Multiplicity-dependent saturation momentum in $p$-Pb collisions at 5.02 TeV

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Semi-inclusive transverse momentum spectra observed in proton-proton and proton-lead nuclear collisions at LHC energies obey a geometric scaling with a scaling variable using multiplicity-dependent saturation momentum. The saturation momentum extracted from the experimental data is proportional to the 1/6 power of the hadron multiplicity in the final state. On the other hand, the system’s transverse size is proportional to the 1/3 power of the multiplicity, and the saturation momentum and the transverse size of the system are strongly correlated with the hadron multiplicity in the final state. Since the saturation momentum is proportional to the average transverse momentum of hadrons, one predicts average transverse momentum is also proportional to the 1/6 power of the multiplicity, which is consistent with experimental results at the LHC energy. We found that a nuclear modification factor $R_{p\text{Pb}}$ calculated by the multiplicity-dependent saturation momentum decreases at $p_T \lesssim 1$ GeV/$c$ and that our model can partially explain the $R_{p\text{Pb}}$’s behavior thought to be caused by nuclear shadowing. On the other hand, Cronin enhancement experimentally observed at $2 \lesssim p_T \lesssim 6$ GeV/$c$ is not reproduced. However, the experimental result, including the Cronin effect, can be reproduced well by introducing $p_T$ dependence as a 4~5% correction to the multiplicity-dependent saturation momentum.

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I. INTRODUCTION

The gluon saturation picture [1–4] has provided us with many hints for a unified understanding of multi-particle production in which strong interactions play a significant role. For example, in a Color Glass Condensate (CGC) model [5–7], which is an effective theory to describe saturated gluons with small $x$, the saturation scale [7, 8] separates the classical gluon field into fast frozen color sources and slow dynamical color fields. The existence of the intrinsic scale of the transverse momentum $Q_s(x)$ is a crucial underlying assumption of the effective theory. Furthermore, by replacing $p_T$ in a Bjorken $x$ of the saturation scale $Q_s(x)$ with some constant characteristic one, we introduce the energy-dependent saturated momentum $Q_{sat}(W)$, which depends only on the collision energy $W$. Then, it is a unique scale that governs $p_T$ spectra of the produced particles, and as a result, geometric scaling [9–11] (GS) is emerges. The authors of Ref. [12–16] confirmed GS certainly holds for inclusive $p_T$ spectra of high-energy $pp$, $pA$, and $AA$ collisions.

In our previous work [17, 18], we confirmed that GS also holds even for semi-inclusive distributions. In these cases, we introduced a saturation momentum $Q_{sat}(W^*)$ that depends on the effective energy $W^*$, which has a one-to-one correspondence with the observed multiplicity in the final state instead of the initial colliding energy $W$. This paper will discuss based on a perspective that the physics of gluon saturation is a fundamental property and should serve as a comprehensive explanation of multi-particle production at different reaction types, energies, and multiplicities.

Recently, a collective motion thought to be characteristic of the hadronic matter produced by the collisions of large systems such as $AA$ has been found in high-multiplicity events by small systems such as $pp$ and $pA$ collisions [19–20]. Therefore, another hint to the unified understanding of multi-particle production in any type of reaction would be seen the similarity in high multiplicity events of $pp$ and $pA$ collisions. The multiplicity dependence on the mean transverse momentum in $pp$, $pA$, and $AA$ collisions is impressive because their dependence is significantly different for each reaction type, especially at high multiplicity [21]. In particular, theoretical studies need to explain a result that the multiplicity dependence on the mean transverse momentum of $p$-$Pb$ collisions is weaker than that of $pp$ for $dn/dy \gtrsim 20$.

The so-called cold nuclear matter effects observed in $pA$ collisions has been investigated by experiments at RHIC [22, 24] and LHC energies [25, 27], and theoretical explanations have been added to those results [28–30]. An important observable, nuclear modification factor $R_{pA}$, is defined by a ratio of the $p_T$ spectrum of $pA$ collisions to that of $pp$ collisions, with particular attention paid to the increase in the yield of $p$-$Pb$ collisions known as the Cronin effect [31, 32]. One considers the deviation of the value of $R_{pA}$ from 1 as the nuclear matter effects on particle production, making it possible to investigate the multiple scattering effects in the nuclear medium including nuclear shadowing [34] and transverse momentum broadening [35]. One may also extract information on the small $x$ gluon distribution of a nucleus in the early stage of collisions [36]. The CGC formalism has been successful in explaining these nuclear shadowing, and transverse momentum broadening in $pA$ collisions at the LHC [37]. For collisions of different system
sizes, such as $pA$, two saturation momentum scales, i.e., $Q_s^p$ for proton and $Q_s^A$ for nucleus, are introduced into theoretical models. However, due to less constrained the initial value of those saturation scales, it gives theoretical uncertainties of the nuclear modification factor $R_{pA}$ at LHC [58]. It has also been pointed out that fluctuations in protons’ saturation momentum play a significant role in the multiplicity distribution of produced particles in $pA$ collisions [59].

