Radio-frequency optomechanical characterization of a silicon nitride drum

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On-chip actuation and readout of mechanical motion is key to characterize mechanical resonators and exploit them for new applications. We capacitively couple a silicon nitride membrane to an off resonant radio-frequency cavity formed by a lumped element circuit. Despite a low cavity quality factor (Qₑ ≈ 7.4) and off resonant, room temperature operation, we are able to parametrize several mechanical modes and estimate their optomechanical coupling strengths. This enables real-time measurements of the membrane’s driven motion and fast characterization without requiring a superconducting cavity, thereby eliminating the need for cryogenic cooling. Finally, we observe optomechanically induced transparency and absorption, crucial for a number of applications including sensitive metrology, ground state cooling of mechanical motion and slowing of light.

Cavity optomechanics boasts a number of tools for investigating the interaction between radiation and mechanical motion and enables the characterization and development of highly sensitive devices⁵. Silicon nitride membranes have been fabricated to exhibit very high tensile stress, resulting in high quality factors, and have been used for a number of applications including measurement of radiation pressure shot noise⁶, optical squeezing⁷, bidirectional conversion between microwave and optical light⁸, optical detection of radio waves⁹,¹⁰, microkelvin cooling¹¹ and cooling to the quantum ground state of motion¹²–¹⁴.

Radio-frequency (rf) cavities allow for sensitive mechanical readout on-chip¹³,¹⁴. We characterize a silicon nitride membrane at room temperature making use of an off-resonant rf cavity¹³,¹⁴. In our approach, the use of lumped elements greatly simplifies the detection circuit in terms of fabrication and allows the integration on chip with the mechanical oscillator. Our circuit has a lower operation frequency than microwave cavities¹¹,¹⁴, and allows for a larger readout bandwidth than previous works¹³. Also, our cavity allows us to inject noise, effectively increasing the mechanical mode temperature. We are able to detect several modes and extract the quality factor and cavity coupling strength for each of them. When the membrane is driven, we are able to resolve the membrane’s motion in real time. We achieve a sensitivity of 0.4 pm/√Hz. A sensitivity of 4.4 pm/√Hz was reported in ref. ¹⁴, although it must be noted that these sensitivities cannot be easily compared, due to the much smaller size of their mechanical resonator. We observe optomechanically induced transparency (OMIT) and optomechanically induced absorption (OMIA) on-chip and deep in the unresolved sideband regime, allowing for the characterization of the membrane’s motion under radiation pressure. OMIT and OMIA are an unambiguous signature of the optomechanical interaction¹⁵ and they can be used to slow or advance light¹⁶. OMIT has also been proposed as a means to achieve ground state cooling of mechanical motion in the unresolved sideband regime¹⁷,¹⁸.

Experiment

Our device consists of a high-stress silicon nitride membrane which is 50 nm thick; it has an area of 1.5 mm × 1.5 mm and 90% of this area is metalized with 20 nm of Al. We suspend this membrane over two Cr/Au electrodes patterned on a silicon chip. A dc voltage Vdc = 15 V is applied to electrode 1, with electrode 2 grounded. Measurements were performed at room temperature and at approximately 10⁻⁶ mbar. For optomechanical read-out and control, electrode 1 is connected to an effective rf cavity. The cavity is realised using an on-chip inductor L and capacitors Cᵣ and Cₛ mounted on the sample holder¹⁹, in addition to the capacitance formed by the membrane Cᵣ. The circuit behaves similarly to a simple LC resonator with total capacitance Cₕ = Cᵣ + Cₛ, where Cᵣ

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accounts for the capacitance between the two sides of the antenna and other parasitic capacitances. This circuit acts as a cavity and can be driven by injecting an rf signal to the input (port 1) via a directional coupler in order to induce an optomechanical interaction between the cavity signal and the mechanical motion. In addition port 3 allows injection of an ac signal to directly excite the membrane’s motion. The entire setup forms a three-terminal circuit with input ports 1 and 3 and output port 2 (Fig. 1(a)). We used a vector network analyzer to measure scattering parameters and a spectrum analyzer to measure power spectra. Figure 1(b) shows the scattering parameter (|S21|) as a function of cavity probe frequency \( f_P \). The cavity resonance is evident in reflection as a minimum in |S21| with quality factor \( Q_E \approx 7.4 \).

