THE MANY USES OF EXCITED HEAVY HADRONS

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I discuss a variety of issues in the physics of excited bottom and charmed hadrons. Recent developments in spectroscopy, strong decays, and production in fragmentation and weak decays are reviewed.

1 Introduction

The experimental and theoretical study of heavy hadrons typically focuses on the ground state $D^{(\ast)}$ and $B^{(\ast)}$ mesons, and on the lightest baryons, $\Lambda_c$ and $\Lambda_b$. This is hardly surprising, since these states are the most copiously produced, and the most long-lived, making detailed experiments possible. Nonetheless, these are but the lightest states in a tower of excitations. These excited states are also interesting, for a variety of reasons. First, one can use them as a laboratory to study the heavy quark (and light flavor) symmetries which are crucial to much of heavy hadron phenomenology. Second, there are new applications of heavy quark symmetry, leading to new questions about QCD, which only arise in the study of the more complicated spin structure of excited heavy hadrons. Third, certain experiments involving these excitations yield new information which is directly applicable to the physics of the ground state heavy hadrons.

In this talk, I will briefly survey a variety of such issues. I begin with a review of hadron dynamics in the heavy quark limit and of the simple spectroscopic predictions which follow from it. Certain of these predictions

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$^a$To appear in the Proceedings of the Twentieth Johns Hopkins Workshop on Current Problems in Particle Theory, “Nonperturbative Particle Theory and Experimental Tests”, Heidelberg, Germany, June 27–29, 1996.

$^b$The reader who desires a more extensive introduction to Heavy Quark Effective Theory, with thorough references to the original literature, may consult a number of excellent reviews.}

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are currently not well satisfied, casting doubt either on the data or on its interpretation. I will then turn to the strong decays of heavy mesons and show that a consistent inclusion of subleading effects can resolve an otherwise puzzling discrepancy. The third topic will be the production of excited heavy hadrons in fragmentation processes, and the fourth will be the production of heavy hadrons in semileptonic decays. In the latter case, we will see that it is possible to extract information from such processes which is useful for improving the extraction of the CKM matrix element $|V_{cb}|$ from semileptonic $B$ decays.

In addressing these four topics, I do not pretend to review the entire field of excited bottom and charmed hadrons. Rather, I hope to illustrate the rich interplay between theory and experiment which these recent developments make possible.

2 The Heavy Quark Limit

Consider a hadron containing a single heavy quark $Q$, where by “heavy” we mean that its mass satisfies the condition $m_Q \gg \Lambda_{\text{QCD}} \sim 500$ MeV. Ultimately, of course, we will apply this analysis to physical charm and bottom quarks, with $m_c \approx 1.5$ MeV and $m_b \approx 4.8$ MeV, which may not be well into this asymptotic regime. Hence, at times it will be important to include corrections which are subleading in an expansion in $\Lambda_{\text{QCD}}/m_Q$. For now, however, let us assume that we are in a regime where this “heavy quark limit” applies.

A heavy hadron is a bound state consisting of the heavy quark and many light degrees of freedom. These light degrees of freedom include valence quarks and antiquarks, sea quarks and antiquarks, and gluons, in a complex configuration determined by nonperturbative strong interactions. These interactions are characterized by the dimensional scale $\Lambda_{\text{QCD}}$, the scale at which the strong coupling $\alpha_s$ becomes of order 1; in particular, $\Lambda_{\text{QCD}}$ is the typical energy associated with the four-momenta carried by the light degrees of freedom. Hence it is also the typical energy of quanta exchanged with the heavy quark in the bound state. Since $m_Q \gg \Lambda_{\text{QCD}}$, the heavy quark does not recoil upon exchanging such quanta with the light degrees of freedom. This is the simple physical content of the heavy quark limit: $Q$ acts as a static source of chromoelectric field, so far as the light degrees of freedom are concerned. In a more covariant language, the four-velocity $v^\mu$ of $Q$ is unchanged by the strong interactions. Because the heavy quark does not recoil from its interactions with the light degrees of freedom, they are insensitive to its mass, so long as $m_Q \gg \Lambda_{\text{QCD}}$. This is analogous to the statement in quantum electrodynamics that the electronic wave function is the same in hydrogen and deuterium.
There is also a condition on the spin of the heavy quark, which couples to the light degrees of freedom primarily through the chromomagnetic interaction. Since the chromomagnetic moment of \( Q \) is given by \( g \hbar / 2 m_Q \), this interaction also vanishes in the heavy quark limit. Not only is the velocity of the heavy quark unchanged by soft QCD, so is the orientation of its spin. Hence, if the light degrees of freedom have nonzero angular momentum \( J_\ell \), then the states with total \( J = J_\ell + \frac{1}{2} \) and \( J = J_\ell - \frac{1}{2} \) are degenerate. This is analogous to the statement in quantum electrodynamics that the hyperfine splitting in hydrogen is much smaller than the electronic excitation energies. Thus we have new symmetries of the spectrum of QCD in the heavy quark limit. These lead to new “good” quantum numbers, the excitation energy and the total angular momentum of the light degrees of freedom, which can be sensibly defined only in this limit.

If we have \( N_h \) heavy quarks, \( Q_1 \ldots Q_{N_h} \), then the heavy quark symmetry group is \( SU(2N_h) \). These symmetries yield relations between the properties of hadrons containing a single heavy quark, including masses, partial decay widths, and weak form factors. These relations can often be sharpened by the systematic inclusion of effects which are subleading in the \( 1/m_Q \) expansion.

3 Spectroscopy

The simplest heavy quark relations are those for the spectroscopy of states containing a single heavy quark. Heavy hadron spectroscopy differs from that for hadrons containing only light quarks because we may specify separately the spin quantum numbers of the light degrees of freedom and of the heavy quark. The constituent quark model can serve as a useful guide for enumerating these states. Of course, we should not take this model too seriously, as it has certain unphysical features, such as drawing an additional distinction between spin and orbital angular momentum of the valence quarks, and including explicitly neither sea quarks nor gluons. So remember in what follows that any mention of constituent quarks is purely for the purpose of counting quantum numbers.

3.1 Heavy mesons

In the constituent quark model, a heavy meson consists of a heavy quark and a light antiquark, each with spin \( \frac{1}{2} \), in a wavefunction with a given excitation energy and a given orbital angular momentum. There is no natural zero-point with respect to which to define energies in this confined system, but differences between energy levels \( E_\ell \) of the light antiquark are well defined. The antiquark can have any integral orbital angular momentum, \( L = 0, 1, 2, \ldots \), with parity \((-1)^L \). Combined with the intrinsic spin-parity \( S_\ell P = \frac{1}{2} \) of the antiquark, we
find states with total spin-parity

\[ J_P^\ell = \frac{1}{2} \pm \frac{3}{2}, \frac{5}{2}, \ldots \]  

This is then added to the spin parity \( S_P^Q = \frac{1}{2}^+ \) of the heavy quark, to yield states with total angular momentum

\[ J_P = 0^\pm, 1^\pm, 2^\pm, \ldots \]  

In the limit \( m_Q \to \infty \), the two states with a given \( J_P^\ell \) are degenerate.

As an example, let us consider the charmed mesons. Our quark model intuition tells us correctly that the ground state light degrees of freedom have the quantum numbers of a light antiquark in an \( s \) wave, so \( J_P^\ell = \frac{1}{2}^- \) and the two degenerate states have \( J_P^0 = 0^- \) and \( 1^- \). This is indeed what is observed: a \( 0^- \) \( D \) meson with mass approximately 1870 MeV, and a slightly heavier \( 1^- \) \( D^* \) at about 2010 MeV. (I will keep to approximate masses for now, as I do not want to concern myself with small isospin splittings which complicate the situation in an unimportant way.) The nonzero splitting between the \( D \) and the \( D^* \) is an effect of order \( 1/m_c \); this splitting scales as \( \Lambda_{QCD}^2/m_c \) in the heavy quark expansion.

As the next excitation, we might expect to find the antiquark in a \( p \) wave. With the antiquark spin, we find light degrees of freedom with \( J_P^\ell = \frac{1}{2}^+ \) and \( J_P^\ell = \frac{3}{2}^+ \), each leading to an (almost) degenerate doublet of states. The doublet with \( J_P^\ell = 0^+ \) and \( 1^+ \) has not been observed, presumably because it is very broad (see Section 3). The other doublet, with \( J_P^\ell = \frac{3}{2}^+ \), consists of the \( D_1(2420) \) and \( D_2^*(2460) \). The splitting between the \( D_1 \) and the \( D_2^* \), again a \( 1/m_c \) effect, is not related to the \( D - D^* \) splitting.

