Cloning SO(N) level 2

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ABSTRACT

For each N an infinite number of Conformal Field Theories is presented that has the same fusion rules as SO(N) level 2. These new theories are obtained as extensions of the chiral algebra of SO(NM^2) level 2, and correspond to new modular invariant partition functions of these theories. A one-to-one map between the c = 1 orbifolds of radius R^2 = 2r and D_r level 2 plays an essential role.

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1. Introduction

Since the discovery of the ADE classification of modular invariant partition functions (MIPF’s) of affine $SU(2)$ [1], the generalization to other affine Lie algebras has continued to fascinate a select group of people (see e.g. [2] [3] [4]). Although completeness proofs were given in a few other cases [5-7], the natural classification problem for affine algebras based on simple Lie algebras remains unsolved, and the generalization to semi-simple algebras looks totally impossible. In the “simple” case there may at some point have a belief that most MIPF’s had been found, but that belief was shaken several times in the last few years. In particular in [3] an infinite series of new MIPF’s of automorphism type was described, for the affine algebras $D_r$ and $B_r$ at level 2.

In this paper I will describe another infinite series occurring for the same algebras, this time of extension type, and apparently not yet known (these new extension invariants should not be confused with another series mentioned in [3], which was shown to be unphysical; the new theories we describe here are definitely physical). The starting point of the analysis is a new interpretation of the partition functions of [3].

It turns out that both types of MIPF (those of [3] and the new ones described here) have a natural interpretation in terms of $c = 1$ orbifolds at arbitrary rational radius. The link between the orbifolds and the affine algebra goes via the coset description of the former. The $c = 1$ orbifolds of a circle are described by the coset CFT’s [8]

$$\frac{SO(N)_1 \times SO(N)_1}{SO(N)_2},$$

where the subscript denotes the level. It turns out that for even $N = 2r$ one obtains all radii $R^2 = 2r$, whereas for odd $N = 2r + 1$ one only gets the radii $R^2 = 2(2r + 1)$. In the latter case the field identification has fixed points [9], so we will focus first on the $D_r$ affine algebras, where the analysis is easier.

We introduce the following notation for $D_r$ Lie-algebra representations in terms of Dynkin labels (the last two of which are the spinor labels).

$$(0) : (0, \ldots, 0, 0)$$

$$(v) : (1, \ldots, 0, 0)$$

$$(s) : (0, \ldots, 0, 1)$$

$$(c) : (0, \ldots, 1, 0)$$

$$(\ell) : (0, \ldots, 0, 1, 0, \ldots, 0, 0).$$

where in the last line the 1 is in position $\ell$. Furthermore we denote by $(vv), (vs), \ldots$ etc. the sum of the corresponding Dynkin labels, e.g. $(vv) = (2, 0, \ldots, 0, 0), \ldots$. The algebra $D_{r,1}$ has four unitary highest weight representations with ground state labels $(0), (v), (s), (c)$, while for $D_{r,2}$ one gets in addition the representations $(vv), (vc), (vs), \ldots$.
(ss), (cc), (sc) and (ℓ), ℓ = 2, r − 2. Hence the total number of primary fields of the $D_{r,2}$ theory is $r + 7$. The coset theory has a four-fold field identification with identification currents [9] $(s, s; ss)$, $(c, c; cc)$ and $(v, v; vv)$. These currents act without fixed points, and the total number of primaries in the coset theory is therefore

$$4 \times 4 \times (r + 7)/4/4 = r + 7.$$ 

This is the same number of fields as for the orbifolds of the $R^2 = 2r$ circle theory [10], and looking at the spectra one concludes that the two theories are indeed the same.

The identification with the orbifold spectrum is as follows for odd rank (column 1) and even rank (column 2).

