Theoretical evaluation of radiation pressure magnitudes and effects in laser material processing

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Abstract

Laser beam radiation and particularly its effects on irradiated materials are commonly specified in terms of intensity. However, laser beam radiation is also a carrier of considerable momentum as a result of high photon flow rates which give rise to a pressure acting at the surface of irradiated probes. Both characteristics, i.e. intensity and radiation pressure are directly related, and resultant effects cannot be easily distinguished from each other under real processing conditions. The current analysis presents theoretical estimations of the different pressure levels that are supposed to be involved in interactions with target surfaces in continuous wave and pulsed-wave laser applications. The results demonstrate that neglecting the radiation pressure seems not to be justified for an appropriate physical characterization and/or interpretation of effects in a number of established laser applications. It is concluded that there is a need to evaluate more profoundly the role of radiation pressure by appropriate experimental and theoretical studies.

Keywords: laser material processing, radiation pressure, photon momentum

(Some figures may appear in colour only in the online journal)

Introduction

Electromagnetic waves and thus also laser radiation are a carrier of energy but also of momentum. This characteristic feature of electromagnetic radiation was already very early supposed by Kepler (1619) and 1883 theoretically proven by Maxwell [1]. The underlying basic relationships can be derived in their simplest form as follows. During a period of time $\Delta t$ the energy $\Delta E_{\text{EM}}$ of an electromagnetic wave is transferred to the irradiated matter, which is related to a corresponding change of momentum $\Delta p$. In the case that the delivered energy is completely absorbed by the matter this change $\Delta p_A$ in momentum is given by:

$$\Delta p_A = \frac{\Delta E_{\text{EM}}}{c} = \frac{P_{\text{EM}} \cdot \Delta t}{c} = \frac{I_{\text{EM}} \cdot A_{\text{IR}} \cdot \Delta t}{c},$$

where $P_{\text{EM}}$ is the transferred power of the wave, $I_{\text{EM}}$ its intensity, $A_{\text{IR}}$ the irradiated interaction area, and $c$ the vacuum speed of light. In the case that the delivered energy is completely reflected, the change in momentum corresponds to the twofold value according to $\Delta p_R = 2 \cdot \Delta p_A$. The resultant force on the irradiated area can be derived from the second Newton’s law according to

$$F_{\text{Abs}} = \frac{\Delta p}{\Delta t} = \frac{P_{\text{EM}}}{c} = \frac{I_{\text{EM}} \cdot A_{\text{IR}}}{c}.$$
completely absorbed radiation by
\[ P_{R,\text{Abs}} = \frac{F_{\text{Abs}}}{A_R} = \frac{I_{\text{EM}}}{c}. \] (3)

Consequently, in the case of completely reflected radiation \( P_{R,\text{Ref}} = 2 \cdot P_{R,\text{Abs}} \) [2]. More sophisticated derivations of these basic principles with consideration of the properties of electro-magnetic waves are given in [3–5] which provide the same final results. Due to the different amounts of momentum transfer for absorption and reflection processes, it seems to be convenient to find generalized formulas for arbitrary degrees of absorptivity and reflectivity. The relationship between absorptivity \( A_{\text{opt}} \) and reflectivity \( R_{\text{opt}} \) of an opaque surface obeys the equation
\[ A_{\text{opt}} + R_{\text{opt}} = 1 \] (4)
due to energy conservation reasons. Considering the different amounts of momentum that are transferred in the cases of absorbed and reflected radiation one gets
\[ \Delta P_{\text{Total}} = A_{\text{opt}} \cdot \Delta E_{\text{EM}} + \frac{2 \cdot R_{\text{opt}} \cdot \Delta E_{\text{EM}}}{c} \]
\[ = \frac{(1 - R_{\text{opt}} + 2 \cdot R_{\text{opt}}) \cdot \Delta E_{\text{EM}}}{c} = \frac{(1 + R_{\text{opt}}) \cdot \Delta E_{\text{EM}}}{c} \] (5)
and for the resultant pressure \( P_{R,\text{Total}} \) acting at a given interaction area
\[ P_{R,\text{Total}} = (1 + R_{\text{opt}}) \cdot \frac{I_{\text{EM}}}{c} \cdot A_R = (1 + R_{\text{opt}}) \cdot \varepsilon_{\text{EM}} \] (6)
where \( \varepsilon_{\text{EM}} \) is the energy density of the electromagnetic wave. The relationship (6) is referred to as the Maxwell–Bartoli equation and its validity was experimentally verified in 1901 by Lebedew [6] who also gave an overview of the very early work related to radiation pressure and its action on objects. With consideration of equation (6), the relationships (1) and (2) can now be given in the generalized formulas
\[ F_{R,\text{Total}} = (1 + R_{\text{opt}}) \cdot \frac{I_{\text{EM}} \cdot A_R}{c} = (1 + R_{\text{opt}}) \cdot \frac{P_{R,\text{Total}}}{c} \] (7)
\[ p_{R,\text{Total}} = F_{R,\text{Total}} \cdot \Delta t_A = (1 + R_{\text{opt}}) \cdot \frac{I_{\text{EM}} \cdot A_R \cdot \Delta t_A}{c} \]
\[ = (1 + R_{\text{opt}}) \cdot \frac{P_{R,\text{Total}} \cdot \Delta t_A}{c} \] (8)
i.e. the radiation force \( F_{R,\text{Total}} \) and the momentum \( p_{R,\text{Total}} \) correlate both with the reflectivity of a surface. In addition, the radiation force is to be considered as proportional to the power of the electro-magnetic wave, and the momentum to their energy. The interaction time \( \Delta t_A \) for a determination of the momentum transfer can be the pulse duration for pulsed-wave (pw) applications or a characteristic interaction interval that is related to the processing speed and the characteristic length of the interaction area in continuous-wave (cw) applications.