In this paper, the multiplicity-dependent saturation momentum is extracted using the geometric scaling property of the semi-inclusive $p_T$ spectra in $pp$ and $p$-Pb collisions. Furthermore, using the experimental results on the nuclear modification factor in the central rapidity region, we further investigate the saturation momentum that governs the multi-particle production process in $p$-Pb collisions.

This article is organized as follows. We briefly explain the geometric scaling for the semi-inclusive distribution and determine its parameters in Section II. Besides, by fitting a universal function of GS to the semi-inclusive $p_T$ spectra observed in $pp$ and $p$-Pb collisions at LHC energies, we determine the multiplicity-dependent saturation momentum $Q_{\text{sat}}(W^*)$ and the effective interaction radius $R_T^{*}$ as a function of the multiplicity density in the central rapidity region. Then, we show that the experimental results on the multiplicity dependence of $\langle p_T \rangle$ are consistent with the GS’s conjecture in Sec III. In the Sec IV, we clarify the role of the saturation momentum in the nuclear modification factor $R_{p\text{Pb}}$ by comparing our model calculations using $Q_{\text{sat}}(W^*)$ obtained for $pp$ and $p$-Pb, respectively. We close with Sec V containing the summary and some concluding remarks.

II. GS IN SEMI-INCLUSIVE TRANSVERSE MOMENTUM SPECTRA

Consider transverse momentum spectra of $pp$ or $p$-Pb collisions with colliding (center of mass) energy $W$ classified by the multiplicity of the charged hadrons in its final state. In the following formulation of our model, except for a determination part of the multiplicity-dependent saturation momentum and the universal functions, we follow the theoretical formulation developed in Ref. [17, 18] for $pp$ collisions.

For each event multiplicity classes, the semi-inclusive transverse spectra of hadrons normalized by a effective crosssectional reaction area $S_T^{*}$ can be scaled to an universal function

$$\frac{1}{S_T^{*}} \frac{1}{2\pi p_T} \frac{d^2n_{ch}}{dp_T dy} = F(\tau),$$  \hspace{1cm} (1a)

with a scaling variable

$$\tau^{1/2+\lambda} = \frac{p_T}{Q_{\text{sat}}(W^*)},$$ \hspace{1cm} (1b)

instead of the merely transverse momentum $p_T$. Here, $Q_{\text{sat}}(W^*)$ denotes a multiplicity-dependent saturation momentum as a function of the effective energy $W^*$ [17, 18]. Equation (1a) is originally for the gluon $p_T$ distribution based on the saturation picture [4, 7, 40]. We assume that the local parton-duality [41] holds in good approximation, and then hadron spectra observed have the same as a gluon distribution but different total multiplicity. The factor of the effective area $S_T^{*}$ absorbs the ratio of the partons and hadrons’ multiplicity as a constant. For an inclusive distribution, the saturation momentum (in literature, it is often referred to as an average saturation momentum or an energy-dependent saturation momentum)

$$Q_{\text{sat}}(W) = Q_0 \left( \frac{x_0 W}{Q_0} \right)^{\lambda/(\lambda+2)},$$  \hspace{1cm} (2)

is uniquely determined by collision energy $W$ with constants $x_0 = 1.0 \times 10^{-3}$, $Q_0 = 1.0$ GeV/$c$, $\lambda = 0.22$ [14, 42]. In our model [18], which deals with GS for the semi-inclusive spectrum, we determine $W^{*}$ and $S_T^{*}$ as fitting parameters to the semi-inclusive spectra for each multiplicity fixed by the event class. Therefore, $Q_{\text{sat}}(W^{*})$ has a one-to-one correspondence with the multiplicity and regarded as a function of the multiplicity. Here, we assume that multiplicity-dependent saturation momentum $Q_{\text{sat}}(W^{*})$ has the same energy dependence as that of Eq. (2), and it is a saturation momentum in which $W$ in Eq. (2) is just replaced by $W^{*}$.

