Torsional Vibration Analysis of Shaft in an Induced Draft Fan Due to Variable Frequency Drive

QINGJIE ZHANG1,2, (Member, IEEE), GUANGXIANG LU1, YOU XU3, AND CHENGYU ZHANG2

1School of Electrical Engineering, Southeast University, Nanjing 210031, China
2College of Engineering, Nanjing Agriculture University, Nanjing 210000, China
3School of Automation, Nanjing Institute of Technology, Nanjing 210000, China

Corresponding author: Qingjie Zhang (zqj518@vip.qq.com)

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ABSTRACT The torsional vibration problem of shaft in an induced draft fan (IDF) caused by frequency conversion driving technology is a result of dynamic electro-mechanical coupling behavior during the energy conversion. Therefore, it is difficult to understand the mechanism of torsional vibration by only analyzing the mechanical structure or the electrical structure. In this paper, eigenvalue structural analysis (ESA) and time-domain transient analysis, which are typically used to examine sub-synchronous resonance (SSR), are employed to resolve this issue. It is difficult to develop a linear model for complex controllers with ESA as ESA can cause certain errors during the linearization and estimation of initial values of equilibrium points. On the contrary, time-domain analysis not only offers a small-step integration technique to simulate a realistic condition but also facilitates transient analysis of complex controllers. Transient analysis has been used to study the influence of elastic parameters of shaft, self-damping coefficients, resistance of the stator side, smooth reactance of the stator side, and voltage-frequency ratio on the control of target frequency converters and cascade multilevel high-voltage frequency converters. In this paper, it is also advised to avoid torsional resonance region with torsional-vibration monitoring during the operation of the system, and stator resistance can be added and the voltage-frequency rate can be adjusted to obtain an integrated control of an active torsional-vibration system.

INDEX TERMS Induced draft fan, variable frequency drive, shaft torsional vibration, eigenvalue structural analysis, time-domain transient analysis, cascade multilevel inverter, surface fitting, sub-synchronous resonance.

I. INTRODUCTION

Induced draft fan (IDF) is one of the equipments with large power consumption in power plants. The implementation of a variable frequency drive (VFD) in IDF not only improves the start-up performance of the unit, the power factor, and the energy efficiency of the fan but also facilitates step-less regulation of motor speed. Therefore, it is widely used for reducing energy consumption and overall cost of the power plant. However, in actual operation, after the IDF is installed with VFD, the frequency changes continuously, which increases the probability of coincidence of electrically generated harmonics with one or more natural frequencies of the rotor components. The harmonics of the frequency converter cause output torque pulsation, which generates impact load and alternating stress on the rotor, resulting in cracks or fracture of the drive shaft, rupture of the coupling diaphragm and bolt, cracks in the impeller hub and blade, etc. Consequently, the safe operation of thermal power plants can lead to hidden dangers. Feese and Maxfield [1] used a dynamic torque meter based on wireless strain gauge to examine torsional vibration of the fan shaft. Based on this
work, Kerkman et al. [2] presented a simulation study and proposed a control method to suppress the torsional vibration. Song-Manguelle et al. [3] [4] studied the relationship between electrical harmonics and torque generated in the air gap. Han and Palazzolo [5] estimated the torsional vibration and shaft life through harmonics of high-voltage inverter. Lu [6] used high elastic rubber coupling to suppress the torsional vibration of fan, while [1] [2] described the coupling failure with higher rigidity after replacement. Nagata et al. [7] and Elshawarby et al. [8] improved the pulse width modulation (PWM) method of frequency converter to suppress the torsional vibration. Muszynski and Deskur [9] designed a proportional–integral–derivative (PID) controller to control the speed of frequency converter for suppressing the torsional vibration. Ma [10] adopted the concept of electrical damping and load torque observer or double closed-loop control to suppress the torsional vibration caused by frequency conversion. All these studies indicated that torsional vibration is the result of dynamic behavior of electromechanical coupling in the process of electromechanical energy conversion, which is a kind of problem associated with sub-synchronous resonance (SSR) in power systems. Compared with the SSR problem in power systems, the VFD system with wide-range speed regulation is more likely to cause torsional vibration, which is a more hidden problem. The torsional vibration problem can be alleviated by simply replacing the coupling or by changing some control methods. The SSR problem causes damage in power systems. Moreover, because the distance between mechanical and electrical equipment is large, this kind of problem is usually analyzed from the perspective of the entire system. Zhu et al. [11] used time-domain analysis to examine the SSR problem. He et al. [12] investigated the characteristics of SSR by using eigenvalue structural analysis (ESA). In this study, time-domain analysis and ESA methods, which are typically used for SSR in power systems, are employed to examine the torsional vibration of VFD, which can boost the application of more analysis methods used for SSR.

