Chiral Surface States in the Bulk Quantum Hall Effect

Leon Balents and Matthew P. A. Fisher
Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106-4030

(October 19, 2018)

In layered samples which exhibit a bulk quantum Hall effect (QHE), a two-dimensional (2d) surface “sheath” of gapless excitations is expected. These excitations comprise a novel 2d chiral quantum liquid which should dominate the low temperature transport along the field (z-axis). For the integer QHE, we show that localization effects are completely absent in the “sheath”, giving a metallic z-axis conductivity. For fractional filling $\nu = 1/3$, the “sheath” is a 2d non-Fermi liquid, with incoherent z-axis transport and $\sigma_{zz} \sim T^3$. Experimental implications for the Bechgaard salts are discussed.

PACS numbers: 73.40.Hm, 75.30.Fv, 71.27.+a

Disorder has a profound effect on transport in two-dimensional (2d) electron systems. In the absence of an applied magnetic field, all the electronic states are believed to be localized due to strong quantum interference effects. With weak disorder, when the conductivity $\sigma$ is larger than $e^2/h$, there is a weak-localization regime described by diffusive behavior with logarithmic temperature corrections. In the low temperature limit, however, a crossover to strongly localized behavior with $\sigma \ll e^2/h$ is always expected. In this regime the conductivity drops rapidly with temperature and vanishes as $T \to 0$.

In the presence of a magnetic field, 2d localization can be circumvented by tuning to the center of Landau levels. At these isolated transitions between quantum Hall plateaus, extended states and a temperature independent conductivity of order $e^2/h$ are predicted, consistent with experiment. There are also other 2d systems which exhibit such “metallic” behavior at isolated transitions, for example 2d films at the superconductor-insulator transition. But away from such transitions, when the conductivity is well below $e^2/h$, it is invariably strongly temperature dependent, and insulating at zero temperature.

In this paper, we describe a novel class of anisotropic 2d electronic phases, which surprisingly can exhibit “metallic” conductivities much smaller than $e^2/h$. These 2d phases arise at the surface of bulk three-dimensional quantum Hall samples. We consider layered samples which exhibit independent quantum Hall states in each layer when a large perpendicular magnetic field is applied. This requires an inter-layer tunneling amplitude $t$ small compared to the 2d quantum Hall gap $E_g$. In such materials, the surface of the sample is enveloped by a sheath of current-carrying states as depicted in Figure 1. This chiral surface phase is the 2d analog of the 1d states at the edges of a single layer quantum Hall fluid. There are currently two candidate experimental realizations of such systems: 2d electron gas multi-layers, and the Bechgaard salts, (TMTSF)$_2$X (where TMTSF is tetramethyltetraselenafulvalene, and X = PF$_6$, ClO$_4$, or ReO$_4$), which exhibit simultaneous field-induced spin density waves (SDWs) and QHE.

Electronic transport parallel to the field direction is a powerful probe of these chiral surface phases. Our analysis reveals that for the integer quantum Hall effect (IQHE), the motion along the z-direction is always diffusive, independent of the impurity scattering strength. Localization effects - weak or otherwise - are completely absent. The 2d resistivity $\rho_{zz}$ is temperature independent as $T \to 0$, with a value which can be much larger than $h/e^2$. But perpendicular to the field, along the x-axis, the transport is ballistic and the resistivity $\rho_{xx}$ vanishes as $T \to 0$. Our predictions for various experimental quantities which should be accessible in the Bechgaard salts, are given in detail at the end of the paper.

The surface sheath in the fractional quantum Hall effect at filling $\nu = 1/3$ is even more anisotropic. In this case, we find that electron transport along the z-axis is always incoherent. At low temperatures insulating behavior is predicted with a resistivity diverging as $\rho_{zz} \sim 1/T^3$, even in the absence of any impurity scattering. At $T = 0$ inter-layer transport is completely absent, and the electrons are “confined” to 1d. This chiral surface sheath for $\nu = 1/3$ is a nice example of a 2d non-Fermi liquid phase.

