Strongly Coupled Chameleon Fields: New Horizons in Scalar Field Theory

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We show that as a result of non-linear self-interactions, scalar field theories that couple to matter much more strongly than gravity are not only viable but could well be detected by a number of future experiments provided they are properly designed to do so.

There is wide-spread interest in the possibility that, in addition to the matter described by the standard model of particle physics, our Universe may be populated by one or more scalar fields. These are a general feature in high energy physics beyond the standard model and are often related to the presence of extra-dimensions. The existence of scalar fields has also been postulated as means to explain the early and late time acceleration of the Universe. It is almost always the case that such fields interact with matter: either due to a direct Lagrangian coupling or indirectly through a coupling to the Ricci scalar or as the result of quantum loop corrections. If the scalar field self-interactions are negligible, then the experimental bounds on such a field are very strong: requiring it to either couple to matter much more weakly than gravity does, or to be very heavy. Recently, a novel scenario was presented by Khoury and Weltman that employed self-interactions of the scalar-field to avoid the most restrictive of the current bounds. They dubbed this feature allows the field to evade local constraints on WEP violation require\(\frac{\lambda B}{M_{pl}}\) to be very heavy. Astrophysical constraints require that \(\frac{\beta}{\lambda B} \lesssim 0.1\) since nucleosynthesis.

We parameterize the matter coupling of the chameleon by a function \(\beta B_{\phi}(\beta \phi/M_{pl})/\rho/M_{pl}\). Astrophysical constraints require that \(\beta M_{pl} / \rho \lesssim 0.1\) since nucleosynthesis.

Preempting this requirement we simplify our calculations by expanding \(B_{\phi}\) about \(\phi = 0\), and scale \(\beta\) so that \(B_{\phi}(0) = 1\). The equation of motion for \(\phi\) then becomes

\[-\Box \phi = V_{,\phi}(\phi) + \beta (\rho + \omega P)/M_{pl},\]

where \(\rho\) is the energy density of matter, \(P\) is its pressure and \(\omega\) parameterises the way in which the chameleon couples to matter. In the simplest models, \(\phi\) couples to the trace of the energy momentum tensor, and so \(\omega = -3\). In what follows we take this to be our fiducial value of \(\omega\) and note that the results for different \(O(1)\) values of \(\omega\) are very similar. We note that the right hand side of eq. \(\text{II}\) vanishes when \(\phi = \phi_c(\rho + \omega P)\):

\[\phi_c(\rho + \omega P) = M (\beta (\rho + \omega P)/(\lambda M_{pl} M^3))^{-\frac{1}{\alpha + 1}}.\]

For \(\phi_c(\rho + \omega P)\) to be real when \(\beta (\rho + \omega P) > 0\), we need either \(n \geq 0\) or for \(n\) to be negative and even; and \(n \neq 0, -2\) for the theory to be non-linear. The mass of small perturbations about \(\phi = \phi_c\) is \(m_c = \sqrt{V_{,\phi\phi}(\phi_c)} = \sqrt{\lambda n (n + 1) M |M/\phi_c|^{n/2+1}}\).

One would expect, in the absence of any chameleon mechanism, the force mediated by \(\phi\) to be \(\beta^2\) as strong as gravity. As a result of quantum corrections \(\beta\) will generally differ slightly for different particle species, which would standardly lead to a composition dependent force that would in turn violate WEP. Solar system bounds on WEP violation require \(\beta \lesssim 10^{-5}\) in non-chameleon
Theories \(8\). Chameleon theories have been shown to be compatible with \(\beta \sim O(1)\) \(1\). In this Letter, however, we will go much further and report how, as a result of non-linear effects, it is possible for a chameleon field to couple to matter much more strongly than gravity does (i.e. \(\beta \gg 1\)) and yet for it to have remained thus undetected. We define \(M_\phi = M_{pl}/\beta\), which is roughly the energy at which chameleon particles would be produced in particle colliders. It would be pleasant in the light of the hierarchy problem if \(M_\phi /\beta \ll M_{pl}\), say of the GUT scale, or, if we hoped to find traces of it at the LHC, maybe even the TeV-scale. We show below that both of these scenarios are allowed for.