The importance of the radiation pressure was particularly recognized for the explanation of a number of cosmological phenomena, such as the directional formation of comet tails or the stability of stars [7]. However, the development of laser technology and the availability of highly focused laser radiation on earth gave also rise to some technological applications of radiation pressure. An overview of scientific and practical application fields is given by Ashkin [8], including light scattering, cloud physics, aerosol science, atomic physics and quantum optics. Fundamental research on the action of radiation pressure was primarily reported by use of dielectric media [9, 10]. Recently, a device has been developed that bases on the action of radiation force. The applied measuring principle uses the power-to-force conversion factor of \( 2/c = 6.67 \times 10^{-8} \text{N \cdot m}^2 \text{\cdot s}^{-2} \) according to equation (7) for normal incidence on a perfectly reflecting mirror \( (R_{\text{opt}} \approx 1) \) to measure the power of laser beams in a range of up to 100 kW, where conventional measuring techniques are not applicable [11]. In fact, the applicability of such a measuring principle can be understood as an additional experimental proof of the formalism being derived for the theoretical description of the action of radiation pressure or radiation force, respectively.

Despite the fact that the concept of radiation pressure is widely accepted and experimentally proven, its importance under conditions of laser material processing with commonly high laser intensities was not thoroughly investigated as yet. Instead, the impact of radiation pressure is considered being negligible in comparison to the recoil pressure as a result of evaporation processes [12]. However, some researchers have at least supposed a possible effect [13] but up to now no quantitative studies were conducted to clarify this issue. It is therefore the aim of this work, to provide a theoretical analysis of achievable radiation pressure levels for cw and pw laser applications for material processing purposes. In addition, possible effects of the radiation pressure on the process dynamics are also discussed.

Fundamentals

Laser radiation and its impact on irradiated matter is commonly described in terms of laser intensity as a continuous field quantity but it can also be described as a flux of photons. These photons can be considered as a carrier of some fundamental properties, such as photon energy \( E_{\text{Ph}} \), photon mass \( m_{\text{Ph}} \) and photon momentum \( p_{\text{Ph}} \). This photon model is well-suited to illustrate the possible impact of laser radiation as a flux of photons during its interaction with matter. The photon energy \( E_{\text{Ph}} \) is given by Planck’s equation as a function of radiation frequency \( \nu_L \) or laser wavelength \( \lambda_L \), respectively, according to
\[ E_{\text{Ph,L}} = h \cdot \nu_L = h \cdot \frac{c}{\lambda_L} \] (9)
with Planck’s constant \( h = 6.625 \times 10^{-34} \text{ J \cdot s} \) and the vacuum speed of light \( c = 3 \times 10^8 \text{ m \cdot s}^{-1} \). For the most prevalent laser sources, i.e. CO2 lasers (index CO2) with \( \lambda_{\text{CO2}} = 10.6 \times 10^{-6} \text{ m} \) and solid-state lasers (index SSL) with a wavelength which is, for the sake of simplicity, assumed to
be $\lambda_{SSL} \approx 1.0 \times 10^{-6}$ m, the photon energies amount to $E_{Ph,CO_2} = 1.875 \times 10^{-20}$ J for CO$_2$ laser radiation and $E_{Ph,SSL} = 1.9875 \times 10^{-19}$ J for solid-state laser radiation. Using Einstein’s formula of mass-energy equivalence, the (equivalent) photon masses $m_{Ph,L}$ can be calculated again as a function of laser wavelength according to:

$$m_{Ph,L} = \frac{E_{Ph,L}}{c^2} = \frac{h}{c^2} \cdot \frac{1}{\lambda_L} = \frac{h}{c \cdot \lambda_L}. \quad (10)$$

The corresponding masses of CO$_2$ laser photons and solid-state laser photons are $m_{Ph,CO_2} = 2.083 \times 10^{-37}$ kg and $m_{Ph,SSL} = 2.208 \times 10^{-36}$ kg. Due to their mass $m_{Ph,L}$ and their velocity $c$, the photons possess a momentum $p_{Ph,L}$ that is given by the relation

$$p_{Ph,L} = m_{Ph,L} \cdot c = \frac{h}{\lambda_L}. \quad (11)$$

which is also referred to as the de-Broglie relation. For CO$_2$ laser and solid state laser photons one gets the corresponding photon momenta $p_{Ph,CO_2} = 6.250 \times 10^{-29}$ Ns and $p_{Ph,SSL} = 6.625 \times 10^{-28}$ Ns. The calculated elementary properties energy $E_{Ph}$, mass $m_{Ph}$ and momentum $p_{Ph}$ of CO$_2$ and solid-state laser photons are finally summarized in table 1 for reasons of clarity. It should be noted that solid-state laser photons possess ten times higher values of photon energy, photon mass and photon momentum compared to CO$_2$ laser photons due to their ten times shorter wavelength.

| Table 1. Elementary properties of CO$_2$ laser and solid-state laser photons. |
|------------------|------------------|
|                  | CO$_2$ laser      | Solid-state laser |
| Energy/J         | $1.8750 \times 10^{-20}$ | $1.9875 \times 10^{-19}$ |
| Mass/kg          | $2.0830 \times 10^{-37}$ | $2.2080 \times 10^{-36}$ |
| Momentum/NS      | $6.2500 \times 10^{-29}$ | $6.6250 \times 10^{-28}$ |

