Application of Principles of Theory of Damage Accumulation to Calculation of Asphalt-Concrete Coatings

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Abstract. It was found that, when exposed to the repeated loads, the asphalt concrete pavement of roads accumulates damage which must be considered in their calculation. By entering the damage measuring theory (L.M. Kachanova continuity, damage Rabotnova Y.N.) and applying the principle of equivalence of a continuous state of stress and damage caused by media the authors modified the criteria proposed by Pisarenko-Lebedev and Drucker-Prager. Such modified criteria take into account the effect of damage accumulation in the calculation of asphalt concrete pavement on the tensile strength in bending and shear. To determine the functional dependence of damage and continuity on the number of applied loads, the authors used the principle of energy equivalence for continuous and damaged media.

The theory of damage accumulation takes into account the effect of fatigue processes on the stress-strain state of the material. The foundations of this theory are developed by L.M. Kachanov and Yu.N. Rabotnov and their generalization performed by Lemaître D. Measures of the theory of accumulation of damages are two scalar quantities the continuity L.M. Kachanova \( \psi \) and the damage Yu.N. Rabotnova \( \omega \). The effect of damage accumulation is taken into account by applying the summation principles: linear (Palmgren - Miner's conjecture), bilinear (S.S. Manson's hypothesis) and nonlinear (Richard - Newmark hypothesis). The application of these principles makes it possible to represent the damage as a function of number of repeated loads and use this dependence when calculating materials subjected to cyclic loads. Thus, the development of methods for calculating road surfaces is based on the theory of damage accumulation is an actual task of the road industry.

For materials in the structure of which there are damages, for example pores, special criteria of strength and plasticity conditions have been developed [1-8], presented in the table 1. These criteria are developed for porous metals, as a result of which they may be inapplicable to the calculation of asphalt-concrete coatings to resistance to fatigue stretching from bending and shear resistance. Therefore, in the works [9-11] we proposed a method for modifying the criteria for the strength and plasticity of solid bodies, based on the principle of the equivalence of the stressed state of a damaged and undamaged body. The essence of the method is, that in accordance with the principle of stress equivalence, any component of the stress tensor of a damaged body \( \sigma_{ijw} \) can be determined by formula [11]:

\[
\sigma_{ijw} = \frac{\sigma_{ij}}{1 - \omega} = \frac{\sigma_{ij}}{\psi}
\]
where \( \sigma_{ij} \) – components of the stress tensor of a continuous body; \( \omega \) – damage Yu.N. Rabotnova, \( \psi \) – continuity L.M. Kachanova.

