Full and Partial Thermalization of Nucleons in Relativistic Nucleus-Nucleus Collisions

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Abstract

We propose a mechanism of thermalization of nucleons in relativistic nucleus-nucleus collisions. Our model belongs, to a certain degree, to the transport ones; we consider the evolution of the system, but we parametrize this development by the number of collisions of every particle in the system rather than by the time variable. We based on the assumption that the nucleon momentum transfer after several nucleon-nucleon (hadron) collisions becomes a random quantity driven by a proper distribution.

Key words: quark-gluon plasma, thermalization, rapidity distribution, transverse mass spectrum

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The Model. Our model is aimed at the description of the nucleon spectra \( dN/dy \), \( dN/m_\perp dm_\perp dy \) for such collision energies, when the number of created nucleon-antinucleon pairs is much less than the number of net nucleons, i.e. it can be applied at AGS and low SPS energies.

We now make three key assumptions about the nucleon system.

1. We separate all nucleons after freeze-out into two groups in accordance with their origination: a) the first group consists of net nucleons that went through hadron reactions; b) the second group includes nucleons which were created in the collective processes, for instance, during the hadronization of a QGP. Then, the nucleon momentum spectrum can be represented as a sum of two different contributions:

\[
\frac{dN}{d^3p} = \left( \frac{dN}{d^3p} \right)_{\text{hadron}} + \left( \frac{dN}{d^3p} \right)_{\text{QGP}}. \tag{1}
\]

In turn, the total number of registered nucleons equals \( N_{\text{total}} = N_{\text{hadron}} + N_{\text{QGP}} \). If this separation can be done, we can define the “nucleon power” of the created QGP as...
\[ P_{\text{QGP}}^{(N)} = \frac{N_{\text{QGP}}}{N_{\text{total}}}. \]

In the present work, we mainly deal with the nucleons from the first group.

2. The collision number for every nucleon (hadron) is finite because the lifetime of the fireball is limited. To determine the maximal number of collisions, \( M_{\text{max}} \), in a particular experiment, we use the results of UrQMD calculations \([1,2]\).

3. Because the colliding nuclei are spatially restricted many-nucleons systems, the different nucleons experience different collision numbers; it is intuitively clear that the collision histories of the inner and surface nucleons will be different. That is why, we partition all amount of nucleons of the first group (nucleons which take part in hadron reactions only) into different ensembles in accordance with a number of collisions before freeze-out. Then the nucleons from every ensemble give their own contribution to the total nucleon spectrum. If we denote the number of particles in a particular ensemble where the particles experienced \( M \) effective collisions by \( C(M) \), then, in correspondence to (1), we can write the total nucleon spectrum as

\[ \frac{dN}{dp^3} = \sum_{M=1}^{M_{\text{max}}} C(M) D_M(p) + C_{\text{therm}} D_{\text{therm}}(p), \]

where \( D_M(p) \) is the spectrum (normalized to unity) of the particles in the \( M \)-th ensemble. The last term on the r.h.s. of (2) corresponds to the possible contribution from the totally thermalized source which we associate with a QGP. Here, \( C_{\text{therm}} \) is the number of nucleons which are created during the hadronization of the QGP, and \( D_{\text{therm}}(p) \) is the thermal distribution.

Consider successive variations of the momentum of the \( n \)-th nucleon from nucleus \( A \) which moves with momentum \( k_0 \) along the collision axis from left to right toward nucleus \( B \). Every \( m \)-th collision induces the momentum transfer \( q_n^{(m)} \) for the \( n \)-th nucleon. So that, after \( M \) collisions, the nucleon acquires the momentum \( k_n = k_0 + Q_n \), where \( Q_n = \sum_{m=1}^{M} q_n^{(m)} \) is the total momentum transfer finally obtained by the \( n \)-th nucleon.

