New mass bound on fermionic dark matter from a combined analysis of classical dSphs

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ABSTRACT

Dwarf spheroidal galaxies (dSphs) are the most compact dark matter-dominated objects observed so far. The Pauli exclusion principle limits the number of fermionic dark matter particles that can compose a dSph halo. This results in a well-known lower bound on their particle mass. So far, such bounds were obtained from the analysis of individual dSphs. In this paper, we model dark matter halo density profiles via the semi-analytical approach and analyse the data from eight ‘classical’ dSphs assuming the same mass of dark matter fermion in each object. First, we find out that modelling of Carina dSph results in a much worse fitting quality compared to the other seven objects. From the combined analysis of the kinematic data of the remaining seven ‘classical’ dSphs, we obtain a new 2σ lower bound of \( m \gtrsim 190 \) eV on the dark matter fermion mass. In addition, by combining a sub-sample of four dSphs – Draco, Fornax, Leo I and Sculptor – we conclude that 220 eV fermionic dark matter appears to be preferred over the standard CDM at about 2σ level. However, this result becomes insignificant if all seven objects are included in the analysis. Future improvement of the obtained bound requires more detailed data, both from ‘classical’ and ultra-faint dSphs.

Key words: dark matter – galaxies: haloes – galaxies: dwarf – galaxies: kinematics and dynamics – methods: statistical

1 INTRODUCTION

The nature of dark matter (DM) is one of the major questions in modern physics. The mass of DM particle candidates, which exist in numerous extensions of the Standard Model, varies in very wide range – from \( \sim 10^{-22} \) eV for ultralight DM (e.g., Hu et al. 2000; Hui et al. 2017; Lee 2018) up to TeVs for WIMPs (see, e.g., Roszkowski et al. 2018; Arcadi et al. 2018, and references therein) or up to \( \sim 10^{13} \) GeV for WIMPZILLAs (e.g. Chung et al. 1999).

The Pauli principle forbids packing too many fermions into a gravitationally bound object. Therefore, the average phase-space density of such an object with mass \( M \) enclosed within a region of radius \( R \), \( \bar{f} \sim \frac{M}{R^3 \sigma^3} \), cannot exceed some maximum \( f_{\text{max}}(m) \), where \( m \) is the mass of fermion, and \( \sigma \) is the particle velocity dispersion. This allows one to obtain the lower bound \( m \gtrsim 0.5 \) keV (Bode et al. 2001; Dalcanton & Hogan 2001; Boyarsky et al. 2009; Horihuchi et al. 2014), based on the extended Tremaine–Gunn (Tremaine & Gunn 1979) approach (see also Gorbunov et al. 2008; Shao et al. 2013; Wang et al. 2017) from the analysis of compact DM dominated objects – dwarf spheroidal satellites (dSphs).

This approach requires an estimator of the dynamical mass \( M \) within a sphere of some radius \( R \) (Wolf et al. 2010; Walker & Peñarrubia 2011; Campbell et al. 2017), see also Kowalczyk et al. (2013) for a detailed study of the mass estimator uncertainties, and Boyarsky et al. (2009) for the estimate of the phase-space volume occupied by the DM particles.

Another method for constraining the mass of the fermionic DM particle uses direct comparison between the detailed prediction of the kinematics of dSph and the observational data (see, e.g., Domcke & Urbano 2015; Randall et al. 2017; Di Paolo et al. 2018). It does not require an estimate of the averaged phase-space density over a spatial region. Direct modelling of kinematics also allows one to incorporate the anisotropy of the velocity dispersion into analysis. Moreover, unlike the Tremaine–Gunn approach, this method allows one to combine the data on several objects to produce better limits on the particle mass. In return, it requires a (semi-)analytical model of the DM density profile and stellar density profile. Many analytical models of fermionic DM halo density profiles have been developed so far; see, e.g., Ruffini & Stella (1983); Bilić & Viollier (1997); Angus (2010); de Vega et al. (2014); de Vega & Sanchez (2016); Merafina & Alberti (2014); Domcke & Urbano (2015); Ruffini et al. (2015); Chavanis et al. (2015);
This paper, we present a new lower bound on the mass of fermionic DM particle, based on the observed kinematics (Bonnivard et al. 2015) and photometry (McConnachie 2012) data of ‘classical’ dSphs, and assuming the DM density model of Rudakovskiy & Savchenko (2018). In comparison to Domcke & Urbano (2015), Di Paolo et al. (2018), this approach allows us not only to analyse individual dSphs, but also to perform combined statistical analysis based on the total $\chi^2$ goodness-of-fit statistics assuming the same dark matter particle mass in all of them. Hereby, we aim at utilising fully the statistical power of the approach.

