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Micro-structural analysis on stress displacement and crack evolution of porous asphalt mixture based on DEM

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Abstract

The purpose of this study is to explore the micro-structure evolution and failure mechanism of porous asphalt mixture under stress. The micro-structural parameters of specimen section are imported into PFC2D, and the change of aggregate displacement is verified by discrete element modeling. Through digital image processing technology, a series of quantitative information of micro structure of mixture section is obtained, and these micro parameters are statistically analyzed, and the statistical results are compared with the macro test. Under the action of stress, porous asphalt mixture produces different degrees of strain. By comparing the meso structural parameters under different strains, the displacement of aggregate and the change process of voids are determined. The results show that with the increase of stress and strain, the specimen is gradually compacted, the aggregate displaces in different degrees, the cracks are gradually generated at the junction of aggregate and cracks, and the specimen is gradually destroyed. The change of void area, void number and void length width ratio can directly show the development process of fracture.

1. Introduction

Porous asphalt mixture adopts skeleton void structure, with the void ratio of 15% to 25%. The large structural voids inside give asphalt mixture good road performance such as noise reduction, skid resistance and drainage [1–4]. The road is always subject to the continuous action of stress in the process of using. With the continuous application of stress, the mixture structure will have different degrees of deformation and failure. Therefore, it is of great significance to study the evolution and failure mechanism of asphalt mixture microstructure to improve the road performance of porous asphalt mixture. Domestic and foreign experts and scholars have made some achievements in the research of porous asphalt mixture. For example, in order to study the creep mechanism of asphalt mixture from the micro level, Wang Hui and others conducted virtual dynamic creep test on asphalt mixture by using two-dimensional discrete element method (DEM). The simulation results are verified by dynamic creep test [5]. Gong ming yang used a multi-scale method based on meso-structure to study the mechanical properties and damage behavior of asphalt pavement with curved concrete deck [6]. Liu Yu et al proposed a new porosity prediction method for granular materials based on DEM and BPNN [7]. Gu, Lin hao established a finite element model of rubber modified asphalt mixture based on Micro-structural to predict the dynamic modulus master curve and phase angle master curve in a large frequency range, so as to study the change of Micro-structural of asphalt mixture [8]. Groozbahany compaction flow test (CFT) is used to roughly simulate the particle flow of asphalt mixture under compaction load [9]. Dai Qinglin used two-dimensional cohesive zone model to simulate the micro scale crack growth under different loading conditions [10]. Wang et al proposed damage parameters, i.e. specific surface area and average size of damage, which can be used to describe the size of interaction between damage and damage surface [11]. Through the combination of indoor test and numerical simulation, Wang Rong used image recognition technology to obtain the micro information of coarse aggregate and pore of rutting board, and revealed the micro mechanical mechanism of uniaxial static...
creep test of porous asphalt mixture [12]. Zhang zeyu used the void content of three asphalt mixtures to characterize the meso-damage of asphalt mixtures under freeze-thaw cycles [13]. The research of porous asphalt mixture by domestic and foreign scholars usually tends to use one method to study the change of micro-structural of asphalt mixture, and does not explore the evolution and failure mechanism of micro-structural under stress from many aspects.

In this study, digital image processing software and two-dimensional modeling software are used to analyze the evolution and failure mechanism of microstructure under stress. In this study, PAC-13 and PAC-16 porous asphalt mixtures were used to exert a constant force at a constant speed rate to produce different degrees of deformation. In order to explore the evolution of microstructure under stress more intuitively, the cylindrical Marshall specimen is split from the middle. The stress of 2 mm min\(^{-1}\) is applied to the specimen to make the specimen deform in different degrees. In this process, the CDD industrial camera is used to collect the image information of the specimen section with the strain of 2 mm, 4 mm and 6 mm. Then, the particle displacement and crack evolution of the specimen are

### 2. Material design and test methodology

#### 2.1. Material design

90\# SBS modified asphalt is used in the test, and its technical indexes are shown in table 1. PAC (porous asphalt concrete) — 13,16 is adopted, and the aggregate gradation is shown in table 2.

Through Marshall test, the optimal asphalt dosage of PAC-13 asphalt mixture is 5.1%, the optimal asphalt dosage of PAC-16 asphalt mixture is 4.5%, the fiber is basalt fiber, the content of PAC-13 and PAC-16 basalt fiber is 0.1%, the technical indexes of basalt fiber are shown in table 3. The target voids for the PAC-13 asphalt mixture are set at 19% and for the PAC-16 asphalt mixture at 22%.

