Theoretical model for the responding time in low permeability reservoirs

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Abstract: As an important index in oilfield development, flood-response time can be used to describe the effect of water flooding in porous media. The responding time in low permeability reservoir is usually calculated by the method of stable state successive substitution neglecting the effect of medium deformation. Numerous studies show that the media deformation has an important impact on the development for low permeability reservoirs and can not be neglected. On the base of streamline tube model, we developed a method to interpret responding time with medium deformation factor. The results show that: the media deformation factor, threshold pressure gradient and well spacing have a significant effect on the flood response time. With the greater the media deformation factor, threshold pressure gradient or well spacing, the flooding response time becomes slower. As the angle with the main streamline increases, the water flooding response time delays as a "parabola" shape.

1. Introduction
Water injection is one of the widest methods used in the development of low permeability reservoirs. After water injection, pressure wave spreads continuously along the streamline from injector to producer. When pressure wave spreads to the production well, the rate of the production well will be affected and this time is called the effective time [1-2]. Effective time is one of the most important parameters to evaluate the effect of water flooding in porous media, and is an important index in oilfield development. So, it is necessary to study the responding time in the development of low permeability reservoirs.

A common method to calculate the responding time is numerical simulation method [3-4]. However, numerical simulation method is susceptible to the influence of numerical dispersion. Therefore, the ability of numerical methods to study the responding time is limited by this constraint. Currently, many scholars have established many models, such as streamline tube model [2,5-8] to calculate the effect time. These methods are convenient to study the responding time, however, these models never consider the impact of the medium deformation. Many studies have indicated that medium deformation is very important in low permeability reservoir and cannot be neglected [9-14]. This paper tested the medium deformation rule through experiments and established a model to calculate the responding time with media deformation factor included.

2. Experimental measurement
In order to ensure the accuracy, we designed the experiment under the stratum condition. For low permeability reservoir, media deformation test under different effective stress has been studied with
OPP high-pressure porosity permeability instrument and constant temperature box. The experiment equipment is shown as Figure 1. The core samples were from low permeability reservoir in Xinjiang oilfield, with the length of 200mm and diameter of 25mm. The maximum working pressure of OPP instrument is 70MPa and the temperature of incubator varies from 0~150℃. The accuracy of volume measurement is 0.001mL.

The normalized permeability under the original formation pressure conditions can be written

\[ K_{De}(p) = \frac{K}{K_e} \]  

(1)

After the normalization processing, rock permeability and effective stresses in different permeability intervals are shown in Figure 2. The regression expression between permeability and effective stress are given in Tab.1. Tab.1 reveals that the permeability and effective stress meet exponential relationship.

![Figure 1. High-pressure OPP poro-permeability equipment](image1)

![Figure 2. The curves of rock normalized permeability versus effective stresses](image2)

| Cores | Initial permeability | Regression expression | R² |
|-------|----------------------|------------------------|----|
| #1    | 5mD                  | \[ K = 0.9768 \exp \left[ 0.0018 \left( p - p_e \right) \right] \] | 0.9467 |
#2 2.88mD \quad K = 0.9106 \exp\left[0.0064\left(p - p_e\right)\right] \quad 0.9439

#3 0.47mD \quad K = 0.8175 \exp\left[0.0123\left(p - p_e\right)\right] \quad 0.9349

#4 0.19mD \quad K = 0.7245 \exp\left[0.0254\left(p - p_e\right)\right] \quad 0.9221

3. Mathematical models

According to the medium deformation experiment, the equation of permeability can be written as

\[ K(p) = K_e \exp\left[\alpha\left(p - p_e\right)\right] \quad (2) \]

According to the thought of stable state successive substitution and the streamline tube model, we can get a steady-state seepage equation with taking threshold pressure gradient and media deformation factor into consideration

\[ \frac{d}{d\xi} \left[ \frac{k(p)}{\mu} A(\xi) \left( \frac{dp}{d\xi} - \lambda \right) \right] = 0 \quad (3) \]

The boundary condition can be written as

\[ p(\xi = 0) = p_{in}; \quad p(\xi = l) = p_{nf} \quad (4) \]

According to the following transformation equation

\[ U(p) = \exp\left[\alpha\left(p - p_e\right)\right] \quad (5) \]

The seepage model can be described as

\[ \left\{ \begin{array}{l}
\frac{d}{d\xi} \left[ A(\xi) \left( \frac{dU}{d\xi} - \alpha \lambda U \right) \right] = 0 \\
U(\xi = 0) = U_1 = e^{\alpha(p_{nf} - p_e)}; \quad U(\xi = l) = U_2 = e^{\alpha(p_{nf} - p_e)}
\end{array} \right. \quad (6) \]

By solving Eq. (6), we can get the following equation

\[ U = e^{\alpha l U_1} + e^{\alpha l U_1 U_2} \int_{r_i}^{l} \frac{e^{-\alpha l}}{A(\xi)} d\xi \quad (7) \]

The cross-sectional area \( A(\xi) \) can be described as

\[ \left\{ \begin{array}{l}
A(\xi) = 2\xi h \tan \theta \quad \xi \leq \frac{l}{2} \\
A(\xi) = 2(l - \xi) h \tan \theta \quad \xi > \frac{l}{2}
\end{array} \right. \quad (8) \]

