Spin-wave filter and Doppler shift in ferrimagnetic domain walls

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[Abstract] Excitation and propagation of spin waves in magnetic domain walls has received intensive attention because of their potential in spintronic and spin-wave applications. In this work, we theoretically and numerically study spin-wave dynamics in ferrimagnetic domain walls. The results demonstrate that the spin-wave dispersion depends on net angular momentum $\delta_s$ and wave handedness. For a positive $\delta_s$, a gapless dispersion is observed for left-handed spin waves, while a forbidden frequency gap appears for right-handed spin waves. To some extent, this work suggests that ferrimagnetic domain wall could serve as a spin-wave filter in which only spin waves with particular handedness and frequencies can be excited and propagated. Importantly, the energy consumption for spin-wave excitation in the domain wall is much lower than in the uniform state, while the group velocity is much faster, demonstrating the advantages of domain walls in serving as waveguides. Moreover, current-induced spin-wave Doppler shift in ferrimagnetic domain wall is also revealed, and the dependence of shift magnitude on $\delta_s$ is investigated. Thus, this work unveils interesting spin-wave dynamics in ferrimagnetic domain walls, benefiting future spin-wave applications.

Keywords: spin waves, domain wall, ferrimagnets, Doppler shift

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I. Introduction

Conventional semiconductor devices transmit and process information with electric current. However, flow of charge causes power consumption due to Joule heating, which also affects operating stability of related devices [1,2]. Recently, spin-wave devices have received much attention as a plausible complement to conventional semiconductor electronics [3–6]. Instead of charge transport, spin wave is used as information carriers in these devices, which is free of the Joule heating [7–10]. Moreover, spin-wave devices allow combining memory and computing into a single hardware unit through combining nonvolatile storage capabilities of magnetic materials with high-frequency data processing in THz regime [11–13]. Thus, it could break the bottleneck of modern CPU and memory architectures and play an essential role in novel information processing [14].

As a matter of fact, spin-wave devices have been proposed theoretically and experimentally, including the nanoscale neural network [15], transistors [6], logic gates [16] and multiplexers [7], where the information is encoded in amplitude, frequency, and phase of spin waves [14,17]. Considering information processing, waveguide structures can guide spin-wave propagation without any disturbances between neighboring channels [17–20], which is promising for future applications. For example, the easy-axis surface anisotropy has been proven to work as a waveguide from the surface spin-wave mode [17,21], while the fabrication could be a high requirement for technology. Along this line, magnetic structures such as domain wall can be alternative choice of waveguides [22,23], noting that these structures are generally with small sizes and can be easily modulated through various methods.

Actually, magnetic domain walls as natural waveguides in various materials have been reported theoretically and experimentally [24–29]. In ferromagnets, spin waves with low frequency, which appear in the gap of bulk spin wave modes, can be excited in domain walls and guided in curved geometries [24]. However, due to the time-reversal symmetry, spin wave can be only right-circularly polarized [14,30], and perturbing stray fields exist in ferromagnets [31–36], which are disadvantageous for high-density device integration. In antiferromagnets, on the other hand, both left- and right-handed spin waves can exist [37], providing additional polarization degrees of freedom, besides their zero stray field and fast magnetic dynamics [37,38]. Interestingly, spin wave dispersion of antiferromagnetic domain wall is also gapless, and the group velocity of spin wave is much higher than in
ferromagnets [25]. Despite these advantages of antiferromagnets in spintronic applications, it is still challenging to experimentally control antiferromagnetic dynamics due to their zero net magnetization [39,40].

Interestingly, ferrimagnets can be used to overcome these disadvantages of ferromagnets and antiferromagnets, due to the fact that they have a limited net magnetic moment and rapid dynamics in the vicinity of angular momentum compensation temperature ($T_A$) [41,42]. So, the magnetic states at $T_A$ can be effectively detected and addressed by conventional methods. Moreover, spin waves also have a full polarization degrees of freedom in ferrimagnets where two inequivalent magnetic sublattices are coupled antiferromagnetically [37,42]. Recently, spin wave propagation in ferrimagnetic uniform state of one-dimensional system has been concerned, which demonstrates that the excitation of spin waves requires a high frequency [37].