#### 2.2. Test methodology

Marshall specimen was made according to the test procedure, and the specimen size was 101.6 mm × 63.5 mm, and the upper and lower parts of the specimen were compacted 50 times [14]. In order to explore the evolution of microstructure under stress more intuitively, the cylindrical Marshall specimen is split from the middle. The stress of 2 mm min\(^{-1}\) is applied to the specimen to make the specimen deform in different degrees. In this process, the CDD industrial camera is used to collect the image information of the specimen section with the strain of 2 mm, 4 mm and 6 mm. Then, the particle displacement and crack evolution of the specimen are

| Table 1. Technical indexes of asphalt. |
|----------------------------------------|
| Technical indicators | Unit | Standard | The test results | Test method |
|-----------------------|------|----------|-----------------|-------------|
| penetration (25°C, 5 s, 100 g) | 0.1 mm | ≥40 | 48 | T0604 |
| Softening point (T&B) | °C | ≥80 | 96.5 | T0606 |
| 5°C degree | cm | ≥20 | 28 | T0605 |
| 15°C degree | cm | ≥50 | 73.2 | T0605 |
| 60°C Dynamic viscosity | Pa·s | ≥20 000 | 37425 | T0620 |

| Table 2. Aggregate gradation. |
|-----------------------------|
| Particle size (mm) | 0.075 | 0.15 | 0.3 | 0.6 | 1.18 | 2.36 | 4.75 | 9.5 | 13.2 | 16 | 19 |
| PAC-13 | 5.2 | 6.4 | 7.5 | 9.5 | 12.5 | 16.5 | 20.6 | 68.5 | 95.9 | 100 |
| PAC-16 | 4.5 | 5.4 | 6.3 | 7.2 | 8.1 | 9 | 15 | 60 | 70 | 95 | 100 |

| Table 3. Technical index of basalt fiber. |
|----------------------------------------|
| Test items | Unit | The technical requirements | Test method |
|-----------------------|------|-----------------------------|-------------|
| Heat resistance(210°C, 2 h) | — | There was no obvious change in volume and color | JT/T 534 |
| Fracture strength | MPa | 2150 | GB/T 7690.3 |
| Elongation at break | % | 3.2 | GB/T 7690.3 |
| Length | mm | 9.5 | JT/T 776.1 |
analyzed by using the discrete element modeling and two-dimensional image processing technology. The test process and method are shown in figure 1. The cutting of the specimen is shown in figure 2.

3. Microstructure analysis of stress displacement and crack evolution of porous asphalt mixture based on DEM

3.1. Establishment of 2D discrete element model
The image file (.tiff) collected by CDD industrial camera is transformed into CAD file (.dxf) with aggregate shape and distribution information by using AutoCAD software. Particles were used to generate a rectangular section of 101.6 mm × 63.5 mm with a constant diameter of 0.4 mm and a regular hexagonal arrangement of particles. The file with aggregate shape and distribution information is imported into the discrete element rectangular section, and the particles are partitioned. The modeling process is shown in figure 3. In this model, there are mainly four kinds of contact between particles (as shown in table 4).

3.2. Microstructure analysis of aggregate displacement and crack evolution
(1) Microstructural analysis of displacement
A series of processing, such as geometric correction, gray scale overall correction and image sharpening, are carried out on the collected cross-section image. In order to observe the changes of the overall meso-structure of
the section and the displacement of particles, the processed section was divided into 16 regions of $4 \times 4$, and a key feature point was selected from each region as the observation point. According to the principle of viscoelasticity, the failure of asphalt mixture is easy to occur at the junction of asphalt and asphalt mixture, so the key points are mainly set at the junction of asphalt and asphalt mixture. The selection of observation points is shown in figure 4. The particle corresponding to the observation point was found in the discrete element model, and the particle number was shown in table 5.

