Abstract

Damage of PC tendons in a prestressed concrete structure need to be detected before such damage accumulates to cause a serious failure. That, however, is quite a difficult task because the PC tendons are invisible from outside and the damage location cannot be known beforehand. Phase space analysis based on vibration data is a novel method for damage detection. An earlier study by the authors demonstrated that Change of Phase Space Topology (CPST) was effective in identifying the existence of PC tendon damage. However, CPST from impact hammer test, which is a more practical method, did not show a trend as obvious as that of the free vibration test. As it is known that PC tendon damage affects several modes of vibrations and that the hammer can be used for excitation to the higher modes, it may be possible to improve the capability of impact hammer test by considering CPST separately in different frequency ranges. As in the previous study, the current study conducted experiments on PC tendon damage, but with an increased number of accelerometers for constructing mode shapes and investigated CPST further in different frequency ranges. The results still revealed that CPST was more sensitive to damage than the parameters from modal-based analysis. CPST in different frequency ranges can improve results from impact hammer test and has the capability to identify roughly the damage locations.

1. Introduction

Damage detection of PC tendon in prestressed concrete structures is a challenging problem. It is inevitable that all structures must suffer from deterioration. For a prestressed concrete bridge, concrete deterioration, fatigue from passing vehicles and overweight trucks can lead to several cracks on the bridge. Depending on the quality of grouting, water, oxygen, and other chemicals can penetrate to PC tendons, cause corrosion, and diminish the structural strength of the bridge (Duke 2002). Degradation or failures of PC tendons in the prestressed concrete bridge are invisible from outside. Therefore, the accumulation of such degradation or failures in the cross-section can lead to a sudden collapse without any warning signs (fib 2003; Sawade and Krause 2010) and cause great loss. Hence, to avoid such problems, early detection of PC tendon damage is necessary.

Several methods have been proposed to detect the PC tendon damage. The method can be classified into two categories; local inspection and global inspection. Examples of local inspection are acoustic emission technique (Yuyama et al. 2007; Ramadan et al. 2008; Shiotani et al. 2013), magnetic inspection (Scheel and Hillemeier 2003), and electromechanical impedance method (Kim et al. 2010; Jiang et al. 2017). However, these methods require prior information of damage location or possible damage location, and without such information, damage detection will be very difficult.

On the other hand, global inspection such as the vibration-based method is also widely used. It is simple, cheaper, and does not need the prior information of damage location. The basic idea is that the changes in modal parameters (frequencies, mode shapes, and modal damping) are related to the changes of structural properties (mass and stiffness). In the view of prestressed concrete structure, many research works, both analytical (Saad et al. 1994; Miyamoto et al. 2000; Hamed and Frostic 2006), and experimental (Hop 1991; Nobel et al. 2015, 2016; Wang 2017), have studied the relation between structural frequencies and magnitude of prestressing force. The results of these studies have revealed that the prestress force magnitude affected the structural frequencies. However, the prestress force in their experiments were uniformly distributed which may not represent the behavior of pre-tension tendon and bonded tendon. Previous work of the authors (Tuttipongsawat et al. 2018) experimented on pre-tension tendon damage.
and found that such damage was not much sensitive to the change of natural frequencies. This characteristic agreed with Nie et al. (2012) that structural frequencies were not sensitive enough to small damage. Moreover, by using other modal-based parameters, they also stated that mode shape changes were generally more sensitive to damage than frequencies but that they required a large number of sensors which were costly and had to process a large amount of data.

Phase space analysis based on vibration data is an alternative method. It is originally from the field of mechanical engineering (Todd et al. 2001; Nichols et al. 2003; Moniz et al. 2005; Chelidze and Liu 2006; Chelidze and Cucumano 2006). The concept is that the method transforms measured data of a time series domain into a spatial domain. A small change of one parameter will affect the entire system. In the field of civil engineering, Nie et al. (2012) first adopted phase space analysis to identify damage in strain history data. An index called Change of Phase Space Topology (CPST) was proposed. Later, they used it in an experiment to indicate damage in concrete slabs using acceleration data (Nie et al. 2013). They found that the CPST index was very effective in identifying the existence of damage because its values increase with damage level regardless of damage location. Other uses of CPST for damage detection can be found in the works of Paul et al. (2017), Nie et al. (2017) and Pamwani and Shelke (2018). For PC tendon damage, the previous work of the authors (Tuttipongsawat et al. 2018) successfully used CPST to detect the damage in PC tendons by considering vibration data. In the experiment, vibrations of the PC girder were obtained by two methods; free vibration test (by releasing a heavy mass) and impact hammer test. The CPST values increased with the increment of damage level and also were more sensitive to damage than the parameters from modal-based analysis. However, the clarity of results seems to depend on the excitation force. The large excitation force of free vibration test resulted in more obvious trend than the low excitation force of impact hammer test. However, on a large structure, obtaining vibration data by releasing a heavy mass is very difficult and inconvenient. Therefore, PC tendon damage detection using vibration data from impact hammer test needs to be improved.

