Modeling the Multi-band Light Curves of the Optical–NIR Afterglows of Four Gamma-Ray Bursts and Their Associated Supernovae

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ABSTRACT

In this paper, we use (broken) power-law plus $^{56}$Ni models to fit the multi-band light curves of the optical and near-infrared (NIR) counterparts of four gamma-ray bursts (GRBs 011121, 100316D, 130702A, and GRB 161219B). We find that the models can account for the light curves of the optical–NIR counterparts which can be divided into the GRB afterglows and their associated supernovae (SNe 2001ke, 2010bh, 2013dx, and 2016jca, respectively). The most parameters we derive are consistent with previous studies. However, the $^{56}$Ni masses we derive are higher than that in the literatures (except for that of GRB 100316D/SN 2010bh). The difference of the $^{56}$Ni masses might be due to the fact that the $^{56}$Ni masses in the literatures are obtained by fitting the quasi-bolometric light curves which are usually (significantly) underestimated, and dimmer than the theoretical bolometric light curves reproduced by the best-fitting parameters we derive. Our results demonstrate that the spectral energy distributions (SEDs) of SNe associated with GRBs can be well described by the blackbody model, and the $^{56}$Ni model can account for their multi-band light curves. We suggest that the $^{56}$Ni masses of a fraction of GRB-SNe have been underestimated.

Keywords: general – supernovae: individual (GRB 011121/SN 2001ke, GRB 100316D/SN 2010bh, GRB 130702A/SN 2013dx, GRB 161219B/SN 2016jca)

1. INTRODUCTION

Gamma ray bursts (GRBs) are the most powerful explosions in the universe. It is widely believed that GRBs come from the relativistic jet launched by the central engine (Woosley 2011). The interactions between the jets with the surrounding medium would produce X-ray, optical, and radio afterglows (see Zhang 2018 and references therein). According to the observation of prompt emission duration, GRBs is divided into long-duration bursts (LGRBs) and short-duration bursts (SGRBs) with a dividing line of ~2 seconds (Kouveliotou et al. 1993). The observations and analysis for some dozen supernovae (SNe) associated with LGRBs (Hjorth et al. 2003; Matheson et al. 2003; Stanek et al. 2003; Malesani et al. 2004; Deng et al. 2005; Modjaz et al. 2006; Mirabal et al. 2006; Sollerman et al. 2006; Campana et al. 2006; Maeda et al. 2007; Chornock et al. 2010; Starling et al. 2011; Olivares et al. 2012; Bufano et al. 2012; Melandri et al. 2012; Singer et al. 2013; Schulze et al. 2014; Melandri et al. 2014; Toy et al. 2016; D’Elia et al. 2015; Cano et al. 2017a; Vovkova et al. 2017b,a; Ashall et al. 2019; Melandri et al. 2019; Hu et al. 2021) indicate that most LGRBs are produced by the explosions of massive stars. On the other hand, the confirmation of SSS17a/AT2017gfo which is a kilonova associated with GW170817 that is a gravitational wave emitted by a merger of a neutron star binary and GRB 170817A that is a SGRB (Arcavi et al. 2017; Drout et al. 2017; Abbott et al. 2017; Coulter et al. 2017; Shappee et al. 2017) supports the conjecture that at least a fraction of SGRBs are produced by the mergers of compact binary stars.

The SNe associated with LGRBs are called GRB-SNe (Woosley & Bloom 2006; Hjorth & Bloom 2012; Cano et al. 2017). On average, one or two GRB-SNe can be found every year. To date, there are about 60 LGRBs that have been confirmed to be associated with SNe. Almost all GRB-SNe are broad-lined Ic (Ic-BL) SNe whose optical spectra
are hydrogen-deficient and show broad absorption line features. The spectral features indicate that the progenitors of GRB-SNe are highly stripped, and might be Wolf-Rayet stars (Price et al. 2002a; Sonbas et al. 2008). The broad absorption lines are indicative of huge ejecta velocities $\gtrsim 2 \times 10^9$ cm s$^{-1}$. Therefore, a major fraction of GRB-SNe (and the SNe Ic-BL without accompanying GRBs) become so-called “hypernovae” (HNe) whose kinetic energy is $\gtrsim 10^{52}$ erg, which is about 10 times that of normal SNe. The explosion mechanisms of GRB-SNe are still elusive.

