A dynamic game on Green Supply Chain Management

Mehrnoosh Khademi $^a$* Massimiliano Ferrara $^{b,c,d}$† Bruno Pansera$^{b,†}$
Mehdi Salimi $^{d,e,§}$

$^a$Department of Industrial Engineering, Mazandaran University of Science and Technology, Babol, Iran
$^b$Department of Law and Economics - University Mediterranea of Reggio Calabria, Italy
$^c$CRIOS - Center for Research in Innovation, Organization and Strategy - Department of Management and Technology - Bocconi University, Italy
$^d$Medalics, Research Centre of the University for Foreigner Dante Alighieri in Reggio Calabria, Italy
$^e$Center for Dynamics, Department of Mathematics, Technische Universität Dresden, Germany

Abstract

In this paper, we establish a dynamic game to allocate CSR (Corporate Social Responsibility) to the members of a supply chain. We propose a model of three-tier supply chain in decentralized state that is including supplier, manufacturer and retailer. For analyzing supply chain performance in decentralized state and the relationships between the members of supply chain, we use Stackelberg game and we consider in this paper a hierarchical equilibrium solution for a two-level game. Specially, we formulate a model that crosses through multi-periods by a dynamic discreet Stackelberg game. We try to obtain an equilibrium point at where both the profits of members and the level of CSR taken by supply chains are maximized.

Keywords: Supply chain, CSR, Game theory, Dynamic game, Stackelberg game.

1 Introduction

In recent years, a growing number of large, medium, and even small-sized companies have increasingly focused on CSR. They have realized the need to develop strategies that extend their traditional corporate governance processes beyond firm boundaries to their supply chain partners [7]. This is chiefly because, along with increasing consumer information about the conditions of

---

* Mehrnoosh Khademi: mehrushkhademi@yahoo.com
† Massimiliano Ferrara: massimiliano.ferrara@unirc.it
‡ Presenter: bruno.pansera@unirc.it
§ Mehdi Salimi: mehdi.salimi@tu-dresden.de & mehdi.salimi@medalics.org
manufacture, they criticize supply chains for several social responsibilities. Moreover, the firms in supply chain have been pressured by regulations and policies related to CSR from governments and organizations. The members of supply chain make their decisions based on maximizing of their individual net benefits. Also, when they have to take a level of CSR; this situation leads to an equilibrium status. Game theory is one of the most effective tools to deal with such a kind of management problems.

A growing number of research papers, use game theoretical applications in supply chain management. Cachon et al. [3] discuss Nash equilibrium in a noncooperative cases in a supply chain where there are one supplier and multiple retailers. Hennet et al. [6] presented a paper to evaluate the efficiency of different types of contracts between the industrial partners of a supply chain. They applied game theory method for decisional purposes.

Tian et al. [12] presented a system dynamics model based on evolutionary game theory for green supply chain management as well.

In this paper, we formulate a model for decentralized supply chain network in CSR conditions in a long term with one leader and two followers. The Stackelberg game model is recommended and applied here to find an equilibrium point in which we maximize the profit of members of supply chain and the level of CSR taken by the supply chain. In this research, the supplier as a leader, can know the optimal reaction of his followers, and regards such processes to maximize his own profit. The manufacturer and the retailer as followers, try to maximize their profits by considering all conditions. We propose a Hamiltonian matrix to solve the optimal control problem to obtain the equilibrium in this game. The paper is organized as follows: Section 2 is devoted to General model. Objective functions, constraints and solving the game are illustrated in Section 3. A Conclusion is provided in Section 4.

2 The general model

We consider a Stackelberg differential game involving two players playing the game over a fixed finite horizon model suggested by He et al. [5]. We consider a dynamic game that goes through multi-periods as a repeated game with complete information. This model is a three-tier, multi-period, decentralized supply chain network. We assume that only one supplier, one manufacturer and one retailer are involved in playing the Stackelberg game as well as allocated social responsibility. A long term Stackelberg game is played between the members of the decentralized supply chain through two levels in which all members take CSR into consideration. We formulate the model by selecting the supplier as the leader and both of the manufacturer and retailer as the follower in Stackelberg game. This model can be solved by considering two levels of Stackelberg game. In first level the manufacturer as the leader and retailer as the follower are considered. In this level, we find equilibrium point and in the second level, we consider the supplier as the leader and the manufacturer as the follower. In fact, we put response functions of followers in the objective function of the leader and we find the final equilibrium point and all of the players make decisions.

As any problem formulated as a dynamic game, this model has a state variable and control variables. We define the state variable as the level of social responsibility taken by companies, and the control variables are the capital amounts invested in taking social responsibility. Specifically,
all of the social responsibility taken by firm \( j \) at period \( t \) can be expressed as investment \( I^j_t \). The level of current supply chain investment in supply responsibility is \( x_t \); therefore the accumulation of level of social responsibility taken by the firms is given by \( x_{t+1} = \alpha x_t + \beta_1 I^S_t + \beta_2 I^M_t + \beta_3 I^R_t \).

