Integrable Systems on Flag Manifold and Coherent State Path Integral

Myung-Ho Kim*

Department of Mathematics, Sung Kyun Kwan University, Suwon 440-746, KOREA

Phillial Oh†

Department of Physics, Sung Kyun Kwan University, Suwon 440-746, KOREA

Abstract

We construct integrable models on flag manifold by using the symplectic structure explicitly given in the Bruhat coordinatization of flag manifold. They are non-commutative integrable and some of the conserved quantities are given by the Casimir invariants. We quantize the systems using the coherent state path integral technique and find the exact expression for the propagator for some special cases.

*E-mail address:mhkim@yurim.skku.ac.kr

†E-mail address:ploh@yurim.skku.ac.kr
It is well known that there is a natural symplectic structure on the coadjoint orbits of Lie group $[1]$ and it can be used to define the generalized Poisson bracket to describe Hamiltonian systems on the coadjoint orbits $[2]$. In fact, there also exists a natural Lie group action on them and they are equipped with Poisson-Lie bracket $[2]$. The geometrical construction of completely integrable systems on such symplectic manifolds is very interesting in view of the recent developments in the theory of finite-dimensional integrable models $[3]$. In this paper, we construct integrable models on flag manifold which is the maximal coadjoint orbit of $SU(N)$ group and quantize the system using the coherent state path integral technique $[4]$. Physically, these systems correspond to $SU(N)$ spin models and they provide another method of classical formulation of non-Abelian Chern-Simons particles $[5]$ and give the framework of geometric quantization for them.

We start with a brief summary of flag manifold which is essential for the presentation. More details can be found in Refs. $[6]$ and $[7]$. $SU(N)$ flag manifold $M_N$ is defined as a set $S$ of all nested sequence of linear subspaces of the standard complex $N$-space $\mathbb{C}^N$,

$$\{(C_1, C_2, \cdots, C_{N-1}) | \dim C_l = l, \ C_l \subset C_{l+1} \subset \mathbb{C}^N\}. \quad (1)$$

Then, there is a natural $SU(N)$ group action on $S$ which is transitive. Let $x_0 = (C^1, C^2, \cdots, C^{N-1}) \in S$. Then, we can see easily that $S = M_N = SU(N)/T^{N-1}$, where the maximal torus group $T^{N-1}$ is the stabilizer group at $x_0$. Let $SU_c(N)$ be the complexification of $SU(N)$ and $B_N$ a Borel subgroup of $SU_c(N)$. Then using the Iwasawa decomposition $SU_c(N) = SU(N)B_N$, we have isomorphism $M_N = SU_c(N)/B_N$. From this complex representation of $M_N$, one can prove that $M_N$ is a complex manifold. Also, there is a natural isomorphism of $M_N = SU(N)/T^{N-1}$ with the coadjoint orbit $\{gT^{-1}g^{-1} | g \in SU(N)\}$ defined by

$$gT^{-1} \rightarrow g\lambda g^{-1} \quad (2)$$

where $\lambda$ is an element of the dual Lie algebra of $T^{N-1}$. Hence, $M_N$ has the symplectic structure inherited from the coadjoint orbit $[1]$. Together with the complex structure, $M_N$ becomes a Kähler manifold.
In this paper, we will concentrate on $M_3$ to make the presentation simple. Analysis of higher $M_N$ will appear elsewhere. In order to construct integrable models explicitly, we have to coordinatize the flag manifold $M_3$. The ideal choice for the explicit construction of the symplectic structure seems to be the Bruhat coordinatization $[8]$. According to Bruhat cell decomposition $[6]$, the flag manifold $M_3$ can be covered with six coordinate patches. The convenient thing about Bruhat cell decomposition is that the largest cell provides a coordinatization $(z_1, z_2, z_3)$ of nearly all of the flag manifold missing only lower-dimensional subspaces. Also on this largest cell, the Kähler metric $ds^2 = \sum_{i,j} g_{ij} dz_i d\bar{z}_j$ and the corresponding symplectic structure

$$\Omega = i \sum_{i,j} g_{ij} dz_i \wedge d\bar{z}_j$$

(3)

can be calculated $[8]$.

