Tachyonic open inflationary universes

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Abstract

We study one-field open inflationary models in a universe dominated by tachyon matter. In these scenarios, we determine and characterize the existence of the Coleman–De Lucia (CDL) instanton. Also, we study the Lorentzian regime, that is, the period of inflation after tunneling has occurred.

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1. Introduction

Cosmological inflation has become an integral part of the Standard Model of the universe. Apart from being capable of removing the shortcomings of the standard cosmology, it gives important clues for structure formation in the universe. The scheme of inflation [1] (see [2] for a review) is based on the idea that at early times there was a phase in which the universe evolved through accelerated expansion in a short period of time at high energy scales. During this phase, the universe was dominated by a potential \( V(\phi) \) of a scalar field \( \phi \), which is called the inflaton.

Normally, inflation has been associated with a flat universe, due to its ability to effectively drive the spatial curvature to zero. In fact, requiring sufficient inflation to homogenize random initial conditions, drives the universe very close to its critical density. In this context, the recent observations from WMAP [3] are entirely consistent with a universe having a total energy density that is very close to its critical value, where the total density parameter has the value \( \Omega = 1.02 \pm 0.04 \). Most people interpret this value as corresponding to a flat universe. But, according to this result, we might take the alternate point of view of a marginally open or closed universe model [4], which at early times in the evolution of the universe presents an inflationary period of expansion. This approach has already been considered in the literature [5–9] in the context of the Einstein theory of relativity, Jordan–Brans–Dicke or brane world cosmology. At this point, we should mention that Ratra and Peebles were the first to elaborate on the open inflation model [10]. The basic idea in an open universe is that a symmetric bubble nucleates in the de Sitter space background, and its interior undergoes a stage of slow-roll inflation, where the parameter \( \Omega_0 \) can be adjusted to any value in the range \( 0 < \Omega_0 < 1 \). Bubble formation in the false vacuum is described by the Coleman–De Lucia (CDL) instantons [11]. Once a bubble has taken place by this mechanism, the inside of the bubble looks like an infinite open universe. The problem with this sort of scenario is that the instanton exists only if the following inequality \( |V_{,\phi\phi}| > H^2 \) is satisfied during part of the tunneling process [12]. On the contrary, during inflation the inequality \( |V_{,\phi\phi}| \ll H^2 \) is satisfied (slow-roll approximation). Linde solves this problem by proposing a simple one-field model in Einstein’s general relativity (GR) theory [6,7] where the crucial point is the very peculiar shape of the effective scalar potential. This scheme has been also successfully used in other more general models [8,9].

On the other hand, as was mentioned before, one normally considers the inflation phase driven by the potential or vacuum energy of a scalar field, whose dynamics is determined by the Klein–Gordon action. However, more recently and motivated by string theory, other non-standard scalar field actions

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have been used in cosmology. In this context the deep interplay between small-scale non-perturbative string theory and large-scale brane-world scenarios has raised the interest in a tachyon field as an inflationary mechanism, especially in the Dirac–Born–Infeld action formulation as a description of the D-brane action [13]. In this scheme, rolling tachyon matter is associate with unstable D-brane. The decay of these D-branes produce a pressureless gas with finite energy density that resembles classical dust. Cosmological implications of this rolling tachyon were first studied by Gibbons [14] and in this context it is quite natural to consider scenarios where inflation is driven by this rolling tachyon.

From the above point of view, we shall study a single-field open inflationary universe, where inflation is driven by a tachyon field. Therefore, we model our potential by a single tachyon field \( \phi \) which presents an exponential term and a barrier generated by a expression which presents a peak with a \( \phi \) tachyon field. Therefore, we model our potential by a single classical dust. Cosmological implications of this rolling tachyon with unstable D-brane. The decay of these D-branes produce a Born–Infeld action formulation as a description of the D-brane field as an inflationary mechanism, especially in the Dirac–scale brane-world scenarios has raised the interest in a tachyon between small-scale non-perturbative string theory and large-scale brane-world scenarios has raised the interest in a tachyon field.

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The mechanism of tachyon decay via quantum tunneling has been studied in the context of brane–antibrane and dielectric brane decays [15], where tachyon potential with similar characteristics of our potential have been derived and studied. In our work we are particularly interested in the cosmological implication of this process and its application to open inflationary models.

