A stableness of resistance model for nonresponse adjustment with callback data

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Summary

Nonresponse arises frequently in surveys and follow-ups are routinely made to increase the response rate. In order to monitor the follow-up process, callback data have been used in social sciences and survey studies for decades. In modern surveys, the availability of callback data is increasing because the response rate is decreasing and follow-ups are essential to collect maximum information. Although callback data are helpful to reduce the bias in surveys, such data have not been widely used in statistical analysis until recently. We propose a stableness of resistance assumption for nonresponse adjustment with callback data. We establish the identification and the semiparametric efficiency theory under this assumption, and propose a suite of semiparametric estimation methods including doubly robust estimators, which generalize existing parametric approaches for callback data analysis. We apply the approach to a Consumer Expenditure Survey dataset. The results suggest an association between nonresponse and high housing expenditures.

KEYWORDS: Callback; Doubly robust estimation; Missing data; Paradata; Semiparametric efficiency.

1. INTRODUCTION

Nonresponse often leads to substantially biased statistical inference and is frequently encountered in surveys and observational studies in many areas of scientific research. It has been a persistent concern of statisticians and applied researchers for many years. The missingness is said to be at random (MAR) or ignorable if it does not depend on the missing values conditional on fully-observed covariates, and otherwise it is called missing not at random (MNAR) or nonignorable. A large body of work for nonresponse adjustment has been based on MAR. However, there is suspicion that the missingness mechanism is MNAR in many situations. For example, social stigma in sensitive questions (e.g., HIV status, income, or drug use) makes nonresponse dependent on unobserved variables. The missing data process and the outcome distribution are identified under MAR, but under MNAR identification in general fails to hold without extra information, which substantially jeopardizes statistical inference. Identification means that the parameter or distribution of interest is uniquely determined from the observed-data distribution; it is crucial for missing data analysis, without which statistical inference may be misleading and is of limited interest. Although one can achieve identification if the impact of the outcome on the missingness is completely known, this should be used

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rather as a sensitivity analysis because in practice such information is seldom available, see e.g. Robins et al. (2000). Otherwise, identification does not hold even for fully parametric models, except for several fairly restrictive ones (e.g., Heckman, 1979; Miao et al., 2016). For semiparametric and nonparametric models, identification and inference under MNAR essentially require the use of auxiliary data, for example, an instrumental variable (Sun et al., 2018; Liu et al., 2020; Tchetgen Tchetgen and Wirth, 2017) or a shadow variable (D’Haultfœuille, 2010; Wang et al., 2014; Miao and Tchetgen Tchetgen, 2016). Researchers have traditionally sought such auxiliary variables from the sampling frame, but it could be difficult in multipurpose studies where multiple survey variables are concerned and multiple auxiliary variables are necessitated. Moreover, such auxiliary variable methods break down if the auxiliary variables also have missing values due to failure of contact in surveys.

In contrast to the paucity of auxiliary variables in the sampling frame, callback data offer an important source of auxiliary information for nonresponse adjustment. In the presence of nonresponse, the interviewer may continue to contact nonrespondents, and the contact process is recorded with callback data, sometimes called level-of-effort data (Biemer et al., 2013). For instance in the 2018 Consumer Expenditure Survey, the maximum number of contact attempts the interviewers made is about 30, and the number of calls made to contact each unit is recorded with the callback data. Callback data have been widely used in epidemiological, economic, social and political surveys for a long time since the 1940’s. Such data are particularly useful to monitor response rates and to study how design features affect the data collection. Examples include Politz and Simmons (1949); Filion (1976); Drew and Fuller (1980); Lin and Schaeffer (1995); Potthoff et al. (1993); Wood et al. (2006); Jackson et al. (2010); Peress (2010); Biemer et al. (2013); Clarsen et al. (2021), and see Groves and Couper (1998); Olson (2013); Kreuter (2013) for a comprehensive review. Although not all surveys could provide callback data, their availability is increasing in modern surveys. The underlying reason is that the response rate in modern surveys is decreasing and follow-ups are essentially required. Examples include the National Health Interview Survey (NHIS), the National Survey of Family Growth (NSFG), the National Survey on Drug Use and Health (NSDUH), the European Social Survey (ESS), the Behavioral Risk Factor Surveillance System (BRFSS), and the Consumer Expenditure Survey (CES).

However, callback data have not been widely used in statistical analysis until recently, although their usefulness for nonresponse adjustment has been recognized since Politz and Simmons (1949). The callback design is analogous to the two/multi-phase sampling (Deming, 1953) by viewing the follow-ups as the second-phase sample, but it differs in that nonresponse may still occur in the follow-up sample and is possibly not at random. Hence, nonresponse adjustment remains difficult even if callbacks are available, due to the challenge for identification and inference under MNAR. In order to do nonresponse adjustment, the notion of “continuum of resistance” assumes that nonrespondents are more similar to delayed respondents than they are to early respondents, so that the most reluctant respondents are used to approximate the nonrespondents. This assumption has been asserted in social and survey researches for decades despite conflicting evidence of its validity (Lin and Schaeffer, 1995; Clarsen et al., 2021). Other approaches include modeling the joint likelihood of callbacks and frame variables (Biemer et al., 2013); Heckman-type models (Chen et al., 2018; Zhang et al., 2018); sensitivity analysis (Rotnitzky and Robins, 1995 unpublished manuscript; Daniels et al., 2015); etc. Most notably, Alho (1990), Kim and Im (2014), Qin and Follmann (2014), and Guan et al. (2018) use callback data to estimate response propensity scores and propose inverse probability weighted and empirical likelihood-based estimation methods, where they impose a fully parametric linear logistic propensity score model and a common-slope assumption that individual characteristics influence the missingness process in the same way across the survey contacts. These previous approaches rest on strong parametric models to achieve identification, which circumvent the underlying sources for identification and limit their use in complex data application.

In order to further investigate the usefulness of callback data and promote their application, we take a
fundamentally nonparametric identification strategy, accompanied with practical parametric/semiparametric estimation methods. Our contributions are threefold. First, we characterize a stableness of resistance assumption and establish the identification under this assumption, which is a nonparametric generalization of the parametric approach of Alho (1990). The stableness of resistance assumption states that the impact of the missing outcome on the response propensity is stable in the first two call attempts. This assumption does not impose parametric functional restrictions on the propensity score, does not restrict the impact of covariates on the missingness, and admits any type (discrete or continuous) of variables. Second, under the stableness of resistance assumption we establish the semiparametric theory, which is for the first time in this field. We characterize the tangent space, the efficient influence function and the semiparametric efficiency bound for estimating a general full-data functional. Third, we propose a suite of semiparametric estimation methods including an inverse probability weighted (IPW) estimator, an outcome regression-based (REG) estimator, and a doubly robust (DR) one. The proposed IPW estimator is a generalization of the calibration estimator of Kim and Im (2014) by allowing for nonlinear propensity score models and call-specific effects of covariates on nonresponse. The IPW and REG estimators rest on correct specification of certain working models; otherwise, they are no longer consistent. However, the doubly robust estimator affords double protection against misspecification of working models: it remains consistent if the working models for either the IPW or the REG estimation is correct, but not necessarily both; moreover, it attains the efficiency bound for the nonparametric model when all working models are correct. To alleviate the concern about misspecification of parametric working models, we further propose a more robust approach that uses more flexible working models to estimate nuisance functions and plugs in the nuisance estimators into the efficient influence function to obtain the estimator of the functional of interest. This approach extends the methods for nonignorable missing data analysis by allowing for flexible estimation of the odds ratio function, instead of requiring it to be known or follow a parametric model. We show that the resultant estimator has second order bias, which affords more robust inference provided that the nuisance estimators converge sufficiently fast. The characterization of the bias involves a more sophisticated analysis rather than directly applying the second-order Taylor expansion technique. This technical contribution may shed some light on the analysis of complicated estimators.

The rest of this paper proceeds as follows. In Section 2, we further discuss the challenge for identification with callback data. In Section 3, we characterize the stableness of resistance assumption for using callback data to identify the full-data distribution and establish the identification results. In Sections 4 and 5, we establish the semiparametric theory and describe the IPW, REG, DR estimators, including one based on flexible working models. We show their asymptotic properties. Section 6 includes extensions to estimation of a general full-data functional and to the setting with multiple callbacks. Sections 7 and 8 include numerical illustrations with simulations and a real data application to the Consumer Expenditure Survey (CES; National Research Council, 2013) dataset, respectively. Section 9 concludes with discussions on other extensions and limitations of the proposed approach.

2. PRELIMINARIES AND CHALLENGES TO IDENTIFICATION

Throughout the paper, we let \((X, Y)\) denote the frame variables that are released in a survey or observational study for investigation of a particular scientific purpose, where \(X\) denotes a vector (possibly a null set) of fully observed covariates and \(Y\) the outcome prone to missing values. The frame variables \(X\) and \(Y\) can be either continuous or discrete, either univariate or multivariate. Suppose in the data collection process the interviewer has continued to contact a unit until he/she responds or the maximum number of call attempts \(K\) is achieved. We let \(R_k\) denote the availability status of \(Y\) after the \(k\)th call, with \(R_k = 1\) if \(Y\) is available after the \(k\)th call and \(R_k = 0\) otherwise. By this definition, we have \(R_{k+1} \geq R_k\); because if \(R_k = 1\), i.e. \(Y\) is already obtained in the \(k\)th call, then by definition \(R_{k+1} = 1 \geq R_k\). The observations of \((R_1, \ldots, R_K)\)
denote a generic probability density or mass function. The full data can be viewed as the monotone missingness or longitudinal setting concerns the missingness of different outcomes. Let callback data represent the availability status of exactly the same outcome in different call attempts, while classical monotone missingness pattern and should not be viewed as a longitudinal setting, because the recording the response process in the survey are known as callback data. This setting differs from the data (the final availability status \( R \) is to make inference about the outcome mean \( \mu \) analysis and then discuss the extension to multiple calls in Section 6 and the supplement. Our primary goal ground ideas, we will focus on the setting with two calls or when only the first two calls are considered for values of \( f \) distribution \( Y \) suffice for identification. Consider a toy example of survey on a binary outcome, which is particularly informative about the nonresponse process. However, callback data still do not suffice for identification. Follmann (2014) consider a common-slope linear logistic model for the propensity scores.