### Table 1. Criteria including measures of damage theory.

| Authors | Mathematical representation of the criterion |
|---------|---------------------------------------------|
| 1. Criteria Shima - Oyane [2] | \[
\frac{3 \cdot J_2}{\sigma^2_{Y_i}} + a \cdot n^b \cdot \left(\frac{I_1}{3 \cdot \sigma^2_{Y_i}}\right)^2 - (1 - n)^3 = 0
\]
| | where \( I_1 \) and \( J_2 \) – the first invariant of the stress tensor and the second invariant of the stress deviator, \( \sigma_{Y_i} \) accordingly; \( a \) and \( b \) – material parameters; \( \sigma_{Y_i} \) – yield strength of a material with zero porosity (Non-porous material); \( n \) – porosity. |
| 2. Gurson's criterion [3] | \[
\frac{3 \cdot J_2}{\sigma^2_{Y_i}} + 2 \cdot n \cdot \cosh \left(\frac{I_1}{2 \cdot \sigma^2_{Y_i}}\right) - (1 + n)^2 = 0
\]
| 3. Tvergaard's criterion [4, 5] | \[
\frac{3 \cdot J_2}{\sigma^2_{Y_i}} + 2 \cdot a \cdot n \cdot \cosh \left(\frac{b \cdot I_1}{2 \cdot \sigma^2_{Y_i}}\right) - (1 + (a \cdot n))^2 = 0
\]
| 4. Lee - Oung criterion [6] | \[
3 \cdot J_2 + 0.25 \cdot n \cdot I_1^2 + (1 - n)(R_{0_c} - R_{0_t}) \cdot (-I_1) = R_{0_c} \cdot R_{0_t} \cdot (1 - n)^2
\]
| where \( R_{0_c} \) and \( R_{0_t} \) – strength of uniaxial compression and tensile equivalent to non-porous (undamaged) material. |
| 5. Sofronis criterion [7] | \[
2 \cdot J_2 \cdot \cos \theta = \left(\frac{1 + n}{1 - n}\right)^{\frac{m}{1 - \psi}} \cdot \left(\frac{1 - \left(\frac{m \cdot n}{1 - \psi}\right)^{\frac{2}{1 - \psi}} \cdot \left(\frac{I_1}{2 \cdot m}\right)^2}{1 + n}\right)^{\frac{m}{1 - \psi}}
\]
| where \( m \) – material parameter |
| 6. Litvinskij G.G. criterion [8] | \[
\tau_{st} = \tau_0 \cdot \psi \cdot \left(\frac{\sigma_{st}}{\sigma_0 \cdot \psi} + 1\right)^{\alpha \psi}
\]

From dependence (1) follows, that the main stresses \( \sigma_{1w}, \sigma_{2w} \) and \( \sigma_{3w} \) in the damaged environment defined by the formulas

\[
\sigma_{1w} = \frac{\sigma_1}{1 - \omega} = \frac{\sigma_2}{\psi} \cdot \sigma_{2w} = \frac{\sigma_3}{1 - \omega} = \frac{\sigma_4}{\psi} \cdot \sigma_{3w} = \frac{\sigma_5}{1 - \omega} = \frac{\sigma_6}{\psi}
\] (2)

where \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) – The main stresses in the undamaged body.

Dependencies (2) can be used to calculate the characteristics of the tensor and stress deviator in a damaged body. The main such characteristics are given in table 2 [9].

Applying the formulas of table 2 in the equations of limiting state of the strength criteria and plasticity criteria of undamaged bodies, they can be modified, that one of the parameters of the material will be damage or continuity. Such strength criteria and plasticity conditions are given in table 3 and table 4.

From the analysis of the equations of limiting states presented in table 3 and table 4 follows, that the task of calculating the asphalt-concrete coating is reduced to calculating the damage or continuity, and also the characteristics of the tensor and deviator stress. When calculating asphalt-concrete coatings and the bases for the bending tensile, the components of the stress tensor can be calculated from the formulas of M.B. Korsunsky. When checking the resistance to shear of asphalt-concrete coating, the main stresses can be determined using the method described in the work [14].
Table 2. Characteristics of the stressed state of a damaged body.

| Name of the characteristic | Formula |
|----------------------------|---------|
| Maximum tangential stresses | \[\tau_{\text{max,\omega}} = \frac{\tau_{\text{max,\omega}}}{1 - \omega} = \frac{\tau_{\text{max,\omega}}}{1 - \omega} = \frac{\sigma_1 - \sigma_3}{2 \cdot (1 - \omega) / \psi} \\]

where \(\tau_{\text{max,\omega}}\) – the maximum tangential stress in an undamaged continuous body, Pa.

| Octahedral normal stress \(\sigma_{\text{oct,\omega}}\) and tangent \(\tau_{\text{oct,\omega}}\) stresses |
|-----------------------------------------------|
| \[\sigma_{\text{oct,\omega}} = \frac{\sigma_{\text{oct,\omega}}}{1 - \omega} = \frac{\sigma_{\text{oct,\omega}}}{1 - \omega} = \frac{\tau_{\text{oct,\omega}}}{\psi} \] |

where \(\sigma_{\text{oct,\omega}}\) and \(\tau_{\text{oct,\omega}}\) – normal and tangential stresses on octahedral platform of solid body, Pa.

