Simple Heuristic for Data-Driven Computational Elasticity  
with Material Data Involving Noise and Outliers:  
A Local Robust Regression Approach  

Yoshihiro Kanno†

Data-driven computing in applied mechanics utilizes the material data set directly, and hence is free from errors and uncertainties stemming from the conventional material modeling. This paper presents a simple heuristic for data-driven computing, that is robust against noise and outliers in a data set. For each structural element, we extract the material property from some nearest data points. Using the nearest neighbors reduces the influence of noise, compared with the existing method that uses a single data point. Also, the robust regression is adopted to reduce the influence of outliers. Numerical experiments on the static equilibrium analysis of trusses are performed to illustrate that the proposed method is robust against the presence of noise and outliers and, hence, is effective for dealing with real-world data.

Keywords
Data-driven computing; model-free computational mechanics; outlier; local regression; robust statistics.

1 Introduction

Recent development and spread of data science are enormously remarkable [4, 9, 10, 24, 29, 33]. It has been widely recognized that the methodology of knowledge extraction from data is extremely useful in diverse fields.

Database methods have also been developed in engineering computation. In the area of computer graphics, data-driven methods were proposed to construct simulation models of elastic fabrics [25, 34], where cloth deformation models are estimated from data of experimental measurements. In computational mechanics, macroscopic constitutive properties of composites are extracted from a data set of the results of numerical material tests [3, 5, 21, 30–32].

This paper is inspired by the work of Kirchdoerfer and Ortiz [18], where the paradigm of data-driven computational mechanics is presented. The awareness of issues raised in [18] is summarized as follows. As an example, consider the static equilibrium analysis of a structure. The boundary value problem consists of (i) the compatibility relation, (ii) the force-balance equation, and (iii) the constitutive law. Among them, (i) is the kinematic constraint, and (ii) is derived from Newton’s laws of motion. Therefore, (i) and (ii) do not possess any uncertainty or error. In contrast, (iii) is a formulation obtained through

---

†Mathematics and Informatics Center, The University of Tokyo, Hongo 7-3-1, Tokyo 113-8656, Japan.  
E-mail: kanno@mist.i.u-tokyo.ac.jp.
a physical modeling based on experiments, and hence is empirical and uncertain. Based on this observation, Kirchdoerfer and Ortiz [18] attempts to directly utilize a data set obtained from the physical experiments, without resorting to empirical modeling of the material in (iii). In the framework of this data-driving computing, we suppose that a material data set in the stress–strain space is given. Then we find a solution, that satisfies (i) and (ii) strictly (in the same manner as the conventional framework requires) and is closest to the data set in the stress–strain space. Thus, the data-driven computing does not require any modeling in (iii), and hence is free from errors and uncertainties stemming from the material modeling.

The method proposed in [18] has recently been extended to static problems with geometrical nonlinearity [26], three-dimensional continua [8], and dynamic problems [20]. Independently, another data-driven approach that makes use of the manifold learning for estimating the material law has been developed [16, 17].

In [18], in the stress–strain space the distance between a point and a data set is defined as the distance between the point and the closest data point in the data set. This means that, for each structural element, the information of only one data point is adopted to extract the material property. Therefore, even if the data set consists of a large number of data points, most information that the data set has is not utilized for computation. Using only a single data point also means that the method has high estimation variance, i.e., it is sensitive to small fluctuation in the data set. In other words, the computational result will be seriously affected by noise and/or outliers in the data set.

Attention of this paper is focused on the robustness of data-driven computational elasticity methods against noise and outliers involved in a data set. Specifically, instead of the single closest data point, we attempt to use the information of the $k$ nearest data points for each structural element, where $k$ is a sufficiently small positive constant compared with the total number of the data points in the data set. It is expected that using several data points reduces the influence of noise involved in the data set. To extract the material property, we apply a robust regression method to the $k$ nearest data points. This reduces the influence of outliers. Thus, the estimation variance of the data-driven solver is reduced. Also, locality of the regression avoids the bias of oversmoothing. Direct comparison of the computational results of the proposed method and the existing method in [18] appears in section 4.1.

