Distributed Containment Control of Fractional-Order Multi-Agent Systems With Unknown Persistent Disturbances on Multilayer Networks

JIANHENG LING, XIAOLIN YUAN, AND LIPING MO

1 Center for Information and Control, School of Automation Science and Electrical Engineering, Beihang University (BUAA), Beijing 100191, China
2 School of Mathematics and Statistics, Beijing Technology and Business University, Beijing 100048, China.

Corresponding author: Xiaolin Yuan (1171681540@qq.com)

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ABSTRACT This work concerns with the distributed containment control problem of fractional-order multi-agent systems (FOMASs) with unknown persistent disturbances on multilayer networks. To solve this problem, observers are designed to estimate the state of each follower and unknown persistent disturbances. Based on estimated information, a novel distributed containment control protocol is designed. And then, some matrix inequalities conditions are deduced for achieving the containment control. Finally, the effectiveness of the obtained theoretical results are demonstrated by several numerical examples.

INDEX TERMS Distributed containment control, fractional-order multi-agent systems, unknown persistent disturbances, multilayer networks.

I. INTRODUCTION

Recently, the coordination control of multi-agent systems (MASs) has been investigated by many researchers, and lots of excellent works have been done, such as leader-following consensus control [1], [2], leaderless consensus control [3]–[7], robust control [8], [9], distributed optimization control [10]–[13] and so on. In practical environments, there may exist multiple followers and leaders, and all followers are required to move into the convex hull spanned by multiple leaders, called containment control. The containment control problem was first proposed in [14]. And then, reference [15] investigated the distributed containment control problem of MASs with both stationary and dynamic leaders. Reference [16] studied the distributed finite-time containment control of MASs with double-integrator dynamics. The containment control problem for both continuous-time and discrete-time MASs with general linear dynamics was studied in [17]. The containment control of heterogeneous and higher-order MASs was respectively studied in [18] and [19] and so on.

Notice that the agents investigated in [14]–[19] are all with integer-order dynamics, however, when agents moving in some complex environments, the dynamics of agents cannot be well described by integer-order dynamics. For example, aircrafts with a high speed flying in rain, snow or dust storm; Underwater vehicles operating in lentic lakes which composed of microbes and viscoelastic materials [20]. Over the past decades, fractional calculus are found to have good properties in memory and hereditary, which can describe the complex phenomena better. The distributed consensus control of FOMASs was first investigated in [20]. Then to achieve the containment control of FOMASs, numerous of good works have been done [21]–[24]. In [21], the quasi-containment control of FOMASs was achieved by taking an event-triggered strategy. In [22], some necessary and sufficient conditions were established for achieving containment control of FOMASs. Moreover, the containment control of uncertain and heterogeneous FOMASs was respectively studied in [23] and [24]. Taking into account the fact that there may exist external disturbances in many practical...
engineering applications, which affect the stability of the investigated system. In [21]–[24], the influence of the external disturbances were not considered. The agreement coordination problem of FOMASs with unknown persistent external disturbances has been studied in [25]. Nevertheless, there is no result on the containment control of FOMASs with unknown persistent disturbances.

In reality, agents may moving on multilayer networks, which consist of various types of connections or multiple subnetworks. Recently, reference [26] made an investigation for the structure and dynamics of multilayer networks. Ref. [27] made an investigation for the structural reducibility of multilayer networks. The synchronization of MASs on multilayer networks was investigated in [28]. The exponential synchronization of hybrid coupled networks with delayed coupling was investigated in [29] and so on [30]. Nevertheless, it is surprise to find that the containment control problem of FOMASs on multilayer networks has been solved.

Motivated by the above discussions, based on output feedback, the distributed containment control problem of FOMASs with unknown persistent disturbances over multilayer networks is studied in this paper. Compared with [14]–[19], where the containment control problems of integer-order MASs were considered, this paper studies the containment control problem of FOMASs. Since the fractional-order calculus has memory property, there are great differences between fractional-order calculus and integer-order calculus, causing the methods proposed in [14]–[19] cannot be directly extended to solve the problem proposed herein. In [21]–[24], the convergence analysis was finished by taking the theories of frequency domain analysis, matrix analysis or bilinear transformation. In contrast, the investigated system in this paper is on multilayer networks and affected by unknown disturbances, therefore, on the one hand, observers need to be designed to make an estimation for the state of each follower, on the other hand, the additive coupling on multilayer networks need to be decoupled. Inspired by [28], the multilayer networks modeled by additive coupling is considered in this paper. It should be noted that the convergence analysis in [28] was finished based on LaSalle’s invariance principle. Different from [28], the dynamics of each agent considered in this paper are fractional-order, then the methods in [28] are unsuitable for the problem proposed herein. To overcome this difficult, the theories of Mittag-Leffler stability are utilized to analyze the convergence of the investigated closed-loop system. The brief contributions of this paper can be summarized as follows:

1) The distributed containment control problem of FOMASs was first studied on multilayer networks;
2) Via output feedback, observers are designed to make an estimation for the state of each follower and unknown persistent disturbances;
3) A novel distributed containment control protocol is designed based on the estimated information;
4) Some matrix inequalities conditions are deduced to guarantee the achievement of the containment control.

Notations: Throughout this paper, $\mathbb{R}$ denotes the sets of real numbers, $\mathbb{R}^{m \times n}$ represents the set of real matrix of dimension $m \times n$, $\mathbb{R}^m$ be the $m$-dimensional Euclidean space, $\mathbb{Z}^+$ denotes the set of positive integer numbers, $I_n$ represents the identity matrix with order $n$, $\otimes$ represents the kronecker product.