Crucial to our ability to constrain chameleon theories is a full understanding of how they behave as field theories. It transpires that when \(\beta \gg 1\), the non-linear nature of the potential, \(V(\phi)\), becomes very important. Even in the, supposedly, simple case of the field produced by a single large body, there might not exist any self-consistent linearisation of eq. \(1\) that is valid everywhere \(2\). Non-linear effects are also non-negligible when calculating the force produced by one body upon another. When linearised theory fails, the solution to the two body problem cannot be found simply by superimposing two copies of the field produced a single body.

Non-linear effects also play a rôle in determining the effective, large-scale or macroscopic theory associated with the chameleon. Eq. \(1\) defines the microscopic, or particle-level, field theory for \(\phi\), whereas in most cases we are interested in the large scale or coarse grained behaviour of \(\phi\). In macroscopic bodies the density is actually strongly peaked near the nuclei of the individual atoms from which it is formed and these are separated from each other by distances much greater than their radii. Rather than explicitly considering the microscopic structure of a body, it is standard practice to define an ‘averaged’ field theory that is valid over scales comparable to the body’s size. If our field theory were linear, then the averaged equations would be the same as the microscopic ones e.g. as in Newtonian gravity. But it is important to note that this is very much a property of linear theories and is not in general true of non-linear ones. Non-linear effects must, therefore, be taken into account. We do this by combining matched asymptotic expansions with exact analytical solution of the full non-linear equations under certain reasonable assumptions. We confirm our results by numerically integrating the field equations.

Firstly we define the concept of a thin-shell. A body is said to have a thin-shell if the coarse-grained value of \(\phi\) (as defined on scales that are large compared to the sizes of the constituent particles of the body) is approximately constant everywhere inside the body, except in a thin-shell near the surface of the body where large changes \((O(1))\) in its value occur. The existence of a thin-shell is related to the presence of non-linear behaviour. Deep inside a body with a thin-shell \(\phi\) is constant, and so we might expect \(\phi = \phi_c(\rho)\), where \(\rho\) is the density of the body (we assume \(\rho \ll \rho_c\)). The effective chameleon mass, \(m_{eff}\), in the body would then be given by \(m_{eff} = m_c(\rho)\). The effect of the non-linearities on the averaging, however, is to limit the averaged value of \(m_\phi\) to be smaller than some critical value, \(m_{crit} \geq 3\). \(m_{crit}\) is a macroscopic quantity but it depends only on the microscopic properties of the body and the index \(n\). It is independent of \(\beta\), \(M\) and \(\lambda\) \(3\). We have modeled the body as being composed of particles of radius \(R_p\) separated by an average distance \(d_p\). The macroscopic mass of the chameleon in the body is then \(m_{eff} = \min(m_c(\rho), m_{crit})\), where:

\[
m_{crit} \approx \sqrt{\frac{3}{2}} (n + 1) d_p^{-1} (R_p / d_p)^{\frac{1}{d_p}} n \neq -4,
\]

where \(q(n) = \min(1, (n + 4)/(n + 1))\) and \(m_{crit} \approx 1.4 / d_p\) when \(n = -4\). Whenever \(m_{eff} = m_{crit}\) it is because the individual particles that make-up the body have themselves developed thin-shells. This critical behaviour emerges from the requirement that non-linear effects are negligible outside of the particle from \(r = R_p\) to \(r = d_p\): this implies a maximal value of \(m_{eff}\), i.e. \(m_{crit}\), that depends only on \(R_p\), \(d_p\) and \(n\). The \(n\) dependence arises because \(n\) determines precisely when linear theory breaks down.

