This talk provides a brief summary of the status of lattice QCD calculations of the light quark masses and the kaon bag parameter $B_K$. Precise estimates of these four fundamental parameters of the standard model, i.e., $m_u$, $m_d$, $m_s$ and the CP violating parameter $\eta$, help constrain grand unified models and could provide a window to new physics.

1. Introduction

It is a great pleasure to be here to help celebrate Pran Nath’s 65th birthday. I first met Pran and Dick in 1982 when I came to Northeastern as a post-doc in their group. They had just developed the first supergravity grand unified model in collaboration with Ali Chamseddine and were very keen to know if grand unified theories could be formulated on the lattice. The first hurdle in this quest was whether supersymmetric non-abelian gauge theories could be formulated on the lattice. Needless to say we did not make much progress on that front. Over the last 22 years there has been significant progress on formulating QCD with chiral symmetry on the lattice (staggered, twisted mass Wilson, overlap and Domain Wall (DW) fermions) and some on chiral gauge theories. However, the original problem of formulating supersymmetric non-abelian gauge theories on the lattice and exploring their strongly coupled sector using lattice simulations still remains.

My three years stay at Northeastern was very productive. Pran and Dick gave me total freedom to pursue lattice QCD and statistical mechanics and were extremely supportive of my work. During this time I continued
my collaboration with Apoorva Patel at Caltech and developed a number of new ones including those with Steve Sharpe and Greg Kilcup at Harvard, Belen Gavela and Rich Brower who were visiting Harvard, Gerry Guralnik and Chuck Zemach at Los Alamos, R. Shankar at Yale, and Bob Cordery and Mark Novotny at Northeastern. Being around Ali, Dick and Pran (in those days they seemed joined at the hip) was very inspirational. They would come in very early in the morning, get the coffee going, lock themselves in the conference room and work through the day taking only food and toilet breaks. Unfortunately, I could not stomach coffee and thus missed induction into the super world.

In this talk I would like to present a status report on two different calculations that were initiated during my stay at Northeastern. The first is on estimates of light quark masses, which is equivalent to validating QCD by reproducing the hadron spectrum. The second is the calculation of the kaon bag parameter $B_K$ which is illustrative of QCD correction to weak matrix elements. Together these calculations address fixing the values of four fundamental parameters of the standard model — the quark masses $m_u$, $m_d$, and $m_s$ and the CP violating parameter $\eta$ in the CKM matrix. Since any grand unified model will have to provide information on their origin and values, and any extension to the standard model will impact their values, determining their precise values is an important step in looking for new physics. There is therefore a deep connection between lattice QCD and the super world.

This talk will focus on providing the current best estimates. For a background on the evolution of these calculations I refer to the proceedings of the yearly lattice QCD conferences. Also, I will use two previous reviews as starting points and update the results here.

2. Light Quark Masses

A self-consistent calculation of light quark masses is equivalent to validating QCD by demonstrating that it reproduces the hadron spectrum. The central question is — do there exist values for the coupling constant $\alpha_s(M_Z)$ and the five quark masses $m_u$, $m_d$, $m_s$, $m_c$, and $m_b$ such that lattice QCD reproduces the masses and decays of all hadrons? Over the last two decades the answer is slowly but surely converging towards YES.

There are three methods that have been used to estimate the masses of the three light quarks — up, down and strange. These are chiral perturbation theory ($\chi$PT), QCD sum rules, and lattice QCD. It is instructive to
assess the strengths and weaknesses of each of these methods.

$\chi PT$ is an effective theory of pseudoscalar mesons. Terms in the chiral Lagrangian have the same symmetries as QCD and are classified in powers of $m^2$ and $p^2$. The expansion is characterized by the Gasser-Leutwyler (GL) coefficients $^7$. Using the $\chi PT$ Lagrangian one derives expressions for the masses and decays as an expansion in the quark masses and momenta, GL parameters, and an unknown scale $\Lambda_{\chi PT}$. An example of such a relation, which is relevant for the discussion of quark masses, is the expansion for $M_K^2$ tailored to staggered fermions on the lattice $^8$

$$\left(\frac{M_{K^+}^{\text{1-loop}}}{\mu (m_x + m_y)}\right)^2 = 1 + \frac{1}{16\pi^2 f^2} \left( \left[-\frac{2a^2\delta_v}{M_{\eta c}^2 - M_{\eta s}^2} (l(M_{\eta c}^2) - l(M_{\eta s}^2)) \right] + [V \rightarrow A] + \frac{2}{3} l(M_{\eta l}^2) \right)$$

$$+ \frac{16\mu}{f^2} (2L_8 - L_5)(m_x + m_y) + \frac{32\mu}{f^2} (2L_6 - L_4)(2m_s + m_x)$$

$$+ a^2C$$

(1)

where the $L_i$ are the GL constants, $l(M^2) = \ln(M^2/\Lambda_{\chi PT}^2)$ are the chiral logarithms, and terms proportional to $a^2$ contain the leading discretization errors.

