Article
The Stability Criterion and Stability Analysis of Three-Phase Grid-Connected Rectifier System Based on Gerschgorin Circle Theorem

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Abstract: With the increasing maturity of new energy technologies, distributed power systems have been widely used in the field of new energy power generation. The grid-connected rectifier is an important device in the distributed power system. When the grid-connected rectifier operates in a weak power grid environment, its operating performance deviates from the design value, endangering the operation safety of the power system. In this paper, the small-signal impedance model of the three-phase LCL grid-connected rectifier in the DQ coordinate system is established. This model considers the influence of the phase-locked loop on the rectifier impedance characteristics and improves the accuracy of the model, and on this basis, the theoretical stability of the system is analyzed. The stability judgment of the system has important engineering guidance significance. The traditional generalized Nyquist stability criterion requires a lot of calculations and is difficult to use. Therefore, a system stability criterion based on the Gerschgorin circle theorem is proposed, which reduces the amount of calculation and provides a higher size for the system design. The simulation results are consistent with the theoretical analysis, which verifies the correctness of the method proposed in this paper.

Keywords: LCL grid-connected rectifier; small-signal impedance modeling; stability criterion; Gerschgorin circle theorem; phase-locked loop

1. Introduction

With the gradual maturity of new energy power generation technology, the development of renewable energy has become an effective way to alleviate the energy crisis. Grid-connected rectifiers, as the main power interface for connecting renewable energy sources such as wind power generation and photovoltaic power generation to the grid, have been more and more widely used in power systems [1]. The grid-connected rectifier is connected to the power grid at the point of common coupling (PCC), and from the PCC to the power grid, there are long transmission and distribution lines. The grid impedance makes the grid appear resistive and inductive [2,3], the interaction between the rectifier impedance and the grid impedance may cause harmonic oscillation [4], and too high a grid impedance may cause instability of the grid-connected rectifier system [5]. In order to control the power delivered by the rectifier to the grid, a phase-locked loop (PLL) is needed to ensure that the output current of the rectifier is synchronized with the grid voltage; however, the increase in the bandwidth of the phase-locked loop has a negative impact on the stability of the grid-connected rectifier system [6]. In recent years, many oscillation problems related to the grid connection of renewable energy have occurred around the world, which poses new challenges for the safe and stable operation of the power system.
In view of the stability of the grid-connected rectifier system, domestic and foreign scholars have proposed a stability analysis method based on the small-signal impedance of the rectifier [7–10]. Regarding the rectifier and the power grid as two independent subsystems for small-signal impedance modeling, the stability of the system is analyzed by the generalized Nyquist stability criterion. According to the different reference coordinate systems used to build the impedance model, small-signal impedance modeling can be divided into two categories: the first type is the harmonic linearization impedance method, which establishes the positive and negative sequence impedance model in the stationary coordinate system [11,12]; the second type is the dq small-signal modeling method, in which the impedance model is established in the dq coordinate system rotating synchronously with the grid frequency [13–15]. The filter and control loop of the rectifier will affect the small-signal impedance characteristics of the rectifier. A study [16] considered the influence of the phase-locked loop in the rectifier control loop and the impedance model of the LCL type grid-connected rectifier was established, but the possible influence of decoupling feedback and capacitor current feedback in the control loop was not considered.

The three-phase grid-connected rectifier system is a MIMO (Multiple-Input Multiple-Output, MIMO) system, where it is difficult to analyze its stability by using the traditional Nyquist stability criterion. This problem can be solved by introducing the generalized Nyquist stability criterion, which is widely used to analyze the stability of the three-phase rectifier or inverter [17]. When applying the generalized Nyquist criterion to analyze system stability, it is necessary to obtain the frequency response of the eigenvalues of the system ratio matrix [18–21], because the impedance model expression of the rectifier is more complicated and the traditional generalized Nyquist criterion requires a high amount of calculation [22], which reduces the practicality of the method in engineering.

In order to make the stability criterion more applicable to engineering, some researchers have improved the generalized Nyquist stability criterion. Belkhayat simplified the generalized Nyquist stability criterion and proposed a stability criterion that is more conservative but easier to calculate [23]. Chandrasekaran analyzed it from the perspective of the transfer function and proposed a similar criterion called a singular-value criterion [24]. Mao proposed the D-channel criterion. Through the analysis of the experimental data, he pointed out that the stability of the system can be given only by the diagonal elements of the d-axis in the ratio matrix, while the amplitude of the other elements is generally small, so it can be ignored [25]. The above methods reduce the amount of calculation, but they still have a certain degree of conservativeness.

