Particle beams in ultrastrong laser fields: direct laser acceleration and radiation reaction effects

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Abstract. Several aspects of the interaction of particle beams with ultrastrong laser fields are discussed. Firstly, we consider regimes when radiation reaction is not essential and it is demonstrated that employing chirped laser pulses, significant improvement of the direct acceleration of particles can be achieved. Results from single- and many-particle calculations of the particle acceleration, in vacuum, by plane-wave fields, as well as in tightly-focused laser beams, show that the mean energies and their spreads qualify them for important applications. Secondly, we investigate the effect of radiation reaction in electron-laser-beam interactions. Signatures of the quantum radiation reaction during the interaction of an electron bunch with a focused superstrong ultrashort laser pulse can be observed in a characteristic behavior of the spectral bandwidth, and the angular spread of the nonlinear Compton radiation on the laser pulse duration. Furthermore, it is shown that the radiation reaction effects can be employed to control the electron dynamics via the nonlinear interplay between the Lorentz and radiation reaction forces. In particular, it is shown that an ultrarelativistic electron bunch colliding head-on with a strong bichromatic laser pulse can be deflected in a controllable way, by changing either the relative phase or the relative amplitude between the two frequency components of the bichromatic field.
1. Introduction

Recent advances in laser techniques have opened up new avenues for the generation of high energy photons, electrons and ions [1]. When strong laser radiation interacts with an atomic gas jet, extreme ultraviolet and x-ray radiation can be generated via high-order harmonic generation processes [2, 3] which, in turn, has led to the field of attoscience [4]. Gamma rays can be created employing Compton scattering of ultrastrong laser radiation by a relativistic electron beam [5]. Moreover, a laser beam can be used to generate relativistic electron beams up to giga-electronvolt energies via wakefield acceleration [6], allowing for the realization of all-optical setups for Compton radiation sources. In strong laser fields the electron radiation can be so strong that it may disturb the electron dynamics. Then, the radiation reaction effects become nonperturbative and the so-called radiation dominated regime sets in [1]. Currently, a lot of efforts are invested also in accelerating ions with ultrastrong laser fields. Thus, for example, with the target normal sheath acceleration mechanism quasi-monoenergetic ion beams are produced with about MeV energy per nucleon [7]. The main aim of the laser acceleration of ions is to develop more cost effective solutions for medical applications.

In this work several aspects of the interaction of particle beams with strong laser fields are discussed. In the first part of the paper we consider the possibility of accelerating the particles by laser means, directly and in the absence of any background like a plasma or boundaries. The main motivation for laser-acceleration is the desire to replace the large and expensive conventional accelerators with compact and cheap laser-based alternatives. Our work on laser acceleration of electrons and ions to energies that may make them useful for various applications involves unchirped as well as chirped laser pulses. In particular, it has been concluded from our unchirped pulse calculations of ion acceleration for tumour therapy [8, 9] that a laser power of 10 PW would be needed. The laser pulse or beam needs to be focused to a sub-wavelength waist radius \(w_0 < \lambda\), which would correspond to the intensity of \(10^{23}\) W/cm\(^2\). By appropriately chirping the frequency of the laser pulse, our simulations [10, 11, 12, 13, 14, 15] have shown that both the power and intensity may be lowered by one order of magnitude and that the beam/pulse need not be focused to a sub-wavelength spot radius \(w_0 > \lambda\). In this paper results from our chirped-pulse laser acceleration will be discussed.

The second part of the paper is devoted to the radiation reaction effects which take place during an electron beam interaction with laser fields. Firstly, we review our results on signatures of radiation reaction in the radiation dominated regime and in a focused laser beam of short duration [16]. We consider a setup where the relativistic electron beam counterpropagates with the laser beam, and investigate the radiation frequency and angle resolved spectra during multiphoton Compton scattering. In particular, the analysis of the dependence of the spectral bandwidth and the angular spread of the radiation on the laser pulse duration, reveals that their behaviour is very specific in the case of the radiation dominated regime in a tightly focused laser beam.

