Maximum Azimuthal Anisotropy of Neutrons from Nb-Nb Collisions at 400 AMeV and the Nuclear Equation of State

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We measured the first azimuthal distributions of triple-differential cross sections of neutrons emitted in heavy-ion collisions, and compared their maximum azimuthal anisotropy ratios with Boltzmann–Uehling–Uhlenbeck (BUU) calculations with a momentum-dependent interaction. The BUU calculations agree with the triple- and double-differential cross sections for positive rapidity neutrons emitted at polar angles from 7 to 27 degrees; however, the maximum azimuthal anisotropy ratio for these free neutrons is insensitive to the size of the nuclear incompressibility modulus $K$ characterizing the nuclear matter equation of state.

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An important goal of relativistic heavy-ion physics is to extract information on the
equation-of-state (EOS) of nuclear matter, which relates density, temperature, and pressure.
This relationship has drastic implications for heavy-ion collisions and also has important
astrophysical consequences [1]. As far as bulk properties are concerned, the vast body of
knowledge associated with nuclear structure physics contributes information on a single
point in the temperature-density plane [viz., at $T = 0$ and $\rho = \rho_0$], where $\rho_0$ is the
density of nuclear matter at equilibrium. Heavy-ion collisions provide a laboratory tool for
investigating the behavior of nuclear matter in regions of temperature and density removed
from equilibrium. In recent years, progress towards this goal has occurred in both theory and
experiment; however, the task of deducing the EOS from high-energy heavy-ion collisions
is a difficult one. One complication arises because nonequilibrium aspects of the collision
are important [2]; this fact reflects on certain observables that were thought originally to
be good probes of the EOS such as, for example, transverse momentum generation [2 4],
pion production [3], kaon production [5], and flow-angle distributions [7]. In this Letter,
we concentrate on another observable associated with the azimuthal distribution about a
reaction plane for particles measured in a heavy-ion collision. Such azimuthal distributions
were measured [8] and calculated [6] previously for charged particles; but here we pursue the
sensitivity to the EOS of the maximum azimuthal anisotropy ratio [9] (defined below) for
neutrons. We might expect the azimuthal distributions of neutrons to differ from those for
charged particles primarily because of the absence of Coulomb effects and different nucleon-
nucleon cross sections. First, we address experimental observations, and then compare the
data with calculations based on nuclear transport theory. We find that the experiment is
insensitive to the nuclear incompressibility modulus $K$ in the EOS.

The experiment was designed to measure triple-differential cross-sections $d^3\sigma/d(cos\theta)
d(\phi - \phi_R) d\alpha$ for neutrons from high-multiplicity collisions of equal-mass nuclei as a function
of mass number and bombarding energy. The symbol $\alpha \equiv (Y/Y_P)_{CM}$ denotes the neutron
rapidity normalized to the projectile rapidity $Y_P$ in the center-of-mass (CM) system. We
obtained an estimate of the azimuthal angle $\phi_R$ of the reaction plane by measuring the
transverse velocities of charged particles emitted with positive rapidities above a cutoff rapidity in the CM system and summing them to obtain a total transverse-velocity vector. The $\phi$-distribution of these summed transverse-velocity vectors is peaked about the reaction plane. This transverse-velocity method \cite{10} is an adaption of the transverse-momentum method of Danielewicz and Odyniec \cite{11}. As described previously \cite{12}, we determined $\phi_R$ with a dispersion $\Delta \phi_R (\geq \alpha_c)$ for charged fragments above a normalized rapidity $\alpha_c$. For 400 AMeV Nb-Nb, we observed $\Delta \phi_R \sim 40^\circ$ for all $\alpha_c$ with uncertainties of about 5%.

Projectile ions traversed a beam telescope before interacting with the target. An array of 18 scintillation detectors was used to detect neutrons emitted from each interaction. The size of each neutron detector and the flight path at each angle were selected to provide approximately equal counting rates and energy resolutions for the highest energy neutrons of interest at each angle. These mean-timed \cite{13,14} detectors spanned the polar angles from $3^\circ$ to $90^\circ$. The flight paths to the 18 detectors ranged from 5.91 to 8.38 m. Charged particles incident on each of the 18 neutron detectors were vetoed with either a 6.3- or 9.5-mm thick anticoincidence plastic scintillator. The time-of-flight (TOF) of each detected neutron was determined by measuring the time difference between the detection of a neutron in one of the neutron detectors and the detection of a Nb ion in a beam telescope. A plastic-wall array, 5-m wide and 4.5-m high, of 184 scintillation detectors (each 9.5-mm thick) was used to detect charged fragments emitted from each interaction. The multiplicity of the detected charged fragments indicated the degree of centrality of the collision. The flight paths to the 184 detectors ranged from 4.0 to 5.0 m. The velocity of each charged fragment detected in the plastic wall in each collision was extracted from the measured flight-time. The uncertainty in the velocity was typically 3% for 400 AMeV charged fragments detected in the plastic wall; about two thirds of the uncertainty came from the intrinsic time dispersion of the detectors, and another one third from the uncertainty in the position of the charged fragments. A thin steel (0.95-mm) sheet, which covered the front side of the 24 inner detectors, was used to absorb the $\delta$-rays produced as the beam traversed the air after it exited the beam pipe just upstream of the target. The Nb target (with a physical thicknesses of 2.04 g/cm$^2$) was
oriented at 60° with respect to the beam. The beam energy at the center of the target was 400 AMeV with an energy spread of ±41 AMeV. Auxiliary measurements with steel shadow shields, typically 1.0-m long, were used to determine target-correlated backgrounds. Each shadow shield was located approximately halfway between the target and the detector. The shadow shields attenuated neutrons by a factor > $10^3$ at all energies.

