Iterated extended Kalman filter based state estimation of diode circuit

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Abstract. This paper presents the state estimation of diode circuit using iterated extended Kalman filter (IEKF). The root mean square error (RMSE) based performance evaluation gives the superiority of the IEKF based estimation over extended Kalman filtering (EKF) based method.

1. Introduction
Diode circuit such as single phase rectifier is widely used in different applications. It is used in voltage ac-dc power conversion voltage clamper [1], pulse width modulation rectifier [2]. The rectifier in combination with dc link circuit and inverter is used for railway electrical traction system [3]. It is also included in power supply [4], power networks [5] and automotive applications [6].
State estimation is important for proper working of circuits as the parameters change with environmental conditions. Estimation of state and parameters is essential for integrated circuit simulation, device modeling and device structure development. It is also required for designing of analog circuits and ICs. Various methods such as recursive least square (RLS) method [7], maximum likelihood estimator [8] and Kalman filter (KF) [9] etc. have been used for parameter estimation in recent years. The disadvantage of Kalman filter is that it can be implemented to linear models only. This constraint has been removed in extended Kalman filter [10]. Sliva et al. [11] proposed an adaptive model of photovoltaic (PV) cell that uses eight parameters. They also proposed pattern search-based optimization algorithm that provides accurate estimation in different environmental conditions. Sahu et al. [12] proposed the maximum power point estimation of PV using Levenberg Marquart method. They also studied the effect of temperature change and irradiation on parameter estimation. Moshksar et al. [13] used convex optimization for parameter estimation of single diode PV system that uses a reduced diode model by expressing three unknown parameters in terms of the other two parameters. The advantage of this method is that it gives precise and unique solution. Cardenas et al. [14] proposed parameter extraction of single diode PV model using I-V characteristics and reduced space search method. Peng et al. [15] investigated the three evolutionary algorithms to improve the conversion efficiency of solar cells. They compared these algorithms in terms of speed of calculation, accuracy and anti-noise capability. Takao et al. [16] proposed loss estimation method of power converters for hybrid circuit of insulated gate bipolar transistor and silicon carbide diode. EKF is applied for state estimation of nonlinear analog circuits [17]-[18]. IEKF has been used for estimation purpose in various applications [19]-[24]. Karamali et al. [19] proposed IEKF design to estimate temperature profile of collector field of solar power plant. They also used EKF which helps to overcome the nonlinear properties of the system by decreasing the disturbance effects and noise of the system. Enayati et al. [20] proposed IEKF in combination with the RLS method for parameter estimation of harmonics of power system’s distorted signal. The superiority of the method is that it can be used for online mode and in presence of large noise also. This method provides good
performance for power quality and protection also. Fang et al. [21] used predictive filter to propose modified IEKF. They used modified IEKF method for state estimation to integrate inertial navigation system and global positioning system. Tian et al. [22] proposed a distributed iterated extended Kalman filter (DIEKF) to estimate speaker’s positions in array of microphone networks. The advantage of their method is that it provides accurate and smooth trajectory of the speaker’s motion. Also, the method is robust to noise and resonant environment. In [23], Zhao et al. proposed the generalized maximum likelihood based IEKF algorithm called GM-IEKF combined with generalized maximum likelihood approach (GM-IEKF). They used it to estimate the state dynamics of power system. The main advantage of GM-IEKF is that it has fast tracking of system transients as compared to EKF and unscented Kalman filter (UKF). Hu et al. [24] proposed stochastic nonlinear system using a generalized iterated Kalman filter (GIKF). It is used for system in which noise is multiplicative function of noise. They also derived important parameters for GIKF. They showed the reliability of the proposed method by evaluating the method with EKF.

In this work, we developed the state space model for the diode circuit. This model has two state variables e.g. capacitor voltage and diode current namely \( v_c(t) \) and \( i_d(t) \) respectively. The \( v_c(t) \) and \( i_d(t) \) have been estimated using IEKF. We compared the estimated states using IEKF with EKF and PSPICE simulated values to illustrate the reliability of the proposed method.

