HITTING TIMES, COMMUTE TIMES, AND COVER TIMES FOR RANDOM WALKS ON RANDOM HYPERGRAPHS

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Abstract. We consider simple random walk on the structure given by a random hypergraph in the regime where there is a unique giant component. Using their spectral decomposition we give the asymptotics for hitting times, cover times, and commute times and show that the results obtained for random walk on random graphs are universal.

1. Introduction

Random walks on random graphs have been an active research area in probability theory for a long time, see e.g. [DS84, Lov93, Woe00]. Besides being a field that poses interesting questions in its own right, they have also been a key tool to understand the properties of random graphs, especially close to the point of phase transition (for a very readable survey see the recent monograph [vdH17]). These so-called exploration processes have been transferred to the investigation of random hypergraphs, see e.g. [BR12, BR17]. This fact may motivate the study of random walks on random hypergraph structures as well.

However, already [CFR11, CFR13] studied the so-called cover time of random walk on a random uniform hypergraph. They considered the following model: Take \( H \) uniformly at random from all \( r \)-regular, \( d \)-uniform hypergraphs. Hence every vertex \( v \in V := \{1, \ldots, n\} \) is contained in \( r \) hyperedges and for all hyperedges \( e \in E \) it holds \( |e| = d \). Colin, Frieze and Radzik analyze simple random walk on the resulting structure, i.e. if the random walk is in a vertex \( v \) at time \( t \in \mathbb{N} \), for the vertex at time \( t + 1 \) it selects a hyperedge \( e \), such that \( v \in e \) and then it selects any \( w \neq v \) in \( e \) with probability \( \frac{1}{d-1} \) and walks there. For this walk the authors analyze the
so-called cover time, i.e. the expected time it takes the walk to see every vertex of $V$. They show that this time $C(H)$ is of order $(1 + \frac{1}{(r-1)(d-1)-r}) n \log n (1 + o(1))$.

Inspired by the results in [LT14] we will study the hitting times, commute times, and cover times for random walks on random hypergraphs. We will refrain from considering regular hypergraphs, but stick with uniform hypergraphs setting. This means, the underlying structure will consist of a realization of a random $d$-uniform hypergraph $H$ on $V = \{1, \cdots, n\}$, i.e. all edges $e \in \binom{V}{d}$ are selected independently and with equal probabilities $p$. This model is known $H(n, p)$ and $E$ is the edge set of the hypergraph. We assume that $p = p_n \gg \frac{\log^4 n}{n^{d-1}}$ (where we write $a_n \gg b_n$, if and only if $\frac{b_n}{a_n} \to 0$), such that, with probability converging to 1, $H$ is connected. All the probabilities considered below are to be understood conditionally on the event that $H$ is connected.

On this structure we will consider simple random walk as described above. This random walk, that we will henceforth call $(X_i)$, can either be considered as a random walk on the multi-graph $G = (V, \tilde{E})$ associated with $H$, i.e. if $v, w \in V$ are in $k$ hyperedges, then there are $k$ edges connecting $v$ and $w$ in $\tilde{E}$. Alternatively, we can consider the random walk on the weighted graph, where the weight of an edge $\{v, w\}$ is the number of hyperedges containing both $v$ and $w$. The invariant measure of the walk is

$$
\pi(i) = \frac{\sum_{e \in E} 1_{\{i \in e\}}}{d|E|} = \frac{d(i)}{\sum_{j \in V} d(j)}
$$

where the degrees $d(i)$ are counted in the multi-graph interpretation.

2. **Hitting times**

For the random walk $(X_i)$ consider the following quantities. Let $H_{ij}$ be the expected time it takes the walk to reach vertex $j$ when starting from vertex $i$. Moreover, let

$$
H_j := \sum_{i \in V} \pi(i) H_{ij} \quad \text{and} \quad H^i := \sum_{j \in V} \pi(j) H_{ij}
$$

be the average target hitting time and the average starting hitting time, respectively (these names are taken from [LPW09]). Note that both, $H_j$ and $H^i$ are expectation values in the random walk measure, but random variables with respect to the realization of the random hypergraph. Also note that, in general, $H_j$ and $H_i$ will be different.