Here, \( \beta_1 \) is the rate of converting the supplier’s capital investment in CSR to the amount of CSR taken by the supply chain; \( \beta_2 \) is the rate of converting the manufacturer’s capital investment in CSR to the amount of CSR taken by the supply chain and \( \beta_3 \) is the rate of converting the retailer’s capital investment in CSR to the amount of CSR taken by the supply chain [11].

For the purpose of the paper, we more specifically assume: function \( B_t(x_t) = \delta x_t \), that represents a social benefit to the firms, where the coefficient \( \delta \) is supposed to be strictly positive [1].

The following functions \( T^S_t = \tau I^S_t \left[ 1 + \theta (I^S_t + I^M_t + I^R_t) \right], \ T^M_t = \tau I^M_t \left[ 1 + \theta (I^S_t + I^M_t + I^R_t) \right] \) measure the amount of money given by the to the supplier and the manufacturer [4]. For retailer \( T^R_t = \tau I^R_t \left[ 1 + \theta (I^S_t + I^M_t + I^R_t) \right] \) both \( \tau \) and \( \theta \) are tax return policy parameters. Specifically, \( \tau \) is the rate of individual post tax return on investment (ROI), and \( \theta \) is rate of supply chain’s post tax return on investment (ROI).

All kinds of social responsibilities are assumed to be expressed as investment \( I_t \).

The market inverse demand is \( P^M(q_t) = a - bq_t \) [8].

## 3 Objective function and constraints

Let the time interval be \([1,T]\). The objective function of the supplier is

\[
J^S = \text{arg max} \sum_{t=1}^{T} P^S_t q_t - c q_t + B^S_t(x_t) + T^S_t(I^S_t, I_t) - I^S_t + dI^M_t
\]

\[
= \text{arg max} \sum_{t=1}^{T} v q_t - c q_t + \delta x^2 + \tau I^S_t \left[ 1 + \theta (I^S_t + I^M_t + I^R_t) \right] - I^S_t + dI^M_t,
\]

subject to \( x_{t+1} = \alpha x_t + \beta_1 I^S_t + \beta_2 I^M_t + \beta_3 I^R_t \), where the coefficients \( \beta \) are positive and with \( \beta < 1 \).

\( P^S_t \) is the price of the supplier’s raw material. Let \( P^S_t = v \). \( B^S_t(x_t) \) is the social benefit of the supplier, \( \delta \) is the parameter of the supplier’s social benefit and \( T^S_t(I^S_t, I_t) \) is the tax return of the supplier. \( d \) is the percentage of investment of the supplier payoff. Similarly, the objective function of the manufacturer is

\[
J^M = \text{arg max} \sum_{t=1}^{T} P^M_t(q_t) q_t - P^S_t q_t + B^M_t(x_t) + T^M_t(I^M_t, I_t) - I^M_t + \hat{d}I^R_t
\]

\[
= \text{arg max} \sum_{t=1}^{T} (a - bq_t) q_t - v q_t + \hat{\delta} x^2 + \tau I^M_t \left( 1 + \theta (I^S_t + I^M_t + I^R_t) \right) - I^M_t + \hat{d}I^R_t,
\]
where \( P^M_t(q_t) \) is the retail price of the product of the manufacturer. \( B^M_t(x_t) \) is the social benefit of the manufacturer, \( \hat{\delta} \) is the parameter of the manufacturer’s social benefit. \( T^M_t(I^M_t, I_t) \) is the tax return of the manufacturer. \( \hat{\delta} \) is the percentage of investment of the manufacturer payoff.

The objective function of the retailer is

\[
J^R = \max \sum_{t=1}^{T} P^R_t q_t - P^M_t(q_t)q_t + B^R_t(x_t) + T^R_t(I^R_t, I_t) - I^R_t,
\]

where \( P^R_t \) is the price of the product the retailer sells to the consumer. Let \( P^R_t = Z \). \( B^R_t(x_t) \) is the social benefit of the retailer, \( \hat{\delta} \) is the parameter of the retailer’s social benefit. \( T^R_t(I^R_t, I_t) \) is the tax return of the retailer.

3.1 Mathematical model

We solve the mathematical model with two levels, in the first level the manufacturer’s optimal function is calculated by reaction function of retailer and the second level, the game is between the supplier as the leader and the manufacturer as the follower. In fact, the reaction functions of two followers (retailer and manufacturer) are placed on the objective function of the leader (supplier), we can find final equilibrium point.

In the level one, we establish a Stackelberg game between manufacturer as the leader and retailer as the follower. In this level, to calculate the equilibrium first we calculate the best reaction function of retailer, then we determine the manufacturer’s optimal decisions based on the retailer’ best reactions. Since we consider this dynamic game as an optimal control problem, the Hamiltonian function is a practical way to find the equilibrium of the game [10]. The manufacturer’s optimal decisions based on the retailer’ best reactions is determined. To obtain the Stackelberg strategy of the manufacturer, we maximize the objective function of the manufacturer by its Hamiltonian function.

We define the firms Hamiltonian as below. For fixed \( I^M_t \) the Hamiltonian function of the retailer is defined by

\[
H^R_t = J^R_t + P^R_{t+1}(x_{t+1}).
\]

By using the conditions for a maximization of this Hamiltonian, we get after some algebras: \( I^R_t \), \( x_{t+1} \) and \( P^R_{t+1} \).

