Type IIB Supersymmetric Flux Vacua

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Abstract

On the Type IIB toroidal $T^6$ orientifolds with generic flux compactifications, we conjecture that in generic supersymmetric Minkowski vacua, at least one of the flux contributions to the seven-brane and D3-brane tadpoles is positive if the moduli are stabilized properly, and then the tadpole cancellation conditions can not be relaxed. To study the supersymmetric Minkowski flux vacua, we simplify the fluxes reasonably and discuss the corresponding superpotential. We show that we can not have simultaneously the positive real parts of all the moduli and the negative/zero flux contributions to all the seven-brane and D3-brane tadpoles. Therefore, we can not construct realistic flux models with the relaxed tadpole cancellation conditions. When studying the supersymmetric AdS vacua, we obtain flux models with the seven-brane and D3-brane tadpole cancellation conditions relaxed elegantly, and we present a semi-realistic Pati-Salam model as well as its particle spectrum. The lifting from the AdS vacua to the Minkowski/dS vacua remains a great challenge in flux model buildings on toroidal orientifolds.

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I. INTRODUCTION

The great challenge in string phenomenology is to construct realistic string models without additional chiral exotic particles at low energy and with the moduli fields stabilized. In particular, the intersecting D-brane models on Type II orientifolds [1], where the chiral fermions arise from the intersections of D-branes in the internal space [2] and the T-dual description in terms of magnetized D-branes [3], have been very interesting during the last decade [4]. Further employing the renormalization group equations in these models, we may test them at the Large Hadron Collider (LHC).

Initially many non-supersymmetric three-family Standard-like models and Grand Unified Theories (GUTs) were constructed on Type IIA orientifolds with intersecting D6-branes [4–7]. However, these models generically suffer uncancelled Neveu-Schwarz-Neveu-Schwarz (NSNS) tadpoles and the gauge hierarchy problem. Later, semi-realistic supersymmetric Standard-like and GUT models were constructed in Type IIA theory on the $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold [8–16] and also other backgrounds [17]. In particular, we emphasize that Pati-Salam like models, the only models that can realize all the Yukawa couplings at the stringy tree level, have been constructed systematically in Type IIA theory on the $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold [12, 16]. The Standard Model (SM) fermion masses and mixings can be generated and the gauge coupling unification can be realized in one of these models [18, 19], however, we are not able to stabilize the modulus fields in this model.

Although some of the complex structure moduli (in Type IIA picture) and the dilaton field might be stabilized due to the gaugino condensation in the hidden sector in some models (for example, see Ref. [20]), the stabilization of all moduli is still a big challenge. Important progresses have been made by introducing background fluxes. In Type IIB theory, the RR fluxes and NSNS fluxes generate a superpotential [21] that depends on the dilaton and complex structure moduli, and then stabilize these moduli dynamically [22, 23]. With non-perturbative effects, one can further determine the Kähler moduli [24]. For model building in such setup, the RR and NSNS fluxes contribute to large positive D3-brane charges due to the Dirac quantization [25, 26]. Thus, they modify the global RR tadpole cancellation conditions significantly and impose strong constraints on the consistent model building [27–31]. Including metric fluxes to RR and NSNS fluxes in Type IIA theory [32–34], we can stabilize the moduli in supersymmetric AdS vacua and relax the RR tadpole cancellation conditions [34, 35]. Interestingly, by relaxing the RR tadpole cancellation conditions, we can construct semi-realistic Type IIA [31, 36] Pati-Salam flux models capable of explaining the SM fermion masses and mixings. However, these models are in the AdS vacua and contain chiral exotic particles that are difficult to be decoupled.

Including the non-geometric and S-dual fluxes [37–39] on the Type IIB toroidal orien-
tifolds, the closed string moduli can be stabilized while the RR tadpole cancellation conditions can be relaxed elegantly in the supersymmetric Minkowski vacua \cite{40}, and the corresponding realistic IIB Pati-Salam flux models were constructed \cite{40}. However, the models in Ref. \cite{40} contain the Freed-Witten anomaly \cite{41} due to its strong constraints on model building. Interestingly, the Freed-Witten anomaly can be cancelled by introducing additional D-branes \cite{42}. In particular, these additional D-branes do not change the major properties of the D-brane models such as the four-dimensional $N = 1$ supersymmetry and the chiral particle spectra \cite{42}. Then we demonstrated a realistic Pati-Salam flux model in a supersymmetric Minkowski vacuum with the RR tadpole cancellation conditions relaxed and the Freed-Witten anomaly free conditions satisfied elegantly \cite{43}. Unfortunately, a mistaken flux algebra in Ref. \cite{38} made all of the interesting flux models \cite{43} gone with the wind.

Concisely, if we want to stabilize the string moduli, one of the most important questions in the realistic intersecting D-brane model building is: Are there supersymmetric Minkowski vacua with flux compatifications where the moduli can be stabilized and the RR tadpole cancellation conditions can be relaxed elegantly? During the last few years, we have been struggling with the search for supersymmetric Minkowski vacua with the RR, NSNS, metric, non-geometric and S-dual fluxes \cite{37–39, 44} compactified on Type IIB toroidal orientifolds. However, we did not find any interesting flux vacuum. Therefore we conjecture that in the generic supersymmetric Minkowski vacua with flux compatifications, at least one of the flux contributions to the seven-brane and D3-brane tadpoles will be positive and then their tadpole cancellation conditions can not be relaxed if the moduli are stabilized properly. In other words, we need to construct realistic flux models in AdS vacua and then lift them to Minkowski vacua similar to the KKLT mechanism \cite{24}. A parallel discussion of no-go theorems for dS vacua on supergravity algebras with generic fluxes can be found in \cite{45}.

In this paper, we first review the flux algebra and flux constraint equations as well as the intersecting D-brane model building setup on the Type IIB toroidal $T^6$ orientifolds with the RR, NSNS, metric, non-geometric and S-dual flux compactifications \cite{37–39, 44}. We simplify the fluxes reasonably, discuss the corresponding superpotential, consider the necessary conditions for supersymmetric Minkowski vacua and present all the concrete flux constraint equations. Because $T^6$ is factorized as $T^6 = T^2 \times T^2 \times T^2$, the real parts of the dilaton, Kähler and complex structure moduli must be positive real numbers when they are stabilized by the fluxes. We show that we can not have positive real parts of these moduli and negative/zero contributions to the seven-brane and D3-brane tadpoles simultaneously in supersymmetric Minkowski vacua, which results in lack of realistic flux models because these tadpole cancellation conditions are not relaxed. The seven-brane and D3-brane tadpole cancellation conditions can be relaxed elegantly in supersymmetric AdS
vacua, and we present a concrete Pati-Salam model as well as its particle spectrum. The lifting from the AdS vacua to the Minkowski/dS vacua is a great challenge and definitely needs further studies.

This paper is organized as what follows. In Section II, we briefly review the flux algebra and flux constraint equations. In Section III, we briefly review the intersecting D-brane model building. In Section IV, we reasonably simplify the fluxes, discuss the necessary conditions for supersymmetric Minkowski vacua, and present all the flux constraint equations in details. We show that there are no consistent supersymmetric Minkowski vacua with flux compactifications where all the seven-brane and D3-brane tadpole cancellation conditions can be relaxed. In Section V. We discuss the supersymmetric AdS vacua with flux compactifications and present a semi-realistic Pati-Salam model in Section VI. The Conclusion is given in Section VII.

II. FLUX ALGEBRA AND FLUX CONSTRAINT EQUATIONS

Let us consider Type IIB orientifold compactifications on $T^6/[\Omega(-1)^{F_L}\sigma]$, where $\Omega$ is the worldsheet parity, $(-1)^{F_L}$ is the left-mover spacetime fermionic number, and $\sigma$ is the involution. From the metric of the internal torus, we can introduce the complex structure moduli $U_k$ and Kähler moduli $T_k$, $k = 1, 2, 3$. In addition, the axion-dilaton modulus $S$ is given by $S = e^{-\phi} + iC_0$ with $C_0$ the RR 0-form. The Kähler potential is

$$K = -\log(S + \bar{S}) - \sum_{k=1}^{3} [\log(U_k + \bar{U}_k) + \log(T_k + \bar{T}_k)].$$

(1)

We can then introduce the non-trivial RR and NSNS 3-form fluxes, $F_3$ and $H_3$. These fluxes deform the moduli space and give a superpotential in the four-dimensional space as

$$W = \int (F_3 - iSH_3) \wedge \omega_3.$$  

(2)

The fluxes induce a D3-brane charge through the Chern-Simons coupling as

$$\frac{1}{2 \cdot 3!} \tilde{F}^{mnp} H_{mnp} = N_{D3/O3},$$

(3)

where $\tilde{F}^{mnp} \equiv \frac{1}{3!} \epsilon^{ijkopq} F_{opq}$, $i, j, k, m, n, o, p, q$ go from 1 to 6.

The 3-form fluxes are not enough to match the superpotentials of Type IIA and Type IIB compactifications under the T-duality, therefore we need to introduce additional fluxes. Consider the IIB NSNS 3-form flux $H_3$ acted under the T-duality

$$H_{mnp} \xrightarrow{T_m} -\omega_{mp} \xrightarrow{T_n} -\omega_{np} \xrightarrow{T_p} R_{mnp},$$

(4)
where \( \omega, Q, \) and \( R \) are introduced as geometric and non-geometric fluxes. Furthermore, to recover the \( SL(2, \mathbb{Z}) \) S-duality in Type IIB theory we introduce one more non-geometric flux \( P \) \cite{38, 44}.

