We explain recent challenging experimental observations of universal scattering rate related to the linear-temperature resistivity exhibited by a large corps of both strongly correlated Fermi systems and conventional metals. We show that the observed scattering rate in strongly correlated Fermi systems like heavy fermion metals and high-\(T_c\) superconductors stems from phonon contribution that induce the linear temperature dependence of a resistivity. The above phonons are formed by the presence of flat band, resulting from the topological fermion condensation quantum phase transition. We emphasize that so-called Planckian limit, widely used to explain the above universal scattering rate, may occur accidentally as in conventional metals its experimental manifestations (e.g., scattering rate at room and higher temperatures) are indistinguishable from those generated by the well-know phonons being the classic lattice excitations. Our results are in good agreement with experimental data and show convincingly that the topological fermion condensation quantum phase transition can be viewed as the universal agent explaining the very unusual physics of strongly correlated Fermi systems.

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Exotic experimentally observable properties of different classes of strongly correlated Fermi systems are still remain largely unexplained due to the lack of universal underlying physical mechanism. It is customary to attribute these properties to so-called non-Fermi-liquid (NFL) behavior. Latter behavior is widely observed in heavy-fermion (HF) metals, graphene, and high-\(T_c\) superconductors (HTSC). Experimental data collected on many of these systems show that at \(T = 0\) a portion of their excitation spectrum becomes dispersionless, giving rise to so-called flat bands, see, e.g., [1–3]. The presence of flat band indicates that the system is close to a special quantum critical point (QCP), in which the topological class of Fermi surface alters. This QCP is coined as topological fermion-condensation quantum phase transition (FCQPT) [1–3], leading to flat bands (\(\epsilon(k) = \mu\), where \(\epsilon(k)\) is the quasiparticle energy and \(\mu\) is the chemical potential) formation. The latter phenomenon is called fermion condensation (FC) and had been predicted long ago [1, 2]. The flat bands are formed by interaction (while a geometric frustration can help the process), and have a special \(T\)-dependence: At rising temperatures, a nonzero slope appears in the \(\epsilon(k)\) dispersion law, while the quasiparticle width \(\gamma \propto T\) [4, 5]. This observation is in accordance with experimental data, see, e.g., [6–8]. Moreover, the FC theory allows one to describe adequately (both qualitatively and quantitatively) the above NFL behavior of strongly correlated Fermi systems [1, 2, 5, 9, 10]. Here, we analyze HF metals and high-\(T_c\) superconductors exhibiting \(\rho(T) \propto T\) at \(T \rightarrow 0\) and, therefore, located near their topological FCQPT [10, 11]. Note that FC theory permits also to consider the systems located relatively far from their FCQPTs, see, e.g., [5, 10].

Recent challenging experimental findings of linear temperature dependence of the resistivity \(\rho(T) \propto T\) collected on HTSC, graphene, HF, and conventional metals, have revealed that the scattering rate \(1/\tau\) of charge carriers reaches the so-called universal Planckian limit \(1/(T\tau) = k_B/h\) \((k_B\) and \(h = h/2\pi\) are the Boltzmann and Plank constants, respectively) [12–15]. Note that above Planckian limit, used to explain the universal scattering rate in the so-called Planckian metals [12–16], can occur accidentally since its experimental manifestations in other (than Planckian met-
In this work, within the framework of the FC theory, we show that the semiclassical physics is still applicable to describe the universal scattering rate $1/\tau$ experimentally observed in strongly correlated metals at their quantum critical region. This is because flat bands, responsible for quantum criticality, generate transverse zero–sound mode, reminiscent of the phonon mode in solids, with the Debye temperature $T_D$ [11, 17]. At $T \geq T_D$ the mechanism of the $T$-linear dependence of the resistivity is the same both in conventional metals and strongly correlated ones, and is represented by electron–phonon scattering. Therefore, it is electron–phonon scattering at $T \geq T_D$ that leads to the near material–independent nature of the lifetime $\tau$ that is expressed as $1/(\tau T) \sim k_B/\hbar$. As a result, we describe and explain recent exciting experimental observations of universal scattering rate related to linear-temperature resistivity of a large number of both strongly correlated Fermi systems and conventional metals [12–15]. Thus, the observed scattering rate is explained by the emergence of flat bands formed by the topological FQCPT, rather than by the so-called Planckian limit at which the assumed Planckian scattering rate takes place. The reason is that the Planckian limit occurs in the conventional metals at relatively high temperatures $T > T_D$, while it is normally takes place at low temperatures $T \to 0$, but it does not [12].