This paper also considers the influence of the phase-locked loop, and the decoupling feedback and capacitive current feedback in the current loop on the impedance characteristics of the rectifier, and establishes a small-signal impedance model of the grid-connected rectifier in the dq coordinate system. The accuracy of the model is improved and more in line with engineering reality. On this basis, an improved stability criterion of the grid-connected rectifier system is proposed according to the generalized Nyquist criterion, and the Geier circle theorem is used to estimate the position of the eigenvalues of the ratio matrix, avoiding the direct calculation of the eigenvalues of the ratio matrix and greatly reducing the amount of calculation. Finally, simulations and experiments verify the validity of the criterion.

2. Small-Signal Model of LCL Grid-connected Rectifier

Figure 1 shows the structure of the three-phase LCL-type grid-connected rectifier system studied in this paper. Switch transistors S1–S6 together with their anti-parallel diodes form a three-phase rectifier bridge, which converts alternating current into direct current, and supplies the resistive DC load R. The rectifier bridge is connected to the grid through an LCL-type filter. The rectifier-side inductor L1, the grid-side inductor L2, and the filter capacitor C form the LCL-type filter, which is connected to the grid at the Point of Common Coupling (PCC).
Previous studies on stability analysis of the power electronic converter have focused on the grid-connected inverter system. Figure 2 shows the typical structure of the three-phase LCL-type grid-connected inverter system, which is similar to Figure 1. The difference is that because the DC input voltage of the grid-connected inverter system is stabilized by the front-stage converter, it is equivalent to an ideal DC source $U_{dc}$. Meanwhile, the DC input voltage of the grid-tied rectifier system $u_{dc}$ is stabilized mainly by the DC regulator $C_{dc}$ and the internal voltage control ring of the rectifier system, and the dynamic feature of $u_{dc}$ cannot be ignored. It is inappropriate to consider DC input voltage $u_{dc}$ as an ideal DC source in the grid-tied rectifier system. Instead, this paper proposes a small-signal model for the output impedance of the three-phase grid-tied rectifier in the d-q frame, considering the dynamic feature of $u_{dc}$.

Compared with the three-phase grid-connected inverter system, the three-phase grid-connected rectifier system has the advantages of small harmonic output current, adjustable power factor, small voltage fluctuation on the DC side, bidirectional energy flow, and fast dynamic response.

The phase-locked loop (PLL) collects the PCC voltage signal to ensure the synchronization between the rectifier output current and the grid voltage. Because of the PCC voltage signal perturbation and the dynamic characteristics of the PLL, the rectifier system has two d-q frames: one is the system d-q frame and another is the controller d-q frame, as shown in Figure 3. The system d-q is defined by the PCC voltage, while the controller d-q frame is defined by the PLL. The PLL estimates the angle and frequency of the grid voltage to make the controller d-q frame approximate to the system d-q frame. In a steady state, the controller d-q frame is aligned with the system d-q frame. When small-signal perturbations appear at the PCC voltage, the position of the system d-q frame is changed. The controller d-q frame is no longer aligned with the system d-q frame because of the PLL dynamics. The angle between two d-q frames is $\delta$, as shown in Figure 3. In order to recognize the difference between two d-q frames, we express those variables (voltage, current, and duty ratio) within the system d-q frame with superscript $s$, while superscript $c$ stands for the controller d-q frame.
When $\delta$ is small, $\sin(\delta) \approx \delta$. The relationship between the different expressions of one vector $x = x_d + jx_q$ in the two d-q frames can be written as:

$$x^e = e^{-j\delta}x^i = (\cos\delta - j\sin\delta)x^i \approx (1 - j\delta)x^i$$  \hspace{1cm} (1)

which can be transferred to matrix form:

$$\begin{bmatrix} x'_d \\ x'_q \end{bmatrix} = \begin{bmatrix} 1 & \delta \\ -\delta & 1 \end{bmatrix} \begin{bmatrix} x'_d \\ x'_q \end{bmatrix}$$  \hspace{1cm} (2)

Adding a small-signal perturbation to Equation (2):

$$\begin{bmatrix} x'_d + \Delta x'_d \\ x'_q + \Delta x'_q \end{bmatrix} \approx \begin{bmatrix} 1 & \delta \\ -\delta & 1 \end{bmatrix} \begin{bmatrix} x'_d + \Delta x'_d \\ x'_q + \Delta x'_q \end{bmatrix}$$  \hspace{1cm} (3)

