The muon g-2 anomaly confronts new physics in Bhabha scattering

Luc Darmé, a,b Giovanni Grilli di Cortona, b and Enrico Nardi b

a Institut de Physique des 2 Infinis de Lyon (IP2I), UMR5822, CNRS/IN2P3, F-69622 Villeurbanne Cedex, France
b Istituto Nazionale di Fisica Nucleare, Laboratori Nazionali di Frascati, C.P. 13, 00044 Frascati, Italy
E-mail: l.darme@ip2i.in2p3.fr, grillidc@lnf.infn.it, Enrico.Nardi@lnf.infn.it

Abstract:

The 4.2σ discrepancy between the standard model prediction for the muon anomalous magnetic moment $a_\mu$ and the experimental result is accompanied by other anomalies. A crucial input for the prediction is the hadronic vacuum polarization $a_\mu^{HVP}$ inferred from $\sigma_{\text{had}} = \sigma(e^+e^- \rightarrow \text{hadrons})$ data. However, the two most accurate determinations of $\sigma_{\text{had}}$ from KLOE and BaBar disagree by almost 3σ. Additionally, the combined data-driven result disagrees with the most precise lattice determination of $a_\mu^{HVP}$ by 2.1σ. We argue that all these discrepancies could originate from a $s$-channel new physics contribution to Bhabha scattering hitting a resonance around the KLOE centre of mass energy. This would affect the KLOE luminosity determination, and hence the inferred value of $\sigma_{\text{had}}$. We discuss in detail this possibility, and we describe a simple dark photon model that can reconcile the KLOE and BaBar results for $\sigma_{\text{had}}$, the data-driven and the lattice determinations of $a_\mu^{HVP}$, and can bring the prediction for $a_\mu$ to agree with the experimental result.
1 Introduction

The experimental value of the muon anomalous magnetic moment measured recently by the FNAL Muon g-2 experiment [1] confirms the old BNL result [2] and adds significance to the long standing discrepancy between the measured value and the standard model (SM) prediction, now raised to $4\sigma$. Currently, the world average for this discrepancy is [1, 3]

$$\Delta a_\mu \equiv a_\mu^{\exp} - a_\mu^{\text{SM}} = (2.51 \pm 0.59) \cdot 10^{-9}.$$  \hspace{1cm} (1.1)

The SM estimate recommended by the Muon g-2 Theory Initiative [3] relies on a data-driven approach that makes use of experimental measurements of the $\sigma_{\text{had}} = \sigma(e^+e^- \rightarrow \text{hadrons})$ cross section to determine the hadronic vacuum polarization contribution $a_\mu^{\text{HVP}}$. This is the most uncertain input in the prediction for $a_\mu$ and, due to its non-perturbative nature, improving in precision is a difficult task. Apart for the uncertainty, one can also wonder to which level the adopted value can be considered reliable, since determinations of $a_\mu^{\text{HVP}}$ using data from different experiments exhibit a certain disagreement. In particular, KLOE [4] and BaBar [5] disagree at the level of $3\sigma$, especially in the $\pi^+\pi^-$ channel that accounts for more than 70\% of the value of $a_\mu^{\text{HVP}}$, and while BaBar data favour smaller values of $\Delta a_\mu$, KLOE data pull to increase the discrepancy.\textsuperscript{1}

\hspace{1cm} \textsuperscript{1}Due to relatively larger errors, there is instead agreement within $1.5\sigma$ between KLOE and CMD-2 [6–8], SND [9], BES-III [10].
The HVP contribution can also be determined from first principles by means of lattice QCD techniques. However, until recently, the uncertainties in lattice results were too large to allow for useful comparisons with the data-driven results. A first lattice QCD determination of $a_H^{\mu}$ with subpercent precision was recently obtained by the BMW collaboration [11] $a_H^{\mu} = 707.5(5.5) \times 10^{-10}$. It differs from the world average obtained from the data-driven dispersive approach by $2.1 \sigma$ [12] and, in particular, it would yield a theoretical prediction for $a_\mu$ only $1.3 \sigma$ below the measurement.\footnote{Other lattice determinations also tend to give larger $a_H^{\mu}$ central values although with considerably larger errors [13, 14].}

We can thus conclude that the muon $g - 2$ anomaly Eq. (1.1) is accompanied by other discrepancies that, although of lesser significance, if are not explained can shed some suspicion on interpreting the $a_\mu$ anomaly as a hint of new physics (NP). In this respect, new high statistics measurements of $\sigma_{\text{had}}$, and in particular in the $\pi^+\pi^-$ channel, that might be soon provided by the CMD-3 collaboration [15], as well as new high precision lattice evaluations, which might confirm or correct the BMW result [11], will be of crucial importance not only to strengthen or resize the evidences for a $(g - 2)_\mu$ anomaly, but also to assess the status of the related additional discrepancies. In the meantime, it is worthwhile wondering if the discordant determinations of $a_H^{\mu}$ (KLOE vs. BaBar and $\sigma_{\text{had}}$ vs. lattice) could be explained, jointly with $\Delta a_\mu$, within a single NP scenario. In this paper we explore the possibility that the origin of the KLOE-BaBar discrepancy could be due to a NP contribution to Bhabha scattering, which is used by the experiments to determine the beam luminosity by comparing the measured number of Bhabha events with the QED prediction. Clearly, not accounting for a NP contribution to Bhabha scattering would result in an incorrect determination of $\sigma_{\text{had}}$. As we will show, if these (hypothetical) NP effects are instead included, the KLOE vs. BaBar discrepancy is solved, and the disagreement between the data-driven and the lattice determination of $a_H^{\mu}$ is also easily explained.

As a concrete example, we discuss a simple model originally proposed as a realisation of the “inelastic dark matter” mechanism [16], in which the contribution to the Bhabha process $e^+e^- \to e^+e^-$ of a new vector boson $V$ of mass $m_V \sim 1$ GeV can result in an overestimation of the luminosity at KLOE, with negligible effects on the other experiments. We show that in our model, the values of the parameters required to account for the KLOE vs. BaBar discrepancy and for reconciling the data-driven vs. lattice determination of $a_H^{\mu}$, do not conflict with other experimental constraints. Furthermore, we show that the direct contributions to $a_\mu$ from loops involving $V$ can also bring the theoretical estimate of the muon anomalous magnetic moment in reasonable agreement with the experimental measurement, thus solving all the $a_\mu$-related anomalies at once. Finally, we discuss various constraints on the
model (mono-photon search in BaBar, \(\phi\)-related observables, etc.). We find that the strongest constraints come from the KLOE measurement of the forward-backward asymmetry in \(e^+e^-\) final states [17]. Yet, these constraints are not sufficiently strong to exclude neither our model, nor the general NP mechanism underlying it.

The paper is structured as follow. In section 2 we first review the main ingredients of the data-driven calculation of \(a_{\mu}^{\text{HVP}}\), and then we illustrate the mechanism through which a shift in the luminosity determination can affect the KLOE result for \(\sigma_{\text{had}}\), and bring it to agree with BaBar. In section 3 we present an explicit realisation of this mechanism based on an inelastic dark matter model, we describe the main phenomenological aspects of the model and we analyse various constraints. In section 4 we resume our main results and we draw the conclusions.

2 The muon magnetic moment and the hadronic cross-section

2.1 Data-driven calculation of \(a_{\mu}^{\text{HVP}}\).

At leading order, the contribution to the muon magnetic moment arising from the hadronic vacuum polarisation can be derived from the data on the hadronic cross-section \(\sigma_{\text{had}} = \sigma(e^+e^- \rightarrow \text{hadrons}(+\gamma))\) by means of the optical theorem [18, 19]

\[
a_{\mu}^{\text{LO,HVP}} = \frac{1}{4\pi^3} \int_{4m_e^2}^{\infty} ds K(s)\sigma_{\text{had}}(s). \tag{2.1}
\]

Here, \(\sigma_{\text{had}}\) is the bare \(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons}(\gamma)\) cross-section, obtained by removing the infinite string of hadronic vacuum polarisation insertions in the photon propagator (that leads to the running of \(\alpha_{\text{QED}}\)), and the kernel function reads

\[
K(x) = \frac{x^2}{2}(1-x^2) + \frac{1}{2} \frac{x}{1-x} \log x + \frac{(1+x^2)(1+x)^2}{x^2} \left(\log(1+x) - x + \frac{x^2}{2}\right), \tag{2.2}
\]

with \(x = (1-\beta_{\mu})/(1+\beta_{\mu})\) and \(\beta_{\mu} = \sqrt{1-4m_e^2/s}\). The numerical value recommended by the Muon g-2 Theory Initiative [3] is

\[
a_{\mu}^{\text{LO,HVP}} = 693.1 \pm 4.0 \cdot 10^{-10}, \tag{2.3}
\]

which directly depends on the accuracy of the hadronic cross-section experimental determination.

Several groups have contributed to estimate the HVP contribution to the muon anomalous magnetic moment (see e.g. the recent works [20–25]). In general, various datasets coming from the same or different experiments and exploiting several final states must be combined. The most important channel is the \(\pi^+\pi^-\) channel which accounts for more than 70\% of \(a_{\mu}^{\text{LO,HVP}}\). Many experimental measurements have been performed, leading to the embarrassing situation in which the two most precise
measurements disagree at the level of $\sim 3\sigma$. Indeed, the values of $a_{\mu}^{LO,HVP}$ reported by KLOE [4] and BABAR [5] show a worrisome difference, while other experiments (CMD-2, BESIII, SND [6–10]) have reported results that lie in between these two, albeit with larger uncertainties. Note that any effect that could lift the value of $\sigma_{\text{had}}$ given by KLOE, besides reducing the discrepancy with BaBar, at the same time would also reduce $\Delta a_{\mu}$.

