Scientific Performance Analysis of the SYZ Telescope Design versus the RC Telescope Design*

Donglin Ma1 and Zheng Cai2,3

1 School of Optical and Electronic Information, Huazhong University of Science and Technology, Wuhan, 430074, China
2 UCO/Lick Observatory, University of California at Santa Cruz, 1156 High Street, Santa Cruz, CA 95064, USA; zcai@ucolick.org

Received 2017 October 4; accepted 2017 November 6; published 2017 December 21

Abstract

Recently, Su et al. propose an innovative design, referred as the “SYZ” design, for China’s new project of a 12 m optical-infrared telescope. The SYZ telescope design consists of three aspheric mirrors with non-zero power, including a relay mirror below the primary mirror. SYZ design yields a good imaging quality and has a relatively flat field curvature at Nasmyth focus. To evaluate the science-compatibility of this three-mirror telescope, in this paper, we thoroughly compare the performance of SYZ design with that of Ritchey–Chrétien (RC) design, a conventional two-mirror telescope design. Further, we propose the Observing Information Throughput (OIT) as a metric for quantitatively evaluating the telescopes’ science performance. We find that although a SYZ telescope yields a superb imaging quality over a large field of view, a two-mirror (RC) telescope design holds a higher overall throughput, a better diffraction-limited imaging quality in the central field of view (FOV < 5") which is better for the performance of extreme Adaptive Optics (AO), and a generally better scientific performance with a higher OIT value.

Key words: telescopes

Online material: color figures

1. Introduction

The development of new technologies yields increasingly advanced astronomical telescopes with novel instruments. These new technologies constantly improve the telescope’s aperture, throughput, angular resolution, spectral resolution, and time resolution of astronomical observations (Nelson et al. 1985; Nelson 2000). Besides, researchers have also explored a variety of configurations for the telescope designs due to their specific applications and requirements. Among them, the two-mirror designs include the Ritchey–Chrétien (RC) configuration and Aplanatic Gregorian (AG) configuration. These RC design telescopes include 8 m Subaru Telescope, Keck (10 m) telescope, and the next generation Thirty-Mirror-Telescope that is currently under construction (Nelson et al. 1985; Kaifu 1998; Nelson 2000). The most typical examples for AG configuration are the Large Binocular Telescope (LBT) (8.4 m × 2) (Hill & Salinari 2003) and the next generation Giant Magellan Telescope (Johns 2008). The RC and AG configurations are concise and contain only two aspheric mirrors with non-zero power: the primary mirror and the secondary mirror. Another plane tertiary mirror with no power is required to direct the incident light beam into the Nasmyth instrument platforms for both RC and AG designs.

Recently, there is a new project of constructing a large optical-infrared telescope (LOT) in China. The aperture of primary mirror is 12 m. Su et al. (2016) propose an innovative design that consists of three aspheric mirrors with non-zero power, including a primary mirror, a secondary mirror, and a relay mirror (referred as SYZ relay mirror) just below the primary mirror. The configuration of the conceptual design along with the comprehensive parameters can be found in Su et al. (2016). According to their statements, the SYZ design yields excellent geometrical image quality over a large field of view (FOV) compared to the traditional 2-mirror design and relatively more flat field curvature at Nasmyth focus. Further, SYZ design can switch between Coude focus and Nasmyth focus conveniently (Su et al. 2016).

To better evaluate the effectiveness of this novel SYZ design, we compare the performance of the SYZ design with that of conventional two-mirror design (e.g., RC design) based on several different performance metrics. Several independent metrics have already been proposed to evaluate the telescope designs, including the calculation of the throughput and the imaging quality metrics (e.g., wavefront error; 80% enclosed energy diameter; equivalent noise area (ENA; King 1983); normalized point source sensitivity (PSS; Seo et al. 2009)). These quantities help to understand the different designs and better budget errors of telescopes. In this paper, we use these metrics and also introduce new metrics to directly and
quantitatively compare the scientific productivity of the SYZ design with that of the RC design. This paper is organized as follows: in Sections 2 and 3, we present the design of a RC and a SYZ system. In Section 4, we first analyze the diffraction image quality and system’s throughput of all designs. Then we compare the adaptive optics (AO) performance of the two-mirror and three-mirror design. Finally, we propose a new performance metric of telescope named as observational information throughput (OIT), which is directly related to the scientific productivity of telescopes with different designs in this paper. Since the Nasmyth focus hold a majority of instruments for current large telescopes, all the performance metrics are principally measured at Nasmyth focus in the following of this paper.

2. SYZ Design and Performance

2.1. SYZ Design

This SYZ design was proposed by Su et al. (2016). The SYZ design contains three mirrors with non-zero power, i.e., primary mirror, secondary mirror and the SYZ relay mirror, as shown in Figure 1. It is an example of 3-mirror telescopes. The parameters of different mirrors are listed in Table 1, and the focal ratio at Nasmyth focus is set to be \( f/12.8 \). The primary mirror (M1) has a ellipsoidal surface. Both M2 surface and M3 surface are aspheric surfaces. The effective focal length (EFFL) of the system is 153, 595 mm. The maximum FOV of the system is set as 20’. In addition, the \( f \)-number of the primary mirror is set to be 1.6.

