Coordination Control and Suitability Evaluation of Multiple Ice-Storage Tanks in an Air-conditioning System Under Unreliable Communication

Meng Yu1,2, Wenbo Hu1,2,*, Weiqiang Huo3, Cui Cui4, Liangliang Zhu1,2, Jiaji Liu1,2, Yao Wang1,2
1Nari Group Corporation/State Grid Electric Power Research Institute, Nanjing 210000, China
2State Grid Electric Power Research Institute Wuhan Efficiency Evaluation Company Limited, Wuhan 430074, China
3State Grid Hubei Electric Power Co., Ltd, Wuhan 430077, China
4State Grid Hubei Energy Comprehensive Service Co. Ltd., Wuhan 430077, China
*Email: huwenbo@sgepri.sgcc.com

Abstract. Electric energy is an important part of building energy consumption, and air-conditioning accounted for most of the consumption of electric energy. Ice storage air conditioning is widely concerned because of its strong adjustability. Due to the limited space, it is often necessary to use multiple ice storage devices simultaneously to meet the application requirements. In this paper, a distributed coordination method for multiple ice storage devices is presented, which can allocate storage tasks according to the capacity of the device. Each device only needs to communicate with its neighbouring cold storage device, and finally achieve the control goal through distributed iteration. The convergence of the proposed algorithm is analysed by rigorous mathematics. If the communication topology between devices is connected, the algorithm can converge to the expected point. Finally, the validity and suitability of the algorithm and the correctness of theoretical analysis are verified and evaluation by digital simulation.

1. Introduction
Energy provides the driving force and material foundation for human survival and social development, and lays a solid foundation for economic development and people's livelihood. Only the proper development and utilization of energy can be conducive to the construction of an environmentally friendly society and the sustainable development of mankind. With the rapid development of economy and society, the demand for energy is also increasing. In the past few decades, coal, as the main energy source, has made great contributions to social progress and economic development, but it has also brought about environmental pollution and ecological imbalance and other problems, thus restricting and affecting sustainable development.

At present, many areas in China are extremely short of electricity during the peak period of power consumption, but there are a lot of electric energy cannot be effectively used during the trough period, which leads to the measures of power restriction have to be taken during the peak period of power consumption, leading to the continuous use of electric energy by users. On the other hand, during the trough period, more power generation, transmission and distribution capacity cannot be utilized effectively, which affects the economic operation of the power grid.
The power consumption of air conditioning accounts for a large proportion of electric energy. The regulation of power supply and demand by air conditioning systems has attracted extensive attention from scholars. For example, a coordinated control has been designed in [1] for frequency regulation, where primary frequency control is implemented by variable frequency air conditioner, and secondary frequency control is realized by constant frequency air conditioner; In the power market environment, the problem of air conditioning aggregators participating in demand response is considered via a novel bilayer interaction strategy [2]; A fuzzy logic and neural networks is used to design in [3] the regulation method for air conditioning systems; Based on the big data from smart meter, in [4], the electrical behaviour of air conditioning system is clustered and subdivided, which lays a foundation for the regulation of air conditioning.

Ice storage technology refers to the application of ice in the main mechanism of electric refrigeration and air conditioning and storage in the ice storage equipment at the low power consumption period when the power load is low. In the daytime of high power load, to avoid the peak electricity price, stop or intermittently run the main engine of electric refrigeration air conditioning, and release the cold quantity stored by the ice storage equipment, so as to meet the requirements of the air conditioning load of the building. Compared with the traditional air conditioning system, the ice storage air conditioning system will have a larger adjustment margin on the basis of ensuring the embodiment of the user. Wei et al., [5] proposed model-free method using adaptive dynamic programming to solve the optimal control of ice-storage air conditioning systems. Huang et al., [6] formulated the mathematical model of optimal operation of ice-storage air conditioning, and an optimal control method was proposed to minimize cooling cost. The concept and development status of ice-storage air conditioning system are briefly introduced in [7]. Lee et al., [8] designed a particle swarm algorithm to solve the optimization problem of ice-storage air-conditioning systems, and the minimal cost was chosen as the objective function. Considering the optimization problem of cycle cost and the use efficiency of ice storage tank at the same time, literature [9] studies the optimization scheduling problem of ice storage air conditioning system. The above mentioned work [5-9] on ice storage air-conditioning system assumes that there is only one ice storage tank in the system. Due to the limitation of space and technology in the practical application scene, the capacity of an ice storage tank is limited and cannot achieve the desired regulation effect. Air conditioning systems with multiple ice storage tanks will have a greater range of adjustment and are eager to achieve better results. Unfortunately, as far as we know, there is no relevant work to analyse the coordination control of the air conditioning system with multiple ice storage tanks.

