AN EFFICIENT AND ROBUST SYSTEM FOR VERTICALLY FEDERATED RANDOM FOREST

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ABSTRACT

As there is a growing interest in utilizing data across multiple resources to build better machine learning models, many vertically federated learning algorithms have been proposed to preserve the data privacy of the participating organizations. However, the efficiency of existing vertically federated learning algorithms remains to be a big problem, especially when applied to large-scale real-world datasets. In this paper, we present a fast, accurate, scalable and yet robust system for vertically federated random forest. With extensive optimization, we achieved $5 \times$ and $83 \times$ speed up over the SOTA SecureBoost model (Cheng et al., 2019) for training and serving tasks. Moreover, the proposed system can achieve similar accuracy but with favorable scalability and partition tolerance. Our code has been made public to facilitate the development of the community and the protection of user data privacy.

1 INTRODUCTION

Machine learning techniques have achieved much success over the past decade. However, serious privacy concerns have also been raised over user data privacy while training these models (Yang et al., 2019; Kairouz et al., 2019). Such concerns are ever-increasing, especially over privacy critical applications such as biomedical engineering (Sheller et al., 2020), self-driving cars (Pokhrel & Choi, 2020), mobile phones (McMahan et al., 2017), smart home devices (Dorri et al., 2017), and finance (Long et al., 2020). Legislative authorities also started taking action to protect user privacy from potential data breaches. For example, in 2012, European Commission released the European Union’s General Data Protection Regulation (GDPR) for digital privacy protection.

To this end, federated learning is proposed as an emerging ideology that performs model training by utilizing data from multiple parties without threatening data privacy (McMahan et al., 2017). Depending on the configuration of data ownership, it can be categorized as vertically federated learning, horizontal federated learning, and federated transfer learning: vertically federated learning corresponds to the case where parties have common users, but each party holds different user features. Horizontal federated learning corresponds to the case where participating parties have different user samples, but feature categories possessed by these parties are generally the same. Federated transfer learning corresponds to the case where few common user and feature categories are shared by participating parties. By bridging multiple parties through federated learning, a much larger and diversified decentralized dataset can be utilized. With better dataset comes more powerful models, which can bring huge benefits to society.

Our implementation can be found at https://github.com/jzForPaper/rf_mlsys2022/tree/main/demos/random_forest

Figure 1. Illustration of the vertically federated learning ideology, which breaks the data barrier between multiple organizations, so that jointly building a more powerful model could be made possible. The same set of users are served by these parties, and each party owns user features of different aspect.
In this paper, we present a highly efficient and robust vertically federated random forest system. The system involves following aspects: (1) A novel federated random forest algorithm is designed for both efficient training and serving. (2) A novel distributed system is designed to utilize the parallelizability of random forest, and to achieve high partition tolerance. (3) An efficient homomorphic cryptosystem is used to provide protection on data privacy.

We highlight that, real-time inference is made possible for the first time through our proposed work. Compared with SOTA SecureBoost (Cheng et al., 2019), the proposed system leads to higher accuracy, less training time, and faster inference speed, as shown in Fig. 2. The proposed federated learning system currently supports dataset size up to millions and has already provided hundreds of millions of calls in production without any system failure, proving its value in solving real-world problems.

2 PRELIMINARIES

2.1 Problem Definition

As a motivating example, we present a real world problem that needs to be solved with federated learning ideology: To reduce the default risk in the credit authorization business, lenders (such as banks) must conduct comprehensive evaluations on the applicants. Furthermore, to make such evaluations more reliable, lenders would like to make use of various external data sources to build more powerful machine learning models.

The challenge to this problem comes from the following aspects: (1) How to utilize user data from other companies or organizations. Direct data copying can be problematic because accessing user data could be highly restricted due to privacy concerns and legislative considerations. (2) How to make the model efficient during both training and inference stages. Faster training speed would be helpful for rapid model iteration, and faster inference speed provides a better customer experience. (3) How to make the online service robust. It is highly desirable to have the system be partition tolerant when serving million-level users, otherwise it may bring a huge risk to the credit authorization business.

To this end, we design an efficient and robust federated learning system that allows multiple parties to jointly build machine learning models without leaking their private user data. Without loss of generality, suppose there are $m$ participating parties, each possesses feature data $X^i \in \mathbb{R}^{n_i \times d_i}$, where $n_i$ and $d_i$ are the number of samples and features on party $i$. We further assume there is an “active party” that contains all the labels (lenders for example). The other participating parties that don’t contain the label information are denoted as the “passive party”. Our goal is to build an efficient and robust system to predict $y$ for the active party based on the features from all participating parties. In the meantime, the exact feature values are not revealed to the active party, and the label information owned by the active party is also kept secret.

