Electric field strength sensor of cylindrical form

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Abstract. An electric-induction cylindrical electric field strength sensor is considered. The aim of this work is to investigate the interaction of a cylindrical sensor with an inhomogeneous electric field of a linear charge in order to determine the parameters of a sensor that affect its error in an inhomogeneous field. Optimization of the set parameters allows creating sensors with known guaranteed metrological characteristics and having an additional error from the inhomogeneity of the field no more than 3% in the spatial range from 0 to 3R to the source of the field. As a result of the study, the formula for estimating the error of a cylindrical sensor caused by the inhomogeneity of the electric field both from its angular and linear dimensions and from the spatial measurement range was obtained for the first time. The range of application of such high-precision sensors of electric fields is wide. They can be used both in production processes and in various areas of society.

1. Introduction

Various conditions for the use of electric field strength sensors [1] make it necessary to constantly improve the designs of both the sensor body and the sensors. The most common physical phenomenon underlying the construction of electric field strength sensors is the phenomenon of electrical induction. The essence of this phenomenon lies in the fact that a redistribution of electric charges occurs in a conducting body placed in an electric field. As a result of this displacement, the intensity of the electric field inside the body and the tangential component of the tension on its surface will vanish. Observance of these two conditions allows us to assume that the potential of the body at all its points will be the same and equal to the potential of an external electric field at some point of the conducting body. A point of a body whose potential is equal to the potential of an external electric field is called a reference-point. The reference-point coincides with the center of the electric charges of the body, the position of which in the body can be determined, according to expression

\[ r = \sqrt{\frac{\sum_{i=1}^{n} q_i r_i^2}{\sum_{i=1}^{n} q_i}}, \]

where \( r \) is distance from center of symmetry to center of electric charges of body; \( r_i \) is distance from charge \( q_i \) to center of electric charges of body.

For symmetric conducting bodies (a sphere, a cube, a cylinder) located in a homogeneous electric field, the reference point coincides with the center of symmetry of the body, and does not coincide in the inhomogeneous field. This feature must be taken into account when constructing and using sensors to measure the strength of the electric field.

It should be noted that the sphere, cube and cylinder are the most common forms of conducting bodies for construction of electric field strength sensors [2]. The effect of non-uniformity of electric fields in test for stability of the measurement result is considered in [3] using various types of sensors, for
example, isotropic and single-coordinate. In [4-7], sensors of the electric field intensity of the spherical shape are examined and analyzed, and also the sensors of a cubic and planar shape, there are practically no work on the sensors of a cylindrical shape. Therefore, the authors of this paper took the trouble to consider and analyze the features of the construction and behavior of the electric field intensity sensor of cylindrical shape in fields of different non-uniformity.

2. Task set

The complex mechanism of the action of electric fields on biological objects has not yet been sufficiently studied. This complexity requires improvement effects metrological characteristics as the electric field sensors, and means of processing signals.

The main contribution to the total error of the sensors and the means of processing their signals is the error of the sensor. The sensor error can be reduced by taking into account the interacting factors of the sensor with the electric field when developing and optimizing the structural elements of the sensor.

The existing need to improve the constructive and, as a result of the metrological characteristics of the sensors, obliges to create all new electric field strength sensors. One of these new sensors is an electric induction cylindrical electric field strength sensor. The sensor will be presented with independence requirements (within the limits of the minimum possible error) of its output signals from the inhomogeneity of the electric field in a wide spatial range of measurements.

3. Theory

The theory of constructing the electric field strength sensor is based on the analysis of the interaction of a conducting cylinder of height $h$ and radius $R$ placed in electric fields of different non-uniform with the intensity $E_0 = E \cdot \sin \omega t$. Further in the text, simply $E_0$.