Example 1 (A common-slope linear logistic model). Alho (1990) assumes that for \( k = 1, \ldots, K \),

\[
\text{logit } f(R_k = 1 \mid R_{k-1} = 0, X, Y) = \alpha_{k0} + \alpha_{k1}X + \gamma_kY, \quad \text{with } \alpha_{k1} = \alpha_1, \quad \gamma_k = \gamma. \quad (1)
\]

Hereafter, we let \( \text{logit } x = \log \{x/(1-x)\} \) be the logit transformation of \( x \) and \( \expit x = \exp(x)/\{1 + \exp(x)\} \) its inverse. Example 1 is a restrictive model: the propensity score for each callback follows a linear (in parameters) logistic model and the effects of both the covariate \( X \) and the outcome \( Y \) do not vary across call attempts. Daniels et al. (2015) mention an identification result that allows for call-specific coefficients of \( X \) but requires constant coefficients of \( Y \) across all call attempts. Guan et al. (2018) relax the common slope assumption by only requiring \( \alpha_{11} = \alpha_{21} \) and \( \gamma_1 = \gamma_2 \). This enables identification with only two call attempts, but their identifying strategy based on a meticulous analysis of the logistic model only admits continuous covariates and outcome. In order to make full use of callback data for nonresponse adjustment, it is of great interest to establish identification for other types of models than the linear logistic model. So far,

| Table 1: Data structure of a survey with callbacks; NA denotes missing values |
|-----------------|-----------------|-----------------|-----------------|
| ID | X | Y | R₁ | R₂ | ⋯ | Rₖ |
| 1  | x₁ | y₁ | 1  | 1  | ⋯ | 1  |
| 2  | x₂ | y₂ | 0  | 1  | ⋯ | 1  |
| :  | :  | NA | 0  | 0  | ⋯ | 0  |
| :  | :  | NA | 0  | 0  | ⋯ | 0  |
| n  | xₙ | yₙ | 0  | 0  | ⋯ | 1  |
3. IDENTIFICATION UNDER A STABILITY OF RESISTANCE ASSUMPTION

3.1 The stability of resistance assumption

An all-important step for identification under MNAR is to characterize the degree to which the missingness departs from MAR. The odds ratio function is a particularly useful measure widely used in the missing data literature to characterize the association between the outcome and response propensity (e.g. Rotnitzky et al., 2001; Osius, 2004; Chen, 2007; Kim and Yu, 2011; Miao and Tchetgen Tchetgen, 2016; Franks et al., 2020; Malinsky et al., 2020). We define the odds ratio functions for the response propensity in the first and second calls as follows,

$$\Gamma_1(X, Y) = \log \frac{f(R_1 = 1 \mid X, Y)f(R_1 = 0 \mid X, Y = 0)}{f(R_1 = 0 \mid X, Y)f(R_1 = 1 \mid X, Y = 0)} \quad (2)$$

$$\Gamma_2(X, Y) = \log \frac{f(R_2 = 1 \mid R_1 = 0, X, Y)f(R_2 = 0 \mid R_1 = 0, X, Y = 0)}{f(R_2 = 0 \mid R_1 = 0, X, Y)f(R_2 = 1 \mid R_1 = 0, X, Y = 0)}. \quad (3)$$

The odds ratio functions $\Gamma_1(X, Y)$ and $\Gamma_2(X, Y)$ measure the impact of the outcome on the missingness in the first and second calls, respectively, by quantifying the change in the odds of response caused by a shift of $Y$ from a common reference value $Y = 0$ to a specific level while controlling for covariates. Probabilities $f(R_1 = 1 \mid X, Y = 0)$ and $f(R_2 = 1 \mid R_1 = 0, X, Y = 0)$ are referred to as baseline propensity scores evaluated at the reference value $Y = 0$. Any other value within the support of $Y$ can be chosen as the reference value. Therefore, the odds ratio function is a measure of the resistance to respond caused by the outcome. A positive odds ratio function evaluated at $(x, y)$ indicates that for participants with $X = x$, those with $Y = y$ are more likely to respond than those with $Y = 0$, and a larger odds ratio means a stronger willingness to respond. The setting with odds ratio functions equal to zero corresponds to MAR where the outcome does not influence the response. Our identification strategy rests on the following assumption.

Assumption 1. (i) Stability of resistance: $\Gamma_1(X, Y) = \Gamma_2(X, Y) = \Gamma(X, Y)$ for some $\Gamma(X, Y)$;

(ii) Positivity: $0 < f(R_1 = 1 \mid X, Y) < 1$ and $0 < f(R_2 = 1 \mid R_1 = 0, X, Y) < 1$ for all $(X, Y)$.

Condition (ii) is a standard positivity assumption in missing data analysis, which ensures sufficient overlap between nonrespondents and respondents in each call attempt. Condition (i) is the key identifying condition, which reveals that the impact of the outcome on the response propensity remains the same in the first two calls. Whether Condition (i) holds or not does not depend on the choice of the reference value in the definition of the odds ratio functions. Although Assumption 1 is concerned with the missingness process, it in fact imposes certain sophisticated restrictions on the outcome distribution. This is because under MNAR the missingness process is not ancillary to the outcome distribution; see Lemma 1 in the supplement for the apparent dependence between the missingness process, the outcome distribution and the odds ratio function.

Similar to various missing data problems, Assumption 1 is untestable based on observed data. Thus, the justification of its validity requires domain-specific knowledge and needs to be investigated on a case-by-case basis. Assumption 1 is motivated from the idea that the individual characteristics may influence the missingness process in the same way across the survey contacts. Similar ideas have been used since Alho (1990), Kim and Im (2014), Qin and Follmann (2014), and Guan et al. (2018). Assumption 1 is plausible if the nonignorable missingness is caused by certain social stigma (or other issues such as difficulty to reach) and the social stigma attached to the units does not change during the first two calls or the contacts are made...
in a short period of time. Besides, sensitivity analysis can be applied to assess how results would change if Assumption 1 were to be violated; see the simulation and application in Sections 7 and 8, respectively.

We provide a generative model based on the discrete choice theory (McFadden, 2001; Train, 2009) to further illustrate the intuition behind Assumption 1. Suppose

\[ Y = \beta_0 + \beta_1 X + C, \]
\[ U_1 = \alpha_{10} + \alpha_{11} X + \gamma_1 C + \varepsilon_1, \quad U_2 = \alpha_{20} + \alpha_{21} X + \gamma_2 C + \varepsilon_2, \]
\[ R_1 = I(U_1 > 0), \quad R_2 = R_1 + (1 - R_1) \times I(U_2 > 0), \]

where \( X \) is the observed covariate and \( C \) is an unmeasured factor (such as social stigma) correlated with both the outcome and missingness mechanism. Variables \( U_1 \) and \( U_2 \) are the utilities (or net benefits) that a subject obtains from responding as opposed to not responding at the first and second calls, respectively, and \( \varepsilon_1 \) and \( \varepsilon_2 \) are independent error terms, both following a logistic distribution and independent of \( X \) and \( C \). The generating process of \( R_1, R_2 \) reveals how the subjects decide to respond: a subject will respond in the first call (i.e., \( R_1 = 1 \)) if utility \( U_1 > 0 \); for those not responded in the first call, he/she will respond at the second call (i.e., \( R_1 = 0, R_2 = 1 \)) if utility \( U_2 > 0 \). We show in Section S1 of the supplement that the above discrete choice model induces the linear logistic propensity score model: logit \( f(R_k = 1 \mid R_{k-1} = 0, X, Y) = \alpha_{k0} + \alpha_{k1} X + \gamma_k Y, \) with \( \alpha_{k0} = \alpha_{k0} - \gamma_k \beta_0 \) and \( \alpha_{k1} = \alpha_{k1} - \gamma_k \beta_1 \) for \( k = 1, 2 \). In this model, the stableness of resistance stating that \( \gamma_1 = \gamma_2 \) in fact means the effects of the unmeasured factor \( C \) on the utilities \( U_1 \) and \( U_2 \) remain the same. However, the stableness of resistance does not require \( \bar{\alpha}_{11} \) and \( \bar{\alpha}_{21} \) to be equal, which admits varying effects of the observed covariate \( X \) on the utilities and the response in different calls.

The biggest difference between Assumption 1 and Model (1) is that, the former is about social reality that researchers can justify based on domain-specific knowledge, whereas the latter further depends on functional form restrictions that may lack real-world justification. Besides, Assumption 1 has several major differences from Model (1), while including the latter as a special case. Under Assumption 1 the propensity scores have the following representation:

\[
f(R_1 = 1 \mid X, Y) = \exp \{ A_1(X) + \Gamma(X, Y) \}, \tag{4}
\]
\[
f(R_2 = 1 \mid R_1 = 0, X, Y) = \exp \{ A_2(X) + \Gamma(X, Y) \}, \tag{5}
\]

where \( A_1(X) = \text{logit} \ f(R_1 = 1 \mid X, Y = 0) \) and \( A_2(X) = \text{logit} \ f(R_2 = 1 \mid R_1 = 0, X, Y = 0) \) are the logit transformations of the corresponding baseline propensity scores which capture impacts of covariates on the missingness, and \( \Gamma(X, Y) \) is the common odds ratio function that measures the impact of the outcome on the missingness in the first and second calls. The functions \( A_1(X), A_2(X), \Gamma(X, Y) \) are unrestricted. Therefore, Assumption 1 admits nonlinear and nonparametric effects of covariates and missing outcomes on the missingness, which is a much larger, and in fact, infinite-dimensional model; it accommodates any types of outcomes. The assessment of Assumption 1 only concerns the first two calls and does not depend on the number of callbacks, while the missingness in the remaining calls are left completely unrestricted; it only requires the effects of missing outcomes, captured by the odds ratio, to be stable, while admitting call-specific impacts of covariates on nonresponse. In contrast, Model (1) is fully parametric and the effects of covariates and missing outcomes on the missingness must be linear, which is a restrictive model indexed by several parameters. Model (1) is a special case of Assumption 1 where \( A_1(X) = \alpha_{10} + \alpha_{11} X, A_2(X) = \alpha_{20} + \alpha_{21} X, \) and \( \Gamma(X, Y) = \gamma Y \) encode the effects of variables on the missingness with the regression coefficients. This is best suited for continuous or binary outcomes, but may not be suitable for other types of outcomes. Model (1) imposes parametric functional restrictions on all contact attempts and assumes stable effects of all variables entering the model; however, this may not hold for the entire time range when \( K \) is large or
callback data are from a long period. In short, Model (1) is a restrictive model making strong parametric
functional restrictions beyond the parameter (e.g. the outcome mean) of interest. Assumption 1 makes much
less restrictive assumptions and has much greater flexibility.