| Intensity of normal \(\sigma_{\text{ln,\omega}}\) and tangential \(\tau_{\text{ln,\omega}}\) stresses |
|-----------------------------------------------|
| \[\sigma_{\text{ln,\omega}} = \frac{\sigma_{\text{ln,\omega}}}{1 - \omega} = \frac{\sigma_{\text{ln,\omega}}}{1 - \omega} = \frac{\tau_{\text{ln,\omega}}}{\psi} \] |

where \(\sigma_{\text{ln,\omega}}\) and \(\tau_{\text{ln,\omega}}\) – intensity of normal and tangential stresses in a continuous body, Pa.

| Invariants of the stress tensor (first \(I_{\text{1,\omega}}\), second \(I_{\text{2,\omega}}\) and third \(I_{\text{3,\omega}}\)) |
|-----------------------------------------------|
| \[I_{\text{1,\omega}} = \frac{I_1}{1 - \omega} = \frac{I_1}{1 - \omega} = \frac{I_2}{\psi^3} \] |

where \(I_1\), \(I_2\) and \(I_3\) – first, second and third invariant of the stress tensor of a continuous body, Pa, Pa² and Pa³ accordingly.

| The second invariant of the stress deviator |
|-------------------------------------------|
| \[J_{\text{2,\omega}} = \frac{1}{(1 - \omega)^2} \left( I_2 - 1 \cdot I_1^2 \right) = \frac{1}{\psi^2} \left( I_2 - 1 \cdot I_1^2 \right) \] |

| The third invariant of the stress deviator |
|-------------------------------------------|
| \[J_{\text{3,\omega}} = \frac{1}{(1 - \omega)^3} \left( I_3 - 1 \cdot I_1 \cdot I_2 + \frac{2}{27} \cdot I_1^3 \right) \] |

Table 3. Modified strength criteria for the calculation of asphalt-concrete coatings and bases at bending tensile: [9,11,12]

| The name of the original criterion | The equation of the limiting state of the modified criterion |
|-----------------------------------|----------------------------------------------------------|
| O. Mora                           | \[\frac{\sigma_1 - R_{\text{c}} \cdot \sigma_3}{1 - \omega} = R_t \text{ or} \] |
| Pisarenko-Lebedev                 | \[\frac{\sigma_1 - R_{\text{c}} \cdot \sigma_3}{\psi} = R_t, \] |

where \(R_c\) and \(R_t\) – tensile strength of the uniaxial compressive and tensile from bending, Pa.

\[\frac{1}{1 - \omega} \left[ \chi \cdot \sigma_t + (1 - \chi) \cdot \sigma_r \right] = R_c; \quad \chi = R_t / R_c, \]

where \(\chi\) – coefficient of material plasticity, characterizing the degree of responsibility for microdamage of shear deformation, creating favorable conditions for the loosening of the material and the formation of cracks [12].
\[
\sigma_2 \leq \frac{\sigma_1 + k \cdot \sigma_3}{1 + k}
\]
\[
\frac{1}{1 - \omega} \left( \sigma_1 - \frac{k \cdot (b \cdot \sigma_2 + \sigma_3)}{1 - b} \right) = R_c, \quad k = \frac{R_i}{R_c};
\]
\[
b = \frac{R_i \cdot \tau_{lim}}{(R_f - \tau_{lim})} \cdot R_c.
\]
\[
\sigma_2 \geq \frac{\sigma_1 + k \cdot \sigma_3}{1 + k}
\]
\[
\frac{1}{1 - \omega} \left( \sigma_1 + \frac{b \cdot \sigma_2 - k \cdot \sigma_3}{1 + b} \right) = R_c
\]

