Variation of fundamental constants and the triple-alpha reaction in Population III stars and BBN

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Abstract. The effect of variations of the fundamental constants on the thermonuclear rate of the triple alpha reaction, $^4\text{He}(\alpha\alpha,\gamma)^{12}\text{C}$, that bridges the gap between $^4\text{He}$ and $^{12}\text{C}$ is investigated. We have followed the evolution of 15 and 60 $M_\odot$ zero metallicity stellar models, up to the end of core helium burning. The calculated oxygen and carbon abundances resulting from helium burning can then be used to constrain the variation of the fundamental constants. To investigate the effect of an enhanced triple alpha reaction rate in Big-Bang Nucleosynthesis, we first evaluated Standard Big-Bang Nucleosynthesis CNO production with a network of more than 400 reactions using the TALYS code to calculate missing rates.

1. Introduction
The equivalence principle is a cornerstone of metric theories of gravitation and in particular of General Relativity. This principle, including the universality of free fall and the local position and Lorentz invariances, postulates that the local laws of physics, and in particular the values of the dimensionless constants, such as the fine structure constant ($\alpha_{em}$), must remain fixed. It follows that by testing the constancy of fundamental constants one actually performs a test of General Relativity, that can be extended on astrophysical and cosmological scales (for a review, see [1]). The $^4\text{He}(\alpha\alpha,\gamma)^{12}\text{C}$, or triple-$\alpha$-reaction, at the origin of the production of $^{12}\text{C}$, sets constraints on the values of the fine structure and strong coupling constants. Its resonant reaction rate is very sensitive to the nuclear interaction: a 1% variation of the Nucleon-Nucleon interaction ($\delta_{NN} = 10^{-2}$) would induce a $\sim 20$ orders of magnitude change in the rate at 0.1 GK. First, we have considered the very first generation of stars which are thought to have been formed a few $10^8$ years after the Big–Bang, at redshifts of $z \sim 10 - 15$, and with zero initial metallicity. For the time being, there are no direct observations of those (Population III) stars but one may expect that their chemical imprints could be observed indirectly in the most metal-poor halo (Population II) stars. Second, Big–Bang nucleosynthesis (BBN) can provide complementary constraints on the variation of constants at a much earlier epoch ($z \sim 10^5$). To extend our previous BBN study[2] that included variations of the weak and $n(p,\gamma)D$ rates to a possibly inflated 3$\alpha$-reaction rate, we first need to calculate the Standard BBN, $^{12}\text{C}$ (and CNO) production, bypassing the A=8 gap. The direct detection of primordial CNO isotopes seems highly unlikely with the present observational techniques but it is important for other applications. Hydrogen burning in low mass Pop. III stars proceeds through the slow pp chains until enough carbon is produced, through the triple-alpha reaction, to activate the CNO cycle.

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The minimum value of the initial CNO mass fraction that would affect Pop. III stellar evolution was estimated to be $10^{-10}$ [3] or even as low as $10^{-12}$ for the less massive ones [4]. This is only two orders of magnitude above the Standard Big-Bang Nucleosynthesis CNO yields obtained using current nuclear reaction rate estimates. Indeed, the main difficulty in BBN calculations up to CNO is the extensive network (>400 reactions) needed, including n, p, α, but also d, t and $^4$He, induced reactions on both stable and radioactive targets. Most of their cross sections cannot be extracted from experimental data only, hence, previous studies (e.g. Ref. [5]) have apparently extensively used the old prescriptions of [6] to estimate those rates. A detailed analysis of all reaction rates and associated uncertainties would be desirable but is impractical for such a large network. So, in this study we have used at first more reliable rate estimates provided by the TALYS [7] code, next we performed a sensitivity study and, finally improve the rate estimates of the most important reactions by dedicated evaluations.