As it is known that the damage can affect several modes of vibrations and that the higher modes of vibrations can be excited by an impact hammer, CPST in different frequency ranges from impact hammer test may show clearer trends and it may be possible to roughly locate the PC tendon damage. Therefore, this paper conducts experiments of PC tendon damage and uses CPST to detect the damage from acceleration data in different frequency ranges. Both free vibration test and impact hammer test are investigated. The experimental program is quite similar to the previous study, but the number of accelerometers is increased and the methods to simulate the PC tendon damage are different. In Section 2, the method of phase space reconstruction and CPST index are reviewed. In section 3, the experimental program is described. The experimental results from modal-based analysis and phase space analysis are presented in Section 4 and CPST in different frequency ranges are discussed. Finally, the conclusions are given in Section 5.

2. Damage detection method based on phase space topology

2.1 Phase space reconstruction

Phase space analysis is a novel method for damage identification. The method is very sensitive to damage because it magnifies changes in time domain by altering them into a spatial domain. Then, all possible state of a system can be represented and plotted as an axis of a multidimensional space. Such plot is called phase space topology. Because it comprises of several dimensions, any change of one parameter will affect the entire system.

Phase space reconstruction can be generated based on delay coordinate and embedding dimension (Takens 1981). From a measurement of time series \( x(i) \) for \( N \) data points (where \( i = 1,...,N \)), phase space can be reconstructed as:

\[
X(n) = [x(n), x(n+1), ..., x(n+(d-1)T)],
\]

(1)

where \( T \) is the delay time and \( d \) is the embedding dimension. Each dimension can be represented as:

\[
x(n) = [x(1), x(2), ..., x(N-(d-1)T)],
\]

\[
x(n+1) = [x(1+T), x(2+T), ..., x(N-(d-2)T)],
\]

\[
\vdots
\]

\[
x((n-(d-1))T) = [x(1+(d-1)T), x(2+(d-1)T), ..., x(N)].
\]

Then, a plot from all dimensions is called phase space topology. A successful reconstruction requires an appropriate choice of time delay and embedding dimension. The delay time \( T \) should be large enough so that each dimension does not contain redundant information. The embedding dimension \( d \) must be large enough to unfold the system. Generally, both parameters can be determined from the first minimum of Average Mutual Information (AMI) (Jiang et al. 2010) and the minimum value of False Nearest Neighbors (FNN) approach (Rhodes and Morari 1997), respectively.

2.2 Change of Phase Space Topology (CPST)

The concept of Change of Phase Space Topology (CPST) index is to measure how much difference of a damage state from a predicted damage state. The index was proposed and used to detect damage of reinforce concrete slab using acceleration data (Nie et al. 2013). In 2018, the authors used CPST index and successfully detected the damage of PC tendons of pretension concrete girder (Tuttipongsawat et al. 2018). The calculation concept is shown in Fig. 1 and summarized herein.
Suppose $X(n)$ and $Y(n)$ are phase space reconstruction of a reference/healthy state and a damage state, respectively. To determine the difference between damage state and predicted damage state, a fiducial point at time index $r$ of the damage state, $Y(r)$ is mapped on to the reference state $X(n)$. The nearest $p$ neighbors of this fiducial point on the reference state are selected base on minimum of Euclidean norm as:

$$X(p_j) : \min \left\| X(p_j) - Y(r) \right\|, \quad j = 1, \ldots, p$$ (3)

where $p$ denotes the nearest neighbors to the fiducial point and the operator $\left\| \right\|$ computes the Euclidean norm. Note that each two nearest neighbor points $X(p_a)$ and $X(p_b)$, $a \leq p, b \leq p$, should not be in the same trajectory. Then the predicted damage state at the next $s$-time step can be calculated from Eq. (4).

