Update in Unit Gradient
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Abstract

In Machine Learning, optimization mostly has been done by using a gradient descent method to find the minimum value of the loss. However, especially in deep learning, finding a global minimum from a nonconvex loss function across a high dimensional space is an extraordinarily difficult task. Recently, a generalization learning algorithm, Sharpness-Aware Minimization (SAM), has made a great success in image classification task. Despite the great performance in creating convex space, proper direction leading by SAM is still remained unclear. We, thereby, propose a creating a Unit Vector space in SAM, which not only consisted of the mathematical instinct in linear algebra but also kept the advantages of adaptive gradient algorithm. Moreover, applying SAM in unit gradient brings models competitive performances in image classification datasets, such as CIFAR - {10, 100}. The experiment showed that it performed even better and more robust than SAM.

Keywords: Deep Learning, Image classification, Optimization, Generalization, Convex loss landscape
1. Introduction

In recent years, machine learning models especially in deep neural network have made a great progress in generalization. In order to understand the phenomenon of generalizing models during the training process, some papers have provided proofs via a theoretical perspective [1]–[5]. Most of the generalization research derived generalization bound which is an upper bound on the test error that be given some quantities, and can be calculated on the training set. By doing arduous works, those bounds had been proved to be tight enough; however, B. Neyshabur et al. (2019) showed that most of bounds performed poorly and were hard to apply in empirical situation [6]. The reason why those generalization bounds failed to perform well was that the deep neural network caused a non-convex loss surface; therefore, there were more and more researchers that emphasized on three conditions to the loss surface [7]–[10]: convexity [11], [12], Lipschitzness [13], and smoothness.

Foret et al. (2020) applied a method of empirical measure [14], sharpness [9], which is a learning algorithm based on PAC-learnable learning algorithm [15]. They tested on the optimizer SGD [16], and proved not only to be more efficient but also to have even better performance than the original optimizer. The model they developed called Sharpness-Aware Minimization (SAM) [14], which promoted state-of-the-arts performances in image classification tasks. Motivated by Path-SGD [17], We introduced Units of Gradient Descent (UGD) which is a basic and instinct way to promote the performance of stochastic gradient descent (SGD). Units of gradient descent (UGD) not only had similar properties of adaptive gradient algorithm (Adagrad) [18], but also shared the robust properties of stochastic gradient descent (SGD). Unfortunately, units of gradient descent (UGD) somehow occurred slight overfitting; therefore, in order to solve this problem, we combined units of gradient descent (UGD) and Sharpness-Aware Minimization (SAM) to a new update method called Sharpness-Aware Minimization in Gradient Norm Space (SAM-GNS), this method had not only better but also more robust performance than Sharpness-Aware Minimization (SAM) on stochastic gradient descent (SGD).

2. Related Work

VC dimension [19]–[21] was proposed to measure the complexity of the learning model, it is defined as the cardinality of the largest set of parameters of the model that can shatter [20]. It is also used to predict a probabilistic upper bound on the test error
$$\mathcal{P}\left( \varepsilon \leq \varepsilon_S + \sqrt{\frac{D \log 2 + 1}{N} \log \frac{1}{\delta}} \right) = 1 - \delta, \text{ given } 0 < \delta \leq 1$$ (2.1)

where $\varepsilon$ is the test error, $\varepsilon_S$ is the training error, $D$ is the VC dimension of the classification model, and $N$ is the size of the training set (Note that if $D$ were larger than $N$, the model may cause over-fitting). Likewise, generalization bounds such as PAC Bayesian Generalization Bound [22] or other generalization bounds [2], [23], those bounds had similar form to the VC generalization bounds [20]. The PAC Bayesian Generalization Bound was proved to be an empirical bound; besides, the learning algorithm, Sharpness-Aware Minimization (SAM) was proved to follow the PAC Bayesian Generalization Bound. Because the proposed method, SAM-GNS is modified base on SAM, the generalization bound of SAM-GNS is also following the PAC Bayesian Generalization Bound.

Optimizers such as SGD, Adagrad, RMSprop [24], Adam [25], etc., were invented to solve specific problems, but in the meanwhile, it also caused some other issues; therefore, there is no a perfect algorithm but a proper algorithm to choose. D. J. Im et al. (2016) showed that different optimizers would lead to the different local minimum on the loss surface [26], and V. Nagarajan et al. (2019) indicated that SGD lied in a flat, wide minimum on the loss surface of an overparameterized deep neural network [3]; moreover, M. Hardt et al. (2016) proved that SGD not only could make the training process faster but also made better generalization in optimization process [27]. As a result, we chose SGD to be the base optimizer to train our models.

