The spin-splitting of collective flows in intermediate-energy heavy-ion collisions

Yin Xia,1,2 Jun Xu*,1 Bao-An Li,3,4 and Wen-Qing Shen1

1Shanghai Institute of Applied Physics, Chinese Academy of Sciences, Shanghai 201800, China
2University of Chinese Academy of Sciences, Beijing 100049, China
3Department of Physics and Astronomy, Texas A&M University-Commerce, Commerce, TX 75429-3011, USA
4Department of Applied Physics, Xi’an Jiao Tong University, Xi’an 710049, China

Abstract

The spin splitting of collective flows in intermediate-energy heavy-ion collisions as a result of the nuclear spin-orbit interaction has been investigated based on a spin- and isospin-dependent Boltzmann-Uehling-Uhlenbeck transport model. We found that the spin splittings of the directed flow and elliptic flow are largest in peripheral heavy-ion collisions at beam energies of about 100-200 MeV/nucleon, and the effect is considerable even in smaller systems and appreciable for nucleons with high transverse momenta. Our study is useful for understanding the spin dynamics in heavy-ion collision and the nature of the nuclear spin-orbit interaction.

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I. INTRODUCTION

Understanding the nucleon-nucleon interaction is one of the main tasks of nuclear physics. It is well known that bare nucleon-nucleon interactions are spin-isospin dependent. As a result, the in-medium nucleon mean-field potentials are expected to be spin-isospin dependent as well. Effects of the isospin-dependence of nuclear interactions on properties of nuclei, nuclear reactions, and neutron stars have been studied extensively and many questions remain to be addressed \cite{1-4}. Similarly, there are also many interesting issues regarding the spin-dependence of nuclear interactions. The nuclear spin-orbit coupling is one of the most important spin-dependent nucleon interactions \cite{5, 6}. The spin-orbit potential, obtained from the effective two-body spin-orbit interaction within Hartree-Fock approach, gives different potentials for nucleons with different spins and modifies the shell structure of finite nuclei, and the effect is more important near the surface of a nucleus. The detailed properties of the spin-orbit potential, such as its strength, isospin dependence \cite{7}, and density dependence \cite{8}, are still under investigating, as they are related to many interesting problems in both nuclear structure and nuclear astrophysics \cite{9-13}. Of course, the spin-isospin correlation is an interesting topic in its own right.

The spin-dependent interactions and mean-fields have been shown to be critical in understanding many structural properties of nuclei and few-body reactions within various models. However, spin effects in heavy-ion collisions are less known. Usually only spin-averaged nucleon interactions are used in transport model studies of heavy-ion collisions mostly due to the difficulties in spin measurements. In recent years, however, the spin-related experiments have been gradually developed and it makes the study of spin dynamics in heavy-ion collisions possible. For example, the spin polarized beam can now be produced with nucleon removal or pickup reactions \cite{14}. In addition, the analyzing power denoting the spin-asymmetry of pp or pA scattering can be measured \cite{15}, and it can be as large as 100\% at certain energies and scattering angles \cite{16}, providing a possible way to disentangle nucleons with different spins. In fact, heavy-ion collisions have unique advantages of varying the density, isospin asymmetry, energy, and momentum current compared with the nuclear spectroscopy and corresponding nuclear structure studies, and might be more powerful in studying the detailed properties of the spin-dependent nucleon interaction, such as the nuclear spin-orbit interaction. Similar to isospin physics, one would expect that
nucleons with different spins will have different dynamics and this may lead to the spin splitting of nucleon collective flows. To study the spin dynamics, we introduced recently the nuclear spin-orbit interaction and nucleon spin degree of freedom to an isospin-dependent Boltzmann-Uehling-Uhlenbeck transport model (SIBUU12) \[17, 18\]. It is worth noting that a similar model based on the quantum molecular dynamical approach was developed more recently in Ref. \[19\]. In the present work, using the SIBUU12 transport model, we investigate in detail the spin splitting of nucleon directed and elliptic flows in heavy-ion collisions at intermediate energies.

