Dynamical models of NGC 3115

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ABSTRACT
We present new dynamical models of the S0 galaxy N3115, making use of the available published photometry and kinematics as well as two-dimensional TIGER spectrography. The models are based on a detailed model of the luminosity distribution built using an MGE fit on HST/WFPC2 and ground-based photometric data. We first examined the kinematics in the central 40 arcsec in the light of two-integral \( f(\mathcal{E}, \mathcal{J}) \) models. Jeans equations were used to constrain the mass-to-light ratio, and the central dark mass, the existence of which was suggested by previous studies. The even part of the distribution function was then retrieved via the Hunter & Qian formalism. We thus confirmed that the velocity and dispersion profiles in the central region could be well-fitted with a two-integral model, given the presence of a central dark mass of \( \sim 10^8 \, M_\odot \). However, no two-integral model could fit the \( h_3 \) profile around a radius of about 25 arcsec where the outer disc dominates the surface brightness distribution.

Three-integral analytical models were therefore built using a quadratic programming technique. These models showed that three-integral components do indeed provide a reasonable fit to the kinematics, including the higher Gauss-Hermite moments. Again, models without a central dark mass failed to reproduce the observed kinematics in the central arcsec. This clearly supports the presence of a nuclear black hole of at least \( 6.5 \times 10^8 \, M_\odot \) in the centre of NGC 3115. These models were finally used to estimate the importance of the dark matter in the outer part of NGC 3115, suggested by the flat stellar rotation curve observed by Capaccioli et al.

This study finally points out the difficulty of integrating independently published data in a coherent and consistent way, thus demonstrating the importance of taking into account the details of the instrumental setup and the reduction processes.

Key words: galaxies: individual: NGC 3115 – galaxies: kinematics and dynamics – galaxies: nuclei.

1 INTRODUCTION

Early-type disc galaxies pose a specific problem for dynamical modelling as neither the bulge nor the disc can be neglected in the computation of the gravitational potential. First, it is generally difficult to disentangle the photometric components according to a priori fixed criteria such as the ellipticity or surface brightness profiles. Although there exist different methods to build corresponding realistic photometric models, they are usually limited to two-component systems (e.g. disc plus spheroid, Byun & Freeman 1995) which often do not reflect the true morphology of the galaxy. Then the different internal dynamics of each component has to be taken into account. Asymmetrical line-of-sight velocity distributions (LOSVDs) observed in disc-like ellipticals (e.g. Scorza & Bender 1995) confirmed the fact that the classically measured velocity \( V \) and dispersion \( \sigma \) do not sufficiently constrain their dynamical structure. Because of these difficulties, the analysis is usually restricted to the central region, or to regions where one component dominates the light (e.g. Jarvis & Freeman 1985).

A number of techniques to build realistic distribution functions of such complex objects have recently been developed. Hunter & Qian (1993) proposed a viable scheme to retrieve the even part of the axisymmetric two-integral distribution function corresponding to a given mass model. It was used by van den Bosch & de Zeeuw (1996) for a set of models including a spheroid and a disc. In principle, this method can be applied to more complex potentials: this will be demonstrated in the present paper. As a further step, three-integral axisymmetric models can be built, using for example the Schwarzschild method in which one derives the best combination of orbits constrained to fit a set of observational data (e.g. van der Marel et al. 1998; Cretton & van den Bosch 1998). In this paper, we wish to present the application of another technique, namely quadratic programming (Dejonghe 1989).

NGC 3115 is interesting in many different aspects. First, it has...
long been assumed to be the prototype of the S0 galaxy type: a bulge-dominated galaxy with an embedded disc and very little gas and dust. With a distance of about 10 Mpc and a nearly edge-on inclination, NGC 3115 is also a perfect target for optical observations. Numerous studies were focused on the understanding of its morphology and kinematics. NGC 3115 contains a double disc structure with an outer Freeman type II disc extending up to ~140 arcsec and a nuclear disc (inside 3 arcsec) clearly revealed by the HST/WFPC2 pictures (Kormendy et al. 1996). Its outer disc was found to exhibit a weak spiral arm structure (Capaccioli, Held & Nieto 1987; Silva et al. 1989) and to thicken outwards (Capaccioli, Vietri & Held 1988). The spheroidal component does not seem to follow the classical r^{1/4} law, and the flattened halo of NGC 3115 extends up to 20 arcmin (Capaccioli et al. 1987).

The kinematics of NGC 3115 also deserved a complete survey: Illingworth & Schechter (1982) presented an extensive series of spectroscopic observations demonstrating that the bulge of this galaxy was nearly consistent with its being an isorotator. NGC 3115 has also been one of the best candidates for the presence of a central dark mass since the spectroscopic observations of Kormendy & Richstone (1992): spherical models suggested an increase of M/L by more than a factor of 10 inside 2 arcsec, which is difficult to account for with normal stellar populations. This conclusion was based on the observed velocity gradient and central Gaussian velocity dispersion σ0, the latter nearly reaching 300 km s^{-1} at a resolution of ~1 arcsec FWHM. This trend has been very recently confirmed with FOS (faint object spectrograph) spectroscopy (Kormendy et al. 1996), yielding σ0 ~ 445 km s^{-1} at HST resolution. Other studies mainly focused on the major-axis kinematics (Carter & Jenkins 1993; van der Marel et al. 1994; Bender, Saglia & Gerhard 1994; Fisher 1997). Finally, Capaccioli et al. (1993) have shown that the stellar rotation and dispersion profiles were flat at large radii (R > 100 arcsec), thus indicating the presence of a massive dark halo.

In this paper, our aim is to make detailed models of this early-type disc galaxy, e.g. constrain its distribution function, fully taking into account the available published data. This includes, for the first time, two-dimensional spectroscopic observations. In Section 2 we describe the photometry and spectroscopic observations used for the modelling, including new HRCAM (high-resolution camera) images and the WFPC2 V-band image as well as original TIGER (Traitement Intégral des Galaxies par l’Etude de leurs Raies) spectrography of the central region. The photometric models based on the MGE (multi-Gaussian expansion) formalism are presented in Section 3 and their corresponding dynamical Jeans models in Section 4. Numerical two-integral distribution functions are derived using the Hunter & Qian method and analysed in Section 5. Two- and three-integral models are computed in Section 6 with the help of the quadratic programming method. Conclusions are drawn in Section 7.

## 2 OBSERVATIONAL DATA

### 2.1 Photometry

Dynamical modelling first implies the availability of a luminosity model. This requires images with high resolution to obtain details of the central region, but also with a large field of view to sample the line of sight correctly at large radii. A wide-field V-band image, obtained with a focal reducer on the 1.23-m telescope at Calar Alto, was kindly put at our disposal by Cecilia Scorza. We then retrieved the V-band WFPC2 image (F555W) of NGC 3115 from the HST archives at ECF (European Coordinating Facility) (P. I. Faber). We finally used a V-band image obtained by Jean-Luc Nieto in April 1992 with HRCAM at the Canada–France–Hawaii Telescope (CFHT). All these images were reduced and normalized using aperture photometry available in the literature (Poulain 1986, 1988; de Vaucouleurs & Longo 1988). These calibrations led to individual errors smaller than 0.05 mag arcsec^{-2} with an excellent overall agreement. The characteristics of the different images are given in Table 1.

### 2.2 Spectroscopy and kinematics

#### 2.2.1 TIGER data

We observed NGC 3115 with the TIGER integral field spectrograph (Bacon et al. 1995) at the CFHT in 1992 April and subsequently in 1996 April. The lens diameter was set to 0.39 arcsec and the spectral domain to 5130–5520 Å for both runs. The spectra were extracted and calibrated in the standard way using the tiger software written at the Lyon Observatory. This includes bias subtraction, flat-fielding, spectrum extraction, wavelength calibration, cosmic ray removal and differential refraction correction (see Emsellem et al. 1996 for more details). The individual exposures were then accurately centred and merged. Variations in the overall atmospheric transparency were smaller than 1 per cent and were corrected. We thus obtained one set of spectra for each run, consisting of 624 and 391 spectra for 'run92' and 'run96' respectively. The final seeing of each data cube was estimated a posteriori by comparing images reconstructed from the spectra with the V-band WFPC2 image. Details of the characteristics of both sets of data are given in Table 2. Note that although the set of data obtained in 1996 April has a significantly better spatial and spectral resolution, we will mainly present the data set obtained in 1992 April as the signal-to-noise ratio is higher and the field of view larger.

