Phase evolution of terms of turbulent kinetic energy transport equation in the boundary layer of a pulsating flow

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Abstract. Experimental data on production, dissipation, turbulent and molecular diffusion of turbulent energy have been obtained in the boundary layer of flow with forced freestream velocity fluctuations. The evolution of these terms over the pulsation phase is presented for streamwise and wall-normal velocity components. The presented results have been obtained on the basis of the dynamics of vector fields measured using a new optical SIV method with a spatial resolution comparable to the Kolmogorov scale.

1. Introduction

A problem of laminar pulsating isotropic incompressible flow of a Newtonian fluid has a well-known analytical solution [1, 2], while a solution for the turbulent particular case has been obtained only recently [3] using DNS. Low resource-intensive URANS models are able to find out some regularities of pulsating flows [4]. However k-ε and k-ω isotropic turbulence models widely used for URANS closure are barely applicable to such flows. This is due to the fact that the significant spatial and temporal anisotropy of Reynolds stress cannot be adequately described by introduction of isotropic eddy viscosity by the Boussinesq hypothesis into k-ε and k-ω models.

Second-order anisotropic turbulence models (Reynolds Stress Models – RSM) obtained in [5] and [6] are based on the differential equation (1) of the turbulent kinetic energy transport equations for each component of the Reynolds stress tensor

\[
\frac{\partial}{\partial t} \left( \rho \bar{u}_i \bar{u}_j \right) + \rho \frac{\partial}{\partial x_k} \left( \bar{U}_i \bar{u}_j + \bar{u}_i \bar{U}_j \right) = -\frac{\partial}{\partial x_k} \left[ \rho \bar{u}_i \bar{u}_j \mu + p \left( \partial_{i} \bar{u}_j + \partial_{j} \bar{u}_i \right) \right] + \frac{\partial}{\partial x_k} \left[ \mu \frac{\partial}{\partial x_k} \left( \bar{u}_i \bar{u}_j \right) \right] - p \left( \frac{\partial}{\partial x_k} \bar{U}_i \bar{U}_j + \frac{\partial}{\partial x_k} \left( \bar{u}_i \bar{u}_j \right) \right) + p \left( \frac{\partial}{\partial x_k} \bar{u}_i \bar{u}_j + \frac{\partial}{\partial x_k} \left( \bar{u}_i \bar{u}_j \right) \right) - 2 \mu \frac{\partial}{\partial x_k} \frac{\partial}{\partial x_k} \left( \bar{u}_i \bar{u}_j \right),
\]

where \( C_{ij} \) is the convective term, \( D_{ij} \) and \( D_{ij} \) are diffusion terms, \( P_{ij} \) is the production term, \( \Phi_{ij} \) is the pressure-strain tensor, \( \varepsilon_{ij} \) is the dissipation term. To close (1), similar to Reynolds stress, transport equations can be written for variables (turbulent diffusion \( D_{ij} \), pressure-strain tensor \( \Phi_{ij} \) and dissipation tensor \( \varepsilon_{ij} \)) as well, but it will add new unknown variables to the system of equations. Thus, only the transport equation for \( \varepsilon_{ij} \) is usually used in practice, while \( D_{ij} \) and \( \Phi_{ij} \) are approximated by models containing empirical parameters. Reliable experimental data are required to estimate these parameters. Such data are extremely difficult to obtain even for steady flows, since estimation of
turbulent energy diffusion and dissipation requires spatial resolution beyond the reach of the majority of measurement methods. For this reason, different approaches were used to derive turbulence models. Some have considerable theoretical basis, relying heavily upon the Navier-Stokes equations for their derivations, but many are strongly heuristic. Most have in common a liberal number of parameters that must be evaluated empirically. Almost none contains the stringent requirements of Galilean and tensor invariance, of parameter universality, and of history dependence, all of which are essential parts of our present theory [7].

In this paper we used the optical Smoke Image Velocimetry (SIV) technique [8] for measuring the flow velocity vector field dynamics, which has extensively tested. Using this method, turbulent characteristics were obtained in a developed turbulent boundary layer at a spatial resolution commensurate with the Kolmogorov scale [9]. The present work is the first step towards experimentally estimating variation of turbulent energy production, diffusion and dissipation in the Reynolds stress transport equation over the phase of forced flow rate pulsations.

2. Experimental setup and procedure

An experimental setup shown in figure 1 was used to study the boundary layer structure in the pulsating flow. Test section 1 of the experimental setup was a 1 m long rectangular channel with a cross section of 75×150 mm² and a 6:1 contraction ratio smooth inlet 10. Turbulence generating grid 9 with square cells was mounted downstream of the smooth inlet. The grid was made of 1.2-mm thick wire; the distance between the wire axes was 5 mm. A 50-mm long strip of abrasive 12 [10] was glued to the channel perimeter. This setup provided the fully developed turbulent boundary layer in the measurement area during the experiments. The channel walls were made of transparent materials (glass and polycarbonate). Stable air flow rate downstream of the test section was provided by a regulating gate 11 and a 1.3 m³ receiver tank 2. The flow rate was measured by an ultrasonic flowmeter IRVIS RS4-Ultra 3 mounted downstream of the receiver tank. The relative error of the flow rate did not exceed 1%. The average flow rate, frequency and amplitude of forced velocity pulsations were adjusted by a static and rotating flap in a pulsator 13.

