Heat transfer coefficient distribution over the inconel plate cooled from high temperature by the array of water jets

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Abstract. The industrial rolling mills are equipped with systems for controlled water cooling of hot steel products. A cooling rate affects the final mechanical properties of steel which are strongly dependent on microstructure evolution processes. In case of water jets cooling the heat transfer boundary condition can be defined by the heat transfer coefficient. In the present study one and three dimensional heat conduction models have been employed in the inverse solution to heat transfer coefficient. The inconel plate has been heated to about 900°C and then cooled by one, two and six water jets. The plate temperature has been measured by 30 thermocouples. The heat transfer coefficient distributions at plate surface have been determined in time of cooling.

1. Introduction

Microstructure evolution processes in metals depend strongly on the temperature and the cooling rate [1]. Numerical simulations have been widely utilized for predicting microstructure and mechanical properties of steels [1,2,3]. In such simulation the boundary conditions of heat transfer play an important role. Recent developments of jet impingement boiling have been widely discussed by Qiu et al. [4]. The researchers are mainly focused on steady state processes typical for heat exchangers. In such cases determination of heat flux at the cooled surface do not requires inverse methods which utilize transient solutions to heat conduction problem. However, for metallurgical processes the heat flux or the heat transfer coefficient (HTC) as functions of the cooled surface temperature under transient cooling are the most important. Further, it has been recognized that the heat transfer during water cooling varies at the cooled surface in space and time [5,6]. In such a case efficient and reliable methods are necessary for identification of the local heat flux or the local HTC.

2. Experimental setup

The plate made of inconel was heated in the electrical furnace to a uniform temperature of about 900 °C. The plate having the thickness of 8 mm was placed horizontally, Fig. 1. The plate was 300 mm long and 210 mm wide. The water jet nozzles have been located 390 mm above the plate, approximately at the centre of the HTC determination domain having dimensions $B = 90$ mm and $L = 100$ mm, Fig. 2. The plate was cooled by one, two and six water nozzles. The nozzle diameter was 2 mm. The water temperature was 18 °C and the water flow rate was equal to: 4.9 dm³/min. for 1 nozzle, 7.2 dm³/min. for 2 nozzles and 10.4 dm³/min. for 6 nozzles, respectively. The water flow was measured with the turbine flow meter having the accuracy ±0.5%. The K type thermocouples with 80 μm diameter wires protected by a 500 μm diameter sheath were used to measure the plate temperature. Measurements were recorded at high frequency of 10 Hz by the data acquisition system equipped with a noise reduction filter having an accuracy class of ±0.5 K. The accuracy of the thermocouples was equal to ±0.4% of the measured temperature. The plate temperature was measured by 30 thermocouples located 2 mm below the cooled surface. The thermocouples locations have been shown in Fig. 3. The influence of the thermocouple on the plate temperature has been simulated with a high accuracy finite element model. The plate temperature with the thermocouple vertical to the cooled
surface is 2 K lower than that obtained for a solid plate. The exact location of the measuring junction of the thermocouple is unknown. In the case of vertical thermocouple the displacement of the measuring junction of 0.25 mm results in an error of 2 K. The simulations have confirmed that the maximum error of the thermocouple having accuracy class of ±0.4% at temperature of 1000°C is about ±4 K. Due to implementation of the noise reduction filter the thermocouple noise is negligible.

3. The inverse problem formulation
The method of the HTC determination over the cooled plate surface proposed by Malinowski at al. [7] has been utilized in the present study. The inverse solutions to the HTC distribution over the plate surface have been obtained by minimising the objective function:

\[ E(p_l) = \frac{1}{NT \cdot NP} \sum_{m=1}^{NT} \sum_{n=1}^{NP} \left\{ \frac{1}{1 + \left( \frac{\Delta T e^{m,n}}{\Delta \tau} \right)^2} \left( T e^{n,m} - T(p_l)^{n,m} \right)^2 \right\} \]

\[ + W_m \left( \frac{\Delta T e^{n,m}}{\Delta \tau} - \frac{\Delta T(p_l)^{n,m}}{\Delta \tau} \right)^2 \]

