We introduce chiral-even and chiral-odd meson wave functions as vacuum-to-meson matrix elements of bilocal quark operators with well-defined (geometric) twist. Thereby, we achieve a Lorentz invariant classification of these distributions which differ from the conventional ones by explicitly taking into account the trace terms. The relations between conventional and new wave functions are given.

I. INTRODUCTION

Recently, we have introduced a group theoretical procedure to decompose local and nonlocal operators [1,2], which are important in different hard scattering processes, into operators of definite twist. This procedure is based on the notion of geometric twist $\tau = d - j$, originally introduced by Gross and Treiman [3]. We are interested in nonlocal light-cone (LC) operators and their matrix elements which describe different phenomenological distribution amplitudes, e.g., parton distribution functions as well as hadronic wave functions. The classification of these functions suffers from the fact, that one and the same operator by its twist decomposition contributes to different wave functions. It is thus necessary to disentangle these various contributions of the original operator and to specify the contributions of the trace terms.

Jaffe and Ji [4] proposed the notion of dynamical twist ($t$) by counting powers $Q^2 - t$ which is directly related to the power by which the corresponding distributions contribute to the scattering amplitudes. They classified the parton distribution functions which correspond to the independent tensor structure of the matrix elements of bilocal quark-antiquark operators. Recently, Ball et al. have used this pattern for the classification of leading and higher twist wave functions of $\rho$-vector mesons in QCD [5–7]. They found eight independent (two-particle) meson wave functions. One key ingredient in their approach was the use of QCD equation of motion in order to obtain integral representations for wave functions that are not dynamically independent. Ball et al. have already pointed out that the geometric twist is more convenient to discuss higher twist effects on a reliable basis [6]. On the other hand, the pion wave functions are earlier investigated in similar way [8,9]. Meson wave functions and form factors have been discussed in the framework of local operator product expansion and in the infinite momentum frame by [10–15] and in the framework of nonlocal operator product expansion [16–18].

However, such different notion of dynamical twist is only defined for the matrix elements of operators, it is not Lorentz invariant and also not simply related to the contributions of (higher) geometric twist. Quit recently, in order to show the relation between the different definitions of twist, we have calculated the forward quark distribution functions by the help of our bilocal quark operators with definite twist [19]. We proved that the two definitions of twist do not coincide at higher orders and we gave relation between Jaffe and Ji’s distribution function and our distributions with geometrical twist.

The aim of this paper is to present the (two-particle) meson wave functions which are related to the nonlocal LC-operators of different geometric twist. In that framework, it is possible to investigate in an unique manner the contributions resulting from the traces of the operators having well-defined twist.

II. TWIST DECOMPOSITION OF BILINEAR QUARK OPERATORS

In [1,2] we explained a procedure of twist decomposition for the bilocal quark operators on the light-cone which are relevant for the meson wave functions, too:

\begin{align*}
O_{\alpha}(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) &= \bar{u}(\kappa_1 \tilde{x})\gamma_{\alpha}U(\kappa_1 \tilde{x}, \kappa_2 \tilde{x})d(\kappa_2 \tilde{x}), \\
M_{[\alpha, \beta]}(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) &= \bar{u}(\kappa_1 \tilde{x})\sigma_{\alpha\beta}U(\kappa_1 \tilde{x}, \kappa_2 \tilde{x})d(\kappa_2 \tilde{x}),
\end{align*}

\begin{align*}
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\end{align*}
with the path ordered gauge factor along the straight line connecting the points $\kappa_1 \tilde{x}$ and $\kappa_2 \tilde{x}$:

$$U(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) = P \exp \left\{ ig \int_{\kappa_1}^{\kappa_2} dw \, \tilde{x}^\mu A_\mu(w \tilde{x}) \right\}. \quad (3)$$

The resulting decomposition for the vector and skew tensor operators is:

$$O_\alpha(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) = O^{tw2}_\alpha(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) + O^{tw3}_\alpha(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) + O^{tw4}_\alpha(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}), \quad \text{(4)}$$

$$M_{[\alpha \beta]}(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) = M^{tw2}_{[\alpha \beta]}(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) + M^{tw3}_{[\alpha \beta]}(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) + M^{tw4}_{[\alpha \beta]}(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) \quad \text{(5)}$$

with

$$O^{tw2}_\alpha(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) = \tilde{x}^\mu O_\mu(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) = \bar{u}(\kappa_1 \tilde{x})(\gamma \tilde{x})U(\kappa_1 \tilde{x}, \kappa_2 \tilde{x})d(\kappa_2 \tilde{x}) \quad \text{(6)}$$