| Coset reps., $r$ odd | Coset reps., $r$ even | Orbifold reps. | Conformal weight |
|----------------------|-----------------------|----------------|-----------------|
| (0,0;0)              | (0,0;0)               | [0]            | 0               |
| (0,0;vv)             | (0,0;vv)              | [V]            | 1               |
| (0,v;ss)             | (0,0;ss)              | [S]            | $r/4$           |
| (0,v;cc)             | (0,0;cc)              | [C]            | $r/4$           |
| (0,c;s)              | (0,s;s)               | $[\sigma]$    | $\frac{1}{16}$ |
| (0,s;c)              | (0,c;c)               | $[\tilde{\sigma}]$ | $\frac{1}{16}$ |
| (0,c;vc)             | (0,s;vc)              | $[\sigma']$   | $\frac{9}{16}$ |
| (0,s;vs)             | (0,c;vs)              | $[\tilde{\sigma}']$ | $\frac{9}{16}$ |
| (0,0;ℓ)              | (0,0;ℓ)               | $[\ell], \ell$ | $\frac{\ell^2}{4r}$ |
| (0, v; ℓ)            | (0, v; ℓ)            | $[\ell], \ell$ | $\frac{\ell^2}{4r}$ |

The representations in columns 1 and 2 are identification orbit representatives chosen by requiring the first entry to be (0). The second entry is then fixed by the selection rules of the coset embedding, given the third entry. The last two columns fix our notation for the orbifold fields, whose weight is listed in the last column.

This table not only implies an equivalence between the coset CFT (1.1) and the $c = 1$ orbifolds, but it also implies an isomorphism between the fusion rings of the $c = 1$, $R^2 = 2r$ orbifold and the affine algebra $D_{r,2}$. This isomorphism is a consequence of the fact that $D_{r,1}$ has simple fusion and that $D_r$ fusion (at any level) preserves conjugacy class charges. The modular transformations and fusion rules of the $c = 1$ orbifolds have been obtained in [10], but I do not know if the isomorphism with $D_{r,2}$ was noticed before.

The modular transformation matrices $S$ and $T$ of $D_{r,2}$ and the orbifold are not the same, but differ by a complex conjugation (since $D_{r,2}$ appears in the denominator in (1.1)) and some phases originating from $D_{r,1}$. Nevertheless the connection is close enough to gain information about modular invariant partition functions in one case and
use it in the other case.

Non-trivial MIPF’s of the \( c = 1 \) orbifold theory can be expected to exist because the diagonal invariant only describes \( R^2 = 2r \) orbifolds, whereas the orbifold theory exists for any \( R \). It was pointed out in [10] that the chiral algebra for the orbifold \( R^2 = 2r/q \) (with \( r \) and \( q \) relative prime) is the same as for \( R^2 = 2rq \). This implies that the former theory must be described by a non-diagonal modular invariant combination of the \( R^2 = 2rq \) characters. This invariant is however not discussed in [10].

Let us first review the partly analogous situation for circle compactifications of a single real boson. Geometrically, such compactifications are described in terms of one modulus, the radius of the circle \( R \). The description in terms of conformal field theory is more complicated, and one has to distinguish three cases: (a) \( R^2 \in 2\mathbb{Z} \), (b) \( R^2 \in 2\mathbb{Q} \) and (c) \( R^2 \) irrational. In case (a) the description is in terms of a chiral algebra generated by the operators “\( \partial X \)” and vertex operators related to the even lattice \( \ell R^2, \ell \in \mathbb{Z} \). The circle compactification corresponds to the diagonal invariant built out of the primary fields of this extended algebra. In case (c) the chiral algebra is just the free boson algebra \( \partial X \) and there is an infinite number of primaries. The other rational circles (case (b)) do not correspond to diagonal invariants of some chiral algebra (case (a) already exhausts all possible chiral algebras), but to automorphism invariants generated by simple currents.

To fix the notation, define \( U_{2r} \) as the \( U(1) \) CFT with \( 2r \) primaries, which has as its extended algebra the one corresponding to \( R^2 = 2r \) above. All the primaries are simple currents, which we will label by \( J, J = 0, \ldots, 2r - 1 \). Suppose \( r = pq \), where \( p \) and \( q \) are relative prime. Now consider the modular invariant partition function generated by the simple current \( J = 2p \); this is an automorphism invariant. It is easy to show that this yields precisely the circle compactification with radius \( R^2 = 2p/q \). The current \( J = 2q \) gives radius \( R^2 = 2q/p \), the T-dual (\( R \leftrightarrow 2/R \)) of the previous case. In particular \( J = 2 \) yields the T-dual of the \( R^2 = 2r \) circle compactification, and corresponds on the other hand to the charge conjugation invariant of \( U_{2r} \).

