Comments on absorption cross section for Chern-Simons black holes in five dimensions

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In this paper we study the effects of black hole mass on the absorption cross section for a massive scalar field propagating in a 5-dimensional topological Chern-Simons black hole at the low-frequency limit. We consider the two branches of black hole solutions ($\alpha = \pm 1$) and we show that, if the mass of black hole increase the absorption cross section decreases at the zero-frequency limit for the branch $\alpha = -1$ and for the other branch, $\alpha = 1$, the behavior is opposite, if the black hole mass increase the absorption cross section increases. Also we find that beyond a certain frequency value, the mass black hole does not affect the absorption cross section.

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I. INTRODUCTION

In general relativity the field equation adopt the following form

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu},$$

where $G$ is a symmetric tensor that depends of the metric $g_{\mu\nu}$ and their first derivatives, furthers its divergences is vanished

$$G^\mu_\mu = 0.$$

In four dimensions these conditions allow to determinate the Einstein tensor, at least of arbitrary multiplicative constant factor

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu}.$$

The Einstein tensor is the only symmetric and conserved tensor depending on the metric and its derivatives, which is linear in the second derivatives of the metric. The invariant action that arise these fields equations, it is the Einstein-Hilbert action with cosmological constant ($\Lambda$). In higher dimensions, the potential problem is to find the most general action that arise a set of second order field equations. The solution to this problems it is the called Lanczos-Lovelock (LL) action. This action is non linear in the Riemann tensor, and differs from the Einstein-Hilbert action only if the space-time has more than 4 dimensions. Therefore, the Lanczos-Lovelock action is the most natural extension of general relativity in higher dimensional space-times, that generate second order field equations. This action in $d$ dimensions can be written as follow

$$S = \sum_{2p<d} \alpha_p S_p,$$

with

$$S_p = \frac{1}{2p!} \int \sqrt{-g} \epsilon^{[\beta_1 \cdots \beta_{2p}]} R_{\alpha_1 \beta_1} \cdots R_{\alpha_{2p-1} \beta_{2p-1}} R_{\alpha_{2p} \beta_{2p}} d^d x,$$
where the \( \alpha_p \) are arbitrary coefficients with dimension of mass\(^{d-2p} \) and the symbols \( \delta^{[\cdots]}_{\cdots} \) are define by

\[
\delta^{[\beta_1 \cdots \beta_{2p}]}_{\alpha_1 \cdots \alpha_{2p}} = \begin{vmatrix}
\delta_{\alpha_1}^{\beta_1} & \cdots & \delta_{\alpha_1}^{\beta_{2p}} \\
\vdots & \ddots & \vdots \\
\delta_{\alpha_2}^{\beta_1} & \cdots & \delta_{\alpha_2}^{\beta_{2p}}
\end{vmatrix},
\]

it represents the generalizes delta Kronecker.

In the particular case when \( d = 4 \), we can identify constants \( \alpha_0 = \frac{\lambda}{8 \pi G_N} \) and \( \alpha_1 = \frac{1}{8 \pi G_N} \) i.e are related to the cosmological constant and gravitational Newton constant respectively.

On the other hand, differential forms allows to characterize LL action and those dynamical equations. In order to do this task we introduce the vielbein \( e^a \) and the spin connection \( \omega^{ab} \) 1-forms. These 1-forms are related to curvature and torsion two-forms as follow \( R^{ab} = d\omega^{ab} + \omega^c_a \omega^{cb} \) and \( T^a = de^a + \omega^c_a e^b \). So, the lagragian can be written as

\[
L_{LL} = \int\sum_{q=0}^{[d/2]} \alpha_q L^q,
\]

where \( \alpha_q \) are arbitrary constants and

\[
L^q = \epsilon_{a_1 \cdots a_d} R^{a_1 a_2} \cdots R^{a_{2q-1} a_{2q}} e^{a_{2q+1}} \cdots e^{a_d}.
\]