2.2 Related Work

The federated learning ideology was first proposed by Google to protect the privacy of mobile phone users during keyboard inputting (McMahan et al., 2017). Great success has been achieved by applying such ideology to its Gboard product: collaboratively learning a shared prediction model across devices, while keeping all user input history data locally. Since then, lots of work has been done to use federated learning to solve existing problems (Dayan et al., 2021; Dorri et al., 2017; Yang et al., 2020). As a special case, the vertically federated learning ideology is more suitable and often used in building cross-enterprise applications (Yang et al., 2019; Liu et al., 2021), where different aspects of user features are owned by different enterprises.

To perform different tasks, various machine learning models have been adapted into federated learning versions, such as linear model (Hardy et al., 2017; Yang et al., 2019), decision tree (Vaidya & Clifton, 2005; Vaidya et al., 2008), random forest (Liu et al., 2020b), kernel methods (Gu et al., 2020), XGBoost (Cheng et al., 2019), and neural networks (Liu et al., 2020a). However, earlier model (such as the ones from (Vaidya & Clifton, 2005) or (Vaidya et al., 2008)) usually suffers from security issues (Cheng et al., 2019), or less efficient and scalable (such as the ones from (Liu et al., 2020b) or (Cheng et al., 2019)).

As federated learning has raised lots of attention over the recent years, researchers also start to investigate building efficient libraries (He et al., 2020; Beutel et al., 2020) and systems (Bonawitz et al., 2019b; Hiessl et al., 2020). Several seminal work has also been done on designing better communication protocol (Konečný et al., 2016), optimization algorithms (Li et al., 2018), and improving model robustness (Konstantidis & Ramamoorthy, 2021). However, most of the existing research is targeting horizontal federated learning systems and vertical federated learning systems are still
waiting to be tailored for industrial production.

3 Federated Random Forest

Federated random forest is built upon classical random forest, whose final predictor is aggregated by multiple single tree predictors:

\[ F(x) = A(F_1(x_1), F_2(x_2), \ldots, F_K(x_K)) \]  

(1)

where \( K \) is the total number of trees that will be trained, and \( \{x_k\}_{k=1}^K \) are randomly sampled from the entire training set \( X \in \mathbb{R}^{n,d} \) with \( n \) samples and \( d \) dimensional features. \( F(\cdot) \) is the random forest predictor, consists of a set of decision tree predictors \( \{F_k\}_{k=1}^K \) each trained with a different sub dataset \( x_k \). The \( A \) operator is the corresponding aggregation strategy, such as mean, median etc. Generally, the result of operator \( A \) will not change if a single (or even a couple of) \( F_i(x_i) \) is missing.

What makes federated random forest different from classical random forest is that its training set \( X \) is distributed on multiple parties and the information leakage between parties will not be allowed. As discussed in Sec.2.1, we denote these data partitions as \( \{X^i\}_{i=1}^m \). Nevertheless, similar to classical random forest, \( \{F_k\}_{k=1}^K \) are still independent of each other and can be trained in parallel under federated learning settings. Such property will be explored in Sec.4.1 to build a robust service system.

In rest of this section, we introduce the training and real-time inference protocols for federated random forest. The discussion on the protocol safety is included in Appendix B.

3.1 Training protocol

The training of federated random forest also involves the training of a collection of federated decision trees, and the training loss may vary for different tasks. Taking federated regression trees for example, choosing to minimize the variance gain at each tree node will lead to the following objective:

\[ L_{reg} = \frac{n_LE[Y_L]^2 + n_RE[Y_R]^2 - (n_L + n_R)E[Y]^2}{n_L + n_R} \]  

(2)

where \( Y_L \) and \( Y_R \) are the labels contained in the left and right sub-spaces created by the tree node, and \( n_L \) and \( n_R \) are the sub-space capacities. Optimization on the training objective will be performed at each tree node level to find the best split threshold. Details on how Eq.2 is derived can be found in the Appendix A.

Similarly, we can optimize for the cross entropy loss for the classification task:

\[ L_{cls} = y\log(p) - (1 - y)\log(1 - p) \]  

(3)

where \( p \) is the predicted probability generated by the model.

