The Energy Representation of World GDP

Boris M. Dolgonosov

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Abstract

The GDP–energy relationship is considered on a global scale. We propose a model which represents world gross product as a power-law function GDP = g( EU ) of current energy consumption E and total energy U consumed over all previous years and materialized in the form of production infrastructure. This energy-based production function has two parameters g and γ that retain their values throughout the years under study (1965–2018), and hence they can be regarded as fundamental characteristics of the world economy within the energy paradigm that considers labor and capital as energy entities.

The model describes empirical data with high accuracy (error 1.2%), despite the fact that energy consumption and GDP increase greatly over the period under study. To provide a robustness check, the production function was fitted to the data for a shortened interval of 1965–2000 with a further projection until 2018, which showed a small error of 1.8% in the target interval of 2001–2018. An additional verification of the model, based on the power-law dependencies of GDP, E and U on world population, confirmed the functional form of the production function and led to almost the same parameter values as those obtained independently.

Keywords World GDP · Production function · Energy consumption · Materialized energy · Population dependencies

Introduction

World economy is driven by energy which is used to build and operate production infrastructure. The global infrastructure has an energy measure equal to the total energy that has been used to create it in all previous years. In essence, this cumulative energy is materialized in the infrastructure, while its work is provided by the current energy consumption. The world economy as a production system is characterized by a production function that can be considered from an energy point of view. This approach will allow drawing parallels between energy and such classical concepts as labor and capital, which, following Cobb and Douglas (1928), determine the production function. More detailed models based on the Solow’s (1956) theory consider labor, capital and energy as independent quantities (Csereklyei et al. 2016; Gozgor et al. 2018). Stern and Kander (2012) introduced a production function in which gross output has the form

$$Y = \left( L^{\gamma_L} (A_L L)^{1-\beta} K^{1-\beta} \right)^{1/\sigma} + E^{1/\sigma} (A_E)^{1/\sigma}$$

(1)

where L = labor, K = capital, E = energy, A_L and A_E are the augmentation indices, σ is elasticity, \( \phi = (\sigma - 1)/\sigma \), \( \gamma_L + \gamma_E = 1 \). This approach is useful if the task requires separate control of labor, capital and energy. However, if the task is to find gross output based on energy consumption for a series of years regardless of how labor and capital change (if necessary, they can be found after solving this task), then this approach is redundant and complicates the description, since it requires information on L, K, E and also on five independent parameters in Eq. (1). The energy representation of gross output is natural within the energy paradigm that considers labor and capital as energy entities.

The objective of this study is to consider the energy representation of world GDP introduced in our previous work (Dolgonosov 2018), analyze it in more details in view of the importance of this topic, provide a rationale and test its consistency with empirical data. The global approach makes it possible to smooth out the differences inherent in different countries, and thereby exclude from consideration energy flows between parts of the world system.

Boris M. Dolgonosov
borismd31@gmail.com

1 Haifa, Israel
Model

We will take as a basis the expression of GDP versus energy proposed in (Dolgonosov 2018) and consider possible ways of justifying it. This expression has the form

\[ G = g(EU)^\gamma, \]  \hspace{1cm} (2)

where \( G \) is world GDP, \( g \) and \( \gamma \) are parameters, \( E \) is total energy consumption, and \( U \) is the materialized energy, defined as the energy consumed over all previous years and used to create production infrastructure:

\[ U(t) = \int_{-\infty}^{t} E(x)dx = U(t_0) + \int_{t_0}^{t} E(x)dx, \] \hspace{1cm} (3)

where \( t_0 \) is a moment chosen as the initial one, \( t > t_0 \). We assume here that energy consumption rapidly decreases with tending to the past, ensuring the existence of the improper integral (3).

Representation (2) is similar to a Cobb–Douglas production function in the form

\[ G = AL^\alpha K^\beta, \] \hspace{1cm} (4)

where \( \alpha, \beta \in [0,1] \). Labor \( L \) uses energy \( E \) to operate and support production infrastructure, while capital \( K \) is embodied in the infrastructure itself, which was created using total energy \( U \) consumed over all previous years. The concept that labor and capital are energy entities is the essence of the energy paradigm. Relation (2) is a special case of Eq. (4) when \( \alpha = \beta, L \propto E \) (labor is proportional to energy consumption) and \( K \propto U \) (capital is proportional to materialized energy).

