STRANGENESS IN SKYRME 
AND IN NAMBU–JONA-LASINIO MODELS

M. PRASZALOWICZ

Institute for Theor. Physcs II, Ruhr-University
4630 Bochum, Germany

ABSTRACT
We discuss the phenomenology of SU(3) Skyrme and Nambu–Jona-Lasinio models. We show that while the Skyrme model, in its simplest version, is unable to reproduce the hyperon spectrum, the NJL model describes both hyperon and isospin splittings with satisfactory accuracy. The difference between the two models lies in new anomalous moments of inertia, which vanish in the Skyrme model and get non-zero contribution from the valence quarks in the NJL model

1. Introduction

Although QCD is now commonly accepted as the ultimate theory of strong interactions, the low energy properties of the hadronic states are usually calculated in terms of various effective models, which are believed to follow from QCD. Since there is however no rigorous direct derivation (see e.g. Ref.[1]), one has some freedom in choosing one’s preferable model and preferable lagrangian.

In this talk we will discuss the phenomenology of the SU(3) Skyrme model1–5 and the Nambu–Jona-Lasinio (NJL) model.6–9 In order to make the discussion transparent we will stick to the simplest versions of the models; the more refined versions are discussed here by other speakers. Our attention will be focused on the splittings both in SU(3) multiplets and in isospin multiplets. We will not consider absolute masses which come out always too high.

We will see that in the simplest version of the Skyrme model the pattern of splittings is generically wrong: Λ−N= Σ−Λ = 2 (Ξ−Σ) in contradiction with experimental situation where Λ−N≈ Ξ−Σ > Ξ−Λ. To cure this disease one usually escapes to more complicated lagrangians and, at the same time, one applies diagonalization procedure of Yabu and Ando.10 Although, as we will see on the example of the explicit perturbative calculation, this method shifts the spectrum in the right direction, one has to express a criticism against it, as it sums up an arbitrary subseries in the strange quark masses, neglecting other terms of the same order.

In the semibosonised NJL model (see e.g. Ref.[11,12]), in which quarks interact via a self-consistent meson field, the spectrum is satisfactorily reproduced6,7 in the linear order in ms. In the NJL model the energy gets contribution from the valence and sea quarks. The classical part, i.e. the energy of the soliton, is exactly the same as in the two flavor case. The quantum corrections are calculated by adiabatical rotation of the

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†Alexander von Humboldt Fellow, on leave of absence from the Institute of Physics, Jagellonian University, ul. Reymonta 4, 30-059 Kraków. Poland
soliton resulting in a hamiltonian analogous to the one of the Skyrmion. A novelty is due to the mixed terms linear in the current quark mass and in the rotational velocity. These terms vanish in an ordinary Skyrme model; in the present model they get main contribution from the valence part. The resulting spectrum fits the data with a 10% accuracy. At the same time the isospin splittings due to the \( m_d - m_u \) mass difference are reproduced within experimental errors. Keeping in mind the simplicity of the starting lagrangian these results are surprisingly accurate.

2. Gell-Mann–Okubo Mass Formulae

Any dynamical model of light baryons has to reproduce and also explain the Gell-Mann–Okubo\(^{13,14}\) mass formulae which are derived assuming that the SU(3) breaking mass operator \( \Delta M \) transforms like a \( Y = 0, \ I = 0 \) and \( I_3 = 0 \) component of the octet tensor operator. Then, due to the Wigner–Eckhart theorem, matrix elements of this operator are given by:

\[
\Delta M_B^{(8)} = F \begin{pmatrix} 8 \\ 000 \\ B & B \end{pmatrix} + D \begin{pmatrix} 8 \\ 000 \\ B & B \end{pmatrix}
\]

for the octet, and

\[
\Delta M_B^{(10)} = C \begin{pmatrix} 8 \\ 000 \\ 10 & 10 \end{pmatrix}
\]

for the decuplet. The reduced matrix elements \( F, \ D \) and \( C \) are free constants. \( B = Y, \ I, \ I_3 \) for the baryon in question. The SU(3) Clebsch–Gordan coefficients can be written in terms of the diagonal SU(3) operators:\(^{15}\)

\[
\Delta M_B^{(8)} = -\frac{F}{2} Y - \frac{D}{\sqrt{5}} (1 - I^2 + \frac{1}{4} Y^2), \quad \Delta M_B^{(10)} = -\frac{C}{2\sqrt{2}} Y.
\]

The predictive power of Eq. (3) consists in the fact, that the number of free parameters, which could in principle parametrize the mass splittings, is reduced from 3 to 2 for the octet and to 1 for the decuplet.

