Atomic Radiative Transitions in Thermo Field Dynamics

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Abstract

In this work we rederive the Lamb-Retherford energy shift for an atomic electron in the
presence of a thermal radiation. Using the Dalibard, Dupont-Roc and Cohen-Tannoudji
-DDC formalism, where physical observables are expressed as convolutions of suitable statistical functions, we construct the electromagnetic field propagator of Thermo Field Dynamics in the Coulomb gauge in order to investigate finite temperature effects on the atomic energy levels. In the same context, we also analyze the problem of the ground state stability.
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I. Introduction

Since the 70’s it has been argued [1] [2] that the physical interpretation of radiative phenomena, in particular the shift of atomic energy levels, rely upon different choices in the ordering of atomic and field operators in the interaction Hamiltonian.

Almost two decades ago Dalibard, Dupont-Roc and Cohen-Tannoudji (DDC) [3] considered the interaction between a non-relativistic atomic electron and the quantized electromagnetic field, showing that the above mentioned arbitrariness can be removed by requiring the observables variation rates to be Hermitian, if we want them to have a physical meaning. They generalized their procedure to the case of a small system $S$ interacting with a large reservoir $R$ (which may be in thermal equilibrium). This construct allowed them to separate the physical processes in two categories, those where $R$ fluctuates and polarizes $S$ (effects of reservoir fluctuations), and those where $S$ polarizes $R$ (effects of self-reaction or radiation reaction).

In the present work we are interested in analyzing the temperature effects in the context of DDC formalism, where the statistical functions, which are defined from two-point correlation functions, play a fundamental role. These functions enable us to obtain expressions, up to second order in perturbation theory, in terms of products of symmetrical correlation functions and susceptibilities [4]. The temperature implementation [3] can be made directly in such statistical functions using the equipartition theorem, leading to a finite temperature description of the relevant phenomena.

In an alternative way, we shall study the problem using Umezawa’s formalism, known as Thermo Field Dynamics (TFD) [5]. In TFD, the quantum statistical average of a physical observable in a given ensemble is identified with its expectation value in a thermal vacuum. In this approach, temperature is introduced as an input in the eigenstates of the number operator associated to the quantized field.

Our idea is to investigate the thermal propagator of the electromagnetic field in the Coulomb gauge in order to identify the symmetric correlation functions and susceptibilities of DDC formalism. This is the matter of section III, after a brief presentation of the main results of DDC construct in section II. In section IV we investigate the temperature dependence of the Lamb-Retherford energy shift of an atomic electron in the presence of a thermal
radiation field. In section V we calculate the variation rate of the mean atomic energy and discuss the stability of the ground state at finite temperature. In both cases, we are assuming that the whole system is in thermodynamic equilibrium. Finally, in section VI, we draw some conclusions.

II. Radiation considered as a Reservoir

In Dalibard, Dupont-Roc and Cohen-Tannoudji [3] construct, the interaction between an atom and the free electromagnetic field can be seen as the interaction of a microscopic system $S$ with a large reservoir $R$, in the sense that $R$ has many degrees of freedom and the correlation time among observables of $R$ is small, allowing a perturbative treatment of the effect due to the coupling of $S$ and $R$.

Considering $S$ an atom fixed at the origin of the coordinate system and $R$ an homogeneous and isotropic broadband radiation field they addressed, among others, the problem of atomic radiative corrections as the Lamb shift and the dynamic AC Stark effect. In particular \(^1\), they showed that the shift in an atomic energy level (say $a$) caused by its interaction with the radiation field can be expressed as

\[
(\delta H_{Sa})^{fr} = -\frac{1}{2\hbar} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \hat{\chi}'_{Sa}(\omega) \hat{C}_R(\omega),
\]

\[
(\delta H_{Sa})^{rr} = -\frac{1}{2\hbar} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \hat{\chi}'_{R}(\omega) \hat{C}'_{Sa}(\omega),
\]

where $\hat{C}_{Sa}(\omega)$ (resp. $\hat{C}_R(\omega)$) and $\hat{\chi}_{Sa}(\omega)$ (resp. $\hat{\chi}_R(\omega)$) are, respectively, the symmetric correlation and the even parity part of the susceptibility functions related to the atomic system (resp. reservoir) in frequency space. Their true meaning is well established in the context of DDC formalism which associates (1) to the reservoir fluctuation effects and (2) to the radiation reaction effects.

