HIGHER TWIST EFFECTS IN NUCLEI

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Abstract

This talk serves as an introduction to higher twist effects in nuclei. We want to discuss how perturbative QCD can be applied to processes involving heavy nuclei by taking into account multiple scattering.

1 Introduction

Perturbative Quantum Chromodynamics (pQCD) has been established as an extremely successful theory to describe phenomena related to scattering reactions off hadrons at large momentum transfer. pQCD calculations combined with the available parameterizations of parton distributions enable us to explain a large set of data including those from deep inelastic lepton nucleon scattering (DIS) \( l + N \rightarrow l + X \) and the famous Drell Yan process (DY) \( N + N \rightarrow l^+ + l^- + X \).

pQCD is in fact a rather strict theory in the sense that no assumptions arising from any model enter. There are quantities of non-perturbative nature which cannot be described by perturbation theory, e.g. the bound states of QCD. Nevertheless there exists a quite rigorous way to separate perturbative (short range) and non-perturbative (long range) physics in a scattering reaction in a proper way. Let us consider DIS as an example. A factorization theorem [1] enables us to shift all non-perturbative physics into a set of well-defined, gauge-invariant (i.e. observable) and universal (i.e. process independent) quantities. These quantities can be expressed by matrix elements of parton operators between hadron states.

In principle an infinite number of such matrix elements could enter our calculation and spoil the usability of the factorization theorem. But we can establish a hierarchy between the matrix elements in terms of an expansion in inverse powers of the momentum transfer. In more detail the quantity we expand in is \( \lambda/Q \) where \( Q \) is the perturbative hard scale, e.g. the virtuality.
of the virtual photon in DIS, and \( \lambda \) (for massless QCD) has to be some non-perturbative (hadronic) scale. The perturbative scale \( Q \) makes the coupling running and has to be large (at least a few GeV) in order to make perturbation theory applicable while \( \lambda \) is of the order of \( \Lambda_{QCD} \approx 200 \text{ MeV} \). Under this circumstances \( \lambda/Q \ll 1 \) and this expansion, called the *twist* expansion, will do well. This is the definition of twist we will use here.

The factorization theorem tells us that the left diagram in Fig. 1 gives the leading contribution to DIS in the twist expansion (and also in the \( \alpha_s \) expansion).

![Fig. 1: Left: leading twist contribution to deep inelastic lepton nucleon scattering. The virtual photon emitted from the lepton (not shown) scatters off one quark inside the nucleon. Right: leading twist contribution to Drell Yan pair production in nucleon nucleon collisions. Each nucleon emits a quark or antiquark which annihilate into a virtual photon, which then decays into a lepton pair (not shown).](image)

Note that the leading twist contribution always consists of one hard scattering on the parton level, here the scattering of the photon off one quark from the nucleon. The non-perturbative part is described by a matrix element which encodes the process of taking one quark out of the nucleon and putting it back (in the complex conjugated graph). It is given by

\[
f_q(\xi) = \int \frac{dy^-}{2\pi} e^{i\xi P^+ y^-} \frac{1}{2} \langle P | \bar{q}(0) \gamma^+ q(y^-) | P \rangle ,
\]

where \( q \) are quark field operators and \( P \) is the momentum of the nucleon. The Bjorken variable \( \xi \) gives the momentum fraction of the nucleon that is carried by the quark in an infinite momentum frame. \( f_q(\xi) \) is then just the famous quark distribution function in the nucleon. It has a parton model interpretation as the probability to find a quark with momentum fraction \( \xi \) in the nucleon. This leading twist contribution to DIS is also called twist-2. The next order (next-to-leading twist) would involve matrix elements of twist-4,

\footnote{We are not going to discuss the subtleties arising from the fact that we have two expansions, one in \( \lambda/Q \) and one in \( \alpha_s \sim \ln \lambda/Q \), which are obviously not independent.}
i.e. $\mathcal{O}(\lambda^2/Q^2)$. The cross section for DIS can then be written as a convolution of a parton distribution $f_a$ with a parton cross section $\sigma_{l+a}$ which itself is a series in $\alpha_s$ plus power corrections which arise from matrix elements which differ from parton distributions.

