A Relativistic Symmetry in Nuclei

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Introduction

More than thirty years ago it was observed that certain quantum energy levels in atomic nuclei were almost degenerate in energy [1]. The states that are almost degenerate (quasi-degenerate) have different radial quantum numbers and different orbital angular momenta, features that made the reason for their degeneracy difficult to penetrate.

The dynamics of neutrons and protons in nuclei have been successfully treated non-relativistically. Therefore it has come as a surprise that this quasi-degeneracy of quantum states in heavy nuclei, which has eluded understanding for about thirty years, can be explained by a relativistic symmetry [2].

The Nuclear Shell Model

Atomic nuclei are well described by nucleons moving in a non-relativistic mean field with residual interactions that induce correlations between the nucleons. The dynamics of the nucleons in the orbits are described by the non-relativistic Schrödinger equation. For spherical nuclei the quantum numbers of the orbits in the mean field are the radial quantum number, \( n \), the orbital angular momentum, \( l \), and the total angular momentum, \( j \), which is the sum of the orbital angular momentum and the spin; \((n, l, j)\) for short. The orbits that are quasi-degenerate in energy are \((1, 0, 1/2)\) and \((0, 2, 3/2)\), \((1, 1, 3/2)\) and \((0, 3, 5/2)\), and so on. That is, the orbit with \((n, l, j= l+1/2)\) will be quasi-degenerate with the orbit \((n−1, l+2, j= l+3/2)\); in other words, is the radial quantum numbers will differ by one unit, the orbital angular momenta will differ by two units, and the total angular momenta by one unit.

For deformed nuclei the orbits that are quasi-degenerate have angular momentum projection along the symmetry axis differing by two units and total angular momentum projection differing by one unit.

The Dirac Hamiltonian

The Dirac equation, not the Schrödinger equation, must be used to describe the relativistic dynamics of nucleons moving in a relativistic mean field. In the limit that the relativistic mean field is small compared to the mass of the nucleon, the non-relativistic limit, then the Schrödinger equation will be a good approximation to the Dirac equation.

The Dirac equation has positive energy eigenfunctions and negative energy eigenfunctions. The former are the eigenfunctions of the particles and the latter are the eigenfunctions of the anti-particles. A Dirac eigenfunction will then have twice as many components as a Schrödinger eigenfunction. In the non-relativistic limit, the “upper” component of the positive energy eigenfunctions will become the Schrödinger eigenfunctions for the particles and the “lower” component will become vanishingly small, whereas the “lower” component of the negative energy eigenfunctions will become the Schrödinger eigenfunctions for the anti-particles and the “upper” component will become vanishingly small.

Likewise, in the Dirac equation two types of potentials are possible, one a relativistic scalar and one a relativistic vector. The sum of the two potentials dominate the dynamics of the particles whereas the difference of the two dominate the dynamics of the anti-particles.

Symmetries of the Dirac Hamiltonian

When the scalar potential and vector potential are equal the Dirac Hamiltonian has spin symmetry. This means that the eigenfunctions that differ in the orientation of the spin will be degenerate in energy. That is, the orbits \((n, l, j= l+1/2)\) and \((n, l, j= l−1/2)\) will have the same energy. These states are spin doublets because the energy does not depend on the orientation of the spin. This symmetry occurs in hadrons [3].

When the scalar potential is equal to the vector potential, but opposite in sign, there is another symmetry of the Dirac equation. This symmetry is called pseudospin symmetry. The states that are degenerate have exactly the radial quantum numbers and orbital angular momenta of the quasi-degenerate states that have been observed in nuclei and these states are pseudospin doublets.

Relativistic Mean Field

Relativistic models of nuclei include nuclear field theories with nucleons interacting by the exchange of mesons on the one hand and nucleons interacting with relativistic interactions.
These models are difficult to solve exactly but have been solved in the relativistic mean field approximation, which reduces to a Dirac Hamiltonian with the scalar and vector potentials determined self-consistently [4]. The resulting scalar and vector potentials are opposite in sign and approximately equal in magnitude. Thus the symmetry, which was observed in the nuclear states more than thirty years ago, is pseudospin symmetry, a symmetry of the Dirac Hamiltonian.