Equations (3) yield the relations:

\[
F = M_\Xi - M_N,
\]

\[
\frac{1}{\sqrt{5}} D = \frac{1}{2} (M_\Sigma - M_\Lambda) = \frac{1}{3} (2M_\Sigma - M_\Xi - M_N) = M_\Xi + M_N - 2M_\Lambda,
\]

for the octet, and equal level spacing for the decuplet:

\[
\frac{1}{2\sqrt{2}} C = M_{\Sigma^*} - M_\Delta = M_{\Xi^*} - M_{\Sigma^*} = M_\Omega - M_{\Xi^*}.
\]

From these relations we can estimate the values of parameters \( F, \ D \) and \( C \):

\[
F = 379, \quad D = 79 \pm 17 \quad \text{and} \quad C = 415 \pm 15 \quad \text{MeV}.
\]
The resulting spectrum is presented in the first column of Fig.1. A small admixture of other operators like $Y^2$ shifts the spectrum to the experimental position.

The mass splittings of the baryons belonging to the same isospin multiplet consist of two parts: hadronic and electromagnetic,\(^{16}\) namely:

$$\Delta m_B = (\Delta m_B)_h + (\Delta m_B)_e. \quad (7)$$

If the isospin breaking is assumed to be driven by an octet isovector tensor operator corresponding to $I_3 = 0$ then, in analogy to the Gell-Mann–Okubo mass formulae (1, 2), one gets:

$$\Delta m_B^{(8)} = \frac{1}{\sqrt{3}} f \begin{pmatrix} 8 & 8 & 8 \\ 0 & 0 & 0 \\ B & B & B \end{pmatrix} + \frac{5}{3} d \begin{pmatrix} 8 & 8 & 8 \\ 0 & 0 & 0 \\ B & B & B \end{pmatrix} \quad (8)$$

for the octet, and

$$\Delta m_B^{(10)} = \frac{2}{3} c \begin{pmatrix} 8 & 8 & 8 \\ 0 & 0 & 0 \\ B & B & B \end{pmatrix} \quad (9)$$

for the decuplet (normalization factors in front of the $f, d$ and $c$ are chosen for future convenience). Evaluating the SU(3) Clebsch-Gordan coefficients gives:\(^{15}\)

$$\Delta m_B^{(8)} = - \frac{1}{3} f I_3 + d Y I_3, \quad \Delta m_B^{(10)} = - \frac{1}{3} c I_3. \quad (10)$$

Electromagnetic part of the isospin splittings was estimated by Gasser and Leutwyler\(^{16}\) for the octet. Their estimate confirms a reasonable assumption that $\Sigma^- - \Sigma^+$ mass difference has no electromagnetic contribution. This assumption allows us to determine coefficient $f$, and also $c$ for the decuplet, where no estimate of the electromagnetic part of $\Sigma^*- - \Sigma^{**}$ exists. Coefficient $d$ can be determined from the hadronic part of the n–p mass difference.\(^{9}\) Altogether we get:

$$f = 12.11 \pm 1.14 \quad , \quad d = 1.73 \pm 0.38 \quad \text{and} \quad c = 6.6 \pm 1.0 \quad \text{MeV.} \quad (11)$$

Symmetry considerations alone are not able to provide us with any relations between the reduced matrix elements. Dynamical models, like Skyrme model or NJL model, make specific predictions for these constants. In the next sections we will calculate coefficients $F, D, C, f, d, \text{ and } c$ within the framework of the simplest version of the SU(3) Skyrme model and subsequently in the simplest version of the semibosonised SU(3) NJL model.

