Decay of neutron with participation of the light vector boson X17

Pham Tien Du\textsuperscript{1}, Nguyen Ai Viet\textsuperscript{2,3}, Nguyen Van Dat\textsuperscript{2,3}

\textsuperscript{1} Department of Physics, ThuyLoi University, Tayson Street, Hanoi, Vietnam
\textsuperscript{2} Information Security Faculty, Information Technology Institute, Xuanhuy Street, Hanoi, Vietnam
\textsuperscript{3} Department of Physics, VNU University of Science, Nguyen Trai Street, Hanoi, Vietnam

E-mail: doandu30111989@gmail.com; dup@tlu.edu.vn

Abstract. The recently discovered light vector boson X17 at ATOMKI can trigger new decay channels of neutron, which can be used to explain the neutron lifetime puzzle. In this article, we calculate all the possible decay widths and discuss the physical implications.

1. Introduction

Krasznahorkay et al. \cite{1} have observed an anomalous internal pair conversion (IPC) in the excited beryllium nucleus transition to its ground state when bombarding a $^7\text{Li}$ target with a low-energy proton beam. The anomaly can be explained by assuming the existence of a vector boson with mass of 17 MeV (X17). In average, there are consistently 5.8 events related to X17 over a total of 1 million ones, mostly dominated by photon.

\[ \frac{\Gamma(^8\text{Be}^*\rightarrow X17)}{\Gamma(^8\text{Be}^*\rightarrow \gamma)} \times Br(X17 \rightarrow e^++e^-) = 5.6 \times 10^{-6}, \]  (1)

where \( Br(X17 \rightarrow e^++e^-) \) is the branching ratio of the decay channel of X17 into the pair \( e^++e^- \). In order to estimate \( \Gamma(^8\text{Be}^*\rightarrow X17) \) and \( Br(X17 \rightarrow e^++e^-) \), one needs a particle model, in which the couplings of the X17 vector boson to electron, neutron and proton are given.

If such couplings exist, it is also possible that X17 will play a role in the electron-nucleon scattering, since in addition to the usual Feynman graphs of such scattering with the intermediate photon, now we have the ones mediated by the virtual X17.

In this article we will show that the existence X17 can solve the neutron lifetime puzzle as it can open up new decay channels of neutron in addition to the usual $\beta$ one.

The puzzle can be described as follows: when neutrons are stripped from atomic nuclei and put in a closed bottle. The neutron decay rate can be measured by counting the number of remaining neutrons in the bottle after some time. The estimation by this method yields the radioactively decaying neutron lifetime as 14 minutes and 39 seconds, in average. There is also another method called beam experiment, by measuring the number of emerging proton in a beam of neutrons. Since today physics knows the $\beta$-decay \( n \rightarrow p + e^- + \bar{\nu}_e \) as the only channel, it implies the average neutron lifetime at around 14 minutes and 48 seconds with a significant
deviation of 9 seconds, which is within the accuracy of today’s experiment capability. So the
discrepancy between the “bottle” and “beam” measurements has become the unsolved neutron
lifetime puzzle. Recently, Fornal and Grinstein [2] suggested that the new decay channels to
a dark neutron in addition to a photon, a pair of electron-positron and a dark scalar particle
can solve the puzzle if the branching ratio of the dark channels is about 1%. However, the
two first channels with photon and electron in the final state have been excluded promptly
by the subsequent experiments by Tang et al. [3] and Sun et al [4]. So, only the dark decay
channel of neutron with the dark scalar remains as the possible solution. Since the theory
of dark neutron and scalar of Fornal and Grinstein is not known, this hypothesis can not be
verified experimentally. Ivanov et al. [5] have proposed a new dark decay channel of neutron
by assuming that the neutron has an additional decay channel beyond the known $\beta$-one. This
channel is $n \rightarrow n_X + \nu + \bar{\nu}$, where $n_X$ is the dark neutron hypothesized by Fornal and Grinstein.
Since the coupling in the effective theory of the dark neutron coupled to neutron and neutrinos
is not known, one can only match it to have the desirable decay rate. So, this work does not
have a quantitative prediction nor a fundamental foundation.

