A Consistent Modeling of Neutrino-driven Wind with Accretion Flow onto a Protoneutron Star and its Implications for $^{56}$Ni Production

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ABSTRACT

Details of the explosion mechanism of core-collapse supernovae (CCSNe) are not yet fully understood. There is now a growing number of examples of reproducing explosions in first-principles simulations, which have shown a slow increase of explosion energy. However, it was recently pointed out that the growth rates of the explosion energy of these simulations are insufficient to produce enough $^{56}$Ni mass to account for observations. We refer to this issue as the ‘nickel mass problem’ (Ni problem, hereafter) in this paper. The neutrino-driven wind is suggested as one of the most promising candidates for the solution to the Ni problem in literature. In this paper, we first built a consistent model of the neutrino-driven wind with an accretion flow onto a protoneutron star (PNS), by connecting a steady-state solution of the neutrino-driven wind and a phenomenological mass accretion model. Comparing the results of our model with the results of first-principles simulations, we find that the total ejectable amount of the neutrino-driven wind is roughly determined within $\sim 1$ sec from the onset of the explosion and the supplementable amount at a late phase ($t_e \gtrsim 1$ sec) remains $M_{ej} \lesssim 0.01 M_\odot$ at most. Our conclusion is that it is difficult to solve the Ni problem by continuous injection of $^{56}$Ni by the neutrino-driven wind. We suggest that the total amount of synthesized $^{56}$Ni can be estimated robustly if simulations are followed up to $\sim 2$ seconds.

Keywords: supernovae: general

1. INTRODUCTION

Core-collapse supernovae (CCSNe) occur at the end of the lives of massive stars, lead to the birth of neutron stars and stellar black holes, and are the production sites of many elements. Details of the explosion mechanism of CCSNe, however, are not yet fully understood. The most promising scenario is the delayed neutrino driven explosion (Bethe & Wilson 1985). On the theoretical studies for the explosion mechanism, there is now an increasing number of examples of reproducing explosions in first-principles simulations. They solve multidimensional hydrodynamics equations, as well as a detailed neutrino transport (see, e.g., Janka 2012, and references therein). Most, if not all, of those state-of-the-art simulations, have shown a slow increase of explosion energy, and the growing rate of the explosion energy is typically $O(0.1)$ Bethe s$^{-1}$ (1 Bethe = $1 \times 10^{51}$ erg), especially for 3D simulations. On the other hand, one of the observational constraints is the amount of $^{56}$Ni synthesized in an explosion, which is very sensitive to the explosion properties and the progenitor core structure, which is very sensitive to the explosion properties and the progenitor core structure (see e.g. Ugliano et al. 2012; Ertl et al. 2016; Sukhbold et al. 2016; Suwa et al. 2019). The amount of $^{56}$Ni has been measured from many SNe through light curves with reasonable accuracy (see e.g. Hamuy 2003). A typical amount of $^{56}$Ni obtained for well-studied SNe is on average $\sim 0.07 M_\odot$ (e.g., SN 1987A, SN 1994I, SN 2002ap; Arnett et al.
of CCSNe has also suggested that the amount of \(^{56}\text{Ni}\) is around \(\sim 0.07M_{\odot}\) at the median (Prentice et al. 2019). It means that, in a canonical CCSNe, on average \(\sim 0.07M_{\odot}\) of \(^{56}\text{Ni}\) should be synthesized. However, recent studies have shown that to reproduce the typical mass \(0.07M_{\odot}\) of \(^{56}\text{Ni}\) by the explosive nucleosynthesis in the ejecta, the growth rate of the explosion energy of \(\mathcal{O}(1)\) Bethe s\(^{-1}\) is required (Suwa et al. 2019; Sawada & Maeda 2019). In other words, the growth rate of the explosion energy of \(\mathcal{O}(0.1)\) Bethe s\(^{-1}\), which is obtained in current typical explosion simulations, is insufficient to produce enough \(^{56}\text{Ni}\) mass. We refer to this issue as the ‘nickel mass problem’ (Ni problem, hereafter) in this paper. While we should note that some models in first-principles simulations have succeeded in producing sufficient amounts of \(^{56}\text{Ni}\) (e.g., Bruenn et al. 2016; Eichler et al. 2018; Burrows et al. 2020), it is unclear whether we can reproduce sufficient \(^{56}\text{Ni}\) amount as a canonical nature.

The neutrino-driven wind, which is subject of this study, is thought to be one of the most promising candidates for the solution to the Ni problem (e.g., Wongwathanarat et al. 2017; Wanajo et al. 2018). The neutrino-driven wind is a phenomenon that blows out of the surface of the proto-neutron star (PNS) after the evacuation of the early ejecta in CCSNe. This wind may solve the issue for the following two reasons. First, the wind can provide \(^{56}\text{Ni}\) in addition to the hydrostatic and explosive nucleosynthesis, since it continues until \(\sim 10\) s after an explosion, which is longer than the converging time of the explosive nucleosynthesis (\(\sim 1\) s). Second, recent detailed simulations have predicted proton-rich ejecta in the post-explosion winds (e.g., Bruenn et al. 2016) and also have indicated that almost all materials come to \(^{56}\text{Ni}\) in the wind with \(Y_e \gtrsim 0.5\) (Wanajo et al. 2018). But recent long-term spherical simulations of the PNS cooling phase show the rapidly decreasing neutrino luminosities and insufficient mass ejection (Fischer et al. 2010; Hüdepohl et al. 2010; Wanajo 2013) so that we may need multi-dimensional simulations, which take into account the mass accretion onto a PNS and ejection by the wind simultaneously, to solve this issue. The problem of these multi-D simulations is that they have a computational time limitation and the possibility of the solution now lies in the phase later than the typical computational time.

