Didactical Design Enrichment of Angle in Geometry

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Abstract. The underlying problem of this research is the lack of student’s competencies in understanding the concept of angle in geometry as the results of the teaching and learning pattern that only to receive the topic rather than to construct the topic and has not paid attention to the learning trajectory. The purpose of this research is to develop the didactical design of angle in space learning activity. The used research method is a method of qualitative research in the form of a didactical design research through three phases of analysis i.e. didactical situation analysis, metapedadidactical analysis, and retrospective analysis, which conducted in students from 10th grade at one of private schools in Bandung. Based on the results of research and discussion, the didactical design that has been made, is capable to change student’s learning habit and quite capable to develop student’s competencies although not optimal.

1. Introduction

Angle in geometry is one of all mathematic topics that learned since elementary school until senior high school even in university like architecture, art, science, and engineering. Concept of angle in geometry learned from the definition, angle in plane, until angle in space. Based from the analysis in student’s book, some problems happen in the teaching material. The problems also happen in student’s books like wrong definition about angle in space that leads to misconception. These are the example:

![Figure 1](left) Form of angle concept definition from student’s book

Figure 1 (left) shows the definition of angle as the intersection between two lines and called the ray AB and AC as a segment even though the angle picture was correct. The other figure (right) shows the definition of angle as the intersection between two segments and called the angular measure as the
angle by pointing it with an arrow as shown above. If we take a look at the geometry book by Edwin E. Moiss ‘…an angle is a figure which is the union of two rays which have the same end point, but do not lie on the same line’ [1]. This misconception and passive form of concept presentation in textbook problem occurs since they study in elementary school until senior high school. These problems are didactical obstacle in learning process. If that continues, the learning difficulties will trigger the emergence of epistemological obstacle, and the similarity within it will caused the ontogenical obstacle [2].

Another problem occurred from the learning activity. Based from the analysis results of learning direct observations and learning video analysis, lot of students just learn about the ‘results of mathematics’, not learning about the ‘process of mathematics’. Lot of student show that they were happy in class when they following the learning activity with some variations of learning models, methods, and strategies that used by teacher. Whereas, the essence of learning mathematics is not just a fun learning as the necessary conditions, but we need to pay attention to the sufficient conditions when students understand the topic and forming a belief system about how they see mathematics carefully [3]. If learning just oriented on the textual understanding, poor meanings and context will be resulted, and if learning just oriented in the scoring results, students become less active and less involve in class [4]. This indicates the need of repersonalization i.e. the process of connecting concept of before, within, and after the concept, before making a learning plan and finding the suitable context. The learning design needs to consider the learning obstacle to determine the proper learning trajectory from beginning until the achieved learning objectives.

2. Experimental Method
This research used the Didactical Design Research through three phases of analysis, prospective analysis, metapedaddidactic analysis, and retrospective analysis. Prospective analysis is analysis of prior learning didactical situation including recontextualisation, repersonalization, and prediction of responses in the form of didactical design hypothesis. Metapedaddidactic analysis is the analysis of learning unity, flexibility, and coherency. Retrospective analysis i.e. the comparison between the prospective analysis and metapedaddidactic analysis. [3] The data analysis was done with the triangulation technique in interpret the data from interview, observation, and documentation (learning video, teaching material, etc.) accompanied by testing with learning obstacle and student’s mathematics competencies test instrument.

Prospective analysis done by using an instrument including test and interview to gather all the possibility of learning obstacles. The instrument test contains questions to examine the spatial sense/ability, conceptual understanding, and procedural prowess of angle in geometry. The test given to 10 students from one senior high school in Bandung, 10 first-year students college in Bandung, who have already studied angle in geometry topics. The interview given to all mathematics teachers in a school and the 20 students that have taken the test. The results from this analysis used to construct the formed design, including the used context, theory, media, and multimedia.

Metapedaddidactic analysis done by direct observation while implementing the formed design. It also done with the help of two observer (triangulation) and recording the activity during implementation. Retrospective analysis done by comparing the analysis results from prospective, metapedaddidactic analysis, and the test results during implementation. The implementation done in two cycle of Didactical Design Research’s phases of analysis in two different class with the same level of competencies.

