Sequential spectrum sensing based on multiple antennas with low computational complexity statistic

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Abstract: This letter presents a multiple antenna based spectrum sensing via a sequential detection technique. A log-likelihood ratio test of a low computational complexity statistic is defined and we develop the sequential detection based spectrum sensing using the test. The presented technique can reduce the computational complexity by overlapping samples for the calculation of each statistic while almost maintaining the accuracy of signal detection. Numerical examples are shown to validate the effectiveness of the presented technique.

Keywords: spectrum sensing, sequential detection, multiple antennas

Classification: Wireless Communication Technologies

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1 Introduction

Recently, the institutionalization of spectrum sharing systems has been focused in mobile wireless communications. For example, Citizens Broadband Radio Service (CBRS) is being considered as a spectrum sharing service by Federal Communications Commission (FCC) in the US [1]. In CBRS, citizens broadband radio service device (CBSD) sensing network provides the spectrum sensing results to priority access license (PAL) users and generalized authorized access (GAA) users to protect the incumbent users. The CBSD sensing network is composed of the sensing dedicated nodes and the GAA users with sensing capability, and therefore, the computational complexity of the spectrum sensing at the GAA users must be reduced. Sequential detection based spectrum sensing is one of the low complexity sensing technique. In sequential detection [2], sensing time is minimized subject to a constraint on the miss detection probability and the number of required samples or the sensing time are not fixed by the resulting likelihood ratio test unlike the Neyman-Pearson criterion based signal detection. Traditionally, a cooperative spectrum sensing technique based on sequential detection has been presented and analyzed [3, 4, 5]. A sequential detection technique for orthogonal frequency division multiplexing (OFDM) signals using its autocorrelation coefficient has been presented [3]. In [4, 5], a cooperative spectrum sensing technique based on sequential detection has been presented and analyzed. Besides, it is well known that the power consumption at signal detection dominates the multiplications [6]. Furthermore, the power consumption also depends on the frequency of logic level changes in field programmable gate array (FPGA) case and it is important to decrease the number of multiplications for signal detection. This letter presents a sequential detection technique based on multiple antennas with statistics which can be obtained with a low computational complexity [7].

2 Preliminary notions

2.1 System model

We assume that one secondary user (SU), such as the GAA user, attempts to sense a PU communication. The spectrum sensing problem can be considered as a binary hypothesis testing problem. Let $\mathcal{H}_1$ and $\mathcal{H}_0$ denote hypotheses which represent the PU that are active and inactive respectively. These can be expressed as

$$\mathcal{H}_1 : r_k(n) = h_k x(n) + v_k(n), \quad k = 1, \cdots, N_R,$$

$$\mathcal{H}_0 : r_k(n) = v_k(n)$$

where $r_k(n)$, $h_k$, $x(n)$, $v_k(n)$ and $N_R$ are the received complex signals at the $k$th receive antenna, the complex channel gain between the transmitter and the $k$th
receive antenna, the transmitted complex PU signal, and the additive white Gaussian noise (AWGN) at the kth receive antenna, respectively. \( x(n) \) and each \( v_k(n) \) are identical and independent distributions with each other.

### 2.2 Conventional technique

In the conventional technique \([5]\), for signal detection, log-likelihood ratio test is executed at each received sample. For the multiple antenna case, signal detection is executed as

\[
\frac{I_{N,N_r}}{I_{\tilde{N},\tilde{N}_r}} = -\frac{NN_r}{N(N-1)} \ln \frac{s_0^2}{s_1^2} + \sum_{n=1}^{N} \sum_{k=1}^{N_r} |r_k(n)|^2, \quad \begin{cases} 
\frac{I_{N,N_r}}{I_{\tilde{N},\tilde{N}_r}} \geq \overline{A} & \text{Accept } \mathcal{H}_1 \\
\frac{I_{N,N_r}}{I_{\tilde{N},\tilde{N}_r}} \leq \overline{B} & \text{Accept } \mathcal{H}_0. 
\end{cases}
\]

where \( N \), \( s_0^2 \), \( s_1^2 \), \( \overline{A} \) and \( \overline{B} \) are the number of samples, variance of \( v_k(n) \) and variance of \( r_k(n) \) for \( \mathcal{H}_1 \), the boundaries for the probability ratios, and they are given by \( \overline{A} = A/(s_0^2 - s_1^2) \), \( \overline{B} = B/(s_0^2 - s_1^2) \). \( A = \ln \left( \frac{1 - P_{\text{MISS}}}{P_{\text{FA}}} \right) \) and \( B = \ln \left( \frac{P_{\text{MISS}}}{1 - P_{\text{FA}}} \right) \).

Further, \( P_{\text{FA}} \) and \( P_{\text{MISS}} \) are the upper bound of false alarm probability and miss detection probability, respectively.

