Finite size effect on the magnon’s correlation functions

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ABSTRACT

We calculate the finite size correction on the three-point correlation function between two giant magnons and one marginal operator. We also check that the structure constant in the string set-up is exactly the same as one of the RG analysis in the gauge theory.

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1 Introduction

The conformal field theory (CFT) is characterized by the conformal dimension of all primary operators and the structure constant included in the three-point correlation functions, because higher point functions may be determined by using the operator product expansion (OPE). The $\mathcal{N} = 4$ super Yang-Mills (SYM) theory in four-dimensional space is an important example to investigate the interacting CFT [1]. After it was shown that there exists an integrable structure in $\mathcal{N} = 4$ SYM theory [2,3,4,5], there was a great progress in finding the spectrum of this theory [6]-[25]. These studies were extended to the ABJM model corresponding to the low energy theory of M-theory or IIA-string theory [26]-[43]. On the contrary, although the structure constant can be evaluated in the weak coupling limit of SYM by computing the Feynman diagrams, at the strong coupling there still remain many things to be done.

After the proposition of the method calculating the three-point correlation function between two heavy operators and one marginal operator [44], there were many interesting works calculating two- and three-point correlation function semi-classically by using the known explicit solutions [45]-[56]. From these works, it was checked that two- and three-point correlation function in the string theory are exactly consistent with the RG calculation in the dual SYM. Another important property of this dual gauge theory is the wrapping effect [57,11], which provided a clue for the all-loop Bethe ansatz. The wrapping effect of the spin chain model was studied by investigating the finite size effect in the dispersion relation of the giant magnon in the string theory. In this paper, as the generalization of Ref. [58,59], we are going to investigate the finite size effect of the three-point correlation function between two magnons and one marginal operator and then compare this result with the structure constant of the dual gauge theory obtained by the RG analysis. Finally, we will show that the finite size effect on the three-point correlation function calculated in the string theory are exactly matched with ones obtained in the gauge theory, which may provide another evident example for the AdS/CFT correspondence.

The rest part is organized as follows. In Sec. 2, we will investigate the finite size correction on
the two-point correlation function of the giant magnon by using the saddle point approximation. In Sec. 3, we will calculate the finite size effect of the three-point correlation function between two magnons and one marginal operator. Finally, we will finish our work with a brief discussion in Sec.4.

**Note added** At the final stage of this work, we noticed that there were overlaps in Ref. [60], in which the different method, so called the Neumann-Rosochatius reduction, with the known solution was used.

## 2 Finite size effect on the dispersion relation of the giant magnon

Consider a solitonic string moving in the $AdS_5 \times S^2$, which is a subspace of $AdS_5 \times S^5$. In the Euclidean Poincare patch

$$ ds^2_{AdS} = \frac{1}{z^2} \left( dz^2 + dx^2 \right), $$

the string solution can be described by a point-particle moving in $AdS$. Especially, in the conformal gauge the integration over the string worldsheet is reduced to the integration over the modular parameter $s$ of the cylinder

$$ \int d^2 \sigma \rightarrow \int_{s/2}^{s/2} d\tau \int_{-L}^{L} d\sigma, $$

where we concentrate on the magnon solution and $\pm L$ imply two ends of the string worldsheet. Notice that the giant magnon, which is dual of the magnon on the open spin chain in the dual gauge theory, is described by the worldsheet solitonic solution on the open string in which one should give up the level matching condition [8, 11].

