Propagation of shock structures in a high density plasma

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A theoretical investigation has been made to study the cylindrical and spherical electron-acoustic shock waves (EASWs) in an unmagnetized, collisionless degenerate quantum plasma system containing two distinct groups of electrons (one inertial non-relativistic cold electrons and other inertialless ultra-relativistic hot electrons) and positively charged static ions. By employing well known reductive perturbation method the modified Burgers (mB) equation has been derived. It is seen that only rarefactive shock waves can propagate in such a quantum plasma system. The effects of degenerate plasma pressure and number density of hot and cold electron fluids, nonplanar geometry, and positively charged static ions are responsible to modify the fundamental properties of EASWs. It is also observed that the properties of planar mB shocks are quite different from those of nonplanar mB shocks. The findings of the present investigation should be useful in understanding the nonlinear phenomena associated with nonplanar EAWs in both space and laboratory plasmas.

Keywords: Electron-acoustic waves, modified Burgers equation, Shock waves, Degenerate pressure, Relativity, Compact objects

I. INTRODUCTION

The apparent interest for quantum plasmas has been retained their interest due to their existence both in laboratory and space plasma environments. Generally, electron-acoustic (EA) waves (EAWs) occur in a plasma environments (white dwarfs, neutron star, etc.) consisting of two distinct temperature electrons (as hot and cold electrons). EAWs are nothing but high-frequency electrostatic mode, for which inertia is provided by the cold electron motion, while the restoring force comes from the hot electron thermal pressure. The positively charged ions may be safely assumed to be stationary, simply maintaining the quasi-neutrality condition of the plasma system. In such acoustic mode, the frequency lies in the range between the plasma frequency of the cold and hot electron fluids. Watanabe and Taniuti have first shown the existence of the electron-acoustic (EA) mode in a plasma of two-temperature (cold and hot) electrons. Some past decades, EA waves has received a great deal of renewed interest not only because of the two distinct group of electron plasma is very common in laboratory experiments and in space but also because of the potential importance of the EA waves in interpreting electrostatic component of the broadband electrostatic noise (BEN) observed in the cusp of the terrestrial magnetosphere, in the geomagnetic tail, in white dwarfs and neutron stars, etc.

Now a days, researchers of plasma community gives great attention to study the nonlinear behavior of astrophysical compact objects e.g. white dwarfs, neutron stars, etc. The plasma particle number density for such compact objects is so high (in white dwarfs it can be of the order of $10^{30} \text{ cm}^{-3}$, even more) that the electron Fermi energy is comparable to the electron mass energy and the electron speed is comparable to the speed of light in a vacuum. Chandrasekhar presented a general expression for the relativistic ion and electron pressures in his classical papers. The pressure for electron fluid can be given by the following equation

$$P_e = K_e n_e^\alpha,$$  \hspace{1cm} (1)

where $n_e$ is the electron number density and

$$\alpha = \frac{5}{3}; \quad K_e = \frac{3}{5} \left( \frac{\pi}{3} \right)^{\frac{1}{2}} \frac{\pi h^2}{m} \simeq \frac{3}{5} \Lambda_e \hbar c,$$  \hspace{1cm} (2)

for the non-relativistic limit (where $\Lambda_e = \pi \hbar/m c = 1.2 \times 10^{-10} \text{ cm}$, and $\hbar$ is the Planck constant divided by $2\pi$). And

$$P_e = K_e n_e^\gamma,$$  \hspace{1cm} (3)

where

$$\gamma = \frac{4}{3}; \quad K_e = \frac{3}{4} \left( \frac{\pi^2}{9} \right)^{\frac{1}{2}} \hbar c \simeq \frac{3}{4} \hbar c,$$  \hspace{1cm} (4)

for the ultra-relativistic limit.

A large number number of works on relativistic degenerate quantum plasma have been accomplished considering different acoustic waves in the recent years. Han et al. investigate the existence of electron-acoustic shock waves and their interactions in a non-Maxwellian plasma with q-nonextensive distributed electrons. Later on, Han et al. theoretically investigated the nonlinear electron-acoustic solitary and shock waves in a dissipative, nonplanar space plasma with superthermal hot electrons. Sahu and Tribeche considered electron acoustic shock waves (EASWs) in an unmagnetized plasma whose constituents are cold electrons, immobile ions and Boltzmann distributed hot electrons and
studied the effects of several parameters and ion kinematic viscosity on the basic features of EA shock waves. By considering quantum plasma El-Labany et al. \[62\] investigated the effects of Bohm potential on the head on collision between two quantum electron-acoustic solitary waves using the extended Pointcar-Lighthill-Kuo method. Mahmoud and Masood \[63\] illustrated that an increase in quantum diffraction parameter broadens the nonlinear structure. Recently, Sah \[67\] demonstrated that the width, the amplitude, and the velocity of electron-

