Since the discovery of the cosmological origin of gamma-ray bursts (GRBs), there has been growing interest in using these transient events to probe the quantum gravity energy scale in the range $10^{16}$–$10^{19}$ GeV, up to the Planck mass scale. This energy scale can manifest itself through a measurable modification in the electromagnetic radiation dispersion relation for high-energy photons originating from cosmological distances. We have used data from the GRB of 2002 December 6 (GRB 021206) to place an upper bound on the energy dispersion of the speed of light. The limit on the first-order quantum gravity effects derived from this single GRB indicates that the energy scale is in excess of $1.8 \times 10^{17}$ GeV. We discuss a program to further constrain the energy scale by systematically studying such GRBs.

**Subject headings:** gamma rays: bursts — gravitation — relativity

### 1. Introduction

The general quantum gravity picture of the vacuum is one of a gravitational medium containing microscopic quantum fluctuations on size scales comparable to the Planck length, $(\hbar G c^3)^{1/2} = 1.6 \times 10^{-33}$ cm. A number of approaches to quantum gravity (noncommutative geometry, loop quantum gravity) have independently been demonstrated to modify the electromagnetic dispersion relation (Amelino-Camelia et al. 1998; Amelino-Camelia et al. 2003), suggesting that first- or second-order spontaneous violation of Lorentz invariance at high photon energies might be a general signature of quantum gravity phenomenology (Sarkar 2002). The effects of this dispersion (reduced propagation speeds at high energies) are expected to be very small, unless the signals travel over very large distances and the photon energies are very different from one another. The magnitude of this in vacuo dispersion is set by an assumed energy scale, $E_{\text{QG}}$, which characterizes the size scale of quantum gravitational effects:

$$v = \frac{\partial E}{\partial p} = c \left[ 1 - \xi \frac{E}{E_{\text{QG}}} - \mathcal{O}\left(\frac{E}{E_{\text{QG}}}^2\right) \right],$$

where $\xi = \pm 1$ but is commonly assumed to be positive (Amelino-Camelia et al. 2002). The value of $E_{\text{QG}}$ is generally assumed to be on the order of the Planck mass ($E_{\text{P}} \sim 10^{19}$ GeV); however, theoretical work has suggested that this energy scale can be as low as $10^{18}$ GeV (Witten 1996) or even as low as $10^{16}$ GeV (Arkani-Hamed et al. 1999). (Note, however, that Lorentz invariance was preserved in both of these models.) With the discovery that gamma-ray bursts (GRBs) are at cosmological distances (van Paradijs et al. 1997), it was identified that GRBs could be sensitive to effective energy scales as high as the Planck mass (Amelino-Camelia et al. 1998). GRBs can combine high-energy photons, millisecond time variability, and very large source distances, making it possible to search for time delays in GRB light curves as a function of energy. The dispersion relation in equation (1) leads to a first-order differential time delay for signals of energy $E$ traveling from a source at cosmological distance $z$ given by (Ellis et al. 2003a)

$$\frac{\partial t}{\partial E} = \frac{1}{H_0 E_{\text{QG}}} \int_0^z \frac{dz}{h(z)},$$

where $t$ is the photon arrival time,

$$h(z) \equiv \sqrt{\Omega_\Lambda + \Omega_\gamma (1 + z)^3},$$

and $\Omega_\Lambda = 0.71$, $\Omega_\gamma = 0.29$, and $H_0 = 72$ km s$^{-1}$ Mpc$^{-1}$ are the current best estimates of the cosmological parameters (Spergel et al. 2003). In some quantum gravity models, the first-order differential time delays vanish, and a second-order delay in $E_{\text{QG}}$ remains. In this case, we would find (Ellis et al. 2003a)

$$\frac{\partial t}{\partial E} = \frac{2E}{H_0 E_{\text{QG}}^2} \int_0^z \frac{(1 + z)dz}{h(z)}.$$

