On the dynamics of a smart tensegrity structure using shape memory alloy

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Abstract. A tensegrity system is composed by two types of elements, tensile and compressive parts, which promotes the structural stability of the system, in this case the tensile parts are cables (steel cables), and the compressive parts are bars made of steel. An attractive characteristic of a tensegrity structure is the capability to be deployable, for this reason, is interesting for space applications due to the small volume that occupy in a possible transport to a station outside the earth. In general, these structures can be large with many repetitive cells. In this work, a tensegrity boom structure consisting of ten periodic cells made of bars and cables is studied. The numerical model, obtained by finite element method, is validated experimentally considering the case of one structural cell. In order to make the system adaptable to external excitation, one of the steel cables in the tensegrity is replaced by a SMA (shape memory alloy) cable (nitinol) allowing the dynamics characteristics of the system to be changed according to an electrical current applied SMA. Various configurations for placing the SMA cable are studied with the objective of reducing the vibration amplitudes for harmonic force excitation.

1. Introduction

Probably one of the first applications considering the use of tensegrity systems was proposed by Buckminster [1] and by Shelnol [2], as structural frameworks with tensile and compressive parts in a lattice form, being separated from each other. From that moment on, several studies with this type of structure were conducted [3], demonstrating diverse applications and studies on the mechanical behavior of these structures [4], and showing that the behavior of self-stressed reticulated spatial systems is non-linear due to their flexibility [5]. Tensegrity structures also receive attention for space applications due to some attractive characteristics, such as the deployability [6] and the good relation to the small volume that occupies in a possible transport outside of the earth, and similarities with regular struts.

The dynamic behavior of some tensegrity structures was examined, and showed that the system can function as a counterpart to conventional trusses, even though some of those systems were ineffective at eliminating vibrations [7, 8]. Linear models can be used to describe the approximate dynamics of tensegrity systems and [9] show that the modal dynamic range commonly increases with pretension. This pretension comes from different elements such as
tendons, membranes and cables [10, 11, 12], and can form a vast type of structures, and also an ample set of applications.

Motivated by these broad applications, the necessity to control also became a need to achieve some determined results like mentioned for space applications [13]. Several types of control can be used to accomplish the best results for those applications [14]. Natural frequencies can be shifted when the self-stress level in the tensegrity structure is modified, using active control [15]. Chan et al. [16] presented a small-scale active three bar structure, with local integral force feedback and acceleration feedback control, achieving effective damping for the next 2 resonant bending modes. Robust active control algorithm ($H_\infty$) in vibration control can be used to provide an efficient performance on the attenuation of the modes, in one degree of freedom [17]. The Linear-quadratic regulator (LQR) can also be integrated into tensegrity system control and offer different actuator placement schemes, similar to the control presented in this paper [18].

The structure used in this work is composed of 40 bars and 102 cables and has 53 nodes localized at the joints. In each cell one of the side cables is a shape memory alloy (SMA), acting as a control [19]. An initial analysis was made on a single cell and was validated by comparing with laboratory tensegrity structure results, in both, heated (SMA cable heated by an electrical current) and room temperature (RT), also named as active and inactive cells respectively. The model SMA wire is connected to the bars edge in the same way as all the others cables due to its malleability, and it was trained (or programmed) to be straight upon heating (transformation to austenite) [20]. In order to control specific modes the elements strain energies can be calculated and a control can be applied on the element with higher value like actuators [21], analogously the control for this work structure is an active cell. The formulation of sensible cost or performance functions was used in this work in order to simplify the results [22].

2. Mathematical Modeling

The tensegrity structure considered in this work is composed of ten repeated cells shown in figure 1 and the unit cell illustration and parameters are presented in section 3. The numbering scheme of the structure is shown in figure 2.

![Figure 1. Tensegrity Boom with 10 equal unit cells.](image1)

In these figures, the thin lines represent the cables and the thick lines represent bars with axial stiffness (bending and torsion of the bars and cables are not considered in the analysis).

During the analysis, the nodes 51, 52 and 53 are considered to be fully clamped and a harmonic force is applied at node 8, as shown in the figure 1. The unit cells are connected by the nodes and share the same from the previous unit cell. The cables are independent of each other, and the SMA is still one of the side cables of each unit cell.
A linearized equation of motion to describe a dynamic model around an equilibrium position as [15] presents, is used to describe the dynamic behavior of the active tensegrity structure, as follow:

\[ M\ddot{u} + C\dot{u} + Ku = f \]  

where \( M, C \) and \( K \), are mass, damping and stiffness global matrices. The term \( f \) is the applied load vector, and \( \ddot{u}, \dot{u} \) and \( u \) are the vectors of nodal acceleration, velocity and displacement respectively. With the development of the tensegrity structure using finite element method, the parameters of mass, damping and stiffness are represented by matrices. The overbar represents variables in the local coordinate system, and the transformation matrix from local to global matrix are described by [22, 23]. The element mass matrix is given by:

\[
M = \frac{\rho AL}{6} \begin{bmatrix} 2I_3 & -I_3 \\ -I_3 & 2I_3 \end{bmatrix}
\]  

