Research on Model Validation and Uncertainty Quantification of Airfoil Flow

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Abstract. Numerical simulations and wind tunnel tests are both basic methods for aerodynamic research and application. They are the modeling and simulation of real world system. There are various uncertainties in the implementation process. In Computational Fluid Dynamics (CFD), the main sources of uncertainty are the model form, model parameters and numerical solution methods. In wind tunnel tests, there are many uncertainties sources, such as geometric model, test environment, test devices and instrumentation, test methods, data recording and processing, etc. This paper takes NACA0012 airfoil as an example. Bayesian model averaging method (BMA) and grid convergence study are used to analyse model form and numerical solution uncertainty in CFD. For the geometric configuration uncertainty in wind tunnel test, regression analysis and bootstrap method are used to carry out data fusion. Finally, model validation research is conducted.

1. Introduction

The application of CFD in transportation, weather forecasting, building bridges and other fields has become more and more extensive, and the discussion about its credibility has also received more and more attention. There are many factors that affect the credibility of CFD. From the source of uncertainties, it can be summarized as model form, model parameter and numerical solution method [1]. According to the amount of knowledge and mathematical representation form, it can be divided into epistemic uncertainty and aleatory uncertainty. CFD validation is generally determined through comparison with experimental data. Compared with the uncertain sources of CFD, the uncertainty of the experiment is reflected in other aspects, such as geometric models, test environments, test devices and instruments, test methods, data recording and processing, etc. How to comprehensively consider these uncertainties and scientifically characterize the consistency of experiments and calculations is an important subject of evaluating the credibility of CFD.

This paper takes NACA0012 airfoil as an example, SA model, SST model and multiple sets of calculation grids are used for numerical simulation. BMA method and Richardson interpolation method are used to quantify the uncertainty of model form and numerical solution. The wind tunnel test generates three sets of data due to the height of the emery, which is treated as the uncertainty of the geometric structure through regression analysis and data fusion methods [2]. For the quantified CFD calculation uncertainty and wind tunnel test uncertainty, similarity measure is used for model validation.
How to effectively identify, characterize and quantify uncertainties requires more research. This paper fully identifies, characterizes and quantifies some uncertain factors in calculations and experiments, and conducts model validation studies. The uncertainty of incoming flow conditions and random errors in the repeated experiment process in wind tunnel experiments are ignored. The second chapter of this article mainly discusses the uncertainty quantification in CFD calculation, the third chapter is the uncertainty quantification of wind tunnel test, the fourth chapter is model validation, and the fifth chapter is the summary.

2. Numerical simulation uncertainty quantification

There are many uncertain factors in CFD calculations. This chapter uses BMA method to analyze common engineering application turbulence models SA model and SST model, uses Richardson interpolation method to quantify the influence of numerical discrete factors.

2.1. Uncertainty quantification of turbulence model form

The turbulence model is an important source of uncertainty in CFD calculations. In order to simulate the turbulence phenomenon more precisely, a series of methods have been developed, divided into RANS, LES and DNS. In this paper, SA model and SST model in the RANS simulation method commonly used in engineering are used to carry out calculations according to the test conditions. The calculation ignores other uncertain factors except the turbulence model and grid. In this section, BMA [3] method is used, and the deviation between the calculated results and the experimental results is the maximum likelihood estimation Standard deviation, calculate the posterior probability of each calculation model. Suppose the test data set \( D \), target variable \( y \), calculation model \( M_i \) \((i = 1, 2, \ldots, n)\), the predicted value \( f_i(x_k) \) of the model \( M_i \) under the input \( x_k \), and the benchmark value \( f(x_k) \) under the input \( x_k \) of the experiment. The calculation steps for the uncertainty of the model form are,

1) For the airfoil calculation examples in this paper, the selected SA and SST models can be well simulated, assuming that the difference \( \varepsilon_{ik} \) between the calculated data and the experimental data satisfies the Gaussian distribution.

\[
y_k - f_i(x_k) = \varepsilon_{ik}, \varepsilon_{ik} \sim N(0, \sigma_i^2)
\]

Which \( \sigma_i^2 \) is determined by the maximum likelihood estimation, \( \sigma_i^2 = \frac{1}{m} \sum_{k=1}^{m} \varepsilon_{ik}^2 \).