This paper is organised as follows: in Sec. 2 our methods are described (a short description of our model of fermionic DM halo is also included), the obtained results are summarised in Sec. 3 and discussed in Sec. 4. We use the recent Planck (Planck Collaboration 2018) cosmological parameters for our calculations.

2 METHODS

We use the semi-analytical method proposed in Rudakovskiy & Savchenko (2018) to obtain the density profile of a dark matter halo. It predicts a cored halo for the general case of warm fermionic dark matter without any extra assumptions about the particle model. Here we briefly summarise this method.

For a fermionic dark matter model with particle mass $m_{\text{DM}}$ and $g$ initial degrees of freedom (hereafter, we assume $g = 2$) the phase space density cannot exceed (Boyarsky et al. 2009)

$$f_{\text{max}} = \frac{g m_{\text{DM}}^2}{2(2\pi)^3 h^3}. \quad (1)$$

For a steady-state isotropic spherically symmetrical dark matter halo (see Rudakovskiy & Savchenko 2018) for a discussion on the applicability of this assumption) the phase space density $f$ is obtained by using the Eddington transformation (Eddington 1916; Binney & Tremaine 2008)

$$f(E) = \frac{1}{\pi^2 V^8} \frac{d}{dE} \int_0^\infty dp \frac{d\Phi}{d\sqrt{E - \Phi}}. \quad (2)$$

where $\Phi$ is the local gravitational potential. We perform the iterative procedure starting from the NFW profile and truncating the phase space density so that it does not exceed the limiting value:

$$f_{\text{NFW}}(E) = \begin{cases} f(E), & f(E) < f_{\text{max}}; \\ f_{\text{max}}, & f(E) \geq f_{\text{max}}. \end{cases} \quad (3)$$

After this, we reconstruct the mass density (Binney & Tremaine 2008)

$$\rho_{\text{NFW}}(r) = 4\pi \int_{\Phi(r)}^0 f_{\text{NFW}}(E) \sqrt{2(E - \Phi(r))} dE \quad (4)$$

for the subsequent step. Rudakovskiy & Savchenko (2018) shows good convergence of this procedure after several iterations. We call the obtained profile tNFW (stands for truncated Navarro–Frenk–White) hereafter.

The density profiles obtained in this model are in a good agreement with numerical N-body simulations (Shao et al. 2013; Macciò et al. 2013a,b), see more in Rudakovskiy & Savchenko (2018).

Given the density distribution of a dark matter halo, we follow the logic of Domcke & Urbano (2015) and Di Paolo et al. (2018) to obtain the velocity dispersion along the line of sight. Specifically, we solve the spherical Jeans equation for the radial velocity dispersion $\sigma_r$,

$$\left( \frac{\partial}{\partial r} + \frac{2\beta}{r} \right) (n_\ast \sigma_r^2) = -n_\ast \frac{GM(r)}{r^2}, \quad (5)$$

with the stellar velocity dispersion anisotropy $\beta = 1 - \sigma_\parallel^2/\sigma_\perp^2$.

In the above, $M(r)$ is the dark matter mass distribution, and $n_\ast$ is the stellar number density, which we represent by the Plummer profile (Plummer 1911)

$$n_\ast(r) = n_0 \left( 1 + r^2/r_h^2 \right)^{-5/2}. \quad (6)$$

The half-light radii $r_h$ for the objects of interest were taken from McConnachie (2012) and are given in Table 1. We then calculate the velocity dispersion along the line of sight:

$$\sigma_{\text{los}}^2(R) = \frac{1}{\Sigma_\ast} \int_{R_\ast}^{\infty} d\Sigma \left( \frac{n_\ast}{\sqrt{\Sigma - R^2}} \right)^2 \left[ 1 - \beta R^2/r_h^2 \right], \quad (7)$$

where $\Sigma_\ast = \int_{R_\ast}^{\infty} d\Sigma n_\ast(\Sigma)/\sqrt{\Sigma - R^2}$ (Binney & Tremaine 2008; Di Paolo et al. 2018).

We model the binned data on the velocity dispersion for eight classical dSphs taken from Bonnivard et al. (2015). For every mass of the dark matter particle in the 100 eV – 900 eV range with logarithmic split we use brute-force grid optimisation over the tNFW profile parameters $c_{200}$, $M_{200}$, and velocity dispersion anisotropy $\beta$ to minimise the objective $\chi^2$ statistic

$$\chi^2 = \sum_i \frac{(\sigma_{\text{los,obs}}(r_i) - \sigma_{\text{los,th}}(r_i))^2}{\delta^2(r_i)}. \quad (8)$$

where $\sigma_{\text{los,obs}}(r_i)$ denotes the $i$’th observational point, $\delta^2(r_i)$ is its 1$\sigma$ error, and $\sigma_{\text{los,th}}(r_i)$ is the predicted value at this point; the summation is performed over the observational points.