The main contributions of this study are as follows:

1. A model for VFD of IDF is established. In several existing studies, the load is often assumed to be a simple constant or linear load. Here, the step-by-step curve method is used to establish the IDF load, and a cascade multilevel high-voltage inverter model is established to systematically examine the torsional vibration of VFD.

2. The ESA method is used to suppress the resonance interval of shaft of IDF caused by VFD. The estimated results are close to the simulation results based on time-domain analysis and the actual test results.

3. The time-domain simulation method is used to study the influence of shaft parameters and electrical parameters on the torsional vibration of VFD.

4. The time-domain analysis revealed that the resistance between the inverter and the motor can suppress the torsional vibration of shaft.

II. MODELING OF IDF UNIT

A. DRIVE SHAFT SYSTEM OF IDF

Axial-flow IDF with adjustable stator blades has a complex structure, and it is mainly composed of inlet box, collector, inlet axial guide valve, impeller, rotor, bearing box, central cylinder, rear guide vane, casing, diffuser, inlet and outlet expansion joints, etc. However, for examining the torsional motion of shaft, the lumped-mass method can be used in which the IDF is considered to be equivalent to one mass block, and the coupled induction motor (IM) is equivalent to another mass block, thereby forming a double mass model, as shown in Fig. 1. Here, \( J_i \) and \( J_e \) are the inertia time constants of IDF and IM, respectively; \( \omega_i \) and \( \omega_e \) are the instantaneous angular frequencies of IDF and IM rotor, respectively; \( \phi_i \) and \( \phi_e \) are the angular torsional displacements of the shaft end of IDF and IM, respectively, \( c_{ie} \) and \( c_{ii} \) are the damping coefficients of mass block of IDF and IM, respectively; \( e_{ie} \) is the mutual resistance coefficient between the mass block of IDF and IM; \( e_e \) is the elastic coefficient of the shaft connecting IDF and IM; \( T_i \) is the input mechanical torque of IDF; \( T_e \) is the electromagnetic torque of IM output.

The Riccati equation of double shaft system is as follows:

\[
\begin{aligned}
    J_i \frac{d\omega_i}{dt} &= e_{ie}(\phi_e - \phi_i) - T_i - c_{ii}\omega_i - c_{ie}(\omega_i - \omega_e) \\
    J_e \frac{d\omega_e}{dt} &= T_e - e_{ee}\omega_e - c_{ie}(\omega_e - \omega_i) - e_i(\phi_e - \phi_i) \\
    \frac{d\phi_i}{dt} &= \omega_e - \omega_i \\
    \frac{d\phi_e}{dt} &= \omega_i - \omega_0
\end{aligned}
\]  

B. TRANSMISSION LOAD OF IDF

The transmission load of IDF is a function of the guide vane opening, rotation speed, and load torque, i.e.,

\[ T_i = f(\beta, n_i) \]  

where \( T_i \) is the mechanical torque input by the IDF, \( \beta \) is the angle of guide vane, and \( n_i \) is the speed of fan.

The fan is generally considered to have a square load, i.e.,

\[ T_i = kn_i^2 \]

where \( k \) is the load factor of fan.

The error of the model is large when the guide vane is adjusted, and the speed is adjusted in a wide range.

In this study, the performance curve of the fan provided by the manufacturer is used to derive the general mathematical model by stepwise curve fitting, and then the transmission model of IDF is obtained according to the similarity rate of fan.

For example, the general performance curve of AN35e6 axial-flow IDF with adjustable stationary blade provided by the manufacturer is shown in Fig. 2. The corresponding specific pressure energy \( (W) \), volume flow \( (Q) \), and guide vane angle of fan \( (\beta) \) can be obtained by quantifying this static characteristic curve by vector drawing tool.
The static performance curve of wind turbine can be regarded as a spatial surface expressed by $W = f(\beta, Q)$. It is a bivariate surface. Here, the problem of multivariate surface fitting is decomposed into one-variable multi-step curve fitting step by step.

1. The curves corresponding to each angle in the static characteristic curve are fitted. The curve fitting functions corresponding to each angle $\beta_1, \ldots, \beta_i$ are $f_1(Q) \ldots f_i(Q)$. For An35e6, the fifth-degree polynomial fitting has higher accuracy.

   The basic fitting function is
   
   \[ f(x) = p_1 \cdot x^5 + p_2 \cdot x^4 + p_3 \cdot x^3 + p_4 \cdot x^2 + p_5 \cdot x + p_6 \]  
   
   where $p_1, p_2, \ldots, p_6$ are the fitting coefficients.

2. According to the above step (1), several sets of fitting functions are obtained, which are then interpolated. In this example, two adjacent groups of curves are selected for interpolation, so group $c$ curves corresponds to $c - 1$ interpolation equations. The relationship is as follows:

   \[ W_i = f_i(Q) \frac{(\beta - \beta_{i+1})}{\beta_i - \beta_{i+1}} + f_{i+1}(Q) \frac{(\beta - \beta_i)}{\beta_{i+1} - \beta_i} \]  

   where $i = 1, 2, \ldots, c$.