In our model, $S_T^{*}$ and $W^{*}$ are fitting parameters, which is equivalent to searching for $S_T^{*}$ and $Q_{\text{sat}}(W^{*})$ directly. In fact, there is the following relationship between $W^{*}$ and $Q_{\text{sat}}$:

$$W^{*} = Q_{\text{sat}} \frac{Q_{\text{sat}}}{x_0} \left( \frac{Q_{\text{sat}}}{Q_0} \right)^{2/\lambda}.$$

The function $F$ in Eq. (1a) is called universal function, and Tsallis type function is often used in GS [14]:

$$F(\tau) = \left[ 1 + (q - 1) \frac{\tau^{1/(2+\lambda)}}{\kappa} \right]^{-1/(q-1)},$$ \hspace{1cm} (4)

where $q$ is a so-called non-extensive parameter and $\kappa$ is a constant parameter which connects $Q_{\text{sat}}(W^{*})$ as an intermediate energy scale and hadronization energy scale, freeze out temperature, for example. In previous analyses, we have determined $Q_{\text{sat}}(W^{*})$ by assuming that a universal function for the inclusive spectra and that for the semi-inclusive spectra are the same. However, the $p_T$ spectra broadens for the high multiplicity events at 7.0 and 13.0 TeV $pp$ collisions. (This tendency already can be seen in Fig. 2 of [18].) Therefore, in this paper, we deal only with $p_T^{\pm}$ spectra to exclude particles with large masses that may be sensitive in the large $p_T$ region. One can also consider light hadrons such as pions to be more suitable for our assumption, such as the saturation picture of gluons and the subsequent particle production in the central rapidity.

Since the saturation picture can be universally applied to gluons inside highly relativistic contracted hadronic
or nuclear matter, GS holds regardless of the collision system and it can be valid in high-energy \( pp \) and \( p\text{-}Pb \) collisions. Therefore, we search for \( Q_{\text{sat}}(W^*) \) and \( R_T^* \) in addition to a universal function \( F(\tau) \) itself, which scales semi-inclusive distributions observed in \( pp \) and \( p\text{-}Pb \) collisions to the common universal function. In Fig. 1, we show two examples of the fit to the semi-inclusive spectra with \( q = 1.145, \kappa = 0.1100 \) for \( p + p \rightarrow \pi^\pm + X \) at energy \( W = 7.00 \text{ TeV} \) and for \( p + \text{Pb} \rightarrow \pi^\pm + X \) at energy \( W = 5.02 \text{ TeV} \) observed by ALICE Collaboration [44, 48]. Besides, Fig. 2 shows that 67 semi-inclusive distributions (947 data points), including the spectra shown in Fig. 1, observed in \( \sqrt{s} = W = 2.76, 7.00, 13.0 \text{ TeV} \) \( pp \) collisions [43, 44, 46], and \( W = 5.02 \text{ TeV} \) \( p\text{-}Pb \) collisions [44, 47, 48] almost perfectly scale to the universal function (1a) with \( q = 1.145 \) and \( \kappa = 0.1100 \). As shown in Fig. 2, we can find suitable \( Q_{\text{sat}}(W^*) \) and an effective radius of the interaction area \( R_T^* \) by fitting the semi-inclusive \( p_T \) spectra to the universal function \( F(\tau) \). Figs 3 (a) and 4 (a) shows \( Q_{\text{sat}}(W^*) \) and \( R_T^* \) extracted from the semi-inclusive \( p_T \) spectra, respectively. Both \( Q_{\text{sat}}(W^*) \) and \( R_T^* \) are functions of the multiplicity \( dn_\pi/dy \) of the final state pion in the central rapidity region. It should be noted that \( Q_{\text{sat}}(W^*) \) and \( R_T^* \) are mutually correlated quantities because they are subject to the fixed multiplicity constraint of the semi-inclusive event as the following:

\[
dn_\pi/dy = \frac{2\pi k^2}{(2-q)(3-2q)} S_T^* Q_{\text{sat}}^2(W^*).\]  

Considering that \( R_T^* \propto [dn_\pi/dy]^{1/3} \) approximately holds as well known in the observation of the HBT effects [49, 51], the saturation momentum should be proportional to the 1/6 power of the multiplicity, \( Q_{\text{sat}}(W^*) \propto [dn_\pi/dy]^{1/6} \). Actually, when \( Q_{\text{sat}}(W^*) \) and \( R_T^* \) are plotted by \([dn_\pi/dy]^{1/6}\) and \([dn_\pi/dy]^{1/3}\), respectively, one confirms such \( dn_\pi/dy \) dependence as shown in Figs 3 (b) and 4 (b), which is consistent with the simple conjecture expected from GS.

