Precision Higgs boson mass determination
at lepton colliders

V. Barger\textsuperscript{a}, M.S. Berger\textsuperscript{b}, J.F. Gunion\textsuperscript{c}, and T. Han\textsuperscript{c}

\textsuperscript{a}Physics Department, University of Wisconsin, Madison, WI 53706, USA
\textsuperscript{b}Physics Department, Indiana University, Bloomington, IN 47405, USA
\textsuperscript{c}Physics Department, University of California, Davis, CA 95616, USA

Abstract

We demonstrate that a measurement of the Bjorken process $e^+e^-, \mu^+\mu^- \rightarrow ZH$ in the threshold region can yield a precise determination of the Higgs boson mass. With an integrated luminosity of 100 fb$^{-1}$, it is possible to measure the Higgs mass to within 60 MeV (100 MeV) for $m_H = 100$ GeV (150 GeV).
One of the triumphs of the LEP program was the measurement of the $Z$-boson mass to two MeV. Expectations are also quite good for the measurement of the $W$-boson mass ($M_W$) and the top quark mass ($m_t$) in the future, perhaps achieving precision of order 10 MeV for $M_W$ and 2 GeV for $m_t$ at the Tevatron and the LHC [1]. Precise values for $M_W$ and $m_t$ can also be obtained at lepton colliders by measuring the $\ell^+\ell^-\rightarrow WW$ and $\ell^+\ell^-\rightarrow t\bar{t}$ ($\ell = e$ or $\mu$) threshold cross sections, as illustrated by measurements of $W$-pair production at LEP center-of-mass energy $\sqrt{s} = 161$ GeV [2]. These measurements will allow an indirect prediction for the Higgs boson mass ($m_H$) and will test the consistency of the Standard Model (SM) at the two-loop level once $m_H$ is known.

In this Letter we point out that, analogously, a very accurate determination of $m_H$ is obtained by measuring the threshold cross section for the Bjorken Higgs-strahlung process $\ell^+\ell^-\rightarrow ZH$; with integrated luminosity $L = 100$ fb$^{-1}$, a 1$\sigma$ precision of order 60 MeV is possible for $m_H = 100$ GeV. This error in $m_H$ is smaller than that achievable via final state mass reconstruction for a typical detector, and would then be the most accurate determination of $m_H$ at an $e^+e^-$ collider.

The SM Higgs boson is easily discovered in the $ZH$ production mode by running the machine well above threshold, e.g. at $\sqrt{s} = 500$ GeV. For $m_H \lesssim 2M_W$ the dominant Higgs boson decay is to $b\bar{b}$ and most backgrounds can be eliminated by $b$-tagging. At the next linear $e^+e^-$ collider (NLC) the accuracy for $m_H$ via reconstruction using final state momenta is strongly dependent on the detector performance and signal statistics: $\Delta m_H \simeq R_{\text{event}}(\text{GeV})/\sqrt{N}$, where $R_{\text{event}}$ is the single-event resolution and $N$ is the number of signal events. At an SLD-type detector, the single event resolution for reconstruction of the Higgs mass is about 4 GeV for most $ZH$ final states (including channels with $Z \rightarrow e^+e^-, \mu^+\mu^-$) [4]. At the “super”-LC detector [3], the Higgs mass measurement would be best performed by examining the mass spectrum of the system recoiling against $Z \rightarrow e^+e^-, \mu^+\mu^-$ decays. The resolution in this spectrum would be about 0.3 GeV [10]. For the SM Higgs boson, the
accuracies of the $m_H$ determination for the two types of detector are

$$\text{SLD : } \Delta m_H \simeq 180 \text{ MeV} \left( \frac{50 \text{ fb}^{-1}}{L} \right)^{1/2}, \quad \text{super-LC : } \Delta m_H \simeq 20 \text{ MeV} \left( \frac{50 \text{ fb}^{-1}}{L} \right)^{1/2}, \quad (1)$$

which take into account the effective branching ratios appropriate in the two different cases.

The super-LC accuracy would be competitive with that we shall obtain via the threshold technique. However, in the not unlikely case that the detector is of the SLD-type, the best means for measuring $m_H$ will be to first determine $m_H$ to within a few hundred MeV in $\sqrt{s} = 500$ GeV running [which will also yield a precise measurement of $\sigma(ZH)$] and then reconfigure the collider for maximal luminosity just above the threshold energy $\sqrt{s} = M_Z + m_H$.

In Fig. 1 we show the cross section for the Bjorken process $\ell^+\ell^- \rightarrow ZH$ for Higgs masses from 50 to 150 GeV. Since the threshold behavior is $S$-wave, the rise in the cross section in the threshold region is rapid, as can be seen for the case of $m_H = 100$ GeV in the inset figure, the cross section being a few tenths of a pb. At LEP II, the few hundred pb$^{-1}$ of luminosity that might be devoted to such a threshold would yield just a handful of events. However, much higher luminosity is possible at threshold at the NLC \cite{7} or a muon collider \cite{8–10}.

In the ideal case that the normalization of the measured $ZH$ cross section as a function of $\sqrt{s}$ can be precisely predicted, including efficiencies and systematic effects, sensitivity to the SM Higgs boson mass is maximized by a single measurement of the cross section at $\sqrt{s} = M_Z + m_H + 0.5$ GeV, just above the real particle threshold. With a $\sim \pm 180$ MeV measurement of $m_H$ from initial running [see Eq. (1)] $\sqrt{s}$ can be set quite close to this optimal point. As an example of the precision that might be achieved, suppose $m_H = 100$ GeV and backgrounds are neglected. The $ZH$ cross section is 120 fb and is rising at a rate of 0.05 fb/MeV. With $L = 50$ fb$^{-1}$ and including an overall ($b$-tagging, geometric and

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\footnote{The LHC collaborations expect that the SM Higgs boson is detectable in the mass range $50 \lesssim m_H \lesssim 150$ GeV via its $\gamma\gamma$ decay mode. The mass resolution is expected to be $\lesssim 1\%.$}
event identification) efficiency of 40%, this yields $2.4 \times 10^3$ events, or a measurement of the cross section to about 2%. From the slope of the cross section one concludes that a $m_H$ measurement with accuracy of roughly 50 MeV is possible.

In practice, there will be systematic errors associated with experimental efficiencies as well as for theoretical predictions of the $ZH$ cross section and $H$ branching ratio(s) that will be very difficult to reduce below the 1% level. The ratio of the cross section measured at $\sqrt{s}$ well-above threshold in the initial $H$ discovery to that measured right at threshold is thus the key to determining $m_H$. The theoretical uncertainties will cancel in the ratio. Given the high luminosity that should be available for measurements both well-above threshold and right at threshold, changes in $b$-tagging, geometrical efficiencies, and jet misidentification as a function of $\sqrt{s}$ may be understood at the < 1% level, provided the final-focus reconfiguration required to optimize luminosity at the lower threshold $\sqrt{s}$ does not impel detector changes that would lead to significant changes in the experimental systematic effects.

For a more precise estimate of the accuracy with which $m_H$ can be measured, we employ $b$-tagging and cuts in order to reduce the background to a very low level. Specifically, we require: 1) tagging of both $b$’s in the event (for which an overall 50% efficiency will be assumed); 2) $|M_{bb} - m_H| < 5$ GeV; 3) $80 < M_{\text{recoil}} < 105$ GeV (i.e. broadly consistent with $M_Z$), where $M_{\text{recoil}} \equiv [p_{\text{recoil}}^2]^{1/2}$ with $p_{\text{recoil}} = p_{\ell^+} + p_{\ell^-} - p_b - p_{\bar{b}}$; 4) $|\cos \theta_{bb,\text{recoil}}| < 0.9$, where $\theta$ is the polar angle with respect to the beam direction. With these cuts the only significant background will be that from $ZZ$ production, where at least one of the $Z$’s decays to $b\bar{b}$. In Fig. 2, we compare the cross section versus $\sqrt{s}$ for the $\ell^+\ell^- \rightarrow Zb\bar{b}$ background to that for the $\ell^+\ell^- \rightarrow ZH$ (with $H \rightarrow b\bar{b}$) signal, where the signal is computed for $m_H = \sqrt{s} - M_Z - 0.5$ GeV. The background is very much smaller than the signal unless $m_H$ is close to $M_Z$.

\footnote{Note that the restriction on $M_{\text{recoil}}$ means that constructive interference of $ZH$ diagrams with $WW$ ($ZZ$) fusion diagrams in the $\nu\bar{\nu}tH$ ($\ell^+\ell^-H$) channels \[1\] will be small.}
The expected precision for the Higgs mass is given in Fig. 3 for an integrated luminosity of 50 fb$^{-1}$. The precision degrades as $m_H$ increases because the signal cross section is smaller (see Fig. 1). The background from the $Z$-peak reduces the precision for $m_H \approx M_Z$. Bremsstrahlung, beamstrahlung and beam energy smearing yield a reduction in sensitivity of 15% at a muon collider and 35% at an $e^+e^-$ collider.

If the Higgs boson is discovered at the Tevatron or LHC prior to construction of the NLC, the NLC could be configured from the beginning for optimal luminosity in the vicinity of the $ZH$ threshold. A motivation for doing so is that, at the NLC, measurements near the peak in the $ZH$ cross section (not far above threshold) would yield the highest rates and, hence, smallest errors possible for determining the branching ratios, couplings and total width of the $H$. In order to determine both these $H$ properties and also $m_H$, a very useful first set of measurements would be to take data at $\sqrt{s} = m_H + M_Z + 20$ GeV and at $\sqrt{s} = m_H + M_Z + 0.5$ GeV. In particular, the $e^+e^- \rightarrow ZH \rightarrow Zb\bar{b}$ rates at these two energies would simultaneously determine $m_H$ and $\sigma(ZH)B(H \rightarrow b\bar{b})$, where $\sigma(ZH) \propto g_{ZZH}^2$, the square of $ZZH$ coupling strength. The inclusive (recoil spectrum) $ZH$ event rate would yield a determination of $\sigma(ZH)$ directly and $B(H \rightarrow b\bar{b})$ could then be computed; deviations in either from SM expectations would be of great interest.

Figure 4 shows the statistical precision that can be obtained in a two parameter fit to $m_H$ and $g_{ZZH}^2B(H \rightarrow b\bar{b})$ (before including smearing effects from bremsstrahlung, beamstrahlung and beam energy spread) using a combined integrated luminosity of 50 fb$^{-1}$ for the above two values of $\sqrt{s}$ in the threshold region. The smallest error in $m_H$ is $\sim \pm 85$ MeV (for $m_H = 100$ GeV) obtained with $L_1 = 30$ fb$^{-1}$ at $\sqrt{s} = m_H + M_Z + 0.5$ and $L_2 = 20$ fb$^{-1}$ at $\sqrt{s} = M_Z + m_H + 20$ GeV. Since $m_H$ is determined by the ratio of the cross sections at

\footnote{If instead a muon collider is the first to be constructed following $H$ discovery, then the appropriate first emphasis might be $s$-channel $H$ production, which allows extremely accurate mass, width and coupling-constant-ratio determinations [8,9].}
the two energies, systematic uncertainties would cancel almost completely for such closely spaced energies, and the error in $m_H$ would be almost entirely statistical. The measurement of $\sigma(ZH)B(H \to b\bar{b})$ would be at the ±2% statistical level (which is better than the precision that can be reached with $L = 200 \text{ fb}^{-1}$ accumulated at $\sqrt{s} = 500 \text{ GeV}$ [12]); at this level of statistical error, the systematic uncertainties on $\sigma B$ from $b$-tagging, geometrical cuts and event-identification efficiencies will probably dominate. Doubling $L = L_1 + L_2$ to 100 $\text{ fb}^{-1}$ (so that $L_1 = 60 \text{ fb}^{-1}$) would yield $\sim \pm 60 \text{ MeV}$ error for $m_H$, i.e. comparable to the error of $\sim \pm 55 \text{ MeV}$ shown in Fig. 3 for $L_1 = 50 \text{ fb}^{-1}$ assuming small statistical error for $\sigma(ZH)B(H \to b\bar{b})$ at $\sqrt{s} = 500 \text{ GeV}$.