Our main concern is the fact that such statistical functions are defined from two point functions of the dynamic operators involved in the interaction Hamiltonian $H_I$. In the case

\(^1\)Here the total hamiltonian is given by $H = H_S + H_R + H_I$, where $H_S$ describes the atomic system, $H_R$ the radiation field and $H_I$ is the interaction Hamiltonian.
we are interested in, the interaction Hamiltonian in the Coulomb gauge is given by

\[ H_I = -\left( \frac{e}{m} \right) \mathbf{p} \cdot \mathbf{A}(0), \]  

where \( \mathbf{p} \) is the momentum associated to the electron’s motion and \( \mathbf{A}(0) \) is the electromagnetic potential in the dipole approximation. The two point function for the spatial component of the field variable \( A_i(0, t) = A(t) \ (i = x, y, z) \) is given by

\[ g(\tau) = \frac{1}{2} \langle \{ A(t'), A(t'') \} \rangle_A + \frac{i}{2} \langle [A(t'), A(t'')/i] \rangle_A , \]  

where \( \tau = t' - t'' \) and \( \langle \cdot \rangle_A \) indicates an average on the reservoir state defined by a given statistical weight. As pointed before, the first term in (4) corresponds to the symmetric correlation function and the second is related to the linear susceptibility of the reservoir. The symmetric correlation function of the observable \( A(t) \),

\[ C_R(\tau) = \frac{1}{2} \langle \{ A(t'), A(t'') \} \rangle_A , \]  

is real and tends to the ordinary correlation function in the classical limit. It gives a physical description of the dynamics of fluctuations of the observable \( A(t) \). The other statistical function is the linear susceptibility \( \chi_R(\tau) \), which characterizes the reservoir response to an external perturbation, defined by

\[ \chi_R(\tau) = \frac{i}{\hbar} \theta(\tau) \langle [A(t'), A(t'')] \rangle_A = \frac{2}{\hbar} \theta(\tau) \text{Im} \ g(-\tau) , \]  

where \( \theta(\tau) \) is the step function.

Since we are interested in analyzing the finite temperature dependency of (1) and (2), we postpone the calculation of the above statistical functions for the field components to the next section, where we shall employ the TFD formalism in order to obtain the finite temperature two point functions for the radiation field.

Nevertheless, we restrict ourselves to the present action of the corresponding correlation and susceptibility functions in frequency space for the \( x \) component\(^2\) of the atomic variable

\(^2\)It can be shown that for \( i \neq j \) the statistical functions vanish \([9]\).
(e\mathbf{p}/m)$, in the situation the atom is found in a given state $|a\rangle$ (with $H_S|a\rangle = E_a|a\rangle$), namely
\begin{equation}
\hat{C}^{xx}_{Sa}(\omega) = \sum_b \frac{e^2}{m^2} |\langle a|p_x|b\rangle|^2 \pi \left[ \delta(\omega_{ab} + \omega) + \delta(\omega_{ab} - \omega) \right],
\end{equation}
\begin{equation}
\hat{\chi}^{\prime xx}_{Sa}(\omega) = \sum_b -\frac{e^2}{\hbar m^2} |\langle a|p_x|b\rangle|^2 \left[ \mathcal{P} \frac{1}{\omega_{ab} + \omega} + \mathcal{P} \frac{1}{\omega_{ab} - \omega} \right],
\end{equation}
\begin{equation}
\hat{\chi}^{\prime\prime xx}_{Sa}(\omega) = \sum_b \frac{e^2}{\hbar m^2} |\langle a|p_x|b\rangle|^2 \pi \left[ \delta(\omega_{ab} + \omega) - \delta(\omega_{ab} - \omega) \right].
\end{equation}
Expressions (8) and (9) are obtained by splitting the atomic susceptibility according to
\[ \hat{\chi}_{Sa}(\omega) = \hat{\chi}^{\prime xx}_{Sa}(\omega) + i \hat{\chi}^{\prime\prime xx}_{Sa}(\omega), \]
where each part characterizes, respectively, the response in phase and in quadrature at the frequency $\omega$. In expression (8), $\mathcal{P}$ denotes the principal value.

