Vibration modes of flexoelectric circular plate

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Beams, plates, and shells, as the fundamental mechanical structures, are widely used in microelectromechanical systems (MEMS) and nanoelectromechanical systems (NEMS) as sensors, actuators, energy harvesters, and among others. Deeply understand the electromechanical coupling of these dielectric structures is of crucial for designing, fabricating, and optimizing practice devices in these systems. Herein we demonstrate the electromechanical coupling in flexoelectric circular plate, in which higher-order strain gradients were considered to extend the classical electromechanical properties to isotropic materials, in which the non-uniform distribution of the electric potential along the radial direction was considered. Analytical solutions for the vibration modes of the flexoelectric circular plates showed that the dynamic modes were totally different from the piezoelectric circular plates owing to the inversion symmetry breaking by the strain gradient. The electromechanical coupling dynamic modes are sensitive to bending, twisting modes owing to the sensitivity of the flexoelectric effect to bending. This work provides a fundamental understanding of the electromechanical coupling in flexoelectric circular plate, which is helpful in designing novel flexoelectric circular plate-based devices, such as flexoelectric mirrors.

Flexoelectricity, Electromechanical, Coupling, Circular plate

1. Introduction

Flexoelectricity, electric polarization in response to elastic strain gradient, is a universal electromechanical coupling phenomenon in all dielectrics [1,2]. Flexoelectricity allows dielectrics to bend in response to an electric field, which provides flexoelectric-based actuators with simple structure but with comparable performance to the state-of-the-art piezoelectric biomorph cantilever actuators [3]. It has been proved that flexoelectricity plays an important role in many physical phenomena, including mechanical writing in ferroelectric thin films [4], polarization rotation [5], rectification of charge transport in strain-graded dielectrics [6], and even flexoelectronic effect in centrosymmetric semiconductors [7]. Since the gradients (strain gradient and electric field gradient) are inversely proportional to the scale of materials, flexoelectricity is expected to match or even dominate over piezoelectricity at nanoscale.

The challenge of developing basic mechanical models for the flexoelectric coupled structures has attracted much attention of many researchers. Inspired by the “giant flexoelectricity” in high dielectric (high-K) materials by Ma and Cross [8-10], a tantalizing concept that fabricating apparent piezoelectric composites without using any piezoelectric constituents has been experimentally and theoretically provided [11,12]. Size-dependent piezoelectricity and elasticity in nanostructures were explained by the flexoelectric effect in the previous studies [13,14]. A continuum theory of flexoelectricity with surface effect for dielectrics was given by Shen and Hu [15], in which the two major bulk static

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flexoelectric effects were included. Based on the above continuum theory of flexoelectricity, Zhang et al. and Yan et al. [16,17] investigated the electroelastic behaviors of dielectric nanostructures. Liang et al. [18] investigated the effect of surface and flexoelectricity on the electromechanical coupling responses of nanostructures. Furthermore, the size-dependent electromechanical behaviors for nanobeams and nanoplates have been investigated by many researchers [19-24]. To the author’s acknowledgment, the electric field in these literatures is assumed only in the thickness direction, and little work is done on the non-uniform distribution of the electric field [25-27]. Moreover, the dynamic mode analysis is not mentioned in the previous studies. In virtue of the wide applications of piezoelectric mirrors in aerospace, solar collectors, and radars, it is essential to derive the fundamental mode shapes of these deformable mirrors [28,29]. This work focuses on the flexoelectric effect on the vibration modes of circular plate, and the model provided herein could be helpful in designing novel flexoelectric-based devices, such as flexoelectric mirrors.

2. Displacement and electric potential in flexoelectric circular plate

The piezoelectric mirror and flexoelectric mirror in Fig. 1a and b can be modeled by a mechanical model with cylindrical coordinates system in Fig. 1c. The key in the analysis is to model the displacement and electric field in the piezoelectric and flexoelectric circular plate, which could also be helpful in electrode designing.

