Displacement memory effect near the horizon of black holes

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ABSTRACT

We study the displacement memory effect and its connection with the extended-BMS symmetries near the horizon of black holes. Considering the near-horizon asymptotic metrics, we show there is a permanent shift in the geodesic deviation vector relating two nearby timelike geodesics placed close to the horizon of black holes, upon the passage of gravitational waves. We also relate this memory effect with the asymptotic symmetries of the near-horizon metric. For black holes, the shift of the relative position of the detectors is shown to be induced by a combination of BMS generators near the horizon. Particularly for extreme black holes, the displacement memory effect near the horizon is quite similar to the same obtained in the far region.
1 Introduction

The direct detection of gravitational wave (GW) \[1, 2\] has enabled researchers to look for various aspects of black hole spacetimes; “Gravitational memory effect” \[3, 4, 5, 6, 7, 8, 9\] is one of such physical effects which manifests the permanent relative separation or displacement between the test masses, initially held at relative rest, upon interacting with GWs. This is nothing but the conventional displacement memory effect. This fascinating field recently has gained considerable attention in the GW physics due to its possible detection in the advanced LIGO or LISA detectors \[10, 11, 12\]. On the other hand, from theoretical perspectives, it has brought interests of many due to its connection with Bondi-Metzner-Sachs (BMS) symmetries \[13\] and soft graviton theorem \[14, 15, 16, 17, 18\]. It has been shown that asymptotic BMS symmetries can be studied in the context of memory effect \[15\]. Further, the fundamental relation between GW-memory and soft graviton theorem has been established by Strominger and Zhiboedov \[14, 19\]. Such connections with memory effect have offered us an intriguing opportunity to delve into the low energy quantum gravity picture \[14, 15, 16, 17\]. Recently, Hawking, Perry and Strominger conjectured that the charges corresponding to BMS symmetries would help to retrieve the information in the Hawking information paradox \[16, 20\]. Therefore, in this respect, BMS symmetries for near-horizon gravitational memory might play a crucial role in understanding the information paradox.

It is known that the supertranslation-like BMS symmetries and the extension of that \[21, 22\]
can also be recovered at the horizon of black hole spacetimes by studying the asymptotic symmetries preserving the near-horizon structure of three and four-dimensional spacetimes [23, 24, 25]. Non-extremal black holes in four-dimensional general relativity exhibit an infinite-dimensional symmetry in their near-horizon region. One can show, by employing suitable boundary conditions that the algebra of asymptotic Killing vectors is infinite-dimensional. It contains two sets of supertranslations and two mutually commuting copies of the Virasoro algebra [23, 24, 25]. For stationary backgrounds, one of the sets of Virasoro algebras freezes off. It should also be noted that the BMS symmetries, including superrotations like transformations, can also be recovered near the black hole horizon demanding the invariance of the induced metric on a null hypersurface situated at the horizon and serving as a boundary of two black hole spacetimes [26, 27]. The memory effect related to these horizon shells containing impulsive gravitational waves and for plane wave spacetimes is recently being studied in [28, 29, 30, 31]. Memory effect near the black hole horizon in terms of BMS charge algebra has been studied in [25]. Interesting studies on the memory effect related to Rindler spacetimes can be found in [32, 33].

Motivated by the Hawking, Perry, Strominger’s proposal [17] and the detection prospect, we provide a detailed analysis of computing the near-horizon displacement memory effect and find its connection with the BMS symmetries recovered near the horizon. Our particular interest is to see the effect of near-horizon BMS symmetries [25] on the test detectors placed near the horizon by computing their permanent shift upon passage of gravitational waves. To compute the displacement memory effect, one starts with the “geodesic deviation equation” (GDE) which measures the deviation or displacement between two nearby timelike geodesics. We focus on estimating the memory effect for the test detectors stationed near the black hole horizon in three and four dimensions. Although the present study can at best be regarded as just a model at the moment, however in future this may provide a window to study the near-horizon memory effect with an advanced detection mechanism.

In order to understand the detection framework, consider two nearby timelike geodesics or test masses of GW detectors positioned near the horizon of a black hole spacetime with a tangent vector $T^\mu$ and deviation or separation vector $S^\mu$. The deviation vector $S^\mu$ between adjacent geodesics evolves according to GDE, given by

$$\frac{D^2}{d\tau^2} S^\mu = -R^\mu_{\alpha \beta \gamma} T^\alpha T^\gamma S^\beta,$$

where $\tau$ is the proper time, $S^\mu$ denotes the spatial separation between two nearby timelike geodesics, and $R^\mu_{\alpha \beta \gamma}$ is the Riemann tensor. We shall adopt Greek letters for spacetime index ($\mu = 0, 1, 2, 3$), Latin lower case letters for hypersurface index ($a = 1, 2, 3$) and Latin upper case letters for the spatial coordinates on the hypersurface, i.e., coordinates for the co-dimension one surface ($A = 2, 3$).
The GDE, in a particular coordinate system, is given by [8]

\[ \ddot{S}^i = -R_{ijl}S^j. \]  

(1.2)

In weak field slow motion approximation, the Riemann tensor depends on the time derivatives of the Quadrupole moment tensor \((Q_{ij})\), \(R_{ijl} = -\frac{1}{r}P[\ddot{Q}_{ij}]\). Where \(P\) function gives ‘projected orthogonally to the radial direction and trace-free’ part of the tensor on which it acts. The interaction between the detectors and GW should induce a permanent change in the spatial separation of the setup. This can be seen by integrating the Eq.\((1.2)\) twice.

\[ \Delta S^i = \frac{1}{r}S^j P[\Delta \ddot{Q}_{ij}], \]  

(1.3)

where \(\Delta\) denotes the difference between the separation vector before and after the passage of GW. This spatial separation between the detectors, to the linear order of \(\frac{1}{r}\), depicts the conventional memory effect. It should be noted here that the Eq.\((1.3)\) is only valid for the linear approximation to general relativity and slowly moving sources. Therefore, this provides a near-horizon counterpart of the effect being depicted in the far region in [15]. In this report, we focus on the near vicinity of the horizon, and should not consider the weak field slow-motion approximation. Nonetheless, we can still obtain an analogous effect given by a change in the metric parameter in the right-hand side of Eq. \((1.3)\) in a suitable coordinate system. Near the horizon, the jump in the separation is governed by the change in the metric parameters instead of Quadrupole moment tensor,

\[ \Delta S^i \sim \rho B_{ij}S^j, \]

where \(B\) denotes a function of the coordinates carried by the metric, and \(\rho\) denotes the radial coordinate. Further, we show that the change in the separation of deviation vector can be realized by a combination of supertranslation and superrotation, revealing the BMS memory effect near the horizon of black hole spacetimes.

