Intrinsic signal processed non-linearity tolerant novel 2-Tier star constellation

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Abstract
In coherent optical systems, optical fiber non-linearity is a consistent limiting factor towards the effective signal-to-noise ratio though being mitigated by various digital signal processing based approaches. In this paper, an intrinsic method of signal processing based on the shape of the input constellation is employed to yield a novel non-linearity tolerant geometric constellation. 16-QAM back-to-back coherent system is optimized for minimum value of non-linear interference, and a novel 2-Tier star constellation using sequential quadratic programming algorithm is proposed. The values of second and fourth order moments of input obtained for the optimized 2-Tier star constellation are 1.19 and 1.70 respectively, resulting in an overall reduction of non-linear interference. The complexity of the system proposed is found to be minimal in comparison to the other existing systems employing digital compensation.

Keywords Constellation shaping · Geometric shaping · Sequential quadratic programming · Non-linearity mitigation · Non-linear interference noise · 16-Quadrature amplitude modulation

1 Introduction

The advent of high-speed digital signal processing has paved the way for coherent optical communication implementing spectral efficient and multi-level modulation formats, thereby increasing the channel capacity. The ever-growing traffic demands are being taken care of by optimal exploitation of the installed optical communication systems (Winzer et al. 2018). To keep up with the demand, various methods have been employed from time to time to exploit the system capacity to Shannon limit, which include advanced modulation formats (Nakazawa...
et al. 2010; Zhou et al. 2011; Zhang et al. 2018a), different multiplexing techniques, coding and shaping techniques (Sakaguchi et al. 2012; Sillard 2014). However, optical fiber non-linearity has been a constant bottleneck to achieve the desired data rate over long-haul communication (Sabapathi et al., 2019). Digital signal processing (DSP) techniques that are robust against fiber non-linearities and offer sensitivity and spectral efficiency improvements have been widely used for the non-linearity mitigation.

To undo with the complexity of the system, non-linearity tolerant systems based on one of the constellation shaping (CS) methods, geometric shaping are being consistently explored (Bülow 2009; Liu et al. 2013; Shiner et al. 2014). The systems employing geometric shaping are intrinsically tolerant to optical fiber non-linearity. 16-QAM employing geometric shaping is found a viable solution to achieve a single-carrier 400G long-haul optical transmission (Qu et al. 2017). Further, to approach the Shannon limit in 16-QAM, coding and CS techniques are adopted, with later gaining more importance lately because coding technique is believed to have been put to an optimal use (Pisek et al. 2017).

2 Geometric Shaping

Geometric shaping (GS) is based on uniform probability distribution of non-equidistant constellation points. It restricts high-energy symbols in the constellation, thereby lowering the peak-to-average power ratio which in turn reduces the non-linear effects (Cartledge et al. 2017). The shaping is yielded by optimizing different criterion which include minimizing Euclidean distance, maximizing mutual information (MI) (Batshon et al. 2010), maximizing generalized mutual information (GMI) (Zhang et al. 2018b), maximizing figure of merit (FOM) (Ren et al. 2019) and minimizing mean square error (Qu and Djordjevic 2017) under a given signal-to-noise ratio (SNR). These different optimization criteria with a constraint of uniform probability distribution to achieve an objective function of improved system capacity or tolerance to fiber non-linearity lead to various geometric constellations.

The basic GS includes square, rectangular and star/circular constellations as shown in Fig. 1.

The overall non-linear interference noise (NLIN) variance depending on the modulation format, and the symbols transmitted over the two channels being independent, uniformly and isotropically distributed, is given as (Dar et al. 2015):

![Fig. 1 Different geometric constellation shapes](image-url)
\[ \sigma_{NLIN}^2 = P^3 \chi_1 + P^3 \chi_2 \left( \frac{\langle |b|^4 \rangle}{\langle |b|^2 \rangle^2} - 2 \right) \]  

where \( P \) is the average input power, \( b \) represents a single-polarization constellation point of the interfering channel, and \( \chi_1 \) and \( \chi_2 \) represent second-order noise (SON) and fourth-order noise (FON) coefficients respectively. It signifies the overall NLIN as a summation of modulation-independent and dependent noises, coined as SON and FON respectively. The fourth moment \( \langle |b|^4 \rangle \) marks the dependence on modulation format, which in turn depends on constellation point arrangement in a spatial domain, in accordance to the equation as:

\[ \hat{\mu}_k = \frac{E[|X - E[X]|^4]}{\left(E[|X - E[X]|^2]\right)^{\frac{3}{2}}} \]  

where \( \hat{\mu}_k \) represents the standardized moment of input constellation \( X \). The values obtained for the standardized moments, \( \hat{\mu}_4 \) and \( \hat{\mu}_6 \), second and fourth-order noise coefficients \( \chi_1 \) and \( \chi_2 \) over the different modulation and shaping formats in a non-linear optical channel are given in Table 1.

