Modeling of optimal scanning for curvilinear survey routes and geometrically complex areas using optoelectronic observation equipment

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Abstract. The models and problems of remote sensing satellite control intended for of the Earth are considered both for optimal scanning of curvilinear survey routes and geometrically complex areas of sensing using optoelectronic observation equipment in the “push broom” mode. A general model for scanning an arbitrary survey route is given, and a variational problem of the synthesis of the optimal control of its scanning is defined. The main tasks related to the optimization of multiroute survey of geometrically complex sensing area are given using a modified model of the theory of optimal coverings.

1. Introduction
The problems of controlling the processes of the Earth remote sensing (ERS) from space are considered [1 – 3] in the paper. At the present time, advances in space technology have led to the widespread use of ERS systems, since the received multispectral images of observation (sensing) areas on the Earth's surface are used to solve the applied problems which encourage socio-economic progress (agriculture, protection and monitoring of special natural areas and hazardous production facilities, development of transport and regional infrastructure, etc.) [3, 4]. Hence, it is important to solve a number of problems of controlling the angular motion of remote sensing satellite (RSS) [5 – 7], which determine the quality and volume of the received sensing information. These include the problems associated with ensuring optimal scanning of curvilinear survey routes using optoelectronic observation equipment operating in the “push broom” mode [2, 3], as well as the scheduling problems [8, 9], which include the development of integrated (continuous) control programs for spacecraft [10], and the implementation of multiroute survey for geometrically complex areas of sensing [9, 11]. The list of such problems was defined and considered in [7, 8] in connection with the problem of the development of integrated control programs for the RSS at multiturn flight intervals. The present paper is mainly focused on the problems of optimal scanning of the curvilinear survey route and one of the possible options for setting up the multiroute scanning task of geometrically complicated sensing areas.

2. The model and conditions for scanning the curvilinear survey route and the variational problem of its optimal scanning
The key approaches to solving the problem of route survey with the help of optoelectronic observation equipment were considered in [12 – 15], where both the mathematical models for the control of scanning of the random route survey in the “push broom” mode and the main approaches to the
optimization of the programmes were presented, taking into account the results of practical application in the development of “Resurs-DK” and “Resurs-P” on-board software [7, 14–16].

The features of the optoelectronic observation equipment utilized in modern RSS [1–4] and the peculiarities of the Earth survey from space with it require the fulfillment of certain conditions that provide the development of the control models for scanning the curvilinear survey route. The first and the main of them is represented by the expression

\[ r_{LS} = r_R - r_{SC}, \]

where \( r_{SC} \) is radius-vector of the spacecraft, \( r_R \) is radius-vector of the central line of the survey route and \( r_{LS} \) is radius-vector of the sight line, which determines the required position of the observation equipment optical axis in space. In expression (1) \( r_{SC} \) is determined by the kinematic equation of motion of the spacecraft \( r_{SC} = r_{SC}(t) \) and \( r_R \) is vector-function of the arc coordinate \( s \), measured along the central line of the given route: \( r_R = r_0(s) \), i.e. \( r_{LS} \) is vector-function of two arguments: \( r_{LS} = r_{LS}(t, s) \). In order to fulfil condition (1), it is necessary to set the law of scanning the survey route \( s = s(t) \) in each time-point, which determines the quality of the survey when combined with the route model. Therefore, in addition to the model of the central route line in the form of a spatial curve \( L = L(s) \), it is necessary to introduce some approximation of the corresponding part of the physical surface of the Earth, for example, its relief by the surface \( \Phi = \Phi(r) \) within the scanning band determined by the ground swath of the spacecraft observation equipment. It is obvious, that the expression \( L = \Phi(r_0(s)) \) should be met, that is \( L \subset \Phi \). The law of scanning \( s = s(t) \) can be obtained as a solution to the initial problem:

\[ \frac{ds}{dt} = v_R(t, s), \quad s(t_0) = 0, \] (2)

