SYMmetry restoration in hot susy

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Abstract

It is by now well known that symmetries may be broken at high temperature. However, in renormalizable supersymmetric theories any internal symmetry gets always restored. In nonrenormalizable theories the situation is far less simple. We review here some recent work which seems to indicate that renormalizability is not essential for the restoration of internal symmetries in supersymmetry.

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I. INTRODUCTION

One of the central issues in high temperature field theory is the question of what happens to the symmetries of the Lagrangian which may (or may not) be spontaneously broken. In spite of one’s intuitive expectations on symmetry restoration [1], based on daily experience and proven correct in the simplest field theory systems [1–3], one can easily find examples with symmetries broken at high temperature [2,4]. This is an important issue, due to its possible role in the production of topological defects in the early universe. Symmetry nonrestoration at high temperature may provide a way out of both the domain wall [5,6] and the monopole problem [7–10].

A simple example of broken symmetry at high temperature is provided by supersymmetry. Due to the different boundary condition for bosons and fermions in thermal field theory, supersymmetry is automatically broken at any non-zero temperature. However, for the issue of topological defects, one would like to know what happens to internal symmetries in the context of supersymmetries. This question is nontrivial due to the highly constrained structure of SUSY models. It has been addressed carefully more than ten years ago [11]: in contrast with the non-supersymmetric case, it was shown that supersymmetry necessarily implies restoration of internal symmetries at high temperature. At least, this is what happens in renormalizable theories.

Recently, this conclusion was questioned by Dvali and Tamvakis [12] precisely by resorting to non-renormalizable potentials. They present an explicit example in which the inclusion of a quartic term in the superpotential allows apparently for non vanishing vevs at high temperature. Stimulated by their interesting suggestion, with A. Melfo we have analyzed carefully their example, arriving however to the opposite conclusion [13]. We review here both the work of Dvali and Tamvakis and our own, and try to explain why there may be a general problem with the idea of Ref. [12]. More precisely, we will see that this program strictly speaking is independent of SUSY, and discuss it in its own merit. Thus in the next section we present a general prototype example (independent of SUSY) along the lines of Ref. [12] and show why it cannot lead to symmetry nonrestoration. In section 3 we apply this to the simple case of a supersymmetric model with a discrete symmetry.

In section 4 we make some remarks on the validity of perturbation theory applied to a general nonrenormalizable potential, and to the SUSY example of section 3. Finally, in the last section we offer some thoughts on the generality of our results. We study a potential counterexample to symmetry restoration based on derivative couplings, which at the first glance could lead to negative T-dependent mass squared for the scalar field. However, in the leading limit studied throughout this work we find it not working. Thus the challenge of proving our results in general SUSY theories of finding a counterexample to symmetry restoration at high $T$ still remains.

II. A PROTOTYPE EXAMPLE

The idea of Dvali and Tamvakis can be exemplified nicely on a simple example of a real scalar field with a symmetry $D: \phi \to -\phi$

$$V = \frac{\mu^2}{2} \phi^2 - \frac{\epsilon}{4} \phi^4 + \frac{\phi^6}{6M^2}, \quad (1)$$
where $\epsilon$ is very small, in order for the nonrenormalizable interaction to play an important role: $\epsilon \ll 1$ (Dvali and Tamvakis take it to be of order $\mu/M$), $M$ is the large scale, imagined to be a GUT or a Planck scale, and $\mu \ll M$. Assuming $M^2 > 0$, the above potential is bounded even for $\epsilon > 0$.

A naive one loop expression for the high temperature potential would then be

$$\Delta V_T(1\text{-loop}) = \frac{1}{24} T^2 \frac{\partial^2 V}{\partial \phi^2} = \frac{1}{24} T^2 \left[ -3\epsilon \phi^2 + 5 \frac{\phi^4}{M^2} \right], \quad (2)$$

where $T^2 \gg \mu M$, but of course $T \ll M$. Notice the well known fact that the sign of $\epsilon$ defines the sign of the temperature induced one-loop mass term (which in turn determines the pattern of symmetry breaking).

At sufficiently high $T$, it is clear that the expression (2) implies a nonvanishing vev for $\phi$:

$$\langle \phi \rangle^2 \simeq \epsilon M^2 \quad (3)$$

as long as $\epsilon > 0$. In the Dvali-Tamvakis case, when $\epsilon \simeq \mu/M$, $\langle \phi \rangle^2 \simeq \mu M$ and $T^2 \gg \mu M$ in order to justify a high $T$ limit for $\phi$. It is easy to see that then a mass term (if there at all) of the order $T^4 \phi^2/M^2$ would dominate the $\epsilon T^2 \phi^2 \simeq (\mu/M) T^2 \phi^2$ one.