Starting from a general magnetized D9-brane in Type I compactification on \( T^6 \) with “wrapping numbers” \((n_1, m_1) \times (n_2, m_2) \times (n_3, m_3)\), under T-duality this is corresponding to the D7\(_k\)-branes in Type IIB. For simplicity, we also assume an underline \( \mathbb{Z}_2 \times \mathbb{Z}_2 \) symmetry. Considering the case of extended \((p, q)\) 7-branes, the flux consistent conditions can be summarized as what follows \cite{44}:

- **Antisymmetry of Commutators**

\[
Q^{ab}_p P^{pc}_m - P^{ab}_p Q^{pc}_m = 0 ,
\]

\[
-Q^{ab}_p H^{clp} + P^{ab}_p \tilde{F}^{clp} + \tilde{H}^{pab} Q^{cd}_p - \tilde{F}^{pab} P^{cd}_p = 0 ,
\]

\[
(F^{I}_{k})^{[a}_{lp} Q^{cd}_p - p^{I}_{k} P^{cd}_p) = 0 .
\]

- **Jocobi Identities**

- **Bianchi Identities**

\[
Q^{[ab}_p Q^{clp}_l = 0 ,
\]

\[
P^{[ab}_p P^{clp}_l = 0 ,
\]

\[
Q^{[ab}_p P^{clp}_l = P^{[ab}_p Q^{clp}_l = 0
\]

\[
Q^{[a}_{lp} \tilde{H}^{bc]p} + Q^{[ab}_p \tilde{H}^{clp} - P^{[a}_{lp} \tilde{F}^{bc]p} - P^{[ab}_p \tilde{F}^{clp} = 0 .
\]

- **Seven-Brane Tadpoles**

\[
(QF_3)_k = - \sum_l (p^I_k)^2 d^I_k ,
\]

\[
(PH_3)_k = - \sum_l (q^I_k)^2 d^I_k ,
\]

\[
(PF_3 + QH_3)_k = -2 \sum_l p^I_k q^I_k d^I_k ,
\]

where \((AB)_3)_k = [AB]_{pm} = \frac{1}{2} A^{ab}_p B^{m}_{ab}, m = p - 3 = k.\]

- **Freed-Witten Anomalies**

\[
Q^{[ab}_p (c^I_k)^{clp} = 0 ,
\]

\[
P^{[ab}_p (c^I_k)^{clp} = 0 ,
\]

where \(c^I_k\) and \(d^I_k\) are coefficients of wrapping numbers of the \( I \)-th stack of seven-branes, and will be discussed in details below.
The individual items of the fluxes are marked into conventional notations \([38, 44]\). In brief, in the Type IIB picture the fluxes contain elements as \(F \supset \{m, q_k, e_k, e_0\}\), \(H \supset \{\bar{h}_0, a_k, a_k, h_0\}\), \(Q \supset \{h_k, b_{ij}, b_{ij}, \bar{h}_k\}\), and \(P \supset \{f_k, g_{ij}, \bar{g}_{ij}, \bar{f}_k\}\). The expansions of the flux constraints by the flux elements can be summarized as

- Antisymmetry of Commutators

From Eq. (5) \(QP - PQ = 0\), we obtain

\[
\begin{align*}
\bar{b}_{ij} f_k + b_{kk} g_{jj} & - b_{jj} g_{kk} - \bar{g}_{ij} h_k = 0, \quad (17) \\
b_{ik} \bar{g}_{ij} + b_{jj} \bar{g}_{jk} - \bar{b}_{ij} g_{ik} - \bar{b}_{jk} g_{jj} &= 0, \quad (18) \\
b_{jk} g_{ij} + b_{jj} \bar{g}_{ik} - \bar{b}_{ij} g_{jk} - \bar{b}_{ik} g_{jj} &= 0, \quad (19) \\
b_{jj} \bar{f}_k + \bar{b}_{kk} g_{ij} - \bar{b}_{ij} g_{kk} - \bar{h}_k g_{jj} &= 0, \quad (20) \\
b_{kj} g_{kk} + \bar{h}_k f_j - \bar{b}_{kk} g_{kj} - \bar{f}_k h_j &= 0, \quad (21) \\
b_{ik} f_j + b_{kj} g_{jk} - b_{jk} g_{kj} - \bar{g}_{ik} h_j &= 0, \quad (22) \\
b_{kk} f_j + b_{kj} f_k - g_{kk} h_j - g_{kj} h_k &= 0, \quad (23) \\
b_{jk} f_j + b_{kj} g_{ik} - b_{ik} g_{kj} - \bar{g}_{jj} h_j &= 0, \quad (24) \\
b_{kk} g_{kj} + \bar{h}_j f_k - b_{kj} g_{kk} - \bar{f}_j h_k &= 0, \quad (25) \\
b_{ik} f_j + b_{kj} \bar{g}_{jk} - b_{jk} g_{kj} - \bar{h}_j g_{ik} &= 0, \quad (26) \\
b_{jk} f_j + \bar{b}_{kj} \bar{g}_{ik} - b_{ik} g_{kj} - \bar{h}_j g_{jk} &= 0, \quad (27) \\
b_{kk} \bar{f}_j + b_{kj} \bar{f}_k - g_{kk} \bar{h}_j - g_{kj} \bar{h}_k &= 0, \quad (28) \\
b_{jj} f_k + b_{kk} g_{ij} - b_{ij} g_{kk} - \bar{g}_{jj} h_k &= 0, \quad (29) \\
b_{ik} \bar{g}_{jj} + b_{jj} \bar{g}_{ik} - b_{ij} g_{ik} - b_{jj} g_{ik} &= 0, \quad (30) \\
b_{ij} \bar{g}_{ik} + b_{jk} \bar{g}_{jj} - b_{ik} g_{ij} - b_{jj} g_{jk} &= 0, \quad (31) \\
b_{ij} \bar{f}_k + b_{kk} \bar{g}_{jj} - b_{jj} \bar{g}_{kk} - \bar{h}_k g_{ij} &= 0. \quad (32)
\end{align*}
\]

From Eq. (6) \(-Q \dot{H} + P \dot{F} + \dot{H} Q - \dot{F} P = 0\), we obtain

\[
\begin{align*}
\alpha_k b_{ik} + a_i b_{kk} + \bar{g}_{jk} e_0 + e_k g_{ik} + e_i g_{kk} + \bar{b}_{jk} h_0 & - \bar{a}_j h_k + f_k q_j = 0, \quad (33) \\
(\bar{a}_i b_{ii} + \bar{f}_i e_0 + e_i \bar{g}_{ii} + h_0 h_i) & + \bar{h}_i h_0 + \alpha_i b_{ii} + m f_i - g_{ii} q_i = 0, \quad (34) \\
(\bar{a}_i b_{ik} - a_j \bar{b}_{jk} + e_i \bar{g}_{ik} + g_{jk} q_j) & - \bar{a}_j b_{jk} + a_i \bar{b}_{ik} - e_j \bar{g}_{jk} - g_{ik} q_i = 0, \quad (35) \\
\bar{a}_k \bar{b}_{jk} + a_i \bar{b}_{kk} + b_{ik} h_0 & - \bar{a}_i h_k - e_i f_k + m g_{ik} - \bar{g}_{kk} q_j - \bar{g}_{jk} q_k = 0. \quad (36)
\end{align*}
\]
From Eq. (7) for $k \neq a, i = 1, 2, 3$, we obtain

\[
\frac{m_a^I}{n_a^I} (f_a p_k^I - h_a q_k^I) = 0 ,
\] (37)

\[
\frac{m_a^I}{n_a^I} (g_a p_k^I - b_a q_k^I) = 0 ,
\] (38)

\[
\frac{m_a^I}{n_a^I} (\bar{g}_a p_k^I - \bar{b}_a q_k^I) = 0 ,
\] (39)

\[
\frac{m_a^I}{n_a^I} (\bar{f}_a p_k^I - \bar{h}_a q_k^I) = 0 ,
\] (40)

- **Jocobi Identities**

- **Bianchi Identities**

From Eq. (8) $QQ = 0$, we obtain

\[
-b_i b_{jk} + \bar{b}_{ki} h_{kk} + h_i \bar{b}_{kk} - b_{ij} b_{ik} = 0 ,
\] (41)

\[
-\bar{b}_i \bar{b}_{jk} + b_{ki} \bar{h}_{kk} + \bar{h}_i \bar{b}_{kk} - \bar{b}_{ij} \bar{b}_{ik} = 0 ,
\] (42)

\[
-b_i \bar{b}_{ij} + \bar{b}_{ji} b_{jj} + h_j \bar{h}_{jj} - b_{ki} \bar{b}_{kj} = 0 ,
\] (43)

\[
\bar{b}_i b_{ij} - b_{ji} \bar{b}_{ij} + h_i \bar{h}_{jj} - b_{kj} \bar{b}_{kj} = 0 .
\] (44)

From Eq. (9) $PP = 0$, we obtain

\[
-g_i g_{jk} + \bar{g}_{ki} \bar{f}_k + f_j \bar{g}_{kk} - g_{ji} g_{kk} = 0 ,
\] (45)

\[
-\bar{g}_i \bar{g}_{jk} + g_{ki} \bar{f}_k + \bar{f}_j g_{kk} - \bar{g}_{ji} \bar{g}_{kk} = 0 ,
\] (46)

\[
-g_{ii} g_{ij} + \bar{g}_{ji} g_{jj} + f_i \bar{f}_j - g_{ki} \bar{g}_{kj} = 0 ,
\] (47)

\[
g_{ii} g_{ij} - g_{ji} g_{jj} + f_i \bar{f}_j - g_{kj} \bar{g}_{kj} = 0 .
\] (48)