The values of the standardized moments, \( \hat{\mu}_4 \) and \( \hat{\mu}_6 \) along with the higher-order noise coefficients, \( \chi_1 \) and \( \chi_2 \) provide an insight of the variation of overall NLIN in an optical fiber channel with respect to the spatial arrangement of constellation points, which in turn leads to the incorporation of GS to minimize the effect. The dependence of NLIN on the spatial arrangement of the constellation points is exploited to obtain an optimized novel 2-Tier star constellation minimizing the values obtained for the standardized moments and the higher-order noise coefficients.

3 Proposed 2-Tier Star Constellation

A novel 2-Tier star constellation is proposed to inhibit the effect of non-linearity in long-haul optical communication systems obtained after optimizing the higher order moments of the input. This GS technique is seen as an effective non-linearity mitigation method as its NLIN performance is better when drawn parallels with the fundamental constellation shapes like square, rectangular or star. An efficient yet simple robust optimization method, sequential quadratic programming (SQP) approach is applied to yield the minimum values of second and fourth order moments of input (Zhou et al. 2012). As the baseline optimization technique, a classic SQP approach is applied to obtain an optimized 2-Tier star constellation. Within each step of the SQP, standardized moments for the constellation are calculated within or on the boundary of the interval of a single constellation point that lead to its optimal or sub-optimal value. After that, the obtained value will replace the original nominal parameter value from the input star constellation. The process continues

| Modulation          | \( \hat{\mu}_4 \) | \( \hat{\mu}_6 \) | \( \chi_1 \)    | \( \chi_2 \)    |
|---------------------|-------------------|-------------------|----------------|----------------|
| Square 16-QAM       | 1.32              | 1.96              | 7.85 \times 10^{-7} | 4.79 \times 10^{-7} |
| Rectangular 32-QAM  | 1.368             | 2.214             | 2.14 \times 10^{-6} | 1.02 \times 10^{-6} |
| Star/Circular 16-QAM | 1.282             | 1.824             | 7.36 \times 10^{-7} | 4.11 \times 10^{-7} |
for the entire range of constellation points to yield an optimized 2-Tier star constellation. The entire process to generate the required optimized constellation with minimum values obtained for the objective function $\hat{\mu}_k$ is shown in an algorithm below, where $X$ represents an array of constellation points, $u_k$ represents $k^{th}$ constellation point and $d_k$ represents the minimum step size.

### 3.1 SQP-based optimized 2-Tier star constellation algorithm

1: Input: $M=16$, number of constellation points.

2: Initialize: Generate star-QAM, an arbitrary constellation to be worked upon.

3: Calculate moments $\mu_k$ of the star constellation using eqn. (2)

4: Iteration $k = 0$, find $\{\mu_k\}$, under constraints $\Re \rightarrow -4 \leq \Re^n \leq 4$ and constellation points being complex conjugates.

5: If $\mu_k > \varepsilon$, find $a_k$ such that $X_k + a_k d_k$ yields:
   
   $\mu_{k+1} < \mu_k$

6: Continue till $\mu_k \leq \varepsilon$.

7: Repeat step-3 to step-7 for entire input constellation.

8: Replace all input constellation points with the local minimum of an objective function.

9: Place all the constellation points in the 2-D space.

The algorithm is defined for pre-defined constraints over a 2-D space to obtain optimal or sub-optimal values of an objective function, optimizing an input star constellation along the dimensions of phase and amplitude to result into a 2-Tier star constellation.

The procedure of SQP to obtain an optimized 2-Tier star constellation is summarized in Fig. 2. The entire structure of the SQP goes through a number of processes to yield the minimum value of an objective function.

It is evident from Fig. 2 that the entire structure of the proposed 2-Tier star constellation is employing a classic SQP with a few additional blocks/processes to minimize the objective function and satisfying the pre-defined constraints.

### 3.2 Optimized 2-Tier star constellation and decision region

The modification over the star constellation resulting in an optimized 2-Tier star constellation is shown in Fig. 3a. The constellation can be visualized as an amalgam of 2-level star and 4-layer circular constellation, with constellation points localized on 4 concentric circles and 16 possible phasors.

