The $pd \rightarrow ^3 H_ΛK^+$ reaction cross section

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Abstract

The one- and two-step mechanisms of the $pd \rightarrow ^3H \Lambda K^+$ reaction in the range of incident proton kinetic energy $T_p = 1.13 - 3.0 \text{ GeV}$ have been investigated for the first time. A remarkable peculiarity of the two-step mechanism which incorporates subprocesses $pp \rightarrow d\pi^+$ and $\pi^+n \rightarrow K^+\Lambda$ is the so called velocity matching providing the presence of all intermediate particles nearly to the on-mass-shell. The differential cross section has been calculated using a realistic model for the hypertritium $^3H_\Lambda$ wave function. The maximum value of the cross section is estimated as $\sim 1\text{nb/sr}$. The contribution of the one-step mechanism with the elementary process $pN \rightarrow NK\Lambda$ into the cross section has been found to be two - three orders of magnitude smaller in comparison with the two-step mechanism.
The $K^+$ meson production in proton-nucleus collisions is of great interest as these reactions allow one to investigate the nuclear structure at short distances between nucleons \[1\]. While the experimental research programs \[2\] are supposed for the target nuclei with $A \geq 12$ the clearest theoretical analysis can be done for the lightest nuclei. The $pd \rightarrow ^3H \Lambda K^+$ reaction investigated here is a process with high momentum transfer. So, at the threshold of this reaction ($T_p = 1132 \text{MeV}$) initial proton and deuteron have momenta $\sim 1 \text{ GeV/c}$ in the c.m.s. but in the final state all nucleons are at rest. At the proton kinetic energy in the laboratory system $T_p$ below 1580 MeV the $p + N \rightarrow N + \Lambda + K$ process on a free nucleon $N$ at rest is forbidden by the energy-momentum conservation. Therefore the $pd \rightarrow ^3H \Lambda K^+$ reaction in this region occurs either through involving high momentum components of the deuteron wave function when incident proton collides with one of its nucleons (one-step mechanism, Fig. 1, a) or by means of active interaction with two nucleons of the deuteron (two-step mechanism, Fig.1, b). It seems less obvious that in the last case the high momentum components of the wave function will be required. In this respect the $pd \rightarrow ^3H \Lambda K^+$ reaction is similar to $pd \rightarrow ^3He \pi^0$ \[3\] and $pd \rightarrow ^3He \eta$ \[4\] reactions for which the two-step mechanism (called a three-body one in literature) was found to dominate \[4\]. Indeed, the $pd \rightarrow ^3H \Lambda + K^+$ and $pd \rightarrow ^3He \eta$ reactions have deeper analogy in the framework of the two-step mechanism with subprocesses $pp \rightarrow d\pi^+$ and $\pi^+n \rightarrow \Lambda K^+$ or $\pi^+n \rightarrow p\eta$ respectively. The relation between masses of initial and final particles in these reactions is such that at the corresponding threshold of the reaction as well as for the angles $\theta_{c.m.} \sim 90^\circ$ which determines the direction of the final meson momentum in respect to the incident beam, all intermediate particles ($\pi-$meson, deuteron, nucleon) are near to on-mass-shell in a very wide energy range above the threshold \[4\]. For this reason the two-step mechanism corresponding to the Feynman graph in Fig.1, b seems to be the most realistic model of this reaction. It should be noted that for production of $\pi-$mesons
and heavier mesons ($\omega, \phi, \eta'$) as well as for target-nuclei with $A \geq 3$ the above mentioned velocity matching does not take the place.

Another interesting aspect of the $pd \rightarrow ^3H\Lambda K^+$ reaction is connected with formation of the hypertritium nucleus $^3H_\Lambda$ in the final state. The $^3H_\Lambda$ nucleus is a loosely bound system with the binding energy $\varepsilon \sim 2.35 MeV$ which probably has a configuration of the $^3H_\Lambda \rightarrow d + \Lambda$ [4]. An investigation of the $pd \rightarrow ^3H\Lambda K^+$ reaction can give a new independent information about the wave function of the $^3H_\Lambda$ nucleus.

