COSMIC STRINGS AND THE STRING DILATON

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Abstract

The existence of a dilaton (or moduli) with gravitational-strength coupling to matter imposes stringent constraints on the allowed energy scale of cosmic strings, $\eta$. In particular, superheavy gauge strings with $\eta \sim 10^{16}$ GeV are ruled out unless the dilaton mass $m_\phi \gtrsim 100$ TeV, while the currently popular value $m_\phi \sim 1$ TeV imposes the bound $\eta \lesssim 3 \times 10^{11}$ GeV. Similar constraints are obtained for global topological defects. Some non-standard cosmological scenarios which can avoid these constraints are pointed out.

Superstring theory predicts the existence of light gauge-neutral scalar fields (the dilaton and the moduli) with gravitational-strength couplings to ordinary matter. Of particular interest among those fields is the model-independent dilaton, whose tree-level couplings are well understood. Because of their weak couplings, the lifetimes of the moduli can be very long. In a cosmological context, if moduli are created in the early universe, their slow decay rate is the source of serious potential conflicts with observations [1], [2], [3], [4]. To simplify
the discussion, we shall refer to moduli as “the dilaton”, but most of the following treatment is applicable, *mutatis mutandis*, to generic moduli.

Several mechanisms of cosmological dilaton production have been discussed in the literature. First, the value of the dilaton field in the early universe can be set away from the minimum of its potential [1], [2], [3], [4]. (This is usually the case because the minima of the dilaton effective potential at late and early times generically differ by $O(m_{\text{Planck}})$). Coherent oscillations of the field about the minimum are then equivalent to a condensate of nonrelativistic particles. Another mechanism is the production of dilatons in binary particle collisions in a hot plasma [5]. A third production mechanism is the amplification of quantum fluctuations of the dilaton field in early cosmology [6], [7]. Requiring consistency between the cosmological production of dilatons and observations leads to very stringent, and *a priori* unnatural, constraints on the dilaton mass and couplings [2]. Several mechanisms have been proposed to solve this cosmological moduli problem: e.g. a late stage of secondary inflation [5], [8], or the presence of a symmetry of moduli space ensuring the coincidence of the minima of the effective potential at early and late times [9], [7].

In this paper we shall discuss another mechanism of dilaton production. Oscillating loops of cosmic string, which could be formed at a symmetry-breaking phase transition in the early universe, will copiously emit dilatons, as long as the characteristic frequency of oscillation is greater than the dilaton mass. Cosmic strings are predicted in a wide class of elementary particle models [10]. Their mass per unit length $\mu$, which is equal to the string tension, is determined by the symmetry breaking energy scale $\eta$, $\mu \sim \eta^2$ [11]. Of particular interest are grand-unification strings with $\eta \sim 10^{16}$ GeV which could be responsible for the formation
of galaxies and large-scale structure. We shall calculate the dilaton density produced by the strings and explore the constraints it imposes on the dilaton and string parameters.

We assume that the string thickness is small compared to the loop size and to the Compton wavelength of the dilaton, so that the string can be regarded as an infinitely thin line. We work in the “Einstein conformal frame” where tensor gravity decouples from the dilaton and is described by the standard Einstein-Hilbert action. Then the interaction of the dilaton field $\phi$ with the string is described by the action

$$S = -\frac{1}{4\pi G} \int d^4x \left[ \frac{1}{2} (\nabla \phi)^2 + V(\phi) \right] - \int \mu(\phi) dS.$$  \hspace{1cm} (1)

Here, $\mu(\phi)$ is the $\phi$-dependent string tension, $dS$ is the surface element on the string world-sheet, $G$ is Newton’s constant, and we have used the Minkowski metric assuming the space-time to be approximately flat.

We choose the origin of $\phi$ so that the minimum of the dilaton potential $V(\phi)$ is at $\phi = 0$. Then, for $\phi$ in the vicinity of the minimum, the dilaton field equation takes the form

$$\left( \nabla^2 - m^2 \phi \right) \phi(x) = -4\pi G \alpha T(x),$$  \hspace{1cm} (2)

where $m^2 = \left[ V''(0) \right]^{1/2}$ is the dilaton mass, and $T(x)$ is the trace of the string energy-momentum tensor. The dimensionless parameter $\alpha \equiv \partial \ln \sqrt{\mu(\phi)} / \partial \phi = \mu'(0) / 2\mu(0)$ measures the strength of the coupling of $\phi$ to cosmic strings. One generically expects $\alpha \sim 1$.

