Milky Way Mass with K Giants and BHB Stars Using LAMOST, SDSS/SEGUE, and Gaia: 3D Spherical Jeans Equation and Tracer Mass Estimator

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ABSTRACT

We measure the enclosed Milky Way mass profile to Galactocentric distances of \(\sim 70\) and \(\sim 50\) kpc using the smooth, diffuse stellar halo samples of Bird et al. The samples are LAMOST and SDSS/SEGUE K giants (KG) and SDSS/SEGUE blue horizontal branch (BHB) stars with accurate metallicities. The 3D kinematics are available through LAMOST and SDSS/SEGUE distances and radial velocities and Gaia DR2 proper motions. Two methods are used to estimate the enclosed mass: 3D spherical Jeans equation and Evans et al. tracer mass estimator (TME). We remove substructure via the Xue et al. method based on integrals of motion. We evaluate the uncertainties on our estimates due to random sampling noise, systematic distance errors, the adopted density profile, and non-virialization and non-spherical effects of the halo. The tracer density profile remains a limiting systematic in our mass estimates, although within these limits we find reasonable agreement across the different samples and the methods applied. Out to \(\sim 70\) and \(\sim 50\) kpc, the Jeans method yields total enclosed masses of \(4.3 \pm 0.95\) (random) \(\pm 0.6\) (systematic) \(\times 10^{11}\) \(M_{\odot}\) and \(4.1 \pm 1.2\) (random) \(\pm 0.6\) (systematic) \(\times 10^{11}\) \(M_{\odot}\) for the KG and BHB stars, respectively. For the KG and BHB samples we find a dark matter virial mass of \(M_{200} = 0.55^{+0.15}_{-0.11}\) (random) \(\pm 0.083\) (systematic) \(\times 10^{12}\) \(M_{\odot}\) and \(M_{200} = 1.00^{+0.67}_{-0.33}\) (random) \(\pm 0.15\) (systematic) \(\times 10^{12}\) \(M_{\odot}\), respectively.

Keywords: galaxies: individual (Milky Way) — Galaxy: halo — Galaxy: mass — Galaxy: kinematics and dynamics — Galaxy: stellar content — stars: individual (BHB) — stars: individual (K giants) — stars: kinematics and dynamics

1. INTRODUCTION

The Milky Way is typical of large spiral galaxies, in possessing a bright disk and bulge embedded in a tenuous, approximately spherical stellar halo, comprised of
stars, globular clusters, satellite galaxies, and a dark matter halo which dominates its mass budget.

The kinematic properties of these halo objects can be used to probe the enclosed mass as a function of Galactocentric radius $r$. Historically, satellite galaxies and globular clusters have allowed us to probe the mass distribution to large distances, beyond 100 kpc from the Galactic center. In an influential study, Battaglia et al. (2005) used 240 halo objects including stars, globular clusters and satellite galaxies to map the mass profile to $\approx 120$ kpc, finding an enclosed mass of $0.8^{+1.2}_{-0.2} \times 10^{12}$ M$_\odot$.

It is clear that large samples of objects are needed to reduce sampling noise and reveal any kinematic substructures which must be removed or otherwise accounted for in the analysis. In the last few years, the total sample size of halo stars with line-of-sight velocities, distances, and stellar parameters has reached of order $10^4$, due to the substantial efforts made in several dedicated facilities to obtain large numbers of medium to high resolution stellar spectra using fiber-fed spectrographs, such as the Large Sky Area Multi-Object Fiber Spectroscopic Telescope (LAMOST) survey (Cui et al. 2012; Deng et al. 2012; Luo et al. 2012; Zhao et al. 2012), Sloan Digital Sky Survey (SDSS) / SEGUE (Yanny et al. 2009), Gaia-ESO Survey (Gilmore et al. 2012; Randich et al. 2013), RAVE (Steinmetz et al. 2006), and Galah (Martell et al. 2017). A range of automated techniques have been developed to find appropriate tracers, including K-giant (KG) stars (Kafle et al. 2014), blue horizontal branch (BHB) stars (Xue et al. 2008; Kafle et al. 2012), RR Lyrae stars (Cohen et al. 2017), reaching in some cases beyond 100 kpc from the Galactic center.

The Milky Way’s enclosed mass is thus being probed with increasingly larger samples of individual stellar halo tracers. Xue et al. (2008) used 2401 SDSS/SEGUE halo BHB stars; Gnedin et al. (2010) used 910 Hypervelocity Star Survey halo BHB stars and blue stragglers, mapping the mass profile out to 80 kpc; Kafle et al. (2012) used 4664 SDSS/SEGUE halo BHB stars to constrain the profile to $\approx 60$ kpc and extended the work in Kafle et al. (2014), using SDSS/SEGUE halo K-giant and BHB stars, with additional constraints provided by the gas terminal velocity curve and Sgr A* proper motion, to measure the virial mass and virial radius of the Milky Way; and Huang et al. (2016) used $\approx 16,000$ outer disk red clump giant stars and $\approx 5700$ SDSS/SEGUE halo K giants to measure the mass profile out to $\approx 100$ kpc. More recently, Zhai et al. (2018) have measured the mass profile to 120 kpc using some 9000 halo K giants.

In most of these studies, only radial velocities have been available for the tracers, and information about the important transverse motions of the tracers need to be gleaned statistically (see, e.g., Kafle et al. (2012), Kafle et al. (2014)) from the samples. At large Galactocentric distances, this becomes increasingly difficult to perform from radial velocities alone. The European Space Agency’s Gaia mission (Gaia Collaboration et al. 2016) is providing the proper motions necessary to measure transverse motions directly; this leads to much improved understanding of the halo tracer kinematics.

Now with the recent availability of high quality full 6D phase-space information for large numbers of sources, much effort has been made to decrease the uncertainties in the Milky Way mass estimate. Recent works using a tracer mass estimator with 6D phase-space information include Sohn et al. (2018, globular clusters), Watkins et al. (2019, globular clusters), and Fritz et al. (2020, satellites). The most recent work using the spherical Jeans equation by Zhai et al. (2018) is very similar to our current investigation in method and data (LAMOST K giants) although only line of sight velocities were included, whereas we additionally make use of proper motions from Gaia to obtain the stellar tangential velocities. Using Bayesian analysis to fit a distribution function to full 6D phase-space data (globular clusters, satellites, halo stars) has been a recent popular choice among many works (Posti & Helmi 2019; Eadie & Jurić 2019; Vasiliev 2019; Deason et al. 2021; Correa Magnus & Vasiliev 2022; Shen et al. 2022; Slizewski et al. 2022; Wang et al. 2022) and a similar distribution function analysis using 5D phase-space data from Gaia (Hattori et al. 2021). In addition to fitting the observational data with a distribution function, several works have incorporated into the fitting a comparison of the observed data with Milky Way-type galaxies from cosmological simulations (Callingham et al. 2019; Li et al. 2020). Newly discovered high velocity stars with full 6D phase-space information have been used to estimate the mass of the Milky Way (Monari et al. 2018; Hattori et al. 2018; Deason et al. 2019; Grand et al. 2019; Koppelman & Helmi 2021; Necib & Lin 2022). Vasiliev et al. (2021) and Craig et al. (2021) have estimated the Milky Way mass by fitting models for the Sagittarius and Magellanic Steams, respectively. Several recent studies have estimated the Milky Way mass using measurements of the rotation curve (Eilers et al. 2019; de Salas et al. 2019; Ablimit et al. 2020; Karukes et al. 2020; Cautun et al. 2020; Jiao et al. 2021). Other works have used 6D satellite phenomenology, characterizing simulated Milky Way-type satellite populations and comparing to the observations of satellites in the Milky Way, to estimate the mass of the Milky Way (Patel et al. 2018; Rodriguez Wilberly et al. 2022; Villanueva-Domingo et al. 2021). Zaritsky...
et al. (2020) apply the timing argument to distant Milky Way halo stars to derive a lower limit to the Milky Way mass.

In this article, we apply the Jeans theorem (Jeans 1915), which relates the density and kinematics of particles moving within a gravitational potential, assuming they are in a steady state. The simplest form is to assume spherical symmetry for the potential, the stellar tracer density, and the tracer velocity dispersion profiles. It has been used to measure the Milky Way gravitational potential (e.g., Battaglia et al. 2005; Dehnen et al. 2006; Xue et al. 2008; Gnedin et al. 2010; Samurović & Lalović 2011; Kafle et al. 2012; Bhattacharjee et al. 2014; Huang et al. 2016; Ablimit & Zhao 2017; Zhai et al. 2018) using line-of-sight velocities, and making assumptions about the transverse velocities of the tracer stars (i.e., their anisotropy). Wang et al. (2020) have recently reviewed Milky Way mass estimates of this type.

Gaia proper motions for tracer stars now make possible the use of the Jeans equation on the full 3D position and velocity data, rather than line-of-sight velocities alone.