The most prevailing model adopted to account for the light curves of GRB-SNe is $^{56}$Ni cascade decay ($^{56}$Ni $\rightarrow^{56}$Co $\rightarrow^{56}$Fe) model (the $^{56}$Ni model, Arnett 1979, 1980, 1982, 1996). Some very luminous GRB-SNe cannot be explained by the $^{56}$Ni model, and alternative or additional energy sources (e.g., the magnetar spinning-down, the fall-back accretion, etc.) are employed to account for the light curves.

Previous studies focusing on GRB-SNe usually construct the quasi-bolometric light curves of the SNe and derive the physical properties of GRB-SNe by fitting the constructed quasi-bolometric light curves. It should be noted that, however, the process constructing the quasi-bolometric light curves might underestimate the luminosities of the SNe and therefore underestimate the $^{56}$Ni masses. Recently, the model directly fit the multi-band light curves (Nicholl et al. 2017) have been adopted to fit the light curves of superluminous SNe (Nicholl et al. 2017; Moriya et al. 2018), the tidal disruption events (Mockler et al. 2019), and ordinary SNe Ib and Ic (Wang et al. 2021).

In this paper, we collect published data of the optical–NIR counterparts (GRB 011121/SN 2001ke, GRB 100316D/SN 2010bh, GRB 130702A/SN 2013dx, GRB 161219B/SN 2016jca) of four GRBs (GRB 011121, GRB 100316D, GRB 130702A, GRB 161219B; here, the GRBs present their afterglows) and use the (broken) power-law plus $^{56}$Ni model to fit their multi-band optical–NIR light curves. In Section 2, we model the multi-band light curves of the four GRB-SNe using the $^{56}$Ni model. In Section 3, we compare parameters and the bolometric properties of the SNe to that in the literatures. We draw some conclusions in Section 4. The values of $E(B-V)$ of the foreground extinction of the Milky Way are from Schlafly & Finkbeiner (2011). The standard cosmological parameters ($\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, and $H_0 = 68$ kms$^{-1}$Mpc$^{-1}$) are adopted throughout this paper.

2. MODELING THE MULTI-BAND LIGHT CURVES OF FOUR GRB-SNE USING THE $^{56}$Ni MODEL

The information of the GRB 011121/SN 2001ke, GRB 100316D/SN 2010bh, GRB 130702A/SN 2013dx, GRB 161219B/SN 2016jca are listed in Table 1. Their optical–NIR flux can be divided into the afterglows of GRBs and the SNe associated with the GRBs. The former follows a power-law ($F_{\nu,AG}(t) = A_{AG} \cdot t^{-\alpha} \cdot \nu^{-\beta}$) or broken power-law decay ($F_{\nu,AG}(t) = A_{AG} \cdot ((t/t_{break})^{-\alpha_1 - n} + (t/t_{break})^{-\alpha_2 - n})^{1/n} \cdot \nu^{-\beta}$) function (Li et al. 2012), while the later can be explained by $^{56}$Ni cascade decay. The details of the $^{56}$Ni model can be found in Wang et al. (2015) are references therein. The total flux is expressed as $F_{\nu,tot}(t) = F_{\nu,AG}(t) + F_{\nu,SN}(t)$, where $F_{\nu,SN}(t)$ is the flux of SNe. To fit the multi-band light curves of the SN components, we suppose that the spectral energy distributions (SEDs) of the SNe can be described by the blackbody model, and adopt the photosphere modulus presented in Nicholl et al. (2017).