On the one hand, in ferrimagnets, the frequency gap could also diminish through domain walls as waveguides, which is definitely meaningful for future applications. Importantly, the gap probably depends on the spin-wave handedness due to the fact that the sign of spin polarization carried by spin waves is determined by the spin-wave handedness. On the other hand, it is worth noting that spin-wave dynamics in ferrimagnetic state can be modulated by current-induced spin-transfer torques [37], which could be used to measure material parameters of ferrimagnets. To some extent, similar property could be available for spin waves channeled in ferrimagnetic domain walls. Thus, considering the advantages of ferrimagnets, dynamics of spin wave propagation in ferrimagnetic domain walls urgently deserves to be clarified. However, as far as we know, few works on this subject have been reported.

In this work, we theoretically and numerically study the excitation and propagation of spin waves in a ferrimagnetic domain wall. It is demonstrated that the dispersion relation of spin waves depends on net angular momentum $\delta_s$ and the spin-wave handedness (left- and right-handed). For a finite $\delta_s$, a gapless dispersion is observed for particular spin-wave handedness, while a forbidden frequency gap of dispersion exists for spin waves with the other handedness. Thus, this property allows ferrimagnetic domain walls serve as spin-wave filter in which only spin waves with a particular handedness can be excited and propagated. Moreover, current-induced spin-wave Doppler shift in ferrimagnetic domain wall is also revealed, and the dependence of the magnitude of shift on $\delta_s$ is discussed.
II. Analytical analysis and numerical simulation

We consider a two-dimensional ultrathin ferrimagnetic film in the $xy$ plane, whose magnetic structure is shown in Fig. 1, where the two inequivalent magnetic sublattices are coupled antiferromagnetically. A domain wall with the axis along $y$ separates two uniformly domains, and a sinusoidal excitation field is applied to excite spin waves which propagate along the $x$ direction.

![Illustration of ferrimagnetic domain walls of Bloch type. External microwave fields are applied to excite spin waves that propagate along the $x$ direction.](image)

2.1. Analytical treatment

We introduce two unit vectors $m_1$ and $m_2$ to denote spins at two sublattices, and define the staggered vector $n = (m_1 - m_2)/2$ and averaged magnetization field $m = (m_1 + m_2)/2$. $\gamma_{1,2}$ and $\alpha_{1,2}$ are the gyromagnetic ratio and the Gilbert damping constant, respectively. Thus, the spin density of sublattice $i$ ($i = 1, 2$) is given by $s_i = M_i/\gamma_i$ with $\gamma_i = g_i\mu_B/\hbar$, where $M_i$ is the saturation magnetization, $g_i$ is the Landé factor, and $\mu_B$ is the Bohr magneton.

Following the earlier work, the Lagrangian density for ferrimagnets is given by [41,42]

$$ L = -s\dot{n} \cdot (n \times m) - \delta_s a(n) \cdot \dot{n} - U, \quad (1) $$

where $s = (s_1 + s_2)/2$ is the staggered spin density, $\delta_s = s_1 - s_2$ is the net angular momentum, and $a(n)$ is the vector potential of a magnetic monopole satisfying $\nabla \times a = n$. The potential energy density $U$ is given by [39]

$$ U = \frac{A}{2} (\nabla n)^2 + \frac{a}{2} m^2 + L m \cdot \partial_z n - \frac{K}{2} (\dot{z} \cdot n)^2, \quad (2) $$

where $A$ is the inhomogeneous exchange, $a$ is the homogeneous exchange, $L$ is the parity-
breaking exchange term, and $K$ is the effective easy-axis anisotropy. The Rayleigh dissipation function is given by $R = s_\alpha \mathbf{a}^2/2$ where $s_\alpha = \alpha_1 s_1 + \alpha_2 s_2$ is a phenomenological parameter quantifying the energy and spin loss due to the magnetic dynamics. For simplicity, we assume $\alpha_1 = \alpha_2 = \alpha$ and neglect nonlocal dipolar interaction considering small net magnetization in the investigated systems. In the presence of spin current, the dynamic governing equations obtained by solving the above equations for $\mathbf{n}$ and $\mathbf{m}$ are given as [37]

$$\dot{\mathbf{n}} = -\frac{1}{s} f_m \times \mathbf{n} + T_{\text{STT}}^n,$$  
(3a)

