Incoherent superconductivity well above $T_c$ in high-$T_c$ cuprates—harmonizing the spectroscopic and thermodynamic data

J G Storey

Robinson Research Institute, Victoria University of Wellington, PO Box 600, Wellington, New Zealand
MacDiarmid Institute, Victoria University of Wellington, PO Box 600, Wellington, New Zealand
E-mail: james.storey@vuw.ac.nz

Keywords: cuprates, scattering, specific heat, superfluid density, Raman spectroscopy, tunneling, optical conductivity

Abstract

Cuprate superconductors have long been known to exhibit an energy gap that persists high above the superconducting transition temperature ($T_c$). Debate has continued now for decades as to whether it is a precursor superconducting gap or a pseudogap arising from some competing correlation. Failure to resolve this has arguably delayed explaining the origins of superconductivity in these highly complex materials. Here we effectively settle the question by calculating a variety of thermodynamic and spectroscopic properties, exploring the effect of a temperature-dependent pair-breaking term in the self-energy in the presence of pairing interactions that persist well above $T_c$. We start by fitting the detailed temperature-dependence of the electronic specific heat and immediately can explain its hitherto puzzling field dependence. Taking this same combination of pairing temperature and pair-breaking scattering we are then able to simultaneously describe in detail the unusual temperature and field dependence of the superfluid density, tunneling, Raman and optical spectra, which otherwise defy explanation in terms a superconducting gap that closes conventionally at $T_c$. These findings demonstrate that the gap above $T_c$ in the overdoped regime likely originates from incoherent superconducting correlations, and is distinct from the competing-order ‘pseudogap’ that appears at lower doping.

1. Introduction

A prominent and highly debated feature of the high-$T_c$ cuprates is the presence of an energy gap at or near the Fermi level which opens above the observed superconducting transition temperature. It is generally known as the ‘pseudogap’. Achieving a complete understanding of the pseudogap is a critical step towards the ultimate goal of uncovering the origin of high-temperature superconductivity in these materials. For example, knowing where the onset of superconductivity occurs sets limits on the strength of the pairing interaction. The community has long been divided between two distinct viewpoints [1]. These can be distinguished by the doping dependence of the so-called $T^*$ line [2], the temperature below which signs of a gap appear. The first viewpoint holds that the pseudogap represents precursor phase-incoherent superconductivity or ‘pre-pairing’. In this case of a single $d$-wave gap the pseudogap opens at $T^*$ and evolves into the superconducting gap below $T_c$. The underlying Fermi surface is a nodal-metal, appearing as an arc due to broadening processes [3, 4]. Here the $T^*$ line merges smoothly with the $T_c$ dome on the overdoped side (see figure 1(a)). The second viewpoint is that the pseudogap arises from some as yet unidentified competing and/or coexisting order. In this two-gap scenario the pseudogap is distinct from the superconducting gap with a different momentum dependence, likely resulting from Fermi surface reconstruction [5]. The $T^*$ line in this case bisects the $T_c$ dome and need not be a transition temperature in the thermodynamic sense or ‘phase transition’, where it would instead mark a crossover region defined by the energy of a second order parameter given by $E_g \approx 2k_B T^*$ (see figure 1(b)). Ironically, the multitude of different techniques employed to study the pseudogap has lead to much confusion over the exact form of the $T^*$ line.
However, an alternative picture is beginning to emerge that encompasses both viewpoints (see figure 1(c)). Small superconducting coherence lengths in the high-$T_c$ cuprates give rise to strong superconducting fluctuations that are clearly evident in many techniques. Thermal expansivity [8], specific heat [7, 8], resistivity [9], Nernst effect [10–12], THz conductivity [13], IR conductivity [14] and Josephson effect [15] measurements show that although the fluctuation regime persists as high as 150 K [8], it is confined to a narrower region above $T_c$ and does not track the $T^*$ line [8, 9, 11, 14] which extends to much higher temperatures at low doping. An effective superconducting gap feature associated with these fluctuations which tails off above $T_c$ can be extracted from the specific heat [16]. And pairing gaps above $T_c$ have been detected by scanning tunneling microscopy in this temperature range [17]. Evidence for a second energy scale, which from here will be referred to specifically as the pseudogap, includes a downturn in the normal-state spin susceptibility [18, 19] and specific heat [19], a departure from linear resistivity [20–22], and a large gapping of the Fermi surface at the antinodes by angle-resolved photoemission spectroscopy (ARPES) [23–26]. The opening of the pseudogap at a critical doping within the $T_c$ dome can be inferred from an abrupt drop in the doping dependence of several properties. These include the specific heat jump at $T_c$ [19], condensation energy [19], zero-temperature superfluid density [19, 27], the critical zinc concentration required for suppressing superconductivity [28], zero-temperature self-field critical current [29], and the Hall number [30], most of which represent ground-state properties. The last signals a drop in carrier density from 1 to $p$ to $p$ holes per Cu, and can be explained in terms of a reconstruction from a large to small Fermi surface [5]. At or above optimal doping the pseudogap becomes similar or smaller in magnitude than the superconducting gap and, since many techniques return data that is dominated by the larger of the two gaps, it has been historically difficult to determine which gap is being observed. In this work it will be demonstrated explicitly that in this doping range it is in fact the superconducting gap persisting above $T_c$ that is being observed, thereby ending the confusion over the shape of the $T^*$ line.