Relevant interactions of laser radiation with opaque matter are commonly described in terms of the macroscopic optical properties absorptivity $A_{opt}$ and reflectivity $R_{opt}$. Applying the photon model and neglecting the fact that in case of opaque materials the absorbed part of the laser radiation is transformed into internal energy within a very thin but finite absorption layer, the interaction processes can be approximately described either as a perfectly elastic collision (reflection) or as a perfectly inelastic collision (absorption) with the irradiated surface whereby the energy (scalar) and momentum (vector) balance has to be fulfilled. For the sake of generality, the reflection and absorption of photons are described for an inclined photon incidence with an inclination angle $\varphi$ relative to the normal of the irradiated surface. Figure 1 initially illustrates the ideal reflection of a photon. In that case the energy $E_{Ph}$ of the photon remains conserved, i.e. $E_{Ph,ref} = E_{Ph,in} = E_{Ph}$ and the momentum balance is given by

$$\left\{ \begin{array}{l}
+ \sin (\varphi) \cdot p_{Ph} \\
+ \cos (\varphi) \cdot p_{Ph}
\end{array} \right\} = \left\{ \begin{array}{l}
p_{S,x} \\
p_{S,z}
\end{array} \right\} + \left\{ \begin{array}{l}
+ \sin (\varphi) \cdot p_{Ph} \\
- \cos (\varphi) \cdot p_{Ph}
\end{array} \right\}. \quad (12)$$

The transferred momentum components $p_{S,x}$ and $p_{S,z}$ to the surface of the irradiated material are consequently:

$$\left\{ \begin{array}{l}
p_{S,x} \\
p_{S,z}
\end{array} \right\} = \left\{ \begin{array}{l}
2 \cdot \cos (\varphi) \cdot p_{Ph} \\
0
\end{array} \right\}. \quad (13)$$

Under the conditions of a perfectly flat surface, the reflected photons are capable of exerting a momentum $p_{S,z}$ only in the normal direction of the surface whereas the tangential component $p_{S,x}$ becomes zero in that case. The momentum in normal direction is at maximum for a perpendicular incidence of the photon ($\varphi = 0$) and decreases with increased inclination.

In contrast, the ideal absorption of a photon is illustrated in figure 2. The photon energy is completely released to the surface, i.e. $E_S = E_{Ph}$, and transferred into internal energy of the material. The momentum balance is given by:

$$\left\{ \begin{array}{l}
p_{S,x} \\
p_{S,z}
\end{array} \right\} = \left\{ \begin{array}{l}
\sin (\varphi) \cdot p_{Ph} \\
\cos (\varphi) \cdot p_{Ph}
\end{array} \right\}. \quad (14)$$

Consequently, it is supposed that the absorption of a photon is capable of implying a tangential momentum component which is zero for a perpendicular incidence ($\varphi = 0$) but grows with the angle of incidence.

Finally, the case of a simultaneous incidence of $n_{Ph}$ photons is considered, whereby $n_{Ph} \cdot A_{opt}$ photons are absorbed and $n_{Ph} \cdot R_{opt}$ photons are reflected from the surface. With consideration of the normalized continuity relation (4),
the total momentum balance can be written as follows:

\[
\begin{pmatrix}
P_{S,x} \\
P_{S,z}
\end{pmatrix} = \begin{pmatrix}
2 \cdot R_{\text{opt}} \cdot \cos(\varphi) \cdot p_{\text{ph}} \\
(1 - R_{\text{opt}}) \cdot \sin(\varphi) \cdot p_{\text{ph}} \\
(1 - R_{\text{opt}}) \cdot \cos(\varphi) \cdot p_{\text{ph}}
\end{pmatrix}
\]

The dimensionless ratio of the tangential to the normal component of photon momentum in dependence of the angle \(\varphi\) of incidence and the reflectivity \(R_{\text{opt}}\). It has to be considered that the reflectivity \(R_{\text{opt}}\) usually changes with inclination angle. This dependence is described for linearly parallel polarized \((R_{\text{p}})\) and perpendicularly polarized radiation \((R_{\text{s}})\) by the Fresnel equations:

\[
R_{\text{p}} = \frac{(n \cdot \cos \varphi - 1)^2 + (k \cdot \cos \varphi)^2}{(n \cdot \cos \varphi + 1)^2 + (k \cdot \cos \varphi)^2}
\]

\[
R_{\text{s}} = \frac{(n - \cos \varphi)^2 + k^2}{(n + \cos \varphi)^2 + k^2}
\]

where \(n\) is the refractive index and \(k\) the extinction coefficient of the irradiated material [14]. For un-polarized or circularly polarized radiation, the reflectivity \(R_{\text{ave}}\) can be commonly estimated as the average value of the reflectivity \(R_{\text{p}}\) and \(R_{\text{s}}\). Possible effects of the polarization state on the normalized components of the photon momentum are discussed for the case of parallel polarized laser radiation which is characterized by a pronounced minimum in reflectivity at a particular angle of incidence, the so-called Brewster angle. Figure 3 shows calculated photon momenta for SSL radiation (figure 3(a)) and CO2 laser radiation (figure 3(b)) for iron as probe material. It is obvious that the normal component of momentum dominates in a wide range but for high inclination angles the tangential component becomes dominant. It should be noted that the calculated values illustrate expected trends but may show some deviations to real values because many factors that may have influence on the reflectivity cannot be considered in such kind of approximate analysis. However, what is regarded as an important outcome is the fact that the transferred momentum to an irradiated surface has to be considered as a function of laser wavelength, optical material properties and inclination angle as well. In combination with the phenomenological relationships (6)–(8) a profound analysis of effects that are probably driven or influenced by photon momentum can be now performed. It is obvious that the small quantities of energy, mass and momentum of a single photon as given in table 1 will have a measurable impact on irradiated surfaces only in the case of high photon rates. Photon rates are usually quite different for cw laser and \(\text{PW}\) laser applications. Therefore, possible effects in both applications fields are separately discussed.

