Parameter identification of Bouc–Wen dynamic model for magnetorheological shimmy damper based on improved simulated annealing algorithm

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Abstract: Magnetorheological (MR) shimmy damper has a good application prospect in aircraft landing gear shimmy control as a semi-active vibration control device; however, its non-linear and hysteretic characteristics bring difficulties to the control and restrict performance. It is necessary to develop a dynamic model of the damper that can effectively show these characteristics. This study is based on the experimental data of the damping force characteristics of MR shimmy damper with different control currents. Bouc–Wen model, apply to describe non-linear and hysteretic characteristics, was selected to establish the dynamic model of damping force, displacement and velocity. This study proposed an improved simulated annealing (SA) algorithm, which can improve the efficiency of identification, to identify the parameters of the model. Comparing with the original algorithm, the improved SA algorithm has the same solution quality and better performance in computational efficiency. The relationships between the identified parameters and the control current were obtained by curve fitting, and the experimental data with different amplitudes and frequencies were used to verify the result. It is shown that the established model can accurately show the dynamic characteristics of the damper under different excitation.

1 Introduction

Shimmy is a kind of self-excited vibration generated by the landing gear wheel of an aircraft, which may cause fuselage shaking and interfere with the pilot controlling, and lead to damage or accidents of fuselage components and landing gear structure [1]. The most common way to control shimmy is to install shimmy dampers on the landing gear. Modern aircraft are widely used with hydraulic shimmy dampers, but they are not able to adjust their own dynamic characteristic, and the control results on shimmy are not good. With the growing working hours of the aircraft, the damping force will decline [2].

The magnetorheological (MR) shimmy damper based on the MR effect, which can change the viscosity of the internal MR fluid by controlling the exciting current, generating different damping force. It has the advantages of rapid response, low energy consumption and good damping effect. Even if the controlling system failed, it could still be used as a passive control device with strong reliability [3]. The design of the MR shimmy damper control strategy should be based on a controllable model, but due to its non-linear and hysteresis characteristics, it is difficult to establish a mathematical model that is accurate and effective. The typical applications of the dynamic model of shimmy damper in a semi-active control system are shown in Fig. 1. Li and Xu [4] try to describe the characteristics of the MR damper with a variety of parametric models, such as Bingham model, Bouc–Wen model, hysteretic model, Jiles–Atherton model, etc. [5–9]. Among them, Bouc–Wen model is widely used in the modelling of a hysteretic system, which can better reflect the dynamic characteristics of the MR damper with low complexity, is conducive to application. Researchers have made some attempts on parameter identification of the Bouc–Wen model. Due to a large number of model parameters and complex factors affecting model parameters, the research results have problems of insufficient accuracy, poor applicability and low identification efficiency. To solve these problems, an algorithm that can efficiently and reliably obtain the global optimal solution of the model parameters is required. Among many of the optimisation algorithms, simulated annealing (SA) algorithm has high reliability in obtaining global solutions, and it does not need auxiliary information such as the initial value of an objective function, derivative value and parameter range, is an ideal algorithm for parameter identification base on experimental data [10].

In this paper, based on the experimental data of the damping force of the MR shimmy damper, the parametric dynamic model of the MR shimmy damper was established based on the Bouc–Wen model, and the model parameters were identified by the improved SA algorithm as well as an original SA algorithm, then the two results are compared. The relationship between excitation current and parameters of identification were obtained by curve fitting, then a group of experimental data was used for verification.

2 Dynamic character experimental design of MR damper

Based on the MR shimmy damper designed for the landing gear of a certain UAV, its structural and appearance are shown in Fig. 2. Experiments are carried out on the damping characteristic testing system shown in Fig. 3 [3].