For flexoelectric circular plate with two electrodes on its surfaces with applied external electric load, according to the Kirchhoff thin plate theory, the displacement field can be expressed as [16,30]

\[
u_r = u_r(r, \theta, t) = -z \frac{\partial w(r, \theta, t)}{\partial r},\]

\[
u_\theta = u_\theta(r, \theta, t) = -z \frac{\partial w(r, \theta, t)}{r \partial \theta},\]

\[
u_z = u_z(r, \theta, t) = w(r, \theta, t),\]

where \(u_r, u_\theta, \) and \(u_z\) are the displacements along the radial, circumferential, and axial direction, respectively. \(w(r, \theta, t)\) is the transverse displacement of the mid-plane of the flexoelectric plate.

For free vibration analysis of piezoelectric coupled structures, a quadratic vibration of the electric potential in the thickness is obtained [28-30]. When an external voltage is applied on the flexoelectric circular plate, the electric potential can be written as [26]

\[
\varphi = \frac{e_{31}}{a_{33}} \left( \frac{r^2}{h} - z \right) \varphi(r, \theta, t) + \frac{\varphi_0}{h}, (4)
\]

where \(\varphi(r, \theta, t)\) is the electric potential on the mid-plane of the flexoelectric circular plate owing to the presence of piezoelectricity, \(\varphi_0\) is the external voltage applied on the flexoelectric circular plate. \(e_{31}\) and \(a_{33}\) are the piezoelectric and dielectric constant, respectively. And \(h\) is the thickness of the circular plate.

From the strain-displacement relation, \(\varepsilon_{kr} = \frac{u_k + u_j}{2}\), and the electric field-electrostatic potential relation, \(E_k = -\varphi_s\), the strain and electric field can be expressed by

\[
\varepsilon_{rr} = \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{u_r}{r}, (5)
\]

\[
\varepsilon_{\theta\theta} = \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_\theta}{r}, (6)
\]

\[
2\varepsilon_{r\theta} = 2\varepsilon_{\theta r} = -z \left( \frac{\partial^2 w}{r \partial \theta \partial r} - \frac{\partial w}{r \partial \theta} \right), (7)
\]

\[
E_r = -e_{31} \left( \frac{r^2}{h^2} - 1 \right) \frac{\partial \varphi}{\partial r}, (8)
\]

\[
E_\theta = -e_{31} \left( \frac{2r}{h} - 1 \right) \frac{\partial \varphi}{r \partial \theta}, (9)
\]

\[
E_z = -e_{31} \left( \frac{2r}{h} - 1 \right) \frac{\partial \varphi}{r \partial \theta}, (10)
\]

where \(\varepsilon_{rr}, \varepsilon_{\theta\theta}\), and \(\varepsilon_{r\theta}\) are the normal strains and shear strain in the circular plate, respectively. \(E_r, E_\theta,\) and \(E_z\) are the electric fields along the radial, circumferential, and thickness direction, respectively. In the previous studies, the electric fields \(E_r\) and \(E_\theta\) are neglected [16,30]. However, the mode shape of the electric potential from free vibration of piezoelectric coupled structures proved the non-uniform distribution in the radial direction. Therefore, we adopted the electric potential in the piezoelectric coupled structures that the non-uniform distribution in the radial direction is taken into account.

The constitutive equations for the flexoelectric plate with piezoelectricity in the Cartesian coordinate are expressed as

\[
\sigma_{ij} = -c_{ijkl} \varepsilon_{kl} - e_{ki} E_k \frac{1}{2} \mu_{klj} \frac{\partial E_j}{\partial x_i}, (11)
\]

\[
\tau_{kl} = 1 \frac{1}{2} \mu_{ijkl} E_l, (12)
\]

\[
D_k = a_k E_i + e_{ki} E_j + \frac{1}{2} \mu_{klj} \frac{\partial E_i}{\partial x_j}, (13)
\]

\[
Q_{kl} = \frac{1}{2} \mu_{klj} E_j, (14)
\]

