Pion-nucleus elastic scattering studies within the microscopic folding potential

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Abstract. The pion-nucleus elastic scattering cross sections are calculated and compared to the data of pion scattering on ²⁸Si, ⁵⁸Ni, ²⁰⁸Pb at energies 162 and 291 MeV by using two kinds of the microscopic optical potentials (OPs). The first one is the folding OP, where the known nuclear density distributions are used while parameters of the elementary pion-nucleon amplitude of scattering are fitted to the data with the aim to estimate the in-medium effect, when pions are scattered not on the free but on the nuclear bounded nucleons. The analysis given when calculations are made using the local Kisslinger type OP, whose parameters are fitted to the data. The cross sections for both OPs are obtained by solving the Klein-Gordon wave equation. The role of a surface region of potentials is discussed.

1. Introduction

When considering the pion-nucleus scattering we apply two models of optical potentials. Thus, we use the local OP presented in Ref.[1] and based on the Krell-Ericson transformation [2] of the ordinary wave function φ(r) that obeys the non-local Kisslinger OP U = a(r) + ∇b(r)∇ [3]. The set of 12 parameters for each of the local OPs were obtained in [1] by fitting the calculated πA cross section to the data at energies of the 3-3 pion-nucleon resonance for scattering on ²⁰⁸Pb at E=162, 180 and 291 MeV and ¹⁶O at 162MeV.

In the other model [4] the optical potential was constructed in the high energy approximation as the folding integral of the πN amplitude and form factor of the nuclear density distribution function. For the pion-nucleus scattering this OP was applied in [5], and unlike the Glauber approach [6] for the scattering amplitude where integration takes place along the straight line trajectory of motion, in our calculations we apply the obtained OPs to solve the Klein-Gordon relativistic wave equation, and therefore the relativistic and distortion effects are accounted for exactly. These both the Kisslinger type and folding OPs were used to calculate differential cross sections and to compare them with experimental data with the aim to establish the meaningful regions of nuclei that reveal themselves in the pion scattering.

In our study we apply the both models of OPs in the region of the πN 3-3-resonance energies and compare the obtained cross sections with the respective set of experimental data. In the second model the fitted parameters are the πN total cross section and the ratio of the real to imaginary parts of the πN amplitude of scattering at zero angles. This makes it possible to...
Figure 1. Comparisons of the calculated $\pi^{\pm} + ^{28}\text{Si}$ elastic scattering cross sections with experimental data from [7]. Solid (red) curves correspond to the folding OP, and dashed (blue) curves are for the local Kisslinger-type OP. The best fit "in-medium" parameters $\sigma, \alpha, \beta$ of the $\pi N$ scattering amplitude are from Table 1.

establish the "in-medium" effect on these characteristics for scattering on bounded nucleons in nuclei as compared to the pion scattering on free nucleons. Below calculations are presented for the pion elastic scattering on nuclei $^{28}\text{Si}$, $^{58}\text{Ni}$ and $^{208}\text{Pb}$ at the energy range around the 33-resonance.

2. Basic equations

In our calculations we use the microscopic folding OP as it done in Ref. [4]

\[
U(r) = -\frac{hv}{(2\pi)^2} \sigma [i + \alpha] \cdot \int j_0(qr) \rho(q) f(q) q^2 dq,
\]

where $v$ is the relativistic velocity of the pion, $f(q)$ – form factor of the $\pi N$-amplitude $F_{\pi N}(q)$, and $\rho(q)$ is the form factor of a nuclear density distribution

\[
\rho(q) = \int e^{iqr} \rho(r) d^3r.
\]

Here we use the charge-averaged $\pi N$-amplitude

\[
F_{\pi N}(q) = \frac{k}{4\pi} \sigma [i + \alpha] \cdot f(q), \quad f(q) = e^{-\beta q^2/2},
\]

dependent on 3 parameters: the total cross section $\sigma$, the ratio of real to imaginary part of the amplitude at forward angles $\alpha$, and the slope parameter $\beta$. The nuclear density distributions
Figure 2. The same as in Fig.1 but for the elastic scattering of $\pi^\pm + {}^{58}\text{Ni}$. Experimental data are from [9].

The radius $R$ and diffuseness parameters $a$ of the target nuclei density distributions are taken from the respective electron-nucleus scattering data. We will fit three parameters $\sigma$, $\alpha$, and $\beta$, and thus the obtained magnitudes of them will characterize the amplitude of scattering of pions on the bound nucleons, i.e. the so-called "in-medium" effect.

