Unconventional quantum Hall effect in Floquet topological insulators

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Abstract
We study an unconventional quantum Hall effect for the surface states of ultrathin Floquet topological insulators in a perpendicular magnetic field. The resulting band structure is modified by photon dressing and the topological property is governed by the low-energy dynamics of a single surface. An exchange of symmetric and antisymmetric surface states occurs by reversing the light’s polarization. We find a novel quantum Hall state in which the zeroth Landau level undergoes a phase transition from a trivial insulator state, with Hall conductivity $\sigma_{\text{yx}} = 0$ at zero Fermi energy, to a Hall insulator state with $\sigma_{\text{yx}} = e^2/2h$. These findings open new possibilities for experimentally realizing nontrivial quantum states and unusual quantum Hall plateaus at $(\pm 1/2, \pm 3/2, \pm 5/2, ... )e^2/h$.

Keywords: Floquet, quantum Hall effect, topological insulator

(Some figures may appear in colour only in the online journal)
Floquet bands were first realized in photonic crystals [22] and have been verified by recent experiments on the surface states of FTIs [23, 24]. These first studies of off-resonant light were limited to the band structure of FTIs and differ from many optical effects in TIs [15]. Also, no magnetic field was involved in these experiments.

In this work we identify a novel quantum Hall state of ultrathin FTIs in a magnetic field when their surface degeneracy is broken due to an off-resonant light. We evaluate their band structure and the longitudinal and Hall conductivities using linear response theory [25, 26].

Model formulation

We consider surface states of ultrathin TIs in the \((x, y)\) plane in the presence of circularly polarized off-resonant light \([18, 28]\) and hybridization \([27]\) between the top and bottom surface states. Extending the 2D Dirac-like Hamiltonian \([18, 28]\) by including an external perpendicular magnetic field \(B\) gives

\[
H'_{l} = v_{F}(\sigma_{l}\Pi_{l} - \sigma_{l}\Pi_{l}) + s_{l}\Delta_{0}\sigma_{3} + \Delta_{l}\sigma_{0},
\]

where \(s = \pm 1\) for symmetric/antisymmetric surface states, \(l = \pm l\) for right-/left-handed circularly polarized off-resonant light, \((\sigma_{x}, \sigma_{y}, \sigma_{z})\) the Pauli matrices and \(v_{F}\) the Fermi velocity. \(\Delta_{l}\) is the hybridization energy between the top and bottom surface states that, depending on the thickness, varies from 20 meV to 120 meV \([29]\). \(\Delta_{l} = e^{2}v_{F}^{2}\hbar^{2}A_{l}^{2}/\hbar^{3}\Omega\) is the mass term induced by the off-resonant light with amplitude \(E_{0}\), \(\Omega\) the light’s frequency, and \(A_{0} = E_{0}/\Omega\). It breaks the time-reversal symmetry and its value is about 50 meV \([23, 24]\).

\(\Pi = p + eA\) is the 2D canonical momentum with vector potential \(A\). In the Landau gauge \(A = (0, Bx, 0)\), diagonalizing the Hamiltonian (1) gives the eigenvalues

\[
E_{n,l}^{\pm} = \lambda|\hbar^{2}\omega_{n}^{2} + \Delta_{l}^{2}|^{1/2}, \quad E_{n,l}^{0} = -\Delta_{l},
\]

where \(\lambda = \pm 1\) represents the electron/hole states, \(\omega_{k} = v_{F}\sqrt{2eB/\hbar}\), and \(\Delta_{l} = l\Omega + s\Delta_{0}\). The corresponding normalized eigenfunctions are

\[
\psi_{n,l}^{\pm}(k) = \frac{e^{ikl}}{\sqrt{L_{y}}}(C_{n,s}^{l}\phi_{n-1}^{l} \pm D_{n,s}^{l}\phi_{n}^{l}), \quad \psi_{0,l}^{0}(0) = \frac{e^{ikl}}{\sqrt{L_{y}}} \phi_{0}^{l}
\]

where

\[
C_{n,s}^{l} = \left[\left(E_{n,s}^{l} + \Delta_{s}^{l}\right)/2E_{n,s}^{l}\right]^{1/2}, \quad D_{n,s}^{l} = \left[\left(E_{n,s}^{l} - \Delta_{s}^{l}\right)/2E_{n,s}^{l}\right]^{1/2}, \quad \phi_{n}^{l}
\]

are the harmonic oscillator eigenfunctions. Notice that equation (1) does not contain the Zeeman term \(gsB\), where \(g = 2\). We neglect it, because we consider only weak \(B\) fields \(\ll 1\ T\), see figure 1. For a \(g\) factor as large as 20 the Zeeman energy at \(B = 1\ T\) is 0.58 meV and much smaller than all other energies.

A close inspection of equation (2) shows that we have a gapped Dirac spectrum, with gap \(\Delta_{0}\) and electron–hole–symmetry for zero off-resonant light, \(\Delta_{0} = 0\). For \(\Delta_{0} > \Delta_{0} > 0\), the eigenvalues of equation (2) show a \(\sqrt{2}\) dependence and the LLs split, see figure 1. We find an exchange of the symmetric (solid curves) and antisymmetric (dotted curves) surface states by changing the light’s polarization from right (red curves) to left (black curves). The parameters used are \(v_{F} = 0.5 \times 10^{6} \text{ m/s}\), \(\Delta_{0} = 20\ \text{ meV}\), and \(\Delta_{0} = 30\ \text{ meV}\) \((e\nu_{F}A_{0} = 0.48\ \text{ eV}, \ h\Omega = 7.5\ \text{ eV})\) \([18]\). The \(n = 0\) LL appears in the hole band for right-handed light (red curves, \(l = 1\)) and in the electron band for left-handed light (black curves, \(l = -1\)), see figure 1. The exchange of surface states induced by the field \(B\) and the off-resonant light in such FTIs is an entirely new phenomenon. The energies of the two surfaces are different for \(B = 0\), since the gap \(l\Delta_{0} + \Delta_{0}\) increases for one surface and decreases for the other.

We emphasize that the band gaps \(l\Delta_{0} + s\Delta_{0}\) at the two surfaces of ultrathin FTIs can be made different by, e.g. varying the light’s frequency or amplitude. This can create, e.g. for \(l = 1\), one surface with a small gap \(\Delta_{0} - \Delta_{0}\) and the other one with a large gap \(\Delta_{0} + \Delta_{0}\); for \(l = -1\) these gaps could be exchanged. Accordingly, only the antisymmetric or symmetric surface contribute to the transport properties depending on the light polarization \((l = \pm 1)\). Such a situation could be realized in experiments on FTIs, similar to those of [23, 24], by varying the sample thickness [29] down to the limit of 6 nm below which different energies \(\Delta_{0}\) have been reported [29]. To our knowledge this is a novel state of matter in FTIs like Bi2Se3, Bi2Te3, HgTe, and related materials.

Longitudinal conductivity

For weak scattering potentials the current is due to hopping between orbit centres as a result of carrier collisions with, e.g. charged impurities [25]. In a normal magnetic field the diffusive contribution \(\sigma_{\text{diff}}^{x}\) to \(\sigma_{xx}\) vanishes and only the collisional contribution \(\sigma_{\text{col}}^{x} = \sigma_{xx}\) is important; it is given by [25, 26].
\[ \sigma_{\alpha \beta} = \frac{e^2 \beta}{2 S_0} \sum_{\xi', \xi} f(E_{\xi'}) \left( 1 - f(E_{\xi}) \right) W_{\xi', \xi}^{\alpha} (X_{\xi'} - X_{\xi})^2, \]  

(4)

where \( f(E_{\xi'}) = (\exp(\beta(E_{\xi'} - E_F)) + 1)^{-1} \) is the Fermi Dirac distribution function, \( \beta = \frac{k_B T}{e} \), \( T \) the temperature, \( k_B \) the Boltzmann constant, and \( E_F \) the Fermi energy. \( W_{\xi', \xi}^{\alpha} \) is the transition rate between the one-electron states \( |\xi'\rangle \) and \( |\xi\rangle \), and \( e \) the charge of the electron. Here \( f(E_{\xi'}) = f(E_{\xi}) \) for elastic scattering and \( X_{\xi'} = \langle \xi' | x | \xi \rangle \) with \( x \) being the position operator.

The scattering rate is given by Fermi's golden rule

\[ W_{\xi', \xi}^{\alpha} = \frac{F}{\hbar \epsilon_F} \sum_{\xi = \xi'} \left| U(\mathbf{q}) \right|^2 J_{\xi}(u) \delta(E_{\xi'} - E_{\xi}) \delta_{\xi, \xi'}, \]  

(5)

with \( F = 2\pi N_0/|S_\mathbf{q}| h \), \( q^2 = q_x^2 + q_y^2 \), \( u = \frac{q_z}{\hbar} \), \( N_0 \) the impurity density. \( J_{\xi}(u) = \langle \xi | \exp(\mathbf{q} \cdot \mathbf{r}) | \xi' \rangle \) are the form factors and \( \xi \equiv (n, s, l, j) \). \( U(\mathbf{q}) = U_0 \exp(q^2 + k_0^2)/2 \) with \( U_0 = e^2 / (2 \epsilon_F \epsilon_0) \). Further, \( k_z \) is the screening wave vector, \( \epsilon_0 \) the relative permittivity, and \( \epsilon_F \) the permittivity of the vacuum. Furthermore, if the impurity potential is short-ranged (of the Dirac \( \delta \)-function type), one may use the approximation \( k_z \gg q \) and obtain \( U(\mathbf{q}) \approx U_0 / k_0 \). Since the scattering is elastic and the eigenfunctions are degenerate in the quantum number \( k_s \), see equation (3), only the \( n \rightarrow n \) transitions are allowed. Further, we have \( (X_{\xi'} - X_{\xi})^2 = \int \rho_{\xi \xi'}^2 \) transform the sums over \( k_t \) and \( q \) into integrals, and evaluate them using cylindrical coordinates. The form factor \( \left| J_{\xi}(u) \right|^2 \) can be evaluated from the matrix element \( \langle \xi | \exp(\mathbf{q} \cdot \mathbf{r}) | \xi' \rangle \). The result is

\[ \left| J_{\xi}(u) \right|^2 = \exp(-u) \left[ C_{\xi, \xi'}^{\alpha \beta} L_{\alpha}(u) + D_{\xi, \xi'}^{\alpha \beta} L_{\alpha}(u) \right]^2 \]  

for \( n = n' \).

With these details equation (4) takes the form

\[ \sigma_{\alpha \beta} = \frac{e^2 N_0}{\hbar \epsilon_F} \sum_{s, n} I_{\alpha \beta}^{n,s}(E_{n,s}) \left[ 1 - f(E_{n,s}) \right], \]  

(6)

where \( f(E_{n,s}) = (\exp(\beta(E_{n,s} - E_F)) + 1)^{-1} \) and \( u_{sc} = \frac{q_z}{\hbar} \). The sum over \( s \) is trivial since the two surfaces can be treated independently due to the different gaps. The factor \( I_{\alpha \beta}^{n,s} \) in equation (6) is the integral

\[ I_{\alpha \beta}^{n,s} = \int_{-\infty}^{\infty} \left| J_{\xi}(u) \right|^2 du \]  

that can be evaluated analytically using the properties of the orthogonal polynomials \( L_{\alpha}(u) \). The result is

\[ I_{\alpha \beta}^{n,s} = (2n+1) \left( C_{\xi, \xi'}^{\alpha \beta} \right)^2 - 2n \left( D_{\xi, \xi'}^{\alpha \beta} \right)^2 \left( 2n-1 \right) \delta_{\alpha, \beta}. \]  

(7)

For \( \Delta\alpha = 0 \), equation (7) reduces to \( 2n/4 \), which means that the minima of \( \sigma_{\alpha \beta} \) occur at the odd factors \( n = 2n + 1 \) in accord with [26].

Since the band gap \( \Delta \alpha + s \Delta_k \) becomes surface dependent, see figure 1, the longitudinal conductivity is dominated by one surface only, that of the symmetric or antisymmetric surface states. As usual, this conductivity, given by equation (6), exhibits Shubnikov–de Haas oscillations. For \( \Delta_k = \Delta_k = 0 \) we must consider both surfaces. The electron–hole spectrum is symmetric with a single peak (solid curve) at the Dirac point, as shown in figure 2(a), using the parameters [6–8]: \( N_0 = 1 \times 10^{13} \text{m}^{-2}, \mu_B = 5.788 \times 10^{-5} \text{eV}^{-1} \text{K}^{-1}, T = 2 \text{K}, B = 1 \text{T}, k_s = 10^{-7} \text{m}^{-1}, v_F = 5 \times 10^5 \text{m s}^{-1}, \) and \( \epsilon_0 = 4 \).

For \( \Delta\alpha > \Delta_k > 0 \), the electron–hole spectrum is asymmetric and we consider only one surface depending on the light’s polarization. We consider only the symmetric surface states \((s = 1, \text{black curves}) \) for left-handed light \((l = -1) \) or the antisymmetric surface states \((s = -1, \text{red curves}) \) for right-handed light \((l = 1) \) and show \( \sigma_{\alpha \beta} \) in figure 2(b). As seen, the \( n = 0 \) LL shifts into the hole or electron band. The shift can be understood from the help of the eigenvalues shown in figure 1: for right-(left-)handed light the \( n = 0 \) LL moves into the hole (electron) band. This is a nontrivial state entirely new in FTIs. We notice in passing that were we to plot the current polarization \( P = \sigma(x(l = 1) - \sigma(x(l = -1)))[\sigma(x(l = 1) + \sigma(x(l = -1))] \) we would have, on account of figure 2(b), only two peaks of height \( P = 1 \) centred at \( E_F \approx -0.01(0.01) \text{eV} \). Also, had we considered the \( s = -1 \) surface with left-handed light \((l = -1) \) or the \( s = 1 \) surface with right-handed light \((l = 1) \), \( \sigma_{\alpha \beta} \) would be zero in the entire range of figure 2 since the corresponding surface states start at \( \pm 0.05 \text{eV} \), see figure 1 for \( B = 1 \text{T} \). This is also corroborated by the fact that at very low temperatures the factor \( \beta f(\ldots)[1 - f(\ldots)] \) in equation (6) behaves as the function \( \delta(E_{n,s} - E_F) \).
Hall conductivity

For linear responses to a weak source-to-drain electric field, the Hall conductivity is given by the Kubo–Greenwood formula [25, 26]

\[
\sigma_{xy} = \frac{i\hbar e^2}{2h} \sum_{\zeta, \zeta'} \frac{(f_\zeta - f_{\zeta'}) \nu_{\zeta\zeta'} \nu_{\zeta'\zeta}}{(E_\zeta - E_{\zeta'})(E_\zeta - E_{\zeta'} + i\Gamma)},
\]

where \(\nu_{\zeta\zeta'}\) and \(\nu_{\zeta'\zeta}\) are the nondiagonal matrix elements of the velocity operator with \(\mu = x, y, \nu = x, y\). The sum runs over all quantum numbers of the states \(\zeta \equiv [n, s, l, k]\) and \(\zeta' \equiv \{n, s', l', k\}'\) provided \(\zeta \neq \zeta'\). Assuming that the level broadening is approximately the same for all LLs, \(\Gamma = \Gamma\), one can show that the imaginary part of equation (8) vanishes. To obtain the most transparent results for the Hall conductivity \(\sigma_{xy}\), we take \(\Gamma = 0\). The relevant velocity matrix elements are obtained from equation (1), for \(\nu = x\) and \(\mu = y\), and the evaluation follows the procedure detailed in [26]. The result for \(\sigma_{xy}\) can be expressed as a sum of two terms, one \((I)\) for \(n \geq 1\) and the other \((II)\) for \(n = 0\), i.e. \(\sigma_{xy} = \sigma_{xy}^I + \sigma_{xy}^{II}\), with

\[
\sigma_{xy}^I = \frac{e^2}{h} \sum_{s,n=1}^{\infty} \left\{ \left( n + 1/2 \right) [f_{n+1,s} - f_{n+1,s}] + f_{n,s} - f_{n,s} \right\}
\]

\[
\sigma_{xy}^{II} = -\frac{\Delta_{\Omega}}{2} \left\{ \left( f_{n+1,s} - f_{n,s} \right) - \left( f_{n+1,s} - f_{n+1,s} \right) \right\}
\]

The sum over \(n\) starts at \(n = 1\), because the \(n = 0\) LL is treated separately. That over \(s\) is trivial, as we consider only one surface at a time. For \(n = 0\) equation (8) gives

\[
\sigma_{xy}^I = \frac{e^2}{h} \sum_{s} \left\{ \left( f_{0,s} - f_{0,s} \right) - \left( f_{1,s}^+ - f_{1,s}^- \right) \right\}/2 + \Delta_{\Omega} \left( f_{1,s}^+ - f_{1,s}^- \right)/2E_{1,s}^{-1/2}
\]

At zero or very low temperature the sum over \(s\) in equations (9) and (10) has the value 1 for \(n = n_p\), since for a single surface the number of filled states is 1. Here \(n_p\) is the LL index at the Fermi energy. Notice also that for \(\Delta_{\Omega} = 0\) equations (9) and (10) take the form \(\sigma_{xy} = 2(e^2/h)(n + 1/2)\) or in terms of the filling factor \(\nu = 2(n + 1/2)\) as \(\sigma_{xy} = \nu e^2/h\) for TIs [6–8, 30]. An important aspect in the QHE in graphene, TIs, silicene, MoS2, etc is the LL at zero energy (gapless graphene) or its modification in gapped systems.

Similar to the band structure and longitudinal conductivity, the unconventional QHE is dominated by one surface. That is, only the symmetric or antisymmetric surface contributes to it for left- or right-handed light, respectively. We show the Hall conductivity \(\sigma_{xy}\) in figure 3, for \(s = 1\), as a function of \(E_F\). In the limit \(\Delta_{\Omega} \rightarrow 0\) (a), solid curve these results reduce to an odd-integer QHE in TIs by multiplying with a factor 2 for surface degeneracy [6, 30] (limit of zero strain in [8]), where the plateaus appear at \((\pm 1, \pm 3, \pm 5, ...)^2/he\), and both surfaces must be considered as in the case of figure 2(a). The results of figure 3(a), dashed curve, can be reduced to those of single-valley gapped graphene for \(\Delta_{\Omega} \neq 0\) [26], irrespective of a factor of 2 due to valley degeneracy, and to those of gapped TIs [31]. However, for \(\Delta_{\Omega} > \Delta_{h} \neq 0\) only one surface must be considered, see figure 2(b) for \(\sigma_{xy}\). The Hall plateaus occur at half-integer values in contrast to previous results for graphene and TIs, as shown in figure 3(b). At the Dirac point we have \(\sigma_{xy} = e^2/2h\) for right-handed light (dashed curve) and \(\sigma_{xy} = -e^2/2h\) for left-handed light (solid curve), due to the occurrence of the \(n = 0\) LL in the hole and electron band, respectively. Again, had we considered the \(s = -1\) surface with left-handed light or the \(s = 1\) surface with right-handed light, \(\sigma_{xy}\) would vanish in the \(E_F\) range of figure 3, since the corresponding states would start at \(\pm 0.05\) eV and the occupation factors \(f(\ldots)\) would be zero.

The electron–hole symmetry is broken and the plateaus appear at \((\pm 1/2, \pm 3/2, \pm 5/2, ...)e^2/h\). This shows a nontrivial transition at the Dirac point which could be experimentally tested. It occurs because the energy term \(\Delta_{\Omega}\) due to the off-resonant light at the Dirac point can be externally tuned to higher values [23, 24]. As for the influence of level broadening, i.e. finite \(\Gamma\), on the results, on the basis of [26] we strongly expect they will not be altered qualitatively. After all, the QHE has already been realized on TIs [8]. These signatures of novel quantum phase transitions in FTTs are distinct from those in graphene or in TIs without light and relate to different values of the Hall conductivity. They could be tested in experiments similar to those performed on semiconducting silicon [32, 33]. Moreover, radiation effects and light-dependent magnetotransport have been observed for Dirac fermions in the presence of on-resonant light [34, 35]. Accordingly, we believe our results can be tested in similar experiments using off-resonant light [23, 24], a regime that is different from that of the optical absorption spectra [15, 36].

![Figure 3](image-url)
Summary

We have identified an unconventional QHE in FTIs, in the presence of a perpendicular magnetic field, by evaluating their band structure and the Hall and longitudinal conductivities. The low-energy dynamics can be governed by a single-surface in a wide range of Fermi energies. This results in a nontrivial phase transition and unusual Hall plateaus at half-integer multiples of $e^2/\hbar$ ($\pm 1/2, \pm 3/2, \pm 5/2, \ldots$). In addition, reversing the light polarization leads to an exchange of surface states, in both the valence and conduction bands, and to a shift of the $n = 0$ LL into the hole and electron bands, respectively. These findings suggest new directions in experimental research and device applications based on FTIs.

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