FIG. 1. (a): Geometry of a 3d quantum Hall sample. z-axis transport is included via the tunneling amplitude $t$, and impurity scattering by the random potential $V$. (b): Associated Fermi surface for the sheath. It differs from that of a conventional open Fermi-surface metal by the absence of the left-moving (dotted) half.

Consider first the IQHE. The electronic states at the edge can be modeled simply as chiral Fermions. In the 3d case of interest here, there is a single edge state per layer ($x$-$y$ plane) per filled Landau level (see Fig. 1). We focus on the case $\nu = 1$; higher integer filling fractions behave similarly. Including a small inter-layer matrix element $t$ leads to the non-interacting Hamiltonian

$$H = \sum_i \int dx [v_\psi \hat{\psi}^\dagger \partial_x \hat{\psi} - t \left( \hat{\psi}^\dagger_i \hat{\psi}^{\dagger}_{i+1} + \hat{\psi}^{\dagger}_{i+1} \hat{\psi}_i \right)]$$

$$= \int_p \left( v_p x_p - 2t \cos p_z \right) \psi_p^\dagger (p_x, p_z) \psi_p (p_x, p_z),$$

where $\int_p \equiv \int_{-\infty}^{+\infty} dp_x/(2\pi) \int_0^{\pi} dp_z/(2\pi)$ and $\psi^\dagger$ is an electron creation operator for the edge state in the $i^{th}$ layer. Here we have set the layer spacing $a$ equal to one. Eq. 2
shows that inter-layer hopping induces a small dispersion along the layering direction, resulting in “half” of an open Fermi surface (see Fig. 1b).

Before including impurity scattering, we re-examine the validity of Fermi-liquid theory (FL) with electron-electron interactions present. Despite the chiral nature of the sheath, the phase-space restrictions which stabilize the FL for a conventional Fermi-surface continue to apply here. In particular, the quasi-particle decay rate \( \text{Im}(\Sigma(\omega)) \sim \omega^2 \ln \omega \), the d.c. conductivity \( \sigma_{zz} \sim 1/T^2 \), and the specific heat \( C \sim T/(\alpha v) \). Long-range Coulomb interactions are screened in the static \( (\omega \to 0) \) limit.

To study d.c. transport at low temperatures, we must include impurity scattering at the surface, which we include by adding to the Hamiltonian a term of the form,

\[
H_{\text{imp}} = \sum_i \int dx V(x, i\alpha) \psi_i^L \psi_i. \tag{3}
\]

For simplicity, we assume the random potential to be Gaussian and uncorrelated, \( V(x, z) V_j(x', z') = \delta(x - x') \delta(z, z') \). In contrast to the results for the pure system, the chiral nature of the dirty sheath leads to dramatic differences from ordinary dirty 2d metallic behavior. In particular, localization along the x-direction is precluded \( \text{a priori} \) since the wavefunctions \( \Phi \) are necessarily de-localized along the z-direction. This follows from the continuity equation \( \partial_t |\Phi|^2 = \nabla_z \cdot J \), where \( J(x, z) = (2t/v) \text{Im}(\Phi^* (x, z) \Phi(x, z + a)) \), valid for arbitrary random potential \( V \). To study possible localization effects along the z-axis, we consider the usual averaged product of advanced and retarded Green’s functions for non-interacting electrons,

\[
D(r; \Omega) = G_{+}(0, r; E) G_{-}(0, r; E + \Omega). \tag{4}
\]

Here \( G_{\pm} = [i\nu \partial_x - t \nabla_z^2 - E \mp i\eta + V(r)]^{-1} \), with \( \nabla_z^2 \) the discrete Laplacian, and \( \eta \) a positive infinitesimal. A standard summation of ladder diagrams, which gives a diffusive form for non-chiral Fermions, yields the approximation

\[
D_{\text{ladder}}(p_z, p_z; \Omega) = \frac{2\pi \rho}{-i(\Omega + v p_z) + D p_z^2 + 2\eta}, \tag{5}
\]

