The effective masses of nucleons in asymmetric nuclear matter in the LOCV framework

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Abstract. The momentum dependence of single-particle potential (SPP) and effective masses of nuclei in asymmetric nuclear matter are studied in the framework of the lowest order constrained variational (LOCV) method at zero temperature. The Av18 interactions including two-body interactions and Urbana type three-body force (TBF) are considered as the input for the nucleon-nucleon potential. We investigate the TBF effect on the momentum-dependence of neutron and proton SPP. Also, we calculate the isospin splitting of the neutron and proton effective masses in neutron-rich nuclear matter.

1. Introduction

One of the major goals of nuclear physics is to obtain the equation of state (EOS) of nuclear matter based on the microscopic many-body approach. One can find the EOS by comparing the experimental data with the results of transport models [1]. The single-particle potential (SPP), which a nucleon feels in the nuclear matter, is one of the main components of transport models. Therefore, in order to obtain the EOS of the nuclear matter from experimental results on heavy ion collisions, we need to calculate the SPP. Note that not only the strength but also the variation of SPP around the Fermi energy is important [2]. Finally, we should solve a semi-classical equation instead of solving N-coupled equations by using the SPP [2].

The single-particle properties of nuclear matter have been studied by using various approaches, such as the Brueckner–Hartree–Fock (BHF) [3], Dirac–Brueckner–Hartree–Fock (DBHF) [4], and variational fermion hypernetted chain (FHNC) approaches [5]. In 1975, a group of scientists at the University of Manchester introduced a purely variational LOCV method for studying the nuclear matter [6]. Modarres used the LOCV method for investigating the asymmetric nuclear matter at finite temperature [7]. Also, Moshfegh et al extended this method for calculating the EOS of beta-stable matter at finite temperature [8]. The SPP and effective mass have been calculated for the symmetric nuclear matter [9]; the LOCV method has been generalized for three-body forces (TBFs) [10]. Goodarzi et al have shown that the input of TBFs plays an important role in the computational results [10]. Of course, it is well known that cold neutron stars contain asymmetric nuclear matter due to the presence of a large neutron fraction. Some important properties of neutron stars depend on the effective masses of nucleons [11]. Therefore, neutron stars can be considered as natural laboratories for studying the nuclear matter.

As far as we know, the TBFs have not been studied in the asymmetric nuclear matter in LOCV framework. Therefore, we intend to calculate the SPP and the effective mass of nucleons...
by analysing the TBFs for the asymmetric nuclear matter.

2. The single-particle potential in the LOCV framework

The nuclear matter Hamiltonian is usually written as

\[ H = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j} V(ij), \]  

(1)

where \( p_i \) is the momentum of \( i \)'th nucleon, \( m \) is the nucleon mass, \( V(ij) \) is the bare two-body nucleon–nucleon potential operators, and \( A \) is the number of nucleons.

The SPP and the effective mass are defined as

\[ e(k_i) = \frac{\hbar^2 k_i^2}{2m} + u(k_i), \quad m^* = m \left( \frac{de(k)}{dk} \right)^{-1}, \]  

(2)

where the single-particle potential \( u(k_i) \) is determined by the equation [9]:

\[ u(k_i) = \sum_j \langle ij|w(12)|ij-j\rangle, \]  

(3)

and the “effective interaction” operator is given by the following equation

\[ w(12) = -\frac{\hbar^2}{2m} \left[ F(12), \left[ \nabla^2_{12}, F(12) \right] \right] + F(12)V(12)F(12). \]  

(4)

Here, \( F(12) \) is the two-body correlation operator [9] and \( V(12) \) is the Argonne Av18 potential [12] supplemented by the UIX three-body force [10]. Algebraic calculations show that the single-particle potential at the lowest order level is

\[ u(k_i) = \frac{4\pi}{\Omega} \sum_{k_j} \sum_{M_{\tau_1}M_{\tau_2}LSJT} (2J+1) \left( \frac{1 - (-1)^{L+S+T}}{2} \right) \times \left| \left\langle \frac{1}{2}m_{\tau_1} \frac{1}{2}m_{\tau_2} | T_{MT} \right\rangle \right|^2 \int r^2 dr W^{LSJT}(r) | j_L(kr) |^2, \]  

(5)

where \( \Omega \) is the system volume, \( W^{LSJT} \) is the expectation value of the effective interaction in each \( \{LSJT\} \) partial wave channel, \( j_L(kr) \) is the standard spherical Bessel function of the order \( L \) and \( k = 2^{-1} |k_i - k_j| \). Note that the correlation functions in the LOCV method have been obtained from the solution of the Euler-Lagrange equation [6].

3. Result and discussion

Figure 1 shows the nuclear matter binding energy per particle versus density for 3 values of the asymmetry parameter (\( \beta = 0 \) red, 0.2 blue, 0.8 black), where the asymmetry parameter is defined as

\[ \beta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}. \]  

(6)

Its limiting values (0 and 1) correspond to the symmetric nuclear matter and pure neutron matter, respectively. The solid and dashed lines in figure 1 refer to the Av18 interaction and the Av18+UIX interaction, respectively.

Figure 2 shows the SPP versus the momentum of neutrons and protons at saturation. According to figure 2a, the magnitude of the SPP increases with increasing the asymmetry.
parameter (adding neutrons to the system). Figure 2b shows that the SPP of neutrons decreases with increasing the asymmetry parameter (adding neutrons to the system). Of course, these results are expected because the asymmetry parameter shifts to one (pure neutron matter) when the potential of the neutrons shifts to the positive values (due to the instability). These results are in good agreement with the results of [13].

The effective mass ratio at the Fermi surface of the nuclear matter versus the asymmetry parameter (calculated with TBFs) are shown in figure 3. As we expect, the effective masses of protons and neutrons are equal for the symmetric nuclear matter. If the asymmetry parameter increases (neutrons are added to the system), the effective mass of the neutron increases and the effective mass of the proton decreases. As a result, in the framework of our computational method, in the asymmetric nuclear matter the effective mass of the neutron is greater than the effective mass of the proton. Our results are in line with the results obtained using the BHF+TBF (for K-mass) method [14]. The splitting of neutron and proton effective masses was a controversial issue, especially between relativistic and nonrelativistic treatments. The discrepancies were found to originate from the use of different definitions [15].

Figure 1. Density dependence of the energy per nucleon for 3 values of the asymmetry parameter ($\beta = 0$ red, 0.2 blue, 0.8 black). The dashed lines refer to the Av18 interaction and the solid lines refer to the Av18+UIX interaction.

Figure 2. The SPP versus the momentum of nucleons at saturation density with TBFs for a) neutrons and b) protons.
4. Conclusion
The momentum and asymmetry parameter of SPP and effective mass of the asymmetric nuclear matter with and without including TBF have been studied in the LOCV framework. The results show that the SPP of proton (neutron) increases (decreases) with increasing the asymmetry parameter (by adding neutrons to the system). Also, as the asymmetry parameter increases, the effective mass of the neutron increases and the effective mass of the proton decreases.

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