An Electron Bunch Compressor Based on an FEL Interaction in the Far Infra Red

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1 Introduction

The scientific case for a short wave length (photon energy 1 keV and more or wavelength 1 nm and less) free electron laser based on the principle of stimulated emission of spontaneous emission (SASE FEL) is significantly enhanced if the optical pulse length allows to resolve molecular vibrations. The fastest oscillation of interest is the C-H stretching mode at a wavelength of 3 $\mu$m corresponding to 10 fs. Larger molecular masses oscillate at lower frequencies. Different proposals exist (at DESY e.g. [1], [2]) to generate pulses of 20 fs and less. In both cases the time structure is derived from “conventional” laser systems while the electron pulse proper is significantly longer.

For an linac based SASE FEL to work electron bunches from the electron source must be compressed. Conventionally this is done by applying an energy chirp along the length of the electron bunch and subsequent dispersion with momentum compaction. The bunch length is a function of the applied longitudinal gradient and the initial energy spread of the bunch. A typical bunch length is 50 $\mu$m.

In a free electron laser the interaction of the optical electric field with the electron beam is mediated by the undulator magnetic field. This leads to a (slow) longitudinal motion. Under appropriate approximations this motion resembles the well known synchrotron motion in the high frequency buckets of synchrotrons or linacs. The combined action of magnetic and radiation field generates an “optical trap” having longitudinal extend of one wavelength $\lambda_1$. The particles execute longitudinal motion governed by an equation which is formally identical to the well known pendulum equation (see eq. 1 below). Saturation of the FEL interaction corresponds to half a synchrotron period in the optical trap.

In this note an electron bunch compressor is proposed based on FEL type interaction of the electron bunch with far infrared (FIR) radiation. This mechanism maintains phase space density and thus requires a high quality electron beam to produce bunches of the length of a few ten $\mu$m.
2 Theory

2.1 Classical FEL Theory

W. Colson [3] summarises the classical one-dimensional FEL theory in the limit of the slowly varying amplitude and phase approximation. The electron dynamics is governed by the pendulum equation

\[ \ddot{\zeta} = |a| \cos(\zeta + \phi) \]  

(1)

and the evolution of the radiation field is given by

\[ \dot{a} = -j < \exp(-i\zeta) > \]  

(2)

< ... > is averaging over the electron distribution.

The longitudinal coordinate \( \Delta z \) of the electron with respect to the synchronous phase is related to the phase \( \zeta \) via the radiation wavelength \( \lambda_1 \) by

\[ \zeta = 2\pi \frac{\Delta z}{\lambda_1} \]  

(3)

The resonance condition is

\[ \lambda_1 = \frac{\lambda_o}{2\gamma^2} (1 + K^2) \]  

(4)

The fractional energy deviation \( \Delta \gamma / \gamma \) from resonance is related to the normalised velocity \( \nu \) by

\[ \nu = \frac{\dot{\zeta}}{\zeta} = 4\pi N \frac{\Delta \gamma}{\gamma} \]  

(5)

Here \( \dot{\zeta} \) denotes the derivative of \( \zeta \) with respect to the normalised time \( \tau \) (see table 1). The electric field of the radiation is \( E \exp i\phi \). The dimensionless field amplitude is given by

\[ a = (4\pi N)^2 \frac{K}{1 + K^2} K_1 \]  

(6)
with
\[ K_1 = \frac{e\lambda_1 E}{2\pi mc^2} \] (7)

The dimensionless current density is
\[ j = 32 \frac{e^2\pi^2 N_e N^2}{mc^2} \frac{K^2}{\gamma (1 + K^2)} \] (8)
\[ = 1.85 \cdot 10^{-2} \left( \frac{I}{\text{Amp}} \right) \frac{N^2}{\gamma} \frac{K^2}{1 + K^2} \] (9)

The Rayleigh range \( z_o \) of an Gaussian mode is related to its waist size \( w_o \) by
\[ w_o = \sqrt{\lambda_1 z_o / \pi} \] (10)

The meaning of most of the other symbols is given in the table.

