We investigate the effects of gas–disk gravity on planetesimal dynamics in inclined binary systems, where the circumprimary disk plane is tilted by a significant angle ($i_B$) with respect to the binary plane. Our focus is on the Lidov–Kozai mechanism and the evolution of planetesimal eccentricity and inclination. Using both analytical and numerical methods, we find that, on one hand, disk gravity generally narrows down the Kozai-on region, i.e., the Lidov–Kozai effect can be suppressed in certain parts of (or even the whole of) the disk depending on various parameters. In the Kozai-off region, planetesimals would move to orbits close to the midplane of the gas–disk, with the relative angle ($i$) following a small amplitude periodical oscillation. On the other hand, when we include the effects of disk gravity, we find that the Lidov–Kozai effect can operate even at arbitrarily low inclinations ($i_B$), although a lower $i_B$ leads to a smaller Kozai-on region. Furthermore, in the Kozai-on region, most planetesimals’ eccentricities can be excited to extremely high values ($\sim 1$), and such extreme high eccentricities usually accompany orbital flipping, i.e., the planetesimal orbit flips back and forth between anterograde and retrograde. Once a planetesimal reaches very high orbital eccentricity, gas drag damping will shrink the planetesimal orbit, forming a “hot planetesimal” on a near circular orbit very close to the primary star. Such a mechanism, if replacing the planetesimals and gas drag damping with Jupiters and tidal damping respectively, may lead to a frequent production of hot Jupiters.

Key words: celestial mechanics – planets and satellites: formation

Online-only material: color figures

1. INTRODUCTION

As of today, over 60 exoplanets have been found in binary star systems, and current observations show that the multiplicity rate of the detected exoplanet host stars is around 17% (Mugrauer & Neuhauser 2009; Eggenberger 2010). Planet formation in binary systems presents numerous challenges, as each stage of the planet formation process can be affected by the binary companion. A crucial stage that may be particularly sensitive to binary effects is the accumulation of 1–100 km sized planetesimals (see the review by Haghighipour 2010 and the references therein). Because of the perturbations from the binary companion, planetesimals will be excited to orbits with high relative velocities, preventing or even ceasing their growth (Heppenheimer 1978; Whitmire et al. 1998). In the past decade, with several discoveries of exoplanets in close binary of separation $\sim 20$ AU (Queloz et al. 2000; Hatzes et al. 2003; Zucker et al. 2004; Correia et al. 2008; Chauvin et al. 2011), the issue of planetesimal growth in binary systems becomes more challenging and therefore attracts many researchers as well as many dynamical and collisional studies (Marzari & Scholl 2000; Moriwaki & Nakazawa 2004; Thébault et al. 2004, 2006, 2008, 2009; Thébault 2011; Paardekooper et al. 2008; Scholl et al. 2007; Paardekooper & Leinhardt 2010; Kley & Nelson 2008; Beaugé et al. 2010; Giuppone et al. 2011; Xie & Zhou 2008, 2009; Xie et al. 2010a, 2010b).

Most of the previous studies have considered only coplanar or near-coplanar cases, where the tilted angle between the binary orbital plane and the circumprimary disk plane was close to zero, i.e., $i_B \sim 0$. In fact, the coplanar case is reasonable only if it is applied to relatively close binary systems with a separation less than $\sim 40–200$ AU (Hale 1994; Jensen et al. 2004), beyond which the distribution of $i_B$ is likely to be random and therefore the highly inclined case is more relevant. Planetesimal dynamics in highly inclined binary systems have only been investigated by Marzari et al. (2009), and most recently (at the time of writing this paper) by Xie et al. (2011), Fragner et al. (2011), and Batygin et al. (2011).

Marzari et al. (2009) found that, due to the perturbations of an inclined binary companion, planetesimals’ nodal lines became progressively randomized, raising their relative velocities to the degree that planetesimal growth by mutual collision was significantly prevented. Nevertheless, the gaseous protoplanetary disk was ignored in their study, where planetesimals were only subject to the gravity of the binary stars. In reality, the gaseous disk can generally have crucial effects on planetesimal dynamics through two factors. One is the hydrodynamic drag force, which has been investigated in detail by Xie et al. (2011). When gas drag is included, it is found that planetesimals from the outer regions (where conditions are hostile to planetesimal accretion) jump inward into an accretion-friendly region and pile up there. This is referred to as the planetesimal jumping-piling (PJP) effect, and its general result, as shown in Xie et al. (2011), is to form a severely truncated and dense planetesimal disk around the primary, providing conditions that are favorable for planetesimal growth and potentially allow for the subsequent formation of planets. Another crucial factor is the gravity of the gaseous disk, which has been studied recently by Fragner et al. (2011) with a hydrodynamical model and by Batygin et al. (2011) with an analytical model. Generally, it is found that gravity can pull the planetesimals back toward the middle plane of the gas–disk. With proper conditions, such as a massive gas disk and/or a large binary distance, the Lidov–Kozai effect could be suppressed regardless of $i_B$. However, Fragner et al. (2011) could only focus...
on several typical cases with a few planetesimals in a relatively short simulation timescale because of the large computational hours, while Batygin et al. (2011) only concentrated on cases of very wide binaries with a separation of \( \sim 1000 \) AU, aiming to only identify the important physical processes at play. In this paper, we investigate the effects of gas–disk gravity on planetesimal dynamics in inclined binary systems through both analytical and numerical fashions. Analytically, we derived the condition at which the Lidov–Kozai effect (Kozai 1962; Lidov 1962) is turned off by disk gravity. Numerically, we confirm our analytical results and provide a global quantitative view of the role of gas–disk gravity in a large parameter space. Furthermore, specific attention is given to the role of disk gravity in shaping the PJP effect found by Xie et al. (2011). The paper is outlined as follows.

In Section 2, we describe our disk model and the initial set up. In Section 3, we analyze the secular motion of a planetesimal under the gravity of the disk and stars, focusing on the evolution of planetesimal inclination and eccentricity. The analytical study is followed by numerical simulations presented in Section 4. In Section 5, we discuss some issues, including the PJP effect, implications for hot Jupiters, and disk precession. Then in Section 6 we present our summary.