Similarly to what is shown in references [15, 24], the local stiffness matrix \( K \) can be decomposed into two other matrices, \( K_E \) and \( K_G \), linear stiffness matrix and the geometrical stiffness matrix correspondingly, \( K = K_E + K_G \) in which:

\[
K_E = \frac{EA}{L} \begin{bmatrix} I_0 & -I_0 \\ -I_0 & I_0 \end{bmatrix}; \quad K_G = \frac{T}{L} \begin{bmatrix} I_3 & -I_3 \\ -I_3 & I_3 \end{bmatrix}; \quad I_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

the terms defining equation 2 and equation 3 are described in table 1. The damping value is treated as equivalent Rayleigh damping, proportional to the stiffness matrix as \( C = \beta K \), in which \( \beta = 3.45 \times 10^{-5} \).

It is assumed that the force applied at node 8 is harmonic, of the type \( f(t) = Fe^{i\omega t} \) where \( \omega \) is frequency of excitation and \( F \) is the magnitude of a complex force. Under this condition, the displacements are also considered harmonic and can be written in the form \( u(t) = Ue^{i\omega t} \) where \( U \) is a vector of complex displacement amplitudes for each degree of freedom in the structure.
2.1. Performance Function for Vibration Reduction

With the objective of understanding the vibrational behavior of such system, a Performance Function $J$ is defined as the sum of the linear squared velocities of the nodes $n_1 = 1$, $n_2 = 2$, and $n_3 = 3$, as shown by Fig 1. This performance function is proportional to the kinetic energy at these nodes and it is calculated by:

$$J(\omega) = v_m^H(\omega)v_m(\omega)$$  \hspace{1cm} (4)

where, $\omega$ is the angular frequency. $v_m^H(\omega)$ is the Hermitian form of the vector $v_m(\omega)$. Abolishing the symbol $\omega$ for simplicity, the vector $v_m$ is defined by:

$$v_m = [v_{n_1}^{n_1} v_{n_1}^{n_2} v_{n_1}^{n_3} v_{n_2}^{n_2} v_{n_2}^{n_3} v_{n_3}^{n_3}]^T$$  \hspace{1cm} (5)

where for an example $v_{n_1}^{n_1}$ is the velocity of the node $n_1$ in the $y$ direction for an $\omega$ frequency, and $T$ is the transpose of the vector.

3. Tensegrity structure unit cell

A single cell of the structure used in this work is composed of four bars and 12 cables, and has 8 nodes localized at the joints, one of the side cables is a shape memory alloy (SMA) and can be represented by figure 3, and figure 4 presents the real model.

![Figure 3. Illustration of the tensegrity system.](image)

![Figure 4. Real tensegrity structure.](image)

In the illustration of figure 3, the thick black lines represent rigid bars, and cables are represented in thinner lines, as commented before. The SMA is represented by the dark red thinner line. In this analysis, the nodes (3, 4, 5, 8) where considered free while nodes (1, 2, 6, 7) were clamped with no rotation allowed. The bars and the cables are made of stainless steel, and the SMA is made by nitinol. The relevant properties for this study using FEM, are the cross-section area ($A$), the material density ($\rho$), the material Young’s modulus ($E$), and tension of the cable($T$). The material and geometric properties of the structure are shown in table 1.
Table 1. Material and Geometric Properties of the Tensegrity Structure.

| Property                              | Value                                |
|---------------------------------------|--------------------------------------|
| Bar Young’s modulus ($E$)             | 200 GPa                              |
| Bar Density ($\rho$)                  | 7850 kg/m$^3$                        |
| Bar Cross Section Area ($A$)          | $2.8274 \times 10^{-5}$ m$^2$        |
| Cable Young’s modulus ($E$)           | 200 GPa                              |
| Cable Density ($\rho$)                | 7850 kg/m$^3$                        |
| Cable Cross Section Area ($A$)        | $3.3183 \times 10^{-7}$ m$^2$        |
| SMA Young’s modulus ($E$)             | 40 GPa                               |
| SMA Density ($\rho$)                  | 6450 kg/m$^3$                        |
| SMA Cross Section Area ($A$)          | $7.854 \times 10^{-7}$ m$^2$         |

3.1. Model Validation

In the experimental procedure, shown in figure 6, two accelerometers (1, 2) and one shaker (3) were used to obtain the dynamic response of the structure. The tensegrity system was connected to the shaker, with one accelerometer in the base of the structure, and the other one coupled in an upper node. Three additional masses (4) equivalent to the accelerometer were included in the experimental structure and model. The structure was submitted to random excitation and after calculating the power spectral density from the accelerometer data, it was possible to obtain the ratio of displacements (top/base) indicating a displacement transmissibility in the frequency domain like results shown by figure 7.

