Can we constrain the evolution of linear bias using configuration entropy?

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ABSTRACT

We study the evolution of the configuration entropy of a biased tracer in the flat ΛCDM model assuming different time evolution of linear bias. We describe the time evolution of linear bias using a simple form $b(a) = b_0 a^n$ with different index $n$. The derivative of the configuration entropy rate is known to exhibit a peak at the scale factor corresponding to the Λ-matter equality in the unbiased ΛCDM model. We show that in the ΛCDM model with time-dependent linear bias, the peak shifts to smaller scale factors for negative values of $n$. This is related to the fact that the growth of structures in the tracer density field can significantly slow down even before the onset of Λ domination in presence of a strong time evolution of the linear bias. We find that the shift is linearly related to the index $n$. We obtain the best fit relation between these two parameters and propose that identifying the location of this peak from observations would allow us to constrain the time evolution of linear bias within the framework of the ΛCDM model.

Key words: methods: analytical - cosmology: theory - large scale structure of the Universe

1 INTRODUCTION

Our knowledge of the present day galaxy distribution in the nearby Universe has been revolutionized by the modern galaxy survey (SDSS, York et al. 2000; 2dFGRS, Colles et al. 2001; 2MRS, Huchra et al. 2012) carried out over the last few decades. However cosmological observations suggest that most of the mass in the Universe is in the form of an unseen dark matter which is yet to be directly detected by observations. The galaxies are known to be a biased tracer of the underlying dark matter distribution. On large scales, it is believed that the fluctuations in the galaxy distribution and the dark matter distribution are linearly related by a bias parameter (Kaiser 1984; Dekel & Rees 1987). The linear bias parameter is known to be scale-independent on large scales (Mann, Peacock & Heavens 1998) but is expected to evolve with time (Fry 1996; Tegmark & Peebles 1998). The time evolution of the linear bias parameter determines the evolution of the large scale distribution of the tracer relative to the underlying mass distribution. However the galaxies have not always been in place. They are the product of the non-linear evolution of the cosmic density field. Thanks to the improvement of computing power and algorithms, modern day N-body simulations (Springel et al. 2005; Vogelsberger et al. 2014) can give us a clear idea about the emergence of structures through non-linear evolution. In fact, the understanding of the process of structure formation has become so good that it has become a standard tool for testing cosmological models.

Early measurements of the two point correlation function for galaxies and galaxy clusters did not match, indicating that both cannot be unbiased tracers of the underlying matter distribution. Kaiser (1984) pointed out that the density field of clusters are the peaks in the galaxy density field and therefore they are more strongly clustered than the galaxies. The relative clustering strength of any tracer with respect to the dark matter distribution is quantified through the linear bias parameter measured from observations. One can employ the two-point correlation function and power spectrum to determine the linear bias parameter (Norberg et al. 2001; Tegmark et al. 2004; Zehavi et al. 2011). The redshift space distortions of the two-point correlation function and power spectrum (Kaiser 1987; Hamilton 1992) can be also employed to measure the linear bias parameter (Hawkins et al. 2003; Tegmark et al. 2004). The other alternatives which have been successfully used to compute the linear bias parameter are the three-point correlation function and bispectrum (Feldman et al. 2001; Verde et al. 2002; Gaztañaga et al. 2005), filamentar-
ity (Pandey & Bharadwaj 2007) and information entropy (Pandey 2017).

