We study the phenomenology of a class of models describing modular invariant gaugino condensation in the hidden sector of a low-energy effective theory derived from the heterotic string. Placing simple demands on the resulting observable sector, such as a supersymmetry-breaking scale of approximately 1 TeV, results in significant restrictions on the possible configurations of the hidden sector.

This talk summarizes recent investigations of the phenomenology of heterotic string-derived supergravity theories invoking gaugino condensation in a hidden sector to break supersymmetry and is a condensation of material more fully presented elsewhere. The philosophy behind the research reported here is to ask whether we can obtain intuition on the nature of the hidden sector matter and gauge content by requiring ever-increasing degrees of agreement with the observed world. The effective Lagrangian we employ incorporates recent developments in string theory and tempers a high degree of realism with sufficient assumptions to preserve tractability.

Supersymmetry breaking is implemented via condensation of gauginos charged under the hidden sector gauge group $G = \prod_a G_a$. For each gaugino condensate a vector superfield $V_a$ is introduced and the gaugino condensate superfields $U_a \equiv \text{Tr}(W^a W_a)$ are then identified as the (anti-)chiral projections of the vector superfields $U_a = - (D_a D^a - 8R) V_a$. The dilaton field (in the linear multiplet formalism used here) is the lowest component of the vector superfield $V = \sum_a V_a$: $\ell = V|_{\theta = \bar{\theta} = 0}$. The vacuum expectation value (VEV) of this field is related to the unified gauge coupling at the string scale by the relation $\langle \ell \rangle = g_{\text{str}}^2/2$. In the class of orbifold compactifications we will be considering there are, in addition to the dilaton, three untwisted moduli chiral superfields $T^I$ which parameterize the size of the compactified space.

The Kähler potential for these moduli and the matter chiral superfields $\Phi^A$ is given by $K = \ln V + \sum_I g^I + \sum_A e^{\sum_I q^I_A g^I} |\Phi^A|^2 + O(\Phi^4)$, where $g^I = -\ln(T^I + \bar{T}^I)$ and the $q^I_A$ are the modular weights of the fields $\Phi^A$. The relevant part of the complete effective Lagrangian is then $\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{KE}} + \mathcal{L}_{\text{GS}} + \mathcal{L}_{\text{VV}} + \mathcal{L}_{\text{pot}}$. The kinetic energy Lagrangian has been ammended to include possible non-perturbative corrections of string-theoretical origin involving the
dilaton: $\mathcal{L}_{\text{KE}} = \int d^4 \theta E \left[ -2 + f(V) \right]$. The parameterization of these effects that we will adopt is of the form

$$f(V) = \left[ A_0 + A_1/\sqrt{V} \right] e^{-B/\sqrt{V}},$$

which was shown to allow dilaton stabilization at weak to moderate string coupling with parameters that are all of $\mathcal{O}(1)$. In the presence of these non-perturbative effects the relation between the string coupling and the VEV of the dilaton becomes $g_{\text{str}}^2/2 = \ell/(1 + f(\ell))$.

The second term in $\mathcal{L}_{\text{eff}}$ is a generalization of the original Veneziano-Yankielowicz superpotential term for the gaugino condensate,

$$\mathcal{L}_{\text{VY}} = \frac{1}{8} \sum_a \int d^4 \theta \frac{E}{R} U_a \left[ b_a' \ln \left( e^{-K/2} U_a \right) + \sum_{\alpha} b_{a\alpha}^0 \ln \left( \left( \Pi^\alpha \right)^{n_{a\alpha}} \right) \right] + \text{h.c.}, \quad (1)$$

which involves the gauge condensates $U_a$ as well as possible gauge-invariant matter condensates described by chiral superfields $\Pi^\alpha \simeq \prod_A \left( \Phi^A \right)^{n_{a\alpha}}$. The coefficients $b_a', b_{a\alpha}^0$ are determined by demanding the correct transformation properties of the expression in (1) under chiral and conformal transformations and yield the following relations:

$$b_a \equiv b_a' + \sum_{\alpha} b_{a\alpha}^0 = \frac{1}{8\pi^2} \left( C_a - \frac{1}{3} \sum_A C^A_a \right), \quad \sum_{\alpha,A} b_{a\alpha}^0 n_{a\alpha} = \sum_A C^A_a/4\pi^2. \quad (2)$$

In (2) the quantities $C_a$ and $C^A_a$ are the quadratic Casimir operators for the adjoint and matter representations, respectively.