(1)

The term \( F^{n,m} \) has been specified as follows:
\[ F_{n}^{m} = \frac{\Delta T_{n}^{m}}{\Delta \tau} \quad \text{for} \quad \left| \frac{\Delta T_{n}^{m}}{\Delta \tau} \right| > 1 \]
\[ F_{nm} = 1 \quad \text{for} \quad \left| \frac{\Delta T_{n}^{m}}{\Delta \tau} \right| \leq 1 \]  

where:
- \( N_P \) number of the temperature measurements performed by one sensor,
- \( N_T \) number of the temperature sensors, \( N_T=81 \),
- \( T_{n}^{m} \) sample temperature measured by the sensor \( m \) at the time \( \tau_n \),
- \( T^{(p_i)}_{n} \) sample temperature at the sensor \( m \) and time \( \tau_n \) obtained from the solution to the heat conduction in the plate for a given vector of \( p_i \) parameters,
- \( p_i \) unknown parameters (the HTC values) to be determined by minimizing the objective function,
- \( W_m \) weighting parameter coupled with the temperature sensor \( m \),
- \( \Delta \tau \) time increment, \( \Delta \tau = 0.1 \) s.

The HTC distribution at the cooled surface has been approximated by 9 surface elements with cubic shape functions from serendipity family. The division of the cooled surface into elements has been shown in Fig. 3. Each element has 12 nodes. Four nodes are located at element’s corners and 8 nodes at element’s sides, two nodes at each side. Since 9 elements have been utilized for the HTC approximation over the cooled surface, it has given 64 nodes at which the HTC values have to be determined from the minimum of the objective function. The number of physical thermocouples is much lower than the number of degrees of freedom. In such a case the inverse solution uncertainty can be high. On the other hand the HTC variation over the cooled surface is high during plate cooling and larger number of elements would give better solution. To resolve this problem the additional temperature sensors shown in Fig. 4 as interpolated thermocouples have been added to the physical thermocouples. It has allowed extending the temperature sensors to 81.
The temperature readings at interpolated sensors have been obtained from the approximation of the nearest four thermocouples indications at a particular time. Four node elements with the linear shape functions shown in Fig. 4 have been used in the approximation. The interpolated thermocouples locations have been selected based on numerical tests in order to achieve a low error of interpolation and to ensure at least 12 temperature sensors for each surface element. The uncertainty of the inverse solution to the HTC and the accuracy of interpolated thermocouples indications have been tested for the specified HTC distribution over the cooled surface varying in time:

$$h(x_2, x_3, \tau) = h_{m}(\tau) \sum_{k=1}^{K=6} \left[ e^{-\left( \frac{(x_2-x_2^k)^2}{c_1(\tau)} + \frac{(x_3-x_3^k)^2}{c_2(\tau)} \right)} \right]$$

(3)

In Eq. (3) $C_s(\tau)$ is a scaling function:

$$C_s(\tau) = 20 \left[ 1 + \left( \frac{2\tau - 60}{60} \right)^2 \right]$$

(4)

The test function (3) models plate cooling with six water jets. The coordinates: $(x_2^k; x_3^k)$ denote the water jets locations. The function $h_{m}(\tau)$ describes a variation of the HTC in time. The cubic-spline functions have been utilized to describe $h_{m}(\tau)$ variation in time. The cooling time has been divided into two periods. The following parameters: $h_{m}(\tau=0) = 100$ W/(m² K); $h_{m}(\tau=30) = 10000$ W/(m² K); $h_{m}(\tau=60) = 400$ W/(m² K); $\partial h_{m}(\tau=0)/\partial \tau = 0$; $\partial h_{m}(\tau=30)/\partial \tau = 0$; $\partial h_{m}(\tau=60)/\partial \tau = -1/30$ have been employed in the approximation of $h_{m}(\tau)$. The boundary condition given by Eq. (3) describes the HTC distribution over the plate similar to that expected during plate cooling with six water jets. At the beginning of cooling nearly an even HTC distribution simulating film boiling over the cooled surface is modeled and function $h_{m}(\tau)$ reaches $10000$ W/(m² K). At the end of cooling the HTC drops to a level typical for natural convection boiling and $h_{m}(\tau)$ = $400$ W/(m² K) at $\tau=60$ s.

