Tidal Decay of Close Planetary Orbits

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ABSTRACT

The 4.2-day orbit of the newly discovered planet around 51 Pegasi is formally unstable to tidal dissipation. However, the orbital decay time in this system is longer than the main-sequence lifetime of the central star. Given our best current understanding of tidal interactions, a planet of Jupiter’s mass around a solar-like star could have dynamically survived in an orbit with a period as short as $\sim 10$ hr. Since radial velocities increase with decreasing period, we would expect to find those planets close to the tidal limit first and, unless this is a very unusual system, we would expect to find many more. We also consider the tidal stability of planets around more evolved stars and we re-examine in particular the question of whether the Earth can dynamically survive the red-giant phase in the evolution of the Sun.

Subject headings: Planets and Satellites: General — Solar System: General — Stars: Planetary Systems — Sun: Solar-terrestrial Relations

1. Introduction

A new era in astronomy has begun recently with the first clear detections of several extra-solar planets, first around a millisecond pulsar (Wolszczan 1994) then around several solar-type stars (Mayor & Queloz 1995; Marcy & Butler 1996). These discoveries will no doubt lead to significant improvements in our understanding of many processes related not

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only to planet formation, structure and evolution, but also, as we illustrate in this paper, to stellar astrophysics.

The new planets have brought many surprises. In particular, the existence of a Jupiter-type planet with a very short orbital period of 4.2 d around 51 Peg (Mayor & Queloz 1995) is very puzzling. Not only is it difficult to fit such an object into accepted scenarios for planet formation but it turns out, as we show below, that its orbit is unstable to tidal dissipation. Orbital decay is therefore inevitable in this system, and the Jupiter-mass companion will ultimately spiral into the star. However, in this system, we will show that the orbital decay timescale is longer than the main-sequence lifetime of the solar-like star, consistent with the planet’s surviving to the present. We will then investigate tidal survival in general for Jupiter-like planets in close orbits around solar-type stars.

In the 51 Peg system, we can be sure that the planet will not survive any post-main-sequence evolution of the central star. By the time the star has grown to about twice its current radius, the orbital decay rate will have exceeded the evolution rate. This last point brings us to reconsider the fate of our own planet Earth. Current stellar evolution calculations (Sackmann, Boothroyd, & Kraemer 1993), taking into account the mass loss from the Sun and the resulting expansion of the Earth’s orbit, predict that the Earth will not be engulfed even when the Sun reaches its maximum radius at the tip of the giant branch. However, as the mass-loss rate increases with the expansion of the solar envelope, so does the tidal decay rate of the Earth’s orbit and we will show here that, as a result, the Earth may well not survive after all.

2. Tidal Evolution of 51 Peg

2.1. Orbital Stability

The combination of tidal torques and viscous dissipation in binary systems act both to circularize the orbit and to synchronize the spin of the stars with the orbital rotation. The circular, synchronized state normally corresponds to a minimum energy for a given total angular momentum. However, when the two components are close, the synchronized state may be unstable (Hut 1980). Instability occurs when the ratio of spin angular momentum to orbital angular momentum $J_{\text{spin}}/J_{\text{orb}} > 1/3$.

If the 51 Peg system were synchronized at its current orbital period, then, in terms of
observable quantities, we would have

$$J_{\text{spin}} / J_{\text{orb}} = 1.47 \left( \frac{M}{M_\odot} \right)^{-1/3} \left( \frac{R}{R_\odot} \right)^2 \left( \frac{k}{k_\odot} \right)^2 \left( \frac{P}{4.23 \text{ d}} \right)^{-5/3} \left( \frac{K}{59 \text{ m s}^{-1}} \right)^{-1} \sin i, \quad (1)$$

where $M$ and $R$ are the mass and radius of the star, $k = (I_*/MR^2)^{1/2}$ is its dimensionless gyration radius, $P$ is the orbital period, $K$ is the projected orbital velocity and $i$ is the unknown inclination angle of the orbit. The star in 51 Peg appears to be nearly identical to our own Sun (mass $M_\odot$, radius $R_\odot$, $k_\odot^2 = 0.08$; see Mayor & Queloz 1995). From line-profile analysis (Soderblom 1983) and given that the star is not active (Mayor & Queloz 1995) we estimate $\sin i \approx 0.4$. Therefore, if synchronized, 51 Peg is certainly very close to the formal stability limit and probably exceeds it.