3.2 Nonparametric identification
We will show that Assumption 1 suffices for identification of the joint distribution \( f(X, Y, R_1, R_2) \). We
first briefly explain why callback data are useful and how to leverage them for identification. Consider
identification of \( f(Y, R_1 \mid X) \), which hinges on identification of \( f(Y, R_1 = 0 \mid X) \), i.e., the missing data
distribution in the first call. In the presence of nonignorable missing data, we need to determine the selection
bias, captured by

\[
\frac{f(Y, R_1 = 1 \mid X)}{f(Y, R_1 = 0 \mid X)} = \frac{f(R_1 = 1 \mid X, Y)}{f(R_1 = 0 \mid X, Y)}. 
\]

With callback data, a natural idea for estimating this selection bias is to approximate \( f(Y, R_1 = 0 \mid X) \)
with \( f(Y, R_1 = 0, R_2 = 1 \mid X) \), where \( f(Y, R_1 = 0, R_2 = 1 \mid X) \) is the distribution of the data
obtained in the second call. This is analogous to the two-phase sampling (Deming, 1953). However, the
challenge here is that nonresponse still occurs in the second call and is possibly not at random, and as a result,
\( f(Y, R_1 = 0, R_2 = 1 \mid X) \) is not necessarily equal to \( f(Y, R_1 = 0 \mid X) \) and this crude approximation does
not suffice for identification. Nonetheless, under Assumption 1 we are able to characterize and can further
identify the bias of this approximation as shown in Proposition 1 and Theorem 1 below.

**Proposition 1.** Letting \( D(X) = A_2(X) - A_1(X) \), then under Assumption 1 we have that

\[
\frac{f(R_1 = 1 \mid X, Y)}{f(R_1 = 0 \mid X, Y)} = \frac{f(Y, R_1 = 1 \mid X)}{f(Y, R_2 = 1, R_1 = 0 \mid X)} - \exp\{-D(X)\}, \\
\frac{f(R_2 = 1 \mid R_1 = 0, X, Y)}{f(R_2 = 0 \mid R_1 = 0, X, Y)} = \exp\{D(X)\} \frac{f(Y, R_1 = 1 \mid X)}{f(Y, R_2 = 1, R_1 = 0 \mid X)} - 1.
\]

We prove this result in the supplement. Proposition 1 also implies an inequality:

\[
\frac{f(Y, R_1 = 1 \mid X)}{f(Y, R_2 = 1, R_1 = 0 \mid X)} > \exp\{-D(X)\}, \tag{6}
\]

i.e., given \( X \) the density ratio on the left hand side is uniformly bounded from zero. This is a restriction on
the observed-data distribution imposed by Assumption 1.

In Proposition 1, \( f(Y, R_1 = 1 \mid X)/f(Y, R_2 = 1, R_1 = 0 \mid X) \) is used to approximate \( f(R_1 = 1 \mid X, Y)/f(R_1 = 0 \mid X, Y) \) and the approximation bias is captured by \( D(X) \). We can further show that
\( D(X) \) is identified under Assumption 1, and thus the selection bias \( f(R_1 = 1 \mid X, Y)/f(R_1 = 0 \mid X, Y) \)
is identified, and then the identification of \( f(Y, R_1 \mid X) \) and \( f(X, Y, R_1, R_2) \) is straightforward.

**Theorem 1.** Under Assumption 1, \( D(X) \) is identified, and as a result, \( f(X, Y, R_1, R_2) \) is identified from
the observed-data distribution.

**Proof.** Proposition 1 implies that

\[
\frac{f(R_2 = 0 \mid R_1 = 0, X)}{f(R_2 = 1 \mid R_1 = 0, X)} = \int \left[ \frac{\exp\{D(X)\} \cdot f(Y, R_1 = 1 \mid X)}{f(Y, R_2 = 1, R_1 = 0 \mid X)} - 1 \right]^{-1} f(Y \mid R_2 = 1, R_1 = 0, X) dY \\
eq L\{D(X)\}. \tag{7}
\]
This is an equation with $D(X)$ unknown while all the other quantities are available from the observed-data distribution. Identification of $D(X)$ can be assessed by checking uniqueness of the solution to this equation. For any fixed $x$ and any $D(x)$ such that (6) is satisfied, $L \{ D(x) \}$ is strictly decreasing in $D(x)$ because

$$
\frac{\partial L \{ D(x) \}}{\partial D(x)} = - \int \frac{\exp \{ D(x) \} f(Y; R_1 = 1 | X = x)}{\left[ \exp \{ D(x) \} f(Y; R_1 = 1, R_1 = 0 | X = x) - 1 \right]^2} f(Y | R_2 = 1, R_1 = 0, X = x) dY < 0.
$$

Therefore, for any fixed $x$ the solution to (7) is unique. Applying this argument to all $x$, then $D(X)$ is identified and $f(R_1 = 1 | X, Y), f(R_2 = 1 | R_1 = 0, X, Y)$ are identified according to Proposition 1. Then it is straightforward to show that $f(X, Y)$ and $f(X, Y, R_1, R_2)$ are identified. □

We achieve nonparametric identification of $f(X, Y, R_1, R_2)$ with callback data under the stableness of resistance assumption. The nonparametric identification elucidates the underlying source for identification with callback data, other than invoking parametric functional restrictions; it extends the application of callback data and opens the way to novel estimation methods. To our knowledge, Assumption 1 is so far the most parsimonious condition characterizing the most flexible model for identification with callbacks, and Theorem 1 is so far the most general identification result. Back to the binary outcome example, Assumption 1 can be viewed as an additional constraint on the parameters so that we have sufficient number of constraints to identify the joint distribution; see Section S1 in the supplement for the details about the parameter count interpretation of identification. Note that Assumption 1 is sufficient but not necessary for identification of the joint distribution. There exist different conditions that can achieve identification. For instance, in a similar spirit, it is of interest and possible to establish identification with stableness of resistance on other scales of the propensity scores, say risk ratio scale, because it also introduces certain constraints on the parameters. In principle, the union of all such conditions constitutes the if and only if condition for identification. However, it may not be worth the chase because such an if and only if condition will be very complicated with difficulty in interpretation and justification in practice, and is thus less preferred in contrast to a sufficient and meaningful condition like the one we propose in the paper. Applying Theorem 1 to the linear logistic model immediately gives the following result.

**Proposition 2.** Assuming that logit $f(R_k = 1 | R_{k-1} = 0, X, Y) = \alpha_{k0} + \alpha_{k1} X + \gamma Y$ for $k = 1, 2$, then $\alpha_{k0}, \alpha_{k1}$, and $\gamma$ are identified.

Proposition 2 generalizes the identification of Model (1) by admitting call-specific coefficients of $X$ in the propensity scores and only requiring the coefficients of $Y$ to be equal for the first two calls. Model (1) also reveals a continuum-of-resistance model where nonrespondents are more similar to late respondents than to early ones due to a common odds ratio for all call attempts. However, in the model in Proposition 2 nonrespondents may depart further from late respondents than from early ones, depending on the form of odds ratio functions in later calls. In this case, continuum-of-resistance models are not suitable.

For estimation, we can in principle first estimate $f(Y | R_2 = 1, R_1 = 0, X), f(Y, R_2 = 1, R_1 = 0 | X), f(Y, R_2 = 1 | X), f(Y; R_1 = 1 | X)$, and $f(R_2 = 1 | R_1 = 0, X)$, then plug them into equation (7) to solve for $D(X)$, and finally obtain the estimate of $f(X, Y, R_1, R_2)$ according to Proposition 1. The first step can be achieved by standard nonparametric estimation and the third step only involves basic arithmetic; however, solving equation (7) in the second step is in general complicated. In the next section, we consider a feasible parametrization for the joint distribution $f(X, Y, R_1, R_2)$ and develop estimation methods.
4. INVERSE PROBABILITY WEIGHTED AND OUTCOME REGRESSION-BASED ESTIMATION

4.1 Parametrization

Under Assumption 1, we introduce the following factorization of the joint distribution as the basis for parametrization and estimation.

$$f(Y, R_1, R_2 | X) = c_1(X) \cdot f(R_1 | X, Y = 0) \cdot \exp\{(R_1 - 1)\Gamma(X, Y)\} \cdot f(Y | R_2 = 1, R_1 = 0, X)$$

$$c_1(X) = \frac{1}{f(R_1 = 1 | X)} \cdot \frac{1}{E\{1/f(R_2 = 0 | R_1 = 0, X, Y) | R_2 = 1, R_1 = 0, X\}},$$

where $c_1(X)$ is a normalizing function of $X$ that makes the right hand side of (8) a valid density function. We prove (8) in Lemma 1 in the supplement. This factorization enables a convenient and congenial specification of four components of the joint distribution:

- two baseline propensity scores $f(R_1 = 1 | X, Y = 0)$ and $f(R_2 = 1 | R_1 = 0, X, Y = 0)$;
- the odds ratio function $\Gamma(X, Y)$;
- and the outcome distribution for the second call $f(Y | R_2 = 1, R_1 = 0, X)$.

This kind of factorization of a joint density into a combination of univariate conditionals and odds ratio is widely applicable; see Osius (2004), Chen (2007), Kim and Yu (2011), and Franks et al. (2020) for examples in missing data analysis and causal inference. For notational convenience, we let $\pi_1(X, Y) = f(R_1 = 1 | X, Y)$ and $\pi_2(X, Y) = f(R_2 = 1 | R_1 = 0, X, Y)$ denote the propensity scores, $A_1(X) = \logit f(R_1 = 1 | X, Y = 0)$ and $A_2(X) = \logit f(R_2 = 1 | R_1 = 0, X, Y = 0)$ the logit transformations of the corresponding baseline propensity scores, and $f_2(Y | X) = f(Y | R_2 = 1, R_1 = 0, X)$ the outcome model for the second call. Note that $\pi_1(X, Y)$ and $\pi_2(X, Y)$ are determined by $A_1(X), A_2(X), \Gamma(X, Y)$ as in equations (4)–(5) and that the joint distribution is determined once given $A_1, A_2, \Gamma, f_2$. We will write $\pi_1, \pi_2, A_1, A_2, f_2, \Gamma$ for short where it does not cause confusion.

