Universal description of radially excited heavy and light vector mesons

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Abstract

A new qualitative stringlike picture for mesons is proposed which leads to a simple and intuitively clear generalization of linear radial Regge trajectories to the case of massive quarks. The obtained universal relation is successfully tested in the sector of unflavored vector mesons, where many radial excitations are known. Some new predictions are given. Our results suggest that the quark masses can be easily estimated from the spectra of radially excited mesons.

1 Introduction

Starting from early times of the hadron spectroscopy it has been widely believed that the dynamics responsible for the formation of hadrons is more or less universal at all scales where the hadron resonances are observed. The discovery of QCD provided a powerful support for this belief. Unfortunately, QCD is still not amenable to analytical calculation of the hadron spectrum and for this purpose one resorts to simplified dynamical models simulating QCD. In practice, for description of the light and heavy hadrons, one exploits usually different models. The main problem in the hadron spectroscopy, however, is the building of a universal solvable model describing in detail the whole meson or baryon spectrum as a function of quark masses. Despite a great deal of interesting attempts (see, e.g., [1–3]), such a quantitative model has not been constructed.

The question, then, is where should we look for the signs of universality? It is hardly believable that we can observe it in decay processes (in decay width) which depend on the number of quark flavors and on the energy scale. But we may hope to find some interesting manifestations of the universality in the mass spectrum of hadron states. Our hope is based on the observation of approximately linear Regge, $M^2 \sim J$, and linear radial Regge, $M^2 \sim n$, trajectories in the heavy mesons (see discussions in [4]). Here $M$ is the meson mass, $J$ denotes the spin, and $n$ means the radial quantum number enumerating the daughter Regge trajectories. The point is that in
the sector of light mesons, such linear trajectories were predicted long ago by the Veneziano dual models and by the hadron string models. The experimental verification appeared much later. The first (and still the only) phenomenological evidence came from the combined analysis of experimental data on proton-antiproton annihilation obtained by the Crystal Barrel Collaboration [5]. The most of light nonstrange mesons discovered in this analysis are still among not confirmed states in the compilation of Particle Data Group [6]. However, according to the review [7], the existence of many of the discovered resonances is quite secure.

The spectrum of light mesons enriched by the numerous states extracted from the Crystal Barrel data has been extensively used in many models for fitting the model parameters and for comparison of theoretical predictions with the real phenomenology. The deviations from linearity in the light mesons were studied within the framework of large-$N_c$ QCD sum rules [8] and in the holographic approach [9]. In the heavy sector, such deviations were analyzed in Ref. [4]. Concerning the recent progress in understanding the global spectral behavior, it was proposed that the spectrum of light nonstrange mesons possesses a large degeneracy of the kind $M^2 \sim J + n$ [10]. Later it was argued that the real degeneracy has the form $M^2 \sim L + n$, where $L$ is the orbital momentum of the valent quark-antiquark pair [11]. This observation was also independently made in Refs. [12, 13]. Physically the degeneracy in question means the existence of a "principal" quantum number in light mesons [14] which coincides with that of the hydrogen atom. The corresponding hydrogenlike classification of meson states was developed in Ref. [15]. The discussions of some further consequences are given in [16]. Recently an alternative analysis gave the averaged spectrum $M^2 \sim J + 1.23n$ [17]. Many illuminating discussions concerning the analysis [17] are contained in Refs. [18, 19].

The extension of these discussions to the heavy mesons would be inevitably very speculative because the variety of established states in this sector is much more restricted. First of all, the highest observed spin of heavy mesons is $J = 2$ while that of the light mesons is $J = 6$. Second, the radial excitations are usually poorly known. The only exception represents the case of unflavored vector states. These resonances have quantum numbers of the photon and therefore are intensively produced in the $e^+e^-$ annihilation. Since the mechanism of creation of the vector mesons in the $e^+e^-$ annihilation is expected to be identical for any flavor and abundant experimental data are available, the sector of unflavored vector mesons is an appropriate place for analyzing the manifestations of universality of the strong interactions in the resonance formation.

In the present paper, we study the spectroscopic universality among the
vector mesons with hidden flavor — the $\varphi$, $\psi$, $\Upsilon$ mesons and their analogues in the sector of $u,d$ quarks — $\omega$ mesons. The paper is organized as follows. The available data on the unflavored vector mesons are summarized in Section 2. In Section 3, we recall a simple string description of the light mesons which leads to the linear radial Regge trajectories. A straightforward inclusion of heavy quarks into this description results in a strong nonlinearity of trajectories. We propose an alternative model based on static quarks. The obtained spectrum is fitted in Section 4 and the results are discussed. Section 5 concludes our analysis.

