Center flux correlation in SU(2) Yang-Mills theory

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By using the method of center projection the center vortex part of the gauge field is isolated and its propagator is evaluated in the center Landau gauge, which minimizes the open 3-dimensional Dirac volumes of non-trivial center links bounded by the closed 2-dimensional center vortex surfaces. The center field propagator is found to dominate the gluon propagator (in Landau gauge) in the low momentum regime and to give rise to an OPE correction to the latter of $\sqrt{\sigma}/p^3$. The screening mass of the center vortex field vanishes above the critical temperature of the deconfinement phase transition, which naturally explains the second order nature of this transition consistent with the vortex picture. Finally, the ghost propagator of maximal center gauge is found to be infrared finite and thus shows that the coset fields play no role for confinement.

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One of the fundamental problems of particle physics is the understanding of confinement of quarks and gluons in QCD. Although confinement has not yet been thoroughly understood, several pictures of confinement have been developed, which received strong support by lattice calculations in recent years. Among these are the dual Meissner effect and the center vortex condensation (for a recent review see [1]). In particular, the center vortex picture is quite appealing: center vortices identified by the method of center projection [2] are physical in the sense of showing the proper scaling [3]. Moreover, the deconfinement phase transition at finite temperature emerges as depercolation transition [4,5,6]. Center vortices also provide a geometric interpretation of topological charge in terms of intersection and writhing of vortex surfaces or loops [7,8].

A different confinement mechanism was proposed by Gribov [9] and further elaborated by Zwanziger [10]. This mechanism is based on the infrared dominance of the field configurations near the Gribov horizon, which gives rise to an infrared singular ghost propagator, which is considered to be a signal of confinement [11]. In Landau gauge this infrared singularity disappears, when center vortices are eliminated from the Yang-Mills ensemble [12]. Since the infrared singularities are caused by field configurations on the Gribov horizon, one expects, that center vortices are on the Gribov horizon, which indeed can be shown to be the case [13]. The results of [12] and [13] show that, in Landau and Coulomb gauge, the center vortices are not only on the Gribov horizon, but they also dominate the infrared physics. This suggests, that the center vortices may be the confiner in any gauge, which is very plausible since center vortices can, in principle, be defined in a gauge invariant way and after all confinement is a gauge independent phenomenon.

In this paper we will further elaborate on the connection of the two confinement scenarios described above, i.e. on the interplay between center vortices and ghosts. Using the method of center projection [2], we separate the confining center degrees of freedom from the remaining (non-confining) coset degrees of freedom and calculate the associate propagators as well as the corresponding ghost propagator. We will find, that in contrast to the familiar Landau gauge, in the maximum center gauge the ghost propagator is infrared finite and thus shows no signal of confinement. This result is not surprising since in this gauge the Faddeev-Popov operator does not feel the center part of the gauge field. The confining center degrees of freedom constitute the center vortex field. We calculate its propagator which is carefully extrapolated to the continuum limit. We will find, that the center field propagator dominates the infrared behavior of the gluon propagator, while the ultraviolet behavior of the latter is exclusively determined by the coset field propagator. However, the center vortex field gives rise to an correction to the gluon propagator of the form $\sqrt{\sigma}/p^3$, which indicates its relevance in the context of the operator product expansion.

Although the lattice provides a gauge invariant approach to Yang-Mills theory the transition to the continuum theory is facilitated by using a gauge, in which the fields are smooth. The prototype of such a gauge is the well known Landau gauge

$$\sum_{x,\mu} tr U_\mu^\Omega (x) \rightarrow \max ,$$

where $U_\mu^\Omega (x) = \Omega(x) U_\mu(x) \Omega(x+\hat{\mu})$ is the gauge transform of the link variable $U_\mu(x)$. This gauge brings the links as close as possible to the unit element of the gauge group and one therefore expects, that the gauge fields $U_\mu(x)$ will be smooth in this gauge. For smooth links near the unity we can extract the continuum gauge field $A_\mu(x)$ defined by $(a \rightarrow 0$, $a$ the lattice spacing)

$$U_\mu(x) = e^{i a A_\mu(x)}$$

in the standard fashion by Taylor expansion. In this (naive) continuum limit the gauge condition [14] reduces
to the usual (continuum) Landau gauge $\partial_\mu A_\mu = 0$. In this paper we will use various modifications of the Landau gauge to identify the center vortex content and the remaining coset part of the gauge field as will be detailed further below.