Recently, Kirchdoerfer and Ortiz [19] has proposed to incorporate a maximum-entropy estimation into their original data-driven solver [18] in order to deal with a data set involving noise. In this method, the solution to be found is defined as a global optimal solution of a nonconvex optimization problem. As a heuristic, Kirchdoerfer and Ortiz [19] used simulated annealing to approximately solve this complicated nonconvex optimization problem. This paper attempts to present an alternative method that is much simpler. This method, based on the local robust regression, is also nothing more than a heuristic, in the sense that convergence is not guaranteed. However, it is simple, easy to understand, and easy to implement. This feature is considered attractive from the viewpoint of the quality management of numerical simulation.

The paper is organized as follows. In section 2, we discuss influence of noise and outliers on a data-driven approach to computational elasticity, and give a brief overview
of the method proposed in this paper. Section 3 presents a full description of the proposed method. Section 4 reports the results of numerical experiments. Some conclusions are drawn in section 5.

In our notation, $\top$ denotes the transpose of a vector or a matrix. For simplicity, we often write the $(n + m)$-dimensional column vector $(x^\top, y^\top)^\top$ consisting of $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$ as $(x, y)$. We use $\text{co} S$ to denote the convex hull of set $S$. For finite set $T$, the cardinality of $T$, i.e., the number of elements in $T$, is denoted by $|T|$. We use $\mathcal{N}(\mu, \sigma^2)$ to denote the normal distribution with mean $\mu$ and variance $\sigma^2$.

# 2 Motivation and overview

In this section, we provide motivation and brief overview of the method presented in this paper.

Consider the truss structure shown in Figure 1. The two members consist of the same material. Suppose that the stress–strain relation of the material is characterized by the experimental data shown in Figure 2. Throughout the paper, we assume the elasticity. The cross-sectional area of each member is $2000 \text{ mm}^2$. As for the external load, the horizontal force of $p_x = 10 \text{ kN}$ and the vertical force of $p_y = 2 \text{ kN}$ are applied at the top node.

As the solution obtained by the method developed in this paper, the two triangles in Figure 2 indicate the pairs of the stress and the strain of the two members. Here, the right triangle corresponds to member 1, and the left one corresponds to member 2. These stresses exactly satisfy the force-balance equation with the specified external load. Also, there exists a nodal displacement vector that exactly satisfies the compatibility relation with these strains. Moreover, each pair of stress and strain seems to agree well with the data in Figure 2. Figure 3 shows a closeup of Figure 2.

---

$^1$ Since this example is a statically determinate truss, for any member strains there exists a compatible nodal displacement vector. What is meant to be explained here is that in the proposed method a solution is always defined so as to satisfy the compatibility relation exactly.
Figure 2: A material data set and the solution obtained by the proposed method.

Figure 3: A closeup of Figure 2. “solid line” The result obtained by the robust regression; “•” the data used for the robust regression; and “dashed line” the result obtained by the least-squares regression.

To obtain the solution above, we may consider that, in a certain sense, the data points that are far from the two triangles in Figure 2 are unnecessary. This observation motivates us to develop a method that utilizes only the data points that are close to the final solution. In contrast, the conventional model-based methods in computational mechanics first assume an empirical single model describing the global behavior of the material. Then the parameters of the model is calibrated so that the model fits the material data. In the example in Figure 2, for instance, this model is required to fit the data points that are far from the solutions, as well as the near data points. However, in
fact, fitting far data points is not necessary for agreement of the final solution with the data set. In contrast, a set of simple local models, which fit well only to some data points that are close to the solution, may probably yield a solution that is likely to reflect more the material data. This is because, from the bias-variance trade-off \[11, 13\], a linear model has low variance and can have low bias for local fitting, while a global nonlinear model has high variance when we reduce its bias. This is an essential idea of the method presented in this paper.

In Figure 3, the solution indicated by “△” is computed by using only 15 data points indicated by “●”. Namely, in the proposed method, we calibrate a linear model depicted by the solid line so as to fit well only these data, and use the calibrated linear model to perform the equilibrium analysis. Constructing such a local model from the data set is known as the \(k\)-nearest neighbor (kNN) local regression; the example in Figure 3 corresponds to \(k = 15\). See, e.g., [1, 6, 7] for fundamentals of kNN local regression, and [12, 22, 27] for recent applications.