$$O^{tw2}_\alpha(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) = \int^1_0 dt \left( \partial_\alpha + \frac{1}{2}(\ln t) x_\alpha \square \right) x^\mu O_\mu(\kappa_1 t \tilde{x}, \kappa_2 t \tilde{x}) \bigg|_{x=\tilde{x}} \quad \text{(7)}$$

$$O^{tw3}(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) = \int^1_0 dt \left( \delta^\alpha_\beta(x \partial) - x^\mu \partial_\alpha - (1 + 2 \ln t) x_\alpha \partial^\mu - (\ln t) x_\alpha x^\mu \square \right) O_\mu(\kappa_1 t \tilde{x}, \kappa_2 t \tilde{x}) \bigg|_{x=\tilde{x}} \quad \text{(8)}$$

$$O^{tw4}_\alpha(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) = \int^1_0 dt \left[ (1 + \ln t) \partial^\mu + \frac{1}{2}(\ln t) x^\mu \square \right] O_\mu(\kappa_1 t \tilde{x}, \kappa_2 t \tilde{x}) \bigg|_{x=\tilde{x}} \quad \text{(9)}$$

$$M^{tw2}_{[\alpha \beta]}(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) = \int^1_0 dt \left\{ 2 t \partial_\partial_\beta \delta^{\mu}_{\alpha} - (1 - t) \left\{ 2 x_\alpha x_\beta \partial^\mu - x_\alpha \delta^{\mu}_{\beta} \square \right\} \right\} x^\nu M_{[\mu \nu]}(\kappa_1 t \tilde{x}, \kappa_2 t \tilde{x}) \bigg|_{x=\tilde{x}} \quad \text{(10)}$$

$$M^{tw3}_{[\alpha \beta]}(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) = \int^1_0 dt \left\{ \left[ x_\alpha \delta^\mu_{\beta} \delta^\nu_{\alpha} - 2 x^\nu \partial_\beta \delta^{\mu}_{\alpha} - \frac{1-t^2}{t} \left( x^\mu \delta^\nu_{\beta} - x^{[\mu} \partial_{\beta]} \partial^\nu \right) \right] M_{[\mu \nu]}(\kappa_1 t \tilde{x}, \kappa_2 t \tilde{x}) \bigg|_{x=\tilde{x}} \right\} \quad \text{(11)}$$

$$M^{tw4}_{[\alpha \beta]}(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) = \int^1_0 dt \left\{ \frac{t-t^2}{t} \left( x^\mu \delta^\nu_{\beta} - x^{[\mu} \partial_{\beta]} \partial^\nu \right) \right\} M_{[\mu \nu]}(\kappa_1 t \tilde{x}, \kappa_2 t \tilde{x}) \bigg|_{x=\tilde{x}} \quad \text{(12)}$$

Let us remark that the vector and skew tensor operators of twist $\tau$, $O^{(\tau)}_\alpha(\kappa_1 \tilde{x}, \kappa_2 \tilde{x})$ and $M^{(\tau)}_{[\alpha \beta]}(\kappa_1 \tilde{x}, \kappa_2 \tilde{x})$, are obtained from the original (undecomposed) operators $O_\alpha(\kappa_1 \tilde{x}, \kappa_2 \tilde{x})$ and $M_{[\alpha \beta]}(\kappa_1 \tilde{x}, \kappa_2 \tilde{x})$, Eqs. (4) and (5), by the application of the corresponding twist projectors (including the $t$–integrations), $P^{(\tau)\mu}_{\alpha}$ and $P^{(\tau)[\mu \nu]}_{[\alpha \beta]}$, defined by Eqs. (7) and (11) – (12), respectively:

$$O^{(\tau)}_\alpha(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) = (P^{(\tau)\mu}_{\alpha}) O_\mu(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) \quad \text{(13)}$$

$$M^{(\tau)}_{[\alpha \beta]}(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) = (P^{(\tau)[\mu \nu]}_{[\alpha \beta]}) M_{[\mu \nu]}(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) \quad \text{(14)}$$

with

$$P^{(\tau)\mu}_{\alpha} = \delta^{\tau \tau'} P^{(\tau')\mu}_{\alpha} \quad \text{(15)}$$

$$P^{(\tau)[\mu \nu]}_{[\alpha \beta]} = \delta^{\tau \tau'} P^{(\tau)[\mu \nu]}_{[\alpha \beta]} \quad \text{(16)}$$

These projection operators contain the corresponding symmetry of the Young pattern.