2.2 Modifying the hadronic cross-section

The experimental results for the hadronic cross-section depend on the specific luminosity $L_{e^+e^-}$ of the colliding beams: $\sigma_{\text{had}} \propto N_{\text{had}}/L_{e^+e^-}$, with $N_{\text{had}}$ the number of hadronic events. The luminosity is estimated by comparing high statistics measurements of $e^+e^- \rightarrow e^+e^-$ events with the SM prediction for Bhabha scattering:

$$L_{e^+e^-}^{\text{SM}} = \frac{N_{\text{Bha}}}{\sigma_{\text{eff}}^{\text{SM}}}$$  \hspace{1cm} (2.4)

where $N_{\text{Bha}}$ is the total number of (background subtracted) Bhabha events. For each experiment, the effective SM Bhabha cross section $\sigma_{\text{eff}}^{\text{SM}}$ is evaluated by inserting into detector simulations the results of high precision Bhabha event generators [30–32]. The presence of NP contributions to $e^+e^- \rightarrow e^+e^-$ unaccounted by $\sigma_{\text{eff}}^{\text{SM}}$ would then yield an incorrect estimate of the beam luminosity. The true luminosity $L_{e^+e^-}$ would be related to the inferred one as

$$L_{e^+e^-} = L_{e^+e^-}^{\text{SM}} \frac{\sigma_{\text{eff}}^{\text{SM}}}{\sigma_{\text{eff}}},$$  \hspace{1cm} (2.5)

where $\sigma_{\text{eff}}$ is the full Bhabha cross-section including the NP contributions, that can be parametrized in terms of a correction to the QED Bhabha cross section as

$$\sigma_{\text{eff}} = \sigma_{\text{eff}}^{\text{SM}} (1 + \delta_R) .$$  \hspace{1cm} (2.6)

The true luminosity would be accordingly smaller (cf. Eq. (2.5)), the true hadronic cross section would become larger $\sigma_{\text{had}} \rightarrow \sigma_{\text{had}} (1 + \delta_R)$, and thus the inferred value of the HVP would be increased as:

$$a_{\mu}^{LO,HVP} \rightarrow a_{\mu}^{LO,HVP} (1 + \delta_R),$$  \hspace{1cm} (2.7)

where it is understood that the luminosity induced correction $\delta_R$ depends on the details of the experimental setup in such a way that while it can be quite important in some experiments, it can be totally negligible in others.

Two remarks are in order. Firstly, we have so far focused only on the indirect effects on $a_{\mu}$ stemming from modifications of the estimated luminosity due to NP

$^{3}$\(a_{\mu}^{HVP}\) can be also estimated from hadronic $\tau$ decays data [23, 26–29]. The resulting value 

$(703.0 \pm 4.4) \times 10^{-10}$ [28] is significantly larger than the result from $e^+e^- \rightarrow \text{hadrons}$, and in agreement with the lattice estimate.
contributions to $e^+e^- \rightarrow e^+e^-$ scattering. More in general, whatever NP is producing such an effect will likely also contribute directly to $a_\mu$, and below we will discuss some direct contributions which arise in a specific model. Secondly, it should be clear that lattice calculations of $a_\mu^{\text{HVP}}$ remain completely unaffected by this type of indirect NP effects.

Fitting the relevant hadronic cross-sections is an intricate process which requires the use of dedicated codes, which are not currently public [20, 21]. In order to obtain an estimate, we therefore rely on the contributions of each experiment as given in Refs. [20, 21] combining them with a simple $\chi^2$. In order to provide a proof-of-principle of the method, we focus on the available results in the energy range $\sqrt{s} \in [0.6, 0.9] \text{ GeV}$. We first take the KLOE data at face value, and combine them with all the other experimental results, as independently fitted in [20]:

\[
a_\mu^{\text{HVP}}(\sqrt{s} \in [0.6, 0.9] \text{ GeV}) = \begin{cases} 
(366.9 \pm 2.1) \cdot 10^{-10} & \text{(KLOE)} \\
(376.8 \pm 2.7) \cdot 10^{-10} & \text{(BABAR)} \\
(372.5 \pm 3) \cdot 10^{-10} & \text{(CMD - 2)} \\
(368.3 \pm 4.2) \cdot 10^{-10} & \text{(BESIII)} \\
(371.8 \pm 5) \cdot 10^{-10} & \text{(SND)} \\
(377.0 \pm 6.3) \cdot 10^{-10} & \text{(CLEO)} .
\end{cases}
\]

This leads to a global fit of $a_\mu^{\text{HVP}}(\sqrt{s} \in [0.6, 0.9] \text{ GeV}) = 371.1$ that matches the results of both the combined fit presented in Ref. [21] and in Ref. [20] (see also table 6 in Ref. [3]), thus corroborating the reliability of our procedure. As a note of caution, we will restrict our analysis to the aforementioned $\sqrt{s}$ interval in which we could validate our procedure. However, since the contribution of the $e^+e^- \rightarrow \pi^+\pi^-$ channel to $a_\mu^{\text{HVP}}$ over the complete $\sqrt{s}$ range is about 35% larger, we expect that a full fit using the private codes from [20, 21] could lead to a larger shift in $a_\mu^{\text{LO,HVP}}$. Therefore our results can be taken as a conservative estimate. The excellent agreement between our naive fit and the full results of Refs. [20, 21] is represented in figure 1 by the precise overlap between the violet and the orange bands labelled respectively “Global + KLOE NP” and “SM-TH initiative” occurring at the left boundary of the figure, where KLOE results are taken at face value and not modified.

We next increase the KLOE value for $\sigma_{\text{had}}$ assuming this is due to some unspecified NP effect which affects negligibly all the other measurements. Fig. 1 shows the impact on the predicted value of $a_\mu$ obtained by increasing the KLOE result, bringing it first in agreement with BaBar (corresponding to “0” on the horizontal axis), and then up to $\sim 3\sigma$ above BaBar (close to the right boundary of the Figure). In order to bring KLOE in agreement with Babar an overall shift in $a_\mu^{\text{LO,HVP}}(\text{KLOE})$ of the order of $\sim 0.5 \times 10^{-9}$ is needed. This also ensures a satisfactory agreement between the full set of data driven determinations. As a result, agreement with the
Figure 1. Theoretical prediction for $a_\mu$ obtained by modifying the KLOE result in the data driven global fit to $a_\mu^{\text{LO,HVP}}$ (oblique violet band). The blue band corresponds to the combined BNL and FNAL experimental results, the red band to the prediction obtained with the BMW lattice estimate of $a_\mu^{\text{LO,HVP}}$, and the orange band to the one obtained from $\sigma_{\text{had}}$ without modifications of the KLOE results. The width of the bands represents 1σ uncertainties.

BMW-lattice determination of $a_\mu^{\text{LO,HVP}}$ within $\sim 1.5\sigma$ is also obtained. On the other hand, in order to reduce down to the 3σ level the tension with the combined BNL and FNAL experimental results, $a_\mu^{\text{LO,HVP}}$ (KLOE) must be increased by an appreciably larger factor of $\sim 1.6 \cdot 10^{-9}$. However, this would bring the KLOE measurement approximately 2σ above BaBar, so that we can conclude that a NP effect acting only indirectly through modifications in the KLOE luminosity is not able to provide a fully satisfactory solution to all the $a_\mu$ related anomalies.

To summarise, in order to bring simultaneously the KLOE and BaBar experimental results as well as the dispersive data-driven and lattice estimates in agreement among them, it is enough to shift by a few percent the KLOE measurement of $\sigma_{\text{had}}$, while keeping unchanged the other results. This can be obtained if the KLOE luminosity estimate is skewed from the presence of a NP contribution. In the next section we will show with an explicit example that this scenario can be realised by assuming that a new Feebly Interacting Particle (FIPs) is produced resonantly at the KLOE CoM energy, provided it decays mostly “semi-visibly” into $e^+e^-$. 
3 Model realisation, constraints and results

In section 3.1 we describe a simple model, that fits in the so-called “inelastic dark matter (iDM) scenario”, which is well suited to increase the number of Bhabha events at KLOE while evading at the same time all other experimental constraints. As discussed above, this will imply that the true value of the KLOE luminosity is lower than the value inferred by the collaboration, and the true value of $\sigma_{\text{had}}$ is accordingly larger. In section 3.2 we first review the details of the luminosity measurements in the different experiments that contributed to the determination of $\sigma_{\text{had}}$, and then we analyse quantitatively the effects on the determination of $a_{\mu,\text{LO}}$ from $\sigma_{\text{had}}$ data that could result from an overestimation of the KLOE luminosity. In section 3.3 we study various constraints on this model. Finally, in section 3.4 we discuss the optimal conditions under which the different $a_{\mu}$ related tensions (KLOE vs. BaBar determination of $\sigma_{\text{had}}$, data-driven vs. lattice estimate of $a_{\mu,\text{LO}}$, HVP $\mu$ from $\sigma_{\text{had}}$ data that could result from an overestimation of the KLOE luminosity) can be maximally reduced simultaneously.

3.1 An inelastic dark matter model

We introduce a “dark” Abelian gauge group $U(1)_D$ with gauge coupling $g_D$, along with a dark Higgs $S$ with $U(1)_D$-charge $q_S = +2$, and two Weil spinors $\eta, \xi$ with charges $q_\eta = -q_\xi = -1$ that can be combined in a Dirac fermion $\chi = (\eta \xi^\dagger)^T$. We furthermore assume that a charge conjugation symmetry extended to the dark sector enforces invariance under the transformations $V \rightarrow -V$, $\eta \rightarrow \xi^\dagger$ and $\xi^\dagger \rightarrow \eta$ [16].