The string world history can be represented as $x^\mu(\zeta^a)$, $a = 0, 1$, where $\zeta^0$ and $\zeta^1$ are the worldsheet coordinates. The choice of these coordinates is largely arbitrary; it is convenient for most purposes to use a “conformal gauge”, specified by the conditions
\[
\dot{x} \cdot x' = 0, \quad \dot{x}^2 + x'^2 = 0, \quad (3)
\]
where dots and primes stand for differentiation with respect to \(\zeta^0\) and \(\zeta^1\), respectively. The residual freedom of coordinate transformations can be used to set \(\zeta^0 = x^0 \equiv t\), which allows us to describe the string trajectory using the three-vector \(x(\zeta, t)\), where \(\zeta \equiv \zeta^1\). The string energy-momentum tensor is then given by

\[
T^{\mu\nu}(x, t) = \mu \int d\zeta (\dot{x}^\mu \dot{x}^\nu - x'^\mu x'^\nu) \delta^{(3)}[x - x(\zeta, t)], \quad (4)
\]
and its trace is

\[
T(x, t) = -2\mu \int d\zeta |x'^2(\zeta, t)\delta^{(3)}[x - x(\zeta, t)], \quad (5)
\]
where \(\mu \equiv \mu(\phi = 0)\). Disregarding dilatonic and gravitational back-reaction, the string equation of motion in the gauge (3) has a simple form,

\[
\ddot{x} - x'' = 0. \quad (6)
\]

It can be shown from Eqs. (3), (6) that the motion of a closed loop of string is periodic with a period \(L/2\), where \(L \equiv M/\mu\) and \(M\) is the loop’s mass. The quantity \(L\) is often called the length of the loop, although the actual length varies with time.

The rates of dilaton energy loss and of dilaton number production by a periodic source of angular frequency \(\omega\) can be found from the following general equations

\[
\dot{E}_\phi = \sum_n P_n, \quad \dot{N}_\phi = \sum_n P_n/\omega_n, \quad (7)
\]

\[
P_n = \frac{G\alpha^2}{2\pi} \omega_n k_n \int d\Omega |T(k, \omega_n)|^2, \quad (8)
\]
\[ T(k, \omega_n) = \frac{1}{T_n} \int_0^{T_n} dt \int d^3x e^{i\omega_n t - \mathbf{i} \mathbf{k} \cdot \mathbf{x}} T(\mathbf{x}, t), \quad (9) \]

where \( \omega_n = n\omega, T_n = 2\pi/\omega_n, \) \( n = 1, 2, \ldots; \) \( k_n \equiv |\mathbf{k}| = (\omega_n^2 - m^2_{\phi})^{1/2}, \) \( d\Omega \) is the solid angle element, and the angular integration is over the directions of \( \mathbf{k}. \) The dilaton momentum \( k_n \) has to be real; hence, only terms with \( \omega_n > m_{\phi} \) are included in the sums \((6)\).

For a loop of length \( L, \) \( \omega_n = 4\pi n/L, \) and the sums are taken over \( n > L/L_c, \) where

\[ L_c = 4\pi/m_{\phi}. \quad (10) \]

For \( L \ll L_c, \) \( \omega_n \gg m_{\phi} \) for all values of \( n, \) and we can approximately set \( m_{\phi} = 0. \) Then, dilaton radiation from specific loop trajectories (described by solutions of Eqs. \((3), (6)\)) can be analyzed using the techniques developed for the gravitational case in Ref. \cite{12}. Details of such an analysis will be given in a separate paper \cite{13}; here we shall only summarize the results. We found that the energy spectrum and angular distribution of the dilaton radiation are very similar to the gravitational case (and very different from the electromagnetic radiation by superconducting strings \cite{14}). The energy and particle radiation rates can be represented as

\[ \dot{E}_{\phi} = \Gamma_{\phi} \alpha^2 G \mu^2, \quad \dot{N}_{\phi} = \tilde{\Gamma}_{\phi} \alpha^2 G \mu^2 / \omega, \quad (11) \]