The structure of the paper is as follows. Section 2 and 3 present brief introductions of EKF and IEKF algorithm. The state space modeling of diode circuit is presented in Section 4. Section 5 gives simulation results. Finally, Section 6 gives concluding remarks.

2. Extended Kalman filter

In general, a discrete time nonlinear system can be represented as:

\[
\begin{align*}
    x_{n+1} &= \varphi_n(x_n, u_n) + G_n w_n \tag{1} \\
    z_n &= h_n(x_n) + v_n \tag{2}
\end{align*}
\]

where \( n \) represents discrete time index. \( \varphi_n(\cdot) \in \mathbb{R}^N \) and \( h_n(\cdot) \in \mathbb{R}^M \) denote the time variant nonlinear functions. All vectors have the following assumptions:

- System state: \( x_n \in \mathbb{R}^N \)
- Initial state: \( x_0 \in \mathbb{R}^N \)
- Input: \( u_n \in \mathbb{R}^N \)
- System noise: \( w_n \in \mathbb{R}^N \)
- Measurement: \( z_n \in \mathbb{R}^M \)
- Measurement noise: \( v_n \in \mathbb{R}^M \).

All the above variables are random excluding \( u_n \), which is deterministic. Here, \( w_n \) and \( v_n \) are uncorrelated to one another with their covariances \( C_{w,n} \) and \( C_{v,n} \) respectively. EKF is a nonlinear version ofKF and has been designed for nonlinear state estimation. It includes updation in time and measurement at time \( n \) after initialization. EKF linearizes the nonlinear function \( \varphi_n(\cdot) \) and \( h_n(\cdot) \) using Taylor series. (3) and (4) have been approximated using Taylor series expansion as:

\[
\begin{align*}
    x_{n+1} &\approx \varphi_n(\hat{x}_n|n) + F_n \delta_n + \text{Higher terms} \tag{3} \\
    z_n &\approx h_n(\varphi_n(\hat{x}_n|n)) + H_n \delta_n + \text{Higher terms} \tag{4}
\end{align*}
\]

where \( \delta_n = x_n - \hat{x}_n \). \( \delta_n \) denotes the priori estimated error with covariance \( \Sigma_n \) at time \( n \). Table 1 summarizes the EKF steps.

3. Iterated Extended Kalman filter

The IEKF technique does the same linearization as the EKF method for \( \varphi_n(\cdot), \hat{x}_{n+1|n} \) and \( \Sigma_{n+1|n} \). The only difference is that in IEKF, linearization of \( g_n(\cdot) \) is based on the updated state estimate \( \hat{x}_{n+1|n} \) rather than the predicted state estimate \( \hat{x}_{n+1|n+1} \). In IEKF, \( v_n \) is formulated as:

\[
    v_n = g_n(z_n, x_n) \tag{5}
\]

where \( g_n(\cdot) \) is nonlinear function. In addition to additive white noise, measurement model becomes
\[ z_n = h_n(x_n) + \zeta_n v_n \]  \hspace{1cm} (6)

Therefore

\[ v_n = \zeta_n^{-1}(z_n - h_n(x_n)) \]  \hspace{1cm} (7)

where \( \zeta_n \) denotes the invertible matrix. IEKF steps have been mentioned in Table 2.

| Table 1. Summary of EKF algorithm |
|-----------------------------------|
| **Step1. Initialization:** Compute initial estimate \( \hat{x}_0 \), initial error covariance \( \Sigma_0 \), process noise covariance \( \Sigma_w \), and observation noise covariance \( \Sigma_v \) |
| **Step 2. State prediction:** Compute Jacobian matrix \( F_n \) and linearized error model \( H_n \) |
| **Step 3. Measurement update:** Compute Kalman gain \( K_n \) and update estimated state \( \hat{x}_{n+1|n+1} \) |

| Table 2. Summary of IEKF algorithm |
|-----------------------------------|
| **Step1. Initialization:** Compute initial estimate \( \hat{x}_0 \), initial error covariance \( \Sigma_0 \), process noise covariance \( \Sigma_w \), and observation noise covariance \( \Sigma_v \) |
| **Step 2. Prediction:** Compute estimated state \( \hat{x}_{n+1|n} \) and covariance \( \Sigma_{n+1|n} \) |
| **Step 3. Update step:** Compute update step \( \hat{x}_{n+1|n+1} \) and \( \Sigma_{n+1|n+1} \) |

