Gravitational Analog of the Aharonov-Casher Effect

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Abstract

The gravitational interaction between a massive particle and a spinning particle in the weak-field limit is studied. We show that a system of a spinning point-like particle and a massive rod exhibit a topological effect analogous to the electromagnetic Aharonov-Casher effect. We discuss the effect also for systems with a cosmic string instead of a massive rod and in the context of 2+1-dimensional gravity.

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1 Introduction

In the Aharonov-Bohm (AB) effect \cite{1}, one considers the motion of a charged particle $e$ around a stationary solenoid enclosing a magnetic flux $\phi$. Aharonov and Casher (AC) \cite{2} studied the possibility that both, the charge and solenoid are in motion, and found that in addition to the usual electromagnetic $eA_{AB}$ vector potential another vector potential coupling must also be invoked. Viewing the solenoid as a collection of magnetic moments $\vec{\mu}$ this ‘Aharonov-Casher vector potential’ is given by $\int \vec{\mu} \times \vec{E}$ where $\vec{E}$ is the electric field of the charged particle. The AC vector potential couples the moving solenoid to the electric field of a charged source. Since the total effective Lagrangian of the system is Galilean invariant, when the charged particle stands still and the solenoid is in motion, the phase shift is given by integration over the AC vector potential alone. A realizable physical setting considered by Aharonov and Casher, that may naturally be regarded as dual to the AB effect, is that of a point-like magnetic moment $\vec{\mu} = \mu \hat{z}$ moving around a homogeneously charged rod, pointing to the $\hat{z}$ direction, with charge density $\lambda$ per unit length. Aharonov and Casher found \cite{2} that in this system, as the magnetic moment travels along a closed path $C$, winding $n$ times around the charged rod, it feels no force\footnote{1 It was argued in \cite{3} that contrary to the AB effect the AC phase can be derived from a classical lag caused by a non-vanishing force. Later it was shown\cite{4} that this force (due to the non-vanishing electric field at the location of the magnetic moment) causes no acceleration, but changes the intrinsic (mechanical) momentum of the current distribution. Particular examples of this subtle effect have been worked out in \cite{5}. It was shown in \cite{6} that a topological AC effect can also be manifested even when the local field components vanish.}, but does accumulate a non-trivial topological phase\footnote{2 The AC phase shift has been first observed in a neutron interferometry experiment \cite{7}.}

$$\Phi_{AC} = \frac{1}{e\hbar} \oint_C (\vec{\mu} \times \vec{E}) \cdot d\vec{l} = 2\pi n \frac{\lambda \mu}{\hbar}.$$  \hspace{1cm} (1)
It has long been noted that the AB effect has a gravitational analog [8]. In the weak field approximation it was shown that a test particle of mass $m$ couples to the metric generated by a rotating source through a vector potential-like term, $mg_{0i}$, analogous to the AB vector potential. Therefore, the mass $m$ plays a role analogous to the charge in the electromagnetic effect. Contrary to the electromagnetic effect, the gravitational AB phase can be understood as due to a classical time delay. [9, 10] The time required to travel along a closed trajectory around the source is longer in the direction opposite to the angular momentum of the source. This classical lag is due to the well known ‘frame dragging’ effect [11] of a rotating source, which is also similar to the Sagnac effect [12] observed by an interference experiment in a rotating frame of reference in Minkowskian space. (For a discussion of experiments using interference of neutrons to measure rotational and gravitational effects see [13].) More recently, the gravitational AB effect was studied in relation to rotating cosmic strings [14, 15, 10] and to Chern-Simons-Witten gauge formulation of 2+1-dimensional gravity [16, 17].

