Inelastic light scattering to probe strongly correlated bosons in optical lattices

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Abstract. We have used inelastic light scattering to study correlated phases of an array of one-dimensional interacting Bose gases. In the linear response regime, the observed spectra are proportional to the dynamic structure factor. In particular we have investigated the superfluid to Mott insulator crossover loading the one-dimensional gases in an optical lattice and monitoring the appearance of an energy gap due to finite particle-hole excitation energy. We attribute the low frequency side of the spectra to the presence of some superfluid and normal phase fraction between the Mott insulator regions with different fillings produced in the inhomogeneous systems. In the Mott phase we also investigated excitations to higher excited bands of the optical lattice, the spectra obtained in this case being connected to the single particle spectral function. In one-dimensional systems the effect of thermal fluctuations and interactions is enhanced by the reduced dimensionality showing up in the dynamic structure factor. We measured the dynamic structure factor of an array of one-dimensional bosonic gases pointing out the effect of temperature-induced phase fluctuations in reducing the coherence length of the system.

1. Introduction
In the last few years the connection between atomic physics and condensed matter physics has become stronger and stronger thanks to the possibility of using ultracold quantum gases to study, in a relatively clean and controllable environment, phenomena that were introduced in the field of condensed matter. Seminal works in this context were the first observation of the superfluid to Mott Insulator transition with ultracold bosons [1] and fermions [2, 3] in optical lattices and the realization of Tonks-Girardeau gases [4, 5]. Soon many suggestions arose to use inelastic light scattering as a probe to investigate the dynamical properties of these strongly correlated systems [6, 7]. Inelastic particle scattering is widely used also in the context of condensed matter. For example, the first experimental observation of a condensate in superfluid $^4$He was obtained by measuring the dynamic structure factor with neutron scattering [8]. In the context of ultracold quantum gases, inelastic scattering experiments are performed illuminating the sample with a pair of far-detuned laser beams with different frequencies ($\omega$ and $\omega'$) and different wavevectors ($\mathbf{k}$ and $\mathbf{k}'$). The atoms can then perform a two-photon transition absorbing a photon from the first laser beam and reemitting a photon with different frequency and wavevector by stimulated
emission induced by the second laser beam. This process allows the transfer of an excitation to the system, with energy $\hbar(\omega - \omega')$ and momentum $\hbar(k - k')$ that can be changed by tuning the laser beam parameters. In the linear response regime the energy transferred to the system is proportional to the dynamic structure factor $S(\omega, k)$ [9].

The first experiments on inelastic light scattering (also called Bragg spectroscopy) from quantum gases were reported in [10, 11]. Bragg spectroscopy allowed measurements of the Bogoliubov excitation spectrum of a weakly-interacting Bose-Einstein condensate (BEC) [12] and phase fluctuations in an elongated condensate [13]. More recently it was extended to study correlated systems as a strongly interacting $^{85}$Rb BEC across a Feshbach resonance [14] and to monitor the BEC-BCS crossover of a sample of interacting $^6$Li fermions [15]. Bragg spectroscopy allowed also the investigation of the band structure of superfluid ultracold samples in optical lattices [16, 17, 18]. Our work extends these measurements to intra-band and inter-band transitions in the superfluid to Mott insulator crossover in a two-dimensional (2D) array of one-dimensional (1D) ultracold bosons loaded into an optical lattice. In the Mott insulator regime intra-band excitations provide access to the two-particle spectral function, the Fourier Transform of which is the dynamic structure factor, while the inter-band excitation should give information on the one-particle spectral function. We will present a detailed investigation of the Bragg spectra obtained across the transition [19] discussing open questions. We also present some preliminary results obtained by performing Bragg spectroscopy on an array of strongly correlated 1D gases pointing out the possibility to use this technique as a monitor to evaluate both phase fluctuations and correlations in 1D systems.

2. Bragg spectroscopy of 1D interacting bosons in a lattice

As suggested in [6, 7] Bragg spectroscopy can be used to measure the energy gap in the Mott insulator phase obtained by loading repulsively interacting ultracold bosons in a periodic potential realized by means of optical lattices. By varying the momentum and the energy given by the Bragg beams, one can address also the superfluid components present in an inhomogeneous wedding-cake structured system as realized in the experiments in [20].

