THE HUBBLE DIAGRAM OF TYPE Ia SUPERNOVAE IN NON–UNIFORM PRESSURE UNIVERSES

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ABSTRACT

We use the redshift-magnitude relation, as derived by Dąbrowski, for the two exact non–uniform pressure spherically symmetric Stephani universes with the observer positioned at the center of symmetry in order to test the agreement of these models with recent observations of high-redshift Type Ia supernovae (SN Ia’s). By a particular choice of model parameters, we show that these models can give an excellent fit to the observed redshifts and (corrected) B-band apparent magnitudes of the data, but for an age of the universe that is typically about 2 Gyr—and may be more than 3 Gyr—greater than in the corresponding Friedmann model, for which nonnegative values of the deceleration parameter appear to be favored by the data. We show that this age increase is obtained for a wide range of the non–uniform pressure parameters of the Stephani models. We claim that this paper is the first attempt to compare inhomogeneous models of the universe with real astronomical data. Several recent calibrations of the Hubble parameter from the Hubble diagram of SN Ia’s and other distance indicators indicate a value of $H_0 \approx 65$ and a Hubble time of $\sim 15$ Gyr. Based on this value for $H_0$ and assuming $\Lambda \geq 0$, the data would imply a Friedmann age of at least 13 Gyr and in fact a best-fit (for $q_0 = 0.5$) age of only 10 Gyr. Our Stephani models, on the other hand, can give a good fit to the data with an age of up to 15 Gyr. The Stephani models considered here could, therefore, significantly alleviate the conflict between recent cosmological and astrophysical age predictions. The choice of model parameters is quite robust: in order to obtain a good fit to the current data, one requires only that the non–uniform pressure parameter $a$ in one of the models be negative and satisfy $|a| \lesssim 3 \text{ km}^2 \text{s}^{-2} \text{Mpc}^{-1}$. This limit gives a value for the acceleration scalar of order $|\ddot{a}| \lesssim 0.66 \times 10^{-10} r \text{ Mpc}^{-1}$, where $r$ is the radial coordinate in the model. Thus, although the pressure is not zero at the center of symmetry, $r = 0$, the effect of acceleration is nondetectable at the center, since the acceleration scalar vanishes there. However, the effect of the non–uniform pressure on the redshift-magnitude relation is clearly seen, since neighboring galaxies are not situated at the center, and they necessarily experience acceleration. By allowing slightly larger negative values of $a$ one may fine-tune the model to give an even better fit to the data.

Subject headings: cosmology: theory — galaxies: distances and redshifts — supernovae: general

1. INTRODUCTION

The standard isotropic Friedmann cosmological models have naturally been the most widely investigated models in studies of the large-scale structure of the universe. This is hardly surprising, in view of their mathematical simplicity and their generic prediction of an approximately linear Hubble expansion at low redshift, which is in excellent agreement with observational data (see Strauss & Willick 1995; Postman 1997). Even in Friedmann models, however, the relation between apparent magnitude and log redshift is in general nonlinear at higher redshift and depends explicitly on the spatial curvature of the universe or, equivalently, on the deceleration parameter $q_0$.

For several decades astronomers have attempted to use the Hubble diagram of some suitable “standard candle” (e.g., first-ranked cluster galaxies) to place constraints on the global geometry of the universe by comparing the observed redshift-magnitude relation of the standard candle with that predicted in Friedmann models with different values of $q_0$ (see Peach 1970; Gunn & Oke 1975; Schneider, Gunn, & Hoessels 1983; Sandage 1988). Results from such analyses have thus far proved inconclusive, however. Due to the intrinsic dispersion in the luminosity function of the standard candles available, one previously had to reach at least $z \approx 1$ before the predictions of models with different values of $q_0$ became sufficiently distinct to be detectable; at the same time, however, the effects of luminosity and number density evolution also become important at these redshifts and are very difficult to correct for. The situation for the Hubble diagram of quasars is equally—if not even more—problematical. Tonry (1993) suggested that the constraints on $q_0$ from such studies were no better than $-1 < q_0 < 1$.

Recently, however, it has been suggested that Type Ia supernovae (hereafter SN Ia’s) represent a standard candle of sufficiently small dispersion to allow meaningful estimates of $q_0$, now to be derived from the SN Ia Hubble diagram at more moderate redshift. In Perlmutter et al. (1997; hereafter P97) a preliminary analysis is presented of seven distant SN Ia’s in the range $0.35 < z < 0.50$. A comparison of the SN Ia magnitudes and redshifts with the predicted relation for various Friedmann models appears to exclude large negative values of $q_0$ and is best fitted by...
values close to $q_0 = 0.5$. This poses a potentially serious problem for Friedmann models. Since many recent determinations of the Hubble constant (including a number of analyses using SN Ia's) suggest that $H_0$ lies in the range 65–70, this would imply an age of the universe of less than 10 Gyr in the “standard” $\Omega_m = 1, \Lambda = 0$ scenario. This result would appear to be in sharp conflict with recent astrophysical age determinations from, e.g., globular clusters and white dwarf cooling (see Chaboyer 1995; Hendry & Tayler 1996)—a conflict that is only slightly alleviated by revisions to globular-cluster age estimates in light of results from the Hipparchos satellite (Chaboyer et al. 1997). Reducing the value of $\Omega_0$ lessens the conflict somewhat, but agreement is still only marginal if one accepts a robust lower bound for the matter density of $\Omega_m = 0.3$, as has been suggested by several different methods of analyzing large-scale galaxy redshift surveys (see Strauss & Willick 1995).

This situation has helped to give a renewed impetus to models with a positive cosmological constant (see Liddle et al. 1996), which contributes an additional component, $\Omega_\Lambda$, to make up the critical density and at the same time extends the age of the universe by up to 2 Gyr—depending on the value of $H_0$ and $\Omega_m$. However, because of the relation

$$q_0 = \frac{1}{5} \Omega_m - \Omega_\Lambda,$$

it is clear that a positive value of $q_0$ is incompatible with a positive value of $\Lambda$, unless the matter density is at least $\frac{1}{5}$ of the critical density. The $q_0 = 0$ case, assuming $\Omega_m = \frac{4}{5}$, $\Omega_\Lambda = \frac{1}{5}$, and $H_0 = 65$, would give an age of the universe of just over 11 Gyr; as $q_0$ increases, the age is decreased still further. Thus, if the results of P97 prove to be correct, and the deceleration parameter is nonnegative, then the conflict between cosmological and astrophysical age predictions remains firmly unresolved—at least if $H_0 \gtrsim 65$. Independent results showing that a positive value of $\Lambda$ is incompatible with the so-called VLBI data (Kellermann 1993), using the angular diameter test, were obtained by Krauss & Schramm (1993) and Stelmacich (1994).

