Unquenched simulations with $N_f = 2$ light quark flavours

qq+q Collaboration
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The quark mass dependence of meson masses and decay constants in $N_f = 2$ QCD is studied at a fixed gauge coupling with different dynamical quark masses. Partially Quenched Wilson Chiral Perturbation Theory (PQW$\chi$PT) is applied to obtain the extrapolation to zero quark mass. It is shown, that in our analysis NNLO-terms play a more important rôle than $O(a)$-terms, which can be neglected. Also we compare our results with recent CP-PACS results.

1. Introduction

To have a reliable extrapolation from lattice-QCD results to physical parameter values, it is important to simulate (dynamical) quark masses, which are light enough in that sense that Chiral Perturbation Theory ($\chi$PT) \cite{1} can be applied. If one enters this quark mass region, it is possible to extract the Gasser-Leutwyler-constants \cite{2} from lattice simulations, which then guide the extrapolation to physical quark masses. The chiral logarithms predicted by this low energy effective theory may serve as a sign that one reached sufficiently light quark masses.

2. Results from PQW$\chi$PT

Our simulations \cite{3} were done at $\beta = 5.1$ with four different sea-quark (i.e. dynamical) masses. The lattice size is $L^3 \times T = 16^3 \times 32$. This work is a continuation of \cite{4} were the lattice size was $16^4$ and only three different sea-quark masses were used.

The lattice spacing was found to be $a = 0.195(4)$fm, which gives a comfortably large volume ($L \approx 3$fm). The dynamical quark masses ($m_{u,d}$) are in the range $\frac{1}{2} m_s < m_{u,d} < \frac{4}{3} m_s$, the pion masses are in $371$MeV $\lesssim m_\pi \lesssim 664$MeV.

The extraction of the Gasser-Leutwyler-constants is done by fitting ratios of masses and decay constants to the formulas predicted by PQW$\chi$PT \cite{5}. We quote our results for the universal low energy scales, assuming $f_0 = 93$MeV:

\begin{align*}
\Lambda_3 &= f_0 \cdot 8.21(27) = 0.76(3)$GeV, \\
\Lambda_4 &= f_0 \cdot 21.4(1.5) = 1.99(14)$GeV.
\end{align*}

These fits also allow to extrapolate the pion decay constant to zero quark mass (for details cf. \cite{3}):

\begin{equation}
Z_A^{-1} f_0 = 121(5)$MeV.
\end{equation}

Comparing the new $16^3 \times 32$ data to the previous $16^4$ one, there are deviations up to 28% in the measured quantities like $r_0/a$, pion and quark masses (which shows the largest deviations). For a more detailed discussion cf. \cite{3}. It is remarkable that almost all these changes are substantially reduced when comparing directly the ratios of meson masses and decay constants as functions of ratios of the (PCAC-)quark masses, fig. 1 serves as an example.

3. $O(a)$ vs. NNLO

Since we are still working at a fixed gauge coupling $\beta$, from our data we only have indirect information about the discretization effects. That means we did not compare our results at different lattice spacings but fitted the $O(a)$-effects in

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the PQW \chi PT, which is possible due to the different functional dependence of these terms on the quark mass. Already in [4] we found the \O(a)-contributions in the considered ratios and double ratios, parametrized by

\eta \equiv \frac{\rho}{\chi S}, \quad \rho \equiv \frac{2W_0cSW}{f_0^2} a, \quad \chi S \equiv \frac{2B_0}{f_0^2} m_q,

to be small: \eta \leq 0.1. When trying global fits (i.e. fitting the different ratios and double ratios of the valence mass dependence for all the four sea quark masses together) we still find the best fit at a small \eta \approx -0.01 (this would correspond to the effect of a quark mass of \approx 0.5\text{MeV}), but a good fit with \eta = 0 (no \O(a)-terms) is also possible. To confirm this result, we looked at the combination of double ratios

\[ RRn + 2RRf - 3 = \begin{cases} \text{NNLO} & \text{for } Rf \equiv \frac{f_0^2}{f_{fS}}, \\ \text{O}(a) & \text{for } Rf \equiv \frac{(r_0m_{PS})^2}{2r_0 Z_q m_q} \end{cases}, \]

which vanishes in the continuum to NLO order and, therefore, it could be either described by NNLO- or \O(a)-terms. The two fits are shown in fig. 2 for our lightest sea quark mass. The \chi^2 of the fit is 1.3 for NNLO and 7.2 for \O(a). This special example also suggests that the NNLO is more important in this fitting procedure.

4. Comparison

Other groups are also working (using different fermion actions, finer lattices etc.) on the same problem. For a review cf. [5], [6]. The CP-PACS collaboration published results at almost the same lattice spacing (\(a = 0.2\text{fm}\)) with a set of light sea quark masses at \(12^3 \times 24\) and \(16^3 \times 24\) [8].

First we compare the ratio \(\frac{r_0 m_{PS}}{r_0 Z_q m_q} vs. (r_0 m_{PS})^2\) (fig. 3). Because we did not compute any renormalization factor for the quark mass, we fitted our data to the CP-PACS data, i.e. assuming \(Z_q = 1\) for CP-PACS. The best fit gave for the \(qq+q\) data: \(Z_q \approx 1.1142\), which is shown in the plot. Of course the errors are large, but there is a reasonable agreement between the two data-sets.

Next one may look at the sea-quark mass dependence of \(Rf \equiv \frac{f_0^2}{f_{fS}}\). By accident it happened that in the two data sets there is a point with almost exactly the same pion mass \((r_0 m_{PS})\): \(\kappa = 0.14585\) (CP-PACS) and \(\kappa = 0.1765\) (qq+q). This is used to set a common scale for comparing the sea-quark mass dependence of the pion decay constant, i.e. looking at the ratio \(Rf\). In fig. 3 the data points are shown together with the (continuum formula) fit from [8]. The second (dashed) curve shows the fit, if one takes the value of \(\Lambda_4 = 2.44(13)\text{GeV}\) from [8] and uses the reference mass parameter \(\chi_R = 35.8(3.3)\). The
Figure 3. Comparison of the ratio $\frac{(r_0 m_{PS})^2}{2 r_0 Z_q m^2}$ vs. $(r_0 m_{PS})^2$ between CP-PACS \cite{8} and $qq+q$ \cite{3}.

$qq+q$ fit gives $\Lambda = 1.99(14)$ GeV, which agrees within errors. The deviation of the CP-PACS data points to the continuum curves should be viewed as (large) $O(a^2)$-effects. Furthermore the $qq+q$ data points show the right curvature due to the chiral logarithms, which is easily seen when connecting the two heaviest $qq+q$ points by a straight line (dashed-pointed line in fig. 3).

5. Discussion & Outlook

In the previous we showed the higher importance of the NNLO-terms compared to $O(a)$-corrections in our data and found a qualitative agreement with recent CP-PACS results.

Recently the phase structure of lattice QCD has been examined by adding a twisted mass term \cite{9} and ref. therein, also \cite{10,11}. Besides the Aoki-phase scenario there exists a second scenario which results in a first order phase transition at $\kappa_{crit}$. The possibility that our two lightest quark masses are already inside the metastable region can not be excluded, as also pointed out in \cite{7}. This may be the reason for the observed strong non-linearity of the quark-mass dependence on $(2\kappa)^{-1}$. We plan to investigate this question in the future.

Apart from the study of the phase structure, the twisted mass formulation also facilitates numerical simulations at light quark masses. The reason is the lower bound to the eigenvalue spectrum provided by the additional mass term. This suggests to continue these types of simulations in the future with twisted quark masses.

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