Soft Photons in $W^+W^-$ Production at LEP200

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Abstract

The pattern of soft photon radiation in $e^+e^- \to W^+W^-$ has a rich structure, with contributions from photon emission off the initial state and off the final state particles both before and after decay. In particular, the interference between the contributions involving the decaying $W$'s depends on the decay width. We review the theoretical result for the radiation pattern, and present predictions for LEP200, i.e. in $e^+e^-$ annihilation just above $W^+W^-$ threshold.

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1 Radiation Pattern Near Threshold

Heavy unstable charged particles such as the $W$ boson can emit photons before and after they decay. The relative size of the two contributions, and consequently the overall radiation pattern, depends sensitively on the timescale of the emission compared to the lifetime of the unstable particle [1]. A particularly important process which exhibits these effects is soft photon production in $e^+e^- \rightarrow W^+W^- \rightarrow f\bar{f}f'\bar{f}'$ at energies just above threshold, which will be studied at LEP200 [2]. The radiation pattern of a soft photon of energy $\omega$ is sensitive to the $W$ decay width for $\omega \sim \Gamma_W$. In a previous study we have derived some general results for the radiation pattern for this process [3], and in this Letter we present specific numerical predictions for LEP200.

The general formalism for calculating the distribution of soft radiation in a process involving the production and decay of unstable particles can be found in references [3–5]. The differential distribution for the production of a soft photon with momentum ($k$) in the process

$$e^-(k_1) + e^+(k_2) \rightarrow W^-(q_1) + W^+(q_2) \rightarrow f(p_1) + \bar{f}(r_1) + f'(p_2) + f'(r_2) ,$$

is given by

$$\frac{1}{N} \frac{dN}{d\omega d\cos \theta d\phi} = \frac{\alpha}{4\pi^2} \omega \mathcal{F} ,$$

where $\omega$ (the photon energy), $\theta$ and $\phi$ are measured in the $e^+e^-$ centre-of-mass frame. The radiation pattern is described by the function $\mathcal{F}$. The result for this is calculated in two steps. First, for the case of soft photon radiation in stable $W^+W^-$ production, we have the well-known result [6]

$$\mathcal{F}_0 = 2\hat{k}_1\hat{k}_2 - \hat{k}_1\hat{k}_1 - \hat{k}_2\hat{k}_2 + 2\hat{k}_1q_1 - 2\hat{k}_2q_1 - 2\hat{k}_1q_2 - 2\hat{k}_2q_2 + 2\hat{q}_1q_2 - q_1q_1 - q_2q_2 ,$$

where the ‘antennae’ are defined by [7]

$$\hat{p}q \equiv \frac{p \cdot q}{p \cdot k \cdot q \cdot k} .$$

Note contributions from initial state radiation, final state radiation, and the interference between them.

For unstable $W$’s, decaying to fermions as in (4), we have $\mathcal{F}_0 \rightarrow \mathcal{F}$ with $\mathcal{F}$ given by (3) with the replacements:

$$\begin{align*}
\hat{k}_1\hat{k}_2, \hat{k}_1\hat{k}_1, \hat{k}_2\hat{k}_2 & \rightarrow \hat{k}_1\hat{k}_2, \hat{k}_1\hat{k}_1, \hat{k}_2\hat{k}_2 \\
\hat{k}_1q_1 & \rightarrow \hat{k}_1q_1 + \chi_1 \left[ Q\hat{k}_1p_1 + (1 - Q)\hat{k}_1r_1 - \hat{k}_1q_1 \right] \\
\hat{k}_1q_2 & \rightarrow \hat{k}_1q_2 + \chi_2 \left[ Q'\hat{k}_1p_2 + (1 - Q')\hat{k}_1r_2 - \hat{k}_1q_2 \right] \\
\hat{k}_2q_1 & \rightarrow \hat{k}_2q_1 + \chi_1 \left[ Q\hat{k}_2p_1 + (1 - Q)\hat{k}_2r_1 - \hat{k}_2q_1 \right]
\end{align*}$$