The accuracy of the interpolated thermocouples indications have been determined based on numerical tests. Cooling of the plate shown in Fig. 2 has been modelled. At the upper plate surface boundary condition given by Eq. 3 has been specified. At the bottom surface of the plate boundary conditions for air cooling has been prescribed [7]. At side surfaces of the plate zero heat fluxes have been assumed. From the direct simulation of the plate cooling the temperature variations at 81 temperature sensors shown in Fig. 4 as physical and interpolated thermocouples have been obtained. The temperature sensor indications 2 mm below the water cooled surface have been obtained. These indications have been assumed as “exact” temperature readings. In the second test only temperature readings at 30 points denoted as “physical thermocouples” have been obtained from the simulation of the plate cooling for the same boundary conditions. Since simulated “physical thermocouples” are located at linear element nodes shown in Fig. 4 readings of the rest of temperature sensors have been obtained from interpolation of the nodal values using linear shape functions. The obtained set of 81 sensors readings which combine “exact” and interpolated sensors have been compared with the “exact” temperature readings. The average temperature error (ATE) between simulated temperature readings and the “exact” temperature readings was below 0.5 K, with the maximum deviation of 1.5 K. Thus, the developed method of the temperature sensor approximation has the accuracy comparable to the physical thermocouple located 2 mm below the cooled surface for the considered cooling conditions. In Fig. 5 the “exact” and the interpolated temperature sensors indications at three points: $P_4(x_2=15; x_3=12.5)$, $P_5(x_2=60; x_3=25)$ and $P_6(x_2=30; x_3=87.5)$ have been compared. The differences are negligible.

For the inverse solution test simulated temperature readings of 30 physical thermocouples have been obtained using the test function (3) as the boundary condition in the finite element method simulation.
of the inconel plate cooling. High accuracy FEM-Re model with linear elements of 9177 degrees of freedom has been employed. But in the inverse solutions the reduced FEM-H2 model with the nonlinear shape functions of 576 degrees of freedom have been employed. Description of the FEM models has been given in [7]. In the direct simulations of the plate cooling the reduced model has given an ATE of 3.5 K with the maximum deviation of 13 K to the data obtained from FEM-Re model. Thus, the inverse solution test has been performed for the data showing essential differences to the reduced FEM model. It is expected that the experimental errors and uncertainties to the measured temperatures will not be higher. In Fig. 6 the variations in time of the HTC at three points: \( P_1, P_2 \) and \( P_3 \) have been presented. The HTC distribution defined by Eq. (3) at \( t = 30 \) s has been presented in Fig. 7. The test function (3) models six HTC maxima simulating the water jets shown in Fig. 1. The location of points at the cooled surface has been shown in Fig. 3.

![Fig. 5. Comparison between the exact and interpolated temperature sensors indications.](image1)

![Fig. 6. Inverse solution to the heat transfer coefficient at points: \( P_1, P_2 \) and \( P_3 \) compared to exact data.](image2)

The inverse solution to the HTC at the cooled surface at \( t = 30 \) s has been presented in Fig. 8. The locations of the HTC maxima have been correctly identified. A local error to the HTC maximum is of 18% and to the HTC minimum of 15%. The ATE between simulated temperature readings and the inverse solution was low, at a level of 0.3 K, with the maximum deviation of 2 K. The inverse solution has given a very high accuracy of 0.1% to the total amount of heat extracted from the cooled surface. In the inverse solution to the simulated temperature readings the HTC distribution in time at nodes of surface elements has been approximated using cubic shape functions. The time of cooling has been divided into 6 periods. It has given a total number of the unknown HTC coefficients \( N_{\text{HTC}} = 1216 \), which have been determined minimizing the objective function (1). In Eq. (1) \( W_m \) is a weighting coefficient coupled with the thermocouple \( m \). The solutions presented in the paper have been obtained for \( W_m = 0.1 \). Extension of the objective function with only 10% of the temperature gradient errors has given smooth solutions close to the exact ones, Fig. 6.
Fig. 7. The HTC distribution over the cooled surface prescribed at $\tau = 30$ s.
$HTC_{\text{max}} = 9908$, $HTC_{\text{min}} = 1119$ W/(m$^2$ K).