The curve equation of guide vane angle between $30^\circ$ and $15^\circ$ is as follows:

\[ W_1 = f_1(Q) \frac{(\beta - 15)}{15} + f_2(Q) \frac{(\beta - 30)}{-15} \]  

According to the similarity rate of fans [13], the following equation can be used for the conversion when the axial-flow fan operates at a fixed angle of rotor (stator) blade and fan speed:

\[ W = \frac{T}{T_m} W_m \]  
\[ Q = \frac{n}{n_m} Q_m \]

where $W_m$ is the rated specific pressure energy, $T_m$ is the rated torque, $Q_m$ is the rated flow, and $n_m$ is the rated speed.

The transmission model of fan can be obtained by substituting equations (7) and (8) into equation (6)

\[ T_i = \frac{T_m}{W_m} f_i(Q) \frac{(\beta - \beta_{i+1})}{\beta_i - \beta_{i+1}} + f_{i+1}(Q) \frac{n}{n_m} Q_m \frac{(\beta - \beta_i)}{\beta_{i+1} - \beta_i} \]  

The curve obtained by step-by-step method is shown in Fig. 3, and the characteristic curve of IDF is shown in Fig. 4.
C. INDUCTION MOTOR

It is assumed that the IM is connected to an infinite bus system. The following assumptions are considered: 1. the spatial and temporal harmonics are ignored; 2. the magnetic saturation is ignored; 3. the core loss is ignored; 4. the influence of temperature and frequency variations on the winding resistance is ignored. According to the motor convention, the transient model equation of IM in rotating orthogonal coordinate system is as follows:

(1) Voltage equation

\[
\begin{bmatrix}
u_{sd} \\
u_{sq} \\
u_{rd} \\
u_{rq}
\end{bmatrix} =
\begin{bmatrix}
R_s & 0 & 0 & 0  \\
0 & R_s & 0 & 0  \\
0 & 0 & R_r & 0  \\
0 & 0 & 0 & R_r
\end{bmatrix}
\begin{bmatrix}
i_{sd} \\
i_{sq} \\
i_{rd} \\
i_{rq}
\end{bmatrix} + \frac{d}{dt}
\begin{bmatrix}
\psi_{sd} \\
\psi_{sq} \\
\psi_{rd} \\
\psi_{rq}
\end{bmatrix}
\]

(2) Flux linkage equation

\[
\begin{bmatrix}
\psi_{sd} \\
\psi_{sq} \\
\psi_{rd} \\
\psi_{rq}
\end{bmatrix} =
\begin{bmatrix}
L_s & 0 & L_m & 0  \\
0 & L_s & 0 & L_m  \\
L_m & 0 & L_r & 0  \\
0 & L_m & 0 & L_r
\end{bmatrix}
\begin{bmatrix}
i_{sd} \\
i_{sq} \\
i_{rd} \\
i_{rq}
\end{bmatrix}
\]

(3) Torque equation

\[
T_e = n_p L_m (i_{sq} i_{rd} - i_{sd} i_{rq})
\]

Considering that the rotor of IM is short circuited, we get

\[
u_{rd} = u_{rq} = 0
\]

Here, \(u_{sd}\) and \(u_{sq}\) are the components of infinite bus voltage on d-q axis. \(u_{rd}\) and \(u_{rq}\) are the components of induction voltage on d-q axis of IM rotor. \(R_s\) and \(R_r\) are the stator and rotor resistance of IM, respectively. \(L_s\) and \(L_r\) are the stator and rotor reactance of IM, respectively. \(L_m\) is the mutual impedance between stator and rotor. \(i_{sd}\) and \(i_{sq}\) are the components of stator current on d-q axis of IM. \(i_{rd}\) and \(i_{rq}\) are the components of rotor current on d-q axis of IM. \(\psi_{sd}\) and \(\psi_{sq}\) are the components of IM stator flux on d-q axis. \(\psi_{rd}\) and \(\psi_{rq}\) are the components of IM rotor flux on d-q axis. \(\omega_1\) is the electrical angular frequency, \(\omega\) is the rotor angular frequency, and \(n_p\) is the number of poles of IM.

