The WZNW Model As An Integrable Perturbation Of The Witten Conformal Point

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Abstract

We show that the WZNW model with arbitrary $\sigma$-model coupling constant may be viewed as a $\sigma$-model perturbation of the WZNW theory around the Witten conformal point. In order for the $\sigma$-model perturbation to be relevant, the level $k$ of the underlying affine algebra has to be negative. We prove that in the large $|k|$ limit the perturbed WZNW system with negative $k$ flows to the conformal WZNW model with positive level. The flow appears to be integrable due to the existence of conserved currents satisfying the Lax equation. This fact is in a favorable agreement with the integrability of the WZNW model discovered by Polyakov and Wiegmann within the Bethe ansatz technique.

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1 Introduction

The Wess-Zumino-Novikov-Witten (WZNW) model [1-4] is a nonlinear two dimensional \( \sigma \)-model extended by the Wess-Zumino term [5]. The action of the theory is written as follows [2]

\[
S = \frac{1}{4\lambda} \int d^2 x \, Tr\left( \partial_\mu g \partial^\mu g^{-1} \right) + k \Gamma, \tag{1.1}
\]

where \( g \) is the matrix field taking its values on the Lie group \( G \); \( \lambda \) is a \( \sigma \)-model coupling constant; \( \Gamma \) is the Wess-Zumino term [5]; \( k \) is a Wess-Zumino coupling constant. For compact groups the Wess-Zumino term is well defined only modulo \( 2\pi \) [2, 6], therefore, the parameter \( k \) must be an integer in order for the quantum theory to be single valued with the multivalued classical action. For noncompact groups the level may be arbitrary.

Polyakov and Wiegmann [6] have solved the WZNW model in eq. (1.1) by means of the Bethe ansatz technique. However the computation of correlation functions remains beyond the powers of the Bethe ansatz method. Much more detailed information about the WZNW model is obtained at its conformal point [2] within the current-current algebraic approach which gives rise to the Ward identities enabling us to compute correlation functions [3]. In spite of the mighty power of the Knizhnik-Zamolodchikov formalism it can say nothing about the theory away from the critical point. Therefore, there are still some reasons for investigating the WZNW model with arbitrary \( \lambda \).

The aim of the present paper is to present the WZNW model with arbitrary \( \lambda \) as a certain integrable perturbation of the WZNW model around its conformal point. We will show that in the theory there is a small parameter which may serve to tune the coupling \( \lambda \) over a large range of values. We find rather surprisingly that the considered perturbation can be performed only around the Witten conformal point with negative level \( k \). Then the renormalization group flow takes the WZNW model to the second critical point which is identified with the Witten conformal point with positive level \( |k| \). In the framework of the perturbative formulation of the WZNW model the integrability of the latter uncovered by Polyakov and Wiegmann within the Bethe ansatz approach originates naturally from the integrability of the model at the Witten conformal point possessing an infinite number of conserved currents. It turns out that the currents of the underlying affine algebra of
the conformal model beyond the criticality continue to be conserved currents along the flow. Due to the integrability of the perturbed theory, the latter can be solved exactly in the framework of the S-matrix approach [7]. On the other hand, the hope is that the explicit form of the perturbation might help one to understand correlation functions of the WZNW model beyond the conformal point.

In section 2 we review the basic features of the WZNW model at the Witten conformal point. In section 3 we describe the $\sigma$-model perturbation of Witten’s fixed point with negative level $k$. It is proved that the suggested perturbation obeys all conditions for relevant and renormalizable perturbations. In section 4 the coefficient in the fusion rule of the perturbation operator is calculated in the large $|k|$ limit. In section 5 we discuss the renormalization group flow between the ultraviolet (starting) and infrared (perturbative) fixed points of the WZNW model. In section 6 we summaries our results and comments on them.

The present paper is an extended version of Ref. 8.

2 Witten’s conformal point

The WZNW model has achieved much attention during the last decade due to its special properties at the conformal point discovered by Witten [2]. It has been realized that the WZNW model at the coupling constant

$$\lambda = \pm 4\pi/k$$

(2.2)

can be quantized nonperturbatively either within the algebraic current-current Hamiltonian approach [3] or by means of the free field representation method in the path integral formalism [9]. Yet, the WZNW model can be explored also as a nonlinear sigma model which is conformal at the fixed point [2, 10, 11].

