Research Article

Estimation of International Gold Price by Fusing Deep/Shallow Machine Learning

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Received 5 January 2022; Revised 1 February 2022; Accepted 7 February 2022; Published 22 March 2022

In this work, we propose a new method that combines the support vector machine (SVM) and the long short-term memory (LSTM) model utilizing the theory of quotient space to predict the price of gold by leveraging the price factors that have supposedly an impact on the gold price. The Pearson correlation coefficient is employed to measure the relations between nine price factors and gold price. The five price factors with larger correlation coefficients are picked. Then, by utilizing the Granger causality test, the gold price may change concerning the two price factors when time is a concern, which results in combining the results of the correlation analysis with the results of Granger causality leading to a total of seven price factors. Also, the gold price can be divided into the quarters of the year according to the theory of the quotient space and temporal attribute. With three granularities per month, a 3-layer quotient space is constructed based on the synthesized and calculated granularities. The proposed method provides the prediction results that are compared with the predicted values of some grey models (GM) and the actual gold price, respectively. The results suggested that the prediction results of gold price have a comparable lower error measurement and perform better.

1. Introduction

The manufacturing industry is the key of large countries such as the United States. Gold has always been a unique exchange material and has gradually fixed its role in the world economy. Moreover, gold has greatly promoted commodity trading and economic development to a certain extent. Although the function of gold currency has weakened since the 1970s, it is still leveraged as a reserve fund by the governments of many countries and has been one of the important components of international reserves. The gold market is a globalized market, which can be conveniently exchanged to any currency across the countries. Since gold is not considered a currency, more and more gold derivatives have appeared in the gold market. This has expanded the trading scale of gold. The price of gold is affected by a variety of factors. Various studies have been conducted on the influencing factors of gold prices from many perspectives. Linna et al. [1] analyzed the relationship between gold prices and short-term influencing factors through descriptive statistics and multiple linear regression. Rong [2] conducted an empirical study on gold prices based on analyzing the influencing factors. Yong [3] discovered the proxy variables of the gold price by considering the attributes of the research object. They also analyzed the role of different influencing factors of gold prices in different periods. Xiaoli [4] investigated the relevant factors affecting the price of gold under different circumstances and the complicated relationships between these factors. Kanjilal and Ghosh [5] analyzed the relationship between the prices of global crude oil and gold by utilizing the error correction model. Gil-Alana et al. [6] applied the concepts of integration, cointegration, and temporal management techniques to model the relationships between oil prices and gold prices. Kamran et al. [7] established the multivariate functional relationships among the gold price, inflation, interest rate, exchange rate, stock market, silver price, per capita income, and domestic savings. Moreover, researchers have focused on investigating the forecasting mechanism of gold prices. Most traditional institutions use statistical models to predict gold prices. This requires mass-scale samples. Therefore, when the number of samples is small
or insufficient, the identification effect would be poor. Sometimes, a “local minimum” problem will occur in the implemeneted models. Yifan and Yuqian [8] conducted a short-term forecast and analysis of gold prices based on the ARMA model. Yanyan and Yanli [9] proposed using the GM (1, 1) model based on equal-dimensional integrals to predict the price of gold. Jie et al. [10] established the DCCM-VGARCH model of the oil, stock, and gold markets to predict the correlation between the oil, stock, and gold markets. Kristjanpoller and Minutolo [11] developed a deep neural network and a generalized autoregressive conditional heteroscedasticity model to predict the fluctuation of gold prices. Dutta et al. [12] used the MFMDA and MFDXA methods to analyze the dynamic correlation between gold prices and SENSEX volatility. Crane et al. [13] formulated the Black–Scholes model to estimate the relevant parameters of gold prices. Yang et al. [14] introduced an empirical decomposition model that combines the support vector machine and proposed the EDM-SVM model to predict the gold price.