In this paper we propose one method to alleviate this age conflict by considering some inhomogeneous cosmological models in which the relation between the age of the universe and a generalized Hubble constant is more general than in the Friedmann case. Despite some theoretical plots of the observational quantities for inhomogeneous models (e.g., Goicoechea & Martin-Mirones 1987; Moffat & Tatarski 1995; Dąbrowski 1995; Humphreys, Maartens, & Matarrese 1997), this paper is—as far as we are aware—the first to compare these models to real astronomical data. In particular, we show that taking an inhomogeneous model into account allows us to obtain a good fit between the predicted redshift-magnitude relation and the P97 data, but for an age of the universe that is several Gyr older than in the Friedmann case. The models under consideration have been discussed before and are known as Stephani universes (see Kramer et al. 1980; Krasiński 1983; Dąbrowski 1993). In these models the energy density $\rho$ depends only on the cosmic time, similarly to the Friedmann models, but the pressure $p$ is a function both of spatial coordinates and of a time coordinate; hence the models are usually referred to as “inhomogeneous-pressure universes.” In the spherically symmetric case under consideration, the pressure is just a function of time and radial coordinates, which means that its values are the same on spheres, $r = \text{constant}$, around the center of symmetry, but differ from sphere to sphere. This essentially means that there is a spatial pressure gradient, and particles are accelerated in the direction from high-pressure regions to low-pressure regions. This effect is usually described by the acceleration vector $\dot{u}$, which in the case of spherical symmetry has only one (radial) component, or by the acceleration scalar $a$ (see eqs. [2.18] and [2.23] of Dąbrowski 1995). The acceleration represents the combined effect of gravitational and inertial forces on the fluid, which in fact, as in Newtonian physics, can not be separated. As a first approximation we assume that the observer is placed at the center of symmetry, which results in no pressure gradient at the observer’s position. This in a sense contradicts the Copernican Principle, but can easily be overcome by applying the formulae for a non-centrally located observer given in § 5 of Dąbrowski (1995)—an appropriate generalization, once larger samples of high-redshift supernovae become available.

In the standard approach we neglect the effect of pressure (i.e., we take pressureless dust, with pressure $p = 0$), as we evaluate chaotic velocities of galaxies to be small. This results in taking the acceleration $\dot{u}$ to be zero as well (see Ellis 1971 for a discussion of the relation between these quantities). However, if there were a large flux of neutrinos or gravitational waves, for instance, this assumption would not be correct, and we would need to take radiation pressure ($p = \frac{2}{3} \dot{\rho}$) into account. This has been, of course, investigated for isotropic cosmologies (Dąbrowski & Stelmacich 1986, 1987), and all the observational quantities have been found. Our main point here is, however, that early universe processes, such as phase transitions (for details see Vilenkin 1985; Kolb & Turner 1990; Vilenkin & Shellard 1994) may result in having different exotic types of matter (e.g., cosmic strings) with many different types of equations of state. In the easiest case (straight cosmic strings) they may end up with the exotic equation of state, $p = -\frac{1}{2} \dot{\rho}$; but in general, the equation of state can be more complicated (e.g., Vilenkin & Shellard 1994 and de Vega & Sanchez 1994 in the context of superstrings) or spatially dependent (e.g., Narlikar, Pecker, & Vigier 1991). The latter case would interest us especially. Of course, the standard energy conditions of Hawking and Penrose might be violated (cf. Hawking & Ellis 1973), which also happens for inflationary models, for instance, and our considerations here are, in a sense, on the same footing as those phenomena. As for the Stephani models, which do not admit any global barotropic equation of state, it has been shown that there exists a consistent nonbarotropic equation of state, and the full thermodynamical scheme exists (Quevedo & Sussman 1994; Krasiński, Quevedo, & Sussman 1997).

Regardless of the physical background of the models under consideration, one of our main tasks here is to draw attention to the entire class of inhomogeneous models that could be a useful alternative to Friedmann models in helping to resolve the apparent incompatibility of measurements of the Friedmann cosmological parameters. Even if the final outcome (after a thorough comparison with data) shows that the universe is indeed isotropic and homogeneous, this conclusion must be drawn by applying some “averaging scale-dependent procedures” (see Ellis 1984; Buchert 1997), since we evidently cannot see the universe as being like that on smaller scales. Being spherically symmetric, Stephani models can also be applied to a local underdense/overdense spherical region embedded in a globo-
ally isotropic Friedmann universe (Moffat & Tatarski 1995) in some analogy to the so-called Swiss cheese model (Kantowski, Vaughan, & Branch 1995).

The reader interested in more generic models should refer to the recent review by Krasinski (1997), as well as to some earlier papers concerning the most popular generalizations of the Friedmann models, such as the spherically symmetric Tolman universes, which are inhomogeneous-density, pressure-free dust shells (Tolman 1934; Bondi 1947; Bonnor 1974). Their properties have been studied quite thoroughly in Hellaby (1984, 1985) and in Hellaby (1987, 1988), and the observational relations for Tolman models were studied by Goiocochea & Martin-Mirones (1987), Moffat & Tatarski (1995), and quite recently by Humphreys et al. (1997). However, in none of these cases has a comparison with real astronomical data been carried out.

The outline of this paper is as follows: In § 2 we reproduce the redshift-magnitude relations for the two Stephani models considered here, as recently derived in Dąbrowski (1995). In § 3 we briefly describe the SN Ia data of In(1995). In § 4 we fit these data to the redshift-magnitude relations of both Friedmann and Stephani models and thus obtain best-fit values for the model parameters. We then discuss the results of these fits and compare the age of the universe given by the best-fit model parameters in the Friedmann and Stephani cases. Finally, in § 5 we summarize our conclusions.