### Table 1. Characteristics of the V-band images of NGC 3115.

|            | Calar Alto | HRCAM | WFPC2 |
|------------|------------|-------|-------|
| pixel size | 1''55      | 0''11 | 0''0455 |
| seeing (σν) | ~3''       | 0''7  | –     |
| field of view | 10'’×15'’ | 70'’×110'’ | 34'’×33'’ |
| exp. time   | 240 s      | 120 s | 1030 s |
| airmass     | 1.65       | 1.13  | –     |

### Table 2. Observational characteristics of the TIGER spectrographic exposures. The fields of view are only indicative, as the coverage of the merged TIGER spectra is not rectangular. All individual frames had an exposure time of 30 min.

|            | 1992 Run | 1996 Run |
|------------|----------|----------|
| Lens diam. | 0''39    | 0''39    |
| Field of view | 14'’5 × 7'’9 | 7'’9 × 7'’9 |
| no. of exposures | 4        | 2        |
| no. of spectra  | 624      | 391      |
| Spect. sampling | 1.8 Å pixel⁻¹ | 1.5 Å pixel⁻¹ |
| Spect. resolution | 3.7 Å (FWHM) | 3.2 Å (FWHM) |
| Inst. broadening (σ) | 94 km s⁻¹ | 78 km s⁻¹ |
| Spect. domain   | 5130–5520 Å | 5130–5520 Å |
| Seeing (FWHM)   | 1''24    | 0''8     |
| Stellar templates | η Cygni | HR 127065 |
|                 | K0III    | K0III    |

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In order to determine the stellar kinematics we also obtained exposures of a few stellar templates: mainly K giants for run92 and G, K and M giants for run96. All spectra were logarithmically rebinned; the stellar continuum was approximated using a fifth degree polynomial and subtracted. We then derived the two-dimensional kinematics of the central region of NGC 3115 using the Fourier Correlation Quotient (FCQ) algorithm (Bender 1990) and an optimal template built with the available stellar spectra. This provides the full LOSVD for each spatial element. The LOSVDs were then parameterized with the Gauss–Hermite functions, as well as the estimations of the four first true moments of the LOSVD $V$, $\sigma$, $h_3$ and $h_4$, which correspond to the Gauss–Hermite parametrization, as well as the estimations of the four first true moments of the LOSVD $\tilde{V}$, $\tilde{\sigma}$, the skewness and the kurtosis.

The mean velocity, velocity dispersion (Gaussian fit), $h_3$ and $h_4$ two-dimensional maps are presented in Fig. 1 (run92). We detect a slight tilt of $10^\circ$ of the velocity minor axis in the 1992 data. The low signal-to-noise ratio of the new TIGER data set prevented us from confirming this; it would therefore be useful to obtain new high spatial resolution kinematical data along the minor axis. Note that the $h_3$ and $h_4$ maps only include spectra with a signal-to-noise ratio higher than 20.

### 2.2.2 Comparison with published kinematics

All kinematical data available were compiled, and the FOS/HST (0.21 × 0.21 arcsec$^2$ aperture) and SIS/CFHT data obtained by Kormendy et al. (1996) scanned. As emphasized by different authors (e.g. van der Marel et al. 1994), the Gaussian velocity $V$ and dispersion $\sigma$ are not always good estimations of the true $\tilde{V}$ and $\tilde{\sigma}$ moments. This is particularly important in the case of NGC 3115 where high values of $h_3$ have been measured (e.g. van der Marel et al. 1994). Therefore, we mainly focused on the data sets that include higher order moments of the LOSVDs: van der Marel et al. (1994, hereafter vdm+94), Bender et al. (1994, hereafter B+94), Fisher (1997, hereafter F97) and the TIGER data. In these cases, we used the Gauss–Hermite expansion of the LOSVDs to estimate $\tilde{V}$ and $\tilde{\sigma}$ using the same method as used by Emsellem et al. (1996). We also made use of other published kinematics such as that in Kormendy & Richstone (1992, hereafter KR92) and Carter & Jenkins (1993), and the recent high spatial resolution FOS/HST and SIS/CFHT data (Kormendy et al. 1996, hereafter K+96). We finally included the data of Illingworth & Schechter (1982, hereafter IS82) for offset axes and Capaccioli et al. (1993, hereafter C+93) for the kinematics at large radii along the major axis. The characteristics of each data set are given in Table 3.

In Fig. 2, we compare the TIGER kinematics along the major axis with the ground-based data with similar resolution: they are all in agreement.
good agreement. The error bars of the TIGER data were determined by the statistics of the fluctuations inside a spatial resolution element, here taken as the full width at half-maximum (FWHM) of the point spread function (PSF). As it turns out, this defines a beam containing on average seven spectra.

2.3 Convolution and pixel integration

The comparison between observed kinematics and theoretical models requires us to take into account the observational characteristics. The seeing smearing and pixel binning have been computed through a double quadrature (see Appendix A). All the comparisons presented in the following sections include this processing, according to the parameters of the relevant data set as tabulated in Table 3.

Note that the PSF of the FOS data was derived using an approximation as given in van der Marel, de Zeeuw & Rix (1997).

2.4 Uncertainties in the kinematics

Additional factors linked with the instrumental setup or the data reduction processes can significantly influence the measured kinematics, and in particular the higher order Gauss–Hermite moments. First, an error in the centring or position angle of the slit induces a change in the observed kinematics. In the case of NGC 3115, an offset of \( \Delta \alpha = 2^\circ \) (major axis) in the position angle of the slit gives \( \Delta \theta_1 \sim 10 \) per cent at \( R = 35 \) arcsec, while the change in velocity amounts to only about \( 3 \) per cent. It is also important to control the spectral filtering applied for the retrieval of the LOSVDS as it could lead to us underestimating the higher order moments. Finally, the obtained kinematics may be sensitive to

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Table 3. Observational characteristics of the kinematical (long-slit) data used in this paper (for the FOS/HST data, see Kormendy et al. 1996).

| Ref.                      | slit width | \( \sigma_v \) (km s\(^{-1}\)) | seeing (FWHM) | \( h_3, h_4 \) |
|--------------------------|------------|---------------------------------|---------------|----------------|
| Illingworth & Schechter (1982) | 2–3”       | 100–130 km s\(^{-1}\)            | 1’5–5”        | no             |
| Capaccioli et al. (1993)  | 1–1’6      | \(-40\) km s\(^{-1}\)           | > 1’5         | no             |
| Kormendy & Richstone (1992) | 0’5        | 68 km s\(^{-1}\)              | 1’–1’6       | no             |
| Carter & Jenkins (1995)   | 0’45       | 10 km s\(^{-1}\)              | 0’8          | no             |
| van der Marel et al. (1994) | 1”        | 38 km s\(^{-1}\)              | 0’96         | yes            |
| Bender et al. (1994)      | 2’1        | 46 km s\(^{-1}\)              | 1’–2’5       | yes            |
| Fisher (1997)             | 2”         | 75 km s\(^{-1}\)              | > 1’5        | yes            |
| Kormendy et al. (1996)    | 0’26       | \(-35\) km s\(^{-1}\)         | 0’37         | –              |

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Figure 2. Comparison between the different published kinematics and the TIGER 1992 (solid circle) data. Each symbol corresponds to a reference as shown in the V panel. F97 (open circles) = Fisher (1997); vdM (open squares) = van der Marel et al. (1994) (WHT data only); BSG (triangles) = Bender et al. (1994); KR (open diamonds) = Kormendy & Richstone (1992) (exposures 29F48, 29F42, 34F16; see KR92); CJ (crosses) = Carter & Jenkins (1993).
the method used (e.g. Fourier fitting, FCQ, etc). In the following paragraph, we discuss such an effect on the central FOS LOSVD.