In this paper, we use the 3D velocity dispersion profiles of Bird et al. (2019); Bird et al. (2021) and, for the first time, use the 3D spherical Jeans equation to measure the Milky Way mass profile. The paper is laid out as follows. In Section 2 we introduce the two methods which we use for Galactic mass estimation, the 3D spherical Jeans equation and the tracer mass estimator (TME) of Evans et al. (2011), both of which assume the simplest case halo dynamics, that of a spherical system traced by a non-rotating, relaxed population in equilibrium. The use of two mass estimate methods allows for comparison to tie the new determination back to previous estimates of the Milky Way’s enclosed mass. After presenting the methods in Section 2, in Section 3 we analyze the sources of random and systematic uncertainty on our mass estimates. These are effects due to the Gaia-Enceladus-Sausage (GES) satellite and Large Magellanic Cloud (LMC), residual substructure in the halo sample, and non-spherical effects. In Section 4, we apply the Jeans and TME methods to our large sample of halo K giants and BHB stars. We extrapolate the mass determination to make an estimate of the Milky Way’s virial mass, virial radius, and concentration parameter, assuming the dark halo has an NFW profile. We discuss and summarize our results and conclusions in Section 5.

2. METHODS

We apply two methods to our data to recover the Milky Way’s enclosed mass profile, firstly, the spherical Jeans equation using full 3D distances and velocities, and secondly a tracer mass estimator (TME) method, which uses the distances, tracer Galactocentric radial velocities, and anisotropy parameter measured from the 3D velocities.

The density fall-off of the tracer stars is modeled as a power-law $\rho \propto r^\alpha$. This assumption remains one of the important sources of uncertainty in our final mass estimates. We discuss this assumption in detail in Sections 4.2 and 5.3.

In order to build the mass profile using our two methods, we bin our stars according to Galactocentric radius, selecting separate radial bins for our KG and BHB samples. We elaborate upon our bin selection later in Section 4.3. Once selected, we use the same binning scheme for both the 3D spherical Jeans method and TME.

We also extrapolate our results to estimate the virial mass and radius of the Milky Way, assuming an NFW dark matter profile.

In this section we describe the details of applying each method and estimating the associated uncertainties propagated from the observed data.

2.1. 3D Spherical Jeans Equation

We use the 3D spherical Jeans equation for a non-rotating, stationary system of stars, to measure the Galactic mass profile $M(< r)$, where $M$ is the mass enclosed within the Galactocentric radius $r$:

$$M(< r) = -\frac{1}{G} \int (r^2 \frac{d\sigma^2}{dr} + r((2 + \alpha)\sigma_r^2 - \sigma_\theta^2 - \sigma_\phi^2)). \quad (1)$$

Here, the gravitational constant is $G = 0.054\times(4\pi \times 10^{-3}) \, (\text{km s}^{-1})^2 \, \text{kpc} \, M_{\odot}^{-1}$, and the velocity dispersions in the radial and two transverse directions are given by $(\sigma_r, \sigma_\theta, \sigma_\phi)$, and $\alpha$ is the slope of the power-law fit to the density fall-off of the tracers $\rho \propto r^\alpha$. In Section 4.2 we discuss $\alpha$ and the density profiles used for the modeling in detail, and in Section 4.3 we detail our selected radial bins which we use to calculate the mass profile.

The 3D velocity dispersion profiles are computed from our sample following Bird et al. (2021). We use the 3D version of the software extreme-deconvolution (Bovy et al. 2011) which takes into account the individual velocity errors on each star in each component $(r, \theta, \phi)$, the covariances between the velocity errors, and the velocity means. This software implements a maximum likelihood density estimation technique in order to model the underlying, error-deconvolved distribution as a mixture of Gaussian components.

The Jeans equation has a term which is the gradient of the radial component of the velocity dispersion. We fit $\sigma_r(r)$ with a smooth, continuously differentiable
function, in order to estimate this term. We find that an exponential form for $\sigma_r(r)$ fits the data well and is bound and well-behaved at the ends of the profile. The choice of an exponential function was determined by trial-and-error in which we also tested linear and spline functions. We select the model as a fitting function only, without direct physical interpretation.

Thus, for $\sigma_r$ we have,

$$
\sigma_r = \sigma_0 \exp \left[ -\frac{r}{h_r} \right],
$$

and the differential form:

$$
\frac{d\sigma_r^2}{dr} = -\frac{2\sigma_0^2}{h_r} \exp \left[ -\frac{2r}{h_r} \right].
$$

We perform a Bayesian Markov chain Monte Carlo (MCMC) maximum likelihood method and make use of the PYTHON package emcee (Foreman-Mackey et al. 2013). We optimize Equation 2 and find the best fit for $\sigma_0$ and $h_r$. We use 1000 walkers, 1500 runs, and 500 runs for the burn-in phase. We select minimal priors, requiring $0 < h_r < 1000$ kpc. We use the 50th percentile as the best fit for $(\sigma_0, h_r)$ and we investigate the goodness-of-fit using the 16th and 84th percentiles as the random uncertainty.

We note that, although we parameterize the $\sigma_r$ profile in order to measure the $\frac{d\sigma_r^2}{dr}$ term needed for the Jeans equation, in our application of the Jeans equation we use a hybrid of binned and parameterized velocity data, i.e., we use the binned velocity dispersion measurements from extreme-deconvolution and the parameterized $\frac{d\sigma_r^2}{dr}$ from our Bayesian MCMC estimate.

Observational errors in each term of the Jeans equation are propagated through to derive uncertainties in the enclosed mass estimate as follows. Firstly, to estimate the error on the velocity dispersions in radial bins, we use the Poisson statistics of the number of stars in each bin (after using extreme-deconvolution to determine the velocity dispersion, taking into account the individual observational errors on the stellar velocities, and the covariances between them). Secondly, to estimate the error on the the gradient of the radial velocity dispersion term in the Jeans equation, we propagate the errors on the radial velocity dispersions, also determined using Poisson statistics of the star numbers in the radial bins. Monte Carlo simulations of the terms in the Jeans equation are then performed with 100 realizations using Gaussian distributed errors as above, to derive a final uncertainty on the enclosed mass profile.

2.2. Tracer Mass Estimator (TME)

Estimation of the mass of a gravitationally bound system typically involves calculating a weighted average of the velocities and distances. Traditionally, mass estimators have been based on the virial theorem with pioneering work by Zwicky (1933); Chandrasekhar (1942); Schwarzschild (1954); Burbidge & Burbidge (1959); and Limber & Mathews (1960). Virial mass estimators take the form of two separate averages for distance and velocity. More convenient forms of the mass estimator, involving the average over the combination of distance and velocity, have been introduced by Holmberg (1937); Page (1952); Hartwick & Sargent (1978); and Bahcall & Tremaine (1981). These authors took differing approaches to derive their mass estimators, including consideration of the orbital properties of the system, use of the collisionless Boltzmann equation and/or the Jeans equation.

The projected mass estimator was introduced by Bahcall & Tremaine (1981), who noted that the virial mass estimator has limitations, being both biased and inefficient (see Section II of Bahcall & Tremaine (1981) and also An & Evans (2011)). The application is to data for which the line-of-sight velocities at projected distances from the center of the mass distribution are available, so as is the case for galaxy clusters.

We use a tracer mass estimator of Evans et al. (2003, 2011). They are the generalization of the projected mass estimator to the case in which the mass (or number) density of the tracer population differs from the overall mass density, and distances and velocities for the tracers are available. The Evans et al. (2011) tracer mass estimator is particularly convenient for this study, as it makes direct use of our observables (distances, line-of-sight velocities, proper motions), and has been tested on simulated data (such as galaxies formed in N-body simulations, Evans et al. 2003, 2011).

As with the Jeans equation, the TME assumes a spherical system of non-rotating relaxed tracers which is in dynamical equilibrium. This holds to a good approximation in our data set except for bulk motion in the outer halo due to the Large Magellanic Cloud.

We use the scale-free tracer mass estimator of Evans et al. (2011, Equation 9)\(^1\). As we will elaborate upon in Section 3.2, we incorporate a correction for the effects of the Large Magellanic Cloud by subtracting the mean radial velocity $\langle V_r \rangle$ from our radial velocities $V_{r,i}$. Thus the equation becomes,

$$
M(<r) \approx \frac{r^0.5(\gamma - \alpha - 2\beta)}{GN} \sum_{i=1}^{N} r_i^{0.5}(V_{r,i} - \langle V_r \rangle)^2.
$$

\(^1\) For a full derivation of the estimator, see An & Evans (2011).
Here, we have \( N \) tracer stars where the \( i \)th tracer has Galactocentric distance \( r_i \) and radial velocity \( V_{r,i} \) to yield the mass \( M(< r) \) of the Galaxy within the radius \( r \) of the most distant star in the sample. To build the \( M(< r) \) mass profile, we apply Equation 4 to our KG and BHB stars in bins of Galactocentric radius, using the same binning scheme as we use for our application of the 3D spherical Jeans method. In this manner, \( \gamma \), \( \alpha \), and \( \beta \) are functions of \( r \). The Galactic potential is represented as a power-law with slope \( \gamma \) as a function of \( r \). The tracer stars are assumed to have a density fall-off as a power-law with slope \( \alpha \) (as adopted for the Jeans equation\(^2\)). The velocity anisotropy profile along Galactocentric radius of the tracer stars is given to be \( \beta \), where

\[
\beta = 1 - (\sigma_v^2 + \sigma_s^2)/(2\sigma_v^2). \tag{5}
\]

To measure \( \beta \), we calculate the 3D velocity dispersion profiles, as in our application of the Jeans equation, by following Bird et al. (2021) using the extreme-deconvolution software (Bovy et al. 2011). Allowing \( \beta \) to vary along \( r \) is appropriate as has been previously shown, e.g., in analyses of simulated Milky Way-type galaxies (Loebman et al. 2018; Cunningham et al. 2019) and in the Milky Way itself (Bird et al. 2019; Cunningham et al. 2019; Bird et al. 2021).