1
\[ k_2 q_2 \rightarrow k_2 q_2 + \chi_2 \left[ Q' k_2 p_2 + (1 - Q') k_2 r_2 - \kappa_2 q_2 \right] \]
\[ q_1 q_2 \rightarrow q_1 q_2 + \chi_2 \left[ Q' q_1 p_2 + (1 - Q') q_1 r_2 - \kappa_1 q_2 \right] \]
\[ + \chi_1 \left[ Q q_2 p_1 + (1 - Q) q_2 r_1 - \kappa_1 q_2 \right] \]
\[ + \chi_{12} \left[ Q Q' p_1 p_2 - Q' q_1 p_2 - (1 - Q') q_1 r_2 - Q q_2 p_1 \right] \]
\[ - (1 - Q) q_2 r_1 + Q (1 - Q') p_1 r_2 + Q' (1 - Q) p_2 r_1 \]
\[ + (1 - Q) (1 - Q') r_1 r_2 + \kappa_1 q_2 \]
\[ q_1 q_1 \rightarrow 2q_1 q_1 + Q^2 p_1 p_1 + (1 - Q)^2 r_1 r_1 \]
\[ + 2Q (1 - Q) p_1 r_1 - 2Q q_1 p_1 - 2(1 - Q) q_1 r_1 \]
\[ + 2\chi_1 \left[ Q q_1 p_1 + (1 - Q) q_1 r_1 - \kappa_1 q_1 \right] \]
\[ q_2 q_2 \rightarrow 2q_2 q_2 + Q^2 p_2 p_2 + (1 - Q)^2 r_2 r_2 \]
\[ + 2Q' (1 - Q') p_2 r_2 - 2Q' q_2 p_2 - 2(1 - Q') q_2 r_2 \]
\[ + 2\chi_2 \left[ Q' q_2 p_1 + (1 - Q') q_2 r_2 - \kappa_2 q_2 \right] \],

(5)

with the charge factors given by \( Q = |Q_f| \) and \( Q' = |Q_{\bar{f}}| \), e.g. 2/3 and 1 for hadronic and leptonic decays respectively. Now we have contributions from additional antennae associated with the decays \( W \rightarrow f \bar{f} \), together with contributions from the interference of photons radiated at the production and decay stages. This interference is controlled by the ‘profile functions’ [4,5]

\[
\chi_i = \frac{M_W^2 \Gamma_W^2}{(q_i \cdot k)^2 + M_W^2 \Gamma_W^2}, \quad i = 1, 2
\]

(6)

\[
\chi_{12} = \frac{M_W^2 \Gamma_W^2}{((q_1 \cdot k)^2 + M_W^2 \Gamma_W^2)((q_2 \cdot k)^2 + M_W^2 \Gamma_W^2)},
\]

(7)

which depend on the \( W \) mass \( (M_W) \) and decay width \( (\Gamma_W) \). They have the (formal) property that \( \chi_i, \chi_{12} \rightarrow 0 \) as \( \Gamma_W \rightarrow 0 \), and \( \chi_i, \chi_{12} \rightarrow 1 \) as \( \Gamma_W \rightarrow \infty \). Note that only soft photons with energy \( \omega \lesssim \Gamma_W \) can lead to significant interference contributions: the emission of energetic photons (either real or virtual) with \( \omega \gg \Gamma_W \) pushes the \( W \) propagators far off their resonant energy and the interference becomes negligible.

This is a well-understood phenomenon, dating back (at least) to the early days of \( J/\psi \) physics\(^2\) [8].

The rather complicated structure implied by (3)-(7) is greatly simplified if we take the ‘threshold limit’ appropriate to LEP 200, i.e. \( \sqrt{s} \sim 2M_W \) so that \( \nu_W \ll 1 \). In this case, the radiation off the almost stationary \( W \) bosons is suppressed [4], and the only contributions which survive are radiation off the initial state leptons, radiation off the two final state \( f \bar{f} \) antennae, and interference between them. Noting also that

