The Super-potential and Holomorphic properties of the MQCD

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Abstract

We study the holomorphic properties of the MQCD by comparing the super-potentials in MQCD and the gauge theory. First we show that the super-potential defined as an integral of three form is NOT appropriate for generic situation with quarks. We report a resolution of the problem which works for the brane configurations of 90 degree rotation, including the true SQCD. The new definition does not need auxiliary surface and can be reduced to a contour integral for some cases. We find relation between the new and old definitions, which is verified by explicit calculation for SU(N), SO(N), Sp(N) simple groups with $F$ of massive quarks.

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1 Introduction

The idea of D-brane[1] opened up a new and surprisingly simple way to communicate between super symmetric (SUSY) gauge theories and the string theories[2, 3, 4, 5, 6, 7, 8]. In a recent paper [5], Witten provided solutions of $N = 2$ SUSY gauge theories in four dimension[9] by reinterpreting web of branes in type II superstring as a single M-brane. In a subsequent paper[8], he showed how some of the long standing problems in particle physics such as quark confinement and chiral symmetry breaking, can be approached from the M-theory point of view. There, he also suggested a way to calculate the super-potential (SP) for MQCD, which immediately gives the tension of the domain wall [10]. Although these are examples of spectacular successes, the reason why it work is not very clear. In fact the region where M-theory is working is quite opposite limit to the region where the gravity is decoupled[13]. In fact, there are evidences[8, 12] that MQCD and the SQCD is not the same theories. However, it is speculated[11, 12] that the holomorphic objects of two theories may be the same. So it is compelling to check whether the holomorphic properties of two theories really agree.

In this paper we study these issues by studying the super-potential for various M-brane configurations and comparing with corresponding gauge theories. In the previous work of the author [14], the Witten’s calculation of the super potential was generalized to the case where gauge group is the product of $SU(k_i)$’s as well as to the case where massless quarks are included. There, it was claimed that the value of $W_3$, the super-potential in MQCD defined in [8], is the same as that of the gauge theory with massive adjoint fields. It turns out that this agreement is limited to very specific cases and also very limited sense that will be described in detail later. Main observation of this paper is that $W_3$ should be replaced by a new definition in order to compare MQCD with SQCD in the presence of matter.

In section 2, we give a review to set up the language. In section 3, we first consider the cases for $SU(N)$ gauge theory with massive adjoint and hyper multiplets and clarify the staement made in the previous work [14]. Namely, in what sense, $W_3$ agrees with gauge theory for non-parallel but non-orthogonal NS five branes. Then
we point out that $W_3$ for the orthogonal NS five branes, which corresponds to the true $N=1$ SQCD with hyper multiplets, disagrees with the minimum value of the gauge theory super-potential. In section 4, we give a modified definition of SP, $W_2$, which does agree with SQCD. The new definition is also ambiguity free since it does not need auxiliary surface. We will find relation between the new and old definitions through these calculations. In section 5, we will give more examples for the theory with $SU(N)$, $SO(N)$, $Sp(N)$ gauge groups with $F$ of massive quarks.

2 Review of the Super-potential in MQCD

Let $\Sigma$ be a surface that describes the five brane as $\mathbb{R}^4 \times \Sigma$. The $\Sigma$ is embedded in the six dimensional internal space-time, $x^4, x^5, x^6, x^{10}, x^8, x^9$. Let $v, w, t$ be the complex co-ordinates defined as $v = x^4 + ix^5$, $w = x^8 + x^9$, $t = e^{-(x^6 + ix^{10})/R}$, where $R$ is the size of eleventh dimension. From the earlier study [7, 6], we know that $v \sim < \Phi >$ hence we assume that $v$ has mass dimension one and $w \sim < \tilde{Q} \bar{Q} >$ hence mass dimension two. Consider a holomorphic top form in the complex three dimensional space whose co-ordinate are $v, w, t$:

$$\Omega_3 = dv \wedge dw \wedge dt/t. \quad (1)$$

Given a brane configuration, the super-potential must give a definite value. The only canonical candidate is integral of the $\Omega_3$ over a three manifolds. To provide such a manifold, Witten introduced an auxiliary surface $\Sigma_0$ in the same homology class of $\Sigma$ such that $\Sigma_0$ is asymptotic to the $\Sigma$. Then there exists a three manifold $B$ enclosed by $\Sigma$ and $\Sigma_0$ and the super-potential is defined by

$$W_3(\Sigma, \Sigma_0) = W(\Sigma) - W(\Sigma_0) = \frac{1}{\pi i} \int_B \Omega_3. \quad (2)$$