$$\sigma_{l+p} = \sum_a f_a \otimes \sigma_{l+a} + \mathcal{O} \left( \frac{\lambda^2}{Q^2} \right)$$  \hspace{1cm} (2)

For the Drell Yan process also a factorization theorem holds which gives the leading twist contribution as a simple annihilation of quarks and antiquarks to produce a virtual photon, see Fig. 1 (right). Note again that leading twist (twist-2) involves only one hard scattering on the parton level (the annihilation), while two matrix elements, one for each nucleon, are in the game. These matrix elements define exactly the same parton distributions as in DIS. This powerful property is called universality. Measuring a parton distribution in one process gives us predictive power for all other processes where this parton distribution enters.

## 2 Multiple Scattering and Nuclear Enhancement

What changes if we replace single hadrons by nuclei and look at $e + A$, $p + A$ or $A + A$ collisions? The factorization theorems should hold also here, but for many observables the picture of one hard scattering without rescattering on the parton level seems not to be the dominant one. E.g. for Drell Yan we expect initial state interactions when the quark and the antiquark participating in the annihilation have to traverse a large piece of nuclear matter. In the framework of pQCD multiple scatterings are exactly higher twist corrections to the single scattering (leading twist) process.

At a given scale $Q^2$ the power corrections by higher twist effects are enhanced because of the nuclear size. This enhancement cannot stem from the hard (short range) part of the cross section, but must arise from the non-perturbative (long range) matrix elements. We will demonstrate later how the matrix elements can test the nuclear size.

It was an idea mainly advocated by Luo, Qiu and Sterman \[2\] that, e.g. on the twist-4 level, there are matrix elements which scale like $\lambda^2 A^{1/3}/Q^2$ where $A$ is the mass number of the nucleus, whereas there are others which are not sensitive to the nuclear size and behave just like $\lambda^2/Q^2$ as usual. For large nuclei we can assume that $A^{1/3} \gg 1$ and conclude that only those higher twist corrections, which show an additional scaling with the nuclear size, called nuclear
enhanced, are important and we safely omit all others. We rearrange our twist series in such a way, that we have an expansion in powers of $\lambda^2 A^{1/3}/Q^2$, i.e. for each power of $Q^{-2}$ we have an additional power of $A^{1/3}$ (maximal nuclear enhancement). Terms with less powers in $A^{1/3}$ than in $Q^{-2}$ are subleading.

Twist corrections normally are a tricky business and very difficult to handle. Starting from the parton distribution in Eq. (1) we can e.g. generate higher twist matrix elements just by inserting covariant derivatives acting on the parton fields. Unfortunately the number of matrix elements contributing at a certain level of twist can be large and in addition most of these matrix elements have no probabilistic interpretation in a parton picture.

This changes dramatically if we demand that a matrix element should be sensible to the size of an extended medium. Extended here means that the extension $L$ should be much larger than the confinement radius $R_0$, $L \gg R_0$, what is true for large nuclei. One can show that only matrix elements with additional pairs of parton operators can contribute. For twist-4 schematically we have e.g. $\langle P | (\bar{q}q)(FF) | P \rangle$ or $\langle P | (FF)(FF) | P \rangle$, where color, spinor and vector indices for each pair are contracted in the same way as in the parton distribution respectively. $F$ here is a gluon field strength. Therefore these matrix elements are limited in number and have a straight forward interpretation as correlators between partons.

In Fig. 2 we give an example for single and double scattering in the DY process at large transverse momentum. Here the quark (antiquark) from the single hadron has the opportunity to scatter off an additional gluon from the nucleus before annihilating with an antiquark (quark).

![Fig. 2: Possible diagrams for single scattering (left) and double scattering (right) contributing to DY pair production in $p + A$ collisions at high transverse momentum. Note that both diagrams are next-to-leading order in $\alpha_s$ respectively, since we radiate an extra gluon in order to generate finite transverse momentum.](image)

The onset of nuclear enhancement can be seen in $p + A$ experiments. The best known example is the famous Cronin effect [3, 4], which is a nuclear enhancement of the production cross section of Drell Yan pairs, $J/\psi$s or other particles at large transverse momentum, whereas at low transverse momentum enhancement is missing. The enhancement is a clear sign of multiple
scattering. Another sign is the transverse momentum broadening of jets or Drell Yan pairs. Fermilab data indicate a rise of the ratio of second and first moment of the transverse momentum spectrum which is compatible with an $A^{1/3}$ behaviour \[^5\].