In generic form the coefficients are arbitrary in the LL action. However, they can be fixed in order to have a unique vacuum or a unique cosmological constant. In this case the coefficient are given by \( \alpha_q := c^k_q \) where \( c^k_q = \binom{2q-k}{d-2q} \) for \( q \leq k \) and vanished for \( q > k \), which \( 1 \leq k \leq \frac{[d-1]}{2} \). Then, the action can be written as follows

\[
I_k = \kappa \int\sum_{q=0}^{[k]} c^k_q L^q.
\]

This action possesses two fundamentals constants \( \kappa \) (related to Newton gravitational constant \( G_k \)) and \( l \), (related to cosmological constant \( \Lambda \)).

\[
\kappa = \frac{1}{2(d-2)! \Omega_{d-2} G_k},
\]

\[
\Lambda = \frac{(d-1)(d-2)}{2l^2},
\]

where \( \Omega_{d-2} \) is the volume of a \( (d-2) \)-unit sphere. For \( k = 1 \) the Einstein- Hilbert action is recovered. It is worth nothing that in odd dimensions the theory is gauge invariant under the (anti-)de Sitter or the Poincare groups and the lagrangian is a Chern-Simons form.

Chern-Simons black holes are special solutions of gravity theories in higher odd dimensions which contain higher powers of curvature. These theories are consistent Lanczos-Lovelock theories resulting in second order field equations for the metric with well defined AdS asymptotic solutions. For spherically symmetric topologies, these black holes are labelled by an integer \( k \) which specifies the higher order of curvature present in the Lanczos-Lovelock action and it is related to the dimensionality \( d \) of spacetime by the relation \( d - 2k = 1 \). If \( d - 2k = 1 \), the solutions are known as Chern-Simons black holes (for a review on the Chern-Simons theories see [3]). These solutions were further generalized to other topologies [4] and they can be described in general by a non-trivial transverse spatial section \( \sum_{\gamma} \) of \( (d-2) \)-dimensions labelled by the constant \( \gamma = +1, -1, 0 \) that represents the curvature of the transverse section, corresponding to a spherical, hyperbolic or plane section, respectively. The solution describing a black hole in a free torsion theory can be written as [4]

\[
ds^2 = -\left( \gamma + \frac{r^2}{l^2} - \alpha \left( \frac{2G_k \mu}{r^{d-2k-1}} \right)^{\frac{1}{2}} \right) dt^2 + \frac{dr^2}{\left( \gamma + \frac{r^2}{l^2} - \alpha \left( \frac{2G_k \mu}{r^{d-2k-1}} \right)^{\frac{1}{2}} \right)} + r^2 d\sigma_\gamma^2,
\]

where \( \alpha = (\pm)^{k+1} \) and the constant \( \mu \) is related to the horizon \( r_+ \) through

\[
\mu = \frac{r_+^{d-2k-1}}{2G_k} (\gamma + \frac{r_+^2}{l^2})^k,
\]
and to the mass $M$ by

$$\mu = \frac{\Omega_{d-2}}{\Sigma_{d-2}} M + \frac{1}{2G_k} \delta_{d-2k,\gamma},$$

(14)

here $\Sigma_{d-2}$ denotes the volume of the transverse space. As can be seen in (12), if $d - 2k \neq 1$ the $k$ root makes the curvature singularity milder than the corresponding black hole of the same mass. At the exact Chern-Simons limit $d - 2k = 1$, the solution has similar structure like the $(2+1)$-dimensional BTZ black hole with a string-like singularity. We are merely interested here for the hyperbolic topology with $\gamma = -1$. In this case $d\sigma^2_{-1}$ in (12) is the line element of the $(d - 2)$-dimensional manifold $\sum_{-1}$, which is locally isomorphic to the hyperbolic manifold $H^{d-2}$.