**Radiation pressure in cw-laser applications**

The photon flux rates for continuous wave laser radiation are given as a function of applied laser power to the energy of a single photon:

\[
\dot{n}_{\text{ph,CW}} = \frac{P_{\text{L,CW}}}{E_{\text{L,ph}}}
\]

If the photon rate is related to the available laser power, one gets the photon flux rates per unit power to \(\dot{n}_{\text{ph,CO2,1W}} = 5.333 \times 10^{19} \text{ Ph s}^{-1} \text{ W}^{-1}\) for CO2 laser radiation and \(\dot{n}_{\text{ph,SSL,1W}} = 5.031 \times 10^{18} \text{ Ph s}^{-1} \text{ W}^{-1}\) for solid-state
laser radiation. The corresponding (equivalent) mass flux rates with consideration of relation (10) according to

\[ \dot{m}_{\text{ph,L}} = \dot{m}_{\text{ph,L}} \cdot m_{\text{ph,L}} = \frac{P_{\text{L,CW}}}{E_{\text{ph,L}}} \cdot \frac{E_{\text{ph,L}}}{c^2} = \frac{P_{\text{L,CW}}}{c^3} \quad (20) \]

are also a function of laser power but independent on laser wavelength. Consequently, the mass flux rate per unit power for laser beams with arbitrary wavelengths is always given by \( m_{\text{ph,L}} = 1.111 \times 10^{-17} \text{kg s}^{-1} \text{W}^{-1} \).

By introducing a photon mass flux continuity relation one gets a relationship for a corresponding photon mass density as a function of the cross-section \( A_L \) of the laser beam with an actual beam radius \( r_L \) according to

\[ \rho_{\text{ph,L}} = \frac{m_{\text{ph,L}}}{c \cdot A_L} = \frac{m_{\text{ph,L}}}{c \cdot \pi \cdot r_L^2} = \frac{P_{\text{L,CW}}}{c \cdot \pi \cdot r_L^2} = \frac{I_{\text{L,CW}}}{c}. \quad (21) \]

Equivalent photon mass densities are even in case of very high intensities (typically in a range of about \( 10^{10} \text{W m}^{-2} \) for high power cw laser material processing) very small due to the proportionality \( \rho_{\text{ph,L}} \sim 1/c^3 \) and thus negligible in comparison to common densities of an ambient gas. However, the resulting equivalent stagnation pressure of photon mass flows will consequently be

\[ P_{\text{ph}} = \rho \cdot c^2 = \frac{1}{c} \cdot \frac{P_{\text{L,CW}}}{\pi \cdot r_{L,0}^2} = \frac{I_{\text{L,CW}}}{c}. \quad (22) \]

with the averaged intensity \( I_{\text{L,CW}} \) of a laser beam with a power of \( P_{\text{L,CW}} \) and a beam radius of \( r_{L,0} \). It is obvious that relation (22) formally corresponds to equation (3). Calculated values of the averaged available radiation pressure levels as a function of \( P_{\text{L,CW}} \) and \( r_{L,0} \) in a well-addressable working area of laser material processing are shown in figures 4(a) and (b). These values are quite noticeable with regard to the reached pressure values but local pressure values in case of focused laser beams even amount to higher values. The radiation pressure distribution in case of a Gaussian intensity distribution is given by

\[ P_{\text{Rad}}(z, r) = \frac{I_{\text{L,CW}}(z, r)}{c} = \frac{1}{c} \cdot \frac{2 \cdot P_{\text{L,CW}}}{\pi \cdot (r_B(z))^2} \cdot \exp \left( -2 \cdot \frac{r^2}{(r_B(z))^2} \right) \quad (23) \]

where \( z \) is the coordinate in propagation direction of the beam, \( r \) is the coordinate in radial direction and \( r_B(z) \) denotes the actual beam radius which is given as a function of \( z \) according to

\[ r_B(z) = r_0 \cdot \sqrt{1 + \frac{z^2}{z_R^2}} \quad (24) \]

with the Rayleigh length \( z_R \) which is given for a beam with a wavelength \( \lambda_L \), a beam quality \( M^2 \) and a focus radius \( r_0 \) by

\[ z_R = \frac{\pi \cdot r_0^2}{M^2 \cdot \lambda_L}. \quad (25) \]

Calculated radiation pressure distributions for diffraction-limited laser beams (\( M^2 = 1 \)) are exemplarily shown in figure 5 for an assumed focus radius of \( r_0 = 50 \mu \text{m} \) and wavelengths of \( 1 \mu \text{m} \) (figure 5(a)) and \( 10.6 \mu \text{m} \) (figure 5(b)) each for an applied beam power of 4000 W. The pressure maxima are \( P_{\text{Rad,Max}} = 3395 \text{Pa} \) in both cases but the longitudinal extension of the high-pressure region is significantly reduced for CO2 laser radiation (\( 10.6 \mu \text{m} \)) due to the shorter Rayleigh length. It should be noted that those pressures are in a comparable range as the arc pressure in plasma arc welding where the arc pressure is assumed to be causing the weld pool deformation [15]. The radiation pressure becomes effective in interaction with the irradiated surface, and is increased by the reflected amount of radiation according to equation (6). Its maximum value is particularly influenced by the focussability of a laser beam. Figure 6 shows radial radiation pressure distributions in the focal plane of a beam for different focus radii, a laser power of 4000 W and an assumed reflectivity of 0.6. It is highly expected that those pressure levels are capable of influencing involved processes in laser beam welding. The applied focus intensities for the considered parameter constellations of beam radius and laser power are well above the threshold of approximately \( 10^{10} \text{W m}^{-2} \) for keyhole welding.
of metals. The keyhole is thought as a vapor channel that penetrates into the material and allows the beam to propagate deeper into the material giving rise to a changed geometrical interaction zone. On the one hand, the lateral surface of the keyhole improves the energy coupling into the material by supporting higher degrees of absorption and multiple reflections, and, on the other hand, the photon density and with that the radiation pressure further increases. It is commonly supposed that the vapor channel is more or less completely filled with laser-generated metal vapor. Since the fundamental work by Semak and Matsunawa [16], the formation and stability of the keyhole are explained as a result of a recoil pressure of evaporating material but recently Seidgazov indicated discrepancies between numerical and experimental data [17]. Being focused on experimental observations, it is well-accepted that most of the molten material flows around the keyhole and only small portions of the molten material do evaporate. Measured outflow velocities at the keyhole opening are in the range between 20 m s\(^{-1}\) for moderate laser powers of 1 kW [18] and 200 m s\(^{-1}\) for high-power laser beam welding with 20 kW laser power [19]. The metal vapor can be commonly treated as an ideal gas under atmospheric pressure conditions. Using thermo-physical properties of pure iron the specific gas constant \(R_{Fe}\) of the vapor is given as:

\[ R_{Fe} = \frac{R_0}{M_{Fe}} = \frac{8.31451 \text{ J mol}^{-1} \text{K}^{-1}}{55.85 \times 10^{-3} \text{ kg mol}^{-1}} = 148 \text{ J kg}^{-1} \text{K}^{-1} \]

with \(R_0\) as the universal gas constant and \(M_{Fe}\) as the molar mass of iron. The density \(\rho_{V,Fe}\) of the generated iron vapor can be calculated from the equation of state of the ideal gas according to:

\[ \rho_{V,Fe} = \frac{P_\infty R_{Fe}}{R_{Fe} T_{BP,Fe}} = \frac{1.0 \times 10^5 \text{ J m}^{-3}}{148.9 \text{ J kg}^{-1} \text{K}^{-1} \cdot 3000 \text{ K}} = 0.224 \text{ kg m}^{-3} \]

with \(P_\infty = 1.0 \times 10^5 \text{ Pa} = 1 \text{ bar}\) as a good approximation of the ambient pressure and \(T_{BP,Fe} = 3000 \text{ K}\) as the corresponding boiling point temperature. An additional useful quantity is the velocity of sound \(v_{\text{Sound,Fe}}\) of iron vapor under the specified conditions. With \(\kappa = 1.6\) as isentropic coefficient of iron vapor one gets

\[ v_{\text{Sound,Fe}} = \sqrt{\kappa \cdot R_{Fe} \cdot T_{BP,Fe}} = 843 \text{ m s}^{-1} \]

i.e. the experimentally measured flow velocities of the iron vapor are far below the velocity of sound and the corresponding vapor flow can be approximately treated by using hydrodynamic laws under the assumption of constant vapor density. The resultant over-pressures that are required to cause the vapor flowing out of the keyhole can be calculated from the Bernoulli equation according to

\[ \Delta P_{\text{Min}} = \frac{\rho_{V,Fe} \cdot v_{V,\text{Min}}^2}{2} = \frac{0.224 \text{ kg m}^{-3} \cdot (20 \text{ m s}^{-1})^2}{2} = 45 \text{ Pa} \]
\[ \Delta P_{\text{Max}} = \frac{\rho_{\text{V,Fe}} \cdot v_{\text{V,Max}}^2}{2} = \frac{0.224 \text{ kg m}^{-3} \cdot (200 \text{ m s}^{-1})^2}{2} = 4480 \text{ Pa}. \]  

As the estimation has shown, those values are reachable by the radiation pressure in the range of characteristic beam configurations, i.e. in principle the flow of vapor out of a laser generated keyhole seems to be explainable by the effect of radiation pressure without consideration of further driving mechanisms.

It has to be admitted that such an agreement in calculated data levels is actually no proof of the relevance of the radiation pressure for the characteristics of deep penetration laser beam welding but it should at least be considered as being worth to check its influence carefully against the established theory of keyhole formation as a result of recoil pressure of evaporating material. It should be noted that the hypothesis of a keyhole completely filled with metal vapor has never truly confirmed by measurements. Instead, experimental evidence was provided that there are keyhole root instabilities which cause pores being primarily composed of gas components from the shielding or ambient atmosphere [20, 21], i.e. the metal vapor has to be treated as a partial component of a keyhole vapor mixture only. This, however, implies the conclusion that the pressure inside the keyhole should not substantially deviate from the ambient pressure. It has been also experimentally revealed that a significant amount of the molten material at the inclined keyhole front flows downwards [22]. This type of motion can be also explained by the action of radiation pressure. With consideration of equation (15) which introduced the different components \( P_{\text{S,S}} \) and \( P_{\text{S,Z}} \) of momentum, it is possible to split the radiation pressure \( P_{\text{Rad}} \) into a shear stress component \( \tau_{\text{Rad}} \) and a normal component \( P_{\text{Rad,N}} \) according to

\[ \begin{pmatrix} \tau_{\text{Rad}} \\ P_{\text{Rad,N}} \end{pmatrix} = \begin{pmatrix} (1 - R_{\text{opt}}) \cdot \sin \varphi \cdot \cos \varphi \cdot I_0/c_0 \\ (1 + R_{\text{opt}}) \cdot \cos^2 \varphi \cdot I_0/c_0 \end{pmatrix} \]

taking into account the reduced laser beam intensity due to an increased interaction area:

\[ I_{0,\varphi} = \frac{P_L}{A_0 / \cos \varphi} = \cos \varphi \cdot \frac{P_L}{A_0} = \cos \varphi \cdot I_0. \]

