Blow-up for Semidiscretisations of a Semilinear Schrodinger Equation with Dirichlet Condition

Konan Firmin N’gohisse¹, Diabate Nabongo², Lassane Traoré¹

¹Department of Biology, University of Peleforo Gon Coulibaly, Korhogo, Côte d’Ivoire
²Departement of Economy, University of Alassane Ouattara, Bouaké, Côte d’Ivoire

Email address:
firmingoh@yahoo.fr (K. F. N’gohisse), nabongo._diabate@yahoo.fr (D. Nabongo), tlassane@yahoo.fr (L. Traoré)

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Abstract: Theoretical study of the phenomenon of blow-up solutions for semilinear Schrödinger equations has been the subject of investigations of many authors (see [1, 3, 9, 15, 17, 18, 23], and the references cited therein). This paper is interested by the numerical study of the above problem. Let I be a positive integer and define the grid 

\[ x_j = jh, 0 \leq j \leq I, \]

where \( h=1/I \). Approximate the solution \( u \) of the problem (1)–(3) by the solution \( u_I(\tau) \). The time \( T \) may be finite or infinite. When \( T \) is infinite, we say that the solution \( u \) exists globally. When \( T \) is finite, the solution \( u \) develops a singularity in a finite time, namely

\[ \lim_{\tau \to T^{-}} \|u(x, \tau)\|_\infty = \infty \]

where \( \|u(x, t)\|_\infty = \sup_{x \in (0, 1)}|u(x, t)| \). In this case, it is say that the solution \( u \) blows up in a finite time and the time \( T \) is called the blow-up time of the solution \( u \).

The theoretical study of the phenomenon of blow-up and in particular blow-up solutions for semilinear Schrödinger equations has been the subject of investigations of many authors (see [1, 3, 9, 15, 17, 18, 23], and the references cited therein).

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\[ u_t = iau_{xx} - ib|u|^p, x \in (0, 1), t \in (0, T) \]  

\[ u(0, t) = 0, u(1, t) = 0, t \in (0, T) \]  

\[ u(x, 0) = u_0(x), x \in [0, 1], \]

which appears in a lot of models of nonlinear optics, energy transfer in molecular systems, quantum mechanics, seismology, plasma physics, see [4, 21, 28], to cite only a few cases. Here \( p>1, a \in \mathbb{R}, a \neq 0, b > 0 \). The initial datum \( u_0(x) \) is a continuous function in \([0, 1]\). The conditions \( u_0(0) = 0 \) and \( u_0(1) = 0 \) mean that the temperature is maintained nil on the boundary \( x=0 \) and \( x=1 \).

Here \((0, T)\) is the maximal time interval of existence of the solution \( u \). The time \( T \) may be finite or infinite. When \( T \) is infinite, we say that the solution \( u \) exists globally. When \( T \) is finite, the solution \( u \) develops a singularity in a finite time, namely

\[ \lim_{\tau \to T^{-}} \|u(x, \tau)\|_\infty = \infty \]

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\[ u(x, 0) = u_0(x), x \in [0, 1], \]
Semidiscrete Blow-up Solutions

In this section, under some assumptions, we show that the solution of the semidiscrete problem blows up in a finite time and estimate its semidiscrete blow-up time. One need the following Lemma.

Lemma 2.1. We have \( \sum_{j=1}^{i-1} \sin(j\pi h) = \cotan \left( \frac{\pi h}{2} \right) \).

Proof. A routine calculation yields

\[
\sum_{j=1}^{i-1} \sin(j\pi h) = i m \left( \sum_{j=1}^{i-1} e^{j\pi h} \right) = i m \left( \sum_{j=1}^{i-1} (e^{\pi h})^j \right)
= i m \left( \frac{e^{i\pi h} - e^{-i\pi h}}{1 - e^{i\pi h}} \right) = i m \left( \frac{-e^{i\pi h} + e^{-i\pi h}}{e^{i\pi h} - e^{-i\pi h}} \right)
= i m \left( \frac{\cot \left( \frac{\pi h}{2} \right) = \cotan \left( \frac{\pi h}{2} \right) \right)
\]

and the proof is complete.