The heavy quark symmetries imply relations between the spectra of the bottom and charmed meson systems. Because the mass of a heavy hadron can be decomposed in the form \( M_H = m_Q + E_\ell \), the entire spectrum of bottom mesons can be determined from the charmed mesons once the quark mass difference \( m_B - m_c \) is known. This difference can be found, for example, from the ground state mesons. Taking a spin average to eliminate the hyperfine energy,

\[ \overline{\mathcal{D}} = \frac{1}{4} (D + 3D^*) , \quad \overline{\mathcal{B}} = \frac{1}{4} (B + 3B^*) \]

and letting the states stand for their masses, we find

\[ \overline{\mathcal{B}} - \overline{\mathcal{D}} = m_b - m_c \approx 3.34 \text{ MeV} . \]

\( ^c \)The difficult question of how properly to define heavy quark masses is not really relevant to heavy hadron spectroscopy. For now, it is best to take \( m_Q \) to denote the pole mass at some fixed order in QCD perturbation theory.
Table 1: The observed charmed and bottom mesons.

| Spin | $D$ system | $B$ system |
|------|------------|------------|
| $J^{P}$ | state | $M$ (MeV) | $\Gamma$ (MeV) | state | $M$ (MeV) | $\Gamma$ (MeV) |
| $\frac{1}{2}^{-}$ | $D^0$ | 1865 | $\tau = 0.42$ ps | $B^0$ | 5279 | $\tau = 1.50$ ps |
| | $D^\pm$ | 1869 | $\tau = 1.06$ ps | $B^\pm$ | 5279 | $\tau = 1.54$ ps |
| | $D_s$ | 1969 | $\tau = 0.47$ ps | $B_s$ | 5375 | $\tau = 1.34$ ps |
| $1^{-}$ | $D^{*0}$ | 2007 | $< 2.1$ | $B^*$ | 5325 |
| | $D^{*\pm}$ | 2010 | $< 0.13$ | $
| | $D_s^*$ | 2110 | $< 4.5$ | $B_s^*$ |
| $\frac{1}{2}^{+}$ | $0^+$ | $D_0^*$ | $B_0^*$ |
| | $1^+$ | $D_1^*$ | $B_1^*$ |
| $\frac{3}{2}^{+}$ | $1^+$ | $D_1$ | $2421 \pm 3$ | $20 \pm 7$ | $B_1$ | 5725 | 20 |
| | $D_1^\pm$ | $2425 \pm 3$ | $26 \pm 9$ |
| | $D_{s1}$ | 2535 | $< 2.3$ | $B_{s1}$ | 5874 | 1 |
| | $2^+$ | $D_2^{*0}$ | $2465 \pm 4$ | $28 \pm 10$ | $B_2^*$ | 5737 | 25 |
| | $D_2^{*\pm}$ | $2463 \pm 4$ | $27 \pm 12$ |
| | $D_{s2}^*$ | $2573 \pm 2$ | $16 \pm 6$ | $B_{s2}^*$ | 5886 | 1 |

One then finds relations for the excited states, such as

$$\overline{B}_1 - \overline{D}_1 = \overline{B} - \overline{D},$$

where $\overline{D}_1 = \frac{1}{2}(3D_1 + 5D_2^*)$ is the appropriate spin average. Including strange quarks, one finds similar relations, such as $\overline{B}_s - \overline{D}_s = \overline{B} - \overline{D}$. There are also relations which exploit the known scaling of the hyperfine splitting in the heavy quark limit. Since $D^* - D \sim \Lambda^2_{\text{QCD}}/m_c$, we find

$$(B^*)^2 - B^2 = (D^*)^2 - D^2,$$

$$(B_{s2}^*)^2 - B_{s1}^2 = (D_{s2}^*)^2 - D_{s1}^2,$$

and so on.

The charmed and bottom mesons which have been identified are listed in Table 1, along with their widths (which will be of interest in Section 4). Given the measured properties of the charmed mesons, we can make a set of predictions for the bottom system,

$$B^* - B = 52 \text{ MeV}$$

$$\overline{B}_1 = 5789 \text{ MeV}$$

Given the measured properties of the charmed mesons, we can make a set of predictions for the bottom system,
The experimental values are given in parentheses. We can estimate the accuracy with which we expect these predictions to hold by considering the size of the largest omitted term in the expansion in $1/m_B$ and $1/m_c$. For relations between spin-averaged quantities, this is

$$
\Lambda_{QCD}^2 \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right) \sim 50 \text{ MeV},
$$

while for relations involving hyperfine splittings, we have

$$
\Lambda_{QCD}^3 \left( \frac{1}{4m_c^2} - \frac{1}{4m_b^2} \right) \sim 5 \text{ MeV}.
$$

These estimates are confirmed by the results given above.

The relations we have derived here follow rigorously from QCD in the heavy quark limit, $m_Q \to \infty$. Of course, they also arise in phenomenological models of hadrons, such as the nonrelativistic constituent quark model. In fact, an important test of any such model is that it have the correct heavy quark limit. Since the constituent quark model has this property, it reproduces these predictions as well. However, unlike the heavy quark limit, the constituent quark model is not in any sense a controlled approximation to QCD, and it is impossible to estimate the error in a quark model prediction in any meaningful way.

One intriguing feature of the quark model is that it makes accurate predictions for many light hadrons, too. It is not clear whether these successes have a clear explanation, or even a single one. Perhaps, at some length scales, nonrelativistic constituent quarks really are appropriate degrees of freedom. Perhaps its success lies in its closeness to the large $N_c$ limit of QCD, in which quark pair production is also suppressed. Whatever the proper explanation, it is important to keep in mind that relations which follow solely from the quark model do not have the same status as those that follow from real symmetries of QCD, such as heavy quark symmetry or light flavor $SU(3)$.

### 3.2 Heavy baryons

Because heavy baryons contain two light quarks, their flavor symmetries are more interesting than those of the heavy mesons; however, because they are
Table 2: The lowest lying charmed baryons Isospin is denoted by $I$, strangeness by $S$.

| Name   | $J^P$ | $s_\ell$ | $L_\ell$ | $J^P_\ell$ | $I$ | $S$ | Decay                  |
|--------|-------|----------|----------|------------|-----|-----|------------------------|
| $\Lambda_c$ | $1^+$ | 0        | 0        | $0^+$      | 0   | 0   | weak                   |
| $\Sigma_c$ | $1^+$ | 1        | 0        | $1^+$      | 0   | 1   | $\Lambda_c\pi$, $\Lambda_c\gamma$, weak |
| $\Sigma_c^*$ | $3^+$ | 1        | 0        | $1^+$      | 1   | 0   | $\Lambda_c\pi$        |
| $\Xi_c$ | $1^+$ | 0        | 0        | $0^+$      | $\frac{1}{2}$ | $-1$ | weak                   |
| $\Xi_c'$ | $3^+$ | 1        | 0        | $1^+$      | $\frac{1}{2}$ | $-1$ | $\Xi_c\gamma$, $\Xi_c\pi$ |
| $\Xi_c^*$ | $3^+$ | 1        | 0        | $1^+$      | $\frac{1}{2}$ | $-1$ | $\Xi_c\pi$            |
| $\Omega_c$ | $1^+$ | 1        | 0        | $1^+$      | 0   | $-2$ | weak                   |
| $\Omega_c^*$ | $3^+$ | 1        | 0        | $1^+$      | 0   | $-2$ | $\Omega_c\gamma$      |

more difficult to produce, less is known experimentally about the spectrum of heavy baryon excitations. For simplicity, let us restrict ourselves to states in which the light quarks have no orbital angular momentum. Then, given two quarks each with spin $\frac{1}{2}$, the light degrees of freedom can be in an antisymmetric state of total angular momentum $J^P_\ell = 0^+$ or a symmetric state with $J^P_\ell = 1^+$. By the Pauli exclusion principle, if neither light quark is a strange quark then the spin and isospin are the same. The exclusion principle also prohibits a $J^P_\ell = 0^+$ state with two strange quarks.

When the spin of the heavy quark is included, the $J^P_\ell = 0^+$ state becomes a baryon with spin-parity $J^P = \frac{1}{2}^+$, while the $J^P_\ell = 1^+$ state becomes a doublet of baryons with $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$. The quantum numbers of the charmed baryons are listed in Table 2 along with their expected decays. Note that the dominant decay channels of the higher mass $J^P = \frac{1}{2}^+$ states $\Sigma_c$ and $\Xi_c$ are determined by the available phase space. If emission of a pion is possible, then they will decay strongly; if not, then they will decay weakly or electromagnetically, depending on their charge.

Again, there are heavy quark symmetry relations between the bottom and charmed systems. The hyperfine interaction between the heavy quark and the $J_\ell = 1$ light degrees of freedom is removed by the spin averages

$$\Sigma_c = \frac{1}{3} (\Sigma_c + 2\Sigma_c^*)$$ \hspace{1cm} (15)

$$\Xi_c = \frac{1}{3} (\Xi_c' + 2\Xi_c^*)$$ \hspace{1cm} (16)

$$\Omega_c = \frac{1}{3} (\Omega_c + 2\Omega_c^*)$$ \hspace{1cm} (17)
Then we find heavy quark relations of the form

\[ \Lambda_b - \Lambda_c = B - D \]  
\[ \Sigma_b - \Lambda_b = \Sigma_c - \Lambda_c \]  
\[ \Sigma_b^* - \Sigma_b = B^* - B \]  
\[ \Sigma_c^* - \Sigma_c = D^* - D. \]  

We can also use light flavor SU(3) symmetry to relate the nonstrange charmed baryons to the charmed baryons with strange quarks. There are three relations which include corrections of order \( m_s \),

\[ \Xi'_c = \frac{1}{2} (\Sigma_c + \Omega_c) \]  
\[ \Xi'_c^* = \frac{1}{2} (\Sigma_c^* + \Omega_c^*) \]  
\[ \Sigma_c^* - \Sigma_c = \Xi_c^* - \Xi_c. \]  

There is another relation in which corrections of order \( m_s \) are not systematically included,

\[ \Xi'_c - \Lambda_c = \Xi'_c - \Xi_c; \]  

however, since the analogous relation in the charmed meson system,

\[ D_{s1} - D_s = D_{1} - D, \]  

works to within a few MeV, we will use this one as well.

