Abstract

We investigate a model with two real scalar fields that minimally generates exponentially different scales in an analog of the Coleman-Weinberg mechanism. The classical scale invariance—the absence of dimensionful parameters in the tree-level action, required in such a scale generation—can naturally be understood as a special case of the multipoint criticality principle. This two-scalar model can couple to the Standard Model Higgs field to realize a maximum multiplicity of criticality for field values around the electroweak scale, providing a generalization of the classical scale invariance to a wider class of criticality. As a bonus, one of the two scalars can be identified as Higgs-portal dark matter. We find that this model can be consistent with the constraints from dark matter relic abundance, its direct detection experiments, and the latest LHC data, while keeping the perturbativity up to the Planck scale. We then present successful benchmark points satisfying all these constraints: The mass of dark matter is a few TeV, and its scattering cross section with nuclei is of the order of $10^{-9}$ pb, reachable in near future experiments. The mass of extra Higgs boson $H$ is smaller than or of the order of 100 GeV, and the cross section of $e^+e^- \rightarrow ZH$ can be of fb level for collision energy 250 GeV, targeted at future lepton colliders.
1 Introduction

The observed Higgs mass is consistent with the assumption that the Standard Model (SM) is not much altered up to the Planck scale. Indeed the critical value of the top-quark pole mass is about \( m_t^{\text{pole}} \approx 171.4 \) GeV for the theoretical border between stability and instability (or metastability) of the effective Higgs potential around the Planck scale [1], which is consistent at the 1.4 \( \sigma \) level with the latest combination of the experimental results \( m_t^{\text{pole}} = 172.4 \pm 0.7 \) GeV [2].

The tremendous success of the standard cosmology requires at least three scales in the SM Lagrangian: the cosmological constant, electroweak, and Planck scales of the order of \( 10^{-12} \) GeV, \( 10^2 \) GeV, and \( 10^{18} \) GeV, respectively. The amount of fine tuning between the bare coupling at the Planck scale and the radiative corrections is roughly of order \( 10^{120} \) and \( 10^{32} \) for the cosmological constant and the Higgs-mass squared, respectively. In this paper, we study the phenomenology of a model that addresses the latter hierarchy.

The Coleman-Weinberg (CW) mechanism naturally generates an exponentially small scalar mass \( m \) from an ultraviolet cutoff \( \Lambda \):

\[
m \sim \Lambda e^{-\lambda/g^2},
\]

where \( \lambda \) and \( g \) are the quartic scalar coupling and the gauge coupling, respectively [3]. The CW mechanism implicitly assumes that the mass-squared parameter, or more precisely the second derivative of the effective potential at the zero field value, is accidentally (or fine-tuned to be) zero.

This assumption, called the classical scale invariance (CSI) [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16], may be justified as a generalization of the multiple-point criticality principle (MPP) [21, 25] because the vanishing point of the second derivative of the effective potential is critical in the sense that the origin of the potential becomes locally stable and unstable for its positive and negative values, respectively [34]. Although the CW mechanism within the particle content of the SM cannot explain the observed Higgs mass, it can be accommodated by adding an extra sector to the latter model [4, 5, 6, 7, 9, 10, 11, 13]. In these models, a new scalar field in the extra sector develops the vacuum expectation value (VEV) by the CW mechanism, which triggers the electroweak symmetry breaking through a coupling between the Higgs field and the new scalar.

Recently, a minimal model with dark matter (DM) implementing the CW mechanism has been proposed in Ref. [34], where only two real scalar fields are added to the SM. Using the generalized MPP, critical points in the model parameter space other than the CSI have also been explored [34]. In this model, the phenomenology of DM corresponds to the Higgs-portal scenario [39, 40, 41, 42], where the DM can interact with SM particles only via Higgs bosons.

It has been known that in the minimal Higgs portal scenario with real singlet scalar DM, a sufficiently large quartic coupling is required in order not to have too much abundance, yet a too large coupling tends to be excluded by the direct detection experiment and also would break the perturbativity of the theory up to the string/Planck scale. Such a dilemma can be relaxed to some extent in our model because an additional neutral Higgs boson can also contribute to the annihilation process.

In this paper, we clarify that the DM candidate is compatible with the observed relic abundance.
abundance under constraints from direct detection experiments and LHC data as well as the perturbativity up to the string/Planck scale. We also discuss the collider phenomenology in several successful benchmark points allowed by all these above constraints. In particular, we focus on the direct search for the additional Higgs boson at future electron-positron colliders.

The organization of the paper is as follows. In Sec. 2, we review the two-scalar dimensional transmutation. In Sec. 3, we show detailed study on the DM phenomenology. In Sec. 4, we discuss the collider phenomenology of the model. Summary and discussion are given in Sec. 5. In Appendix A, we list the renormalization group equations that we use.

2 Minimal dimensional transmutation

In this section, we briefly review the minimal dimensional transmutation model based on the MPP that naturally realizes the analog of the CW mechanism \cite{34}. The model is composed of additional two real scalar fields $\phi$ and $S$ that are singlet under the SM gauge symmetry. These new fields $\phi$ and $S$ play the role of the scalar and gauge fields in the original CW mechanism, respectively. That is, a loop of $S$ induces an effective potential of $\phi$ to generate its VEV $\langle \phi \rangle$. Throughout this paper, we impose a $Z_2$ symmetry $(\phi, S) \rightarrow (\phi, -S)$ which is assumed to be unbroken, i.e., $\langle S \rangle = 0$, so that $S$ can be a candidate of DM.

The MPP in short is “the higher the multiplicity of critical points of effective potential is, it is more likely to be realized.” The $Z_2$-symmetric point in the theory space is a simple choice among various criticalities. Depending on patterns of criticality, we consider both the cases where another $Z'_2$ symmetry $(\phi, S) \rightarrow (-\phi, +S)$ exists and does not exist in the action.

In the following, we first show how the class of models discussed in this paper fits in the broader context of the MPP. Then in the subsequent subsections, we will separately discuss the cases with and without the $Z'_2$ symmetry, and classify critical points (CPs) for each case.

The CSI, with all the dimensionful parameters being zero, realizes a certain multipoint criticality: A mass-squared parameter such as $m_\phi^2$ gives a boundary in the parameter space at $m_\phi^2 = 0$ between the local stability and instability at the origin of the field space $\phi = 0$; similarly, a vanishing cubic coupling, e.g. $\mu_{\phi S}$ of the $\phi S^2$ term, gives a border (in the parameter space) for stability and meta-stability at $S = 0$ (in $S$-field space) if we switch on a non-zero $\phi$, hence realizing a multipoint criticality at $\mu_{\phi S} = 0$. The same argument holds for $\mu_{\phi H} \phi^3 H^\dagger H$, where $H$ is the SM Higgs doublet field.