2 The vector spectrum

The masses of known unflavored vector mesons are given in Table 1. According to the quark model, the vector two-quark states can represent the $S$ or $D$-wave resonances. We tried to keep the typical assignment to the $S$ or $D$ states accepted in the literature.

Some doubtful states are omitted. The $\rho(1570)$ is not included since it seems to be a OZI-violating decay mode of $\rho(1700)$ (see the discussions on this state in Particle Data [6]). We omit $\omega(2330)$ because it likely represents $\omega(2290)$ obtained by other collaboration (in Table 1, we use Bugg’s analyses of the Crystal Barrel data [6]). We also exclude $\Upsilon(11020)$ whose assignment is uncertain (its electromagnetic coupling is suspiciously small in comparison with the coupling of $\Upsilon(10860)$ thus indicating on a strong $D$-wave admixture in this resonance).

The $\phi(2175)$ is regarded as the 3rd radial excitation of $\phi(1020)$ since in this case we have a natural mass splitting between the 1st and 3rd excitations, about 500 MeV (a natural splitting between the 1st and 2nd excitations lies...
Table 2: The radial Regge trajectories (1) in Figs.(1a) and (1b).

| $M_n^2$ | $a(n + b)$ |
|---------|------------|
| $M_\psi^2$ | $3.26(n + 3.03)$ |
| $M_\Upsilon^2$ | $6.90(n + 13.28)$ |

near 300–350 MeV). This assignment will be also justified by our fits in Section 4.

The reliable $D$-wave $\phi$ and $\Upsilon$ are not known. For this reason we will look for a unified description of the $S$-wave sector only. The established $S$-wave radial spectra of $\psi$ and $\Upsilon$ are richer than the spectra of light vector mesons. They are displayed in Figs.(1a) and (1b). These spectra reveal a remarkable Regge behavior

$$M_n^2 = a(n + b),$$

which is very similar to the one observed in the light nonstrange mesons [5,7]. However, the fitted slope $a$ and intercept $b$ are much larger than those in the light mesons [5,7], see Table 2. While for the intercept this looks natural, the slope is proportional to the tension of hadron string (or energy per unit length in the potential models with linearly rising confinement potential) and should have a value of about 1 GeV$^2$.

Below we motivate a linear parametrization for the whole spectrum of unflavored vector mesons with universal parameters $a$ and $b$. The only varying parameter will be the quark mass.
3 Hadron strings

It is instructive to see how the relation (1) emerges in the stringlike models. There exist plenty of models of this kind in the literature but some common basic steps can be extracted and exposed in a simplified manner. One assumes that a massless quark-antiquark pair (produced, e.g., in the $e^+e^-$-annihilation) moves back to back creating an elongated flux-tube of chromoelectric field which one treats as a string. The energy of the system (the meson mass) is

$$M = 2p + \sigma r,$$

where $p$ denotes the quark momentum, $r$ is the distance between the quark and antiquark, and $\sigma$ represents a constant string tension. One assumes also that the quarks perform the oscillatory motion inside the flux-tube and that the semiclassical Bohr-Sommerfeld quantization condition can be applied to this motion,

$$\int_0^l pdr = \pi(n + b), \quad n = 0, 1, 2, \ldots$$

(3)

Here $l$ is the maximal quark-antiquark separation and the constant $b$ depends on the boundary conditions. Substituting the momentum $p$ from (2) to (3) and using the definition $\sigma = M/l$ one arrives at the expression

$$M_n^2 = 4\pi\sigma(n + b),$$

(4)

which has the form of (1).