Both $\chi PT$ and lattice QCD rely on relations like Eq. 1 to relate hadron masses to quark masses. The advantage of lattice QCD over $\chi PT$ is that one can carry out simulations dialing the quark masses and thus validate Eq. 1 over a range of quark masses. In $\chi PT$ one is limited by the number of physical meson masses and, unfortunately, even at NLO there are more unknown GL coefficients than there are pseudoscalar mesons. The key point made by Sharpe and Shoresh $^9$ is that as long as one simulates with 3 flavors of dynamical quarks (with equal or unequal valence and sea quark masses) that are small enough for NLO $\chi PT$ to be reliable, the GL coefficients are the same as in physical QCD so extrapolations to the physical point can be made using Eq. 1 and the GL coefficients so extracted are those of the physical world.

The second limitation of $\chi PT$ is that since it is based on the symmetries of QCD it does not provide an absolute scale for quark masses but does well in predicting ratios. At the 1-loop level it provides two ratios (one, due to the Kaplan-Manohar symmetry, requires some additional but reasonable
Estimates of these ratios from chiral perturbation theory are more accurate than present lattice results. The main uncertainties in lattice calculations of $m_u$ and $m_d$ are due to lack of complete control over chiral extrapolations down to a few MeV and due to ignoring electromagnetic effects in the simulations. So this talk will focus on lattice estimates of $m_s$ and knowing it $m_u$ and $m_d$ can be estimated using the ratios given in Eq. 1.

Lattice simulations until 2000 were done mostly in the quenched approximation due to limitations of computer power. In this approximation the effects of virtual quark loops on background gauge configurations are neglected. This approximation, in principle, is drastic as the quenched theory is non-unitary and one relied on phenomenological arguments to assume that estimates are reliable to within 10%. Nevertheless, since calculations of observables on an ensemble of quenched background gauge configurations are, in most cases, the same as in the full theory, the community obtained valuable understanding and control over statistical and the following systematic errors

- Finite volume corrections
- Extrapolation to the continuum limit
- Chiral extrapolation to physical $m_u$ and $m_d$.
- The calculation of renormalization constants using improved perturbation theory and/or non-perturbative methods.

What was missing was control over quenching errors and estimates of light quark masses using different states to set the lattice scale and fix their values varied by $10-30\%$.

In the last year two collaborations, HPQCD-MILC-UKQCD\textsuperscript{11} and CP-PACS/JLQCD\textsuperscript{12}, have reported results based on simulations with $2 + 1$ flavors of dynamical quarks. These simulations explore a range of quark mass values (as low as $m_s/8$ in the MILC simulations) and yield

\begin{align*}
    m_s(\overline{MS}, 2 \text{ GeV}) &= 76(0)(3)(6)(0) \text{ MeV} \quad (HPQCD - MILC - UKQCD) \\
    m_s(\overline{MS}, 2 \text{ GeV}) &= 80.4(1.9) \text{ MeV} \quad (CP - PACS/JLQCD) \quad (M_K) \\
    m_s(\overline{MS}, 2 \text{ GeV}) &= 89.3(2.9) \text{ MeV} \quad (CP - PACS/JLQCD) \quad (M_\phi).
\end{align*}
The CP-PACS/JLQCD collaboration use the estimate from $M_K$ for their central value and increase the upper limit of the error to accommodate the estimate from $M_\phi$. Last year, based on preliminary results from these two collaborations, I had concluded that $m_s(M_S, 2 \text{ GeV}) = 75(15)$ \(^5\). Both groups have tightened control over some of their uncertainties and the new estimate, averaging the two results based on $M_K$, is

$$m_s(M_S, 2 \text{ GeV}) = 78(10)$$ \(^4\)

*i.e.* mostly the change is in a reduction in the error estimate.

This value is significantly lower than the estimate from QCD sum rules which until 1996 was $m_s(M_S, 2 \text{ GeV}) = 125(40) \text{ MeV}$ \(^10\). Since then estimates from QCD sum rules have been getting lower and tracking those from lattice QCD \(^5\). The two main uncertainties in QCD sum rule analysis are the quality of experimental information on spectral functions in the scalar and pseudoscalar channels on one side and the convergence of perturbation theory on the other \(^5\). The most hopeful channel for precise determination of $m_s$ is hadronic $\tau$ decays. New precision data in this channel have been reported by the CLEO \(^13\) and OPAL \(^14\) collaborations. These have lead to better understanding of the SU(3) breaking effects in the hadronic $\tau$-decay sum rules and better resolution of the scalar and vector spectral functions. Incorporating these results Gamiz et al. derive the estimate \(^15\)

$$m_s(M_S, 2 \text{ GeV}) = 81(22).$$ \(^5\)

The analysis remains sensitive to resolving SU(3) breaking, *i.e.* the cancellations between $ud$ and $us$ parts, as pointed out by Maltman \(^16\). Hopefully the precision of sum rule analysis will improve as more experimental data is collected.

