Game Theory Approach on Modeling of Residential Electricity Market by Considering the Uncertainty due to the Battery Electric Vehicles (BEVs)

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Abstract—Based on sophisticated metering infrastructure (AMI), one can use big data to provide demand response (DR) solutions. There is a need to develop optimized cost structures for consumers. In this paper, Stackelberg game approaches are utilized, and residential loads are considered including battery electric vehicles (BEVs) equipped with BEV communication controllers and vehicle-to-grid (V2G) technologies. Efficient and effective optimized algorithms are developed for users (followers) based on time dependent pricing schemes. In the “games,” besides the followers, other participant is an electricity retailer company (leader), with a two-way bilateral communication procedure accepted and established by all participants. The user side of the games is related to the demand side management (DSM). Real-time pricing (RTP) from time-of-use (TOU) companies is used to achieve better results. Monte Carlo simulations (MCS) represent uncertain behaviors of BEV drivers. Results indicate that customers’ demands can be met while reaching the best efficiency.

Keywords—internet of things; real-time pricing; demand response; game theory; battery electric vehicle; vehicle-to-grid

I. INTRODUCTION

In a traditional power grid, proper metering may decrease demand response (DR) non-scheduled loads slightly. In a smart grid, several advanced techniques can be integrated, including advanced metering infrastructure, energy management systems, distributed energy systems, intelligent electronic devices, internet of things (IoTs), and battery electric vehicles (BEVs) [1]. When a smart metering infrastructure is combined with DR programs, efficiency can be improved [2]. Two types of DR have been discussed previously: 1. The retailer has all the power to directly control consumers’ usage which decreases the satisfaction of the users; and 2. The retailer reshapes DR programs through dynamic pricing such as critical pricing and real time pricing (RTP) [3]. The second type is preferred in this study.

Modern energy managements via smart grids may take advantages of following technology developments: bi-directional energy flows, price-responsive loads, intelligent electronic devices (IEDs), and phasor measurement units (PMUs). Thus, sophisticated smart metering facilities and advanced two-ways communication technologies enable more flexible energy generations, deliveries and consumptions [4-7]. In a self-scheduling model simulation, consumers can participate “day-ahead” energy markets in order to minimize costs and/or maximize profits [8]. Price uncertainty was studied for generation company as a leader in game theory framework [9-10]. Recently, game theories are being considered in DR solutions and load peak shavings. In references [11-12], a Stackelberg game approach was used to deal with DR scheduling under load uncertainty based on real-time pricing in a power grid. In references [13-14], similar approaches were used to stimulate a power company and its customers to “play the game” in order to maximize their benefits and to eventually flatten aggregated load curves. With bi-level hybrid multi-objective evolutionary algorithms, utility companies’ profits may be optimized [15]. Additionally, various approaches have been developed to find the clearing price in retail electricity markets with high penetrations of renewable energies. Based on a time-of-use (TOU) pricing structure, providers or retailers can charge a calculable fee for the fixed price depending on an amount of consumption during a specific time interval [16-17]. Commonly, electric vehicles are connected to grids to receive energy, and limited savings can be achieved when charging schedules are adjusted to avoid peak load periods [18].

In this study, a mixed integer nonlinear programming scheme is considered to achieve benefits for both “energy providers” and “consumers.” A new approach is considered here to flatten peak loads by scheduling home appliances and BEVs which can be charged and discharged via two way technologies by combing grid-to-vehicle (G2V) and vehicle-to-grid (V2G) connectivity. By using V2G technologies, retailers can buy energy from consumers if such practices are mutually beneficial. Thus, the following sections will study advanced two-way communication links and BEVs charging/discharging schedules to maximum utility companies’ profits via peak-shaving, and to satisfy consumers by elevating their roles to participants and to minimizing
costs. Using a price-based model to guarantee an efficient algorithm between an electricity retailer and users owning BEVs with the aim of balancing the load. Main contributions of this paper include following key points:

- Using a price-based model to develop an efficient algorithm between retailers and users (with BEVs), and aiming for balanced supply-load by sharking peaks;
- Generating optimized results for both retailers and users with an iterative algorithm, and finding Stackelberg equilibrium (SE) to achieve optimal loads for both parties;
- Formulating 1-N leader-follower Stackelberg relationship between one retailer and N users; and adopting the RTP function of the retailer and the utility function of N users; and
- Handling uncertainty of different driving habits of BEV owners by Monte Carlo simulations (MCS).