Lemma 2.2 Let \( U_h, V_h \) two vectors such that

\[
U_0 = 0, U_j = 0, V_0 = 0, V_j = 0
\]

Then we have

\[
\sum_{j=1}^{i-1} h U_j \delta^2 v_j = \sum_{j=1}^{i-1} h V_j \delta^2 u_j
\]

Proof. A straightforward computation reveals that

\[
\sum_{j=1}^{i-1} h U_j \delta^2 v_j = \sum_{j=1}^{i-1} h V_j \delta^2 u_j = \sum_{j=1}^{i-1} \frac{1}{h} (\delta^2 u_j - \delta^2 v_j)
\]

and

\[
A = \sum_{j=1}^{i-1} \tan \left( \frac{\pi h}{2} \right) \sin(j\pi h)Re(\varphi_j)
\]

Then the solution \( U_h \) of (4)—(6) blows up in a finite time \( T_h \) which is estimated as follows

\[
T_h \leq \frac{1}{\lambda_h} \arccos \left( 1 - \frac{a h A^{1-p}}{b(p-1)} \right)
\]

Proof. Since \( (0,T_h) \) is the maximal time interval on which \( \| U_h(t) \|_\infty \) is finite, our aim is to show that \( T_h \) is finite and obeys the above inequality. Introduce the functions \( v \) and \( w \) defined as follows

\[
v(t) = \sum_{j=1}^{i-1} \tan \left( \frac{\pi h}{2} \right) \sin(j\pi h) U_j(t)
\]

\[
w(t) = \sum_{j=1}^{i-1} \tan \left( \frac{\pi h}{2} \right) \sin(j\pi h) \bar{U}_j(t)
\]

Taking the derivative of \( v \) in \( t \) and using (4), we get
\[v'(t) = ia \sum_{j=1}^{i-1} \tan \left( \frac{nh}{2} \right) \sin(jnh) U_j(t)\]

\[-ib \sum_{j=1}^{i-1} \tan \left( \frac{nh}{2} \right) \sin(jnh) |U_j(t)|^p\]

One observes that \(\delta^2 \sin(jnh) = -\lambda_h \sin(jnh)\). Due to Lemma 2.2, we arrive at

\[v'(t) = -ia\lambda_h v(t) - ib \sum_{j=1}^{i-1} \tan \left( \frac{nh}{2} \right) \sin(jnh) |U_j(t)|^p\]

which implies that

\[\frac{d}{dt} e^{-ia\lambda_h t} v(t) = -ibe^{-ia\lambda_h t} \sum_{j=1}^{i-1} \tan \left( \frac{nh}{2} \right) \sin(jnh) |U_j(t)|^p\]

We also observe that, taking the derivative of \(w\) in \(t\) and using (4), we discover that

\[b_i = \frac{1}{L \tan SHIcD/6E + Hn^6} \]

Reasoning as above, we find that

\[Z'(t) = b \sin(a\lambda_h t) Z(t) |Z(t)|^p \]

where \(Z(t) = Z(t) = e^{ia\lambda_h t U(t) + e^{-ia\lambda_h t} w(t)}\). From Lemma 2.1, we see that \(\sum_{j=1}^{i-1} \tan \left( \frac{nh}{2} \right) \sin(jnh) |U_j(t)|^p\) equals one. Thus applying Jensen’s inequality, we find that

\[\sum_{j=1}^{i-1} \tan \left( \frac{nh}{2} \right) \sin(jnh) |U_j(t)|^p\]

is bounded from below by \(\left( \sum_{j=1}^{i-1} \tan \left( \frac{nh}{2} \right) \sin(jnh) |U_j(t)|^p\right)^p\). Applying the triangle inequality, we discover that \(|Z(t)|\) is bounded from above by \(\sum_{j=1}^{i-1} \tan \left( \frac{nh}{2} \right) \sin(jnh) |U_j(t)|^p\). Since \(\sin(a\lambda_h t)\) is nonnegative when \(t\) is between 0 and \(\pi / a\lambda_h\), we deduce that

\[Z'(t) \geq b \sin(a\lambda_h t) |Z(t)|^p \text{ for } t \in \left(0, \frac{\pi}{a\lambda_h}\right)\]

This inequality implies that the function \(Z(t)\) is increasing. Since \(Z(0)\) is positive, we find that

\[
\frac{dZ}{Z^p} \geq b \sin(a\lambda_h t) dt \text{ for } t \in \left(0, \frac{\pi}{a\lambda_h}\right).
\]

Let \(T_h' = \min \left(\frac{\pi}{a\lambda_h}, T_h\right)\). Integrating this inequality over \((0, T_h')\), we conclude that

\[
\frac{(Z(0))^{1-p}}{p-1} \geq \frac{b}{a\lambda_h} (1 - \cos(a\lambda_h T_h')).
\]