Then, separating small-signal perturbations and ignoring high-order infinitesimals:

$$\begin{bmatrix} \Delta x'_d \\ \Delta x'_q \end{bmatrix} \approx \begin{bmatrix} \Delta x'_d + x'_d\delta \\ \Delta x'_q - x'_q\delta \end{bmatrix}$$  \hspace{1cm} (4)

$$\begin{bmatrix} \Delta x'_d \\ \Delta x'_q \end{bmatrix} \approx \begin{bmatrix} \Delta x'_d - x'_d\delta \\ \Delta x'_q + x'_q\delta \end{bmatrix}$$  \hspace{1cm} (5)

The angle between two d-q frames $\delta$ is measured by the PLL. Figure 4 shows the control block diagram of the PLL: The q-channel component of the PCC voltage in the controller d-q frame $u_{PCCq}$ is the input of the PI controller $T_{\text{ PLL}}(s)$. The output of $T_{\text{ PLL}}(s)$ is $o_0$, which is the angular frequency of the controller d-q frame. By integrating $o_0$, the phase angle of the controller d-q frame $\theta_c$ is acquired, which is used for the d-q transformation of the three-phase sampling value.
When small-signal perturbations appear at the PCC voltage, small-signal perturbation \( \Delta u_{PCC} \) is added to \( u_{PCC} \). Meanwhile, \( \theta_s \), which is the phase angle of the system d-q frame, is no longer aligned with \( \theta_c \). The phase difference between \( \theta_s \) and \( \theta_c \) is \( \delta \), which can be acquired by the following equation:

\[
\delta = \frac{T_{PLL}(s)}{s} \Delta u_{PCC} = \frac{k_{PLL} + k_{PLL}/s}{s} \Delta u_{PLL_q} \tag{6}
\]

According to Equation (4), \( \Delta u_{PCC} = \Delta u_{PCC} - u_{PCC_d} \delta \). By substituting it into Equation (6), the relation between phase difference \( \delta \) and the q channel PCC voltage in the system d-q frame can be derived as

\[
\delta = \frac{T_{PLL}}{s + u_{PCC} T_{PLL}} \Delta u_{PCC} = G_{PLL} \Delta u_{PCC} \tag{7}
\]

According to Equations (4) and (5), when small-signal perturbations appear at the PCC voltage, the relationship of small-signal variables in two dq frames is as shown in Equation (8).

\[
\begin{bmatrix}
\Delta u_{PCC_d} \\
\Delta u_{PCC_q}
\end{bmatrix}
= \begin{bmatrix}
\Delta u_{PCC_d} + u_{PCC_d} \delta \\
\Delta u_{PCC_q} - u_{PCC_q} \delta
\end{bmatrix}
\begin{bmatrix}
\Delta i_{L_2d} \\
\Delta i_{L_2q}
\end{bmatrix}
= \begin{bmatrix}
\Delta i_{L_2d} + i_{L_2d} \delta \\
\Delta i_{L_2q} - i_{L_2q} \delta
\end{bmatrix}
\begin{bmatrix}
\Delta d_d \\
\Delta d_q
\end{bmatrix}
\tag{8}
\]

By substituting Equation (7) into Equation (8), the phase difference \( \delta \) in Equation (8) can be eliminated as follows:

\[
\begin{bmatrix}
\Delta u_{PCC_d} \\
\Delta u_{PCC_q}
\end{bmatrix}
= T_1 \begin{bmatrix}
\Delta u_{PCC_d} \\
\Delta u_{PCC_q}
\end{bmatrix}
+ \begin{bmatrix}
\Delta i_{L_2d} \\
\Delta i_{L_2q}
\end{bmatrix}
= T_1 \begin{bmatrix}
\Delta u_{PCC_d} \\
\Delta u_{PCC_q}
\end{bmatrix}
+ \begin{bmatrix}
\Delta i_{L_2d} \\
\Delta i_{L_2q}
\end{bmatrix}
\tag{9}
\]

where
Figure 5 depicts the small-signal model of the three-phase LCL-type rectifier in the system d-q frame, where \( \omega_0 \) is the grid fundamental frequency, and \( \omega_0 L_1 \Delta i_{L1d}^s, \omega_0 C \Delta u_{Cd}^s, \omega_0 C \Delta u_{Cq}^s, \omega_0 L_2 \Delta i_{L2d}^s, \) and \( \omega_0 L_2 \Delta i_{L2q}^s \) are the coupling terms between the d-channel and q-channel in the system d-q frame.