3 RESULTS

The dependence of the best-fitting $\chi^2$ statistics on the particle mass for every individual object is plotted in Fig. 1, and the best-fitting model parameters are summarised in Table 1.

The goodness-of-fit is acceptable for every object except Carina dSph, which is the only dSph from our selection that has best-fitting $\chi^2$ higher than two standard deviations ($2\sqrt{2N_{\text{dof}}}$) above the mean value $\chi^2_{\text{mean}} = N_{\text{dof}}$ of the chi-squared distribution. Therefore, we exclude Carina dSph from the subsequent combined analysis.

Apart from the individual fits, we are interested in the combined goodness-of-fit. We consider the overall $\chi^2$ to be the sum of chi-squared statistics of the individual fits for every dark matter particle mass. The overall best fit is obtained for the particle mass of 342 eV with $\chi^2 = 124.7$ for 134 degrees of freedom. This value of mass, however, cannot be statistically distinguished from the higher values, as the
Figure 1. Minimal values of $\chi^2$ statistics as functions of the DM particle mass for the tNFW profile model for each of the eight ‘classical’ dSphs studied in this paper. Also given is the number of the degrees of freedom of the fits. Notice the clearly visible minimums in four objects: Draco, Fornax, Leo I, Fornax.
The particle mass is the value for which Sec. 15.6 of Press et al. (2007). The lower bound on the parameters of the particle mass, we can build the confidence  

models is preferred by our analysis. The best-fitting statistics is 125.1 for 134 degrees of freedom, so none of this dark matter model. The best-fitting χ₂ differences between the corresponding chi-squares are negligible. For comparison, we fitted the data using the Navarro–Frenk–White profile, as the tNFW halo model approaches that of the NFW in this limit. The dashed line shows the 2σ confidence bound on the particle mass. 

Using the dependence of the overall best-fitting statistics on the particle mass, we can build the confidence range for the mass via the standard approach, described in Sec. 15.6 of Press et al. (2007). The lower bound on the particle mass is the value for which $\chi^2 = \chi^2_{\text{best-fit}} + \Delta \chi^2$, where for 2σ confidence level $\Delta \chi^2 = 4$. The resulting mass bound of $m_{2\sigma} \approx 190$ eV is shown in Fig. 2. 

In Fig. 4 we show the effect of particle mass on the velocity dispersion profile in all objects. It is clearly seen that small particle masses strongly modify this profile. We also combine four objects that show notable local minimum on $\chi^2$ vs mass dependence, namely, Draco, Fornax, Leo1 and Sculptor. The combined fitting statistics in this case is plotted in Fig. 3. The minimum $\chi^2_{\text{min}} = 87.4$ for 99 degrees of freedom is obtained for particle mass $m = 220$ eV, whereas the Navarro–Frenk–White profile fits the data with $\chi^2 \approx 91.4$. Thus one can conclude that $220$ eV fermionic dark matter is preferred over CDM with $\Delta \chi^2 = 4$. However, this observation should not be treated as a strict result, because inclusion of the rest of objects into analysis reduces the local minimum to a statistically insignificant depth of $\Delta \chi^2 = 0.4$. 

### Table 1. The best-fitting parameter values for the modelled objects and goodness-of-fit statistics. Also provided is the half-light radii used in our fits.

| Object   | $r_h$, kpc | $\chi^2/N_{\text{df}}$ | $m_{\text{DM}}$, eV | $M_{200}$, $10^8 M_{\odot}$ | $c_{200}$ | $\beta$ |
|----------|------------|------------------------|----------------------|-------------------------------|-----------|---------|
| Carina   | 0.25       | 37.5/21                | 561                  | 111.7                         | 5         | 0.21    |
| Draco    | 0.221      | 4.1/7                  | 255                  | 177.8                         | 10        | 0.34    |
| Fornax   | 0.71       | 28.7/46                | 171                  | 9.57                          | 53        | -0.05   |
| Leo1     | 0.251      | 10.4/13                | 310                  | 155.7                         | 8         | 0.44    |
| Leo2     | 0.176      | 5.5/8                  | 650                  | 127.6                         | 9         | 0.61    |
| Sculptor | 0.283      | 43.2/33                | 220                  | 6.01                          | 59        | 0.10    |
| Sextans  | 0.695      | 16.1/13                | 650                  | 875.6                         | 2         | -0.38   |
| Ursa Minor | 0.181    | 11.8/14                | 561                  | 4.92                          | 36        | -1.32   |

### Figure 2. Overall best-fitting $\chi^2$ statistics as a function of the dark matter particle mass. In the limit of high mass the curve approaches the value obtained in the fit with the Navarro–Frenk–White profile, as the tNFW halo model approaches that of the NFW in this limit. The dashed line shows the 2σ confidence bound on the particle mass.