Galaxies do not exist at high redshift whereas the neutral Hydrogen (HI) is present throughout the history of the Universe since its formation after the recombination at \( z \sim 1100 \). The redshifted 21 cm line from neutral Hydrogen would reveal a wealth of information about the formation and evolution of structures in the Universe. A number of surveys (HIPASS, Zwaan, et al. 2005; ALFALFA, Martin, et al. 2012) have been designed to map the HI content of galaxies in the nearby Universe. A significant effort has been also directed to detect the redshifted 21 cm signal using different ongoing and upcoming radio interferometric facilities (GMRT, Paciga, et al. 2013; LOFAR, van Haarlem, et al. 2013; MWA Bowman, et al. 2013; SKA, Mellema, et al. 2013). The redshifted 21 cm line can be used as a promising probe of the large scale structures over a wide redshift range (Bharadwaj, Nath & Sethi 2001; Bharadwaj & Sethi 2001). The knowledge about the HI bias and its time evolution is also important in understanding the uncertainties associated with the measured intensity fluctuation power spectrum. Several studies have been carried out to measure the HI bias (Martin, et al. 2012; Masui, et al. 2013; Switzer, et al. 2013) at low redshifts \((z < 1)\) but presently the evolution of HI bias with redshift is not known. Some theoretical and observational constraints on the evolution of HI bias over the redshift range \( 0 – 3.5 \) is summarized in Padmanabhan, Choudhury & Refregier 2015 and references therein.

Recently, it has been suggested that the measurement of the configuration entropy (Pandey 2017, 2019) of the mass distribution in the Universe can be used to test the different cosmological models (Das & Pandey 2019a), determine the mass density parameter and cosmological constant (Pandey & Das 2019) and constrain the dark energy equation of state parameters (Das & Pandey 2019b). In the present work, we propose a theoretical framework based on the study of configuration entropy which would allow us to probe the evolution of linear bias from future redshifted 21 cm observations.

2 THEORY

2.1 Evolution of configuration entropy

We consider a large comoving volume \( V \) of the Universe and divide it into sub-volumes \( dV \). Let the density of the tracer in each of these sub-volumes at time \( t \) be \( \rho_T(\vec{x}, t) \) where \( \vec{x} \) is the comoving coordinate of the sub-volume defined with respect to an arbitrary origin. The configuration entropy of the matter density field can be defined as (Pandey 2017),

\[
S_t(\vec{x}, t) = - \int \rho_T(\vec{x}, t) \log \rho_T(\vec{x}, t) \ dV.
\]  (1)

The definition of configuration entropy is motivated from the definition of information entropy (Shannon 1948).

The mass distribution of the Universe is often treated as an ideal fluid to a good approximation. The continuity equation of this fluid in an expanding Universe can be written as,

\[
\frac{\partial \rho_T}{\partial t} + \frac{\dot{a}}{a} \rho_T + \frac{1}{a} \nabla \cdot (\rho_T \vec{v}_T) = 0.
\]  (2)

In Equation 2, \( a \) is the cosmological scale factor and \( \vec{v}_T \) is the peculiar velocity of the tracer fluid element. We can combine Equation 1 and Equation 2 to get,

\[
\frac{dS_t(t)}{dt} + 3 \frac{\dot{a}}{a} S_t(t) - \frac{1}{a} \int \rho_T(3a + \nabla \cdot \vec{v}_T) \ dV = 0.
\]  (3)

We rewrite Equation 3 as,

\[
\frac{dS_t(a)}{da} \dot{a} + 3 \left( \frac{\dot{a}}{a} - \frac{\dot{a}}{a} \right) S_t(a) - \frac{1}{a} \int \rho_T(\vec{x}, a) \ dV
\]

\[
- \frac{1}{a} \int \rho_T(\vec{x}, a) \nabla \cdot \vec{v}_T \ dV = 0,
\]  (4)

where the variable of differentiation has been changed from \( t \) to \( a \). Here \( \int \rho_T(\vec{x}, a) \ dV = M_T \) is the total mass of the tracer contained inside the comoving volume \( V \). The density of tracer at comoving location \( \vec{x} \) can be expressed as \( \rho_T(\vec{x}, a) = \bar{\rho}_T(1 + \delta_T(\vec{x}, a)) \), where \( \bar{\rho}_T(\vec{x}, a) \) is the density contrast at location \( \vec{x} \) and \( \delta_T = \frac{M_T}{\bar{\rho}_T(a) V} \) is the average density of tracer. In linear perturbation theory, one can write \( \delta_m(\vec{x}, a) = D(a) \delta_m(\vec{x}) \) and \( \nabla \cdot \vec{v}_T = -\frac{\partial \bar{\rho}_T}{\partial a} \vec{x} \). Here, \( D(a) \) is the growing mode and \( \delta_m(\vec{x}) \) is the initial mass density perturbation at location \( \vec{x} \). We simplify Equation 4 using these relations to get,