**Predictions of Pseudospin Symmetry**

*Amplitudes*

One of the predictions of this pseudospin symmetry is that the spatial amplitudes of the lower components for the two states in the degenerate doublets should be equal in magnitude. We have tested this condition by examining the lower amplitudes of the Dirac eigenfunctions determined in relativistic mean field calculations of nuclear spectra using realistic vector and scalar potentials [5]. In Figure 1 we show an example of the amplitudes of the lower components of two states of a pseudospin doublet in in the spherical nucleus $^{208}$Pb. in Figure 1a is the upper amplitude, $g(r)$, for the $(n=1, l=0, j=1/2)$ state (solid line) and the $(n=0, l=2, j=3/2)$ state (dashed line). These radial amplitudes have very different in shapes. However, the lower amplitudes, $f(r)$, are almost identical as seen in Figure 1b. In Figure 1c is the upper amplitude, $g(r)$, for the $(n=2, l=0, j=1/2)$ state (solid line) and the $(n=1, l=2, j=3/2)$ state (dashed line). Again these radial amplitudes have very different shapes. However, the lower amplitudes, $f(r)$, are almost identical, as seen in Figure 1d. As the radial quantum number increases, the lower amplitudes become more similar, implying that pseudospin conservation improves as the binding energy decreases.

Pseudospin symmetry also imposes conditions on the upper amplitudes but these are more complicated, involving differential equations between the amplitudes. However, these conditions are approximately satisfied as well and improve as the binding energy decreases [6] just like the lower amplitudes.

A survey of other states in both deformed and spherical nuclei for pseudospin symmetry in both upper and lower components show that pseudospin symmetry is approximately conserved and the conservation increases as the binding energy decreases [7].

![Figure 1](image_url) *The upper amplitudes, $g(r)$, and lower amplitudes, $f(r)$, versus the radius $r$.  

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Magnetic Dipole and Gamow Teller Transitions

Magnetic dipole transitions between pseudospin doublets are forbidden non-relativistically because the states in the doublets have angular momentum differing by two units and the dipole can only change the angular momentum by at most one unit. However, these transitions are allowed relativistically. If pseudospin symmetry is conserved, then, if the magnetic moment of the states in the doublet is known, the magnetic dipole transition between the pseudospin doublets can be determined [8]. For example, using the magnetic moment of the ground state of \(^{39}\text{Ca}\), the predicted magnetic dipole transition agrees with the measured transition within experimental error. A global analysis of such transitions for many nuclei shows that these predictions are approximately valid [9]. Similar relations hold for Gamow Teller transitions in beta decay as well.

Nucleon-Nucleus Scattering

The elastic scattering of medium energy nucleons from nuclei can described successfully with a relativistic optical model with complex scalar and vector potentials. The scalar and vector potentials determined by fitting the scattering data are approximately equal in magnitude but different in sign even though they are complex [10]. The scattering amplitude consists of two parts, one independent of pseudospin symmetry and one pseudospin dependent. The pseudospin dependent amplitude has been extracted from the measured spin polarization and the spin rotation [11,12]. For the scattering angles measured the pseudospin dependent amplitude is only 10% of the pseudospin independent amplitude. For lower energy nucleons, however, the pseudospin breaking increases [13].

Antinucleon-Nucleus Scattering

A nucleon changes into an antinucleon under charge conjugation. Under charge conjugation the scalar potential remains unchanged but the vector potential changes sign. Thus, an antinucleon in a nuclear environment will experience vector and scalar potentials that are approximately equal. This implies spin symmetry. Indeed, spin polarization measured in antinucleon nucleus scattering is consistent with zero, implying spin symmetry [14].

Fundamental Theory of the Strong Interactions and Pseudospin Symmetry

Quantum Chromodynamics (QCD), the fundamental theory of the strong interactions, predicts that the vector

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and scalar potentials in nuclei are almost equal in magnitude and opposite in sign [15], which is consistent with approximate pseudospin symmetry. The difference in sign comes from the fact that the quark condensate of the vacuum is negative.

**Future Study**

This connection with QCD suggests that there may exist a more basic rationale for pseudospin symmetry in nuclei based on the interaction between quarks that needs to be explored. For example, one question is “Why is pseudospin symmetry valid for nuclei, whereas spin symmetry is valid for hadrons?” [16]

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