The largest cell on $M_3 = SU_c(3)/B_3$ is represented as follows:

$$[g_c(z)]_{B_3} = \begin{pmatrix} 1 & 0 & 0 \\
 z_1 & 1 & 0 \\
 z_2 & z_3 & 1 \end{pmatrix}_{B_3} \mapsto (z_1, z_2, z_3)$$

(4)

with $g_c \in SU_c(3)$. Symplectic structure is given by the Kähler potential $W$ which was calculated explicitly by the holomorphic line bundle approach $[8]$ with the result

$$W = \log(1 + |z_1|^2 + |z_2|^2)^m (1 + |z_3|^2 + |z_2 - z_1 z_3|^2)^n$$

(5)

where $m, n$ are integers. Using the symplectic structure expressed in terms of $W$,

$$\Omega = i \partial \bar{\partial} W,$$

(6)

we obtain the Kähler metric given by

$$g_{ij} = \frac{\partial^2 W}{\partial z_i \partial \bar{z}_j}.$$
\{F, G\} = -i \sum_{i,k} g^{ki} \left( \frac{\partial F}{\partial z_k} \frac{\partial G}{\partial \bar{z}_i} - \frac{\partial G}{\partial z_k} \frac{\partial F}{\partial \bar{z}_i} \right) \quad (8)

where the inverse metric \( g^{ki} \) satisfies \( g_{ik} g^{kj} = \delta^j_i \).

Using the above symplectic structure, we calculate the Hamiltonian function \( F_a \) associated with the generator \( T_a \) satisfying \([T_a, T_b] = -f^c_{ab} T_c \) with the structure constants given in the Gell-Mann basis \[4\]. We first perform this calculation for \( T_3 \) which is given by

\[
T_3 = \frac{1}{2} \begin{pmatrix} i & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & 0 \end{pmatrix}.
\quad (9)
\]

Recall the definition of the Hamiltonian vector field \( X_3 \) associated with \( T_3 \):

\[
X_3 = \frac{d}{dt} \bigg|_{t=0} (\exp tT_3) \circ [g_c(z)]_B.
\quad (10)
\]

Now, by multiplying a suitable \( b \in B \), we have

\[
(\exp tT_3) \circ [g_c(z)]_B = [g_c(z')]_B = \begin{pmatrix} 1 & 0 & 0 \\ z_1' & 1 & 0 \\ z_2' & z_3' & 1 \end{pmatrix}_B.
\quad (11)
\]

A simple calculation shows

\[
z_1' = (1 - it)z_1, \quad z_2' = (1 - \frac{t}{2})z_2, \quad z_3' = (1 + \frac{t}{2})z_3,
\quad (12)
\]

from which we obtain

\[
X_3 = -i\frac{t}{2} \left( 2z_1 \frac{\partial}{\partial z_1} + z_2 \frac{\partial}{\partial z_2} - z_3 \frac{\partial}{\partial z_3} \right) + \text{c.c.}
\quad (13)
\]

The Hamiltonian function associated with the vector field \( X_3 \) is defined by \[2\]

\[
X_3|\Omega = dF_3.
\quad (14)
\]

After a little bit of algebra, we get

\[
F_3 = m \frac{2|z_1|^2 + |z_2|^2}{2L_1} + n \frac{|z_2 - z_1z_3|^2 - |z_3|^2}{2L_2}
\quad (15)
\]
where we defined

\[ L_1 = (1 + |z_1|^2 + |z_2|^2) \]
\[ L_2 = (1 + |z_3|^2 + |z_2 - z_1 z_3|^2). \] (16)

Repeating the same procedure, we get

\[ X_8 = \frac{-i}{2\sqrt{3}} (z_2 \frac{\partial}{\partial z_2} + z_3 \frac{\partial}{\partial z_3}) + c.c \]
\[ F_8 = \frac{m}{2\sqrt{3}} \frac{|z_2|^2}{L_1} + \frac{n}{2\sqrt{3}} \frac{|z_2 - z_1 z_3|^2 + |z_3|^2}{L_2}. \] (17)

We can calculate the remaining Hamiltonian functions in a similar manner and they generate Poisson-Lie algebra homomorphism [2]:

\[ \{ F_a, F_b \} = -f_{c a b} F_c, \] (18)

from which we deduce

\[ \{ F_3, F_8 \} = 0 \] (19)

and no other Hamiltonian functions commute with both \( F_3 \) and \( F_8 \).