The massive quarks are introduced via the replacement

$$p \rightarrow \sqrt{p^2 + m^2}.$$  

(5)

If we make the substitution (5) in the simplistic model above (for two quarks of equal mass $m$) we obtain

$$M_n \sqrt{M_n^2 - 4m^2} + 4m^2 \ln \frac{M_n - \sqrt{M_n^2 - 4m^2}}{2m} = 4\pi\sigma(n + b).$$

(6)

In the relativistic limit, $M_n \gg m$, the relation (6) reduces to (4) while in the nonrelativistic one, $M_n - 2m \ll 2m$, the relation (4) yields the spectrum of linearly rising potential, $M_n \sim n^{2/3}$. In the general case of polynomially rising potential $V \sim r^\alpha$ ($\alpha > 0$) one has the spectrum $M_n \sim n^{2/\alpha + 2}$ at large enough $n$ [22]. The spectrum of $\psi$ and $\Upsilon$-mesons cannot be well fitted by the relation (3) with universal parameters $\sigma$ and $b$ because of strong nonlinearity at typical masses of heavy quarks. This nonlinearity originates from the substitution (5). The use of this substitution is common in the stringlike [23] and
semirelativistic potential models \cite{2,3,24,25} as well as in models based on the Bethe-Salpeter equation \cite{26}. All these models share the nonlinearity at large \( m \), although the form of this nonlinearity is model-dependent. In essence, the quarks are treated as if they were almost free on-shell particles. If we want to have a Regge-like behavior for any quark mass, the substitution (5) should not be exploited.

In reality, even the light quarks are always massive. Hence, we can choose a reference frame where one of quarks is at rest. Consider the large-\( N_c \) limit. The hadrons do not decay and represent bound states. We assume that the meson bound state corresponds to the situation when two quarks are situated at a fixed relative distance. They are bound by exchange of some massless particle. The physical interpretation for this particle may be different — it could be the massless pion or gluon. The relation (2) is then replaced by

\[ M = m_1 + m_2 + p + \sigma r. \]

(7)

Here \( p \) is the momentum of the particle mediator, \( m_1 \) and \( m_2 \) are quark masses, \( m_1 = m_2 \equiv m \) for the unflavored mesons. Let us apply the quantization condition (3) to the exchanged particle. Taking into account that now \( \sigma = (M - 2m)/l \), this will result in the mass relation

\[ (M_n - 2m)^2 = 2\pi \sigma (n + b). \]

(8)

The slope \( 2\pi \sigma \) in (8) coincides with the slope of rotating Nambu string \cite{27}. Thus the model is able to explain the large degeneracy \( M^2 \sim J + n \) discussed in Introduction. A more important point for us is that the spectrum has the Regge-like form (1) with universal parameters \( a = 2\pi \sigma \) and \( b \). The only varying parameter is the quark mass \( m \). In the next Section, we estimate the free parameters from the experimental data.

4 Discussions, fits, and predictions

Our main result, Eq. (8), was deduced in the large-\( N_c \) limit of QCD. This relation predicts infinitely many excited states as expected in the large-\( N_c \) world. In the real \( N_c = 3 \) world, the hadron string breaks down at some point (for sufficiently large radial number); hence, the relation (8) should have a limit of application. The rigorous derivation of this limit is a difficult task. But we can propose a qualitative estimate. The onset of the continuum spectrum may be identified with the point where the total widths of neighboring resonances overlap almost completely. Let us accept the following criterion:

\[ \text{6} \]
If since some excitation number \( k \), the half-width is comparable with the mass difference between the \( k \)th resonance and the \((k + 1)\)th one,

\[
\Gamma_k/2 \approx \sqrt{a(k + 1 + b)} - \sqrt{a(k + b)},
\]

the \((k + 1)\)th state is indistinguishable from continuum. The total decay width (inverse life-time) is proportional to the probability of creation of extra quark-antiquark pair along the string which, in turn, is proportional to the string length \( l \). As long as \( l \) is proportional to the meson mass in this scheme, \( M_k = \sigma l_k \), we conclude \[28\]

\[
\Gamma_k = c_k \sqrt{a(k + b)}. \tag{10}
\]

The averaged empirical value of constants \( c_k \) for the excited light nonstrange mesons is \( c \equiv \langle c_k \rangle \approx 0.1 \) (see, e.g., the second reference in \[10\]). Let us insert \[10\] into \[9\] and treat \( c \) as a small parameter. Keeping only linear in \( c \) contribution, we obtain the estimate

\[
k \approx \left[ \frac{1}{c} - b \right], \tag{11}
\]

where the square brackets denote the integer part. Since \( c = \mathcal{O}(1/N_c) \), the number of observable resonances is \( \mathcal{O}(N_c) \). For the light vector states, the constant \( c \) lies in the interval \( c = 0.1 - 0.2 \) while \( 0 \lesssim b \lesssim 1 \) (see, e.g., the Fit A in Table 3). Then the number of radially excited states described by the relation \[8\] ranges from \( k = 4 \) (pessimistic estimate) to \( k = 9 \) (optimistic estimate).