Multi-agent system is a powerful tool for designing distributed coordination control. The consensus problem is fundamental topic in studying coordination of multi-agent system. A lot of research progress has been made in this field [10-12]. There is also a lot of research work applied the consensus of multi-agent system to the control and optimization of smart grid [13-15]. The control and optimization method based on consensus also has the potential to be applied in the control and optimization of the air conditioning system with multiple ice storage tanks.

Motivated by above discussion, this paper studied a distributed method to allocation the task of ice storage, where each ice storage tank undertakes the task of ice storage according to its own capacity. Each ice storage tank is assumed to have independent computing and communication capabilities with adjacent nodes. Ice storage tanks communicate with each other to form a network of ice storage tanks. Assume that ice storage tanks are connected to each other by wireless communication. Due to the unreliability of the network, it will lead to the loss of packets and other problems. The proposed algorithm is robust to the unreliability of communication networks. We show that as long as the network can maintain joint connectivity, the proposed distributed algorithm can ensure convergence to the distribution of ice storage tank capacity. The key influencing factors of multi-dimensional ice storage air conditioning suitability evaluation, such as user demand, policy adaptation, economic benefits and supporting facilities, were analyzed, and the evaluation method of cooling and heating suitability based on various factors was studied. Based on the numerical simulation, the suitability of the proposed algorithm is analyzed.
The outline of the rest paper is given as follows. Section 2 provides preliminaries and problem formulation of this paper. Section 4 is the main results. Finally, we give the case study in section 5.

2. Preliminaries and Problem Formulation
This section will review some results on graph theory and consensus algorithm, which will be used in this paper.

2.1. Graph Theory
A graph is denoted by \( G = (V, E) \) where \( V = \{1, 2, \ldots, N\} \) is node set and \( E = V \times V \) is edge set. \( N_i = \{j \mid (j, i) \in E\} \) denotes the neighbor set of node \( i \). The Laplacian matrix \( \Gamma \) is defined as

\[
\begin{cases}
    l_{ij} < 0, & \text{if } (j, i) \in N_i, i \neq j \\
    l_{ij} = 0, & \text{if } (j, i) \notin N_i, i \neq j \\
    l_{ii} = -\sum_{j=1}^{N} l_{ij},
\end{cases}
\]

(1)

2.2. Consensus Algorithm
Consider \( N \) individuals with state \( s_i, i \in V \). Each individual updates its state as follows,

\[
\dot{s}(t) = -\sum_{j=1}^{N} l_{ij} \left( s_j(t) - s_i(t) \right)
\]

(2)

Consensus of algorithm (2) is achieved, i.e.,

\[
\lim_{t \to \infty} \left\| s_i(t) - s_j(t) \right\| = 0
\]

(3)

for any \( i, j \in V \), if and only if the corresponding graph contains a spanning tree.

If the communication is unreliable, the corresponding graph is time-varying. Then, each individual updates its state via time-varying interaction,

\[
\dot{s}(t) = -\sum_{j=1}^{N} l_{ij}(t) \left( s_j(t) - s_i(t) \right)
\]

The consensus of algorithm (2) can be reached if the corresponding graph is connected jointly.

2.3. Problem Formulation
In this paper, we consider an air conditioning system with \( N \) ice storage tanks. The current remaining capacity of ice storage tank \( i \) is denoted by \( \hat{S}_i \). The total volume of the ice storage mission is denoted by \( S \). Decision variable \( S_i \) denotes the task load of this dispatching of ice storage tank \( i \). Obviously,

\[
S = \sum_{j=1}^{N} S_i
\]

(4)

\[
0 \leq S_i \leq \hat{S}_i
\]

(5)

Assume that
This ensures that the mission can be completed by the ice tank. Assume that one of the tanks can obtained the total task $S$.

The objective of this paper is to design a distributed algorithm such that

$$\left\| \frac{S_i}{S_j} - \frac{S_j}{S_i} \right\| = 0,$$  \hspace{1cm} (6)

i.e., The larger the capacity, the more tasks are assigned. The communication among tanks is described by $\mathcal{I}$.

### 3. Distributed Allocation of Multiple Ice Storage Tanks

The following distributed allocation algorithm is proposed to realize the allocation given in (6).

$$S_i = -\sum_{j=1}^{N} l_{ij} \left( \frac{S_i}{S_j} - \frac{S_j}{S_i} \right)$$ \hspace{1cm} (7)

where

$$\bar{S}_i = \frac{S_i}{S_j}, \quad S_1(0) = S, \quad S_2(0) = S_1(0) = \cdots = S_N(0) = 0.$$  

Next, we will prove that

$$\left\| \frac{S_i}{S_j} - \frac{S_j}{S_i} \right\| = 0$$ \hspace{1cm} (8)

as $t$ goes to infinite.