This defines $W(\Sigma)$ up to an additive constant. \#1 We included the $\frac{1}{\pi i}$ factor since it is the right normalization when we compare with the gauge theory results. Notice that we do not have the factor $R$ in the above definition unlike ref. [8], due to our mass.

\#1 It is very similar to the Wess-Zumino term in field theory in the sense that it require one more dimension.
dimension of $v, w$. Namely, the super-potential must have mass dimension three and the sum of the mass dimension of $v$ and $w$ is already three, hence there is no room for $R$. The super symmetry R charge is carried by $v$. In order to assign a unique value $W(\Sigma)$ to a given brane configuration $\Sigma$, $W(\Sigma_0)$ must be universal. In order to achieve this goal, Witten required $\Sigma_0$ be invariant under chiral rotation so that one can put $W(\Sigma_0) = 0$. In case this is impossible, we have to regards that $W_3$ depends on the pair of surfaces $\Sigma, \Sigma_0$.

3 Difficulties of $W_3$

First we consider MQCD corresponding to $SU(N)$ gauge theory with $F$ (massive) quarks as well as quadratic super potential for the adjoint field $\Phi$. The curve $\Sigma$ is given in parametric form by:

\begin{align*}
  v(\lambda) &= -m_f + \frac{1}{\mu} \left( \frac{\lambda - \lambda_+ \lambda - \lambda_-}{\lambda} \right) \\
  w(\lambda) &= \lambda \\
  t(\lambda) &= \lambda^{N-F}(\lambda - \lambda_+)^r(\lambda - \lambda_-)^{F-r}.
\end{align*}

To construct a three volume, we need an auxiliary curve $\Sigma_0$ which we define by

\begin{align*}
  v &= \left( \frac{\lambda_+ \lambda_-}{\mu \lambda} - m_f \right) f_0(|\lambda|) + \left( \frac{\lambda - \lambda_+ - \lambda_-}{\mu} \right) f_\infty(|\lambda|) \\
  w &= \lambda f_\infty(|\lambda|) \\
  t &= \lambda^{N-F}(\lambda - \lambda_+)^r(\lambda - \lambda_-)^{F-r},
\end{align*}

where $f_0(|\lambda|) = 1$ for $|\lambda| < \epsilon$ for small enough $\epsilon$, and $f_\infty(|\lambda|) = 1$ for $|\lambda| > \frac{1}{2}\min(|\lambda_-, |\lambda_+|)$ and vanish rapidly outside. The subtlety and problems in this choice will be discussed after the calculation. Now $B$ is defined as the volume interpolating $\Sigma$ and $\Sigma_0$. It can be parameterized by $g_\alpha(|\lambda|, \sigma)$’s which interpolate $f_\alpha$ and 1 as $\sigma$ varies from 0 to 1.

\begin{align*}
  v &= \left( \frac{\lambda_+ \lambda_-}{\mu \lambda} - m_f \right) g_0(|\lambda|, \sigma) + \left( \frac{\lambda - \lambda_+ - \lambda_-}{\mu} \right) g_\infty(|\lambda|, \sigma).
\end{align*}
From this, \[ W_3 := W_3(\Sigma) - W_3(\Sigma_0) = -\frac{\lambda_+\lambda_-}{\mu}(N - \frac{1}{2}F) + \frac{1}{2}m_f (r\lambda_+ + (F - r)\lambda_-). \] (11)