We discuss somewhat peculiar multiplicity dependence on \( Q_{\text{sat}}(W^*) \) extracted from \( p_T \) spectra of \( p\text{-}Pb \) collisions at 5.02 TeV observed by the ALICE Collaboration [44]. (See, Fig. 3 (a) and (b).) It is observed that the saturation momentum extracted from the spectra is significantly less multiplicity-dependent than the case of \( pp \) collisions. On the other hand, the extraction of \( Q_{\text{sat}}(W^*) \) from spectra observed by CMS Collaboration [47] with the same collision system and the same energy gives almost the same results as that obtained in \( pp \) collisions. While ALICE Collaboration has published data on the transverse momentum spectra for \( p_T < 20 \text{ GeV} / c \), we used it for \( 0 < p_T < 3.0 \text{ GeV} / c \) to rule out hadron jet effects in the extraction of \( Q_{\text{sat}}(W^*) \). Choosing the maximum \( p_T = 2.0 \text{ GeV} / c \), which is the same as the CMS, did not significantly affect results obtained. However, the rapidity range is slightly different between two collaborations, where CMS is \([y] < 1 \), whereas ALICE is \(0 < y < 0.5\), and ALICE has observed pions for the more central rapidity region. It is still unclear whether the rapidity window of the semi-inclusive \( p_T \) spectra affects the evaluation of \( Q_{\text{sat}}(W^*) \) and \( R_T^* \).

We fitted \( Q_{\text{sat}}(W^*) \) and \( R_T^* \) shown in Figs 3 (a) and Fig. 4 (a) by the following fitting formulae of 1/6 and 1/3 power of \( dn_\pi/dy \), respectively:

\[
Q_{\text{sat}}(W^*) = a_Q + b_Q \left( \frac{dn_\pi}{dy} \right)^{\frac{1}{6}},
\]

and

\[
R_T^* = a_R + b_R \left( \frac{dn_\pi}{dy} \right)^{\frac{1}{3}}.
\]

The values of the coefficients \( a_Q, b_Q, a_R \) and \( b_R \) fitted to the data of Figs 3 (a) and 4 (a) are shown in Table I. Here, if the constant terms \( a_Q \) and \( a_R \) can be ignored, the following relation is derived from Eq. (5):

\[
\sqrt{\frac{(2-q)(3-2q)}{2\pi^2}} = \frac{\kappa b_Q b_R}{0.197 \text{ [GeV fm]}}.
\]
As shown in Table II, Eq. (7a) is approximately satisfied by the LHC energies of pp and p-Pb collisions. At the initial stage of collisions, gluon number density saturates due to their non-linear interactions. The inverse of saturation momentum $1/Q_{sat}$ gives a transverse cross-sectional size scale where saturated gluons are packed (one may consider it as a color flux tube size) \( \frac{1}{|Q_{sat}|} \geq 2.4 \) by CMS Collaboration (the pseudo rapidity window \( |\eta| < 2.4 \)). The \( pT \) spectra in pp collisions for 9 event classes with multiplicity range of 3.98 \( \leq dN_{ch}/dy \leq 20.1 \) at 7.00 TeV \[46\] by CMS Collaboration are also shown. For \( p-Pb \) collisions (red symbols) at 5.02 TeV for multiplicity class with track number 19 \( \leq n_{tracks} \leq 235 \) by CMS \[47\] and ALICE Collaboration 7 event classes with multiplicity range of 4.4 \( \leq dN_{ch}/dy \leq 45 \) \[44,48\] are shown.

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FIG. 2. Geometric scaling of the semi-inclusive \( pT \) spectra in pp collisions (black symbols) for multiplicity class with track number 16 \( \leq n_{tracks} \leq 98 \) at $\sqrt{s}$=2.76 TeV, 16 \( \leq n_{tracks} \leq 131 \) at $\sqrt{s}$=7.00 TeV \[45\], and 28 \( \leq n_{tracks} \leq 172 \) at $\sqrt{s}$=13.0 TeV \[46\] by CMS Collaboration (the pseudo rapidity window \( |\eta| < 2.4 \)). The \( pT \) spectra in pp collisions for 9 event classes with multiplicity range of 3.98 \( \leq dN_{ch}/dy \leq 20.1 \) at 7.00 TeV \[46\] by CMS Collaboration are also shown. For \( p-Pb \) collisions (red symbols) at 5.02 TeV for multiplicity class with track number 19 \( \leq n_{tracks} \leq 235 \) by CMS \[47\] and ALICE Collaboration 7 event classes with multiplicity range of 4.4 \( \leq dN_{ch}/dy \leq 45 \) \[44,48\] are shown.