One may have concern that inequality (6) makes the range of $f(R_2 = 1 | R_1 = 0, X, Y)$ dependent on the value of $f(R_1 = 1 | X, Y)$. However, we do not model these two propensity scores separately, and we model $A_1(X), A_2(X)$ and $\Gamma(X, Y)$ instead. We show in Lemma 1 in the supplement that these four components $A_1, A_2, f_2, \Gamma$ of the joint distribution are indeed variationally independent. In the next, we consider semiparametric estimation under correct specification of a subset of these four models. If one would like to consider a different parametrization of the joint distribution, we refer to Richardson et al. (2017) about how to address the issue of variational dependence.

4.2 Inverse probability weighting

We specify parametric working models for two baseline propensity scores $A_1(X; \alpha_1), A_2(X; \alpha_2)$, and the odds ratio function $\Gamma(X, Y; \gamma)$. By definition, we require $\Gamma(X, Y = 0; \gamma) = 0$. This is equivalent to specifying propensity score models $\pi_1(X; \alpha_1, \gamma)$ and $\pi_2(X; \alpha_2, \gamma)$. The logistic model in Proposition 2 is an example. The following equations characterize the propensity scores,

$$0 = E\left\{\frac{R_1}{\pi_1} - 1 \mid X, Y\right\}, \tag{9}$$

$$0 = E\left\{\frac{R_2 - R_1}{\pi_2} - (1 - R_3) \mid X, Y\right\}, \tag{10}$$

$$0 = E\left\{\frac{R_2 - R_1}{\pi_2} - \frac{1 - \pi_1}{\pi_1} R_1 \mid X, Y\right\}. \tag{11}$$
The first equation follows from the definition of $\pi_1$ and the other two echo the definition of $\pi_2$ by noting that $E\{ (R_2 - R_1)/\pi_2 - (1 - R_1) \mid X, Y \} = E\{ (1 - R_1)(R_2/\pi_2 - 1) \mid X, Y \} = 0$. These three conditional moment equations motivate the following marginal moment equations for estimating $(\alpha_1, \alpha_2, \gamma)$:

$$
0 = \hat{E} \left\{ \frac{R_1}{\pi_1(\alpha_1, \gamma)} - 1 \right\} \cdot V_1(X),
$$

$$
0 = \hat{E} \left\{ \frac{R_2 - R_1}{\pi_2(\alpha_2, \gamma)} - (1 - R_1) \right\} \cdot V_2(X),
$$

$$
0 = \hat{E} \left\{ \frac{R_2 - R_1}{\pi_2(\alpha_2, \gamma)} - 1 - \pi_1(\alpha_1, \gamma) \cdot R_1 \right\} \cdot U(X, Y),
$$

where $\hat{E}$ denotes the empirical mean operator and $V_1(X) = \partial A_1(X; \alpha_1) / \partial \alpha_1$, $V_2(X) = \partial A_2(X; \alpha_2) / \partial \alpha_2$, $U(X, Y) = \partial \Gamma(X, Y; \gamma) / \partial \gamma$. For instance, in the linear logistic model with $A_1(X; \alpha_1) = (1, X^T)\alpha_1$, $A_2(X; \alpha_2) = (1, X^T)\alpha_2$, $\Gamma(X, Y; \gamma) = Y\gamma$, one may use $V_1(X) = V_2(X) = (1, X^T)^T$, $U(X, Y) = Y$. Note that $V_1, V_2, U$ can be chosen as other user-specified functions; see Tsiatis (2006, page 30).

Equations (12)–(14) only involve the observed data. The generalized method of moments (Hansen, 1982) can be implemented to solve these equations. Letting $(\hat{\alpha}_{1,ipw}, \hat{\alpha}_{2,ipw}, \hat{\gamma}_{ipw})$ be the nuisance estimators obtained from (12)–(14), $\hat{\pi}_1 = \pi_1(\hat{\alpha}_{1,ipw}, \hat{\gamma}_{ipw})$ and $\hat{\pi}_2 = \pi_2(\hat{\alpha}_{2,ipw}, \hat{\gamma}_{ipw})$ the estimated propensity scores, and $\hat{p}_2 = \hat{\pi}_1 + \hat{\pi}_2(1 - \hat{\pi}_1)$ an estimator of $p_2 = f(R_2 = 1 \mid X, Y)$, we propose the following IPW estimator of the outcome mean,

$$
\hat{\mu}_{ipw} = \hat{E} \left\{ \frac{R_2}{\hat{p}_2} Y \right\}.
$$

Preceding our proposal, Kim and Im (2014) developed a calibration estimator under the common-slope logistic model (1). Our IPW estimator can be viewed as a generalization of the calibration estimator, which can in fact be obtained from our IPW estimator by choosing appropriate functions $V_1, V_2, U$. However, the calibration estimator may be biased if the slopes of covariates $X$ differ in the two propensity score models, while our IPW estimator works in this case. We include a numerical simulation comparing these two estimators in Section S6.1 of the supplement. Under the common-slope linear logistic model, Qin and Follmann (2014) and Guan et al. (2018) developed empirical likelihood-based estimation that is convenient to incorporate auxiliary information to achieve higher efficiency; it is of interest to extend their approach to the setting with call-specific slopes for covariates.

4.3 Outcome regression-based estimation

Alternatively, the outcome mean can be obtained by estimation or imputation of the missing values. We specify and fit working models $\pi_1(\alpha_1, \gamma), f_2(Y \mid X; \beta)$ for the first-call propensity score $f(R_1 = 1 \mid X, Y)$ and the second-call outcome distribution $f(Y \mid X, R_2 = 1, R_1 = 0)$, and impute the missing values with

$$
f(Y \mid X, R_2 = 0) = \frac{\exp(-\Gamma)f(Y \mid X, R_2 = 1, R_1 = 0)}{E\{\exp(-\Gamma) \mid X, R_2 = 1, R_1 = 0\}},
$$

which is known as the exponential tilting or Tukey’s representation (Kim and Yu, 2011; Vansteelandt et al., 2007; Franks et al., 2020). To obtain the nuisance estimators $(\hat{\beta}, \hat{\alpha}_{1,reg}, \hat{\gamma}_{reg})$, we solve

$$
0 = \hat{E} \left\{ (R_2 - R_1) \cdot \frac{\partial \log f_2(Y \mid X; \beta)}{\partial \beta} \right\},
$$

$$
0 = \hat{E} \left\{ \frac{R_1}{\pi_1(\alpha_1, \gamma)} - R_2 \right\} U(X, Y) - (1 - R_2) E\{U(X, Y) \mid X, R_2 = 0; \beta, \gamma\},
$$
where \( E(\cdot \mid X, R_2 = 0; \beta, \gamma) \) is evaluated according to (16) and \( U(X, Y) = \{ \partial A_1(X; \alpha_1)/\partial \alpha_1, \partial U(X, Y; \gamma)/\partial \gamma \} \). Note that \( U(X, Y) \) can be chosen as other user-specified functions; see Tsiatis (2006, page 30). Equation (17) is the score equation of \( f_2(Y \mid X; \beta) \). Equation (18) is motivated by the equation \( \hat{E}[(1 - R_2) \{ U(X, Y) - E\{ U(X, Y) \mid R_2 = 0, X \} \}] = 0 \), but the evaluation of \( \hat{E}\{ U(X, Y) \} \) is untenable due to missing values of \( Y \), and thus we replace with \( \hat{E}\{ R_1U(X, Y)/\pi_1 \} \). An estimator of the outcome mean by imputing the missing outcome values with \( E(Y \mid X, R_2 = 0; \hat{\beta}, \hat{\gamma}_{reg}) \) is

\[
\hat{\mu}_{reg} = \hat{E}\{ R_2Y + (1 - R_2)E(Y \mid X, R_2 = 0; \hat{\beta}, \hat{\gamma}_{reg}) \}.
\] (19)

We refer to this approach as the outcome regression-based (REG) estimation, although the first-call propensity score model is also involved.

Guan et al. (2018) have previously proposed an estimator that involves the full-data outcome regression \( E(Y \mid X) \) whereas our REG estimation involves \( E(Y \mid R_2 = 1, R_1 = 0, X) \) that only concerns the observed data and enjoys the ease for model specification and estimation. Moreover, estimation of their outcome regression model requires estimating the propensity scores for all call attempts, but our REG estimation only requires estimating the first-call propensity score.

Note that (12)–(15) are unbiased estimating equations for \((\alpha_1, \alpha_2, \gamma, \mu)\) in model

\[
M_{ipw} = \{ f(X, Y, R_1, R_2) : \text{Assumption 1 holds; } \pi_1(\alpha_1, \gamma) \text{ and } \pi_2(\alpha_2, \gamma) \text{ are correct} \},
\]

and (17)–(19) are unbiased estimating equations for \((\beta, \alpha_1, \gamma, \mu)\) in model

\[
M_{reg} = \{ f(X, Y, R_1, R_2) : \text{Assumption 1 holds; } \pi_1(\alpha_1, \gamma) \text{ and } f_2(Y \mid X; \beta) \text{ are correct} \}.
\]

As a consequence, consistency and asymptotic normality of \((\hat{\alpha}_{1,ipw}, \hat{\alpha}_{2,ipw}, \hat{\gamma}_{ipw}, \hat{\mu}_{ipw})\) in model \( M_{ipw} \) and of \((\hat{\beta}, \hat{\alpha}_{1,reg}, \hat{\gamma}_{reg}, \hat{\mu}_{reg})\) in model \( M_{reg} \) can be established under standard regularity conditions (see e.g., Newey and McFadden, 1994) by following the theory of estimating equations, which we will not replicate here. The choice of user-specified functions \( V_1(X), V_2(X), U(X, Y) \) depends on the working models and influences the efficiency of the estimators. In principle, the optimal choice can be obtained by deriving the efficient influence function for the nuisance parameters in models \( M_{ipw} \) and \( M_{reg} \), respectively. However, the potential prize of attempting to attain local efficiency for nuisance parameters may not always be worth the chase because it depends on correct modeling of additional components of the joint distribution beyond the working models, and as pointed out by Stephens et al. (2014) such additional modeling efforts seldom deliver the anticipated efficiency gain. Besides, consistency of the estimators is undermined if the required working models are incorrect. Therefore, we next propose a doubly robust and locally efficient approach.