2. THE REDSHIFT-MAGNITUDE RELATION FOR INHOMOGENEOUS PRESSURE MODELS

Recently Dąbrowski (1995) considered the redshift-magnitude relation for Stephani universes. Two exact cases were presented, and the predicted relations were plotted for a range of different parameter values. The relations were defined following the method of Kristian & Sachs (1966) of expanding all relativistic quantities in power series and truncating at a suitable order. Approximate formulae, to first order in redshift \( z \), for models I and II, respectively, were given by

\[
m_B = M_B + 25 + 5 \log_{10} \left[ \frac{cz}{2a \tau_0 + b} \right] + 1.086 \left[ 1 + 4a \left( \frac{a \tau_0 + b}{2a \tau_0 + b} \right)^2 \right] z, \tag{2}
\]

and

\[
m_B = M_B + 25 + 5 \log_{10} \left( \frac{2}{3 \tau_0} + \frac{c}{3} \right) z + 1.086 \left( \frac{9}{8} + \frac{c^2 a^4 \tau_0^3}{3} \right) z, \tag{3}
\]

which are essentially equations (5.6) and (5.10), respectively, of Dąbrowski (1995).\(^1\) Here \( m_B \) and \( M_B \) denote apparent and absolute magnitude, respectively, in the \( B \) band, \( \tau_0 \) denotes the current age of the universe, and \( c = 3 \times 10^8 \) km\( \text{s}^{-1} \) is the velocity of light. The constants \( a, b, \) and \( d \) are parameters of model I, and \( z \) is a parameter of model II. Convenient units for these parameters are \( [a] = \text{km}^2 \text{s}^{-2} \text{Mpc}^{-1}, [b] = \text{km} \text{s}^{-1}, [d] = \text{Mpc}, \) and \( [z] = \text{km} \text{s}^{-1} \text{Mpc}^{-1} \). From the definition of the acceleration scalar (see Dąbrowski 1995) we conclude that the parameters that relate directly to the nonuniformity of the pressure are \( a \) in model I and \( z \) in model II. In model I, \( b \) plays a similar role to that of the coefficient of time in the expression for the scale factor \( (R \propto \tau^p, \) where \( p \) is any power) in Friedmann models, and it can be considered as an immanently Friedmannian parameter of the Stephani models, while \( a \) and \( z \) are completely non-Friedmannian.

The models considered here are spherically symmetric, which means that we can have both centrally placed and noncentrally placed observers. For simplicity the redshift-magnitude relations reproduced above correspond to a centrally placed observer. Dąbrowski (1995) also derived relations for the case of a noncentrally placed observer, which, although more general, introduced several additional free parameters. The main difference in this more general case is that the apparent magnitude depends on the position of the source in the sky and renders comparison with the Friedmann case more complicated. We thus consider only centrally placed observers in this paper.

Note that these formulae are truncated at first order in \( z \) and thus would become increasingly inaccurate if applied to redshifts greater than or equal to unity. Since the redshifts SN Ia\'s grows, we will extend the approximate redshift-magnitude relations to higher order, as required.

The reader is referred to Dąbrowski (1995) for a detailed discussion of the derivation of the above formulae. Note, however, that for model I the expression for the generalized scale factor \( R(\tau) \) as a function of cosmic time \( \tau \) is given by equation (2.11) of Dąbrowski (1995):

\[
R(\tau) = a \tau^2 + b \tau + d. \tag{4}
\]

However, \( R(\tau) \) does not have to be positive (§ 4.1 and Fig. 6 of Dąbrowski 1993) for the Stephani models. For the subclass under consideration, \( R(\tau) \) easily relates to the spatial curvature of the models:

\[
k(\tau) = -4 \frac{a}{c^2} R(\tau), \tag{5}
\]

and the curvature index is not constant in time, as it is for Friedmann models. In principle, one can restrict \( R(\tau) \) to be positive (this is especially reasonable if we want to obtain the full Friedmann limit), which ends up with the simple relation for the spatial curvature of the models being positive for negative non-uniform pressure parameter \( a \) and negative for positive \( a \). Since \( R(\tau) = 0 \) at the singularity (the big bang), we can require that \( R(\tau) \to 0 \) as \( \tau \to 0 \) (i.e., we set the origin of our time coordinate at the big bang) and thus demand that \( d \) be identically zero. Therefore, according to condition (2.13) of Dąbrowski (1995), which, in fact, allows one to have the Friedmann limit for the Stephani models under consideration, we have

\[
b^2 = 1 \tag{6}
\]

for the values of \( b \). Without loss of generality we assume
that $b = +1$ (see the discussion above about the meaning of $b$ in Friedmann models), leaving only one free parameter of model I: $a$.

Other important physical quantities of model I are the following: the energy density

$$\frac{8\pi G}{c^2} \varrho(t) = \frac{3}{(ar^2 + b\tau)^2},$$

the pressure

$$\frac{8\pi G}{c^2} p(t) = \frac{1}{(ar^2 + b\tau)^2} \left[1 + 2a(ar^2 + b\tau)\tau^{-2}\right],$$

and the acceleration scalar

$$\dot{u} = -2 \frac{a}{c^2} r.$$

From the above one can see that the finite-density singularities of pressure appear at $r \to \infty$, where there is the antipodal center of symmetry. We assume that we are placed at the center of symmetry at $r = 0$, so that we have these singularities far away from us (see Fig. 6 of Dąbrowski 1993). Of course, we cannot live at the singularity of pressure.

At the center of symmetry the fluid fulfills the barotropic equation of state $p = -\frac{\varrho}{\tau}$ (as for straight cosmic strings, see Vilenkin 1985), while at $r \to \infty$ the pressure goes either to $+\infty$ or to $-\infty$. Then, assuming $R(t) > 0$, it diverges to $-\infty$ if $a > 0$ and to $+\infty$ if $a < 0$. In such a case, the fluid is accelerated away from a high-pressure region at $r = 0$ to low-pressure regions at $r \neq 0$ if $a > 0$, and toward a low-pressure region at $r = 0$ from high-pressure regions at $r \neq 0$ if $a < 0$. Of course, if $R(t) < 0$ the situation is the opposite. The acceleration scalar is zero at $r = 0$, and it diverges at $r \to \infty$.

In the case of model II, the time-dependent curvature index is given by ($\beta$ here plays the same “Friedmannian” role as $b$ in model II)

$$k(t) = -a\beta^2 c^{-2}\tau^{2/3},$$

while the energy density, pressure, and acceleration scalar are given, respectively, by (see Dąbrowski 1993, Appendix C)

$$\frac{8\pi G}{c^4} \varrho(t) = \frac{4}{3} \frac{1}{\tau^{2/3}} - \frac{3x}{\tau^{2/3}},$$

$$\frac{8\pi G}{c^4} p(t) = \frac{2x}{\tau^{2/3}} - \frac{4}{3} \frac{x\beta^2}{\tau^{4/3}} r^2 + a^2 \beta^2 r^2,$$

and

$$\dot{u} = -\frac{1}{2} a\beta r.$$

3. THE SN Ia OBSERVATIONS OF PERLMUTTER ET AL. (1997)

Type Ia supernovae are thought to be the result of the thermonuclear disruption of a white dwarf star that has accreted sufficient matter from a binary companion to reach the Chandrasekhar mass limit. For several decades they have been considered as suitable (nearly) standard candles for the testing of cosmological models, because of the relatively small dispersion of their luminosity function at maximum light and the fact that they are observable at very great distances. In recent years the Hubble diagram of SN Ia’s has been used by a number of authors to obtain estimates of the Hubble constant (cf. Riess, Press & Kirshner 1996; Hamuy et al. 1995, 1996; Branch et al. 1996) and the motion of the Local Group (Riess, Press, & Kirshner 1995).

