ABSTRACT
We review some recent results and open problems on four fermion physics at LEP2 and beyond.

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LEP2 4f review

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Abstract
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1. Introduction
LEP2 has opened the era of four fermion physics. These processes have shown to be essential for deeper insight in the SM and their precise knowledge is fundamental for limits and searches of signals of New Physics. Important experimental results in this field have already been achieved (see other talks in this session).

On the theory side, a strong effort has led in recent years to a number of dedicated 4f MC generators. Among them, some are capable to compute (with different methods, features, approximations etc.) the whole set of 4f final states: EXCALIBUR [1], which has been the first complete code, ALPHA [2], COMPHEP [3], KORALW [4], GRC4F [5], WPHACT [6], WTO [7], WGENPV [8]. The only 4f code to use a semianalytical approach is GENTLE [9], which can compute all final states with no identical particles, e's or νe's. Good technical agreement among these generators has been tested during the first LEP2 workshop [10]. Despite the very high technical precision of the codes, the theoretical uncertainty for the 4f processes is normally $\mathcal{O}(1\div2\%)$. But of course it depends very much on final states and on cuts and for some configurations (eg. arbitrarily low invariant masses and angles) we do not even have a safe estimate. Already with the present LEP2 luminosity ($L \approx 1200$ pb$^{-1}$) many statistical errors are comparable or even smaller than theoretical ones. Therefore, in view of the foreseen total luminosity and of the many different physical processes explored, new requests come from the experimental community for a higher precision. The Linear Collider (LC) will pose further severe requests on the reliability of 4f computations. In the following, after recalling gauge invariance issues, we will give a short review of recent developments in this sense and of some relevant open problems.

2. The gauge invariance issue
As it is well known, perturbative calculations should be gauge invariant ($gi$), otherwise the result can depend in an uncontrolled way on the chosen gauge. 4f calculations and their radiative corrections, involve in general three types of possible gauge invariance ($GI$) problems related to: the use of an incomplete set of diagrams, the non $GI$ of some initial (ISR) or final (FSR) state radiation treatment, the fact that one has to deal with unstable particles.

As an example of the first type, we may recall that the double resonant diagrams for WW (CCO3) or ZZ (NCO2) production are the dominant contributions but they are not $gi$ as other diagrams (single or non resonant ones) contribute to the same final state. The latter are however $\mathcal{O}(\Gamma/M)$ and suppressed for invariant mass cuts near $M$.

For $GI$ and ISR/FSR, let us recall that to have a $gi$ quantity, one must consider the emission of a photon from all possible parts of a charged line. So for instance in WW production diagrams one cannot just consider the emission from incoming or outgoing fermions, as the charged line passes through the W which can also emit. As a consequence, ISR and FSR alone are $gi$ only in LL approximation, and this implies for instance that accounting for $p_T$ γ's via ISR/FSR might in principle be dangerous.

The presence of unstable particles can easily lead to violating gauge invariance: intermediate W or Z give rise at tree level to poles $\frac{1}{p^2-M^2}$. This is cured by introducing finite decay width: $\frac{1}{p^2-M^2} \rightarrow \frac{1}{p^2-M^2+i\Pi}$, with $\Pi=\Gamma M$ (fixed width) or $\Pi=\Gamma M^2$ (running width). It corresponds to the Dyson resummation of self energy graphs. In this way part of higher order corrections are accounted for and the procedure breaks $GI$. Some see order by order in perturbation theory. In general this results in negligible effects $\mathcal{O}(\frac{1}{\pi})$, but they can be
dramatically enhanced by gauge cancellations or small scales, as it happens for instance in so called single W processes. Different possible solutions are known:

- Fixed width. It makes use of fixed width also in t channel Vector Boson propagators.
- Overall scheme. All tree level diagrams (not only the resonant ones) are multiplied by factors \( \frac{p^2 - M^2}{p^2 - M^2 + \Gamma M} \).
- Pole scheme [1]. Expand the amplitudes in \( \frac{1}{\Gamma} \) around the complex poles. Each order of the expansion will be separately \( g_1 \).
- Fermion loop (FL) scheme [2]. Use Dyson resummed resonant propagator with (at least) all other one loop corrections necessary to satisfy the Ward identities. This is considered to be the most satisfactory from a theoretical point of view.