In our experiment, we load with an exponential ramp in 140 ms a $^{87}$Rb BEC in a 2D optical lattice created by two orthogonal standing waves ($\lambda=830$ nm) aligned in the $x-z$ plane. The height of the periodic potentials barriers created by the two optical lattices is 35 $E_R$ (where $E_R = \hbar^2/(2m\lambda^2)$ is the recoil energy acquired by the atom with mass $m$ absorbing one photon from the optical lattice laser with wavelength $\lambda$). This creates a 2D array of independent 1D atomic tubes (as shown schematically in Fig. 1a) longitudinally trapped by a harmonic potential. Along the $y$ direction a third optical lattice is ramped on (Fig. 1b) with a variable height $V_y = s_y E_R$ (with $s_y$ ranging from 0 to 15) in order to drive the superfluid to Mott insulator transition [21]. After a few milliseconds a pair of Bragg beams (shown in Fig. 1c) detuned by $\sim 200$ GHz from the D2 transition of Rb are pulsed on. The direction of propagation of the Bragg beams is fixed and chosen in order to transfer a momentum $\hbar\delta k$ along the atomic tubes of $0.96\hbar k_L$ where $k_L = 2\pi/\lambda$ is the wavenumber of the optical lattice beams. This choice has been made to maximize the excitation strength of the Mott insulator phase which is proportional to $\sin^2(\pi\delta k/(2k_L))$ [7] allowing us to perform the measurements in the linear response regime. Bragg spectroscopy previously used to detect the Mott insulator with ultracold atoms, has been performed at zero momentum transfer in the non-linear regime [21]. After the Bragg pulse, we ramp the height of the three lattices down to 5 $E_R$ where the ground state is superfluid and let the system thermalize for 5 ms [21]. We then suddenly switch off all the confining potentials and take an absorption image of the atomic density distribution after time of flight. The measured density distribution after free expansion shows an interferogram reflecting the coherence of the superfluid phase [1]. The interference pattern consists of a series of peaks separated by a distance proportional to the lattice wavevector $k_L$ and the width of the central peak is proportional to the
energy of the atomic sample. For a more detailed description of the experimental procedure the reader is referred to [22]. We measure the width of the interferogram central peak changing the frequency difference $\delta \omega = (\omega - \omega')$ of the Bragg beams and this provides us with an evaluation of the energy $\Delta E$ transferred to the system as a function of the frequency of the excitation. As $\Delta E = N\hbar \delta \omega$ ($N$ being the number of excitations) and $N \propto S(\omega, k)$, the energy acquired by the system after the Bragg pulse is $\Delta E \propto \omega S(\omega, k)$ [9].

As the array of 1D samples is loaded into an optical lattice, a band structure arises and at fixed $k$, by varying the energy difference of the Bragg beams, excitations belonging to different energy bands can be induced [17]. In particular, at lower frequencies, the excitations still belong to the lattice lower energy band (intra-band spectroscopy). In the following we will report first the measurements we have performed in this regime of frequencies, and then extend our Bragg spectroscopy measurements to the higher energy bands (inter-band spectroscopy).

### 2.1. Spectra across the superfluid to Mott insulator transition (intra-band spectroscopy – two-particle spectral function)

In Fig. 2 we show the energy transferred to the system of an array of 1D atomic tubes as a function of the frequency of the Bragg excitation for different values of the longitudinal lattice height $s_y$. With increasing $s_y$ the ground-state of the system is expected to go from a superfluid (Fig. 2a) to the inhomogeneous Mott insulator phase (Fig. 2b,c,d) where, due to the variation of the density across our sample, we expect to have alternating regions of Mott insulator with different fillings separated by superfluid or normal regions [20]. In particular, the appearance of the first Mott insulator region (in the outer part of the trapped atomic cloud) is expected to occur at $s_y \approx 5.5$ [23]. As shown in Fig. 2 both the amplitude of the response and the lineshape of the spectra substantially change when entering the strongly correlated Mott insulator phase. As expected, in the insulator phase the amplitude drops (Fig. 2b and c) and a multiple resonance structure arises with two clear resonances (respectively around 2 kHz and 4 kHz). The first resonance corresponds to the energy gap $\Delta$ of the insulator phase due to the finite energy necessary to promote a particle from one lattice site with $n$ bosons to the next one with the same filling $n$ [1]. The second resonance, at twice the frequency, can be interpreted in terms of particle-hole excitations between sites with different fillings ($n$ and $n + 1) due to inhomogeneity in the sample and the presence of defects across the lattice [24]. Finally, the spectra exhibit a region at low frequencies (<1 kHz) where the system can be excited. These
excitations can partly be explained as due to the presence of superfluid layers between Mott insulator regions with different fillings [25, 26]. As detailed in [19], we think that when strongly in the Mott regime (s=13) finite-temperature effects have to be considered in order to interpret the measured excitations in the region $500 \, \text{Hz} < \omega/(2\pi) < 1000 \, \text{Hz}$ suggesting the possibility to use Bragg spectroscopy to estimate temperature for ultracold atoms loaded in optical lattices. The complex multi-resonance structure observed in Mott insulator phases can thus be used to monitor different components present in an inhomogeneous trapped gas system.