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\(^2\)VAK thanks V.S. Fadin for reminding him of this.
near threshold
\[ \chi_1, \chi_2, \chi_{12} \to \chi(\omega) = \frac{\Gamma^2_W}{\omega^2 + \Gamma^2_W}, \] (8)
we find [3]
\[
\mathcal{F} = \frac{1}{\omega^2} N_{\text{tot}} \equiv \frac{1}{\omega^2} (N_{\text{IS}} + N_{\text{FS}} + N_{\text{I/F}})
\] (9)
where
\[
N_{\text{IS}} = \frac{4}{\sin^2 \theta_0}
\] (10)
\[
N_{\text{FS}} = \frac{[(2Q - 1) + \cos \theta_1]^2}{\sin^2 \theta_1} + \frac{[(2Q' - 1) + \cos \theta_2]^2}{\sin^2 \theta_2} - 2\chi(\omega)
\times (\cos \Theta_{12} - \cos \theta_1 \cos \theta_2) \frac{[(2Q - 1) + \cos \theta_1][(2Q' - 1) + \cos \theta_2]}{\sin^2 \theta_1 \sin^2 \theta_2}
\] (11)
\[
N_{\text{I/F}} = \chi(\omega) N_{\text{IS}} \left[ (\cos \theta_0 \cos \theta_1 - \cos \Theta_1) \frac{(2Q - 1) + \cos \theta_1}{\sin^2 \theta_1} \right.
\] (12)
\[
- (\cos \theta_0 \cos \theta_2 - \cos \Theta_2) \frac{(2Q' - 1) + \cos \theta_2}{\sin^2 \theta_2} \right].
\]

The angles are defined as follows (see Fig. 1): \( \theta_1 \) is the angle between the photon and the fermion \( f \), \( \theta_2 \) is the angle between the photon and the fermion \( f' \), \( \theta_0 \) is the angle between the photon and the incoming electron \( e \), \( \Theta_{12} \) is the angle between \( f \) and \( f' \), \( \Theta_1 \) is the angle between \( f \) and \( e \), and \( \Theta_2 \) is the angle between \( f' \) and \( e \). In deriving this above result we have assumed that all fermions are massless; the corresponding result for massive fermions is given in reference [3].

We see from (8)-(12) that the radiation pattern of a soft photon of energy \( \omega \) has a rich structure depending on the relative orientation of the charged particles, and in particular is sensitive to the \( W \) decay width through the profile function \( \chi(\omega) \). The sensitivity is evidently largest for \( \omega \sim \Gamma_W \). For larger \( \omega \) values \( \chi \) becomes small, and the pattern of radiation is simply that of three independent antennae: the initial \( e^+e^- \) antenna and two final state \( f\bar{f} \) antennae. In contrast, for \( \omega \) smaller than \( \Gamma_W \) we have \( \chi \sim 1 \), and the interference between the emission off the different antennae becomes large (‘coherent emission’) [3].

### 2 Examples

The remainder of this Letter is devoted to a brief practical study of the photon distribution implied by the above results. In particular, we focus on the role of the \( W \) width in determining the shape of the distributions. Finally, we address the question
of whether the effects we describe are observable, and whether the sensitivity to $\Gamma_W$ is large enough to be measurable.

Obviously an important issue is the overall number of events. From both a theoretical and experimental point of view, the cleanest final state ($ll$) is $l\bar{l}\nu\bar{\nu}$, but this also has the smallest branching ratio. The event rates for final states involving at least one hadronic $W$ decay, $(ql)$ and $(qq)$, are of course larger, but here we encounter the problem of being unable to distinguish the quark jet from the antiquark jet. (We assume that it will always be possible to pair the jets in a four-jet event according to which come from the same $W$ decay. This is certainly true just above threshold, where the $W$’s decay to almost back-to-back jets.) We must therefore symmetrize the above result when applied to $q\bar{q}$ decays to allow for this ambiguity. In addition, with final jets we have additional ‘secondary’ sources of photons from within the jet (from $\pi^0$ decay etc.), and so it will be necessary in practice to isolate the photons from the jet axes. Some (less stringent) isolation of the photons from the charged leptons may also be required.

In the present context, the radiation from the initial state electrons can be regarded as a background, whose effect is minimized by keeping all final state particles including the photon well away from the beam direction. We can even imagine an idealized situation where all the final state particles are transverse to the beam direction, in which case the initial state contributions are simply $N_{IS} = 4$, $N_{IF} = 0$. Apart from the overall constant contribution from the initial state radiation, this situation is very similar to the analysis of soft gluon radiation in $t\bar{t}$ production [4,3].