### Figure 3. Overall best-fitting $\chi^2$ as a function of dark matter particle mass for the combined analysis of only four selected objects: Draco, Fornax, Leo1 and Sculptor. The minimum at 220 eV indicating the preferred particle mass is clearly visible. The depth of the dip corresponds to 2σ significance ($\Delta \chi^2 = 4$). However, it becomes negligibly small ($\Delta \chi^2 = 0.4$) when the rest of objects are included into analysis, see Fig. 2.

### 4 CONCLUSIONS & DISCUSSION

In this paper, we derive a new maximally model-independent bound on the mass of fermionic dark matter particle. We use the halo model of Rudakovskyi & Savchenko (2018) and the Jeans equation for modelling the line-of-sight velocity dispersion. We obtained the conservative 2σ lower bound $m \gtrsim 190$ eV on the mass of fermionic dark matter particle. Fermionic DM with higher particle mass cannot be distinguished from the CDM.

Using only the data on four selected objects, namely, Draco, Fornax, Leo1 and Sculptor, we obtain that fermionic DM with $m = 220$ eV particle mass is preferred over CDM on 2σ level. The significance decreases to a negligible value when the rest of objects are included into the analysis.

The main advantage of our analysis is the combined study of several objects: we simultaneously fit the data for seven classical dSphs. While the fits of the data of individual objects show different preferred particle masses (see Fig. 1) and lead to different bounds, the combined analysis ensures...
Figure 4. Velocity dispersion along the line of sight versus the distance from the object centre. The dots with error bars represent the binned observational data, taken from Bonnivard et al. (2015). The lines show the best-fitting dependence obtained in the tNFW model with different particle masses, namely, $m = 100$ eV which is below the obtained 2σ bound, $m = 220$ eV preferred by the combined analysis of four selected dSphs (Draco, Fornax, Leo I, Sculptor), and the mass which provides the minimal $\chi^2$ in the fit of the corresponding individual object. One can see that for most of objects the behaviour of $\sigma_{\text{los}}$ in the case of low particle mass strongly changes whereas variation of mass above the 190 eV bound has small impact on this behaviour.
robustness of the results. Moreover, when modelling several object, we are able to produce stronger bound. For example, the strongest limit of 100 eV in Di Paolo et al. (2018) is obtained by analysing the smallest dwarfs, whereas the analysis of the classical dwarfs only leads to the mass limit of few tens of electron-volts.

In general, the dark matter halo profile of Di Paolo et al. (2018) systematically prefers lower particle masses due to its fully degenerate nature, which produces sharp cut-off in the density profile. Unlike in Di Paolo et al. (2018), in our model the DM halo has two regions: a fully degenerate core and non-degenerate dispersed outskirts. Fig. 5 shows the fast clipping of this profile and the smaller core size, compared with more “blurred” tNFW profile.

Recent direct measurements of 3D stellar kinematics in Sculptor (Massari et al. 2018) and kinematics data modelling via the Schwarzschild method (Kowalczyk et al. 2019) revealed that the stellar velocity dispersions in the dwarf spheroidal galaxies are likely to be non-isotropic, but the uncertainties in the value of $\beta$ are very large. Therefore, we assume, for simplicity, that this quantity is constant on all radii. Inclusion of non-zero stellar velocity anisotropy into the analysis leads to a lower DM mass bound compared to the previous findings (e.g. Boyarsky et al. 2009). In the case of non-zero $\beta$, we found that DM particle masses in wide range are statistically indistinguishable. This agrees qualitatively with the results of Di Paolo et al. (2018) and Randall et al. (2017) for models of non-fully degenerate fermionic halos. This $\beta$-degeneracy could be overcome by assuming multiple stellar sub-populations (Battaglia et al. 2008; Walker & Peñarrubia 2011; Agnello & Evans 2012; Amorisco et al. 2013) or by using the Virial equations instead of the Jeans equations (Richardson & Fairbairn 2014). However, the existing data, which does not include proper 3D stellar kinematics with possible asphericity of stellar populations, is not enough to completely break this degeneracy (Kowalczyk et al. 2013; Genina et al. 2018; Hayashi et al. 2018).

We also did not include the effects of the supernova feedback (Navarro et al. 1996a; Pontzen & Governato 2012; Oh et al. 2011; Teyssier et al. 2013; Zolotov et al. 2012), other stellar feedback mechanisms (Chan et al. 2015), and dynamical friction (El-Zant et al. 2004; Sánchez-Salcedo et al. 2006; Romano-Díaz et al. 2008; Del Popolo & Pace 2016). These effects could cause additional flattening of the dark matter profile and reduction of the central phase-space density. However, inclusion of these effects could increase the lower mass bound.

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