Abstract

Generic members of the observational results depend upon both the quantum state and the rules for extracting the probabilities from it. It is often argued that inflation may make our observations independent of the quantum state. In a framework in which one considers the state and the rules as logically separate, it is shown how it is possible that the probabilities are indeed independent of the state, but the rules for achieving this seem somewhat implausible.
**Introduction**

A goal of science is to produce complete theories $T_i$ that each predict normalized probabilities $P_j(i)$ of observations $O_j$,

$$P_j(i) \equiv P(O_j|T_i) \text{ with } \sum_j P_j(i) = 1. \quad (1)$$

Assuming that a complete physical theory of the universe is quantum, I would argue [1] that it should contain at least the following elements:

1. Kinematic variables (wavefunction arguments)
2. Dynamical laws (‘Theory of Everything’ or TOE)
3. Boundary conditions (specific quantum state)
4. Specification of what has probabilities
5. Probability rules (analogue of Born’s rule)
6. Specification of what the probabilities mean

Here I shall call elements (1)-(3) the quantum state (or the “state”), since they give the quantum state of the universe that obeys the dynamical laws and is written in terms of the kinematic variables, and I shall call elements (4)-(6) the probability rules (or the “rules”), since they specify what it is that has probabilities (here taken to be the results of observations, $O_j$, or “observations” for short), the rules for extracting these observational probabilities from the quantum state, and the meaning of the probabilities. What I shall write below is largely independent of the meaning of the probabilities, though personally I view them in a rather Everettian way as objective measures for the set of observations with positive probabilities.

Usually it is implicitly believed that the observational probabilities depend strongly upon the quantum state. (Sometimes the Everett interpretation [2] is taken to mean that all of physical reality is determined purely by the quantum state, without the need for any additional rules to extract probabilities, but this extreme view seems untenable [4] and will not be adopted here. Instead, I shall discuss the opposite view, that the probabilities are independent of the quantum state.) However, some advocates of inflation[5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22] often claim that our observations do not depend upon the quantum state at all, but rather that inflation acts as an attractor to give the same statistical distribution of observations from any state.

In this note, I shall use the framework of state plus rules to discuss this possibility that observational probabilities might be independent of the quantum state. I shall show that this indeed is logically possible, but apparently only if the probability rules are rather *ad hoc*. If indeed the rules are this *ad hoc*, so that the probabilities of our observations do not depend upon a quantum state at all, it would seem to leave it mysterious why many of our observations can be simply interpreted as if our universe really were quantum.
1 States and rules

Let me first discuss the logical independence of the quantum state and the probability rules. I shall assume that even if one fixes the kinematic variables and the dynamical laws, there remains freedom in what the quantum state is (e.g., many different solutions to the same Schrödinger equation with the same arguments and the same Hamiltonian, or many different solutions to the constraint equations of quantum gravity). The set of all quantum states obeying whatever kinematic and dynamical constraints one might impose I shall call the state space; it might or might not be a Hilbert space. The states themselves might be pure states, density matrices, or $C^*$-algebra states, but I shall assume that they are at least $C^*$-algebra states, so that each state gives the expectation value of the kinematically allowed quantum operators. For simplicity, I shall often assume that the quantum state is a pure state in a finite-dimensional Hilbert space, though most of the discussion should be generalizable to any $C^*$-algebra states.

In traditional quantum theory, observational results are eigenvalues of a certain self-adjoint operator called an observable, which in the finite-dimensional case at least can be written as a sum of orthonormal projection operators (formed from the eigenstates of the observable, or from the eigenspaces of eigenstates for degenerate eigenvalues) multiplied by coefficients that are the eigenvalues of the observable. Then the observational probability of each eigenvalue is given by Born’s rule [23] as the expectation value of the corresponding projection operator in the quantum state of the system. In this case, the logical freedom of the probability rules is the freedom to choose the observable whose eigenvalues represent the observational results.

In the case of a pure state in a finite-dimensional Hilbert space, the state by itself does not determine the observational probabilities, since the probabilities also depend upon the orthonormal projection operators corresponding to the observable. Furthermore, any other pure state in the same Hilbert space would give the same probabilities for another observable obtained simply by transforming the original observable by the same unitary transformation used to transform the state from the original one to the final one. (This unitary transformation is not uniquely defined, since only its action on the original quantum state is specified, so there is a large set of different transformed observables that all give the same probabilities as well.) Then the probability of an eigenvalue of the new observable in the new state would be the same as that of the eigenstate of the original observable in the original state. Thus states by themselves do not determine probabilities, and all pure states give the same probabilities when they are paired with corresponding observables. It is only the relation between the state and the observable that determines unique probabilities by Born’s rule.