5. SEMIPARAMETRIC THEORY AND DOUBLY ROBUST ESTIMATION
5.1 Semiparametric theory
We consider the model \( M_{np} \) characterized by Assumption 1,

\[
M_{np} = \{ f(X, Y, R_1, R_2) : \text{Assumption 1 holds; } A_1(X), A_2(X), \Gamma(X, Y), f_2(Y \mid X) \text{ are unrestricted} \}.
\]

Although the stableness of resistance assumption does impose an inequality constraint (6) on the observed-data distribution, we refer to \( M_{np} \) as the (locally) nonparametric model because no parametric models are imposed in \( M_{np} \) and as established in the following, the observed-data tangent space under \( M_{np} \) is the entire Hilbert space. We aim to derive the set of influence functions for all regular and asymptotically linear (RAL) estimators of \( \mu \) and the efficient influence function under \( M_{np} \). To achieve this goal, the primary step is to derive the observed-data tangent space for model \( M_{np} \). Hereafter, we let \( O = (X, Y, R_1, R_2) \) if \( R_2 = 1 \) and \( O = (X, R_1, R_2) \) if \( R_2 = 0 \) denote the observed data.
**Proposition 3.** The observed-data tangent space for \( \mathcal{M}_{np} \) is

\[
\mathcal{T} = \{ h(O) = R_1 h_1(X, Y) + R_2 h_2(X, Y) + h_3(X); E\{ h(O) \} = 0, E\{ h^2(O) \} < \infty \},
\]

where \( h_1(X, Y), h_2(X, Y) \) and \( h_3(X) \) are arbitrary measurable and square-integrable functions.

This proposition states that the observed-data tangent space for \( \mathcal{M}_{np} \) is the entire Hilbert space of observed-data functions with mean zero and finite variance, equipped with the usual inner product. Hence, the stableness of resistance assumption does not impose any local restriction on the observed-data distribution. As a result, there exits a unique influence function for \( \mu \) in model \( \mathcal{M}_{np} \), which must be the efficient one. We have derived the closed form for the efficient influence function.

**Theorem 2.** The efficient influence function for \( \mu \) in the nonparametric model \( \mathcal{M}_{np} \) is

\[
\text{IF}(O; \mu) = \left\{ \frac{R_2 - R_1}{\pi_2} - \frac{1 - \pi_1}{\pi_1}\frac{R_1}{\pi_2} + \frac{R_1}{\pi_1} \right\} \left\{ Y - \frac{E(Y/\pi_2 \mid X, R_2 = 0)}{E(1/\pi_2 \mid X, R_2 = 0)} \right\} + \frac{E(Y/\pi_2 \mid X, R_2 = 0)}{E(1/\pi_2 \mid X, R_2 = 0)} - \mu.
\]

If we know a priori that the missingness mechanism is at random, i.e., in the submodel of \( \mathcal{M}_{np} \) with \( \Gamma(X, Y) = 0 \), or equivalently \( R_1 \perp Y \mid (X, R_2) \) and \( R_2 \perp Y \mid (X, R_1) \), we can show that the efficient influence function is

\[
\text{IF}_{aipw}(O; \mu) = \frac{R_2}{f(R_2 = 1 \mid X)} X - \left\{ \frac{R_2}{f(R_2 = 1 \mid X)} - 1 \right\} E(Y \mid X) - \mu.
\]

**Proposition 4.** \( \text{IF}_{aipw}(O; \mu) \) is the efficient influence function for \( \mu \) in model \( \mathcal{M}_{mar} = \{ f(X, Y, R_1, R_2) : f(X, Y, R_1, R_2) \in \mathcal{M}_{np} \text{ and } \Gamma(X, Y) = 0 \} \).

The tangent space of \( \mathcal{M}_{mar} \) is no longer the entire Hilbert space and Proposition 4 does not contradict Theorem 2. It is well known that \( \text{IF}_{aipw}(O; \mu) \) is the efficient influence function for \( \mu \) in the model \( \{ f(X, Y, R_2) : R_2 \perp Y \mid X \} \) when the final availability \( R_2 \) is MAR. The availability of callback data \( R_1 \) does not change this efficiency bound when missingness in both calls are at random.

From Theorem 2, the semiparametric efficiency bound for estimating \( \mu \) in \( \mathcal{M}_{np} \) is \( E\{ \text{IF}(O; \mu)^2 \} \). A locally efficient estimator attaining this semiparametric efficiency bound can be constructed by plugging nuisance estimators of \( \pi_1, \pi_2, E(\cdot \mid R_2 = 0, X) \) that converge sufficiently fast into the efficient influence function and then solving \( E\{ \text{IF}(O; \mu) \} = 0 \). However, one may not be confident that these nuisance parameters can be modeled correctly. It is therefore of interest to develop a doubly robust estimation approach, which delivers valid inferences about the outcome mean provided that a subset but not necessarily all low dimensional models for the nuisance parameters are specified correctly.

### 5.2 Doubly robust estimation

To construct a doubly robust (DR) estimator, we specify working models \{ \( A_1(X; \alpha_1), A_2(X; \alpha_2), \Gamma(X, Y; \gamma), f_2(Y \mid X; \beta) \} \) and estimate the nuisance parameters by solving

\[
0 = \hat{E} \left\{ \left( R_2 - R_1 \right) \cdot \frac{\partial \log f_2(Y \mid X; \beta)}{\partial \beta} \right\}, \tag{20}
\]

\[
0 = \hat{E} \left\{ \left\{ \frac{R_1}{\pi_1(\alpha_1, \gamma)} - 1 \right\} \cdot V_1(X) \right\}, \tag{21}
\]

\[
0 = \hat{E} \left\{ \left\{ \frac{R_2 - R_1}{\pi_2(\alpha_2, \gamma)} - (1 - R_1) \right\} \cdot V_2(X) \right\}, \tag{22}
\]

\[
0 = \hat{E} \left\{ \left\{ \frac{R_2 - R_1}{\pi_2(\alpha_2, \gamma)} - \frac{1 - \pi_1(\alpha_1, \gamma)}{\pi_1(\alpha_1, \gamma)} R_1 \right\} \cdot \{ U(X, Y) - E(U(X, Y) \mid X, R_2 = 0; \beta, \gamma) \} \right\}. \tag{23}
\]
where $V_1(X) = \partial A_1(X; \alpha_1)/\partial \alpha_1$, $V_2(X) = \partial A_2(X; \alpha_2)/\partial \alpha_2$, $U(X, Y) = \partial \Gamma(X, Y; \gamma)/\partial \gamma$. Note that $V_1, V_2, U$ can be chosen as other user-specified functions; see discussions in Section 4.3 and Tsiatis (2006, page 30). Let $(\hat{\beta}, \hat{\alpha}_{1, \text{dr}}, \hat{\alpha}_{2, \text{dr}}, \hat{\gamma}_{\text{dr}})$ denote the solution to (20)–(23) and $\hat{\pi}_1 = \pi_1(\hat{\alpha}_{1, \text{dr}}, \hat{\gamma}_{\text{dr}})$, $\hat{\pi}_2 = \pi_2(\hat{\alpha}_{2, \text{dr}}, \hat{\gamma}_{\text{dr}})$, then an estimator of $\mu$ motivated from the influence function in Theorem 2 is

$$
\hat{\mu}_{\text{dr}} = E \left[ \left\{ \frac{R_2 - R_1}{\hat{\pi}_2^2} - \frac{1 - \hat{\pi}_1}{\hat{\pi}_2} \frac{R_1}{\hat{\pi}_1} \right\} Y \right] - \tilde{E} \left[ \left\{ \frac{R_2 - R_1}{\hat{\pi}_2^2} - \frac{1 - \hat{\pi}_1}{\hat{\pi}_2} \frac{R_1}{\hat{\pi}_1} + \frac{R_1}{\hat{\pi}_1} - 1 \right\} \frac{E(Y | X, R_2 = 0; \hat{\beta}, \hat{\gamma}_{\text{dr}})}{E(1/\hat{\pi}_2 | X, R_2 = 0; \hat{\beta}, \hat{\gamma}_{\text{dr}})} \right].
$$

Equations (20)–(22) for estimating $(\beta, \alpha_1, \alpha_2)$ remain the same as (17), (12), and (13) in the IPW and REG estimation, respectively; equation (23) is a doubly robust estimating equation for $\gamma$. We summarize the double robustness of the estimators.

**Theorem 3.** Under Assumption 1 and regularity conditions described by Newey and McFadden (1994, Theorems 2.6 and 3.4), $(\hat{\alpha}_{1, \text{dr}}, \hat{\gamma}_{\text{dr}}, \hat{\mu}_{\text{dr}})$ are consistent and asymptotically normal provided one of the following conditions holds:

- $A_1(X; \alpha_1), \Gamma(X, Y; \gamma)$, and $A_2(X; \alpha_2)$ are correctly specified; or
- $A_1(X; \alpha_1), \Gamma(X, Y; \gamma)$, and $f_2(Y | X; \beta)$ are correctly specified.

Furthermore, $\hat{\mu}_{\text{dr}}$ attains the semiparametric efficiency bound for the nonparametric model $M_{\text{np}}$ at the intersection model $M_{\text{ipw}} \cap M_{\text{reg}}$ where all models $\{A_1(X; \alpha_1), A_2(X; \alpha_2), \Gamma(X, Y; \gamma), f_2(Y | X; \beta)\}$ are correct.