In this paper, we will investigate the possibility of having the interaction between the neutron-
dark neutron and neutrino being mediated by X17. Therefore, we can find a solution for the
neutron lifetime puzzle with the decay channel depicted in the following diagram.

![Decay of neutron via X17.](image)

The underlying theory to explain the existence of X17 together with the dark neutron can be
found in Viet’s discrete extra dimension model [6], where X17 and dark neutron are interpreted
as Kaluza-Klein partners of photon and neutron. Additionally, we make few further “reasonable
assumptions”. Firstly, the mass splitting between the neutron and its Kaluza-Klein partner is
less than $1.102 \text{ MeV}$, which suppresses the neutron decay channel into electron-positron pairs
to satisfy the recent experimental observation by Tang [3]. Secondly, the mass of neutrino’s
Kaluza-Klein partner is assumed to be as small as in the $eV$ range. Thirdly, the masses of
Kaluza-Klein partners of proton and electron are at least in the $TeV$ range to explain why those
particles have not been observed.

This article is organized as follows: In Sec.2, we present a brief introduction of the Discretized
Kaluza-Klein theories developed by Viet and collaborators [6, 7, 8]. Then, we use Viet’s model
to compute decay of neutron with participation of the light vector boson X17 (see Figure 1).

2. Coupling constant of X17 to fermions from a discretized extra dimension

Today, the extra dimension is a de facto standard for construction of unified theories. So,
searching for those is one of the main tasks of modern physics. We can mention some remarkable
ideas proposed by many authors. The large extra dimension (LED) proposed by Arkani-Hamed,
Dimopoulos and Dvali [9] postulates a large extra dimension of size 1 mm to bring the value of
the Planck mass to the TeV energy scale. The size of LED can also be chosen around 1 fermi,
if several LEDs are considered. The universal extra dimension (UED) proposed by Appelquist, Cheng and Dobrescu [10], postulates a size of $10^{-10}$ m which is much larger than the traditional Planck scale but smaller than LED, to bring the Planck scale down to 1000 $TeV$. The Randall-Sundrum model (RS1) [11] also postulates an extra dimension with two specific branes of TeV and Planck energy scales with a warping factor to solve the hierarchy problem. Both LED and RS1 require an assumption that the physical fields are localized on some membranes.

The discrete extra dimension has been proposed by Viet and Wali as an intuitive interpretation of Connes-Lott’s model noncommutative geometric space-time. [12]. Since 1994, Viet and Wali have developed the Discretized Kaluza-Klein theory (DKKT) [7], with the fifth dimension having only two points. One might consider this theory either as the discretized approximation of the continuous space dimension or as an alternative one.

Recently, Viet has put forward a model with the extended space-time with the fifth discrete extra dimension having only two-points and the following metric structure [6]

$$ds^2 = G_{MN}dx^Mdx^N = \eta_{\mu\nu}dx^\mu dx^\nu + \lambda^4 dx^5 dx^5,$$

(2)

$$G_{\mu\nu} = \text{diag}(-1,1,1,1),$$

$$G_{\mu 5} = G_{5\mu} = 0, G_{55} = \lambda^4,$$

where $M, N, L = \mu, 5$ are the indexes of the non-normal frame and $\lambda$ is a warping factor similar to the one in the RS1 model [11]. The metric in Eq.(2) corresponds to the extended vierbein

$$E^A_M = E^A_M DX^M, DX^M = E^M_A E_A,$$

(3)

$$G^{AB} = E^A_M G^{MN} E^B_N = \text{diag}(-1,1,1,1),$$

$$G^{MN}G^{NL} = \delta^M_L,$$

where $A, B = a, 5$ (with $a = 0, 1, 2, 3$) are the indexes of the locally orthonormal frame.

In this space-time, the extended fermion field in can be represented by two column spinor

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix},$$

(4)

and the extended Dirac operator is

$$\bar{D} = \bar{\theta}.1 + \Theta = \begin{bmatrix} \bar{\theta} & -im\theta \\ im\theta & \bar{\theta} \end{bmatrix},$$

(5)

where $\theta$ satisfying $\theta^2 = 1$, is a Clifford element.

