Contextual Latent-Movements Off-Policy Optimization for Robotic Manipulation Skills

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Abstract—Parameterized movement primitives have been extensively used for imitation learning of robotic tasks. However, the high-dimensionality of the parameter space hinders the improvement of such primitives in the reinforcement learning (RL) setting, especially for learning with physical robots. In this paper we propose a novel view on handling the demonstrated trajectories for acquiring low-dimensional, non-linear latent dynamics, using mixtures of probabilistic principal component analyzers (MPPCA) on the movements’ parameter space. Moreover, we introduce a new contextual off-policy RL algorithm, named LAtent-Movements Policy Optimization (LAMPO). LAMPO can provide gradient estimates from previous experience using self-normalized importance sampling, hence, making full use of samples collected in previous learning iterations. These advantages combined provide a complete framework for sample-efficient off-policy optimization of movement primitives for robot learning of high-dimensional manipulation skills. Our experimental results conducted both in simulation and on a real robot show that LAMPO provides sample-efficient policies against common approaches in literature. Code available at https://github.com/SamuelePolimi/lampo

I. INTRODUCTION

Learning manipulation skills is essential for enabling robots to execute many tasks, both in home and industrial environments. An essential aspect of future robots is their ability to acquire, adapt or improve their skills, using demonstrations from non-expert users [1]. The provision of demonstrations is essential for robots to learn fast new tasks that do not have a concrete description, goal, or reward function [2]. Arguably, despite significant progress in learning to manipulate [3], the acquired skills do not generalize well across different tasks, and domains [4]. Manipulation tasks are described by various motions related to multiple objects (i.e., contexts) [5]. Robotics research has investigated different solutions to robot manipulation relative to different specifications of the problem. Notably, motion planning methods are designated for solving problems with access to an accurate dynamics model and precise goal specification [6], [7]. When the model is unknown or imperfect, but there is a partial task description through a reward function, reinforcement learning (RL) methods are the most suitable for acquiring complex skills [8]. Imitation learning (IL) has been extensively used when neither a perfect model nor a good task description is available [2].

IL methods employ sets of expert demonstrations on the task to be considered [9], [10]. Usually, these demonstrations represent a set of robotic joint configurations at specific time intervals. The most prominent methods in robot learning maps the set of demonstrated motions to a parametric model of the movements [11], [12]. Movement Primitives (MP) represent a broad family of dynamical system descriptors used to parameterize robot movements [12]. Dynamic MPs (DMPs) have been thoroughly explored as a robot control policy, as they are stable and robust, however, they lack generalization properties [13]. Due to their modularity, MPs emerged as a generic framework for IL, as they are sample efficient, they can be used with non-expert demonstrations, and provide safe learning [2]. The framework of Probabilistic Movement Primitives (ProMPs) [14], [15] enjoys favorable properties, like time modulation and conditioning to different contexts, which makes it a well-suited tool for imitation and improvement of robotic movements. Contexts are typically considered as vectors, describing, for example, a goal position, the position of the object to be picked, etc. A promising approach towards generalization of skills is the improvement of the learned policies through RL. Hence, researchers have opted for combining policy optimization with the learned MPs to generalize over different tasks, and related contexts [16]–[19].

However, the usual high-dimensional representation of robotic movements complicates the application of RL techniques since learning on the real robot requires much more sample-efficiency than learning in simulation [3]. To overcome this difficulty, researchers have proposed the application of dimensionality reduction (DR) techniques in movement primitives (MPs) [10], [20]–[24]. The resulting latent representation can help us decode the inter-dependencies between movements and task-contexts, allowing us to use...
contextual RL [5], [25], [26] to optimize over the task-related parameters.

