Production of photons in a bouncing universe

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Abstract

Using a new non-singular solution, it is shown that the production of photons in dilaton electrodynamics in a cosmological setting is greatly increased if the effect of matter creation on the geometry is taken into account. We show that this increment can be related to the problem of the origin of magnetic fields in the universe.

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1. Introduction

The origin, evolution and structure of the magnetic fields present in galaxies and clusters of galaxies are amongst the most important open issues in astrophysics and cosmology. Typically, the magnetic field present in galaxies is of the order of a few $\mu$G, and is coherent on the galactic scale. The observed field in the case of clusters is correlated over 10–100 kiloparsecs, and can be as high as tens of $\mu$G. The standard mechanism to account for these fields is the dynamo [1], which amplifies small seed fields to the above-mentioned values [2].

The seeds that fuel the dynamo may have an astrophysical origin or may be primordial. An astrophysical mechanism for the generation of these pre-galactic magnetic fields is the Biermann battery [3], which works for the generation of seeds to be amplified by the dynamo in galaxies, but can hardly account for the fields in clusters [4, 5]. Other astrophysical mechanisms (involving for instance starbursts, jet-lobe radio sources or accretion discs of black holes in AGNs [6]) seem to need pre-existing magnetic fields. The current prevalent view is that the magnetic fields observed in galaxies and clusters have a primordial origin (see [7] for a review).

The processes that may account for a primordial origin can be divided into two types: causal (those in which the seeds are produced at a given time inside the horizon, like QCD and EW phase transitions [7]) and inflationary (where correlations are produced outside the horizon [8]). In the latter, vacuum fluctuations of the electromagnetic field are ‘stretched’ by the evolution of the background geometry to super-horizon scales, and they could appear
today as large-scale magnetic fields. However, since Maxwell’s equations are conformally invariant in the FRW background, the amplification of the vacuum fluctuations (which amounts to particle production) via inflation can work only if conformal invariance is broken at some stage of the evolution of the universe.

There are several ways in which conformal invariance can be broken [9]: non-minimal coupling between gravitation and the electromagnetic field [8, 10, 11], quantum anomaly of the trace of the stress–energy tensor of electrodynamics [12], coupling of the EM field to a charged scalar field [8, 13], exponential coupling between a scalar (whose potential drives inflation) and the EM field [14] and a non-zero mass for the photon [15]. Yet another possibility is dilaton electrodynamics [16], in which there is a scalar field (the dilaton) which couples exponentially to an Abelian gauge field. In this model, the inflationary expansion is driven and not by the potential but by the kinetic term of the scalar field. The exponential coupling is naturally implemented in the low-energy limit in string theory [17], and in Weyl integrable spacetime (WIST) [18], and can also be viewed as a time dependence of the coupling constant, an idea considered first by Dirac [19]. This avenue has been pursued by Giovannini in a series of articles [20]. More recently, a model with both a dilaton and an inflaton was analysed in [21], and later generalized to a noncommutative spacetime [22].

In all the above-mentioned articles, different aspects of photon production have been analysed. We will focus in this contribution on two definite items. First, we shall address the creation of photons in a non-singular universe described by an exact solution of the field equations. More specifically, in this solution the passage through the bounce is described by the equations of the model without resorting to unknown (Planck scale) physics. The results of this calculation are to be compared with those of the second item, coming from a more complete model. We shall take into account in this second part the influence of the creation of matter on the squeezing of the vacuum state, a problem that seems to have received little attention in the literature. It will be shown that as a result of matter creation, there is an increment in the number of photons originating in the stretching of the vacuum fluctuations. Thus, this effect may be of importance in the generation of primordial magnetic fields.

The analysis will be done in the framework of a phenomenological model, in which the creation of ultrarelativistic matter is restricted to an interval centred in the bounce. Two interesting features of the solution representing this model are that it goes automatically into a radiation regime after a short time, and it displays a constant value for the dilaton after entering the radiation era.