The overall training protocol of the proposed algorithm can be viewed as finite automata, whose control flow is given in Alg. 1. The feature of instance space \( J \) is distributed across all participating parties. The termination criteria \( S \) is used to perform early stopping and prevent over-fitting. It could include the maximum depth of the tree, the minimum number of samples required to split an existing node, and the minimum number of samples required to be at a leaf node.

\begin{algorithm}
\caption{Main Control Flow (for active party)}
1: \textbf{Input:} Instance space \( I \); encrypted label \( \langle Y \rangle \); termination criteria \( S \); number of trees \( J \);
2: \textbf{Output:} Federated random forest \( \{T_j\}_{j=1}^J \)
3: Perform ID alignment;
4: \textbf{Send encrypted label} \( \langle Y \rangle \) \textbf{to all parties};
5: \textbf{while} True \textbf{do}
6: Call Alg. 2 with encrypted label \( \langle Y \rangle \);
7: Call Alg. 3 to retrieve encrypted data statistics \( \langle Y \rangle \) and find the best split feature and threshold for current node;
8: if \( S \) is satisfied then
9: break;
10: \textbf{else}
11: Call Alg. 4 to split the instance space into \( I_L \) and \( I_R \), create and broadcast the new node to all participating parties;
12: \textbf{end if}
13: Call Alg. 1 recursively to establish left subtree with \( I_L \) and right subtree with \( I_R \);
14: \textbf{end while}
\end{algorithm}

In the initialization stage, the encrypted label will be broadcasted to all participating parties, and ID alignment will be performed. Specifically, Diffie-Hellman algorithm (Li, 2010) will be used to perform private set intersection (Debnath et al., 2021) (PSI) and fulfill the ID alignment task. During the training process, Alg. 1 calls different subroutines located at each participating party. Such a remote calling process is done with gRPC under the proposed distributed system, which will be discussed in Sec.4.1. For better visualization, steps that involve data transfer are marked with the tilted font.

The main purpose of Alg. 2 is to obscure the raw data information by computing the statistics over a bin of data. The input includes hyper-parameter \( P \) which specifies the number of bins to use for data obfuscation, local feature \( X^i \) on the \( i \)-th party, and encrypted label \( \langle Y \rangle \). In the rest of this paper, we will use \( \langle * \rangle \) to represent encrypted variables. Such computation will be performed on all participating parties. It first loops over all features and sorts the encrypted label by each feature, then the aggregated feature mean \( Y \) will be
Algorithm 2 Statistics Computation (for $i$-th party)

1: Input: Homomorphically encrypted label $\langle Y \rangle$; local feature $X^i$; number of bins $P$.
2: Output: Homomorphically encrypted statistics $\langle Y \rangle$.
3: for $k = 0$ to $P$ do
4: Sort $X^i_k$ (the $k$-th feature component of $X^i$);
5: Compute the percentile: $V_k = \{ v_{k,p} \}_{p=1}^P$, where $v_{k,p}$ are the $p$-th percentile value of $X^i_k$;
6: for $p = 0$ to $P$ do
7: Get a bin of samples: $B = \{ i \mid v_{k,p-1} \leq x_j,k < v_{k,p}, x_j \in X^i \}$
8: Get the bin size: $n_{kp} = |B|$;
9: Compute homomorphically encrypted statistics: $\langle Y_{kp} \rangle = \frac{1}{n_{kp}} \sum_{j \in B} \langle y_j \rangle$;
10: end for
11: end for

The best split feature and threshold will be selected in Alg. 3. The first step is to transfer all homomorphically encrypted data statistics $\langle Y \rangle$ obtained from Alg. 2 to the active party. Since only (encrypted) mean (or sum) for each bin will be sent from passive party to active party, the communication cost is actually not significant compared to the entire dataset.

Algorithm 3 Find Optimal Split (for active party)

1: Input: Instance space of the current node $I$, encrypted statistics $\{ \langle Y^i \rangle \}_{i=1}^m$, Minimum information gain $\epsilon$
2: Output: Optimal splitting feature $k_{opt}$ and the splitting threshold $v_{opt}$
3: for $i = 0$ to $m$ do
4: // enumerate all parties
5: Transfer $\langle Y^i \rangle$ from $i$-th party to the active party;
6: for $k = d_0$ to $d_i$ do
7: // enumerate all features
8: for $v = 0$ to $v_k$ do
9: // enumerate all threshold values
10: Decrypt $y^i_{kv} \leftarrow Y^i_{kv}$;
11: Compute loss value $L_{kv}$, use Eq. 2 or Eq. 3;
12: Update $L_{opt}$, $k_{opt}$, $v_{opt}$ if $L_{kv}$ is lower;
13: end for
14: end for
15: end for
16: Current node will be split if $L_{opt} > \epsilon$;

Finally, Alg. 4 splits the current node based on the feature and threshold obtained. The instance space of the left and right child will be used to establish new nodes.

