Improvement of radar detection capabilities for fluctuating targets using convolutional error control coding technique

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Abstract. Fluctuating targets are important phenomena that affect detection probability. Several investigations about this topic have been carried on with good results to radar capabilities improvement. However, the increasing development of stealth technology has weakened traditional surveillance systems. This work has as main objective to improve radar detection of fluctuating targets using error control codes techniques. The proposed novel idea consists on a long pulse divided into a number of shorter sub-pulses, equally spaced in time duration. The binary phases of these sub-pulses are selected in accordance with Barker codes. Convolutional coding and Binary Phase Shift Keying (BPSK) modulation techniques are used. We determine the probability of detection computing the bit error rate values. Matlab software is used to make several simulations that back up this theory. The results demonstrate that the probability of detection for Swerling I and Swerling III fluctuating target models is increased with the use of convolutional coding technique to detect and correct sub-pulse errors. This work offers a preliminary analysis of the detection improvement method with the use of error-control coding on radar pulsed with quite generalized assumptions thereby left a bigger space for future work with assumptions to fit practical implementation of the method.

1. Introduction

Radar systems, in general, use modulated waveforms and directive antennas to transmit electromagnetic energy into a specific volume in space to search for targets. Objects within a search volume will reflect portions of this energy (echoes) back to the radar. These echoes are then processed by the radar receiver to extract target information such as range, velocity, angular position, and other target identifying characteristics.

In practice, some part of the transmitted energy is absorbed and the reflected energy is not distributed equally in all directions. This phenomenon causes fluctuations in the received signal based on the complicated diagram of the radar cross section (RCS). Swerling models are used to describe the statistical properties of the RCS of objects with complex formed surface.

Stealth technology, also termed low observable technology, is a sub discipline of military tactics and passive and active electronic countermeasures, which covers a range of methods used to make personnel, aircraft, ships, submarines, missiles, satellites, and ground vehicles less visible (ideally
invisible) to radar, infrared, sonar and other detection methods. The use of this technology to reduce radar cross section increases the survivability and decreases the target detection of military aircraft. During the last decades, this technology has proven to be one of the most effective approaches as far as the endeavour to hide from radar systems is concerned. For this reason, investigators concentrate their efforts in to find novel methods to counter measure stealth technology with the main idea of increasing radar’s probability of detection. A method improving capabilities of fluctuating targets detection in radar systems using error control codes is presented in this work.

2. Radar detection
In radars design the main purpose is to get the maximum detection probability for a given false alarm probability. Thus, the equations of these two probabilities need to be found.

2.1 Non coherent detection

The probability of false alarm $P_{fa}$ is defined as the probability that a sample of the signal $r(t)$ will exceed the threshold voltage $V_t$ when noise alone is present in the radar, so it can be written as:

$$P_{fa} = \int_{V_t}^{\infty} \frac{r^2}{2\Psi^2} \exp \left(-\frac{r^2}{2\Psi^2}\right) dr = \exp \left(-\frac{V_t^2}{2\Psi^2}\right)$$

In equation (1) $\Psi^2$ is the noise variance, for this case is the variance of white Gaussian noise. The probability of detection $P_d$ is the probability that a sample value will exceed the threshold voltage in the case of noise plus signal:

$$P_d \approx 0.5 \text{erfc} \left(\sqrt{-\ln P_{fa} - \sqrt{SNR + 0.5}}\right)$$

$$\text{erfc}(z) = 1 - \frac{2}{\sqrt{\pi}} \int_{0}^{z} \exp(-v^2) dv$$

The term $SNR$ is the Signal to Noise ratio.

2.2 Coherent detection.

In coherent detection it is possible to calculate the false alarm probability with the threshold value and vice-versa, according to:

$$P_{fa} = \frac{1}{2} \text{erfc} \left(\frac{V_t}{\sqrt{2N\beta^2}}\right)$$

$$V_t = \sqrt{2\pi N\beta^2} \text{erfc}^{-1} \left(1 - 2P_{fa}\right) = \sqrt{2N\beta^2} \cdot \text{erfc}^{-1} \left(2P_{fa}\right)$$

The threshold can be computed from the number $N$ of samples, the variance $\beta^2$ of the noise, is assumed known, and the desired false alarm probability should also be known. Thus it is necessary only to determinate the probability of detection:

$$P_d = \frac{1}{2} \text{erfc} \left(\text{erfc}^{-1} \left(2P_{fa}\right) - \sqrt{N \cdot SNR}\right)$$

2.3 Detection of fluctuating targets.

The probability of detection of a target by a pulse search radar has been treated in considerable detail by J. I. Marcum for the case in which the amplitude of the returned signal pulses is not fluctuating. Swerling extended Marcum’s work and four different models of target fluctuating are considered.
Those models chosen for consideration are felt to be typical of situations which are likely to be met in practice, or, at least, to bracket a wide range of practical cases. In this work we only analyze Swerling I and Swerling III models.

When only a single pulse is used, the detection threshold is related to the probability of false alarm as defined in equation. (3) DiFranco and Rubin[6] derived a general form relating the threshold and for any number of pulses when non-coherent integration is used. It is:

$$P_{fa} = 1 - \Gamma_I \left( \frac{V_T}{\sqrt{n_p}}, n_p - 1 \right)$$

For this equation, $\Gamma_I$ is used to denote the Incomplete Gamma function and $n_p$ is the number of integrated pulses. The threshold value can then be approximated by the recursive formula used in the Newton-Raphson[7] method.