Let us emphasize that single scattering contributions in principle scale with $A$, i.e. just they scale with the volume the reaction can take place. The nuclear enhanced double scattering picks up an additional power of the nuclear radius and therefore scales like $A^{4/3}$ and so on. With this mind one would expect an overall enhancement of all cross sections but experimentally this is clearly not the case. Indeed most cross sections are not enhanced like e.g. DY at low transverse momenta. If pQCD intends to explain the enhancement by the introduction of new matrix elements, it should also account for the absence of enhancement in other observables. Again this works fine. We will see that interference seems to be very important. This can spoil the enhancement in certain observables.

3 An example: double scattering in Drell Yan

Let us discuss the Drell Yan process in $p + A$ as an example. We would like to calculate the effect of double scattering at large transverse momentum \[^6\], \[^7\], \[^8\].

The DY cross section with full kinematic dependence on Mass $Q$, transverse momentum $q_\perp$ and rapidity $y$ of the lepton pair and on the angular distribution of the lepton pair can be decomposed into four structures parametrized by four helicity amplitudes $W_{TL}$, $W_L$, $W_\Delta$ and $W_{\Delta\Delta}$.

\[
\frac{d\sigma}{dQ^2 dq_\perp^2 dy d\Omega} = \frac{\alpha^2_{em}}{64\pi^3 S Q^2} \left( W_{TL} \left( 1 + \cos^2 \theta \right) + W_L \left( \frac{1}{2} - \frac{3}{2} \cos^2 \theta \right) \right) + W_\Delta \left( \sin 2\theta \cos \phi \right) + W_{\Delta\Delta} \left( \sin^2 \theta \cos 2\phi \right) \right)
\]  

(3)

If we integrate over the angles, only the term proportional to $W_{TL}$ contributes to the cross section. Fig. 3 shows two typical examples for graphs for twist-4. At this order of twist we not only have to take into account double scattering but also interference of single and triple scattering (which we will include into the term double scattering in the following since the matrix elements are the same).
Fig. 3: Two diagrams contributing to twist-4 at large transverse momentum. The left diagram shows an ordinary double scattering with a quark gluon pair from the nucleus on the left and right (complex conjugated) part of the diagram. The right diagram shows a possible interference between triple scattering (two gluons and one quark on the left part) and single scattering (one quark on the right side).

It turns out that there are two different contributions to double scattering. Both partons from the nucleus can be hard (double hard scattering) or one can be hard and one soft (soft hard scattering). The final formulae for the helicity amplitudes $W_i$ including the partons $a$ and $b$ from the nucleus and parton $c$ from the single nucleon are

$$ W_{iDH,ab+c}^i = \int_B \frac{d\xi_2}{\xi_2} f_c(\xi_2) T_{ab}^{DH}(x_a, x_h) H_{iDH,ab+c}(x_a, x_h, \xi_2), \quad (4) $$

$$ W_{iSH,ab+c}^i = \int_B \frac{d\xi_2}{\xi_2} f_c(\xi_2) \left( -\frac{g^{\lambda\kappa}}{2} \frac{d^2}{dK^\lambda dK^\kappa} \right)_{K_\perp=0} T_{ab}^{SH}(x_b) H_{iSH,ab+c}(x_b, x_s, \xi_2) \quad (5) $$

for double hard and soft hard scattering respectively. $f_c$ are the usual parton distributions for the nucleon, $H^{ab+c}$ is the cross section on the parton level (the hard part of the cross section) and the $T_{ab}$ are the new matrix elements describing two partons in the nucleus.