From Eq. (10) $QP = 0$, we obtain

\[
b_{kk} \bar{g}_{kj} - h_k \bar{f}_j - \bar{b}_{jk} g_{jj} + b_{ik} \bar{g}_{ij} = 0 ,
\] (49)

\[
b_{kk} g_{ij} - h_k g_{jj} - \bar{b}_{jk} f_j + b_{ik} g_{kj} = 0 ,
\] (50)

\[
\bar{b}_{kk} \bar{g}_{kj} - \bar{h}_k g_{jj} - b_{jk} \bar{f}_j + \bar{b}_{ik} \bar{g}_{kj} = 0 ,
\] (51)

\[
\bar{b}_{kk} g_{ij} - \bar{h}_k f_j - b_{jk} g_{jj} + \bar{b}_{ik} g_{ij} = 0 .
\] (52)

From Eq. (10) $PQ = 0$, we obtain

\[
g_{kk} \bar{b}_{kj} - f_k \bar{h}_j - \bar{g}_{jk} b_{jj} + g_{ik} \bar{b}_{ij} = 0 ,
\] (53)

\[
g_{kk} b_{ij} - f_k b_{jj} - g_{jk} h_j + g_{ik} b_{kj} = 0 ,
\] (54)

\[
\bar{g}_{kk} \bar{b}_{ij} - \bar{f}_k b_{jj} - g_{jk} \bar{h}_j + \bar{g}_{ik} \bar{b}_{kj} = 0 ,
\] (55)

\[
\bar{g}_{kk} b_{kj} - \bar{f}_k h_j - g_{jj} \bar{b}_{jj} + g_{ik} b_{ij} = 0 .
\] (56)
From Eq. (11), we have
\[ Q^i \tilde{H}^i + Q^j \tilde{H}^j - P^i \tilde{F}^i - P^j \tilde{F}^j = 0, \]
and
\[ a_k b_{ik} + a_i b_{ik} + \bar{g}_{jk} e_0 + e_k g_{jk} + e_i g_{ik} + \bar{b}_{jk} h_0 - \bar{a}_j h_k + f_k q_j = 0, \]  
\[ (\bar{a}_i b_{ii} + \bar{h}_0 h_i + e_0 f_i + e_i g_{ii}) - a_j \bar{b}_{ji} + \bar{a}_j b_{ki} + e_k g_{ki} + q_j g_{ji} + (h_0 \bar{h}_i + a_i \bar{b}_{ii} + m f_i - q_i g_{ii}) + a_k \bar{b}_{ki} - a_j \bar{b}_{ji} - e_j \bar{g}_{ji} - q_k g_{ki} = 0. \]  

- Seven-Brane Tadpole Constraint Equations

Let \((n_i', m_i')\), \(i = 1, 2, 3\) be the wrapping numbers, then \(d_i' = -n_i' m_j' m_k'\), \(i \neq j \neq k\).

From Eq. (12), we obtain
\[ \sum_i (p_i')^2 d_i' + \frac{1}{2} [m h_i - e_0 \bar{h}_i - \sum_j (q_j b_{ji} + e_j \bar{b}_{ji})] = 0. \]  
From Eq. (13), we obtain
\[ \sum_i (q_i')^2 d_i' + \frac{1}{2} [h_0 f_i - \bar{h}_0 f_i - \sum_j (\bar{a}_j g_{ji} - a_j \bar{g}_{ji})] = 0. \]  
From Eq. (14), we obtain
\[ -2 \sum_i (p_i' q_i') d_i' + \frac{1}{2} [\bar{h}_0 h_i - \bar{h}_i h_0 + \sum_j (\bar{a}_j b_{ji} - a_j \bar{b}_{ji})]
+ e_0 f_i - m f_i + \sum_j (q_j g_{ji} + e_j \bar{g}_{ji}) = 0. \]

- D3-Brane RR Tapole Constraint Equation

From Eq. (13), we obtain
\[ \sum_i n_i' n_2' n_3' + \frac{1}{2} [m h_0 - e_0 \bar{h}_0 + \sum_i (q_i a_i + e_i \bar{a}_i)] = 16. \]

- Freed-Witten Anomalies

From Eq. (15), for \(i \neq j \neq k\), \(a = 1, 2, 3\), we obtain
\[ (h_j m_j' n_i' + h_i m_j' n'_j) n_k = 0, \]  
\[ (b_{aj} m_j' n_i' + b_{ai} m_j' n'_j) n_k = 0, \]  
\[ (\bar{b}_{aj} m_j' n_i' + \bar{b}_{ai} m_j' n'_j) n_k = 0, \]  
\[ (\bar{h}_j m_j' n_i' + \bar{h}_i m_j' n'_j) n_k = 0. \]  
From Eq. (16), for \(i \neq j \neq k\), \(a = 1, 2, 3\), we obtain
\[ (f_j m_j' n_i' + f_i m_j' n'_j) n_k = 0, \]  
\[ (g_{aj} m_j' n_i' + g_{ai} m_j' n'_j) n_k = 0, \]  
\[ (\bar{g}_{aj} m_j' n_i' + \bar{g}_{ai} m_j' n'_j) n_k = 0, \]  
\[ (f_j m_j' n_i' + \bar{f}_i m_j' n'_j) n_k = 0. \]
III. INTERSECTING D-BRANE MODEL BUILDING ON TYPE IIB ORIENTIFOLD

We now consider the Type IIB string theory compactified on a $T^6$ orientifold where $T^6$ is a six-torus factorized as $T^6 = T^2 \times T^2 \times T^2$ whose complex coordinates are $z_i, i = 1, 2, 3$ for the $i$-th two-torus, respectively [25, 26, 29]. The orientifold projection is implemented by gauging the symmetry $\Omega R$, where $\Omega$ is world-sheet parity, and $R$ is given by

$$R: (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, -z_3).$$

(72)

Thus, the model contains 64 O3-planes. In order to cancel the negative RR charges from these O3-planes, we introduce the magnetized D$(3+2n)$-branes which are filling up the four-dimensional Minkowski space-time and wrapping 2n-cycles on the compact manifold. Concretely, for one stack of $N_a$ D-branes wrapped $m^i_a$ times on the $i$-th two-torus $T^2_i$, we turn on $n^i_a$ units of magnetic fluxes $F^i_a$ for the center of mass $U(1)_a$ gauge factor on $T^2_i$, such that

$$m^i_a \frac{1}{2\pi} \int_{T^2_i} F^i_a = n^i_a,$$

(73)

where $m^i_a$ can be half integer for tilted two-torus. Then, the D9-, D7-, D5- and D3-branes contain 0, 1, 2 and 3 vanishing $m^i_a$s, respectively. Introducing for the $i$-th two-torus the even homology classes $[0_i]$ and $[T^2_i]$ for the point and two-torus, respectively, the vectors of the RR charges of the $a$ stack of D-branes and its image are

$$[\Pi_a] = \prod_{i=1}^{3} (n^i_a[0_i] + m^i_a[T^2_i]),$$

$$[\Pi'_a] = \prod_{i=1}^{3} (n^i_a[0_i] - m^i_a[T^2_i]),$$

(74)

respectively. The “intersection numbers” in Type IIA language, which determine the chiral massless spectrum, are

$$I_{ab} = [\Pi_a] \cdot [\Pi_b] = \prod_{i=1}^{3} (n^i_a m^i_b - n^i_b m^i_a).$$

(75)

Moreover, for a stack of $N$ D$(2n+3)$-branes whose homology classes on $T^6$ is (not) invariant under $\Omega R$, we obtain a $(U(N))\ USp(2N)$ gauge symmetry with three (adjoint) anti-symmetric chiral superfields due to the orbifold projection. The physical spectrum is presented in Table II.

The flux models on Type IIB orientifolds with four-dimensional $N = 1$ supersymmetry are primarily constrained by the RR tadpole cancellation conditions that will be given
TABLE I: General spectrum for magnetized D-branes on the Type IIB $T^6$ orientifold.

| Sector | Representation |
|--------|----------------|
| $aa$   | $U(N_a)$ vector multiplet |
|        | 3 adjoint multiplets |
| $ab + ba$ | $I_{ab} (N_a, N_b)$ multiplets |
| $ab' + b'a$ | $I_{ab'} (N_a, N_b)$ multiplets |
| $aa' + a'a$ | $\frac{1}{2}(I_{aa'} - I_{O3})$ symmetric multiplets |
|        | $\frac{1}{2}(I_{aa'} + I_{O3})$ anti-symmetric multiplets |

later, the four-dimensional $N = 1$ supersymmetry condition, and the K-theory anomaly free conditions. For the D-branes with world-volume magnetic field $F^i_a = n^i_a / (m_a \chi_i)$ where $\chi_i$ is the area of $T^2_i$ in string units, the condition to preserve the four-dimensional $N = 1$ supersymmetry is

$$
\sum_i \left( \tan^{-1}(F^i_a)^{-1} + \theta(n^i_a) \pi \right) = 0 \mod 2\pi.
$$

(76)

where $\theta(n^i_a) = 1$ for $n^i_a < 0$ and $\theta(n^i_a) = 0$ for $n^i_a \geq 0$. The K-theory anomaly free conditions are

$$
\sum_a N_a m_a^1 n_a^2 m_a^3 = \sum_a N_a n_a^1 m_a^2 n_a^3 = \sum_a N_a n_a^1 m_a^2 n_a^3
$$

$$
= \sum_a N_a n_a^1 n_a^2 m_a^3 = 0 \mod 2.
$$

(77)

And the holomorphic gauge kinetic function for a generic stack of D($2n+3$)-branes is given by

$$
f_a = \frac{1}{\kappa_a} \left( n_a^1 n_a^2 n_a^3 s - n_a^1 m_a^2 m_a^3 t_1 - n_a^2 m_a^1 m_a^3 t_2 - n_a^3 m_a^1 m_a^2 t_3 \right),
$$

(78)

where $\kappa_a$ is equal to 1 and 2 for $U(n)$ and $USp(2n)$, respectively.