In cosmology with a universe large enough that there may be copies of an observer, no matter how precisely it is described locally, Born’s rule does not work [24, 25, 26, 1] and must be replaced by another set of rules for extracting observational probabilities from the quantum state. Generically then the ambiguity in the rules is even greater than in traditional quantum theory in which one needed to specify just one quantum operator, a single observable, in addition to the state.
Here for simplicity I shall focus on cases in which the set of rules are that observational probabilities are obtained by normalizing a set of unnormalized measures that are each given by the expectation value of a positive operator in the quantum state,

\[ P_j(i) = \frac{p_j(i)}{\sum_k p_k} \quad \text{with} \quad p_j(i) = \langle q_j \rangle_i, \quad (2) \]

where \( q_j \) is the positive operator corresponding to the observational result \( O_j \) (or observation \( j \), for short), and where \( \langle \ldots \rangle_i \) denotes the quantum expectation value, of whatever operator replaces the \( \ldots \) inside the angular brackets, in the quantum state \( i \) given by the theory \( T_i \). Then, instead of the single observable required to give the probability rule in traditional quantum theory by Born’s rule, one needs a whole set of positive operators \( q_j \), one for each observation \( j \).

Quantum theories of this form may be axiomatized by the following two axioms:

**State:** There is a quantum state that gives expectation values of operators.

**Rules:** Each possible observation has a corresponding positive operator whose expectation value in the quantum state is the measure for that observation.

When we want to do a Bayesian analysis and compare different theories \( T_i \) for which we have assigned prior probabilities \( P(T_i) \), we would like to normalize the measures for observations by dividing by the total measure and then interpret the normalized measures as the likelihoods or the probabilities of the observation given the theory, \( P(O_j|T_i) \). Then if we had a complete set of theories for which we assigned nonzero prior probabilities, so \( \sum_i P(T_i) = 1 \), then the posterior probability of theory \( T_i \), given the observation \( O_j \), would be given by Bayes’ formula as

\[ P(T_i|O_j) = \frac{P(T_i)P(O_j|T_i)}{\sum_l P(T_l)P(O_j|T_l)}. \quad (3) \]

Under the assumption that observations are conscious perceptions, the operators \( q_j \) whose expectation values would then give the measures of the corresponding conscious perceptions were called *awareness operators* in my previous work [27, 28, 29, 30], but in [24, 25, 26, 1] and here I am not restricting to the assumption that observations must be conscious perceptions (though I have not given up my personal belief that the most fundamental observations are indeed conscious perceptions). The only restriction on observations I am making here is that each of them should be a complete observation in the sense that no observation is a proper subset of another observation. Here, let us call the \( q_j \) *observation operators*, since it is their expectation values that give the ratios of observational probabilities.

## 2 Rules giving state-independent probabilities

Now let us consider whether we can have probability rules giving observational probabilities independent of the quantum state, as is often claimed or wished to be the case for inflationary universes. It is clear that if \( \langle q_j \rangle_i \) is to be independent of the quantum state, the observation operator \( q_j \) must be proportional to the identity
operator, $q_j = p_j(i)I$, with each nonnegative $p_j(i)$ that can be chosen arbitrarily and independently of the quantum state. Then indeed the observational probabilities are independent of the quantum state. Therefore, there is no logical difficulty in defining probability rules such that the probabilities of observations are independent of the quantum state, as is often claimed or wished to be for inflation.

On the other hand, it seems quite ad hoc to have the observation operators all be proportional to the identity, so that the observational probabilities are independent of the quantum state. If that were the case, what would be the point of having a quantum state at all in the theory? One could just say that the theory consisted of directly giving the observational probabilities $P_j(i)$ (perhaps from unnormalized probabilities $p_j(i)$ if they are intrinsically simpler). If our observations are indeed independent of the quantum state, why have our observations been taken to support quantum theory? That is, why has it been so successful to unify and simplify the description of our observations by assuming that they arose from quantum aspects of the universe, if they come from the expectation values of operators that all commute?

Therefore, although I have shown here that it is logically possible for our observations (meaning their probabilities) to have arisen from probability rules that make them independent of the quantum state, the way to do this seems highly ad hoc and implausible. Surely a much simpler explanation of our observations will use both a non-random quantum state and a non-random set of rules for extracting the probabilities of observations from that quantum state.

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