SUPERSYMMETRIC DARK MATTER – A PHYSICS GODOT?¹

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Where is the long–awaited one? A supersymmetric neutralino has been a favored candidate for the WIMP dark matter but, so far, it has not been found. One way to locate it is to identify where it can be hiding in the vast supersymmetric parameter space. A combination of theoretical, experimental and cosmological constraints lead to remarkably well–defined allowed regions which favor lighter neutralino, and other superpartner, mass ranges. Nevertheless the neutralino can still be as heavy as roughly 800 GeV. The resulting scalar direct detection cross sections range from some $10^{-7}$ pb down to $10^{-10}$ pb, which is at least one order of magnitude below the sensitivity of today’s experiments but which will be almost fully explored by a new generation of planned detectors.

1 Why SUSY and Why WIMPs

You may wonder why I have pointed a finger on supersymmetric dark matter (SUSY DM) as a guest who has failed to show up, the “Godot” that we have been waiting and waiting for so long, and so far with no luck. After all, one can list many more “Godot’s” in physics, so why pick on SUSY DM. Without looking too far, no unambiguous signal of SUSY itself has been found yet, after some twenty years of searching. Nor have we managed to find out what the real nature of the dark matter in the Universe is since some seventy years ago when the DM problem was first identified in the Coma cluster by Zwicky.

We do not know for sure what the SUSY DM is but we have a rather good idea what it is likely to be. The main suspect is of course the LSP, the lightest SUSY particle, which is stable due to an assumed $R$–parity. Simulations of large structure formation and CMB studies point towards cold DM. Astrophysical constraints tell us that such a particle should not be electrically charged, nor (preferably) should it interact strongly, hence another acronym has been coined for the favorite class of DM: a WIMP, the weakly interacting massive particle. SUSY’s LSP has long been known to fit the bill pretty well. In fact, if anything, the list could only be shorter. A commonly considered candidate for the CDM is the lightest neutralino. The neutralino is known to often provide a sizeable contribution to the relic density for reasonable ranges of other superpartner masses. But the list does not end there. The axino (a superpartner of the axion)¹ and the gravitino are also very well-motivated and attractive. The only problem with them would be that, as “Godots”, one would have to wait for them even longer due to their exceedingly faint interactions with ordinary matter, means detectors.

In this talk I will focus on the neutralino “Godot”. I will try to see where this guest who has failed to show up, may be hiding. The SUSY parameter space is known to be large. It is

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still rather mildly constrained from below by collider searches, while from above superpartner masses are not expected to exceed a few TeV due to a somewhat vague ‘naturalness criterion’. Fortunately, much more constraining information is provided by non-collider searches, especially $BR(B \to X_s\gamma)$ and the anomalous magnetic moment of the muon $(g-2)_\mu$, as well as cosmological considerations: the age of the Universe and direct determinations of the WIMP relic abundance $\Omega_{\chi}h^2$. I will present here results of a recent comprehensive analysis. Due to the lack of space, I will quote here only the papers presented here - extensive sets of references can be found there.

2 Constrained MSSM

The framework I will adopt is that of the Constrained MSSM (CMSSM), as defined in the original paper where the acronym was introduced and what most theorists mean by it. (This is in contrast to what LEP experimentalists would refer to as CMSSM today, or rather, yesterday.)

In the [theorist’s] CMSSM, in addition to the requirement of a common gaugino mass $m_{1/2}$ at the unification scale $M_{\text{GUT}}$, which is usually made in the more generic Minimal Supersymmetric Standard Model (MSSM), one further assumes that the soft masses of all scalars (sfermion and Higgs) are equal to $m_0$ at $M_{\text{GUT}}$, and analogously that the trilinear soft terms unify at $M_{\text{GUT}}$ at some common value $A_0$. These parameters are run using their respective Renormalization Group Equations (RGEs) from $M_{\text{GUT}}$ to some appropriately chosen low-energy scale $Q_0$ where the Higgs potential (including full one-loop corrections) is minimized, while keeping the usual ratio $\tan \beta$ of the Higgs VEVs fixed. The Higgs/higgsino mass parameter $\mu$ and the bilinear soft mass term $B_\mu$ are next computed from the conditions of radiative electroweak symmetry breaking (EWSB), and so are the Higgs and superpartner masses. The CMSSM thus has a priori only the usual $\tan \beta, m_{1/2}, m_0, A_0, \text{sgn}(\mu)$ as input parameters. However, in the case of large $m_{1/2}, m_0 \gtrsim 1$ TeV and/or large $\tan \beta \sim \mathcal{O}(m_1/m_0)$ some resulting masses will in general be highly sensitive to the assumed physical masses of the top and the bottom (as well as the tau), and they will also strongly depend on the correct choice of the scale $Q_0$. This in particular will affect the impact of the cosmological constraints as I will discuss below.