Then, we have a system of non-commutative integrability [10]. This can be seen from the fact that

\[ F_1 = -m \frac{z_1 + \bar{z}_1}{2L_1} + n \frac{(z_2 - z_1 z_3) \bar{z}_3 + (\bar{z}_2 - \bar{z}_1 \bar{z}_3) z_3}{2L_2} \]
\[ F_2 = m i \frac{z_1 - \bar{z}_1}{2L_1} - m i \frac{(z_2 - z_1 z_3) \bar{z}_3 - (\bar{z}_2 - \bar{z}_1 \bar{z}_3) z_3}{2L_2}, \] (20)

and \( F_3 \) and \( F_8 \) generate \( \mathcal{G} = su(2) \times u(1) \) Poisson-Lie algebra with the property that

\[ \dim \mathcal{G} + \text{rank } \mathcal{G} = 6, \] (21)

which is equal to \( \dim M_3 \). In this case, the level set \( M_c = \{ x \in M : F_i = c_i, \ i = 1, 2, 3, 8 \} \) is a smooth 2-dimensional torus \( T^2 \) and it is invariant under \( F_3 \) and \( F_8 \) for some \( c_i \). Furthermore, one can find a second commutative algebra \( \mathcal{G}' \) such that \( \dim \mathcal{G}' = 3 \) [10]. It is easy to see that \( \mathcal{G}' \) is generated by \( F_3, F_8 \) and the Casimir invariant \( \frac{1}{2}(F_1^2 + F_2^2 + F_3^2) \equiv C_2. \)
The above feature of non-commutative integrability generalizes to higher flag manifold \(M_N = SU(N)/T^{N-1}\). Consider the generators of \(su(N)\) algebra \(T_a\) satisfying the commutation relations \([T_a, T_b] = -f_{ab}^c T_c\) in Gell-Mann basis. Then, repeating the same procedure as in the case of \(su(3)\), we can calculate all the Hamiltonian functions \(F_a\) associated with each of the generators \(T_a\) and these \(F_a\)'s satisfy the Poisson-Lie algebra \([18]\). Obviously, there exist \(N - 1\) commuting Hamiltonian functions \(F_3, F_8, F_{15}, \ldots, F_{N^2-1}\) which are far less than \((1/2) \dim M_N\) for higher \(N\). However, \(SU(N-1) \times U(1)\) group actions on \(M_N\) satisfy the criteria for the non-commutative integrability \([10]\):

\[
\dim \mathcal{G} + \text{rank } \mathcal{G} = \dim M_N. \tag{22}
\]

The level set \(M_{N_c} = \{x \in M_N : F_i = c_i, \ T_i \in su(N-1) \times u(1)\}\) is a smooth \(N - 1\) dimensional torus \(T^{N-1}\). Furthermore, we can find a second commutative algebra \(\mathcal{G}'\) such that \(\dim \mathcal{G}' = \frac{1}{2} \dim M_N\). The construction goes as follows: Denote rank \(n\) Casimir invariant of \(su(m)\) algebra by \(C_n(m)\) \([3]\). So, for example, \(C_2(3) = \frac{1}{2}(F_1^2 + F_2^2 + \cdots + F_8^2)\). Then, \(\mathcal{G}'\) is generated by \(F_3, F_8, \ldots, F_{N^2-1}, C_p(q) - C_p(q - 1), \ p \leq q = 2, 3, \ldots, N - 1\) where we take \(C_p(q) = 0\) for \(p > q\). For example, in \(SU(4)\) case, we have six commuting functions \(F_3, F_8, F_{15}, C_2(2) = 1/2(F_1^2 + F_2^2 + F_3^2), C_2(3) - C_2(2) = 1/2(F_4^2 + F_5^2 + F_6^2 + F_7^2 + F_8^2), C_3(3) = d_{abc}F_a F_b F_c\), where \(d_{abc}\) is the symmetric structure constant of \(su(3)\) \([3]\).

We consider the integrable system with \(F_3, F_8, \) and \(C_2\) in involution with Hamiltonian given by

\[
H \equiv H(F_3, F_8, C_2). \tag{23}
\]

According to the Liouville theorem, we can find action-angle variables \((I_1, I_2, I_3, \phi_1, \phi_2, \phi_3)\) such that the original symplectic two form Eq.(3) can be expressed as a Darboux form:

\[
\Omega = \sum_{i=1}^{3} dI_i \wedge d\phi_i \tag{24}
\]

They are given by \([11]\)

\[
I_1 = F_3, \quad I_2 = F_8, \quad I_3 = \sqrt{C_2}. \tag{25}
\]
Note that $I_1$ and $I_2$ are global Hamiltonian functions, but $I_3$ is a local Hamiltonian function. It can not be extended to the entire $M_3$. In other words, $I_1$ and $I_2$ generate global torus symplectic actions, whereas $I_3$ generates a local one.