We assume for the moment that the elastic scattering gives the main contribution to the two-nucleon collision amplitude. The initial momentum of every nucleon in nucleus \( A \) is \( k_a = k_0 = (0, 0, k_0z) \), while the initial momentum of every nucleon in nucleus \( B \) is \( k_b = -k_0 = (0, 0, -k_0z) \), in the c.m.s. of colliding nuclei. The energy and momentum are conserved in every separate collision of two particles, \( \omega(k_a) + \omega(k_b) = \omega(p_a) + \omega(p_b) \), \( k_a + k_b = p_a + p_b \), where \( p_a \) and \( p_b \) are the momenta of the particles after collision. We assume that the particles are on the mass shell, so that \( \omega(k) = \sqrt{m^2 + k^2} \) (the system of units \( \hbar = c = 1 \) is adopted). Determining the six unknown quantities, \( p_a \) and \( p_b \), from four equations is straightforward but two quantities (e.g., \( (p_a)_x \) and \( (p_b)_x \)) remain uncertain and can be considered as such which accept random values driven by the scattering probability. After the third collision, every component of the particle momentum becomes completely random. If the initial momentum is fixed, this means the full randomization of the momentum transfer after three successive collisions. So, if we follow the elastic scattering of a nucleon from the first collision to the last one, we would see the full randomization of the momentum transfer after every three successive scattering acts. As for the inelastic collisions, the nucleon momentum transfer undergoes the even faster randomization \([3]\).

First, we would like to determine the density distribution function \( f_{2N} \) in the momentum space, which describes \( 2N \) nucleons after \( M \) collisions per particle. The whole
consideration is carried out in the c.m.s. of two identical colliding nuclei. Let us write down a density distribution function in the form \( f_{2N} = C \tilde{f}_{2N} \), where \( C \) is the normalization constant. The unnormalized distribution function \( \tilde{f}_{2N} \) can be defined in a two-fold way: first, we follow all collisions of a particular nucleon by the integration with respect to all nucleon random momentum transfer, and, second, we fix the total energy of the 2N-nucleon system after freeze-out in a microcanonical-like way: \( E_{\text{tot}} = \sum_{n=1}^{2N} \omega(k_n) \).

The unnormalized integral measure of the momentum transfer for the \( n \)-th particle in a series of \( M \) collisions is determined as

\[
dQ_n \equiv \prod_{m=1}^{M} J_m(q_n^{(m)}) \frac{d^3q_n^{(m)}}{(2\pi)^3} \quad \text{with} \quad \int \frac{d^3q}{(2\pi)^3} J_m(q) = 1,
\]

where the distribution of the momentum transfer is characterized by the presence of the form-factor \( J_m(q) \). For the sake of simplicity, we assume the independence of \( J_m(q) \) on the collision number \( m \), i.e. \( J_m(q) \to J(q) \). Hence, we adopt an approximation where just one form-factor \( J(q) \) characterizes a distribution of the momentum-transfer in a series of collisions which are experienced by a nucleon during its traveling through the fireball.

Then the unnormalized 2N-particle distribution function reads

\[
\tilde{f}_{2N}(E_{\text{tot}}; k_1, \ldots, k_{2N}) = \delta \left( E_{\text{tot}} - \sum_{n=1}^{2N} \omega(k_n) \right) \prod_{n=1}^{N} \frac{dQ_n}{V} \ldots \frac{dQ_{2N}}{V} \times \prod_{n=1}^{N} \left[ (2\pi)^3 \delta^3 \left( k_n - k_0 - \sum_{m=1}^{M} q_n^{(m)} \right) \right] \prod_{n=N+1}^{2N} \left[ (2\pi)^3 \delta^3 \left( k_n + k_0 - \sum_{m=1}^{M} q_n^{(m)} \right) \right],
\]

where \( V \) is the volume of the system in the coordinate space. Making the Laplace transformation, we determine the unnormalized distribution function

\[
\tilde{F}_{2N}(\beta; k_1, \ldots, k_{2N}) = \int_{E_{\text{min}}}^{\infty} dE_{\text{tot}} e^{-\beta E_{\text{tot}}} \tilde{f}_{2N}(E_{\text{tot}}; k_1, \ldots, k_{2N}).
\]