On the other hand, the theory of the quotient space mainly discusses the representation and properties of domains, attributes, and structures at different granularities as well as the interdependence and mutual conversion of these representations and properties. It is pervasively used in data mining and pattern recognition as well as cross-covering algorithms. Even though the classification is considered particularly challenging and it cannot directly solve the data fitting problem [15, 16], support vector machine (SVM) is a new type of machine learning technique based on the Vapnik–Chervonenkis (VC) dimension theory of statistical learning and the principle of structural risk minimization. It has advantages to find global optimization and strong generalization ability.

In this manuscript, we combine the LSTM model with SVM utilizing the theory of the quotient space to build a hybrid model to tackle the prediction problem. By doing so, the data domain is divided into multiple granularities utilizing the theory of quotient space [17–27] since the advantage of this theory has not been utilized yet.

The Pearson correlation coefficient and the Granger causality analysis are concurrently utilized to uncover the more important factors that have an impact on the gold price. Subsequently, the LSTM and SVM are combined regarding the theory of the quotient space. Thus, the hybrid model is finally leveraged to predict the effectiveness of factors on the prediction of gold price. The comparison results suggested that the proposed method generates better forecasts. Figure 1 depicts the proposed method.

The rest of the manuscript is organized as follows. Section 2 presents the related work. Section 3 introduces the proposed method by underlying the fundamental information that contributes to the development of the proposed method. Section 4 presents experimental results. Section 5 concludes the research.

2. Related Work

The economic theory claims that commodity prices are determined by the sophisticated supply and demand relationships. However, the gold price appears differently in various supply and demand relationships. As a commodity, there should be a certain pattern in the price fluctuation of gold. In the past few years, the top five countries in global gold production have been China, the United States, Russia, Australia, and South Africa. On the other hand, the producer index and consumer index have an impact on the gold production of these countries. These two indexes show that there is a close correspondence between the indexes and the gold production with a trend of steady growth. We compare the gold price changes in 12 countries (China, the United States, Europe, Canada, Australia, Russia, South Africa, Turkey, Saudi Arabia, the UAE, South Korea, and Japan) covering 2006 through 2015 based on the statistics of gold prices. It can be observed that the price of gold in most countries fluctuates with various ranges as price cycles were monitored. However, the gold price has generally increased in the long run. After a sharp drop in the gold prices occurring in some countries, the price of gold rose again. In the past 10 years, the price of gold in China reached its highest value in 2011, and the price of gold showed a tortuous increase.

As a value-preserving instrument, gold plays an irreplaceable role when global inflation has a huge impact on the financial market. In this way, the price of gold is closely related to the level of inflation. The commodity price index (CPI) can also reflect the global inflation level to a certain extent, and the changes in the CPI are in turn related to the West Texas Intermediate (WIT) crude oil futures price. The variation in the price of WIT crude oil futures will cause fluctuations in the CPI under different circumstances. This also leads to fluctuations in the gold price. The trends of WIT crude oil futures prices are closely related to the gold prices too. For example, since the international spot gold price is based on US dollars, the price of gold has not only been affected by its supply and demand conditions but also been impacted by the value of the US dollar. In addition, the gold price will also be affected by the value of the currencies of other countries. When the global five (G5) currency index is under consideration, including the US dollar, the Euro, the Japanese Yen, the British Pound, and the Canadian Dollar onwards 2012, the price of gold has been related to the US dollar index and the G5 currency index too. Some of the key findings determined by research can be summarized as follows: the factors affecting the gold price include WIT crude oil futures, Dow Jones index, interest rate index, federal funds rate (FFRM) US, US dollar index, stock index, CPI index, US GDP growth rate, county risk premium (CRP) index, and gold reserves [17–27]. Besides, social factors, cultural customs, and political factors would also affect the price of gold. Generally speaking, the factors that have a greater impact on the price of gold include nine factors, namely, commodity index, consumer price index, US dollar index, WIT crude oil futures, Dow Jones index, inflation rate, G5 currency index, producer index, and consumer index. These nine factors as influencing factors of the gold price were selected for this research.

3. The Proposed Method

The proposed prediction model of the gold price consists of three key constituents: quotient space theory, SVM, and
LSTM. We will briefly introduce them in the following sections.

