Spectral matrix methods for partitioning power grids: Applications to the Italian and Floridian high-voltage networks

Ibrahim Abou Hamad, Brett Israel, Per Arne Rikvold, Svetlana V. Poroseva

Abstract

Intentional islanding is used to limit cascading power failures by isolating highly connected “islands” with local generating capacity. To efficiently isolate an island, one should break as few power lines as possible. This is a graph partitioning problem, and here we give preliminary results on islanding of the Italian and Floridian high-voltage grids by spectral matrix methods.

Keywords: power grid, intentional islanding, cascading failure, blackout prevention, network theory, graph partitioning, spectral matrix methods

1. Introduction

Large-scale blackouts have devastating effects on the economy and welfare of any modern society [1, 2]. One of the reasons [3, 4] that make such catastrophic events possible is the lack of a pre-planned strategy for splitting a power grid into separate parts with independent generation, also called islands [5]. This defensive strategy, called planned, intentional, controlled, or defensive islanding, is a last-resort, but effective means to prevent cascading outages [2, 6].

Intentional islanding splits a power system into islands by breaking selected transmission lines. Multiple approaches (see, e.g., [1, 2, 6, 7, 8, 9]) have been suggested for optimizing the selection of the lines to be cut. Most analyze the system state, steady or dynamic. A useful contribution to these studies can be an analysis of the system topology based on a representation of the network as a graph [10, 11, 12, 13]. Identification of “weak” links, whose removal can split a given network into independent islands can be beneficial for i) initiating fast predetermined intentional islanding and ii) preventing unintentional islanding. If one knows in advance the minimal set of links that must be broken to create a separate island, a decision on intentional islanding can be made very fast. These links should also be closely monitored, as their removal (e.g., by accident or sabotage) will result with certainty in unintentional islanding. The advantages of this strategy are that i) the islanded areas can be planned and analyzed in advance with regard to their generating capacity and necessary load-shedding if an island has to be formed, ii) islands do not
depend on the system disturbance and coherency of generators [14], iii) depending on the scale of the event, several islands can be formed. Furthermore, this approach is fully compatible with other techniques [1, 2, 6, 7, 8, 9].

Here we present some preliminary results using spectral matrix methods for intentional islanding of utility power grids, illustrated by applications to the Italian and Floridian high-voltage grids. The methods are briefly outlined in Sec. 2, numerical results are presented in Sec. 3, and some conclusions are drawn in Sec. 4.

2. Methods

We represent a power grid by an undirected graph [10, 12], defined by the \( N \times N \) symmetric weight matrix \( W \), whose elements \( w_{ij} \geq 0 \) represent the capacities of the transmission lines (edges) between the \( N \) locations (vertices) \( i \) and \( j \). (If all \( w_{ij} \) are either 0 or 1, \( W \) is known as the adjacency matrix.) Examples of the graph representations of the Italian [13] and Floridian [15] high-voltage grids at various levels of islanding are shown in Figs. 1 and 2, respectively.

The row sums of \( W \), \( w_i = \sum_j w_{ij} \), are the vertex strengths, and \( w = \sum_i w_i \) is the total strength of the graph. (In the unit-weight case, the \( w_i \) are known as vertex degrees, and \( w \) is twice the total number of edges.) The list of vertex strengths, \( \{w_i\} \), defines a diagonal matrix \( D \). Spectral graph analysis is usually not performed directly on \( W \), but rather on one of several matrices derived from it. The most common ones are the Laplacian matrix and the Normal matrix [10, 11]. The Laplacian is defined as \( L = D - W \) and is symmetric with vanishing row sums. It embodies Kirchhoff’s laws and represents a simple resistor network with conductances \( w_{ij} \). Multiplied with a column vector \( \phi \) of vertex potentials, it yields the vector of currents entering the circuit at each vertex. The eigenvalue problem, \( L \phi = \lambda \phi \), has (at least) one zero eigenvalue, whose eigenvector corresponds to equal potentials at each vertex. If the zero eigenvalue is \( k \)-fold degenerate, the graph has \( k \) disjoint parts. The signs of the components of the eigenvector corresponding to the smallest nonzero eigenvalue \( \lambda_2 \) (the Fiedler vector \( |\phi_2\rangle \) [10]) provide a partition of the network into two almost disconnected parts (“min-cut theorem” [10, 13]).

The normal matrix \( N \) is defined by its elements, \( n_{ij} = w_{ij}/w_i \), so that all its row-sums equal unity (i.e., it is a row-stochastic matrix). Left multiplication by a vector representing a probability distribution, \( (p(i))N = (p(i) + 1) \), describes a discrete-time random walk along the edges of the graph. An eigenvalue problem is now given by \( (\phi)N = (\phi)\mu \). The largest eigenvalue of \( N \) equals unity, and the corresponding left eigenvector (properly normalized) corresponds to the equilibrium distribution, \( p(i) = w_i/w \). (If the unit eigenvalue is \( k \)-fold degenerate, the graph has \( k \) disjoint parts.) The eigenvector corresponding to the second-largest eigenvalue represents the most slowly relaxing perturbation away from the equilibrium distribution, and the signs of its components identify two almost disconnected sets. While the Laplacian depends only on the off-diagonal part of \( W \), the Normal matrix also depends on the diagonal terms, \( w_{ii} \), which represent self loops in the graph that may endow vertices with internal structure.

The islanding problem is one of partitioning the power grid into communities of vertices that are highly interconnected among themselves, but only sparsely connected to the rest or the graph. So, ideally one would like to find a partitioning into a “suitable” number of communities while maximizing the number of intra-community edges and minimizing the number of inter-community edges. This problem is NP-hard [11], and so one has to resort to heuristics producing “reasonably good,” approximate solutions. For the islanding to be useful, each island should contain at least one generating plant.
Figure 1: Modularities and islanding configurations for the Italian grid, shown vs. eigenvector number, as obtained by bisection methods (a) and from single eigenvectors (b). See details in the text.