These time delays hold for any astrophysical source, not just GRBs, so several high-energy sources exhibiting time variability have been used to set a lower limit on $E_{\text{QG}}$ for first-order corrections to the dispersion. Pulsed emission from the Crab pulsar in the GeV photon range has been used to set a lower limit of $E_{\text{QG}} > 1.8 \times 10^{15}$ GeV (Kaaret 1999). Initial analysis of GRB timing in the MeV photon range for bursts at known redshifts set a lower limit of $10^{15}$ GeV (Ellis et al. 2000), while a more detailed wavelet analysis extended this limit to $6.9 \times 10^{15}$ GeV (Ellis et al. 2003a). TeV observations of flares in the active galactic nucleus Mrk 421 increased this limit to $6 \times 10^{16}$ GeV (Biller et al. 1999). The current limit using this method is set at $8.3 \times 10^{16}$ GeV by observations of GRB 930131 (Schaefer 1999), but the lack of a distance measurement makes this subject to considerable uncertainty. Other astrophysical methods have placed more stringent constraints on $E_{\text{QG}}$ assuming that the electron dispersion relation is modified as well. For example, observations of TeV $\gamma$-rays emitted by blazars place a limit on $E_{\text{QG}} > 3.4 \times 10^{16}$ GeV by constraining the decay of photons into electron-positron pairs (Stecker 2003). Also, the discovery of polarized $\gamma$-ray emission (Coburn & Boggs 2003) from the same GRB discussed in this Letter led to limits of $E_{\text{QG}} > 10^{33}$ GeV from birefringence constraints (Jacobson et al. 2004; Mitrofanov
This page contains a scientific discussion on the observation and analysis of a gamma-ray burst (GRB) detected by the Reuven Ramaty High-Energy Solar Spectroscopic Imager (RHESSI). The burst, GRB 021206, was observed on December 6, 2002, and had a peak energy extending to the MeV range. The analysis focused on setting limits on the energy scale associated with the GRB, $E_{QG}$, using the observed light curve and spectral properties.

The discussion begins with an introduction to the burst, noting its brightness and the presence of a single, fast flare of photons extending above 10 MeV. The limits on $E_{QG}$ set by this method are compared to previous limits and absorption methods.

The observations were made using RHESSI, which has an array of large-volume detectors designed to study solar X-ray and gamma-ray emission. The spacecraft has high angular resolution and is sensitive to gamma rays between 3 keV and 17 MeV.

The light curve of GRB 021206 is divided into three energy bands, spanning 0.2–17 MeV. The fast flare seen above 3 MeV mixes with lower energy flares below 3 MeV. The 1–2 MeV range is the lowest energy band where this flare is resolved, although at these energies it is surrounded by a number of neighboring peaks. Below 1 MeV, this feature is completely lost in the noise of the other low-energy flares.

The analysis focuses on the 1–17 MeV energy range, and a second-order limit is derived. The reader is cautioned about the potential for a factor of 2 shift in the redshift uncertainty with this novel method.

The light curve is also presented in finer energy bands and with finer temporal resolution. The number of flare counts in the 7–17 MeV band is significantly smaller than at lower energies, but the combined significance of the 7–17 MeV flare is large, with a chance of $52.6\#10^{-5}$ of being a random Poisson fluctuation in the background rate.

The burst was observed with the Interplanetary Network (IPN) and the Ulysses spacecraft, indicating a 25–100 keV fluence of $4.8 \times 10^{-4}$ ergs cm$^{-2}$, making this an extremely bright GRB. The IPN localized the burst to an 8.6 arcmin$^2$ error ellipse located 18° from the Sun.

Follow-up optical observations have yet to measure the redshift of the GRB host galaxy, which is quite faint. However, the redshift can be estimated from the GRB spectral and temporal properties, yielding a redshift uncertainty of up to a factor of 2.

The figure shows the GRB light curve divided into three energy bands, spanning 0.2–17 MeV, and an expanded view with finer energy bands and temporal resolution. The 7–17 MeV flare is significant, with a chance of $52.6\#10^{-5}$ of being a random Poisson fluctuation in the background rate.
peak counts in a single 7.8125 ms time bin about once every 8.7 hr of RHESSI background data.

For each of the energy bands shown in Figure 2, we analyzed the peaks with two separate methods to determine the peaking time of the flare in each band. The first method was to bin the event data into the histogrammed light curves shown in Figure 2 and then fit the flare to a Gaussian profile in order to characterize the peaking time and the uncertainty. For this analysis, we chose 7.8125 ms wide bins, which are narrow enough to resolve the flare in the 7–10 and 10–17 MeV energy ranges. The second method we used to determine the peak times was to use the event data directly and to determine the average (peak) time and standard deviation for all events in a 50 ms time window centered on the short flare. The results of this analysis were relatively insensitive to variations in the window size and center as long as the flare dominated the total counts in the window. These two methods placed comparable limits on the dispersion, and in Figure 3 we show the results from averaging the results of these two separate analysis techniques.