2.2. System Performance at Nasmyth (Nas.) Focus

For the imaging quality of SYZ design, Figure 2 shows the spot diagrams at different field positions. The fraction of encircled energy (EE) as a function of image spot radius is shown in Figure 4. From these results, we can see that the SYZ system has an exceptional geometrical image quality: the diameter of 80% encircled energy \( \text{EE}_{80} \) less than 0.002 for the full FOV. Besides, the curvature radius of the Nasmyth focal surface of SYZ system is about 8.85 m, which is relatively flat and thus easier for implementation of instruments with large FOV. These results are all consistent with that reported in Su et al. (2016). However, it is well-known that the geometrical image spot size cannot fully reflect the performance of a diffraction-limited optical system since the diffraction effect will dominate the final image quality. For

![Figure 1.](image.png)

Figure 1. Nasmyth system of 12 m SYZ telescope (M1: primary mirror; M2: secondary mirror; M3: SYZ relay mirror; M4: flat fold mirror). Different colors represent the different field radii. The field radius is defined as the angle between incident light and telescope optical axis. (A color version of this figure is available in the online journal.)
the wavelength of 0.55 μm, the Airy disk’s full width at half-
maximum (FWHM) diameter for diffraction-limited condition
is specified as (Airy 1835)

\[ \theta_{\text{Airy}} = 1.22 \frac{\lambda_{0.55}}{D}, \]  \hspace{1cm} (1)

where \( \lambda_{0.55} \) is the wavelength at 550 nm, \( D \) is the effective
entrance aperture diameter of the optical system. The ideal Airy
disk size of the LOT optical system with an aperture of 12 m is
0′0115 by Equation (1). A linear relation between EE80 and
FWHM: EE80 = 1.6 × FWHM. Thus, the EE80 diameter of
ideal Airy disk for LOT telescope is 0′018, which is much
larger than the geometric image spot size. For the diffraction-
limited scenario, diffraction dominates the
final image quality,
and the actual image spot size can be approximately derived by

\[ \theta_{\text{Spot}} = \sqrt{\theta_{\text{Airy}}^2 + \theta_{\text{Geo}}^2}, \]  \hspace{1cm} (2)

Since \( \theta_{\text{Geo}} \) is ignorable (less than 0′002 in full FOV) in the
SYZ design, the size of SYZ spot size in the diffraction-limited
scenario is close to that of the Airy disk.
2.3. The Vignetting and Imaging Quality due to M4’s Central Hole

To further evaluate SYZ telescope, we used ZEMAX to take a quantitative evaluation of image spots’ EE distribution for SYZ systems with different central obscuration (CO) ratios. We find that the CO ratio is defined as the relative linear size of the M4’s central hole to the M4’s linear size. Actually, the size of M4’s central hole plays a key role for optical system’s performance of SYZ designs: on one hand, it degrades the image quality due to its diffraction effect; on the other hand, vignetting effect caused by the central hole occurs when FOV increases. As we have stated above, if we need unvignetted full FOV of 14′, then the relative size of the central hole to the size of M4 (denoted as \(\epsilon\)) must be equal to or larger than 0.35 (see Figure 3). The relative size of M4’s hole is 0.5 corresponding to an unvignetted FOV of 20′. Thus, the CO ratio \(\epsilon\) determines the FOV at Nasmyth focus. But one should note that the light loss will be larger with the increasing of M4’s hole size. Since the M4 location is closely conjugate to the position of the M2 position, the light coming from M3 nearly uniformly illuminates M4. Thus, the light loss due to M4’s hole is equal to \(\epsilon^2\).

Further, M4’s hole will bring a imaging quality degradation (Figure 4). Due to the diffraction effect of M4’s central hole, the image quality of central FOV (within 5′ for \(\epsilon = 0.35\); within 7′ for \(\epsilon = 0.5\)) for SYZ designs is not as good as that for the two-mirror designs. We present the EE as a function of image spot radius in Figure 4: Figure 4(a) showing EE versus radius with \(\epsilon = 0.35\), corresponding to a FOV of 14′, and Figure 4(b) showing EE versus radius with \(\epsilon = 0.5\), corresponding to a FOV of 20′. The 80% enclosed energy (EE80) is about 0.036 when the relative size of M4’s central hole is \(\epsilon = 0.35\); and the EE80 is about 0.055 when \(\epsilon = 0.5\).