With this new framework, we can extend the usual vector field 1-form $\gamma^\mu b_\mu(x)$ into the hermitian 2 x 2 matrix containing the photon field $A_\mu(x)$ and its Kaluza-Klein partners [6]

$$B = \Gamma^\mu B_\mu(x) + \Gamma^5 \phi(x) = \begin{bmatrix} b_1 & i\theta \phi(x)/\lambda^2 \\ -i\theta \phi(x)/\lambda^2 & b_2 \end{bmatrix}$$

(6)

$$= \begin{bmatrix} gAQ + g'XQX & i\theta \phi(x)/\lambda^2 \\ -i\theta \phi(x)/\lambda^2 & gAQ \end{bmatrix},$$

where the vector field $X_\mu(x)$ and the scalar one $\phi(x)$ are Kaluza-Klein partners of the photon. $Q$ and $Q_X$ are, respectively, the electric and dark charge operators. $g$ and $g'$ are, respectively, the electromagnetic and dark couplings. In this model, these coupling constants are not independent.

From the above point of view, we must also extend the particle model of nuclear physics. Neutron, proton, electron and neutrino can also have their own Kaluza-Klein partners. Following
Viet, we identify X17 with the vector field $X_\mu(x)$. The interaction of X17 with fermions then can be obtained from a generalization of the Yukawa coupling of photon with fermions. Therefore, we can have an explicit interaction terms of neutrino, neutron and dark neutron as in [6] as follows

\begin{align}
  g_{n\bar{n}X} &= \sqrt{2}g \sin 2\theta_n, \\
  g_{\bar{\nu}nX} &= g \sin^2 \theta_\nu, \\
  g_{n\bar{n}X} &= g \sin^2 \theta_\nu, \\
  g_{e^-e^+X} &= g \sin^2 \theta_e,
\end{align}

where $\theta_e, \theta_n, \theta_\nu$ are respectively the mixing angles of electron, neutron and neutrino with their Kaluza-Klein partners.

The mass splitting between any given fermion $\psi$ and its Kaluza-Klein partner $\psi_X$ is given by

\begin{align}
  m_{\psi_X} - m_\psi = \frac{2m}{\sin 2\theta_\psi \lambda^2},
\end{align}

where $\theta_\psi$ is the mixing angle of the fermion $\psi$ and its Kaluza-Klein partner.

3. Dark decay of neutron to neutrino via a virtual X17 exchange

Based on our assumptions, the only kinetically possible dark decay channels of neutron are the ones into neutrino and/or its Kaluza-Klein partners

\begin{align}
  n \rightarrow n_X + \nu + \bar{\nu}, \\
  n \rightarrow n_X + \nu + \bar{\nu}_X, \\
  n \rightarrow n_X + \bar{\nu} + \nu_X, \\
  n \rightarrow n_X + \nu_X + \bar{\nu}_X.
\end{align}

All the above decay channels of neutron with X17 can be depicted by the Feynman diagram in Figure 1, where $\psi$ can be either neutrino or its Kaluza-Klein partner. The generic amplitude for these processes can be represented as follows

\begin{align}
  \mathcal{M} = \left[ \bar{u}(n_X) i g_1 \gamma^\mu u(n) \right] \frac{g_\mu - q_u q_\nu / m_X^2}{q^2 - m_X^2} \left[ \bar{u}(\psi) i g_2 \gamma^\nu u(\bar{\psi}) \right],
\end{align}

where $q_\mu$ is the transfer 4-momentum of the virtual X17. $g_1$ and $g_2$ are coupling constants at the corresponding vertexes to be determined later. $m_X = 17$ MeV is the mass of X17.

In the neutron rest frame $p_1 = (m_n, 0)$, we can neglect the mass of the particle $\psi$ and find the generic decay amplitude

\begin{align}
  \langle |\mathcal{M}|^2 \rangle = \frac{16 g_1^2 g_2^2 m_n}{(m_n^2 + m_{n_X}^2 - 2m_nE_3 - m_{n_X}^2)^2} \left[ \begin{array}{c}
  \frac{m_{n_X}^2 - m_{n}^2}{m_{n_X}^2} (E_2 + E_4) \\
  -2m_n \left( \frac{E_3^2 + E_4^2 - m_{n_X}E_3}{m_{n_X}} \right) \\
  -m_{n_X} \left( \frac{m_{n_X}^2 + m_{n_X}^2}{m_{n_X}} \right)
  \end{array} \right],
\end{align}

where $E_2$ and $E_4$ are energies of the outgoing $\psi$ and $\bar{\psi}$ particles, which must be integrated over to obtain the contribution to the neutron lifetime. $E_3$ is the energy of the dark neutron $n_X$.

