Comments on “Overdemodulation for High-Performance Receivers with Low-Resolution ADC”

Tristan Afortiori

Abstract—In the above-titled paper by Stein, Theiler, and Nossek (see ibid., vol. 4, no. 2, p. 169-172, April 2015), the design of analog demodulator for receivers with low-resolution analog-to-digital converters (ADCs) was addressed. The authors proposed to increase the number (M) of analog demodulation channels in the radio front-end beyond that of the classical quadrature demodulation (with M = 2 orthogonal sinusoidal functions). The proposed design was called overdemodulation. The estimation performance was evaluated in terms of a lower bound for the Fisher Information with respect to an unknown parameter \( \theta \). It was demonstrated via simulations that the lower bound could be improved as \( M \) increases. In this comment, we point out that this result trivially follows (not for the lower bound but for the actual Fisher Information) from the chain rule for the Fisher Information. More specifically, since each channel in the overdemodulation framework provides a 1-bit random variable (RV) parametrized by \( \theta \), the Fisher Information increases with the addition of each new channel unless the 1-bit RV from the new channel conditioned on all the 1-bit RVs from the remaining channels is independent of the parameter \( \theta \). Furthermore, we argue that overdemodulation does not make sense as a radio front-end from an engineering perspective. Additional channels in the overdemodulation framework are deterministically related to the in-phase (I) and quadrature (Q) channels of the classical quadrature demodulation. This, in turn, implies that the signals over the auxiliary \( M - 2 \) channels can be synthesized at the baseband without any connection to the physical world outside and are therefore redundant. We also point out that the lower bound for Fisher Information employed in the letter, which was credited to an earlier letter by the first and third authors, is a well-known result in statistics.

Index Terms—quadrature demodulation, overdemodulation.

In order to expound the claims raised above, we first recall from equations 9 and 10 in [1] that the received samples at the output of the proposed overdemodulation (i.e., \( M > 2 \)) system are given by

\[
\tilde{y}_n = A(\varphi)(\gamma B(\psi)x_n(\tau) + \eta_n),
\]

where

\[
A(\varphi) = \begin{bmatrix}
\cos(\varphi_1) & \sin(\varphi_1) \\
\cos(\varphi_2) & \sin(\varphi_2) \\
\vdots & \vdots \\
\cos(\varphi_M) & \sin(\varphi_M)
\end{bmatrix},
\]

\[
B(\psi) = \begin{bmatrix}
\cos(\psi) & \sin(\psi) \\
-\sin(\psi) & \cos(\psi)
\end{bmatrix},
\]

\[
\tilde{y}_n = \begin{bmatrix}
y_1 \left( \frac{n-1}{f_s} \right), y_2 \left( \frac{n-1}{f_s} \right), \ldots, y_M \left( \frac{n-1}{f_s} \right)\end{bmatrix}^T,
\]

\[
x_n(\tau) = \begin{bmatrix} x_1 \left( \frac{n-1}{f_s} - \tau \right), x_2 \left( \frac{n-1}{f_s} - \tau \right) \end{bmatrix}^T,
\]

\[
\eta_n = \begin{bmatrix} \eta_1 \left( \frac{n-1}{f_s} \right), \eta_2 \left( \frac{n-1}{f_s} \right) \end{bmatrix}^T.
\]

The classical quadrature demodulation with \( M = 2 \) orthogonal sinusoidal functions can be obtained from (1) by setting \( A(\varphi) = I_2 \), where \( I_2 \) denotes the \( 2 \times 2 \) identity matrix. More explicitly, we substitute \( \varphi_1 = 0, \varphi_2 = \pi/2 \), and remove all but the first two rows in (2). In this case, the signal model given in (1) would simplify to

\[
y_n = \gamma B(\psi)x_n(\tau) + \eta_n,
\]

where \( B(\psi), x_n(\tau) \) and \( \eta_n \) remain the same as above, and the in-phase (I) and quadrature (Q) outputs of the quadrature demodulator are denoted as

\[
y_n = \begin{bmatrix} y_I \left( \frac{n-1}{f_s} \right), y_Q \left( \frac{n-1}{f_s} \right) \end{bmatrix}^T.
\]