Specifically, the rest of the paper is organized as follows. Section II discusses the system model and formulates the Stackelberg game theory. In Section III, we use data to generate load profiles of BEVs. Some key aspects of Stackelberg game theory and the logic are presented in Section IV. Results are provided in Section IV, and conclusions are drawn in Section V.

II. SYSTEM MODEL

In Fig. 1, a model is established for one utility company and N users with BEVs. These users will adjust their electricity usages by using advanced metering infrastructure and heterogeneous communication technologies, with the ultimate goals to reduce overall costs. The utility company (retailer) can provide hourly price structures to these users to encourage peak shaving and ultimately to maximize profits.

Advanced communication technologies enable the cloud computing environment and real-time sensing/controlling including IoTs [19]. In this environment, an entity represents an electric load (consumer) or a prosumer such as a BEV. In the cloud environment, the real-time access to each entity is guaranteed. The system operator can categorize users into different groups. As shown in Fig.1, all prosumers can be placed in one group, and other conventional loads in other. By utilizing instantaneous two-way communication links, real-time electricity prices will be broadcasted, conventional users’ consumption behaviors will be adjusted, and BEVs charging/discharging schedules can be changed. To maintain system stabilities and guarantee maximum profits, the retailer can impose load curtailment during peak hours.

A. Retailer Company Model

In this model, the cost function for the retailer is labelled as \( C_t(g_t) \), and depends on the amount of electricity provided \((g_t)\) during an interval \( t \); where \( t \in T, T = [T] \) that is a strictly convex and monotonically increasing function.

\[
C_t(g_t) = \frac{a_t}{2} g_t^2 + b_t g_t + c_t
\]  

(1)

Where \( a_t, b_t \) and \( c_t \) are the generating coefficients. During different time intervals, each coefficient has a different value. Taking a differential operation against (1), one obtains the marginal cost function \( C_t' \):

\[
C_t'(g_t) = a_t g_t + b_t
\]  

(2)

which is the cost to produce one more unit of electricity. Such marginal cost must be lower than the actual cost in order to guarantee profits for the retailer company. So, market price equation can be defined:

\[
P_t(g_t) = \lambda_2 C_t'(g_t) \Rightarrow \lambda_2 \geq 1
\]  

(3)

where \( P_t \) is price at each time slot and \( \lambda_2 \) is a time-wise profit coefficient. In reference [15], a similar profit coefficient was introduced, minimized and validated. In this paper, there are 24 time intervals (slots), corresponding to 24 prices in a day. The retailer provides electricity and its price to the consumers who will decide how much to use in each time interval. Thus, a user can move his/her electricity usages to off-peak intervals to minimize the total costs. The retailer wants the most profit and the least aggregated peak load in order to avoid expensive backup generators. Flattening the demand load should be two-ways as a user has critical needs for electricity. To determine the optimal generation vector, one should minimize variations in generation, and match supply with demand. Therefore, the retailer problem can be formulated as follows:

\[
\begin{align*}
\min \quad & U_{RC}(g_t) = \sum_{t \in T} \left( g_t - \bar{g} \right)^2 \\
\text{s.t.} \quad & L_t \leq g_t \leq \min \left( g_t^+, L_t^{\max} \right) \\
& L_t = \sum_{n=1}^{N} l_{i,n,t} \\
& L_t = \sum_{i=1}^{N} l_{i,n,t} + l_{n,j,t} \quad i \neq n
\end{align*}
\]  