In the CMSSM, the LSP neutralino is often a nearly pure bino because the requirement of radiative EWSB typically gives $|\mu| \gg M_1$ where $M_1$ is the soft mass of the bino. This often (albeit not always) allows one to impose strong constraints from $\Omega_{\chi}h^2 \lesssim \mathcal{O}(1)$ on $m_{1/2}$ and $m_0$ (and therefore also on heaviest Higgs and superpartner masses) in the ballpark of 1 TeV. This was originally shown in Refs. and later confirmed by many subsequent studies.

3 Mass Spectra and Experimental Constraints

In the case of the CMSSM, the most important experimental constraints from LEP are those on the masses of the lightest chargino $m_{\chi_1^\pm}$, which includes the standard model (SM) $h$ and Higgs boson $h$. For a Standard Model-like Higgs, the bound is $m_h > 113.5$ GeV but one should keep in mind that, due to large radiative corrections, the theoretical uncertainty in $m_h$ in the CMSSM is probably of the order of 2–3 GeV. I will therefore show the Higgs mass contours corresponding to the value given above and also to 111 GeV as a more conservative value.

Let’s next turn to non-accelerator constraints. First, there has been much activity in determining $BR(B \to X_s\gamma)$. A recent combined experimental result, which incorporates the new CLEO result, gives $BR(B \to X_s\gamma) = (3.23 \pm 0.42) \times 10^{-4}$. This allows for some, but not much, room for contributions from SUSY when one compares it with the updated prediction for the Standard Model (SM) $BR(B \to X_s\gamma) = (3.73 \pm 0.30) \times 10^{-4}$. Second, at large $\tan \beta$ next-to-leading order supersymmetric corrections to $b \to s\gamma$ become important. In our analysis we adopt the full expressions for the dominant terms. We add the two 1σ errors (the experimental and SM) in quadrature and further add linearly 0.2 to accommodate the theoretical uncertainty...
in SUSY contributions which is roughly 5% of the SM value for branching ratio. Altogether we conservatively allow our results to be in the range \( BR(B \rightarrow X_{\gamma}) = (3.23 \pm 0.72) \times 10^{-4} \) for SM plus two-Higgs doublets plus superpartner contribution. The excluded regions of SUSY masses will not however be extremely sensitive to the choice of these error bars but instead to the underlying assumption of minimal flavor violation.\

The first measurement by the Brookhaven experiment E821 of the anomalous magnetic moment of the muon last year gave \( a_\mu = (g_\mu - 2)/2 \). After some ups and downs with correcting sign errors in theory calculations, the result implies a mild 1.6σ discrepancy between the experimental value and the SM prediction \( a_\mu^{\text{expt}} - a_\mu^{\text{SM}} = (25.6 \pm 16.6) \times 10^{-10} \). As we will see, this will provide an upper limit on the plane of \((m_{1/2}, m_0)\) at the 1σ level (also disfavoring negative \( \mu \)), but it will quickly evaporate if one takes a slightly more conservative approach.

The details of our procedure for obtaining the mass spectra can be found in Ref.\(^3\). We calculate superpartner and Higgs mass spectra using the two-loop RGEs for the gauge, Yukawa and soft mass parameters. Appropriate QCD corrections are included which become important especially at large \( \tan \beta \). Of particular importance is a correct treatment of the Higgs sector and the conditions for the EWSB. We include full one-loop corrections to the Higgs potential and minimize it at \( Q^-_t \sim \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \). The mass of the pseudoscalar will play a crucial role in computing \( \Omega_\chi h^2 \), especially at very large \( \tan \beta \sim 50 \). This is so for three reasons: \( m_A \) becomes now much smaller than at smaller \( \tan \beta \) due to the increased role of the bottom Yukawa coupling; because the \( A \)-resonance in \( \chi \chi \rightarrow f \bar{f} \) is dominant since the coupling \( A f \sim \tan \beta \) for down-type fermions; and because, in contrast to the heavy scalar \( H \), this channel is not \( p \)-wave suppressed. In ISASUGRA \( m_A \) is computed as \( m_A^2 = (\tan \beta + \cot \beta) (-B\mu + \Delta_{\lambda}^2) \) where \( \Delta_{\lambda}^2 \) stands for the full one-loop corrections which can be significant.