Using only a few data points has, however, a disadvantage that the computational results can possibly be sensitive to outliers (or large errors). Indeed, if we use a conventional least-squares regression to calibrate a linear model, then we obtain the model depicted by the dotted line in Figure 3. This model gives a lower stress level than the main locus of the data points, which shows the effect of one (or, possibly two) outlier(s) with small stress level(s). To avoid such undesired perturbation due to outliers, in this paper we adopt the robust regression. The solid line in Figure 3, which is actually less affected by the outlier(s), is obtained by the robust regression.

Because our local models are constructed by the linear regression, each model may be considered a parametric model. However, we do not know \(a\ priori\) which data points will be used to construct a local model. The set of data points to be used depends on the structural system, the external load, etc., and hence a set of the local models is adaptive. Therefore, in general, the set of the local models used in this method does not reflect the \emph{global} behavior of the material. This is a significant difference of this method from the conventional model-based approach.

3 Data-driven equilibrium analysis with robustness against noise and outliers

The method overviewed in section 2 is formally described in this section. In section 3.1 we apply the local robust linear regression to material data. In section 3.2 we present the overall framework of data-driven equilibrium analysis.

3.1 \(k\)-nearest neighbor local robust regression for material data

As for the characterization of the material, suppose that the experimental data of the uniaxial strain, \(\varepsilon\), and the uniaxial stress, \(\sigma\), are given. The data set, denoted \(D\), consists of pairs \((\varepsilon, \sigma)\) as

\[
D = \{(\check{\varepsilon}_1, \check{\sigma}_1), \ldots, (\check{\varepsilon}_d, \check{\sigma}_d)\},
\]
where $d$ is the number of data points. For simplicity, we assume that the structure consists of a single material. An example of a data set is shown in Figure 2. Define $E$ by

$$E = \{\tilde{\varepsilon}_1, \ldots, \tilde{\varepsilon}_d\},$$

which is the set of strain data points.

For point $\varepsilon \in \mathbb{R}$, let $N_k(\varepsilon) \subseteq E$ denote the set of $k$ data points that are closest to $\varepsilon$, i.e.,

$$N_k(\varepsilon) = \arg\min_{\mathcal{N} \subseteq E, |\mathcal{N}| = k} \left\{ \sum_{\tilde{\varepsilon} \in \mathcal{N}} |\tilde{\varepsilon} - \varepsilon| \right\}.$$

(1)

Namely, $N_k(\varepsilon)$ is the $k$-nearest neighbor of $\varepsilon$. We use $J_k(\varepsilon)$ to denote the set of indices of the data points in $N_k(\varepsilon)$, i.e.,

$$J_k(\varepsilon) = \{j \in \{1, \ldots, d\} | \tilde{\varepsilon}_j \in N_k(\varepsilon)\}.$$

Following the concept of the kNN local regression [1, 6, 7], we assume that, in the neighborhood of a given point $\tilde{\varepsilon} \in \mathbb{R}$, the stress $\sigma$ is given by a deterministic linear function of $\varepsilon$ with additive noise as

$$\sigma = w \varepsilon + v + \epsilon, \quad \forall \varepsilon \in \text{co} N_k(\tilde{\varepsilon}),$$

(2)

where $w \in \mathbb{R}$ and $v \in \mathbb{R}$ are parameters, and $\varepsilon$ and $\sigma$ are considered the explanatory variable and the dependent variable, respectively. If $\epsilon$ is Gaussian noise with 0 mean and $\mu^2$ variance (i.e., $\epsilon \sim \mathcal{N}(0, \mu^2)$), then the maximum likelihood estimation of the parameters of the regression model in (2) coincides with the least-squares approximation for the data points in $N_k(\tilde{\varepsilon})$ (and the corresponding stress data). This local least-squares method can be formulated as follows:

Minimize $$\sum_{j \in J_k(\tilde{\varepsilon})} (w \tilde{\varepsilon}_j + v - \tilde{\sigma}_j)^2.$$

(3)

Here, $w$ and $v$ are variables to be optimized. Since we suppose that the data set $D$ includes some outliers, we replace the quadratic penalty function in (3) with a penalty function (or a loss function) for robust regression, denoted $\phi : \mathbb{R} \rightarrow \mathbb{R}$, as follows:

Minimize $$\sum_{j \in J_k(\tilde{\varepsilon})} \phi(w \tilde{\varepsilon}_j + v - \tilde{\sigma}_j).$$

(4)

An example of $\phi$ is the Huber penalty function [13, 23]

$$\phi(t) = \begin{cases} 
t^2 & \text{if } |t| \leq M, \\
M(2|t| - M) & \text{otherwise},
\end{cases}$$

where $M > 0$ is a constant. The Huber penalty function works in the manner same as the least-squares penalty function for a residual smaller than $M$, and behaves as the $\ell_1$-norm penalty function for a residual larger than $M$. It is worth noting that problem (4) is an unconstrained convex optimization problem [2].
Remark 3.1. In (1), to define the \( k \)-nearest neighbor, we define the distance between two data points as the difference of their strain values, \( |\hat{\epsilon} - \epsilon| \). Other definition of the distance can certainly be adopted. For instance, following the idea in [18], we may use 
\[
\frac{1}{2}c_\epsilon(\hat{\epsilon} - \epsilon)^2 + \frac{1}{2}c_\sigma(\hat{\sigma} - \sigma)^2
\]
with positive constant \( c_\epsilon \) instead of \( |\hat{\epsilon} - \epsilon| \).

3.2 Equilibrium analysis of truss structures

Consider a truss structure consisting of \( m \) members. We use \( n \) to denote the number of degrees of freedom of the nodal displacements.

Let \( u \in \mathbb{R}^n \) denote the vector of the nodal displacements. We use \( \epsilon_i \in \mathbb{R} \) to denote the uniaxial strain of member \( i \). Under the assumption of the small deformation, the compatibility relation can be written in the form
\[
\epsilon_i = b_i^T u,
\]
where \( b_i \in \mathbb{R}^n \) is a constant vector.

For member \( i \), let \( a_i \) and \( l_i \) denote the cross-sectional area and the undeformed member length, respectively, which are considered constants. We use \( \sigma_i \in \mathbb{R} \) to denote the uniaxial stress. The force-balance equation can be written as
\[
\sum_{i=1}^{m} a_i l_i \sigma_i b_i = p,
\]
where \( p \in \mathbb{R}^n \) is the vector of the nodal external forces.

The relation between \( \epsilon \) and \( \sigma \) can be retrieved from the experimental material data in the manner proposed in section 3.1. Therefore, our data-driven solver predicts the nodal displacements at the equilibrium state as the solution to the following system:
\[
\epsilon_i = b_i^T u, \quad i = 1, \ldots, m, \quad (6)
\]
\[
\sum_{i=1}^{m} a_i l_i \sigma_i b_i = p, \quad (7)
\]
\[
\sigma_i = \hat{w}_i \epsilon_i + \hat{v}_i, \quad i = 1, \ldots, m, \quad (8)
\]
\[
(\hat{w}_i, \hat{v}_i) = \arg \min \left\{ \sum_{j \in J(\epsilon_i)} \phi(w_i \epsilon_j + v_i - \bar{\sigma}_j) \mid (w_i, v_i) \in \mathbb{R}^2 \right\}, \quad i = 1, \ldots, m. \quad (9)
\]

It is worth noting that (6), (7), and (8) are linear equations. In contrast, it is not easy to deal with (9) directly, since its right-hand side involves unknown \( \epsilon_i \). This requires an iterative method. We adopt the following simple heuristic: Let \( \epsilon_i^{(l)} \) denote the incumbent solution of the strain of member \( i \). For each \( i = 1, \ldots, m \), we solve the optimization problem in (9) with \( \epsilon := \epsilon_i^{(l)} \) to obtain the optimal solution denoted by \( (\hat{w}_i^{(l)}, \hat{v}_i^{(l)}) \). With \( (\hat{w}_i, \hat{v}_i) := (\hat{w}_i^{(l)}, \hat{v}_i^{(l)}) \) in (8), we solve the system of linear equations, (6), (7), and (8), to find the solution \( (u^{(l+1)}, \epsilon^{(l+1)}, \sigma^{(l+1)}) \). We terminate the iteration if \( \|u^{(l+1)} - u^{(l)}\| \) is small enough.
Figure 4: An example such that the proposed problem formulation with \( k = 2 \) has no solution. (a) A material data set; (b) incumbent solution \( \varepsilon(l) \); and (c) solution \( \varepsilon(l+1) \) obtained from the local linear regression for \( N_2(\varepsilon(l)) \).