In [LT14] the same quantities were studied for random graphs instead of random hypergraphs and it was shown that $H_j = n(1 + o(1))$ asymptotically almost surely (a.a.s., for short), which means that the probability that a vertex $j$ admits $H_j$ that is not of this order, vanishes for $n \to \infty$. This result confirmed a prediction in the physics literature (see [SRBA04]). The aim of the present note is to generalize this result to our random hypergraph setting. Our results can hence be understood as a universality statement about random graphs and hypergraphs. They also may
be interpreted as a generalization of the results in [LT14] to weighted graphs and multigraphs. A key difference between the random graph case and our situation, however, is not only that we may have multiple edges connecting two nodes, but also that these edges are no longer independent. Moreover, a key tool in [LT14] is the analysis of the spectrum of a random graph taken from [EKYY13]. This is not available in our setting.

We will thus to give asymptotic results for $H_j$ and $H^i$. To this end, we will derive a different representation of $H_j$ and $H^i$ as in [Lov93]. Let $B := \sqrt{DA} \sqrt{D}$ be the graph Laplacian of the hypergraph structure we realize. Here $D := \left( \text{diag} \left( \frac{1}{d_i} \right) \right)_{i=1}^n$ and $A = (a_{ij})$ is the adjacency matrix of the multi-graph $\tilde{G} = (V, \tilde{E})$. Thus,

$$a_{ij} = \sharp \{ e \in \tilde{E} : e = \{i, j\} \}$$

and

$$B = \left( \frac{a_{ij}}{\sqrt{d_i} \sqrt{d_j}} \right).$$

Let

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$$

be the eigenvalues of $B$. $w := (\sqrt{d_1}, \ldots, \sqrt{d_n})$ satisfies

$$Bw = \sum_{i=1}^n a_{ij}w_j = \sum_{i=1}^n a_{ij}d_i = \frac{d_i}{\sqrt{d_i}} = \sqrt{d_i}.$$  

Thus, $\lambda_1 = 1$ is an eigenvalue for the matrix $B$ and by the Perron-Frobenius theorem it is the largest one. We will always normalize the eigenvectors $v_k$ to the eigenvalues $\lambda_k$ to length one such that, in particular,

$$v_1 := \frac{w_*}{\sqrt{2|\tilde{E}|}} = \left( \frac{\sqrt{d_j}}{2|\tilde{E}|} \right)_{j=1}^n.$$

In general, the matrix of the eigenvector is orthogonal and the scalar product of two eigenvectors $v_i$ and $v_j$ satisfies $\langle v_i, v_j \rangle = \delta_{ij}$. In particular, for $v_1$ we obtain:

$$0 = \langle v_k, v_1 \rangle = \frac{1}{2|\tilde{E}|} \sum_{j=1}^n v_{k,j} \sqrt{d_j} \text{ for } k \neq 1 \text{ and } \sum_{j=1}^n v_{k,j}^2 = \sum_{k=1}^n v_{k,j}^2 = 1.$$

A key observation for our context is that hitting times possess a spectral decomposition as was given by Lovász (see [Lov93]) in the following theorem.

**Theorem 1.** [Lov93, Theorem 3.1] The expected hitting times have the following spectral decomposition

$$H_{ij} = 2|\tilde{E}| \sum_{k=2}^n \frac{1}{1 - \lambda_k} \left( \frac{v_{k,j}^2}{d_j} - \frac{v_{k,i}v_{k,j}}{\sqrt{d_i d_j}} \right). \tag{1}$$
As a matter of fact, Lovász proves this theorem just for ordinary graphs. It is, however, simple matter to check that it easily translates to multi-graphs. Theorem 1 allows to also give a spectral representation of the average target hitting time and the average starting hitting time $H_j$ and $H^i$. Indeed, using Theorem 1 together with the orthogonality of the eigenvectors gives

