T*: A Heuristic Search Based Algorithm for Motion Planning with Temporal Goals

Danish Khalidi and Indranil Saha

Abstract—Motion planning is the core problem to solve for developing any application involving an autonomous mobile robot. The fundamental motion planning problem involves generating a trajectory for a robot for point-to-point navigation while avoiding obstacles. Heuristic-based search algorithms like A* have been shown to be extremely efficient in solving such planning problems. Recently, there has been an increased interest in specifying complex motion plans using temporal logic. In the state-of-the-art algorithm, the temporal logic motion planning problem is reduced to a graph search problem and Dijkstra’s shortest path algorithm is used to compute the optimal trajectory satisfying the specification.

The A* algorithm when used with a proper heuristic for the distance from the destination can generate an optimal path in a graph efficiently. The primary challenge for using A* algorithm in temporal logic path planning is that there is no notion of a single destination state for the robot. In this thesis, we present a novel motion planning algorithm T* that uses the A* search procedure in temporal logic path planning opportunistically to generate an optimal trajectory satisfying a temporal logic query. Our experimental results demonstrate that T* achieves an order of magnitude improvement over the state-of-the-art algorithm to solve many temporal logic motion planning problems.

I. INTRODUCTION

Motion planning is the core problem in robotics where we design algorithms to enable an autonomous robot to carry out a real world complex task successfully. A basic motion planning task consists of point-to-point navigation while avoiding obstacles and satisfying some constraints. Many methods exist to solve this problem that use graph search algorithms [1] and also sampling based motion planning techniques such as rapidly exploring random trees [2].

Recently, there has been an increased interest in specifying complex motion plans using temporal logic (e.g. [3], [4], [5], [6], [7], [8], [9]). Using temporal logic [10], one can specify requirements that involve temporal relationship between different operations performed by robots. Such requirements arise in many robotic applications, including persistent surveillance [8], assembly planning [11], evacuation [12], search and rescue [13], localization [14], object transportation [15], and formation control [16].

A number of algorithms exist for solving Linear Temporal logic (LTL) motion planning problems [17], [18]. In case of continuous state space and large transition systems representing the workspace of the robot we resort to algorithms that do not focus on optimality of the path, rather they focus on reducing the computation time to find the path. Sampling based motion planning algorithms are faster and compute the LTL motion planning path for continuous state space (e.g. [19], [20], [21]), while for large discrete workspace we can use graph search algorithms (e.g. [22], [8]).

Traditionally, the LTL motion planning problem for the robots with discrete dynamics is reduced to the problem of finding the shortest path in a weighted graph and Dijkstra’s shortest path algorithm is employed to generate an optimal trajectory satisfying an LTL query [22]. However, for a large workspaces and a complex LTL specification, this approach is merely scalable. Heuristics based search algorithms such as A* [23] have been successfully used in solving point to point motion planning problems and is proven to be significantly faster than Dijkstra’s algorithm. The A* algorithm when used with a proper heuristic for distance from the destination node can generate an optimal path in a graph efficiently. A* algorithm have not been used in temporal logic path planning as there is no notion of a single destination state in LTL motion planning. We attempt to use the A* algorithm in LTL path planning opportunistically to generate an optimal trajectory satisfying an LTL query with a significantly improved running time than that of the solution obtained using Dijkstra’s algorithm. Our experimental results demonstrate that in many cases, T* achieves an order of magnitude better computation time than that of the traditional approach to solve LTL motion planning problems.

II. PROBLEM

The input given to the algorithm consists of robot workspace W, the set of robot actions Act and a Linear Temporal Logic Query ϕ.

A. Workspace and Robot Actions and Trajectory

In this work, we assume that the robot operates in a two-dimensional workspace W which we represent as a grid map. The grid divides the workspace in square shaped cells. Each of these cells denote a state in the workspace W which is referenced by its coordinates. Some cells in the grid are marked as obstacles.

We capture the motion of a robot using a set of Actions Act. The robot changes its state in the workspace by performing the actions in Act. An action act ∈ Act is associated with a cost which captures the energy consumption or time delay to execute it.

A robot can move to satisfy a given specification by executing a sequence of actions in Act. The sequence of states followed by the robot is called the trajectory of the robot. The cost of a trajectory is defined as the sum of costs of the actions that are utilized to realize the trajectory.
B. Linear Temporal Logic

The motion planning query in our work is given in terms of formulas written using Linear Temporal Logic (LTL). LTL formulae over the set of atomic propositions $AP$ are formed according to the following grammar [10]:

$$\Phi ::= true \mid a \mid \phi_1 \land \phi_2 \mid \neg \phi \mid X\phi \mid \phi_1 U \phi_2.$$ 

The basic ingredients of an LTL formulae are the atomic propositions, the Boolean connectives like conjunction $\land$, and negation $\neg$, and two temporal operators $X$ (next) and $U$ (until). The semantics of an LTL formula is defined over an infinite trajectory $\sigma$. The trajectory $\sigma$ satisfies a formula $\xi$, if the first state of $\sigma$ satisfies $\xi$. For an LTL formula $\phi$, $X\phi$ is true in a state if $\phi$ holds at the next step. The Until operator $U$ has two operands. The formula $\phi_1 U \phi_2$ denotes that $\phi_1$ must remain true until $\phi_2$ becomes true at some point in future. The other LTL operators that can be derived are $\text{Always}$ $\Box$ and $\text{Eventually}$ $\Diamond$. The formula $\Box \phi$ denotes that the formula $\phi$ has to hold for all the time in the future. The formula $\Diamond \phi$ denotes that the formula $\phi$ has to hold sometime in the future.

C. Problem Description

A trajectory $\sigma$ satisfying an LTL formula $\Phi$ and synthesized using the actions from $Act$ is optimal if there does not exist another trajectory $\sigma'$ satisfying $\Phi$ and synthesized using the same set of actions $Act$ such that $\text{cost}(\sigma') < \text{cost}(\sigma)$.