We can also arrive at relation (2) in another way, using dimensional analysis. Consider the case when GDP growth occurs over a certain characteristic time \( \tau \) (for example, if the process develops exponentially). Then from the dimensional quantities \( E, U, \tau \) that characterize the production system, we can compose the only combination in units of energy consumption (power): \((EU/\tau)^{1/2}\). Multiplying this combination by a coefficient \( g_1 \), which converts energy units into financial ones, we get

\[ G = g_1(EU/\tau)^{1/2}, \] \hspace{1cm} (5)

that is a special case of Eq. (2) with \( \gamma = 1/2 \) and \( g = g_1/\tau^{1/2} \).

The situation changes if GDP growth does not have a characteristic time (for example, if the process develops according to a power law). This possibility follows from the relationship \( G = pN^q \) (Dolgonosov 2018) and the hyperbolic law

\[ N = C/(t_s - t), \]

where \( N \) is world population, \( q \approx 2 \), \( p \) and \( C \) are constants, \( t_s \) is the singularity moment, which according to Foerster et al. (1960) falls at the end of 2026. In this case, GDP obeys the law \( G \propto (t_s - t)^{-q} \). It is clear that the interval \( t_s - t \) cannot represent the characteristic time of the process, since it is not constant. The singularity is the result of idealization; in fact, hyperbolic growth should cease as it approaches \( t_s \). Demographic data show that the hyperbolic law has acted for a long time (about 1000 years), and is currently being violated, showing a decrease in growth rate (Akaev and Sadovnichii 2010; Korotayev et al. 2015; Dolgonosov 2016). In the absence of characteristic time, the process is significantly extended in time, amplifying the effect of energy on GDP that leads to a higher value of the exponent in Eq. (2): \( \gamma > 1/2 \).

Results and discussion

To find the parameters of Eq. (2), we used the literature data on world GDP (WB 2020), energy consumption (BP 2020), and population (US 2017), which are shown in Fig. 1a. We use the following units:

- Time \( t \), year;
- Materialized energy \( U \), Gtoe (= Gigaton of oil equivalent);
- Energy consumption \( E \), Gtoe/year;
- GDP, TS/year (TS = Trillion $ US 2010).

There are systematic data on energy consumption for the years 1965–2018; this period is considered in our analysis. In calculations, integral (3) is replaced by summation over the years:

\[ U_t = U_0 + \sum_{i=t_0}^{t} E_i, \] \hspace{1cm} (6)

where \( t_0 = 1965 \) is the initial year, \( U_0 \) is the cumulative energy materialized in the previous years (until year \( t_0 \)). Calibration of model (2), (6) shows that the minimum standard deviation of \( G \) from GDP data (equal to 0.52 TS/year or 1.2% of the average) is achieved at

\[ U_0 = 141.25 \text{ Gtoe} \] \hspace{1cm} (7)

According to Eq. (2), the relationship between the logarithms of \( G \) and \( EU \) is linear,

\[ \ln G = \ln g + \gamma \ln (EU), \] \hspace{1cm} (8)

which is confirmed with high accuracy: linear regression (8) fits perfectly with empirical data (Fig. 1b). Based on the indicated regression, one can find the parameter values in (2):

\[ g = e^{-1.2459} = 0.2877, \quad \gamma = 0.6258; \] \hspace{1cm} (9)

recall that \( g \) is a dimensional quantity expressed here in the system of units (Gtoe, TS, year).

The GDP calculated by Eqs. (2) and (6)–(9) is in good agreement with empirical data (Fig. 1c). The invariability
of $g$ and $\gamma$ throughout the entire time interval means that there is a time-independent relationship between GDP and energy (in a special aggregate form) despite the fact that the basic quantities vary significantly during this time (Table 1). Meanwhile, energy intensity, defined as $E$/$\text{GDP}$ and commonly used when analyzing the relationship between energy consumption and GDP, is not constant but decreases markedly over time (by 1.5 times for the period under consideration) as follows from Table 1 (see also: Csereklyei et al. 2016; Stern 2018). Thus, in contrast to variable energy intensity, the conservative parameters $g$

![Image of graphs](image_url)

**Fig. 1** a World population, energy consumption, and GDP over 1965–2018 (for population from the US (2017) database—until 2015). b log (GDP) versus log (EU). c Comparison of calculated GDP with data; calibration of model (i) over 1965–2018, and (ii) over 1965–2000 with projection until 2018. d, e, f Energy consumption ($E$), materialized energy ($U$), and GDP as functions of world population ($N$). Data sources: $N$—(US 2017), $E$—(BP 2020), GDP—(WB 2020)

| Year  | E    | GDP  | E/GDP | U    |
|-------|------|------|-------|------|
| 1965  | 3.73 | 14.84| 0.251 | 145.0|
| 2018  | 13.86| 82.46| 0.168 | 600.7|
| 2018 to 1965 | 3.72 | 5.56 | 0.669 | 4.14 |

**Table 1** Changes in basic quantities
and $\gamma$ are fundamental characteristics of the world economy in terms of the energy paradigm.