Remark 3.2. Since this paper is intended to be the first attempt to make use of the local robust regression in the data-driven computational mechanics, the proposed method admits of improvement. Particularly, a drawback is that the system (6), (7), (8), and (9) may possibly have no solution. For instance, consider the material data set in Figure 4(a). Suppose that a statically determinate truss is of interest, and the stress of one of members, satisfying the force-balance equation, is \( \sigma \). We set \( k = 2 \). Suppose that the 2-nearest neighbor of the incumbent solution consists of the left two data points. Then the local linear regression results in the solid line in Figure 4(b), and the corresponding strain is obtained as \( \varepsilon(l) \). In the next iteration, \( N_2(\varepsilon(l)) \) consists of the two right data points. Therefore, the local linear regression results in the solid line in Figure 4(c). Accordingly, the strain is updated to \( \varepsilon(l+1) \). Since \( N_2(\varepsilon(l+1)) \) consists of the two left data points, we go back to the situation in Figure 4(b). Thus, the system (6), (7), (8), and (9) has no solution for the example in Figure 4(a) with \( k = 2 \). We leave resolution of this issue as future work, and attention of this paper is focused on the effectiveness of the presented method in avoiding the influence of noise and outliers when a solution is successfully found. In the numerical experiments in section 4, a solution is found for almost all problem instances. In the case that the proposed method does not converge, we increase the value of \( k \) as a heuristic way of coping with this issue.

4 Numerical experiments

The method proposed in section 3 was implemented on Matlab ver. 9.0.0. A Matlab built-in function \texttt{robustfit} was used to solve the robust regression with the Huber penalty function in (9). In section 4.1 we compare the proposed method with the existing data-driven solver proposed in [18]. In section 4.2 we perform the comparison with the method using the least-squares regression, which is not robust against the presence of outliers.
4.1 Example (I): One-bar truss

As for the simplest example, consider a bar in Figure 5. The bar cross-sectional area is \( a = 200 \text{mm}^2 \). The external load is given as \( p = \lambda \bar{p} \) with \( \bar{p} = 10 \text{N} \), where \( \lambda \) is the load multiplier.
In the proposed method, the size of the neighborhood is set to \( k = 15 \). The \texttt{tune} parameter of the Matlab built-in function \texttt{robustfit} is set to \( 10^{-3} \), which was determined by preliminary numerical experiments.

Figure 6 shows the material data set, which consists of 100 data points. This data set might be interpreted as the linear elasticity with additive noise.

The computational results of the proposed method are shown in Figure 7. The equilibrium states were computed for \( \lambda = 0, 2, 4, \ldots, 38 \). The triangles in Figure 7(a) indicate the values of \((\varepsilon, \sigma)\) at these equilibrium states. Figure 7(b) plots the variation of the displacement with respect to the load multiplier. The linearity of the structural behavior is drawn out despite the presence of noise in the material data.

Figure 8 shows the results obtained by the method proposed in [18]. In contrast to Figure 7(b), it is clearly seen that the equilibrium path in Figure 8(b) is strongly affected by noise. It is worth noting that, both in the methods proposed in [18] and in this paper, the member stresses of a statically determinate truss are uniquely determined from the force-balance equation (in this example, the stress is determined as \( \lambda \bar{p}/a \)). Moreover, in this example the compatibility relation is automatically satisfied for any strain. Therefore, the method in [18] adopt the data point the stress value of which is closest to \( \lambda \bar{p}/a \). The displacement is then determined from the strain value of that data point. Use of information of only a single data point makes the method in [18] sensitive to noise.

4.2 Example (II): 27-bar truss

Consider the planar truss shown in Figure 9. The undeformed length of each of the vertical and horizontal members is 1 m. The cross-sectional area of every member is
Figure 9: A 27-bar truss.