To visualize the flow pattern, the air-aerosol mixture was supplied from the preparation chamber 4 to the channel inlet. This mixture represented MT-Gravity fluid with medium fog density and average particle size of 0.1…5 μm; Safex aerosol generator 5. The measurement area 6 was illuminated by a continuous diode-pumped solid-state laser KLM-532/5000-h 7. The flow pattern in the channel symmetry plane at the distance of $L = 0.7$ m from the turbulence generating grid was recorded by a monochrome high-speed camera Fastec HiSpec 8 with the frame resolution of 665×110 pixel (scaling factor of 0.0625 mm/pixel), frame rate $f = 7083$ 1/s, and recording time of 3 s.

Flow velocity fields were measured by the SIV optical method [8, 9], in which velocity fields are obtained from the analysis of turbulent structure displacements visualized by smoke. Period-averaged frame resolution in $y'$-coordinates (wall units) was 1 pixel = 0.8 y’, interrogation window size was 16×16 pixel, the period-averaged ratio of the resolved scale to the Kolmogorov length scale was 1.6.

![Figure 1. Experimental setup.](image-url)
3. Results and discussion

There is currently no conventional representation of turbulent characteristics in the boundary layer of pulsating flow. Instantaneous normalized turbulent production, diffusion and dissipation can be written in wall coordinates:

\[ P^* = P \frac{v}{U_* (\phi)}, \quad \varepsilon^* = \varepsilon \frac{v}{U_* (\phi)}, \quad D_T^* = D_T \frac{v}{U_* (\phi)}, \quad D_L^* = D_L \frac{v}{U_* (\phi)}. \]  

(2)

The phase-averaged friction velocity, \( U_*(\phi) \), is used for normalization in (2). Such a representation enables to analyze the effect of forced unsteadiness on turbulent energy transport in terms of comparison with the steady flow with its established concept of the budget of turbulent production, diffusion and dissipation in the turbulent kinetic energy transport equation.

Figure 2 shows the variation of the normalized production, diffusion and dissipation (ordered top-bottom) in the Reynolds stress transport equation over the period of flow rate pulsation (the zero value of phase angle was taken as the angle when the freestream velocity reaches the average velocity in acceleration phase) for tensor components \( u'u' \), \( u'v' \), and \( v'v' \) (where \( u \) is streamwise velocity component and \( v \) is wall-normal velocity component). Similar results obtained in steady boundary layer are shown to the right of unsteady flow images with the same color palette for comparison.

Maximum values of production and dissipation terms of the turbulent kinetic energy transport equation (Figure 2) correspond to the phase angle range of 250° to 315°, i.e. to the range within the streamwise pressure gradient (flow acceleration) changes its sign. Turbulent production \( P^*(u'u') \) and dissipation \( \varepsilon^*(u'u') \) attain their maxima around these phase angles. These maxima can reach almost five times the values observed in steady boundary layer. On the other hand, \( P^*(u'u') \) and \( \varepsilon^*(u'u') \) are minimal in the phase of flow rate maximum (90°). Nevertheless, the ratio \( P^*/\varepsilon^* \) is generally constant over \( y' \)-coordinate during the whole period, and moreover, it is similar to the distribution in steady turbulent boundary layer.

From Figure 2 it follows that forced flow pulsations can lead to drastic redistribution of turbulent diffusion of velocity fluctuations, \( D_T^* \), over the boundary layer thickness and pulsation phase. For example, while turbulent transport of \( u'u' \) fluctuation energy in steady flow is directed from the peak
production zone $y^+ = 10...15$ (sign of $D_T^+$ changes in this zone) towards both the wall and the core flow, the sign of $D_T^+$ in pulsating flow changes with the phase of forced pulsations.

4. Conclusions

It has been shown in experiments that forced velocity pulsations at the external edge of the boundary layer can lead to drastic redistribution of terms of the turbulent kinetic energy transport equation over the phase of pulsations and boundary layer thickness. During the period of the forced flow oscillations, the values of the production and dissipation terms of turbulence energy change more than five times, reaching a maximum level in the phase of the flow deceleration, and the diffusion term of the turbulent pulsations changes sign. It is hoped that the obtained results on the evolution of the terms of turbulent energy transport equation over the phase of forced flow pulsations will be useful to theorists in their work on improving turbulence models with respect to unsteady flows.

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