Evaluation and analysis of bias tunnel diseases based on fuzzy mathematics theory

Huajiang Liang
Beibu Gulf Vocational-Technical School. Qinzhou, Guangxi, 535000, China
Correspondence should be addressed to Huajiang Liang; 2451819631@qq.com

Abstract: Biased tunnels are subject to the combined effects of vehicle loads, groundwater, atmosphere, and geology during a long period of operation, resulting in the gradual incubation and development of tunnel diseases and tunnel damage. In order to effectively comprehensively evaluate the disease level of the bias tunnel in the operation stage, fuzzy mathematical theory is introduced in this paper. By determining the evaluation indexes of the tunnel disease at all levels, and considering the deterioration effect of the bias on the disease, the bias index is added, and the comprehensive and objective fuzzy operator is used to calculate. The tunnel is evaluated in sections to achieve a more accurate assessment of disease levels. The theory is applied to the disease evaluation of an operating biased tunnel, and a good evaluation result is obtained. This method furtherly perfects the fuzzy comprehensive evaluation system for the disease of biased tunnels.

1. Introduction
China's new transportation tunnel project is in the rapid development stage. At the same time, a large number of old tunnels are in the process of long-term operation. Due to the combined effects of vehicle loads, atmosphere, groundwater, and geological effects, the diseases of operating tunnels will inevitably go from the incubation to the development until destruction. Therefore, it is very important to maintain the operating tunnel to ensure the normal operation of the tunnel, and it is particularly important to accurately evaluate the damage level of the tunnel and clear the damage degree of the tunnel.

A large number of researches have been done on the evaluation of tunnel diseases at home and abroad, and a variety of modern evaluation theories have been introduced. Especially in the introduction of fuzzy mathematics to evaluate tunnel diseases, many results have been obtained \cite{1-7}. Among them, Feng Shi \cite{8} based on the rough set and gray-fuzzy theory to establish a tunnel structure health status evaluation model for traffic tunnels, to quantitatively evaluate the tunnel health value. Lei Jia \cite{9} mainly used the geological radar to perform quality inspection on the second lining. Base on this, he used the analytic hierarchy process and fuzzy mathematical theory to establish a fuzzy evaluation theory. Bojian Zhao \cite{10} established a quantitative evaluation system for railway tunnel diseases based on the analytic hierarchy process theory by applying the fuzzy evaluation model with the evaluation indicators of various types of diseases in Yiwan railway tunnel. At the same time, grade standard was used to evaluate the safety of Yiwan Railway Tunnel. Zhou Qin \cite{11} established a safety calculation model of the lining structure based on the load calculation principle of the load-structure method, and derived the law of the influence of diseases on the lining structure.

The research on fuzzy evaluation of tunnel diseases at home and abroad mainly adopts conventional disease type evaluation indicators, but in the evaluation of biased tunnel diseases, the influence of bias factors has not been considered. This will definitely affect the reliability of the judgment. This paper is
mainly based on the establishment of fuzzy theory of biased tunnel diseases, and considers the influence of bias factors to furtherly improve the method of fuzzy comprehensive evaluation of tunnel diseases.

2. **Fuzzy comprehensive evaluation theory**

2.1. **Fundamental of fuzzy comprehensive evaluation**

Fuzzy comprehensive evaluation theory is based on fuzzy mathematical theory to quantify the unclear and difficult to quantify problems in nature. It is a method to comprehensively evaluate the subject's degree of affiliation level from multiple factors.

Firstly, determining the domain \( U \) of evaluation indicators at all levels of the subject and comment set domain \( V \). Then, determine the weight of each factor separately to obtain the evaluation vector matrix. By using appropriate fuzzy operators, the weights of each index and the fuzzy matrix are synthesized, and the fuzzy evaluation matrix and the index's weight vector are fuzzy-calculated and then normalized to obtain the fuzzy comprehensive evaluation result.