The lightest observed heavy baryons are listed in Table 3, along with their masses and the decay channels in which they have been identified. I identify the observed states by provisional names, while in the penultimate column I give the conventional assignment of quantum numbers to these states. These assignments are motivated primarily by the quark model.

Let us compare the predictions of heavy quark and flavor SU(3) symmetry to these experimental results. The heavy quark constraints (18) and (19) are both satisfied to within 10 MeV. However, the hyperfine relation (20) is badly violated. One finds \((\Sigma_b^* - \Sigma_b)/(\Sigma_c^* - \Sigma_c) \approx 0.84 \pm 0.21\), too large by more than a factor of two! (I have ignored the correlation between the errors on the masses of the \( \Sigma_b \) and the \( \Sigma_b^* \), thereby overestimating the total uncertainty.) Clearly, if these data are correct then there is a serious crisis for the application of heavy quark symmetry to the charm and bottom baryons. On the other hand, this crisis rests entirely on the reliability of the DELPHI measurement of these states.

The situation is somewhat better for the SU(3) relations, although not perfect. The first equal spacing rule (21), yields the prediction \( \Xi_c = 2577 \text{ MeV} \),
Table 3: The observed heavy baryon states, with their conventional and alternative identities. Isospin multiplets have been averaged over. Statistical and systematic errors have, for simplicity, been added in quadrature. The approximate masses of the proposed new states are given in parentheses.

| State | Mass (MeV) | Ref. | Decay | Conventional | Alternative |
|-------|------------|------|-------|--------------|-------------|
| Λ_c  | 2285 ± 1   | weak | Λ_c  | weak         | Λ_c         |
|       | (2380)     | weak | absent| absent       | Σ_c^9++, Σ_c^9++ |
| Λ_c  | 2285 ± 1   | weak | Λ_c  | Λ_c + γ     | Σ_c         |
|       | (2380)     | weak | Λ_c  | Λ_c + π     | Σ_c^+       |
| Σ_c1 | 2453 ± 1   | weak | Σ_c1 | Σ_c         | Σ_c^+       |
| Σ_c2 | 2519 ± 2   | weak | Σ_c2 | Σ_c         | Σ_c^0       |
| Σ_c  | 2468 ± 2   | weak | Σ_c  | Σ_c         | Σ_c         |
| Σ_c  | 2563 ± 15 (?) | Σ_c + γ | Σ_c  | Σ_c         | Σ_c         |
| Σ_c1 | 2563 ± 15 (?) | Σ_c + γ | Σ_c  | Σ_c         | Σ_c         |
| Σ_c2 | 2644 ± 2   | weak | Σ_c2 | Σ_c         | Σ_c         |
| Ω_c  | 2700 ± 3   | weak | Ω_c  | Ω_c         | Ω_c         |
| Ω_c  | not yet seen |      |      |              |             |
| Λ_b  | 5623 ± 6   | weak | Λ_b  | weak         | Λ_b         |
|       | (5760)     | weak | Λ_b  | Λ_b + γ     | Λ_b         |
| Λ_b  | (5760)     | weak | Λ_b  | Λ_b + π     | Λ_b         |
| ΢_b1 | 5796 ± 14  | weak | ΢_b1 | ΢_b1         | ΢_b1         |
| ΢_b2 | 5852 ± 8   | weak | ΢_b2 | ΢_b2         | ΢_b2         |

somewhat large but probably within the experimental error. The second rule (22) cannot be tested, as the Ω_c state has not yet been found. The third rule (23) yields the prediction Ξ_c = 2578 MeV, again, reasonably consistent with experiment. (In fact the precise agreement of these two sum rules might lead one to expect that, when confirmed, the mass of the Ξ_c will be somewhat higher than its present central value.) By contrast, the final SU(3) relation (24) fails by approximately 60 MeV, almost an order of magnitude worse than for the charmed mesons. However, this relation is not on the same footing as the others, so its failure is not as significant as that of the heavy quark relation (21).

What is going on here? One possibility is that the heavy quark relations are simply no good for the spectroscopy of charmed baryons. Of course, we would like to avoid this glum conclusion, because it would call into question other applications of heavy quark symmetry to charmed hadrons, such as the treatment of exclusive semileptonic B decays used to extract |V_{cb}|. Another possibility is that the data are not correct. This may not be unlikely, particularly as the discrepancy rests primarily on the single DELPHI measurement.
However, let us look for an alternative resolution, in which we take the reported data seriously, within their reported errors. As the data change in the future, so perhaps will the motivation for such an alternative.

Let us, then, reinterpret the data under the constraint that the heavy quark and $SU(3)$ symmetries be imposed explicitly, including the dubious relation (24). Then if we identify the observed $\Xi_c'$ with the $\Xi_c^*$ state, the $SU(3)$ relations lead to the prediction $\Sigma_c = 2380\text{ MeV}$. If this is true, then it cannot be correct to identify the $\Sigma_c$ with the observed $\Sigma_{c1}$; rather, the $\Sigma_c$ would correspond to a state below threshold for the decay $\Sigma_c \to \Lambda_c + \pi$, which is yet to be seen. The observed $\Sigma_{c1}$ must then be the $\Sigma_{c*}$, while the observed $\Sigma_{c2}$ is some more highly excited baryon, perhaps an orbital excitation. The new assignments are given in the final column of Table 3.

A similar reassignment must be applied to the bottom baryons as well. The $\Sigma_b$ is now assumed to be below $\Lambda_b + \pi$ threshold, while the $\Sigma_{b1}$ is identified as the $\Sigma_b^*$. Then the poorly behaved symmetry predictions improve remarkably. For example, let us take the masses of the new states to be $\Sigma_c = 2380\text{ MeV}$ and $\Sigma_b = 5760\text{ MeV}$. Then the hyperfine splitting ratio (20) improves to $(\Sigma_b^* - \Sigma_b) / (\Sigma_c^* - \Sigma_c) = 0.49$, and the $SU(3)$ relation (24) between the $s_t = 0$ and $s_t = 1$ states is satisfied to within 5 MeV. The heavy quark relation (18) is unaffected, while the constraint (19) for the $\Sigma_Q$ excitation energy is satisfied to within 20 MeV, which is quite reasonable. Only the $SU(3)$ equal spacing rules (21) and (23) suffer from the change. The former relation now fails by 23 MeV. The latter now fails by 8 MeV, but the discrepancies are in opposite directions, and the two relations cannot be satisfied simultaneously by shifting the mass of the $\Xi_c'$. With these new assignments, intrinsic $SU(3)$ violating corrections of the order of 15 MeV seem to be unavoidable. In this context, a confirmation of the $\Xi_c'$ state is very important. If the mass were to be remeasured to be approximately 2578 MeV, then $SU(3)$ violation under the conventional assignments would be extremely small and we might be more disinclined to relinquish them.

Still, with respect to the symmetry predictions as a whole, the new scenario is quite an improvement over the old. The heavy quark and $SU(3)$ flavor symmetries have been resurrected. We can improve the agreement further if we allow the measured masses to vary within their reported $1\sigma$ errors. One set of allowed masses is $\Sigma_c = 2375\text{ MeV}$, $\Sigma_c^* = 2453\text{ MeV}$, $\Xi_c' = 2553\text{ MeV}$, $\Xi_c^* = 2644\text{ MeV}$, $\Sigma_b = 5760\text{ MeV}$, and $\Sigma_b^* = 5790\text{ MeV}$. For this choice, the $SU(3)$ relations (21), (23) and (24) are satisfied to within 15 MeV, 13 MeV and 4 MeV, respectively. The hyperfine ratio (20) is $(\Sigma_b^* - \Sigma_b) / (\Sigma_c^* - \Sigma_c) = 0.38$, and $\Sigma_b - \Lambda_b$ is equal to $\Sigma_c - \Lambda_c$ to within 15 MeV. This is better agreement with the symmetries than we even have a right to expect.
Of course, this new proposal implies certain issues of its own. The most striking question is whether the new $\Sigma_c$ and $\Sigma_b$ states are already ruled out. Consider the $\Sigma_c$, since much more is known experimentally about charmed baryons. The $\Sigma_c$ is an isotriplet, so it comes in the charge states $\Sigma_c^0$, $\Sigma_c^+$ and $\Sigma_c^{++}$. With the proposed mass, these states are too light to decay strongly, to $\Lambda_c^+ + \pi$. Instead, the $\Sigma_c^+$ will decay radiatively,

$$\Sigma_c^+ \rightarrow \Lambda_c^+ + \gamma,$$

while the others decay weakly, via channels such as

$$\Sigma_c^{++,0} \rightarrow \Sigma_c^{\pm} + \pi^+$$
$$\rightarrow p + \pi^\pm + K_S$$
$$\rightarrow \Sigma_c^{\pm} + \ell^\pm + \nu.$$

The challenge, then, is either to find these states or conclusively to rule them out.