In a paper with Melfo [13], we found out that at the two-loop level such term is generated with a positive coefficient, more precisely we get

$$\Delta V_T(2\text{-loops}) = \frac{5T^4}{96M^2} \phi^2. \quad (4)$$

This implies clearly that for $T^2 \gg \epsilon M^2$ or $T^2 \gg \mu M$ the symmetry is necessarily restored.

A comment is noteworthy here. The above result should not be viewed as the breakdown of perturbation theory, as one may think at first sight. After all we are saying that the two-loop effect is larger than the one-loop one. However, this is a common situation in theories with more than one coupling. The prototype example of such a situation is the Coleman-Weinberg [14] one-loop effective potential that may dominate the tree-level potential.

The contribution (3) has its origin in the nonrenormalizable coupling $\phi^6/M^2$ in (1) and it appears for the first time at the two-loops level. In this sense, the one-loop approximation fails, but not the perturbation theory itself, for after the inclusion of (3), the higher-loop effects are down by $\phi/M$, $T/M$ or $\epsilon$. The essential ingredient in all of the above is the fact that $M$ is a large decoupled scale much above all the other scales in question.

**III. A SUSY EXAMPLE**

We take here the prototype model for symmetry nonrestoration of Ref. [12], which is basically a Wess-Zumino model with a discrete symmetry $D : \Phi \to -\Phi$ and the addition of a non-renormalizable interaction term:
\[ W = -\frac{1}{2}\mu \Phi^2 + \frac{1}{4M} \Phi^4, \]  

where \( M \gg \mu \). This leads to the scalar potential

\[ V = |\phi|^2 - \mu + \frac{\phi^2}{M}, \]  

\[ = \frac{\mu^2}{2}(\phi_1^2 + \phi_2^2) - \frac{\mu}{2M}(\phi_1^4 - \phi_2^4) + \frac{1}{8M^2}(\phi_1^2 + \phi_2^2)^3, \]

where \( \phi = (\phi_1 + i\phi_2)/\sqrt{2} \) is the scalar component of the chiral Wess-Zumino superfield \( \Phi \). Notice that \( \phi_1 \) has a negative quartic self coupling. At \( T = 0 \), as usual, one finds a set of two degenerate minima: \( \langle \phi \rangle = 0 \) and \( \langle \phi \rangle^2 = \mu M \). The usual 1-loop induced correction to the effective potential at high \( T \) is now

\[ \Delta V_{1\text{-loop}}(T) = \frac{T^2}{8} \left| \frac{\partial^2 W}{\partial \phi^2} \right|^2 = \frac{T^2}{8} \left| -\mu + 3\frac{\phi^2}{M} \right|^2 \]  

or

\[ \Delta V_{1\text{-loop}}(T) = \frac{T^2}{8} \left[ \mu^2 - 3\frac{\mu}{M}(\phi_1^2 - \phi_2^2) \right. \]

\[ + \left. \frac{9}{4M^2}(\phi_1^2 + \phi_2^2)^2 \right]. \]

Notice that here the field \( \phi_1 \) plays precisely the role of the field \( \phi \) in section II. Thus, we would again conclude (erroneously) that the symmetry is not restored at high \( T \). However, as shown in section II, at two loops one finds the dominant \( (T^4/M^2)\phi^2 \) term.

Using the superpotential (5) and the usual rules for the evaluation of Feynman diagrams in thermal field theory [2,3], it is straightforward to calculate this contribution as [13]

\[ \Delta V_{2\text{-loops}}(T) = \frac{9T^4}{32M^2} |\phi|^2 = \frac{9T^4}{64M^2}(\phi_1^2 + \phi_2^2). \]

Obviously, just as in the example of the real field \( \phi \) in section II, the symmetry \( \phi \to -\phi \) gets clearly restored at high temperature.

**IV. GENERAL DISCUSSION**

We would like to address here the issue of the convergence of the perturbation theory at high temperature for a general nonrenormalizable theory. Inspired by the application to SUSY, we take a case of a complex scalar field in the limit of the vanishing \( T = 0 \) mass, and for simplicity we imagine a larger \( U(1) \) global symmetry. Then the general form of the non-renormalizable potential is

\[ V = \sum_{n=0}^{\infty} C_{2n} |\phi|^{2n+4}/M^{2n}. \]
Now, of course, one has to go to all the loops and it is not a hard exercise to obtain the following leading $T$–dependent mass for $\phi$ (along the lines of Ref. [13])

$$m_T^2 = \sum_{n=0}^{\infty} C_{2n} \frac{(n + 2)(n + 2)!}{M^{2n}} \left( \frac{T^2}{12} \right)^{n+1}. \quad (11)$$

For large orders of perturbation theory (at least if $C_{2n}$ is not vanishing fast enough), one has the usual factorial growth which indicates the nonconvergence of the perturbation theory. This is reminiscent of all the known field theories (see e.g the recent review with references [13]). However, in some cases, such as the case in which the $C_{2n}$ alternate the signs and do not grow too fast, the perturbation theory can be given a meaning through the prescription of Borel.