From the peaking times plotted in Figure 3, we can see that the measured slope would be strongly affected by the 1–2 and 2–3 MeV data points, and we cannot preclude the possibility that an additional unresolved flare at energies less than 3 MeV is biasing these two points to earlier peaking times. Therefore, we performed a fit of the dispersion for just the data greater than 3 MeV. For the 3–17 MeV band, the time drift of the peak is measured to be $\Delta t = 0.0 \pm 4.8$ ms, yielding

$$\frac{\Delta t}{\Delta E} = 0.00 \pm 0.34 \text{ s GeV}^{-1} (3–17 \text{ MeV}).$$

(5)

This fit is consistent with a 95% confidence upper limit on the dispersion of

$$\frac{\Delta t}{\Delta E} < 0.7 \text{ s GeV}^{-1}.$$  

(6)

If we include the 1–3 MeV data in this fit, the upper limit remains comparable at $\Delta t/\Delta E < 0.8$ s GeV$^{-1}$.

Given our upper limit on the dispersion in equation (6) and the estimated source redshift, we can calculate the limit on $E_{\text{GRB}}$ for first-order dispersion effects from equation (2). This yields a lower limit of $E_{\text{GRB}} > 1.8 \times 10^{17}$ GeV. For second-order dispersion effects from equation (4), we can set a lower limit of $E_{\text{GRB}} > 5.5 \times 10^{17}$ GeV. It has been widely speculated that GRBs are detectable to redshifts of 10 and beyond (Lamb & Reichart 2000). If the same dispersion were measured for a burst at redshift 10, the lower limit would be 33 times higher, or $6 \times 10^{19}$ GeV, which is only slightly smaller than $E_{\gamma}$.

3. DISCUSSION

Many GRB energy spectra display hard-to-soft evolution (Preece et al. 1998). However, this refers to a global trend across the entire GRB time history and across the ~25–1000 keV spectrum. In contrast, our results use the behavior of ~millisecond peaks at energies greater than 1000 keV. In another study (Norris et al. 2000), the lag as a function of energy was examined for individual pulses in GRBs. A spectral lag was found, characterized by pulses peaking at high energy before they peaked at low energy. However, the pulses in question had durations of ~seconds, the low and high energies were tens of keV and hundreds of keV, and the resulting lags had magnitudes of up to several hundred milliseconds. This same study also confirmed an earlier result (Fenimore et al. 1995), namely, that pulse widths are narrower at higher energies. Here, too, however, the pulse durations are ~seconds. We also note that this earlier study, which extended only up to ~1000 keV, made no mention of spectral lag (Fenimore et al. 1995). The pulses that we are concerned with here are orders of magnitude shorter and orders of magnitude higher in energy. The fact that pulses tend to be narrower with increasing energy is an advantage, since our estimate of $\Delta t$ is not based on rise times or fall times but rather on the times of the peaks, which are better defined for narrower pulses. To our knowledge, no studies have focused on such high-energy, short-duration pulses.

A reliable measurement of $E_{\text{GRB}}$ will require a systematic study of the dispersion as a function of source redshift in order to separate out any residual GRB source geometry or emission mechanism effects that can bias the results. The ideal instrument to study $E_{\text{GRB}}$ using GRBs would have coverage to high energies (>10 MeV) and fine time resolution (<0.1 ms). RHESSI, designed to study solar flares in the 3 keV–17 MeV range with 1 µs photon timing, provides a unique, all-sky GRB monitor for these studies. RHESSI nicely complements the HETE-2 and upcoming Swift missions (Ricker et al. 2001; Gehrels 2000), which are able to localize GRBs for follow-up redshift determinations but do not have the spectral range for these studies. RHESSI will also provide a low-energy complement to the upcoming GLAST mission, which will also be sensitive for constraining $E_{\text{GRB}}$ (Norris et al. 1999). We have established a program to study the high-energy timing of the hundreds of bursts seen in the RHESSI detectors, with a goal of further constraining $E_{\text{GRB}}$. The best GRBs for this will have the high-energy emission as seen in GRB 021206 and, ideally, even faster flare peaks.

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