To compare our experimental results with transport model calculations, it is necessary to match the impact parameter range in the model with the observed multiplicity values. Well-known geometrical arguments \[15\] for estimating the impact parameter assume a correlation between the impact parameter and the fragment multiplicity $M$. Measurements of the multiplicity distribution with the target removed showed that collisions with a charged multiplicity $M \geq 26$ contained only about 5% of the background contamination from collisions of Nb with the air or the material in the beam telescope. This multiplicity threshold selects about 22% of the total geometric cross-section which corresponds to a value of 0.47 for the ratio of the maximum impact parameter $b_0$ to the nuclear diameter $2R$.

Welke et al. \[9\] discussed the maximum azimuthal anisotropy ratio as a testing ground for the EOS. At each polar angle, the maximum azimuthal anisotropy ratio $r(\theta, \alpha)$ for neutrons in a given rapidity bin is the ratio of the maximum value of the triple-differential cross section $\sigma_3^{MAX}$ to the minimum value $\sigma_3^{MIN}$. For positive (negative) CM rapidities, we observed that $\sigma_3$ reaches a maximum (minimum) at $\phi = 0° (180°)$ and a minimum (maximum) at $180° (0°)$. To evaluate the ratio $r(\theta)$ for each rapidity bin, we fit the measured $\sigma_3$ at each $\theta$ with a function of the form $\sigma_3(\phi, \theta) = a(\theta) + b'(\theta)\cos(\phi - \phi_R)$, and obtain an estimate $r'(\theta)$. For positive rapidities, the parameter $b'(\theta)$ is positive; thus, $r'(\theta, \alpha > 0) = [a(\theta) + b'(\theta)] / [a(\theta) - b'(\theta)]$. To correct for the finite rms dispersion in $\phi_R$, we use the fact that the parameter $b'(\theta) = b(\theta)\exp[-\frac{1}{2}(\Delta\phi_R)^2]$ for a Gaussian distribution of $\phi_R$ with an rms dispersion $\Delta\phi_R$. Finally, from the observed $b'(\theta)$ and the measured dispersion $\Delta\phi_R$, we calculate $b(\theta)$ and obtain the dispersion-corrected ratio $r(\theta) = [a(\theta) + b(\theta)] / [a(\theta) - b(\theta)]$.

For theoretical interpretation, we rely here on the BUU approach \[10\] with a momentum–dependent nuclear mean field, $U(\rho, \vec{p})$, as parameterized in Ref. \[2\]. The momentum-
dependent interaction is essential not only from a theoretical standpoint [17], but also has important observable implications [18,19]. The BUU calculations here have implemented a new algorithm, which considers the altered in-medium phase space density that occurs with a momentum-dependent interaction. This algorithm amounts to having a scattering cross-section that depends on the effective mass [20] of the nucleon. This correction turns out to be non-negligible in our case and a systematic investigation of its experimental consequences will appear elsewhere [21]. Transport theories used to extract the EOS must account for the polar-angle dependence of the maximum azimuthal anisotropy ratio $r(\theta)$; furthermore, at each polar angle the calculations should fit the absolute triple-differential cross-sections and the absolute double-differential cross sections (obtained by integrating the triple-differential cross-sections over the azimuthal angle). It is important to point out that such complete data provide a crucial test for theoretical models.

The full one-body BUU theory considers all nucleons emitted in a nucleus-nucleus collision: free nucleons and those carried away in composite fragments. The initial application of the full model revealed that the maximum neutron azimuthal anisotropy ratio is sensitive to the value of K, as in the original proposal [9]; however, we observed that these analyses were dominated by a trend not present in the experimental data [22] which include only free neutrons. In an attempt to gain further understanding, we proceeded to correct for cluster contamination in our comparison with free neutron data. We subtracted contributions to the cross section from composite fragments by rejecting neutrons when the distance between the neutron and any other nucleon from the same BUU ensemble [16] is less than a critical distance $d$ [23], which we find to be 2.5 fm. By restricting the analysis to free neutrons, the BUU calculations of both the double- and triple-differential cross sections agree generally with the data and are insensitive to K; the comparison is shown in Fig. 1 for the triple-differential cross section at $24^\circ$, and in Fig. 2 for the double-differential cross section. The double-differential cross sections are significantly higher (lower) than the data with $d = 2.0(3.0)\ fm$.