Additionally, the twist–2 vector operator, Eq. (4), is related to the corresponding scalar operator, Eq. (6). This leads to relations of the corresponding meson wave functions, Eqs. (18) and (21) below.

**III. THE $\rho$-MESON MATRIX ELEMENTS OF LC–OPERATORS WITH TWIST**

In this section we define the meson wave functions for the bilinear LC-quark operators with definite twist sandwiched between the vacuum and the meson state. As usual, the matrix elements of the meson are related to the meson
momentum $P_\mu$ and meson polarization vector $e^{(\lambda)}_\mu$, respectively, with $P^2 = m^2_\rho$, $e^{(\lambda)} \cdot e^{(\lambda)} = -1$, $P \cdot e^{(\lambda)} = 0$, $m_\rho$ denoting the meson mass.

Taking meson matrix elements of Eqs. (3) – (12) we see that, observing the correct tensor structure by the use of $P_\mu, e^{(\lambda)}_\mu$ and $\hat{x}_\mu$, we may introduce any parametrization for the matrix elements of the undecomposed operators, e.g.,

$$
\langle 0| O_{\alpha}^{(\tau)}(\kappa_1 \hat{x}, \kappa_2 \hat{x})|\rho(P, \lambda)\rangle = f_{\rho} m_\rho \int_0^1 du \left( e^{(\lambda)}_\alpha \Phi_1(u, \mu^2) + P_\alpha \left( e^{(\lambda)}_2 \hat{x}/P \right) \Phi_2(u, \mu^2) + \hat{x}_\alpha m^2_\rho \left( e^{(\lambda)}_3 \hat{x}/(\hat{x}P)^2 \right) \Phi_3(u, \mu^2) \right) e^{i\kappa \xi(\hat{x}P)},
$$

(17)

where $\mu^2$ denotes the renormalization scale and $f_{\rho}$ is the vector meson decay constant. We defined $\kappa = (\kappa_1 - \kappa_2)/2$. According to the above projection properties we are able to introduce one (and only one) wave function for any operator of definite twist.

We start with the chiral-even scalar operator. The matrix element of this nonlocal twist-2 operator, Eq. (3), taken between the vacuum (0) and the meson state $|\rho(P, \lambda)\rangle$ reads

$$
\langle 0| O^{tw2}(\kappa_1 \hat{x}, \kappa_2 \hat{x})|\rho(P, \lambda)\rangle = f_{\rho} m_\rho \int_0^1 du \Phi^{(2)}(u, \mu^2) e^{i\kappa \xi(\hat{x}P)} = f_{\rho} m_\rho \sum_{n=0}^{\infty} \frac{(i\kappa(\hat{x}P))^n}{n!} \Phi^{(2)}_\alpha(\mu^2).
$$

(18)

Here, $\Phi^{(2)}(u, \mu^2)$ is the twist-2 meson wave function which is related to the corresponding moments

$$
\Phi^{(2)}(\mu^2) = \int_0^1 du \xi^n \Phi^{(2)}(u, \mu^2).
$$

For brevity, we use the shorthand notation

$$
\xi = u - (1 - u) = 2u - 1.
$$

(19)

The wave functions describe the probability amplitudes to find the $\rho$-meson in a state with the minimal number of constituents, e.g., quark which carries momentum fractions $u$ and antiquark with $1 - u$, respectively.

Now we consider the chiral-even vector operator. Using the projection properties (13), together with (12), we introduce the meson wave functions $\Phi^{(\tau)}(u, \mu^2)$ of twist $\tau$ by

$$
\langle 0| O^{(\tau)}_{\alpha}(\kappa_1 \hat{x}, \kappa_2 \hat{x})|\rho(P, \lambda)\rangle \equiv \langle 0| (P_{\alpha}^{(\tau)}|\kappa_1 \hat{x}, \kappa_2 \hat{x})|\rho(P, \lambda)\rangle = f_{\rho} m_\rho P_{\alpha}^{(\tau)} \int_0^1 du \Phi^{(\tau)}(u, \mu^2) e^{i\kappa \xi(\hat{x}P)},
$$

