The numerical calculation of the viscous incompressible fluid transfer between contacting surfaces

L G Varepo\textsuperscript{1}, A V Panichkin\textsuperscript{2} and O V Trapeznikova\textsuperscript{1}

\textsuperscript{1} Omsk State Technical University, 11, Mira ave., Omsk, 644050, Russia
\textsuperscript{2} S L Sobolev Omsk branch of Institute of mathematics of the Siberian Branch of the Russian Academy of Sciences, 13, Pevtsova st., Omsk, 644043, Russia

E-mail: larisavarepo@yandex.ru

Abstract. The movement of the thin layer of the viscous incompressible fluid (VIF) between two cylinders is analysed. The numerical calculations results of VIF transfer from the engaged zone of two cylinders to porous substrates are presented. The VIF (ink) is moved along the rubberized top blanket of the first cylinder. The surface of the second cylinder contacts the substrate with some part of the VIF layer transferred from the first cylinder. The fluid is double bounded by the free surface. Images of cylinders boundary deformation and VIF flow areas are shown.

1. Introduction
The process of image printing is VIF (ink) transfer from engaged surfaces of the offset and the printing cylinders.

The known modelling methods of the VIF transfer process onto substrate are characterized by limited functional features: dates are reduced to half splitting of ink with zero absorptive capacity. So, it is conventional that the VIF layer (ink) is divided in a bisected way [1-8].

Therefore, the quantitative evaluation of the VIF transfer ratio onto a substrate in the engaged zone is actual at the moment due to wide and novel assortment of substrates and their different surface parameters.

There are not numerous literature sources about ink dusting evaluation. The information about ink dusting is experimental [9-10].

2. Task definition
As the problem, concerning the part of the VIF (ink) that remains on the ink-carried surface after each cycle of dividing, has not obtained one-valued solution it is necessary to develop the evaluation algorithm of this parameter.

Also we need to formulate the quantitative evaluation algorithm of ink dusting and the ratio of the ink amount immobilized by the printed surface.

3. Experimental methods
A simulated model of ink transfer onto the printed surface in the engaged zone was proposed for evaluation of parameters of VIF (ink) transfer between contacted zones.
The modelling was carried out using a developed algorithm of numerous solving of incompressible Navie-Stokes equations by finite-difference approximation on the two-dimensional grid and the compact template.

The flow is of the laminar nature.

4. Results and discussion

The Navie-Stokes equations for the postulated conditions of printed simulation of coordinates in accordance with system \((x, y) = (R\theta, R\cdot r)\) and system angular acceleration \(\varepsilon\) are as follows:

\[
\frac{\partial U_r}{\partial t} + U_r \frac{\partial U_r}{\partial r} + \frac{U_\theta R}{r} \frac{\partial U_r}{\partial \theta R} - \frac{(U_\theta + \omega r)^2}{r} = -\frac{1}{\rho} \frac{\partial P}{\partial r} + \nu \left( \frac{\nabla^2 U_r}{r^2} - \frac{2R}{r^2} \frac{\partial U_\theta}{\partial \theta R} \right),
\]

\[
\frac{\partial U_\theta}{\partial t} + U_r \frac{\partial U_\theta}{\partial r} + \frac{U_\theta R}{r} \frac{\partial U_\theta}{\partial \theta R} + 2U_r \omega + \varepsilon r = -\frac{1}{\rho} \frac{\partial P}{\partial \theta R} + \nu \left( \frac{\nabla^2 U_\theta}{r^2} + \frac{2R}{r^2} \frac{\partial U_r}{\partial \theta R} \right),
\]

(1)

\[
\frac{\partial U_r}{\partial r} + \frac{U_r}{r} + \frac{R}{\partial \theta R} = 0,
\]

where

\[
\nabla^2 V = -\frac{\partial U}{\partial r} + \frac{\partial^2 U}{\partial r^2} + \frac{R^2}{\partial (\partial \theta R)^2}
\]

– Laplace operator, \(\nu\) – kinematic viscosity, \(\rho\) – fluid density, \(P\) – pressure, \(\omega\) – angular rotating.

The finite-difference methods with input of moving boundary units for the second cylinder boundary and free-boundary fluid, which at the initial moment is placed on the first cylinder without relative movement, are used on the fixed grid.