We begin with considering the schematic $T$–$B$ phase diagram of strongly correlated Fermi system [18] depicted in Fig. 1. The magnetic field $B$ plays a role of the control parameter, driving the system towards its QCP represented by FCQPT. The FCQPT occurs at $B = B_c$, yielding new strongly degenerate state at $B = B_c$. To lift this degeneracy, the system forms either superconducting (SC), magnetically ordered (ferromagnetic (FM), antiferromagnetic (AFM), etc.), or nematic states [5]. In the case of CeCoIn$_5$, this state is located at $B_c = 4.9$ T and covered with superconducting “dome” with the critical field $B = B_c = 5$ T [19]. In case of Sr$_3$Ru$_2$O$_7$, $B_c = 7.9$ T, while the SC state is absent [12]. It is seen in Fig. 1 that at fixed temperature the increase in $B$ drives the system along the horizontal arrow from NFL state to LFL one. On the contrary, at fixed magnetic field and raising temperatures the system transits along the vertical arrow from LFL state to NFL one. The region shown by the arrow and labeled by $T_{\text{cross}}(B \sim T)$ signifies a transition regime between the LFL part with almost constant effective mass and NFL one at which $\rho(T) \propto T$.

At low temperatures, the observed resistivity in HTSC and HT metals located near their QCPs, obeys linear law (so-called linear $T$-resistivity)

$$\rho(T) = \rho_0 + AT. \quad (1)$$

This law demonstrates their quantum criticality and new state of matter [11, 20]. Here, $\rho_0$ is the residual resistivity and $A$ is a $T$-independent coefficient. Explanations based on quantum criticality for the $T$-linear resistivity have been given in the literature, see, e.g., [11, 17, 21–26] and references therein. On the other hand, at room temperature the $T$-linear resistivity is exhibited by conventional metals such as Al, Ag or Cu. In case of a simple metal with a single Fermi surface pocket the resistivity reads $e^2 n \rho = p_F/(\tau v_F)$ [27], where $e$ is the electronic charge, $\tau$ is the lifetime, $n$ is the carrier concentration, and $v_F$ and $p_F$ are the Fermi momentum and velocity, respectively. Representing the lifetime $\tau$ (or inverse scattering rate) of quasiparticles in the form [26, 28]

$$\frac{\hbar}{\tau} \approx a_1 + \frac{k_BT}{a_2}, \quad (2)$$

![Fig. 1. (Color online) Schematic $T$–$B$ phase diagram of a strongly correlated Fermi system. The vertical and horizontal arrows crossing the transition region marked by the thick lines depict the LFL–NFL and NFL–LFL transitions at fixed $B$ and $T$, respectively. At $B < B_{c2}$ (dashed line beginning at solid rectangle) the system is in its possible SC state, with $B_{c2}$ being a critical magnetic field. The hatched area with the solid curve $T_{\text{cross}}(B \sim T)$ represents the crossover separating the NFL and LFL domains. A part of the crossover is hidden inside possible SC state. The boxes $\rho(T) \sim T$ and $\rho(T) \sim T^2$ demonstrate the NFL and LFL behavior of resistivity, respectively.](image-url)
we obtain [11]

\[ a_2 e^2 \hbar \frac{\partial \rho}{\rho_T k_B \partial T} = \frac{1}{v_F}, \]

(3)

where \( a_1 \) and \( a_2 \) are \( T \)-independent parameters. A challenging point for a theory is that experimental facts corroborate Eq. (3) in case of both strongly correlated metals (HF metals and HTSC) and ordinary ones, provided that these demonstrate the linear \( T \)-dependence of their resistivity [12], see Fig. 2.