Under these premises, we can write the following Lagrangian terms:

$$L_V = -\frac{1}{4} F'^{\mu\nu} F'^{\mu\nu} - \frac{g'\varepsilon}{\cos \theta_w} V^\mu J_Y^\mu , \quad (3.1)$$

$$L_S = (D^\mu S)^\dagger (D_\mu S) + \frac{\mu_S^2}{2} |S|^2 - \frac{\lambda_S}{2} |S|^4 - \frac{\lambda_S H}{2} |S|^2 |H|^2 , \quad (3.2)$$

where we have introduced the customary kinetic mixing term $\varepsilon$ corresponding to a small interaction between the dark photon $V_\mu$ and the hypercharge current $J_Y^\mu$ [33, 34]. The Lagrangian for the dark fermions can be written as

$$L_{\text{DM}} = \bar{\chi} \left( i \partial^\mu - m_\chi \right) \chi - \frac{1}{2} y_S S (\eta^2 + \xi^2) + \text{h.c.} \quad (3.3)$$

where the second term describes the coupling between the Weil fermions and the dark Higgs boson $S$. Note that the charge conjugation symmetry mentioned above enforces that both $\eta$ and $\xi^\dagger$ couple to $S$ with the same Yukawa coupling. The dark Higgs boson mass $m_S$ and the dark photon mass $m_V$ read

$$m_S = \sqrt{2\lambda_S} v_S , \quad (3.4)$$

$$m_V = 2g_D v_S = \left( \sqrt{\frac{2g_D}{\sqrt{\lambda_S}}} \right) m_S . \quad (3.5)$$
The diagonalisation of the fermion mass matrix is straightforward and leads to two states \( \chi_1 = \frac{1}{\sqrt{2}}(\eta - \xi) \) and \( \chi_2 = \frac{1}{\sqrt{2}}(\eta + \xi) \) with masses

\[
M_{1,2} = m_\chi \mp \frac{1}{\sqrt{2}}y_S v_S .
\]  

(3.6)

Note that the phase of \( \chi_1 \) has been fixed to obtain a positive mass term under the assumption \( m_\chi > y_S v_S / \sqrt{2} \). In the mass basis, the dark Higgs \( S \) couples diagonally to \( \chi_{1,2} \) (expressed in term of the Majorana spinors) via the term:

\[
\mathcal{L}_{S\chi} = -\frac{1}{2} y_S S (\chi_2^2 - \chi_1^2) ,
\]  

(3.7)

where the Yukawa coupling can be expressed as

\[
y_S = \frac{g_D M_2 - M_1}{\sqrt{2} m_V} .
\]  

(3.8)

In contrast, the dark photon interacts with the fermions via an off-diagonal coupling

\[
\mathcal{L}_{V\chi} = -ig_D V_\mu \bar{\chi}_2 \gamma^\mu \chi_1 .
\]  

(3.9)

This construction can satisfy the main conditions to generate a significant shift in the KLOE luminosity estimate while escaping detection in other experiments:

- the dark photon mass must be very close to the KLOE CoM energy \( \sqrt{s} \simeq 1.02 \text{GeV} \), in order to produce \( V \) resonantly;

- dark photon decays must contribute non-negligibly to Bhabha scattering events, and therefore they need to include \( e^+e^- \) pairs with invariant mass close to 1 GeV

- in order to escape bump searches, the dark photon main decay channel must be multibody and must include some missing energy.

We can choose \( m_V \sim 1 \text{GeV} \gtrsim M_2 \gg M_1 \) by fixing \( v_S \) and by requiring the approximate relations

\[
y_S v_S \sim \sqrt{2} m_\chi , \quad \sqrt{2} y_S \lesssim g_D .
\]  

(3.10)

When this arrangement of mass values is satisfied, the dark photon decays mainly proceed through the chain \( V \rightarrow \chi_1 \chi_2 \rightarrow \chi_1 \chi_1 e^+e^- \). That is, \( V \) decays with almost a 100% branching ratio into \( \chi_1 \chi_2 \) while direct \( V \rightarrow e^+e^- \) decays are suppressed as \( e^2 \) and thus subdominant. The lightest new fermion \( \chi_1 \), which may play the role of the dark matter particle, is stable and escapes undetected, while the heavier fermion \( \chi_2 \) decays into \( \chi_1 e^+e^- \), providing the main contribution to the NP-related additional electron-positron pairs. The \( V \) width that is almost saturated by
the $V \to \chi_1 \chi_2$ process will be an important quantity in the rest of this work. In the limit $\delta m = m_V - (M_1 + M_2) \ll m_V$ we have:

$$\Gamma_V \simeq \frac{2\alpha D \sqrt{\delta m (\delta m + 2M_1)^{3/2}}}{m_V}.$$  \hspace{1cm} (3.11)

In particular, in the benchmark points used in the next section, phase space suppression due to $M_2 \sim m_V$ leads to a width of order MeV. Note that since there are charged particles in the decay chain final state, the $V$ boson escapes invisible decays searches. At the same time, since the three body decay $\chi_2 \to \chi_1 e^+ e^-$ is characterised by a continuous and smooth $e^+ e^-$ energy spectrum, it also escapes standard "bump" searches.

The mass of the dark Higgs boson depends on its quartic coupling $\lambda_S$, which is a free parameter in the theory. The particular case $m_S \lesssim 2M_1$, which ensures that the $S \to \chi_1 \chi_1$ decay channel is closed, represents an interesting possibility to render $\chi_1$ a good DM candidate [35–37]. The argument proceeds in two steps:

1. The $t$-channel $p$-wave $\chi_1 \chi_1 \leftrightarrow SS$ process is enhanced by a relatively large Yukawa coupling (see Eq. (3.8)). As a consequence, it keeps $\chi_1$ in equilibrium with the $S$ population up to a temperature $T_{fo}$. This temperature can be modified by changing the $S$ mass in the regime $m_S \gtrsim M_1$.

2. At $T \gtrsim m_S$ thermal equilibrium between the dark sector composed of $S$ and $\chi_1$ and the lightest SM particles is typically maintained via the standard $\chi_2 \chi_1 \to e^+ e^-$ process, with the $\chi_1$ relativistic. Moreover, at $T < m_S$, the contribution of $S$ decays and inverse decays mediated by a triangle loop involving $VVe$ maintains $S$ in thermal equilibrium. Consequently, $S$ continues to be coupled to the thermal bath down to temperatures $T \ll m_S$ and in particular throughout the $\chi_1 \chi_1 \to SS$ annihilation process.

Arranging for freezing-out the $\chi_1 \chi_1 \to SS$ process around $T_{fo} \sim M_1/20$ then allows to reproduce the correct dark matter relic density. This corresponds to the “forbidden annihilation” regime of the iDM model which can be realised when $M_S/2 \lesssim M_1 \lesssim M_S$.\footnote{Note that these loop processes typically dominate over the Higgs-portal induced interactions due to the large kinetic mixing considered in this study (see e.g. [35]).}

Finally, let us briefly discuss the direct contribution of $V$ to the muon $g - 2$. Any vector particle with a direct interaction with muons will give rise to a one-loop contribution to $a_\mu$. For a pure vector coupling, this contribution can be written as:

$$\Delta a_\mu = \frac{\alpha_{em} \varepsilon^2}{2\pi} x_\mu^2 \mathcal{F}(x_\mu),$$  \hspace{1cm} (3.12)

\footnote{In the case where the $S$ boson has a longer life-time, additional effects such as the dilution of the $\chi_1$ relic density from the entropy injection from $S \to e^+ e^-$ could also lead to the proper relic density, see the recent work [38], although additional constraints from BBN then apply (see e.g. [35, 39, 40]).}
where \( x_\mu = m_\mu/m_V, \) \( g_{eV} = \varepsilon e \) with \( e \) the electromagnetic coupling constant, and the loop function \( \mathcal{F} \) is given by:

\[
\mathcal{F}(x) = \int_0^1 \frac{2z^2(1-z)}{x^2z + (1-z)(1-x^2z)} dz.
\] (3.13)

For \( m_V \) in the GeV range, the loop function simplifies and one has approximately

\[
\Delta a_\mu \sim 2 \cdot 10^{-9} \times \left( \frac{e\varepsilon}{0.005} \right)^2 \times \left( \frac{1\text{ GeV}}{m_V} \right)^2.
\] (3.14)

The direct contribution in Eq. (3.14), together with the indirect corrections from the luminosity determination, can reconcile the theoretical predictions for \( a_\mu \) with the experimental results.

### 3.2 Numerical analysis

We have implemented the iDM model in FEYNRULES/UFO [41–43] files, and we have used the MADGRAPH5_aMC@NLO platform [44] in order to generate \( s \)- and \( t \)-channel \( e^+e^- \) events\(^6\) that would contribute to the KLOE, BaBar and BESIII measurements of the Bhabha cross section.\(^7\) We have simulated the process \( e^+e^- \rightarrow \chi_1\chi_1e^+e^- \) and we have applied the relevant cuts directly on the generated final states. For the luminosity measurement, the different experiments exploited various kinematic ranges:

- At KLOE, with CoM energy \( \sqrt{s} = 1.02 \text{ GeV} \), the following cuts were applied \([48]\): \( \cos \theta \in [-0.57, 0.57] \), \( E_e \in [0.3, 0.8] \text{ GeV} \), \( p \geq 400 \text{ MeV} \) and a cut on the polar angle acollinearity \( \zeta < 9^\circ \). KLOE reported the Bhabha cross section \( \sigma_{\text{vis}}^{ee} = (431 \pm 0.3) \text{ nb} \).

- At BaBar, with asymmetric collisions of 9 GeV electrons and 3.1 GeV positrons corresponding to \( \sqrt{s} = 10.58 \text{ GeV} \), the following cuts were applied \([49]\): the polar angles in the centre of mass are required to satisfy \( |\cos \theta| < 0.7 \) for one track and \( |\cos \theta| < 0.65 \) for the other; the scaled momentum \( P_i = 2p_i/\sqrt{s} \), where \( p_i \) are the momenta of the track \( i \) and \( \sqrt{s} \) is the CoM energy, were required to

---

\(^6\)We simulate \( e^+e^- \rightarrow \chi_1\chi_2, \chi_2 \rightarrow \chi_1e^+e^- \), including the leptonic branching ratios of the \( \chi_2 \). The hadronic branching ratios are still small at this mass, see Fig. 1 of \([45]\).\(^7\)The two experiments CMD-2 \([6, 46]\) and SND \([9, 47]\) carried out measurements in the energy range for \( \pi^+\pi^- (\gamma) \) production by scanning directly with the beam energy. For each point, Bhabha events were used to calibrate the luminosity. The angular cuts applied are respectively \( \cos \theta \in [-0.83, 0.83] \) (SND \([47]\)) and \( \cos \theta \in [-0.45, 0.45] \) (CMD-2 \([6]\)) . These experiments differ in two main aspects from the previous ones. First they cannot distinguish easily \( \pi \)’s from \( \mu \)’s, so that the final hadronic cross-section can be obtained only after subtracting the \( e^+e^- \rightarrow \mu^+\mu^- \) component by relying on the theoretical estimation. Second, each energy point in the scan has its own luminosity measurement, so that the effects of NP would need to be estimated independently for each \( \sqrt{s} \).
satisfy $P_1 > 0.75$ and $P_2 > 0.5$ ($i = 1$ denotes the track with the higher CoM momentum); the cut on the acollinearity angle was $\zeta < 30^\circ$. The luminosity was estimated also by exploiting the $e^+e^- \rightarrow \mu^+\mu^-$ process and, according to the uncertainties reported in Table 3 of [49], the muon process slightly dominates the average. Furthermore, BaBar associated a 0.7% uncertainty on the Bhabha estimates due to "Data-MC differences". After accounting for the detector efficiency, the reported measurement for the Bhabha cross section is $\sigma_{e^+e^- \rightarrow e^+e^-}^{\text{vis}} = (6.169 \pm 0.041) \text{ nb}$ and that for the $e^+e^- \rightarrow \mu^+\mu^-$ process is $\sigma_{e^+e^- \rightarrow \mu^+\mu^-}^{\text{vis}} = (0.4294 \pm 0.0023) \text{ nb}$.