where \(\sigma_{ij}, \tau_{kl}, D_k,\) and \(Q_{kl}\) are the Cauchy stress, higher-
order moment stress, electric displacement, and higher-order electric displacement, respectively. \( \partial E_k / \partial x_i \) and \( \partial E_{kl} / \partial x_j \) are the electric field gradient and strain gradient, respectively. \( a_{kl}, c_{ijkl}, e_{ijkl}, \) and \( \mu_{ijkl} \) are material properties, particularly, \( a_{kl} \) is the second-order dielectric constant, \( c_{ijkl} \) is the fourth-order elastic constant, \( e_{ijkl} \) is the third-order piezoelectric constant, and \( \mu_{ijkl} \) is the fourth-order flexoelectric constant, respectively.

The physical stress and physical electric displacement are defined by \([32,33]\)

\[
\sigma_{ij}^{\text{phys}} = \sigma_{ij} - \tau_{ijl} = c_{ijkl}E_k - e_{ijkl}E_k - \mu_{ijkl}\frac{\partial E_{ij}}{\partial x_l},
\]

(15)

\[
D_k^{\text{phys}} = D_k - Q_{kl,l} = a_{kl}E_l + e_{ijkl}E_k + \mu_{ijkl}\frac{\partial E_{ij}}{\partial x_l},
\]

(16)

Recalling the relationship between electric displacement, electric polarization, and electric field in the dielectric materials, \( P_k = P_k + \varepsilon_k E_k \), one has

\[
P_k = \chi_{kl}E_l + e_{ijkl}E_k + \mu_{ijkl}\frac{\partial E_{ij}}{\partial x_l},
\]

(17)

where \( \chi_{kl} \) is the dielectric susceptibility of the material. Equation (17) demonstrates that the presence of strain gradient in dielectric material will induce electric polarization without external electric field, even in those materials without piezoelectricity.

According to the classical Kirchhoff thin plate theory, the normal stress \( \sigma_z \) and the shear stresses \( \sigma_{r\theta} \) and \( \sigma_{\theta r} \) are negligible, so that plane-stress condition was used. However, for the flexoelectric Kirchhoff thin plate theory, the physical stresses, \( \sigma_z^{\text{phys}} \), \( \sigma_{r\theta}^{\text{phys}} \), and \( \sigma_{\theta r}^{\text{phys}} \) rather than the classical Cauchy stresses, are negligible. The physical stresses are defined by \([32]\)

\[
\sigma_z^{\text{phys}} = \sigma_z - \left( \frac{\partial \tau_{zr}}{\partial r} + \frac{\partial \tau_{z\theta}}{\partial \theta} + \frac{\tau_{rz}}{r} + \frac{\tau_{z\theta}}{r} \right),
\]

(18)

\[
\sigma_{r\theta}^{\text{phys}} = \sigma_{r\theta} - \left( \frac{\partial \tau_{rr}}{\partial r} + \frac{\tau_{r\theta}}{r} + \frac{\tau_{rr}}{r} \right),
\]

(19)

\[
\sigma_{\theta r}^{\text{phys}} = \sigma_{\theta r} - \left( \frac{\partial \tau_{rr}}{\partial \theta} + \frac{\tau_{\theta r}}{r} + \frac{\tau_{rr}}{r} \right).
\]

(20)

In fact, the in-plane dimension in the flexoelectric Kirchhoff thin plate is much larger than that of the out-of-plane dimension so that the gradients along the thickness are much stronger than the strain gradient along in-plane direction (radial and circumferential).

