Optimization of CW Fiber Lasers With Strong Nonlinear Cavity Dynamics

O.V. Shtyrina$^{1,2}$, S.A. Efremov$^{1,2}$, I.A. Yarutkina$^{1,2}$, A.S. Skidin$^{1,2}$, M.P. Fedoruk$^{1,2}$

$^1$Novosibirsk State University, 630090, 2 Pirogova str., Novosibirsk, Russia
$^2$Institute of Computational Technologies SB RAS, 630090, 6 Lavrentiev ave., Novosibirsk, Russia

E-mail: efremov.math@gmail.com

Abstract. In present work the equation for the saturated gain is derived from one-level gain equations describing the energy evolution inside the laser cavity. It is shown how to derive the parameters of the mathematical model from the experimental results. The numerically-estimated energy and spectrum of the signal are in good agreement with the experiment. Also, the optimization of the output energy is performed for a given set of model parameters.

1. Introduction

In general, the optimization of the signal evolution inside the strong nonlinear laser cavity cannot be performed using the analytical approximation. To carry out such a multiparametric optimization, the numerical modeling based on the coupled modified Schrodinger equations is employed [1]. However, not all the model parameters can be derived from the experimental results. Because of the large number of parameters and the numerical modeling complexity, there is a need in developing the simplified models of the saturation gain.

To describe the spatial-temporal evolution of the electromagnetic field, the following coupled modified Schrodinger equations with the gain saturation are used:

$$\pm \frac{\partial A^\pm}{\partial z} = -i\beta_1 \frac{\partial A^\pm}{\partial t} - i\beta_2 \frac{\partial^2 A^\pm}{\partial t^2} + i\gamma \left( |A^+|^2 + 2|A^-|^2 \right) A^\pm + \frac{g}{2(1 + P/P_{SAT})} A^\pm. \quad (1)$$

Here the signs "±" denote the forward and backward field directions, respectively, $\beta_1$, $\beta_2$, $\gamma$ are dispersion and nonlinear coefficients [1]. The power is denoted by $P = P^+ + P^- = \frac{1}{T_R} \left( \int |A^+|^2 \, dt + \int |A^-|^2 \, dt \right)$, $T_R$ is the round-trip time, $g$ and $P_{SAT}$ are the small signal gain and saturation power, respectively. The values of $g$ and $P_{SAT}$ are derived from the experimental measurements using the method described below. To numerically solve the system, the split-step Fourier method was used.

Besides the system of NLS equations, the one-level gain media model is used:

$$\frac{dP^\pm}{dz} = \pm \left( \frac{g}{1 + P/P_{SAT}} \right) P^\pm, \quad P(z) = P^+(z) + P^-(z). \quad (2)$$
The experimental setup is shown in figure 1 [1]; it represents itself the Yb-doped fiber laser of the Fabry-Perot type that consists of the high-reflectivity fiber Bragg grating ($R^-$) and the 4% back reflection ($R^+$) from the fiber edge at the left and the right sides respectively. The active fiber length is $L = 35$ m. From the left side of the cavity the pump is produced by the 23 W laser diode at 910 nm. The output signal power of such setup can be up to 12 W with spectrum width 0.5 nm and the maximum spectral power at 1084 nm. The experimental results are shown on figure 2.

Figure 2 shows the energy balance inside the laser cavity. Here one can notice that at the free end the output energy is much higher, than at the FBG: it means that FBG has much higher reflectivity. The FBGs refraction spectrum is shown on figure 3.

To perform the numerical simulation (particularly, to perform the optimization), it is necessary to know the parameters of the model. However, in many cases some parameters cannot be
derived directly from the experimental results. Such parameters can be obtained by means of the theoretical approximation.

2. Saturated gain equation

Figure 4. A simple scheme of the energy balance in the laser cavity.

Let us derive a signal gain coefficient. When considering the signal gain, it is normally believed that the gain of the signal propagating in forward and backward directions is the same ([1],[2]). Therefore, the signal gain can be expressed as follows:

$$G = \frac{P^+(L)}{P^+(0)} = \frac{P^-(0)}{P^-(L)}.$$

It should be noted that the experimental results do not normally contain the information about the signal gain. But in mathematical modeling and the energy balance analysis the gain of the radiation through the laser cavity is one of the most important characteristics. Below in this section we derive the equation of the experimental gain coefficient that takes into account the structure of the linear laser cavity.