$$H_j = \sum_{i=1}^{n} \pi(i) H_{ij} = \sum_{i=1}^{n} \sum_{k=2}^{n} \frac{1}{1 - \lambda_k} \left( v_{k,j}^2 \frac{d_i}{d_j} - v_{k,i} v_{k,j} \sqrt{\frac{d_i}{d_j}} \right)$$

$$= \left( \frac{1}{d_j} \sum_{i=1}^{n} d_i \right) \left( \sum_{k=2}^{n} \frac{1}{1 - \lambda_k} v_{k,j}^2 \right) - \sum_{k=2}^{n} \frac{1}{\sqrt{d_j}} \frac{1 - \lambda_k}{1} \sum_{i=1}^{n} v_{k,i} \sqrt{d_i}$$

$$= \frac{2|E|}{d_j} \sum_{k=2}^{n} \frac{1}{1 - \lambda_k} v_{k,j}^2 \sqrt{2|E|} \frac{v_{k,j}}{\sqrt{d_j}} < v_k, v_1 >$$

$$= \frac{1}{\pi(j)} \sum_{k=2}^{n} \frac{1}{1 - \lambda_k} v_{k,j}^2$$

Similarly we obtain,

$$H^i = \sum_{j=1}^{n} \pi(j) H_{ij} = \sum_{j=1}^{n} \sum_{k=2}^{n} \frac{1}{1 - \lambda_k} \left( v_{k,j}^2 \frac{d_i}{d_j} - v_{k,i} v_{k,j} \sqrt{\frac{d_i}{d_j}} \right)$$

$$= \sum_{k=2}^{n} \frac{1}{1 - \lambda_k} \left( \sum_{j=1}^{n} v_{k,j}^2 - v_{k,i} \sqrt{\frac{1}{d_i} \sum_{j=1}^{n} v_{k,j} \sqrt{d_j}} \right) = \sum_{k=2}^{n} \frac{1}{1 - \lambda_k}$$

Note, that by orthogonality we have

$$\sum_{k=2}^{n} v_{k,j}^2 = 1 - v_{1,j}^2 = 1 - \pi(j).$$

On the other hand

$$\sum_{k=2}^{n} (1 - \lambda_k) v_{k,j}^2 = \sum_{k=1}^{n} (1 - \lambda_k) v_{k,j}^2 = 1 - B_{jj} = 1$$

since $B = \sum_{k=1}^{n} \lambda_k v_k v_k^T$ (by the spectral theorem and the fact that the adjacency matrix has zeros on the diagonal). Therefore, employing the inequality between arithmetic and harmonic means

$$\frac{\sum_{k=2}^{n} \frac{1}{1 - \lambda_k} v_{k,j}^2}{\sum_{k=2}^{n} v_{k,j}^2} \geq \frac{\sum_{k=2}^{n} v_{k,j}^2}{\sum_{k=2}^{n} (1 - \lambda_k) v_{k,j}^2}.$$
Thus

\[ H_j = \frac{1}{\pi(j)} \sum_{k=2}^{n} \frac{1}{1 - \lambda_k} v_{k,j}^2 \geq \frac{1}{\pi(j)} \frac{(\sum_{k=2}^{n} v_{k,j}^2)^2}{(1 - \lambda_k) v_{k,j}} = \frac{1}{\pi(j)} (1 - \pi(j))^2 \geq \frac{2|\tilde{E}|}{d_j} - 2 \]

On the other hand,

\[ H_j = \frac{2|\tilde{E}|}{d_j} \sum_{k=2}^{n} \frac{1}{1 - \lambda_k} v_{k,j}^2 \leq \frac{2|\tilde{E}|}{d_j} \frac{1}{1 - \lambda_2} (1 - \pi(j)) = \frac{2|\tilde{E}|}{d_j} \frac{1}{1 - \lambda_2} (1 - \frac{d_j}{2|\tilde{E}|}). \]

It thus suffices to analyze the behaviour of $|\tilde{E}|$, $d_j$, and the size of the spectral gap $1 - \lambda_2$.