3. Result and Discussion

3.1. Learning Obstacle Findings
Based from the results of learning obstacle test, interview, and observations for three months, there are six learning obstacles found. First, the lack of student’s spatial ability. Second, the missing of angular measure concept in student’s book, teaching material, teacher’s explanation. Third, the lack of student understands in prerequisites topic or concept. Fourth, student mistakenly use the procedure in determining the angle between two intersect objects. Fifth, student used to solve three-dimensional
problem only if it was the same with the example problem. Sixth, teacher did not involve their teaching with constructing concept activity.

The most surprising from these learning obstacle findings is the second learning obstacle, the lost angular measure concept. Whereas this concept is the essence and the most important link in learning the concept of angle in geometry, all procedure and steps in determining the angle between two objects are using the definition of angular measure. The lost angular measure concept alleged to have already occurred long ago, because based on the test results and interview against students in junior and senior high school even first year students that majoring mathematics in university, tells that they did not know the definition of angular measure, and its difference with angle.

![Figure 2. Learning Obstacle’s result representation](image)

Figure 2 shows the confirmation of the missing concept. All the questions on the test were just asking the students to determine the formed angle. Instead of using the vertex of each angle name, they point the used angular measure symbol in the picture (left) and state that the pointed part was the formed angle (right). It clearly shows the misconception that they did not know the difference between angle and angular measure, or they might be never know about the angular concept all this time.

![Figure 3. Student’s answer in defining the angular measure](image)

Figure 3 shows how they explain the concept of angular measure. From the first picture the student stated that ‘the angular measure is the area that was formed by two rays with the same end point’ and we all know, an angle in plane divided the plane into two area which means, this definition was not correct, more over if the angle was in space. From the second picture the student stated that ‘angular measure is measure that most important, practical, and generally used degree or radian as the unit’, he only explain for a question that asking the unit of angular measure. These results confirm the missing angular concept in students’ learning experience.

These learning obstacles also resulting in the lack of student’s mathematics proficiency [5] in concept of angle in geometry. Learning obstacle 2 has a big effect in the learning trajectory, because missing concept also means missing link in the topic’s structural learning trajectory and the development of student’s competencies. Students cannot learn angle in geometry concept well, because the lack of their competencies including the spatial ability, but the lack of spatial ability caused by the lack of geometry education [6]. Learning obstacle 6 confirmed the truth of the statement based from learning video analysis that students just learn about the ‘results of mathematics’, not learning about the ‘process of mathematics’ and also resulting the other learning obstacle.
3.2. Didactical Design Preparation & Implementation

Before creating the didactical design of angle in geometry concept and topic, it needs a learning trajectory needed consist with the aims, development trajectory, and instructional task [7]. Based from the learning obstacle findings and the concept map, this is the formed learning trajectory,

![Learning trajectory of angle in geometry](image)

Figure 4. Learning trajectory of angle in geometry

Figure 4 shows the position of angular measure concept and its connection with the other concept. The learning trajectory divided into two framework. First, finding the angular measure concept using game (Figure 5). Second, using the angular measure concept to form the finding angle procedure and using the formed procedure to solve angle and angular measure problems. Angular measure concept plays an important role in findings the angle between two intersected object by finding the shortest rotating track between two rays of angle as the representation for each object.

![Spinning Soccer Game way of working](image)

Figure 5. Spinning Soccer Game way of working

Figure 5 shows the game framework that included in the learning design. The main idea of the Spinning Soccer Game is using their habit when they played it by spin the pawn to kick the ball and make them realize that when they spin it. They tried to spin it in the most efficient way, which means spin it through the shortest rotating track that also the angular measure of the formed angle between
the pawn and the ball. Game used based from the results’ analysis that didactical games was an effective way of mathematics’ teaching. Didactical games have an effect in non-cognitive area such as feelings, motivation and behaviour, and cognitive area. Didactical games possible main usage areas were including discovering of new knowledge, exercising and fastening of teaching, and development of thinking and applying of knowledge [8]. Using games as the enrichment of the didactical design gives hope for the development of students’ competencies in understanding the angle in geometry concept.