The reception of a constellation point (a symbol) at the receiver depends upon the decision region which is pre-defined for every constellation point in a constellation. The decision regions are not straight-forward for 2-Tier star constellation, but based on the decision
boundaries of a basic star constellation, consisting of inclined regions with all possible phase angles. The decision making for 2-Tier star constellation requires the entire constellation divided into 8 sub-regions with phase angle $\frac{2l+1}{8}$, with each sub-region further divided almost at the mid-point to obtain triangular regions for inner and quadrilateral regions for outer constellation points as shown in Fig. 3b. For inner constellation points, if the amplitude of lower and upper threshold rings is $r_i$ and $r_0$ respectively and at time instance $t$ and $t+1$, the received symbol amplitude are $r_t$ and $r_{t+1}$ respectively, then the bit is set to 1, if it meets the condition

\[
\frac{r_{t+1}}{r_t} > \frac{r_i + r_0}{2} \quad \frac{r_{t+1}}{r_t} < \frac{2}{r_i + r_0}
\]

otherwise set to 0. The same decision making is applied for outer constellation points as well resulting in a 2-Tier system. Similarly, to retrieve the phases, if $\theta_t$ and $\theta_{t+1}$ are the
The phase angles of the subsequent received symbols, then the demodulated angle will be (Singya et al. 2020)

$$\theta_{\text{dem}} = (\theta_{t+1} - \theta_t) \mod 2\pi$$ \hspace{1cm} (4)

The decoded angle is quantized to the defined angle set of the constellation.

Fig. 3  Optimized 2-Tier star constellation for 16-QAM and its decision region

Fig. 4  Block diagram of 16-QAM back-to-back coherent optical system implementing geometric shaping
4 Set-up and Modeling

The experimental set-up for 16-QAM back-to-back coherent system is simulated in Optisystem-16 software. The set-up consists of a transmitter which includes a QAM sequence generator to generate multi-dimensional QAM symbols from the input binary sequence. QAM sequence generator decides about the GS and accordingly it is set as regular, circular/star or 2-Tier star constellation. The QAM sequence of the particular GS obtained from the generator is modulated using Mach-Zender modulator (MZM) and M-ary raised cosine pulse generator. The optical power of the signal is set as 0 dB and transmitted over non-linear optical fiber channel. The transmission over the link is done in accordance with the Manakov equations and split-step fourier method (SSFM). Using an optical amplifier, erbium doped fiber amplifier (EDFA), the signal is amplified in each span of the fiber link, before it is fed to the coherent receiver where continuous wave (CW) laser acts as a local oscillator (Khaki et al. 2021). Using coherent detection, the transmitted symbol of the particular QAM format and GS is decided in accordance to pre-defined decision thresholds. The performance of the set-up is evaluated in terms of bit error rate (BER) and symbol error rate (SER), and non-linearity tolerance is calculated as NLIN variance. The block diagram of the set-up employing GS is shown in Fig. 4.

BER test set calculates probability of BER in accordance to:

\[
BER = \frac{1}{\log_2(M)} \cdot 2 \left( 1 - \frac{1}{\sqrt{M}} \right) \text{erfc} \left( \sqrt{\frac{3 \cdot \log_2(M) \cdot E_b}{2(M-1)N_o}} \right) - \left( 1 - \frac{2}{\sqrt{M}} + \frac{1}{M} \right) \text{erfc}^2 \left( \sqrt{\frac{3 \cdot \log_2(M) \cdot E_b}{2(M-1)N_o}} \right)
\]

where \( E_b \) is energy of a bit and \( N_o \) is the noise power.

The equation for probability of symbol being in error, given that the signal (symbol)to-noise ratio is \( \frac{E_s}{N_0} \) is given as:

\[
SER = 2 \left( 1 - \frac{1}{\sqrt{M}} \right) \text{erfc} \left( \sqrt{\frac{3E_s}{2(M-1)N_o}} \right) - \left( 1 - \frac{2}{\sqrt{M}} + \frac{1}{M} \right) \text{erfc}^2 \left( \sqrt{\frac{3E_s}{2(M-1)N_o}} \right)
\]

In GS, SER is upper bounded as

\[
SER \leq \frac{1}{M} \sum_{i=0}^{M-1} \sum_{j \neq i} \sum_{n=0}^{1} \frac{1}{2} \text{erfc} \left( \frac{d_{ij}}{2\sqrt{N_0}} \right)
\]

where \( d_{ij} \) is the Euclidean distance between any two constellation points \( c_i \) and \( c_j \), \( \text{erfc}(x) \) is the complementary error function equal to \( \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} \exp(-t^2) \, dt \) and \( \frac{N_0}{2} \) is the variance in each dimension of the Gaussian noise. Equation (7) is an approximate upper bound, considered a true value only when \( N_0 \) is equal to zero (Mirani et al. 2021).