Finally, we demonstrate that the radiation reaction effects can provide a route to the control of the electron dynamics in the interaction with a strong few-cycle [17] and bichromatic laser pulse [18]. In particular, an ultrarelativistic electron bunch initially counterpropagating with respect to a bichromatic laser pulse, can be deflected in a controllable way, by changing the relative phase or the relative amplitude of the two frequency components of the laser pulse field [18]. In addition, the effect is independent of the initial electron energy, as long as the quantum corrections remain suppressed because a larger initial electron energy, in turn, implies a larger radiation reaction force and, thus, a larger energy loss.

2. Direct laser-acceleration of particles

The focus of our earlier work on direct laser acceleration was on electrons, employing plane-wave and tightly-focused laser beams and pulses. In more recent investigations, attention was
Exit kinetic energy \( K_{\text{exit}} = mc^2[\gamma_f - 1] \), where \( \gamma_f = \gamma(\eta_f) \), vs. the excursion distance \( z \), following interaction with linear and quadratic chirps, \( \omega = \omega_0(1 + b\eta) \) and \( \omega = \omega_0(1 + b\eta^2) \), respectively, in which \( \eta = \omega_0(t - z/c) \). The pulse is modeled by a finite-duration pulse of \( \cos^2 \) shape (in place of the Gaussian in Eq. (3), see [20]). In both cases, the dimensionless intensity parameter \( a = eE_0/(mc\omega) = 3 \) (which corresponds to the intensity \( I \sim 1.23 \times 10^{19} \text{ W/cm}^2 \)), the scaled injection energy \( \gamma_0 = 10 (K_0 \sim 4.6 \text{ MeV}) \), \( \lambda_0 = 1 \mu \text{m} \), and the initial phase \( \psi_0 = 0 \). Temporal width of the \( \cos^2 \) pulse is \( \tau = 50 \text{ fs} \).

Directed at the use of chirped laser pulses to accelerate electrons [19] and then protons. A further shift from linearly- to quadratically-chirped pulses has also been the subject of a more recent publication [20].

The single-particle interaction may be handled by solving the relativistic equations of motion in the electric and magnetic fields of the pulse, \( E \) and \( B \), respectively, subject to the appropriate initial conditions. With the relativistic momentum \( p = \gamma mv \) and relativistic energy \( \epsilon = \gamma mc^2 \), where \( m \) is the particle’s mass and \( \gamma = (1 - \beta^2)^{-1/2} \) its Lorentz factor, the equations of motion read

\[
\frac{dp}{dt} = q[E + \beta \times B] \quad (1)
\]

\[
\frac{dc}{dt} = qv \cdot E. \quad (2)
\]

In the above equations, \( q \) is the charge of the particle and \( \beta = v/c \) is its velocity, scaled by the speed of light.

### 2.1. Direct laser acceleration of electrons

With \( m \) and \( q \) in the equations of motion replaced by those of an electron, dynamics of single and many electrons have been studied within the context of many schemes involving a single laser beam, as well as crossed-beam and beat-wave configurations. In those investigations, unchirped linearly- and radially-polarized laser systems were employed. Later work [20, 21] also employed linearly- and quadratically-chirped laser pulses.

From a purely theoretical point of view, the basic idea of a chirped pulse is to make its frequency a function of the time. In a linear chirp, the central frequency \( \omega_0 \) would be replaced by \( \omega \approx \omega_0(1 + b\eta) \), where \( \eta = \omega_0 t - k_0 z \), with \( c k_0 = \omega_0 \) and \( c \) the speed of light in vacuum, for a pulse propagating along \( z \). The constant \( b \), here, is a dimensionless chirp parameter. With this
Figure 2. (a) Evolution of the kinetic energy $K = (\gamma - 1)mc^2$ of a single proton during interaction with a single chirped plane-wave pulse whose parameters are given. (b) Electric field of the same pulse seen by the proton.

transformation, the electric field of a plane-wave Gaussian pulse $E = E_0 \cos(\eta) \exp(-\eta^2/2\sigma^2)$ becomes

\[ E = E_0 \cos(\eta + b\eta^2) \exp\left[ -\frac{\eta^2}{2\sigma^2} \right], \quad \sigma = \frac{\omega_0\tau}{\sqrt{2\ln 2}} \]

where $\tau$ is the temporal width of the pulse.

