Method of quasienergies and transparency of multilevel atomic systems

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Abstract. We consider a medium of multilevel atomic systems interacting with radiation pulses. A relatively simple technique of analytic calculations is proposed, which allows revealing all necessary conditions (with sufficient conditions to be checked separately) imposed on the interaction parameters, for which the mean dipole moment of a multilevel atomic medium vanishes, i.e., the medium becomes transparent via adiabatic interaction. The proposed technique is based on the method of quasienergies and illustrated for three- and five-level atomic systems.

1. Introduction

Adiabatic processes are a specific sub-class of coherent interactions, which rely on adiabatic evolution of the quantum system. As the evolution is slow, the system typically remains in a certain laser-controlled quantum state, which is an eigenstate of the interaction Hamiltonian. This yields a stable situation. An example of such states is the so-called dark-state, formed in three-level systems; it lies in the basis of many widely studied coherent phenomena [1]. Dark-state involves, due to quantum interference, only two of three atomic states; in particular, in a Λ-system it does not involve the upper atomic state leading thus to the medium transparency.

Note that the dark state is not the only state resulting in the medium transparency. For example, for V-systems, a medium is well-known [2] to become transparent when interacting with degenerate pumping; this transparency is not associated with the dark state, but is caused by interference of coherences leading to compensation for the medium dispersion.

In the present work we apply a relatively simple technique of analytic calculations of propagation of light pulses in a medium consisting of multilevel atomic systems; the technique allows us to determine necessary conditions which should be imposed on the interaction parameters, for which the mean dipole moment of the medium vanishes, i.e., the medium becomes transparent via adiabatic interaction. The proposed technique is illustrated for a five-level atomic system.

2. Formalism

We assume that considered pulses have durations much shorter than all times of relaxation and at the same time much longer than the inverse frequency distance between the closest quasienergies, in order to ensure the adiabaticity of interaction.

Adiabatic propagation of pulses in an atomic medium is described by a self-consistent system of truncated equations in running coordinates $x$ and $t-x/c$, which may be represented in the form [3]
\[
\frac{\partial E_j}{\partial x} = -i \frac{2\pi N \omega_j \hbar}{c} \frac{\partial \lambda}{\partial E_j} \tag{1}
\]

Here \( \lambda \) is the eigenvalue of the interaction Hamiltonian, \( E \) the complex amplitude of the pulse field, and other notations are conventional. If the system has a quasienergy which remains constant during the overall time of interaction, i.e., \( \partial \lambda / \partial E = 0 \), then according to (1), the induced dipole moment in such a system is zero and the pulse propagates in such medium without change in shape at the group velocity equal to \( c \). It is known [4] that the quasienergies (eigenvalues) of the system are the roots of characteristic equation

\[
\det(H - \lambda I) = 0 \tag{2}
\]

where \( I \) is the unit matrix and \( H \) the interaction Hamiltonian which may in the resonance approximation be represented in the form

\[
H = \sum \sigma_{ii} \delta_{i-1} + (\sum \sigma_{ii} + \Omega_i + h.c.) \tag{3}
\]

with \( \sigma_i \) being the projection matrices, \( \Omega_i \) the Rabi frequencies of corresponding transitions \( i \rightarrow i+1 \), and \( \delta_{i-1} \) the \((i-1)\)-photon detuning \((\delta_0 = 0)\). By differentiating the characteristic equation with respect to \( E_j \) and setting zero the derivatives of quasienergy, we obtain a system of algebraic equations, from which the needed values of interaction parameters may be determined. The problem can, however, be simplified. Really, at turning off the fields the interaction Hamiltonian is diagonalized and the roots of equation (2) go to the following constant values:

\[
\lambda_i \rightarrow \delta_i \tag{4}
\]

The problem of determination of necessary (but not always sufficient) conditions for medium transparancy is thus reduced to the search of such parameters of adiabatic interaction for which the system quasienergy remains equal to one of resonance detunings always during interaction. By substituting these values successively into equation (2) we obtain the needed conditions for the interaction parameters.