This theorem states that $(\hat{\alpha}_{1, \text{dr}}, \hat{\gamma}_{\text{dr}}, \hat{\mu}_{\text{dr}})$ are doubly robust against misspecification of the second-call baseline propensity score $A_2(X; \alpha_2)$ and the second-call outcome distribution $f_2(Y | X; \beta)$, provided that the first-call propensity score $\pi_1(X, Y; \alpha_1, \gamma)$ (i.e., $A_1(X; \alpha_1)$ and $\Gamma(X, Y; \gamma)$) is correctly specified. Moreover, $\hat{\mu}_{\text{dr}}$ is locally efficient if all working models are correct, regardless of the efficiency of the nuisance estimators. The outcome mean estimator $\hat{\mu}_{\text{dr}}$ has an analogous form to the conventional augmented inverse probability weighted (AIPW) estimator: the first part is an IPW estimator and the second part is an augmentation term involving the outcome regression model. Compared to the IPW and REG estimators in the previous section, the doubly robust estimator offers one more chance to correct the bias due to model misspecification. However, the DR estimator is not anticipated to have a smaller variance than the IPW or the REG estimator, because $M_{\text{np}}$ is a larger model with a semiparametric efficiency bound no smaller than that of $M_{\text{ipw}}$ or $M_{\text{reg}}$. Besides, if both $A_2(X; \alpha_2)$ and $f_2(Y | X; \beta)$ are incorrect, the proposed doubly robust estimator will generally also be biased. The odds ratio $\Gamma(X, Y; \gamma)$ is essential for all three proposed estimation methods, because it encodes the degree to which the outcome and the missingness process are correlated. In order to estimate the outcome mean, one must be able to account for this correlation. The first-call baseline propensity score $A_1(X; \alpha_1)$ needs to be correct for estimation of the odds ratio, and as a result, $\hat{\alpha}_{1, \text{dr}}$ is also doubly robust. If $\pi_1(\alpha_1, \gamma)$ is incorrect, $\hat{\mu}_{\text{dr}}$ is in general not consistent even if both $A_2(X; \alpha_2)$ and $f_2(Y | X; \beta)$ are correct—because the latter two models only concern the second call. Variance estimation and confidence intervals for the doubly robust approach also follow from the general theory for estimation equations (see e.g., Newey and McFadden, 1994), which can be constructed based on the normal approximation or bootstrap under standard regularity conditions.

Doubly robust methods have been advocated in recent years for missing data analysis, causal inference, and other problems with data coarsening. Previous proposals have assumed that the odds ratio function
\( \Gamma(X, Y) \) is either known exactly with the special case of MAR (e.g., Scharfstein et al., 1999; Lipsitz et al., 1999; Tan, 2006; van der Laan and Rubin, 2006; Vansteelandt et al., 2007; Okui et al., 2012), or can be estimated with the aid of an extra instrumental or shadow variable (e.g., Miao and Tchetgen Tchetgen, 2016; Sun et al., 2018; Ogburn et al., 2015). We offer an alternative approach that achieves doubly robust estimation of both the odds ratio model and the outcome mean with the aid of callback data. The double robustness of the DR estimator in Theorem 3 requires \( A \) estimation of both the odds ratio model and the outcome mean with the aid of callback data. The doubly robust estimation is anticipated to be more efficient than that in the nonparametric model \( M_{np} \), the validity of this efficient influence function relies on the correctness of parametric working models for \( A_1(X) \) and \( \Gamma(X, Y) \). Instead, it is preferable to use flexible or nonparametric working models to estimate all the nuisance functions and then plug in them into the efficient influence function \( \text{IF}(O; \mu) \) in the nonparametric model \( M_{np} \) to estimate \( \mu \). The following theorem establishes that such an estimator can deliver valid \( n^{1/2} \) inference for the functional of interest even if the nuisance estimators could have convergence rates considerably slower than \( n^{1/2} \). Let \( \hat{A}_1, \hat{A}_2, \hat{\Gamma} \) and \( \hat{f}_2 \) denote the nuisance estimators using some flexible estimation methods, with the following norms that measure the convergence rates, \( \| \hat{A}_1 - A_1 \|_2 = \| \{ \hat{A}_1(x) - A_1(x) \}^2 f(x)dx \|^{1/2}, \| \hat{A}_2 - A_2 \|_2 = \| \{ \hat{A}_2(x) - A_2(x) \}^2 f(x)dx \|^{1/2}, \| \hat{\Gamma} - \Gamma \|_2 = \| \{ \int (\hat{\Gamma}(x, y) - \Gamma(x, y))^2 f_2(y | x)dyf(x)dx \|^{1/2}, \| \hat{f}_2 - f_2 \| = \| \{ \int \hat{f}_2(y | x) - f_2(y | x)dy \}^2 f(x)dx \|^{1/2}. \)

**Theorem 4.** *Under Assumption 1, suppose \( c \leq \pi_1, \pi_2 \leq 1 - c \) for some constant \( c > 0 \), \( |Y| \leq M \) for some constant \( M > 0 \), and the nuisance estimators satisfy that (i) \( c \leq \pi_1, \pi_2 \leq 1 - c \); (ii) \( \| \hat{A}_1 - A_1 \|_2 = o_p(1), \| \hat{A}_2 - A_2 \|_2 = o_p(1), \| \hat{\Gamma} - \Gamma \|_2 = o_p(1), \| \hat{f}_2 - f_2 \| = o_p(1) \); and (iii) \( \hat{A}_1, \hat{A}_2, \hat{\Gamma}, \hat{f}_2 \) and \( A_1, A_2, \Gamma, f_2 \) are in a Donsker class. Letting*

\[
\hat{\mu}_{dr2} = \hat{E} \left[ \left\{ \frac{R_2 - R_1}{\pi_2} - \frac{1 - \hat{\pi}_1 R_1}{\pi_2} + \frac{R_1}{\pi_1} \right\} Y \right] - \hat{E} \left[ \left\{ \frac{R_2 - R_1}{\pi_2} - \frac{1 - \hat{\pi}_1 R_1}{\pi_2} + \frac{R_1}{\pi_1} - 1 \right\} \int \frac{y e^{-\hat{\Gamma}/\hat{\pi}_2 \hat{f}_2}dy}{\int e^{-\hat{\Gamma}/\hat{\pi}_2 \hat{f}_2}dy} \right],
\]

*then we have*

\[
\hat{\mu}_{dr2} - \mu = \hat{E} \{ \text{IF}(O; \mu) \} + O_p(\text{Rem}) + o_p(n^{-1/2}),
\]

*where*

\[
\text{Rem} = \| \hat{A}_1 - A_1 \|^2 + \| \hat{\Gamma} - \Gamma \|^2 + \| \hat{A}_1 - A_1 \|_2 \cdot \| \hat{\Gamma} - \Gamma \|_2 + \| \hat{A}_2 - A_2 \|_2 \cdot \| \hat{f}_2 - f_2 \|^2 + (\| \hat{A}_1 - A_1 \|_2 + \| \hat{\Gamma} - \Gamma \|_2) \cdot (\| \hat{A}_2 - A_2 \|_2 + \| \hat{f}_2 - f_2 \|_2). \]

**Theorem 4** generalizes the parametric doubly robust estimation. **Theorem 4** implies that \( \hat{\mu}_{dr2} \) has influence function \( \text{IF}(O; \mu) \) if the bias term \( \text{Rem} \), a function of estimation errors of nuisance estimators, is of order \( o_p(n^{-1/2}) \). The bias of \( \hat{\mu}_{dr2} \) is of second order, i.e., it only depends on squared errors of nuisance estimators. This result opens the way to implement more sophisticated estimation methods built on flexible, data-adaptive machine learning or nonparametric nuisance models to achieve \( n^{1/2} \)-consistent estimation of the functional of interest, provided that the nuisance estimators have estimation error of order
smaller than $n^{-1/4}$. For example, one can use sieve estimation with polynomials as basis functions to approximate nuisance functions. Suppose $A_1(X)$ and $A_2(X)$ are unrestricted, $\Gamma(X, Y) = G(X)Y$, $f_2(Y \mid X) \sim N(B_1(X), \exp\{B_2(X)\})$ for some unknown functions $G, B_1, B_2$ of $X$, and let $\{\psi_j(X)\}_{j=1}^{\infty}$ denote a sequence of polynomials of $X$ and $V_{i,n}(X) = \{\psi_1(X), \ldots, \psi_{J_{i,n}}(X)\}^T$ for some $J_{i,n}$, $i = 1, \ldots, 5$, then we approximate the nuisance functions by $A_{1,n}(X) = \alpha_{1,n}^T V_{1,n}(X)$, $A_{2,n}(X) = \alpha_{2,n}^T V_{2,n}(X)$, $G_n(X) = \gamma_n^T V_{3,n}(X)$, $B_{1,n}(X) = \beta_{1,n}^T V_{4,n}(X)$, $B_{2,n}(X) = \beta_{2,n}^T V_{5,n}(X)$. The nuisance estimators are obtained with $\alpha_{1,n}, \alpha_{2,n}, \gamma_n, \beta_{1,n}, \beta_{2,n}$ estimated by solving equations (20)-(23). Finally, $\hat{\mu}_{dr2}$ is obtained by plugging in the nuisance estimators into (25). Theorem 4 imposes the Donsker condition (van der Vaart and Wellner, 1996) on the complexity of the nuisance models. It could be relaxed to admit more complicated working models by employing the cross-fitting method (e.g., Robins et al., 2008; Chernozhukov et al., 2018).

Estimation methods with second-order bias have been extensively studied by Benkeser et al. (2017); Kennedy et al. (2017); Athey et al. (2018); Tan (2020); Rotnitzky et al. (2021); Dukes and Vansteelandt (2021); Chernozhukov et al. (2018), and Theorem 4 contributes to this literature with an instance in non-ignorable missing data analysis. Theorem 4 reveals the mixed bias property (Rotnitzky et al., 2021) in the nonignorable missing data setting when the odds ratio function needs to be estimated: if $A_1$ and $\Gamma$ are estimated parametrically, the bias of $\hat{\mu}_{dr2}$ only depends on $||\hat{A}_2 - A_2||_2$ and $||\hat{f}_2 - f_2||$ through their product. The justification of this result involves a more sophisticated analysis of the bias term instead of directly applying the second-order Taylor expansion technique, and our technical contribution may shed some light on characterizing the bias terms of estimators with multiple nuisance components.

