In this letter we show novel features of the neutrino halo [1] that may impact cutting edge research on neutrino signals and nucleosynthesis in compact objects and core-collapse supernovae (CCSNe). During the collapse and subsequent supernova explosion of a massive progenitor star, some $\sim 3 \times 10^{52}$ erg of energy is released as neutrino radiation, a tiny fraction of which undergoes wide angle scattering to form a halo of neutrinos on trajectories which do not originate on the proto-neutron star (PNS) at the core.

Wide angle neutrino scattering is also responsible for electron lepton number (ELN) crossings, where the net lepton number of neutrinos emitted along a given trajectory changes sign. ELN crossing is understood to be key to the phenomenon of fast flavor conversion (FFC) [2,3] which are of great theoretical interest due to the potential to alter the flavor evolution of the entire neutrino population and that correcting for time of flight can produce conditions which may trigger fast neutrino flavor conversion. We also find that there exists a window of time early in all CCSNe where the neutrino halo population is sufficiently small that it may be negligible. This suggests that collective neutrino oscillation calculations which neglect the Halo may be well founded at sufficiently early times.

We argue that the neutrino halo, a population of neutrinos that have undergone direction-changing scattering in the stellar envelope of a core-collapse supernova (CCSNe), is sensitive to neutrino emission history through time of flight. We show that the constant time approximation, commonly used in calculating the neutrino halo, does not capture the spatiotemporal evolution of the halo neutrino population and that correcting for time of flight can produce conditions which may trigger fast neutrino flavor conversion.

In analogy to the technique used in classical electrodynamics, we identify two “retarded” time scales relevant to enforcing causality between the radially emitted neutrino, $\nu_i$, and the halo neutrino, $\nu_k$: the time elapsed $t_{\nu_k} = |\vec{r} - \vec{r}_k| / c$.
since $\nu_k$'s scattering, $t_s$, and the total time elapsed since $\nu_k$'s emission, $t_r$. Within CCSNe, emitted neutrino luminosities, flavors, neutrino spectral energy distributions, entropy and mean nuclear masses, shock propagation, and turbulent mixing all can evolve on short $\sim 10 - 100$ ms time scales. The retarded timescales are significant so long as either of two conditions are met: that the timescale for evolution of neutrino emission from the PNS at the time of emission, $t_{em}$, is short compared to $t_s$, or that the timescale for the evolution of the density and composition of the envelope of CCSNe at $\vec{r}$ is short compared to $t_s$. In either of these cases, the CT approximation will lead to a-causal calculations of the distribution of halo neutrinos arriving at $\vec{r}$.

The ToF corrected expression for the number density of neutrinos in the halo (omitting factors which have no temporal dependence) is,

$$n_{\nu} (\vec{r}, \theta_{ik}, t_{em}) \propto \int dE_k d^3\vec{r} f_{\nu} (E_k, t_{em} - t_s) \rho (\vec{r}, t_{em} - t_s) \times \mathcal{M} [E_k, \vec{r} - \vec{r}', A (\vec{r}', t_{em} - t_s), Z (\vec{r}', t_{em} - t_s)]$$

(1)

where $E_k$ is the neutrino energy, $f_{\nu} (E_k, t)$ is the emitted neutrino spectral energy distribution, the envelope matter density is $\rho (\vec{r}, t)$, the nuclear composition is given by the average atomic number $Z (\vec{r}, t)$ and average nucleon number $A (\vec{r}, t)$, and $\mathcal{M}$ is the scattering kernel. Previous work characterizing the effects of the halo neutrinos have treated Equation[1] using the CT approximation, taking $t = t_{em}$ identically [1, 25, 27-37]. Similarly, prior work considering FFC [9-20, 23, 24] made use of 1D or ray-by-ray Boltzmann neutrino transport schemes which include neutrino propagation time delays only in 1D (along the axis of the ray). This approach explicitly omits any ToF correction for neutrino transport ray-to-ray, implicitly employing the CT approximation at first order in the number of direction changing scatterings to compute neutrino distributions within each ray.