As fields of different non-uniform, we choose boundary fields - a uniform electric field and a field of a linear charge with a strong non-uniform. A uniform field acts as an exemplary, reference field. In relation to it, the error of the sensor operating in an non-uniform field of linear charge will be estimated. We assume that the error of the sensor in other non-uniform fields is less than in the field of a linear charge. By a linear charge we mean an infinitely long uniformly charged thread. The linear charge field is chosen from the condition of the greatest non-uniform, which can be modeled when analyzing the behavior of the sensor in an non-uniform field.
3.1. Description of a single-coordinate cylindrical sensor

The sensor is based on the dielectric cylinder 1 with radius \( R \) and height \( h \). On the external surface of the sensor on one coordinate axis diametrically opposite and two conducting sensors 2 and 3 of semi-cylindrical shape with an angular size \( \theta_0 \) and a concave part to cylinder axis are diametrically opposed and isolated from each other. Sensitive elements 2 and 3 are a thin conductive layer with a thickness of the order of \( 10 \div 100 \mu m \), deposited by nanotechnology methods on the surface of a dielectric cylinder, and their radius and length coincide with the dimensions of the dielectric cylinder, as shown in FIG. 1.

If the radius \( R \) of the dielectric cylinder 1 is much larger than the thickness of the sensors 2 and 3, then we can assume that the radius of curvature of the sensing elements is also equal to \( R \). Making gaps between the sensor elements \( 2b << R \), we can assume that the sensor elements 2 and 3 have equal potentials (special measures will be taken for this), and the sensor is a single conducting cylindrical surface.

3.2. Single-coordinate cylindrical sensor in uniform field

In considering this question, we use FIG. 2.

The solution of this problem reduces to finding the surface density of the induced charge on the conducting cylinder through the potential of an arbitrary point \( A \) lying outside the cylinder

\[
\varphi = -E_0 \cdot \rho \cdot \left(1 - \frac{R^2}{\rho^2}\right) \cdot \cos \theta, \tag{1}
\]

vector of the intensity of the resultant field through the potential gradient

\[
\vec{E} = -\text{grad} \varphi = -\left(\vec{e}_\rho \cdot \frac{\partial \varphi}{\partial \rho} + \vec{e}_\theta \cdot \frac{1}{\rho} \frac{\partial \varphi}{\partial \theta} + \vec{e}_z \cdot \frac{\partial \varphi}{\partial z}\right) \tag{2}
\]

and the components of the intensity vector of the resultant electric field:

\[
E_\rho = E_0 \cdot \left(1 + \frac{R^2}{\rho^2}\right) \cdot \cos \theta, \quad E_\theta = -E_0 \left(1 - \frac{R^2}{\rho^2}\right) \cdot \sin \theta, \quad E_z = 0. \tag{3}
\]
where in expressions (1), and (3) \( E_0 \) - is intensity of external uniform field; \( R \) is radius of the cylinder; \( \rho \) and \( \alpha \) are the polar coordinates lying in the plane of cylinder cross-section, and the value of the longitudinal axial coordinate \( z \) is indifferent. Then it follows from expressions (3) that, on the surface of the conductive cylinder, \( \rho=R \) the tangential components of the voltage vector of the electric field are reversed to zero, and the radial component \( E_\rho \) determines the value of the surface density of the induced electric charge:

\[
\sigma = \sigma(\alpha) = 2\varepsilon\varepsilon_0 E_0 \cdot \cos \theta ,
\]

(4)

where \( \varepsilon \) is the dielectric constant of the medium surrounding the conductive plate; \( \varepsilon_0 \) is the dielectric constant.

Analysis of the expression (4) shows that the surface density of the charges on the conductive cylinder in the homogeneous field is, is constant but varies according to cosine law depending on polar angle \( \alpha \). Therefore, in this case, the charge is non-uniformly distributed over the surface of the cylinder. For example, at \( \theta=0 \) and \( \theta=\pi \), on the generators of the cylinder, there arise maximal but opposite charges of charge density \( \sigma=\pm 2\varepsilon\varepsilon_0 E_0 \), and at \( \theta=\pi/2 \) and \( \theta=3\pi/2 \) -zero densities. Thus, the lower half of the cylinder (FIG. 2) becomes a charged positive charge and the upper half is negative. The plane defined by the divided cylinder into two oppositely charged parts may be referred to as the plane of the electric neutral. In the homogeneous field this plane coincides with the plane of geometric symmetry, i.e. the plane dividing the cylinder into two equal parts.