This is equal to the minimum value of \[ W_{\text{eff}} = (N - F) \left( \frac{\Lambda_+^{3N-F}}{\det M} \right)^{1/(N-F)} + \frac{1}{2\mu} \left( \text{Tr}(M^2) - \frac{1}{N}(\text{Tr}M)^2 \right) + \text{Tr}(m_f M), \] (12)

if we identify the \( \lambda_\pm \) with two distinguished eigenvalues of the meson matrix M. A technical remark: The appearance of weighting factor \( \frac{1}{2} \) in eq. 11 is a characteristic feature of \( W_3 \). This factor gives the possibility to find a choice of \( \Sigma_0 \) that gives the gauge theory value when there is a quadratic super potential \( \text{Tr}\Phi^2 \). In the limit \( \mu \to \infty \) [6], \( \lambda_+/\mu \to -m_f \) and \[ W \to N\zeta, \] (13)

where \( \zeta = m_f\lambda_- \). This is equal to the minimum values of the gauge theory super-potential without \( 1/\mu \) term.

Apparently, this looks good. However, we should notice that the value of \( W_3 \) depends on \( \Sigma_0 \) very sensitively. \( \Sigma_0 \) is not chirally invariant and in fact it is impossible to choose chirally invariant \( \Sigma_0 \) that is asymptotic to \( \Sigma_0 \) since the chiral symmetry is explicitly broken by the mass. Therefore one can not set its super-potential zero and the value \( W_3(\Sigma, \Sigma_0) \) can not be attributed to the brane configuration only. This is the first difficulty. If we ask whether one can find a class of auxiliary surface \( \Sigma_0 \), then from the above calculation the answer is yes. In fact the conclusion of ref.[14] should be considered only in this sense.

In fact a much more natural choice of \( \Sigma_0 \) defined as
\[
\begin{align*}
  w &= \lambda g_\infty(|\lambda|, \sigma) \\
  t &= \lambda^{N-F}(\lambda - \lambda_+)^r(\lambda - \lambda_-)^{F-r},
\end{align*}
\] (9)
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\[
\begin{align*}
  v &= v(\lambda)f_0(|\lambda|) \\
  w &= w(\lambda)f_\infty(|\lambda|) \\
  t &= t(\lambda)
\end{align*}
\] (14)
\[
\begin{align*}
  t &= t(\lambda)
\end{align*}
\] (15)
fails to fit the gauge theory value. Apart from the simplicity and the generality it satisfies all the asymptotic conditions by choosing $f_\infty(\lambda) = 1$ for $|\lambda| > \max(|\lambda_-|, |\lambda_+|)$ . However, according to the definition of $W_3$ it gives

$$W_3(\Sigma, \Sigma_0) = \frac{\lambda_+ \lambda_-}{\mu} (N - F) + \frac{1}{2} m_f(r\lambda_+ + (F - r)\lambda_-)$$

which is different from the gauge theory value.

Now we calculate $W_3$ for M-brane configuration that correspond to the 90 degree rotation ($\mu \to \infty$). Namely, if we take the limit $\mu \to \infty$ of the eq. $4$, we get $\[6\]$

$$v = -\zeta/\lambda$$
$$w = \lambda$$
$$t = \lambda^{N-F}(\lambda - \lambda_-)^F,$$

Using the method described above, we get

$$W_3 = \zeta(2N - F)$$

This value is not consistent with the gauge theory value of eq.$13$. The same result is obtained for the configuration where not all the eigenvalues of the meson matrix are degenerate. Therefore there is a discontinuity at $1/\mu = 0$:

$$\lim_{\mu \to \infty} W_3(\Sigma(\mu)) \neq W_3 \left( \lim_{\mu \to \infty} \Sigma(\mu) \right).$$

For orthogonal NS branes $vw = \zeta$, so there is no room to play with in choosing the $\Sigma_0$, therefore there is no $\Sigma_0$ such that it is asymptotic to $\Sigma$ and $W_3$ fits the gauge theory value. So it is much worse than the case of non-orthogonal NS branes.