Note what chirping the frequency typically does to the scaled electric field. The pulse shape and field oscillations get distorted and the electric field appears to develop a low-frequency (almost quasi-static) portion. Interaction of a charged particle with the unchirped pulse results in no net gain or loss, because each field cycle consists of two almost identical half-cycles, one positive and the other negative. Interaction with the quasi-static part of the chirped pulse, on the other hand, allows for an extended period of energy gain without loss. Eventually, the particle is left behind the pulse, retaining some of the energy it has gained from the field.

Employing a plane-wave model for the pulse fields, the equations of motion admit analytic solutions in closed form. While the details may be found in the references, it suffices here to display some of the most recent results graphically. In Fig. 1 we show variation of the exit kinetic energy of a single electron subjected to plane-wave linearly- and quadratically-chirped pulses, as functions of the excursion distance of travel along the pulse’s propagation direction. It is evident from these model calculations that the same set of parameters (apart from the chirp parameter $b$) lead to energy gains in the case of a linear chirp that are almost twice what would be gained from a quadratic chirp. This result may be model dependent, though.

2.2. Direct laser acceleration of protons

Figure 2 (a) shows evolution of the kinetic energy of a single proton, from rest at the origin of coordinates, during interaction with a plane-wave 300 fs, 1 PW Gaussian laser pulse of central wavelength $\lambda_0 = 1 \mu m$. Fig. 2 (b) shows evolution of the scaled electric field during interaction with the proton. The kinetic energy evolution exhibits synchronization with the field oscillations. Interaction with the highly-symmetrical parts of the field (on the left and right) results in no net gain, whereas interaction with the quasi-static part (to the left of the center) does lead to
In all our calculations, the following field components will be employed, with each multiplied by a Gaussian temporal envelope. But a plane-wave is only an idealization, suitable for modeling a Gaussian temporal envelope. However, the initial conditions, otherwise, were assumed to be the same as for the ensemble of 3000 protons. In further calculations, a code based on the single-particle equations of motion has been used to solve for the particle dynamics in the ensemble.

As was pointed out in the Introduction, particle bunches containing $10^7 - 10^{11}$ particles/bunch are needed for medical and other applications. In recent simulations [10, 11], dynamics of an ensemble of 3000 non-interacting protons was investigated, whose initial positions were distributed randomly within a cylinder centered at the origin of coordinates and of radius $R = 0.3 \lambda_0$ and height $L = 0.2 \lambda_0$. On the other hand, the initial conditions on the velocities of the particles were derived from a normal distribution of initial kinetic energies of mean value $K_0 = 10 \text{ keV}$ and spread $\Delta K_0 = 0.5 \text{ keV}$. A code based on the single-particle equations of motion has been used to solve for the particle dynamics in the ensemble. In further calculations, the 3000 protons were assumed to be taken from a hydrogen gas jet or an expanding hydrogen cluster.

Analytic solutions to the equations of motion are, unfortunately, not possible when the above field components are used. Inevitably, one resorts to numerical simulations [10, 11] based on Eqs. (1) and (2) instead. Some results of simulations are displayed in Fig. 3.
of non-interacting protons. When the pulse impinges upon the target, an under-dense plasma of electrons and protons is generated quickly. The electrons get swiftly accelerated and, being the lighter component, they immediately leave the scene and drag the much more massive, and sluggish, protons behind them. The protons will also be accelerated further by interaction with the pulse. A particle-in-cell (pic) code has been used to solve the equations of motion self-consistently for the particle dynamics in the plasma. Unlike the proton ensemble calculations, in the pic simulations all of the Coulomb particle-particle interactions are taken into account.