In PFC2D, the ball trace command can be used to detect the particle trajectory. Figure 5 shows the specific application of particle trajectory monitoring. The ball with id 1 is given a constant velocity to the right, and the time step is set so that the ball moves at a displacement of 1 m. The red line is the trajectory of the ball as it moves.
In order to make the model closer to the macro process, the model is servoed. After the servoing is completed, the model is loaded. The load is accomplished by the movement of the wall. The velocity was applied to the top wall of the discrete element specimen to simulate the unconfined compression test under real conditions, and the final displacement of the wall was controlled to be 6 mm. Since the particle radius in the discrete element model was set to 0.4 mm, and the particle size was significantly different from that of the specimen, in order to make the particle trajectory more clearly expressed, visual processing such as stroke and boldness was performed on the trajectory of the ball. The processing results are shown in figure 6. In the macroscopic aspect, the
displacement changes of the key points of each specimen section at 2 mm, 4 mm and 6 mm were observed. The original position of a key point in a strain of 0 mm is taken as the reference point. Since the particles will not move upwards, the following values are set: positive downward, positive right and negative left. Take A as an example, when the strain is 2 mm, A moves 1.62 mm downward and 0.42 mm to the left, which is recorded as $(-1.62, -0.42)$. Figures 6 and 7 show the displacement changes of key points of each specimen section with strain at 2 mm, 4 mm and 6 mm.

Figure 6 shows the movement trajectory of particles corresponding to the key points in the discrete element model, and figure 7 shows the displacement change of key points under different strains. Generally speaking, asphalt mixture is a kind of granular viscoelastic material. Asphalt mixture has the following characteristics:

1. The material is composed of many particles;
2. The strength of the particles themselves is greater than that of the bonding material between the particles;
3. Under the action of external force, relative displacement occurs between particles [15].

Firstly, in the elastic stage and local failure stage, the displacement of PAC-16 is larger than that of PAC-13 under the same strain condition, which is because the nominal particle size of PAC-16 is larger and the voids are larger. In the elastic stage and local failure stage, the aggregate mainly produces vertical displacement. In the overall failure stage, the aggregate produces both vertical displacement and transverse displacement. In the elastic stage and local failure stage, the voids of the specimen are compressed continuously, and the structure of the specimen is not damaged. In the overall failure stage, with the increase of deformation, most of the voids are compressed, the aggregates begin to squeeze each other, the aggregates begin to rotate while moving downward, and the structure of the specimen is damaged. Compared with the displacement changes of the key points at the same height of the same specimen under the same strain condition, the displacement changes of the key points
near the aggregate are smaller than those near the void. This is because in the process of increasing stress and strain, the void is compressed first, and the key point near the nearby aggregate is impacted by the surrounding aggregate in the process of moving, so the displacement is small. Under the same strain condition of the same specimen, the lateral displacement of the key point near the two sides is greater than that of the key point near the middle. The transverse displacement of aggregate is produced by the transverse extrusion between aggregates. The aggregate near the middle needs to extrude the surrounding aggregate to produce the transverse displacement. Therefore, the transverse displacement of the key points near both sides is greater than that of the key points near the middle. The higher the original height of each point, the greater the displacement. This is because the higher the initial height, the more compressible voids and the greater the displacement change.

(2) Microstructural analysis of fracture evolution
The generation of cracks is shown in the discrete element model, and the cracks are shown by DFN. In PFC2D, DFN is a discrete, finite size element [16]. Figures 8 and 9 show the simulation of cracks in the discrete element model under different strains, and the cracks are mainly concentrated at and near the junction of asphalt and aggregate.

Because asphalt mixture is a kind of granular viscoelastic material, and the particle strength is greater than the bonding strength, the cracks are mainly located at the junction of particles and bonding materials. Discrete element modeling is used to analyze the generation and evolution of fractures. The simulation results are shown in figures 8 and 9. Figure 9 is the process of fracture generation and evolution by extracting local data from
figure 8. It can be seen that the cracks are mainly located at the junction of the particle and the bonding material. In the discrete element method, DFN usually appears at the junction individually. With the increasing of the damage, DFN gradually increases, and the adjacent DFN gradually connect to form larger cracks.

(3) The evolution process of displacement and fracture

The evolution process of displacement and fracture is shown in figure 10. Under the same strain, the number of cracks in PAC-16 is always greater than that in PAC-13. For the same kind of asphalt mixture, under different strains, the growth rate of crack number of asphalt mixture is 4 mm to 6 mm > 2 mm to 4 mm > 0 mm to 2 mm. The results show that the growth rate of asphalt mixture cracks in the overall failure stage is higher than that in the local failure stage and higher than that in the elastic stage. According to the discrete element analysis and macro analysis, the critical points of local failure stage and overall failure stage of PAC-13 and PAC-16 are located in the blue oval region. With the increase of strain, the critical points of PAC-13 in local failure stage and overall failure stage lag behind the critical points of PAC-16 in local failure stage and overall failure stage. This phenomenon shows that the crack resistance performance of PAC-13 asphalt mixture is better.