This work was inspired by two recent studies. The first by Reber et al [31] fitted the ARPES-derived tomographic density of states using the Dynes equation [32]

$$I_{\text{DOS}} = \text{Re} \frac{\omega - i\Gamma_{\text{c}}}{(\omega - i\Gamma_{\text{c}})^2 - \Delta^2}$$

(1)

to extract the temperature dependence of the superconducting gap $\Delta$ and the pair-breaking scattering rate $\Gamma_{\text{c}}$. They found that $\Delta$ extrapolates to zero above $T_c$ while $\Gamma_{\text{c}}$ increases steeply near $T_c$. They also found that $T_c$ occurs when $\Delta = 3\Gamma_{\text{c}}$. Importantly, these parameters describe the filling-in behavior of the gapped spectra with temperature (originally found in tunneling experiments e.g. [33–35] and also inferred from specific heat and NMR [36]), as opposed to the closing behavior expected if $\Delta$ was to close at $T_c$ in the presence of constant scattering.

The second study, by Kondo et al [37], measured the temperature dependence of the spectral function around the Fermi surface using high-resolution laser ARPES. This was fitted using the phenomenological self-energy proposed by Norman et al [38]

$$\Sigma(\mathbf{k}, \omega) = -i\Gamma_{\text{single}} + \frac{\Delta^2}{\omega + \xi(\mathbf{k}) + i\Gamma_{\text{pair}}}.$$  

(2)

where $\xi(\mathbf{k})$ is the energy-momentum dispersion, $\Gamma_{\text{single}}$ is a single-particle scattering rate and $\Gamma_{\text{pair}}$ is a pair-breaking scattering rate. The gap is well described by a $d$-wave BCS temperature dependence with an onset temperature $T_{\text{pair}}$ above the observed $T_c$. $\Gamma_{\text{pair}}$ increases steeply near $T_c$, with $T_c$ coinciding with the temperature

---

**Figure 1.** Candidate phase diagrams for the hole-doped cuprates. For simplicity only the superconducting and pseudogap phases are shown. (a) Precursor pairing scenario. (b) Competing or coexisting order parameter scenario. (c) Combined phase diagram proposed in this work.
2.1. Specific heat

The Green’s function with the above self-energy (2) is given by

$$ G(\xi, \omega) = \frac{1}{\omega - \xi + \Gamma_{\text{single}} - \frac{\Delta^2}{\omega + \xi + \Gamma_{\text{pair}}}}. $$

(3)

The superconducting gap is given by $\Delta = \Delta_0 e^{\theta(T)} \cos 2\theta$, where $\Delta_0 = 2.14 k_B T_p$ and $\theta(T)$ is the $d$-wave BCS temperature dependence. $\theta$ represents the angle around the Fermi surface relative to the Brillouin zone boundary and ranges from 0 to $\pi/2$. The density of states $g(\omega)$ is obtained by integrating the spectral function

$$ A(\xi, \omega) = \pi^{-1} \text{Im} G(\xi, \omega) $$

(4)

The electronic specific heat coefficient $\gamma(T) = \partial S / \partial T$ is calculated from the entropy

$$ S(T) = -2k_B \int [ f \ln f + (1 - f) \ln (1 - f)] g(\omega) d\omega, $$

(5)

where $f$ is the Fermi distribution function. The temperature dependence of $\Gamma_{\text{pair}}$ is extracted by using it as an adjustable parameter to fit specific heat data under the following assumptions: (i) the superconducting gap opens at $T_p = 120$ K, at the onset of superconducting fluctuations; and (ii) a linear-in-temperature $\Gamma_{\text{single}}$ ranging from 5 meV at 65 K to 14 meV at 135 K, similar to values reported by Kondo et al. There is difficulty in applying this approach over the whole temperature range is that the $T$-dependence of the underlying normal-state specific heat $\gamma_n$ must be known. Therefore attention will be focused close to $T_c$ on Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ data with a doping of 0.182 holes/Cu, where $\gamma_n$ can be taken to be reasonably constant. In practice the quantity fitted is the dimensionless ratio of superconducting- to normal-state entropies $S_c(T) / S_n(T)$.