The CSI scalar potential invariant under the $Z_2 \times Z'_2$ symmetry consists of the following terms at tree level:

$$V^\text{tree}_0 = \frac{\lambda_{H}}{2} (H^\dagger H)^2 - \frac{\lambda_{\phi H}}{2} \phi^2 H^\dagger H + \frac{\lambda_{S H}}{2} S^2 H^\dagger H + \frac{\lambda_{\phi S}}{4!} \phi^4 + \frac{\lambda_{\phi S}}{4!} \phi^2 S^2 + \frac{\lambda_{S}}{4!} S^4. \quad (1)$$

In general, when we do not assume CSI nor $Z'_2$ symmetry, the tree-level potential can have the following additional terms:

$$V^\text{tree} = V^\text{tree}_0 + \frac{m_\phi^2}{2} \phi^2 + \frac{m_S^2}{2} S^2 + m_H^2 H^\dagger H + \frac{\mu_{\phi}}{3!} \phi^3 + \frac{\mu_{\phi S}}{2} \phi S^2 + \mu_{\phi H} \phi H^\dagger H, \quad (2)$$

where we have removed the linear term of $\phi$ by the field re-definition of its constant shift, without loss of generality. The $\mu_{\phi}, \mu_{\phi S},$ and $\mu_{\phi H}$ terms softly break the $Z'_2$ symmetry. Notice here that we cannot write down hard breaking terms of the $Z'_2$ symmetry at renormalizable level due to the unbroken $Z_2$ symmetry. Most generally, one should examine each multipoint
criticality including these six dimensionful parameters, which will be an interesting research work in itself. Here instead, we examine possible multipoint criticality by turning on either the $m_\phi^2$ or $\mu_\phi$ term separately, for cases without the CSI.

2.1 Case with exact $Z'_2$ symmetry

We first consider the $Z'_2$ invariant Lagrangian under two classes of criticality: the CSI (CP 1-1) and the degeneracy (CP 1-2). The shape of the scalar potential is shown in Fig. 1 for each critical point.

2.1.1 CSI

The Lagrangian is given by

$$L_0 = L_{SM} + \frac{1}{2} \partial_\mu S \partial^\mu S + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V_0,$$

where $L_{SM}$ is the SM Lagrangian without the Higgs potential, and $V_0$ is the one-loop effective potential:

$$V_0 = V_0^{tree} + \frac{\lambda_\phi^2 \phi^4}{256 \pi^2} \left[ \ln \frac{\lambda_\phi \phi^2}{2 \mu^2} - \frac{1}{2} \right] + \frac{\lambda_{\phi S} \phi^4}{256 \pi^2} \left[ \ln \frac{\lambda_{\phi S} \phi^2}{2 \mu^2} - \frac{1}{2} \right].$$

In the above expression, we have included the relevant one-loop corrections to the effective potential of $\phi$, and assumed $\lambda_{\phi H} \ll \lambda_{\phi S}$ and $\lambda_{SH} \ll \lambda_{\phi S}$ such that we neglect the $H$ loop contribution to the $\phi^4$ term as well as the field dependent masses of $\phi$ and $S$ coming from $H$. We choose a renormalization scale $\mu_*$ at which the running quartic coupling of $\phi$

\footnote{For such a purpose, it would be more convenient to remove the $\phi H^\dagger H$ term rather than the linear $\phi$ term by the field redefinition of the constant shift of $\phi$.}

\footnote{In the following analysis, it is enough to consider only the tree level terms for $S$.}

\footnote{Here we adopt the renormalization scheme in Ref. [31]. If wanted, one may trivially switch to the $\overline{MS}$ scheme whose scale $\mu$ is related to the current choice by $\mu = \mu/\sqrt{\pi}$.}
vanishes: \( \lambda_\phi(\mu_*) = 0 \). For later purpose, it is convenient to rewrite the potential given in Eq. (4) at \( \mu_* \) into the following form:

\[
V_0 = \frac{\lambda_H}{2} \left( \mathcal{H}^\dagger \mathcal{H} - \frac{\lambda_{\phi H}}{2\lambda_H} \phi^2 \right)^2 + \frac{\lambda_{SH}}{2} S^2 \mathcal{H}^\dagger \mathcal{H} + \frac{\lambda_{\phi S}}{4} \phi^2 S^2 + \frac{\lambda_S}{4!} S^4 + \frac{\lambda^2_{\phi S}}{256\pi^2} \left[ \ln \frac{\lambda_{\phi S}}{2\mu_*^2} - \frac{1}{2} \right] - \frac{\lambda^2_{\phi H}}{8\lambda_H} \phi^4. \tag{5}
\]

As we consider the case \( \langle S \rangle = 0 \), the VEV of the Higgs doublet \( v = \sqrt{\langle \mathcal{H}^0 \rangle} \) is determined from the first term of Eq. (5) for a given \( v \phi = \langle \phi \rangle \) as

\[
v = \frac{\lambda_{\phi H}}{\lambda_H} v_\phi. \tag{6}
\]

The VEV of \( \phi \) can solely be determined at \( \mu = \mu_* \) from the second line in Eq. (5) as

\[
v_\phi = v_* , \tag{7}
\]

where

\[
v_* = \sqrt{\frac{2}{\lambda_{\phi S}}} \mu_* e^{\frac{16\pi^2 \lambda_{\phi H}}{\lambda_H \lambda_{\phi S}}}. \tag{8}
\]

We use the same definition of \( v_* \) given by Eq. (8) for the different critical points discussed below, in which \( v_* \) does not necessarily mean the VEV, but behaves just as a parameter. Solving \( \mu_* \) with respect to \( v_\phi \), the potential for \( \phi \) can be rewritten in terms of \( v_\phi \) as

\[
V^\phi_0 = \frac{\lambda^2_{\phi S}}{256\pi^2} \left( \ln \frac{\phi^2}{v_\phi^2} - \frac{1}{2} \right). \tag{9}
\]

The shape of the potential given in Eq. (9) is depicted as the solid curve in Fig. 1.

