Point-by-point extraction of parton distribution functions from SIDIS single transverse–spin asymmetries

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We show how some parton distribution functions related to the transverse spin of nucleons can be extracted point by point from combinations of proton and deuteron observables. In particular, we present a determination of the valence and sea Sivers functions from the single-spin asymmetries measured by COMPASS.

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The transverse–spin structure of the nucleon is presently one of the most relevant topics of hadronic physics (for reviews, see [1–4]). On the experimental side, the semi-inclusive deep inelastic scattering (SIDIS) measurements have provided a wealth of data on single spin asymmetries, which shed light on the transversity distribution and on the leading-twist transverse-momentum dependent distribution functions (TMDs). In most phenomenological studies, these data are analyzed using specific functional forms for the transversity and the TMDs, with a certain number of free parameters determined by fits to the measured asymmetries. Alternatively, one can adopt a simpler approach consisting in using simultaneously the proton and deuteron asymmetries measured at the same \( x \) and \( Q^2 \), and performing a point-by-point extraction of the parton distribution functions directly from the data, with a very limited set of assumptions.

In [5] we applied this method to extract the transversity distributions from the Collins and di-hadron asymmetries on proton and deuteron measured by the COMPASS Collaboration, using also the corresponding \( e^+e^- \) asymmetries from the Belle experiment. The results of our determination of the valence and sea transversity are shown in Fig. 1.

The same method can be used to extract a very important TMD, the Sivers function \( f_{T}^{1} \) [6–9], which encodes the correlation between the transverse momentum \( k_T \) of quarks in a transversely polarized nucleon and the spin of the parent nucleon. Here we briefly report on this extraction, presented in detail in [10].

The asymmetry related to the Sivers function has been found to be different from zero for positive charge hadrons produced on protons first by the HERMES experiment [11, 12] and a few years later, at higher beam energy, by the COMPASS experiment [13–16]. The first COMPASS measurements, performed using a deuteron target, showed no clear signal [13, 17, 18]. Measurements on pion production on a transversely polarized \(^3\)He target and 6 GeV electron beam have been performed more recently by

![Figure 1](image-url)
the Hall A Collaboration at JLab \[^{13}\].

The Sivers asymmetry is proportional to a convolution over transverse momenta of the Sivers function \(f^{\perp}_{1T}\) and of the unpolarized fragmentation function \(D_1\). It can be factorized using a Gaussian Ansatz \[^{20\,22}\] and becomes

\[
A_h(x, z, Q^2) = G \frac{\sum_{q, \bar{q}} e_q^2 x f^{\perp,(1)q}_{1T}(x, Q^2) z D_{1q}(z, Q^2)}{\sum_{q, \bar{q}} e_q^2 x f^{q}_{1T}(x, Q^2) D_{1q}(z, Q^2)}.
\]  

(1)

where

\[
f^{\perp,(1)}_{1T}(x, Q^2) = \int d^2k_T \frac{k_T^2}{2M^2} f^{\perp}_{1T}(x, k_T^2, Q^2)
\]

(2)

is the first \(k_T^2\) moment of the Sivers function. The \(G\) factor, resulting from the integration over transverse momenta, is given by

\[
G = \frac{\sqrt{\pi M}}{\sqrt{\langle p_T^2 \rangle + z^2 \langle k_T^2 \rangle_S}}.
\]

(3)

where \(\langle p_T^2 \rangle\) and \(\langle k_T^2 \rangle_S\) are the widths of the transverse-momentum parts of the fragmentation function and of the Sivers function respectively. A good approximation is to set \(G \approx \pi M/2 \langle P_{h\perp}\rangle\), where \(\langle P_{h\perp}\rangle\) is the mean value of the final hadron transverse momentum, and take it as a constant, since the measured \(z\) dependence of \(\langle P_{h\perp}\rangle\) is smooth in the range of interest. This approximation, which should give systematic corrections well within the overall uncertainties, allow us to the Sivers asymmetry as a function of \(x\) as

\[
A_h(x, Q^2) = G \frac{\sum_{q, \bar{q}} e_q^2 x f^{\perp,(1)q}_{1T}(x, Q^2) \tilde{D}^{(1)}_{1q}(Q^2)}{\sum_{q, \bar{q}} e_q^2 x f^{q}_{1T}(x, Q^2) \tilde{D}_{1q}(Q^2)}.
\]

(4)

where

\[
\tilde{D}_1(Q^2) = \int dz D_1(z, Q^2), \quad \tilde{D}^{(1)}_{1}(Q^2) = \int dz z D_1(z, Q^2).
\]

(5)

The fragmentation functions and the unpolarized distribution functions appearing in eq. (4) can be obtained from standard parametrizations, so we can extract the transverse moments of the Sivers function \(f^{\perp,(1)}_{1T}\) by properly combining the asymmetries on proton and deuteron, for charged pions and kaons.