At $s_y < 5$ the array of 1D bosons in the optical lattice is still in the superfluid phase but close enough to the Mott phase that correlations are expected to build up. At $s_y = 4$ (spectrum in Fig.2a) the first band in the Bogoliubov approximation extends up to $\omega/(2\pi) = 3700 \, \text{Hz}$. This value has been calculated by considering a single atomic tube with 300 atoms which we expect to be an upper limit for the central tube in our sample [27]. The measured spectrum clearly shows excitations well above that limit in the gap between the lower band and the first excited band starting at $\omega/(2\pi) = 8100 \, \text{Hz}$. We suggest that the observed excitations in the band gap between the lower energy band and the first excited band in the strongly correlated superfluid phase can be a first experimental evidence of the extra gapped mode predicted in [28, 29].

2.1.1. Inter-band spectroscopy – single-particle spectral function

Increasing the frequency of the Bragg excitation, we investigated a region of the spectra where $\hbar \omega > \Delta$ ($\Delta$ being the energy gap of the Mott insulator phase). In this case the excitation creates a hole in the correlated Mott insulator phase and populates single particle states in excited bands. The spectra obtained can be related to the single-particle spectral function of the system [30] if the excited single particle state is decoupled from the ground state. In Fig. 3 we show two spectra obtained populating the first and the second excited band for $s_y = 8$ (left panel) and $s_y = 10$ (right panel). The shaded areas in Fig.3 correspond to the extension of the first and second excited bands calculated using

![Figure 2. Bragg spectra obtained with four different values of $s_y$ (respectively $s_y=4, 7, 10, 13$) across the superfluid to Mott insulator transition.](image_url)
the Bogoliubov mean-field approximation for a 1D single atomic sample with 300 atoms. In particular the second excited band spectra (22 kHz < \omega/(2\pi) < 40 kHz) show a structured profile and we think that a detailed theoretical analysis would provide information on the dispersion relation of the hole and the density of states in the Mott phase [31].

![Figure 3. Bragg spectra in the MI phase (s_y = 8 and s_y = 10 for the left and right panel respectively) showing excitations to the first and second excited bands of the periodic system. Dashed areas correspond to the frequency intervals of the first and second excited bands calculated in the Bogoliubov approximation for a single 1D atomic tube with 300 atoms.]

3. Bragg spectroscopy of 1D interacting bosons
In this section we present Bragg spectroscopy measurements performed on a BEC loaded into a two dimensional optical lattice (Fig. 1 a)) as a function of the height of the two lattices s_\perp. With increasing s_\perp, the system crosses from a 3D BEC, reaches an intermediate regime where an array of coupled 1D atomic tubes forms. Finally for s_\perp > 10 the tunneling time between the lattice sites becomes longer than the experimental time scale and we obtain an array of decoupled 1D atomic tubes.