The definitions, the units, and the priors of the parameters of the models are listed in Table 2. Throughout this paper, the value the opacity ($\kappa$) of the ejecta is set to be 0.07 cm$^2$g$^{-1}$. We adopt the Markov Chain Monte Carlo (MCMC) method by using emcee of Python package (Foreman-Mackey et al. 2013) to fit the data to obtain the best-fitting parameters and 1 $\sigma$ parameter range.

We find that the light curves of GRB 011121/SN 2001ke can be fitted by the power-law afterglow plus $^{56}$Ni model, while the rest three GRB-SNe (GRB 100316D/SN 2010bh, GRB 130702A/SN 2013dx, and GRB 161219B/SN 2016jca) can be fitted by broken power-law afterglow plus $^{56}$Ni model. We note that, while almost all bands of the four GRB-SNe can be fitted by the multi-band model, the $z$–band light curve of GRB 130702A/SN 2013dx, and the $z$–, $J$–, and $H$–band light curves of GRB 161219B/SN 2016jca cannot be well fitted by the multi-band model (see the bottom panels of Figure 1).

There are two (possible) reasons that can explain the bad quality of the fits for the bands. (1). The $z$–band light curve of GRB 130702A/SN 2013dx and the $J$–, and $H$–band light curves of GRB 161219B/SN 2016jca show fluctuation features that cannot be fully fitted by the theoretical light curves which are smooth. (2). The SEDs of GRB 161219B/SN 2016jca might deviate from the blackbody function in $z$–band.

The fits of the four GRB-SNe and the best-fit parameters are presented in Figure 1 and Table 3, respectively. The corresponding corner plots are shown in Figures A1-A4. We find that the masses of $^{56}$Ni of GRB 011121/SN 2001ke,
GRB 100316D/SN 2010bh, GRB 130702A/SN 2013dx, and GRB 161219B/SN 2016jca are $0.43^{+0.01}_{-0.01}$, $0.12^{+0.00}_{-0.00}$, $0.62^{+0.00}_{-0.00}$, and $0.73^{+0.00}_{-0.00}$, respectively. The ejecta masses of the four GRB-SNe are $2.05^{+0.07}_{-0.03}$, $1.67^{+0.08}_{-0.08}$, $2.19^{+0.08}_{-0.11}$, and $5.06^{+0.12}_{-0.11}$, respectively. The respective velocity of the ejecta of the four GRB-SNe are $2.19^{+0.05}_{-0.05} \times 10^9$ cm s$^{-1}$, $1.56^{+0.03}_{-0.03} \times 10^9$ cm s$^{-1}$, $1.96^{+0.02}_{-0.02} \times 10^9$ cm s$^{-1}$, and $3.12^{+0.00}_{-0.00} \times 10^9$ cm s$^{-1}$. The parameters are roughly consistent with the parameter ranges in the literatures.

3. DISCUSSION

Here, we compare the values of the $^{56}$Ni masses, the ejecta masses, the ejecta velocity, and the kinetic energy of the ejecta of the four GRB-SNe to that in the literature and discuss the reasons causing the discrepancies. Moreover, we discuss the theoretical bolometric light curves of the four GRB-SNe.

3.1. The $^{56}$Ni Masses of the Four GRB-SNe

The $^{56}$Ni mass of GRB 011121/SN 2001ke is $0.43$ M$_\odot$, which is higher than that of SN 1998bw which is $0.10 \pm 0.01$ M$_\odot$, and Bufano et al. (2012) (0.12 $\pm$ 0.02 M$_\odot$), but lower than that obtained by Cano et al. (2011) (0.21 $\pm$ 0.03 M$_\odot$). The $^{56}$Ni mass of GRB 161219B/SN 2016jca is $0.73$ M$_\odot$, which is $0.22^{+0.08}_{-0.08}$ M$_\odot$ and Ashall et al. (2019) (0.27 $\pm$ 0.05 M$_\odot$), respectively.