The BESIII collaboration also delivered an initial state radiation (ISR) measurement [10], although with larger uncertainties, at the CoM energy $\sqrt{s} = 3.773 \text{ GeV}$. This measurement is in better agreement with KLOE data. The luminosity estimate [50] relies solely on Bhabha scattering, with the relatively loose cuts $\cos \theta \in [-0.83, 0.83]$. They reported a Bhabha cross section of $\sigma_{e^+e^- \rightarrow e^+e^-}^{\text{vis}} = (147.96 \pm 0.74) \text{ nb}$.

Because of the strong suppression of the Bhabha scattering due to the angular cuts and the $\varepsilon^4$ suppression, the order percent NP contribution required to shift significantly $a_\mu^{\text{HVP}}$ cannot be obtained from the off-shell exchange of a $V$ boson. However, for $s$-channel on-shell $V$ production the suppression is partly compensated by the resonant nature of the process. The direct consequence is that given a vector boson $V$ of a certain mass, at most one among the above experiments can be affected non-negligibly in its luminosity determination.

Since the KLOE measurement is the main responsible of reducing the data driven value of the HVP, we need to increase $\sigma_{\text{had}}$ around the CoM energy of $\sim 1 \text{ GeV}$. Accordingly, we require a dark photon mass $m_V \sim 1 \text{ GeV}$. In this case NP effects on the luminosity measurement will translate into a large contribution to the hadronic cross section as derived from the KLOE data. On the other hand, for the other experiments the corresponding effects are off-resonance, and we have explicitly checked that they are negligible for the relevant parameter space of our model as expected.

Figure 2 shows the relative enhancement $\delta_R$ of Eq. (2.6) of the inferred Bhabha cross section as a function of the mass of the dark photon. This directly translates into an equal enhancement of the hadronic cross section. The dashed curves are obtained for $m_{\chi_2} = 0.90 m_V$, while the solid curves for $m_{\chi_2} = 0.95 m_V$. The different colors denote different values of the dark coupling $\alpha_D = 0.05$ (blue), 0.1 (yellow) and 0.5 (green). In addition, we fixed $m_{\chi_1} = 25 \text{ MeV}$ and the dark photon couplings to electrons and muons to $\varepsilon_{ee} = 5 \times 10^{-3}$. This value for the couplings is allowed by current experimental results, as we will discuss in section 3.3. The red region corresponds to the values of $\delta_R$ needed to bring the KLOE and BaBar cross sections to agree within $2\sigma$. This also implies that in this region the data-driven results based on the dispersive method agree with the lattice determination of $a_\mu^{\text{HVP}}$ at a level
Figure 2. Relative enhancement $\delta R$ of the inferred Bhabha cross section as a function of $m_V$ for $m_{\chi_2} = 0.95 m_V$ (solid curves) and $m_{\chi_2} = 0.9 m_V$ (dashed curves), and $\alpha_D = 0.05$ (blue), 0.1 (orange) and 0.5 (green). In this plot $m_{\chi_1} = 25 \text{MeV}$ and $\epsilon = 1.65 \times 10^{-2}$. The red shaded area shows the region where the KLOE and BaBar cross sections are compatible at 2σ.

below 2σ, while all the experimental results on $\sigma_{\text{had}}$ remain mutually compatible within $\sim 3\sigma$. In particular, closer to the upper part of the red shaded band, the largest tension ($\sim 3\sigma$) would be between the BESIII and the KLOE results.

3.3 Relevant Constraints

In this section we study the most relevant constraints on the iDM scenario outlined in section 3.1.

**BaBar dark photon searches.** The BaBar experiment made two searches relevant to our case. The first one focuses on resonance in the $e^+e^-$ spectrum with an energetic initial state radiation (ISR) photon [51]. They applied the following selection cuts (with all variables in the CoM frame): $\cos \theta_{e^+} > -0.5$, $\cos \theta_{e^-} < 0.5$, $E_\gamma > 0.2 \text{GeV}$ and $E_\gamma + E_{e^+} + E_{e^-} \simeq 10.58 \text{GeV}$.

The polar angles are defined with respect to the electron beam direction, and the last selection cut requires the centre-of-mass energy of the candidate event to be
within the beam energy spread, that is around $5 - 10$ MeV [52] (see also [53], which quotes the spread $5.5$ (2.7) MeV for the $e^-$ ($e^+$) beam). This search is ineffective in constraining the iDM model because the emission of two $\chi_1$ states always results in missing energy larger than 20 MeV, that is larger than the beam energy spread.

In the second search, BaBar analysed single photon events with large missing energy searching for bumps in the photon spectrum [54] (which superseded the older search for invisible final states in single-photon decays employed in $\Upsilon(1S)$ [55]). We reinterpret the mono-photon sensitivity exploiting the same strategy of [56, 57]. Since this search does not use cuts on simple quantities but a multivariate analysis, we use the following cuts: the events satisfy the mono-photon selection criteria if $E_{\gamma}^{\text{CMS}} > 2$ GeV, $-0.16 \lesssim \cos(\theta_\gamma) \lesssim 0.84$ and $p(e^-) < 150$ MeV or $p(e^+) < 150$ MeV. The expected sensitivity in terms of the kinetic mixing $\varepsilon = g_{\nu V}/e$ for the iDM model is obtained as [56]

$$
\varepsilon_{\text{iDM}}^{\text{exp}} = \varepsilon_{\text{mono-\gamma}}^{\text{exp}} \sqrt{\frac{N_{\gamma \text{ cuts}}}{N_{\text{all}}}},
$$

where $\varepsilon_{\text{mono-\gamma}}^{\text{exp}}$ is the current bound from the BaBar mono-photon analysis [54], $N_{\gamma \text{ cuts}}$ are the number of events selected using only the photon cuts, and $N_{\text{all}}$ are the number of events that pass all the cuts.

Although BaBar results do not constrain significantly our scenario, the semi-visible final states could be probed in the future by Belle-II [56, 58], possibly from longer decay chain involving also the dark Higgs boson [59]. One key factor compared to the analysis in Ref. [56] is that here we are concerned with a parameter space region in which the $V$ decay does not lead to a displaced vertex. A complete background study is thus required to derive a proper projection of the foreseeable constraints on our scenario.

**Effects on the $\Phi$-meson properties.** Since the dark photon mass must be close to the KLOE CoM energy, it will also be close to the $\phi$-meson mass, and thus the two widths $\Gamma_\phi$ and $\Gamma_V$ will overlap, possibly affecting the properties of the $\phi$. In particular, if the dark photon couples universally to all SM fermions proportionally to their electric charge, then its coupling to the strange quark will unavoidably induce $\phi - V$ mixing (instead, in the case of a more specific lepto-philic vector boson, this would not occur, and the analysis below would not apply).

The mixed mass term $M_{V\phi}$ can be derived from the real part of the off-diagonal self-energy of the $(V, \phi)$ system. Assuming $m_V \sim M_\phi$ we have

$$
M_{V\phi} = \frac{1}{2M_\phi} \langle V| \frac{e\varepsilon}{3} V^* s^\gamma s|\phi\rangle = f_\phi \frac{e\varepsilon}{6} \sim 0.25 \text{ MeV} \times \left(\frac{\varepsilon}{0.01}\right),
$$

where $f_\phi$ is the $\phi$ meson decay constant defined as

$$
\langle 0| s^\gamma s|\phi\rangle = f_\phi M_\phi e^{*\rho},
$$
and we have factorised the amplitude and used the fact that mixing can only occur between states with the same polarisation \( \epsilon^* \). The evolution in time of the mixed states is governed by the time-dependent Heisenberg equation:

\[
i \frac{\partial}{\partial t} \left( |\phi\rangle \langle V| \right) = \left( \begin{array}{cc} M_\phi - i \frac{\Gamma_\phi}{2} & M_{V\phi} \\ M_{V\phi} & m_V - i \frac{\Gamma_V}{2} \end{array} \right) \left( |\phi\rangle \langle V| \right).
\]

(3.18)

Mixing effects are strongly suppressed when the diagonal entries in the effective Hamiltonian differ sizeably, that is when \( |M_\phi - m_V| \gg M_{V\phi} \) or \( |\Gamma_\phi - \Gamma_V| \gg M_{V\phi} \).

For instance, in the case \( m_V \approx M_\phi \) but \( \Gamma_V \neq \Gamma_\phi \), the mass difference \( M_2 - M_1 \) between the two mass eigenstates is given by

\[
M_2 - M_1 = \frac{2\pi \alpha_{em}(M_\phi - m_V)\epsilon^2 f_\phi^2}{9(\Gamma_V - \Gamma_\phi)^2}.
\]

(3.19)

For the range of parameters relevant for this work, the mass shift is both smaller than the experimental uncertainty on the \( \phi \) mass (\( M_\phi^{\text{exp}} = 1019.461 \pm 0.016 \text{MeV} \)) and orders of magnitude lower than the uncertainty on the lattice prediction (\( M_\phi^{\text{th}} = 1018 \pm 17 \text{MeV} [60] \)). Similarly, the shifts in the width of the mass eigenstates are safely suppressed by \( \epsilon^2 \).