### 2.1.2 Degenerate true vacua

We have seen above that the case with CSI can be regarded as a generalization of the MPP. Instead we may add a mass term for \( \phi \) in order to realize degenerate minima, which might fit better in the original proposal of the MPP \[24\] \[8\]

\[
V_1 = V_0 + \frac{1}{2} m_\phi^2 \phi^2. \tag{10}
\]

We require that the potential has two degenerate minima at \( \phi = 0 \) and \( \phi = v_\phi \):

\[
V^\phi_1(\phi = 0) = V^\phi_1(\phi = v_\phi) = 0, \quad \frac{dV^\phi_1(\phi)}{d\phi} \bigg|_{\phi=0} = \frac{dV^\phi_1(\phi)}{d\phi} \bigg|_{\phi=v_\phi} = 0, \tag{11}
\]

\[\text{8} \] The one-loop correction to the effective potential in the second line in Eq. (5) is modified such as \( \lambda^2_{\phi} \ln \lambda_{\phi} \phi^2 \to (\lambda_{\phi} \phi^2 + 2m_\phi^2)^2 \ln (\lambda_{\phi} \phi^2 + 2m_\phi^2) \), etc.; but this modification merely results in a constant shift of the potential at the scale \( \mu_* \) where \( \lambda_{\phi} = 0 \).
where $V_1^\phi = V_0^\phi + m_\phi^2 \phi^2/2$. From these two equations, the mass parameter $m_\phi^2$ and the VEV $v_\phi$ are determined as

$$m_\phi^2 = \frac{\lambda_\phi^2 S}{128\pi^2} v_\phi^2, \quad v_\phi = \frac{v_s}{e^{1/4}}.$$  \hspace{1cm} (12)

The potential is then rewritten as

$$V_1^\phi = \frac{\lambda_\phi^2 S}{256\pi^2} \phi^2 \left[ \phi^2 \ln \frac{\phi^2}{v_\phi^2} + v_\phi^2 - \phi^2 \right].$$  \hspace{1cm} (13)

The shape of the potential is depicted as the dashed curve in Fig. 2.

### 2.2 Case without $Z'_2$ symmetry

In the above, the $Z'_2$ symmetry is spontaneously broken by the VEV of $\phi$, which causes the cosmological domain wall problem [43, 44]. A simple solution to avoid the problem is to introduce soft breaking terms of the $Z'_2$ symmetry such as the $\phi^3$ term:

$$V_2 = V_0 + \frac{\mu_\phi}{3!} \phi^3.$$  \hspace{1cm} (14)

In this case, we can consider two critical points having degenerate false vacua (CP 2-1) or a saddle point (CP 2-2). The shape of the potential with these criticalities is shown in Fig. 2.

#### 2.2.1 Degenerate false vacua

As shown in Fig. 2 with the solid curve, two degenerate false minima can appear at $\langle \phi \rangle = 0$ and $\langle \phi \rangle = v_{\text{deg}}$ by tuning $\mu_\phi$: Imposing Eq. (14) with the replacement of $V_1^\phi \leftrightarrow V_2^\phi$ with
\[ V_2^\phi = V_0^\phi + \mu_\phi \phi^3/3! , \] we obtain

\[ \mu_\phi = \frac{3\lambda^2_{\phi S}}{64\pi^2} v_{\text{deg}}, \quad v_{\text{deg}} = -\frac{v_*}{e^{3/4}}, \quad (15) \]

The VEV at the true vacuum \( \langle \phi \rangle = v_\phi \) is determined by

\[ v_\phi = \exp \left[ W \left( \frac{3}{4e^{3/4}} \right) \right] v_*, \quad (16) \]

where \( W \) is the Lambert \( W \) function that satisfies \( x = W(xe^x) \). The potential becomes

\[ V_{\text{deg}}^2 = \frac{\lambda^2_{\phi S}}{256\pi^2} \phi^4 \left\{ \ln \frac{\phi^2}{v_\phi^2} - \frac{1}{2} + 2W \left( \frac{3}{4e^{3/4}} \right) \right\} - \frac{2v_\phi}{\exp \left[ \frac{3}{4} + W \left( \frac{3}{4e^{3/4}} \right) \right]} \phi^3. \quad (17) \]

### 2.2.2 Saddle point

Another critical point in the parameter space, having a saddle point in the field space, can be found as in Fig. 2 with the dashed curve: Imposing the vanishing of the first and second derivative \[45, 26\]:

\[ \frac{\partial^2 V_2^\phi}{\partial \phi^2} \bigg|_{\phi = v_{\text{sadd}}} = \frac{\partial V_2^\phi}{\partial \phi} \bigg|_{\phi = v_{\text{sadd}}} = 0, \quad (18) \]

we obtain

\[ \mu_\phi = \frac{\lambda^2_{\phi S}}{16\pi^2} v_{\text{sadd}}, \quad v_{\text{sadd}} = -\frac{v_*}{e}. \quad (19) \]

The true vacuum \( \langle \phi \rangle = v_\phi \) can be determined by substituting Eq. (19) into the potential:

\[ v_\phi = e^{W(1/e)} v_* \quad (20) \]

The potential becomes

\[ V_{\text{saddle}}^2 = \frac{\lambda^2_{\phi S}}{256\pi^2} \phi^4 \left\{ \ln \frac{\phi^2}{v_\phi^2} - \frac{1}{2} + 2W \left( \frac{1}{e} \right) \right\} - \frac{8v_\phi}{\exp \left[ 1 + W(1/e) \right]} \phi^3. \quad (21) \]

### 2.3 Summary of the critical points

Let us summarize four critical points of our model discussed in the previous subsections. The basic properties of each critical point are given in Table I. In the following, we discuss the mass formulae for the scalar bosons.

We parametrize the fluctuations of the Higgs doublet \( \mathcal{H} \) and the singlet filed \( \phi \) at around the VEVs as

\[ \mathcal{H} = \begin{bmatrix} \chi^+_{v+h+i\lambda^0} \\ \sqrt{2} \end{bmatrix}, \quad \phi = v_\phi + \hat{\phi}, \quad (22) \]

\[ \text{Without loss of generality, we have chosen the negative value of } \mu_\phi \text{ to let the true vacuum located at a positive value: } v_\phi > 0. \]
where $\chi^\pm$ and $\chi^0$ are the Nambu-Goldstone modes which are absorbed by the longitudinal component of the $W$ and $Z$ boson, respectively. The squared mass matrix for the physical Higgs bosons is given in the basis of $(\hat{h}, \hat{\phi})$ as

$$M^2 = (\lambda_{\phi H} v_\phi^2) \times \begin{pmatrix} \frac{1}{\sqrt{\lambda_{\phi H} / \lambda_H}} & -\sqrt{\frac{\lambda_{\phi H}}{\lambda_H}} \frac{C}{2\pi^2 \lambda_{\phi H}} \\ -\sqrt{\frac{\lambda_{\phi H}}{\lambda_H}} \frac{C}{2\pi^2 \lambda_{\phi H}} & \frac{\lambda_{\phi H}}{\lambda_H} + \frac{1}{\sqrt{\lambda_{\phi H} / \lambda_H}} \frac{C}{2\pi^2 \lambda_{\phi H}} \end{pmatrix},$$

where the factor $C$ depends on the critical points as given in Table 1. The mass eigenstates of the Higgs bosons are written as

$$\begin{pmatrix} \hat{h} \\ \hat{\phi} \end{pmatrix} = \begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix},$$