This exhausts the set of rational circles, but not the set of simple currents. One may in fact use any simple current \( J = 2j \) (odd \( J \)'s are not in the effective center [11]), and in general one obtains \( R^2 = 2r/N^2 \), where \( N \) is the order of \( J \). If \( r \) contains \( N^2 \) as a factor this operation reduces the theory to one with \( R^2 = 2\tilde{r} \), \( \tilde{r} = r/N^2 \in \mathbb{Z} \). This should therefore be a diagonal theory. Indeed, in that case the current \( J \) has integer spin and generates an extension of the chiral algebra (thus reducing \( r \) to \( \tilde{r} \)), and not an automorphism (there are also mixed cases where a factor \( \tilde{N}^2 \) of \( N^2 \) divides \( r \)).

One would expect a similar situation to occur in the orbifold theory, but there is a clear difference: the orbifold has only four simple currents, the fields \([0],[V],[S]\) and \([C]\) (except for \( r = 1 \), when the orbifold coincides with the \( R^2 = 8 \) circle). This is clearly not sufficient, and we are forced to conclude that at least the invariants required to go from \( R^2 = 2pq \) to \( R^2 = 2p/q \) must be exceptional (i.e. not simple current) invariants. Because of the isomorphism, their existence then implies the existence of analogous invariants for \( D_{r,2} \).
At this point we make contact with [3], where precisely these invariants were discovered from an entirely different point of view, namely Galois symmetry. They exist for $D_r$ and also for $B_r$. The invariants described in [3] occur whenever $2r - 1 = \Pi_{i=1}^K p_i$ (for $B_r$) or $r = \Pi_{i=1}^K p_i$ (for $D_r$), where the factors $p_i$ are distinct primes. In total there are $2^{K-1}$ distinct invariants, including the identity, precisely one for each way of writing $2r - 1$ (resp. $r$) as a product of two factors. It is easy to check that these automorphisms do indeed yield the partition function one expects for the $R^2 = 2p/q$ orbifolds. On closer inspection the results of [3] can be generalized to all cases where $r = pq$ (or $2r - 1 = pq$) if $p$ and $q$ are relative prime. This exhausts all rational orbifolds, and gives us a raison d’être of these exceptional $D_r$ automorphisms.

In addition to these automorphisms the circle theories at suitable radii also have extensions by integer spin simple currents. Although nothing requires their orbifold equivalent to exist (we already found all rational orbifolds) it turns out that they do nevertheless exist. They are given by the following general formula for the $D_r, 2$ invariant, if $r = \tilde{r}M^2$ and $M$ is odd.

\begin{align*}
|X_0|^2 + \sum_{m=1}^{(M-1)/2} |X_{2m\tilde{r}M}|^2 + |X_{vv}|^2 + \sum_{m=1}^{(M-1)/2} |X_{2m\tilde{r}M}|^2 \\
+ |X_{ss}|^2 + \sum_{m=1}^{(M-1)/2} |X_{(2m-1)\tilde{r}M}|^2 + |X_{cc}|^2 + \sum_{m=1}^{(M-1)/2} |X_{(2m-1)\tilde{r}M}|^2 \\
+ \sum_{l=1}^{\tilde{r}-1} \sum_{m=0}^{M-1} |X_{r-lM-2m\tilde{r}M}|^2 \\
+ |X_s|^2 + |X_c|^2 + |X_{vc}|^2 + |X_{vs}|^2
\end{align*}

(1.2)

There is a completely analogous invariant for the $c = 1$ orbifold, but of course it merely describes the $c = 1$ orbifold at a reduced radius, which is nothing new. The $D_{r,2}$ invariant, however, describes a conformal field theory with fusion rules that are isomorphic to $D_{\tilde{r},2}$, but with a different spectrum. Hence for any $\tilde{r}$ we get an infinite series of CFT’s that are “fusion-isomorphic” to $D_{\tilde{r},2}$. These new CFT’s are labelled by an odd integer $M$, and have a chiral algebra that is an extension of $D_{\tilde{r}M^2,2}$. We will call this new theory $D_{\tilde{r},2}^M$.

