Quantum message-passing algorithm for optimal and efficient decoding

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Simple quantum decoding problem

Classical message

Classical encoder

$W$ $W$ $W$ $W$

Quantum decoder

Decoded message

Uniformly random

Linear code

CQ channel

???
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W W W W

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Follow BP and try to decode bitwise…
BPQM algorithm

- Introduced at ISIT 2017: “Belief propagation decoding of quantum channels by passing quantum messages”
- Studied by Rengaswamy et al. at ISIT 2020
  - Simplification in sequential decoding
  - Block optimality in a 5-bit example
- What’s new this year?
  - Actual message passing version — original does not pass all info!
  - Efficient implementation — above flaw means original algorithm not efficient!
  - Application to non-tree codes via approximate cloning
  - Proof of block optimality for all tree codes
Outline

- Variation of classical BP
- BPQM: Passing quantum messages for single bit estimation
- Successive BPQM for entire codewords
- Loopy BPQM
- Summary and open questions
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Belief propagation decoding as tensor network contraction

Contract to find estimate of $X_2$ given observed $y_1y_2y_3y_4$.

Run in parallel to estimate all other codeword bits.
Belief propagation decoding acting on output bits: BSC

- Associate a bit $b$ and likelihood $\ell = \frac{\delta}{1-\delta}$ to each node
- Traverse tree from leaves to root, generating node $(b, \ell)$ data from children node data.
Belief propagation decoding acting on output bits: BSC

- Associate a bit \( b \) and likelihood \( \ell = \frac{\delta}{1-\delta} \) to each node
- Traverse tree from leaves to root, generating node \((b, \ell)\) data from children node data.

- Leaf nodes: \( b \) is channel output, \( \delta \) from \( W \)
- At + nodes: \( b = b_1 \oplus b_2 \) and \( \ell = \frac{\ell_1 + \ell_2}{1 + \ell_1 \ell_2} \).
- At = nodes: \( b = b_1 \). Determine parity \( k = b_1 \oplus b_2 \), set \( \ell_2 \leftarrow \ell_2^{(-1)^k} \) and then \( \ell = \ell_1 \ell_2 \)
- At root, generate estimate given the root bit \( b \) and \( \ell \).
Belief propagation decoding acting on output bits: BSC

- Message passing: $b$ and $\ell$
- The operations add to the factor graph, but then it simplifies by channel combining rules.
- Results in a single input to a BSC whose output is the root bit $b$, with channel param. $\ell$
- Completely unnecessary, of course: LLR processing in BP includes both $b$ and $\ell$
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BP for decoding CQ channel outputs

Pick the simplest possible quantum extension:

Channel with symmetric pure state outputs $\left| \varphi_x \right>$
BP for decoding CQ channel outputs

Pick the simplest possible quantum extension:
Channel with symmetric pure state outputs $|\varphi_x\rangle$

Need to construct a measurement to estimate $X_2$ from $Q_1Q_2Q_3Q_4$

Tensor network contraction method not possible!
CQ channel output description

Bloch vector:
\[ \hat{n} = z \hat{z} + (-1)^x \sqrt{1 - z^2} \hat{x} \]

Like \( \ell' \) from BSC:
Small value indicates a reliable channel
Quantum message passing algorithm: BPQM

- Associate a qubit and \( z \) parameter to each node
- Traverse the tree from \( W \) leaves to root
Quantum message passing algorithm: BPQM

- Associate a qubit and $z$ parameter to each node
- Traverse the tree from $W$ leaves to root
- At $=$ nodes: Apply unitary $U(z_1, z_2)$ and keep just 1st qubit. Set $z = z_1 z_2$.
- At $+$ nodes: Apply CNOT, measure 2nd qubit $\rightarrow k$. Reset $z_2 \leftarrow (-1)^k z_2$ and set param to $\frac{z_1 + z_2}{1 + z_1 z_2}$.
- Measure root qubit in $\hat{x}$ basis.
Quantum message passing algorithm: BPQM

- $\Rightarrow$: Apply unitary $U(z_1, z_2)$, discard 2nd qubit. Set param to $z_1 z_2$.
- $\oplus$: Apply CNOT, measure 2nd qubit $\rightarrow k$. Discard 2nd qubit. Reset $z_4 \leftarrow (-1)^k z_4$ and set param to $\frac{z_3 + z_4}{1 + z_3 z_4}$.
- Measure last qubit in $\hat{x}$ basis.
Quantum message passing algorithm: BPQM

- Implements optimal bitwise measurement: operations are actually reversible
- Factor graph simplifies as before, to a single classical input and pure state output.
- Messages passed are one part classical ($z$), one part quantum (qubit)
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Successive BPQM for decoding entire codeword
Successive BPQM for decoding entire codeword

- **Problem**: Intermediate measurements.
  **Solution**: Perform BPQM coherently (“deferred measurement”). Rewind the circuit after measuring the output qubit.
Successive BPQM for decoding entire codeword

- **Problem**: Intermediate measurements.
  **Solution**: Perform BPQM coherently.
  Rewind the circuit after decoding each bit.

- **Problem**: Exponential overhead from + controls.
  **Solution**: Quantize $z$ register. Uncompute after use.

- **Problem**: Need infinite dimensions for $z$ register.
  **Solution**: Discretize to finite precision.
  For target error $\varepsilon$, register size only $O(\log 1/\varepsilon)$.

- All messages passed are now quantum!
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Loopy BPQM: Setup

Unroll Tanner graph to computational graph

Run BPQM:
Initialize leaves with approximately cloned qubits and appropriate z
Loopy BPQM: Performance

![Graphs showing the performance of Loopy BPQM with different channel parameters.](image)

**Figure 17:** Numerical results from decoding $X_1$, $X_5$ and the complete codeword in the 8-bit code.
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Summary & Open questions

- **BPQM**: efficient bitwise-optimal quantum message passing decoder
- Also blockwise optimal!
- Applications to capacity-achieving polar codes:
  - BPSK on pure loss Bosonic channel for transmitting classical information
  - CSS codes for amplitude damping channel for transmitting quantum information
- LDPC codes?
- Codes with loops?
- BPQM for mixed state output channels?