Fig. 8. The inverse solution to the HTC over the cooled surface at $\tau = 30$ s.
$HTC_{\text{max}} = 11675$, $HTC_{\text{min}} = 1291$ W/(m$^2$ K).

More detailed description of the FEM models, boundary conditions and the previous uncertainty analysis have been given in [6] and [7]. In the present paper only new test for plate cooling with six water jets has been presented.

4. Inverse solutions to inconel plate cooling

The HTC distribution in the inverse solutions to the measured temperatures has been approximated by 9 surface elements with cubic shape functions shown in Fig. 3. Expansion in time of the HTC at nodes of surface elements has been interpolated using: 14, 10 and 7 periods with cubic shape functions over each period for the plate cooling by: one, two and six water jets, respectively. It has given: 2752, 1984 and 1408 $p_i$ parameters to be determined in the inverse solution to the boundary condition at the water cooled surface. The inverse solution to the plate cooling by one water jet has converged to an ATE of 1.6 K, with the maximum deviation of 11 K. For two water jets ATE of 0.9 K with the maximum deviation of 6 K has been obtained. In the case of six water jets the solution has given ATE of 0.6 K, with the maximum deviation of 3 K. One of the important problems in the inverse solutions which utilize gradient methods for the objective function minimization is a choice of a prior solution. The developed method of the HTC approximation in space and time offers a natural choice for that. The vector of unknown $p_i$ parameters is composed of 64 subvectors which define the HTC distribution in time at nodes of surface elements. The HTC distributions at nodes of surface elements can be readily obtained from one dimensional inverse solution (1D) to the heat conduction in plate. Such prior solutions have been improved using three dimensional models (3D) of heat conduction in the plate. In the 3D models of heat conduction prism elements with nonlinear shape functions of third degree have been utilized. The HTC determination zone has been divided into 25 prism elements. Only one element has been used over the plate thickness. The tests of the finite element model presented in [7] have shown that the developed model of heat conduction offers a good accuracy at a low number of freedoms. In the case of 25 prism elements the heat conduction model has only 576 unknowns.
In Fig. 9 the inverse solutions at points: \( P_1 \), \( P_2 \) and \( P_3 \) for the inconel plate cooling by one water jet have been presented. The plate cooling by one water jet results in a very fast increase of the HTC at point \( P_2 \) which is located near the stagnation point. The plate temperature below the water jet (at stagnation point) decreases rapidly and the wetting zone visible as a dark spot in Fig. 11 grows. As the wetting zone grows the HTC maximum moves to points \( P_1 \) and \( P_3 \). After about 20 s the HTC reaches a maximum value of 9115 W/(m\(^2\) K), Fig. 12. The heat flux at point \( P_2 \) reaches a maximum value of 1733 kW/m\(^2\) at surface temperature of 380 °C, Fig. 10. As the distance from the stagnation point grows heat flux maxima reach lower values.

The plate cooling by one water jet results in a highly heterogeneous heat transfer over the cooled surface. In such a case the implementation of 1D heat conduction model for the inverse solutions to...

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**Fig. 9.** The HTC distributions at points: \( P_1 \), \( P_2 \) and \( P_3 \) obtained from 1D and 3D solutions for the inconel plate cooling by one water jet.

**Fig. 10.** Inverse solution to the heat flux at points: \( P_1 \), \( P_2 \) and \( P_3 \) for the inconel plate cooling by one water jet.