In this note we will show that a gravitational effect analogous to the electromagnetic AC exists as well. We find that in the weak field approximation, $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu} \ll 1$, the phase accumulated by a point-like particle with intrinsic angular momentum $\vec{J}$, which travels in the background metric $g_{\mu\nu}$, is expressed by integrating along the path the gravitational analog of the AC vector potential $\vec{J} \times \vec{\nabla} (h_{00} - \frac{1}{2} \eta_{\mu\nu} h_{\mu\nu})$. The gravitational analog of the AC effect appears in a system of a spinning particle and a massive rod. The gravitational attraction between a massive rod and the point-like mass can be eliminated by adding suitable compensating electric charge, which cancel (to the leading order) the local forces but do not alter the vector potentials effect. The remaining vector potential effect yields a gravitational AC phase. The same AC vector potential yields a topological force-free effect in the case of a spinning test particle in the presence of a non-rotating
(vacuum) cosmic string source. In this case however, the AC phase derived using the weak field approximation turns out to be exact, and applies also for a relativistic Dirac particle. Finally, we show that in the case of Chern-Simons-Witten gauge theory formulation of 2+1 gravity, both the AB and AC phases are ‘unified’ and are derived from a single elementary vector potential.

As in the case of the gravitational AB effect, the gravitational AC effect can be viewed as due to a classical lag. In this case a string-like source, say in the \( \hat{z} \) direction, causes a rotation around the \( \hat{z} \) axis of a vector undergoing a parallel transport around the string. The rotation is different for trajectories going in opposite directions. This effect has been noted for a special case in reference \([18]\).

The paper continues as follows. In Section 2 we examine the basic spin-mass interaction in the weak field approximation. In Section 3 we construct the gravitational analog of the AC effect. Finally, in Section 4 we discuss the AC effect for the case of a spinning particle in motion around a cosmic (vacuum) string and in the context of 2+1-dimensional gravity. We adopt the units \( c = G = \hbar = 1 \).

## 2 Gravitational Spin-Mass Interaction

Consider two localized matter distributions of total rest masses \( m, M \) and intrinsic (orbital) angular momentum \( \vec{J}_m = 0, \vec{J}_M = \vec{J} \), respectively. The sources interact via the gravitational field. We denote by \( \vec{r} \) and \( \vec{R} \) the locations of the center of mass of \( m \) and \( M \), and the velocities by \( \vec{v} = \dot{\vec{r}}, \vec{V} = \dot{\vec{R}} \). In the tree-level approximation, the Lagrangian of the system is given by

\[
L = L_{\text{matter}} - \frac{1}{2} \int T_{\mu\nu} h_{\mu\nu} d^3x, \tag{2}
\]
where \( h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu} \) (\( \eta_{\mu\nu} = diag(1, -1, -1, -1) \)) is the linear superposition of the metric generated by each of the two sources.

From now on we chose to work in the harmonic gauge

\[
\partial^\nu \bar{h}_{\mu\nu} = 0, \tag{3}
\]

where

\[
\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h, \quad h = \eta^{\mu\nu} h_{\mu\nu}. \tag{4}
\]

We still have the gauge freedom to perform small coordinate transformations

\[
h'_{\mu\nu} = h_{\mu\nu} - \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu. \tag{5}
\]

In the weak field approximation Einstein Equations read

\[
\eta^{\alpha\beta} \partial_\alpha \partial_\beta \bar{h}_{\mu\nu} = 16\pi T_{\mu\nu}. \tag{6}
\]

In analogy to the electromagnetic case we define a ‘vector potential’

\[
h_\mu \equiv (\bar{h}_{00}, h_{0i}). \tag{7}
\]

\( h_\mu \) transforms like a vector only for \( \partial_0 \Lambda_i = 0 \).

The generic metric produced by a source at \( x_i = 0 \) with total mass \( M \) and total angular momentum \( \vec{J} \), is given in the weak field approximation (or sufficiently large \( x, x^2 = x_i x^i \)) by \[11\]

\[
ds^2 = \left(1 - \frac{2M}{x}\right) dt^2 - \left(1 + \frac{2M}{x}\right) (dx^i)^2 - 4\epsilon_{ijk} \frac{x^j J^k}{x^3} dt dx^i + O(1/x^3). \tag{8}
\]

The coupling of \( m \) with the gravitational field produced by the spinning source \( M \) yields

\[
-\frac{1}{2} \int d^3r T_{\mu\nu}(m) h_{\mu\nu}(M, J) = -\frac{1}{4} m h_0 (\vec{r} - \vec{R}) - m \vec{v} \cdot \vec{h}(\vec{r} - \vec{R}) + O(v^2) \tag{9}
\]