where \( t_0 \) is the initial time of scanning, \( v_R \) is the module of scanning speed vector \( v_R = v_R \frac{dr_R}{ds} \). Vector \( v_R \) is defined by the following conditions required for scanning [8, 9, 12, 13]: firstly, by the alignment of the optical axis of observation equipment with the visual line vector \( r_{LS} \); secondly, by the proportionality of the vector \( M_v \) projection on the focal plane of the observation equipment and the speed of the current image to central route line \( w \) which should be orthogonal to the CCD detector line [2, 13]. The following relationship between \( M_v \) and \( w \) is as follows [13]:

\[ w = f \frac{\sin \alpha}{D} v_R, \]
(3)

where \( f \) is parameter of the observation equipment, \( D(t) = \tilde{D}(t, s(t)) \), \( \tilde{D}(t, s) = |r_{LS}(t, s)| \), \( \alpha(t) = \tilde{\alpha}(t, s) \) is the angle between vectors \( \tau(s) \) and \( e_{LS}(t) = \tilde{e}_{LS}(t, s) \) and \( \tau(s) \) is the tangential unit vector to the central line of survey route. Taking into account the content of the problem under consideration, the speed of image running \( w \) is a control parameter which values are determined by the values of \( v_M \) according to (3). In general, based on the definition of the values occurring in (3), it can be rewritten in the following form:

\[ \tilde{v}_R = -\frac{\tilde{D}(t, s)}{f \sin \tilde{\alpha}(t, s)} w(\cdot), \] (4)

where the control parameter \( w(\cdot) \) must be determined from the solution of the corresponding synthesis problem of optimal control as \( w(\cdot) = w(t) \) or \( w(\cdot) = w(t, s) \). Thus, \( \tilde{v}_R = \tilde{v}_R(t, s, w) \) in (4), but then the equation (2) should be rewritten as:
\[ \frac{d s}{d t} = P(t, s), \quad s(t_0) = 0, \quad w(t) = \frac{\hat{D}(t, s)}{s \sin \alpha(t, s)}, \]

where \( P(t, s) \) and it is additionally assumed that \( w > 0 \), which corresponds to a strictly monotonically increasing solution of equation (5) \( s = s(t) \ \forall t \geq t_0 \).

The orientation of the scanning sector plane of the observation equipment in space is specified by the unit vector \( e_D \), the required position of which is determined by the velocity vector \( v_M \) according to the above scanning conditions, namely:

\[ \hat{e}_D(t, s, w) = \frac{\hat{v}_D(t, s, w)}{v_D(t, s, w)}, \quad (6) \]

where \( \hat{v}_D(t, s, w) = \hat{e}_{LS}(t, s) \cdot [\hat{v}_E(t, s, w) \times \hat{e}_{LS}(t, s)] \). The required position of CCD is defined by unit vector:

\[ \hat{e}_a(t, s, w) = \hat{e}_{LS}(t, s) \times \hat{e}_D(t, s, w). \quad (7) \]

The required orientation of spacecraft in space relative to the scanned survey route is determined using a triple of unit vectors, determined by the scanning conditions (1), (3), (6) and (7): \( \hat{e}_{LS}(t, s) \), \( \hat{e}_D(t, s, w) \), \( \hat{e}_a(t, s, w) \). The indicated unit vectors are defined in the Greenwich coordinate system as well as vectors from the expression (1). To generate the angular motion programs of the RSS which provide scanning of the specified survey routes, in addition to its orientation it is also necessary to determine the components of the angular velocity vector of the spacecraft

\[ \hat{\omega}_{SC}(t, s, w) = \omega_{LS} e_{LS} + \omega_{e} e_{e} + \omega_{t} e_{t}, \]

which are calculated from the formulas given in [8, 13]. Thus, when defining the trajectory of the spacecraft \( r_{SC} = r_{SC}(t) \) and the central line of the survey route as a curve \( L = L(s) \), including \( \Phi = \Phi(r) \) as the corresponding approximation of the sensing area surface, and taking into account the necessary scanning conditions for this route (1), (4), (6), (7), a general mathematical model for scanning an arbitrary curvilinear survey route has been obtained. In addition, the scanning law \( s = s(t) \) is an integral part of this model, with which the resulting model is assumed to be closed.