Moreover, the analysis of literature data for the various compounds with the linear dependence of \( \rho(T) \) shows: the coefficient \( a_2 \) is always close to unity, \( 0.7 \le a_2 \le 2.7 \), notwithstanding huge distinction in the absolute value of \( \rho \), \( T \), and Fermi velocities \( v_F \), varying by two orders of magnitude [12]. As a result, it follows from Eq. (2) that the \( T \)-linear scattering rate is of universal form, \( 1/(\tau T) \sim k_B/\hbar \), regardless of different systems displaying the \( T \)-linear dependence with parameter entering Eq. (3), \( a_2 = 1 \) [10–12]. Indeed, this dependence is demonstrated by ordinary metals at temperatures higher than the Debye one, \( T \ge T_D \), with an electron–phonon mechanism and by strongly correlated metals which are assumed to be fundamentally different from the ordinary ones, in which the linear dependence at their quantum criticality and temperatures of a few Kelvin is assumed to come from excitations of electronic origin rather than from phonons [12]. We note that in some of the cuprates the scattering rate has a momentum and doping dependence omitted in Eq. (3) [29–31]. According to Fig. 2, this scaling relation spans two orders of magnitude in \( v_F \), attesting to the robustness of the observed empirical law [12]. This behavior is explained within the framework of the FC theory, since both for conventional metals and strongly correlated ones the scattering rate is defined by phonons [11]. In the case of conventional metals at \( T > T_D \), it is well known that phonons make the main contribution to the linear dependence of the resistivity, see, e.g., [27]. On the other hand, it was shown that the semiclassical physics describes the \( T \)-linear dependence of the resistivity of strongly correlated metals at \( T > T_D \), since flat bands, forming the quantum criticality, generate transverse zero–sound mode with the Debye temperature \( T_D \) located within the quantum criticality area [11, 17, 26]. Therefore, the \( T \)-linear dependence is formed by electron–phonon scattering in both ordinary metals and strongly correlated ones. As a result, it is electron–phonon scattering that leads to the near material–independence of the lifetime \( \tau \) that is expressed as

\[ \tau T \sim \frac{\hbar}{k_B}. \]

(4)

Now we turn to the twisted bilayer graphene exhibiting the universal scattering rate [13] and having flat band [3]. Recent calculations have shown that under the application of pressure the graphene produces increasing correlated behavior, identified by the presence of flat bands at twist angles that increase with growing pressure [32]. Such a behavior signals that it is the correlations that induce the flat bands, and is in accordance with the scenario of FC [1, 5]. We can qualitatively figure out this observation: The twisted bilayer graphene can be represented by a quasicrystal that is long-range ordered and yet non-periodic. In that case, both the flat band and the corresponding NFL behavior emerge [10, 33]. In fact, without inter-layer coupling, a monolayer twisted graphene is not a quasicrystal, for it remains a periodic system, while it is the coupling that makes the quasicrystal, and the application of pressure strengthens the coupling. To support the scenario, we predict that under the application of magnetic field the observed \( T \)-linear dependence of the resistivity is changed to the \( T^2 \) one, for the system transits from the NFL state to LFL one as it is seen from phase diagram 1 and the universality of the scattering rate is violated. In the same way, the asymmetrical tunneling conductivity vanishes in the LFL state, as it was predicted within the framework of the
As a result, we can safely surmise that it is the transverse sound that forms the universal scattering rate rather than the Planckian limit does.

The next example is HF metal Sr$_3$Ru$_2$O$_7$ having universal scattering rate related to the linear-temperature resistivity [12], see Fig. 2. This metal represents a useful example because of numerous experimental measurements taken on it, see, e.g., [12, 37–39]. This HF metal is tuned to its quantum critical line under the application of magnetic field, as shown in Fig. 3a. The ordered nematic phase highlighted by the horizontal lines is to remove the entropy excess that emerges at $T = 0$. At relatively low temperatures $S(T) = S_0$ and becomes larger than that at LFL state and the second order phase transition converts to first one at the tricritical points $T_{c1}$ and $T_{c2}$, as it is shown in Fig. 3a. Within the nematic phase at $T \leq T_D$ and at $T_D \leq T$ the system exhibits the NFL behavior with $\rho(T) \propto T$, as shown in Figs. 3a and 3b. We note that at $T_D \geq T \rho(T) \propto T$, but the Planckian limit does not take place, for $d\rho(T \geq T_D)/dT = d\rho(T \leq T_D)/dT$, see Fig. 3b. Thus, despite the $\rho(T) \propto T$, the universal scattering rate does not occur. This fact shows that the Planckian limit, quantum critical behavior and the $T$-linear resistivity are not directly related.

Figure 4 shows the temperature dependence $\rho(T)$ of Sr$_3$Ru$_2$O$_7$ in magnetic fields. As magnetic field $B \rightarrow B_{c1}$, the temperature range with the LFL behavior characterized by $\rho(T) \propto T^2$ shrinks, and at $B = B_{c1} \approx 7.9$ T, the resistivity $\rho(T) \propto T^2$ over the whole measurement range; at $B \geq B_{c2}$ the magnetoresistivity exhibits a small negative magnetoresistance.
and the LFL behavior at low temperatures [12], see Fig. 3a. We note that such a Planckian metal does exhibit the Planckian limit in its LFL state, and even it does not demonstrate the limit at $T < T_D$, as we have seen above. All these experimental observations are in accordance with general behavior of strongly correlated Fermi systems and can be explained within the FC theory framework [5, 10].