![Figure 5. Experimental scheme](image)

![Figure 6. Experimental setup.](image)

At first, the analysis was conducted using the SMA at room temperature (RT), further, the temperature of the wire was elevated until occurs the transformation from martensite to austenite. In figure 7 it can be seen the first resonance frequency peak shift, and even though the heated SMA Young’s modulus is higher, the resonance frequency is lower. The SMA used in the experiment was straight annealed. When heated, it tends to expand, causing a reduction on the tension on the other cables of the structure, resulting on an overall reduction of the first natural frequency of the system.
As described in reference [5], the tension on the cables are determined according to their lengths. The tension used on the finite element model was adjusted based on the experimental procedure described in this section. The tension at RT for the upper, side and lower cables are 3.8 N, 3.8 N and 2.69 N, respectively and when heated 2.3 N, 2.3 N and 1.63 N. The unit cell resonance frequency using the SMA at RT and heated are 12.5 and 9.8 Hz, respectively.

![Figure 7. Comparison Experimental and Numerical, RT and heated SMA.](image)

4. Results
For the next analysis it was considered that the structure is fixed-free, by fixing the nodes 51, 52, and 53. The efficiency of the control proposed in this work, combining active and inactive cells, can be measured by calculating the attenuation in the performance function. This attenuation expression is given by:

\[ a(n)[\text{dB}] = 10 \log_{10} \frac{\bar{J}_n}{\bar{J}_0} \]  

(6)

where, \( n \) is the setup number and \( \bar{J}_0 \) is the performance function arithmetic mean for the setup with no active cell.

![Figure 8. Frequency Response Function (Mobility). dB ref. 1 Ns/m](image)

![Figure 9. Frequency Response Function (Mobility - low frequency detail). dB ref. 1 Ns/m](image)

Two different ranges of frequencies were tested. Focused on the lower frequency, the first range is from 7 Hz to 15 Hz (Range 1), and the second range from 200 to 300 Hz (Range 2).
The frequency response functions (FRF) were obtained by applying a harmonic force of 1 N at node 8, in the z direction. The figures 9 and 8, presents the response for the inactive structure.

The metric used in this section to compare the vibrations levels, uses the same principle as $\mathcal{H}_2$ [25].

4.1. Single Active cell allocation

Different setups were tested in this subsection, with different positions of the active cells, in order to verify the behavior of the tensegrity boom with the change of stiffness on diverse locations. The active cells were activated in order of position in the structure, from cells 1 to 10, activating one at a time.

![Figure 10. First range amplitude attenuation for single active cell setups.](image1)

![Figure 11. Second range amplitude attenuation for single active cell setups.](image2)

The relation between no active cell and the setups using Eq. 6, for the lower frequency is shown by figure 10, and exhibits that, setups 3 to 9 have lowered the mean FRF amplitude. The figure 11 present the results for the higher frequency, which is possible to see that all setups lowered the mean FRF amplitude. Notably, for range 1 and 2, setups 5 and 3 lowered the mean amplitude by 3.4 and 6.2 dB respectively.

4.1.1. First Range Response

The two best results for the first range are presented by figures 12 and 13.

![Figure 12. 5th active cell - FRF Range 1. dB ref. 1 Ns/m](image3)

![Figure 13. 8th active cell - FRF Range 1. dB ref. 1 Ns/m](image4)
For both examples mostly resonance peaks were not just shifted but also lowered, even with a tiny reduction of the damping caused by the reduction of the system stiffness. Lower amplitude between peaks also were created with the active cell leading to a lower performance function. Overall, in the two best results the mean reduction was around 3.2 dB.

4.1.2. Second Range

The two best results for the second range are presented by figures below.

**Figure 14.** 3rd active cell - FRF
Range 2. dB ref. 1 Ns/m

**Figure 15.** 8th active cell - FRF
Range 2. dB ref. 1 Ns/m

For this range some peaks disappeared and a gap is made between 240 and 270 Hz. It is shown that the active cells like for the first range lowered the amplitude between peaks, and in most cases lowered the peaks. Overall, in the two best results the attenuation was 5.6 dB.

4.2. Multiple Active Cells

The second analysis is based on the use of multiple active cells at the same time in positions through the boom. The following figures, presents the configurations of the four setups tested in this part, with active cells represented by red color.