### 2.4.1 The central \( h_4 \) value

The central observed FOS LOSVD is one of the best data sets we can use to constrain the central dark mass in NGC 3115, as it has the highest available spatial resolution, includes the higher order kinematical terms and does not depend on the odd part of the distribution function (or very weakly since \( r = 0.01 \) arcsec). K+96 quotes a central value of \( h_4 = 0.10 \pm 0.03 \). However, higher order Gauss–Hermite moments may significantly depend on the instrumental setup and reduction procedures. We thus wish here to assess the robustness of the central \( h_4 \) value before attempting any detailed modelling.

We have therefore retrieved and reduced the FOS spectra and tried to reproduce the analysis mentioned in K+96. Our velocity and dispersion profiles obtained with the Fourier Quotient method (FQ) are compatible (within 15 km s\(^{-1}\)) with the ones quoted by K+96: e.g. we find a value of \( \sigma(0) = 448 \) km s\(^{-1}\) to be compared with \( \sigma(0) = 443 \pm 18 \) km s\(^{-1}\) as given by K+96. However, the shape of the LOSVD, and thus the dispersion and \( h_4 \) values, obtained using the FCQ can be severely influenced by template mismatch, a bad continuum subtraction and the spectral filtering (see also vdM+94). These would in turn affect the high frequencies of the LOSVD, thus diminishing its peakedness, and/or the low frequencies smoothing out its wings. Moreover, large \( h_4 \) values may anyway be difficult to obtain, as already emphasized by van der Marel (1994).

We have thus simulated artificial FOS spectra by convolving a FOS stellar template with different numerical LOSVDs, including the noise level present in the observed FOS spectra\(^2\) of NGC 3115. We then calculated the LOSVDs via FCQ, as in K+96, and parametrized them using Gauss–Hermite functions. These simulations clearly demonstrate that large \( h_4 \) values are underestimated (and as a consequence, Gaussian dispersion obtained via FCQ probably overestimated): the dominant effect is a strong filtering of the central peak, although the large wings of the LOSVDs are also significantly affected. Considering the central LOSVD presented in K+96 and the results of our simulations, we can estimate the true value of the central \( h_4 \) to be \( \sim 0.17 \) with a possible range \( 0.14 < h_4 < 0.21 \). As discussed above, however, these values are ill-constrained and an accurate estimation would require additional observations. We should finally note that this sensitivity is particularly important for the FOS/HST data: at a lower spatial resolution, gradients are not as extreme and the LOSVDs are closer to Gaussians.

### 3 The MGE luminosity model

We applied the MGE technique (Monnet, Bacon & Emsellem 1992; Emsellem, Monnet & Bacon 1994; Emsellem 1995) to the available V-band images to build a complete photometric model of NGC 3115. The individual PSFs were taken into account to obtain a deconvolved model which fits the data from the central \( HST/WFPC2 \) pixel (0.0455 arcsec) to 300 arcsec. In Fig. 3 we present the isophotes of NGC 3115 at three different scales compared with the MGE V-band model. The fit is excellent at all scales (see also Fig. 4). The only significant discrepancies are observed at \( R \sim 5 \) arcsec where the isophotes are boxy because of a detected spiral-like structure, and at \( R \sim 30 \) arcsec where the galaxy slightly departs from axisymmetry. The largest component has a FWHM of 895 arcsec and an axis ratio of 0.85.

We did not attempt to go beyond this scale as the photometry becomes rather uncertain. Hence, at radii larger than 600 arcsec along the major-axis, the MGE model decreases more rapidly (exponentially) than the \( B \)-band profile published by Capaccioli et al. (1987). However we tested that this does not significantly influence the dynamics up to 150 arcsec, radius of the last observed kinematic data point.

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\(^2\) This noise does not appear in fig. 3 of K+96, so we assumed that the spectra have been significantly filtered for the plot.
In Fig. 5, we show the isophotes of the deprojected MGE model for an inclination angle of 86°. The following different components clearly appear in this plot:

(i) the flattened extended halo which dominates the light of the galaxy at radii larger than 100 arcsec ($\epsilon \sim 0.65$ at 300 arcsec); 
(ii) the outer disc which extends up to $\sim100$ arcsec and is truncated at $R \sim 5$ arcsec (Freeman type II disc); 
(iii) the inner spheroid which exhibits a boxy shape and an ellipticity in the range 0.4–0.6; 
(iv) the inner or nuclear disc which continues up to the centre and has a radial extent of about 3 arcsec; 
(v) finally, the point-like nuclear source with magnitude $m_V = 16.63$ mag arcsec$^{-2}$ and half-light radius $r_h \sim 0.054$ arcsec, values consistent with the ones given by Kormendy et al. (1996). Note that it is still impossible to disentangle between a flattening of the surface brightness profile of this nucleus inside 0.1 arcsec and a still-increasing power law (Fig. 4).

In the following modelling we will always use a distance of 10 Mpc for NGC 3115: this value is intermediate between 11.0, recently given by Elson (1997), and the previously used value (9.24 Mpc – e.g. KR92).

4 TWO-INTEGRAL JEANS MODELS

In this section we wish to constrain the mass-to-light ratio in the central 40 arcsec of NGC 3115 by comparing the observed kinematics with Jeans dynamical models. We assumed a constant $M/L_V$ for the luminous mass of the galaxy, and added any required additional dark mass. We computed the second order non-centred projected velocity moment $\mu_2 = \sqrt{\bar{V}^2 + \bar{\sigma}^2}$ ($\bar{V}$ and $\bar{\sigma}$ are the projected mean velocity and velocity dispersion respectively) by solving the Jeans equations for a two-integral distribution function $f(E,J)$. This is particularly straightforward in the context of the MGE formalism, as $\mu_2$ can be evaluated through a single quadrature (see Emsellem et al. 1994). Moreover, in the frame of a $f(E,J)$ dynamical model, $\mu_2$ does not depend on the dispersion tensor anisotropy. Observed $\mu_2$ can only be determined with reasonable accuracy for data including higher order moments.

4.1 The inclination angle

Taking into account the axis ratios of the disc components, the inclination angle $i$ of the galaxy is restricted to a small range between about 83° and 90° (edge-on). Note that the lower limit is slightly model-dependent as the MGE model cannot exactly reproduce the thickness of the outer disc, which is constant or slightly rising outwards (see Capaccioli et al. 1988). We have built Jeans models assuming different values for $i$, sampling the interval between 83° and 90°. Varying the value of $i$ mostly affects the major-axis kinematics, where the flattened components dominate the light distribution. The minor-axis $\mu_2$ profile remains nearly unchanged. We can thus use the observed ratio between the value of $\mu_2$ along the minor and major axes as an indicator for the inclination angle, as it does not depend on the mass-to-light ratio and anisotropy. A value of $i = 86°$ gives the best fit, and will be used for all following models.
4.3 An estimation of the central dark mass

As emphasized already in the previous section and by previous studies (see K+96 and references therein), the central dispersion gradient calls for an increase of the mass-to-light ratio in the central few arcsec of NGC 3115. The FOS kinematics presented by K+96 then implies that this takes place inside a radius of ∼0.2 arcsec. According to our Jeans models, the observed gradients in the kinematics would lead to \(ML_v\) greater than 60 for the nucleus. This strongly suggests the presence of a central dark mass, which we denote by \(M_{BH}\), that is larger than \(10^8 M_\odot\) (the mass of the nucleus with \(ML_v = 6.5\) would be \(\sim 6.3 \times 10^8 M_\odot\)). In the following paragraphs we now provide some constraints on the central mass concentration in the frame of a \(f(E,J)\) dynamical model.