Given a discrete robot workspace $W$, the set of actions $Act$ of the robot and an LTL query $\Phi$ with the description of the propositions with respect to the states in $W$, our goal is to design an algorithm to compute an optimal trajectory for a single robot to satisfy a given LTL task specification.

III. Basic Solution Approach

Given the workspace description $W$, the set of actions $Act$ of the robot, and the set of propositions $AP$ used to define the LTL specification $\Phi$, we represent the behavior of the robot as a transition graph.

Definition 3.1 (Robot Transition Graph $G_i$): The robot transition graph $G_i = (S, s_0, E, L)$ is a directed graph with a set of vertices $S$ and a set of edges $E$. Each vertex in $S$ represents a robot state $s$, where $s_0$ is the initial state of the robot. Each edge $(s_i, s_j) \in E$ connects two robot states $s_i$ and $s_j$ if the robot can reach state $s_j$ by performing an action $act \in Act$ at state $s_i$. The weight corresponding to the edge is equal to the cost of the action $act$. $L : S \rightarrow 2^{AP}$ maps a state $s \in S$ to a subset of atomic propositions true at it. A state $s$ satisfies Boolean formula $\phi$, i.e. $s \models \phi$, if $L(s) \models \phi$.

The basic algorithm for finding a path in a transition graph $G_i$ satisfying an LTL query $\Phi$ is as follows. The first step is to generate a Buchi automaton $B$ from the LTL query $\Phi$. We describe the Buchi automaton as follows:

Definition 3.2 (Buchi Automaton): A Buchi automaton $A$ can be represented as $(Q, Q_0, \Sigma, \delta, Q_f)$, where: (a) $Q$ is the set of finite states in $B$, (b) $Q_0 \subseteq Q$ is the set of initial states (c) $\Sigma$ denotes the set of input symbols accepted by the automaton, (d) $\delta \subseteq Q \times \Sigma \times Q$ is a transition relation, and (e) $Q_f \subseteq Q$ is the set of accepting states. An accepting state in the Buchi automaton is the one that needs to occur infinitely often on an infinite length string consisting of symbols from $\Sigma$ to get accepted by the automaton. For the sake of simplicity, we assume that $Q_0$ contains a single initial state $q_{init}$ in our problem.

The second step of the solution is to construct a product graph $G_p$ of the transition graph $G_i$ and the Buchi automaton $B$.

Definition 3.3 (Product Automaton Graph): The product graph of transition system $G_i$ and Buchi automaton $B$ is denoted by $G_p = (G_i \otimes B)$. A node in the product graph is denoted as $(s, q)$ where $s \in S$ of $G_i$ and $q \in Q$ of $B$. An edge from a node $(s_i, q_i)$ to another node $(s_j, q_j)$ exists in $G_p$ if there exists an edge $(s_i, s_j) \in E$ in $G_i$ and a transition $\delta(q_i, c) = q_j$ in $B$, and $L(s_j) \models c$. The node $(s, q_f)$ in $G_p$ represents an accepting or final state if $q_f \in Q_f$ in automaton $B$. We denote the set of all accepting states in $G_p$ by $Q_f'$.

The final step of the solution is to find a path in $G_p$ satisfying the LTL query $\Phi$. If the trajectory for the LTL query is finite, then we can find a path from the initial state to an accepting state in the product graph $G_p$. If the trajectory for the LTL query is infinite then it can be finitely represented as a prefix path followed by a suffix cycle which can be repeated to obtain the infinite trajectory.

An infinite length trajectory can be generated by finding a path $P_{pre}$ from an initial state $(s_0, q_{init})$ to an accepting state $(s_f, q_f)$ and then checking whether there exists a cycle $P_{suf}$ in the product graph consisting of the accepting state $(s_f, q_f)$. There can be multiple such accepting states. If such cycle and a prefix path from the source exists for an accepting state $(s_f, q_f)$, then the path $P_{pre} \odot P_{suf}$ satisfies the LTL query. The symbol $\omega$ denotes that the suffix cycle is repeated infinite times. We will focus on finding infinite length trajectory $\xi : P_{pre} \cdot P_{suf}^\omega$ in product graph $G_p$. The optimal trajectory $\xi$ can be generated using Dijkstra’s shortest path algorithm for computing suffix and prefix paths for every accepting states in $G_p$ and print a trajectory $\xi$ with the smallest suffix cycle satisfying the LTL query.

Since we run Dijkstra’s algorithm for finding shortest suffix cycle for each point in $V_{pf}$, the complexity of the above solution is $|V_{pf}| \times (E_p \times \log V_p)$ using a binary heap data structure [22]. In the following section, we present $T^*$ algorithm that provides a significantly improved running time over the Dijkstra’s algorithm based solution for generating an optimal trajectory satisfying a given LTL query.

IV. $T^*$ Algorithm

The basic motivation behind the algorithm is that if we apply Dijkstra’s algorithm on the product graph $G_p$ of the transition graph of the robot and Buchi automaton, it would consume a lot of memory for larger workspaces and automaton generated for LTL query while maintaining priority queue in running Dijkstra’s shortest path Algorithm. To solve the problem, our approach is to use a reduced product graph $G_h$. The steps of constructing the graph $G_h$ are described later in this section.

In the product graph $G_p$, we take a transition from a node $(s_i, q_j)$ to a node $(s_j, q_j)$ changing the automaton state and the system state ensuring that $s_j \models c$ in transition $\delta(q_i, c) =
Algorithm 1 T* Algorithm

INPUT: The robot transition system graph $G_t = (S, s_0, E, L)$, LTL query $\Phi$.
OUTPUT: A trajectory $\xi$ of the robot satisfying the LTL query $\Phi$.