(20)

which, for $\tau = 2$, is consistent with (18). Using the twist projection operators as they are determined by Eqs. (3) – (12) we obtain for the twist-2 operator (from now on we suppress $\mu^2$)

$$
\langle 0| O^{tw2}_{\alpha}(\kappa_1 \hat{x}, \kappa_2 \hat{x})|\rho(P, \lambda)\rangle = f_{\rho} m_\rho \int_0^1 dt \left[ \frac{1}{2}(\ln t)x_\alpha \square \right] (e^{(\lambda)}_2 \hat{x}) \int_0^1 du \Phi^{(2)}(u, \mu^2) e^{i\kappa \xi(t\hat{x}P)}
$$

(21)

and for the higher twist operators

$$
\langle 0| O^{tw4}_{\alpha}(\kappa_1 \hat{x}, \kappa_2 \hat{x})|\rho(P, \lambda)\rangle = f_{\rho} m_\rho \int_0^1 dt \int_0^1 du \Phi^{(4)}(u) \left[ \left( e^{(\lambda)}_\alpha(\hat{x}P) - P_\alpha (e^{(\lambda)}_2 \hat{x}) \right) i\kappa t - \hat{x}_\alpha m^2_\rho (e^{(\lambda)}_2 \hat{x}) (i\kappa(t))^2 \ln t \right] e^{i\kappa \xi(t\hat{x}P)}
$$

(23)

$$
\langle 0| O^{tw6}_{\alpha}(\kappa_1 \hat{x}, \kappa_2 \hat{x})|\rho(P, \lambda)\rangle = \frac{1}{2} f_{\rho} m_\rho \hat{x}_\alpha (e^{(\lambda)}_2 \hat{x}) m^2_\rho \int_0^1 dt \int_0^1 du \Phi^{(4)}(u) (i\kappa(t))^2 (\ln t) e^{i\kappa \xi(t\hat{x}P)}
$$

(25)

$$
\langle 0| O^{tw8}_{\alpha}(\kappa_1 \hat{x}, \kappa_2 \hat{x})|\rho(P, \lambda)\rangle = -f_{\rho} m_\rho \int_0^1 dt \int_0^1 du \Phi^{(6)}(u) (i\kappa(t))^2 \left( \frac{n(n-1)}{2(n+1)^2} \hat{x}_\alpha \right) e^{i\kappa \xi(t\hat{x}P)}
$$

(26)
In the first line of any equation we have given the nonlocal matrix element of definite twist, and in the second line, after expanding the nonlocal expression, we introduced the moments of the meson wave functions; thereby, the \( t \)-integrations contribute the additional \( n \)-dependent factors. Obviously, the trace terms which have been explicitly subtracted are proportional to \( m_\rho^2 \). According to the terminology of Jaffe and Ji, they contribute to dynamical twist-4.

For the twist–2 operator we observe that the terms proportional to \( e^{(\lambda)}_\alpha \) and \( \bar{x}_\alpha \) have contributions starting with the zeroth, first and second moment, respectively. The twist–3 operator starts with the first moment, and the twist–4 operator starts with the second moment. Analogous statements also hold for the twist–\( \tau \) operators below.

Putting together the different twist contributions we obtain, after replacing \( i\kappa \xi(t\bar{x}P) \) by \( t\partial/\partial t \) and performing partial integrations, the following matrix element of the original operator (\( \zeta = \kappa \bar{x} P \))

\[
\langle 0 | O_\alpha (\kappa_1 \bar{x}, \kappa_2 \bar{x}) | \rho (P, \lambda) \rangle = f_\rho m_\rho \left[ P_\alpha \frac{e^{(\lambda)}_\alpha \bar{x}}{\bar{x}P} \int_0^1 \frac{du}{u} \left( \Phi^{(2)} (u) - \Phi^{(3)} (u) \right) \left[ e_0 (i\kappa \xi) - e_1 (i\kappa \xi) \right] \right]
\]

\[
+ e^{(\lambda)}_\alpha \int_0^1 du \left( \Phi^{(2)} (u) e_1 (i\kappa \xi) + \Phi^{(3)} (u) \left[ e_0 (i\kappa \xi) - e_1 (i\kappa \xi) \right] \right)
\]

\[- \frac{1}{2} \bar{x}_\alpha \frac{e^{(\lambda)}_\alpha \bar{x}}{\bar{x}P} \int_0^1 \frac{du}{u} \left( \Phi^{(2)} (u) - 2 \Phi^{(3)} (u) + \Phi^{(4)} (u) \right) \left[ e_0 (i\kappa \xi) - 3e_1 (i\kappa \xi) + 2 \int_0^1 dt e_1 (i\kappa \xi t) \right] ,
\]

where we used the following “truncated exponentials”

\[
e_0 (i\kappa \xi) = e^{i\kappa \xi}, \quad e_1 (i\kappa \xi) = \int_0^1 dt e^{i\kappa \xi t} = \frac{e^{i\kappa \xi} - 1}{i\kappa \xi}, \quad \ldots, \quad e_{n+1} (i\kappa \xi) = \frac{(-1)^n}{n!} \int_0^1 dt t^n e^{i\kappa \xi t} .
\]