We shall start in section 2 by deducing a Hamiltonian for the EM field in a curved background coupled to the scalar field. The quantization of this Hamiltonian system will be presented in section 3, following the method of the squeezed states. In section 4, we shall study the production of photons in the absence of matter (i.e., with only the scalar field present). Variation of the photon number in a model that takes into account matter creation will be discussed in section 5. We conclude with a discussion in section 6.

2. Field equations

A time-dependent gauge coupling is a generic feature of four-dimensional theories obtained by compactification of some theory formulated in a spacetime with more than four dimensions, such as Kaluza–Klein theory [23] and string theory [24]. Time-dependent gauge couplings are also present in WIST [18]. In all these cases, the action can be conveniently written in the form

$$S = \frac{1}{2} \int d^4x \sqrt{-g} f(\omega) F_{\mu \nu} F^{\mu \nu},$$

(1)
where \( \omega \) is the dilaton (in the case of string theory) or the scalar field associated with Weyl geometry (in the case of WIST) and \( F_{\mu\nu} \) is an Abelian field. The function \( f(\omega) \) will be left unspecified for the time being, but later we will set \( f(\omega) = e^{-2\omega} \), which corresponds to the case of string theory and WIST. The equations of motion (EOM) that follow from this action are

\[
(f(\omega)F^{\mu\nu})_{\parallel\nu} = 0, \tag{2}
\]

\[
F_{\parallel\beta} = 0, \tag{3}
\]

where the derivative w.r.t. the background geometry is denoted by ‘\( \parallel \)’, and the dual of the electromagnetic tensor \( \ast F_{\mu\nu} \) is defined as

\[
\ast F_{\alpha\beta} \equiv \frac{1}{2} \eta_{\alpha\beta}^{\mu\nu} F_{\mu\nu}.
\]

Advantage will be taken in the following of the formal equivalence of equations (2) and (3) and the equations for the electromagnetic field in a material medium, an equivalence that is fulfilled if we define

\[
P^{\mu\nu} \equiv f(\omega)F^{\mu\nu}. \tag{4}
\]

A reference frame must be chosen in order to define the electric and magnetic fields from \( P_{\mu\nu} \) and \( F_{\mu\nu} \). Since the cosmological model to be used in this paper is described by the Friedmann–Robertson–Walker (FRW) metric, we can use the 4-velocity vector field \( V^\mu = \delta^\mu_0 \), which is orthogonal to the three-dimensional surfaces of homogeneity of the FRW geometry. The electric and magnetic parts of the electromagnetic tensor are defined as

\[
D_\alpha \equiv P^{\alpha\beta} V_\beta, \quad H_\alpha \equiv \ast P^{\alpha\beta} V_\beta, \tag{5}
\]

\[
E_\alpha \equiv F^{\alpha\beta} V_\beta, \quad B_\alpha \equiv \ast F^{\alpha\beta} V_\beta. \tag{6}
\]

In the comoving frame, the EOM along with equation (4) become

\[
D_\alpha |_{\parallel\alpha} = 0, \tag{7}
\]

\[
B_\alpha |_{\parallel\alpha} = 0, \tag{8}
\]

\[
(h_\alpha^\beta D_\alpha) + \frac{2}{3} \theta D_\alpha + \eta_{\lambda}^{\beta\rho\sigma} V_\rho H_\sigma |_{\parallel\beta} = 0, \tag{9}
\]

\[
h_\alpha^\beta B_\alpha + \frac{2}{3} \theta B_\alpha + \eta_{\lambda}^{\beta\rho\sigma} V_\rho E_\sigma |_{\parallel\beta} = 0, \tag{10}
\]

with \( \theta = 3\dot{a}/a \), \( a \) the scale factor of the FRW metric, a dot represents derivative w.r.t. cosmological time and \( h_\alpha^\beta \) is the metric of the 3-space orthogonal to \( v_\mu \). As shown in the pioneering work by Lifshitz [25], it is useful to expand the solutions of linear differential equations defined on a curved manifold in the corresponding spherical harmonics basis. In order to do so, we will first split the equations into space and time parts. The vectors describing the electromagnetic field are already space vectors, since \( E_\alpha V^\alpha = D_\alpha V^\alpha = B_\alpha V^\alpha = H_\alpha V^\alpha = 0 \). From now on we shall work in conformal time, with the FRW metric given by

\[
dx^2 = a(\eta)^2[\dy^2 - \eta_{ij}(\tilde{x}) \dx^i \dx^j],
\]

where \( \omega \) is the dilaton (in the case of string theory) or the scalar field associated with Weyl geometry (in the case of WIST) and \( F_{\mu\nu} \) is an Abelian field. The function \( f(\omega) \) will be left unspecified for the time being, but later we will set \( f(\omega) = e^{-2\omega} \), which corresponds to the case of string theory and WIST. The equations of motion (EOM) that follow from this action are