2. The triple–alpha reaction

The $^4$He(αα, γ)$^{12}$C reaction is very sensitive to the position of the ”Hoyle state” [8] as the corresponding resonance width is very small (a few ~eV) as compared with the competing reaction $^{12}$C(α, γ)$^{16}$O, dominated by broad (~100 keV) resonances and subthreshold levels. Hence, in this work, we only considered variations of the $^{4}$He(αα, γ)$^{12}$C reaction rate. Assuming i) thermal equilibrium between the $^4$He and $^8$Be nuclei, so that their abundances are related by the Saha equation and ii) the sharp resonance approximation for the alpha capture on $^8$Be, the $^4$He(αα, γ)$^{12}$C rate, can be expressed [9] as:

$$N_A^2 (\sigma v)^{\alpha\alpha\alpha} = 3^{3/2} 6N_A^2 \left(\frac{2\pi}{M_\alpha k_B T}\right)^3 \hbar^5 \omega \gamma \exp\left(\frac{-Q_{\alpha\alpha\alpha}}{k_B T}\right)$$  \hspace{1cm} (1)

with $\omega \gamma \approx \Gamma_{\gamma}(^{12}\text{C})$, the radiative partial width of the ”Hoyle level” and $Q_{\alpha\alpha\alpha} = 380$ keV its energy relative to the triple alpha threshold. The variation of $Q_{\alpha\alpha\alpha}$ with the nucleon nucleon interaction dominates the variation of the reaction rate in Eq. (1). However, a more accurate calculation [10] requires a numerical integration, as done by [11], to take into account i) the two step process, two alpha particle fusion into the $^8$Be ground state at a resonance energy of $E_R(\kappa\text{Be})$ followed by another alpha capture to the Hoyle state at a resonance energy of $E_R(^{12}\text{C})$ \(Q_{\alpha\alpha\alpha} = E_R(\kappa\text{Be}) + E_R(^{12}\text{C})\) and ii) the energy dependent finite widths of those two resonances. In order to analyze their variation with the nuclear interaction, we have used a microscopic cluster model (see [12] and references therein). The nucleon-nucleon interaction $V(\mathbf{r})$ depends on the relative coordinate $\mathbf{r}$, and is written as:

$$V(\mathbf{r}) = V_C(\mathbf{r}) + (1 + \delta_{NN})V_N(\mathbf{r}).$$  \hspace{1cm} (2)

where $V_C(\mathbf{r})$ is the Coulomb force, $V_N(\mathbf{r})$ the nuclear interaction [13], and $\delta_{NN}$ the parameter that characterizes the change in the nucleon-nucleon interaction. (The variation of the Coulomb interaction is assumed to be negligible compared to the nuclear interaction). We use a cluster approximation in which the wave functions of the $^8$Be and $^{12}$C nuclei are approximated by a cluster of respectively two and three α particle wave functions. This approach has been shown to be well adapted to cluster states, and in particular to the $^8$Be and $^{12}$C levels of interest [14]. We obtained: \(\Delta B_D/B_D = 5.7701 \times \delta_{NN}\) for the binding energy of deuterium, $E_R(\kappa\text{Be}) = (0.09208 - 12.208 \times \delta_{NN})$ MeV, and $E_R(^{12}\text{C}) = (0.2877 - 20.412 \times \delta_{NN})$ MeV, for the resonance energies. The strong energy dependence of the particle widths $\Gamma_{\lambda}(E)$ follows the Coulomb penetrability factor [9] while the radiative width $\Gamma_{\gamma}(E)$ scales as $E^{2\lambda+1}$ where $\lambda$ is the multipolarity (here 2 for $E2$) of the electromagnetic transition. Taking into account the variation of $E_R(\kappa\text{Be})$ and $E_R(^{12}\text{C})$ and of the partial widths, as a function of $\delta_{NN}$, after numerical integration, we obtain the variation of the rate spanning many orders of magnitude for $|\delta_{NN}| \lesssim 1\%$. 

2
3. Stellar evolution

We considered Population III stars with typical masses, 15 and 60 $M_\odot$ and zero metallicity and used the Geneva code, assuming no rotation (see [4] and references therein). The computations were stopped at the end of the core helium burning (CHeB). Note that the $^4\text{He}(\alpha\alpha,\gamma)^{12}\text{C}$ reaction is also important during the hydrogen burning phase at zero metallicity. It is clearly visible in the HR diagram [15] that in the absence of initial CNO, stars keep contracting while core hydrogen burning proceeds through the slower pp-chain until sufficient $^{12}\text{C}$ has been produced by the triple alpha reaction. Figure 1 shows the central abundances at the end of core He burning for the 15 and 60 $M_\odot$ models. From these characteristics, we can distinguish four different cases:

1) During He burning, $^{12}\text{C}$ is produced, until the central temperature is high enough for the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reaction to become efficient: in the last part of the CHeB, $^{12}\text{C}$ is processed into $^{16}\text{O}$. The star ends the CHeB phase with a core composed of a mixture of $^{12}\text{C}$ and $^{16}\text{O}$.