Another point that we need to check concerns the partial width for \( \phi \) decays into leptons, that is strongly suppressed with respect to hadronic decay channels, and which could be affected by mixing with the \( V \). The experimental value is \( \Gamma_\phi^{\text{exp}} = 1.27 \pm 0.04 \text{keV} \) [61] while recent QCD lattice estimates have a theoretical error of the same order, dominated by the theoretical error on the \( \phi \) meson decay constant \( f_\phi = 241 \pm 9 \pm 2 \text{MeV} \) [61]. It is straightforward to solve numerically Eq. (3.18) and derive the time-dependent evolution of the mixed states: \( |\phi(t)\rangle = a(t)|\phi\rangle + b(t)|V\rangle \), where the kets in the right-hand side corresponds to the states at \( t = 0 \). Since both the mixing term \( b \) and the \( V \) decay width in \( e^+e^- \) are \( \epsilon^- \)-suppressed, the dominant modification to the \( \phi \) leptonic width will arise from the time-integration of the \( |a(t)| \) factor. By taking \( m_V \approx M_\phi \) and \( \Gamma_V \neq \Gamma_\phi \), the change in the leptonic width is well reproduced by the following scaling

\[
\frac{\delta \Gamma_\phi^{\text{ee}}}{\Gamma_\phi^{\text{ee}}} \sim \epsilon^2 \frac{f_\phi}{|\Gamma_\phi - \Gamma_V|}.
\]

(3.20)

For all relevant points in our parameter space, this correction remains safely below the experimental and theoretical uncertainties which are both of a few percent.\(^8\)

\(^8\)The mixing is maximal when \( m_V = M_\phi \) and \( \Gamma_V = \Gamma_\phi \), in which case the mass splitting is simply \( \delta m = 2M_{V\phi} \). This shift is still an order of magnitude lower than the theoretical estimate for \( M_\phi \) [60] so that we cannot derive useful constraints from the mass shift. Note that the splitting is also smaller than \( \Gamma_\phi \), so that at KLOE both \( \phi \) and \( V \) can be simultaneously produced on resonance.

\(^9\)We have cross-checked numerically this result by adding also the direct \( V \rightarrow e^+e^- \) contribution. In this case both the \( \phi \rightarrow e^+e^- \) and \( V \rightarrow e^+e^- \) amplitudes must be estimated and their interference must be included in the time-integrated decay rate. We have found that the correction saturates the experimental uncertainty only for rather large values of kinetic mixing, exceeding \( \sim 0.05 \).
Altogether, we can conclude that both the measurements and the theoretical predictions for \( \phi \)-meson related observables do not have a sufficient level of precision to constrain effectively the new physics scenario outlined above. Eventually, the main reason underlying this conclusion is that both the \( \phi \) and the \( V \) boson have very suppressed leptonic branching ratios, while their main decay channels \( \phi \to \text{hadrons} \) and \( V \to \chi_1\chi_2 \) are to completely different final states.

**KLOE forward-backward asymmetry.** The KLOE collaboration performed an analysis of the forward-backward asymmetry \( A_{FB} \) in \( e^+e^- \to e^+e^- \) around the \( \phi \) resonance in order to extract the leptonic widths \( \Gamma_{\phi ee} \) and \( \Gamma_{\phi \mu\mu} \) and to carry out a test of lepton flavour universality [17]. They measured \( A_{FB} \) for three different CoM energies:

\[
A_{FB}(\sqrt{s}) = \begin{cases} 
0.6275 \pm 0.0003 & (\sqrt{s} = 1017.17 \text{ MeV} \simeq M_{\phi} - \Gamma_{\phi}/2) \\
0.6205 \pm 0.0003 & (\sqrt{s} = 1019.72 \text{ MeV} \simeq M_{\phi}) \\
0.6161 \pm 0.0004 & (\sqrt{s} = 1022.17 \text{ MeV} \simeq M_{\phi} + \Gamma_{\phi}/2) 
\end{cases} 
\tag{3.21}
\]

where all three data-points share a common systematic uncertainty of 0.002. The collaboration then fitted these three experimental values to the interference pattern expected from the \( \phi \) to obtain the measurement \( \Gamma_{\phi ee} = 1.32 \pm 0.05 \pm 0.03 \text{ keV} \). An off-shell \( V \) exchange will also induce an interference pattern in the asymmetry, we can re-interpret this measurement as being a limit on the \( V \) contribution. However, we also must include the direct contribution from the \( e^+e^- \to V \to \chi_1\chi_1 e^+e^- \) process. Let us denote the measured number of forward/backward events as \( N_F \) and \( N_B \). The asymmetry can be decomposed as

\[
A_{FB}(\sqrt{s}) = \frac{N_F - N_B}{N_F + N_B} = A_{FB}^{\text{bhabha}} \times (1 - \delta_R(\sqrt{s})) + A_{FB}^{\phi} + A_{FB}^{V}. \tag{3.23}
\]

The first term of the second line includes a correction which is due to the fact that the process \( e^+e^- \to V \to \chi_1\chi_1 e^+e^- \) contributes to the denominator of the Bhabha asymmetry while, since it has a negligible asymmetry, it does not contribute to the numerator. The second and third contributions correspond to the interference between virtual photon exchange and respectively the \( \phi \) and the \( V \) vector boson. Since the \( V \) is produced resonantly, the correction \( \delta_R \) depends strongly on \( \sqrt{s} \) and on the value of the \( V \) width. The effect of \( \delta_R \) is to reduce the asymmetry, while both the interference terms contribute positively.

In order to assess the complete \( V \) contribution, we have simulated the full process \( e^+e^- \to e^+e^- \) in \textsc{MadGraph5}\_\textsc{aMC@NLO}, implementing the cuts given in [17]. It is clear that the fit performed by the collaboration cannot be applied to the scenario at hand with multiple interference terms plus the “inverse resonance” effect. Hence
we instead adopt a more conservative proxy, the difference between the lowest and the highest measurements of \( A_{FB} \) in \( \sqrt{s} \):

\[
\Delta A_{FB} \equiv \frac{A_{FB}(M_{\phi} - \Gamma_{\phi}/2) - A_{FB}(M_{\phi} + \Gamma_{\phi}/2)}{A_{FB}(M_{\phi} - \Gamma_{\phi}/2) + A_{FB}(M_{\phi} + \Gamma_{\phi}/2)}
\]

\[
= \Delta A_{FB}^{\phi} + \Delta A_{FB}^{V} - \frac{\delta R(\sqrt{s^-}) - \delta R(\sqrt{s^+})}{2},
\]

where \( s^\pm \simeq M_{\phi} \pm \Gamma_{\phi}/2 \) are the CoM energies of the KLOE measurements, cf. Eq. (3.21). KLOE measured \( \Delta A_{FB}^{\text{exp}} = (9.17 \pm 0.35) \cdot 10^{-3} \). In either the vector meson dominance\(^{10}\) or in a simple factorisation approach, the \( \phi \)-mediated interference cross-section can be obtained as

\[
\sigma_{\text{int}} = \frac{3\alpha_{\text{em}}\Gamma_{\phi}\epsilon}{M_{\phi}} \frac{s - M_{\phi}^2}{(s - M_{\phi}^2)^2 + s\Gamma_{\phi}^2} \int_{c_{\text{min}}}^{c_{\text{max}}} dc_\theta \left[ \pi \left( c_\theta^2 - \frac{(c_\theta + 1)^2}{1 - c_\theta} + 1 \right) \right],
\]

where \( c_\theta \) is the outgoing electron angle and \( c_{\text{min}}, c_{\text{max}} \) are either the acceptance or 0 for the forward/backward case. Estimating the theoretical uncertainties on this expression is delicate, although it is clear that the theoretical prediction for \( \Gamma_{\phi} \) plays the leading role. We can get an estimate of this effect by using the lattice result \( f_\phi \). The most recent estimate [60] gives \( f_\phi = 241 \pm 9 \pm 2 \text{ MeV} \). However, an earlier lattice estimate [66] found \( f_\phi = 308 \pm 29 \text{ MeV} \) which, in spite of the much larger error, differs from the result of Ref. [60] by more than 2\( \sigma \). Another earlier estimate [67] found a central value in agreement with Ref. [60] but with twice the error \( f_\phi = 241 \pm 18 \text{ MeV} \). This latter uncertainty would correspond to a \( 1.8 \cdot 10^{-3} \) uncertainty on \( \Delta A_{FB} \), while the difference between the central values of Refs. [60] and [66] would translate into an uncertainty of \( \approx 6.7 \cdot 10^{-3} \).

From our simulations, we obtain a theoretical prediction for the contribution from Bhabha and the \( \phi \) resonance of \( \Delta A_{FB}^{\text{th}} = 12.0 \cdot 10^{-3} \) which also differs from the experimental measurement by \( 2.8 \cdot 10^{-3} \). Altogether, it is clear that additional theoretical input on the \( \phi \) interference contribution would be required to match the experimental precision. We will therefore present in the rest of this work both a conservative estimate based on the discrepancy between the lattice estimate of Refs. [60] and [66] (corresponding to a \( \sim 0.5 \cdot 10^{-3} \) error), and a more aggressive limit based only on the difference between our estimate of the SM \( \phi \) contribution and the measurement (\( \sim 3 \cdot 10^{-3} \)). If we neglect the interference term and consider the small width regime \( \Gamma_V < \Gamma_{\phi} \), then for the case of maximum negative shift, \( M_V \simeq \sqrt{s^-} \), we have \( \Delta A_{FB}^{\text{NP}} \simeq -\delta R(\sqrt{s^-})/2 \). For the “aggressive” limit this translates into a maximum shift on the luminosity of \( \delta R \simeq 1.2\% \). As shown in Fig. 2, this still allows to bring KLOE in agreement within 2\( \sigma \) with BaBar.

\(^{10}\)See e.g. Ref. [62] for more details on the Vector Meson Dominance approach (VMD) which has been widely used in recent dark sector literature [63-65] for the \( \phi \) and dark photon-related amplitudes.
In practice, the interference contribution is also important and allows for somewhat larger shifts due to cancellations between both terms, as is confirmed by the full numerical results. Furthermore, the above limit is strongly reduced for larger width with a $\Gamma_\phi^2/\Gamma_V^2$ suppression of $\Delta A_{FB}^{NP}$ when $\Gamma_\phi \ll \Gamma_V$. Finally, while the analysis in Ref. [17] relied on a very precise calibration of the CoM energy, this was not the case for the study used to derive the $a_\mu^{HVP}$ KLOE contribution. As can be seen e.g. from Fig. 7 of Ref. [48], the spread in the CoM energy is larger, of the order of MeV, and this is related to the method used to measure the hadronic cross-section via ISR. Since we are primarily interested in this measurement, then we need to consider an uncertainty on $m_V$ of the same order.\footnote{This prevents us to use the asymmetries corresponding to the $\sqrt{s_0} \simeq M_\phi$ bin, since the two points are separated by an energy interval of the order of the uncertainty on $m_V$.}

Although limited by theoretical uncertainty, the forward-backward asymmetry measurement can provide significant constraints on our scenario. This mostly occurs because in the particular iDM model we have considered, the $V$ width (see Eq. (3.11)) is always in the MeV range. It would certainly be interesting to have a more complete experimental dataset, leveraging this observable to constrain this type of physics, for example from the CMD-3 experiment [15].