D. HIGH-VOLTAGE INVERTER

The high-voltage inverter with Robincon structure is called “perfect harmonic-free high-voltage inverter” [14]–[16]. A phase-shifting transformer is used as the input transformer of this inverter, which changes high-voltage electrical signal into multiple groups of low-voltage signal with different phases. Each group of low-voltage signal is used as the input of a common low-voltage inverter. The output terminals of each power unit are connected in series, i.e., the low-voltage modules are superposed. High-voltage output forms multi-pulse, multi-rectification mode. The output harmonic of such an inverter is very small and it exhibits wide speed range. Therefore, it has been widely used in the field of high-voltage frequency conversion in recent years [14]–[16].
In this paper, the cascading of five-stage power units is taken as an example. As shown in Fig. 5, the primary side of the multiple phase-shifting transformer is connected with 6 kV power grid. The secondary side is divided into five groups with 15 windings in total. When the five-stage cascading is adopted, the phase shift angle of the secondary winding is \(-24^\circ, -12^\circ, 0^\circ, 12^\circ, 24^\circ\). A, B, and C in each group are still staggered by 120\(^\circ\), and the output low-voltage of each group is converted into low-voltage at specified frequency after rectification and inversion of power unit. The low-voltage of each group is connected in series to obtain high-voltage AC frequency conversion output. In the entire inverter system, the input side contains 30 pulse rectifiers. Theoretically, the input current of the grid side does not contain harmonics below order 29, so the harmonic pollution to the power grid is very small, and there is no need for external input filter device.

III. EIGENVALUE STRUCTURAL ANALYSIS OF IDF UNIT

In the case of small disturbance, the ESA method is used to determine the stability of the system by obtaining the eigenvalues of the coefficient matrix after linearizing the system. This is based on second Lyapunov stability theorem [17], [18].

A. NATURAL TORSIONAL FREQUENCY (NTF) ANALYSIS

If the continuous effects of damping and external moment are neglected, the system is free from vibration without any damping, i.e., assuming that \(c_{ee}, c_{ii}, c_{ie}, T_i, T_e\) are 0, equation (1) can be linearized as follows:

\[
\begin{align*}
Jp^2 \Delta \omega_i &= e_{ie}(\Delta \phi_e - \Delta \phi_i) \\
Jp^2 \Delta \omega_e &= -e_{ie}(\Delta \phi_e - \Delta \phi_i) \\
p^2 \Delta \phi_e &= \Delta \omega_i \\
p^2 \Delta \phi_i &= \Delta \omega_e
\end{align*}
\] (14)

The intermediate variables \(\omega_i\) and \(\omega_e\) can be eliminated to obtain

\[
\begin{align*}
Jp^2 \Delta \phi_i &= e_{ie}(\Delta \phi_e - \Delta \phi_i) \\
Jp^2 \Delta \phi_e &= -e_{ie}(\Delta \phi_e - \Delta \phi_i)
\end{align*}
\] (15)

The characteristic roots of the equation of state are as follows:

\[
x_{1,2} = \pm \sqrt{e_{ie}(J_i + J_e)/J_iJ_e}i
\] (16)

The imaginary part represents the NTF of the system

\[
\Delta \omega = \sqrt{e_{ie}(J_i + J_e)/J_iJ_e}
\] (17)

B. VIBRATION ANALYSIS OF ELECTROMECHANICAL COUPLING

Ignoring the system damping, the load torque can be obtained from the transmission model of the IDF. In this case, the fixed blade angle is considered for the fan load, and the load torque is linearized.

\[
\Delta T_i = \frac{60 Q_m}{\pi n_0 \Delta \omega_i} \Delta \omega_i
\] (18)

where \(n_0\) is the speed of the fan at the equilibrium point of the system.

The output electromagnetic torque of IM can be linearized by equation (12) to obtain

\[
\Delta T_e = n_p L_m(i_{dq0} \Delta \omega_d + i_{dq0} \Delta i_{sq} - i_{d0} \Delta i_{q0} - i_{q0} \Delta i_{sd})
\] (19)

where \(i_{dq0}, i_{d0}, i_{dq03}, \) and \(i_{q0}\) are the values of the corresponding current at the equilibrium point.

Substituting the flux equation (11) into the voltage equation (10), the intermediate flux variables \(\psi_{sd}, \psi_{sq}, \psi_{rd}, \) and \(\psi_{rq}\) are eliminated. After linearization, the equation of state vector \([\Delta \psi_{d}, \Delta \psi_{q}, \Delta \omega_d, \Delta \omega_q, \Delta i_{sd}, \Delta i_{sq}, \Delta i_{rd}, \Delta i_{rq}]^T\) can be obtained by using equations (14), (18), and (19).