In this paper, we investigate the potential of the neutrino-driven wind to solve Ni problem, especially at later phases than a few seconds after a successful explosion. For this purpose, we first build a consistent model of the neutrino-driven wind with the accretion flow onto the PNS, by connecting a steady-state solution of the neutrino-driven wind and a phenomenological mass accretion. We then compare the results of our model with the results of first-principles simulations and discuss the possibilities to solve the Ni problem. In Section 2, we describe the treatment of three important equations in our modeling: the neutrino-driven winds, mass accretion flows onto the PNS, and combining them. Our results are given in Section 3 before summary in Section 4.

2. MODELS

In this section, we aim to build a consistent model of the neutrino-driven wind with the accretion flow onto the PNS, as illustrated in Fig. 1. It is known that the neutrino-driven wind are very successfully described by the steady-state semi-analytical solutions (e.g., Qian & Woosley 1996; Wanajo et al. 2001; Thompson et al. 2001; Wanajo 2013; Bliss et al. 2018). To build this consistent model, we first solve the steady-state equations of the wind and derive relations between the wind \(M_{\text{wind}}\) and the PNS profiles; the neutrino luminosities \(L_{\nu}\), the gain radius \(R_{\text{gain}}\) and the PNS mass \(M_{\text{PNS}}\) (§2.1). The next step is to formulate the phenomenological accretion flow model \(M_{\text{acc}}\) (§2.2). We then model the evolution of the gain radius \(R_{\text{gain}}\) and the neutrino luminosity \(L_{\nu}\) with accretion rates \(M_{\text{acc}}\) and PNS masses \(M_{\text{PNS}}\), and finally derive the wind model with an accretion flow, taking into account geometric effects (§2.3).

2.1. Steady-state wind model

In this study, we use the spherically symmetric and general relativistic semi-analytic wind model in Wanajo (2013). Previous works studied the physical state of the neutrino wind for a wide range of neutron star masses and neutrino luminosities. Based on these results, they then studied the behavior of the neutrino-driven wind with the PNS evolution was studied by stitching this semi-analytic wind model (e.g., Otsuki et al. 2000; Bliss et al. 2018). Based on this result, the behavior of the neutrino-driven wind with the PNS evolution was studied by stitching this semi-analytic wind model (e.g., Wanajo et al. 2001; Wanajo 2013).

The basic equations to describe the spherically-symmetric and steady-state winds in the Schwarzschild geometry are

\[
\dot{M} = 4\pi r^2 \rho v, \quad (1)
\]
\[
\frac{dv}{dr} = -\frac{1 + (v/c)^2 - (2GM/c^2r)P}{\rho(1 + \epsilon/c^2) + P/c^2} - \frac{GM_{\text{PNS}}}{r^2}, \quad (2)
\]
\[
\dot{Q} = v \left( \frac{dv}{dr} - P \frac{d\rho}{\rho^2 dr} \right), \quad (3)
\]
in which we assume electron/positron captures are in equilibrium and an initial composition consists mainly of neutrons and protons. Then, we get

\[ Y_e = \left[ 1 + \frac{L^\nu_{\nu_e} \langle \sigma_{\nu_e} \rangle}{L^\nu_{\nu_e} \langle \sigma_{\nu_e} \rangle} \right]^{-1} = 0.5, \]

where \( L^\nu_{\nu_e} = L_{\nu_e}/(E_{\nu_e}) \) is the number luminosity and is assumed to be the same for electron neutrinos and antineutrinos. The cross-sections of electron neutrino absorption at neutrons (\( \langle \sigma_{\nu_e} \rangle \)) and electron anti-neutrino absorption at protons (\( \langle \sigma_{\nu_e} \rangle \)) depend on the average neutrino and anti-neutrino energies. Thus, given \( L^\nu_{\nu_e}, (E_{\nu_e}) \) and \( Y_e \) as parameters, Eq. (4) and the assumption of \( L^\nu_{\nu_e} = L^\nu_{\bar{\nu}_e} \) leads to the anti-neutrino energy and luminosity. With respect to the choice of \( Y_e \), we are interested in an environment that maximizes the production of \(^{56}\text{Ni}\). Previous studies have shown that the abundance of \(^{56}\text{Ni}\) is extremely suppressed at \( Y_e < 0.5 \) in nuclear statistical equilibrium (NSE) (Seitenzahl et al. 2008), and the same behavior is known to occur in the neutrino-driven wind environment (Wanajo et al. 2018). In order to focus on maximizing \(^{56}\text{Ni}\) production, we here fix \( Y_e \) value to be 0.5 throughout this paper.

The mass outflow rate \( \dot{M} \) determines how much material is ejected by the neutrino-driven wind. The solutions of Eqs. (1)–(3) depend on this mass outflow rate \( \dot{M} \) (Qian & Woosley 1996). Note that \( \dot{M} = \dot{M}_{\text{tran}} \) corresponds to the transonic solution, which gives the maximum mass ejection. For instance, for large enough mass outflow (\( \dot{M} > \dot{M}_{\text{tran}} \)), one has no physical solution since the mass outflow experiences an infinite acceleration. The so-called breeze (or subsonic) solutions are found for \( \dot{M} < \dot{M}_{\text{tran}} \). In the following, we focus our discussion on transonic solutions which give the maximum ejected amount of the wind.