2. DISK MODEL

In our model, planetesimals are assumed to be initially moving on circular orbits in the midplane\(^5\) of a gaseous disk around the central star with one solar mass (\(M_\odot\)). A companion star with mass of \(M_B\) (a free parameter) is orbiting around the central star–disk system with an orbital semimajor axis of \(a_B\) (a free parameter), inclination of \(i_B\) (a free parameter, relative to the midplane of the disk), and eccentricity of \(e_B = 0\) (constant). In this paper, for simplicity, we only consider the circular case (\(e_B = 0\)) and focus on the effect of gas gravity. For the eccentric case \(e_B \neq 0\), the Lidov–Kozai mechanism itself is more complicated (Katz et al. 2011; Lithwick & Naoz 2011), and thus this case\(^6\) is not addressed in this paper.

For the gas disk, we use a three-dimensional steady model as in Takeuchi & Lin (2002). In cylindrical coordinates \((r, z)\), the disk density profile is

\[
\rho_c(r, z) = \rho_0 f_g \left( \frac{r}{\text{AU}} \right)^\beta \exp \left( -\frac{z^2}{2 h_g^2} \right),
\]

and the gas rotation rate is

\[
\Omega_g(r, z) = \Omega_{K, \text{mid}} \left[ 1 + \frac{1}{2} \left( \frac{h_g}{r} \right)^2 \left( \beta + \gamma + \frac{\gamma z^2}{2 h_g^2} \right) \right],
\]

where \(\Omega_{K, \text{mid}}\) is the Keplerian rotation in the midplane, \(h_g(r) = h_0(r/\text{AU})^{(\gamma + 3)/2}\) is the scale height of the gas disk, \(f_g\) is a scaling number with respect to the minimum mass of solar nebulae (Hayashi 1981, hereafter MMSN), \(\rho_0 = 2.83 \times 10^{-10} \text{ g cm}^{-3}\), \(\gamma \approx -0.5\), \(h_0 = 0.33 \times 10^{-2}\), and \(\beta\) is a free parameter.

The surface density of the disk has a power-law form of \(\Sigma = \sqrt{2\pi \rho_0 f_g h_0 (r/\text{AU})^k}\), where \(k = \beta + 1.25\). Namely, in this paper, we set \(k = -1\) as the standard case. The inner and outer boundaries of the disk are set as \(r_{in} = 0.1 \text{ AU}\) and \(r_{out} = 12.5 \text{ AU}\). Their values have little effect on the final results as long as they are not very close to the planetesimals.

Our disk model is a very simple one that ignores the reaction of the gas disk to the binary perturbations. In more realistic situations, as shown in the simulations of Larwood et al. (1996) and Fragner & Nelson (2010), the disk will become eccentric, develop a warp, and precess under the perturbations of the companion star. Nevertheless, as pointed out by Fragner et al. (2011, also see our discussion in Section 5.2), planetesimals’ secular dynamical behaviors are similar both in the evolving and non-evolving disk models, and thus our choice of a steady model can be reasonable to at least a zeroth-order approximation. Furthermore, using such a simple gas disk model is much less time consuming in computing the disk gravity compared to using a hydrodynamical code, allowing us to see the effect of disk gravity on a much longer timescale. In addition, our simple gas disk model is convenient for making some analytical studies.

It also worth noting that the gaseous disk would slowly relax to the binary orbital plane on the viscous evolution timescale (Fragner & Nelson 2010). Thus the assumption of a constant and relatively large \(i_B\) in our model is only relevant if the viscous timescale, \(t_{\text{vis}} \sim r^2/(\alpha h^2 \Omega_{K, \text{mid}})\), is larger than the secular perturbation timescale, \(t_{\text{sec}} \sim 2\pi/r\). Here \(B \sim 0.75GM_B/(na_B^3)\) (see also in Equations 8 and 9) is the characteristic frequency, where \(G\) and \(n\) are the gravity constant and the local orbital mean frequency around the primary star, respectively. Equating these two timescales, the critical viscous parameter can be derived as

\[
\alpha_c \sim 5 \times 10^{-2} \left( \frac{r}{50 \text{ AU}} \right)^{5/2} \left( \frac{a_B}{50 \text{ AU}} \right)^{-3} \left( \frac{M_B}{M_\odot} \right)^{1/2}.
\]

Therefore, a highly inclined case, which is studied in this paper, is relevant only for \(\alpha < \alpha_c\). If otherwise \(\alpha > \alpha_c\), it is likely to reduce to a near-coplanar case, which has been studied in many previous works (Marzari & Scholl 2000; Thébault et al. 2006; Paardekooper et al. 2008; Xie & Zhou 2008, 2009).

3. ANALYTIC STUDY

In this section, we analytically study the secular dynamics of a planetesimal under the gravitational perturbations from both the companion star and the disk. Our interests focus on the evolution of the planetesimal’s orbital eccentricity and inclination, aiming to see how the Lidov–Kozai effect operates if disk gravity is included. For the sake of this derivation, we introduce two coordinate systems: (1) the disk coordinate, where the \(XY\)-plane is set as the disk midplane with the \(X\) direction toward the ascending node of the binary orbit and (2) the binary coordinate, where the \(XY\)-plane is set as the orbital plane of the binary star with the \(X\) direction the same as \(X\). In the disk coordinate system, angular elements are marked with a superscript \((')\). For example, \(i\) and \(\Omega\) denote the orbital inclination and longitude of the ascending node in the disk coordinate system, respectively, while \(i\) and \(\Omega\) are those in the binary coordinate system.

3.1. The Disturbing Function

The disturbing function of the star–disk–planet system can be expressed as

\[
R = R_D + R_B.
\]
where $R_D$ and $R_B$ are contributions from the gravity of disk and binary stars, respectively.