### III. Thermal Correlation Functions and Susceptibilities

In this section we study the thermal propagator of the electromagnetic field of Thermo Field Dynamics (TFD). Our idea is to define the statistical functions $C_R(\omega)$ and $\chi'_R(\omega)$ from the appropriate propagator of QED, implementing temperature at the beginning. We start by writing the quantized electromagnetic potential $A_i(t)$ as
\[ A_i(t) = A_i^{(+)}(t) + A_i^{(-)}(t), \]
where $A_i^{(+)}(t)$ and $A_i^{(-)}(t)$ are the components with positive and negative frequency, defined respectively as
\begin{equation}
A_i^{(+)}(t) = \sum_{k,r} \alpha_k \epsilon^r_i(k) a^r_k e^{-i\omega_k t},
\end{equation}
\begin{equation}
A_i^{(-)}(t) = \sum_{k,r} \alpha_k \epsilon^r_i(k) a^{r\dagger}_k e^{i\omega_k t},
\end{equation}
with
\[ \alpha_k = \left( \frac{\hbar}{2\varepsilon_0 L^3 \omega_k} \right)^{1/2}. \]

In TFD we double the field degrees of freedom introducing the tilde conjugated of $A_i(t)$ [5] [6]. Using the thermal doublet notation [6] [7], we obtain
\[ A_i(t) = \left( \begin{array}{c} A_i(t) \\ \tilde{A}_i(t) \end{array} \right), \quad \tilde{A}_i(t) = (A_i(t), -\tilde{A}_i(t)), \]
\[ \tilde{A}_i(t) = \left( \begin{array}{c} \tilde{A}_i(t) \\ -A_i(t) \end{array} \right). \]
\[ ^3 \text{As in the last section, we assume that the atom is at rest at the origin of the coordinate system (r = 0) and that we are in the dipole approximation.} \]
where (−) denotes the transposed and

\[ A_i(t) = \sum_{k,r} \alpha_k e_i^r(k) \left( a_k^r e^{-i\omega_k t} + a_k^{r\dagger} e^{i\omega_k t} \right) = A_i^{(+)}(t) + A_i^{(-)}(t), \]  
(15)

\[ \tilde{A}_i(t) = \sum_{k,r} \alpha_k e_i^r(k) \left( \tilde{a}_k^r e^{i\omega_k t} + \tilde{a}_k^{r\dagger} e^{-i\omega_k t} \right) = \tilde{A}_i^{(+)}(t) + \tilde{A}_i^{(-)}(t). \]  
(16)

By construction, both fields \( A_i \) and \( \tilde{A}_i \) are independent; the corresponding absorption and emission operators satisfy the algebra [6]

\[ [a_k^r, a_s^{r\dagger}] = [\tilde{a}_k^r, \tilde{a}_s^{r\dagger}] = \delta_{k,k'} \delta_{r,s}. \]  
(17)

At zero temperature, the vacuum state is given by the direct product \( |0\rangle_A \otimes |0\rangle_{\tilde{A}} \equiv |0\rangle \). Using (17), it follows that

\[ A_i^{(+)} |0\rangle = 0, \quad \tilde{A}_i^{(+)} |0\rangle = 0. \]  
(18)

In order to find the thermal propagator associated with the statistical functions, we must calculate the commutator

\[ [A_i(t'), \tilde{A}_j(t'')]_{\mu\nu} = \Delta_{ij}^{\mu\nu}(\tau), \]  
(19)

where \( \mu, \nu = 1, 2 \) and \( i, j = x, y, z \). The anti-diagonal components of the above quantity are identically zero when their expectation value in the \( |0\rangle \) state is taken. The component \( \mu = \nu = 1 \) can be written as

\[ \Delta_{ij}^{11}(\tau) = \Delta_{ij}^{11}(+) (\tau) + \Delta_{ij}^{11}(−) (\tau), \]  
(20)

where

\[ \Delta_{ij}^{11}(+) (\tau) \equiv [A_i^{(+)}(t'), A_j^{(-)}(t'')] \],  
(21)

\[ \Delta_{ij}^{11}(−) (\tau) \equiv [A_i^{−}(t'), A_j^{(+)}(t'')] \].  
(22)