For the first two quantitites, consider any vertex $j \in H$. Then

\[ d_j = (d - 1) \sharp \{ e : j \in e \} \quad \text{i.e.} \quad d_j = \sum_{i_1 < i_2 < \ldots < i_{d-1}, \forall k = 1, \ldots, d-1} X_{i_1, i_2, \ldots, i_{d-1}, j} \]

where $X_{i_1, \ldots, i_d}$ is the indicator for the presence of the edge $(i_1, \ldots, i_d)$. Note that $\mathbb{E}(d_j) = \binom{n}{d-1} p$ tends to $\infty$ by definition of $p$. By Chernoff’s inequality:

\[ \mathbb{P}(d_j \leq \mathbb{E}(d_j) - \lambda) \leq e^{-\frac{\lambda^2}{2\mathbb{E}(d_j)}} \quad \text{and} \quad \mathbb{P}(d_j \geq \mathbb{E}(d_j) + \lambda) \leq e^{-\frac{\lambda^2}{2\mathbb{E}(d_j) + \frac{\lambda}{3}}} \]

Choosing $\lambda = c\sqrt{\binom{n}{d-1} p}$ for some constant $c > 0$ leads to:

\[ \mathbb{P} (\mathbb{E}(d_j) - \lambda < d_j < \mathbb{E}(d_j) + \lambda) = \mathbb{P} (\{ d_j \leq \mathbb{E}(d_j) - \lambda \} \cup \{ d_j \geq \mathbb{E}(d_j) + \lambda \}) \]

\[ = 1 - \mathbb{P} (\{ d_j \leq \mathbb{E}(d_j) - \lambda \} \cup \{ d_j \geq \mathbb{E}(d_j) + \lambda \}) \]

\[ \geq 1 - \mathbb{P}(d_j \leq \mathbb{E}(d_j) - \lambda) - \mathbb{P}(d_j \geq \mathbb{E}(d_j) + \lambda) \]

\[ \geq 1 - e^{-\frac{\lambda^2}{2\mathbb{E}(d_j)}} - e^{-\frac{\lambda^2}{2\mathbb{E}(d_j) + \frac{\lambda}{3}}} \]

\[ \geq 1 - e^{-\frac{\lambda^2}{2\mathbb{E}(d_j)}} - e^{-\frac{\lambda^2}{2\mathbb{E}(d_j) + \frac{\lambda}{3}}} \geq 1 - 2e^{-\frac{\lambda^2}{3\mathbb{E}(d_j)}} \]

for $n$ sufficiently large.

On the other hand, $|\tilde{E}| = \binom{d}{2} \sharp \{ e : e \in E \}$ where $E$ is the set of hyperedges. Thus $\mathbb{E}(|\tilde{E}|) = \binom{d}{2} \binom{n}{d} p$. If we consider a deviation of $c\sqrt{\binom{d}{2} \binom{n}{d} p}$ for some $c > 0$ we again obtain by an application of Chernoff’s inequality as above that with probability $1 - 2e^{-\frac{\lambda^2}{3\mathbb{E}(d_j)}}$:

\[ \left( \binom{d}{2} \binom{n}{d} p - c\sqrt{\binom{d}{2} \binom{n}{d} p} < \tilde{E} < \left( \binom{d}{2} \binom{n}{d} p + c\sqrt{\binom{d}{2} \binom{n}{d} p} \right) \right) \]

If we choose $c = \log n$ we obtain that for every fixed $j$ with probability at least $1 - 4e^{-\frac{(\log n)^2}{4}}$:

\[ \frac{2|\tilde{E}|}{d_j} \leq \frac{2\binom{d}{2} \binom{n}{d} p + \log n \sqrt{\binom{d}{2} \binom{n}{d} p}}{(d-1)\binom{n}{d} p - \log n \sqrt{n^{d-1} p}} = n(1 + o(1)) \]
(due to our choice of \( p \)). Similarly we see that \( \frac{2|\tilde{E}|}{d_j} \geq n(1 - o(1)) \) with probability at least \( 1 - 4e^{-\frac{(\log n)^2}{4}} \). Since \( ne^{-\frac{(\log n)^2}{4}} \) converges to 0, we see that \( \frac{2|\tilde{E}|}{d_j} = n(1 + o(1)) \) a.a.s. simultaneously for all \( j \).