### Helical polarisation is assumed.

| Symbol | Description | Value |
|--------|-------------|-------|
| \( e \) | electric charge | \( 4.8 \cdot 10^{19} \text{esu} \) |
| \( m \) | electron mass | \( 0.911 \cdot 10^{-27} \text{g} \) |
| \( c \) | speed of light | \( 3 \cdot 10^{10} \text{cm/sec} \) |
| \( \lambda_1 \) | radiation wavelength | \( \frac{e\lambda_0 B}{2\pi mc^2} \) |
| \( N \) | number of undulator periods | Gauss |
| \( K \) | wiggler parameter | \( e\lambda_0|E|/2\pi mc^2 \) |
| \( \lambda_0 \) | undulator period length | statVolt/cm |
| \( B_0 \) | undulator field amplitude | \( t c \beta \) |
| \( K_1 \) | radiation field parameter | \( |E|/(N \lambda_0) \) |
| \( E \) | electric field amplitude of radiation | \( \frac{\partial}{\partial \tau} \) |
| \( N_e \) | number of electrons within one \( \lambda_1 \) | |
| \( \gamma \) | electron energy | |
| \( \tau \) | dimensionless time 0 ... 1 | |
| \( (\ldots) \) | derivative with respect \( \tau \) | |
Particles move in longitudinal phase space \((\zeta, \nu)\) along trajectories with a constant of motion

\[2H_o = \nu_o^2 - 2|a|\sin(\zeta_o + \phi)\]  

Particles with a constant of motion \(H_o \leq a\) stay inside the optical trap defined by

\[H_o = a\]  

Equation 12 defines the separatrix with longitudinal extend of \(2\pi\) corresponding to one wavelength, and vertical extend of \(\pm 2\sqrt{a}\). Particles near the fixed point at the center of the trap at \((\zeta, \nu) = (\pi/2, 0)\) execute harmonic oscillations.

### 2.2 The FEL Buncher

The principle of the FEL buncher is to exploit the synchrotron motion in the optical trap of a suitably designed FEL interaction to rotate the electron bunch in longitudinal phase space by 90°.

In the FEL buncher particles are made to execute a quarter of a synchrotron period in the optical trap due to an FEL interaction with a superimposed radiation field with wavelength \(\lambda_1\). A particle distribution with initial bunch length \(\sigma_z\) and initial fractional energy spread \(\Delta\gamma/\gamma\) is transferred into a particle distribution of final length \(\sigma'_z\) and final energy spread \(\Delta\gamma'/\gamma\). Provided the initial energy distribution is small enough and the initial bunch length is short compared to the extend of the trap this causes bunching of the beam (see fig. 1).

We assume that all motions in the optical trap are harmonic, i.e. the initial phase space distribution of the beam occupies only a small fraction of the optical trap near its center. The equation of motion of the field amplitude eq. 2 is not considered here assuming no change of the electric field.

Conservation of phase space density implies

\[\frac{\Delta\gamma'}{\Delta\gamma} = \frac{\sigma_z}{\sigma'_z}\]  

The bunching condition requires a quarter of a synchrotron period in the optical trap. From eq. 1 we note that the oscillation frequency for small amplitudes
Figure 1: Longitudinal phase space $\nu$ versus $\zeta$ of FEL interaction with initial (open symbols) and final (full symbols) particle distribution. The initial bunch length is $\sigma_z = 0.16 \lambda_1$, the initial energy spread is $\Delta \gamma = 5 \cdot 10^{-4}$, the number of periods is $N = 64$, the normalised field strength is $a = 1.35 \cdot (\pi/2)^2$. 

is $\sqrt{a}$ and the phase advance is $\sqrt{a} \tau$. At the end of the interaction $\tau = 1$. The bunching condition thus is

$$\sqrt{a} = \pi/2$$

(14)
Note: Eq. [14] refers to the final synchrotron phase advance. A variation of the field amplitude during the interaction can be tolerated.