Summarizing, Witten’s definition of the super-potential$[8]$ involves an auxiliary surface and this cause a lot of difficulties for the case including (massive) quarks especially for orthogonal five branes. The ambiguity in the super-potential coming from the dependence on $\Sigma_0$ was stressed both in $[14]$ and $[11]$. Therefore it is reasonable to conclude that $W_3$ is not appropriate to use to compare with gauge theory for generic situation. So we need some other method to calculate the M-theoretic super-potential. In the next section, we report a resolution to the problem which works for the brane configuration of 90 degree rotation, including the true SQCD.
4 New definition of super-potential in MQCD

We try to modify the definition of the super potential such that it depends only on $\Sigma$. Since $\Sigma$ is a surface, we look for a 2-form $\Omega_2$ such that $d\Omega_2 = \Omega_3$ to define

$$W_2 \sim \int_{\Sigma} \Omega_2.$$  \hfill (20)

For a closed surface $\Sigma$, it is just rewriting by Stokes’ theorem. However, since $\Sigma$ is not a compact manifold in our application, there is an ambiguity corresponding to the gauge transformation:

$$\Omega_2 \rightarrow \Omega_2 + df_1.$$  \hfill (21)

That is, for any one form $f_1$, $d(\Omega_2 + df) = d\Omega_3$. But the integral of $df_1$ over the $\Sigma$ is non-zero since it is not a compact manifold. This apparent ambiguity is resolved by noticing the symmetry of the $\Omega_3$. It is antisymmetric under the exchange of $v$ and $w$. Requiring the same symmetry to $\Omega_2$ fixes the two form $\Omega_2$ uniquely,

$$\Omega_2 = \frac{1}{2}(vdw - wdv) \wedge dt/t.$$  \hfill (22)

Therefore it is reasonable to take

$$W_2 = \frac{1}{\pi i} \int_{\Sigma} \frac{1}{2}(vdw - wdv) \wedge dt/t$$  \hfill (23)

as the new definition of the super-potential: it is independent on the auxiliary surface $\Sigma_0$ and it respects many of the symmetry of $W_3$. However, the pull back of the holomorphic two form on a holomorphic curve is simply zero. Therefore one should ask how to get non-zero value of the the super-potential. What saves us from the triviality is precisely the non-compactness (singularity) of the Riemann surface. In the presence of the singularities, one should take out small disks around them. Once the singularities are cut out, the integrand becomes either identically zero or total derivative term, which in turn becomes a line integral over the sum of the boundaries of the small disks. In this way our definition reduces to a contour integral.

We now illustrate these idea for the cases where NS and NS’ branes are orthogonal. First, take a pull-back of the surface to the $v$- or $w$- plane using the equations
that define $\Sigma$:

\begin{align*}
  w &= w(v), \quad t = t(v) \text{ for the pull-back to v-plane} \\
  v &= v(w), \quad t = t(w) \text{ for the pull-back to w-plane}.
\end{align*}

(24)

According to which pull-back we make, the algebraic form of the curve look different. It has been known [6, 7] that the co-ordinate $v$ corresponds to the adjoint field $\Phi$ in the effective action and $v$’s values are eigenvalues of the quark mass matrix $m_{ij}$ due to the coupling $\hat{Q}\Phi Q$. Similarly the co-ordinate $w$ corresponds to the meson fields $M_{ij} = \hat{Q}_i Q_j$ and $w$’s values are eigenvalues of the meson matrix which again corresponds to the mass of the dual quarks via $\hat{q}Mq$. Therefore it is appropriate to call the pull-back to the $w$-plane $w$- or meson-picture and call the pull-back to the $v$-plane $v$- or $\Phi$-picture.