Now, we turn to the spectral gap. Fortunately most of the work has already been done by Lu and Peng (see [LP12]) consider \( d \)-uniform hypergraphs \( H \) and for every pair of sets \( I \) and \( J \) with cardinality \( s \) they associate a weight \( w(I, J) \), which is the number of edges in \( H \) passing through \( I \) and \( J \) if \( I \cap J = \emptyset \), and 0, otherwise. The \( s \)-th Laplacian of \( H \) is defined to be the normalized Laplacian of the thus obtained weighted graph. As a special case, for \( s = 1 \) we can thus consider the Laplacian \( L := I - D^\frac{1}{2}AD \frac{1}{2} \). As shown in [LP12] the ordered eigenvalues of \( L \) fulfill

\[
0 = \tilde{\lambda}_0 \leq \tilde{\lambda}_1 \leq \cdots \leq \tilde{\lambda}_{n-1} \leq 2
\]

and:

**Theorem 2.** (cf. [LP12], Theorem 2] of which this is a special case) Denote by 
\( \lambda = \max \{ 1 - \lambda_1, \lambda_{n-1} \} = \lambda(H^d(n, p)) \). If \( p(1-p) \gg \frac{\log^4 n}{n^4} \) and \( 1-p \gg \frac{\log n}{n^2} \) then a.a.s.

\[
\lambda(H^d(n, p)) \leq \frac{1}{n-1} + (3 + o(1)) \sqrt{\frac{1-p}{\binom{n-1}{d-1}p}}.
\]

**Remark 1.** The second condition on \( p \), \( 1-p \gg \frac{\log n}{n^2} \), may be omitted for our purposes because just serves to control the smallest eigenvalue of \( L \). Also note that \( \sqrt{\frac{1-p}{\binom{n-1}{d-1}p}} \) is at most of order \( \frac{1}{\log n} \).

Translated to our problem, Theorem 2 implies that the eigenvalues \( \lambda_1, \ldots, \lambda_{n-1} \) for the matrix \( D^\frac{1}{2}AD^\frac{1}{2} = I - L \) satisfy \( \lambda_1 = 1 \) and

\[
1 - \lambda_2 \geq 1 - \frac{1}{n-1} - (3 + o(1)) \sqrt{\frac{1-p}{\binom{n-1}{d-1}p}}.
\]

Thus we get the following upper bound

**Corollary 1.** If \( p(1-p) \gg \frac{\log^4 n}{n^4} \) a.a.s.

\[
\frac{1}{1-\lambda_2} \leq \frac{1}{1 - \frac{1}{n-1} - (3 + o(1)) \sqrt{\frac{1-p}{\binom{n-1}{d-1}p}}} = 1 + o(1)
\]

Thus we have seen

**Theorem 3.** If \( p(1-p) \gg \frac{\log^4 n}{n^4} \) then a.a.s.

\[
H_j = n(1 + o(1)).
\]

On the other hand, we have already seen that \( H^i = \sum_{k=2}^{n} \frac{1}{1-\lambda_k} \). We therefore obtain
Theorem 4. If \( p(1 - p) \gg \frac{\log^4 n}{n^{d-1}} \) then a.a.s.
\[
\frac{1}{2} n(1 + o(1)) \leq H^i \leq n(1 + o(1)).
\]

Proof. The key observation is that under the given conditions we have that
\[
\frac{1}{2} \leq \frac{1}{1 - \lambda_k} \leq \frac{1}{1 - \lambda_2} = 1 + o(1)
\]
for all \( k \). This proves the assertion.