II. MODEL DESCRIPTION

In this section we summarize briefly the main ingredients of the SIBUU12 transport model \[17\] where the spin-orbit coupling and the spin degree of freedom have recently been explicitly incorporated. We begin with the Skyrme-type effective two-body nuclear spin-orbit interaction \[20\]

\[
V_{so} = iW_0(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{k} \times \delta(\vec{r}_1 - \vec{r}_2)\vec{k}',
\]

where \(W_0\) is the strength of the spin-orbit coupling, \(\vec{\sigma}_1(2)\) is the Pauli matrix, \(\vec{k} = (\vec{p}_1 - \vec{p}_2)/2\) is the relative momentum operator acting on the right with \(\vec{p} = -i\nabla\), and \(\vec{k}'\) is the complex conjugate of \(\vec{k}\). Within the framework of Hartree-Fock calculation, the above nuclear interaction leads to the spin-dependent mean-field potentials including the time-even term

\[
U_{q}^{s-even} = -\frac{W_0}{2}[\nabla \cdot (\vec{J} + \vec{J}_q)] + \frac{W_0}{2}(\nabla \rho + \nabla \rho_q) \cdot (\vec{p} \times \vec{\sigma})
\]

and the time-odd term

\[
U_{q}^{s-odd} = -\frac{W_0}{2}\vec{p} \cdot [\nabla \times (\vec{s} + \vec{s}_q)] - \frac{W_0}{2}\vec{\sigma} \cdot [\nabla \times (\vec{j} + \vec{j}_q)].
\]

In the above, \(q = n\) or \(p\) is the isospin index, and \(\rho, \vec{s}, \vec{j},\) and \(\vec{J}\) are respectively the number, spin, momentum, and spin-current densities, whose definitions in terms of the nucleon wave function can be found in Ref. \[21\]. It is noteworthy that although the time-odd potential is negligible in studying spherical nuclei, it is important in dealing with deformed nuclei. In heavy-ion collisions with all kinds of collision geometries, both time-even and time-odd potentials should be included.
In our newly developed spin- and isospin-dependent Boltzmann-Uehling-Uhlenbeck transport model, $\rho$, $\vec{s}$, $\vec{j}$, and $\vec{J}$ are calculated by the test particle method. The expectation value of the spin of each nucleon is represented by a unit vector $\vec{\sigma}$, and the probability of spin projection on an arbitrary direction can then be easily calculated. The equations of motion in the presence of the spin-orbit coupling can be written as

$$\frac{d\vec{r}}{dt} = \frac{\vec{p}}{m} + \frac{W_0}{2} \vec{\sigma} \times (\nabla \rho + \nabla \rho_q) - \frac{W_0}{2} \nabla \times (\vec{s} + \vec{s}_q), \quad (4)$$

$$\frac{d\vec{p}}{dt} = -\nabla U_q - \nabla U_q^{s\text{--even}} - \nabla U_q^{s\text{--odd}}, \quad (5)$$

$$\frac{d\vec{\sigma}}{dt} = W_0[(\nabla \rho + \nabla \rho_q) \times \vec{p}] \times \vec{\sigma} - W_0[\nabla \times (\vec{j} + \vec{j}_q)] \times \vec{\sigma}, \quad (6)$$

where $U_q$ is the Skyrme-type momentum-independent mean-field potential, which is chosen to give an incompressibility of $K_0 = 230$ MeV for symmetric nuclear matter and a symmetry energy $E_{\text{sym}}(\rho_0) = 30$ MeV with the slope parameter $L$ of 60 MeV at saturation density $\rho_0 = 0.16$ fm$^{-3}$. Since the spin of a nucleon may flip after nucleon-nucleon scatterings but its dependence on the energy and isospins of the colliding nucleons is poorly known, to focus on effects from the spin-dependent nuclear mean-field potential we randomize the spins of nucleons after their scatterings. The strength of the spin-orbit coupling is set to be $W_0 = 150$ MeVfm$^5$ in the following calculations unless stated. The initial density distributions of projectile and target nuclei are obtained from a modified Skyrme-like interaction based on Hartree-Fock calculation, and the spins of initial nucleons are randomly distributed for reactions with unpolarized nuclei.