The main results from the many particle calculations, employing pulse parameters the same as in Fig. 3, apart from \( w_0 = 2.2 \lambda_0 \) and \( \tau = 350 \text{ fs} \), are as follows [10]. The two calculations (interacting and non-interacting) return exit energy gains within 5% of each other (245.3 MeV and 258.3 MeV, respectively). In both cases, the spreads in the exit kinetic energies are about 1%, which qualifies the protons accelerated in this fashion for use in tumor therapy. The spread in the distribution of the ensemble of non-interacting protons is larger than that of the protons from the plasma. This should come as no surprise in light of the fact that the pic code takes into account motion of, and all effects associated with, the electrons. Attraction of the electrons prevents the protons from spreading out spatially and pulls further on them in the direction of their collective motion, which results ultimately in less exit energy spread.

3. radiation reaction effects in ultrastrong laser fields

3.1. Signatures of radiation reaction in the interaction of electron and laser beams

We consider a relativistic electron beam moving in the field of an ultrastrong laser field opposite to the laser field propagation direction. Then, in the electron rest frame the laser field is Lorentz-boosted and the radiation dominated regime is achievable when the parameter \( R \equiv \alpha \xi \) is of order of one, where \( \alpha \) is the fine structure constant, \( \xi = cE_0/mc\omega_0 \) is the relativistic invariant parameter of the laser field strength, \( \chi = E'/E_{cr} \) is the quantum strong field parameter, \( E_{cr} = m^2c^3/\hbar \) is the QED critical field, \( E' \approx 2\gamma E_0 \) is the laser field in the rest frame of the electron and \( \gamma \) is the Lorentz-factor. When \( \chi \gtrsim 1 \), the photon emission recoil is not negligible and the radiation dynamics is quantum mechanical. If additionally \( \alpha \xi \gtrsim 1 \), then radiated energy during the electron motion in a single laser period is comparable with the electron total energy and the interaction is in the quantum radiation dominated regime. We
calculated in [23] the Compton radiation spectra in the quantum radiation reaction regime taking into account the effects of the radiation reaction. The radiation reaction effect is exhibited in the radiation spectrum as decreasing the photon piling effect at the maximal photon energy, shifting the peak of the spectra towards lower photon energies [23] and, consequently, decreasing the emitted photon numbers in the high energy region. However, detection of these effects in an experiment requires accurate measurement of the emitted photon numbers which is not an easy task. That is why in [24] it was proposed to measure the electron parameters (energy) after the end of interaction as a measure of the radiation reaction in the experiment. Yet it is disappointing, because the electron energy decrease is a trivial signature of the radiation reaction. Then, we asked ourselves if one can find non-trivial signatures of the radiation reaction in the spectra of emitted radiation which have qualitative character and do not require very accurate measurement.

We calculated angle resolved spectra of the electron radiation during the Compton scattering and investigated its dependence on the laser pulse duration. For the calculation of the radiation spectra we assumed that the electron dynamics in the laser field is classical. This is justified because for the ultrarelativistic motion of the electron in the infrared laser field, the electron de-Broglie wavelength is much smaller than the laser wavelength. For the calculation of the radiation spectra the full quantum mechanical formulas have been used in the asymptotic limit $\xi \gg 1$ [25]. In this limit the coherence length of the radiation $l_{coh} \sim \lambda_0/\xi$ is much smaller than the laser wavelength and the emission intensity depends only on the local field values, i.e., one can assume that the radiation is emitted from a local point on the electron trajectory. At each point of the trajectory one can calculate the emitted radiation and deduct from the electron energy and the momentum the energy and momentum carried by the emitted Compton photons (and by the absorbed laser photons). One should take into account that the emission of the Compton photons is possible only when the electron absorbs from the laser wave a certain number of laser photons (as required by the energy-momentum conservation). The accurate accounting of the electron momentum change due to the radiation yields an additional radiation reaction term in the classical equation of motion which is derived in [26, 27]:

$$\frac{dp^\alpha}{d\tau} = \frac{e}{mc} F^{\alpha\beta} p_\beta - \frac{I}{mc^2} p^\alpha + \tau_c \frac{I}{I_c} F^{\alpha\beta} F_{\beta\gamma} p^\gamma, \quad (10)$$

where $\tau$ is the proper time, $\tau_c \equiv 2e^2/(3mc^3)$ and $I_c = 2e^2\omega^2\xi^2/(3c)$ is the classical radiation loss rate and $I$ is the total quantum mechanical radiation loss rate.