II. GOVERNING EQUATIONS

We consider a cylindrical and spherical EA waves in an unmagnetized, collisionless plasma, which is composed of non-relativistic inertial cold electrons, both non-relativistic and ultra-relativistic degenerate hot electron fluids, and static positive ions. Thus at equilibrium, we consider the combine effects of nonplanar geometry, effects of relativistic limits (i.e., both non-relativistic and ultra-relativistic) and degenerate plasma pressure which can significantly modify the propagation of solitary and shock waves.

\( \omega_{\mu i} = (4\pi n_{e0}e^2/m_e)^{1/2} \), and the space variable \((x)\) is normalized by \( \lambda_s = (m_e c^2/4\pi n_{e0}e^2)^{1/2} \). The coefficient of viscosity \( \eta \) is a normalized quantity given by \( \omega_{\mu i}^2 \lambda_s m_s n_0 \). We have defined \( K_1 = n_{e0}^{-1} K_i/m_e c^2 \) and \( K_2 = n_{h0}^{-1} K_e/m_e c^2 \).

III. DERIVATION OF MODIFIED BURGERS EQUATION

We derive a dynamical modified Burgers (mB) equation for the nonlinear propagation of the EA waves by using equations \[3[64-69]. \) To do so, we employ a reductive perturbation technique to examine electrostatic perturbations propagating in the relativistic degenerate dense plasma system due to the effect of dissipation, we first introduce the stretched coordinates \[71\]:

\[ \xi = -\epsilon(r + V_p t), \quad \tau = \epsilon^2 t, \] (10)

where \( V_p \) is the wave phase speed \((\omega/k)\) with \( \omega \) being the angular frequency and \( k \) being the wave number, and \( \epsilon \) is a smallness parameter measuring the weakness of the dissipation \((0 < \epsilon < 1)\). We expand the parameters \( n_c, n_h, u_c, \phi, \) and \( \rho \) in power series of \( \epsilon \) as:

\[ n_c = 1 + \epsilon n_c^{(1)} + \epsilon^2 n_c^{(2)} + \cdots, \] (11)

\[ n_h = 1 + \epsilon n_h^{(1)} + \epsilon^2 n_h^{(2)} + \cdots, \] (12)

\[ u_c = \epsilon u_c^{(1)} + \epsilon^2 u_c^{(2)} + \cdots, \] (13)

\[ \phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots, \] (14)

\[ \rho = \epsilon \rho^{(1)} + \epsilon^2 \rho^{(2)} + \cdots, \] (15)

Now, expressing equations \[3[64-69] \] (using equation \[10\]), in terms of \( \xi \) and \( \tau \), and substituting equations \[11\]-\[15\], one can easily develop different sets of equations in various powers of \( \epsilon \). To the lowest order in \( \epsilon \), we have:

\[ u_c^{(1)} = V_p \phi^{(1)}/(V_p^2 - K_1^2), n_c^{(1)} = -\phi^{(1)}/(V_p^2 - K_1^2), n_h^{(1)} = \phi^{(1)}/K_2', V_p = \sqrt{K_2'/\mu - K_1'}, \] where \( K_1' = 0 K_1 \) and \( K_2' = \gamma K_2 \). The relation \( V_p = \sqrt{K_2'/\mu - K_1'} \) represents the dispersion relation as well as the phase speed for the EA type electrostatic waves in the degenerate quantum plasma under consideration.

To the next higher order in \( \epsilon \), we obtain a sets of equations:

\[ \frac{\partial n_c^{(1)}}{\partial \tau} - V_p \frac{\partial n_c^{(2)}}{\partial \xi} - \frac{\partial}{\partial \xi} [u_c^{(2)} + n_c^{(1)} u_c^{(1)}] - \frac{\mu u_c^{(1)}}{V_p \tau} = 0 \] (16)

\[ \frac{\partial u_c^{(1)}}{\partial \tau} - V_p \frac{\partial u_c^{(2)}}{\partial \xi} - u_c^{(1)} \frac{\partial u_c^{(1)}}{\partial \xi} + \frac{\partial \phi^{(2)}}{\partial \xi} - K_2' \frac{\partial}{\partial \xi} \frac{n_c^{(2)} + (\gamma - 2)n_h^{(2)}}{2} - \mu \frac{\partial^2 u_c^{(2)}}{\partial \xi^2} = 0, \] (17)

\[ \frac{\partial \phi^{(2)}}{\partial \xi} - K_2' \frac{\partial}{\partial \xi} \frac{n_h^{(2)} + (\gamma - 2)n_h^{(2)}}{2} = 0, \] (18)

\[ n_c^{(2)} + (\mu - 1)n_h^{(2)} = 0, \] (19)
FIG. 1: (Color online) Variation of phase speed $V_p$ with ion-to-cold electron number density ratio $\mu$ for $u_0 = 0.1$.