Let us briefly discuss how the paper has been organized. We start by reviewing some basic details in Sec.(2). In Sec.(3), we provide a detailed analysis of displacement memory effect for three-dimensional near-horizon extreme and non-extreme black holes. The similar analysis, we have provided for the four-dimensional black holes in Sec.(4). We also establish the connection between memory effect and BMS symmetries. Finally, we conclude our findings by summing them up in Sec.(5) of discussion. For simplicity, we have done a perturbative (in surface gravity \(\kappa\)) analysis. In the appendix, we have provided an exact analysis.
2 Memory effect at null infinity in flat spacetimes

The gravitational memory effect for the asymptotically flat spacetimes near future null infinity has been a well-studied area. The inertial detectors positioned near the future null infinity get permanently displaced after the passage of GW. This is regarded as the conventional gravitational memory effect [3, 4, 6, 14, 15, 34, 35, 36, 37]. It has also been established that there is a direct relation between this displacement and the BMS symmetries [14, 34]. Recently, it has been explicitly shown in [15] how the displacement memory effect at the far region is related to a supertranslation. To see this, consider the general form of the asymptotically flat metric

\[ ds^2 = -du^2 - 2dudr + 2r^2\gamma_{zz}dzd\bar{z} + \frac{2m_B}{r}du^2 + D^zC_{zz}dudz + D^{\bar{z}}C_{\bar{z}z}dud\bar{z} + \]

\[ rC_{zz}dz^2 + rC_{\bar{z}z}d\bar{z}^2 + \frac{1}{r}\left(\frac{4}{3}(N_z + u\partial_z m_B) - \frac{1}{4}\partial_z(C_{zz}C^{zz})\right)dudz + c.c. + ..., \] (2.1)

where \( m_B, N_z \) and \( C_{zz} \) in general depend on \((u, z, \bar{z})\); and also called as “Bondi mass”, “angular momentum aspect” respectively. Whereas \( C_{zz} \) describes GWs. It is, in fact, the free data available near asymptotic null infinity. It is related to the “Bondi news tensor” \((N_{zz})\), written as \( N_{zz} = \partial_u C_{zz} \).

We can see that the first three terms in the metric (2.1) represent the flat Minkowski spacetime and other terms represent the leading order correction to the metric. “…” tells about the subleading terms at large \( r \). For large \( r \) fall-offs, the metric (2.1) components are

\[ g_{uu} = -1 + O(\frac{1}{r}) ; \quad g_{ur} = -1 + O(\frac{1}{r}) ; \quad g_{uz} = O(1) \]
\[ g_{zz} = O(r) ; \quad g_{\bar{z}\bar{z}} = r^2\gamma_{\bar{z}\bar{z}} + O(r) ; \quad g_{rr} = g_{rz} = 0 \] (2.2)

The geodesic deviation equation for the metric (2.1) takes the following form

\[ r^2\gamma_{\bar{z}\bar{z}}\partial_u^2 S^{\bar{z}} = -R_{uzuz}S^{\bar{z}}, \] (2.3)

where \( R_{uzuz} = -\frac{r}{2}\partial_u^2 C_{zz} \). Therefore, the change in the displacement is given by

\[ \Delta S^{\bar{z}} = \frac{\gamma_{\bar{z}\bar{z}}}{2r}\Delta C_{zz}S^{\bar{z}}. \] (2.4)

From the \( uu \)-component of the Einstein field equation, and for a stress tensor with shockwave profile of the form

\[ T_{uu}(u, z, \bar{z}) = \mu\delta(u - u_{rad})\frac{\delta^2(z - z_{rad})}{\gamma_{\bar{z}\bar{z}}}, \] (2.5)
the leading order change in the $C_{zz}$ is given by

\[ \Delta C_{zz}(z, \bar{z}) = C_{zz}(z, \bar{z})|_{u=u_f} - C_{zz}(z, \bar{z})|_{u=u_0} \]

\[ \Delta C_{zz}(z, \bar{z}) = 2\mu D_z^2 G(z, \bar{z}; z_{\text{rad}}, \bar{z}_{\text{rad}}) - \frac{\mu}{2\pi} \int d^2 z' \gamma_{zz'} D_z^2 G(z, \bar{z}; z', \bar{z}') , \quad (2.6) \]

where $G(z, \bar{z}; z_{\text{rad}}, \bar{z}_{\text{rad}})$ is the Green’s function can be found in [34]. The above expression contributes to the deviation equation as a memory effect for far region spacetimes.

Further, to relate the memory effect with BMS symmetry, consider the supertranslation of type $u \rightarrow u + f(z, \bar{z})$. One can find a $f(z, \bar{z})$ which would give rise the same change in $C_{zz}$ of (2.6). For this, taking Lie derivative of $C_{zz}$ along the supertranslation parameter $f$,

\[ \mathcal{L}_f C_{zz} = f N_{zz} - 2D_z^2 f. \quad (2.7) \]

Eq.(2.7) is directly related to change in the $C_{zz}$. If before and after the passage of GW the $N_{zz}$ is zero, then

\[ \Delta C_{zz} = -\mathcal{L}_f C_{zz} = 2D_z^2 f. \quad (2.8) \]

Now one can choose $f(z, \bar{z})$ to be

\[ f(z, \bar{z}) = \mu G(z, \bar{z}; z_{\text{rad}}, \bar{z}_{\text{rad}}) - \frac{\mu}{4\pi} \int d^2 z' \gamma_{zz'} D_z^2 G(z, \bar{z}; z', \bar{z}'). \quad (2.9) \]

This choice of $f$ produces the same change in the $\Delta C_{zz}$ as appears in (2.6). This establishes the connection between memory effect and BMS symmetries near the future null infinity for asymptotic flat spacetimes. Now, we provide a similar study and connection with BMS symmetries for the near-horizon black holes.