Regarding the inner boundary condition, we assume, for simplicity, that the neutrino-driven wind starts at the gain radius \( R_{\text{gain}} \), which the heating and cooling due to neutrino interactions are in equilibrium (\( Q \approx 0 \)). It is because the wind blows from the heating region outside the gain radius. It allows us to determine the temperature of the inner boundary, \( T_0 \). While the inner boundary is set to be \( \rho_0 = 10^{10} \text{ g cm}^{-3} \) in the ordinary neutrino-driven wind models (e.g., Otsuki et al. 2000; Wanajo et al. 2001; Wanajo 2013), we set the density at the inner boundary in our model to \( \rho_0 = 10^{10} \text{ g cm}^{-3} L_{\nu_e,51}^{1/2} \), following the method discussed by Fujibayashi et al. (2015), in order to solve with the very high neutrino luminosity. Given the density of the inner boundary \( \rho_0 \), Eq. (1) determines the initial velocity \( v_0 \) at \( r = R_{\text{gain}} \) for each \( \dot{M} \).
we consider to model the relation between the maximum mass ejection of the wind $\dot{M}_{\text{wind,iso}}$ and the three PNS parameters. Figures 2 illustrate mass ejection rates $\dot{M}_{\text{wind,iso}}$ as a function of each PNS parameters. We confirmed that our results are in close agreement with the results of the previous 1D numerical (Sumiyoshi et al. 2000) and semi-analytical studies (Otsuki et al. 2000; Wanajo et al. 2001) with the same set of parameters, except for the treatment of $Y_e$.

It is found that ejection rates $\dot{M}_{\text{wind,iso}}$ have a relation with the PNS parameters, which is approximated by a power-law function as

$$\dot{M}_{\text{wind,iso}} \approx 8.3 \times 10^{-3} M_\odot s^{-1} \left( \frac{L_{\nu_e}}{10^{52} \text{erg s}^{-1}} \right)^{\alpha} \left( \frac{R_{\text{gain}}}{4 \times 10^6 \text{cm}} \right)^{\beta} \left( M_{\text{PNS}} / 1.4 M_\odot \right)^{\gamma}, \tag{5}$$

where the index we adopted is as follows; $\alpha = 7/4$, $\beta = 5/2$ and $\gamma = -7/2$. To credit this modeling, we should mention two points in comparison to previous studies. First, our model focuses on a mainly larger region for the gain radius $R_{\text{gain}}$ ($10 - 60$ km), compared to previous studies ($10 - 30$ km; Qian & Woosley 1996; Otsuki et al. 2000). We confirmed that almost the same relation can be found as in Qian & Woosley (1996) when we focus on the same parameter region of $R_{\text{gain}}$ as the literature. Second, while the result of Wanajo et al. (2001) tends to deviate the power-law relation at high luminosity, our solutions do not. It is due to our treatment of the density at the inner boundary (gain radius) that follows the method of Fujibayashi et al. (2015). This method takes into account the physical equilibrium conditions, especially at high neutrino luminosity, which is different from those of the Wanajo et al. (2001). We also confirmed that the same trend as in Wanajo et al. (2001) when using the same boundary conditions.

Since we employ this power-law relation to construct our wind model in a later section, here we discuss the error between this equation and the numerical solution. Table 1 shows the wind solution from the numerical calculations ($\dot{M}_{\text{wind,cal}}$), the value estimated from the power-law relation ($\dot{M}_{\text{wind,model}}$), and the error of $\dot{M}_{\text{wind,cal}}$ with respect to $\dot{M}_{\text{wind,model}}$, for typical values of each PNS parameters. The error values in Table 1 indicate that our subsequent analytical discussion using the power-law relation keep an error within 30% from the more precise numerical calculations.

### 2.2. Mass accretion model

In this section, we construct a phenomenological mass accretion model based on Müller et al. (2016). We assume that matter reaches on the PNS with a free-fall
Table 1. The wind solution from the numerical calculations \(\dot{M}_{\text{wind,cal}}\), the value estimated from the power-law relation \(\dot{M}_{\text{wind,model}}\), and the error of \(\dot{M}_{\text{wind,cal}}\) with respect to \(\dot{M}_{\text{wind,model}}\), for typical values of each PNS parameters.

| \(L_\nu\) | \(R_{\text{gain}}\) | \(M_{\text{PNS}}\) | \(\dot{M}_{\text{wind,cal}}\) | \(\dot{M}_{\text{wind,model}}\) | error \(^a\) |
|---|---|---|---|---|---|
| \(10^{51}\) erg s\(^{-1}\) | \(10^{40}\) km | \(M_\odot\) | \(M_\odot\) s\(^{-1}\) | \(M_\odot\) s\(^{-1}\) | \% |
| 10 | 40 | 1.4 | \(8.25 \times 10^{-3}\) | \(8.25 \times 10^{-3}\) | 0.0 |
| 100 | 40 | 1.4 | \(5.98 \times 10^{-1}\) | \(4.64 \times 10^{-1}\) | 28.9 |
| 1 | 40 | 1.4 | \(1.23 \times 10^{-4}\) | \(1.47 \times 10^{-4}\) | -16.3 |
| 10 | 50 | 1.4 | \(1.79 \times 10^{-2}\) | \(1.44 \times 10^{-3}\) | 24.4 |
| 10 | 10 | 1.4 | \(3.00 \times 10^{-4}\) | \(2.58 \times 10^{-3}\) | 16.2 |
| 10 | 40 | 2.0 | \(2.40 \times 10^{-3}\) | \(2.37 \times 10^{-3}\) | 1.3 |
| 10 | 40 | 1.2 | \(1.54 \times 10^{-2}\) | \(1.42 \times 10^{-2}\) | 8.5 |

Note—\(^a\)We denote the difference between the value of the numerical solution to the model value, normalized by the model value \(\frac{\dot{M}_{\text{wind,cal}} - \dot{M}_{\text{wind,model}}}{\dot{M}_{\text{wind,model}}}\), as a percentage.