Finally, we did not include possible limits from the value of the $\phi$ partial width into muons reported in the same reference [17]. This is because firstly $\Gamma_{\phi\mu\mu}$ was inferred from cross-sections measurements only and thus has significantly larger experimental uncertainties, and moreover it is also sensitive to the Bhabha luminosity shift we have described above. Secondly, it depends on the $V$ coupling to muons, which do not directly enter our mechanism to shift $a_\mu^{HVP}$, and also on the mass $m_{\chi_1}$ of the lightest dark matter particle, since for the $\mu^+\mu^-$ channel the experimental cuts on the missing energy are significantly tighter. Similar considerations can be applied to the estimates of $\Gamma_{\phi\mu\mu}$ derived from fitting measurements of cross-sections into hadronic final states.

**Indirect effects on LEP precision measurements.** A new vector boson of $\sim 1$ GeV mass can give rise to different types of indirect effects on LEP electroweak precision measurements. A first effect is a modification of the value of the electromagnetic coupling constant extrapolated at the large CoM energy scale of the LEP $e^+e^-$ collisions, that can be written as
\[
\alpha(s) = \frac{\alpha}{1 - \Delta\alpha_\ell(s) - \Delta\alpha_{\text{top}}(s) - \Delta\alpha_{\text{had}}^{(5)}(s)},
\] (3.27)
where $\Delta\alpha_\ell$ and $\Delta\alpha_{\text{top}}$ are the contribution to the photon vacuum polarization from the leptons and the top quark, which can be computed perturbatively with good accuracy, while the five-flavor hadronic contribution $\Delta\alpha_{\text{had}}^{(5)}$ has to be extracted from
data, and is determined by the dispersion relation

$$\Delta \alpha_{\text{had}}^{(5)}(s) = \frac{s}{4\pi^2\alpha} P \int ds' \frac{\sigma_{\text{had}}(s')}{s - s'}, \quad (3.28)$$

where $P$ denotes the integral principal value. The important difference with the otherwise similar expression in Eq. (2.1) is the $1/s$ factor in that equation which implies that low energy data play a dominant role for $\alpha_\mu$, while e.g. for $s \approx M_Z^2$ the low energy contribution to the integral in Eq. (3.28) is much less relevant. Hence, an increased value of the KLOE result for $\sigma_{\text{had}}$ in the [0.6,0.9] GeV energy range is unlikely to affect the overall agreement with the LEP electroweak precision data (see also the dedicated analysis in Ref. [68]).

Additional subtle effects are related to corrections to luminosity measurements. The importance of a reliable determination of the LEP luminosity is well exemplified by the recent reassessment of the LEP measurement of the number of light active neutrino species $N_\nu$ [69]. During the first phase (LEP-1) high statistics data were collected at and around the $Z$ pole, providing a wealth of measurements with sub-percent precision [70]. In particular, the value of $N_\nu$ was estimated from the measurement of the hadronic peak cross section $\sigma_{\text{peak, had}}$ by means of the relation

$$N_\nu \left( \frac{\Gamma_{\nu, SM}^{\ell}}{\Gamma_{\nu, SM}^{\ell}} \right) = \left( \frac{12\pi}{m_\tau^2} \frac{R_\ell^0}{\sigma_{\text{peak, had}}} \right) - R_\ell^0 - 3 - \delta_\tau \quad (3.29)$$

where $\Gamma_{\nu, SM}^{\ell}$ are the SM predictions for the partial $Z$ decay width into neutrinos and (massless) charged leptons, $R_\ell^0$ is the ratio between the $Z$ branching fractions into hadrons and leptons and $\delta_\tau \sim 2.26 \times 10^{-3}$ accounts for a small correction from finite $m_\tau$ effects on the $\Gamma_{\nu, SM}^{\ell}$ partial width. The combination of the measurements made by the four LEP experiments [70] led to: $N_\nu = 2.9840 \pm 0.0082$ which had a $2\sigma$ tension with the canonical SM value $N_\nu^{SM} = 3$. The measurement is directly affected by biased errors in estimating the integrated luminosity only through $\sigma_{\text{peak, had}}$, all other quantities in Eq. (3.29) instead do not depend on the absolute luminosity. Indeed, the uncertainty on the integrated luminosity represents the largest contribution to the uncertainty on $N_\nu$. The LEP luminosity was determined by comparing the measured rate of Bhabha-scattering process at small angles with the SM prediction. Recently this determination has been reassessed first by correcting for a bias in the luminosity measurement due to the large charge density of the particle bunches which modify (decrease) the effective acceptance of the luminometer [69], and next by using an updated and more accurate prediction of the Bhabha cross section which is found to reduce its value by about 0.048% [71]. Both effects go in the direction of decreasing the Bhabha cross section with respect to that used in the LEP fit [70], thus increasing the effective luminosity, decreasing $\sigma_{\text{had}}^0$, and eventually raising the inferred value of active neutrino species to $N_\nu = 2.9963 \pm 0.0074$, in perfect agreement with the SM.
The direct effect of a (constructive) NP contribution to the Bhabha cross section would instead go in the opposite direction, decreasing the estimated luminosity, and hence increasing $\sigma_{\text{peak}}$ had and reducing $N_\nu$. However, at LEP energies the $m_V \sim 1$ GeV vector boson grossly behaves like a massive photon with $\varepsilon$-suppressed couplings to electrons/positrons. The most relevant effect will then come from $t$-channel $\gamma$-$V$ interference, which in our case is suppressed by a relative $\varepsilon^2 \sim 10^{-4}$, and thus negligible. An additional indirect effect is again related to adjustments in the value of the hadronic vacuum polarization that enters the $t$-channel photon propagator, and that could modify the value of the $\alpha(t)$ input to the LEP Bhabha event generators [30]. Yet, given that at the small Bhabha scattering angles/small momentum transfer ($\theta \lesssim 60$ mrad, $t \lesssim (2.8 \text{ GeV})^2$) specific of the LEP luminosity measurements the hadronic contribution remains small ($\Delta \alpha^{(5)}_{\text{had}} \lesssim 0.008$ and at most $\sim 30\%$ of the total vacuum polarization, see e.g. Ref. [30]) the corresponding correction remains below the systematic uncertainties. It is interesting to note that, although both the direct contribution of $V$ to Bhabha scattering and the indirect effect of increasing the value of $\Delta \alpha^{(5)}_{\text{had}}$ affect negligibly the LEP luminosity measurement, they go in the same direction and, for example, both would tend to decrease the central value of $N_\nu$.

**LEP limit on $V-Z$ mixing** In the inelastic dark matter model, the interaction between the dark photon and the SM particles proceed via kinetic mixing between the $U(1)_D$ and the $U(1)_Y$ field strength. This implies that after electroweak symmetry breaking, the $Z$ boson mixes with the dark photon, leading to a small modification of the SM electroweak coupling. A complete fit for this effect in the electroweak precision observables was performed in [72, 73] leading to the relatively model independent bound:

$$\varepsilon < 0.027 \quad (\text{LEP - EW fit}) \quad (3.30)$$

### 3.4 Joint solution to the $a_\mu$-related anomalies

In this section we study in which range of parameters for the model outlined in section 3.1 the indirect Bhabha-induced effects on $a_\mu$, together with the direct loop contributions of the new vector boson $V$ can optimally resolve the various $a_\mu$ related discrepancies.

First we present in Fig. 3 the parameter space in which one can obtain a simultaneous fit to both the BaBar/KLOE discrepancy and the $a_\mu$ at the $2\sigma$-level. The blue region denotes the values of $\varepsilon$ where the KLOE result is within $2\sigma$ of the BaBar measurement, while the green region is the area where $a_\mu$ fit the experimental result at the $2\sigma$-level (with both the shift in data-driven estimate and the new physics contribution included). As expected from the resonant nature of the production mechanism, the shift in KLOE data brings it in agreement with BaBar only in a narrow region around the $\phi$ mass. Depending on the strength of the shift, it
Figure 3. Parameter range compatible at 2σ with the experimental measurement of \( \Delta a_\mu \) (green region) resulting from a redetermination of the KLOE luminosity, for \( \alpha_D = 0.5, m_{\chi_2} = 0.95 m_V \) and \( m_{\chi_1} = 25 \) MeV. In the blue region the KLOE and BaBar results for \( \sigma_{\text{had}} \) are brought into agreement at 2σ. The red region corresponds to a shift of the KLOE measurement in tension with BaBar (and with the other experiments) at more than 2σ. The limit from the electroweak fit at LEP (gray band), the projection for LHC run-3 [73] (violet dashed line), and the recasting of the BaBar limit [51] (orange band) are also shown (see text). The hatched magenta region corresponds to the conservative 2σ exclusion using the \( \Delta A_{FB} \) measurement as presented in section 3.3, while the dashed magenta line is the “aggressive” exclusion.

In Fig. 4 we show in purple the \( \pm 1\sigma \) band for the theoretical prediction of \( a_\mu \) as a function of \( \varepsilon \) for \( m_{\chi_1} = 25 \) MeV, \( m_{\chi_2} = 0.9 \) GeV, \( m_V = 1.019 \) GeV and \( \alpha_D = 0.5 \). Since our analysis only includes the correction to \( a_\mu^{\text{HVP}} \) in the \( \sqrt{s} \in [0.6, 0.9] \) GeV range, we use a theoretical uncertainty on our result of 35% to account for the missing contribution. The dashed curve denotes the values of \( \varepsilon \) where the KLOE...
Figure 4. Theoretical prediction (purple) for $a_\mu$ as a function of $\varepsilon$ for a dark photon model with $m_{\chi_1} = 25$ MeV, $m_{\chi_2} = 0.9$ GeV, $m_V = 1.019$ GeV and $\alpha_D = 0.5$. The dashed purple curve denotes the region where the KLOE and BaBar results are more than $2\sigma$ away. The blue band corresponds to the combined BNL and FNAL experimental results after subtracting the direct NP contribution from the dark photon. The red band shows the prediction obtained with the BMW lattice estimate of $a_\mu^{\text{LO}, \text{HVP}}$. The width of the bands represents $1\sigma$ uncertainties. The grey region is excluded by LEP.

result is above $2\sigma$ of the BaBar measurement. The red region shows the $\pm 1\sigma$ BMW-lattice computation and the blue band shows the $\pm 1\sigma$ band for the BNL and FNAL experimental results after subtracting the direct contribution to $a_\mu$. The grey area is excluded by a fit to the electroweak SM couplings at LEP, see eq. (3.30). Therefore, this figure shows the interplay between the contribution from the modification of the luminosity of KLOE (purple curve) and the direct NP one (blue curve) needed to bring the experimental result and the lattice and data-driven computation to be compatible below $\sim 2\sigma$. The figure demonstrates also that with this choice of the model parameters it is possible to have a complete agreement between the experimental result and the data-driven computation, having a discrepancy of only $\sim 1\sigma$ with the lattice computation.