As it should be the application of the projection operators \( P_\rho^{(\gamma)\beta} \) onto \( (27) \) reproduces the matrix elements \( (21) - (25) \). In comparison with \( (13) \) we also observe that the wave functions are accompanied not only by the exponentials, \( e_0 (i\kappa \xi) \), but by more involved combinations whose series expansion directly leads to the representations with the help of moments. Let us mention that the twist-2 part of Eq. \( (27) \) and the local expression \( (22) \) are in agreement with Stoll’s result \( (20) \) for the light-cone twist-2 operator.

Now we consider the chiral-even vector operator \( O_{5\alpha} (\kappa_1 \bar{x}, \kappa_2 \bar{x}) = \bar{u} (\kappa_1 \bar{x}) \gamma_\alpha \gamma_5 U (\kappa_1 \bar{x}, \kappa_2 \bar{x}) d (\kappa_2 \bar{x}) \), which obeys relations Eqs. \((13) - (15)) \) with the same projection operator as the vector operator. Let us introduce the corresponding meson wave functions \( \Xi^{(\tau)} (\bar{x}) \) of twist \( \tau \) by

\[
\langle 0 | O_{5\alpha}^{(\tau)} (\kappa_1 \bar{x}, \kappa_2 \bar{x}) | \rho (P, \lambda) \rangle = \frac{1}{2} \left( f_\rho - f_\rho \frac{m_u + m_d}{m_\rho} \right) m_\rho \sigma_\alpha \beta \gamma \mu \nu \epsilon \lambda P_\mu \bar{x}_\nu \int_0^1 \frac{du}{u} \Xi^{(\tau)} (u) e^{i\kappa \xi (\bar{x}P) \epsilon \lambda}
\]

\[
where \( f_\rho \) denotes the tensor decay constant. The vacuum-to-meson matrix elements of these vector operators of twist \( \tau \) are obtained as follows:

\[
\langle 0 | O_{5\alpha}^{tw3} (\kappa_1 \bar{x}, \kappa_2 \bar{x}) | \rho (P, \lambda) \rangle = \frac{1}{2} \left( f_\rho - f_\rho \frac{m_u + m_d}{m_\rho} \right) m_\rho \sigma_\alpha \beta \gamma \mu \nu \epsilon \lambda P_\mu \bar{x}_\nu \int_0^1 \frac{du}{u} \Xi^{(3)} (u) \left( 1 + i\kappa \xi \epsilon \lambda (\bar{x}P) \right) e^{i\kappa \xi (\bar{x}P) \epsilon \lambda}
\]

\[
\frac{1}{2} \left( f_\rho - f_\rho \frac{m_u + m_d}{m_\rho} \right) m_\rho \sigma_\alpha \beta \gamma \mu \nu \epsilon \lambda P_\mu \bar{x}_\nu \sum_{n=0}^{\infty} \frac{(i\kappa \xi (\bar{x}P))^n}{n!} \Xi^{(3)} .
\]

We see that only the twist-3 operator give a contribution and the twist-2 and twist-4 operator as well as all trace terms vanish. Thus, the matrix element of the original operator reads

\[
\langle 0 | O_{5\alpha} (\kappa_1 \bar{x}, \kappa_2 \bar{x}) | \rho (P, \lambda) \rangle = \frac{1}{2} \left( f_\rho - f_\rho \frac{m_u + m_d}{m_\rho} \right) m_\rho \sigma_\alpha \beta \gamma \mu \nu \epsilon \lambda P_\mu \bar{x}_\nu \int_0^1 \frac{du}{u} \Xi^{(3)} (u) e_0 (i\kappa \xi) .
\]