II. PRELIMINARIES

Definition 1 [32]: The Caputo’s fractional order derivative of order $\alpha \in \mathbb{R}$ for a function $f(t) \in \mathbb{R}^n[[t_0, t]], \mathbb{R}$ is defined as follows:

$$C^\alpha_{t_0} D^\alpha_t f(t) = \frac{1}{\Gamma(n - \alpha)} \int_{t_0}^t \frac{f^{(n)}(\xi)}{(t - \xi)^{\alpha + 1 - n}} d\xi,$$

where $t_0 \leq t$ and $n = [\alpha] + 1$. Particularly, when $0 < \alpha < 1$,

$$C^\alpha_{t_0} D^\alpha_t f(t) = \frac{1}{\Gamma(1 - \alpha)} \int_{t_0}^t \frac{f'(\xi)}{(t - \xi)^\alpha} d\xi.$$

Definition 2 [32]: A two-parameters Mittag-Leffler function is defined as

$$E_{\alpha, \beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(k\alpha + \beta)}.$$

where $\alpha > 0$, $\beta > 0$ and $z \in \mathbb{R}$. For $\beta = 1$, its one-parameter form is written as

$$E_{\alpha}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(k\alpha + 1)}.$$

Lemma 1 [33]: Consider $C^\alpha_{t_0} D^\alpha_t u(t) = g(t, x)$ with $\bar{x} = 0$ be an equilibrium point. Let $U(t, x(t)) : [0, \infty) \times \Omega \rightarrow \mathbb{R}$ be a continuously differentiable function and satisfy locally Lipschitz with respect to $x$ such that

$$\alpha_1 \frac{\|x\|^a}{U(t)} \leq U(t) \leq \alpha_2 \frac{\|x\|^{ab}}{U(t)} \leq -\alpha_3 \|x\|^{ab},$$

where $\alpha_1, \alpha_2, \alpha_3, a, b$ are arbitrary positive constants. Then $\bar{x} = 0$ is Mittag-Leffler stable. If the assumptions hold globally on $\mathbb{R}^n$, then $\bar{x} = 0$ is globally Mittag-Leffler stable.

Lemma 2 [34]: If $\xi(t) \in \mathbb{R}^n$ is a continuous derivable function and $Q \in \mathbb{R}^{n \times n}$ is a symmetric positive definite matrix, then

$$\frac{1}{2} \int_{t_0}^t C^{\alpha}_{t_0} D^\alpha_t \xi^T(t) Q\xi(t) \leq \xi^T(t) Q C^{\alpha}_{t_0} D^\alpha_t \xi(t),$$

where $t_0 \leq t$, $\alpha \in (0, 1)$.

Lemma 3 [35]: The Kronecker product $\otimes$ has the following properties:

1) $(kA) \otimes B = A \otimes (kB) = k(A \otimes B)$, where $k \in \mathbb{R}$,
(2) \((A + B) \otimes C = A \otimes C + B \otimes C\);
(3) \((A \otimes B) \otimes C = A \otimes (B \otimes C)\);
(4) \((A \otimes B)(C \otimes D) = (AC) \otimes BD\);
(5) \((A \otimes B)^T = A^T \otimes B^T\),

where matrices \(A, B, C, D\) with appropriate dimensions.

### III. SYSTEM MODEL AND PROBLEM STATEMENT

Consider the distributed containment control of FOMASs with \(N\) followers and \(M\) leaders over a multilayer undirected network which consist of \(K\) layers, \(K \in \mathbb{Z}^+\). Suppose that there exists additive coupling between different layers. Let \(\mathcal{V}_F = \{1, 2, \ldots, N\}\) and \(\mathcal{V}_L = \{N + 1, N + 2, \ldots, N + M\}\) respectively denotes the set of followers and leaders, with \(N \geq 1, M > 1\). Take \(\mathcal{V} = \mathcal{V}_F \cup \mathcal{V}_L\) represents the set of all nodes. Let \(G^{(s)} = (\mathcal{V}, E^{(s)}, A^{(s)})\), \(s = 1, 2, \ldots, K\) represents the topology structure of the \(s\)th layer, \(E^{(s)} \subseteq \mathcal{V} \times \mathcal{V}\) represents the set of edges on the \(s\)th layer. \(E^{(s)}_j = (i, j)\) denotes an edge of \(G^{(s)}\), where \(i \in \mathcal{V}\). \(A^{(s)} = \{a^{(s)}_{ij}\}\) represents the weighted adjacency matrix on the \(s\)th layer. \(a^{(s)}_{ij}\) is defined as the communication quantity between node \(i\) and node \(j\) that satisfies \(a^{(s)}_{ii} = 0\), \(a^{(s)}_{jj} > 0\) when \(e^{(s)}_{ij} \in E^{(s)}\), otherwise \(a^{(s)}_{ij} = 0\). Let \(N^{(s)}_i = \{j \in \mathcal{V} : (j, i) \in E^{(s)}\}\) represents the neighbors set of node \(i\) on the \(s\)th layer. The Laplacian \(L^{(s)} = [l^{(s)}_{ij}]\) of graph \(G^{(s)}\) is defined as \(l^{(s)}_{ii} = \sum_{j \neq i} a^{(s)}_{ij}\) and \(l^{(s)}_{ji} = -a^{(s)}_{ij}\) for \(i \neq j\). A sequence of edges \((i_1, i_2-1), (i_2-1, i_3-2), \ldots, (i_2, i_1)\) is called a directed path from node \(i_k\) to node \(i_1\), where \(i_k \in \mathcal{V}\).

To continue, we make the following assumptions:

**Assumption 1:** In every graph \(G^{(s)}, s = 1, 2, \ldots, K\), for each follower, there exists at least one leader that has a directed path to the follower.

**Assumption 2:** The communication between followers is undirected. There is no communication between leaders. Besides, followers can receive information from leaders, but leaders cannot receive information from followers.

Under Assumptions 1 and 2, The Laplacian matrix \(L^{(s)}\) of graph \(G^{(s)}\) on the \(s\)th layer can be partitioned as \(L^{(s)} = \begin{bmatrix} L^{(s)}_{1(N \times N)} & L^{(s)}_{2(N \times M)} \\ O_{(M \times N)} & O_{(M \times M)} \end{bmatrix}\).