There are two further requirements:

The electron bunch must be short compared to the wavelength. This is needed to achieve the desired bunch length. Otherwise non-linear motion far off the center of the trap dilutes the phase space distribution and reduces the bunching effect. The **short bunch requirement** reads

\[
\frac{2\pi \sigma_z}{\lambda_1} \leq 1
\]  

(15)

The induced energy spread is

\[
\frac{\Delta \gamma'}{\gamma} \geq \frac{1}{4\pi N} \frac{\sigma_z}{\lambda_1/2\pi} = \frac{1}{2N} \frac{\sigma_z}{\lambda_1}
\]  

(16)

The **cold beam requirement** is needed to get the desired pulse length

\[
\sigma_z' \geq 4\pi N \frac{\Delta \gamma}{\gamma} \frac{\lambda_1}{2\pi} = 2N \frac{\Delta \gamma}{\gamma} \lambda_1
\]  

(17)

or

\[
N \leq \frac{1}{2} \frac{\sigma_z'}{\lambda_1} \frac{\gamma}{\Delta \gamma}
\]  

(18)

### 2.3 Power Requirement and Jitter

A major limitation of the FEL buncher is the required optical power to drive the FEL interaction.

The FEL interaction strength depends on the focusing of the optical mode. The interaction strength is maximised for a Gaussian mode with a Rayleigh range \( z_o \) of

\[
z_o = \frac{1}{2} N \lambda_o
\]  

(19)

The minimum optical cross section is

\[
w_o = \sqrt{z_o \lambda_1/\pi}
\]  

(20)
The cross section varies along the axis $z$ as

$$w^2(z) = w_o^2 (1 + (\frac{z}{z_o})^2)$$ (21)

The power in the optical driving field is using eq. 19, 20 and 4

$$P = \frac{c^2}{2} E^2 \cdot \pi w_o^2$$ (22)

$$= \left(\frac{1}{4\pi}\right)^4 \frac{c}{2} \left(\frac{2\pi mc^2}{e}\right)^2 \frac{1 + K^2}{K^2} \frac{\gamma^2}{N^3} a^2$$ (23)

The field strength $E$ follows from the bunching condition eq. 14. We obtain

$$P = \frac{c^2}{2^{13}} \left(\frac{2\pi mc^2}{e}\right)^2 \frac{\gamma^2}{N^3} \frac{1 + K^2}{K^2}$$ (24)

$$= 42.2\; MW \frac{\gamma^2}{N^3} \frac{1 + K^2}{K^2}$$ (25)

Using eq. 18 the power becomes

$$P = \frac{c^2}{2^{10}} \left(\frac{2\pi mc^2}{e}\right)^2 \frac{\gamma^2}{\sigma_x^2} \frac{1 + K^2}{K^2}$$ (26)

$$= 340\; MW \left(\frac{\gamma^2}{\sigma_x^2}\right) \frac{1 + K^2}{K^2}$$ (27)

Using the equality in eq. 15 shows the scaling of the optical power with the third power of the compression ratio times the energy spread:

$$P = \frac{\pi^3}{2^7} c \left(\frac{2\pi mc^2}{e}\right)^2 \left(\frac{\sigma_z}{\sigma'_z}\right)^3 \frac{1 + K^2}{K^2}$$ (28)

$$= 84\; GW \left(\frac{\sigma_z}{\sigma'_z}\right)^3 \frac{1 + K^2}{K^2}$$ (29)

This power must be increased to compensate for non-harmonic particle motion in the optical trap.

A higher electron energy linearly decreases the power requirement at the expense of a longer interaction region (cf. eq. 4).
The resulting electric field strength of a beam with cross section $A$ is

$$E = \sqrt{\frac{2P}{\varepsilon_o c A}}$$  \hspace{1cm} (30)

($\varepsilon_o = 8.85 \cdot 10^{-12} \text{ F/m}$)

At the waist we have $A = \pi w_o^2 = z_o \lambda_1$.

The electron bunch slips back by one optical wavelength per undulator period.

The optical puls must have a length of at least this slippage distance $N \cdot \lambda_1$.

