Decomposition and reconstruction of ore-caused anomalies information by airborne gamma-ray spectrum based on fractal singular value decomposition method

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Abstract. Identification method research on ore-caused anomalies information is one of the core technologies of airborne gamma-ray spectrum data post-processing. This paper proposed a decomposition and reconstruction method of anomaly information called fractal singular value decomposition (FSVD), aiming at the current ubiquitous truncation error and boundary effect problems in filter design in ore-caused anomaly information identification methods in airborne gamma-ray spectrum. Based on the multi-fractal theory, this method decomposes the singular value of the specific nucleus activity data of the airborne gamma-ray spectrum under time series, and quantitatively calculates the fractal singular value truncation parameter according to residual square sum minimum principle. According to the calculation result, the original data is partially reconstructed to rebuild the ore-caused anomalies information. This paper used the 1:50000 airborne gamma-ray spectrum trial production flight data of a certain area in Inner Mongolia (the surveying area contains proved metal ore occurrences) to verify the method. The results show that: after the original signals are decomposed by FSVD, the high, medium and low singular values can represent the background, anomaly, and noise signal, respectively. At the same time, the false anomaly signals caused by flight error exist in high singular values. After reconstructing the decomposed signals according to median singular values, background noises can be effectively separated, the ore-caused anomaly in the known ore-caused anomalies can be identified and band-like false anomalies caused by time domain batches can be removed.

1. Introduction
Airborne gamma-ray spectrum survey (AGS) can measure the distribution of radioactive elements in the primary halo of superficial soils. With fast measuring speed, large measuring area and ability to avoid unfavorable terrain, it has become an important part of China’s mineral resources exploration technology [1]. Since radioactive elements can directly indicate the distribution of uranium deposits, areas with anomaly distribution of radioactive elements have higher metallogenic potential. The extraction effect of anomaly information in the processing of airborne gamma spectrum data directly affects the quality of the following work.

Nowadays, filtering original signals according to the nature of geological signal frequency domain has been widely recognized as one of the effective methods for extracting anomaly signals. Fourier transform has been widely used in petroleum exploration, seismic exploration, ground penetrating radar, geochemistry [2-4] and other fields. By filtering signals, the signal-to-noise ratio can be improved, and
ore-caused anomalies can be divided. Although this method can effectively separate ore-caused anomaly signals, due to the mathematical principle of the Fourier transform (fundamental space is sine wave), truncation errors and boundary effects cannot be avoided.

Singular Value Decomposition (SVD) is an emerging method for analyzing signals in time series, and has been widely used in telecommunications, ecology, machinery industry, meteorology, geological exploration and other fields [5]. SVD uses the reduced-rank principle to achieve signal separation, which avoids truncation errors and boundary effects caused by Fourier transform. After being decomposed by SVD, original signals can form several singular values, and different singular values represent signals of different energies. By reconstructing part of the signals, the effect similar to filtering can be achieved.

Fractal theory is an effective method to deal with nonlinear signals, which has been widely used in mathematics, materials science, ecology, geological exploration, computer information science and other fields [6]. Because the ore-forming process is affected by various elements such as different stratums and structures, the geologic body and corresponding geophysical field will exhibit spatial self-similarity characteristic. Fractal theory is an effective method to deal with information with this characteristic. In recent years, fractal theory has been applied to describe the distribution regularities with multiple measures in geophysics and geochemistry [6-8].

In this paper, a method for decomposing and reconstructing ore-caused anomaly information of airborne gamma-ray spectrum is proposed from the angle of energy domain in time series, namely FSVD method, which is an improved method for processing nonlinear signals, compared with SVD method. The method re-divides the original data from the angle of energy domain, processes the matrix formed by the interpolated original signals by SVD, piecewise fits the fractal curve of the singular values according to the principle of minimum residual square sum, separates the singular values corresponding to different energies, and reconstructs the singular values of each segment. In this way, errors caused by using naked eyes to determine critical singular values in the conventional SVD method can be avoided. FSVD separates the ore-caused anomaly from the background according to the difference and law of the critical singular value, and restores the geophysical field background at the energy scale, and extracts the ore-caused anomaly information of airborne gamma-ray spectrum.