Figure 10: Two typical computational results of example (II). (a), (b) The material data sets; and (c), (d) the obtained equilibrium paths. “solid line” The result of the proposed method; and “dotted line” the result of the least-squares regression.
Figure 11: The statistics of the computational results of example (II). (a) The means; and (b) the coefficients of variation. “△” The proposed method; and “▽” the least-squares regression.

As for the external load, \( p = \lambda \bar{p} \), vertical downward force of 50\( \lambda \) in N are applied at the bottom two nodes as shown in Figure 9. In the proposed method, the size of neighborhood is set to \( k = 40 \). The tune parameter of \texttt{robustfit} is set to 10\(^{-4} \). For comparison, the method using the least-squares regression in (3) is also examined. A Matlab built-in function \texttt{regress} was used for this purpose.

Figure 10(a) and Figure 10(b) show typical data sets, each of which consists of 400 data points. The computational results for these data sets are shown in Figure 10(c) and Figure 10(d), respectively. The solid line depicts the equilibrium path (the variation of the horizontal displacement of the bottom rightmost node with respect to the load multiplier) obtained by the proposed method, while the dotted line depicts the equilibrium path obtained by using the least-squares regression. It is observed in Figure 10(c) that that the magnitudes of the displacements obtained by the least-squares regression are smaller than the ones obtained by the proposed method. In contrast, in Figure 10(d) the magnitudes of the displacements obtained by the least-squares regression are larger than the ones obtained by the proposed method. Thus, the results of the least-squares regression are more strongly affected by outliers, and seem to have larger variation depending on given data sets. This observation is verified in Figure 11 as explained in the following.

We generated a data set \( D \) consisting of 400 observations as follows. The data set \( E \) of the strain is drawn from the uniform distribution on interval \([-5 \times 10^{-3}, 5 \times 10^{-3}]\). From among them, we randomly choose 360 data points, and set the stress values as

\[
\sigma = \tilde{\sigma} + 0.1 \epsilon_1
\]
with
\[
\hat{\sigma} = \frac{10^6}{1 + \exp(-10^3 \cdot \varepsilon)}
\]
in Pa and the noise \(\varepsilon_1 \sim \mathcal{N}(0, 1)\). The remaining 40 data points are possibly outliers, the stress values of which are set as
\[
\sigma = \hat{\sigma} + 0.1\varepsilon_1 + 0.8\varepsilon_2
\]
with \(\varepsilon_2 \sim \mathcal{N}(0, 1)\). One data set is generated in this way. We generated 100 data sets independently, for each of which we performed the equilibrium analysis with the proposed method, as well as the method with the least-squares regression. The computational results are shown in Figure 11. Figure 11(a) plots the means of the obtained displacements. It is observed that the results of the two methods agree well with each other. Figure 11(b) shows (the absolute values of) coefficients of variation. It is observed that the coefficients of variation are reduced by using the robust regression instead of the least-squares regression. This illustrates that the proposed method is robust against the presence of outliers. It is worth noting that this robustness stems from the definition of Huber penalty function in (5), where we can see that, for large \(t\), the increase of \(\phi(t)\) with respect to the increase of \(t\) is less drastic than the increase of the quadratic penalty function used in the least-squares regression.

It is worth noting that additional computational cost required by the proposed method, compared with a conventional method for problems with material nonlinearity, is not very large. In fact, a standard method for the equilibrium analysis with material nonlinearity may be application of the Newton–Raphson method, which usually takes several iterations before convergence. In the numerical experiments above, the proposed method with the robust regression requires 3.6 iterations in average to obtain the results shown in Figure 10(d), and the one with the least-squares regression requires 4.9 iterations in average. Additional computational task of the proposed method is sorting the material data points for identifying \(J_k(\varepsilon_i^{(l)})\) and performing the robust regression for finding \((\hat{w}_i^{(l)}, \hat{v}_i^{(l)})\). Among then, the sorting can be carried out efficiently with time complexity \(O(d \log d)\). Also, the robust regression problem considered in this paper is recast as a convex quadratic problem, which can be solved within the polynomial time of \(k^2\), where \(k \ll d\). Thus, the two additional computational tasks of the proposed method can be performed efficiently.