In case of the linear cavity, there are two reflection coefficients at the left ($R^-$) and the right ($R^+$) sides, the radiation propagates in both directions; hence, the following equations take place:

$$P^-(L) = GR^+P^+(0), \quad P^+(0) = GR^-P^-(L).$$

These expressions indicate that to achieve a stable generation, the signal gain should compensate the intracavity losses. It means that

$$G = \frac{1}{\sqrt{R^+R^-}}. \tag{3}$$

Thus, the output power can be expressed as follows:

$$P_{out}^+ = G \left(1 - R^+\right) P^+(0),$$
$$P_{out}^- = G \left(1 - R^-\right) P^-(L).$$

After some simple transformations one can obtain that

$$\frac{P^+(L)}{P^+(0)} = G = \frac{P_{out}^+}{P_{out}^-} \cdot \frac{R^+G^2 - 1}{1 - R^+},$$
and the following saturated gain equation can be obtained:

\[ G^2 - \frac{P_{\text{out}}}{P_{\text{out}}^+} \left( \frac{1}{R^+} - 1 \right) G - \frac{1}{R^+} = 0. \] (4)

A physically consistent (positive) solution always exists and is derived from the following expression:

\[ G = \frac{1}{2} \left[ \frac{P_{\text{out}}^-}{P_{\text{out}}^+} \left( \frac{1}{R^+} - 1 \right) + \sqrt{\left( \frac{P_{\text{out}}^-}{P_{\text{out}}^+} \left( \frac{1}{R^+} - 1 \right) \right)^2 + 4} \right]. \]

Here one can notice that in the final equation there is no left-end reflection (\( R^- \)); for this reason there is no need to estimate this coefficient.

On figure 5 the signal gain estimated using equation 4 is shown. It can be seen that the gain slightly declines when the pump power is \( P_{\text{pump}} = 1.7 \) W. The reason for such behaviour may lie in a measurement error, this fact shows that although theory shown in previous section allows to exclude some parameters measured in experiment, equation 4 is still sensitive to the measurement accuracy.

The equation derived in this section enables to estimate the signal gain as a function of only the output signal power measured experimentally.

3. Saturated gain model

In present work the one-level gain model was chosen. Such model correctly describes the energy dynamics inside a laser cavity of chosen configuration. This model includes background losses, but here they can be neglected. Experimental results show, that generation area of such laser for a big range of inward pump (from 0.6 W up to 23 W) includes tiny variations of the signal gain (given in 4), and gain value corresponds to saturated regime. Therefore there is no need to take into consideration the background losses. Moreover in the experiment discussed here the active fiver length was always kept constant, so the small signal gain can be considered as

![Figure 5. The signal gain computed from the experimental data.](image-url)
invariable. Here the small signal gain value was chosen as \( gL = 10 \) dB.

After reasoning given above one comes to the system of balance equations

\[
\frac{dP^\pm}{dz} = \pm \left( \frac{g}{1 + P/P_{SAT}} \right) P^\pm
\]

which appears to be two differential equations with separable variables and can be easily integrated

\[
\ln G + \frac{G - 1}{P_{sat}} (P^+(0) + P^-(L)) = gL.
\]

So, one can rewrite it in terms of the output power:

\[
\ln G + \frac{G - 1}{GP_{sat}} \left( \frac{P^{out+}}{1 - R^+} + \frac{P^{out-}}{1 - R^-} \right) = gL.
\]  

It is known that the saturation power depends on the pump power linearly [3]. It means that the following equation holds:

\[
P_{sat}[W] = A + B \cdot P_{pump}[W],
\]

where \( A \) and \( B \) are constants. These parameters were found by the method of least squares, and finally we obtain the following:

\[
P_{sat}[W] = 0.0648 + 0.0642 \cdot P_{pump}[W],
\]

On figure 6 one can see the experimental and the analytically-estimated values of the saturation power. The analytical approximation and experimental results show good agreement.