6. EXTENSIONS

6.1 Estimation of a general full-data functional

We extend the proposed IPW, REG, and DR methods to estimation of a general smooth full-data functional $\theta$, defined as the unique solution to a given estimating equation $E\{m(X, Y; \theta)\} = 0$. Familiar examples include the outcome mean with $m(X, Y; \mu) = Y - \mu$; the least squares coefficient with $m(X, Y; \theta) = X(Y - \theta^T X)$; and the instrumental variable estimand in causal inference with $m(X, Y; \theta) = Z(Y - \theta^T W)$, where $W$ and $Z$ are subvectors of $X$, $\theta$ is the causal effect of $W$ on $Y$ and is subject to unmeasured confounding and $Z$ is a set of instrumental variables used for confounding adjustment in causal inference. Given estimators of the nuisance parameters, the IPW and REG estimators of $\theta$ can be obtained by solving

$$0 = \hat{E} \left\{ \frac{R_2}{\hat{p}_2} m(X, Y; \theta) \right\},$$

$$0 = \hat{E}[R_2 m(X, Y; \theta) + (1 - R_2) E\{m(X, Y; \theta) \mid X, R_2 = 0; \hat{\beta}, \hat{\gamma}_{\text{reg}}\}],$$

respectively. Letting $m(\theta) \equiv m(X, Y; \theta)$, the efficient influence function for $\theta$ in model $\mathcal{M}_{np}$ is

$$IF(O; \theta) = - \left[ \frac{\partial E\{m(\theta)\}}{\partial \theta} \right]^{-1} \left[ \left\{ \frac{R_2 - R_1}{\pi_2^2} - \frac{1 - \pi_1}{\pi_1} \frac{R_1}{\pi_2} + \frac{R_1}{\pi_1} - 1 \right\} \left\{ m(\theta) - \frac{E(m(\theta)/\pi_2 \mid X, R_2 = 0)}{E(1/\pi_2 \mid X, R_2 = 0)} \right\} + m(\theta) \right].$$

(27)

We prove this result in the supplement. Then a doubly robust and locally efficient estimator of $\theta$ can be constructed by solving $\hat{E}\{IF(O; \theta)\} = 0$, after obtaining nuisance estimators.

6.2 Estimation with multiple callbacks

When multiple calls ($K \geq 3$) are available, identification of $f(X, Y, R_1, \ldots, R_K)$ equally holds under Assumption 1, even if the propensity scores and outcome distributions for the third and later call attempts
are completely unrestricted. Besides, identification can be achieved if Assumption 1 holds for at least two known adjacent calls. To see this, suppose that the stableness of resistance holds for the \( k \)th and \( k+1 \)th calls. By viewing the \( k \)th and \( k+1 \)th calls as the first two calls in a subsurvey on nonrespondents from the first \( k-1 \)th calls (i.e., \( R_{k-1} = 0 \)) and by applying the proposed approach, we can identify \( f(X, Y, R_k, R_{k+1} | R_{k-1} = 0) \). Noting that \( f(X, Y, R_k, R_{k+1} | R_{k-1} = 1) \) is available from the observed data, we can identify \( f(X, Y, R_{k-1}, R_k, R_{k+1}) \) and thus \( f(X, Y) \). Identifying \( f(X, Y) \) suffices to identify \( f(X, Y, R_1, \ldots, R_K) \). In this case, the proposed IPW, REG, and DR estimation methods developed for the setting with two calls still work but they are agnostic to the data obtained in later calls. In Section S4 of the supplement, we extend the proposed IPW, REG, and DR estimators to the setting with multiple callbacks, which can incorporate all observations on \( (X, Y) \).

7. SIMULATION

We evaluate the performance of the proposed estimators and assess their robustness against misspecification of working models and violation of the key identifying assumption via simulations. We conduct simulations for both continuous and binary outcome settings. Simulation results for these two settings are analogous, and to save space, here we only present simulations for the continuous setting and defer the binary setting to Section S6.3 of the supplement.

Let \( X = (1, X_1, X_2)^T \) and \( \bar{X} = (1, X_1^2, X_2^2) \) with \( X_1, X_2 \) independent following a uniform distribution \( \text{Unif}(-1, 1) \). We consider four data generating scenarios according to different choices for the second-call baseline propensity score and the second-call outcome distribution. The following Table 2 presents the data generating mechanisms and the working models for estimation.

| Scenario | Data generating model | Working model for estimation |
|----------|-----------------------|-----------------------------|
| TT       | \( \pi_1 = \expit(\alpha_1 X + \gamma Y) \), \( \pi_2 = \expit(\alpha_2 W_1 + \gamma Y) \), \( f_2(Y | X) \sim N(\beta^T W_2, \sigma^2) \) | Working model for estimation: \( \pi_1 = \expit(\alpha_1^T X + \gamma Y) \), \( \pi_2 = \expit(\alpha_2^T X + \gamma Y) \), \( f_2(Y | X) \sim N(\beta^T X, \sigma^2) \) |
| FT       | \( W_1 = X, W_2 = X \) \quad \pi_1 = \expit(\alpha_1^T X + \gamma Y) \), \( \pi_2 = \expit(\alpha_2^T W_1 + \gamma Y) \), \( f_2(Y | X) \sim N(\beta^T W_2, \sigma^2) \) | Working model for estimation: \( \pi_1 = \expit(\alpha_1^T X + \gamma Y) \), \( \pi_2 = \expit(\alpha_2^T X + \gamma Y) \), \( f_2(Y | X) \sim N(\beta^T X, \sigma^2) \) |
| TF       | \( W_1 = \bar{X}, W_2 = X \) \quad \pi_1 = \expit(\alpha_1^T \bar{X} + \gamma Y) \), \( \pi_2 = \expit(\alpha_2^T W_1 + \gamma Y) \), \( f_2(Y | X) \sim N(\beta^T W_2, \sigma^2) \) | Working model for estimation: \( \pi_1 = \expit(\alpha_1^T \bar{X} + \gamma Y) \), \( \pi_2 = \expit(\alpha_2^T X + \gamma Y) \), \( f_2(Y | X) \sim N(\beta^T X, \sigma^2) \) |
| FF       | \( W_1 = X, W_2 = \bar{X} \) \quad \pi_1 = \expit(\alpha_1^T X + \gamma Y) \), \( \pi_2 = \expit(\alpha_2^T X + \gamma Y) \), \( f_2(Y | X) \sim N(\beta^T W_2, \sigma^2) \) | Working model for estimation: \( \pi_1 = \expit(\alpha_1^T X + \gamma Y) \), \( \pi_2 = \expit(\alpha_2^T X + \gamma Y) \), \( f_2(Y | X) \sim N(\beta^T X, \sigma^2) \) |

Hence, the working model for the second-call baseline propensity score is correct in Scenarios (TT) and (TF), the second-call outcome model is correct in Scenarios (TT) and (FT), and they both are incorrect in Scenario (FF). The first-call baseline propensity score and the odds ratio models are correct in all scenarios. For estimation, we implement the proposed IPW, REG, and DR methods \( (\hat{\mu}_{ipw}, \hat{\mu}_{reg}, \hat{\mu}_{dr}) \) based on parametric working models to estimate the outcome mean and the odds ratio parameter. We compute the variance of these estimators and then construct the confidence interval based on the normal approximation and evaluate the coverage rate. For comparison, we also implement standard IPW_<sub>mar</sub>, REG_<sub>mar</sub>, DR_<sub>mar</sub> estimators \( (\hat{\mu}_{ipw,mar}, \hat{\mu}_{reg,mar}, \hat{\mu}_{dr,mar}) \) that are based on MAR, with the number of callbacks included as an additional covariate.

For each scenario, we replicate 1000 simulations at sample size 5000. Figures 1 and 2 show the bias for estimators of the outcome mean and the odds ratio parameter, respectively, and Table 3 shows the coverage rates for the corresponding confidence intervals. In Scenario (TT) where all working models are correct,
all three proposed estimators have little bias and the 95% confidence intervals have coverage rates close to 0.95; in Scenario (FT) the second-call baseline propensity score model is incorrect but the second-call outcome model is correct, then the IPW estimator has large bias with coverage rate well below 0.95 and the REG estimator has little bias with the coverage rate close to 0.95; conversely, in Scenario (TF) the second-call baseline propensity score model is correct but the second-call outcome model is incorrect, then the IPW estimator has little bias with an appropriate coverage rate and the REG estimator has large bias with an undersized coverage rate. The DR estimator has little bias with appropriate coverage rates in all the three scenarios when at least one of these two working models is correct. In Scenario (FF) where both working models are incorrect, all three proposed estimators are biased. These numerical results show the robustness of $\hat{\mu}_{dr}$ against partial misspecification of working models. We compute the standard deviations of the three proposed estimators in Table S.1 in the supplement, which are close when all working models are correct. However, the three standard MAR estimators have large bias in all four scenarios. Therefore, we recommend the proposed methods for nonresponse adjustment with callback data, and we suggest to use different working models to gain more robust inferences.

In Section S6.2 of the supplement, we conduct additional simulations to compare the performance of $\hat{\mu}_{dr2}$ using flexible nuisance models and the IPW, REG and DR estimators that are based on parametric working models. We consider six data generating scenarios, including two scenarios where all parametric working models are correct or incorrect, respectively, and four scenarios where each of $A_1, A_2, f_2, \Gamma$ is incorrect, respectively. The estimator $\hat{\mu}_{dr2}$ has little bias with appropriate coverage rates in all six scenarios. But the IPW, REG and DR estimators could have large bias if the required working models are incorrect; in particular, these three estimators are biased if the odds ratio model is incorrectly specified. The numerical results show the robustness of $\hat{\mu}_{dr2}$, which delivers valid inference in scenarios where the nuisance models are complicated and parametric working models are more likely to be misspecified.

In addition to model misspecification, we also evaluate sensitivity of the inference against violation of the identifying assumption. We include a sensitivity analysis to evaluate the performance of the proposed estimators when call-specific odds ratios arise, i.e., the stableness of resistance assumption is not met. The difference between the odds ratios is used as the sensitivity parameter to capture the degree to which the stableness of resistance assumption is violated. The proposed estimators exhibit small bias when the difference of odds ratios varies within a moderate range but it could become severe if there were a big gap between the odds ratios. To obtain reliable inferences in practice, we recommend such sensitivity analysis for assessing robustness of inference against violation of the stableness of resistance assumption.
Figure 1: Bias for estimators of $\mu$ in the continuous outcome setting.