We have also apply them in calculations for comparison with experimental data the local model of Kisslinger-type OP, developed in Ref. [1] in the form

$$U(r) = \frac{(\hbar c)^2}{2E} \frac{1}{1 - \alpha(r)} \left\{ q(r) - k^2 \alpha(r) - \frac{1}{2} \nabla^2 \alpha \left[ 1 + \frac{(1/2) \nabla^2 \alpha}{1 - \alpha} \right] \right\},$$

where the first term arises from the s- and p-wave $\pi N$ interaction, while the other smaller terms are from the p-wave alone. Here $q(r)$ and $\alpha(r)$ are rather complicated functions dependent on the nuclear density and its derivatives. Their expressions are done in [1] together with the respective set of 12 parameters, for each energy $E$.

After calculations of the respective folding and Kisslinger OPs, we use them to obtain the corresponding differential cross sections. To this aim we it was solved the Klein-Gordon equation in its form at conditions $E \gg U$ (below $\hbar = c = 1$) [8]

$$\left( \Delta + k^2 \right) \psi(\vec{r}) = 2\mu U(r) \psi(\vec{r}), \quad U(r) = U^H(r) + U_C(r).$$

$$\rho(r)$$ are taken in the form of a symmetrized Fermi-function, whose form factor is presented as

$$\rho_{SF}(r) = \rho_0 \frac{\sinh (R/a)}{\cosh (R/a) + \cosh (r/a)}, \quad \rho_0 = \frac{A}{1.25\pi R^3} \left[ 1 + \left( \frac{\pi a}{R} \right)^2 \right]^{-1},$$

$$\rho_{SF}(q) = -\rho_0 \frac{4\pi^2 a R}{q} \frac{\cos qR}{\sinh(\pi a q)} \left[ 1 - \left( \frac{\pi a}{R} \right) \coth(\pi a q) \tan qR \right].$$

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We have also apply them in calculations for comparison with experimental data the local model of Kisslinger-type OP, developed in Ref. [1] in the form
Figure 3. The same as in Fig.1 but for the elastic scattering of $\pi^\pm + ^{208}$Pb. Experimental data are from [9].

Here $k$ is the relativistic momentum of pions in center-of-mass system, and $\bar{\mu} = \frac{E_M}{E+M_A}$ is the relativistic reduced mass, $E = \sqrt{k^2 + m^2}$ — total energy, $m_\pi$ and $M_A$ are the pion and nucleus masses.

When comparing calculated cross sections to experimental data, we obtain the fitted parameters $\sigma, \alpha, \text{and } \beta$ which have a meaning of the "in-medium" parameters for the amplitude of scattering of pions on the bound nucleons. The procedure of fitting is estimated by the magnitude $\chi^2$-function

$$\chi^2 = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{d\sigma^{\text{exp}}/d\Omega(\theta_i) - d\sigma^{\text{theor}}/d\Omega(\theta_i)}{s_i^2} \right|^2,$$

(8)

where there are shown experimental and theoretical differential cross sections, and the $s_i = (\text{top}_i - \text{bottom}_i)$ values are done for the respective experimental cross sections to define error in a given point number $i$.

For the Kisslinger type OP, the respective 6x2=12 parameters of fitting were made in Ref. [1] for each set of colliding systems and energies.

3. Results of calculations

Figures 1, 2 and 3 demonstrate a reasonable agreement of the calculated and experimental differential cross sections of elastic scattering of $\pi^\pm$-mesons on nuclei $^{32}$Si, $^{58}$Ni and $^{208}$Pb at energies 162 and 291 MeV. Experimental data are taken from [7] and [9]. In this figures the solid red curves are calculated cross sections for the folding OPs obtained using the nuclear point-density-distributions in the form of a symmetrized fermi-function (4). The radius $R$ and
diffuseness $a$ parameters established as $R = 3.134$ fm and $a = 0.477$ fm for $^{28}$Si [10], 4.2 fm and 0.475 for $^{58}$Ni [11], 6.654 and 0.475 respectively for $^{208}$Pb [12]. The ”in-medium” parameters $\sigma, \alpha, \beta$ of a $\pi N$ scattering amplitude have been fitted to the experimental data [7, 9], and the corresponding $\chi^2$ magnitudes per one experimental point for the region of scattering angles up to 80 degrees are presented in Table 1.