Fits and parameters are shown in figure 2 for data measured at zero and 13 T applied magnetic field. $\Gamma_{\text{pair}}$ increases steeply near $T_c$ in a very similar manner to the scattering rates found from the ARPES studies mentioned above. No particular relationship between $\Gamma_{\text{pair}}$, $\Gamma_{\text{single}}$, and $T_c$ is observed, however the peak of the specific heat jump occurs when $\Gamma_{\text{pair}} = \Delta$. In other words, once the pair-breaking becomes of the order the superconducting gap the entropy changes less rapidly with temperature, which intuitively makes sense. This appears to differ significantly with the result $\Delta(T_c) = 3\Gamma(T_c)$ from Reber et al, but note that fitting with the Dynes equation returns a smaller scattering rate $\Gamma_c$ equal to the average of $\Gamma_{\text{single}}$ and $\Gamma_{\text{pair}}$. A puzzling feature of the cuprate specific heat jump is its non-mean-field-like evolution with magnetic field. Rather than shifting to lower temperatures, it broadens and reduces in amplitude with little or no change in onset.

where $\Gamma_{\text{pair}} = \Gamma_{\text{single}}$. The aim of the present work is to investigate whether other experimental properties are consistent with this phenomenology. The approach is to fit the bulk specific heat using (2) then, using the same parameters, calculate the superfluid density, tunneling and Raman spectra, and optical conductivity. To reiterate, the focus here is the overdoped regime near $T_c$ where the pseudogap and subsidiary charge-density-wave order are absent [39].

2. Results

Figure 2. Fits (blue and magenta lines) made to the electronic specific heat of slightly overdoped Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ ($p = 0.182$ holes/Cu) at zero and 13 T fields (black lines) using the self-energy given by (2). $\Gamma_{\text{pair}}$ is the adjustable parameter and $\Delta(T)$ and $\Gamma_{\text{single}}(T)$ are assumed.
temperature. The fits explain this in terms of an increase in $\Gamma_{\text{pair}}$ with field, without requiring a reduction in gap magnitude. Note that taking $\Delta(H) = \Delta_0 \sqrt{1 - (H/H_c)^2}$ from Ginzburg–Landau theory [42], the estimated reduction in the gap at 13 T near $T_c$ is only 7%–2% for upper critical fields in the range 50–100 T. Other properties will now be calculated using the parameters in figure 2.

2.2. Superfluid density

The two scattering rates, $\Gamma_{\text{pair}}$ and $\Gamma_{\text{single}}$ are inserted into the anomalous Green’s function $F$ as follows

$$F(\xi, \omega) = \frac{\Delta}{(\omega + \xi + i\Gamma_{\text{pair}})(\omega - \xi + i\Gamma_{\text{single}} - \Delta^2/\omega)}}. \quad (6)$$

The superfluid density $\rho_s$ is proportional to the inverse square of the penetration depth ($\lambda$) calculated from [43]

$$\frac{1}{\lambda^2(T)} = \frac{16\pi e^2}{c^4V} \sum_k v_k^2 \int d\omega d\omega' \lim_{\theta \to 0} \left[ \frac{f(\omega') - f(\omega)}{\omega' - \omega} \right] \times B(k + \mathbf{q}, \omega') B(k, \omega), \quad (7)$$

where the anomalous spectral function $B$ is given by the imaginary part of $F$. For a free-electron-like parabolic band $\xi(k) = \hbar^2(k_x^2 + k_y^2)/2m - \mu$, $v_k = \hbar k/m = \sqrt{2(\xi + \mu)/m} \cos \theta$ and changing variables to $\xi$ and $\theta$ gives

$$\frac{1}{\lambda^2(T)} \propto \int (\xi + \mu) \cos^2 \theta \int \left[ \frac{f(\omega') - f(\omega)}{\omega' - \omega} \right] \times B(\xi, \theta, \omega') B(\xi, \theta, \omega) d\omega' d\omega d\xi d\theta. \quad (8)$$