2.2. **Determining the Comprehensive Evaluation Index and Commentary Field of Biased Tunnel**

The biggest difference between a biased tunnel and an ordinary tunnel is that the terrain of the cave is asymmetric. Therefore, one side of the tunnel is biased to be loaded, and the geostress on the larger side of the cover layer must cause the tunnel lining structure to be damaged. Series of tunnel diseases. Therefore, for a tunnel subjected to a large deflection load, the bias press is likely to be an important factor causing the tunnel disease. Therefore, this paper mainly focuses on the fuzzy comprehensive evaluation for the biased tunnel disease, by adding the bias evaluation index, to furtherly improve the fuzzy evaluation system of the tunnel disease.

The evaluation index of the bias tunnel should be multi-level evaluation index, in order to obtain a more accurate evaluation result, that is:

\[
\sum_{i=1}^{k} U_j = U \\
U_j \cap U_j = \emptyset (i \neq j)
\]  

(2-1)

First level evaluation factor set: \( U = \{U_1, U_2, ..., U_k\} \)  
\( (k\) represents the number of indicators in the evaluation object) ;  
Second-level evaluation factor set: \( U_i = \{u_{i1}, u_{i2}, ..., u_{im}\} \)  
\( (m\) represents the number of indicators in the secondary evaluation object) ;

According to this, the discussion domain of fuzzy comprehensive evaluation of bias tunnel diseases is divided into two levels, the first level is:

\( U_1 = \text{Cave Disease Detection}, \ U_2 = \text{Core Sampling Detection}, \)  
\( U_3 = \text{bias test}, \ U_4 = \text{geological radar test} \)

that is:

\( U = \{\text{Cave Disease Detection; Core Sampling Detection; Bias Detection; Geological Radar Detection}\} \)

Among them, \( U_3 = \text{bias test}. \) Based on section squeezed offset is the evaluation index by applying lining contour laser section device

The secondary indicators are mainly classified for cave diseases:

\( U_{13} = \text{cracking of lining}, \ U_{12} = \text{seepage} , \)  
\( U_{13} = \text{lining surface disease}, \ U_{14} = \text{pavement cracking} \)

\( U_1 = \{\text{cracking of lining; seepage; lining surface disease; pavement cracking}\} \)

According to the characteristics of the bias tunnel disease, the evaluation factors are divided into four
levels $V = \{v_1, v_2, v_3, v_4\}$:
that is:
$$V = \{\text{Normal}, \text{Poor}, \text{Bad}, \text{Extremely Bad}\}\]$$

2.3. Determining the weight of the bias tunnel evaluation index
Tunnels are in a certain geological body, and the geological environment in which they are located is not specific. Different tunnels relying on the same set of standards for evaluation are inappropriate. Therefore, the evaluation index of the tunnel should combine with the local engineering experience. Among the evaluation index weights of the bias tunnel, it is more reasonable to use the expert estimation method, which mainly is depended on the local tunnel experts to evaluate and quantify the impact weight of an evaluation index.
that is:
$$A =(A_1,A_2,\ldots,A_m)$$
($A_i$ represents the weight of the $i$th evaluation factor among the evaluation factors. And satisfies $a_i>0$, $\sum a_i = 1$)

The weight value of the first-level evaluation factors of the biased tunnel disease is set according to the characteristics of the evaluation factors. There are a total of four:
$$A = (A_1,A_2,\ldots,A_4)$$
There are four secondary evaluation indicators (only four weight values for disease detection):
$$A_2 = (a_{21},a_{22},a_{23},a_{24})$$

2.4. Determining the fuzzy relation matrix
The determining of the matrix starts with the lowest-level evaluation index $U_i$. This level of single factor is evaluated, in order to determine the degree of affiliation of each evaluation index in the discourse $U$ to the comment discourse $V$, and achieve the purpose of quantifying the evaluation index. Finally, iterative evaluation is performed step by step to the next higher evaluation index. Finally, the first-level fuzzy relation matrix $R$ is obtained.

$$R = \begin{bmatrix}
    r_{11} & r_{12} & \cdots & r_{1n} \\
    r_{21} & r_{22} & \cdots & r_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    r_{m1} & r_{m2} & \cdots & r_{mn}
\end{bmatrix} \quad (2-2)$$

($r_{ij}$ indicates the degree of affiliation of a certain factor in the evaluation index domain $U$ to the comment domain $V$.)