2) If the $3\alpha$ rate is weaker, $^{12}\text{C}$ is produced at a slower pace, and $T_c$ is high from the beginning of the CHeB phase, so the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reaction becomes efficient very early: as soon as some $^{12}\text{C}$ is produced, it is immediately transformed into $^{16}\text{O}$. The star ends the CHeB phase with a core composed mainly of $^{16}\text{O}$, with very little $^{12}\text{C}$ and an increasing fraction of $^{24}\text{Mg}$ for decreasing $\delta_{NN}$. 3) For still weaker $3\alpha$ rates, the central temperature during CHeB is such that the $^{16}\text{O}(\alpha,\gamma)^{20}\text{Ne}(\alpha,\gamma)^{24}\text{Mg}$ chain becomes efficient, reducing the final $^{16}\text{O}$ abundance. The star ends the CHeB phase with a core purely composed of $^{24}\text{Mg}$. 4) If the $3\alpha$ is strong ($\Delta B_D/B_D > 0$), the $^{12}\text{C}$ is very rapidly produced, but the $T_c$ is so low that the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reaction can hardly enter into play: the $^{12}\text{C}$ is not transformed into $^{16}\text{O}$. The star ends the CHeB phase with a core almost purely composed of $^{12}\text{C}$.

![Figure 1.](image-url) Central abundances at the end of CHe burning as a function of $\delta_{NN}$ for 15 (solid lines) and 60 (dashed lines) $M_\odot$ stars. The dotted line corresponds to the $-0.0005 < \delta_{NN} < 0.0015$ adopted interval for which both C and O are produced.

4. Limits on the variations of the fundamental constants in Pop III

The results of the 15 and 60 $M_\odot$ models [15] shows that the variation of the N–N interaction should be in the range $-0.0005 < \delta_{NN} < 0.0015$ (dotted lines in Fig. 1) to insure conservatively that the C/O ratio be of the order of unity. The parameter $\delta_{NN}$ is not directly related to a set of fundamental constants such as the gauge and Yukawa couplings. In order to make this connection, we use our computation of the deuterium binding energy $B_D$ to obtain $-0.003 < \Delta B_D/B_D < +0.009$. Using a potential model, the dependence of $B_D$ on the nucleon, $\sigma$-meson and $\omega$-meson mass has been estimated [16] so that it can be related to the
We found that for all but a few reactions, a three order of magnitude change in their rate ≥ larger than 20% were flagged. (Mass fractions of isotopes with A≥ 12 are added up together.)

procedure. Reactions whose rate variation induce a relative change in at least one isotope yield of magnitudes assumed here. However, to keep the procedure simple, we applied the same because of experimental works, that these uncertainties are much lower than the three orders variations, ∆Λ/Λ = R∆αem/αem, and is in the range -20 ≤ R ≤ +40 [18] while S reflects the sensitivity of the Higgs vev to the parameters of the supersymmetric model and is in the range 80 ≤ S ≤ 500 [19]. The factor in Eq. (3) is hence very model dependent and can take values between 0.1 and 1000. For typical values of R=36 and S=240, the limits given by BBN (i.e. at a redshift of z ∼ 10^8) on the variation of the fine structure constant are given by -3.2 × 10^{-5} < ∆αem/αem < 4.2 × 10^{-5}[2]. Within the same model and parameter values, the constraints obtained above would lead to -3. × 10^{-6} < ∆αem/αem < 10^{-5} at z = 10–15.

5. CNO production in Standard Big-Bang Nucleosynthesis

In this study, we have included 59 nuclides from the neutron to 23Na, linked by 391 reactions involving n, p, d, t and 3He induced reactions and 33 beta decay processes. Reaction rates were taken primarily from [11, 20, 21] and other evaluations when available. The complete list of reactions with associated references to the origin of the rates can be found in Ref. [22]. In comparison with previous works, the present study includes two specific features, namely the introduction of new evaluations of experimental reaction rates and the extensive use of the TALYS reaction code [7] to evaluate the unknown reaction rates. This code is based on the Hauser-Feshbach statistical model which is, a priori, not well suited for the description of the reaction involving light nuclei. However, it was found to provide rather fair estimates of those rates that can be used as a first guess for a sensitivity analysis. Indeed, comparison with experimentally determined reaction rates[11, 21] shows differences rarely exceeding three orders of magnitude at BBN relevant temperatures. The results of the calculation, for the stable CNO isotopes, are displayed in Fig. 2: the total CNO mass fraction is ≈ 0.5 × 10^{-14}. This is in overall agreement with Ref. [5] (with some differences in the isotopic ratios e.g. 14C) and two orders of magnitude below the abundance needed to influence Pop. III stellar evolution. Our results on 4He, D, 3He and 7Li are within 2% of the central value of our latest Monte–Carlo study[23] of BBN nuclear uncertainties, partly due to WMAP most recent η value [24].

