Optimization of Profile Shift Co-efficient for Highest Contact Ratio in Non-standard Gearing

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Abstract. The power transmission in mechanical drives is undoubtedly handled by gears. Tooth profile of a gear characterizes the power transmission. Standard gears mostly employ involute tooth for reasons of simplicity, while non-standard gears employ involute tooth profile shifting either positive or negative to modify the tooth geometry. Profile shifting is practiced in two ways, first known as S-gearing, either \( S_0 \) or \( S_{\pm} \) without change in tooth-sum while the second known as Altered tooth-sum gearing, aptly called \( Z_{\pm} \) gearing with change in tooth-sum \( Z_s \), tooth-sum being the sum of teeth on pinion and gear. In \( S_0 \) gearing the center distance will change while in \( S_{\pm} \) gearing and \( Z_{\pm} \) gearing the center distance remains unchanged. Altered tooth-sum gearing is a novel method of increasing (\( Z^+ \)) or decreasing (\( Z^- \)) the tooth-sum of standard gearing either by adding or removing one or more teeth (\( \pm Z_s \)) in either pinion or gear or both and eventually distributing the resulting total profile shift co-efficient among the gear pair, thus maintaining the same center distance. In designing Non-standard gearing the amount of profile shift is expressed as ‘\( X_m \)’, where ‘\( X \)’ is known as Profile shift co-efficient and ‘\( m \)’ is module of the gear. This has a significant influence on the gearing in terms of load carrying capacity, contact ratio (CR), load sharing etc. In this study contact ratio of altered tooth-sum gearing is focused, as it affects the tooth load sharing with influence on vibration and noise levels as well. Optimization of the profile shift co-efficient will be meaningful in designing gears to give highest contact ratio. Load sharing is observed to be beneficial in Altered tooth-sum gearing using increased tooth-sum (\( Z^+ \) gearing), besides this method offering unparalleled benefits to gear engineering.

Key words: Profile shift, Altered tooth-sum gearing, High contact ratio, Vibration, Noise, Load sharing.

1. Introduction

Gears used in power transmission mostly have involute tooth profile. Most of the involute gears have their tooth profile shifted constituting Non-standard gears. The amount of profile shift is expressed in terms ‘\( X_m \)’, ‘\( X \)’ being the profile shift co-efficient and ‘\( m \)’ is module of the gear. Profile shift is common in gear design to modify the tooth geometry in order to meet certain requirements that are not
fulfilled by standard gears. Such Non-standard gears perform better in terms of tooth strength, contact ratio and many other parameters. Profile shifting in S-gearing may be $S_0$ gearing or $S_\pm$ gearing. $S_0$ gearing involves shifting of the tooth profiles in opposite direction such that the center distance remains unaltered, while $S_\pm$ gearing involves shifting of the tooth profiles independently such that their center distance alters. Non-standard gearing can also be obtained using a novel and unique way by altering the tooth-sum of a standard gear without change in center distance. In this, tooth-sum $Z_s$ of a considered standard gearing is the sum of the number of teeth $Z_1+Z_2$ of the gear pair operating between a standard centre distance $a_o$. Altered tooth-sum gearing is different to $S_\pm$ gearing with the possibility of conforming different tooth-sums $Z'_s$ (or $Z'_1+Z'_2$) involving eventual profile shifting without change in center distance, this necessitates the gearing to work on a different pressure angle known as operating (working) pressure angle, $\alpha_w$. The tooth-sum $Z_s$ of a standard gear pair is altered by adding ($Z+$ gearing) or removing ($Z-$ gearing) one or more teeth $\pm Z_e$ to give altered tooth-sum $Z'_s$ with eventual negative or positive profile shift respectively, while still working on the same centre distance for a given module and pressure angle. The required amount of total profile shift co-efficient $X_e$ is computed for each number of teeth altered which must be distributed among the meshing gears. Further tooth topping has also to be done to ensure backlash free contact.