The limitations of the scanning law \( s = s(t) \) are determined by the technical characteristics of the observation equipment and, in some cases, by the geometric characteristics of the survey route. First of all, the following restrictions must be met:

\[ 0 < w_{\min} \leq w \leq w_{\max} < \infty. \quad (8) \]

Choosing a value \( w = w_0 \) from this range, one can obtain the corresponding scanning law \( s = s_0(t) \) from the solution of equation (5). It should be noted that the solution of the problem of parametric optimization, for which it is necessary to introduce a certain criterion of scanning quality, various models of which, for example, were considered in [8, 9, 13], is not excluded even in this case. One of the main factors that determines the quality of scanning the survey route is associated with the deviation of the vectors of the image running speed in the focal plane of the observation equipment at the points of the “elementary scan band” \( P_M \) from the required values \( w_0 = w_0 e_D \), which leads to the image motion aberration [8]. The “band” \( P_M \) is formed when the capture sector of the spacecraft observation equipment, to which the unit vector \( e_D \) is the normal line, meets the surface \( \Phi \), and its points correspond to the side sight lines given by the unit vectors: \( e_{LS} = \cos \varepsilon + e_{\pi} \sin \varepsilon \), where \( -e_0 \leq \varepsilon \leq e_0 \) (\( e_0 \) is the half-angle of the scanning sector; when \( \varepsilon = 0 \) we get the unit vector of the sight line of the point of the route central line \( e_{LS}^{(0)} = e_{LS} \)). In addition, the quality of surveying the
scanned route is also influenced by the spacecraft orientation errors, geometric characteristics of $\Phi$ and some other factors.

As a final result, for some given function $w_0 = w_0(t, s)$ and the image speeds of the points of the “elementary band” $P_M$, obtained for it as $w_e(t, s, w_0, \varepsilon)$, the quality of its scanning can be estimated by the integral:

$$G(t, s, w_0) = \frac{1}{2\varepsilon_0} \int_{-\varepsilon_0}^{+\varepsilon_0} R d\varepsilon,$$

where $R$ is some function of the type of a suitable vector norm for vector differences $w_e(t, s, w_0, \varepsilon) - w_0 \varepsilon_D(t, s)$; by using it, one can additionally take into account a number of effects that affect the quality of scanning, for example: illumination and contrast of the subject, cloudiness, defocusing of the observation equipment, etc. Accordingly, the scan quality criterion for the entire survey route can be determined as follows:

$$J = \int_{0}^{s_f} G(t, s, w) ds,$$

where $s_f$ is the length of the shooting route in arc coordinate units. Taking into account equation (5), this criterion can be rewritten in the following form:

$$J = \int_{0}^{t_f} F(t, s, w) w dt,$$

where $F(t, s, w) = G(t, s, w) P(t, s)$ and $t_f$ is scan completion time, i.e. $s_f = s(t_f)$.

The differential relation between $s$, $t$ and $w$ (5) is the basic equation for controlling the scanning process of the survey route specified using the model described above. Taking into account this equation and the scan quality index (9), we can formulate the following variational problem [13]: it is required to find an admissible control $w = w_{opt}(\cdot)$ that satisfies the restrictions on the control parameter and $w$ (8) and delivers a minimum to the functional (9) taking into account the differential coupling (5) and the boundary conditions for it:

$$s(t_0) = 0; \quad s(t_f) = s_f.$$  

Various approaches to the solution of the variational problem (5), (8) - (10) were considered in [8, 9, 12 – 16]. The optimization problem of the pattern, which includes pairs of maneuvers for controlling the orientation of the RSS during retargeting of its optoelectronic observation equipment and subsequent scanning of arbitrary survey routes was described in [15] in the framework of synthesis of the integrated control programs for the RSS. The “retargeting-scanning” pattern, which was included as part of an integrated control program, was considered as an “elementary operation” for controlling the orientation of the spacecraft. Thus, the main aim of its optimization was increasing not only the quality of the received information but also the performance of the RSS at a given flight interval. The problem of optimal conjugation of control programs for pattern maneuvers was also solved, which is necessary in the case of multi-route scanning optimization of a geometrically complex sensing area, which is associated with the formation of a pattern involving up to several of the “elementary operations”.