where the numerical coefficients \( \Gamma_{\phi} \) and \( \tilde{\Gamma}_{\phi} \) depend on the loop trajectory (but not on its size). Typically, \( \Gamma_{\phi} \sim 30 \) and \( \tilde{\Gamma}_{\phi} \sim 13. \) The total radiation power from the loop is \( \dot{E} = \Gamma G \mu^2 \) with \( \Gamma = \Gamma_g + \alpha^2 \Gamma_{\phi}, \) where \( \Gamma_g \sim 65. \) The high-frequency asymptotic of the spectrum for a loop with cusps is \( P_n \propto n^{-4/3}, \) and for a cuspless loop with kinks is \( P_n \propto n^{-2}. \) This can be used to estimate the radiation rates from loops with \( L \gg L_c, \)

\[ \dot{E}_{\phi} \sim \Gamma_{\phi} \alpha^2 G \mu^2 (L/L_c)^{-1/3}, \quad \dot{N}_{\phi} \sim \tilde{\Gamma}_{\phi} \alpha^2 G \mu^2 m_{\phi}^{-1} (L/L_c)^{-1/3}. \quad (12) \]
Here, we used a ‘cuspy’ loop spectrum, $P_n \propto n^{-4/3}$, and introduced a lower cutoff at $n \sim L/L_c$.

To estimate the cosmological density of dilatons produced by oscillating string loops, we shall adopt a simple model in which the loops radiate all $\phi$-quanta in the fundamental mode, $\omega_1 \equiv \omega = 4\pi/L$. This approximation has been proven to give accurate results (within a factor of $\sim 3$) for the gravitational wave power spectrum. Moreover, the large-$n$ contribution to $\dot{N}_\phi$ in Eq. (7) converges faster than that to the power $E_\phi$, and thus we expect our estimate for the particle density to be no less accurate than that for the power spectrum.

Loops of initial length $L$ are chopped off the string network at a cosmic time $t_i \sim L/\beta$ and decay at time $t_f \sim (\Gamma G\mu)^{-1}L$. They have number density $n_i \sim \zeta \beta^2/L^3$ at the time of birth and

$$n_f \sim \left(\frac{t_i}{t_f}\right)^{3/2} n_i \sim \frac{\kappa^{1/2} \zeta}{\Gamma G\mu t_f^3}$$

at the time of decay. Here, $\zeta$ is a parameter characterizing the density of long strings (its definition can be found in Ref. [10]), $\kappa \equiv \beta/\Gamma G\mu$, and we have used the radiation era expansion law, $a(t) \propto t^{1/2}$. Numerical simulations of string evolution indicate that $\zeta \sim 14$ and $\beta \lesssim 10^{-3}$. The exact value of $\beta$ is not known, but it is bounded from below by $\beta \gtrsim \Gamma G\mu$, so that $\kappa \gtrsim 1$. From Eq. (11), the total number of dilatons emitted by a loop decaying at $t \sim t_f$ is

$$N \sim \dot{N} t_f \sim (4\pi)^{-1} \Gamma \dot{\Gamma}_\phi \alpha^2 G^2 \mu^3 t_f^2.$$  

(14)

The quantity of interest to us will be $Y_\phi = n_\phi(t)/s(t)$. Here, $n_\phi(t)$ is the dilaton density, $s(t)$ is the entropy density, which during the radiation era is given by $s(t) = 0.0725 [\mathcal{N}(t)]^{1/4} (m_p/t)^{3/2}$. $\mathcal{N}(t)$ is the effective number of spin degrees of freedom at time $t$,
and $m_p$ is the Planck mass. Apart from dilaton production and decay and out-of-equilibrium phase transitions (such as thermalization after inflation), $Y_\phi$ is conserved in the course of the cosmological evolution. The contribution to $Y_\phi$ from loops decaying at $t \sim t_f$ can be estimated as

$$Y_\phi(t_f) \sim n_f N/s(t_f) \sim \kappa^{1/2} \zeta \tilde{\Gamma}_\phi \alpha^2 (G\mu)^2 (m_p t_f)^{1/2} N_f^{-1/4},$$  \hfill (15)

where $N_f \equiv N(t_f)$. Eqs. (14) and (15) are valid as long as $t_f \lesssim t_c \equiv 4\pi/\Gamma G\mu m_\phi$, so that the loop sizes are smaller than the critical size (11). From Eq. (13) we see that larger values of $t_f$ give a greater contribution, and thus the dominant contribution to $Y_\phi$ is given by $t_f \sim t_c$ [16]. With $\zeta \sim 14$, $N(t_c) \sim 100$, $\tilde{\Gamma}_\phi \sim 13$ and $\Gamma \sim 100$, we have

$$Y_\phi \sim Y_\phi(t_c) \sim 20 \kappa^{1/2} \alpha^2 (G\mu)^{3/2} (m_p/m_\phi)^{1/2}. \hfill (16)$$

Eq. (14) is the main result of the present paper.