### In Mathematics

Unit vector was often used to control the steps of directions. S. Lipschutz et al. (2009) also applied those basic concepts to tensor analysis [28].

### In Generalization Researches

B. Neyshabur et al. (2019) tested over 40 complexity measures to show which had positive relation to the specific hyperparameter in hyperparameter space [6]; in conclusion of that, sharpness-based measure won the highest correlation score; therefore, we started to study Sharpness-Aware Minimization (SAM). In the period of studying Sharpness-Aware Minimization (SAM), we found that there were still unsolvable questions. Since those questions remained unsolved, we chose to improve the algorithm in the perspective of basic and instinct mathematical analysis.
3. Preliminary

Given a training set \( \mathcal{S} \equiv \bigcup_{i=1}^{n} \{(x_i, y_i)\} \) where \( x_i \in \mathcal{X}, y_i \in \mathcal{Y} \), which distributed i.i.d from a population distribution \( \mathcal{D} \), and let a single data-generation model \( f : \mathcal{X} \rightarrow \mathcal{Y} \) parametrized by a weight vector \( w \in \mathcal{W} \subseteq \mathbb{R}^d \); given the loss function \( l : \mathcal{Y} \times \mathcal{Y} \rightarrow \mathbb{R}_+ \), we defined the training loss \( \mathcal{L}_\mathcal{S}(w) \equiv \frac{1}{n} \sum_{i=1}^{n} l(y_i, f(x_i; w)) \) and the population loss \( \mathcal{L}_\mathcal{D}(w) \equiv \mathbb{E}_{(x,y) \sim \mathcal{D}}[l(y, f(x; w))] \); the ultimate goal is to minimize \( \mathcal{L}_\mathcal{D} \), but unfortunately, the model \( f \) knows nothing about the population distribution \( \mathcal{D} \), so we minimize \( \mathcal{L}_\mathcal{S}(w) \) to solve the optimization problem [29].

3.1 Sharpness-Aware Minimization (SAM)

In the Sharpness-Aware Minimization (SAM), the population loss \( \mathcal{L}_\mathcal{D} \) they used, had been proved to follow the minimization of PAC-Bayesian generalization bound [3],

\[
\mathcal{L}_\mathcal{D}(w) \leq \max_{\|\epsilon\|_2 \leq \rho} \mathcal{L}_\mathcal{S}(w + \epsilon) + h\left(\frac{\|w\|_2}{\rho^2}\right),
\]

where \( h : \mathbb{R}^+ \rightarrow \mathbb{R}^+ \) is a strictly increasing function. Given the definition of the sharpness of the loss function as

\[
\max_{\|\epsilon\|_p \leq \rho} \mathcal{L}_\mathcal{S}(w + \epsilon) - \mathcal{L}_\mathcal{S}(w),
\]

the Sharpness-Aware Minimization problem could be defined as the following

\[
\min_w \mathcal{L}^{SAM}_\mathcal{S}(w) \equiv \min_w \max_{\|\epsilon\|_p \leq \rho} \mathcal{L}_\mathcal{S}(w + \epsilon).
\]

In order to evaluate \( \epsilon \), first-order Taylor expansion at the center of \( w \) had been used to approximate \( \mathcal{L}_\mathcal{S}(w + \epsilon) \), obtaining

\[
\mathcal{L}_\mathcal{S}(w + \epsilon) \approx \mathcal{L}_\mathcal{S}(w) + w^T \nabla_w \mathcal{L}_\mathcal{S}(w) + \epsilon(w) \approx \arg\max_{\|\epsilon\|_p \leq \rho} w^T \nabla_w \mathcal{L}_\mathcal{S}(w).
\]
Then we approximate \( \text{argmax} \ |\|\epsilon\|_{p} \leq \rho \) using the solution to a dual norm problem:

\[
\epsilon(w) \approx \rho \text{sign} \left( \nabla_w L_S(w) \right) \frac{\left| \nabla_w L_S(w) \right|^{q-1}}{\left( \| \nabla_w L_S(w) \|_q \right)^{p-1}}, \text{where} \ 1/p + 1/q = 1. \quad (3.7)
\]

It was suggested that select \( p = q = 2 \), so the algorithm of SAM could be expressed to the following Algorithm 1.

