Charge-exchange quasi-elastic process $nd \rightarrow p(nn)$ under $0^\circ$ in the frame of elastic $np \rightarrow np$ scattering to $180^\circ$

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Abstract

It is considered the problem of spin physics related with the difference of representation of the elastic interaction between the neutron and proton. In the first case the charge-exchange reaction $np \rightarrow pm$ under the angle $\theta$ is supposed, in the second — the simple elastic scattering of $np \rightarrow np$, when the neutron is going in opposite direction $\pi - \theta$. The transition from one representation to another is provided by the Majorana operator. In the framework of impulse approximation it is twice calculated the quasi-elastic charge-exchange reaction of a neutron on a deuteron. In the frame of $nd \rightarrow p(nn)$ scattering of proton to the angle $\theta$ it gives the well-known Dean formula. Using other representation $nd \rightarrow (nn)p$ as a neutron elastic scattering under the angle $\pi - \theta$ (together with neutron-spectator in $nn$-pair) the alternative formula is presented.
1 Introduction

The formalism of NN-interaction repeats the method intended to describe the electron scattering on the particles with half-integer spin, for example, on other electron or on the atom with one electron outside of closed shell. Therefore in the beginning the definitions of spin and space fermion functions will be considered.

The spin-vector is an analog of mechanical moment but its projection to any direction has a discrete values. In the case of particle with spin $\frac{1}{2}\hbar$ the projection will be equal to $s_z = +\frac{1}{2}\hbar$ or $-\frac{1}{2}\hbar$. To account this duality the three Hermitian $2 \times 2$ matrices are used that forms the Pauli-operator $\hat{\sigma}$:

$$\hat{\sigma} = \hat{i}\sigma_x + \hat{i}\sigma_y + \hat{k}\sigma_z,$$

where

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. $$

Since only $\sigma_z$ has a diagonal view its own vectors or spinors are pure states:

$$\chi_z(s_z = +\frac{1}{2}\hbar) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi_z(s_z = -\frac{1}{2}\hbar) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. $$

For each fermion there is own polarization axis $\vec{s}$ where its state is defined by the spinor $|\vec{s}\rangle$. Along any other directions the fermion spin state will be mixed. Own vectors of matrix $\sigma_r = (\hat{\sigma} \cdot \vec{r})$ (Fig. 1) will have both components $|\vec{s}\rangle$ and $|\vec{r}\rangle$ which define two variants of spin polarization. If fermion is polarized along $z$ its state along $\vec{r}$ expresses as follows:

$$\chi_r(s_z = +\frac{1}{2}\hbar) = \left(\frac{\cos \frac{\theta}{2}}{e^{i\varphi} \sin \frac{\theta}{2}}\right) = \cos \frac{\theta}{2} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + e^{i\varphi} \sin \frac{\theta}{2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}. $$

Here $\cos \frac{\theta}{2}$ and $e^{i\varphi} \sin^2 \frac{\theta}{2}$ are the amplitudes of probability to have the spin projections $s_z = +\frac{1}{2}\hbar$ and $-\frac{1}{2}\hbar$. In our case there are two fermions and each of them can have own spin direction. Therefore more suitable to use the definition of Bloch sphere where the spin states of particles are considered in one coordinate system and quantized along $z$-axis (Fig. 2).

To distinguish both particles the follows definition will be used:

$$\chi_{z,n}(\vec{s}_m) \equiv |\vec{s}_m\rangle_n = \left(\frac{\cos \frac{\theta}{2}}{-e^{i\varphi} \sin \frac{\theta}{2}}\right)_n, \quad n, m = \{1, 2\}. $$
\[ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]

\[ \chi_z(\vec{s}) = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \]

\[ \chi_r(\vec{r}) = \begin{pmatrix} \cos\theta/2 \\ e^{i\phi}\sin\theta/2 \end{pmatrix} \]

\[ \Sigma \text{-sphere} \]

Figure 1: Projection of the Pauli operator \( \hat{\sigma} = i\sigma_x + j\sigma_y + k\sigma_z \) to any direction \( \vec{r} \) equals to unit: \( |\sigma_r|^2 = 1 \). That allows to present its like a \( \Sigma \)-sphere. Each point of the \( \Sigma \)-sphere corresponds to operator \( \sigma_r \) which has two own spinors \( \chi_r(s_z = +\frac{\hbar}{2}) \) and \( \chi_r(s_z = -\frac{\hbar}{2}) \).

\[ \chi_z(\vec{s}) = \begin{pmatrix} \cos\theta/2 \\ -e^{i\phi}\sin\theta/2 \end{pmatrix} \]

\[ \chi_z(\vec{s}_2) = \begin{pmatrix} \cos\theta_2/2 \\ -e^{i\phi_2}\sin\theta_2/2 \end{pmatrix} \]

Figure 2: The Bloch sphere. The spin direction \( \vec{s} \equiv \vec{s}(\theta, \phi) \) is defined by the angles \( \theta \) and \( \phi \). For each \( \vec{s} \) the point on the Bloch sphere corresponds to the single spin state along \( z \)-axis: \( \chi_z(\vec{s}) = \cos\theta \vec{1}_{0} + e^{i\phi}\sin\theta \vec{0}_{1} \). The sign minus before low spinor element arises due to the inverse count of angle \( \theta \) in comparison with the case on Fig. 1.

The index \( m = 1, 2 \) expresses the number of states. The order numbers of particles \( n = 1, 2 \) are arbitrary. Thus the vectors \( |\vec{s}_1\rangle_1 \) and \( |\vec{s}_1\rangle_2 \) present one fermion but named its either first or second particle.

In most cases so detailed definitions of spin states \( (1) \) are superfluous and enough to define them as spinors \( (\alpha \beta) \) and \( (\gamma \delta) \) where the \( \alpha, \beta, \gamma, \delta \) have complex values and satisfy to the conditions: \( |\alpha|^2 + |\beta|^2 = |\gamma|^2 + |\delta|^2 = 1 \). Independence of particles leads to factorization of their function: \( \chi_{12} = \chi_1\chi_2 \). The freedom of order numbers provides the following:

\[ |\vec{s}_1, \vec{s}_2\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}_1 \begin{pmatrix} \gamma \\ \delta \end{pmatrix}_2, \quad |\vec{s}_2, \vec{s}_1\rangle = \begin{pmatrix} \gamma \\ \delta \end{pmatrix}_1 \begin{pmatrix} \alpha \\ \beta \end{pmatrix}_2. \]

\[ (2) \]
To describe a wave properties of particle it is suitable to use the exponent \( e^{i(\vec{p}\vec{r} - Et)/\hbar} \) where \( \vec{p} \) and \( E \) — momentum and full energy of particle, \( \vec{r} \) and \( t \) — space and time variables. Such harmonic is an approximation and presents the infinite plane wave. For our purposes the stationary functions \((t = \text{const})\) will be enough. To distinguish both particle the analogical \([11]\) definitions are chosen:

\[
\varphi_{\vec{p}_m}(\vec{r}_n) \equiv |\vec{p}_m\rangle_n = Ce^{i\vec{p}\vec{r}/\hbar}, \quad n, m = \{1, 2\}, \quad C = (2\pi\hbar)^{-3/2}.
\]

(3)

Momentum has own index \( m \) which does not depend from the number \( n \). Thus the vectors \( |\vec{p}_1\rangle_1 \) and \( |\vec{p}_2\rangle_1 \) show that first particle can have momentum \( \vec{p}_1 \) either momentum \( \vec{p}_2 \). Functions \([3]\) are orthogonally among themselves that expresses the independence of particles and provides the factorization:

\[
|\vec{p}_1, \vec{p}_2\rangle = |\vec{p}_1\rangle_1 |\vec{p}_2\rangle_2, \quad |\vec{p}_2, \vec{p}_1\rangle = |\vec{p}_2\rangle_1 |\vec{p}_1\rangle_2.
\]

In the c.m.s. the momenta permutation is equivalent to the conjugation:

\[
|\vec{p}_1, \vec{p}_2\rangle \rightarrow \varphi_{12} = Ce^{i\vec{p}\vec{r}/\hbar}, \quad |\vec{p}_2, \vec{p}_1\rangle \rightarrow \varphi^*_{12} = Ce^{-i\vec{p}\vec{r}/\hbar},
\]

(4)

where \( \vec{r} = \vec{r}_1 - \vec{r}_2 \) — relative radius-vector.