The analysis of NLIN is taken up with at least two channels for inter-channel and a unitary channel for intra-channel non-linear impairments. In an optical fiber, electric field vector is given as (Dar et al. 2015)

\[
u(0, t) = \sum_{n} a_n g(0, t - nT) + e^{-i\Omega t} \sum_{n} b_n g(0, t - nT)
\]

where the symbols \( a_n \) and \( b_n \) represent the transmitted data in the channel of interest (COI) and adjacent interfering channel, respectively, in the n-th time-slot, having a symbol
duration of T. g(0, t) is the fundamental injected waveform, with energy normalized and orthogonal in time, i.e. \( \int g^*(0, t - nT)g(0, t - n'T) = \delta_{n,n'} \) (Dar et al. 2015). The waveform of a single pulse reaching any arbitrary point z along the fiber is given as

\[
g(z, t) = \exp \left( -\frac{i}{2} \beta'' \frac{d^2}{dt^2} \right) g(0, t)
\]

where \( \beta'' \) coefficient of fiber dispersion.

In order to detect the data symbols \( a_0 \) under the influence of NLIN, a matched filter is used prior to coherent detection in order to isolate the COI and mitigate the effect of linear noise. The received signal is expressed as \( r_0 = a_0 + \Delta a_0 \), where \( \Delta a_0 \) represents the NLIN contribution. The NLIN contribution over the dual-polarization is extended as

\[
\Delta a_0 = \frac{8}{9} \sum_{l,k,m} S_{l,k,m} a_k a_m a_l + i \frac{8}{9} \sum_{l,k,m} X_{l,k,m} (b_k b_m I + b_m b_k) a_l
\]

where \( \gamma \) is the fiber non-linearity coefficient, \( I \) is an identity matrix of order 2 × 2.

The higher order noise coefficients \( S_{l,k,m} \) and \( X_{l,k,m} \) are given by

\[
S_{l,k,m} = \int_0^L dz f(z) \int_{-\infty}^\infty dt g^*(z, t)g(z, t - IT) \times g^*(z, t - kT)g(z, t - m T)
\]

\[
X_{l,k,m} = \int_0^L dz f(z) \int_{-\infty}^\infty dt g^*(z, t)g(z, t - IT) \times g^*(z, t - kT - \beta'' \Omega_z)g(z, t - mT - \beta'' \Omega_z)
\]

where \( L \) is the length of the optical fiber link and \( f(z) \) is the function of its loss/gain profile, representing both intra-channel and inter-channel interference (Dar et al. 2015). A non-linearity component, phase-noise exists within NLIN, which in turn depends on the modulation format of the co-propagating channels. The phase-noise is contributed by the diagonal elements of the 2 × 2 matrices \( X_{0,k,m} (b_k b_m I + b_m b_k) \) which are further divided into two subgroups. The first subgroup consists of the terms where \( k = m \), whereas the second subgroup consists of the terms where \( k \neq m \). The coefficients \( X_{0,m,m} \) are real-valued, the diagonal contributions from the first group are at a complex angle of \( \pi/2 \) from the detected symbol \( a_0 \), constituting phase-noise. The same holds good for the terms where \( k \neq m \), such that \( X_{0,m,m} = X_{0,m,k} \) (Dar et al. 2015). In a single-polarization transmission, NLIN coefficients \( \chi_1 \) and \( \chi_2 \), from Eq. (1) are given by Eqs. (13) and (14) multiplied by \( T^3 \), whereas in case of dual polarizations the equations need to be multiplied by \( \frac{8}{27} \) and \( \frac{20}{81} \), respectively.

\[
\chi_1 = \frac{32}{27} \gamma^2 \sum_{l,k,m} |X_{l,k,m}|^2
\]

\[
\chi_2 = \frac{80}{81} \gamma^2 \sum_{l,m} |X_{l,m,m}|^2
\]