**Algorithm 1** SAM algorithm \((p = q = 2)\)

**Input:** Training set \( \mathcal{S} \triangleq \bigcup_{i=1}^{n} \{ (x_i, y_i) \} \), the loss function \( l : \mathcal{Y} \times \mathcal{Y} \to \mathbb{R}_+ \), batch size \( n \), annealing learning rate \( \eta \), forced updating value \( \rho \), random initial weight \( w_0 \) by given \( t = 0 \).

**Output:** Trained weight \( w \), randomly initialized weight \( w_0 \)

while not converged do

1. Sample a set \( \mathcal{B} \) of batch size \( n \) from \( \mathcal{S} \)
2. Compute the gradient \( \nabla_w L_{\mathcal{B}}(w) \) of the set \( \mathcal{B} \)
3. Compute \( \epsilon(w) := \rho \frac{\nabla_w L_S(w)}{\| \nabla_w L_S(w) \|_2} \)
4. Update \( w_t \) temporarily to \( w_t^{SAM} = w_t + \epsilon(w_t) \)
5. Back propagation on \( w_t^{SAM} \) to get the gradient \( \nabla_w L_{\mathcal{B}}(w_t^{SAM}) \)
6. Call back to \( w_t \) and update \( w_t \) to \( w_{t+1} \) using the gradient \( \nabla_w L_{\mathcal{B}}(w_t^{SAM}) \)
   \[
   w_{t+1} = w_t - \eta \nabla_w L_{\mathcal{B}}(w_t^{SAM})
   \]
7. \( t = t + 1 \)

end while

return \( w_t \)

### 3.2 Gradient Descent

Gradient descent method was invented by Cauchy in 1847, a first-order iterative learning algorithm. The method was developed to find the local minimum of a differentiable function. Since the learning process in Machine Learning could be regarded as the optimization problem, it could apply gradient descent to minimize the loss function \( L(\theta) \) parameterized by parameters \( \theta \in \mathbb{R}^\ell \) in the model; further, the formula of gradient descent as the following
\[ \theta_{t+1} = \theta_t - \eta \nabla_{\theta} \mathcal{L}(\theta_t). \]  \hspace{1cm} (3.8)

In the process of minimizing the loss, the parameters \( \theta \) being updated in the opposite direction of the gradient of the loss function \( \nabla_{\theta} \mathcal{L}(\theta) \) with respect to the parameters \( \theta \). The learning rate \( \eta \) represents the size of each step that taking us to the (local) minimum. In short, we followed the direction of loss surface [30] downhill until reaching a valley.

### 3.3 Unit vector in vector space

Unit vector is a basic concept in linear algebra, it is a vector of length 1 in the normed vector space, the definition could be defined as following.

**Definition**
Let \( \mathbf{v} \in \mathcal{V} \) be a vector in vector space \( \mathcal{V} \), and \( \|\cdot\| \) be the norm of the vector, then the vector \( \mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|} \) is called the unit vector of \( \mathbf{v} \).

### 3.4 Unit gradient loss

Applying the concept of unit vector in linear algebra, we defined the unit of the gradient of the loss as the following:

**Definition**
Let \( \nabla_{\mathbf{w}} \mathcal{L}_s(\mathbf{w}) \) be the gradient of the loss function with respect to the weight vector \( \mathbf{w} \). The unit of gradient of the loss was given by \( \hat{\mathbf{g}} = \frac{\nabla_{\mathbf{w}} \mathcal{L}_s(\mathbf{w})}{\|\nabla_{\mathbf{w}} \mathcal{L}_s(\mathbf{w})\|} \), and then we defined the unit of gradient descent

\[
\begin{align*}
\theta_{t+1} &= \theta_t - \eta \hat{\mathbf{g}} \\
&= \theta_t - \eta \frac{\nabla_{\mathbf{w}} \mathcal{L}_s(\mathbf{w})}{\|\nabla_{\mathbf{w}} \mathcal{L}_s(\mathbf{w})\|} \\
&= \theta_t - \eta \nabla_{\mathbf{w}} \mathcal{L}_s(\mathbf{w}) \| \end{align*}
\hspace{1cm} (3.9)
\hspace{1cm} (3.10)

thus, the unit of gradient descent algorithm could be expressed as **Algorithm 2**.
Algorithm 2 Unit of Gradient Descent (UGD)

Input: Training set $\mathcal{S} \triangleq \bigcup_{i=1}^{n}\{(x_i, y_i)\}$, the loss function $l : \mathcal{Y} \times \mathcal{Y} \rightarrow \mathbb{R}_+$, batch size $n$, annealing learning rate $\eta$, random initial weight $w_0$ by given $t = 0$.