### 3. Setting the problem of the multi-route survey optimization for geometrically complex sensing areas

Modern RSS solve a wide range of tasks, the main one of which is related to the implementation of various types of surveys [3 – 8, 10, 11, 15, 17, 18]. For example, the RSS “Resurs” developed by the Space rocket center “TsSKB-Progress” with optical-electronic observation equipment can perform
object and route survey, as well as complex types of shooting: stereophotography, site survey, etc. [19]. Stereophotography and site surveying are special cases of multi route surveying of geometrically complex sensing areas [11]. Carrying out such a survey has its own peculiarities and significantly differs (in controlling the angular motion of the RS) from the route survey of long-range sensing objects, for example: coastlines, river valleys, transport networks, etc. In case of the use of optoelectronic observation equipment operating in the “push broom” mode [2], as a rule, repeated coverage by overlapping scanning bands is required for geometrically complex and large-scale sensing regions, which is implemented with the help of separate survey routes brought into accordance in a certain way. In this respect, the problem of optimization of multi route shooting in terms of the theory of best cover [20, 21] was set for the first time in [11], and a modified model of the theory of optimal coverings, which includes a couple of limited sets X and Y, as well as a nonnegative function ρ(x, y) ∀x ∈ X and ∀y ∈ Y, was considered in [11]. Function ρ(x, y) is convex on X for any y ∈ Y.

Hence, if a certain system of n elements, called “centers” [21], is singled out from Y, y_k ∈ Y, k = 1, 2, ..., n, or \{y_k\}_n ∈ Y, then \(E_c(y_k) = \{x ∈ X : ρ(x, y_k) ≤ c\}\) may be assigned to each \(y_k \in Y\) and some values of \(c > 0\) from X subset. According to the definition [20, 21], the system \(\{y_k\}_n \in Y\) “covers” X with the radius \(c > 0\), if

\[X ⊂ \bigcup_{k=1}^{n} E_c(y_k),\]  

i.e. the condition \(\min_{k=1, 2, ..., n} ρ(x, y_k) ≤ c\) is met for each \(x ∈ X\). Function ρ(x, y) is interpreted here as “distance” from the point \(x ∈ X\) to the “center” \(y ∈ Y\) and, consequently, the “distance” from \(x ∈ X\) to the nearest “center” will not exceed the value of number \(c > 0\), which is called “coverage range” (of the set X by the system of “centers” \(\{y_k\}_n \in Y\)). For each \(y_p ∈ \{y_k\}_n ∈ Y\) \((1 ≤ p ≤ n)\) in X, one can distinguish the Dirichlet domain or the region of its “influence”:

\[D(y_p) = \{x ∈ X : ρ(x, y_p) ≤ ρ(x, y_k), ∀k = 1, 2, ..., n, k ≠ p\},\]

the dimensions of which for the \(p\)-th “centers” are defined as \(A_p = A_p(y_p) = \max_{x ∈ D(y_p)} ρ(x, y_p)\). If there is no other “center” arbitrarily close to the \(p\)-th “center”, then it is called unimprovable [20], otherwise, \(y_p \in Y\) is an improvable “center”. Accordingly, it is proved in [20] that any system \(\{y_k\}_n \in Y\) \((n > 1)\) that contains at least one improved “center” \(y_q ∈ \{y_k\}_n \in Y\) can be replaced, starting from \(y_q\), by a system of unimprovable “centers” \(\{y_k\}_n \in Y\) using some procedure of (continuous) improvement.