The string action on $AdS_5 \times S^2$ is given by [34, 36]

$$ S_{st} = \int d^2 \sigma \mathcal{L}_{st} $$

$$ = -\frac{T}{2} \int d^2 \sigma \left[ -\frac{(\partial_\tau x)^2 + (\partial_\tau z)^2}{z^2} - (\partial_\tau \theta)^2 + (\partial_\sigma \theta)^2 - \sin^2 \theta \left\{ (\partial_\tau \phi)^2 - (\partial_\sigma \phi)^2 \right\} \right], $$

where $\frac{T}{2}$ is a string tension, $T = \frac{\sqrt{\lambda}}{2\pi}$ for $AdS_5 \times S^5$. The solutions of the equations of motion for $AdS$ coordinates, $z(\tau)$ and $x(\tau)$ are given by

$$ z(\tau) = \frac{R}{\cosh \kappa \tau}, $$

$$ x(\tau) = R \tanh \kappa \tau + x_0, $$

(4)
which is the specific parameterization of a geodesic in \( AdS \), \((x(\tau) - x_0)^2 + z(\tau)^2 = R^2 \). From these solutions, the action of the \( AdS \) part simplifies to

\[
\frac{T}{2} \int_{-s/2}^{s/2} d\tau \int_{-L}^{L} d\sigma \frac{(\partial_\tau x)^2 + (\partial_\tau z)^2}{z^2} = \frac{T}{2} \int_{-s/2}^{s/2} d\tau \int_{-L}^{L} d\sigma \kappa^2.
\]

(5)

Imposing the boundary conditions

\[(x(-s/2), z(-s/2)) = (0, \epsilon) \quad \text{and} \quad (x(s/2), z(s/2)) = (x_f, \epsilon),\]

(6)

in which \( \epsilon \) is very small and corresponds to an appropriate UV cut-off, we can find a relation between \( \kappa \) and \( x_f \)

\[
\kappa \approx \frac{2}{s} \log \frac{x_f}{\epsilon},
\]

(7)

with \( x_f \approx 2R \approx 2x_0 \).

Now, consider the equations of motion for \( S^2 \) coordinates. Under the following parameterization

\[
\theta = \theta(y), \quad \phi = \nu \tau + g(y), \quad \text{and} \quad y = a\tau + b\sigma,
\]

(8)

the equations of motion for \( \phi \) reads off

\[
0 = \partial_y \left\{ \sin^2 \theta \left( a\nu + (a^2 - b^2)g' \right) \right\},
\]

(9)

where the prime means the derivative with respect to \( y \). So \( g' \) can be rewritten in terms of \( \theta \) as

\[
g' = \frac{1}{b^2 - a^2} \left( a\nu - \frac{c}{\sin^2 \theta} \right),
\]

(10)

where \( c \) is an integration constant. The equation of motion for \( \theta \) after multiplying \( 2\theta' \) can be rewritten as the following form

\[
0 = \partial_y \left( \theta'^2 + \frac{b^2\nu^2}{(b^2 - a^2)^2} \sin^2 \theta + \frac{c^2}{(b^2 - a^2)^2 \sin^2 \theta} \right).
\]

(11)

From the above, we can also rewrite \( \theta' \) in terms of \( \theta \)

\[
\theta'^2 = \frac{b^2\nu^2}{(b^2 - a^2)^2 \sin^2 \theta} \left[ -\sin^4 \theta + \frac{w^2}{b^2\nu^2} \sin^2 \theta - \frac{c^2}{b^2\nu^2} \right].
\]

(12)

where \( \frac{w^2}{(b^2 - a^2)^2} \) is introduced as another integration constant. If we solve (10) and (12), we should introduce two additional constants, which fix the position of the giant magnon on \( S^2 \).

The integration constants, \( c \) and \( w \), in the above determine the velocity of the giant magnon in \( \theta \)- and \( \phi \)-directions. As a result, totally four integration constants appear in the exact solution. However, since we are interested in the magnon’s dispersion relation, which is described by
the conserved charges including one derivative, the additional two integration constants are irrelevant.