**Lemma 4:** [36] Under Assumption 1, \(L^{(s)}_{1}\) is invertible and all eigenvalues of \(L^{(s)}_{1}\) have real-parts. Besides, \(-\lambda^{(s)}_1 L^{(s)}_2\) is non-negative and \(-\lambda^{(s)}_2 L^{(s)}_2\) is non-negative.

**Assumption 3:** \((A, C)\) is detectable.

**Assumption 4:** The set \([L^{(s)}_{1}, s = 1, 2, \ldots, K]\) is a family of commutable matrices in which every pair of matrices commutes.

**Lemma 5:** [28] Under Assumptions 1, 2 and 4 there exists an orthogonal matrix \(U \in \mathbb{R}^{N \times N}\) such that \(U^T L^{(s)} U = \Lambda^{(s)} = \text{diag}(\lambda^{(s)}_1, \lambda^{(s)}_2, \ldots, \lambda^{(s)}_N)\), where \(\lambda^{(s)}_i (i = 1, 2, \ldots, N)\) are the eigenvalues of \(L^{(s)}_1\) and satisfy \(0 < \lambda^{(s)}_1 \leq \lambda^{(s)}_2 \leq \ldots \leq \lambda^{(s)}_N\).

**Remark 1:** Under Assumption 3, there must exist matrices \(F \in \mathbb{R}^{N \times n}, G \in \mathbb{R}^{n \times m}\) such that \(F = A - GC\), and \(F\) is Hurwitz.

**Remark 2:** Assumption 4 is similar to the assumption that taken in [28], which is necessary for Lemma 5. However, in some situations, Assumption 4 cannot be satisfied. In other words, the theoretical results obtained in this paper only suitable for the system which satisfies Assumption 4, this is a conservativeness issue of this paper, and we will study this problem in our future work.

The dynamics of each follower are given as follows:

\[
\begin{align*}
0 &= D^\alpha_0 \xi_i(t) = L^{(s)}_2 \xi_i(t) + u_i(t) + \xi_i(t), \\
\xi_i(t) &= C \xi_i(t),
\end{align*}
\]

where \(i \in \mathcal{V}_F, 0 \leq \alpha < 1, \xi_i(t) \in \mathbb{R}^n, u_i(t) \in \mathbb{R}^n, \gamma_i(t) \in \mathbb{R}^m\) and \(\xi_i(t) \in \mathbb{R}^n\) respectively represents the state, control input, control output and unknown persistent disturbance of the \(i\)th follower, \(A \in \mathbb{R}^{n \times n}\) and \(C \in \mathbb{R}^{m \times n}\) respectively represents the system matrix and output matrix.

The dynamics of each leader are given as follows:

\[
\begin{align*}
0 &= D^\alpha_0 \hat{\xi}_i(t) = L^{(s)}_2 \hat{\xi}_i(t),
\end{align*}
\]

where \(i \in \mathcal{V}_L, 0 \leq \alpha < 1, \xi_i(t) \in \mathbb{R}^n\) represents the state of the \(i\)th leader. The states of leaders are denoted by \(\hat{\xi}_{N+1}, \hat{\xi}_{N+2}, \ldots, \hat{\xi}_{N+M}\). Let \(Q\) represents the convex hull formed by leaders, specifically, \(Q = \{ \sum_{i=N+1}^{N+M} \theta_i \hat{\xi}_i(t) \geq 0, \sum_{i=N+1}^{N+M} \theta_i = 1 \}\). The objective of this paper is to design a distributed containment control protocol to force all followers asymptotically move into the convex hull spanned by multiple leaders, that is, \(\lim_{t \to +\infty} \| \hat{\xi}_i(t) - Q \| = 0, \forall i \in \mathcal{V}_F\).

### IV. MAIN RESULTS

Based on output feedback information, the following state and disturbances observers are introduced for each follower:

\[
\begin{align*}
0 &= D^\alpha_0 \hat{\xi}_i(t) = L^{(s)}_2 \hat{\xi}_i(t) + u_i(t) + \hat{\xi}_i(t) + Gy(t), \\
&+ H_1 \sum_{j \in N_i} \mu_j a^{(s)}_{ij}(\hat{y}_j(t) - \hat{y}_j(t)),
\end{align*}
\]

\[
\begin{align*}
0 &= D^\alpha_0 \hat{\xi}_i(t) = H_2 \sum_{j \in N_i} \mu_j a^{(s)}_{ij}(\hat{y}_j(t) - \hat{y}_j(t)),
\end{align*}
\]

where \(i \in \mathcal{V}_F, \hat{y}_j(t) = C \hat{\xi}_j(t) - y_j(t), \hat{\xi}_j(t) \in \mathbb{R}^n, \hat{y}_j(t) \in \mathbb{R}^m\) respectively represent the estimated values of \(\xi_j(t)\) and \(\hat{\xi}_j(t)\), \(\hat{\xi}_j(t) \in \mathbb{R}^m\) is the observed output. In view of Remark 1, \(F \in \mathbb{R}^{n \times n}, G \in \mathbb{R}^{n \times m}\), such that \(F = A - GC\), and \(F\) is Hurwitz. \(H_1 \in \mathbb{R}^{n \times m}, H_2 \in \mathbb{R}^{n \times m}\) are feedback gain matrices that will be determined in the following analysis. \(\mu_s\) is the coupling strength on the \(s\)th layer.

Furthermore, the following distributed containment control protocol is proposed for each follower:

\[
\begin{align*}
u_i(t) &= \frac{h}{\sum_{j \in N_i} \mu_j a^{(s)}_{ij}} \Delta_s(\hat{\xi}_j(t) - \hat{\xi}_j(t)) - \hat{\xi}_j(t),
\end{align*}
\]
where \( i \in \mathcal{V}_F, h \in \mathbb{R} \) is feedback gain that will be determined in the following analysis, \( \Delta_i = \text{diag}(\delta_{i1}^h, \delta_{i2}^h, \ldots, \delta_{in}^h) \in \mathbb{R}^{n \times n} \) is positive semi-definite inner coupling matrix on the \( h \)th layer.

