Pentaquark $\Theta^+$ in nuclear matter and $\Theta^+$ hypernuclei

H. Shen

Department of Physics, Nankai University, Tianjin 300071, China

H. Toki

Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki, Osaka 567-0047, Japan

Abstract

We study the properties of the $\Theta^+$ in nuclear matter and $\Theta^+$ hypernuclei within the quark mean-field (QMF) model, which has been successfully used for the description of ordinary nuclei and $\Lambda$ hypernuclei. With the assumption that the non-strange mesons couple only to the $u$ and $d$ quarks inside baryons, a sizable attractive potential of the $\Theta^+$ in nuclear matter is achieved as a consequence of the cancellation between the attractive scalar potential and the repulsive vector potential. We investigate the $\Theta^+$ single-particle energies in light, medium, and heavy nuclei. More bound states are obtained in $\Theta^+$ hypernuclei in comparison with those in $\Lambda$ hypernuclei.

PACS numbers: 21.65.+f, 21.30.Fe, 21.60.-n, 21.80.+a

Keywords: Pentaquark, Quark mean-field model, Hypernuclei

*Electronic address: songtc@public.tpt.tj.cn
†Electronic address: toki@rcnp.osaka-u.ac.jp
I. INTRODUCTION

The signature of the pentaquark was found in the experiment with GeV photons on $^{12}\text{C}$ at LEPS of SPring8\cite{1, 2}. The $\gamma n \rightarrow K^+ K^- n$ reaction was analyzed and a narrow resonance state was identified with the mass of 1.54 GeV and the width smaller than 25 MeV. Such a narrow state was predicted by Diakonov et al.\cite{3} as a pentaquark state at nearly the same energy. There were many experimental works published after the original study by using various reactions\cite{4}. Recently, many further experiments, particularly at very high energy, report negative results except for a few positive results\cite{4}. At this moment, the spin and parity of the state are not known. Even the width is not precisely identified experimentally.

There are many theoretical works\cite{4}. It seems those models, which emphasize the importance of the chiral symmetry, suggest the positive parity ($J^P = 1/2^+$) as the original predication of Diakonov et al.\cite{3}. On the other hand, the naive constituent quark models tend to provide the negative parity ($J^P = 1/2^-$). The QCD originated works as the lattice QCD and the QCD sum rule approaches seem to provide the negative parity, although these methods face the difficulty of handling the nearby $K^+ n$ threshold and the small pion mass. Hence, we are at this moment not sure about the presence of the pentaquark state, the spin-parity and even the width of this state\cite{4}.

We should make further efforts to get more information for the pentaquark state. There are several interesting theoretical works to study the pentaquark in nuclear medium\cite{5, 6, 7, 8, 9}. In Ref.\cite{7}, the authors study the interaction of the pentaquark with nucleons using the meson exchange model. They have obtained a large binding potential with the width to be small to take the pentaquark nucleus seriously. This work then motivated a theoretical work to calculate the ($K^+, \pi^+$) reaction for the formation of the pentaquark nuclei\cite{10}. There is a good possibility to see $\Theta^+$ hypernucleus in such a reaction. It is then very interesting to study the $\Theta^+$ hypernucleus from different viewpoint.

We studied nuclei and hypernuclei in terms of the quark mean-field (QMF) model\cite{11, 12}. We assume in the QMF model that up and down quarks in the baryons interact with $\sigma$, $\omega$, and $\rho$ mesons (mesons made of up and down quarks) in nuclear matter and in nuclei. Hence, the interaction of the $\Theta^+$ with other nucleons is obtained without introducing new parameters in the QMF model. The only assumption needed here is the isospin of the $\Theta^+$ to be $I = 0$. The interactions of quarks with the meson fields change the properties of the
baryons in hadronic matter and hence the hadronic matter property is obtained from these interactions self-consistently.