\[
\frac{dS_t(a)}{da} \dot{a} + 3 \left( \frac{\dot{a}}{a} - \frac{\dot{a}}{a} \right) S_t(a) - \frac{1}{a} \int \delta_T(\vec{x}, a) \nabla \cdot \vec{v}_T \ dV = 0.
\]  (5)

In the linear bias assumption,

\[
\delta_T(\vec{x}, a) = b(a) \delta_m(\vec{x}, a),
\]  (6)

where \( b(a) \) is the scale-independent linear bias parameter and \( \delta_m(\vec{x}, a) \) and \( \delta_m(\vec{x}) \) are the density contrast corresponding to the tracer and the underlying mass density field respectively. So,

\[
\nabla \cdot \vec{v} = -a \dot{a} \left[ D(a) \frac{db(a)}{da} + b(a) \frac{dD(a)}{da} \right] \delta_m(\vec{x}).
\]  (7)

We combine Equation 7 and Equation 5 and simplify to get,

\[
\frac{dS_t(a)}{da} + 3 \frac{\dot{a}}{a} (S_t(a) - M_T) + \bar{\rho}_T B(a) \int \delta_m^2(\vec{x}) \ dV = 0.
\]  (8)

Here, \( B(a) = b(a) D(a) [D(a) \frac{db(a)}{da} + b(a) \frac{dD(a)}{da}] \) where \( f(a) = \frac{a \frac{db(a)}{da}}{dD(a)/da} \) is the dimensionless linear growth rate. This equation governs the evolution of configuration entropy of the tracer in presence of time evolution of linear bias. One can integrate Equation 8 to get

\[
\frac{S_t(a)}{S_t(a_1)} = \frac{M_T}{S_t(a_1)} + \left[ 1 - \frac{M_T}{S_t(a_1)} \right] \left( \frac{a}{a_1} \right)^3
\]

\[- \bar{\rho}_T \int \delta_m^2(\vec{x}) \ dV \int_{a_1}^a da \ a^3 B(a').
\]  (9)

Here \( a_1 \) is some initial scale factor and \( S_t(a_1) \) is the initial configuration entropy. In our analysis we have chosen \( a_1 = 10^{-3} \).

We find the evolution of the ratio of configuration entropy to its initial value by numerically calculating the integral in the third term for different time evolution of bias and substituting back at Equation 9. We set the product \( \bar{\rho}_T \int \delta_m^2(\vec{x}) \ dV = 1 \) for simplicity. The choices of \( S_t(a_1) \) and \( \bar{\rho}_T \) are fixed by observational constraints.
Equation 9 shows the entropy evolution. The present value of the density parameter corresponding to cosine conditions. The integral in the third term of Equation 9 involves evolution of growing mode, time dependent bias and their derivatives which are discussed in the next two subsections.

2.2 Growth rate of density perturbations

The CMBR observations suggest that the Universe was highly isotropic at $z \sim 1100$. But the present day Universe is not homogeneous and isotropic on small scales. We find galaxies and clusters of galaxies where huge mass is accumulated over a small region whereas there are large empty regions or voids with very little amount of mass. The linear perturbation theory provides a theoretical framework to understand the growth of structures from tiny fluctuations seeded in a homogeneous and isotropic distribution in the early Universe. In the currently accepted paradigm, gravitational instability is the primary mechanism behind the formation of structures in the Universe. CMBR observations indicate that inhomogeneities of very small magnitude were present in the matter distribution at the time of recombination. These tiny inhomogeneities get amplified by the gravitational instability over time. When the density contrast is much smaller than 1, its evolution can be described by the following differential equation,

$$\frac{\partial^2 \delta_m(\vec{x}, t)}{\partial t^2} + 2H(a) \frac{\partial \delta_m(\vec{x}, t)}{\partial t} - \frac{3}{2} \Omega_m H_0^2 \frac{1}{a^3} \delta_m(\vec{x}, t) = 0. \quad (10)$$