Let \( e_{\xi}(t) = \xi(t) - \xi_i(t), e_{\zeta}(t) = \zeta(t) - \zeta_i \), \( i \in \mathcal{V}_F \), define \( \xi(t) = (\xi_1^T(t), \xi_2^T(t), \ldots, \xi_n^T(t))^T \), \( \zeta(t) = (\zeta_1^T(t), \zeta_2^T(t), \ldots, \zeta_n^T(t))^T \), \( \hat{\xi}_i(t) = (\hat{\xi}_1^T(t), \hat{\xi}_2^T(t), \ldots, \hat{\xi}_n^T(t))^T \), \( \hat{\zeta}(t) = (\hat{\zeta}_1^T(t), \hat{\zeta}_2^T(t), \ldots, \hat{\zeta}_n^T(t))^T \), \( e_{\xi}(t) = (e_{\xi_{i1}}(t), e_{\xi_{i2}}(t), \ldots, e_{\xi_{in}}(t))^T \), \( e_{\zeta}(t) = (e_{\zeta_{i1}}(t), e_{\zeta_{i2}}(t), \ldots, e_{\zeta_{in}}(t))^T \).

Then we have
\[
C_0 D^\alpha_t \hat{\xi}_i(t) = (I_M \otimes A) \hat{\xi}_i(t).
\]

Besides, we have
\[
C_0 D^\alpha_t \hat{\zeta}(t) = A \hat{\xi}(t) - \hat{\zeta}(t) + \zeta_i
\]
\[
+ h \sum_{s = 1}^n \sum_{j \in N_i} \mu_{s, i j}(\Delta_s \xi_j(t) - \hat{\xi}_j(t))
\]
\[
= A \hat{\xi}(t) + h \sum_{s = 1}^n \sum_{j \in N_i} \mu_{s, i j}(\Delta_s (\xi_i(t) - \hat{\xi}_i(t))
\]
\[
+ h \sum_{s = 1}^n \sum_{j \in N_i} \mu_{s, i j} \Delta_s (e_{\xi}(t) - e_{\xi}(t) - e_{\zeta}(t),
\]
\[
C_0 D^\alpha_t e_{\xi}(t) = \sum_{s = 1}^n \sum_{j \in N_i} \mu_{s, i j} \Delta_s (e_{\xi}(t) - e_{\xi}(t) - e_{\zeta}(t)
\]
\[
F \hat{\xi}(t) - A \hat{\xi}(t) + \zeta_i + G y_i(t)
\]
\[
+ h \sum_{s = 1}^n \sum_{j \in N_i} \mu_{s, i j} \Delta_s (\xi_j(t) - \hat{\xi}_j(t))
\]
\[
= h \sum_{s = 1}^n \sum_{j \in N_i} \mu_{s, i j} \Delta_s (e_{\xi}(t) - e_{\xi}(t))
\]
\[
+ Fe_{\xi}(t) + e_{\zeta}(t),
\]
\[
C_0 D^\alpha_t e_{\xi}(t) = H_2 \sum_{s = 1}^n \sum_{j \in N_i} \mu_{s, i j} \Delta_s (e_{\xi}(t) - e_{\xi}(t)).
\]

Describe the above equations in compact form as
\[
C_0 D^\alpha_t \hat{\xi}_i(t) = (I_N \otimes A) \hat{\xi}_i(t) - (I_N \otimes I_n) e_{\zeta}(t)
\]
\[
+ \sum_{s = 1}^K h \mu_s (L_1^{(s)} \otimes \Delta_s) \hat{\xi}_i(t) + ((L_1^{(s)})^{-1} L_2^{(s)}) \otimes I_n \hat{\xi}_i(t)
\]
\[
+ \sum_{s = 1}^K h \mu_s (L_1^{(s)} \otimes \Delta_s) e_{\xi}(t),
\]
\[
C_0 D^\alpha_t e_{\xi}(t) = (I_N \otimes F) e_{\xi}(t) + (I_N \otimes I_n) e_{\zeta}(t)
\]
\[
+ \sum_{s = 1}^K \mu_s (L_1^{(s)} \otimes H_1 C) e_{\xi}(t),
\]
\[
C_0 D^\alpha_t e_{\zeta}(t) = \sum_{s = 1}^K \mu_s (L_1^{(s)} \otimes H_2 C) e_{\xi}(t).
\]

Let \( \bar{\zeta}_f^{(s)}(t) \) = \( \xi_i(t) + ((L_1^{(s)})^{-1} L_2^{(s)}) \otimes I_n \bar{\xi}_i(t), s = 1, 2, \ldots, K \). Under Assumption 2, we can get from Lemma 4 that \( -((L_1^{(s)})^{-1} L_2^{(s)} \otimes I_n) \bar{\xi}_i(t) \) is within the convex hull spanned by the multiple leaders on the \( h \)th layer. Hereafter, the containment control problem can be solved if \( \bar{\zeta}_f^{(s)}(t) \) converge to zero asymptotically.

Therefore, we could get
\[
C_0 D^\alpha_t \bar{\zeta}_f^{(s)}(t)
\]
\[
= (I_N \otimes A) \bar{\zeta}_f^{(s)}(t) + ((L_1^{(s)})^{-1} L_2^{(s)} \otimes I_n) \bar{\xi}_i(t)
\]
\[
- (I_N \otimes I_n) e_{\zeta}(t) + \sum_{s = 1}^K h \mu_s (L_1^{(s)} \otimes \Delta_s) \bar{\zeta}_f^{(s)}(t)
\]
\[
+ \sum_{s = 1}^K h \mu_s (L_1^{(s)} \otimes \Delta_s) e_{\xi}(t)
\]
\[
+ (I_N \otimes A) \bar{\zeta}_f^{(s)}(t) + \sum_{s = 1}^K h \mu_s (L_1^{(s)} \otimes \Delta_s) \bar{\zeta}_f^{(s)}(t)
\]
\[
+ \sum_{s = 1}^K h \mu_s (L_1^{(s)} \otimes \Delta_s) e_{\xi}(t) - (I_N \otimes I_n) e_{\zeta}(t),
\]
\[
\text{where } \kappa = 1, 2, \ldots, K. \text{ According to Lemma 5, when Assumptions 1, 2 and 4 are all satisfied, there exists an orthogonal matrix } U \in \mathbb{R}^{n \times n} \text{ such that } U^T L_1^{(s)} U = \Lambda^{(s)} := \text{diag} (\lambda_1^{(s)}, \lambda_2^{(s)}, \ldots, \lambda_n^{(s)}).
\]