In the next section, we would like to present the formulation of the Θ⁺ hypernuclei in the relativistic quark mean-field model. In section III we provide the numerical results for various Θ⁺ hypernuclei. Section IV will be devoted to the conclusion and discussions.

II. QUARK MEAN-FIELD MODEL FOR THE Θ⁺ IN MEDIUM

We start with the description of the Θ⁺ in nuclear medium, which is comparable to the treatment of Λ hypernuclei in our previous work [12]. The Θ⁺ is known to be an exotic baryon containing five quarks (uudd̄s) with strangeness \( S = +1 \), isospin \( I = 0 \), and charge \( Q = +1 \). However, its parity is still ambiguous experimentally. In this work, we use the constituent quark model to describe the Θ⁺ and nucleons in medium, while the exchanged mesons couple directly to the quarks inside baryons. The constituent quarks satisfy the Dirac equation:

\[
\left[ i \gamma_\mu \partial^\mu - m_q - \chi_c - g_\sigma^q \sigma - g_\omega^q \omega \gamma^0 - g_\rho^q \rho \tau_3 \gamma^0 \right] \phi^q(r) = 0, \tag{1}
\]

where \( \chi_c \) is the confining potential taken as the quadratic form, \( \chi_c = \frac{1}{2} k r^2 \). Because the Lorentz structure of the confining interaction is not well known yet, we here use a scalar confinement. In our previous studies for finite nuclei and Λ hypernuclei [11, 12], both scalar and scalar-vector confinements have been taken into account, but it is known that antiquarks cannot be confined by a scalar-vector confinement when the vector part is equal or larger than the scalar part. Therefore we just adopt the scalar confinement in the present calculation of Θ⁺. We assume that the non-strange mesons, \( \sigma, \omega, \) and \( \rho \), couple exclusively to the up and down quarks and not to the strange quark (or antiquark) according to the OZI rule, hence the strange antiquark inside Θ⁺ satisfies the Dirac equation in which the terms coupled to the non-strange mesons vanish \( (g_\sigma^{s, \bar{s}} = g_\omega^{s, \bar{s}} = g_\rho^{s, \bar{s}} = 0) \). We note that the Dirac equation for the strange antiquark \( \bar{s} \) in Θ⁺ is identical to the equation for the strange quark \( s \) in Λ when a scalar confining potential is adopted in Eq. (1). We follow Ref. [11] to take into account the spin correlations and remove the spurious center of mass motion, and then obtain the effective mass of the Θ⁺ in medium as

\[
M_{\Theta}^* = \sqrt{(4 e_q + e_\bar{s} + E_{\text{spin}}^\Theta)^2 - (4 \langle p_q^2 \rangle + \langle p_{\bar{s}}^2 \rangle)}, \tag{2}
\]
while the effective mass of the nucleon is expressed as

\[ M_N^* = \sqrt{(3e_q + E_{\text{spin}}^N)^2 - 3\langle p^2_q \rangle}. \]  

(3)

Here, the subscript \( q \) denotes the \( u \) or \( d \) quark. The energies (\( e_q \) and \( e_{\bar{s}} \)) and momenta (\( \langle p^2_q \rangle \)) can be obtained by solving the corresponding equations. The spin correlations (\( E_{\text{spin}}^\Theta \) and \( E_{\text{spin}}^N \)), which are taken as parameters in the QMF model, are determined by fitting the baryon masses in free space (\( M_\Theta = 1540 \text{ MeV} \) and \( M_N = 939 \text{ MeV} \)). The effective masses of baryons are influenced by the \( \sigma \) mean-field in nuclear medium, which provides a scalar potential to the \( u \) and \( d \) quarks in baryons and as a consequence reduces the constituent quark mass to \( m_q^* = m_q + g_q^3 \sigma \) (\( q = u, d \)). However, the \( \omega \) and \( \rho \) mean fields could not cause any change in the baryon masses, and they appear merely as the energy shift. Therefore, we obtain the effective masses of baryons as functions of the \( \sigma \) mean-field, \( M_{\Theta}^*(\sigma) \) and \( M_N^*(\sigma) \), expressed by Eqs. (2) and (3).