Here we have considered perturbation to only component density. $\Omega_m$ and $H_0$ are the present value of density parameter for matter and Hubble parameter, respectively. This equation governs the growth of density perturbation in the underlying matter distribution. The equation has a growing mode solution of the form $\delta_m(\vec{x}, t) = D(t) \delta_m(\vec{x})$. The growing mode solution can be expressed as (Peebles 1980)

$$D(n) = \frac{5}{2} \Omega_{\Lambda 0} X^5(n) \int_0^n \frac{da}{a^3 X^5(a')} \left[1 - \frac{1}{5} \Omega_m(a)(1 + \Omega_m(a)) \right]. \quad (11)$$

where $X(a) = H(a)^2 / H_0^2 = [\Omega_m a^{-3} + \Omega_{\Lambda 0}]$. Here $\Omega_{\Lambda 0}$ is the present value of the density parameter corresponding to cosmological constant.

The dimensionless linear growth rate $f(a) = \frac{\partial \ln D(n)}{\partial \ln a}$ in a universe with no curvature can be approximated as (Lahav et al. 1991)

$$f(a) = \Omega_m(a)^{0.6} + \frac{1}{70} \left[1 - \frac{1}{2} \Omega_m(a)(1 + \Omega_m(a)) \right]. \quad (12)$$

Here $\Omega_m(a) = \Omega_{\Lambda 0} a^{-3} X(a)$. We have used $\Omega_{\Lambda 0} = 0.3$ and $\Omega_{\Lambda 0} = 0.7$ throughout the present work.

2.3 Evolution of linear bias

The time evolution of the linear bias parameter is expected to affect the time evolution of the configuration entropy of the tracer density field. We consider a simple power law of the form $b(a) = b_0 a^n$ with different possible values of $n$. The functional form is motivated by Bagla, Khandai & Datta (2010) where $b(z) \propto (1 + z)^{0.5}$ was reported to give a reasonably good description of the evolution of HI bias in the simulated HI distributions from the N-body simulations. We consider the following values of $n$ in our analysis: $n = -1, -0.75, -0.5, -0.25, 0.5, 1$. We also incorporate the unbiased ΛCDM model in this framework by putting $b(a) = b_0$. We set $b_0 = 1$ in all the models considered here.

3 RESULTS AND CONCLUSIONS

We show the evolution of the linear bias with scale factor for different values of $n$ in the top left panel of Figure 1. The amplitude of the bias at any given scale factor depends on the index $n$. The linear bias monotonically decreases with increasing scale factor for negative $n$. A negative value of $n$ indicates that the tracer density field was strongly biased in the past which decreases with time and eventually reaches unity at present. The decrease in bias corresponds to an overall dilution in the clustering of the tracer mass distribution. The evolution of $S_{\text{bias}}$ with scale factor for all these models is shown in the top right panel of Figure 1. The evolution of the configuration entropy is primarily governed by the growth of density perturbations which is in turn is affected by the dynamics of the expansion of the Universe. Expansion of the Universe slows down the growth of perturbations. Besides the expansion, the time evolution of bias would also play an important role in controlling the dissipation of the configuration entropy of the Universe. For example, all the models with negative $n$ show a decrease in the configuration entropy at earlier times. However the dissipation slows down with time and in some cases it may even reverse its behaviour and starts to grow with time. The time of reversal from dissipation to growth depends on the index $n$. More negative index leads to an early reversal in the behaviour of the configuration entropy.