The spherical topologies Chern-Simons black holes have similar causal structure as the $(2+1)$-dimensional BTZ black hole [3], and they have positive specific heat and therefore are thermodynamical stable. For hyperbolic topologies, the Chern-Simons black holes resemble to the topological black holes [4] in their zero mass limit, and their thermodynamic behavior was studied in [3] and the quasi-normal modes (QNMs), the reflection coefficients, the transmission coefficients and the absorption cross section of scalar perturbations was studied in [2]. The QNMs of the Chern-Simons black holes depends on the black hole parameters and on the fundamental constants of the system, also it depends on the curvature parameter $k$. Besides this curvature parameter is related to the number of symmetries in the theory, taking his maximum value for Chern-Simons theories. Also, at low frequency limit that there is a range of modes with high angular momentum which contributes at the absorption cross section. An illustrative example of Chern-Simons black holes is provided by the Gauss-Bonnet theory for $d = 5$ and $k = 2$. Static local solutions of this theory are well studied over the years [8]. This theory has two branches of solutions. If there is a fine tuning between $k$ and the Gauss-Bonnet coupling constant $\alpha$, the two solutions coincide to the Chern-Simons black hole solution which has maximum symmetry. This is known as the Chern-Simons limit (for a review see [9]). The stability of these solutions has also been studied [10]. It was found in Ref. [11] that one of these solutions suffers from ghost-like instability up to the strongly coupled Chern-Simons limit where linear perturbation theory breaks down.

The Hawking radiation is a semiclassical effect and it gives the thermal radiation emitted by a black hole. At the event horizon, the Hawking radiation is in fact blackbody radiation. However, this radiation still has to traverse a non-trivial curved spacetime geometry before it reaches a distant observer who detects it. The surrounding spacetime thus works as a potential barrier for the radiation giving a deviation from the blackbody radiation spectrum, seen by an asymptotic observer [12, 13]. So, the total flux observed at infinity is that of a d-dimensional greybody at the Hawking temperature.

In the present work, we focus our study on the effect of the mass of topological Chern-Simons black hole in five dimensions on the absorption cross section for scalar fields and mainly we show that from a certain frequency the mass black hole mass does not affect at the absorption cross section.

II. QUASINORMAL MODES AND ABSORPTION CROSS SECTIONS OF SCALAR PERTURBATIONS

The metric of Chern-Simons black hole under consideration\(^1\) is

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\sigma^2_{\gamma},$$

(15)

where,

$$f(r) = \gamma + \frac{r^2}{l^2} - \alpha(2\mu G_k)^{\frac{1}{k}},$$

(16)

and the horizon is located at

$$r_+ = l\sqrt{\alpha(2\mu G_k)^{\frac{1}{k}} - \gamma}.$$  

(17)

Considering the horizon geometry with a negative curvature constant, $\gamma = -1$, the allowed range of $\mu$ for $r_+ \geq 0$ are: if $k$ is odd, $\alpha = 1, \mu \geq \frac{1}{2G_k}$; if $k$ is even and $\alpha = 1, \mu \geq 0$ and if $k$ is even and $\alpha = -1, \frac{1}{2G_k} \geq \mu \geq 0$ [3].

\(^1\) See Ref. [2] for details and discussions.
Performing the change of variables $v = 1 - l^2 p/r^2$, where $p = 1 + \alpha (2 \mu G_0)^{\frac{1}{d}}$ and $t = lt$, and written the Klein-Gordon equation for a minimally coupled scalar field in the background of a Chern-Simons black hole (in d-dimensions) we obtain

$$v(1-v)\partial_v^2 R(v) + \left[1 + \left(\frac{d-5}{2}\right)v\right]\partial_v R(v) + \left[\frac{\omega^2}{4pv} - \frac{Q}{4p} - \frac{m^2 l^2}{4(1-v)}\right] R(v) = 0,$$

where $Q$ corresponds to eingenvalues for Laplace operator in the hyperbolic manifold $\sum_{d-2}$, and it is given by $Q = (\frac{d-3}{2})^2 + \xi^2$, without any identifications of the pseudosphere the spectrum of the angular wave equation is continuous, thus $\xi$ takes any real value $\xi \geq 0$. Since the $(d-2)$-dimensional manifold $\sum$ is a quotient space of the form $H^{d-2}/\Gamma$ and it is a compact space of constant negative curvature, the spectrum of the angular wave equation is discretized and thus $\xi$ takes discrete real values $\xi \geq 0$ \cite{4}. Now, the radial function $R(v)$ becomes under the decomposition $R(v) = v^\alpha (1 - v)^\beta K(v)$, Eq. (18) can be written as a hypergeometric equation for $K$