Borel’s prescription says the following. If the series

$$f(z) = \sum_{n=1}^{\infty} a_n z^n \quad (12)$$

diverges, try instead

$$f(z) = \int_0^\infty db e^{-b/z} \tilde{f}(b), \quad (13)$$

where

$$\tilde{f}(b) = \sum_{n=0}^{\infty} a_{n+1} b^n \frac{b^n}{n!} \quad (14)$$

is called the Borel transform of $f(z)$. Of course, when $f(z)$ in (12) converges, the expressions (12) and (13) are completely equivalent.

To make this more transparent, we take a simple example

$$V = \frac{|\phi|^6/M^2}{1 + |\phi|^2/M^2}. \quad (15)$$

This is the potential that one obtains at the tree-level in the renormalizable version of the SUSY example of section II with $\mu = 0$:

$$W = MX^2 + X\phi^2. \quad (16)$$

After integrating out the heavy field $X$, one obtains the effective potential in (15). In this case we have

$$C_0 = 0$$
$$C_{2n} = (-1)^{n+1}, \quad n > 0 \quad (17)$$

and the Borel prescription gives

$$m_T^2 = \frac{T^2}{12} g(x), \quad (18)$$
where
\[ g(x) = \int_0^\infty db e^{-b/x} \frac{18 - 6b}{(1 + b)^5} \quad (19) \]
and \( x = T^2/(12M^2) \). Since \( x \ll 1 \), it is easy to see that in (18) most of the contribution comes from \( b \) in the vicinity of the origin, where \( 18 \gg 6b \).

Then \( g(x) > 0 \) and we get a positive leading high \( T \) mass term
\[ m_T^2 > 0 \quad (20) \]

This is to be expected, since it is known that the asymptotic series converges to the Borel result up to terms \( \sim 1/x \). Since \( x \) is extremely small, the sum converges up to very high orders in perturbation theory. In other words, up to an error of order \( x^2 \), the Borel summed result in (18)-(19) is equal to the leading two-loop result in section III (in order to compare with (9), the result (18) must be multiplied by a factor of \((3/2)^2\), since in (9) also the fermion loops were considered). However, it is reassuring to know that one can give a meaning to an infinite diverging sum for (11).

V. SUMMARY AND OUTLOOK

As clear from the above, the internal symmetry in supersymmetric theories seem to get restored at high temperature, even in the presence of non-renormalizable interactions. However, one must admit that the proof offered is valid only for a single chiral superfield. We suspect that the above is true in general, but we have not been able to come up with an explicit proof. It remains a challenge to do so or to find a counterexample [16]. A potential counterexample could result from derivative couplings [16]. In fact derivative couplings ought to be present in any nonrenormalizable field theory. In SUSY, even assuming only terms with maximally two derivatives, one can show that the bosonic part of a nonrenormalizable Lagrangian for one chiral superfield takes the form [17]
\[ L = \frac{\partial^2 K}{\partial \phi \partial \phi^*} |\partial \phi|^2 - \frac{1}{\partial K} \left| \frac{dW}{d\phi} \right|^2 \quad (21) \]
where \( K = K(\phi, \phi^*) \) is the Kähler potential and \( W = W(\phi) \) is the superpotential. For the example of Section III, after integrating out the heavy field \( X \), one gets from (14) (valid for \( \mu = 0 \))
\[ \frac{\partial^2 K}{\partial \phi \partial \phi^*} = 1 + \frac{\phi^2}{M^2} \quad (22) \]
\[ W = \frac{\phi^4}{4M} \quad (23) \]
which from (21) gives
\[ L_{INT} = \frac{|\phi|^2 |\partial \phi|^2}{M^2} - \frac{|\phi|^6/M^2}{1 + |\phi|^2/M^2} \quad (24) \]
The second term has been already discussed above, so let us now concentrate on the first term. It contributes to the 1-loop mass correction as (for $\mu \neq 0$)

$$\Delta m^2_T = \frac{i}{M^2} \int_T \frac{d^4 k}{(2\pi)^4} \frac{k^2}{k^2 - \mu^2}$$

$$= -\frac{T}{M^2} \sum_n \int \frac{d^3 \vec{k}}{(2\pi)^3} \frac{(2\pi n T)^2 + \vec{k}^2}{(2\pi n T)^2 + \vec{k}^2 + \mu^2},$$

(25)

which is potentially of the same order $T^4/M^2$, that we kept before. However, it can be shown after some thought that in the limit $\mu = 0$ (which is our leading approximation) it vanishes.

If it were true that internal symmetries in SUSY are always restored, one would remain with the cosmological problem of domain walls and monopoles. Of course, inflation remains as a possible way out, however it must take place after the creation of these topological defects. In the case of domain walls, it may be that the quantum gravitational effects break discrete symmetries and thus provide a way of preventing these objects from dominating the energy density of the universe [18].

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