Based on our calculations in Appendix A for the obscured diffraction pattern (e.g., Airy 1835; Sacek 2006), the EE80 diameter of image spot with CO of 0.5 (0.25 for the area obscuration) is more than twice of the Airy disk size, consistent with our previous ZEMAX simulation results. We conclude that the SYZ’s on-axis resolution is relatively lower than the two mirror systems. Further, the larger size of M4’s hole for the SYZ system will result in a worse on-axis image quality. In next section, we will provide more discussions about the effect of M4’s central hole such as central obstruction, FOV vignetting, and extra diffraction degradation.

2.4. SYZ System Performance at Cassegrain (Cas.) Focus

SYZ is optimally designed for Nasmyth focus, and thus the image quality at Cassegrain focus is relatively worse because of lacking symmetry between the two foci. In the current SYZ design (Su et al. 2016), the EE80 diameter of image spot in the central FOV is about 1.87′, which can be reduced to 1.0′ with further optimization at Cassegrain focus by increasing the telescope tube length (equivalently increasing the Cassegrain focal ratio). Considering the image quality issue as well as other instrumentation problems (such as location of this focus), the SYZ design will be difficult to be compatible with Cassegrain focus. However, Cas. focus is important for some two-mirror telescopes because of its superb throughput and symmetric optics. For example, Keck telescope supports LRIS, ESI and MOSFIRE (Oke et al. 1995; Sheinis et al. 2002; McLean et al. 2012) at Cas. focus. Lacking a good Cas. focus may be a potential drawback for SYZ design.
3. RC Design and Performance

As we have stated above, the most widely used designs for large telescopes are two-mirror designs, such as RC and AG systems. Compared to a three-mirror system, a two-mirror telescope has simpler structure which probably yields less construction cost. In this section, we design a Ritchey–Chrétien telescope. The design parameters are listed in Table 2 and the 2D layout of the RC telescope is shown in Figure 5. We set

![Figure 4](image-url)

**Figure 4.** Fraction of encircled energy as a function of image spot radius in the diffraction limited scenario: (a) central obscuration ratio \( \varepsilon = 0.35 \); (b) central obscuration ratio \( \varepsilon = 0.5 \). The different colors represent the different field radius. The field radius is defined as the angle between the incident light and the telescope axis.

(A color version of this figure is available in the online journal.)

| f-ratio | Element | Curvature Radius (mm) | Thickness (mm) | Aperture Diameter (mm) | Conic |
|---------|---------|-----------------------|----------------|------------------------|-------|
| \( f/15 \) | M1 | \(-38400\) | \(-1.696E+4\) | \(12000\) | \(-1.0030048\) |
| | M2 | \(-5014.97\) | \(12000\) | \(1540\) | \(-1.5707753\) |
| | M3 | \(\infty\) | \(9000\) | \(1760 \times 1245\) | \(-\) |
| | Image | \(2860.89\) | \(\ldots\) | \(1046\) | \(-\) |
| \( f/12.8 \) | M1 | \(-38400\) | \(-1.645E+4\) | \(12000\) | \(-1.0051125\) |
| | M2 | \(-5014.97\) | \(12000\) | \(1540\) | \(-1.7064614\) |
| | M3 | \(\infty\) | \(9000\) | \(1740 \times 1230\) | \(-\) |
| | Image | \(2641.18\) | \(\ldots\) | \(1045\) | \(-\) |
both Nasmyth focus and Cassegrain focus at $f/12.8$ and thus the effective focal length of the system is 153.595 m. The $f$-number of the primary mirror is set as 1.60, which is the same as current SYZ design. The maximum FOV of RC design is set to be 20′.

In Figure 6, we present the fractional EE distribution of image spots with diffraction effect for the RC design at $f/12.8$. Obviously, image spots at different field positions have very different sizes, which, however, are relatively uniform for the SYZ telescope design. The diameters of image spots with
diffraction (EE80) for RC design vary from 0\(^\circ\)0191 (central field position) to 0\(^\circ\)3913 (marginal field position). In addition, the curvature radius of the focal surface for RC design is estimated to be 2.86 m, which is less flat compared to that of the SYZ telescope.

### 4. Comparison of System Performance between the Syz Design and RC Design

Compared to two-mirror telescope systems, the SYZ telescope has relatively small field curvature and excellent geometrical image quality for full FOV. However, the SYZ system has a more complex telescope structure, higher central obstruction ratio, higher reflection loss introduced by an additional mirror, and worse on-axis image quality caused by annular pupil diffraction effect due to the central obstruction. In the following sections, we will provide a systematic comparison between RC design and SYZ design. We will focus on the diffraction image quality, system throughputs, and the actual scientific performance. We will apply several telescope evaluation criteria, such as EE80, central intensity ratio (CIR), ENA, and observing information throughput (OIT) to evaluate the performance of various telescopes.

#### 4.1. Diffraction Image Quality

Again, we use EE80 diameter in diffraction limited scenario to evaluate the image qualities of different designs. In Table 3, we show the comparison results of image spot sizes at Nasmyth focus for different telescope design, where \(\epsilon\) specifies the relative size of CO of M4 in SYZ design. From Table 3, we can also see that the imaging quality of \(f/12.8\) RC design has a imaging quality similar to but slightly higher \((\leq 5\%)\) than that of the \(f/15\) RC design.