The purpose of the experiment is to measure the output damping force of the damper corresponding to excitation displacement under different current and excitation conditions. According to the properties of MR materials, damping force

Fig. 1 Typical applications of the dynamic model
increases with the increase of applied current, and there is also an obvious saturation phenomenon. When the input current of the damper reaches 1.2 A, it is close to the saturation state. Therefore, four typical current values of 0, 0.4, 0.8, and 1.2 A are selected for the experiment. The displacement excitation used sinusoidal wave excitation with frequency $f = 3$ Hz, amplitude $A = 1.4$ mm, and frequency $f = 5$ Hz, amplitude $A = 3$ mm. Each group of experiments recorded the values of the output damping force and displacement under different current and displacement excitation during the time interval from the beginning to the sixth cycle, and then calculated the velocity values.

The experimental data when $f = 3$ Hz and $A = 1.4$ mm were used to participate in parameter identification, and the variation of damping force under different current conditions was measured, as shown in Fig. 4. It can be observed that an obvious magnetic saturation appeared under current conditions of 0.8 and 1.2 A. The data during the third and fourth cycle, which is under a stable condition in the middle part of the experiment is taken as the target of parameter identification, and the number of sampling points in each period reaches 333 by interpolation, which improves the smoothness of the curve and the accuracy of the calculation results of the optimisation algorithm. The curves of experimental damping force–displacement and damping force–velocity after treatment are shown in Figs. 5 and 6.

### 3 Bouc–Wen dynamic model and parameters of MR shimmy damper

Bouc–Wen model was first proposed by Wen in 1976 [11]. It is composed of a hysteretic system, spring and damper in parallel. The model is shown in Fig. 7, and the mathematical expression is shown below:

\[
F = c_i x + k_i x + a_i z
\]

where $F$ is the damping force output by the MR damper, $c_i$ is the viscous damping coefficient of the MR fluid in the damper, $k_i$ is the elastic coefficient, $a_i$ is the proportional coefficient of hysteresis force, $x$ is the relative displacement of the damper, $z$ is the hysteresis displacement caused by the hysteresis characteristic of the MR material. The hysteretic displacement $z$ is calculated by

\[
z = -\gamma |x|^n - 1 - \beta |x|^l + A x
\]

According to experience, the eight parameters of Bouc–Wen model can be divided into two categories: one is the parameter determined by the properties of MR materials and the structure of dampers, and its value does not change with the magnetic field strength, the other is a function of the exciting current. $c_i$, $k_i$ and $a_i$ are functions of excitation current [9, 11], which can be approximately expressed as third-order polynomials, as shown in (3). $\gamma$, $n$, $\beta$ and $A$ are parameters independent of the current. Referring to the research...
results of the same model of MR shimmy damper in previous literature [9]

\[
\begin{align*}
\gamma &= 150, \quad n = 2, \quad \beta = 150, \quad A = 6 \\
c_i &= c_o + c_i + c_i^2 + c_i^3 \\
k_i &= k_o + k_i + k_i^2 + k_i^3 \\
\alpha_i &= \alpha_o + \alpha_i + \alpha_i^2 + \alpha_i^3
\end{align*}
\] (3)

Therefore, the parameter identification of the MR damper can be divided into two parts. The fixed parameter value should be obtained from the empirical value summarised from the test data. The parameter identification of the parameters which changes with the excitation current is a typical multi-objective optimisation problem, which is suitable for the optimisation algorithm.

4 Design and application of improved SA algorithm

The idea of the SA algorithm was first proposed by Metropolis in 1953. It is a stochastic optimisation algorithm based on Monte Carlo iterative solution strategy [12]. Its advantages include the algorithm is simple and easy to implement, high reliability of obtaining the global optimal solution, and less requirements on the objective function, which overcomes the defect of other optimisation algorithms such as easily falling into local minimum value and dependence on the initial value. The fundamental reason for the algorithm to get the global optimal solution out of the local optimal solution is to comply with the Metropolis criterion [13].