3 Near-Horizon Memory for Three-Dimensional Black Holes

Three-dimensional gravity theories sometimes give us a good hint of what should be expected in the four-dimensional scenario. In many cases, analysing three-dimensional models become easy. Therefore, to start with, we analyse the three-dimensional near-horizon metric for anti-de Sitter (AdS) spacetime, and compute the displacement memory restored in the GW setup installed near the horizon. We are interested in measuring the change in the relative displacement or deviation vector between two nearby timelike geodesics or test detectors, which is induced via the interaction with GWs generated from the black hole spacetime. Let us consider the near-horizon metric for a
three-dimensional black hole in Gaussian null coordinates [23, 24, 25],

\[ ds^2 = \xi dv^2 + 2kdvdp + 2hdvd\phi + R^2d\phi^2, \quad (3.1) \]

where \( v \) is the retarded time coordinate, \( \rho \geq 0 \) represents the radial distance to the horizon and \( \phi \) is the angular coordinate of period \( 2\pi \). Functions \( \xi, k, h, \) and \( R \) are expected to comply with the following fall-off conditions near the horizon \( \rho = 0 \):

\[
\begin{align*}
\xi &= -2\kappa \rho + O(\rho^2) \quad ; \\
k &= 1 + O(\rho^2) \\
h &= \theta(\phi)\rho + O(\rho^2) \quad ; \\
R^2 &= \gamma(\phi)^2 + \lambda(v, \phi) + O(\rho^2) \quad (3.2)
\end{align*}
\]

where \( \kappa(v, \phi), \theta(\phi), \gamma(\phi) \) and \( \lambda(v, \phi) \) are arbitrary functions of the coordinates. The metric components \( g_{\rho\phi} \) and \( g_{\rho\rho} \) decay rapidly as \( O(\rho^2) \). We also observe that the \( \rho^2 \) order terms will vanish or fall more faster than the terms with first order in \( \rho \) for near-horizon geometry. This ensures that the metric is consistent with the near-horizon approximation.

The asymptotic boundary conditions are being preserved by asymptotic Killing vectors, given by [24]

\[
\begin{align*}
Z^v &= f(v, \phi) \\
Z^\rho &= -\partial_v f \rho + \partial_\phi f \frac{\theta}{2\gamma^2} \rho^2 + O(\rho^3) \\
Z^\phi &= Y(\phi) - \partial_\phi f \frac{\mu}{\gamma^2} + \partial_\phi f \frac{\lambda}{2\gamma^4} \rho^2 + O(\rho^3),
\end{align*}
\]

where \( f(v, \phi) \) and \( Y(\phi) \) are arbitrary functions, and prime denotes the derivative with respect to \( \phi \). In addition to this, \( Z^\rho \) may contain \( O(1) \) term \( \tilde{z}(v, \phi) \). However, we set such terms to be zero provided \( Z = 0, \partial_\phi Y = 0 \). Under such transformations, the arbitrary functions transform along the killing direction as following

\[
\begin{align*}
\delta_{Z^\kappa} &= Y \partial_\phi \kappa + \partial_v (\kappa f) + \partial_\phi^2 f \\
\delta_{Z^\gamma} &= \partial_\phi (Y \gamma) + f \partial_\phi \gamma \\
\delta_{Z^\theta} &= \partial_\phi (Y \theta) + f \partial_\phi \theta - 2\kappa \partial_\phi f - 2\partial_v \partial_\phi f + 2\partial_\phi f \frac{\partial_\phi \gamma}{\gamma} \\
\delta_{Z^\lambda} &= Y \partial_\phi \lambda + 2\lambda \partial_\phi Y + 2\partial_\phi f - 2\partial_\phi \gamma + f \partial_\phi \lambda - \lambda \partial_\phi f.
\end{align*}
\]

Introducing the modified Lie bracket

\[
[Z_1, Z_2] = \mathcal{L}_{Z_1} Z_2 - \delta_{Z_1} Z_2 + \delta_{Z_2} Z_1,
\]
and for fixed temperature configuration, i.e., for $\kappa$ fixed, we get

$$f(v, \phi) = T(\phi) + e^{-\kappa v}X(\phi), \quad (3.6)$$

where $T(\phi)$ and $X(\phi)$ are two sets of supertranslation generators. Under this consideration, the algebra turns out to be

$$[Z(T_1, X_1, Y_1), Z(T_2, X_2, Y_2)] = Z(T_{12}, X_{12}, Y_{12}), \quad (3.7)$$

where

$$T_{12} = Y_1 \partial_\phi T_2 - Y_2 \partial_\phi T_1$$

$$X_{12} = Y_1 \partial_\phi X_2 - Y_2 \partial_\phi X_1 - \kappa(T_1 X_2 - T_2 X_1) \quad (3.8)$$

$$Y_{12} = Y_1 \partial_\phi Y_2 - Y_2 \partial_\phi Y_1$$

One can further establish the algebra between generators $T_n, X_n$ and $Y_n$ by defining Fourier modes,

$$T_n = Z(e^{in\phi}, 0, 0), \quad X_n = Z(0, e^{in\phi}, 0) \quad \text{and} \quad Y_n = Z(0, 0, e^{in\phi}),$$

given by

$$i[Y_m, Y_n] = (m - n)Y_{m+n}; \quad i[Y_m, T_n] = -nT_{m+n}$$

$$i[Y_m, X_n] = -nX_{m+n}; \quad i[T_m, X_n] = -\kappa X_{m+n} \quad (3.9)$$

The generator $T_n$ appears as a supertranslation from the symmetry of type $v \rightarrow v + T(\phi)$ and $Y_n$ causes superrotations. The conserved charges corresponding to the symmetries have also been found in [23, 24, 25] and form a representation of (3.9). The zero modes of the generators produce the Bekenstein-Hawking entropy of stationary black holes [23, 24].