The discrepancy might be due to the facts that Toy et al. (2016), Cano et al. (2017a), and Ashall et al. (2019) derived the $^{56}$Ni masses by fitting the quasi-bolometric light curves, which are dimmer than the bolometric light curves, and that our blackbody multi-band fits correspond to the bolometric light curve.

The $^{56}$Ni masses of GRB 130702A/SN 2013dx and GRB 161219B/SN 2016jca are comparable, but smaller than the parameter values in the literature. Therefore, we suggest that the $^{56}$Ni masses of the two GRB-SNe are reasonable.

3.2. The Properties of the Ejecta

The ejecta mass of GRB 011121/SN 2001ke is $2.05^{+0.07}_{-0.03}$ M$_\odot$; the ejecta mass of GRB 100316D/SN 2010bh is $1.67^{+0.09}_{-0.03}$ M$_\odot$, which is slightly lower than that in the literatures (2.24 $\pm$ 0.08 M$_\odot$, Cano et al. 2011; 2.6 $\pm$ 0.2 M$_\odot$, Olivas et al. 2012; 3.2 $\pm$ 1.6 M$_\odot$, Bufano et al. 2012); the ejecta mass of GRB 130702A/SN 2013dx is $2.15^{+0.06}_{-0.06}$ M$_\odot$, which is lower than the value derived by Toy et al. (2016) (3.1 $\pm$ 0.1 M$_\odot$) and D’Elia et al. (2015) ($\sim$ 7 M$_\odot$); the ejecta mass of GRB 161219B/SN 2016jca is $4.93^{+0.1}_{-0.1}$ M$_\odot$, which is slightly lower than that derived by Cano et al. (2017a) (5.8 $\pm$ 0.3 M$_\odot$) and that derived by Ashall et al. (2019) (6.5 $\pm$ 1.5 M$_\odot$).

The ejecta velocity of GRB 011121/SN 2001ke is $2.15^{+0.06}_{-0.05} \times 10^9$ cm s$^{-1}$; the ejecta velocity of GRB 100316D/SN 2010bh is $1.56^{+0.03}_{-0.03} \times 10^9$ cm s$^{-1}$, which is (slightly) lower than that in the literatures ($\sim 2.5 \times 10^9$ cm s$^{-1}$, Cano et al. 2011; $\approx 2.5 - 3.1 \times 10^9$ cm s$^{-1}$, Olivas et al. 2012; up to $\approx 5 \times 10^9$ cm s$^{-1}$, Bufano et al. 2012); the ejecta velocity of GRB 130702A/SN 2013dx is $1.96^{+0.02}_{-0.02} \times 10^9$ cm s$^{-1}$, which is lower than the values in the literatures ($3.0 \times 10^9$ cm s$^{-1}$, Toy et al. 2016; $2.7 \times 10^9$ cm s$^{-1}$, D’Elia et al. 2015); the ejecta velocity of GRB 161219B/SN 2016jca is $3.12^{+0.00}_{-0.00} \times 10^9$ cm s$^{-1}$, which is slightly higher than that derived by Cano et al. (2017a) (2.97 $\pm$ 0.15 $\times 10^9$ cm s$^{-1}$, but consistent with the value derived by Ashall et al. (2019) (3.5 $\pm$ 0.7 $\times 10^9$ cm s$^{-1}$).