### Table II: The values of the parameters used in Eqs. (6a) and (6 b) for fitting $Q_{sat}(W)$ and $R_t^+$ extracted from the semi-inclusive $pT$ spectrum, respectively. We also show the right and left hand sides of Eq. (7a) to check the GS conjecture.

| Event Class | \( a_Q + b_Q(dN/dy)^{1/6} \) | \( a_R + b_R(dN/dy)^{3/4} \) | l.h.s. | r.h.s. |
|-------------|------------------------------|------------------------------|--------|--------|
| pp $\rightarrow$ $\pi^\pm$ + X | | | | |
| 2.76 TeV \[45\] | -1.0 < y < 1.0 | -0.019 0.854 | 0.006 0.371 | 0.175 0.177 |
| 7.00 TeV \[45\] | -1.0 < y < 1.0 | -0.149 0.954 | 0.051 0.345 | 0.175 0.184 |
| 7.00 TeV \[43\] | -0.5 < y < 0.5 | -0.225 0.985 | 0.073 0.345 | 0.175 0.190 |
| 13.0 TeV \[46\] | -1.0 < y < 1.0 | -0.472 1.164 | 0.156 0.302 | 0.175 0.196 |
| pp $\rightarrow$ $h^\pm$ + X | | | | |
| 5.02 TeV \[52\] | -0.8 < $\eta$ < 0.8 | -0.311 1.160 | 0.078 0.296 | 0.181 0.211 |
| 13.0 TeV \[52\] | -0.8 < $\eta$ < 0.8 | -0.390 1.323 | 0.072 0.246 | 0.181 0.200 |
| p-Pb $\rightarrow$ $\pi^\pm$ + X | | | | |
| 5.02 TeV \[47\] | -1.0 < y < 1.0 | 0.078 0.899 | 0.030 0.358 | 0.175 0.180 |
| 5.02 TeV \[44\] | 0.0 < y < 0.5 | 0.315 0.600 | -0.130 0.446 | 0.175 0.149 |

As shown in Table II, Eq. (7a) is approximately satisfied by the LHC energies of pp and p-Pb collisions. At the initial stage of collisions, gluon number density saturates due to their non-linear interactions. The inverse of saturation momentum $1/Q_{sat}$ gives a transverse cross-sectional size scale where saturated gluons are packed (one may consider it as a color flux tube size) \( \frac{1}{|Q_{sat}|} \geq 2.4 \) by CMS Collaboration (the pseudo rapidity window \( |\eta| < 2.4 \)). The \( pT \) spectra in pp collisions for 9 event classes with multiplicity range of 3.98 \( \leq dN_{ch}/dy \leq 20.1 \) at 7.00 TeV \[46\] by CMS Collaboration are also shown. For \( p-Pb \) collisions (red symbols) at 5.02 TeV for multiplicity class with track number 19 \( \leq n_{tracks} \leq 235 \) by CMS \[47\] and ALICE Collaboration 7 event classes with multiplicity range of 4.4 \( \leq dN_{ch}/dy \leq 45 \) \[44,48\] are shown.

If we evaluate the tube size scale from inclusive spectra, it is determined by solely the collision energy $W$ and does not depend on the event multiplicity. For example, when the collision energy is $W = 7.0, 13.0$ TeV, the saturation momentum is $Q_{sat}(W) = 1.213$ and 1.289 GeV /c, respectively. (These give flux tube size scale 0.162 and 0.153 fm, respectively.) However, the satura-
FIG. 3. (a) The multiplicity-dependent saturation momentum \( Q_{\text{sat}}(W^+) \) obtained by the fitting Eq. (b) to the semi-inclusive \( \pi^+ \) transverse spectra as a function of \( dn_\pi/dy \).
(b) The same as (a) but as a function of \( [dn_\pi/dy]^{1/6} \).

FIG. 4. (a) Effective transverse radii \( R_\pi^* \) of interaction cross sectional area obtained by the fitting Eq. (b) to the semi-inclusive \( \pi^+ \) transverse spectra as a function of \( dn_\pi/dy \).
(b) The same at (a) but as a function of \( [dn_\pi/dy]^{1/3} \).

FIG. 5. \( dn_\pi/dy \) dependence of (a) \( 1/Q_{\text{sat}}(W^+) \) which can be interpreted as a scale of the radius of color flux tube and (b) \( S_\pi^*Q_{\text{sat}}^2(W^+) \) which is the total number of the tubes produced in the interaction area.