$$\dot{\mathbf{m}} = -\frac{1}{s} f_n \times \mathbf{n} + 2\alpha \mathbf{m} \times \mathbf{m} - \frac{\delta}{s} \mathbf{n} + T_{\text{STT}}^m,$$  
(3b)

where $f_n$ and $f_m$ are the effective fields associated with $\mathbf{n}$ and $\mathbf{m}$, respectively. The effective fields with the aid of suitable Lagrangian multipliers subject to the aforementioned constraints leads to $f_m = -\partial \mathcal{U}/\partial \mathbf{m} = -a \mathbf{m}$ and $f_n = -\partial \mathcal{U}/\partial \mathbf{n} = A \mathbf{n} \times (\nabla^2 \mathbf{n} \times \mathbf{m}) + K(\mathbf{n} \cdot \hat{z}) \mathbf{n} \times (\hat{z} \times \mathbf{n})$. The spin-transfer torques (STT) modulating $\mathbf{n}$ and $\mathbf{m}$ dynamics read $T_{\text{STT}}^n = -(b_j^+ \partial \mathbf{n}/\partial x + \beta b_j^+ \mathbf{n} \times \partial \mathbf{n}/\partial x)/2$ and $T_{\text{STT}}^m = -b_j^+ \partial \mathbf{n}/\partial x - \beta b_j^+ \mathbf{n} \times \partial \mathbf{n}/\partial x$, respectively, where $b_j^+ = -\mu_b (P_1 g_1/M_1 \pm P_2 g_2/M_2) J_e^2 e$ is the magnitude of adiabatic spin torque, $P_i$ is the spin polarization, $J_e$ is the current density, and $\beta$ is the nonadiabaticity.

Subsequently, we study the propagation of spin waves in ferrimagnetic domain walls. We ignore the damping team and consider a small fluctuation of the staggered vector $\mathbf{n}$ around initial $\mathbf{n}_0$, $\mathbf{n} = \mathbf{n}_0 + \delta \mathbf{n}(x, t)$ with spin position $x$ and time $t$. Then, the equations of motion for $\mathbf{n}$ and $\mathbf{m}$ are obtained as

$$\ddot{\mathbf{n}} = \frac{a}{s} \mathbf{m} \times \mathbf{n}_0,$$  
(4a)

$$\ddot{\mathbf{m}} = -\frac{\delta}{s} \mathbf{n} - b_j^+ \frac{\partial \mathbf{n}}{\partial x} - \beta b_j^+ \mathbf{n} \times \frac{\partial \mathbf{n}}{\partial x} - \frac{1}{s} \left[A(\nabla^2 \mathbf{n}_0 \times \mathbf{n}_0 + \nabla^2 \mathbf{n}_0 \times \delta \mathbf{n} + \delta \mathbf{n} \times \mathbf{n}_0) - K \left((\mathbf{n}_0 \cdot \hat{z}) \hat{z} \times \mathbf{n}_0 + (\mathbf{n}_0 \cdot \hat{z}) \hat{z} \times \delta \mathbf{n} + (\delta \mathbf{n} \cdot \hat{z}) \hat{z} \times \mathbf{n}_0)\right]\right]$$  
(4b)

We consider the Walker ansatz for a Bloch-type domain wall profile centered at $y = 0$ as shown in Fig. 1: $\mathbf{n}_0 = [\text{sech}(y/\Delta), 0, \tanh(y/\Delta)]$ [25] with the domain wall width $\Delta = (A/K)^{1/2}$. Similarly, we transform the coordinates with mutually orthogonal basis vectors $\mathbf{j} = (0, 1, 0)$ and $\mathbf{i} = \mathbf{j} \times \mathbf{n}_0 = [\tanh(y/\Delta), 0, -\text{sech}(y/\Delta)]$ to express the fluctuating fields $\delta \mathbf{n}$ and $\mathbf{m}$ in the plane perpendicular to $\mathbf{n}_0$. Considering the monochromatic waves propagate along the $x$ direction, one obtains $\delta \mathbf{n} = [n(y) \mathbf{i} + n(y) \mathbf{j}] e^{ikx - i\omega t}$, $\mathbf{m} = [m(y) \mathbf{i} + m(y) \mathbf{j}] e^{ikx - i\omega t}$. By arranging Eq. (4) on the
basis of \( \mathbf{i} \) and \( \mathbf{j} \), one obtains
\[
-i\omega n_i (-n_j), \quad (5a)
\]
\[
-i\omega n_i (-n_j) = \mathcal{A}_i \n_i (n_j) + \mathcal{A}_j \n_j (n_i) + s\beta \mathcal{B}_i \n_i (n_j) - s\beta \mathcal{B}_j \n_j (n_i)
\]
\[
\begin{align*}
&= \left[ A \frac{d^2 n_i (n_j)}{dy^2} - Ak^2 n_i (n_j) + K \left( \text{sech}^2 \left( \frac{y}{\Delta} \right) - \tanh^2 \left( \frac{y}{\Delta} \right) \right) n_j (n_i) \right],
\end{align*}
\]
\[(5b)\]