At this point, it can be seen easily that the samples taken at the outputs of the \( M \) channels of the proposed overdemodulation scheme can as well be obtained by forming linear combinations of the in-phase and quadrature outputs of the classical quadrature demodulator, i.e.,

\[
\tilde{y}_n = A(\varphi)y_n = \begin{bmatrix}
\cos(\varphi_1) & \sin(\varphi_1) \\
\cos(\varphi_2) & \sin(\varphi_2) \\
\vdots & \vdots \\
\cos(\varphi_M) & \sin(\varphi_M)
\end{bmatrix} \begin{bmatrix} y_I \\ y_Q \end{bmatrix}.
\]

Hence, no new information is gained with the addition of auxiliary channels to the demodulation stage. As long as the demodulator has access to two linearly independent channels (i.e., two linearly independent rows of \( A(\varphi) \)), that is sufficient (since \( A(\varphi) \) is at most rank 2).

Remark 1: Only at most 2 of the \( M \) channels in the proposed radio-frequency (RF) front-end (as depicted in Fig. 1) can carry independent information. The rest of the channels is related deterministically and hence does not provide novel information. The signals over the remaining \( M - 2 \) channels can be synthesized from the I and Q channels without any connection to the physical world outside. From an engineering perspective, implementing an \( M \)-channel RF front-end, as described in Fig. 1, is a waste of resources for \( M > 2 \).

Remark 2: In practice, the RF processing required by the overdemodulation framework (as depicted in Fig. 1) would render overdemodulation a very unlikely choice especially since the same samples can be obtained according to (5) at the output of the quadrature demodulator with the ease of baseband processing (e.g., summing amplifier circuits may be
The overdemodulation framework is much more demanding because it is hard to generate and maintain the phase offsets of the correlator signals for all $M$ channels at the RF stage. Furthermore, one needs to take into account gain and phase imbalance among $M$ channels as well as the nonlinear effects of additional mixers and RF amplifiers.

Next, we focus on the improvement in estimation performance attributed to the overdemodulation framework in $\textbf{1}$. The outputs from $M$ demodulation channels are quantized using 1-bit ADCs, i.e., $r_n = \text{sign}(\tilde{y}_n) = \text{sign}(A(\varphi)y_n)$, where $\text{sign}(\cdot)$ is the element-wise signum-function. The quantized version of the received signal $r_n$ has dimension $M$, i.e., $r_n \in \{-1, 1\}^M$. An unknown parameter $\theta$ is estimated from a sequence of values of $r_n$.

Now, suppose that another channel is added and the quantizer output from that channel is denoted as $q_n$. From the Chain Rule for Fisher Information Matrix [2, Lemma 1], it follows that

$$I(r_n, q_n; \theta) = I(r_n; \theta) + I(q_n; \theta|r_n)$$

(6)

where $I(\cdot; \theta)$ denotes the Fisher Information Matrix with respect to parameter $\theta$. Due to the nonnegativity of the Fisher Information, we get

$$I(r_n, q_n; \theta) \geq I(r_n; \theta),$$

(7)

with equality if $I(q_n; \theta|r_n) = 0$, i.e., the conditional distribution of $q_n$ given $r_n$ is independent of $\theta$ (See also [2, Lemma 2]). The inequality in (7) indicates that the Fisher Information contained in all the $M+1$ quantizer outputs about the parameter $\theta$, denoted by $I(r_n, q_n; \theta)$, dominates the Fisher Information of any subset of $M$ quantizer outputs, denoted by $I(r_n; \theta)$. The inequality in (7) is strict unless the conditional distribution of $q_n$ given $r_n$ is independent of $\theta$.

**Remark 3:** The main contribution of $\textbf{1}$, which is to demonstrate via simulations that employing more than two demodulation channels can improve the estimation performance follows trivially from the chain rule and nonnegativity of the Fisher Information. Besides, the authors’ analysis to demonstrate the performance improvements relies on a lower bound mentioned in Remark 5 instead of the actual Fisher Information.