Now, using (11), (12), (17) and (18), we obtain,

\[ \Delta_{ij}^{11}(+) (\tau) = \sum_{k,r} \alpha_k^2 e_i^r(k) e_j^r(k) e^{-i\omega_k \tau}, \]  
(23)
\[ \Delta^{11}_{ij}(-\tau) = -\sum_{k,r} \alpha_k^2 e_i^r(k) e_j^r(k) e^{i\omega_k\tau}. \tag{24} \]

From (23) and (24), we can define two functionals:

\[ \Delta^{11}_{ij(\text{ret})}(\tau) \equiv \theta(\tau)\Delta^{11}_{ij(+)}(\tau) + \theta(\tau)\Delta^{11}_{ij(-)}(\tau) = \Delta^{11}_{ij(\text{ret})(+)}(\tau) + \Delta^{11}_{ij(\text{ret})(-)}(\tau), \tag{25} \]

and

\[ \Delta^{11}_{ij (1)}(\tau) \equiv \Delta^{11}_{ij(+)}(\tau) - \Delta^{11}_{ij(-)}(\tau). \tag{26} \]

It can be easily shown that, at zero temperature,

\[ \Delta^{11}_{ij (1)}(\tau) = \langle 0|\{A_i(t'), A_j(t'')\}|0 \rangle. \tag{27} \]

So, we should point out that the quantity \( g(\tau) \), defined before as a two time average of a given observable, is associated in the present case with the functional

\[ g_{ij}(\tau) = \langle 0|A_i(t')A_j(t'')|0 \rangle = \frac{1}{2}\Delta^{11}_{ij(1)}(\tau) + \frac{1}{2}\Delta^{11}_{ij(-)}(\tau), \tag{28} \]

according to expression (4).

By taking the Fourier transform of (25) and (26) we obtain, respectively,

\[ \Delta^{11}_{ij(\text{ret})}(\omega) = \sum_{k,r} \alpha_k^2 e_i^r(k) e_j^r(k) \left[ \left( \frac{i}{\omega - \omega_k + i\epsilon} \right) - \left( \frac{i}{\omega + \omega_k + i\epsilon} \right) \right]. \tag{29} \]

\[ \Delta^{11}_{ij (1)}(\omega) = \sum_{k,r} \alpha_k^2 e_i^r(k) e_j^r(k) \pi \left[ \delta(\omega + \omega_k) + \delta(\omega - \omega_k) \right]. \tag{30} \]

Adopting the same procedure, we can extend the above calculation to the component \( \mu = \nu = 2 \). As a result, we have

\[ \Delta^{22}_{ij(\text{ret})}(\omega) = \sum_{k,r} \alpha_k^2 e_i^r(k) e_j^r(k) \left[ \left( \frac{i}{\omega - \omega_k - i\epsilon} \right) - \left( \frac{i}{\omega + \omega_k - i\epsilon} \right) \right], \tag{31} \]

\[ \Delta^{22}_{ij (1)}(\omega) = -\sum_{k,r} \alpha_k^2 e_i^r(k) e_j^r(k) \pi \left[ \delta(\omega + \omega_k) + \delta(\omega - \omega_k) \right]. \tag{32} \]

We may write expressions (29) and (31) in a more compact notation, i.e.,

\[ \Delta_{ij (ret)}(\omega) = \sum_{k,r} \alpha_k^2 e_i^r(k) e_j^r(k) \left\{ \frac{i}{k_0 - \omega_k + i\tau_3\epsilon} - \frac{i}{k_0 + \omega_k + i\tau_3\epsilon} \right\} \tag{33} \]
and, in the same way, we write (30) and (32) as

\[ \Delta_{ij}^{(1)}(\omega) = -\sum_{k,r} \alpha_k^2 e_i^r(k)e_j^r(k) \pi \tau_3 [\delta(\omega + \omega_k) + \delta(\omega - \omega_k)], \]  