We now derive a self-consistent treatment for a \( \Theta^+ \) hypernucleus, which contains a pentaquark \( \Theta^+ \) and many nucleons. The baryons in this system interact through the exchange of \( \sigma, \omega, \) and \( \rho \) mesons, just like in the case of a single \( \Lambda \) hypernucleus. The effective Lagrangian at the hadron level within the mean-field approximation can be written as

\[
\mathcal{L} = \bar{\psi} \left[ i\gamma_\mu \partial^\mu - M_N^*(\sigma) - g_\omega \omega \gamma^0 - g_\rho \rho \tau_3 \gamma^0 - e \frac{(1 + \tau_3)}{2} A^\gamma_0 \right] \psi \\
+ \bar{\psi}_\Theta \left[ i\gamma_\mu \partial^\mu - M_{\Theta}^*(\sigma) - g_\omega^\Theta \omega \gamma^0 - e A^\gamma_0 \right] \psi_\Theta \\
- \frac{1}{2} (\nabla \sigma)^2 - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{4} g_3 \sigma^4 + \frac{1}{2} (\nabla \omega)^2 + \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{4} \sigma_3 \omega^4 \\
+ \frac{1}{2} (\nabla \rho)^2 + \frac{1}{2} m_\rho^2 \rho^2 + \frac{1}{2} (\nabla A)^2, \]

where \( \psi \) and \( \psi_\Theta \) are the Dirac spinors for the nucleon and \( \Theta^+ \). The mean-field values are denoted by \( \sigma, \omega, \) and \( \rho \), respectively, while \( m_\sigma, m_\omega, \) and \( m_\rho \) are the masses of these mesons. The electromagnetic field, denoted by \( A \), couples both to the proton and \( \Theta^+ \), which carry a positive charge. The influences of the \( \sigma \) mean-field on baryons are contained in the effective masses \( M_N^*(\sigma) \) and \( M_{\Theta}^*(\sigma) \). The \( \omega \) meson, which couples directly to the \( u \) and \( d \) quarks inside baryons, provides an interaction at the hadron level with the coupling constant \( g_\omega^i = n_i g_\omega^n \), where \( n_i \) is the number of the \( u \) and \( d \) quarks in the baryon \( i \). When \( i = \Theta^+ (i = N) \), we get \( g_\omega^\Theta = 4 g_\omega^N \) (\( g_\omega^N = g_\omega = 3 g_\omega^d \)). The \( \rho \) meson doesn’t couple to the \( \Theta^+ \) which is an isoscalar particle, but it couples to the nucleon with the coupling constant \( g_\rho^N = g_\rho = g_\rho^d \), as given
In Ref. [11]. In the QMF model, the basic parameters are the quark-meson couplings \( g_\sigma, g_\omega, \) and \( g_\rho \), the nonlinear self-coupling constants \( g_3 \) and \( c_3 \), and the mass of the \( \sigma \) meson \( (m_\sigma) \), which have been determined by fitting the properties of nuclear matter and finite nuclei in Ref. [11]. Therefore, no more adjustable parameters exist when it is extended to the calculations for \( \Theta^+ \) or \( \Lambda \) hypernuclei. From the Lagrangian given in Eq. (4), we obtain the following Euler-Lagrange equations:

\[
\begin{align*}
&\left[ i\gamma_\mu \partial^\mu - M_N^*(\sigma) - g_\omega \omega \gamma^0 - g_\rho \rho \tau_3 \gamma^0 - e \frac{(1 + \tau_3)}{2} A\gamma^0 \right] \psi = 0, \\
&\left[ i\gamma_\mu \partial^\mu - M_\Theta^*(\sigma) - g_\Theta \Theta \omega \gamma^0 - e A\gamma^0 \right] \psi_\Theta = 0, \\
&\left( -\Delta + m_\sigma^2 \right) \sigma = -\frac{\partial M_N^*(\sigma)}{\partial \sigma} \rho_s - \frac{\partial M_\Theta^*(\sigma)}{\partial \sigma} \rho_\Theta^s - g_3 \sigma^3, \\
&\left( -\Delta + m_\omega^2 \right) \omega = g_\omega \rho_v + g_\Theta \Theta \omega \rho_\Theta^v - c_3 \omega^3, \\
&\left( -\Delta + m_\rho^2 \right) \rho = g_\rho \rho_3, \\
&-\Delta A = e \left( \rho_p + \rho_\Theta^v \right),
\end{align*}
\]

where \( \rho_s (\rho_\Theta^s), \rho_v (\rho_\Theta^v), \rho_3, \) and \( \rho_p \) are the scalar, vector, third component of isovector, and proton densities, respectively. The above coupled equations are solved self-consistently with the effective masses \( M_N^*(\sigma) \) and \( M_\Theta^*(\sigma) \) obtained at the quark level.

The application of the QMF model to the description of \( \Lambda \) hypernuclei has been reported in our previous work [12]. Without adjusting any parameters, the properties of \( \Lambda \) hypernuclei could be described reasonably well. The \( \Lambda \) single-particle energies obtained in the QMF model are slightly underestimated in comparison with the experimental values. In the present work, we investigate the \( \Theta^+ \) hypernuclei within the QMF model, and then provide the predictions for \( \Theta^+ \) single-particle energies in light, medium, and heavy nuclei.

### III. NUMERICAL RESULTS

In this section, we will present the results calculated in the QMF model for the \( \Theta^+ \) in nuclear medium. We need first to specify the parameters used in the present calculation. For the parameters at the quark level, we take the constituent quark masses \( m_u = m_d = 313 \) MeV and \( m_s = m_\bar{s} = 490 \) MeV, and the strength of the confining potential \( k = 700 \) MeV/fm\(^2\), as those used in the case of \( \Lambda \) hypernuclei [12]. The meson masses are taken as \( m_\sigma = 470 \) MeV, \( m_\omega = 783 \) MeV, and \( m_\rho = 770 \) MeV. The quark-meson couplings, \( g_\sigma^3 = 3.14, \)
$g_\omega^q = 4.20$ and $g_\rho^q = 4.3$, and the nonlinear self-coupling constants, $g_3 = 50.7$ and $c_3 = 53.6$, have been determined by fitting the following equilibrium properties of nuclear matter: equilibrium density $\rho_0 = 0.145 \text{ fm}^{-3}$; binding energy $E/A = -16.3 \text{ MeV}$; incompressibility $k = 280 \text{ MeV}$; effective mass $M_N^* = 0.63 M_N$; symmetry energy $a_{\text{sym}} = 35 \text{ MeV}$. With the above parameters, the QMF model has provided quite satisfactory results for finite nuclei and $\Lambda$ hypernuclei [11, 12]. Now, we take the same parameters as above to perform the calculations for the $\Theta^+$ in nuclear medium.

It is very interesting to see how the $\Theta^+$ properties change in nuclear matter. In Fig. 1, we present the variation of the effective mass of the $\Theta^+$, in comparison with those for the nucleon and $\Lambda$, as a function of the quark mass correction due to the presence of the $\sigma$ mean-field, $\delta m_q = m_q - m_q^* = -g_\omega^q \sigma$ ($q = u, d$). It is obvious that the reduction of $M_\Theta^*$ is larger than those of $M_N^*$ and $M_\Lambda^*$, since there are four $u$ and $d$ quarks in the $\Theta^+$ which are influenced by the $\sigma$ mean-field, but only three or two quarks influenced in the nucleon or $\Lambda$.