The distribution function in the canonical ensemble reads \( F_{2N}(\beta) = \tilde{F}_{2N}(\beta)/Z_{2N}(\beta) \)

\[
F_{2N}(\beta, k_1, \ldots, k_{2N}) = \prod_{n=1}^{N} f_a(k_n) \prod_{n=N+1}^{2N} f_b(k_n),
\]

where

\[
f_{a(b)}(k) = \frac{1}{z_{a(b)}(\beta)} e^{-\beta z_{a(b)}(\beta)} I_M(k \pm k_0), \quad z_{a(b)}(\beta) = \int \frac{d^3k}{(2\pi)^3} e^{-\beta z_{a(b)}(k)} I_M(k \pm k_0)
\]

are the single-particle distribution functions attributed to nucleus “A” for subindex \( a \) or to nucleus “B” for subindex \( b \), respectively. Note that \( z_a(\beta) = z_b(\beta) = z(\beta) \) for identical nuclei. We define the “multi-scattering form-factor”

\[
I_M(Q) \equiv \frac{1}{V} \int d^3r e^{-iQ \cdot r} [J(r)]^M \quad \text{with} \quad J(r) = \int \frac{d^3q}{(2\pi)^3} J(q) e^{iq \cdot r}.
\]

For a large enough number \( M \) of the effective collisions and for the form-factor which possesses the spherical symmetry, \( J(q) = J(|q|) \), we can calculate the first integral in within the saddle-point method.
\[ I_M(Q) = \left( \frac{6\pi}{M\langle q^2 \rangle} \right)^{3/2} e^{-\frac{3q^2}{2M\langle q^2 \rangle}}, \quad \text{where} \quad \langle q^2 \rangle = \int \frac{d^3q}{(2\pi)^3} q^2 J(q). \] (9)

In the limit case \( M \to \infty \), the dependence on the initial momentum \( k_0 \) is washed out, and both single-particle distributions \( f_{a(b)}(k) \) take the same “thermal” limit: \( f_{a(b)}(k) \to f_{\text{therm}}(k) = e^{-\beta \omega(k)}/z_{\text{therm}}(\beta) \), where \( z_{\text{therm}}(\beta) = \int d^3k e^{-\beta \omega(k)} \).

Using a “two-source” single-particle distribution function \( f(k_a, k_b) = f_a(k_a) f_b(k_b) \) to average the quantity \( W(k_a, k_b) = \frac{1}{2} [\delta^3(p - k_a) + \delta^3(p - k_b)] \), we obtain

\[ D_M(p) = \left( \frac{1}{2N} \frac{dN}{d^3p} \right)_M = \frac{1}{2z(\beta)} e^{-\beta \omega(p)} [I_M(p - k_0) + I_M(p + k_0)]. \] (10)

It is evident that the spectrum has two items which can be attributed to the first and second colliding nuclei, respectively, and hence it can be named a “two-source single-particle spectrum”.

**Nucleon Rapidity Distribution and Transverse Spectrum.** To obtain the transverse mass and rapidity distributions, we pass to new variables: \( m_\perp = (m^2 + p_\perp^2)^{1/2} \), \( p_\perp^2 = p^2_\perp + p_y^2 \), tanh \( y = p_z/\omega_p \), then \( dp = d\phi \omega_p m_\perp dm_\perp dy \). In accordance with (2),

\[ \frac{d^2N}{m_\perp dm_\perp dy} = 2\pi m_\perp \cosh y \left[ \sum_{M=1}^{M_{\text{max}}} C(M) D_M(m_\perp, y) + C_{\text{therm}} D_{\text{therm}}(m_\perp, y) \right]. \] (11)

