SO(3) vs. SU(2) Yang-Mills theory on the lattice: an investigation at non-zero temperature *

A. Barresi

Dipartimento di Fisica, Università di Pisa e I.N.F.N. Sezione di Pisa, Via Buonarroti 2, 56127 Pisa, Italy
E-mail: barresi@df.unipi.it

G. Burgio
School of Mathematics, Trinity College, Dublin 2, Ireland
E-mail: burgio@maths.tcd.ie

M. Müller-Preussker
Humboldt-Universität zu Berlin, Institut für Physik, Newtonstr. 15, 12489 Berlin, Germany
E-mail: mmp@physik.hu-berlin.de

(Dated: November 30, 2021)

Abstract

The adjoint $SU(2)$ lattice gauge theory in 3+1 dimensions with the Wilson plaquette action modified by a $\mathbb{Z}_2$ monopole suppression term is reinvestigated with special emphasis on the existence of a finite-temperature phase transition decoupling from the well-known bulk transitions.

* Based on contributions by G. Burgio and M. Müller-Preussker at CONFINEMENT 2003, RIKEN, Tokyo and LATTICE 2003, Tsukuba, Japan
I. INTRODUCTION AND MOTIVATION

The evidence and our detailed understanding of the deconfinement phase transition in $SU(N)$ gauge theories at finite temperature mainly comes from lattice gauge theories (LGT) formulated in the fundamental representation \cite{1,2}. For pure LGT the transition is associated with the spontaneous breaking of the global center $\mathbb{Z}_N$ symmetry \cite{3,4}:

$$U_4(\vec{x}, x_4) \rightarrow z \cdot U_4(\vec{x}, x_4), \quad z \in \mathbb{Z}_N \quad \text{for all } \vec{x} \text{ at } x_4 = \text{fixed},$$

which leaves the lattice gauge action invariant but flips the Polyakov loop variables

$$L_F(\vec{x}) = \frac{1}{N_c} \text{Tr}_F \prod_{x_4=1}^{N_\tau} U_4(\vec{x}, x_4)$$ \hspace{1cm} (1)

as $L_F \leftrightarrow z L_F$. As a consequence the standard order parameter for the deconfinement transition is defined as

$$\langle |L_F| \rangle = \left\langle \left| \frac{1}{N^3} \sum_{\vec{x}} L_F(\vec{x}) \right| \right\rangle,$$ \hspace{1cm} (2)

where the ensemble average is taken with the Boltzmann distribution represented by the lattice-discretized path integral with periodic boundary conditions for the gauge fields in the imaginary time direction $x_4$. The above mentioned global symmetry breaking mechanism provides a close analogy to spin models. In particular, the universality class of $SU(2)$ LGT is that of the 3d Ising model \cite{5}. On the other hand the origin of quark and gluon confinement as well as of the occurrence of the finite-temperature phase transition has been seen in the condensation of topological excitations like Abelian monopoles \cite{6} and center vortices \cite{7}.

Lattice gauge theories can be formulated in different group representations of the gauge fields, e.g. in the center blind adjoint representation. In this case (extended) vortices and Abelian monopoles are still present, but the mechanism of spontaneous $\mathbb{Z}(N)$ breaking is obviously not realized. Moreover, the adjoint representation LGT’s at strong coupling are strongly affected by bulk phase transitions \cite{8,9} driven by lattice artifacts \cite{10}. A finite temperature transition - if it exists - seems to be completely overshadowed by these bulk transitions.

Therefore, the question of universality in particular of the existence of the finite temperature phase transition remains an important issue. If the existence of this transition turns out to be independent of the group representation, then the question remains whether the driving mechanism related to the condensation of topological excitations is the same.
In the past this principally important question has been studied by several groups mainly in the case of the mixed $SU(2) - SO(3) = SU(2)/\mathbb{Z}_N$ theory realized with the Villain action\textsuperscript{11–17}. Still we have not yet reached a completely satisfying answer. Nevertheless, over the last years there has been an interesting progress\textsuperscript{18–21} worth to be reviewed at this conference.

In the following Section 2 we shall shortly review $SU(2)$ lattice gauge theories with different mixed fundamental-adjoint actions. In Section 3 we introduce the center-blind model we have further investigated, i.e. the adjoint representation Wilson lattice action with a $\mathbb{Z}_2$ monopole suppression term. In Section 4 we discuss the results of our investigations based on twist variables, the fundamental Polyakov loop distributions as well as on the Pisa disorder operator providing evidence for the existence of a distinct finite-temperature transition in the center-blind theory. Our conclusions are drawn and an outlook is given in Section 5.