The addition of a central point-like mass in the MGE model is straightforward and the formalism is described in appendix A of Emsellem et al. (1994). We built a number of models with masses \(M_{BH}\) ranging from \(10^8\) to \(2 \times 10^9 M_\odot\) and for different \(ML_v\). In the following, we give the values of \(M_{BH}\) which best fitted each individual data set, as well as an indicative range in an attempt to take into account the formal error bars of the measured kinematics. Note that fitting the central kinematics requires us to vary both \(M_{BH}\) and \(ML_v\). Fig. 7 presents the comparison between these models and the data inside 7 arcsec along the major axis.

Estimations of the true moments are sensitive to errors in the measured high-order Gauss–Hermite moments (e.g. \(h_1\) and \(h_2\), see Bender et al. 1994): this may be the case of the low central dispersion value in vdM+94’s data. If we ignore the two central points in the fit we find \(M_{BH} \sim 6 \pm 1 \times 10^8 M_\odot\) and \(ML_v = 6.1 \pm 0.4\). The overall best fit to the vdM+94 data is obtained with \(M_{BH} \sim 4 \pm 1.5 \times 10^8 M_\odot\) and \(ML_v = 6.2 \pm 0.3\). The TIGER data indicate a slightly higher value for the central dark mass of \(M_{BH} = 6.6 \pm 1 \times 10^8 M_\odot\) with \(ML_v = 6.5 \pm 0.5\). Similar comparisons have been made with the data of Fisher (1997) and Bender et al. (1994), which both give similar values of \(M_{BH} \sim 7.0 \pm 3 \times 10^8 M_\odot\) and \(ML_v = 6.8 \pm 0.5\).

For the global set of kinematical data, the best estimation for the black hole mass is then \(M_{BH} = 6.5 \pm 3.5 \times 10^8 M_\odot\) with \(ML_v = 6.5 \pm 0.7\). Values as high as \(2 \times 10^9\) quoted by K+96 are excluded at more than the 3σ level for the simple \(f(E,J)\) case.\(^3\)

4.4 Dark matter at large radii

At radii larger than 40 arcsec along the major axis the observed velocity and dispersion profiles are almost flat out to the last measured point with \(\langle V \rangle \sim 250 \text{ km s}^{-1}\) and \(\langle \sigma \rangle \sim 100 \text{ km s}^{-1}\) (Capaccioli et al. 1993). The Jeans model with \(ML_v = 6.5\) predicts a dispersion profile that is consistent with the observed one at large radii, but under-estimates the velocity profile for \(R \geq 70\) arcsec by as much as 60 per cent at \(R = 120\) arcsec. As will be seen in Section 6, this implies a strong increase of the mass-to-light ratio at large radii.

5 TWO-INTEGRAL DISTRIBUTION FUNCTIONS

5.1 The method

In Section 4, we have restricted the range of values for the central dark mass using Jeans models and estimations of the true moments, and found a best fit with \(M_{BH} = 6.5 \pm 3.5 \times 10^8 M_\odot\). We wish now \(^3\)The distance used in this paper for NGC 3115 is slightly higher than that of K+96, which increases this discrepancy.
to compare the models directly with the measured values, namely \( V \), \( \sigma \) and the higher order Gauss–Hermite moments. This requires the knowledge of the full distribution function. We therefore applied the method of Hunter & Qian (1993) to derive the even part of the distribution function \( f(E,J) \) corresponding to the axisymmetric MGE mass model of NGC 3115.

The even part \( f_{\ell} \) of \( f(E,J) \) was calculated with a grid of 49 × 38 points in \((E,J)\) space designed to sample properly the gradients of \( f \) [particularly at \( J/E_{\text{max}}(E) \approx 1 \), because of the presence of the discs]. The odd part \( f_{o} \) remains as a free function in our models with the constraint that \( f(E,J) = f_{\ell}(E,J) + f_{o}(E,J) \) is positive everywhere. We have separated the contributions of the discs and the bulge in the computation of the distribution function: this is achieved by using the mass density of each component but including the total potential.

In order to derive the odd part \( f_{o} \) we used the parametrization introduced by van der Marel et al. (1994) following the work of Dejonghe (1986, 1987): we define a function \( h_{\ell} [\eta = JJ_{\text{max}}(E)] \) such that

\[
h_{\ell}(\eta) = \begin{cases} 
\tanh(\eta \alpha/2)/\tanh(\alpha/2), & (\alpha > 0), \\
\eta, & (\alpha = 0), \\
(2/\alpha) \arctanh [\eta \tanh(\alpha/2)], & (\alpha < 0),
\end{cases}
\]

so that \( f_{\ell}(E,J) = h_{\ell}(\eta) f_{\ell}(E,J) \). In practice, we used different values of \( \alpha \) for the disc and bulge components. This was essential to fit the detailed observed kinematics.

The LOSVDs were derived from the obtained numerical distribution function on a fine grid of more than 1000 points to allow a proper sampling of the sky plane in the central 45 arcsec. The areas of the projected LOSVDs (normalized by the projected luminosity density) were always equal to 1 within 1 per cent: this is an a posteriori check of the validity of the overall computations. This grid of LOSVDs was then used to compute (via an interpolation) the corresponding LOSVDs for each data set including the effects of seeing convolution, pixel integration and spectral resolution. Finally, the resulting LOSVDs were parametrized using the Gauss–Hermite moments as well as the true velocity moments.

A number of models have thus been built, including different cusp slopes and a range of values for the mass-to-light ratio \( M/L_V \) and the central dark mass \( M_{\text{BH}} \). In Fig. 8, we present one example of such a distribution function obtained for \( \rho \propto r^{-1.3} \) and \( M_{\text{BH}} = 1.0 \times 10^5 \, M_\odot \) as a function of the normalized energy \( E_{\text{max}} \) corresponding to the central potential of the model excluding the central dark mass) and angular momentum \( J = J_{\text{max}} \) corresponds to circular orbits at a given energy). The double disc structure is clearly visible as two bumps near the line of maximum angular momentum, and the plateau at high energy corresponds to the cusp (for a cusp slope \( \gamma = -1.5 \) and a Keplerian potential the distribution function is constant with energy, see Qian et al. 1995).

5.2 The inner 45 arcsec

As shown in Section 4, an overall good fit to the data inside 40 arcsec requires a rather high mass-to-light ratio \( M/L_V \sim 6.5 \). For the best sets of data (namely vdm+94, CJ93 and KR92) that cover these radii, we built models with different central dark mass concentrations. In Fig. 9 we present the KR92 data set which extends further than 40 arcsec and overimpose the best-fitting model which includes a central dark mass of \( M_{\text{BH}} = 0.94 \times 10^5 \, M_\odot \) and has a mass-to-light ratio \( M/L_V = 6.1 \). The major-axis velocity and dispersion (best Gaussian \( V \) and \( \sigma \) profiles are well-fit by this model even in the inner part. The fit is also good along the minor axis apart from a significantly lower dispersion around 3 arcsec where the central dark mass starts to dominate the observed kinematics. The fit to the data of CJ93, which has a slightly higher spatial resolution, is also excellent.

In Fig. 10 we present the fit to the vdm+94 data set. In this case, the higher order Gauss–Hermite moments could also be included. We could not obtain a reasonable fit to the \( h_3 \) parameter, for which the 2I model predicts values around \( 0.25 \) at \( R \sim 25 \) arcsec while the observed value is \( \sim 0.16 \). It is possible to reduce (in absolute value) the predicted \( h_3 \) by changing the odd part of the distribution function, but at the expense of a significantly lower velocity \( V \). Note that the predicted \( h_3 \) does not depend on the mass-to-light ratio (assuming it is constant with radius).