Therefore, we have

\[\cos(a\lambda_h T_h') \geq 1 - \frac{a\lambda_h (Z(0))^{1-p}}{b - p-1}.
\]

Since the quantity on the right-hand side of the above inequality is positive, we see that the time \(T_h^*\) is estimated as follows

\[T_h^* \leq \frac{1}{a\lambda_h} \arccos \left(1 - \frac{a\lambda_h (Z(0))^{1-p}}{b - p-1}\right).
\]

Since \(1 - \frac{a\lambda_h (Z(0))^{1-p}}{b - p-1}\) is positive, we deduce that \(T_h^* \leq \frac{\pi}{2a\lambda_h}\). Consequently \(T_h^* = T_h\) is finite. Use the fact that \(Z(0) = A\) to complete the rest of the proof.

Now, we consider the following initial-boundary value problem

\[u_t - iau_{xx} = b|u|^p, x \in (0, 1), t \in (0, T)\]

\[u(0, t) = 0, u(1, t) = 0, t \in (0, T)\]

\[u(x, 0) = u_0(x), x \in [0, 1]\]

where \(p > 1\), \(u_0(0) = 0\) and \(u_0(1) = 0\).

Approximate the solution \(u\) of (9)—(11) by the solution \(u_k(t) = (U_k(t), \ldots, U_l(t))^T\) of the following semidiscrete equations

\[
\frac{d}{dt} U_j(t) = i\alpha \delta^2 U_j(t) + b|U_j(t)|^p, 1 \leq j \leq l - 1, t \in (0, T)\]

\[U_l(t) = 0, U_j(t) = 0, t \in (0, T)\]

\[U_j(0) = \varphi_j, 0 \leq j \leq l\]

where \((0, T)\) is the maximal time interval on which \(\|U_k(t)\|_\infty\) is finite. Our second result on blow-up is the following.

Theorem 2.2 Assume that \(\alpha = \frac{a\lambda_h A^{1-p}}{b(p-1)} \leq \frac{1}{2}\) where

\[\lambda_h = \frac{2 - 2 \cos nh}{h^2}\]

and
Then the solution \( U_h \) of (12)—(14) blows up in a finite time \( T_h \) which is estimated as follows

\[
T_h \leq \frac{1}{a\lambda_h} \arcsin \left( 1 - \frac{a\lambda_h A^{1-p}}{b(p-1)} \right)
\]

Proof. Since \((0, T_h)\) is the maximal time interval on which \( \| U_h(t) \|_\infty \) is finite, our aim is to show that \( T_h \) is finite and obeys the above inequality. Introduce the functions \( v \) and \( w \) defined as follows

\[
v(t) = \sum_{j=1}^{l-1} \tan \left( \frac{\pi h}{2} \right) \sin(j\pi h) U_j(t)
\]

and

\[
w(t) = \sum_{j=1}^{l-1} \tan \left( \frac{\pi h}{2} \right) \sin(j\pi h) \bar{U}_j(t).
\]

Taking the derivative of \( v \) in \( t \) and using (12), we get

\[
v'(t) = ia \sum_{j=1}^{l-1} \tan \left( \frac{\pi h}{2} \right) \sin(j\pi h) U_j(t) + b \sum_{j=1}^{l-1} \tan \left( \frac{\pi h}{2} \right) \sin(j\pi h) |U_j(t)|^p,
\]

which implies that

\[
d \left( e^{ia\lambda_h t} v(t) \right) = be^{ia\lambda_h t} \sum_{j=1}^{l-1} \tan \left( \frac{\pi h}{2} \right) \sin(j\pi h) |U_j(t)|^p.
\]

We also observe that, taking the derivative of \( w \) in \( t \) and using (12), we have

\[
w'(t) = -ia \sum_{j=1}^{l-1} \tan \left( \frac{\pi h}{2} \right) \sin(j\pi h) \delta^2 \bar{U}_j(t) + b \sum_{j=1}^{l-1} \tan \left( \frac{\pi h}{2} \right) \sin(j\pi h) |U_j(t)|^p
\]

Reasoning as above, we find that

\[
d \left( e^{-ia\lambda_h t} w(t) \right) = be^{-ia\lambda_h t} \sum_{j=1}^{l-1} \tan \left( \frac{\pi h}{2} \right) \sin(j\pi h) |U_j(t)|^p.
\]