We assumed that the pressure terms \( T_{ij}, (i, j = 1, 2, 3) \), are negligible.
$M$ couples only to $h_{00}(m)$. However in the rest frame of $M$, $T_{00}^{(0)} = \rho \left( \int \rho = M \right)$ and a non-zero $T_{0i}^{(0)}$ generates the rest frame angular momentum $\vec{J}$:

$$T_{0i}^{(0)} = (\nabla \times \mathbf{J})_i, \quad \vec{J} = \int d^3x \vec{J}. \quad (10)$$

where $\mathbf{J}$ is the local ‘spin density’. Boosting to a moving frame of velocity $\vec{V}$ we find

$$T_{00} = \rho - 2V_i T_{0i}^{(0)} + O(V^2), \quad (11)$$

$$T_{ii} = -2V_i T_{0i}^{(0)} + O(V^2). \quad (12)$$

There is no summation in equation (12).

The coupling to the rotating mass $M$ is thus

$$\frac{1}{2} \int T_{\mu\nu}(M) h_{\mu\nu}(m) d^3R = -\frac{1}{4} \int \rho h_0(\vec{R} - \vec{r}) d^3R + \int h_0 \vec{V} \cdot \vec{\nabla} \times \vec{J} d^3R$$

$$= -\frac{1}{4} M h_0(\vec{R} - \vec{r}) - \vec{V} \cdot \vec{J} \times \vec{\nabla} h_0(\vec{R} - \vec{r}) \quad (13)$$

Therefore, the total Lagrangian reads

$$L = \frac{1}{2} mv^2 + \frac{1}{2} MV^2 - m M \phi(\vec{r} - \vec{R}) - m \vec{v} \cdot \vec{h} + \vec{V} \cdot \vec{J} \times \vec{\nabla} h_0 \quad (14)$$

where $\phi(\vec{r} - \vec{R})$ stands for the ordinary Newtonian gravitational potential, $\vec{h}$ is generated by the spinning mass $M$, and $h_0$ is generated by $m$. The last two terms are in a complete analogy to the vector potentials obtained previously [2] for a system of a charged particle $e$ and a neutral source with a magnetic moment $\vec{\mu}$. $\vec{h}$ plays the role of the electromagnetic Aharonov-Bohm vector potential $\vec{A}$ and $\vec{J} \times \vec{\nabla} h_0$ is the gravitational analog of the Aharonov-Casher vector potential $\vec{\mu} \times \vec{E}$. It is also apparent that the rest mass $m$ and the intrinsic angular momentum $\vec{J}$ are the analogous quantities to $e$ and $\vec{\mu}$ in the electromagnetic effect.
The necessity of the gravitational Aharonov-Casher vector potential is clear from the required translation and Galilean invariance of our non-relativistic Lagrangian. Using the metric (8) the ‘gravitational Aharonov-Bohm’ vector potential is given by

$$\vec{v} \cdot \vec{h} = 4m \vec{v} \cdot (\vec{r} - \vec{R}) \times \vec{J} / |\vec{r} - \vec{R}|^3$$

(15)

that does not satisfy Galilean invariance. Only by combining the two vector potential terms on the right hand side of Eq. (14) we get a proper invariant interaction term

$$4m(\vec{v} - \vec{V}) \cdot (\vec{r} - \vec{R}) \times \vec{J} / |\vec{r} - \vec{R}|^3.$$ 

(16)

As we will show in the next section by replacing one of the sources by a string-like source we obtain via this vector potential interaction a topological effect.

### 3 Topological Aharonov-Casher Effect

Instead of the localized mass $m$ consider a rod at rest with constant mass density $\mu$ per unit length, and $T_{ij} \simeq 0$. In the linear approximation the metric produced by such a rod located at $\rho^2 = x^2 + y^2 = 0$ is given by

$$h_{00} = h_{ii} = -4\mu \ln \rho / \rho_0.$$ 

(17)

The gravitational coupling of the non-relativistic particle-like source of mass $M$ and intrinsic angular momentum $J$ is according to the effective Lagrangian (13) given by a Newtonian attraction term $1/4Mh_0(\rho)$, and by the gravitational Aharonov-Casher vector potential term

$$\vec{A}_{ac} = -\vec{V} \cdot \vec{J} \times \vec{\nabla}h_0$$

(18)

