Behavior of Hadrons at Finite Density
– Lattice Study of Color SU(2) QCD –

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Abstract

Using two-color lattice QCD with Wilson fermions, we report a study of the finite baryon number density system with two-flavors. First we investigate the Polyakov line and thermodynamical quantities in the \((\kappa, \mu)\) plane, where \(\kappa\) and \(\mu\) are the hopping parameter and chemical potential in the fermion action, respectively. Then we calculate propagators of meson \((q\Gamma q)\) and baryon \((q\Gamma q)\) states. We find that the vector meson propagators are strongly modified in large \(\mu\) regions, indicating the reduction of the mass. This anomalous behavior of the vector meson is observed for the first time in lattice QCD.

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1 Introduction

QCD has confinement and deconfinement phases when the temperature, \(T\), is varied. When an additional parameter, the density or the chemical potential, is added, QCD may have a much richer structure [1, 2]. Experimentally, a wide region of the \((T, \mu)\) plane has been investigated by AGS, SPS and RHIC, and a higher density realm might be covered by GSI and JHF in the future. There have been experiments which suggest that vector meson masses are modified at finite density [3, 4].

Lattice QCD is expected to provide nonperturbative information of the hadronic world at finite density as the first principle calculation [5]. However, numerical study of lattice QCD with chemical potential is extremely difficult, because at finite \(\mu\), the fermion matrix does not satisfy the usual condition,

\[ D^\dagger = \gamma_5 D \gamma_5, \] (1)
and the fermion determinant $\det D$ becomes complex, which appears in the Euclidean path integral measure,

$$Z = \int \mathcal{D}U \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{-\beta S_G - \bar{\psi} D \psi} = \int \mathcal{D}U \det D e^{-\beta S_G}.$$  \hspace{1cm} (2)

In order to circumvent the above difficulty, two-color QCD has been investigated \cite{6, 7, 8, 9, 10, 11, 12}. Although recent progress of lattice calculations with the finite chemical potential has been prominent \cite{13, 14, 15, 16, 17}, it is still very difficult to study regions around critical $\mu$ at low temperature by lattice QCD simulations. We report here, for the first time, a lattice study of hadron propagators together with thermodynamical quantities for finite density SU(2) QCD with Wilson fermions. Two-flavor case is studied.

The two-color SU(2) theory qualitatively has most of the important features of QCD, such as deconfinement transition at finite temperature. The essential difference between two- and three-color systems is that in the SU(2) case baryons are made of two quarks, i.e., they are bosons. Therefore we must be careful in the study of two-color QCD in order to gain some insight into a subset of the phenomena expected in QCD. It is indispensable to compare results for mesons and diquarks which have a special relation in the SU(2) case. Recent theoretical progress has greatly improved our understanding of the two-color QCD system \cite{18, 19, 20, 21}.

## 2 Actions and algorithm

We introduce the chemical potential in the conventional manner,

$$D(x, x') = \delta_{x,x'} - \kappa \sum_{i=1}^{3} \left\{ (1 - \gamma_i)U_i(x)\delta_{x',x+i} + (1 + \gamma_i)U_i^\dagger(x')\delta_{x',x-i} \right\}$$

$$- \kappa \left\{ e^{+\mu}(1 - \gamma_4)U_4(x)\delta_{x',x+4} + e^{-\mu}(1 + \gamma_4)U_4^\dagger(x')\delta_{x',x-4} \right\}.$$  \hspace{1cm} (3)

For the gauge action, we employ the plaquette and Iwasaki improved actions. Here, we report the improved action case only, and for analyses with the plaquette gauge action, we refer to Ref. \cite{22}. Little is known about dynamical fermion simulations in which the chemical potential is introduced. We therefore employ an algorithm where the ratio of the determinant,

$$\frac{\det D(U + \Delta U)}{\det D(U)} = \det(I + D(U)^{-1} \Delta D)$$  \hspace{1cm} (4)

is evaluated explicitly in each Metropolis update process, $U \rightarrow U + \Delta U$, where $\Delta D \equiv D(U + \Delta U) - D(U)$. For details of the algorithm, see Refs. \cite{23, 24}. This algorithm has a long Markov step and is very reliable \cite{25}. Numerical costs are, however, huge and we are restricted to small lattices. In the following studies, therefore, we check that the results obtained are not sensitive to the boundary conditions.
3 Study of the $(\kappa, \mu)$ Parameter Space

Since there are few color $SU(2)$ lattice studies using Wilson fermions with finite $\mu$ in the literature except Ref. [6], we first investigate the relevant parameter space. We measure the Polyakov line, $\langle L \rangle$ on a $4^4$ lattice by changing $\beta$ for $\mu = 0$ and $\kappa = 0.150$ and choose the region where $\langle L \rangle$ is small, i.e., the system is in the confinement phase at zero baryon number density. We set $\beta = 0.7$ on the basis of this analysis.