\[
\begin{bmatrix}
P \Delta \omega_i \\
P \Delta \omega_e \\
P \Delta \phi_e \\
P \Delta \phi_i \\
P \Delta i_{sd} \\
P \Delta i_{sq} \\
P \Delta i_{rd} \\
P \Delta i_{rq}
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} & A_{17} & A_{18} \\
A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} & A_{27} & A_{28} \\
A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} & A_{37} & A_{38} \\
A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} & A_{47} & A_{48} \\
A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} & A_{57} & A_{58} \\
A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} & A_{67} & A_{68} \\
A_{71} & A_{72} & A_{73} & A_{74} & A_{75} & A_{76} & A_{77} & A_{78} \\
A_{81} & A_{82} & A_{83} & A_{84} & A_{85} & A_{86} & A_{87} & A_{88}
\end{bmatrix} \begin{bmatrix}
\Delta \omega_i \\
\Delta \omega_e \\
\Delta \phi_e \\
\Delta \phi_i \\
\Delta i_{sd} \\
\Delta i_{sq} \\
\Delta i_{rd} \\
\Delta i_{rq}
\end{bmatrix}
\] (20)
\[ A_{42} = A_{43} = A_{44} = A_{45} = A_{46} = A_{47} = A_{48} = 0 \]
\[ A_{51} = 0 \]
\[ A_{52} = \left(i_0L_m^2 + i_0L_rL_m\right)/\left(L_m^2 - L_r^2\right) \]
\[ A_{53} = 0 \]
\[ A_{54} = 0 \]
\[ A_{55} = -R_sL_r/\left(L_m^2 - L_r^2\right) \]
\[ A_{56} = R_sL_m/\left(L_m^2 - L_r^2\right) \]
\[ A_{57} = N_p\left(\omega_1L_m^2 - \omega_1L_m^2 + \omega_0L_m^2\right)/\left(L_m^2 - L_r^2\right) \]
\[ A_{58} = N_p\omega_0L_rL_m/\left(L_m^2 - L_r^2\right) \]
\[ A_{61} = 0 \]
\[ A_{62} = \left(i_0L_mL_s + i_0L_sL_s\right)/\left(L_m^2 - L_r^2\right) \]
\[ A_{63} = 0 \]
\[ A_{64} = 0 \]
\[ A_{65} = -R_sL_m/\left(L_m^2 - L_r^2\right) \]
\[ A_{66} = R_sL_s/\left(L_m^2 - L_r^2\right) \]
\[ A_{67} = N_p\omega_0L_mL_s/\left(L_m^2 - L_r^2\right) \]
\[ A_{68} = N_p\left(\omega_1L_m^2 - \omega_1L_m^2 + \omega_0L_m^2\right)/\left(L_m^2 - L_r^2\right) \]
\[ A_{71} = 0 \]
\[ A_{72} = \left(-i_0L_mL_m - \omega_0L_rL_m\right)/\left(L_m^2 - L_r^2\right) \]
\[ A_{73} = 0 \]
\[ A_{74} = 0 \]
\[ A_{75} = -\omega_1L_rL_r + \omega_1L_m^2 - \omega_0L_m^2 \]
\[ A_{76} = -N_p\omega_0L_rL_m/\left(L_m^2 - L_r^2\right) \]
\[ A_{77} = -R_sL_r/\left(L_m^2 - L_r^2\right) \]
\[ A_{78} = R_sL_m/\left(L_m^2 - L_r^2\right) \]
\[ A_{81} = 0 \]
\[ A_{82} = \left(-i_0L_mL_m - i_0L_rL_m\right)/\left(L_m^2 - L_r^2\right) \]
\[ A_{83} = 0 \]
\[ A_{84} = 0 \]
\[ A_{85} = -N_p\omega_0L_mL_s/\left(L_m^2 - L_r^2\right) \]
\[ A_{86} = N_p\left(-\omega_1L_m^2 + \omega_1L_m^2 - \omega_0L_m^2\right)/\left(L_m^2 - L_r^2\right) \]
\[ A_{87} = -R_sL_m/\left(L_m^2 - L_r^2\right) \]
\[ A_{88} = R_sL_s/\left(L_m^2 - L_r^2\right) \]

When the system is balanced, we have

\[
\begin{align*}
T_i &= T_e \\
\omega_i &= \omega_e \\
\frac{di_{at}}{dt} &= \frac{di_{at}}{dt} = \frac{di_{at}}{dt} = \frac{di_{at}}{dt} = 0
\end{align*}
\]

Accordingly, the state of the equilibrium point can be estimated, and the eigenvalue can be determined by substituting it into the characteristic equation. The real and imaginary parts of each eigenvalue vary with frequency. The frequency varies from 5 to 50 Hz with an interval of 0.1 Hz. Equation (20) represents the eighth-order equation of state with eight eigenvalues. Its real part and imaginary parts are shown in Fig. 6 and Fig. 7, respectively.

The calculated parameters of the IDF system are shown in Table 1.