4 Results

In Figs. 1a and b I present two typical distinct cases. The first applies to values of \( \tan \beta \) up to around 45, the second to larger values. All the grey, red and light orange regions are excluded as described in the captions. In particular, the last one \((\Omega_\chi h^2 > 0.3)\) comes from the lower limit on the age of the Universe and clearly provides an extremely impressive constraint. The narrow white bands are allowed by cosmology \((\Omega_\chi h^2 < 0.3)\) while the [even narrower] green strips correspond to the favored range \( 0.1 < \Omega_\chi h^2 < 0.2 \). The relic abundance \( \Omega_\chi h^2 \) can now be evaluated at a per cent level.\(^4\)

It is clear that in the left window only two very thin regions are allowed. The horizontal region at \( m_0 \sim \) few hundred GeV is, at lower \( m_{1/2} \), excluded by the chargino and Higgs mass bounds at smaller \( \tan \beta \) and by the \( BR(B \rightarrow X_{\gamma}) \) constraint at large \( \tan \beta \). In fact, were it not for the coannihilation of the neutralino with the lighter stau \( \tilde{\tau}_1 \), much of it (on the right side) would be excluded by \( \Omega_\chi h^2 > 0.3 \). The one at \( m_0 \gg m_{1/2} \) is disfavored by the current value of \( (g - 2)_\mu \) and, to some extent, by naturalness arguments.

The visible difference between the two windows comes from the very wide pseudoscalar Higgs resonance in the annihilation process \( \chi \chi \rightarrow A \rightarrow f \bar{f} \). Since \( m_A \) decreases with increasing \( \tan \beta \), at some point, this opens up a corridor in the plane of \((m_{1/2}, m_0)\) along \( m_A = 2m_\chi \).

The regions consistent with all the theoretical, experimental and cosmological constraints are indeed remarkably small, when compared to the whole available parameter space, especially at \( \tan \beta \leq 45 \). This has clear implications for the Higgs and superpartner masses and other properties. Unfortunately, the “Godot” we are after, the (bino-like) neutralino LSP, while confined to the allowed regions, is not all that well-constrained. Its mass \( m_\chi \sim 0.4m_{1/2} \) can still be as large as some 800 GeV (at very large \( \tan \beta \geq 50 \)), or even more, but only because of the coannihilation with the stau and because of the wide \( A \)-resonance. The resulting direct detection WIMP-proton cross sections, ranges
Figure 1: The plane \((m_{1/2}, m_0)\) for \(\tan \beta = 10\) (left window), \(\tan \beta = 50\) (right window) and for \(A_0 = 0, \mu > 0, m_t \equiv m_{\text{pole}} = 175\) GeV and \(m_b \equiv m_{\text{MS}} = 4.25\) GeV. The light red bands on the left are excluded by chargino searches at LEP. In the grey wedge in the left-hand corner electroweak symmetry breaking conditions are not satisfied. The dark red region denoted ‘\(\chi\) NOT LSP’ corresponds to the lighter stau being the LSP. The large light orange regions of \(\Omega_\chi h^2 > 0.3\) are excluded by cosmology while the narrow green bands correspond to the expected range \(0.1 < \Omega_\chi h^2 < 0.2\). Also shown are the semi-oval dark yellow contours of \(a_{\mu}^{\text{SUSY}} = \frac{\Delta a_{\mu}^{\text{SUSY}}}{\Delta m_{\text{SUSY}}} \times 10^{-10}\) favored by the anomalous magnetic moment of the muon measurement at 1\(\sigma\) CL \(a_{\mu}^{\text{SUSY}} = 9, 42.2\). The lines of the lightest Higgs scalar mass \(m_h = 111\) GeV and 113.5 GeV are denoted by short and long-dash lines, respectively.

from some \(10^{-7}\) pb down to \(10^{-10}\) pb, or so, but I have no time to explain it.

So, instead of waiting for the SUSY DM “Godot” the strategy would be to go and get him. We theorists have pointed a finger on where he may be hiding. It is now up to our experimentalist colleagues to do the rest of the job. This is what the next talk is going to address.

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