**Fig. 11.** The inconel plate after about 20 s of cooling by one water jet.

**Fig. 12.** The inverse solution at \( \tau = 20 \) s to the HTC over the plate cooled by one water jet.

\[ \text{HTC}_{\text{max}} = 9115, \ \text{HTC}_{\text{min}} = 49 \ \text{W/(m}^2\text{K)} \]
the HTC at the thermocouple locations does not gives acceptable results, Fig. 9. The HTC maximum at point $P_2$ reaches only 5370 W/(m$^2$K) and the error to 3D inverse solution is of 40%. At the end of cooling, the heat conduction at point $P_1$ is nearly one dimensional and the 1D and 3D models have given similar results. The inconel plate cooling by two water jets results in a more effective heat transfer in the HTC determination domain in comparison to one water jet.

![Fig. 13. The inverse solution at $t = 20$ s to the plate surface temperature cooled by two water jets. $T_{max}=829$, $T_{min}=112 ^\circ$C](image1)

![Fig. 14. The inverse solution at $t = 20$ s to the HTC over the plate cooled by two water jets. $HTC_{max}=13675$, $HTC_{min}=19$ W/(m$^2$K).](image2)

![Fig. 15. The HTC distributions at points: $P_1$, $P_2$ and $P_3$ obtained from 1D and 3D solutions for the inconel plate cooling by two water jets.](image3)

![Fig. 16. Inverse solution to the heat flux at points: $P_1$, $P_2$ and $P_3$ for the inconel plate cooling by two water jets.](image4)

The temperature field shown in Fig. 13 indicates that after 20 s of cooling more than 50 % of the plate surface has temperature below 700 °C. Below this temperature the wetting zone develops and the HTC raises rapidly, Fig. 14. The HTC reaches the maximum value over 13 kW/(m$^2$K). The HTC variations in time presented in Fig. 15 confirm that heat conduction is far from one dimensional and the 1D
model has given the HTC maxima lover of 25 % to 40 % than the 3D model. The heat flux distributions presented in Fig. 16 as functions of the plate surface temperature also confirm essential differences in heat transfer at points: $P_1$, $P_2$ and $P_3$.

Fig. 17. The inverse solution at $r = 20$ s to the plate surface temperature cooled by six water jets. $T_{\text{max}} = 498$, $T_{\text{min}} = 93$ °C.

Fig. 18. The inverse solution at $r = 20$ s to the HTC over the plate cooled by six water jets. $\text{HTC}_{\text{max}} = 20723$, $\text{HTC}_{\text{min}} = 17$ W/(m² K).

Fig. 19. The HTC distributions at points: $P_1$, $P_2$ and $P_3$ obtained from 1D and 3D solutions for the inconel plate cooling by six water jets.

Fig. 20. Inverse solution to the heat flux at points: $P_1$, $P_2$ and $P_3$ for the inconel plate cooling by six water jets.

The inconel plate cooling by 6 water jets has allowed covering nearly the whole HTC determination domain by water streams. The plate temperature drops rapidly but varies from 90 to 500 °C over the surface after 20 s of cooling, Fig. 17. It causes high differences in the HTC shown in Fig. 18. The HTC reaches the maximum value of 20 kW/(m² K). However, heat conduction at points: $P_1$, $P_2$ and $P_3$ is nearly one dimensional and 1D and 3D models have given similar results, Fig. 19. Also the heat flux variations versus surface temperature at these points decreases. The heat flux maximum values are similar, Fig. 20. But the critical heat flux temperature varies from 300 to 500 °C.
5. Another section of your paper
The inverse solutions to water jets cooling of the inconel plate have shown high differences in heat transfer over the cooled surface. The plate temperature varies significantly and it can be important in modelling microstructure evolution processes and development of thermal stresses. Implementation of 1D heat conduction models for the inverse determination of the local HTC results in most cases in the essentially lower (up to 40%) HTC values. The inverse solutions which utilise 3D heat conduction model have shown local errors to the HTC of about 20%. The extension of the physical thermocouples with the interpolated temperature sensors has allowed obtaining higher resolution to the HTC distribution over the plate surface for stationary locations of the HTC maxima in space. However, the conducted physical experiments have shown that the HTC maxima move essentially over the plate surface due to several reasons. In such a case the distance between physical thermocouples should be reduced. The arrangement of the physical thermocouples in an array of 7×7 sensors should be considered in further research.

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