The bottom line is that lattice QCD has revised our thinking regarding light quark masses, *i.e.*, $m_s$ is significantly lower than the estimate used by phenomenologists until 1996 $m_s(M_S, 2 \text{ GeV}) = 125(40) \text{ MeV}$ \(^10\). A lower estimate has two important consequences. One, it increases the standard model estimate for $\epsilon'/\epsilon$ \(^17\) and, second, it poses a challenge to grand unified model builders \(^9\).

### 2.1. Is $m_u = 0$?

As the quality of lattice data improve with respect to both the number of quark masses and lattice scales simulated one can make increasingly precise fits to relations like Eq. 1. Through these fits one can extract various GL low
energy couplings in the chiral Lagrangian. Here I summarize the extraction of \(2L_8 - L_5\) by the MILC collaboration \(^{11}\) who find

\[
2L_8 - L_5 = -0.2(1)(2) \times 10^{-3},
\]

which is significantly different from the range

\[
-3.4 \times 10^{-3} \lesssim 2L_8 - L_5 \lesssim -1.8 \times 10^{-3}
\]

allowed by \(\chi PT\) for \(m_u = 0\). So current lattice data rule out \(m_u = 0\) which would have provided a convenient solution to the strong CP problem.

3. \(B_K\)

The kaon bag parameter \(B_K\) measures the QCD corrections to the weak mixing between \(\bar{K}^0\) and \(K^0\). It is defined as the dimensionless ratio of the matrix element of the \(\Delta S = 2\) effective weak operator between a \(\bar{K}^0\) and \(K^0\) to its vacuum saturation approximation

\[
B_K = \frac{\langle \bar{K}^0 | \bar{s} \gamma_\mu (1 - \gamma_5) d \bar{s} \gamma_\mu (1 - \gamma_5) d | K^0 \rangle}{\Delta \langle \bar{K}^0 | \bar{s} \gamma_\mu \gamma_5 d | 0 \rangle \langle 0 | \bar{s} \gamma_\mu \gamma_5 d | K^0 \rangle}. \tag{8}
\]

Its measurement gives a constraint in the form of a hyperbola in the \(\tilde{\eta} = \eta(1 - \lambda^2/2), \tilde{\rho} = \rho(1 - \lambda^2/2)\) plane \(^{17}\)

\[
\tilde{\eta} \left[ (1 - \tilde{\rho}) A^2 \eta_2 S_0(x_t) + P_0(\epsilon) \right] A^2 \hat{B}_K = 0.226
\]

where \(\eta, \rho\) and \(A\) are parameters in the CKM matrix.

Over time different approaches, including chiral perturbation theory, the large \(N_c\) expansion, QCD sum rules and lattice QCD, have been used to estimate \(B_K\). Current phenomenology uses the lattice result obtained by the JLQCD collaboration with unimproved staggered quarks in the quenched approximation \(^{18}\)

\[
B_K(M_{\overline{MS}}, NDR, 2 \text{ GeV}) = 0.63(4) \tag{9}
\]

or the corresponding renormalization group invariant quantity \(^6\)

\[
\hat{B}_K = 0.86(6)(14), \tag{10}
\]

where the second error is an estimate of the quenching and SU(3) breaking \((m_d, m_u\) are degenerate in these calculations) uncertainty.

The JLQCD calculation \(^{18}\) showed that there are significant (i) \(a^2\) discretization errors in unimproved staggered fermions and (ii) the unknown \(O(\alpha_s^2)\) corrections to the one-loop perturbative renormalization constants could be as large as \(\sim 10\%\). The continuum extrapolation involved a subtle
interplay between these two effects. For this reason there has been concern whether the continuum extrapolation leading to Eq. 9 is under control. To test this, and because the calculation of $B_K$ provides a laboratory for evaluating the efficacy of improved fermion formulations, there are a number of new simulations with different lattice formulations.

I will focus on three sets of new calculations, all within the quenched approximation, to elucidate our current understanding of the continuum limit. (i) Fermion formulations that incorporate chiral symmetry $a$ la Ginsparg-Wilson (Overlap and Domain Wall fermions); (ii) improved staggered fermions; and (iii) twisted mass lattice QCD. These results are summarized in Fig. 1.