 Canonical quantization of the Hamiltonian Eq.(23) could be rather simple, because the Hamiltonian is diagonalized by construction. However, path integral quantization is non-trivial as was pointed out in Ref. [12,13,11]. We perform coherent state path integral of our integrable system, especially by restricting our Hamiltonian to be a linear function of the global torus actions $I_1$ and $I_2$,

$$H_t = \omega_1 Q_1 + \omega_2 Q_2$$

(26)

where we defined

$$Q_1 = F_3 + \sqrt{3} F_8 = m \frac{|z_1|^2 + |z_2|^2}{L_1} + n \frac{|z_2 - z_1 z_3|^2}{L_2},$$

$$Q_2 = F_3 - \sqrt{3} F_8 = m \frac{|z_1|^2}{L_1} - n \frac{|z_3|^2}{L_2},$$

(27)

as a suitable linear combination of $I_1$ and $I_2$ to achieve the calculational simplicity. $Q_1$ and $Q_2$ generate the following symplectic torus actions:

$$Q_1 : (z_1, z_2, z_3) \mapsto (e^{i\theta_1} z_1, e^{i\theta_1} z_2, z_3), \quad Q_2 : (z_1, z_2, z_3) \mapsto (e^{i\theta_2} z_1, z_2, e^{-i\theta_2} z_3).$$

(28)

This Hamiltonian is special in the sense that the semiclassical approximation to the path integral gives the exact expression for the quantum mechanical propagator due to the Duistermaat-Heckman (D-H) integration formula [14] which found many applications in physics and mathematics recently [15–17].

To see this, we perform the coherent state path integral explicitly [4]. Let us define

$$|Z\rangle = \sum_{i=1}^{3} \exp(z_i E_i)|0\rangle$$

(29)

where $Z = (z_1, z_2, z_3)$, $E_i$ are the three positive roots and $|0\rangle$ is the highest weight vector. The normalization for Eq.(29) is chosen so that

$$\langle Z'|Z \rangle = (1 + \bar{z}_1' z_1 + \bar{z}_2' z_2)^m (1 + \bar{z}_3' z_3 + (\bar{z}_2' - \bar{z}_1' z_3')(z_2 - z_1 z_3))^n$$

(30)
Notice that this definition differs from the usual one by the normalization factor \( N = L_1^{-m} L_2^{-n} \). We have chosen this definition here because in the subsequent analysis, \( \bar{Z} \) and \( Z \) can be treated independently and the over-specification problem can be side-stepped. Then, the resolution of identity is expressed as

\[
I = N \int d\mu(Z, Z) |Z\rangle \langle Z| \quad \text{(31)}
\]

where \( d\mu(Z, Z) \) is the Liouville measure.

Our main interest lies in the evaluation of the propagator

\[
K(\bar{\xi}', \xi'; t) = \langle \bar{\xi}' | e^{-i\hat{H}t} | \xi' \rangle. \quad \text{(32)}
\]

Divide the time \( T \equiv t'' - t' = S \epsilon \) into \( S \) equal intervals, \( Z \equiv Z(p), \ p = 0, 1, \cdots, S \) and the boundary condition is given by

\[
\bar{Z}(t'') = \bar{\xi}', \ Z(t') = \xi'. \quad \text{(33)}
\]

Inserting Eq.(31) repeatedly, we have

\[
\langle \bar{\xi}' | e^{-i\hat{H}t} | \xi' \rangle = \int \prod_{p=1}^{S-1} d\mu(p) N(p) \prod_{p=1}^{S} \langle Z(p) | e^{-i\hat{H} \epsilon} | Z(p-1) \rangle \quad \text{(34)}
\]