4. **Modeling of Diode Circuit Using State Space Model**

Figure 1 shows the circuit diagram of a diode circuit. The sinusoidal signal is used for input voltage \( v_i(t) \), which is in series with the inductor \( L_s \) and resistor \( R_s \). \( C \) is a blocking capacitor. \( R_l \) is the load resistance. Diodes \( D_1 \) to \( D_4 \) are assumed to be identical diode with voltage drop \( v_D \). \( i_D(t) \) and \( v_c(t) \) are the diode current flowing in circuit and voltage drop across the capacitor \( C \) respectively.

![Diode circuit diagram](image-url)
Using the Kirchhoff’s laws, we have

\[ C \frac{d}{dt} v_C(t) + \frac{v_C(t)}{R_s} = i_D(t) \]  \hspace{1cm} (8)

\[ v_i(t) = R_s i_D(t) + L_s \frac{d}{dt} i_D(t) + 2v_D + v_C(t) \]  \hspace{1cm} (9)

where \( i_D(t) = I_0 (e^{v_C/v_D} - 1) \).

Representing equations (8) and (9) in terms of state equations, we have

\[ \frac{d}{dt} v_C(t) = -\frac{1}{R_s C} v_C(t) + \frac{1}{C} i_D(t) \]  \hspace{1cm} (10)

\[ \frac{d}{dt} i_D(t) = -\frac{1}{L_s} v_C(t) - \frac{(R_s + 2V_T / I_0)}{L_s} i_D(t) \]

\[ + \frac{V_T}{L_s I_0^2} i_D^2(t) - \frac{1}{L_s} v_C(t) \]  \hspace{1cm} (11)

Here, \( V_T \) and \( I_0 \) are the thermal voltage and reverse saturation current of diode respectively.

Representing (10) and (11) in state space form as:

\[ \frac{d}{dt} x(t) = F(x(t)) + B v_C(t) + w(t) \]  \hspace{1cm} (12)

where \( x(t) \) is the state vector consisting of two state variables \( v_C(t) \) and \( i_D(t) \) respectively.

\[ F(x(t)) = \begin{bmatrix} \frac{1}{R_s C} & \frac{1}{C} \\ \frac{1}{L_s} & -\frac{(R_s + 2V_T / I_0)}{L_s} \end{bmatrix} \]

\[ B = \begin{bmatrix} 0 & 0 \\ \frac{V_T}{L_s I_0^2} & \frac{1}{L_s} \end{bmatrix} \]  \hspace{1cm} (13)

The right side of (13) represents the linear and nonlinear part respectively. The measurement model is:

\[ z(t) = H(x(t)) + v(t) \]  \hspace{1cm} (15)

where \( H = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \). The discrete time equations are:

\[ x_{n+1} = F_n x_n + B_n u_n + w_n \]  \hspace{1cm} (16)

\[ z_n = H_n (x_n) + v_n \]  \hspace{1cm} (17)

The matrices \( F_n \), \( B_n \) and \( H_n \) are

\[ F_n = \begin{bmatrix} 1 - \frac{T_s}{R_s C} & \frac{T_s}{C} \\ -\frac{T_s}{L_s} & 1 - T_s \left( \frac{R_s + 2V_T / I_0}{L_s} + \frac{2V_T^2}{I_0^2} \right) \end{bmatrix} \]

\[ B_n = \begin{bmatrix} 0 \\ \frac{T_s}{L_s} \end{bmatrix} \] and \( H_n = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \)