Equation (5) helps to understand the improvement in the Fisher Information attributed to overdemodulation in $\textbf{1}$. With only $y_1$ and $y_Q$, the quantized data provides information only on the signs of $y_1$ and $y_Q$. On the other hand, when the overdemodulation outputs $\tilde{y}_n = A(\varphi)y_n$, as given in (5) are input to 1-bit ADCs, additional information is revealed about the signs of various linear combinations of $y_1$ and $y_Q$. This, in turn, facilitates better localization of $y_1$ and $y_Q$ over $\mathbb{R}^2$.

**Remark 4:** With each additional channel in the overdemodulation architecture, an additional 1-bit ADC is implicitly included into the demodulation stage. In the light of (7), it can be concluded that the Fisher Information is a nondecreasing function of the number of channels ($M$) in the overdemodulation framework, and consequently the number of ADCs, which is also equal to $M$. The improvement in Fisher Information presented in $\textbf{1}$ is due to higher number of bits used employed in the overdemodulation ($M > 2$) framework in comparison to a quadrature demodulation scheme followed by two 1-bit ADCs.

In an $M$–channel overdemodulation scheme, $M$ 1-bit ADCs are employed. Therefore, the proposed architecture allocates $M$ bits for each observation. However, the overdemodulation framework proposed in $\textbf{1}$ does not offer an efficient use of the $M$ available bits in comparison with the classical quadrature demodulation. In the overdemodulation framework, the outputs from $M$ demodulation channels are quantized using 1-bit ADCs, i.e., $r_n = \text{sign}(\tilde{y}_n) = \text{sign}(A(\varphi)y_n)$. The only information provided by $r_n$ is about the phase of $y_n$. More explicitly, if the $i$th component of $r_n$ is equal to 1, this means that the phase of $y_n$ resides in the interval $[\phi_i - \pi/2, \phi_i + \pi/2]$, where $\phi_i$ is the phase of correlator signal of the $i$th channel as depicted in $\textbf{1}$ Fig. 1. Since $A(\varphi)$ does not have full row-rank, not all the components of $r_n$ carry independent information about $y_n$ either. With $M$ channels, the 2-dimensional Euclidean space is actually divided into at most $2M$ sectors.

**Remark 5:** The proposed $M$–channel overdemodulation framework followed by 1-bit quantizers at the output of each channel provides only phase information and no magnitude information about the underlying infinite precision sample. However, with a total of $M$ bits and uniform quantization over $I$ and $Q$ axes, the 2-dimensional Euclidean space can be divided into $2^M$ rectangular regions, as is usually the case in practice. Hence, the proposed overdemodulation architecture is inferior to classical communications systems that rely on quadrature demodulation followed by quantization in the utilization of the available bits.

Finally, we note that the lower bound for Fisher Information given in $\textbf{1}$ Eq. 9 is wrongly credited to an earlier paper by the first and third authors [3]. It is a simple restatement of the well-known Fisher Information Inequality (also known as the Cramer-Rao Lower Bound - CRLB) corresponding to the trivial estimator $\theta(r_n) = r_n$. This moment-based bound (which can be traced back to the origins of the CRLB) as well as its generalizations were already published in [3] (see Eq. 2.1, the line preceeding Eq. 2.5, and Theorem 2.2 in [3]).

---

1 So that there is a total of $M + 1$ channels.

---

**REFERENCES**

[1] M. Stein, S. Theiler, and J. A. Nossek, “Overdemodulation for high-performance receivers with low-resolution ADC,” IEEE Wirel. Commun. Lett., vol. 4, no. 2, pp. 169–172, April 2015.

[2] R. Zamir, “A proof of the Fisher information inequality via a data processing argument,” IEEE Trans. Inform. Theory, vol. 44, no. 3, pp. 1246–1250, May 1998.

[3] M. Stein, A. Mezghani, and J. A. Nossek, “A lower bound for the Fisher information measure,” IEEE Signal Proc. Lett., vol. 21, no. 7, pp. 796–799, July 2014.

[4] R. G. Jarrett, “Bounds and expansions for Fisher information when the moments are known,” Biometrika, vol. 71, no. 1, pp. 101–113, April 1984.

---

$^2$ $M$ non-coincident lines all passing through the origin divides $\mathbb{R}^2$ into $2^M$ sectors.