For the case that $\vec{J} = J\hat{z}$ (18) reduces to

$$\vec{A}_{ac} = 4\mu J \hat{z} / \rho$$

(19)
where \( \hat{\theta} \) is the cylindrical angular unit vector (\( \hat{\theta} \times \hat{\rho} = 1 \)). It is clear that this interaction will lead to a topological path independent term. The Aharonov-Casher phase collected by \( M \), moving in a closed path \( C \) about the rod, while the direction of \( \vec{J} \) is kept fixed is given by (restoring \( G \) and \( \hbar \))

\[
\Phi_{ac} = \frac{1}{\hbar} \oint \vec{A}_{ac} \cdot d\vec{l} = 2\pi n \frac{4G\mu J}{\hbar}
\]  

(20)

with \( n \) as the winding number of \( C \) around the road.

Using the Galilean invariance of the system can also derive this phase by looking at the motion of the rod around the spinning mass \( M \). Viewing the rod as the sum of small mass we need to integrate the interaction of each piece of the rod with the Gravitational AB vector potential \( \mu \vec{v} \cdot \vec{h}(M,J) \). The result is in agreement with (20).

By the effective Lagrangian we find that

\[
M \ddot{\vec{V}} = -m \ddot{\vec{V}} = -mM \vec{\nabla}R \phi(\vec{r} - \vec{R}).
\]  

(21)

The vector potentials do not generate forces, but there is a Newtonian attraction. Therefore, it may be argued that the analogy with the electromagnetic force free effect breaks. However, since this force is due to a potential effect, we can easily eliminate it, by adding on-top of the gravitational interaction another potential. For example, we could charge the road with a uniform charge distribution of linear density \( \lambda \) and add in the center of \( M \) a charge \( q \). When \( \lambda q = \mu M \) the electric force compensates exactly the Newtonian force. (If the charge \( q \) is not a point-like it induces a magnetic moment and in addition to the gravitational vector potentials we need to invoke also the usual electromagnetic AC and AB vector potentials. A mixing of the gravitational and electromagnetic effects is obtained.) Of course, the charge modifies the metric used above, but to the lowest order the corrections are negligible.
4 Cosmic Strings and Reduction to 2+1 Gravity

It has been noted that an exact gravitational analog for the AB effect exists in the context of cosmic string and 2+1-D Einstein gravity. The space-time around a (vacuum) cosmic string (in the \( \hat{z} \) direction) is flat and the metric reads

\[
ds^2 = (dt - 4Sd\theta)^2 - d\rho^2 - \alpha^2 \rho^2 d\theta^2 - dz^2,
\]

where \( \alpha = 1 - 4\mu \) and \( \mu, S \) are the mass and intrinsic spin per unit length, respectively. In this coordinate system \( \rho \) and \( \theta \) take the usual values \( \rho \in (0, \infty) \), and \( \theta \in (-\pi, +\pi) \). With an appropriate coordinate transformation the metric can be written in a Minkowskian form but has a non-trivial topology of a cone with a helical time structure. It is straightforward to see that for a particle of mass \( m \) that encircles the cosmic string the \( g_{\theta \theta} \) term in (22) produces an exactly the AB phase numerically identical (when \( J \to 0 \)) to that obtained in equation (20). In the limit of \( \alpha \to 1 \) the scattering amplitude coincides with that of the electromagnetic effect where \( S \) and \( m \) replacing the magnetic flux and the charge in the AB cross-section amplitude.