In the present paper we will actively use the properties of the WZNW model at the Witten conformal point given by eq. (2.2). Therefore, we would like to sketch some of them in this section.
For our purposes it is more convenient to use complex coordinates

\[ z = x^1 + ix^2, \quad \bar{z} = x^1 - ix^2, \] (2.3)

with the metric having the form

\[ ds^2 = dz d\bar{z}. \] (2.4)

In these coordinates the action \( S^* \) of the WZNW model at the Witten conformal point is written as follows

\[ S^*(g) = - \frac{k}{4\pi} \left\{ \int d^2 z \left[ Tr|g^{-1} dg|^2 + \frac{i}{3} d^{-1} Tr(g^{-1} dg)^3 \right] \right\}. \] (2.5)

The functional \( S^*(g) \) obeys the Polyakov-Wiegmann formula [6]

\[ S^*(gh) = S^*(g) + S^*(h) - \frac{k}{2\pi} \int d^2 z Tr(g^{-1} \partial_z g \cdot \partial_{\bar{z}} hh^{-1}). \] (2.6)

By using this formula one can easily establish the symmetry of the theory \( S^* \) under the transformations

\[ g(z, \bar{z}) \to \bar{\Omega}(\bar{z}) g(z, \bar{z}) \Omega, \] (2.7)

where \( \Omega(z) \) and \( \bar{\Omega}(\bar{z}) \) are arbitrary \( G \)-valued matrices analytically depending on the complex coordinates (2.3).

The symmetry gives rise to an infinite number of conserved currents which can be derived from the basic currents \( J \) and \( \bar{J} \),

\[ J = J^a t^a = -\frac{1}{2} k g^{-1} \partial g, \] (2.8)

\[ \bar{J} = \bar{J}^a t^a = -\frac{1}{2} k \bar{\partial} g g^{-1}, \]

satisfying the equations of motion

\[ \bar{\partial} J = 0, \quad \partial \bar{J} = 0. \] (2.9)

In eqs. (2.8) \( t^a \) are the generators of the Lie algebra \( \mathcal{G} \) associated with the Lie group \( G \),

\[ [t^a, t^b] = f^{abc} t^c, \] (2.10)

with \( f^{abc} \) the structure constants.
Knizhnik and Zamolodchikov [3] have shown that the existence of an infinite number of conserved currents forming an affine algebra $\hat{G}$, together with the Virasoro algebra, leads to the Ward identities for correlation functions. We will make use of the following one

$$
\langle J^a(z)\phi_1(z,\bar{z}_1)\cdots\phi_N(z_N,\bar{z}_N)\rangle = \sum_{j=1}^{N} \frac{t^a_j}{z-z_j} \langle \phi_1(z_1,\bar{z}_1)\cdots\phi_N(z_N,\bar{z}_N) \rangle.
$$

(2.11)

Here the matrices $t^a_i$ correspond to the left representation of the affine-Virasoro primary fields $\phi_i(z,\bar{z})$. The given Ward identity is a direct consequence of the following operator product expansion (OPE)

$$
J^a(w)\phi_i(z,\bar{z}) = \frac{t^a_i}{w-z}\phi_i(z,\bar{z}) + \text{reg.},
$$

(2.12)

which is determined by the transformation property of the field $\phi_i$ with respect to the infinitesimal transformations

$$
\Omega(z) = I + \omega^a(z)t^a,
$$

(2.13)

$$
\bar{\Omega}(\bar{z}) = I + \bar{\omega}^a(\bar{z})\bar{t}^a.
$$

Note that the current $J$ itself is not an affine primary field since its OPE with itself is

$$
J^a(z)J^b(w) = \frac{k\delta^{ab}}{(z-w)^2} + \frac{f^{abc}}{z-w}J^c(w) + \text{reg.}
$$

(2.14)

The last equation fixes our normalization of the affine currents.