where the density of states \( \rho = 1/2\pi v a \), and \( D = t^2 \tau a^2 \), with the relaxation time \( \tau = 2\nu/\Delta \). The anisotropic form of the denominator in Eq. \( 5 \) reflects the difference in propagation along x (ballistic) and z (diffusive). The Einstein relation \( \sigma_{zz} = e^2 \rho D \) can be verified by explicit computation. The derivation of the diffusive transverse motion in Eq. \( 5 \) has neglected possibly important quantum interference corrections. Indeed, for isotropic 2d systems, such corrections invalidate diffusive behavior at low temperatures, due to localization. To study possible interference effects here, we use the \( Q \)-matrix approach, which allows a systematic treatment of corrections to the simple ladder result in Eq. 5. Following McKane and Stone \( 8 \), averaged correlation functions are obtained from the replicated partition function \( Z = \int [d\phi][d\phi][dQ] \exp(-S) \), where

\[
S = \sum_i \int dx \left\{ i\phi \Lambda \left[ iv \partial_x - t \nabla_z^2 - E - \Delta Q \right] \phi + \frac{\Delta}{2} \text{Tr} Q^2 + \eta\phi \right\}. \tag{6}
\]

Here \( \phi \) is a 2n-component complex vector, \( Q \) is a 2n by 2n hermitian matrix, and \( \Lambda = 1 \otimes \sigma_z \). Eq. 5 follows from introducing \( \phi \) fields to generate retarded and advanced Green’s functions, averaging the \( n \)th moment of the original generating function \( Z_0 \), and decoupling the resulting quartic interaction using the \( Q \) matrix. For \( \eta = 0 \), this action has a non-compact \( O(n, n) \) symmetry, reflecting the physical equivalence of replicas. In many respects, \( \eta \) acts as a symmetry breaking field, but is also essential to obtain convergence of the functional integral. Physical correlators, which are derived from the logarithm of \( Z_0 \) (\( \ln Z_0 = \lim_{n \to 0} \ln(Z_0^n - 1)/n \)), must be computed in the limit \( n \to 0 \). Writing \( Q = \left( Q_{++} \ Q_{+-} \right) \left( Q_{-+} \ Q_{--} \right) \), the density of states is, e.g., \( \rho = (1/\pi) \text{Im}(Q_{++\alpha\alpha}) \), where no sum is implied over the (arbitrary) replica index \( 1 \leq \alpha \leq n \). Up to non-singular contact terms, the correlator \( D(r; 0) = \left( Q_{++\alpha\alpha}(r)Q_{-+\alpha\alpha}(0) \right) \).

The ladder approximation is recovered as the leading term in a saddle point expansion of the effective action for \( Q \), obtained by integrating out the bosonic fields. At the mean-field level, \( Q_{++} = -Q_{--} = i/(2\nu a) \), reflecting the finite density of states.

Expanding \( Q = Q + \delta Q \), the \( Q_{++} \) and \( Q_{--} \) fields have only massive fluctuations around the saddle, but the \( Q_{+-} \) and \( Q_{-+} \) fields are massless Goldstone bosons for \( \eta = 0 \). Following the conventional non-linear sigma model approach, we ignore the massive fluctuations and focus on the Goldstone modes, staying within the saddle point manifold by eliminating \( Q_{++} \) and \( Q_{--} \) via the zero momentum, tree equations of motion. The resulting theory may be expressed solely in terms of the off-diagonal submatrices of \( Q \), and is governed by an action of the form

\[
S_{\text{eff}} = \frac{1}{2\pi \rho} \int \left\{ \left( -ivp_x + D p_x^2 \right) \text{Tr}[Q_{++}(p)Q_{--}(-p)] \right\}
+ \prod_{p_i} \int \Gamma(p_i) \text{Tr}[Q_{+-}Q_{--}Q_{++}Q_{-+}], \tag{7}
\]

where \( \Gamma(p) = \delta(\sum p_i) f(p_i) \), with \( f(p) \sim c_1 p_x + c_2 p_z^2 \) for small momenta, and all higher vertices must have similar momentum structure owing to the continuous \( O(n, n) \) symmetry. Truncating at quadratic order leads to the ladder result for \( D \), while retaining the anharmonic corrections is analogous to the usual expansion of the appropriate non-linear sigma model.