The Letter is organized as follows. In Section 2, we write and numerically solve the tachyon field equations in an Euclidean space–time. Here the existence of the CDL instanton for a tachyon model is studied, together with the basic properties of the bubble which is created during the tunneling process. In Section 3, we study the characteristics of the open inflationary universe model that is produced after tunneling has occurred. We determine the corresponding scalar density perturbations

We are going to consider \( \lambda \) and \( V_0 \) as free parameters, and \( \alpha \), \( \beta \) and \( v \) as arbitrary constants, which will be set by phenomenological considerations, as we shall show below. In the following, we shall take \( \lambda > 0 \). The first term of the effective potential controls inflation after quantum tunneling has occurred. Its form coincides with that used in the simplest tachyonic inflationary universe model where \( V(\phi) = V_0 e^{-\lambda \phi} \) [17]. In the same spirit of Ref. [6], the second term controls the bubble nucleation, whose role is to create an appropriate shape in the tachyon potential, \( V(\phi) \), where its local maximum occurs at the top of the barrier \( \phi = \phi_t \) (see Fig. 1). After the quantum tunneling, we should note that the stable vacuum to which the tachyon condenses, is at \( \phi \rightarrow + \infty \), when \( V(\phi) \rightarrow 0 \).

Tachyon models which are classically stable but decay by quantum tunneling have been used in the context of non-standard D-brane decays. In these models it has been reported that together with normal annihilation of branes via gravitational forces, branes can decay in another way, that is through the tunnel effect by creation of a throat between the brane and the antibrane [15,18,19]. In these models, tachyon potentials with similar characteristic as (2) appear, due for example, to the \( U(1) \) charge of the tachyon field in the brane–antibrane configuration.

Nevertheless our model has been motivated by string theory, we consider it just at a phenomenological level without claiming any direct identification of \( \phi \) with the string tachyon field. Indeed, as it was mentioned in Refs. [20,21], there are problems with inflation paradigm when the origin of \( \phi \) is traced in string theory. Then, we take the approach of Refs. [17,22,23] and set the parameters of the model according to phenomenological consideration.

The \( O(4) \)-invariant Euclidean space–time metric is described by

\[
\begin{align*}
\Delta s^2 &= \Delta \tau^2 + a^2(\tau) \left[ d\psi^2 + \sin^2(\psi) d\Omega_2^2 \right],
\end{align*}
\]
where \( a(\tau) \) is the scale factor of the universe and \( \tau \) represents the Euclidean time.

The equation governing the evolution of factor \( a(\tau) \) is

\[
\left( \frac{a'}{a} \right)^2 = \frac{1}{a^2} - \frac{\kappa}{3} \left[ \frac{V(\phi)}{\sqrt{1 + \phi^2}} \right],
\]

and, after varying (1) with respect to \( \phi \), we obtain the equation of motion of the tachyon field

\[
\frac{\phi''}{1 + \phi^2} = -3\frac{a'}{a} \phi' + \frac{1}{V} dV, 
\]

where the primes denote derivatives with respect to \( \tau \).

From Eqs. (4) and (5) we obtain

\[
a'' = -\frac{\kappa}{3} a \frac{V(\phi)}{\sqrt{1 + \phi^2}} \left( 1 + \frac{3}{2} \phi^2 \right). 
\]

Now, we proceed to find a criterion for the existence of a CDL instanton in a tachyon model. This criterion could be found in a similar way as it is done in an universe described by Einstein’s general relativity, where the matter content is described by a scalar field [6]. In this approach, the bubbles have size greater than the Compton wavelength of the scalar field, \( r \gtrsim m^{-1} \sim (V,\phi \phi)^{-1/2} \), where \( m \) is the mass associated of the scalar field. In this way, the instanton CDL solution can exist only if the bubble can fit into de Sitter sphere of radius \( r \sim H^{-1} \), therefore the CDL instantons are possible only if \( H^2 < V,\phi \phi \). In our case, according to Ref. [24] the mass of the tachyon field is given by

\[
m^2 = \frac{V,\phi \phi}{V} - \left( \frac{V,\phi}{V} \right)^2 \left( 3 - \frac{V^2}{\rho^2} \right),
\]

where \( \rho \) is the energy density of the tachyon field. Therefore, the CDL instantons are possible in the tachyonic case if the condition

\[
m^2 = \frac{V,\phi \phi}{V} - \left( \frac{V,\phi}{V} \right)^2 \left( 3 - \frac{V^2}{\rho^2} \right) > H^2,
\]

is satisfied during the tunneling process.