Consider the line of the above-described single-coordinate sensor in a uniform field.

Let in some volume \( V \) space filled with dielectric with relative permittivity \( \varepsilon \) (in particular air) there is a time-varying uniform \( E_0 \) with the intensity \( E_0 \) generated by the external sources. In order to measure \( E_0 \), cylindrical sensor is introduced into it. It is necessary to establish a mathematical relationship between the electric charges induced on the sensitive electrodes of the Sensor And the intensity of \( E_0 \). All geometric relationships, dimensions of electrodes forming the field of the source, and location of the sensor in this system of electrodes are considered to be known.

Let's select one of the sensitive elements, for example 2 on the \( x \)-axis, to be considered on the surface of the sensor and determine the induced electric charge (FIG. 1).

It follows from expression 3 that when the conductive insulated cylinder is introduced into the \( E_0 \) on the surface thereof, there will be only the normal component of the field strength \( E_\rho \), the parameters of the field, the size of the cylinder and the parameters of the cylinder material are determined. The electric charge that has acquired the insulated conductive cylinder is defined by [8-10]

\[
Q = \int \int_S \sigma \cdot dS ,
\]

(5)

where \( \sigma \) is the surface charge density defined by (4);

\[
dS = R \cdot d\theta \cdot dz
\]

(6)

is element of cylindrical surface expressed in polar coordinate system; \( R \) is radius of cylindrical electrode; \( \alpha \) is angle of polar coordinate system; \( dz \) - the element of the \( z \)-axis coinciding with the axis of symmetry of the cylinder and varying from \( 0 \) to \( h \); \( h \) - the height of the cylinder.

The area of the cylindrical sensitive electrode is determined from expression (6), after substituting the corresponding integration limits (FIG. 1)

\[
S = \int_{\theta_0}^{\theta_b} R \cdot d\theta \cdot dz = 2\theta_0 Rh .
\]

(7)

Taking into account the expressions (4), (5) and (6), we find the charges induced by a homogeneous electric field on the surfaces of cylindrical sensitive electrodes 2 and 3 (FIG. 1) oriented along the direction of the field.
As can be seen from the expressions (8) and (9) the charges induced on the conductive surface of the sensitive electrodes 2 and 3 are proportional to the electric field strength. Therefore, they can act as a measure of field strength. In this case, the sensor sensitivity $G$ of the electrodes 2 and 3 will be equal to

$$G = \frac{dQ}{dE_0} = \pm 4\varepsilon\varepsilon_0 R \cdot h \cdot \sin \theta_0,$$

where "+" corresponds to sensor element 2, and "-" corresponds to sensor sensitive element 3.

It presence of two sensitive electrodes oppositely located on one coordinate axis allows us to speak of a dual sensor. At its differential inclusion, the total charge $Q_{\text{diff}}^0$ of the sensor, and, consequently, its sensitivity $G_{\text{diff}}^0$ is doubled

$$Q_{\text{diff}}^0 = (Q_2^0) - (-Q_3^0) = 8\varepsilon\varepsilon_0 R \cdot h \cdot \sin \theta_0.$$

$$G_{\text{diff}}^0 = \frac{dQ}{dE_0} = 8\varepsilon\varepsilon_0 R \cdot h \cdot \sin \theta_0.$$

It follows from expressions (10) and (12) that the sensitivity of the sensor depends on the geometric dimensions of the cylindrical sensor body, namely, the radius $R$ and height $h$, as well as angular $\alpha_0$ and linear $h$ dimensions of sensitive electrodes. At the same time, the sensitivity of the sensor does not depend on the distance to the source of the field, since it is believed that the field source is at infinity. If the geometric dimensions of the cylindrical body and sensor sensitive electrodes do not change during the measurement, the sensor sensitivity remains constant. This condition is satisfied when the sensor is placed in a uniform electric field.

