Latest results from lattice QCD for the Roper resonance

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The present status of the Roper resonance in lattice QCD is reviewed. Some of the latest lattice results are discussed with particular emphasis on a large systematic error stemming from the finite size effect. These results suggest that the Roper resonance can be described by the simple three quark excitation of sizable extent.

§1. Introduction

In the excited baryon spectrum, of particular interest is the level order of the positive-parity excited nucleon, so-called the Roper resonance $N'(1440)$ and the negative-parity nucleon $N^*(1535)$. It is worth mentioning that this pattern of the level order between positive and negative-parity excited states can be found universally in the $\Delta$, $\Sigma$ and flavor-octet $\Lambda$ channels. It is also interesting to note that quark confining models such as either the harmonic oscillator quark model or the MIT bag model have some difficulty reproducing the correct level order, as eigenstates in each model alternate in parity. This wrong ordering problem does not seem to be easily alleviated. Hence, the first principle calculation, lattice QCD is demanded to resolve such a long-standing puzzle.

In lattice QCD, computations of the hadron masses rely on the asymptotic behavior of the two-point hadron correlator in the large Euclidean time region as $G(t) \sim \exp(-M_0 t)$. This approach mainly accesses the mass of the lowest-lying states, $M_0$. As for the higher-lying states, we need more sophisticated analysis such as the matrix-correlator method. The old calculation suggested the large mass of the first positive-parity excited nucleon and reported the possibility of missing the Roper resonance in lattice calculation. Recently, a first systematic calculation for the nucleon excited states in both parity channels showed that the wrong ordering between $N'$ and $N^*$ actually happens in the relatively heavy-quark mass region whereas the $N^*$ state can be reproduced well in lattice QCD (see Fig. 1). What one can see in this figure closely resembles the wrong ordering problem of the excited nucleon spectrum in the confining models. After this work, many lattice calculations confirmed this particular issue.

The question arises whether or not lattice calculations fail to reproduce the Roper resonance. An answer to this question provides some perspective to understand the structure of the mysterious Roper resonance since above lattice calculations basically utilize the simple description of the Roper resonance as three valence quarks. However, we should not conclude anything from the previous lattice results without examining possible systematic errors.

From the phenomenological point of view, the mass splitting between the ground state and the radial excited state, which has the same quantum number of the ground
Fig. 1. The low-lying nucleon spectrum in both positive parity and negative parity channel. Note the large mass splitting of \(N\) (diamonds) and \(N^*\) (squares) which is within 15% (in the chiral limit) of the experimental value (burst). The mass of the positive parity excited state \(N'\) (circle) is too high, however. All data are from Ref. 1 where the domain wall fermions are utilized. The corresponding experimental value for \(N(940), N'(1440)\) and \(N^*(1530)\) are marked with lower, middle and upper stars.

state, is almost independent of the quark masses. For instance, such a feature is easily confirmed among spin-1/2 octet baryons as \(M_{N(1440)} - M_{N(940)} \approx M_{\Sigma(1660)} - M_{\Sigma(1190)} \approx M_{A(1600)} - M_{A(1115)} \approx 0.5\text{GeV}\). In the case of the vector mesons, charm and bottom sectors reveal more clear evidence such as: \(M_{\rho(1450)} - M_{\rho(770)} \approx M_{\phi(1680)} - M_{\phi(1020)} \approx M_{\psi(3690)} - M_{J/\psi(3110)} \approx M_{\Upsilon(10020)} - M_{\Upsilon(9460)} \approx 0.6 - 0.7\text{GeV}\).

On the contrary, Fig. 1 shows that the data points corresponding to the two lightest quark masses for \(N'\) seem to behave against this empirical fact. In addition, the mass splitting between \(N\) and \(N'\) at three heaviest quark mass points is roughly consistent with experiment value \(\approx 0.5\text{GeV}\). Indeed, authors of Ref. 1 remarked that their utilized matrix-correlator method is no longer helpful in the lighter quark mass region due to systematic uncertainties from non-negligible higher-lying contribution.

We also remark that the simulation for the light-quark mass requires large lattice size since the “wave function” of the bound state enlarges as the quark mass decreases. Once the “wave function” is squeezed due to the small volume, the kinetic energy of internal quarks increases and thus the total energy of the bound state should be pushed up. This is an intuitive picture for the finite size effect on the mass spectrum. Such effect is expected to become serious for the radial excited state rather than the ground state. Indeed, Fig. 1 shows that the \(N'\) mass in the light-quark region is significantly heavier than the mass extrapolated from the heavy quark region. Their lattice simulation was performed on relatively small volume (\(L a \approx 1.5\text{fm}\)).

Unless taking into account these possible systematic errors, namely the effect
of high-lying states and that of finite volume, we cannot rule out the possibility of level switching between \( N' \) and \( N^* \) near the chiral limit. To resolve the remaining puzzle, we utilize the maximum entropy method (MEM), instead of the conventional analysis such as the matrix-correlator method. The MEM analysis can take into account non-negligible contribution from higher-lying states through the reconstruction of the spectral functions (SPFs), \( A(\omega) \) from given Monte Carlo data of the two-point hadron correlator \( G(t) = \int d\omega A(\omega) \exp(-\omega t) \). Recently the MEM analysis is widely employed on various problems in lattice simulations after the first success in our research area.\(^9\)\(^{10}\) Of course, we should also perform the finite size study for evaluating another systematic error.