Other than higher order moments and noise coefficients, some real coefficients generally called as additional noise terms contribute to the variation of NLIN. Mathematically, the dependence of NLIN on these additional terms is given as (Fehenberger et al. 2016):

\[
\sigma_{NLIN}^2 = P_3 \left[ \gamma_0 + (\hat{\mu}_4 - 2) \cdot \chi_4 + (\hat{\mu}_4 - 2) \cdot \hat{\chi}_4 + \hat{\mu}_6 \cdot \hat{\chi}_6 \right]
\]
where $\chi_0$, $\chi_4$, $\chi_4'$ and $\chi_6$ are real coefficients that contribute to the fiber non-linearities. Though, the value of these terms is very small yet considered during the non-linear analysis of fiber-optic systems. The standard moments $\hat{\mu}_4$ and $\hat{\mu}_6$, NLIN coefficients $\chi_1$ and $\chi_2$ together with the modulation-dependent coefficients $\chi_0$, $\chi_4$, $\chi_4'$ and $\chi_6$ define the value of NLIN in a system.

The variation of modulation-dependent coefficients $\chi_0$, $\chi_4$, $\chi_4'$ and $\chi_6$ over different modulation formats is given in Table 2.

The NLIN variance increases with the increase in the value of standard moments $\hat{\mu}_4$ and $\hat{\mu}_6$, NLIN coefficients $\chi_1$ and $\chi_2$ and modulation-dependent coefficients $\chi_0$, $\chi_4$, $\chi_4'$ and $\chi_6$. It is evident that the values obtained for the standard moments and modulation-dependent coefficients vary over different modulation schemes and geometric shapes which in turn lead to the variation in overall NLIN of the fiber-optic system.

A multi-span 16-QAM back-to-back coherent system is simulated over a non-linear fiber employing GS with the main parameters listed in Table 3.

### Table 2: Probability distribution and NLI terms

| Modulation          | $P_X$   | $\chi_0$      | $\chi_4$      | $\chi_4'$     | $\chi_6$   |
|---------------------|---------|---------------|---------------|---------------|------------|
| Square 16-QAM       | Uniform | $1.143 \times 10^{-7}$ | $1.171 \times 10^{-7}$ | $1.171 \times 10^{-7}$ | $2.143 \times 10^{-7}$ |
| Rectangular 32-QAM  | Uniform | $1.137 \times 10^{-7}$ | $1.103 \times 10^{-7}$ | $2.117 \times 10^{-7}$ | $1.167 \times 10^{-7}$ |
| Star/Circular 16-QAM| Uniform | $1.136 \times 10^{-7}$ | $1.116 \times 10^{-7}$ | $2.114 \times 10^{-7}$ | $1.163 \times 10^{-7}$ |

5. Results and Discussion

The proposed GS based 2-Tier star 16-QAM is simulated and analysed over a non-linear fiber channel, under the input parameters as given in Table 3. The analysis is based on the parameters like bit error rate (BER), symbol error rate (SER) and NLIN. BER and SER are symbolic to the overall performance of the system and NLIN gives a measure of non-linearity tolerance.

5.1. BER and SER analysis

In Fig. 5, the BER performance of 16-QAM undergoing optimized geometric constellation shaping, 2-Tier star is studied under the static spectral efficiency and filter roll-off conditions. The analysis is carried out over a non-linear fiber having the total propagation distance of 80 km. It is observed that at low input power, when there is minimal effect of non-linearity, regular (square) constellation shaping undergoes minimal BER with respect to star/circular constellation shaping, higher order modulation format, rectangular 32-QAM or proposed 2-Tier star constellation.

Beyond a particular threshold of signal-to-noise ratio, approximated at around 8 dB, the variation in BER with respect to different geometric shaping changes is found minimal for 2-Tier star constellation underpinning the geometric shaping as a non-linearity tolerant method for high power input. The minimum BER of $1.46 \times 10^{-4}$ is obtained for 2-Tier star constellation with respect to $4.27 \times 10^{-4}$ and $1.77 \times 10^{-3}$ for star/circular and square constellation respectively for the maximum signal-to-noise ratio of 12 dB. The variation is.
attributed to Eq. (5), resulting into waterfall curves, with 2-Tier star constellation standing out as best choice for higher power inputs or higher values of non-linear interference. Since star constellation gives better BER performance over other fundamental shapes at higher values of SNR, 2-Tier star constellation, an optimized star constellation a modification over it yields even better values.

In Fig. 6, the SER performance is found to be at an optimum level for 2-Tier star constellation, thereby encouraging geometric shaping as an important method of non-linearity tolerance in a defined range of SNR. SER is found to be minimum for square constellation bounded at 12 dB of SNR in accordance with Eq. (7), maintaining maximum possible minimum Euclidean distance for square constellation. However, the high data-rates for the same constellation at very high input power (or SNR) requires strict SER/BER requirements, undergoes cell-to-cell interference (Khaki et al., 2021), thereby maintaining 2-Tier star constellation as an optimal solution at high input powers.