In Section 4.2 we discuss \( \alpha \) and the density profiles used for the modeling in detail, and in Section 4.3 we detail our selected radial bins which we use to calculate the mass profile.

In an analysis of NFW potentials covering a wide range of virial radii and concentrations, Watkins et al. (2010) found that a scale-free host potential with a power-law index of \( \gamma = 0.5 \) well represents the NFW potential for large galaxies like the Milky Way. We adopt \( \gamma = 0.5 \) to analyze our sample, since many studies have shown the success of using an NFW model to approximate dark halo mass distributions. We estimate the random uncertainty on the TME mass by bootstrapping the stellar sample. For each radial bin, we bootstrap 100 new samples, calculate the TME mass using the newly sampled \((r, V_r)\), and take the quadrature sum of the 16th and 84th percentiles as the random uncertainty on the mass.

2.3. Estimation of the Virial Mass

The Jeans and TME methods yield the enclosed mass of the Milky Way, which represents the sum of baryonic and dark components. In order to estimate the mass of the dark component, we fit the enclosed mass profiles after subtracting a model of the radial distribution of the Milky Way’s baryons.

Specifically, we fit the Jeans and TME enclosed mass profiles (Sections 4.4 and 4.5, respectively) with a model of the Milky Way baryons (consisting of a bulge and disk component Bovy & Rix 2013; Bovy 2015) and an NFW dark matter profile.

The bulge and disk models are taken from the MWPotential2014 model within the galpy.potential module of the PYTHON library for Galactic dynamics galpy (Bovy 2015). We adopt the values \( R_\odot = 8.3 \) kpc and \( v_{\text{circ}}(r_\odot) = 220 \) km s\(^{-1}\) for the distance of the Sun from the Galactic center and the solar circular velocity (Bovy et al. 2012; Bovy 2015).

The bulge component is modeled as a power-law density profile that is exponentially cut-off (PowerSphericalPotentialCutoff). We adopt a power-law exponent of \( \alpha_{\text{bulge}} = -1.8 \), mass \( M_b = 0.5 \times 10^{10} \) M\(_\odot\), and cut-off radius of \( r_{\text{bulge}} = 1.9 \) kpc:

\[
\rho(r) = \frac{GM_b}{r^{\alpha_{\text{bulge}}} - (r/r_{\text{bulge}})^2}. \tag{6}
\]

The disk component is modeled as a Miyamoto-Nagai (1975) disk (MiyamotoNagaiPotential) with a disk scale length \( a = 3 \) kpc, disk scale height \( b = 0.28 \) kpc, and mass \( M_d = 6.8 \times 10^{10} \) M\(_\odot\):

\[
\Phi(R,z) = -\frac{GM_d}{\sqrt{R^2 + (a + \sqrt{z^2 + b^2})^2}}. \tag{7}
\]

Compared to the total radial force at \( R_\odot \), the relative force contribution from the bulge is 0.05 and from the disk is 0.6. These relative amplitudes have been fit to the data in Bovy (2015).

The NFW dark matter component as a function of \( r \), as presented in Navarro et al. (1995, 1996, 1997) and Binney & Tremaine (2008, p. 71), takes the form

\[
M_{\text{NFW}}(r) = 4\pi \rho_0 r_s^3 \left( \ln(1 + r/r_s) - \frac{r/r_s}{1 + (r/r_s)} \right), \tag{8}
\]

where

\[
\rho_0 = \rho_{\text{crit}} \delta_c, \tag{9}
\]

\[
\delta_c = \frac{200}{3} \left[ \ln(1 + c) - c/(1 + c) \right], \tag{10}
\]

\[
\rho_{\text{crit}} = 3 H^2/(8\pi G)/1000^3, \tag{11}
\]

\[
r_s = r_{200}/c. \tag{12}
\]

\( \rho_{\text{crit}} \) is the critical density of the universe, \( r_{200} \) is the radius within which the mean density is 200 times the critical density, and \( c \) is the speed of light.

\( \delta_c \) is the deviation of the density from the critical density at \( r \), and \( r_s \) is the scale radius of the NFW profile.

\( \rho_0 \) is the dark matter density at the virial radius of the Milky Way.

\( H \) is the Hubble constant.

\( G \) is the gravitational constant.

\( \rho_{\text{crit}} \) is the critical density of the universe.

\( r_{200} \) is the radius within which the mean density is 200 times the critical density.

\( c \) is the speed of light.

\( r_s \) is the scale radius of the NFW profile.

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\( \rho_0 \) is the dark matter density at the virial radius of the Milky Way.

\( H \) is the Hubble constant.

\( G \) is the gravitational constant.

\( \rho_{\text{crit}} \) is the critical density of the universe.

\( r_{200} \) is the radius within which the mean density is 200 times the critical density.
Here, \( r_{200} \) is the distance at which the mean density of the Galaxy is 200 times the present critical density \( \rho_{\text{crit}} \) of the Universe.

We adopt a Hubble constant of \( H = 67.8 \text{ km s}^{-1} \text{ Mpc}^{-1} \) (Planck Collaboration et al. 2016). The uncertainty in this parameter affects our virial mass estimates negligibly compared to other sources of error.

The concentration \( c \) and characteristic overdensity \( \delta_c \) are dimensionless parameters and \( r_s \) is the NFW scale radius. The concentration and scale radius set the value of \( \rho_0 \). The virial mass \( M_{200} \) is the dark mass enclosed within the spherical volume of radius \( r_{200} \),

\[
M_{200} = \frac{4\pi}{3} 200 \rho_{\text{crit}} r_{200}^3. \tag{13}
\]

We set the virial radius \( r_{200} \) and concentration parameter \( c \) of the NFW profile as free parameters in our fitting. We perform a Bayesian MCMC method and make use of the PYTHON package emcee (Foreman-Mackey et al. 2013). We optimize Equation 8 and find the best fit for \( r_{200} \) and \( c \). We use 500 walkers, 400 runs, and 250 runs for the burn-in phase. We select minimal priors, requiring \( c > 0 \) and \( 200 < v_{\text{circ}}(R_\odot) < 250 \) km s\(^{-1}\). For discussion and comparison, we use the software package numpy.average to calculate the average mass and uncertainties of our mass estimates together with other virial mass \( M_{\text{vir}} \) estimates from the literature. These averages are weighted by the errors quoted for each measurement.

3. ITEMIZING SOURCES OF UNCERTAINTY

In the past, measuring the mass profile of the Milky Way has been strongly affected by random and systematic errors, particularly due to small sample sizes.

The advent of large automated radial velocity and proper motion surveys have brought sampling errors down substantially, and also allowed us to probe other important sources of error, such as the validity of the well-mixed assumption in applying kinematic methods to measuring mass. That the assumption that the halo is roughly spherical and in a stationary dynamical state, even with the careful removal of substructure, will be incorrect at some level.

In this section we itemize such sources of error on our analysis. We first consider the uncertainties involved in our mass estimation and in the following Section 4 we present our results. In Section 3.1 we discuss the effects of the ancient merger Gaia-Enceladus-Sausage, in Section 3.2 we detail our correction for the effects of the LMC, and in Section 3.3 we explain our adopted 15 percent systematic uncertainty on our mass estimate due to non-spherical and non-virial effects. These are in addition our estimation of the random sampling uncertainty which we have detailed previously in Section 2.

3.1. Effects of the Gaia-Enceladus-Sausage Satellite (GES)

The structure known as the Gaia-Enceladus-Sausage (GES, Belokurov et al. 2018; Deason et al. 2018b; Hayward et al. 2018; Helmi et al. 2018; Koppelman et al. 2018) permeates the inner population of halo stars. As this large satellite merged at an early time in the Galaxy’s evolution, most of its stars have likely virialized and cannot be recognized as originating in the GES satellite from kinematics alone. Since we clean substructure from our sample to get at the smooth, virialized background halo, the GES will remain as part of the sample.

On the other hand, Lancaster et al. (2019) are able to adequately model the velocity ellipsoid of BHB halo stars using two Gaussian components, one representing the GES and one an ‘isotropic stellar halo.’ As we use our smooth, diffuse halo sample from which we have removed the streams and substructure via the stellar orbital parameters (Xue et al. 2022, in preparation), we use a single Gaussian model for the velocity ellipsoid to describe our sample. Further work on fitting the model of Lancaster et al. (2019) to our smooth, diffuse halo sample is presented in Wu et al. (2022). If the smooth, diffuse halo is a composition of radial GES stars and isotropic stars combined from other mergers and events, it is possible measuring the density profile for each component and using these in the 3D Jeans mass estimation may further improve the mass estimate. This would be significantly more complex modeling than what we use in this paper, concentrating on the diffuse, smooth component only.