According to Nagasawa et al. (2000, see the appendix of their paper), taking the second-order approximation, $R_D$ can be expressed as

$$R_D = -\frac{na^2}{2}[Te^2 + Si'^2],$$

where $n$, $a$, $e$, and $i'$ are the orbital mean frequency, semimajor axis, eccentricity, and inclination (in the disk coordinate) of the planetesimal, respectively. $T$ and $S$ are two characteristic frequencies (see Appendix A of this paper for details of their definition and calculation) which, under the disk model assumed in Section 2, can be approximately fit by the following formulas:

$$T(a) = f_S \times 4.5 \times 10^{-4} \left(\frac{a}{\text{AU}}\right)^{k+1} \text{rad yr}^{-1},$$

$$S(a) = f_S \times 1.7 \times 10^{-2} \left(\frac{a}{\text{AU}}\right)^{k+1/4} \text{rad yr}^{-1}.$$  

Note, $T$ is actually the apsidal recession rate of a planetesimal if the planetesimal is affected only by the disk gravity in the coplanar case ($i' = 0$). Our calculation of $T$ is generally consistent with that of Batygin et al. (2011, here correspond to a disk mass of $\sim 0.02 M_\odot$ in Figure 2 of their paper) who used a similar disk model but different computing techniques. However, we emphasize that $T$ should be scaled with the local surface density as in Equation (6) rather than with the total mass of the disk (as was done in Figure 2 of Batygin et al. 2011 and Equation (30) in Fragner et al. 2011).

Following Innanen et al. (1997), the binary part of the disturbing function can be expressed as

$$R_B = \frac{na^2}{2}B[e^2 - (1 + 4e^2 - 5e^2\cos^2\omega)\sin^2 i],$$

where $i$ and $\omega$ denote the orbital inclination and pericenter (in the binary coordinate) of the planetesimal. The characteristic frequency $B$ is actually the precession rate of the planetesimal caused by the secular binary perturbation in the coplanar case ($i = 0$), and in the first order it can be expressed as

$$B \sim B_1 = \frac{3GM_B}{4na_b^2(1 - e_b^2)^{3/2}}.$$  

However, such a first-order expression can be rather inaccurate unless one uses the second-order correction ($B_2$) as suggested by Thébault et al. (2006) and Giuppone et al. (2011),

$$B \sim B_2 = B_1 \left[1 + \frac{32M_B}{M(1 - e_b^2)}\left(\frac{a}{a_b}\right)^2\right].$$

Hereafter, we adopt $B = B_2$ if there is no specific explanation.

We plot $T$, $S$, and $B$ in Figure 1 for the standard case, where the companion has a mass of $M_B = 0.5 M_\odot$, semimajor axis of $a_B = 50$ AU, and the disk surface density slope of $k = -1$. The blue dotted line, red dashed line, and black solid line indicate $T$, $S$, and $B$ as a function of the semimajor axis of the planetesimal ($a$), respectively. As can be seen, $S$ is much greater than $T$ and $B$ in the whole of the plotted region of the disk, while $T$ is greater (less) than $B$ in the inner (outer) region. We will show in the following subsections that such a picture of $T$, $B$, and $S$ determines the dynamical evolution of the planetesimal’s orbit.

3.2. Evolution of the Planetesimal Inclination

As the planetesimal is initially moving on a circular orbit in the midplane of the disk, the initials $e$ and $i'$ are approximately zero; thus we ignore quantities that are on an order higher than $o(e^2)$, $o(i'^2)$, or $o(e'i')$. The disturbing function relating to the inclination can then be reduced to

$$R_{rd} \sim \frac{na^2}{2}[-Si'^2 - B\sin^2 i].$$

Considering the relation between $i$, $i'$, and $\Omega'$ and introducing two new variables, $p = i'\sin\Omega'$ and $q = i'\cos\Omega'$, then Lagrange’s planetary equations (relating to $i'$ and $\Omega'$) can be written as (see Appendix B for a detailed derivation)

$$\frac{dp}{dt} = -B\cos 2i_B + S, \quad \frac{dq}{dt} = B \sin 2i_B,$$

where $i_B$ is the angle between the disk plane and the binary orbital plane. Note $S > B > 0$ and the initial condition $p_0 = q_0 = 0$, thus the solution of $p$ and $q$ can be written as

$$p = \frac{B \sin(2i_B)}{2f} \sin(ft), \quad q = \frac{B \sin(2i_B)}{2B \cos 2i_B + 2S}[1 - \cos(ft)],$$

where $f = \sqrt{(B\cos^2 i_B + S)(B\cos 2i_B + S)}$. The maximum value of $i'$ (note that $i' = \sqrt{p^2 + q^2}$) is

$$i'_{\text{max}} = \frac{B \sin(2i_B)}{B \cos 2i_B + S}.$$  

As $S \gg B$ shown in Figure 1, thus $f \sim S$ and $i'_{\text{max}}$ is as small as on an order of $o(B/S)$. It means that the planetesimal will keep its orbital plane close to the disk midplane, having the relative tilted angle $i'$ oscillating with a frequency of $f \sim S$ and
an amplitude of \( \sim B/S \). Such an analytical result is consistent with the hydrodynamical simulation performed by Fragner et al. (2011), which has shown that disk gravity would try to pull the planetesimal orbit back to the disk midplane, maintaining a small relative angle (see Figures 3 and 10 in their paper).

Recalling the approximation (quantities that are \( O(e^2), O(i^2) \) or \( O(ei) \) or higher are ignored) adopted before our derivation, we thus emphasize that our analytical results about the evolution of planetesimal inclination remain valid only if the planetesimal eccentricity is not excited or remains at a low value. Such an assumption, however, will break down if the Lidov–Kozai effect kicks in. In the following subsection, we will address this issue, deriving the conditions in which the Lidov–Kozai effect takes over and planetesimal eccentricity is excited.

3.3. Evolution of the Planetesimal Eccentricity

Following Innanen et al. (1997), the Lagrange planetary equations describing the evolution of the planetesimal’s orbital eccentricity (\( e \)) and pericenter (\( \omega \)) can be written as

\[
\frac{de}{dt} \sim \frac{SB \cos^2 i_B}{B \cos 2i_B + S}.
\]

(14)

\[
\frac{d\omega}{dt} \sim B(2 - 5\sin^2\omega \sin^2 i_B) + D.
\]

(15)

Compared to Equation (5) in the paper of Innanen et al. (1997), here we add the term of contribution from the disk (\( D \)), ignoring quantities that are on an order of \( O(e^2) \) or higher because of the initial circular planetesimal orbit, and take \( i \sim i_B \) because \( i \) is very small before \( e \) is excited according to Equation (13). The disk contribution term (\( D \)) can be written as (see Appendix C for the detail of derivation)

\[
D \sim -T - \frac{SB \cos^2 i_B}{B \cos 2i_B + S}.
\]

(16)

Note,\(^7\) as \( i = 0 \) is a singular point in the Lagrange planetary equation, thus Equations (15) and (16) cannot be applied to the case of \( i_B = 0 \).