### 5.2 Performance and complexity analysis

The various DSP techniques to compensate non-linearities over any optical communication channel include Digital back-propagation (DBP), Phase conjugation (PC), Volterra series
based non-linear equalizer (VNLE), Perturbation-based non-linear compensation (PB-NLC) and Inter-subcarrier non-linear interference canceler (INIC). DBP is generally being used to compensate almost all deterministic effects and provides high performance, but at the cost of high computational load. VNLE has relatively lower complexity due to parallel implementation, but complex enough for any commercial implementation. The limitation with these two techniques is the non-linear interference observed over multi-channel systems. The lesser complex techniques, PB-NLC and OPC techniques have their own limitations. NLC needs to compute large number of perturbation terms whereas OPC has a problem of flexibility in meeting the precise positioning and a symmetric communication channel (Amari et al. 2017).

The performance and complexity analysis of different digital signal processing techniques vis-a-vis 2-Tier star constellation is given in Fig. 7.

PC and PB-NLC show limited performance because of the high impact of non-linear and linear interference. DBP and VNLE exhibit better performance in comparison to PC and PB-NLC techniques. The gain of DBP and VNLE is about 0.84 dB and 0.38 dB in comparison with PC, respectively. INIC strongly outperforms both DBF and VLNE. The maximum gain of INIC is about 0.56 dB with respect to DBP. At high input powers, the performance of all digital compensation techniques is comparable. The intrinsic 2-Tier star constellation yields gain of about 0.14 dB and 0.08 dB over VNLE and DBP compensation techniques. At high input power, the performance of 2-Tier star constellation is equivalent and comparable to that of VNLE and DBP compensation respectively. 2-Tier star constellation yields higher values of Q-factor than both PC and PB-NLC for the input power ranging from -4 dBm to 3 dBm.

The higher values of Q-factor obtained for different digital compensation techniques like INIC, DBP and VLNE are at the cost of cumbersome mathematical operations which generally employ Fast Fourier Transform (FFT), implementing a huge number of multiplications. The complexity of the system can be evaluated based on the required number of real multiplications to be used for the non-linearity compensation. Electronic dispersion compensation requires $4N_f \log_2(N_f) + 4N_f$ real multiplications (Liu et al. 2012), where $N_f$ corresponds to the FFT size. The number of multiplications required for DBP and VNLE
are $4N_sN_f\log_2(N_f) + 10.5N_sN_f$ and $2N_sN_f\log_2(N_f) + 4.25N_sN_f$, respectively where $N_s$ is the number of spans. The INIC method of compensation, though yielding the highest Q-factor roughly triples the complexity of the system because of the three-step implementation based on decision feedback equalizer. Therefore, a trade-off is evident between the performance, Q-factor and complexity of the system for compensating the non-linearities. The intrinsic signal processed 2-Tier star constellation is tolerant to the non-linearities with its performance comparable to DBP and VLNE, and simultaneously being the simplest technique among all the aforementioned methods which does not require any complex digital compensation circuitry.

5.3 NLIN analysis

In order to attain high data rates, the system is subjected to high input powers resulting into severe NLIN. As observed 2-Tier star constellation yields minimum BER beyond a certain input power, NLIN of the different geometric constellation shapes is calculated, using the standardized moments for input, second and fourth-order noise and modulation-dependent coefficients.

The values of standard moments, $\mu_4$ and $\mu_6$, higher order noise coefficients, second-order, $\chi_1$ and fourth-order, $\chi_2$ over different GS and modulation formats (16, 64 and 256-QAM) are given in Table 4.

The values of standard moments, $\hat{\mu}_4$ and $\hat{\mu}_6$ are obtained using an Eq. (2). Likewise, higher order noises coefficients, second-order $\chi_1$ and fourth-order $\chi_2$ are obtained using Eqs. (11) and (12).

Modulation-dependent coefficients $\chi_0$, $\chi_4$, $\chi_4'$ and $\chi_6$ for the different geometric constellation shapes of 16, 64 and 256-QAM i.e. regular (square), circular and 2-Tier star are obtained using Eqs. (11), (12) and (15), given in Table 5.