Let
\[
\begin{align*}
\bar{\zeta}_f^{(s)}(t) &= (U^T \otimes I_n) \bar{\xi}_f^{(s)}(t), \\
\kappa &= 1, 2, \ldots, K, \\
\bar{\xi}_f^{(s)}(t) &= (U^T \otimes I_n) e_{\xi}(t), \\
\bar{\zeta}_f^{(s)}(t) &= (U^T \otimes I_n) e_{\zeta}(t).
\end{align*}
\]

Then the following equations could be obtained:
\[
C_0 D^\alpha_t \bar{\zeta}_f^{(s)}(t) = (I_N \otimes A) \bar{\zeta}_f^{(s)}(t) + \sum_{s = 1}^K h \mu_s (\Lambda^{(s)} \otimes \Delta_s) \bar{\xi}_f(t)
\]
\[
+ \sum_{s = 1}^K h \mu_s (\Lambda^{(s)} \otimes \Delta_s) \bar{\xi}_f^{(s)}(t) - (I_N \otimes I_n) \bar{\xi}_f^{(s)}(t),
\]
\[
\text{where } \kappa = 1, 2, \ldots, K.
\]

\[
C_0 D^\alpha_t \bar{\xi}_f^{(s)}(t) = (I_N \otimes F) \bar{\xi}_f^{(s)}(t) + (I_N \otimes I_n) \bar{\xi}_f^{(s)}(t)
\]
\[
+ \sum_{s = 1}^K \mu_s (\Lambda^{(s)} \otimes H_1 C) \bar{\xi}_f(t),
\]
\[
C_0 D^\alpha_t \bar{\xi}_f^{(s)}(t) = \sum_{s = 1}^K \mu_s (\Lambda^{(s)} \otimes H_2 C) \bar{\xi}_f(t).
\]

**Theorem 1:** Under Assumptions 1, 2, 3 and 4, by taking the state observer (3), disturbances observer (4) and the distributed containment control protocol (5), the distributed containment control of FOMASs (1) and (2) could be achieved, if there exist symmetric positive-definite matrices.
\[ P_1, P_2, \ldots, P_K \in \mathbb{R}^{n \times n}, Q \in \mathbb{R}^{n \times n}, R \in \mathbb{R}^{n \times n}, \] 
matrices \[ H_1 \in \mathbb{R}^{n \times m}, H_2 \in \mathbb{R}^{n \times m} \] and constant \[ h \in \mathbb{R} \] such that

\[
\Omega^{(i)} = \begin{pmatrix}
\Omega_{11}^{(i)} & \Omega_{12}^{(i)} & \cdots & \Omega_{1(K+2)}^{(i)} \\
\Omega_{21}^{(i)} & \Omega_{22}^{(i)} & \cdots & \Omega_{2(K+2)}^{(i)} \\
\vdots & \vdots & \ddots & \vdots \\
\Omega_{(K+1)1}^{(i)} & \Omega_{(K+1)2}^{(i)} & \cdots & \Omega_{(K+1)(K+2)}^{(i)}
\end{pmatrix} < 0, \quad (7)
\]

where \( i = 1, 2, \ldots, N, \)

\[ \Omega_{11}^{(i)} = P_1 + A^T P_1 + h \mu_1 \lambda_1^{(1)}(P_1 \Delta_1 + \Delta_1^T P_1), \]
\[ \Omega_{12}^{(i)} = h \mu_1 (\lambda_1^{(2)} P_1 \Delta_2 + h \mu_2 (\lambda_1^{(1)} \Delta_1^2 P_2), \]
\[ \vdots \]
\[ \Omega_{1K}^{(i)} = h \mu_1 \lambda_1^{(K)} (P_1 \Delta_K + \Delta_1^2 P_K), \]
\[ \Omega_{21}^{(i)} = \sum_{s=1}^{K} h \mu_s (\lambda_1^{(s)} P_1 \Delta_s), \]
\[ \Omega_{22}^{(i)} = -P_2, \]
\[ \vdots \]
\[ \Omega_{KK}^{(i)} = P_K A + A^T P_K + h \mu_K \lambda_1^{(K)} (P_K \Delta_K + \Delta_1^2 P_K), \]
\[ \Omega_{K(K+1)}^{(i)} = \sum_{s=1}^{K} h \mu_s (\lambda_1^{(s)} P_K \Delta_s), \]
\[ \Omega_{K(K+2)}^{(i)} = -P_K, \]
\[ \Omega_{(K+1)(K+1)}^{(i)} = QF + F^T Q + \sum_{s=1}^{K} \mu_s \lambda_1^{(s)} (QH_1 C + C^T H_1^T Q), \]
\[ \Omega_{(K+1)(K+2)}^{(i)} = Q, \]
\[ \Omega_{(K+2)(K+2)}^{(i)} = \sum_{s=1}^{K} \mu_s \lambda_1^{(s)} (RH_2 C + C^T H_2^T R). \]

**Proof:** Construct the following Lyapunov function

\[
V(t) = \sum_{s=1}^{K} \tilde{\xi}_s^{(i)}(t) (I_N \otimes P_s) \tilde{\xi}_s^{(i)}(t) + \varepsilon_1^T(t)(I_N \otimes Q) \tilde{\varepsilon}_1(t) + \varepsilon_2^T(t)(I_N \otimes R) \tilde{\varepsilon}_2(t). \quad (8)
\]