Eqs. (5a) describes the dynamics of small variations \( \delta \mathbf{n} \) in the \( \mathbf{i} \) and \( \mathbf{j} \) directions, and Eqs. (5b) describes to the dynamics of the canted magnetization \( \mathbf{m} \) in the \( \mathbf{i} \) and \( \mathbf{j} \) directions. Moreover, we eliminate \( m_i \) and \( m_j \) through coupling Eq. (5a) to Eq. (5b), we obtain
\[
\begin{align*}
&-\delta_i n_i (n_j) - sb_j n_i (n_j) + s\beta \mathcal{B}_j n_i (n_j) \\
&= -A \frac{d^2 n_i (n_j)}{dy^2} + Ak^2 n_i (n_j) - K \left( 2 \text{sech}^2 \left( \frac{y}{\Delta} \right) - 1 \right) n_j (n_i) - \rho \omega^2 n_j (n_i),
\end{align*}
\]
\[(6a)\]
\[
\begin{align*}
&-\delta_j n_i (n_j) + sb_j n_i (n_j) + s\beta \mathcal{B}_i n_i (n_j) \\
&= A \frac{d^2 n_i (n_j)}{dy^2} - Ak^2 n_i (n_j) + K \left( 2 \text{sech}^2 \left( \frac{y}{\Delta} \right) - 1 \right) n_i (n_j) + \rho \omega^2 n_i (n_j),
\end{align*}
\]
\[(6b)\]

where \( \rho = s^2/a \) is the inertia of the dynamics, \( d \) is lattice spacing. By defining a complex field as \( \psi_{\pm} = n_i \mp in_j \) for right- and left- handed spin waves, Eq. (6a) and Eq. (6b) can be linearized and updated to
\[
\begin{align*}
p^2 \psi_{\pm} (\xi) &= \left( -\frac{d^2}{d\xi^2} - 2 \text{sech}^2 \xi \right) \psi_{\pm} (\xi),
\end{align*}
\]
\[(7)\]

with \( \xi = y/\Delta \) and \( p^2 = (\rho \omega^2 \pm \delta_{\pm} \omega_{\pm} - Ak^2 \pm sb_j k)/K - 1 \). Equation (7) has two types of solutions: the exact solutions for the uniform state (ferrimagnetic state) and the domain wall, which correspond to \( p = 0 \) and \( p = -i \), respectively [27,30]. Then, we obtain the dispersion relation given by
\[
\begin{align*}
\omega_{\pm}^{\text{uni}} &= \frac{\pm \delta_{\pm} + \sqrt{\delta_{\pm}^2 + 4 \rho (Ak^2 \mp sb_j k + K)}}{2 \rho},
\end{align*}
\]
\[(8a)\]
\[
\begin{align*}
\omega_{\pm}^{\text{dw}} &= \frac{\pm \delta_{\pm} + \sqrt{\delta_{\pm}^2 + 4 \rho (Ak^2 \mp sb_j k)}}{2 \rho}.
\end{align*}
\]
\[(8b)\]

Eqs. (8a) and (8b) describe the dispersion relation for propagation of the spin waves in ferrimagnets with the uniform state and domain wall, respectively, where the upper (lower) sign corresponds to the right-circularly (left-circularly) polarized spin waves.

Obviously, for the uniform state, there is a frequency gap caused by the magnetic anisotropy,
which suppresses the excitation of spin waves. Importantly, the hidden frequency gap does not exist in the domain wall state for spin waves with particular handedness (left-handed spin wave for a positive $\delta_s$, for example, $\omega_{DW} = 0$ for $k = 0$), allowing the excitation of spin waves with a finite frequency. Thus, ferrimagnetic domain walls can also serve as waveguides, which provides channels for the propagation of spin wave. Moreover, spin-wave Doppler shift is available because finite $b_j$ is related to current.

