Equilibrium initial data for luminous matter on top of a BEC dark matter halo

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Abstract. Considering dark matter is a Bose Einstein Condensate modeled with the Gross-Pitaevskii equation, and assuming luminous matter is an n-body system, we construct initial data for the luminous matter on top of a BEC dark matter halo. The conditions on the luminous matter are that the peculiar velocities are random, that the density profile is nearly spherical and that the configuration of luminous plus dark matter is a long living system. We analyze the life-time using the full numerical evolution of the configuration.

1. Introduction
The Bose Einstein Condensate (BEC) dark matter model has recently acquired more attention, specially due to the recent advances related to structure formation simulations, under the different name of wave dark matter [1]. In the past, some interesting properties of BEC dark matter were analyzed, specially its properties at galactic core scale helping to solve the cusp-cure problem shown by the more popular CDM model [2, 3, 4, 5]. The main idea of the model is that it assumes dark matter is a condensate of ultralight spinless bosons that can potentially solve the two basic problems of CMD, namely, the cusp-core problem and the over abundance of small structures [1].

In this contribution we focus on the role of the BEC at local scales, specifically in galaxies. It has been very common during the development of the BEC dark matter model, that the interaction of dark and luminous matter has been underestimated. In the first versions of the model, considering it was an exploratory phase of the hypothesis, the main problem was that of the galactic rotation curves (RCs) (see e.g. [6]). Some very strong hypotheses were assumed, for instance, that luminous matter are test particles and travel on circular trajectories. These hypotheses looked reasonable from a pure theoretical point of view, however in practice these are very strong idealizations. On the one hand, there have not been any arguments supporting the first of the hypothesis, it is well known that the process of RCs measurements involves a highly non-trivial data averaging, and that RCs are not measured for exactly symmetric galaxies with the correct orientation as to observe the peculiar red and blue shift of the gas. On the other hand, assuming luminous matter is only test particles can only be applied to galaxies with a high mass to light ratio.

The most recent advances considering the interaction of BEC dark matter with luminous matter can be found in [4], where the luminous matter is evolved on top of the gravitational potential due to the BEC dark matter halo of a disk galaxy, but the BEC dark matter is fixed in
time. More recently, in [7] two structures collide and the offset of luminous versus dark matter is analyzed; the luminous particles are considered test particles though. In [8] the same problem of galactic encounters is analyzed, but the gravitational interaction among luminous particles is included.

The idea is now to consider all the interactions. The task is non-trivial and some small steps are necessary before addressing the most general scenario. As a step toward including the full interaction between BEC dark matter and luminous matter, in this proceeding we present the construction of an isolated system made of BEC dark matter and luminous matter modeled with N-particles.

Instead of tackling once and for all the general problem of a system made of BEC dark matter and luminous matter, we present a test that the model should pass before being used in the case of many structures interacting among each other. Thus, our result at this point is restricted to a spherically symmetric structure. The structure has two components, the BEC dark matter halo and the luminous matter modeled with N-point particles.

We therefore construct a nearly virialized spherically symmetric structure that can be evolved for a sufficiently long time (millions of years). We include the interaction that the BEC has on the luminous particles, the interaction among the particles themselves and the back reaction of the luminous particles on the BEC gravitational potential.

2. Model of the structure

The BEC halo. The system ruling the BEC is the Gross-Pitaevskii-Poisson coupled system of equations

\[
\begin{aligned}
\frac{i\hbar}{\partial t} \partial \Psi &= -\frac{\hbar^2}{2m} \nabla^2 \Psi + V \Psi + \frac{2\pi \hbar^2 a}{m^2} |\Psi|^2 \Psi, \\
\nabla^2 V &= 4\pi G m |\Psi|^2,
\end{aligned}
\]

where in general \( \Psi = \Psi(t, x) \), \( m \) is the mass of the boson, \( V \) is the gravitational potential acting as the condensate trap, \( a \) is the scattering length of the bosons. This is a coupled system consisting of an evolution equation for \( \Psi \) with a potential that is solution of Poisson equation sourced by \( |\Psi|^2 \).

We choose a BEC dark matter halo to be a ground state equilibrium configuration of the above system (see [9] for a detailed construction), which has the very desirable properties of being stable under radial and axial perturbations [10], and that it is a late-time attractor for the evolution of an initial profile [11]. This configuration has shown to model very well rotation curves of dwarf galaxies with and without a small rotation [5].