II. $SU(2)$ LATTICE GAUGE THEORIES WITH MIXED FUNDAMENTAL-ADJOINT ACTION

Among the “first day” lattice gauge theory models were also those with a mixture of different group representations for the plaquette contribution, e.g. for $SU(2)$

- the Wilson-type mixed action \textsuperscript{8}

\begin{equation}
S = \beta_A \sum_P \left( 1 - \frac{1}{3} \text{Tr}_A U_P \right) + \beta_F \sum_P \left( 1 - \frac{1}{2} \text{Tr}_F U_P \right),
\end{equation}

\begin{equation}
\frac{1}{g^2} = \frac{\beta_F}{4} + 2 \frac{\beta_A}{3},
\end{equation}

- the Villain-type mixed action \textsuperscript{10}

\begin{equation}
S = \beta_V \sum_P \left( 1 - \frac{1}{2} \sigma_P \text{Tr}_F U_P \right) + \beta_F \sum_P \left( 1 - \frac{1}{2} \text{Tr}_F U_P \right),
\end{equation}

where $\sigma_P = \pm 1$ is an auxiliary dynamical $\mathbb{Z}_2$ plaquette variable.

The non-trivial phase structure with first order bulk transitions (see Fig. \textsuperscript{11}) is governed by lattice artifacts: $\mathbb{Z}_2$ magnetic monopoles and electric vortices the densities of which can be defined as follows ($N_c$ and $N_l$ being the number of 3-cubes and lattice links, respectively) \textsuperscript{11}

\begin{equation}
M = 1 - \left\langle \frac{1}{N_c} \sum c \rho_c \right\rangle, \quad \rho_c = \prod_{P \in \partial c} \sigma_P \quad \text{or} \quad \prod_{P \in \partial c} \text{sign}(\text{Tr}_F U_P)
\end{equation}
\[ E = 1 - \langle \frac{1}{N_l} \sum_l \rho_l \rangle, \quad \rho_l = \prod_{p \in \partial l} \sigma_p \quad \text{or} \quad \prod_{p \in \partial l} \text{sign}(\text{Tr}_F U_P). \quad (6) \]

These lattice excitations can be suppressed by modifying the action with suppression terms like \[ \lambda_V \sum_c (1 - \rho_c), \quad \gamma_V \sum_l (1 - \rho_l). \quad (7) \]

For the Villain-type action the equivalence between \( SO(3) \) and \( SU(2) \) has been proven in the limit of complete \( \mathbb{Z}(2) \) monopole suppression \( \lambda_V \rightarrow \infty \) for \( \beta_F = \gamma_V = 0 \) \cite{22,25,18} in the following form

\[
\sum_{\text{twist sectors}} Z_{SU(2)} \equiv A \sum_{\sigma_P = \pm 1} \int DU e^{\beta_V \sum_P \sigma_P \frac{1}{2} \text{Tr}_F U_P} \prod_c \delta(\prod_{p \in \partial c} \sigma_p - 1)
\]

where on the l.h.s. the twist sectors are imposed by twisted boundary conditions \( U_\nu(x + L_\mu) = z_{\mu\nu} U_\nu(x), \quad z_{\mu\nu} \in \pi_1[SU(2)/\mathbb{Z}_2] = \mathbb{Z}(2) \). On the r.h.s. the twist sectors are dynamically encountered, under circumstances separated by large barriers.

The case \( T \neq 0 \) has been mostly studied with the modified Villain action but always with a non-vanishing admixture of the fundamental representation \( (\beta_F \neq 0) \). Lines of a finite-\( T \) phase transition presumably of second order have been found in the \( \beta_V - \beta_F \) plane for \( \lambda_V \geq 1 \) and \( \gamma_V \geq 5 \) \cite{16}. Above the finite-\( T \) transition the adjoint Polyakov line \( \langle L_A \rangle \) has been seen trapped into metastable states \cite{14,15}.

\[
\langle L_A \rangle \rightarrow \begin{cases} 
1 & \text{as} \quad \beta_V \rightarrow \infty \\
\frac{1}{3} & \end{cases}
\quad (8)
\]
Jahn and de Forcrand related the negative $L_A$ states to non-trivial twists. For demonstrating this they introduced $SO(3)$ – i.e. center-blind – twist variables 
\[ z_{\mu\nu} \equiv \frac{1}{N_{\rho}N_{\sigma}} \sum_{P \in \mu,\nu-plane} \text{sign} \ (\text{Tr}_F U_P) \in [-1, +1], \quad \epsilon_{\mu\nu\rho\sigma} = 1. \tag{9} \]