Table 4. Modified plasticity conditions for the calculation of asphalt-concrete coatings by shear resistance. [9-11,13]

| The name of the original condition | The equation of the limiting state of the modified criterion |
|----------------------------------|----------------------------------------------------------|
| Coulomb - Mora                  | \( \left( \frac{1}{\cos \phi} - \frac{\sigma_1 - \sigma_3}{2} - \tan \phi \cdot \frac{\sigma_1 + \sigma_3}{2} \right) \cdot \frac{1}{1 - \omega} = c \) , |
| where \( c \) and \( \phi \) – adhesion and angle of internal friction asphaltic, Pa and rad. accordingly. |
| Drucker - Prager                | \( \frac{1}{(1 - \omega)} \cdot \left( I_2 - \frac{1}{3} \cdot I_1^3 \right) - a \cdot \frac{I_1}{1 - \omega} - k = 0 \) |
| where \( a \) and \( k \) – material parameters, associated with the angle of internal friction and adhesion. |
| Lade - Duncan                   | \( \frac{I_1}{1 - \omega} - k_{L-D} \cdot \frac{I_3}{(1 - \omega)^2} = 0 \) |
| Matsuoka - Nakai                | \( \frac{I_1}{1 - \omega} \cdot \frac{I_2}{(1 - \omega)^2} - k_{M-N} \cdot \frac{I_3}{(1 - \omega)} = 0 \) |
| Modified three-parameter criterion Coulomb-Mora | \( \frac{1}{2 \cdot (1 - \omega)} \cdot \left( \sigma_1 \cdot \left( \frac{1 - \sin \phi}{1 + \sin \phi} \right)^d - \left( \frac{1 + \sin \phi}{1 - \sin \phi} \right)^d \cdot \sigma_3 \right) = c \) |
| where \( d \) – material parameter, depends on the deformation, accepted for the maximum value when performing triaxial tests. |

Currently, there are two known approaches to the calculation of damage. The first approach is based on the application of various hypotheses of summation of damages (linear, bilinear and nonlinear), it is used when calculating materials for the effect of cyclic loading. The principle of linear summation of Palmgren-Miner was applied by E. V. Uglova [15] to calculate the damage to asphalt concrete pavement from the impact of mobile loads. The first approach is designed to predict the increase in damage when subjected to a prolonged load. It is based on the application of various principles of the equivalence of states of a continuous and damaged medium. Among these principles, the most frequently used principle is the equivalence of deformations and energy equivalence [16,17]. In accordance with these principles, the damage is a function of the ratio of the modulus of elasticity of the damaged \( E_D \) and undamaged \( E \) material and is determined by the formulas [16,17]:

\[
\omega = 1 - \frac{E_D}{E}, \quad \psi = 1 - \sqrt{\frac{E_D}{E}} \tag{3}
\]

To determine continuity, it is necessary to use relations [18]:

\[
\psi + \omega = 1; \quad \omega = 1 - \psi; \quad \psi = 1 - \omega \tag{4}
\]
Substituting dependencies (3) in the third expression (4), we get:

$$\psi = \frac{E_D}{E}, \quad \psi = \sqrt{\frac{E_D}{E}}$$  \hspace{1cm} (5)

The first dependency formulas (3) and (5) are based on the application of the principle of equivalence of deformations of a damaged and intact body, and the second formulas of these dependences are a consequence of the application of the principle of energy equivalence.

In this paper, the authors will attempt to develop a calculation of the modulus of elasticity of a damaged material under the action of a repeated (cyclic) load and the subsequent application of the obtained dependence in the fundamental formulas (3) and (5). In such a relationship, the modulus of elasticity of the damaged medium must be a function of the number of design loads.

For mathematical modeling, we introduce the assumptions, that all loads are the same in magnitude, applied pressure, and duration of exposure. In the practice of designing road clothes, this assumption is realized by reducing all transport loads to the estimated, under which the load is taken $A_1$, $A_2$ or $A_3$. Therefore, the assumption made by us corresponds to the requirement of normative documents, and therefore can be used in mathematical modeling.