Let us shortly discuss where the distinction between soft hard and double hard arises from. We have to integrate over longitudinal momenta of parton lines connecting soft and hard parts of the diagram and we have propagators in the hard part which can provide poles in the integrands. An example is given in Fig. 4. Both propagators marked by circles give poles for the integration of the gluon momentum fraction $x$. 
The upper pole fixes $x$ to a large (finite) value $x_{\text{hard}}$, whereas the lower pole gives $x_{\text{soft}} \sim K^2 / S \sim 0$ where $K_\perp$ is the intrinsic transverse momentum of the gluon. Carrying out the pole integration gives a sum of both residues:

$$M \sim \int dx \left( \frac{1}{x - x_{\text{soft}} + i\epsilon} \right) \left( \frac{1}{x - x_{\text{hard}} + i\epsilon} \right) F(x, k_t, \xi_1)$$

$$= \left( \frac{F(x_{\text{soft}}, k_t, x_{\text{tot}} - x_{\text{soft}})}{x_{\text{soft}} - x_{\text{hard}}} \right) - \left( \frac{F(x_{\text{hard}}, k_t, x_{\text{tot}} - x_{\text{hard}})}{x_{\text{soft}} - x_{\text{hard}}} \right)$$

$$= M_{\text{soft,hard}} - M_{\text{hard,hard}} \quad (6)$$

Note the minus sign between both terms. To get the cross section we have to square the term above. The hierarchy $x_{\text{hard}} \gg x_{\text{soft}}$ can only be ensured as long as the transverse momentum $q_\perp$ is large. In this case the interference between both residues is negligible and the cross section is given just by the sum of squares $M^2_{\text{soft,hard}} + M^2_{\text{hard,hard}}$. The first term is the soft hard contribution (soft gluon) the second term is the double hard contribution (hard gluon).

However if $q^2_\perp \ll Q^2$ then $x_{\text{hard}}$ starts to approach $x_{\text{soft}}$ and both residues start to cancel each other. This is the explanation of the important statement already given above: for small transverse momenta the double scattering contribution undergoes destructive interference and there is no nuclear enhancement from double scattering in that kinematic region.

Let us now discuss the twist-4 matrix elements. The double hard matrix elements depend on the momentum fractions $\xi$ and $x$ of both hard partons. The quark gluon correlator e.g. is

$$T_{qg}^{\text{DH}}(\xi, x) = \frac{1}{x} \int \frac{dz_1}{2\pi} \frac{dz_3}{2\pi} \Theta(z_1 - z_3) \Theta(-z_4) \Theta(z_1 - z_3) \Theta(-z_4) \quad (7)$$

$$e^{i\xi P_1^+ z_1^-} e^{i\eta P_1^+(z_3^- - z_4^-)} \frac{1}{2} \langle P_1 \mid F^{\omega_+}(z_4^-) F^{\omega}(z_3^-) \bar{q}(0) \gamma^\mu q(z_1^-) \mid P_1 \rangle .$$

The same correlator for soft hard scattering reads

$$T_{qg}^{\text{SH}}(\xi) = \int \frac{dz_1}{2\pi} \frac{dz_3}{2\pi} \Theta(z_1 - z_3) \Theta(-z_4) \quad (8)$$
\[ e^{i\xi P^1 z_1^-} \frac{1}{2} \langle P_1 | F^{\omega^+} (z_4^-) F^{\omega} (z_3^-) \gamma^+ q(0) | P_1 \rangle. \]

It depends only on the momentum of the hard parton. The \( \Theta \)-functions ensure causality. PQCD is not able to predict these matrix elements from first principles and they are not measured up to now. In order to make any numerical statements about the size of the nuclear enhanced corrections we have to rely on models at this stage. However the important point is, we can extract the scaling with the nuclear size. To do this we have to take into account the colour structure of the operators, the oscillating exponential factors and the boundaries of the integrals. An analysis gives that both matrix elements above have one free integration which can test the extension of the nucleus. Pictorially this is the distance between the two parton pairs \( \bar{q}q \leftrightarrow FF \). From that we assume that the matrix elements can be modelled by ordinary parton distributions by setting

\[ T^{DH}_{ab} (\xi, x) = CA^{4/3} f_a (\xi) f_b (x), \quad (9) \]
\[ T^{SH}_{ab} (\xi) = \lambda^2 A^{4/3} f_a (\xi). \quad (10) \]

\( C \) and \( \lambda^2 \) are normalization constants of the order of \( \Lambda_{QCD} \).