The luminous matter. We assume the luminous matter is a system of N-bodies that interact with each other and the gravitational potential due to the BEC halo. These \( N \) particles have equal mass \( M_L/N \), where \( M_L \) is the mass of the luminous matter of the structure. We spatially distribute the particles such that they satisfy the Plummer density profile

\[
\rho(r) = \frac{M_L}{4\pi} \frac{3r_0^2}{(r^2 + r_0^2)^{5/2}},
\]

corresponding to the potential \( \Phi(r) = \frac{GM_L}{(r^2 + r_0^2)^{1/2}} \). To do that, we start with the cumulative mass distribution

\[
m(r) = \int_0^r 4\pi r^2\rho(r)dr,
\]
and using Eq. (2) we integrate for \( m(r) \), which gives 

\[ m(r) = r^3(r^2 + 1)^{-3/2}. \]

Inverting this expression for \( r \) we obtain [12]

\[ r(m) = (m^{-2/3} - 1)^{-1/2}. \] (4)

The procedure to distribute the particles in a three dimensional domain is the following: for every particle we need to find the coordinates \( r, \theta \) and \( \phi \) of the particle’s position. We choose a random number \( m \) such that \( 0 \leq m \leq M_L \) and identify it with the cumulative mass contained within the radius \( r \) for that particle. Using Eq (4) we determine the coordinate \( r \) for the position of the particle. To determine the angles \( \theta \) and \( \phi \) we choose two random numbers. The first one between \(-1\) and \(1\) corresponding to the value of \( \cos(\theta) \) and the second one between \(0\) and \(2\pi\) corresponding to the value of \( \phi \).

Now, we proceed to choose the magnitude and direction for the velocity of one of the particles with respect to the center of the distribution. First, the maximum velocity allowed for a particle at a given \( r \) is the escape velocity \( v_e \). This velocity can be obtained equating the potential and kinetic energies and solving for \( v_e \) to obtain 

\[ v_e(r) = \sqrt{2gM_L(r^2 + r_n^2)^{-1/4}} \] [12].

Next, the probability \( g(v) \) to have a velocity \( v = ||\vec{v}|| \) at a radial position \( r = ||\vec{r}|| \) is given by

\[ g(v)dv \propto (-E(r,v))^7/2v^2dv. \] (5)

In terms of the escape velocity and defining \( q = v/v_e \), the distribution function for \( v \) becomes:

\[ g(q) = (1 - q^2)^{7/2}q^2 \text{ with } 0 \leq q \leq 1. \] (6)

Instead of inverting Eq.(6) to estimate \( v \), we use a rejection technique. That is, choose a random number \( x \) between \(0\) and \(1\) and a random number \( y \) between \(0\) and \( g_{max} = 0.1 \). If \( y < g(x) \) we accept the pair \((x, y)\) and compute the velocity as 

\[ v = \sqrt{2gM(r^2 + r_n^2)^{-1/4}}, \]

otherwise, repeat the process choosing another pair \((x, y)\). In this way the particles have randomized peculiar velocities around the center and the configuration is nearly stationary.

3. Tests

The way to know whether the interplay between these two components is correctly implemented is measuring the time-life of the structure. For this, it is necessary to carry out the evolution of the initial configuration and determine the life time of the structure. In Fig. 1 we show various snapshots of an evolved configuration in code units. Each time code unit for a boson mass \( m = 10^{-23}\)eV/c\(^2\) corresponds to \(1.58 \times 10^9\)yr, (for a detailed translation of all the units for a given boson mass see [5]).

The key advance in this paper is the contribution of the luminous matter to the total gravitational potential due to the BEC. In Fig. 2 we show the gravitational potential of a BEC halo with and without including the luminous matter, for a case with \( M_L = 10\% \) of the mass of the BEC configuration and a core radius of luminous matter \( r_n = 0.25R_{BEC.95} \), where \( R_{BEC.95} \) is the radius that contains \(95\%\) of the BEC matter integrated in the whole numerical domain.

A second and more demanding tests is the evolution of the same structure but this time in motion. The structure formation analysis in [1] indicates that high density cored clumps show the same profile as the solitonic solutions of (1), a property that was already shown to happen in [10]. However, one of the most interesting properties of solitonic solutions is that their density does not change with motion and in the most extreme case their profile is retained even after a collision with other soliton.