3.1. The Theory of the Quotient Space. In the theory of the quotient space [13], the prediction problem of the gold price is expressed by a triple set:

$$(X, f, T),$$

where $X$ is the universe of discourse, $f$ is the attribute (vector) function, and $T$ is the topological structure on $X$. The equivalence relation, denoted by $R$, is represented by the set of $(X, f, T)$. The theory of the quotient space investigates the quotient set determined by $R$, including the quotient structure and quotient attributes [14]. $T$ represents the transformation from a coarse level to a fine-grained level. For multiple coarse-grained granularities, we construct an appropriate granularity level of attribute and structure, which can fulfill the description of the granularity problem corresponding to the specific situation and the synthesis of quotient space completely. In the theory of the quotient space, the division of the universe of discourse is calculated by the equivalence relation $R$ that divides the domain of discourse $X$ into several particles. The notation $X/R$ is used to represent a set of equivalence classes on a given domain of $X$. Each equivalence class contains particles. The size of the particles is a measurable quantity. The structure of the quotient space of a given problem obtains the granularity from these three aspects: domain, attribute, and structure [7].

The granularity of the attribute is pertinent to the granularity of the quotient set corresponding to the range of the values of $Y$ [15]. The corresponding relation $i$ is denoted by $R_i$. The attribute function is denoted by $f_i(x) = (f_i(x), f_2(x), \ldots, f_n(x))$. $G_i$ is a relation defined on the attributes of $x$ and $y$ implying that

$$G_i: x \sim y \iff f_i(x)R_i f_i(y),$$

where $G_i$ is defined as an equivalence relation on $X$, which generates the corresponding quotient space. The method utilizing the granularity of the structure is based on utilizing $T$ to obtain the coarse topology $T_i$.

3.2. Support Vector Machine. The linear regression method in the support vector machine [13, 14] is leveraged to solve the prediction problem of gold price. The sample data is denoted by $(x_i, y_i)$, where $i = 1, 2, \ldots, k; x_i, y_i \in R$. The linear regression function is represented by

$$f(x) = w \cdot x + b,$$

where $w$ is the weight vector of the hyperplane; $x$ is the sample data; and $b$ is the bias term. If there is a function $f$ that satisfies the accuracy requirement $\varepsilon$, then the minimum $w$ can be solved by a convex optimization problem defined by

$$\min \frac{1}{2} w^2.$$

Herein, the constraint is represented by $y_i - w \cdot x_i - b \leq \varepsilon$ $wx_i - y_i - b \leq \varepsilon$. If the fitting error is allowed, the relaxation factor $\xi_i \geq 0$ is introduced, and the convex optimization problem can be solved based on the objective function defined by

$$\min \frac{1}{2} \|w\|^2 + C (\xi_i + \xi_i^*),$$

where the constant $C$ is the degree of punishment for the error sample and $\xi_i$ and $\xi_i^*$ are the relaxation factors. The fitting error of the function is within the following range denoted by
\[ y_i - wx - t \leq \varphi + \xi_i, \]  

(6)

Only a small part of \( \alpha_i \) and \( \alpha_i^* \) is not zero. The kernel function \( K(x_i, x_j) \) is used to replace the inner product operations in (6), and the nonlinear fitting function is defined by

\[ f(x) = w \cdot x + b_1 ky_i(a) \cdot i = i \cdot x_i \cdot x_j + b, \]  

(7)

where the kernel functions in SVMs include both local and global kernel functions. In practice, there are many types of kernel functions such as linear, polynomial, radial basis, and sigmoid functions. The elements in the set are divided into attributes based on the element sets of different granularities that are obtained. Hence, the element labels are unchanged, and the element attributes and the value space are reserved. The granularity of different granularity sets is obtained regarding the equivalence relation defined by the clustering and the value space are reserved. Hence, the element labels are unchanged, and the element attributes and the value space are reserved.

\[ D_x = \{(x_i \in R^d, y_i \in \{1, -1\}, i = 1, 2, \ldots, k)\}, \]  

