Gauge/gravity correspondence  
in accelerating universe

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Abstract

We discuss time-dependent backgrounds of type IIB supergravity realizing gravitation duals of gauge theories formulated in de Sitter space-time as a tool of embedding de Sitter in a supergravity. We show that only the gravitational duals to non-conformal gauge theories are sensitive to a specific value of a Hubble parameter. We consider two nontrivial solutions of this type: a gravity dual to six-dimensional $(1,1)$ little string theory, and to a four-dimensional cascading $SU(N+M) \times SU(N)$ supersymmetric gauge theory (related to fractional D3-branes on a singular conifold according to Klebanov et al), in accelerating universe. In both cases we argue that the IR singularity of the geometry is regulated by the expansion of the gauge theory background space-time.
1. Introduction

Gauge theory – gravity duality relates a gauge theory on the world volume of large number of D-branes to purely supergravity backgrounds where the branes are replaced by the corresponding fluxes. In the simplest case the duality is realized by a system of $N$ D3-branes in a flat type IIB string theory background. At small ’t Hooft coupling $g_s N \ll 1$, the system is best described by open strings and realizes $SU(N) \ N = 4$ supersymmetric gauge theory. In the limit of strong ’t Hooft coupling this gauge theory has a perturbative description as type IIB supergravity compactified on $AdS_5 \times S^5$, with $N$ units of the RR 5-form flux through the $S^5$. If this is a genuine equivalence, then phenomena observed on the gauge theory side should have dual description in string theory on $AdS_5 \times S^5$. In particular, any deformation on the gauge theory visible in the large $N$ limit should have a counterpart in the dual gravitational description, and vice versa.

As in we use “deformations” in a generalized sense. For example, Klebanov-Witten duality describing regular D3-branes placed at a conical singularity in type IIB string theory can be thought of as a $Z_2$ orbifold of the original duality of Maldacena along with a certain mass deformation that leaves only a quarter of the original supersymmetries unbroken. One could go a step further and consider deformations of a background space-time in which one formulates gauge dynamics. In a gauge-gravity correspondence was considered in which Minkowski background space-time of the Klebanov-Strassler (KS) cascading gauge theory was replaced with $R \times S^3$ or (in a Euclidean case) $S^4$. It was argued there that the curvature of the background geometry provided an infrared cutoff on the gauge theory dynamics and resolved the Klebanov-Tseytlin (KT) naked singularity.

A natural extension to proposal of is to ask the question: what the gauge-gravity duality of Maldacena would look like when the gauge theory space-time background is de Sitter? This is a perfectly valid “deformation” of the gauge theory background where one “turns on” a Hubble parameter. And thus, provided the original gauge-gravity correspondence was exact, one should be able to map this deformation onto the dual supergravity. In this paper we describe such a map. We would like to emphasize that much like in the original Maldacena correspondence, the gauge theory space-time is not dynamical on the gauge theory side of the correspondence. In other words, on the gauge theory side of the correspondence we completely neglect the backreaction of the gauge theory dynamics on the background, we ignore background fluctuations as well. The story on the gravity side

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1 For reviews and references see, e.g., [1].
of the correspondence is drastically different: here, as in the original Maldacena correspondence, what was the gauge theory background becomes a part of a dynamical type IIB supergravity background. Thus, finding a gravity dual to a gauge theory on a (decoupled) dS background would provide an embedding of this space-time into dynamical supergravity. Put differently: we want to view “cosmological” deformation of the gauge-gravity correspondence as a tool of embedding a de Sitter space-time into a supergravity.

The paper is organized as follows. In the next section we describe a motivation for a time-dependent metric ansatz of type IIB supergravity background dual to a gauge theory in accelerating universe. We observe that de Sitter deformation applied to the $\mathcal{N} = 4$ $SU(N)$ supersymmetric Yang-Mills gauge/gravity correspondence does not give rise to a different geometry on the dual supergravity side: all we get is a de Sitter slicing of the AdS factor in the original Maldacena duality. Nonetheless, we expect physically the deformed gauge theory to be rather different from the undeformed one. In particular, because of the conformal coupling of the gauge theory scalars to the scalar curvature, in the $H \neq 0$ ($H$ is the expansion rate of the universe) case the SYM would not have a moduli space. We further show that conformal gauge theories are the only examples for which supergravity duals for nonzero Hubble parameter $H$ are related by some coordinate reparametrization to their $H = 0$ supergravity duals.