The third term in $\mathcal{L}_{\text{eff}}$ is a modular-invariant superpotential term for the matter condensates: $\mathcal{L}_{\text{pot}} = \left\{ \frac{1}{2} \int d^4 \theta E \sum \left[ W \left( \left( \Pi^\alpha \right)^{p_\alpha}, T \right) \right] + \text{h.c.} \right\}$. We adopt the simplifying assumptions that there are no unconfined hidden sector matter fields, each condensate is charged under only one hidden sector gauge group and that each term in the hidden sector superpotential is effectively of dimension three. This allows a simple factorization of the superpotential of the form $W \left( \left( \Pi^\alpha \right)^{p_\alpha}, T \right) = \sum_c c_a W_a \left( T \right) \left( \Pi^\alpha \right)^{p_\alpha}$, and we require $p_\alpha \sum_A n_{a\alpha} = 3 \quad \forall \alpha$.

The remaining term in $\mathcal{L}_{\text{eff}}$ is the Green-Schwarz (GS) counterterm introduced to ensure modular invariance. In this note we consider its simplest possible form $\mathcal{L}_{\text{GS}} = b \int d^4 \theta EV \sum_t g^t$ where $b \equiv C_{E_8}/8\pi^2 \approx 0.38$ is proportional to the beta-function coefficient for the group $E_8$.

Having described the effective Lagrangian we are in a position to turn our attention to the observable sector phenomenology. The equations of motion

\(^{a}\text{As an example, the dilaton potential can be minimized with vanishing cosmological constant and } \alpha_{\text{str}} = 0.04 \text{ for } A_0 = 3.25, A_1 = -1.70 \text{ and } B = 0.4 \text{ in } f(V).\)
for the auxiliary fields of the condensates $U^a$ give

$$
\rho_a^2 = e^{-2\beta_a} e^K e^{-(1+\ell) \phi_a} e^{-\frac{\beta_a}{4} \sum_f \left| \eta(t_f) \right|^4 \prod_\alpha \left| b^\alpha_a / 4c_\alpha \right|^2},
$$

(3)

where $t_f \equiv T_I|_{\theta=\bar{\theta}=0}$, $u_a = U_a|_{\theta=\bar{\theta}=0} \equiv \rho_a e^{i\omega_a}$ and $\eta(t_f)$ is the Dedekind function. The scalar potential for the moduli $t_I$ is minimized at the self-dual points $\langle t_I \rangle = 1$ or $\langle t_I \rangle = \exp(i\pi/6)$, where the corresponding F-components $F_I$ of the chiral superfields $T^I$ vanish – thus allowing for the “dilaton-dominated” supersymmetry-breaking pattern with naturally suppressed flavor-changing neutral currents.

To disentangle the complexity of (3) it is convenient to assume that all of the matter in the hidden sector which transforms under a given subgroup $G_a$ is of the same representation, such as the fundamental representation, and then make the simultaneous variable redefinition

$$
\sum_\alpha b^\alpha_a \equiv (b^\alpha_a)_{\text{eff}} = N_c b^\text{rep}_a; \quad (c_\alpha)_{\text{eff}} \equiv N_c \left( \prod_{\alpha=1}^{N_c} c_\alpha \right)^{\frac{1}{N_c}}.
$$

(4)

In the above equation $b^\text{rep}_a$ is proportional to the quadratic Casimir operator for the matter fields in the common representation and $N_c$ is the number of condensates.

From a determination of the condensate value $\rho$ the supersymmetry-breaking scale can be found by solving for the gravitino mass, given by $M_{3/2} = 1/4 \left( \langle |M| \rangle = 1/4 \langle |\sum_b b_a u_a| \rangle \right)$. In the case of multiple gaugino condensates the scale of supersymmetry breaking is governed by the condensate with the largest one-loop beta-function coefficient. Hence we will here consider the case with just one condensate with beta-function coefficient denoted $b_+ = M_{3/2} = \frac{1}{4} b_+ \langle |u_+| \rangle$.

Now for given values of $(c_\alpha)_{\text{eff}}$ and $g_{\text{str}}$ the condensation scale $\Lambda_{\text{cond}} = (M_{\text{Planck}}) \langle \rho^2 \rangle^{1/6}$ and gravitino mass can be plotted in the $\{b_+, (b^\alpha_a)_{\text{eff}}\}$ plane, as in Figure 1. Also shown is the variation of the gravitino mass as a function of the Yukawa parameters $(c_\alpha)_{\text{eff}}$, where a very generous spread in the supersymmetry-breaking scale and hidden sector Yukawas is allowed.