(8)

where \( i \) is equal to the category label of the sample in the subset. Denoting \( |\pi_i| \) as the number of samples at coarse-grained, \( x_i \) corresponds to fine-grained. If \( |\pi_i| \) becomes larger, then the empirical risk in the support vector machine method corresponding to fine-grained representation would be greater. The support vector at the coarse-grained acts similarly. The empirical risk of the support vector machine must consider the number of samples included in each division. In this manner, the original problem can be expressed by

\[ \min \frac{1}{2} w^2 + C \sum \pi_i |\xi_i|. \]  

(9)

The constraints are denoted by \( w, y_i, \varphi(u_i) \geq 1 - \xi_i \). The dual model is defined by

\[ \min_{\pi} \tau G(\alpha) = \frac{1}{2} \alpha^T \pi Q \pi - \alpha^T \tau. \]  

(10)

\( 0 \leq \alpha_i \leq |\pi_i|, \forall i = 1, 2, \ldots, k \), where \( \alpha_i \) is a vector composed of elements \( \alpha_i \), \( e_i \) is a k-dimensional vector with all ones; and \( Q \) is a \( K \times K \) semidefinite matrix denoted by \( Q_{\pi i} = \varphi(u_i), \varphi(u_i) \). Given the training set of gold prices at different times, the SVM is used to predict the gold price of a test set in the proposed method.

3.3. Long Short-Term Memory (LSTM). LSTM is a deep neural network developed to solve the problem of the disappearance of gradients caused by the recurrent neural network model due to the long input sequence [12, 13]. LSTM is composed of memory cells, input gates, output gates, the forget gate, and the activation functions of the three gates that are all sigmoid functions. The input gate controls the input information of the neural unit at the current time, whereas the forget gate is leveraged to control the historical information stored in the neural unit at the previous time. Meanwhile, the output gate is utilized to control the output information of the neural unit at the current time [14]. Figure 2 presents an expanded view of the network structure of LSTM. While part "3" represents the input at the current time, part "2" represents the state value of the cell at the current time.

The stock prediction network model based on LSTM-CNN-CBAM is built under the LINUX operating system, whose GPU version is GTX 2080 under the PyTorch framework. By incorporating the CBAM attention mechanism into the time series classification model that combines the long short-term memory neural network, the LSTM model can automatically learn and extract the local features and long-memory features in the time series. As elaborated in Figure 2, the first is the LSTM module, which uses the time-series features in the learning data of the 3-layer LSTM neural network. Each layer of LSTM has 128 hidden neurons, the learning rate is set to 0.001, the number of iterations (epochs) is set to 200, and subsequently, the extracted features are passed through. The convolutional neural network performs feature learning and extraction and then incorporates an attention mechanism. Finally, a five-layer backpropagation neural network calculates the predicted prices. The number of neurons in each fully connected layer is set to 1,024, 128, 64, 20, and 1, respectively. The activation function uses the ReLu function. Since the LSTM neural network can capture the features at the temporal level, we leverage the first 85% of the data set as the training set data, while the rest of the 15% as the test set data. In the LSTM-CNN-CBAM stock prediction network model, the experimental results can be obtained by setting different temporal steps for experimental comparison. It is observable that setting different time steps can accurately predict the results. When the time step is assigned to 5, the global factors do not need to be considered. Thus, the prediction results have large deviations, and the data have certain fluctuations. On the other hand, when the time step is assigned to 30, the considered time range is too large, which results in ignoring the influence of public opinion in a shorter period. Therefore, the prediction result will be inaccurate. When the time step is assigned to 20, the error would be the smallest and the accuracy would be the highest. Thus, we finally set the time step to 20 and further utilize the data of nine attributes concerning the previous 20 days as the input layer of the neural unit. The closing price of the 21st day is used as the label to train the model. Finally, the LSTM model is used to predict the price of gold at a given time. Then, the prediction results of LSTM are combined with those of SVM to
calculate the gold price. The steps of the proposed method are presented in algorithm 1.