In the framework of the two-step mechanism the amplitude $A^{twost}(pd \rightarrow ^3H\Lambda K^+)$ of the $pd \rightarrow ^3H\Lambda K^+$ reaction can be written in the full analogy with the amplitude of the $pd \rightarrow ^3He\eta$ reaction [5]. As a result, we get

$$A^{twost}(pd \rightarrow ^3H\Lambda K^+) = C \frac{\sqrt{3}}{2m} A_1(pp \rightarrow d\pi^+) A_2(\pi^+n \rightarrow K^+\Lambda) \mathcal{F}(P_0, E_0) \quad (1)$$

where $A_1$ and $A_2$ are the amplitudes of the processes $pp \rightarrow d\pi^+$ and $\pi^+n \rightarrow K^+\Lambda$ respectively, $m$ is the nucleon mass, $C = 3/2$ is the isotopic spin factor allowing for the summation over isotopic spin indices in the intermediate state; the nuclear formfactor in exp. (1) is defined as

$$\mathcal{F}(P_0, E_0) = \int \frac{d^3q_1}{(2\pi)^3} \frac{d^3q_2}{(2\pi)^3} \frac{\Psi_d(q_1)\Psi_H(q_2)}{E_0^2 - (P_0 + q_1 + q_2)^2 + i\epsilon}. \quad (2)$$

Here $\Psi_d(q_1)$ is the wave function of the deuteron and $\Psi_H(q_2)$ is the wave function of the $^3H_\Lambda$ nucleus in the $^3H_\Lambda \rightarrow d + \Lambda$-channel in momentum space; $E_0$ and $P_0$ are the energy and momentum of the intermediate $\pi-$ meson at zero momenta of nucleons in the nuclear vertices $q_1 = q_2 = 0$:

$$E_0 = E_K + \frac{1}{3}E_H - \frac{1}{2}E_d, \quad P_0 = \frac{2}{3}P_H + \frac{1}{2}P_d, \quad (3)$$

where $E_j$ is the energy of the jth particle in the c.m.s., $P_d$ and $P_H$ are the momenta in the initial deuteron and the $^3H_\Lambda$ nucleus in the c.m.s. respectively.

According to the paper [3], when deriving exp. (2) we neglect zero components
\( q_{10} \) and \( q_{20} \) of the 4-momenta \( q_1 \) and \( q_2 \) in the 4-dimensional propagator of \( \pi^{-} \)-meson

\[
(p_{\pi}^2 - m_{\pi}^2 + i\varepsilon)^{-1} = ((p_K + \frac{1}{3}P_H - \frac{1}{2}P_d + q_1 - q_2)^2 - m_{\pi}^2 + i\varepsilon)^{-1}
\]

in comparison with the energies \( E_k, E_H, E_d \). The 3-momenta \( q_1 \) and \( q_2 \) are taken exactly. Recently there has appeared a calculation \([5]\) for the \( pd \rightarrow ^3He\eta \) reaction near the threshold in the two-step model which is very similar to that developed in paper \([3]\) and used here. The authors of paper \([5]\) apply the 3-dimensional diagram technique and instead of the 4-dimensional \( \pi^{-} \)-meson propagator \((p_{\pi}^2 - m_{\pi}^2 + i\varepsilon)^{-1}\) they deal with the energy denominator \((\sqrt{s_{pd}} - E_{\pi} - E_n - E_d + i\varepsilon)^{-1}\). The linearization procedure over Fermi momenta \( q_1 \) and \( q_2 \) is used in order to perform integration over \( dq_1 \) and \( dq_2 \). Exp. \( (2) \) for the nuclear formfactor differs from that in paper \([3]\) while in the both cases it is a rather smooth function of kinematic variables. It is obvious that the reasons for this difference are different means for consideration of relativistic effects in the two-step models \([3]\) and \([5]\).