According to Lemma 2, taking the \( \alpha \)-order Caputo’s fractional derivative of (8),

\[
\frac{C_0^\alpha D_t^\alpha V(t)}{t^\alpha} \leq \sum_{s=1}^{K} 2 \tilde{\xi}_s^{(i)}(t) (I_N \otimes P_s) \frac{C_0^\alpha D_t^\alpha \tilde{\xi}_s^{(i)}(t)}{t^\alpha} + 2 \varepsilon_1^T(t)(I_N \otimes Q) \frac{C_0^\alpha D_t^\alpha \tilde{\varepsilon}_1(t)}{t^\alpha} + 2 \varepsilon_2^T(t)(I_N \otimes R) \frac{C_0^\alpha D_t^\alpha \tilde{\varepsilon}_2(t)}{t^\alpha}.
\]

Let \( \Xi(t) = (\tilde{\xi}_1^{(i)}(t), \tilde{\xi}_2^{(i)}(t), \ldots, \tilde{\xi}_K^{(i)}(t), \varepsilon_1^T(t), \varepsilon_2^T(t))^T, \) then we could obtain that

\[
\frac{C_0^\alpha D_t^\alpha V(t)}{t^\alpha} \leq \Xi^T(t) \Omega \Xi(t), \quad (9)
\]

where

\[
\Omega = \begin{pmatrix}
\Omega_{11} & \Omega_{12} & \cdots & \Omega_{1(K+2)} \\
\Omega_{21} & \Omega_{22} & \cdots & \Omega_{2(K+2)} \\
\vdots & \vdots & \ddots & \vdots \\
\Omega_{(K+1)1} & \Omega_{(K+1)2} & \cdots & \Omega_{(K+1)(K+2)}
\end{pmatrix},
\]

\[
\Omega_{11} = I_N \otimes (P_1 A + A^T P_1) + h \mu_1 (\Lambda^{(1)} \otimes (P_1 \Delta_1 + \Delta_1^2 P_1)), \]
\[ \Omega_{12} = h \mu_1 (\Lambda^{(2)} \otimes P_1 \Delta_2 + h \mu_2 (\Lambda^{(1)} \otimes \Delta_1^2 P_2), \]
\[ \vdots \]
\[ \Omega_{K1} = h \mu_1 \lambda_1^{(K)} (P_1 \Delta_K + \Delta_1^2 P_K), \]
\[ \Omega_{K(K+1)} = \sum_{s=1}^{K} h \mu_s (\Lambda^{(s)} \otimes P_1 \Delta_s), \]
\[ \Omega_{K(K+2)} = -I_N \otimes P_1, \]
\[ \Omega_{21} = I_N \otimes (P_2 A + A^T P_2) + h \mu_2 (\Lambda^{(2)} \otimes (P_2 \Delta_2 + \Delta_1^2 P_2)), \]
\[ \vdots \]
\[ \Omega_{2K} = h \mu_2 (\Lambda^{(K)} \otimes P_2 \Delta_K + h \mu_K (\Lambda^{(1)} \otimes \Delta_1^2 P_K), \]
\[ \Omega_{2(K+1)} = \sum_{s=1}^{K} h \mu_s (\Lambda^{(s)} \otimes P_2 \Delta_s), \]
\[ \Omega_{2(K+2)} = -I_N \otimes P_2. \]
The conditions in Theorem 1 are satisfied, the containment to \(\lim_{t \to \infty} \lambda(t) = 0\), the th follower, and take \(\Omega \preceq \lambda(t) < 0\). Therefore,

\[
C_0 D_t^\alpha V(t) = \Xi^T(t) \Omega \Xi(t) \leq \lambda_{\max}(\Omega) \Xi^T(t) \Xi(t). \tag{10}
\]

Noted that \(V(t) = \Xi^T(t) \delta \Xi(t)\), where \(\delta = \text{diag}[I_N \otimes P, I_N \otimes Q, I_N \otimes R].\) Let \(\lambda_{\max}(\delta)\) represents the maximum eigenvalue of \(\delta\), then we have \(\lambda_{\max}(\delta) > 0\) and \(V(t) \leq \lambda_{\max}(\delta) \Xi^T(t) \Xi(t)\). Furthermore, together with (10), we have

\[
C_0 D_t^\alpha V(t) \leq \frac{\lambda_{\max}(\Omega)}{\lambda_{\max}(\delta)} V(t). \tag{11}
\]

According to Lemma 1, \(\lim_{t \to \infty} V(t) = 0\), which is equivalent to \(\lim_{t \to \infty} \xi(t) = 0\). \(\xi(t) = 0\), \(\cdots, \xi^{(K)}(t) = 0\) at \(t \to \infty\), \(\xi(t) = 0\), \(\lim_{t \to \infty} \xi(t) = 0\). By the inverse transformation of (6), we have

\[
\lim_{t \to \infty} \xi(t) = 0, \quad \lim_{t \to \infty} e(t) = 0, \quad \lim_{t \to \infty} e(t) = 0.
\]

Therefore, the distributed containment control of the FOMASs (1) and (2) is achieved. The proof of Theorem 1 is completed.

Remark 3: The conditions in Theorem 1 are sufficient conditions not necessary conditions. In other words, when the conditions in Theorem 1 are satisfied, the containment control of the investigated closed-loop system must could be achieved, but the opposite is not necessarily true.

