Performance of probabilistic amplitude shaping with BICM-ID

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It is numerically shown that bit error rate (BER) performance of probabilistic amplitude shaping (PAS) applying a bit-to-symbol mapping of non-Gray natural binary code assigned to 64-quadrature amplitude modulation is significantly improved by bit-interleaved coded modulation with iterative detection (BICM-ID). The obtained BER performance outperforms the PAS using a standard Gray code mapping. Furthermore, it has been numerically verified that the non-Gray natural binary code mapping is suitable for BICM-ID by analysing the extrinsic information transfer chart. In addition, two degradation factors, the shaping and the coding gaps, are estimated, from the Shannon limit by analysing mutual information characteristics. The obtained results quantitatively show that BICM-ID can effectively suppress the coding gap of the PAS applying non-Gray mapping in BER performance.

Introduction: Probabilistic amplitude shaping (PAS) modulation is one of the coded modulation formats that uses the non-uniformly distributed symbols on a conventional quadrature amplitude modulation (QAM) constellation with both flexible transmission capacity and high signal-to-noise ratio (SNR) sensitivity. The PAS is a promising format to construct high-speed optical transmission systems and has been massively investigated in recent years [1–3].

Bit-interleaved coded modulation with iterative detection (BICM-ID), which is another coded modulation scheme, offers remarkable improvements of bit error rate (BER) performance by iteratively exchanging bitwise soft information between a demodulator and a decoder [4]. In this process, the detection accuracy of a certain bit at the demodulator is enhanced by the soft information of bits other than itself that compose the same symbol. It is known that the performance improvement characteristics of BICM-ID largely depends on a bit-to-symbol mapping and that the characteristics can be predicted by extrinsic information transfer (EXIT) chart that characterises a flow of soft information between a demodulator and a decoder [4, 5]. Moreover, optimal bit-to-bit symbol mapping is investigated for probability shaping with BICM without iteration detection [6].

This study proposes the PAS combined with BICM-ID, which applies a bit-to-symbol mapping of non-Gray natural binary code assigned to 64-QAM, and their BER performances are numerically evaluated. The obtained results, for the first time to the best of the author's knowledge, demonstrate that the PAS with natural binary code using BICM-ID outperforms the PAS with Gray code under the same decoder characteristics. Furthermore, the EXIT chart analysis and mutual information (MI) analysis, respectively, show that the non-Gray natural binary code mapping has suitable characteristics for BICM-ID unlike Gray code mapping for PAS.

Overall System configuration: Figure 1 shows an overall system configuration of PAS systems to be evaluated composed of a distribution matcher (DM) and inverse DM (DM−1), forward error correction (FEC) encoder and decoder and two-dimensional 64-QAM. The encoder and the decoder use low-density parity-check code (LDPC) defined by Digital Video Broadcasting–Satellite–Second Generation with codeword length of 64,800. The number of LDPC decoding iterations is 20 for its inner loop in this work.

After additive white Gaussian noise is added to the modulated signal, the demodulator calculates log-likelihood ratios (LLRs) using bit-metric decoding technique, where the supposed probability density of each symbol is weighted with the probability of occurrence of a constellation point on 64-QAM [1]. In addition, BICM-ID is applied to exchange bitwise soft information of LLR up to three outer iterations between a demodulator and a decoder. BER performance is evaluated after BICM-ID process.

Modulation format and bit-to-symbol mapping: Two kinds of the bit-to-symbol mapping assigned to the 8-ary PAM symbol are illustrated in Figure 2. Figure 2(a) is the Gray code mapping that differs from adjacent symbols by only 1 bit, while Figure 2(b) is the non-Gray natural binary code mapping where each binary number decreases by 1 in decimal when each respective amplitude increases. A symbol of 64-QAM is formed by combining an in-phase (I) 8-ary PAM symbol with another quadrature-phase (Q) 8-ary pulse-amplitude modulation (PAM) symbol.

Distribution matcher: A DM transforms a uniformly distributed binary data block into a distributed amplitude data block. Constant composition DM (CCDM) based on an arithmetic coding is used in our numerical calculations, in which all amplitude data blocks have a predefined probability distribution [1].

CCDM defines two amplitude probability distributions of (k, n) = (720,600) and (600, 600), where k and n are, respectively, a binary data block length and an amplitude data block length. Each distribution sets to form quantised Gaussian distribution as much as possible. Specific probability distributions of amplitude elements, expressed as a percentage, are (64.17%, 29.17%, 6.00%, 0.67%) and (71.50%, 25.17%, 3.17%, 0.17%), respectively, for CCDM corresponding to 64-QAM amplitude of (1, 3, 5, 7) in I or Q direction. Each entropy is 4.44 or 4.04 bits/symbol (per two dimensions) for each probability distribution, respectively, calculated using the probability of every constellation point [2, 3].