There is, however, one important way we can take advantage of the initial state radiation. We are interested in sensitivity to $\chi(\omega)$, and we see from (12) that the initial–final state interference is proportional to $\chi(\omega)$. Furthermore, we see from (10)-(12) that when the final state fermions are in the transverse plane ($\cos \Theta_1 = \cos \Theta_2 = 0$), the $N_{IF}$ contribution is antisymmetric with respect to the forward and backward directions, $\cos \theta_0 > 0$ and $< 0$ respectively, the other two contributions being symmetric. The forward-backward asymmetry in this case is then

$$\Delta N_{tot} = 2N_{IF} = \chi(\omega) \frac{8 \cos \theta_0}{\sin^2 \theta_0} \left[ \frac{1 + (2Q - 1) \cos \theta_1}{\sin^2 \theta_1} - \frac{1 + (2Q' - 1) \cos \theta_2}{\sin^2 \theta_2} \right].$$

(13)

The existence of the asymmetry is easy to understand physically. The interference is linear in the initial and final state charges; the electron and positron pieces correspond to interchanging the forward and backward directions, and contribute with opposite signs. Since the asymmetry is directly proportional to $\chi(\omega)$, it provides, at least in principle, a method for extracting $\Gamma_W$. This will be illustrated below.

As an aside, we note that in the general case, $N_{IF}$ is antisymmetric under the interchanges $\cos \theta_0 \leftrightarrow -\cos \theta_0$, $\cos \Theta_{1,2} \leftrightarrow -\cos \Theta_{1,2}$, and therefore vanishes after integration over the angles between the initial state and final state antennae (keeping the relative angles between the pairs of decay products fixed).
The next step, then, is to recast the results of (11)-(12) into the appropriate form for the three types of final state. This is a straightforward process, and the explicit expressions for the three functions $N_{tot}^{(ll)}, N_{tot}^{(ql)}$ and $N_{tot}^{(qq)}$ are given in reference [3] (Eqs. (D.6),(D.7)). In particular, the double leptonic decay result $N_{tot}^{(ll)}$ is obtained by setting $Q = Q' = 1$. For one leptonic decay and one hadronic decay ($N_{tot}^{(ql)}$) we set $Q = 2/3, Q' = 1$, and symmetrize between the quark and antiquark directions, i.e. $Q \leftrightarrow 1 - Q$. This is equivalent to $\cos \theta_1 \leftrightarrow -\cos \theta_1, \cos \Theta_1 \leftrightarrow -\cos \Theta_1$, and $\cos \Theta_{12} \leftrightarrow -\cos \Theta_{12}$, or dropping terms linear in $(2Q - 1)$. Finally, setting $Q = Q' = 2/3$ and symmetrizing in both the ‘1’ and ‘2’ angles gives $N_{tot}^{(qq)}$.

The following examples illustrate the effect of $\Gamma_W$ on the radiation pattern. In each case we take the final state fermions to be in the plane transverse to the beam. This is done merely for convenience; the effect of the forward-backward asymmetry discussed above is straightforward to see in such configurations.

First consider the case of a four-jet final state where all four jets are in the transverse plane and $\Theta_{12}$ is the (acute) angle between a jet from each $W$, labelled “1” and “2”. After the appropriate symmetrization we have

$$N_{tot}^{(qq)} = \frac{4}{\sin^2 \theta_0} + \frac{\frac{1}{3} + \cos^2 \theta_1 + \frac{1}{3} + \cos^2 \theta_2}{\sin^2 \theta_1}$$

$$+ 2 \chi(\omega) \left[ \frac{\cos \theta_1 \cos \theta_2 (\cos \theta_1 \cos \theta_2 - \cos \Theta_{12})}{\sin^2 \theta_1 \sin^2 \theta_2} \right]$$

$$+ \frac{2 \cos \theta_0}{\sin^2 \theta_0} \left( \frac{1}{\sin^2 \theta_1} - \frac{1}{\sin^2 \theta_2} \right) \right]. \quad (14)$$

The second example is for the one hadronic – one leptonic $W$ decay final state. With the jets and leptons again in the transverse plane we have

$$N_{tot}^{(ql)} = \frac{4}{\sin^2 \theta_0} + \frac{\frac{1}{3} + \cos^2 \theta_1 + \frac{1}{3} + \cos^2 \theta_2}{\sin^2 \theta_1} + \frac{\cos \theta_2}{1 - \cos \theta_2} + 2 \chi(\omega)$$

$$\times \left[ \frac{\cos \theta_1 (\cos \theta_1 \cos \theta_2 - \cos \Theta_{12})}{\sin^2 \theta_1 (1 - \cos \theta_2)} \right] + \frac{2 \cos \theta_0}{\sin^2 \theta_0} \left( \frac{1}{\sin^2 \theta_1} - \frac{1}{1 - \cos \theta_2} \right) \right]. \quad (15)$$