\[\square\]

3. Commute times and Cover times

We turn now to the study of the commute time \( \kappa(i, j) = H_{ij} + H_{ji} \). An elementary computation using Theorem 1 gives that
\[
\kappa(i, j) = 2|\tilde{E}| \sum_{k=2}^{n} \frac{1}{1 - \lambda_k} \left( \frac{v_{k,i}}{d_i} - \frac{v_{k,j}}{d_j} \right)^2
\]
(also see [Lov93, Corollary 3.2]). Using this representation we obtain:

Proposition 3.1. For all \( i, j \in V \) we obtain the following bounds for the commute time
\[
|\tilde{E}| \left( \frac{1}{d_i} + \frac{1}{d_j} \right) \leq \kappa(i, j) \leq \frac{2|\tilde{E}|}{1 - \lambda_2} \left( \frac{1}{d_i} + \frac{1}{d_j} \right).
\]

Proof. The proof follows the ideas in the of an unweighted simple graph (see [Lov93]). Again \( \frac{1}{2} \leq \frac{1}{1 - \lambda_k} \leq \frac{1}{1 - \lambda_2} \). Hence
\[
|\tilde{E}| \sum_{k=2}^{n} \left( \frac{v_{k,i}}{\sqrt{d_i}} - \frac{v_{k,j}}{\sqrt{d_j}} \right)^2 \leq \kappa(i, j) \leq \frac{2|\tilde{E}|}{1 - \lambda_2} \sum_{k=2}^{n} \left( \frac{v_{k,i}}{\sqrt{d_i}} - \frac{v_{k,j}}{\sqrt{d_j}} \right)^2.
\]

But
\[
\sum_{k=2}^{n} \left( \frac{v_{k,i}}{\sqrt{d_i}} - \frac{v_{k,j}}{\sqrt{d_j}} \right)^2 = \frac{1 - \pi(i)}{d_i} + \frac{1 - \pi(j)}{d_j} - 2 \sum_{k=1}^{n} \frac{v_{k,i}v_{k,j}}{\sqrt{d_i}d_j} + 2 \frac{v_{1,i}v_{1,j}}{\sqrt{d_i}d_j}
\]
\[= \frac{1}{d_i} + \frac{1}{d_j} - \frac{1}{2|\tilde{E}|} \left( \frac{d_i}{\sqrt{2|\tilde{E}|}} \right)^2 + \frac{d_j}{2|\tilde{E}|} \sqrt{d_i}d_j,
\]
\[= \frac{1}{d_i} + \frac{1}{d_j}.
\]

This gives the following bound on \( \kappa(i, j) \).

Theorem 5. For \( p(1 - p) \gg \frac{\log^4 n}{n^{d-1}} \) a.a.s. in \( i \) and \( j \)
\[
n(1 + o(1)) \leq \kappa(i, j) \leq 2n(1 + o(1)).
\]
Finally, we also want to give a bound the cover time $C(H)$. From Theorem 2.7 in Lovázs (see [Lov93]) we have that:

**Theorem 6.** The cover time from any vertex $i$ of a graph with $n$ vertices is bounded as follows:

$$\min_{i,j} H_{i,j} \sum_{k=1}^{n} \frac{1}{k} \leq C(H) \leq \max_{i,j} H_{i,j} \sum_{k=1}^{n} \frac{1}{k}$$

Thus we obtain

**Theorem 7.** For $p(1-p) \gg \frac{\log^4 n}{n}$ we have a.a.s $\frac{n}{2} \log n \leq C(H) \leq n \log n$.

**Proof.** By (1) and $\frac{1}{2} \leq \frac{1}{1-\lambda_k} \leq \frac{1}{1-\lambda_2}$ we get:

$$|\hat{E}| \sum_{k=2}^{n} \left( \frac{v_{k,j}^2}{d_j} - \frac{v_{k,i}v_{k,j}}{\sqrt{d_id_j}} \right) \leq H(i, j) \leq \frac{2|\hat{E}|}{1-\lambda_2} \sum_{k=2}^{n} \left( \frac{v_{k,j}^2}{d_j} - \frac{v_{k,i}v_{k,j}}{\sqrt{d_id_j}} \right)$$