We should also note that nonrelativistic constituent quark models typically do not favor such light $\Sigma_c^{(*)}$ and $\Sigma_b^{(*)}$ as I have suggested here. (See, for example, recent papers by Lichtenburg and Franklin.) These models often have been successful at predicting hadron masses, and are thus not unreasonably, quite popular. However, despite common misperceptions, they are less general, and make substantially more assumptions, than a treatment based solely on heavy quark and SU(3) symmetry. A reasonable quark model respects these symmetries in the appropriate limit, as well as parametrizing deviations from the symmetry limit. Such models therefore cannot be reconciled simultaneously with the heavy quark limit and with the reported masses of the $\Sigma_b$ and $\Sigma_b^{*}$. Hence, the predictions of this analysis follow experiment in pointing to physics beyond the constituent quark model. While the historical usefulness of this model for hadron spectroscopy may deepen one’s suspicion of the DELPHI data on $\Sigma_{b1,2}$, such speculation is beyond the scope of this discussion. To reiterate, I have taken the masses and errors of all states as they have been reported to date; as they evolve in the future, so, of course, will the theoretical analysis.

4 Strong Decays of Excited Charmed Mesons

Let us turn now from the spectroscopic implications of heavy quark symmetry to its implications for the strong decays of excited hadrons. We will focus on the system for which there is the most, and most interesting, data available, the excited charmed mesons.
As we saw in Section 3, there are two doublets of $p$-wave charmed mesons, one with $J^P = \frac{1}{2}^+$ and one with $J^P = \frac{3}{2}^+$. The former correspond to the physical states $D_0^*$ and $D_1'$, the latter to $D_1$ and $D_2^*$. Note that the $D_1$ and $D_1'$ both have $J^P = 1^+$, being distinguished by their light angular momentum $J_\ell^P$, which is a good quantum number only in the limit $m_c \to \infty$.

The $D_0^*$ and $D_1'$ decay via $s$-wave pion emission,

$$
D_0^* \to D + \pi \\
D_1' \to D^* + \pi.
$$

If their masses do not lie close to the threshold for this decay, then these states can easily be quite broad, with widths of order 100 MeV or more. Hence they could be very difficult to identify experimentally, and in fact no such states have yet been found. By contrast, the $D_1$ and $D_2^*$ are constrained by heavy quark symmetry to decay via $d$-wave pion emission. The channels which are allowed are

$$
D_1 \to D^* + \pi \\
D_2^* \to D^* + \pi \\
D_2^* \to D + \pi.
$$

Because their decays rates are suppressed by a power of $|p_\pi|^5$, these states could be much narrower than the $D_0^*$ and $D_1'$. In fact, resonances decaying in these channels have been identified, and the properties of the $D_1(2420)$ and the $D_2^*(2460)$ are given in Table 1.

Since pion emission is a transition of the light degrees of freedom rather than of the heavy quark, all of the decays (27) are really a single nonperturbative process, differentiated only by the relative orientation of the spins of the heavy quark and the initial and final light degrees of freedom. Hence the three transitions are related to each other by heavy quark symmetry. In the strict limit $m_c \to \infty$, both the $D_1$ and $D_2^*$ and the $D$ and $D^*$ are degenerate doublets, so the factor of $|p_\pi|^5$ is the same in all three decays. The finite hyperfine splittings $D^* - D \approx 150$ MeV and $D_2^* - D_1 \approx 40$ MeV are effects of order $1/m_c$, but their influence on $|p_\pi|^5$ is substantial. Hence we will account for this factor explicitly by invoking heavy quark symmetry at the level of the matrix elements responsible for the decays, and using the physical masses to compute the phase space. A straightforward calculation then yields two predictions for the full and partial widths:

$$
\Gamma(D_2^* \to D + \pi)/\Gamma(D_2^* \to D^* + \pi) = 2.3
$$

$$
\Gamma(D_1)/\Gamma(D_2^*) = 0.30.
$$
For comparison, the experimental ratios are
\[
\Gamma(D_2^* \rightarrow D + \pi) / \Gamma(D_2^* \rightarrow D^* + \pi) = 2.2 \pm 0.9 \quad (30)
\]
\[
\Gamma(D_1) / \Gamma(D_2^*) = 0.71. \quad (31)
\]
We see that the first relation works very well, while the second fails miserably. This unfortunate prediction raises a similar question as we faced earlier: is this a sign of a general failure of heavy quark symmetry as applied to charmed mesons, or can it be understood within the heavy quark expansion? Naturally, we would much prefer this latter outcome, for the familiar reason that we want very much to believe we can trust this expansion for the charmed mesons in other contexts.

Explanations for the failure for the prediction (29) have been offered in the past. One is to suppose a small mixing, of order $1/m_c$, of the narrow $D_1$ with the broad $s$-wave $D_1'$. Since these states have the same total angular momentum and parity, $J^P = 1^+$, such mixing is allowed when corrections for finite $m_c$ are included. A small mixing, of the size one might reasonably expect at this order, could easily double the width of the physical $D_1$. This is a plausible explanation, and could well contribute at some level, but for two reasons it is somewhat unlikely to be the dominant effect. First, there is no evidence for an $s$-wave component in the angular distribution of $p_\pi$ in the decay $D_1 \rightarrow D^* + \pi$. Although such a component could have escaped undetected by a conspiracy of unknown final interaction phases, such a situation is certainly not the generic one. Second, there is no evidence for an equivalent mixing between the strange analogues $D_{s1}$ and $D_{s1}'$, which would broaden the observed $D_{s1}$ unacceptably. Of course, light flavor $SU(3)$ might do a poor job of predicting a mixing angle, which is actually a ratio of matrix elements both of which receive $SU(3)$ corrections. So, while this explanation is not ruled out, neither does this evidence give one particular confidence in it.

Another possibility is that the width of the $D_1$ receives a large contribution from two pion decays to the $D$, either nonresonant,
\[
D_1 \rightarrow D + \pi + \pi, \quad (32)
\]
or through an intermediate $\rho$ meson,
\[
D_1 \rightarrow D + \rho \rightarrow D + \pi + \pi. \quad (33)
\]
Again, the problem is that there is no experimental evidence for such an effect. Also, it is somewhat difficult, within the schemes in which such decays are discussed, to broaden the $D_1$ enough to match fully the experimental width.
Hence, we are motivated to continue to search for a more elegant and plausible explanation, which does not force us to give up heavy quark symmetry for charmed mesons.

The answer, it turns out, lies in studying the heavy quark expansion for the excited charmed mesons at subleading order in $1/m_c$. In this case, we need a theory which contains both charmed mesons and soft pions, coupled in the correct $SU(3)$ invariant way. Such a technology is heavy hadron chiral perturbation theory. While the formalism is in some ways more than we need, as it includes complicated pion self-couplings which will play no role here, it is useful in that it allows us to keep track of all the symmetries in the problem mechanically (and correctly).

Heavy hadron chiral perturbation theory accomplishes three things. First, it builds in the heavy quark and chiral $SU(3)$ symmetries explicitly. Second, it implements a momentum expansion for the pion field, in powers of $\partial_{\mu} \pi/\Lambda_\chi$, where the chiral symmetry breaking scale is $\Lambda_\chi \approx 1$ GeV. Finally, and very important in the present context, it allows one to include symmetry breaking corrections in a systematic way.

To implement the symmetries, the Lagrangian must be built out of objects which carry representations not just of the Lorentz group, but of the heavy quark and $SU(3)$ symmetries as well. Clearly, these objects must contain both members of a single heavy meson doublet of fixed $J^{P}\ell$, and depend explicitly on the heavy meson velocity. For the ground state mesons $D$ and $D^*$, this the “superfield” \[ H_a = \frac{1 + \beta}{2\sqrt{2}} \left[D^*_a \gamma_{\mu} - D_a \gamma^5\right], \] where the index on the $D^*$ is carried by the polarization vector. Under heavy quark spin rotations $Q$, Lorentz transformations $L$, and $SU(3)$ transformations $U$, $H_a$ transforms respectively as

\[ H_a \rightarrow S_Q H_a \] \[ H_a \rightarrow S_L H_a S_L^\dagger \] \[ H_a \rightarrow H_a U_{ab}. \]

Here $S_Q$ and $S_L$ are the spinor representations of the Lorentz group, and $U_{ab}$ is the usual matrix representation of the vector subgroup of spontaneously broken chiral $SU(3)$ symmetry. There are similar superfields for the excited mesons, \[ S_a = \frac{1 + \beta}{2\sqrt{2}} \left[D^\mu_{1a} \gamma_{\mu} \gamma^5 - D^*_{0a}\right], \]
transforming in analogous ways. The superfields $H_a$, $S_a$ and $T^\mu_a$ all have mass dimension $\frac{3}{2}$. The pion fields appear as in ordinary chiral perturbation theory; since we will not be interested in pion self-couplings, we will just recall the linear term in the exponentiation of the pion fields,

$$A_\mu = \frac{1}{f_\pi} \partial_\mu \Pi + \ldots ,$$

where $\Pi$ is the matrix of Goldstone boson fields and $f_\pi \approx 132$ MeV.