Using the Fermi’s Golden rule, we can determine the differential decay width as follows

\begin{align}
  d\Gamma = \frac{\langle |\mathcal{M}|^2 \rangle}{(4\pi)^3 m_n} dE_2 dE_4.
\end{align}
In order to integrate over the variable $E_2$, we use the following approximation

$$\ln(1 + x) = x - \frac{1}{2}x^2 + 0(x^3),$$  \hspace{1cm} (19)

$$\frac{1}{1 + x} = 1 - x + x^2 + 0(x^3),$$  \hspace{1cm} (20)

with the following range of the $E_2$ integral

$$E_{2-} = \frac{m_n^2 - m_{nX}^2 - 2m_nE_4}{2m_n} < E_2 < \frac{m_n^2 - m_{nX}^2 - 2m_nE_4}{2m_n - 4E_4} = E_{2+},$$  \hspace{1cm} (21)

we obtain

$$\frac{d\Gamma}{dE_4} \left( \frac{g_1^2 g_2^2}{4\pi^3} \right)^{-1} = -\frac{32}{m_X^6}E_4^6 - \frac{48D - 8m_X^2E_4^5}{m_nm_X^6} + \frac{1}{m_X^2m_X^4} \left( -2m_n^2 + 10m_{nX}^2 - 4m_n m_{nX} - \frac{24D^2}{m_X^4} \right) E_4^4$$

$$+ \frac{4D}{m_X^2m_X^4} \left( m_n^2 + m_{nX}^2 - m_n m_{nX} - \frac{D^2}{m_X^2} \right) E_4^3$$

$$+ \frac{D^2}{2m_n^2m_X^4} \left( 3m_n^2 + m_{nX}^2 - 2m_n m_{nX} \right) E_4^2,$$  \hspace{1cm} (22)

where $D = -m_n^2 + m_{nX}^2$.

Now we can integrate over the variable $E_4$. Note that $E_4 < m_n - m_{nX} << m_X, m_n$, we can carry out the integration in a good approximation and obtain the neutron decay rate via X17 as follows

$$\Gamma_\psi \approx \frac{g_1^2 g_2^2}{30\pi^3 m_X^4}(m_n - m_{nX})^5.$$  \hspace{1cm} (23)

The total decay width $\Gamma_X$ of all four new neutron decay channels given in Eq.(7) is independent of the neutrino mixing angle $\theta_\nu$

$$\Gamma_X = \sum \Gamma_\psi = \frac{g_4^4 \sin^2 2\theta_\nu}{30\pi^3 m_X^4}(m_n - m_{nX})^5.$$  \hspace{1cm} (24)

By using the mass splitting formula in Eq.(11), we can replace also the neutron mixing angle by the mass difference between neutron’s Kaluza-Klein partner and the “warping factor” $\lambda^2$ in Eq.(24) as follows

$$\Gamma_X = \frac{2g_4^4}{15\pi^3 m_X^4 \lambda^4}(m_n - m_{nX})^3.$$  \hspace{1cm} (25)

On the other hand, we have the $\beta$ decay rate of neutron via intermediate $W$ boson [13]

$$\Gamma_W = \frac{1.633m_e^5}{2\pi^3}v^2G_F^2 \left( 1 + \frac{3g_4^2}{g_W^2} \right),$$

$$= \frac{9.504g_4^4}{64\pi^3\sin^4 \theta_W m_W^4}m_e.$$  \hspace{1cm} (26)
where $G_F, g_A, g_V$ and $g$ are respectively the Fermi, axial vector, vector weak coupling constants and the electromagnetic one. $\theta_W$ is the Weinberg angle, $m_W \sim 80\text{GeV}$ is the $W$ boson mass, $V_{ud}$ is the CKW matrix element.