3.2. Effects of the Large Magellanic Cloud (LMC)

Petersen & Penarrubia (2021) and Erkal et al. (2021) have used distant halo stars to measure the present-day reflex (or sloshing) motion of the stellar halo which is induced by the LMC. The LMC’s mass induces systematic motions in the outer halo, affecting the measured velocities in Galactic longitude \( V_L \) (and in the vertical direction, \( V_Z \)), with the largest influence occurring for \( r > 50 \) kpc.

Sloshing of the halo by the LMC has been examined using simulations (Gómez et al. 2015; Erkal et al. 2020; Garavito-Camargo et al. 2019; Cunningham et al. 2020; Petersen & Penarrubia 2020; Wang et al. 2022). Specifically, Erkal et al. (2020) run Milky Way models to quantify the bias introduced in mass determinations of the Milky Way for this effect, since the assumption of equi-
librium dynamics is broken. This bias can cause an overestimation of the Milky Way mass by 25 percent. They use the TME as presented by Watkins et al. (2010) and find that, by incorporating a correction to the velocity term by subtracting the mean velocity, the bias is greatly reduced. They find that the bias changes depending on the observed quadrant of the sky. In several regions, the bias is negligible after applying the correction to the mass estimate.

Deason et al. (2021) apply their distribution function mass measurement method to the Milky Way models of Erkal et al. (2020) and estimate the correction to the velocities needed for their method. They then apply their method to a sample of distant halo stars (their distant halo K-giant and BHB stars (r > 50 kpc) and our halo star samples are taken from the same catalogs). They note that the sample stars (fortuitously) occupy a region of configuration space in the halo which is least affected by the LMC, so for the sample stars here, the correction is small. Failing to account for the LMC’s sloshing results in overestimating the Milky Way enclosed mass by ≈ 6 – 7 percent for the region of the sky in our halo star sample.

Evidence in support of a massive LMC on its first infall (e.g., Besla et al. 2007; Kallivayalil et al. 2013; Laporte et al. 2018) was one of the main motivations driving Slizewski et al. (2022) to revisit the distribution function mass estimation method of Eadie & Jurić (2019). Slizewski et al. (2022) measure the Galactic mass using dwarf galaxies separated into different distance intervals and, as investigated by Erkal et al. (2020), different spatial octants. The largest difference they find is the mass measured from stars within the range of 30 < r < 45 kpc. They conjecture a possible cause is the influence of the LMC on the velocities of these stars such as caused by the sloshing (Petersen & Peñarrubia 2021; Erkal et al. 2021) or the wake (Conroy et al. 2021).

Following the recommended velocity correction of Erkal et al. (2020), we take the mean velocity into account for both the velocity dispersions (using extreme-deconvolution) of the 3D spherical Jeans and the velocity component in the Evans et al. (2011) TME used in this work. Applying the mean velocity corrections, we find that the changes in the mass profile are very small (few percent) compared to other sources of error. We include the mean velocity when calculating the enclosed mass profiles.

3.3. Non-Virialization, Non-Spherical, and Non-NFWness Effects

Mass estimators generally assume that the dark matter and/or stellar halo tracer is kinematically virialized and stationary (i.e., has a steady state distribution function) and that the mass distribution is spherical. Frequently the dark matter halo is assumed to follow an NFW profile. Biases in mass estimates occur when these assumptions do not adequately characterize the galaxy and stellar halo being studied (e.g., Yencho et al. 2006; Wang et al. 2015; Han et al. 2016b; Sanderson et al. 2017; Wang et al. 2017; Eadie et al. 2018; Wang et al. 2018; Grand et al. 2019; Deason et al. 2021).

Studies find that non-virialization and non-spherical effects tend towards underestimating the mass when using stellar tracers (e.g. Yencho et al. 2006; Wang et al. 2018; Grand et al. 2019; Deason et al. 2021; Rehemtulla et al. 2022). Recently Deason et al. (2021) apply the distribution function method to stellar halos from Auriga (Grand et al. 2017) and find that the mass is typically underestimated by ∼ 10 percent, with a scatter of ∼ 25 percent.

Wang et al. (2017) analyze the extent to which these assumptions affect the mass estimates of dark halos in cosmological simulations. They find that spherical, NFW mass profile fits to the dark matter halos can typically induce errors in the mass estimates of 25 percent.

The NFW concentration c parameter (Equation 12) depends on the scale radius which can be from one to a few ×10 kpc for Milky Way-type galaxies. Thus, the inner halo data are needed to constrain c, as noted by, e.g., Kafle et al. (2014) and Wang et al. (2017).

On the other hand, the addition of baryons to simulations can cause the inner halo to deviate from an NFW profile (e.g., Schaller et al. 2015; Dutton et al. 2016; Lovell et al. 2018) which can warrant for excluding the innermost halo samples. In such cases, Wang et al. (2017) find that the use of stellar halo tracers can yield systematically underestimated galaxy masses and overestimated concentration. The bias is highest when they use stars with r > 1 kpc but progressively decreases as they exclude inner halo stars, e.g., by keeping stars with r > 10 kpc or r > 20 kpc to measure the mass.

In this study, we adopt an error in our mass estimates due to non-spherical and non-virialization effects of 15 percent following Deason et al. (2021). In attempts to alleviate possible deviations from the NFW profile in the inner halo, we subtract the baryon mass profile from our measured total mass profile before fitting with an NFW model; additionally, we require the circular velocity near the Sun to be 200 < v_{circ} < 250 km s^{-1}. We further discuss the effects of the inner halo stars on the mass estimate in Section 5.3.

4. DATA ANALYSIS AND RESULTS

4.1. Data: Two Smooth, Diffuse Stellar Halo Samples
We use the LAMOST K-giant, SDSS K-giant, and SDSS BHB halo star samples of Bird et al. (2021). These consist of line-of-sight velocities and metallicities obtained with LAMOST and SDSS/SEGUE, star distance estimates using the method of Xue et al. (2014) and Xue et al. (2008) for K giants and BHB stars (respectively), and finally proper motions from Gaia DR2 (Gaia Collaboration et al. 2018). The LAMOST/SDSS KG and BHB samples initially contain 12728, 5248, and 3982 stars (Table 2 Bird et al. 2021), respectively.

Our LAMOST KG line-of-sight velocities have been corrected by adding 8 km s$^{-1}$ to account for the systematic bias found by comparing stars in common with our SDSS/SEGUE KG sample (Bird et al. 2021). This bias has been previously detected and discussed in the literature although its origin remains unknown (Luo et al. 2015; Tian et al. 2015; Xiang et al. 2015; Schönrich & Aumer 2017; Yang et al. 2019; Wang et al. 2020).

The Galactocentric Cartesian Coordinates follow the conventions in Astropy Collaboration et al. (2018) and Galactocentric spherical coordinates follow Bird et al. (2019) and Bird et al. (2021). The Sun is located at $X_\odot = -8.3$ kpc and $Z_\odot = 29$ pc. The local standard of rest (LSR) velocity is $v_{LSR} = 220$ km s$^{-1}$, and the motion of the Sun with respect to the LSR is $(U,V,W)_\odot = (11.1,12.2,7.3)$ km s$^{-1}$ (Schönrich et al. 2010). These are similar to the more recent measurements of the LSR parameters (Huang et al. 2015; Tian et al. 2015).

As expressed in Bird et al. (2021), our samples include the propagation of (1) uncertainties in the distances of the stars, (2) line-of-sight radial velocity uncertainties, (3) two Gaia DR2 proper motion error estimates, and (4) the Gaia DR2 proper motion covariances. These four uncertainties are propagated via 1000 Monte Carlo runs per star with the python package pyia (Price-Whelan 2018) to derive the uncertainties on the 3D kinematics of each star in the Galactocentric Cartesian and spherical coordinate systems.

Both the 3D spherical Jeans equation and TME assume the population tracing the Galactic potential is a relaxed and virialized system; we, thus, use the smooth, diffuse halo samples from which streams and substructure have been removed using the method of Xue et al. (2022, in preparation) who identify substructure through clustering in integral-of-motion space.

As shown in Bird et al. (2021), the two KG samples are very similar in their properties, and we combine them for our current study.

The final sample sizes are 10762 K giants and 2526 BHB stars covering a distance range from $3 < r < 105$ kpc and $4 < r < 60$ kpc, respectively. Fig-

![Figure 1](image-url) Cumulative probability (upper panel), one dimensional kernel density estimation (middle panel), and probability density distribution of [Fe/H] (lower panel) using KG (red) and BHB (blue) smooth, diffuse halo stars. The z-axis of the upper panel is in log scale and of the middle panel linear. More than 50 percent of our smooth halo KG and BHB samples lie within 25 kpc; this is within the break radius where the fall in the density profile changes as discussed in Section 4.2 and shown in Figure 3. Our KG sample peaks at higher metallicities compared to our BHB sample.