Figure 3 shows the mass accretion rates \(\dot{M}_{\text{acc,iso}}\) as a function of \(t\), which is identified with \(t_f\). In our model, the time origin is when the neutrino-driven wind starts to blow, corresponding to the time of the SN shock revival, which is given by the time at the mass shell of \(M_{s=4k_B}\) accreting onto the PNS. Here \(M_{s=4k_B}\) gives then mass coordinate with the entropy being \(4k_B\) baryon\(^{-1}\). This is because recent hydrodynamics simulations show that the shock launch takes place when a mass element with \(s = 4k_B\) baryon\(^{-1}\) accretes onto the shock (Ertl et al. 2016; Suwa et al. 2016). Gray lines are the mass accretion rates of progenitor stars from Sukhbold et al. (2018) (on \(0.1 M_\odot\) steps over the range \(M_{\text{ZAMS}} = 12.0 - 20.0 M_\odot\)). We approximate our accretion model as the red line, which is given by

\[
\dot{M}_{\text{acc,iso}}(t) = \dot{M}_{\text{acc,0}} \left(\frac{t}{t_0} + 1\right)^{-2},
\]

where \(\dot{M}_{\text{acc,0}}\) and \(t_0\) are free parameters.

2.3. Consistent wind model with mass accretion

Our description above is based on a one-dimensional radial flow. In order to construct a consistent model of the wind with mass accretion, it is necessary to take into account the geometric structure. Hereafter, we present the collimation of the wind and the accretion flow due to the asymmetric structure of the supernova explosion with a geometrical factor \(f_\Omega\) \((\leq 1)\). \(f_\Omega\) relates the wind intrinsic properties and isotropic equivalents as

\[
\dot{M}_{\text{wind}} = f_\Omega \dot{M}_{\text{wind,iso}},
\]

\[
\dot{M}_{\text{acc}} = (1 - f_\Omega) \dot{M}_{\text{acc,iso}}.
\]

We next rewrite the outflow rate \(\dot{M}_{\text{wind}}\), which is obtained for a given set of the three parameters \((L_\nu, R_{\text{gain}},\)
$M_{\text{PNS}}$ in Section 2.1, to the equation which is approximately determined by a given set of the two parameters ($M_{\text{acc}}, M_{\text{PNS}}$). We need the model of the gain radius $R_{\text{gain}}$ and the neutrino luminosity $L_{\nu}$ as function of $M_{\text{acc}}$ and $M_{\text{PNS}}$. Müller et al. (2016) found that, in the condition of $M_{\text{acc}} \gg 10^{-3} M_{\odot} s^{-1}$, the gain radius $R_{\text{gain}}$ can be described as

$$R_{\text{gain}} \approx 40 \text{ km} \left( \frac{M_{\text{acc}}}{0.1 M_{\odot} s^{-1}} \right)^{1/3} \left( \frac{M_{\text{PNS}}}{1.4 M_{\odot}} \right)^{-1}. \quad (11)$$

This approximation has been confirmed to show reasonably consistent results with the contraction of the PNS in hydrodynamics simulations.

In our system, the neutrino luminosity $L_{\nu}$ is assumed to be dominated by accretion luminosity $L_{\nu, \text{acc}}$. The accretion luminosity $L_{\nu, \text{acc}}$ is roughly given by the mass accretion rate $M_{\text{acc}}$ and the gravitational potential at the neutron star surface (Fischer et al. 2009),

$$L_{\nu} \approx L_{\nu, \text{acc}} = \frac{G M_{\text{PNS}} \dot{M}_{\text{acc}}}{R_{\text{PNS}}}, \quad (12)$$

where $R_{\text{PNS}} = 5 R_{\text{gain}}/7$ (Müller et al. 2016) and $\eta$ is an efficiency parameter, which specifies the conversion of accretion energy into neutrino luminosity. Note that the neutrino luminosity includes two components: accretion luminosity and diffusion luminosity (Fischer et al. 2009). It is difficult to completely separate these components. By taking into account the contribution of the diffusion luminosity $L_{\nu, \text{diff}}$, $\eta$ would exceed unity (see Müller & Janka 2014). In this study, we calibrate our model with the electron neutrino luminosity at $\sim 1$ sec after the SN shock revival, which is well studied in the first-principles calculations of CCSN explosions (Müller & Janka 2014), and use $\eta = 1$.

