Time Reversal ESPRIT Imaging with Frequency-domain Sampling of Single Target

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Abstract. A time-reversal imaging method, based on frequency-domain data sampling and the estimation of signal parameters via rotational invariance techniques (ESPRIT), is proposed to obtain imaging with higher resolution in this paper. The proposed algorithm is obtained, which firstly applies the coarse and fine frequencies sampling to all received scatter signal, thus forming the plurality of frequency-frequency multistatic data matrices (FF-MDM); subsequently, the signal subspace is obtained through the application of singular value decomposition (SVD) to the FF-MDM; finally, the imaging pseudo-spectrum can be acquired by processing to the signal subspace with ESPRIT. The simulation results show that the proposed method has better resolution for imaging as compared with conventional imaging methods.

1. Introduction

Time Reversal (TR) has recently become the focus of the imaging research because of its remarkable anti-multipath and space-time focusing ability, which was first proposed by Mathias Fink in 1989. Based on array signal processing techniques, Mathias Fink made some processing in sound waves, such as sampling, transforming, storing, time reversals and re-emits. Therefore incident sound waves can be focused at the location of the acoustic wave emission in the inhomogeneous medium [1]. Subsequently, the research team led by Fink introduced TR technology into the field of UWB electromagnetic waves [2].

The TR operator (TRO) is, plays an essential role in time reversal imaging (TRI), obtained from the scattering multistatic data matrix (MDM). The TRO is generally formed from a space-space MDM (SS-MDM) at a single frequency (such as a central frequency). The decomposition of the TR operator (DORT) and the TR multiple signal classification (TR-MUSIC) can be obtained by performing eigenvalue decomposition on the TRO. When the central frequency (CF) of operation is employed, DORT and TR-MUSIC are considered as CF-DORT method and CF-TR-MUSIC method respectively, which can also be applied to the entire UWB band, called time domain DORT (TD-DORT) [3] and UWB-TR-MUSIC [4]. However, after eigenvalue decomposition of TRO at different frequencies, the feature vector will produce a random phase dependenting on frequency. Therefore, TD-DORT needs to perform a pre-processing on the feature vector to obtain the coherent time-domain pulse before returning the signal [5].

In [6], the coherent time domain vector is directly obtained through the singular value decomposition (SVD) of space-frequency MDM (SF-MDM). The SF-MDM columns and rows include the space and frequency components of the received signals, respectively. The left singular
vector contains the target position information which could be obtained by applying SVD to the SF-MDM. But it has the same phase shift information at different frequency components, weakening the coherence of the return signal at the target position. In addition, a large number of echo signals need to be measured by DORT and TR-MUSIC in order to form SS-MDM. There is an increase in the amount of computation due to performing SVD on SS-MDMs of different frequencies. As for the SF-DORT method, although it avoids repetitive matrix decomposition at different frequency points, its positioning accuracy is low. It still needs to measure a large number of echo signals for processing when full SF-MDM is used.

In this paper, a new frequency-frequency ESPRIT (FF-ESPRIT) time-reversal imaging method based on the estimation of signal parameters via rotational invariance techniques (ESPRIT) is proposed, due to the limitations of the above methods. A single transmitter and moving receiver are used, instead of a set of transceivers at different locations in the [8]. The FF-MDM is obtained by using the fine and coarse frequency sampling to echo scatter signal [7-9]. And the inconsistent modification phase will be automatically eliminated by FF-MDM. Then the SVD is applied to the FF-MDM. Finally, the imaging pseudo-spectrum is obtained by using the signal subspace rotation invariance [10]. Simulation results show that FF-ESPRIT has higher imaging resolution than SF-DORT and FF-DORT.

2. FF-MDM Imaging

In this paper, an array of electromagnetic wave transceiver (transmitter and receiver) is considered, which is transmitting an excitation signal through a transmitter to the detection area and recording the received signals on the moving receiver. Then, the single received signal can be cast into a $N \times P$ matrix composed of fine ($\omega_f$) and coarse ($\omega_c$) frequency samples given as:

$$K_{FF}^n = \begin{bmatrix} k_{f1}^n(\omega_f) & \cdots & k_{fF}^n(\omega_f) \\ \vdots & \ddots & \vdots \\ k_{c1}^n(\omega_c) & \cdots & k_{cP}^n(\omega_c) \end{bmatrix}$$  \hspace{1cm} (1)

where $n$ is the $n$-th location of the receiver, $n = 1, \cdots, L$, $\omega_f = \omega_0 + (i-1)\omega_c + j\omega_f$ with $\omega_c$ being the starting frequency of sampling, $i = 1, \cdots, N$, $j = 1, \cdots, P$. The rows and columns of the aforementioned matrix correspond to the fine ($\omega_f$) and coarse ($\omega_c$) frequency samples, respectively [9].

Distortion waveform Born approximation (DWBA) is a standard approximation used in most time inversion and inverse scattering studies, and it ignores all multiple scattering between various targets. The two central assumptions for DWBA are that the scatterers are point-like and well resolved; one can use DWBA and write $k_{\theta}^n$ as:

$$k_{\theta}^n = \sum_{m=1}^{M} \tau_m G(s, x_m, \omega_f) G(x_m, r_n, \omega_f) s(\omega_f)$$  \hspace{1cm} (2)

where

$$G(r, x_m, \omega) = \frac{i}{4} H_0(k|\mathbf{r} - \mathbf{x}_m|)$$  \hspace{1cm} (3)

is the free space Green’s function and $H_0$ is the zeroth-order Hankel function, $k$ is the wave number at frequency $\omega$, $G(s, x_m, \omega_f)$ is the background Green’s function between the transmitting $s$ and the $m$-th scatterer location $x_m$, and the $G(x_m, r_n, \omega_f)$ is the background Green’s function of the $m$-th scatterer to the receiver location $r_n$ [11], $M$ is the number of scatterers, $\tau_m$ is the scattering strengths of the $m$-th scatterer, and $s(\omega)$ is the input excitation signal in the frequency domain. Green’s function can be decomposed into two parts. One part is the function of $\omega_f$, and the other one is the function of $\omega_f$. So, $K_{FF}^n$ can be written as:


\[ K_{FF}^{\omega} = \sum_{m=1}^{M} Z_n(x_m, \omega) g_n(x_m, \omega) g_n^T(x_m, \omega_f) \]

where

\[ g_n(x_m, \omega) = \left[ G(s, x_m, \omega_1), \cdots, G(s, x_m, \omega_{N_1}) \right] \times \left[ G(x_m, r_n, \omega_1), \cdots, G(x_m, r_n, \omega_{N_1}) \right]^T \]

\[ g_n(x_m, \omega_f) = \left[ G(s, x_m, \omega_1), \cdots, G(s, x_m, \omega_{P_1}) \right] \times \left[ G(x_m, r_n, \omega_1), \cdots, G(x_m, r_n, \omega_{P_1}) \right]^T \]

are the \( N \times 1 \) and \( P \times 1 \) background Green’s function vectors that connect the \( m \)-th scatterer location to the \( n \)-th receiver and \( X_n(x_m) \) is a coefficient in accordance with Green’s function approximation and the reflection strength of the \( m \)-th scatterer, \( \times \) is expressed as cross products. The SVD of FF-MDM yields

\[ K_{FF}^{\omega} = U_{FF}^n A_{FF}^n (V_{FF}^n)^H . \]

\( U_{FF}^n \) is a \( N \times N \) left singular vector corresponding to the coarse frequency portion of the signal, \( V_{FF}^n \) is the right singular vector of \( P \times P \) corresponding to the fine frequency part, \( A_{FF}^n \) is a singular value matrix of \( N \times P \). The singular vectors corresponding to the non-zero singular values in \( U_{FF}^n \) and \( V_{FF}^n \) can represent the signal subspace, and the singular vectors corresponding to the zero singular values can represent the noise subspace. In the single target case, the signal subspace can be represented by the singular vector corresponding to the largest singular value. Therefore, using the singular vectors of FF-MDMs for the receiver in all different locations, indicates the FF-DORT imaging functions as:

\[ D^{\omega}(x_s) = \prod_{n=1}^{N_1} \left\langle g_n(x_s, \omega), u_{FF}^{n} \right\rangle \left\langle g_n(x_s, \omega), v_{FF}^{\nu} \right\rangle \]

where \( \left\langle \cdot, \cdot \right\rangle \) represents the standard inner product, and \( L \) represents the number of movements of the receiving antenna.

Using the linear phase of the signal subspace, ESPRIT is the following application in our paper. The phase of signal subspace \( u_{FF}^{np} \) can be divided into two sub-arrays \( u_{FF}^{np}{1} \) and \( u_{FF}^{np}{2} \) :

\[ u_{FF}^{np} = \begin{bmatrix} u_{FF}^{np}{1} \\ u_{FF}^{np}{2} \end{bmatrix} \]

where sub-array \( u_{FF}^{np}{1} \) and \( u_{FF}^{np}{2} \) are \( N \) column vectors. Define the rotation operator corresponding to the subspace is \( \Phi \), which is the diagonal \( \frac{N}{2} \times \frac{N}{2} \) matrix. Then the relationship between sub-arrays \( u_{FF}^{np}{1} \) and \( u_{FF}^{np}{2} \) can be expressed as a rotation operator:

\[ u_{FF}^{np}{2} = \Phi u_{FF}^{np}{1} \]

Similarly, the phase of the background Green's function also has the following conversion:

\[ g_{n}(x_m, \omega) = \begin{bmatrix} g_{n1}(x_m, \omega) \\ g_{n2}(x_m, \omega) \end{bmatrix} \]

by introducing the corresponding rotation operator \( \Psi \), the above sub-array can be expressed as:

\[ g_{n}(x_m, \omega) = \Psi' g_{n}(x_m, \omega) \]

Definitions \( \varphi_i \) and \( \psi_i \) are the \( i \)-th primary diagonal elements of \( \Phi \) and \( \Psi' \), respectively. Accumulate the errors between \( \varphi_i \) and \( \psi_i \) in the imaging area, \( e_{nu} \) is

\[ e_{nu} = \frac{\sum_{i=1}^{N/2} |\psi_i| - |\varphi_i|}{\sum_{i=1}^{N/2} |\varphi_i|} \]
Similarly, the phase of subspace \( v^{n}_{FFm} \) and \( g_{n}(x_{m}, \omega_{n}) \) can be processed as \( e_{nm} \) to obtain the \( e_{nv} \), so the imaging pseudo-spectrum is expressed as:

\[
E^{\Omega}(\mathbf{x}) = \left\{ \prod_{n=1}^{L} \left( e_{nm} \overline{e_{mv}} \right) \right\}^{-1}
\]  

(13)

3. Simulation Results

In this section, the echo signal was generated by the finite difference time domain (FDTD) method. A grid of \( N_{x} \times N_{y} = 400 \times 400 \) cells is used with perfectly matched layers and a spatial cell size of \( \Delta x = \Delta y = \frac{\lambda_{c}}{32} \) is chosen where \( \lambda_{c} \) is the wavelength of the central frequency. Set a fixed transmitter and a moving receiver, as shown in Figure 1.

\[ \text{Target} \]

\[ \text{Transmitter} \]

\[ \text{Receiver} \]

**Figure 1.** The receiver and a point target are indicated by a black triangular marker and circular marker, and the transmitter is indicated by a blue triangular marker.

The Perfect Electric Conductor (PEC) point-like target is located at (1.8m, 2.7m) in the probed domain. The excitation signal is a 1 GHz Gaussian pulse with a central frequency, and effective bandwidth 1GHz. Set \( \omega_{c} = 250 \text{MHz} \) and \( \omega = 25 \text{MHz} \). Then apply the SVD to FF-MDM \( K_{FF}^{n} = U_{FF}^{n} A_{FF}^{n} (V_{FF}^{n})^H \) and target imaging using the singular vector corresponding to the largest singular value.

Figure 2 plots an image result by the general SF-DORT method, and Figure 3 and Figure 4 are showed that images of the above FF-DORT and FF-ESPRIT methods. Figure 4 is zoomed in to show the focusing details clearly. Compared the results of FF-ESPRIT methods with others, and showing that the proposed FF-ESPRIT method has higher imaging resolution than the standard SF-DORT and FF-DORT.

\[ \text{(a)} \]

\[ \text{(b)} \]

**Figure 2.** (a) SF-DORT imaging pseudo-spectrum. (b) SF-DORT imaging.
4. Conclusion
In this paper, a new TR imaging method is proposed, which is the combination of FF-MDM singular value decomposition and ESPRIT algorithm. The theoretical derivation is given in detail. The simulated results show that the proposed method improves the imaging resolution compared with the similar TR imaging methods, and the new algorithm can potentially be employed in synthetic aperture radar (SAR) application.

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