Any affine-Virasoro primary field $\phi_i$ in the WZNW model is degenerate and its dimensions are given by [3]

$$
\Delta_i = \frac{c_i}{c_V+k}, \quad \bar{\Delta}_i = \frac{\bar{c}_i}{c_V+k},
$$

(2.15)

where $c_i = t^a_it^a_i$, $\bar{c}_i = \bar{t}^a_i\bar{t}^a_i$ and $c_V$ is defined according to

$$
f^{acd}f^{bed} = c_V\delta^{ab}.
$$

(2.16)
3 The $\sigma$-model perturbation

It is natural to try to consider the WZNW model around the Witten conformal point. To this end we present the $\sigma$-model coupling constant in the following form

$$\frac{1}{\lambda} = \frac{1}{\lambda^*} + \epsilon \frac{k^2}{4},$$

(3.17)

where $\lambda^*$ is the Witten conformal point; $\epsilon$ is a small parameter. With the given reparametrization of $\lambda$ the action of the WZNW model is written as follows

$$S = S^* + \epsilon \int d^2 z \text{Tr} \left( \frac{k^2}{4} \partial g \cdot \bar{\partial} g^{-1} \right).$$

(3.18)

Here $S^*$ is the action of the exact conformal theory. So, the second term in eq. (3.18) appears to be a certain perturbation to the conformal system $S^*$.

At the quantum level we have to define a quantum perturbation associated with the classical expression in eq. (3.18). Obviously the expansion of the partition function in the perturbation around Witten’s conformal point allows us to use normal ordering with respect to the affine currents [12, 13]. To set the perturbation to the normal ordered form, we use the following identity

$$\text{Tr} \left( \frac{k^2}{4} \partial g \cdot \bar{\partial} g^{-1} \right) = \phi^{\bar{a} \bar{a}} \cdot J^{a} \cdot \bar{J}^{\bar{a}},$$

(3.19)

where the currents $J^{a}$, $\bar{J}^{\bar{a}}$ are given by eqs. (2.8) and the field $\phi^{\bar{a} \bar{a}}$ is as follows

$$\phi^{\bar{a} \bar{a}} = \text{Tr} (g^{-1} t^{a} g \bar{t}^{\bar{a}}).$$

(3.20)

In the theory $S^*$ the field $\phi^{\bar{a} \bar{a}}$ is both an affine and Virasoro primary field with scaling dimensions

$$\Delta_{\phi} = \bar{\Delta}_{\phi} = \frac{c_V}{c_V + k},$$

(3.21)

where $c_V$ is as in eq. (2.16).

Thus the normal ordered perturbation can be defined according to the rule

$$O(z, \bar{z}) \equiv: \phi^{\bar{a} \bar{a}} \cdot J^{a} \cdot \bar{J}^{\bar{a}}: (z, \bar{z}) = \oint_{2\pi i} \oint_{2\pi i} \frac{dw \ J^{a}(w) \cdot \bar{J}^{\bar{a}}(\bar{w}) \cdot \phi^{\bar{a} \bar{a}}(z, \bar{z})}{|z - w|^2}. $$

(3.22)

Here in the numerator the product is understood as an OPE. Keeping in mind eq. (2.12), it is easy to see that the given product does not contain singular terms. Correspondingly
the WZNW model can be described as a perturbed conformal model with the action

\[ S = S^* - \epsilon \int d^2 z O(z, \bar{z}). \tag{3.23} \]

Note that the proposed perturbation \( O \) is not a product of analytical operators.

Let us now turn to the large \( |k| \) limit. It is not hard to see that in this limit the perturbation \( O \) becomes a quasimarginal operator with anomalous dimensions

\[ \Delta = \bar{\Delta} = 1 + \frac{c_V}{k} + \mathcal{O}(k^{-2}). \tag{3.24} \]

Hence, when \( k \) is negative the given perturbation is to be classified as relevant; whereas for positive \( k \) one should refer to an irrelevant perturbation. Note that \( k \) must be lesser than \(-c_V\).

In the case of the relevant perturbation the perturbing operator \( O \) possesses an important property. Namely the OPE of \( O \) with itself does not lead to new relevant operators but \( O \). Indeed, the perturbation by the operator \( O \) in eq. (3.23) preserves explicitly the global \( G \times G \) symmetry of the affine \( \hat{G} \times \hat{G} \) invariance of the conformal theory \( S^* \). Due to the \( G \times G \) symmetry, the operator \( O \) has to obey the following fusion rule

\[ O \cdot O = [O], \tag{3.25} \]

where the square brackets denote the contributions of \( O \) and the corresponding descendants of \( O \).