We deduce that

\[
Z'(t) = b \cos(a\lambda_h t) \sum_{j=1}^{l-1} \tan \left( \frac{\pi h}{2} \right) \sin(j\pi h) |U_j(t)|^p,
\]

where \( Z(t) = e^{ia\lambda_h t} w(t) + e^{-ia\lambda_h t} w(t) \). Arguing as in the proof of Theorem 2.1, we deduce that

\[
Z'(t) \geq b \cos(a\lambda_h t) (Z(t))^p \quad \text{for } t \in \left(0, \frac{\pi}{a\lambda_h}\right),
\]

which implies that

\[
\frac{dZ}{Z^p} \geq b \cos(a\lambda_h t) dt \quad \text{for } t \in \left(0, \frac{\pi}{a\lambda_h}\right).
\]

Let \( T_h^* = \min \left( \frac{\pi}{2a\lambda_h}, T_h \right) \). Integrating this inequality over \((0, T_h^*)\), we obtain

\[
\frac{(Z(0))^{1-p}}{p-1} \geq \frac{b}{a\lambda_h} (\sin(a\lambda_h T_h^*)),
\]

which implies that

\[
\sin(a\lambda_h T_h^*) \leq \frac{a\lambda_h (Z(0))^{1-p}}{b}.
\]

We deduce that

\[
T_h^* \leq \frac{1}{a\lambda_h} \arcsin \left( \frac{a\lambda_h (Z(0))^{1-p}}{b} \right)
\]

Since \( \frac{a\lambda_h (Z(0))^{1-p}}{p-1} \leq \frac{1}{2} \) we have \( T_h^* \leq \frac{\pi}{2a\lambda_h} \). This implies that \( T_h^* = T_h \) is finite. Therefore \( T_h \) is finite and use the fact that \( Z(0) = A \) to complete the rest of the proof.

Remark 2.1 Consider the following initial-boundary value problem

\[
u_t - i a u_{xx} = (c - ib) |u|^p, x \in (0, 1), t \in (0, T)
\]

\[
u(0, t) = 0, u(1, t) = 0, t \in (0, T)
\]

\[
u(x, 0) = u_0(x), x \in [0, 1]
\]

where \( c > 1, b > 0 \) and approximate the solution \( u \) of (16)—(18) by the solution \( U_j(t) = (U_0(t), ..., U_{j-1}(t), U_j(t), ..., U_{l-1}(t)) \) of the following semidiscrete equations

\[
\frac{dU_j(t)}{dt} = ia\delta^2 U_j(t) + (c - ib) |U_j(t)|^p, 1 \leq j \leq l - 1, t \in (0, T)
\]

\[
U_0(t) = 0, U_{j-1}(t) = 0, t \in (0, T)
\]

\[
U_j(0) = \varphi_j, 0 \leq j \leq l
\]

Combining the methods developed in the proofs of Theorems 2.1 and 2.2, we easily prove that if \( \sum_{j=1}^{l-1} \tan \left( \frac{\pi h}{2} \right) \sin(j\pi h) \Re(\varphi_j) \) is large enough, the solution \( U_j(t) \) of the above semidiscrete problem blows up in a finite time.
3. Numerical Results

In this section, one present some numerical approximations of the blow-up time for the solution of the problem (1)-(3). Consider the following explicit and implicit schemes

Scheme I

\[
\frac{U_{j}^{(n+1)} - U_{j}^{(n)}}{\Delta t_{n}} = ia \left( U_{j+1}^{(n)} - 2U_{j}^{(n)} + U_{j-1}^{(n)} \right) / h^2 - ib|U_{j}^{(n)}|^p, \quad 1 \\
\leq i \leq l - 1,
\]

\[
U_{0}^{(n)} = 0, U_{l}^{(n)} = 0,
\]

\[
U_{j}^{(0)} = \varphi_{j}, 0 \leq i \leq l.
\]

Scheme II

\[
\frac{U_{j}^{(n+1)} - U_{j}^{(n)}}{\Delta t_{n}} = ia \left( U_{j+1}^{(n+1)} - 2U_{j}^{(n+1)} + U_{j-1}^{(n+1)} \right) / h^2
\]

\[
- ib|U_{j}^{(n)}|^p, \quad 1 \leq i \leq l - 1,
\]

\[
U_{0}^{(n+1)} = 0, U_{l}^{(n+1)} = 0,
\]

\[
U_{j}^{(0)} = \varphi_{j}, 0 \leq i \leq l,
\]

where \( n \geq 0, \Delta t_{n} = \min \left\{ \frac{h^2}{2|a|}, \tau \right\} \left\| U_{h}^{(n)} \right\|_{\infty}^{1-p} \) with \( \tau = \text{const} \in (0,1) \). We need the following definition.