(4)

(5)

(6)

(7)

Where \( U_{RC} \) is the utility function of the retailer company and \( \bar{g} \) is an average power generation during a day. \( L_t \) is the summation of electricity demands of all \( N \) users during the time interval \( t \). \( g_t^+ \) is the maximum capacity of the retailer company’s generation at the interval \( t \). \( L_t^{\max} \) is the overall upper bound of the total power demands for slot \( t \). \( l_{i,n,t} \) is the
power consumed by user \( n \) in the interval \( t \). One must notice that the above-mentioned function is different from profit maximization. In (4), minimizing utility function may lead to maximized profits. In (5), the generation should always meet the total demand of all users, and be lower than the smallest threshold for generation capacity and upper load ranges. In (6) and (7), \( L_t \) can be obtained by asynchronous user’s adjustment of consumption, reflecting the human nature that no two users will react to real-time pricing schemes due to different needs. In (7), a user adjusts his/her usage, while others don’t. In such non-simultaneous framework, no two users will negate their effects by increasing and decreasing their demands at the same time.

B. User Model

A user utility function is illustrated in (8). The user can be an entity representing an electric load (consumer) as the first term, or a prosumer such as a BEV as the second term. Function \( \psi_{n,t}(x_{n,t}) \) models the satisfaction of the electricity consumers; in which \( x_{n,t} \) is the general power consumption variable, \( I_{n,t} \) being the residential demand and \( S_{n,t} \) being BEV demand. More electricity consumptions represent more satisfactions. The third term is the total amount of money to be paid by the consumers, which leads to less satisfaction as represented by a negative sign. Besides, (8) demand side problems can be formulated in (9-14).

\[
I_{n,t} = \arg \max_{I_{n,t}} U_o(I_{n,t}, S_{n,t}) = \sum_{i=1}^{24} \psi_{n,t}(I_{n,t}) + \sum_{t=1}^{24} P_{n,t} \left( g_{n,t} \right) \left( \left( I_{n,t} + S_{n,t} - P_{n,t}^{BEV} \right) \right) \quad (8)
\]

\[
\psi_{n,t}(x_{n,t}) = \omega_{n,t} x_{n,t} - \frac{\theta_{n,t}}{2} x_{n,t}^2, \quad \omega_{n,t} > 0, \quad \theta_{n,t} > 0
\quad (9)
\]

\[
s.t. \quad I_{n,t}^{-} \leq I_{n,t} \leq I_{n,t}^{+}
\quad (10)
\]

\[
\sum_{t=1}^{24} I_{n,t} = L_n
\quad (11)
\]

\[
\sum_{t=1}^{24} S_{n,t} = T_n^{req}
\quad (12)
\]

\[
\sum_{t=1}^{24} \left| S_{n,t} \right| \leq T_n^{max}
\quad (13)
\]

\[
SOC_{n,t}^{dep} = BC_n
\quad (14)
\]