2) Calculate the edge likelihood of the model \( M_i \),

\[
P(D|M_i) = \left( \frac{1}{2\pi \sigma_i^2} \right)^{n/2} \exp \left( -\frac{\sum_{k=1}^{n} \varepsilon_{ik}^2}{2\sigma_i^2} \right)
\]

3) Calculate the posterior probability of the model \( M_i \). When there is no more information, assume that the prior probability of each calculation model is equal, \( P(M_i) = 1/n \), and the posterior probability is,

\[
P(M_i|D) = \frac{P(M_i)P(D|M_i)}{\sum_{j=1}^{n} P(M_j)P(D|M_j)}
\]

4) Calculate the distribution of the output response of the average model under any input,

\[
p(y|D) = \sum_{j=1}^{n} P(M_j|D)P(y|M_j,D)
\]
5) Calculate the mean and variance of the output response.

\[ E(y|D) = \sum_{j=1}^{n} P(M_j|D) E(y|M_j, D) \]

\[ \text{var}(y|D) = \sum_{j=1}^{n} P(M_j|D) \text{var}(y|M_j, D) + \sum_{j=1}^{n} P(M_j|D) \left[ E(y|M_j, D) - E(y|D) \right]^2 \]

With reference to the test data released by NASA, the numerical simulation results of the lift coefficient Cl and drag coefficient Cd are shown in Figure 1 and Figure 2.

![Figure 1. The lift coefficient changes with the angle of attack](image1)

![Figure 2. The drag coefficient changes with the angle of attack](image2)

The posterior probability of the BMA can be given as shown in Table 1.

| Table 1. Posterior probability of two turbulence models |
|----------------|----------------|
|               | Cl             | Cd             |
| SA             | 0.4742         | 0.9975         |
| SST            | 0.5258         | 0.0025         |

It can be seen from Table 1 that for the lift coefficient Cl, the posterior probability of the SA model and the SST model are equivalent, and there is no significant difference in calculation. The significant difference in the calculation of the drag coefficient may be due to the difference at high angles of attack. The SST calculation results are more consistent with the experimental results than the SA model. In order to avoid too much influence of a few calculated positions on the overall posterior probability, this paper further studies the local posterior probability of different turbulence models at each angle of attack, and the probability curves are shown in Figure 3 and Figure 4. The data shown can be averaged to obtain the posterior probabilities of the two turbulence models, as shown in Table 2.

![Figure 3. The local posterior probability of different turbulence models for lift coefficient](image3)

![Figure 4. The local posterior probability of different turbulence models for drag coefficient](image4)
2.2. Numerical solution uncertainty estimation
Numerical simulation of NACA0012 airfoil is performed on multiple sets of grids. The grid size is given in Table 3, and the results are shown in Figure 5. Using Richardson interpolation analysis [4], the numerical solution of the finer grid is in the asymptotic convergence region, and the least square method is used to complete the error estimation of the numerical solution. Assuming that the discrete solution satisfies Taylor expansion,

\[ f_i = f_0 + a h_i^p \quad (p > 0) \]  

Where \( h_i \) is the measure of the grid scale, the nominal accuracy of the format is \( p_{\text{format}} = 2 \), and the error estimate of the numerical solution is,

\[ f_{\text{exact}} \leq [f_i - \Delta], f_i + \Delta \]

\[ \Delta = F_i \left| f_i - f_0 \right| + \sigma + \left| f_i - f_{\text{fit}} \right| \]  

The first term of the expression is the GCI process [5], and the last two terms represent the fitting error, which \( F_i \) is 3 or 1.25.

Table 3. Different computing grid scales

| Number | grid size (radial & spanwise) |
|--------|------------------------------|
| grid0  | 1792*512                     |
| grid1  | 896*256                      |
| grid2  | 448*128                      |
| grid3  | 224*64                       |
| grid4  | 112*32                       |

The result of the error estimation is shown in Figure 6, and the numerical error estimation of the lift coefficient of the two common engineering turbulence models is given.
3. Uncertainty quantification of wind tunnel test
The airfoil experiment is divided into three groups of experiments, grit80, grit120 and grit180, according to the height of the emery at the leading edge of the wing. Each group of experimental data shows discrete points that vary with the angle of attack, as shown in Figures 1 and Figure 2. In this paper, the influence of different emery heights on the airfoil is regarded as the uncertainty of the geometric structure. The regression analysis is performed on each set of experiments, and then the three sets of experimental data are fused.

3.1. Regression analysis method
A polynomial is used to perform regression analysis on each group of experimental data. The polynomial order P adopts a cross-validation method, comprehensively considering the mean and variance of error, and adopts the best order.

\[ y = \beta_0 + \beta_1 \alpha + \cdots + \beta_p \alpha^p + \epsilon \]  

(8)

Where \( y \) is the output response, \( \alpha \) is the angle of attack, \( \epsilon \) and is the error.