\(\beta\)-independent critical behaviour is also seen in the \(\phi\)-force between two bodies. The onset of this critical behaviour is linked to the emergence of a thin-shell. A body of radius \(R\) and density \(\rho_c\) in a background of density \(\rho_b \ll \rho_c\) has a thin-shell if:

\[
m_{eff} R \gtrsim \sqrt{\frac{3}{2}} n + 1 \left| 1 - \frac{\rho_c / \rho_b}{n + 1} \right|^{1/2}, n \neq -4.
\]

The existence of a thin-shell is essentially due to non-linearities being strong near the surface of a body but weak in other regions. When \(n = -4\), a thin-shell occurs for \(m_{eff} R \gtrsim 4\), whereas linearised theory fails to be accurate for \(m_{eff} R \gtrsim 1.4\). When \(n > 0\), \((\rho_c / \rho_b)^{1/n+1} \gg 1\) and so the thin-shell condition, eq. \(4\), depends greatly upon the background density. The same is not true when \(n \leq -4\) since here \((\rho_c / \rho_b)^{(1/n+1)} \ll 1\). Therefore, \(n > 0\) theories can behave differently in space-based experiments than they do in laboratory ones, because the thin-shell condition is more restrictive in low-density background of space than it is in the lab \(3\). In contrast, theories with \(n \leq -4\) will exhibit no big difference in their behaviour in space-based tests to that seen on Earth.

The existence of a thin-shell in the test-masses used in experimental searches for deviations from general relativity is vital if we are to evade their bounds. Whereas the force between two non-thin-sheeted bodies with separation \(r\) is \(\beta^2 (1 + m_{eff}) c^2 m_{eff} r\) times the gravitational force between them \((m_b\) is the chameleon mass in the region between the bodies), the force between two bodies, of masses \(M_1\) and \(M_2\), with thin-shells is found to be independent of the coupling \(\beta\) \(\geq 3\). When \(d \gg R_1, R_2\), where
R₁ and R₂ are the respective radii of the two bodies, this force is found to be α₁₂ times the strength of gravity, where for \( n \neq -4 \):

\[
α₁₂ = \frac{S(n, mₖ)M₂^{2}(1 + mₖ r)e^{−mₖ r}}{M₁ M₂} (M² R₁ R₂)²(ₙ(n))ⁿ,
\]

where \( S(n, mₖ) \) is \( (3/ₙ)ⁿ²⁺² \) for \( n < -4 \), whereas for \( n > 0 \) it equals \( (n(n + 1))² / (mₖ²/n⁺²) \). When \( n = -4 \):

\[
α₁₂ = \frac{M₂^{2}(1 + mₖ r)e^{−mₖ r}}{8λM₁ M₂ \sqrt{ln(r/R₁)² ln(r/R₂)}}.
\]

For \( d \ll R₁, R₂ \) a different value for \( α₁₂ \) applies and is given below. This \( β \)-independence was first noted in [8], in the context of \( φ^4 \) theory. However, the authors were mostly concerned with region of parameter space \( β < 1 \), \( λ \ll 1 \); in our analysis we go further: considering a wider range of theories and also the possibility that \( β \gg 1 \).

We can understand the \( β \)-independence as follows: just outside a thin-shelled body, the potential term in eq. (1) is large and negative (\( \sim O(−β/ₚₗ) \)), and it causes \( φ \) to decay very quickly. At some point \( φ \) will reach a critical value, \( φ_{c_r-it} \), that is small enough so that non-linearities are no longer important. Since this all occurs outside the body, \( φ_{c_r-it} \) can only depend on the size of the body, the choice of potential \( (M, λ, n) \) and the mass of \( φ \) in the background, \( mₖ \). This is precisely what was found above.