4. Experimental Results and Analysis

When both gold market and world economic trends in the past decade are under consideration, we select samples disseminated by the World Gold Council between 2006 through 2015 (including commodity price index, consumer price index, U.S. dollar index, WTI crude oil futures, Dow Jones index, inflation rate, G5 currency index, production consumer index, consumer index, and gold price). While the samples from 2006 through 2015 are utilized as the training samples, the samples in 2016 are used as the test samples. The explanatory variables are called commodity index, consumer price index, U.S. dollar index, WTI crude oil futures, Dow Jones index, inflation rate, G5 currency index, producer index, and consumer index. These attributes are assumed to be factors that have an impact on gold prices. Figure 3 depicts the trends of some factors.

The training and test samples are normalized to speed up the calculation process. After the normalization step, the values are transformed into $[−1, 1]$. The normalization method is defined by

$$a = \frac{x^k - \min(X)}{\max(X) - \min(X)}$$

where $a$ is the normalized value, $x^k$ is the sample data, $\max(X)$ is the maximum value of the sample data, and $\min(X)$ is the minimum value of the sample data. Then, the Pearson correlation coefficient is computed to quantitatively measure the linear relationship between the impact factors and the gold price. The correlation coefficient is defined by

$$r = \frac{N \sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sqrt{\left[ N \sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2 \right] \left[ N \sum_{i=1}^{n} y_i^2 - (\sum_{i=1}^{n} y_i)^2 \right]}}$$

where $N$ is the sample size and $x_i$ and $y_i$ are variables. Table 1 presents the results.

The variables $x_1$ and $x_2$ seem more likely to be the explanatory variables of $y$, which is based on the constructed causal relationship. However, the values in the table show that $y$ is more likely to be explained by the variable of $x_3$, which is inconsistent with objective economic reality. Nevertheless, $x_3$ is utilized as an explanatory variable of $y$. With a 90% probability, $x_3$, $x_5$, $x_7$, $x_9$, $x_9$, and $y$ that are mutually causal are relatively small, and $x_7$, $x_9$, and $x_9$ are not considered in the gold prediction model. In the forecast model based on the quotient space theory, the U.S dollar index, WIT crude oil futures, G5 currency index, producer index, consumer index, commodity index, and consumer price index will be utilized as price factors affecting the gold prices at the final stage. To study their impacts on the price of gold, the initial granularity of domain $X$ in the database is picked as a month. Herein, the above factors could be divided regarding month, season, and year between 2006 through 2015. It is possible to devise different granularities such as 1080 monthly, 360 quarterly, and 90 annually. The spatial structure for the quotient of the gold price is calculated. Each

granularity is divided into levels, and each granularity is composed of gold price and price factors. After selecting the appropriate sample set, the appropriate kernel function in the SVM space should be determined. The characteristics of the gold price data can be considered as a linear problem. Herein, the kernel functions commonly used in the SVM model for such problems are divided into polynomial kernel functions and Gaussian kernel functions. The Gaussian kernel function has fewer parameters and can reduce the number of calculations when the parameters are optimized.

It is also convenient to adjust the aforementioned parameters. The polynomial kernel function will increase the computational complexity when the polynomial order is higher. Therefore, the Gaussian kernel function selected for the SVM method is denoted by

$$k(x_i, x_j) = \exp\left(-\frac{\delta}{x_i - x_j}\right),$$

where $\delta$ is the width parameter of the function and $\delta > 0$ controls the radial range of the function. We determine the training samples and observe the optimal parameters of the SVM model. The 10-fold cross-validation strategy is used to select the optimal cost parameter. Then, we utilize the training samples to obtain (6) and (7). Based on the values of $\epsilon$, $b$, and the support vector, we calculate the optimal SVM prediction model. After the SVM is determined, the test sample is substituted into the prediction model to calculate the prediction value, as shown in Table 2.