A probabilistic DR technique of ProMPs in the joint space was presented in [23], using expectation-maximization (EM) over the context variables, for representing the movements in a linear low-dimensional subspace of the full configuration space. Then, the policy is further optimized via relative entropy policy search (REPS) [27]. In [24], the authors introduced a DR technique of the exploration parameters of DMPs when using a path integral policy improvement algorithm [28]. In [9] a low dimensional latent variable model for ProMPs is extracted using fully Bayesian hierarchical models, which is used only for imitation learning. Autoencoded DMPs are proposed in [21], which uses deep autoencoders to find a latent representation of the movement from the robot’s task space, towards generalizing the performance of the DMPs. In [22] the authors proposed the use of a time-dependent variational autoencoder to address the generalization challenges. Auto-encoders with ProMPs for efficient human motion prediction were proposed in [29]. In [30], a reduction of the parametric space was proposed, as more appropriate for learning MPs. In [31] the authors use parametric DR to learn a mapping from the MPs latent space to a reward for policy improvement.

This paper introduces a novel RL algorithm for optimizing over MPs, named LAtent-Movements Policy Optimization (LAMPO). This algorithm’s primary focus is to gain sample efficiency by 1. performing the policy updates in a reduced latent space, and 2. using off-policy gradient estimations, which re-use samples collected from previous learning episodes. In robot learning, the reduced number of samples and the use of contextual policies are important for acquiring manipulation skills through demonstrations and, subsequently, improving them so that they generalize across various contexts. For learning the latent representation, we use a Mixture of Probabilistic Principal Component Analysis (MPPCA) [32]. This method, differently from other DR techniques, is fully probabilistic, allowing us to perform conditioning. MPPCA can be seen as a Gaussian mixture model (GMM), enabling us to represent multimodalities and non-linear dependencies in the demonstrated data, while it can perform de-noising, as it can extract isotropic noise contained in the data. Our method does not use demonstrations as means of initialization. We pre-train a structured latent space of the MPs through MPPCA, which is essential for exploring and optimizing through RL.

For the off-policy estimation we use self-normalized importance sampling [33]. Differently from other approaches [34], [35], we perform a full-gradient estimation, also considering the normalization factor, which further lowers the variance, similarly to the baseline subtraction method [36], [37]. Off-policy evaluation and improvement are core problem in RL. The off-policy estimation is in fact hard to obtain, due to distribution mismatch [34], [38]. Recent model-free deep RL methods employ off-policy estimation [8]; though, such methods require a vast amount of interactions of the agents with the environment. Notably, our method combines off-policy estimation together with trust-region regularization that contains the estimation variance [39]–[41]. We apply a forward Kullback-Leibler (KL) divergence bound between the subsequent policies, which, jointly to a KL-regularization of the context distribution, provides a robust policy optimization. We evaluated our algorithm both in simulated environments provided by RLBench [42] and on a real manipulator robot. The experimental analyses show that LAMPO outperforms the state of the art techniques in sample efficiency on challenging high-dimensional robotic manipulation tasks.

II. PROBLEM STATEMENT

We consider contextual problems where the robot must adapt its behaviour based on a context $c$. The context can be seen either as a description of the current state of the task (e.g., position of an object), or as a goal (e.g., a desired quantity that the robot should pour in a bowl) and is described by a $d_c$-dimensional vector (i.e., $c \in \mathbb{R}^{d_c}$). At each episode, the robot observes a new context distributed according to $p(c)$. In our work, we consider robotic movements parametrized by a vector of parameters $\omega \in \mathbb{R}^m$. Since the movements depend on the context $c$, the behavior of the human demonstrator can be described as a stochastic mapping $q(\omega|c)$ between the movements’ parameters $\omega$ and the context $c$. A user-defined reward $R(\omega, c)$ indicates how well a particular movement parametrized by $\omega$ performed according to the context $c$. Our objective is to find the optimal policy which maps movements and contexts

$$\theta^* = \arg\max_{\theta} \int R(\omega, c)p_0(\omega|c)q(\omega|c)\, dc\, d\omega. \quad (1)$$