The driving laser beam is circularly polarized. We have applied laser pulses of different duration from 0.5 to 9 cycles. The circular polarization has an advantage that the spectra have no dependence on the carrier-to-envelope phase of the short laser pulse. The first panel in Fig. 4 shows the angle resolved total radiation intensity (color coded), $\theta$ and $\phi$ are the polar and azimuthal angles, respectively. For each azimuthal angle there is contribution from different polar angles which correspond to different laser cycles with different magnitudes of the laser field amplitude. We choose the azimuthal angle where the emission intensity is maximal and plot the corresponding spectrum, resolved on the polar angle, in the right column of Fig. 4. The radiation intensity averaged by the azimuthal angle is shown in the middle column of Fig. 4. We inspect the spectral and angular bandwidths of the radiation. The latter is defined as the spectral and angular region where the emission intensity drops to one-half its peak value.

First of all we note from the last column of Fig. 4 that the emission is not uniform with respect to the polar angle: there is a spike at a polar angle far from 180° (direction of the electron motion). The main emission region near 180° arises at the peak of the parameter $\chi$, which takes place before the electron reflection in the laser field. This can be deduced from Fig. 5 (the bottom panel). This figure shows the electron velocity direction, which coincides with the emission direction (as the relativistic electron radiates forward). At those moments $\eta$
Figure 4. (Color online) The radiation spectrum: (left column) the angle resolved total radiation energy (color coded), $\theta$ is the polar and $\phi$ azimuthal angles; (middle column) the radiation spectrum averaged by the azimuthal angle; (right column) the radiation spectrum at the azimuthal angle where the emission energy is maximal. $\xi = 230$, $\gamma = 1000$, the laser pulse duration is 1 cycle (first row), 1.5 cycle (second row), and 5 cycles (third row).

Figure 5. (Color online). The electron dynamics in counterpropagating laser pulses of various durations: (a) The parameter $\chi$; (b) The electron energy, (c) the radiation loss, (d) the emission angle, (e) the electron coordinate along the propagation direction vs the laser phase; (f) The electron trajectory. The black, red, blue, and green curves (from left to right) correspond to the laser pulse durations $\tau_0 = T_0$, $\tau_0 = 1.5T_0$, $\tau_0 = 3T_0$, and $\tau_0 = 5T_0$, respectively.
when $\chi$ is maximal, the electron direction $\theta$ is still close to 180°. The electron reflection point corresponds to the minimum of the angle $\theta$ after which the radiation is negligible. Although near the minimum $\theta$ the parameter $\chi$ is small and the radiation rate is also small, the total emitted energy is not small because it is accumulated along a longer time interval than in the case of the peak of the parameter $\chi$. The radiation emitted at the minimum of $\theta$ is seen as the spike in the spectral distribution of the last column of Fig. 4.

Concentrating on the main emission region and neglecting the spike of the radiation, we plot in Fig. 6 the dependence of the radiation angular and spectral bandwidth on the laser pulse duration. While the spectral bandwidth is decreasing monotonously with increasing laser pulse duration, the angular spread of the radiation shows non-monotonous behavior. The monotonous decrease of the spectral bandwidth is due to the decrease of the maximum of the $\chi$-parameter with increasing laser pulse duration. The latter is due to more decrease of the electron energy because of radiation in a longer laser pulse. The non-monotonous behavior of the angular spread is due to two competing effects: radiation reaction and focusing. The radiation angle is determined by the electron velocity direction at the point where the $\chi$-parameter is maximal. Note that $\chi \sim \xi \gamma$ is proportional to the laser field and the electron energy. If there were no focusing of the laser pulse then in the longer laser pulse the largest $\chi$ is reached at a lower field strength $\xi$ (the energy loss due to radiation dominates) and the emission angle is closer to 180°. While when there were no radiation reaction effects, in the longer laser pulse the larger $\chi$ is reached at the larger $\xi$ value and the emission angle is more remote from 180°.