The $z_{\mu\nu}$'s measure the $Z(2)$ fluxes through $\mu\nu$-planes. Then the state 
\[ L_F = 0 \iff L_A \equiv \frac{1}{3} \text{Tr}_A L = -\frac{1}{3}, \quad \text{Tr}_A L = (\text{Tr}_F L)^2 - 1 \tag{10} \]
is related to electric twist $z_{i,4} = -1, \quad i = 1, 2, 3$.

Having these observations in mind we are going now to check and to illustrate this scenario for a center-blind modified adjoint Wilson action. We ask how to establish a finite $T$ transition for the center-blind theory and what rôle do play the different twist sectors in this case.

### III. ADJOINT SU(2) MODEL WITH $Z_2$ MONPOLE SUPPRESSION

In our investigations we have considered the Wilson plaquette action with link variables $U_\mu(x) \in SU(2)$
\[ S = \beta_F \sum_P \left( 1 - \frac{1}{2} \text{Tr}_F U_P \right) + \beta_A \sum_P \frac{4}{3} \left( 1 - \frac{1}{4} (\text{Tr}_F U_P)^2 \right) + \lambda \sum_c (1 - \rho_c) \tag{11} \]
where $\rho_c = \prod_{P \in \partial c} \text{sign} \text{Tr}_F U_P$. For $\beta_F = 0$ the action $S$ becomes center-blind
\[ U_\mu(x) \to -1 \cdot U_\mu(x) \implies \rho_c \to \rho_c. \]

Fig. 2 shows the phase diagram in the $\beta_F - \beta_A$-plane for varying chemical potential $\lambda$ for $T = 0$. Obviously, the suppression of $Z_2$ monopoles ($\lambda > 0$) shifts the horizontal line down to smaller $\beta_A$-values. At a first glance the phase II seems to be disconnected from phase I (the ordinary confinement phase) in the range $0 \leq \lambda \leq 1$. But see the discussion further below.

If we put $\beta_F = 0$ the emerging $\beta_A$-$\lambda$ diagram looks as shown in Fig. 3. Phase I – which at $\beta_F \neq 0$ is connected with the ordinary confinement phase – is characterized by a non-zero $Z(2)$ monopole density and by twist variables fluctuating close to zero. On the contrary in phase II the monopoles become suppressed and the twist variables (meta)stable at $\pm 1$. The phase transition line has been established by studying the average plaquette, the adjoint
Polyakov loop variable $< L_\lambda >$, the average density $M$ of $\mathbb{Z}(2)$ monopoles and additionally the twist variables $\langle \bar{z} \rangle$ as well as their 'susceptibility'

$$\chi_{\text{twist}} = N_\sigma^3 \cdot (\langle \bar{z}^2 \rangle - \langle z \rangle^2)$$

with $\bar{z} \equiv \frac{1}{3}(|z_{xt}| + |z_{yt}| + |z_{zt}|)$.

We found out that the bulk transition line in a certain range $0 < \lambda < \lambda_c$ (with $\lambda_c \simeq 0.7$ for the lattice size $12^3 \times 4$) shows a discontinuous behaviour of the monopole density $M$, of the average plaquette as well as of the adjoint Polyakov loop across the transition. At the same time tunneling between different twist sectors becomes strongly suppressed when passing the border from phase I to phase II. Along this line as long as $\lambda < \lambda_c$ the twist sectors are clearly related to metastable states of the adjoint Polyakov loop. We interpret
the transition likely to be a first order transition in this range.

On the contrary, for \( \lambda_c < \lambda < 0.95 \) the monopole density \( M \) and the average plaquette turn out to behave smoothly across the bulk transition line. For the average adjoint Polyakov loop one finds \( < L_A > \simeq 0 \) on both sides of the line, i.e. phase II in this range seems to be also confining. This already is an indication that for larger \( \lambda \)-values and increasing \( \beta_A \) there should be a further (more or less horizontal) transition line hopefully behaving as a finite temperature transition. The ‘tricritical’ value \( \lambda_c \) seems to indicate the position, where this additional line might join the bulk transition. We come back to this question in the next section. The bulk transition itself in the range \( \lambda_c < \lambda < 0.95 \) is visible because of the enhanced tunneling between different twist sectors. This we have observed for all lattice sizes (so far up to \( V = 12^4 \)). The twist susceptibility (12) has a strong peak which increases with increasing lattice size (see Fig. 4). A rough finite-size scaling test shows the transition to resemble the 4D Ising one and, therefore, seems to be of second order.