If the cosmic string is in (slow) motion around the point-like source \( m \) at rest then by the Galilean invariance we expect that the phase accumulated by the string, via the interaction with the gravitational field of \( m \), should reproduce the same phase. Indeed, this can readily be verified. The cosmic string may be viewed as a line of rotating masses. The string has a mass density \( \mu \) with additional negative stress \( T_{zz} = p_z = -\mu \), in the \( \hat{z} \) direction. Repeating the analysis leading to equation (13) and computing the interaction of each of the small segments of the rod with the Schwarzschild field of the mass \( m \) in equation (8), we find that the total interaction of the moving cosmic string is given by

\[
-\frac{1}{2} \int (h_{00} T_{00} + h_{zz} T_{zz}) + \int \vec{V} \cdot \vec{J} \times \vec{\nabla} h_0.
\]

The first two terms above vanish, due to the negative pressure in the string. This means
that as expected the cosmic string does not feel a Newtonian force. Integrating along the string the Aharonov-Casher vector potential along the string (the last term on the right in (23) above) we get the effective vector potential

\[ \vec{A}_{\text{eff}} = 4mS \frac{\hat{\theta}}{\rho}, \]  

where \( \rho \) is the location of the string in cylindrical coordinates and \( \theta \) the related orthogonal angular unit vector. Consequently, although the mass \( m \) (analogous to \( e \) in the e-m effect) produces a non-zero curvature (analogous to \( \vec{E} \) in the e-m effect) it does not produce any force on the string and the topological phase is generated by the effective vector potential above.

In the last example of a moving (non-rotating) cosmic string we reproduced the topological phase shift by integrating over the gravitational AC vector potential in (23). We can also reproduce the gravitational analog of AC effect by considering the motion of a spinning (localized) mass \( M \) moving around a spinless (static) cosmic string of linear mass density \( \mu \). The metric of the cosmic string is given by (22) with \( S = 0 \). This metric can be written in harmonic coordinates by transforming to the new radial coordinate \( \rho' \) defined by

\[ \left(1 - 8\mu \ln(\rho')\right)\rho'^2 = (1 - 8\mu)\rho^2. \]  

In this gauge we have

\[ ds^2 = dt^2 - (1 - 8\mu \ln(\rho')(d\rho'^2 + \rho'^2d\theta^2)) - dz^2, \]  

and therefore,

\[ h_0 = 8\mu \ln(\rho'), \quad h_i = 0. \]  

Repeating the calculation leading to (14) we find that the interaction in the Lagrangian is given by

\[ L_{\text{int}} = \vec{V} \cdot \vec{A}_{ac} = \frac{1}{2} \vec{V} \cdot \vec{S} \times \vec{\nabla} h_0. \]
This expression for the effective coupling of the spinning particle with the AC vector potential $\vec{S} \times \vec{\nabla}h_0$ differs by factor of 1/2 from the coupling obtained in equation (18) for a ordinary massive rod. However, the resulting phase shift $\int \vec{A}_{ac} \cdot d\vec{l}$ is identical to that in equation (20). Therefore, the topological AC effect induced by a massive rod and by a cosmic string are identical. The factor 1/2 arises because in the case of the cosmic string $h_{00} = 0$ and $h_0 = -\frac{1}{2} \eta^{\alpha\beta} h_{\alpha\beta} = 2h_0(rod)$.

The AC effect in this case of a cosmic string is force free without the necessity of adding a compensating scalar potential. The same topological phase is obtained if the cosmic string is circling a stationary spinning particle. This phase is obtained by repeating the calculation using the gravitational AB vector potential generated by the metric of a spinning source in equation (8). The local curvature at the location of the cosmic string is non-zero but as before in equation (23) it induces zero force on the cosmic string due to the negative pressure in the $\hat{z}$ direction.

We have derived the gravitational Aharonov-Casher in the weak coupling with the spinning particle. We now notice that in the case of a cosmic string the result is exact without appealing to a week field approximation. The Dirac equation in curved space-time is

$$\gamma^a e^\mu_a \left(-i\partial_\mu - \frac{1}{2} S_{\alpha\beta} \omega^{\alpha\beta}_\mu \right) \psi + m\psi = 0,$$

where $e^\mu_a$ and $\omega^{\alpha\beta}_\mu$ are the dreibein and spin connection, respectively, and $S_{\alpha\beta} = \{\gamma_a, \gamma_b\}$. For the metric (22) with $S = 0$ the only non-vanishing component of the spin connection is given by

$$\omega^{12}_i = 4m \frac{\epsilon_{ij} \hat{x}_j}{\rho^2}.$$  \hspace{1cm} (30)

The coupling therefore corresponds to a vector potential

$$A_\theta = 4m \frac{\hat{S}_z}{\rho^2}.$$  \hspace{1cm} (31)
It was recently shown [20] that the Lagrangian for a classical spinning test particle contains the coupling \( \frac{1}{2} \frac{d\rho^a}{d\tau} \omega_{\mu}^{ab} S_{ab} \). Therefore, this result is expected to hold also for the general case of a spinning test particle with spin constantly pointing to the \( \hat{z} \) direction.

