Gravitational Lensing Statistics as a Probe of Dark Energy

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Abstract

By using the comoving distance, we derive an analytic expression for the optical depth of gravitational lensing, which depends on the redshift to the source and the cosmological model characterized by the cosmic mass density parameter $\Omega_m$, the dark energy density parameter $\Omega_x$ and its equation of state $\omega_x = p_x/\rho_x$. It is shown that, the larger the dark energy density is and the more negative its pressure is, the higher the gravitational lensing probability is. This fact can provide an independent constraint for dark energy.

I. INTRODUCTION

The standard cosmological model is based on three cornerstones: the Hubble expansion, the Cosmic Microwave Background Radiation (CMBR) and the primordial Big Bang Nucleosynthesis. Now these three kinds of observations, such as Hubble’s relation for fifty-some Ia type supernovae (SNeIa) out to redshifts of nearly one [1], the anisotropy of the CMBR [2] and the deuterium abundance measured in four high redshift hydrogen clouds seen in absorption against distant quasars [3] (combined with baryon fraction in galaxy clusters from [4]).

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X-ray data [4]), have made a strong case for the existence of a nearly uniform component of dark energy with negative pressure. It seems that determining the amount and nature of the dark energy is emerging as one of the most important challenges in cosmology.

One apparent plausible candidate for the dark energy is the cosmological constant or vacuum energy density [5,6]. The possibility of a nonzero cosmological constant Λ has been advocated and then discarded several times in the past for theoretical and observational reasons [7]. Due to this checked history and the difficulty in understanding the observed Λ in the framework of modern quantum field theory [8], Most physicists and astronomers believe that the cosmological constant should be zero because of some unknown physics. Other candidates for the dark energy include: a frustrated network of topological defects (such as strings or walls) [8] and an evolving scalar field (referred to by some as quintessence) [9]. As shown in literatures, it is difficult to discriminate well against these different possibilities either by the SNIa data alone [10] or only by the CMBR data [11]. This led some authors to consider the combination of the SNIa measurements with the anisotropy of CMBR [12] or the large scale structure [13].

In this paper, an independent means for probing the amount and nature of dark energy is proposed. It relies on the gravitational lensing statistics, which has been shown to be an efficient tool for determining the cosmological parameters [14]. Some authors even have given the general expressions for the optical depth and mean image separation in general Friedman-Robertson-Walker (FRW) cosmological models [15]. But these expressions are complicated and thereby hard to apply in practice. By using the comoving distance we derive an analytic and simple expression for the optical depth of gravitational lensing that depends on the redshift to the source and the cosmological model characterized by the cosmic mass density parameter Ω_m, the dark energy density parameter Ω_x and its equation of state ω_x = p_x/ρ_x. It is shown that for a flat universe the lensing probability is very sensitive to Ω_x and ω_x, and hence provides an independent probe for the dark energy.
II. KINEMATICS WITH DARK ENERGY COMPONENT

We assume a homogeneous and isotropic universe with Robertson-Walker metric (in the $c = 1$ unit):

$$ds^2 = -dt^2 + R^2(t) \left[ d\chi^2 + f^2(\chi)(d\theta^2 + \sin^2 \theta d\phi^2) \right],$$

where $f(\chi) = \chi$ for a flat universe ($k = 0$), $f(\chi) = \sin \chi$ for a closed universe ($k = +1$), and $f(\chi) = \sinh \chi$ for an open universe ($k = -1$). Defining the scale factor $a(t) = R(t)/R_0$, $a = 1$ today, and the Friedmann equation takes the form