where $c_\theta = \cos \theta$ and $s_\theta = \sin \theta$. The squared masses of $h$ and $H$ and the mixing angle $\theta$ are then expressed in terms of the mass matrix elements:

$$m_h^2 = M_{11}^2 c_\theta^2 + M_{22}^2 s_\theta^2 - M_{12}^2 s_\theta c_\theta,$$

$$m_H^2 = M_{11}^2 s_\theta^2 + M_{22}^2 c_\theta^2 + M_{12}^2 c_\theta s_\theta,$$

$$\tan 2\theta = \frac{2 M_{12}^2}{M_{11}^2 - M_{22}^2}.$$

The squared mass of $S$ is given by

$$m_S^2 = \frac{\lambda_{SH}}{2} v^2 + \frac{\lambda_{\phi S}}{2} v_\phi^2.$$
respectively. In terms of these parameters, the quartic couplings are expressed as

\[ \lambda_H = \frac{m_h^2}{v^2}, \quad \lambda_{\phi H} = \frac{m_{\phi}^2}{v_{\phi}^2}, \quad \lambda_{\phi S} = \frac{2m_S^2 - v^2 \lambda_{SH}}{v_{\phi}^2}. \]  

(30)

Furthermore, the squared mass of \( H \) and the mixing angle \( \theta \) may be expanded as

\[ m_H^2 = C \frac{\left(2m_S^2 - v^2 \lambda_{SH}\right)^2}{32\pi^2v_{\phi}^2} + \mathcal{O} \left(\frac{m_h^4}{v_{\phi}^4}\right), \quad \tan 2\theta = \frac{2m_h^2}{m_H^2 - m_h^2} \frac{v}{v_{\phi}} + \mathcal{O} \left(\frac{m_h^5}{v_{\phi}^5}\right). \]  

(31)

From the above expression, we see that \( m_H \) is much smaller than \( m_h \) for \( v_{\phi} \gg v, m_S \), while it can be larger than \( m_h \) for \( m_S \gg v_{\phi} \). The mixing angle \( \theta \) is typically very small, being given as \( |\theta| \approx v/v_{\phi} \), and can be sizable only at around \( m_H = m_h \). These properties turn out to be essentially important for the phenomenology of DM discussed in the next section.

3 Dark matter

In this section, we discuss the phenomenology of DM, i.e., the relic abundance and the constraint from direct search experiments.

In our model, the singlet scalar field \( S \) can be a candidate of DM because it cannot decay into SM particles due to the unbroken \( Z_2 \) symmetry. Our DM candidate can interact with SM particles only via the Higgs boson \( h \) or \( H \) so that it corresponds to the so-called Higgs portal scenario. All the annihilation channels are shown in Fig. 3, where the annihilation

\[ \text{Figure 3: Feynman diagrams for the DM annihilation.} \]

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occurs via the scalar cubic and quartic couplings. These couplings are expressed as

\[
\lambda_{SSh} = \frac{1}{v_\phi} \left[ \frac{v \lambda_{SH}}{2} (v s_\theta - v_\phi c_\theta) - m^2_S s_\theta \right], \quad \lambda_{SSH} = \frac{1}{v_\phi} \left[ \frac{v \lambda_{SH}}{2} (v c_\theta + v_\phi s_\theta) - m^2_S c_\theta \right],
\]

\[
\lambda_{SShh} = \frac{1}{4} \left[ -\lambda \lambda_{SH} c_\theta^2 + \frac{2 \theta}{v_\phi} (v^2 \lambda_{SH} - 2 m^2_S) \right], \quad \lambda_{SSHH} = \frac{1}{4} \left[ -\lambda \lambda_{SH} \theta^2 + \frac{2 \theta}{v_\phi} (v^2 \lambda_{SH} - 2 m^2_S) \right],
\]

all of which are determined by fixing three parameters in Eq. (29). The relic abundance of \( S \), \( \Omega_S h^2 \), can then be calculated by assuming the cold DM scenario as follows:\[46\]:

\[
\Omega_S h^2 = 1.1 \times 10^9 \frac{x_S}{M_P \sqrt{g_* (\sigma_{v_{rel}})}} \text{ GeV}^{-1},
\]

where \( M_P \) is the Planck mass, \( g_* \) is the effective relativistic degrees of freedom in the thermal bath, \( \langle \sigma v_{rel} \rangle \) is the thermally averaged cross section for the DM annihilation process multiplied by the relative velocity \( v_{rel} \), and \( x_S \equiv m_S / T_D \) with \( T_D \) being the decoupling temperature which can be estimated by solving the Boltzmann equation. On the other hand, the \( \lambda_{SSh} \) and \( \lambda_{SSH} \) couplings also contribute to the scattering cross section of DM and nucleon as follows

\[
\sigma_N \simeq \frac{g_N^2 m_N^2}{\pi (m_S + m_N)^2} \left| \frac{\lambda_{SSh} c_\theta - \lambda_{SSH} \theta}{m_H^2} \right|^2,
\]

where \( g_N \) is the effective nucleon-nucleon-DM coupling given by \( g_N \simeq 1.1 \times 10^{-3} \). In the following, we use the micrOMEGAs version 5 for numerical evaluations of the DM relic abundance and the DM scattering cross section with the nucleus. We note that the basic property of DM discussed above is common to all the four critical points defined in Table [11] but \( \Omega_S h^2 \) and \( \sigma_N \) can be different among the critical points mainly because of the difference of \( m_H \). We shall specify the critical point as needed in the following discussion.

It is important that in the limit of \( v_\phi \to \infty \) the DM annihilation effectively becomes the same as that in the minimal Higgs portal model, having only single additional real scalar field. In this limit, all the DM couplings with \( H \) and the mixing angle \( \theta \) become zero as we can see from Eqs. (31) and (32), so that \( H \) no longer contributes to the annihilation cross section. On the other hand, the contribution of the \( H \) mediation to the DM cross section with nuclei does not disappear in the \( v_\phi \to \infty \) limit because \( s_\theta^2 / m_H^2 \) approaches to a constant. Due to this contribution, our model tends to receive a severer constraint from the direct detection experiments as compared with the minimal Higgs portal scenario as we will see below.