From the above equations of motion, nucleons with different spins are expected to have different dynamics, and the orientation of the spin is also affected by the spin-related potentials. In the transport simulation the beam direction is set to be in z direction, and x-o-z plane forms the reaction plane. It is natural to consider the spin polarization in y direction, i.e., the direction of the total angular momentum of the collision, and we thus define the nucleons with spin projection in +y (-y) direction as spin-up (spin-down) nucleons. We will illustrate in the following that despite the large cancellation of the effects from the time-even and time-odd spin-related potentials, the residue potentials for spin-up and spin-down nucleons are still different, and this leads to the spin splitting of the nucleon collective flows.
III. RESULTS AND DISCUSSIONS

To ease the following discussions, let’s first illustrate the density evolution in \( \text{Au+Au} \) collisions at a beam energy of 100 MeV/nucleon and an impact parameter of \( b = 8 \text{ fm} \) in Fig. 1. In the present configuration of projectile and target, local polarization is observed and participant (spectator) matter is slightly spin polarized in \(+y\) (-\(y\)) direction. However, one can estimate that this spin polarization leads to negligible contribution to the time-even and time-odd spin-dependent mean-field potentials, compared to the second terms in Eqs. (2) and (3). The third and the fourth row in Fig. 1 display the contributions from the dominating terms in \( U_{s-\text{even}} \) and \( U_{s-\text{odd}} \), respectively. It is seen that the time-odd spin-dependent potential is much larger than the time-even one, giving spin-up (spin-down) nucleons an additional net attractive (repulsive) potential. \(^1\)

The distribution of nucleons with respect to the rapidity \( y_r \) and transverse momentum \( p_T \) in heavy-ion collisions can be expressed as \([23, 24]\)

\[
\frac{d^3N}{p_T dp_T dy_r d\phi} = \frac{d^2N}{2\pi p_T dp_T dy_r} \left[ 1 + 2v_1(y_r, p_T) \cos(\phi) + 2v_2(y_r, p_T) \cos(2\phi) + \ldots \right],
\]

(7)

where \( \phi = \tan^{-1}(p_y/p_x) \) is the azimuthal angle,

\[ v_1 = \langle \cos(\phi) \rangle = \left\langle \frac{p_x}{p_T} \right\rangle \]

(8)

is the directed flow, and

\[ v_2 = \langle \cos(2\phi) \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle \]

(9)

is the elliptic flow. Both the directed flow and the elliptic flow are important observables for the study of dynamics as well as the equation of state of produced matter and particle interactions in heavy-ion collisions. In the present study we investigate the collective flows of free nucleons that leave the system with a freeze-out density of \( \rho_0/8 \).

The directed flow is a result of the participant squeeze-out and spectator bounce-off dynamics. Figure 2 shows the directed flows of all the nucleons as well as those for spin-up

\(^1\) According to the present configuration of projectile and target, the total angular momentum of the system is in \(+y\) direction. If initially the target (projectile) nucleus is put on the -x (+x) side like the conversional configuration used by others in non-central collisions, the total angular momentum is in -\(y\) direction. In that case the spin-up (spin-down) nucleons with spin in \(+y\) (-\(y\)) direction have a net repulsive (attractive) potential as a result of the spin-orbit coupling.
FIG. 1: (Color online) Contours of the nucleon reduced density $\rho/\rho_0$ (first row), $y$ component of the spin density $s_y$ (second row), product of the $x$ component of the density gradient $(\nabla \rho)_x$ and the averaged $z$ component of nucleon momentum (third row), and the $y$ component of the curl of the momentum density $(\nabla \times \vec{j})_y$ (fourth row) in the reaction plane at different stages in Au+Au collisions at a beam energy of 100 MeV/nucleon and an impact parameter of $b = 8$ fm.

and spin-down nucleons in Au+Au collisions at different beam energies and impact parameters. Because of the different initializations of the projectile and the target in momentum-coordinate space in simulations, the slope of the directed flow in our simulations might be opposite in sign compared to results obtained using other conventions of initializing nuclei. Due to the competition between the attractive mean-field potential in the energy range considered and the repulsive nucleon-nucleon scattering, the directed flow increases with increasing beam energy, and the slope is largest at $b = 4$ fm and changes sign at $b = 12$ fm. Moreover, one can observe the flow splitting of spin-up and spin-down nucleons, and the
FIG. 2: (Color online) Directed flows for all the nucleons, spin-up nucleons, and spin-down nucleons as a function of reduced rapidity in Au + Au collisions at different beam energies and impact parameters.

flow difference is mainly due to the residue potential from the cancellation of the time-even and the time-odd potential as discussed above, with spin-up (spin-down) nucleons a more attractive (repulsive) potential and thus leading to a smaller (larger) or more (less) negative directed flow.