In the case of the present structure, the matter involved is not only that of the solution of (1) but also involves matter. We want to show how capable our tool is at evolving the initial
Figure 1. Snapshots of $\rho_{\text{BEC}}$ and the distribution of luminous matter $\rho_L$ measured using an SPH method at various times in code units. Both, the wave function and luminous particles are evolving, however remain in a nearly stationary distribution. For $m = 10^{-23} \text{eV}/\text{c}^2$ the snapshots correspond to 0, 3.16 and 6.32 millions of years.

Figure 2. Gravitational potential with and without considering the luminous matter for the parameters of Fig. 1. As expected, the contribution of luminous matter makes the potential well deeper. Also shown is a view from the top of the particles distribution on the $xy$ plane and isocontours of $\rho_{\text{BEC}}$. 
configuration when it has received a boost. The boost that is applied to the luminous matter is an addition to the peculiar randomized velocities of the particles of a given velocity, in our case along the $z$-direction. For the BEC it is slightly different, and the velocity is added as a phase of the wave function $\Psi$. Both, the phase for the wave function and the velocity of luminous particles in code units is $v_z = 1.5$. For a boson mass $m = 10^{-23}\text{eV}/c^2$ the velocity is 2160km/s, which is a very high speed for a galaxy.

In Fig. 3 we show snapshots of the luminous and BEC dark matter, showing the profiles remain nearly time independent, even when the boost is of such magnitude as to expect solitonic behavior according to [10]. In this case, the luminous matter distorts its initial shape and gets more compact. However, after an initial transient the shape stabilizes and evolves following the BEC.

![Figure 3. Snapshots of a boosted configuration. In this case the interplay deforms the gravitational potential, unlike the case in which $\rho_L$ does not source Poisson equation in [8].](image)

4. Final comments

We have presented the first BEC dark matter plus luminous matter evolution with full interaction, that is, interaction between the luminous matter, and interaction between the BEC dark matter and luminous matter. Our structure is spherically symmetric but contains all the ingredients for a generalization: randomized peculiar velocities of luminous particles and N-body type of interaction. In a second case, this structure is boosted in order to know whether or not the luminous matter follows the dark matter in a lonely structure scenario. Our result here indicates that luminous matter is a good dark matter tracker, although a more quantitative analysis is required.
This is a small step compared to the advancements in CDM models that include not only luminous particles playing the role of stars, but also include gas, radiation pressure and various other ingredients. Nevertheless, within the BEC dark matter model, this is a significant advance.

This type of initial data in a more general scenario of structure is expected to be very important since it includes now galaxies with a low mass to light ratio, where luminous matter cannot be considered test particles. It will also allow to study the behavior of luminous matter fully coupled during and after the collision of two BEC dark matter structures, which eventually will provide restrictions to the BEC dark matter model.

In order to describe some of the properties of the exampled worked out here, some properties are in turn. The mass of the ground state configuration for the boson mass $m = 10^{-23} eV/c^2$ is of 1.75 x $10^{10} M_\odot$ with a radius of 3.93kpc, parameters appropriate for a dwarf galaxy. More elaborate configurations of luminous matter and bigger galaxies would involve better BEC halo models, for which there are some possibilities like adding rotation that disperses away the density of the BEC [5] or multi-state configurations that consider various superposed modes of the solutions of the Gross-Pitaevskii equation and show a less compact BEC matter distribution [13].

Aside of the dark matter distribution, our approach is easily generalizable to more arbitrary distributions both, in the density and the velocity field. One would only need to break the isotropy of the case in this paper and use the same selection methods to populate the luminous particles and distribute the components of the velocity field corresponding to -for instance- those of a disk.

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6. Appendix: units
In order to show the units treatment and scaling laws of the system we follow [5]. One does not solve eq. (1) directly, before that it is important to remove the constants using the following change of variables $\tilde{\Psi} = \sqrt{\frac{4\pi G}{m c^2}} \Psi, \tilde{x}_i = \frac{m c^2}{\hbar} x_i$ (for cartesian coordinates $x_i$), $\tilde{t} = \frac{m c^2}{\hbar} t, \tilde{V} = \frac{V}{m c^2}, \tilde{a} \rightarrow \frac{c^2}{2m c^2} a$, so that the numerical coefficients $\hbar, h^2/m, 2\pi h^2/m^2, 4\pi G m$ do not appear in (1). Thus by fixing the boson mass $m$ the hatted variables are fixed. Moreover, the system (1) is invariant under the transformation $t = \lambda^2 \tilde{t}, x_i = \lambda \tilde{x}_i; \Psi = \Psi/\lambda^2, V = V/\lambda^2, a = \lambda^2 \tilde{a}$, for an arbitrary value of the parameter $\lambda$ [9]. This rescaling reduces the original system (1) to the following one