GLOBAL: node_list, adj

1: $B(Q, q_{init}, \Sigma, \delta, Q_f) \leftarrow \text{lTL_to_buchi_automata} (\Phi)$
2: $v_{src} \leftarrow (s_0, q_{init})$
3: for all $q_i, q_j \in Q$ do
   4:   $S_{q_i,q_j} \leftarrow \{(s \in S | \delta(q_i, c) = q_j \land L(s) = c)\}$
5:   $G_h \leftarrow (\emptyset, \emptyset), adj \leftarrow \emptyset$
6:   $V_{adj} \leftarrow \{(s, q) | s \in S$ and $q \in Q\}$
7:   for all $v_f \in V_{adj}$ do
   8:      $path \leftarrow \text{find_path} (G_h, G_t, B, v_f, v_f)$
   9:      $\text{suf_path_list}.append (path)$
10:     $\text{suf_path_list}.sort \_increasing(\text{suf_path_list})$
11:     for $count = 1$ to $\text{size}(\text{suf_path_list})$ do
12:        $\text{suf} \leftarrow \text{suf_path_list}[\text{count}]$
13:        $\text{pre} \leftarrow \text{find_path} (G_h, G_t, B, v_{src}, v_{src})$
14:     if $\text{pre} \neq \emptyset$ then return $\text{pre.suf}$
15: else
16:     return $\text{NULL}$

Algorithm 2 find_path

INPUT: $G_h, G_t, B$. Source vertex $v_{src}$, Destination vertex $v_{dest}$.
OUTPUT: Trajectory $\zeta_{fin}$ from from $v_{src}$ to $v_{dest}$ in Product graph $G_h$.

1: path_found $\leftarrow$ false, cangetpath $\leftarrow$ true, $V_h \leftarrow V_h \cup \{v_{src}\}$
2: while $\neg \text{path_found} \land \neg \text{cangetpath}$ do
   3:      cangetpath $\leftarrow \text{false}, \text{queue} \leftarrow \emptyset, \text{node_list} \leftarrow \emptyset$
   4:      if $v_{src} = v_{dest}$ then
   5:         add_neighbour($G_h, \text{queue}, \text{get_node}(v_{src}, 0, \text{NULL}))$
   6:      else
   7:         queue.push(get_node($v_{src}, 0, \text{NULL}$))
   8:      while queue $\neq$ EMPTY do
   9:         $node \leftarrow \text{extract_min} (\text{queue})$
10:     if $node = v_{dest}$ then
11:         cangetpath $\leftarrow$ true
12:         updates $\leftarrow \text{modify_edges} (G_h, G_t, \text{node})$
13:     if updates $= 0$ then
14:         path_found $\leftarrow$ true
15:     return print_path ($G_h, G_t, \text{node}$)
16:     break
17:     add_neighbour($G_h, \text{queue}, \text{node}$)
18:     return $\text{NULL}$

Algorithm 3 get_node

INPUT: vertex $(s, q), \text{dist}, \text{parent}$.
OUTPUT: node.

1: if node $(s, q) \in \text{node_list}$ then return node $(s, q)$
2: node.vertex $\leftarrow (s, q)$
3: node.dist $\leftarrow \text{dist}, \text{node.parent} \leftarrow \text{parent}$
4: node_list $\leftarrow \text{node_list} \cup \{\text{node}\}$
5: return node

$q_j$. The transition condition $c$ can be satisfied by a small subset of $S$ or a large subset of $S$. To distinguish between these two scenarios we introduce a threshold $\eta \ll |S|$. Let $S_c$ denote the set of all states satisfying $c$. A transition condition $c$ where $|S_c| \leq \eta$ is denoted as $c_{pos}$ while a transition condition $c$ where $|S_c| > \eta$ is denoted as $c_{neg}$.

The algorithm $T^*$ maintains a product graph $G_h = (V_h, E_h)$ which is grown incrementally throughout the algorithm. The algorithm uses Dijkstra’s algorithm for computing suffix and prefix paths on the graph $G_h$. Also, since $G_h$ includes edges between non-neighbour nodes in $G_p$, we use $A^*$[24] to compute the edge costs in $G_h$. $A^*$ allows us to compute edge costs faster than Dijkstra’s algorithm. However, the actual edge costs between nodes in $G_h$ are computed using $A^*$ if they lie on the shortest path computed on $G_h$. But before that the edge costs in $G_h$ are initialized with a heuristic edge cost. The heuristic edge cost between a node $(s_i, q_{i})$ to $(s_j, q_{j})$ is always less than or equal to actual distance between $s_i$ and $s_j$ in $G_t$. $T^*$ computes the shortest trajectories in $G_h$ by running Dijkstra’s shortest path algorithm on $G_h$ to get the outline of the path denoted as $\zeta$. It then computes the actual path in $G_p$ corresponding to $\zeta$ by running $A^*$ on a subset of transition system graph $G_t$.

The algorithm is described in the form of procedures. $G_t$ is the robot transition graph representing the robot environment. $S_c$ denotes set of system states satisfying the automaton transition $\delta(q_i, c) = q_j$ and $adj$ denotes the dynamic size array having the tuple (cost, updated) as elements. Each tuple represents information for an edge in $G_h$, where cost denotes the edge cost and updated denotes that the edge cost has been updated by $A^*$. The node $v_{src} = (s_0, q_{init})$ denotes the source node of product graph $G_h$ initialized with initial automaton state $q_{init}$ and $s_0$ denotes the initial state of robot in $G_t$.