In light of our 3+1-dimensional results, we would finally like to comment on the somewhat different nature of the analogous topological effect in 2+1 gravity. We first note that two point-like particles in 2+1-dimensions are represented in 3+1 dimensions by two parallel (vacuum) strings. The (2+1) mass \( M \) and spin \( S \) obviously represent the mass and spin per unit length of the corresponding strings. Therefore, a two particle system in 2+1-dimensions is not equivalent to the string/rod + point particle generic system studied above. (In fact the phase accumulated by such a system is ill defined). In the 2+1 setting the local curvature (outside the sources) is identically zero. Nevertheless, one may rederive in complete analogy to the treatment above the AC and AB phases using the weak field method (in the harmonic gauge). However, in the case of 2+1 gravity a more exact treatment is available.

As is well known, pure gravity in 2+1-dimensions can be formulated as a Chern-Simons gauge theory of the ISO(2,1) Poincaré group. [21] In this formalism of gravity the basic (independent) variables are the dreibein \( e^a_{\mu} \) and the spin connection \( \omega_{\mu}^a = -\frac{1}{2} \epsilon^{abc} \omega_{\mu ab} \). The non-abelian vector potential is given by

\[
A_\mu = e^a_{\mu} P_a + \omega_{\mu}^a J_a, \tag{32}
\]

where \( P_a \) and \( J_a \) are the generators of translation and Lorentz-rotation transformations, respectively. Although it is not possible to couple Chern-Simons-Witten gravity to arbitrary matter fields (since then a non-gauge invariant metric is required), one still can couple gravity to point-like particle sources. [22] Such a source would be described by a non-abelian charge

\[
Q(x) = p^a(x) J_a + j^a(x) P_a, \tag{33}
\]
generating a current \( Q(x) \frac{dx^\mu}{dt} \) along a world line \( x(\tau) \). The test particle couples to the gauge field through the invariant interaction term \[ \tag{17} \]

\[ S_{int} = \int \langle Q, A_\mu \rangle dx^\mu = \int (\epsilon_\mu^a p^a + \omega_\mu^a j^a) dx^\mu, \] (34)

where \( \langle , \rangle \) is the invariant inner product defined by \[ \tag{21} \]

\[ \langle J_a, P_b \rangle = \eta_{ab}, \langle J_a, J_b \rangle = \langle P_a, P_b \rangle = 0. \] Consider now the 2+1-dimensional background generated by a static point particle of mass \( M \) and intrinsic spin \( S \), and located at \( \rho = 0 \). (The related metric is given by (22) with one dimension reduced). The only non-vanishing (one-form) dreibein and spin connection are \( e^0 = 4GSd\theta \) and \( \omega^0 = 4GMd\theta \). Therefore, for a world line \( C \) winding around the source \( n \) times, (34) reduces to

\[ S_{int} = \oint_C e^0 p_0 dx^\mu + \oint_C \omega^0 j_0 dx^\mu = 2\pi n(4GSm + 4GMs). \] (35)

In the last equality above we have assumed that for an adiabatic slow motion of the test particle, \( p_0 \) and \( j_0 \) can be replaced by \( m \) and \( s \), the rest mass and intrinsic spin of the test particle, respectively. As could be anticipated, since both the source and the test particle have mass and intrinsic spin, the action (35) yields simultaneously both the AB and AC phases, which are identical to the topological phases we have obtained previously in the analogous 3+1-dimensional case. Note that the AB phase is obtained by integrating over the dreibein part of the vector potential, and that the AC part is derived from the spin connection part. Amusingly, in this gauge theory formalism of 2+1 gravity, both the AB and AC phases are derived from a single basic vector potential. Of course, this separation of the total phase is not unique. In another coordinate (gauge) system the topological phase will be represented by a different mixing of the two phases.
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