Pion-Nucleon Scattering in the Infinite Momentum Frame

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Abstract

A light front field theory treatment of a chiral lagrangian is applied to pion-nucleon and nucleon-nucleon scattering.

Introduction

Another title of this talk could be “Light Front Treatment of Nuclei-Chiral Symmetry and Pion Nucleon scattering”. A light front treatment is almost the same as the infinite momentum frame.

The motivation for this approach begins with a desire to understand the EMC effect. The observed structure function depends on $x_{Bj}$, which in the parton model is the ratio of the quark plus momentum to that of the target. If one regards the nucleus as a collection of nucleons, $x_{Bj} = p^+/k^+$, where $k^+$ is the plus momentum of a bound nucleon. A direct relationship between nuclear theory and experiment occurs by using a theory in which $k^+$ is one of the canonical variables. Since $k^+$ is conjugate to a spatial variable $x^- \equiv t - z$, it is natural to quantize the dynamical variables at the equal light cone time variable of $x^+ \equiv t + z$. This is light front quantization. More generally, one expects to be able to profit from using light cone quantization in any situation which involves a a large momentum.

Redoing Nuclear Physics on the Light Front

The use of a new quantization procedure requires that one redo all of nuclear physics and check the most important features. The necessary steps include the following list.

1. Reproduce well-known results for nuclear matter in the mean field approximation[1, 2].

2. Provide a light front mean field treatment of finite nuclei. P. Blunden (Manitoba) and I are undertaking this task now.
3. Provide a light front Bruckner theory of nuclear matter to include the effects of correlations. R. Machleidt (Idaho) and I are working on this.

4. Obtain a light front chiral treatment of the NN force, for use in the Bruckner theory.

5. Obtain a light front chiral treatment of πN scattering.

Here I present a low-order use of a chiral hadronic Lagrangian to obtain a light front treatment of pion-nucleon scattering. This shows that the formalism works and indicates it is possible to ultimately obtain a fully relativistic chiral treatment of πN and NN scattering.

**Light Front Quantization**

Here is a primer aimed at providing the essentials of the light front formalism. One uses the energy momentum tensor $T^{\mu\nu}$ to construct the momentum $P^\mu$ with the relation:

$$P^\mu = \frac{1}{2} \int d^2x_\perp dx^- T^{+\mu}. \tag{1}$$

The light front Hamiltonian is $P^-$. The nucleon fields $\psi$ are a four-component spinor, but there are only two independent degrees of freedom. In the light front formalism it is easy to separate the dependent and independent variables using the projection operators $\Lambda^\pm$ with $\Lambda^\pm = \gamma^0 (\gamma^0 \pm \gamma^3)/2 = \gamma^0 \gamma^\pm$. The independent field is $\psi_+ = \Lambda_+ \psi$, and the dependent field is $\psi_- = \Lambda_- \psi$. The procedure is to use the field equations to obtain $\psi_-$ as a function of $\psi_+$, and then express $P^-$ in terms of independent fields. There is a similar treatment for vector mesons. The details of the formalism can be found in Refs. [4, 9], and especially the references therein.

**Lagrangian**

We use a non-linear chiral model in which the nuclear constituents are nucleons $\psi$ (or $\psi'$), pions $\pi$ scalar mesons $\phi$ and vector mesons $V^\mu$. The pion and nucleon part of the Lagrangian $\mathcal{L}_{\pi N}$ is given by

$$\mathcal{L}_{\pi N} = \frac{1}{4} f^2 Tr (\partial_\mu U \partial^\mu U^\dagger) + \frac{1}{4} m_\pi^2 f^2 Tr (U + U^\dagger - 2) + \bar{\psi}' \left( \gamma^\mu \left( \frac{i}{2} \gamma_\mu - MU \right) \right) \psi' \tag{2}$$
where the bare masses of the nucleon, scalar and vector mesons are given by $M, m_s, m_v$, and $V^{\mu\nu} = \partial^{\mu}V^{\nu} - \partial^{\nu}V^{\mu}$. The unitary matrix $U$ can be chosen as $U \equiv e^{i\gamma_5 \tau \cdot \pi / f}$.

The pion-nucleon coupling here is chosen as that of linear representations of chiral symmetry used by Gursey [3], with the Lagrangian approximately invariant under a chiral transformation. Another transformation can be used to convert this to a Lagrangian of the non-linear representation [4]. In this case the early soft pion theorems are manifest in the Lagrangian, and the linear pion-fermion coupling is of the pseudovector type. However, the use of light front theory, requires that one find an easy way to solve the constraint equation that governs the fermion field. The constraint can be handled in a simple fashion by using the linear representation [2].

Equation (2) may be thought of as a low energy effective theory for nuclei under normal conditions. Here we use its light front Hamiltonian.

The aim here is to understand low energy pion nucleon scattering. The effects of pair suppression must emerge even though the linear pion-nucleon coupling is given by the pseudoscalar $\gamma_5$ operator.

