Calculating the Finite-Speed-of-Light Effect in Atom Gravimeters with General Relativity

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This work mainly presents a relativistic analytical calculating method for the finite speed-of-light effect in atom gravimeters, which can simplify the derivation and give a more complete expression for the associated correction.

1. Introduction

The finite-speed-of-light (FSL) effect has been studied by other research groups, and they gave different results.\textsuperscript{1,2} In this article, we present an analytical relativistic study method to recalculate this effect.

2. Starting point of the method

In an atom-gravimeter system, the total phase shift can be written as the sum of three components: \( \Delta \phi_{\text{tot}} = \Delta \phi_{\text{propagation}} + \Delta \phi_{\text{laser}} + \Delta \phi_{\text{separation}} \).\textsuperscript{2} For calculating the propagation phase shift \( \Delta \phi_{\text{propagation}} \), one should first perform integrals of the lagrangian along the upper and lower paths over time to obtain the actions, and then take the difference between them. When one considers that the speed of light is finite, the calculation is complex, since the integral intervals for the two paths are different (see Fig. 1 left). To simplify the calculation, we propose analyzing the system in a new coordinate system, where the integral intervals can be synchronized. Then, taking the first pulse separation for example, the difference between the actions for the two paths will undergo the change below:

\[
\int_{t_a}^{t_n} L_{\text{upper}} \, dt - \int_{t_a}^{t_c} L_{\text{lower}} \, dt \to \int_{t_a}^{t_n} (L'_{\text{upper}} - L'_{\text{lower}}) \, dt',
\]

which can save a lot of unnecessary calculation. Thus, the crucial step for our method is making a coordinate transformation for the laser beam.
Fig. 1. The spacetime diagram of a light-pulse in an atom interferometer before (left) and after (right) the coordinate transformation.

3. Coordinate transformation

We assume light $\vec{k}_1$ is the one reflected by the mirror in the bottom of the experimental setup. Based on the analysis in Ref. 2, light $\vec{k}_1$ determines the change of the atom’s state, and it can be considered as the “control light.”

First, we should solve the geodesic equation of the photon to derive its trajectory $t = f(z)$, and then make a coordinate transformation for light $\vec{k}_1$:

$$
\begin{align*}
  t' &= t - f(z), \\
  z' &= z.
\end{align*}
$$

After this transformation, the coordinate velocity of $\vec{k}_1$ undergoes the change $c \rightarrow +\infty$ (see Fig. 1 right), and the lagrangian $L'$ of the atom can be written as the sum of a quadratic part and a nonquadratic part: $L' = L'_{\text{quad}} + L'_{\text{nonquad}}$. Thus, the propagation phase shift can be further expressed as:

$$
\Delta \phi_{\text{propagation}} = \Delta \phi_{L'_{\text{quad}}} + \Delta \phi_{L'_{\text{nonquad}}}. 
$$

4. Calculating the phase shift and the measured $g$

For $\Delta \phi_{L'_{\text{quad}}} + \Delta \phi_{\text{separation}}$, one can calculate it by combining the Bordé ABCD matrix method with quantum mechanics, which was previously studied in Ref. 3: first derive the quadratic hamiltonian through the
quadratic lagrangian, and further solve the motion equation of the atoms, and finally insert them into the action of the atoms (in the ABCD matrix form) to get the related phase shift.

For $\Delta \phi_{\text{nonquad}}$, one can refer to Ref. 4: treat $L'_{\text{nonquad}}$ as a perturbation for $L'_{\text{quad}}$, and use the perturbation approach to calculate the phase shift.

For $\Delta \phi_{\text{laser}}$, the frequency chirps should be taken into consideration. Taking light $\vec{k}_1$, for example, one can simply introduce the phase as:

$$-\omega_1(t - z/c) - \frac{1}{2}\alpha_1(t - z/c)^2,$$

where $\alpha_1$ is the frequency chirp for light $\vec{k}_1$. Then, combining the interaction between the atoms and the Raman light field, one can obtain the phase shift introduced by the laser beams.

Consequently, through summing the phase shifts analyzed above, one can derive the total phase shift and further the measured acceleration due to gravity, which can be kept to some high-order terms including some general relativistic effects. Considering only the FSL correction, the measured $g$ can be written as:

$$g \approx g_0 \left( 1 + \frac{v(T)}{c} + 2 \frac{v(T)}{c} \frac{\alpha_1 + \alpha_2}{\alpha_1 - \alpha_2} \right),$$

with $v(T)$ the atom velocity at the $\pi$ pulse, and $\alpha_1$ and $\alpha_2$ the frequency chirps for the two Raman beams.

5. Summary

This analytical study method can be used to calculate the relativistic effects and present an analytical derivation process. From the result, one can separate the FSL effect and obtain a more complete expression for the FSL correction.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (Grant No. 11275075).

References

1. A. Peters, K.Y. Chung, and S. Chu, Metrologia 38, 25-61 (2001).
2. S. Dimopoulos, P.W. Graham, J.M. Hogan and M.A. Kasevich, Phys. Rev. D 78, 042003 (2008).
3. Ch. Antoine and Ch.J. Bordé, J. Opt. B: Quantum Semiclass. Opt. 5, S199 (2003).
4. P. Storey and C. Cohen-Tannoudji, J. Phys. II France 4, 1999 (1994).