In general, this kind of D-brane models possesses Freed-Witten anomalies [25, 41]. In the world-volume of a generic stack of D-branes we have a $U(1)$ gauge field whose scalar partner parameterizes the D-brane position in the compact space. Such kind of $U(1)$’s usually obtain Stückelberg masses by swallowing RR scalar fields and then decouple from the low-energy spectrum. At the same time these scalars participate in the cancellations of $U(1)$ gauge anomalies through a generalized Green-Schwarz mechanism [48].
IV. SIMPLIFYING THE FLUX CONSTRAINT EQUATIONS AND CONDITIONS FOR SUPERSYMMETRIC MINKOWSKI VACUA

The system with general fluxes is very complicated, therefore, we intend to simply the system by some isotropy conditions. First we assume the three complex structure moduli the same. For simplicity, we redefine the dilaton $S$, the three Kähler moduli $T_i$, and the three complex structure moduli $U_i$ as

$$ S \equiv -i\sigma, \quad T_i \equiv -i\tau_i, \quad U_i \equiv U \equiv -i\rho. \quad (79) $$

Because the real parts of $S$, $T_i$, and $U_i$ must be positive real numbers, the imaginary parts of $\sigma$, $\tau_i$ and $\rho$ must be positive as well. The fluxes can be simplified as

$$ e_i = e, \quad q_i = q, \quad a_i = a, \quad \bar{a}_i = \bar{a}, $$
$$ b_{ij} = b_{ij}, \quad b_{ii} = \beta_i, \quad \bar{b}_{ij} = \bar{b}_j, \quad \bar{b}_{ii} = \bar{\beta}_i, \quad (80) $$
$$ g_{ij} = g_{ij}, \quad g_{ii} = \gamma_i, \quad \bar{g}_{ij} = \bar{g}_j, \quad \bar{g}_{ii} = \bar{\gamma}_i. \quad (81) $$

The corresponding superpotential including the fluxes is

$$ W = e_0 + 3e\rho + 3q\rho^2 - m\rho^3 $$
$$ + \sigma[h_0 + 3a\rho - 3\bar{a}\rho^2 - \bar{h}_0\rho^3] $$
$$ + \sum_i \tau_i[-h_i + (2b_i + \beta_i)\rho - (2\bar{b}_i + \beta_i)\rho^2 + \bar{h}_i\rho^3] $$
$$ + \sigma \sum_i \tau_i[f_i - (2g_i + \gamma_i)\rho + (2\bar{g}_i + \gamma_i)\rho^2 - \bar{f}_i\rho^3]. \quad (82) $$

The Kähler moduli are not simplified yet because we want to keep some degrees of freedoms for model building. For convenience we assume $\tau F = \tau_i F_i$, $\tau_i = k_i\tau$ and $F_i = F/k_i$, where $k_i$ are real and positive constants depending on the models, and $F \in \{b, \bar{b}, \beta, \bar{\beta}, f, \bar{f}, g, \bar{g}, \gamma, \bar{\gamma}, h, \bar{h}\}$. Thus, the real part of $\tau$ must be also a positive real number, and the superpotential $W$ turns out

$$ W = E_1 + \sigma E_2 + \tau E_3 + \sigma\tau E_4 $$
$$ = e_0 + 3e\rho + 3q\rho^2 - m\rho^3 $$
$$ + \sigma[h_0 + 3a\rho - 3\bar{a}\rho^2 - \bar{h}_0\rho^3] $$
$$ + 3\tau[-h + (2b + \beta)\rho - (2\bar{b} + \beta)\rho^2 + \bar{h}\rho^3] $$
$$ + 3\sigma\tau[f - (2g + \gamma)\rho + (2\bar{g} + \gamma)\rho^2 - \bar{f}\rho^3]. \quad (83) $$

If we pursue AdS vacua, then it is required that

$$ \frac{\partial W}{\partial \sigma} = \frac{\partial W}{\partial \rho} = \frac{\partial W}{\partial \tau} = 0. \quad (84) $$
If \( E_4 \neq 0 \), it implies
\[
\sigma = \frac{E_3}{E_4}, \quad \tau = \frac{E_2}{E_4}, \quad (85)
\]

On the other hand, if we look for Minkowski vacua, the additional condition is \( W = 0 \), so we can define a new polynomial \( E \) as
\[
E = E_1E_4 - E_2E_3 = 0, \quad \frac{\partial E}{\partial \rho} = 0. \quad (86)
\]

The function must have a double root \( \rho_0 \), and its complex conjugate. Therefore, the function \( E = 0 \) can be written as
\[
E = (\rho - \rho_0)^2(\rho - \rho_0^*)^2(A\rho^2 + B\rho + C). \quad (87)
\]

The simplified flux constraint equations are:

1. Antisymmetry of Commutators

From Eq. (5) \( QP - PQ = 0 \), we obtain
\[
\begin{align*}
\text{(17), (22), (24)} & \rightarrow \bar{b}f = g\bar{h}, & (88) \\
\text{(18)} = \text{(19)} & \rightarrow \bar{b}\bar{g} - \bar{b}g + \beta\bar{g} - \bar{b}\gamma = 0 , & (89) \\
\text{(26), (27), (32)} & \rightarrow b\bar{f} = g\bar{h}, & (90) \\
\text{(30)} = \text{(31)} & \rightarrow \bar{b}\bar{g} - \bar{b}g - \bar{b}\gamma = 0 , & (91) \\
\text{(20)} & \rightarrow \beta\bar{f} - h\gamma + \beta\bar{g} - \bar{b}\gamma = 0 , & (92) \\
\text{(21)} & \rightarrow f\bar{h} - \bar{f}h - \beta g + b\gamma = 0 , & (93) \\
\text{(23)} & \rightarrow \beta f - h\gamma + bf - gh = 0 , & (94) \\
\text{(25)} & \rightarrow f\bar{h} - \bar{f}h + \beta\bar{g} - \bar{b}\gamma = 0 , & (95) \\
\text{(28)} & \rightarrow \beta\bar{f} - h\gamma + \bar{b}\bar{f} - \bar{g}\bar{h} = 0 , & (96) \\
\text{(29)} & \rightarrow \beta f - h\gamma + \beta g - b\gamma = 0 . & (97)
\end{align*}
\]

From Eq. (6) \( -Q\bar{H} + P\bar{F} + \bar{H}Q - \bar{F}P = 0 \), we obtain
\[
\begin{align*}
\text{(33)} & \rightarrow ab + a\beta + e_0\bar{g} + eg + e\gamma + \bar{b}h_0 - \bar{a}h + fq = 0 , & (98) \\
\text{(34)} & \rightarrow \bar{a}\beta + a\bar{\beta} + e_0\bar{f} + e\gamma + h_0\bar{h} + \bar{h}_0h + mf - \gamma q = 0 , & (99) \\
\text{(35)} & \rightarrow 0 = 0 , & (100) \\
\text{(36)} & \rightarrow \bar{a}\bar{b} + a\bar{\beta} - e\bar{f} + \bar{b}\bar{h}_0 - \bar{a}\bar{h} + mg - \bar{g}q - \gamma q = 0 . & (101)
\end{align*}
\]
From Eq. (7), $i \neq k$, we obtain

$$\frac{m_i^l}{n_i}(f_i p_k^l - h_i q_k^l) = 0, \quad (102)$$

$$\frac{m_i^l}{n_i}(g_i p_k^l - b_i q_k^l) = 0, \quad \frac{m_i^l}{n_i}(\gamma_i p_k^l - \beta_i q_k^l) = 0, \quad (103)$$

$$\frac{m_i^l}{n_i}(\bar{g}_i p_k^l - \bar{b}_i q_k^l) = 0, \quad \frac{m_i^l}{n_i}(\bar{\gamma}_i p_k^l - \bar{\beta}_i q_k^l) = 0, \quad (104)$$

$$\frac{m_i^l}{n_i}(\bar{f}_i p_k^l - \bar{h}_i q_k^l) = 0. \quad (105)$$

2. Jocobi Identities

• Bianchi Identities

From Eq. (8) $QQ = 0$, we obtain

$$\rightarrow - \beta b + \bar{b}h + h\bar{\beta} - bb = 0 , \quad (106)$$

$$\rightarrow - \bar{\beta} \bar{b} + b\bar{h} + \bar{h}\beta - \bar{b}b = 0 , \quad (107)$$

$$\rightarrow h\bar{h} = bb . \quad (108)$$

From Eq. (9) $PP = 0$, we obtain

$$\rightarrow - \gamma g + \bar{g}f + f\bar{\gamma} - gg = 0 , \quad (109)$$

$$\rightarrow - \bar{\gamma} \bar{g} + g\bar{f} + \bar{f}\gamma - \bar{g}\bar{g} = 0 , \quad (110)$$

$$\rightarrow f\bar{f} = g\bar{g} . \quad (111)$$

From Eq. (10) $QP = 0$, we obtain

$$\rightarrow \beta \bar{g} - h\bar{f} - \bar{b}\gamma + b\bar{g} = 0, \quad \oplus (89) \rightarrow \bar{f}h = \bar{b}g , \quad (112)$$

$$\rightarrow - \beta g + \bar{b}f + h\bar{\gamma} - bg = 0 , \quad (113)$$

$$\rightarrow - \bar{\beta} \bar{g} + b\bar{f} + \bar{h}\gamma - \bar{b}g = 0 , \quad (114)$$

$$\rightarrow \beta g - \bar{h}f - b\bar{\gamma} + \bar{b}g = 0, \quad \oplus (91) \rightarrow \bar{f}h = \bar{b}g . \quad (115)$$