Regression analysis can give the predicted value and the corresponding confidence interval under the new input conditions. Figure 7 shows the fitting accuracy of different polynomials for grit80 emery test data. After comprehensive consideration, the optimal 4th-order polynomial is selected. Figure 8 reflects the 95% confidence of the lift coefficient of the regression curve under the new predicted angle of attack.

3.2. Data Fusion
Using the Bootstrap [6] residual method, try to give the predicted distribution of each set of experimental data. The process of this method is briefly described as follows:

1) Using test data to fit regression model, the fitting response value \( \hat{y}_i \) and residual \( \hat{e}_i \) are obtained.
2) For the sequence, the Bootstrap residual set is a sampling with replacement method.
3) By adding the fitting results and the Bootstrap residuals, the pseudo test data can be obtained.
4) Based on the pseudo test data set, the fitting regression is carried out again and the response output of interest is got. The process is repeated for 2-4 times.

Figure 7. Grit80 emery regression polynomial order selection  
Figure 8. Lift coefficient prediction value and 95% confidence interval
Figure 9. Distribution of predicted values of lift coefficients in each group at 4° angle of attack

Figure 10. Fusion results of predicted lift coefficients at 4° angle of attack

The cumulate probability distribution of the three sets of data is obtained, and two methods are used for data fusion. One is the unified sample method, and the other is the sample feature preprocessing method [7]. The first method regards the distribution of the three sets of data as equal weights, and the new distribution obtained is the result of the addition and average. The second method introduces a weighting factor through distance, and the weight ratio of data with large deviation is relatively low. Figure 9 shows the cumulate probability distribution of the predicted values of the lift coefficients at an angle of attack of 4°. Figure 10 shows the corresponding data fusion results. The red curve corresponds to the first method with equal weight, and the blue curve shows the second method introduces a weighting factor. The fusion cumulative distribution of the weighting factor introduced in Figure 10 has a higher cumulative distribution on the lower lift coefficient than the fusion with equal weights. This also reflects that the red and green distributions are closer in Figure 9. The blue distribution is away from the red and green distributions. We believe that data fusion with the introduction of weighting factors can better reflect the information of concentrated data and reduce the impact of distant data.

4. Model validation

In Section 2 and Section 3, the calculation output response interval and the experimental data interval with quantified uncertainty are obtained respectively. The consistency comparison between the two is regarded as part of the validation metric work. Introduce similarity coefficient as a metric to measure the similarity of two sets,

\[ J(A, B) = \frac{|A \cap B|}{|A \cup B|} \]  

(9)

In the formula, A and B respectively represent the calculated output response interval and the experimental data interval, and the coefficient calculation results are shown in Figure 11 and Figure 12. It can be seen from the figure that for the lift coefficient, the SST model combined with the experimental data has a higher similarity, and for the drag coefficient, the SA model has a higher similarity. But it must be pointed out that the similarity measure does not consider the influence of the distance between intervals. For example, when the calculated and experimental interval similarity measure value is 0, but the two are close in one case, and the two are far apart in one case, the meanings of the two cases are obviously different in engineering.

However, the similarity coefficient has a clear meaning for measuring the similarity of two intersecting intervals, as shown in Figure 12 for the case of a smaller angle of attack. The effect of this measurement cannot be reflected in Figure 2 alone. So in summary, we believe that scientific model validation should consider multiple factors. In this case, in addition to the measurement results in Figure 11 and Figure 12, we also need to consider the intuitive difference in Figure 1 and Figure 2, and the posterior probability of the model in Figure 3 and Figure 4. For different output responses, the
performance of the calculation model is not consistent, and the opposite conclusion may be got. For the naca0012 airfoil in this paper, it is better to use SST model for lift coefficient and SA model for drag coefficient.

5. Conclusion
In this article, for the naca0012 airfoil, the uncertainty of CFD calculation and wind tunnel test is considered, and the research of model validation is initially performed. The important content is,

1) Use the Bayesian model average method to measure the posterior probability of the calculation model, and guide the project to use the calculation data interpolation when the experimental data is insufficient;
2) Use polynomial regression to analyze each set of experimental data, and give an approximate uncertainty band;
3) The introduction of weighting factors combines multiple sets of experimental data, which improves the utilization of experimental data;
4) The similarity measurement method compares experimental and calculated intervals.

In the research of this article, there are also some shortcomings, such as insufficient consideration of experiment and calculation uncertainty, ignoring the random changes of incoming flow conditions, and the model validation method does not consider the effect of the result interval distance. We believe that when the calculation and experimental data are sufficiently abundant, they can be effectively processed by the methods of probability box and area measurement, which also presents directions and challenges for future research.

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