We now build a Lagrangian out of invariant combinations of these elements. At leading order we get the terms responsible for the $d$-wave decay of the $D_1$ and $D_2^*$, 

$$f \frac{h}{\Lambda_X} \text{Tr} \left[ \overline{H} T^\mu \gamma_5 (i D_\mu A_\nu + i D_\nu A_\mu) \right] + \text{h.c.} ,$$

and for the $s$-wave decay of the $D_0^*$ and $D_1'$,

$$f \text{Tr} \left[ \overline{H} S \gamma_5 A_\mu \right] + \text{h.c.} .$$

Expanding these interactions in terms of the individual fields, we find the same symmetry predictions (28) and (29) as before.

However, now we would like to go further, and include the leading corrections of order $1/m_c$ in the effective Lagrangian. To understand how to do this, we turn to the expansion of QCD in the heavy quark limit, given by the heavy quark effective theory. This Lagrangian is written in terms of an effective HQET field $h(x)$, which satisfies the conditions

$$\frac{1 + \frac{\gamma}{2}}{2} h(x) = h(x)$$

and

$$i \partial_\mu h(x) = k_\mu h(x) ,$$

where $k^\mu = p^\mu - m_c v^\mu$ is the “residual momentum” of the charm quark. Including the leading corrections, the HQET Lagrangian takes the form

$$\mathcal{L}_{\text{HQET}} = \bar{h} i v \cdot D h + \frac{1}{2 m_c} \bar{h} (i D)^2 h + \frac{1}{2 m_c} \bar{h} \sigma^{\mu \nu} \left( \frac{1}{2} g_{\mu \nu} \right) h + \ldots .$$

The effect of the subleading terms $\bar{h} (i D)^2 h$ and $g \bar{h} \sigma^{\mu \nu} G_{\mu \nu} h$ on the chiral expansion may be treated in the same manner as other symmetry breaking perturbations to the fundamental theory such as finite light quark masses. Namely,
we introduce a “spurion” field which carries the same representation of the symmetry group as does the perturbation in the fundamental theory, and then include this spurion in the chiral lagrangian in the most general symmetry-conserving way. When the spurion is set to the constant value which it has in QCD, the symmetry breaking is transmitted to the effective theory. In the case of finite light quark masses, for example, the symmetry breaking term in QCD is \( \bar{q} M_q q \), where \( M_q = \text{diag}(m_u, m_D, m_s) \). Introducing a spurion \( M_q \) which transforms as \( M_q \to L M_q R^\dagger \) under chiral \( SU(3) \), we then include terms in the ordinary chiral lagrangian such as \( \mu \text{Tr} \left[ M_q \Sigma + M_q \Sigma^\dagger \right] \).

In the present case, only the second of the two correction terms in \( \mathcal{L}_{HQET} \) violates the heavy spin symmetry. We include its effect in the chiral lagrangian by introducing a spurion \( \Phi_{\mu\nu} \) which transforms as \( \Phi_{\mu\nu} \to S_Q \Phi_{\mu\nu} S_Q^\dagger \) under a heavy quark spin rotation \( S_Q \). This spurion is introduced in the most general manner consistent with heavy quark symmetry, and is then set to the constant \( \Phi_{\mu\nu} = \sigma_{\mu\nu}/2m_c \) to yield the leading spin symmetry violating corrections to the chiral lagrangian. We will restrict ourselves to terms in which \( \Phi_{\mu\nu} \) appears exactly once.

The simplest spin symmetry violating effect is to break the degeneracy of the heavy meson doublets. This occurs through the terms

\[
\lambda_H \text{Tr} \left[ \overline{H} \Phi_{\mu\nu}^H H \sigma_{\mu\nu} \right] - \lambda_S \text{Tr} \left[ \overline{S} \Phi_{\mu\nu}^S S \sigma_{\mu\nu} \right] - \lambda_T \text{Tr} \left[ \overline{T} \Phi_{\mu\nu}^T T_\alpha \sigma_{\mu\nu} \right].
\]

The dimensionful coefficients are fixed once the masses of the mesons are known. For the ground state \( D \) and \( D^* \), for example, we find

\[
\lambda_H = \frac{1}{8} \left[ M_{D^*}^2 - M_D^2 \right] = (260 \text{ MeV})^2.
\]

This value is entirely consistent with what one would obtain, instead, with the \( B \) and \( B^* \) mesons. For the \( D_1 \) and \( D_2^* \), we find

\[
\lambda_T = \frac{3}{16} \left[ M_{D_2^*}^2 - M_{D_1}^2 \right] = (190 \text{ MeV})^2.
\]

Note that \( \sqrt{\lambda_H} \) and \( \sqrt{\lambda_T} \) are of order hundreds of MeV, the scale of the strong interactions.

We are interested in the spin symmetry violating corrections to transitions in the class \( T^\mu \to H + \pi \), which will arise from terms analogous to \( \mathcal{L}_d \) but with one occurrence of \( \Phi_{\mu\nu}^\nu \). The spin symmetry, along with the symmetries which constrained \( \mathcal{L}_d \), requires that any such term be of the generic form

\[
\frac{1}{\Lambda_X} \text{Tr} \left[ \overline{T} \Phi_{\mu\nu}^\nu T_\alpha C_{\mu\nu\alpha\beta\gamma} \gamma^5 \left( iD^\beta A^\gamma + iD^\gamma A^\beta \right) \right] + \text{h.c.},
\]

where \( \Lambda_X \) is the typical scale of the strong interactions.
where \( C_{\mu\nu\alpha\beta\kappa} \) is an arbitrary product of Dirac matrices and may depend on the four-velocity \( v^\lambda \). This would seem to allow for a lot of freedom, but it turns out that there is only a single spin symmetry-violating term which respects both parity and time reversal invariance:

\[
L_{d1} = \frac{h_1}{2m\Lambda}, \quad \text{Tr} \left[ \bar{H} \sigma^{\mu\nu} \sigma^{\mu\nu} \gamma^5 (iD_\alpha A_\kappa + iD_\kappa A_\alpha) \right] + \text{h.c.} .
\]

(50)

We expect the new coefficient \( h_1 \), which has mass dimension one, to be of order hundreds of MeV.

The mixing of \( D'_1 \) and \( D'_1 \) is also a spin symmetry violating effect which arises at order \( 1/m_c \). There is a corresponding operator in the chiral lagrangian which is responsible for this,

\[
L_{\text{mix}} = g_1 \text{Tr} \left[ \bar{S} \Phi^{\mu\nu}_s T^\mu_{\alpha\nu} v^\alpha \right] + \text{h.c.} .
\]

(51)

However, we will neglect this term for now. It is straightforward to include both \( L_{d1} \) and \( L_{\text{mix}} \) in a more complete analysis.

We now compute the partial widths for the decays of the \( D'_1 \) and the \( D'_2 \) at subleading order in the \( 1/m_c \) expansion. We find

\[
\Gamma(D'_2^0 \to D\pi) = \frac{1}{10\pi} \frac{m_{D'_2}}{M_{D'_2}} \left( \frac{|p_{\pi}|}{f_\pi} \right)^5 \left[ h - \frac{h_1}{m_c} \right]^2
\]

(52)

\[
\Gamma(D'_2^0 \to D^*\pi) = \frac{3}{20\pi} \frac{m_{D'_2}}{M_{D'_2}} \left( \frac{|p_{\pi}|}{f_\pi} \right)^5 \left[ h - \frac{h_1}{m_c} \right]^2
\]

(53)

\[
\Gamma(D_1 \to D^*\pi) = \frac{1}{4\pi} \frac{m_{D_1}}{M_{D_1}} \left( \frac{|p_{\pi}|}{f_\pi} \right)^5 \left[ h + \frac{5h_1}{3m_c} \right]^2 + \frac{8h_1^2}{9m_c^2}
\]

(54)

where in each expression \( |p_{\pi}|^5 \) is computed using the actual phase space for that decay. Setting \( h_1 = 0 \) would reduce these results to the leading order predictions. Note that the ratio of partial widths of the \( D'_2 \) is independent of \( h_1 \), and so is unchanged by the inclusion of \( 1/m_c \) effects. However, the ratio of the widths of the \( D_1 \) and the \( D'_2 \) receives a large correction,

\[
\frac{\Gamma(D_1)}{\Gamma(D'_2)} = 0.30 \left[ 1 + \frac{16}{3 \frac{h_1}{m_c h}} + \ldots \right] .
\]

(55)

From the width of the \( D'_2 \), and taking \( \Lambda = 1 \text{ GeV} \), we find \( h \approx 0.3 \). Then we see that even for a modest coefficient \( h_1 \approx 100 \text{ MeV} \), we get a correction to the ratio of widths of order 100%!
What we have learned, then, is that a $1/m_c$ correction of the canonical size, with no tuning of parameters, naturally leaves one of these predictions alone while destroying the other. In this sense, we understand the failure of the bad prediction within the heavy quark expansion. This is what we mean by saying that heavy quark symmetry (or any symmetry) “works”. It need not be the case that every prediction of the symmetry limit be well satisfied by the data. Rather, it is crucial that deviations from the symmetry limit can be understood within a systematic expansion in the small parameters which break the symmetry. When a symmetry works in this sense, we retain predictive power even in cases when the symmetry predictions behave poorly.