From the comparison results and from Section 2.3, the size of M4’s central hole plays a key role for optical system’s performance of SYZ designs: on one hand, it degrades the image quality due to its diffraction effect; on the other hand, vignetting effect caused by the central hole occurs when FOV increases. As we have stated above, if we need unvignetted full FOV of 14’, then the relative size of the central hole of M4 must be larger than 0.35. The relative size of M4’s hole is 0.5 for an unvignetted FOV of 20’. However, the larger the central hole of M4, the more degradation it will bring to the image quality. Due to the diffraction effect of M4’s central hole, the image quality of central FOV (within 5’ for \(\epsilon = 0.35\); within 7’ for \(\epsilon = 0.5\)) for SYZ designs is not as good as that for the two-mirror designs. However, we can reach diffraction-limited observation with the help of AO technologies such as extreme adaptive optics (ExAO) and multi-conjugate adaptive optics (MCAO), whose FOVs are currently restricted to less than 1’ (Rigaut et al. 2000; Macintosh et al. 2006). For GLAO, although its FOV may reach 10’ (the maximum GLAO FOV is current 4’ at LBT), it can only correct the ground-layer turbulence. At Mauna Kea, the theoretical predictions indicate that GLAO can improve the FWHM by a factor of \(\sim 2\) (Andersen et al. 2006; Hart et al. 2010). All other telescope sites are expected to have GLAO performance worse than Mauna Kea (e.g., Figure 1 in Schock et al. 2009). Even we assume the superb \(\sim 0.2\) FWHM GLAO can be achieved by the LOT telescope over a 10’ FOV, such an excellent FWHM is still much larger than the aberration at the field edge of 10’ FOV, which is 0’0.09. Thus, we conclude as follows. First, the image quality of the SYZ telescope design is worse than that of the RC telescope design at the central field, especially within central FOV of 1’. Second, at the edge of the large FOV (e.g., 10’), the FWHM of point sources are dominant by the seeing condition, in regardless of the RC or SYZ telescope designs.

#### 4.2. Throughput Analysis

To maximize the throughputs of the telescope optical systems, we need to apply reflective metal coating to all mirror surfaces to increase the reflectivity. Generally, aluminum, silver and gold are the three most common metal coatings. Aluminum is the most popular coating material for most of the astronomical observatories. Aluminum is reflective over the full range of wavelengths from 300 nm to 25 \(\mu\)m. However, the reflectivity of aluminum coating in the 300–1000 nm range is only about 90%. Silver is a better coating choice especially for wavelength that is longer than 340 nm. Nevertheless, it tarnishes fast, so that its reflectivity drops rapidly if coated naked. Proper protective coatings are required. The throughput of the gold coating is dropped in the green and blue wavelengths.

For Aluminum coating, we assume that its lifetime is 2 years. The reflectivity of aluminum coating in the optical wavelength range will degrade from 90% to about 87% after 1 year, and to 84% after two years (Magrath 1997). For simplicity, we take the reflectivity of aluminum coating for the full optical wavelength range as 87% during a 2 year lifetime. However,
if we do not consider the ultraviolet band and adopt the enhanced silver coating, then the average reflectivity in optical wavelength range can reach as high as 95% (Vucina et al. 2006). That means if we use Al coating, then an extra mirror of SYZ telescope yields 13% extra light loss. If one adopts enhanced silver coating, then an extra mirror of SYZ yields 5% extra light loss.

As stated above (Section 2.3), another important light loss is from the CO effect caused by the optical elements along the optical path (see Figure 3(b) and Section 2.3). In this paragraph, we quantitatively evaluate this effect. For the SYZ design, secondary mirror, central hole of primary mirror, and central hole of M4 will obscure light near the optical axis. Moreover, the obscuration ratio for the three elements are 0.212, 0.117, and 0.35 (FOV = 15°) or 0.50 (FOV = 20°) respectively. Typically, the central hole of M4 in either design dominates the CO effect along the optical path for Nasmyth focus of SYZ design. For RC design, only the secondary mirror has a central obstruction effect along the optical path. Thus, considering reflection loss and central obstruction effect, the final throughput of all designs can be denoted by

$$\eta = r_C^N \cdot (1 - e_M^2),$$

(3)

where $r_C$ is the reflectivity of mirror coating, $N$ is the number of mirrors along the optical path, and $e_M$ is the central obstruction ratio of telescope system, which is generally decided by the element with maximum CO along the optical path. Based on Equation (3), the total throughputs of all designs can be concluded in Table 4. From both Tables 3 and 4, we know that the $f/12.8$ RC and $f/15$ RC designs (Keck $f/ratio$) have very similar imaging quality and CO. Thus, in the later discussion, we use $f/12.8$ RC which has the same $f/ratio$ with the SYZ design.