Now, we determine two integrations constants, $c$ and $w$, by imposing the appropriate boundary conditions. First, we impose that $\theta$ has a maximum value $\theta_{\text{max}}$ at which $\theta'$ is zero. In the infinite size limit $\theta_{\text{max}} = \pi/2$, this boundary condition makes the giant magnon have the infinite energy and angular momentum, which is the typical structure of the magnon’s dispersion relation. By imposing this boundary condition, (12) can be rewritten as

$$\theta'^2 = \frac{b^2 \nu^2}{(b^2 - a^2)^2 \sin^2 \theta} \left[ (\sin^2 \theta_{\text{max}} - \sin^2 \theta) (\sin^2 \theta - \sin^2 \theta_{\text{min}}) \right],$$

with

$$\sin^2 \theta_{\text{max}} + \sin^2 \theta_{\text{min}} = \frac{w^2}{b^2 \nu^2},$$
$$\sin^2 \theta_{\text{max}} \sin^2 \theta_{\text{min}} = \frac{c^2}{b^2 \nu^2}. \quad (14)$$

Another important structure of the magnon’s dispersion relation in the infinite size limit is that the difference between the energy and angular momentum of magnon is finite, which can be achieved by imposing the second boundary condition, $\partial_{\sigma} \phi = 0$ at $\theta = \theta_{\text{max}}$. For the finite size case, we can also apply these two boundary conditions to determine the magnon’s dispersion relation. These two boundary conditions fix two integration constants, $c$ and $w$, as

$$\sin^2 \theta_{\text{max}} = \frac{c}{\nu a} \quad \text{and} \quad \sin^2 \theta_{\text{min}} = \frac{ac}{b^2 \nu},$$

with

$$w^2 = \frac{c \nu (a^2 + b^2)}{a}. \quad (16)$$

Following the prescription of Ref. [44], we should consider the evolution of the wave function. Then, the new action $\tilde{S}$ including the convolution with the wave function is given by

$$\tilde{S} \equiv S - \int d^2 \sigma \Pi_{\theta} \dot{\theta} - \int d^2 \sigma \Pi_{\phi} \dot{\phi} = -\frac{T}{2} \int_{s/2}^{s/2} d\tau \int_{-L}^{L} d\sigma \rho^2,$$

with

$$\rho \equiv \sqrt{\frac{\nu c}{a}}, \quad (18)$$

where $2L$ is the length of the worldsheet string. Using this action on $S^2$ together with (5) on $AdS_5$, the total action for the magnon is given by

$$iS_{\text{tot}} = i \left( S_{\text{AdS}} + \tilde{S} \right) = i \left( \frac{4}{3 \pi} \log \frac{x_f}{\epsilon} - \rho^2 \right) sLT. \quad (19)$$
From this total action, the saddle point of the modular parameter $s$ reads

$$\bar{s} = -\frac{2i}{\rho} \log \frac{x_f}{\epsilon},$$

which corresponds to the Virasoro constraint for the einbein. At this saddle point, $\kappa$ and $\rho$ are related by $\kappa = i\rho$ and the semi-classical partition function of the giant magnon becomes

$$e^{iS_{\text{tot}}} = \left(\frac{\epsilon}{x_f}\right)^{2E},$$

where $E$ corresponds to the magnon’s energy. Notice that the form of the semi-classical partition function for the finite size giant magnon is the same as the result of the infinite one. So, the finite size effect comes from the definition of the conserved charges. The corresponding conserved charges are given by

$$E = 2T \frac{z_{\text{max}}^2 - z_{\text{min}}^2}{z_{\text{max}} \sqrt{1 - z_{\text{min}}^2}} K(x),$$

$$J = 2T z_{\text{max}} \left[K(x) - E(x)\right],$$

$$\frac{\Delta \phi}{2} = \sqrt{1 - z_{\text{min}}^2} \Pi \left(\frac{z_{\text{max}}^2 - z_{\text{min}}^2}{\sqrt{z_{\text{max}}^2 - 1}}; x\right) - \sqrt{1 - z_{\text{max}}^2} K(x),$$

with the elliptic integrals of the first, second and third kinds

$$K(x) = \int_{z_{\text{min}}}^{z_{\text{max}}} dz \frac{z_{\text{max}}}{\sqrt{(z_{\text{max}}^2 - z^2)(z^2 - z_{\text{min}}^2)}},$$