However, it is almost certain that the star in this system is in fact not synchronously rotating. Instead, it appears to be slowly spinning, much as we would expect an ordinary solar-like star (Mayor & Queloz 1995). Stars like the Sun are believed to have been spun down by magnetic braking (Skumanich 1972). Magnetic dynamo models indicate that the braking rate decreases with decreasing angular velocity. Skumanich's surface velocity $v_{\text{surf}} \propto t^{-1/2}$, where $t$ is the age of the star, leads to a spin-down timescale of $\tau_{\text{sd}} \propto \Omega_{\text{spin}}^{-2}$, while the theory of Tout & Pringle (1992) for fully convective, rapidly rotating stars leads to the weaker, but still inverse, dependence of $\tau_{\text{sd}} \propto \Omega_{\text{spin}}^{-1/2}$. In general the tidal synchronization rate increases with the lack of synchronicity, $\tau_{\text{su}} \propto \Delta \Omega^{-1}$, where $\Delta \Omega$ is the difference between stellar spin and orbital angular velocities. If $\tau_{\text{su}}$ and $\tau_{\text{sd}}$ were both short compared with the age of the star then an equilibrium would be set up in which $\tau_{\text{su}} \approx \tau_{\text{sd}}$. This is the case for cataclysmic variables, in which tidal spin up and magnetic braking (or gravitational radiation of angular momentum) are in equilibrium (Verbunt & Zwaan 1981). These systems are unstable because their orbits decay on a timescale $\tau_{\text{sd}}$.

In the case of 51 Peg, because the star appears to be spinning slowly we can conclude that $\tau_{\text{sd}} \ll \tau_{\text{su}}$ over the lifetime of the star and magnetic braking in this system has proceeded in a similar way to the Sun. For the Sun the current braking timescale is of the order of its age $\tau_{\text{sd}} \approx 10^{10}$ yr so that $\tau_{\text{su}} \gg 10^{10}$ yr. Further, because tides transfer orbital angular momentum to the stellar spin, $\tau_{\text{su}} \approx (I_{\text{orb}}/I_*) \tau_a$, where $\tau_a$ is the orbital decay timescale for the system and $I_{\text{orb}}$ and $I_*$ are the moments of inertia the orbit and star respectively, we may write

$$\tau_{\text{su}} = \frac{ma^2}{k^2 M R^2} \tau_a \approx \tau_a, \quad (2)$$

where $m \approx 10^{-3} M$ is the mass of the planet and $a \approx 10 R$ is the radius of its orbit. Thus we expect $\tau_a \gg 10^{10}$ yr so that, although it is unstable, this particular orbit will not decay during the star’s main-sequence lifetime. We shall proceed to calculate the timescale $\tau_a$. 
from detailed tidal theory both to test this expectation and to place limits on what other systems might be found in the future.

### 2.2. Orbital Decay Rate

The rate of tidal transfer of angular momentum is rather uncertain, particularly in solar-like stars where the dissipation is provided by the eddy viscosity in a relatively deep but not very massive convective envelope. However, these uncertainties do not affect our main conclusions. Assuming standard tidal dissipation theory (see, e.g., Zahn 1977; Verbunt & Phinney 1995) we write the orbital decay rate as

\[
\frac{1}{\tau_a} = \frac{|\dot{a}|}{a} = \frac{f}{\tau_c} \frac{M_{\text{env}}}{M} q(1 + q) \left( \frac{R}{a} \right)^8,
\]

where \(\tau_c \approx (MR^2/L)^{1/3}\) is the eddy turnover timescale. Here we will actually use a more precise estimate for the turnover time of the largest eddies at the base of the convective zone (cf. §3),

\[
\tau_c = \left[ \frac{M_{\text{env}} R_{\text{env}} (R - R_{\text{env}})}{3L} \right]^{1/3}
\]

where \(M_{\text{env}}\) is the mass in the convective layer, \(R_{\text{env}}\) is the radius at its base, and \(M, R\) and \(L\) are the mass, radius and luminosity of the star. The mass ratio \(q = m/M\), where \(m\) is the mass of the planet and \(a\) is its orbital radius. The numerical factor \(f\) is obtained by integrating viscous dissipation of tidal energy throughout the convective zone. Theoretically (Zahn 1977) and observationally (Verbunt & Phinney 1995) \(f \approx 1\) as long as \(\tau_c \ll P\), the orbital period.