4. Macroscopic demonstration of displacement and crack evolution of porous asphalt mixture under stress

4.1. Variation of displacement and crack of section structure under axial compression

(1) Stress strain relationship under axial compression

In order to better demonstrate the change of Micro-structural displacement and crack of porous asphalt mixture, axial compression is carried out on the cut specimen to produce 0 mm, 2 mm, 4 mm and 6 mm strains (as shown in figure 11). On this basis, the change law of stress displacement and crack of porous asphalt mixture is further analyzed and demonstrated.

In order to further explore and demonstrate the microstructure evolution of porous asphalt mixture under stress, the cutting specimen is applied with a constant rate of 2 mm min$^{-1}$ to produce different degrees of deformation, and the stress-strain curves of PAC-13 and PAC-16 specimens are obtained (as shown in figure 12). It can be seen from the figure that the peak stress of PAC-13 specimen is about 2 mm, and the peak stress of PAC-16 specimen is between 1.5 mm and 2 mm.

Generally speaking, asphalt mixture is a kind of uniform viscoelastic material. By analogy with the stress-strain curve of the material, the stress process of the asphalt mixture can be simplified into viscoelastic stage (ab and $a'b'$), local failure stage ($bd$ and $b'd'$), and global failure stage ($de$ and $d'e'$). In the elastic stage, the stress is proportional to the strain of the specimen, and the specimen is in the viscoelastic deformation stage... In the local failure stage, the plastic deformation of the specimen is obvious. Compared with the elastic stage, the slope of the stress-strain curve becomes smaller, and the ability of the specimen to resist stress begins to decline. The local failure stage of PAC-13 and PAC-16 is near the peak of stress-strain curve. During the global failure stage, the specimen begins to be damaged, and obvious cracks appear between asphalt and aggregate, and the ability of
resisting stress of the specimen decreases significantly. The global failure stage of PAC-13 and PAC-16 is after the peak of stress-strain curve.

In the viscoelastic state, micro cracks appear in a single form at the junction of aggregate and asphalt. With the continuous application of stress and strain, the number of cracks increases gradually, and the macro cracks are formed by the combination of adjacent micro cracks. When the global failure stage, the adjacent micro cracks gradually merge to form macro visible macro cracks.

In conclusion, PAC-13 and PAC-16 are in elastic phase when the strain is within 0 mm to 2 mm; When the strain is within 2 mm to 4 mm, the specimens of PAC-13 and PAC-16 are in local failure stage; When the strain is greater than 4 mm, the whole structure of PAC-13 and PAC-16 specimens is damaged.

(2) Comparative analysis of crack formation and macroscopic stress-strain diagram

As the analysis in figure 9 shows the generation and evolution of cracks with PAC-16 4 mm and PAC-16 6 mm, it is more appropriate to use PAC-16 for the comparison of the generation of cracks with macro stress and strain. It can be seen from figure 12 that when the strain is 4 mm, the PAC-16 specimen is in the overall failure stage. Therefore, the macro stress-strain of PAC-16 4 mm is used in figure 13 to compare with the generation of cracks. It can be seen from figure 13 that, as described in 3.2-(2), by comparing the crack generation of the same specimen under different strains, it can be seen that with the increase of strain, a single crack is gradually generated at the junction of aggregate and asphalt, with the continuous increase of strain, a single crack near asphalt and aggregate is gradually increased, and several adjacent single cracks are gradually developed into larger cracks. This process explains the specific process that with the increase of strain, micro cracks gradually develop into macroscopic cracks. By comparing the crack generation of different specimens under the same strain, it can be seen that under the same strain, the cracks of PAC-13 are less than those of PAC-16. This is
because the nominal particle size of PAC-13 asphalt mixture is smaller than that of PAC-16 asphalt mixture, and there are more fine aggregates in PAC-13, so the large voids formed by PAC-13 are much less than that of PAC-16. Under the same strain, the deformation resistance of PAC-13 is much better than that of PAC-16. In the stress-strain curve, after the stress reached the peak, the stress curve of PAC-13 and PAC-16 decreased to different degrees with the continuous increase of strain. In the initial descent stage, the stress curve of PAC-16 decreased faster, because the void of PAC-16 was more than that of PAC-13, and the void was not completely compressed. The PAC-13 has fewer voids and is primarily resistant to damage due to the intercalation between aggregates. In the subsequent descent stage, the stress curve of PAC-16 gradually slows down. At this time, the voids are completely compressed, and the intercalation between aggregates plays a major role in resisting the failure.