### 3.2. Electron dynamics controlled via radiation reaction

#### 3.2.1. Analytical analysis

In the classical framework, radiation reaction effects can be taken into account by the so called Landau-Lifshitz (LL) equation [28]:

$$\frac{dp^\mu}{d\tau} = eF^\mu\nu \frac{p_\nu}{mc} + \frac{2e^4}{3m^3c^3} \left[ F^\mu\nu F^\nu\alpha p^\alpha + (F^{\nu\beta } p_\beta)^2 \frac{p^\mu}{m^2c^2} \right].$$

(11)

The LL equation is equivalent to the Sokolov equation (10) in the realm of classical electrodynamics, the difference between the two equations being smaller than than quantum effects. Also, the term of the RR force containing the derivatives of the field tensor $F^\mu\nu$ [28] has been dropped from Eq. (11) since its contribution is negligible [29].

Let us consider an ultrarelativistic electron with initial momentum $p_0 = (0, 0, p_{z0})$ interacting with a counterpropagating laser pulse which, in first approximation, is modeled as a plane-wave pulse. Since the exact analytical solution of Eq. (11) for a plane-wave pulse with arbitrary shape, frequency and polarization is known [30], the final momentum $p$ after the electron passes...
through a pulse propagating along the positive z-axis and polarized along the x-axis is [18, 30]:

\[ p_x = \frac{I}{\hbar}, \quad p_y = 0, \quad p_z = \frac{p_{z_0}}{\hbar} + \frac{(h^2 - 1)mc + T^2/mc}{2\rho_0 \hbar}, \]

with:

\[ h = 1 + \frac{2e^4 \rho_0}{3m^3 c^5} \int_{-\infty}^{+\infty} d\varphi E_x^2(\varphi), \]

\[ I = \frac{2e^5 \rho_0}{3m^3 c^5} \int_{-\infty}^{+\infty} d\varphi E_x(\varphi) \int_{-\infty}^{+\infty} d\theta E_x^2(\theta), \]

where \( E_x(\varphi) \) is the plane-wave electric field as a function of the phase \( \varphi = (t - z/c) \), \( \rho_0 = (\gamma_0 - p_{z_0}/mc) \) is the initial Doppler factor, and \( \gamma_0 = \sqrt{1 + p_{z_0}^2/m^2c^2} \) is the initial relativistic factor. Note that the quantity \( h \) accounts for the energy losses associated with the emission of high frequency radiation and appears in the denominator in Eqs. (12)-(14), while the quantity \( I \) appears in the numerator and accounts for the work performed by the Lorentz force in the presence of the radiation reaction force, i.e. along the radiation-reaction-altered electron trajectory [18].

In the following we consider the effect induced by the interaction of an electron bunch with a second-harmonic enriched laser pulse with electric field: \( E_x(\varphi) = g(\varphi)[E_1 \sin(\omega\varphi + \theta_1) + E_2 \sin(2\omega\varphi + \theta_2)] \), where \( \omega \) is the lower frequency component, \( g(\varphi) \) is the pulse envelope, \( E_1, E_2 \) are the field amplitudes of each frequency component, and \( \theta_1 \) and \( \theta_2 \) are two constant initial phases.

For the sake of comparison, first we briefly discuss the case of a quasimonochromatic plane-wave pulse. In this case (\( E_2 = 0 \) and duration of several periods) the integrand of Eq. (16) basically contains only odd frequencies, which arise from the cubic nonlinearity with respect to the plane-wave field. Hence, \( I \) averages out to zero and no transverse momentum gain is possible [see Eqs. (12) and (16)]. On the contrary, if both frequencies are simultaneously present with the higher frequency being twice the lower frequency mode then a zero-frequency term arises in the integrand of Eq. (16). As a consequence, \( I \) diverges in the limit of infinite pulse duration. In fact, from Eq. (16) and in the limit of long pulse duration, we obtain \( I \propto E_1^2 E_2 \Delta t \cos(\theta_2 - 2\theta_1) \), where \( \Delta t \) is the laser pulse duration. Hence, in the interaction with a bichromatic laser pulse the electron can gain a transverse momentum along the polarization axis [see Eq. (12)], which can be controlled by changing either \( E_1, E_2 \) or the relative phase \( \theta_2 - 2\theta_1 \). In fact, it was shown that for an ultrarelativistic electron the deflection angle with respect to the initial propagation direction is [18]:

\[ \zeta \approx \arctan \left( \frac{2I/mc\rho_0}{1 - I^2/m^2c^2\rho_0^2} \right) \]

if \( |I|/mc\rho_0 < 1, \zeta - \pi \) if \( I/mc\rho_0 < -1 \) and \( \zeta + \pi \) if \( I/mc\rho_0 > 1 \). Note that \( I/mc\rho_0 \) and the deflection angle \( \zeta \) are independent of the initial electron energy because higher initial energies imply larger RR effects. As a result, for \( |I|/mc\rho_0 > 1 \) all electrons with arbitrarily high initial energy are back reflected as long as quantum effects do not noticeably alter the predictions of classical electrodynamics. We mention that such controlled deflection is also achievable by employing carrier-envelope-phase (CEP) tunable few-cycle laser pulses, due to the very broad bandwidth and the consequent nonlinear frequency combination analogous to the case of a bichromatic pulse. In this latter case, the deflection angle can be controlled by changing the
Figure 7. (Color online) Illustration of the emission process in the presence of a bichromatic laser pulse by changing the relative phase between the two frequency components. (a) An electron emits a photon and the recoil reduces its momentum and shifts its position inside the laser field. (b) A change in the relative phase between the two frequency components alters the direction of photon emission. In addition, in (b) the total field is stronger at the electron position because the two frequency components interfere constructively. As a consequence, the radiation reaction force, i.e. the recoil due to the photon emission, is larger in (b) compared to (a). In turn, this results in a larger shift with respect to the laser pulse field.

CEP of the few-cycle pulse [18]. Such ultraintense and CEP-tunable few-cycle laser pulses can be generated, e.g., via the relativistic mirror technique [17].

Physically, the energy and momentum losses associated with the emission of high-frequency radiation modulate the position of the electron within the plane-wave pulse. In the case of a quasimonochromatic plane-wave pulse there are no free parameters to control the effect of the emission and the consequent modulation. On the contrary, if a second higher frequency component is present, its frequency and relative phase can be chosen in such a way that the resulting modulation is resonant with the lower frequency component of the bichromatic field. Hence, the electron emits radiation prevalently along the positive (or negative) direction of the polarization axis, and the effect can be controlled by changing the relative phase between the two frequency components (see Fig. 7 for an illustration).

3.2.2. Simulation results In our simulation we considered a focused bichromatic laser pulse which is 100 fs long between its first and last half maximal intensity. Its total peak intensity is $4.2 \times 10^{21}$ W/cm$^2$ with 37% contribution from the second harmonic component, 7 $\mu$m waist radius and 1.55 eV energy of the lower frequency component. Initially, the electron bunch is distributed according to a six-dimensional Gaussian probability distribution:

$$f_e(x,p) = \frac{1}{N_e} \frac{1}{(2\pi)^3 \sigma_T^2 \sigma_L^2 \sigma_{pt}^2 \sigma_{px}^2} e^{-\frac{x^2 + y^2 + (z-z_0)^2}{2\sigma_T^2} - \frac{p_z^2 + p_x^2 + (p_x - p_{x,0})^2}{2\sigma_{px}^2}}.$$

with $N_e = 400$ being the total number of electrons and $\sigma_T = 0.2$ $\mu$m and $\sigma_L = 0.5$ $\mu$m ($\sigma_{pt} = 1$ mc and $\sigma_{px} = 12$ mc) being the transverse and the longitudinal position (momentum) widths, respectively. For the sake of simplicity, we set the constant phase $\theta_1 = 0$ so the deflection is controlled by $\cos(\theta_2)$ at fixed field amplitudes $E_1$, $E_2$.