Information rate: Information rate (IR) of the PAS per symbol assigned to 64-QAM is described as follows [2]:

$$R = 2 \cdot \left( \frac{k}{n} + 1 - (1 - R_s) \right)$$

(1)

where Rs is the code rate of LDPC, set to 2/3. Specific IR values of (k, n) = (720, 600) and (600, 600) are, respectively, 2.4 and 2.0 bits/symbol.

Note that the first two terms of Equation (1) or 2 ⋅ (k/n + 1) represent the IR before subtracting LDPC overhead capacity. The specific IR value before the subtraction becomes 4.4 bits/symbol for (k, n) = (720, 600) or 4.0 bits/symbol for (600, 600). The shaping gaps, which quantitatively describes the incompleteness of the probability distribution, are calculated by subtracting the ratio of these IRs and the entropy values obtained in the last part of the former section from 1.0 [2]. They are 0.0090 and 0.0099 for (720,600) and (600, 600). This preliminary evaluation of the
shaping gap quantitatively indicates that two probability distributions by CCDM are close to ideal.

**BER performance:** Figure 3 shows the obtained BER performances after LDPC decoding. The red and green circles and crosses, respectively, show BERs for Gray code mapping without outer iteration and with three times outer iterations at 2.0 and 2.4 bits/symbol. Since each circle has almost the same value as the cross at the same SNR, BICM-ID does little to improve the BER of PAS with a Gray code mapping assigned to two-dimensional 64-QAM.

On the other hand, the blue and pink triangles, diamonds and squares, respectively, show that the BER values of non-Gray natural binary code mapping are successfully improved by BICM-ID and that the SNR at BER less than $1 \times 10^{-5}$ after three times outer iterations finally becomes about 0.4 dB smaller than the SNR of Gray code. Note that no improvement in BER was observed after more than three times outer iterations.

It can be noted that BER after DM$^{-1}$ becomes worse than the results shown in Figure 3 due to burst error in DM$^{-1}$ operation [1]. The BER should be improved to be less than $1 \times 10^{-15}$ in combination with an appropriate outer FEC. In that case, amounts of IR become smaller, and Shannon limit described in broken lines slightly shift to the left in accordance with the outer FEC code rate.

**EXIT chart and MI analysis:** Figure 4(a) shows the EXIT charts of PAS. The brown circles represent mutual MI input/output characteristics of LDPC decoder at $R_c = 2/3$ or 0.67. When the MI value of the LDPC input is about 0.74, the MI value of the LDPC output becomes 1.0. Therefore, the coding gap representing the performance of the LDPC used in this calculation is calculated to be about 0.07 [2].

The red, green, blue and pink marks, respectively, represent input/output MI characteristics of demodulator at SNR conditions that offer BER of less than $1 \times 10^{-5}$. Among these marks, green and red circles, which represent the LDPC decoder characteristics of PAS with Gray mapping, form almost straight lines that are approximately parallel to the horizontal axis in both figures. These results show that no improvement is expected with BICM-ID.

On the contrary, the curves formed by blue and pink squares have some slopes and do not intersect the curve formed by the brown circles in Figure 4(a). That is, even if the MI value of demodulator output is 0.71 for $(k, n) = (720, 600)$ or MI for (600, 600) without MI from the LDPC decoder, the MI of the LDPC becomes 1.0 by BICM-ID, and three times outer iterations are enough to achieve BER-free if MI trajectory analysis is performed [4].

Figures 4(b) and (c), respectively, show MI output characteristics of the demodulator as a function of SNR for the PASs with Gray code and natural binary code at $(k, n) = (720, 600)$ and (600, 600). The SNR penalties for Gray code due to the shaping gap are about 0.4 dB in both figures, which can be estimated by comparing the SNR values at MI values of 0.67 with the SNR value at Shannon limit. Moreover, the SNR penalties due to the coding gap are about 1.7 dB, which can be estimated by comparing the SNR values with MI values of 0.67 and 0.74. On the other hand, the MI characteristics of the natural code are inferior to the MI characteristics of the Gray code. However, BICM-ID allows the input MI to the LDPC decoder to be set to 0.69 or 0.71, resulting in an SNR gain of about 0.4 dB for each probability distributions of (720,600) and (600, 600).

**Conclusion:** We evaluated BER performance of PAS applying a bit-to-symbol mapping of non-Gray natural binary code assigned to 64-QAM and confirmed the performance, which is significantly improved by BICM-ID. The SNRs at BER less than $1 \times 10^{-5}$ of PAS format with the natural binary code is about 0.4 dB smaller than that with Gray code mapping. Furthermore, we numerically verified that the non-Gray natural binary code mapping is suitable for BICM-ID by analysing the EXIT chart. In addition, MI characteristics analysis quantitatively indicates that BICM-ID provides SNR gain by reducing the coding gap for a bit-to-symbol mapping of non-Gray natural binary code.

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