From Eq. (10) $PQ = 0$, we obtain

$$\rightarrow \bar{b}\gamma - f\bar{h} - \beta \bar{g} + b\bar{g} = 0, \quad \oplus (89) \rightarrow \bar{f}h = \bar{b}g , \quad (116)$$

$$\rightarrow b\gamma - f\bar{\beta} - h\bar{g} + bg = 0 , \quad (117)$$

$$\rightarrow \bar{b}\gamma - \bar{f} \bar{\beta} - \bar{h}g + \bar{b}g = 0 , \quad (118)$$

$$\rightarrow b\bar{\gamma} - \bar{f}h - \bar{\beta} g + \bar{b}g = 0, \quad \oplus (91) \rightarrow \bar{f}h = \bar{b}g . \quad (119)$$
From Eq. (11) \[ Q[\tilde{H}] + Q[\tilde{H}] - P[\tilde{F}] - P[\tilde{F}] = 0, \] we obtain
\[(57) \rightarrow (98), \quad (58) \rightarrow (101), \quad (59) \rightarrow (99).\]

V. SUPERSYMMETRIC MINKOWSKI FLUX VACUA

A. Scenarios with Flux \( P = 0 \)

It is hard to satisfy the third anti-symmetric conditions in Eqs. (102)-(105) for a model consisting of more than two stacks of D-branes. Therefore, a reasonable choice is setting \( P = 0 \), which implies \( E_4 = 0 \). Then the superpotential turns out to be
\[ W = E_1 + \sigma E_2 + \tau E_3. \] (120)

To obtain Minkowski vacua, we can conclude the conditions as
\[ E_1 = E_2 = E_3 = 0, \quad \sigma = -\frac{E_1'}{E_2'}, \quad \tau = -\frac{E_1'}{E_3'}. \] (121)

The dilaton modulus \( \sigma \) is determined by \( E_2 \) while the Kähler modulus \( \tau \) is controlled by \( E_3 \). For simplicity, we will temporarily ignore \( E_2 \) by setting the corresponding fluxes \( H : \{a, \bar{a}, h_0, \bar{h}_0\} \) zero because they are related to fewer constraints and can be included independently later. Then we obtain the following constraints of the fluxes in \( E_3 \) from the Bianchi identities
\[ b(b + \beta) = h(\bar{b} + \bar{\beta}), \quad \bar{h}(b + \beta) = \bar{b}(\bar{b} + \bar{\beta}), \quad \bar{b} = h\bar{h}. \] (122)

Rewriting these flux conditions by introducing two parameters \( \xi \) and \( \chi \), we obtain
\[ h = b\xi, \quad b + \beta = h\chi = b\xi\chi, \quad \bar{b} = \bar{h}\xi, \quad \bar{b} + \bar{\beta} = b\chi, \] (123)

Therefore, \( E_3 \) can be factorized as
\[ E_3 = 3 \left[ -h + (b + h\chi)\rho - (\xi\bar{h} + b\chi)\rho^2 + \bar{h}\rho^3 \right] \] (124)
\[ = 3(\rho - \xi)(\bar{h}\rho^2 - b\chi\rho + b). \] (125)

The complex roots of \( E_3 \) are
\[ \rho_0 = \frac{b\chi \pm \sqrt{b^2\chi^2 - 4bh}}{2h} \Rightarrow b^2\chi^2 < 4bh, \quad bh > 0, \] (126)
and
\[ \text{Re}(\rho_0) = \frac{b\chi}{2h} \equiv R, \quad \text{Im}(\rho_0) = \pm \frac{\sqrt{b^2\chi^2 - 4bh}}{2h} \equiv \pm I. \] (127)
\( E_1 \) has the same complex roots, so we assume
\[
E_1 = e_0 + 3e \rho + 3q \rho^2 - m \rho^3 = (A \rho + B)(\bar{h}\rho^2 - b \chi \rho + b).
\] (128)

Comparing the coefficients, we obtain
\[
A = -\frac{m}{h}, \quad B = \frac{e_0}{b}, \quad 3q = \frac{e_0 \bar{h}}{b} + \frac{mb \chi}{h}, \quad 3e = -\frac{mb}{h} - e_0 \chi.
\] (129)

Then, we have
\[
\tau_0 = \frac{E_1'}{E_3'}|_{\rho_0} = -\frac{(A \rho_0 + B)}{3(\rho_0 - \xi)},
\]
\[
\text{Im}(\tau_0) = \frac{\pm I}{3[(R - \xi)^2 + I^2]} \left( \frac{e_0 \bar{h} - mh}{\bar{h}b} \right) = \frac{\text{Im}(\rho_0)}{3[(R - \xi)^2 + I^2]} \left( e_0 \bar{h} - mh \right). \] (130)

1. \((p, q) = (1, 0)\)

First we consider the case with only D-branes where \(p_i = 1\) and \(q_i = 0\). Recall that \(b \bar{h} > 0\). The D7-brane tadpole contribution from the fluxes is
\[
N_{\text{fluxD7}} = \frac{1}{2} \left[ mh - e_0 \bar{h} - (2b + \beta)q - (2\bar{b} + \bar{\beta})e \right]
= \frac{1}{6} (mh - e_0 \bar{h}) \left( 4 - \frac{b \chi^2}{h} \right). \] (132)

In this paper, we denote and emphasize that \(N_{\text{fluxD7}}\) is the number of D7-branes that we need to introduce for D7-brane tadpole cancellations due to the flux contributions. In other words, \(-N_{\text{fluxD7}}\) is the flux contribution to the D7-brane tadpoles.

We have learned \(b^2 \chi^2 < 4b \bar{h}\) for \(\rho_0\) being complex, so \(4 - \frac{b \chi^2}{h} > 0\). It turns out that \(N_{\text{fluxD7}}\) has always negative sign to \(\text{Im}(\tau_0)\). Thus, we do not have supersymmetric Minkowski flux vacua that the D7-brane tadpole cancellation condition is relaxed and the moduli are stabilized properly.

2. \((p, q) = (0, 0)\)

The D7-brane tadpole contribution from the fluxes has to be zero, therefore
\[
N_{\text{fluxD7}} = \frac{1}{6} (mh - e_0 \bar{h}) \left( 4 - \frac{b \chi^2}{h} \right) = 0.
\] (133)

For \(\rho_0\) to be a complex number, we must require \(4 - \frac{b \chi^2}{h} > 0\). So we have \(mh - e_0 \bar{h} = 0\), which implies that \(\tau_0\) has only real part. Thus, the moduli \(\tau_i\) can not be stabilized properly.
Finally we consider the general case with \( p \cdot q \neq 0 \). Similarly to the \((p, q) = (1, 0)\) case, \( N_{\text{fluxD7}} \) has always negative sign to \( \text{Im}(\tau_0) \). And since \( P = 0 \), the flux contribution to the NS7-brane tadpole is zero. Thus, we do not have supersymmetric Minkowski flux vacua with the D7-brane tadpole cancellation condition relaxed and the moduli stabilized properly.

B. General Discussions for \( P \neq 0 \)

The constraint condition of the flux \( P = 0 \) is too stringent, thus, we consider the cases with \( P \neq 0 \). With the fluxes in \( E_4 \) non-zero, the Bianchi identities in Eqs. (109)–(111) give

\[
g(g + \gamma) = f(g + \bar{\gamma}), \quad \bar{f}(g + \gamma) = \bar{g}(g + \bar{\gamma}), \quad \bar{g}g = \bar{f}f.
\]

Similarly, we can rewrite these flux conditions by introducing parameters \( \xi' \) and \( \chi' \) as

\[
f = g\xi', \quad g + \gamma = f\chi', \quad \bar{g} = \bar{f}\xi', \quad \bar{g} + \bar{\gamma} = g\chi'.
\]

In addition, there are also the Bianchi conditions between fluxes \( P \) and \( Q \):

\[
f\bar{h} = b\bar{g}, \quad \bar{f}h = bg.
\]

After factorization and using Eq. (90), we can rewrite \( E_4 \) as follows

\[
E_4 = -3(\rho - \xi')(\bar{f}\rho^2 - g\chi'\rho + g) = -3\frac{g}{b}(\rho - \xi')(\bar{h}\rho^2 - b\chi'\rho + b).
\]

From the previous subsection we learned that \( E_3 \) can be rewritten as \( E_3 = 3(\rho - \xi)(\bar{h}\rho^2 - b\chi\rho + b) \), and \( \chi^2 - \frac{4b}{b} < 0 \) for \( \rho \) being complex. Plugging Eqs. (123) and (135) into the antisymmetry conditions, we find that it must be \( \chi' = \chi \). In other words, \( E_3 \) and \( E_4 \) have the same factor. Recall the Minkowski condition in Eq. (86), we obtain

\[
E = E_1E_4 - E_2E_3
\]

\[
= -\frac{3}{b}(\bar{h}\rho^2 - b\chi\rho + b)[g(\rho - \xi')(e_0 + 3e\rho + 3g\rho^2 - m\rho^3)
\]

\[
+ b(\rho - \xi)(h_0 + 3a\rho - 3\bar{a}\rho^2 - \bar{h}_0\rho^3)],
\]

and

\[
\sigma = -\frac{E_3}{E_4} = \frac{b(\rho - \xi)}{g(\rho - \xi')}, \quad \tau = -\frac{E_2}{E_4}.
\]