Algorithm 4 Update Instance Space (for all parties)

1: Input: Instance space of current node $I$; selected feature $k_{opt}$ and threshold $v_{opt}$
2: Output: $I_L$ and $I_R$
3: Split $I$ into $I_L$ and $I_R$ according to $k_{opt}$ and $v_{opt}$
4: Generate a record for the current branching, and broadcast to all participating parties;

Algorithm 5 Optimized Federated Inference (for all parties)

1: Active party send inference initialization signal to passive parties.
2: Passive parties compare their local features with the corresponding thresholds at each node, and send the result to the active party.
3: Active party ensemble all trees and perform inference.

We propose a new inference strategy to solve this problem. Instead of performing the evaluation on the fly, the passive party will evaluate all possible paths in the decision trees it has, and send the information to the active party through a single communication. Based on the information received, the active party can ensemble the decision tree to perform inference locally. There are only 2 rounds of communications throughout the process, which corresponds to around 100ms delay. This strategy can be viewed as trading communication complexity with computational complexity, which is summarized in Alg. 5. As a result, the inference could run in real-time without dropping any accuracy.
4 System Design

The overall system performance comes from not only novel algorithm design but also from the high-performance system design. There are two major design considerations: (1) high partition tolerance, which ensures the service to be reliable, (2) low latency, which reduces customer waiting time. In this section, we will explore how such a system can be established.

4.1 Distributed Service System

Similar to the classical random forest, federated random forest can easily be trained in parallel. There are two folds of parallelism: (1) all trees are independent of each other and can be built in parallel; (2) while splitting a node, the evaluation of each feature is independent of each other and can be done in parallel. As shown on the left part of Fig. 3, each tree in the forest is built in parallel by different computational units. Furthermore, while building each tree, each feature can be evaluated in parallel using different CPU cores owned by each computational unit (shown in the right part of Fig. 3). Since the computation of building each tree and computing the statistics of each feature is completely independent of each other, the entire computation is fully decoupled and can be performed very well with MapReduce schema.

Furthermore, such parallelism offers great isolation between the computation of different trees, which means only one tree will be affected if a computation unit is down. Since the performance of the forest will not be influenced much by the absence of a single tree, high partition tolerance could be achieved. Furthermore, since our system is initially designed for financial applications, high partition tolerance is required. With carefully designed system, off-line computational unit(s) should not impact the overall system performance.

4.1.1 Optimized communication

Heavy cross-party communication can be the major bottleneck in federated learning. To further improve the system efficiency, we optimize its communication process in this subsection.

Compared with plaintext, encryption significantly increases the message size, which leads to more time consumption in data transfer and eventually the overall training time. Thus, we send $\langle Y \rangle$ in the initialization stage, which only happens once. During the training process, we only need to send encrypted statistics $\langle Y \rangle$, whose size is much smaller.

When a new node is being created in the forest in Alg. 4, $I_L$ and $I_R$ need to be broadcasted to all participating parties. As the sample space and the number of tree nodes can be very large, this step could potentially become another potential bottleneck to the training process. Here we discuss several ways to efficiently encode the instance space. A naive way to serialize the index space is to directly jsonify the indices into a string, which we denote as the vanilla approach. Alternatively, we can use an array of 1 and 0 bits to denote whether a sample has been selected or not, which we denote as bit encoding. When the sampling rate is very small (close to leaf nodes), the bit string obtained from bit encoding can be super sparse. In this case, we can further encode the number of consecutive 0 bits instead based on the bit encoding result. We call this approach extreme bit encoding, which can be more efficient for an even smaller sampling rate.

Furthermore, we adopt an asynchronous communication strategy: instead of waiting for a response in Alg. 2, the system continues sending messages to sort different features. As illustrated in Fig. 5, with this strategy, Alg. 2 can run in parallel on different parties and system idle time can be minimized.
4.1.2 Optimized cryptosystem

Homomorphic encryption is an ideal tool for designing federated learning algorithms. As a special form of encryption, it enables certain computations to be done on encrypted data without been decrypted first (such as RSA (Rivest et al., 1978), Paillier (Paillier, 1999) or Gentry (Gentry et al., 2009)).

The fact that data lives in different parties (usually different geo locations) and the usage of homomorphic encryption in federated random forest brings extra time consumption in both communication and computation. Now we target at alleviating these issues to increase the system efficiency with modified cryptosystem and high performance computation library.

We adopt the Randomized Iterative Affine Cipher (RIAC)\(^1\) cryptosystem in our algorithm. The algorithm is included in the appendix for completeness.

4.1.3 High performance multi-precision computation

Computation with homomorphically encrypted ciphertext is generally composed of two operations: modular multiplication (mulmod) and modular exponentiation (powmod). These operations are applied to large integers (potentially several hundred digits). To improve the computational efficiency of mulmod and powmod, we integrate the Multiple Precision Arithmetic (GMP) Library (Granlund, 1996) into our system. GMP brings extra performance gain, which is analyzed in the Appendix.