As for the cross sections calculated within the local Kisslinger-type optical potentials, they are shown in Figs.1,2 and 3 by dotted blue curves. The respective 12 parameters of OPs for every target nucleons have already been fitted in Ref. [1] in dependence of an energy.

Their applications for our case lead to about the same $\chi^2$ magnitudes that were obtained for the folding OPs. Thus one can conclude that in spite of the so different constructions

**Table 1.** The best-fit parameters $\sigma$, $\alpha$, $\beta$ and respective $\chi^2$ quantities per one experimental point estimated at $\Theta_{\text{CM}}$ between 0 and 80 degrees.

| reaction | $T_{\text{lab}}, \text{MeV}$ | $\sigma, \text{fm}^2$ | $\alpha, \text{fm}^2$ | $\beta, \text{fm}^2$ | $\chi^2/k$ |
|----------|-------------------------------|----------------------|-----------------------|-----------------------|------------|
| $\pi^- + ^{28}$Si | 162 | 9.05 | 0.49 | 0.50 | 0.398 |
| $\pi^+ + ^{28}$Si | 8.15 | 0.68 | 0.81 | 0.776 |
| $\pi^- + ^{58}$Ni | 10.53 | 0.09 | 0.99 | 3.685 |
| $\pi^+ + ^{58}$Ni | 8.03 | 0.49 | 0.87 | 1.111 |
| $\pi^- + ^{208}$Pb | 10.00 | 0.30 | 1.01 | 0.644 |
| $\pi^+ + ^{208}$Pb | 6.22 | 0.61 | 1.28 | 0.719 |
| $\pi^- + ^{28}$Si | 291 | 4.86 | -0.80 | 0.39 | 1.169 |
| $\pi^+ + ^{28}$Si | 5.24 | -0.76 | 0.45 | 0.874 |
| $\pi^- + ^{58}$Ni | 4.55 | -0.86 | 0.32 | 0.863 |
| $\pi^+ + ^{58}$Ni | 5.47 | -0.65 | 0.37 | 0.558 |
| $\pi^- + ^{208}$Pb | 4.97 | -0.93 | 0.64 | 0.592 |
| $\pi^+ + ^{208}$Pb | 6.04 | -0.43 | 0.64 | 0.574 |
of potentials in the Kisslinger approach and in the folding model and sometimes so different behavior of them as functions of $r$, both OPs yield cross sections in reasonably good agreement with the experimental data at energies of 3-3 resonance. In our calculations it was found out that at energy 291 MeV both the Kisslinger and folding OPs are close to each other. However, at lower energies they have rather different behaviour in its internal region. As a rule the folding potential is rather smooth while the Kisslinger OP can be of the wave behaviour there.

### Table 2

|    | $\sigma^{av}$ 162 MeV | $\sigma^{av}$ 291 MeV |
|----|----------------------|----------------------|
| $^{28}$Si | 8.60                 | 5.05                 |
| $^{58}$Ni | 9.28                 | 6.37                 |
| $^{208}$Pb | 8.11                 | 5.51                 |
| $\sigma^{free}$ | 13.37                | 4.88                 |

In table 2 we present the result for the fitted total cross section parameter $\sigma^{av}$ of the pion scattering on the free and the bounded nucleons. One can see that the pion interaction with bound nucleons in nuclear matter at energies near the maximum of the 3-3-resonance at about 160 MeV is weaker than in the case of free nucleons.

### 4. Conclusion

Our approach based on 3-parameter OP, allows one to explain fairly well the experimental data on pion-nucleus elastic scattering. We show that both microscopic folding and local Kisslinger-type optical potentials provide a good agreement with the experimental data of pion-nucleus elastic scattering at intermediate energies at 162 and 291 MeV. Such an agreement takes place in spite of the fact that the Kisslinger-type potential has some variations in its inner part. At the same time both potentials are close to each other in the surface region and thus one can conclude that this region plays a decisive role in the mechanism of scattering.

In this connection, the data on inelastic $\pi A$-scattering may help us to reduce this ambiguity. Indeed, in the models of such processes one applies the derivative of the already fitted elastic scattering OP, that reveals itself mainly in the surface region. So, in Ref.[14] the data on the inelastic scattering cross sections with excitations of $2^+$ and $3^-$ collective states of nuclei were explained fairly well basing on the known folding OPs and on the only additional parameter, the deformation of a nuclear surface.

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