The $T$-dependence of $\Gamma_{\text{pair}}$ causes a clear steepening of $\rho_s$ away from the BCS $T$-dependence, with the main onset being pushed down from $T_p$ to $T_c$, see figure 3(a). The same result can be obtained using one scattering rate equal to the average of $\Gamma_{\text{single}}$ and $\Gamma_{\text{pair}}$ at each temperature. When plotted in terms of reduced temperature $T/T_c$, there is a very good match with experimental data from optimally doped cuprates (figure 3(b)). The data, taken by different techniques, includes a YBa$_2$Cu$_4$O$_7$–$\delta$ (YBCO) crystal [44] and film [45] with $T_c$’s near 90 K, as well as a (BiPb)$_2$(SrLa)$_2$CuO$_{8+p}$ crystal [46] with a $T_c$ of 35 K. This raises the question as to whether the mooted Berezinskii–Kosterlitz–Thouless universal jump in superfluid density may not simply be attributable to the rapid increase in pair breaking scattering rate near $T_c$ arising from fluctuations on a pairing scale that exceeds $T_c$ [47]. Although the tail above $T_c$ is not evident in the selected experimental data, it is observed elsewhere in the literature [48]. There is a resemblance to an approximate strong-coupling $T$-dependence (dotted line in figure 3(a)), calculated from a rescaled BCS gap of magnitude $\Delta_0 = 2.9k_B T_c$ closing at $T_c = 94$ K, in the absence of strong pair-breaking. However, as will be seen in the following sections, this interpretation of $\rho_s(T)$ is inconsistent with other observations. The suppression in superfluid density with field bears a qualitative similarity to field-dependent measurements on a YBCO thin film [49], but because of that sample’s apparent low upper critical field the calculated suppression is much smaller in magnitude.

2.3. Tunneling

The current–voltage curve for a superconductor–insulator–superconductor tunnel junction is calculated from [50]

$$I(V) \sim \int g(E) g(E - eV) [f(E) - f(E - eV)] dE, \quad (9)$$

where $g(E)$ is the density of states given by (4). The tunneling conductance $dl/dV$ is plotted in figure 4 for several temperatures around $T_c$. The evolution of the spectra with temperature is very consistent with experimental observations [35, 51–54]. These show a filling-in of the gap with temperature and a broadening and suppression of the peaks at $2\Delta$, with little or no shift in their positions. This is contrary to the expected shift toward zero voltage that would occur for a strong coupling gap closing at $T_c$ in the absence of pair-breaking scattering. A depression persists above $T_c$ and vanishes as $T_p$ is approached, where the superconducting gap closes. Remember that a pseudogap is not included in these calculations. The linearly sloping background seen in the experimental data can be reproduced by adding a linear-in-frequency term, as seen in ARPES [55], to $I_{\text{single}}$.

2.4. Raman spectroscopy

Another property that supports the persistence of $\Delta$ above $T_c$ is the Raman $B_{ig}$ response given by [56]

$$\chi''(\omega) = \sum_k (\gamma_{B_{ig}}^k)^2 \int \frac{d\omega'}{4\pi} \left[ f(\omega') - f(\omega' + \omega) \right] \times [A(k, \omega' + \omega)A(k, \omega') - B(k, \omega' + \omega) B(k, \omega')].$$

The Raman $B_{ig}$ vertex $\gamma_{B_{ig}}^k \propto \cos k_x - \cos k_y \approx \cos 2\theta$ probes the antinodal regions of the Fermi surface where $\Delta(k)$ is largest. Changing variables from $k$ to $\xi$ and $\theta$ gives
Figure 3. (a) Normalized superfluid density calculated using the parameters from fits to the specific heat in figure 2 (red and green lines). (b) Comparison of the calculated zero-field superfluid density with experimental data from [44-46].

Figure 4. SIS junction tunneling conductance at several temperatures around $T_c$ calculated using the parameters from fits to the specific heat in figure 2. Inset: experimental data reproduced from [51]. Copyright 2001, with permission from Elsevier.

$$\gamma''(\omega) \propto \int d\xi d\theta \cos^2(2\theta) \int d\omega'[f(\omega') - f(\omega' + \omega)]$$

$$\times [A(\xi, \theta, \omega' + \omega)A(\xi, \theta, \omega') - B(\xi, \theta, \omega' + \omega)B(\xi, \theta, \omega')] .$$

(11)
The superconducting Raman $B_{1g}$ response function with the normal-state response at 122 K subtracted is shown in figure 5(a) for several temperatures around $T_c$. The resemblance to experimental data, reported in [57–59], is striking. Like the tunneling results above, the peak at $2\Delta$ broadens and reduces in amplitude and barely shifts with temperature indicating that the gap magnitude is still large at $T_c$ [58]. Figure 5(b) shows the normalized area under the curves in (a) compared with experimental values from [57] for dopings $p = 0.19$ and 0.21.