$$Y(r+s) = \frac{1}{p} \sum_{j=1}^{p} X(p_j + s), \quad j = 1, \ldots, p$$ (4)

Therefore, the difference between damage state $Y$ and the predicted damage state $\hat{Y}$ of the fiducial point at time index $r$ can be determined as:

$$CPST_r = \frac{1}{p} \left\| Y(r+s) - \hat{Y}(r+s) \right\|, \quad r = 1, \ldots, N_f$$ (5)

where $N_f = N - (d-1)T - s$ is total number of fiducial points. This calculation will be repeated for $N_f$ times. An average value of such difference will be CPST defined by Eq. (6).

$$CPST = \frac{1}{N_f} \sum_{r=1}^{N_f} CPST_r.$$ (6)

In general, CPST of the reference state is not always zero because it is the difference between itself and its predicted value. To eliminate such error, the index is normalized by the reference state as:

$$\Delta CPST = \frac{CPST - CPST^{ref}}{CPST^{ref}},$$ (7)

where $CPST^r$ is the CPST value of the $r^{th}$ event and $CPST^{ref}$ is the CPST value of the reference state.

3. Experimental program

3.1 Specimen and experimental setup

In this experiment, a pretension concrete girder taken from an existing PC bridge was used as a specimen, likewise that of the previous experiment (Tuttipongsawat et al. 2018). The bridge was located in Kyushu area. It was a part of supporting road used during the construction of the Kyushu Expressway since 1989 and was not used until recently. The bridge was removed in 2016 and its PC girders were used for research work. The material properties of the PC girder are shown in Table 1. It has I-shape cross-section with a depth of 425 mm and a span length of 8.5 m. Nine of seven-wire strands of nominal diameter 10.8 mm were used as the PC tendons, two being assigned to the upper part and seven to the lower part. The schematic of the PC girder is shown in Fig. 2 and Fig. 3. PC tendon damage would be located at sections A, B, and C. It was assumed that the concrete parts covering on the damaged PC tendons simultaneously deteriorated with the damage of PC tendons. Therefore, the concrete parts of those sections were removed before starting the experiment. The PC tendons of section A and C would be artificially cut. For convenient cut, concrete parts of $105 \times 65 \times 105$ mm at section A and C were removed at both sides in order to avoid geometric asymmetry. On the other hand, PC tendons of section B would be broken by a loading/unload system. However, it is necessary to make notches on the PC tendons during experiment. Therefore, the concrete parts of section B were drilled with a diameter of 33 mm. All of the removed concrete parts are shown by dashed lines in Fig. 3.

To measure vibrations and further investigate in mode shapes, nine accelerometers were attached along the PC girder at both left side and right side of cross-section as shown in Fig. 2. They are ±2g tri-axis accelerometers with the specifications shown in Table 2. Only the vertical direction was considered. Sampling rates were set at 3 kHz for free vibration test and 4 kHz for impact hammer test.
3.2 PC tendons breakage sequence

At section A and C, two out of five PC tendons at the lowest part would be artificially cut during the experiment. For section B, all PC tendons at the lowest part would be notched (partially cut) in order to allow breakage by the loading machine. The red circles in Fig. 3 show the damaged PC tendons, and their breakage sequences are shown in Fig. 4 by the blue arrows. The name of each PC tendon is defined by its section (A, B, or C) followed by the order of the tendons (1 to 5). The asterisk mark refers to a partial cut in order to make notches on the PC tendon. For example, B1* means partially cutting the first PC tendon at section B. It should be noted that the letter ‘T’ refers to ‘total’, therefore BT* means total of PC tendons at section B were partially cut. According to Fig. 4, the PC girder was subjected to four levels of loading; dead load, live load, elastic limit load, and load beyond the elastic limit. The elastic limit load was calculated from FEM analysis where live load represented 90% of the elastic limit and the dead load was 50% of live load. The PC girder was loaded and unloaded to simulate the real behavior when vehicles pass through the bridge. The damage of the PC tendons at section A and C were artificially made at both dead load and live load levels. An example of PC tendon damage in these sections can be seen in Fig. 5c. After that, the PC tendons of section B were partially cut in order to facilitate breakage. The equipment used for the partial cut and an example of the cut can be seen in Fig. 5d. The PC girder was then loaded and unloaded at the elastic limit and beyond the elastic limit until rupture.