It is possible to define the main problems within the framework of the above model of the optimal coverings theory [20, 21].

**Problem 1** (on the minimum radius of coverage). It is required to choose such a system of n “centers” \(\{y_k\}_n \in Y\) \((n ≥ 1)\) so that condition (10) is satisfied for the smallest coverage radius \(c > 0\).

**Problem 2** (on the minimum system of “centers”). It is required for a given coverage radius \(c > 0\) to choose a system of “centers” \(\{y_k\}_n \in Y\) such that condition (10) is satisfied for the smallest number of n.

The particular properties of problems 1 and 2 depend: firstly, on the nature of the sets X and Y, and also on the form and properties of the function ρ(x, y), that is, on the adopted model of optimal coverings and, secondly, on the content of the problems being solved. The example of problems 1 and 2, in which sets X and Y and function ρ(x, y) are given as follows: the set X ⊂ V is the unit square of the plane V ∈ R², the system of “centers” is a system of points \(\{y_k\}_n \in V\), and the function
\( \rho(x, y) \) is the Euclidean norm on the plane \( V \), is considered in [21]. Despite the fact that this model is simple, a number of important applied problems can be reduced to it [21]. Therefore, an example of generalization of the simplest model is provided for multitire survey problem of the RSS considered above [11]. Let the set \( X \) be a limited surface area of a common terrestrial ellipsoid \( G \) which image is the plane domain \( V_X \subset V \) which uniquely and mutually continuously mapped into \( X \), where \( V \) is the plane representing, for example, one of the main cartographic projections in the form of a surface projection \( G \) onto a cylindrical surface, and \( Y \) is given in the form of a set that coincides with \( X \). Then the function \( \rho(x, y) \) can be specified by the length of the geodetic curve connecting the points \( x, y \in X \). Despite the fact that in this case the solution of the main problems in the theory of optimal coverings proves to be considerably more laborious in comparison with the existing approaches to solving the multitire survey problem for spacecraft, this allows one to consider new approaches to the solution of this problem, since it contributes to the list of applied problems of the optimal coverings theory due to the fact that sets \( X \) and \( Y \) are introduced into consideration which interact with their elements as objects of different types in a more complex way. The following model for the main problems in the theory of optimal coverings was considered as an illustration of this approach in [11]. Namely, \( X \subset V \) is a bounded domain of the plane \( V \subset R^2 \), and its elements are the points \( x \in V \), and, accordingly, the set \( Y \) is the set of straight lines \( P \in V \). In this case, the problem of optimal coverage of \( X \) by the bands, the boundaries of which are parallel to the “centers” in the form of straight lines \( P \in V \) and separated from them at a distance equal to the half-width of the bands, is considered. It should be noted that such a model, although it is the simplest one, is identical to the optimal cover theory model which is required for the general formulation of the optimal multitire survey problem in case of geometrically complex sensing areas.

Turning to the statement of the problem, let us introduce the required models. First of all, let us consider the procedure for the development of a scanning band model, which is formed on the surface \( \Phi = \Phi(r) \) by “push brooming” it with the scanning sector \( S = S(r) \) of the spacecraft observation equipment, the plane of which is specified by the normal \( e_D \), and the half-angle \( \varepsilon_0 \) is its main parameter. It is obvious that the left and right borders of the scan band for the given survey route \( L \subset \Phi \) will be drawn on \( \Phi \) in the direction of scanning \( L \) by the extreme sight lines of the sector \( S = S(r) \). In this case, the scan sector intersects with the surface \( \Phi \) along the “elementary scan band” \( P_M \). Thus, for a given survey route \( L(s) \); initial time of its scanning \( t_0 \); the scanning law \( s = s(t) \), as well as the spacecraft equation of motion \( r_{sc} = r_{sc}(t) \) and according to the model of survey route scanning of the surface \( \Phi \) given in p. 2, we obtain the respective corresponding scanning band \( H \subset \Phi \), which width is determined by the angle \( \varepsilon_0 \ll \frac{\pi}{2} \) [3].