Calculated values of the shear stress and the normal component of radiation pressure as a function of the inclination angle are exemplarily shown in figure 7 for a beam configuration (un-polarized) with \( P_L = 4000 \text{ W} \) laser power and a beam radius of \( r_0 = 50 \mu\text{m} \) and taking into account the optical properties of pure iron for the considered wavelengths of \( \text{CO}_2 \) laser (figure 7(a)) and SSL radiation (figure 7(b)). The calculated values are lower limits of achievable shear stress and pressure levels because the real values are supposed to be higher due to the superposition of multiple reflections inside a laser-induced keyhole. It should be noted that a shear stress in the estimated range of a few hundred Pa is in the same size of magnitude or even higher as other driving melt pool flow mechanisms, such as surface-tension driven flows [23] or the metallic vapor ejection effect [24]. The calculated shear stress levels acting at a free surface of the melt with a dynamic viscosity of \( \eta_{\text{Fe}} = 4.54 \times 10^{-3} \text{ Pa s} \) should be able to cause high surface flow velocities in the range of several meters per second which are consequently in the range of experimentally observed flow velocities.

**Radiation pressure in pw-laser applications**

The emitted number \( n_{\text{PH,PW}} \) of photons during one single pulse is given as the ratio of pulse energy \( E_{\text{Pulse}} \) and photon energy \( E_{\text{PH,L}} \) of a single photon. The pulse momentum \( P_{\text{Pulse}} \) is given as the product of \( n_{\text{PH,PW}} \) times momentum \( p_{\text{PH,L}} \) of the
single photon:

\[ P_{\text{pulse}} = n_{\text{ph, PW}} \cdot \frac{P_{\text{ph, L}}}{c} \cdot \frac{h}{\lambda_L} = \frac{\lambda_L \cdot E_{\text{pulse}}}{h \cdot c} \cdot \frac{h}{\lambda_L} = \frac{E_{\text{pulse}}}{c} \]  

revealing the proportionality of pulse momentum to pulse energy as already stated with the fundamental relationship (1). The resultant radiation pressure \( P_{R, PW} \) follows to

\[ P_{R, PW} = \frac{F_{\text{Pulse}}}{A_{\text{Spot}}} = \frac{P_{\text{pulse}}}{A_{\text{Spot}}} = \frac{E_{\text{pulse}}}{c \cdot \Delta P_{\text{pulse}} \cdot A_{\text{Pulse}}} = \frac{l_{\text{pulse}}}{c} \]  

where \( F_{\text{Pulse}} \) denotes the pulse fluence which is defined as the product of the averaged intensity of the laser radiation and the pulse duration. The fluence \( F_{\text{Pulse}} \) is introduced here because most published data on material processing with pulsed lasers uses this quantity for the characterization of the applied laser source. Figure 8 shows calculated radiation pressure values for ns-laser pulses (figure 8(a)) and ps-laser pulses (figure 8(b)). These pressure levels are much higher than radiation pressures in cw-laser applications and worth to be considered for a theoretical description of involved processes. The high pressures in pw-laser applications are a result of the high intensities of single pulses. These high intensities are simultaneously able of causing anomalous thermal phenomena during and after the pulse phase, which are quite different from those observed in cw-laser applications. In particular, an explosive ablation mechanism was identified in a wide range of short and ultra-short pulse laser processes. This phenomenon was explained by Miotello and Kelly [25] as phase explosion or explosive boiling that takes place if the target surface reaches temperatures close to the thermodynamic critical temperature. In that case a homogeneous bubble nucleation causes a rapid transition from a superheated state to a mixture of vapor and equilibrium liquid droplets which is assumed to be linked to a dramatic rise of the pressure over the liquid [26, 27]. A rough estimation of corresponding ablation pressures can be made under the assumption that heat conduction into the bulk material is negligible for short and ultra-short pulse interaction intervals. In that case, the mass \( m_{\text{IA}} \) of the thermally activated interaction zone is approximately given as

\[ m_{\text{IA}} = \rho_{\text{Bulk}} \cdot A_{\text{Spot}} \cdot l_{\text{opt}} = \rho_{\text{Bulk}} \cdot A_{\text{Spot}} \cdot \frac{\lambda_L}{4\pi \cdot k} \]  

with the absorption length \( l_{\text{opt}} \), the laser wavelength \( \lambda_L \), the extinction coefficient \( k \), the spot size area \( A_{\text{Spot}} \) and the density \( \rho_{\text{Bulk}} \) of the bulk material. The absorption length of metals is commonly much lower than the emission wavelength; for pure iron with \( k_{\text{Fe},\lambda} = 4 \) one gets

\[ l_{\text{opt,Fe}} = 21.2 \times 10^{-9} \text{ m} \]  

for a laser wavelength of \( \lambda_L = 1.064 \mu\text{m} \). The required energy to cause the phase explosion can be thought as an excess energy, which is given as

\[ E_{\text{Excess}} = A_{\text{opt}} \cdot E_{\text{Pulse}} = m_{\text{IA}} \cdot \Delta h_{\text{CP}} \]  

with the increase \( \Delta h_{\text{CP}} \) of specific enthalpy to reach a thermal state near the critical point. This enthalpy can be roughly approximated by summarizing the amounts of heating, melting and evaporating the material according to

\[ \Delta h_{\text{CP}} \approx c_p \cdot (\vartheta_{\text{CP}} - \vartheta_{\infty}) + \Delta h_{\text{M}} + \Delta h_{\text{V}} \]  

with the heat capacity \( c_p \), the critical point temperature \( \vartheta_{\text{CP}} \) which is roughly estimated about 3/2 of the boiling point temperature under atmospheric conditions (Guldberg rule), the temperature \( \vartheta_{\infty} \) of the environment, and the specific enthalpies \( \Delta h_{\text{M}} \) and \( \Delta h_{\text{V}} \) of melting and boiling. In case of pure iron, the amount of specific enthalpy to reach a state near the critical point should be consequently lower than