The total optical energy in the puls thus is

$$E_{opt} = \frac{1}{c} P N \lambda_1$$  \hspace{1cm} (31)

A further limitation is due to the short bunch requirement eq. 15. It refers to an effective bunch length including jitter between the phase $\phi$ of the optical field and the electron bunch position $\zeta_o$. For reliable operation one must have $jitter \ll \lambda_1$. This limits the usable wavelength and thus the achievable effective gradient.

3 A Numerical Example

Designing an FEL buncher at this stage consists of the following steps:

1. Select a wavelength $\lambda_1$ such that the short bunch requirement is met.

2. Select a position along the linear accelerator and get the power from eq. 26. The short bunch must be transportable to its final destination without unacceptable bunch lengthening. If needed the electron bunch can be cooled by scraping off particles with a large energy deviation prior to the FEL buncher.

3. Check that the resulting energy spread eq. 16 is acceptable.

4. Check that the number of periods from eq. 18 is acceptable.

5. Check the technical feasibility of the (helical) undulator.

6. Design a suitable source for the optical field.
As an example we choose the situation downstream of the third bunch compressor of the TTF-FEL as described in the proposal [4].

The wavelength is chosen to be $\lambda_1 = 314 \mu \text{m}$, which is $2\pi$ the initial bunch length. This choice is dictated by the short bunch requirement and the desire to avoid unharmonic motion of the bunch in the optical trap. The jitter must be well below 1 psec. This is believed to be achievable if the optical phase is synchronized to the main electron beam via the FIR generator (cf. below).

The electron beam energy is 516 MeV ($\gamma = 1010$) with an initial energy spread of 1000 keV. In the interest to limit the power we scrap off particles such that the energy spread is reduced to $\Delta \gamma/\gamma = 5 \cdot 10^{-4}$ and $\Delta \gamma = 0.5$ ($\sigma_\nu = 4\pi N \frac{\Delta \nu}{\gamma} = 0.20$). This reduces the bunch charge by a factor of about 4. – The initial bunch length is $\sigma_z = 50 \mu \text{m}$.

Aiming at final bunch length of $\sigma'_z = 20 \mu \text{m}$ corresponding to 60 fs the power must be $P = 300 \text{ MW}$. This number includes a factor of 1.8 to compensate for the non-harmonic motion of particles in the optical trap.

The number of periods from eq. (18) is $N = 64$. The energy in the optical pulse of length $N \cdot \lambda_1$ is $E_{opt} = 0.020 \text{ J}$. The final energy spread is $\frac{\Delta \gamma}{\gamma} = 1.2 \cdot 10^{-3}$.

The helical undulator has a period of $\lambda_o = 400 \text{ mm}$ and a field amplitude of $B_o = 10.6 \text{ kGauss}$ resulting in a wiggler strength $K = 40$. The interaction length is thus about 26 m. There is a copious amount of spontaneous hard radiation.

The Rayleigh range of the optical $TEM_{oo}$ mode is $z_o = 13 \text{ m}$ leading to a waist of $w_o = 36 \text{ mm}$.

The electric field strength at the waist $E = 6.1 \text{ MV/m}$.

Fig. 1 shows a one-dimensional simulation (constant field amplitude throughout the interaction) integrating eq. (1) for 200 particles with the parameters given above.

The discussion of a possible generator is deferred to the next section.

The requirement will be relaxed if the energy spread and/or bunch length of the initial electron beam is further reduced. The interaction length is reduced for an earlier location in the linac.
4 The Generator

The required power of the order of GigaWatt increases rapidly if the energy spread is bigger than assumed and/or if stronger bunching is required. These power levels suggest to use a FEL type generator. At these FIR wavelength an electron beam is easily pre-bunched, i.e. the bunch length is much shorter than the wavelength.

4.1 Secondary Electron Beam

We first consider a generator operating on a secondary electron beam which can be discarded once the power is generated.

Both beams, the secondary one and the main one, consist of a single electron bunch. The secondary bunch is synchronized to the main bunch. The secondary bunch comes from a laser driven photo cathode rf gun much like the main beam. The two guns a synchronized by deriving the laser pulses from a common laser oscillator system.