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\[
dx^2 = a(\eta)^2[\dy^2 - \eta_{ij}(\tilde{x}) \dx^i \dx^j],
\]
where \( \eta \) is the metric of the hypersurface \( \eta = \text{constant} \), and \( i = 1, 2, 3 \). In terms of 3-vectors, equations (7)–(10) become

\[
\nabla_i D^i = 0, \\
\nabla_i B^i = 0, \\
(D^i)' = \frac{a'}{3a} D^i + \eta^{ij} \nabla_j H_n = 0, \\
(B^i)' = \frac{a'}{3a} B^i - \eta^{ij} \nabla_j E_n = 0,
\]

where the prime denotes derivative w.r.t. conformal time.

Since we are dealing with vector quantities we will use a vector basis \( P^i \) defined by the following relations [26]:

\[
P^i = P^i (x'), \\
\gamma^{ij} \nabla_i \nabla_j P^l = -m^2 P^l, \\
\gamma^{ij} \nabla_i P_j = 0.
\]

The eigenvalue \( m \) denotes the wave number of the corresponding vector eigenfunction of the Laplacian operator on the spatially homogeneous hypersurfaces. The spectrum of eigenvalues depends on the 3-curvature \( \epsilon \), and is given by

\[
m^2 = s^2 + 2, \quad 0 < s < \infty, \quad \epsilon = -1, \\
m = s, \quad 0 < s < \infty, \quad \epsilon = 0, \\
m^2 = s^2 - 2, \quad s = 2, 3, \ldots, \quad \epsilon = 1.
\]

The pseudovector basis \( \tilde{P}^i \), convenient to work with the magnetic field, is defined by

\[
\tilde{P}^i = \eta^{ij} \nabla_j P^j.
\]

In terms of the basis \( P^i \) and of the associated basis \( \tilde{P}^i \), the electric and magnetic fields can be expanded as follows:

\[
E^i (\eta, \vec{x}) = \sum_{l,m,\sigma} E^{(\sigma)}_{ml} (\eta) P^{(\sigma) l i} (\vec{x}), \\
B^i (\eta, \vec{x}) = \sum_{l,m,\sigma} B^{(\sigma)}_{ml} (\eta) \tilde{P}^{(\sigma) l i} (\vec{x}).
\]

These expressions are valid in the case \( \epsilon = 1 \). For \( \epsilon = 0 \) and \( -1 \), the sum must be replaced by an integration \((2\pi)^{-3} \int d^3 x\). The index \( l = 1, 2 \) describes the two transverse degrees of freedom of the electric and magnetic fields. Since we are using an expansion in standing waves\(^3\), the index \( \sigma \) takes the values ‘+’ or ‘−’. Consequently, the fields in the case \( \epsilon = 0 \) can be written as

\[
E^i (\eta, \vec{x}) = \sum_{l,m} \left( E^{(+)}_{ml} (\eta) \cos (\vec{m} \cdot \vec{x}) + E^{(-)}_{ml} (\eta) \sin (\vec{m} \cdot \vec{x}) \right) e_j^i,
\]

(where the \( e_j^i \) are modulus-one polarization vectors), and an analogous expression for \( B^i \). In the more general case of \( \epsilon = 1, -1 \), the (considerably more involved) analytic expression for