To identify the center vortex content of a gauge field, we use the method of center projection [2], which is based on the so-called maximum center gauge defined by

$$\sum_{x,\mu} \left[ \text{tr} U^{\Omega}_\mu(x) \right]^2 \rightarrow \text{max}, \quad (3)$$

This gauge fixes the gauge group only up to center gauge transformations, i.e. it fixes only the coset $SU(2)/Z(2) = SO(3)$ and brings a given link $U_\mu(x)$ as close as possible to a center element (±1 for $SU(2)$). Once, this gauge is implemented, center projection implies to replace a link $U_\mu(x)$ by its closest center element, which is given by

$$Z_\mu(x) = \text{sign tr} U^{\Omega}_\mu(x). \quad (4)$$

The center projected configurations $Z_\mu(x)$ form 3-dimensional volumes of links $Z_\mu(x) = -1$, the closed boundaries of which represent the center vortices. These vortex surfaces are formed by dual plaquettes $P_{\mu\nu}(x) = -1$. We separate the center projected vortices from the original gauge fields, by writing [14]

$$U^{\Omega}_\mu(x) = Z_\mu(x)U_\mu(x). \quad (5)$$

The maximal center gauge is known to be just the minimal Landau gauge for the adjoint representation, which does not feel the center. Thus, the links $\bar{U}_\mu(x)$, defined in the MCG, should be sufficiently closed to 1 and we can extract a continuum field $\bar{A}_\mu(x)$ in the standard fashion $\bar{U}_\mu(x) = \exp (i A_\mu(x))$. With the parameterization $U^{(3)}_\mu(x) = a_\mu^0(x) + i\bar{a}_\mu(x)^\tau$, we find $a A^b_\mu(x) = 2 a^0_\mu(x) a^b_\mu(x)^b$. In view of [3] we interpret the $\bar{A}_\mu(x)$ as the gluonic radiative fluctuations around the center vortex “background”.

Since $\bar{U}_\mu$ satisfies the adjoint Landau gauge, the radiation field $\bar{A}_\mu(x)$ is transversal and its propagator can be expressed as

$$\langle \bar{A}_\mu(p)\bar{A}_\nu(-p) \rangle = \left( \frac{\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}}{p^2} \right) F_{\text{ad}}(p^2), \quad (6)$$

where $F_{\text{ad}}(p^2)$ is the adjoint gluon form factor.

We have performed a large scale study of SU(2) lattice gauge theory using $16^4$ and $24^4$ lattices and $\beta$ values of the Wilson action in the range $\beta \in [2.15, 2.5]$. The result for the (renormalized) adjoint gluon form factor is shown in figure[4] (details of the numerical approach will be published elsewhere). For large momenta this quantity reproduces the perturbative result for the full gluon form factor in Landau gauge. Like the full gluon form factor, $F_{\text{ad}}(p^2)$ vanishes for $p \rightarrow 0$, signaling a mass gap in the excitation spectrum of $\bar{A}_\mu(x)$. Finally, $F_{\text{ad}}(p^2)$ deviates essentially from the gluon form factor of Minimal Landau Gauge in the intermediate momentum regime.

It has been well established by now, that for SU(2) the center projected vortices, defined by the $Z_\mu(x)$ [14], basically produce the full string tension, while the radiative coset field $\bar{U}_\mu$ (or $\bar{A}_\mu$) does not contribute to confinement.
Since, the MCG condition \( \Sigma \) depends, only on the coset field \( \tilde{U}_\mu \), we expect the corresponding ghost propagator to show no signal of confinement. The Faddeev-Popov operator of the MCG is given by

\[
M^{ab}(x, y) = \sum_\mu \left\{ - \left[ f^{ab}(x - \mu) + f^{ab}(x) \right] \delta(x, y) + \left[ f^{ab}(x - \mu) - g^{ab}(x - \mu) \right] \delta(x - \mu, y) + \left[ f^{ab}(x) + g^{ab}(x) \right] \delta(x + \mu, y) \right\},
\]

where

\[
f^{ab}(x) = (a_\mu^0(x))^2 \delta^{ab} - a_\mu^a(x)a_{\mu}^b(x), \]
\[
g^{ab}(x) = a_\mu^0(x)e^{abc}a_{\mu}^c(x).
\]

Figure 2 shows the ghost form factor \( J(p^2) \) of the MCG defined by

\[
\left\langle M^{-1}\right\rangle^{ab}(p) = \delta^{ab}J(p^2)/p^2 .
\]

At high momenta it approaches the ghost form factor of the Minimal Landau Gauge but differs drastically from the latter in the infrared: While the ghost form factor of the Minimal Landau gauge is infrared divergent (what is considered as a signal of confinement), the one of MCG, which does not feel the center, seems to be infrared finite. This is consistent with the result obtained in [12], where it was shown, that the infrared divergent behavior of the ghost form factor in minimal Landau gauge disappears, when center vortices are removed from the Yang-Mills ensemble.