### 3.1 Geodesic Deviation Equation and Memory Effect

Now, at first, we would concentrate on the memory effect part. As mentioned earlier that $\kappa$ depends on $(v, \phi)$, the deviation equation takes the following form

$$\gamma^2(\partial^2 S^\phi - \kappa(v, \phi)\partial_v S^\phi) = -R_{\phi\nu\phi\nu} S^\phi, \quad (3.10)$$

where the Riemann tensor component is given by

$$R_{\phi\nu\phi\nu} = \rho \left( \frac{1}{2} \kappa(v, \phi)\theta(\phi)^2 + \kappa(v, \phi)^2 \lambda(v, \phi) + \frac{\kappa(v, \phi)\theta(\phi)\gamma(\phi)'}{\gamma(\phi)} - \kappa(v, \phi)\theta(\phi)' - \frac{\gamma(\phi)'}{\gamma(\phi)} \partial_\phi \kappa(v, \phi) + \partial^2_\phi \kappa(v, \phi) - \frac{1}{2} \lambda(v, \phi) \partial_\nu \kappa(v, \phi) + \frac{1}{2} \kappa(v, \phi) \partial_\nu \lambda(v, \phi) - \frac{1}{2} \partial^2_\nu \lambda(v, \phi) \right) + O(\rho^2). \quad (3.11)$$

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4Here, we have considered terms leading order in $\rho$ for the Riemann tensor.
Here, prime denotes the derivative of the function with respect to $\phi$ coordinate. However, as our motivation is to find the memory near black holes, we consider only fixed temperature configurations i.e. cases where $\kappa$ is constant. We also set $\partial v \gamma = 0$. Under these assumptions, the Riemann tensor component becomes

\[
R_{\phi v\phi v} = \rho \left( \frac{1}{2} \kappa \theta(\phi)^2 + \kappa^2 \lambda(v, \phi) + \frac{\kappa \theta(\phi) \gamma(\phi)}{\gamma(\phi)} - \frac{1}{2} \kappa \partial_v \lambda(v, \phi) - \frac{1}{2} \partial_v^2 \lambda(v, \phi) \right) + O(\rho^2),
\]

(3.12)

where $\kappa = \frac{1}{l(r_+ - r_-)}$; $r_+$ and $r_-$ denoting the outer and inner horizons respectively. $l$ is the length scale of AdS spacetime.

### 3.1.1 Extremal Case

Above equations (Eq. 3.10, 3.12) considerably simplify in the case when $\kappa = 0$. In this case except the $g_{vv}$ component of the asymptotic metric, the boundary conditions do not get altered and will be similar to (3.2). The metric component $g_{vv}$ becomes $O(\rho^3)$ with an arbitrary function $L(v, \phi)$, $g_{vv} = L(v, \phi) \rho^2 + O(\rho^3)$ [25] and therefore does not contribute to the leading order. Now, for this extremal form of the metric (3), the geodesic deviation equation (1.1) takes the following form

\[
\gamma(\phi)^2 \partial_v^2 S_E^\phi = \frac{\rho}{2} \partial_v^2 \lambda S_E^\phi,
\]

(3.13)

where $S_E^\phi$ depicts the $\phi$ component of the deviation vector for extremal case.

Now, we can measure the change in the deviation vector which can be written as

\[
\Delta S_E^\phi = \frac{\rho}{2 \gamma(\phi)^2} (\Delta \lambda) S_E^\phi + O(\rho^2).
\]

(3.14)

Here, we observe that our goal is to find the change in $\lambda(v, \phi)$ in order to estimate the jump in the deviation vector. For this, we solve the $vv$-component of the Einstein field equation, given by

\[- \frac{\rho}{2 \gamma(\phi)^2} \partial_v^2 \lambda = 8 \pi T_{vv}^M.\]

(3.15)

Further, define $T_{vv} = \lim_{\rho \rightarrow 0} 8 \pi \frac{T_{vv}^M}{\rho}$ and consider a shock-wave type profile for the stress tensor as,

\[
T_{vv}(v, \phi) = s \delta(v - v_0) g(\phi),
\]

(3.16)

where $s$ is just a constant. The solution of the field equation is

\[
\lambda(v, \phi) = c_1(\phi) + \left( c_2(\phi) - 2 s \gamma(\phi)^2 g(\phi) \right) v.
\]

(3.17)
To compute the change in $\lambda(v, \phi)$, i.e., $\Delta \lambda(\phi)$ for some initial and final $v$, we set the boundary conditions as following
\[
\lambda(v = v_i, \phi) = \lambda_0(\phi) \quad ; \quad \lambda(v = v_f, \phi) = \lambda_f(\phi).
\] (3.18)

Hence, the full solution can be written as,
\[
\lambda(v, \phi) = C(\phi) + F(\phi)v,
\] (3.19)
for some function $F(\phi)$ that contains the metric parameters only depending on $\phi$. Therefore, the change in $\lambda(v, \phi)$ can be written as
\[
\Delta \lambda = \lambda(v = v_f, \phi) - \lambda(v = v_i, \phi) = F(\phi)\Delta v,
\] (3.20)
where $\Delta v = v_f - v_i$. We plug this expression of $\Delta \lambda$ into (3.14). This gives us the change in the deviation vector $S^\phi_E$.
\[
\Delta S^\phi_E = \frac{\rho}{2\gamma(\phi)} F(\phi)\Delta v S^\phi_E + O(\rho^2).
\] (3.21)

This provides the change in the deviation vector between the test geodesics near the horizon for three-dimensional extreme black holes. This is the permanent change in the setup induced by the interaction with GWs and regarded as the displacement memory effect. Figure (1) provides an illustration of the situation we are examining.