When \(A = O_{n \times n}\), the dynamics of each follower are given as follows:

\[
C_0 D_t^\alpha \xi_i(t) = u_i(t) + \xi_i(t),
\]

where \(i \in \mathcal{V}_F\), \(y_i(t) \in \mathbb{R}^n\) represents the control output of the ith follower, and take \(C = I_n\).
where $\kappa = 1, 2, \ldots, K$,

$$\begin{align*}
C_0^D\tilde{e}_\xi(t) &= (I_N \otimes I_n)\tilde{e}_\xi(t) + \sum_{s=1}^{K} \mu_s (A_s^{(s)} \otimes H_3)\tilde{e}_\xi(t), \\
C_0^D\tilde{e}_{\xi}(t) &= \sum_{s=1}^{K} \mu_s (A_s^{(s)} \otimes H_4)\tilde{e}_\xi(t).
\end{align*}$$

**Corollary 1:** Under Assumptions 1, 2, 3 and 4, by taking the state observer (14), disturbances observer (15) and the distributed containment control protocol (5), the distributed containment control of FOMAs (12) and (13) could be achieved, if there exist symmetric positive-definite matrices $P_1, P_2, \ldots, P_K \in \mathbb{R}^{n \times n}$, $Q \in \mathbb{R}^{n \times n}$, $R \in \mathbb{R}^{n \times n}$, matrices $H_3 \in \mathbb{R}^{n \times n}$, $H_4 \in \mathbb{R}^{n \times n}$ and constant $h \in \mathbb{R}$ such that

$$\hat{\Omega}^{(i)} = \begin{pmatrix}
\hat{\Omega}_{11}^{(i)} & \hat{\Omega}_{12}^{(i)} & \cdots & \hat{\Omega}_{1(K+2)}^{(i)} \\
\hat{\Omega}_{21}^{(i)} & \hat{\Omega}_{22}^{(i)} & \cdots & \hat{\Omega}_{2(K+2)}^{(i)} \\
\vdots & \vdots & \ddots & \vdots \\
\hat{\Omega}_{K1}^{(i)} & \hat{\Omega}_{K2}^{(i)} & \cdots & \hat{\Omega}_{K(K+2)}^{(i)}
\end{pmatrix} < 0, \quad (16)
$$

where $i = 1, 2, \ldots, N$.

$$\begin{align*}
\hat{\Omega}_{11}^{(i)} &= \mu_1 \lambda_1^{(i)}(P_1 \Delta_1 + \Delta_1^TP_1), \\
\hat{\Omega}_{12}^{(i)} &= \mu_1 (\lambda_2^{(i)}P_1 \Delta_2 + \mu_2 \lambda_2^{(i)} \Delta_1^TP_2), \\
\vdots & \vdots \\
\hat{\Omega}_{1K}^{(i)} &= \mu_1 \lambda_1^{(i)}(P_K \Delta_K + \Delta_1^TP_K), \\
\hat{\Omega}_{1(K+1)}^{(i)} &= \sum_{s=1}^{K} \mu_s \lambda_s^{(i)}(P_1 \Delta_3), \\
\hat{\Omega}_{1(K+2)}^{(i)} &= -P_1, \\
\hat{\Omega}_{21}^{(i)} &= -\mu_2 \lambda_2^{(i)}(P_2 \Delta_2 + \Delta_1^TP_2), \\
\vdots & \vdots \\
\hat{\Omega}_{2K}^{(i)} &= \mu_2 \lambda_2^{(i)}(P_K \Delta_2 + \Delta_1^TP_K), \\
\hat{\Omega}_{2(K+1)}^{(i)} &= \sum_{s=1}^{K} \mu_s \lambda_s^{(i)}(P_2 \Delta_3), \\
\hat{\Omega}_{2(K+2)}^{(i)} &= -P_2, \\
\hat{\Omega}_{K1}^{(i)} &= \mu_K \lambda_K^{(i)}(P_K \Delta_K + \Delta_1^TP_K), \\
\hat{\Omega}_{K(K+1)}^{(i)} &= \sum_{s=1}^{K} \mu_s \lambda_s^{(i)}(P_K \Delta_3), \\
\hat{\Omega}_{K(K+2)}^{(i)} &= -P_K, \\
\hat{\Omega}_{(K+1)1}^{(i)} &= \sum_{s=1}^{K} \mu_s \lambda_s^{(i)}(QH_3 + H_3^TQ), \\
\hat{\Omega}_{(K+1)(K+1)}^{(i)} &= Q, \\
\hat{\Omega}_{(K+2)1}^{(i)} &= \sum_{s=1}^{K} \mu_s \lambda_s^{(i)}(RH_4 + H_4^TR), \\
\hat{\Omega}_{(K+2)(K+2)}^{(i)} &= \sum_{s=1}^{K} \mu_s \lambda_s^{(i)}(QH_3 + H_3^TQ),
\end{align*}$$

**V. SIMULATIONS**

In reality, lots of vehicles may move in an extreme environment with dangerous obstacles, but only several vehicles have the ability to detect the hazardous obstacles. Those vehicles who could detect the hazardous obstacles are normally designed as “leaders” while the rest are designated as “followers”. Since a convex hull could be spanned by multiple leaders and all leaders could detect the hazardous obstacles, then the convex hull could be seen as a safety area. All vehicles could arrive at the destination safety, if all followers always stay within the safety area [15]. In this section, several numerical examples are given to show the effectiveness of the obtained theoretical results.

**A. EXAMPLE 1**

Consider the containment control of FOMAs (1) and (2) over the multilayer networks that shown in Fig. 1, where node 1, 2, 3, 4 represent the follower, node 5, 6, 7 represent the leader, i.e., $V_F = \{1, 2, 3, 4\}$, $V_L = \{5, 6, 7\}$. It is obvious that Assumptions 1, 2, 4 are all satisfied. Take $K = 2$, the coupling strength are chosen as $\mu_1 = 1, \mu_2 = 0.7$, and the inner coupling matrices are shown as $A_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 \end{pmatrix}, G = \begin{pmatrix} 2.5 \end{pmatrix}, h = -2, H_1 = \begin{pmatrix} 1 & 0 \\ -2 & 0 \end{pmatrix}, H_2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The initial values are chosen as $\xi_1(0) = (4.5, 4)^T, \xi_2(0) = (-28, 5)^T, \xi_3(0) = (2.5, -3)^T, \xi_4(0) = (-4.5, 2.5)^T, \xi_5(0) = (3, 5)^T$. $\xi_6(0) = (-4.5, -3)^T, \xi_7(0) = (1, -2)^T, \xi_8(0) = (1, 2, 0.2, 0.2)^T, \xi_9(0) = (0.4, 0.4)^T, \xi_{10} = (0.6, 0.6)^T, \xi_{11} = (0.8, 0.8)^T, \xi_{12} = (0, 0)^T, \xi_{13} = (0, 0)^T, \xi_{14} = (0, 0)^T, \xi_{15} = (0, 0)^T$. The communication subnetworks are shown in Fig. 1.
FIGURE 2. Position trajectories of all agents.