For example, in case of a three-level system the root \( \lambda = 0 \) is realized at exact two-photon resonance \( \delta_2 = 0 \) and under the same condition the root \( \lambda = \delta_2 = 0 \) is realized. The wave function corresponding to these quasienergies (dark state) is well known [5]:

\[
|\psi\rangle = \cos \theta |1\rangle - \sin \theta |3\rangle \tag{5}
\]

where \( \tan \theta = \Omega_1 / \Omega_2 \). As the population of the level \( |2\rangle \) is zero, the dipole moments of \( 1 \rightarrow 2 \) and \( 2 \rightarrow 3 \) transitions also vanish.

![Fig.1. Three-level Λ-, V-, and ladder-systems.](image)

The third root \( \lambda = \delta_1 \) is realized at degenerate pumping \((\Omega_1 = \Omega_2)\) and the condition for detunings \( \delta_2 = 2\delta_1 \). As distinct from the previous case, the wave function involves all bare atomic states, however
the medium becomes transparent because of destructive interference of dipole moments of transitions $1 \rightarrow 2$ and $2 \rightarrow 3$. Similar scheme of interaction may be realized also in V-system (see Fig.1), if the atom is prepared in state 2. It should be noted that in case of non-degenerate pumping the root $\lambda=\delta_1$ does not provide transparency, since the dipole moments of the transitions $1 \rightarrow 2$ and $2 \rightarrow 3$ do not equal zero individually. So, the condition $\lambda=\delta_1$ is indeed necessary, but not sufficient condition.

3. Five-level atomic system

We use now the proposed technique for a five-level atomic system and the pulses resonant with only the adjacent transitions. Several examples of such diagrams are demonstrated in Fig.2.

![Fig.2. Several five-level configurations and relevant Rabi frequencies.](image)

Under conditions of exact two-photon resonances ($\delta_3=0$, $\delta_4=0$) a dark state is realized in the system (see, e.g., [7]) which corresponds to the root $\lambda=0$:

$$|\psi\rangle = \cos \theta_1 \cos \theta_2 |1\rangle - \sin \theta_1 \cos \theta_2 |3\rangle + \sin \theta_1 \sin \theta_2 |5\rangle$$  \hspace{1cm} (6)

where $\tan \theta_1 = \Omega_1/\Omega_3: \tan \theta_2 = \Omega_2/\Omega_4$.

The value $\lambda=\delta_1$ can be realized under conditions $\delta_1=\delta_2=\delta_3/2$ and degenerate pumping at the transition $1 \rightarrow 3$ ($\Omega_1=\Omega_2=\Omega$). The wave function does not contain the level 4 and the dipole moments of transitions $1 \rightarrow 2$ and $2 \rightarrow 3$ are in antiphase leading to their destructive interference:

$$|\psi\rangle = \frac{1}{R} (\sin \Phi \cos \theta_2 |1\rangle + \cos \Phi \cos \theta_2 |2\rangle - \sin \Phi \cos \theta_2 |3\rangle + \sin \Phi \sin \theta_2 |5\rangle)$$  \hspace{1cm} (7)

where $R=(\sin^2 \Phi + \cos^2 \theta_2)^{1/2}$ and $\tan 2\Phi = \Omega/\delta_1$. For realization of this state the atom should be initially prepared in state $|2\rangle$. This state may efficiently be used in atomic level configurations of Fig.2b or of Fig.2c. So, with use of counterintuitive sequence of pulses at transitions $3 \rightarrow 4$ and $4 \rightarrow 5$ with a longer pulse at transition $1 \rightarrow 3$ (see Fig.3), it is possible to transfer the population from level 2 to level 5. In this case a population inversion is produced between the levels 5 and 4, which may be used for amplification of radiation in case of the last diagram of Fig.2.
The value \( \lambda = \delta_1 \) may be realized also under other conditions, e.g., at \( \delta_1 = \delta_2 = \delta_3 \), \( \delta_4 = 2\delta_1 \) and degenerate pumpings of transitions \( 1 \rightarrow 3 \) and \( 3 \rightarrow 5 \) (\( \Omega_1 = \Omega_2 \) and \( \Omega_3 = \Omega_4 \)). In this case the wave function involves all bare atomic states, but dipole moments of both degenerate transitions are zero:

\[
|\psi\rangle = \frac{1}{R} (\sin \Phi_1 \sin \Phi_2 |1\rangle + \cos \Phi_1 \sin \Phi_2 |2\rangle - \sin \Phi_1 \sin \Phi_2 |3\rangle - \sin \Phi_1 \cos \Phi_2 |4\rangle + \sin \Phi_1 \sin \Phi_2 |5\rangle)
\]

where \( R = (\sin^2 \Phi_1 + \sin^2 \Phi_2 + \sin^2 \Phi_1 \sin^2 \Phi_2)^{1/2} \) and \( \tan 2\Phi_1 = \Omega_1 / \delta_1, \tan 2\Phi_2 = \Omega_3 / \delta_3 \).

The atom should be initially in state \( |2\rangle \) and the pulses turn on in counterintuitive sequence (see Fig.3). A similar state may be obtained when \( \Omega_1 = \Omega_4 \) and \( \Omega_3 = \Omega_2 \).

The value \( \lambda = 0 \) also may be realized in different ways. For example, instead of \( \delta_2 = \delta_4 \) we can require vanishing of three-photon detuning, \( \delta_3 = 0 \), and \( \delta_2 = -\delta_4 \neq 0 \). In this case, if pumping of the \( 3 \rightarrow 5 \) transition is degenerate (\( \Omega_3 = \Omega_4 \)), we obtain

\[
|\psi\rangle = \frac{1}{R} (\sin \Phi_3 \cos \theta_1 |1\rangle - \sin \Phi_3 \sin \theta_1 |3\rangle + \cos \Phi_3 \sin \theta_1 |4\rangle + \sin \Phi_3 \sin \theta_1 |5\rangle)
\]

where \( R = (\cos^2 \Phi_3 + \sin^2 \theta_1)^{1/2} \) and \( \tan \Phi_3 = -\Omega_3 / \delta_3 \). The atom is initially in the state \( |4\rangle \), while the state \( |2\rangle \) does not enter the superposition.

Fig.3. Sequence of pulses for efficient transfer of population from level 2 to 5.

In addition to the cases considered above the value \( \lambda = \delta_2 \) can be obtained under conditions \( 2\delta_2 = \delta_1 + \delta_3 = \delta_4 \), \( \Omega_1 = \Omega_4, \Omega_3 = \Omega_2 \) and the value \( \lambda = \delta_3 \) under conditions \( \delta_1 = \delta_2, \ 3\delta_3 = \delta_4, \ \Omega_1 = \Omega_2 \).

4. Conclusion

We have developed and analyzed, for the example of a five-level system, a relatively simple method for finding necessary conditions under which adiabatic short light pulses can travel in the medium without distortion of the shape and phase. It was shown that the necessary conditions for such bleaching of medium (i.e., vanishing of dipole moments induced in the medium at the frequencies of all interacting fields) is the equality of system quasienergies to one of the values of single- or multi-photon detunings of resonances. In this case the medium bleaching may be caused by both interference of quantum states and interference of dipole moments of different transitions if pumping is degenerate. The existence of such regimes means that at the propagation lengths where the interaction adiabaticity does not break, stable superposition states are produced all over the medium and these states may easily be controlled by adjusting the parameters of pulses. Finding of the sufficient conditions requires determination of roots of algebraic equations which may be done only numerically for the order of
equation higher than three. However, the sufficiency of found necessary conditions may easily be checked directly by calculation of the induced dipole moments.

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