### 3.1.2 Relation with BMS symmetry

To see the connection between memory effect and BMS symmetry, we consider the change in the parameter $\lambda$. The other parameters may also undergo changes but those are not important here. For extreme case, the $v$ component of the Killing vector $f(v, \phi)$, generating two sets of supertranslations $T(\phi)$ and $X(\phi)$, becomes
\[
f(v, \phi) = T(\phi) + vX(\phi)
\] (3.22)
In general, the $\lambda(v, \phi)$ can be written as, $\lambda(v, \phi) = C(\phi) + vF(\phi)$ (3.19). If $\Delta v$ is the difference in advanced-time before and after the passage of GW, then
\[
\Delta \lambda = F(\phi)\Delta v.
\] (3.23)
This is an equivalent way of writing (3.20). Now, we consider the change in $\lambda(v, \phi)$ induced by asymptotic Killing vector $Z$ [25] given by

$$\mathcal{L}_Z \lambda = 2 \lambda \partial_\phi Y + Y \partial_\phi \lambda + 2 \theta \partial_\phi f - 2 \partial_\phi f \frac{\partial_\phi \gamma}{\gamma(\phi)} + f \partial_v \lambda - \lambda \partial_v f.$$  \hspace{1cm} (3.24)

We now specialize for the case where $f$ becomes independent of $v$, and the set of supertranslations generated by $X(\phi)$ does not exist. Only $T(\phi)$ contributes. This is analogous to the Killing supertranslations introduced in [38]. Further, the $v$ dependence of $\lambda$ for extreme case is linear in $v$ as seen in (3.19), [24]. To make sure the whole right hand side of the Eq. (3.24) matches with the displacement in $\lambda$ derived in (3.20), we choose the superrotations

$$Y = Y^\phi \partial_\phi \sim F^{-1/2} \partial_\phi.$$  \hspace{1cm} (3.25)

This will ensure no $v$ dependent terms arise from the first two terms of (3.24). As a result, Eq. (3.24) becomes entirely $v$ independent and a function of $\phi$ only. Now, the remaining four terms (which are functions of $\phi$ only) can be arranged in such a way that they become equal to $\Delta v F(\phi)$. Therefore, the displacement memory effect can be accomplished by a supertranslation $T(\phi)$ as well as superrotation $Y$ here.
3.2 Non-Extremal Case

For nonzero but constant $\kappa$ configuration, we again need to find the change in $\lambda(v, \phi)$, and $\lambda$ can be obtained by solving the $vv$-component of the Einstein’s field equation (to the leading order in $\rho$)

$$-\frac{\rho}{2l^2\gamma^3}\left(-4\kappa\gamma^3 + 2l^2\kappa\theta\gamma' + l^2\gamma(\kappa\theta^2 + 2\kappa^2\lambda(v, \phi) - 2\kappa\theta' + 3\kappa\partial_v\lambda(v, \phi) + \partial_v^2\lambda(v, \phi))\right) = 8\pi T_{vv}^M. \quad (3.26)$$

Prime denotes the derivative of the function with respect to $\phi$ coordinate. Further, we shall again define $T_{vv} = \lim_{\rho \to 0} 8\pi T_{vv}^M$ and consider the terms, other than $\lambda(v, \phi)$, as a function $\kappa F(\phi)$, defined by,

$$F(\phi) = \rho\left(\frac{\theta(\phi)\gamma(\phi)'}{\gamma(\phi)^2} + \frac{\theta(\phi)^2}{2} - \theta(\phi)'ight).$$

Since we are interested to solve the field equation (3.26) with respect to $v$ coordinate, $\phi$-dependent functions would be unaffected while solving the equation. Now Eq.(3.26) becomes

$$-\frac{1}{2\gamma(\phi)^2}\frac{\partial^2}{\partial v^2}\lambda(v, \phi) - \frac{3\kappa}{2\gamma(\phi)^2}\frac{\partial}{\partial v}\lambda(v, \phi) - \frac{\kappa^2}{\gamma(\phi)^2}\lambda(v, \phi) + \kappa F(\phi) - T_{vv}(v, \phi) = 0. \quad (3.27)$$

Next, we consider a shock wave type stress tensor, i.e., $T_{vv}(v, \phi) = s\delta(v - v_0)g(\phi)$ for solving the field equation. Where ‘$s$’ is a constant or can also be a function of $\phi$. The equations still describe a fixed temperature but not a stationary black hole configuration. For stationary black holes, the $v$ dependence of $\lambda$ goes away and the analysis becomes quite similar to the extreme case.

For non-stationary background, let us first try to solve (3.10) together with (3.12) perturbatively in $\kappa$ to gain insight into the problem. We do an exact analysis for all values of $\kappa$ in the Section (A).

Upto leading order in $\kappa$, we get from (3.10) and (3.12),

$$\gamma^2(\partial^2_v S^\phi - \kappa\partial_v S^\phi) = -\rho\left(\frac{1}{2}\kappa\theta(\phi)^2 + \frac{\theta(\phi)\gamma(\phi)'}{\gamma(\phi)} - \kappa\theta(\phi)' + \frac{1}{2}\kappa\partial_v\lambda(v, \phi) - \frac{1}{2}\partial_v^2\lambda(v, \phi)\right)S^\phi + O(\kappa^2). \quad (3.28)$$

Integrating this twice, we get

$$\Delta S^\phi(1 - \kappa \Delta v) = \frac{\rho}{\gamma^2}\left(\frac{1}{2}\Delta \lambda - \frac{1}{2}\kappa \Delta v \Delta \lambda - \kappa(\Delta v)^2\Theta(\phi)\right)S^\phi + O(\kappa^2), \quad (3.29)$$

where

$$\Theta(\phi) = \frac{\theta(\phi)^2}{2} + \frac{\theta(\phi)\gamma(\phi)'}{\gamma(\phi)} - \theta(\phi)'. \quad (3.30)$$

$^5\Delta v = v_f - v_i$ as defined earlier.
Finally, we get
\[ \Delta S^\phi = \frac{p}{2\gamma} \left( \frac{1}{2} \Delta \lambda - \kappa (\Delta v)^2 \Theta (\phi) \right) S^\phi + \mathcal{O}(\kappa^2), \]  
(3.31)

Now from (3.27) up to \( \mathcal{O}(\kappa) \), we get integrating twice
\[ \Delta \lambda \simeq 2\gamma^2 \left[ - s \Delta v g(\phi) + \kappa (\Delta v)^2 (F + 3 s g(\phi)) \right]. \]
(3.32)

Inserting Eq. (3.32) in (3.31) describes an expression of shift in deviation vector due to passage of a gravitational wave pulse near the asymptotic horizon of a non-extreme black hole.