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\(^3\) Note that the expansion in standing waves is completely equivalent to that of travelling waves (see for instance [30]).
the vector base can be found in [27]. The result of substituting into equations (11)–(14) the expansions given in equations (16) and (17) is

\[
B_{ml}^{(m)} + \frac{\alpha}{\alpha} B_{ml}^{(m)} - f^{-1}(\omega) D_{ml}^{(m)} = 0, \tag{19}
\]

\[
D_{ml}^{(m)} + \frac{\alpha}{\alpha} D_{ml}^{(m)} + (m^2 + 2 \epsilon) f(\omega) B_{ml}^{(m)} = 0. \tag{20}
\]

The system described by equations (19) and (20) is not Hamiltonian when written in the variables \( D \) and \( B \), but a Hamiltonian can be introduced in terms of a new set of variables defined by

\[
p_{ml}(\eta) \equiv a(\eta) D_{ml}(\eta), \tag{21}
\]

\[
q_{ml}(\eta) \equiv a(\eta) B_{ml}(\eta). \tag{22}
\]

In terms of these, equations (19) and (20) can be written as

\[
p_{ml}^{(m)} + (m^2 + 2 \epsilon) f(\omega) q_{ml}^{(m)} = 0, \tag{23}
\]

\[
q_{ml}^{(m)} - f(\omega)^{-1} p_{ml}^{(m)} = 0. \tag{24}
\]

Since the new variables constitute a pair of canonical variables, the dynamical system given by equations (19) and (20) can now be described by the Hamiltonian

\[
\mathcal{H}_{ml}^{(m)}(p, q) = \frac{1}{2} b_2 (2 N_{ml}^{(m)} + 1) + \frac{1}{4} b_1 q_{ml}^{(m)}^2, \tag{25}
\]

where

\[
b_1(\eta) = f(\omega)^{-1}, \quad b_2(\eta) = (m^2 + 2 \epsilon) f(\omega). \tag{26}
\]

The Hamiltonian for the mode characterized by \((\sigma, m, l)\) given in equation (25) is identical to that of a single harmonic oscillator problem with time-dependent coefficients. We shall review in the following section how to quantize this system.

3. Quantization

The expression for \( \mathcal{H} \) obtained in the previous section appears in many different branches of physics, ranging from quantum optics to gravitational waves [28]. In order to quantize it, we shall follow a standard procedure of quantum optics. First, we define creation and annihilation operators by the expressions

\[
\hat{p}_{ml}^{(m)} = -i \sqrt{\frac{\gamma}{2}} (\hat{a}_{ml}^{(m)} - \hat{a}_{ml}^{(m)}\dagger), \tag{27}
\]

\[
\hat{q}_{ml}^{(m)} = \frac{\hat{a}_{ml}^{(m)} + \hat{a}_{ml}^{(m)}\dagger}{\sqrt{2\gamma}}, \tag{28}
\]

where \( \gamma \) is a constant to be determined later. In terms of \( \hat{a} \) and \( \hat{a}^\dagger \), the Hamiltonian operator can be written as

\[
\hat{\mathcal{H}}_{ml}^{(m)} = \frac{1}{4} k(\eta) (2 \hat{N}_{ml}^{(m)} + 1) + \frac{1}{4} h(\eta) \hat{a}_{ml}^{(m)} + \frac{1}{4} h(\eta) \hat{a}_{ml}^{(m)}\dagger, \tag{29}
\]

where

\[
k(\eta) = \gamma b_1(\eta) + \frac{b_2(\eta)}{\gamma}, \quad h(\eta) = \frac{b_2(\eta)}{\gamma} - \gamma b_1(\eta). \]
and \( N^{(\sigma)}_{ml} = \hat{a}^{(\sigma)\dagger}_{ml} \hat{a}^{(\sigma)}_{ml} \) is the number operator. The first term in equation (29) looks like the free Hamiltonian for the mode \((\sigma, m, l)\) with a time-dependent function instead of the frequency. This part conserves the number of photons in each mode, hence the total number of photons and the total energy. The second and third terms represent instead a time-dependent interaction which does not conserve the photon number.