In the absence of electric current $b_j = 0$, the group velocity of spin wave can be derived from the dispersion relation and given by

$$v_{UNI} = \frac{2Ak}{\sqrt{\delta_s^2 + 4\rho(Ak^2 + K)}}, \quad (9a)$$

$$v_{DW} = \frac{2Ak}{\sqrt{\delta_s^2 + 4\rho(Ak^2)}}, \quad (9b)$$

It is clearly shown that for any possible $k$, spin waves in domain wall propagate faster than in the uniform state, which demonstrates again the advantage of domain walls as waveguides. Moreover, for $\delta_s = 0$ at $T_A$, $v_{DW}$ is independent of the spin-wave frequency and vector, the same as the cases of antiferromagnets.

2.2. Numerical calculation

To verify the above theoretical analysis, we also perform numerical simulations based on the atomistic Landau-Lifshitz-Gilbert (LLG) equation. Here, the corresponding atomistic Hamiltonian is given by

$$\mathcal{H} = A_{sim} \sum_{i,j} \mathbf{S}_i \cdot \mathbf{S}_j - K_{sim} \sum_i (\mathbf{S}_i \cdot \hat{z})^2, \quad (10)$$

where $A_{sim}$ is the exchange coupling constant between the nearest neighbors, $\mathbf{S}_i$ is the normalized magnetic moment at site $i$, $K_{sim}$ is the easy-axis anisotropy along the $z$ axis.

In the presence of spin current in the current-in-plane geometry, the dynamics is investigated by solving the LLG equation

$$\frac{\partial \mathbf{S}_i}{\partial t} = -\gamma \mu_0 \mathbf{S}_i \times \mathbf{H}_{eff,i} + \alpha_s \mathbf{S}_i \times \frac{\partial \mathbf{S}_i}{\partial t} - b_{j,j} \frac{\mathbf{S}_{i+1} - \mathbf{S}_{i-1}}{2d} - \beta b_{j,j} \mathbf{S}_i \times \frac{\mathbf{S}_{i+1} - \mathbf{S}_{i-1}}{2d}, \quad (11)$$

where $\mathbf{H}_{eff,i} = -(1/\mu_i)\partial \mathcal{H}/\partial \mathbf{S}_i$ is the effective field with the magnetic moment $\mu_i$, and $b_{j,j} = -(g_e P_{\mu B}/2eM_i) J_c$ is the magnitude of adiabatic STT.
The system size is 1000 nm × 200 nm × 7 nm with a cell size of 2 nm × 2 nm × 7 nm. Absorbing boundary conditions are considered in the simulations. The initial magnetization state was obtained by relaxation with a Bloch-type domain-wall configuration for sufficiently long time. The external field \( \mu_0 H = \mu_0 H_0(0, \pm \sin 2\pi ft, -\cos 2\pi ft) \) is applied to excite spin waves. By taking the well-known rare-earth transition-metal (RE-TM) ferrimagnet GdFeCo as an example, we use the following simulation parameters [43]: \( A_{\text{sim}} = 15 \times 10^{-12} \text{ J/m, } K_{\text{sim}} = 2 \times 10^5 \text{ J/m}^3, \mu_0 H_0 = 10 \text{ mT, the current density } J_c = 1 \times 10^{13} \text{ A/m}^2, \text{the spin polarization } P_{\text{RE}} = 0.1, P_{\text{TM}} = 0.4, \) and the gyromagnetic ratio \( \gamma_{\text{RE}} = 1.76 \times 10^{11} \text{ rad/sT} \) and \( \gamma_{\text{TM}} = 1.936 \times 10^{11} \text{ rad/sT} \) (corresponding to the Landé factors \( g_{\text{RE}} = 2 \) and \( g_{\text{TM}} = 2.2 \)). We consider the damping constant \( \alpha_{\text{RE}} = \alpha_{\text{TM}} = 0.003 \), and the nonadiabaticity \( \beta = 0.03 \). The magnetic moments \( M_{\text{RE}} \) and \( M_{\text{TM}} \) are listed in Table I.

| Parameter        | 1       | 2       | 3       |
|------------------|---------|---------|---------|
| \( M_{\text{TM}} \) (kA/m) | 950.8   | 1100    | 1280    |
| \( M_{\text{RE}} \) (kA/m)   | 938     | 1000    | 1090    |
| \( \delta_s \) (10^{-6}Js/m^3) | -2.6    | 0       | 2.6     |

### III. Results and discussion

3.1. **Channeling of spin waves in ferrimagnetic domain walls**

![Simulated (solid symbols) and calculated (solid lines) dispersion relation for (a) right- and (b) left-handed waves](image-url)

Fig. 2 (color online). Simulated (solid symbols) and calculated (solid lines) dispersion relation for (a) right-
handed, and (b) left-handed spin waves in ferrimagnetic domain walls for various $\delta$. For comparison, the simulated (empty symbols) and calculated (dashed lines) results in the uniform state are also given.

In this section, we investigated the spin-wave dynamics in the absence of STT and addressed the effect of $\delta$. Fig. 2(a) presents the numerically simulated (symbols) and Eq. (8)-based calculated (lines) dispersion relations of the right-handed spin waves in the uniform state (dashed lines and empty symbols) and domain wall (solid lines and symbols) for various $\delta$. The simulated data fit the calculations perfectly, confirming the validity of the theoretical analysis. For $\delta = 0$ at $T_A$, a frequency gap of spin waves in the uniform state is observed, and spin waves can be only excited when the frequency is higher than the forbidden frequency gap (~137 GHz). Interestingly, the frequency gap can be modulated through tuning $\delta$, and a positive (negative) $\delta$ enhances (suppresses) the gap. Thus, the net angular momentum could be used to suppress the gap in ferrimagnets, noting that the gap should be reduced and even eradicated to enhance the spin wave velocity and decrease the exciting power.

More importantly, for left-handed spin waves and a positive $\delta$, a gapless dispersion is observed, as shown in Fig. 2(b) (solid blue line and points) where gives the dispersion relations of the left-handed spin waves. It demonstrates the channeling of left-handed spin waves in ferrimagnetic domain walls with positive $\delta$. However, a forbidden frequency gap for right-handed spin waves $\sim\delta/\rho$ is also generated in ferrimagnetic domain walls attributing to the inequivalent moments at two sublattices. This behavior is rather different from ferromagnetic and antiferromagnetic domain walls where no frequency gap appears in domain wall spin wave dispersion. To some extent, it is suggested that at low frequency, ferrimagnetic domain walls could be used as spin-wave filters, in which only spin waves with a particular handedness can be excited and propagated. Moreover, in the uniform state, a positive (negative) $\delta$ suppresses (enhances) the frequency gap of the left-handed spin waves, which is in contrast to the right-handed spin waves because of the symmetry of ferrimagnets.
From Eq. 9, the group velocity of spin wave depending on $k$ and $\delta_s$ is predicted, while it is independent of the spin wave chirality. Fig. 3 presents the theoretically calculated group velocity of spin waves in the uniform state (dashed lines) and domain wall (solid lines) as a function of $k$ for various $\delta_s$. For $\delta_s = 0$, the group velocity of spin wave in the domain wall $v_{DW}$ is independent of $k$. For a finite $\delta_s$, with the increase of $k$, $v_{DW}$ quickly increases to the maximum value which is estimated to be $\sim 1.414 A/ds$ in the limit $k \to \infty$. In the uniform state, the group velocity of spin wave $v_{UNI}$ gradually increases as $k$ increases. As a result, the spin waves propagate in the domain wall much faster than in the uniform state, especially in the low frequency regime.

Furthermore, we compute energy consumptions in exciting spin waves in the uniform state and domain wall. The instantaneous power is written as $P(t) = \int_{V_{sw}} M \cdot h(t) \, dr$ with $V_{sw}$ is the volume of microwave field sources, $M$ is the local magnetic moment in $V_{sw}$, and $h(t)$ is the derivative of microwave field with respect to time $t$. Generally, the energy consumption to excite spin waves in domain wall is much lower than in the uniform state. For $\delta_s = 0$ and $f = 150$ GHz, as an example, the exciting power of spin waves reaches up to 224.35 nW in the uniform state, while only is $\sim 13.25$ nW in domain wall. Notably, the spin wave propagation in ferrimagnetic domain wall not only has a larger velocity, but also consumes a much lower energy than in the uniform state. Thus, spin-wave channeling in ferrimagnetic domain wall significantly facilitates transmission of information carried by spin waves.
3.2. Spin-wave Doppler shift in Channeling of ferrimagnetic domain walls

Earlier work has reported the current-induced spin-wave Doppler shift in the uniform state of compensated ferrimagnets. In this section, we reveal the Doppler shift of spin waves in ferrimagnetic domain wall.

As a matter of fact, the spin-wave Doppler shift $\Delta \omega_\pm$ can be estimated from $\omega_\pm - \omega_{\pm,0}$ with the frequency under zero current $\omega_{\pm,0} = \left( \pm \delta_s + \sqrt{\delta_s^2 + 4\rho Ak^2} \right) / 2\rho$. Then, the current-induced Doppler shift $\Delta \omega_\pm$ is given by

$$
\Delta \omega_\pm = \frac{sb_\pm k}{\sqrt{\delta_s^2 + 4\rho Ak^2}},
$$

(12)

![Fig. 4](color online) Current-induced Doppler shift of right-handed spin waves in ferrimagnet for (a) $\delta_s > 0$, (b) $\delta_s = 0$, and (c) $\delta_s < 0$, and of left-handed spin waves for (d) $\delta_s > 0$, (e) $\delta_s = 0$, and (f) $\delta_s < 0$.

Eq. (12) shows that the sign of spin-wave Doppler shift depends on the electric current, spin-wave handedness, and wave vector. For example, for the right-handed spin wave and $k > 0$, a positive (negative) current with $b_\pm < 0$ ($b_\pm > 0$) increases (decreases) the spin wave frequency. This behavior has been confirmed in our simulations, and the corresponding results are given in Fig. 4 (a) where presents the calculated and simulated dispersion relation for right-handed spin waves for a positive $\delta_s$. The simulated results coincide well with the theoretical calculations, demonstrating again the validity of the theory. Moreover, the sign of the Doppler
shift is independent of $\delta_s$, as shown in Figs. 4(b) and 4(c) where give the spin-wave dispersion for $\delta_s = 0$ and a negative $\delta_s$, respectively.

On the other hand, the sign of the Doppler shift of the left-handed spin wave is opposite to that of the right-handed spin wave, as demonstrated in Figs. 4(d)-4(f). Specifically, for the left-handed spin wave and $k > 0$, a positive (negative) current decreases (increases) the spin wave frequency. Thus, the Doppler shift of the spin wave in ferrimagnetic domain wall is similar to that in the uniform state.

At last, we discuss quantitatively the Doppler shift for spin waves. The calculated $\Delta \omega$ for right- and left-handed spin waves as a function of $\delta_s$ for various $k$ is shown in Fig. 5, which demonstrates a significant dependence of the shift on $\delta_s$ value. To some extent, the dependence of $\Delta \omega$ on $\delta_s$ could allow one to estimate the net spin angular momentum by measuring the current-induced spin-wave Doppler shift in future experiments. Moreover, in the vicinity of $T_A$ for $\delta_s \sim 0$, $\Delta \omega$ is estimated to be $\Delta \omega = sh^2/4 \rho A$, which is independent of the wave vector $k$. The maximum of the Doppler shift appears at $\delta_s < 0$, and the peak position shifts toward high $k$ side as $k$ increases.

IV. Conclusion

In conclusion, we have studied theoretically and numerically the excitation and propagation of spin waves in ferrimagnetic domain wall. Our study shows that the dispersion relation of
spin waves depends on the net angular momentum $\delta_s$ and the spin-wave handedness. For a positive $\delta_s$, the dispersion of left-handed spin waves is gapless, while that of right-handed spin waves is still with a forbidden frequency gap. Thus, it is suggested that ferrimagnetic domain wall could serve as a filter in which only spin wave with a particular handedness can be excited and propagated. Importantly, spin-wave dynamics in domain wall is much faster than in the uniform state and with much less energy consumption, demonstrating again the advantages of domain walls as waveguides. Moreover, the current-induced spin-wave Doppler shift is also revealed in ferrimagnetic domain walls. It is demonstrated that $\delta_s$ can effectively modulate the frequency shift magnitude, which like that in the uniform state. Thus, this work unveils interesting roles of ferrimagnetic domain walls in propagating spin waves, benefiting future experiments design and spin wave applications.

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