### 3.1 Light dark matter scenario

We first consider the scenario with a light DM particle \( m_S < m_h \). As in the Higgs portal scenario, the dominant annihilation process is given by the \( SS \to f \bar{f} \) channel in this mass region. The cross section can be expressed as

\[
\langle\sigma v_{rel}\rangle \simeq \sum_{f \neq t} \frac{N_c^f m_f^2}{4 \pi} \frac{|\lambda_{SSH}|^2 c_\theta^2}{(4 m_S^2 - m_h^2)^2 + m_h^2 \Gamma_h^2},
\]

\[11\] We define these couplings by the coefficient of the corresponding vertex in the Lagrangian.
Figure 4: Spin independent scattering cross section with a nucleon $N$ as a function of the mass of DM $m_S$ in CP 2-2 with $v_φ = 3$ TeV. The black, blue and red curve show the case with $\lambda_{SH} = 0.2, 0.3$ and 0.4, respectively. The dashed curve denotes the upper limit on the cross section at 90% confidence level given by the XENON1T experiment.

where $Γ_h (\simeq 4$ MeV) is the width of $h$, and $N_f^f$ is the color factor. In the above expression, the contribution from the $H$ mediation is neglected, because its effect is highly suppressed by the factor of $m_f^2/v_φ^2$. From Eq. (35), we see that at $m_S \simeq m_h/2$ the observed value of $Ω_S h^2 \simeq 0.12$ can be accommodated by an arbitrarily small value of $\lambda_{SSH}$ because of the resonance of the Higgs boson. In the minimal Higgs portal scenario, such a solution at $m_S \simeq m_h/2$ works to explain the relic abundance under the constraint from the direct search experiment. In our scenario, however, it does not work. From Eq. (34), it is clear that even if we take a small enough value of the $\lambda_{SSH}$ coupling, typically $\lambda_{SSH}/v < \mathcal{O}(10^{-2})$, by tuning the $\lambda_{SH}$ parameter, we cannot take a small value of the $\lambda_{SSH}$ coupling, because there is no more free parameter to tune $\lambda_{SSH}$, see Eq. (32). Therefore, the light DM scenario is difficult to simultaneously satisfy the relic abundance and the bound from the direct search experiment in our model.

3.2 Heavy dark matter scenario

Let us consider the scenario with heavier DM, i.e., $m_S \gg m_h$. For concreteness, we first focus on CP 2-2 as the representative case, and then discuss the other three critical points later.

We first discuss the constraint from the direct detection. In Fig. 4 we show the scattering cross section of the $NS \rightarrow NS$ process with $v_φ = 3$ TeV. We take $\lambda_{SH} = 0.2$ (black), 0.3 (blue) and 0.4 (red). The dashed curve is the current upper limit on the cross section at 90% confidence level given by the XENON1T experiment [50]. From this figure, we can extract the lower limit of $m_S$ to be about $m_S = 1.6\sim 1.7$ TeV depending on the value of $\lambda_{SH}$. We note that the small dip at $m_S \simeq 1.7$ TeV appears due to the enhancement of the mixing angle as explained above. From this result, we typically need to take a few TeV for the mass of DM in order to avoid the constraint from the direct search experiment.

We then consider the relic abundance of the DM, in which the main annihilation channels are $SS \rightarrow VV/hh/HH$ $(V = W^\pm, Z)$. In the left panel of Fig. 5 we show the relic abundance of DM as a function of $m_S$ with the parameter choice same as in Fig. 4. It is seen that the abundance increases until $m_S \simeq 1.5, 1.7$, and 2 TeV for the respective values of $\lambda_{SH}$,
and then goes down as $m_S$ becomes larger. This behavior can be understood by looking at the right panel of Fig. 5 which shows, for $\lambda_{SH} = 0.3$, the relative contribution of each annihilation channel to $(\Omega_S h^2)^{-1}$, i.e. the relative magnitude of the thermally averaged cross section $\langle \sigma v \rangle_{\text{rel}}$. We see that the $SS \rightarrow HH$ ($SS \rightarrow VV$ and $SS \rightarrow hh$) channel becomes dominant (subdominant) when $m_S \gtrsim 1.7$ TeV, in which the contact diagram for $SS \rightarrow HH$ shown in Fig. 3 is enhanced by the factor of $m_S^2$, see also Eq. (32). Such enhancement does not occur for the $SS \rightarrow hh$ channel, because the $m_S^2$ term in the $\lambda_{SShh}$ coupling is highly suppressed by the factor of $s^2$. We note that at around $m_S = 1.7$ TeV the mass of $H$ gets close to $m_h$, so that the mixing angle $\theta$ becomes significant, and the $SS \rightarrow hH$ channel can be dominant at around this point. From these results, we learn that we can obtain two solutions of $m_S$ satisfying $\Omega_S h^2 \simeq 0.12$ for a fixed value of $\lambda_{SH}$ as long as $\lambda_{SH}$ does not exceed a certain critical value, e.g., $\lambda_{SH} \simeq 0.42$ for the case with $v_\phi = 3$ TeV. This critical value depends on the choice of $v_\phi$ as we will see below.

In order to extract the set of input parameters (29) that satisfy the relic abundance and the direct search experiment simultaneously, we scan $\lambda_{SH}$ and $m_S$ with several fixed values of $v_\phi$. We numerically find that the condition to reproduce the observed relic abundance, $\Omega_S h^2 \simeq 0.12$, is fitted by a function

$$4 \lambda_{SH}^2 + \lambda_{S\phi}^2 = \left( \frac{m_S}{m_{th}} \right)^2,$$

where $m_{th} = 1590$ GeV, and $\lambda_{S\phi}$ is related to $\lambda_{SH}$ and $m_S$ through Eq. (30). The margin of error is less than 10 percent. In the case with $m_S \gg m_h$, this equation is consistent with the fact that the annihilation cross section is mainly determined by the contact diagrams of $SS \rightarrow hh$/$HH$ and the $s$ channel diagram of $SS \rightarrow VV$. The first term of the left hand side of Eq. (36) comes from the contact diagram of the $SS \rightarrow hh$ process and the $s$ channel $SS \rightarrow VV$ process. Because the latter can be replaced by three times the former due to the equivalence theorem (namely, by the contact interactions of the $SS \rightarrow \chi^+\chi^-$ and $SS \rightarrow \chi^0\chi^0$...
processes), we have the factor of 4 in front. On the other hand, the second term of Eq. (36) solely comes from the contact diagram of the $SS \rightarrow HH$ process.

In Fig. 6, we show the correlation between the values of $\lambda_{SH}$ and $m_S$ to satisfy $\Omega_S h^2 = 0.12$ in the cases of CP 1-1 (upper left panel), CP 1-2 (upper right), and CP 2-2 (lower) for $v_\phi = 2.5$ (red curve), 3 (magenta), 4 (green), 5 (blue), and 10 TeV (black). We do not display the result of CP 2-1 because it is almost the same as that of CP 2-2. We note that the region between each curve can be filled by scanning the value of $v_\phi$. The dotted and solid curves correspond to the result using micrOMEGAs and the fitting function Eq. (36), respectively. The blue shaded region is excluded by the XENON1T experiment.

