A Multi-proxy Signature Scheme for Partial delegation with Warrant

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Abstract

In some cases, the original signer may delegate its signing power to a specified proxy group while ensuring individual accountability of each participant signer. The proxy signature scheme that achieves such purpose is called the multi-proxy signature scheme and the signature generated by the specified proxy group is called multi-proxy signature for the original signer. Recently such scheme has been discussed by Lin et al. Lin’s scheme is based on partial delegation by Mambo et al. In present chapter we introduce a new multi-proxy signature scheme, which requires less computational overhead in comparison to Lin et al, and also fulfill the requirement of partial delegation with warrant simultaneously.

1 Introduction

Mambo et al introduced a proxy signature scheme in which an original signer delegates his signing power to another signer called the proxy signer. They also proposed three proxy signature schemes for partial delegation based on ElGamals, Okamotos and Schnorrs scheme. Since then several proxy-signature schemes have been developed [?, ?, ?, ?, ?, ?, ?]. One well-known variation of the proxy signature scheme by Kim et al includes a warrant in the proxy signature. The warrant is a message, which includes information of the messages types that are delegated and proxy signers identity that prevent the transfer of proxy power to another party. Another variation, viz, threshold proxy signature scheme was proposed in [?] in which original signer delegates its signing power to a set of proxy signers in such a way that any t out of these n proxy signers may produce a valid proxy signature.

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multi-proxy signature scheme, which requires less computational overhead in comparison to Lin et al, and also fulfills the requirement of partial delegation with warrant simultaneously. Our scheme is based on Kim et al’s scheme. For ordinary signing operation we use Schnorrs scheme.

The rest of the chapter is summarized thus: In section 2 we give review of literature. In section 3 Kim’s scheme is discussed. Section 4 contains the proposed scheme. Section 5 contains proofs for correctness of the scheme and in section 6 we discuss performance. Finally, Section 7 concludes the discussion.

2 Related Previous Works

Mambo et al. [?] developed a systematic approach to proxy or delegated signatures. After the introducing the proxy problem they firstly divide the delegation in three kinds; which was further extended by Kim et al [?] by adding one more kind partial delegation by warrant. Till now we have the following four types of delegations discussed in previous chapters.

Yi et al, in 2000, proposed proxy multisignature scheme which allows a group of original signers to delegate its signing power to a single proxy signer. Proxy signer can sign any message on behalf of whole group. Further, Hwang et al, in 2001, introduced a new proxy multi-signature scheme.

On the other hand Lee et al’s scheme states a multi proxy signature scheme which fulfills similar criteria discussed as before. This scheme also allows a group of original signers to delegate its signing capability to a single proxy signer. We feel the nomenclature used by Lee et al is not proper. To remove this ambiguity we define these terms: multi proxy signature and proxy multi signature as follows:

**Definition 1.** An original signer delegates its signing power to the specified proxy group while ensuring individual accountability of each participant signer. The proxy signature that is generated by such a specified proxy group, we call, multi proxy signature.

**Definition 2.** A specified original signers group delegate the groups signing power to a single proxy signer. The proxy signature that is generated by such a proxy signer, we call, proxy multi-signature.

3 Kim’s Scheme for Partial Delegation [?]

Before introducing our proposed scheme it is necessary to introduce Kim et al’s scheme. Here \( p \) denotes a large prime with \( 2^{511} < p < 2512 \) and \( g \) denotes a generator for \( \mathbb{Z}_p^* \). Each user selects a secret key \( x_u \in \mathbb{Z}_q^* \) and computes a public key \( y_u = g^{x_u} \mod p \). The system parameters are -
• $p$: A large prime number
• $q$: A prime factor of $(p - 1)$
• $g$: Element of $\mathbb{Z}_p^*$ of order $q$
• $x_u$: Secret key of the original signer $S$, here $x_u \in \mathbb{Z}_q$
• $y_u$: Public key of the original signer $S$, such that $y_u = g^{x_u} \mod p$

Basic protocol consists of the following:

3.0.1 Proxy Generation

1. (Key Generation)- An original signer selects a random number $k \in \mathbb{Z}_q$, $k \neq 1$ and computes
   
   $$r = g^k \mod p \text{ and } e = h(m_w, r)$$

   where $m_w$ is warrant message having the information about delegation and $h$ is publicly known hash function. Now he/she computes the proxy signature key $s = xe + k \mod q$

2. (Proxy Key Delivery)- The original signer sends $(m_w, s, r)$ to a proxy signer in a secure way.