In line 1 of $T^*$ Algorithm, Buchi Automaton $B$ is generated from the LTL query $\Phi$. In line 2 to 3 we associate system states to an automaton edge $\delta(q_i, c) = q_j$. In line 4 we initialize the product graph $G_h$ with zero nodes and zero edges and the array $\text{adj}$ as $\text{NULL}$. In line 6 we compute the set of accepting states $V_{adj}$ in $G_p$. In lines 7 to 9 we compute the suffix cycle for every state $v_f \in V_{adj}$. In line 10 we sort all the suffix cycles based on increasing cost of the path. In lines 11 to 15 we find the shortest suffix cycle for which a prefix path $\text{pre}$ from the source node in $G_p$ exists. The $\text{node_list}$ and $\text{adj}$ are global data structures used throughout all modules in the algorithm. $\text{node_list}$ refers to all the nodes visited while running $\text{find_path}$ algorithm between two nodes.

Algorithm 2 $\text{find_path}$, finds the shortest path from $v_{src}$ to $v_{dest}$ in product graph $G_p$ by running Dijkstra’s shortest path algorithm on graph $G_h$. $\text{queue}$ is a min priority queue that extracts nodes based on distance from $v_{src}$ denoted by $\text{dist}$. In line 3 to 7 in case the path $\zeta$ is found in $G_h$, $\text{modify_edges}$ is called in line 8 to compute the actual edge cost between the nodes in the path $\zeta$. If no updates is made by $\text{modify_edges}$ in edges of path $\zeta$, actual path $\zeta_{fin}$ in $G_p$ is generated corresponding to the outline path $\zeta$ through $\text{print_path}$ procedure.

Algorithm 3 $\text{get_node}$, finds and relaxes the $\text{dist}$ of the adjacent nodes of the current node $(s, q)$ in the product graph $G_h$. Line 1 checks whether there exists a self transition $\delta(q_i, c_0) = q_i$ where $|S_c| > \eta$ in automaton state $q_i$. Line 3 to 7 finds all nodes $(s_i, q_i)$ in product graph $G_p$ where $L(s_i)$ satisfies an automaton transition condition $\delta(q_i, c) = q_i$ and $|S_c| \leq \eta$. In line 4 for all the nodes $s_i \in S$ satisfying such automaton transitions, we run a loop and those nodes
Algorithm 4 add_neighbour

```
1: if selfloop ← false
2: if δ(q,c) = q′ in B where |S_c| > η then selfloop ← true
3: for all δ(q,c) = q′ in B where |S_c| ≤ η do
4: for all s_i ∈ S, do
5: if selfloop and (s_i, s) ∉ E in G then continue
6: v ← get_node((s_i, q_i), NULL)
7: relax_edge(G, queue, u, v)
8: if ∃ δ(q,c) = q′ and q_i ≠ q′ then δ(q,c) = q_i
9: for all δ(q,c) = q′ in B where |S_c| > η do
10: if all such (s_i, s) ∈ E and L(s_i) = c do
11: v ← get_node((s_i, q_i), NULL)
12: relax_edge(G, queue, u, v)
```

Algorithm 5 relax_edge

```
1: if (s_i, q_i) ∉ V_t then V_q ← V_q ∪ {(s_i, q_i)}
2: if ((s_i, q_i) ≠ (∗, q_i)) ∈ E_h then
3: E_h ← E_h ∪ {(s_i, q_i)}
4: adj[(s_i, q_i), (s_j, q_j)].cost ← heuristic_distance(s_i, s_j)
5: adj[(s_i, q_i), (s_j, q_j)].updated ← false
6: if v.dist = ∞ then push(v)
7: if u.dist + adj[(s_i, q_i), (s_j, q_j)].cost < v.dist then
8: v.dist ← u.dist + adj[(s_i, q_i), (s_j, q_j)]
9: v.parent ← u
```

are added in G_h. Line 5 checks if there is no selfloop (as mentioned in line 2) then in (s_i, q_i), s_i must be a neighbour of s in G_l since path finding to a non-neighbour node is done through A* only when there is a self loop in current automaton state of type c_neg. Lines 8 to 12 add nodes (s_i, q_i) to G_h as neighbour of (s, q) on automaton transition δ(q,c) = q′ where |S_c| > η and s_i is a neighbour of s in G_l and s_i = c. Since c is satisfied by a large number of points in G_l we just look to the neighbours of s in G_l. Also, in case of q_i equal to q, neighbour s_i = c will only be added in case there exists another automaton transition δ(q,c) = q_j where |S_c| > η.

Algorithm 5 relax_edge, checks whether there exists a shorter path to the neighbour node v through the current node u and updates the dist and parent of the node v accordingly. In lines 2 to 5 if the edge between u and v is not in E_h, we initialize it with a heuristic distance and mark that the edge cost is not updated. The heuristic distance calculated is less than equal to actual distance between s and s_i in G_l. In lines 6 to 9 if a shorter path to v through u, the distance of v from source node is updated and parent pointer of v is set to u.

Algorithm 6 modify_edges

```
1: if v = NULL then return 0
2: updates ← modify_edges(G_h, G_l, v.parent)
3: if v.parent ≠ NULL then
4: (s_i, q_i) ← v.parent
5: if adj[(s_i, q_i), (s_j, q_j)].updated = false then
6: C_s ⇒ [c | δ(q,c) = q_j].
7: src ← ∪ {s | s ≠ c ∧ s ∈ S}
8: δ′ ← δ \ src
9: E′ ← E \ {(v → v′) | v ∈ src ∧ v′ ∈ obs}
10: G′ ← (S′, E′)
11: adj[(s_i, q_i), (s_j, q_j)].cost ← A_search(s_i, s_j, G′)
12: adj[(s_i, q_i), (s_j, q_j)].updated ← true
13: updates ← updates + 1
14: return updates
```

Algorithm 7 print_path

```
1: if v = NULL then return NULL
2: path ← print_path(G_l, G_vsrc, v.parent)
3: if v.parent ≠ NULL then
4: (s_i, q_i) ← v.parent
5: C_s ⇒ [c | δ(q,c) = q_i].
6: src ← ∪ {s | s ≠ c ∧ s ∈ S}
7: S′ ← S \ src
8: E′ ← E \ {(v → v′) | v ∈ src ∧ v′ ∈ obs}
9: G′ ← (S′, E′)
10: path.append(A_path(s_i, s_j, G′))
11: return path
```

check whether there are any special conditions to be satisfied in between. This is done by looking at the self transition δ(q,c) = q_i in automaton state q_i in B. If a vertex in G_l does not satisfy any of the transition condition c, we mark them as obstacles in the workspace and create graph G_l’ in line 7 to 10. We then compute the edge cost using A* in G_l’ and mark the edge has been updated so that each edge is updated only once.