From Figure 5, the AC-side small-signal expression of the three-phase LCL-type rectifier in the system d–q frame is derived as

\[
\begin{align*}
\begin{bmatrix}
\Delta u_{Cd}^s \\
\Delta u_{Cq}^s \\
\Delta i_{L1d}^s \\
\Delta i_{L1q}^s \\
\Delta i_{L2d}^s \\
\Delta i_{L2q}^s \\
\Delta u_{PCCd}^s \\
\Delta u_{PCCq}^s
\end{bmatrix} &=
\begin{bmatrix}
A_1 & A_2 & A_3 \\
A_2 & A_4 & A_5 \\
A_3 & A_4 & A_6 \\
A_3 & A_5 & A_6 \\
A_4 & A_5 & A_6 \\
A_4 & A_5 & A_6 \\
A_5 & A_6 & A_6 \\
A_5 & A_6 & A_6
\end{bmatrix}
\begin{bmatrix}
\Delta d_d^s \\
\Delta d_q^s \\
\Delta i_{L1d}^s \\
\Delta i_{L1q}^s \\
\Delta i_{L2d}^s \\
\Delta i_{L2q}^s \\
\Delta u_{PCCd}^s \\
\Delta u_{PCCq}^s
\end{bmatrix}
\end{align*}
\]

where
According to Figure 5, the DC-side small-signal expression of the three-phase rectifier in the system d-q frame is obtained as

\[
\begin{align*}
(A_1 = & \begin{bmatrix} u_{dc} & 0 \\ 0 & u_{dc} \end{bmatrix} \\
A_2 = & \begin{bmatrix} d'_d \\ d'_q \end{bmatrix} \\
A_3 = & \begin{bmatrix} sL_1 + R_{L1} & -\omega_L L_1 \\ \omega_L L_1 & sL_1 + R_{L1} \end{bmatrix} \\
A_4 = & \begin{bmatrix} sC & -\omega_L C \\ \omega_L C & sC \end{bmatrix} \\
A_5 = & \begin{bmatrix} sL_2 + R_{L2} & -\omega_L L_2 \\ \omega_L L_2 & sL_2 + R_{L2} \end{bmatrix} \\
\end{align*}
\]

(12)

Substituting Equations (13) to (11), the following equation shows the small-signal expression of the three-phase LCL-type rectifier.

\[
B_1 \begin{bmatrix} \Delta i_{L1d} \\ \Delta i_{L1q} \end{bmatrix} + B_2 \begin{bmatrix} \Delta d'_d \\ \Delta d'_q \end{bmatrix} = \Delta u_{dc}
\]

(13)

where

\[
B_1 = \begin{bmatrix} 1.5d'_d R & 1.5d'_q R \\ \frac{sC}{sC_{dc}} R + 1 & \frac{sC}{sC_{dc}} R + 1 \end{bmatrix} \\
B_2 = \begin{bmatrix} 1.5i_{L1d} R & 1.5i_{L1q} R \\ \frac{sC}{sC_{dc}} R + 1 & \frac{sC}{sC_{dc}} R + 1 \end{bmatrix}
\]

(14)

Substituting Equations (13) to (11), the following equation shows the small-signal expression of the three-phase LCL-type rectifier.

\[
N_1 \begin{bmatrix} \Delta d'_d \\ \Delta d'_q \end{bmatrix} + N_2 \begin{bmatrix} \Delta u_{PCCd} \\ \Delta u_{PCCq} \end{bmatrix} = N_3 \begin{bmatrix} \Delta i'_{L2d} \\ \Delta i'_{L2q} \end{bmatrix}
\]

(15)

where

\[
\begin{align*}
N_1 = & -(A_1 + A_2 B_2) \\
N_2 = & I + A_2 B_1 A_4 + A_1 A_4 \\
N_3 = & A_3 A_2 A_4 + A_3 + A_2 B_1 A_4 A_5 + A_2 B_1 + A_3
\end{align*}
\]

(16)

The PI double closed-loop control, as shown in Figure 6, is a common control strategy for the three-phase rectifier system. G_i and G_v in Figure 6 are PI controllers of the current-inner-loop and voltage-outer-loop, respectively. Equation (17) shows the expressions of G_i and G_v. The output of the voltage loop is used as the reference value of the d-channel current-loop. The feedback decoupling strategy and grid voltage feedforward strategy are considered in the current-loop design, where \( L_f = L_1 + L_2 \), \( k_{pi} \) is the proportional gain.
of the current loop, \( k_i \) is the integral gain of the current loop, \( k_{pv} \) is the proportional gain of the voltage loop, and \( k_{iv} \) is the integral gain of the voltage loop.