According to the Pauli principal the wave function \( \Psi \) of identical fermions should be antisymmetric relative the permutation of their spins and momenta:

\[
\Psi = \frac{1}{\sqrt{2}} \left( |\vec{p}_1, \vec{s}_1\rangle_1 |\vec{p}_2, \vec{s}_2\rangle_2 - |\vec{p}_2, \vec{s}_2\rangle_1 |\vec{p}_1, \vec{s}_1\rangle_2 \right),
\]

(5)

where \( |\vec{p}_m, \vec{s}_m\rangle_n = |\vec{p}_m\rangle_n |\vec{s}_m\rangle_n \).

The full permutation don’t change the particles configuration (Fig. 3). Thus the momentum of fermion with spin polarization \( \vec{s}_1 \) anyway equals to \( \vec{p}_1 \).

Figure 3: On the left the states of two particles 1 and 2 are defined by the function \( |\vec{p}_1, \vec{s}_1\rangle_1 |\vec{p}_2, \vec{s}_2\rangle_2 \). On the right the order numbers of particles are changed to correspond to the function \( |\vec{p}_2, \vec{s}_2\rangle_1 |\vec{p}_1, \vec{s}_1\rangle_2 \).
The formulas (2) and (4) allow to express the wave function (5) as follows:

\[ \Psi = \frac{1}{\sqrt{2}} \left[ (\alpha \beta) \begin{pmatrix} 1 \\ 1 \end{pmatrix} \varphi_{12} - (\gamma \delta) \begin{pmatrix} 2 \\ 1 \end{pmatrix} \varphi^*_{12} \right] \quad \text{or} \quad (\alpha \beta)_p \begin{pmatrix} 1 \\ -1 \end{pmatrix} \varphi_{12} , \tag{6} \]

\[ \int \int \Psi^+(\vec{p}')\Psi(\vec{p}) d^3\vec{r} d^3\vec{p} = \int \delta(\vec{p} - \vec{p}') d^3\vec{p} = 1 . \tag{7} \]

Taking into account the rule (7) we will suppose for simplification that the function (6) is normalized to unit: \( |\Psi|^2 = 1 \).

## 2 Fermions elastic scattering

The result of interaction of two particles can be presented as a transformation of initial wave function \( \Psi_{\text{in}} \) to final \( \Psi_{\text{fin}} \) (Fig. 4) and both of them are supposed as the plane waves (6). To change the momentum direction \( \vec{p} \rightarrow \vec{p}' \) let us to define the special unitary operator \( \hat{P}(\theta) \):

\[ \hat{P}(\theta) \varphi_{12} = \varphi'_{12} = e^{i \frac{\hbar}{\epsilon} \vec{p}' \cdot \vec{r}} , \quad \cos \theta = \frac{(\vec{p}, \vec{p}')}{|\vec{p}| \cdot |\vec{p}'|} , \]

\[ |\hat{P}(\theta)|^2 = 1 , \quad \hat{P}(\theta)\hat{P}(\omega) = \hat{P}(\theta + \omega) , \quad \hat{P}(\theta) = \hat{P}(\theta + 2\pi n) . \tag{8} \]

\[ \Psi_{\text{fin}} = \hat{P}(\theta) \times \Psi_{\text{in}} . \]

Figure 4: The plane of interaction of two particles. Circular arrows show the freedom of rotation of this plane around the direction \( \vec{p} \).

\(^1\)The operator \( \hat{P}(\theta) \) can be presented as \( 3 \times 3 \) matrix \( A \) which rotates the momentum: \( \vec{p}' = A\vec{p} \). The transformation \( \hat{P}(\theta)\varphi \rightarrow \varphi' \) carry out in the c.m.s. If we go into the laboratory coordinates the action of this operator reduces to multiplying the wave function \( |\vec{p}_1, \vec{p}_2\rangle \) by the exponent \( e^{-\frac{i\hbar}{\epsilon} \vec{q} \cdot (\vec{r}_1 - \vec{r}_2)} \) where the \( \vec{q} \) is a momentum of recoil particle.

5
The momentum \( \vec{p}' \) lies between the angles \( \theta \) and \( \theta + d\theta \) on the reaction plane therefore in the space the scattered wave \( \Psi_{fin} \) propagates to the solid angle \( d\Omega = 2\pi \sin \theta \, d\theta \). Returning to the definition of probability of elastic interaction it is enough to multiply the wave \( \Psi_{fin} \) by an amplitude:

\[
\Phi = A(\theta)\Psi_{fin}, \quad \frac{d\sigma(\theta)}{d\Omega} = |\Phi|^2 = |A(\theta)|^2.
\]

Using the well known solution of scattering problem [1, 2] take it as spin matrix \( M(\theta) \) in the Goldberger-Watson representation [3, 4]:

\[
M(\theta) = a I_{1,2} + b \sigma_1 \sigma_2 n + c (\sigma_1 n + \sigma_2 n) + e \sigma_{1m} \sigma_{2m} + f \sigma_{1l} \sigma_{2l}.
\]

\( I_{1,2} \) — multiplying of two unit 2 \times 2 matrices of first and second particles, amplitudes \( a, b, c, e, f \) are the complex functions of scattering angle \( \theta \) and kinetic energy of incident particle in the lab system. New spin states are defined by the Pauli-operators which acts along the vectors \( \vec{m}, \vec{l}, \vec{n} \):

\[
\sigma_m = (\hat{\sigma} \cdot \vec{m}), \quad \sigma_l = (\hat{\sigma} \cdot \vec{l}), \quad \sigma_n = (\hat{\sigma} \cdot \vec{n}),
\]

\[
\vec{m} = \frac{\vec{p} - \vec{p}'}{|\vec{p} - \vec{p}'|}, \quad \vec{l} = \frac{\vec{p} + \vec{p}'}{|\vec{p} + \vec{p}'|}, \quad \vec{n} = \frac{\vec{p} \times \vec{p}'}{|\vec{p} \times \vec{p}'|}.
\]

Since the operator \( \hat{P}(\theta) \) turns the momenta and the matrix \( M(\theta) \) changes the spin projections their order does not matter and the simultaneous action of them provides the full determination of scattering process:

\[
\hat{P}(\theta) \ M(\theta) \equiv M(\theta) \ \hat{P}(\theta), \quad \Phi = \hat{P}(\theta) \ M(\theta) \times \Psi_{in}.
\]

### 2.1 Flip and Non-Flip parts of differential cross-section

The arbitrarity condition of quantization axis \( z \) allows to direct it along the vector \( \vec{n} \), two other axes \( x \) and \( y \) will be along the vectors \( \vec{m} \) and \( \vec{l} \). The elastic scattering matrix [9] becomes more definite. The spin transformation is performed by the operator \( \hat{R}_t^+(\varphi) = E \cos \frac{\varphi}{2} + i(\hat{\sigma} \cdot \vec{l}) \sin \frac{\varphi}{2} \) where the \( \vec{l} \) and \( \varphi \) are the axis and angle of rotation. Since \( \hat{R}_t^+(\pi) = i\sigma_t \) the sigma-matrices \( \sigma_x, \sigma_y \) and \( \sigma_z \) are the rotation operators by 180° around their axes (Fig. 5).