For the Lidov–Kozai effect to kick in, we expect \( d\omega/dt \approx 0 \). Using this condition to eliminate the variable \( \omega \) in Equations (15) and (16), we then have

\[
\frac{de}{dt} \sim 5eB \sqrt{\frac{2B + D}{5B}} \left( \sin^2 i_B - \frac{2B + D}{5B} \right).
\]

(17)

In order to increase \( e \), we need \( de/dt > 0 \), which then leads to

\[
0 < 2 + D/B < 5\sin^2 i_B.
\]

(18)

This is the condition for the Lidov–Kozai effect to operate under the gravity from both binary stars and the disk. For a disk-free case, i.e., \( D = 0 \), Equation (18) is reduced to the classical one, i.e., \( i_B > \arcsin(\sqrt{2}/5) \approx 39.2^\circ \).

Equation (18) is equivalent to \( a_{11} < a < a_{22} \), where \( a_{11} \) and \( a_{22} \) can be derived from \( 2 + D/B = 0 \) and \( 2 + D/B = 5\sin^2 i_B \). If the disk is not very tenuous, such as \( f_g > 0.1 \), then \( S \gg B \) holds and thus Equation (16) can be reduced to

\[
D \sim -T - B \cos^2 i_B.
\]

In such a case, we can solve \( a_{11} \) and \( a_{22} \) analytically if \( k = -1,8\)

\[
a_{11} \sim 4.17 \text{ AU} \left[ \frac{1}{f_g M_B \left( \sin^2 i_B + 1 \right)} \right]^{-2/3} \left( \frac{a_B}{50 \text{ AU}} \right)^2,
\]

(19)

\[
a_{22} \sim 4.17 \text{ AU} \left[ \frac{1}{f_g M_B \left( 1 - 4\sin^2 i_B \right)} \right]^{-2/3} \times \left( \frac{a_B}{50 \text{ AU}} \right)^2, \quad \text{if } i_B < 30^\circ, \sim \infty \quad \text{if } i_B \geq 30^\circ.
\]

(20)

Compared to the classical disk-free case, where the Lidov–Kozai effect takes place only if \( i_B > 39.2^\circ \), here the Lidov–Kozai effect (or eccentricity excitation) can occur for an arbitrary \( i_B \), and the value of \( i_B \) only determines the disk range \((a_{11} < a < a_{22})\) that is subject to the Lidov–Kozai effect.

4. NUMERICAL STUDY

In this section, we perform numerical simulations to test our analytical results presented in Section 3. Planetesimals are only subject to gravity from the binary stars and the disk.\(^9\) We calculate the disk’s gravity at lattice points in the \( r-z \) plane before orbital integrations and obtained the gravitational force at an arbitrary point by bicubic interpolation (see Appendix D for a detail description about computing disk gravity). The equations of planetesimal motion are integrated using a fourth-order Hermite method (Kokubo et al. 1998).

4.1. Examples

As a first example (hereafter referred to as the standard case), we assume that \( M_B = 0.5 M_\odot, a_B = 50 \text{ AU}, i_B = 50^\circ \) for the binary and \( f_g = 1, k = -1 \) for the disk. The results of this case are plotted in Figures 2 and 3.

In Figure 2, we plot the maximum orbital eccentricity \((\epsilon_{\text{max}})\) and inclination \((i_{\text{max}})\) in disk coordinates that the planetesimal achieved during its evolution as a function of its orbital semimajor axis. As can be seen from Figure 2, in the inner region planetesimal eccentricities are not excited, and they remain at very low inclinations with \( i_{\text{max}} \) fitting well with our analytical result (Equation (13)). In the outer region, the Lidov–Kozai effect is switched on, and thus leads to large planetesimal eccentricities \((\epsilon_{\text{max}} \sim 1)\) and inclinations \((i_{\text{max}} \sim 90^\circ)\). The boundary that separates the inner Kozai-off region and the outer Kozai-on region is roughly consistent with the analytical estimate (Equation (19)). In addition, we also plot the results of the disk-free case as shown in the red dashed line in Figure 2. Comparing the two cases with and without the disk, we see that \( \epsilon_{\text{max}} \) is much larger (close to unity) in the former case.

In Figure 3, for a specific planetesimal with semimajor axis of 6.5 AU where the Lidov–Kozai effect should be switched on according to Figure 2, we plot the temporal evolution of its orbital eccentricity \( e \), inclination \( i \), longitude of periastron \( \Omega \), and ascending node \( \Omega \) for the two cases with and without the disk. Note, here all the angular elements plotted in Figure 3

---

\(^7\) Setting \( i_B = 0 \) in Equations (15) and (16), the binary and disk’s contributions to \( d\omega/dt \) are \( 2B \) and \(-B-T\), respectively, which are obviously wrong though their sum \((B-T)\) is correct.

\(^8\) In order to analytically derive \( a_{11} \) and \( a_{22} \) we assume \( B = (1 + \eta)B_1 \) and \( \eta = 0.4 \) is used for deriving Equation (19). Note that Equations (19) and (20) cannot be applied to the disk-free case by just setting \( f_g = 0 \) because we have presupposed that \( f_g > 0.1 \). And for the case of \( k \neq -1, a_{11} \) and \( a_{22} \) should be solved numerically from Equation (18).

\(^9\) In fact, planetesimals are also subject to the hydrodynamical drag force from the gas disk. See Section 5.1 for a discussion of gas drag or see the paper of Xie et al. (2011) for a detailed study of the effects of gas drag.
Figure 2. Planetesimal’s maximum eccentricities (panel (a)) and inclinations (panel (b)) as a function of its initial semimajor axis in the standard case. The red dashed line indicates the results without including disk gravity. In panel (a), the vertical black dash-dotted line indicates the analytical boundary of the Kozai effect (Equation (19)). In panel (b), the black dashed line shows the analytical result from Equation (13).

(A color version of this figure is available in the online journal.)

Figure 3. Orbital evolution of a planetesimal with the semimajor axis at 6.5 AU. All the orbital elements are in the binary coordinate. The two left panels are results of the standard case, while the two right panels are results for the case with the same binary configuration but without including disk gravity. (Note the different scales for the eccentricity and inclination scales in the two plots.)