\[
\begin{align*}
G_i &= k_{pi} + \frac{k_i}{s} \\
G_e &= k_{pv} + \frac{k_{iv}}{s}
\end{align*}
\]

(17)

**Figure 6.** The small-signal model of the double-loop control loop of the rectifier.

The reference values of the voltage-loop and q-channel current-loop can be considered as constant, which means \( \Delta U_{dc}\text{ref} = 0 \) and \( \Delta i_{Ld}\text{ref} = 0 \). According to Figure 6, the small-signal expression of the PI double closed-loop control circle is obtained as follows:

\[
\begin{bmatrix}
\Delta d_d^c \\
\Delta d_q^c
\end{bmatrix} = C_1 \Delta u_{dc} + C_2 \begin{bmatrix}
\Delta i_{Ld}^c \\
\Delta i_{Lq}^c
\end{bmatrix} + \Delta u_{PCCd}^c
\]

(18)

where

\[
C_1 = \begin{bmatrix}
-G_i & G_i \\
0 & 0
\end{bmatrix}
\]

\[
C_2 = \begin{bmatrix}
-G_i & 0 \\
\omega_i L_T & -\omega_i L_T
\end{bmatrix}
\]

(19)

By substituting Equations (9) and (13) into Equation (18), the following equation can be derived:

\[
N_4 \begin{bmatrix}
\Delta d_d^c \\
\Delta d_q^c
\end{bmatrix} + N_5 \begin{bmatrix}
\Delta i_{Ld}^c \\
\Delta i_{Lq}^c
\end{bmatrix} = N_6 \begin{bmatrix}
\Delta u_{PCCd}^c \\
\Delta u_{PCCq}^c
\end{bmatrix}
\]

(20)

where
From Equations (15) and (20), the small-signal admittance dynamic of the three-phase LCL-type rectifier system can be derived as

\[
Y_{rec} \begin{bmatrix}
\Delta i_{pccd} \\
\Delta i_{pccq}
\end{bmatrix} = \begin{bmatrix}
\Delta u_{pccd} \\
\Delta u_{pccq}
\end{bmatrix}
\]  (22)

where

\[
Y_{rec} = (N_3 - N_4 N_4^{-1} N_3^{-1})^{-1} (N_3 N_4^{-1} N_6 + N_2)
\]  (23)

The matrix \( Y_{rec} \) can be defined as the small-signal admittance matrix of the three-phase LCL-type rectifier.

3. System Stability Analysis

When the DC load is connected to a weak grid via a rectifier, the influence of the grid impedance on system stability cannot be ignored. In the system d-q frame, the grid impedance matrix can be written as

\[
Z_g = \begin{bmatrix}
sL_g & \omega_L L_g \\
-\omega_L L_g & sL_g
\end{bmatrix}
\]  (24)

where \( L_g \) is the equivalent inductance of the power grid. For system stability analysis, it can be assumed that the power grid is stable when unconnected with the rectifier, and the DC load with the rectifier is stable when powered from an ideal source. In this case, the stability of the grid-connected rectifier system can be identified by applying a Generalized Nyquist Criterion (GNC) to the return ratio matrix \( L(s) \) of the system:

when the net sum of anti-clockwise encirclements of the critical point \((-1, j0)\) by the set of characteristic loci of the return ratio matrix \( L(s) \) is equal to the total number of right-half plane poles of \( Y_{rec} \) and \( Z_g \), the grid-connected rectifier system will be closed-loop stable. The following equation shows the return ratio matrix of the grid-connected rectifier system that is defined by the GNC:

\[
L = Z_g Y_{rec} = \begin{bmatrix}
L_{dd} & L_{dq} \\
L_{qd} & L_{qq}
\end{bmatrix}
\]  (25)

where \( L_{dd}, L_{dq}, L_{qd}, \) and \( L_{qq} \) are four elements of the return ratio matrix. The two characteristic loci of the return ratio matrix \( L(s) \) can be derived as

\[
\lambda_1 = \frac{1}{2} (L_{dd} + L_{qq}) + \sqrt{\frac{1}{4} (L_{dd} + L_{qq})^2 - L_{dd} L_{qq} + L_{dq} L_{qd}}
\]

\[
\lambda_2 = \frac{1}{2} (L_{dd} + L_{qq}) - \sqrt{\frac{1}{4} (L_{dd} + L_{qq})^2 - L_{dd} L_{qq} + L_{dq} L_{qd}}
\]  (26)

As the power grid is stable when unconnected with the rectifier, and the DC load with the rectifier is stable when powered from an ideal source, the poles of \( Y_{rec} \) and \( Z_g \)
are always on the left-hand s-plane. When the loci of $\lambda_1$ and $\lambda_2$ do not encircle the critical point (−1, j0), the system is stable according to the GNC.