From Eqs. (5)–(10), we can write the ejection rate of the neutrino-driven wind with the accretion flow onto the PNS as

$$M_{\text{wind}} \approx 1.3 \times 10^{-2} M_{\odot} s^{-1} \times f_{\Omega} \left( \frac{1 - f_{\Omega})M_{\text{acc,iso}}}{0.1 M_{\odot} s^{-1}} \right)^{2 \alpha + \beta} \left( \frac{M_{\text{PNS}}}{1.4 M_{\odot}} \right)^{2 \alpha - \beta + \gamma}, \quad (13)$$

where the value of the indices estimated by adopting our model are $2 \alpha + \beta = 2$ and $2 \alpha - \beta + \gamma = -5/2$.

3. TOTAL MASS EJECTION BY WINDS

3.1. Possible Contribution to the Ni problem

In this section, we describe a potential of the neutrino-driven wind to solve the Ni problem. A summary of the claim of the Ni problem is that while on average $\sim 0.07 M_{\odot}$ of $^{56}$Ni should be synthesized in a canonical CCSNe, the first-principles simulations have been able to reproduce less than half of it. Our purpose is to investigate whether the wind can eject $0.07 M_{\odot}$ of $^{56}$Ni or not.

In the present study, by assuming $Y_c = 0.5$ as described in §2.1, we consider all material ejected by the wind to be $^{56}$Ni. Namely, we estimate the ejectable maximum amount (which corresponds directly to the maximum amount of $^{56}$Ni) by integrating the mass ejection rate of our wind model.

We integrate Eq. (13) with Eq. (8) from the shock revival time ($t = 0$) as follows,

$$M_{\text{ej}, \infty} = \int_0^\infty dt \dot{M}_{\text{wind}}(t)$$

$$\approx 1.3 \times 10^{-2} M_{\odot} s^{-1} \times f_{\Omega} \left( \frac{1 - f_{\Omega})M_{\text{acc,iso}}}{0.1 M_{\odot} s^{-1}} \right)^2 \times \left( \frac{M_{\text{PNS}}}{1.4 M_{\odot}} \right)^{-5/2} \int_0^\infty dt \left( \frac{t}{t_0} + 1 \right)^{-4} \times f_{\Omega} \left( \frac{t_0}{1 \text{ s}} \right)^2 \left( \frac{1 - f_{\Omega})M_{\text{acc,iso}}}{0.1 M_{\odot} s^{-1}} \right)^2 \left( \frac{M_{\text{PNS}}}{1.4 M_{\odot}} \right)^{-5/2}, \quad (14)$$

where we neglect the mass evolution of the PNS within the integration, which decreases the mass ejection by $O(10)^\%$. Moreover, when we adopt $f_{\Omega} = 1/3$, which gives the geometric effect term $f_{\Omega}(1 - f_{\Omega})^2$ maximum, Eq. (14) is written as

$$M_{\text{ej}, \infty}^\text{max} = 6.4 \times 10^{-4} M_{\odot} \times \left( \frac{t_0}{1 \text{ s}} \right)^2 \left( \frac{M_{\text{acc,iso}}}{0.1 M_{\odot} s^{-1}} \right)^2 \left( \frac{M_{\text{PNS}}}{1.4 M_{\odot}} \right)^{-5/2}. \quad (15)$$

The main goal of this paper is to find out the ejectable maximum mass of our wind model from Eq. (15). When we adopt the initial mass of the PNS to be $1.4 M_{\odot}$ (e.g., Müller & Janka 2014), then two free parameters, $M_{\text{acc,iso}}$ and $t_0$, remain. We first adopt $M_{\text{acc,iso}} = 1 M_{\odot} s^{-1}$ for $M_{\text{acc,iso}}$ as a phenomenological upper limit from Fig. 3. $t_0$ is constrained by the maximum PNS mass as follows. Taking into account the geometric effects (Eq. 10), and ignoring the mass decreases due to ejected wind because it is relatively small, the mass of the PNS can be written as

$$M_{\text{PNS}} \approx 1.4 M_{\odot} \times \left( \frac{t_0}{1 \text{ s}} \right)^2 \left( \frac{M_{\text{acc,iso}}}{0.1 M_{\odot} s^{-1}} \right)^2 \left( \frac{M_{\text{PNS}}}{1.4 M_{\odot}} \right)^{-5/2}. \quad (16)$$
as
\[
M_{\text{PNS}}(t) \approx M_{\text{PNS,0}} + \int_{0}^{t} dt' \dot{M}_{\text{acc}}(t')
\]
\[
= M_{\text{PNS,0}} + (1 - f_{\Omega}) \dot{M}_{\text{acc,0}} t_{0} \left( \frac{t}{t + t_{0}} \right),
\]
where \(M_{\text{PNS,0}}\) is the initial mass of the PNS (at the time of the SN shock revival). We assume that the wind ceases when the PNS mass exceeds a black hole mass. These assumptions of the total accretion mass (Eq. 16 and the PNS mass up to \(2.1 M_{\odot}\)) gives us \((1 - f_{\Omega}) \dot{M}_{\text{acc,0}} t_{0} \leq 0.7 M_{\odot}\). To conclude, the ejectable maximum mass of our wind model is given with \(\dot{M}_{\text{acc,0}} = 1 M_{\odot} s^{-1}\), \(t_{0} = 1.05 \text{ s}\), and \(f_{\Omega} = 1/3\) in Eq. (15) as follows,
\[
M_{\text{ej,\infty}}^{\text{max}} = 0.067 M_{\odot}.
\]
If most of this compensation from the wind is added at late phase, which is later than the computation time of the first-principles simulations, this value is then sufficient to compensate for the lack of \(^{56}\text{Ni}\) in the recent Ni problem. In the following, we investigate the time evolution of the cumulative ejected mass of the wind and discuss the nature of the explosion that could solve Ni problem.