There are several ways that the eigenvectors of $L$ or $N$ can be used to partition a graph in such an approximate fashion. Two are based on successive bisections. At the first step they are identical: one divides the graph into two parts or “communities” according to the signs of the components of the first nontrivial eigenvector (i.e., the one with the smallest nonzero eigenvalue for $L$, or the one with the eigenvalue closest below unity for $N$). In the following steps one can either a) continue to successively bisect the network into quadrants, octants, etc., according to the signs of the components of the next following eigenvectors (“soft bisection”), or b) remove the edges connecting the two parts obtained in the first step, calculate the first nontrivial eigenvectors of each part separately, bisect each according to the signs of their respective eigenvector components, and then repeat this procedure with the individual parts as often as desired [12] (“hard bisection”). A third possible method could use each eigenvector separately, labeling each vertex +1 or −1 according to the sign of the corresponding eigenvector component, and then identify communities with the separate “Ising clusters” generated by that particular eigenvector.

The quality of a particular partitioning of the graph into $M$ communities, $C = \{C_1, ..., C_M\}$ can be quantified by Newman’s modularity [12]. It gives the difference between the proportion of edges that are internal to a community in the particular graph, and the average of the same proportion in a null-model that preserves the individual vertex strengths, $w_i$ (and consequently also the total strength, $w$), but is otherwise randomly connected. It is defined as follows:

$$Q = \frac{1}{w} \sum_{ij} \left( w_{ij} - \frac{w_i w_j}{w} \right) \delta(C(i), C(j)), \quad (1)$$

where $\delta(C(i), C(j)) = 1$ if vertices $i$ and $j$ are in the same community, and vanishes otherwise.

3. Results

3.1. Italy

The Italian 380 kV grid [13] was modeled as an undirected graph of 127 vertices and 169 edges of unit weight. We had no information to distinguish between vertices representing generating plants, substations, etc., so they are all treated as equivalent. International connections are
Figure 2: Results for the Floridian grid, shown vs. eigenvector number. (a) Modularities and islanding configurations obtained by bisection with all vertices equivalent. (b) The number of islands without generators for several schemes that distinguish the generators by self loops. Circles: no self loops. Diamonds: 6 self loops per generator. Squares: self loops for generators and their nearest neighbors according to strength. Star: 6 self loops for generators and 3 for each of their nearest neighbors. The inset represents the three-level soft bisection with 6 self loops per generator. See details in the text.

ignored. Modularity and partitionings for the Italian grid are shown vs. eigenvector number in Fig. 1. The results of soft bisection based on $L$ and $N$ and hard bisection based on $N$ are shown in Fig. 1(a), and partitionings based on higher-order, single eigenvectors are shown in Fig. 1(b).

A maximum modularity of $Q \approx 0.72$ was obtained at the third level of hard bisection based on $N$ (8 islands with at least 7 vertices each). Among the bisection methods, this is followed by $Q \approx 0.69$ at the fourth level of soft bisection based on $L$ (12 islands including one single vertex), and $Q \approx 0.65$ at the third level of soft bisection based on $N$ (7 “islands” including two that are internally disconnected). The modularities obtained by all the bisection methods decrease rapidly beyond the third or fourth level of bisection.

The modularities obtained from the “Ising clusters” defined by individual eigenvectors are somewhat irregular and also decrease much more slowly with the number of the eigenvector used, than do the results from the bisection methods. The maxima are $Q \approx 0.69$ for the 10th eigenvector (8 islands with at least 4 vertices each), and $Q \approx 0.68$ for the 15th eigenvector (11 islands with at least 3 vertices each), respectively. The 60th eigenvector still yields a partition with $Q \approx 0.45$, but it produces approximately 40 islands.

3.2. Florida

The map of the Floridian high-voltage grid [15] is a composite of three networks (500, 230, and 138 kV) with 84 vertices, 31 of which are generating plants. We have modeled it as an undirected graph with 137 edges. The edges have integer weights between 1 and 4, according to the actual number of direct lines between pairs of connected vertices. Interstate connections are ignored (except for including two generating plants and four substations in southern Georgia).

Modularity and partitionings for the Floridian grid based on bisection with all vertices equivalent are shown vs. eigenvector number in Fig. 2(a). Maximum modularities of $Q \approx 0.66$ were obtained at the third level of hard bisection based on $N$ (8 islands with at least 3 vertices each, all
with generating plants) and the fourth level of soft bisection based on $N$ [11 islands, including one single vertex (not a generating plant) and one internally disconnected “island” in which the smaller part has no generator]. This is followed by $Q \approx 0.64$ at the third level of soft bisection based on $L$ (7 islands with at least 6 vertices each, all with generators). As for Italy, the modularities decrease rapidly beyond the third or fourth level of bisection.

Bisections according to $N$ offer the opportunity to give extra weight to generating plants by attaching self loops. Several schemes were tested, as described in the caption of Fig. 2(b). In some cases this enables us to increase the highest level of bisection that ensures generators in all islands from two to three. An example for three-level soft bisection with six self loops per generator (7 islands with a minimum of 6 vertices, $Q \approx 0.70$) is shown as an inset.

### 4. Conclusions

Our results indicate that spectral matrix method can be used, at least to obtain an initial partitioning of a power grid into islands. As bisection methods (Fig. 1(a) and Fig. 2) require evaluation of only a few, dominant eigenvectors, they are computationally much more economical than methods based on higher-order, single eigenvectors (Fig. 1(b)). With the normal matrix method, generating plants can be weighted by the introduction of self-loops to increase the probability that each island has at least one generator.

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