We note that the dependence of the effective masses on the $\sigma$ mean-field must be calculated self-consistently within the quark model, therefore the effective masses of baryons obtained in the QMF model are not simply linear functions of $\sigma$, as given in the relativistic mean-field (RMF) models [13, 14]. We show in Fig. 2 the ratios of the baryon radii in medium to those in free space as functions of the nuclear matter density. It is found that all of the baryons, $\Theta^+$, $\Lambda$, and $N$ (nucleon), show significant increase of the radius in nuclear medium, which is consistent with the famous EMC effect. The nucleon gains larger increase of the radius than the $\Theta^+$ and $\Lambda$, because all three quarks in $N$ are involved in the interactions with mesons while only four-fifths in $\Theta^+$ and two-thirds in $\Lambda$ involved. The baryon radii increase by about $4\% \sim 6\%$ at normal matter density in the present model. In Fig. 3, we plot the scalar and vector potentials of the $\Theta^+$ in medium as functions of the nuclear matter density, and the results of the nucleon ($N$) and $\Lambda$ are also shown for comparison. At $\rho = \rho_0 = 0.145 \text{ fm}^{-3}$, we get $U_S^\Theta = -420 \text{ MeV}$ and $U_V^\Theta = 370 \text{ MeV}$, therefore an residual attractive $\Theta^+$ potential, $U^\Theta = U_S^\Theta + U_V^\Theta = -50 \text{ MeV}$, is predicted in the present model. For $\Lambda$ at $\rho = \rho_0$, we have got the potential $U^\Lambda = U_S^\Lambda + U_V^\Lambda = -24 \text{ MeV}$, which is about half of the $\Theta^+$ potential. In Ref. [4], a large attractive $\Theta^+$ potential ranging from $-60$ to $-120 \text{ MeV}$ at normal matter density was predicted by calculating the $\Theta^+$ selfenergy tied to the $KN$ decay of the $\Theta^+$ and the two meson baryon decay channels of the $\Theta^+$ partners in an antidecuplet of baryons. A QCD sum rule calculation [8] predicted an attractive $\Theta^+$
potential of about $-40 \sim -90$ MeV. Therefore, the prediction of the $\Theta^+$ potential in the QMF model is compatible with the results obtained by other groups \[7, 8\].

We now present the results of self-consistent calculations for the $\Theta^+$ hypernuclei containing a $\Theta^+$ bound in nuclei. In Fig. 4, we show the predicted $\Theta^+$ single-particle energies in $^{17}$O, $^{41}$Ca, and $^{209}$Pb. It is found that there are many bound states of the $\Theta^+$ in light, medium, and heavy nuclei. We can see that the separation between the deep $\Theta^+$ energy levels is comparable with the $\Theta^+$ width. The theoretical prediction of the free $\Theta^+$ width in the chiral soliton model is less than 15 MeV \[3\], and the $\Theta^+$ width in medium might be reduced to about one third or less of the free width due to Pauli blocking and binding, as pointed out in Ref. \[7\]. It is clear that large separation of the deep $\Theta^+$ levels is helpful to observe distinct peaks in the experiment to produce $\Theta^+$ bound states \[10\]. It is very interesting to compare the results of $\Theta^+$ hypernuclei to those of $\Lambda$ hypernuclei. We can see that the single-particle energy of $1s_{1/2}$ $\Theta^+$ in $^{209}$Pb is smaller than the one in $^{41}$Ca, which is contrary to the results of $\Lambda$ hypernuclei. This is mainly because $\Theta^+$ carrying a positive charge behaves like a proton in nuclei, while $\Lambda$ is a charge neutral particle more like a neutron. The Coulomb energies of $\Theta^+$ in heavy nuclei are much larger than those in light nuclei, that $\Theta^+$ single-particle energies in heavy nuclei can be reduced by the large positive Coulomb energy. We plot in Fig. 5 the scalar, vector, and Coulomb potentials in $^{41}$Ca with a $\Theta^+$ bound at $1s_{1/2}$ state. At the center of the $\Theta^+$ hypernucleus, the attractive scalar potential ($U_\Theta^S \sim -559$ MeV) is partly cancelled by the repulsive vector potential ($U_\Theta^V \sim 515$ MeV) and Coulomb potential ($U_C \sim 11$ MeV), and as a consequence, an attractive $\Theta^+$ potential ($U_\Theta \sim -33$ MeV) remains in this case.