After obtaining a comprehensive evaluation of indicators at all levels, a comprehensive evaluation of multiple indicators is carried out. Selecting a appropriate fuzzy synthesis operator ($\circ$) to synthesize the first-order fuzzy weight vector $A$ with the final synthetic fuzzy relation matrix $R$.

Obtaining the fuzzy comprehensive evaluation result that is finally used for the second stage object level to $B$:

$$B = A \circ R = (a_1,a_2,\ldots,a_n)$$

$$B = (b_1,b_2,\ldots,b_n) \quad (2-3)$$
2.5. Analyzing the evaluation vector to determine the damage level of the bias tunnel

Through the above synthetic calculation, the evaluation vector B for target level is finally obtained, and the result of the fuzzy evaluation is performed by the evaluation vector. By applying the principle of maximum degree of affiliation, the comment corresponding to the largest proportion value in the evaluation vector B is used as the evaluation level of the entire evaluation object.

According to the principle of maximum degree of affiliation, for the evaluation result vector \( B = (b_1, b_2, b_3, \ldots, b_n) \), when \( b_k = \text{Max} \{b_j\} \), the evaluated object belongs to the k-th rank as a whole.

3. Engineering example application

3.1. Project Overview

A mountain tunnel has a simple geological structure, no large faults, rock strata obvious and 50° to 70° angle between the rock formation trend and the tunnel axis. The lithology of the strata from the entrance to the exit is: fine sandstone, calcareous siltstone, marl, limestone, mudstone, and thin-layered sandstone. It is a bias tunnel with a total length of 656m and is a secondary highway tunnel. Start and end mileage: K45+862 ~ K46+518. The entrance and exit doors are in the form of wing walls, and the support form of the cave is curved wall, as shown in Figure 3-1. The tunnel is designed as a two-way two-lane road. The designed traffic flow is 650 vehicles/h (two-lane) and limited speed is 40 km/h. The width of the tunnel is 7.5m and the sidewalk width is 2 × 0.75m. Ten years after it was opened to traffic, diseases such as large-scale seepage and cracking of some structures occurred.

In order to clarify the situation of tunnel diseases, the main first-level work items for disease detection of the tunnel include: cave disease detection; core sampling detection; bias detection; geological radar detection. The cave disease detection mainly includes the following two sub-projects: cracking of lining; seepage; lining surface disease; pavement cracking. As shown in Figure 3-2.

Core sampling detection is based on the actual sampling core reaction lining thickness, morphology, etc, as shown in Figure 3-3. The tunnel outline is detected by the perimeter of the tunnel, as shown in Figure 3-4. It reflects the deformation of the cave caused by the current tunnel compression damage and compares it with the original cave to define the effect of the bias on the tunnel. The geological radar uses non-destructive testing to reflect the thickness of the lining of the cave segment without core detection, to achieve complementary and perfect detection.
Cave disease detection is mainly based on the density of the disease; core sampling detection is based on compressive strength; geological radar detection is based on core integrity. It is worth noting that when the bias detection index item is added in the paper, the relative shrinkage area of the cave is used as the evaluation index. The relationship between the original cross-sectional area of the cave and the current cross-sectional area and mileage is shown in Figure 3-5. Obviously, the current cross-sectional area of the tunnel in segments K46 + 062 ~ K46 + 482 is smaller than the original section. Part of the cave segment is severely contracted, indicating that this segment is most heavily biased.

Due to the tunnel being a long structure, the strata they pass through are different. The evaluation of the disease level of the entire tunnel should not be carried out generally in the entire tunnel, but should be carried out in several segments. In this example, the fuzzy evaluation of the disease is mainly based on the 100m segment of the tunnel with a mileage of K46 + 262 ~ K46 + 362. The disease in this segment is more serious and the disease density is higher. For other cave segments, the same evaluation process can be used to evaluate in stages, in order to achieve a more accurate disease assessment. The detection results of this project are summarized in Table 3.1.
### Table 3.1 Summary of engineering detection result levels