Figure 2: Bias for estimators of $\gamma$ in the continuous outcome setting.
Table 3: Coverage rate of 95% confidence interval in the continuous outcome setting

| Scenarios | IPW  | REG  | DR   | IPW_{mar} | REG_{mar} | DR_{mar} | IPW  | REG  | DR   |
|-----------|------|------|------|-----------|-----------|----------|------|------|------|
| TT        | 0.955| 0.955| 0.950| 0.109     | 0.362     | 0.363    | 0.955| 0.955| 0.956|
| FT        | 0.629| 0.948| 0.952| 0.000     | 0.442     | 0.426    | 0.322| 0.957| 0.945|
| TF        | 0.944| 0.125| 0.952| 0.000     | 0.000     | 0.000    | 0.947| 0.501| 0.944|
| FF        | 0.286| 0.446| 0.287| 0.000     | 0.000     | 0.000    | 0.489| 0.614| 0.675|

8. REAL DATA APPLICATION

The Consumer Expenditure Survey (CES; National Research Council, 2013) is a nationwide survey conducted by the U.S. Bureau of Labor Statistics to find out how American households make and spend money. It comprises two surveys: the Quarterly Interview Survey on large and recurring expenditures such as rent and utilities, and the Diary Survey on small and high frequency purchases, such as food and clothing. The survey data are released annually since 1980, which contain detailed callback history. We analyze the public-use microdata from the Quarterly Interview Survey in the fourth quarter of 2018, available from https://www.bls.gov/cex/pumd_data.htm#csv. The nonresponse of frame variables is concurrent due to contact failure or refusal and no fully-observed baseline covariates are available in this dataset. For illustration, we analyze this dataset to study the expenditures on housing and on utilities, fuels and public services. This survey contains 9986 households and 277 of them with extremely large or small expenditures are removed in our analysis. The maximum number of contact attempts the interviewers made in this survey is about 30. Figure 3 (a) shows the cumulative response rate. The overall response rate is about 0.6 and 80% of the respondents completed the survey within the first five contact attempts. Let \((Y_1, Y_2)\) denote the logarithm of the expenditure on housing and on utilities, fuels and public services, respectively. Figure 3 (b) shows the mean of \((Y_1, Y_2)\) for respondents to each call attempt. The mean of \(Y_1\) for respondents gradually increases with the number of contacts, and the mean of \(Y_2\) has a sharp increase in the second and third contacts and fluctuates in later contacts. These results suggest that the delayed respondents are likely to have higher expenditures and the nonresponse is likely dependent on the expenditures, in particular, on \(Y_1\).

![Figure 3: Response rates and outcomes mean for respondents in the CES application.](image-url)

We apply the proposed methods to estimate the outcomes mean \(\mu = (\mu_1, \mu_2) = (E(Y_1), E(Y_2))\). Analogous to previous survey studies (e.g., Qin and Follmann, 2014; Boniface et al., 2017), we split the
contact attempts into two stages: 1–2 calls (early contact) and 3+ calls (late contact). Among the 9709 households we analyze, 1992 responded in the first stage, 3287 responded later, and 4430 never responded. We fit the data with working models \( \pi_1 = \expit(\alpha_1 + \gamma_1 Y_1 + \gamma_2 Y_2) \), \( \pi_2 = \expit(\alpha_2 + \gamma_1 Y_1 + \gamma_2 Y_2) \) and \( f_2(Y) \sim N(\mu, \Sigma) \), and apply the proposed IPW, REG, and DR methods to estimate the outcomes mean. The common odds ratio parameters \((\gamma_1, \gamma_2)\) reveal that the resistance to respond caused by the outcomes remain the same in these two contact stages. We also compute the complete-case (CC) sample mean and apply standard inverse probability weighting, regression and doubly robust estimation methods that are based on MAR and include the number of contacts as a covariate, which are respectively denoted by IPW_{mar}, REG_{mar}, and DR_{mar}.

Table 4 presents the estimates. The DR point estimate of \( \mu_1 \) is 7.842 with 95% confidence interval (7.800, 7.884), and of \( \mu_2 \) is 6.345 with (6.296, 6.393). The CC estimate of \( \mu_1 \) is 7.756 (7.734, 7.778) and of \( \mu_2 \) is 6.285 (6.264, 6.306); the DR_{mar} estimate of \( \mu_1 \) is 7.767 (7.744, 7.790) and of \( \mu_2 \) is 6.287 (6.266, 6.309). The IPW and REG methods produce estimates close to DR; however, the CC and standard MAR estimates of the outcomes mean, in particular of \( \mu_1 \), are well below the DR estimate. As shown in Figure 3 (b), this can be partially explained by the fact that the outcomes mean in early respondents is lower than in the delayed, and therefore, the outcomes mean in the respondents is likely lower than in the nonrespondents. The estimation results of the odds ratio parameters reinforce this conjecture: the IPW, REG, and DR estimates of the odds ratio parameters are all negative, suggesting that high-spending people are more reluctant to respond or more difficult to contact. The odds ratio estimate for expenditure on housing is statistically significant at level 0.01, although it is not significant for expenditure on utilities, fuels and public services. This is evidence for missingness not at random. These results indicate that the expenditure on housing play a more important role in the response process; this may be because the survey takes personal home visit as one of the main modes of interview and people with high expenditure on housing are more difficult to reach. Increasing the variety of interview modes may potentially alleviate such nonignorable nonresponse.

| \( \mu_1 \) | IPW | REG | DR | IPW_{mar} | REG_{mar} | DR_{mar} | CC | IPW | REG | DR |
|---|---|---|---|---|---|---|---|---|---|---|
| Estimate | 7.859 | 7.861 | 7.842 | 7.767 | 7.769 | 7.767 | 7.756 | -0.269 | -0.258 | -0.238 |
| CI or p-value | (7.785, 7.932) | (7.810, 7.912) | (7.800, 7.884) | (7.744, 7.778) | (7.746, 7.791) | (7.744, 7.778) | 0.004 | 0.001 | 0 |

| \( \mu_2 \) | IPW | REG | DR | IPW_{mar} | REG_{mar} | DR_{mar} | CC | IPW | REG | DR |
|---|---|---|---|---|---|---|---|---|---|
| Estimate | 6.346 | 6.350 | 6.345 | 6.287 | 6.288 | 6.287 | 6.285 | -0.028 | -0.042 | -0.056 |
| CI or p-value | (6.269, 6.423) | (6.297, 6.403) | (6.296, 6.393) | (6.266, 6.309) | (6.267, 6.310) | (6.266, 6.309) | 0.812 | 0.658 | 0.521 |

We further conduct sensitivity analysis to assess robustness of the above results against violation of the stableness of resistance assumption. The sensitivity analysis shows that our results are not sensitive to mild violations of the stableness of resistance assumption. In particular, when the sensitivity parameter varies within a moderate range, the DR estimate of \( \mu_1 \) remains larger than the complete-case (CC) sample mean and the estimates based on MAR, and the estimate of \( \gamma_1 \) remains significantly negative. Such results reinforce our finding that high-spending people are more reluctant to respond or more difficult to contact. Details of the sensitivity analysis are relegated to Section S7 of the supplement.
9. DISCUSSION

We establish a novel framework for nonresponse adjustment with callback data, which further illustrates the usefulness and extends the application of callback data. Although not all surveys provide callback data, their availability is increasing in modern surveys. The stableness of resistance assumption is key to our framework. Under this assumption, we establish nonparametric identification and propose a suite of novel estimators including a doubly robust one, which extend previous parametric approaches and further elucidate the underlying source for nonresponse adjustment with callback data. We caution that the stableness of resistance assumption is untestable based on observed data. Therefore, analogous to various missing data problems (Molenberghs et al., 2008; Miao and Tchetgen Tchetgen, 2016; Sun et al., 2018), its validity should be justified based on domain-specific knowledge and needs to be investigated on a case-by-case basis. We have clarified the motivation, implication and limitation of the stableness of resistance assumption to facilitate the justification in practice. Even if the assumption does not hold, our approach constitutes a valid test of whether the missingness is entirely MAR—because the stableness of resistance assumption naturally holds under the null hypothesis of MAR. Besides, sensitivity analysis is warranted to assess robustness of inference against violation of the assumption.

We have employed an odds ratio parametrization and adopted practical parametric working models in the IPW, REG and DR estimators. There exist other parametrizations and we describe estimation under an alternative parametrization in Section S2 of the supplement. Kang and Schafer (2007) cautioned for potentially disastrous bias of certain DR estimators under MAR when all parametric working models are incorrect. However, previous authors have proposed alternative constructions of nuisance estimators and DR estimators to alleviate this problem, see e.g. Tan (2010); Vermeulen and Vansteelandt (2015); Tsiatis et al. (2011) and the discussions alongside Kang and Schafer (2007). Besides, multiply robust estimation in the sense of Vansteelandt et al. (2007) is also of interest. In addition, our nonparametric identification and estimation results open the way to more sophisticated estimation methods built on complex models, such as series or sieve estimation. In particular, our proposal in Theorem 4 allows for flexible estimation of the odds ratio function, without requiring it to be known or follow a parametric model, which extends the methods for nonignorable missing data analysis. For large to high-dimensional covariates, a heuristic approach for variable selection is to include penalties (e.g. LASSO) into the optimization of estimating equations for the nuisance parameters (e.g. Garcia et al., 2010; Fang and Shao, 2016). However, there remain challenges to the variable selection in the presence of nonignorable missing data and callbacks. It is of interest to incorporate these approaches to improve the proposed estimation methods.

We considered a single action response process—a call attempt either succeeds or fails; however, there may exist several dispositions, e.g., interview, refusal, other non-response or final non-contact (Biemer et al., 2013). Concurrent nonresponse is often the case when the missingness is due to failure of contact, but in practice different frame variables may be observed in different call attempts. In this case one may combine the callback design and the graphical model (e.g., Sadinle and Reiter, 2017; Malinsky et al., 2020; Mohan and Pearl, 2021) to account for complex patterns of missingness. In addition to nonresponse adjustment, callback data are also useful for the design and organization of surveys, e.g., allocation of time and staff resources. The integration of the callback design and other tools (e.g., instrumental variables) may be useful for handling simultaneous problems of nonresponse and confounding or other deficiencies in survey and observational studies. It is of interest to pursue these extensions.

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SUPPLEMENTARY MATERIAL
Supplementary material online includes further illustration for the stableness of resistance assumption, estimation under an alternative parametrization, the efficient influence function under the semiparametric model that \( A_1(X; \alpha_1), \Gamma(X, Y; \gamma) \) are correctly specified, estimation with multiple callbacks, proof of theorems, propositions, and important equations, additional simulations and real data analysis results, and codes and data for reproducing the simulations and application.

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