4 Near-Horizon Memory for Four-Dimensional Black Holes

Now, we consider a more realistic scenario by extending our analysis to four-dimensional black holes. We shall again adopt a similar strategy as implemented in the Sec. (3). We consider the same configuration, i.e., two nearby timelike geodesics or test detectors are being positioned near the black hole horizon. Let us start by writing the general form of the near-horizon four-dimensional metric [23, 24, 25]
\[ ds^2 = g_{\nu\nu} dv^2 + 2\kappa dvdp + 2g_{\nu A} dv dx^A + g_{AB} dx^A dx^B, \]
(4.1)

with following fall-off conditions for the horizon \( \rho = 0 \):
\[ g_{\nu\nu} = -2\kappa + \mathcal{O}(\rho^2) \quad ; \quad k = 1 + \mathcal{O}(\rho^2) \]
\[ g_{\nu A} = \rho \theta_A + \mathcal{O}(\rho^2) \quad ; \quad g_{AB} = \Omega \gamma_{AB} + \rho \lambda_{AB} + \mathcal{O}(\rho^2) \]

where \( \theta_A \) and \( \Omega \) are functions of \( x^A \); \( \lambda^{AB} \) is also a function of coordinates, \( \lambda^{AB} = \lambda^{AB}(v, x^A) \). \( \gamma_{AB} \) represents the 2-sphere metric. In stereographic coordinates \( x^A = (\zeta, \bar{\zeta}) \), the spherical part of the metric is \( \gamma_{AB} dx^A dx^B = \frac{4}{(1+\zeta \bar{\zeta})^2} d\zeta d\bar{\zeta} \). While metric components \( g_{\rho A} \) and \( g_{\rho \rho} \) fall as \( \mathcal{O}(\rho^2) \). We will be concentrating on non-rotating configurations here, hence we set \( g_{\nu \zeta} \) and \( g_{\nu \bar{\zeta}} \) to be zero. The resultant metric reduces to the following form
\[ ds^2 = g_{\nu\nu} dv^2 + 2\kappa dvdp + g_{AB} dx^A dx^B. \]
(4.2)
Following the similar method of three-dimensional case, the asymptotic Killing vectors preserving fall-off boundary conditions, given by

\[ \chi^v = f(v, x^A) \]
\[ \chi^\rho = -\partial_v f \rho + \frac{1}{2} g^{AB} \partial_B f \rho^2 + O(\rho^3) \]
\[ \chi^A = Y^A(x^B) + g^{AC} \partial_C f \rho + \frac{1}{2} g^{AD} g^{CB} \lambda_{DB} \partial_C f \rho^2 + O(\rho^3), \]

where \( f(v, x^A) \) and \( Y(x^A) \) are arbitrary functions, and \( g^{AB} \) is inverse of \( g_{AB} \). One can find the transformation of the functions \( \theta_A, \lambda_{AB}, g_{AB} \) and \( \kappa \), and establish the algebra between supertranslation and superrotation as an extension of three-dimensional analysis. The explicit details can be seen in [23, 24, 25]. Next, we consider the memory effect analysis.

4.1 Extreme Case

Let us consider the extremal case first. The deviation equation for the metric (4.1) is

\[ \frac{2}{(1 + \zeta \bar{\zeta})^2} \partial^2_{\bar{v}} S_{E}^{\bar{\zeta}} = -(R_{\zeta \bar{\zeta}} S_{E}^{\bar{\zeta}} + R_{\zeta \zeta} S_{E}^{\zeta}), \]

where \( S_{E}^{\bar{\zeta}} \) and \( S_{E}^{\zeta} \) depict the \( \bar{\zeta} \) and \( \zeta \) components of the deviation vector respectively for extremal case. One can compute the geodesic deviation equation for the metric (4.2) also. The form of the deviation equation does not change as the left hand side of the expression (4.4) is written upto \( O(\rho) \). The corresponding Riemann tensors are

\[ R_{\zeta \zeta} = -\frac{\rho^2}{2} \partial^2_{\bar{v}} \lambda_{\bar{\zeta}} + O(\rho^2) ; \quad R_{\zeta \bar{\zeta}} = -\frac{\rho^2}{2} \partial^2_{\bar{v}} \lambda_{\zeta} + O(\rho^2). \]

Therefore, the change in the deviation vector can now be written as

\[ \Delta S_{E}^{\bar{\zeta}} = \frac{\rho}{4} (1 + \zeta \bar{\zeta})^2 (\Delta \lambda_{\bar{\zeta}} S_{E}^{\bar{\zeta}} + \Delta \lambda_{\zeta} S_{E}^{\zeta}) + O(\rho^2). \]

Similarly, one can also write the change in the displacement vector for \( S_{E}^{\zeta} \), given by

\[ \Delta S_{E}^{\zeta} = \frac{\rho}{4} (1 + \zeta \bar{\zeta})^2 (\Delta \lambda_{\bar{\zeta}} S^{\bar{\zeta}} + \Delta \lambda_{\zeta} S^{\zeta}) + O(\rho^2). \]

We first focus on the \( \bar{\zeta} \) component here. For our computational purpose, we can consider diagonal components of \( \Delta \lambda_{AB} \) to be zero. Under this assumption, Eq.(4.6) reduces to a single term on the right-hand side, and now we need to find the change in \( \lambda_{\bar{\zeta} \bar{\zeta}} \) in order to estimate the jump in the deviation vector. This would require to solve the Einstein field equation as we did for the
three-dimensional extremal case, and the relevant component of the field equation is given by

$$-\frac{\rho}{2}(1 + \zeta \bar{\zeta})^2 \partial^2_{\zeta \bar{\zeta}} \lambda_{\zeta \bar{\zeta}} = 8\pi T_{vv}^M. \quad (4.8)$$

Again, by employing similar initial conditions we get,

$$\Delta S_{\bar{\zeta}} = \frac{\rho}{4}(1 + \zeta \bar{\zeta})^2 G(\zeta, \bar{\zeta}) \Delta v S_{\bar{\zeta}} + O(\rho^2). \quad (4.9)$$

This again evaluates the change in the deviation vector between the test geodesics near the horizon for four-dimensional extreme black holes and describes the displacement memory effect. Similar analysis can be done for $\Delta S_{\zeta}$.