Algorithm 7 print_path simply prints the actual path corresponding to the path computed by the algorithm in graph G_h by recursively moving to the parent node. The actual path between two adjacent nodes (s_i, q_i) and (s_j, q_j) in G_h can be found by computing shortest path from s_i to s_j in G_l using A* as written in line 10. Before running A* to compute the actual path from s_i to s_j in G_l we mark all the vertices in G_l not satisfying any of the transition condition δ(q,c) = q_i, as obstacles in line 6. A new graph G_l’ is created by removing all the edges and nodes containing obstacles in G_l. We then run A* from s_i to s_j in G_l’ to find the path from (s_i, q_i) to (s_j, q_j) in G_p.

A. An Illustrative Example

The following example shows the working of the algorithm for a query □ (p_1 ∧ □ p_2 ∧ ¬ p_3). The grid can be represented as a graph G_l with each cell as a robot state. All the diagonal and non-diagonal neighbours in the grid
have edges between them in graph \( G_t \) with diagonal edge cost as 1.5 and non-diagonal edge cost as 1.

The steps solving the problem in \( T^* \) are listed below:

- Using this formula we computed the heuristic distances defined as cost as 1.5 and non-diagonal edge cost as 1.

• First we calculate suffix cycle containing \( s_3 \).

• In this we execute Dijkstra’s algorithm on graph \( G_t \).

• We add a neighbour node edge \((s_4 q_0)\) following lines 10-13 of algorithm [4] since the transition condition from \( q_1 \) to \( q_0 \) is of type \( c_{\text{neg}} \) and push the new node in queue.

• On extracting \((s_4 q_0)\) from the priority queue, the edges added are \((s_4 q_0) \rightarrow (s_1 q_2), (s_4 q_0) \rightarrow (s_3 q_2)\) with cost from source 11.5 and 7 respectively. The transition condition from \( q_0 \) to \( q_2 \) is of type \( c_{\text{pos}} \) while exists a transition condition of type \( c_{\text{neg}} \) from \( q_0 \) to \( q_0 \).

• Next step we extract \((s_3 q_2)\) from the priority queue which will add edge \((s_3 q_2) \rightarrow (s_2 q_1)\) and update \((s_2 q_1)\) cost from the source \( v_f \) as 14.

• Next step we extract \((s_1 q_2)\) from queue and add edge \((s_1 q_2) \rightarrow (s_2 q_1)\) in add_neighbour. Finally \((s_2 q_1)\) is extracted from queue which is the accepting state \( p_f \).

• On reaching the accepting state \( p_f \) we call modify_edges to calculate the edge cost in path \((s_2 q_1) \rightarrow (s_4 q_0) \rightarrow (s_3 q_2) \rightarrow (s_2 q_1)\).

• In modify_edges we calculate the actual cost of each edge. To calculate cost from \((s_4 q_0)\) to \((s_3 q_2)\) we first mark all the points not satisfying \( \delta(q_0, \neg p_3) = q_0 \) as obstacles. Then run \( A^* \) from \( s_4 \) to \( s_3 \) in \( G_t \) following lines 10-15 of algorithm [6]. The cost from \((s_4 q_0)\) to \((s_3 q_2)\) is 12.5. Similarly we compute distance from \((s_3 q_2)\) to \((s_2 q_1)\) which is equal to 13.5.

• Since modify_edges has updated the edges, pathfound is false, and since we reached \( p_f \), can_check_path=true. So the while loop of method find_path will be executed again. This time we will get a suffix path \((s_2 q_1) \rightarrow (s_4 q_0) \rightarrow (s_1 q_2) \rightarrow (s_2 q_1)\) where the cost of the edges is computed by modify_edges. The actual cost and heuristic cost of the edges in the path matches and hence no updates.

• Thus the suffix path generated is \((s_2 q_1) \rightarrow (s_4 q_0) \rightarrow (s_1 q_2) \rightarrow (s_2 q_1)\). To generate the actual path corresponding to this suffix trajectory we call the print_path procedure which prints the intermediary nodes between the nodes in the suffix path computed. Similarly, we find the prefix path from \( p_{\text{src}}(s_0, q_0) \) to \( p_{\text{dest}}(s_2 q_1)\) using the method find_path.

### B. Computational Complexity

We implement the Dijkstra’s shortest path and \( A^* \) using priority queue. Let \( S_u \) denote the set of system states in \( G_t \) satisfying a transition condition \( c \) in automaton \( B \), where \(|S_u| \leq \eta \) and \( V_u \) be the nodes in \( G_h \) where the transition system state \( s \in S_u \). Since the states satisfying such automaton transition condition can add edge between two non-neighbour nodes of \( G_p \) in \( G_h \), the number of such states denotes the number of times \( A^* \) has to be called to compute the distance between two non-neighbour nodes of \( G_p \) connected through an edge in \( G_h \). Let the total number of subformulas in LTL formula \( \Phi \) be denoted as \(|\Phi|\). So maximum states in automaton will be \( 2^{|\Phi|} \). So LTL to Buchi automaton conversion will have complexity \( O(2^{|\Phi|}) \) as mentioned in [25]. Computing the set \(|S_u|\) for every automaton edge \( \delta(q_1, c) = q_1 \) will take \( O(2^{|\Phi|}) \). Since we run Dijkstra’s algorithm for every accepting state in \( G_p \) and the number of maximum updates possible are \( V_h \times V_u \), it will have time complexity \( O(|P_f| \times V_h \times V_u \times E_h \times \log V_h) \). Also, the distance between non-neighbour nodes in \( G_h \) was computed using \( A^* \). Assuming the \( A^* \) complexity to be same as Dijkstra’s algorithm and the number of edges to non-neighbour nodes for each vertex in \( G_h \) can be \( V_u \), complexity for \( A^* \) computations is \( O(V_h \times V_u \times E \times \log(|S|)) \). So the
C. Correctness and Optimality

In this section, we prove the correctness and optimality of our algorithm.