It can be seen from Fig. 6 that among the eight modes, the real parts of the characteristic roots of C and D are greater than zero in the range of 19.9-22.1 Hz. According to the ESA method, if the real part of eigenvalue is greater than zero, the system loses stability. The amplitude of the imaginary part corresponding to modes C and D varies from 110.9 to 112.3, which is the angular frequency of oscillation that is close to the NTF.
IV. TIME-DOMAIN ANALYSIS OF TORSIONAL VIBRATION OF IDF UNIT

A. FIXED FREQUENCY ANALYSIS OF IDEAL FREQUENCY-CONVERSION POWER SUPPLY

An infinitely adjustable three-phase power supply is used as the variable-frequency power supply. The main parameters of the system are shown in Table 1. Steady state test was conducted in different frequency ranges. Figs. 8, 9, 10, and 11 show the torque oscillation waveforms corresponding to constant frequency test with resonant modes at 19.2, 19.3, 22.8, and 22.9 Hz, respectively. The oscillation waveform is attenuated at 19.2 Hz and is gradually divergent at 19.3 Hz. It is close to the critical constant amplitude oscillation at 22.8 Hz and begins to converge at 22.9 Hz. Fig. 12 shows the details of torque oscillation waveform (magnified section) during constant frequency test at 22.8 Hz. The estimated oscillation frequency is nearly 17.5 Hz, which is close to the NTF. Through time-domain analysis, it is found that the torsional vibration of the system is amplified at 19.3-22.8 Hz, which is close to the oscillation range obtained by the characteristic structural analysis.

B. SPEED-UP AND SPEED-DOWN ANALYSIS OF IDEAL FREQUENCY-CONVERSION POWER SUPPLY

In the torsional vibration test of the shaft, the resonance region is obtained by analyzing the torsional vibration in the uniform speed-up and speed-down operations. Fig. 13 shows the variation in torque as a function of frequency. It is clear that when the frequency decelerates at a constant rate of 0.1 Hz/s, there is no significant region with torque oscillation. Fig. 14 shows that when the frequency decelerates at a rate of 0.01 Hz/s, the torque diverges and oscillates after entering the resonance area and obviously converges after 20.8 Hz. Fig. 15 shows that when the frequency accelerates at a rate of 0.01 Hz/s, the torque starts to diverge and oscillate at nearly 20.5 Hz, and then converges after 21 Hz. Therefore, the oscillation characteristics and interval change under different frequency variation rates. Combined with the results of ESA, it can be inferred that the frequency variation rate affects the damping of the system. When the frequency variation rate is high, the operation time of the system in the resonance region is short, and the torsional vibration mode...
may not be fully excited. In the actual torsional vibration test, to find the resonance region, the frequency variation rate should be reduced as much as possible. The influence of frequency rise and fall rates should be considered while dividing the speed forbidden zone. The speed control must be established in such a way it passes through the resonance area as soon as possible.

C. ANALYSIS OF TORSIONAL VIBRATION SUPPRESSION

When the shaft is damaged by torsional vibration, the maintenance process usually involves replacing it with a new shaft, strengthening the shaft strength, or improving the frequency converter. When the elastic coefficient of the shaft is increased to $1.68 \times 10^8$ Nm/rad, the torque oscillation waveform for rising frequency at a rate of 0.01 Hz/s is shown in Fig. 16. After the elastic strength of the shaft increases, it can be seen from equation (17) that the NTF increases, and the resonance region is narrowed and shifts to the right. Nonetheless, the resonance region exists, and it is still
possible to operate in the resonance region under the condition of wide-range speed regulation.

For suppressing the vibration of shaft, silicon oil damper and other related schemes can be used to increase the system damping. After adjusting the self-damping coefficient of shaft to 50 N.ms/rad and 500 N.ms/rad, the torque oscillation waveforms are shown in Fig. 17 and Fig. 18, respectively. The resonance region of torque expands and shifts to the left.

The torsional vibration is suppressed by changing the medium-voltage frequency ratio of the frequency converter. The torque waveform obtained after increasing the voltage frequency ratio by 1.2 times is shown in Fig. 19, where the amplitude of the resonance region is reduced. However, in the actual situation, the insulation voltage requirements and voltage frequency limits of the motor should be considered.

When the reactance of the inverter is 0.01 H, the torque oscillation waveform is shown in Fig. 20. Here, the resonance region is not improved significantly. When the reactance value is further increased, the resonance area is widened. Therefore, it is difficult to suppress the torsional vibration by adding reactance.

In the SSR suppression, a vital component of blocking filter is the resistance component, which can also be suppressed by changing the equivalent access resistance of synchronous generator. When a resistance of 0.01 W and 0.5 W is connected between the IM and the frequency converter, the corresponding torque waveforms are shown in Fig. 21 and 22, respectively. As the access resistance increases, the resonance
region becomes narrow and shifts to the right, and the peak torque decreases.

It is clear from the above analysis that it is difficult to use a single method to damp the torsional vibration under wide range of speed regulation. By increasing the elastic coefficient, the cost of the system increases, and the resonance transfer range becomes limited. When the self-damping coefficient is increased, the resonance region is shifted to the low-frequency band. When the voltage frequency ratio is modified, the torsional vibration can be partially suppressed, but the reactance can increase the risk of torsional vibration. When the resistor is connected in series, the resonance region shifts to the right and the resonance amplitude decreases.