Now, the manufacturer is faced with the optimal control problem. To obtain the Stackelberg strategy of the manufacturer, we maximize the objective function of the manufacturer by its Hamiltonian function. We fix the value of \( I^R_t \), then get the Hamiltonian function of the manufacturer

\[
H^M_t = J^M_t + P^M_{t+1}(x_{t+1}) + u_t(P^R_t).
\]
So, we obtain, $I_t^M$, $x_{t+1}$, $P_{t+1}^M$ and $u_{t+1}$.

In this level, the game is between supplier as the leader and the manufacturer as the follower. In fact, the reaction functions of two followers (retailer and manufacturer) are placed on the objective function of the leader (supplier). We can find final equilibrium point.

For fixed $I_t^M$ Hamiltonian function of the supplier is defined by

$$H_t^S = J_t^S + P_{t+1}^S(x_{t+1}) + u_t'(P_t^M).$$

We place reaction functions of followers into the leaders Hamiltonian function and we obtain $I_t^S$, $P_{t+1}^S$, $u_t'(P_t^M)$ and $x_{t+1}$.

For solving the above optimal control problem, we chose an algorithms given by Medanic and Radojevic which is an augmented discrete Hamiltonian matrix [9].

First, we assume

$$\begin{bmatrix} \tilde{x}_{t+1} \\ \tilde{P}_{t+1} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \tilde{x}_t \\ \tilde{P}_{t+1} \end{bmatrix} + \begin{bmatrix} D \\ E \end{bmatrix} = \begin{bmatrix} A\tilde{x}_t + B\tilde{P}_{t+1} + D \\ C\tilde{x}_t + A\tilde{P}_{t+1} + E \end{bmatrix},$$

where $\tilde{x}_{t+1} = \begin{bmatrix} x_{t+1} \\ u_{t+1} \end{bmatrix}$ and $\tilde{P}_{t+1} = \begin{bmatrix} P_{t+1}^M \\ p_{t+1}^R \end{bmatrix}$.

$A$, $B$, and $C$ are $2 \times 2$ matrices, and $D$ and $C$ are $2 \times 1$ matrices.

We solved the above problem by sweep method [2], by assuming a linear relation between $\tilde{p}_t$ and $\tilde{x}_t$; thus, we can compute the value of $\tilde{p}_t$ and $\tilde{x}_t$ and we can obtain the values of the other variables for all points in time by backward and forward loop.

4 Conclusion

This paper investigated a decentralized three-tier supply chain consisting of supplier, manufacturer and retailer for the allocating CSR to members of supply chain system in over time. We considered a two-level Stackelberg game consisting of two followers and one leader. The members of a supply chain play games with each other to maximize their own profits; thus, the model used to be a long-term co-investment game model. The equilibrium point at which members make their decisions to maximize profits by implementing CSR among members of the supply chain in a time horizon was determined. We applied control theory and used an algorithm (augmented discrete Hamiltonian matrix) to obtain an optimal solution for the dynamic game model.

References

[1] Batabyal, A., Consistency and optimality in a dynamic game of pollution control II: Monopoly, Environmental and Resource Economics, 8, 315-330, (1996).
[2] Bryson, A.E., Ho, Y.C., Applied Optimal Control, John Wiley and Sons, New York, 1975.

[3] Cachon, G.P., Zipkin, P.H., Competitive and cooperative inventory policies in a two-stage supply chain, Management Science, 45, 936-953, (1999).

[4] Feibel, B.J., Investment Performance Measurement, Hoboken, NJ: John Wiley and Sons, Inc, 2003.

[5] He, X., Gutierrez, G., Sethi, S.P., A survey of Stackelberg differential game models in supply and marketing channels, Journal of Systems Science and Systems Engineering, 16, 385-413, (2007).

[6] Hennet, J.C., Arda, Y., Supply chain coordination: A game-theory approach, Engineering Applications of Artificial Intelligence, 21, 399-405, (2008).

[7] Kytle, B., Ruggie, J.G., Corporate social responsibility as risk management: a model for multinationals, Corporate Social Responsibility Initiative Working Paper No. 10. Cambridge, MA: John F. Kennedy School of Government, Harvard University, (2005).

[8] Mankiw, N.G., Principles of Microeconomics, Mason, Ohio: Thomson/South- Western, 2004.

[9] Medanic, J., Radojevic, D., Multilevel Stackelberg strategies in linear-quadratic systems, Journal of optimization theory and applications, 24, 485-497, (1978).

[10] Sethi, S.P., Dimensions of corporate social responsibility, California Management Review, 17, 58-64, (1975).

[11] Shi, H., A Game theoretic approach in green supply chain management, M.S. Thesis, University of Windsor, 2011.

[12] Tian, Y., Govindan, K., Zhu, Q., A system dynamics model based on evolutionary game theory for green supply chain management diffusion among Chinese manufacturers, Journal of Cleaner Production, 80, 96-105, (2014).