1) Correctness: We have proved the following two theorems to establish the correctness of our algorithm and the optimality of the trajectory generated by it.

**Theorem 4.1:** The trajectory generated by the T* algorithm satisfies the given LTL query.

**Proof:** Let \( B \) be the Buchi automaton representing the LTL query \( \Phi \) and \( G_t \) the transitions system capturing the workspace and actions of the robot. \( L(s_i) \) denotes the proposition true at state \( s_i \) of \( G_t \). AP is the set of atomic propositions and \( w \) is the set of all words formed from atomic propositions (total - 2\(^{|AP|}\)). Trace of a path \( \xi = (s_1, s_2, \ldots, s_k) \) is denoted by \((w_1, w_2, \ldots, w_k)\) where each \( w_i \) denotes the atomic propositions true at state \( s_i \).

An infinite LTL query is satisfied by a trace if it leads to the accepting state of the automaton infinitely often. Such trace is of infinite length which can be written in the format of \( \text{pre}.(\text{suffix})^\omega \), where \( \text{pre} \) is the prefix part of the path and \( \text{suffix} \) part is repeated infinitely often. The \( \text{pre} \) part leads to the accepting state of the automaton. The \( \text{suffix} \) part starts from the accepting state and leads to the accepting state of the automaton again.

To satisfy a suffix trace \((w_1, w_2, \ldots, w_k)\), we need to find a path in \( G_t \) satisfying this trace. To achieve this, we create a product graph \( G_h \) of \( B \) and \( G_t \) incrementally in \( T^* \). A state in the product graph is denoted as \((s_i, q_i)\), where \( s_i \) denotes system state in \( G_t \), \( q_i \) denotes an automaton state. If we move from point \( s_i, q_i \) to state \( s_j \) in \( G_t \) we check whether there exists a transition \( \delta(q_i, c) = q_j \) where \( s_j \) satisfies \( c \). If it exists, we form a transition from \((s_i, q_i)\) to \((s_j, q_j)\) in \( G_h \). Also in case the transition from \((s_i, q_i)\) to \((s_j, q_j)\) is added between two non-neighbour nodes, it is checked whether there exists an actual path from \( s_i \) to \( s_j \) in \( G_t \) satisfying the automaton transitions \( \delta(q_i, c) = q_j \).

We start at an accepting state of the product graph \((s_f, q_f)\) where \( q_f \) must be an accepting state of automaton \( B \). If we find a cycle \((s_f, q_f), \ldots, (s_f, q_f)\) in the product graph \( G_h \), this means it satisfies a suffix trace \((w_1, w_2, \ldots, w_k)\) since we are changing the state of automaton in \( G_h \) according to the proposition true at a system state when it satisfies a transition condition in \( B \). Thus by finding a cycle containing accepting state we find a suffix trace. This is done in step 4 of Algorithm 1.

Now we need to find a path \( \xi_{\text{pre}} \) in the product graph from initial state \((s_0, q_0)\) to the accepting state \((s_f, q_f)\) for which a suffix cycle \( \xi_{\text{suffix}} \) exists. This path will satisfy the prefix trace since it leads from \((s_0, q_0)\) to \((s_f, q_f)\) i.e from automaton state \( q_0 \) to \( q_f \). This is done in step 6 of Algorithm 1.

Hence, our algorithm finds the prefix path \( \xi_{\text{pre}} \) satisfying the prefix trace and suffix path \( \xi_{\text{suffix}} \) satisfying the suffix trace which together form the trace \( \text{pre}.(\text{suffix})^\omega \) satisfying the given LTL query.

2) Optimality: The algorithm computes a suffix cycle for an accepting state in the product graph. When we compute the suffix cycle for an accepting state \((s_i, q_f)\) where \( q_f \) is an accepting state of the Buchi automaton, using Dijkstra’s shortest path algorithm we ensure that it will find the shortest cycle containing \((s_i, q_f)\).

**Lemma 4.2:** The algorithm \( T^* \) adds edges in the product graph \( G_h \) by looking at the automaton transition conditions of the outgoing edges from the current automaton state in \( B \).

**Proof:** The algorithm adds edges in product graph \( G_h \) by looking at the automaton transition conditions of the outgoing edges from the current automaton state in \( B \).

1) If the number of states \( s \in S \) satisfying the transition condition \( c \) from \( q_i \) to \( q_j \) is less than or equal to \( \eta \) i.e \( \delta(q_i, c) = q_j \) where \( |S_{c_i}| \leq \eta \). Automaton state \( q_f \) can be equal or different from \( q_i \). Current state is \((s_i, q_f)\). Transition from \((s_i, q_i)\) to \((s_j, q_j)\) can take place in \( G_h \) if \( s_j = c_i \). The transition system state \( s_i \) need not be neighbour of \( s_i \) as shown in lines 4 to 9 in Algorithm 1. The edge from \((s_i, q_i)\) to \((s_j, q_j)\) will be added based on two cases:

a) There exists a transition \( \delta(q_i, c_i) = q_i \) where \( |S_{c_i}| > \eta \). Then mark all the nodes not satisfying the Boolean condition \( c \) in any of the self transitions \( \delta(q_i, c) = q_i \) as obstacles in \( G_t \) and run \( A^* \) from \( s_i \) to \( s_j \) in \( G_i \) to compute the distance between the nodes \((s_i, q_i)\) to \((s_j, q_j)\) in \( G_h \).