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**Figure 1.** Cumulative probability (upper panel), one dimensional kernel density estimation (middle panel), and probability density distribution of [Fe/H] (lower panel) using KG (red) and BHB (blue) smooth, diffuse halo stars. The z-axis of the upper panel is in log scale and of the middle panel linear. More than 50 percent of our smooth halo KG and BHB samples lie within 25 kpc; this is within the break radius where the fall in the density profile changes as discussed in Section 4.2 and shown in Figure 3. Our KG sample peaks at higher metallicities compared to our BHB sample.
ures 1–2 show the distributions for our two smooth, diffuse stellar halo samples in Galactocentric distance, metallicity, and velocity. In this work, to perform the kernel density estimation, we use the python package scipy.gaussian_kde; we allow the software to automatically determine the bandwidth which it does so using Scott’s Rule (Scott 1992).

We measure the enclosed mass profiles and estimate the virial mass using our KG and BHB samples separately.

We have sufficiently large samples of both KG and BHB stars to estimate the 3D velocity dispersion and enclosed mass profiles separately.

We note that our KG and BHB smooth diffuse stellar halo samples peak at different metallicities as seen in the lower panel of Figure 1. We have shown previously that the metallicities and velocity anisotropy $\beta$ show a relation such that the more metal rich stars have more radial orbits compared to the more metal poor stars (Bird et al. 2019; Bird et al. 2021). Since our smooth, diffuse KG sample peaks at higher metallicities than our BHB sample, we expect the velocity dispersions of the samples to also differ, which we see in Figure 2 where the dispersion is slightly larger in radial velocity for KG stars and in tangential velocities for BHB stars. The different velocity distributions will influence our Jeans and TME mass estimations. Differences in the tracer density profiles will correspond to differences in the velocity dispersion profiles, since the tracer stars occupy a common gravitational potential. As the uncertainties are large for our estimated density profiles, we prefer to measure the mass from the KG and BHB samples separately and, as we detail in the following Section 4.2, use the density profiles from the literature measured separately for KG and BHB samples. In this way, differences in the Galactic mass profile measured between the KG and BHB tracers will give us an estimation of the mass uncertainties.

4.2. Data: Tracer Density Profile

The density fall-off $\rho(r)$ of the tracer stars is an important assumption in the mass determination, both for the Jeans equation and TME.

We have chosen to use the stellar density models of Xu et al. (2018) and Das et al. (2016) for KG and BHB stars, respectively, (also see Xue et al. (2015) and Deason et al. (2011a) for similar density profiles based on KG and BHB stars, respectively). For the KG stellar halo we use the model which includes flattening of the stellar halo that changes with radius and a single power law density; this is the best model determined by Xu et al. (2018) as compared to other models tested. Das et al. (2016) use a broken power law density model with constant flattening of the stellar halo.
These authors use similar halo star samples as we use in the current work, but the density profiles are derived based on a flattened stellar halo.

Since we use the spherical Jeans equation and TME (which also assumes a spherical system) in this work, we have computed the density profiles for spherical shells along Galactocentric radius $r$ using the Xu et al. (2018) and Das et al. (2016) models.

We fit a broken power law density model using a Bayesian MCMC maximum likelihood method and make use of the PYTHON package emcee (Foreman-Mackey et al. 2013). We use 300 walkers, 700 runs, and 200 runs for the burn-in phase. We use the 50th percentile as the best fit and we investigate the goodness-of-fit using the 16th and 84th percentiles as the random uncertainty.

Our resulting broken power law density profile fits are $\alpha_{\text{in}} = -3.1$, $r_{\text{break}} = 17$ kpc, and $\alpha_{\text{out}} = -4.6$ for our combined LAMOST and SDSS KG stars, and $\alpha_{\text{in}} = -3.5$, $r_{\text{break}} = 22$ kpc, and $\alpha_{\text{out}} = -4.7$ for SDSS BHB stars. The uncertainties on $\alpha_{\text{in}}$ and $\alpha_{\text{out}}$ are $\sim \pm 0.001 \sim \pm 0.004$ and on $r_{\text{break}} \pm 1$ kpc. As seen in Figure 1, over half of our KG and BHB samples are located within their respective $r_{\text{break}}$. We show the fitting results in Figure 3.

Using $\alpha_{\text{in}} = -3.1$, we find that the circular velocity near the Sun for our KG sample is $> 250$ km s$^{-1}$. As this somewhat high compared to measurements found in the literature, we adjust our selected value to $\alpha_{\text{in}} = -2.8$ which gives $v_{\text{circ}} < 250$ km s$^{-1}$ near the Sun. This effectively sets a lower limit on $\alpha_{\text{in}}$. We further discuss uncertainties due to the density profile in Section 5.3.

4.3. Radially Binning the Data

To calculate the enclosed mass profile by applying the 3D spherical Jeans method and TME, we bin the data in Galactocentric radial bins. Several factors influence the bin selection. We select bin sizes taking into account the distance uncertainty of our KG and BHB samples. The KG sample has a 16 percent relative distance uncertainty and the BHB sample has 10 percent. We thus allow the BHB bins to be up to half as many as the KG bins. We also consider the number density of our samples. Over 60 percent of our KG and BHB samples are located within 20 kpc. When selecting the most distant bins, we check that over 10 stars reside in each bin, although the majority of the most distant bins have over 50 stars. Finally, we select the ends of each bin as $r = 15, 25, 35, 45, 55, 65,$ and 125 for our KG sample and $r = 7, 8, 10, 13, 20, 25, 30, 35, 43, 50,$ and 125 kpc for our BHB sample. For plotting purposes, we use the median distance of stars within the bin.

4.4. Results: 3D Spherical Jeans Milky Way Mass

To apply the Jeans equation, we first measure the velocity dispersions ($\sigma_r, \sigma_\phi, \sigma_\theta$) as functions of Galactocentric radius $r$ from the sample stars. The velocity dispersions in the three components $V_r, V_\phi, V_\theta$ have been computed using the extreme-deconvolution algorithm (Bovy et al. 2011), as described in (Bird et al. 2021). This takes into account the individual velocity errors on each star in each component, as well as the covariances between the velocity errors (which are mostly driven by the covariance between the Gaia DR2 proper motions,
Figure 4. Profiles for the 3D spherical $\sigma$ (upper panel), $\sigma^2$ (middle panel), and $\frac{d\sigma^2}{dr}$ (lower panel) using KG (red) and BHB (blue) stars. Markers (circle, triangle, square) are $\sigma$ and $\sigma^2$ (in Galactocentric spherical coordinates $r, \theta, \phi$, respectively) measured from the data, whereas dashed lines are the exponential fit for $\sigma_r$. The uncertainties propagated from the observed data for each bin (explained in Section 2.1) are included, although many markers are larger than the uncertainty. In the lower panel, the shaded regions mark the statistical uncertainty for $\frac{d\sigma^2}{dr}$. The fit to the $\sigma_r$ velocity term is used to calculate the $\frac{d\sigma^2}{dr}$ term that go into our 3D spherical Jeans mass estimate. Each marker represents the median radius of the stars within our selected radial bins.

Figure 5. Profiles for Milky Way enclosed mass $M(<r)$ (upper panel) and circular velocity $v_{\text{circ}}$ (lower panel) using KG (red) and BHB (blue) stars. Jeans (filled markers) and TME (open markers) $M(<r)$ and $v_{\text{circ}}$ are plot for our KG (circles) and BHB (squares) samples. Each marker represents the median radius of the stars within our selected radial bins. The x-axis of the upper panel is in log scale and of the lower panel linear. Using the Bovy & Rix (2013); Bovy (2015) mass model consisting of a disk and bulge component and an NFW profile, we fit the data (markers) and plot the models for the Jeans (solid lines) and TME (dotted lines) methods. The best fit NFW profile yields a dark matter mass $M_{200}$ [$\times10^{12} M_\odot$] of 0.55 (KG Jeans), 1.00 (BHB Jeans), 0.46 (KG TME), and 0.67 (BHB TME). In the upper panel, the most distant markers of each KG (circles) and BHB (squares) mass profile are the corresponding total mass $M_{\text{tot}}$ estimates within the virial radius $r_{200}$ from the model fits. Gray triangle markers are the $M(<r)$ and $v_{\text{circ}}$ estimates from the literature using various methods and are summarized in Wang et al. (2020).

but also by the distance error for each star). We use the Poisson statistics of the number of stars in each bin (after using extreme-deconvolution to determine the velocity dispersion, taking into account the individual observational errors on the stellar velocities, and the covariances between them). We detail the propagation of the observational uncertainties in Section 2.1. As men-
Table 1. Enclosed total mass estimates.

| Method (star type) | \(< r^a\) | \(M(< r)\) \(\times 10^{11} M_\odot\) |
|-------------------|---------|---------------------------------|
| Jeans (KG)        | 73      | \(4.3 \pm 0.95^b\)             |
| Jeans (BHB)       | 52      | \(4.1 \pm 1.2\)                |
| TME (KG)          | 73      | \(4.3 \pm 1.1\)                |
| TME (BHB)         | 52      | \(3.6 \pm 1.6\)                |

aMedian Galactocentric distance of stars in the last radial bin.

bUncertainties propagated from the observed data are included, as described in Section 2.1. We detail the systematic uncertainty within the text (namely, in Section 3.3 we elaborate upon our adopted 15 percent systematic uncertainty due to non-spherical and non-virial effects).