(A color version of this figure is available in the online journal.)

Figure 4. Similar to Figure 2 but with $i_B = 20^\circ$.

(A color version of this figure is available in the online journal.)

mechanism (Lithwick & Naoz 2011). While in the present paper we assume a zero eccentricity of the outer perturbing body ($e_B = 0$), the orbital flip observed in Figure 3 should be due to the effect of disk gravity.

As a second example (hereafter referred to as the low inclination case), we only change the binary orbital inclination to $i_B = 20^\circ$ and keep all the other parameters the same as in the standard case. The result of this low inclination case is plotted in Figure 4. In contrast to the $i_B = 50^\circ$ case, here the Lidov–Kozai effect can only take place within the region $a_{c1} < a < a_{c2}$. This is consistent with our analytical results in Equations (19) and (20).

4.2. Parameter Exploration

In this subsection, we extend the standard case above by numerically investigating the effects of other parameters, including $i_B$, $a_B$, $M_B$, $f_\ast$, and $k$. We adopt the following strategy: to investigate the effect of a given parameter, we set this parameter as the only free one and fix all the other parameters to be the same as in the standard case. The results are then plotted in Figures 5
and 6. The former shows the radial distribution of planetesimal’s maximum eccentricity and its dependency on \(i_B\) (panel (a)), \(M_B\) (panel (b)), \(a_B\) (panel (c)), and \(f_e\) (panel (d)). Red regions mean the planetesimals’ eccentricities \(\sim 0\), namely the Lidov–Kozai effect is switched off. The black dashed lines indicate the analytical boundaries of the Lidov–Kozai effect described by Equation (18). For all four panels, \(k = 1\). (A color version of this figure is available in the online journal.)

As shown in Figure 5, (1) the Lidov–Kozai effect can be switched on even with an \(i_B\) as small as \(\sim 5^\circ\), although the width of the Kozai-on region decreases as \(i_B\) decreases. (2) The Lidov–Kozai effect can be suppressed over a larger region if either the mass of the companion star decreases or the separation of the companion and/or the density of the disk increases. (3) The analytical results (dashed and solid lines; see also in Equations (19) and (20)) approximate the numerical results in the inner region with \(a < 9–10\) AU. Beyond this, in the region close to the disk outer boundary and the orbital stability boundary, the deviation is large, indicating that our analytical approximation is not valid there. (4) The boundaries that separate the Kozai-on and Kozai-off regions are very steep; most planetesimal eccentricities are either very high (close to 1) or very low (close to 0) with planetesimals of moderate eccentricities being very rare. The effect of the disk density slope \(k\) can be seen from Figure 6, which shows that (5) the Kozai-off region extends outward more and more as \(k\) increases, i.e., the disk radial density profile becomes more flat. In the case of \(k = -1/2\), the Lidov–Kozai effect turns off in the whole disk.

5. DISCUSSION

5.1. Planetesimal Jumping and Pile-up

In the early stage, there must be a gas disk around the primary star. The gas disk has crucial effects on the dynamics of planetesimals through two factors. One is gravity, which was studied in detail in previous sections of this paper. The other one is the hydrodynamic drag force, whose role has been investigated in detail by Xie et al. (2011). In general, Xie et al. (2011) find that if planetesimals are excited to orbits with very high inclinations (relative to the disk plane) and eccentricities, they will be subjected to very strong hydrodynamic drag forces from the gas disk, letting them jump inward and pile up, i.e., the so-called PJP effect. Nevertheless, the disk gravity is not included by Xie et al. (2011). In the following, we show how the PJP effect is modified if both the gas drag and disk gravity are included.

We consider four cases, (a) the standard case as described in Section 4.1, (b) a more compact case—similar to the standard case but with \(a_B\) decreasing to 40 AU—(c) a low inclination case—similar to the standard case but with \(i_B\) decreasing to 20°—and (d) a disk gravity free case—similar to the standard case but the disk gravity is not included. In each case, gas drag force is calculated by assuming a single planetesimal radial size of 5 km and following the procedure as described in Section 2.2 of Xie et al. (2011). The results are plotted in Figures 7 and 8.

In Figure 7, we plot the evolution of the orbital semimajor axis (also periastron, top panel) and the eccentricity (bottom panel) of three planetesimals in case (a), i.e., the standard case. Three planetesimals with near circular orbits start from 5, 6, and 7 AU. Beyond \(a_c = 5.4\) AU, where the Lidov–Kozai effect is switched on, the two planetesimals’ eccentricities are excited and thus they suffer significant gas drag force, leading to rapid inward migration. The one starting from 7 AU is excited to extremely high eccentricity and directly jumps to the innermost orbit, and the one starting from 6 AU with modest eccentricity quickly migrates to 2–3 AU from the central star. On the other side, for \(a_c < 5.4\) AU, the Lidov–Kozai effect is suppressed, and thus the planetesimal starting from 5 AU does not suffer eccentricity excitation and hence does not migrate.

In Figure 8, we plot the evolution of the local surface density enhancement (\(\Sigma/\Sigma_0\)) of the planetesimal disk for the four cases. In each case, 5000 planetesimals treated as test particle tracers are initially distributed uniformly from 0.2 AU to 10 AU in the midplane of the gas disk with circular
orbits and thus the initial profile follows a power law with a semimajor axis dependence equal to $-1$. By tracing the radial distribution of those planetesimals, we can calculate the local surface density enhancement ($\Sigma/\Sigma_0$) of the planetesimal disk. The results are shown in Figure 8 and can be summarized as follows.

1. As can be seen in the top panel (case (a)), planetesimals in the outer Kozai-on region migrate into the inner Kozai-off region (in fact, most are "jumping" as shown in Figure 7 and pile up there, leading to surface density enhancement in the innermost region ($a < 0.2$ AU)). This pile-up effect increases when the binary separation, $a_B$, decreases (case (b)), because the outer Kozai-on region is larger and thus more planetesimals can move in and pile up. Conversely, when we reduce $i_B$ (case (c)), the pile-up effect is reduced, and the pile-up region shifts outward to 1–2 AU, because the outer Kozai-on region shrinks and less particles are excited to eccentricity close to 1 (see Figure 4).