5 Numerical Results

In this section, we investigate the speed and accuracy of our federated random forest with several public datasets and datasets collected from our online business. Ablation studies are also conducted to help better understand the performance gains.

5.1 Experimental Setup

Five datasets were used in our experiments: a9a, gisette, and epsilon from LIBSVM (Chang & Lin, 2011), and FK and HB gathered from our credit authorization business. The latter two datasets reflect real-world classification problems and include features such as user’s credit history, purchase frequency, retailer distribution, etc. These datasets are binary labeled, representing user default or not. A summary of these datasets are listed in Tab. 1. All datasets are split into three parties: one active party with label and part of the feature, and two passive parties with the rest of the features randomly partitioned.

Table 1. Dataset size (n) and feature dimension (d) of different datasets used in the experiment.

| Datasets | Feature Dimension | Dataset Size |
|----------|------------------|--------------|
| a9a      | 123              | 48,842       |
| gisette  | 5,000            | 6,000        |
| epsilon  | 2,000            | 400,000      |
| HB       | 115              | 77,000       |
| FK       | 113              | 362,000      |

The hardware we used to run the experiment has one Intel Xeon Gold 6130 CPU and 32G RAM. Without further specification, each participating party is equipped with one such machine. The network connection between participating parties has a speed of 300kB/s and a delay of 50 ms. To fully exploit the parallelizability, the number of computation units in the distributed system is set to be the same as the number of trees in random forest.

5.2 Results and Discussions

5.2.1 Model training

We first compare the training time between the proposed algorithm and SOTA SecureBoost(Cheng et al., 2019). As shown in Tab. 2, the proposed algorithm takes much less time on almost all datasets. Moreover, for larger datasets like epsilon and FK, SecureBoost is not able to converge after the excessively long waiting time (15 hours and 5 hours...
respectively). With much less training time, the proposed system enables more extensive hyper-parameter tuning and rapid model iteration, which could potentially lead to higher performance in accuracy.

Table 2. Benchmark on training time (in minutes). We use “>X” to denote the training process does not converge after X minutes.

| DATA SET | SECUREBOOST | OURS | BETTER? |
|----------|-------------|------|---------|
| A9A      | 4           | 5    |         |
| GISETTE  | >900        | 26   | ✓       |
| EPSILON  | >900        | 24   | ✓       |
| HB       | 117         | 21   | ✓       |
| FK       | >300        | 60   | ✓       |

To fully understand the benefits of different strategies applied in our work, here we perform an ablation study with the FK dataset. With all optimization techniques applied, the training time can be reduced from 35 hours to an hour. A breakdown of the performance improvement can be seen from Tab. 3.

Table 3. Training time consumption on FK dataset with different levels of optimization applied. "+O1" means adding RIAC, "+O2" means further adding MPC, "+O3" means further adding optimized communication.

| MODEL   | TRAINING TIME | SPEEDUP |
|---------|---------------|---------|
| BASELINE| 2100          | 1×      |
| +O1     | 420           | 5×      |
| +O2     | 210           | 10×     |
| +O3     | 60            | 35×     |

In the next experiment, we compare the testing accuracy with AUC as the performance metric. As shown in Tab. 4, the proposed algorithm and SecureBoost lead to very similar performance. The biggest difference is that, for SecureBoost, the AUC on the epsilon dataset is not obtained after 15 hours’ training, thus labeled as “N/A” in the table. And SecureBoost also has a lower AUC on FK dataset as well, which is due to the fact that more training time would be needed to fully converge. Due to much faster training speed, the proposed algorithm is still in favor compared with SecureBoost as fast model iteration could be achieved. A more through investigation on the individual optimization techniques is included in the Appendix D.

Furthermore, the impact that the proposed federated learning system has on model performance is also investigated. For the three public datasets, the AUC of models trained locally with the entire dataset is also listed in Tab. 4. As shown in the “centralized” column, compared to the models trained locally, our federated random forest could achieve comparable performance while being able to protect data privacy.

Table 4. Performance comparison of different models. Benchmark on epsilon dataset. Numbers are measured in testing AUC. Unconverged models were denoted with “N/A”, numbers adopted from (Lei & Ying, 2021) are denoted with *, and “-” denotes the numbers couldn’t be obtained due to privacy issues.

| DATASET | SECUREBOOST | OURS | CENTRALIZED |
|---------|-------------|------|-------------|
| A9A     | 0.89        | 0.89 | 0.90*       |
| GISETTE | 0.99        | 0.99 | 0.99*       |
| EPSILON | N/A         | 0.79 | 0.79        |
| HB      | 0.67        | 0.67 | –           |
| FK      | 0.64        | 0.66 | –           |

5.2.2 Model serving

Once the federated random forest has been trained, it needs to be served efficiently and reliably with our system. To investigate the inference speed, another experiment is performed with the proposed system, SecureBoost (Cheng et al., 2019) and unoptimized federated forest (similar to (Liu et al., 2020b)). For random forest, we set the forest to have 50 trees, and the maximum depth is set to be 10. As for SecureBoost, it has 25 trees and a depth of 6.