b) If the condition in (a) is not true then the transition from \((s_i, q_i)\) to \((s_j, q_j)\) will only take place if \( s_j \) is a neighbour of \( s_i \) in \( G_t \). This is because all the transitions conditions \( c \) in automaton where \(|S_{c_i}| \leq \eta \) are allowed to add transitions in \( G_i \). If (a) is not true then all the self loop transition conditions \( c_i \) for state \( q_i \) in automaton has \(|S_{c_i}| \leq \eta \) and will add nodes and edges in \( G_{h_i} \) according to lines 4 to 9 of Algorithm 1. This will lead us to the node \((s_j, q_j)\) since the path to a state \( s \) in \( G_t \) satisfying \( c_i \) will be through the nodes satisfying self loop transitions of \( q_i \).

2) Current state is \((s_i, q_i)\) and \( q_i \neq q_j \). There is an automaton transition \( \delta(q_i, c) = q_j \) with \( |S_{c_i}| > \eta \). Transition \((s_i, q_i)\) to \((s_j, q_j)\) can take place in \( G_h \) if \( s_j = c_i \) and \( s_i \) is a neighbour of \( s_i \) in \( G_t \). In this case transition from \((s_i, q_i)\) to \((s_k, q_k)\) can also take place in \( G_h \) on transition \( \delta(q_i, c_k) = q_k \), where \( s_k = c_i \) and \( s_k \) is a neighbour of \( s_i \) in \( G_t \). This is covered in lines 10 to 13 in Algorithm 1.

**Theorem 4.3:** The algorithm finds the shortest path between two accepting states and from source to accepting states in the product graph \( G_p \).

**Proof:** We run Dijkstra’s shortest path Algorithm on product graph \( G_h \) which gives shortest path between two nodes in \( G_h \). Let the suffix cycle computed by \( T^* \) for a point \((s_f, q_f) \in V_h \) in product graph \( G_h \) be \( Path_{\text{suffix}} \). For the sake of contradiction we assume there exists a shortest suffix cycle \( Path_v \) in \( G_p \) containing \((s_f, q_f)\) which is shorter than \( Path_{\text{suffix}} \). There must be a point in \( Path_v \) which is lying in \( G_h \) since \((s_f, q_f)\) also lies in graph \( G_h \).
1) Let \((s_j, q_i)\) be the first point in \(Path_v\) not lying in \(G_h\) and the previous point denoted by \((s_i, q_i)\) in \(G_h\). If \(\delta(q_i, c) = q_i\) where \([S_c] \leq \eta\) and \(s_j \models c\), \((s_j, q_i)\) will be included in \(G_h\) using transition type 1 in Lemma 4.2. If \(\delta(q_i, c) = q_i\) where \([S_c] > \eta\) and \(s_j \models c\), \((s_j, q_i)\) will be included in \(G_h\) using transition 2 in Lemma 4.2. Hence if a point \((s_i, q_i)\) is in \(G_h\), then the neighbour \((s_j, q_j)\) in \(G_h\) is also neighbour in \(G_h\). This contradicts the fact \((s_j, q_j)\) does not lie in \(G_h\) and there exists a shorter suffix cycle than \(Path_{suf}\).

2) Let \((s_j, q_i)\) be the first point in \(Path_v\) not lying in \(G_h\) and \(Path_{suf}\), the previous point to \((s_j, q_i)\) in \(Path_v\) denoted by \((s_i, q_i)\) is in \(G_h\).

a) If \(\delta(q_i, c) = q_i\) where \([S_c] \leq \eta\) and \(s_j \models c\), \((s_j, q_i)\) will be included in \(G_h\) according to transition 1(b) in Lemma 4.2.

b) If \(\delta(q_i, c) = q_i\) where \([S_c] > \eta\) and \(s_j \models c\), \((s_j, q_i)\) will be included in \(G_h\) if there exists another transition \(\delta(q_i, c) = q_j\) where \([S_c'] > \eta\) according to transition 2 in Lemma 4.2.

c) If \(\delta(q_i, c) = q_i\) where \([S_c] > \eta\). All automaton transitions of \(q_i\) to neighbour nodes in \(B\) are of type \(\delta(q_i, c) = q_j\) where \([S_c] \leq \eta\), then \((s_j, q_i)\) will not be added to \(G_h\). The point \((s_j, q_i)\) must be leading to a state \((s_k, q_j)\) where automaton state changes from \(q_i\) to \(q_j\) in \(Path_v\), i.e., a node with some other automaton state \(q_j\) since \(q_i\) is not the accepting automaton state. Since all outgoing automaton transitions from \(q_i\) are of type \(\delta(q_i, c) = q_j\) where \([S_c] \leq \eta\), there will be an edge in \(G_h\) from node \((s_i, q_i)\) to \((s_k, q_j)\) where \(s_k \models c_t\) according to transition 1(a) in Lemma 4.2. Since according to \(Path_v\), \((s_j, q_i)\) lies in the shortest path between \((s_i, q_i)\) and \((s_k, q_j)\), \((s_j, q_i)\) will be included in shortest path computed using \(A^*\) from node \((s_i, q_i)\) to \((s_k, q_j)\), and hence will be part of \(Path_{suf}\).

Hence this contradicts the fact that \((s_j, q_j)\) does not lie in \(G_h\) and it is not part of the suffix cycle \(Path_{suf}\) computed by \(T^*\) by running Dijkstra’s algorithm on \(G_h\). Thus, there does not exist a shorter suffix cycle than \(Path_{suf}\) computed by running Dijkstra’s shortest path algorithm in \(G_h\).