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \sum_i \rho_i - \frac{k}{a^2 R_0^2},$$

where $i$ includes all components of matter and energy in the universe. If the effective equation of state for the $i$-component is parametrized as $\omega_i = p_i/\rho_i$, its density scales as $\rho_i \propto a^{-n_i}$ where $n_i = 3(1 + \omega_i)$. For instance, nonrelativistic matter scales as $\rho_m \propto a^{-3}$ while relativistic matter, such as radiation, changes as $\rho_r \propto a^{-4}$, and vacuum energy (cosmological constant) is invariant ($\rho_\Lambda \equiv \Lambda/(8\pi G) \propto a^0$) as the universe expands. For the purpose of studying gravitational lensing statistics, we need only consider two important components: one is the nonrelativistic matter ($\rho_m$), the other is a nearly uniform dark energy ($\rho_x, \omega_x = p_x/\rho_x$) with a constant $\omega_x$. In order to avoid interfering with structure formation, the dark energy component must be less important in the past than matter although it may dominates the universe today. So $\omega_x$ and hence the pressure of dark energy must be negative [16]. Defining

$$\Omega_m = \frac{8\pi G}{3H_0^2} \rho_{m0}, \quad \Omega_x = \frac{8\pi G}{3H_0^2} \rho_{x0}, \quad \Omega_k = \frac{-k}{R_0^2 H_0^2},$$

where $H_0$ is the Hubble constant, $\rho_{m0}$ and $\rho_{x0}$ are the nonrelativistic matter density and the dark energy density at present respectively, then Eq.2 becomes

$$\frac{1}{a} \frac{da}{dt} = H_0 \left( \frac{\Omega_x}{a^{3(1+\omega_x)}} + \frac{\Omega_m}{a^3} + \frac{\Omega_k}{a^2} \right)^{1/2}.$$  

Note that we have a relation, $1 = \Omega_x + \Omega_m + \Omega_k$. The distance a light ray travels can be calculated as following. Light rays follow null geodesics where $ds^2 = 0$ so that $dt^2 = R_0^2 a^2 d\chi^2$. 

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It has been shown that the natural cosmological distance for the analysis of gravitational lensing statistics is the comoving distance \([19]\). With Eq. (4) and the relation \(a(t) = (1 + z)^{-1}\), the comoving distance is

\[
\chi = \begin{cases} 
\int_0^z \frac{dz}{\sqrt{\Omega_x(1+z)^3(1+\omega_x) + (1-\Omega_x)(1+z)^3}} & (k = 0) \\
|\Omega_k|^{1/2} \int_0^z \frac{dz}{E(z)} & (k = \pm 1)
\end{cases}
\]

(5)

\(E(z) \equiv \sqrt{\Omega_x(1+z)^3(1+\omega_x) + \Omega_m(1+z)^3 + \Omega_k(1+z)^2}\).

The comoving volume \(dV\) of the shell \(d\chi\) at \(\chi\) reads

\[
dV = 4\pi R^3(t_0)f^2(\chi)d\chi.
\]

(6)

### III. OPTICAL DEPTH OF GRAVITATIONAL LENSING IN GENERAL FRW COSMOLOGIES

Now, let’s calculate the optical depth of gravitational lensing by all galaxies with different luminosities and redshifts in the universe. Following Turner et al. \([17]\), we model the mass density profile of a galaxy matter as the singular isothermal sphere (SIS) parameterized by its velocity dispersion \(\sigma\). The dimensionless cross-section of multiple images for a point source located at \(z_s\) produced by a single SIS galaxy at \(z_d\) is \([15,19]\)

\[
\hat{\sigma} = 16\pi^3\sigma^4 \frac{f(\chi_s - \chi_d)}{f(\chi_s)}^2.
\]

(7)

By accumulating the contributions of various galaxies, the total dimensionless cross-section by galaxies at redshifts ranging from 0 to \(z_s\) for distant sources like quasars at \(z_s\) is

\[
\hat{\Sigma}(z_s) = 4\pi \times F \times T(z_s), \quad F = \sum_{i=E,S0,S} F_i.
\]

(8)

The parameter \(F_i\), which depends only on the intrinsic and statistical properties of galaxies in the universe, represents the effectiveness of the \(i\)-th morphological type of galaxies in
producing multiple images \[17\]. Due to its uncertainties discussed widely in literatures \[14,15,19\], we treat \(F\) as a normalized factor hereon. The \(z_s\) dependent factor \(T(z_s)\) is

\[
T(z_s) = (H_0R_0)^3 \int_0^{z_s} \left[ \frac{f(x_s - x_d)}{f(x_s)} \right]^2 f^2(x_d) dx_d
\]