The optimization results of the gold price based on SVM model parameters are computed by utilizing R-Studio software when constructing the model. Through the optimized SVM model, the forecast value of the gold price in 2016 is obtained. The GM (1, 1) model is built utilizing the data between 2006 through 2015 to predict the gold price in 2016 to compare the real value of gold price in the same year. The true value of the gold price was 8,306.0 yuan/troy ounce and was different than the predicted value. Therefore, the absolute error is calculated. Through comparative analysis, it is observable that combining the quotient space theory with the constructed three-layer granularity model for the prediction of gold price makes each learning sample more prominent. The GM (1, 1) model predicts gold prices with an absolute error greater than 10% when compared with the actual gold price. On the other hand, SVM plus LSTM based on the theory of the quotient space generates a lower error rate than GM (1, 1). The predicted value of the optimized proposed model was 8,053.1 yuan/troy ounces in 2016.

The GM (1, 1) model is also built using China’s gold price from 2006 to 2015 to predict the gold price in 2016. By comparing the predicted gold price in 2016 with the true value of the gold price in 2016 (8,306.0 yuan/troy ounce) and calculating the absolute error, the proposed model has a better absolute error rate than does the GM (1, 1) model. Thus, we conclude that the characteristics of each learning sample are more prominent. Since the gray model does not consider the role of price factors, the constructed GM (1, 1) model has an absolute error of more than 10% when compared with the actual gold price in the same year. Also,
the SVM plus LSTM based on the theory of the quotient space has a lower absolute error rate in the same year.

On the other hand, when comparisons are conducted among the grey SCGM (1, 1) model, the equal-dimensional dynamic SCGM (1, 1) model, and the equal-dimensional dynamic Markov SCGM (1, 1)C model, while the SCGM (1, 1) model has the lowest accuracy, the accuracy level of the equal-dimensional dynamic SCGM (1, 1)C model is found to be better. The accuracy level of the equal-dimensional dynamic Markov SCGM (1, 1) C model is relatively the best by exhibiting the best fitting and is called the optimal model. Therefore, the model is leveraged to predict the gold price in

\begin{algorithm}
Input: the gold price at multiple times, parameters of the SVM and LSTM; output: the gold price at a test time.

1. Project the gold price at different times onto the quotient space.
2. Determine an SVM classifier from the training samples, and use it for gold price prediction at a test time.
3. Determine LSTM from the training samples, and then leverage it for predicting gold price at a test time.
4. Combine the predicted results from SVM and LSTM to obtain the final results.

\end{algorithm}
May 2019, and the predicted value is USD 1,314.78/ounce. The comparisons based on the predicted values of the SVM plus LSTM model and the predicted values of the equal-dimensional dynamic Markov SCGM (1, 1) models suggested that the proposed method has better forecast accuracy than the others. However, the equal-dimensional dynamic Markov SCGM (1, 1) model exhibits a slightly better prediction accuracy among the GM models. The reason is based on the utilization of the fixed parameters. This determines the forecast result that changes according to a certain trend, and it does not take into account sudden changes timely since the gold price is affected by multiple external factors. Also, the prediction value of the equal-dimensional dynamic Markov SCGM (1, 1)C model is only related to the previous state. Therefore, the proposed method is more usable when the predicted value is required to be as close to the actual value as possible.

5. Conclusion

When the current international market conditions of gold prices are under consideration, we research the affecting price factors on gold prices and leverage the Person correlation and Granger causality to quantitatively analyze and select them. Thus, we select nine variables that are expected to have a greater impact on gold prices. Finally, the gold price is predicted by the a hybrid model, which combines the theory of the quotient space with support vector machine customized to the LSTM model. The R-Studio software is utilized to optimize the relevant parameters of the proposed model to be utilized for the prediction of gold prices. When predicted results of the gold price are compared between the proposed model and the traditional GM (1, 1) model and its modified versions, even though the equal-dimensional dynamic Markov SCGM (1, 1)C model outperforms the other gray models, it does not perform better than the proposed model. Therefore, we conclude that the proposed method has better prediction results with a lower absolute error rate, generates more accurate results, and does it faster.

Data Availability

Data will be provided on request with the consent of the author of this paper.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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