Strings of energy scale $\eta$ are typically formed at time $t_s \sim t_p/G\mu$, where $t_p = m_p^{-1}$ is the Planck time. Long strings are initially overdamped and begin to move relativistically at time $t_* \sim t_p/(G\mu)^2$. Small loops become relativistic at an earlier time, but damping due to interaction with the surrounding plasma remains a significant energy loss mechanism until $t_*$. In the derivation of Eqs. (11), (16) we assumed damping to be negligible, and thus the condition of validity of (16) is $t_c > t_*$, which gives

$$m_\phi/m_p < 4\pi G\mu/\Gamma. \hfill (17)$$

The analysis of the cosmological implications of the dilaton density (16) is similar to that for any weakly-interacting relic particles [2], [7]. The resulting constraints are sensitive to
the lifetime of the dilaton $\tau$, which is determined by its mass and couplings. The dilaton couples (in the Einstein frame) to spin-0 and spin-1/2 particles only through the mass terms, so that decays into such particles are suppressed by powers of their mass [18]. The interaction Lagrangian responsible for decays into light gauge bosons is $\mathcal{L}_{\text{int}} = \frac{1}{2} \alpha_F \phi F_{\mu\nu}^2$, and the corresponding lifetime is

$$\tau = \frac{4 m_p^2}{N_F \alpha_F^2 m_\phi^3} = 3.3 \times 10^{13} (12/N_F) \alpha_F^{-2} m_G^{-3} \text{ s.}$$

(18)

Here, $m_G = m_\phi/1 \text{ GeV}$, $N_F$ is the number of gauge bosons with masses $\ll m_\phi$, and the value of $\alpha_F^2$ is averaged over all such bosons. For $m_\phi \gtrsim 1 \text{ TeV}$, all standard-model gauge bosons should be included ($N_F = 12$). The coupling constant $\alpha_F$ is normalized so that $\alpha_F = 1$ for a tree-level superstring dilaton. It is generically expected that $\alpha_F \sim 1$ for all moduli. For numerical estimates below we set $\alpha_F = \alpha = \kappa = 1$. (Note that since $Y_\phi \propto \kappa^{1/2}$ and $\kappa \gtrsim 1$, setting $\kappa = 1$ will result in conservative bounds on $\mu$ and $m_\phi$).

A multitude of astrophysical constraints on unstable relic particles have been discussed in the literature. Short-range Cavendish experiments [19] exclude ultra-light dilatons of mass smaller than $1.6 \times 10^{-3} \text{eV}$ [20]. For quasi-stable dilatons, with lifetimes larger than the present age of the universe, $\tau > t_0 \approx 4 \times 10^{17} \text{s}$ (corresponding to $m_\phi \lesssim 40 \text{MeV}$), one has the usual upper bound on the cosmological dilaton mass density $\Omega_\phi h^2 < 1$ [21], where $\Omega_\phi = n_\phi m_\phi/\rho_{\text{critical}}$ and $h \equiv H_0/100 \text{km s}^{-1} \text{Mpc}^{-1}$. This yields $Y_\phi < 3.6 \times 10^{-9} m_G^{-1}$. For $\tau \gtrsim t_{\text{dec}} \sim 10^{13} \text{ s}$, very stringent constraints follow from the limits on the diffuse $\gamma$-ray background that would result from dilaton decays [21]: $Y_\phi < 2.9 \times 10^{-16} m_G^{-1}$ for $t_{\text{dec}} \lesssim \tau \lesssim t_0$ and $Y_\phi < 1.3 \times 10^{-20} m_G^{-4}$ for $\tau \gtrsim t_0$. For $10^{-1} \text{s} \lesssim \tau \lesssim t_{\text{dec}}$, the bounds are obtained by requiring that the decay products do not significantly change the abundances of $^4\text{He}$, $^3\text{He}$ and
D. The relevant processes are the interaction of ambient nucleons with the hadronic showers resulting from hadronic decays for $0.1 \, \text{s} \lesssim \tau \lesssim 10^7 \, \text{s}$ [22], and photodissociation and photoproduction of light elements by electromagnetic cascades initiated by the decay products for $10^4 \, \text{s} \lesssim \tau \lesssim t_{\text{dec}}$ [24, 17] (both processes being important for $10^4 \, \text{s} \lesssim \tau \lesssim 10^7 \, \text{s}$ [22]). The $\tau$-dependence of the resulting bound on the dilaton density is rather complicated, but roughly $Y_\phi \lesssim 1.4 \times 10^{-12} \mu^{-1}$ for $10^7 \, \text{s} \lesssim \tau \lesssim t_{\text{dec}}$ and $Y_\phi \lesssim 10^{-13} - 10^{-14}$ for $1 \, \text{s} \lesssim \tau \lesssim 10^7 \, \text{s}$. For $\tau < 0.1 \, \text{s}$, dilatons decay well before the onset of nucleosynthesis, and the bound is rapidly weakened as we move towards smaller values of $\tau$.