Output: Trained weight $w$, randomly initialized weight $w_0$

while not converged do
    1. Sample a set $\mathcal{B}$ of batch size $n$ from $\mathcal{S}$
    2. Compute the gradient $\nabla_w \mathcal{L}_\mathcal{B}(w)$ of the set $\mathcal{B}$
    3. Compute the unit of gradient of the loss $\tilde{g} = \frac{\nabla_w \mathcal{L}_\mathcal{S}(w)}{||\nabla_w \mathcal{L}_\mathcal{S}(w)||}$
    4. Do the gradient step by using the unit of gradient of the loss $\tilde{g}$.
        $w_{t+1} = w_t - \eta \tilde{g}$
        $= w_t - \eta \frac{\nabla_w \mathcal{L}_\mathcal{S}(w_t)}{||\nabla_w \mathcal{L}_\mathcal{S}(w_t)||}$
    5. $t = t + 1$
end while

return $w_t$

4. The Proposed Method - Sharpness-Aware

Minimization in Gradient Norm Space (SAM-GNS)

In Algorithm 1, Sharpness-Aware Minimization (SAM) forced to update the weight $w_t$ to $w_t^{\text{SAM}} = w_t + \epsilon(w_t)$, and then computed the gradient $\nabla_w \mathcal{L}_\mathcal{B}(w_t^{\text{SAM}})$ on $w_t^{\text{SAM}}$. After obtaining the new gradient on $w_t^{\text{SAM}}$, it called back to $w_t$, also it used the new gradient $\nabla_w \mathcal{L}_\mathcal{B}(w_t^{\text{SAM}})$ to do a gradient descent update. Since it still remained unclear why the new gradient $\nabla_w \mathcal{L}_\mathcal{B}(w_t^{\text{SAM}})$ is "better" (i.e. will lead to a flatten minimum [31]) than the original gradient $\nabla_w \mathcal{L}_\mathcal{B}(w_t)$, we changed the new gradient $\nabla_w \mathcal{L}_\mathcal{B}(w_t^{\text{SAM}})$ to $\nabla_w \mathcal{L}_\mathcal{B}(w_t^{\text{SAM-GNS}}) = \frac{\nabla_w \mathcal{L}_\mathcal{B}(w_t^{\text{SAM}})}{||\nabla_w \mathcal{L}_\mathcal{B}(w_t^{\text{SAM}})||_2}$, which could be considered of using Unit of the Gradient of the loss to update the weight $w_t$. By scaling the gradient $\nabla_w \mathcal{L}_\mathcal{B}(w_t^{\text{SAM}})$ into $\ell_2$ gradient norm space, it avoided the risk of dropping into the sharp minimum, and had more opportunity to choose the different paths in the early training steps, thus it increased the chance of updating to the convex minimum.
Compare with Adaptive Gradient Algorithm (Adagrad)

In $\ell_2$ gradient norm space, the update process had the same properties as adaptive gradient algorithm (Adagrad)

$$ w_{t+1}^{Adagrad} = w_t - \eta \frac{\nabla \theta (\theta_t)}{\sum_{i=1}^{n} \left( \frac{\partial L}{\partial w_i}(w_t) \right)^2 + \epsilon}, \text{ where } \epsilon \sim N(0,1), \quad (4.1) $$

particularly in adaptive gradient algorithm (Adagrad), the learning rate was scaled to

$$ \frac{\eta}{\sum_{i=1}^{n} \left( \frac{\partial L}{\partial w_i}(w_t) \right)^2 + \epsilon}, \quad (4.2) $$

here $\frac{\partial L}{\partial w_i}(w_t)$ represented the partial derivative of $L$ with respect to $w_i$, and also it is regarded as the gradient of the loss with respect to $w_t$. Let us look into the following SAM-GNS algorithm (Algorithm 3), the update process

$$ w_{t+1}^{SAM-GNS} = w_t - \eta \frac{\nabla w L_B(w_t^{SAM})}{\|\nabla w L_B(w_t^{SAM})\|_2} \quad (4.3) $$

would not depend on $w_t$, so that it kept robustness of SGD.
Algorithm 3 SAM-GNS algorithm