There is another possibility to obtain Eq. (2) considering $E, U,$ and GDP as functions of population (Fig. 1d, e, f), which can be approximated by a power law with good accuracy:

$$E = kN^d, U = rN^s, G = pN^q. \tag{10}$$

Parameters of these functions were obtained for population time series that were retrieved from the three demographic databases referred in Table 2. Equation (10) are consistent with the functional form of the production function (2); their substitution in (2) gives the relationships between the parameters

$$p = g(kr)^\gamma, q = \gamma(d+s); \tag{11}$$

this implies

$$g = p/(kr)^\gamma, \gamma = q/(d+s). \tag{12}$$

Thus, $g$ and $\gamma$ are expressed through the parameters of Eq. (10).

Table 2 shows that the parameter values corresponding to the specified databases are slightly different. As for the parameters $g$ and $\gamma,$ their values found from Eq. (12) are very close to those obtained independently, cf. (9): $g = 0.2877$ and $\gamma = 0.6258.$ This is an additional evidence in favor of the production function (2).

To provide a robustness check, we fitted Eq. (2) to a shortened GDP time series over 1965–2000 and then calculated a projection of GDP for 2001–2018. The parameters estimated in this way are as follows:

$$g = 0.3026, \gamma = 0.6187 \tag{13}$$

The GDP growth over time, calculated in both cases (9) and (13), is shown in Fig. 1c. The model curves are fairly close to the actual data. Standard deviations over the entire period 1965–2018 for both cases are 0.52 and 0.76 T$/year, respectively, or 1.2 and 1.8% of the average 42.4 T$/year. Over the period 2001–2018, standard deviations are 0.71 and 1.18 T$/year, or 1.1 and 1.8% of the average 66.0 T$/year. An error of 1.8% is small enough for a projection to a depth of 18 years, which confirms the robustness of the model.

The model is developed for a global scale. When studying smaller scales, such as regions or countries, it is necessary to consider the flows of labor, capital, goods and energy across borders. Converting these dissimilar entities into energy requires additional information.

Conclusions

In this study, the energy representation of world GDP (2) has received versatile confirmation:

(i) By transforming the Cobb–Douglas production function into an energy form in accordance with the energy paradigm that considers labor and capital as energy entities;

(ii) By applying dimensional analysis to determine the form of the energy-based production function; and,

(iii) By using the power-law dependences of energy consumption, materialized energy and GDP on world population in order to verify the production function developed and show how to calculate its parameters through the parameters of the specified population dependencies.

The analysis showed that world GDP is completely determined by two energy quantities: current energy consumption $E$ and total energy $U$ which was consumed over all previous years and materialized in the form of production infrastructure.

GDP is a power-law function of the aggregate variable which is the product of energy consumption and materialized energy, $G = g(EU)^\gamma.$ The conservative parameters $g$ and $\gamma$ are fundamental characteristics of the world economy.

Calibration of the model according to the data for 1965–2018 gave the following parameter values for the world production system: $U_0 = 141.25 \text{ Gtoe (to top 1965),}$ $\gamma = 0.6258, g = 0.2877$ ($g$ is measured in the system of units: Gtoe, T$/US 2010, year)$).

Energy consumption, materialized energy, and GDP are power-law functions of world population. Calculation of $g$ and $\gamma$ through the parameters of these population

| Database | $E$ | $U$ | $G$ | Equation (12) |
|----------|-----|-----|-----|---------------|
|          | $k$ | $d$ | $r$ | $s$ | $p$ | $q$ | $g$ | $\gamma$ |
| US (2017) | 0.7136 | 1.4582 | 16.628 | 1.7512 | 1.3587 | 2.0054 | 0.2896 | 0.6248 |
| UN (2020) | 0.7491 | 1.4237 | 17.356 | 1.7200 | 1.4313 | 1.9678 | 0.2874 | 0.6259 |
| WB (2020) | 0.7539 | 1.4242 | 17.489 | 1.7206 | 1.4435 | 1.9686 | 0.2872 | 0.6260 |
dependencies gives values close to the above ones, which indicates the consistency of the model.

Model (2), (6) describes empirical data with high accuracy (error 1.2%), despite a significant increase in energy consumption and GDP (by 3.72 and 5.56 times, respectively) over the period under study.

Calibration of the model using the shortened GDP time series over 1965–2000 and a projection of GDP for the period 2001–2018 shows an error of 1.8% against GDP data. The small error value allows the model to be considered robust.

Compliance with Ethical Standards

Conflict of interest The author declares no conflict of interest.

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