If we look at the curve for $v_\phi = 3$ TeV shown in the lower panel, we can reproduce the results given in Fig. 5. Namely, for e.g., $\lambda_{SH} = 0.4$ there are two solutions of $m_S$ at around $m_S = 1.5$ TeV and 2.5 TeV to satisfy $\Omega_S h^2 = 0.12$. For $\lambda_{SH} \gtrsim 0.42$, the solution disappears because the DM abundance becomes smaller than the observed value. It can also be seen that...
the case with smaller values of $m_S$ is excluded by the direct search experiment, as we have seen it in Fig. 4. For the larger values of $v_\phi$, the values of $m_S$ and $\lambda_{SH}$ to satisfy the relic abundance become larger. This is because the amplitude of the dominant DM annihilation processes, the contact diagram shown in Fig. 3, is suppressed by the factor of $1/v_\phi^2$, and thus larger values of $m_S$ or $\lambda_{SH}$ are required to compensate such suppression. As aforementioned, our scenario effectively becomes the minimal Higgs portal one in the large $v_\phi$ limit for the DM relic abundance. In fact, for the case with $v_\phi = 10$ TeV, the result (black curve) is in good agreement with the result reported in Ref. [42]. On the other hand, the constraint from the XENON1T experiment is stronger than the minimal Higgs portal model [51]. The red shaded region is excluded because there are no solutions satisfying (36). Explicitly, we need

$$2\lambda_{SH}m_{th} \leq m_S, \quad (37)$$

for the existence of a solution.

In addition to the constraints from the relic abundance and the direct search, we can impose a perturbativity bound as a theoretical constraint. By using the RGEs presented in Appendix [A] we compute the dimensionless couplings at high energy scales. Specifically, we require the absence of the Landau pole up to $\mu = 10^{17}$ GeV (this scale is supposed to be around the string scale). The gray region in Fig. 4 is then further excluded by the perturbativity bound. For comparison, we also show a black dashed curve which corresponds to the stronger criteria defined as

$$\max (|\lambda_{\phi H}(\mu)|, |\lambda_{SH}(\mu)|, |\lambda_{\phi S}(\mu)|, |\lambda_{S}(\mu)|, |\lambda_{\phi}(\mu)|, |\lambda_{H}(\mu)|) \leq 10 \quad \text{for} \quad \mu \leq 10^{18} \text{GeV}. \quad (38)$$

Note that, at high energy scales, typically $\lambda_{\phi S}$ or $\lambda_S$ becomes large so that a wider parameter region is excluded by imposing the condition (38). Furthermore, the constraint from the LHC data is imposed, by which the green shaded region is excluded. Detailed discussions for the LHC constraint will be given in the next section.

By taking into account all these constraints explained above, the white region in Fig. 6 is left allowed. It is seen that in CP 1-2 almost all the parameter region is excluded, while in CP 1-1 (CP 2-2) the region with $2.0 \text{ TeV} \lesssim m_S \lesssim 2.5 \text{ TeV}$ ($1.5 \text{ TeV} \lesssim m_S \lesssim 2.5 \text{ TeV}$) is allowed if we impose the milder constraint of the perturbativity bound. If we impose the stronger one defined in Eq. (38), the allowed region in CP 1-1 disappears, while a quite tiny region with $1.7 \text{ TeV} \lesssim m_S \lesssim 2.0 \text{ TeV}$ survives in CP 2-2.

The similar figure but for the correlation between $m_S$ and $m_H$ is shown in Fig. 7. The meaning for the shaded region except for the red one is the same as in Fig. 6. The red shaded region is excluded by the upper limit on $m_H$ and $m_S$ determined by Eqs. (28) and (36) as

$$m_H \leq \frac{1}{2\pi \sqrt{C/2}} \left( \frac{v_\phi}{2m_{th}} \right)^2 v_\phi, \quad m_S \leq \frac{v_\phi^2}{2m_{th}}. \quad (39)$$

Again, the white region is left allowed after taking into account all these constraints, and no solution is found in CP 1-2. For CP 1-1 and CP 2-2, the region without the degeneracy $m_H \simeq 125$ GeV is allowed if we use the milder constraint from the perturbativity bound.

To conclude, we find the parameter region satisfying the observed relic abundance under the constraints from the DM direct search experiment, LHC data and the perturbativity bound in CP 1-1, CP 2-1, and CP 2-2, among which CP 2-1 and CP 2-2 can further satisfy the stronger condition of the perturbativity defined by Eq. (38).
Figure 7: Same as Fig. 6 but for the correlation between $m_S$ and $m_H$.

4 Collider phenomenology

In this section, we discuss the collider phenomenology, particularly focusing on CP 2-2 as the representative one in which the largest region of the parameter space among the four CPs is allowed by the constraints discussed in Sec. 3.

As we have seen in the previous section, the mass of DM $m_S$ has to be typically a few TeV in order to explain the relic abundance and to avoid the constraint from the direct search experiment. On the other hand, the mass of the extra Higgs boson $m_H$ is typically of order 100 GeV or smaller as a consequence of the CW mechanism. Therefore, the collider phenomenology of our model is similar to that of the Higgs singlet model, see for a recent study e.g., [52], with a light singlet-like Higgs boson.

As in the Higgs singlet model, a discovery of the singlet-like Higgs boson $H$ can be direct evidence of the model. At collider experiments, $H$ can be produced by the same mechanism as that for the SM-like Higgs boson $h$ via the mixing. Thus, the production cross section is given by $\sigma_h \times s^2_\theta$ with $\sigma_h$ being the production cross section of the SM Higgs boson at the Higgs boson mass to be $m_H$. For $m_H < m_h/2$, $H$ can also be produced via the decay of $h$, i.e., $h \rightarrow HH$. However, such a light $H$ is almost excluded by the constraint from the direct search experiments for DM as we have seen in Fig. 7. If we take $v_\phi = 10$ TeV, a small window of $50$ GeV $\lesssim m_H < 62.5$ GeV is allowed by the constraint, in which the branching ratio of $h \rightarrow HH$ can maximally be about 1.5% at $m_H = 50$ GeV. In Ref. [53], the search for an exotic decay of the 125 GeV Higgs boson has been performed, in which the cross section
times branching ratio for the $h \to aa \to b\bar{b}\mu\mu$ ($a$ being a light CP-odd Higgs boson) process has been constrained. The observed upper limit on $\sigma_h/\sigma_{h}^{SM} \times BR(h \to aa \to b\bar{b}\mu\mu)$ at 95% confidence level is between $1.0 \times 10^{-4}$ and $6.0 \times 10^{-4}$ depending on the mass of $a$, where $\sigma_{h}^{SM}$ is the cross section of the SM Higgs boson. In our scenario with $v_\phi = 10$ TeV, $s_\theta^2$ is given to be of order $10^{-3}$, so that the ratio of the cross section $\sigma_h/\sigma_{h}^{SM}$ is $\mathcal{O}(10^{-3})$, and as mentioned above the branching ratio $BR(h \to H H)$ is given to be one percent level. Thus, even at the stage of $pp \to h \to H H$ (before the decay of $H$), the cross section is typically one order of magnitude smaller than the current limit given by the LHC data, so that we can safely avoid such constraints.