Using \( e^{-i\hat{H} \epsilon} = I - i\epsilon \hat{H} \) and the normalization condition Eq.(30) to evaluate \( \langle Z(p) | Z(p-1) \rangle \), we get in the limit \( \epsilon \to 0 \),

\[
K = \int d\mu \ \exp \left\{ m \log L_1(\bar{\xi}', Z(t'')) + n \log L_2(\bar{\xi}', Z(t'')) + i \int_{t'}^{t''} L dt \right\} \quad \text{(35)}
\]

where we defined

\[
L_1(\bar{\xi}', Z(t'')) = (1 + \bar{\xi}_1' z_1(t'') + \bar{\xi}_2' z_2(t''))
\]

\[
L_2(\bar{\xi}', Z(t'')) = (1 + \bar{\xi}_3' z_3(t'') + (\bar{\xi}_2' - \bar{\xi}_1' \bar{\xi}_3')(z_2(t'') - z_1(t'') z_3(t''))), \quad \text{(36)}
\]

and the Lagrangian is given by

\[
L = m \frac{\dot{z}_1 + z_2 \dot{z}_2}{L_1} + n \frac{z_3 \dot{z}_3 + (z_2 - z_1 z_3)(\dot{z}_2 - \dot{z}_1 \dot{z}_3 - \ddot{z}_1 z_3)}{L_2} - H_t \quad \text{(37)}
\]
The equations of motions are

\[ i\dot{z}_i = \hat{g}^{ij} \frac{\partial H_t(\bar{Z}, Z)}{\partial \bar{z}_j}, \quad i\dot{\bar{z}}_i = -\hat{g}^{ij} \frac{\partial H_t(\bar{Z}, Z)}{\partial z_j}. \]  

The solution of equations of motion is linearized completely: with the boundary conditions given by Eq. (33), we have

\[ z_1(t) = \xi_1' e^{i(\omega_1 + \omega_2)(t-t')}, \quad z_2(t) = \xi_2' e^{i\omega_1(t-t')}, \quad z_3(t) = \xi_3' e^{-i\omega_2(t-t')} \]

\[ \bar{z}_1(t) = \xi_1'' e^{-i(\omega_1 + \omega_2)(t-t')}, \quad \bar{z}_2(t) = \xi_2'' e^{-i\omega_1(t-t')}, \quad \bar{z}_3(t) = \xi_3'' e^{i\omega_2(t-t')} \]  

Denoting the above classical solutions by \( \bar{Z}_c \) and \( Z_c \) and expanding around the classical solutions

\[ \bar{Z} = \bar{Z}_c + \delta \bar{Z}, \quad Z = Z_c + \delta Z, \]  

with boundary conditions \( \delta \bar{Z}(t'') = \delta Z(t') = 0 \), we find the following expression for the propagator:

\[ K(\tilde{\xi}'', \xi'; T) = a(T) L_1(\tilde{\xi}'', Z_c(t''))^m L_2(\tilde{\xi}'', Z_c(t''))^n \exp(iS(Z_c, Z_c, T)). \]  

Here \( a(T) \) is the Van Vleck determinant coming from the Gaussian integration of the fluctuations \( \delta \bar{Z} \) and \( \delta Z \). Substituting the classical solutions into the above equations and calculating the Van Vleck determinant as in Ref. [16], we find

\[ K = (1 + \tilde{\xi}_1'' \xi_1' e^{i(\omega_1 + \omega_2)T} + \tilde{\xi}_2'' \xi_2' e^{i\omega_1 T})^m (1 + \tilde{\xi}_3'' \xi_3' e^{-i\omega_2 T} + (\xi_2' - \xi_1'' \xi_3'')(\xi_2' - \xi_1'' \xi_3')) e^{i\omega_1 T} \]  

which is guaranteed to be exact due to D-H formula. The exactness of the above propagator can also be checked by solving the time-dependent Schrödinger equation set up through the geometric quantization of the Hamiltonian [20]. Then, wave function in the anti-holomorphic polarization at arbitrary time is given by

\[ \Psi(\tilde{\xi}'', t) = \int \frac{d\mu(\tilde{Z}, Z)}{L_1^m L_2^n} K(\tilde{\xi}'', Z; t) \Psi(Z, 0) \]
In summary, we constructed integrable spin models on flag manifold by using the symplectic structure explicitly given in Bruhat coordinatization of flag manifold. We find that the systems are non-commutative integrable and some of the conserved quantities are given by the Casimir invariants. We quantized the systems using the coherent state path integral technique and found the exact expression for the propagator for a special case where the Hamiltonian is given by the global symplectic torus action on the flag manifold, and semiclassical approximation gives the exact results due to the D-H integration formula. Analysis of higher $N$ case, many spin systems and geometric quantization of these systems will be discussed in a forthcoming paper [21].

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REFERENCES

[1] A. A. Kirillov, *Elements of the Theory of Representations* (Springer-Verlag, Berlin, 1976).

[2] V. I. Arnold, *Mathematical Methods of Classical Mechanics* (Springer-Verlag, Berlin, 1978); R. Abraham and J. E. Marsden, *Foundations of Mechanics* (Addison Wesley, 1978).