3. Radiative corrections - DPA

Full EW RC to on shell W production and decay are known. Complete tree level for 4f (+ ISR/FSR, Coulomb, hadronization, PS, hadronization, ...) are implemented in MC event generators. Complete EW one loop corrections to full ME are incredibly complicated and at present not available. A consistent, gauge invariant way of evaluating the most relevant corrections to “WW (or ZZ) like” full processes has been studied in recent years (DPA) [3, 4] and completed recently [5]. It corresponds to the above mentioned Pole scheme and it is referred to as Double Pole Approximation. With these corrections \( \approx 5\% \) theoretical precision will hopefully be achieved in WW cross sections and similar methods can be extended to ZZ.

In DPA one expands the amplitude around complex poles of unstable W’s. Only double resonant terms are retained in tree level and \( O(\alpha) \) corrections. It corresponds to an expansion both in \( \alpha \) and \( \frac{\Gamma_W}{M_W} \). Terms \( O(1) \), \( O(\alpha) \), \( O(\frac{\Gamma_W}{M_W}) \) are kept; \( O(\frac{\Gamma_W^2}{M_W^2}) \) is neglected. Resonant propagators can be safely be resummed and the various orders are separately \( g_1 \). The method is reliable only above threshold and for searches. Moreover, e\( ^+ e^- \rightarrow 4f \gamma \) processes with both visible and invisible \( \gamma \) are important. Even if \( 4f^+ \) visible \( \gamma \) events are not as abundant at LEP2 as they will be at LC, they involve triple and quartic gauge coupling and are relevant for W mass measurement and for searches. Moreover, e\( ^+ e^- \rightarrow 4f \gamma \) calculations are a fundamental step towards having a description of radiation that goes beyond \( p_T \) ISR/FSR and its GI problems. The goal is to match them with radiative corrections and collinear radiation.

Calculations with massive fermions + \( \gamma \) have already been performed for CC10 (\( \mu u d \gamma \)) and CC20 (\( e u d \gamma \)) processes. Recently a new massive computation of (\( \mu u d \gamma \)) has been completed [6], which makes use of multichannel MC techniques. Results from a complete \( 4f + \gamma \) MC (RACOONWW) have appeared this year [7]. This is at present

\[ d\sigma = d\sigma^0_{DPA}(1 + \delta_DPA) + (d\sigma^0 - d\sigma^0_{DPA}) \]

In fig. 1 is reported the W invariant mass spectrum computed in DPA, where the complete calculation show a sizeable shift. Exact complete DPA are at present not yet implemented in a MC.

4. 4f + \( \gamma \)

e\( ^+ e^- \rightarrow 4f \gamma \) processes with both visible and invisible \( \gamma \) are important. Even if \( 4f^+ \) visible \( \gamma \) events are not as abundant at LEP2 as they will be at LC, they involve triple and quartic gauge coupling and are relevant for W mass measurement and for searches. Moreover, e\( ^+ e^- \rightarrow 4f \gamma \) calculations are a fundamental step towards having a description of radiation that goes beyond \( p_T \) ISR/FSR and its GI problems. The goal is to match them with radiative corrections and collinear radiation.
the only code which can account for all $4f$ and $4f + \gamma$ final states with all tree level Feynman diagrams. Other important RACOONWW features are: the multichannel integration, the non linear gauge fixing which simplifies the calculations and the gauge restoring scheme which preserves $SU(2)\times U(1)$ using everywhere complex boson masses ($M^2 \rightarrow M^2 - i\Gamma M$). The code assumes massless fermions, therefore collinear and soft regions have to be excluded with cuts and so far it has no ISR/FSR and interface to hadronization.