2. If the disk gravity is not included (case (d)), then the situation reduces to the situation considered in Xie et al. (2011). In such a case, the Lidov–Kozai effect can only be suppressed by the gas damping and this only in the very inner region within 1–2 AU, where gas density is high. Beyond 1–2 AU, planetesimals experience the Lidov–Kozai effect and most (if not all) will migrate inward and pile up within $\sim 0.2–1.0$ AU, leading to an average local density enhancement of $\Sigma/\Sigma_0 \sim 10$ (see also in Figure 9 of Xie et al. 2011). However, we note that there are far fewer planetesimals piling up within the region $<0.2$ AU in case (d) than in case (a). The reason is that the planetesimal eccentricity (see Figure 3) in case (d) is not high enough to let planetesimals directly jump into the innermost region $<0.2$ AU.

In a word, the role of the disk gravity playing in the PJP effect can be summarized as the following. On one hand, disk gravity reduces the average PJP effect because it reduces the Kozai-on region in the outer disk. However, on the other hand, the disk gravity significantly enhances the PJP effect in the innermost region ($<0.2$ AU) as it increases the orbital eccentricities of planetesimals in the Kozai-on region to values close to 1.

5.2. Effects of Planetesimal Collisions

In this paper, planetesimals are treated as test particles and their mutual collisions are ignored. As planetesimals jump inward, their orbital eccentricities are very high and thus they are potentially subject to collisions of very high relative velocities, which can entirely disrupt themselves. To figure out how relevant the collisions could be, we estimate the collisional timescale ($t_{col}$) first. Following Xie et al. (2010b), $t_{col}$ in an inclined binary system can be estimated as

$$t_{col} \approx \frac{4}{3} \times 10^4 f_g^{-1} f_{ice}^{-1} \left( \frac{f_B}{f_0} \right) \left( \frac{a}{\text{AU}} \right)^3 \times \left( \frac{M_0}{M_\odot} \right)^{-1/2} \left( \frac{R_p}{\text{km}} \right) \text{yr},$$

where $f_{ice}$ is a solid density enhancement beyond the ice line and $R_p$ is the planetesimal radii. Taking the typical parameters, i.e., $f_g = 1.0$, $f_{ice} = 4.2$, $f_B = 50^\circ$, $M_0 = M_\odot$, and $R_p = 5 \text{ km}$, gives us $t_{col} \sim 8 \times 10^5 (a/\text{AU})^3$ yr. Because planetesimals typically complete their jumps in a timescale of $10^4–10^5$ yr, as shown in Figures 7 and 8, we thus conclude that collisions have little effect before or under the process of planetesimal jumping, but they do play an important role after planetesimal jumping.

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10 Combine Equations (3), (6), and (7) in Xie et al. (2010b).
inside 1 AU. Actually, this expectation is confirmed by the simulations in Xie et al. (2011). As shown in Figure 13 of their paper, in the first 10 years, the collisional velocity is very high but the collisional frequency is rather low. Afterward, collisions become more frequent as more planetesimals pile up in the inner region. At the same time, planetesimals are damped to near coplanar and circular orbits, leading to a friendly condition for subsequent planetesimal growth by mutual collisions.

5.3. Implication for the Formation of Hot Jupiters

The Lidov–Kozai effect induced by a companion star in a binary system has been suggested as an important mechanism for the formation of hot Jupiters (Wu & Murray 2003; Fabrycky & Tremaine 2007). If a planet’s eccentricity is high enough that its periastron is very close to the star (say <0.1 AU) during the Kozai cycle, then tidal dissipation can kick in, which may circularize and shrink the planet’s orbit, finally letting it become a hot planet. However, to induce such a high eccentricity through the classical Lidov–Kozai effect, it needs an extremely misaligned configuration11 (say \(i_B > 85^\circ\), according to Wu & Murray 2003) that is not common and thus lowers the chance of forming a hot Jupiter. As estimated by Wu et al. (2007), such a “stellar Kozai” mechanism can only produce 10% hot Jupiters.

Nevertheless, the situation will be different if disk gravity is included in the Lidov–Kozai effect. In such a case, almost in the whole Kozai-on region the eccentricity can be excited to an arbitrarily high value even with very low initial binary inclination \(i_B\), which produces many more “hot planetesimals” \((a < 0.2 \text{ AU})\) as shown in Figure 8. Similarly, if our model and results can be applied to a Jupiter-like planet12 (by replacing the gas drag damping with tidal damping in Figure 8), it should also produce many more hot Jupiters. The key issue is to what degree the production rate of hot Jupiter can increase via the above “modified stellar Kozai” mechanism. We will address this in detail in a forthcoming paper.

5.4. Disk Precession

In this paper, we assume that the gas disk is non-evolving and axisymmetric, which is apparently a crude approximation. In fact, the gas disk (if it is not entirely disrupted) should undergo a near rigid body precession (Larwood et al. 1996; Fragner & Nelson 2010), and the precession rate can be estimated as

\[
\dot{\Omega}_d = -\left(\frac{3GM_B}{4a_B^3}\cos i_B - \frac{\int \Sigma g r^3 \, dr}{\int \Sigma k \Omega_k \mu m^3 \, dr}\right).
\]

For the standard case considered in this paper, Equation (22) gives \(\dot{\Omega}_d \sim \frac{-2.6 \times 10^{-4}}{a_B^3} \text{ rad yr}^{-1}\). Adding a rigid precession to the gas disk, we re-run the simulations shown in the top left panel of Figure 5 and plot the results in Figure 9. The two black solid curves in Figure 9 are two critical semimajor axes \(a_{c1}\) and \(a_{c2}\) derived from Equation (18) (not from Equations (19) and (20)) by assuming

\[
D \approx T + B \cos^2 i_B + \cos i_B \dot{\Omega}_d.
\]

Although Equation (23) is a very crude approximation, it produces reasonable \(a_{c1}\) and \(a_{c2}\) that fit the numerical results shown in Figure 9. Furthermore, both the cases with and without disk precession (comparing the two panels of Figure 9) produce some similar features, such as (1) in the central regions of the disk, the Lidov–Kozai effect can be switched on at very low inclinations and (2) once the Lidov–Kozai effect is switched on, the planetesimal eccentricities can be much higher (most are close to 1) than those in the case without disk gravity.