At this value of $\beta$, we measure $\langle L \rangle$, its susceptibility, $\frac{\partial \langle L \rangle}{\partial \mu}$, the gluon energy density, $\langle E_g \rangle = < \frac{1}{V} \frac{\partial}{\partial (1/T)} S_G >$ and the number density, $\langle n \rangle = \frac{T}{V} \frac{\partial}{\partial n} \log Z$, as a function of $\mu$ and $\kappa$. Here $S_G$ is the gauge action and $V = N_x N_y N_z$ is the spatial volume of the lattice. In Fig.1, we show $\langle L \rangle$, $\langle n \rangle$ and $\langle E_g \rangle$ on a $4^4$ lattice as a function of $\mu$ and $\kappa$. They increase as $\mu$ becomes large and show the deconfinement behavior. We observe that the simulation always breaks down when we increase $\mu$ further.

![Figure 1](image)

Figure 1: The Polyakov line $\langle L \rangle$, the number density $\langle n \rangle$ and the gluon energy $\langle E_g \rangle$ as a function of $\kappa$ and $\mu$. Lattice size is $4^4$.

In Fig.2, we show the Polyakov line susceptibilities $\kappa = 0.150$ and 0.175 under the antiperiodic spatial boundary condition and for $\kappa = 0.160$ under the periodic boundary condition. All exhibit a peak when $\mu$ increases, which indicates a deconfinement transition. All quantities support the picture that at large $\mu$ the system undergoes the transition from the confinement to the deconfinement phase. In addition to the increase of $\langle L \rangle$ and $\langle E_g \rangle$, the rapid increase of $\langle n \rangle$ is observed. The
instability at large $\mu$ may be an indication of a new phase with the diquark condensation.\footnote{Since our fermion action includes only bilinear terms of $\bar{\psi}$ and $\psi$ and not those of $\psi$ and $\bar{\psi}$, or $\bar{\psi}$ and $\bar{\psi}$, the diquark condensation cannot emerge.}

Although the behavior of all quantities supports the existence of the deconfinement phase at large $\mu$, there are some indications that suggest a more complicated phase. In many cases, we observe a second peak in the Polyakov line susceptibility at large $\mu$.

## 4 Hadron propagators

We calculate correlations of color singlet hadron operators, $M(x) = \bar{\psi}(x)\Gamma\psi(x)$ and $B(x) = \epsilon^{ab}\bar{\psi}(x)(CT)_{\alpha\beta}\hat{\tau}\psi_{\beta}(x)$, where $\Gamma$ is the product of Dirac matrices and $\hat{\tau}$ is a Pauli matrix acting on flavor indices. $C$ is the charge conjugation matrix and $a$ and $b$ are color indices. We set $\hat{\tau} = \tau_2$ or $\tau_2\vec{\tau}$ so that the wave function is totally antisymmetric.

To our knowledge, no study of the behaviors of hadrons including vector mesons at finite baryon density has been performed using lattice QCD. Vector mesons are important since they provide information at several stages of heavy ion collision in the form of lepton pairs.

In Fig. 3, we show propagators of the pseudoscalar and vector mesons and those of the scalar ($\Gamma = \gamma_5$) and pseudovector ($\gamma_\nu$) baryons for $\mu = 0$ and $\mu a = 0.8$, where $a$ is the lattice spacing. At $\mu = 0$, the propagator of pseudoscalar mesons is equivalent to that of scalar baryons, and that of the vector meson is equivalent to that of the pseudovector baryon. We see that this relation is satisfied in the numerical calculation. The most prominent feature here is that the vector meson propagator is strongly modified at $\mu a = 0.8$. Its slope is more gradual than that for the pseudoscalar, i.e., the vector meson becomes lighter.