The dependence of the capacitor formed between the electrodes and the metalized membrane \( C_C \) on the mechanical displacement \( u \) leads to coupling between the cavity and the mechanical motion. This coupling is given by \( \frac{\partial}{\partial u} \approx \frac{1}{(4\pi^2 LC_1^2)^{1/2}} \), where \( f_c \) is the cavity resonance frequency, \( C_1 \) the total circuit capacitance and \( u \) the membrane’s displacement from its equilibrium position. The coupling causes phase shifts of the cavity reflection, allowing us to monitor the membrane’s motion in the unresolved sideband limit. The single-photon coupling strength, which measures the interaction between a single photon and a single phonon, is therefore,

\[
\frac{g_0}{2\pi} \approx \frac{1}{4\pi^2 LC_1^2} \frac{\partial C_C}{\partial u} \mu_Z P, \tag{1}
\]

where \( \mu_Z P \) is the amplitude of the membrane’s zero-point motion.

Using a simple circuit model (See Supplemental Material for details of the circuit simulation), we fit \( |S_{21}| \) and extract \( C_C \approx 1.6 \) pF. Within the parallel plate capacitor approximation, \( C_C \approx \frac{e_0 d}{d} \), where \( e_0 \) is the vacuum permittivity.
activity, $a$ is the metallised area of the membrane and $d$ is the membrane-electrode gap. From this expression, we extract $d \sim 9 \mu m$.

**Results**

To find the mechanical resonances, we drove the cavity on resonance ($f_C \approx 209.2 \text{ MHz}$) with input power $P_C = 5 \text{ dBm}$ at port 1. Meanwhile, we excited the membrane's motion via port 3, using a sinusoidal signal at frequency $f_E$ and with amplitude $V_M = 3.6 \text{ Vrms}$ at electrode 1. In order for the mechanical response to appear broader in the frequency spectrum, facilitating the detection of the mechanical modes, the excitation frequency at port 3 was modulated with a white noise pattern with a deviation of 200 Hz. The power spectrum $P$ of the reflected signal at port 2 shows sidebands at $f_C \pm f_E$ due to non-linearities of the rf circuit giving rise to frequency mixing. The mechanical signal appears when $f_E$ is close to a mechanical resonance, and is evident as a pronounced increase in sideband amplitude and width (Fig. 1(c)).

Figure 1(d) shows the sideband at $f_C \pm f_E$ as a function of $f_E$. The fundamental mode frequency $f_{0,1,0}$, which we will call $f_0$, is $\sim 77.9 \text{ kHz}$, giving an unresolved sideband ratio of $2\pi f_0/\kappa \sim 3 \times 10^{-3}$, with $\kappa = 2\pi f_C/Q_E$ the cavity linewidth. As well as the fundamental mode, we observe less strong harmonics $f_{i,j}$ near the expected frequencies. The expected frequencies for higher harmonics theoretically satisfy the ratios $f_{i,j}/f_0 = \frac{1}{\sqrt{i^2 + j^2}}$ for integers $i$ and $j$ with symmetric roles as expected for a square membrane. Two of the sidebands are double peaked, evidencing nearly degenerate mechanical modes (insets Fig. 1(d)). The broken degeneracy could be due to imperfections in how the membrane was fabricated and fixed in place or uneven binding/deposition of the Al layer.