3. SU(3) Skyrme Model

Let us start by specifying the effective lagrangian proposed by Skyrme\(^{17,18}\) and later generalized by Witten:\(^{19,20}\)

$$\int dt L_{Sk} = \frac{F_\pi^2}{16} \int dt \int d^3 r \text{Tr}(\partial_{\mu}U^\dagger \partial^\mu U) + \frac{1}{32e^2} \int dt \int d^3 r \text{Tr}([\partial_{\mu}U U^\dagger, \partial_{\nu} U U^\dagger]^2)$$

$$+ \int dt \int d^3 r \text{Tr} \left\{ a \{U + U^\dagger - 2\} + b \{(U + U^\dagger) \lambda_8\} \right\} + N_c \Gamma_{WZ}. \quad (12)$$
Here \( N_c \) is a number of colors, parameters \( F_\pi \) and \( e \), if taken from meson physics, are equal to \( F_\pi \sim 186 \text{ MeV} \) and \( e \sim 5.5 \) respectively. \( \Gamma_{WZ} \) denotes the Wess-Zumino term\(^{21,22} \) and

\[
a = \frac{F_\pi^2}{32} (m^2_\pi + m^2_\eta), \quad b = \frac{\sqrt{3} F_\pi^2}{24} (m^2_\pi - m^2_K).
\]  

(13)

A time-independent Ansatz for the SU(3) matrix \( U_0 \) takes the form of a hedgehog:

\[
U_0 = \begin{bmatrix}
\cos P(r) + i\vec{n}\tau \sin P(r) & 0 \\
0 & 0 & 1
\end{bmatrix},
\]

(14)

where \( P(r) \) is a profile function. For the purpose of this talk we will take

\[
P(r) = 2 \arctan \left( \frac{r}{r_0} \right)^2
\]

(15)

and calculate analytically all relevant quantities as functions of the soliton size \( r_0 \).

The energy of the solution \((14)\) is then given by:\(^{23}\)

\[
M_{\text{cl}} = \frac{F_\pi}{e} \pi^2 \frac{3\sqrt{2}}{16} (4x_0 + \frac{15}{x_0}) + \frac{\mu^2}{e^3 F_\pi} \pi^2 \frac{\sqrt{2}}{2} x_0^3.
\]

(16)

with

\[
\mu^2 = \frac{m^2_\pi + 2m^2_K}{3} \approx (412 \text{ MeV})^2,
\]

(17)

where \( x_0 = e F_\pi r_0 \).

Minimizing Eq.\((14)\) with respect to \( x_0 \) we find in the chiral limit \((\mu^2 = 0)\) \( x_0 = \sqrt{15/4} \) and \( M_{\text{cl}} = 40.54 \text{ F}_\pi / e \) i.e. approximately 1370 MeV. This certainly unsatisfactory result (going off chiral limit shifts \( M_{\text{cl}} \) further up by about 280 MeV) is common for all chiral models which tend to overestimate the classical soliton mass. There is a hope that various other effects like gluonic corrections\(^{24} \) or Casimir effect\(^{25,26} \) may bring this value down to less than 1 GeV. In what follows we will abandon the idea to fit the absolute masses, instead we will concentrate on the mass splittings.

The splittings are calculated by rotating the static solution

\[
U_0 \rightarrow A(t) U_0 A^\dagger(t),
\]

(18)

where \( A \in SU(3)/U(1) \), since \([\lambda_8, U_0] = 0\). Therefore matrix \( A \) is defined up to a local \( U(1) \) factor \( h = \exp(i\lambda_8 \phi) \), i.e. \( A \) and \( Ah \) are equivalent. This leads to a constraint which has to be imposed on the physical spectrum. Introducing 8 collective coordinates:\(^{2-4}\)

\[
A^\dagger(t) \frac{d}{dt} A(t) = \frac{i}{2} \sum_{\alpha=1}^{8} \lambda_\alpha \Omega_\alpha;
\]

(19)
one gets the following quantum mechanical lagrangian:  

\[ L = -M_{cl}[P] + \frac{I_A[P]}{2} \sum_{i=1}^{3} \Omega_i^2 + \frac{I_B[P]}{2} \sum_{k=4}^{7} \Omega_k^2 + \frac{N_c}{2\sqrt{3}} \Omega_8 + \Delta m[A, P], \]  