4.3. Central Intensity Ratio (CIR) Comparison

We have compared the optical performance of RC design and SYZ design under seeing limited observation and GLAO-corrected observations. Most large telescopes nowadays aim to achieve diffraction-limited performance up to a FOV of a few tens of arcseconds with extreme adaptive optics (ExAO). In this case, we generally use CIR to evaluate the telescope performance. The CIR is to evaluate the adaptive optics (AO) performance, because it combines the central intensities of PSF in different Strehl ratios (SRs; Dierickx 1992).

In the RC or SYZ design, the SR degradation of PSF at the center of FOV is largely dependent on the diffraction effect of central obstruction. Based on our derivation of CO diffraction pattern in Appendix A, one can see that SR of a telescope with CO ratio of $\epsilon$ can be estimated as $(1 - \epsilon^2)^2$. Considering the reflection loss, telescope’s CIR at the center of FOV under ExAO can be expressed by

$$\text{CIR} = r_C^N \cdot (1 - e_M^2)^2.$$

(4)

Based on Equation (4), comparison of RC design and SYZ design under ExAO corrected observations can be summarized in Table 5. Following the comparison results, one can see that RC design generally has a higher CIR and thus has a better ExAO performance.

4.4. OIT Evaluation

In this section, we introduce a new evaluation criterion, the OIT, to evaluate telescope’s observing capability for full FOV. OIT is a quantity that is directly related to the scientific productivity. This is a dimensionless quantity that combining imaging quality (ENA), throughput and the field of view. These three parameters mainly determine the speed of the observations given the same signal-to-noise ratio. Later in this section, we can see that the integration of $\eta/\text{ENA}$ over the field of view is directly proportional to 1/exposure time (see Equations (7) and (8) later in this section). Before doing this, we first introduce another general telescope’s performance metric, i.e., ENA. ENA has been first proposed in King (1983), and further adopted as a crucial performance metric for large telescopes (Nelson & Sanders 2008; Angeli et al. 2011). The ENA has a physical meaning of the smallest aperture that can be used to extract all the information of a faint source, which can be

| Table 4 | The Throughput of SYZ Designs and RC Designs |
|---------|---------------------------------------------|
| Design  | Central Obscuration | Total Throughput |
|         |                  | New Al | Two-year Al | Enhanced Ag |
| SYZ($\epsilon = 0.35$) | 0.35 | 57.6% | 50.3% | 71.5% |
| SYZ($\epsilon = 0.50$) | 0.50 | 49.2% | 43.0% | 61.1% |
| RC(Nas. $f/12.8$) | 0.14 | 71.7% | 64.8% | 84.3% |
| RC(Cas. $f/12.8$) | 0.14 | 79.6% | 74.4% | 88.7% |
| RC(Nas. $f/15$) | 0.13 | 71.8% | 64.9% | 84.4% |
| RC(Cas. $f/15$) | 0.13 | 79.7% | 74.5% | 88.8% |

| Table 5 | The Central Intensity Ratio (CIR) of SYZ Design and RC Design Under ExAO |
|---------|---------------------------------|
| Design  | Central Intensity Ratio |
|         | New Al | Two-year Al | Enhanced Ag |
| SYZ($\epsilon = 0.35$) | 0.51 | 0.44 | 0.63 |
| SYZ($\epsilon = 0.50$) | 0.37 | 0.32 | 0.46 |
| RC(Nas. focus) | 0.70 | 0.64 | 0.83 |
| RC(Cas. focus) | 0.78 | 0.73 | 0.87 |
defined as
\[
ENA = \frac{1}{\iint \phi^2 dx dy},
\]  
(5)

where \(\phi\) is the point-spread function (PSF). Detailed derivations of ENA can be found in Appendix B. Typically, ENA is the inverse of the PSS, which is another telescope performance metric proposed by Seo et al. (2009). Assuming a source with \(E\) as the total photons per second, we can define the SNR for the observation of the source as
\[
\frac{S}{N} = \frac{E \cdot t \cdot \eta}{\sqrt{E \cdot t \cdot \eta + B \cdot t \cdot ENA \cdot \eta}},
\]  
(6)

where \(\eta\) is the throughput of the telescope, \(E\) is the stellar intensity in photons, and \(B\) is the observed background sky level in photons. For the background limited case, the exposure time can be represented by
\[
t = \frac{ENA \cdot S \cdot B}{\eta \cdot N \cdot E^2}.
\]  
(7)

Based on Equation (7), we can see that the integration time is proportional to ENA for observing faint sources. As long as the total energy of the source (\(E\)), and background (\(B\)) is fixed, the integration time is proportional to the ENA/\(\eta\) given the same \(S/N\). Thus, \(\eta/ENA\) can be used as a direct parameter to evaluate the science productivity given a FOV and a given exposure time. In addition, we consider that the survey speed is proportional to FOV area of telescope. Thus, we suggest the integration of \(\eta/ENA\) over the whole FOV of telescope as an independent, dimensionless criterion for any telescope design. We call this new criterion as OIT, which can be defined as
\[
OIT = \int_0^{2\pi} \int_0^{FOV/2} \frac{\eta}{ENA} r dr d\theta,
\]  
(8)

where \(r\) is the field radius of telescope measured by radian (or arcminutes), and ENA is a quantity to evaluate the image quality on the focal surface of telescope. OIT directly measures the total science output of telescopes with a given exposure time.

The total ENA of the telescope can be approximated by
\[
ENA = ENA_{Seeing} + ENA_{Astig} + ENA_{Error},
\]  
(9)

where \(ENA_{Seeing}\) is the ENA caused by atmosphere seeing, \(ENA_{Astig}\) is the ENA caused by astigmatism of telescope design, and \(ENA_{Error}\) is the ENA caused by the error due to the telescope instrumentation. Based on our definition of ENA, the value of \(ENA_{Seeing}\) for atmosphere seeing of 0.7 arcmin is about 1.20 arcsec\(^2\) assuming PSF is a Gaussian profile. Note the Gaussian approximation of the PSF results in only a very small (<1%) difference in the ENA calculation compared to that calculated using the realistic PSF profile (Moffat profile). Our numerical calculation has proved that the ENA is about 1.21 arcsec\(^2\) if we use a Moffat PSF profile. Since it is hard to quantify the instrumentation error, we assume that we can achieve a perfect instrumentation of telescope for all designs, i.e., \(ENA_{Error} = 0\). The SYZ design is diffraction-limited design for whole FOV, so its \(ENA_{Astig}\) can be taken as zero too. While for the two-mirror design, \(ENA_{Astig}\) can be calculated as follows:
\[
ENA_{Astig} = \kappa \cdot AAST^2,
\]  
(10)

where \(\kappa\) is a constant with value of \(\pi/4\) that can be derived based on the PSF caused by astigmatism and the definition of ENA, and \(AAST\) is the value of astigmatism for RC design at a given field radius of \(r\), which can be expressed by
\[
AAST = AAST_{15} \cdot \left(\frac{2r}{15}\right)^2,
\]  
(11)

where \(AAST_{15}\) is the astigmatism of RC design at field diameter of 15' and can be estimated as 0.0243 based on our simulation result in ZEMAX. Thus, through a simple calculation, over a field diameter of 15', the total ENA is dominated by atmosphere seeing rather than telescope design aberrations, which once again demonstrates our previous conclusion in Section 4.1.

After figuring out the ENAs of all telescope designs proposed above, we can achieve related OITs for these telescope configurations with different coatings separately. The results are as shown in Table 6. Based on the OIT results, we can see that SYZ designs have a lower OIT value compared to RC design at both Nasmyth focus and Cassegrain focus.

To make a systematic comparison between RC design and SYZ design in any given FOV, we further quantify the \(OITs\) of all optical systems designed with a wider range of FOVs. For the RC design, the central obstruction ratio caused by the secondary mirror for zero FOV is 0.117, which is 0.128 for 20' FOV. There is a linear relationship between the required size of secondary mirror and the designed maximum FOV, and the central obstruction ratio caused by secondary mirror in a RC system designed for a given FOV can be derived by:
\[
\epsilon_{M2-RC} = 0.117 + 0.011 \cdot \frac{FOV}{20'},
\]  
(12)

While for the SYZ design, the CO ratio caused by the secondary mirror for zero FOV is 0.205, which is 0.213 for 20' FOV; the CO ratio caused by M4 for a given FOV is proportional to the size of M4's central hole, which is also directly proportional to the designed FOV of SYZ telescope. Thus, the CO of the SYZ system at a given FOV can follow the
By plugging Equations (12) and (13) into Equation (3) separately, we can estimate the total throughputs for both designs and further calculate OITs of all telescopes with the enhanced silver coating adopted, (as shown in Figures 7(a) and (b)). As seen from this figure, we conclude that the SYZ telescope potentially has a less scientific productivity compared to the two-mirror telescope designed for any given FOV. Based on the OIT curve for the RC design as shown in Figure 7(b), OIT is increasing with FOV. However, for the SYZ system, although the ENA of SYZ is smaller than RC system over a large FOV, the SYZ’s OIT will decrease with the increase of FOV after 30′, because of a very significant loss of throughput due to M4’s central hole.

5. Conclusion

In this paper, we systematically compare the scientific performance of telescopes between the two-mirror (RC) design and the SYZ design. Based on our simulation, although SYZ design has a better imaging quality over a large FOV and more flat field curvature, the RC performs better performance under the Ex-AO observations, because RC telescope can have better diffraction-limited image quality and higher photon throughput. Also, we have found that the SYZ design does not work as well as RC design under the seeing-limited observations, where both
designs have almost same performance of image quality due to
the atmosphere effect, but RC design can have much higher
photon throughput compared to SYZ design. Furthermore, we
propose a new performance evaluation criterion, i.e., the OIT to
evaluate the scientific productivity of telescopes. Using this
OIT metric, we compare the two-mirror design with the SYZ
design. The RC telescope has a better scientific performance
compared to the innovative SYZ design. In future, we will
study whether an updated SYZ telescope design can be
improved, especially in the telescope throughput. For example,
we will study whether a slightly off-axis M4 can reduce the
central obstruction; the error budget, monetary budget analysis
of the SYZ design compared to that of RC design. All these
can guide us to further investigate whether the LOT or future
telescope project should adopt an SYZ design.

We acknowledge the anonymous referee for a careful
reading of the article and insightful comments that led to a
great improvement of this manuscript. We acknowledge
valuable discussions from Jerry Nelson and Sandra Faber.
We also acknowledge the great helps, supports and encourage-
ments from Jiansheng Chen, Suijian Xue, Luis Ho, Lei Hao, Lu
Feng. Finally, the authors dedicate this work to the memory of
Jerry Nelson, without whom this work would not be possible.

This work is supported by Wuhan Science and Technology
Bureau (Wuhan Intellectual Property Bureau) under grant
number 2017010201010110. This work is supported by Huazhong
University of Science and Technology under grant number 2017KFYXJJ026. Support for part of this work was
also provided by NASA through the Hubble Fellowship grant
HST-HF2-51370 awarded by the Space Telescope Science
Institute, which is operated by the Association of Universities
for Research in Astronomy, Inc., for NASA, under contract
NAS 5-26555.

Appendix A
Central Obstruction (CO) Diffraction

In the SYZ telescope design, there are several central
obstruction apertures along the optical path of SYZ telescope as
shown in Figure 8, which include central obstruction caused by
secondary mirror, central hole of primary mirror and central
hole of fold mirror M4. In this appendix, we try to explore the
diffraction patterns of the annular pupil with various CO ratios,
and analyze their effect on final image quality of optical
systems.

Any obstruction placed in the light path of an imaging system
will block a portion of the wavefront reaching the final focal
surface. The consequence is a change in wave contribution at every
point of the diffraction pattern, leading to a so-called obscured
Airy pattern. The new intensity distribution of the obscured Airy
pattern can be described by Airy (1835), Sacek (2006) (also see

\[
I(\theta) = \frac{I_0}{(1 - \epsilon^2)^2} \left( \frac{2J_0(x)}{x} - \frac{2\epsilon J_1(\epsilon x)}{x} \right)^2, \tag{14}
\]

where \( \epsilon \) is the annular aperture linear obscuration ratio, \( I_0 \) is the
maximum intensity of the pattern at the Airy disc center with
no obscuration, \( J_1 \) is the Bessel function of the first kind of
order 1, and \( x \) is defined as:

\[
x = ka \sin \theta \approx \frac{\pi R}{\lambda f/#}, \tag{15}
\]

where \( R \) is radial distance in the focal plane from the optical
axis, \( \lambda \) is the wavelength, \( f/# \) is the f-number of the system.
In Equation (14), the central intensity with a given CO ratio of
\( \epsilon \) is still normalized to \( I_0 \) by dividing a normalization factor of
\((1-\epsilon^2)^2\). As a result, the SR of an optical system’s PSF with CO
ratio of \( \epsilon \) can be written as

\[
SR = (1 - \epsilon^2)^2. \tag{16}
\]

Then by integrating Equation (14) without the normalization
coefficient over image spot radius, the fraction of EE is provided by

\[
E(R) = \frac{1}{1 - \epsilon^2} \left[ 1 - J^2_0(x) - J^2_1(x) + \epsilon^2 \right. \\
\left. \times \left[ 1 - J^2_0(\epsilon x) - J^2_1(\epsilon x) \right] \\
- 4\epsilon \int_0^\infty J_0(t)J_1(\epsilon t)dt \right]. \tag{17}
\]

Therefore, based on Equation (17), we can get the diffraction
pattern of central obstruction and the corresponding PSF and
EE distribution curve as shown in Figure 9. Due to the
obstruction effect, the 80% encircled energy (EE80) diameter
of the spot with relative central obstruction of 0.5 is almost
twice of the Airy disk size.

\[ \text{Figure 8. Central obstruction (CO): } D \text{ = aperture diameter; } \epsilon = \text{relative size of CO in units of } D. \]

(A color version of this figure is available in the online journal.)

http://www.telescope-optics.net-obstruction.html)
Appendix B
Derivation of Equivalent Noise Area (ENA)

Our derivation of ENA largely follows the procedures in King (1983), and Mighell (2003, 2005). Consider a CCD photometry of point sources. Assuming that we know the PSF of the observations, then a simple model of the observation can be written using the following parameters: stellar intensities ($E_i$) in photons, the coordinate of the point-source positions ($x, y$) in pixels, and observed background sky level in photons ($B$). If there are not only one star, and further, if some stars are overlapping, then the parameters of each star are dependent variables. The reasonable model of multiple overlapping PSFs will be nonlinear. With the nonlinear least-square fitting algorithm, the nonlinear function can simultaneously determine any dependent or independent parameters in the nonlinear model functions (Mighell 2003).

Following Mighell (2005), assuming that we have a CCD images with $N$ pixels and that $n_i$ is the number of photons recorded in the $i_{th}$ pixel. The $i_{th}$ pixel resides in the position of $(x_i, y_i)$ of this CCD, and this pixel has an error of $\sigma_i$ photons. Further, let us denote $m(x, y; n_1, ..., n_M)$ to be an observational model of the pixel values in CCD that has the coordinate $(x, y)$. Let us denote vector $p$ to represent all the model parameters $[p \equiv (p_1, ..., p_M)]$. The observational model of $i_{th}$ pixel can be written as $m_i \equiv m(x, y; p)$.

We use $\chi^2$ to quantitatively measure the goodness of the fit between a nonlinear model and data, in which $\chi^2$ can be expressed as follows:

$$\chi^2(p) = \sum_{i=1}^{N} \frac{1}{\sigma_i^2}(m_i - E_{i})^2. \quad (18)$$

Assuming $p_0$ is the optimal parameter vector, let us consider a parameter $p_i$, and then the standard errors according to the nonlinear least-square fitting is

$$\delta_i = \left(\sum_{i=1}^{N} \frac{1}{\sigma_i^2} \frac{\partial m_i}{\partial p_i}\right)^{-1/2}. \quad (19)$$

The observational model for $i_{th}$ pixel would be

$$N_i = B + E\phi_i, \quad (20)$$

where $E$ is the total number of photons received from the source, and $\phi_i$ is the value of $i_{th}$ pixel of the normalized PSF. For faint limit, the error of $E$ can be expressed as follows based on Equation $(19)$:

$$\sigma_E^2 = \left(\sum_{i=1}^{N} \frac{1}{\sigma_{rms}^2} \frac{\partial E}{\partial \phi_i}\right)^{-1}. \quad (21)$$

Consider the large array of CCD and the discrete CCD array can be approximated as continuous, then Equation $(19)$ can be written as follows:

$$\sigma^2 = \sigma_{rms}^2 \int \phi^2 dxdy. \quad (22)$$

Thus, to get a complete information of the photon from the source ($E$), we need to figure out $\int \phi^2 dxdy$. Thus, the $\int \phi^2 dxdy$ is defined as Equivalent-Noise-Area (ENA).

References

Airy, G. B. 1835, TCaPS, 5, 283
Andersen, D. R., Stoesz, J., Morris, S., et al. 2006, PASP, 118, 1574
Angeli, G. Z., Seo, B., Nissly, C., & Troy, M. 2011, Proc. SPIE, 8127, 812709
Dierickx, P. 1992, JMOs, 39, 569
Hart, M., Milton, N. M., Baranec, C., et al. 2010, Natur, 466, 727
Hill, J. M., & Salinari, P. 2003, Proc. SPIE, 4837, 140
Johns, M. 2008, Proc. SPIE, 6986, 698603
Kaifu, N. 1998, Proc. SPIE, 3352, 14
King, I. R. 1983, PASP, 95, 163
Macintosh, B., et al. 2006, Proc. SPIE, 6272, 62729N
Magrath, B. 1997, PASP, 109, 303
McLean, I. S., et al. 2012, Proc. SPIE, 8446, 84460J
Mighell, K. 2003, in ASP Conf. Ser. 295, Astronomical Data Analysis Software and Systems XII, ed. H. E. Payne, R. I. Jedrzejewski, & R. N. Hook (San Francisco, CA: ASP), 395
Mighell, K. 2005, MNRAS, 361, 861
Nelson, J. E. 2000, Proc. SPIE, 4004, 282
Nelson, J. E., Mast, T. S., & Faber, S. M. 1985, Keck Observatory Report No. 90
Nelson, J. E., & Sanders, G. 2008, Proc. SPIE, 7012, 70121A
Oke, J. B., Cohen, J. G., Carr, M., et al. 1995, PASP, 107, 375
Rigaut, F. J., Ellerbroek, B. L., & Flicker, R. 2000, Proc. SPIE, 4007, 432
Sacek, V. 2006, Notes on Amateur Telescope Optics, Web-published at website: http://www.telescope-optics.net/
Schöck, M., Els, S., Riddle, R., et al. 2009, PASP, 121, 384
Seo, B. J., Nissly, C., Angeli, G., et al. 2009, ApOpt, 48, 5997
Sheinis, A. I., Bolte, M., Epps, H. W., et al. 2002, PASP, 114, 851
Su, D., Liang, M., Yuan, X., Bai, H., & Cui, X. 2016, MNRAS, 460, 2286
Vucina, T., Boccas, M., Araya, C., & Ahhee, C. 2006, Proc. SPIE, 6273, 62730W