It is interesting and important to compare the results of the QMF model with those obtained by other groups \[7, 9\], and discuss the origin of the differences in the $\Theta^+$ single-particle energies. More bound states of the $\Theta^+$ in nuclei were predicted in some recent works \[7, 9\]. In Ref. \[7\], the authors obtained more deeply bound states by solving the Schrodinger equation with two potentials: $V(r) = -60\rho(r)/\rho_0$ (MeV) and $V(r) = -120\rho(r)/\rho_0$ (MeV). It is obvious that larger $\Theta^+$ single-particle energies should be achieved due to the deeper $\Theta^+$ potentials used in their calculations. In Ref. \[9\], the $\Theta^+$ hypernuclei were studied by using the RMF model with the $\Theta^+$ couplings, $g_\sigma^\Theta = \frac{4}{3}g_\sigma$ and $g_\omega^\Theta = \frac{4}{3}g_\omega$, which are the quark model predictions close to those given in the quark-meson coupling (QMC) model. It may be helpful to look at the case of $\Lambda$ hypernuclei, because of the similarity of the $\Theta^+$ hypernuclei.
and Λ hypernuclei. It is well known that the properties of Λ hypernuclei are very sensitive to the effective coupling constants at the hadron level, especially the two relative couplings $R_\sigma^\Lambda = g_\sigma^\Lambda / g_\sigma$ and $R_\omega^\Lambda = g_\omega^\Lambda / g_\omega$ [15]. The quark model prediction, $R_\sigma^\Lambda = R_\omega^\Lambda = 2/3$, usually gives large overbinding of Λ single-particle energies in comparison with the experimental values. For example, by taking TM1 parameter set in the RMF model with $R_\sigma^\Lambda = R_\omega^\Lambda = 2/3$, the $1s_{1/2}$ Λ single-particle energy in $^{41}\Lambda$Ca is calculated to be $-35.2$ MeV, while in the case of the NL-SH parameter set used, it is $-36.8$ MeV. We note that the experimental value of the $1s_{1/2}$ Λ binding energy in $^{40}\Lambda$Ca is about $-18.7$ MeV. Usually some small deviations from $R_\sigma^\Lambda = R_\omega^\Lambda = 2/3$ should be taken into account in the studies of Λ hypernuclei in order to reproduce the experimental values [15]. It has been pointed out that about 10% increase from $R_\omega^\Lambda = 2/3$ seemed to be needed to improve the overestimated Λ single-particle energies in the QMC model, since the $R_\sigma^\Lambda$ calculated at the quark level in the QMC model was very close to $2/3$ [16, 17]. On the other hand, the QMF model yielded slightly underestimated Λ single-particle energies due to smaller value than $2/3$ of the $R_\sigma^\Lambda$ obtained in the QMF model, which required about 3% reduction from $R_\omega^\Lambda = 2/3$ in order to reproduce the experimental values. It should be due to the same reason that the QMF model predicts relatively small Θ$^+$ single-particle energies in comparison with those obtained in the RMF model with the quark model value of the Θ$^+$ coupling, $g_\sigma^\Theta / g_\sigma = g_\omega^\Theta / g_\omega = 4/3$. In the QMF model, $g_\omega^\Theta = 4g_\omega^q = \frac{4}{3}g_\omega$ is adopted according to the assumption that non-strange mesons couple only to the $u$ and $d$ quarks inside baryons, but $g_\sigma^\Theta = \frac{\partial M^\ast_\Theta}{\partial \sigma}$ has to be worked out self-consistently at the quark level. We notice that the effective σ–baryon couplings in the QMF model are not simply constants as in the RMF model. Therefore, the relative coupling obtained in the QMF model, $R_\sigma^\Theta = \left[ \frac{\partial M^\ast_\Theta}{\partial \sigma} \right] / \left[ \frac{\partial M^\ast_\sigma}{\partial \sigma} \right]$, depends on the σ mean-field in medium, and we get smaller values than $4/3$ for the $R_\sigma^\Theta$ in the QMF model. This is considered as the dominant origin of the differences between the predictions given by the QMF model and those obtained in the RMF model [9].