As shown in Tab. 5, our system leads to a large improvement in inference speed over the existing work. We observe that the optimization technique O4 reduces our system delay from 1000 ms to 120 ms, which makes real-time service possible. Besides, through batch processing of 100 queries per batch, our system composed of single physical machine for each party could achieved QPS up to 3000. Furthermore, it is worth noting that this system can be scaled up easily for a larger random forest. Due to the fact that the trees are perfectly isolated, such tasks can be fulfilled by simply adding more machines.

Table 5. Benchmark on inference speed.

| MODEL              | QPS   | LATENCY (s) |
|--------------------|-------|-------------|
| SECUREBOOST        | 0.1   | 10          |
| UNOPTIMIZED        | 1     | 1           |
| OPTIMIZED (STREAMING) | 8     | 0.12        |
| OPTIMIZED (BATCHING) | 3,000 | 0.2         |

6 CONCLUSION

An efficient and robust vertically federated random forest system is presented in this paper to preserve data privacy. For higher training and inference efficiency, several optimization techniques are utilized on the system design, cryptosystem, communication, and parallelization. Novel system design is also made to further improve the efficiency and robustness. Ablation studies are provided to show the
benefits of proposed optimization techniques, which we hope could help the community building even better practical federated learning systems in the future. After extensive optimization, the proposed implementation for federated random forest algorithm could achieve favorable performance but much higher speed than SOTA benchmark on both open benchmark datasets as well as real-world business datasets.

**References**

Beutel, D. J., Topal, T., Mathur, A., Qiu, X., Parcollet, T., de Gusmão, P. P., and Lane, N. D. Flower: A friendly federated learning research framework. *arXiv preprint arXiv:2007.14390*, 2020.

Bonawitz, K., Eichner, H., Grieskamp, W., Huba, D., Ingerman, A., Ivanov, V., Kiddon, C., Konečný, J., Mazzocchi, S., McMahan, H. B., et al. Towards federated learning at scale: System design. *arXiv preprint arXiv:1902.01046*, 2019a.

Bonawitz, K., Eichner, H., Grieskamp, W., Huba, D., Ingerman, A., Ivanov, V., Kiddon, C., Konečný, J., Mazzocchi, S., McMahan, H. B., et al. Towards federated learning at scale: System design. *arXiv preprint arXiv:1902.01046*, 2019b.

Chang, C.-C. and Lin, C.-J. LIBSVM: A library for support vector machines. *ACM Transactions on Intelligent Systems and Technology*, 2:27:1–27:27, 2011. Software available at [http://www.csie.ntu.edu.tw/~cjlin/libsvm](http://www.csie.ntu.edu.tw/~cjlin/libsvm).

Cheng, K., Fan, T., Jin, Y., Liu, Y., Chen, T., and Yang, Q. Secureboost: A lossless federated learning framework. *arXiv preprint arXiv:1901.08755*, 2019.

Dayan, I., Roth, H. R., Zhong, A., Harouni, A., Gentili, A., Abidin, A. Z., Liu, A., Costa, A. B., Wood, B. J., Tsai, C.-S., et al. Federated learning for predicting clinical outcomes in patients with covid-19. *Nature medicine*, pp. 1–9, 2021.

Debnath, S. K., Stânică, P., Kundu, N., and Choudhury, T. Secure and efficient multiparty private set intersection cardinality. *Advances in Mathematics of Communications*, 15(2):365, 2021.

Dorri, A., Kanhere, S. S., Jurdak, R., and Gauravaram, P. Blockchain for iot security and privacy: The case study of a smart home. In *2017 IEEE international conference on pervasive computing and communications workshops (PerCom workshops)*, pp. 618–623. IEEE, 2017.

Gentry, C. et al. A fully homomorphic encryption scheme, volume 20. Stanford university Stanford, 2009.