\[ \Delta h_{\text{CP,Fe}} = 8.6 \times 10^6 \text{ J kg}^{-1} \]  

with \( c_{p,\text{Fe}} = 450 \text{ J kg}^{-1} \text{ K}^{-1} \), \( \Delta h_{\text{M,Fe}} = 274 \times 10^6 \text{ J kg}^{-1} \) and \( \Delta h_{\text{V,Fe}} = 6340 \times 10^6 \text{ J kg}^{-1} \). Both sides of equation (37) can be divided by the thermally activated volume (as product of spot size \( A_{\text{Spot}} \) and absorption length \( l_{\text{opt}} \)) to simplify the analysis.
One gets
\[ e_{\text{Excess}} = \frac{A_{\text{opt}} \cdot F_{\text{Spot}}}{t_{\text{opt,Fe}}} - \rho_{\text{Bulk,Fe}} \cdot \Delta h_{\text{CP,Fe}}. \] (40)

It is evident that for pulse fluences above 1 J cm\(^{-2}\) = \(10^4\) J m\(^{-2}\) and absorption lengths in the range of equation (36) the first term dominates the amount of excess energy which can be estimated in this fluency range as a primary function of the applied pulse parameters according to
\[ E_{\text{Excess}} \approx A_{\text{opt}} \cdot E_{\text{Pulse}} = A_{\text{opt}} \cdot F_{\text{Spot}} \cdot A_{\text{Spot}} \] (41)
and derived quantities must be regarded as upper limits. The corresponding momentum \(p_{\text{Excess}}\) is given under the assumption that the expelling mechanism acts on the whole interaction mass \(m_{IA}\) by
\[ P_{\text{Excess}} = \sqrt{F_{\text{Excess}} \cdot 2 \cdot m_{IA}} \]
\[ = \sqrt{A_{\text{opt}} \cdot F_{\text{Spot}} \cdot A_{\text{Spot}} \cdot 2 \cdot \rho_{\text{Bulk,Fe}} \cdot t_{\text{opt}}} \]
\[ = A_{\text{Spot}} \cdot \sqrt{A_{\text{opt}} \cdot F_{\text{Spot}} \cdot 2 \cdot \rho_{\text{Bulk,Fe}} \cdot t_{\text{opt}}} \] (42)

Calculated momenta as a function of fluence and beam spot area are shown in figure 9 for an assumed bulk density \(\rho_{\text{Bulk,Fe}} = 7800\) kg m\(^{-3}\) of iron and an assumed coupling efficiency of \(A_{\text{opt}} = 0.1\). The resultant equivalent pressure follows to
\[ P_{\text{Spot,Excess}} = \frac{p_{\text{Excess}}}{A_{\text{Spot}} \cdot \Delta t_{\text{Pulse}}} = \frac{\sqrt{A_{\text{opt}} \cdot F_{\text{Spot}} \cdot 2 \cdot \rho_{\text{Bulk,Fe}} \cdot t_{\text{opt}}}}{\Delta t_{\text{Pulse}}} \] (43)
which can be now evaluated as a function of laser fluence \(F_{\text{Spot}}\) and pulse duration \(\Delta t_{\text{Pulse}}\) in comparison to the calculated radiation pressure levels. Calculated values of the excess pressure are shown in figures 10(a) and (b). With respect to magnitudes the calculated values are qualitatively in agreement with experimentally determined values for other metals, such as aluminum or silicon. Fishburn \textit{et al.} [28] measured recoil momenta in the range of \(10^{-9}\) Ns for nanosecond laser ablation of aluminum in a fluency range of 2–10 J cm\(^{-2}\). Lee and Jeong [29] measured recoil pressures in the range of 50–200 GPa for nanosecond laser ablation of silicon in a very high fluency range of up to 300 J cm\(^{-2}\). It is therefore expected that the estimated levels lie in a meaningful range and can be consequently applied as a basis for a comparison with radiation pressure levels. The ablation pressures are three to four orders of magnitude higher than the corresponding radiation pressure levels. It is consequently not far to conclude that the radiation pressure despite the high absolute values might be of lower relative importance in pw-laser applications. Nevertheless, it is supposed that the radiation pressure should have some impact on the time-dependent process dynamics of the pulse phase and as a possible interaction mechanism with ablated particles if the pulse repetition rates are sufficiently high. This conclusion probably holds also true in cases where the ablation mechanism should be stemmed from a Coulomb explosion effect which is also considered to play an important role in laser treatment with ultra-short laser pulses [30, 31].

\[ \text{Figure 9. Recoil momentum as function of beam spot size for different fluence values.} \]

\[ \text{Figure 10. (a) Excess pressure levels for ns-laser pulses and different fluence values. (b) Excess pressure levels for ps-laser pulses and different fluence values.} \]
Summary and conclusions

Radiation pressure levels were estimated for cw and pw laser applications based on fundamental relationships concerning photon energy, mass and momentum. The calculated values demonstrate that neglecting the radiation pressure as possibly driving force during laser-matter interaction seems to be not justified for an appropriate physical characterization and/or interpretation of effects in a number of established procedures of laser material processing. Particularly in the case of cw-laser applications such as deep penetration welding, the radiation pressure should be high enough to play an important role for the characteristics of a number of experimentally observed phenomena above the threshold for keyhole formation. It is expected that deeper insights into the involved mechanisms can be only achieved by a careful spatial and temporal modeling of these processes with consideration of beam propagation characteristics inside a laser-generated keyhole and its coupling to the hydrodynamics of the produced vapor in interaction with the heat and mass flow of the melt. There are currently no appropriate models available which take into account the addressed effect mechanisms proposed by the conducted approximate analysis. In the case of pw-laser applications, the radiation pressures are found to be significantly higher and dramatically increasing with decreased pulse duration. However, the results of the analysis suggest that the ablation pressures as a possible result of an important amount of excess energy are assumed to be some orders of magnitudes higher. This is currently a surprising result of the analysis that the relative importance of the radiation pressure is found to be more pronounced in cw-laser applications rather than in pw-laser applications.