Figure 4 shows the time evolution of the cumulative ejected mass of the wind model. The cumulative ejected mass is given as
\[
M_{\text{ej}}(t_{e}) = \int_{0}^{t_{e}} dt \dot{M}_{\text{wind}}(t)
\]
\[
\approx 6.4 \times 10^{-4} M_{\odot} \left[ 1 - \left( \frac{t_{0}}{t_{0} + t_{e}} \right)^{3} \right]
\]
\[
\times \left( \frac{t_{0}}{1 \text{s}} \right) \left( \frac{\dot{M}_{\text{acc,0}}}{0.1 M_{\odot} s^{-1}} \right)^{2} \left( \frac{M_{\text{PNS,0}}}{1.4 M_{\odot}} \right)^{-5/2},
\]
where \(t_{e}\) is time after the neutrino-driven wind starts to blow, corresponding to the time after the SN shock revival. We adopt \(f_{\Omega} = 1/3\), which gives the geometric effect term \(f_{\Omega}(1 - f_{\Omega})^{2}\) maximum. When we fix \(M_{\text{PNS,0}} = 1.4 M_{\odot}\) for simplicity, then two free parameters, \(\dot{M}_{\text{acc,0}}\) and \(t_{0}\), remain to determine the trajectory of the time evolution. As with the condition for Eq. (17), these two parameters are given by the conditions \(\dot{M}_{\text{acc,0}} \leq 1.0 M_{\odot} s^{-1}\) and \((1 - f_{\Omega}) \dot{M}_{\text{acc,0}} t_{0} \leq 0.7 M_{\odot}\), respectively, from the phenomenological accretion model (Fig. 3) and the limits of the total accretion mass (Eq. 16 and the PNS mass up to \(2.1 M_{\odot}\)). A degenerate set of parameters that converge to the same \(M_{\text{ej,\infty}}\) are shown in the same color in Figure 4. We further compare the time evolution with the multi-D first-principles simulations especially at \(t_{e} \lesssim 1.0\) (Wanajo et al. 2018), which are shown in Figure 4 as rhombus points. It indicates that, within parameter ambiguities, the total ejectable amount of the neutrino-driven wind is roughly determined within 1 sec from the onset of the blowing, which is reachable for first-principles simulations. Moreover, we also find that the supplementable amount from the wind at a later phase \((t_{e} \gtrsim 1 \text{ sec})\) remains \(\lesssim 0.01 M_{\odot}\). It also shows that, comparing with first-principles simulations, the expected total amount from the wind in a canonical CCSN explosion is about \(\lesssim 0.03 M_{\odot}\) at most.

We should mention that, in fact, some models in first-principles simulations of CCSNe have produced a sufficient amount of \(^{56}\text{Ni}\) in total of explosive and the wind nucleosynthesis (e.g., Bruenn et al. 2016; Eichler et al. 2018; Burrows et al. 2020). However, the claim of the Ni problem is that we are now struggling to reproduce sufficient \(^{56}\text{Ni}\) amount as a canonical nature of the CCSN explosion, and many previous studies place their hopes on the supplement from the neutrino-driven wind at a later phase (e.g., Wongwathanarat et al. 2017; Wanajo et al. 2018). Our conclusion on this issue is first that, in
order to compensate for sufficient $^{56}\text{Ni}$ by the neutrino-driven wind, it is preferred to have an active ejection in the early phase rather than a continuous ejection until the later phase. We note that while the wind driven by magneto-rotational explosions may be more energetic, it is not expected to solve the Ni problem due to low $^{56}\text{Ni}$ as it is ejected with a low electron fraction (Nishimura et al. 2015). It should also be emphasized that the total amount of synthesized $^{56}\text{Ni}$ can be estimated robustly if the first-principles simulations are performed up to $\sim 2$ seconds.

### 3.2. Effects of PNS mass evolution

To further expand the discussion in Eqs. (14) and (18) about the ejectable amount of the wind, here we discuss the effect of the time evolution of the PNS mass on the results above. We first introduce the ratio $\epsilon$ of the total accretion mass to the initial PNS mass as

$$
\epsilon = \frac{1}{M_{\text{PNS},0}} \int_0^\infty dt \dot{M}_{\text{acc}}(t) .
$$

Using this ratio $\epsilon$, the time evolution of the PNS mass expressed in Eq. (16) is written as

$$
M_{\text{PNS}}(t) \approx M_{\text{PNS},0} \left(1 + \epsilon \frac{t}{t + t_0}\right) ,
$$

where we note that this ignores the mass loss due to wind, as in Eq. (16). No matter how light the initial PNS mass is assumed at SN shock revival (e.g., $M_{\text{PNS},0} \approx 1.2M_\odot$), this $\epsilon$ is less than unity, since this mass will never grow more than twice as large due to the constraint that the wind stops when it becomes a black hole. Then we can update the time evolution term of the integral in Eq. (14) as follows

$$
\int_0^\infty dt \left(\frac{t}{t_0} + 1\right)^{-4} \left(1 + \epsilon \frac{t}{t + t_0}\right)^{-5/2} = \frac{2}{3\epsilon^3} (\epsilon^2 - 4\epsilon + 8\sqrt{\epsilon + 1} - 8) t_0
$$

$$
\approx \left(\frac{1}{3} - \frac{5}{24} \epsilon + \frac{7}{48} \epsilon^2\right) t_0 .
$$

where $\epsilon = 0$ corresponds to the result in Eq. (14). We find that, for $\epsilon < 1$, the effect of mass dependence only decreases the wind mass loss. The same can be argued for the time evolution of the cumulative mass, i.e., the discussion in Eq. (18) and Fig. 4.