With this formulation, the stability of the “diffusive” metal can be evaluated from simple power counting. To make the quadratic action dimensionless, we rescale \( z \to b z, x \to b x, \) and \( Q \to b^{-1/2} Q \) (with \( b > 1 \) to focus on long-wavelength behavior). Under this transformation the quartic coupling amplitude \( \Gamma_0 \to \Gamma_0/b \),
implying the irrelevance of anharmonic terms in $Q_{+-}$ and the stability of the metallic phase. This is in contrast to the usual case of a non-chiral 2d dirty conductor, in which the metallic fixed point is marginally unstable. Physically, the absence of localization effects may be attributed to the ballistic motion along the $x$ axis, which suppresses multiple scattering. Although it is possible that strong enough disorder could induce some kind of transverse localization, we have been unable to find evidence of such a phase, and suspect that metallic behavior persists for all impurity concentrations.

In other 2d electron models which exhibit diffusion at $T = 0$, the de-localized wavefunctions typically exhibit multi-fractal scaling [2]. A classic example is the IQHE plateau transition, which manifests multi-fractal behavior via anomalous scaling of $D(p, \Omega)$, in the limit $Dp^2 < \Omega$. But in the present case $D(p, \Omega)$ scales trivially with $\Omega$ and $p_2$, implying that the wavefunctions are not multi-fractal.

Having established the stability of the metallic phase in the non-interacting case, we turn to the combined influence of interactions and disorder. As shown by Altschuler and Aronov (see Ref. [3] and Refs. therein), interaction effects are much enhanced by diffusive motion in conventional metals. For example, diffusive relaxation of density fluctuations leads to singularities in the tunneling density of states. For our semi-ballistic metal, one expects this effect to be absent, since any charge buildup is swept away at the chiral Fermi velocity. Indeed, one can show that the diagrams responsible for the singularity yield only smooth contributions in the chiral case.

The $z$-axis transport for the surface sheath in the IQHE behaves much as a conventional metal. Dramatically different results are obtained for fractional filling factors, as we now describe. We focus on the odd-denominator fractional states, particularly $\nu = 1/3$. The edge states are then chiral Luttinger liquids of charge $\nu$ Laughlin quasiparticles [2]. Because these quasiparticles have integrity only within a single quantum Hall plane, inter-layer transport must involve the tunneling of physical electrons, or equivalently 1/$\nu$ quasiparticles. The surface “sheath” can be described using the bosonized Euclidean action

$$S = \sum_i \int_{-\infty}^{\infty} dx \int_0^\beta d\tau \left\{ \frac{1}{4\pi \nu} \partial_x \phi_i (i \partial_x \phi_i + v \partial_x \phi_i) - t \cos( (\phi_i - \phi_{i+1})/\nu ) \right\},$$

where $\phi$ is a boson field, and $\beta = 1/T$. The charge density is $n_i(x) = \partial_x \phi_i / (2\pi)$, and the operator $e^{i \phi(x)}$ creates a quasiparticle ($2\pi$ soliton in $\phi$) at position $x$. The cosine term describes inter-layer tunneling with amplitude $t$. In the ideal $H \parallel \hat{z}$ geometry, no flux penetrates between successive edge modes; oscillatory phase factors are therefore absent from this term. To study the effects of tunneling, we employ standard renormalization group methods perturbative in $t$. After introducing a cutoff $\Lambda$ in $q_x$, one integrates out modes with $\Lambda e^{-dt} < |q_x| < \Lambda$.

Upon rescaling $x \to e^{dt} x$ and imaginary time $\tau \to e^{dt} \tau$, the quadratic action is brought back to its original form. The tunneling amplitude is, however, renormalized