[3] See, for example, O. Babelon, P. Cartier and Y. Kosmann-Schwarzbach, eds., *Lectures on Integrable Systems* (World Scientific, Singapore, 1994); M. A. Olshanetsky and A. M. Perelomov, Phys. Reports 71 (1981) 313 and 94 (1983) 313.

[4] J. R. Klauder and B. S. Skagerstam, *Coherent States: Applications in Physics and Mathematical Physics* (World Scientific, Singapore, 1985); A. Perelomov, *Generalized Coherent States and Their Applications* (Springer-Verlag, Berlin, 1986); W. M. Zhang, D. H. Feng and R. Gilmore, Rev. Mod. Phys. 62, 867 (1990).

[5] T. Lee and P. Oh, Phys. Lett. B 319, 497 (1993), Phys. Rev. Lett. 72, 1141 (1994), and Ann. Phys. (N. Y.) 235, 413 (1994); M. Kim and P. Oh, J. Math. Phys. 35, 3959 (1994); D. Bak, R. Jackiw and S.-Y. Pi, Phys. Rev. D 49, 6778 (1994); W. T. Kim and C. Lee, Phys. Rev. D 49, 6829 (1994).

[6] S. Helgason, *Differential Geometry, Lie Groups, and Symmetric Spaces* (Academic Press, New York, 1978).

[7] A. Pressley and G. Segal, *Loop Groups* (Clarendon Press, Oxford, 1986).

[8] R. F. Picken, J. Math. Phys. 31, 616 (1990).

[9] J. F. Cornwell, *Group Theory in Physics* Vol. I and II (Academic Press, London, 1984).

[10] A. T. Fomenko and V. V. Trofimov, *Integrable Systems on Lie Algebras and Symmetric Spaces* (Gordon and Breach, New York, 1988).
[11] K. Johnson, Ann. Phys. (N. Y.) **192**, 104 (1989).

[12] H. B. Nielsen and D. Rohrlich, Nucl. Phys. B **299**, 471 (1988).

[13] A. Alekseev, L.D. Faddeev and S. L. Shatashvili, J. Geom. Phys. **3**, 1 (1989).

[14] J. J. Duistermaat and G. J. Heckman, Invent. Math. **69**, 259 (1982) and **72**, 153 (1983); N. Berline and M. Vergne, Duke Math. J. **50**, 539 (1983); M. F. Atiyah and R. Bott, Topology **23**, 1 (1984); J.-M. Bismut, Commun. Math. Phys. **98**, 213 (1985) and **103**, 127 (1986); F. Kirwin, Topology **26**, 37 (1987); E. Witten, J. Geom. Phys. **9**, 303 (1992).

[15] M. Stone, Nucl. Phys. B **314**, 557 (1989); M. Blau, E. Keski-Vakkuri and A. J. Niemi, Phys. Lett. B **246**, 92 (1990); M. Blau, Int. J. Mod. Phys. A **6**, 365 (1991); E. Keski-Vakkuri, A. J. Niemi, G. Semenoff and O. Tirkkonen, Phys. Rev. D **44**, 3899 (1991); R. J. Szabo and G. W. Semenoff, Nucl. Phys. B **421**, 391 (1994); S. G. Rajeev, S. Kalyana Rama and S. Sen, J. Math. Phys. **35**, 2259 (1994); K. Funahashi, T. Kashiwa, S. Sakoda and K. Fujii, J. Math. Phys. in press.

[16] P. Oh and M.-H. Kim, Mod. Phys. Lett. A **9**, 3339 (1994).

[17] M. Blau and G. Thompson, preprint IC/95/5, [hep-th/9501075](http://arxiv.org/abs/hep-th/9501075).

[18] J. R. Klauder, Phys. Rev. D **19**, 2349 (1979).

[19] L. D. Faddeev and A. A. Slavnov, *Gauge Fields: Introduction to Quantum Theory* (Benjamin/Cummings Pub., MA, 1980); L. S. Brown, *Quantum Field Theory* (Cambridge Univ. Press, 1992).

[20] L. Schulman, *Techniques and Applications of Path Integration* (John Wiley & Sons, New York, 1981); H. Kleinert, *Path Integrals in Quantum Mechanics, Statistics and Polymer Physics* (World Scientific, Singapore, 1990).

[21] M.-H. Kim and P. Oh, in preparation.