The fusion rule given by eq. (3.25) guarantees the renormalizability of the perturbed conformal model. Therefore, given the perturbation one can try to calculate the renormalization beta function associated with the coupling \( \epsilon \).

Away of criticality, where \( \epsilon \neq 0 \), the beta function is defined according to [14–17]

\[ \beta = [2 - (\Delta + \bar{\Delta})\epsilon - \pi C \cdot \epsilon^2 + \mathcal{O}(\epsilon^3)], \tag{3.26} \]

where \((\Delta, \bar{\Delta})\) are given by eq. (3.24). The constant \( C \) is taken here to be the coefficient of the three point function

\[ \langle O(z_1)O(z_2)O(z_3) \rangle = \frac{C}{|z_{12}|^{\Delta+\bar{\Delta}} |z_{13}|^{\Delta+\bar{\Delta}} |z_{23}|^{\Delta+\bar{\Delta}}} \tag{3.27} \]
when the two point functions are normalized to unity.

One can easily solve equation (3.26) to find fixed points of the beta function. There are two solutions

\[ \epsilon_1 = 0, \quad \epsilon_2 = -2c_V/(\pi Ck). \]  

(3.28)

The first one is nothing but Witten’s conformal point of the \( S^* \) model; whereas the second solution signifies a new nontrivial conformal point in the WZNW model. In order to derive the value of the second conformal point, one has to compute the coefficient \( C \) in eq. (3.27). This task will be addressed in the next section.

\[ 4 \quad \text{The coefficient } C \]

Bearing in mind the definition of the operator \( O \) (3.22) we obtain the following expression for the three point function

\[
\langle O(z_1, \bar{z}_1)O(z_2, \bar{z}_2)O(z_3, \bar{z}_3) \rangle = \oint \frac{dw_1}{2\pi i} \oint \frac{d\bar{w}_1}{2\pi i} \oint \frac{dw_2}{2\pi i} \oint \frac{d\bar{w}_2}{2\pi i} \oint \frac{dw_3}{2\pi i} \oint \frac{d\bar{w}_3}{2\pi i}
\]

\[
\langle J^a(w_1)J^\bar{a}(\bar{w}_1)\phi^{a\bar{a}}(z_1, \bar{z}_1)J^b(w_2)J^\bar{b}(\bar{w}_2)\phi^{b\bar{b}}(z_2, \bar{z}_2)J^c(w_3)J^\bar{c}(\bar{w}_3)\phi^{c\bar{c}}(z_3, \bar{z}_3) \rangle.
\]

(4.29)

The correlator in the r.h.s. of eq. (3.29) can be readily calculated. By using the Ward identity given by eq. (2.11) as well as the OPE in eq. (2.14) we find

\[
\langle O(z_1, \bar{z}_1)O(z_2, \bar{z}_2)O(z_3, \bar{z}_3) \rangle = \frac{k^2 f^{abc} f^{\bar{a}\bar{b}\bar{c}} C_{\phi\phi\phi}^{a\bar{a} b\bar{b} c\bar{c}}}{|z_{12}|^{2\Delta_a} |z_{13}|^{2\Delta_a} |z_{23}|^{2\Delta_a}}.
\]

(4.30)

where \( C_{\phi\phi\phi} \) is the coefficient of the three point function

\[
\langle \phi^{a\bar{a}}(z_1, \bar{z}_1)\phi^{b\bar{b}}(z_2, \bar{z}_2)\phi^{c\bar{c}}(z_3, \bar{z}_3) \rangle = \frac{C_{\phi\phi\phi}^{a\bar{a} b\bar{b} c\bar{c}}}{|z_{12}|^{2\Delta_a} |z_{13}|^{2\Delta_a} |z_{23}|^{2\Delta_a}}.
\]

(4.31)

For our aims it is sufficient to estimate \( C \) in leading order in \( 1/k \). Therefore, we can consider the limit \( |k| \to \infty \). In this limit the operator

\[
K^a =: \phi^{a\bar{a}}J^\bar{a} := Tr(-\frac{k}{2} g^{-1} \tilde{g} t^a)
\]

(4.32)

acquires the canonical dimension \((0,1)\). Hence, in the large \(|k|\) limit \( K^a \) must behave as a current. In particular, the OPE of \( K^a \) with \( \phi^{a\bar{a}} \) has to be as follows

\[
K^a(w, \bar{w})\phi^{b\bar{b}}(z, \bar{z}) = -\frac{f^{abc}}{\bar{w} - \bar{z}} \phi^{c\bar{c}}(z, \bar{z}) + O(1/k).
\]