**Chiral Symmetry and Pion-Nucleon Scattering**

The first test for any chiral formalism is to reproduce the early soft pion theorems. Here we concentrate on low energy pion nucleon scattering because of its relation to the nucleon-nucleon force. We work to second order in $1/f$ in this first application. In this case, $U$ takes the form:

$$U = 1 + i\gamma_5 \frac{\tau \cdot \pi}{f} - \frac{\pi^2}{2f^2},$$

which is used to construct the Hamiltonian $P^-$. The second order scattering graphs are of the three types shown as time $x^+$ ordered diagrams in Fig. 1. The kinematics are such that $\pi_i(q)N(k) \rightarrow \pi_f(q')N(k')$. The direct and crossed graphs of Fig. 1a involve matrix elements of $\gamma_5$ between $u$ spinors, which vanish near threshold. The terms of Fig. 1b are generated by $\bar{u}\gamma_5v$ terms. In any light front time-ordered graph the plus-momentum is conserved, and the plus momentum of every particle line is greater than zero. This means that the first of Fig.1b vanishes identically, and the second vanishes for values of the initial pion plus momentum that are less than twice
Fig. 1 $x^+$-ordered graphs for low energy pion-nucleon scattering.

the nucleon mass. The net result is that only the instantaneous terms (which arise from replacing $\psi_-$ by a function of $\psi_+$) and the $\pi^2$ term of (shown in Fig. 1c) remain to be evaluated.

Proceeding more formally, the result is

$$M = \tau_i \tau_f \frac{M^2}{f^2} \frac{\bar{u}(k')\gamma^+ u(k)}{2(k^+ + q^+)} + \tau_f \tau_i \frac{M^2}{f^2} \frac{\bar{u}(k')\gamma^+ u(k)}{2(k^+ - q^+)} - \delta_{if} \frac{M}{f^2} \frac{\bar{u}(k')u(k)}{2} \quad (4)$$

where the three terms here correspond to the three terms of Fig. 1c. The role of cancellations in the reduction of the term proportional to $\delta_{if}$ is already apparent. To understand the threshold physics take $k'^+ = k^+ = M$ and $q'^+ = q^+ = m_\pi$. Then one finds

$$M = \delta_{if} \frac{2m_\pi^2}{f^2} + 2i\epsilon_{fin}\tau_n \frac{m_\pi M}{f^2} \quad (5)$$

to leading order in $m_\pi/M$. The weak nature of the $\delta_{if}$ term and the presence of the second Weinberg-Tomazowa term is the hallmark of chiral symmetry.
Chiral Nucleonic Two Pion Exchange Potential

We discuss the two pion exchange contribution (of order \((M/f)^4\)) to the nucleon nucleon potential. The property that a sum of light cone time-ordered diagrams equals a single Feynman graph can be used to simplify the calculation. The relevant Feynman graphs are displayed in Fig. 2; the terms originating from the linear \(\gamma_5 \tau \cdot \pi\) coupling (a,b), from the quadratic \(\pi^2 - N\) coupling (c) and from a combination of the linear and quadratic interactions (d) are indicated. The line through the two-nucleon intermediate state of Fig. 2a indicates the subtraction of the contribution of the iterated the one pion exchange interaction.

The sum of the terms of Fig. 2a and 2b is equal to the Partovi-Lomon two pion exchange potential, as they used the pseudoscalar pion-nucleon interaction. This interaction certainly simplifies the calculation; in particular the diagrams of Fig. 2a, b and d are convergent (whereas they would be strongly divergent if pseudovector coupling were used). One can use such a pseudoscalar coupling, and include the effects of chiral symmetry, provided
one also includes the effects of the $\pi^2 - N$ coupling shown in Fig. 2c, and the combined effects of the linear and quadratic interactions, Fig. 2d. The quadratic interaction term cancels the large pair terms in pion-nucleon scattering and should also play a significant role here in reducing the size of the computed potential. Thus we expect that the Partovi-Lomon potential contains too large an attraction.

Next turn to the procedure used in constructing the full Bonn potential. This potential is constructed by ignoring all of the $Z$-graphs and including the effects of the two-nucleon intermediate states which arise from the crossed graph, Fig. 2b, as well as the parts of Fig. 2a arising from time ordered terms in which two pions exist at the same time. (For such contributions to the TPEP the linear pseudoscalar and pseudovector interactions are evaluated between on shell positive energy nucleon spinors, and are therefore equivalent.) The resulting contribution to the TPEP is small, but is comparable to that of the iterated OPEP. The neglect of the $Z$ graphs goes a long way towards including the effects of chiral symmetry. However, terms involving the Weinberg-Tomazowa interaction at one or two vertices are ignored. The computation of the graphs of Fig. 2 would include such effects implicitly as well as that of pair suppression. Thus a detailed comparison would be useful. The small nature of the effects that we discuss now indicate that the dominance of the TPEP by effects of intermediate $\Delta$'s will remain unchallenged.

Summary and Discussion

The present paper argues that the light front quantization of a chiral Lagrangian can handle pion-nucleon scattering. One can go beyond the present tree approximation and obtain new relativistic chiral treatments of both pion-nucleon and nucleon scattering. The resulting amplitudes can be used to obtain a new treatment of nuclear physics which includes the effects of correlations and relativity.

References

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