Refined assessment of the state of the power system for its effective management

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Abstract. The technique of assessing the state of the power network with two independent nodes of the radial and magistral structure is given. The advantages of the proposed approach in comparison with numerical methods are the efficiency of calculations. The theoretical significance lies in the possibility of expanding the presented approach on networks with a large number of independent nodes. Practical significance lies in the possibility of applying the results obtained to calculate the parameters of the electric network during its preliminary decomposition. The obtained formulas are tested on models of the electric network of the radial and magistral configuration.

1. Introduction

Assessment of the condition of the electrical system allows determining important indicators of quality and efficiency of its operation. Existing methods of calculation allow determining the parameters only using numerical methods. The equation of balance of the nodal capacities of the system has the form:

\[ \mathbf{U}^* \mathbf{Y} \mathbf{U} + \mathbf{U} \mathbf{J} + \mathbf{S} = 0 \]  

where \( \mathbf{U} \) - diagonal matrix of complex amplitudes of nodal voltages; \( \mathbf{Y} \) - matrix of nodal conductivity; \( \mathbf{J} \) - column vector of currents of current sources; \( \mathbf{S} \) - nodal power loads, «*» - symbol of conjugate.

The matrix multiply is generally non-commutative, which does not allow expressing \( \mathbf{U} \) from (1).

The use of numerical methods involves iterative calculation of nodal voltages. Such an approach will allow assessing the state of the network with the maximum residual power balance \( \Delta P \). With high requirements for the accuracy of determining the parameters of the regime, the number of required iterations will also increase. This will negatively affect the increased requirements for the efficiency of calculation, for example, in SCADA systems, and when applying heuristic optimization algorithms [1].

The direct method of solution (1), proposed in this article, allows increasing the efficiency of calculations not only due to the absence of iterations, but also due to the lack of the need to evaluate parameters in uncontrolled areas [2-5]. So, for example, direct calculation will allow determining the node voltage only in the nodes specified by the user, eliminating the need to calculate nodes in the entire circuit. Direct calculation is also able to increase the efficiency of network sensitivity calculations.
The article gives and tested formulas for calculating the parameters of an electric network with two independent nodes.

2. Implementation

Let us consider open network structures with two independent nodes - radial (fig.1, a) and magistral (figure 1, b) structure.

![Figure 1. Considered schemes.](image)

For a radial structure the matrix $\tilde{Y}$ is a 2x2 square matrix in which the intrinsic nodal conductivities are on the main diagonal, and the other elements are equal to zero due to the absence of coupling between the nodes. Then the multiply of the matrices $\tilde{Y}_d \tilde{Y} U$ from (1) is commutative, which allows us to reduce (1) to an equation of the form:

$$\dot{Y} U^2 + \dot{J} U + \dot{S} = 0$$  \hspace{1cm} (2)

Passing to the scalar form (2) for any $i (i=1,2)$ node, we obtain the equation:

$$\dot{Y}_{ii} u_i^2 + J_i \dot{u}_i + S_i = 0$$  \hspace{1cm} (3)

We introduce the known coefficients for each element (3):
- $Y_{ii}$ - module of self-conductivity, Ohm$^1$;
- $\varphi_{Y_{ii}}$ - argument of self-conductivity, rad;
- $J_i$ - module of head current, kA;
- $\varphi_{J_i}$ - argument of head current, rad;
- $S_i$ - module of nodal load, MVA;
- $\varphi_{S_i}$ - argument of nodal load, rad.

We also introduce quantities requiring calculation:
- $U_i$ - module of nodal voltage, kV;
- $\varphi_{U_i}$ - argument of nodal voltage, rad.

We also introduce coefficients requiring calculation:
- $k_1 = 2 Y_{ii} S_i \cos (\varphi_{Y_{ii}} - \varphi_{S_i})$;
- $k_2 = \left[ J_i^4 + 4 Y_{ii} S_i^2 \cos (2 (\varphi_{Y_{ii}} - \varphi_{S_i}))-1 \right] - 4 Y_{ii} J_i S_i \cos (\varphi_{Y_{ii}} - \varphi_{S_i}) \right]^{1/2}$

Then solution (3) is presented in compact form:

$$U_i = \frac{\sqrt{2} (J_i^2 - k_1 + k_2)^{1/2}}{2 Y_{ii}}$$  \hspace{1cm} (4)

$$\varphi_{U_i} = -\pi \cdot \varphi_{J_i} + \arcsin \left[ \frac{\sqrt{2} \sin (\varphi_{Y_{ii}}) k_2 + J_i \sin (\varphi_{Y_{ii}}) + J_i S_i \sin (\varphi_{S_i})}{2 \sqrt{J_i^2 + k_1 + k_2}^{1/2}} \right]$$  \hspace{1cm} (5)

Expressions (4) and (5) make it possible to calculate the nodal voltages and their phases for the radial configuration circuit with known power parameters. These expressions are valid both for a circuit with two independent nodes, and for a circuit with a large number of nodes.
For the magistral configuration the matrix $\tilde{Y}$ is represented by the sum of the matrices $\tilde{Y}_1$ and $\tilde{Y}_2$.

$$
\tilde{Y}_1 = \begin{pmatrix} Y_{1,1} & 0 \\ 0 & Y_{2,2} \end{pmatrix}; \quad \tilde{Y}_2 = \begin{pmatrix} 0 & Y_{1,2} \\ Y_{1,2} & 0 \end{pmatrix}
$$