The undulator of the generator is of the same length as the buncher undulator. Thus the emitted radiation pulse has the proper length for the FEL buncher.

The spontaneous radiation generated in this undulator is much stronger than the usual undulator radiation since the electron is already pre-bunched. The spontaneous radiation is amplified in further undulator modules of the same length. Phasing, dispersion, and focussing of the radiation is achieved using retarders in between the modules.

The electron beam should be defocussed in the undulators to efficiently couple to the lowest optical mode \( (TEM_{oo}) \), and should be focussed to stay clear from the components of the retarder.

There are at least two options for the retarder.

The first one consists of plates of appropriate thickness and material to slow down the radiation. To achieve a retardation by \( \Delta = N \cdot \lambda_1 = 20 \text{ mm} \) the plate should have a thickness of \( \Delta/n \) where \( n \) is the index of refraction. These plates have curved surfaces introducing focussing to counteract diffraction. They contain a hole in the center to transmit the (focused) electron beam. Further units are needed to control phasing, i.e. a phase modulator acting on the electron beam much like in an optical klystron. The undulator modules are tapered to match energy extraction.
A major problem with this design is the power density in the focussing retarders.

The second option for the retarder consists in a zig-zag arrangement of the undulators. The electron beam is bent by magnets while the light beam is reflected by a focussing mirror. The straight path of the light is longer than the curved path of the electrons. This path lengthening must be equal to \( N \cdot \lambda_1 \). Steerer magnets associated to the bending magnets can vary the electron beam path length.

The induced path length difference \( \Delta \) at a bending radius \( R \) and a deflection angle \( \theta \) is

\[
\Delta = 2R \left( \tan \frac{\theta}{2} - \frac{\theta}{2} \right)
\]

At \( R = 1.6 \, m \) a deflection angle of \( \theta = 30^\circ \) leads to a path difference of \( \Delta = N \cdot \lambda_1 = 20 \, \text{mm} \).

The separation \( x \) between the optical beam and the electron path is

\[
x = R \left( \frac{1}{\cos \frac{\theta}{2}} - 1 \right)
\]

The maximum seperation between the optical beam and the electron beam becomes \( x = 56 \, \text{mm} \).

4.2 Main Electron Beam FEL

In this case the main electron beam is used to generate the required FIR power in a FEL cavity. The electron beam consists of a sequence of bunches. The first few bunches of the bunch train are used to build up the intracavity power. The trailing bunches are compressed in the FEL interaction and simultaneously provide for enough optical gain to maintain the power level corresponding to optimum bunching.

To do this the beam has to have a repetition rate matched to the cavity length. The choice of the cavity length is probably dictated by the consideration of the power density on the mirror surfaces (see below).

This FEL uses a prebunched electron beam. The losses in the cavity and thus the intra-cavity power density is regulated such that in the steady state the electrons perform a quarter of a synchrotron oscillation inside the optical
Figure 2: Longitudinal phase space $\nu$ versus $\zeta$ of FEL interaction with initial (open symbols) and final (full symbols) particle distribution. Initial distribution off center. The initial bunch length is $\sigma_z = 0.125 \cdot \lambda_1$, the initial energy spread is $\Delta \gamma = 1 \cdot 10^{-3}$, the number of periods is $N = 16$, and the normalised field strength is $a = 1.25 \cdot (\pi/2)^2$.

It is speculated that the phase of the electron bunch can be stabilised by choice of cavity length and magnetic field amplitude as qualitatively shown in fig. 2. Also optical dispersion in the cavity can help. It needs to be verified
by (numerical) simulations including the nonlinear electron dynamics.

We note that the coupling between electron beam and the optical mode is rather poor. The filling factor i.e. the ratio of the cross sections of the electron beam and the optical mode is much less than 1. In other words the spatial distribution of the generated radiation does not match the field distribution in the cavity very well.