$$v(1-v)K''(v) + [c - (1 + a + b)v] K'(v) - abK(v) = 0.$$ 

Where the coefficients are given by

$$a = -\left(\frac{d-3}{4}\right) + \alpha + \beta + \frac{i}{2}\sqrt{\frac{\xi^2}{p} + \left(\frac{d-3}{2}\right)^2 \left(\frac{1}{p} - 1\right)},$$

$$b = -\left(\frac{d-3}{4}\right) + \alpha + \beta - \frac{i}{2}\sqrt{\frac{\xi^2}{p} + \left(\frac{d-3}{2}\right)^2 \left(\frac{1}{p} - 1\right)},$$

$$c = 1 + 2\alpha,$$

where $c$ cannot be an integer and the exponents $\alpha$ and $\beta$ are

$$\alpha = \pm \frac{i\omega\sqrt{p}}{2p},$$

$$\beta = \beta_{\pm} = \left(\frac{d-1}{4}\right) \pm \frac{1}{2}\sqrt{\left(\frac{d-1}{2}\right)^2 + m^2 l^2}.$$

Without loss of generality, we choose the negative signs for $\alpha$. The general solution of Eq. (19) takes the form

$$K = C_1 F_1(a, b, c; v) + C_2 v^{1-c} F_1(a-c+1, b-c+1, 2-c; v),$$

which has three regular singular point at $v = 0$, $v = 1$ and $v = \infty$. Here, $F_1(a, b, c; v)$ is a hypergeometric function and $C_1, C_2$ are constants. Then, the solution for the radial function $R(v)$ is

$$R(v) = C_1 v^{\alpha}(1 - v)^\beta F_1(a, b, c; v) + C_2 v^{-\alpha}(1 - v)^\beta F_1(a-c+1, b-c+1, 2-c; v).$$

To obtain an exact expression for the quasi-normal modes of scalar perturbations of a Chern-Simons black hole in $d-$dimensions we need to impose boundary conditions on asymptotically AdS spacetime. First, we have to impose our boundary conditions on the horizon that there exist only ingoing waves. This fixes $C_2 = 0$. Then the radial solution becomes

$$R(v) = C_1 e^{\alpha \ln v}(1 - v)^\beta F_1(a, b, c; v) = C_1 e^{-i\omega\sqrt{p}}\ln v(1 - v)^\beta F_1(a, b, c; v).$$

In order to implement boundary conditions at infinity ($v = 1$), we shall apply in Eq. (27) the Kummer’s formula for the hypergeometric function \cite{13}, with this expression the radial function results in

$$R(v) = C_1 e^{-i\omega\sqrt{p}}\ln v(1 - v)^\beta \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} F_1(a, b, a+b-c, 1-v)$$

$$+ C_1 e^{-\frac{i\omega}{\sqrt{p}}\ln v}(1 - v)^{c-a-b+\beta} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} F_1(c-a, c-b, c-a-b+1, 1-v).$$
The solution of Eq. (32) is a linear combination of the Bessel function \[15\] given by
\[\omega = \mp \sqrt{\xi^2 + \left(\frac{d-3}{2}\right)^2 (1-p)} - i \sqrt{p} \left[2n + 1 \pm \sqrt{\left(\frac{d-1}{2}\right)^2 + m^2 l^2}\right], \tag{29}\]
for \(\beta_+\) and \(\beta_-\), respectively.