Uncertainties caused by errors in the positioning of the slit or by spectral filtering (see Section 2.4) of the LOSVDs are far too small to account fully for this discrepancy. The effect of the latter is shown in Fig. 10, where the predicted LOSVDs have been slightly Fourier filtered before measuring the Gauss–Hermite moments: the fit is then significantly better for all \( h_3, h_4, h_5, h_6 \) profiles, without changing \( V \) and \( \sigma \) significantly. However, the filtering required to explain the data is large considering the signal-to-noise ratio \( S/N \sim 20 \) per 10 km s\(^{-1}\)) and spectral resolution \( \sigma_{\text{fwhm}} \sim 23 \) km s\(^{-1}\)) quoted by vdm+94. We then performed some extensive simulations to derive the effect of the deconvolution method (Fourier fitting of the LOSVD) on \( V \) and \( h_3 \): it is indeed significant for \( R \sim 25 \) arcsec, where it can reach 15 km s\(^{-1}\) and \( \sim 0.04 \) respectively. The data of vdm+94 are thus consistent with \( h_3 = -0.2 \) at \( R \sim 25 \) arcsec, still too low in absolute value to be reconciled with our two-integral model. Finally, there are uncertainties associated with the data as illustrated in Fig. 11: the same 2I model is compared with the kinematics obtained by vdm+94 but this time derived using a one-sided Fourier-fitting technique (see vdm+94 for details). The fits to the higher order moments are good, although the predicted \( V/\sigma \) is now too low.
5.3 The central arcsec

The observed kinematics in the central arcsec is very sensitive to the spatial resolution, particularly in the case of NGC 3115 for which both the photometry and kinematics exhibit large central gradients. As we have shown in Section 2.4.1, however, the procedures used in the analysis of the data do also affect the final result. In the following paragraphs, we used the data provided by the FOS/HST and SIS spectrographs in parallel with the (lower resolution but) two-dimensional TIGER kinematics to constrain the free parameters of the dynamical models. We again examined a large range of different models, varying the central dark mass, the mass-to-light ratio and the central cusp slope as well as the odd part of the two-integral distribution function. For reasons of clarity, we only present here a few of these models which fit the data well.

5.3.1 Best-fitting models

All our two-integral models predict rather high $h_4$ values ($> 0.08$) when $M_{\text{BH}}$ is a few $10^8 M_\odot$. Our best-fitting model to the FOS data only has a cusp with $\rho \propto r^{-1.5}$ and a flattening of 0.7, a central mass $M_{\text{BH}} = 1.0 \times 10^9 M_\odot$ and $M/L_V = 6.5$ (model M1, see Fig. 12), although the cusp slope and flattening are not well constrained by the present data. In Fig. 12, we present the kinematical profiles obtained from the FOS spectra and model M1. The overall agreement is good, but our two-integral model predicts a slightly higher central velocity dispersion with $\sigma(0) = 480$ km s$^{-1}$. We should note, however, that the best Gaussian fit to the LOSVD obtained by K+96 and presented in their fig. 3 has $\sigma(0) \sim 489$ km s$^{-1}$, significantly larger than the one derived via FQ, but consistent with our value. This point is emphasized in Fig. 13 where we now present the comparison between the FOS LOSVD obtained by K+96 and the one predicted by model M1. The agreement is excellent, considering that the ‘wiggles’ present in the FOS central LOSVD are almost certainly not real (as are the negative points), but result from the presence of noise in the FOS spectrum as well as template mismatching. It is striking to see the difference between the LOSVD directly predicted from model M1 and the one retrieved via FCQ using the same spectral filtering as for the FOS data: this forces us here to underestimate $h_4$ and correspondingly to overestimate the Gaussian dispersion (see Section 2.4.1).

Model M1 includes a rapidly rotating nuclear disc with $a = 5$ (see Section 5.1). This model predicts velocities which are clearly too high at the SIS resolution, as shown in Fig. 14. Kormendy et al. (1998) noticed the fact that, for SIS data of NGC 3377, FCQ seems to provide slightly higher velocities than FQ by about 10 per cent (e.g. see their fig. 15). If this is also the case for NGC 3115, it could reconcile our model M1 with the SIS data.

We calculated other models in which the parameter $a$ was significantly increased (up to $a = 100$) for stars with high energy.
(close to the centre as we deal with positive energies): in this region
the total distribution function tends towards the maximum stream-
ing two-integral model. Figs 12 and 14 present two other models,
respectively M2, which corresponds to
\[ M_{\text{BH}} \approx 1.24 \times 10^9 M_\odot \]
and \[ M/L_V = 5.4 \], and M3, which has \[ M_{\text{BH}} = 0.94 \times 10^9 M_\odot \]
and \[ M/L_V = 6.1 \]. Although model M2 is a better fit to the FOS velocity
and dispersion profiles, the higher black hole mass implies
\[ h_4 \approx 0.134 \], a value at the upper limit of the error bar mentioned
by K+96. Model M3 is the best fit to the SIS data, and represents the
best compromise for the \textit{HST} data with a measured central
\[ h_4 = 0.094 \]. These models also show that the detailed kinematical
structure inside 0.3 arcsec remains ill-constrained. Although stellar
orbits may be biased towards near-maximum angular momentum
values in the central 10 pc, it is still improper to discuss the internal
kinematical structure of the point-like nucleus detected in the
photometry (Section 3), all the more because its luminosity dis-
tribution is uncertain as well (see Fig. 4).

5.3.2 The TIGER data

We finally compare the three best fits (M1, M2 and M3) with the
kinematics obtained with TIGER. Although the resolution of these
data is significantly lower than that of the SIS data, they still
represent a good constraint as they homogeneously cover a two-
dimensional field on the sky. As seen in Fig. 1, the TIGER
isovelocities are significantly flattened. This suggests again that
the nuclear disc is rapidly rotating. The contrast between the disc
and the central spheroid is, however, weaker than in the case of the
Sombrero galaxy, for which very asymmetric LOSVDs (and
therefore high \( h_3 \) values) are observed at the centre (Wagner,
Dettmar & Bender 1989; Emsellem et al. 1996). All three models,
M1, M2 and M3, fit the TIGER kinematical maps reasonably well,
with M1 giving the smallest residuals. The kinematical maps
corresponding to model M1 are presented in Fig. 15.

The results presented in the previous sections, using two-integral
\( f(E, J) \) models to fit the observed kinematics, can thus be sum-
morized in a few points.

(i) There is a significant discrepancy between the observed and
predicted \( h_3 \) values at a radius of about 25 arcsec.

(ii) The fit of the central FOS kinematics requires the nuclear
disc to have a nearly maximum streaming distribution function
inside 1 arcsec.

(iii) All the available kinematics in the central arcsec can be
rather well fitted by a two-integral model with constant mass-to-
light ratio. The best-constrained parameters are the central dark
mass and the mass-to-light ratio: considering the error bars in
the different data sets, our best estimations are \[ M_{\text{BH}} = 0.94^{+0.35}_{-0.29} \times 10^9 M_\odot \]
and \[ M/L_V = 6.1^{+0.7}_{-0.5} \]. These values can be
refined using the higher order Gauss–Hermite moments in the
central arcsec. However, the observed kinematics are very sensitive
to the details of the instrumental setup and the analysis procedures.

We could in principle reconcile our 2I model with the observed
kinematics along the major axis. The input mass model has indeed
been derived from a V-band image that may not reflect the true mass
distribution. We have for example assumed that the bulge and disc
components have the same mass-to-light ratio. This is certainly not
true, because they do not share the same colour profiles (Fisher,
Franx & Illingworth (1996). A difference of 0.1 mag in $B-R_c$ as typically observed between the bulge and the disc regions in NGC 3115 (Fisher et al. 1996) corresponds to a maximum change of about 5 per cent in $M/L_V$ (Worthey 1994), too small an effect to account for the discrepancy in $h_3$. In the following, we wish to examine another explanation: the distribution function depends on a third integral of motion. A two-integral distribution function $f(E,J)$ imposes $\sigma_R = \sigma$, everywhere. This may not be the case for the S0 galaxy NGC 3115, as is the case for our own Galaxy. This is also supported by derivations kindly made by Roeland van der Marel, using the formalism described in de Bruijne, van der Marel & de Zeeuw (1996) which allows a rather general geometry of the velocity ellipsoid. Although these models cannot mimic the complex morphology of NGC 3115, they clearly suggest that, for very flattened components, low $h_3$ values of the order of $-0.2$ could be obtained by relaxing the constraint of a two-integral distribution function.