In what picture should we work? In a gauge theory, the super-potential was always evaluated in terms of the meson fields, hence one should work in meson-picture to compare the value of the super potentials of MQCD with that of SQCD. In meson picture, $v(w)dw \wedge dt(w)/t$ is zero due to the holomorphic dependence of $dt(w)$ on $w$. Hence the super-potential becomes

\begin{align*}
  W_2(\Sigma) &= -\frac{1}{2\pi i} \int_{R^2_w} wdv(w) \wedge dt(w)/t \\
  &= -\frac{1}{2\pi i} \int_{\cup_j C_j} wv(w)dt(w)/t \\
  &= \frac{1}{2\pi i} \zeta \int_{C_\infty} dt(w)/t \\
  &= \zeta (\text{No. of zeroes - No. of poles of $t$}) 
\end{align*}

(25)

where $C_j$’s are the small curves around the singularities of the curve $\Sigma$ and $C_\infty$ is the circle around the $w = \infty$. We have used $vw = -\zeta$. Therefore the value of the super-potential, when NS and NS’ brane are orthogonal, is proportional to the asymptotic bending power of NS’ brane, defined as $t \sim w^{\text{Power}}$ as $w \to \infty$.

One should also notice that we cut out the disks around all the singularities of the curves, namely those of $dt(w)/t$ as well as those of $v(w)$. Therefore the tubes

\#2These are not yet the same as the ‘electric’ and ‘magnetic’ theory, since even for the $N > F$ case, this classification is valid.
corresponding to the semi-infinite four branes as well as asymptotic circle of the
NS’ brane contributes to the super-potential directly. As we will see in the explicit
element below, the tubes’ contribution is equal to the mass term $TrmM$ in the gauge
theory.

So far we succeeded to find an expression that is non-trivial and we want to see
that the precise relation between the super-potential in SQCD and MQCD is:

$$W_{SQCD} = W_2(\Sigma)$$

Later, we will verify this relation for several examples which cover all the physically
important cases. For a moment, we will ask more general questions: what happen if
we define the super-potential in the v-picture as

$$W'^v(\Sigma) = \frac{1}{2\pi i} \int_{\mathbb{R}^2} vdw(v) \wedge dt(v)/t$$

$$= \frac{1}{2\pi i} \int_{C_i} vw(v) dt(v)/t$$

$$= -\frac{1}{2\pi i} \zeta \int_{C_{v,\infty}} dt(v)/t$$

(27)

where $C_i$’s are the small curves around the singularities in $v$-plane, and $C_{v,\infty}$ is a large
circle around the $v = \infty$. Notice that this is the same as the the contribution of the the
circle near $w = 0$ to $W'^w(\Sigma)$ as it should be, since the circle near $w = 0$ is equivalent
to the circle near $v = \infty$. Therefore by adding $W'^v$ and $W'^w$, the contribution of the
circle at the asymptotic region of NS brane is weighted by two relative that of the
tubes. Therefore we we prove that

$$W_3(\Sigma) = W_2^{(v)} + W_2^{(w)},$$

(28)

which gives the relation of ’old’ and new super-potential at least for the orthogonal
NS five branes. This relation can be proved case by case in the examples below.

Now we give some explicit examples. The first example is the brane configuration corresponding to the $SU(N)$ super Yang-Mill theory. The curve is given by

$$vw = -\zeta, \ t = v^N$$

(29)
and we get the value of the super-potential $N\zeta$ both in $v$- and $w$-pictures. This is consistent to the value of SQCD as well as verifying the relation eq.28.

The second and more interesting example is the case of $SU(N)$ with $F$ hyper multiplets in the fundamental representation. The curve is given by

$$vw = -\zeta, \quad t = w^{N-F} \prod_{i=1}^{F} (w - w_i)$$

(30)

The value of the super-potential in this case is

$$W_2^{(w)} = \zeta((N - F) + F) = \zeta N,$$

(31)

which agrees with the known value in gauge theory[15]. Here $(N-F)$ is contribution from the infinite circle of NS brane ($w = 0$) and $F$ is the sum of those from infinitesimal circles in the tubes ($w = w_i$’s). In the $v$-picture, we get $(N - F)R\zeta$. Notice that in both examples, $W_3$, the value of the super-potential calculated in generalized Witten’s method is the sum of the values in electric and magnetic pictures in the new definition: namely,

$$W_3(\Sigma) = W_2^{(v)} + W_2^{(w)} = \zeta(2N - F).$$

(32)

One should also notice the semi-topological character of the super-potential: The value of the super-potential does not depend on the location of the semi-infinite D4 branes only through the value of $\zeta$. It is consequence of $wv = -\zeta$, i.e, the orthogonality of the two NS branes. This is not surprising, since the value of the super-potential depends only on the vacuum configuration where no massive excitation are created. Therefore we do not expect explicit dependence on the quark masses.