The SNR performance of an optical communication system with varying GS is given in Fig. 8a. It is observed from the figure that the SNR is influenced by both SON approximated as Gaussian and non-linear FON noise. 2-Tier star GS results in increased value of SNR over varying input power ranging from $-5$ dBm to $1$ dBm. Based on the results
obtained for higher order moments, higher order noise and modulation-dependent coefficients for different geometric shapes, NLIN is calculated according to Eq. (1) and its variation with respect to the range of input powers, from $-8$ dBm to $2$ dBm is observed in Fig. 8b.

Since the overall distribution of SON is Gaussian and the value of FON obtained over low input powers is very small, the variation of NLIN is linear for low input powers, ranging from $-8$ dBm to $-2$ dBm which eventually becomes non-linear over the increased values of input power, attributed to higher values of FON. 2-Tier star constellation is found to be most tolerant to non-linearity, well in conformity with the values obtained for higher order moments and noise coefficients using Eqs. (2) and (15) respectively. The values of NLIN obtained for 2-Tier star constellation range from $-62.6$ dBm to $-45$ dBm. The overall gain, with respect to square and star constellation is found to be $1.12$ and $0.89$ dBm respectively.

Variation of higher order noises, SON and FON for different geometric shapes with respect to number of spans over a fixed optical link length is observed in Fig. 9. It can be observed from Fig. 9a that SON behaves completely Gaussian, increases linearly with the increase in number of spans. The variation goes well in accordance with Eq. (1) with 2-Tier star constellation standing out as the best solution in comparison to square and star constellation. The minimum values of SON obtained over the number of spans ranging from 1 to 20 spans are $-74$ dBm and $-32$ dBm respectively. However, it
is observed from Fig. 9b that the behaviour of FON is completely non-Gaussian. FON is calculated using Eqs. (1) and (14).

The difference in the values of FON over different GS is maximum for a single span which considerably reduces over the increasing values of spans, attributed to increased number of incomplete collisions of non-linear interacting pulses. The variation of 2-Tier star constellation stands out with minimal values of FON. In 2-Tier star constellation, the maximum gain of 1.12 dB is obtained over the other constellation shapes. The variation of FON for all three geometric constellation shapes taper towards its far end over...
the range of spans, attributed to the decreased number of incomplete collisions occurring over a large number of spans.

The variation of SON and FON with respect to number of spans is extended to the variation of NLIN for different geometric shapes under different fixed span lengths. The variation of NLIN with respect to number of spans is shown in Fig. 10.

With the minimum number of spans, NLIN behaves completely Gaussian, which is attributed to minimal collisions over the fiber length ranging from 25 to 200 km. However, with the increase in number of spans, the increased values of non-linearity shift the overall behaviour of the system from being completely Gaussian, to non-linear attributed to increased value of FON and consequently NLIN.

The variation of NLIN is further extended to different modulation formats viz 16-QAM, 64-QAM and 256-QAM over a varying length. In Fig. 11, it is observed that 2-Tier star constellation yields lowest of NLIN over different modulation formats as well. NLIN increases with higher modulation format at an expense of increased mutual

Fig. 10 NLIN vs number of spans for different geometric constellation shapes, with fixed span length (a) 25 km to (b) 50 km (c) 100 km (d) 200 km
information, but within a particular modulation format, the novel 2-Tier star yields the lowest values of NLIN in comparison to regular or star/circular constellation.

2-Tier star constellation stands out with minimal values obtained for NLIN ranging from -38.4 dBm to -30.8 dBm over the fixed span length of 25 km and increasing number of spans which consequently increases with the increase in span length. The variation of NLIN for all three geometric constellation shapes is in coherence with FON. Figure 12. shows the variation of the mitigation gain of 2-Tier star constellation with respect to SNR of the optical signal.

The maximum value of the gain obtained is 1.12 dB when calculated for the highest OSNR value of 24 dB. The received OSNR in an optical fiber link is given as:

\[
\text{OSNR} \approx 58 - 10 \log_{10} \left( N_{\text{EDFA}} \right) + P_{\text{Launch}} - \text{Loss} - NF
\]

(16)

where \( N_{\text{EDFA}} \) is the chain of EDFA optical amplifiers, \( P_{\text{Launch}} \) is input optical power and \( NF \) is the noise figure of an EDFA.

The simulation results based on autocorrelation function (ACF) obtained over one of the diagonal elements from Eq. (10), and extended to almost the entire range of symbols, give an insight into the overall non-linearity tolerance. The behavior of ACF over different GS obtained from the higher order noise coefficients is shown in Fig. 13.