Since the coefficients of \( E_2 \) are real and \( \tau_0 \) is limited with \( \rho_0 \), \( E_2 \) must also contain the factor \( (\bar{h}\rho^2 - b\chi\rho + b) \). In addition, \( E \) must have double roots for \( E' = 0 \), therefore \( E_1 \) has the same factor \( (\bar{h}\rho^2 - b\chi\rho + b) \) as well. In summary, \( E_1 \) and \( E_2 \) can be rewritten as

\[
E_1 = -\frac{m}{h}\tau - \frac{e_0}{b}(\bar{h}\rho^2 - b\chi\rho + b), \quad E_2 = -\frac{h_0}{\bar{h}}(\bar{h}\rho^2 - b\chi\rho + b),
\]

\[
(139)
\]
with the flux relations

\[ q = \frac{1}{3} \left( \frac{e_0 h}{b} + \frac{mb}{h} \chi \right), \quad e = -\frac{1}{3} \left( \frac{mb}{h} + e_0 \chi \right), \]

\[ a = -\frac{1}{3} \left( \frac{h_0 h}{b} + \frac{h_0 b}{h} \chi \right), \quad a = -\frac{1}{3} \left( \frac{h_0 b}{h} + h_0 \chi \right). \] (140)

The imaginary parts of moduli \( \sigma \) and \( \tau \) must have the same sign as the imaginary part of \( \rho \). From the above equations we obtain

\[ \sigma = \frac{b(\rho - \xi)}{g(\rho - \xi')}, \quad \Rightarrow \quad \frac{b}{g}(\xi - \xi') = \frac{gh - bf}{g^2} > 0; \] (141)

\[ \tau = \frac{-fh_0 \rho + \bar{g}h_0}{3g(g \rho - f)}, \quad \Rightarrow \quad -\frac{1}{3gf}(fh_0 - f\bar{h}_0) > 0. \] (142)

The antisymmetry constraints from Eqs. (98), (99), and (101) can be written as

\[ (hh_0 + e_0 f)(4 - \frac{b\chi^2}{h}) = 0, \]

\[ (\bar{h}_0 \bar{h} + mf)(4 - \frac{b\chi^2}{h}) = 0, \] (143)

\[ (h\bar{h}_0 + e_0 \bar{f} + h_0 \bar{h} + mf)(4 - \frac{b\chi^2}{h}) = 0. \]

Since we require \( \chi^2 - \frac{4h}{b} < 0 \) for \( \rho \) to be complex, it turns out that

\[ hh_0 + e_0 f = 0, \quad \bar{h}_0 \bar{h} + mf = 0, \quad h\bar{h}_0 + e_0 \bar{f} + h_0 \bar{h} + mf = 0. \] (144)

From Eqs. (60)-(63), the D7, NS7, I7, and D3 flux contributions to the corresponding tadpoles are

\[ N_{\text{fluxD7}} : \frac{1}{2}[mh - e_0 \bar{h} - (2b + \beta)q - (2b + \bar{\beta})e] = \frac{1}{6}(mh - e_0 \bar{h})(4 - \frac{b\chi^2}{h}), \] (145)

\[ N_{\text{fluxNS7}} : \frac{1}{2}[\bar{f}h_0 - f\bar{h}_0 - \bar{a}(2g + \gamma) + a(2\bar{g} + \gamma)] = \frac{1}{6}(\bar{f}h_0 - f\bar{h}_0)(4 - \frac{b\chi^2}{h}), \] (146)

\[ N_{\text{fluxI7}} : \frac{1}{2}[\bar{h}_0 h - \bar{h}_0 \bar{h} + \bar{a}(2b + \beta) - a(2\bar{b} + \bar{\beta}) + e_0 \bar{f} - mf + q(2g + \gamma) + e(2\bar{g} + \gamma)] \]

\[ \quad = \frac{1}{6}(\bar{h}_0 h - \bar{h}_0 \bar{h} + e_0 \bar{f} - mf)(4 - \frac{b\chi^2}{h}), \] (147)

\[ N_{\text{D3}} : 16 - \frac{1}{2}[mh_0 - e_0 \bar{h}_0 + 3qa + 3e\bar{a}] = 16 - \frac{1}{6}(mh_0 - e_0 \bar{h}_0)(4 - \frac{b\chi^2}{h}). \] (148)

The third antisymmetry condition in Eq. (7) has not been confined, and we will consider it in two cases, \( (p, q) = (0, 0) \) and \( p \cdot q \neq 0 \).

Here, we emphasize again that \( N_{\text{fluxD7}}, N_{\text{fluxNS7}}, N_{\text{fluxI7}}, \) and \( N_{\text{D3}} \) are respectively the numbers of D7-branes, NS7-branes, I7-branes, and D3-branes that we need to introduce for their tadpole cancellations due to the flux contributions. In other words, \( -N_{\text{fluxD7}}, -N_{\text{fluxNS7}}, -N_{\text{fluxI7}}, \) and \( -N_{\text{D3}} \) are the flux contributions to the D7-brane tadpoles, NS7-brane tadpoles, I7-brane tadpoles, and D3-brane plus O3-plane tadpoles, respectively.
1. \((p, q) = (0, 0)\)

The seven-brane tadpole contributions of the fluxes must be zero because we set \((p, q) = (0, 0)\) in this analysis. From Eqs. (145)-(147), we can conclude, for \(b \neq 0\),

\[
m h - e_0 \bar{h} = 0, \quad \bar{f} h_0 - f \bar{h}_0 = 0, \quad h \bar{h}_0 - \bar{h} h_0 + e_0 \bar{f} - m f = 0. \quad (149)
\]

From the tadpole condition of the NS7-branes it results in zero imaginary part of \(\tau\). Thus, the moduli \(\tau_i\) can not be stabilized properly.

2. \(p \cdot q \neq 0\)

Recall the third antisymmetry condition in Eq. (7) for \(k \neq a, i = 1, 2, 3\), we have

\[
\frac{m^I_a}{n^I_a} (f_a p^I_k - h_a q^I_k) = 0, \quad (150)
\]
\[
\frac{m^I_a}{n^I_a} (g_{ia} p^I_k - b_{ia} q^I_k) = 0, \quad (151)
\]
\[
\frac{m^I_a}{n^I_a} (g_{ia} p^I_k - \bar{b}_{ia} q^I_k) = 0, \quad (152)
\]
\[
\frac{m^I_a}{n^I_a} (f_a p^I_k - \bar{h}_a q^I_k) = 0. \quad (153)
\]

From Eqs. (123) and (135) it is required that \(\xi = \xi'\) if \(b, g, \bar{h}, \bar{f}, \xi, \xi'\) are non-zero. It turns out that the imaginary part of \(\sigma\) is zero. Thus, the real part of dilaton \(S\) can not be stabilized properly. If we can tolerate \(\text{Im}(\sigma) = 0\), we can continue this analysis to see if there is a solution for the remaining conditions. Let \(p^I = p, q^I = q\) for simplicity, the third antisymmetry condition in Eq. (7) implies

\[
\frac{f}{h} = \frac{\bar{f}}{\bar{h}} = \frac{g}{b} = \frac{\gamma}{\beta} = \frac{\bar{\gamma}}{\bar{\beta}} = \frac{q}{p}. \quad (154)
\]

We have known \(b \bar{h} > 0\) for \(\rho_0\) complex, and this implies \(g \bar{f} > 0\). Therefore for \(\tau\) condition in Eq. (142), it is required that \(\bar{f} h_0 - f \bar{h}_0 < 0\). However, this makes the flux NS7-brane tadpole contribution Eq. (146) always negative, \(i.e.,\) the flux contribution to the NS7-brane tadpole is positive. Thus, we can not obtain supersymmetric Minkowski flux vacua where the NS7-brane tadpole cancellation condition is relaxed and the moduli are stabilized properly. For the \((p, q) = (1, 0)\) case, the discussions are similar.
3. A Special Case with \( b = 0 \)

Considering the case \((b, \bar{b}) = (0, 0)\), we obtain the following conditions from the antisymmetry conditions and Jocobi identities:

\[
\beta = 0, \quad \bar{\beta} = -\bar{b}, \quad \gamma = -g, \quad \bar{\gamma} = -\bar{g};
\]
\[
\bar{b}f = \bar{g}h, \quad \bar{b}g = \bar{f}h, \quad f \bar{f} = gg.
\]  

(155)

We can rewrite \(E_3\) and \(E_4\) as

\[
E_3 = -3(h + \bar{b} \rho^2) = -\frac{3h}{f} (f + \bar{g} \rho^2),
\]
\[
E_4 = 3(f - g \rho + \bar{g} \rho^2 - \bar{f} \rho^3) = -\frac{3}{f} (g \rho - f)(f + \bar{g} \rho^2).
\]  

(156)

(157)

\(\rho\) can have a pure imaginary root if \(f \bar{g} > 0\). The moduli \(\sigma\) and \(\tau\) are

\[
\sigma = -\frac{E_3}{E_4} = -\frac{h}{g \rho - f},
\]
\[
\tau = -\frac{E_2}{E_4} = \frac{f h_0 + 3a \rho - 3\bar{a} \rho^2 - \bar{h}_0 \rho^2}{3 (g \rho - f)(f + \bar{g} \rho^2)}.
\]  

(158)

(159)

For a finite \(\tau\), \(E_2\) also has the factor \((f + \bar{g} \rho^2)\), so we obtain two additional constraints

\[
h_0 \bar{g} + 3\bar{a} f = 0, \quad \bar{h}_0 f + 3a \bar{g} = 0.
\]  

(160)