where \( \omega_{n,t} \) is the preference parameter with indices \( n \) (user) and \( t \) (time), \( \theta_{n,t} \) is a predetermined constant integer, \( L_n \) is the total daily energy usage, \( S_{n,t} \in \{-1, 0, 1\} \) is a discrete variable which is multiplied by rated battery power \( P_{n,t}^{BEV} \) of electric vehicles and indicates BEV charging \( \{1\} \), discharging \( \{-1\} \) status. If it is charging, it will have a positive effect on satisfaction of the utility function, and if it is discharging, it has a negative effect. Although discharging has a negative effect on satisfaction, it is losing the power obtained in the past charging periods. However, by selling discharged electricity, it has positive effect on the third term of utility function. \( T_n^{req} \) is the required time to fully charge the BEV, \( T_n^{max} \) is the maximum number of hours that the battery is interacting actively with the network. \( SOC_{n,t}^{dep} \) shows the BEV state of charge, and \( BC_n \) is the battery capacity, \( T_n^{dep} \) is the vehicle’s departure time from the house. (9) Denotes the satisfaction function \( \psi_{n,t}(x_{n,t}) \) of the user \( n \) by consuming \( l_{n,t} \) amount of electricity. (10) provides the upper and lower boundaries for electricity demand of \( n \)th user \( n \) for interval \( t \). (11) expresses the temporally-coupled constraint which wouldn’t reduce the total daily consumption of the user \( n \), so we can apply load shifting only in terms of pick shaving and load shifting. (12) states the cumulative charging time in order to fully charge the BEV before leaving one’s residence. (13) limits the maximum number of hours for a BEV connected to the grid system, either in G2V or V2G mode. Therefore, by limiting the hours that the battery is being charged or discharged, we reduce the detrimental effect of constant battery usage. In (14) the sequence of charging/discharging should be in a way that by the time that the BEV owner decides to leaves the house, the SOC should be 100%.

In the Stackelberg equilibrium context and the hierarchical process, our aim is to maximize the leader’s (retailer) utility function by having the data of the follower’s (user) rational reaction set (RRS). The existence of the Stackelberg equilibrium has been discussed and shown in [4]. In addition, the model in (8) is formulated as a mixed integer nonlinear programming problem, and can be solved via MATLAB-GAMS interface.

III. BEV LOAD PROFILE

To model BEV load profiles, data from National Household Travel Survey (NHTS) for rural New York State were utilized to estimate average driving mileages and optimize battery usages. The SOC of the vehicles upon their home arrival time is of crucial importance [20]. Table I shows the data according to different types of cars and different characteristics. Also, in Figs. 2-4, probability distribution functions (PDFs) for driving behavior, including arrival time, departure time, and traveled distance in rural areas in New York state has been drawn from NHTS data. The reason why we chose rural areas in New York State is that Alfred University is located in rural areas.

Moreover, configuring the time of fully charged BEV depends on the distance it travels daily, and we assume that BEV is a constant power prosumer. From Fig. 5, we obtain the required charging time based on daily driven distance [21-22].

IV. STACKELBERG GAME THEORY ALGORITHM

In Fig. 6, iterative DR algorithm is illustrated via a flow chart. At the beginning of the computation, the retailer broadcasts hourly price one hour ahead. According to (3), marginal cost function is calculated by power generation need
and initial price broadcasted. As the relationship between the price equation and power is linear, the algorithm uses the generation value. Final prices can be found once the total power generation value is known. Afterwards, Monte Carlo simulation emulates driving behaviors of BEV owners.

Users respond to the prices broadcasted, and shift their usages to non-peak time slots. BEV discharging features can help the retailer company to offset some loads during peak hours. The peak-shaving and the demand aggregation can be accomplished continuously instead.

Table I: Different Characteristics of BEVs

| Vehicle Type   | Commercial model | Expected Market Share [%] | Energy [kW-hr] | Rated Battery Power [kW] | Single Charge Drive Range [mile] |
|----------------|------------------|---------------------------|----------------|--------------------------|----------------------------------|
| Compact sedan  | i3, BMW          | 51.48                     | 33             | 7                        | 114                              |
| Mid-size sedan | Model S, Tesla   | 10.35                     | 75             | 11.5                     | 259                              |
| Mid-size SUV   | Model X, Tesla   | 38.17                     | 100            | 17.2                     | 295                              |

Fig. 2. Arrival time pdf for BEV owners in rural areas in New York State

Fig. 3. Departure time pdf for BEV owners in rural areas in New York State

Fig. 4. Average travelled distance pdf for BEV owners in rural areas in New York State

TABLE I: Different Characteristics of BEVs

Fig. 5. Required time to fully charge a BEV with respect to distance driven and car size