P97 consider the redshift-magnitude relation of SN Ia’s at high redshift, observed by the “Supernova Cosmology Project,” as a means of constraining $q_0$. In P97, SN Ia’s are not treated as precise standard candles, a “stretch factor” correction being applied to account for the correlation between SN Ia luminosity and the shape of their light curve.

In this paper we use the redshifts and $B$-band magnitudes—with and without light-curve shape corrections—as presented in Table 1 of P97, to which the reader is referred for details of their observing strategy, data reduction procedures, and magnitude error estimates.

4. COMPARISON OF THE DATA WITH FRIEDMANN AND STEPHANI MODELS: RESULTS AND DISCUSSION

4.1. Friedmann Models

Figure 4 of P97 shows the Hubble diagram of their SN Ia’s, compared to the theoretical magnitude-redshift relations for a Friedmann model with different combinations of $\Omega_m$ and $\Omega_\Lambda$. While P97 argue correctly that one should generally express the Friedmann magnitude-redshift relations in terms of $\Omega_m$ and $\Omega_\Lambda$ separately, and not just in terms of their combination via $q_0$, for the redshift range of the P97 data, one may adequately approximate the relation by

$$m_B = M_B + 5 \log_{10} cz + 1.086(1 - q_0) z,$$

where

$$M_B = M_B - 5 \log_{10} H_0 + 25,$$

with the corresponding expression for the corrected $B$-band magnitudes. For reasons that will become clear when we consider the Stephani models, it is useful for us to write equation (14) in this form, in terms of $q_0$. Since we will make use of similar expressions for the Stephani universes, we construct for the Friedmann case the (reduced) $\chi^2$ statistic

$$\chi^2 = \frac{1}{n - 1} \sum_{i=1}^n \left[ \frac{m_B^{\text{obs}}(i) - m_B^{\text{pred}}(i)}{\sigma(i)} \right]^2,$$

where $n$ is the number of SN Ia’s, $m_B^{\text{obs}}(i)$ and $\sigma(i)$ are, respectively, the observed $B$-band apparent magnitude and error estimate of the $i^{th}$ SN Ia, and $m_B^{\text{pred}}(i)$ is the predicted $B$-band apparent magnitude of the $i^{th}$ SN Ia, for a given value of $q_0$, derived from equation (5) (or its equivalent for the corrected magnitudes). Following P97 we adopt $M_B = -3.17 \pm 0.03$ and $M_B,0 = -3.32 \pm 0.05$.

From equations (14) and (16) it follows that $\hat{q}_0$, the maximum likelihood (equivalently, the minimum $\chi^2$) estimate of $q_0$, is given by

$$\hat{q}_0 = -\left[ \sum_{i=1}^n \frac{x_i y_i}{\sigma^2(i)} \right]^{-1} \left[ \sum_{i=1}^n \frac{x_i^2}{\sigma^2(i)} \right],$$
cosmological parameters $\Omega_m$, $\Omega_\Lambda$, and the age of the universe $t_0$, all for a Hubble constant $H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Column (3) shows the corresponding value of $q_0$, calculated from equation (1). Column (5) gives the reduced $\chi^2$ of the fit to all seven SN Ia’s, while column (6) gives the reduced $\chi^2$ obtained using the five SN Ia’s with corrected magnitudes.

It is clear from Table 1 that one cannot obtain, with $H_0 \approx 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$, an acceptable fit either to the corrected or to the uncorrected data and at the same time ensure an age of the universe in excess of 14 Gyr. We discuss the situation for other values of $H_0$ below.

4.2. Stephani Model II

We now compare the SN Ia data to the predicted magnitude-redshift relations of the Stephani models. We first consider model II and the relation given by equation (3). If we compare equations (3) and (14), we see that in the limit as $z \to 0$, these equations are identical if and only if

$$\tau_0 = \frac{z}{2} H_0^{-1}. \quad (20)$$

In other words, for nearby SN Ia’s Stephani model II predicts the same linear redshift-magnitude relation as do Friedmann models, and with an age of the universe equal to $\frac{z}{2}$ times the inverse of the Friedmann Hubble constant. This is precisely the age of a Friedmann universe that is flat with a zero cosmological constant. In particular, if $H_0 \approx 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$, then independent of the value of the parameter $\varpi$, the age of the universe $\tau_0$ in model II is approximately 10 Gyr, which certainly appears to be too low to be consistent with astrophysical age determinations. Hence it would seem that model II is not particularly useful in resolving the current age conflict, since the age is inextricably linked to the value of the Friedmann Hubble constant: as soon as the latter is specified, then so too is the age of model II.

The link between the magnitude-redshift relation for model II and the Friedmann case is, nonetheless, interesting for the following reason. Note that equation (3) may be rewritten as

$$m_B = M_B + 25 - 5 \log_{10} \frac{2}{3 \tau_0} + 5 \log_{10} cz + 1.086(1 - q_0)z, \quad (21)$$

which means that for any given age of the universe $\tau_0$, we can choose the parameter $\varpi$ so that the magnitude-redshift relation for model II is identical in form to equation (14), with $\tau_0 = \frac{z}{2} H_0^{-1}$. The crucial difference is that whereas in the Friedmann case with $\Lambda = 0$, equation (20) implies that $q_0 = 0.5$, in the Stephani case we still retain the freedom to specify a relation that is equivalent to any value of $q_0$ by suitable choice of $\varpi$.

In particular, by choosing $\varpi < 0$ one can obtain a magnitude-redshift relation that corresponds to a Friedmann model with $q_0 > 0.5$. This is in full analogy to Friedmann models if the relation for the curvature of the Stephani models is taken into account (Dąbrowski 1995, eq. [2.14]). It shows that the time-dependent curvature index (eq. [10]) for $\varpi < 0$ is positive (if cosmic time $\tau > 0$), while for $\varpi > 0$ it is negative. The pressure (eq. [12]) is positive or negative for $\varpi$ being positive or negative, respectively, at the center of symmetry $r = 0$, and it diverges either to $+\infty$ or
to $-\infty$ (depending on the values of other parameters) at the antipodal center of symmetry $r \to \infty$, and the particles are either accelerated away or toward $r = 0$. Bearing in mind the effect of curvature (eq. [10]) of the models, one can roughly say that the inclusion of non-uniform pressure mimics a flat Friedmann model $q_0 = \frac{1}{2}$ to look curved: positively curved for $\alpha < 0$ and negatively curved for $\alpha > 0$.