**Figure 6.** The saturation power computed and approximated from the experimental data.
4. Optimization

In the previous sections the total gain equation was obtained and the original system was integrated. The proposed analytical estimation has enabled to obtain all parameters of the system, and thus has made possible to perform the optimization of the output signal power. To carry out the optimization, the Split-Step Fourier method was used to solve the Schrodinger equations (equations 1) with two initial conditions: \( A^+(t,0) \) and \( A^-(t,0) \); from the conditions the forward and backward signals power can be derived (\( P^+(0) \) and \( P^-(0) \)). To solve the problem one need also to know the total signal gain \( G \), which is derived by back reflections at each side of the cavity (equation 3), and the back reflection (\( R^- \)) on the FBG. In this section we propose the algorithm that allows iteratively find a solution with a lower computational time spend.

The initial backward signal at the left end \( \tilde{A}^- (t,0) \) can be represented as a Gaussian noise with a low power. The forward signal at the left end \( \tilde{A}^+ (t,0) \) is the reflected backward signal, therefore it can be computed using the following expression:

\[
\tilde{A}^+ (t,0) = F^{-1} \left[ f^- (\omega) F \left[ \tilde{A}^- (t,0) \right] (\omega) \right],
\]

where \( F[A(t,z)] (\omega, z) \) and \( F^{-1}[A(\omega,z)] (t,z) \) denote forward and backward Fourier transforms respectively, and \( f^- (\omega) \) is the FBG spectrum reflectivity, shown on Figure 3. The backward reflection coefficient is estimated as follows:

\[
R^- = \frac{P^+(0)}{P^-(0)} = \frac{\int |\tilde{A}^+ (t,0)|^2 dt}{\int |\tilde{A}^- (t,0)|^2 dt},
\]

and then the total signal gain is (equation 3)

\[
G = \frac{1}{\sqrt{R^+ R^-}}.
\]

The next step is to find the sum \((P^+(0) + P^-(L))\) of the forward and backward signal power:

\[
P^+(0) + P^-(L) = \frac{P_{sat} \cdot (gL - \ln G)}{G - 1},
\]

where the signal power for each direction can be computed using the following equations:

\[
P^+(0) = \frac{P^+(0) + P^-(L)}{1 + R^+ G}, \quad P^-(L) = \frac{P^+(0) + P^-(L)}{1 + R^- G}.
\]

After all manipulations described above the initial conditions \( A^+ (t,0) \) and \( A^- (t,0) \) should be transformed with respect to computed signal power (the multiplication by a constant does not affect the spectrum width, that is why this transform is correct):

\[
A^- (t,0) = \frac{P^- (L) \cdot G \cdot T_R}{\int |\tilde{A}^- (t,0)|^2 dt} \tilde{A}^- (t,0), \quad A^+ (t,0) = \frac{P^+ (0) \cdot T_R}{\int |\tilde{A}^+ (t,0)|^2 dt} \tilde{A}^+ (t,0).
\]

After the total gain is estimated, one can obtain its distribution along the z-axis, which can be found from equation 5, where the integration limit \( L \) is replaced by the variable \( z \) which is varied between 0 and \( L \):

\[
\ln[G(z)] + \frac{G(z) - 1}{P_{sat}} \left( P^+(0) + \frac{P^-(0)}{G(z)} \right) = gz.
\]
When the signal power stabilization process and the gain distribution computation algorithm were described, one should make the Schrodinger equations simulation process clear. As it represents the Cauchy problem, the initial conditions are set for the left side of the computational area, so the integration direction derived as left-to-right. But while the roundtrip the signal also reflects from the right side of the cavity and starts the backward propagation which direction is opposite to the direction of integration, therefore the original problem was divided into two parts: at first the forward and the backward signals are computed in forward (left-to-right) direction (respectively to the system 1) and at the second part one considers that:

\[ A^- (t, L) = R^+ \cdot A^+ (t, L). \]