The physics of 1D systems is very peculiar due to the fact that the interplay between strong interactions and confinement to a low-dimensional geometry amplifies the effects of quantum fluctuations, thermal fluctuations and correlations giving rise to interesting effects [32]. In a 1D homogeneous system Bose-Einstein condensation is prevented by thermal fluctuations hampering long range order. For trapped gases, the situation changes and three regimes become possible [33], namely a true condensate, a quasicondensate, and a strongly interacting regime of a trapped Tonks-Girardeau gas of impenetrable bosons [34, 35]. The study of the dynamic structure factor through Bragg spectroscopy would allow an investigation of the properties of these systems. The increase in correlations towards the fermionized Tonks-Girardeau regime is expected to strongly modify S(\omega,k). The Lieb-Liniger model of interacting bosons in 1D predicts the presence of two types of elementary excitations [35] instead of one, as predicted by the Bogoliubov theory, and the dynamic structure factor broadens as these two modes arise [36] [37]. This effect becomes important in the strongly interacting limit which is approached when the interaction energy becomes much larger than the kinetic energy. In a 1D system the ratio of the interaction energy and kinetic energy (\gamma=E_{int}/E_{kin}) is given by [38]

$$\gamma = \frac{mg}{\hbar^2 n_{1D}}$$

(1)

where g is the interaction strength and n_{1D} is the one-dimensional atomic density.
The broadening of $S(\omega, k)$ for a 1D system can also be related to finite-temperature effects causing phase fluctuations on a typical length scale called “phase coherence length” $L_\phi$ [39, 13]

$$L_\phi = \frac{\hbar^2 n_{1D}}{mk_B T}$$

(2)

where $k_B$ is the Boltzmann constant and $T$ is the temperature. Phase fluctuations result in a momentum spread

$$\Delta p = \frac{\hbar}{L_\phi}$$

(3)

which contribute to the broadening of the Bragg spectrum.

In Fig. 4 we present three Bragg spectra obtained for an inhomogeneous array of 1D atomic tubes for $s_\perp = 10, 20$ and $45$, respectively. As $s_\perp$ increases we observe a shift of the centre of the resonance towards higher frequency and a broadening. An estimation of $\gamma$ [40] for our system gives $\gamma \simeq 0.3$ at $s_\perp = 10$ and $\gamma \simeq 0.9$ at $s_\perp = 45$ and thus we expect correlations to play a minor role in the observed spectra. We think that the broadening of the Bragg spectra in the range of parameters we have investigated would thus provide a measurement of the reduced coherence length of our sample caused by the increase in thermal fluctuations. An estimation of the temperature would be possible after a detailed modeling of the system taking into account the fact that we are investigating an inhomogenous array of 1D atomic tubes each of them having a different $L_\phi$ [41].

4. Conclusions

We have used inelastic light scattering to probe dynamical properties of strongly correlated phases of bosons loaded into optical lattices. In particular we monitored the formation of an energy gap across the superfluid to Mott insulator transition of an array of 1D atomic gases loaded in a longitudinal optical lattice $s_y$. Signatures of correlations were obtained also in the superfluid phase. In the Mott regime the spectra show different peaks corresponding to different phases present in our inhomogeneous sample. We also induced Bragg excitations to higher excited bands pointing out the possibility to use this method to access the single particle spectral function of the Mott phase. In the last section, we have presented some preliminary results obtained performing Bragg spectroscopy in an array of 1D atomic samples for different transverse confinement and discussed the perspective to use Bragg spectroscopy to investigate the effect of correlations and temperature fluctuations in interacting 1D Bose systems.
5. References