3.3. Single-coordinate cylindrical sensor in non-uniform field of linear charge

We place the cylindrical sensor in the electric field of the linear charge. As a linear charge, we will consider a uniformly charged rectilinear filament with an electric charge density per unit length $\tau$ (Figure 3).
We shall find the charges induced by the electric field of a linear charge on the surfaces of cylindrical sensitive electrodes 2 and 3 oriented along the direction of the field. To do this, we use expressions (5) and (6), as well as an expression for the electric charge density on a conducting cylindrical surface located near a linear charge

\[
\sigma = \sigma(\theta) = \varepsilon \varepsilon_0 E_\rho (a, \theta) = -2 \varepsilon \varepsilon_0 \frac{1}{2a} \left[ 1 - \frac{(1 - a^2)}{(1 - 2a \cos \theta + a^2)} \right] E_0,
\]

where \( E_0 = \tau / 2 \pi \varepsilon_0 d \) is the voltage of the initial EF produced by the uniformly charged rectilinear thread with the surface density of the charge \( \tau \) in the point with the coordinates \( \rho = 0, \theta = 0, z = 0 \) in the absence of conductive cylinder; \( a = R/d \) is the relative distance from the cylinder center to the Power Source EF (characterizes the degree of field non-uniformity). By taking the tables of integrals [11] and considering this, the electric charges on the sensing electrodes 2 and 3 (see FIG.1) are defined by expressions

\[
Q_2^H = \int_{-\theta_0}^{\theta_0} \int_{0}^{h} \sigma(\theta) \cdot R \cdot d\theta \cdot dz = 4 \varepsilon \varepsilon_0 R \cdot h \cdot \frac{1}{2a} \left[ \theta_0 - 2 \arctan \left( \frac{1 + a}{1 - a} \tan \frac{\theta_0}{2} \right) \right] E_0;
\]

\[
Q_3^H = \int_{\theta_0}^{\theta_0} \int_{0}^{h} \sigma(\theta) \cdot R \cdot d\theta \cdot dz = -4 \varepsilon \varepsilon_0 R \cdot h \cdot \frac{1}{2a} \left[ \theta_0 - 2 \arctan \left( \frac{1 - a}{1 + a} \tan \frac{\theta_0}{2} \right) \right] E_0.
\]

It follows from expressions (14) and (15) that the charges induced by an inhomogeneous field on the conducting surface of the sensitive electrodes 2 and 3 are proportional to the strength of the initial electric field. Therefore, as in the uniform field, they may be a measure of the strength \( E_0 \). However, in an inhomogeneous field of sensitivity \( G \) of the sensor with respect to electrodes 2 and 3 will be different, and determined, according to expressions

![Cylindrical sensor in linear charge field](image-url)
\[ G_2 = \frac{dQ}{dE_0} = 4\varepsilon_0 R \cdot h \cdot \left\{ \frac{1}{2a} \left[ \theta_0 - 2 \arctan \left( \frac{1+a}{1-a} \tan \frac{\theta_0}{2} \right) \right] \right\}, \quad (16) \]

\[ G_3 = \frac{dQ}{dE_0} = -4\varepsilon_0 R \cdot h \cdot \left\{ -\frac{1}{2a} \left[ \theta_0 - 2 \arctan \left( \frac{1-a}{1+a} \tan \frac{\theta_0}{2} \right) \right] \right\}. \quad (17) \]