Therefore, the total decay rate of neutron via $X_{17}$, in case $m_{\nu X} < m_e$, and the $\beta$-decay rate of neutron via the $W$ boson

$$\frac{\Gamma_X}{\Gamma_W} = \frac{3.59m_Xm_W^4}{\lambda^4m_e^2} \sin^4\theta_W \sin^32\theta_n.$$  \hspace{1cm} (27)

As mentioned by Fornal and Grinstein, if there are new decay channels which contribute to a ration at least 1% of the beta decay width, then the neutron puzzle can be considered as solved. In fact, in our model, there are new neutron channels in addition to the traditional beta one. Following Fornal-Grinstein’s resolution, we require that $\Gamma_X / \Gamma_\beta \sim 1\%$ to obtain the constraint between the metric warping factor $\lambda^4$

$$\frac{8m_X^3\lambda^4}{(m_{\nu X} - m_n)^3} = \sin^32\theta_n\lambda^{10} = -0.261 \times 10^{24}. \hspace{1cm} (28)$$

The constraint (28) can be satisfied by choosing a large value of $\lambda^2$ and a reasonable value of the neutron mixing angle $\theta_n$.

Since $\sin^3\theta_n \geq -1$ we have the lower bound of $\lambda^2$

$$\lambda^2 \geq 4.82 \times 10^4. \hspace{1cm} (29)$$

Since electrons have not been found in Tang’s experiment, we can assume that the creation of electron-positron pairs as final product of the neutron decay is suppressed kinetically. That is to say the mass difference between neutron and its Kaluza-Klein partner must be smaller than twice of electron mass $m_{\nu X} - m_n < 1.102 \text{MeV}$, from which we can imply the upper bound of $\lambda^2$

$$\lambda^2 \leq 8.9 \times 10^9. \hspace{1cm} (30)$$

4. Conclusion and discussion

In this article, based on a model of discrete extra dimension, we demonstrate that the dark neutron decay channels into neutrino and its KK-sibling via a virtual $X_{17}$ exchange can reasonably explain quantitatively the neutron lifetime puzzle. The quantitative determination of the mixing angles can be determined by the scattering processes involving Kaluza-Klein partners of neutron, which are under consideration [14].

Acknowledgment

The supports of Department of Physics, ThuyLoi University toward to work of Pham Tien Du are greatly appreciated. The research is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 103.01-2017.319.

References

[1] Krasznahorkay A, Csatlós M, Csige L, Gácsi Z, Gulyás J, Hunyadi M, Kuti I, Nyakó B, Stuhl L, Timár J et al. 2016 Physical review letters \textbf{116} 042501

[2] Fornal B and Grinstein B 2018 \textit{Phys. Rev. Lett.} \textbf{120}(19) 191801

[3] Tang Z et al. 2018 \textit{Physical Review Letters} \textbf{121} 022505

[4] Sun X et al. 2018 \textit{Physical Review} \textbf{C97} 052501

[5] Ivanov A N, Höllwieser R, Trötskaya N I, Wellenzohn M and Berdnikov Y A 2019 \textit{Nucl. Phys.} \textbf{B938} 114–130 (Preprint 1808.09805)
[6] Viet N A 2019 Extra dimension of space-time exposed by anomalies at low energy (Preprint hepph/1907.04517)
[7] Landi G, Viet N A and Wali K C 1994 Physics Letters B326 45
[8] Viet N A and Wali K C 1996 International Journal of Modern Physics A 11 533–551
[9] Arkani-Hamed N, Dimopoulos S and Dvali G 1998 Physics Letters B 429 263–272
[10] Appelquist T, Cheng H C and Dobrescu B A 2001 Physical Review D 64 035002
[11] Randall L and Sundrum R 1999 Physical Review Letters 83 3370
[12] Connes A and Lott J 1991 Nucl. Phys. B 18 29–47
[13] Pei C T and Fong L L 1984 Gauge Theory of Elementary Particle Physics (Clarendon Press)
[14] Dat N V, Viet N A and Du P T 2019 (Preprint Presentation at National Conference of Theoretical Physics, Quang Binh, Vietnam)