FIG. 2: (Color online) Nonlinear shock waves are shown for different values of $\nu$ when cold electron and hot electron fluids both are nonrelativistic degenerate with parameters $u_0 = 0.01$ and $\mu = 0.93$.

FIG. 3: (Color online) Effects of cylindrical geometry on EA shock waves when both cold electron and hot electron fluids are nonrelativistic and degenerate ($\nu = 1$, $u_0 = 0.01$ and $\mu = 0.93$).

FIG. 4: (Color online) Effects of cylindrical geometry on EA shock waves when both cold electron and hot electron fluids are nonrelativistic and degenerate ($\nu = 1$, $u_0 = 0.01$ and $\mu = 0.93$).

FIG. 5: (Color online) Effects of spherical geometry on EA shock waves when both cold electron and hot electron fluids are nonrelativistic and degenerate ($\nu = 2$, $u_0 = 0.01$ and $\mu = 0.93$).

Now, combining equations (16)-(19) we deduce Burgers equation

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + \frac{\nu \phi^{(1)}}{2\tau} = B \frac{\partial^2 \phi^{(1)}}{\partial \xi^2},$$

where

$$A = \left( \frac{V_p^2 - K_1'}{2V_p} \right)^2 \left[ (\gamma - 2)(\mu - 1) - 3V_p^2 + K_1'(\alpha - 2) \right]$$

$$B = \frac{\eta}{2}.$$

Here $A$ and $B$ are two constants and may be defined as nonlinearity and dissipative constants respectively.

IV. DISCUSSION AND RESULTS

In this section, our first intention to numerically analyze the Burgers equation. However, for clear understanding, we first briefly discuss about the stationary shock wave solution for equation (20) with $\nu = 0$, though the solution is similar for both IA and EA waves (excluding the values of $A$ and $B$). We should note that for a large value of $\tau$, the term $\frac{\nu \phi^{(1)}}{2\tau}$ is negligible. So, in our numerical analysis, we start with a large value of $\tau$ (viz. $\tau = -20$), and at this large (negative) value of $\tau$, we choose the stationary shock wave solution of equation (23) [without the term $\frac{\nu \phi^{(1)}}{2\tau}$] as our initial pulse. The stationary shock wave solution of this standard Burgers equation is obtained by considering a frame $\xi = \zeta - u_0\tau$ (moving with speed $u_0$ which is the ion fluid speed at equilibrium) and the solution is [72, 74].
The ion-to-cold electron density ratio (\(\eta\)) and the (cold electron) kinematic viscosity (\(\nu\)) are important to note that the dissipation term only depends on the electron kinematic viscosity (\(\nu\)).

The cylindrical and spherical variation of the amplitude of shock structures for both non-relativistic and ultra-relativistic limits is shown in Figs. 4-7. Finally, the results that we have found in this investigation can be summarized as follows:

1. The cylindrical and spherical plasma system under consideration supports only rarefactive shock waves with negative potential, but no compressive shock waves exist.

2. The fundamental properties of EASWs are found to be significantly modified by the relativistic parameters, nonplanar geometry and plasma particle number densities.

3. It is observed that the phase speed \((V_p)\) of these EA shocks inversely proportional to the square root of ion to cold electron number densities ratio \(\mu\).

4. It is also found that the phase speed \((V_p)\) of EA waves decreases with the increasing values of \(\mu\) (see Fig. 1).

5. It is observed that the amplitude of the shock is maximum for the spherical geometry, intermediate for cylindrical geometry, while it is minimum for the planar geometry (see Figs 2-3).

6. The amplitude of shocks proportional to the fluid speed \(u_0\) but inversely proportional to the constant \(A\).

7. From Figs. 3-7 we observed that the amplitude of the nonplanar EA rarefactive shock waves is lower for ultra-relativistic case than for non-relativistic case.

In conclusion, our simplified theoretical model represents a small yet steady step towards the rigorous understanding of the behavior of cylindrical and spherical EA shocks in degenerate plasma environments, which appear to be of fundamental importance in a wide range of astrophysical and laboratory scenarios.
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