At differential connection of the sensor, the total charge \( Q'^{\text{diff}} \) from the sensor electrodes and its sensitivity \( G'^{\text{diff}} \) will be correspondingly equal

\[ Q'^{\text{diff.}} = (Q'^{\text{2''}}_2) - (Q'^{\text{3''}}_3) = 8\varepsilon_0 R \cdot h \cdot \left\{ \frac{1}{2a} \left[ \arctan \left( \frac{1+a}{1-a} \tan \frac{\theta_0}{2} \right) - \arctan \left( \frac{1-a}{1+a} \tan \frac{\theta_0}{2} \right) \right] \right\} E_0 \quad (18) \]

\[ G'^{\text{diff.}} = \frac{dQ}{dE_0} = 8\varepsilon_0 R \cdot h \cdot \left\{ \frac{1}{2a} \left[ \arctan \left( \frac{1+a}{1-a} \tan \frac{\theta_0}{2} \right) - \arctan \left( \frac{1-a}{1+a} \tan \frac{\theta_0}{2} \right) \right] \right\}. \quad (19) \]

Figure 4. Dependence of field non-uniformity error depending on relative distance \( a \) to linear charge at \( \alpha_0 = \pi/2 \)

It follows from the expressions (16), (17) and (19) that the sensitivity of the sensor of a linear charge placed in an non-uniform field is determined not only by the geometric dimensions of the cylindrical body and the sensor's sensitive electrodes, but also by the relative distance \( a = R/d \) to the source of the field (\( R \) - radius of the cylindrical sensor housing, \( d \) is distance from axis of symmetry of sensor to linear charge). Consequently, the sensitivity of the sensor in the non-uniform field will not remain constant, but depends on the distance to the field source. The presence of this dependence leads to an additional error of the sensor from the inhomogeneity of the electric field. This error is estimated. For this, the expressions (11) and (18) and the well-known formula for the relative error:
\[ \delta(a) = \frac{Q_{\text{diff.}}^H - Q_{\text{diff.}}^O}{Q_{\text{diff.}}^O} \times 100 = \left[ \frac{\arctan\left(\frac{1 + a \tan \frac{\theta_0}{2}}{1 - a \tan \frac{\theta_0}{2}}\right) - \arctan\left(\frac{1 + a \tan \frac{\theta_0}{2}}{1 - a \tan \frac{\theta_0}{2}}\right)}{2a \cdot \sin \theta_0} \right] \times 100 \] (20)

In the expression for the error, there is a parameter \( a = \frac{R}{d} \) characterizing the proximity of the sensor to the source of the field. Using the mathematical editor MathCAD 14, we plot the error of the sensor from the inhomogeneity of the field as a function of the parameter \( a \). The plot of this error is shown in Fig. 4.

4. Test results
The investigations made it possible to establish the relationship between the parameters of the cylindrical electric field strength sensor and the spatial measurement range from the field non-uniformity. This relationship is reflected in the form of the first obtained expression for the error of the sensor caused by the non-uniformity of the field from the angular dimensions \( \theta_0 \) of the sensor's sensitive electrodes and the spatial measurement range \( a \). Analysis of this error shows (FIG 4) that sensor with angular dimensions of sensitive electrodes \( \theta_0 = \frac{\pi}{2} \) in the entire spatial measurement range has a negative error and even at \( a > 0.3 \) this error exceeds 3%.

5. Conclusions
The mathematical dependence (20) of the error of a cylindrical sensor caused by the non-uniformity of the electric field on the angular and linear dimensions, both the sensor and the spatial measurement range \( a = \frac{R}{d} \), limiting the range of the use of the sensor, was obtained for the first time. From the analysis of the study of the graphical dependence of the indicated error (FIG. 4) it follows that the above-mentioned voltage sensor has a negative error from field non-uniformity to -3% in the spatial range from 0 to 3R from the field source, where \( R \) is radius of cylindrical body of sensor. The sensor gives underestimated values of charges in an non-uniformity field, this can lead to a non-objective evaluation of the effect of the electric field on technical and biological objects.

The next stage of the study of the sensor in this question will be related to the solution of the problem of optimizing the dimensions of its sensitive elements in order to minimize the error from the non-uniformity of the electric field.

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