The Pauli-operators [10] transform initial wave [6] and give the 6 final wave functions which are orthogonal on the Bloch sphere [4], i.e. when the both particles

\[\forall \ \vec{t} \in \mathbb{C}, \chi = \left(\begin{array}{c} \cos \theta_m/2 \\ -e^{i\varphi_m} \sin \theta_m/2 \end{array}\right) \Rightarrow \frac{1}{4\pi} \int_0^{2\pi} \int_0^\pi \chi^+(\hat{\sigma} \cdot \vec{l}) \chi \, d\theta_m \, d\varphi_m = 0.\]
Figure 5: Four directions of the fermions polarization are shown which are related among themselves with rotations by 180° around the axes (x, y, z).

of beam and target are unpolarized:

\[
\Psi_a = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} p, \quad \Psi'_c = \begin{pmatrix} \alpha \\ -\beta \end{pmatrix} p, \\
\Psi''_c = \begin{pmatrix} \alpha - \beta \\ \beta \end{pmatrix} p, \quad \Psi_b = \begin{pmatrix} \alpha \\ -\beta \end{pmatrix} p,
\]

The action of matrix \( M(\theta) \) can be considered with the position of spin changing of scattering particle. The transition \((\alpha \beta) \rightarrow (\alpha \beta)\) is provided by two operators \( I_1, I_2 \) and \((\sigma_1 + \sigma_2)\). The differential cross-section of these processes is defined as follows:

\[
\frac{d\sigma(\theta)}{d\Omega}^{\text{Non-Flip}} = |a|^2 + |c|^2. \quad (12a)
\]

The operator \((\sigma_1 + \sigma_2)\) also performs the transition \((\alpha \beta) \rightarrow (-\alpha \beta)\) and analogic changing is given by the operator \(\sigma_1 \sigma_2\). The physical sense of this transformation is the spin rotation by 180° around the z-axis. The transitions \((\alpha \beta) \rightarrow (-\alpha \beta)\) are performed by the operators \(\sigma_1 \sigma_2\) and \(\sigma_1 \sigma_2\) respectively that corresponds to the spin rotations by 180° around the axes x and y. The differential cross-section of these transitions calculated by the formula:

\[
\frac{d\sigma(\theta)}{d\Omega}^{\text{Flip}} = |b|^2 + |c|^2 + |e|^2 + |f|^2. \quad (12b)
\]
2.2 Two representations of elastic scattering

The main task of the theory is the prediction what a spin states of particles will be after the interaction (Fig. 6). If both fermions are identical it is impossible to predict which of them is scattered to the angle $\theta$ and who goes in the opposite direction $\pi - \theta$. In this case the definition of scattered particle is conditionally.

Figure 6: Elastic scattering of two identical fermions. One of them in the $(\alpha \beta)$ state flies from the left with momentum $\vec{p}$. The state of another fermion with momentum $-\vec{p}$ is given by the spinor $(\gamma \delta)$. Secondary particles going from the point of interaction with momenta $\vec{p}'$ and $-\vec{p}'$ have states $\chi(\vec{s}_{sc})$ and $\chi(\vec{s}_{rec})$ respectively.

Supposing that the initial wave function of two identical fermions describes by the form (6). The scattering matrix $M(\theta)$ determines the 6 secondary waves (11) and each of them has own amplitude. They form a system of wave functions along which the final state vector of two particles is decomposed:

$$\Phi = \sum_{i=1}^{6} A_i \Psi_{fin,i} \, .$$

(13)

Anyway another wave system $\{\Psi^*_{fin,i}\}$ can be used but the transition from the first system should be unitary. Then the vector (13) will receive new coordinates $\Phi = \sum A^*_i \Psi^*_{fin,i}$. If elastic ineraction is considered like a scattering along the momentum $-\vec{p}'$ then the angle is changed $\theta \rightarrow \theta - \pi$ and instead of matrix $M(\theta)$ we need to use the matrix $M(\theta - \pi)$. For observer in the c.m.s. of two particles it will be look as if the detector takes inverse position and catches other particle whom named before as a recoil particle. But the configuration of two particles does not change and hence our measurement should provide the same result, i.e. the vector of final state $\Phi$ (13) which have been defined by the first approach. Therefore both representations of elastic scattering are equivalent:

$$\Phi = \hat{P}(\theta)M(\theta) \times \Psi_{fin} = \hat{P}(\theta - \pi)M(\pi - \theta) \times \Psi_{fin} \, .$$

(14)
Here we take into account that $M(\theta - \pi) = M(\pi - \theta)$ according to the space isotropy. Using the properties of operator (8) it provides: $\hat{P}(\theta - \pi) = \hat{P}(\theta)\hat{P}(\pi)$. But the $\hat{P}(\pi)$ operator performs the momenta inversion $\hat{P}(\pi) \times |\vec{p}_1, \vec{p}_2\rangle = |\vec{p}_2, \vec{p}_1\rangle$. and it is known as the Majorana operator $\hat{P}_M \equiv \hat{P}(\pi)$ [5]. Thus the expression (14) allows to find:

$$M(\theta) = \hat{P}(\pi) \times M(\pi - \theta).$$  \hspace{1cm} (15)

Since the Bartlett operator $\hat{P}_B = \frac{1}{2}(1 + \hat{\sigma}_1 \hat{\sigma}_2)$ changes the spin states then the combination $\hat{P}_M \hat{P}_B$ performs the full permutation:

$$\hat{P}_M \hat{P}_B \times \Psi = -\Psi \Rightarrow \hat{P}_M = -\hat{P}_B = -\frac{1}{2} (1 + \hat{\sigma}_1 \hat{\sigma}_2).$$  \hspace{1cm} (16)

Although in this analysis the fermions were considered but the definition of spin $\frac{1}{2} \hbar$ did not use absolutely therefore the formula (15) is suitable for any two identical particles with any quantum numbers. From this issue only the expression of Majorana operator should be dependent.

3 Neutron-proton elastic scattering

The $NN$-formalism repeats the method of spin physics. The isotopic spin $T$ is defined and two its projections $T_3 = +\frac{1}{2}$ and $T_3 = -\frac{1}{2}$ mean the proton and neutron. They are entered like the isotopic spinors $^{1}_{0}$ and $^{0}_{1}$ which can be write as $p$ and $n$ for simplification. Linear combinations

$$\chi^T_0 = \frac{1}{\sqrt{2}} (p_1 n_2 - n_1 p_2) \quad \text{and} \quad \chi^T_{1,0} = \frac{1}{\sqrt{2}} (p_1 n_2 + n_1 p_2)$$

present the nucleons in the isotopic states $T = 0$ and $T = 1$ (with projection $T_3 = 0$) respectively and both of them are own vectors of $\hat{\tau}_1 \hat{\tau}_2$ operator:

$$\hat{\tau}_1 \hat{\tau}_2 = \tau_{1,3} \tau_{2,3} + 2 \left[ \tau_{1+} \tau_{2-} + \tau_{1-} \tau_{2+} \right],$$

$$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \tau_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \tau_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix},$$

$$\hat{\tau}_1 \hat{\tau}_2 \times \chi^T_0 = -3 \chi^T_0, \quad \hat{\tau}_1 \hat{\tau}_2 \times \chi^T_{1,0} = + \chi^T_{1,0}.$$  \hspace{1cm} (15)

The neutron-proton wave function is build using the same scheme (6):

$$\Psi_{in} = \frac{1}{\sqrt{2}} \left[ n_1 p_2 \begin{pmatrix} \alpha \gamma \\ \beta \delta \end{pmatrix}_1 \varphi_{12} - p_1 n_2 \begin{pmatrix} \gamma \alpha \\ \beta \delta \end{pmatrix}_2 \varphi^*_{12} \right].$$  \hspace{1cm} (17)
Since in the nuclear interaction full isospin $T$ and its projection $T_3$ are saved\footnote{Weak processes $n \rightarrow p + e^+ \tilde{\nu}$ and $p \rightarrow n + e^+ + \nu$ lead to a violation of isotopic invariance with probability $\sim 10^{-8}$, i.e. in the nuclear interactions the neutron does not converted to proton at all and vice versa, therefore $T = \text{const}$ and $T_3 = \text{const}$.} the nucleons elastic scattering matrix has the next form:

$$M(\theta) = M_0(\theta) \frac{1 - \hat{\tau}_1 \hat{\tau}_2}{4} + M_1(\theta) \frac{3 + \hat{\tau}_1 \hat{\tau}_2}{4}. \quad (18)$$

Here the $M_0$ and $M_1$ are the spin matrixes like (9) but present the elastic scattering of nucleons in pure isotopic states $T = 0$ and $T = 1$. Thus the $M_1$ describes the $np$-scattering in the state $T = 1$ and has the 5 own amplitudes $(a_1, b_1, c_1, e_1, f_1)$. The matrix $M_0$ with amplitudes $(a_0, b_0, c_0, e_0, f_0)$ is intended to the $T = 0$ state.