The process of accumulating damage is continuous because the damage increases monotonically with each subsequent load. Therefore, for mathematical modeling of the increase in damage and the reduction of continuity, it is possible to apply integral equations of hereditary theories. Analyzing formulas (3) and (5) we note, that the modulus of elasticity of the damaged medium should decrease as the number of repeated loads increases.

Therefore, to determine the function of change the modulus of elasticity, we can also use the integral equations of heredity. The integrand function, determining increment reduce the elastic modulus from the load application, having a sequence number $n$, is given by the exponent equation:

$$\Delta E_D = a \cdot n^b$$  \hspace{1cm} (6)

where $a$ and $b$ – parameters materials, depending on the type of asphalt concrete and being a complex function of the temperature of asphalt concrete, coefficient of compaction and etc.

Composing an integral equation, we obtain a certain improper integral in the form

$$E_{DN} = E \cdot \left(1 - a \cdot \frac{1}{b+1} \right)$$  \hspace{1cm} (7)

In that case, if $b\neq1$, we get

$$E_{DN} = E \cdot \left(1 - a \cdot \frac{N^{b+1} - 1}{b+1} \right)$$  \hspace{1cm} (8)

Substituting (8) in depending (3), we get, that the damage is determined by the formulas:

$$\omega = a \cdot \frac{N^{b+1} - 1}{b+1}; \quad \omega = 1 - \sqrt{1 - a \cdot \frac{N^{b+1} - 1}{b+1}}$$  \hspace{1cm} (9)

Analyzing these works [16,17] we will point out, that currently the preference is given to the dependencies of the calculation of the damage, obtained using the principle of energy equivalence. Therefore, in the future it is expedient to apply the second formula (9). Substituting this dependence in the third expression of formulas (4), we get:

$$\psi = \sqrt{1 - a \cdot \frac{N^{b+1} - 1}{b+1}}$$  \hspace{1cm} (10)
In this way, to calculate the damage it is necessary to apply the second dependence of the expressions (9), and to calculate the continuity of the formula (10).

The material parameters must be established by laboratory tests. The purpose of such tests should be to determine the modulus of elasticity when subjected to repeated loads. And for definition of parameters of a material of layers on resistance to fatigue of stretching from bending, in the laboratory it is necessary to experiment on the bending of asphalt concrete beams by cyclic loading. To define the parameters material, used in the calculation of asphalt concrete coatings on the resistance to shear, it is necessary to perform testing of cylindrical samples repeatedly applied by a triaxial load. Such tests are realized in dynamic triaxial compression devices.

Determination the parameter of materials is the task of further research by the authors, and their results will be given by us in the following publications.

In conclusion, we note, that as a result of theoretical research:
1. Established, that for mathematical modeling of the change in continuity and damage during the action of repeated loads, may be used the principles of the equivalence of deformations of a damaged and continuous medium, and also the principle of their energy equivalence, in which the integral equations of hereditary theories are used to calculate the modulus of elasticity of a damaged material.
2. Established, that the increment of the decrease in the elastic modulus as a result of the action from the n-th load is described by the power function.
3. Integrating equations obtained exponential mathematical models (9) and (10), which allow with sufficient accuracy to determine the damage and continuity when subjected to repeated loads.
4. The tasks of further publications are:
   • development of experimental techniques and their application for the determination parameters of material, depending on the type of asphalt concrete, temperatures, coefficient of compaction and etc., and their use in models (9) and (10).
   • modification of the calculation of asphalt-concrete coatings and bases by the criterion of resistance to fatigue stretching in which use the criteria, presented in table 3, and dependencies (9) and (10).
   • the development of a new method for calculating asphalt-concrete coatings for shear resistance, in which the plasticity conditions are applied, presented in table 4, as well as dependencies (9) and (10).

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