The operators \( \hat{a}^{(\sigma)}_{ml}\) and \( \hat{a}^{(\sigma)\dagger}_{ml}\) obey the commutation relations

\[
\left[ \hat{a}^{(\sigma)}_{ml}, \hat{a}^{(\sigma)\dagger}_{m'l'} \right] = \delta_{ll'} \delta_{mm'} \delta_{\sigma\sigma'}, \quad \text{for } \epsilon = 1,
\]

\[
\left[ \hat{a}^{(\sigma)}_{ml}, \hat{a}^{(\sigma)\dagger}_{m'l'} \right] = \delta_{ll'} \delta_{\sigma\sigma'} \delta(m - m'), \quad \text{for } \epsilon = 0, -1,
\]

where we have settled \( \gamma = m \) in order to have vacuum as the initial state. Since we shall work with a single mode (and in order to avoid clumsy expressions) the indices \( m, l \) and \( \sigma \) will be omitted from now onwards.

We will proceed to solve Schrödinger’s equation for the Hamiltonian given in equation (29) by writing the time evolution operator as a product of the rotation and the single-mode squeezed operators, along with a phase factor:

\[
U(\eta, \eta_0) = e^{i\phi S(\eta)R(\eta)},
\]

where \( S(\eta) \) and \( R(\eta) \) are defined as

\[
S(\eta) = \exp(A(\eta)\hat{a}^2 - A^*(\eta)\hat{a}^\dagger^2),
\]

\[
R(\eta) = \exp(i\Gamma(\eta)\hat{a}^\dagger\hat{a}).
\]

The function \( \Gamma(\eta) \) is called the rotation angle, and \( A(\eta) \) is usually defined as

\[
A(\eta) = \frac{1}{2} r(\eta) \exp(-2i\varphi(\eta)),
\]

where \( r \) is the squeeze factor (defined in the range \( 0 \leq r < \infty \)) and \( \varphi \) the squeeze angle (defined in the range \(-\pi/2 \leq \varphi < \pi/2\)). The parameter \( r \) determines the strength of the squeezing while \( \varphi \) gives the distribution of the squeezing between conjugate variables. Schrödinger’s equation with the Hamiltonian given in equation (29) reduces, after a direct but somewhat long calculation, to the following system of first-order coupled differential equations valid for each mode of the field:

\[
ir' + (\varphi' + \Gamma') \sinh 2r = e^{2\varphi} \frac{h}{2},
\]

\[
-ir' + (\varphi' + \Gamma') \sinh 2r = e^{-2\varphi} \frac{h}{2},
\]

\[
(\varphi' + \Gamma') \cosh 2r = \varphi' + k,
\]

\[
\varphi' = -\frac{\Gamma'}{2}.
\]

Similar equations were obtained by Albrecht et al [29] in the case of the scalar degrees of freedom of metric perturbations in cosmology. Grishchuk [31] derived analogous equations for the case of gravitational waves, and Matacz [32], for a scalar field. To solve the system, we shall follow the method presented in [32]. With the change of variables

\[
a(\eta) = e^{-i\Gamma(\eta)} \cosh r(\eta),
\]

\[
b(\eta) = -\exp(-2i(\varphi(\eta) + \Gamma/2) \sinh r(\eta)),
\]