3. (Key verification)- After receiving the secret key $(m_w, s, r)$ the proxy signer computes $e = h(m_w, r)$ and checks the validity of the key with the following equation
   
   $$g^s = y^e r \mod p$$

   If this congruence passes, he/she accepts it as secret key otherwise rejects it and request another one, or simply stops the protocol.

3.0.2 Proxy Signing

When the proxy signer signs a message $m$ on behalf of the original signer, he computes a signature $s_p$ using any original signature scheme and $s$ as the secret key. Then the pair $(m, m_w, s_p, r)$ is the proxy signature,

3.0.3 Verification

The verification of the proxy signature is carried out by same checking operation as in original signature scheme except for extra computation

$$e = h(m_w, r) \text{ and } y' = y^e r \mod p$$

The value $y'$ may be dealt with as a new public key, which shows the involvement of Alice.
4 Proposed Scheme

4.0.4 System Setup

Choose two large primes $p$ and $q$ such that $q | (p - 1)$, a generator $g \in \mathbb{Z}_p^*$ with order $q$. $h(.)$ is a one way hash function. The original signer O has its private ket as $x_0 \in \mathbb{Z}_q$ and public $y_0 = g^{x_0} \mod p$. $P = \{P_1, P_2, ..., P_n\}$ is the set of the delegated signers. Then each $P_i$ has its private key $x_i$ and public key $y_i$ where $y_i = g^{x_i} \mod p$.

4.0.5 Proxy Generation

1. (Key Generation)- The original signer selects $n$ random numbers $k_i \in \mathbb{Z}_q, k_i \neq 1$ (for $i = 1, 2, ..., n$) and computes a commitment

$$r_i = g^{k_i} \mod p$$

where $m_w$ is warrant message having the information about delegation and $r = \Pi r_i \mod p$. Now he/she computes the proxy signature key

$$\sigma = x r + k \mod q$$

2. (Proxy Key Delivery)- The original signer sends $(\sigma, r)$ to a proxy signer in a secure way and makes $m_w$ public.

3. (Key verification)- After receiving the tuple $(m_w, s, r)$ the proxy signer computes $m_w = g^{-\sigma} y_0^r \mod p$ and checks the $m_w$. If this holds, he/she accepts it as secret key otherwise rejects it and request another one, or simply stops the protocol.

4.0.6 Proxy Signing

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4.0.7 Verification

The verification of the proxy signature is carried out by same checking operation as in original signature scheme except for extra computation

$$e = h(m_w, r) \text{ and } y' = y^e r \mod p$$

The value $y'$ may be dealt with as a new public key, which shows the involvement of Alice.
Table 1: Lin et al’s scheme

| Phases                          | Computational Load       |
|---------------------------------|--------------------------|
| Proxy generation with verification | $(n + 3)E + (2n)M + H$    |
| Multi proxy signature generation (including proxy verification) | $(2n + 2)E + (n^2 + 2n)M + (n + 1)H$ |
| Multi-signature verification    | $2E + M + H$             |

Table 2: Our Scheme

| Phases                          | Computational Load       |
|---------------------------------|--------------------------|
| Proxy generation with verification | $(n + 4)E + (2n + 4)M + 2I$ |
| Multi proxy signature generation (including proxy verification) | $(5n + 2)E + (4n + 4)M + 2H$ |
| Multi-signature verification    | $3E + 3M + H$             |

5 Performance

Let $E$, $M$ and $I$ respectively denote the computational load for exponentiation, multiplication and inversion. Then following table shows the computational load of our scheme. Each phase in our scheme has less computational load than Lin et al’s scheme. In our scheme computational load in verification phase is $2E + M + H$, whereas in Lin et al scheme it is $3E + 3M + H$. In some applications digital information is signed once but verified more than once. In such situation the efficiency of our scheme increases with the number of times verification is done. Security of scheme is inherited from original scheme used. Further the total computation cost in our scheme is less than other existing schemes. Thus, our scheme has computational advantage over other schemes.

6 Correctness of the Scheme

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7 Conclusion

In present paper we introduced a new multi-proxy signature scheme, which requires less computational overhead in comparison to Lin et al, and also fulfill the requirement of partial delegation with warrant simultaneously. Our scheme is based on Kim et al’s scheme. We also reviewed the ambiguity in nomenclature of multi proxy signature and proxy multi signature. and redefine these both signatures on the basis of literature in existence.