Proton polarizabilities: status, relevance, prospects

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Abstract. This is a brief review of the status of understanding the proton polarizabilities in chiral perturbation theory and of their relevance to the “proton charge radius puzzle”.

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1. INTRODUCTION

A nucleon immersed in an external electromagnetic field acquires the electric and magnetic dipole moments which size is given, respectively, by the electric and magnetic polarizabilities $\alpha_E$ and $\beta_M$. These static quantities, together with quantities such as the anomalous magnetic moment and charge radius, reflect the complexity of the nucleon structure. Their empirical determination is very important for at least two reasons: first is that we would like to test our understanding of the nucleon structure based on microscopic calculations of these quantities, and second is that it enables us to evaluate the polarizability effects in phenomena such as hydrogen Lamb shift, properties of nuclear matter, etc. Here I shall illustrate these two motivations by looking at the description of proton polarizabilities in chiral perturbation theory (Sect. 2), and at their relevance to the muonic-hydrogen Lamb shift measurement (Sect. 3). I shall conclude with some words on prospects for better measurements of proton polarizabilities and of their momentum-transmster dependencies.

2. STATUS

In Chiral Perturbation Theory (ChPT) [1, 2], the nucleon polarizabilities should largely come as a prediction since the leading chiral-loop contribution is of order $p^3$, while the unknown low-energy constants (LECs) come only at order $p^4$. Recall that $p$ is of order $m_n/(4\pi f_p)$ and hence $p^4$ contribution is expected to be no greater than 15 percent of $p^3$. The result, however, depends on how or whether one includes the relativistic effects as well as the effects due to the $\Delta(1232)$-isobar excitation. The leading relativistic corrections carries an extra factor of $m_n/M_N$ and hence nominally is of order $p^4$. The scheme which consistently shuffles the relativistic corrections to the order where they nominally should appear is called Heavy-Baryon ChPT (HBChPT) [3], in contradistinction with Baryon ChPT (BChPT) which simply follows from the manifestly Lorentz-invariant Lagrangian of ChPT with baryons fields [4, 5]. The leading $\Delta(1232)$ contribution to polarizabilities is of order $p^4/\Delta$, where $\Delta = M_\Delta - M_N \approx 300$ MeV is the Delta-nucleon mass difference and hence, depending on counting, had been considered to be of order $p^3$ (\epsilon-expansion [6]) or $p^4$ (“resonance saturation”), or in between ($\delta$-expansion [7]). By now, all the relevant contributions have been calculated in both HBChPT and BChPT and their numerical values are given as follows (in units of $10^{-4}$ fm$^3$):

$0(p^3)$ BChPT [8]: $\alpha_{E1} = 6.8, \quad \beta_{M1} = -1.8; \quad O(p^3)$ HBChPT [9]: $\alpha_{E1} = 12.2, \quad \beta_{M1} = 1.2$.

$O\left(\frac{E}{\Delta}\right)$ BChPT [10]: $\alpha_{E1} = 4.0, \quad \beta_{M1} = 5.8; \quad O\left(\frac{E^4}{\Delta}\right)$ HBChPT [11]: $\alpha_{E1} = 8.6, \quad \beta_{M1} = 13.5$.

$O(p^3 + p^4/\Delta)$ BChPT: $\alpha_{E1} = 10.8, \quad \beta_{M1} = 4.0; \quad O(p^3 + p^4/\Delta)$ HBChPT: $\alpha_{E1} = 20.8, \quad \beta_{M1} = 14.7$

with a relatively small uncertainty due to higher-order ($p^5$) contribution. Modern evaluations of the Baldin sum rule [12] yield for the sum of polarizabilities a value of 13.8(5) which compares well with either the total $O(p^3 + p^4/\Delta)$ BChPT value or with $O(p^3)$ HBChPT value. This shows that in HBChPT the $\Delta$ contributions should only be treated together with $O(p^4)$. If the deference between the BChPT and HBChPT numbers comes indeed from recoil corrections, then they are too significant to be neglected, and hence $O(p^4)$ is to be mandatorily included in HBChPT. In case of
proton Compton scattering, where these polarizabilities prominently appear, the calculations show that upon inclusion of $O(p^4)$ contributions the HBChPT achieves roughly the same results as $O(p^3 + p^4/\Delta)$ BChPT [13], albeit with a loss of some predictive power due to the appearance of two new LECs.

The present status of the BChPT, HBChPT, as well as “more empirical” extractions of proton polarizabilities, as summarised in [14], is shown in Fig. 1. Note the significant discrepancy of the BChPT prediction with the current Particle Data Group values, which thus far has been attributed to a sizeable underestimate of uncertainty in the TAPS and subsequently PDG values.

3. RELEVANCE: HYDROGEN LAMB SHIFT

The electric polarizability of the proton is responsible for a zero-range force in atoms, which lead to a shift in the $S$-levels:

$$
\Delta E_{nS}^{(pol.)} = -4 \alpha_{em} \phi_n^2(0) \int_0^{\infty} dQ \left[ \sqrt{1 + \frac{Q^2}{4m^2}} - \frac{Q}{2m} \right] \alpha_{E1}(Q^2),
$$

where $\alpha_{em}$ is the fine-structure constant, $\phi_n^2(0) = \alpha_{em}^3 m_n^3/(\pi n^3)$ is the square of the hydrogen wave-function at the origin, $m$ is the lepton mass and $m_r$ is the reduced mass: $m_r = M_p m_\ell/(M_p + m_\ell)$. The effect of magnetic polarizability is suppressed.