5 More examples: The cases including orientifold

The curves for brane configuration that correspond to $N = 1$ SQCD with $SO(N)$, $Sp(N)$ gauge theories were given in [16]. First, $Sp(N)$ with $F$ flavor in fundamental representation, the curve is given by

$$vw = -\zeta$$

$$t = \xi w^{N+2-F} \prod_{i=1}^{F/2} (w^2 - w_i^2),$$

(33)
where
\[ \xi = (\text{Pf}M)^{-2(N+2)/F}, \quad \text{and} \quad \zeta = \Lambda_{\text{Sp}}^{3(N+2)/(N+2-F)}(\text{det} M)^{F/(F-N-2)}. \]

The value of the super-potential is
\[ W_2^w = \zeta (N + 2) \]

Notice that this is the asymptotic bending power of the NS brane where the $F$ semi-infinite brane is attached. This is in electric picture. In the v-picture,
\[ W_2^v = \zeta (N + 2 - F) \]

Next SO(N) with $F$ flavor in fundamental representation: The curve is given by
\[ vw = -\zeta \]
\[ t = \xi w^{N-2-F} \prod_{i=1}^{F/2} (w_i^2 - w_1^2), \]

where
\[ \xi = (\text{Pf}M)^{-2(N-2)/F}, \quad \text{and} \quad \zeta = \Lambda_{\text{Sp}}^{3(N-2)/(N-2-F)}(\text{det} M)^{F/(F-N+2)}. \]

The value of the super-potential is
\[ W_2^w = \zeta (N - 2), \]

in the meson picture. In the v-picture,
\[ W = \zeta (N - 2 - F) \]

In all of these examples $W_{\text{SQCD}} = W_2^w(\Sigma)$ and the relation $W_3 = W_2^w + W_2^v$ holds.

Finally we go back to the non-orthogonal case Now it is time to ask what happen to the non-orthogonal case with the new definition of the super-potential. The curve, $\Sigma$, for the gauge theory with massive quarks can be written as
\[ v + m_f = \frac{1}{\mu} \left( w - w_+ \right) \left( w - w_- \right) \]
\[ t(w) = w^{N-F}(w - w_+)^r(w - w_-)^{F-r}, \]
Upon the pull back to the w plane, we again have $v(w)dw \wedge dt/t = 0$.

$$W_2(\Sigma) = -\frac{1}{2\pi i} \int v(w)wdt/t = -\frac{w^-w^+}{\mu}(N - F) + m_f \text{Tr}M.$$  \hspace{1cm} (43)

This value is not the same as the value of the gauge theory. The difference is $-\frac{w^-w^+}{\mu} N$. We expect that this is the contribution from the asymptotic region of the NS five brane, although it is not clear how to compute the $W_2^w$ in this non-orthogonal case. It would be very interesting to prove that this is the case.

6 discussion

In this paper we showed that $W_3$, Witten’s definition of the super potential, does not give the same value to that of the gauge theory for the orthogonal NS and NS’ branes. We gave a modified definition of MQCD super-potential, $W_2$, which gives the same value as that of SQCD. We give the relation between $W_3$ and $W_2$. For non-orthogonally rotated brane configuration which corresponds to the gauge theory with mass of the adjoint field, $W_3$ fits the value of the gauge theory. It is still unclear how to understand the gauge theory super-potential for the non-orthogonal case from the point of new definition. Why $W_2^w$ fails for non-orthogonal case? It is also interesting to check the relation eq.28 continues to hold in the presence of the more general tree level super-potential $W_{\text{tree}}(\Phi)$. These issues are under investigation.

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