Now, in order to obtain a numerical solution of the field equations (5) and (6), we will choose a particular election of the parameters that appears in the scalar potential (2). In particular, we use the COBE normalized value for the amplitude of the scalar density perturbations in order to estimate \( \lambda \) and \( V_0 \) [17].

Thus we have \( \lambda = 10^{-3} \kappa^{-1/2} \) and \( V_0 = 10^{-7} \kappa^{-2} \). We also consider \( \beta^2 = 2 \alpha^2 \) with \( \beta = 6.67 \times 10^3 \kappa^{1/2} \) and \( v = 3 \times 10^5 \kappa^{1/2} \), which, as we will see, will provide about 46 e-folds of inflation after the tunneling. The lower value of the e-folding is not a problem, since in the context of the tachyonic-curvaton reheating, it could be of the order 45 or 50, since the inflationary scale can be lower [25].

We solve Eqs. (4) and (6) considering the following boundary conditions: \( \phi = \phi_T \), \( \phi' = 0 \), \( a = 0 \), \( a' = 1 \), at \( \tau = 0 \) and working in units where \( \kappa = 1 \). Actually, the instanton has the topology of a four-sphere and there are two places where the scale factor \( a(\tau) \) vanishes. These are the points at which \( \tau = 0 \) and \( \tau = \tau_{\text{max}} \). Then, the boundary conditions on \( \phi \) arise from the requirement that the term in the scalar field Eq. (5) \( 3\phi' a'/a \), be finite at these point i.e., \( \phi'(0) = \phi' (\tau_{\text{max}}) = 0 \). From Eq. (4), we get that at the points where of the scalar factor vanishes we have, \( a' = \pm 1 \). Fig. 2 shows how the scale factor evolves during the tunneling process. A numerical solution which corresponds to the CDL instanton is showed in Fig. 3. In that case tunneling occurs from \( \phi_T \approx 2.95 \times 10^5 \) to \( \phi_T \approx 3.043 \times 10^5 \). Notice that the tachyon potential considered in our model satisfies the condition for the existence of the instanton, i.e. satisfies the inequality (8). In order to see this, in Fig. 4 we have plotted \( m^2/H^2 \) as a function of the Euclidean time \( \tau \) for our model. From this plot we observe that most of the time during the tunneling, we obtain \( |m^2| > H^2 \).

In the following, we calculate the instanton action for the quantum tunneling between the values \( \phi_F \) and \( \phi_T \) in the effective tachyon potential, see Fig. 1. By integrating by parts and using the Euclidean equations of motion, we find that the ac-
where we have defined $V_F = V(\phi_F)$. Thus, the total reduced bounce-action for the thin-wall bubble results to be:

$$B = \frac{6\pi^2}{\kappa} RS_1 + \frac{12\pi^2}{\kappa^2} \left( \frac{1}{V_F} \left[ (1 - \kappa V_F R_b^2/3)^{3/2} - 1 \right] \right) - \frac{1}{V_F} \left[ (1 - \kappa V_F R_b^2/3)^{3/2} - 1 \right],$$

(14)

where, we have taken into account the contributions from the wall (first term) and the interior of the bubble (second and third terms).

The radius of curvature of the bubble could be obtained demanding that the bounce-action (14) is an extremum. Then, the radius of the bubble is determined by setting $dB/dR_b = 0$, which gives

$$\frac{S_1}{2R_b} = \left[ \left( 1 - \kappa V_F R_b^2/3 \right)^{1/2} - \left( 1 - \kappa V_F R_b^2/3 \right)^{1/2} \right].$$

(15)

Thus, the radius of the bubble, $R_b$, is found by solving this equation. Also, from Eq. (15) and following similar arguments as those found in Ref. [26], we could obtain the following condition for our configuration to describe the $O(4)$-symmetric false vacuum decay

$$\Delta s = \frac{S_1}{2R_b} < 1,$$

(16)

where, the dimensionless quantity, $\Delta s$, represents the strength of the wall tension in the thin-wall approximation in our model.