The kinetic energy ($E_k$) of the ejecta can be derived by $E_k = 0.3 M_{ej} v_{ej}^2$. The $E_k$ of the ejecta of GRB 011121/SN 2001ke is $0.56^{+0.09}_{-0.08} \times 10^{52}$ erg; the $E_k$ of GRB 100316D/SN 2010bh is $0.24^{+0.02}_{-0.02} \times 10^{52}$ erg, which is significantly lower than that in the literatures ($1.39 \pm 0.06 \times 10^{52}$ erg, Cano et al. 2011; $2.4 \pm 0.7 \times 10^{52}$ erg, Olivas et al. 2012; $0.97 \pm 0.55 \times 10^{52}$ erg, Bufano et al. 2012); the smaller value of $E_k$ of GRB 100316D/SN 2010bh is due to a smaller velocity. The kinetic energy of the ejecta of GRB 130702A/SN 2013dx is $0.49^{+0.02}_{-0.02} \times 10^{52}$ erg, which is slightly lower than the value derive by (Toy et al. 2016, 0.82 $\times 10^{52}$ erg) and significantly lower than the value inferred by D’Elia et al. (2015) (3.5 $\times 10^{52}$ erg). We note that Toy et al. (2016) derived the kinetic energy using $E_k = 0.5 M_{ej} v_{ej}^2$; the value of $E_k$ must be $0.49 \times 10^{52}$ erg which is equal to our derived value, if $E_k = 0.3 M_{ej} v_{ej}^2$ was adopted. The kinetic energy of the ejecta of GRB 161219B/SN 2016jca is $2.93^{+0.00}_{-0.08} \times 10^{52}$ erg, which is slightly lower than that derived by (Cano et al. 2017a, 5.1 $\pm$ 0.8 $\times 10^{52}$ erg).

D’Elia et al. (2015) construct the the quasi-bolometric light curve of SN 2013dx and derive the $^{56}$Ni mass by scaling the quasi-bolometric light curve of SN2003dh.
3.3. The Theoretical Bolometric Light Curves

We use the derived best-fitting parameters to yield the bolometric light curves of the four GRB-SNe we study, see Figure 2. We find that the peak bolometric luminosities of SN 2001ke, SN 2010bh, SN 2013dx, and SN 2016jca are $1.26 \times 10^{43}$ erg s$^{-1}$, $3.56 \times 10^{42}$ erg s$^{-1}$, $1.74 \times 10^{43}$ erg s$^{-1}$, and $1.81 \times 10^{43}$ erg s$^{-1}$, respectively.

For comparison, the peak (quasi-)bolometric luminosities of the four GRB-SNe derived by the literatures are $6 \times 10^{42}$ erg s$^{-1}$ (Cano et al. 2017b), $3.0 \times 10^{42}$ erg s$^{-1}$ (Bufano et al. 2011) or $\sim 3.6 \times 10^{42}$ erg s$^{-1}$ (Cano et al. 2011) or $4.3 \times 10^{42}$ erg s$^{-1}$ (Olivares et al. 2012), $1 \times 10^{43}$ erg s$^{-1}$ (Toy et al. 2016), and $6.3 \times 10^{42}$ erg s$^{-1}$ (Ashall et al. 2019) or $4.6 \times 10^{42}$ erg s$^{-1}$ (Cano et al. 2017a), respectively.

By comparing our derived peak bolometric luminosities of SN 2001ke, SN 2010bh, SN 2013dx, and SN 2016jca to their peak (quasi-)bolometric luminosities in the literatures, we find that the former are respectively 1.85, 1.26 (or 1.08, or 0.91), 1.8, and 3.17 (or 4.34) times that the latter.

The discrepancies of the peak luminosities of bolometric light curves we derive and that of the quasi-bolometric light curves might be due to the fact that the latter omit the flux in UV and/or IR bands. Cano et al. (2011) construct the quasi-bolometric light curve of SN 2010bh by integrating the flux in $UBV'RIJH$-bands, so the quasi-bolometric light curve is slightly dimmer than the bolometric light curve. Toy et al. (2016) construct the quasi-bolometric light curve of SN 2013dx by integrating the flux in $g'r'\prime i'\prime z'yJ$ bands, more flux are neglected. Cano et al. (2017a) use the griz band data to construct the quasi-bolometric light curve of SN 2016jca, the flux might be severely underestimated.