In the pion case we use the favored and unfavored fragmentation functions defined as

\[
D_{1,\text{fav}}^\pi = D_{1u}^\pi = D_{1d}^\pi = D_{1u}^\pi = D_{1d}^\pi,
\]

\[
D_{1,\text{unf}}^\pi = D_{1u}^{-\pi} = D_{1d}^{+\pi} = D_{1u}^{+\pi} = D_{1d}^{-\pi},
\]

(6)

and for the strange quark we assume

\[
D_{1s}^{+\pi} = D_{1d}^{+\pi} = N D_{1,\text{unf}}^\pi,
\]

(7)

with the constant factor \(N \approx 0.8\) evaluated in \[^{23}\]. The asymmetries can then be expressed in terms of the ratios of fragmentation functions

\[
\beta_\pi(Q^2) = \frac{\tilde{D}^{\pi}_{1,\text{unf}}(Q^2)}{\tilde{D}^{\pi}_{1,\text{fav}}(Q^2)}, \quad \beta^{(1)}_\pi(Q^2) = \frac{\tilde{D}^{(1)\pi}_{1,\text{unf}}(Q^2)}{\tilde{D}^{(1)\pi}_{1,\text{fav}}(Q^2)}, \quad \rho_\pi(Q^2) = \frac{\tilde{D}^{(1)\pi}_{1,\text{unf}}(Q^2)}{\tilde{D}^{(1)\pi}_{1,\text{fav}}(Q^2)},
\]

(8)

and the valence Sivers distributions turn out to be given by

\[
x_{f^{(1)}_{1T}}^{u,v} = \frac{1}{5G\rho_\pi(1 - \beta^{(1)}_\pi)} \left[ (x f_{p}^{+ A_{p}^{u} - x f_{p}^{- A_{p}^{v}}}) + \frac{1}{3} (x f_{d}^{+ A_{d}^{u} - x f_{d}^{- A_{d}^{v}}}) \right],
\]

\[
x_{f^{(1)}_{1T}}^{d,v} = \frac{1}{5G\rho_\pi(1 - \beta^{(1)}_\pi)} \left[ \frac{4}{3} (x f_{d}^{+ A_{d}^{u} - x f_{d}^{- A_{d}^{v}}}) - (x f_{p}^{+ A_{p}^{u} - x f_{p}^{- A_{p}^{v}}}) \right],
\]

(9)
where \( f_{p,d}^{\pm} \) are linear combinations of the unpolarized distribution functions (for their explicit expressions see [10]). From the measured asymmetries one can also obtain directly the difference of the sea distributions \( x f_{1T}^{(1)u} - x f_{1T}^{(1)d} \):

\[
x f_{1T}^{(1)u} - x f_{1T}^{(1)d} = \frac{1}{15 G \rho \pi \left(1 - \beta_{\pi}^{(12)}\right)} \left[ 2(1 - 4 \beta_{\pi}^{(1)}) x f_{p}^{\pi^+} A_{p}^{\pi^+} + 2(4 - \beta_{\pi}^{(1)}) x f_{p}^{\pi^-} A_{p}^{\pi^-} - (1 - 4 \beta_{\pi}^{(1)}) x f_{d}^{\pi^+} A_{d}^{\pi^+} - (4 - \beta_{\pi}^{(1)}) x f_{d}^{\pi^-} A_{d}^{\pi^-} \right],
\]

(10)

In the case of charged kaons, following the same procedure, we introduce

\[
D_{1u,\text{fav}}^{K} = D_{1u}^{K^+} = D_{1u}^{K^-}, \quad D_{1u,\text{unf}}^{K} = D_{1u}^{K^+} = D_{1u}^{K^-},
\]

\[
D_{1d}^{K} = D_{1d}^{K^+} = D_{1d}^{K^-} = D_{1d}^{K^-} = D_{1d}^{K^-},
\]