The radius of the bubble associated to the numerical solution displayed in Fig. 3, could be found from Eq. (15) yielding $R_b = 664.547$. We also can determine the strength of the wall tension, $\Delta s = 3.302 \times 10^{-5}$, result that satisfies the consistency condition for the false vacuum decay Eq. (16).

3. Inflation after tunneling and scalar perturbation spectra

Let us study what happens after the tunneling. In order to do that we make an analytical continuation to the Lorentzian space–time and study the time evolution of the tachyon field $\phi(t)$ and scale factor $a(t)$. The equations of motion are:

$$\frac{\dot{\phi}}{1 - \phi^2} = -\frac{3}{a} \dot{\phi} - \frac{1}{V} \frac{dV}{d\phi},$$

(17)

$$\ddot{a} = \frac{\kappa}{3} \frac{V(\phi)}{\sqrt{1 - \phi^2}} \left( 1 - \frac{3}{2} \phi^2 \right),$$

(18)

where the dots represents derivative with respect to the cosmological time $t$.

These equations are solved numerically. This requires to consider the parameters of the potential defined in the previous section and the following boundary conditions $\phi(0) = \phi_T$, $\phi(0) = 0$, $a(0) = 0$, $\dot{a}(0) = 1$. We also consider units where $\kappa = 1$. Solutions to these equations are shown in Figs. 5 and 6, where we have taken $\phi_T \approx 3.043 \times 10^5$, a value obtained from the Euclidean numerical solution. We note in Fig. 5 that after the tunneling, during the period $0 \leq t < t_T$, the tachyon field satisfies the slow roll condition $\phi^2 < 2/3$. Then, we con-
The behavior of the tachyon field after the tunneling as a function of the cosmological time. Notice that the different values of $\phi$ are such that the slow-roll condition is satisfied during inflation ($0 \leq t < t_f$). Also we can note that the scalar field after inflation ($t \geq t_f$) does not oscillate.

The growth of the logarithm of the scale factor as a function of the cosmological time. Here $H_0^2 = V_T / 3$.

clude that after the tunneling our model generates a consistent inflationary era, where the end of inflation happen at the cosmological time $t = t_f$. On the other hand, the scale factor expands approximately $e^{46}$ times during this period, fact shown in Fig. 6.

Even though the study of scalar density perturbations in open universes is quite complicated [7], it is interesting to give an estimation of the standard quantum scalar field fluctuations inside the bubble for our scenario. In particular, the spectra of scalar perturbations for a flat space, generated during tachyon inflation, expressed in terms of the slow-roll parameters defined in Ref. [27] becomes [22]:

$$\frac{\delta \rho}{\rho} = \left[ 1 - 0.11 \epsilon_1 + 0.36 \epsilon_2 \right] \frac{\kappa H}{2 \pi \sqrt{2} \epsilon_1},$$

where the slow-roll parameters are given by:

$$\epsilon_1 \simeq \frac{1}{2 \kappa} \left( \frac{V_{\phi}}{V} \right)^2,$$

$$\epsilon_2 \simeq \kappa^{-1} \left[ -2 \frac{V_{\phi}^2}{V^2} + 3 \left( \frac{V_{\phi}}{V^2} \right)^2 \right].$$

Certainly, in our case, Eq. (19) is an approximation and must be supplemented by several different contribution in the context of an open inflationary universe [6]. However, one may expect that the flat-space expression gives a correct result for $N > 3$. Fig. 7 shows the magnitude of the scalar density perturbations $\delta \rho / \rho$ for our model as a function of the $N$ e-folds of inflation, after the open universe was formed. Notice that $\delta \rho / \rho$ has a deep minimum at $N \lesssim 3$ (mechanism of suppression of large scale density perturbation) and then approaches to its maximum at $N \sim 7$. The shape of the graph showed in Fig. 7 seems to be a generic feature of the one-field open inflation models based on tunneling and bubble formation [6,8,9]. Even though in our model $\delta \rho / \rho$ has a maximum at small $N$, as opposed to the standard single scalar inflationary models, where the maximum is located at $N \sim 10$. If $N = O(1)$ corresponds to density perturbations on the horizon scale $\sim 10^{28}$ cm, then the maximum of the spectrum appears on a scale which is about three orders of magnitude smaller $\sim 10^{25}$ cm, in our model.