3.3 Vibration testing

Excitation of vibrations of the PC girder at each state of damage was by its free vibration and by impact hammer. The free vibration response was obtained by hanging and releasing a mass of 50 kg at the middle span. Due to a large excitation force of the free vibration test, it was expected to give a clear decay curve for analysis and can be used as a reference. On the other hand, in spite of

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Table 2 Accelerometer specifications.

| Parameters                      | Units | Value |
|--------------------------------|-------|-------|
| Measurement range              | g     | ±2    |
| Sensitivity                    | mV/g  | 660   |
| Sensitivity variation over     | %     | ±0.1(xy), 0.02(z) |
| temperature                    | °C    | -40 to 85 |
| Operating temperature range    | °C    | -40 to 85 |
| Zero-g offset                  | V     | 1.650 |
| Bandwidth                      | Hz    | 800   |
| Noise density                  | μg√Hz| 45    |
| Module dimension               | mm    | 5 x 5 x 1.2 |

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Fig. 2 Schematic of the PC girder on a loading machine, showing the accelerometer locations and hitting locations for impact hammer.

Fig. 3 Cross-section of PC girder.

(a) Side view

(b) Top view

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(Units: mm)
the low excitation force of an impact hammer, the impact hammer test is also necessary because it is a more practical method for structural inspection. The specifications of the impact hammer used in this experiment are shown in Table 3. A soft tip made of a rubber insert in aluminum was selected. It is known that hitting at different locations results in different vibration characteristics. In order to capture most characteristics, 19 hitting points were set along the top surface of PC girder with the spacing of 300 to 400 mm as shown in Fig. 2b. Three impact hammer hits were made in the vertical direction for each hitting location. Both free vibration test and impact hammer test were conducted at all states of damage when the PC girder was unloaded to 0 kN and the distributing girder was detached from the specimen. As in Fig. 4, the free vibration test is shown by the yellow arrows and the impact hammer test is shown by the letter 'IM' in green color. The events (E1 to E15) of the vibration tests are shown in Table 4.

### 4. Results and discussion

Photographs taken during the experiment are shown in Fig. 5. Self-breaking events of the PC tendon at section B caused by loading machine are shown by the text in red color in Fig. 4. The first self-breaking of PC tendon and the first crack of the PC girder were found at 53 kN after event E7 where all PC tendons at section B were partially cut. The second self-breaking was found at one cycle later at a load of 54.8 kN. After that, for a few cycles of loading, the third self-breaking was found. Then the cyclic load level was increased to larger than the elastic limit, several self-breakings of PC tendons were found, and the PC girder was ruptured after subjected to 4 times of such cyclic load. Acceleration data measured at each damage event were used in analysis.

#### 4.1 Data processing

Acceleration data for 5 s of free vibration test and for 1.25 s of impact hammer test were used for analysis. These window sizes are large enough for analysis because not only the measured acceleration data decayed very near to zero, but also because the measurement was set at high sampling rates (3 kHz for free vibration test and 4 kHz for impact hammer test). For the data obtained from impact hammer test, it was normalized by the maximum acceleration of impact hammer in order to remove the effect of different excitation forces as:

![Fig. 4 Sequence of loading, PC tendon cutting, and vibration tests. The text in red color indicates damage events during experiment.](image-url)
\[ x_{\text{HM}} = \frac{x_{\text{girder}}}{\max(x_{\text{hammer}})} \]  

where \( x_{\text{girder}} \) is the measured acceleration data and \( x_{\text{hammer}} \) is the acceleration of hammer calculated from measured impact force divided by hammer mass. Moreover, in order to have data on the same scale, before use in phase space analysis, both acceleration data from free vibration test and impact hammer test were further standardized by Eq. (9):

\[ \hat{x} = \frac{x - \bar{x}}{\sigma}, \]

where \( \bar{x} \) and \( \sigma \) are mean and standard deviation of the measured time series data, respectively.

### 4.2 Modal-based analysis

In this part, the first three modes of natural frequencies at each damage event are investigated by Fast Fourier Transform for both free vibration test and impact hammer test. Then, due to having several sensor locations, mode shape of each damage event is also determined using Eigensystem Realization Algorithm (ERA) (Juang and Pappa 1985). However, only the first mode shape due to free vibration test is presented in this study because the system needs a large matrix size and it is time-consuming.