The curves are obtained by computing the higher order noise coefficients \( S_{l,k,m} \) and \( X_{l,k,m} \) given in Eqs. (11) and (12) over a non-linear optical fiber channel. It can be seen from the figure that the 2-Tier star constellation exhibits the longest temporal correlation whereas for square constellation, the values obtained are comparatively shorter. The results signify that the long temporal correlation obtained for 2-Tier star constellation intrinsically offer better tolerance to non-linearity in comparison to square and star constellations. NLIN mitigation gain for 2-Tier star constellation is calculated as the ratio of NLIN variance of 2-Tier star constellation to square and star constellations. It is evident from the figure that the non-linearity tolerant 2-Tier star constellation yields a gain of 1.12 and 0.89 over square and star constellations respectively.
In order to consider the variation of non-linear interference over a time-varying communication channel, the channel model given by Eq. (10) is modified. The equation is given as

$$\Delta a_n = \sum H_l^{(n)} a_{n+l}$$

where the ISI coefficients are the $2 \times 2$ matrices $H_l^{(n)}$ given by

$$H_l^{(n)} = i \frac{8}{9} \gamma \sum_{k,m} (b_{n+k} b_{n+m} I + b_{n+m} b_{n+k}) X_{l,k,m}$$

$H_l^{(n)}$ depends on the data symbols that were transmitted over the interfering channel.
The ISI matrices are time-dependent based on the fact that all the interfering symbols given in the Eq. (18) change with n. This dependence of the symbols is therefore given as a subscript (n).

In long-haul optical communication, when large intra and inter-channel interference is considered, $H_l^{(n)}$ depends very weakly on n, suggesting a minimal variation with respect to time. The underlying factor for the weak dependence is the interaction of each and every symbol from the COI with a large number of symbols from adjacent interfering channels. The reason being the different velocities at which WDM channels propagate, particularly in presence of linear and non-linear noises. Therefore, adjacent symbols in the COI are essentially affected by non-linearity in a highly correlated manner.

The autocorrelation function (ACF) calculated for the top diagonal element of the given matrix is given as $R_l(k) = \langle H_{l,i}^{(n+k)} H_{l,i}^{(n)} \rangle$. The ACF is calculated over a discrete range of $l$, where $l$ is the order of interference. The behavior can be extended to all the elements of the adjacent interfering matrices (Dar et al. 2015). Figure 14 shows the ACF curves computed over the coefficients $X_{l,k,m}$ with $l = 0$, $l = 1$, and $l = 2$.

It is observed from the figure, the elements of the zeroth interference matrix $H_0^{(n)}$ are having the longest of the temporal correlations in comparison to the higher-order ISI elements. It is inferred from the results obtained that the sufficiently long temporal correlations are only obtained for some of the lowest orders of interference but the higher orders are predominantly having very short temporal correlations. Therefore, it can be concluded that over a time-varying channel, the effect of geometric shaping is predominantly determined by the modulation order.

Fig. 14 Temporal autocorrelation function
6 Conclusion and Future Scope

In this paper, GS is employed over a long-haul communication system and the behaviour of the system is studied both analytically and experimentally. The numerical simulations are carried out for both Gaussian and non-Gaussian behaviour of the channel to include linear and non-linear impairments. It is found that geometric constellation shaping has a strong bearing to BER and NLIN for a coherent optical communication system employing higher order modulation format. Square constellation yields minimum value of BER for a maximum bounded input power attributed to minimum Euclidean distance, but beyond the maximum bound of input power, the proposed novel 2-Tier star constellation provides an optimal solution. In long-haul optical communication system operated at high input powers, constellation shaping has proved to be an intrinsic solution to mitigate the non-linearity effect. 2-Tier GS, a modified star constellation has an improved non-linearity tolerance with minimal receiver sensitivity requirements. 2-Tier star constellation yields minimal values of NLIN over a varying distance across different modulation formats, 16-QAM, 64-QAM and 256-QAM. The values of SON and FON obtained are minimal for optimized 2-Tier star constellation resulting in lowering the values of NLIN. Therefore, the system is seen tolerant to non-linearity in comparison to the rest of geometric constellation shapes.

The complexity of the system employing intrinsic signal processing is minimal in comparison to the systems employing digital signal processing. DSP compensation requires a lot of mathematical calculations resulting in an overall a very complex system.

In future, alongside geometric shapes, other shaping techniques like probabilistic shaping and hybrid shaping techniques can be exploited and synergistically used along digital signal processing to increase the overall capacity of the system, with minimum values of BER and NLIN.

Authors contribution  All authors contributed extensively to the work presented in the paper.

Declarations

Conflict of interest  There is no such potential conflict of interest involved in this work.

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