5. Simulation results

The simulations have been performed in MATLAB software. IEKF estimates the capacitor voltage and diode current. IEKF based estimates are compared with EKF based method. A sinusoidal input of 10 volts and frequency 50 Hz has been used at input for simulation purpose as shown in Figure 2. The system and measurement noise used are white Gaussian with zero mean and variance 0.5 and 0.01 respectively. The thermal voltage (\( V_T \)) and saturation current (\( I_0 \)) are 0.025 volts and \( 10^{-9} \) amperes respectively. The diode model used for PSPICE simulation is D1N4002. The circuit component values are: \( R_s = 17.5 \Omega, R_l = 750 \Omega, L_s = 91.9 \text{ mH} \) and \( C = 100 \mu \text{F} \). Figure 3 to 10 show the comparison of
estimated capacitor voltage and diode current using IEKF with EKF and PSPICE simulated values. The PSPICE simulated values are used as the actual value. The root mean square error (RMSE) and signal to noise ratio (SNR) parameters have been used to evaluate the performance of IEKF method. Table 3 and 4 present the comparison of the estimated parameters using IEKF method with EKF method and PSPICE simulated values. The estimated currents are identically mapped to the transients over the first half period, whereas deviations are noted between the estimated and actual values in succeeding half periods due to change in initial value of the capacitor voltage.

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{n}(\hat{y}_i - y_i)^2}{n}}
\]

\[
SNR = \frac{\sum_{i=1}^{n}(\hat{y}_i)^2}{\sqrt{\sum_{i=1}^{n}(\hat{y}_i - y_i)^2}}
\]

where \( \hat{y} \) = estimated value, \( y \) = actual value, and \( n \) = total number of observations.

Figure 2. Input sinusoidal signal.

Figure 3. Estimated voltage using IEKF and EKF method for noiseless input signal.
Figure 4. Estimated voltage using IEKF and EKF method for noisy input signal (zero mean and variance 0.1).

Figure 5. Estimated voltage using IEKF and EKF method for noisy input signal (zero mean and variance 0.5).

Figure 6. Estimated voltage using IEKF and EKF method for noisy input signal (zero mean and variance 1.0).
Figure 7. Estimated current using IEKF and EKF method for noiseless input signal.

Figure 8. Estimated current using IEKF and EKF method for noisy input signal (zero mean and variance 0.1).

Figure 9. Estimated current using IEKF and EKF method for noisy input signal (zero mean and variance 0.5).
Figure 10. Estimated current using IEKF and EKF method for noisy input signal (zero mean and variance 1.0).

Table 3. Comparison of SNR (dB) for $v_C$ and $i_D$ estimation.

| Input signal with variance ($\sigma^2$) | $v_C$       | $i_D$       | $v_C$       | $i_D$       |
|---------------------------------------|-------------|-------------|-------------|-------------|
|                                       | Using IEKF  | Using EKF   | Using IEKF  | Using EKF   |
| 0.0                                   | 1.42        | 1.07        | 1.55        | 1.29        |
| 0.1                                   | 1.39        | 1.01        | 1.20        | 1.05        |
| 0.5                                   | 1.15        | 1.00        | 1.16        | 1.01        |
| 1.0                                   | 1.06        | 0.90        | 1.10        | 1.00        |

Table 4. Comparison of RMSE for $v_C$ and $i_D$ estimation.

| Input signal with variance ($\sigma^2$) | $v_C$       | $i_D$       | $v_C$       | $i_D$       |
|---------------------------------------|-------------|-------------|-------------|-------------|
|                                       | Using IEKF  | Using EKF   | Using IEKF  | Using EKF   |
| 0.0                                   | 0.24        | 0.86        | 0.19        | 0.336       |
| 0.1                                   | 0.50        | 0.96        | 0.40        | 0.66        |
| 0.5                                   | 0.89        | 1.01        | 0.50        | 1.56        |
| 1.0                                   | 1.10        | 1.57        | 1.53        | 1.82        |

6. Conclusions
The diode states are estimates using IEKF based method. These estimations are compared with EKF based estimations. Simulation results validate the superiority of IEKF based estimation over the EKF based method, as the linearization error is taken into account by IEKF method.

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