**Figure 16.** Structure with interleaved active cells - Setup 1

**Figure 17.** Structure with active cells at the extremities - Setup 2

**Figure 18.** Structure with 3 active cells at the central region - Setup 3

**Figure 19.** Structure with 3 interleaved cells - Setup 4
The setups have at least 3 and 5 maximum active cells. The attenuation for the two ranges is shown by figures 20 and 21, were for the first range, both setups 1 and 2 presents a increased amplitude in response of the harmonic force applied for the analysis, when compared to the structure with no active cell. The setup 3 have a mitigation about 8.5 dB and is the best of the four setups for the range. For the second range the first setup have the best results with attenuation of 7.9 dB. In all setups for this case the amplitude diminishes, different from the first range with just 2 better results.

![Figure 20. First range amplitude attenuation for multiple active cells setups.](image1)

![Figure 21. Second range amplitude attenuation for multiple active cells setups.](image2)

4.2.1. Results for the First Frequency Range

The first range setups 3 and 4 presents the best results, and the frequency response function is represented by figures 22 and 23.

![Figure 22. 3rd Setup FRF - Range 1. dB ref. 1 Ns/m](image3)

![Figure 23. 4th Setup FRF - Range 1. dB ref. 1 Ns/m](image4)

Similar to the single active cell, the setup 3 created deeper valleys in the response, notably from 10 to 13 Hz, and also did not just shift the peaks, shown by figure 22 in the last resonance peak. For the setup 4, is visible that was a shift to the left, and like all the setups tested, shows an amplitude reduction. Overall, in the two best results the mean reduction was around 16.5 dB.

4.2.2. Results for the Second Frequency Range

The best results for the second range are presented by figures 24 and 25. It is visible that the results are similar to the single active cell
as well, and reducing the amplitude for the range, with a shift for specific natural frequencies. Overall, in the two best results the mean reduction was around 5.2 dB.

**Figure 24.** 1st Setup FRF - Range 2.

**Figure 25.** 3rd Setup FRF - Range 2. dB ref. 1 Ns/m

### 4.3. Vibration Control of specific modes and frequencies

Analogously to what is proposed in reference [21], the strain energy was calculated using Equation 7, for all the elements of the structure.

\[
v_i = \frac{\phi_i^T B_i^T T_i^T K_i T_i B_i \phi_i}{\phi_i^T K_G \phi_i}
\]

Where, \( \phi_i \), is the collection matrix containing the \( i^{th} \) mode shape components associated with the degree of freedom of the element. \( B_i \), is the Boolean matrix associated with the element \( i \). \( T_i \) is the coordinate transformation matrix for the element \( i \), \( K_i \) is the stiffness matrix of the element \( i \) and \( K_G \) is the global stiffness matrix.

The purpose of this calculation is to do a similar analysis to place an active cell, to control the structure for a specific frequency. This cell strain energy was calculated by a simple sum of the strain energy of all elements that compose a cell in the tensegrity boom, so for a modal analysis, each mode will have the response for the whole cell, not just an element. Applying this concept for 3 specific modes, the cells with higher strain energy were activated, and the mobility is presented by figures 26, 27 and 28.

**Figure 26.** 2nd Mode FRF comparison. dB ref. 1 Ns/m

**Figure 27.** 3rd Mode FRF comparison. dB ref. 1 Ns/m

**Figure 28.** 5th Mode FRF comparison. dB ref. 1 Ns/m

The vertical dashed red line presents the natural frequency of the mode for a structure with no active cell, and the blue dashed line presents the natural frequency for the same mode with
the active cell, with the active cell on the cell with higher strain energy for the corresponding mode. The “x” marker shows the amplitude of the natural frequency in both, active and not active structures, indicating the response change, becoming clear that the strain energy cell is effective for a specific mode control.

5. Conclusions
This paper has shown the capability of controlling a tensegrity structure using shape memory alloy. A system model was obtained by a linearized equation of motion around an equilibrium configuration and compared with experimental results showing satisfactory agreements. The finite element method is used to calculate the response to harmonic excitation first for a single structure cell, in order to compare with the experimental system, and then, applied to the whole structure.

Three different control approaches were studied, a single cell activation, multiple cell activation, and active cells for specific modes, for two frequency ranges. The first one, all cells of the structure are activated one at a time, and for both ranges the two best results had an amplitude reduction about 4.4 dB, with not just peak shifts but also with deeper valleys, similar results were obtained with multiple active cells in different configurations with an amplitude reduction around 8.2 dB. The cells strain energy showed also to be efficient to control specific modes, mainly by shifting the peaks.

The simulation results showed that the dynamics of a tensegrity structure can be controlled by a shape memory alloy, using similar control methods as used in regular truss structures, and presented good amplitude reductions in order of about 8.5 dB in some cases.

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