The different \( dn_\pi/dy \) dependence on the average transverse momentum \( \langle p_T \rangle \) in \( pp \) and \( p\text{-Pb} \) collisions has been reported in Ref. [21]. In GS model, since the average transverse momentum of the charged hadrons is proportional to the multiplicity-dependent saturation momentum \( Q_{\text{sat}}(W^+) \), one expects \( \langle p_T \rangle \) also proportional to the 1/6 power of the multiplicity fixed for the semi-inclusive events [17-18]:

\[
\langle p_T \rangle = \frac{2\pi \kappa^2}{(2-q)(3-2q)} \left( \frac{0.197 [\text{GeV fm}]}{b_Q b_R} \right)^2
\]

Experimental data on the mean transverse momentum of \( \pi^\pm \) observed by CMS and the mean transverse momentum of charged hadron \( h^\pm \) observed by ALICE are re-plotted in Fig. 4 as a function of \( dn_\pi/dy \) to the
1/6 power. As shown in Fig. 6, the experimental results are substantially proportional to the multiplicity to the 1/6 power. It is worth noting that the multiplicity dependence of \( \langle p_T \rangle \) for \( pp \) data observed by the two experimental groups shows similar changes around \( [dn/dy]^{1/6} \approx 1.6 \) (\( dn/dy \sim 20 \)), although the absolute value of it differs due to the difference in acceptance of the measurement. These experimental facts suggest that the multiplicity dependence of \( Q_{\text{sat}}^{W^*} \) changes at \( dn/dy \sim 20 \) in central rapidity region. Here, ignoring the contribution from \( a_Q \) and \( a_R \) in Eq. (6) as small and using Eqs. (7a) and (8), we obtain the following for the slope of the graph shown in Fig. 6:

\[
\frac{\langle p_T \rangle}{[dn/dy]^{1/6}} = \frac{0.197 \text{ [GeV-fm]} \cdot \sqrt{2(2 - q)(3q - 2)}}{b_R} \cdot \frac{\pi}{(4 - 3q)} \approx 0.3 \sim 0.4 \text{ [GeV/c]} .
\]

The multiplicity dependence of \( \langle p_T \rangle \) given by Eq. (9) well explain the behavior of the experimental results. Moreover, one may explain the behavior of \( \langle p_T \rangle \) for \( pp \) collisions observed by ALICE Collaboration for \( [dn/dy]^{1/6} \geq 1.6 \) is due to the change of behavior of the \( Q_{\text{sat}}^{W^*} \). Namely, the decrease of \( b_Q \) and the increase of \( b_R \) (See, Table III) change the slope of \( \langle p_T \rangle \) vs. \( [dn/dy]^{1/6} \). Note that there is no change in the dependence proportional to the multiplicity’s 1/6 power, just a change in the coefficients. As pointed out in Sec II the saturation momentum \( Q_{\text{sat}}^{W^*} \) extracted from the semi-inclusive \( p_T \) spectra in \( pp \) collisions changes its slope at \( [dn/dy]^{1/6} \geq 1.6 \) (See, Fig. 3 (b)). Interestingly, both \( Q_{\text{sat}}^{W^*} \) and \( \langle p_T \rangle \) show a qualitative change around the almost same \( dn/dy \) in their multiplicity dependence. Thus, the saturation momentum that governs the \( p_T \) spectra increases in proportion to the 1/6 power of multiplicity with the same proportional coefficient for low multiplicity events in both \( pp \) and \( pp \) collisions. However, for high multiplicity events in \( pp \) collisions, \( b_Q \) extracted from the ALICE data changes its value at \( [dn/dy]^{1/6} \approx 1.6 \). Furthermore, as can be seen from Fig. 4 the coefficient of \( [dn/dy]^{1/3} \) for \( R_{T}^{*} \) also changes at the same multiplicity as \( Q_{\text{sat}}^{W^*} \). This is precisely what Eq. (9) expresses. On the other hand, for \( pp \) collisions, there are no indications that the multiplicity dependence of \( \langle p_T \rangle \) changes up to the maximum multiplicity observed.

This multiplicity dependence change in \( \langle p_T \rangle \) may be interpreted as follows. As can be seen from Fig. 3 (b), the number of flux tubes is proportional to the event multiplicity regardless of the reaction and energy. (Approximately four pions are produced per unit rapidity from one tube.) In the case of \( pp \), the multiplicity increases as the tube’s diameter decrease simultaneously as the system’s reaction size increases. (more tubes are packed in the interaction area.) In \( pp \), the multiplicity is increased by the same mechanism as the \( pp \) collision when the multiplicity is small. However, at a certain multiplicity, the flux tube’s size becomes difficult to become thin, and instead, the reaction region becomes large, so that the multiplicity increases.