$$\frac{dt}{d\tau} = (2 - 1/\nu) t,$$

and is technically an irrelevant perturbation. To extract the $z$-axis conductivity imagine renormalizing until the rescaled temperature is of order the quantum Hall gap $E_g$, which implies a rescaling factor $e^{t} = E_g/T$. From scaling one then obtains an inter-layer tunneling time $\tau_{in} \sim \tau_0 |E_g/t(\ell)|^2$, with $t(\ell) \sim (T/E_g)^{1/\nu - 2}$. Here $\tau_0 \sim 1/T$ is a thermal de-phasing time. Since $t(\ell)$ scales to zero at low temperatures, $\tau_{in} >> \tau_0$ and the inter-layer transport is incoherent. The $z$-axis conductivity can then be obtained from the diffusion constant $D \sim a^2/\tau_{in}$ and the Einstein relation $\sigma_{zz} \sim \epsilon^2 \rho D$. This gives $\sigma_{zz} \sim (e^2/\hbar)(a/v \tau_{in}) = (const)T^{2/\nu - 3}$, which vanishes as $T \to 0$. We now summarize our predictions for transport and thermodynamic quantities. For temperatures well below the bulk QHE gap, the surface sheath dominates the $z$-axis transport. In this regime, for $\nu = 1$, a temperature independent (sheet) conductivity is predicted: $\sigma_{zz}(\nu = 1) \sim (e^2/h)(a/v \tau_{in}) = (const)T^{2/\nu - 3}$. Each additional full Landau level will give another surface “sheath”, which contributes additively to the sheet conductance, so $\sigma_{zz}(\nu = N) = N \sigma_{zz}(1)$ (see Fig. 2). This behavior is roughly consistent with recent data on (TMTSF)$_2$ClO$_4$ [4]. As discussed above, the $\nu = 1/3$ state, if present, has a vanishing $z$-axis conductivity, $\sigma_{zz} \sim T^3$. In contrast to the quantized Hall plateaus, the “plateaus” in $\sigma_{zz}$ are unquantized, and will probably exhibit small non-vanishing slopes even at the lowest temperatures, as depicted in the Figure. However, the “step-like” features, which arise from the 3d phase transitions between successive bulk Hall phases, can be sharp at $T = 0$. The low temperature behavior of the “steps” depends upon the order of the bulk phase transition. In (TMTSF)$_2$X, these transitions might be first order, with the SDW period changing discontinuously. In this case, we expect a discontinuous jump in $\sigma_{zz}$. However, experience with the usual QHE leads us to expect continuous critical behavior in 2d electron gas multi-layers; sufficient disorder may also drive the SDW transitions second order. In this case, the “steps” will exhibit critical singularities, as we now describe.

FIG. 2. Predicted behavior of $\sigma_{zz}$. We have indicated a small with $\delta B$, such as would be present at finite temperature in the case of an isotropic bulk critical point.

A continuous bulk plateau transition is worthy of study in its own right. Here we restrict the discussion to scaling properties which influence the surface sheath. Near the transition one expects an (in-plane) localization length which diverges as $\xi \sim |\delta B|^{\nu}$. Allowing for anisotropic scaling, the $z$-axis localization length is as-
sumed to vary as $\xi_z \sim \xi^\zeta$ (we expect $\zeta \leq 1$ on physical grounds), whereas characteristic energy scales vanish as $\omega \sim \xi^{-z}$ with $z$ a dynamical exponent. Following standard methods, we find that the bulk conductivity $\sigma_{\text{bulk}} \sim \xi^{-2\zeta(T/\xi^2)}$. This form connects activated behavior away from the transition (with energy scale $T_0 \sim E_g(\ell_B/\xi)^2$, where $\ell_B$ is the magnetic length) to the power-law temperature dependence $\sigma_{\text{bulk}} \sim T^{(2-\zeta)/z}$ at the critical point. As an aside, we point out that the surface sheath develops a thickness $\xi$ at the critical point. As an aside, we point out that $\sigma_{zz} \sim T^{\zeta/2} \to 0$ at the 3d critical point, in contrast with the prediction of a universal finite value in 2d.