(20)

where \( \Delta m[A, P] \) corresponds to the symmetry breaking piece. The moments of inertia can be again calculated analytically:  

\[ I_A = \frac{1}{e^3 F_\pi} \pi^2 \sqrt{2} \left( \frac{6x_0^3 + 25x_0}{12} \right) \approx \frac{107}{e^3 F_\pi}, \]  

\[ I_B = \frac{1}{e^3 F_\pi} \pi^2 \sqrt{2} \left( \frac{4x_0^3 + 9x_0}{16} \right) \approx \frac{40.5}{e^3 F_\pi}. \]  

(21)

The standard quantization procedure leads to the following hamiltonian:  

\[ M_B = M_{cl} + H_{SU(2)} + H_{SU(3)} + H_{br}, \]  

(22)

with  

\[ H_{SU(2)} = \frac{C_2(SU(2)_R)}{2I_A}, \]  

\[ H_{SU(3)} = \frac{C_2(SU(3)_L) - C_2(SU(2)_R) - \frac{N_c^2}{12}}{2I_B}, \]  

(23)

where \( C_2 \) denotes the quadratic Casimir operator of the right \( SU(2) \) symmetry corresponding to spin and of the left \( SU(3) \) corresponding to flavor.

The wave function of the baryon state can be written as an \( SU(3) \) rotation matrix:  

\[ \psi_B(A) = \sqrt{\text{dim}(p, q)} \ D_{A}^{(p,q)}(A) = \sqrt{\text{dim}(p, q)} \ \langle Y, I, I_3 | D^{(p,q)}(A) | Y_R, S, -S_3 \rangle, \]  

(24)

where quantum numbers \( B \) and \( S \) denote now hypercharge, isospin and its third component, and right hypercharge, spin and its third component (with minus sign), respectively. \( SU(3) \) representations are labeled by \( (p, q) \), however not all \( p \) and \( q \) are allowed. The system is constrained since lagrangian (20) does not contain terms quadratic in the 8-th velocity. The constraint \( Y_R = \frac{N_c}{3} \) selects the representations of triality zero:  

\[ 8, \ 10, \ \bar{10}, \ 27, \ 35, \ \bar{35}, \ 64, \ldots \]  

(25)

for \( N_c = 3 \). The success of the model is the prediction that the lowest baryonic states belong to the octet and decuplet representations of \( SU(3) \).

The symmetry breaking Hamiltonian:  

\[ H_{br} = \alpha \ D_{8,8}^{(s)}(A) \]  

(26)

splits the hyperon spectrum (here representation (1,1) is denoted as \( (8) \) and index 8 corresponds to \( Y = 0 \) and \( I = 0 \)). Constant \( \alpha \) is given as:  

\[ \alpha = \frac{\Delta \mu^2}{e^3 F_\pi} \pi^2 \sqrt{2} \frac{x_0^3}{2} \approx -\frac{7.7 \ \text{GeV}^2}{e^3 F_\pi}, \]  

(27)
Fig. 1. Spectrum of light baryons. Dashed lines represent experimental data, solid lines correspond to theoretical predictions described in the text.
where we have used
\[ \Delta \mu^2 = \frac{2}{3}(m_K^2 - m_{\pi}^2) \approx (388 \text{ MeV})^2. \] (28)

The first and probably the only good news about the breaking hamiltonian (26) is that it automatically fulfills the Gell-Mann–Okubo mass relation, since the left SU(3) Clebsch-Gordan coefficients coincide with the ones of Eqs[1, 2]. The right C-G coefficients give specific predictions for the reduced matrix elements:
\[ F = -\frac{1}{2} \alpha, \quad D = -\frac{1}{2\sqrt{5}} \alpha, \quad C = -\frac{1}{2\sqrt{2}} \alpha. \] (29)

The spectrum however looks rather odd since the two ratios:
\[ \frac{F}{D} = \sqrt{5} = 2.24 \quad \text{(exp. 4.40)}, \quad \frac{F}{C} = \sqrt{2} = 1.41 \quad \text{(exp. 0.91)} \] (30)

are far from their experimental values. If we choose \( \alpha = -758 \text{ MeV} \) in such a way that \( F \) is reproduced then we get spectrum shown in the second column of Fig.1. However \( \alpha \) is not a free parameter. The 10–8 splitting (230 MeV) requires \( e^3F_\pi = 16.41 \text{ GeV} \). Through Eqs(21, 27) we get that \( 1/I_B = 405 \text{ MeV} \) and \( \alpha \approx -470 \text{ MeV} \), approximately 40 % smaller than the value required to fit \( F \). One could in principle enlarge \( \alpha \) by tuning the kaon mass, but the problem of Eq.(30) remains.

Guadagnini\(^2\) in the first paper on the SU(3) quantization of the Skyrme model proposed to add by hand a term proportional to the hypercharge:
\[ H_G = \alpha D^{(8)}_{s,s}(A) + \beta Y. \] (31)

\( \beta \) is a free parameter and the reduced matrix elements read:
\[ F = -\frac{1}{2} \alpha - 2 \beta, \quad D = -\frac{1}{2\sqrt{5}} \alpha, \quad C = -\frac{1}{2\sqrt{2}}(\alpha + 8 \beta). \] (32)

An excellent fit to the data (column 3 in Fig.1) is obtained with \( \alpha = -382 \text{ MeV} \) and \( \beta = -94 \text{ MeV} \). This time \( \alpha \) is smaller than the value required by 10–8 splitting, but only by about 20 %.

One way to generate effective \( \beta \) is to calculate the \( O(\alpha^2) \) correction to baryon energy (this approach for not too large \( \alpha \) is in fact equivalent to the diagonalization procedure of Yabu and Ando\(^10\) which sums up the perturbative series in \( \alpha \)):
\[
\Delta M^{(8)}_B = \left( \frac{1}{4} \alpha - \frac{1}{60} \alpha^2 I_B \right) Y + \left( \frac{1}{10} \alpha + \frac{2}{75} \alpha^2 I_B \right) \left( 1 - I^2 + \frac{1}{4} Y^2 \right) \\
- \frac{1}{750} \alpha^2 I_B Y^2 - \frac{47}{750} \alpha^2 I_B,
\]
\[
\Delta M^{(10)}_B = \left( \frac{1}{8} \alpha - \frac{29}{672} \alpha^2 I_B \right) Y - \frac{1}{168} \alpha^2 I_B Y^2 - \frac{13}{168} \alpha^2 I_B.
\] (33)
Constants $F$, $D$ and $C$ can be immediately read off from Eqs.(33), moreover new terms not present in the original Gell-Mann–Okubo mass formula (3) are generated. Numerically they are small and we will neglect them in the present discussion.

In order to fit $F$ and $D$ one needs $\alpha \approx -677$ MeV and $1/I_B = 378$ MeV; with these values one gets a bit too small $C = 327$ MeV. These values of $\alpha$ and $I_B$ are again larger than the ones required by $10^{-8}$ splitting.

This was the status of the SU(3) Skyrme model in the mid eighties. A philosophical question whether one should trust perturbative expansion in the strange quark mass arose. Some people decided to work within an approach which breaks the SU(3) flavor symmetry already at the level of the Anzatz for the $U_0$ field, others tried to push up $\alpha$ and $I_B$ either by enlarging the original Skyrme lagrangian or by employing another model (like NJL model discussed in the next section).

4. Semibosonised SU(3) Nambu–Jona-Lasinio Model

It was already shown in the talk of Alkofer that the solitonic solutions of the semibosonised NJL model are studied in terms of an effective action:

$$S_{\text{eff}} = -Sp \log(i\phi - m - M U^\gamma).$$

This time, however, $U$ is an SU(3) matrix of Eq.(14). $M$ is the constituent quark mass, which is in fact the only free parameter of the model. The bare quark mass matrix can be written in a form:

$$m = \mu_0 \lambda_0 - \mu_8 \lambda_8 - \mu_3 \lambda_3,$$

where $\lambda_i$ are Gell-Mann SU(3) matrices ($\lambda_0 = \sqrt{2/3} 1$) and

$$\mu_0 = \frac{1}{\sqrt{6}}(m_u + m_d + m_s), \quad \mu_8 = \frac{1}{\sqrt{12}}(2 m_u - m_d - m_s), \quad \mu_3 = \frac{1}{2}(m_d - m_u).$$

The energy of the soliton consists of two parts: the energy of the continuum, i.e. the energy corresponding to the effective action (34), and the energy of the valence level. In what follows, for simplicity, we confine our discussion to the continuum part only, but one always has to remember that the pertinent valence contribution has to be added.

The effective action (34) can be rewritten in terms of the Euclidean spectral representation:

$$S_{\text{eff}} = -N_c T \int \frac{d\omega}{2\pi} \text{Tr} \log(i\omega + H) \left[ 1 + \frac{1}{i\omega + H}(-i\gamma_4 A^\dagger m A + A^\dagger A) \right],$$

where $H$ is the hermitean static hamiltonian: $H = \gamma_4 (\gamma_i \partial_i + M U_0)$, and appropriate regularization is understood. The static soliton solution for $H$ reduces to the one found in the SU(2) case. Formula (37) is already written in a form ready to be
expanded in a power series in $m$ and in generalized velocities $\Omega$. Let us for the moment forget about the mass matrix $m$ and expand (37) in powers of $\Omega$. We get (back in Minkowski metric) familiar lagrangian (20) with:

$$I_{ab} = \frac{N_c}{4} \int \frac{d\omega}{2\pi} \text{Tr} \left[ \frac{1}{i\omega + H} \lambda_a \frac{1}{i\omega + H} \lambda_b \right] = \begin{cases} I_A \delta_{ab} & \text{for } a, b = 1...3 \\ I_B \delta_{ab} & \text{for } a, b = 4...7 \\ 0 & \text{for } a, b = 8 \end{cases} \quad (38)$$

The above expression is assumed to be properly regularized and the full moments of inertia have also a valence part.

The quantization proceeds exactly as in the case of the Skyrme model,\textsuperscript{2–4} and as a result one arrives at the hamiltonian (22).

The novelty comes from the expansion in powers of the rotated matrix $m$:

$$L_m = -\sigma[\sqrt{6}\mu_0 - \sqrt{3}(\mu_8 D_{88}^{(8)} + \mu_3 D_{38}^{(8)})] - 2(\mu_8 D_{8a}^{(8)} + \mu_3 D_{3a}^{(8)})K_{ab}\Omega_b, \quad (39)$$

where constant $\sigma$

$$i\frac{N_c}{4} \int \frac{d\omega}{2\pi} \text{Tr} \left[ \frac{1}{i\omega + H} \gamma_1 \lambda_a \right] = \begin{cases} 6 \sigma & \text{for } a = 0 \\ 3 \sigma & \text{for } a = 8 \\ 0 & \text{for } a = 1...7 \end{cases} \quad (40)$$

is related to the pion-nucleon sigma term $\Sigma = 3/2(m_u + m_d)\sigma$ and anomalous tensor $K_{ab}$ is defined as:

$$K_{ab} = i\frac{N_c}{4} \int \frac{d\omega}{2\pi} \text{Tr} \left[ \frac{1}{i\omega + H} \gamma_2 \lambda_a \frac{1}{i\omega + H} \lambda_b \right] = \begin{cases} K_A \delta_{ab} & \text{for } a, b = 1...3 \\ K_B \delta_{ab} & \text{for } a, b = 4...7 \\ 0 & \text{for } a, b = 8 \end{cases} \quad (41)$$

We call $K_{ab}$ anomalous since it comes from the imaginary part of the effective action, which is related to anomaly, and as such does not require regularization. In fact $K_{ab}$ gets contribution almost entirely from the valence level.

The quantized hamiltonian $H_{br}$ is not as simple as in the Skyrme model and reads:

$$H_{br} = \alpha_8 D_{88}^{(8)}(A) + \beta_8 \gamma_8 + \sum_{a=1}^3 D_{8a}^{(8)}(A)S_a$$

$$H_{br} = \alpha_3 D_{38}^{(8)}(A) + \beta_3 \frac{2}{\sqrt{3}} I_3 + \gamma_3 \sum_{a=1}^3 D_{3a}^{(8)}(A)S_a, \quad (42)$$

where

$$\alpha_i = \sqrt{3} \left( -\sigma + \frac{K_B}{T_B} \right) \mu_i, \quad \beta_i = -\sqrt{3} \frac{K_B}{T_B} \mu_i, \quad \gamma_i = 2 \left( \frac{K_A}{T_A} - \frac{K_B}{T_B} \right) \mu_i. \quad (43)$$

Index $i = 8$ corresponds to $Y = 0, I = 0$ and index $i = 3$ to $Y = 0, I = 1, I_3 = 0$. 9
Fig. 2. Constants $F, D$ and $1/2 C$. Solid lines represent model predictions as functions of $m_s$. Dashed line correspond to the error bars of Eq.(6).