Then the matrix equation (1) takes the form:

$$
U_0 \tilde{Y}_1 + U_0 \tilde{Y}_2 + \tilde{J} + \tilde{S} = 0
$$

(6)

$$
\begin{pmatrix} Y_{1,1} & 0 \\ 0 & Y_{2,2} \end{pmatrix} \begin{pmatrix} U_1^2 \\ U_2^2 \end{pmatrix} + \begin{pmatrix} Y_{1,2} \end{pmatrix} \begin{pmatrix} U_1 \end{pmatrix} + \begin{pmatrix} J_1 \end{pmatrix} + \begin{pmatrix} S_1 \\ S_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}
$$

(7)

We will replace the variables:

$$
\tilde{\delta} = \frac{U_2}{U_1}
$$

$$
\begin{pmatrix} Y_{1,1} & 0 \\ 0 & Y_{2,2} \end{pmatrix} \begin{pmatrix} U_1 \end{pmatrix}^2 + \begin{pmatrix} Y_{1,2} \end{pmatrix} \begin{pmatrix} U_2 \end{pmatrix} + \begin{pmatrix} J_1 \end{pmatrix} + \begin{pmatrix} S_1 \\ S_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}
$$

(8)

Add to the already accepted notation:
- $Y_{1,2}$ - module of relative conductivity 1-2, Ohm$^{-1}$;
- $\varphi_{1,2}$ - argument of relative conductivity 1-2, rad.

Reveal (8). Considering that the elements of the matrices are complex numbers, we separately equate their real and imaginary parts to zero.

$$
\begin{pmatrix} Y_{1,1} \cos (\varphi_{Y_{1,1}}) + Y_{1,2} \delta \cos (\varphi_{Y_{1,2}} - \varphi_{\delta}) \end{pmatrix} U_1^2 + U_1 J_1 \cos (\varphi_{J_1} + \varphi_{U_1}) + S_1 \cos (\varphi_{S_1}) = 0
$$

$$
\begin{pmatrix} Y_{1,1} \sin (\varphi_{Y_{1,1}}) + Y_{1,2} \delta \sin (\varphi_{Y_{1,2}} - \varphi_{\delta}) \end{pmatrix} U_1^2 + U_1 J_1 \sin (\varphi_{J_1} + \varphi_{U_1}) + S_1 \sin (\varphi_{S_1}) = 0
$$

$$
\begin{pmatrix} Y_{1,2} \cos (\varphi_{Y_{1,2}}) \delta^2 + Y_{1,2} \delta \cos (\varphi_{Y_{1,2}} + \varphi_{\delta}) \end{pmatrix} U_1^2 + S_2 \cos (\varphi_{S_2}) = 0
$$

$$
\begin{pmatrix} Y_{1,2} \sin (\varphi_{Y_{1,2}}) \delta^2 + Y_{1,2} \delta \sin (\varphi_{Y_{1,2}} + \varphi_{\delta}) \end{pmatrix} U_1^2 + S_2 \sin (\varphi_{S_2}) = 0
$$

(9)

To solve (9), we use the software package of computer mathematics. When solving, we express and substitute in the equations sequentially $U_1$, $\delta$, $\varphi_{U_1}$. As a result, we obtain one equation for $\varphi_{\delta}$, which taking into account the coefficients $f=\frac{15}{11}$ will be:

$$
f_1 \sin 4 \varphi_{\delta} - f_2 \cos 4 \varphi_{\delta} + f_3 \sin 2 \varphi_{\delta} \cos 2 \varphi_{\delta} = 0
$$

(10)

The solution for $\varphi_{\delta}$ allows one to recover successively the coefficients $\varphi_{U_1}$, $\delta$, $U_1$, and, therefore, $U_1$ and $\varphi_{U_2}$.

3. Results of application

The correctness of the obtained formulas proves by comparing the modules and phases of the nodal voltages calculated by the derived formulas and in the application software package. The rated voltage of the power grid is 110 kV. We accept the voltage of the base $U_0=116.8e^{j45deg}$, kV.

Parameters of power grid showed in tables 1 - 2.

**Table 1.** Nodes parameters.

| Node | P, W | Q, Var | b, μOhm$^{-1}$ |
|------|------|--------|----------------|
| SST 1 | 35   | 16     | -165.3         |
The calculation results are shown in the table 3.

| Parameter                  | Radial structure | Magistral structure |
|----------------------------|------------------|---------------------|
|                            | formula          | software package    | Δ       | formula          | software package    | Δ       |
| $U_1$, kV                  | 113.149          | 113.149             | -1.4E-14| 109.32           | 109.32              | -1.1E-15|
| $\varphi_{U_1}$, °        | -6.1             | -6.1                | 0       | -7.35            | -7.35               | 0       |
| $U_2$, kV                  | 111.188          | 111.188             | 1.4E-14 | 103.307          | 103.307             | 1.1E-15|
| $\varphi_{U_2}$, °        | -6.4             | -6.4                | 0       | -9.96            | -9.96               | 0       |

Below are figures of the considered structures.

4. Conclusion
Analysis of table 3 showed that the calculation results differ slightly. The presence of deviations can be explained by the specified accuracy of the calculation adopted in the calculation in the software package, and the accuracy of the calculation of formulas with a floating point.

For the adopted calculation accuracy, the maximum absolute deviation of the nodal voltage modules for the radial and main circuits is 1.4E-14 and 1.1E-15 kV respectively. The calculations of the voltage phases in both cases completely converged.

The advantage of the formulas obtained is the ability to determine the dependencies between the individual network parameters and the mode. A direct connection of the parameters will ensure the efficiency of calculating the sensory elements of the electric network.

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