### IV. NUCLEAR MODIFICATION FACTOR

We have confirmed in Sec II that the semi-inclusive \( p_T \) spectra of both \( pp \) and \( pp \) scale to the same universal function. The saturation momentum, which plays a central role in the GS, behaves differently from \( pp \), especially in high-multiplicity events of \( pp \) collisions. In the case of \( pp \) collisions, nuclear matter may affect the multiplicity-dependent saturation momentum. To investigate the nuclear matter effects on the saturation momentum in \( pp \) collisions, the nuclear modification factor experimentally observed is compared with the model calculations. The modification factor is a ratio of \( p_T \) differential yield relative to the \( pp \) reference and it is defined by:

\[
R_{\text{exp}}^{p_{\text{Pb}}}(p_T) = \frac{d^2N_{pp}^{p_{\text{Pb}}}/d\eta dp_T}{(T_{pp})^2p_{\text{Pb}}^\sigma_{\text{ch}}/d\eta dp_T}. \tag{10a}
\]

where \( (T_{pp}) = 0.0983\text{mb}^{-1} \) \( 48 \) is an average nuclear overlap function. In experiments, \( R_{\text{exp}}^{p_{\text{Pb}}} \) is defined by the inclusive spectra, but we substitute it with the following equation using the semi-inclusive spectra to clarify a role of the saturation momentum:

\[
R_{p_{\text{Pb}}^{pp}}(p_T) = \frac{d^2n_{pp}^{p_{\text{Pb}}}/d\eta dp_T}{C d^2n_{pp}^{p_{\text{Pb}}}/d\eta dp_T}. \tag{10b}
\]

Here, \( C \) in Eq. (10b) is a constant factor and is related to the experimental data of \( (T_{pp}) \) and the total inelastic nucleon-nucleon cross section \( \sigma_{\text{inel}}^{NN} = 67.6 \text{mb} \) \( 53 \) as
The nuclear modification factor $R_{pp\text{b}}$ calculated by Eqs. (12a) and (12b) for the multiplicity-dependent saturation momentum of $pp$ and $p$-Pb collisions, respectively, is shown by the broken line in Fig. 7(a). We can partially reproduce $R_{pp\text{b}}$, such as suppression in the low $p_T$ region and asymptotic behavior in the high $p_T$ region. However, the simple calculation using Eqs. (12a) and (12b) overestimates $R_{pp\text{b}}$ in the low $p_T$ region compared to the experimental data and cannot reproduce the so-called Cronin enhancement.

Let us introduce $p_T$ dependence as a phenomenological side effect on the saturation momentum $Q_{\text{sat}}(W^*)$, which has been regarded as a function of effective energy $W^*$ (or average multiplicity $dN/dy$) only. Recall that the saturation momentum $Q_{\text{sat}}(W)$ is derived from an intermediate energy scale $Q^a(x) \equiv Q_0^2(x_0/x)^\lambda$ given by Bjorken $x = p_T/W$. Then $Q_{\text{sat}}$ is defined with the solution $p_T$ satisfying $Q_{\text{sat}}(W) = Q^a(p_T/W)$. Therefore, this is a good approximation for $p_T \sim Q_{\text{sat}}$ and neglects the weak $p_T$ dependence in $p_T \gg Q_{\text{sat}}$ and $p_T \ll Q_{\text{sat}}$ region, resulting in deviations from the original intermediate energy scale $Q_s(x)$ (See Fig. 1 in Ref. [18]). This weak $p_T$ dependence may need to be taken into account, especially for observables such as Eq. (10b), which is sensitive to the behavior of $p_T$. Another reason to introduce this effect is to investigate the in unknown detail of gluon recombination effect in multi-particle production from experimental data. (Of course, it is not possible to distinguish whether recombination occurred before or after the color flux tube formation.) We introduce such an effect phenomenologically and investigate whether it contributes to explaining $R_{pp\text{b}}$ obtained in the experiment [18]. Thus, instead of Eq. (12b), we introduce modification for $Q_{\text{sat}}(p_T)$ given by Eq. (12c) as the following:

$$Q_{\text{sat}}^{pp\text{b}}(W^*) = a_Q + b_Q \left[ \frac{dN_{pp\text{b}}}{dy} + \delta \right]^{1/6},$$

where

$$\delta = \alpha \left( p_T - \beta Q_{\text{sat}}^{pp\text{b}} \right) \exp \left[ -\frac{p_T}{\gamma Q_{\text{sat}}^{pp\text{b}}} \right]$$