Calculating our ToF corrected neutrino number densities using the discrete time evolution of several CCSNe progenitor models [27, 38, 40], we can directly construct the neutrino-neutrino Hamiltonian, $H_{\nu \nu}$, which couples the flavor histories for neutrinos on intersecting trajectories [29, 47, 48]. As shown in Fig.[1] a neutrino $\nu_i$ leaving the core will experience a potential given by a sum over neutrinos located at the same point as neutrino $\nu_i$:

$$H_{\nu \nu} = \sqrt{2} G_{\nu \nu} \sum_\alpha \int (1 - \cos \theta_{ik}) \times$$

(2)

$$|n_{\nu_{\alpha}} (\theta_{ik}) |\psi_{\nu_{\alpha}} \rangle \langle \psi_{\nu_{\alpha}} | - |n_{\bar{\nu}_{\alpha}} (\theta_{ik}) |\psi_{\bar{\nu}_{\alpha}} \rangle \langle \psi_{\bar{\nu}_{\alpha}} |] d \cos \theta_{ik}$$

where the flavor state of neutrino $\nu_{\alpha}$ is represented by $|\psi_{\nu_{\alpha}} \rangle$ and $\theta_{ik}$ is the angle of intersection between $\nu_i$ and neutrino or antineutrino $\nu_k/\bar{\nu}_k$. Here, $n_{\nu_{\alpha}}$ is the local number density of neutrinos in state $\alpha$, and the $1 - \cos \theta_{ik}$ factor disfavors small intersection angles, thereby suppressing the contribution of the forward-scattered-only neutrinos [39, 40]. Halo neutrinos may have larger intersection angles as shown in Fig.[1] and therefore can contribute significantly to the flavor-changing potentials despite their small numbers.

In the mean-field, coherent approximation, neutrino flavor evolution is governed by the equation $i \hbar \partial |\psi_{\nu_{\alpha}} \rangle / \partial t = [H_{\nu \nu}, |\psi_{\nu_{\alpha}} \rangle]$, where $t$ is the Affine parameter along $\nu_i$’s world line, and $H = H_V + H_e + H_{\nu \nu}$ is the appropriate neutrino propagation Hamiltonian, with vacuum and matter components $H_V$ and $H_e$, respectively. At any point within the envelope, $H_{\nu \nu}$ can be split into two pieces: the contribution from outward directed neutrinos $H_{\nu \nu}^\text{out} = [H_{\nu \nu}]^\text{out}_{0}$, and the contribution from inward directed neutrinos $H_{\nu \nu}^\text{in} = [H_{\nu \nu}]^\text{in}_{1/2}$, with $H_{\nu \nu} = H_{\nu \nu}^\text{out} + H_{\nu \nu}^\text{in}$. We make this split because of the intrinsic asymmetry of neutrino emission in the CCSNe environment, with the radially directed neutrino emission from the PNS core providing the preponderance of neutrino number density in the envelope.

The extent to which CT is a reasonable approximation can be quantified by comparing the ratio $|H_{\nu \nu}^\text{in}|/|H_{\nu \nu}^\text{out}|$ as a function of $(r, t_{em})$ to the same quantity when calculating the halo neutrino population including the ToF correction in Equation[1]. Fig.[2] shows the result of this comparison for a single time snapshot for neutrinos emitted 5 ms post core bounce for a CCSN simulation of a 40M⊙ progenitor star [38]. We can see that the CT approximation significantly overestimates the magnitude of $H_{\nu \nu}^\text{in}$ at this early time. Under the CT approximation, the entirety of the envelope is taken to be scattering neutrinos into the halo assuming the neutrino emission luminosity is $L_\nu (5\text{ ms})$, which is quite large early in the neutrino burst. During the core infall epoch neutrino emission which populates the halo is considerably less luminous and lower energy. From Fig.[1] it can be seen...
that the volume of the stellar envelope which is illuminated by the neutrino burst at $t_{\text{em}} = 5\,\text{ms}$ is greatly reduced, bounded spatially and temporally by the requirement that $t_e \leq 5\,\text{ms}$. Taking the PNS to be a point source in Fig. 1, the region of the stellar envelope which is contributing to the halo neutrino population which was emitted from the core at time $t_{\text{em}} - t_e$ is given by the ellipsoid of revolution with the PNS as one focus and the position $\vec{r}$ as the second focus. The semi-major axis of this ellipsoid is then $a = c t_e / 2$ with eccentricity $e = |\vec{r}| / c t_e$. As the eccentricity of this ellipsoid increases, the discrepancy between CT and ToF calculations of each shell’s contribution to $|H_{\nu\nu}^\text{in}| / |H_{\nu\nu}^\text{out}|$ will grow.