Fig. 5 shows a comparison of the results between analytical integration without the effect of the PNS masses and the numerical integration with the effect. This figure shows that the result of Eq. (14) gives a robust upper limit on the ejectable mass of the wind. This comparison indicates that the results of Eqs. (14) and (18) give robust upper limits on the ejectable mass of the wind.

![Figure 5](image)

**Figure 5.** Same as Figure 4, but a comparison of the analytical results ignoring the effect of the time-evolution of PNS mass (Eq. 18) and the numerical results taking into account the effect (see Eq. 21).

### 4. SUMMARY

In this paper, we have constructed a consistent model of the accretion flow onto the PNS and the neutrino-driven wind, and then estimate the ejectable $^{56}\text{Ni}$ mass as a supplement to the Ni problem of the CCSN explosions. We first derived the ejecta amount for the spherical-symmetric steady-state neutrino-driven wind with the PNS parameters as follow

$$
\dot{M}_{\text{wind,iso}} \approx 8.3 \times 10^{-3} M_\odot s^{-1}
$$

$$
\times \left(\frac{L_{\nu_\mu}}{10^{52}\text{erg} \text{ s}^{-1}}\right)^{7/4} \left(\frac{R_{\text{gain}}}{4 \times 10^9 \text{cm}}\right)^{5/2} \left(\frac{M_{\text{PNS}}}{1.4M_\odot}\right)^{-7/2} .
$$

We then derived a consistent model of the neutrino-driven wind with the accretion flow onto the PNS by modeling the evolution of the gain radius $R_{\text{gain}}$ and the neutrino luminosity $L_{\nu_\mu}$. The wind mass loss rate (Figure 1) is

$$
\dot{M}_{\text{wind}} \approx 1.3 \times 10^{-2} M_\odot s^{-1}
$$

$$
\times f_\Omega \left(\frac{(1 - f_\Omega)\dot{M}_{\text{acc,iso}}}{0.1M_\odot s^{-1}}\right)^2 \left(\frac{M_{\text{PNS},0}}{1.4M_\odot}\right)^{-5/2} .
$$

Based on this equation and adopt $f_\Omega = 1/3$, which gives the geometric effect term $f_\Omega(1 - f_\Omega)^2$ maximum, the
of our wind model is derived to be

\[ M_{\text{ej},\infty} = 6.4 \times 10^{-4} \, M_{\odot} \]

\[ \times \left( \frac{t_0}{1 \text{ s}} \right) \left( \frac{M_{\text{acc},0}}{0.1 M_{\odot} \text{s}^{-1}} \right)^2 \left( \frac{M_{\text{PNS},0}}{1.4 M_{\odot}} \right)^{-5/2} \]

(24)

where \( M_{\text{acc},0} \) and \( t_0 \) are the parameters which characterize the mass accretion rate \( M_{\text{acc},\infty} \). Eqs. (22)–(24) are important results in a consistent model of the neutrino-driven wind with the accretion flow onto the PNS (Fig. 1), which is one of the goals of this paper.

Based on these equations, when we take the upper limit as far as possible within the range of the parameter constraint \( M_{\text{PNS},0} \geq 1.4 M_{\odot} \), \( M_{\text{acc},0} \leq 1.0 M_{\odot} \text{s}^{-1} \) and \((1 - f_0)M_{\text{acc},0}t_0 \leq 0.7 M_{\odot} \), the ejectable maximum mass of our wind model is derived to be \( M_{\text{ej},\infty} = 0.067 M_{\odot} \).

If most of this compensation from the wind is added at late phase, which is later than the computation time of the first-principles calculations, this value is then sufficient to compensate for the lack of \( ^{56}\text{Ni} \) in the recent Ni problem. However, we found that, within parameter ambiguities, the total ejectable amount of the neutrino-driven wind is roughly determined within 1 sec from the start of the blowing, which is reachable by first-principles simulations. Moreover, we also found that the supplementable amount from the wind at a later phase \((t_e \gtrsim 1 \text{ s})\) remains \( M_{\text{ej}} \lesssim 0.01 M_{\odot} \) at most, independent of the parameter choice, i.e., the nature of the explosion.

Our conclusions on the Ni problem are first that in order to compensate for sufficient \( ^{56}\text{Ni} \) by the neutrino-driven wind, it is preferred to have an active ejection in the early phase rather than a continuous ejection until the later phase. It is also an important suggestion that the total amount of synthesized \( ^{56}\text{Ni} \) can be estimated robustly if the first-principles simulations are followed up to 2 seconds.

In summary, it is difficult to solve the Ni problem in a way that continuously supplements for \( ^{56}\text{Ni} \) at the late phase of the explosion by the neutrino-driven wind. Therefore, requiring an explosion capable of active ejection in the early phase of shock revival is a simple and straightforward solution for the SN mechanism to satisfy the Ni problem as canonical, without fine-tuning.