5.5. Comparison to the Hydrodynamical Results

In order to further examine the validity of our disk model, we compare the results of our model to the hydrodynamical results given by Fragner et al. (2011). We adopt the same initial set up as in the simulations shown in Figure 10 of Fragner et al. (2011) and run the simulation with our model and numerical method described in Appendix D. The comparison results are plotted in Figure 10. As can be seen, the results computed by our model are generally consistent with the hydrodynamical results of Fragner et al. (2011). Given such a comparison, we feel confident of the results shown in other places of this paper.

6. SUMMARY

In this paper, we investigated the effects of gas–disk gravity on planetesimal dynamics in inclined binary systems using both analytical and numerical methods. Our major conclusions are summarized as the following.

Analytically, we derive that the planetesimal inclination follows a small amplitude oscillation around the midplane of the disk (see Equations (12) and (13)) if the Lidov–Kozai effect is suppressed and thus planetesimal eccentricity is not excited. Furthermore, we derive the threshold condition (see Equations (18)–(20)) in which the Lidov–Kozai effect switches on. We find that the Lidov–Kozai effect can operate at very low inclinations if the disk gravity is considered, although the radial extent of the Kozai-on region is much smaller.

Numerically, we confirm our analytical results over a very large parameter space by considering the variation of \(i_B, a_B, M_B,\) and \(f_s\). We find that the disk gravity narrows down the Kozai-on region, but at the same time significantly increases the maximum eccentricity (close to 1) of planetesimals in the Kozai-on region (see Figure 2). Such high planetesimal eccentricities usually accompany orbital flipping (see Figure 3), i.e., planetesimal orbits flip back and forth between prograde to retrograde.

Applying the effects of disk gravity to the PJP process, we find that, on average over the disk, disk gravity reduces the PJP effect. However, the PJP effect is significantly enhanced in
Figure 10. Comparison between our results (left three panels) and those in Figure 10 of Fragner et al. (2011; right three panels). From top to bottom, they are evolutions of planetesimals’ eccentricities, orbital inclinations (relative to the binary orbital plane), and nodal precession, respectively. The line color indicates the planetesimal’s initial semimajor axis. The initial setup including the configuration of the binary and disk is adopted from model 3 of Fragner et al. (2011, see Section 3.2 and Table 2 of their paper).

(A color version of this figure is available in the online journal.)

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APPENDIX A
THE DISTURBING FUNCTION OF THE DISK

According to Nagasawa et al. (2000), taken to the second order in $e$ and $i'$, the disturbing function caused by the disk can be expressed as

$$ R_D = -\frac{na^2}{2} \left[ T(a)e^2 + S(a)i'^2 \right], \quad (A1) $$

$T(a)$ and $S(a)$ are given using an integral of cylindrical coordinates ($r', \phi', z'$):

$$ T(a) = \frac{1}{2n} \int_{r_{in}}^{r_{out}} \int_{-\infty}^{\infty} \int_{0}^{2\pi} \left[ \frac{3 - 2r' \cos \phi'/a}{\Delta^3} \right] \frac{3(a - r' \cos \phi')^2}{\Delta^5} G\rho_g(r', z') r'dr'd\phi'dz', $$

$$ S(a) = \frac{1}{2n} \int_{r_{in}}^{r_{out}} \int_{-\infty}^{\infty} \int_{0}^{2\pi} \left( \frac{r' \cos \phi'/a}{\Delta^3} - \frac{3z'^2}{\Delta^5} \right) \times G\rho_g(r', z') r'dr'd\phi'dz', \quad (A2) $$

where $\Delta = (a^2 + r'^2 + z'^2 - 2ar' \cos \phi')^{1/2}$. 

the innermost region within 0.2 AU (see Figure 8). In addition, given the extremely high eccentricity under the effects of disk gravity, we believe that the production rate of hot Jupiters via the “stellar Kozai” mechanism could be increased.
APPENDIX B

INCLINATION EVOLUTION EQUATION

Ignoring $i^2$ and higher order terms in the disturbing function $R$, the perturbation function relating to inclination has the form

$$ R = \frac{na^2}{2}[-(S(a)i^2 - B(a)\sin^2 i)], \quad (B1) $$

where $i$ is the inclination in the binary coordinate and $i'$ is that in the disk coordinate. In the binary coordinate system, the $xy$-plane is the binary’s orbital plane, and the $x$-axis is the ascending node of the companion with respect to the disk. In the coordinate system of the disk, the $x$-axis is the same as that of the binary coordinate system, and the $xy$-plane is the midplane of the disk.

According to the geometrical relationship between the two coordinates, we have

$$ \begin{pmatrix} \sin i \sin \Omega \\ \cos i \sin \Omega \\ \cos i \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos i_B & \sin i_B \\ 0 & -\sin i_B & \cos i_B \end{pmatrix} \begin{pmatrix} \sin i' \sin \Omega' \\ \cos i' \sin \Omega' \\ \cos i' \end{pmatrix}. \quad (B2) $$

It is easy to obtain

$$ \begin{pmatrix} \sin i \sin \Omega \\ \sin i \cos \Omega \\ \cos i \end{pmatrix} = \begin{pmatrix} \sin i' \sin \Omega' \\ \sin i' \cos \Omega' \cos i_B - \cos i' \sin i_B. \\ \sin i' \cos \Omega' \sin i_B + \cos i' \cos i_B \end{pmatrix}. \quad (B3) $$

Then

$$ \sin^2 i = 1 - \sin^2 i' \cos^2 \Omega' \sin^2 i_B - \cos^2 i' \cos^2 i_B $$

$$ - 2 \sin i' \cos \Omega' \sin i_B \cos i' \sin i_B $$

$$ = \sin^2 i_B + \sin^2 i' \cos^2 i_B - \sin^2 i' \cos \Omega' \sin^2 i_B $$

$$ - \sin i' \cos \Omega' \sin i_B $$

$$ = \sin^2 i_B + \sin^2 i' \cos^2 \Omega' \sin^2 i_B + \sin^2 i' \cos^2 \Omega' \cos 2i_B $$

$$ - \sin i' \cos \Omega' \sin i_B $$

$$ = i^2 \sin^2 \Omega' \cos^2 i_B + i^2 \cos^2 \Omega' \cos 2i_B $$

$$ - \sin^2 i' \sin \Omega' \sin i_B $$

$$ = (2 \sin^2 i' \cos i - 2 \sin i_B \sin i' \cos \Omega' \sin i_B)/\sin i. \quad (B4) $$