\section*{§2. Latest lattice results}

Our numerical simulations are performed on three different lattice sizes, \( L^3 \times T = 16^3 \times 32, 24^3 \times 32 \) and \( 32^3 \times 32 \), to determine the finite size effect and to take the infinite volume limit for the observed masses. We generate quenched QCD configurations with the standard single-plaquette action at \( \beta = 6.0 \) \( (a^{-1} \approx 1.9 \text{GeV}) \). The quark propagators are computed using the Wilson fermion action at four values of the hopping parameter \( \kappa \), which cover the range \( M_\pi \approx 0.6-1.0 \text{GeV} \). Our preliminary results are analyzed on 444 configurations for the smallest lattice \( (La \approx 1.5 \text{fm}) \), 350 configurations for the middle size lattice \( (La \approx 2.2 \text{fm}) \) and 200 configurations for the largest lattice \( (La \approx 3.0 \text{fm}) \).

We use the conventional nucleon interpolating operators, \( \varepsilon_{abc}(u^T_a C\gamma_5 d_b)u_c \). Correlators constructed from those operators are supposed to receive contributions from both positive and negative-parity states. More details of the parity projection are described in Ref. 1. In this article, we focus only on the positive parity states. Details of the MEM analysis can be found in Ref. 11.

First we show dimensionless spectral functions of the nucleon for the smallest size (dotted line) and the largest size (solid line) in Fig. 2. At glance, the gross feature in both volume is similar. There are two sharp peaks and two large bumps. The crosses on each peak or bump represent the statistical significance of SPF obtained by the MEM. Clearly the peak positions at each peak or bump are relatively shifted to the right as the lattice size decreases. The peak shift indicates the direct observation on the finite size effect. One can see a large finite size effect on the second peak as compared with the first peak and two bumps. Here we comment that two large bumps might be the unphysical bound states of a physical quark and two doublers, which have been found in the mesonic case.\(^{12}\) We actually confirmed this speculation for baryons through the additive simulations at different lattice spacing. Those bumps appear at the same frequency \( \omega \) in lattice units. This means that those states are infinitely heavy in the continuum limit. We now conclude that the first two peaks are physical states but two large bumps are unphysical states. Thus, Fig. 2 indicates that the (physical) radial excited state is significantly affected by the finite size effect in comparison to the ground state.

Next we plot the masses of the ground state (open circle) and the first excited state (solid circle), which correspond to the peak positions of first two peaks, as
The dimensionless spectral function $\rho(\omega) = A(\omega)\omega^5$ in the nucleon channel as function of the frequency $\omega$ in lattice units. The second peak corresponding to the first excited state is significantly affected by the finite size effect in comparison to the first peak (the ground state).

Fig. 2. The dimensionless spectral function $\rho(\omega) = A(\omega)\omega^5$ in the nucleon channel as function of the frequency $\omega$ in lattice units. The second peak corresponding to the first excited state is significantly affected by the finite size effect in comparison to the first peak (the ground state).

Fig. 3. Masses of the ground state (open circle) and the first excited state (solid circle) in the positive-parity nucleon channel as function of the spatial lattice size $L$ in lattice units. Infinite volume extrapolation (dotted line) of those masses is guided by a power law behavior.

function of the spatial lattice size $L$ in Fig. 3. The errors are estimated by the jackknife method. Again, one can find the significantly large finite size effect appears in the first excited state as compared with the ground state. Clearly, the finite-size effect is rather severe in the lighter quark mass region. The infinite volume limit of the mass of each state at each quark mass is guided by a power law behavior as $M_L - M_\infty \propto c/L^3$.

Finally, we show the masses of the nucleon (diamonds) and the Roper state (circles) in the infinite volume as function of the pseudo pion mass squared in Fig. 4. Square symbols denote the negative parity nucleon, of which data is borrowed from the previous DWF calculation for your eyes’s guide. An important observation is that the mass splitting between the ground state and its radial excited state does
not seem to depend on the pseudo pion mass squared as expected. Clearly, the level switching between the negative parity nucleon and the corresponding Roper state reveal after removing the possible large systematic errors from non-negligible higher-lying contribution and the large finite size effect. In addition, the finite size effect on the negative parity nucleon state is known to be relatively small.\(^5\)

Recently, F.X. Lee and his collaborators also reported that a dramatic level switching between the negative parity nucleon and the corresponding Roper state is found in their calculation where the lightest pion mass is achieved to be close to the physical pion mass by using overlap fermions.\(^14\) However, there are several essential differences between our results and their results. They observed the strong quark mass dependences on the mass splitting between the ground state and the radial excited state, which is against the empirical fact as well as our results. The level switching in their calculation takes place in the region of \(M_\pi \sim 0.3\) to \(0.4\) GeV, which is far from our observation \(M_\pi \sim 0.6\) GeV. Finally, we remark that our lightest mass for the radial excited state is still below the threshold of the quenched \(\eta'\)-\(N\) intermediate state, which may contribute a dramatic spectral change such as a non-analytic behavior as function of the pion mass squared.

§3. Summary

We explored the level order of the positive-parity excited nucleon and the negative-parity nucleon. Based on the systematic analysis utilizing three different lattice size, we confirmed the large finite size effect on the Roper state in the light quark mass region originally pointed out in Ref. 1. The level switching between the \(N^*\) state and the Roper state should happen in lattice simulations with large spatial size which is...
larger than 3.0 fm. Our lattice calculation suggests that the Roper resonance can be described by the simple three quark excitation of sizable extent.

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