An interesting example of photon spectra calculation by RACOONWW is reported in fig. 2

5. Single W

The charged current processes with an electron and its neutrino in the final state ($e\bar{\nu}f_1f_2$) have, besides the usual diagrams of the corresponding process $\mu\bar{\nu}f_1f_2$, all diagrams obtained exchanging the incoming $e^+$ with the outgoing $e^-$. These contributions become dominant for $\theta_e \rightarrow 0$ for the presence of the t-channel $\gamma$ propagator. These four fermion events with $e$ lost in the pipe are often referred to as single W, and they are relevant for triple gauge studies and as background to searches.

The t channel $\gamma$ propagator diverges at $\theta_e = 0$ in the $m_e \rightarrow 0$ limit, so fermion masses have to be exactly accounted for. Moreover, the $1/\sin \theta_e$ behaviour is reduced to $1$ by gauge cancellations. This implies that even a little violation of gauge conservation can have dramatic effects and the use of some gauge conserving scheme is unavoidable. Finally, the phase space integration has to be studied carefully for the logarithmic behaviour in $m_e$ and also in other masses for low $m(f_1f_2)$ (multiperipheral diagrams).

Two strategies have been used so far: Improved Weizsacker Williams (IWW)[19] and completely massive codes.

In the first case one separates the 4 t-channel photon diagrams, evaluates them analytically in equivalent photon approximation with exact dependence on all masses, and then adds the rest of diagrams + interference in the massless approximation. An example of results obtained with this approach is given by the m(ud) mass distribution of fig. 3.

In the fully massive MC numerical approach COMPHEP, GRC4F, KORALW and WPHACT have recently compared their results and found a very good agreement reported in fig. 4. An agreement of the order of .5% has been obtained by COMPHEP, GRC4F, WPHACT also at 800 GeV. It has to be noticed that the version of WPHACT used in these comparison is a new one in which all completely massive matrix elements have been added, so that one can choose between these and the fastest massless ones.

In fig. 4 are reported also the results obtained by the massless codes EXCALIBUR and ERATO[20] using a fictitious minimal angle to avoid the divergence for $m_e = 0$.

In view of the relevance of using a gauge restoring scheme in these computations, Massive FL (Im part) scheme has recently been implemented [21]. The comparison among this scheme and the others reported in fig. 5 shows a substantial independence from the $gi$ scheme.

Figure 2. $d\sigma/dE_e$ (fb/GeV) photon spectra from the complete calculation and from triple gauge boson production (from ref.[17])

Figure 3. Invariant mass $m(ud)$ distribution for the process $e^+e^- \rightarrow e^-\nu ud$ computed by W0
Figure 4. Comparison on the total cross section for $e^+e^- \rightarrow e^-\nu ud$ with $|\cos \theta_e| > .997$, $m(u,d) > 5$ GeV, $E_{u,d} > 3$ GeV.

Figure 5. Total cross sections for different gauge restoring schemes computed by WPHACT for the same process and cuts as in fig. 4.

6. Conclusions

Recent developments show that 4f physics is at present leaving the percent era. The new precision requires better knowledge of RC to full 4f final states. Important results in this sense will probably allow a sensible diminution of the theoretical error for WW or double resonant physics. However, also other channels, processes and selection cuts are under active experimental investigation and in many cases the actual theoretical precision is much worse than 1%. In LEP2 MC workshop, 4f subgroup, these topics are at present under discussion.

Some of the main open problems which still need a definitive answer regard

- Inclusion of photons and ISR/FSR: beyond $p_T$ ISR/FSR; radiative and non radiative events; how to match the two regimes.
- 4F contribution to monojets
- “Single W” ($e\nu 2q$ with lost e) for 2q’s with low invariant masses: resolved photons.
- Single Z
- Gamma Gamma physics with 4f and relationship with dedicated gamma gamma MC

- $t$ channels and ISR/FSR
- Radiative corrections: DPA for ZZ; for MIXED processes; for non WW or ZZ physics; complete $O(\alpha)$; LC and radiative corrections.

Needless to say, their solution is in some case far from easy but one must realize that these investigations, important for LEP2, will become essential in the future for all physics studies at Linear Collider.

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