To check that the proposed partition function is indeed modular invariant we need the transformation matrix $S$. It can be computed either using the Kac-Peterson formula for the affine algebras, but can be more easily obtained from the results of [10], adding a few $D_{r,1}$ phases where needed. Checking the invariance is then straightforward. Note that the partition function has a somewhat unusual form: the characters of the extended

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* The $c = 1$ theories, just as $D_r$ level 2, have (at least) one additional automorphism, corresponding in $D_r$ to spinor conjugation (or charge conjugation for $r$ odd). This automorphism invariant may be interpreted as the $T$-dual of the orbifold theory. For $r = 1$, the three-critical point, it becomes the usual $T$-duality of the circle.
theory are linear combinations of either $\frac{1}{2}(M+1)$, $M$ or one character of the original theory, and furthermore the first four blocks are not “orthogonal”, e.g. the currents that extend the algebra occur also in a non-identity character. Furthermore the short blocks have multiplicity 1, unlike fixed point characters of simple current invariants. The fact that all multiplicities are 1 makes it straightforward to compute the matrix $S$ of the extended theory. As expected, for the orbifold theory it is again the one given in [10], whereas for $D_{r,2}$ it differs by the $D_{r,1}$ phases. Since the matrices $S$ for $D_{\tilde{r}M^2,1}$ and $D_{\tilde{r}}$ are identical if $M$ is odd (because $M^2 = 1 \mod 8$), we may replace the $D_{r,1}$ by $D_{\tilde{r},1}$ phases, which shows that $D_{\tilde{r},2}^M$ has the same matrix $S$ as $D_{r,2}^1 = D_{\tilde{r},2}$.

The foregoing results can be extended from odd $M$ to all integer $M$. Consider first $M = 2$. In this case one does not require an exceptional invariant, but instead the simple current $[S]$ (or $[C]$) achieves the reduction of $r$ by a factor 4. The invariant generated by this simple current is

$$
|\mathcal{X}_0 + \mathcal{X}_{ss}|^2 + |\mathcal{X}_{uv} + \mathcal{X}_{cc}|^2 + 2 |\mathcal{X}_s|^2 + 2 |\mathcal{X}_{cc}|^2 + 2 |\mathcal{X}_{r/2}|^2 + \sum_{\ell = 2}^{r/2-2} |\mathcal{X}_\ell + \mathcal{X}_{r-\ell}|^2
$$

(1.3)

Here each fixed point splits into two distinct fields, and hence we get a total of $2 + 6 + (r/4 - 1) = \tilde{r} + 7$ fields in the new theory. In this case a fixed point resolution procedure is needed to compute the new matrix $S$, but the required formalism is available [9] [12]. Fixed point resolution then yields the matrix $S$ of $D_{\tilde{r},2}$, $\tilde{r} = r/4$ (up to a few phase changes if $\tilde{r} \not\equiv 0 \mod 4$ due to the fact that the matrices $S$ for $D_{r,1}$ and $D_{\tilde{r},1}$ are not identical; there are no such phase changes in the orbifold case, because then (1.3) produces a theory that is identical to the $R^2 = 2\tilde{r}$ orbifold).

Since we always land on another $D_{r,2}$ theory these extensions can be performed successively, which allows a reduction of $r$ by any factor $4^n$. A reduction by a factor 16 or more is however not a standard simple current invariant. It turns out that the first reduction by a factor 4 promotes the primary field $(r/2)$ to a simple current, which is then used in the second stage. This is possible because $(r/2)$ is a fixed point in the first stage. Finally we note that when removing the last factor of 4 from $r$ we pass from the $r$ even to $r$ odd. In [10] it was found that $S$ is real for $r$ even, but complex for $r$ odd. The imaginary part comes from the fixed point resolution. Without fixed points the new matrix elements of $S$ are real linear combinations of the old ones, and an imaginary part cannot be generated. Using the table in [12] one readily finds that an imaginary fixed point resolution matrix occurs precisely for $D_r$, $r = 0 \mod 4$, $r \not\equiv 0 \mod 8$.