As $Y_{rec}$ includes information of the control circuit dynamic, the expressions of $\lambda_1$ and $\lambda_2$ are typically complex, which makes it difficult to analyze the system stability by applying the traditional GNC.

This paper estimates the characteristic loci of $L(s)$ in the s-plane by applying the Gershgorin Circle Theorem, which simplifies the traditional GNC method and decreases the computational complexity of system stability analysis.

4. System Stability Criterion Based on Gershgorin Circle Theorem

The Gershgorin Circle Theorem provides a method to determine the distribution area of the matrix eigenvalue. According to the Gershgorin Circle Theorem, the eigenvalues of the return ratio matrix $L(s)$ are limited to two Gershgorin circles in the s-plane, which is depicted in Figure 7. Equation (27) shows the relationship between the elements and eigenvalues of $L(s)$.

\[
\begin{align*}
|\lambda_1 - L_{dd}| &< |L_{dq}| \\
|\lambda_2 - L_{qq}| &< |L_{qd}|
\end{align*}
\] (27)

Figure 7. Eigenvalue interval.

As shown in Figure 8, if two Gaelic circles are contained in the unit circle on the s-plane for any frequency point, the eigenvalue trajectory of $L(s)$ must not surround the point (−1, j0), and the system must then be stable. The sufficient condition of this stable condition can be written as:

\[
\begin{align*}
|L_{dd}| + |L_{dq}| &< 1 \\
|L_{qq}| + |L_{qd}| &< 1
\end{align*}
\] (28)
In order to increase the allowable domain of the Gaelic circle on the s plane, as shown in allowable domain one in Figure 9, define the system stability margin parameter $A$ ($0 < A \leq 1$) if, for any frequency point, both Gaelic circles are on the right side of the parallel line $\text{Re}(s) = -A$ of the imaginary axis, that is, when only two Gaelic circles are restricted from entering the shaded area in Figure 9, the eigenvalue trajectory of $L(s)$ must not surround the point $(-1, j0)$, and the system must be stable. The expression of the stability condition can be written as:

$$\left\{ \begin{array}{l}
\text{Re}(L_{s_1}) - |L_{q_1}| > -A \\
\text{Re}(L_{q_2}) - |L_{q_2}| > -A 
\end{array} \right.$$

(29)

In order to further reduce the area of the forbidden area on the s-plane, that is to increase the probability of system stability, the system stability margin parameters $A$ ($0 < A \leq 1$) and $P$ ($0 < p \leq 90^\circ$) can be introduced to construct allowable domain two, as shown in Figure 10. When the two Gaelic circles do not enter the shaded area in Figure 10 at any frequency point, the eigenvalue trajectory of $L(s)$ must not surround the point $(-1, j0)$, so it can be determined that the system is stable. The expression of the stability condition can be written as:

$$\left\{ \begin{array}{l}
|\text{Im}(L_{s_1})| \times \cos P + |\text{Re}(L_{s_1}) + A| \times \sin P - |L_{q_2}| > 0 \\
|\text{Im}(L_{q_2})| \times \cos P + |\text{Re}(L_{q_2}) + A| \times \sin P - |L_{q_2}| > 0 
\end{array} \right.$$

(30)
Figure 10. S-plane margin after increasing the value interval.

5. Simulation

In order to verify the validity of the stability criterion proposed in this chapter, a three-phase LCL grid-connected rectifier system was simulated. The parameters of the three-phase LCL rectifier are shown in Table 1.

| Control Loop Parameters                  | Numerical Value |
|-----------------------------------------|-----------------|
| PLL proportional gain $K_{pPLL}$        | 0.28            |
| PLL integral gain $K_{iPLL}$            | 8               |
| Current loop proportional gain $K_{pi}$ | 4.21            |
| Current loop integral gain $K_{ii}$     | 300             |
| DC voltage loop proportional gain $K_{pv}$ | 0.16           |
| DC voltage loop integral gain $K_{iv}$  | 6               |
| Sampling frequency $f_s$                | 10 kHz          |

| Power transmission loop parameters      | Numerical value |
|-----------------------------------------|-----------------|
| Rectifier rated power $P_0$             | 1.5 kW          |
| DC bus voltage $U_{dc}$                 | 230 V           |
| DC filter capacitor $C_{dc}$            | 3 mF            |
| Resistive DC load $R_{load}$            | 30 Ω            |
| Rectifier side AC filter inductance $L_1$ | 1.2 mH        |
| AC filter inductance on the grid side $L_2$ | 0.6 mH     |
| AC filter capacitor C                   | 50 μF           |
| Grid connection voltage $U_{PCC}$ (RMS) | 80 V            |
| Grid frequency $f_0$                    | 50 Hz           |
| operating frequency $f_{sw}$            | 5 kHz           |