On the other hand, the reflection and the transmission coefficients are defined by
\[\Re := \left|\frac{F_{\text{out} \text{ asym}}} {F_{\text{asymp}}}\right|, \quad \text{and} \quad \Im := \left|\frac{F_{\text{in} \text{ asym}}}{F_{\text{asymp}}}\right|, \tag{30}\]
so we need to know the behavior of the radial function both at the horizon and at the asymptotic infinity. The behavior at the horizon is given by Eq. (27). Thus, to obtain the asymptotic of the \(R(v)\) we use \(1 - v = \frac{r^2}{\xi^2}\) and taking into account the limit of \(R(v)\), Eq. (26), when \(v \to 1\), we have
\[R(r) = C_1 \left(\frac{1}{r} \sqrt{\frac{\beta}{r}}\right)^{2\beta} \frac{\Gamma(c) \Gamma(c - a - b)}{\Gamma(c - a) \Gamma(c - b)} + C_1 \left(\frac{1}{r} \sqrt{\frac{\beta}{r}}\right)^{d-1-2\beta} \frac{\Gamma(c) \Gamma(a + b - c)}{\Gamma(a) \Gamma(b)}. \tag{31}\]

On the other hand, when \(r \to \infty\), Klein-Gordon equation approximates to
\[\partial^2_r R(r) + \frac{d}{r} \partial_r R(r) + \left(\frac{\omega^2 l^2}{r^4} - \frac{Q^2}{r^4} - \frac{m^2 l^2}{r^2}\right) R(r) = 0. \tag{32}\]
The solution of Eq. (32) is a linear combination of the Bessel function \[15\] given by
\[R(r) = \left(\frac{\sqrt{A}}{2r}\right) \frac{d+1}{2} \left[D_1 \Gamma(1 - C) J_{-C} \left(\frac{\sqrt{A}}{r}\right) + D_2 \Gamma(1 + C) J_C \left(\frac{\sqrt{A}}{r}\right)\right], \tag{33}\]
where
\[A = l^2 (\xi^2 - Q^2), \tag{34}\]
\[C = \frac{1}{2} \sqrt{(d - 1)^2 + 4m^2 l^2}. \tag{35}\]
Now, using the expansion of the Bessel function \[15\] we find the asymptotic solution in the polynomial form
\[R_{\text{asymp.}}(r) = D_1 \left(\frac{\sqrt{A}}{2r}\right)^{\frac{d+1}{2} - C} + D_2 \left(\frac{\sqrt{A}}{2r}\right)^{\frac{d+1}{2} + C}, \tag{36}\]
for \(\sqrt{r} \ll 1\). Introducing,
\[\tilde{D}_1 \equiv D_1 \left(\frac{1}{r}\right)^{\frac{d+1}{2} - C}, \quad \tilde{D}_2 \equiv D_2 \left(\frac{1}{r}\right)^{\frac{d+1}{2} + C}, \tag{37}\]
we write Eq. (36) as
\[R_{\text{asymp.}}(r) = \tilde{D}_1 \left(\frac{1}{r}\right)^{\frac{d+1}{2} - C} + \tilde{D}_2 \left(\frac{1}{r}\right)^{\frac{d+1}{2} + C}. \tag{38}\]
Comparison of Eqs. (31) and (38), regarding \(\beta = \beta_-\), it allows us to immediately read off the coefficients \(\tilde{D}_1\) and \(\tilde{D}_2\) which can be decomposed in terms of the incoming and the outgoing coefficients \(D_{\text{in}}\) and \(D_{\text{out}}\), Refs. (7, 16–20).
Defining, $\hat{D}_1 = D_m + D_{\text{out}}$ and $\hat{D}_2 = i\hbar(D_{\text{out}} - D_m)$. In this way, the reflection and transmission coefficients are given by

$$\Re = \frac{|D_{\text{out}}|^2}{|D_m|^2},$$  

(39)

$$\Im = \frac{\omega l^{d-1} p^{\frac{d-4}{2}} |C_1|^2}{2 |h| C |D_m|^2},$$  

(40)

and the absorption cross section, $\sigma_{\text{abs}}$, is given by

$$\sigma_{\text{abs}} = \frac{\Im}{\omega} = \frac{l^{d-1} p^{\frac{d-4}{2}} |C_1|^2}{2 |h| C |D_m|^2},$$  

(41)

where, the coefficients $D_m$ and $D_{\text{out}}$ are given by

$$D_m = \frac{C_1}{2} \left[ (l\sqrt{p})^{2\beta - 1} \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} + \frac{i}{\hbar} (l\sqrt{p})^{d-1-2\beta} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} \right],$$  