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Suppose $I$ is the instance space that contains data label $y$, $I_L$ and $I_R$ are the sub-spaces split by the tree node, $n_L$ and $n_R$ are the corresponding space capacities, $y_L$ and $y_R$ are the labels contained in the left and right sub-spaces. The regression loss for variance reduction can be formulated as:

\[
Score = \frac{\|I_L\|}{|I|} \text{Var}[y|y_L] - \frac{\|I_R\|}{|I|} \text{Var}[y|y_R]
\]

\[
= E[y^2] - E[y_L]^2 - \frac{n_L}{n_L + n_R}(E[y_L]^2 - E[y_L^2])
\]

\[
- \frac{n_R}{n_L + n_R}(E[y_R^2] - E[y_R]^2)
\]

\[
= \sum Y^2 - \frac{(\sum Y)^2}{(n_L + n_R)^2} - \frac{\sum Y^2_R}{n_L + n_R}
\]

\[
+ \frac{\sum Y_L^2}{n_L(n_L + n_R)} - \frac{\sum Y^2_R}{n_R(n_R + n_L)} + \frac{\sum Y_R^2}{n_R(n_L + n_R)}
\]

\[
= \frac{1}{N_L + N_R} \left[\frac{(\sum Y_L)^2}{N_L} + \frac{(\sum Y_R)^2}{N_R} - \frac{(\sum Y)^2}{N_L + N_R}\right]
\]

\[
= \frac{1}{n_L + n_R} [n_L E[Y_L]^2 + n_R E[Y_R]^2 - (n_L + n_R) E[Y]^2]
\]

B Protocol Safety

In this part, we will analyze if the proposed federated random forest is secure. The honest-but-curious (HBC) adversary will be considered: the participant does not deviate from the protocol but will attempt to learn all possible information from the received messages. The system safety will be analyzed from two aspects: (1) Label security: will the passive party be able to derive the label information owned by the active party, and (2) Feature security: will the active party be able to derive feature information owned by the passive party.

There are four data transfers during the training process: (1) In Alg. 1, the active party will send the encrypted label to passive party upon training starting; (2) In Alg. 3, passive parties will send $\langle Y \rangle$ (accumulated encrypted label value by percentiles) to the active party; (3) In Alg. 4, active party will send the best split feature, threshold, and the samples space of left and right subtree to all participating parties.

Label security: Since labels are encrypted before sending out by the active party, brutal force decryption can be difficult or infeasible. However, a possible threat was pictured in (Cheng et al., 2019): Consider a node owned by one passive party whose children nodes are tree leaves owned by the active party. Since the sample space of the leaf nodes is decided by the parent node (in the passive party), then the passive party knows the sample space of these two leaf nodes. Under such situations, the passive party can easily
guess the prediction result of these two leaf nodes, and the guessing accuracy is based on the purity of the leaf nodes. In the meantime, since random forest tends to over-fit on each individual tree, the risk that the prediction of a single tree gets detected is also higher. We adopted the "Completely SecureBoost" (Cheng et al., 2019) ideology to alleviate the above security concerns.

**Feature security**: The active party has the following information about the passive party: feature dimension $d$, sample space $I$, and accumulated label sum by percentile ($Y$). Generally speaking, it is not sufficient for the active party to derive the feature information on such information. However, if the active party keeps sending a small sample space to the passive party, the feature rank in the passive party could be learned by the active party. A simple strategy that can be used to protect passive party’s feature information is to randomly shuffle the left and right subtree while splitting. In this way, the active party will not be able to detect the passive party feature information even under the extreme case we constructed above.

### C CRYPTO SYSTEM

We choose RIAC as the cryptosystem in our work mainly due to its efficiency. RIAC is built on the classical affine cipher, which uses a linear transformation and a mode operation to encrypt and decrypt the plaintext:

$$E(x) = (ax) \mod n$$
$$D(x) = (a^{-1}x) \mod n$$

where $a$ and $n$ are coprime integers, $a^{-1}$ is the multiplicative inverse of $a$ under $\mod n$. Affine cipher is fast to compute; however, it has a major flaw as its ciphertext preserves the frequency property of plaintext to some extent, making it vulnerable under certain attacks. To this end, RIAC made two improvements: (1) iteratively apply multiple rounds of affine cipher encryption, (2) use auxiliary random number $r$ and $s$ to randomize the frequency of ciphertext:

$$E_{i+1}(x) = (a_{i}E_{i}(x)) \mod n_{i}$$
$$D_{i+1}(x) = (a_{i}^{-1}D_{i}(x)) \mod n_{i}$$

where $i$ is the current round of RIAC computation, and:

$$E_{0}(x) = [(y \cdot s) \mod n_{0}, (x + y \cdot h) \mod n_{0}]$$
$$h = (s \cdot r) \mod n_{0}$$
$$D_{0}(x) = (c_{2} - r \cdot c_{1}) \mod n_{0}$$

Here $c_{1}, c_{2}$ are two components of the ciphertext obtained from the last decryption round. And $y$ is an extra randomly generated large integer to further randomize the computation, so that every time the encoding of the same number will lead to different ciphertext.