The lattice size of $4^3 \times 8$ is too small to extract information of the mass pole.
Nevertheless, we fit the data to clarify the chemical potential effect qualitatively, and in Fig. 4, we plot the scalar and vector meson masses together with those of corresponding baryons, as a function of the chemical potential. We find that the vector meson mass drops as \( \mu \) reaches the critical region. In order to confirm this unexpected result, we calculate both periodic and antiperiodic boundary conditions and several \( \kappa \)'s (\( \kappa = 0.150, 0.160, 0.175 \)); the reduction of vector meson mass is always observed. Although our lattice size is too small and the statistics are insufficient for extrapolation to the chiral limit, the signal of the anomalous behavior of the vector channel is clearly seen.

At \( \mu = 0 \), because of the QCD inequality \cite{19}, the lightest mass should be in the pseudoscalar channel. The inequality holds under two assumptions, i.e., (i) there is no disconnected diagram and (ii) Eq. (1) is valid. The second condition is not satisfied for the finite density state. Indeed, there are several conjectures in the literature. Brown and Rho first proposed the scaling law \( m^*/m = f^*_\pi/f_\pi \) to explain the large low mass lepton pair enhancement observed in CERES \cite{3}, where \( m^* \) and \( f^*_\pi \) are the mass and the pion decay constant in the medium \cite{26}. Based on the QCD sum rule, Hatsuda and Lee predicted a decrease of \( \rho \) meson mass as a function of \( \mu \) \cite{27}. Harada et al. showed that vector meson mass vanishes at the critical density as a consequence of an effective theory with hidden local symmetry \cite{28}. Yokokawa et al. proposed simultaneous softening of \( \sigma \) and \( \rho \) mesons associated with the chiral restoration \cite{29}. If the sudden drop of the vector meson mass is not a special feature of the color SU(2) model, this may be the first lattice QCD result to show the reduction of the vector meson mass in the medium.

We do not observe any special feature in baryon (diquark) channels. We fit the baryon propagators as

\[
G(t) = C_1 e^{-(m-2\mu)t} + C_2 e^{-(m+2\mu)(N_t-t)}, \tag{5}
\]

where \( C_1 \) and \( C_2 \) are not independent because of the boundary condition, \( G(0) = G(N_t) \). At \( \mu > 0 \), the pseudovector baryon propagator is not equivalent to the vector meson, but their masses are similar.
Figure 4: Meson mass (left) and diquark mass (right) as a function of \( \mu \) at \( \kappa = 0.160 \) with the periodic boundary condition.

5 Concluding remarks

In this study, we have investigated the Polyakov line and its susceptibility together with thermodynamical quantities for Wilson fermions in \((\kappa, \mu)\) parameter space. Since there has been no lattice study using Wilson fermions and improved gauge action for the two-color finite density system, this step is necessary in order to understand in which region hadron propagators in the medium change their behavior.

We observed a sudden reduction of the vector meson mass when the chemical potential was increased. If this is due to the mixing of meson and diquark states, the mass of diquark partner, \( \psi_c \gamma_\mu \psi \), should increase, but this is not the case. Rather, \( \psi_c \gamma_\mu \psi \) behaves in a similar way to the vector. As an urgent project, we will perform simulations on larger lattices, which will allow us to perform the chiral limit extrapolation to estimate the physical scale.

Our lattice here is small, but results for the case of periodic and antiperiodic boundary conditions in the spatial directions show the same qualitative behavior.

The behavior of the thermodynamic quantities together with that of the Polyakov line supports the standard picture, i.e., QCD undergoes a transition from the confinement to the deconfinement phase. We have observed several indications in the susceptibility and gluon propagators, which may suggest a more complicated phase at finite baryon number density. The number density may play an essential role in confirming the situation; it should increase from \( \mu = M_B/N_c \), where \( M_B \) is the lightest baryon mass and \( N_c \) is the number of colors. At the deconfinement transition point, \( \mu_c \), particle which bears the baryonic charge is changed from hadrons to quarks, therefore the mass of particles corresponding to the chemical potential changes. This results in the jump of the number density at \( \mu_c \). In the following
paper, we will perform detailed study of \( \langle n \rangle \) and its susceptibility.

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