In Figure 4, we show the velocity terms as functions of \(r\), their uncertainties, and the corresponding exponential fit to \(\sigma_r\). The Bayesian MCMC best fits \((\sigma_0, h_r,\) Equations 2–3\) are \((187 \pm 3 \text{ km s}^{-1}, 68 \pm 3 \text{ kpc})\) and \((143 \pm 5 \text{ km s}^{-1}, 111 \pm 20 \text{ kpc})\) for our KG and BHB samples, respectively.

Applying the Jeans equation yields the enclosed mass \(M(< r)\) and circular velocity \((v_{\text{circ}}(< r)) = \sqrt{GM(< r)/r}\) profiles shown in Figure 5.

Note that the error estimates displayed in Figures 4 and 5 are the uncertainties propagated from the observational data only (discussed in Sections 2.1–2.2. These random statistical errors are of order 10 to 30 percent. As discussed in Section 3.3, we add 15 percent systematic uncertainty to our mass estimate to account for non-virial and non-spherical effects.

We summarize our mass measurements in Table 1. We find that the 3D spherical Jeans equation yields the enclosed Milky Way mass profiles shown in Figure 5 (upper panel) with \(M(< 73 \text{ kpc}) = 4.3 \pm 0.95 \times 10^{11} M_\odot\) and \(M(< 52 \text{ kpc}) = 4.1 \pm 1.2 \times 10^{11} M_\odot\) for the KG and BHB samples, respectively. To these estimates for KG and BHB stars, we add \(\pm 0.6\) and \(\pm 0.6 \times 10^{11} M_\odot\), respectively, to account for systematic uncertainties.

We also display in Figure 5 individual mass estimates from various methods which are summarized by Wang et al. (2020); specifically, these include the measurements from Peñarrubia et al. (2014), Küpper et al. (2015), Malhan & Ibata (2019), McMillan (2011, 2017), Nesti & Salucci (2013), Posti & Helmi (2019), Watkins et al. (2019), Kafle et al. (2012), and Eadie & Jurić (2019).

4.5. Results: TME Milky Way Mass

We find that the Evans et al. (2011) mass estimator yields the enclosed Milky Way mass profiles shown in Figure 5 (upper panel) with \(M(< 73 \text{ kpc}) = 4.3 \pm 1.1 \times 10^{11} M_\odot\) and \(M(< 52 \text{ kpc}) = 3.6 \pm 1.6 \times 10^{11} M_\odot\) for the KG and BHB samples, respectively. We estimate the random uncertainty on the TME mass by bootstrapping the stellar sample as described in Section 2.2. As with the Jeans method, these random statistical uncertainties are of order 10 to 30 percent.
Taking into account the 15 percent systematic uncertainty (outlined in Section 3.3), we add $\pm 0.6 \times 10^{11} M_\odot$ to the KG (BHB) mass estimate.

We summarize our enclosed mass measurements in Table 1 and plot our mass and circular velocity profiles in Figure 5.

Figure 7. Quartiles for the Bayesian MCMC NFW fitting for $r_{200}$ and $c$ for the KG (upper panel) and BHB (lower panel) TME mass profiles in the upper panel of Figure 5.

4.6. Results: Virial Mass

We can extrapolate our results in the previous sections for the directly measured enclosed mass (which reaches distances of $\sim 50$ to $\sim 70$ kpc) out to the virial radius of the dark halo, by subtracting the baryonic component and fitting an NFW profile to the residual mass distribution.

Specifically, using the mass model and Bayesian MCMC method described in Section 2.3, we present the best fitting virial mass, radius, and concentration parameter to the KG and BHB 3D spherical Jeans and TME mass profiles in Table 2. The total mass estimates (baryonic mass plus dark mass $M_{200}$) at the virial radius for our KG and BHB Jeans and TME mass profiles are shown in Figure 5 (most distant markers for each corresponding profile). The quartiles are shown in Figure 6 and Figure 7 for the 3D spherical Jeans and TME methods, respectively. Together our two samples and methods give a weighted average of $M_{200} = 0.53 \pm 0.05 \times 10^{12} M_\odot$, $r_{200} = 172 \pm 5$ kpc, and $c = 20 \pm 3$.

Note that our constraint on $c$ is dominated by our inner data points, since the inflection point of the NFW profile is the scale radius $r_s = r_{200}/c$. This is as expected, as noted by Kafle et al. (2014), who show that tracer stars beyond about 40 kpc add little to our knowledge of the mass and concentration parameter for the NFW dark halo: rather, the inner data (in the Galactocentric range 5 to 40 kpc) dominate the fitting.

We compare our dark matter halo $M_{200}$ values with other virial mass estimates in the literature also using similar methods involving tracer samples in addition to the most recent mass estimates in Figure 8. Among the estimates shown which use either Jeans or TME (namely, Battaglia et al. 2005; Xue et al. 2008; Gnedin et al. 2010; Watkins et al. 2010; Kafle et al. 2012, 2014; Huang et al. 2016; Zhai et al. 2018; Sohn et al. 2018; Watkins et al. 2019; Fritz et al. 2020, and including our estimates) the weighted average virial mass is $\sim 0.83 \times 10^{12} M_\odot$ with a scatter of $< 10$ percent. The weighted average virial mass for all those shown in Figure 8 is $\sim 0.85 \times 10^{12} M_\odot$ with a scatter of $< 5$ percent; this excludes Zaritsky et al. (2020, lower limit $M_{200}$).

Table 2. Best fit parameters for an NFW dark matter profile.

| Method (star type) | $M_{200}$ $\times 10^{12} M_\odot$ | $r_{200}$ kpc | $c$ |
|-------------------|------------------------------------|--------------|-----|
| Jeans (KG)        | $0.55^{+0.15}_{-0.11}$            | $173^{+15}_{-12}$ | $18.2^{+7.1}_{-6.8}$ |
| Jeans (BHB)       | $1.00^{+0.67}_{-0.33}$            | $211^{+39}_{-26}$ | $9.4^{+4.4}_{-3.2}$ |
| TME (KG)          | $0.46^{+0.04}_{-0.04}$            | $162^{+4.9}_{-4.9}$ | $28.5^{+2.4}_{-2.2}$ |
| TME (BHB)         | $0.67^{+0.24}_{-0.14}$            | $185^{+17}_{-14}$ | $19.0^{+4.7}_{-4.1}$ |

All estimates are from the Bayesian MCMC fitting, we take the 50th percentile as the best fit and the 16th and 84th percentiles as the random uncertainty.
We find that the outer density power law slopes for our KG and BHB samples are similar ($\alpha_{\text{out}} = -4.6$ and $-4.7$, respectively). The break radius is slightly further out for the BHB sample ($r_{\text{break}} = 17$ and 22 kpc for our KG and BHB samples, respectively). We find that in order to maintain our criteria of $v_{\text{circ}} < 250$ km s$^{-1}$, the KG sample requires a more shallow inner density power law slope than we recover from our fit to the data and compared to the BHB sample; we must increase $\alpha_{\text{in}}$ from $-3.1$ to $-2.8$, both of which are more shallow than that of the BHB sample $\alpha_{\text{in}} = -3.5$.

Studies also find that the flattening of the density distribution of the stellar halo (deviation from sphericity) shows a dependency with metallicity (e.g., Chiba & Beers 2000; Carollo et al. 2007; Sato & Chiba 2022) such that the more metal rich stellar halo samples are more flattened compared to the more metal poor. As our KG sample peaks at higher metallicities, they likely represent a more flattened stellar halo system than our BHB sample. This is important because in our application of the Jeans equation and TME, we assume that the tracer sample is a spherical system tracing a spherical dark matter halo. If our KG sample if more flattened (an investigation which is beyond the efforts of this current study), our mass estimates from the KG sample will suffer from a slightly different bias compared to estimates using the BHB sample. As we elaborate upon in Section 3.3, we add a 15 percent systematic uncertainty which accounts for, e.g., non-sphericity of the density distribution of our halo tracer sample.

The LAMOST K giants are selected on the basis of effective temperature and surface gravity estimates (Liu et al. 2014) which are based on LAMOST spectra. The SDSS/SEGUE K giants are a collection of three main KG samples with slightly different spectroscopic selection criteria (see, e.g., Xue et al. 2014). The BHB stars are selected similarly, but via very different spectral features (predominantly due to hydrogen, whereas the K giants predominantly use Mg lines, and molecular bands due to TiO, CN, and CH).

BHB stars have smaller distance uncertainties than the KG stars; their absolute luminosities are much easier to calibrate, because of the nearly constant luminosity of the horizontal branch, compared to the wide range of luminosities covered by K giants. On the other hand, the number of BHB stars in our sample is fewer than K giants, so the sampling uncertainties are larger for our BHB sample.