2. Principle and Method

2.1. SVD (Singular Value Decomposition)

Assuming that the element content measured by airborne gamma-ray spectrometry can form matrix $A$ ($m \times n$, $m$ is the number of lines, $n$ is the number of points measured by each line), rank of $A$ is $r(r \leq \min(m,n)$, and there are orthogonal matrix $V$ of size $n \times n$, and orthogonal matrix $U$ of size $m \times m$,

$$U^T A V = B$$

In the formula (1), $B$ is a non-negative diagonal matrix,

$$B = \begin{bmatrix} S & 0 \\ 0 & 0 \end{bmatrix}, \quad S = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_r), \quad r \leq \min(m,n)$$

$\sigma_1, \sigma_2, \ldots, \sigma_r$ is called the singular values of $A$, $\sigma_1 > \sigma_2 > \cdots > \sigma_r > 0$, and the column vectors $\mu_i$, $\nu_i$ of $U$ and $V$ is the left and right singular vectors of $A$ respectively ($\mu_i$ is the $i$-th eigenvector of $AA^T$, and the $i$-th eigenvector of $A^T A$ is $\nu_i$).

Expression $A = U B V^T$ is called the singular value decomposition of $A$. The product of the matrix $U$ and $V^T$ constitutes an orthogonal complete space, denoted as $\phi(\sigma_i) = U \otimes V^T$. The matrix product of the $k$-th column vector $\mu_k$ of $U$ and the $k$-th row vector $\nu_k$ of $A$ is
\[
A_k = \begin{bmatrix} 
\mu_{1,k} \times V_{k,1} & \cdots & \mu_{1,k} \times V_{k,n} \\
\vdots & \ddots & \vdots \\
\mu_{m,k} \times V_{k,1} & \cdots & \mu_{m,k} \times V_{k,n} 
\end{bmatrix} \quad k=1,2,\ldots,r
\]

\(A_k\) constitutes a complete 2D orthogonal space. The projection coefficient of matrix \(A(m \times n)\) at \(A_k\) is singular value \(\sigma_k\). \(\sigma_k\) forms the spectral density or feature space decomposition coefficient of matrix \(A\), namely:

\[
A(m \times n) = \sum_{k=0}^{r} \sigma_k \mu_k V_k
\]

In formula (3), like the coefficients in the Fourier transform, \(\sigma_k\) represents the coefficients projected or mapped by matrix \(A(m \times n)\) on the feature space \(\mu_k V_k\). The accumulation of multiple characteristic matrices with weighting coefficients can be used to characterize the matrix \(A\). The contribution of the singular value in matrix reconstruction is positively correlated with the magnitude of its corresponding singular value, and the larger singular value represents the main component; The energy of uranium geophysical field represented by matrix \(A\) is closely related to the singular value. The sum of eigenvalues \(2\lambda_{ii} = \sigma_i^2\) can reflect the sum of uranium geophysical field energy. Therefore, the greater the singular value (or eigenvalue) component, the greater the contribution power of the geophysical field signal.

Taking \(k\) as abscissa and \(\sigma_k\) as ordinate, the energy density curve of \(A\) after SVD can be obtained. In the \(k\)-th eigenspace, the energy distribution percentage of the geophysical field of the original uranium element is as follows:

\[
M_k = \frac{\sigma_k^2}{\sum_{i=1}^{r} \sigma_i^2} , \quad k=0,1,\ldots,r
\]

In formula (4), \(M_k\) is also called as the percentage of the spectral density of the matrix \(A(m \times n)\) in the \(k\)-th feature space. Normally, the normalized energy integral curve is plotted with \(k\) as the abscissa and \(M_k = \sum_{i=1}^{k} M_i\) (cumulative energy percentage) as the ordinate. When \(k=r\), \(M_r = 1\).

If \(A(m \times n)\) is a real matrix, the range of \(p\) singular values is \(0-\lambda_1^{1/2}\). The singular value is defined as its spectral radius. Since the integral power spectral density at the spectral radius can be replaced by eigenvalues, the energy from the minimum spectral radius (0) to the current spectral radius \(\lambda_i^{1/2}\) can be defined as the energy measure \(E_i\) in the sense of scale \(\lambda_i\):

\[
E_i = \sum_{k=1}^{P} \lambda_k , \quad 1 \leq i \leq P
\]

Formula (5) is the energy corresponding to a spectral radius, which can also be understood as an energy measure that can be obtained at a spectral scale. If the rank of the matrix is \(P\), a sequence pair with length \(P\) can be obtained. The energy measure of the sequence pair varies with the energy spectrum scale. Similar to the relationship between energy spectrum and frequency in Fourier transform, there is a fractal law between the scale defined in this way and the energy measure [9]. Using measure to characterize the energy distribution of ore-caused elements can quantitatively express complex geological mineralization in the form of energy redistribution. Therefore, the analysis of the measurement curve is a new way to quantitatively divide the energy boundary scale of mineralization.
2.2. Calculating Critical Singular Value Based on Multi-fractal Quantification