Another important test is to measure deviations from the SM predictions in properties of the discovered Higgs boson such as decay branching ratios and cross sections. Similar to the Higgs singlet model, couplings of $h$ with the fermions and the gauge bosons are universally suppressed by $c_\theta$ at tree level, which can be modified with one percent level by one-loop corrections. Thus, the cross section of $h$ can be estimated by $\sigma_h \times c_\theta^2$, while the branching ratios are the same as those of the SM Higgs boson at tree level, because of the universal suppression of the decay rates by the factor of $c_\theta^2$, as long as $h \to H H$ does not open. By detecting this characteristic pattern of the deviation in the cross section and the branching ratios for $h$, the model can be indirectly tested. Therefore, for both the direct search for $H$ and the indirect test, the mixing angle $\theta$ plays a crucial role.

The mixing angle $\theta$ is constrained from the measurement of the signal strength $\mu_h$ of the discovered Higgs boson at the LHC. From the Run II data, the ATLAS [56] and CMS [57] experiments have measured $\mu_h = 1.11 \pm 0.09$ and $\mu_h = 1.17 \pm 0.10$, respectively, so that upper limit on $s_\theta^2$ is given to be about 0.07 and 0.03 at the 2$\sigma$ level. This rather strong bound $s_\theta^2 < 0.03$ essentially comes from the fact that the central value of $\mu_h$ is observed to be larger than one, not due to the accuracy of the measurement of $\mu_h$. The size of the mixing angle can also be constrained by direct searches for the singlet-like scalar boson at the LHC. Statistically, no clear signature of $H$ has given a weaker bound $s_\theta^2 < 0.25$ ($s_\theta^2 < 0.16$) in the region of $80 \text{ GeV} < m_H < 600 \text{ GeV}$ ($100 \text{ GeV} < m_H < 150 \text{ GeV}$) [58]. We note that the LEP experiment has also provided a severe bound $s_\theta^2 \lesssim 0.01$ especially for the case with $m_H \lesssim 90 \text{ GeV}$ [59]. We have checked, however, that no further region is excluded by this LEP bound, namely, the region excluded by the LEP limit is already excluded by the constraints from the signal strength measured at the LHC or the DM direct search experiment.

The upper limit on $s_\theta^2$ can further constrain the region of the parameter space in our model. In Figs. [3] and [7], the green shaded region is excluded by the constraint from the signal strength, i.e., $s_\theta^2 < 0.03$. The mixing angle becomes significant at around $m_H = m_h$ so that the region with $m_H \simeq m_h$ is excluded as seen in Fig. [7] The corresponding exclusion on the $\lambda_{HS}$ and $m_S$ plane is also shown in Fig. [6].

Finally, we would like to comment on the possibility to test our model at future $e^+e^-$ colliders such as the International Linear Collider (ILC) [60], the Future Circular Collider (FCC-ee) [62], and the Circular Electron Positron Collider (CEPC) [63].

At the center of mass energy of 250 GeV, the main production channel of $H$ is the $Z$ boson strahlung $e^+e^- \to Z H$ similar to the $h$ production. The production cross section is given by [64]:

$$\sigma(e^+e^- \to Z H) = s_\theta^2 \frac{G_F^2 m_Z^4}{96\pi s(1-x_Z)} (v_e^2 + a_\tau^2) [12 x_Z + \lambda(x_Z, x_H)] \lambda^{1/2}(x_Z, x_H), \quad (40)$$

where $G_F$ and $\sqrt{s}$ are the Fermi constant and the center of mass energy of the electron and
positron collision, respectively. In addition, we have introduced \( x_Z = m_Z^2/s \), \( x_H = m_H^2/s \), \( v_e = -1 + 4s_W^2 \), \( a_e = -1 \), and \( \lambda(x, y) = (1 - x - y)^2 - 2xy \) with \( s_W \) being sine of the Weinberg angle. The cross section is numerically evaluated as

\[
\sigma(e^+e^- \to ZH) \simeq s_\theta^2 \times 417 (293) \text{ [96] fb for } m_H = 50 (100) \text{ [150] GeV},
\]

at \( \sqrt{s} = 250 \text{ GeV} \). Thus, we can obtain the cross section of \( \mathcal{O}(1) \text{ fb level in the typical case of our scenario.} \)

| BP | \( v_\phi \) | \( m_S \) | \( \lambda_{SH} \) | \( s_\theta^2 \) | \( m_H \) | \( \sigma_N \) | \( \sigma_{ZH} \) |
|----|----------|--------|-------------|------------|--------|--------|--------|
| BP1 | 2.5 \text{ TeV} | 1.76 \text{ TeV} | 0.24 | 0.025 | 159 \text{ GeV} | 2.1 \times 10^{-9} \text{ pb} | 0.40 \text{ fb} |
| BP2 | 3 \text{ TeV} | 1.9 \text{ TeV} | 0.43 | 0.025 | 154 \text{ GeV} | 2.1 \times 10^{-9} \text{ pb} | 1.8 \text{ fb} |
| BP3 | 4 \text{ TeV} | 2.2 \text{ TeV} | 0.60 | 0.014 | 154 \text{ GeV} | 2.0 \times 10^{-9} \text{ pb} | 0.98 \text{ fb} |
| BP4 | 5 \text{ TeV} | 2.0 \text{ TeV} | 0.59 | 0.020 | 101 \text{ GeV} | 2.0 \times 10^{-9} \text{ pb} | 5.7 \text{ fb} |
| BP5 | 10 \text{ TeV} | 2.0 \text{ TeV} | 0.61 | 8.7 \times 10^{-4} | 51 \text{ GeV} | 2.5 \times 10^{-9} \text{ pb} | 0.36 \text{ fb} |

Table 2: Benchmark points (BPs) satisfying the DM relic abundance, the bounds from the DM direct search, and the perturbativity in CP 2-2. For each point, we show the predictions of \( m_H, \sigma_N \) and \( \sigma_{ZH} \) (\( \equiv \sigma(e^+e^- \to ZH) \)) are shown. As expected, the cross section can be one fb, so that \( \mathcal{O}(1000) \) signal events can be expected at the ILC assuming 2000 fb\(^{-1} \) which might be enough large number for the detection of the second Higgs boson \( H \); see Ref. [65] for the detailed simulation study at the ILC.