One interesting parameter to consider is the so-called spectral index $n$, which is related to the power spectrum of density perturbations $P_{\delta \rho / \rho}^{1/2}(k)$. For modes with a wavelength much larger than the horizon ($k \ll aH$), the spectral index $n$ is an exact power law, expressed by $P_{\delta \rho / \rho}^{1/2}(k) \propto k^{-n}$, where $k$ is the comoving wave number. Also it is interesting to give an estimate of the tensor spectral index $n_T$. In tachyon inflationary models the scalar spectral index and the tensor spectral index are given by $n = 1 - 2 \epsilon_1 - \epsilon_2$ and $n_T = -2 \epsilon_1$, in the slow-roll approximation [22]. From the numerical solution we can obtain their values. In particular for $N \sim 7$ we have $n \approx 0.98$ and $n_T \approx -0.033$. Notice that those indices are very closed to the Harrison–Zel’dychovitch spectrum [28].
4. Conclusion and final remarks

Since we still do not know the exact value of the $\Omega$ parameter, it is convenient to count on an inflationary universe model in which $\Omega < 1$. In this sense, we could have single-bubble open inflationary universe models, which may be consistent with a natural scenario for understanding the large scale structure. However, open inflationary models have a more complicated primordial spectrum than the one obtained in flat universes. Here, extra discrete modes and possibly large tensor anisotropies spectrum could be found, especially those related to supercurvature modes, which are particular to open inflationary universes. Forthcoming astronomical measurements will determine if this extra terms are present in the scalar spectrum. Here, we have studied a one-field open inflationary universe model, where inflation is driven by a tachyonic field. In particular, we focus on the viability of the model in order to generate a consistent inflationary scenario, compatible with observations (number of e-folding, density perturbations, spectral and tensor spectral indices). The idea of consider a tachyon field to drive inflation is a natural choice since the tachyon is an unstable particle. On the other hand, the possibility that this tachyon generates an open universe via bubble nucleation by quantum tunneling appears as an interesting possibility to explore, given that, in the context of non-standard brane decays [15], tachyon potentials with similar characteristics as our potential appear.

We have found that the generalized condition for the existence of the CDL instanton in a tachyonic universe is satisfied together with the requirement of tachyonic inflation (slow-roll condition). In this sense, our models are different to that corresponding to the scalar inflaton case [6], where both conditions are appositive to each other. In this sense, we have provided a model in which both conditions are satisfied simultaneously, and thus, a slow-roll open inflationary tachyonic dominated universe could be realized.

We have generalized the CDL instanton action to tachyonic dominated universe. This action is described by expression (9) and was used to study the probability of nucleation of a bubble. We found that this probability differs from the usual obtain in the context of open scalar inflation. We determined the corresponding wall tension, the strength of the wall tension and the radius of the bubble and found that these results are in agreement with the consistency condition for the false vacuum decay Eq. (16).

In addition, we computed what happens after the tunneling. In order to do this we solved numerically the Lorentzian field equation in our theory. Essentially, we found that after the tunneling, during the period $0 \leq t < t_f$ (see Fig. 5), the tachyon field satisfies the slow-roll condition and then an inflationary era is realized. On the other hand, the scale factor expands approximately $e^{46}$ times during this period. The lower value of the e-folding is not a problem, since in the context of the tachyonic curvaton reheating, the e-folding could be of the order 45 or 50, since the inflationary scale can be lower [25].

We gave an estimate of the scalar density perturbation, where we found that the values calculated during the slow-roll inflation coincide with the result of COBE normalized value, $\delta_H \sim 2 \times 10^{-5}$ [29]. Also we have to mention that the indices computed in the tachyonic model are very closed to the Harrison–Zel’dovich spectrum.

In this way, we have shown that one-field open inflationary universe models can be realized in the tachyonic theory in the context of the standard Einstein general relativity. We hope, that the most natural study for tachyonic theory would be realized for the brane world cosmology scenarios and we leave this task for a near future.

Finally, in principle we could compare our model, i.e. the tachyonic model, with that referred to the standard case i.e. the no-tachyon case. However, at the moment we must say that in both cases a fine tuning of the parameters appearing in the scalar potential is required in order to get a suitable model which may describe an inflationary period for the evolution of the universe. In fact, as far as we know, we could not say which of these models need more fine tuning that the other one. Perhaps when new measurements of astronomical observations be available we will be able to say which one of these models needs less fine tuning of its parameters.

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