After this caveat, we should test the Eq. \[8\] using the available experimental data. A traditional problem emerging at this point is the choice of data; in our case this is determining which states should be treated as reliable \( S \)-wave vector resonances. Another source of uncertainty comes from the fact that formally we should use the values of meson masses in the large-\( N_c \) limit of QCD which are unknown. The hadron masses presented in the PDG \[6\] refer to \( N_c = 3 \) world. They have uncertainty falling into the interval \( M \pm \Gamma/2 \) since the resonance mass \( M \) depends on the reaction channel and on the method used to extract it \[17\]. The lattice calculations of meson masses at large values of \( N_c \) could help but a little is known on this subject. The lattice simulations of Ref. \[29\] resulted in the following interpolating formula for the \( \rho \)-meson mass in the chiral limit: \( m_\rho/\sqrt{\sigma} = 1.54(1) + 0.92(21)/N_c^2 \), where \( \sigma \) represents the universal string tension measured on the lattice. This relation seems to predict that the \( \rho \)-meson mass diminishes by about 50 MeV when going to \( N_c = \infty \) world plus small chiral corrections. The large-\( N_c \) studies in the unitarized chiral perturbation theory \[30\] led to the opposite effect — the
enhancement of $m_\rho$ by 40-60 MeV in the large-$N_c$ limit. But both estimates fall into the uncertainty interval $m_\rho \pm \Gamma_\rho/2$ ($\Gamma_\rho = 147.8(9)$ MeV \[^6\]).

Since our analysis is rather qualitative and we do not pretend to the accuracy better than the accuracy of large-$N_c$ limit in QCD, it is enough to present one reasonable fit based on the relation \[^5\]. We have tried many possibilities and assumptions. Below two typical fits are provided. In both cases the spectrum of $\omega$ mesons is used for the light nonstrange vector resonances ($\omega$ is the direct analogue of unflavored quarkonia as the flavor part of its wave function is symmetric). Within the approximate relation \[^5\], the $\omega$ and $\rho$ are degenerate. Phenomenologically this is consistent with the accuracy of the large-$N_c$ limit.

In the first case (Fit A), we assume the $M^2 \sim L + n$ degeneracy in the light nonstrange mesons \[^11\]. According to the analysis of the first reference in \[^11\], the phenomenological spectrum is (in GeV^2)

\[ M^2(L, n) \approx 1.10(L + n + 0.62). \]  

The given degeneracy allows us to replace the masses of unknown $S$-wave $\omega$ mesons in Table 1 by the masses of corresponding known $D$-wave states. The masses of $u$ and $d$ quarks are set to zero (the chiral limit is accepted for simplicity). It is of course implied that in reality the $u$ and $d$ quarks have small masses (this was assumed in the derivation of relation \[^5\]), we just neglect the $\mathcal{O}(m_{u,d})$ shifts in the meson masses which are expected to be much smaller than the errors induced by the departure from the large-$N_c$ limit of QCD.

The results of global fit are displayed in Table 3. We see a striking agreement of the obtained parameters both with the phenomenological spectrum \[^12\] and with the current-quark masses in Particle Data \[^6\]: $m_s = 95(5)$ MeV, $m_c = 1.275(25)$ GeV, $m_b = 4.18(3)$ GeV. It should be taken into account that these masses are given at a scale 2 GeV. Since the typical scale of excited $\phi$ mesons is rather 1 GeV per quark, another value of $m_s$ should be used for comparison: $m_s(1$ GeV) $\approx 1.35m_s(2$ GeV) $\approx 128(7)$ GeV. It should be noted also that if we ascribe $\phi(2175)$ to the 2nd radial excitation of $\phi(1020)$, the global fit is worse.