The original SA algorithm will calculate the same number of iterations in each temperature and drop the same amplitude of cooling coefficient \(T\) and random disturbance decline coefficient \(k\). For short, the objective function value is smaller than the temperature can be taken as the stopping condition of the algorithm. When the object function value is smaller than acceleration error limit \(P\), run accelerate function, acceleration cooling coefficient \(T_d\) and Markov chain attenuation coefficient \(N_d\) are used to change the temperature attenuation coefficient and Markov chain length, in order to speed up the annealing process. According to experience and objective function parameters, the determined parameters are shown in Table 1.

4.2 Establishment of an objective function

The target of the optimisation algorithm is to find the acceptable optimal parameters to minimise the objective function, which is applied to this paper is to find the parameters of the model to make the calculated output damping force close to the output damping force measured by the experiment under the same condition. So, the objective function can be set as

\[ f = \frac{1}{n} \sum_{i=1}^{n} (F_{si} - F_{mi})^2 \] (4)

In (4), \(f\) is the value of the objective function, \(n\) is the total number of data involved in the calculation, \(F_{si}\) is the \(i\)th output damping force calculated by Bouc–Wen model, and \(F_{mi}\) is the \(i\)th output damping force measured by experiment. When the value of (4) is 0, \(F_{si} = F_{mi}\) which is an ideal model. The optimisation algorithm is introduced to search for the objective function's minimum value. The objective function value is also the mean square error between the model calculated damping force and the experimentally measured damping force, when the objective function value is small enough, the identification is a success.

4.3 Calculation results processing and analysis

Develop a program of model parameter identification using both improved SA and original SA algorithm in MATLAB, run the program, the results are shown in Table 2, obviously, the results between the two algorithms are extremely close. The searching process of the objective function and parameters \(c_o, k_i, \alpha_i\) is shown in Fig. 9. It can be seen from the iteration process that, with the growing number of iterations, all parameters and objective functions tend to be stable, and the objective function also knowns as the mean square error between the calculated value of the model and the experimental results are close to 0. Indicating that the process has fully completed and obtained the optimal solution, the identification result can be used as the parameter value of the desired model.

Since \(c_i, k_i, \text{and } \alpha_i\) are functions of current, the fitting tool is used in ORIGIN according to (3), and the fitting results are shown in Table 3 and Fig. 10.

Substitute the fitting results into (3)

\[
\begin{align*}
0.0154 + 0.0109i + 0.0366i^2 - 0.0203i^3 \\
0.0241 - 0.0373i + 0.1897i^2 - 0.1003i^3 \\
0.6612 + 4.1160i + 1.1680i^2 + 0.2943i^3
\end{align*}
\] (5)
Because of the number of random disturbance iterations declined and the temperature cooled faster when the value of objective function reach acceleration error limit $P$. The improved SA is more efficient than an original SA algorithm, during the searching process, the iteration times of inner cycle decrease from 800 to 750, which is fixed at 800 in the original SA algorithm. The total

**Fig. 8 Flow chart of parameter identification using improved SA algorithm**

**Table 1 Parameters of improved SA algorithm**

| Parameter                  | Value   |
|----------------------------|---------|
| initial temperature, $T$   | 4000    |
| cut-off temperature, $T_s$ | 0.00005 |
| Markov chain length, $L$   | 800     |
| initial disturbance step, $d$ | 0.1    |
| step attenuation coefficient, $m$ | 0.96   |
| temperature attenuation coefficient, $K$ | 0.9 |
| acceleration error limit, $P$ | 5      |
| acceleration cooling coefficient, $T_d$ | 0.97 |
| Markov chain attenuation coefficient, $N_d$ | 0.99 |

**Table 2 Parameter identification result**

| Current, A | $c_i$   | $k_i$   | $a_i$   | $f$      |
|------------|---------|---------|---------|----------|
| 0          | 0.0153  | 0.0241  | 0.6612  | 0.000295 |
| 0.4        | 0.0156  | 0.0331  | 2.1020  | 0.001062 |
| 0.8        | 0.0197  | 0.0643  | 3.0560  | 0.002538 |
| 1.2        | 0.0199  | 0.0792  | 3.4102  | 0.002902 |
iteration time of the outer cycle is 129 times instead of 174 times in the original SA. The efficiency of the algorithm is improved >25%.