This \( β \)-independence is of great of importance if one wishes to design an experiment to detect the chameleon through WEP violations. Since the \( φ \)-force is independent of the coupling, \( β \), for bodies with thin-shells, any microscopic composition dependence in \( β \) will be hidden on macroscopic length scales. The only ‘composition’ dependence in \( α₁₂ \) is through the masses of the bodies and their dimensions (\( R₁ \) and \( R₂ \)). The strength of WEP violations is quantified by the Eötvös parameter, \( η \). If we measure the differential accelerations of two test masses, \( M₁ \) and \( M₂ \), of radii \( R₁ \) and \( R₂ \) towards a third body, mass \( M₃ \) and radius \( R₃ \), then: \( η = α₁₁₃ − α₁₂₃ \). Taking the third body to be the Sun or the Moon, experiments searches for WEP violations have up to date found that \( η \lesssim 10⁻¹³ \) [6]. In most of these searches, although the composition of the test-masses is different, they are made to have the same mass \( (M₁ = M₂) \) and the same size \( (R₁ = R₂) \). Therefore, if the test-masses have thin-shells we have \( η = 0 \) and no WEP violation will be detected. The only implicit dependence of this result on \( β \) is that the larger the coupling is, the more likely it is that the test-masses will satisfy the thin-shells conditions. The first important consideration for future experiments is that: if one wishes to detect a chameleon field through WEP violations one must either ensure that test-masses do not satisfy the thin-shell conditions or that they are of different masses and/or dimensions.

We shall assume that such an experiment as been conducted, using two spherical test bodies both with a mass of 10 g, where one is made entirely of copper and the other of aluminum. The strongest bounds on chameleon fields would then come from measuring the differential acceleration of these bodies towards the Moon. We indicate in FIG. 1 the restrictions that finding \( η \lesssim 10⁻¹³ \) in such an experiment would place on these chameleon theories. The Moon is a better choice of attractor than the Earth or the Sun for such experiments since \( α₁₁₃ \) is proportional to \( M₃²/M₁ M₃ \) and so the smaller mass of the test-bodies, \( M₁ \), and the attractor, \( M₃ \), the larger \( η \) will be compared to gravity. The corollary of this result is that if we are unable to detect \( φ \) in lab-based, micro-gravity experiments where both \( M₁ \) and \( M₂ \sim O(10 g) \) (such as the Eöt-Wash experiment) then the \( φ \)-force between larger (say human-sized) objects, would also be undetectably small. For this reason measurements of the differential acceleration of the Earth and Moon towards

![FIG. 1: The whole of shaded area shows the allowed parameter space with all current bounds. For some values of \( M \) and \( λ \) we need the BeCu sheet to have a thin-shell; this results in a region near \( β \sim O(1) \) being ruled out. Future space-based tests could detect the more lightly shaded regions. The solid horizontal lines indicate the case where the chameleon field behaves like dark energy. Plots for theories with \( n < -4 \) or \( n > 0 \) are similar to cases \( n = -8 \) and \( n = 4 \) respectively.](image-url)
the Sun, e.g. lunar laser ranging, are not competitive with lab-based experiments.

Future, space-based tests of WEP promise to be able to detect $\eta$ up to a precision of $10^{-18}$, we indicate on FIG. 1 the regions of parameter space that such experiments would be able to detect. The $\phi$-mediated force will also produce effective corrections to the $1/r^2$ behaviour of gravity. The best bounds on such corrections come from the Eötvös experiment performed by Hoyle et al. which employs a torsion balance to measure the torque induced on a pendulum by rotating an attractor at a separation $d$. For $d \gtrsim 0.1\text{mm}$, they find that $\alpha_{12} \lesssim 10^{-2}$.

For a chameleon theory to satisfy this bound we need both the attractor and pendulum to have thin-shells. In this scenario $d$ is small compared to the size of test-masses ($d < R_1, R_2$) and the previous formula for $\alpha_{12}$ does not apply. When the mass of the chameleon inside the attractor and pendulum, $m_a$, obeys $m_a d \gg 1$ (as is the case for $\beta \gtrsim 1$) we find that the $\phi$-force is $\alpha$ times the strength of gravity, where $\alpha_{12}$ is:

$$5 \times 10^{-4} \left( \frac{M}{0.1\text{mm}} \right)^{\frac{2(n+3)}{n+2}} \left( \frac{\lambda^{1/n} B \left( \frac{d}{d - 0.1\text{mm}} \right)}{n} \right)^{\frac{3 n + 3}{2 n + 2}},$$