The conditions for \(\text{Im}(\sigma)\) and \(\text{Im}(\tau)\) are

\(\text{Im}(\sigma) : \ h \bar{f} > 0, \quad \text{Im}(\tau) : \ h_0 \bar{f} + 3a \bar{g} < 0.\)

(161)

On the other hand, \(E\) has double roots and can be rewritten in terms of \(E_3\) and \(E_4\) as

\[
E = \frac{3}{f} (f + \bar{g} \rho^2)(h E_2 - (g \rho - f) E_1) = \frac{3}{f} (f + \bar{g} \rho^2)^2 (A \rho^2 + B \rho + C).
\]  

(162)

where the coefficients \(A\), \(B\), and \(C\) are

\[
A = \frac{m g}{g}, \quad B = -\frac{1}{g} (h \bar{h}_0 + m f + 3 g q), \quad C = e_0 + \frac{h h_0}{f}.
\]  

(163)

Combining with the antisymmetry constraints in Eqs. (98), (99), and (101), we obtain the following relations:

\[
e_0 \bar{g} + \bar{b} h_0 - \bar{a} h + f q = 0, \quad \bar{h}_0 h + e_0 \bar{f} - a \bar{b} + g q = 0,
\]
\[
e \bar{f} = mg, \quad e \bar{g} = mf, \quad e g = f q - \bar{a} h, \quad e_0 \bar{f} = 2a \bar{b} + 2e \bar{g}.
\]  

(164)
The seven-brane tadpole contributions of the fluxes are

\[
N_{\text{fluxD7}} : \frac{1}{2} [m h - e_0 \bar{h} - (2b + \beta)q - (2\bar{b} + \bar{\beta})\bar{e}] = \frac{1}{2} (m h - e\bar{b}) = 0, \tag{165}
\]

\[
N_{\text{fluxNS7}} : \frac{1}{2} [\bar{f} h_0 - f \bar{h}_0 - a(2g + \gamma) + a(2\bar{g} + \bar{\gamma})] = \frac{2}{3} [3a\bar{g} + h_0 \bar{f}], \tag{166}
\]

\[
N_{\text{fluxI7}} : \frac{1}{2} [\bar{h}_0 h - \bar{h} h_0 + a(2b + \beta) - a(2\bar{b} + \bar{\beta}) + e_0 f - m f + q(2g + \gamma) + e(2\bar{g} + \bar{\gamma})]
\]

\[
= \frac{1}{2} [\bar{h}_0 h - a\bar{b} + e_0 \bar{f} + q g] = 0. \tag{167}
\]

It is generic that \( N_{\text{fluxD7}} = 0 \) and \( N_{\text{fluxI7}} = 0 \) from the conditions above. We consider two possible cases for the NS7-brane tadpole in the following discussion:

- \((p, q) = (0, 0)\)

  The condition \( q^I = 0 \) implies \( N_{\text{fluxNS7}} = 0 \), which turns out \( 3a\bar{g} + h_0 \bar{f} = 0 \). Then there is no imaginary part for \( \tau \). Thus, the moduli \( \tau \) can not be stabilized properly.

- \( p \cdot q \neq 0\)

  A nonzero \( q^I = 0 \) implies \( N_{\text{fluxNS7}} > 0 \), however it violates the condition of \( \text{Im}(\tau) \) for it to have the same sign as \( \text{Im}(\rho) \). Thus again, we do not have supersymmetric Minkowski flux vacua where the NS7-brane tadpole cancellation condition can be relaxed and the moduli can be stabilized properly.

VI. SUPERSYMMETRIC AdS VACUA AND A SEMI-REALISTIC PATI-SALAM MODEL

To relax the constraints we shall consider an AdS vacuum. For simplicity, we choose \( q^I_i = 0 \) so that \( P = 0 \) for the third antisymmetry condition relaxed. In addition, we assume \( E_2 = 0 \) by ignoring the dilaton modulus at the current stage because there are enough degrees of freedom to compute it at any time. By the AdS conditions, we obtain

\[
E_3 = 0, \quad \tau = -\frac{E_1'}{E_3'}. \tag{168}
\]

Again with the same setup, we have

\[
h = b\xi, \quad b + \beta = h\chi = b\xi\chi, \quad \bar{b} = \bar{h}\xi, \quad \bar{b} + \bar{\beta} = b\chi, \tag{169}
\]

and \( E_3 \) is factorized as \( E_3 = 3(\rho - \xi)(\bar{h}\rho^2 - b\chi \rho + b) \). The complex roots of \( \rho \) have the following properties

\[
\rho_0 = \frac{bt \pm \sqrt{b^2 \chi^2 - 4bh}}{2h}, \quad b^2 \chi^2 - 4bh < 0, \quad b^2 \chi^2 < 4bh, \quad b\bar{h} > 0. \tag{170}
\]
The modulus \( \tau_0 \) and the condition for its imaginary part to have the same sign as \( \rho_0 \) are
\[
\tau_0 = -\frac{E'_1}{E_3}|\rho_0| = -\frac{3e + 6q\rho_0 - 3mp_0^2}{3(\rho_0 - \xi)(2\rho_0 - b\chi)},
\]
\[
\frac{1}{\hbar}(mb^2\xi\chi^2 - mb^2\chi - 2qb\bar{h}\xi\chi + 4qb\bar{h} - 2mb\bar{h}\xi + eb\bar{h}\chi - 2e\bar{h}^2\xi) > 0.
\] (172)

The D7-brane tadpole contribution of the fluxes is
\[
N_{\text{fluxD7}} = \frac{1}{2}[mh - e_0\bar{h} - (2b + \beta)q - (2\bar{b} + \bar{\beta})e] = \frac{1}{2}[mb\xi - e_0\bar{h} - qb - qb\xi\chi - eb\chi - e\bar{h}\xi].
\] (174)

Therefore, for the case of \( N_{\text{fluxD7}} > 0 \), \( \text{Im}(\tau_0) \) can still be positive, which implies the existence of AdS solutions. The flux contributions to the D3-brane, NS7-brane, and I7-brane tadpoles are
\[
N_{\text{fluxD3}} : 16 - \frac{1}{2}[mh_0 - e_0\bar{h}_0 + 3qa + 3e\bar{a}] = 16 = \sum n_i^1 n_j^1 n_k^1,
\] (175)
\[
N_{\text{fluxNS7}} : \frac{1}{2}[(f\bar{h}_0 - f\bar{h}_0 - \bar{a}(2g + \gamma) + a(2\bar{g} + \bar{\gamma})) = 0,
\] (176)
\[
N_{\text{fluxI7}} : \frac{1}{2}[(\bar{h}_0h - \bar{h}_0h + \bar{a}(2b + \beta) - a(2\bar{b} + \bar{\beta}) + e_0\bar{f} - mf + q(2g + \gamma) + e(2\bar{g} + \bar{\gamma})]
= 0.
\] (177)

Table II: The choices of fluxes.

| \( b_1 \) | \( b_2 \) | \( b_3 \) | \( m \) | \( e_0 \) | \( e \) | \( q \) |
|---|---|---|---|---|---|---|
| 4 | 12 | 12 | -2 | 2 | -2 | -2 |

Next, let us construct a semi-realistic Pati-Salam model. We assume \( \chi = 1 \), \( \xi = 1 \), \( b = h = \bar{h} = \bar{b} \), and \( \beta = \bar{\beta} = 0 \). From Eq. (172), for \( \text{Im}(\tau) \) having the same sign as \( \text{Im}(\rho) \), we obtain
\[
b(2q - 2m - e) > 0.
\] (178)

The D7-brane tadpole from flux contribution is
\[
N_{\text{fluxD7}} = \frac{1}{2}b(m - e_0 - 2q - 2e) = \frac{1}{2}[-b(2q - 2m - e) - b(m + e_0 + 3e)] > 0.
\] (179)

We assume \( E_2 = 0 \) by setting \( a = \bar{a} = h_0 = \bar{h}_0 = 0 \), so then \( N_{D3} = 16 \). We shall present an example by choosing \( N_{D7} = 8 \) and \( (e_0 + m + 3e) \neq 0 \). Recall that we assumed \( \tau b \equiv (\tau_i/k_i)(b_i k_i) \), with the Freed-Witten anomaly condition imposed, we obtain the choices of the fluxes which are given in Table II. We present the D-brane configurations and intersection numbers of our Pati-Salam model in Table III. The corresponding particle spectrum is given in Table IV.
TABLE III: D-brane configurations and intersection numbers for a model on $T^6$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{\text{observable}} \times [U(2) \times USp(2)^2]_{\text{hidden}}$, the SM fermions and Higgs fields arise from the first two torus.

VII. DISCUSSIONS AND CONCLUSION

We conjectured that in generic supersymmetric Minkowski vacua, at least one of the flux contributions to the seven-brane and D3-brane tadpoles are positive if the moduli can be stabilized properly on the Type IIB toroidal $T^6$ orientifolds with the RR, NSNS, metric, non-geometric and S-dual flux compactifications. Therefore, these tadpole cancellation conditions can not be relaxed for realistic model building. To study the supersymmetric Minkowski vacua, we started from the reasonably simplified fluxes and then discussed the corresponding superpotential. We showed that we are not able to have the positive real parts of all the moduli and the negative/zero flux contributions to all the seven-brane and D3-brane tadpoles simultaneously. In the supersymmetric AdS vacua, we can have the flux vacua where the seven-brane and D3-brane tadpole cancellation conditions are relaxed elegantly, and we presented a concrete semi-realistic Pati-Salam model as well as its particle spectrum. The lifting from the AdS vacua to the Minkowski/dS vacua remains a great challenge in flux model building and is dedicated to the future work. On the other hand, some directions of searching for Minkowsky/dS vacua with fluxes on general geometries are interesting, for example, the recent development with structure manifolds [49].