**Figure 5. Benchmark on optimized communication. "Baseline", "Comp", and "Comp+Async" denotes model without any communication optimization, model with data compression along, and model with both data compression and asynchronous communication, respectively.**

### D ABLATION STUDY

A benchmark to communication optimization is done with the FK dataset, where the results are visualized in Fig. 5. Without any optimization in communication, the training would take 210 min to finish. With data compression applied, the training time is reduced to 120 min. With both compression and async communication, the time cost is further reduced to 60 min.

#### D.1 Optimized Communication

A comparison of the speed difference for the above methods is listed in Tab. 6. The result is obtained with different sampling rates on 1 million samples, and the network speed is assumed to be 300 kB/s. It can be seen that bit encoding performs the best under large and medium sampling rates, while extreme bit encoding performs the best under small sampling rates. And both methods are much faster than the vanilla approach under all settings. In our final system, we choose to transfer the data index through bit encoding when the sampling rate is larger than 15%, and through extreme bit encoding otherwise.

To understand the scalability with more participating parties, we conduct the following experiment on the Epsilon dataset: Assuming the features are evenly distributed on all parties, we increase the number of participating parties from 3 to 5 to 10 and train the system. Without the proposed O3 (optimized communication), the training time is 66.1 min, 129.4 min, 292.6 min, respectively. However, with O3, the training time becomes 23.7 min, 27.6 min, and 32.5 min, which is 2.79x, 4.69x, and 9.00x faster. This empirically shows that the proposed O3 is not only helpful at reducing the time consumption at the same number of parties but also significantly reduces the growth of time consumption to sublinear. Thus making the system more scalable in terms of participating parties.
Table 6. Comparison on the time consumption of RIAC and Paillier protocols under Python implementation. It is assumed that the dataset has 1 million samples in total.

| SAMPLE RATE & METHOD | mssg size (kB) | TRANSFER COST (s) | serialize cost (s) | SPEEDUP RATIO | BEST? |
|---------------------|---------------|------------------|-------------------|---------------|-------|
| 10%                 |               |                  |                   |               |       |
| V0                  | 788.9         | 5.136            | 5.329             | 1.0×          |       |
| V1                  | 333.3         | 2.170            | 2.340             | 2.3×          |       |
| V2                  | 112.2         | 0.732            | 0.874             | 6.1×          |       |
| 50%                 |               |                  |                   |               |       |
| V0                  | 394.4         | 25.680           | 26.034            | 1.0×          |       |
| V1                  | 333.3         | 2.170            | 2.296             | 11.3×         | √     |
| V2                  | 112.5         | 3.662            | 3.934             | 6.6×          |       |
| 90%                 |               |                  |                   |               |       |
| V0                  | 7888          | 51.360           | 52.208            | 1.0×          |       |
| V1                  | 333.3         | 2.170            | 2.272             | 23.0×         | √     |
| V2                  | 2125          | 13.834           | 14.151            | 3.7×          |       |

Table 7. Comparison on the time consumption of RIAC and Paillier protocols at performing different computations.

| PROTOCOL | PAILLIER (µS) | RIAC (µS) | SPEEDUP |
|----------|---------------|-----------|---------|
| SCALAR MULT. | 53.17       | 3.66      | 14×     |
| ADDITION  | 148.38       | 2.54      | 58×     |
| DECRYPTION | 4.76        | 0.0036    | 130×    |
| ENCRYPTION | 16570       | 53.86     | 307×    |

Table 8. GMP Benchmark on mulmod and powmod operations.

| OP      | W. GMP(µS) | W/O GMP (µS) | SPEEDUP |
|---------|------------|--------------|---------|
| MULMOD  | 1.17       | 4.67         | 4×      |
| POWMOD  | 3.95       | 60.8         | 15×     |

D.2 RIAC

As a comparison, Tab. 7 shows the speedup of RIAC over the popular Paillier cryptosystem (Paillier, 1999). The time consumption is obtained from an average of 1000 plaintext numbers randomly drawn from \(N(0, 10^8)\). As a reference, similar addition and multiplication operations in plaintext take 7.15 ns and 7.22 ns, respectively. It can be clearly seen that, although both RIAC and Paillier are much more time consuming than plaintext operations, RIAC is much faster than Paillier and can help to reduce the overall training time of federated random forest.

D.3 MPC

As an example, we compute \((c_1 \cdot c_2) \mod n\) and \((c_1^{c_2}) \mod n\) where \(c_1, c_2, n\) are big integers of 300 digits. The speed comparison of mulmod and powmod operators with and without GMP is shown in Tab. 8. Although the actual run time may vary with different integer digits and machines, it is obvious that GMP could make the cryptosystem much faster.