where $B(p, q)$ is the beta function. We note that $\alpha$, as before, is independent of $\beta$. The Eötvös bound is strongest for $n = -4$ where it appears to rule out a ‘natural’ value for $\lambda$ of $1/4!$: $0.56 \lambda^{-1} \lesssim 1$. However this is not the whole story. In this experiment a uniform $10\mu\text{mm}$ thick BeCu membrane is placed between the pendulum and attractor to shield electromagnetic forces. For $O(1)$ values of $\beta$ and $\lambda \sim 1/4!$ or $M \sim (0.1\text{mm})^{-1}$ this sheet does not have a thin-shell and makes little difference to the analysis. For slightly larger values of $\beta$ however ($\beta \gtrsim 10^4$ and $\lambda = 1/4!$ for $n = -4$) it will develop a thin-shell. Taking the mass of the chameleon inside the sheet to be $m_a$, the effect of this membrane is then to attenuate $\alpha_{12}$ by a factor of $\exp(-m_a d_s)$, where $d_s$ is the thickness of the sheet. The Eötvös-bound is then easily satisfied even for $\lambda \sim 1/4!$. The larger $\beta$ becomes, the larger $m_a$ is and the less restrictive this bound becomes. Experiments such as this must therefore be redesigned if they are to be able to detect chameleon theories with $\beta \gg 1$.

The prospect that couplings with $\beta \gg 1$ could be allowed is exciting. But to be taken seriously we must also consider bounds coming from astrophysical constraints, such as the stability and mass-radius relationship of white dwarfs and neutron stars as well as bounds coming from big-bang nucleosynthesis (BBN) and the Cosmic Microwave Radiation temperature anisotropies. These bounds can be summarised as requiring $|\beta \phi/M_{pl} | \lesssim 0.1$ over the whole universe since the BBN epoch.

This condition is enough to ensure that there has been no more than a 10% change in particle masses since BBN and in the redshift of the surface of last-scattering. Whilst we satisfy the same physical constraints as Amendola for non-chameleon, coupled quintessence, the chameleon mechanism ensures a significantly less restrictive bound on $\beta$ than was found there. Astrophysical constraints only place a weak upper bound on $\beta$ which is strongest for $n = -4$, e.g. if $\lambda = 1/4!$ we need $M_{pl}/\beta \gtrsim 10\text{GeV}$. However, realistically, we probably require $M_{pl}/\beta \gtrsim 200\text{GeV}$ for it not to have been seen so far in particle colliders.

In summary, we have considered a wide spectrum of scalar field theories with a chameleon mechanism and for the first time, the non-linear structure of these theories has been properly taken into account. We have found a surprising result that the chameleon force between two bodies with thin-shell is independent of their coupling to the field $\phi$, and that as a result the bounds on the coupling, $\beta$, can be exponentially relaxed. We have also noted that some laboratory experiments should be redesigned to detect the chameleon. For ‘natural’ values of $M \sim (0.1\text{mm})^{-1}$ or $\lambda \sim 1/4!$, the strongest upper bounds on $\beta$ probably come from particle colliders and $200\text{GeV} < M_{pl}/\beta \lesssim 10^{15}\text{GeV}$ is allowed for all $n$. If $M_{pl}/\beta \sim 1\text{TeV}$ we might even hope to see chameleon production at the LHC; although without a renormalisable quantum theory of the chameleon it is hard to say for sure if this happen. Planned space-based tests such as STEP, MICROSCOPE and SEE, promise improved precision and, when $n > 0$ there is also still the possibility that WEP violations in space can be stronger than the level already ruled out by laboratory based experiments. As noted in [1, 2], the chameleon field is a good candidate for dark energy if $M \sim (\rho_\Lambda)^{1/4} \approx (0.1\text{mm})^{-1}$; this result is unchanged for $\beta \gg 1$.

In conclusion: scalar field theories that couple to matter much more strongly than gravity are not only viable but could well be detected by a number of future experiments provided they are properly designed to do so. This result opens up an altogether new window which might lead to a completely different view of the rôle played by scalar fields in particle physics and cosmology.

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