In order to quantitatively describe the spin splitting of the directed flow, we defined the spin up-down differential directed flow

\[ v_{1}^{ud}(y_r) = \frac{1}{N(y_r)} \sum_{i=1}^{N(y_r)} \sigma_i \left( \frac{p_x}{p_T} \right)_i, \]  

where \( N(y_r) \) is the number of nucleons with rapidity \( y_r \), and \( \sigma_i \) is 1\((-1\) for spin-up (spin-down) nucleons. The above differential directed flow largely cancels the effect from spin-
FIG. 3: (Color online) Spin up-down differential directed flows as a function of reduced rapidity in Au + Au collisions at different beam energies and impact parameters.

independent potentials while highlights the spin splitting of the flow as a result of spin-dependent potentials. Results for the spin up-down differential directed flow in Au+Au collisions at different beam energies and different impact parameters are shown in Fig. 3. It is seen that the differential directed flow is generally stronger with increasing impact parameter, different from the total directed flow as show in Fig. 2. This is because that the spin-dependent potentials are mostly related to the gradient of density or current, and it is thus a surface effect which is more important at large impact parameters due to the diffusive density distribution of finite nuclei. In addition, as the angular momentum of heavy-ion collisions increases with increasing beam energy, the differential directed flow as a result of the nucleon spin-orbit coupling increases with increasing beam energy. On the other hand, the violent nucleon-nucleon scatterings, which play a more important role at higher collision energies, randomize the nucleon spin and wash out part of the effects from the spin-related potential, so we found that the spin up-down differential directed flow becomes weaker at even higher energies. The competition between the above two effects leads to a maximum of the differential directed flow at beam energies around 100-200 MeV/nucleon in mid-central and peripheral Au+Au collisions. One sees from Figs. 2 and 3 that the magnitude of the
differential directed flow is as large as 20% that of the total directed flow in mid-central and peripheral collisions.

![Graph showing directed flows and spin up-down differential directed flows as a function of reduced rapidity.](image)

FIG. 4: (Color online) Directed flows (left column) and spin up-down differential directed flows (right column) as a function of reduced rapidity in mid-central $^{197}$Au + $^{197}$Au and $^{124}$Sn + $^{124}$Sn collisions at beam energies of 50, 100, and 200 MeV/nucleon at the same centrality. The density profiles for $^{197}$Au and $^{124}$Sn are shown in the inset.

We have also studied the system-size dependence of both the total directed flow and the spin up-down differential directed flow at different beam energies, and the results are shown in Fig. 4. Here we choose an impact parameter of $b = 8$ fm for $^{197}$Au + $^{197}$Au collisions and $b = 6.9$ fm for $^{124}$Sn + $^{124}$Sn collisions so that the two colliding systems have the same $b/b_{\text{max}}$ and can be compared at the same centrality. It is seen that the directed flow is larger for $^{197}$Au + $^{197}$Au collisions than for $^{124}$Sn + $^{124}$Sn collisions at different beam energies due to the higher density reached in the heavier system and thus a higher pressure which leads to a larger directed flow. On the other hand, the spin up-down differential directed flow
for $^{197}$Au + $^{197}$Au collisions and $^{124}$Sn + $^{124}$Sn collisions are nearly the same at the beam energy of 50 and 100 MeV/nucleon. This is due to the similar density gradient near the nucleus surface for $^{197}$Au and $^{124}$Sn as shown in the inset of Fig. 4, which leads to a similar strength of the spin-dependent potentials. At the beam energy of 200 MeV/nucleon, the spin up-down differential directed flow in $^{124}$Sn + $^{124}$Sn collisions is slightly larger than that in $^{197}$Au + $^{197}$Au collisions. The reason is that for a heavier system at relatively higher beam energies, the nucleon-nucleon scatterings are more violent and they reduce more effects from the spin-orbit coupling compared to a smaller system. From the above discussions, it is seen that the spin splitting of the directed flow is a robust phenomena even in smaller systems.

![Graph](image.png)

**FIG. 5:** (Color online) Elliptic flows of all the nucleons, spin-up nucleons, and spin-down nucleons in Au + Au collisions at different beam energies and impact parameters.