[1] Greiner M, Mandel O, Esslinger T, Hänsch TW and Bloch I 2002 Nature 415 39
[2] Jördens R, Strohmaier N, Günter K, Moritz H and Esslinger T 2008 Nature 455 204
[3] Schneider U, Hackermüller L, Will S, Best Th, Costi T A, Helmes R W, Rasch D and Rosch A 2008 Science 322 1520
[4] Paredes B, Widera A, Mandel O, Fölling S, Cirac I, Shlyapnikov G V, Hänsch T W and Bloch I 2004 Nature 429 277
[5] Jördens R, Strohmaier N, Günter K, Moritz H and Esslinger T 2008 Nature 455 204
[6] van Oosten D, Dickerscheid D B M, Farid B, van der Straten P and Stoof H T C 2005 Phys. Rev A 71 021601
[7] Rey A M, Blakie B, Pupillo G, Williams C J and Clark C W 2005 Phys. Rev A 72 023407
[8] Henshaw D G and Woods A D B 1961 Phys. Rev. 121 1266
[9] Kinoshita T, Wenger T and Weiss D S 2004 Science 305 1125
[10] Kozuma M, Deng L, Hagley E W, Wen J, Lutwak R, Helmerson K, Rolston S L and Phillips W D 1999 Phys. Rev. Lett 82 871
[11] Stenger J, Inouye S, Chikkatur A P, Stamper-Kurn D M, Pritchard D E and Ketterle W 1999 Phys. Rev. Lett 82 4569
[12] Steinhauser J, Ozeri R, Katz N and Davidson N 2002 Phys. Rev. Lett 88 120407
[13] Richard S, Gerbier F, Thywissen J H, Huckart M, Bouyer P and Aspect A 2003 Phys. Rev. Lett 91 010405
[14] Papp S B, Pino J M, Wild R J, Ronen S, Wieman C E, Jinn D S, and Cornell E A 2004 Phys. Rev. Lett 101 135301
[15] Veeravalli G, Kuhnle E, Dyke P, Vale C J 2005 Phys. Rev. Lett 101 250403
[16] Du X, Xan S, Yesilada E, Ryu C and Heinen D J 2007 arXiv:0704.023c0
[17] Fabbi N, Clément D, Fallani L, Fort C, Modugno M, van der Stam K M R and Inguscio M 2009 Phys. Rev. A 79 043623
[18] Ernst P T, Götze S, Krauser J S, Pyka K, Lühmann D S, Pfannkuche D and Sengstock K 2009 Nat. Phys. 5 1
[19] Clément D, Fabbri N, Fallani L, Fort C and Inguscio M 2009 Phys. Rev. Lett 102 155301
[20] Jaksh D, Bruder C, Cirac J I, Gardiner C W, and Zoller P 1998 Phys. Rev. Lett 81 3108; Batrouni G G, Rousseau V, Scalettar R T and Denteneer P J H 2005 Phys. Rev. Lett 97 050402
[21] Battabhi N, Clément D, Fabbri N, Fallani L, Fort C and Inguscio M 2009 Phys. Rev. Lett 102 155301
[22] Kühn T D and Monien H 1998 Phys. Rev. B 58 R14 742
[23] Kollath C, Iucci A, Giamarchi T, Holstetter W and Schollwöck U 2006 Phys. Rev. Lett. 97 050402
[24] Batrouni G G, Assaad F F, Scalettar R T and Denteneer P J H 2006 Phys. Rev. A 72 031601R
[25] Papp S B, Pino J M, Wild R J, Ronen S, Wieman C E, Jinn D S, and Cornell E A 2004 Phys. Rev. Lett 101 135301
[26] Fabbi N, Clément D, Fallani L, Fort C, Modugno M, van der Stam K M R and Inguscio M 2009 Phys. Rev. A 79 043623
[27] Ernst P T, Götze S, Krauser J S, Pyka K, Lühmann D S, Pfannkuche D and Sengstock K 2009 Nat. Phys. 5 1
[28] Clément D, Fabbri N, Fallani L, Fort C and Inguscio M 2009 Phys. Rev. Lett 102 155301
[29] Jaksh D, Bruder C, Cirac J I, Gardiner C W, and Zoller P 1998 Phys. Rev. Lett 81 3108; Batrouni G G, Rousseau V, Scalettar R T and Denteneer P J H 2005 Phys. Rev. Lett 97 050402
[30] Kühn T D and Monien H 1998 Phys. Rev. B 58 R14 742
[31] Kollath C, Iucci A, Giamarchi T, Holstetter W and Schollwöck U 2006 Phys. Rev. Lett. 97 050402
[32] Batrouni G G, Assaad F F, Scalettar R T and Denteneer P J H 2006 Phys. Rev. A 72 031601R
[33] Papp S B, Pino J M, Wild R J, Ronen S, Wieman C E, Jinn D S, and Cornell E A 2004 Phys. Rev. Lett 101 135301
[34] Michele Modugno, private communication.
[35] Huber S D, Altman E, Böhlter H P, and Blatter G 2007 Phys. Rev. B 75 085106
[36] Menotti C and Trivedi N 2008 Phys. Rev. B 77 235120
[37] Clément D, Fabbri N, Fallani L, Fort C and Inguscio M 2010 J. Low Temp. Phys. 158 5
[38] Work is in progress to extract temperature estimation from the measured Bragg spectra.
[39] Note that our system consists of an array of inhomogeneous 1D atomic sample each of them characterized by a different density and thus a different γ. The values of γ given in the text are averaged over the whole array.
[40] Work is in progress to extract temperature estimation from the measured Bragg spectra.