III. LATENT MOVEMENTS POLICY OPTIMIZATION

Finding the optimal policy of the objective (1) is non-trivial. In our setting, we acquire an initial set of $N$ demonstrations, from which we learn an initialization of the parameters of the ProMPs. We start with IL, where we aim to find an initial policy parameter set $\theta_0$ for which the initial policy $p_{\theta_0}(\omega|c)$ approximates the demonstrator’s behavior $q(\omega|c)$. By maximizing

$$\theta_0 = \arg\max_{\theta} \sum_{i} \log p_{\theta_0}(\omega_i, c_i), \quad (2)$$

where $\omega_i, c_i \sim q(\omega_i|c_i)q(c_i)$, we can then find the policy $p_{\theta_0}(\omega|c)$ by conditioning. Subsequently, we aim to use RL for policy improvement. This requires the use of a generative model of $p(\omega|c)$ for generating trajectories over context. At every iteration $T$, we collect a set of rewards $\{R_t, \omega_t, c_t\}_{t=(T-1)n}$ for $n$ number of episodes using the $T^{th}$
set of parameters \( \theta_T \) and optimize objective (1) to acquire a better set of parameters \( \theta_T \) (Alg. 1). In our framework, we replace the high-dimensional \( \omega \) with a better suited low-dimensional latent representation. Fig. 2 is a graphical description of the proposed algorithm, which we detail in the following.

### A. Imitation Learning with Dimensionality Reduction

Consider the specific parametrization of the MP in [14], which allows to obtain a specific smooth trajectory \( \tau \) depending on a parameter vector \( \omega \). Given a specific trajectory \( \tau \), it is possible to infer the most likely set of parameters \( \omega \) that generated such trajectory (via Ridge regression). The ProMPs framework introduces the density estimate of the parameters \( p(\omega) \) given a set of demonstrations \( \{ \tau \} \). Furthermore, using probabilistic conditioning, one can provide a set of demonstrations that depends on a context \( \{ \tau_i, c_i \} \) and infer what are the most likely parameters given a new unseen context \( c \), i.e., \( p(\omega|c) \). To overcome the high dimensionality of the ProMPs, we introduce a mixture of probabilistic movements to find a latent space to represent jointly the movement parameters and the context,

\[
\begin{align*}
  k & \sim \text{Categorical}(k|\pi) \quad (k \in \{1, \ldots, K\}) \\
  z & \sim \mathcal{N}(z|\mu_k, \Sigma_k) \quad (z \in \mathbb{R}^d) \\
  e_\omega, e_c & \sim \mathcal{N}(e_\omega, e_c|0, I) \\
  \omega & = \Theta_k z + \bar{\omega}_k + \sigma^2 e_\omega \quad (\omega \in \mathbb{R}^m) \\
  c & = C_k z + \bar{c}_k + \sigma^2 e_c \quad (c \in \mathbb{R}^{d_c})
\end{align*}
\]

where \( k \) is a categorical latent variable representing the selection of a particular Gaussian, its parameter \( \pi \) denotes the probabilities describing this selection; \( z \) is a Gaussian latent variable representing a specific movement-context pair in each cluster. The isolation of the isotropic noises \( e_\omega, e_c \) helps to compress only the useful information, and act as a denoiser in the generative model.

Groping the two last equations together

\[
\begin{align*}
  \omega | c & = \begin{vmatrix} \Theta_k \\ C_k \end{vmatrix} z + \bar{\omega}_k + \sigma^2 e_\omega \\
  c | & \sim \mathcal{N}(c|\mu_k, \Sigma_k) \quad \quad \quad \quad \quad \quad (3)
\end{align*}
\]

we notice the equivalence with the mixture of principal component analyzers.

The parameters \( \{ \pi, \Theta_k, C_k, \bar{\omega}_k, \bar{c}_k, \sigma^2 \} \) can be inferred via EM [32], assuming, without loss of generality, that \( \mu_k = 0, \Sigma_k = I \). Once the maximum-likelihood parameters are obtained, to generate new movements conditioned on a context \( c \), we first sample the latent variables \( z, k \) conditioned on \( c \), and then we establish the relation \( \omega = \Theta_k z + \bar{\omega}_k \) removing the isotropic noise on the movement (as it is an unnecessary perturbation).