(34)

where, in the last two expressions,

\[ \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \]  

(35)

In TFD, it is known that the propagator at zero temperature is related to the one calculated in the thermal vacuum through a Bogoliubov transformation [8]. Applying this result to (33) and (34), we obtain, respectively,

\[ \Delta_{ij}^{\mu\nu}(\omega) = \{ B_k^{-1}(\beta) \Delta_{ij}^{(ret)}(\omega) B_k(\beta) \}^{\mu\nu}, \]  

(36)

\[ \Delta_{ij}^{\mu\nu}(\omega) = \{ B_k^{-1}(\beta) \Delta_{ij}^{(1)}(\omega) B_k(\beta) \}^{\mu\nu}, \]  

(37)

where \( B_k(\beta) \) is given by

\[ B_k(\beta) = (1 - n_k)^{1/2} \left( \frac{1}{f_k^{1-\alpha}} - \frac{f_k^\alpha}{1} \right), \]  

(38)

with \( \alpha = 1/2, \ f_k = \exp[-\hbar \omega_k/\beta] \) and

\[ n_k = \frac{1}{f_k^{-1} - 1} = \frac{1}{e^{\hbar \omega_k/\beta} - 1}, \]  

(39)

(\( \beta = 1/kT, \) where \( k \) is the Boltzmann constant and \( T \) the equilibrium temperature).

The \( \mu = \nu = 1 \) component of (36) is found to be

\[ \Delta_{ij}^{\beta}(ret)(\omega) = -i \sum_{k,r} \alpha_k^2 e_i^r(k)e_j^r(k) \left\{ \slashed{\mathcal{P}} \frac{1}{\omega_k - \omega} + \slashed{\mathcal{P}} \frac{1}{\omega_k + \omega} + i \pi \left[ \delta(\omega_k - \omega) - \delta(\omega_k + \omega) \right] (1 + 2n(\omega_k)) \right\}, \]  

(40)

and, from (37),

\[ \Delta_{ij}^{\beta}(1)(\omega) = \sum_{k,r} \alpha_k^2 e_i^r(k)e_j^r(k) \pi \left[ \delta(\omega - \omega_k) + \delta(\omega + \omega_k) \right] (1 + 2n(\omega_k)). \]  

(41)

Now, relating (5) to (41) and (6) to (40), we are in position to define the thermal correlation function and susceptibilities,

\[ C_{ij}^{\beta}(\omega) \equiv \Delta_{ij}^{\beta}(1)(\omega), \]  

(42)
\[ \chi_{ij}^\beta(\omega) = \frac{i}{\hbar} \Delta_{ij}^{11}(\omega) \]  
(43)

where, again, we split (43) as

\[ \chi_{ij}^\beta(\omega) = \chi_{ij}'^\beta(\omega) + i \chi_{ij}''^\beta(\omega), \]  
(44)

\[ \chi_{ij}'^\beta(\omega) = \frac{1}{\hbar} \sum_{k,r} \alpha_k^2 e_i^k(k) e_j^r(k) \left( \mathcal{P} \frac{1}{\omega_k - \omega} + \mathcal{P} \frac{1}{\omega_k + \omega} \right), \]  
(45)

\[ \chi_{ij}''^\beta(\omega) = -\frac{1}{\hbar} \sum_{k,r} \alpha_k^2 e_i^k(k) e_j^r(k) \pi \left( 1 + 2n(\omega_k) \right) \left[ \delta(\omega_k + \omega) - \delta(\omega_k - \omega) \right]. \]  
(46)

Choosing \( i = j = x \) and substituting the summation over modes by a polarization sum and an integral in \( k \), we obtain

\[ C_{xx}^\beta(\omega) = \frac{1}{3\pi \epsilon_0 c^3} \int_0^{\omega_M} d\omega' \ h \omega' (n(\omega') + 1/2) \left[ \delta(\omega' - \omega) + \delta(\omega' + \omega) \right] \]  
(47)

\[ \chi_{xx}'^\beta(\omega) = \frac{1}{6\pi \epsilon_0 c^3} \int_0^{\omega_M} d\omega' \omega' \left( \mathcal{P} \frac{1}{\omega' - \omega} + \mathcal{P} \frac{1}{\omega' + \omega} \right), \]  
(48)

\[ \chi_{xx}''^\beta(\omega) = \frac{1}{6\pi \epsilon_0 c^3} \int_0^{\omega_M} d\omega' \omega' \left( 2n(\omega') + 1 \right) \left[ \delta(\omega' + \omega) - \delta(\omega' - \omega) \right] \]  
(49)