By the help of the resultant moment in the plate,

\[
M_{rr} = \int_{-h/2}^{h/2} z \sigma_{rr}^{\text{phys}} \, dz,
\]

(21)

\[
M_{r\theta} = \int_{-h/2}^{h/2} z \sigma_{r\theta}^{\text{phys}} \, dz,
\]

(22)

\[
M_{\theta r} = \int_{-h/2}^{h/2} z \sigma_{\theta r}^{\text{phys}} \, dz.
\]

(23)

And the resultant shear forces are defined by

\[
q_r = \frac{\partial M_{rr}}{\partial r} + \frac{\partial M_{r\theta}}{\partial \theta} + \frac{M_{r\theta} - M_{\theta r}}{r},
\]

(24)

\[
q_\theta = \frac{\partial M_{r\theta}}{\partial r} + \frac{\partial M_{\theta r}}{\partial \theta} + \frac{2M_{\theta r}}{r}.
\]

(25)

The governing equation for the flexoelectric Kirchhoff thin plate can be written as

\[
\frac{\partial q_r}{\partial r} + \frac{\partial q_\theta}{\partial \theta} + \frac{q_r}{r} = \rho h \frac{\partial^2 W}{\partial t^2}.
\]

(26)

Substituting Eqs. (1)-(10) into the constitutive equations, and then into Eqs. (21)-(26), one has

\[
D \nabla^2 W + \frac{2\mu_{13} c_{31}}{a_{33}} \nabla^2 \phi + \rho h \frac{\partial^2 W}{\partial t^2} = 0,
\]

(27)

where \( \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial \theta^2} \) is the two-dimension Laplace operator in polar coordinate. Equation (27) is the mechanical governing equation for the flexoelectric plate, and it is reduced to the pure elastic case if the flexoelectric effect is neglected (i.e., \( \mu_{13} = 0 \)).

In the absence of free charges in the flexoelectric circular plate, the Maxwell equation for the electrostatic field gives...
\[ \frac{\partial \text{e} \text{D Phys}}{\partial r} + \frac{\partial \text{D Phys}}{\partial \theta} + \frac{\partial \text{D Phys}}{\partial z} = 0. \]  

(28)

By substituting the displacements and electric field into the above equations, the electric potential is governed by

\[ (\mu_{12} + 2\mu_{44})z \nabla^2 w + a_{11} \nabla^2 \varphi + a_{11} \frac{\partial^2 \varphi}{\partial z^2} = 0. \]  

(29)

Assuming a harmonic vibration of the flexoelectric thin plate, \( w(r, \theta, t) = W(r, \theta) e^{i\omega t} \), the solution for the amplitude of the displacement can be written as

\[ W(r, \theta) = \sum_{m=1}^{\infty} G_m f_m(\sqrt{s_1} r) \cos m\theta \]

\[ + \sum_{m=1}^{\infty} G_{m^2} f_{m^2}(\sqrt{s_2} r) \cos m\theta \]

\[ + \sum_{m=1}^{\infty} G_{m^3} f_{m^3}(\sqrt{s_3} r) \cos m\theta, \]  

(30)

where \( s_1, s_2, \) and \( s_3 \) are dependent of material properties and vibration frequency.

The resonant frequencies and mode shapes can be solved by the eigenvalue problem of the following equation:

\[ \sqrt{s_2} s f_{m^2}(\sqrt{s_1} r_0) I_{m^2}(\sqrt{s_1} r_0) \sqrt{s_3} I_{m^3}(\sqrt{s_2} r_0) \]

\[ + \sqrt{s_1} s f_{m^3}(\sqrt{s_3} r_0) I_{m^3}(\sqrt{s_3} r_0) \sqrt{s_2} I_{m^2}(\sqrt{s_2} r_0) \]

\[ - \sqrt{s_1} s f_{m^2}(\sqrt{s_1} r_0) I_{m^2}(\sqrt{s_1} r_0) \sqrt{s_3} I_{m^3}(\sqrt{s_3} r_0) \]

\[ - \sqrt{s_2} s f_{m^3}(\sqrt{s_2} r_0) I_{m^3}(\sqrt{s_2} r_0) \sqrt{s_1} I_{m^2}(\sqrt{s_1} r_0) \]

\[ - \sqrt{s_3} s f_{m^2}(\sqrt{s_3} r_0) I_{m^2}(\sqrt{s_3} r_0) \sqrt{s_2} I_{m^3}(\sqrt{s_2} r_0) \]

\[ = 0. \]  

(31)

The vibration modes rather than the vibration frequency are more important for practice device application such as piezoelectric mirror; therefore, the vibration modes are analyzed in the next section.