Figure 6. The second framework in didactical design

Figure 6 shows the important role of using the angular measure concept to determine the angle between two intersection objects by finding the shortest rotating track between the two objects and can be used to compose the procedure in the didactical design second framework. Realistic tools such as flashlight to help the orthogonal projection, modelling compound to form a point, stick as line (including segment and ray), and thin stiff paper as plane.

Considering the learning obstacle findings, the observation results from observing 26 class meetings of mathematics learning from different classroom, and the problem-solving continuum, most students classified as ‘Naive’ [9]. This considerations implies a prediction of responses that the students responses are mostly in the form where they always need a stimulus but not a direct stimulus in every step of the didactical design. The stimulus should be in the form of Socratic Questioning as the reinforcement for their thinking processes in order to develop their competencies as the anticipation of their responses. 3D software such as Geogebra and Cabri to develop student’s spatial ability with the addition of teaching the angle in geometry preconditions.

Based from the formed learning trajectory, frameworks, and student’s responses’ prediction and anticipation, the didactical design divided into four part. First part is to learn the basic concepts i.e. angle, angular measure as the most essence, line, segment, ray, and plane. Second to fourth part is to find the procedure to determine angle between different objects (lines, line and plane, planes), evaluate it, and using it. During the implementation, the first part of the didactical design that begin with remembering the definition and what angle mean and then finding the difference between angle and angular measure by playing the Spinning Soccer Game (Figure 5) went very well and effective. The students did not need so many stimulus in every step in this first part. The implementation of second to fourth part of the didactical design did not go really well. The second to fourth part was a big challenge for them since they did not have any habit and experience in constructing concept, procedure, and connection, this habit of them was really a big wall for the didactical design to go really well and already affect their belief in mathematics as calculating and equations. Even the learning activity was a big new challenge for students, during the processes, they look curious and try to understand it in the way they could do.

3.3. Student’s Competencies Development

The development of student’s competencies after the implementation was not very significant, but the formed design clearly shows the ability to develop it. During the process in using the formed
procedure, most students understand the used concept very well and able to use and connect it during the activity but not so flawless. The way they think to understand the concept, improve as the activity goes on and it indicates that their conceptual understanding improved very well.

Students’ procedural fluency and strategic competence well improved by using the formed learning design. Students can make a representation, thinking of strategy, and solving with more than one solution after they quite flexibly used the formed procedure. Their thought in mathematical reasons did not meet the adaptive reasoning’s indicators, just a sign that they began to meet the indicators.

Student’s productive disposition clearly visible during implementation the formed design and well improved. Students’ persistence clearly increased during the learning and problem solving process and they began to realize that this topic was useful to those who would study architecture or when they just want to design something, they said ‘Which means Sir, geometry was used when we want to study architecture, art, and design? And building workers too?’ and ‘Sir, it means that when we being taught by Mr Yayat in art lessons, where there were some kind of one, two, three vanishing point as the point of view was geometry as well?’. The didactical design clearly change some part of their belief in mathematics, their habit in learning mathematics also gradually changed. Their behaviours in problem solving also change, most students ‘Naive’ behaviours change to ‘Routine’ behaviours where some few students start to get in the ‘Sophisticated’ behaviours [9].

4. Conclusion
The enrichment in the formed didactical design are changing the main framework of learning angle in geometry, adding a didactical games, realistic tools, 3D software, and list of responses prediction with a correspondence anticipations. The formed design quite capable to overcome the learning obstacle findings, develop student’s competencies and able to change their behaviour in solving geometric problems. The main problem for the capability to develop student’s competencies was their habit in only receiving rather than constructing the topic and the lack of their learning independence. These big obstacles in the process of design’s implementation was quite difficult to passed, so the development of competencies is only up to the level enough. The didactical design was better able to fix student’s habits and change their point of view in learning mathematics. The formed didactical design still need more enrichment and development to be more able to develop student’s competencies more significant. The formed didactical design have enough flexibility to adjust with student’s thinking level ability but still need more improvement to be more flexible and able to develop student’s competencies. Inclusion of geometry software such as GeoGebra and Cabri in it also help students to improve their spatial ability.

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