In order to examine the sensitivity of Θ$^+$ single-particle energies to the Θ$^+$ couplings, we perform the same calculation with a reduced coupling $0.97 \times g_\omega^\Theta$. In this case, the $1s_{1/2}$ Θ$^+$ single-particle energy in $^{41}\Lambda$Ca changes to be $-38.7$ MeV from the value $-28.1$ MeV as shown in Fig. 4. Hence, 3% reduction of $g_\omega^\Theta$ can lead to $\sim 10.6$ MeV decrease of the $1s_{1/2}$ Θ$^+$ single-particle energy in $^{41}\Lambda$Ca, and it provides $\sim 10$ MeV more attraction for the Θ$^+$ potential in nuclear matter at normal matter density. In the present work, we have
performed self-consistent calculations for both $\Theta^+$ and nucleons in $\Theta^+$ hypernuclei. By comparing the results of a normal nucleus containing $A$ nucleons with those of the $\Theta^+$ hypernucleus containing $A$ nucleons and one $\Theta^+$, the effects of the $\Theta^+$ on nucleons can be found. The existence of the $\Theta^+$ does enlarge the scalar and vector potentials of nucleons, but the enlargements are mostly cancelled with each other.

IV. CONCLUSIONS

We have studied the properties of the $\Theta^+$ in nuclear matter and performed self-consistent calculations for $\Theta^+$ hypernuclei in the framework of the QMF model, which has been successfully applied to the descriptions of stable nuclei and $\Lambda$ hypernuclei. In the present work, we have used the constituent quark model to describe the $\Theta^+$ and nucleons, which naturally allows the direct coupling of exchanged mesons with quarks inside baryons. With the parameters determined by the equilibrium properties of nuclear matter and the assumption that non-strange mesons couple only to the $u$ and $d$ quarks, an attractive $\Theta^+$ potential has been achieved as a consequence of the cancellation between the attractive scalar potential and the repulsive vector potential in nuclear matter. The QMF model provides an attractive $\Theta^+$ potential of $\sim -50 \text{ MeV}$, and yields a reduction of effective $\Theta^+$ mass of $\sim 420 \text{ MeV}$ as well as $\sim 5\%$ increase of the $\Theta^+$ radius at normal matter density.

We have investigated the $\Theta^+$ single-particle energies in light, medium, and heavy nuclei. It is found that there are many bound states in $\Theta^+$ hypernuclei, and the $\Theta^+$ seems to be more deeply bound than the $\Lambda$ in hypernuclei. We have discussed the sensitivity of $\Theta^+$ single-particle energies to the $\Theta^+$ couplings. The comparison between the results calculated in the QMF model and those obtained by other groups has been done, and the origin of the differences has also been discussed by referring to the case of $\Lambda$ hypernuclei. The results obtained in the QMF model provide encouragement for the experimental work to produce $\Theta^+$ hypernuclei.