To define the result of elastic $np$-interaction it is suitable to share the matrix (18) by the direct and exchange parts:

$$M(\theta) = \frac{1}{2} (M_1(\theta) + M_0(\theta)) + \frac{1}{2} (M_1(\theta) - M_0(\theta)) \hat{P}^T_B, \quad (19)$$

$$\hat{P}^T_B = \frac{1}{2} (1 + \hat{\tau}_1 \hat{\tau}_2) \quad \text{isotopic exchange Bartlett operator.}$$

One half of sum $\frac{1}{2} (M_1(\theta) + M_0(\theta))$ gives the simple scattering $np \rightarrow np$ under the angle $\theta$ either describes the symmetric reaction $pn \rightarrow pn$. The second part (19) has the exchange isotopic operator $\hat{P}^T_B$ and adds to scattering the isospins permutation. Therefore the half of difference $\frac{1}{2} (M_1(\theta) - M_0(\theta))$ is the spin matrix of charge-exchange $np \rightarrow pn$ or $pn \rightarrow np$ reactions under the angle $\theta$.

### 3.1 Change representation of $np$ elastic scattering

Let the wave function of two nucleons is defined by the formula (17). If the proton in the c.m.s. system of elastic $np$-interaction is scattered under the angle $\theta$ then the neutron flies in inverse direction $\pi - \theta$. This reaction can be presented as charge-exchange $np \rightarrow pn$ process when the spin states of neutron and proton are described by the matrix $\frac{1}{2} (M_1(\theta) - M_0(\theta))$. Either the matrix $\frac{1}{2} (M_1(\theta) + M_0(\theta))$ can be used to define the elastic $np \rightarrow np$ scattering under the angle $\pi - \theta$. Since the vector $\Phi$ of final states of neutron and proton can not depend from our view which of these particles are registered the both representations are equivalent but can differ among themselves by the set of basis wave functions of vector $\Phi$. According to (18) the transition from one representation to another should be performed by the Majorana operator:

$$\frac{1}{2} (M_1(\pi - \theta) + M_0(\pi - \theta)) = \hat{P}_M \times \frac{1}{2} (M_1(\theta) - M_0(\theta)) \hat{P}^T_B. \quad (20)$$
Because the nucleons have the variables of spin, isospin and space the full permutation is performed by three operators $\hat{P}_M$, $\hat{P}_B$ and $\hat{P}_B^T$:

$$\hat{P}_M \hat{P}_B \hat{P}_B^T \times \Psi = -\Psi \Rightarrow \hat{P}_M = -\hat{P}_B \hat{P}_B^T = -\frac{1}{2} (1 + \hat{\sigma}_1 \hat{\sigma}_2) \frac{1}{2} (1 + \hat{\tau}_1 \hat{\tau}_2).$$

The unitarity $|\hat{P}_B^T|^2 = 1$ transforms the (20) to the next:

$$\frac{1}{2} (M_1(\pi - \theta) + M_0(\pi - \theta)) = -\hat{P}_B \times \frac{1}{2} (M_1(\theta) - M_0(\theta)). \quad (21)$$

For the first this expression (21) was defined in the work [6]. Before this time all amplitudes transformations were limited in the frame of symmetry rules [7] using scattering angle replacing $\theta \rightarrow \pi - \theta$. Passing to the Goldberger-Watson amplitudes [4] we get the canonical transformation [8, 9, 10]:

$$c^{\text{exch}}(\theta) = \frac{1}{2} (c_1(\theta) - c_0(\theta)) = \frac{1}{2} (c_1(\pi - \theta) + c_0(\pi - \theta)) = c(\pi - \theta),$$

$$\begin{pmatrix}
  a^{\text{exch}}(\theta) \\
  b^{\text{exch}}(\theta) \\
  c^{\text{exch}}(\theta) \\
  f^{\text{exch}}(\theta)
\end{pmatrix} =
\begin{pmatrix}
  -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\
  -\frac{1}{2} & -\frac{1}{2} & +\frac{1}{2} & +\frac{1}{2} \\
  -\frac{1}{2} & +\frac{1}{2} & +\frac{1}{2} & +\frac{1}{2} \\
  -\frac{1}{2} & +\frac{1}{2} & -\frac{1}{2} & +\frac{1}{2}
\end{pmatrix}
\begin{pmatrix}
  a(\pi - \theta) \\
  b(\pi - \theta) \\
  c(\pi - \theta) \\
  f(\pi - \theta)
\end{pmatrix}. \quad (22)$$

If the initial states of neutron and proton are defined by the spinors $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ and $\begin{pmatrix} \gamma \\ \delta \end{pmatrix}$ respectively then the amplitude $a^{\text{exch}}(\theta)$ show the correlation between the spinor $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ and final state $\chi_p$ of proton. In the same time the amplitude $a(\pi - \theta)$ say about the $\chi_p$ and spinor $\begin{pmatrix} \gamma \\ \delta \end{pmatrix}$ correlation. These amplitudes can be equal only if the proton spin state $\chi_p$ is symmetric (if at least it is average symmetric relative to all directions of spin polarizations of neutron and proton) between the initial states $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ and $\begin{pmatrix} \gamma \\ \delta \end{pmatrix}$. However the theory does not require this.
4 Quasielastic reaction \( nd \rightarrow p(nn) \)

4.1 Dean formula

The Pomeranchuk-Chew hypothesis \([11, 12, 13, 14]\) as well as the A. Migdal idea \([15]\) supposes to consider the charge-exchange \( nd \rightarrow p(nn) \) reaction using the free-nucleons scattering formalism. Following to this approach the initial wave function of three nucleons will be presented in the next form:

\[
\Psi_{3N} = \frac{1}{\sqrt{3!}} \begin{vmatrix}
|\xi_1\rangle_1 & |\xi_2\rangle_1 & |\xi_3\rangle_1 \\
|\xi_1\rangle_2 & |\xi_2\rangle_2 & |\xi_3\rangle_2 \\
|\xi_1\rangle_3 & |\xi_2\rangle_3 & |\xi_3\rangle_3
\end{vmatrix}.
\] (23)

Here \( \xi \) is the set of nucleon states including momentum, spin and isotopic spin. Let the \( \xi_1 \) expresses the beam neutron state and the \( \xi_2 \) and \( \xi_3 \) will be the proton and neutron of deuterium nucleus. Decomposing the determinant (23) by the elements of the first column gives:

\[
\Psi_{3N} = \frac{1}{\sqrt{3}} \sum |\xi_i\rangle_i \frac{1}{\sqrt{2}} \left( |\xi_2\rangle_3 |\xi_3\rangle_k - |\xi_3\rangle_j |\xi_2\rangle_k \right),
\] (24)

\( \{i, j, k\} = \{1, 2, 3\}, \{2, 3, 1\}, \{3, 1, 2\} \).