![Adjoint Polyakov Loop](image)

**FIG. 4:** Electric twist histories at \( \beta_A = 0.65, \lambda = 0.858 \) for \( 12^4 \) (left) and twist susceptibility for \( \beta_A = 0.65 \) at varying \( \lambda \) and lattice sizes \( 8^4, 10^4 \) and \( 12^4 \) (right).

**IV. EVIDENCE FOR A FINITE-TEMPERATURE TRANSITION**

We observed that for sufficiently large chemical potential \( \lambda \geq 1.0 \) tunneling between twist sectors becomes completely suppressed. We decided to run the simulations in this range within fixed twist sectors (mostly the trivial one). That is, we suppress the generation or annihilation of extended vortices (up to numbers modulo two).
For fixed $\lambda$ we carried out measurements with varying $\beta_A$. The adjoint Polyakov loop average $\langle L_A \rangle$ as a function of $\beta_A$ in the zero twist sector is drawn in Fig. 5. Its rise clearly indicates a transition close to $\beta_A \simeq 1$. Frequency distributions of the local fundamental Polyakov loop variable $L_F(\vec{x})$ are plotted for two temporal lattice extensions in Fig. 6. One sees that the shape of the distributions clearly changes from a phase, where they peak at zero, to a phase, where they have two symmetric maxima away from zero. The critical $\beta_A$, where two maxima just occur, changes to larger values as $N_t$ is increasing from 4 to 6 in agreement with scaling. Therefore, we can conclude that a finite-temperature transition is really seen. In order to check the existence of the transition at finite $T$ with an independent measurement we have computed also the Pisa disorder parameter [26, 27] which on the basis of the dual superconductor model [6] allows to test for a condensate of Abelian monopoles in the confinement phase. The order operator for the condensation of magnetic charges $\mu(t)$ is defined by modifying the action at a given time-slice $t$ by a classical Dirac monopole field insertion $\Phi$ [26, 27]:

$$\mu(t) = \exp(-\beta \Delta S(t))$$

$$\Delta S(t) = \frac{1}{2} \sum_{i, \vec{x}} \text{Tr} \left[ U_{i4}(\vec{x}, t) - U_{i4}'(\vec{x}, t) \right]$$

$$U_{i4}'(\vec{x}, t) = U_i(\vec{x}, t) \Phi_i(\vec{x} + \hat{i}, \vec{y}) U_4(\vec{x} + \hat{i}, t) U^\dagger_i(\vec{x}, t + 1) U^\dagger_4(\vec{x}, t)$$
\[ \beta_A = 0.9 \quad \beta_A = 1.2 \quad \beta_A = 1.4 \]

FIG. 6: Frequency distributions of the local fundamental Polyakov loop variable \( L_F(\vec{x}) \). Upper row for lattice size \( V = 4 \times 16^3 \), lower row for \( V = 6 \times 16^3 \), all for \( \lambda = 1.0 \) and zero twist.

The operator can be generalized to the adjoint \((SO(3))\) action case. In the thermodynamic limit one expects

\[
\langle \mu \rangle \begin{cases} 
\neq 0 & T < T_c \\
= 0 & T > T_c 
\end{cases}
\]

In practice, one measures instead

\[
\rho = \frac{d}{d\beta} \log \langle \mu \rangle = -\langle S + \Delta S \rangle|_{S+\Delta S} + \langle S \rangle|_S. \quad (16)
\]

The experience for the \( SU(2) \) and \( SU(3) \) cases tells that the latter quantity exhibits a sharp dip signalling the deconfinement phase transition to be driven by the breaking of a dual magnetic symmetry.

Some of our results for the \( SO(3) \) case are collected in Fig. 7.

The computations show that for \( \lambda = 0.7 \) the bulk phase transition \( I \leftrightarrow II \) is correctly localized. For increasing \( \lambda \), e.g. at \( \lambda = 0.85 \), we see simultaneous, distinct signals for the bulk and the finite-\( T \) transitions. At even larger chemical potential \( (\lambda = 1.0) \) we find only a signal for the finite-\( T \) transition. The localizations of the bulk and finite temperature transitions agree reasonably with those obtained with the other methods mentioned before. The universality class of the transition to be determined e.g. via the critical indices has still to be investigated.