In total, altered tooth-sum gearing will undergo the following geometrical changes that are computed using the equations given in table 1:

i. The number of teeth either on pinion or gear or both changes, this alters the size of base circles defining a new common tangent that introduces operating (working) pressure angle $\alpha_w$.
ii. Eventual profile shifting such that $X_e=X_1+X_2$, here $X_1$ is decided by designer and $X_2=X_e-X_1$.
iii. Equal amount of tooth topping 'ym' on both the meshing gears.

| Parameters | Operating (Working) pressure angle (deg) | Sum of profile shift co-efficients | Tooth topping |
|-----------|------------------------------------------|-----------------------------------|--------------|
| Equations | $\alpha_w = \cos^{-1} \left( \frac{Z_s \cos(\alpha)}{Z_e} \right)$ | $X_e = \frac{Z_e [\ln \sec \alpha - \ln \cos \alpha]}{2 \tan \alpha}$ | $ym = a_o + (X_e)m - a$ |

From the computed value of total profile shift $X_e$, $X_1$ is decided by the designer, while $X_2$ is obtained using $X_e=X_1+X_2$ (i.e., $X_2=X_e-X_1$). As same center distance can be specified, altered tooth-sum gears will be subjected to tooth topping 'ym' ensuring backlash-free contact. For the purpose of visualization of altered tooth-sum gearing Figures 1.1 (a, b and c) show the images of complete gear belonging to GR=1:1, $m=4$mm, $\alpha=20^\circ$ deg, $Z_e=100$ (50x50), altered with $Z_e=\pm2$, i.e., $Z_e=\pm2$ so that $Z'_e=48(24x24)$ constituting $Z-$ gearing, and $Z_e=+2$ so that $Z'_e=52(26x26)$ constituting $Z+$ gearing.

![Images](a) Standard gear (25 teeth) (b) $Z-$ gear (24 teeth) (c) $Z+$ gear (26 teeth)

Figures 1.1 (a), (b) and (c) CAD models of complete gear.
Contact ratio (CR) refers to the number of pairs of teeth in mesh during the contact period. It is a very important parameter to be considered while designing gears because it decides the smoothness of engagement and also its noise behaviour. A CR between 1 and 2 is referred as normal contact ratio while that between 2 and 3 is referred as High Contact Ratio (HCR). In gear design, practically a minimum contact ratio of 1.4 is specified for satisfactory performance of gears involved in power transmission in order to take care of any misalignments.

A geometrical representation of a gear pair in mesh is shown in Figure 1.2 in which a pinion is driving a gear. Referring to geometry of the figure, CR which is denoted by $\varepsilon$ is given by equation (1).

\[
\varepsilon = \frac{\sqrt{r_{a1}^2 - r_{b1}^2} + \sqrt{r_{a2}^2 - r_{b2}^2}}{p_b} - a_o \sin \alpha
\]

Figure 1.2

It can be seen that the contact ratio is dependent on addendum radius $r_a$, base radius $r_b$, centre distance $a_o$, pressure angle $\alpha$ and base pitch $p_b$, all these parameters in turn are influenced by the profile shift. Hence profile shifting has a significant effect on contact ratio which is discussed in sections ahead.

2. Literature review

Considerable amount of research has been carried in regard to standard gears and Non-standard gears. The amount of profile shift required, its calculation and use are discussed by Niemann,G [1]. Merritt H.E [2] has mentioned that S± gearing can be used to confirm different tooth-sums to a specified centre distance. Maitra [3] has suggested altering the tooth-sum for correcting small deviations in gear ratio. Joseph Gonsalvis and Sachidananda H.K [4] have shown using large number of monographs that altered tooth-sum design can tailor the gear drive either for normal or high contact ratio. Dr.Gonzalo Gonzalez Rey [5] has opined that gear noise is an indication of a flawed design or inferior product. It is reported that gear noise in standard gears can be reduced by increasing the contact ratio by adding one or two teeth subjected to negative profile shift. Moldovean Gheorghe et al. [6] have shown that CR of a gearing can be increased either by increasing the number of teeth, lowering the pressure angle or by increasing the addendum factor. These studies reveal that no attempts are made to optimize the profile shift co-efficient for achieving highest contact ratio.