The $r(\theta)$ vs $\theta$ curves (for $0.7 < \alpha \leq 1.2$) in Fig. 3 are BUU results for free neutrons.
Comparison with the data reveals that $r(\theta)$ for free neutrons is also insensitive to $K$. This result indicates that the azimuthal anisotropy of free neutrons is not a sensitive probe of the EOS, contrary to what was hoped previously [9]; however, the fact that full one–body calculations of $r(\theta)$ exhibit considerable structure and sensitivity to $K$ [22] points to composites as the carriers of the information. An important consequence of our analysis is that the azimuthal anisotropy ratio as a technique to probe the EOS is not invalidated by the data on free neutrons; instead, it is necessary to concentrate on the application of this technique to composites. This assertion is presently under close quantitative scrutiny and preliminary results are encouraging [24]. Because of large statistical uncertainties in both the data and the model calculations in regions of phase space where composite formation is small, $K$ is still constrained poorly in these regions.

The unique features of this experiment include (a) the first measurement of neutron flow; (b) simultaneous determination of absolute triple- and double-differential cross sections; and (c) a successful demonstration of a new approach to “$4\pi$” physics at intermediate energies with separation and economical optimization of two previously combined detector functions (viz., measurement of $\Phi_R$ and sampling of fragment momenta). We simplified the task of filtering models to simulate experimental acceptances (with a relatively low cost detector) and measured absolute triple-differential cross sections over a wide energy region.

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FIGURES

FIG. 1. Triple-differential cross sections for neutrons emitted at a polar angle of 24 degrees with rapidities \(0.7 < \alpha \equiv (Y/Y_p)_{CM} \leq 1.2\) from semi central Nb-Nb collisions at 400 AMeV with a reaction plane dispersion of 40 degrees. The ratio of the maximum impact parameter to the nuclear radius \(b_0/2R = 0.47\). The broken lines represent results of BUU calculations for free neutrons with three values of the incompressibility modulus \(K\); the statistical uncertainties are typically \(\sim 10\%\) for each \(\Phi\)-bin of 18\(^\circ\) at polar angles from 7\(^\circ\)-15\(^\circ\); increase to \(\sim 14\%\) for for \(\phi - \phi_R > 90^\circ\) at \(\theta = 18^\circ\)-24\(^\circ\), and become progressively worse as \(\theta\) increases from 27\(^\circ\) to 36\(^\circ\). The solid line represents a least-squares fit to the data.

FIG. 2. The polar-angle dependence of the double-differential cross sections for free neutrons emitted with rapidities \(0.7 < \alpha \equiv (Y/Y_p)_{CM} \leq 1.2\) from semi central Nb-Nb collisions at 400 AMeV. The circles are the data. The open symbols are BUU calculations.

FIG. 3. The polar-angle dependence of the maximum azimuthal anisotropy ratio \(r(\theta)\) for neutrons emitted with rapidities \(0.7 < \alpha \equiv (Y/Y_p)_{CM} \leq 1.2\) from multiplicity-selected Nb-Nb collisions at 400 AMeV with impact parameters \(0 < b(fm) \leq 4.8\). The circles represent ratios determined from experiment and corrected to zero dispersion. The ratio of the maximum impact parameter to the nuclear radius \(b_0/2R = 0.47\). The lines represent results of BUU calculations for free neutrons \((d = 2.5 \text{ fm})\) for four values of the incompressibility modulus \(K\).
 Nb 400 AMeV, 0.7 < \alpha \leq 1.2

\frac{d^3 \sigma}{d(\cos \theta) \cdot d(\phi - \phi_R) \cdot d\alpha} (b/sr)

BUU
\begin{align*}
d &= 2.5 \text{ fm} \\
K \text{ (MeV)} &= 380, 215, 100
\end{align*}

24^\circ
400 AMeV Nb–Nb

$0 < b_{(fm)} \leq 4.8$

$0.7 < \alpha \leq 1.2$

E848H

$\frac{d^2\sigma}{d(\cos\theta)} \, d\alpha \, (b/\text{sr})$

BUU

d = 2.5 fm

K MeV

☐ 100

◇ 215

△ 380

Polar Angle, $\theta$ (deg)
Maximum azimuthal anisotropy, $r(\theta)$

- BUU K (MeV), $d = 2.5$ fm
- E848H $M \geq 26, b \leq 4.8$ fm

$0.7 \leq \alpha \leq 1.2$