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References

[1] Maxwell J C 1883 Lehrbuch der Electricität und des Magnetismus (Berlin: Springer) 792 p 547
[2] Halliday D, Resnick R and Walker J 2003 Physik (Weinheim: Wiley-VCH GmbH & Co. KGaA) p 977
[3] Laue M V 1913 Strahlung—Thermodynamik der strahlung ed E Korschell et al Handwörterbuch der Naturwissenschaften (Jena: Verlag von Gustav Fischer) pp 789–96
[4] Orear J 1991 Physik (Augsburg: Weltbild Verlag GmbH) pp 446–7
[5] Gertshen C and Vogel H 1993 Physik (Berlin: Springer) 17th ed 658
[6] Lebedew P 1901 Untersuchungen über die druckkräfte des lichts Ann. Phys., Lpz. 4 433–58
[7] Atkins K R 1986 Physik—Die Grundlagen des physikalischen Weltbildes 2nd Auflage (Berlin, New York: Walter de Gruyter) p 434
[8] Ashkin A 1980 Applications of laser radiation pressure Science 201 1081–8
[9] Wunenburger R, Casner A and Delville J P 2006 Light-induced deformation and instability of a liquid interface: I. Statics Phys. Rev. E 73 036314
[10] Wunenburger R, Casner A and Delville J P 2006 Light-induced deformation and instability of a liquid interface: II. Dynamics Phys. Rev. E 73 036315
[11] Williams P A, Hadler J A, Lee R, Maring F C and Lehman J H 2013 Use of radiation pressure for measurement of high-power laser emission Opt. Lett. 38 4248–51
[12] von Allmen M and Blatter A 1998 Laser-Beam Interactions with Materials 2nd Auflage (Berlin: Springer) p 128
[13] Steen W M and Mazumder J 2010 Laser Material Processing 4th edn (London: Springer) p 79
[14] Modest M F 2001 Reflectivity and absorptivity of opaque surfaces JIA Handbook of Laser Materials Processing ed J F Ready and D F Farson (Orlando: Magnolia Publishing) pp 175–81
[15] Trautmann M, Hertel M and Füssel U 2018 Numerical simulation of weld pool dynamics using a SPh approach Weld. World 62 1013–20
[16] Semak V and Matsunawa A 1997 The role of recoil pressure in energy balance during laser material processing J. Phys. D: Appl. Phys. 30 2541–52
[17] Seidgazov R D 2009 Thermocapillary mechanism of melt displacement during keyhole formation by the laser beam J. Phys. D: Appl. Phys. 42 175501
[18] Tenner F, Brock C, Gürtler F, Klämpf F and Schmidt M 2014 Experimental and numerical analysis of gas dynamics in the keyhole during laser metal welding Phys. Proc. 56 1268–76
[19] Beyer E 1995 Schweißen mit Laser (Berlin: Springer)
[20] Matsunawa A, Kim J D, Seto N, Mizutani M and Katayama S 1998 Dynamics of keyhole and molten pool in laser welding J. Laser Appl. 10 247–54
[21] Seto N, Katayama S and Matsunawa A 2001 Porosity formation mechanism and suppression procedure in laser welding of aluminium alloys Weld. Int. 15 191–202
[22] Matsunawa A 2001 Understanding physical mechanisms in laser welding for mathematical modelling and process monitoring Proc. 1st Int. WLT-Conf. on Lasers in Manufacturing, LIM 2001 (Munich) pp 79–93
[23] Mahle A and Beyer E 2008 Derivation of optimal processing parameters for conduction mode laser beam welds by simulation Proc. 3rd Pacific Int. Conf. on Application of Lasers and Optics (Beijing, China) p P223
[24] Fabбро R, Hamadou M and Coste F 2004 Metallic vapor ejection effect on melt pool dynamics in deep penetration laser welding J. Laser Appl. 16 16–9
[25] Miotello A and Kelly R 1999 Laser-induced phase explosion: new physical problems when a condensed phase approaches the thermodynamic critical temperature Appl. Phys. A 69 (Suppl.) S67–73
[26] Miotello A and Kelly R 1995 Critical assessment of thermal models for laser sputtering at high fluxes Appl. Phys. Lett. 67 3535–7
[27] Kelly R and Miotello A 1996 Comments on explosive mechanisms of laser sputtering Appl. Surf. Sci. 96–98 205–15
[28] Fishborn J M, Withford M J, Coutts D W and Piper J A 2006 Study of the influence dependent interplay between laser induced material removal mechanisms in metals: vaporization, melt displacement and melt ejections Appl. Surf. Sci. 252 5182–8
[29] Lee D J and Jeong S H 2004 Analysis of recoil force during Nd:YAG laser ablation of silicon Appl. Phys. A 79 1341–4
[30] Stoian R, Ashkenasi D, Rosenfeld A and Campbell E E B 2000 Coulomb explosion in ultrashort pulsed laser ablation of Al₂O₃ Phys. Rev. B 62 167–73
[31] Dachraoui H, Husinsky W and Betz G 2006 Ultra-short laser ablation of metals and semiconductors: evidence of ultra-fast Coulomb explosion Appl. Phys. A 83 333–6