The amplitude \((1)\) is connected to the differential cross section of the \( pd \rightarrow ^3H\Lambda K^+ \) reaction by the following expression

\[
d\sigma = \frac{1}{64\pi^2 s_{pd}} \frac{|P_H|}{|P_d|} |A(pd \rightarrow ^3H\Lambda K^+)|^2,
\]

where \( s_{pd} \) is the invariant mass of the initial p+d state. The amplitudes \( A_1(pp \rightarrow d\pi^+) \) and \( A_2(\pi^+n \rightarrow \Lambda K^+) \) are related to the corresponding differential cross sections by analogous relations. One should note that the amplitudes \( A_1 \) and \( A_2 \) are factored outside the integral sign at the point \( q_1 = q_2 = 0 \). As mentioned in paper \([3]\), factorisation of the \( pd \rightarrow ^3HeX \) cross section in the product of \( pp \rightarrow d\pi^+ \) and \( \pi^+n \rightarrow \eta p \) cross sections takes place if only one of two invariant forward \( pp \rightarrow d\pi^+ \) amplitudes dominates. For simplicity we assume here that this condition is fulfilled.

The amplitude of the one-step mechanism corresponding to the Feynman graph
in Fig.1, \( a \) can be written as
\[
A^{\text{onest}}(pd \to ^3H\Lambda K^+) = \sqrt{\frac{3}{m}} A_3(pN \to N\Lambda K^+) \Phi(Q), \tag{5}
\]
where \( A_3 \) is the \( pN \to N\Lambda K^+ \) process amplitude which is factored outside the two-loop integration sign. The nuclear formfactor \( \Phi(Q) \) is defined by
\[
\Phi(Q) = \int d^3r \varphi_d(r) \varphi_d^+(r) \psi_H^+(r) \exp(iQr), \tag{6}
\]
where
\[
Q = \frac{1}{3} P_H - \frac{1}{2} P_d. \tag{7}
\]
One should note that integral (6) has a meaning of the deuteron elastic formfactor \( F_d(2Q) \) at the transferred momentum \( \Delta = 2Q \) modified by the presence of the hypertritium wave function \( \psi_H^+(r) \) in the integrand. It is obvious that the formfactor \( \Phi(Q) \) decreases fast with growing \( Q \).

The one-step amplitude has been numerically calculated here using both \( S- \) and \( D- \) components of the deuteron wave function for the RSC potential in parametrisation [9]. Using the experimental data on the total cross section \( \sigma_{NN \to K^+\Lambda N} \) [10] we estimated here the squared amplitude \( |A_3(pN \to N\Lambda K^+)|^2 \) as \( \sim 250 \div 450 GeV^{-2} \) in the initial proton energy range \( 1.6 \div 3.0 GeV \). The numerical calculations for the two-step mechanism are performed in the s-wave approximation for the deuteron wave function [9]. (As was shown by our calculations, the contribution of the deuteron D-component to the cross section is about 10 %). For the wave function of the \( ^3H\Lambda \) nucleus the \( d + p \)-model developed in Ref. [8] on the basis of separable \( \Lambda N \)-interaction is used. In this model the \( ^3H\Lambda \) wave function only contains the \( S \)-component. In the S-wave approximation the factor (2) takes the form
\[
\mathcal{F}_{000}(P_0, E_0) = \frac{1}{4\pi} \int_0^\infty j_0(P_0r) \exp(iE_0r)\varphi_d(r)\varphi_H(r)r \, dr. \tag{8}
\]
For the differential cross section of the reaction \( pp \to d\pi^+ \) the parametrisation of Ref. [11] is used here. For the \( \pi^+n \to \Lambda K^+ \) differential cross section the parametrisation
of the total cross section from Ref. [12] is used and isotropic behaviour of the cross section is assumed.