By using Fast Fourier Transform, examples of acceleration data in frequency domain of the initial event E1 are shown in Figs. 6 and 7 for free vibration test and impact hammer test, respectively. Note that the measured acceleration data in this experiment contains some

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**Fig. 5 Photographs taken during experiment.**
amount of electrical noise at 60 Hz. In the case of free vibration test, results show that the 1st mode of vibrations is obviously dominant while the other modes are quite difficult to observe. On the other hand, in the case of impact hammer test, several modes of vibrations can be detected where the dominant modes depend on hitting locations and sensor locations. As shown in Fig. 7, the 1st mode is more dominant when hitting at Pt.10, the 2nd mode is more dominant when hitting at Pt.5, and similarly the 3rd mode is more dominant when hitting at Pt.1. Moreover, some modes of vibration are difficult to be detected when sensor locations are located near their antinodes of vibrations such as the 2nd mode from CH02, CH04, CH05 and the 3rd mode from CH01, CH03.
CH06, CH07. According to this reason, averages of natural frequencies were calculated from all sensors except those at the antinodes of vibrations.

The first three modes of natural frequencies from both free vibration test and impact hammer test are shown in Fig. 8 with the values shown in Table 5. The 1st mode of natural frequency slightly decreases. The decrease is up to -1% at event E12 where the PC girder has already cracked, and it is the 4th self-breaking of PC tendons. The reductions of the 1st mode of frequency at event E12 to E14 are faster than that of previous events. However, the reduction is still less than -3%. Then, at event E14, the PC girder damage suddenly and the natural frequency dramatically drops by -10%. The 2nd mode of frequency has decreasing trend with some variations. However, the percentages of reduction of all damage events are less than -1%. For the 3rd mode of frequency, the data from impact hammer test is considered because their peaks in frequency domain as in Fig. 7 are clearer than those of free vibration test (Fig. 6). The 3rd mode of frequency also has a decreasing trend. The reduction seems faster than that of the 1st mode of frequency. It is up to -1% at event E7 where it is the first self-breaking of PC tendon. It continuously decreases with the maximum reduction of -9% when the PC girder is ruptured. Results from the first three modes of natural frequencies imply that, as the levels of PC tendon damage increase, the natural frequencies slightly decrease, and they suddenly drop when the PC girder ruptures (except the 2nd mode because the damage of PC girder occur at the middle span).

To construct mode shapes, Eigensystem Realization Algorithm (ERA) is adopted with the Hankel matrix size of 200 × 200 and the minimum system order of 150. The 1st mode shapes due to the data from free vibration test are shown in Figs. 9 and 10 for both 2D and 3D, where the dashed line in magenta color refers to the mode shape of event E1. Mode shapes of all events will be compared to the reference event (E1) using Modal Assurance Criterion (MAC) proposed by Pastor et al. (2012) which can be determined as:

$$MAC = \frac{\phi^H \phi_{ref}}{(\phi^H \phi)(\phi_{ref}^H \phi_{ref})},$$

where $\phi$ is mode shape of the $i^{th}$ event, $\phi_{ref}$ is mode shape of the reference event (E1), and * denotes conjugate transpose. Then, all MAC values for free vibration