| First-level evaluation factors | Second-level evaluation factor | Normal ($V_1$) | Poor ($V_2$) | Bad ($V_3$) | Extremely Bad ($V_4$) | Total |
|-------------------------------|--------------------------------|----------------|-------------|-------------|---------------------|------|
| **cave disease detection**    | cracking of lining (set)      | 8              | 39          | 74          | 14                  | 135  |
|                               | Seepage (set)                 | 5              | 14          | 31          | 9                   | 59   |
|                               | lining surface disease (set)  | 4              | 18          | 29          | 16                  | 67   |
|                               | pavement cracking (set)       | 8              | 6           | 17          | 5                   | 36   |
| **core Sampling detection**   | Compressive strength (Pieces) | 20             | 15          | 11          | 2                   | 48   |
| **bias detection**            | Tunnel area shrinkage (Pieces)| 0              | 3           | 13          | 2                   | 18   |
| **Geological radar detection**| Core integrity (Pieces)       | 41             | 35          | 22          | 2                   | 100  |

#### 3.2. Determining the construction of fuzzy evaluation indicators and the value of comment domain

In the geological environment of the bias tunnel, bias is an important influencing factor, so this evaluation index is added in this article. According to the detection conditions of the tunnel, it is determined that the first-level evaluation index of the biased tunnel disease evaluation is $U = \{U_1, U_2, U_3, U_4\}$; For the second-level evaluation index, it is divided only for the detection of cave disease, that is, $U_1 = \{u_{11}, u_{12}, u_{12}, u_{14}\}$.

According to the fuzzy comprehensive evaluation theory and the actual characteristics of the tunnel, the comment set of the tunnel is divided into 4 levels. That is, $V=\{v_1, v_2, v_3, v_4\}=\{\text{Normal, Poor, Bad, Extremely Bad}\}$. The comment ratio conversion is performed for each level of indicators in Table 3.1. The results are shown in Table 3.2.

### Table 3.2 Table of Evaluation Indicators at Different Levels

| First-level evaluation factors | Second-level evaluation factor | Normal ($V_1$) | Poor ($V_2$) | Bad ($V_3$) | Extremely Bad ($V_4$) |
|-------------------------------|--------------------------------|----------------|-------------|-------------|-----------------------|
| **cave disease detection**    | cracking of lining (set)      | 0.06           | 0.29        | 0.55        | 0.10                  |
|                               | Seepage (set)                 | 0.08           | 0.24        | 0.53        | 0.15                  |
|                               | lining surface disease (set)  | 0.06           | 0.27        | 0.43        | 0.24                  |
|                               | pavement cracking (set)       | 0.22           | 0.17        | 0.47        | 0.14                  |
| **core Sampling detection**   | Compressive strength (Pieces) | 0.42           | 0.31        | 0.23        | 0.04                  |
| **bias detection**            | Tunnel area shrinkage (Pieces)| 0.00           | 0.17        | 0.72        | 0.11                  |
| **Geological radar detection**| Core integrity (Pieces)       | 0.41           | 0.35        | 0.22        | 0.02                  |

According to the proportion of each indicator in Table 3.2, the fuzzy theory is used to determine the secondary fuzzy matrix. Obtaining the secondary fuzzy matrix $R_2$ of the disease evaluation cave disease detection for this project:

$$
R_2 = \begin{bmatrix}
0.06 & 0.29 & 0.55 & 0.10 \\
0.08 & 0.24 & 0.53 & 0.15 \\
0.06 & 0.27 & 0.43 & 0.24 \\
0.22 & 0.17 & 0.47 & 0.14
\end{bmatrix}
$$
3.3. Determine fuzzy comprehensive evaluation weights

According to 2.3, the weight of the fuzzy comprehensive evaluation of the tunnel adopts the expert scoring method. The biased evaluation index is added in this paper, so its evaluation weight is taken as an important influence factor to take a larger proportion. Therefore, considering the actual situation, the first and second fuzzy evaluation weights of the tunnel are as follows:

- First-level weight vector value:
  \[ A = \begin{bmatrix} 0.32 & 0.21 & 0.34 & 0.13 \end{bmatrix} \]  
  \[(3-2)\]

- Second-level weight vector value of core sampling detection:
  \[ A_2 = \begin{bmatrix} 0.38 & 0.32 & 0.10 & 0.20 \end{bmatrix} \]  
  \[(3-3)\]