Fig. 2 shows that the infall epoch neutrino emission from the core, emitted in the time between the onset of gravitational instability and core-bounce at nuclear densities ($\sim 0.5\,\text{s}$), contributes to the halo neutrino population, accounting for an overall magnitude of $\sim (\text{a few})\%$ of the ratio $|H_{\nu\nu}^\text{in}| / |H_{\nu\nu}^\text{out}|$. This raises the point that the ToF correction for the halo neutrino potential is coupled to the progenitor star and its subsequent CCSN evolution. Progenitor properties such as pre-collapse mass, neutronization history, core-compactness, composition and density structure play a role in infall neutrino emission and pre-population of the halo neutrinos. After core bounce the hydrodynamic evolution of the CCSN affects the emitted neutrino luminosity history through accretion and through halo neutrino scattering on nuclei/nucleons in the envelope. Shock propagation and turbulence influences these issues.

Thus the impact of ToF on the neutrino flavor transformation can be affected by the progenitor structure. A generic effect of ToF at early times is to reduce the ratio $|H_{\nu\nu}^\text{in}| / |H_{\nu\nu}^\text{out}|$ by reducing the volume of brightly illuminated stellar envelope contributing to $H_{\nu\nu}^\text{in}$ ahead of the ellipsoid focus at $\vec{r}$. So long as $|H_{\nu\nu}^\text{in}| / |H_{\nu\nu}^\text{out}|$ is much less than unity, it may be reasonable to treat the flavor evolution of neutrinos using established techniques. The ratio must inevitably rise as more of the stellar envelope is illuminated with high luminosity neutrino radiation. To quantify the window of time in which ToF reduction of the magnitude of $H_{\nu\nu}^\text{in}$ is significant in the calculation of neutrino flavor evolution, we define the timescale $t_{\nu\nu}(1)$ to be the minimum time at which $|H_{\nu\nu}^\text{in}| / |H_{\nu\nu}^\text{out}| \geq 1$ for any radius $\vec{r}$ in the stellar envelope.

Table I shows the results of this calculation for a variety of progenitors of varying mass. At the low end of the mass scale, 8.8 $M_\odot$ [39, 52] and 9.6 $M_\odot$ [27, 42], we find that with ToF corrections the halo neutrino population is not sufficient to push $|H_{\nu\nu}^\text{in}| / |H_{\nu\nu}^\text{out}| \geq 1$ over the course of the simulation, and so these simulations potentially have $t_{\nu\nu}(1) \rightarrow \infty$. This opens the possibility that conventional techniques may be used to calculate the neutrino flavor transformation for stars in this mass range.