The matrix element of the simplest bilocal scalar operator arises as

\[
\langle 0 | \bar{u} (\kappa_1 \bar{x}) U (\kappa_1 \bar{x}, \kappa_2 \bar{x}) d (\kappa_2 \bar{x}) | \rho (P, \lambda) \rangle = -\frac{1}{2} \left( f_\rho - f_\rho \frac{m_u + m_d}{m_\rho} \right) m_\rho \frac{(i\kappa \xi \bar{x})}{m_\rho} \int_0^1 \frac{du}{u} \frac{\epsilon \lambda}{\epsilon \lambda} \Xi^{(3)} (u) \Xi^{(3)} .
\]

\[
= -\frac{1}{2} \left( f_\rho - f_\rho \frac{m_u + m_d}{m_\rho} \right) m_\rho \frac{(i\kappa \xi \bar{x})}{m_\rho} \sum_{n=0}^{\infty} \frac{(i\kappa \xi (\bar{x}P))^n}{n!} \frac{\epsilon \lambda}{\epsilon \lambda} \Xi^{(3)} ,
\]
In Eq. (40) only the twist–4 operator contributes which result from the trace terms of (10). After multiplication of remark that only those terms of the operator (11) contribute to the twist–3 wave function which result from the trace twist-four contribution.

The matrix elements of the twist–2 wave functions of mesons are related to true traceless operators they differ from the trace terms at least by the contributions from the trace terms. As far as the scalar LC–operators are concerned which definitely are of twist–2 the new and the old wave functions coincide. However, for the vector and (skew) tensor operators also the contributions of dynamical twist–2 differ from those of geometric twist–2.

IV. RELATIONS BETWEEN NEW AND CONVENTIONAL MESON WAVE FUNCTIONS

Obviously, since these new wave functions of mesons are related to true traceless operators they differ from the conventional ones (11) for higher twist at least by the contributions from the trace terms. This is analogous to the vector case the forward matrix element of the ‘true’ twist–3 part of (11) vanishes. In Eq. (41) only the twist–4 operator contributes which result from the trace terms of (11). After multiplication of (11) with $\delta_{\alpha \alpha}$ (or $\delta_{\beta \beta}$) the matrix element vanishes because the corresponding vector operator does not contain any twist-four contribution.

The matrix element of the original skew tensor operator is obtained as

$$\langle 0 | M_{\alpha \beta}^{T(4)}(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) | \rho(P, \lambda) \rangle = i f^T_{\rho} \int_0^1 d\lambda \int_0^1 du \Psi^{(2)}(u) \left( \left( e^{(\lambda)}_{\alpha} P_{\beta} - e^{(\lambda)}_{\beta} P_{\alpha} \right) \right) \int_0^1 d\xi \Psi^{(3)}(u, \xi) e^{i\kappa \xi (\tilde{x} P)}.$$  (35)
Now, we are able to compare our new and the conventional meson wave functions. We rewrite the matrix elements\cite{27,32} and (42) by choosing
\[\tilde{x}_\alpha = x_\alpha - \frac{P_\alpha}{m_\rho^2} ((xP) - \sqrt{(xP)^2 - x^2m_\rho^2}),\]
\[P_\alpha = p_\alpha + \frac{1}{2} \tilde{x}_\alpha m_\rho^2 \frac{m_\rho^2}{(xP)},\]
\[\xi^{(\lambda)} = e^{(\lambda)} + \alpha e^{(\lambda)}_{\perp} - \frac{1}{2} \tilde{x}_\alpha m_\rho^2 \frac{e^{(\lambda)}_\perp}{(xP)^2},\] (43)
where \(p_\alpha\) is a light-like vector \((p^2 = 0\) and \(p \cdot \tilde{x} = P \cdot \tilde{x}\) and \(e^{(\lambda)}_{\perp}\) is the transversal polarization vector of the \(\rho\)-meson. Our result is
\[\langle 0 | O_{\alpha}(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) | \rho(P, \lambda) \rangle = \int_{0}^{1} \frac{du}{\frac{1}{2}} \Phi^{(2)}(u)e_{0}(i\lambda\xi) + \Phi^{(3)}(u)e_{0}(i\lambda\xi)\]
\[- \frac{1}{2} \tilde{x}_\alpha m_\rho^2 \frac{e^{(\lambda)}_\perp}{(xP)^2} \int_{0}^{1} \frac{du}{\frac{1}{2}} \Phi^{(4)}(u)\]
\[e_{1}(i\lambda\xi) - 2 e_{1}(i\lambda\xi) + 2 \int_{0}^{1} \frac{dt}{\frac{1}{2}} e_{1}(i\lambda\xi)\]
\[+ 4 \Phi^{(3)}[e_{1}(i\lambda\xi) - \int_{0}^{1} \frac{dt}{\frac{1}{2}} e_{1}(i\lambda\xi)]\] , (44)
\[\langle 0 | O_{\alpha}(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) | \rho(P, \lambda) \rangle = \int_{0}^{1} \frac{du}{\frac{1}{2}} \Phi^{(2)}(u)e_{0}(i\lambda\xi)\]
\[+ 2 \tilde{x}_{\alpha} \lambda_{\perp} m_\rho^2 \frac{e^{(\lambda)}_\perp}{(xP)^2} \int_{0}^{1} \frac{du}{\frac{1}{2}} \Phi^{(3)}(u)e_{2}(i\lambda\xi) - \Phi^{(3)}(u)e_{0}(i\lambda\xi) + 2 e_{2}(i\lambda\xi)\]
\[- \tilde{x}_{\alpha} e^{(\lambda)}_{\perp} \frac{m_\rho^2}{(xP)^2} \int_{0}^{1} \frac{du}{\frac{1}{2}} \{2 \Phi^{(2)}(u)e_{1}(i\lambda\xi) + 2 e_{2}(i\lambda\xi)\} - \Phi^{(3)}(u)[1 + 2 e_{2}(i\lambda\xi)] + \Phi^{(4)}(u)[1 + e_{0}(i\lambda\xi) - e_{2}(i\lambda\xi)]\} . (45)