To find the memory, similar approach is adopted for four-dimensional extreme black holes as has been done for the three dimensional case. $\lambda(v, \zeta, \bar{\zeta})$ can be written as, $\lambda_{\zeta \bar{\zeta}}(v, \zeta, \bar{\zeta}) = C(\zeta, \bar{\zeta}) + vH(\zeta, \bar{\zeta})$. Again, if before and after the passage of GW means an interval of $\Delta v$, then

$$\Delta \lambda_{\zeta \bar{\zeta}} = H(\zeta, \bar{\zeta}) \Delta v. \quad (4.10)$$

Now considering a transformation in $\lambda_{\zeta \bar{\zeta}}(v, \zeta, \bar{\zeta})$ along killing direction $\chi$ to preserve the change in $\lambda(v, \zeta, \bar{\zeta})$ together with the shift in the deviation vector (4.9), given by [25]

$$\mathcal{L}_\chi \lambda_{AB} = f \partial_v \lambda_{AB} - \lambda_{AB} \partial_v f + \mathcal{L}_Y \lambda_{AB} + \theta_A \partial_B f + \theta_B \partial_A f - 2\nabla_A \nabla_B f, \quad (4.11)$$

where $\chi$ is the Killing vector, and $\chi^v$, is chosen so that it becomes $\sim f(\zeta, \bar{\zeta}) = T(\zeta, \bar{\zeta})$ [25], for $\kappa = 0$. Again almost identical analysis can be followed as depicted in (3.1.2), the right hand side of Eq. (4.11) becomes $v$ independent. Therefore, here also we can find a $T(\zeta, \bar{\zeta})$ and $Y$ that will induce the desired shift in the deviation vector components.

### 4.2 Non-Extreme Case

Now, we analyze our approach for non-extremal case when $\kappa$ is nonzero. The relevant components of the Riemann tensor are

$$R_{\zeta \bar{\zeta}v} = \left(\kappa^2 \lambda_{\zeta \bar{\zeta}} + \frac{1}{2} \kappa \partial_v \lambda_{\zeta \bar{\zeta}} - \frac{1}{2} \partial_{v}^2 \lambda_{\zeta \bar{\zeta}}\right) \rho + O(\rho^2); \quad R_{\zeta \zeta v} = \left(\kappa^2 \lambda_{\zeta \zeta} + \frac{1}{2} \kappa \partial_v \lambda_{\zeta \zeta} - \frac{1}{2} \partial_{v}^2 \lambda_{\zeta \zeta}\right) \rho + O(\rho^2). \quad (4.12)$$

The deviation equation in this case becomes

$$\frac{2}{(1 + \zeta \bar{\zeta})^2} \left(\partial^2_{\zeta \bar{\zeta}} S_{\bar{\zeta}} - \kappa \partial_v S_{\bar{\zeta}}\right) = -(R_{\zeta \zeta v} S_{\bar{\zeta}} + R_{\zeta \zeta v \zeta} S_{\zeta}), \quad (4.13)$$
For, $\lambda_\zeta \zeta = 0$, we get
\[
\frac{2}{(1 + \zeta \bar{\zeta})^2} (\partial^2_v S^{\zeta} - \kappa \partial_v S^{\zeta}) = -R_{\zeta \bar{v} \zeta v} S^{\zeta}.
\] (4.14)

Like the three-dimensional case, we first try to solve (4.14) pertubatively in $\kappa$.

At $O(\kappa^0)$, we get,
\[
\partial^2_v S^{\zeta} = \frac{\rho}{4} (1 + \zeta \bar{\zeta})^2 \partial^2_v \lambda_\zeta \bar{\zeta} S^{\zeta}.
\] (4.15)

This is same as the extremal case mentioned in (4.6). Then we can add the $O(\kappa)$ correction to it and get
\[
\partial^2_v S^{\zeta} - \kappa \partial_v S^{\zeta} = \frac{\rho}{4} (1 + \zeta \bar{\zeta})^2 \partial^2_v \lambda_\zeta \bar{\zeta} S^{\zeta} - \frac{\rho}{4} (1 + \zeta \bar{\zeta})^2 \kappa \partial_v \lambda_\zeta \bar{\zeta} S^{\zeta}.
\] (4.16)

Integrating twice the above expression, we get
\[
\Delta S^{\zeta} (1 - \kappa \Delta v) = \frac{\rho}{4} (1 + \zeta \bar{\zeta})^2 (1 - \kappa \Delta v) \Delta \lambda_\zeta \bar{\zeta} S^{\zeta}.
\] (4.17)

This can be simplified to get
\[
\Delta S^{\zeta} = \frac{\rho}{4} (1 + \zeta \bar{\zeta})^2 \Delta \lambda_\zeta \bar{\zeta} S^{\zeta}.
\] (4.18)

Next, we need to find $\Delta \lambda_\zeta \bar{\zeta}$ by integrating the relevant component of the Einstein equations. Again upto $O(\kappa)$, we get
\[
-\frac{1}{2} (\zeta \bar{\zeta} + 1)^2 (3 \kappa \partial_v \lambda_\zeta \bar{\zeta} + \partial^2_v \lambda_\zeta \bar{\zeta}) + 2\kappa - T_{vv}^{M} (v, \zeta, \bar{\zeta}) = 0.
\] (4.19)