* The invariant is written here for $D_{r,2}$, but the corresponding invariant for the $R^2 = 2r$ orbifold is completely analogous.
1.1. The series for $B_{r,2}$

For the algebra $B_{r,2}$ we can also derive the matrix $S$ from the results of [10]. In this case we use the coset theory

$$\frac{B_{s,1} \times B_{s,1}}{B_{s,2}}$$

with identification current $(v, v; vv)$. The identification of the spectrum is now slightly more difficult because this current has a fixed point, but is not difficult to show that this coset gives the $c = 1$ orbifold with $r = 2(2s + 1)$. Hence using $B_s$ (i.e. $SO(L), L = 2s + 1$) we only get cosets with even radii $r = 2L$. The $B_{s,2}$ spectrum consists of the representations $(0), (vv), (s), (sv), (\ell), \ell = 1, \ldots, (L - 1)/2$, where we use a notation analogous to $D_r$. The representation $(ss)$ is in fact an anti-symmetric tensor of rank $(L - 1)/2$ and is more conveniently denoted as such. The identification of the spectrum is

| Coset repr. | $c = 1$ repr. | Coset repr. | $c = 1$ repr. |
|-------------|---------------|-------------|---------------|
| $(0,0;0)$   | $[0]$         | $(0,0;\ell)$| $[2\ell], \ell$ even |
| $(0,0;vv)$  | $[V]$         | $(0,0;\ell)$| $[2L - 2\ell], \ell$ odd |
| $(0,v;0)$   | $[S]$         | $(0,v;\ell)$| $[2L - 2\ell], \ell$ even |
| $(v,0;0)$   | $[C]$         | $(0,v;\ell)$| $[2\ell], \ell$ odd |
| $(0,s;s)$   | $[\sigma]$    | $(s,s;0)$   | $[L]$         |
| $(s,0;s)$   | $[\tilde{\sigma}]$ | $(s,s;\ell)_{1}$ | $[L - 2\ell]$ |
| $(0,s;vs)$  | $[\sigma']$   | $(s,s;\ell)_{2}$ | $[L + 2\ell]$ |
| $(s,0;vs)$  | $[\tilde{\sigma}']$ |               |               |

Choosing a basis $(0, vv, \ell, s, vs)$ for the $B_{s,2}$ representations we get then the following result for $S$

$$S = \begin{pmatrix}
  a & a & 2a & \frac{1}{2} & \frac{1}{2} \\
  a & a & 2a & -\frac{1}{2} & -\frac{1}{2} \\
  2a & 2a & 4a \cos \frac{2\pi \ell' L}{L} & 0 & 0 \\
  \frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} \\
  \frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{2} & \frac{1}{2}
\end{pmatrix}$$

(1.5)

where $a = \frac{1}{2\sqrt{L}}$. Now consider $L = \tilde{L}M^2$. In that case the following is a modular...
The subscript of the last term may be simplified by doubling the range of the anti-symmetric tensors from $1 \leq l \leq (L - 1)/2$ to $1 \leq l \leq (L - 1)$, identifying $l$ with $L - l$ (note that the formula for $S$ is invariant under this). Then the subscript is simply $lM + m\tilde{L}$ (a similar remark applies to (1.2)).

The matrix $S$ that transforms the new, extended characters turns out to be precisely the one of $SO(\tilde{L})$. In the limiting case $\tilde{L} = 1$ this requires an extension to “$SO(1)$”, but (1.5) is well-defined in that limit. In the limit one gets a $D_{r,1}$-type matrix, and this implies that all four representations are simple currents. In particular $(s)$ and $(sv)$ become simple currents after the extension of $SO(M^2)$. The conformal weights of these two representations are $\frac{1}{16}(M^2 - 1)$ and $\frac{1}{16}(M^2 + 7)$ respectively, and for any odd $M$ one of these weights is integer and the other half-integer. The integer spin simple current can be used to extended the algebra even further, leaving only a single representation. The result is a meromorphic CFT [13][4]. Indeed, the first examples of this phenomenon are already known, and correspond to the conformal embedding $B_{4,2} \subset E_{8,1}$ and the $c = 24$ meromorphic CFT based on $B_{12,2}$ (Also the $D_{9,2}$ extension can be found on the list of $c = 24$ meromorphic CFT’s, although less directly).