V. Evaluation

In this section, we present the results to establish the computational efficiency of \(T^*\) algorithm. The results have been obtained on a desktop computer with a 3.4 GHz quadcore processor with 16 GB of RAM. We use LTL2TBGA tool[25] as the LTL query to Buchi automaton converter.

A. Workspace Description and LTL Queries

The robot workspace is represented as a 2D grid. Each cell in the grid is denoted by integer coordinates \((x, y)\). Each of these cells have 2 horizontal, 2 vertical and 4 diagonal neighbours. The cost of an edge connecting diagonal neighbours is 1.5 and the non-diagonal neighbours is 1. A grid cell can have a proposition true or false at it. We consider several data gathering tasks performed by a robot by visiting some locations.

We evaluated \(T^*\) algorithm for seven LTL queries borrowed from [26]. The LTL queries are denote by \(\Phi_A, \Phi_B, \ldots, \Phi_C\). Here, we mention two of those LTL specifications, \(\Phi_C\) and \(\Phi_D\), in detail. Here \(p_1, p_2, p_3\) denote data gathering locations and propositions \(p_4\) and \(p_5\) denote data upload locations.

1) We want the robot to gather data from all the three locations and upload the gathered data to one of the data upload locations. Moreover, after visiting an upload location, robot must not visit another upload location until it visits a data gathering location. The query can be represented as

\[
\Phi_C = \Box (\Diamond p_1 \land \Diamond p_2 \land \Diamond p_3) \land \Diamond (\Diamond p_4 \lor \Diamond p_5) \land (\Diamond (p_4 \lor p_5) \rightarrow X ((\neg p_4 \land \neg p_5) \lor (p_4 \lor p_5)))
\]

2) It can happen that the data of each location has to be uploaded individually before moving to another gathering place. We can put an until constraint that once visited a data gathering location do not visit another gathering location until the data is uploaded. This can be captured as

\[
\Phi_D = \Phi_C \land \Box ((p_1 \lor p_2 \lor p_3) \rightarrow X ((\neg p_1 \land \neg p_2 \land \neg p_3) \lor (p_4 \lor p_5)))
\]

B. Results on Comparison with Dijkstra’s Algorithm

We Compare \(T^*\) algorithm with Dijkstra’s algorithm based implementation using LTL2TBGA tool on a couple of workspaces. The workspaces which have been borrowed from [26] and [27] are shown in Figure 5 and Figure 6 respectively. The size of workspace is 100x100.

Table I shows the speedup of \(T^*\) over Dijkstra’s algorithm planned for several workspaces. Each data point in Table I is the average of data obtained from 10 experiments. From the table, it is evident that for both the workspaces, \(T^*\) provides over an order of magnitude improvement in running time with respect to a Dijkstra’s shortest path based algorithm.

![Table I](image)

Trajectories for queries \(\Phi_C\) and \(\Phi_D\) in workspace 1 and 2 as generated by \(T^*\) algorithm are shown in Figure 7 and Figure 8 respectively.
2) Complexity of LTL Query: We consider the LTL query $\Phi_D$ for this experiment. We define a template or the LTL formula of the form $\Phi_s = \Box\Diamond(gather) \land \Box\Diamond(upload) \land \Box((upload \rightarrow X((\neg upload)U(gather))) \land \Box(gather \rightarrow X((\neg gather)U(upload))))$. Starting with 2 gather and 1 upload locations, the gather locations were incremented by 2 and upload locations by 1 for 4 instances. In the template LTL query $\Phi_s$, gather holds when all the gather locations are visited in any order and upload holds when one of the upload locations is visited. The speedup is shown in Figure 5(b) for the workspace shown in Figure 6. Figure 6 also shows the trajectories for the robot for two instances of $\Phi_s$.

As we scale the number of propositions in the query with the same workspace and obstacle density, the length of trajectory satisfying $\Phi_s$ increases as shown in Figure 6. The trajectory for the given query is a sequence of $(g_i, u_i)$ i.e visiting gathering location $g_i$ and then an upload location $u_i$. The ratio of the number of states explored by Dijkstra’s algorithm on $G_p$ and $T^*$ algorithm on $G_h$ remains almost the same for each $(g_i, u_i)$ part. However, with the increase in the length of this sequence, the speedup increases with every increase in number of data gathering and upload locations.

3) The size of the workspace: We experimented with query $\Phi_D$ on Workspace 1 shown in Figure 3 by increasing the size of the square grid. We considered the following sizes for the side length of the square grid: 20, 100, 200, 300, 400 and 500. As shown in the Figure 5 the scaling of the grid does not affect the speedup factor of $T^*$ after reaching side length 100 and remains close to 20 in the given graph.

Keeping the LTL query and the obstacle density the same, the ratio of length of the trajectory satisfying the LTL query remains the same in comparison to size of the grid. Hence time taken to compute the trajectory by $T^*$ and Dijkstra algorithm based solution increase almost in the same ratio with the increase in size of the workspace. Hence the speedup factor of $T^*$ remains almost the same on scaling the workspace.

D. Experiments with Robot

We used the trajectory generated by $T^*$ algorithm to carry out experiments on Turtlebot on a grid of size $5 \times 5$ with four non-diagonal movements to the left, right, forward and backward direction. The cost of the forward and backward movement is 1, whereas the cost of the left and right movement is 1.5 as it involves a rotation followed by a forward movement.
movement. Two queries $\Phi_C$ and $\Phi_D$ were executed by the Turtlebot. The location of Turtlebot in the workspace was tracked using Vicon localization system [28]. The video of our experiments is submitted as a supplementary material.

VI. CONCLUSION

In this work, we have developed a static motion planning algorithm for LTL goals that scales order of magnitude better than the state-of-the-art algorithm. Our algorithm is an extension of A* algorithm which expands less number of nodes and thus is faster than the algorithm based on Dijkstra’s shortest path algorithm. Our future work include evaluating our algorithm for 3-D workspace and extending it for multi-robot systems and dynamic environments.

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