The BHB stars are only the blue part of the horizontal branch; the red horizontal branch is better represented by redder stars such as RR Lyrae. Although the K giants have larger distance uncertainties compared to...
Figure 8. Comparison of our KG and BHB dark matter virial mass $M_{200}$ estimates with virial mass $M_{\text{vir}}$ estimates from the literature. We plot those measured using similar Jeans and TME methods involving tracer samples, as well as the recent virial mass estimates using various different methods. Colors represent the method as specified in the legend. The $\dagger$ marks virial mass estimates using definitions other than $M_{200}$. As the following exceptions, we plot the enclosed mass estimate within 200 kpc of Watkins et al. (2010), the lower limit $M_{200}$ estimate from Zaritsky et al. (2020), the best estimated range of mass measured by Rodriguez Wimberly et al. (2022), and we do not show uncertainties for Hattori et al. (2018) and Cunningham et al. (2020) since none are quoted for these mass estimates. The $x$-axis is in log scale. The weighted average virial mass is $\sim 0.85 \times 10^{12} M_\odot$ with a scatter of < 5 percent; this includes all mass estimates shown excluding Zaritsky et al. (2020), Rodriguez Wimberly et al. (2022), Hattori et al. (2018), and Cunningham et al. (2020) for the reasons above.
BHB stars, they represent a more complete sample of old populations because they later evolve to both blue and red parts of the horizontal branch.

BHB stars are selected spectroscopically, strongly relying on Balmer line cuts, thus they may suffer from larger selection effects than K giants as the large majority of our KG sample are selected by effective temperature and surface gravity criteria. As shown by Liu et al. (2014), their proposed KG selection criteria (the method we use for our LAMOST sample) select red-giant-branch stars over their full range of metallicities, and the contamination by stars other than K giants is small, < 2.5 percent.

Taking these into consideration, we expect that the KG sample is more complete and the better sample to measure the stellar halo density, kinematics, and mass profiles in future efforts.

5.2. Comparison of 3D Spherical Jeans and TME

The 3D spherical Jeans equation requires an assumption about the form of the potential, i.e., that it is spherical, that the velocity distribution can be well modeled with a single 3D velocity ellipsoid, and that the effects of bulk velocity flow are negligible.

The TME makes these assumption as well, and two additional assumptions for simplicity, that the dark matter halo follows an NFW profile and that the gravitational potential is well-described by a power law $\gamma$. Watkins et al. (2010) find the gravitational potential for NFW halos are well-described by a power-law of $\gamma = 0.5$, which is what is assumed in the Evans et al. (2011) TME that we use in this work.

Any difference between the Jeans method and the TME will likely be ascribed to these two assumptions.

Generally speaking in any analysis, the fewer the assumptions the better, and in this sense the Jeans equation is the preferred of our two methods. Additionally, it has significant advantages over the TME as we make full use of the 3D velocity distribution of the tracers (in particular the transverse velocities), whereas the TME parameterizes the behavior of the transverse velocities into the scaling factor using $\beta$.

On the other hand, in our application of the Jeans equation we assume a functional form to the radial velocity dispersion, which is entirely obviated in the TME method. The gradient of the radial velocity dispersion profile is required by the Jeans equation, and, although direct differentiation can be used, we have chosen a smoothly differentiable fit to this profile in order to compute this term. We choose an exponential function since it is easily differentiable and well behaved at the limits of the data. We explored linear and spline fits as well, and in several recent works, for example, B-splines have been used (Rehemtulla et al. 2021, 2022).

5.3. Systematic Uncertainty in Distance and Density of Tracers

Systematic uncertainty in the distances of the tracer stars leads to systematic error in the enclosed mass estimate: this is straightforward to estimate. Less straightforward is the form of the tracer density profile, which we here model as a power-law fall-off.

We show in Bird et al. (2019), that the systematic errors on the distances to the K giants and BHB stars is of order 10 percent. This affects the density profile fall-off, but also the transverse velocities of the stars through the proper motions (particularly at large Galactocentric distances). In Bird et al. (2019) and Bird et al. (2021) we show the effects on velocity anisotropy due to 10 percent distance uncertainties in our samples. We find that a ±10 percent systematic distance uncertainty causes systematic uncertainties in the mass estimates of 20 percent. For full details, see the Appendix.

More problematic is what uncertainty to adopt for the power-law fall-off index $\alpha$ in the density of the tracer stars. We are guided by the significant range of $\alpha$ in the literature, with values populating the range $-2 < \alpha < -5$, depending on the methods used and the Galactocentric radii probed.

Additionally, there is good evidence for a break in the power-law slope at around 20 kpc from the Galactic center. These we denote $\alpha_{in}$ and $\alpha_{out}$ within and without this break radius respectively.

We constrain the uncertainties on $\alpha_{in}$ by requiring that the enclosed mass at 10 kpc must lie in the range of 0.9 to 1.5 × 10^{11}M_\odot in order to be consistent with a circular velocity of $200 < v_{circ}(10\text{kpc}) < 250$ km s$^{-1}$ (we choose a larger uncertainty range compared to that measured by Bovy et al. (2012) to allow for a generous budget of possible values for $\alpha$). This leads to an uncertainty of 20 percent error in the mass due to uncertainty in $\alpha_{in}$. As mentioned earlier, observational constraints on the slope of the outer power-law fall-off, $\alpha_{out}$ are quite poor and could easily be as large as ±1. Adopting this uncertainty for $\alpha_{out} = -4.6 \pm 1$ yields an uncertainty of 20 percent on the enclosed mass at the limit of the data ($\sim 70$ kpc). This is similar to the uncertainty in the enclosed mass of 27 percent at 100 kpc found by Deason et al. (2021) (see their Equation 6) for the same uncertainty of ±1 in the power-law slope of the outer halo.

The observational errors in $v_{los}$ and proper motions add negligible uncertainty to the mass estimate compared to other sources of error.
5.4. Comparison with Previous Studies

The tracer mass estimator of Evans et al. (2003) has been tested by several authors using simulations (e.g. Wilkinson & Evans 1999; Yencho et al. 2006; Watkins et al. 2010; Deason et al. 2011b). Yencho et al. (2006) find a fundamental limit on the accuracy of the tracer mass estimator of 20 percent, due to systematic errors from substructure in the tracer population, regardless of sample size and the accuracy of the parameter measurements.

Wang et al. (2017) and Han et al. (2016b) have analyzed the stellar halo component of Milky Way type galaxies found in cosmological simulations. They use the Han et al. (2016a) steady-state spherical system and find an intrinsic systematic bias in the mass measurement of up to 30 percent, which is caused by the lack of equilibrium of the Milky Way size halos. This large error exists before observational error or uncertainty in the state of the system or potential are included.

Wang et al. (2015, 2016) find that as Galactocentric distance increases, mass estimators become increasingly less accurate and the leading source of error differs from halo to halo. They conclude the mass is best constrained within $0.2 r_{200}$ from the studied simulations. For the Milky Way, the mass estimate even at this radius may be complicated as we know the Sagittarius Stream is strongly prominent in the halo star counts in the range $30 < r < 50$ kpc.

Wang et al. (2015, 2016) investigate many of the possible biases involved with the tracer mass estimator such as the one used in the current study: correlation between model parameters, deviation from the NFW model, and violation of model assumptions including dynamical equilibrium, spherical symmetry, and infinite halo boundaries.

Rehemtulla et al. (2022) have developed a novel application of the 3D spherical Jeans method, for the specific case of the Galactic stellar tracer density and velocity distributions modeled non-parametrically using B-splines. They test their method on stellar halos in Milky Way-type galaxies in cosmological simulations, incorporating typical observational uncertainties for current surveys, and are able to recover the enclosed mass profiles with accuracies of around 20 percent or better depending on the amount of substructure or streaming motion in the galaxy. This uncertainty is largely systematic.

In contrast to their method, our approach is to partly parameterize the fitting. We do not measure the density profile of the tracer stars from our sample directly, due to the insufficient sample size and the many selection bias corrections that would be needed to do so. Instead, we have modeled our KG and BHB stellar density profiles using broken power laws based on the extensive work of Xu et al. (2018) and Das et al. (2016) for KG and BHB stars in the halo. As the Jeans equation requires the derivative of the radial velocity dispersion profile, we chose to fit it parametrically (using an exponential) to $\sigma_r$, since the function is easily differentiable and provides an adequate fit of the data. Finally, we use the binned velocity dispersion profiles directly in the Jeans Equation – no parametric fitting is required. We thus have a hybrid approach to the enclosed mass determinations. We are able to achieve mass estimates with random uncertainties of order 20 percent; our uncertainties are not systematic as in those measured by Rehemtulla et al. (2022).

Our results for the halo mass and concentration compare favorably with Kafle et al. (2014), who found $M_{\text{vir}} = 0.80^{+0.31}_{-0.16} \times 10^{12} M_\odot$ and $c = 21.1^{+14.8}_{-8.3}$, and another similar recent study by Huang et al. (2016), who find $M_{\text{vir}} = 0.9 \pm 0.1 M_\odot$ and $c = 18 \pm 1$. We find similar uncertainties to those found by Kafle et al. (2014), as we have followed a similarly careful analysis of random and (particularly) systematic errors.

Our mass estimate is in good accord with the mass estimates of previous studies applying similar 3D spherical Jeans methods to halo stars (e.g. Battaglia et al. 2005; Xue et al. 2008; Gnedin et al. 2010; Kafle et al. 2012, 2014; Huang et al. 2016, note that these authors define differently the virial mass). These mass estimates are compared in Figure 8.

