of the bunch (possibly) generated with controlled trapping. The probe pulse propagates against the simulated electron bunch ($\alpha^L = \pi$), is 1ps long, has energy $E = 0.1J$ and waist size $w = 50 \mu m$.

The simulation of the detected photon yield is performed with a Monte Carlo method by using Eq. 1, assuming an acceptance angle of the photon detector (a CCD camera, see below) of $30 mrad$ and a detection efficiency in the range $0.2 - 1$ (see [18]). The angular and spectral distribution of the $\approx 10^6$ detected photons is shown in Fig. 1.

Since the probe and bunch parameters allow the application of the simplified analysis (see Eq. 7), to analyze the ‘experimental’ results by using Eq. 8 we proceed by computing the deconvolution of the photon yield with the angular distribution of the bunch. The result of the analysis is shown in Fig. 2. It is clear that the estimated energy spectrum well reproduces the simulated one also at large energies, where the effect of the bunch divergence is significant. We note also that the results of the analysis of the simulated data clearly show that the electron spectrometer is capable of measuring, in a single shot, the very wide energy spectrum with a resolution good enough to detect a narrow peak.
A possible setup of the spectrometer is shown in Fig. 3. The probe pulse is focused with an off-axis parabola and directed with a mirror towards the electron bunch. The Thomson scattered radiation is detected with an energy calibrated CCD camera working in a single-photon mode \([17]\). In order to ensure the acquisition of a large number of photons for each shot, the CCD camera should have both a large number of pixel and a reasonable high quantum efficiency \((QE)\) at large energies \([18]\).

Finally, we briefly discuss the main sources of uncertainty of the electron spectrometer. Let \((\gamma_{\text{min}}, \gamma_{\text{Max}})\) be the range of measure of the electron spectrometer, and \(\Delta E, \delta(\Delta \theta_e)\) the uncertainties on the energy of a scattered photon and on the bunch divergence, respectively. Taking into account Eq. 5, we get

\[
\frac{\delta \gamma}{\gamma} \approx \frac{1}{2} \frac{\Delta E}{E} \approx \frac{1}{2} \left( \frac{1}{\gamma^2} \frac{\Delta E_{\text{Exp}}}{4E_{\text{Laser}}} + \frac{1}{\Omega T} + \gamma^2 \frac{\delta(\Delta \theta_e)^2}{4} \right)
\]

provided that \(\delta(\Delta \theta_e) \ll 1/\gamma_{\text{Max}}\). Eq. (9) shows that the range of measurable energy spectrum is limited on the low energy side by the energy resolution of the photon detector and on the high energy side by the error on the beam divergence. Supposing reasonable values for \(\Delta E_{\text{Exp}}\) and \(\delta(\Delta \theta_e)\) of 100eV and 0.5mrad (see Ref. \([1]\)), we obtain that estimation of the \(\gamma\) factor are available within (say) 5 percent error in the very wide band \(10^{-3} - 1000\). The uncertainty on the amount of electrons having \(\gamma\) fixed depends critically on the experimental setup. Apart from counting errors, strong error sources are the uncertainty on the spatial bunch parameters, while errors in the estimations of the laser parameters are usually low. The main uncertainty can be, in fact, on the envelope of the intersection between the electron and laser beams, which is described by the functions \(H, K\). We stress immediately that, if the experimental environment let the laser pulse counterpropagate against the electron bunch \((\alpha_L = \pi)\), this error source can be made negligible, provided that the laser waist is large enough (see Eqq. (3,7)). In this case the largest error sources on \(F(\gamma)\) should realistically be linked to the statistics on the photon counting, so if a single shot electron spectrometer is wanted, a large number of photons for each shot should be detected, as in the example just analyzed.

To conclude, we have shown that Thomson Scattering of a laser beam onto a relativistic electron bunch at non relativistic laser intensities can be used to generate a beam of scattered photons and that the distribution of the photon yield can be linked to the energy distribution of the electron bunch. In the case of probe waist size much higher than the transversal...
size of the counterpropagating electron beam, a strong simplification of the analysis of the experimental data occurs and a very accurate spectrometer can been implemented.

One of the authors (PT) wish to acknowledge support from Italian M.I.U.R (Project “Metodologie e diagnostiche per materiali e ambiente).

* Also at Dipartimento di Fisica Università di Pisa, Unità INFM, Pisa Italy
† Also at Dipartimento di Fisica Università di Bologna, Bologna Italy

[1] W.P. Leemans et. al., PRL 77 20, 4182-4185 (1996)
[2] W.P. Leemans et. al., PRL 67 11, 1434-1437 (1991)
[3] N. Renard et. al., PRL 77 18, 3807-3810 (1996)
[4] K. Krushelnick et. al., PRL 78 21, 4047-4050 (1997)
[5] T.N. Carlstrom et. al., Rev. Sci. Instrum. 63 10, 4901-496 (1992)
[6] K. Muraoka, K. Uchino and M.D. Bowden, Plasma Phys. Control. Fusion 40, 1221-1239 (1998)
[7] P. Catravas, E. Esarey and W. P. Leemans, Meas. Sci. Technol. 12 (2001) 1828-1834
[8] A. Ts. Amatuni and M. L. Petrossian, Nucl. Meth. in Phys. Res. A 455, 128-129 (2000)
[9] J. J. Livingood,., The Optics of Dipole Magnets, Academic press N.Y. and London, 1969
[10] D. Giulietti et. al., submitted to Phys. of Plasmas, 2002
[11] D. Gordon et. al., PRL 80, 2133 (1998)
[12] J.D. Jackson, Classical Electrodynamics, (Wiley, New York, 1975
[13] E. Esarey, S. K. Ride and P. Sprangle, PRE 48, 4 3003-3021 (1993)
[14] S. K. Ride, E. Esarey and M. Baine, PRE 52, 5 5425-5442 (1995)
[15] T. Tajima and J. M. Dawson, PRL 43, 267 (1979)
[16] S. Bulanov, N. Naumova, F. Pegoraro and J. Sakai, PRE 58 5, R5257-R5260 (1998)
[17] L.A. Gizzi et. al., PRL 76 2278 (1996)
[18] C.M. Castelli and G.W. Fraser, Nucl. Instr. Meth. in Phys. Res. A 376 (1996) 298-300
FIG. 1: Numerical simulation of the scattered photon yield integrated onto the azimuthal angle $\phi$. 
FIG. 2: Comparison between the bunch energy spectrum estimated with the Thomson spectrometer and the simulated spectrum (full line).
FIG. 3: Example of a possible experimental setup. The probe pulse is focussed onto the electron bunch with an off-axis parabola and the scattered photons are detected with an ADC-calibrated CCD camera.