While in the Friedmann case $q_0 > 0.5$ would imply an age of the universe $\tau_0 < \frac{2}{3}H_0^{-1}$, in the case of model II we still have $\tau_0 = \frac{3}{2}H_0^{-1}$. Model II would therefore be of considerable interest if SN Ia’s (or other) observations were to suggest that $q_0 > 0.5$, which certainly cannot be ruled out on the basis of the data alone. As an illustrative (if somewhat extreme) example, consider the case in which $\Omega_m = 2$ and $\Omega_s = 0$, so that $q_0 = 1$ for the Friedmann model. As can be seen from equation (22), the Friedmann and model II magnitude-redshift relations are identical when

$$\alpha = -\frac{4}{5}c^2 \tau_0^{-4/3}. \quad (23)$$

Whereas the age of the Friedmann model with $q_0 = 1$ would be reduced by $\sim 15\%$, compared to the Einstein–de Sitter age (i.e., $\tau_s \approx 0.57H_0^{-1}$), for the Stephani model II we still have $\tau_s = \frac{3}{2}H_0^{-1}$. Although the scenario of $q_0 > 0.5$ appears highly unlikely in view of a variety of other observations of large-scale structure and CMBR anisotropies, this serves as an interesting example of how the Stephani models can be compatible with high-redshift observations over a larger region of parameter space than Friedmann models support.

4.3. Stephani Model I

One of the reasons why model II is not particularly useful as an extension of the Friedmann case is that the effect of the nonuniform pressure (manifest via the parameter $\alpha$) only becomes apparent at high redshift. The situation with Stephani model I is different, however. We can see from equation (2) that the effect on the magnitude-redshift relation of the non–uniform pressure parameter $\alpha$ is immediate. In particular, therefore, even at low $z$, model I does not in general reduce trivially to a specific Friedmann case.

Note that after setting the parameter $b$ from model I equal to unity (provided $d = 0$), we can rewrite equation (2) to depend only on the non–uniform pressure parameter $\alpha$. Thus

$$m_B = M_B + 25 + 5 \log_{10} \left[ cz \frac{\alpha r_0^2 + \tau_0}{2 \alpha r_0 + 1} \right] + 1.086 \left[ 1 + 4 \frac{\alpha r_0^2 + \tau_0}{(2 \alpha r_0 + 1)^2} \right] z. \quad (24)$$

As a means of estimating what range of values of $\alpha$ and $\tau_0$ will give an acceptable fit to the P97 data, it is useful to note further that we may recast equation (24) in the form

$$m_B = M_B + 25 + 5 \log_{10} cz - 5 \log_{10} \tilde{H}_0 + 1.086(1 - \tilde{q}_0)z, \quad (25)$$

where

$$\tilde{H}_0 = \frac{2 \alpha r_0 + 1}{\alpha r_0^2 + \tau_0}, \quad (26)$$

and

$$\tilde{q}_0 = -4\alpha \frac{\alpha r_0^2 + \tau_0}{(2 \alpha r_0 + 1)^2}. \quad (27)$$

Equation (24) now takes the same functional form as equation (14), as was similarly pointed out in Dąbrowski (1995), with $\tilde{H}_0$ and $\tilde{q}_0$ replacing $H_0$ and $q_0$. We can think of $\tilde{H}_0$ (which is $\frac{1}{2}$ of the expansion scalar $\Theta$ of the model) and $\tilde{q}_0$ as a generalized Hubble parameter and deceleration parameter that are related to the age of the universe in a different way from that of the Friedmann case. The key question of interest here is therefore whether one can construct generalized parameters $\tilde{H}_0$ and $\tilde{q}_0$ that are in good agreement with the P97 data, but which correspond to a value of $\tau_0$ that exceeds that Friedmann age with $H_0 = \tilde{H}_0$ and $q_0 = \tilde{q}_0$. The fact that we can write the model I redshift-magnitude relation in the form of equation (25) confirms, however, that our choices of $\tau_0$ and $\alpha$ are certainly not arbitrary. Combinations of $\tau_0$ and $\alpha$ that give a large negative value of $\tilde{q}_0$, for example, would clearly be incompatible with the SN Ia Hubble diagram—just as was the case for Friedmann models with $q_0 < 0$.

In order to estimate the parameters $\tau_0$ and $\alpha$, we construct the reduced $\chi^2$ statistic

$$\chi^2 = \frac{1}{n - 2} \sum_{i=1}^{n} \frac{\left[ m^\text{obs}(i) - m^\text{pred}(i; \tau_0, \alpha) \right]^2}{\sigma(i)}, \quad (28)$$

where $n$ is the number of SN Ia’s and $m^\text{pred}(i; \tau_0, \alpha)$ is obtained for the $i$th SN Ia from equations (25), (26), and (27). We determine $M_B$ from Hamuy et al. (1996), adopting their best-fit value of $H_0 = 63.1 \text{ km s}^{-1} \text{ Mpc}^{-1}$, determined from four local calibrating SN Ia’s. Thus,

$$M_B \equiv \mathcal{M}_B + 5 \log H_0 - 25 = -19.17 \pm 0.03, \quad (29)$$

and

$$M_{B,\text{corr}} \equiv \mathcal{M}_{B,\text{corr}} + 5 \log H_0 - 25 = -19.32 \pm 0.05. \quad (30)$$

The dependence of equation (28) on $\tau_0$ and $\alpha$ is nonlinear, making a plot of the surface $z = \chi^2(\tau_0, \alpha)$ difficult to interpret. We therefore consider slices through this surface. Moreover, for plots of $\chi^2$ at constant $\tau_0$, it is useful to plot $\chi^2$ as a function of $a^{-1}$. Figure 2 shows $\chi^2$ as a function of $a^{-1}$ for $\tau_0 = 13$ Gyr, using the uncorrected P97 data. The behavior of $\chi^2$ is seen to be rapidly varying for values of $a^{-1}$
around zero, but is essentially flat for all \( a^{-1} \gtrsim 0.3 \). Thus, provided \( |a| \lesssim 3 \text{ km}^2 \text{ s}^{-2} \text{ Mpc}^{-1} \), we see that the goodness of fit of model I to the data is essentially independent of the value of \( a \) and depends only on \( \tau_0 \). A very similar curve is obtained for the corrected magnitudes.