Fig. 6. Stackelberg game theory algorithm
V. SIMULATION AND RESULTS

Residential consumers are divided into three groups as tabulated in Table II. Group 1 corresponds to utility function coefficient $\omega_{n,t}$ of 5.0, with 50 BEVs. Groups 2 and 3 do not have BEVs, but have different utility coefficients. The minimum and the maximum values refer to the lower and upper bounds of the targeted power demands. For example, Group 1 consumption varies between 70% and 150% of its nominal load. The price coefficient is kept constant, i.e., $\lambda_t = 1.2$. In Fig. 7, load profiles are shown for Group 2 and 3; and benefits from the game theory applications can be visualized by examining a profile with or without the algorithm. In general, the load profiles are flattened due to users’ participations. In comparison with Group 2, Group 3 shows less eagerness to participate as it has larger $\omega_{n,t}$. For group 1, as illustrated in Table I, managing the vehicle’s V2G or G2V status is of great importance. If a large number of BEV owners begin to charge their vehicles immediately returning home, the sudden power load may be undesirable. By applying the algorithm in Fig. 6, the uncertainty related to random behaviors of BEV owners can be managed by optimizing charging and discharging activities. In Fig. 8, wide bands of load profile for BEVs are shown for each time slot in a day. Without game theory applications, the users’ peak load can coincide with that of the BEV charging peaks. In Fig. 9, modified BEV load profile is obtained after applying the algorithm. The negative values represent the V2G feature of the BEVs, or discharging. After coming back to home (between 17:00 or so and midnight), most vehicles are either sending back power to the grid or in standing by mode. The charging period mainly happens between 1:00 and 6:00. Comparing the time interval between 14:00 to 16:00 in Fig. 7 and Fig. 10, more peak shaving has been achieved for Group 1 due to the presence of V2G technology. Fig. 11 shows the total generation over 24 hours for users’ aggregated demand with and without implementing the DR program. The expected value at each hour is used to obtain

| Scenario | Peak demand [kw] | Total Energy Usage [kwh] | Total Payments [$] | Generation Cost [$] |
|----------|------------------|--------------------------|--------------------|--------------------|
| Without RTP and Without V2G | 3841 | 59092 | 36135 | 15115 |
| RTP and V2G | 2755 | 59092 | 30244 | 12661 |

Table III. Numerical performance evaluation
the profiles in Fig. 10 and Fig. 11. From Table IV, the standard deviation shows that more fluctuations will happen mainly from 22:00 to 24:00 and 1:00 to 5:00. Therefore, the power system operator should consider more spinning reserve after midnight; this will cause an increase in power system operation cost. Finally, the real-time prices over 24 hours that has been derived from the algorithm is compared with the conventional price in Fig. 12. The performance of the algorithm is evaluated by two scenarios (with and without RTP and V2G) from various aspects. The numerical results are shown in Table III. The peak demand is significantly decreased by almost 1 MW. Total energy usage has not been changed, which proves that the retailer cannot reduce the whole amount of energy that a user expects to consume per day. This reality is shown in (11). Using algorithm in Fig. 6, the total user payments decreased about 16%, which demonstrates the efficiency of game theory application. Additionally, the generation costs of both scenarios can be less than the total payments, i.e., making profits for the retailer.

**Conclusion**

Using game theory algorithm, one can reduce costs to consumers and potentially reshape generating profiles. In this paper, a model included one retailer and many users (with some of them owning BEVs). In particular, an optimized approach can shave peaks with DR managements. The game theory and Stackelberg equilibrium have been utilized to illustrate an algorithm for three different user groups. By comparing the results with and without DR managements, the efficiency of the algorithm can be shown. Using MCS in MATLAB-GAMS, the stochastic behavior of BEVs are simulated. According to the statistical analysis, such algorithm based on the game theory can reduce the peak loads. We are currently investigating possibilities to include more leaders and followers.

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