Finally, the double-leptonic decay, while less interesting from the practical point of view, shows the most dramatic $\chi$ dependence. For the transverse configuration described above we have

$$N_{tot}^{(ll)} = \frac{4}{\sin^2 \theta_0} + \left[ \frac{1 + \cos \theta_1}{1 - \cos \theta_1} + \frac{1 + \cos \theta_2}{1 - \cos \theta_2} \right] + 2 \chi(\omega)$$

$$\times \left[ \frac{\cos \theta_1 \cos \theta_2 - \cos \Theta_{12}}{(1 - \cos \theta_1)(1 - \cos \theta_2)} \right] + \frac{2 \cos \theta_0}{\sin^2 \theta_0} \left( \frac{1}{1 - \cos \theta_1} - \frac{1}{1 - \cos \theta_2} \right) \right]. \quad (16)$$
To illustrate the effects that can arise due to the $W$ width, we show the photon angular distributions implied by Eq. (15) for one particular final state configuration in which the $W^+$ decays leptonically and the $W^-$ decays to jets. As stated above, we take the final state fermions to be in the plane transverse to the beam, and for this example we choose $\Theta_{12} = 90^\circ$. Our angular convention is such that the electron beam defines the polar axis (so that the leptons and quarks have $\theta = 90^\circ$) and the charged lepton direction defines the photon azimuthal angle $\phi_0 = 0^\circ$. Thus the jet directions are given by $\phi = 90^\circ$ and $270^\circ$.

First we show how the antisymmetry of $N_{\text{I/F}}$ gives rise to a $\chi$-dependent asymmetry in $N_{\text{tot}}^{(q)}$. Figures 2 (a-c) show the radiation pattern $N_{\text{tot}}^{(q)}$ as a function of $\theta_0$ for fixed values of $\phi_0$, for $\chi = 0$ (solid lines) and $\chi = 1$ (dashed lines). Fig. 2(a) corresponds to $\phi_0 = 0^\circ$, i.e. the photon is in the plane defined by the beam and the final state lepton, so that $\theta_2 = |\theta_0 - 90^\circ|$. The singularities at $\theta_0 = 0^\circ, 90^\circ, 180^\circ$ correspond to the incoming electron, final charged lepton, and incoming positron, respectively. Notice the non-zero asymmetry about $\theta_0 = 90^\circ$ for $\chi \neq 0$. In fact for $\chi = 1$ the interference is so large that the radiation is almost totally suppressed at angles close to $50^\circ$. At the minimum, there is a two orders of magnitude difference between the $\chi = 0$ and $\chi = 1$ distributions. In practice, such a configuration could provide enhanced sensitivity to $\Gamma_W$. As we shall see, there is sensitivity to $\Gamma_W$ in other configurations as well.

In Fig. 2(b) we again show $N_{\text{tot}}^{(q)}$ as a function of $\theta_0$ at fixed $\phi_0$, this time for $\phi_0 = 45^\circ$, halfway between the lepton and the closer jet. Again we see an asymmetry, but much smaller in size than in Fig. 2(a). When we increase $\phi_0$ to $90^\circ$ in Fig. 2(c) to correspond to the jet direction, we see the asymmetry return, but this time with the opposite sign. (This explains the small size of the effect in Fig. 2(b).) The sign of the asymmetry changes because the quarks’ charges are opposite in sign to that of the lepton, and for this value of $\phi$ the $2Q - 1$ term dominates in the interference; see e.g. Eq. (13). Note also that the asymmetry here is less pronounced than in Fig. 2(a): the (absolute value of the) average charge of the quarks is less than the charge of the lepton.

The forward–backward asymmetry is not the only manifestation of width dependence. As indicated in Eq. (11), there are $\chi$-dependent interference terms in the final state radiation contribution as well; in the case of soft gluon emission in $e^+e^- \to t\bar{t}$, these terms constituted all of the $\chi$ dependence [3,4]. To illustrate these final-state width effects, we show in Figure 3 distributions in $\phi_0$ for fixed values of $\theta_0$. Recall that the final state particles are in the $\theta_0 = 90^\circ$ plane.