The computational region is represented as a rectangle with a regular grid and similar steps. For equation (1) in changed coordinate system \((x, y) = (R\theta, R\cdot r)\) we used the stabilize error-correction scheme with inertial time step \(\tau\) and grid steps \(h_x\) and \(h_y\):

\[
\frac{V^{n+1/2} - V^n}{\tau} = \Lambda_1 \left( V^{n+1/2} - V^n \right) + \Lambda_\theta \frac{G_\theta^{n+1}}{\rho} + F^n(\mathbf{\sigma}, \rho),
\]

(2)

\[
\frac{V^{n+1} - V^{n+1/2}}{\tau} = \Lambda_2 \left( V^{n+1} - V^n \right),
\]

(3)

\[
\Lambda_1 = u \frac{\Lambda_1 + \Lambda_2}{2h_x} + v \frac{\Lambda_1 + \Lambda_2}{2h_y}, \quad \Lambda_2 = v \frac{\Lambda_2 + \Lambda_2}{2h_y} + \nu \frac{\Lambda_2 h_2}{2h_y},
\]

(4)

\(\Lambda_1, \Lambda_2, \Lambda_2, \Lambda_2\) – operators of function shift to grid step \(h_x\) or \(h_y\) up or down to axes \(x\) and \(y\); \(\Lambda_0 = \Lambda_1 + \Lambda_2; V = (u, v)\) – speed vector with components \(u, v; \quad G_\theta\) – field gradient; \(\mathbf{\sigma}\) – vector of angular rotation; \(F\) – acceleration component from external forces and from the changed coordinate system; \(n\) – time iteration number at moment \(t = n\tau\).

If approximation of these operators is of the order that does not exceed the second one \((O(h_x^2 + h_y^2))\), the pressure gradient approximation \((G_\theta)\) in (2)) will be realized by the fourth order as follows:
for which approximation of the third-order derivatives with respect to \( x \) and \( y \) from pressures \( p_x^{n+1} \) and \( p_y^{n+1} \) has been formulated on the compact template as the equation of the second-order after \( p_{xxx}^{n+1} \) and \( p_{yy}^{n+1} \) has been changed.

The pressure determination equation has been determined as a divergence-free transformation equation of a low compressive fluid model. After replacing a partial derivative of pressure with respect to time, the equation is:

\[
\varepsilon_p \frac{p_x^{n+1, k+1, l}}{r} + \nabla \cdot p^{n+1, k} = 0,
\]

where \( \varepsilon_p > 0 \) – parameter which was optimally chosen for numerical calculations of the convergence and approximation to the initial equation, \( k \) – number of additional iteration for pressure calculation, the speed divergence was presented with the forth-order approximation as:

\[
\nabla \cdot p_{ij} = \left( \frac{M_1}{12h_{ij}} \right) + O(h_x^2 + h_y^2),
\]

where

\[
M_1 = u_{i+1,j} + u_{i+1,j+1} + 4u_{i+1,j+1} - u_{i,j+1} - 4u_{i,j} - u_{i,j-1},
\]

\[
M_2 = v_{i+1,j} + v_{i+1,j+1} + 4v_{i+1,j+1} - v_{i,j+1} - 4v_{i,j} - v_{i,j-1},
\]

with main indexes:

\( (i, j), i = 1, \ldots, N_x - 1, j = 1, \ldots, N_y - 1, \)

\( N_x, N_y \) – parameters of the computational grid with respect to \( x \) and \( y \).

During movement of fluid through the contact zone, some part of fluid is penetrated into the substrate structure due to absorption. The dynamical equation of deformation was defined for some elastic layer (a substrate, a rubberized top blanket) according to arising elastic strains of cylinders.

The parameters of the VIF flow between cylinders were calculated according to:

- the deformation of surface layers of the rubberized top blanket of the first cylinder and the substrate contacting the second cylinder surface due to pressure changing inside the fluid on contact with them;
- the deformatonal influence on final fluid distribution parameters (ink) along cylinders including absorption into the substrate and dusting;
- the contact width.

The deformation of centres \( \chi_c \) (through the thickness) of the thin layers of rubber and paper were calculated using shortcut evaluation according to a low speed of relative movement of compressed surfaces compared with wave-induced stresses (several hundred times)

\[
H S \chi_c \sim 2 - \frac{E}{H} (\Delta u/2 + x_c) S + (P_l - P_0) \cdot S
\]

where \( \Delta u \) – boundary deformation value at some moment \( t_1 \), \( H \) – initial thickness of the stressed layer, \( S \) – small square near a desirable point of layer deformation, \( E \) – Young’s modulus of rubber or paper elasticity, \( P_0 \) – atmosphere pressure, \( P_l \) – pressure on the stressed layer, \( \rho \) – fluid density.

Solution (9) with accuracy at current rotating speed for choosing a boundary point with velocity \( \dot{\chi}_{c,1} \) at moment \( t_1 \) is:
\[
x_c = \frac{v^2}{Hp} \left[ (P_f - P_0) \frac{E\Delta r}{H^2} \right] \left( 1 + \frac{E v^2}{Hp^2} \right) \left[ \frac{\frac{\Delta^2}{H^2}}{1 + \frac{E v^2}{Hp^2}} \right]^{\frac{1}{2}}.
\]

(10)

Signs ‘+’ or ‘−’ for (10) are chosen according to the sign of \( \Delta r \); it shows the boundary deformation direction with value \( \Delta r + 2x_c \).