Our derived rise time of SN 2001ke, SN 2010bh, SN 2013dx, and SN 2016jca are 11.8 days, 12.3 days, 12.5 days, and 14.5 days, respectively. The respective rise time of the four SNe in the literatures are $\sim$17.5 days (Cano et al. 2017b), $\sim$8 days (Cano et al. 2011; Olivares et al. 2012), $\sim$14 days (Toy et al. 2016), and $\sim$10 days (Ashall et al. 2019).

4. CONCLUSIONS

In the past two decades, a few dozen LGRBs have been confirmed to be associated with SNe Ic, most of which are SNe Ic-BL and HNe. While the kinetic energy of most GRB-SNe is $\gtrsim 10$ times that normal SNe Ic, their average peak luminosities are not significantly higher than that of SNe Ic. Therefore, the $^{56}$Ni model adopted to account for the light curves of normal SNe Ic have also been used to explain the light curves of GRB-SNe. However, many studies exploring the energy sources of GRB-SNe construct the quasi-bolometric light curves and fit them. This method might underestimdate the $^{56}$Ni masses needed to power the light curves of SNe.

We collect the optical-NIR data of four well-observed GRB-SNe (GRB 011121/SN 2001ke, GRB 100316D/SN 2010bh, GRB 130702A/SN 2013dx, GRB 161219B/SN 2016jca) and use the (broken) power law plus $^{56}$Ni model to fit the multi-band light curves of the total flux which is the sum of that of the optical afterglows of the GRBs and the SNe. We find that the multi-band light curves of GRB 011121/SN 2001ke can be fitted by the power-law plus $^{56}$Ni model, while the multi-band light curves of the rest three GRB-SNe can be fitted by the broken power-law plus $^{56}$Ni model, demonstrating that the SEDs of SNe associated with GRBs can be well described by the blackbody model.

The $^{56}$Ni mass of GRB 100316D/SN 2010bh we derive (0.12 M$_{\odot}$) is consistent with that (0.10 M$_{\odot}$) of Cano et al. (2011) and that (0.12 M$_{\odot}$) of Bufano et al. (2012), but about half of that of Olivares et al. (2012). This is mainly because that the quasi-bolometric light curve of SN 2010bh constructed by Cano et al. (2011) neglect a minor fraction of the flux, being approximately equal to the bolometric light curve.

In contrast, the $^{56}$Ni masses of SN 2013dx and SN 2016jca are 0.62 M$_{\odot}$ and 0.73 M$_{\odot}$, respectively. The former is about $\sim 1.7$ times that of the values derived by Toy et al. (2016) and D’Elia et al. (2015), while the latter is $\sim 3$ times that of the values derived by Cano et al. (2017a) and Ashall et al. (2019). This is because that the constructed quasi-bolometric light curves of SN 2013dx and SN 2016jca omit a larger fraction of the total flux.

The ejecta masses of GRB 100316D/SN 2010bh, GRB 130702A/SN 2013dx, GRB 161219B/SN 2016jca are all slightly lower than that in the literatures. The ejecta velocities of GRB 100316D/SN 2010bh and GRB 130702A/SN 2013dx we derive are (significantly) smaller than that in the literature. This is because the velocities we derive are the photosphere velocity, while the velocities in the literatures are that of the line forming regions which are outside the photospheres and have larger velocities. As pointed by Wang et al. (2021), the derived photosphere velocities of almost all SNe Ic are (significantly) smaller than that inferred from the spectra.