After some algebra calculation, we get an analytically simple expression for the optical depth (probability) of gravitational lensing for a point source at \(z_s\) in general FRW cosmologies with dark energy

\[
p(z_s; \Omega_m, \Omega_x, \omega_x) = \begin{cases} 
\frac{F}{30} x_s^3 & (k = 0) \\
\frac{F}{|\Omega_k|^2} \left[ \frac{1}{8} (1 + 3 \cot^2 x_s) x_s - \frac{3}{8} \cot x_s \right] & (k = +1) \\
\frac{F}{|\Omega_k|^2} \left[ \frac{1}{8} (-1 + 3 \coth^2 x_s) x_s - \frac{3}{8} \coth x_s \right] & (k = -1) 
\end{cases}
\]

where \(\Omega_k = 1 - \Omega_m - \Omega_x\) and \(x_s\) can be calculated through Eq. 5. In the above derivations, the comoving number density of galaxies is assumed to be constant. However, this may not hold true for the realistic situation. If the galaxy evolution depends on cosmological models, one can hardly divide the optical depth into \(F\)-term and \(T\)-term \[21\]. Fortunately, by using the galaxy merging model proposed by Broadhurst et al. \[18\] which can successfully account for both the redshift distribution and the number counts of galaxies at optical and near-infrared wavelengths, Zhu and Wu \[20\] has shown that the galaxy merging doesn’t affect the optical depth of lensing.

IV. RESULTS AND DISCUSSIONS

Knowing the analytic expressions for the lensing optical depth in general FRW cosmologies, it is easy to demonstrate how the lensing probability depends sensitively on the amount and nature of dark energy. For sake of simplicity, we now concentrate on flat universe models, which is strongly supported by various observational evidences \[1-3\] and preferred by the inflationary scenario. Fig.1 shows how the probability \(p\) depends on a vacuum energy
component (cosmological constant) for a flat universe. In our calculations, the source has been set at $z_s = 1, 2, 3, 4, 5$ respectively. Indeed, the probability $p$ depends sensitively on the cosmological constant, this is why astronomers take it as a promising means to determine $\Lambda$ [14].

Generally, the dark energy is parameterized by $(\Omega_x, \omega_x = p_x/\rho_x)$. Therefore the optical depth of gravitational lensing depends on the amount of dark energy $\Omega_x$ as well as its equation of state $\omega_x$. The larger the dark energy amount is, the higher the gravitational lensing probability is; the more negative the dark energy pressure is, the higher the optical depth is (see Fig.2). This fact can provide an independent constraint for dark energy, as a complement to other methods. Fig.2 is the contour plots for gravitational lensing probability (normalized to the parameter $F$) in the $(\omega_x, \Omega_x)$ plane. Along each contour, the dark energy model is degenerated for a lensing optical depth. Fortunately, this degeneracy can be resolved when combining the lensing data with SNIa and CMBR measurements [22]. Thus the gravitational lensing statistics can serve as an efficient but independent tool for probing the dark energy.

In summary, by using the comoving distance we derive an analytic and simple expression for the optical depth of gravitational lensing that depends on the redshift to the source and the cosmological model characterized by the cosmic mass density $\Omega_m$, the dark energy density $\Omega_x$ and its equation of state $\omega_x = p_x/\rho_x$. It is shown that, for a flat universe, the lensing probability is very sensitive to $\Omega_x$ and $\omega_x$ and hence provides an independent probe for the dark energy.

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FIGURES

FIG. 1. The lensing probability (in unit of the effectiveness of galaxies in producing multiple images $F$) as a function of the cosmological constant for different source redshifts. The universe is assumed to be flat.

FIG. 2. The contour plots for lensing probability (in unit of the effectiveness of galaxies in producing multiple images $F$) in the $(\omega_x, \Omega_x)$ plane, where the universe is assumed to be flat and the source is set at $z_s = 3$. 
\( \Omega_m + \Omega_x = 1 \)