$$\frac{i}{\hbar} \frac{\partial \Psi}{\partial \tilde{t}} = -\frac{1}{2} \nabla^2 \Psi + V \Psi + a |\Psi|^2 \Psi$$

$$\nabla^2 V = |\Psi|^2, \quad \nabla^2 V = |\Psi|^2,$$

which is the one we solve numerically and is written in what we call code units. If one calculates one solution for the non-hatted variables, say for a given $\lambda$, other solutions are automatically found for any other value of $\lambda$, that is, a new solution for the hatted and tilde variables is found just by using a new value of this parameter.

The code and physical units are related once the scale invariance parameter $\lambda$ is fixed. This can be achieved by fixing for instance the mass of the configuration to the value of a given galaxy mass, or by choosing a spatial scale in given units. Here we choose the second approach following [5]. Specifically, since $\lambda = \frac{\hbar}{m c^2 \tilde{x}_i}$ (or equivalently the spherical coordinate $r$), if physical measurements of space $\tilde{x}_i$ are given in kpc, we choose the code coordinates $x_i$ to represent kpc.
as well. Thus it suffices to write the factor $\frac{\hbar}{m c}$ in kpc. For $m = 10^{-23}\text{eV}/c^2$ its value is

$$\lambda = \frac{\hbar}{m c} \left[\frac{\text{kpc}}{\text{[kpc]}}\right] \tilde{x} = 0.0006389.$$

In this paper we use only one value of the boson mass. If one chooses another value of the boson mass, say $m = 10^{-22}\text{eV}/c^2$ as suggested in [14], the effect would be that $\lambda = \frac{\hbar}{m c} \left[\frac{\text{kpc}}{\text{[kpc]}}\right] \tilde{x} = 0.006389$. With this, the spatial and time scales would change according to the scaling relations described above.

References

[1] H-Y Schive, T. Chiueh and T. Broadhurst, Nat. Phys. 10, 496 (2014).
[2] T. Harko, JCAP 1105, 22 (2011).
[3] D. J. E. Marsh and A-R. Pop, MNRAS, 451, 2479 (2015).
[4] L. A. Martínez-Medina and T. Matos, MNRAS 444, 185-191 (2014).
[5] F. S. Guzmán and F. D. Lora-Clavijo, Gen. Rel. Grav. 47, 21 (2015).
[6] S-J. Sin, Phys. Rev. D 50, 3650 (1994). S.U. Ji and S-J. Sin, Phys. Rev. D 50, 3655 (1994). F. E. Schunk, astro-ph/9802258. F. S. Guzmán, T. Matos and H. Villegas-Brena, Astron. Nachr. 320, 97 (1999). F. S. Guzmán and T. Matos, Class. Quantum Grav., 17, L9-L16 (2000). A. Arbey, J. Lesgourges, P. Salati, Phys. Rev. D 64, 123528 (2001). A. Arbey, J. Lesgourges, P. Salati, Phys. Rev. D 68, 023511 (2003). C. G. Boehmer and T. Harko, JCAP 0706, 025 (2007). L. A. Ureña-López and A. Bernal, Phys. Rev. D 82, 123535 (2010). F. S. Guzmán and F. D. Lora-Clavijo, Gen. Rel. Grav. 47, 21 (2015).
[7] A. Paredes and H. Michinel, Physics of the Dark Universe 12, 5055 (2016).
[8] F. S. Guzmán, J. A. González, J. P. Cruz, Phys. Rev. D 93, 103535 (2016).
[9] F. S. Guzmán and L. A. Ureña-López, Phys. Rev. D 69, 124033 (2004).
[10] A. Bernal and F. S. Guzmán, Phys. Rev. D 74, 103002 (2006).
[11] F. S. Guzmán and L. A. Ureña-López, ApJ 645, 814-819 (2006).
[12] S. J. Aarseth, M. Hénon and R. Wielen, A & A 37, 183-187 (1974). P. Hut and J. Makino, The Art of Computational Science: The Kali Code.
[13] L. A. Ureña-López and A. Bernal, Phys. Rev. D 82, 123535 (2010).
[14] S-R Chen, H-Y Schive, T. Chiueh, arXiv:1606.09030 [astro-ph.GA]