Upon $Z_N$ orbifold compactification the $E_8$ gauge group of the hidden sector is presumed to break to some subgroup(s) of $E_8$. For each such subgroup the equations in (4) define a line in the $\{b_+, (b^\alpha_a)_{\text{eff}}\}$ plane. In Figure 2 we have overlaid these gauge lines on a plot similar to the previous one. We restrict the effective Yukawa couplings of the hidden sector to a more reasonable range and give three different values of the string coupling at the string scale. The choice of string coupling is made when specifying the boundary conditions for solving the dilaton potential, and hence $f(\langle t_f \rangle)$ in equation (3).
A typical matter configuration would be represented in Figure 2 by a point on one of the gauge group lines. The number of possible configurations consistent with a given choice of \( \{ \alpha_{\text{str}}, \langle c_{\alpha} \rangle_{\text{eff}} \} \) and supersymmetry-breaking scale \( M_{3/2} \) is quite restricted. For example, Figure 2 immediately rules out hidden sector gauge groups smaller than SU(6) for weak coupling at the string scale \( (g_{\text{str}}^2 \approx 0.5) \). Furthermore, even moderately larger values of the string coupling at unification become difficult to obtain as it is necessary to postulate a hidden sector with very small gauge group and particular combinations of matter to force the beta-function coefficient to small values.

Additional constraints on the model arise from demanding an acceptable pattern of electroweak symmetry breaking, super-partner masses consistent with search results from LEP and the Tevatron and cosmology. The spectra of soft-terms arising from these models was investigated at tree-level and recently at the one-loop level. To focus on the gaugino sector, the gaugino masses at the condensation scale are given by

\[
m_{\lambda_{\alpha}|\mu=\Lambda_{\text{cond}}} = -\frac{g_{\alpha}^2(\mu)}{2} \left[ \frac{3b_{+}(1 + b'_{\alpha}\ell)}{1 + b_{+}\ell} - 3b_{\alpha} \right] M_{3/2}, \tag{5}
\]

when the matter fields do not couple to the Green-Schwarz counterterm. The first term in (5) arises from the presence of the gaugino condensate directly, while the second term is the one-loop contribution arising from the conformal...
anomaly. When the condensing group beta-function coefficient $b_+$ is sufficiently small this anomaly-induced piece can be the dominant contribution to gaugino masses, giving rise to a phenomenology typical of so-called “anomaly-mediated” models. Such situations generally produce a neutralino as the lightest supersymmetric particle (LSP) with significant wino-like content, as is displayed in Figure 3.

In models where the scalars decouple from the Green-Schwarz term the scalar masses are equal to the gravitino mass and typically are larger than the gaugino masses by an order of magnitude. Thus, avoiding LEP search limits on charginos generally requires scalar masses in the TeV range and low $\tan \beta$. In standard supergravity scenarios such heavy scalars generally prove catastrophic as relic neutralino LSPs fail to annihilate rapidly enough in the early universe to be consistent with current knowledge of the age of the universe (i.e. $\Omega_\chi h^2 \leq 1$). However, if the LSP has a small but significant wino-like content this problem can be avoided and the relic LSP can again be an excellent candidate for cold dark matter. In the right panel of Figure 3 we have overlaid those points in parameter space for which the relic density of neutralinos is in the cosmologically allowed range of $0.1 \leq \Omega_\chi h^2 \leq 1$. Note that the scalars can
be multi-Tev in mass provided the condensing group beta-function coefficient lies in a particular range – suggestively the range in which the highlighted point of Figure 2 lies. In this cosmologically preferred region of the parameter space the wino content of the LSP is 10-20%. Additional analysis of models with heavy scalars and satisfactory relic densities is underway.

A more realistic model may alter these results to some degree and uncertainty remains in the general size and nature of the Yukawa couplings of the hidden sector of these theories. Nevertheless this survey suggests that eventual measurement of the size and pattern of supersymmetry breaking in our observable world may well imply a very limited choice of hidden sector configurations (and hence string compactifications) compatible with low energy phenomena.

Acknowledgments

This work was performed in collaboration with Mary K. Gaillard and was supported in part by the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY-95-14797 and PHY-94-04057.
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