IV. The Lamb-Retherford Shift via TFD

We now apply the above results to the case of Lamb-Retherford shift and discuss the related thermal radiative effects. As mentioned in section I, this is done by substituting (8) and (47) into expression (1) which, according to [4], represent the desirable radiative correction,

\[ h(\delta H_{Sa})^{\text{fr}} = \frac{e^2}{6\pi \epsilon_0 m^2 c^3} \sum_b \langle a|p|b \rangle^2 \times \]  
\[ \times \int_0^{\omega_M} d\omega' \omega' \langle n(\omega') + 1/2 \rangle \left[ \mathcal{P} \frac{1}{\omega_{ab} + \omega'} + \mathcal{P} \frac{1}{\omega_{ab} - \omega'} \right]. \]  
(50)

The atomic energy shift due to the field fluctuations appears as a sum of the effects \( (\delta H_{Sa})^{\text{fr}} \) of the “thermal photons”, proportional to \( \langle n(\omega) \rangle \), and that of the vacuum fluctuations \( (\delta H_{Sa})^{\text{fv}} \), corresponding to the “\( \hbar \omega/2 \) by mode”. This last term can be manipulated using the relations

\[ \int_0^{\omega_M} \omega' d\omega' \mathcal{P} \frac{1}{\omega' \pm \omega_0} = \omega_M \mp \omega_0 \ln \frac{\omega_M}{\omega_0} + O \left( \frac{\omega_0}{\omega_M} \right) \]  
(51)
\[
\int_0^{\omega_M} \omega' \, d\omega' \left[ P \frac{1}{\omega' + \omega_0} + P' \frac{1}{\omega_0 - \omega'} \right] = -2\omega_0 \ln \frac{\omega_M}{\omega_0}.
\] (52)

Hence, we obtain

\[
\hbar (\delta H_a)^{nv} = \frac{e^2}{6\pi^2 \varepsilon_0 m^2 c^3} \sum_b |\langle a|p|b \rangle|^2 (-\omega_{ab}) \ln \frac{\omega_M}{|\omega_{ab}|},
\] (53)

or

\[
\hbar (\delta H_a)^{nv} = \frac{\alpha}{3\pi} \left( \frac{\hbar}{mc} \right)^2 \left( \ln \frac{\omega_M}{c K_a} \right) |a| \frac{e^2}{\varepsilon_0} \delta(r)|aangle,
\] (54)

where \( \alpha \) is the fine structure constant and \( \hbar c K \) is the mean atomic excitation energy. Expression (54) corresponds to the (pure) Lamb-Retherford shift as found in literature [10]. It is well known that its physical origin comes from the vacuum fluctuation of the radiation field (reservoir). The presence of \( \hbar \) in (54) shows the quantum character of this effect, just as the vacuum fluctuation which gives rise to it.

The contribution \((\delta H_{Sa})^{nr'}\) proportional to \(\langle n(\omega')\rangle\) correspond to a stimulated radiative correction due to the “thermal photons”. It resembles the AC Stark effect when the thermal radiation field is substituted by a quantized electromagnetic field. In the present context \((\delta H_{Sa})^{nr'}\) corresponds to thermal radiative correction to the (pure) Lamb-Retherford shift and its effect vanishes as the temperature approach to zero.