Two comments are particularly relevant. First, a ±60 MeV uncertainty on $m_H$ would allow almost immediate centering on the $s$-channel Higgs resonance peak at a muon collider (thereby avoiding expending luminosity on a scan location of the peak). For $m_H \lesssim 2M_W$, a fine scan over the Higgs peak at the muon collider would then yield an extraordinarily precise determination of $m_H$ along with a determination of the total $H$ width that is far more accurate [8,9] than achievable by other means [12] in this mass region. Second, a ±60 MeV level of accuracy for $m_H$ should prove to be of great value for constraining parameters entering into radiative corrections to the Higgs mass. In the minimal supersymmetric standard model, the leading one-loop correction to the tree-level prediction for the mass of the light SM-like $h^0$ is [13]:

$$\Delta m_{h^0}^2 = 3g^2m_t^4\ln\left(\frac{m_T^2}{m_t^2}\right)/[8\pi^2M_W^2],$$

where $m_T$ is the top-squark mass and we have simplified by neglecting top-squark mixing and non-degeneracy. From this formula one finds $dm_{h^0}/dm_t \sim 0.6$, and $dm_{h^0}/dm_T \sim 0.05$, for $m_{h^0} = 100 \text{ GeV}$, $m_t = 175 \text{ GeV}$ and $m_T \sim 500 \text{ GeV}$. Thus, a ±60 MeV measurement of $m_{h^0}$ would translate into very tight constraints on $m_t$ and $m_T$ of about ±100 MeV and ±1.2 GeV, respectively. Important squark mixing parameters would be similarly constrained. The challenge will be to compute higher loop corrections to $m_{h^0}$ to the ±60 MeV level.

In conclusion, we have shown that with sufficient luminosity it is possible to determine the Higgs boson mass to a high and very valuable level of precision by measuring the $\ell^+\ell^- \to ZH \to Zb\bar{b}$ cross section just above threshold and normalizing to a second measure-
ment either well above threshold or near the $ZH$ cross section peak. One simultaneously determines $\sigma(ZH)B(H \to b\bar{b})$ at a level of accuracy that could distinguish between the Standard Model Higgs sector and its many possible (e.g. supersymmetric) extensions.

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FIG. 1. The cross section vs. $\sqrt{s}$ for the process $\ell^+\ell^- \rightarrow Z^* H \rightarrow f\bar{f}H$ for a range of Higgs masses. The inset figure shows the detailed structure for $m_H = 100$ GeV in the threshold region.

FIG. 2. The $\ell^+\ell^- \rightarrow ZH \rightarrow Zb\bar{b}$ signal and the irreducible $\ell^+\ell^- \rightarrow Zb\bar{b}$ background vs. $\sqrt{s}$, including $b$-tagging and cut requirements 1)-4), see text.
FIG. 3. The precision $\Delta m_H$ attainable from a 50 fb$^{-1}$ measurement of the $Zb\bar{b}$ cross section at $\sqrt{s} = M_Z + m_H + 0.5$ GeV as a function of $m_H$, including $b$-tagging and cuts 1)-4). Bremsstrahlung, beamstrahlung, and beam energy smearing are neglected. A precise measurement of the cross section well above threshold is presumed available.

FIG. 4. The $\Delta \chi^2 = 1$ contours for determining the Higgs mass and $g_{ZZH}^2 B(H \to b\bar{b})$ by devoting $L_1 + L_2 = 50$ fb$^{-1}$ to two points along the threshold curve: $L_1$ at $\sqrt{s} = M_Z + m_H + 0.5$ GeV and $L_2$ at $\sqrt{s} = M_Z + m_H + 20$ GeV. We have assumed $m_H = 100$ GeV; $b$-tagging and cuts 1)-4) are imposed.