We have investigated here numerically the behaviour of the formfactor $F_{000}(P_0, E_0)$ as a function of incident proton kinetic energy $T_p$ at different $K^+$-meson scattering angles $\theta_{c.m.}$. The momentum $P_0$ is a rather fast decreasing function of $T_p$ at $\theta_{c.m.} = 180^{\circ}$ ($P_0 = 0.5 - 0.1 GeV/c$ in the range $T_p = 1.1 - 3.0 GeV$). On the contrary, at the scattering angles $\theta_{c.m.} = 0^{\circ}$ and $90^{\circ}$ both the energy $E_0$ and momentum $P_0$ are increasing functions of $T_p$ ($E_0, P_0 \sim 0.5 - 1.2 GeV$). This behaviour of $P_0$ results in a large value of the formfactor $|F_{000}(P_0, E_0)|^2$ at $\theta_{c.m.} = 180^{\circ}$ in comparison to the ones at $\theta_{c.m.} = 0^{\circ}$ and $90^{\circ}$. If one substitutes the wave function of the $^3$He nucleus in the $d+p-$ channel [13] instead of the $^3$H$_\Lambda$ hypernucleus in exp. (8) then the squared formfactor $|F_{000}(P_0, E_0)|^2$ corresponds to the one for the $pd \rightarrow ^3H\eta$ reaction and it turns out to decrease faster with growing incident energy $T_p$ and its value at the threshold increases by a factor of 3 - 5.

The calculated differential cross sections of the $pd \rightarrow ^3H\Lambda K^+$ reaction are presented in Fig.2. One can see from this picture that for any scattering angle the differential cross section has a sharp maximum at the proton energy $T_p \sim 1.2 GeV$, which displays the corresponding sharp peak observed in the total cross section of the $\pi^+N \rightarrow \Lambda + K^+$ reaction (see Ref. [12] and references therein). On the whole, the relations between differential cross sections at the angles $\theta_{c.m.} = 0^{\circ}, 90^{\circ}$ and $180^{\circ}$ follow from corresponding relations between formfactors $|F_{000}(P_0, E_0)|^2$.

The differential cross section of the $pd \rightarrow ^3H\Lambda K^+$ reaction predicted by the two-step model differs from that for the $pd \rightarrow ^3H\eta$ reaction in two respects [1]. First, the maximum value of the $K^+$-meson production cross section $\sim 1 nb/sr$ is about 50 times smaller than that for the $\eta-$meson production. Secondly, the $pd \rightarrow ^3H\Lambda K^+$ reaction cross section is a smoother decreasing function of incident proton energy in comparison with the cross section of the $pd \rightarrow ^3H\eta$ reaction. As
follows from the behaviour of the formfactor $|\mathcal{F}_{000}(P_0, E_0)|^2$ both these peculiarities are in part connected to the form of the wave function of the $^3H_\Lambda$ nucleus.

The results of calculation in the framework of the one-step mechanism are presented in Fig.3. One can see that the contribution of this mechanism is two-three orders of magnitude smaller than that following from the two-step model.

In conclusion, we note that the two-step mechanism of the $pd \rightarrow ^3H_\Lambda K^+$ reaction is used owing to the velocity matching. In the case of $\eta$–meson production this mechanism explains qualitatively the energy dependence of the cross section above the threshold [5]. However, just at the threshold this model is in strong contradiction with the experimental data on the $pd \rightarrow ^3He\eta$ reaction [5]. One of a reason for it is probably a strong attractive interaction in the final $\eta – ^3He$ state caused by an excitation of the nucleon $N^*(1535)$ resonance [5, 14]. At present there are no experimental data pointing to the presence of strong coupling of the $K^+$–meson to any nucleon resonance in the resonance mass region of $1.2 – 2.0 \, GeV$. Therefore one can suppose that final state interaction in the $pd \rightarrow ^3H_\Lambda K^+$ reaction will not be of great importance in contrast to the $\eta$–production.
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Figure 1:

**Figure captions**

Fig.1 The one-step (a) and two-step (b) mechanisms of the $pd \rightarrow ^3 H\Lambda K^+$ reaction.

Fig.2. The differential cross section of the $pd \rightarrow ^3 H\Lambda K^+$ reaction calculated for the two-step mechanism as a function of incident proton kinetic energy at different angles of $K^+$-meson $\theta_{c.m.} = 0^\circ, 90^\circ, 180^\circ$

Fig.3. The same as in Fig.3 but for the one-step mechanism
Figure 2:
Figure 3: