An Introduction to the Massive Helicity Formalism with applications to the Electroweak SM

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Abstract. The power of the helicity formalism has been appreciated recently from its application to the massless case, where plenty of formal aspects of multi-legs amplitudes have been derived. However, in order to extend the formalism to the realistic cases, such as the electroweak Standard Model and QCD with massive quarks, some extra inputs are needed. We discuss first the formalism needed to evaluate amplitudes expressed with the modern notation for massive fermions and vectors based on a thorough treatment of Proca vector fields. Then, we present some examples of elementary processes where it is shown how the formalism leads to tremendous simplifications, these include 2-body decays $Z \rightarrow ff, h \rightarrow ff$ and $h \rightarrow W^- W^+$ as well as the 3-body decay $h \rightarrow V f f'$.

1. Introduction

The spinor helicity formalism (SHF) is a tool for calculating scattering amplitudes much more efficiently than the traditional approach. Its versatility is based on the fact that all objects appearing in the Feynman rules for gauge theories (QED, QCD, EW and SUSY) can be written in terms of two-component Weyl spinors (and Pauli sigma matrices). Dotted and undotted Weyl spinors are used, and in fact they are more fundamental than Minkowski four-vectors since they form irreducible representations of the Lorentz group, and unlike Dirac spinors they are not a “mixture” (i.e. a direct sum) of two different representations, furthermore Weyl spinors are easier to handle.

When dealing with a massless fermion, the solutions of the momentum-space Dirac equation have only two non-vanishing components, in that case the distinction between a 4-component Dirac spinor and a 2-component Weyl spinor essentially disappears. These Weyl spinors, which are helicity eigenstates in the massless case, can then be used to rewrite the Feynman rules for external legs (both fermionic and bosonic), vertices and propagators. The expressions obtained for the helicity amplitudes are in general very simple, and they can be squared directly, without any need of spinor completeness relations and Casimir tricks. This is one of the most marvelous advantages of both massless and massive SHF.

The SHF is usually introduced only at the massless level (see references [1]-[5]), while the massive formalism is less studied in the current literature. The only notable exception is Ref. [6], although it is written in a somewhat old-fashioned notation. Other papers, such as [7]-[12], study scattering amplitudes of several theories using mostly BCFW recursion (Britto-Cachazo-Feng-Witten) [13], creating a huge gap between the well-understood massless SHF and the work...
done in the massive SHF even for “simpler” theories such as QED.

The SHF brings huge simplifications (as compared to the usual Feynman’s completeness-relations-trace-evaluation method) to the calculation of scattering amplitudes. In the massive case, although the simplifications are not that large, the method is in general a little bit more tricky than the massless case \[16\]. Finally, we must say that one of the main applications of the modern amplitude calculation techniques is to QCD gluon processes. Using recursion relations such as BCFW, one can calculate multi-gluon scattering amplitudes which would be impossible (in practice) to obtain using “traditional” methods. Of course, gluons are massless, so one does not need to extend the SHF to include massive particles in this case.

Computational tools have been developed to deal with both massless and massive spinor helicity amplitudes (see Refs. \[14\] and \[15\]), which of course comes very handy when dealing with complicated processes. However, one can notice that a self-contained theory of massive helicity amplitudes and detailed applications to simple processes of realistic theories is lacking, which is one of the motivations of this paper.

In this work we show some practical examples where we have applied the massive SHF to different processes (at tree level) in the Electroweak SM without using BCFW or other on-shell (or off-shell) recursion scheme, nor are we using symbolical or numerical evaluation methods.

This paper is structured as follows: Section 2 contains the solutions of the Dirac equation (massive case), as well the solutions of the Maxwell and Proca equations in order to derive the Feynman rules for external fermionic and bosonic lines, then we use the so-called light cone decomposition (LCD) \[15, 16, 17\] to express massive fermions in terms of massless ones. In Section 3 we present some calculations in Electroweak SM. Finally in Section 4 we present our conclusions and final comments of this work.

2. Helicity Method for QED

Consider a theory with a massive Dirac fermion, which is described by Dirac equation. Writing the momentum space Dirac spinor \(u\) in terms of two Weyl spinors the equation of motion takes the form

\[
(\slashed{p} + m)u(p) = \left( \begin{array}{c} m \\ p_\mu \sigma_\mu a \\ m \\ \end{array} \right) \left( \begin{array}{c} \chi_a(p) \\ \xi^a(p) \end{array} \right) = 0. \tag{1}
\]

Through all the paper we shall use the following spacetime metric \(g_{\mu\nu} = \text{diag}(−1, 1, 1, 1)\), also we have used the base where the Dirac matrices take the following form

\[
\gamma^\mu = \left( \begin{array}{cc} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{array} \right), \tag{2}
\]

with \(\sigma^\mu = (1, \vec{\sigma})\) and \(\bar{\sigma}^\mu = (1, -\vec{\sigma})\).

Weyl spinor \((\xi^a, \chi_a)\) could be expressed in terms of bracket labels, additionally the spinor indices could be omitted, this reads as \(\chi = |r\rangle\), \(\xi^a = \frac{m}{(rq)}|q\rangle\), where \(|r\rangle\) and \(|q\rangle\) are 2-component Weyl spinors linked to the light-like momenta \(r^\mu\) and \(q^\mu\) respectively, these arise when one uses LCD to express a massive momentum \(p^\mu\) in term of 2 massless momenta \((p^\mu = r^\mu + \frac{p^2 q^\mu}{2r^\nu q^\nu})\), this approach will be used throughout the paper to express massive spinors in terms of massless ones.

Equation (1) has 8 solutions, (see Ref. \[16\] for a complete direction), these are:

\[
u_+(p) = |r\rangle + \frac{m}{(rq)}|q\rangle \quad \text{and} \quad v_-(p) = -\frac{m}{(rq)}|q\rangle + |r\rangle; \tag{3}
\]

\[
u_+(p) = |r\rangle - \frac{m}{(rq)}|q\rangle \quad \text{and} \quad v_-(p) = -\frac{m}{(rq)}|q\rangle + |r\rangle; \tag{4}
\]
\( \bar{u}_-(p) = \frac{m}{\langle qr \rangle} |q⟩ + ⟨r| \), \( \bar{u}_+(p) = |r⟩ + \frac{m}{\langle qr \rangle} ⟨q| \); \hspace{1cm} (5)

\( \bar{v}_+(p) = -\frac{m}{\langle qr \rangle} |q⟩ + ⟨r| \), \( \bar{v}_-(p) = |r⟩ - \frac{m}{\langle qr \rangle} ⟨q| \); \hspace{1cm} (6)

Spinor products satisfy \([rq] ≡ -[qr]\), similarly \(⟨rq⟩ = -(⟨qr⟩\). For real momenta we have \(⟨pk⟩ ≡ [kp]^*\). When \(m = 0\), one can verify that \(\bar{u}_± = \bar{v}_±\). Equations (3)-(6) will allow us to compute scattering amplitudes in a more practical way than the traditional approach.

For massless spin-1 particles, one can write the transversal polarization vectors as follows \(\epsilon^\mu_+(p) = -\frac{[q r ]}{\sqrt{2}⟨qr⟩} p^\mu\) and \(\epsilon^\mu_-(p) = -\frac{[q r ]}{\sqrt{2}⟨qr⟩} q^\mu\), see Ref.[1, 2], where \(p\) is the spin-1 momentum, and \(q\) is a reference momentum defined such that \(q^2 = 0\). Note that \((\epsilon^\mu_\pm)^*(p) = -\epsilon^\mu_\pm(p)\). The slashed versions of these polarization vectors will also be useful to compute scattering amplitudes in the examples presented later, these are \(\bar{f}_+(p) = \sqrt{2} ⟨p|q⟩⟨q|p|⟩\) and \(\bar{f}_-(p) = \sqrt{2} ⟨p|q⟩⟨q|q|p|⟩\).

Finally, for massive spin-1 bosons (see Ref. [15, 16]) making the identification \(\epsilon^\mu_+,\epsilon^\mu_0(p) \equiv \epsilon^\mu_+,\epsilon^\mu_0(p)\), we can write \(\epsilon^\mu_0(p) = \frac{1}{m} p^\mu + \frac{m}{2 p^2} q^\mu\), where now \(p^2 = -m^2\), the momenta \(r\) and \(q\) are related with the momentum \(p\) by LCD.

3. Elementary Processes in the Standard Model

In this section we will use the SHF to compute several elementary processes in the Standard Model of particle physics. Considering massive particles but using LCD, we shall express the massive 4-component Dirac spinor in terms of the massless 2-component Weyl spinors. This will allow us to exploit all the available identities from the massless SHF.

3.1. 2-body Higgs decay \(h(p_1) → W^+(p_2)W^-(p_3)\)

The helicity amplitude (HA) for the process \(h → W^+W^-\) is as follows

\[ \mathcal{M}_{\lambda_2\lambda_3}(p_1, p_2, p_3) = \frac{2M_h^2}{v} e^\mu_\lambda(p_2) e^\mu_\lambda(p_3), \]

(7)

where \(p_1^2 = -M_h^2\), \(p_2^2 = p_3^2 = -M_W^2\) and the \(\lambda\)'s represent the helicity of the particles. We shall use simultaneous light cone decomposition (SLCD) to express the massive momenta \(p_2\) and \(p_3\) in terms of massless momenta \(r_2, q_2, r_3, q_3\). The massive momenta take the form:

\(p_2 = r_2 - \frac{M_W^2}{2 r_2 q_2} q_2\), and \(p_3 = q_2 - \frac{M_W^2}{2 r_2 q_2} r_2\), here SLCD imposes that \(r_3 = q_2\) and \(q_3 = r_2\). For this process there are \(3^2\) HA’s, \(M_{++}, M_{+-}, M_{-+}, M_{--}\), \(M_{0+}, M_{00}, M_{-0}, M_{0-}\). Using SLCD to fix the massless momenta will be crucial to reduce the number of HA’s, in this case HA’s: \(M_{++}, M_{+-}, M_{00}, M_{-0}, M_{0-}\) vanish. The nonzero HA’s are shown in Table 1.

| \(\lambda_2\lambda_3\) | \(\mathcal{M}_{\lambda_2\lambda_3}\) |
|---------------------|---------------------|
| −−                  | \(2M_h^2 \frac{v}{v} [q_2 q_2^\dagger]\) |
| ++                  | \(2M_h^2 \frac{v}{v} [q_2 q_2^\dagger]\) |
| 00                  | \(-\frac{(2M_h^2)(r_2^\dagger q_2 + M_W^2)}{v M_W^2 (r_2 q_3 + q_2 r_3)}\) |

Table 1. Helicity Amplitudes for the 2-body Higgs decay \(h → W^+W^-\).

In the expressions of Table 1 we have used the following spinor relations

\( [q_2 |\gamma^\mu| q_2] = [q_3 |\gamma^\mu| q_2] \), \hspace{1cm} (8)

\( [q_2 |\gamma^\mu| q_3] = [q_3 |\gamma^\mu| q_2] \), \hspace{1cm} (9)
as well as Fierz identity:

$$\langle q_2 | \gamma^\mu | r_2 \rangle \langle r_3 | \gamma_\mu | q_3 \rangle = 2 \langle q_2 | r_3 \rangle \langle r_2 | q_3 \rangle. \quad (10)$$

The averaged and squared amplitude for the 2-body Higgs decay ($h \to W^+W^-$) is:

$$\langle |M|^2 \rangle = 2|\mathcal{M}_{++}|^2 + |\mathcal{M}_{00}|^2, \quad (11)$$

$$= \left( \frac{2M_W^2}{v} \right)^2 \left( 2 + \left( \frac{1}{2M_W^2 s_{q_2q_3}} \right)^2 (s_{q_2q_3}^2 + M_W^4) \right), \quad (12)$$

$$= \frac{M_h^4}{v^2} \left( 1 - 4x^2 + 12x^4 \right), \quad (13)$$

with $s_{q_2q_3} = -(q_2 + q_3)^2 = -2q_2 \cdot q_3$ and $x = \frac{M_h}{M_W}$, the momenta $q_2$ and $q_3$ are defined as follows [15, 16].

$$q_2 = \frac{\text{sgn}(p_2 \cdot p_3)\sqrt{\Delta} + p_2 \cdot p_3 - p_3^2 p_2}{2 \text{sgn}(p_2 \cdot p_3)\sqrt{\Delta}} \quad \text{and} \quad q_3 = \frac{\text{sgn}(p_2 \cdot p_3)\sqrt{\Delta} + p_2 \cdot p_3 - p_2^2 p_3}{2 \text{sgn}(p_2 \cdot p_3)\sqrt{\Delta}}, \quad (14)$$

with $\Delta = (p_2 \cdot p_3)^2 - p_2^2 p_3^2$. Furthermore we have used in Eq. (11) that $|\mathcal{M}_{--}|^2 = |\mathcal{M}_{++}|^2$. From Eq. (13) we find the decay width $\Gamma$ for the process $h \to W^+W^-:

$$\Gamma(h \to W^+W^-) = \frac{\lambda(1, x, x)^{1/2}}{16\pi M_h} \langle |M|^2 \rangle = \frac{\alpha_W M_h}{16\pi^2} \left( 1 - 4x^2 + 12x^4 \right) \sqrt{1 - 4x^2}, \quad (15)$$

where $\alpha_W = \frac{M_h^2}{v^2}$ and the term $\sqrt{1 - 4x^2}$ is the W velocity in the Higgs rest reference frame.

### 3.2. 2-body Higgs decay $h(p_1) \to f(p_2)\bar{f}(p_3)$

The HA for the process $h \to f\bar{f}$ is the following:

$$\mathcal{M}_{\lambda_2\lambda_3}(p_1, p_2, p_3) = \frac{1}{v} \bar{u}_{\lambda_2}(p_2) v_{\lambda_3}(p_3), \quad (16)$$

where $p_1^2 = -M_h^2$, $p_2^2 = p_3^2 = -m_f^2$. There are 2^2 HA’s, $\mathcal{M}_{--}$, $\mathcal{M}_{-+}$, $\mathcal{M}_{+-}$ and $\mathcal{M}_{++}$, but using SLCD for momenta $p_2$ and $p_3$, $\mathcal{M}_{-+}$ and $\mathcal{M}_{+-}$ vanish. The nonzero HA’s are shown in Table 2.

| $\lambda_2\lambda_3$ | $\mathcal{M}_{\lambda_2\lambda_3}$ |
|---------------------|-------------------------------|
| $- -$               | $\frac{m_f}{v_{q_2q_3}}(s_{q_2q_3} - m_f^2)$ |
| $+ -$               | $\frac{m_f}{v_{q_2q_3}}(s_{q_2q_3} - m_f^2)$ |

**Table 2.** Helicity Amplitudes for the Higgs decay $h \to f\bar{f}$.

The averaged squared amplitude is then:

$$\langle |M|^2 \rangle = 2|\mathcal{M}_{--}|^2 = 2|\mathcal{M}_{++}|^2 = \frac{2m_f^2}{v^2 s_{q_2q_3}} (s_{q_2q_3} - m_f^2)^2 = \frac{y^2}{v^2} (1 - 4y^2)^{3/2}, \quad (17)$$

with $y = \frac{m_f}{M_h}$. Then the decay width $\Gamma$ goes as follows

$$\Gamma(h \to f\bar{f}) = \frac{\alpha_W M_h y^2}{8} (1 - 4y^2)^{3/2}. \quad (18)$$
The decay width for this channel is then:

\[ \mathcal{M}_{\lambda_1 \lambda_2 \lambda_3} = \frac{1}{2} g_Z \epsilon_{\mu \lambda_1} (p_1) \bar{u}_{\lambda_2}(p_2) \gamma^\mu (v_f - a_f \gamma^5) v_{\lambda_3}(p_3). \]  

We shall assume that \( m_f \ll M_Z \equiv M \), that could not true when the fermion is the quark top, while for the other cases in practice it will be equivalent to consider massless fermions (\( p_f^2 = p_{\bar{f}}^2 = 0 \)). Then the amplitude will vanish unless \( f \) and \( \bar{f} \) have opposite helicities. We choose the arbitrary reference momentum \( q_1 = p_2 \), then by momentum conservation (\( \sum_{i=1}^{n} [q_i] \langle ik \rangle = 0 \))

\[ \langle p_3 r_1 | r_{1p_2} | 0 \rangle, \]  

Using Eq.(20), the independent HA’s are shown in Table 3.

| \( \lambda_1 \lambda_2 \lambda_3 \) | \( \mathcal{M}_{\lambda_1 \lambda_2 \lambda_3} \) |
|---|---|
| ++ - | \( \frac{1}{2} g_Z \frac{[p_2]_{\gamma_\mu} [r_1]}{\sqrt{2}} (v_f - a_f) [p_2] [\gamma^\mu] [p_3] = \frac{g_Z (v_f - a_f)}{\sqrt{2} M} \langle r_{1p_2} | r_1p_2 \rangle \) |
| 0 + - | \( - \frac{1}{2} g_Z \left( \frac{M}{p_{2p_2}} p_{2\mu} - \frac{M}{p_{2p_2}} p_{1\mu} \right) (v_f - a_f) [p_2] [\gamma^\mu] [p_3] = \frac{g_Z (v_f - a_f)}{\sqrt{2} M} \langle r_{1p_2} | r_1p_2 \rangle \) |
| - + - | \( \frac{1}{2} g_Z \frac{[r_1]_{\gamma_\mu} [p_2]}{\sqrt{2}} (v_f - a_f) [p_2] [\gamma^\mu] [p_3] = 0 \) |

**Table 3. Helicity Amplitudes for the Higgs decay \( h \to f \bar{f} \).**

The rest of the HA’s are obtained by complex conjugation. Besides, because of the \( \gamma^5 \) matrix, the sign of the coefficient \( a_f \) will change. This is because \( \gamma^5 v_- (p) = v_- (p) \) but \( \gamma^5 v_+ (p) = -v_+ (p) \). Then we have

\[ \mathcal{M}_{- - -} = \frac{g_Z (v_f + a_f)}{\sqrt{2}} \frac{[p_2 p_3] [r_1 p_2]}{[r_1 p_2]} \].

\[ \langle |\mathcal{M}|^2 \rangle = \frac{1}{3} \left( |\mathcal{M}_{++-}|^2 + |\mathcal{M}_{--+}|^2 \right) = \frac{g_Z^2 M^2}{3} \left( |v_f|^2 + |a_f|^2 \right). \]

The decay width for this channel is then:

\[ \Gamma (Z \to f \bar{f}) = \frac{g_Z^2 M}{48 \pi} \left( |v_f|^2 + |a_f|^2 \right). \]

### 3.4. 3-body Muon Decay \( \mu(p_1) \to \bar{\nu}_e (p_2) \nu_\mu (p_3) e^- (p_4) \)

The HA for the process \( \mu \to \bar{\nu}_e \nu_\mu e^- \) is as follows

\[ \mathcal{M}_{\lambda_1 \lambda_2 \lambda_3 \lambda_4} = \left( \frac{g_W}{\sqrt{8 M_W}} \right)^2 \left[ \bar{u}_{\lambda_3}(p_3) \gamma^\mu (1 - \gamma_5) u_{\lambda_1}(p_1) \right] \left[ \bar{u}_{\lambda_4}(p_4) \gamma_\mu (1 - \gamma_5) v_{\lambda_2}(p_2) \right], \]

\[ = \left( \frac{g_W}{\sqrt{8 M_W}} \right)^2 A_{\lambda_3 \lambda_1} \mathcal{B}_{\mu \lambda_4 \lambda_2}, \]
where $p_1^2 = -m_e^2$, $p_2^2 = -m_e^2$, $p_3^2 = 0$ and $p_4^2 = 0$. We have defined $A_{\lambda_3\lambda_1}^\mu$ and $B_{\mu\lambda_3\lambda_2}$ as follows

$$A_{\lambda_3\lambda_1}^\mu = 2\bar{u}_{\lambda_3}(p_3)\gamma^\mu \hat{P}_L u_{\lambda_1}(p_1),$$

$$B_{\mu\lambda_3\lambda_2} = 2\bar{u}_{\lambda_4}(p_4)\gamma_{\mu} \hat{P}_L v_{\lambda_2}(p_2),$$

From equations (28) and (29) one obtains the following HA’s: $A^{+-} = 0$, $A^{--} = 0$, $A^{++} = 2m_e(q_1\gamma^\mu|q_1)$, $A^{++} = 2[p_3|\gamma^\mu|p_1]$, and $B^{++} = 0$, $B^{--} = 0$, $B^{--} = 2m_e(q_4|\gamma_{\mu}|p_2)$, $B^{++} = 2[r_4|\gamma_{\mu}|p_2]$, in Table 4 we show the nonzero products for the partial helicity amplitudes $A_{\lambda_3\lambda_1}^\mu$ and $B_{\mu\lambda_3\lambda_2}$.

| $\lambda_1\lambda_2\lambda_3\lambda_4$ | $A_{\mu\lambda_3\lambda_1}$ | $B_{\mu\lambda_3\lambda_2}$ | $A_{\mu\lambda_3\lambda_1} B_{\lambda_3\lambda_2}$ | $M(p_2 = q_1, p_3 = q_4)$ |
|--------------------------------|-----------------|-----------------|-----------------|-----------------|
| $++-+$ | $2[p_3|\gamma^\mu|p_1]$ | $2[r_4|\gamma|p_2]$ | $4(p_2 r_1)\langle p_3 r_4 \rangle$ | $2\sqrt{2}\bar{m}_e$ |
| $+-+-$ | $2[m_e(q_1\gamma|q_1)]$ | $2[m_e(q_1\gamma|q_1)]$ | $4\bar{m}_e(q_2 r_1)\langle p_3 q_4 \rangle$ | $0$ |
| $--++$ | $2[m_e(q_1\gamma|q_1)]$ | $2[r_4|\gamma|p_2]$ | $4\bar{m}_e(q_2 r_1)\langle p_3 q_4 \rangle$ | $0$ |
| $-+-+$ | $2[m_e(q_1\gamma|q_1)]$ | $2[m_e(q_1\gamma|q_1)]$ | $4\bar{m}_e(q_2 r_1)\langle p_3 q_4 \rangle$ | $0$ |

Table 4. Helicity Amplitudes for Muon decay ($\mu \rightarrow e^- \bar{\nu}_e - \nu_\mu$) with the momentum assignment $r_2 = q_1$ and $r_3 = q_4$.

We have used in the fourth column of the Table 4 the reflection property Eq. (8) and Fierz identity Eq. (10), while in the last column it was chosen $r_2 = q_1$ and $r_2 = q_4$, this reduces all the helicity amplitudes to one ($M^{++--}$). The squared and averaged amplitude for the muon decay is:

$$\langle |M^{++--}|^2 \rangle = \frac{1}{2} |M^{++--}|^2 = 2 \left( \frac{g_W}{M_W} \right)^4 (p_1 \cdot p_2)(p_3 \cdot p_4)$$

From this result we can arrive to the decay width, which agrees with results of textbooks.

3.5. 3-body Higgs decay $h(p_1) \rightarrow Z(p_2) f(p_3) \bar{f}(p_4)$

The HA for this process is the following

$$M_{\lambda_2\lambda_3\lambda_4} = \frac{M_Z^2}{v} \frac{e}{c_W s_W} \epsilon_{\lambda_2}^\mu(p_2) \left( g_{\mu\nu} + \frac{k_{\mu} k_{\nu}}{M_Z^2} \right) \frac{1}{k^2 + M_Z^2} \bar{u}_{\lambda_3}(p_3)\gamma^\nu(v_f - a_f \gamma^5)v_{\lambda_4}(p_4).$$

We will consider just the case when the fermions are leptons ($e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau$), with this approximation the leptons are consider as massless, so they must have opposite helicities. Then, remembering that $k = p_3 + p_4$, we have

$$k_{\mu}\bar{u}_+(p_3)\gamma^\nu(v_f - a_f \gamma^5)v_-(p_4) = (v_f - a_f)[k_3(p_3 + p_4)]p_4) = 0,$$

$$k_{\mu}\bar{u}_-(p_3)\gamma^\nu(v_f - a_f \gamma^5)v_+(p_4) = (v_f + a_f)[k_3(p_3 + p_4)]p_4 = 0;$$

it is very hard to find this kind of simplification (Equations (32) and (33)) with the traditional approach of computing scattering amplitudes, but now the amplitude (31) becomes simply

$$M_{\lambda_2\lambda_3\lambda_4} = C \bar{u}(p_3)\bar{f}(p_2)(v_f - a_f \gamma^5)v(p_4),$$

with $C = C_{\lambda_2\lambda_3\lambda_4}$. 

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where we have defined \( C \equiv \frac{M_Z^2}{v} \frac{e}{c} \frac{1}{k^2 + M^2_Z} \). Choosing \( q_2 = p_3 \), we get

\[
\begin{align*}
\phi^+(p_2) &= \sqrt{2} \langle r_2 | (p_3 | p_3) | p_2 \rangle, \\
\phi^-(p_2) &= \sqrt{2} \langle r_2 | (p_3 | p_3) | p_2 \rangle, \\
\phi_0(p_2) &= \frac{1}{M_Z} \phi^+ + \frac{M_Z}{2p_{23}} p_3.
\end{align*}
\]

The nonzero HA’s are show below in the Table 5.

| \( \lambda_2 \lambda_3 \lambda_4 \) | \( |\mathcal{M}|^2 \) |
|---|---|
| ++ -- | \( C |p_3| \langle \frac{\sqrt{2}}{\langle r_2 | p_3 \rangle} \rangle |p_3| |p_3| |r_2| |p_4| = \sqrt{2}C |v_f |a_f |p_3| |p_3| |p_4| \) |
| -- ++ | \( C \langle \frac{\sqrt{2}}{\langle r_2 | p_3 \rangle} \rangle |p_3| |p_3| |r_2| |p_4| = \sqrt{2}C |v_f |a_f |p_3| |p_3| |p_4| \) |
| 0 + -- | \( C |p_3| (\frac{1}{M_Z} \phi^+ + \frac{M_Z}{2p_{23}} p_3) |p_3| |r_2| |p_4| = \frac{C v_f |a_f |p_3| |r_2| |p_4|}{M_Z} \) |
| 0 -- + | \( C |p_3| (\frac{1}{M_Z} \phi^+ + \frac{M_Z}{2p_{23}} p_3) |p_3| |r_2| |p_4| = \frac{C v_f |a_f |p_3| |r_2| |p_4|}{M_Z} \) |

**Table 5.** Helicity Amplitudes for the 3-body Higgs decay \( h \rightarrow Z f f \).

Therefore

\[
\langle |\mathcal{M}|^2 \rangle = 8C^2 \left( |v_f|^2 + |a_f|^2 \right) \left[ -p_{34} + \frac{r_2 \cdot p_3}{M_Z^2} \right],
\]

From momentum conservation we can obtain; \( 2p_{23} = M_Z^2 - M_H^2 - 2p_{24} - 2p_{34}. \) Substituting everything in Eq.(38) we obtain

\[
\langle |\mathcal{M}|^2 \rangle = \sum |\mathcal{M}|^2 = 8C^2 \left( |v_f|^2 + |a_f|^2 \right) \left[ -p_{34} + \frac{2p_{24}}{M_Z^2} \left( p_{24} + p_{34} + \frac{M_H^2}{2} - p_{24} \right) \right].
\]

4. Conclusions

In this paper we have presented a summary of the basic formulae of the SHF. In order to appreciate the value of the methods, we studied the phenomenology of the Electroweak sector of the Standard Model of Particle Physics, including the evaluation of the decays: \( Z \rightarrow ff \), \( h \rightarrow ff \), \( h \rightarrow W^- W^+ \) and \( h \rightarrow V f f \). Although these results are well known, it can be appreciated that the simplification obtained by using SHF, make it worth to use them in teaching of Particle Physics with a modern approach.

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