| Event | Mode 1 (Hz) | Mode 2 (Hz) | Mode 3 (Hz) | % difference from E1 |
|-------|------------|------------|------------|---------------------|
|       | ERA | FFT | ERA | FFT | ERA | FFT | ERA | FFT | ERA | FFT | ERA | FFT | ERA | FFT |
| E01   | 12.93 | 12.92 | 47.06 | 101.23 | 0.00 | 0.00 | 0.00 | 0.00 |
|       | (12.99) | (17.13) | (101.42) | 0.31 | (0.00) | (0.00) | (0.00) |
| E02   | 12.90 | 12.91 | 46.95 | 99.18 | -0.29 | -0.08 | 0.23 | -2.03 |
|       | (12.95) | (17.12) | (100.84) | -0.31 | (-0.02) | (-0.47) | (-0.57) |
| E03   | 12.89 | 12.89 | 46.97 | 101.18 | -0.34 | -0.20 | 0.18 | -0.05 |
|       | (12.94) | (17.10) | (100.96) | -0.37 | (-0.06) | (-0.45) | |
| E04   | 12.91 | 12.91 | 46.94 | 100.54 | -0.16 | -0.08 | 0.24 | -0.68 |
|       | (12.99) | (17.02) | (100.61) | -0.00 | (-0.24) | (-0.79) | |
| E05   | 12.88 | 12.86 | 46.81 | 100.20 | -0.41 | -0.43 | -0.52 | -1.01 |
|       | (12.98) | (16.98) | (100.45) | -0.05 | (-0.32) | (-0.95) | |
| E06   | - | - | - | - | - | - | - | - |
| E07   | 12.89 | 12.91 | 46.86 | 98.87 | -0.33 | -0.08 | 0.42 | -2.33 |
|       | (12.87) | (17.06) | (100.40) | -0.89 | (-0.15) | (-1.00) | |
| E08   | 12.88 | 12.86 | 46.81 | 99.73 | -0.39 | -0.43 | 0.54 | -1.48 |
|       | (13.01) | (16.96) | (100.12) | 0.16 | (-0.37) | (-1.28) | |
| E09   | 12.86 | 12.86 | 46.86 | 99.61 | -0.62 | -0.43 | 0.42 | -1.60 |
|       | (12.95) | (16.98) | (99.98) | -0.31 | (-0.32) | (-1.42) | |
| E10   | 12.87 | 12.86 | 46.78 | 100.12 | -0.50 | -0.43 | 0.60 | -1.10 |
|       | (12.93) | (16.93) | (100.05) | -0.42 | (-0.43) | (-1.35) | |
| E11   | 12.84 | 12.86 | 46.72 | 100.54 | -0.75 | -0.43 | 0.71 | -0.68 |
|       | (12.86) | (16.88) | (100.12) | -0.94 | (-0.54) | (-1.28) | |
| E12   | 12.80 | 12.82 | 46.68 | 98.70 | -1.01 | -0.79 | 0.81 | -2.50 |
|       | (12.82) | (16.79) | (99.57) | -1.31 | (-0.71) | (-1.82) | |
| E13   | 12.72 | 12.73 | 46.65 | 98.76 | -1.65 | -1.50 | 0.88 | -2.44 |
|       | (12.76) | (16.82) | (98.74) | -1.72 | (-0.65) | (-2.64) | |
| E14   | 12.65 | 12.68 | 46.69 | 98.30 | -2.18 | -1.85 | 0.78 | -2.89 |
|       | (12.63) | (16.75) | (98.23) | -2.72 | (-0.80) | (-3.14) | |
| E15   | 11.60 | 11.58 | 46.81 | 92.25 | -10.34 | -10.35 | -0.54 | -8.87 |
|       | (11.66) | (16.95) | (91.94) | -10.23 | (-0.39) | (-9.34) | |

*Note: Results from the free vibration tests are shown without parentheses, while those from the impact hammer tests are shown within parentheses.
test are shown in Fig. 11. It is shown that MAC drops at event E7 where it is the first self-breaking of PC tendons and the 1st crack occurs. After that, MAC drops again when the PC girder ruptures. However, the changes in MAC values are quite less.

4.3 Phase space analysis

(1) Phase space reconstruction

For damage detection using phase space analysis, both acceleration data from free vibration test and impact hammer test are used in this part. Due to the high sampling rate and longer decay time of the data from free vibration test, there are a large number of data points (15,000 data points) which uses a large memory to calculate CPST. Therefore, acceleration data from free vibration test is resampled from 3 kHz to 1 kHz. Then, the number of data reduces to 5,000 data points. For acceleration data from impact hammer test, it is recorded at 4 kHz and decays to zero within 1.25 s. The number of data is also 5,000 data points and it is not necessary to resample.

As frequency ranges of the first three modes are interested, the acceleration data is band-pass-filtered into 3 ranges which cover the first three modes of vibrations: 10 - 15 Hz, 45 - 50 Hz, and 97 - 102 Hz. In order to reconstruct phase space topology, the appropriate delay time and embedding dimensions based on the first minimum of Average Mutual Information (AMI) and the lowest of False Nearest Neighbors (FNN) are selected as shown in Table 6. The delay time are chosen as 0.019 s, 0.005 s, and 0.003 s for the three frequency ranges, respectively, and the embedding dimension is chosen as 20 dimensions for all cases.