However, if the tidal pumping period \(P/2\) is less than \(\tau_c\) then the largest convective cells can no longer contribute to viscosity, because the velocity field that they are damping will have changed direction before they can transfer momentum. If convection is modelled as a turbulent cascade, then only eddies which turn over in a time less than the pumping timescale will contribute (Goldreich & Keeley 1977) and the average length \(l\) and velocity \(v_c\) of these cells will both be smaller by the same factor \(2\tau_c/P\). The eddy viscosity \(\nu \approx v_c l/3\) is then reduced by a factor \((2\tau_c/P)^2\). We may therefore write in general

\[
f = f' \min \left[ 1, \left( \frac{P}{2\tau_c} \right)^2 \right],
\]

where now \(f' \approx 1\). In a main-sequence solar-type star \(\tau_c \approx 20\) d, so that \(f \approx 0.01f'\). The planet around 51 Peg has a mass similar to that of Jupiter (Mayor & Queloz 1995) so that
\[ q \approx M_1/M_\odot \approx 10^{-3}. \] The period of 4.23 d then corresponds to a separation \( a \approx 11 R_\odot \) and we get

\[ \tau_a \approx 4 \times 10^{13} \text{ yr} \frac{1}{f^\prime} \left( \frac{\tau_c}{20 \text{ d}} \right)^3 \left( \frac{M_{\text{env}}}{0.028 M_\odot} \right)^{-1} \left( \frac{M}{M_\odot} \right)^{-1} \left( \frac{q}{10^{-3}} \right)^{-1} \left( \frac{a}{11 R_\odot} \right)^8 \left( \frac{R}{R_\odot} \right)^{-8}, \] (6)

where we have used the value \( M_{\text{env}} = 0.028 M_\odot \) from our own solar models discussed below (§3). The star in 51 Peg may be slightly more evolved, and therefore slightly larger than the Sun at present, with the timescale accordingly shorter. Mayor & Queloz quote a radius \( R = 1.29 R_\odot \), which would give \( \tau_a = 5 \times 10^{12} \text{ yr} \). However, we must compare this orbital decay time with the main-sequence lifetime \( \tau_{\text{ms}} \approx 10^{10} \text{ yr} \). Over most of this lifetime the star was smaller, and had a radius closer to \( 1 R_\odot \) if its mass is close to \( 1 M_\odot \). Clearly, equation (6) shows that \( \tau_a \gg \tau_{\text{ms}} \), consistent with the fact that this system has survived to the present.

We can define a minimum period for survival \( P_{\text{min}} \) by setting \( \tau_a = \tau_{\text{ms}} \) in equation (6) and solving for \( P \). This gives

\[ P_{\text{min}} = 8.6 \text{ hr} \left[ f^\prime \left( \frac{\tau_{\text{ms}}}{10^{10} \text{ yr}} \right) \left( \frac{M_{\text{env}}}{0.028 M_\odot} \right) \left( \frac{m}{M_J} \right)^{3/10} \left( \frac{\tau_c}{20 \text{ d}} \right)^{-9/10} \left( \frac{M}{M_\odot} \right)^{-7/5} \left( \frac{R}{R_\odot} \right)^{12/5} \right] \] (7)

We see that for 51 Peg we would need \( P < P_{\text{min}} \approx 9 \text{ hr} \) before the orbital decay rate becomes significant. At this period a synchronized Jupiter of radius \( R_p \approx R_J \approx 0.1 R_\odot \) would just fill its Roche lobe (of radius \( R_L \approx 0.49 a q^{1/3} \) for \( q \ll 1 \)) and would therefore begin to be tidally disrupted anyway. Even if convective viscosity were not limited by the turnover timescale (i.e., if \( f \approx 1 \) in equation (3)), the minimum period would become \( P_{\text{min}} \approx 2 \text{ d} \) and 51 Peg’s planet would still be safe.