Since the wave functions (24) are orthogonal it is enough to take one variant, for example \( \{i=1, j=2, k=3\} \), reducing the coefficient \( \frac{1}{\sqrt{3}} \). The incident neutron can be presented by the vector: \( |\xi_1\rangle_1 = \eta_1(\gamma_3)\eta_1|\vec{p}_n\rangle_1 \). The state of deuteron nucleons is subject to several conditions. For the first, the spins of proton and neutron should be parallel because \( S_d = \hbar \). Denote their states by the spinors \( (\gamma_2 \gamma_3)_2 \), here \( \gamma = \cos \lambda/2, \delta = -e^{i\mu} \sin \lambda/2, \lambda \) and \( \mu \) are zenith and asimutal angles of deuteron polarization. For the second, the nucleons are joint by the isosinglet function \( \chi_T^0 = \frac{1}{\sqrt{2}}(p_2 n_3 - n_2 p_3) \). The space part of their wave should be symmectric \( \varphi_d(\vec{r}) = \varphi_d(-\vec{r}) \) here \( \vec{r} = \vec{r}_2 - \vec{r}_3 \). Therefore:

\[
\Psi_{3N} = \Psi_{nd} = |\xi_1\rangle_1 \Psi_d = n_1 \left( \begin{array}{c} \alpha \\ \beta \end{array} \right) \frac{1}{\sqrt{2}} \left( \begin{array}{c} p_2 n_3 - n_2 p_3 \\ \delta \end{array} \right) \left( \begin{array}{c} \gamma \\ \delta \end{array} \right) \varphi_d.
\] (25)

In the c.m.s. of beam neutron and proton of deuterium nucleus the exchange of charges and spin states is described by the matrix \( \frac{1}{2}(M_1(\theta) - M_0(\theta)) \hat{P}_d^T \). The scattering to the angle \( \theta \) is defined by the operator \( \hat{P}(\theta) \) \([8]\). In the deuteron system this changing is equivalent to the momentum transfer \( \vec{p}_n \rightarrow \vec{p}'_n = \vec{p}_n - \vec{q} \).
that adds the exponent $e^{-\frac{\vec{q} \cdot \vec{r}}{\hbar}}$ to the wave function of scattered particle. Recoil particle obtains the $e^{\frac{\vec{q} \cdot \vec{r}}{\hbar}}$ or $e^{-\frac{\vec{q} \cdot \vec{r}}{\hbar}}$ depending from the proton number.

$$\Phi_{p(nn)} = \frac{1}{2}(M_1(\theta) - M_0(\theta)) \hat{P}_B^{\top} \hat{P}(\theta) \times \Psi_{nd} =$$

$$= \sum A_t^{exch} p_1(\alpha_t, \beta_t) |\vec{r}_{23}^n| \frac{n^2 n_3}{\sqrt{2}} \left[ \left( \frac{\gamma_t}{\delta_t} \right) \left( \frac{\gamma}{\delta} \right) e^{\frac{\vec{q} \cdot \vec{r}}{\hbar}} - \left( \frac{\gamma}{\delta} \right) \left( \frac{\gamma_t}{\delta_t} \right) e^{-\frac{\vec{q} \cdot \vec{r}}{\hbar}} \right] \varphi_d =$$

$$= \sum A_t^{exch} p_1(\alpha_t, \beta_t) |\vec{r}_{23}^n| \frac{n^2 n_3}{\sqrt{2}} \left[ \chi(-) \cos \frac{\vec{q} \cdot \vec{r}}{2\hbar} + i \chi(+) \sin \frac{\vec{q} \cdot \vec{r}}{2\hbar} \right] e^{\frac{\vec{q} \cdot \vec{r}}{\hbar}} \varphi_d , \quad (26)$$

$$\vec{r}_{nn} = \frac{1}{2}(\vec{r}_2 + \vec{r}_3) - \text{radius-vector of two neutrons system},$$

$$\chi(\pm) = \left( \frac{\gamma_t}{\delta_t} \right) \left( \frac{\gamma}{\delta} \right) \pm \left( \frac{\gamma}{\delta} \right) \left( \frac{\gamma_t}{\delta_t} \right) \cdot \quad (27)$$

$A_t^{exch}$ is amplitude of charge-exchange $np \to pn$ elastic scattering to the angle $\theta$: $A_t^{exch} = a^{exch}(\theta) = \frac{1}{2}(a_1(\theta) - a_0(\theta))$ etc. (Tab. 1). The spinors $(\alpha_i, \beta_i)$ are defined by the operators which follow with their amplitudes (11). The function $\chi(-)$ corresponds to the spin singlet state and the $\chi(\pm)$ is a linear combination of three spin functions $\chi_{1, -1}$, $\chi_{1, 0}$ and $\chi_{1, 1}$. It is obviously that the $\chi(-)$ enters to the (26) with even space function $\cos \frac{\vec{q} \cdot \vec{r}}{2\hbar}$ and the $\chi(\pm)$ goes with odd function $\sin \frac{\vec{q} \cdot \vec{r}}{2\hbar}$, i.e. the wave of two neutrons is antisymmetric. Differential cross-section of $nd \to p(nn)$ reaction is defined as $|\Phi_{p(nn)}|^2$. Since the $\Phi_{p(nn)}$ is dependent from the $\vec{r} = \vec{r}_2 - \vec{r}_3$ it is nesessary to take the integral over the deuteron space. If the particles were unpolarized the (26) needs averaged over all spins directions:

$$\frac{d\sigma(\theta)}{d\Omega_{nd \to p(nn)}} = \oint |\Phi_{p(nn)}|^2 dV =$$

$$= \frac{1}{2} \sum |A_t^{exch}|^2 \left[ |\chi(-)|^2 \oint |\varphi_d|^2 \cos^2 \frac{\vec{q} \cdot \vec{r}}{2\hbar} dV + |\chi(\pm)|^2 \oint |\varphi_d|^2 \sin^2 \frac{\vec{q} \cdot \vec{r}}{2\hbar} dV \right]. \quad (28)$$

Calculation provides the next results (Tab. 1):

$$|\chi(-)|^2 = 2|\delta \gamma - \gamma \delta|^2, \quad |\chi(\pm)|^2 = 4 - |\chi(-)|^2 ,$$

$$|\delta \gamma - \gamma \delta|^2 = \begin{cases} 0, & \text{if } \left( \frac{\gamma_t}{\delta_t}, \frac{\gamma}{\delta} \right) = \left( \frac{\gamma}{\delta}, \frac{\gamma_t}{\delta_t} \right) ; \\
2/3, & \text{if } \left( \frac{\gamma_t}{\delta_t}, \frac{\gamma}{\delta} \right) = \left( \frac{\gamma}{\delta}, \frac{\delta}{\gamma} \right) \text{ or } \left( \frac{\delta}{\gamma}, \frac{\gamma}{\delta} \right) \end{cases}.$$
Table 1: Weight fractions of the spin states $\chi(-)$ and $\chi(+)\) of two neutrons system from the quasi-elastic charge-exchange reaction $nd \rightarrow p(nn)$.

| $A_{i}^{\text{exch}}$ | $t$ | $\gamma_{i}$ | $\delta_{i}$ | $|\chi(-)|^{2}$ | $|\chi(+)|^{2}$ |
|------------------------|-----|-------------|-------------|----------------|----------------|
| $a_{i}^{\text{exch}}(\theta)$ | 1   | $\gamma$   | $\delta$   | 0              | 4              |
| $b_{i}^{\text{exch}}(\theta)$ | 2   | $\gamma$   | $-\delta$  | $4/3$          | $8/3$          |
| $c_{i}^{\text{exch}}(\theta)$ | 3   | $\gamma$   | $\delta$   | 0              | 4              |
| $d_{i}^{\text{exch}}(\theta)$ | 4   | $\gamma$   | $-\delta$  | $4/3$          | $8/3$          |
| $e_{i}^{\text{exch}}(\theta)$ | 5   | $\delta$   | $\gamma$   | $4/3$          | $8/3$          |
| $f_{i}^{\text{exch}}(\theta)$ | 6   | $\delta$   | $-\gamma$  | $4/3$          | $8/3$          |

Summing over the contributions of all amplitudes of scattering matrix using the definition of Flip (12b) and Non-Flip (12a) parts of the differential cross-section of elastic $np \rightarrow pn$ charge-exchange process that gives:

$$\frac{d\sigma(\theta)}{d\Omega}_{nd\rightarrow p(nn)} = \frac{2}{3} \frac{d\sigma(\theta)}{d\Omega}_{np\rightarrow pn}^{\text{Flip}} \int |\varphi_d|^2 \cos^2 \frac{q_r}{2\hbar} dV +$$

$$+ \left(2 \frac{d\sigma(\theta)}{d\Omega}_{np\rightarrow pn}^{\text{Non-Flip}} + 4 \frac{d\sigma(\theta)}{d\Omega}_{np\rightarrow pn}^{\text{Flip}} \right) \int |\varphi_d|^2 \sin^2 \frac{q_r}{2\hbar} dV . \quad (29)$$