(4.33)
Equation (4.33) gives rise to the useful relation

$$C_{\phi \phi \phi}^{abc} f^{\bar{a}b\bar{c}} f^{\bar{b}c\bar{d}} = f^{abc} \delta^{\bar{a}b} + \mathcal{O}(1/k).$$

(4.34)

Taking this identity in eq. (3.30) we obtain the following expression

$$\bar{C} = k^2 f^{abc} f^{\bar{a}b\bar{c}} C_{\phi \phi \phi}^{abc} = c_V (k \dim G)^2.$$  

(4.35)

The relation between $\bar{C}$ and $C$ is as follows

$$\bar{C} = ||O||^2 C,$$

(4.36)

where $||O||$ is a norm of the operator $O$

$$||O||^2 = \langle O(1) O(0) \rangle.$$  

(4.37)

Taking into account the definition of $O$ given by eq. (3.22) we find

$$||O||^2 = (k \dim G)^2.$$  

(4.38)

Note that $O$ has a positive norm.

Thus,

$$C = \bar{C} / (k \dim G)^2 = c_V + \mathcal{O}(1/k).$$

(4.39)

5 A flow

With the given expression for $C$ the second solution in (3.28) is found as follows

$$\epsilon_2 = -2/(\pi k).$$

(5.40)

Correspondingly the $\sigma$-model coupling constant $\lambda$ at the second critical point is given by

$$1/\lambda = (k/4\pi) - (k/2\pi) = -(k/4\pi).$$

(5.41)

Thus, we established that the critical WZNW model $S^*(k)$ with negative $k = -|k|$, being perturbed by the $\sigma$-model term, arrives at the critical WZNW model $S^*(l)$ with positive level $l = -k = |k|$. In the case of the compact group $G$ the critical point $\epsilon_1$ in
(3.28) corresponds to the nonunitary WZNW theory $S^*(k)$; whereas the second solution $\epsilon_2$ appears to be the unitary system $S^*(l)$. Therefore, the flow from $\epsilon_1$ to $\epsilon_2$ ought to be a unitary flow. This conjecture can be justified by the Zamolodchikov’s $c$-theorem [18]. According to the theorem, if the flow between ultraviolet and infrared fixed points is unitary, then the Virasoro central charge corresponding to the perturbative (infrared) fixed point must be lesser than the central charge at the ultraviolet conformal point. In the case under consideration, we know $c(\epsilon_1), c(\epsilon_2)$ exactly. It is easy to check that

$$c(\epsilon_2) - c(\epsilon_1) = \frac{-2k c_V \dim G}{c_V^2 - k^2}. \quad (5.42)$$

Now it becomes clear that when $k < -c_V$ the difference written above is less than zero in full agreement with the $c$-theorem. Interestingly, in the limit $k \to -\infty$ the critical points $\epsilon_1, \epsilon_2$ collide so that the corresponding central charges become equal.

The point to be made is that the Virasoro central charge at the perturbative conformal point $\epsilon_2$ might be also estimated by perturbation theory. There is available, for example, the Cardy-Ludwig formula [14-16]. However, in the case under consideration there are two obstacles obstructing the use of this formula. The first one is that the perturbation operator $O$ given by eq. (3.22) is not a primary operator; it belongs to $[\phi^{\alpha\bar{a}}]$. Namely,

$$O = J^a_{-1} J^a_{-1} \phi^{\alpha\bar{a}}. \quad (5.43)$$

The second problem is that the $c$-function may turn out to be a nonanalytical function of $\epsilon$ in the interval $]\epsilon_1, \epsilon_2[$. Indeed, in this interval the coupling constant $1/\lambda$ changes its sign and, hence, passes through zero so that $\lambda \to \pm\infty$. Therefore, the perturbative formula for the Virasoro central charge needs to be carefully investigated.

It is instructive to compute anomalous dimensions of the operator $O$ at the second conformal point. We can use the perturbative formula due to Redlich [19]. To given order in $1/k$ the formula yields

$$\Delta(\epsilon_2) = \bar{\Delta}(\epsilon_2) = 1 - c_V/k. \quad (5.44)$$

The given expression is consistent with the exact result

$$\Delta(\epsilon_2) = \bar{\Delta}(\epsilon_2) = 1 + \frac{c_V}{c_V - k}. \quad (5.45)$$
where $k$ is negative.