As the grid inductance $L_g$ increases, the characteristics of the weak grid gradually become more prominent, and the system will gradually become unstable. Therefore, the parameters of the rectifier should be kept unchanged, $L_g$ should gradually increase from 2 mH, and the dynamic characteristics of the system should be observed. First, using the stability criteria corresponding to the allowable domains in Figures 8–10, respectively, the system stability should be analyzed, their respective results should be compared, and then a simulation model of the three-phase grid-connected rectifier should be built in MATLAB/Simulink to verify the validity of the prediction results of the stability criterion.

Using Figure 8 to determine system stability, define:
Substituting the parameters of Table 1 into Equation (31), the grid inductance $L_g$ is gradually increased from 2 mH to 4 mH, and the trends of the frequency response of $M_1(s)$ and $N_1(s)$ are shown in Figures 11a,b, respectively. According to the definition of the permissible domain of the unit circle, when $M_1(s)$ and $N_1(s)$ are below the black dotted line at any frequency point, the system is stable. It can be seen from Figure 11 that the criterion can reflect the tendency of the system stability margin to decrease as the grid inductance increases. The critical point for determining the system stability is about $L_g = 3.17$ mH.

![Figure 11](image)

**Figure 11.** Judging the stability of the system by using the allowable unit circle: (a) frequency response of $M_1(s)$; (b) frequency response of $N_1(s)$.

Using Figure 9 to determine the system stability, and setting the system stability margin parameter $A = 1$, define:

$$
\begin{align*}
  M_2(s) &= \text{Re}(L_{dd}^{-1}) - |L_{dq}| \\
  N_2(s) &= \text{Re}(L_{dq}^{-1}) - |L_{qq}|
\end{align*}
$$

(32)

Substituting the parameters of Table 1 into Equation (32) increases the grid inductance $L_g$ from 2 mH to 8 mH, and the frequency response trends of $M_2(s)$ and $N_2(s)$ are shown in Figures 12a,b, respectively. According to the definition of allowable domain 1, when $M_2(s)$ and $N_2(s)$ are above the black dotted line at any frequency point, the system is stable. As can be seen from Figure 12, this criterion can also reflect the trend of the system stability margin decreasing with the increase in the grid inductance, and the critical point for determining the system stability is about $L_g = 6.23$ mH. It can be seen that compared with the circular allowable domain, the conservativeness brought by allowable domain 1 to the system parameter value range is significantly reduced.
Figure 12. Determination of the system stability by using allowable domain 1: (a) frequency response of $M_3(s)$; (b) frequency response of $N_3(s)$.

If the allowable domain 2 shown in Figure 10 is applied, set the system stability margin parameters $A = 1$, $P = 10^6$, and define:

\[
\begin{align*}
    M_3(s) &= \text{Im}(L_\omega) \cdot \cos P + [\text{Re}(L_\omega) + A] \cdot \sin P - \left| L_\omega \right|
    \\
    N_3(s) &= \text{Im}(L_\omega) \cdot \cos P + [\text{Re}(L_\omega) + A] \cdot \sin P - \left| L_\omega \right|
\end{align*}
\]  

(33)

Substituting the parameters in Table 1 into Equation (33) increases the grid inductance $L_g$ from 2 mH to 8 mH, and the frequency response change trends of $M_3(s)$ and $N_3(s)$ are shown in Figure 13a and Figure 13b, respectively. According to the definition of allowable domain two, when $M_3(s)$ and $N_3(s)$ are above the black dotted line at any frequency point, the system is stable. It can be seen from Figure 13 that the critical point of the criterion to judge the stability of the system is about $L_g = 6.71$ mH, and it can be seen that the conservativeness of the allowed criterion corresponding to the stability criterion of domain 2 is somewhat lower than that of allowable domain 1.

Figure 13. Determination of the system stability by using allowable domain 2: (a) frequency response of $M_3(s)$; (b) frequency response of $N_3(s)$.