2.5. Optical conductivity

The final property considered in this work is the $ab$-plane optical conductivity calculated from [60]

$$\sigma(\omega) = \frac{e^2}{\hbar} \sum_k \nu_{ab}(k) \int \frac{\omega'}{\pi} \left[ f(\omega') - f(\omega' + \omega) \right] \times [A(k, \omega')A(k, \omega' + \omega) + B(k, \omega')B(k, \omega' + \omega)],$$

(12)

where $\nu_{ab}(k) = \sqrt{v_x^2 + v_y^2}$. Again a change of variables is made from momentum to energy and Fermi surface angle as follows

$$\sigma(\omega) \propto \frac{1}{\omega} \int d\xi d\theta (\xi + \mu) \int d\omega' [f(\omega') - f(\omega' + \omega)] \times [A(\xi, \theta, \omega')A(\xi, \theta, \omega' + \omega) + B(\xi, \theta, \omega')B(\xi, \theta, \omega' + \omega)].$$

(13)

Spectra at several temperatures around $T_c$ are plotted in figure 6. A suppression is visible below $2\Delta$ at low temperature that fills in as temperature is increased. A gap closing at $T_c$ would result in the onset of this suppression shifting to lower frequency. The calculations bear a strong qualitative resemblance to the overdoped data reported by Santander-Syro et al [61].

3. Discussion

As summarized in table 1 only the superfluid density, and more approximately the zero-field specific heat, can be interpreted by a strong-coupling gap closing at $T_c$ in the absence of scattering. The non-mean-field $T$-dependence of all properties examined in this work is instead well described in terms of a superconducting gap that persists above $T_c$, in the presence of a steep increase in scattering. This result is insensitive to the addition of linear-in-frequency terms or a $\cos 2\theta$ momentum dependence to $\Gamma_{\text{pair}}$ and $\Gamma_{\text{single}}$. The scattering is further enhanced by magnetic field. What is the origin of the scattering and can it be suppressed to bring $T_c$ up to $T_p$? A rapid collapse in quasiparticle scattering below $T_c$, also found in microwave surface impedance measurements [62], is expected when inelastic scattering arises from interactions that become gapped or suppressed below $T_c$. 

Figure 5. (a) Difference between the superconducting and normal-state (i.e. just above $T_p$) antinodal ($B_{1g}$) Raman response functions at temperatures around $T_c$, calculated using the parameters from fits to the specific heat in figure 2. (b) Normalized area under the curves in (a) compared with experimental values from [57] for dopings $p = 0.19$ and 0.21.
The spin fluctuation spectrum is a plausible candidate and has been investigated extensively [64, 65], although those calculations assumed that the superconducting gap closes at $T_c$.

The work presented here illustrates that the merging of the $T^*$ line on the lightly overdoped side of the $T_c$ dome is not a product of the pseudogap per se, but rather the persistence of the superconducting gap into the fluctuation region between $T_c$ and $T_p$. As doping increases, this region becomes narrower and experimental properties become more mean-field-like. Switching direction, as doping decreases the pseudogap opens, grows, and eventually exceeds the magnitude of the superconducting gap at the antinodes. When this occurs, the gap associated with $T^*$ changes to the pseudogap. In other words, $T^*$ is given by the larger of $T_p$ and $E_k/2k_B$ (see figure 1(c)). Such an interpretation makes immediate sense of the phase diagram presented by Chatterjee et al [66].

Acknowledgments

Supported by the Marsden Fund Council from Government funding, administered by the Royal Society of New Zealand. The author acknowledges helpful discussion with J L Tallon.