The entire set of mechanical resonances can be observed in a single measured power spectrum by driving the cavity at $f_C$ and injecting white noise via port 3. White noise excites the motion of the membrane at all frequencies, which is equivalent to raising the effective mechanical temperature. In this way, the root mean square (rms) displacement is increased, thereby facilitating the detection of mechanical modes. The noise signal has a bandwidth of 1 MHz, larger than the spectral width of the mechanical modes, and an amplitude $V_M = 3.6 \text{ Vrms}$. Figure 2(a) shows the mechanical sidebands at $f_C \pm f_E$. To distinguish mechanical sidebands from other parasitic signals, we increase $P_C$ until we observe a frequency shift (See Supplemental Material for further details). The fundamental mode of the membrane is at $f_0 = 78.573 \pm 0.002 \text{ kHz}$. We fit each mechanical sideband with a Lorentzian (Fig. 2(b–d)). As in Fig. 1(d), we observe double peaked sidebands (Fig. 2(c,d)).
The broad cavity bandwidth allowed us to measure the actuated membrane’s motion in real time. We excite the membrane with white noise whilst driving the cavity at $f_c$. In order to record the motion in real time, the cavity output signal is mixed with a local oscillator. The output signal (Fig. 2(c)) shows clear sinusoidal oscillations evidencing the membrane’s motion.

We plot $f_i/f_0$ as a function of $f_i$ in Fig. 3(a) for all mechanical resonances observed in Figs. 1(d) and 2(a). Lower frequency modes show better agreement with theoretical ratios than higher frequency modes. From the Lorentzian fits of each mechanical sideband, we extract the mechanical quality factors $Q_{i,j}$ and single photon coupling strengths $g_{i,j}$ (Fig. 3(b)). These values of $Q_{i,j}$ measured in the spectral response, are sensitive both to dissipation and dephasing. The fundamental mode has a quality factor $Q_0 = (1.6 \pm 0.1) \times 10^5$ and the highest quality factor measured was $(20 \pm 4) \times 10^5$ for the mode at 241 kHz. Different modes and even nearly degenerate mechanical modes have significant differences in their values of $Q_{i,j}$ as previously reported. The values of $Q_{i,j}$ vary slightly as a function of $V_M$ ranging from 0.1 to 3.6 $V_{rms}$ but they do not show a specific trend. These values of $Q_{i,j}$ can be compared with the predictions of Eq. 1. Taking $C_f$ and $d$ from the circuit model, and using that $\frac{\partial S_v}{\partial S_i} = -|S_{21}|^2$ for a parallel-plate capacitor (with a known prefactor to account for the mechanical mode profile of the membrane), gives a coupling strength $g_{i,j}/2 \pi \approx 2$ mHz for the fundamental mode. The estimated values of $g_{i,j}$ are similar to the ones extracted from the sideband powers.

We can estimate the vibrational amplitude sensitivity $S_v$ given an amplifier voltage sensitivity $S_a = \sqrt{k_B T_0 Z_0}$, where $k_B$ is the Boltzmann constant, $T_0 = 300$ K and $T_N \sim 293$ K the system noise temperature. We can write $S_v = S_a |\frac{\partial V_{in}}{\partial V_{in}}|$ where $V_{in}$ is the voltage at electrode 1 corresponding to $P_1$. Taking $\frac{\partial V_{in}}{\partial C_f}$ from the circuit model and $C_f = 1.7 \times 10^{-9}$ F/m extracted from the sideband’s area corresponding to the fundamental mode, we obtain $S_v = 0.4$ pm/\sqrt{Hz}. This value is comparable to that obtained for a suspended carbon nanotube device at cryogenic temperatures.