\[
C(\eta) = \sinh r(\eta).
\]
equations (36)–(39) can be written as
\[
2\alpha' = -ih\beta - i\alpha, \quad (42)
\]
\[
2\beta' = ik\beta + i\alpha. \quad (43)
\]
Introducing the function \(\mu(\eta)\) defined by
\[
\mu = \frac{\beta^* - \alpha^*}{\beta^* + \alpha^*}, \quad (44)
\]
equations (42) and (43) are equivalent to
\[
2\mu' - i(k - h)\mu^2 + i(k + h) = 0. \quad (45)
\]
Finally, setting
\[
\mu = \frac{i}{y} b_1 \frac{g'}{g}, \quad (46)
\]
we obtain the equation
\[
g'' - b_1' g' + b_1 b_2 g = 0.
\]
The problem has been reduced then to solving a single second-order differential equation for the function \(g(\eta)\), which in terms of \(f(\omega)\) reads (see equation (26))
\[
g'' + \frac{1}{f} \frac{df}{d\omega} \omega' g' + (m^2 + 2\epsilon)g = 0. \quad (47)
\]
We can see from this equation that there is no photon creation if the scalar field is constant, a fact that justifies the identification of \(\omega\) with the laser ‘pump’ used in experiments devised to observe squeezed states [28]. With the function \(g\) we can get the squeeze parameter \(r\) through equation (46) as follows. From equations (40), (41) and (44) we get
\[
\mu(\eta) = \frac{1 + e^{2\nu(\eta)} \tanh r(\eta)}{-1 + e^{2\nu(\eta)} \tanh r(\eta)}, \quad (48)
\]
which can be inverted to get \(r\), given by
\[
\tanh^2 r(\eta) = \frac{1 + \mu(\eta) + \mu^*(\eta) + |\mu(\eta)|^2}{1 - \mu(\eta) - \mu^*(\eta) + |\mu(\eta)|^2}. \quad (49)
\]
All the functions appearing in the evolution operator can be written in terms of \(\mu\) (we refer the reader to [32] for details). A quantity of interest in the following is the mean number of photons per mode as a function of the conformal time, given by [35]
\[
\langle N(\eta) \rangle = \sinh^2 r(\eta). \quad (50)
\]
In the following sections, we shall calculate \(\langle N \rangle\) for two different models.

4. A simplified model

Before going into the details of the model which takes into account the effect of matter creation on the metric, we shall study the production of photons in a simpler case, namely that in which the sole matter content is given by the scalar field \(\omega\). As discussed in [33], there are non-singular solutions in the theory of WIST that describe a FRW geometry plus a Weyl scalar field. Non-singular solutions are also present in string theory [34]. Let us briefly review these
solutions. The EOM for gravitation plus scalar field written in conformal time are [33]

\[ a'^2 + \epsilon a^2 + \frac{\lambda^2}{6} (\omega' a)^2 = 0, \quad \omega' = \sigma a^{-2}, \tag{51} \]

where \( \sigma = \text{constant} \) and \( \lambda^2 \) is the coupling constant of the scalar field to gravity. Note that the equation of state of the scalar field is given by \( \rho_\omega = p_\omega \), where

\[ \rho_\omega = -\frac{\lambda^2}{2} \left( \frac{\omega'}{a} \right)^2. \]

From these equations, we get

\[ a'^2 = -\epsilon a^2 - \frac{b^2}{a^2}, \tag{52} \]

where we have defined \( b^2 = \frac{\lambda^2 \sigma^2}{6} \). Equation (52) shows that only solutions with \( \epsilon = -1 \) are possible. Hence, from now on we shall restrict to the negative curvature case, for which equation (52) can easily be integrated. Although this choice is at odds with observation, we emphasize that we are using this exact solution only as a toy model to study the effect of matter creation on the production of photons. The result of the integration for the scale factor is

\[ a(\eta) = \sqrt{|b|} \sqrt{\cosh(2\eta + \delta)}. \tag{53} \]

From equation (51),

\[ \omega(\eta) = \pm \frac{\sqrt{6}}{2\lambda} \arctan(e^{2\eta+\delta}) + \frac{\pi}{4} \sqrt{6} \frac{s}{\lambda}, \tag{54} \]

where \( \delta \) is an integration constant. The plots for these functions are given in figure 1. The scale factor displays a bounce, produced by the violation of the strong energy condition by the scalar field [37]. The number of photons created by the expansion in this model must be calculated numerically, since equation (47) has no analytical solution for the scalar field given in equation (54) with the coupling \( f(\omega) = e^{-2\omega} \). The results are given by the dashed curve in figure 2, for a wave number typical of the size of the intergalactic scale (1 Mpc).

\[ a(\eta) = \sqrt{|b|} \sqrt{\cosh(2\eta + \delta)}. \tag{53} \]
5. A model with matter creation

We shall consider in this section the production of photons in a background of the system composed of the scalar field plus matter and geometry, using a non-singular new solution that incorporates the effect of the creation of matter on the geometry. Friedmann’s equation in conformal time for this case is given by

\[ a'^2 - a^2 = -\frac{\lambda^2}{6} (\omega' a)^2 + \frac{a^4}{3} \rho_m, \]  