### 6 A THREE-INTEGRAL DISTRIBUTION FUNCTION

In this section, we will construct a three-integral model using a quadratic programming technique. With the three-integral modeling, we wish to approach two questions.

(i) Is it possible to fit the data without a central dark mass, with a distribution function that has the appropriate three-integral
structure? In this context, we recall the well-known case of M87, where Binney & Mamon (1982) succeeded in fitting the (then available) data with a distribution function that was very anisotropic.

(ii) What is the phase-space structure of the best global 3I model?

This is an opportunity to present the use of the quadratic programming technique on a complex object, using a complete set of two-dimensional kinematical data. It should be clear from the outset, however, that we chose to produce analytical distribution functions. This implies that our third integral must be approximate. In that sense, the three-integral models we present here must be seen as belonging to the class of models with approximate third integrals.

6.1 The method

The essence of the method relies on the fact that most, if not all, observable kinematic quantities can be expressed as linear functions of the distribution function, if the potential is given. This means that if we approximate the distribution function \( f \) as a sum of some conveniently chosen basis functions \( f_i \),

\[ f = \sum_i c_i f_i(I_1, I_2, I_3), \]

then the same relation holds between the (e.g.) projected mass density \( \rho_p \) and the projected mass density of the components \( \rho_{p,j} \),

\[ \rho_p = \sum_i c_i \rho_{p,i}; \]

there is a similar relation for the projected pressures,

\[ \rho_p (\sigma_p^2 + \langle v_p \rangle^2) = \sum_i c_i \rho_{p,i} (\sigma_i^2 + \langle v_i \rangle^2); \]

If we stick to the mass density \( \rho \) for simplicity, we can construct a \( \chi^2 \) variable with the observed values \( \rho_i \) as follows:

\[ \chi^2 = \sum w_i \left( \rho_i - \sum c_i \rho_{i,j} \right)^2, \]

with \( \rho_{i,j} \) the value of the mass density at the \( j \)th point for component \( i \), and the \( w_i \) the associated weights. Clearly this \( \chi^2 \) is quadratic in the coefficients \( c_i \). Inclusion of other observables is trivial.

The minimization of such a \( \chi^2 \) will produce a set of coefficients, which will yield a distribution function (2) that will not necessarily be positive everywhere. Therefore, one must include constraints

\[ \sum_i c_i F_i[(I_1, I_2, I_3)_\text{tot}] \geq 0, \quad m = 1, \ldots, M, \]

expressing the positivity of the distribution function on a grid in phase space with points \((I_1, I_2, I_3)_\text{tot}\). This formulates a problem of quadratic programming, which is a well-known problem for which efficient algorithms exist.

In the present case, we use the luminosity model presented in Section 3. As discussed in Section 4, a constant \( M/L \) model will not work at large radii. This is indicated by the rotation curve, which is almost flat up to 200 arcsec (Capaccioli et al. 1993). We therefore construct a model for the dark mass that preserves the form of the luminous mass (say, for simplicity of the argument, ellipses with fixed axis ratios and semi-major axes \( a \)) and that has the radial dependence

\[ \rho_{\text{tot}}(a) = \rho_{\text{lum}}(a)(1 + C a^p). \]

We used \( p = 1.5 \), and the coefficient \( C \) is such that

\[ \frac{\rho_{\text{lum}}(a)}{\rho_{\text{tot}}(a)} = 30, \quad a = 30 \text{ kpc}. \]

This prescription produces a flat rotation curve. In practice, we adopt the same shape for the dark matter as for the luminous mass density, by specifying the relation (7) for all the radial functions in the harmonic expansion of \( \rho_{\text{lum}} \). In Table 4, we present the dependence of the mass-to-light ratio of the three-integral model on the radius in the equatorial plane. For the sake of comparison, we will also present in Section 6.2 a model where the mass-to-light ratio was forced to stay constant (apart from the central dark mass).

**Figure 13.** Central LOSVDs: observed FOS (thin solid line, shifted to zero velocity), that directly predicted from model M1 (dotted line), and that obtained via FCQ and using the characteristics of the FOS central spectrum (thick solid line).

**Figure 14.** Observed SIS kinematics along the major axis (K+96, circles), with the thin solid, dashed and thick solid lines corresponding to predictions of models M1, M2 and M3 respectively.
The calculation of the potential is straightforward, when the harmonic expansion of the total mass density is given. In practice, we interpolate the potential on a grid. As it is our goal to explore what the three-integral dynamical model contributes to the modelling of NGC 3115, we need an expression for the third integral \( I_3 \). This is done by fitting the potential with a Stäckel potential, and adopting the third integral from the Stäckel case as our approximation of the third integral. Thus, our models belong to the class of 3I models that use an approximation for the third integral. The procedure is outlined in more detail in Dejonghe, De Bruyne & Vauterin (1996).

Finally, we include a central singularity. It is a softened \( 1/r \) singularity (hence a Plummer sphere), with a softening length of the order of 50 000 au (~0.25 pc or ~5 mas, about forty times smaller than the FOS aperture). In the transition region where the gravitational force of the black hole is of comparable strength to the gravitational force of the core of the galaxy, there is no third integral and the Stäckel approximation of the potential is probably invalid. However, as argued in Section 4, we adopt as our working hypothesis that the spheroidal central component is two-integral. Hence, it will suffice if we produce three-integral models that do not penetrate too deep into the centre, where the model thus remains two-integral. It is, however, important to note that because orbits are non-local, the three-integral nature of our models significantly extends the probed space of solutions.

The data we use to build the \( \chi^2 \) are photometric and kinematical. As for the photometry, we produce a series of data points taken from a (logarithmically spaced) grid of deprojected luminous mass density as obtained in Section 3. The kinematical data for this modelling are twofold.

(i) Outside 50 arcsec along the major axis and 8 arcsec along the minor axis, the effect of pixel integration and seeing convolu-

| \( R \) (kpc) | \( R \) (arcsec) | \( M/L_\nu \) | \( M_{\text{sum}} \) (per cent) |
|-------------|----------------|-------------|----------------------|
| 0.1         | 21             | 5.8         | 100                  |
| 1.5         | 30             | 6.1         | 90                   |
| 2.9         | 60             | 9.9         | 79                   |
| 4.4         | 90             | 13.9        | 68                   |
| 5.8         | 120            | 19.9        | 59                   |
| 7.3         | 150            | 26.0        | 52                   |
| 8.7         | 180            | 32.5        | 46                   |
| 10.2        | 210            | 39.5        | 41                   |
| 11.6        | 240            | 46.9        | 38                   |
| 13.1        | 270            | 54.4        | 35                   |
| 14.5        | 300            | 63.1        | 33                   |

Table 4. Mass-to-light ratio of the three-integral model versus the radius in arcsec or kpc. The contribution of the luminous mass is given in the right column.
The Gaussian velocity and dispersion are good approximations for the two first true moments. We thus selected a set of points from the above-mentioned sources. It is impractical (at present) to include all data points, while some are mildly inconsistent.

(ii) The rest of the galaxy (mainly the central part and the major axis inside 50 arcsec) was sampled using the estimations of the true moments given by the TIGER data presented in Section 2.2 and the published kinematics of vdM+94. This of course required us to include the instrumental setup in the $\chi^2$.