Fig. 2 shows a one dimensional simulation with the following parameters: The wavelength is $\lambda_1 = 400 \, \mu m$; the undulator has $N = 16$ periods with a period length of $\lambda_o = 500 \, mm$; the wiggler strength is $K = 40$ corresponding to a field amplitude of $B_o = 0.86 \, Tesla$; the Rayleigh range is $z_o = 4 \, m$ and the waist size is $w_o = 22.5 \, mm$. The electron beam has an energy of 516 MeV ($\gamma = 1010$) and is scraped to an energy spread of $\Delta \gamma/\gamma = 1 \cdot 10^{-3}$. A bunch charge of 0.5 $nCb$ and a bunch length of $\sigma_z = 0.125 \cdot \lambda_1 = 50 \, \mu m$ represents a current of 3 $kAmp$. This translates into a dimensionless current density of $j = 14$; thus the neglect of the field driving term in equ. 6 is a crude approximation. The intracavity circulating power is $P = 16 \, GW$. The power corresponds to $a = 1.25 \cdot (\pi/2)^2$. This increase beyond equ. 13 compensates for the non-harmonic particle motion in the optical trap. The electric field strength at the waist reaches $E = 88 \, MV/m$; the optical pulse energy is $E_{opt} = 350 \, mJ$.

The parameters leading to the situation in fig. 2 cause a change in the electron energy of $\Delta \gamma = \Delta \nu = 2.8 \cdot 10^{-3}$. At a bunch charge of 0.5 $nCb$ the optical gain at an optical power level of 16 GW is $g = \frac{\text{electron energy change}}{\text{optical energy}} = 2 \cdot 10^{-3}$. It thus takes about 500 electron bunches to increase the power by a factor $e$ (assuming no losses) at the desired steady state. It is speculated here that the FIR power in the cavity has built up from the spontaneous emission to the desired power level before the end of the linac bunch train.

The observed bunch length is $\sigma'_z = 10 \, \mu m$ in this simulation and the final energy spread is $\Delta \gamma'/\gamma = 4 \cdot 10^{-3}$.

At the time of the writing the principal limitation seems to be the power density on the cavity mirrors which can only be reduced by designing a long cavity. When this problem can be solved the bunching can be enhanced by using a shorter interaction length and a higher power level which at the same time tolerates a larger initial energy spread.
5 Summary and Outlook

The proposed FEL buncher uses the transvers electric field of a freely propagating radiation field and couples it to the electron beam via the FEL interaction. Since the wavelength can be much shorter than in an RF device the potential for a high longitudinal gradient is given.

However, the requirement for the optical power density inside the FEL interaction is very significant. A more realistic study than the one done here will further increase the power requirement. The calculations presented here are based on the pendulum equation eq. 1 which depends on the slowly varying amplitude and phase approximation. The simulation assumes a constant optical field, i.e. no diffraction. It also assumes perfect matching of the electron beam cross section to the optical mode size. In reality a mismatch of the order of 1000 is probable. This does not change the coupling of the electron beam to the optical field in the buncher, but it reduces the gain in the generator, in particular in the case of fig. 2 where the main beam is used to generate the FIR power.

Handling the high optical power in the GW range requires special attention. It is believed that the power handling capability of mirrors is limiting. Extending the cavity length does distribute the power over a larger area but requires an increased mirror size. Consider the example of fig. 2. The power density in the waist is about $1\text{ GW/cm}^2$. When a deduction of the power density by a factor of 100 is required, the cavity length must be 80 m and the mirror diameter becomes 1.4 m.

Spiking is undesirable from the users’ point of view. Spiking in a high power SASE FEL is due to the combination of synchrotron motion in the optical trap and the lethargy of the FEL interaction. While seeding improves the spectral line width by correlating the optical phase of different spikes it does not in general eliminate spiking as such. Short electron bunches may not only be beneficial from the point of view of doing time resolved experiments but also from the point of view of controlling the spiking. Spiking is expected to be reduced by an electron bunch length comparable to the total slippage distance. An electron bunch length of 20 $\mu$ may influence spiking up to a photon energy of about 100 eV.

Alternatively, the synchrotron motion in the optical trap is significantly altered by tapering the magnetic field along the length of the SASE undulator thus influencing spiking.
References

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