This Dean’s formula\(^4\) for the first time was published in [16, 17]. The same result was obtained in [18, 19]. The idea of method presented here is that the final state of three nucleons is determined by the direct action of nucleons matrix without any considerations about the properties of wave function of two recoil neutrons. The formula (29) show how to use the deuterium nucleus as an amplitudes filter [11, 12, 13, 14]. If the proton is scattered to zero angle the transfer momentum $\vec{q}$ is going to zero too. It gives $\cos^2 \frac{q_r}{2\hbar} \approx 1$, $\sin^2 \frac{q_r}{2\hbar} \approx 0$. Therefore:

$$\frac{d\sigma(0)}{d\Omega}_{nd\rightarrow p(nn)} = \frac{2}{3} \cdot \frac{d\sigma(0)}{d\Omega}_{np\rightarrow pn}^{\text{Flip}} . \quad (30)$$

\(^4\)Original Dean’s formula use the deuteron form-factor $F(q) = \int |\varphi_d|^2 \cos \frac{q_r}{\hbar} dV$:

$$\int |\varphi_d|^2 \cos^2 \frac{q_r}{2\hbar} dV = \frac{1}{2} \int |\varphi_d|^2 (1 + \cos \frac{q_r}{\hbar}) dV = \frac{1}{2}(1 + F(q)) ,$$

$$\int |\varphi_d|^2 \sin^2 \frac{q_r}{2\hbar} dV = \frac{1}{2}(1 - F(q)) .$$
Measuring the proton yields of elastic $np \rightarrow pn$ and quasi-elastic $nd \rightarrow p(nn)$ scatterings under zero angles we define their ratio $R_{dp}(0)$ that allow to calculate the relation $r^{\text{np}/\text{fl}}_{np \rightarrow pn}(0)$ between the Non-Flip and Flip parts of differential cross-section of elastic $np \rightarrow pn$ charge-exchange process:

$$r^{\text{np}/\text{fl}}_{np \rightarrow pn}(0) = \frac{d\sigma(0)^{\text{Non-Flip}}_{np \rightarrow pn}}{d\Omega} / \frac{d\sigma(0)^{\text{Flip}}_{np \rightarrow pn}}{d\Omega} = \frac{2}{3} \frac{1}{R_{dp}(0)} - 1. \quad (31)$$

4.2 Alternative formula

Let us to consider the quasi-elastic $nd \rightarrow p + nn$ charge-exchange reaction using another representation as the elastic $np \rightarrow np$ scattering. Initial wave of three nucleons is taken in the same form (25):

$$\Psi_{nd} = \frac{1}{\sqrt{3}} \sum n_i \left( \frac{\alpha}{\beta} \right) \langle \vec{p}_n \rangle_i \xi_n \frac{p_j n_k - n_j p_k}{\sqrt{2}} \left( \frac{\gamma}{\delta} \right) \varphi_{d,jk}, \quad (32)$$

$$\{i, j, k\} = \{1, 2, 3\}, \{2, 3, 1\}, \{3, 1, 2\}, \quad \varphi_{d,jk} = |\vec{p}_{p^*} \rangle_j |\vec{p}_{n^*} \rangle_k.$$  

The isotopic and spin variables are determined now by the scattering matrix $\frac{1}{2}(M_1(\pi - \theta) + M_0(\pi - \theta))$. The space part of wave $|\vec{p}_n \rangle_i \varphi_{d,jk}$ is changed by the operator $\hat{P}(\pi - \theta)$ which action can be shared by two steps. The $\hat{P}(\pi)$ provides the momenta permutation $\vec{p}_{n,cm} \leftrightarrow \vec{p}_{p^*,cm}$ and the operator $\hat{P}(\theta)$ gives the rotation by the angle $\theta$. In the deuteron coordinate system the incident neutron knocks out the proton and passes the momentum $\vec{p}'_p = \vec{p}_n - \vec{q}$. To the deuton wave function the exponent $e^{i\vec{q} \cdot \vec{r}'_d}$ is added by the momentum transfer $\vec{q}$ which remains after the impact. If the proton has number $j$ the action of $\hat{P}(\pi - \theta)$ operator can be presented as follows:

$$\hat{P}(\theta)\hat{P}(\pi) \times |\vec{p}_n \rangle_i |\vec{p}_{p^*} \rangle_j |\vec{p}_{n^*} \rangle_k = \hat{P}(\theta) \times |\vec{p}_{p^*} \rangle_i |\vec{p}_n \rangle_j |\vec{p}_{n^*} \rangle_k =$$

$$= e^{-i\vec{q} \cdot (\vec{r}'_j - \vec{r}'_i)} |\vec{p}_n \rangle_j |\vec{p}_{p^*} \rangle_i |\vec{p}_{n^*} \rangle_k = |\vec{p}_p \rangle_j e^{i\vec{q} \cdot \vec{r}'_d} \varphi_{d,ik}. \quad (33)$$

In the $np \rightarrow np$ representation the neutron is the scattered particle (forming with the neutron-spectator an $nn$-pair) and the proton is the recoil particle

---

\(\frac{1}{2}(M_1(\pi - \theta) + M_0(\pi - \theta))\) defines the instantaneous transfer momentum $\vec{q} (\vec{p'})$ of $nn$-pair but the observing value of $\vec{q} = \vec{p}_n - \vec{p}'_p$ can be supposed as an average $\langle \vec{q} \rangle$: $\hat{P}_{12}(\theta) \times |\vec{p}_n \rangle_1 \varphi_{d,23} = |\vec{p}_n \rangle_1 \int e^{i\vec{q} \cdot (\vec{r}_2 - \vec{r}_1)} \Psi_H(p) e^{i\vec{q} \cdot (\vec{r}_2 - \vec{r}_3)} d^3\vec{p} = |\vec{p}_p \rangle_1 e^{i\vec{q} \cdot \vec{r}_2} \varphi_{d,23}$.

where the $\Psi_H(p)$ is the momentum representation of Hulthen’s $S$-wave function [23].
therefore the final vector will be defined by the next formula:

\[
\Phi_{(nm)p} = \frac{1}{2}(M_1(\pi - \theta) + M_0(\pi - \theta)) \hat{P}(\pi - \theta) \times \Psi_{nd} = \\
= \frac{1}{\sqrt{3}} \sum_{i,j,k} A_i \left[ \frac{n_i n_p n_k}{\sqrt{2}} \left( \frac{\gamma_i}{\delta_i} \right) \left( \frac{\alpha_t}{\beta_i} \right) \hat{\varphi}_{t,p} + \right. \\
\left. - \frac{n_i n_p n_k}{\sqrt{2}} \left( \frac{\gamma_i}{\delta_i} \right) \left( \frac{\gamma_i}{\delta_i} \right) \hat{\varphi}_{t,p} \right] e^{i \frac{4}{3} \hat{\varphi}_{d,ik}} \varphi_{d,ik} \tag{34}
\]

Here \( A_i \) is one of fifth amplitudes of the \( np \rightarrow np \) scattering to the angle \( \pi - \theta \): 
\( A_1 = a(\pi - \theta) = \frac{1}{2}(a_1(\pi - \theta) + a_0(\pi - \theta)) \) etc. The spin states \( (\frac{\gamma_i}{\delta_i}) \) and \( (\frac{\gamma_i}{\delta_i}) \) are defined by the Pauli-operators which follow with their amplitudes. The sum over the indexes \( \{i, j, k\} \) can be ordered\(^6\) by the proton’s numbers:

\[
\Phi_{(nm)p} = \frac{1}{\sqrt{3}} \sum_{i,j,k} A_i p_i \left[ \left( \frac{\gamma_i}{\delta_i} \right) \left( \frac{\alpha_t}{\beta_i} \right) \hat{\varphi}_{t,p} \right] \\
\times \frac{n_j n_k}{\sqrt{2}} \left[ \left( \frac{\gamma}{\delta} \right) \left( \frac{\alpha_t}{\beta} \right) - \left( \frac{\gamma}{\delta} \right) \left( \frac{\gamma}{\delta} \right) \hat{\varphi}_{d,jk} \right] \varphi_{d,jk} \tag{35}
\]