Fig. 3(a) shows $N_{\text{tot}}^{(q)}$ for $\theta_0 = 50^\circ$, roughly halfway between the electron beam and the final charged lepton, again for $\chi = 0$ (solid line) and $\chi = 1$ (dashed line). The difference is striking. The $\chi = 0$ curve is nearly featureless except for slight increases in the beam–lepton plane at $\phi_0 = 0^\circ, 360^\circ$. In contrast, the interference suppresses the $\chi = 1$ distribution in that same plane, but enhances it at $\phi_0$ values corresponding to the
beam–jet planes. As we increase $\theta$ and move toward the transverse plane containing the final state particles (Fig. 3(b), $\theta_0 = 75^\circ$), we see similar effects, although the $\chi = 0$ and 1 distributions are less distinct.

Fig. 3(c) shows the distribution for $\theta_0 = 105^\circ$, corresponding to the backward plane. We see that the width effect is reversed from the forward plane. Now the $\chi = 1$ curve is enhanced in the beam–lepton plane while the $\chi = 0$ curve exhibits peaks in the beam–jet planes. Finally, in Fig. 3(d) we take $\theta = 130^\circ$ (between the backward jet and the positron beam) and the $\chi = 0$ curve is again featureless while the interference induces structure in the dashed curve for $\chi = 1$.

Note that although we have chosen the $ql$ case to illustrate $\Gamma_W$ effects, the $ll$ and $qq$ exhibit significant sensitivity to $\Gamma_W$ as well. In fact the interference effects in the $ll$ case are even more dramatic than those shown here because there is no charge symmetrization. In the same way, we expect slightly less sensitivity in the $qq$ case because the symmetrization applies to both $W$ decays.

Of course, a detailed study of the $ll$ case along the lines of the $ql$ case described above is likely to be restricted in practice by the lack of events. As pointed out in [3], however, there is a more inclusive quantity which utilizes all the events and exhibits sensitivity to $\chi(\omega)$. If we integrate over all photon angles, the total yield can be written

$$\omega \frac{dN_{(\alpha\beta)}}{d\omega} = \frac{\alpha}{\pi} \left[ R_{\text{ind}}^{(\alpha\beta)} + 2\chi(\omega) F^{(\alpha\beta)}(\Theta_{12}) \right],$$

(17)

where $(\alpha\beta) = (ll), (ql), (qq)$. The important point is that the first, $\chi$-independent term does not depend on $\Theta_{12}$. The second term is given by

$$F^{(ql)} = F^{(qq)} = \log \left( \frac{\sin \Theta_{12}}{2} \right) + 1,$$

$$F^{(ll)} = \log \left( \frac{1 - \cos \Theta_{12}}{2} \right) + 1.$$

(18)

These can be either positive (smaller $\Theta_{12}$) or negative (larger $\Theta_{12}$) and the critical angles where the sign change occurs are readily shown to be [3] $\Theta_{\text{crit}} \approx 47.4^\circ$ and $74.7^\circ$ for the $(ql), (qq)$ and $(ll)$ cases respectively. If we integrate over events with $\Theta_{12}$ above and below the critical angle and take the difference

$$\delta^{(ll)} = \frac{1}{1 + \cos \Theta_{\text{crit}}} \int_{\Theta_{\text{crit}}}^{\pi} \omega \frac{dN^{(ll)}}{d\omega} \sin \Theta_{12} d\Theta_{12} \left[ 1 - \frac{1}{1 - \cos \Theta_{\text{crit}}} \int_{\Theta_{\text{crit}}}^{\pi/2} \omega \frac{dN^{(ql,qq)}}{d\omega} \sin \Theta_{12} d\Theta_{12} \right],$$

(19)

$$\delta^{(ql,qq)} = \frac{1}{1 - \cos \Theta_{\text{crit}}} \int_{\Theta_{\text{crit}}}^{\pi/2} \omega \frac{dN^{(ql,qq)}}{d\omega} \sin \Theta_{12} d\Theta_{12} \left[ 1 - \frac{1}{ \cos \Theta_{\text{crit}}} \int_{\Theta_{\text{crit}}}^{\pi} \omega \frac{dN^{(ql,qq)}}{d\omega} \sin \Theta_{12} d\Theta_{12} \right].$$