Let

$$p_i = \frac{S_i}{S_j}$$ \hspace{1cm} (9)

Then, from (7), one has

$$\dot{p}_i = \frac{\dot{S}_i}{S_j} = -\sum_{j=1}^{N} l_{ij} \left( \frac{S_i}{S_j} - \frac{S_j}{S_i} \right) = -\sum_{j=1}^{N} l_{ij} \left( p_i - p_j \right)$$ \hspace{1cm} (10)

Let

$$\bar{L} = \begin{bmatrix} \frac{l_{11}}{S_1} & \frac{l_{12}}{S_1} & \cdots & \frac{l_{1N}}{S_1} \\ \frac{l_{21}}{S_2} & \frac{l_{22}}{S_2} & \cdots & \frac{l_{2N}}{S_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{l_{N1}}{S_N} & \frac{l_{N2}}{S_N} & \cdots & \frac{l_{NN}}{S_N} \end{bmatrix}_{N \times N}$$ \hspace{1cm} (11)

which can also be regarded as Laplacian matrix of $\mathcal{I}$.

From (8),
\[ \dot{p}_i = - \sum_{j=1}^{N} l_{ij}(p_i - p_j). \]  
(12)

Then, it can be rewritten as matrix form as

\[ \dot{p} = - \bar{L} p \]  
(13)

According to classic consensus theory, one has

\[ \lim_{t \to 0} \left\| p(t) - \frac{1}{N} \mathbf{1}^T p(t) \right\| = 0 \]  
(14)

if and only if \( \Gamma \) contains a spanning tree.

Next, we will consider the communication is unreliable, which can be model as a time-varying graph \( G(t) = \{V, E(t)\} \). The corresponding Laplacian matrix is

\[
L(t) = \begin{cases}
    l_{ij}(t), & \text{if } (j,i) \in E_i(t), i \neq j \\
    0, & \text{if } (j,i) \notin E_i(t), i \neq j \\
    -\sum_{j=1}^{N} l_{ij}(t).
\end{cases}
\]  
(15)

Similarly, one has

\[ \dot{p}_i = - \sum_{j=1}^{N} \bar{l}_{ij}(t)(p_i - p_j) \]  
(16)

where

\[
\bar{L}(t) = \left(\bar{l}_{ij}(t)\right)_{N \times N} = \begin{bmatrix}
\frac{l_{11}(t)}{\bar{S}_1} & \frac{l_{12}(t)}{\bar{S}_1} & \ldots & \frac{l_{1N}(t)}{\bar{S}_1} \\
\frac{l_{21}(t)}{\bar{S}_2} & \frac{l_{22}(t)}{\bar{S}_2} & \ldots & \frac{l_{2N}(t)}{\bar{S}_2} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{l_{N1}(t)}{\bar{S}_N} & \frac{l_{N2}(t)}{\bar{S}_N} & \ldots & \frac{l_{NN}(t)}{\bar{S}_N}
\end{bmatrix}_{N \times N}
\]  
(17)

Then, according to classic consensus theory with switching topology, one has

\[ \lim_{t \to 0} \left\| p(t) - \frac{1}{N} \mathbf{1}^T p(t) \right\| = 0 \]  
(18)

if and only if \( G(t) \) contains a spanning tree jointly.

4. Simulation

In this section, a simulation example will give to show the effectiveness and suitability of the proposed method. An air-conditioning system with six ice-storage tanks is considered. The communication between tanks is unreliable. Hence, the communication topology can be modelled as a time-varying graph shown in Figure 1. The communication topology switches back and forth between the two topologies. The capacity of tanks is shown in the Table 1.
Figure 1. communication topology.

Table 1. Capacity of Tanks.

|   | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S_6$ |
|---|---|---|---|---|---|---|
|   | 74 | 54 | 99 | 45 | 75 | 72 |

The simulation results are shown in Figure 2, the control objective is achieved.

Figure 2. Simulation results.

5. Conclusion
In this paper, a distributed coordination method for multiple ice storage devices has been presented, which can allocate storage tasks according to the capacity of the device. Each device only needs to communicate with its neighbouring cold storage device, and finally achieve the control goal through distributed iteration. The convergence of the proposed algorithm was analysed by rigorous mathematics. If the communication topology between devices is connected, the algorithm can converge to the expected point.

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