On the other hand:

$$\sum_{k=2}^{n} \left( \frac{v_{k,j}^2}{d_j} - \frac{v_{k,i}v_{k,j}}{\sqrt{d_id_j}} \right) = \sum_{k=1}^{n} \left( \frac{v_{k,j}^2}{d_j} - \frac{v_{k,i}v_{k,j}}{\sqrt{d_id_j}} \right) - \frac{v_{1,j}^2}{d_j} + \frac{v_{1,i}v_{1,j}}{\sqrt{d_id_j}}$$

$$= \frac{1}{d_j} - \frac{d_j}{2|\hat{E}|} + \frac{\sqrt{d_j}}{\sqrt{2|\hat{E}|}} \frac{\sqrt{d_j}}{\sqrt{2|\hat{E}|}} = \frac{1}{d_j}.$$ 

Now $\frac{1}{1-\lambda_2} = 1 + o(1)$ and $\frac{2|\hat{E}|}{d_j} = n(1 + o(1))$ a.a.s. uniformly in $j$. This, together with $\sum_{k=1}^{n} \frac{1}{k} \sim \log n$ finishes the proof. \qed

**Remark 2.** We remark that the vertex cover time $C(H)$ in the case of random walk on $d$-uniform hypergraphs is smaller than the vertex cover time in the case of random walk on $r$-regular $d$-uniform hypergraphs.

**References**

[BR12] Béla Bollobás and Oliver Riordan. Asymptotic normality of the size of the giant component in a random hypergraph. Random Structures Algorithms, 41(4):441–450, 2012.

[BR17] Béla Bollobás and Oliver Riordan. Exploring hypergraphs with martingales. Random Structures Algorithms, 50(3):325–352, 2017.

[CFR11] Colin Cooper, Alan Frieze, and Tomasz Radzik. The cover times of random walks on hypergraphs. In Structural information and communication complexity, volume 6796 of Lecture Notes in Comput. Sci., pages 210–221. Springer, Heidelberg, 2011.

[CFR13] Colin Cooper, Alan Frieze, and Tomasz Radzik. The cover times of random walks on random uniform hypergraphs. Theoret. Comput. Sci., 509:51–69, 2013.

[DS84] Peter G Doyle and J Laurie Snell. Random walks and electric networks, volume 22. Mathematical association of America, 1984.

[EKYY13] László Erdős, Antti Knowles, Horng-Tzer Yau, and Jun Yin. Spectral statistics of erdős-rényi graphs i: Local semicircle law. Ann. Probab., 41(3B):2279–2375, 2013.
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[Lov93] Laszlo Lovász. Random walks on graphs: A survey. Combinatorics, Paul Erdös is eighty, 2(1):1–46, 1993.

[LP12] Linyuan Lu and Xing Peng. Loose Laplacian spectra of random hypergraphs. Random Structures Algorithms, 41(4):521–545, 2012.

[LPW09] David Asher Levin, Yuval Peres, and Elizabeth Lee Wilmer. Markov chains and mixing times. AMS Bookstore, 2009.

[LT14] Matthias Löwe and Felipe Torres. On hitting times for a simple random walk on dense Erdös-Rényi random graphs. Statist. Probab. Lett., 89:81–88, 2014.

[SRBA04] Vishal Sood, Sidney Redner, and Dani Ben-Avraham. First-passage properties of the erdős–renyi random graph. Journal of Physics A: Mathematical and General, 38(1):109, 2004.

[vdH17] Remco van der Hofstad. Random graphs and complex networks. Vol. 1. Cambridge Series in Statistical and Probabilistic Mathematics, [43]. Cambridge University Press, Cambridge, 2017.

[Woe00] Wolfgang Woess. Random walks on infinite graphs and groups, volume 138. Cambridge university press, 2000.

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