It should be stressed that these new MIPF’s are neither simple current invariants nor conformal embeddings, i.e. their chiral algebras contain neither simple currents nor spin-1 currents (apart from those of the original affine Lie algebra). Nevertheless they are closely related to both types. The new theories all contain a representation $[\mathcal{V}]$ which is a spin-1 simple current. They can therefore be further extended, and then their $SO(N)_2$ algebra is extended to $SU(N)_1$. This additional extension is both a simple current invariant and a conformal embedding. The resulting theory is a non-trivial MIPF of $SU(N)_1$, and is in fact a simple current invariant of $SU(N)_1$. Hence the new invariants are related to already known ones in the following way

$$SU(N)_1 \xrightarrow{S.C} SU(N)_{1}^{\text{ext}}$$

$$\uparrow \text{C.E} \quad \longrightarrow \quad \downarrow$$

$$SO(N)_2 \xrightarrow{H.S.E} SO(N)^{\text{ext}}_2$$

Here “S.C.”, “C.E” and “H.S.E” stand for simple current, conformal embedding and higher spin extension respectively. The new MIPF’s are in the lower right corner. The
inverse of the conformal embedding is a $\mathbb{Z}_2$ orbifold (inversion of the Cartan sub-algebra of $SU(N)_1$). Applying this same orbifold procedure to the simple current extensions corresponds to the fourth, unmarked arrow in the diagram: the new $SO(N)_2$ invariants are $\mathbb{Z}_2$ orbifolds of simple current extensions of $SU(N)_1$. This proves that a sensible CFT corresponding to these MIPF’s exists. The $\mathbb{Z}_2$-orbifold was in fact used in [13] to construct – among others – the meromorphic $B_{12,2}$ theory, described above, from the $A_{24}$ self-dual lattice.

The diagram (1.6) suggests a generalization to other cases of conformal subalgebras $H \subset G$, where $G$ can be extended by simple currents. The non-trivial issue is the existence of the unmarked arrow in the diagram. This corresponds to “undoing” the conformal embedding for the simple current extension of $G$. In the example discussed in this paper, this undoing is achieved by a simple and well-understood orbifold procedure, but this is not true in general. Furthermore $H$ has representations not present in $G$ (corresponding to twisted states in an orbifold), and the matrix $S$ of $G$ gives no information about $S$ on these states. A partial inspection of the list of conformal embeddings [14] indicates that (1.6) does not generalize, or at least not to all cases.

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REFERENCES

[1] A. Cappelli, C. Itzykson and J.-B. Zuber, Nucl. Phys. B280 (1987) 445; Commun. Math. Phys. 113 (1987) 1.

[2] T. Gannon and M. A. Walton, Heterotic Modular Invariants and Level–Rank Duality, hep-th/9804040; M. R. Abolhassani and F. Ardalan, Int. J. Mod. Phys. A9 2707 (1994); J. Fuchs, B. Gato-Rivera, A.N. Schellekens and C. Schweigert, Phys. Lett. B334 (1994) 113; A. Font Mod.Phys.Lett.A6 (1991) 3265. For older references see: J. Fuchs, Affine Lie Algebras and Quantum Groups [Monographs on Mathematical Physics], Cambridge University Press (1992).

[3] J. Fuchs, A.N. Schellekens and C. Schweigert, Nucl. Phys. B437 (1995) 667.

[4] A.N. Schellekens, Commun. Math. Phys. 153 (1993) 159.

[5] T. Gannon, Commun. Math. Phys. 161(1994)233; Annales Poincare Phys.Theor.65 (1996) 15-56.

[6] T. Gannon, The Classification of $SU(N)_k$ automorphism invariants, hep-th/9408119; T. Gannon, P. Ruelle and M.A. Walton, Automorphism Modular Invariants of Current Algebras, hep-th/9503141.
[7] T. Gannon and Q. Ho-Kim, Int. J. Mod. Phys. A9 (1994) 2667.
[8] L. Dixon, cited in P. Ginsparg, Applied Conformal Field Theory, in the Proceedings of the Les Houches school on Fields, Strings and Critical Phenomena, 1988.
[9] A.N. Schellekens and S. Yankielowicz, Nucl. Phys. B334 (1990) 67.
[10] R. Dijkgraaf, C. Vafa, E. Verlinde and H. Verlinde, Commun. Math. Phys. 123 (1989) 16.
[11] A.N. Schellekens and S. Yankielowicz, Nucl. Phys. B334 (1990) 67; Int. J. Mod. Phys. A5 (1990) 2903.
[12] J. Fuchs, A.N. Schellekens and C. Schweigert, Nucl. Phys. B473 (1996) 323.
[13] L. Dolan, P. Goddard and P. Montague, Phys. Lett. B236 (1990) 165.
[14] F. Bais and P. Bouwknegt, Nucl. Phys. B279 (1987) 561; A.N. Schellekens and N.P. Warner, Phys. Rev. D34 (1986) 3092.