**Swerling I**: The probability of detection for Swerling I model is calculated as follow:

$$P_d = \exp \left( - \frac{V_T}{1 + SNR} \right) n_p = 1$$

$$P_d = 1 - \Gamma_I (V_T, n_p - 1) + \left( 1 + \frac{1}{n_p SNR} \right)^{n_p - 1} \cdot \Gamma_I \left( \frac{V_T}{1 + \frac{1}{n_p SNR}} \right) \exp \left( - \frac{V_T}{1 + \frac{1}{n_p SNR}} \right) n_p > 1$$

**Swerling III**: The exact formulas for the probability of detection for this model when $n_p = 1, 2$ is:

$$n_p = \exp \left( - \frac{V_T}{1 + n_p \frac{SNR}{2}} \right) \left( 1 + \frac{2}{n_p SNR} \right)^{n_p - 2} \cdot K_o$$

$$K_o = 1 + \frac{V_T}{1 + n_p \frac{SNR}{2}} - \frac{2}{n_p SNR} (n_p - 2)$$

For $n_p > 2$ the expression is:

$$P_d = \frac{V_T \cdot e^{-V_T}}{(1 + n_p \frac{SNR}{2})(n_p - 2)} + 1 - \Gamma_I (V_T, n_p - 1) + K_o \Gamma_I \left( \frac{V_T}{1 + n_p \frac{SNR}{2}}, n_p - 1 \right)$$

### 2.4 Convolutional codes performance

In communication systems employing forward error-correction coding, digital information source sends bits comprised data sequence to encoder. Encoder inserts redundant (or parity) bits, outputting a longer sequence of code bits called codeword. On the receiving end, codewords are used by a suitable decoder to extract the original data sequence. A convolutional code generates redundant bits by using modulo-2 convolutions, hence the name.

A convolutional code is specified by three parameters $(n, k, K)$ or $(k / n, K)$. We already saw that $k$ is the number of input data bits and $n$ is the output code bits. In practice, we usually use $k = 1$. The coding rate is $r = k / n$, determining the number of data bits per coded bit. The code rate can be simplified to $r = 1 / m$. The constraint length of a convolutional code $K$, expressed in terms of messages bits is a vector whose length is the number of inputs in the encoder diagram. We can define $n$ generator polynomials, one for each modulo-2 adder. Each polynomial describes the connection of the shift registers to the corresponding modulo-2 adder.
As a convolutional decoder, we use a Viterbi algorithm which walks through the Trellis\textsuperscript{[8]} diagram by computing a metric for every possible path. It is a maximum likelihood decoder, which is optimum for an additive white Gaussian noise (AWGN) channel. We use soft decoding and several values of the Viterbi traceback length $L$ to analyze its effect on the system performance.

Convolutional codes performance can be quantified through analytical means or by computer simulation. In this work we analyze its performance using computer simulation. Due to convolutional coding performance depends on the coding rate and the constraint length we use different values of these parameters during simulation.

With BER values it is possible to calculate the probability of detection using the following equation\textsuperscript{[9]}:

$$Pd = 1 - \sum_{i=0}^{E} \frac{N!}{i!(N-i)!}p^i(1-p)^{N-i}$$

In this equation, $N$ is the length of the message and in this work it is assumed to be equal to the Barker code length. The average probability of error in transmission is $p = BER$, while $E$ is the maximum number of errors allowed.

3. Novel idea and results

Figure 1 shows the general block diagram of the novel idea. In this process, a long pulse is divided into a number of subpulses with equal time durations, according to binary phase Barker code. The phase of the transmitted signal alternates between 0 and 180 degrees in dependence with the sequence of elements in the phase code. Then, the channel encoder uses convolutional encoding and after that the signal is modulated using BPSK modulation.

![Figure 1. Block diagram of the purposed novel idea using convolutional codes in radar systems.](image)

In accordance with Swerling I or Swerling III model, fluctuations are added to transmitted signal before been received. In the receiving side, noise is added to simulate the channel noise. The received signal is then decoded using Viterbi algorithm and finally its BER is computed. With the BER we can get the probability of detection according to equation (13). Probability of detection values resulting from simulation are compared for non-coherent and coherent detection, for both fluctuating Swerling I and III target model for traditional detection; and also using convolutional coding. The system performance is analysed for several values of rate, constraint length and Viterbi traceback length. It is important to clear that for every value of rate and constraint length will be many possibilities of generator polynomial. For that reason, several simulations were done with the main objective of analyse their behaviour in a deeply way.
For the system simulation we assume the following parameters: the Barker code length is 5 (1,1,0,1,0), the rate values used are \( r = 3, 4 \); the constraint length is \( K = 4, 5 \) and the Viterbi traceback length is \( L = 4 \). Also, the false alarm probability is \( P_{fa} = 10^{-4} \).

Matlab software allowed us to compute the theoretical and simulated values of the probability of detection. Figure 2, shows the performance of detection proved by probability of detection and signal to noise ratio.

![Figure 2](image)

Figure 2. Analysis of the system performance: (a) Swerling I model coherent detection; (b) Swerling III model coherent detection; (c) Swerling I model non-coherent detection and (d) Swerling III model non-coherent detection.

After several simulations we can demonstrate that the probability of detection is improved with the use of convolutional codes for fluctuating targets detection in radar systems. For the same SNR value, the probability of detection using this error control technique is higher than the one for traditional radar fluctuating target detection models.

4. Conclusions

This new approach is entirely based on coding-decoding binary phase Barker codes using convolutional codes to improve radar detection. We use Barker codes for intrapulse modulation, add some redundant bits in the convolutional encoder and modulate with binary phase-shift keying modulation technique. In the receiver side, the Viterbi algorithm detects and corrects some errors occurred during transmission. Simulation results demonstrate that the designed method improves fluctuating targets radar detection. Some assumptions are made for the sake of making comparisons and finding justification for the proposed method.
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