ORCID

J G Storey @ https://orcid.org/0000-0001-9995-7109

References

[1] Norman M R, Pines D and Kallin C 2005 The pseudogap: friend or foe of high $T_c$? Adv. Phys. 54 715–33
[2] Tallon J L and Loram J W 2001 The doping dependence of $T^*$—what is the real high-$T_c$ phase diagram? Physica C 349 53–68
[3] Kanigel A et al 2006 Evolution of the pseudogap from Fermi arcs to the nodal liquid Nat. Phys. 2 447–51
[4] Reber T J et al 2012 The origin and non-quasiparticle nature of Fermi arcs in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ Nat. Phys. 8 606–10
[5] Storey J G 2016 Hall effect and Fermi surface reconstruction via electron pockets in the high-$T_c$ cuprates Europhys. Lett. 113 27003
6. Meingast C, Pasler V, Nagel P, Rykov A, Tajima S and Olsson P 2001 Phase fluctuations and the pseudogap in $\text{YBa}_2\text{Cu}_3\text{O}_6$. Phys. Rev. Lett. 86 1606–9
7. Wen H-H, Mu G, Luo H, Yang H, Shan L, Ren C, Cheng P, Yan J and Fang L 2009 Specific-heat measurement of a residual superconducting state in the normal state of underdoped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ cuprate superconductors Phys. Rev. Lett. 103 067002
8. Tallon J, Storey J and Loram J W 2011 Fluctuations and critical temperature reduction in cuprate superconductors Phys. Rev. B 83 092502
9. Alloul H, Rullier-Albenque F, Vignolle B, Colson D and Forget A 2010 Superconducting fluctuations, pseudogap and phase diagram in cuprates Europhys. Lett. 91 37005
10. Xu Z A, Ong N P, Wang Y, Kakeshita T and Uchida S 2000 Vortex-like excitations and the onset of superconducting phase fluctuations in underdoped $\text{La}_2\text{Sr}_x\text{Cu}_3\text{O}_y$ Nature 406 486–8
11. Wang Y, Li L and Ong N P 2006 Nernst effect in high-$T_c$ superconductors Phys. Rev. B 73 024510
12. Chang J et al 2012 Decrease of upper critical field with underdoping in cuprate superconductors Nat. Phys. 8 751–6
13. Bilbro L S, Valdés Aguilar R, Logvnen G, Pellegr O, Bozorth R V and Armitage N P 2011 Temporal correlations of superconductivity above the transition temperature in $\text{La}_2\text{Sr}_x\text{Cu}_3\text{O}_y$ probed by terahertz spectroscopy Nat. Phys. 7 298–302
14. Dubroka A et al 2011 Evidence of a precursor superconducting phase at temperatures as high as 180 K in $\text{Rb}_x\text{Cu}_2\text{O}_y$ ($R = Y, \text{Gd, Eu}$) superconducting crystals from infrared spectroscopy Phys. Rev. Lett. 106 047006
15. Bergeal N, Lesueur J, April M, Fanti G, Contour J P and Leridon B 2008 Pairing fluctuations in the pseudogap state of copper-oxide superconductors probed by the Josephson effect Nat. Phys. 4 608
16. Tallon J, Barber F, Storey J G and Loram J W 2013 Coexistence of the superconducting energy gap pseudogap and above and below the transition temperature of cuprate superconductors Phys. Rev. B 87 144508
17. Gomes K K, Pasupathy A N, Pushp A, Ono S, Ando Y and Yazdani A 2007 Visualizing pair formation on the atomic scale in the high-$T_c$ superconductor $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ Nature 447 569–72
18. Williams G V M, Tallon J L, Haines E M, Michalak R and Dupree R 1997 NMR evidence for a $d$-wave normal-state pseudogap Phys. Rev. Lett. 78 721–4
19. Loram J W, Luo J, Cooper J R, Liang W Y and Tallon J L 2001 Evidence on the pseudogap and condensate from the electronic specific heat J. Phys. Chem. Solids 62 59–64
20. Ando Y, Komiya S, Segawa K, Ono S and Kurita Y 2004 Electronic phase diagram of high-$T_c$ cuprate superconductors from a mapping of the in-plane resistivity curvature Phys. Rev. Lett. 93 267001
21. Naqib S H, Cooper J R, Tallon J L, Islam R S and Chakalov R A 2005 Doping phase diagram of $\text{YBa}_2\text{Cu}_3\text{O}_6–\delta$ from transport measurements: tracking the pseudogap below $T_c$ Phys. Rev. B 71 054502
22. Daou R et al 2009 Linear temperature dependence of resistivity and change in the Fermi surface at the pseudogap critical point of a high-$T_c$ superconductor Nat. Phys. 5 31–4