Input: Training set $S \triangleq \bigcup_{i=1}^{n}\{(x_i, y_i)\}$, the loss function $l : \mathcal{Y} \times \mathcal{Y} \to \mathbb{R}_+$, batch size $n$, annealing learning rate $\eta$, forced updating value $\rho$, random initial weight $w_0$ by given $t = 0$.

Output: Trained weight $w$, randomly initialized weight $w_0$

while not converged do
  1. Sample a set $\mathcal{B}$ of batch size $n$ from $S$
  2. Compute the gradient $\nabla_w L_{\mathcal{B}}(w)$ of the set $\mathcal{B}$
  3. Compute $\epsilon(w) := \rho \frac{\nabla_w L_S(w)}{\|\nabla_w L_S(w)\|_2}$
  4. Update $w_t$ temporarily to $w_t^{SAM} = w_t + \epsilon(w_t)$
  5. Back propagation on $w_t^{SAM}$ to get the gradient $\nabla_w L_{\mathcal{B}}(w_t^{SAM})$
  6. Compute the unit gradient of the loss $\nabla_w L_{\mathcal{B}}(w_t^{SAM-\text{GNS}}) = \frac{\nabla_w L_{\mathcal{B}}(w_t^{SAM})}{\|\nabla_w L_{\mathcal{B}}(w_t^{SAM})\|_2}$
  7. Call back to $w_t$ and use the gradient $\nabla_w L_{\mathcal{B}}(w_t^{SAM})$ to update $w_t$
     $w_{t+1} = w_t - \eta \nabla_w L_{\mathcal{B}}(w_t^{SAM-\text{GNS}})$
  8. $t = t + 1$
end while

return $w_t$

5. Simulation Experiment

The experimental design was setting to train different networks on \{CIFAR10, CIFAR100\} using the same base optimizer – SGD, and then we compared SAM-GNS to same SAM-type optimizers on the same hyperparameter settings.

Computational Equipment

We trained our models on different computational equipment. In Table 1, No.1, No.2, No.3 were used to train on CIFAR10 and CIFAR100; No.4 was used to finetune the pretrained models.
Table 1 The Specification of Computational Equipment.

| No. | GPU                     | CPU                          | RAM   |
|-----|-------------------------|------------------------------|-------|
| 1   | NVIDIA GeForce RTX 2070| AMD R7 3700X 8-core          | 16GB  |
| 2   | NVIDIA GeForce RTX 2080 SUPER | AMD R7 2700X 8-core     | 32GB  |
| 3   | NVIDIA GeForce RTX 2080 Ti | AMD R7 2700X 8-core      | 32GB  |
| 4   | NVIDIA TITAN RTX      | Intel(R) Core(TM) i9-9900KF | 32GB  |

5.1 Image Classification

We evaluated the performance of SAM-GNS, and also compared with SAM and Adaptive Sharpness-Aware Minimization (ASAM) [32] on base optimizer SGD across benchmark datasets (e.g., CIFAR-10, 100) [33]). We expected to use the same hyperparameter settings on different optimizers. Since Adagrad performed poorly by setting learning rate (lr) = 0.1; furthermore, [6], [34] showed that if we tested on different optimizers by using different hyperparameter settings, it may affect the optimization algorithm by implicit generalization terms. As the results, we preferred not to compare SGD with Adagrad using different hyperparameter settings.