$$E(x) = \int_{z_{\text{min}}}^{z_{\text{max}}} dz \frac{z^2}{z_{\text{max}} \sqrt{(z_{\text{max}}^2 - z^2)(z^2 - z_{\text{min}}^2)}},$$

$$\Pi \left(\frac{z_{\text{max}}^2 - z_{\text{min}}^2}{\sqrt{z_{\text{max}}^2 - 1}}; x\right) = \int_{z_{\text{min}}}^{z_{\text{max}}} dz \frac{z_{\text{max}}(1 - z_{\text{max}}^2)}{(1 - z^2) \sqrt{(z_{\text{max}}^2 - z^2)(z^2 - z_{\text{min}}^2)}},$$

where $z = \cos \theta$, $z_{\text{max}}^2 \equiv \cos^2 \theta_{\text{min}}$, $z_{\text{min}}^2 \equiv \cos^2 \theta_{\text{max}}$ and $x = \sqrt{1 - \frac{z_{\text{min}}^2}{z_{\text{max}}^2}}$. In the case of the large but finite angular momentum, $J/T \gg 1$, since $z_{\text{min}}$ is very small, we can expand the elliptic integrals around the limit $z_{\text{min}} \to 0$

$$K(x) = \log \left(4 \frac{z_{\text{max}}}{z_{\text{min}}}\right) + \frac{1}{4} \frac{z_{\text{min}}^2}{z_{\text{max}}^2} \left[\log \left(4 \frac{z_{\text{max}}}{z_{\text{min}}}\right) - 1\right] + \cdots,$$

$$E(x) = 1 + \frac{1}{4} \frac{z_{\text{min}}^2}{z_{\text{max}}^2} \left[2 \log \left(4 \frac{z_{\text{max}}}{z_{\text{min}}}\right) - 1\right] + \cdots,$$
\[
\Pi\left(\frac{z_{\text{max}}^2 - z_{\text{min}}^2}{\sqrt{z_{\text{max}}^2 - 1}}; x\right) = (1 - z_{\text{max}}^2) \left[ \log \left( \frac{z_{\text{max}}}{z_{\text{min}}} \right) + \frac{1}{4} \frac{z_{\text{min}}}{z_{\text{max}}} \left( 2z_{\text{max}}^2 + 1 \right) \log \left( \frac{z_{\text{max}}}{z_{\text{min}}} \right) - \left( z_{\text{max}}^2 + 1 \right) \right] + \left(1 + \frac{1}{2} z_{\text{min}}^2\right) z_{\text{max}} \sqrt{1 - z_{\text{max}}^2} \arcsin z_{\text{max}} + \cdots.
\]

(26)

From the above, \(z_{\text{max}}\) and \(\log \left( \frac{4z_{\text{max}}}{z_{\text{min}}} \right)\) can be rewritten in terms of \(J\) and \(p\) up to \(z_{\text{min}}^2\) order as

\[
\begin{align*}
\frac{z_{\text{max}}}{z_{\text{min}}} &= \sin \frac{p}{2} + \frac{1}{4} \left( 1 - \sin^2 \frac{p}{2} \right) \left( \frac{J}{T \sin \frac{p}{2}} + 3 \right) \frac{z_{\text{min}}^2}{2} \\
\log \left( \frac{4z_{\text{max}}}{z_{\text{min}}} \right) &= \frac{J}{2T \sin \frac{p}{2}} + 1 + \left[ \frac{1}{4} \frac{z_{\text{min}}^2}{z_{\text{max}}} - \frac{J}{8T \sin^3 \frac{p}{2}} \left( \frac{2}{3} \frac{J}{T \sin \frac{p}{2}} - 3 \sin^2 \frac{p}{2} - \frac{J \sin \frac{p}{2}}{T} \right) \right] \frac{z_{\text{min}}^2}{2}.
\end{align*}
\]

(27)

(28)

Furthermore, since \(z_{\text{min}}\) is already the small value, from (28) it is given at the leading order by

\[
\frac{z_{\text{min}}}{2} = 4 \sin \frac{p}{2} e^{-\frac{J}{2T \sin \frac{p}{2}}},
\]