3. Objectives

In the light of the available literature the following objectives are identified:

i. To understand profile shifting in non-standard gearing by altered tooth-sum design.
ii. To optimize the profile shift co-efficient in order to achieve highest contact ratio.
iii. To identify the high contact ratio cases in non-standard gearing and its useful discussion.
4. Contact ratio in Non-standard gearing
Contact ratio is the average number of pairs of teeth in mesh at any given instance. If CR is between 1 and 2 it is normal CR whereas if it is between 2 and 3 it is HCR. For smooth and continuous meshing the CR must be high. A minimum value of contact ratio specified is 1.4 for practically satisfactory performance. Equation (2) is the modified form of equation (1) that suits non-standard gearing.

\[ c = \frac{(r_{b1}^2 - r_{b2}^2) + (r_{b2}^2 - r_{b1}^2) - 2r_{b1}r_{b2}}{p_b} \]  

Low noise behavior in standard industrial gear units is becoming an important selection criterion, it is a factor indicating gear quality to the customer. In several gear tests and practical research the gear contact ratio has been reported to have large effect on noise level, especially in spur gear applications. Lower noise levels are generally associated with gears having higher contact ratio. For this reason gear design leading to high contact ratio is an important key for reduced noise levels. In this study, guidelines to design non-standard gears generated with standard tools of 20° pressure angle and tooth modification using profile shift to have high contact ratio are presented.

5. Optimization of Profile shift co-efficient
In gears contact ratio is undoubtedly an important issue. Standard gearing can fetch a highest CR of 1.94 when meshed with a rack, but still it is less than 2. High Contact Ratio (HCR) is desirable in applications that are characterized by low load carrying capacity, quieter operation and vibration free running. This is possible by profile shift, because the CR increases with increase in profile shift up to certain limit beyond which it decreases. If the GR is 1:1, highest CR can be obtained by equal distribution of resulting total profile shift between the mating gears \((X_1=X_2)\), but for other GRs, this does not holds good. Hence there is a need to optimize the profile shift co-efficient in order to achieve highest CR. In profiles shifted gears the geometrical equations involve the terms \(X_1\) and \(X_2\), besides the operating pressure angle \(a_e\). Optimization of profile shift co-efficient \(X_1\) for highest CR is done using \(\frac{dx}{dx} = 0\), the optimized value of \(X_1\) is given by equation (3).

\[ X_1 = \frac{(a+m)(r_{b2} - r_{b1}) + m(r_{b2} - r_{b1}) + r_{b1}r_{b2}}{m(r_{b1} + r_{b2})} \]  

Where, \(a = \) specified centre distance \(m = \) module \(r_{b1}, r_{b2} = \) base circle radii of pinion, gear \(r_{1}, r_{2} = \) pitch circle radii of pinion, gear \(X_e = \) total profile shift co-efficient

6. Results and discussion
The CRs obtained using profile shift co-efficient \(X_1\) from equation (3) for different altered tooth-sums and GRs are tabulated in table 2. For illustration \(Z_e=100\) \((50 \times 50)\), \(m=2\ mm\ \alpha = 20°,\ GR\ 1:1\ and\ 1:2\) are considered. It shows \(Z_e\) gearing offering a family of altered tooth-sums for a given standard tooth-sum, the amount of profile shift \(X_1\) to be allowed on the pinion that gives highest contact ratio for each tooth altered is determined using equation (3). The designer can choose a particular altered tooth-sum (among the family) that gives the highest value. It is seen that altered tooth-sum gearing altered with positive values of \(Z_e\) yield contact ratios exceeding 2. In gears with HCR the length of contact will be longer than two times the base pitch, thus accommodating more pairs of teeth to mesh (a minimum of two pairs of teeth will always be in mesh). Besides smooth engagement, the prescribed GR \((\frac{Z_e}{Z_{1}})\) can also be achieved along with reduced tooth load resulting in lower stresses. The last column indicates the optimum value of \(X_1\) computed using equation (3). For a GR of 1:1 with \(Z_e=+5\) \((Z_{1}=105)\), \(X_1=-0.964\) the highest CR achieved is 2.177 which is 24.11% higher and for a GR of 1:2 with \(Z_e=+4\)
(Zs=104), X1=−0.60 it is 2.075 which is 19.4% higher compared to standard gearing (both are Z+ gearing shown in italics). The limitation of this equation is that it holds good only for number of teeth greater than 41 either on the pinion or on the gear, because this number of teeth decides whether the root circle is greater or base circle is greater that affects the effective length of involute or length of path of contact.