5 Production of Heavy Hadrons via Fragmentation

Before heavy hadrons can decay, they must be produced. The production of a heavy hadron proceeds in two steps. First, the heavy quark itself must be created; because of its large mass, this process takes place over a time scale which is very short. Second, some light degrees of freedom assemble themselves about the heavy quark to make a color neutral heavy hadron, a process which involves nonperturbative strong interactions and typically takes much longer. If the heavy quark is produced with a large velocity in the center of mass frame, and if there is plenty of available energy, then production of these light degrees of freedom will be local in phase space and independent of the light degrees of freedom in the initial state. This is the fragmentation regime. We will see that heavy quark symmetry simplifies the description of heavy hadron production via fragmentation, because, as before, it allows us to separate certain properties of the heavy quark from those of the light degrees of freedom. This is particularly important in the production of excited heavy hadrons, for which the behavior of the spin of the light degrees of freedom can be quite interesting.

Our consideration of heavy quark fragmentation will lead us to consider two related questions:

1. What are the nonperturbative features of the fragmentation process? In particular, can we exploit heavy quark symmetry to isolate and study the spin of the light degrees of freedom?
2. What is the fate of a polarized heavy quark created in the hard interaction? Is any initial polarization preserved until the heavy quark undergoes weak decay?

We will see that an understanding of the first question will cast a useful light on the second. In the latter case, the excited heavy baryons will play a significant role.
The analysis depends on following the spins of the heavy quark and the light degrees of freedom separately through the three phases of fragmentation, life of the state, and decay. The net interaction of the heavy and light angular momenta $S_Q$ and $J_\ell$ depends both on the strength of the coupling between them and on the length of time they have to interact. Of course, the coupling between the spins is small in the heavy quark limit, because it is mediated by the chromomagnetic moment of the heavy quark. This moment scales as $1/m_Q$, so the time $\tau_s$ it take for the heavy and light spins to precess once about each other is of order $m_Q/\Lambda_{\text{QCD}}^2$, much longer than typical time scales associated with the strong interactions.

This fact is enough to assure that the heavy quark spin is essentially frozen during the process of fragmentation itself. Since fragmentation is purely a phenomenon of nonperturbative QCD, it takes place on a time scale of order $1/\Lambda_{\text{QCD}} \ll \tau_s$. Hence there is not enough time for the relatively weak spin-exchange interactions to take place.

Naively, one can say something similar when the heavy quark fragments to an excited hadron which decays via a strong transition of the light degrees of freedom. The time scale of a strong transition is set by nonperturbative QCD and should be comparable to the fragmentation time. Thus, one might expect generically that the lifetime $\tau$ of the state satisfies $\tau \ll \tau_s$, and the heavy quark spin continues to be frozen in place during the life of the excited hadron. However, if the energy available in the decay is not much larger than $m_\pi$, the lightest hadron which can be emitted in a strong transition, then $\tau$ can be increased by the limited phase space. The most dramatic example is $D^*$ decay, which is so close to threshold that the strong ($D^* \rightarrow D + \pi$) and electromagnetic ($D^* \rightarrow D + \gamma$) widths are almost equal.

So we must treat excited hadrons on a case by case basis, depending on the relative sizes of $\tau$ and $\tau_s$. For simplicity, we will consider here only two extreme cases. Let the excited heavy doublet be composed of a hadron $H$ of spin $J$ and mass $M$ and a hadron $H^*$ of spin $J + 1$ and mass $M^*$. The first possibility is the “naive” one $\tau_s \gg \tau$, where $H$ and $H^*$ are formed and then decay before the angular momenta $S_Q$ and $J_\ell$ have a chance to interact. In this case, there is no depolarization of the heavy quark spin $S_Q$, if one was present initially. Similarly, when $H$ and $H^*$ decay strongly, the light degrees of freedom in the decay carry any information about the spin state in which they were produced. Note that the very spin-exchanges interaction which is inhibited here is the one responsible for the hyperfine splitting between $H$ and $H^*$. Hence, under these conditions the resonances are almost completely overlapping, with a width $\Gamma = 1/\tau$ satisfying $\Gamma \gg |M^* - M|$. This is another consequence of the effective decoupling of $S_Q$ and $J_\ell$, which are independent
good quantum numbers of the resonances.

The second possibility is the opposite extreme, $\tau \gg \tau_s$. This corresponds to heavy hadrons which decay weakly or electromagnetically, or to strong decays which are severely suppressed by phase space. Here the spins $S_Q$ and $J_\ell$ have plenty of time to interact, precessing about each other many times before $H$ and $H^*$ decay. There is at least a partial degradation of any initial polarization of $Q$, as well as a degradation of any information about the fragmentation process which may be carried by the light degrees of freedom. The signature of this situation is that the states $H$ and $H^*$ are well separated resonances, since the chromomagnetic interactions have ample opportunity to produce a hyperfine splitting much larger than the width, $|M^* - M| \gg \Gamma$. In contrast with the first case, here the heavy and light spins are resolved into states of definite total spin $J$.

5.1 Production and decay of $D_1$ and $D_2^*$

We will consider two examples, the first of which is the production and decay of the excited charmed mesons $D_1$ and $D_2^*$. We see from Table 1 that the splitting between these states is 35 MeV, while their widths are approximately 20 MeV. This makes them somewhat of an intermediate case; however, for simplicity let us treat them in the “widely separated resonances” limit. A more precise treatment which takes into account their finite widths is straightforward but not very pedagogically enlightening.

We must follow the orientations of the spins $S_Q$ and $J_\ell$ through the following sequence of events:
1. The charm quark is created in some hard interaction.
2. Light degrees of freedom with $J_\ell^P = \frac{3}{2}^+$ are created in the process of fragmentation.
3. The spins $S_Q$ and $J_\ell$ precess about each other, resolving the states $D_1$ and $D_2^*$ of definite total angular momentum $J$.
4. The $D_1$ or the $D_2^*$ decays via $d$-wave pion emission. We can measure the direction of this pion with respect to the spatial axis along which the fragmentation took place.

The light degrees of freedom can be produced with helicity $h = \pm \frac{3}{2}$ or $h = \pm \frac{1}{2}$ along the fragmentation axis. While parity invariance of the strong interactions requires that the probabilities for helicities $h$ and $-h$ are identical, the relative production of light degrees of freedom with $|h| = \frac{3}{2}$ versus $|h| = \frac{1}{2}$ is determined in some complicated and incalculable way by strong dynamics. Let the quantity $w_{3/2}$ denote the probability that $|h| = \frac{3}{2}$,

$$w_{3/2} = P(h = \frac{3}{2}) + P(h = -\frac{3}{2}).$$

(56)
Then $1 - w_{3/2}$ is the probability that $|h| = \frac{1}{2}$. Completely isotropic production corresponds to $w_{3/2} = \frac{1}{2}$. We have identified a new nonperturbative parameter of QCD, which is well defined only in the heavy quark limit.

This new parameter can be measured in the strong decay of the $D_2^*$ or $D_1$. For example, consider the angular distribution of the pion with respect to the fragmentation axis in the decay $D_2^* \to D^+ \pi$. This is a decay of the light degrees of freedom in the excited hadron, so it will depend on their initial orientation (that is, on $w_{3/2}$) and on the details of the precession of $J_\ell$ around $S_Q$ during the lifetime of the $D_2^*$. Following the direction of $J_\ell$ through fragmentation, precession and decay, we find the distribution

$$
\frac{1}{\Gamma} \frac{d\Gamma}{d\cos \theta} = \frac{1}{4} \left[ 1 + 3 \cos^2 \theta - 6 w_{3/2} (\cos^2 \theta - 1) \right].
$$

This distribution is isotropic only when $w_{3/2} = \frac{1}{2}$, that is, when the light degrees of freedom are produced isotropically in the fragmentation process. Similar distributions are found in the decays $D_2^* \to D^* \pi$ and $D_1 \to D^* \pi$.

A fit of ARGUS data[29] to the expression (57) seems to indicate that a small value of $w_{3/2}$ is preferred; while the errors are large, we find that $w_{3/2} < 0.24$ at the 90% confidence level[24].

5.2 Polarization of $\Lambda_b$ at SLC/LEP

After warming up with the excited charmed mesons, we are set to address a somewhat more practical question: What is the polarization of $\Lambda_b$ baryons produced at the $Z$ pole? This question is motivated by the fact that $b$ quarks produced in the decay of the $Z$ are 94% polarized left-handed. Since the $\Lambda_b$ is composed of a $b$ quark and light degrees of freedom with zero net angular momentum, the orientation of a $\Lambda_b$ is identical to the orientation of the $b$ quark inside it. Similarly, the $b$ quark spin does not precess inside a $\Lambda_b$. Hence if a $b$ quark produced at the $Z$ fragments to a $\Lambda_b$, then those baryons should inherit the left-handed polarization of the quarks and reveal it in their weak decay.