The factor in the square brackets of Eq. (1) acts as a soft cutoff at the scale of order of the lepton mass $m_\ell$, and hence the proton polarizability contribution in $\mu H$ is expected to be bigger than in $e H$. How much bigger?

The transfer-momentum dependence of $\alpha_{E1}$ is inferred from the forward doubly-virtual Compton scattering, and hence is not accessible in a direct experiment. Only the sum, $\alpha_{E1}(Q^2) + \beta_{M1}(Q^2)$, is accessible through a generalized Baldin sum rule. The Baldin sum rule has been evaluated in several works leading to the so-called ‘inelastic’
Substituting this into Eq. (4) we obtain

$$\Delta E_{2S}^{(\text{incl.})} = -4\alpha_{em} \phi_2^2(0) \int_0^\infty dQ \left[ \sqrt{1 + \frac{Q^2}{4m_\mu^2}} - \frac{Q}{2m_\mu} \right] \left\{ \alpha_{E1}(Q^2) + \beta_{M1}(Q^2) \right\} \approx -13 \mu eV.$$  \hspace{1cm} (2)

One then need subtract $\beta_{M1}(Q^2)$ to obtain the energy shift as defined in Eq. (1), i.e.: $\Delta E_{2S}^{(\text{pol.})} = \Delta E_{2S}^{(\text{incl.})} + \Delta E_{2S}^{(\text{subt.})}$, with

$$\Delta E_{2S}^{(\text{subt.})} = 4\alpha_{em} \phi_2^2(0) \int_0^\infty dQ \left[ \sqrt{1 + \frac{Q^2}{4m_\mu^2}} - \frac{Q}{2m_\mu} \right] \beta_{M1}(Q^2).$$  \hspace{1cm} (3)

In other words the problem is shifted to finding $\beta_{M1}(Q^2)$ which seems to be just as unknown as $\alpha_{E1}(Q^2)$. This uncertainty of the polarizability contribution has been exploited by Miller [28] to suggest that it could be as large as $-310 \mu eV$ needed to resolve the charge radius puzzle.

An insight can be gained by using ChPT, which should work well for momenta probed in atomic systems. Based on general analytic properties of the momentum-transfer dependence (i.e., analyticity in the complex $Q^2$ plane, except for the negative real axis—timelike region) infer a dispersion relation of the type:

$$\int_{Q^2} \frac{dt}{t + Q^2 - i0^+} \text{Im} \left\{ \frac{\alpha_{E1}(-t)}{\beta_{M1}(-t)} \right\},$$  \hspace{1cm} (4)

where $0^+$ is an infinitesimal positive number. An explicit $p^3$ calculation in HBCPT yields:

$$\text{Im} \beta_{M1}^{(3)}(-t) = \frac{\alpha_{em} g_A^3}{16f_\pi^2} \frac{m_\pi^2}{t^{3/2}} \theta(t - 4m_\pi^2),$$  \hspace{1cm} (5)

where $g_A \approx 1.27, f_\pi \approx 92.4$ MeV are respectively the axial and pion-decay constant; $m_\pi$ is charged-pion mass. Substituting this into Eq. (4) we obtain

$$\beta_{M1}^{(3)}(Q^2) = \frac{\alpha_{em} g_A^3}{16\pi f_\pi^2} \frac{m_\pi}{Q^2} \left[ 1 - \frac{2m_\pi}{Q} \arctan \frac{Q}{2m_\pi} \right],$$  \hspace{1cm} (6)

which reproduces the result of Birse and McGovern [27]. Substituting this into Eq. (3) and setting for simplicity $m_\mu = m_\pi$, we obtain the following subtraction contribution:

$$\Delta E_{2S}^{(\text{subt.})} = \frac{\alpha_{em} m_\pi^3 g_A^3}{2(4\pi f_\pi)^2} \left( \frac{1}{8} - \frac{1}{4}C + \frac{1}{2} \ln 2 \right) \simeq 1.4 \mu eV,$$  \hspace{1cm} (7)

where $C \simeq 0.9160$ is the Catalan number. We conclude that the outcome is tiny and is very unlikely to change by orders of magnitude upon refining the ChPT calculation. A calculation in BChPT is nevertheless forthcoming [29].

4. PROSPECTS

There is a still big room for improvement of our knowledge of nucleon polarizabilities, most notably in the empirical knowledge of their static values, as well as of their momentum-transfer dependence. Even the static electric and magnetic polarizabilities of the proton are not pinned down to the accuracy of less than 10 percent. Such accuracy seems well within the reach of current experimental capabilities and we shall definitely see a progress in this direction in a very near future. The beam asymmetry of Compton scattering looks especially promising for a precise determination of the small magnetic polarizability [14]. A new round of real- and virtual-Compton scattering experiments at low energies on the proton and light nuclei targets has recently commenced at Mainz Microtron (MAMI) at the University of Mainz. There is real-Compton scattering program running at the High Intensity Gamma Source (HIGS) facility at Duke University. The new high-intensity beam facilities, such as MESA, will bring new opportunities in this field.
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