The multi-fractal model can qualitatively solve the problem of irregular nonlinear feature extraction related to mineralization [10]. In general, the critical singular value in airborne gamma-ray spectroscopy data can be determined by directly observing the singular value \( \sigma_i \) and the energy measurement curve \( E_i \), but it has a large observation error which can be avoided by calculating the critical eigenvalue quantitatively by mathematical means. The distribution of singular value-energy measure curve has multi-fractal similarity under different observation scales. Critical singular value represents the quantization of different observation scales. The change of critical singular value can directly determine the scale of different energy signals being filtered. Therefore, when the multi-fractal model appears in the data studied, the geometric characteristics of the singular value-energy measurement curve will not change with the critical singular value. The curve with the above properties must obey the following power law distribution:

\[
E_i = C ? \sigma_i^{-D} (\sigma_i, 0)
\]  

(6)

In formula (6), \( \sigma_i \) is a singular value; \( C \) is a non-negative constant, \( D \) is a fractal dimension; \( E_i \) is an energy measure corresponding to \( \sigma_i \). Take logarithms on both sides of Formula (6), A simple linear equation between \( E_i \), \( D \) and constant \( C \) can be obtained.

\[
\log E_i = -D \log(\sigma_i) + \log(C)
\]  

(7)

The slope \( D \) obtained by least square estimation is the fractal dimension.

Specific steps of FSVD:

1. Perform Kriging interpolation on the Airborne gamma-ray spectrum data to form regular gridding data;
2. Perform singular value decomposition on the gridding data to obtain a couple of non-zero singular values arranged in a large to small manner;
3. Calculate the energy measure \( E_i \) corresponding to the first \( n \) singular values \( \sigma_i \);
4. Take \( \sigma_i \) as the horizontal axis and \( E_i \) as longitudinal axis, project scatter points on the X-Y plane coordinates.
5. With the principle that the sum of residual squares is the smallest, fit scatter points into multiple linear segments, and singular values corresponding to intersections of the fitted segments are the critical singular values.
6. Reconstruct the singular values corresponding to the ore-caused anomaly.

3. Experiment and Result

3.1. General Situation of the Survey Area

This paper selects the AGS863 airborne gamma spectroscopy system independently developed by Chengdu University of Technology as the detection tool, and selects the uranium content in the 1:50000 airborne radioactivity field test data within the scope of a metal mine in Inner Mongolia Autonomous Region as the data source. The location of the survey area is shown in Figure 1(A), and the geological map is shown in Figure 1(B). There are known lead-zinc mineralized spots in the survey area. The uranium element is high in lead-zinc ore, and target prospecting area can be identified by the anomaly area of uranium ore. Ground verification was carried out in lead-zinc mining area, and the geological
map of the verification area is shown in Figure 1(C). AGS863 airborne gamma spectrometry detector consists of two boxes of crystals and each of them consists of 5 4L arrayed NaI crystal detectors (one of the crystals placed in the upper part is used to monitor the concentration of atmospheric radon and cosmic rays). The detector is installed on a Y-12 conveyor. The measurement platform is equipped with data acquisition system, radio altimeter, and global satellite positioning system, etc. With 21 design survey lines and 6705 survey points, the area of the survey area is about 300 km². The average value of uranium content is \( 3.968 \, \mu g/g \) with the highest value being \( 37.204 \, \mu g/g \), the lowest value being \( 0.049 \, \mu g/g \), and the standard deviation being \( 3.186 \, \mu g/g \).

![Diagram showing map of the verification area](image)

**Figure 1.** A is the sketch map of the location of the survey area; B is the geological map of the survey area; C is the geological map of the ground verification area.

### 3.2. FSVD Anomaly Information Extraction

By formula (1), use SVD to interpolate the data of the survey area, calculate the relevant parameters and draw Table 1. It can be seen from Table 1 that the signals corresponding to the large singular values account for most of the energy of the total signals, and the small singular values account for a small proportion of energy. Only the first nine singular values contribute 93% of the total energy to the total energy.

| No. | Singular Value | Energy Density (%) | Cumulative Energy (%) |
|-----|----------------|--------------------|-----------------------|
| 1   | 117.80         | 59.87              | 59.87                 |
| 2   | 43.49          | 8.16               | 78.96                 |
| ... | ...            | ...                | ...                   |
| 8   | 17.09          | 1.26               | 92.58                 |
| 9   | 15.87          | 1.09               | 93.67                 |
| ... | ...            | ...                | ...                   |
| 31  | 2.81           | 0.34               | 99.50                 |
| ... | ...            | ...                | ...                   |
By analyzing the relationship between the sequence of singular values and the cumulative energy percentage, it is found that when the singular value sequence is 31, the cumulative energy percentage reaches 99.5%. It can be considered that the first 31 singular values can reflect most of the energy of the total signals. At this point, linearly fit the singular value curve and quantitatively calculate critical singular values. The fitting curve is shown in Figure 2.