(42)

$$D_{\text{out}} = \frac{C_1}{2} \left[ (l\sqrt{p})^{2\beta - 1} \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} - \frac{i}{\hbar} (l\sqrt{p})^{d-1-2\beta} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} \right].$$  

(43)

Now, we will carry out a numerical analysis of the absorption cross section given by the Eq. (41), for a five-dimensional topological Chern-Simons black hole, $k = 2$. We consider without loss of generality, $m^2l^2 = -15/4$, $l = 1$ and we fix $\hbar = -1$. Then, we analyze the behavior of the absorption cross section for various values of $p$, which it is given by $p = 1 + \alpha(2\mu G_k)^{\frac{1}{k}}$ with $\mu = \frac{\Omega_s^{-1}}{2\pi^2} M$, for a topological black holes. The two branches of black hole solutions are given by $\alpha = \pm 1$. Therefore, for $k = 2$ and $\alpha = 1$, then $\mu \geq 0$ and $p \geq 1$ and for $k = 2$ and $\alpha = -1$, then $\frac{1}{2\pi^2} \geq \mu \geq 0$ and $0 \leq p \leq 1$. Massless black hole corresponds to $p = 1$. Besides, we restricted values of $p$ and $\xi$, such as the quasi-normal modes exhibited real and imaginary part, i.e. $p < \xi^2 + 1$. Thus, we are leaving out of our analysis the pure damping modes. Our results for $\xi = 0$ and $\alpha = -1$ are shown in Fig. (1). Other branch define by $\alpha = 1$, it does not satisfy the restriction between $\xi$ and $p$ described above. Then, for $\xi = 1$ and $\alpha = \mp 1$, our results are shown in Figs. (2) and (3), respectively. Finally, for $\xi = 2$ and $\alpha = \pm 1$ our results are shown in Figs. (4) and (5), respectively. In general, for the branch $\alpha = -1$, the absorption cross section decreases if the mass of black hole increase, Figs. (1) and (4), at the zero-frequency limit. For the other branch, $\alpha = 1$, the behavior is opposite, the absorption cross section increases if the black hole mass increase, Figs. (3) and (5). Also, we can see from the figures that the values of the coefficients converge at the values of coefficients for the topological massless black hole. Therefore, we can see that beyond a certain frequency the mass black hole does not affect the absorption cross section.

III. SUMMARY

In this work we have studied the absorption cross section for a massive scalar field propagating in a 5-dimensional topological Chern-Simons black hole. In this sense, at the low-frequency limit, we analyzed the effect of the mass black hole. We considered the two branches of black hole solutions $\alpha = \pm 1$ and we showed that, for the branch $\alpha = -1$, the absorption cross section decreases if the mass of black hole increase, at the zero-frequency limit. For the other branch, $\alpha = 1$, the behavior is opposite, the absorption cross section increases if the black hole mass increase. We would like to note that the absorption cross section shows two characteristic behaviors. The first one, it shows a maximum value for the absorption cross section near to zero frequency and the second it shows a maximum for a finite non-vanishing value. Besides, beyond a certain frequency the mass black hole does not affect the absorption cross section.
FIG. 1: Absorption cross section $v/s \omega; d = 5, m^2l^2 = -15/4, l = 1, h = -1, \alpha = -1$ and $\xi = 0$.

FIG. 2: Absorption cross section $v/s \omega; d = 5, m^2l^2 = -15/4, l = 1, h = -1, \alpha = -1$ and $\xi = 1$.

FIG. 3: Absorption cross section $v/s \omega; d = 5, m^2l^2 = -15/4, l = 1, h = -1, \alpha = 1$ and $\xi = 1$. 
FIG. 4: Absorption cross section $v/s \omega$; $d = 5$, $m^2l^2 = -15/4$, $l = 1$, $h = -1$, $\alpha = -1$ and $\xi = 2$.

FIG. 5: Absorption cross section $v/s \omega$; $d = 5$, $m^2l^2 = -15/4$, $l = 1$, $h = -1$, $\alpha = 1$ and $\xi = 2$.

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