The lower left panel of Figure 1 shows the entropy rate as a function of scale factor in models with different $n$. The entropy rate is decided by the function $B(n) = b(a) D(n) \frac{\partial a}{\partial a} + b(a) \frac{\partial a}{\partial a} \left[1 + \Omega_m(a) \right]$ which consists of two terms and the combined contribution from these two terms decides the behaviour of the entropy rate at any given time for any specific model. The two terms are separately plotted as function of the scale factor for different models in the lower left and right panels of Figure 2. Clearly a growth in entropy is expected when $B(n)$ is negative and a positive $B(n)$ is associated with entropy dissipation. For example $B(n)$ is negative at all scale factor for $n = -1$ and this implies that there will be no dissipation of entropy in this model. On the other hand the model with $n = 1$ and $n = 0.5$ have positive $B(n)$ at all scale factors and there is a continuous dissipation of entropy in these models. All the other models considered here show dissipation of entropy at some scale factors and growth of entropy at some other scale factors. A zero up crossing in the entropy rate corresponds to a local minimum in the configuration entropy. Clearly this zero up crossing appears at a smaller scale factor for more negative values of $n$.

We show the derivative of the entropy rate in these models in the lower right panel of Figure 1. The derivative of the entropy rate exhibits a peak in all the models with negative
We find that the location of the peak is sensitive to the index $n$ and it appears at a smaller scale factor for models with smaller index. In an earlier work, Pandey & Das (2019) noted that in the unbiased $\Lambda$CDM model, this peak is exactly located at the scale factor corresponding to the $\Lambda$-matter equality. We have used $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ in the $\Lambda$CDM model. So in the unbiased $\Lambda$CDM model the peak is expected to appear at $a = 0.754$. This can be clearly seen in the result shown for the unbiased $\Lambda$CDM model in the same panel. Now the location of this peak is shifted towards a smaller scale factor when time evolution of bias is considered within the $\Lambda$CDM model. The shift is measured with reference to the location of the peak in the unbiased $\Lambda$CDM model. The magnitude of the entropy rate slows down after the onset of $\Lambda$ domination. The bias models with negative value of $n$ dilute the clustering and slows down the structure formation even before the $\Lambda$-matter equality. This effect would manifest in a more prominent way in the models with more negative $n$. So the peak in the derivative of the entropy rate is expected to exhibit a larger shift in these models. We measure the location of the peak in the models with different negative index and find them to be linearly related. We show the index $n$ as a function of the location and the shift of the peak in the left and right panels of Figure 3 respectively. The best fit relations between these parameters are also provided in the same figure.

We also consider two positive values of $n$ in the time evolution of bias. A positive value of $n$ in this case ($b_0 = 1$) indicates that the tracer density field is anti biased with respect to the underlying mass density field and the bias slowly increases from a very small positive value to unity at present. A decrease in anti biasing with time would enhance the clustering of the tracer leading to a continuous dissipation of the configuration entropy. In these models, initially entropy show a slower decrease than that of $\Lambda$CDM model but then decrease quite quickly in the later part. We do not observe the peak in the derivative of the entropy rate in these models and they can be easily distinguished from the models with negative values of $n$. These models are not realistic and we consider them only for the sake of completeness.

In this work, we calculate the evolution of the configuration entropy for a biased tracer of the density field (e.g. neutral Hydrogen) assuming different time evolution of bias. We consider the flat $\Lambda$CDM model as the benchmark model of the Universe and within it consider the time evolution of linear bias as, $b(a) = b_0 a^n$ with different values of the index.
We show that the time evolution of bias alters the position of the peak in the derivative of entropy rate. The peak shifts towards a smaller scale factor for negative index and is absent when the index is positive. We find that the shift is linearly related with the index $n$ and a larger shift is observed for a smaller index. We find the best fit relation between these two parameters and propose that identifying the location of this peak from observations would allow us to constrain the time evolution of linear bias within the framework of the $\Lambda$CDM model. Finally we conclude that the analysis presented in this work provides an alternative method to constrain the evolution of linear bias using configuration entropy.

4 ACKNOWLEDGEMENT

BP acknowledges financial support from the Science and Engineering Research Board (SERB), Department of Science & Technology (DST), Government of India through the project EMR/2015/001037. BP would also like to acknowledge IUCAA, Pune for providing support through the associateship programme.

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