For the components, we use two families. The first one is an extension of the Fricke components, and has distribution functions of the form

$$F_{\mu,0} = (E - E_0)^p (J - J_0)^q,$$

wherever $\pm J \geq J_0$ and $E > E_0$. The distribution functions are zero

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Figure 16. Data points from B+94 (triangles) and vdM+96 (crosses), the best fit (3I models) with a $6.5 \times 10^5 M_\odot$ black hole (solid line) and best fit for a model without black hole (dotted line). The models have been convolved to correspond to B+94 data.

Figure 17. A comparison of the luminous mass density, as obtained by the MGE technique (solid lines), and the 3I fit (dotted or dashed lines). The upper left panel shows the log of the luminous mass density along the major axis, the upper right panel the same along the minor axis.
elsewhere. The exponents $m$ and $n$ are integers, but $p$ can be real. In general the parameter $p$ controls the central concentration of the component and the parameter $q$ controls the amount of angular momentum, while $n$ is indicative of the dependence on the third integral. The cut-off binding energy $E_0$ is needed to limit the extent of the components, for further enhanced flexibility. Also, because the potential is almost singular at the centre, large values of $p$ (thus components that are concentrated) produce impossibly high mass densities at the centre. Hence, we instead control the central concentration with the parameter $E_0$, and adopt low or even fractional values for $p$. Finally, the cut-off angular momentum $J_0$ prohibits the orbits from penetrating too deep into the potential well, where the third integral may not exist.

The second family is designed to produce very thin 2I discs in the equatorial plane of the galaxy (Batsleer & Dejonghe 1995), and has members of the form

$$F_{6pq} = [S_0(J)]^p J^q [E - S_0(J)]^p.$$

wherever $-J \geq 0, E > S_0(J)$. The distribution functions are zero elsewhere. The parameters $p, q$ and $s$ can be real. The quantity $S_0(J)$ is the maximum binding energy that a star with angular momentum $J$ can have if its distance from the equatorial plane remains below $z_0$. If $z_0 = 0, S_0(J)$ is the binding energy of a circular orbit with angular momentum $J$. The parameters $p$ and $q$ have the same meaning as in the first family, while $s$ controls the concentration of the component towards the equatorial plane.

Figure 18. The TIGER maps of the first true velocity moments $\tilde{V}$ (upper panel) and $\tilde{\delta}$ (lower panel), compared with the three-integral model with $M_{\text{BH}} = 6.5 \times 10^8 M_\odot$ (contours: step of 20 km s$^{-1}$ for $\tilde{V}$ and 25 km s$^{-1}$ for $\tilde{\delta}$).
6.2 Results

In Fig. 16 we compare two models on the major axis, within the central 8 arcsec. The solid line corresponds to a model with a $6.5 \times 10^8 M_\odot$ black hole, the dashed line has no black hole. Clearly, even in the three-integral case, the latter model fails dismally, especially in the inner 2 arcsec. This is to be expected of course: the large dispersion in the centre exceeds the escape velocity if no black hole is present. Any reshuffling of orbits in order to obtain a favourable viewing position is insufficient in the absence of a black hole, because there is simply not enough kinetic energy present. Although new components not included in our library may help, this is a strong suggestion that three-integral models without a black hole cannot fit the data.

In fact, if we make the integral space compact by considering the representation in $(E, J \sqrt{E}, L_1 E)$ then powers, such as Fricke components, form a complete set, if one allows negative coefficients. In practice, of course, one is limited by finite computing resources. At least all orbits are present, however, with relative weights that vary continuously, and thus not necessarily with arbitrarily flexible relative weights. However, this practical incompleteness is not fundamentally different from the one present in, say, the Schwarzschild method. In the latter case, phase space cannot be covered completely, also for practical reasons, but the orbits considered can have arbitrarily flexible relative weights, at least without the (necessary) smoothing. So both methods are incomplete in practice, be it to varying degree, but the incompleteness will certainly manifest itself differently.

Regarding our second question, we first compare in Fig. 17 the luminous mass density as obtained with the MGE technique (see Section 3) with the best 3I model that has a black hole of $6.5 \times 10^8 M_\odot$ (see also Fig. 4). The fit is quite reasonable, certainly considering the dynamic range that is to be covered (see upper panels).

In Figs 18, 19 and 20 we next compare the input kinematical data with our best model that has a black hole of $6.5 \times 10^8 M_\odot$ and includes a dark halo. The fit is good, except perhaps at the centre along the cut at $y = 20$ arcsec and the one at $x = 40$ arcsec (Fig. 20). In fact, the data (Illingworth & Schechter 1982) presented in this plot are mildly inconsistent with other published data along the minor or major axis (e.g. vdM+94, see Fig. 19). The model with a constant mass-to-light ratio ($M/L_V = 10$, dotted lines in Figs 19 and 20) does however show significant discrepancies along the major axis and the cut at $y = 20$ arcsec, with an overestimate of the velocity in the inner 50 arcsec. This shows, as already stated, that the mass-to-light ratio must significantly increase outwards, by nearly a factor of 2 between the inner parts ($R < 50$ arcsec, $M/L \sim 6$) and the last measured point ($R \sim 200$ arcsec, $M/L_V \sim 10$). We therefore confirm what was suggested by C+93 via simple dynamical arguments, but here using general three-integral dynamical models. Although a stellar mixture with a global mass-to-light ratio $M/L_V \sim 10$ is not excluded, it would have to be reconciled with the nearly constant (but decreasing) colours in the outer parts of NGC 3115 (for $R > 120$ arcsec, Strom et al. 1977) and the metal-poor populations observed in the halo by Elson (1997). The major-axis velocity and dispersion profiles are also more consistent with flat curves than with falling ones (see Figs 19 and 20): this suggests that $M/L_V$ is still significantly increasing outwards for $R > 150$ arcsec. In view of these two points, we favour the hypothesis of the presence of a dark halo to explain the observed kinematics of NGC 3115 at large radii.

For efficiency reasons, we included values for the Hermite coefficients at only two radii. At 7 arcsec, the model convolved at the resolution of vdM+94 data has $h_1 = -0.0025$, close to the observed $h_1 = -0.05$, and in fact consistent with other published $h_1$ at this radius (Bender et al. 1994; F96). At 26 arcsec we obtain $h_1 = -0.15$, a much better fit than the 21 result, now perfectly consistent with the observed $h_1 = -0.16$. In fact all observed higher order moments are now reasonably fitted by the three-integral model.

Finally, we present in Figs 21 and 22 the mean rotation and velocity dispersions of the model in a meridian. The inner disc is tangentially anisotropic with $\sigma_\phi/\sigma_r$ reaching a value of $\sim 0.6$ in the inner 2 arcsec. This confirms the finding obtained via the two-integral model in Section 5 that the inner disc seems to be close to maximum rotation. Between 3 arcsec and 15 arcsec in the equatorial plane, the model is nearly isotropic. The inner spheroid ($R < 60$ arcsec) is close to two-integral and has a nearly constant $\sigma_\phi/\sigma_r \sim 0.65$. Going outwards, the distribution function becomes significantly three-integral and more radially anisotropic. Already at a radius of 30 arcsec in the equatorial plane ($R \sim 1.5$ kpc), $\sigma_\phi/\sigma_r \sim 1.1$ and $\sigma_\phi/\sigma_r \sim 0.8$. The ratio $\sigma_\phi/\sigma_r$ reaches a local maximum of 0.92 at a radius of $\sim 100$ arcsec, where the outer disc contribution disappears and the dark matter becomes dominant.

7 Conclusions

We have presented a detailed analysis of the kinematics of NGC 3115 using different modelling techniques. As far as possible, we have made use of the photometric and kinematical data available to us, in order to build dynamical models following realistic light and mass distributions. For the first time, we have included two-dimensional spectroscopic data in order to constrain the models in the central few hundred pc.
Jeans equations were used to define reasonable ranges for the mass-to-light ratio and the central dark mass in the frame of a two-integral dynamical model. Fitting the first two velocity moments in the central 40 arcsec gives $M_{\text{BH}} \approx 6.5 \times 10^6 M_\odot$ and $M/L_V = 6.5 \pm 0.7$. These estimations were then tested by retrieving the corresponding distribution functions via the Hunter & Qian method (Section 5). We would like to emphasize the fact that a simple two-integral model with a constant mass-to-light ratio can fit the velocity and dispersion profiles of NGC 3115 inside ~45 arcsec surprisingly well (see Section 5).