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REFERENCES

Arnett, W. D., Bahcall, J. N., Kirshner, R. P., & Woosley, S. E. 1989, ARA&A, 27, 629,
doi: 10.1146/annurev.aa.27.090189.003213
Bethe, H. A., & Wilson, J. R. 1985, ApJ, 295, 14,
doi: 10.1086/163343
Bliss, J., Witt, M., Arcones, A., Montes, F., & Pereira, J. 2018, ApJ, 855, 135,
doi: 10.3847/1538-4357/aaadbe
Bruenn, S. W., Lentz, E. J., Hix, W. R., et al. 2016, ApJ, 818, 123,
doi: 10.3847/0004-637X/818/2/123
Burrows, A., Radice, D., Vartanyan, D., et al. 2020, MNRAS, 491, 2715,
doi: 10.1093/mnras/stz3223
Eichler, M., Nakamura, K., Takiwaki, T., et al. 2018, Journal of Physics G Nuclear Physics, 45, 014001,
doi: 10.1088/1361-6471/aa8891
Ertl, T., Janka, H. T., Woosley, S. E., Sukhbold, T., & Ugliano, M. 2016, ApJ, 818, 124,
doi: 10.3847/0004-637X/818/2/124
Fischer, T., Whitehouse, S. C., Mezzacappa, A., Thielemann, F. K., & Liebendörfer, M. 2009, A&A, 499, 1,
doi: 10.1051/0004-6361/200811055
—. 2010, A&A, 517, A80,
doi: 10.1051/0004-6361/200913106
Fujibayashi, S., Yoshida, T., & Sekiguchi, Y. 2015, ApJ, 810, 115,
doi: 10.1088/0004-637X/810/2/115
Hamuy, M. 2003, ApJ, 582, 905,
doi: 10.1086/344689
Hüdepohl, L., Müller, B., Janka, H. T., Marek, A., & Raffelt, G. G. 2010, PhRvL, 104, 251101,
doi: 10.1103/PhysRevLett.104.251101
Iwamoto, K., Nomoto, K., Höflich, P., et al. 1994, ApJL, 437, L115,
doi: 10.1086/187696
Janka, H.-T. 2012, Annual Review of Nuclear and Particle Science, 62, 407,
doi: 10.1146/annurev-nucl-102711-094901
Mazzali, P. A., Deng, J., Maeda, K., et al. 2002, ApJL, 572, L61,
doi: 10.1086/341504
Müller, B., Heger, A., Liptai, D., & Cameron, J. B. 2016, MNRAS, 460, 742,
doi: 10.1093/mnras/stw1083
Müller, B., & Janka, H.-T. 2014, ApJ, 788, 82,
doi: 10.1088/0004-637X/788/1/82
Nishimura, N., Takiwaki, T., & Thielemann, F.-K. 2015, ApJ, 810, 109,
doi: 10.1088/0004-637X/810/2/109
Otsuki, K., Tagoshi, H., Kajino, T., & Wanajo, S.-y. 2000, ApJ, 533, 424,
doi: 10.1086/308632
Prentice, S. J., Ashall, C., James, P. A., et al. 2019, MNRAS, 485, 1559, doi: 10.1093/mnras/sty3399
Qian, Y. Z., & Woosley, S. E. 1996, ApJ, 471, 331, doi: 10.1086/177973
Sawada, R., & Maeda, K. 2019, ApJ, 886, 47, doi: 10.3847/1538-4357/ab4da3
Seitenzahl, I. R., Timmes, F. X., Marin-Laflèche, A., et al. 2008, ApJL, 685, L129, doi: 10.1086/592501
Sukhbold, T., Ertl, T., Woosley, S. E., Brown, J. M., & Janka, H.-T. 2016, ApJ, 821, 38, doi: 10.3847/0004-637X/821/1/38
Sukhbold, T., Woosley, S. E., & Heger, A. 2018, ApJ, 860, 93, doi: 10.3847/1538-4357/aa2eda
Sumiyoshi, K., Suzuki, H., Otsuki, K., Terasawa, M., & Yamada, S. 2000, PASJ, 52, 601, doi: 10.1093/pasj/52.4.601
Suwa, Y., Tominaga, N., & Maeda, K. 2019, MNRAS, 483, 3607, doi: 10.1093/mnras/sty3309
Suwa, Y., Yamada, S., Takiwaki, T., & Kotake, K. 2016, ApJ, 816, 43, doi: 10.3847/0004-637X/816/1/43
Thompson, T. A., Burrows, A., & Meyer, B. S. 2001, ApJ, 562, 887, doi: 10.1086/323861
Timmes, F. X., & Swesty, F. D. 2000, ApJS, 126, 501, doi: 10.1086/313304
Ugliano, M., Janka, H.-T., Marek, A., & Arcones, A. 2012, ApJ, 757, 69, doi: 10.1088/0004-637X/757/1/69
Wanajo, S. 2013, ApJL, 770, L22, doi: 10.1088/2041-8205/770/2/L22
Wanajo, S., Kajino, T., Mathews, G. J., & Otsuki, K. 2001, ApJ, 554, 578, doi: 10.1086/321339
Wanajo, S., Müller, B., Janka, H.-T., & Heger, A. 2018, ApJ, 852, 40, doi: 10.3847/1538-4357/aa9d97
Wongwathanarat, A., Janka, H.-T., Müller, E., Plumbi, E., & Wanajo, S. 2017, ApJ, 842, 13, doi: 10.3847/1538-4357/aa72de
Woosley, S. E., & Heger, A. 2015, ApJ, 806, 145, doi: 10.1088/0004-637X/806/1/145