V. TORSIONAL VIBRATION ANALYSIS OF IDF UNIT DRIVEN BY HIGH-VOLTAGE INVERTER

A. TIME-DOMAIN ANALYSIS OF TORSIONAL VIBRATION DRIVEN BY HIGH-VOLTAGE INVERTER

In ESA, it is difficult to establish the small-signal linearization model of high-voltage inverter with cascade multilevel structure. The model described in Section 2.4 is established to replace the ideal variable-frequency power supply in Section 3 for time-domain analysis. Fig. 23 and Fig. 24 show the diverging torque waveforms for constant operating frequencies of 18.23 and 19.84 Hz, respectively. Through frequency sweeping, it is found that the resonance region is 18.23-19.84 Hz, which is close to the ideal frequency conversion region and is slightly shifted to the left.

Similar to the ideal high-voltage inverter, a resistor is connected after the converter to suppress the torsional vibration. As shown in Fig. 25, the torque waveform for constant frequency operation at 19.5 Hz after connecting 0.5 W resistor is convergent. As shown in Fig. 26, the torque waveform for constant frequency operation at 35 Hz after connecting 0.5 W resistor is decentralized. When the resistance is increased, the resonance region shifts to the right. The voltage frequency ratio can be controlled within a certain
range by adjusting the PWM. When the voltage frequency ratio is increased, the resonance region is narrowed. However, when combined with the resistance, it can facilitate a better damping. As shown in Fig. 27, after the frequency converter is adjusted, the equivalent access resistance is actually changed, thereby changing the electrical damping of the system.

The time-domain analysis shows that the resonance region of torsional vibration for the IDF shaft system driven by frequency converter is related to the actual internal parameters, adjustment system, and control scheme of the frequency converter. The resonance region can be changed by connecting the resistance at the stator side. Further, better suppression of torsional vibration effect can be obtained by connecting the resistance and improving the regulation.

B. FIELD TORSIONAL VIBRATION TEST OF HIGH-VOLTAGE INVERTER DRIVE

In the power plant, the IDF unit was in power frequency operation mode initially. During the operation, the vibration test of the unit met the requirements, i.e., the horizontal, vertical, and axial vibration displacements were not more than 0.03 mm, 0.04 mm, and 0.04 mm, respectively. Further, the winding temperature of the motor met the qualification criterion, i.e., the maximum temperature for long-term operation was 86 °C, and the sound of the motor was normal and stable. After retrofitting the IDF with frequency converter, the vibration value, winding temperature, and motor noise increased significantly. Nearly three months after the VFD transformation, the welding seam between the intermediate shaft of the IDF near the motor side flange and the intermediate shaft cracked, and the bearing bush at the motor drive end was burned. After welding and strengthening the cracked intermediate shaft, it was put into frequency conversion operation again, and the vibration value and noise were still large. After three months of reinforcement operation, the intermediate shaft of the induced draft fan broke during variable frequency operation, which led to bending of the motor spindle, as shown in Fig. 28.

During the test, two measuring points were deployed: one at the intermediate shaft near the IDF end, and the other at the intermediate shaft near the IM end. The arrangement of the measuring points is shown in Fig. 29.

Each point was tested using power frequency light load, power frequency heavy load, frequency conversion heavy
load speed-up, frequency conversion light load speed-up, fre-
quency conversion heavy load speed-down, frequency con-
version light load speed-down, and constant frequency load
experiments to observe the influence of the frequency con-
verter on the torsional vibration of IDF unit. The working cur-
rents under light and heavy load were approximately 174 and
460 A, respectively. The intensity of the harmonic signal was
mainly 2 times and 2.5 times that of the input first-harmonic
signal, and the specific torsional vibration test under vari-
ous conditions was performed. The harmonic-speed curve
of torque angle speed under heavy-load acceleration test is
shown in Fig. 30.

According to the experimental analysis, the torsional vibra-
tion of the shaft does not change with the load under pow-
er frequency operation, and the overall torsional angle is
extremely small. According to the harmonic test analyzer test,
the current harmonic component under power frequency is
less than 1%. In frequency conversion operation under light
load, the overall torsional angle is not large, but compared
with the power frequency operation, the torsion angle is sig-
ificantly increased, and the 2 and 2.5 harmonics are excited.
Further, the current harmonic component is increased. Under
heavy load, the resonance region of torsional vibration is
basically unchanged, and the value of torsion angle is sig-
ificantly increased. The proportion of non-integer har-
monics in the current harmonic is considerably increased.

Based on the above experiments, it can be inferred that in
the speed range of 400-640 r/min, the torsional angle of 2
and 2.5 harmonics increases significantly, and the frequency
of torsional vibration remains basically the same. Further,
the fractional harmonic component of current harmonic com-
ponent is relatively consistent, and the proportion of har-
monic component is related to load variation. The resonance
speed region is transformed into torsional resonance region,
which is close to the interval obtained by the ESA and
time-domain analysis but is slightly shifted to the left. In the
field, by enhancing the voltage frequency ratio by 1.2 times
and inserting 0.5 W resistance in series after the frequency
converter, the torsional vibration can be greatly alleviated,
and the harmonic-speed curve is shown in Fig. 31.