Figure 14 shows the grid-connected voltage and current waveforms at the PCC when the three-phase rectifier with the parameters shown in Table 1 is connected to the grid with three different grid inductance parameters $L_g$, as can be seen from Figure 14a. When $L_g = 3$ mH, the grid-connected rectifier system is stable, the harmonics contained in the
output voltage and current waveform are small, and the PCC voltage THD of phase A is 3.68%. Referring to Figures 11–13, this is consistent with the predictions made by the stability criteria corresponding to the three allowable domains given above.

Figure 14. The voltage and current output waveforms at grid connection points under different grid inductances: (a) grid inductance $L_g = 3$ mH; (b) grid inductance $L_g = 6$ mH; (c) grid inductance $L_g = 8$ mH.

When $L_g = 6$ mH, according to the results obtained in Figures 11–13, the unit circle allows the stability criterion corresponding to the domain to determine the system as unstable, while the stability criterion corresponding to domain 1 and domain 2 still determines the system to be stable. It can be clearly seen from Figure 14b that the system can still maintain stability at this time, which is consistent with the results obtained by the system stability criterion proposed in this paper; note that compared with the existing unit circle allowable domain, the newly proposed stability criterion can make the value range of system parameters less conservative. At this time, the phase A PCC voltage THD is 4.31%, and the harmonic content is slightly improved compared to Figure 14a.

When $L_g = 8$ mH, as can be seen from Figure 14c, there was obvious oscillation in the grid-connected rectifier system, which entered an unstable state. It can be seen from the results obtained above that this is in accordance with the prediction given by the new stability criterion proposed in this paper, indicating that the criterion can ensure the stability of the three-phase grid-connected rectifier system and prevent accidents.
When using the generalized Nyquist stability criterion method to calculate the Nyquist curve of the return rate matrix eigenvalues, the results are as shown in Figure 15.

![Nyquist Stability Diagram](image)

**Figure 15.** Stability analysis of generalized Nyquist stability criterion.

It can be seen that when \( L_g \) increases, the curve gradually becomes larger. When \( L_g = 2 \) mH and \( L_g = 5 \) mH, the curve does not enclose the point \((-1, j0)\), and it shows that the system is stable. When \( L_g = 9 \) mH, the curve encloses the point \((-1, j0)\), which means the system is in an unstable state. This result is very close to the result obtained by the proposed method. It shows that the proposed method is correct and effective. Compared with the generalized Nyquist stability criterion, the results of the proposed method are relatively conservative, while the method reduces the calculation and is more suitable for engineering.

6. Conclusions

In this paper, the small-signal analysis of the three-phase grid-connected rectifier system is carried out, and the small-signal model of the grid-connected rectifier system considering the compensation factors of the phase-locked loop and the current loop in the dq coordinate system is established, which improves the accuracy of the model. On this basis, a simplified stability criterion based on the generalized Nyquist theory is proposed, the eigenvalue of the return rate matrix \( L(s) \) is estimated by the Gaelic circle theory, and the system is guaranteed by setting different forbidden zones in the s plane stable. Compared with the existing generalized Nyquist stability criterion, this criterion avoids the direct calculation of the eigenvalue of \( L(s) \) at the cost of a little increase in conservatism, and greatly reduces the amount of calculation. In the actual application environment, the stability criterion proposed in this paper can give full play to its advantage of small calculation. As a practical method for estimating the critical point of system stability, the system stability region is quickly estimated when the system environmental parameters are constantly changing, and by introducing the system stability margin parameter, the size of the forbidden zone on the s-plane can be controlled, which provides higher flexibility for the system design. Simulation and experimental results verify the validity of the stability criterion.

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Symbols

\( \delta \) angle between controller d-q frame and system d-q frame

\( u_{PCC} \) PCC voltage

\( k_{pPLL} \) PLL proportional gain

\( k_{iPLL} \) PLL integral gain

\( \omega_0 \) grid fundamental frequency

\( k_{pi} \) current loop proportional gain

\( k_i \) current loop integral gain

\( k_{pv} \) voltage loop proportional gain

\( k_{iv} \) voltage loop Integral gain

\( u_{dc} \) DC bus voltage

\( C_{dc} \) DC filter capacitor

\( L_1 \) AC filter inductance on the rectifier side

\( L_2 \) AC filter inductance on the grid side

\( C \) AC filter capacitor

\( L_g \) equivalent inductance of power grid

\( T_{PLL}(s) \) PI controller of phase lock loop

\( G_i(s) \) PI controller of current-inner-loop

\( G_v(s) \) PI controller of voltage-outer-loop

\( Y_{rec}(s) \) small-signal admittance matrix of three-phase LCL-type rectifier system

\( Z_s(s) \) grid impedance matrix

\( L_g(s) \) return ratio matrix of the system

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