- The domain wall simulations by CP-PACS \(^\text{19}\) and RBC \(^\text{20}\) collaborations agree with each other and give \(B_K(\overline{\text{MS}}, \text{NDR}, 2 \text{ GeV}) = 0.57(3)\) after extrapolation to the continuum limit.

- There are two calculations using overlap fermions: DeGrand \(^\text{21}\) finds \(B_K(\overline{\text{MS}}, \text{NDR}, 2 \text{ GeV}) = 0.55(3)\) at \(\beta = 5.9\) and 6.1 whereas Garron \textit{et al.} \(^\text{22}\) find \(B_K(\overline{\text{MS}}, \text{NDR}, 2 \text{ GeV}) = 0.63(6)\) at \(\beta = 6.0\).

- Improved staggered fermion simulations by Lee \textit{et al.} \(^\text{23}\) at \(\beta = 6.0\) yield \(B_K(\overline{\text{MS}}, \text{NDR}, 2 \text{ GeV}) = 0.58(2)(4)\). This calculation shows that both discretization errors and perturbative corrections to renormalization constants are small with HYP smeared staggered fermions, alleviating the two most serious systematic errors in this approach, and making it computationally attractive.

- Estimate from simulations of Twisted mass QCD by Dimopoulos \textit{et al.} \(^\text{24}\) is \(B_K(\overline{\text{MS}}, \text{NDR}, 2 \text{ GeV}) = 0.592(16)\) in the continuum limit.

All these new estimates are, within errors, consistent and covered by the range

\[
B_K(\overline{\text{MS}}, \text{NDR}, 2 \text{ GeV}) = 0.58(4)
\]

shown in Fig. 1. Even though no single collaboration has obtained sufficient data to do a reliable continuum extrapolation, what has become clear from these calculations is that by improving the lattice formulation (and in some cases supplementing it with non-perturbative determination of the renormalization constants) the dependence on \(a\) has been reduced very significantly. What is less clear is whether the difference from the JLQCD result 0.63(4) is significant. With hindsight one can look at the data in Fig. 1 and conclude that the extrapolation of the unimproved staggered estimates to the continuum limit is at fault because the fit parameters were not well determined by the data. My take on this issue is that, given the data, JLQCD did the best possible extrapolation incorporating the leading two corrections, \(a^2\) and \(\alpha_s^2\) (since they used one-loop matching between the lattice and \(\overline{\text{MS}}\) schemes), and requiring that gauge-invariant and non-invariant operators give the same estimate in the continuum limit. Their analysis including only \(a^2\) errors gave 0.598(5). On adopting a better motivated procedure their error estimate increased considerably and explains the difference at roughly 1\(\sigma\) level if indeed the final quenched value settles at 0.58.

Computationally, the simplest of these approaches to extend to dynamical quarks is the improved staggered, however, there is a caveat. Dynamical
staggered simulations require taking the fourth root of the determinant in order to simulate one flavor of quarks. It has not been shown rigorously whether the action of the resulting theory is local or in the same universality class as QCD. Simulations by the MILC collaboration \cite{11} of a number of observables suggest that the continuum limit is approached smoothly and this fourth-root trick does reproduce QCD. Work on clarifying this issue is in progress.

Dynamical simulations are just beginning. The Riken-Brookhaven-Columbia collaboration have presented first results based on three ensembles of \( n_f = 2 \) dynamical lattices with domain wall fermions at scale \( \alpha^{-1} \approx 1.7 \) GeV and quark masses in the range \( m_s - 0.5m_s \). Their result is

\[
B_K(\overline{MS}, NDR, 2 \text{ GeV}) = 0.509(18)
\]  
(12)

for degenerate quarks \( (m_d = m_s) \) and

\[
B_K(\overline{MS}, NDR, 2 \text{ GeV}) = 0.495(18)
\]  
(13)

with \( m_d = 0.5m_s \). Compared to their quenched estimate 0.57(3) they find a \( \sim 15\% \) decrease. It remains to be seen how much more this estimate will change when a third flavor is added to the simulations and \( m_d \) is varied over a larger range.

4. Conclusions

Much has changed since I was at Northeastern. The campus has been improved beyond recognition. Many things are the same. All my friends and colleagues are still thriving and Pran is as productive, dedicated and driven as ever. He continues to be an inspiration for all.

Lattice QCD has progressed tremendously. Gone are the days when one stayed up nights trying to harness all possible VAX computers running at a fraction of a megaflop to generate one quenched background configuration in a month. One now talks of sustained teraflops on dedicated massively parallel computers. As a result of this increase in computing power and better algorithms and theoretical understanding we are now simulating QCD without any approximations (with 2+1 dynamical flavors). Thus, from now on the community will provide increasingly precise estimates and hopefully one day soon the effort will, without doubt, validate QCD and yield a glimmer of new physics.
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