Comparing these expressions with the meson wave functions with dynamical twist given by Ball et al.\cite{6} we observe that it is necessary to re-express the truncated exponentials and perform appropriate variable transformations. After such manipulations we obtain the following relations, which allow us to reveal the interrelations between the different twist definitions of meson wave functions: (For simplicity, we give here only the integral relations of the meson wave functions in a state with quarks which carry momentum fraction \(u\).)
\[\phi_{||}(u) = \Phi^{(2)}(u),\] (47)
\[g^{(v)}_{\perp}(u) = \Phi^{(3)}(u) + \int_{u}^{1} \frac{dv}{v} \left(\Phi^{(2)} - \Phi^{(3)}\right)(v),\] (48)
\[g_{3}(u) = \Phi^{(4)}(u) - \int_{u}^{1} \frac{dv}{v} \left\{\left(\Phi^{(2)} - 4 \Phi^{(3)} + 3 \Phi^{(4)}\right)(v) + 2 \ln \left(u \frac{v}{v}\right)\left(\Phi^{(2)} - 2 \Phi^{(3)} + \Phi^{(4)}\right)(v)\right\},\] (49)
\[g_{\perp}^{(v)}(u) = \Xi^{(3)}(u),\] (50)
\[h^{(s)}_{\perp}(u) = \Upsilon^{(3)}(u),\] (51)
\[\phi_{\perp}(u) = \Psi^{(2)}(u),\] (52)
\[h^{(t)}_{||}(u) = \Psi^{(3)}(u) + 2 u \int_{u}^{1} \frac{dv}{v^2} \left(\Psi^{(2)} - \Psi^{(3)}\right)(v),\] (53)
\[h_{3}(u) = \Psi^{(4)}(u) + \int_{u}^{1} \frac{dv}{v} \left\{2 \left(\Psi^{(2)} - \Psi^{(4)}\right)(v) - 2 u \frac{v}{v}\left(\Psi^{(2)} - \Psi^{(3)}\right)(v) - \delta\left(\frac{u}{v}\right)\right\},\] (54)

We observe that both decompositions coincide in the leading terms, but differ at higher twist. For instance, the meson wave functions \(g_{\perp}^{(v)}(u)\) and \(h^{(t)}_{||}(u)\) with dynamical twist \(t = 3\) contain contributions with geometrical twist \(\tau = 2\) and 3. Additionally, dynamical twist \(t = 4\) meson wave functions \(g_{3}(u)\) and \(h_{3}(u)\) contain contributions with geometrical twist \(\tau = 2, 3\) as well as 4.