Figure 8 displays the electron density distribution $n_e(p_z, p_x)$ as a function of the longitudinal $p_z$ and transverse $p_x$ momentum for the interaction of 400 electrons with the focused bichromatic laser pulse both for $\cos(\theta_2) = 0$ and $\cos(\theta_2) = 1$, with and without RR. No appreciable difference
Figure 8. (Color online) Electron density distribution $n_e(p_z, p_x)$ as a function of the longitudinal $p_z$ and transverse $p_x$ momentum after the interaction of 400 electrons with a focused bichromatic laser pulse with $4.2 \times 10^{21}$ W/cm$^2$ peak intensity (37% contribution from the second harmonic component) and 7 $\mu$m waist radius. Panel (a): $\cos(\theta_2) = 0$ without RR. Panel (b): $\cos(\theta_2) = 0$ with RR. Panel (c): $\cos(\theta_2) = 1$ without RR. Panel (d): $\cos(\theta_2) = 1$ with RR.

between $\cos(\theta_2) = 0$ and $\cos(\theta_2) = 1$ is found if only the Lorentz force is taken into account. Furthermore, if the RR force is neglected, the mean of the momentum distribution remains unaltered after the electron bunch has passed through the laser pulse $\bar{p}_z \approx 0$ and $\bar{p}_z \approx -165 mc$ [see Figs. 8(a), 8(c)]. However, if the RR force is taken into account, for $\cos(\theta_2) = 0$ the electrons still move along their initial propagation direction and are distributed symmetrically in the transverse momentum space with $\bar{p}_x \approx 0$ and $\bar{p}_z \approx -76 mc$ [see Fig. 8(b)] in good agreement with the plane wave prediction $p_x,f \approx 0$ and $p_z,f \approx -73 mc$. On the other hand, for $\cos(\theta_2) = 1$ all the electrons are deflected in the transverse direction independently of their initial energy, the mean of the momentum distributions being $\bar{p}_x \approx -6.6 mc$ and $\bar{p}_z \approx -76 mc$ [see Fig. 8(d)] which implies a deflection angle of about $5^\circ$. For the corresponding plane-wave pulse, we obtain $p_{x,f} \approx -5.6 mc$ and $p_{z,f} \approx -73 mc$ and $4.4^\circ$ deflection, in fair agreement with the above mentioned focused pulse results.

4. Conclusions
Firstly, we have presented in the paper our results on laser acceleration in vacuum of electrons and protons by chirped pulses, for energies that make them useful for applications. Results from single-particle calculations and many-particle simulations have been discussed. In particular, the work has demonstrated feasibility of the process of vacuum laser acceleration of protons for the tumor therapy, including mean exit energies within the required range of 20 – 580 MeV and spread of about 1%. The calculations have also demonstrated utility of the plane-wave model (in representing the accelerating fields of the pulses employed) for most cases considered. Effect of the particle-particle interactions, when taken into account within the context of a many-particle calculation, on the exit energy spread, has been shown to be small but must be considered seriously in any medical applications.

Secondly, we have discussed the signatures of the radiation reaction during interaction of the relativistic electron beam with a counterpropagating focused short laser pulse. The spectral bandwidth decreases monotonously with increasing laser pulse duration caused by radiation losses. The angular spread of radiation shows different dependence on the pulse duration for short and long laser pulses which results from competition of the radiation reaction effect (in the quantum radiation dominated regime) and the laser field focusing effect. While due to the former the angular spread decreases with increase of the laser pulse duration, due to the latter,
inversely, the angular spread increases with increase of the pulse duration.

Thirdly, we have shown that radiation reaction effects provide a route to the control of the electron dynamics in the interaction of an ultrarelativistic electron bunch colliding head-on with a bichromatic laser pulse. In this case the electron bunch is deflected with respect to the initial propagation direction, and the deflection angle can be tuned by changing the relative amplitude or the relative phase between the two frequency components of the bichromatic field. In addition, the deflection depends on the properties of the laser field only. Indeed, for ultrastrong bichromatic pulses the deflection angle can be so large that all electrons are back-reflected independently of their initial energy as long as quantum corrections are relatively small.

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