We suggest that the $^{56}$Ni masses of the at least a fraction of GRB-SNe have been underestimated, and the multi-band $^{56}$Ni model can make it possible to avoid underestimating the luminosities of SNe and therefore the $^{56}$Ni masses. Furthermore, the model can be used to the SNe observed in only two or three bands at some or all epochs.
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Table 1. The information of GRB 011121/SN 2001ke, GRB 100316D/SN 2010bh, GRB 130702A/SN 2013dx, and GRB 161219B/SN 2016jca.

| Transient         | RA.             | Dec.             | $z$   | $E_{B-V,\text{fore}}$ | $E_{B-V,\text{host}}$ | data sources |
|-------------------|-----------------|------------------|-------|-----------------------|------------------------|--------------|
| GRB 011121/SN 2001ke | 11h34m29.640s   | -76d01m41.51s    | 0.36  | 0.4197                | 0.117                  | 1, 2, 3      |
| GRB 100316D/SN 2010bh | 07h10m31.800s   | -56d15m20.20s    | 0.0593| 0.1                   | 0.0434                 | 4, 5         |
| GRB 130702A/SN 2013dx | 14h29m14.780s   | +15d46m26.40s    | 0.145 | 0.0368                | 0.1                    | 6, 7, 8      |
| GRB 161219B/SN 2016jca | 06h06m51.430s   | -26d47m29.50s    | 0.1475| 0.0283                | 0.0527                 | 9, 10        |

1. Bloom et al. (2002); 2. Price et al. (2002b); 3. Garnavich et al. (2003); 4. Bufano et al. (2012); 4. Olivares et al. (2012); 6. Toy et al. (2016); 7. Volnova et al. (2017b); 8. D’Elia et al. (2015) 9. Cano et al. (2017a); 10. Ashall et al. (2019).
Table 2. The definitions, the units, and the prior ranges of the parameters of model.

| Parameters | Definitions | Unit | Posterior |
|------------|-------------|------|-----------|
| $A$        | Parameters describing the intensity of afterglow | $[10^1, 10^{10}]$ |
| $\alpha$  | Afterglow powerlaw time exponent | $[-2, 4]$ |
| $\beta$   | Power-law Spectral index | $[-2, 4]$ |
| $\alpha_1$| The post-broken decay index | $[-2, 4]$ |
| $\alpha_2$| The initial decay index | $[-2, 4]$ |
| $n$        | the parameter describing the smoothness of breaking | $[0.01, 500]$ |
| $t_b$      | the break time | $[0.005, 50]$ |
| $M_{ej}$   | the ejecta mass | $M_{\odot}$ | $[0.1, 25]$ |
| $v$        | the ejecta velocity | $10^9$ cm s$^{-1}$ | $[0.5, 4.0]$ |
| $M_{Ni}$   | the $^{56}$Ni mass | $M_{\odot}$ | $[0.01, 0.8]$ |
| $\kappa_\gamma$ | gamma-ray opacity of $^{56}$Ni -cascade-decay photons | cm$^2$ g$^{-1}$ | $[10^{-1.6}, 10^3]$ |
| $T_f$      | the temperature floor of the photosphere | K | $[1000, 20,000]$ |

(a). If the best-fitting velocity exceed that derived by the spectra, the later would be set to be the new upper limits before re-fitting.
Table 3. The Best-fitting Parameters of multi-band lightcurves of GRB 011121/SN 2001ke, GRB 100316D/SN 2010bh, GRB 130702A/SN 2013dx, and GRB 161219B/SN 2016jca.