The elliptic flow is a result of the expansion of almond-shaped medium in heavy-ion collisions and the dynamics is more complicated than the directed flow. The in-plane hydrodynamical flow leads to a positive elliptic flow while the out-of-plane squeeze-out results in a negative elliptic flow. Figure 5 displays the elliptic flows of all the nucleons, spin-up nucleons, and spin-down nucleons in Au + Au collisions at different beam energies and im-
pact parameters. One sees that the sign of the elliptic flow changes from positive at lower energies to negative at high energies, as a result of blocking effects from the spectator on the expansion of the participant \([25]\), and it is seen that the energy at which the elliptic flow changes sign depends on the impact parameter. Similar to the case for the directed flow, the spin splitting of the elliptic flow is observed and the effect is more appreciable in peripheral collisions at beam energies of about 100-200 MeV/nucleon. However, the dynamics is more complicated as the magnitude of the elliptic flow depends on the nucleon potential as well as the shadowing from the spectator, and in this case a more attractive (repulsive) potential for spin-up (spin-down) nucleons somehow leads to a larger \(v_2\). We have also observed that in this collision situation neutrons with more repulsive potential than protons have a smaller \(v_2\) when Coulomb potential is turned off, confirming the validity of our argument on the relative \(v_2\) splitting and the potential difference.

![FIG. 6: (Color online) Spin up-down differential elliptic flow as a function of reduced rapidity in Au+Au collisions at different beam energies and impact parameters.](image)

To quantitatively describe the spin splitting of the elliptic flow, we can similarly define the spin up-down differential elliptic flow

\[
v^{ud}_2(y_r) = \frac{1}{N(y_r)} \sum_{i=1}^{N(y_r)} \sigma_i \left( \frac{p_{x_i}^2 - p_{y_i}^2}{p_T^2} \right),
\]

and the rapidity dependence of \(v^{ud}_2\) is shown in Fig. 6. Consistent with Fig. 5 it is seen
that the differential elliptic flow increases with increasing impact parameter and collision energy, and its magnitude can be as large as 20% that of the total elliptic flow in peripheral collisions at beam energies of 100-200 MeV/nucleon.

![Graph showing the transverse momentum dependence of the elliptic flow for mid-rapidity spin-up and spin-down nucleons in peripheral Au+Au collisions at different beam energies.]

FIG. 7: (Color online) Transverse momentum dependence of the elliptic flow for mid-rapidity spin-up and spin-down nucleons in peripheral Au+Au collisions at different beam energies.

We have also studied the transverse momentum dependence of the elliptic flow for mid-rapidity spin-up and spin-down nucleons, and the results in peripheral Au+Au collisions at beam energies of 50, 100, and 200 MeV/nucleon are shown in Fig. 7. The elliptic flow for spin-up nucleons are larger than that for spin-down nucleons, and the difference is larger at higher nucleon transverse momenta. This is understandable as the strength of the spin-related potential increases with increasing nucleon momentum from Eqs. (2) and (3).

The spin up-down differential flows defined above can be good probes of the nuclear spin-dependent interaction. As an illustration, we compared the differential directed flows and elliptic flows from different values of nuclear spin-orbit coupling constant $W_0$ in Fig. 8. It is seen that when the value of $W_0$ decreases from 150 MeVfm$^5$ to 80 MeVfm$^5$, the slope of the differential directed flow as well as the magnitude of the differential elliptic flow are also reduced by approximately a factor of two. It thus confirms that the spin-related mean-field potentials induced by the nuclear spin-dependent interaction are the source of the spin dynamics of heavy-ion collisions, and both the spin up-down differential directed flow and elliptic flow are sensitive candidates for studying the in-medium nuclear spin-orbit interaction.
FIG. 8: (Color online) Spin up-down differential directed flow (a) and elliptic flow (b) with different spin-orbit coupling strength in peripheral Au+Au collisions at a beam energy of 200 MeV/nucleon.

interaction and the spin dynamics in intermediate-energy heavy-ion collisions.

IV. SUMMARY

In summary, we have investigated the spin splitting of collective flows based on the spin and isospin-dependent Boltzmann-Uehling-Uhlenbeck transport model. It is found that the spin up-down splitting of both the directed flow and elliptic flow are largest in Au+Au reactions at beam energies of about 100-200 MeV/nucleon, and they are generally more appreciable in peripheral collisions. The effect is also considerable in smaller collision systems and becomes larger for energetic nucleons. The nucleon spin up-down differential directed flow and elliptic flow are sensitive and robust probes of the in-medium nuclear spin-orbit interaction, which is related to many interesting problems in nuclear structure as well as nuclear astrophysics. Our study shed new light on the spin dynamics in intermediate-energy heavy-ion collisions and may stimulate interests in carrying out more spin-related
experiments in the near future.

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