4 Conclusions

In this work, we have explored the intriguing possibility that a feebly interacting particle with mass around the KLOE CoM energy, produced in $e^+e^-$ collisions and contributing to the measured number of final state $e^+e^-$ events, could significantly affect the luminosity estimate of this experiment. Such a possibility implies a cascade
of related effects, eventually increasing the data-driven estimate of $a_H^{\mu}$ by a few percent. We find that while the indirect effects of the luminosity re-determination cannot fully account for the discrepancy between the theoretical prediction for $a_\mu$ and the experimental measurement, it suffices to solve the tension between the KLOE and BaBar experimental determinations of $\sigma_{\text{had}}$, and to reconcile the estimate of the hadronic vacuum polarisation contribution $a_H^{\mu}$ based on the data-driven dispersive method, with the latest lattice calculation, and with the estimate of $a_H^{\mu}$ from $\tau$ hadronic decays.

We have constructed an explicit model based on an iDM paradigm which includes a vector particle mediator with a $\sim 1$ GeV mass, that can realise this scenario. The main decay of the vector mediator proceeds via a semi-visible channel, and can mimic effectively the Bhabha signature used to calibrate the experimental luminosity. However, in order to give rise to a sizeable effect, the dark photon mass must lie close to the KLOE CoM energy, so that it can be produced resonantly. Although the dark photon and the $\phi$ meson masses will then have rather close values, we have found that this does not yield strong enough constraints to rule out the model. As mentioned above, the luminosity related indirect effects by themselves are not sufficient to solve completely the discrepancy between the experimental determination of $a_\mu$ and the prediction. However, the vector boson mediator will also contribute directly to $a_\mu$ via genuine NP loop contributions, and we find that when both effects are considered together, the $a_\mu$ discrepancy can indeed be solved (with about $2/3$ of the effect from loops and $1/3$ from the shift in the KLOE luminosity).

The simple model that we have put forth also provides an adequate light dark matter candidate with a rich phenomenology which might be worthwhile exploring further. On the long run, we hope that our work can provide further motivations to the search for similar “stealthy” dark photons characterised by the required semi-visible decays, especially considering the fact that to shift significantly the KLOE luminosity the required couplings cannot be too suppressed. Eventually, since the most important effect in our construction is that of shifting $a_H^{\mu}$ to larger values, a crucial test of the whole idea could come from the MuonE experiment [74, 75], as well as from new high precision determinations of $a_H^{\mu}$ on the lattice.

Finally, although we have derived our numerical result by focusing on the peculiar case of an iDM scenario, we stress that the mechanism of accommodating the $(g - 2)_\mu$ anomalies by hypothesising a new physics effect in the KLOE luminosity determination is completely generic. The only requirement is that the final state should contribute sufficiently to large angle Bhabha scattering, while at the same time being characterised by a multi-body final state yielding certain amount of missing energy, in order to escape standard bump searches.
Note added  As this paper was being finalised, the work [76] appeared which studied the possibility of reconciling the data driven and the lattice determinations of $a_{\mu}^{\text{HVP}}$ by invoking a light NP mediator that directly modifies $\sigma_{\text{had}}$. The authors concluded that such a possibility is excluded by a number of experimental constraints. The NP effect explored in this work is of a completely different nature. As we have shown, it can not only reconcile the data driven and lattice determination of $a_{\mu}^{\text{HVP}}$, but it can also bring into agreement the KLOE and BaBar results for $\sigma_{\text{had}}$.

Acknowledgments

We thank M. Raggi and T. Spadaro for discussions, and F. Piccinini and C.M. Carloni Calame for providing us with details on the BabaYaga event generator. LD thanks Olcyr Sumensari and Diego Guadagnoli for useful discussions. This work has received support by the INFN Iniziativa Specifica Theoretical Astroparticle Physics (TAsP). G.G.d.C. is supported by the Frascati National Laboratories (LNF) through a Cabibbo Fellowship, call 2019. This project has received funding from the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 101028626.

References

[1] Muon g-2 collaboration, Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm, Phys. Rev. Lett. 126 (2021) 2021 [2104.03281].

[2] Muon g-2 collaboration, Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL, Phys. Rev. D 73 (2006) 072003 [hep-ex/0602035].

[3] T. Aoyama et al., The anomalous magnetic moment of the muon in the Standard Model, Phys. Rept. 887 (2020) 1 [2006.04822].

[4] KLOE-2 collaboration, Combination of KLOE $\sigma (e^+e^- \rightarrow \pi^+\pi^-\gamma(\gamma))$ measurements and determination of $a_{\mu}^{\pi^+\pi^-}$ in the energy range 0.10 < s < 0.95 GeV$^2$, JHEP 03 (2018) 173 [1711.03085].

[5] BABAR collaboration, Precise measurement of the $e^+e^- \rightarrow \pi^+\pi^- (\gamma)$ cross section with the Initial State Radiation method at BABAR, Phys. Rev. Lett. 103 (2009) 231801 [0908.3589].

[6] CMD-2 collaboration, Reanalysis of hadronic cross-section measurements at CMD-2, Phys. Lett. B 578 (2004) 285 [hep-ex/0308008].
[7] V. M. Aul’chenko et al., Measurement of the $e^+e^- \rightarrow \pi^+\pi^-$ cross section with the CMD-2 detector in the 370 - 520-MeV c.m. energy range, JETP Lett. 84 (2006) 413 [hep-ex/0610016].

[8] CMD-2 collaboration, High-statistics measurement of the pion form factor in the rho-meson energy range with the CMD-2 detector, Phys. Lett. B 648 (2007) 28 [hep-ex/0610021].

[9] M. N. Achasov et al., Update of the $e^+e^- \rightarrow \pi^+\pi^-$ cross-section measured by SND detector in the energy region 400-MeV < s^{1/2} < 1000-MeV, J. Exp. Theor. Phys. 103 (2006) 380 [hep-ex/0605013].

[10] BESIII collaboration, Measurement of the $e^+e^- \rightarrow \pi^+\pi^-$ cross section between 600 and 900 MeV using initial state radiation, Phys. Lett. B 753 (2016) 629 [1507.08188].

[11] S. Borsanyi et al., Leading hadronic contribution to the muon magnetic moment from lattice QCD, Nature 593 (2021) 51 [2002.12347].

[12] A. Keshavarzi, K. S. Khaw and T. Yoshioka, Muon $g - 2$: current status, 2106.06723.

[13] C. Lehner and A. S. Meyer, Consistency of hadronic vacuum polarization between lattice QCD and the R-ratio, Phys. Rev. D 101 (2020) 074515 [2003.04177].

[14] A. Gérardin, The anomalous magnetic moment of the muon: status of Lattice QCD calculations, Eur. Phys. J. A 57 (2021) 116 [2012.03931].

[15] A. E. Ryzhenenkov et al., Overview of the CMD-3 recent results, J. Phys. Conf. Ser. 1526 (2020) 012009.

[16] A. Berlin and F. Kling, Inelastic Dark Matter at the LHC Lifetime Frontier: ATLAS, CMS, LHCb, CODEX-b, FASER, and MATHUSLA, Phys. Rev. D 99 (2019) 015021 [1810.01879].

[17] KLOE collaboration, Measurement of the leptonic decay widths of the phi-meson with the KLOE detector, Phys. Lett. B 608 (2005) 199 [hep-ex/0411082].

[18] S. J. Brodsky and E. De Rafael, SUGGESTED BOSON - LEPTON PAIR COUPLINGS AND THE ANOMALOUS MAGNETIC MOMENT OF THE MUON, Phys. Rev. 168 (1968) 1620.

[19] B. E. Lautrup and E. De Rafael, Calculation of the sixth-order contribution from the fourth-order vacuum polarization to the difference of the anomalous magnetic moments of muon and electron, Phys. Rev. 174 (1968) 1835.

[20] M. Davier, A. Hoecker, B. Malaescu and Z. Zhang, A new evaluation of the hadronic vacuum polarisation contributions to the muon anomalous magnetic moment and to $\alpha(m_Z^2)$, Eur. Phys. J. C 80 (2020) 241 [1908.00921].

[21] A. Keshavarzi, D. Nomura and T. Teubner, $g - 2$ of charged leptons, $\alpha(M_Z^2)$, and the hyperfine splitting of muonium, Phys. Rev. D 101 (2020) 014029 [1911.00367].
[22] G. Colangelo, M. Hoferichter and P. Stoffer, *Two-pion contribution to hadronic vacuum polarization*, *JHEP* 02 (2019) 006 [1810.00007].

[23] F. Jegerlehner, *The Anomalous Magnetic Moment of the Muon*, vol. 274. Springer, Cham, 2017, 10.1007/978-3-319-63577-4.

[24] M. Benayoun, L. Delbuono and F. Jegerlehner, *BHLS2, a New Breaking of the HLS Model and its Phenomenology*, *Eur. Phys. J. C* 80 (2020) 81 [1903.11034].

[25] B. Ananthanarayan, I. Caprini and D. Das, *Pion electromagnetic form factor at high precision with implications to $a_\mu^{\pi\pi}$ and the onset of perturbative QCD*, *Phys. Rev. D* 98 (2018) 114015 [1810.09265].

[26] M. Davier, A. Hoecker, B. Malaescu and Z. Zhang, *Reevaluation of the Hadronic Contributions to the Muon g-2 and to $\alpha(MZ)$*, *Eur. Phys. J. C* 71 (2011) 1515 [1010.4180].