Upon approaching such a continuous bulk transition, the surface sheath develops a thickness $\xi \gg \ell_B$, and we may estimate the z-axis sheet conductivity as $\sigma_{zz} \sim \sigma_{\text{bulk}}^z \xi$. Using the bulk scaling results, this implies singular behavior at zero temperature for the “steps” in $\sigma_{zz}$:

$$\sigma_{zz} \sim \xi^{\zeta-1} \sim |\delta B|^{(1-\zeta)\nu}. \quad (10)$$

Experience with other similar disordered systems suggests that the critical behavior may be isotropic, so that $\zeta = 1$. If so, we again find a jump in $\sigma_{zz}$. A jump due to an isotropic but continuous bulk transition can be differentiated from a bulk first order transition by its behavior at finite temperature: in the former case, the step develops a width $\delta B \sim T^{1/(z\nu)}$, while for a first order transition the discontinuity should be present even at $T \neq 0$.

The SDW compounds provide a unique opportunity to study the specific heat of quantum Hall edge modes, in this case those of the whole sheath. In 2d electron gases, the edge contribution to $C$ is masked by that of the localized states in the interior. While this should also be the case in multi-layers, the SDW gap implies that the bulk contribution to $C$ is activated at low temperatures. One can then measure the specific heat from the gapless surface modes. They contribute a linear temperature dependence to the specific heat per area: $C_A = N(xk_B^2T/6h\nu)$. Since the ratio $C/T$ is linear in the number $N$ of full Landau levels, one expects “steps” in this quantity as well. Recent specific heat data in the ClO$_4$ Bechgaard salt appears roughly consistent with this variation. The evidence for a latent heat peak near the transitions is less clear.

Finally, we consider the effects of a transverse magnetic field ($B_y \neq 0$) on the integer surface sheath, induced (albeit non-uniformly) by tilting the sample. In the gauge $A_x = -B_y z$, this field shifts the relative Fermi momenta between edge modes in adjacent layers by $\delta k_F = e\Delta B_y$. In the absence of impurities, momentum conserving interlayer tunneling is not possible below an energy scale, $\Delta_B \sim e\Delta B_y$. This leads to activated z-axis transport, $\sigma_{zz} \sim \sigma_{zz,0} \exp(-\Delta_B/T)$, characteristic of decoupled 1d conductors. With impurity scattering, a finite conductivity will be restored at $T = 0$. But for a relatively clean surface, a positive magneto-resistance for z-axis transport is expected.

We are grateful to P. Chaikin for helpful conversations. This work has been supported by the National Science

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[1] See, e.g. P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. 57, 287 (1985).
[2] B. Huckestein, Rev. Mod. Phys. 67, 357 (1995).
[3] For recent experiments, see, e.g. L. Balicas, G. Kriza, and F. I. B. Williams, Phys. Rev. Lett. 75, 2000 (1995);
S. M. McKernan, S. T. Hannahs, E. M. Scheven, G. M. Danner, and P. M. Chaikin, Phys. Rev. Lett. 75, 1630
(1995); and references therein.
[4] For theoretical background, see, e.g., L. P. Gor’kov and A. G. Lebed’, J. de Phys. Lett., 45, L433 (1994);
D. Poilblanc, G. Montambaux, M. Hérétier, and P. Ledrer, Phys. Rev. Lett. 58, 270 (1987);
K. Machida, Y. Hasegawa, V. M. Yakovenko, Y. Horii, and M. Kishig, Phys. Rev. B 50, 921 (1994); and references therein.
[5] B. I. Halperin, Phys. Rev. B 25, 2185 (1982).
[6] A. J. McKeon and M. Stone, Annals of Phys. 131, 36 (1981).
[7] X. G. Wen, Phys. Rev. B 43, 11025 (1991); Phys. Rev. Lett. 64, 2206 (1990); Phys. Rev. B 44, 5708 (1991).
[8] See, e.g., C. L. Kane and M. P. A. Fisher, Phys. Rev. B 51, 13449 (1995).
[9] P. Chaikin, private communication.
[10] M. P. A. Fisher, P. B. Weichman, G. Grinstein, and D. S. Fisher, Phys. Rev. B 40, 546 (1989).
[11] J. D. Reeger, T. A. Tokuyasu, A. P. Young, and M. P. A. Fisher, Phys. Rev. B 44, 7147 (1991).
[12] U. M. Scheven, E. I. Chashechkina, E. Lee, and P. M. Chaikin, Phys. Rev. B 52, 3484 (1995).