Combining these bounds on $Y_\phi$ with the expression (16) for the dilaton density produced by cosmic strings, we obtain constraints on $m_\phi$ and $\mu$ which are represented in Fig.1. We see that the excluded domain cuts deeply into the region of physically interesting values of the parameters. In particular, the most popular values of $G\mu \sim 10^{-6}$ and $m_\phi \sim 1 \, \text{TeV}$ are incompatible with one another. If the dilaton mass is indeed $\sim 1 \, \text{TeV}$, then the string tension is bounded by $G\mu \lesssim 6 \times 10^{-16}$, which corresponds to symmetry breaking scales $\eta \lesssim 3 \times 10^{11} \, \text{GeV}$. On the other hand, if GUT-scale strings are discovered, then the dilaton mass must satisfy $m_\phi \gtrsim 100 \, \text{TeV}$.

These conclusions are rather robust with respect to the variation of the numerical coefficient in Eq. (16) (which we expect to be accurate only within a factor of $\sim 3$). If, for example, the coefficient is changed by one order of magnitude, then the bound on $G\mu$ at a fixed $m_\phi$ is modified by a factor of $\sim 5$, and the bound on $m_\phi$ with $G\mu$ in the grand unification range remains essentially unchanged.

In the derivation of Eq. (16) for $Y_\phi$ we assumed that gravitational and dilaton radiation
were the dominant energy loss mechanisms of strings. This is justified for gauge strings, formed as a result of a gauge symmetry breaking. In the case of global strings, oscillating loops lose most of their energy by Goldstone boson radiation at the rate $\dot{E} \sim \Gamma \eta^2$. Here, $\eta$ is the global symmetry breaking scale and $\Gamma$ is about the same as in the gravitational case, $\Gamma \sim 65$. The mass per unit length of a global string has a logarithmic length-dependence, $\mu = 2\pi \eta^2 \ln(L/2\pi \delta)$, where $\delta \sim \eta^{-1}$ is the string thickness, and the lifetime of a loop is $\tau \sim E/\dot{E} \sim KL$ with $K = (2\pi/\Gamma) \ln(L/2\pi \delta)$. If we take, for example, a GUT-scale string with $\eta \sim 10^{15}$ GeV of length $L \sim L_c = 4\pi/m_\phi$ with $m_\phi \sim 1$ TeV, then $K \sim 3$, and a loop will make $\sim 6$ oscillations before losing half of its energy. The loops are expected to form with sizes $L \sim t/K$ and decay in about a Hubble time: $t_i \sim t_f \sim KL$. Once again, the main contribution to $Y_\phi$ comes from $t_f \sim t_c \sim 4\pi K/m_\phi$, and it is easily verified that Eq. (16) is replaced by

$$Y_\phi \sim 350\alpha^2 (G\mu)^2 (m_p/m_\phi)^{1/2},$$  \hspace{1cm} (19)$$

and its condition of validity (17) by

$$m_\phi/m_p < 40(G\mu)^2,$$  \hspace{1cm} (20)$$

where we have used $K \sim 3$. For $m_\phi \sim 1$ TeV the bound on the dilaton density is $Y_\phi \lesssim 3 \times 10^{-14}$, and the constraint on $G\mu$ following from Eq. (13) (with $\alpha \sim 1$) is $G\mu \lesssim 10^{-12}$. However, according to (20), with this value of $m_\phi$ Eq. (19) is valid only for $G\mu \gtrsim 2 \times 10^{-9}$, and thus we can conclude only that $G\mu \lesssim 2 \times 10^{-9}$.