In Figure 1 we show the survival boundary for planets around a solar-like star, as a function of period and planetary mass. It follows from the figure that we cannot significantly constrain the mass of this planet by tidal effects. If planets exist close to this boundary then they are likely to be found first in radial velocity searches, because the radial velocity increases with decreasing period. In the same figure, we show lines of constant radial velocity, above which detections are likely. We note that, although this particular planet is in the corner of detectability and survivability, it is not so close to either boundary as to make it unlikely. The positions of the more recently discovered planets around 70 Vir and 47 UMa (Marcy & Butler 1996) are also included in the figure. Both are further from the tidal boundaries than 51 Peg.
Fig. 1.— The dynamical limits for a planet of mass \( m \) and period \( P_{\text{orb}} \) around a solar-like star. The timescale \( \tau_a \) is the orbital decay timescale obtained from equations (2)–(5). The survival boundary is defined by setting \( \tau_a = \tau_{\text{ms}} \), where \( \tau_{\text{ms}} \approx 10^{10} \) yr is the main-sequence lifetime. The dashed line for \( \tau'_a = 10^{10} \) yr applies if the tidal dissipation is not reduced by the long convective turnover timescale (i.e., when setting \( f = 1 \) rather than using eq. [4]). Also shown is the Roche limit where a planet of Jupiter’s radius would overflow its critical tidal lobe \( (R_L < R_J) \). The boundary where \( K/\sin i = 10 \) m s\(^{-1} \) represents an optimistic detection threshold for current spectroscopic searches. The dashed line at \( K/\sin i = 100 \) m s\(^{-1} \) is a more conservative estimate. The \( M \sin i \) values for the planets around 70 Vir and 47 UMa are also plotted against their periods. They are both much further from the tidal boundaries.
2.3. Orbital Circularization

The current measurement of the orbital eccentricity of 51 Peg gives $e = 0.09 \pm 0.06$, indicating a nearly-circular orbit (Mayor & Queloz 1995). We can be fairly certain that tidal dissipation in the star has not played any role in the circularization of the orbit during the main-sequence phase of the evolution. This is because the timescales for circularization and orbital decay are the same within a factor of 2 (see, e.g., Zahn 1977). Therefore, the same mechanism would have brought about a substantial decay of the orbit, which would now be shrinking rapidly, making it highly improbable that the system would be found in its current state.

However, the star also raises tides on the planet. These tides can synchronize the planet’s spin with the orbit in a timescale (Guillot et al. 1995)

$$\tau_s \approx Q \left( \frac{R_p^3}{G m} \right) \omega \left( \frac{a}{R_p} \right)^6,$$

where $\omega = |\omega_{\text{spin}} - \omega_{\text{orb}}|$ is the difference between the spin and orbital angular velocities. The factor $Q$ is inversely proportional to the dissipation and has been determined observationally (Ioannou & Lindzen 1993) for Jupiter to be about $10^5$. Therefore the timescale to synchronize a Jupiter-like planet originally spinning as fast as Jupiter is $\tau_s \approx 2 \times 10^6$ yr. The same tides can also circularize the orbit, even after synchronization, because radial tides can dissipate energy without transferring angular momentum. The circularization timescale is (Goldreich & Soter 1966)

$$\tau_e = \frac{4}{63} Q \left( \frac{a^3}{GM} \right)^{1/2} \left( \frac{m}{M} \right) \left( \frac{a}{R_p} \right)^5.$$

Now $Q$ is proportional to the tidal pumping period, so that, with the same dissipation mechanism, it will be ten times larger after synchronization of the planet. The circularization timescale becomes $\tau_e \approx 2 \times 10^9$ yr. Since this is less than the age of the system, the fact that the observed orbit is circular is not surprising.

3. Implications for Planet Formation

Clearly, if planets commonly form at short periods, and we assume that they can survive the effects of thermal evaporation (Guillot et al. 1996), we may now expect to find many more. Such discoveries can affect our ideas of planet formation (see, e.g., Lissauer 1993 and references therein), which require Jovian planets to form at distances of a few astronomical units or more.
Indeed, models of the proto-solar nebula suggest that the temperature increases very rapidly close to the center. This rapid rise in temperature has been invoked to explain the difference in composition between rocky terrestrial planets and ice-rich satellites in the outer solar system, as well as the absence of objects inside Mercury’s orbit. The high temperature is thought to prevent the condensation of high-Z material into grains (Barshay & Lewis 1976), thereby preventing planetesimal formation. In addition, any planetesimals that would form in the innermost region of a protoplanetary disk are expected to spiral-in rapidly because of gas drag, and to be destroyed. Alternatively, models in which the formation of a giant planet occurs directly by gravitational collapse in a massive disk have also been discussed, but recently these models have fallen out of favor for a variety of reasons (Lissauer 1993).