(29)

where the small correction proportional to \(T/J\) in the exponent is ignored because we consider the large angular momentum limit, \(J \gg T\). Using these results, we can write \(E - J\) in terms of \(J\) and \(p\) up to \(z_{\text{min}}^2\) order

\[
E - J = 2T \left[ \frac{z_{\text{max}}^2 - z_{\text{min}}^2}{z_{\text{max}} \sqrt{1 - z_{\text{min}}^2}} K(x) - z_{\text{max}} \left( K(x) - E(x) \right) \right] = 2T \sin \frac{p}{2} - 8T \sin^3 \frac{p}{2} e^{-\frac{J}{T \sin \frac{p}{2}}},
\]

(30)

which is the same as the result in Ref. [11]. If we take the limit \(J \to \infty\), we can ignore the second term, so the above is reduced to the dispersion relation of the giant magnon in the infinite size limit.

3 Finite size effect on the three-point correlation function

In this section, we will investigate the finite size effect on the three-point correlation function between two magnons and one marginal scalar operator. To calculate this correlation function in the string set-up, we introduce a massless scalar field which is dual to the marginal scalar operator. The bulk-to-boundary propagator of a massless scalar field \(\chi\) in \(AdS\) is given by [61]

\[
K_\chi(x^\mu, z; y^\nu) = \frac{6}{\pi^2} \left( \frac{z}{z^2 + (x - y)^2} \right)^4.
\]

(31)
Then, the three-point function between two magnon operators denoted by $O_m$ and marginal scalar operator $D_\chi$ is given by

$$\langle O_m(0)O_m(x_f)D_\chi(y) \rangle \approx \frac{I_\chi[X(s,y)]}{|x_f|^{2E}}, \quad (32)$$

with

$$I_\chi[X, s; y] = i \int_{-s/2}^{s/2} d\tau \int_{-L}^{L} d\sigma \left. \frac{\delta S_p[X, s, \chi]}{\delta \chi} \right|_{\chi=0} K_\chi(X(\tau, \sigma); y), \quad (33)$$

where $\chi$ corresponds to the massless dilaton fluctuation and $S_p[X, s, \chi]$ represents the Polyakov action including the dilaton fluctuation

$$S_p[X, s, \chi] = -\frac{T}{2} \int d^2\sigma \sqrt{-\gamma} \gamma^\alpha_\beta \partial_\alpha X^A \partial_\beta X^B G_{AB} e^{\chi/2} + \cdots. \quad (34)$$

For the magnon case, $I_\chi[X, s; y]$ becomes

$$I_\chi[X, s; y] = i \frac{3T}{2\pi^2} \int d^2\sigma L_{st} \times \left( \frac{z}{z^2 + (x - y)^2} \right)^4, \quad (35)$$

where $L_{st}$ corresponds to the Polyakov action in the absence of the dilaton field, which is given in (3). Inserting solutions obtained in the previous section, the above integration is reduced to

$$I_\chi[X, s; y] = -\frac{6T}{\pi^2} \rho \frac{z_{\min}^{2} E(x) - z_{\min}^{2} K(x)}{z_{\max} \sqrt{1 - z_{\min}^{2}}} \int_{-s/2}^{s/2} d\tau \left( \frac{z}{z^2 + (x - y)^2} \right)^4. \quad (37)$$

For $\kappa s \gg 1$, performing the $\tau$-integration using the solution in (4) gives at the leading order

$$\int_{-s/2}^{s/2} d\tau \left( \frac{z}{z^2 + (x - y)^2} \right)^4 = \frac{1}{12\rho} \frac{x_f^4}{y^4 (x_f - y)^4}. \quad (38)$$