Having convinced ourselves that there is a finite temperature transition also in the completely center-blind adjoint \( SU(2) \) LGT we are tempted to ask whether there is a sponta-
neous breaking of an appropriate global symmetry. If yes, what is the corresponding order parameter?

For $SO(3)$ the question means, whether there is a globally defined operator flipping the adjoint Polyakov loop $L_A = 1 \iff L_A = -\frac{1}{3}$ while leaving the action invariant. There is an approximate solution based on the dual superconductor picture assuming that the long-distance properties are carried by Abelian degrees of freedom.

Let us consider flip operators with $P \in SU(2)$ or $SO(3)$ satisfying the conditions $P^2 = \pm 1, \quad P^\dagger = \pm P$. The only ones fulfilling these conditions are for

\[SU(2): \quad P = \pm \mathbb{I}_2, \quad \hat{P} = \pm \vec{n} \cdot \vec{\sigma},\]
\[SO(3): \quad P = +\mathbb{I}_3, \quad \hat{P} = \mathbb{I}_3 + 2(\vec{n} \cdot \vec{T})^2, \quad (n^2 = 1).\]

Let us assume the $SU(2)$ or $SO(3)$ fields to be Abelian (diagonal) with respect to a fixed ‘isospin’ $\vec{n}$-direction, then $\hat{P}$ applied to a given time sheet, indeed, leaves the action invariant, and the Polyakov loop is flipped to the other state (if its value is different from zero). In practice, this is realized only approximately by fixing the 3D maximally Abelian gauge (MAG) on each time slice $x_4$ separately. This is achieved by maximizing the gauge functional with respect to gauge transformations $g$, e.g. for $SU(2)$:

\[F_{x_4}(g) = \sum_\vec{x} \sum_{i=1,2,3} \text{Tr} \left( (\vec{n} \cdot \vec{\sigma}) U_i^\beta(\vec{x}, x_4) (\vec{n} \cdot \vec{\sigma}) \left( U_i^\beta(\vec{x}, x_4) \right)^\dagger \right). \quad (17)\]

The ‘order parameter’ is then defined as

\[\langle \Delta_{F,A} \rangle = c_{F,A} \langle |L_{F,A} - \bar{L}_{F,A}| \rangle, \quad (18)\]
where
\[
\bar{L}_{F,A} = \frac{1}{N^3} \sum_{\vec{x}} \text{Tr}_{F,A} \left( \hat{P} \prod_{x_4=1}^{N_4} U_4(\vec{x}, x_4) \right).
\] (19)

The ‘order parameter’ should work as for the standard $SU(2)$ case as well as for $SO(3)$. We have checked this. Indeed, for $SU(2)$ we obtained the results drawn in Fig. 8 for the order parameter itself as well as for its susceptibility. The susceptibility develops a peak around $\beta_F = \beta_c \approx 2.3$ as it should be. The corresponding Binder cumulant computed for varying 3-volume has been seen to provide intersecting lines at the same $\beta_F$ value. If one computes the corresponding ‘order parameter’ $\Delta$ for the adjoint $SO(3)$ action case at $\lambda = 1$. and in the zero-twist sector one gets a similar picture as the left one in Fig. 8.

V. CONCLUSIONS AND OUTLOOK

We summarize our studies by drawing the phase diagram for the modified adjoint $SU(2)$ theory at $T > 0$ as shown in Fig. 9. We see the existence of a finite temperature transition decoupling from the bulk transition at a position which scales in $\beta_A$ as one would expect. The finite temperature transition was also localized with the help of the Pisa disorder parameter indicating that the dual superconductor scenario seems to work also for $SO(3)$. Whether also the determination of the free energy of an extended center vortex (see references [28, 29]) would also point to the same result remains to be seen. Our numerical study presented here is in many respects still preliminary. Larger lattices, the use of an ergodic algorithm allowing
FIG. 9: Phase diagram of the center-blind $SO(3)$ model with $Z(2)$ monopole suppression term. The horizontal lines indicate the position of the finite temperature transition for varying time extension $N_t = 4, 6$.

to enhance tunneling between different twist sectors and the determination of critical indices in order to determine the universality class require more extensive computations, until we can draw final conclusions.