However, as many reaction rates are uncertain due to the lack of experimental data, we performed a sensitivity study for each of 271 reactions whose rates are obtained either from TALYS or whose uncertainties have not been estimated. Based on our comparison between TALYS calculated rates on the one hand and experimental rates on the other hand (see Fig. 3 and Ref.[22]) we consider three orders of magnitude variations around the standard reaction rates. Hence to estimate the impact of reaction rates on Standard Big-Bang Nucleosynthesis for each reaction, we performed six calculations after changing its rate by factors of 0.001, 0.01, 0.1, 10., 100. and 1000. and calculate the relative change in isotopic abundances. In many cases, even if the rate uncertainties have not been explicitly calculated it is clear that, e.g. because of experimental works, that these uncertainties are much lower than the three orders of magnitudes assumed here. However, to keep the procedure simple, we applied the same procedure. Reactions whose rate variation induce a relative change in at least one isotope yield larger than 20% were flagged. (Mass fractions of isotopes with A≥12 are added up together.) We found that for all but a few reactions, a three order of magnitude change in their rate
had no significant impact on BBN. We recall that we excluded the known important reactions for which uncertainties are small and whose impact, e.g. on $^4\text{He}$, D, $^3\text{He}$ and $^7\text{Li}$, have been extensively studied. Following this study, very few TALYS calculated reaction rates were found to be crucial for CNO production in BBN. Examples are the $^{11}\text{B}(d,p)^{12}\text{B}$ and $^{11}\text{B}(d,n)^{12}\text{C}$ whose rate variations induced a factor of up to 10 and 300 respectively in CNO production. A detailed analysis of the available experimental data supplemented by theoretical calculations enabled us to reduce the rate uncertainties of the flagged reactions. As a result, changes in reaction rates induced effects nearly compensating each other, confirming the value of $\approx 0.5 \times 10^{-14}$ CNO mass fraction. We are now in position to compare this value with the CNO production in Standard Big-Bang Nucleosynthesis by an inflated $^4\text{He}(\alpha\alpha,\gamma)^{12}\text{C}$ rate.

6. CNO production in BBN with an inflated $^4\text{He}(\alpha\alpha,\gamma)^{12}\text{C}$ rate

To disentangle standard CNO production as described in the previous section with the one proceeding through the triple–alpha reaction, we reduced the network to the 15 reactions involved in $A<8$ nucleosynthesis plus $^4\text{He}(\alpha\alpha,\gamma)^{12}\text{C}$. As we are investigating a possible enhancement of CNO production we only considered positive $\delta_{NN}$ values that lead to a higher triple–alpha reaction rate. We limited ourselves to $\delta_{NN} \leq 0.007$ as above, $^8\text{Be}$ would be bound requiring a different treatment for the calculation of the rate. This is not a limitation as the CNO yield displays a maximum at $\delta_{NN} \approx 0.006$ which is ($\approx 10^{-20}$ in mass fraction) six orders of magnitude below the standard production. This maximum is due to the sharp resonances (both $^8\text{Be}$ g.s. and “Hoyle state”) mechanism dominating the cross section becomes highly inefficient when their positions are far from the Gamow window. We can hence conclude that CNO production in BBN cannot be increase through a modification of the triple–alpha reaction.

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Figure 4. Most important reactions for the production of $^4\text{He}$, $^3\text{He}$, $^7\text{Li}$, $^9\text{Be}$, $^{10}\text{B}$, $^{11}\text{B}$ and CNO isotopes at $\eta = \eta_{\text{W,MAP}}$.

Figure 5. $^{12}\text{C}$ production, in number of atoms relative to H, by the triple alpha reaction as a function of the NN-interaction.

Yi Xu.

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