At the moment our goal must be to gather experimental information about the new matrix elements and therefore our study for RHIC \[4, 5\] intends to look for observables where this can be done. Independent of the models which one plugs in for the matrix elements one can establish some results: Double hard scattering has trivial angular dependence like the leading twist, leading \( \alpha_s \) process. It seems to confirm the picture of two independent binary collisions, first \( q + g \rightarrow q + g \) and then \( q + \bar{q} \rightarrow \gamma^* \). Confer the right diagram of Fig. 4: the pole splits the diagram into two independent subprocesses. Also the so called Lam Tung sum rule \( 2W_{\Delta \Delta} = W_L \), a long standing leading twist prediction \[4\], is respected by double hard scattering. On the other hand the results on soft hard scattering are more complicated and violate the Lam Tung relation. Another important point is that the leading twist result for \( W_\Delta \) is nearly zero (it vanishes for \( p + p \) from symmetry reasons), while there is a non vanishing twist-4 contribution. These are the most important model independent results which could allow a glance at the new matrix elements in forthcoming data.

At the end of this section we show two results for proton gold collisions at RHIC. In Fig. 5 the rapidity distributions of the amplitudes \( W_{\tau L} \) and \( W_\Delta \) are given. Note that at RHIC energies soft hard seems to be dominated by the double hard process.
Fig. 5: Results for 100 GeV Au + 250 GeV p collisions at RHIC. Cross sections for helicity channels $W_{TL}$ (left) and $W_{\Delta}$ (right) as a function of rapidity $y$ at $Q = 5$ GeV and $q_\perp = 4$ GeV. Single scattering (solid lines), double hard (dashed) and soft hard (dot dashed) for two different choices of parton distribution parameterizations (CTEQ3M and CTEQ3M+EKS98) are shown.

4 Further topics

The transverse momentum broadening of DY pairs was also explained recently by X. Guo within this formalism [10]. For this observable one is sensible to small transverse momenta and one has to calculate graphs like in Fig. 6 (left) with no additional parton in the final state. The ratio of second and first moment of the transverse momentum spectrum is surprisingly easy

$$\frac{\langle q_{\perp}^2 \rangle}{\langle q_{\perp}^0 \rangle} = \frac{4\pi^2\alpha_s}{3} f_c \otimes T_{ag}^{SH} \frac{4\pi^2\alpha_s}{3} \frac{A^{1/3} \chi^2}{f_a}$$

with partons $a$ and $c$ being quark and antiquark or vice versa. The last equal sign holds for our model for the soft hard matrix elements. One may note that the pQCD result indeed rises with $A^{1/3}$ compatible with the data.

In the low $q_\perp$ regime one would like to sum the effect of multiple scattering which would be a resummation of higher twist. Diagrams like in Fig. 6 (right) introduce matrix elements which are correlators with an arbitrary number $n$ of gluon fields $\langle P|\bar{q}q|(FF)^n|P \rangle$. If we extrapolate the model above we can establish connections between matrix elements of different $n$ and are able to carry out the summation.

Fig. 6: DY graphs with scattering off one (left) and $n$ (right) additional gluons from the nucleus.
For the transverse momentum spectrum we obtain a shift and smearing. The shift is given by $4\pi^2\alpha_s A^{1/3}\lambda^2/3$. Some other topics addressed recently include the medium modification of fragmentation functions \[11\] which can explain energy loss and the medium modification of parton distributions via higher twist modifications internal to the nucleus \[12\] which can account for shadowing.

Let us close with some remarks about the limitations of pQCD \[12\]. It is well known that QCD factorization breaks down in Drell Yan beyond the order of twist-4 \[13\]. Practically this means that there are non-factorizable contributions that make the twist expansion no longer reasonable beyond twist-4. However the nuclear enhancement argument saves us for $p + A$ collisions since it allows us to stay at twist-2 for the single proton. In that case we can go to arbitrary twist for the nucleus, like we can do also for $e + A$. For $A + A$ however the problem cannot be waived and we are limited to twist-4 accuracy, which this still gives us the leading medium effect.

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