We also use the analog of the Landau gauge for the center projected fields

\[
\sum_{x, \mu} Z_\mu(x) \xrightarrow{\Omega_2} \text{max} ,
\]

where \( \Omega_2 \) are \( Z_2 \) gauge transformations. Each link \( Z_\mu(x) \) defines an elementary cube on the dual lattice and the total number of non-trivial center links \( Z_\mu(x) = -1 \) defines open hypersurfaces \( \Sigma \) bounded by closed center vortex surfaces \( \partial \Sigma \). The Landau center gauge condition, \( \partial \Sigma \), minimizes the number of \( Z_\mu(x) = -1 \) links and thus minimizes the volume of the open 3-dimensional hypersurfaces \( \Sigma \). Since, the minimal open hypersurfaces are completely determined by their boundary \( \partial \Sigma \), it is expected that they scale properly towards the continuum limit, if the vortex surfaces \( \partial \Sigma \) do. Let \( \mathcal{P} \) be the probability that a given link element \( Z_\mu(x) \) is negative. If the ratio between the minimal cubic volume and the spacetime volume, i.e.,

\[
\frac{\mathcal{P}}{N^3 N_t a^4} = \frac{6 \mathcal{P}}{a} ,
\]

is a physical quantity in the continuum limit \( a \to 0 \), the probability must scale according \( \mathcal{P} = \kappa a \), with \( \kappa \) being independent of \( a \). This is indeed confirmed by lattice calculations [14]. Our numerical calculations yield \( \kappa \approx 0.26(1) \sqrt{\sigma} \).

Consider now the connected center field correlation function

\[
a^2 c_{\mu\nu}(x - y) = \langle Z_\mu(x)Z_\nu(y) \rangle - \langle Z_\mu(x) \rangle \langle Z_\nu(y) \rangle .
\]

The factor \( a^2 \) was introduced for later convenience. Although \( Z_\mu(x) \) are integer valued fields, the function \( c_{\mu\nu}(x - y) \), emerging from an ensemble average, appears to be smooth. Since the center Landau gauge condition \( \Omega_2 \) is equivalent to

\[
\sum_\mu \left[ Z_\mu(x + \hat{\mu}) - Z_\mu(x) \right] = 0 ,
\]
the center field propagator is transverse: \(\partial_\nu \, c_{\mu \nu}(x-y) = 0\). We therefore introduce the center field form factor \(F_{\text{cen}}(p)\) as usual by

\[
C_{\mu \nu}(p) = \left(\delta_{\mu \nu} - \frac{p_\mu p_\nu}{p^2}\right) C(p^2), \quad C(p^2) = \frac{F_{\text{cen}}(p)}{p^2}.
\]

One of our important findings is that the center field correlation function \(C_{\mu \nu}(p)\) is independent of the lattice spacing. The consequences are two-fold: Firstly, the propagator \(C_{\mu \nu}(p)\) is not subjected to wave function renormalization. Secondly, \(C_{\mu \nu}(p)\) behaves as a genuine gluonic correlation function with mass dimension two.

Our numerical results for the propagator \(C_{\mu \nu}\) in momentum space are shown in figure 3. Most striking is that the high momentum tail is well fitted by the power law (see figure 3)

\[
C(p)/4 = \frac{3.7(1) \sqrt{\sigma}}{p^3}, \quad p \geq 2 \text{ GeV}.
\]

This implies that the center field correlator is sub-leading in the high momentum regime compared with the perturbative correlator (see also figure 1). The center field correlator, however, contributes to the operator product corrections. Hence, the center vortex “background” field could serve a natural explanation for the condensates entering the operator product expansion. Finally note that \(F_{\text{cen}}(p^2)\) is enhanced in the low momentum regime, where it has the same shape as the form factor in Minimal Landau gauge (see figure 1).

Let us finally consider the deconfinement phase transition at high temperatures. This transition is well understood in the vortex picture where it appears as vortex depercolation transition [1][2][3]. Since the transition is of 2nd order, it is accompanied by the occurrence of a massless excitation. However, it is known that neither the gluonic mass gap nor the color singlet states hardly change significantly and thus cannot be identified with the excitation that becomes massless at the transition. Because of the success of the vortex picture in describing this transition, one might suspect that the center field correlator contains the desired information. We have therefore studied the correlator \(\sum_x c(t, \vec{x})\) as function of (Euclidean) time \(t\) for several temperatures. Simulations have been carried out using a \(30^3 \times 10\) lattice. \(\beta\) was varied from 1.91 to 2.69 to adjust the temperature. The result is shown in fig. 4. At temperatures \(T\) below the critical temperature \(T_c\) we find an exponential decrease of the correlator \(C(t)\) in accordance with the zero temperature result [17]. For \(T > T_c\), the correlation is compatible with a power-law indicating a vanishing mass gap for the center field propagator.

In conclusions, MCG supplemented with \(Z_2\)-Landau gauge allows for a meaningful identification of the \(Z_2\) center field and \(SU(2)/Z_2\) coset parts of the gluon propagator in the continuum limit. While the center field contribution dominates the infrared behavior of the gluon propagator, it does not contribute to the high energy tail of the latter. Furthermore, the center field propagator embodies the excitations which become massless at the finite temperature \(SU(2)\) deconfinement phase transition. The ghost form factor of the MCG is infrared finite and thus shows that the “adjoint” part does not play a role for confinement.

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