Zhai et al. (2018) have measured the mass profile to 120 kpc using some 9000 halo K giants from LAMOST, in a study similar to our own. They use the Jeans equation to derive mass profiles for the Milky Way under assumptions about the velocity isotropy/anisotropy of the tracer stars. For NFW dark matter profiles, they derive a virial mass of $M_{200} = 1.08^{+0.17}_{-0.14} \times 10^{12} M_\odot$, $r_{200} = 205^{+10}_{-9}$ kpc, and a concentration parameter of $c = 18.5^{+3.6}_{-2.9}$, which is comparable within the uncertainties with our estimates for our 3D spherical Jeans KG mass profile with $M_{200} = 0.55^{+0.15}_{-0.11} \times 10^{12} M_\odot$ and best fit values of an NFW profile of $r_{200} = 173^{+15}_{-12}$ kpc and $c = 18.2^{+7.1}_{-6.8}$, as shown in Figure 8.

Most recently Sohn et al. (2018), Watkins et al. (2019), and Fritz et al. (2020) have applied the 3D TME method of Watkins et al. (2010) to various data sets with high quality proper motions. Sohn et al. (2018) use globular clusters with Hubble Space Telescope proper motions. Watkins et al. (2019) and Fritz et al. (2020) use globular clusters and satellites, respectively, with Gaia proper motions. Our mass estimates are in agreement with these three recent studies also using tracers (of dif-
fert types) with 3D velocities and a TME method. We include these works for comparison in Figure 8.

With the results from Gaia and the large number of stars collected from dedicated surveys, a plethora of Galactic mass estimates have recently been made using different methods and data samples. We summarize these most recent results in Figure 8. Grouping together the works applying a distribution function method (Callingham et al. 2019; Posti & Helmi 2019; Eadie & Jurić 2019; Vasiliev 2019; Li et al. 2020; Deason et al. 2021; Hattori et al. 2021; Correa Magnus & Vasiliev 2022; Shen et al. 2022; Slizewski et al. 2022; Wang et al. 2022) the weighted mean mass estimate is $\sim 0.93 \pm 0.047 \times 10^{12} M_\odot$. The methods using high velocity stars with full 6D phase-space information have been used to estimate the mass of the Milky Way (Monari et al. 2018; Deason et al. 2019; Grand et al. 2019; Koppelman & Helmi 2021; Necib & Lin 2022) have a weighted mean mass of $\sim 0.93 \pm 0.099 \times 10^{12} M_\odot$.

Two recent works using either the Sagittarius or Magellanic Steams (Vasiliev et al. 2021; Craig et al. 2021) give a weighted mean mass of $\sim 0.98 \pm 0.15 \times 10^{12} M_\odot$. The lightest mass measurements have come from rotation curve based methods (Eilers et al. 2019; de Salas et al. 2019; Ablimit et al. 2020; Karukes et al. 2020; Cautun et al. 2020; Jiao et al. 2021) with a weighted mean of $\sim 0.76 \pm 0.023 \times 10^{12} M_\odot$. The works which use 6D satellite phenomenology compared to Milky Way-type simulations (Patel et al. 2018; Villanueva-Domingo et al. 2021) infer a weighted mean Galactic mass of $\sim 0.97 \pm 0.0091 \times 10^{12} M_\odot$ and Rodriguez Wimberly et al. (2022) find their results are consistent with a Galactic mass in the range of $1 - 1.2 \times 10^{12} M_\odot$. Zaritsky et al. (2020) apply the timing argument to distant Milky Way halo stars to derive a lower limit to the Milky Way mass of $0.91 \times 10^{12} M_\odot$. Within the uncertainty, our BHB mass estimates are in agreement with Zaritsky et al. (2020), although our KG mass estimates and a few of the recent mass estimates which we plot in Figure 8 are lower than this limit.

In summary, we use two mass estimate methods on LAMOST and SEGUE K giants, and on SDSS BHB stars, finding good agreement between the methods and samples within the limits of our methods such as the systematic uncertainty in the tracer density profile.

The rotation curve we find is also in good agreement with the Huang et al. (2016) rotation curve measured from a mixed sample of HI gas, disk primary red clump stars and SEGUE halo K giants. We find our rotation curve is likewise in good agreement with the recent works of Eilers et al. (2019), Mróz et al. (2019), and Ablimit et al. (2020) who use various tracer stars in the disk, including red giants and Cepheids.

A dark matter virial mass of $M_{200} = 0.53 \pm 0.05 \times 10^{12} M_\odot$ (where we have taken the weighted average of the values in Table 2), when compared to the Milky Way’s total baryonic mass of $8 \times 10^{10} M_\odot$ (Bland-Hawthorn & Gerhard 2016, in this review of Milky Way properties, they include the stellar disk, bulge, cold disk gas, and hot halo gas in their total baryonic mass budget), yields a total baryonic mass fraction for the Milky Way of 0.15.

Systematic errors still dominate the uncertainty budget. In this study, the dominant one is the uncertainty in the density profile of the tracer stars. The immediate future looks bright however, as improved analyses of the stellar halo density profile are being made (see, e.g., Thomas et al. (2018) and references therein). Improved mass estimates for the Milky Way halo will result from these efforts in the near future. In two very interesting studies, Kaflé et al. (2018) and Wang et al. (2018) have shown that with appropriate tracers, the accuracy and precision of mass profile determinations for Milky Way-type galaxies (in simulations) can be constrained to better than 12 percent via the Jeans equation.

5.5. Conclusions

We use the 3D spherical Jeans equation and the Evans et al. (2011) TME to measure the Milky Way mass profile out to a Galactocentric distance of $\sim 50$ and $\sim 70$ kpc with the smooth, diffuse stellar halo samples of LAMOST and SDSS/SEGUE K giants and SDSS/SEGUE BHB stars from Bird et al. (2021). The extent of this sample reaches to past 100 kpc. For our two star types and their corresponding mass profiles derived from the two methods, we find a weighted average dark matter virial mass of $M_{200} = 0.53 \pm 0.05 \times 10^{12} M_\odot$ with weighted average virial radius and concentration parameter $r_{200} = 172 \pm 5$ kpc and $c = 20 \pm 3$.

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Data Availability Statement:
The data presented in the figures are available upon request from the authors.

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Figure 9. Test of distance uncertainties on the enclosed mass $M(<r)$ profile. We systematically change our distance scales for halo LAMOST K giants (left panels) and SDSS BHB stars (right panels) by ±10 percent and remeasure the mass using 3D spherical Jeans (upper panels) and TME (lower panels). Black symbols show the mass profile for the smooth, diffuse halo LAMOST K giants and SDSS BHB stars using the original distances used for the main results in this work. Shaded regions show the results of the test for overestimated mass (blue, due to overestimating distances by 1.1$d_{\text{helio}}$) and underestimated mass (red, due to underestimating distances by 0.9$d_{\text{helio}}$). The width of the shaded regions are based on the estimated uncertainties in mass. Each marker represents the median radius of the stars within our selected radial bins.

APPENDIX

In Bird et al. (2019) and Bird et al. (2021) we have investigated the accuracy and effects due to uncertainty in heliocentric distances $d_{\text{helio}}$ for KG and BHB stars. We here test the effects of a ±10 percent systematic distance uncertainty for both our KG and BHB stars. This is a generous budget in uncertainty to test. In Bird et al. (2019) we detected an approximate 10 percent underestimation for our LAMOST halo KG distance scale when compared to that of Bailer-Jones et al. (2018) for halo KG stars within 4 kpc of the Sun. We prefer to use the Xue et al. (2014) distance method since it uses globular cluster fiducials observed by SDSS for calibration whereas Bailer-Jones et al. (2018) uses open clusters. The more metal poor globular cluster fiducials are more appropriate for our stellar halo sample than more metal rich open clusters. Yang et al. (2019), using the same KG and BHB sample as our study, select stars with high quality parallaxes within 4 kpc and compare the KG and BHB stars with inverted parallax. Although they find the Xue et al. (2014) KG distances are closer on average by ∼15 percent, this bias decreases with fainter Gaia $G$ magnitudes; thus our preference remains to use the Xue et al. (2014) method as the large bulk of our halo KG stars have fainter $G$ magnitudes. Yang et al. (2019) find no bias in BHB distances. Although we explore the effects on mass estimation due to biased BHB distances, this is a generous test as we are more confident in the accuracy of BHB distances.
As in Section 4.2, we compute the density profiles for spherical shells along Galactocentric radius $r$ using the Xu et al. (2018) and Das et al. (2016) models for our KG and BHB samples, respectively. The ±10 percent distance uncertainties introduce variations of a few tenths in the power law (both inner and outer) and ±1 kpc in break radius. When needed, we adjust the power law to ensure $200 < v_{\text{circ}}(10 \, \text{kpc}) < 250 \, \text{km} \, \text{s}^{-1}$.

In Bird et al. (2019) and Bird et al. (2021) we have shown that a ±10 percent distance bias induces changes in the velocity anisotropy $\beta$ of less than one or two tenths.

The changes due to ±10 percent systematic distance uncertainties are shown in Figure 9 where we present the test results for the Jeans and TME enclosed mass profiles for the LAMOST KG and SDSS BHB smooth halo samples. The changes induced are of order 20 percent.

We estimate the virial mass from these tests and find differences of order 50 percent.