We can understand the rapidly varying behavior of \( \chi^2 \) for small values of \( |a^{-1}| \) by considering the behavior of \( H_0 \) and \( \dot{q}_0 \) in equations (26) and (27). We see that when \( a^{-1} = -2\tau_0 \), we have \( H_0 = 0 \) and \( |\dot{q}_0| \to \infty \), so that \( \chi^2 \to \infty \). It therefore follows that for \( \tau_0 = 13 \text{ Gyr} = 13.26 \times 10^{-3} \text{ s Mpc}^{-1} \), a singular value of \( \chi^2 \) occurs when \( a^{-1} \approx -0.026 \), and \( \chi^2 \) varies very rapidly close to this value. However, the range of very small \( a^{-1} \) is not of interest to us, since it deviates too far from Friedmann models.

Figures 3a and 3b show plots of \( \chi^2 \) as a function of \( a^{-1} \), but now with \( \tau_0 = 15 \text{ Gyr} \), for the corrected and uncorrected magnitudes, respectively. A narrower range of values of \( a^{-1} \) is shown in order to illustrate better the behavior of \( \chi^2 \) for small \( |a^{-1}| \). We find that \( \chi^2 \) again tends to infinity when \( a^{-1} = -2\tau_0 \) and is again essentially flat for all \( |a^{-1}| \gtrsim 0.3 \). Note also that the asymptotic value of \( \chi^2 \) is a little smaller than that of \( \tau_0 = 13 \text{ Gyr} \), and moreover, that there exists a narrow range of values of \( a \) for which \( \chi^2 \) dips appreciably below its asymptotic value.

Figures 4a and 4b, on the other hand, show plots of \( \chi^2 \) as a function of \( \tau_0 \) for \( a = -1.0 \) for the uncorrected and corrected magnitudes, respectively. This value of \( a \) is chosen to be representative of the asymptotic behavior of \( \chi^2 \); essentially the same plots would be obtained for all \( a \lesssim 3 \text{ km}^2 \text{ s}^{-2} \text{ Mpc}^{-1} \). We can see that model I gives a good fit to the data for \( \tau_0 \) in the range 13–15 Gyr.

Figures 5a and 5b show the values of \( H_0 \) and \( \dot{q}_0 \), respectively, as a function of \( \tau_0 \), again for the representative value of \( a = -1.0 \). Also shown for comparison are the best-fit values of \( H_0 = 63.1 \text{ km} \text{ s}^{-1} \text{ Mpc}^{-1} \) and \( \dot{q}_0 = 0.5 \), obtained from Hamuy et al. (1996) and § 4.1, respectively. We see from Figure 5b that \( \dot{q}_0 \) is almost independent of \( \tau_0 \) over the range shown, increasing from \( q_0 \approx 0.04 (\tau_0 = 10 \text{ Gyr}) \) to \( q_0 \approx 0.08 (\tau_0 = 20 \text{ Gyr}) \). The dependence of \( H_0 \) on \( \tau_0 \) is rather more pronounced, however: good agreement with the Hamuy et al. value is found in the age range 14–16 Gyr.

The behavior of \( H_0 \) and \( \dot{q}_0 \) in Figures 5a and 5b makes sense when we consider the form of equations (26) and (27) for \( a\tau_0 \ll 1 \). To first order in \( a\tau_0 \), these reduce to

\[
\dot{H}_0 = \frac{1}{\tau_0} \left( 1 + a\tau_0 \right), \tag{31}\]

and

\[
\dot{q}_0 = -4a\tau_0. \tag{32}\]

Thus we see that as \( a\tau_0 \to 0 \), \( \dot{H}_0 \to \tau_0^{-1} \) and \( \dot{q}_0 \to 0 \).
The potential usefulness of model I is now apparent. In the limit where $\alpha \tau_0 \to 0$, the age of the universe in this model is increased by 50%, compared to the Einstein–de Sitter age, giving, for example, $\tau_0 = 15$ Gyr (compared to only 10 Gyr) for $H_0 \sim 65$. Of course, when $\alpha \tau_0 \to 0$, $q_0 \to 0$ also, so that the more meaningful comparison is between model I and a Friedmann model with $q_0 = 0$. Taking $\Omega_m = 0.3$, $q_0 = 0$, and $H_0 = 65$, however, gives a Friedmann age of only 12.5 Gyr, so that the model I age is still 2.5 Gyr greater than the Friedmann age. Taking larger values for the matter density gives, for example, $9.8 \times 10^8$ Gyr (compared to only 10 Gyr) giving, for example, $q_0 \to 0$, away from which par-

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Figures 5a and 5b are indicative of the limiting behavior of $\dot{H}_0$ and $\ddot{q}_0$, for small $\alpha \tau_0$. As we already remarked for the $\chi^2$ plots, one obtains similar plots for all $\alpha \leq 3$ km$^2$ s$^{-2}$ Mpc$^{-1}$, with $\dot{H}_0$ strongly varying as a function of $\tau_0$, but with $\ddot{q}_0$ much more weakly dependent on $\tau_0$. The smaller the value of $|\alpha|$, the closer lies $\ddot{q}_0$ to zero at a given $\tau_0$—as is obvious from equation (32). The sign of $\alpha$ does have some bearing on the goodness of fit of the model to the P97 data, however. Although it can be seen from Figure 1 and Table 1 that the current data give an acceptable fit for $q_0 = 0$, the fit rapidly deteriorates for negative values of $q_0$. Thus, if $\alpha > 0$ (i.e., a high-pressure region at $r = 0$, away from which particles are accelerated), then $\ddot{q}_0 \to 0$ from below, and a value of $a = 3$ km$^2$ s$^{-2}$ Mpc$^{-1}$ would imply that $\ddot{q}_0 \approx -0.2$ for $\tau_0 = 15$ Gyr, which gives only a marginally acceptable fit to the P97 data. If $\alpha < 0$, on the other hand (i.e., a low-pressure region at $r = 0$ toward which particles are accelerated), then one can obtain much better fits: e.g., $\alpha = -3$ km$^2$ s$^{-2}$ Mpc$^{-1}$ implies that $\ddot{q}_0 \approx 0.2$ for $\tau_0 = 15$ Gyr.

We have emphasized the limiting behavior of model I for small $\alpha$ in order to make clear that fact that the usefulness of the model is a fairly robust result and is not too sensitive to the exact value of $\alpha$ that is chosen—although it is true that negative values of $\alpha$ are favored. It is particularly noteworthy that the limit $|\alpha| \leq 3$ may be considered as the restriction on this parameter from the observational data and allows us not to be too far from the range in which Friedmann models are valid. In other words one can obtain a good fit to the P97 data with a significantly larger age, but without requiring that model I depart too much from a Friedmann model.