If the new planet in 51 Peg had formed, like our own Jupiter, at a large distance from the central star, some angular-momentum-loss mechanism must have brought it in. Any dissipative mechanism, such as friction in a primordial stellar nebula, would, like the tides, increase rapidly with decreasing separation. It would have had to switch off at a critical moment for the planet to end up so close to the star and, in this case, this kind of system ought to be extremely rare.

An interesting formation process based on a dissipative mechanism has been discussed by Lin, Bodenheimer & Richardson (1996). The mechanism they consider is the dissipative interaction between the planet and a protoplanetary disk. Spiraling-in of the planet could be halted at a small distance from the star when the tidal interaction becomes important. For this to work, Lin et al. must invoke a somewhat shorter timescale for tidal interaction on the pre-main-sequence. In addition, the star must be spinning faster than the planet is orbiting while the external torque acts. We note that, since the star is spinning much more slowly today, its spin rate must have passed through a point of synchronous rotation and become unstable some time in the past. The system could only have survived this if the star had already contracted or altered its structure sufficiently that \( \tau_a \) were already long at that time. This would require a rather delicate balance between the spin-down, contraction and tidal timescales of the protostellar 51 Peg. For instance, a spin-down timescale of the order of \( 10^4 \) yr, deduced from the theory of Tout & Pringle (1992), would have left the structure and size of the protostar virtually unchanged at corotation and the planet would inevitably have fallen in.

Alternatively, a dynamical mechanism for angular momentum loss could also be invoked. One possibility (Rasio & Ford 1996) is that two (or more) Jupiter-like planets had initially formed (in the conventional way) at a large distance from the central star, and later interacted dynamically. This could happen because the planets’ orbits evolved secularly at
different rates, or because their masses increased, resulting in a dynamical instability of the
orbits and a close interaction between the two planets (Gladman 1993). The interaction
can lead to the ejection of one planet, leaving the other in a highly eccentric orbit. If the
pericenter distance of the inner planet is sufficiently small, its orbit can later circularize at
an orbital separation of a few stellar radii, as seen in 51 Peg. Otherwise, the planet would
be left in a highly eccentric orbit at some intermediate distance, as seen in another recently
discovered system, 70 Vir (Marcy & Butler 1996).

4. Implications for Tidal Dissipation Theory

If common, planets such as the one around 51 Peg will be invaluable for improving
our understanding of tidal dissipation. Their advantage over more equal mass binary
systems lies in the asynchronicity of the star leading to orbital decay, rather than just to
circularization. For instance, we might envisage a more effective source of viscosity than
convective turbulence, such as magnetic fields generated by the tidal disturbance. This
would move the tidal boundary in Figure 1, and the discovery of planets well to the left of
the new boundary could be used to rule out that particular mechanism.

Even with this system we can already comment on a much more efficient mechanism
for circularization discussed by Tassoul and Tassoul (Tassoul & Tassoul 1992). In their
model, tidal forces induce circulations which are viscously damped at the effectively rigid
surface of the star. The timescale for orbital decay is (Livio 1994)

$$\tau_a \approx 9 \times 10^6 \text{ yr} \ (1 + q)^{-11/8} \left( \frac{L}{L_\odot} \right)^{-\frac{1}{2}} \left( \frac{M}{M_\odot} \right)^{-\frac{1}{8}} \left( \frac{M_{\text{env}}}{0.028 \ M_\odot} \right)^{-1} \left( \frac{R}{R_\odot} \right)^{\frac{9}{8}} \left( \frac{a}{11 \ R} \right)^{\frac{40}{9}},$$

(10)

for 51 Peg. Even if a factor of 100 (from eq. 11) is applicable here too, this is still much
less than the age of the system. Therefore we can deduce observationally that the Tassoul
mechanism does not operate in this system.

5. Planets around Red Giants

5.1. The Fate of the Earth

The star 51 Peg is already slightly more evolved than the Sun (Mayor & Queloz 1995). When it becomes a red giant of about twice the Sun’s radius, it will already possess a deep,
massive convective envelope in which the critical turnover time is not much longer than it
is now. The orbital decay time $\tau_a$ will then be reduced by a factor of about $10^4$, becoming
shorter than the stellar evolution time for the red giant. Inevitably, the planet’s orbit will then decay into the stellar envelope.