Inserting Eq. (8) into Eq. (7)

\[ U = e^{\alpha l U_1} + e^{\alpha l U_1 U_2} \frac{2h \tan \theta (U_2 - e^{\alpha l U_1})}{\int_{r_i}^{l/2} \frac{e^{-\alpha l}}{\xi} d\xi + \int_{l/2}^{l - \xi} \frac{e^{-\alpha l}}{l - \xi} d\xi} \int_{r_i}^{l} \frac{e^{-\alpha l}}{A(\xi)} d\xi \quad (9) \]

Then, the pressure distribution can be written

\[ p(\xi) = p_e + \frac{1}{\alpha} \ln \left\{ e^{\alpha l U_1} + e^{\alpha l U_1 U_2} \frac{2h \tan \theta (U_2 - e^{\alpha l U_1})}{\int_{r_i}^{l/2} \frac{e^{-\alpha l}}{\xi} d\xi + \int_{l/2}^{l - \xi} \frac{e^{-\alpha l}}{l - \xi} d\xi} \int_{r_i}^{l} \frac{e^{-\alpha l}}{A(\xi)} d\xi \right\} \quad (10) \]

According to the rate formula,
we have

\[
q = A(\xi) \frac{k(p)}{\mu} \left( \frac{dp}{d\xi} - \lambda \right) \tag{11}
\]

Due to the elasticity theory and the material balance principle, the flow rate can be written as

\[
p(\xi) = p_e + \frac{1}{\alpha} \ln \left( \frac{e^{2\alpha l U_1} + e^{\alpha l (\xi - l)}}{e^{2\alpha l U_1} + 1} \right) \int_{r_e}^{\xi} \frac{e^{-\alpha l \xi}}{A(\xi)} d\xi
\]

\[
+ \frac{1}{\alpha l} \int_{r_e}^{\xi/2} e^{-\alpha l \xi} d\xi + \int_{\xi/2}^{\xi} e^{-\alpha l \xi} d\xi \frac{q(\xi) - p_{\text{eff}}}{A(\xi) - \lambda k_e} \tag{12}
\]

Due to the elasticity theory and the material balance principle, the flow rate can be written as

\[
q_r = \int_{r_e}^{\xi} A(\xi) \frac{\phi C_r (p(\xi) - p_{\text{eff}})}{q} d\xi \tag{13}
\]

then the responding time can be calculated as

\[
t_{\text{eff}} = \int_{r_e}^{\xi} A(\xi) \frac{\phi C_r (p(\xi) - p_{\text{eff}})}{q} d\xi \tag{14}
\]

4. Results and Discussion

Employing the data in Table 2 and using the formula of the responding time by Eq. (14), we can study the responding time in detail.

| Table 2 Based data and actual production data |
|-----------------------------------------------|
| Initial permeability 2.0×10^{-3}μm² | Water injection rate 30m³/d |
| Effective thickness 4.5m | Radius 0.12m |
| Compressibility 4.34×10^{-3}MPa⁻¹ | Injection pressure 25MPa |
| Initial porosity 12% | Viscosity 4.34mPa·s |
| Threshold pressure gradient 0.01MPa/m | Media deformation factor 0.01MPa⁻¹ |
| Well spacing 150m | Bottom hole pressure 8MPa |
| Initial pressure 20MPa |

The responding time varies with the change of media deformation factor. The relationship between the effect time and media deformation factor is shown as Figure 3. From Fig.3, it can be seen that the greater media deformation coefficient is, the slower the responding time is. The main reason for this is that permeability and conductivity coefficient decreases when the rock pores deformed. With the larger the media deformation coefficient, the rock permeability becomes smaller and he propagation speed of pressure wave becomes slower. The responding time delays due to the threshold pressure gradient. The relationship between the effect time and threshold pressure gradient is shown as Fig. 4.
Figure 3. Relationship between media deformation factor and responding time

Figure 4. Relationship between threshold pressure gradient and responding time

Figure 5 reveals that the responding time delaying with the increase of the well spacing. The larger well spacing is, the longer the distance for propagation of pressure wave is. The responding time varies with different streamlines (shown as Fig.6). The main streamline (angle with the main streamline equals zero) has the fastest response time. The larger the angle with the main streamline is, the slower the response time along the streamline is. With the increase of the angle, the water flooding response time delays as a "parabola" shape.
5. Conclusion
1. A medium deformation experiment has been studied under the stratum condition with the OPP high-pressure porosity permeability instrument and constant temperature box. Through it, we verified that permeability and effective stress met exponential relationship.

2. Based on the model of the streamline tube, a new model with medium deformation factor to calculate water flood response time has been established. In addition, the influence of different parameters on the water flood effective time has been studied.

3. Many parameters such as media deformation factor, threshold pressure gradient and injector spacing have a significant impact on the effective time. The larger the media deformation factor is, the smaller the permeability becomes, and the slower the responding time is.

4. Waterflooding effective time varies with different streamlines. The pressure wave along the main streamline has the fastest propagation velocity and the shortest propagation path. With the angle with the main streamline increases, the water flooding response time delays as a "parabola" shape.
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