Additionally, the conventional wave functions can be written in terms of our new functions. The nontrivial relations...
are:

\[
\Phi^{(3)}(u) = g^{(v)}_{\perp}(u) + \frac{1}{u} \int_{u}^{1} dv (g^{(v)}_{\perp} - \phi_{\parallel})(v),
\]

\[
\Phi^{(4)}(u) = g_{3n}(u) + \frac{1}{u^2} \int_{u}^{1} dv (3g_{3n} - 4g^{(v)}_{\perp} + \phi_{\parallel})(v) + \frac{1}{u^2} \int_{u}^{1} dv v \left(1 - \frac{u}{v}\right) (3g_{3n} - 4g^{(v)}_{\perp} + 3\phi_{\parallel})(v),
\]

\[
\Psi^{(3)}(u) = h^{(t)}_{\parallel}(u) + \frac{2}{u} \int_{u}^{1} dv (h^{(t)}_{\parallel} - \phi_{\perp})(v),
\]

\[
\Psi^{(4)}(u) = h_{3n}(u) + \frac{2}{u^2} \int_{u}^{1} dv (h_{3n} - h^{(t)}_{\parallel})(v) - \frac{2}{u^2} \int_{u}^{1} dv v \left(1 - \frac{u}{v}\right) (h^{(t)}_{\parallel} - \phi_{\perp})(v).
\]

The relation between the moments may be read off from Eqs. (44) – (56) as follows:

\[
\phi^{(v)}_{\parallel n} = \Phi^{(2)}_{n},
\]

\[
g^{(v)}_{\perp n} = \Phi^{(3)}_{n} + \frac{1}{n+1} (\Phi^{(2)}_{n} - \Phi^{(3)}_{n}),
\]

\[
g_{3n} = \Phi^{(4)}_{n} - \frac{1}{n+1} (\Phi^{(2)}_{n} - 4\Phi^{(3)}_{n} + 3\Phi^{(4)}_{n}) + \frac{2}{(n+1)^2} (\Phi^{(2)}_{n} - 2\Phi^{(3)}_{n} + \Phi^{(4)}_{n}),
\]

\[
h^{(a)}_{\perp n} = \Xi^{(3)}_{n},
\]

\[
h^{(s)}_{\parallel n} = \Upsilon^{(3)}_{n},
\]

\[
\phi_{\perp n} = \Psi^{(2)}_{n},
\]

\[
h^{(t)}_{\parallel n} = \Psi^{(3)}_{n} + \frac{2}{n+2} (\Psi^{(2)}_{n} - \Psi^{(3)}_{n}),
\]

\[
h_{3n} = \Psi^{(4)}_{n} + \frac{2}{n+1} (\Psi^{(2)}_{n} - \Psi^{(4)}_{n}) - \frac{2}{n+2} (\Psi^{(2)}_{n} - \Psi^{(3)}_{n}) - \delta_{n0} (\Psi^{(3)}_{n} - \Psi^{(4)}_{n}).
\]

In terms of the moments the relations between old and new wave functions may be easily inverted; for the wave functions itself the expression of the new wave functions through the old ones is more involved. The inverse relations are:

\[
\Phi^{(3)}_{n} = g^{(v)}_{\perp n} + \frac{1}{n} (g^{(v)}_{\perp n} - \phi_{\parallel n}), \quad n > 0
\]

\[
\Phi^{(4)}_{n} = g_{3n} + \frac{1}{n-1} \left(3g_{3n} - 4g^{(v)}_{\perp n} + \phi_{\parallel n}\right) + \frac{1}{n(n-1)} \left(g_{3n} - 4g^{(v)}_{\perp n} + 3\phi_{\parallel n}\right), \quad n > 1
\]

\[
\Psi^{(3)}_{n} = h^{(t)}_{\parallel n} + \frac{2}{n} (h^{(t)}_{\parallel n} - \phi_{\perp n}), \quad n > 0
\]

\[
\Psi^{(4)}_{n} = h_{3n} + \frac{2}{n-1} (h_{3n} - h^{(t)}_{\parallel n}) - \frac{2}{n(n-1)} (h^{(t)}_{\parallel n} - \phi_{\perp n}), \quad n > 1.
\]

V. CONCLUSIONS

In this letter, we have discussed the calculation of the vacuum-to-meson matrix elements for nonlocal LC-operators using the notion of geometric twist. We have found eight meson wave functions with geometric twist \(\tau\). From the field theoretical point of view this Lorentz invariant classification is the most appropriate frame of introducing wave functions since the separation of different (geometric) twist is unique and independent from the special kinematics of the process. An important result of our calculations are the relations between the new meson wave functions and those given by Ball et al. [6,7]. These integral relations reveal the connection between the geometric and dynamical twist definitions for the meson wave functions.

An advantage in our approach was that we use operators with well-defined twist. Therefore, we have not used operator relations for different geometric twist in order to isolate contributions of (geometric) twist-3 and 4 (without gluonic contributions and neglecting quark masses) what is used in [8,9].
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