Finally, we measure optomechanically induced transparency (OMIT) and absorption (OMIA), which are signatures of optomechanical coupling and demonstrate that the membrane’s motion can be actuated by radiation-pressure alone. OMIT and OMIA are characterized by the emergence of a transparency or absorption window in $S_{21}$ when a strong drive tone $f_D$ and a weak probe tone $f_p$ are injected into the cavity, and...
the frequency difference between these tones $\delta f = f_P - f_D$ coincides with the frequency of a mechanical mode. When this condition is met, the beat between the drive and the probe field excites the membrane’s motion and the destructive (constructive) interference of excitation pathways for the intracavity probe field results in a transparency (absorption) window in $|S_{21}|$. To show optomechanical control, we measured OMIT and OMIA by injecting a strong tone at frequency $f_D$ and a weak tone at frequency $f_P$ through a directional coupler at port 1. We injected three different drive frequencies (Fig. 4(b)); $f_D \sim f_C - \kappa/2$ (red detuned), $f_D \sim f_C$ (resonant) and $f_D \sim f_C + \kappa/2$ (blue detuned). When $f_D \sim f_C$, a peak (dip) is observed at $f_D - \pm f_0$ (Fig. 4(c)). When $f_D - f_C \pm \kappa/2$, we observe Fano-like features at $f_D \pm f_0$.

Figure 4. (a) Schematic of signal injection and detection for OMIT/OMIA measurements. A tone with frequency $f_D$ and a tone with frequency $f_P$ are injected through a directional coupler via port 1. We monitor $|S_{21}|$ with a vector network analyzer. (b) Schematic showing each of the drive frequencies injected and a cartoon of the mechanical sidebands appearing at $f_D \pm f_0$. Drive frequencies were $f_D \sim f_C$ (green), $f_D \sim f_C - \kappa/2$ (red) and $f_D \sim f_C + \kappa/2$ (blue). The corresponding mechanical sidebands are shown in panels (c–e) respectively. (c–e) Measured $|S_{21}|$ as a function of $\delta f$ showing mechanical sidebands at $\delta f = -f_0$ (left panel) and $\delta f = f_0$ (right panel) for each of the values of $f_D$ considered in (b). At port 1, the drive power was 5 dBm and the probe power $-27.5$ dBm. Black curves are a fit to the data.

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$\delta f = f_P - f_D$
We fitted OMIT and OMIA features by modelling the transmission of the probe field (See Supplemental Material for details of the estimation of the single-photon coupling from OMIT measurements). From the fit of the spectral features, we extract $f_0 = 77.2 \pm 0.1$ kHz, $Q_0 = (1.2 \pm 0.1) \times 10^4$ and $g_0/2\pi = 2.3 \pm 0.3$ mHz. Error bars were obtained by combining fit results of the six curves in Fig. 4(c–e). The values obtained for $f_0$ and $Q_0$ are similar to those extracted from the Lorentzian fits as in Fig. 2(b). The value of $g_0$ is in agreement with that extracted from Fig. 2(a) and estimated from the parallel plate capacitor approximation.

**Discussion**

To conclude, we have characterized several modes of a silicon nitride membrane with an off-resonant rf circuit at room temperature, deeper in the unresolved sideband regime than previously explored. Our cavity allows for the injection of noise to actuate the motion of the membrane and effectively increase its mechanical mode temperature. We achieve a sensitivity of 0.4 pm/√Hz, a tenfold improvement to that reported in ref. 14; although the smaller size of their mechanical resonator makes direct comparison difficult. Our results show that our on-chip platform can be used for membrane actuation and characterization. The readout circuit operates at a convenient frequency and does not require the cavity to be tuned into resonance with the membrane as in other approaches. It therefore has applications in mechanical sensing and microwave-to-optical conversion. Thanks to the large bandwidth of the cavity we have also measured the actuated membrane's motion in real time. Finally we have observed OMIT and OMIA, from which we obtained a separate measure of the frequency, quality factor and coupling strength of the fundamental mode.

**Data availability**

The data analysed during the current study are available from the corresponding author on reasonable request.

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**Author contributions**
A.N.P. and N.A. performed the experiment, analysed the data and prepared the manuscript with contributions from E.A.L. K.E.K. contributed to the theory, M.M. helped with device preparation. N.A. conceived the experiment. A.N.P., K.E.K., M.M., E.A.L., G.A.D.B. and N.A. discussed results and commented on the manuscript.

**Competing interests**
The authors declare no competing interests.

**Additional information**
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