5 Summary and discussion

We have discussed the model [34] including two real scalar fields \( \phi \) and \( S \) in addition to the SM fields, with a \( Z_2 \) symmetry \( \phi \to +\phi \) and \( S \to -S \) while all the SM fields are even. This is a minimal setup to realize an analog of the CW mechanism which generates the hierarchy between the Planck and electroweak scales. Assuming the \( Z_2 \) symmetry to be unbroken, \( S \) can be a candidate for DM. A non-zero VEV of \( \phi \) turns out to be the origin of the electroweak scale. We have classified four special critical points of the model, which we denote by CPs, motivated by a generalization of the MPP. Two of the four CPs are based on the scenario with the exact \( Z_2' \) symmetry, \( \phi \to -\phi \) and \( S \to +S \), in the action. The other two are without the \( Z_2' \) symmetry, and hence the domain-wall problem can be avoided.

We then have investigated the constraints from the relic abundance and direct searches of DM on three independent parameters, i.e., the mass of DM, the quartic coupling between DM and the Higgs doublet field, and the VEV of \( \phi \). Differently from the minimal Higgs portal scenario with a real singlet scalar filed, DM in our model can also annihilate into the additional Higgs boson \( H \) which is mainly composed of the singlet field \( \phi \). We have clarified that our DM can satisfy the thermal relic abundance \( \Omega_{\chi h^2} \simeq 0.12 \) measured by the Planck experiment when the mass of DM is taken to be multi-TeV region without the confliction to
the DM direct search experiment at XENON1T. We also have imposed the constraints from collider experiments among which the signal strength of the discovered Higgs boson measured at the LHC gives the most stringent bound on the parameter space. Furthermore, we have required perturbativity condition up to the string scale. Consequently, we have found that three of four CPs can satisfy all these constraints, in which the mass of DM is typically given to be around 2 TeV. If we impose a stronger constraint on the perturbativity bound, requiring that all dimensionless couplings do not exceed 10 up to the energy scale of $10^{18}$ GeV, two CPs without the $Z'_2$ symmetry can still satisfy all the constraints, while the CP with the exact $Z'_2$ is excluded.

Finally, we have discussed testability of our model at collider experiments. Since the DM should be as heavy as a TeV range in order to satisfy the relic abundance and the constraint from the direct searches, detection of $H$ can be an important probe of our model similarly to the Higgs singlet model. We have particularly focused on the production of $H$ at the ILC with the collision energy of 250 GeV, where $H$ can mainly be produced in association with the $Z$ boson $e^+e^- \rightarrow ZH$. In the benchmark parameter points which are allowed by all the constraints discussed above, we have found that the mass of $H$ can be in the range of 50–150 GeV barring the region around 125 GeV, and the cross section can be of the order of fb level. Therefore, our model would be tested at the ILC or future measurements of the direct search experiment such as XENON1T.

In the critical points without $Z'_2$, there is a possibility of having a first order electroweak phase transition in the early universe. It will be interesting to study future detectability of the gravitational waves produced through the phase transition. In this paper, we have applied the fact that minimally two scalar fields suffice for the dimensional transmutation analogous to the CW mechanism such that one of the scalars plays the role of the Higgs-portal DM. Instead, one may give up providing DM and identify one of the two scalars directly the SM Higgs doublet [4]. It would be interesting to analyze such a model for all the possible criticalities along the line of the current work. These possibilities will be pursued in separate publications.

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Appendix

A Renormalization Group Equations

The renormalization group equations are

\[
16\pi^2 \beta_{\lambda_{PH}} = 6\lambda_{PH} \lambda_{H} + \lambda_{PH} \lambda_{\phi} - 4\lambda_{PH}^2 + \lambda_{PH} \lambda_{SH} + 6\lambda_{PH} g_t^2 - \frac{3}{2} \lambda_{PH} g_Y^2 - \frac{9}{2} \lambda_{PH} g_2^2,
\]

\[
16\pi^2 \beta_{\lambda_{SH}} = 6\lambda_{SH} \lambda_{H} + \lambda_{SH} \lambda_{S} + 4\lambda_{SH}^2 - \lambda_{PH} \lambda_{PH} + 6\lambda_{SH} g_t^2 - \frac{3}{2} \lambda_{SH} g_Y^2 - \frac{9}{2} \lambda_{SH} g_2^2,
\]

\[
16\pi^2 \beta_{\phi_S} = \lambda_{\phi_S} \lambda_{PH} + \lambda_{\phi_S} \lambda_{S} + 4\lambda_{\phi_S}^2 - 4\lambda_{PH} \lambda_{HS},
\]

\[
16\pi^2 \beta_{\lambda_S} = 3\lambda_{S}^2 + 3\lambda_{\phi_S}^2 + 12\lambda_{SH}^2,
\]

\[
16\pi^2 \beta_{\lambda_{PH}} = 3\lambda_{PH} + 3\lambda_{\phi_S}^2 + 12\lambda_{PH}^2,
\]

\[
16\pi^2 \beta_{g_{Y23,4}} = (\text{Same as the SM}),
\]

\[
16\pi^2 \beta_{\lambda_{H}} = \lambda_{S}^2 + \lambda_{PH}^2 + 12\lambda_{H}^2 - 3\lambda_{H} g_Y^2 + \frac{3}{4} g_Y^4 - 9\lambda_{H} g_2^2 + \frac{3}{2} g_Y^2 g_2^2 + \frac{9}{4} g_2^4 + 12\lambda_{H} g_t^2 - 12 g_t^4 + \frac{1}{16\pi^2} \left( - 4 \lambda_{S}^2 \lambda_{H} - 5 \lambda_{S}^2 \lambda_{S} - 78 \lambda_{H}^2 + 18 \lambda_{H} \left( g_Y^2 + 3 g_2^2 \right) + \lambda_{H} \left( \frac{629}{24} g_Y^4 + \frac{39}{4} g_Y^2 g_2^2 - \frac{73}{8} g_2^4 \right) \right) + \frac{305}{8} g_2^4 - \frac{289}{24} g_Y^2 g_2^4 - \frac{559}{24} g_Y^4 g_2^2 - \frac{379}{24} g_Y^6 - 64 g_2^2 g_Y^4 - \frac{16}{3} g_Y^2 y_t^4 - \frac{9}{2} g_2^4 y_t^2 + \lambda_{H} g_t^2 \left( \frac{85}{6} g_Y^2 + \frac{45}{2} g_2^2 + 80 g_5^2 \right) - \frac{19}{2} g_Y^2 y_t^2 + 21 g_Y^2 g_2^2 y_t^2 - 72 \lambda_{H} y_t^2 + 3 \lambda_{H} y_t^4 + 60 y_t^6 \right),
\]

where \( g_3, g_2 \) and \( g_Y \) are SU(3), SU(2) and U(1) gauge couplings in the SM, respectively. See Ref. [66] for the SM beta functions and initial values of the couplings at the electroweak scale. We choose the top mass to be 172GeV.

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