Unfortunately, life is not that simple. Two recent measurements of $\Lambda_b$ polarization from LEP are[32,33]

$$
P(\Lambda_b) = 0.08^{+0.35}_{-0.29} \text{(stat.)}^{+0.18}_{-0.16} \text{(syst.)} \quad \text{(DELPHI)},
$$

$$
P(\Lambda_b) = 0.26^{+0.20}_{-0.25} \text{(stat.)}^{+0.12}_{-0.13} \text{(syst.)} \quad \text{(ALEPH)},
$$

(57)
both a long way from $P(\Lambda_b) = 0.94$. The reason is that not all $b$ quarks which wind up as $\Lambda_b$ baryons get there directly. In particular, they can fragment to the excited baryons $\Sigma_b$ and $\Sigma_b^*$, which then decay to $\Lambda_b$ via pion emission. If the excited states, which have light degrees of freedom with $S_\ell = 1$, live long enough, then the $b$ quark will precess about $S_\ell$ and the polarization will be degraded. The result will be a net sample of $\Lambda_b$’s with a polarization less than 94%, as is in fact observed.

In addition to the requirement that $\tau > \tau_s$ for the $\Sigma_b^{(\ast)}$, any depolarization of $\Lambda_b$’s by this mechanism depends on two unknown quantities:
1. The production rate $f$ of $\Sigma_b^{(\ast)}$ relative to $\Lambda_b$. Isospin and spin counting enhance $f$ by a factor of nine, while the mass splitting between $\Sigma_b^{(\ast)}$ and $\Lambda_b$ suppresses it; studies based on the Lund Monte Carlo indicate $f \approx 0.5$ with a very large uncertainty.
2. The orientation of the spin $S_\ell$ with respect to the fragmentation axis. This orientation, which is nonperturbative in origin, reflects the possible helicities $h = 1, 0, -1$. By analogy with the treatment of the heavy mesons, we define

\[ w_1 = P(h = 1) + P(h = -1). \]  

In this case, isotropic production corresponds to $w_1 = \frac{2}{3}$. We may measure $w_1$ from the angle of the pion with respect to the fragmentation axis in the decay $\Sigma_b^* \to \Lambda_b + \pi$,

\[ \frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta} \equiv \frac{1}{4} \left[ 1 + 3 \cos^2 \theta - \frac{9}{2} w_1 (\cos^2 \theta - \frac{1}{3}) \right]. \]  

(59)

It turns out that the decay $\Sigma_b \to \Lambda_b + \pi$ is isotropic in $\cos \theta$ for any value of $w_1$.

The polarization retention of the $\Lambda_b$ may be computed in terms of $f$ and $w_1$. As before, it is more tedious than instructive to present the general case in which the $\Sigma_b$ and the $\Sigma_b^*$ may partially overlap, so let us restrict to the extreme situation $\tau \gg \tau_s$. Then the polarization of the observed $\Lambda_b$’s is $P(\Lambda_b) = R(f, w_1)P(b)$, where $P(b) = 94\%$ is the initial polarization of the $b$ quarks, and

\[ R(f, w_1) = \frac{1 + \frac{3}{5}(1 + 4w_1)f}{1 + f}. \]  

(60)

Note that for $f = 0$ (no $\Sigma_b^{(\ast)}$’s are produced), $R(0, w_1) = 1$ and there is no depolarization. For the Lund value $f = 0.5$, $R$ ranges between 0.70 and 0.85.

Can the very low measured values of $P(\Lambda_b)$ be accommodated by the present data on the $\Sigma_b^{(\ast)}$? The situation is still unclear. On the one hand,
the same DELPHI analysis which found such surprising masses for the excited bottom baryons reported \( w_1 \approx 0 \) and \( 1 < f < 2 \) with large uncertainty. If this is confirmed, and if the conventional identification of the bottom baryons is correct, then a polarization in the range \( P(\Lambda_b) \approx 40\% - 50\% \) is easy to accommodate. On the other hand, CLEO’s recent announcement of the \( \Sigma_c^* \) was accompanied by a measurement \( w_1 = 0.71 \pm 0.13 \), consistent with isotropic fragmentation. Recall that by heavy quark symmetry, \( w_1 \) measured in the charm and bottom systems must be the same, so this result is inconsistent with the report from DELPHI. Clearly, further measurements are needed to resolve this situation.

6 Weak Decays

We now turn to our final topic, the production of excited charmed hadrons in semileptonic \( B \) decays. The branching fraction of

\[
B \to (D_1, D_2^*) + \ell + \nu
\]

has been measured by two groups with roughly consistent results:

\[
\begin{align*}
\text{OPAL} & : \quad (34 \pm 7)\% \\
\text{CLEO} & : \quad < 30\% \text{ at } 90\% \text{ c.l.}
\end{align*}
\]

It is not unreasonable to assume that this measurement will eventually be improved, and any discrepancies resolved. The question is, what can we learn from it? How useful would an effort to improve this measurement really be?

I will propose that it would be extremely useful. First, because studying the production of excited charmed mesons in \( B \) decay gives us direct information about QCD, and second, because through this insight into QCD we can dramatically reduce the single most nettlesome theoretical uncertainty in the extraction of \( |V_{cb}| \) from inclusive semileptonic \( B \) decays, namely the dependence on the \( b \) quark mass.

The heavy quark expansion and perturbative QCD may be used to analyze semileptonic and radiative \( B \) decays in a systematic expansion in powers of \( 1/m_b \) and \( \alpha_s(m_b) \). Since the energy \( m_b - m_c \) which is released in such a decay is large compared to \( \Lambda_{QCD} \), we may invoke the duality of the partonic and hadronic descriptions of the process. The idea is that sufficiently inclusive quantities may be computed at the level of quarks and gluons, if the interference between the short-distance and long-distance physics may be neglected. Except near the boundaries of phase space, this is usually the case if the ratio of typical long wavelengths (\( \sim 1/\Lambda_{QCD} \)) to typical short wavelengths
(\sim 1/m_b) is sufficiently large. While it is reasonable to expect parton-hadron duality to hold for arbitrarily large energy releases, its application at the $b$ scale requires a certain amount of faith.

Consider a $B$ meson with initial momentum $p_B^\mu = m_B v^\mu$, which decays into leptons with total momentum $q^\mu$ and a hadronic state $X_c$ with momentum $p_X^\mu = p_B^\mu - q^\mu$. Since we are interested in the properties of the hadrons which are produced, we define the kinematic invariants:

$$s_H = p_X^2 \quad (64)$$
$$E_H = p_X \cdot p_B / m_B, \quad (65)$$

which are, respectively, the invariant mass of the hadrons and their total energy in the $B$ rest frame. We then compute the doubly differential distribution $d\Gamma/ds_H dE_H$ using the heavy quark expansion. First, we use the optical theorem to relate the semileptonic decay rate for fixed $q^\mu$ to the imaginary part of the forward scattering amplitude:

$$\sum_{X_c} \int dq \langle X_c(p_X)(\ell\nu)(q)|O_W|B\rangle^2 = \frac{1}{2} G_F^2\int dq e^{iq\cdot x} L_{\mu\nu}(q) \langle B|T\{J^\mu_{bc}(x), J^\nu_{bc}(0)\}|B\rangle. \quad (66)$$

Here $O_W = (G_F/\sqrt{2}) J^\mu_{\alpha\beta} J^\nu_{\alpha\beta}$ is the product of left-handed currents responsible for the semileptonic decay $b \to c\ell\nu$, and

$$L_{\mu\nu} = \frac{1}{3} (q_\mu q_\nu - q^2 g_{\mu\nu}) \quad (67)$$

is the tensor derived from the squared leptonic matrix element. The next step is to expand the time-ordered product $T\{J^\mu_{bc}(x), J^\nu_{bc}(0)\}$ in inverse powers of $1/m_b$, using the operator product expansion and the heavy quark effective theory. This yields an infinite sum of operators written in terms of the effective field $h(x)$, which we will truncate at order $1/m_b^2$. Finally, we write the matrix elements of the form $\langle B|h\cdots h|B\rangle$ in terms of parameters given by the heavy quark expansion.

Once we have the differential distribution $d\Gamma/ds_H dE_H$, we can weight with powers of the form $s_H^n E_H^m$ and integrate to compute moments of $s_H$ and $E_H$. Of course the $(n, m) = (0, 0)$ moment is just the semileptonic partial width $\Gamma$. The moments of $s_H$, which will be of particular interest, are sensitive to the production of excited charmed hadrons such as the $D_1$ and $D_2^*$.

Our results will be in terms of four QCD and HQET parameters, since we keep only terms up to order $1/m_b^2$:

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1. The strong coupling constant \( \alpha_s(m_b) \). We get powers of \( \alpha_s(m_b)/\pi \) when we compute the radiative corrections to the time-ordered product.

2. The “mass” \( \bar{\Lambda} \) of the light degrees of freedom, defined by

\[
\bar{\Lambda} = \lim_{m_b \to \infty} [m_B - m_b].
\]  

Because the quark mass which appears in this expression is the pole mass \( m_b = m_{b\text{pole}} \), the quantity \( \bar{\Lambda} \) suffers from an infrared renormalon ambiguity of order \( \sim 100 \text{ MeV} \). This ambiguity affects the interpretation of \( \bar{\Lambda} \), and so we must treat with caution any expression in which it appears. For comparison with data, it is preferable to use expressions in which the renormalon ambiguity can be shown to cancel.