Global monopoles and textures have also been suggested as possible seeds of galaxy formation [25, 26]. The energy density of these defects varies on the horizon scale $R \sim t$. 

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The corresponding field gradients are $\dot{\Phi} \sim |\nabla \Phi| \sim \eta/t$ and $T'_\mu \sim \eta^2/t^2$. The dilaton density produced by the relativistic evolution of the field $\Phi$ in a Hubble time can be estimated from Eq. (2): $\phi/t^2 \sim 4\pi G \alpha \eta^2/t^2$, which gives $\phi \sim 4\pi G \alpha \eta^2$ and $n_\phi \sim m_\phi \phi^2/8\pi G$. With $s \sim 0.2(m_p/t)^{3/2}$ and $t \sim t_c \sim 4\pi/m_\phi$, we have

$$Y_\phi \sim 10^3 \alpha^2 (\eta/m_p)^4 (m_p/m_\phi)^{1/2}. \quad (21)$$

With $\alpha = 1$ and $m_\phi \sim 1$ TeV, the resulting constraint on $\eta$ is $\eta \lesssim 10^{13}$ GeV. [Damping is unimportant for global monopoles and textures [27], and there is no analogue of the condition (20)]. Hence, gauge cosmic strings and global strings, monopoles and textures are excluded as seeds for structure formation if the dilaton mass is $m_\phi \sim 1$ TeV.

We finally mention some ways of avoiding the above constraints. The main contribution to the dilaton density in Eqs. (11), (19) and (21) comes from the time $t \sim t_c$, which corresponds to the temperature $T_c \sim 10^9 (G\mu)^{1/2} m_\mu^{1/2}$ GeV for gauge strings and $T_c \sim 10^8 m_\mu^{1/2}$ GeV for global defects. Our analysis, therefore, is not directly applicable to models in which the universe has never been heated up to such temperatures. For example, in inflationary scenarios the thermalization temperature after inflation can be below $T_c$. Alternatively, string formation can be delayed until after $T_c$: in some supersymmetric models GUT-scale strings can be formed at temperatures as low as the electroweak scale [28]. Another possibility is to invoke models where topological defects are produced during inflation [29]. Then the defects begin emitting dilatons only after their characteristic scale comes within the horizon, which can happen at $t > t_c$. In all three cases the resulting dilaton density is very model-dependent.

Once dilatons are produced, they can be diluted by a brief period of inflation. Models of this kind have been suggested [5], [8] to resolve the usual Polonyi-moduli problem: overpro-
duction of dilatons and other moduli due to a mismatch of the minima of their potential at early and late times. The same models can be used to relax the constraints on topological defects discussed here. We note, however, that another proposed solution to the moduli problem will not work in our case. Dine, Randall and Thomas [3] suggested that moduli production during inflation can be suppressed if the potential in moduli space has some symmetry which enforces that the potential minima before and after inflation coincide. Clearly, this does not resolve the conflict between moduli and defects: all defects formed after inflation will produce dilatons, and thus the defect parameters are subject to all constraints we discussed earlier in this paper. The only exception is the model suggested in Ref. [20] (whose cosmological consequences were further studied in Ref. [7]) in which the minimum of the potential is a point of enhanced symmetry for all dilaton couplings. Then, near the minimum, the dilaton is essentially decoupled from all other fields (in particular $\alpha \ll 1$), and dilaton production by topological defects is suppressed.

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Figure 1: Constraints on $\log_{10}(G\mu)$ versus $\log_{10}(m_\phi/1\text{Gev})$. The region above the solid curve is forbidden. Labels indicate the source of the various constraints: Cavendish experiments (C), $\Omega_\phi h^2 < 1(O)$, gamma-ray background (G), photodissociation (P), combined hadroproduction and photodissociation (HP), hadroproduction (H). The dashed line indicates the condition of validity (17). [The constraints apply only above the dashed line.]