5 Conclusions

In this paper, we have presented a simple heuristic for data-driven computational elasticity with data involving noise and outliers. In contrast to the conventional methods that assume a single model describing the entire stress–strain relationship, the proposed method selects some data points adaptively and construct a set of local models each of which corresponds to the material property of one structural element. The material
data points that are not close to the final solution are ignored in the proposed method, and, hence, the set of local models can reflect the data more directly compared with a single entire model.

The existing data-driven approach to computational elasticity [18] uses information of a single data point for each structural element. To avoid the influence of noise included the data, the proposed method uses information of the \(k\)-nearest neighbor to construct a local model. Since only quite a small number of data points are used for constructing each local model, we adopt the robust regression to make the proposed method robust against the presence of outliers. Furthermore, the method is simple, easy to understand, and easy to implement.

This paper has been intended to be the first attempt to develop a robust data-driven approach to computational mechanics. Much remains to be explored. For example, as already mentioned in Remark 3.2, a solution does not necessarily exist to the proposed formulation, although in the numerical experiments a solution was found successfully for almost all instances. Also, it is not clear how one can choose an appropriate size of the neighborhood, \(k\). An adaptive scheme to adjust the value of \(k\) might be able to be explored. Extensions of the proposed method to inelastic problems might be challenging. In contrast, it seems to be straightforward to apply the proposed method to large deformation problems. Although we have restricted to truss structures for simple presentation, the proposed method can be applied to frame structures in a straightforward manner. Extension to continua remains to be studied.

**Acknowledgments**  This work is partially supported by JSPS KAKENHI 17K06633.

**References**

[1] N. S. Altman: An introduction to kernel and nearest-neighbor nonparametric regression. *The American Statistician*, 46, 175–185 (1992).

[2] S. Boyd, L. Vandenberghe: *Convex Optimization*. Cambridge University Press, Cambridge (2004).

[3] M. A. Bessa, R. Bostanabad, Z. Liu, A. Hu, D. W. Apley, C. Brinson, W. Chen, W. K. Liu: A framework for data-driven analysis of materials under uncertainty: Countering the curse of dimensionality. *Computer Methods in Applied Mechanics and Engineering*, 320, 633–667 (2017).

[4] H. Chen, R. H. L. Chiang, V. C. Storey: Business intelligence and analytics: From big data to big impact. *MIS Quarterly archive*, 36, 1165–1188 (2012).

[5] A. Clément, C. Soize, J. Yvonnet: Computational nonlinear stochastic homogenization using a nonconcurrent multiscale approach for hyperelastic heterogeneous microstructures analysis. *International Journal for Numerical Methods in Engineering*, 91, 799–824 (2012).
[6] W. S. Cleveland: Robust locally weighted regression and smoothing scatterplots. *Journal of the American Statistical Association*, **74**, 829–836 (1979).

[7] W. S. Cleveland, S. J. Devlin: Locally weighted regression: An approach to regression analysis by local fitting. *Journal of the American Statistical Association*, **83**, 596–610 (1988).

[8] S. Conti, S. Müller, M. Ortiz: Data-driven problems in elasticity. *Archive for Rational Mechanics and Analysis*, **229**, 79-123 (2018).

[9] A. Fahad, N. Alshatri, Z. Tari, A. Alamri, I. Khalil, A. Y. Zomaya, S. Foufou, A. Bouras: A survey of clustering algorithms for big data: Taxonomy and empirical analysis. *IEEE Transactions on Emerging Topics in Computing*, **2**, 267–279 (2014).

[10] W. Fan, C. Hu: Big graph analyses: From queries to dependencies and association rules. *Data Science and Engineering*, **2**, 36–55 (2017).

[11] P. Flach: *Machine Learning: The Art and Science of Algorithms that Make Sense of Data*. Cambridge University Press, Cambridge, (2012).

[12] M. R. Gupta, E. K. Garcia, E. Chin: Adaptive local linear regression with application to printer color management. *IEEE Transactions on Image Processing*, **17**, 936–945, (2008).

[13] T. Hastie, R. Tibshirani, J. Friedman: *The Elements of Statistical Learning: Data Mining, Inference, and Prediction (2nd ed.)*. Springer, New York (2009).

