Numerical analysis on target strength of the air filled cylindrical elastic shell

Wang Jihui, Li Guijuan, Jia Bing, Wang Zhenshan, Wang Rui
Science and Technology on Underwater Test and Control Laboratory, Dalian, China
wangjihuihgc@163.com

Abstract. Aimed at simulating the air filled finite cylindrical shell target strength, the numerical calculation method of scattering form function and target strength is given in this paper using the theory of Stanton T K. Cylindrical shells of different material properties, thickness, length are calculated, and the results indicate that factor of the material property and dimension is very important. Compared with other kinds of material properties, the gum rubber cylindrical shell has much more greater target strength for low frequency. Factors of radius and thickness have big effect to its fundamental resonance and the choice should be made in accordance with the actual needs. The numerical study provides theoretical basis and design reference for engineering application.

1. Introduction
Study on acoustic scattering characteristics of target includes the scattering cross section, target strength and form function. The method of separation of variables and the boundary element equation are mainly adopted. As the cylindrical shell form is used widely, so acoustic scattering of the shell with medium filled is one of the hot issues in the study of many scholars.

Payton has given a normal mode solution for the sound pressure generated by the cylindrical shell under the action of plane sound wave. The scattering form function of the infinite cylindrical shell is driven by Faran【1】. The derivation of the formula for calculating the target strength has been got by Stanton T K. In engineering applications, test results of American undersea warfare office have shown that low cost and high target strength reflectors can be fabricated using gas-filled compliant cylindrical shell well【2】. In this paper, scattering form function and target strength of air filled cylindrical shell are calculated on the basis of Stanton theory, and cylindrical shells of different material properties, thickness, radius are analyzed.

2. Numerical methods
It is essential to study the target scattering strength. Marston has given the concept that partial wave can be used approximately to express the back scattering form function of plane wave on a typical elastic body.

\[ f = \sum_{n=0}^{\infty} f_n \]  

Here \( f \) is the form function, and \( f_n \) is the \( n \) order harmonic component.
Figure 1. Schematic graph of the air filled cylinder
As shown in figure 1, L is cylindrical shell length of air inside and water outside.

2.1 Scattering form function
The scattering form function of infinite cylinder is calculated at first. Assume that the finite cylindrical shell has the same dimension with the infinite cylindrical shell, and these scattering field are the same in near field.

For the infinite cylindrical shell, the scattering field is expressed as \( P_0 = \sum_{n=0}^{\infty} P_n \cos n\theta \) \( \cos n\theta \)\( \cos n\theta \) (2)

\( P_0 \) is the incident wave pressure, and neuman coefficient is:

\[ e_n = \begin{cases} 1, & n = 0 \\ 2, & n \geq 1 \end{cases} \]

\( J_n (kr) \) is n order bessel function. The scattering field is:

\[ P_n = P_0 \sum_{n=0}^{\infty} e_n B_n J_n (kr) \cos n\theta \]

\( B_n \) is the six order determinant, every element is decided by the boundary condition, the material of the cylindrical shell, and the air inside. And it is composed of n order Bessel, Hankel, Neumann functions.

\[ D_n = \begin{vmatrix} d_1 & \partial_{d_2} & \partial_{d_3} & \partial_{d_4} & \partial_{d_5} & \partial_{d_6} \\ d_2 & d_3 & d_4 & d_5 & d_6 & 0 \\ 0 & \partial_{d_2} & \partial_{d_3} & \partial_{d_4} & \partial_{d_5} & \partial_{d_6} \\ 0 & 0 & \partial_{d_2} & \partial_{d_3} & \partial_{d_4} & \partial_{d_5} \\ 0 & 0 & 0 & \partial_{d_2} & \partial_{d_3} & \partial_{d_5} \\ 0 & 0 & 0 & 0 & \partial_{d_2} & \partial_{d_5} \end{vmatrix}, \quad V_n = \begin{vmatrix} v_1 & \partial_{v_2} & \partial_{v_3} & \partial_{v_4} & \partial_{v_5} & \partial_{v_6} \\ v_2 & v_3 & v_4 & v_5 & v_6 & 0 \\ 0 & v_2 & v_3 & v_4 & v_5 & v_6 \\ 0 & 0 & v_2 & v_3 & v_4 & v_5 \\ 0 & 0 & 0 & v_2 & v_3 & v_5 \\ 0 & 0 & 0 & 0 & v_2 & v_5 \end{vmatrix} \]

\( D_n \), \( V_n \) is the six order determinant, every element is decided by the boundary condition, the material of the cylindrical shell, and the air inside. And it is composed of n order Bessel, Hankel, Neumann functions.

\[ d_1 = a^2 J_0 (k a) \]

\[ d_2 = k a J_1 (k a) \]

\[ v_1 = -a^2 H_0^{(1)} (k a) \]

\[ v_2 = 2 \mu k^2 a^3 J_0 (k a) - \lambda k^2 a^3 J_1 (k a) \]
2.2 Target strength
The scattering sound pressure of the infinite cylindrical shell is integrated on the finite cylindrical shell surface \([4]\).

\[
f(\theta_n, \phi) = \frac{iL}{\pi} \sin(kL \cos \theta_n) \sum_{n=1}^\infty B_n \cos(n\phi)
\]

Eqn.(10) is the finite cylindrical shell scattering form function. \(L\) is the length, \(\theta_n\) is the incidence angle, \(\phi\) is the scattering angle, \(n\) is Bessel function order number.