**Definition 3.1** One say that the solution \( U_{h}^{(n)} = (U_{0}^{(n)}, ... , U_{l}^{(n)}) \) of Scheme I or II blows up in a finite time if \( \lim_{n \to \infty} \left\| U_{h}^{(n)} \right\|_{\infty} = \infty \) and the series \( \sum_{n=0}^{\infty} \Delta t_{n} \) converges. The quantity \( \sum_{n=0}^{\infty} \Delta t_{n} \) is called the numerical blow-up time of \( U_{h}^{(n)} \).

In the tables 1 and 2 in rows, we present the numerical blow-up times, the numbers of iterations, the CPU times and orders of the approximations corresponding to meshes of 16, 32, 64, and 128. We take the numerical blow-up time \( t_{n} = \sum_{n=1}^{1} \Delta t_{j} \) which is computed at the first time when \( \Delta t_{n} = |t_{n+1} - t_{n}| \leq 10^{-16} \). The order (s) of the method is computed from

\[
S = \frac{\log((T_{4n} - T_{2n})/(T_{2n} - T_{h}))}{\log(2)}.
\]

For the numerical values, we take \( p=2 \), \( U_{j}^{(0)} = 20 \sin(\pi j h) \) \( a=1, b=1 \) and \( \tau = h^{3/2} \).

**Table 1.** Numerical blow-up times, numbers of iterations, CPU times (seconds) and orders of the approximations obtained with Scheme I.

| I  | \( t_{n} \)  | N    | CPUt | S   |
|----|----------------|------|------|-----|
| 16 | 0.078223       | 18837| -    | -   |
| 32 | 0.078229       | 72612| 4    | -   |
| 64 | 0.078232       | 279341| 45  | 1.00|
| 128| 0.078233       | 6962549 | 14611| 1.58|

In this graphics, one can see that the norm of the solution \( u \) of the problem (1)—(3) is increasing and develops a singularity in a finite time. Also, we see that the blow-up rate occurs at the middle of the solution for the mesh \( i=1/2 \). This graphics respect \( U_{0}(t) = 0, U_{l}(t) = 0, t \in (0, T_{h}) \). But this condition doesn’t prevent the blow-up of the solution.

**Table 2.** Numerical blow-up times, numbers of iterations, CPU times (seconds) and orders of the approximations obtained with Scheme II.

| I  | \( t_{n} \)  | N    | CPUt | S   |
|----|----------------|------|------|-----|
| 16 | 0.078280       | 14807| 1    | -   |
| 32 | 0.078244       | 56510| 6    | -   |
| 64 | 0.078236       | 214935| 95  | 2.1 |
| 128| 0.078234       | 6962549 | 14611| 2.0 |

4. Conclusion

Under some assumption, and using a method based on a modification of the method of Kaplan, it is show that the semidiscrete solution of the semilinear solution of the problem (1)—(3) blows up in a finite time and the semidiscete blow-up time is estimate. The result obtains with the problem (1)—(3) is generalize considering a reaction term more complex. At the end, two schemes proposed, permit to illustrate the estimation of the numerical blow-up time which converge to 0.0782 (see Tables 1 and 2). But the convergence of the schemes proposed was not proof and can be the subject of another investigation.

**References**

[1] R. Adami, G. D. Antonio, R. Figari and A. Teta, Blow-up solution for the Schrödinger equation in dimension three with a concentration nonlinearity, Ann. I. H. Poincaré, 21 (2004), 121-137.

[2] A. Ambrosetti, E. Colorado, Standing waves of some coupled nonlinear Schrödinger equations, J. London. Math. Soc., 75 (2007), 67-82.

[3] G. D Akrivis, V. A. Dougalis, O. A. Krakashian and W. R. McKinney, Numerical approximation of blow-up of radially symmetric solutions of the nonlinear Schrödinger, SIAM. J. Sci. Comput., 25 (2003), 186-212.

[4] O. Bang, P. L. Christiansen, K. O. Rasmussen and Yu. B. Gaididei, The role of nonlinearity in modeling energy transfer in scheibe aggregates in nonlinear excitation in Bimoleculare, M. Peyrard, ed., Les Editions de Physique, Springer, Berlin, (1995), 317-336.
[5] C. Besse, N. J. Mauser, H. P. Stimming, Numerical study of the Davey-Stewartson system, Modélisation Mathématique et Analyse Numérique, Tome 38 (2004) no. 6, p. 1035-1054.