As shown in the following Table 2, all of SAM-type optimizers were using the same base optimizer: SGD. It is obvious that SAM-GNS had the best performances applying to each neural network on base optimizer SGD. Table 2 presented test accuracies of various models trained on CIFAR10 without data augmentation (input size = 32 × 32). The hyperparameters were set as: learning rate (lr) = 0.1, weight decay = 0.0005, momentum = 0.9, rho (ρ) = 0.05, nesterov = False. Also, we downloaded the datasets from pytorch modulo: torchvision.datasets.{CIFAR10, CIFAR100}, batch size = 100, and learning rate annealing was set to cosine annealing [35].
Table 2 Testing accuracies of each optimizer on various kinds of neural networks on CIFAR10. VGG [36]. Resnet [37]. CPA: Channel Pixel Attention [38]. UPA: Universal Pixel Attention Networks [38]. DLA: Deep Layer Aggregation [39].

| CIFAR10 | Model (Size M) | SGD   | SAM   | ASAM  | SAM-GNS  |
|---------|----------------|-------|-------|-------|----------|
| Resnet18 (≈11.17M)  | 95.61          | 95.57 | 95.16 | 95.96  |
| Resnet18_CPA (≈11.71M) | 96.35          | 96.4  | 95.98 | 96.44'  |
| Resnet50 (≈23.52M)   | 95.62          | 95.59 | 95.25 | 96      |
| VGG16 (≈14.728M)     | 93.66          | 94.13 | 94.05 | 94.46   |
| DLA (≈16.29M)        | 95.86          | 96.09 | 95.09 | 96.16   |
| UPA16 (≈1.51M)       | 95.3           | 95.21 | 94.67 | 95.47   |
| DenseNet121 (≈6.95M) | 95.51          | 95.8  | 95.36 | 96.05   |

Figure 1. The Results of Testing accuracies on CIFAR10. (SAM-GNS has a stellar record on different neural networks.)
According to Figure 1, CPA layers adding in ResNet18 has the highest performances; VGG has the worst performances since the network itself has only original CNN structure; SAM-GNS performs the best among all optimizers in the experiment.

As shown in the following Table 3, SAM-GNS has almost the best performances comparing with all optimizers on CIFAR100 without data augmentation (input size = 32 × 32). The hyperparameters were set as same as the CIFAR10 settings.

Table 3 Testing accuracies of each optimizer on various kinds of networks on CIFAR100.

| CIFAR100          | SGD  | SAM  | ASAM | SAM-GNS |
|-------------------|------|------|------|---------|
| Resnet18 (≈11.17M) | 78.16 | 78.05 | 76.28 | 80.09   |
| Resnet18 CPA (≈11.71M) | 80.77 | 80.11 | 79.83 | 81.45   |
| Resnet50 (≈23.52M) | 79.09 | 78.6  | 78.15 | 81.5    |
| VGG16 (≈14.728M)  | 75.09 | 73.25 | 73.54 | 75.07   |
| UPA16 (≈1.51M)    | 75.28 | 75.18 | 74.17 | 77.06   |
| UPA64 (≈23.59M)   | 80.22 | 80.82 | 79.96 | 82.39*  |

Figure 2. The Results of Testing accuracies on CIFAR100.
Overviewing all performances on CIFAR100, the main reason why SAM has worse performance than SGD may be the choice of the value of rho ($\rho$). The paper [14] showed that they chose the value of rho ($\rho$) = 0.1 on WRN 28-10 [40] without shakedrop regularization [41]; although the value of rho ($\rho$) does not change to a proper value to SAM-type optimizers, SAM-GNS still performs better and more robust than other optimizers.

5.2 Finetuning

The goal of transfer learning [42] is to improve the performance of the target learner by training a previous model on a large datasets and then transfer the knowledge from the previous model to the target learner. Transfer learning [42] is a powerful technique; as the result, we used the pretrained models (on 1K-ImageNet [43]) to finetune on CIFAR10 dataset; hyperparameters were set as: learning rate (lr) = {0.01, 0.001}, weight decay = 0.0005, momentum = 0.9, rho ($\rho$) = 0.05, nesterov = False; a smaller learning rate was appropriate to a heavier model. As shown in the following Table 4, we applied SAM-GNS to finetune on several pretrained models (resolution 224×224) and got brilliant performances.