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TABLE IV: The chiral and vector-like superfields, and their quantum numbers under the gauge symmetry $U(4)_C \times U(2)_L \times U(2)_R \times U(2) \times USp(2)^2$.

| Field | Quantum Number | $Q_A$ | $Q_{2L}$ | $Q_{2R}$ | Field |
|-------|----------------|-------|---------|---------|-------|
| $ab$  | $4 \times (4, 2, 1, 1, 1)$ | 1     | -1      | 0       | $F_L(Q_L, L_L)$ |
| $ac$  | $4 \times (\bar{4}, 2, 1, 1, 1)$ | -1    | 0       | 1       | $F_R(Q_R, L_R)$ |
| $ae$  | $2 \times (4, 1, 1, 1, 2, 1)$ | 1     | 0       | 0       |       |
| $af$  | $2 \times (\bar{4}, 1, 1, 1, 2, 1)$ | -1    | 0       | 0       |       |
| $ac'$ | $3 \times (4, 1, 2, 1, 1, 1)$ | 1     | 0       | 1       |       |
|       | $3 \times (\bar{4}, 1, \bar{2}, 1, 1, 1)$ | -1    | 0       | -1      |       |
| $ad$  | $2 \times (4, 1, 1, 2, 1, 1)$ | 1     | 0       | 0       |       |
|       | $2 \times (\bar{4}, 1, 1, 2, 1, 1)$ | -1    | 0       | 0       |       |
| $ad'$ | $2 \times (4, 1, 1, 2, 1, 1)$ | 1     | 0       | 0       |       |
|       | $2 \times (\bar{4}, 1, 1, 2, 1, 1)$ | -1    | 0       | 0       |       |
| $bd$  | $1 \times (1, 2, 1, 1, 2, 1)$ | 1     | 0       | 0       |       |
| $bf$  | $2 \times (1, 2, 1, 1, 2, 1)$ | 0     | 1       | 0       |       |
| $bc'$ | $1 \times (1, 2, 2, 1, 1, 1)$ | 0     | 1       | 1       |       |
|       | $1 \times (1, 2, \bar{2}, 1, 1, 1)$ | 0     | -1      | -1      |       |
| $bd'$ | $1 \times (1, 2, 1, 2, 1, 1)$ | 0     | 1       | 0       |       |
|       | $1 \times (1, \bar{2}, 1, \bar{2}, 1, 1)$ | 0     | -1      | 0       |       |
| $be$  | $1 \times (1, 2, 1, 1, 2, 1)$ | 0     | 1       | 0       |       |
|       | $1 \times (1, \bar{2}, 1, 1, 2, 1)$ | 0     | -1      | 0       |       |
| $cd'$ | $2 \times (1, 1, 2, 2, 1, 1)$ | 0     | 0       | 1       |       |
| $ce$  | $2 \times (1, 1, \bar{2}, 1, 2, 1)$ | 0     | 0       | -1      |       |
| $cd$  | $1 \times (1, 1, 2, 2, 1, 1)$ | 0     | 0       | 1       |       |
|       | $1 \times (1, 1, \bar{2}, 2, 1, 1)$ | 0     | 0       | -1      |       |
| $cf$  | $1 \times (1, 1, 2, 1, 1, 2)$ | 0     | 0       | 1       |       |
|       | $1 \times (1, 1, \bar{2}, 1, 1, 2)$ | 0     | 0       | -1      |       |
| $bc$  | $3 \times (1, 2, 2, 1, 1, 1)$ | 0     | 1       | -1      | $H_u, H_d$ |

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[1] J. Polchinski and E. Witten, Nucl. Phys. B 460, 525 (1996).
[2] M. Berkooz, M. R. Douglas and R. G. Leigh, Nucl. Phys. B 480, 265 (1996).
[3] C. Bachas, arXiv:hep-th/9503030.
[4] R. Blumenhagen, M. Cvetič, P. Langacker and G. Shiu, Ann. Rev. Nucl. Part. Sci. 55, 71 (2005), and the references therein.
[5] R. Blumenhagen, L. Goerlich, B. Kors and D. Lust, JHEP 0010, 006 (2000).
[6] C. Angelantonj, I. Antoniadis, E. Dudas and A. Sagnotti, Phys. Lett. B 489, 223 (2000).
[7] L. E. Ibanez, F. Marchesano and R. Rabadas, JHEP 0111, 002 (2001).
[8] M. Cvetič, G. Shiu and A. M. Uranga, Phys. Rev. Lett. 87, 201801 (2001).
[9] M. Cvetič, G. Shiu and A. M. Uranga, Nucl. Phys. B 615, 3 (2001).
[10] M. Cvetič and I. Papadimitriou, Phys. Rev. D 67, 126006 (2003).
[11] M. Cvetič, I. Papadimitriou and G. Shiu, Nucl. Phys. B 659, 193 (2003) [Erratum-ibid. B 696, 298 (2004)].
[12] M. Cvetić, T. Li and T. Liu, Nucl. Phys. B 698, 163 (2004).
[13] M. Cvetić, P. Langacker, T. Li and T. Liu, Nucl. Phys. B 709, 241 (2005).
[14] C.-M. Chen, G. V. Kranoti, V. E. Mayes, D. V. Nanopoulos and J. W. Walker, Phys. Lett. B 611, 156 (2005); Phys. Lett. B 625, 96 (2005).
[15] C. M. Chen, T. Li and D. V. Nanopoulos, Nucl. Phys. B 732, 224 (2006).
[16] C. M. Chen, V. E. Mayes and D. V. Nanopoulos, Phys. Lett. B 648, 301 (2007).
[17] R. Blumenhagen, L. Görlich and T. Ott, JHEP 0301, 021 (2003); G. Honecker, Nucl. Phys. B666, 175 (2003); G. Honecker and T. Ott, Phys. Rev. D 70, 126010 (2004) [Erratum-ibid. D 71, 069902 (2005)].
[18] C.-M. Chen, T. Li, V. E. Mayes, D. V. Nanopoulos, Phys. Lett. B665, 267-270 (2008).
[19] C.-M. Chen, T. Li, V. E. Mayes, D. V. Nanopoulos, Phys. Rev. D77, 125023 (2008).
[20] M. Cvetić, P. Langacker and J. Wang, Phys. Rev. D 68, 046002 (2003).
[21] S. Gukov, C. Vafa and E. Witten, Nucl. Phys. B 584, 69 (2000) [Erratum-ibid. B 608, 477 (2001)].
[22] S. B. Giddings, S. Kachru and J. Polchinski, Phys. Rev. D 66, 106006 (2002).
[23] S. Kachru, M. B. Schulz, P. K. Tripathy and S. P. Trivedi, JHEP 0303, 061 (2003).
[24] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, Phys. Rev. D 68, 046005 (2003).
[25] J. F. G. Cascales and A. M. Uranga, JHEP 0305, 011 (2003).
[26] R. Blumenhagen, D. Lüst and T. R. Taylor, Nucl. Phys. B 663, 319 (2003).
[27] F. Marchesano and G. Shiu, Phys. Rev. D 71, 011701 (2005); JHEP 0411, 041 (2004).
[28] M. Cvetić and T. Liu, Phys. Lett. B 610, 122 (2005).
[29] M. Cvetić, T. Li and T. Liu, Phys. Rev. D 71, 106008 (2005).
[30] J. Kumar and J. D. Wells, JHEP 0509, 067 (2005).
[31] C. M. Chen, V. E. Mayes and D. V. Nanopoulos, Phys. Lett. B 633, 618 (2006).
[32] T. W. Grimm and J. Louis, Nucl. Phys. B 718, 153 (2005).
[33] G. Villadoro and F. Zwirner, JHEP 0506, 047 (2005).
[34] P. G. Camara, A. Font and L. E. Ibanez, JHEP 0509, 013 (2005).
[35] C. M. Chen, T. Li and D. V. Nanopoulos, Nucl. Phys. B 740, 79 (2006).
[36] C. M. Chen, T. Li and D. V. Nanopoulos, Nucl. Phys. B 751, 260 (2006).
[37] J. Shelton, W. Taylor and B. Wecht, JHEP 0510, 085 (2005).
[38] G. Aldazabal, P. G. Camara, A. Font and L. E. Ibanez, JHEP 0605, 070 (2006).
[39] G. Villadoro and F. Zwirner, JHEP 0603, 087 (2006).
[40] C. -M. Chen, T. Li, Y. Liu, D. V. Nanopoulos, Phys. Lett. B668, 63-66 (2008).
[41] D. S. Freed and E. Witten, arXiv:hep-th/9907189.
[42] J. F. G. Cascales and A. M. Uranga, JHEP 0305, 011 (2003).
[43] C. M. Chen, T. Li and D. V. Nanopoulos, arXiv:0812.2089 [hep-th].
[44] G. Aldazabal, P. G. Camara, J. A. Rosabal, Nucl. Phys. B814, 21-52 (2009).
[45] B. de Carlos, A. Guarino and J. M. Moreno, JHEP 1001, 012 (2010); JHEP 1002, 076 (2010).
[46] D. Cremades, L. E. Ibanez, F. Marchesano, JHEP 0207, 009 (2002).
[47] D. Lust, P. Mayr, R. Richter, S. Stieberger, Nucl. Phys. B696, 205-250 (2004).
[48] G. Aldazabal, S. Franco, L. E. Ibanez, R. Rabadan and A. M. Uranga, J. Math. Phys. 42, 3103 (2001).
[49] U. H. Danielsson, S. S. Haque, P. Koerber, G. Shiu, T. Van Riet and T. Wrase, arXiv:1103.4858 [hep-th].