The generative model of \( p(z, k|c) \) is a GMM, \( p(z, k|c) = p(z|k, c)p(k|c) \), where the conditional responsibility is

\[
p(k|c) = \frac{p(c|k)p_k}{\sum_k p(c|k)p_k} \quad (4)
\]

and \( p(c|k) \) is obtained marginalizing \( z \) from \( p(c|k, z) \) in (3) and

\[
p(z|k, c) = \mathcal{N}(z|B_k (\sigma^2 C_k^{-1} c - \bar{c}_k) + \Sigma_k \mu_k), B_k) \quad (5)
\]

with \( B_k = (\Sigma_k + \sigma^2 C_k^{-1})^{-1} \). In the generative model, we assume no isotropic noise on the movement’s parameter \( \omega = \Theta_k z + \bar{\omega}_k \), with \( z \sim p(z|k, c), \quad k \sim p(k|c) \). (6)

The described generative model captures non-linear dependencies between the context and the robotic movement, both defining a convenient latent representation and maintaining mathematical tractability.

### B. Off-Policy Reinforcement Learning

The variables \( \{ C_k, \bar{\omega}_k, \Omega_k, \bar{c}_k, \sigma_k \}_{k=1}^K \) describe how to project, for each component \( k \), a variable \( z \) to a movement-context pair. To maintain a plausible representation of the movement, this projection should remain fixed, while we optimize the distributions of \( k \) and \( z \) that represent how the movements and the context are distributed in the latent space. The variables \( k \) and \( z \) are described by \( \{ \pi \} \cup \{ \mu_k, \Sigma_k \}_{k=1}^K \).

Since the model’s variables \( \{ \Omega_k, \bar{\omega}_k, C_k, \bar{c}_k \}_{k=1}^K \) are now fixed, they can be considered a part of the reward signal, changing the initial problem (1) into

\[
J(\theta) = \int R(\omega, c)p_\theta(\omega|c|q(c) \cdot d\omega \, dc
\]

\[
= \int \sum_{k=1}^m R(\Omega_k z + \bar{\omega}_k, c)p_\theta(z, k|c)q(c) \cdot dc \quad (7)
\]

The above formulation provides a desirable latent representation of the policy, as the latter is depended on the latent parameters \( z, k \) conditioned on the underlying context \( c \). LAMPO interleaves policy-optimization and data-gathering. At each iteration \( T \) we consider all samples collected from all past policies \( \theta_1, \ldots, \theta_{T-1} \). For each new set of parameters \( \theta_T \), we obtain a new conditional model \( p_{\theta_T}(\omega|c) \), and a new context distribution \( p_{\theta_T}(c) \) that allows to obtain new samples. However, when \( p_{\theta_T}(c) \) diverges from \( q(c) \), the computation of \( p_{\theta_T}(\omega|c) \) can suffer from numerical instabilities. To prevent this issue, we use a KL-regularization to keep the two distributions close enough. Furthermore, to prevent premature convergence of the policy to a local optimum, we use a KL-constraint between the previous and the current policy distribution i.e., \( KL(p_{\theta_T}(\cdot|c)\|p_{\theta_{T-1}}(\cdot|c)) \).

1) Off-Policy Estimate: To reuse all the past experience, we propose to use Self-Normalized Importance Sampling (SNIS) [33]. SNIS estimation has usually lower variance than pure importance sampling, at the price of a small bias. Its usage in the context of RL is well established [34], [35].

At the iteration \( T+1 \), the dataset is composed of samples generated by \( T \) different policies. We compute the importance ratio \( \rho \) for each pair of samples \( z_i, k_i \) by using the current policy \( p_\theta(z_i, k_i|c_i) \) at the numerator and the mixture of the past policies at the denominator. The SNIS estimate results to be

\[
J(\theta) \approx \hat{J}(\theta) = \sum_{i=1}^n \frac{\rho_i R_i}{\nu} \quad \text{with} \quad \nu = \sum_{i=1}^n \rho_i \quad (8)
\]

2) Context Regularization: To compute \( p(z, k|c) \), we need the ratio \( p(z, k, c)/p(c) \). Let us rewrite (7) as

\[
\int \sum_{k=1}^K \int \int \int \int R(\Omega_k z + \bar{\omega}_k, c)p_\theta(z, k|c)q(c) \cdot d\omega \, dc \, d\theta \quad (9)
\]