If we ignore $i^3$ and higher-order terms, the relationship becomes

$$ \sin^2 i = p^2 \cos^2 i_B + q^2 \cos 2i_B - q \sin(2i_B) + \sin^2 i_B, \quad (B5) $$

where $p = i' \sin \Omega'$ and $q = i' \cos \Omega'$. Thus the perturbation function becomes

$$ R = \frac{na^2}{2}[-(B \cos^2 i_B + S)p^2 - (S + B \cos 2i_B)q^2 $$

$$ + B \sin(2i_B)q - B \sin^2 i_B]. \quad (B6) $$

Using Lagrange’s equations of motion, the evolution of the inclination is given by

$$ \frac{dp}{dt} = -(B \cos 2i_B + S)q + \frac{B}{2} \sin(2i_B), $$

$$ \frac{dq}{dt} = (B \cos^2 i_B + S)p. \quad (B7) $$

APPENDIX C

THE CONTRIBUTION OF THE DISK TO THE PERIASTRON PRECESSION

The disturbing function of the disk has the form

$$ R_D = -\frac{na^2}{2} [T(a)e^2 + S(a)i^2]. \quad (C1) $$

Using Lagrange’s equations of motion and ignoring the $i^2$ term, we can give the expression for the evolution of $\omega$ caused by the disk

$$ \left( \frac{d\omega}{dt} \right)_{\text{disk}} = -T + \frac{1}{2} S \cot i (\partial i^2/\partial i) \equiv D, \quad (C2) $$

where $i$ and $i'$ are the inclinations in the companion coordinate system and disk coordinate system. Proceeding with the same method as used in Appendix A, we have

$$ \sin^2 i' = \sin^2 i \sin^2 \Omega \cos^2 i_B + \sin^2 i \cos^2 \Omega \cos 2i_B $$

$$ + \sin i \cos i \cos \Omega \sin 2i_B + \sin^2 i_B, \quad (C3) $$

then we can obtain that

$$ \partial(\sin^2 i'/\partial i) = \sin 2i \sin^2 \Omega \cos^2 i_B + \sin 2i \cos^2 \Omega \cos 2i_B $$

$$ + \sin 2i \cos \Omega \sin 2i_B $$

$$ = 2 \cot i \sin^2 i \sin^2 \Omega \cos^2 i_B $$

$$ + \sin^2 i \cos^2 \Omega \cos 2i_B + \sin i \cos i \cos \Omega \sin 2i_B $$

$$ + \sin^2 i_B - \cos \Omega \sin 2i_B - 2 \cot i \sin^2 i_B $$

$$ = (2 \sin^2 i' \cos i - 2 \sin i_B \sin i' \cos \Omega' \sin i_B)/\sin i. \quad (C4) $$

Because initially $i' = 0$, we ignore $o(i^2)$ and have

$$ \partial(i^2)/\partial i = -2i' \cos \Omega' = -2q. \quad (C5) $$

We have obtained previously that

$$ q = i' \cos \Omega' = \frac{B \sin(2i_B)}{2B \cos 2i_B + 2S} [1 - \cos(f t)]. \quad (C6) $$

For the case $S \gg B$, the timescale of the evolution of $i'$ and $\Omega'$ is much shorter than the Kozai timescale. Thus we replace $q$ with its average value:

$$ \langle q \rangle = \frac{B \sin(2i_B)}{2B \cos 2i_B + 2S} = \frac{i_{\text{max}}^2}{2}. \quad (C7) $$

and $D$ becomes

$$ D = -T - \frac{1}{2} S \sin(2i_B) \cot i = -T - \frac{S B \cos^2 i_B}{(B \cos 2i_B + S)}. \quad (C8) $$

APPENDIX D

GRAVITATIONAL FORCE OF THE DISK

According to Nagasawa et al. (2000), the potential of the disk at $(r, \phi, z)$ is

$$ V = G \int_{r_{\text{in}}}^{r_{\text{out}}} \int_{-\infty}^{\infty} \int_{0}^{2\pi} \rho(r', z') r' d\phi' d'z' dr' \left( \frac{r^2 + r'^2 - 2rr' \cos \phi + (z - z')^2 + \epsilon^2}{r^2 + r'^2 - 2rr' \cos \phi + (z - z')^2 + \epsilon^2} \right)^{1/2}. \quad (D1) $$

where $r_{\text{in}}$ and $r_{\text{out}}$ are the inner and outer edge of the disk, respectively, and $\epsilon$ is a softening parameter used to avoid a singularity. For reasons of efficiency and precision, we set it to be $1 \times 10^{-7}$. The derivative of the potential with respect to $r$ or $z$ yields the $r$ or $z$ component of the disk’s gravity.

$$ F_r = G \int_{r_{\text{in}}}^{r_{\text{out}}} \int_{-\infty}^{\infty} \int_{0}^{2\pi} \rho(r', z') (r - r' \cos \phi') r' d\phi' d'z' dr' \left( \frac{r^2 + r'^2 - 2rr' \cos \phi + (z - z')^2 + \epsilon^2}{r^2 + r'^2 - 2rr' \cos \phi + (z - z')^2 + \epsilon^2} \right)^{1/2}. \quad (D2) $$

$$ F_z = G \int_{r_{\text{in}}}^{r_{\text{out}}} \int_{-\infty}^{\infty} \int_{0}^{2\pi} \rho(r', z') (z - z' \cos \phi') r' d\phi' d'z' dr' \left( \frac{r^2 + r'^2 - 2rr' \cos \phi + (z - z')^2 + \epsilon^2}{r^2 + r'^2 - 2rr' \cos \phi + (z - z')^2 + \epsilon^2} \right)^{1/2}. \quad (D2) $$
We numerically integrated Equation (D2) using closed Newton–Cotes formulas with Bode’s rule (Press et al. 1992). Since the integration costs too much CPU time, we cannot do it for each orbital integration step. Instead, we calculated the disk’s gravity at lattice points in the $r$–$z$ plane before starting the orbital integrations and obtained the gravitational force at arbitrary points during the orbital integration by performing bicubic interpolations (Press et al. 1992) using the value at lattice points.

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