[27] M. Davier, A. Hoecker, G. Lopez Castro, B. Malaescu, X. H. Mo, G. Toledo Sanchez et al., *The Discrepancy Between tau and $e^+e^-$ Spectral Functions Revisited and the Consequences for the Muon Magnetic Anomaly*, *Eur. Phys. J. C* 66 (2010) 127 [0906.5443].

[28] M. Davier, A. Höcker, B. Malaescu, C.-Z. Yuan and Z. Zhang, *Update of the ALEPH non-strange spectral functions from hadronic $\tau$ decays*, *Eur. Phys. J. C* 74 (2014) 2803 [1312.1501].

[29] M. Bruno, T. Izubuchi, C. Lehner and A. Meyer, *On isospin breaking in $\tau$ decays for $(g-2)_\mu$ from Lattice QCD*, *PoS LATTICE2018* (2018) 135 [1811.00508].

[30] S. Jadach et al., *Event generators for Bhabha scattering*, in *CERN Workshop on LEP2 Physics (followed by 2nd meeting, 15-16 Jun 1995 and 3rd meeting 2-3 Nov 1995)*, 2, 1996, hep-ph/9602393, DOI.

[31] A. B. Arbuzov, G. V. Fedotovich, F. V. Ignatov, E. A. Kuraev and A. L. Sibidanov, *Monte-Carlo generator for $e^+e^-$ annihilation into lepton and hadron pairs with precise radiative corrections*, *Eur. Phys. J. C* 46 (2006) 689 [hep-ph/0504233].

[32] G. Balossini, C. M. Carloni Calame, G. Montagna, O. Nicrosini and F. Piccinini, *Matrix elements and Parton Shower in the event generator BABAYAGA*, *Nucl. Phys. B Proc. Suppl.* 162 (2006) 59 [hep-ph/0610022].

[33] B. Holdom, *Two U(1)’s and Epsilon Charge Shifts*, *Phys. Lett. B* 166 (1986) 196.

[34] P. Fayet, *The light U boson as the mediator of a new force, coupled to a combination of Q, B, L and dark matter*, *Eur. Phys. J. C* 77 (2017) 53 [1611.05357].

[35] L. Darmé, S. Rao and L. Roszkowski, *Light dark Higgs boson in minimal sub-GeV dark matter scenarios*, *JHEP* 03 (2018) 084 [1710.08430].

[36] L. Darmé, S. Rao and L. Roszkowski, *Signatures of dark Higgs boson in light fermionic dark matter scenarios*, *JHEP* 12 (2018) 014 [1807.10314].
[37] G. N. Wojcik and T. G. Rizzo, Forbidden Scalar Dark Matter and Dark Higgses, 2109.07369.

[38] P. Asadi, T. R. Slatyer and J. Smirnov, WIMPs Without Weakness: Generalized Mass Window with Entropy Injection, 2111.11444.

[39] A. Fradette and M. Pospelov, BBN for the LHC: constraints on lifetimes of the Higgs portal scalars, Phys. Rev. D 96 (2017) 075033 [1706.01920].

[40] M. Kawasaki, K. Kohri, T. Moroi and Y. Takaesu, Revisiting Big-Bang Nucleosynthesis Constraints on Long-Lived Decaying Particles, Phys. Rev. D 97 (2018) 023502 [1709.01211].

[41] N. D. Christensen, P. de Aquino, C. Degrande, C. Duhr, B. Fuks, M. Herquet et al., A Comprehensive approach to new physics simulations, Eur. Phys. J. C 71 (2011) 1541 [0906.2474].

[42] C. Degrande, C. Duhr, B. Fuks, D. Grellscheid, O. Mattelaer and T. Reiter, UFO - The Universal FeynRules Output, Comput. Phys. Commun. 183 (2012) 1201 [1108.2040].

[43] A. Alloul, N. D. Christensen, C. Degrande, C. Duhr and B. Fuks, FeynRules 2.0 - A complete toolbox for tree-level phenomenology, Comput. Phys. Commun. 185 (2014) 2250 [1310.1921].

[44] J. Alwall, R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer et al., The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations, JHEP 07 (2014) 079 [1405.0301].

[45] B. Batell, J. Berger, L. Darme and C. Frugiuele, Inelastic dark matter at the Fermilab Short Baseline Neutrino Program, Phys. Rev. D 104 (2021) 075026 [2106.04584].

[46] CMD-2 collaboration, Measurement of $e^+e^-\rightarrow \pi^+\pi^-$ cross-section with CMD-2 around rho meson, Phys. Lett. B 527 (2002) 161 [hep-ex/0112031].

[47] M. N. Achasov et al., Study of the process $e^+e^-\rightarrow \pi^+\pi^-$ in the energy region 400 $<s^{1/2}<1000$-MeV, J. Exp. Theor. Phys. 101 (2005) 1053 [hep-ex/0506076].

[48] KLOE collaboration, Measurement of the DAFNE luminosity with the KLOE detector using large angle Bhabha scattering, Eur. Phys. J. C 47 (2006) 589 [hep-ex/0604048].

[49] BaBar collaboration, Time-Integrated Luminosity Recorded by the BABAR Detector at the PEP-II $e^+e^-$ Collider, Nucl. Instrum. Meth. A 726 (2013) 203 [1301.2703].

[50] BESIII collaboration, Measurement of the integrated luminosities of the data taken by BESIII at $\sqrt{s}=3.650$ and 3.773 GeV, Chin. Phys. C 37 (2013) 123001 [1307.2022].

[51] BaBar collaboration, Search for a Dark Photon in $e^+e^-$ Collisions at BaBar, Phys. Rev. Lett. 113 (2014) 201801 [1406.2980].
[52] BABAR collaboration, *The BABAR Detector: Upgrades, Operation and Performance*, *Nucl. Instrum. Meth. A* **729** (2013) 615 [1305.3560].

[53] J. Seeman et al., *Status report on PEP-II performance*, in 7th European Particle Accelerator Conference (EPAC 2000), 8, 2002.

[54] BABAR collaboration, *Search for Invisible Decays of a Dark Photon Produced in e^+e^- Collisions at BaBar*, *Phys. Rev. Lett.* **119** (2017) 131804 [1702.03327].

[55] BABAR collaboration, *Search for Production of Invisible Final States in Single-Photon Decays of Υ(1S)*, *Phys. Rev. Lett.* **107** (2011) 021804 [1007.4646].

[56] M. Duerr, T. Ferber, C. Hearty, F. Kahlhoefer, K. Schmidt-Hoberg and P. Tunney, *Invisible and displaced dark matter signatures at Belle II*, *JHEP* **02** (2020) 039 [1911.03176].

[57] M. Duerr, T. Ferber, C. Garcia-Cely, C. Hearty and K. Schmidt-Hoberg, *Long-lived Dark Higgs and Inelastic Dark Matter at Belle II*, *JHEP* **04** (2021) 146 [2012.08595].

[58] BELLE-II collaboration, *The Belle II Physics Book*, *PTEP* **2019** (2019) 123C01 [1808.10567].

[59] S. Dreyer et al., *Physics reach of a long-lived particle detector at Belle II*, 2105.12962.

[60] χQCD collaboration, *Charmed and φ meson decay constants from 2+1-flavor lattice QCD*, *Chin. Phys. C* **45** (2021) 023109 [2008.05208].

[61] Particle Data Group collaboration, *Review of Particle Physics*, *PTEP* **2020** (2020) 083C01.

[62] T. Fujiwara, T. Kugo, H. Terao, S. Uehara and K. Yamawaki, *Nonabelian Anomaly and Vector Mesons as Dynamical Gauge Bosons of Hidden Local Symmetries*, *Prog. Theor. Phys.* **73** (1985) 926.

[63] S. Tulin, *New weakly-coupled forces hidden in low-energy QCD*, *Phys. Rev. D* **89** (2014) 114008 [1404.4370].

[64] P. Ilten, Y. Soreq, M. Williams and W. Xue, *Serendipity in dark photon searches*, *JHEP* **06** (2018) 004 [1801.04847].

[65] L. Darmé, L. Di Luzio, M. Giannotti and E. Nardi, *Selective enhancement of the QCD axion couplings*, 2010.15846.

[66] ETM collaboration, *Meson masses and decay constants from unquenched lattice QCD*, *Phys. Rev. D* **80** (2009) 054510 [0906.4720].

[67] HPQCD collaboration, *Vcs from Ds → φℓν semileptonic decay and full lattice QCD*, *Phys. Rev. D* **90** (2014) 074506 [1311.6669].

[68] A. Keshavarzi, W. J. Marciano, M. Passera and A. Sirlin, *Muon g−2 and Δα connection*, *Phys. Rev. D* **102** (2020) 033002 [2006.12666].
[69] G. Voutsinas, E. Perez, M. Dam and P. Janot, *Beam-beam effects on the luminosity measurement at LEP and the number of light neutrino species*, Phys. Lett. B 800 (2020) 135068 [1908.01704].

[70] ALEPH, DELPHI, L3, OPAL, SLD, LEP ELECTROWEAK WORKING GROUP, SLD ELECTROWEAK GROUP, SLD HEAVY FLAVOUR GROUP collaboration, *Precision electroweak measurements on the Z resonance*, Phys. Rept. 427 (2006) 257 [hep-ex/0509008].

[71] P. Janot and S. Jadach, *Improved Bhabha cross section at LEP and the number of light neutrino species*, Phys. Lett. B 803 (2020) 135319 [1912.02067].

[72] A. Hook, E. Izaguirre and J. G. Wacker, *Model Independent Bounds on Kinetic Mixing*, Adv. High Energy Phys. 2011 (2011) 859762 [1006.0973].

[73] D. Curtin, R. Essig, S. Gori and J. Shelton, *Illuminating Dark Photons with High-Energy Colliders*, JHEP 02 (2015) 157 [1412.0018].

[74] G. Abbiendi et al., *Measuring the leading hadronic contribution to the muon g-2 via μe scattering*, Eur. Phys. J. C 77 (2017) 139 [1609.08987].

[75] MUONE collaboration, *Status of the MUonE experiment*, PoS ICHEP2020 (2021) 223 [2012.07016].

[76] L. Di Luzio, A. Masiero, P. Paradisi and M. Passera, *New physics behind the new muon g-2 puzzle?*, 2112.08312.