[1] L. D. McLerran and B. Svetitsky, Phys. Lett. B98, 195 (1981).
[2] J. Kuti, J. Polonyi and K. Szlachanyi, Phys. Lett. B98, 199 (1981).
[3] A.M. Polyakov, Phys. Lett. B72, 477 (1978).
[4] L. Susskind, Phys. Rev. D20, 2610 (1979).
[5] B. Svetitsky and L. G. Yaffe, Nucl. Phys. B210, 423 (1982).
[6] Y. Nambu, Phys. Rev. D10, 4262 (1974);

G. 't Hooft, in 'High Energy Physics', Proceedings of the EPS International Conference, Palermo 1975, ed. A. Zichichi, Editrice Compositori, Bologna 1976;

S. Mandelstam, Phys. Rep. 23, 245 (1976);

A.S. Kronfeld, M.L. Laursen, G. Schierholz and U.J. Wiese, Phys. Lett. B198, 516 (1987);

V. Bornyakov, E.-M. Ilgenfritz, M.L. Laursen, V.K. Mitrjushkin, M. Müller-Preussker, A.J. van der Sijs, A.M. Zadorozhny, Phys. Lett. B261, 116 (1991);

G. Bali, V. Bornyakov, M. Müller-Preussker and K. Schilling, Phys. Rev. D54, 2863 (1996).

[7] G. 't Hooft, Nucl. Phys. B138, 1 (1978);
L. Del Debbio, M. Faber, J. Greensite and S. Olejnik, *Phys. Rev.* **D55**, 2298 (1997);

L. Del Debbio, M. Faber, J. Giedt, J. Greensite and S. Olejnik, *Phys. Rev.* **D58**, 094501 (1998);

K. Langfeld, H. Reinhardt and O. Tennert, *Phys. Lett.* **B419**, 317 (1998);

M. Engelhardt, K. Langfeld, H. Reinhardt and O. Tennert, *Phys. Lett.* **B452**, 301 (1999);

*Phys. Rev.* **D61**, 054504 (2000)

[8] G. Bhanot and M. Creutz, *Phys. Rev.* **D24**, 3212 (1981).

[9] J. Greensite and B. Lautrup, *Phys. Rev. Lett.* **47**, 9 (1981).

[10] I.G. Halliday and A. Schwimmer, *Phys. Lett.* **B101**, 327 (1981).

[11] I.G. Halliday and A. Schwimmer, *Phys. Lett.* **B102**, 337 (1981).

[12] R.V. Gavai, M. Grady and M. Mathur, *Nucl. Phys.* **B423**, 123 (1994).

[13] R.V. Gavai, *Nucl. Phys.* **B474**, 446 (1996).

[14] S. Cheluvaram and H. S. Sharathchandra, hep-lat/9611001 (1996).

[15] S. Datta and R.V. Gavai, *Phys. Rev.* **D57**, 6618 (1998).

[16] S. Datta and R.V. Gavai, *Phys. Rev.* **D60**, 034505 (1999).

[17] R.V. Gavai and M. Mathur, *Phys. Lett.* **B458**, 331 (1999).

[18] Ph. de Forcrand and O. Jahn, *Nucl. Phys.* **B651**, 125 (2003).

[19] A. Barresi, G. Burgio and M. Müller-Preussker, *Nucl. Phys. B (Proc. Suppl.)* **106&107**, 495 (2002).

[20] A. Barresi, G. Burgio and M. Müller-Preussker, *Nucl. Phys. B (Proc. Suppl.)* **119**, 571 (2003).

[21] A. Barresi, G. Burgio and M. Müller-Preussker, hep-lat/0309010 (2003).

[22] E. Tomboulis, *Phys. Rev.* **D23**, 2371 (1981).

[23] G. Mack and V.B. Petkova, *Z. Phys.* **C12**, 177 (1982).

[24] T. Kovacs and E. Tomboulis, *Phys. Rev.* **D57**, 4054 (1998).

[25] A. Alexandru and R.W. Haymaker, *Phys. Rev.* **D62**, 074509 (2000).

[26] A. Di Giacomo, B. Lucini, L. Montesi and G. Paffuti, *Phys. Rev.* **D61**, 034503 (2000).

[27] A. Di Giacomo, B. Lucini, L. Montesi and G. Paffuti, *Phys. Rev.* **D61**, 034504 (2000).

[28] T. Kovacs and E. Tomboulis, *Phys. Rev. Lett.* **85**, 704 (2000).

[29] Ph. de Forcrand and L. Von Smekal, *Phys. Rev.* **D66**, 011504 (2002).