(2) Change of phase space topology in different frequency ranges

CPST for both vibration tests are calculated using 5 nearest neighbor points and 5 s - time step. These parameters are selected based on previous researches that the number of nearest neighbors should be between 10^{-4} and 10^{-3} of total data points and the s - time step should be in a range of 1 \leq s \leq T/2 (Torkamani et al. 2012; Nichols 2003). According to several hitting locations in impact hammer test, data from all hitting locations are calculated and demonstrated by average values. The normalized CPST of the free vibration test and impact hammer test for the three frequency ranges are shown in Figs. 12, 13 and 14.

For the first range of frequency (10 - 15 Hz), the normalized CPST of free vibration test (Fig. 12a) gradually increase from event E1 to E10 where it is the 2nd self-breaking of PC tendons. After that, the values obviously
increase. It is noticed that the normalized CPST from all channels are quite the same except those of CH08 and CH09 which are closer the antinodes of vibrations. For impact hammer test (Fig. 12b), the normalized CPST also show increasing trend and obviously increase from E11 where it is the 3rd self-breaking of PC tendons. However, the trend is not much clear as that of free vibration test. Similar to the previous work, this may be because the impact hammer has lesser excitation force.

For the second range of frequency (45 - 50 Hz), the normalized CPST of free vibration test (Fig. 13a) show unclear trends. This is because the excitation location of the free vibration test is close to the antinodes of the 2nd mode of vibrations. Then, the peaks of the 2nd mode in the frequency domain as in Fig. 6 is not clear and causes variations in phase space topology. On the other hand,

| Test                | Sampling rate (Hz) | Time (s) | Numbers of data (samples) | Delay time (samples) | Embedding dimension (samples) |
|---------------------|--------------------|----------|---------------------------|----------------------|-------------------------------|
| Free vibration     | 3000               | 1000     | 5                         | 5000                 | 19 19 10 20                   |
| Impact hammer      | 4000               |          | 1.25                      | 5000                 | 78 21 10 20                   |

Table 6 Parameters for phase space reconstruction.

Fig. 9 3D Mode shapes from each damage event due to free vibration test. The dashed lines in magenta color refer to mode shape of the reference event E1.
the normalized CPST of impact hammer test (Fig. 13b) show a better trend especially in the events of E2 to E5 where the damage locations of PC tendons are close to the point of maximum amplitude of the 2nd mode of vibrations. The normalized CPST obviously increase. After that, the normalized CPST from E6 to E15 have both increasing and decreasing trends. This is because the damage of E6 to E15 located at the middle of span which is the antinode of the 2nd mode of vibrations and the damage are difficult to be detected. Note that the dashed lines in grey color refer to the channels located at the antinode of vibrations and do not need to be considered.

For the third range of frequency (97 - 102 Hz), the normalized CPST of free vibration test (Fig. 14a) show increasing trend. The values are obvious when the PC girder is ruptured but the values caused by PC tendon damage (E2 to E14) still have high variations. On the other hand, the normalized CPST of impact hammer test (Fig. 14b) show a smooth increasing trend. The normalized CPST increase gradually until the event E11 (the 3rd self-breaking of PC tendons), and then they obviously increase. It should be noted that the trend of normalized CPST from impact hammer test is more obvious than that of the free vibration test because the impact hammer test can excite the higher modes of vibrations and has several hitting locations to excite the PC girder. Therefore, it has high possibility to obtain more information of higher modes of vibrations.

According to the three ranges of frequency, it is ob-

Fig. 10 2D Mode shapes from each damage event due to free vibration test. The dashed lines in magenta color refer to mode shape of the reference event E1.
served that the normalized CPST from free vibration test is advantage to detect damage at the 1st mode of vibrations because the PC girder is excited by the high energy while the normalized CPST from impact hammer test is advantage to detect damage at higher modes of vibrations.

4.4 Discussion on damage sensitivity

Results from modal-based analysis and phase space analysis have been compared for sensitivity to the PC tendon damage. However, before the comparison, the modal frequencies are normalized with the reference event E1 as Eq. (11).

$$\Delta f^i = \frac{f^i - f_{ref}^i}{f_{ref}^i}.$$  \hspace{1cm} (11)

![Fig. 11 Modal Assurance Criterion (MAC) for mode shape comparison.](image)

![Fig. 12 Normalized CPST of 1st frequency range.](image)

![Fig. 13 Normalized CPST of 2nd frequency range. The lines in grey color refer to the channels located near antinodes of vibrations.](image)
The normalized indices (normalized frequency, normalized CPST from impact hammer test, and normalized CPST from free vibration test) are separately plotted in Fig. 15a to show their own orders and plotted together in Fig. 15b for comparison. As the PC girder damage is gradually increased with the increase in the number of damaged PC tendons, the values of reliable index should also increase with the damage. As in Fig. 15a, all normalized indices can reflect the damage but their sensitivities to the damage are different. Similar to the previous work, the normalized CPST is more sensitive to the PC tendon damage than the parameters from modal frequencies. Moreover, by considering in different frequency ranges, the damage location can be roughly identified. During the event E2 to E5 where the damage locations are at section A and C, although the normalized CPST of the second and the third frequency ranges from impact hammer test (blue cross mark and empty blue circle) and the normalized CPST of the first frequency range from free vibration test (filled red circle) are more sensitive to damage than others, the normalized CPST of the second frequency range from impact hammer test (blue cross mark) has monotonic trend with the increasing of damage level. This imply that the normalized CPST of the second frequency range from impact hammer test is more sensitive to damage at section A and C. For the event after E6 where the damage locations are at the middle of span (section B), the normalized CPST of the third frequency range from free vibration test (filled red circle) and the normalized CPST of the third frequency range from impact hammer test (empty blue circle) are more sensitive to PC tendon damage. This is because the damage locations are at the middle of span which is the maximum amplitude of the 1st and the 3rd modes of vibrations. It should be noted that the normalized CPST of the first frequency range from impact hammer test is less sensitive than that of the third frequency range because the impact hammer does not much excite the lower mode of vibrations.

In summary, it can be concluded that the normalized CPST is an effective index to detect the PC tendon damage and by considering the index in different frequency ranges the damage locations can be roughly identified using impact hammer test.

5. Conclusions

PC tendon damage can be detected based on Change of Phase Space Topology (CPST) using acceleration data. This paper further investigated the capability of CPST index in different frequency ranges in order to use benefits and improve the results of impact hammer test. In the experiment, PC tendon damage was simulated by artificial cuts at both quarter sides of the PC girder and by self-breaks caused by a loading machine at the middle of the PC girder. Two types of vibration tests, free vibration test and impact hammer test, were performed after each damage state. Several accelerometers were attached along the PC girder in order to further investigate in mode shapes. Parameters from modal-based analysis and the change of phase space topology (CPST) in different frequency ranges were discussed and compared.

For modal-based analysis, the first three modes of natural frequencies and the first mode shapes were investigated. The first three modes of natural frequencies decreased slightly when the PC tendon damage levels increased. The 1st and 3rd modes of frequency suddenly dropped when the PC girder ruptured but the 2nd mode of frequency did not show such behaviour. For mode shapes, Modal Assurance Criterion (MAC) has been used to compare the 1st mode shape of each damage event. The MAC showed decreasing trend according to the increment of PC tendon damage level. However, the reduction was small.
For phase space analysis, CPST from both free vibration test and impact hammer test in different frequency ranges were investigated. Three frequency ranges, which covered the first three modes of natural frequencies, have been selected. The results revealed that, at the first frequency range, both CPST from free vibration test and impact hammer test showed increasing trend especially after the 3rd self-breaking of PC tendons where the damage locations were at the middle of span. The CPST from the free vibration test was more obvious than that of the impact hammer test. For the second frequency range, which covered the 2nd mode of vibrations, CPST from impact hammer test showed a clearer trend than that of free vibration test, especially when the damage locations were at the quarter of span. A monotonic increasing trend of CPST could be observed. For the third frequency range, which covered the 3rd mode of vibrations, both CPST from free vibration test and impact hammer test showed the increasing trends but the trend of CPST from impact hammer test was smoother.

Finally, the results from modal-based analysis and phase space analysis have been compared. It is obvious that CPST was more sensitive to PC tendon damage than the parameters from modal-based analysis. Moreover, by considering in different frequency ranges, CPST from impact hammer test has been improved by showing more obvious trends. The sensitivity in different frequency ranges can roughly identify the PC tendon damage location.

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