[14] P. J. Huber: Robust estimation of a location parameter. *The Annals of Mathematical Statistics*, **35**, 73–101 (1964).

[15] P. J. Huber, E. M. Ronchetti: *Robust Statistics (2nd ed.)*. John Wiley & Sons, 2009.

[16] R. Ibañez, E. Abisset-Chavanne, J. V. Aguado, D. Gonzalez, E. Cueto, F. Chinesta: A manifold learning approach to data-driven computational elasticity and inelasticity. *Archives of Computational Methods in Engineering*, **25**, 47–57 (2018).

[17] R. Ibañez, D. Borzacchiello, J. V. Aguado, E. Abisset-Chavanne, E. Cueto, P. Ladeveze, F. Chinesta: Data-driven non-linear elasticity: Constitutive manifold construction and problem discretization. *Computational Mechanics*, **60**, 813–826 (2017).

[18] T. Kirchdoerfer, M. Ortiz: Data-driven computational mechanics. *Computer Methods in Applied Mechanics and Engineering*, **304**, 81–101 (2016).

[19] T. Kirchdoerfer, M. Ortiz: Data driven computing with noisy material data sets. *Computer Methods in Applied Mechanics and Engineering*, **326**, 622–641 (2017).

[20] T. Kirchdoerfer, M. Ortiz: Data-driven computing in dynamics. *International Journal for Numerical Methods in Engineering*, **113**, 1697–1710 (2018).
[21] B. Klusemann, M. Ortiz: Acceleration of material-dominated calculations via phase-space simplicial subdivision and interpolation. *International Journal for Numerical Methods in Engineering*, **103**, 256–274 (2015).

[22] T. Lee, T. B. M. J. Ouarda, S. Yoon: KNN-based local linear regression for the analysis and simulation of low flow extremes under climatic influence. *Climate Dynamics*, **49**, 3493–3511 (2017).

[23] R. Maronna, D. Martin, V. Yohai: *Robust Statistics: Theory and Methods*. John Wiley & Sons, Chichester (2006).

[24] C. A. Mattmann: Computing: A vision for data science. *Nature*, **493**, 473–475 (2013).

[25] E. Miguel, D. Bradley, B. Thomaszewski, B. Bickel, W. Matusik, M. A. Otaduy, S. Marschner: Data-driven estimation of cloth simulation models. *Computer Graphics Forum*, **31**, 519–528 (2012).

[26] L. T. K. Nguyen, M.-A. Keip: A data-driven approach to nonlinear elasticity. *Computers and Structures*, **194**, 97–115 (2018).

[27] J. R. Prairie, B. Rajagopalan, T. J. Fulp, E. A. Zagona: Statistical nonparametric model for natural salt estimation. *Journal of Environmental Engineering*, **131**, 130–138 (2005).

[28] R. Ruiters, C. Schwartz, R. Klein: Data driven surface reflectance from sparse and irregular samples. *Computer Graphics Forum*, **31**, 315–324 (2012).

[29] S. Siuly, Y. Zhang: Medical big data: Neurological diseases diagnosis through medical data analysis. *Data Science and Engineering*, **1**, 54–64 (2016).

[30] İ. Temizer, P. Wriggers: An adaptive method for homogenization in orthotropic nonlinear elasticity. *Computer Methods in Applied Mechanics and Engineering*, **196**, 3409–3423 (2007).

[31] İ. Temizer, T. I. Zohldi: A numerical method for homogenization in non-linear elasticity. *Computational Mechanics*, **40**, 281–298 (2007).

[32] K. Terada, J. Kato, N. Hirayama, T. Inugai, K. Yamamoto: A method of two-scale analysis with micro-macro decoupling scheme: Application to hyperelastic composite materials. *Computational Mechanics*, **52**, 1199–1219 (2013).

[33] C.-W. Tsai, C.-F. Lai, H.-C. Chao, A. V. Vasilakos: Big data analytics: A survey. *Journal of Big Data*, **21**, Article No. 21 (2015).

[34] H. Wang, J. F. O’Brien, R. Ramamoorthi: Data-driven elastic models for cloth: Modeling and measurement. *ACM Transactions on Graphics*, **30**, Article No. 71 (2011).