The numerical solutions on 80 \( \times \) 80 grid are shown in figure 1 calculated for the following conditions: the angular rotation is 10 radians per second at different moments, Young’s modulus of rubber or paper elasticity is \( E_1 = 2.9 \times 10^7 \) N/m\(^2\) and \( E_2 = 4.8 \times 10^6\) N/m\(^2\), cylinders radiuses \( R_1 = R_2 = 0.30 \) m, \( P = P_{atm} = 10^5\) N/m\(^2\), kinematic viscosity \( \nu = 0.012\) m\(^2\)/sec, density \( \rho = 10^3\), fluid surface tension \( \sigma = 0.03\) N/m.

The algorithm works with data describing a two-dimensional fluid flow within the required range [11-12]. The presented equation of the ink transfer process was limited by fixed time value during different printing stages (Figure 1).

Images of stress boundaries of contacting surfaces in the VIF transfer areas onto the substrate are presented in the figures below: (1) – ink input in the engaged zone; (2), (3) – ink transfer onto the substrate in the engaged zone; (4), (5), (6) – ink output from the engaged zone. The results of the numerous modeling show that the ink quantity on the offset cylinder is less than 50 % (Table 1).

| Roughness \( (R_a) \), \( \mu m \) | Ink amount on offset cylinder, % | Misting ink amount, % | Total ink quantity on print after engagement zone, % |
|-------------------------------|---------------------------------|----------------------|---------------------------------------------------|
| 0.427                         | 42.52                           | 2.07                 | 55.31                                             |
| 2.310                         | 35.03                           | 2.24                 | 62.73                                             |
| 3.320                         | 32.13                           | 3.66                 | 64.21                                             |

The results of the numerical exam of the ink misting are presented in Table 1. The dependence of the ink quantity on the arithmetical mean deviation value of the profile is \( R_a \) (Table 1).

The analysis of the relationship of the ink quantity located on the printed surface in the engaged zone and microgeometry parameters has revealed their high correlation (Figure 2). Results of the dusting evaluation are presented in Table 1.

![Figure 2](image-url)

**Figure 2.** The dependence of the ink quantity transferred onto the printed \( k_{ink} \) surface from average profile deviation \( R_a \).
Figure 1. Boundary geometry of VIF and cylinders at different $t$. Graphical visualization of the ink splitting process at the outlet of the printed contact zone.
5. Conclusion

The simulated model of the VIF transfer process between contacting surfaces with different microgeometry according to deformation within the engaged zone has been developed.

The quantitative evaluation of VIF (ink) transfer parameters – the ink ratio on the offset cylinder after the engaged zone; the ink transfer ratio onto the printed material including ink absorbed inside paper pores and the ink quantity immobilized by the printed surface; dusting ratio – has been performed.

Reference:
[1] Zettelmoyer A C, Scarr R F and Schaeffer W D 1958 *International Bulletin for the Printing and Allied Trades* **13** 88–94
[2] Meyers R R, Miller J C and Zettelmoyer A C 1959 *J. Colloid Sci.* **14** 3 287–299
[3] Fetsko J M, Schaeffer W D and Zettelmoyer A C 1962 13th *TAPPI Testing Conf.* 1–6
[4] Truman A B and Hudson F 1962 *Pulp and Paper Magazine of Canada* **6** 299–306
[5] Mac Phee J 1998 *Fundamentals of Lithographic Printing* (Pittsburgh: GATFPress) p 365
[6] Ozaki Y and Kimura M 2000 *Appita J.* **3** 216–219
[7] Koivula H, Preston J S, Heard P J and Toivakka M 2008 *Colloids and Surfaces A: Physicochemical and Engineering Aspects* **317** 1-3 557–567
[8] Reis Jr. N C, Griffiths R F and Santos J M 2004 *J. Comput. Phys.* **198** 747–770
[9] Vlachopoulos G, Claypole T and Bould D 2010 *IARIGAI 2010 Proceedings: Advances in Printing and Media Technology* **37** 227–234
[10] Claypole J, Williams P R and Deganello D 2012 *IARIGAI 2012 Proceedings: Advances in Printing and Media Technology* **39** 207–211
[11] Panichkin A and Varepo L 2014 *International Multidisciplinary Microscopy Congress, Springer Proceedings in Physics* **154** 79–83
[12] Varepo L and Panichkin A 2014 *Dynamics of Systems, Mechanisms and Machines* 7005689