(55)

while the second Einstein equation is

\[-3 \left( \frac{2 a''}{a^3} - \frac{a'^2}{a^2} - \frac{1}{a^2} \right) = \rho_m + 3 \rho_\omega. \]

(56)

The conservation of the stress–energy tensor gives

\[ \frac{d}{d\eta} (a^3 (\rho_m + \rho_\omega)) + (p_m + p_\omega) \frac{da^3}{d\eta} = 0. \]

(57)

In the case of ultra-relativistic matter, this equation takes the form

\[ (a^2 \rho_m)' + \frac{1}{a^2} (a^2 \rho_\omega)' = 0. \]

(58)

We would like to have a solution that describes creation of relativistic matter only around the bounce, and enters a radiation phase with a constant scalar field in a short time. Clearly, an asymmetry is to be expected both in the scale factor and in \( \omega \), since the evolution of this universe starts from the vacuum and enters a radiation-dominated epoch. An expression for \( a \) that fulfills these requirements is

\[ a(\eta) = \beta \sqrt{\cosh(2\eta) + k_0 \sinh(2\eta) - 2k_0 (\tanh(\eta) + 1)}, \]

(59)

with \( \beta = a_0 \sqrt{\frac{1}{1 - 2k_0}} \) and \( 0 < k_0 \leq 1/7 \). This \( a(\eta) \) is a solution of Einstein’s equations (55) and (56) in the case of ultra-relativistic matter as a source (see equation (58)). The expression
Figure 3. Plot of $a$ and $\omega$ for $k_0 = 1/7$ and $a_0 = 0.93$, values chosen by imposing that the solution in equation (59) enters the radiation era for $t \approx 10^{-8}$ s.

Figure 4. Plot of $H$ for the vacuum case (full line) and for the solution given by equation (59) as a function of the conformal time for $\lambda = 1$ in $c = 1$ units.

for $\omega$ can be obtained using this scale factor in equations (55) and (56). We shall not give the explicit expression, but the plots for $a(\eta)$ and $\omega(\eta)$ (see figure 3). Note that the plot shows the announced asymmetry. The evolution of the Hubble parameter for the vacuum case (studied in the previous section) and for the case with matter creation is shown in figure 4. Let us emphasize that since the scalar field tends rapidly to a constant value, the production of matter (controlled by $\omega'$, see equation (58)) soon stops, and the model enters a radiation phase without the need of a potential. In this sense, this solution describes a hot bounce, as opposed to cold bouncing solutions, which do not enter the radiation era unless they are heated up [36]. Another nice feature of this solution is that the dilaton goes automatically to a constant
value for $\eta \to \infty$, in such a way that the solution could be taken as the leading order of a perturbative development (as is the case in string theory). Again, no potential was needed in order to display this feature. The mean number of photons per mode can be calculated as in the previous cases, after numerical integration of equation (47). The full line plot in figure 2 shows that the production of photons is increased in the case of the model with matter creation. The number of photons is directly related to the fraction of electromagnetic energy stored per mode, relative to the background radiation energy $\rho_{\gamma}$ through the expression

$$r(\nu) \approx \frac{\nu^4}{\rho_{\gamma}} \langle N \rangle_\nu,$$

where $\nu = \sqrt{m^2 - 2j}/a$. Consequently, the increment in the number of photons may be of importance in the problem of the generation of magnetic field seeds.

6. Discussion

Using a covariant description for the electromagnetic field coupled to a scalar in a curved background, we obtained the Hamiltonian for each mode of the field. With the aid of the formalism of the squeezed states, we have calculated the number of photons in a non-singular universe using first an already known solution [33] with no matter present and afterwards a new solution which takes into account the effect of the creation of ultrarelativistic matter on the evolution of the scale factor and of the scalar field. This new solution presents several interesting features, namely the transition of an expanding phase to a radiation era and the constancy of the scalar field a short time after the bounce. The results for the mean photon number per mode show that the production is increased in the case of matter production. This increment may be relevant for the creation of seeds of the magnetic field. We hope to discuss this issue in detail in a forthcoming publication.

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