[6] T. K. Boni and F. K. Ngohisse, Numerical blow-up for a nonlinear heat equation, In Acta Math. Sinica English serie, Vol. 27 (5) (2011), 845-862.

[7] J. Colliander, M. Grillakis, and N. Tzirakis, Remarks on global a priori estimates for the nonlinear Schrödinger equation, Proc. Amer. Math. Soc. 138 (2010), no. 12, 4359-4371.

[8] W. Dai, An unconditionally stable three-level explicit difference scheme for the Schrödinger equation with a variable coefficient, SIAM. J. Numer. Anal., 29 (1992), 174-181.

[9] V. D. Dinh, Blowup of $H^1$ solutions for a class of the focusing inhomogeneous nonlinear Schrödinger equation, Nonl. Analysis, 174 (2018), 169-188.

[10] W. Dai, Absolute stable explicit and semi-explicit schemes for Schrödinger equations, Math. Num. Sinica, 11 (1989), 128-131.

[11] V. V. Drits, Conservative difference schemes for problems of nonlinear optics, 1, Differential Equation, 27 (1991), 1153-1161.

[12] R. ~ Ferreira, P. ~ Groisman and J. ~ D. ~ Rossi, Numerical blow-up for a nonlinear problem with a nonlinear boundary condition, Math. Models Methods. Appl. Sci., 12 (2002), 461-483.

[13] F. Ivanauskas and M. Radzianas, On convergence and stability of the explicit difference method for solution of nonlinear Schrödinger equations, SIAM J. Numer. Anal., 36 (1999), 1466-1481.

[14] S. Kaplan, On the growth of solutions of quasilinear parabolic equations, Communs pure Appl. Math., 16 (1963), 305-330.

[15] O. Kavian, A remark on the blowing up of solutions to the Cauchy problem for nonlinear Schrödinger equations, Trans. Amer. Math. Soc., 193 (1987), 193-203.

[16] S. Klainerman and G. Ponce, Global, small amplitude solutions to nonlinear evolution equations, Communs pure Appl. Math., 36 (1983), 133-141.

[17] Z. Lu and Z. Liu, $L^2$-concentration of blow-up solutions for two-coupled nonlinear Schrodinger equations, J. Math. Anal. Appl., 380 (2) (2011), 531-539.

[18] F. Merle, P. Raphael, Profiles and quantization of the blow up mass for critical nonlinear Schrödinger equation, Comm. Math. Phys., 235 (2005), 675-704.

[19] T. Nakagawa, Blowing up on the finite difference solution to parabolic equation, Appl. Math. Optim., 2 (1976), 337-350.

[20] K. Nguesan, N. Diabate and A. K. Toure, Blow-up for semidiscretization of some nonlinear parabolic equation with a convection term, In J. of progressive reachach in Math. Vol. 5 (2) (2015), 499-518.

[21] B. E. A. Saleh and M. C. Teich, Fundamentals of fotonics, Interscience Publication, New-York., (1991).

[22] T. R. Taha, A numerical scheme for the nonlinear Schrödinger equation, Comput. Math. Optim., 2 (1976), 337-350.

[23] Y. Tourigny and J. M. Sanz Serna, The numerical study of blow-up with application to a nonlinear Schrödinger equation, J. Comp. Phys., 102 (1992), 407-416.

[24] M. Tsutsumi, On global solutions to the initial-boundary value problem for the nonlinear Schrödinger equations in exterior domains, Comm. Part. Diff. Equat., 16 (1991), 885-907.

[25] L. Wu, Dufort-Frankel-type methods for linear and nonlinear Schrödinger equations, SIAM. J. Numer. Anal., 33 (1996), 1526-1533.

[26] J. Zhang, On an initial-boundary value problem for a class of nonlinear Schrödinger equations, Comm. Part. Diff. Equat., 21 (1996), 687-692.

[27] S. Zhu, Blow-up solutions for inhomogeneous Schrodinger equation with $L^2$ supercritical nonlinearity, J. Math. Anal. Appl., 409 (2) (2014), 760-776.

[28] V. E. Zakharov and A. B. Shabat, An $L^{-}$ Exact theory of two dimensional self focusing and one dimensional self modulation of waves in nonlinear media, Soviet Physics JETP., 34 (1972), 62-69.