4.2. Analysis of crack failure mechanism of porous asphalt mixture under stress

(1) Acquisition of meso damage parameters of specimens
On the basis of the above research, the microstructure evolution and damage mechanism are further analyzed by image processing technology. Visual C++ 6.0 is used as the development tool for Geo-Image image processing program. The pre-processing includes image smoothing, denoising and enhancement, brightness adjustment and contrast adjustment. Then, the gray level of each image is processed and the microstructure feature parameters are extracted. After binary statistical calculation, the quantitative information of microstructure such as the total area of voids, the number of voids, the shape of voids and the perimeter of voids are obtained, and these micro parameters are statistically analyzed [17].

(2) Meso crack damage analysis of specimens
The microstructure parameters of PAC-13 and PAC-16 porous asphalt mixture cutting surface under different strains are shown in Figure 14. It can be seen from Figure 14 that with the increase of strain, a series of parameters such as the total number of voids, the total area of voids and so on decrease when the specimen is not completely destroyed, that is, in the elastic stage and local failure stage (as shown in Figure 14). When the strain is from 0 mm to 2 mm, the specimens of PAC-13 and PAC-16 are in the elastic stage most of the time. At this time, the change of displacement is caused by the decrease of voids. A series of parameters, such as the total number of voids, the total area of voids and so on, are reduced. When the strain is from 2 mm to 4 mm, the specimens of PAC-13 and PAC-16 are in the stage of local failure. Due to the large displacement at this time, the specimens begin to produce local failure. A series of parameters such as the total number of voids and the total area of voids also decrease, and the reduction rate becomes slower. In this process, the total number of voids and the total area of voids in PAC-13 and PAC-16 were only reduced by 65.1% and 74.9% of those in the range of 0 mm to 2 mm. When the strain is from 4 mm to 6 mm, the specimens of PAC-13 and PAC-16 are in the overall failure stage most of the time. At this time, the structure begins to appear large-scale failure and obvious cracks. When the strain is 6 mm, the total number of voids and the total area of voids are higher than that when the strain is 4 mm. This is because with the increase of strain, the specimen is constantly compressed and the interior of the specimen is constantly damaged. Therefore, in the process of strain increase, a series of parameters such as the total number of voids and the total area of voids also decrease, but the decrease rate is less than the increase rate. Therefore, the total number of voids and the total area of voids when the strain is 6 mm is greater than that when the strain is 4 mm.
(3) Fracture evolution under stress

After further processing, the gap was divided into six groups according to the size, which were \((0,0.001]\), \((0.001,0.01]\), \((0.01,0.1]\), \((0.1,1]\), \((1,10]\), \((10,100]\). According to the image analysis, the gap on the cutting surface of asphalt mixture specimen is shown in figure 15 according to the gap area.

From the data in figure 16, it can be seen that with the increasing load, the voids are compressed and new voids are generated. The decrease of average void area and the change of void number can well explain this. The aspect ratio of a single void is usually small. When the adjacent voids develop and merge, the void shape is obviously slender [18]. Therefore, the change of aspect ratio can be used to describe the development of cracks (the calculation formula of aspect ratio is shown in formula (1)).

\[
\text{Aspect Ratio} = \frac{\text{Length of major axis}}{\text{Length of min or axis}}
\]

The length and width under different strains are shown in figure 16. It can be seen from figures 14–16 that with the increase of strain, the number of voids gradually decreases and the length width ratio of voids gradually increases. This phenomenon indicates that with the increase of strain, single voids merge with adjacent voids and cracks gradually develop.

5. Conclusions

(1) By introducing the micro structural parameters of specimen section into PFC2D and using the method of discrete element modeling, more damage information can be revealed, and the position change of aggregate
under stress, the generation of cracks and the evolution of micro structure of the whole section can be more intuitively understood.

(2) Generally speaking, asphalt mixture is a kind of homogeneous viscoelastic material. Compared with the stress-strain curve of material, the stress process of asphalt mixture can be simplified into elastic stage, local failure stage and overall failure stage.

(3) The quantitative information of micro structure, such as the total area of voids, the number of voids, the shape of voids and the perimeter of voids, can well describe the microstructure evolution and failure mechanism of specimens under stress, and the aspect ratio of voids can directly represent the development process of cracks.

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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