We then define $T_{vv} = \lim_{\rho \to 0} 8\pi \frac{T_{vv}^{M}}{\rho}$ and $T_{vv} = q \delta (v - v_0)Z(\zeta, \bar{\zeta})$. Integrating (4.19), we get
\[
\Delta \lambda_\zeta \bar{\zeta} (1 + 3 \kappa \Delta v) = -\frac{2 q}{(1 + \zeta \bar{\zeta})^2} \Delta v Z(\zeta, \bar{\zeta}) + \frac{4 \kappa (\Delta v)^2}{(1 + \zeta \bar{\zeta})^2},
\] (4.20)

which upto $O(\kappa)$ reads
\[
\Delta \lambda_\zeta \bar{\zeta} = -\frac{2 q}{(1 + \zeta \bar{\zeta})^2} \Delta v (1 - 3 \kappa \Delta v) Z(\zeta, \bar{\zeta}) + \frac{4 \kappa (\Delta v)^2}{(1 + \zeta \bar{\zeta})^2}.
\] (4.21)

Combining (4.18) and (4.21), we get
\[
\Delta S^{\zeta} = \left[ -\frac{q \rho}{2} \Delta v Z(\zeta, \bar{\zeta}) + \rho \kappa (\Delta v)^2 \left( 1 + \frac{3 q}{2} Z(\zeta, \bar{\zeta}) \right) \right] S^{\zeta} + O(\kappa^2).
\] (4.22)
This is the full shift in deviation vector components for the case when we have a large black hole (background stationary solution) i.e. for small $\kappa$. Here also we may seek for a BMS symmetry that induces the shift given in the above equation. This is again in principle be determined from (4.11), by choosing $\lambda$ to be linear in $v$ and setting $Y$ to be zero. As the entire change in $\lambda$ then would be determined by functions that are dependent on $\zeta, \bar{\zeta}$ only.

5 Discussion

The primary objective of this report is to study how GWs affect inertial test detectors near the horizon of black hole spacetimes. This analysis could serve as a model for estimating the deviation between the test masses of a GW detector. We also provide an explicit connection between BMS symmetries and memory effect near the horizon of black holes.

There are crucial differences between memory effect and BMS symmetries obtained in the future null infinities and the same obtained near the horizon of a black hole. As the algebra of asymptotic symmetries contains two mutually commuting Virasoro generators, we have two sets of supertranslation and a set of superrotation generators near the horizon. The extra set of supertranslation generator is something that we do not recover at the null infinities. Another crucial difference seems to be in the structure of the geodesic deviation equation (GDE). The near-horizon GDE contains a linear derivative term of the deviation vector with respect to time apart from a double-time derivative, whereas the far region GDE contains only a double-time derivative of deviation vector to the leading order. The function $\lambda(\lambda_{AB}$ in four dimensions) mimics the derivative of News tensor, $C_{zz}$, in the far region. The Riemann tensor in GDE also contains a double derivative and a single derivative of $\lambda$, which is absent in the GDE of far region (2.4). These differences make the analyses of near-horizon memory significantly non-trivial than the far region case. To show a direct effect of BMS symmetries with the displacement memory, we have considered the extremal case. The analysis for an extreme black hole horizon quite remarkably matches with the far region non-extreme case. We also see the memory in the near region is due to the dual effect of supertranslation and superrotation generators. This is also a striking difference from what is known for far region (null infinity) memory effect. The non-extreme case seems to be analytically difficult to analyze. We have provided an analytic expression for the memory effect in small $\kappa$ approximation. Determining an explicit relation between a BMS transformation and the memory for the non-extreme case seems to be difficult, although it can be achieved under some assumptions. We wish to report more on this in upcoming studies.

In the near future, the observational aspects of GW will serve a great purpose in measuring the memory effect, and this could allow us to examine BMS symmetries in more depth as a direct evidence [39]. The detection of near-horizon memory in the far region may be possible if the test detectors or particles near the black hole region interact with some matter or radiation that can
be detected in the far region and encode the near-horizon physics including memory. It will be interesting to investigate the existence of such a phenomenon.

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A Near-Horizon Memory: An exact analysis

Here we provide an exact analysis for the three-dimensional case.

One can directly solve Eq. (3.27) for finding the change in the \( \lambda \) with respect to the \( v \) coordinate. It is given by

\[
\lambda(v, \phi) = c_1 e^{-\kappa v} + c_2 e^{-2\kappa v} + \frac{\gamma^2}{\kappa} F + \frac{2s\gamma^2}{\kappa} \left( e^{-2\kappa(v-v_0)} - e^{-\kappa(v-v_0)} \right) g(\phi),
\]

where \( c_1 \) and \( c_2 \) are functions of \( \phi \). Now, if we put this solution in the Riemann tensor expression (3.12) and solve for the geodesic deviation equation (3.10), we get the final solution, after extracting the real part, as following

\[
S_\phi = 2^{\frac{1}{2}} \left( \frac{\sqrt{w}}{2\kappa\gamma^3} + 1 \right) \left( e^{-2\kappa v} \right) \frac{\sqrt{w}}{2\kappa\gamma^3} - \frac{1}{4} \left( \rho e^{-2\kappa v} \left( c_2 \kappa + 2 g(\phi) s\gamma^2 e^{2\kappa v_0} \right) \right) - \frac{\sqrt{w}}{4\kappa^2\gamma^4}
\]

\[
C_1 I \frac{\sqrt{w}}{2\kappa\gamma^3} \left( \sqrt{2e^{-2\kappa v} \rho \left( 2e^{2\kappa v} g(\phi) s\gamma^2 + c_2 \kappa \right)} \right),
\]

where \( w \rightarrow \kappa^2\gamma^3 \left( \gamma^3 \left( \kappa^2 - 4F \rho \right) - 2\kappa\rho\gamma \left( \theta^2 - 2\theta(\phi') \right) - 4\kappa\rho\theta\gamma(\phi') \right) \). \( C_1 \) is another function of \( \phi \) and contributes from the real part of the solution. \( I \) is the Bessel’s function. It is expected that setting appropriate boundary conditions on \( \lambda(v, \phi) \), would give rise the nonzero finite change in the deviation vector \( S^\phi \). This would also depict a permanent change in the deviation vector, and will be regarded as memory effect in non-extreme case. For the four-dimensional case, we have analysed the general case with \( \lambda_\zeta = 0 \), and have got a rather long expression in terms of Bessel’s functions which we are not displaying here.

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