Since all waves are orthogonal it is enough to take one variant \( \{i=1, j=2, k=3\} \) reducing the coefficient \( \frac{1}{\sqrt{3}} \). Denoting the variables \( \vec{r} = \vec{r}_2 - \vec{r}_3 \) and \( \vec{r}_{mn} = \frac{1}{2}(\vec{r}_2 + \vec{r}_3) \) that gives:

\[
\Phi_{(nm)p} = \sum A_i p_i \left[ \left( \frac{\gamma}{\delta} \right) \left( \frac{\alpha_t}{\beta} \right) \hat{\varphi}_{t,p} \right] \frac{n_j n_k}{\sqrt{2}} \left[ \chi(-) \cos \frac{\hat{\varphi}}{2\hbar} - i \chi(+) \sin \frac{\hat{\varphi}}{2\hbar} \right] e^{i \frac{4}{3} \hat{\varphi}_{d,mn}} \varphi_{d,23} \tag{36}
\]

\[\chi(\pm) = \left( \frac{\gamma}{\delta} \right) \left( \frac{\alpha_t}{\beta} \right) \pm \left( \frac{\gamma}{\delta} \right) \left( \frac{\gamma}{\delta} \right) \frac{1}{3} \tag{37}\]

The functions \( \Phi_{(nm)p} \) (36) and \( \Phi_{p(nn)} \) (26) have the same structure. Their difference lies only in how the spin functions are defined (37) and (27). The differential cross-section of the \( nd \rightarrow (nn)p \) reaction equals to \( |\Phi_{(mn)p}|^2 \) with the integration over the deuteron space and with the averaging of all spin polarizations of three particles:

\[
\frac{d\sigma(\pi - \theta)}{d\Omega} \bigg|_{nd \rightarrow (nn)p} = \int \left[ |\Phi_{(mn)p}|^2 dV \right] = \\
= \frac{1}{2} \sum |A_i|^2 \left[ \left| \chi(-) \right|^2 \int |\varphi_d|^2 \cos^2 \frac{\hat{\varphi}}{2\hbar} dV + \left| \chi(+) \right|^2 \int |\varphi_d|^2 \sin^2 \frac{\hat{\varphi}}{2\hbar} dV \right] . \tag{38}
\]

\(^6\)To regroup the elements of (34) need use the cyclic substitutions: \( i \rightarrow j, j \rightarrow k, k \rightarrow i. \)
The spins of incident neutron and neutron-spectator are arbitrary directed therefore between the spinors \((\alpha_{\ell t}, \beta_{\ell t})\) and \((\gamma\delta),\) the correlation is absent. The substitutions \((\alpha_{\ell t}, \beta_{\ell t}) = (\alpha\beta),\) \((\alpha\beta)\) or \((\beta, \alpha)\) to the \((37)\) gives exactly values:

\[
|\chi(+)|^2 = 2 |\gamma\delta| = 2, \quad |\chi(-)|^2 = 1, \quad |\chi| = 3.
\] (39)

Then:

\[
d\sigma (\pi - \theta) \left. \frac{d\Omega}{nd\rightarrow(np)p} \right|_{nd\rightarrow(np)p} = \left. \frac{d\sigma (\pi - \theta)}{d\Omega} \right|_{np\rightarrow(np)} \left( \frac{1}{2} + \int |\varphi_d|^2 \sin^2 \frac{q}{2m} dV \right) = \left. \left( 1 - \frac{1}{2} F(q) \right) \frac{d\sigma (\pi - \theta)}{d\Omega} \right|_{np\rightarrow(np)}.
\] (40)

The representation \(np \rightarrow np (\pi - \theta)\) does not share the differential cross-section of quasi-elastic \(nd \rightarrow (nn)p\) reaction by the Flip and Non-Flip parts of elastic \(np \rightarrow np\) scattering. Since the transition from one representation to another is unitary it allows to change simultaneously in the formula \((40)\) the values of angles \((\pi - \theta) \rightarrow \theta\) and denoting the differential cross-sections by the charge-exchange reactions \(nd \rightarrow p(nn)\) and \(np \rightarrow pn:\n
\[
\frac{d\sigma (\theta)}{d\Omega} \left. \frac{d\Omega}{nd\rightarrow(p(nn)} \right|_{nd\rightarrow(p(nn)} = \left( 1 - \frac{1}{2} F(q) \right) \left. \frac{d\sigma (\theta)}{d\Omega} \right|_{np\rightarrow(pn)}.
\] (41)

The most interest related with the case when the proton is scattered forward therefore the momentum transfer \(\vec{q} = \vec{p}_n - \vec{p}'_p\) closes to zero and the deuteron form-factor \(F(q)\) reaches unit. Hence:

\[
\frac{d\sigma (0)}{d\Omega} \left. \frac{d\Omega}{nd\rightarrow(p(nn)} \right|_{nd\rightarrow(p(nn)} = \frac{1}{2} \left. \frac{d\sigma (0)}{d\Omega} \right|_{np\rightarrow(pn)}.
\] (42)

Because the formulas \((41)\) and \((22)\) give the same definition of differential cross-sections \(d\sigma (\theta)/d\Omega_{nd\rightarrow(p(nn)}\) removing from them the similar terms and reducing the same factors it allows to define:

\[
\frac{d\sigma (\theta)}{d\Omega} \left. \frac{d\Omega}{np\rightarrow(pn)} \right|_{np\rightarrow(pn)} = 3 \left. \frac{d\sigma (\theta)}{d\Omega} \right|_{np\rightarrow(pn)}.
\] (43)

The expression \((43)\) should be regarded more as a hypothesis, as an indication obtained within the impulse approximation. On the other hand the elastic \(np \rightarrow pn\) charge-exchange reaction does not depend on how the neutron is scattered on the deuteron. To verify \((43)\) it need to be done the direct reconstruction of scattering amplitudes (DRSA) that requires the complete data set of \(np\)-observables \([1, 7]\).
4.3 Equivalent formulas of two representations

\( nd \to p(nn) \) \( \mu \) \( nd \to (nn)p(\pi) \)

Let us to consider in more detail the case when the secondary protons fly to zero angle that gives \( F(q) \approx 1 \) and allows to transform the formulas (28) and (38) to the next view:

\[
\frac{d\sigma(0)}{d\Omega}_{nd\to p(nn)} = \frac{1}{2} \sum |A_{t}^{exch}|^2 \left( \frac{\gamma}{\delta} \right)_2 \left( \frac{\gamma_t}{\delta_t} \right)_3 - \left( \frac{\gamma_t}{\delta_t} \right)_2 \left( \frac{\gamma}{\delta} \right)_3 \right|^2, \quad (44)
\]

\[
\frac{d\sigma(\pi)}{d\Omega}_{nd\to (nn)p} = \frac{1}{2} \sum |A_{t}|^2 \left( \frac{\gamma}{\delta} \right)_2 \left( \frac{\alpha_t}{\beta_t} \right)_3 - \left( \frac{\alpha_t}{\beta_t} \right)_2 \left( \frac{\gamma}{\delta} \right)_3 \right|^2. \quad (45)
\]

These definitions of differential cross-sections (44) and (45) have the same structure. Their difference lies in how the spin of the neutron is defined forming the singlet state \( S_{nn} = 0 \) with the neutron-spectator in both cases. If the quasi-elastic \( nd \)-interaction is named as the \( nd \to p(nn) \) reaction (44) the neutron gets out as a recoil particle and its spin \( \left( \frac{\gamma}{\delta} \right) \) is presented by the one of four variants of changed spin state of deuteron’s proton. When the \( \left( \frac{\gamma}{\delta} \right) = \left( \frac{\delta}{\gamma} \right) \) the yield of Non-Flip amplitude \( a^{exch}(0) \) disappears. In inverse the all Flip spinors \( \left( \frac{\gamma}{\delta} \right), \left( \frac{\delta}{\gamma} \right) \) and \( \left( \frac{\delta}{\gamma} \right) \) enter with weight \( \frac{4}{3} \) (Tab. 1). After the multiplying by the factor of \( \frac{1}{2} \) the only \( \frac{2}{3} \) remains from the Flip-part of differential cross-section of charge-exchange elastic reaction \( np \to pn (0) \). In the other hand if the \( nd \to (nn)p \) representation (15) is used the beam neutron in the \( np \)-system is considered as a particle scattered under the angle 180°. All spin states \( \left( \frac{\alpha}{\beta} \right) \) enter with equal weight 1 (39) therefore it does not give the separation of Flip and Non-Flip parts and from the differential cross-section of elastic reaction \( np \to np (\pi) \) only one half is remained (45).