VI. CONCLUSION

The torsional vibration caused by variable-frequency speed
regulation of IDF is a result of dynamic electromechanical
coupling behavior in the process of electromechanical energy
conversion. In this paper, a complete model of the system was
established to investigate the mechanism of torsional vibra-
tion using ESA and time-domain analysis. In ESA, if the load
torque and electromagnetic torque were assumed to be zero,
the NTF of the shafting could be obtained by determining the
eigenvalues of the state matrix of the shaft. After considering
the linearized small-signal model of the entire system, the real
and imaginary parts of the state matrix with the frequency
variation could be obtained through the continuous variation
of frequency, and then the resonance region of the torsional
vibration of shaft was obtained. In the time-domain analy-
isis, the oscillation and divergence of the torque waveform
were examined through the fixed-frequency scanning test,
and the resonance region of torsional vibration was obtained,
which was consistent with the result of ESA. The method
of increasing and decreasing speed was used to excite tor-
sional vibration. If the frequency variation rate was high,
the damping coefficient of the system was affected. More-
over, the operation time of the system in the torsional reso-
nance region was short, and the torsional vibration of the shaft
was not necessarily excited. Several methods for suppressing
the torsional vibration of shaft were considered. Changing
the elastic coefficient of shaft could change the NTF and
restrained the torsional vibration to a certain extent, but the
resonance region of torsional vibration could not be com-
pletely eliminated. When the damping coefficient of shaft
was changed, the resonance region of torsional vibration was
shifted, and the amplitude of torsional vibration was reduced,
which inhibited the torsional vibration of shaft to a certain
extent. If reactance was connected behind the frequency
converter, the torsional vibration could not be significantly
suppressed. By connecting a resistor to the stator side of the
IM, the torsional resonance region was shifted, and the tor-
sional vibration was significantly suppressed. The torsional
vibration could also be suppressed by adjusting the voltage
frequency ratio. It is difficult to obtain the linearized small-
signal model of cascaded multilevel high-voltage inverter
while using ESA. The detailed model of the converter could be established using time-domain analysis, and the influence of internal parameters and control methods of the inverter on the torsional vibration of the shaft was fully considered. In the time-domain analysis of cascaded multilevel high-voltage inverter, the resonance region was slightly shifted to the left, and the torsional vibration could be suppressed by adjusting the speed regulation and series resistance.

With the increasing popularity of VFD technology, the speed of several rotating shafting systems can be adjusted in a wide range, but they are likely to operate in the resonance region of torsional vibration for a long time. Using the proposed analysis and calculation techniques in the initial stage of system design, the resonance region can be actively avoided during the operation. In the VFD system, according to the actual conditions, the test experiment of torsional vibration can be conducted to identify the resonance region in the system. In the speed regulation experiment, the influence of frequency variation rate on the resonance interval should be considered. Further, a better understanding of the existing shaft torsional vibration systems can be obtained from the analysis and suppression methods of SSR.

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Q. Zhang et al.: Torsional Vibration Analysis of Shaft in an IDF Due to Variable Frequency Drive

QINGJIE ZHANG (Member, IEEE) was born in Jiangsu, China. He received the B.S. degree in electrical engineering from the Yancheng Institute of Technology, China, in 2004, and the M.S. degree in electrical engineering from Southeast University, Nanjing, China, in 2007, where he is currently pursuing the Ph.D. degree in electrical engineering. He is also a Lecturer with the College of Engineering, Nanjing Agricultural University. His current research interests include torsional vibration measurement, power electronics for renewable energy generation, power system stability control, and microgrid technology.

GUANGXIANG LU was born in Jiangsu, China, in October 1956. He received the B.S., M.S., and Ph.D. degrees in electrical engineering from Southeast University, Nanjing, China, in 1982, 1991, and 2000, respectively. He is currently a Professor with the Department of Electrical Engineering, Southeast University. His current research interests include micro-grid power quality, electric power savings, and relay protection.

YOU XU was born in Jiangsu, China. He received the B.S. degree in electrical engineering from the China University of Mining and Technology, Xuzhou, China, in 2002, and the M.S. and Ph.D. degrees in electrical engineering from Southeast University, Nanjing, China, in 2007 and 2013, respectively. He is currently a Lecturer with the School of Automation, Nanjing Institute of Technology. His current research interests include electric power converters for distributed generation, fault ride through for grid-connected inverters, and the application of embedded system for power electronics in power systems.

CHENGYU ZHANG was born in Jiangsu, China. He received the Ph.D. degree in physics from Nanjing University, Nanjing, China, in 2007. He is currently a Lecturer with the College of Engineering, Nanjing Agricultural University. His current research interests include soft matter and electromagnetic field.