According to equation (9), $|\alpha| \leq 3$ km$^2$ s$^{-2}$ Mpc$^{-1}$ translates to a limit on the value of acceleration scalar of $|\ddot{u}| \leq 0.66 \times 10^{-10}$ Mpc$^{-1}$, where $r$ is the radial coordinate in the model. Notice that although the pressure is different from zero at the center $r = 0$, the effect of acceleration is not detectable at the center, since the acceleration scalar (and of course vector) vanishes there.

### Table 2

| $\tau_0$ (Gyr) | $a$ (km$^2$ s$^{-2}$ Mpc$^{-1}$) | $\dot{H}_0$ (km$^2$ s$^{-1}$ Mpc$^{-1}$) | $\ddot{q}_0$ | $\chi^2$ | $\chi^2_{\text{corr}}$ (Gyr) | $\tau_{\phi}(\Lambda = 0)$ (Gyr) | $\tau_{\phi}(\Omega_m + \Omega_\Lambda = 1)$ (Gyr) |
|----------------|-------------------------------|-----------------------------|--------------|--------|----------------|----------------|------------------|
| 13.00……       | $-10.00$                      | 63.7                        | 0.86         | 1.67   | 1.07           | 9.1             | 9.5              |
| 13.25……       | $-8.33$                       | 64.4                        | 0.67         | 1.40   | 0.79           | 9.5             | 9.8              |
| 13.50……       | $-7.14$                       | 64.5                        | 0.55         | 1.24   | 0.64           | 9.9             | 10.0             |
| 13.75……       | $-6.67$                       | 63.8                        | 0.51         | 1.15   | 0.56           | 10.2            | 10.2             |
| 14.00……       | $-6.25$                       | 63.0                        | 0.48         | 1.12   | 0.58           | 10.4            | 10.3             |
| 14.25……       | $-5.55$                       | 62.6                        | 0.42         | 1.14   | 0.65           | 10.8            | 10.6             |
| 14.50……       | $-5.00$                       | 62.0                        | 0.38         | 1.22   | 0.77           | 11.1            | 10.8             |
| 14.75……       | $-4.55$                       | 61.4                        | 0.34         | 1.32   | 0.94           | 11.4            | 11.0             |
| 15.00……       | $-4.17$                       | 61.3                        | 0.27         | 1.48   | 1.14           | 11.9            | 11.2             |
| 15.25……       | $-3.85$                       | 60.0                        | 0.29         | 1.66   | 1.42           | 12.0            | 11.4             |
| 15.50……       | $-3.33$                       | 59.6                        | 0.25         | 1.88   | 1.72           | 12.4            | 11.6             |
| 15.75……       | $-3.12$                       | 58.8                        | 0.23         | 2.14   | 2.07           | 12.7            | 11.8             |
| 16.00……       | $-0.35$                       | 58.1                        | 0.22         | 2.43   | 2.45           | 12.9            | 12.0             |

**Notes.** Cols. (7) and (8) show the age $\tau_{\phi}$ of the universe in a Friedmann model with the same values of $H_0$ and $q_0$, with $\Lambda = 0$ and $\Omega_m + \Omega_\Lambda = 1$, respectively.
If we allow slightly larger negative values for the non-uniform pressure parameter \( a \), then by careful choice of \( a \) and \( \tau_0 \), we can obtain fits that give significant positive values of \( \tilde{q}_0 \) while retaining a significant difference between the Stephani and Friedmann ages. Although these fits require slightly more fine-tuning, they are clearly in much closer agreement with the best-fit Friedmann value of \( q_0 = 0.5 \). Some examples of fits of this type are given in Table 2.

The final two columns of Table 2 show the age \( \tau_{fp} \) of the universe in a Friedmann model with \( \Lambda = 0 \) and \( \Omega_m + \Omega_{\Lambda} = 1 \), respectively.

Some general trends are evident from Table 2. Note that in all cases, we see that as \( \tau_0 \) increases and \( |a| \) becomes smaller, the values of \( \tilde{H}_0 \) and \( \tilde{q}_0 \) are both reduced, and the goodness of fit to the corrected and uncorrected data gradually deteriorates. For \( \tau_0 \geq 16 \) Gyr, the goodness of fit quickly becomes unacceptably large: although by a suitable choice of \( a \) one can ensure that \( \tilde{q}_0 \) remains positive, the value of \( \tilde{H}_0 \) also reduces, and overall, the fit deteriorates. Further decrease in \( |a| \) for \( \tau_0 \geq 16 \) Gyr increases the value of \( \tilde{H}_0 \) but pushes \( \tilde{q}_0 \) closer to zero, so that the goodness of fit remains poor. This behavior can also be easily seen from equations (31) and (32).

It would seem, therefore, that an age of \( \tau_0 = 15 - 16 \) Gyr represents the upper age limit from model I with the P97 data—at least if one adopts the SN Ia calibration of \( H_0 \). Moreover, if subsequent analyses of larger samples of SN Ia’s serve to tighten the limits on a positive value of \( \tilde{q}_0 \), then this limiting age could perhaps be reduced to \( \tau_0 \sim 14 \) Gyr.

The important point to note, however, is that in this case the age limits on Friedmann models would also be reduced. As can be seen from Table 2, a value of \( \tilde{q}_0 \sim 0.5 \) can be well fitted by model I with \( \tau_0 = 14 \) Gyr, which still represents an increase in the age of the universe of more than 3 Gyr compared to the \( q_0 = 0.5 \) Friedmann model with either zero cosmological constant or critical density.

5. CONCLUSIONS

In this paper we have considered the two exact non-uniform pressure spherically symmetric Stephani universes and have compared the redshift-magnitude relations derived for these models in Dąbrowski (1995) to the recent SN Ia observations of P97. We have investigated the extent to which, with a suitable choice of the Stephani model parameters, we may obtain good fits of the P97 data to the predicted redshift-magnitude relations, but for universes that are older than their Friedmann counterparts.

We emphasize that we have considered only the case of centrally placed observers, which results in having zero pressure gradient in our location. Although this is clearly a special case, it is mathematically the simplest possibility and merits consideration first. It can be extended relatively simply to the case of a non-centrally placed observer using the formulae given in § 5 of Dąbrowski (1995). However, such a generalization introduces additional model parameters that make apparent magnitude a function both of redshift and of direction in the sky. In principle, we could have estimated these parameters in this paper, but the small size of the P97 sample would make such a parameter fit statistically meaningless. Indeed, even for the case of a centrally placed observer, one should ideally consider a much larger supernova sample. We will consider the more general case in the future, when the number of observed supernovae has significantly increased.