Table 2. Family of Z± gearing offering High Contact Ratio (HCR) for Zs=100, GR 1:1 and 1:2 (considering module m=2mm and pressure angle \(\varphi=20^\circ\) deg).

| GR and tooth-sum Zs (Z1xZ2) | CR for Std. gearing | No. of teeth altered Ze  | Profile shift co-efficient, X1 | X1 obtained by equation (3) for highest CR |
|-----------------------------|---------------------|--------------------------|-------------------------------|------------------------------------------|
| Zs=100 (50x50)             | 1.755               | +3                       | 103 (51x52)                  | -                                        | 2.013 (X1=−0.659)                               |
| GR=1:1                     |                     | 2.023                    | 2.076                        | 2.086 (X1=−0.829)                          |
|                            | +4                   | 104 (52x52)              | 2.084                        |                                           |
|                            | +5                   | 105 (52x53)              | 2.152                        | 2.177 (X1=−0.964)                          |
| Zs=100 (33x67)             | 1.738               | +3                       | 103 (34x69)                  | -                                        | 2.013 (X1=−0.710)                               |
| GR=1:2                     |                     | 2.012                    | 2.001                        | 2.075 (X1=−0.60)                           |
|                            | +4                   | 104 (34x70)              | -                            |                                           |
|                            | +5                   | 105 (35x70)              | 2.017                        | 2.070 (X1=−0.35)                           |

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|                            | +4                   | 104 (34x70)              | -                            |                                           |
|                            | +5                   | 105 (35x70)              | 2.017                        | 2.070 (X1=−0.35)                           |

Bold indicates value of highest CR obtained using X1 (in parenthesis) computed from equation (3).

7. Conclusion

In view of the fact that contact ratio has been reported to have significant effect on tooth load sharing, noise and vibration levels, an alternative design approach is presented to design non-standard gearing with high CR by altering the tooth-sum. This produces quieter gears without expensive manufacturing costs or structural modifications to the gear housing, especially in spur gear applications. A gear pair with a tooth sum of 100 is altered with +1 tooth to +5 teeth resulting in altered tooth-sums 101 to 105 for GRs 1:1 and 1:2. With judicious selection of number of teeth altered and distribution of resulting total profile shift \(X_e\) such that \(X_1\) being computed using equation (3) and \(X_2=X_e−X_1\), it is possible to design gears having highest contact ratio to address the above issue. However this equation comes with a limitation that it holds good for gears having number of teeth greater than 41 either on the pinion or on the gear. Since highest CR can be achieved from the proposed equation without resorting to any structural changes, it may be concluded that altered tooth-sum design can be considered as a novel, promising and unique approach to gear design.

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9. References

[1] Niemann. G, “Machine Elements”, Vol. II, Allied publishers, New Delhi, 1980.

[2] Merritt. H.E, “Gear Engineering”, 3rd Edition, Pitman, London, 1962, pp 124-125.

[3] Gitin M. Maitra, “Hand Book of Gear Design”, TMH, New Delhi.

[4] Dr. Joseph Gonsalvis and Sachidananda. H.K, “Altered Tooth-sum Gearing for High Contact Ratio Gearing”, Proceedings of the ETIME - 2006, BMSCE, Bangalore.

[5] Dr. Gonzalo Gonzalez Rey, “Higher Contact Ratios for Quieter Gears”, Gear solutions, January 2009, pp 22-27.

[6] Moldovean Gheorghe, Gavrila C Catalin and Huidan Livia, “Ways to Increase the Contact Ratio for Spur Gears”, Annals of Oradea University, Fascicle of Management and Technological Engineering, Vol. XI (XXI), 2012, NR1, pp 2.82-2.88.