3. Mode analysis of piezoelectric and flexoelectric circular plate

In this section, the vibration mode analysis of piezoelectric and flexoelectric circular plate were numerically investigated. For circular thin plate fabricated by barium titanate, the flexoelectric coefficient of such material has been determined experimentally, the elastic stiffness \( c_{11} \) of the barium titanate is 173 GPa, and the dielectric constant \( \varepsilon_{33} \) is 53.1 nF/m. The flexoelectric coefficient \( \mu_{12} \) of the barium titanate is 10 μC/m, as that determined in the experiment [33]. And the mass density of the material is 5.9 × 10^3 kg/m^3. The radius of the flexoelectric circular plate is 20 μm, and the thickness is 1 μm. The resonant frequencies for the flexoelectric circular thin plate are shown in Table 1, in which the mode (0,1) is on the level of 10 MHz. Obviously, the vibration modes \((d, c)\) are divided according to pitch diameter \(d\) and pitch circle \(c\), which are related to the \(m\)th and \(c\)th roots of the eigenvalue. It is clear that when \(d\) equals 1, 3, 5 ···, the vibration modes disappear, which is different from the piezoelectric circular thin plate. The mechanical governing equation is influenced by a coupling between piezoelectricity and flexoelectricity, as in Eq. (27), while the electrical governing equation is influenced by flexoelectricity. Since the non-uniform distribution of the electric potential was considered in this work, the obtained results were totally different from that in the previous studies [22,24].

In order to illustrate the difference in the mode shape, the first eight mode shapes were calculated and plotted in Figs. 2 and 3. We demonstrated that the mode 2 (2,1) for the flexoelectric circular thin plate is the same as that the mode 3 (2,1) for the piezoelectric circular plate without flexoelectricity, and the mode 2 (1,1) disappeared in the flexoelectric circular thin plate owing to the symmetry breaking by the strain gradient in the vibration modes.

The results of Figs. 2 and 3 proved that: (1) the non-uniform distribution of the electric potential in the radial direction, and (2) the differences of the mode shapes in flexoelectric circular thin plate with piezoelectric circular thin plate.

4. Conclusion

A vibration modes analysis for flexoelectric coupled circular thin plate structure is proposed in this work. Solutions for the vibration of flexoelectric circular thin plate were obtained based on the Kirchhoff thin plate theory, in which the non-uniform distribution of the electric potential in the radial direction was considered. The electromechanical coupling dynamic modes for the flexoelectric circular thin plate were derived. It is clear that the mode shape for the flexoelectric circular plate is different from that of the piezoelectric circular plate. This work provides a fundamental understanding of the electromechanical coupling behavior.
of the flexoelectric structures, and it could be helpful in designing novel flexoelectric devices, such as flexoelectric mirrors.

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### 拱曲圆板的振动模态

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**摘要** 梁、板和壳作为基本力学结构在微机电系统和纳机电系统中广泛用作传感器、驱动器、能量俘获器等。深入理解这些电介质结构的力电耦合有助于微纳机电系统中结构的设计、制备和优化。本文中，我们阐述了拱曲圆板的力电耦合行为，其中通过引入应变梯度将经典力电耦合理论拓宽到中心对称性，并考虑了电势沿径向非均匀分布的影响。拱曲圆板的振动模态的解析解表明其与压电圆板的振动模态完全不同，这是由于应变梯度局部破坏晶体的反演对称从而在圆板中引起的力电耦合导致的。力电耦合振动模态对弯曲和扭转模态更加敏感，本文给出了拱曲圆板的基本认知，其有助于设计新型拱曲圆板器件，例如拱曲电反射镜等。