### 3 Presented technique

#### 3.1 Sequential detection with low computational complexity statistic

Traditionally, the statistic with low computational complexity for signal detection with multiple antennas has been presented \([7]\). In this section, we discuss the sequential detection using the statistic. We let \( N_0 \) denote the number of samples, and the statistic is given by

\[
T_{N_0,N_r}(n) = \sum_{m=n-N_0+1}^{n} U_{N_r}(m),
\]

where \( U_{N_r}(m) = \left\{ \sum_{k=1}^{N_r} |\text{Re}\{r_k(m)\}| \right\}^2 + \left\{ \sum_{k=1}^{N_r} |\text{Im}\{r_k(m)\}| \right\}^2 \). In this letter, we attempt to employ the statistic for the sequential detection. Note that \( U_{N_r}(m) \) follows not Gaussian but folded normal distribution at a signal sample \([7]\). For the employing log-likelihood test, a large \( N_0 \) is required for \( T_{N_0,N_r}(n) \) to follow Gaussian because of the central limit theorem \([8]\). However, a large \( N_0 \) reduces the signal detection opportunities because signal detection is executed only once for \( N_0 \). As a result, it may increase the number of samples for signal detection. Therefore, we consider the use of overlapping statistics for signal detection. Concretely, we attempt to execute signal detection using \( N_0 \) samples at \((1 - F_0)N_0\) intervals where \( F_0 \) is an overlap factor and \( 0 \leq F_0 \leq 1 \). To represent this, \( T_{N_0,N_r}(n) \) is rewritten as,

\[
T_{N_0,N_r}(n) = \sum_{m=n-F_0N_0+1}^{n} U_{N_r}(m) + \sum_{m=n-(1-F_0)N_0+1}^{n} U_{N_r}(m) + \sum_{m=n-F_0N_0+1}^{n} U_{N_r}(m).
\]

Note that the first term is calculated at the previous signal detection and the third term is also used for the next signal detection. Further, the first and third terms equal to zero for \( F_0 = 0 \) which means the case without \( F_0 \). The introduction of \( F_0 \) can expect to reduce the number of samples for signal detection. However,
this makes each $T_{N_0,N_R}(n)$ statistically non-independent. The effect of this on the performances of the presented technique is evaluated later. Based on Eq. (4), we define a log-likelihood test based on $T_{N_0,N_R}(n)$ as shown in the following equation,

$$
\lambda_{N_0,N_R}(n) = \sum_{m=0}^{n_c-1} \ln \left( \frac{p_1}{p_0} \left( \frac{T_{N_0,N_R}(n-m(1-F_0)N_0)}{T_{N_0,N_R}(n-m(1-F_0)N_0)} \right) \right)
$$

$$
= \frac{1}{2} \sum_{m=0}^{n_c-1} \left[ \left( \frac{T_{N_0,N_R}(n-m(1-F_0)N_0) - \mu_{0,N_R}}{\sqrt{2}\sigma_{0,N_R}} \right)^2 - \left( T_{N_0,N_R}(n-m(1-F_0)N_0) - \mu_{1,N_R} \right)^2 \right] - \ln \left( \frac{\sigma_{1,N_R}^2}{\sigma_{0,N_R}^2} \right), \tag{5}
$$

where $p_y[X]$, $\mu_{0,N_R}$, $\sigma_{0,N_R}^2$, $\mu_{1,N_R}$, $\sigma_{1,N_R}^2$, and $n_c$ are the probability density function of $X$ in $\mathcal{H}_y$, the mean and variance of $T_{N_0,N_R}(n)$ in $\mathcal{H}_0$, the mean and variance of $T_{N_0,N_R}(n)$ in $\mathcal{H}_1$ and a positive integer which represents the number of computation for the internal of $[\cdot]$ in Eq. (5), i.e., $n_c$ represents the number of samples used for the decision at $n$-th calculation, respectively. Note that these variables are algebraically obtained by $\mu_{X,2} = 2N_0s_X^2(1+2/\pi)$, $\sigma_{X,2}^2 = \left(8N_0s_X^2/\pi\right)(2+1/\pi)$, $\mu_{X,3} = 6N_0s_X^2(1/2+2/\pi)$ and $\sigma_{X,3}^2 = 12N_0s_X^2(3/4+4/\pi-6/\pi^2)$ [7] where $X \in \{0,1\}$. Eq. (5) can be rewritten for actual signal detection as

$$
\lambda_{N_0,N_R}(n) = \frac{1}{N_0} \sum_{m=0}^{n_c-1} \frac{\sigma_{1,N_R}^2}{\sigma_{0,N_R}^2} \left( \frac{T_{N_0,N_R}(n-m(1-F_0)N_0) - \mu_{0,N_R}}{\sqrt{2}\sigma_{0,N_R}} \right)^2
$$

$$
- \left( T_{N_0,N_R}(n-m(1-F_0)N_0) - \mu_{1,N_R} \right)^2, \tag{6}
$$

$$
\begin{cases}
\lambda_{N_0,N_R}(n) \geq \alpha = 2\sigma_{1,N_R}^2 \ln \left( \frac{\sigma_{1,N_R}^2}{\sigma_{0,N_R}^2} \right) + n_c \ln \left( \frac{\sigma_{1,N_R}^2}{\sigma_{0,N_R}^2} \right) & \text{Accept } \mathcal{H}_1 \\
\lambda_{N_0,N_R}(n) \leq \beta = 2\sigma_{1,N_R}^2 \ln \left( \frac{\sigma_{1,N_R}^2}{\sigma_{0,N_R}^2} \right) + n_c \ln \left( \frac{\sigma_{1,N_R}^2}{\sigma_{0,N_R}^2} \right) & \text{Accept } \mathcal{H}_0 \\
\beta < \lambda_{N_0,N_R}(n) < \alpha & \text{Go on}
\end{cases} \tag{7}
$$