After calculation, $\sigma_1=13.53$, $\sigma_2=49.52$. In Figure 2, the fitting line segment A corresponds to the smallest singular value, and the corresponding energy is the lowest, which represents the perturbation anomaly caused by the high frequency noise in the survey area; segment B corresponds to relatively small singular values, and its corresponding energy is higher, which represents the anomaly information of element enrichment caused by hydrothermal mineralization and other geological processes in the survey area. Segment C corresponds to the largest singular value and the highest energy, representing the stable geological background information in the survey area.

![Figure 2](image)

**Figure 2.** Figure with short caption (caption centred).

### 4. Result and Discussion

Using the singular values in segment B, the decomposed data is reconstructed, and the raw data of survey area and the anomaly data extracted by FSVD are respectively plotted into the contour map. The result is shown in Figure 3A and Figure 3B.

In Figure 3A, due to noise and different time domain batches, different degrees of irregular halo anomalies appear in the survey area. Among them, due to the metal mines in area a, the uranium content is extremely high, and thus a large number of irregular anomalies appear in the southeast corner of the survey area; there are banding anomalies along the route direction in areas b and c; large-scale weak anomalies exist in the $J_{mn}$ in area d; A step type of positive-negative anomaly crossover occurred in area e where the $J_\gamma$ outcrop. The above anomalies are superimposed on ore-caused anomalies and interfering noises, causing interference to ore-caused anomaly identification and ground verification work.

In Figure 3B, after the raw data is processed by FSVD, the background is suppressed, negative anomaly area is reduced, and the ore-caused anomaly position of the ore occurrence is highlighted. A “crescent” anomaly occurs in area a, which coincides with the height of the ore shape in Figure 1B. The banding false anomalies along the route direction of the b and c areas are basically filtered out, showing a sporadic distribution of scatter-like anomalies; area d shows that there is a weak anomaly; the step type of positive-negative anomalies disappear and the weak anomalies are highlighted.
Comparing area a with ground verification map (Figure 3C), it is found that compared with the anomaly information presented by the raw data, the information about shape, location and scope of anomalies decomposed and reconstructed by are consistent with the ground verification result, and the anomaly intensity and background value are basically unchanged, which shows that FSVD method can filter out false anomaly information and eliminate interfering noise, suppress background, highlight anomalies, and accurately identify ore-caused anomalies from data doped with interference.

Figure 3. A Raw Data Contour Map; B FSVD Processing Effect contour map; C Ground Verification Regional Geological Map.

5. Conclusion
FSVD uses SVD to decompose the original signals. Based on fractal theory, the singular value-energy measure curve after decomposition is analyzed. According to the principle of minimum residual square sum, the curve is fitted and the intersection points are obtained. The critical singular values can be calculated quantitatively, and the complicated geological structures can be distinguished. The partial singular values are reconstructed, and the anomaly signals caused by uranium ore in the survey area are successfully extracted. Through the experimental analysis of the airborne gamma-ray spectrum production data, it is found that the original data is decomposed by singular values, and different singular value sections represent different anomaly information: The high singular value section (>50) represents the background signal, the medium singular value (13-50) represents the ore-caused anomaly information, the low singular value part (<13) represents other noises; and the banding anomaly signal caused by flight error exists in the high singular value section. The method can suppress the background better and highlight the anomaly. The banding false anomalies caused by time domain batches are weakened, and the positive and negative false anomalies appearing in the southern part of the survey area disappear. Combined with the ground verification map, it is found that the anomaly range of the
known mine points is reduced, and the anomaly shape is consistent with the actual mine site, which provides reliable data support for the ground anomaly verification work.

FSVD is used to decompose, filter and reconstruct the aerial gamma energy spectrum data, which can effectively extract the anomaly and provide an indicative role for the selection of the prospecting target areas. However, when interpolating the original data, the interpolation method will affect the result. In the next step, the interpolation method suitable for the aviation gamma spectroscopy data should be further analyzed to improve the accuracy of the anomaly information extraction.

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