23. Tanaka K et al 2006 Directnet Fermi momentum–dependent energy gaps in deeply underdoped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ Phys. Rev. Lett. 97 156401
24. Lee W S, Vishik I M, Tanaka K, Lu D H, Sasagawa T, Nagaosa N, Devereaux T P, Hussain Z and Shen Z X 2007 Abrupt onset of a second energy gap at the superconducting transition of underdoped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ Nature 450 81–4
25. Vishik I M, Lee W S, He R-H, Hashimoto M, Hussain Z, Devereaux T P and Shen Z-x 2010 ARPES studies of cuprate Fermiology: pseudogap and quasiparticle dynamics New J. Phys. 12 105008
26. Matt C et al 2015 Electron scattering, charge order, and pseudogap physics in $\text{La}_{1.6–x}\text{Nd}_x\text{Sr}_0.2\text{Cu}_3\text{O}_7$; an angle-resolved photoemission spectroscopy study Phys. Rev. B 92 134524
27. Bernhard C, Tallon J L, Blasius T, Golnik A and Niedermayer C 2001 Anomalous peak in the superconducting condensate density of cuprate high-$T_c$ superconductors at a unique doping state Phys. Rev. Lett. 86 1614–7
28. Tallon J, Loram J W, Cooper J R, Panagopoulos C and Bernhard C 2003 Superfluid density in cuprate high-$T_c$ superconductors: a new paradigm Phys. Rev. B 68 180501
29. Naamneh M, Campuzano J C and Kanigel A 2014 Doping dependence of the critical current in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$. Phys. Rev. B 90 224501
30. Badoux S et al 2016 Change of carrier density at the pseudogap critical point of a cuprate superconductor Nature 531 210–4
31. Reber T et al 2015 Pairing, break-up, and their roles in setting the $T_c$ of cuprate high temperature superconductors arXiv:1508.06252
32. Dynes R C, Narayanamurti V and Ganno J P 1978 Direct measurement of quasiparticle-lifetime broadening in a strong-coupled superconductor Phys. Rev. Lett. 41 1509–12
33. Miyakawa N, Zasadzinski J F, Ozyuzer L, Guptasarma P, Hinks D G, Kendziora C and Gray K E 1999 Predominantly superconducting origin of large energy gaps in underdoped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ from tunneling spectroscopy Phys. Rev. Lett. 83 1018–21
34. Matsuda A, Sugita S and Watanabe T 1999 Temperature and doping dependence of the $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ pseudogap and superconducting gap Phys. Rev. B 60 1377–81
35. Ozyuzer L, Zasadzinski J F, Gray K E, Kendziora C and Miyakawa N 2002 Absence of pseudogap in heavily underdoped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ from tunneling spectroscopy of break junctions Europhys. Lett. 58 589
36. Williams G V M, Tallon J L and Loram J W 1998 Crossover temperatures in the normal–state phase diagram of high-$T_c$ superconductors Phys. Rev. B 58 15053–61
37. Kondo T, Maalew W, Ishida Y, Sasagawa T, Sakamoto H, Takeuchi T, Toyohama T and Shin S 2013 Point nodes persisting far beyond $T_c$ in $\text{Bi}_2\text{Sr}_2\text{CuO}_4$ Nat. Commun. 6 7699
38. Norman M R, Randeria M, Ding H and Campuzano J C 1998 Phenomenology of the low–energy spectral function in high-$T_c$ superconductors Phys. Rev. B 57 R11093–6
39. Badoux S et al 2016 Critical doping for the onset of Fermi–surface reconstruction by charge–density-wave order in the cuprate superconductor $\text{La}_2\text{Sr}_x\text{CuO}_4$ Phys. Rev. X 6 021084
40. Junod A, Bonjour E, Calemczuk R, Henry J Y, Muller J, Triscione G and Vallier J C 1993 Specific heat of an $\text{YBa}_2\text{Cu}_3\text{O}_{y}$ single crystal in fields up to 20 T Physica C 211 304–18
41. Inderhees E, Salamon M B, Rice J P and Ginsberg D M 1993 Heat capacity of $\text{YBa}_2\text{Cu}_3\text{O}_{y–\delta}$ crystals along the $H_2$ line Phys. Rev. B 47 1053–63
42. Douglass D H 1961 Magnetic field dependence of the superconducting energy gap Phys. Rev. Lett. 6 346–8
43. Carbottte J P, Fisher K A G, LeBlanc J P F and Niccol E J 2010 Effect of pseudogap formation on the penetration depth of underdoped high-$T_c$ cuprates Phys. Rev. B 81 014522