(11)

the ratios \( \beta_K, \beta_K^{(1)} \) and \( \rho_K \) defined as in eq. [5], but for kaons, and

\[
\gamma_K(Q^2) = \frac{D_{1u,\text{fav}}^{K}(Q^2)}{D_{1u,\text{fav}}^{K}(Q^2)}, \quad \gamma_K^{(1)}(Q^2) = \frac{D_{1u,\text{fav}}^{K^{(1)}}(Q^2)}{D_{1u,\text{fav}}^{K^{(1)}}(Q^2)}.
\]

(12)

In this case, assuming that the difference of strange sea distributions \( x f_{1T}^{(1)s} - x f_{1T}^{(1)s} \) is negligible, we obtain

\[
x f_{1T}^{(1)u} = \frac{1}{4 G \rho_K \left(1 - \beta_K^{(1)}\right)} \left[ (x f_{p}^{K^+} A_{p}^{K^+} - x f_{p}^{K^-} A_{p}^{K^-}) \right],
\]

\[
x f_{1T}^{(1)d} = \frac{1}{4 G \rho_K \left(1 - \beta_K^{(1)}\right)} \left[ (x f_{d}^{K^+} A_{d}^{K^+} - x f_{d}^{K^-} A_{d}^{K^-}) - (x f_{p}^{K^+} A_{p}^{K^+} - x f_{p}^{K^-} A_{p}^{K^-}) \right],
\]

(13)

where the quantities \( f_{p,d}^{\pm} \) are linear combinations of the unpolarized distribution functions.

To extract the Sivers functions from eqs. [9], [10] and [13] we used the COMPASS measurements of the Sivers asymmetries in SIDIS of 160 GeV muons on proton [16] and deuteron targets [13] for charged pions and kaons. The x binning is the same for all series of data. Concerning the momentum transfer \( Q^2 \), it ranges from 1.2 GeV\(^2\) for the lowest \( x \) point to 20 GeV\(^2\) for the highest \( x \) point. We have used the unpolarized distribution functions from the CTEQ5D global fit [24], and the unpolarized fragmentation functions from the DSS parametrization [28]. Finally, the quantity \( G = \pi M/2\langle P_{h,\perp}\rangle \) has been calculated using the measured \( \langle P_{h,\perp}\rangle \), which is \( \sim 3 \) for pions and \( \sim 2.5 \) for kaons with a slight \( x \) dependence.

Figure [2] shows the extracted values of the Sivers distribution \( x f_{1T}^{(1)u} \) (left) and \( x f_{1T}^{(1)d} \) (right), as obtained from pion and kaon data. The error bars indicate the statistical uncertainties only. The \( u_v \)
distribution is clearly positive and different from zero over most of the covered $x$ range. The statistical errors for $x f_{1T}^{u(1)}$, are much larger because of the unbalanced proton–deuteron statistics in the COMPASS data. Still, the $d_v$ distribution appears to be negative in the valence region and the values are compatible with $x f_{1T}^{d(1)} \approx -x f_{1T}^{u(1)}$. The agreement between the independent results obtained from pion and kaon data is quite good, as expected. Also, our results agree rather well with previous extractions (for instance, with the fits of [25, 26]).

The sea difference $x f_{1T}^{u(1)u} - x f_{1T}^{d(1)d}$ obtained from the pion asymmetries is shown in Fig. 3 as one can see, it is compatible with zero, with small statistical uncertainties.

To summarize, the first $k_2T$ moments of the Sivers distributions have been extracted directly from the Sivers asymmetries for charged pions and kaons measured by COMPASS using proton and deuteron targets in the same kinematical region and in the same $x$ bins. The main advantage of this point-by-point determination is that no specific parametrization of $f_T^{u(1)}$ is required. Our results clearly show a non-vanishing and positive $u_v$ Sivers function different from zero, and an isotriplet Sivers sea $x f_{1T}^{u(1)u} - x f_{1T}^{d(1)d}$ compatible with zero. As for the $d_v$ Sivers function, it has opposite sign with respect to the $u_v$ distribution, but to improve its knowledge more precise deuteron data are clearly needed.

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