The projections of the central line of the survey route \( L \) and the band \( H \) on the surface of a common terrestrial ellipsoid \( G \) determine the curve \( \hat{L} \) and the domain \( \hat{H} \) restricted by the corresponding projections of the boundaries \( H \). Since \( r = r(B, L, H) \), where \( B, L, H \) are the geodesic coordinates of the point given by the radius vector \( r \in \Phi \) (in the Greenwich coordinate system), then the projection of this point to the surface \( G \) will be provided by the radius vector \( \hat{r} = \hat{r}(B, L, 0) \in G \). The relationship between \( r \) and \( \hat{r} \) is established using the surface model \( \Phi \) as a particular model of the sensing area relief: \( H = \Phi(B, L) \). This model is the surface equation \( \Phi = \Phi(r) \) in explicit form and the corresponding parametrization of this surface. The model of the survey route (for given \( r_{sc} = r_{sc}(t) \) and \( s = s(t) \)), represented by its central line \( L \) and scan band \( H \), can be considered as a “physical model”, and the survey route model in the form of projections \( L \) and \( H \) on an ellipsoid \( G \), i.e. \( \hat{L} \) and \( \hat{H} \), can be regarded as a geodetic model (or model in geodesic coordinates), which should also include the model of the sensing area relief. Lines \( L \) on \( \Phi \) in plane domain \( V \) will correspond
to its image in the form of a parameterized curve \( l \), and a curve \( \tilde{l} \) will correspond to the image \( \tilde{L} \) in the domain \( V \). It is obvious that the curves \( L \) and \( \tilde{l} \) in the region \( V \) coincide. Ultimately, assigning any of them together with some model of the sensing area relief \( H = \Phi (B, L) \) is enough for defining \( L \) and \( \tilde{L} \). It should be noted that the boundaries of the scan band for the survey route will depend both on the choice of the initial time of scanning \( t_0 \) and the law of scanning \( s = s(t) \). Therefore, when the arc coordinate changes the width of the scanning band \( \Pi \) varies accordingly, namely, its transverse dimensions will be determined, first of all, by the angle of the capture sector of the observation equipment \( e_0 \), i.e. it is advisable to choose this parameter as the coverage radius.

Thus, to formulate the problem under consideration, we introduce [11]: firstly, the model of the sensing region in the form of a given domain \( \tilde{X} \subset G \), in which \( V_X \subset V \) is the image in the domain \( V \); secondly, the set of “centers” \( \tilde{Y} \subset G \) with elements in the form of smooth curves without self-intersections and limited curvature \( M \in \tilde{Y} \subset G \), on which the “initial” points \( m_0 \in M \) are indicated, from which arc coordinates \( \tilde{s} \) are measured in the direction of scanning along the curves \( M = M(\tilde{s}) \).

Let us introduce the distance function from \( x \in \tilde{X} \) to curves \( M \in \tilde{Y} \) in the form of the geodesic curve length (on the surface \( G \)) that connects \( x \in \tilde{X} \) with the point \( m(\tilde{s}) = M(\tilde{s}) \in M \), i.e. \( \rho(x, M) = \rho(x, m(\tilde{s})) \) where \( \tilde{s} \) is the arc coordinate of the point \( m(\tilde{s}) \) closest to the point \( x \in \tilde{X} \subset G \). If \( \xi \in V_X \) is the image of the point \( x \in \tilde{X} \), and \( \mu(0) \in V_Y \subset V \) is the image of the curve \( m(\tilde{s}) \in M \), then one can also introduce the “distance” function into the domain \( V \), assuming \( \rho(\xi, \mu) = \rho(\xi, \mu(\theta)) = \rho(x, M) \) where the value of the parameter \( \theta \) corresponds to the value of \( \tilde{s} \).