As progenitor mass increases, $t_{\nu\nu}(1) = 95\,\text{ms}$ for a 10.8 $M_\odot$ progenitor [38, 41] and rapidly converges to $t_{\nu\nu}(1) \sim 60\,\text{ms}$ for higher mass progenitors [38, 40, 43, 46, 53]. The near independence of $t_{\nu\nu}(1)$ on progenitor mass seems to come from a counter balance of competing effects. The more massive progenitors tend to be more centrally condensed prior to collapse, which increases the baryon (and scattering target) density near the PNS and increases $|H_{\nu\nu}^\text{in}|$. This same increase in central condensation tends to increase the neutrinosphere radius, $R_{\nu}$, through rapid accretion on the PNS. Because $|H_{\nu\nu}^\text{out}| \propto R_{\nu}^2$ for neutrinos which stream out of the envelope, this buoys the magnitude of the outward directed Hamiltonian contribution. It is not apparent why these two effects should cancel, but it is intriguing that the high mass progenitor models in Table I show similar $t_{\nu\nu}(1)$ timescales.

Another feature of the ToF corrected neutrino halo population can be seen when comparing maps of the ratio $H_{\nu\nu}^\text{in} / H_{\nu\nu}^\text{out}$ (without absolute values) between ToF and CT calculations, 2D examples of which are shown in Fig. 3 for the $t_{\text{em}} = 55\,\text{ms}$ snapshot of the 11.2 $M_\odot$ progenitor CCSN simulation. We expect disagreement between CT and ToF to be larger early in the CCSN history because of ToF reduction in the illuminated volume of the envelope, and greater at large radius because of the increased eccentricity of the ellipsoid which populates the ToF corrected halo neutrino population. Both of these trends can be seen in Fig. 3 which disagree by so much as the overall sign of $H_{\nu\nu}^\text{in} / H_{\nu\nu}^\text{out}$ ($r$) beyond $\sim 4500\,\text{km}$.

This sign change in $H_{\nu\nu}^\text{in} / H_{\nu\nu}^\text{out}$ is a new phenomenon not found within the framework of the CT approximation. Because of the energy dependence of neutrino direction changing scattering processes (e.g., the neutral current scattering cross section $\propto G_F^2 (E_\alpha^2)$), it is possible under the CT approximation that $H_{\nu\nu}^\text{in}$ has a relative sign difference compared to $H_{\nu\nu}^\text{out}$ if,

$$\text{Sign} \sum_{\alpha} \left[ \frac{L_{\alpha}}{\langle E_\alpha \rangle} - \frac{L_{\bar{\alpha}}}{\langle E_{\bar{\alpha}} \rangle} \right] \neq 0,$$

and

$$\text{Sign} \sum_{\alpha} \left[ \frac{L_{\alpha} G_F^2 (E_\alpha^2)}{\langle E_\alpha \rangle} - \frac{L_{\bar{\alpha}} G_F^2 (E_{\bar{\alpha}}^2)}{\langle E_{\bar{\alpha}} \rangle} \right] \sigma,$$

(3)

where $L_{\alpha}/\bar{\alpha}$ is the neutrino luminosity for each flavor and $\sigma$ is the integrated column density of scattering targets (which also scales with the composition of scattering targets through the mean nuclear mass squared dependence of coherent neutral current scattering). Here the left hand side is the limiting case for the radially emitted net lepton number, and the right hand side is proportional to the limiting case for the wide angle scattered neutrino net lepton number. Satisfaction of Eq. (3) is sufficient to guarantee the existence of an ELN crossing in the outward directed neutrinos. However, because the

| Mass ($M_\odot$) | 8.8 | 9.6 | 10.8 | 11.2 | 15.0 | 40.0 | 50.0 |
|----------------|-----|-----|------|------|------|------|------|
| $t_{\nu\nu}(1)$ (ms) | > 335 | > 230 | 95 | 53 | 64 | 60 | 66 |
The relative sign change in $H_{\nu\nu}^{in}/H_{\nu\nu}^{out}$ as a function of position in the envelope for a 11.2 $M_\odot$ progenitor star 55 ms after core bounce. Left panel shows results for the CT approximation. Center panel shows results for the ToF corrected halo. Right panel: ToF corrected ELN crossing angle for $\nu_e - \bar{\nu}_e$ for the same 55 ms post core bounce snapshot. The ELN crossing angle, $\theta$, is defined such that 0 is radially outward and $\pi$ is radially inward.

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