Then as the direction of propagation is opposite to the integration direction the signs in original system of Schrodinger equations should be changed \[1\], so the backward propagation system is:

\[ \mp \frac{\partial A^\pm}{\partial z} = -i \beta_1 \frac{\partial A^\pm}{\partial t} - i \beta_2 \frac{\partial^2 A^\pm}{\partial t^2} + i \gamma (|A^\pm|^2 + 2|A^\mp|^2) A^\pm + \frac{g}{2(1 + P/P_{SAT})} A^\pm, \]

with the initial conditions \( A^+ (t, L) \), \( A^- (t, L) \). When one comes back to the left side of the computational area, one obtains the value:

\[ \tilde{A}^- (t, 0) = A^- (t, 2L) \]

to correct the gain distribution and the signal power. Described algorithm should be repeated iteratively until the signal spectrum and power stabilize, the stopping condition can be expressed by the following inequality:

\[ |(R^-)_{i+1} - (R^-)_i| \leq \epsilon. \]

Using the described method, the optimization of the output signal power as a function of the forward back reflection and the inward pump power has been carried out. These two parameters were selected, because they can be easily changed in the experiment while keeping unchanged the laser configuration. The left picture on figure 7 shows a good accordance of the computed signal spectrum to the experimental one. On the right picture the output signal power optimization results are represented. One can see that the maximum output power is observed for high pump powers and low coupler reflectivities.

**Figure 7.** (left) The comparison of experimental (black curve) and numerical (blue curve) spectra, \( P_{pump} = 23W, R^+ = 0.04 \); (right) The results of optimization of the output power. The maximum power 15W is observed for low losses.
5. Conclusion
In this research the one-level gain media model has been analyzed. The signal gain equation has been analytically described, and the original system has been integrated. Also the balance equation to derive the power characteristics has been obtained. The theoretical results have shown good agreement with the experimental results. This indicates that it is possible to estimate the small signal gain and the saturation power based on the experimental data. The numerical optimization based on the analytical equations and experimental data has been performed. The analysis of experimental measurements shows that the simplified model with the saturated gain is applicable to the modeling of CW strong nonlinear dynamics in the Fabry-Perot cavity.

6. Funding
The authors acknowledge the support of the Russian Scientific Foundation (project 17-71-20082).

References
[1] S. K. Turitsyn, A. E. Bednyakova, M. P. Fedoruk, A. I. Latkin, A. A. Fotiadi, A. S. Kurkov, and E. Sholokhov. Modeling of CW Yb-doped fiber lasers with highly nonlinear cavity dynamics. Optics Express Vol. 19, No. 9, 25 April 2011.
[2] C. Barnard, P. Myslinski, J. Chrostowski, and M. Kavehrad. Analytical model for rare-earth-doped fiber amplifiers and lasers. IEEE J. Quantum Electron. 30, 1817-1830 (1994).
[3] O. V. Shtyrina, A. V. Ivanenko, I. A. Yarutkina, A. V. Kemmer, A. S. Skidin, S. M. Kobytskyy, and M. P. Fedoruk. Experimental measurement and analytical estimation of the signal gain in an Er-doped fiber. Journal of the Optical Society of America B Vol. 34, No. 2, February 2017.
[4] A. E. Siegman. Lasers (University Science Books, 1986).
[5] A. S. Kurkov and E. M. Dianov. Moderate-power CW fiber lasers. Quantum Electron. 34(10), 881-900 (2004).
[6] B. G. Bale, O. G. Okhotnikov, and S. K. Turitsyn. Modeling and technologies of ultrafast fiber lasers. In Fiber Lasers, O. G. Okhotnikov, ed. (Wiley, 2012).
[7] S. K. Turitsyn. Theory of energy evolution in laser resonators with saturated gain and non-saturated loss. Opt. Express 17, 11898-11904 (2009).
[8] I. A. Yarutkina, O. V. Shtyrina, A. Skidin, and M. P. Fedoruk. Theoretical study of energy evolution in ring cavity fiber lasers. Opt. Commun. 342, 2629 (2015).
[9] T. Pfeiffer and H. Bullow. Analytical gain equation for erbium-doped fiber amplifiers including mode field profiles and dopant distribution. IEEE Photon. Technol. Lett. 4, 449451 (1992).