We see that (42) contains a term proportional to $Y$ which naturally arises in this model, moreover there is another term proportional to the product of a $D$ function and the spin operator.

We adopt the following numerical procedure: first we find the solitonic solution for a range of constituent masses $M$, then we find the optimal value of $M$ which reproduces the 10–8 splitting due to the rotational hamiltonian $H_{SU(2)}$ (23). It turns out$^{6,7}$ that $M = 390$ MeV and the corresponding moments of inertia take the following values: $1/I_A = 157$, $1/I_B = 234$, $1/K_A = 469$ and $1/K_B = 704$ MeV, and $\sigma = 3.07$.\footnote{\textsuperscript{20}} We will see that not only hyperon splittings but also isospin splittings are well reproduced. To this end let us let us define the following quantities:

$$\varphi = \sigma + 2 \frac{I_B}{K_B} + \frac{I_A}{K_A}, \quad \gamma = \sigma + 2 \frac{I_B}{K_B} + 5 \frac{I_A}{K_A}, \quad \delta = \sigma + 2 \frac{I_B}{K_B} - 3 \frac{I_A}{K_A}. \quad (44)$$

Then we get:

$$F = \frac{1}{2} \varphi m_s, \quad D = \frac{1}{2\sqrt{5}} \delta m_s, \quad C = \frac{1}{2\sqrt{2}} \gamma m_s, \quad f = \frac{3}{4} \varphi (m_d - m_u), \quad d = \frac{3}{20} \delta (m_d - m_u), \quad c = \frac{3}{8} \gamma (m_d - m_u). \quad (45)$$

In Figs 2 and 3 we plot the splitting constants of Eqs(45, 46) as functions of $m_s$ and $m_d - m_u$ respectively, together with the error bars corresponding to Eqs(8, 11). For $m_s \approx 175$ MeV the $N-\Xi$ splitting (\textit{i.e.} constant $F$) is reproduced. For the same value of $m_s$ constant $D$ corresponding to $\Sigma-\Lambda$ splitting is overestimated by 35 MeV,
whereas constant $C$ is underestimated by 60 MeV. The resulting spectrum is shown in column 4 of Fig.1. On the other hand, the isospin breaking constants $f$, $d$ and $c$ are reproduced within experimental errors for $m_d - m_u \approx 3.5$ MeV (see Fig.3).

The present model makes specific prediction for the ratio of hadronic to isospin breaking constants:

$$\frac{f}{F} (2.99 \pm 0.3) = \sqrt{5} \frac{d}{D} (4.90 \pm 2.15) = \sqrt{2} \frac{c}{C} (2.25 \pm 0.42) \times 10^{-2}, \quad (47)$$

where the numbers in brackets correspond to the experimental values of Eqs(6, 11). Certainly the central values are fairly scattered. We would like to offer the following explanation of this discrepancy. The isospin splittings are proportional to a tiny parameter, namely $m_d - m_u$, and therefore the first order of the perturbation theory is legitimate. On the contrary, for the hyperon splittings which are proportional to the much larger parameter, namely strange quark mass, one may expect some corrections from the higher order of the perturbative expansion in $m_s$. Indeed, already the second order brings the splittings to their experimental values with an accuracy of a few MeV. Last column in Fig.1 shows the spectrum obtained by applying the Yabu Ando procedure to the present model.

To summarize: we have studied the symmetry breaking effects due to the quark masses in the Skyrme model and in the solitonic sector of the semibosonised NJL model. In the simplest version of the Skyrme model the splitting operator is too
simple to account for hyperon splittings. On the contrary, in the simplest version of the NJL model, a satisfactory description of the hadronic mass spectrum including both the hyperon and the isospin splittings was found. The new terms in the splitting operator, which are not present in the Skyrme model, are proportional to the anomalous moments of inertia which get the main contribution from the valence level. The absolute masses are too big, but there exist several mechanisms which may bring them down, namely gluon corrections, rotational and translational band subtraction and Casimir energies of quantum fluctuations.

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