5 Experimental data analysis of ratio \( R_{dp} \)

In four runs of 2003-07 using the Delta-Sigma Setup at LHE JINR facility its were performed the measurements of \( R_{dp} \)-ratio between the protons yields of quasi-elastic \( nd \to p(nn) \) and elastic \( np \to pn \) charge-exchange reactions when the protons are scattered under zero angle at energies \( T_n = 0.5 \div 2.0 \text{ GeV} \) [8, 9, 10, 20, 21, 22]. The results of this experiment and other world data are given in Att. A1 (Tab. 2) and shown on the Fig. 7. The ratio \( R_{dp} \) over the energy range \( T_n = 0.55 \div 2.0 \text{ GeV} \) behaves like a constant on the level 0.56.

Returning to the question about the representation choosing it should be noted that the first calculations of \( R_{dp} \)-ratio which made using the solutions of
phase shift analysis (PSA) were in sharp contradiction with the experiment [26]. The formula (29) was used also but according to [27] the amplitudes of elastic np → np(θ = π) reaction and transformed to the np → pn(θ = 0) representation using (22). Values of $R_{dp}$-ratio are calculated by the formula (30). The curve SP07* is obtained using the Non-Flip and Flip parts (12a, 12b) of reaction np → np(θ = π), i.e. ignoring a difference between two representations. Line “1:2” show the true approach using formalism np → np(θ = π) reaction (42).

Our calculations by the formula (42) leads to prediction $R_{dp} = 1/2$ (Рис. 7). The experimental data are higher by 12 % which can be explained by several reasons. First, in the impulse approximation the neutron-spectator is ignored but this is not true for large Fermi-momenta inside deuteron. For example, if the Hulthen’s solution [28] is taken and $\langle P_F \rangle \approx 110 \text{MeV}/c$ then at energy $T_n = 800 \text{MeV}$ the ratio of interaction time of incident neutron and deuteron to the period of its nucleons motion will about $\sim 1/10$. It means that in 10 cases out of 100 in the reaction nd → p(nn) the deuteron should be considered as a whole.
Secondly, starting with the meson production threshold $\sim 290\text{MeV}$ the reactions of intermediate $\Delta$-resonance excitation are possible and these channels can change the form of three nucleons wave function in the final state. Thirdly, in our calculations the share of $D$-wave state inside deuteron ($\sim 4\%$) was neglected. But if the mixing of $S$ and $D$-waves carry out at once after the charge-exchange $d \rightarrow nn$ it should give an additional yield too.

The formula (42) does not depend from PSA solutions and it is an advantage in comparison with the formula (29) allowing to find the discrepancy between theory and experiment. Because the method of obtaining of formulas (29) and (41) is the same the additive of $12\%$ should be true for both of them. Therefore the calculated $R_{dp}$-values need taken with the factor of 1.12 anyway.

6 Conclusion

1. The problem of difference between two representations of elastic interaction of two identical particles is investigated. It leads to two equivalent but not identical expressions of wave function in the final state. The transition between these representations is given by the Majorana operator.

2. The method is proposed to calculation the Dean formula for quasi-elastic charge-exchange reaction $nd \rightarrow p(nn)$ where the wave function of proton and two neutrons in the final state is defined as a direct action of nucleon scattering matrix. Using alternative $np \rightarrow np$ backward scattering representation the new formula is established. The differential cross-sections of quasi-elastic $nd \rightarrow p(nn)$ and elastic $np \rightarrow pn$ scattering under zero angles should be related among themselves like $1:2$.

3. In the energy range $T_n = 0.55 \div 2.0\text{GeV}$ the experimental ratio $R_{dp}$ is like a constant on the level 0.56. It is more higher then calculated value $R_{dp} = 1/2$ by $12\%$ and related mainly with an accuracy of impulse approximation.

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A1 World Data

Table 2:
Ratios $R_{dp}$ and $r_{np\rightarrow pn}^{nf/ff}$ and their errors.
Experiment Delta Sigma, 2003-07 [8, 9, 10, 20, 21, 22].

| $T_n$, ГэВ | 0.55 | 0.8 | 1.0 | 1.2 | 1.4 | 1.7 | 1.8 | 2.0 |
|-------------|------|-----|-----|-----|-----|-----|-----|-----|
| $R_{dp} (0)$ | 0.608 | 0.546 | 0.553 | 0.554 | 0.574 | 0.550 | 0.584 | 0.557 |
| $\varepsilon$ | 0.035 | 0.024 | 0.012 | 0.010 | 0.027 | 0.034 | 0.024 | 0.024 |
| $r_{np\rightarrow pn}^{nf/ff} (0)$ | 0.097 | 0.222 | 0.204 | 0.204 | 0.162 | 0.155 | 0.142 | 0.197 |
| $\varepsilon$ | 0.062 | 0.053 | 0.026 | 0.023 | 0.054 | 0.074 | 0.046 | 0.052 |

Table 3:
World experimental data on ratios $R_{dp}$ and $r_{np\rightarrow pn}^{nf/ff}$.

| $T_{kin}$, МэВ | $R_{dp} (0)$ | $r_{np\rightarrow pn}^{nf/ff} (0)$ | Laboratory | Year, Ref |
|---------------|--------------|-------------------------------|------------|----------|
| 90            | 0.397 ± 0.044 | 0.679 ± 0.186 | UCRL       | 1951, [29] |
| 95            | 0.480 ± 0.030 | 0.389 ± 0.087 | Harvard U. | 1953, [30] |
| 96            | 0.587 ± 0.029 | 0.136 ± 0.056 | Harwell    | 1967, [31] |
| 135           | 0.652 ± 0.154 | 0.022 ± 0.241 | Harwell    | 1965, [32] |
| 144           | 0.601 ± 0.057 | 0.109 ± 0.105 | Harwell    | 1967, [31] |
| 152           | 0.650 ± 0.100 | 0.026 ± 0.158 | Harvard U. | 1966, [30] |
| 200           | 0.553 ± 0.030 | 0.205 ± 0.065 | JINR LNP  | 1962, [33] |
| 270           | 0.710 ± 0.021 | −0.061 ± 0.278 | UCRL       | 1952, [34] |
| 380           | 0.200 ± 0.035 | 2.333 ± 0.583 | INP Dubna  | 1955, [35] |
| 647           | 0.600 ± 0.080 | 0.111 ± 0.148 | LAMPF      | 1976, [36] |
| 710           | 0.483 ± 0.080 | 0.380 ± 0.229 | LRL        | 1960, [37] |
| 794           | 0.560 ± 0.040 | 0.190 ± 0.085 | LAMPF      | 1978, [38] |
| 800           | 0.660 ± 0.080 | 0.010 ± 0.122 | LAMPF      | 1978, [36] |
| 997           | 0.550 ± 0.080 | 0.212 ± 0.176 | JINR LHE   | 2002, [39] |
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