We comment here a possible method to produce $\Theta^+$ hypernuclei. The gamma induced reaction on nucleus for the formation of the $\Theta^+$ leads to the $\Theta^+$ in a scattering state due to the large momentum transfer. On the other hand, the $K^+$ induced reaction with a nucleus could produce the $\Theta^+$ with a reasonable momentum of order 500 MeV, when an outgoing pion is observed at the forward angle. This momentum transfer would lead to high angular
momentum transfer of order $L \approx 10$, when the $\Theta^+$ is produced at the nuclear surface with a medium heavy nuclei. Hence, it is highly plausible to observe a sharp peak with high angular momentum; high spin $\Theta^+$ - high spin nucleon hole state, popping up in the beginning of the continuum region as the case of the $(p,\pi^-)$ reaction [18].

**Acknowledgments**

This work was supported in part by the National Natural Science Foundation of China (No. 10135030) and the Specialized Research Fund for the Doctoral Program of Higher Education (No. 20040055010).
[1] T. Nakano et al. (LEPS collaboration), Nucl. Phys. A721, 112c (2003).
[2] T. Nakano et al. (LEPS collaboration), Phys. Rev. Lett. 91, 012002 (2003).
[3] D. Diakonov, V. Petrov, and M. Polyakov, Z. Phys. A 359, 305 (1997).
[4] http://www.rcnp.osaka-u.ac.jp/~penta04/
[5] G.A. Miller, Phys. Rev. C 70, 022202 (2004).
[6] H.C. Kim, C.H. Lee, and H.J. Lee, J. Kor. Phys. Soc. 46, 393 (2005).
[7] D. Cabrera, Q.B. Li, W. Magas, E. Oset, and M.J. Vicente Vacas, Phys. Lett. B 608, 231 (2005).
[8] F.S. Navarra, M. Nielsen, and K. Tsushima, Phys. Lett. B 606, 335 (2005).
[9] X.H. Zhong, Y.H. Tan, L. Li, and P.Z. Ning, Phys. Rev. C 71, 015206 (2005).
[10] H. Nagahiro, S. Hirenzaki, E. Oset, and M.J. Vicente Vacas, nucl-th/0408002
[11] H. Shen and H. Toki, Phys. Rev. C 61, 045205 (2000).
[12] H. Shen and H. Toki, Nucl. Phys. A707, 469 (2002).
[13] B.D. Serot and J.D. Walecka, Adv. Nucl. Phys. 16, 1 (1986).
[14] Y. Sugahara and H. Toki, Prog. Theor. Phys. 92, 803 (1994).
[15] C.M. Keil, F. Hofmann, and H. Lenske, Phys. Rev. C 61, 064309 (2000).
[16] K. Tsushima, K. Saito, and A.W. Thomas, Phys. Lett. B 411, 9 (1997); (E) Phys. Lett. B 421, 413 (1998).
[17] K. Tsushima, K. Saito, J. Haidenbauer, and A.W. Thomas, Nucl. Phys. A630, 691 (1998).
[18] A.B. Brown, O. Scholten, and H. Toki, Phys. Rev. Lett. 51, 1952 (1983).
FIG. 1: The variations of the baryon effective masses, $\delta M_i^* = M_i^* - M_i (i = N, \Lambda, \Theta^+)$, as functions of the quark mass correction, $\delta m_q = m_q - m_q^* = -g_3^2 \sigma$. 
FIG. 2: The ratios of the baryon radii in nuclear matter $R_i$ to that in free space $R_i^0$ as functions of the nuclear matter density $\rho$. 
FIG. 3: The scalar and vector potentials, $U_S$ and $U_V$, as functions of the nuclear matter density $\rho$. 
FIG. 4: The $\Theta^+$ single-particle energies in $^{17}$O, $^{41}$Ca, and $^{209}$Pb.
FIG. 5: The scalar potential $U_i^S$, the vector potential $U_i^V$, and the Coulomb potential $U_C$ for the nucleon ($i = N$) and the $\Theta^+$ ($i = \Theta$) in $^{41}_{\Theta}Ca$ with a $\Theta^+$ bound at $1s_{1/2}$ state.