Table 4 Finetuning several models on CIFAR10. ViT-B/16 [44]. WRN [40].

| SAM-GNS finetuned | learning rate | epoch | Best testing accuracy |
|-------------------|---------------|-------|-----------------------|
| Resnet18          | 0.01          | 100   | 97.46                 |
| Resnet50          | 0.01          | 100   | 97.95                 |
| Resnet101         | 0.01          | 100   | 98.4                  |
| Resnet152         | 0.01          | 100   | 98.5                  |
| WRN101_2          | 0.01          | 100   | 98.56                 |
| ViT-B/16          | 0.001         | 100   | 99.13                 |

According to Table 4, the best score reached 99.13 by ViT-B/16(SAM-GNS), also it reached rank 5 on CIFAR10 leaderboard (Table 5) (recorded date on 2021/7/29) (https://paperswithcode.com/sota/image-classification-on-cifar-10). Since it had not been uploaded, comparing with Table 5, the performance of ViT-B/16(SAM-GNS) outperformed ViT-B/16(SAM) as well.
Table 5 CIFAR10 leaderboard recorded on the date: 2021/7/29. Efficientnets: [45], [46]. CaiT: [47]. CvT: [48]. BiT: [49]. CeiT: [50]. TNT: [51]. DeiT: [52]

| Rank | Model | Accuracy | Using Extra training data |
|------|-------|----------|---------------------------|
| 1    | Efficientnet-L2 (SAM) | 99.70    | Yes                       |
| 2    | CaiT-M-36 U 224 | 99.4     | Yes                       |
| 3    | CvT-W24 | 99.39    | Yes                       |
| 4    | BiT-L (ResNet) | 99.37    | Yes                       |
| 5    | TNT-B | 99.1     | Yes                       |
| 6    | DeiT-B | 99.1     | Yes                       |
| 7    | EfficientnetV2-L | 99.1     | Yes                       |
| 8    | CeiT-S (384 finetuned resolution) | 99.1 | Yes |
| ...  | ... | ... | ... |
| 22   | ViT-B/16(SAM) | 98.6     | Yes                       |
| 35   | ResNet152(SAM) | 98.2     | No                        |
| 60   | ResNet50(SAM) | 97.4     | No                        |

Finally, we applied pretrained model - ViT-B/16 (on 1K-ImageNet) to finetune ViT-B/16(SAM-GNS) on CIFAR100 dataset. The testing accuracies reached even better rank than CIFAR10 dataset. As seen as the following Table 6, ViT-B/16 (SAM-GNS) reached 93.95 of rank 4, and outperformed ViT-B/16 (SAM) as well.

Table 6 Comparison the result of ViT-B/16(SAM-GNS) finetuned on CIFAR100 and the result of CIFAR100 leaderboard (recorded on 2021/7/29). (https://paperswithcode.com/sota/image-classification-on-cifar-100)

| Rank | Model | Accuracy | Using Extra training data |
|------|-------|----------|---------------------------|
| 1    | Efficientnet-L2(SAM) | 96.08    | Yes                       |
| 2    | ViT-B/16 (ImageNet-21K-P pretrained) | 94.2      | Yes                       |
| 3    | CvT-W24 | 94.09    | Yes                       |
| 4    | BiT-L(ResNet) | 93.51    | Yes                       |
| 5    | CaiT-M-36 U 224 | 93.1     | Yes                       |
| ...  | ... | ... | ... |
| 27   | ViT-B/16(SAM) | 89.1     | Yes                       |
| ViT-B/16(SAM-GNS) | 93.95 | Yes |
5.3 Summary

Experiments show that SAM-GNS has brilliant performances, and even performed stronger on the more complex distribution dataset; the difference between SAM-GNS and SAM are the update step – one used SGD update, the other used UGD update, thus SAM-GNS performed better than other optimizers in the experiments. Unfortunately, we were not able to train Efficientnet-L2 because of the computational equipment limitation; We hoped to have more sophisticated equipment to train a heavier model in the future.

6. Conclusion

Because of the unit gradient descent update, it always keeps the gradient as a unit vector in the vector space; hence, it also solves the problem of Adagrad: it may lead to zero gradient in the process of the late training steps. Moreover, the gradient of the loss in SAM-GNS algorithm is taken from the process of Sharpness-Aware Minimization (SAM), thus it leads to a flatten minimum with the step of unit vectors. It also outperforms Sharpness-Aware Minimization (SAM) applying on various kinds of neural networks on SGD with the same hyperparameter settings.

Discussion & Future Work

We introduced SAM-GNS, which is a simple modified work consists of simple mathematical instinct, but in fact, there were still a lot of works to research in the future; such as changing the definition of sharpness or changing the value of $\epsilon(w_t)$ and the gradient norm. Besides, this work was only testing on image classification task, it was expected to apply to other tasks. Last but not least, the reason why new gradient would lead to a proper direction still remained unclear, we hoped to have solutions in the near future.
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