3. The “kinetic energy” \( \lambda_1 \) of the heavy quark, defined by

\[
\lambda_1 = \lim_{m_b \to \infty} \langle B| \bar{b}(iD)^2 b|B \rangle / 2m_B.
\] 

Note that \( \lambda_1 \) is not exactly the \( b \) quark kinetic energy (or rather, its negative), since there are gauge fields in the covariant derivative. Relative to the \( b \) quark’s rest energy, its nonrelativistic kinetic energy is suppressed by \( 1/m_b^2 \).

4. The energy of the \( b \) quark due to its hyperfine interaction with the light degrees of freedom, given by

\[
\lambda_2 = \lim_{m_b \to \infty} \langle B| \frac{1}{4} g \sigma^{\mu\nu} G_{\mu\nu} b|B \rangle / 6m_B.
\] 

This is the only one the four parameters where the spin of the \( b \) enters directly. We can extract \( \lambda_2 \) from the \( B^* - B \) mass splitting, which yields

\[
\lambda_2 = 0.12 \text{ GeV}.
\] 

We will present results which include heavy quark corrections up through order \( 1/m_b^2 \), and radiative corrections up through two loops. Actually, the two loop corrections are only partially computed, with just those pieces proportional to \( \beta_0 \alpha_s^2 \), where \( \beta_0 = 11 - \frac{2}{3} n_f \) is the leading coefficient in the QCD beta function. We may hope that this piece dominates the two loop term, because of the large numerical coefficient \( \beta_0 \); in fact, for other calculations for which the full two loop result is known, this is usually the case. For semileptonic \( B \) decay, the full two loop calculation has not been completed.

We will present results for the semileptonic partial width, and for the first moment of the hadronic invariant mass spectrum. It is convenient to substitute

\[\frac{d\lambda_2}{d\mu} \quad \text{or} \quad \lambda_2(\mu)\] 

Because the chromomagnetic operator is renormalized, \( \lambda_2(\mu) \) actually depends slightly on the renormalization scale. The number we give here is \( \lambda_2(m_b) \).

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all appearances of the charm and bottom quark masses with spin-averaged meson masses, using the expansion

$$\overline{m}_B = m_B + \bar{\Lambda} - \frac{\lambda_1}{2m_B} + \ldots,$$

(72)

and analogously for charm. Then the coefficients which appear below are functions of the measured ratio $\overline{m}_D/\overline{m}_B$, with no hidden dependence on unknown quark masses. For the semileptonic partial width, we find

$$\Gamma(B \to X_c e\nu) = \frac{G_F^2|V_{cb}|^2}{192\pi^3}m_B^5 \left[ 1 - 1.54\frac{\alpha_s(m_b)}{\pi} - 1.43\beta_0\frac{\alpha_s^2(m_b)}{\pi^2} - 1.65\frac{\bar{\Lambda}}{m_B} \left( 1 - 0.87\frac{\alpha_s(m_b)}{\pi} \right) - 0.95\frac{\bar{\Lambda}^2}{m_B} - 3.18\frac{\lambda_1}{m_B^2} + 0.02\frac{\lambda_2}{m_B^2} + \ldots \right],$$

(73)

and for the average hadronic invariant mass,

$$\langle s_H - \overline{m}_D^2 \rangle = m_B^2 \left[ 0.051\frac{\alpha_s(m_b)}{\pi} + 0.096\beta_0\frac{\alpha_s^2(m_b)}{\pi^2} + 0.23\frac{\bar{\Lambda}}{m_B} \left( 1 + 0.43\frac{\alpha_s(m_b)}{\pi} \right) + 0.26\frac{\bar{\Lambda}^2}{m_B} + 1.01\frac{\lambda_1}{m_B^2} - 0.31\frac{\lambda_2}{m_B^2} + \ldots \right].$$

(74)

We include a subtraction of $\overline{m}_D^2$ in the invariant mass so that the theoretical expression will start at order $\alpha_s$ and $\bar{\Lambda}$. The heavy quark expansion seems to be under control, as the corrections proportional to $\lambda_1$ and $\lambda_2$ are at the level of a few percent. However, this not true of the expansion in perturbative QCD. Since $\beta_0\alpha_s/\pi \approx 0.6$, we see that the two loop corrections to (73) and (74) are as large as the one loop terms.

This is real trouble! With such a poorly behaved perturbation series, these expressions are not trustworthy. Actually, there is a problem with the nonperturbative corrections, too, since they contain the ambiguous parameter $\bar{\Lambda}$. How, then, can we use this theory to do reliable phenomenology?

Remarkably, these two problems are actually connected, and can be used to solve each other. The renormalon ambiguity of $\bar{\Lambda}$ arises from the poor behavior of QCD perturbation theory at high orders in the series for $m_b^{pole}$. Perhaps it is the same poor behavior which manifests itself in the perturbation
series for $\Gamma$ and $\langle s_H \rangle$. If so, then the solution is to eliminate $\bar{\Lambda}$ in favor of some unambiguous physical quantity, solving both problems as once.

In fact, it can be shown that this is precisely the case. The bad perturbation series in $\Gamma$ arises from the indirect dependence of the theoretical expression on the pole mass $m^\text{pole}_b$, through $\bar{\Lambda}$. One way to eliminate $\bar{\Lambda}$ is to write it in terms of $\langle s_H - m^2_D \rangle$, which can be measured. We then find

$$
\Gamma(B \to X_c \ell \nu) = \frac{G_F^2 |V_{cb}|^2}{192\pi^3} m_B^5 0.369 \left[ 1 - 1.17 \frac{\alpha_s(m_b)}{\pi} - 0.74 \frac{\alpha_s^2(m_b)}{\pi^2} - 7.17 \frac{\langle s_H - m^2_D \rangle}{m_B^2} + \ldots \right],
$$

omitting the small terms of order $1/m_b^2$. Note that the size of the two loop term has shrunk by a factor of two with this rearrangement. We have regained some measure of control over the perturbation series.

The moral of this exercise is that while it is perfectly fine to keep $\bar{\Lambda}$ in intermediate steps in calculations, it should be eliminated from predictions of physical quantities. By the same token, any extraction of $\bar{\Lambda}$ from the data is ambiguous, in the sense that it is necessarily polluted with an infrared renormalon ambiguity and a corresponding poorly behaved perturbation series.

We can use the data (62) to derive an experimental lower bound on $\langle s_H - m^2_D \rangle$. Taking the relative branching ratio to be 27%, consistent with all measurements, we find

$$
\langle s_H - m^2_D \rangle \geq 0.49 \text{ GeV}^2.
$$

We can translate this into a bound on $\bar{\Lambda}$, which at one loop yields

$$
\bar{\Lambda}_{\text{one loop}} > \left[ 0.33 - 0.07 \left( \frac{\lambda_1}{0.1 \text{ GeV}^2} \right) \right] \text{ GeV}. \tag{77}
$$

Note that our prejudice is that $\lambda_1 < 0$, so it is probably conservative to ignore the small $\lambda_1$ term. When two loop corrections (proportional to $\beta_0 \alpha_s^2$) are included, the bound is weakened to

$$
\bar{\Lambda}_{\text{two loop}} > \left[ 0.26 - 0.07 \left( \frac{\lambda_1}{0.1 \text{ GeV}^2} \right) \right] \text{ GeV}. \tag{78}
$$

The instability of these bounds when radiative corrections are included is a direct reflection of the renormalon ambiguity.

This improvement may be interpreted as an increase in the BLM renormalization scale from $\mu_{\text{BLM}} = 0.16 m_B$ to $\mu_{\text{BLM}} = 0.38 m_B$. 

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Of more interest is the bound on $|V_{cb}|$ from the improved relation (75), which has no (leading) renormalon ambiguity. Including two loop corrections, we find [4]

$$|V_{cb}| > \left[ 0.040 - 0.00028 \left( \frac{\lambda_1}{0.1 \text{ GeV}^2} \right) \left( \frac{\tau_B}{1.60 \text{ps}} \right)^{-1/2} \right].$$

(79)

We have left explicit the dependence on the lifetime $\tau_B$ of the $B$ meson. The contribution of the two loop correction to this bound is 0.002, well within reason. If, to be conservative, we take $\langle s_H - \bar{m}_D^2 \rangle = 20\%$, then the bound becomes $|V_{cb}| > 0.038$.

From this point of view, of course, the ideal experiment would measure $\langle s_H - \bar{m}_D^2 \rangle$ directly, as well as higher moments such as $\langle (s_H - \bar{m}_D^2)^2 \rangle$. Such a program could lead to the best possible measurement of $|V_{cb}|$, with theoretical uncertainties at the level of a few percent.

7 Conclusions

We have seen that excited heavy hadrons have a lot to teach us about both QCD and physics at short distances. The phenomenology of these hadrons is extremely rich. We have illustrated their potential by discussing their spectroscopy, strong decays and production in fragmentation and semileptonic decay, but by no means need this exhaust the possibilities. Dedicated theoretical and experimental study of these states will pay real physics dividends in the upcoming Factory Era.

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