3.4. Fuzzy Evaluation of Biased Tunnel

The results of fuzzy comprehensive evaluation, strive to be objective and reliable, and minimize the impact of uncertain factors. Therefore, for the selection of fuzzy operators, a comprehensive fuzzy operator that can reflect this characteristic should be used, that is,
\[ M = (\cdot, \oplus) \]

("\cdot" means multiply; "\oplus" means plus). Then, the comprehensive evaluation vector of cave disease detection can be obtained:

Among them, the operator 
\[ M = (\cdot, \oplus) \]
, operation rules shown as:
\[ B_k = \sum_{j=1}^n a_{jk} r_{jk}, \quad k = 1, 2, \ldots, n \]  
\[(3-4)\]

\[ B_2 = A_2 \circ R_2 = \begin{bmatrix} 0.38 & 0.32 & 0.10 & 0.20 \end{bmatrix} \circ \begin{bmatrix} 0.06 & 0.29 & 0.55 & 0.10 \\ 0.08 & 0.24 & 0.53 & 0.15 \\ 0.06 & 0.27 & 0.43 & 0.24 \\ 0.22 & 0.17 & 0.47 & 0.14 \end{bmatrix} \]
\[ = \begin{bmatrix} 0.0984 & 0.248 & 0.5156 & 0.138 \end{bmatrix} \]  
\[(3-5)\]

The evaluation vector \( B_2 \) is normalized to be obtained:
\[ B_2 = \begin{bmatrix} 0.10 & 0.24 & 0.52 & 0.14 \end{bmatrix} \]  
\[(3-6)\]

In the same way, the second-level evaluation result \( B_2 \) above is iterated into the first-level fuzzy evaluation factor domain. The first-level fuzzy comprehensive evaluation as:

Determining the first-order fuzzy matrix:
\[ R = \begin{bmatrix} 0.10 & 0.24 & 0.52 & 0.14 \\ 0.42 & 0.31 & 0.23 & 0.04 \\ 0.00 & 0.17 & 0.72 & 0.11 \\ 0.41 & 0.35 & 0.22 & 0.02 \end{bmatrix} \]  
\[(3-7)\]

\[ B = A \circ R = \begin{bmatrix} 0.32 & 0.21 & 0.34 & 0.13 \end{bmatrix} \circ \begin{bmatrix} 0.10 & 0.24 & 0.52 & 0.14 \\ 0.42 & 0.31 & 0.23 & 0.04 \\ 0.00 & 0.17 & 0.72 & 0.11 \\ 0.41 & 0.35 & 0.22 & 0.02 \end{bmatrix} \]  
\[(3-8)\]
Vector B normalization: 
\[ B = \begin{bmatrix} 0.17 & 0.24 & 0.49 & 0.1 \end{bmatrix} \] (3.9)

Through the above calculation, the evaluation vector B is obtained, and according to the fuzzy comprehensive evaluation theory, the principle of maximum membership is adopted. The "bad" comment level corresponding to the largest proportion of 0.49 was selected as the rating standard for the entire disease evaluation. Therefore, the disease degree of segments $K46 + 262 \sim K46 + 362$ of this project are “Bad” level.

4. Conclusion

(1) Bias is an important factor influencing tunnel diseases. It is necessary and reasonable to add bias indicators to the evaluation of bias tunnel diseases. Using the tunnel section shrinkage to determine the bias index level has a good applicability. At the same time, it is necessary to appropriately increase or decrease the evaluation weight according to the magnitude of the biased ground stress, so that the fuzzy evaluation of the bias tunnel is more consistent with the actual situation.

(2) The fuzzy evaluation of the disease in the tunnel should be carried out in stages according to the actual situation to obtain a more accurate disease level. The four independent detection which including cave disease detection, core Sampling detection, bias detection, geological radar detection, indicators are related through fuzzy theory, which makes the tunnel disease evaluation comprehensive and relevant.

(3) Through the application of the bias tunnel project, it is possible to comprehensively evaluate the damage level of the tunnel more effectively, and it has a good fit with the actual damage situation of the bias tunnel. It can provide reliable guidance for the follow-up disease treatment, and also can provide a reference for similar bias tunnel projects.

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