### V. Energy Exchange

In order to analyze the effects of the thermal reservoir on the stability of the atomic ground state, we now consider the energy exchange between a bound electron and the thermal radiation field using the results of section III. Following [4], the variation rate of the mean atomic energy when the system is in its ground state (say \(a\)) is given by

\[
\frac{d}{dt} \langle H_S \rangle_a^\beta = \sum_b (E_b - E_a) \Gamma_{a \rightarrow b},
\] (55)

where \( \Gamma_{a \rightarrow b} \) represents the transition rate between the ground state \(a\) and an excited state \(b\) due to the interaction with the reservoir. It is shown in reference [3] that (55) can be written as

\[
\frac{d}{dt} \langle H_S \rangle_a^\beta = \dot{Q}_{a}^{\text{ii}} + \dot{Q}_{a}^{\text{rr}},
\] (56)
where

\[ \dot{Q}^f_\beta = \int \frac{d\omega}{2\pi} \omega \hat{C}_R^\beta(\omega) \hat{\chi}''_{S\alpha}(\omega), \quad (57) \]

\[ \dot{Q}^{rr}_\beta = -\int \frac{d\omega}{2\pi} \omega \hat{\chi}''_{R}(\omega)\hat{C}_{S\alpha}(\omega). \quad (58) \]

The last two expressions have a clear meaning: (57) is associated with the energy absorption by the system when it is affected by reservoir fluctuations and (58) is related to the damping of the atomic motion caused by the reservoir.

Using expressions (9) and (47) and taking into account the spatial components \(x, y\) and \(z\) of the electromagnetic potential, we find that (57) can be written as

\[ \dot{Q}^f_\beta = \dot{Q}^f_r + \dot{Q}^{fv} \]

\[ = \sum_b (E_b - E_a) \Gamma^{sp}_{ab}[\langle n(|\omega_{ab}|) \rangle + 1/2], \quad (59) \]

where

\[ \Gamma^{sp}_{ab} = \frac{e^2 |\langle a|p|b \rangle|^2 |\omega_{ab}|}{3\pi \varepsilon_0 \hbar m^2 c^3} \]

(60)

is the rate of spontaneous emission related to the transition between the levels \(b\) and \(a\).

The quantity \(\dot{Q}^{rr}_\beta\) is calculated in the same way from expressions (7), (49) and (58). As a result, we find

\[ \dot{Q}^{rr}_\beta = \dot{Q}^{rr}_r + \dot{Q}^{fv}_r \]

\[ = -\sum_b (E_b - E_a) \Gamma^{sp}_{ab}[\langle n(|\omega_{ab}|) \rangle + 1/2]. \quad (61) \]

Substituting (59) and (61) in (56) we conclude that

\[ \frac{d}{dt} \langle H_S \rangle^\beta_a = 0. \]

(62)

This result is what we must expect since the whole system is in thermal equilibrium at temperature \(T\). In the present context, one can say that, in thermodynamic equilibrium, the bound electron reaches a new ground state which corresponds to the original one shifted by the amount \((\delta H_{S\alpha})^\beta\). For \(T = 0\), the ground state stability still holds, since the effects of radiation reaction, \(\dot{Q}^{rr}_\infty\), are cancelled by the effects of thermal vacuum fluctuations, \(\dot{Q}^{fv}_\infty\).
VI. Concluding Remarks

In the present work we have used the structure of DDC construct to implement temperature effects via TFD. After a brief review of the main DDC results, we have investigated the propagators of the electromagnetic field in the context of TFD and derived the symmetric correlation functions and susceptibilities for the field variable $A(0, t)$. Applying the results to the case of an atomic electron interacting with a thermal radiation field, we calculate the Lamb-Retherford energy shift and the corresponding corrections due to thermal photons.

In the last section we have analyzed the energy exchange between the atomic electron and the thermal radiation field and concluded that, once the whole system is in thermodynamic equilibrium at a given temperature $T$, the stability of the ground state is maintained, even when $T$ approaches to zero.

We must point out that the original DDC formalism includes the case where the reservoir is a thermal radiation field. As remarked in [3], this is done by replacing the mean number of particles ($n(|\omega|)$) by a Bose-Einstein distribution in the resulting statistical functions. However, such procedure differs from ours in the sense that the detailed balance principle become meaningless in the context of TFD where the population dynamics between two given atomic states is not accessible.

Finally, we mention that the applicability of TFD in the scope of DDC formalism is not restricted to the problem we have just revisited. Among the physical phenomena we intend to investigate in the near future are those related to the dissipative processes in quantum optics [11] [12].

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