Let us introduce \( \tilde{Y} \) into some system of “centers” \( M_k \in \tilde{Y} \) with “initial” points \( m_0 = M_k \) and reference directions of the arc coordinates. In addition, let us introduce the intervals \([t_0, t_f]\) and laws of scanning \( s(t) \) taking into account the conditions of their physical realizability: \( \bigcap_{k=1}^{n} [t_0, t_f] = \emptyset \); \( \sum_{k=1}^{n-1} (t_k - t_{k-1}) \geq T_{\text{min}} \) where \( T_{\text{min}} \) is the total minimum permissible time necessary for retargeting the observation equipment at inter-route intervals. It is obvious that, taking into account the above limitations for \( \{M_k\} \in \tilde{Y} \), the given spacecraft flight trajectory \( r_{SC} = r_{SC}(t) \) and given \( e_0 > 0 \), considered as the “coverage radius” for \( \tilde{X} \), each “center” can be associated with a subset in \( \tilde{X} \) as the intersection of the sensing area and the \( k \)-th scan band: \( E_{e_0}(M_k) = \{x \in \tilde{X} \cup \tilde{N}_k\} \) and indicate \( M_p = \{M_k\} \in \tilde{Y} \) \( (1 \leq p \leq n) \) in the corresponding Dirichlet domains and their dimensions for each “center” [11]. By definition, the system \( \{M_k\} \in \tilde{Y} \) covers a set \( \tilde{X} \) with radius \( e_0 > 0 \) if the following condition is met:

\[
\tilde{X} \subset \bigcup_{k=1}^{n} E_{e_0}(M_k),
\]

(11)

that is, each point \( x \in \tilde{X} \) belongs to at least one of the bands \( \tilde{N}_k \) \( (k = 1,2,...,n) \).

Taking into account the modification of the optimal cover theory model proposed above, it is possible to formulate the problems of optimal multiroute scanning of the sensing region [11], identical to the problem of the minimal coverage radius (problem 1) and the problem 2.

**Problem 3.** For given \( \tilde{X} \), it is required to choose such system of \( n \geq 1 \) “centers” that the condition (11) is satisfied with the smallest value \( e_0 > 0 \).
Problem 4. For a given parameter $\varepsilon_0 > 0$, it is required to select such system of “centers” \( \{M_k\}_n \in \mathcal{Y} \) that condition (11) is satisfied for the smallest number \( n \).

Thus, the main problems of optimal multiroute scanning of an arbitrary sensing area, the characteristic dimensions of which substantially exceed the width of the capture band of the optoelectronic observation equipment of the RSS, have been defined in terms of the optimal coverings theory. It should be noted that the commonality of problems 3 and 4 is redundant for solving the corresponding applied problems. Nevertheless, the analysis of these problems when additional restrictions (for example, for curves used to define the central lines of survey routes) are introduced shows that they can be relevant both during the design stage of the RSS and the development of the optimal scanning plans. This technique allows one to develop new approaches, for example, to the assessment and development of requirements to the dynamic characteristics of motion control systems of the RSS [22, 23]. The results of the studies related to the problems 3 and 4 have been applied to the development of the “Resurs” on-board software installed in the RSS which allowed the autonomous formation of angular motion control programs for the RSS [17, 24].

4. Conclusion

The models and main problems of remote control of the RSS provided with optoelectronic observation equipment which functions in the “push broom” mode have been considered. A general model of scanning of the curvilinear survey route, with the help of which the variational problem of synthesizing the optimal control of its scanning are set, has been defined. A modified model of the optimal coverings theory, with the help of which problems of optimization of multiroute survey of geometrically complex sensing areas are defined, namely: the problem of the minimum radius of coverage of the sensing area by scanning bands and the problem of minimizing the number of survey routes required for optimal coverage of the sensing area with a given radius, has been proposed. It is noted that in spite of very high complexity of tasks, their consideration and analysis allows one to form new approaches of multiroute survey, including the development of “Resurs-DK” and “Resurs-P” on-board software required for autonomous formation of angular motion control programs of the RSS.

5. References

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