We have found that the age of the universe in Stephani model II is, in fact, independent of the non-uniform pressure parameter \( a \) and is equal to the age of an Einstein–de Sitter–Friedmann model, i.e., \( \tau_0 = \frac{3}{2} H_0^{-3} \). This model would be of considerable interest if the total density of the universe were greater than the critical density, since the age of the corresponding Friedmann model would then be less than the Einstein–de Sitter age. Since there exists no compelling observational evidence to suggest that the universe is closed, however, model II is of limited use, as it would in general predict a smaller age than would its Friedmann counterpart with the same value of the Hubble constant.

We have shown that Stephani model I would be of considerably greater interest, however. We have found that the redshift-magnitude relation predicted for model I can be expressed in terms of two parameters: the age \( \tau_0 \) of the universe and the non-uniform pressure parameter \( a \). One can write the redshift-magnitude relation in exactly the same form as in the Friedmann case, introducing an effective Hubble parameter \( \tilde{H}_0 \) and deceleration parameter \( \tilde{q}_0 \), which are nonlinear functions of \( a \) and \( \tau_0 \). We have shown that for a wide range of different values of \( a \), we can obtain good fits to the P97 data for a universe of age up to 15 Gyr, which is typically 2 or 3 Gyr greater than the corresponding Friedmann model. These fits are quite robust, requiring only that \( |a| \lesssim 3 \text{ km}^2 \text{ s}^{-2} \text{ Mpc}^{-1} \), which gives the value of the acceleration scalar \( \dot{a} \) of order \( |\dot{a}| \lesssim 6.6 \times 10^{-10}r \text{ Mpc}^{-1} \), where \( r \) is the radial coordinate of the model. Then, although the pressure is not zero at the center of symmetry \( r = 0 \), the effect of acceleration is nondetectable at the center, since the acceleration scalar vanishes there. However, this effect is easily extracted from the redshift-magnitude relation, since neighboring galaxies are not situated at the center, and they necessarily experience acceleration.

The above robust fits are for the limiting case in which the product \( a \tau_0 \) is small, and they imply an effective deceleration parameter \( \tilde{q}_0 \) that is close to zero—a value that is certainly not as yet ruled out by the P97 data, although the fit to a value of \( \tilde{q}_0 \sim 0.5 \) is currently somewhat better. By some fine-tuning of the model parameters, one can obtain good fits with \( \tilde{q}_0 \sim 0.5 \) and \( \tau_0 \sim 14 \) Gyr. While an age of only 14 Gyr may still be in conflict with astrophysical age determinations, the conflict is considerably worse for Friedmann models: the age of an \( H_0 \sim 65 \), \( q_0 \sim 0.5 \), critical-density Friedmann universe is only \( \sim 10 \) Gyr, and for closed models with \( q_0 \sim 0.5 \), the age is even smaller.

Thus, we find that model I can give an age of the universe that is consistently and robustly between 2 and 3 Gyr older than the oldest acceptable open or flat Friedmann models.

Since the preliminary results of P97 were first presented, there have been several important developments in the measurement of fundamental cosmological parameters. The recalibration of the RR Lyrae distance scale has revised age estimates of the oldest globular clusters to \( t_0 = 11.7 \pm 1.4 \) Gyr (see Chaboyer et al. 1997). This undoubtedly lessens the conflict with the standard (\( \Omega_m = 1, \Lambda = 0 \)) cosmological model, particularly if one argues for a value of \( H_0 \sim 55 \) (see Tammann 1996). If one requires a “gestation period” of around 1 Gyr between the big bang and the formation of the first globular clusters, however, then agreement with the standard model is still only marginal—even for \( H_0 = 55 \)—and open Friedmann models would appear to be favored. Since we have argued in this paper that the P97
data do not yet exclude models with $q_0 \sim 0$, it is only fair to point out that open Friedmann models with $\Lambda \neq 0$, $q_0 \sim 0$, and $H_0 \sim 55$ offer a comfortable resolution of the age problem, in light of the Chaboyer et al. results. Adopting, for example, $\Omega_m = 0.5$, $\Omega_\Lambda = 0.25$, and $H_0 = 55$ gives a Friedmann age of $t_F = 14.0$ Gyr.

It is important to recognize, however, that this agreement rests crucially upon the value of $H_0$. If instead one adopts $H_0$ from the Hubble Space Telescope (HST) distance scale Key Project, $H_0 = 73 \pm 6 \pm 8$ km s$^{-1}$ Mpc$^{-1}$ (Freedman 1996), the above Friedmann age reduces to only 10.6 Gyr, and for the standard model (with $q_0 = 0.5$), it is only 8.9 Gyr. Moreover, the impact on $H_0$ of the Hipparcos recalibration of the LMC distance modulus has recently been shown by Madore & Freedman (1997) to be less significant than had previously been reported (see Feast & Catchpole 1997). Thus it would seem that the rumors of the end of the age problem are perhaps somewhat premature. If the value of $H_0$ does indeed lie close to that obtained by the HST Key Project, then we note that the Stephani models considered here can still give an age of up to $\sim 12.5$ Gyr with $q_0 \sim 0$ and up to $\sim 13.4$ Gyr with $q_0 \sim 0$. While the data clearly do not yet present a case for abandoning Friedmann models, they equally do not rule out the possible need to do so in the future—when bounds on $H_0$ and $q_0$ are tightened—and the Stephani models investigated here could indeed prove to be very important.

In this paper we have considered only a particular class of inhomogeneous models in order to illustrate their potential usefulness in addressing the apparent conflict between the observed values of the Friedmann model parameters. In future work we will extend our treatment to a wider class of non-Friedmann models and test their compatibility with the Hubble diagram of high-redshift objects and other cosmological observations. Such a comparison will be greatly enhanced by having larger samples of distant SN Ia—a development that the modern observing strategies adopted by P97 and other groups will shortly provide.

As for further progress in our comparison of the Stephani models with astronomical data, one should of course investigate such standard tests as galaxy number counts, the angular size-redshift relation, or microwave background anisotropies so far as to adopt the second-order terms in redshift $z^2$. This can indeed be done after some relatively tedious calculations, again using the powerful power series methods originally given by Kristian & Sachs (1966; see also Ellis 1971 for more detailed discussion). Many of these issues have been studied previously for Friedmann models (e.g., Dąbrowski & Stelmach 1986, 1987; Stelmach, Byrka, & Dąbrowski 1990) and will be the subject of future work.

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