During the policy optimization, \( p_\theta(c) \) can diverge from \( q(c) \) causing high variance of the estimate. To avoid this issue, we encourage \( p_\theta(c) \) to stay close to \( q(c) \) using the KL-divergence, where \( \theta_0 \) are the initial policy’s parameters
acquired with IL. Since \( p_\theta(c) \) is a mixture of Gaussians, we cannot compute in closed form the KL between \( p_\theta(c) \) and \( p_{\theta_0}(c) \). To overcome this issue, we compute the KL between \( p_\theta(c, k) \) and \( p_{\theta_0}(c, k) \), which is an upper-bound of the previous term

\[
\int \sum_k p_\theta(c, k) \log \frac{p_\theta(c, k)}{p_{\theta_0}(c, k)} dc = \int p(c) \sum_k p_k(c | k) \log \frac{p_k(c | k)}{p_{\theta_0}(k|c)} dc + \int p_\theta(c) \log \frac{p_\theta(c)}{p_{\theta_0}(c)} dc.
\]

Always non-negative

The analytical expression of the KL-divergence between \( p_\theta(c, k) \) and \( p_{\theta_0}(c, k) \) takes the form of

\[
\eta_\theta = KL(p_\theta(c, k) || p_{\theta_0}(c, k)) = \sum_k p_k(c) \left( \mathcal{H}(p_\theta(c | k), p_{\theta_0}(c | k)) - \mathcal{H}(p_\theta(c | k), p_{\theta_0}(c | k)) \right)
\]

where \( \mathcal{H}(\cdot) \) is the entropy, and since \( p_\theta(c | k) \) and \( p_{\theta_0}(c | k) \) are Gaussian distributions, and \( p_k(c) \) and \( p_{\theta_0}(c, k) \) are categorical distributions, (10) is computable in closed form. Using the divergence in (10) as a regularization term in our objective (8) stabilizes the optimization process.

3) Trust Region: To obtain a robust optimization process, it is a common practice to introduce a KL-constraint (or regularization) between two subsequent policies during the optimization [27], [40]. These methods avoid the premature convergence to a local optimum and they guarantee a smooth optimization process. Usually, the KL-constraint is approximated via the samples generated by the previous policy [39]. In our case the KL between the current and the previous policy defined on the latent variables \( k \) and \( z \) have closed form, as shown in the previous subsection. Hence, the KL divergence between the optimized policy and the policy at iteration \( T - 1 \) can be computed as

\[
g_\theta(c) = KL(p_{\theta_{T-1}}(\cdot | c) || p_\theta(\cdot | c))
\]

C. Off-policy improvement

We have introduced so far the off-policy objective, the context, and the thrust region regularization. We can formulate the gradient estimation as a constrained optimization problem. To ensure the parameters consistency, we encode the categorical distribution as \( \pi = e^{\theta_i} / \sum_{\theta_i} e^{\theta_i} \), and the covariances \( \Sigma_k \) as positive-definite diagonal matrices. By combining (8) and (10), and (11), we obtain

\[
\max_\theta J(\theta) - \gamma \eta_\theta s.t. (nT)^{-1} \sum_{i=1}^{Tn} g_\theta(c_i) \leq \chi
\]

which turns out to be equal to the SNIS of the classic gradient with baseline subtraction, yielding lower variance in the gradient estimation [36], [43], [44]. We could compute \( \nabla_\theta \log p_\theta(z_i, k_i | c_i) \) by taking the gradient of (5), but due to the matrix inversion in \( B_k \), the computation presents numerical instability. We propose, instead, the derivation of the gradient from

\[
\log p_\theta(z, k | c) = \log p_\theta(c, z, k) + \log p_\theta(z | k) + \log \pi_k
\]

which is solvable in closed form, where \( p_\theta(z | k) = \mathcal{N}(z | C_j \mu_j + \bar{c}_j, \sigma_j^2 I + C_j \Sigma_k C_j^{-1}) \), does not require the inversion of \( B_k \). The gradient can be computed via automatic differentiation tools, like PyTorch or TensorFlow. An overall description of LAMPO is presented in Alg. [1]

IV. EXPERIMENTAL ANALYSIS

A. Experimental setup

In the following, we analyze the performance of LAMPO. We study the effect of motion multimodalities, comparing against baseline approaches, i.e., planning, deepRL and policy improvement methods. We also test whether LAMPO scales to different contexts and dimensionalities of a given problem. Finally, we train and validate our proposed method on a real robotic platform.