In principle, the demonstration of this fit is sufficient to justify the viability of the relation \[^5\]. But we would like to show an alternative interpretation for the parameter $m$ in \[^5\]. Let us leave the assumption about large degeneracy \[^12\] in the light mesons and determine the parameters $a$, $b$, $m_c$, $m_b$ from the sector of heavy quarkonia where the radial spectrum is better established. After that we obtain the parameters $m_{u,d}$ and $m_s$ from the best fit to the known $S$-wave $\omega$ and $\phi$ mesons. The results are given in Table 3.
Table 3: The quark masses (in GeV), the slope $a$ (in GeV$^2$) and the dimensionless intercept parameter $b$ in the relation (8).

|        | Fit A | Fit B |
|--------|-------|-------|
| $m_{u,d}$ | 0     | 0.28(4) |
| $m_s$    | 0.12(8) | 0.40(3) |
| $m_c$    | 1.20(7) | 1.48(5) |
| $m_b$    | 4.32(6) | 4.59(5) |
| $a$      | 1.06(3) | 0.67(7) |
| $b$      | 0.63(7) | 0.00(0) |

(Fit B). According to this fit, it looks natural to interpret the parameter $m$ in (8) as the mass of constituent quark. Indeed, the constituent mass of $u,d$ quarks lies around 300 MeV, the difference $m_s - m_{u,d} = 120$ MeV is close to the current mass of the strange quark as it is expected in the potential models\footnote{This is not the case if we ascribe $\phi(2175)$ to the 2nd radial excitation of $\phi(1020)$. We would then obtain $m_s = 0.49(3)$ GeV. The given observation is another one justification for our interpretation of $\phi(2175)$.}, the so-called 1S-mass of $b$-quark is 4.66(3) GeV \cite{6}. It should be noted that masses of constituent quarks can be quite different in different potential models. For instance, $m_{u,d} = 220$ MeV in Ref. \cite{3} and $m_{u,d} = 313$ MeV in \cite{25}.

If we relax the condition $m_{u,d} = 0$ in the Fit A and take $m_{u,d}$ as a free parameter, the ensuing fits resemble the Fit B, the parameter $m$ acquires then typical values of constituent masses in the potential models.

A graphical comparison of the relation (8) with the experimental spectra is plotted in Figs. (2a) and (2b). Some meson masses predicted by the Fits A and B are demonstrated in Tables 4–7. The measure of quality of the fits is $\chi^2 = \sum_n \left( \frac{M_n^2 - M_{n,\exp}^2}{M_{n,\exp}^2} \right)^2$.

As we have mentioned, other assumptions for fitting the experimental data are possible. For example, one can notice that the ground state in Figs. (1a) and (1b) lies systematically below the linear trajectory. This observation may motivate the exclusion of the ground state in our analysis. However, excluding the ground state does not lead to a noticeable improvement of the global fits and predictions. In any case, the relation (8) remains reasonable from the phenomenological point of view, albeit parametrically dependent on the choice of data.

The previous version of our analysis, Ref. \cite{31}, motivated the authors of Ref. \cite{32} to study the universality of radial meson trajectories in a different...
Table 4: The theoretical and experimental masses of $S$-wave $\omega$ mesons (in GeV) [6]. The experimental error is not displayed if it is less than 1 MeV. The question mark stays at the states whose assignment is questionable.

| $M_\omega(n)$ \ $n$ | 0       | 1       | 2       | 3       | 4       | 5       | $10^3 \chi^2$ |
|---------------------|---------|---------|---------|---------|---------|---------|---------------|
| Fit A               | 0.82(5) | 1.32(3) | 1.67(3) | 1.96(3) | 2.22(3) | 2.44(3) | 39            |
| Fit B               | 0.56(8) | 1.38(9) | 1.72(10)| 1.98(11)| 2.20(12)| 2.39(12)| 249           |
| Experiment          | 0.783   | 1.425(25)| 1.67(30)$^?$| 1.960(25)$^?$| 2.205(30)| 2.330(30)$^?$|               |

Table 5: The theoretical and experimental masses of $S$-wave $\phi$ mesons (in GeV) [6].

| $M_\phi(n)$ \ $n$ | 0       | 1       | 2       | 3       | 4       | 5       | $10^3 \chi^2$ |
|---------------------|---------|---------|---------|---------|---------|---------|---------------|
| Fit A               | 1.06(16)| 1.56(16)| 1.91(16)| 2.20(16)| 2.46(16)| 2.68(16)| 26            |
| Fit B               | 0.80(6) | 1.62(7) | 1.96(8) | 2.22(10)| 2.44(10)| 2.63(11)| 155           |
| Experiment          | 1.020   | 1.680(20)| —       | 2.175(15)| —       | —       |               |

Table 6: The theoretical and experimental masses of $S$-wave $\psi$ mesons (in GeV) [6].