(20)
then the $\chi$ independent term in (17) cancels and we are left with

\[ \delta(\ell\ell) = \frac{\alpha}{\pi} \frac{4\chi(\omega)}{1 + \cos \Theta_{\text{crit}}} = \frac{\alpha}{\pi} 3.164 \chi(\omega), \tag{21} \]

\[ \delta(q\ell) = \delta(qq) = \frac{\alpha}{\pi} \frac{2\chi(\omega)}{\cos \Theta_{\text{crit}} (1 - \cos \Theta_{\text{crit}})} \left[ -\cos \Theta_{\text{crit}} + \frac{1}{2} \log \left( \frac{1 + \cos \Theta_{\text{crit}}}{1 - \cos \Theta_{\text{crit}}} \right) \right] \]

\[ = \frac{\alpha}{\pi} 1.343 \chi(\omega). \tag{22} \]

Fig. 4 shows these quantities as functions of the ratio $\omega/\Gamma_W$. Several remarks are appropriate. First, the term $\mathcal{R}_{\text{ind}}^{(a\beta)}$ in (17) contains logarithmic singularities when the photon is collinear with the (massless) fermions, but these cancel in the difference $\delta^{(a\beta)}$. Second, contributions from ‘secondary’ photons in the quark jet $s$ are also expected to cancel in the difference. In practice, the photons can be isolated from the final state jets and leptons (assuming $\theta_{\text{iso}} \ll \Theta_{\text{crit}}$) without significantly weakening the dependence of the $\delta^{(a\beta)}$ on $\omega/\Gamma_W$.

We now return to the question of event rates with which this section began. The probability of finding an additional soft photon in a $WW$ event is just given by the radiation probability given in (2), integrated over photon angles and energies. This probability depends on the velocities of the final state particles (the velocities must be kept to avoid collinear singularities) as well as the range of photon energies included, but is typically $5 - 10\%$. With an integrated luminosity of $500 \text{ pb}^{-1}/\text{yr}$, LEP200 will produce about 8000 $W$ pairs each year. Combining these numbers with the appropriate branching ratios ($1/81, 12/81$, and $36/81$ for the $\ell\ell, q\ell$, and $qq$ modes, respectively) implies event rates ranging from on the order of 10 (for $\ell\ell$) to 100 (for $qq$) per year. Although these numbers are small, we emphasize again that the asymmetries $\delta$ defined above make use of all events while retaining sensitivity to $\Gamma_W$. However, many years’ running at LEP200 will probably be required for the effects described above to be measurable with any precision. Future linear colliders with higher luminosity would certainly fare better.

3 Concluding Remarks

A measurement of the total $W$ width, independent of decay modes (and of the $Z$ width) is by no means easy to obtain. The method described here, using soft photon radiation, is limited by statistics; alternatives either are not direct measurements of the total width, or have systematic difficulties of their own. In any case, a direct measurement of the $W$ width, at LEP200 or at future colliders if necessary, is worth pursuing.

In summary, we have shown that soft photon radiation in $WW$ production near threshold is sensitive to $\Gamma_W$ due to width-dependent interference effects. We have
illustrated this sensitivity with some numerical results for angular distributions for one particular configuration of final state particles. The configuration we chose was by no means unique, or even optimal, in its sensitivity. Since soft photons may be a promising way to get at the $W$ width directly, more detailed studies of these effects, which take proper account of event rates and detector capabilities, should prove worthwhile.

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Figure Captions

[1] The process $e^+e^- \rightarrow f \bar{f} f' \bar{f}' \gamma$ illustrating the angles used to define the radiation pattern.

[2] The radiation pattern $N_{\text{tot}}^{(qf)}$ as a function of $\theta_0$ for fixed values of $\phi_0$: (a) $\phi_0 = 0^\circ$, (b) $\phi_0 = 45^\circ$ and (c) $\phi_0 = 90^\circ$, for $\chi = 0$ (solid lines) and $\chi = 1$ (dashed lines).

[3] The radiation pattern $N_{\text{tot}}^{(qf)}$ as a function of $\phi_0$ for fixed values of $\theta_0$: (a) $\theta_0 = 50^\circ$, (b) $\theta_0 = 75^\circ$, (c) $\theta_0 = 105^\circ$ and (d) $\theta_0 = 130^\circ$, for $\chi = 0$ (solid lines) and $\chi = 1$ (dashed lines).

[4] The normalized integrated photon yield differences $\delta^{(\alpha\beta)}$ defined in Eqs. (19,20) as functions of $\omega/\Gamma_W$, in units of $\alpha/\pi$. 