Using these results, we can expand $I_\chi[X, s; y]$ up to $z_{\min}^{2}$ order as

$$I_\chi[X, s; y] = -\frac{T}{2\pi^2} \left[ \sin \frac{p}{2} - \frac{J}{4T} z_{\min}^{2} - \frac{1}{4} \sin \frac{p}{2} z_{\min}^{2} \right] \frac{x_f^4}{y^4 (x_f - y)^4}. \quad (39)$$
After substituting the above together with (29) into (32), we can finally find the three-point correlation function between two magnons and one marginal operator

\[
\langle O_m(0) O_m(x_f) D\chi(y) \rangle = \frac{1}{2\pi^2} \left[ -T \sin \frac{p}{2} + \left( 4J \sin^2 \frac{p}{2} + 4T \sin^3 \frac{p}{2} \right) e^{-\frac{J}{T \sin \frac{p}{2}}} \right] \frac{1}{x_f^{2E-4} y^4(x_f - y)^4}.
\]

So the structure constant \( a_{D, AA} \) in the string theory side reads off

\[
2\pi^2 a_{Dmm} = -T \sin \frac{p}{2} + \left( 4J \sin^2 \frac{p}{2} + 4T \sin^3 \frac{p}{2} \right) e^{-\frac{J}{T \sin \frac{p}{2}}} \quad (40)
\]

If taking \( J \to \infty \), the above is reduced to the structure constant between two infinite size magnons and one marginal operator. So the second term in (41) corresponds to the leading finite size correction in the large \( J/T \) limit.

In the dual conformal field theory, the three-point correlation function between two magnons and one marginal operator is given by

\[
\langle O_m(0) O_m(x_f) D\chi(y) \rangle = \frac{a_{Dmm}}{x_f^{2E-4} y^4(x_f - y)^4},
\]

where the denominator is fixed by the global conformal transformation. The unknown structure constant, which is not determined by the conformal symmetry, can be fixed by the formular obtained from the RG analysis

\[
a_{Dmm} = -g^2 \frac{\partial}{\partial g^2} \Delta = -\frac{T}{2} \frac{\partial}{\partial T} \Delta, \quad (43)
\]

where \( \Delta \) is the conformal dimension of the magnon and \( T = 2g \). Since at the large 't Hooft coupling regime the magnon’s conformal dimension is the same as the energy of the giant magnon in (30), the structure constant between two magnons having finite size and a marginal operator are given by

\[
a_{Dmm} = \frac{T}{2} \frac{\partial}{\partial T} \left( J + 2T \sin \frac{p}{2} - 8T \sin^3 \frac{p}{2} e^{-\frac{J}{T \sin \frac{p}{2}}} \right) = -T \sin \frac{p}{2} + \left( 4J \sin^2 \frac{p}{2} + 4T \sin^3 \frac{p}{2} \right) e^{-\frac{J}{T \sin \frac{p}{2}}} \quad (44)
\]

which is the same as the result obtained from the string calculation. Notice that though the angular momentum \( J \) in the string theory is proportional to the string tension \( T \), \( J \) in \( \mathcal{N} = 4 \) SYM corresponds to the number of scalar fields, so \( J \) is independent of the coupling \( g^2 \), which means \( \frac{\partial}{\partial T} J = 0 \) in (44).
4 Discussion

We calculated the finite size correction on the two- and three-point correlation functions of the giant magnon. By the saddle point approximation, we rederived the finite size effect for the dispersion relation of the giant magnon. We also calculated the finite size effect on the three point correlation function between two giant magnons and one marginal operator, whose result is exactly the same as one obtained by the RG analysis. The calculation of the finite size correction on the giant magnon can be easily extended to the dyonic magnon case moving on $AdS_5 \times S^3$.

It is interesting to investigate the three point correlation function of heavy operators like the giant magnon with the relevant or irrelevant light operator instead of the marginal one. We hope to report these issues elsewhere.

Acknowledgement

This work was supported by the National Research Foundation of Korea(NRF) grant funded by the Korea government(MEST) through the Center for Quantum Spacetime(CQUeST) of Sogang University with grant number 2005-0049409. C. Park was also supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education, Science and Technology(2010-0022369).

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