We can define the distribution functions in the rapidity space as

\[ \Phi_M(y) = \pi \cosh y \int_m^\infty dm_\perp m_\perp^2 \left[ f_a(m_\perp, y) + f_b(m_\perp, y) \right] \quad \text{with} \quad \int dy \Phi_M(y) = 1. \] (12)

Then, the rapidity distribution looks like

\[ \frac{dN}{dy} = \sum_{M=1}^{M_{\text{max}}} C(M) \Phi_M(y) + C_{\text{therm}} \Phi_{\text{therm}}(y). \] (13)

**Toy Model.** To present the explicit results of our approach, we consider a toy model: the form-factor \( J(q) \) is chosen as a homogeneous distribution in the sphere of finite radius \( q_{\text{max}} \),

\[ J(q) = \frac{(2\pi)^3}{V_q} \theta(q_{\text{max}} - |q|), \quad V_q = \frac{4}{3} \pi q_{\text{max}}^3 \quad \text{with} \quad \int \frac{d^3q}{(2\pi)^3} J(q) = 1. \] (14)

In the proposed model, the maximum number of collisions, \( M_{\text{max}} \), is assumed to be finite and determined by the nucleus number \( A \), initial energy, and centrality. With the help of the UrQMD transport model [12], it was found that, under the AGS conditions [4] with a centrality of (0-3)\%, \( M_{\text{max}} = 13 \) and \( q_{\text{max}} = 0.8 \text{ GeV/c} \) (for the toy model \( \frac{3}{2} q_{\text{max}}^2 = \langle q^2 \rangle_{\text{UrQMD}} \)). Utilizing the thermal distribution, we extract a slope parameter from experimental data on the proton \( m_\perp \)-spectra [4]. \( T = 280 \text{ MeV} \). Note, the proton data are of interest, first of all, because we know an exact value of the initial nucleon momentum.

**Table 1**

| \( C(1) \) | \( C(2) \) | \( C(3) \) | \( C(4) \) | \( C(5) \) | \( C(6) \) | \( C(7) \) | \( C(8) \) | \( C(9) \) | \( C(10) \) | \( C(11) \) | \( C(12) \) | \( C(13) \) |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 1.3       | 23.2      | 4.7       | 5.3       | 6.6       | 8.2       | 9.5       | 10.6      | 11.6      | 12.3      | 12.9      | 13.5      | 13.9      | 16.6      |
The results of the fit to experimental data \cite{4} on the rapidity distribution and $m_\perp$-spectra of the net protons are depicted in Figs. 1, 2. The fit was done with making use of the maximum entropy method \cite{5}. The set of coefficients $C(M)$ (see Table 1) are nothing more as the absolute number of protons in every collision ensemble. This description was carried out with account for the contribution of the thermal source, which cannot appear due to the nucleon-nucleon (-hadron) rescattering (see the partial contributions in Fig. 1). We assume that this source is a thermalized multiparton system (QGP) which emits totally thermalized nucleons through the hadronization process. The knowledge of the number of protons, $C_{therm}$, which come from the QGP, gives us a possibility to evaluate the “nucleon power” of the QGP, $P_{QGP}^{(N)}$, created in a particular experiment on the nucleus-nucleus collision. We find that, under the AGS conditions \cite{4} (a centrality of 0-3%), $P_{QGP}^{(N)} \approx 11\%$. So, in the framework of the proposed criterion, it could be claimed that the QGP (as a nucleon source) was created not only at SPS energies \cite{6}, but it was also created in the central collisions at AGS energies.

Meanwhile, the partial expansion, $dN/dy = \sum_{M=1}^{M_{max}} C(M) \varphi_M(y)$ (see (13)), where we do not use the thermal contribution, makes a good description (the same $\chi^2$) of the experimental data on both the rapidity distribution and $m_\perp$-spectra. So, we cannot resolve unambiguously the presence of the thermal source. In fact, to overcome the problem, we need a more detailed experimental information for the central rapidity region.

All this encourages us to apply the model to other experiments and problems.

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