This last point brings us to re-examine the fate of our own Earth. In the case of a planetary system resembling our own, the planets can get close enough to the central star only when the star evolves to become a giant. The innermost planets may be completely engulfed inside the stellar envelope during this phase. Early stellar evolution calculations suggested that this would in fact happen to the Earth. First, on the red giant branch, the Sun would swell up, almost but not quite reaching the Earth’s orbit. Then, on the asymptotic giant branch, after the exhaustion of core helium, the Sun would engulf the Earth altogether.

However, theoretical considerations (Faulkner 1972) and observations (Reimers 1975) of giants and supergiants indicate that these stars suffer mass loss at high rates. As a result of mass loss, the Earth’s orbit expands owing to conservation of angular momentum. An inclusion of this effect in the evolutionary calculations for the Sun (Sackmann et al. 1993) show that its mass will be reduced to $0.59 M_\odot$ when it reaches its maximum radius of $0.99 \text{AU}$. By this time, however, the earth’s orbit will be at $1.69 \text{AU}$ and so the current prediction is that the Earth will in fact survive this phase (see also Maddox 1994). A reduction in the mass-loss rate (which is somewhat uncertain) would work in the direction of increasing the probability of engulfing (by increasing the Sun’s final radius and reducing the expansion of the Earth’s orbit), but it appears that for mass-loss rates which are consistent with those deduced from star clusters (Weidemann & Koester 1983), the Earth would always survive. The important point, however, is that the calculations of Sackmann et al. (1993) neglected the orbital decay driven by the tidal interaction between the Earth and the Sun.

5.2. Effects of Tidal Dissipation

We have recomputed the evolution of the Earth–Sun system, using the evolutionary code of Eggleton (Pols et al. 1995), taking into account the effects of tidal dissipation. Mass loss was included according to the Reimers formula (Kudritzki & Reimers 1978)

$$\dot{M} = -4 \eta 10^{-13} M_\odot \text{yr}^{-1} \frac{(L/L_\odot)(R/R_\odot)}{(M/M_\odot)},$$

(11)

with $\eta = 0.6$, as in Sackmann et al. (1993). First, we confirm their results: when we neglect the tidal interaction, we find that the Earth always remains safely outside the Sun’s expanding envelope. At the tip of the giant branch we find $a = 315 R_\odot$, while the evolved
solar radius $R'_\odot = 195 R_\odot$. The evolution of the radius of the Sun and the radius of the Earth’s orbit are shown in Figure 2.

Next, we include the tidal interaction by integrating equation (3) in parallel with the stellar evolution equations. Here we can assume $f \approx 1$ since $P > 1 \, \text{yr} > \tau_c$. For $f = 1$ (as favored for example by the results of Verbunt & Phinney 1995), we find that the effects of tidal dissipation are already quite noticeable at the tip of the giant branch. Note that Venus’ orbit would be engulfed by the Sun already in this case (see Fig. 2). For $f = 2$ the Earth’s orbit decays rapidly enough for our planet to be completely engulfed as well. Our current understanding of tidal dissipation is sufficiently uncertain that $f$ could well be as large as this, especially if magnetic fields are important. Clearly, any additional observational evidence from nearby planets in close orbits could be invaluable in determining the fate of our own.

We thank Doug Lin for useful discussions. F.A.R. is supported by an Alfred P. Sloan Research Fellowship. S.L. acknowledges support from NASA Grant NAGW-4156 and M.L. from NASA Grant NAGW-2678.

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Note, however, that our formulation of eq. 3 will slightly overestimate the tidal effects relative to the results of Verbunt & Phinney (1995) with the same numerical value for $f$. 
Fig. 2.— The evolution of the orbital separation of the Earth $a_\oplus$ and the radius of the Sun $R'_\odot$ as functions of its mass $M'_\odot$, which is falling (increasingly rapidly) as a result of mass loss in a stellar wind. The top line shows the Earth’s orbit in the absence of tidal dissipation. The next two lines correspond to tidal dissipation with $f = 1$ and $f = 2$ in equation (2). The orbit of Venus is also shown with tidal effects included for $f = 1$. When the orbit curves intersect the solar-radius curve the Sun’s expanding envelope engulfs the planets. For $f = 2$ this will happen to the Earth in 8 Gyr from now.
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