where $\alpha$, $\beta$ and $\sigma_{1,N_R}^2$ are the boundaries of the probability ratios and $\sigma_{1,2}^2 = (8\sigma_1^2/\pi)(2+1/\pi)$ and $\sigma_{1,3}^2 = 12\sigma_1^2(3/4+4/\pi-6/\pi^2)$, respectively.

### 3.2 Computational complexity of presented technique

Next, we consider the complexity of the statistical tests. The presented statistical test calculates $\lambda_{N_0,N_R}(n)$ at $N_0$ samples whereas the conventional one calculates $T_{N_0,N_R}$ at every sample. From Eqs. (2), (6) and (7), we can obtain that the complexities of the presented technique and the conventional technique when signal detection is executed at every $N_0$ sample. The number of multiplications of the presented technique is $M_p = 2n_c(N_0 + 2)$ whereas the number of that of the conventional one is $M_c = 2N_0N$. Note that the number of multiplications of the presented technique does not depend on $N_R$ because of the statistic as shown in Eq. (3). In the next section, we numerically evaluate the number of multiplications for signal detection.

### 4 Numerical examples

We assume that the target signal is the OFDM signal where the number of samples for a data duration and guard interval are 512 and 36, and the data symbols are
modulated with quadrature phase shift keying and are conveyed by 300 subcarriers. To realize uncertainty, we introduce $s_1^2$ with a lower bound of $s_1^2$, i.e., we assume that $s_1^2 \leq s_1^2$ can be achieved, and we assume that $s_0^2$ representing a noise floor is known. These assumptions are the same as the ones in the general spectrum sensing problem. In the evaluation for the practical case, all $s_1^2$s are replaced with $s_1^2$s in Eqs. (2), (6) and (7). This is similar to the evaluation in [4]. Results shown in this section are obtained under that the lower bound of SNR is $-14$ dB whereas the actual SNR is randomly determined from $-14$ dB to $-12$ dB.

First, we evaluate the miss detection probability $P_{\text{MISS}}$ and false alarm probability $P_{FA}$ for the presented technique and the conventional technique. Figures 1(a) and 1(b) show the performance of the miss detection probability and the false alarm probability for the presented technique and the conventional one, respectively. As shown in figures, the performances of the presented technique satisfy the $P_{\text{FA}}$ and $P_{\text{MISS}}$ as the decision interval $N_D$ increases. Furthermore, it can be seen that a large $F_0$ deteriorates the performance of $P_{\text{MISS}}$ and $P_{FA}$, respectively.

Next, we show the average number of samples for signal detection in Fig. 2. Results in Fig. 2 are obtained for $N_0 = 128$ and $N_R = 2, 3$. Figure 2(a) shows the performance in $H_1$ case. As shown in Fig. 2(a), the performances of the presented technique with $N_0 = 128$ are slightly inferior to those of the conventional one in $H_1$. Figure 2(b) shows the performance in $H_0$ case. It can be seen that the performances of the presented technique are almost the same as those of conventional one. Furthermore, it can be seen that $F_0 \neq 0$ can improve the performances in $H_1$ and $H_0$ cases. Finally, we show the average number of multiplications for the presented technique and the conventional technique required for one signal detection. Figures 3(a) and 3(b) show the average number of multiplications in $H_1$ and $H_0$ case, respectively. As shown in both figures, the average numbers of multiplications for the presented technique are less than those for the conventional one.

From these, it can be said that the relationship between the presented technique and the conventional technique is similar to that between the least mean square (LMS) algorithm and the recursive least square (RLS) algorithm [9]. It is well known that the relationship with advantages and disadvantages; the LMS has a low complexity but a slow convergence whereas the RLS has a fast convergence but a high complexity, and the complexity depends on the number of multiplications. As
shown in Figs. 2 and 3, the average numbers of samples for the presented technique are just only slightly more than those of the conventional technique despite that the average numbers of multiplications for the presented technique are less than those of the conventional one. This indicates that disadvantages of the presented technique are few. Furthermore, as shown in these figures, the presented technique is particularly effective in low channel occupancy rate (COR) environment. This is also obviously from the fact that the $P_{\text{FA}}$ performances of the presented technique are superior to those of the conventional one as shown in Fig. 1(b).

5 Conclusion

This letter presented the sequential spectrum sensing technique based on multiple antennas using the low computational complexity statistic. We developed the sequential detection method using the log-likelihood ratio test by employing the statistic. The presented technique can reduce the computational complexity by overlapping samples for the calculation of each statistic while almost maintaining the accuracy of signal detection. Numerical examples were shown to validate the effectiveness of the presented technique.

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