| Parameter | GRB 011121/SN 2001ke | GRB 100316D/SN 2010bh | GRB 130702A/SN 2013dx | GRB 161219B/SN 2016jca |
|-----------|-----------------------|------------------------|------------------------|------------------------|
| log A     | 2.19$^{+0.14}_{-0.14}$ | 1.02$^{+0.04}_{-0.02}$ | 10.83$^{+0.43}_{-0.43}$ | 4.66$^{+0.11}_{-0.11}$ |
| $\alpha_1$ or $\alpha$ (a) | 1.7$^{+0.01}_{-0.01}$ | 2.66$^{+0.20}_{-0.23}$ | 1.2$^{+0.02}_{-0.01}$ | 3.99$^{+0.01}_{-0.02}$ |
| $\alpha_2$ | -0.11$^{+0.01}_{-0.01}$ | 0.6$^{+0.02}_{-0.02}$ | 0.69$^{+0.00}_{-0.00}$ | 1.82$^{+0.12}_{-0.1}$ |
| n         | 79.81$^{+66.22}_{-73.33}$ | 250.3$^{+169.79}_{-169.77}$ | 1.82$^{+0.1}_{-0.1}$ | 1.82$^{+0.1}_{-0.1}$ |
| $t_b$ (days) | 8.3$^{+0.17}_{-0.22}$ | 1.26$^{+0.05}_{-0.05}$ | 8.32$^{+0.13}_{-0.13}$ | 8.32$^{+0.13}_{-0.13}$ |
| $\beta$  | 0.82$^{+0.01}_{-0.01}$ | -0.04$^{+0.00}_{-0.00}$ | 0.59$^{+0.03}_{-0.03}$ | 0.22$^{+0.01}_{-0.01}$ |
| $M_{ej}$ (M$_\odot$) | 2.05$^{+0.07}_{-0.03}$ | 1.67$^{+0.08}_{-0.09}$ | 2.15$^{+0.06}_{-0.06}$ | 5.06$^{+0.12}_{-0.11}$ |
| $v_{ej}$ ($10^9$ cm s$^{-1}$) | 2.15$^{+0.00}_{-0.5}$ | 1.56$^{+0.03}_{-0.03}$ | 1.96$^{+0.02}_{-0.02}$ | 3.12$^{+0.00}_{-0.00}$ |
| $M_{Ni}$ (M$_\odot$) | 0.43$^{+0.01}_{-0.01}$ | 0.12$^{+0.00}_{-0.00}$ | 0.62$^{+0.00}_{-0.00}$ | 0.73$^{+0.00}_{-0.00}$ |
| log$\kappa$ | -1.95$^{+0.05}_{-0.04}$ | -1.55$^{+0.01}_{-0.01}$ | -1.29$^{+0.02}_{-0.02}$ | -1.35$^{+0.02}_{-0.02}$ |
| $T_f$ (K) | 5593.18$^{+139.56}_{-139.28}$ | 4885.97$^{+69.07}_{-69.49}$ | 4138.66$^{+36.26}_{-40.25}$ | 4758.48$^{+25.62}_{-25.21}$ |
| $\chi^2$/dof | 3.28 | 13.13 | 18.42 | 44.99 |

(a). For power-law function, the parameter is $\alpha$; for broken power-law function, the parameter is $\alpha_1$. 
Figure 1. The fits of the multi-band light curves of GRB 011121/SN 2001ke (the top-left panel), GRB 100316D/SN 2010bh (the top-right panel), GRB 130702A/SN 2013dx (the bottom-left panel), and GRB 161219B–SN 2016jca (the bottom-right panel). The solid, dotted, and the dashed lines present the total flux, the afterglow flux, and the SN flux, respectively.

Figures A1, A2, A3, and A4 show the corner plots of the $^{56}\text{Ni}$ model for GRB 011121/SN 2001ke, GRB 100316D/SN 2010bh, GRB 130702A/SN 2013dx, and GRB 161219B/SN 2016jca in the main text.
Figure 2. The bolometric light curves reproduced by the best-fitting parameters of the $^{56}\text{Ni}$ model.
Figure A1. The corner plot of the $^{56}\text{Ni}$ model for multi-band light curves of GRB 011121/SN 2001ke
Figure A2. The corner plot of the $^{56}\text{Ni}$ model for multi-band light curves of GRB 100316D/SN 2010bh
Figure A3. The corner plot of the $^{56}\text{Ni}$ model for multi-band light curves of GRB 130702A/SN 2013dx
Figure A4. The corner plot of the $^{56}$Ni model for multi-band light curves of GRB 161219B/SN 2016jca