FIGURE 3. Trajectories of all agents (the first component).

FIGURE 4. Trajectories of all agents (the second component).

FIGURE 5. Positions of all agents at some specific time.

FIGURE 6. Trajectories of $\hat{\xi}_i(t) - \xi_0(t)$ and $\hat{\xi}_i(t) - \xi_1(t)$, $i = 1, 2, 3, 4$.

Suppose that the convex hull spanned by leader 5, 6, 7 is represented by black dashed line. To begin with, not all followers are located in the convex hull. Afterwards, it is obvious that a convex hull which contains all followers must could be find. Trajectories of $\hat{\xi}_i(t) - \xi_0(t)$ and $\hat{\xi}_i(t) - \xi_1(t)$, i.e., $e_\xi(t)$, $i = 1, 2, 3, 4$ are given in Fig. 6, from which we could see that $e_\xi(t)$ converge to zero asymptotically. Therefore, the observers and distributed containment control protocol are all effectiveness.

B. EXAMPLE 2
Consider the containment control of FOMASs (12) and (13) over the multilayer networks that shown in Fig. 1. Based on Example 1, the positions of the stationary leaders are chosen as $\xi_5 = (1, 2)^T$, $\xi_6 = (-3, -1)^T$, $\xi_7 = (3, -2)^T$. Take $H_3 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, $H_4 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$. 
By taking the state observer (14), disturbances observer (15) and the distributed control protocol (5), the position trajectories of all agents are shown in Fig. 7 and Fig. 8, from which we could see that follower 1-4 converge to the convex hull spanned by node 5-7, i.e., leader 1-3. For more intuitive, the position trajectories of all agents in phase plane are given in Fig. 9. Suppose that the red dashed line in Fig. 9 represents the convex hull that spanned by the multiple leaders, it is obvious that all followers converge to the convex hull asymptotically. The positions of all followers and leaders at time 0.01s, 3s, 6s and 9s are depicted in Fig. 10. Suppose that the black dashed line represents the convex hull spanned by node 5-7, i.e., leader 1-3. From Fig. 10 we could see that all followers are in the convex hull when time is later. Furthermore, trajectories of $\hat{\xi}_i(t) - \xi_i(t)$ and $\hat{\xi}_2(t) - \xi_2(t)$, i.e., $e_\xi(t)$, $i = 1, 2, 3, 4$ are shown in Fig. 11, from which we could see that $e_\xi(t)$ converge to zero asymptotically. Therefore, the observers and distributed containment control protocol are all effectiveness.

C. EXAMPLE 3

Base on Example 2, take $\zeta_1 = (2, 2)^T$, $\zeta_2 = (4, 4)^T$, $\zeta_3 = (6, 6)^T$, $\zeta_4 = (8, 8)^T$. The position trajectories of all agents in phase plane are shown in Fig. 12, from which we could see that all followers converge to the convex hull spanned by the multiple stationary leaders.
To further show the effectiveness of the proposed methods, a comparison is made with the following traditional distributed containment control protocol [22]–[24]:

\[
    u_i(t) = \sum_{s=1}^{K} \sum_{j \in N_i} d_{ij}^{(s)} \Delta_s (\xi_i(t) - \xi_j(t)).
\]  

(17)
Based on Example 2, consider the containment control of FOMASs (12) and (13) with the protocol (17). The position trajectories of all agents in phase plane are shown in Fig. 13, from which we could see that all followers converge to the convex hull spanned by multiple stationary leaders. Therefore, it could conclude that the protocol (17) is robust. However, when take \( \zeta_1 = (2, 2)^T \), \( \zeta_2 = (4, 4)^T \), \( \zeta_3 = (6, 6)^T \), \( \zeta_4 = (8, 8)^T \), by taking the protocol (17), the containment control of the investigated system cannot be achieved, as shown in Fig. 14. Compared with the results that given in Example 3, we could conclude that the methods proposed in this paper are more effective.

### VI. CONCLUSION

This paper investigated the distributed containment control problem of FOMASs with unknown persistent disturbances on multilayer networks. Via output feedback, observers were proposed to get the estimation information of each follower and unknown persistent disturbances. A novel distributed containment control protocol was proposed based on the estimated information. And then, some matrix inequalities conditions were deduced to guarantee the achievement of the desired objective. Finally, simulations results were given to show the effectiveness of the obtained theoretical results. In reality networks, some agents might not receive its neighbors’ information instantaneously due to the existence of time-delays. Therefore, we will focus on the distributed containment control of FOMASs with time-delays over multilayer networks in our future work.

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JIANHENG LING received the B.S. degree in control engineering from Central South University, China, in 2009, and the M.S. degree in control theory and control engineering from Beihang University, China, in 2012, where he is currently pursuing the Ph.D. degree in control theory and control engineering with the Center for Information and Control, School of Automation Science and Electrical Engineering, Seventh Research Division. His current research interests include dynamics, and adaptive control and consensus control of multimechanical systems.

XIAOLIN YUAN received the B.S. degree from the School of Science, Beijing Technology and Business University, China, in 2017, where she is currently pursuing the M.S. degree with the School of Mathematics and Statistics. Her current research interests include stochastic systems, coordination control of multiagent systems, and distributed optimization.

LIPO MO was born in Hebei, China, in 1980. He received the B.S. degree in mathematics and applied mathematics from Shihezi University, in 2003, and the Ph.D. degree in control theory from the School of Mathematics and Systems Science, Beihang University, in 2010. He is currently a professor with Beijing Technology and Business University. His research interests include stochastic systems, coordination control of multiagent systems, and distributed optimization.

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