The target strength of the finite cylindrical shell under the incident plane wave is defined as:

\[
TS = 10 \log_{10} \left| f(\theta_n = \pi/2, \phi = \pi) \right|^2
\]

At the fundamental resonance of the cylinder \(n=0\), Eqn (11) evaluates as:

\[
f(\theta_n = \pi/2, \phi = \pi, n = 0) = \frac{iL}{\pi}
\]

gas-filled cylinder at resonance becomes simply as:

\[
TS = 20 \log_{10} \left( \frac{L}{\pi} \right)
\]

3. Model simulating and analysis

3.1 Computing model
The length of the gas filled cylindrical shell is 6 meters and the water is outside. The acoustic properties of aluminum, rubber, glass, and steel are listed in table 1 \([5]\). The frequency is from 10 Hz to 5kHz. And in order to observe the results of low frequency calculation further, the results between 10 Hz to 1kHz are displayed contrast especially.

| Table 1. Sound parameters of materials. |
|----------------------------------------|
|                                | Longitudinal sound speed (m/s) | Transverse sound speed (m/s) | Densitity (kg/m³) |
|----------------------------------------|
| Air                                    | 343                            | 1500                          | 1.21*1.5          |
| Water                                  | 6260                           | 3080                          | 2700              |
| Aluminum                               | 6260                           | 3080                          | 2700              |
| Rubber                                 | 2300                           | 940                           | 1200              |
| Glass                                  | 5640                           | 3280                          | 2230              |
| Steel                                  | 5790                           | 3100                          | 8000              |

3.2 Simulation results and analysis
Outer radius a is 0.2 meter and inner radius is 0.18 meter. The acoustic property of the material is selected from table 1. The target intensity frequency response curves between 10Hz and 5kHz are presented in figure 2(a). It can be seen that the rubber shell quickly reach the resonance frequency \((n=0)\), and has bigger target strength. Figure 2(b) shows the feature more obvious below 1kHz frequency, and also it maintain high target strength at a larger frequency range after the resonance point.
Figure 2. Cylindrical shell TS with different materials

(a) 10Hz-5kHz
(b) 10Hz-1kHz

Figure 3. Cylindrical shell TS with different radius

Figure 3(a) shows that in the case of the same diameter and thickness ratio, the size of the radius has a great influence on the target strength especially on low frequency. From figure 3(b) it can be seen that the bigger radius and there is the lower resonance frequency.

Figure 4. Cylindrical shell TS with different thickness

Figure 4(a) shows that in the case of the same outer radius, the size of the thickness has a great influence on the air filled cylindrical shell target strength. When the radius is selected, the smaller the thickness and the smaller the resonance frequency is. Figure 4(b) shows this in detail on low frequency below 1kHz.
Figure 5 shows that the internal air has little effect on the cylindrical shell target strength. However, it also has to consider if it can maintain the normal shape under the pressure of the deep water.

4. Conclusion
In this paper the air filled cylindrical target strength computation model is set up, and the factor of the influence of material and size factors is analyzed. Different material air filled cylindrical shells have little difference in the target strength of high frequency. But on the low frequency especially below 1kHz, the rubber shell has significantly higher target strength. The radius and thickness are important for the fundamental resonance frequency of the cylindrical shell. It has to select proper dimensions to meet the actual needs of the project.

References
[1] Hua dacheng. Acoustic scattering study of elastic cylinder, Master’s degree thesis of Ocean University of China, 2010
[2] Malme C I. Development of high target strength passive acoustic reflector for low frequency sonar applications. IEEE Journal of Oceanic engineering, 1994, 19(3):438～448
[3] Stanton T K. Sound scattering by cylinders of finite length I: fluid cylinders. J Acoust Soc Am, 1988, 83(1):55～63
[4] Marston Philip L. GTD for backscattering from elastic spheres and cylinders in water and the coupling of surface elastic waves with the acoustic field. J Acoust Soc Am, 1988, 83:25～37
[5] Zhang Xiaofeng, Zhao Junwei, Bai Yinsheng, A study on underwater passive sound reflectors, ACTA ARMAMENTARII, 2004, 25(5)