We show the results of fitting of Eq. (12c) with Eq. (11) and (12) to the experimental data $R_{pp\text{b}}^{pp\text{b}}(p_T)$ by the solid line in Fig. 7(a), and the values of the parameters of Eq. (12d) in Table III. As shown in Fig. 7(b), one can well reproduce the experimental data of the nuclear modification factor within 4% change for saturation momentum $p_T \lesssim 20$ GeV/c. In particular, the Cronin effect, in which an enhancement peak appears around $p_T = 2 \sim 6$ GeV/c in $R_{pp\text{b}}^{pp\text{b}}$, is explained by being about 1% larger than the multiplicity-dependent saturation momentum of $p$-Pb, Eq. (12b). Note that the original saturation scale $Q_s(x)$ with fixed collision energy $W$ depends on the power of the gluon’s transverse momentum: $Q_s(x) \propto p_T^{-\lambda/2}$. For example, when comparing the value of the saturation scale at $p_T \approx 1$ GeV/c and 5 GeV/c, the value at 5 GeV/c is about 15% smaller than the value at $p_T \approx 1$ GeV/c. Hence, the direction of the correction that reintroduces the weak $p_T$ dependence of $Q_s(x)$ is the opposite direction of the correction required to explain the experimental result of $R_{pp\text{b}}^{pp\text{b}}$.

There are two possibilities to explain this variation in the saturation momentum $Q_{\text{sat}}(W^*)$. One possibility is that the $p_T$ dependence in saturation momentum may be explained as the effects of interactions such as absorption and emission of gluons after flux tubes decay. The other is that such fluctuation may have already existed around the average saturation momentum at the time the color flux tube was formed. The former suggests that the application of parton-hadron duality requires caution. It may be possible to study these two possibilities by similar analyzing a nuclear modification factor by using prompt photons [20] in the same way as discussed in this article.

V. SUMMARY AND CONCLUDING REMARKS

Semi-inclusive spectra of $pp$ and $p$-Pb collisions normalized by the 1/3 power of the multiplicity scales to the same universal function using a saturation momentum proportional to the 1/6 power of the multiplicity. The agreement between the experimental results and GS conjecture suggests that the saturation momentum, determined by a multiplicity of the final state (by assuming local parton hadron duality, it also depends on the gluon’s initial state) dominates the multi-particle production regardless of the reaction type and collision energy. The existence of geometric scaling across different reactions such as proton-proton and proton-nucleus collisions strongly suggests the gluon saturation mechanism in the early stage of the reaction, which should be a physics common to the elementary processes of phenomena on the multi-particle production. One of the advantages of GS analysis is that one can derive information on color flux tube formation in the early stage.
FIG. 7. (a) Comparison of the nuclear modification factor $R_{pPb}$ with ALICE data for $p$-Pb collision at 5.02 TeV by saturation momentum with Eq. (12b) and (12c) shown in the right panel (b).

of hadronic or nuclear collisions from the multiplicity dependent saturation momentum $Q_{\text{sat}}(W^*)$. The information may be carried by the coefficients $b_Q$ and $b_R$ in Eqs. (6a) and (6b). Moreover, there is a constraint condition between them as Eqs. (7) or (9). Based on a color flux tube picture with these equations, we pointed out a reason why the multiplicity dependence of mean transverse momentum in $p$-Pb collisions varies compared to $pp$ collisions is a change of the multiplicity dependence of the diameter of the color flux tube and the size of the area packed tubes. Furthermore, observations of the nuclear modification factor suggest that the multiplicity-dependent saturation momentum needs to introduce small transverse momentum dependence. However, the physical origin of this correction for the multiplicity-dependent saturation momentum is still unclear.

The next step in our work is to analyze the semi-inclusive $p_T$ spectrum obtained from $AA$ collisions to determine a multiplicity dependent saturation momentum. Furthermore, the multiplicity dependence of the mean transverse momentum of Pb-Pb collisions is even weaker than that of $p$-Pb [21]. We need to investigate whether multiplicity-dependent saturation momentum extracted from the $AA$ collisions also satisfies the 1/6 power law of the multiplicity, and clarify the difference between $p$-A and $AA$ from the viewpoint of a multiplicity-dependent saturation momentum. We plan to investigate those issues at some other opportunity.

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