Virtual environments

2d-reacher: toy simulated environment where a robotic manipulator is composed of two revolute joints, and two links of length one. The task consists in reaching four different target areas, as depicted in Fig. [3] left. The user can select the number of clusters from which one can generate the respective goals, to acquire diverse and of different complexity movements in the dataset. The demonstrations are provided using inverse kinematics (IK). A variation of the previous task includes also an obstacle (Fig. 2). The purpose is to compare LAMPO performance against RRT*. The demonstrations are also provided using RRT*. The movements are encoded by 20 radial basis functions, for a total of 40 parameters. We add to this set an extra-parameter

\[
\text{Algorithm 1 LAtent Movement Policy Optimization}
\]

1: input: Dataset \( \{\tau_i, c_i\}_{i=1}^{nT} \) of trajectories and contexts.
2: for each trajectory \( \tau_i \) compute the parameter vector \( \omega_i \).
3: EM to find the MPPCA’s parameters \( \pi, \Omega, \omega, C_k, \Sigma_k \) [32]
4: assuming \( \mu_k = 0, \Sigma_k = I \).
5: Set \( \theta_0 \equiv \{\pi \} \cup \{\mu_k, \Sigma_k\}_{k=1}^{K} \).
6: for \( T \in \{1 \cdots N_T\} \) do
7: for \( i \in \{n(T-1) \cdots nT\} \) do
8: observe context \( c_i \)
9: perform movement \( \omega_i \) with [6]
10: collect reward \( R_i \)
11: end for
12: compute a differentiable model of \( J(\theta), \eta_\theta \) and \( g(\theta) \) using [8]
13: solve (12) for \( \{\pi\} \) \cup \{\mu_k, \Sigma_k\}_{k=1}^{K} \) using SLSQP
14: update \( \theta_T \equiv \{\pi\} \cup \{\mu_k, \Sigma_k\}_{k=1}^{K} \)
15: end for
### B. Evaluation in Virtual Environments

For providing insights regarding the performance of LAMPO, we have compared it against the most relevant algorithm by Colomé and Torras (CT) [30], that uses GMMs in the parameter space of ProMPs only after performing DR, and optimizes the policy using REPS, but also with IK-based and RRT* planning. We decided, first, to compare on the toy experiment of 2d-reacher for the policy improvement setting, depicted in Fig. 6. In this evaluation, we keep the latent space dimension fixed for CT and LAMPO, but we test the performance for different number of goal-clusters (from 1 to 4). For fair comparison, we have also included another baseline method, GMM+REPS, in which we do not apply DR on the collected data. As expected, while for unimodal movements ($K = 1$) LAMPO and CT have similar performance, for multimodal movements ($K > 1$), CT performance is suboptimal. The simple GMM+REPS approach performs poorly in all cases, showing the importance of acquiring a low-dimensional latent representation that preserves the motion’s non-linearities. Furthermore, we tested LAMPO on a more challenging setting of the 2d-reacher with an obstacle, and compared with RRT*, as depicted in Fig. 4. LAMPO achieves a good performance of 0.93 success rate of reaching the target goal positions. We argue that our method can scale quite well in high-dimensional tasks, which allows us to trade-off the agent’s performance with the possibility of applying our algorithm when there is not an accurate description of the model or the task’s objective.

After the above observations, we evaluated LAMPO on high dimensional problems. Starting with the...
ONCLUSIONS

We introduced LAMPO, a novel contextual RL algorithm for optimizing over MPs while learning high-dimensional robotic manipulation tasks with different contexts in a safe way. The main advantage of LAMPO is twofold: first, it offers a reduced latent representation of non-linear dynamics, while encoding the dependencies between movements and task-related contexts introducing the MPPCA graphical model on MPs. Second, it gains in sample-efficiency with off-policy gradient estimations, re-using samples collected from previous learning episodes. We evaluated our algorithm both on simulated environments and on a real manipulator robot. The experimental analyses show that LAMPO outperforms the state of the art techniques in terms of sample efficiency on high-dimensional robotic manipulation tasks. Our future work will focus on introducing a decomposition of demonstrated tasks into sub-policies, to improve modularity and task performance.

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