44. Hardy W N, Bonn D A, Morgan D C, Liang R and Zhang K 1993 Precision measurements of the temperature dependence of $\lambda$ in $\text{YBa}_2\text{Cu}_3\text{O}_{y–\delta}$ strong evidence for nodes in the gap function Phys. Rev. Lett. 70 3999–4002
[45] Boyce B R, Skinta J A and Lemberger T R 2000 Effect of the pseudogap on the temperature dependence of the magnetic penetration depth in YBCO films Physica C 341–348 561–2
[46] Khasanov R, Kondo T, Strässle S, Heron D O G, Kaminski A, Keller H, Lee S I and Takeuchi T 2009 Zero-field superfluid density in a d-wave superconductor evaluated from muon-spin-rotation experiments in the vortex state Phys. Rev. B 79 180507
[47] Hetel I, Lemberger T R and Randeria M 2007 Quantum critical behaviour in the superfluid density of strongly underdoped ultrathin copper oxide films Nat. Phys. 3 700
[48] Jacobs T, Sridhar S, Li Q, Gu G D and Koshizuka N 1995 In-plane and c-axis microwave penetration depth of Bi2Sr2CaCu2O8±δ crystals Phys. Rev. Lett. 75 4516–9
[49] Pesetski A A and Lemberger T R 2000 Experimental study of the inductance of pinned vortices in superconducting YBa2Cu3O7−δ films Phys. Rev. B 62 11826–33
[50] Miyakawa N, Guptasarma P, Zasadzinski J F, Hinks D G and Gray K E 1998 Strong dependence of the superconducting gap on oxygen doping from tunneling measurements on Bi2Sr2CaCu2O8−δ Phys. Rev. Lett. 80 157–60
[51] Dipasupil R M, Oda M, Nanjo T, Manda S, Momono M and Ido M 2001 Pseudogap in the tunneling spectra of slightly overdoped Bi2212 Physica C 364–365 604–7
[52] Ren J K, Zhu X B, Tian Y, Yang H F, Gu CZ, Wang N L, Ren Y F and Zhao S P 2012 Energy gaps in Bi2Sr2CaCu2O8±δ cuprate superconductors Sci. Rep. 2 2248
[53] Ren J K, Wei Y F, Yu H F, Tian Y, Ren Y F, Zheng D N, Zhao S P and Lin C T 2012 Superconducting gap and pseudogap in near-optimally doped Bi2Sr2La1−xCaCu2O8+δ Phys. Rev. B 86 014520
[54] Benseman T M, Cooper J R and Balakrishnan G 2015 Interlayer tunnelling evidence for possible electron–boson interactions in Bi2Sr2CaCu2O8−δ arXiv:1503.00335
[55] Kaminski A et al 2005 Momentum anisotropy of the scattering rate in cuprate superconductors Phys. Rev. B 71 014517
[56] Valenzuela B and Bascones E 2007 Phenomenological description of the two energy scales in underdoped cuprate superconductors Phys. Rev. Lett. 98 227002
[57] Blanc S, Gallais Y, Cazayous M, Méasson M A, Sacuto A, Georges A, Wen J S, Xu Z J, Gu G D and Colson D 2010 Loss of antinodal coherence with a single d-wave superconducting gap leads to two energy scales for underdoped cuprate superconductors Phys. Rev. B 82 144516
[58] Guyard W, Le Tacon M, Cazayous M, Sacuto A, Georges A, Colson D and Forget A 2008 Breakpoint in the evolution of the gap through the cuprate phase diagram Phys. Rev. B 77 024524
[59] Guyard W, Sacuto A, Cazayous M, Gallais Y, Le Tacon M, Colson D and Forget A 2008 Temperature dependence of the gap size near the Brillouin-zone nodes of HgBa22Cu3O8+δ superconductors Phys. Rev. Lett. 101 097005
[60] Yanase Y and Yamada K 2001 Pseudogap state and superconducting state in high-Tc cuprates: anomalous properties in the observed quantities J. Phys. Chem. Solids 62 219–20
[61] Santander-Syro A F, Lobo R P S M, Bontemps N, Konstantinovic Z, Li Z and Raffy H 2002 Absence of a loss of in-plane infrared spectral weight in the pseudogap regime of Bi2Sr2CaCu2O8±δ Phys. Rev. Lett. 88 097005
[62] Bonn D A et al 1993 Microwave determination of the quasiparticle scattering time in YBa2Cu3O6+δ Phys. Rev. B 47 11314–28
[63] Hosseini A, Harris R, Kamal S, Dosanjh P, Preston J, Liang R, Hardy W N and Bonn D A 1999 Microwave spectroscopy of thermally excited quasiparticles in Yba2Cu3O6+δ Phys. Rev. B 60 1349–59
[64] Quinlan S M, Scalapino D J and Bulut N 1994 Superconducting quasiparticle lifetimes due to spin-fluctuation scattering Phys. Rev. B 49 1470–3
[65] Duffy D, Hirschfeld P J and Scalapino D J 2001 Quasiparticle lifetimes in a dxy superconductor Phys. Rev. B 64 224522
[66] Chatterjee U et al 2011 Electronic phase diagram of high-temperature copper oxide superconductors Proc. Natl Acad. Sci. 108 9346–9