| $M_\psi(n)$ \ $n$ | 0       | 1       | 2       | 3       | 4       | 5       | $10^3 \chi^2$ |
|---------------------|---------|---------|---------|---------|---------|---------|---------------|
| Fit A               | 3.21(15)| 3.71(14)| 4.06(14)| 4.36(14)| 4.61(14)| 4.84(14)| 6             |
| Fit B               | 2.96(10)| 3.78(11)| 4.12(12)| 4.38(12)| 4.60(13)| 4.79(14)| 12            |
| Experiment          | 3.097   | 3.686   | 4.039(1)| 4.361(9)(9)| 4.634(8)(8)| —       |               |

Table 7: The theoretical and experimental masses of $S$-wave $\Upsilon$ mesons (in GeV) [6].

| $M_\Upsilon(n)$ \ $n$ | 0       | 1       | 2       | 3       | 4       | 5       | $10^3 \chi^2$ |
|------------------------|---------|---------|---------|---------|---------|---------|---------------|
| Fit A                  | 9.46(13)| 9.96(13)| 10.31(13)| 10.61(13)| 10.86(13)| 11.09(13)| 0.4           |
| Fit B                  | 9.18(10)| 10.00(11)| 10.34(12)| 10.58(12)| 10.82(13)| 11.01(14)| 3.5           |
| Experiment             | 9.460   | 10.023  | 10.355(1)| 10.579(1)| 10.876(11)| 11.019(8) |               |
large-\(N_c\) framework. The best fit was obtained after exclusion of the ground states. The results turned out to be very close to our Fit A except the intercept parameter \(b\) \((b = 0.93(50)\) in Ref. [32]). Our aim was not to find the best fit but rather to demonstrate the viability of the phenomenological relation [5] within the accuracy of the large-\(N_c\) approximation. The Fit A meets our goal. Unfortunately, many details are missing in the short report [32] that hampers a comparison of our analysis with that of Ref. [32].

The mass of the ground state is seriously underestimated in the Fit B where this mass is the sum of constituent quark masses. This makes the Fit B worse than Fit A if we use our \(\chi^2\) criterion. However, if the ground states are excluded from \(\chi^2\) then \(\chi^2\) is less (by a factor of 3) for the Fit B in all cases except the \(\psi\) mesons.

Concluding our discussions, we make some remarks on the predictions displayed in Tables 4–7. Our fits indicate the existence of a new \(\phi\) meson in the mass interval 1900–2000 MeV. The resonance \(\psi(4361)\) observed by Belle Collaboration [20] seems to be a better candidate for the role of the 3rd radial excitation of the \(J/\psi\)-meson than the resonance \(\psi(4415)\) (although the latter was used as an input in our fits). It looks natural to assign the next excitation to \(\psi(4634)\) also observed by Belle Collaboration some time ago [21] and interpret \(\psi(4415)\) as a \(D\)-wave state. This assignment agrees with the results of Ref. [25]. The \(\Upsilon(11020)\) is compatible with the role of the 5th excitation of \(\Upsilon(9460)\).

5 Conclusions

We put forward a new generalization of linear radial Regge trajectories to the case of massive quarks. Within this generalization, the form of contribution to the meson masses due to confinement is universal for any quarkonia. We demonstrated that the ensuing mass relation [5] can be well consistent with
the experimental data on the unflavored vector mesons. In addition, it leads
to some interesting predictions in the meson spectroscopy. Although our
considerations were simple and did not take into account many effects (such
as the relativistic mixing of $S$ and $D$ wave states resulting in some mass
shifts [33] and the mass shifts caused by the transition from the large-$N_c$ limit
to the real $N_c = 3$ world) the quality of final fits is comparable with typical
results of semirelativistic potential models and other technically nontrivial
approaches.

A natural question emerging after our analysis is whether the relation (8)
can be extended to other types of mesons? Unfortunately, the available
experimental data are too scarce for making any definite conclusions. Among
the possible extensions of (8) are

\[(M_n - m_1 - m_2)^2 = a(n + x + b), \tag{13}\]

where we may have $x = \alpha L$ or $x = \alpha J$ (here $L$ and $J$ mean the orbital
momentum of valent quarks and total spin). The constant $\alpha$ should be
fixed from the phenomenology. An intriguing possibility $\alpha = 1$ would lead
to a large degeneracy observed in the light nonstrange mesons. Needless to
say, establishing a consistent form for the relation (13) is very interesting
since it is able to give many predictions for new experiments in the hadron
spectroscopy. This problem is left for future work.

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