However, no two-integral model could fit the value of $V$ and $h_3$ simultaneously as observed by vDM+94 around 25 arcsec along the major axis, where the outer disc significantly contributes to the surface brightness. Models built using the quadratic programming technique confirmed that three-integral components solve this problem (Section 6). The dynamical structure of the outer part of NGC 3115 is therefore very probably three-integral.

In Section 5, we showed that there are no two-integral axisymmetric models that can fit the kinematics of NGC 3115 in the central region, without the addition of a central dark mass of about $10^9 M_\odot$. The best fit, which makes use of FOS/HST (Kormendy et al. 1996) and Gauss–Hermite moments, was obtained for $M_{\text{BH}} = 0.94 \times 10^9 M_\odot$ and $M/L_V = 6.1$. This estimation of the central dark mass is at the upper limit of the
Figure 21. Some moments of the best-fitting 3I distribution function in a meridian. Upper panel: $v_i$; lower panel: $\sigma_i$.

Figure 22. Some moments of the best-fitting 3I distribution function in a meridian. Upper panel: $\sigma_r/\sigma_z$; lower panel: $\sigma_z$. 

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range derived from simple Jeans models. This shows the need of high spatial resolution kinematics including the higher order moments in order to constrain the mass distribution more accurately.

The quadratic programming technique confirmed the need of a central dark mass, as even the three-integral models without a black hole failed to fit the central rise in the velocity dispersion. In principle, one could object that, since a library of 3I distribution functions is always limited, no matter what method is considered for the modelling, this is no definite proof that a roughly constant MIL model cannot fit the observables. However, such a model would have to populate many stars on radial orbits in the very centre. This would produce a galaxy core that is extremely prone to radial orbit instabilities, which, given the dynamical time of the order of \(8 \times 10^6\) yr at a radius of 10 arcsec \((\sim 500\) pc), would have relaxed into a more stable configuration a long time ago, or would have produced a bar, evidence of which is not (yet) present.

Previous estimations of the central dark mass in NGC 3115 are surprisingly different from one paper to another. Kormendy & Richstone (1992) derived a value of \(M_{\text{BH}}\) from \(1-2 \times 10^8\) \(M_\odot\) using spherical models corrected for flattening. Then Kormendy et al. (1996), using the same formalism, confirmed the upper limit of \(\sim 2 \times 10^8\) \(M_\odot\), by including the SIS and HST/FOS data. Both papers used a mass-to-light ratio of \(M/L_V \sim 4\). Finally, Magorrian et al. (1998) found a best-fitting value of \(M_{\text{BH}} \sim 4.8 \times 10^8\) \(M_\odot\) with a \(M/L_V \sim 7.65\) (both values scaled for \(D = 10\) Mpc), noting that their value for \(M_{\text{BH}}\) was probably an underestimate. This is consistent with the picture drawn in the present work, as we find that a value of \(M_{\text{BH}}\) as low as \(6.5 \times 10^8\) \(M_\odot\) (with a corresponding \(M/L_V \sim 6.8\)) is compatible with the existing data. However, our best-fitting model has \(M_{\text{BH}} = 0.94 \times 10^8\) \(M_\odot\), and values as high as \(M_{\text{BH}} = 2 \times 10^8\) \(M_\odot\) are excluded at more than the 3\(\sigma\) level. Our models, because they follow the observed light distribution more precisely and include the derivation of the full distribution function, give improved estimations of the central dark mass and mass-to-light ratio.

We finally showed that the mass-to-light ratio should increase by a factor of 2 between the inner parts \((R < 50\) arcsec) and the outer parts \((R > 100\) arcsec) of NGC 3115. We produced a model that fits the flat rotation and dispersion profiles along the major axis and thus gave the first evaluation of the amount of dark matter required to explain the observed kinematics at large radii: in our model the dark matter represents about 50 per cent of the total mass inside 150 arcsec and nearly 70 per cent inside 300 arcsec. These values are obviously still uncertain as we did not fully examine alternative spatial distributions or radial profiles.

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APPENDIX A: CONVOLUTION AND PIXEL BINNING USING A KERNEL

It is possible to combine pixel integration and PSF smearing in only two integrals (as otherwise there would be four: two for the seeing and two for the pixel integration). This is described below in the case of circular or rectangular pixels (see also Qian et al. 1995).

A1 Circular pixel

For a circular pixel (TIGER-like lenses) of radius \(d\), the observable \(S_0\) at a position \((x', y')\) after convolution with a single Gaussian of
dispersion $\sigma$ and integration on the pixel is
\[ S_0(x', y') = \frac{1}{\pi \sigma^2} \int_0^\infty r dr \int_0^{2\pi} d\theta K(r) \]
\[ \times S[x' + r \cos (\theta + \theta_0), y' + r \sin (\theta + \theta_0)], \tag{A1} \]
where
\[ K(r) = \int_0^{\pi/2} s \, ds \, \psi_0 \left( \frac{sr}{\sigma} \right) e^{-sr^2} e^{-\frac{1}{2}(s-r)^2}. \tag{A2} \]
Note that if the integrals are derived with fixed quadratures, the kernel has only to be calculated once, and only depends on the radius $r$.

### A2 Rectangular pixel

For a rectangular pixel of dimension $2l \times 2w$, the observable $S_0$ at a position $(x', y')$ after convolution with a single Gaussian of dispersion $\sigma$ and integration on the pixel is
\[ S_0(x', y') = \frac{1}{4lw} \int_0^\infty r dr \int_0^{2\pi} d\theta K(r, \theta) \]
\[ \times S[x' + r \cos (\theta + \theta_0), y' + r \sin (\theta + \theta_0)], \tag{A3} \]
where
\[ K(r, \theta) = \frac{1}{4} \left[ \text{erf} \left( \frac{l + r \cos \theta}{\sqrt{2} \sigma} \right) - \text{erf} \left( \frac{-l - r \cos \theta}{\sqrt{2} \sigma} \right) \right] \]
\[ \times \left[ \text{erf} \left( \frac{w + r \sin \theta}{\sqrt{2} \sigma} \right) - \text{erf} \left( \frac{-w - r \sin \theta}{\sqrt{2} \sigma} \right) \right]. \tag{A4} \]

Note again that if the integrals are derived with fixed quadratures, the kernel can be calculated only once on a fixed grid.

### APPENDIX B: CUSPS IN MGE PHOTOOMETRIC MODELS

As we wish to build photometric models that can reproduce central power laws, it has been necessary to extend the MGE formalism (Emsellem et al. 1996) to include components with central cusps. The adopted form for the central component is then
\[ \rho(m_c^2) = I_c \times \left( \frac{m_c^2}{2\sigma_c^2} \right)^{-\gamma/2} \exp \left( -\frac{m_c^2}{2\sigma_c^2} \right). \tag{B1} \]
where $m_c^2 = R^2 + Z^2 l q_c^2$. This is the natural generalized form of the three-dimensional Gaussian function.

The integrated luminosity of this component is $I_c \times 2\pi (\sqrt{2} \sigma_c)^3 q_c \Gamma(1 - \gamma/2)$, and the gravitational potential contribution is simply
\[ \Phi(R, Z) = 4\pi G q_c \sigma_c^2 P_c \tag{B2} \]
\[ \times \int_0^1 \frac{1}{1 - \gamma/2} \frac{T^2}{2\sigma_c^2} \left[ R^2 + Z^2 l (1 - e_c^2 T^2) \right] \frac{dT}{(1 - e_c^2 T^2)^{3/2}}. \tag{B3} \]
with $e_c^2 = 1 - q_c^2$ and $P_c = M / L \times I_c$.