Thus, the fundamental picture outlined by Eq. (3) is strongly supported by measurements of the resistivity on Sr$_3$Ru$_2$O$_7$ for wide range of temperatures. At $T \geq 100$ K, the resistivity becomes again the $T$-linear at all applied magnetic fields, as it does at low temperatures and under the application of magnetic fields $B_1 \geq B \geq B_2$, see Fig. 3a. In the latter case, the coefficient $A$ is lower than that seen at high temperatures [12]. This is because the coefficient $A$ is the composition of two contributions coming from the transverse zero sound and the FC state, see Fig. 3b. If we subtract the FC contribution, $A$ becomes approximately the same at $T \geq T_D$ and at $T \geq 100$ K [11]. Thus, similar strongly correlated compounds exhibit the same behavior of the resistivity at both quantum critical regime and high temperature one, allowing us to expect that the same physics governs the $T$-linear resistivity of the strongly correlated Fermi systems and of conventional metals.

We note that there can be another mechanism supporting the $T$-linear dependence even at $T < T_D$, which lifts the constancy of $\tau$ regardless the presence of $T$-linear dependence of the resistivity [11, 26]. The mechanism comes from flat bands that is formed by the FC state, and contributes both to the linear dependence of the resistivity and to the residual resistivity $\rho_0$, see Eq. (1). Note that these observations are in good agreement with experimental data [11, 26]. The important point here is that under the application of magnetic field the system in question transits from NFL behavior to LFL one and both flat bands and the FC state are destroyed [5, 10], see the $T-B$ phase diagrams in Figs. 1 and 3a. Therefore, the resistivity is $\rho(T) \propto T^2$, magnetoresistance becomes negative, while the residual resistivity $\rho_0$ decreases abruptly [10, 11, 26]. Such a behavior is in accordance with experimental data, see, e.g., the case of the HF metals CeCoIn$_5$ [19] and Sr$_3$Ru$_2$O$_7$ [12] that also demonstrate the universal scattering rate, see Fig. 2.

We mention here similar recent experimental observations of the linear resistivity $\rho(T) \propto T$ at low temperatures, which relates the slope of the linear-$T$-dependent resistivity to the London penetration depth $\lambda_0$ indicating a universal scaling property

$$\frac{d\rho}{dT} \propto \lambda_0^2,$$  \hspace{1cm} (5)

for numerous high-$T_c$ superconductors [40]. This scaling relation spans several orders of magnitude in $\lambda_0$, attesting to the robustness of the empirical law (5). This law, in turn, can be explained within the framework of the FC theory [35] rather than addressing the quantum diffusion that assumed to be the origin of this scaling relation [40]. It is important to note that the FC theory presented here is insensitive to and transcends the microscopic, non-universal features of the substances under study. This is attributed to the fact that the FC state is protected by its topological structure and therefore represents a new class of Fermi liquids [2, 10]. In particular, consideration of the specific crystalline structure of a compound, its anisotropy, its defect composition, etc. do not change our predictions qualitatively. In other words, the fermion condensation of charge carriers in the vast body of considered strongly correlated compounds, engendered by a quantum phase transition, is indeed the primary physical mechanism responsible for their observable universal scaling properties. This mechanism can be extended to a broad set of substances with a very different microscopic characteristics, as discussed in detail in [5, 10].

In summary, we have explained recent challenging experimental observations that the scattering rate $1/\tau$ of charge carriers collected on high $T_c$ superconductors, graphene, heavy fermion and conventional metals exhibits the universal behavior [12–15] generated by the semiclassical properties of above strongly correlated materials. While the Planckian limit may occur accidentally: It is highly improbable that it would be realized in conventional metals, which, obviously, cannot be recognized as Planckian metals with quantum criticality at high or low temperatures. Finally, the fact that we observe the same universal behavior of the scattering rate in microscopically different strongly correlated compounds like HTSC, HF and conventional metals, suggests that some general theory is needed to explain the above body of materials and their behavior in the uniform manner. We may conclude that the FC theory is the suitable candidate.

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