As the level $k$ goes to $-\infty$, the $\sigma$-model coupling constant $\lambda$ is very sensitive to small changes of the perturbative parameter $\epsilon$. Therefore, by doing the perturbation with $\epsilon$ being smaller than the critical value $\epsilon_2$, we should be able to gain insight into the behaviour of the WZNW model with arbitrary $\lambda$ in the intervals $]-\infty, -4\pi/|k|[$ and $]4\pi/|k|, +\infty[$.

Let us look at the integrability of the WZNW model in the intervals just mentioned above. Once we are dealing with the perturbation around conformal model, we can try to disclose the existence of nontrivial conserved currents in the model by following in the manner of Ref. [20]. Namely, we may try to discover what is happening to the conserved currents of the unperturbed model in the course of the perturbation. In the WZNW model at the critical point $\epsilon_1$ there are infinite number of conserved currents forming the affine algebra. Away of the criticality the affine currents are no longer analytical but yet may continue to be conserved. To clarify this point, we have to consider the following correlator

$$
\langle J^a(z, \bar{z}) . . . \rangle = \langle J^a(z, \bar{z}) . . . \rangle_{S^*} + \epsilon \int \langle J^a(z) O(z_1, \bar{z}_1) . . . \rangle_{S^*} d^2 z_1 + O(\epsilon^2).
$$

(5.46)

When $(z_1, \bar{z}_1)$ approaches the point $(z, \bar{z})$ the integral in eq. (5.46) should be regulated. When this is done, $\partial \bar{z} J^a$ is no longer zero. By using the definition of the operator $O$ given by eq. (3.22) we obtain the formula

$$
\int \langle J^a(z) O(z_1, \bar{z}_1) . . . \rangle_{S^*} d^2 z_1 = \epsilon k \int \langle K^a(z_1, \bar{z}_1) . . . \rangle_{S^*} d^2 z_1,
$$

(5.47)

where $K^a$ is as in eq. (4.32). The last equation gives rise to the following relation

$$
\bar{\partial} J^a = -\eta \partial K^a + O(\epsilon^2),
$$

(5.48)

where

$$
\eta = \pi \epsilon k / 2
$$

(5.49)

So, to first order in $\epsilon$ the current $J$ continues to be conserved. Moreover, it satisfies the Lax equation

$$
\bar{\partial} J = \pi \epsilon \left[ K, J \right].
$$

(5.50)
This establishes the integrability of the WZNW model to lowest order in the perturbation.

When $\epsilon$ approaches the critical value $\epsilon_2$, one has to go to higher orders in $\epsilon$ since the ratio $\epsilon k$ is no longer small but goes to a finite constant. Nevertheless, we can argue that the current $J$ must be conserved up to the fixed point $\epsilon_2$. Indeed, Polyakov and Wiegmann [6] have managed to prove the integrability of the WZNW model with arbitrary $\lambda$ within the Bethe ansatz technique. Therefore, the system has to possess an infinite number of conserved currents. At the second critical point the model becomes again a conformal system with an affine symmetry. Of course, it is very interesting to realize the deformation of the affine algebra under the perturbation.

6 Conclusions

We have seen that the WZNW model with arbitrary $\sigma$-model coupling constant $\lambda$ can be presented as the conformal WZNW model perturbed by the $\sigma$-model term. The perturbation turns out to be relevant around the Witten conformal point provided the level underlying affine algebra is negative. In the course of the perturbation, the model arrives at the infrared fixed point corresponding to the conformal WZNW model with positive level. The flow from the nonunitary conformal point to the unitary conformal point appears to be integrable. It is rather amusing that the smaller the size of the perturbation one takes, the more information of the WZNW model one can get. In the limit $|k|$ goes to infinity the theory seems to be solvable for the whole line of values of the coupling constant $\lambda$.

Given the perturbative description of the WZNW model one can try to explore correlation functions beyond the Witten conformal point. In this connection it is very important to understand the transition from the nonunitary phase to the unitary phase.

We expect also that within the perturbative formulation it will be possible to study the deformation of the affine algebra along the integrable renormalization group flow.

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