5 Inspection and analyse of the model

The sinusoidal excitation signals of 3 Hz and 1.4 mm used in the experiment were, respectively, applied to four models with the different current to calculate the output forces. The model calculation results were compared with the damping force–displacement and damping force–velocity relations of the experimental data, as shown in Figs. 11 and 12. It can be seen that the identified model can accurately represent the output characteristics of the MR shimmy damper.

The sinusoidal signals of 5 Hz and 3 mm in the reference group are used as excitation, and the damping force–displacement and damping force–velocity relations are compared under the condition of 0 A current, as shown in Fig. 13. It can be seen that the Bouc–Wen dynamic model is applicable and can accurately represent the output characteristics of the MR shimmy damper under different frequency and displacement conditions.

Table 3 Parameter curve fitting results

| Order | $c_i$   | $k_i$   | $\alpha_i$ | $f$    |
|-------|---------|---------|------------|--------|
| 0     | 0.0153  | 0.0247  | 0.6670     | 0.000301 |
| 0.4   | 0.0156  | 0.0335  | 2.1110     | 0.001089 |
| 0.8   | 0.0196  | 0.0691  | 3.0497     | 0.002611 |
| 1.2   | 0.0198  | 0.0779  | 3.4612     | 0.002902 |

6 Conclusion

In this paper, a dynamic experiment is designed based on the self-made MR shimmy damper; the experimental data is used to identify the Bouc–Wen dynamic model parameters of the MR

It can be seen from the comparison of experimental results that the dynamic model established in this paper has a good applicability. Under different current, frequency and displacement conditions, the hysteresis characteristics and dissipation capacity of the shimmy damper can be clearly demonstrated.

By observing the calculation results of the model and experimental results, it can be found that the error increase with the current. By analysing the experiment and identification process, the main reasons for the error are (i) the limitations of Bouc–Wen model, this problem can be improved by adding variables to the model, but it also increases the complexity of the model, (ii) the identification ignores the factors such as temperature that have a minor influence on the experimental results, and (iii) the shimmy damper itself has to influence factors such as bubbles and impurities in the MR fluid.
shimmy damper. The identification used an improved SA algorithm, which is more efficient and has the same reliability comparing with the original SA algorithm to find the optimal solution of the object model parameters. Identification results are compared with the experimental data; the mean square error is in $10^{-3}$ level, proved the validity of the identification method and results, Bouc–Wen model using a few parameters which can accurately describe the MR damper's character between vibration frequency, vibration amplitude, exciting current and damping force, also can show the dynamic hysteresis characteristics of dampers and dissipation ability, is suitable for promotion in the engineering application.

This paper proved that the improved SA algorithm could effectively use in MR shimmy damper's Bouc–Wen model parameter identification, compared with an original SA algorithm, the new algorithm improved the efficiency >25%, by adjusting the algorithm parameters, improved SA algorithm can be applied to another MR damper's dynamic model identification works.

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Fig. 11 Damping force–displacement comparison curve between experimental data and model
(a) 0 A damping force–displacement curve, (b) 0.4 A damping force–displacement curve, (c) 0.8 A damping force–displacement curve, (d) 1.2 A damping force–displacement curve

Fig. 12 Damping force–velocity comparison curve between experimental data and model
(a) 0 A damping force–velocity curve, (b) 0.4 A damping force–velocity curve, (c) 0.8 A damping force–velocity curve, (d) 1.2 A damping force–velocity curve

Fig. 13 Comparison between experimental data and model under 5 Hz and 3 mm sinusoidal signal
(a) Damping force–displacement comparison curve, (b) Damping force–velocity comparison curve