We then move on to consider non-conformal examples in sections 3 and 4. In section 3 we present supergravity dual to (1,1) little string theory (LST) in an inflationary patch of the $dS_6$. The $H = 0$ solution reproduces the BPS system of $\mathcal{N} \gg 1$ NS5-branes, and thus has curvature singularity (in the Einstein frame) at the branes core. From the dual gauge theory perspective this singularity is generated by the zero modes of the $d = 6$ SYM theory, which is the infrared limit of (1,1) LST. We explicitly demonstrate that de Sitter deformation of LST regulates this curvature singularity.

In section 4 we briefly discuss gravitational dual to Klebanov-Tseytlin/Klebanov-Strassler (KT/KS) cascading gauge theory in accelerating universe. We show that the KT deformation is related by a Wick rotation plus some scaling of the KT gauge theory on $S^4$ previously considered in [3] . Thus the infrared singularity of the extremal KT geometry is resolved for $H \neq 0$ as explained in [3] . We conclude in section 5.

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2 Related ideas of realizing de Sitter gravity in warped compactifications of type IIB string theory were discussed in [7] .

3 A similar phenomenon for the $R \times S^3$ deformation of the $\mathcal{N} = 4$ SYM was emphasized in [3] .

4 It would be very interesting to study de Sitter deformations of gauge theories from purely field theoretical perspective. In this paper we focus on the supergravity part of the de Sitter deformed gauge-gravity correspondence.
2. Supergravity duals of gauge theories in accelerating universe

We mentioned in the introduction that given the original gauge-gravity duality of Maldacena there is a simple way to embed dS space-time into supergravity. The reason for this is that since we can deform a background space-time of the gauge theory from Minkowski to a flat Robertson-Walker universe by simply “turning on” a Hubble parameter, we should be able to do this in the supergravity dual to this gauge theory.

Typically, in a gauge-gravity correspondence the dual supergravity metric\(^5\) can be written as

\[
    ds_{10E} = c_1^2 (dM_d)^2 + c_2^2 dr^2 + (d\mu_{9-d})^2,
\]

where \(M_d\) is a \(d\)-dimensional Minkowski space-time, which is related to the space-time background of the dual gauge theory, and \(\mu_{9-d}\) (for a fixed \(r\)) is a compact \((9-d)\)-dimensional Riemannian manifold that encodes the gauge theory dynamics at energy scale \(E \sim \rho\) with \(c_2 dr \sim \frac{d\rho}{\rho}\) as \(\rho \to \infty\). From now on we consider only the cases where \(c_i\) depend only on \(r\)\(^6\). The metric on \(M_d\) does not depend on the angles of \(\mu_d\) while \((d\mu_{9-d})^2\) does not depend on the \(M_d\) coordinates, though both \(M_d\) and \(\mu_d\) can have explicit \(r\) dependence\(^7\). It seems natural to assume that such “separation of variables” would hold even when we start deforming the gauge theory space-time \(M_d\). Specifically, taking the \(d\)-dimensional gauge theory in an accelerating universe

\[
    (ds^H_d)^2 = -dt^2 + e^{2Ht} d\bar{x}^2,
\]

which for \(H = 0\) has a dual supergravity background with the metric (2.1), we assume the metric ansatz of the dual supergravity for general \(H\) to be

\[
    (dM_d)^2 \to (dM^H_d)^2 \equiv (ds^H_d)^2.\]

In the orthonormal frame

\[
    e^1 = c_1 dt, \quad e^{i+1} = e^{Ht} c_1 dx_i, \quad i = 1 \cdots d - 1, \quad e^{d+1} = c_2 dr, \quad e^j, \quad j = d + 2, \cdots, 10 \quad \text{such that} \quad e^j e^j \equiv (d\mu_{9-d})^2,
\]

the ricci components of the metric are time independent; so in this frame the supergravity fluxes and the dilaton would be time-independent as well.

We begin explicit examples by considering the case of the \(H \neq 0\) deformation of the gauge-gravity correspondence discussed in \([4]\) where the gauge theory is conformal, namely D3 branes at a conical singularity\(^8\). We observe that the dual supergravity background for

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\(^5\) We always work in the Einstein frame.

\(^6\) This is not always the case, as for example in Polchinski-Strassler gauge-gravity correspondence\(^\text{[8]}\).

\(^7\) The examples where \((dM_d)^2\) has an \(r\) dependence correspond to gauge theories formulated on compact manifolds as in \([8]\).

\(^8\) There is an obvious generalization to \(AdS_5 \times S^5\), and other conformal cases.
$H \neq 0$ still remains $AdS_5 \times S^5$: the only difference is that now we are doing a de Sitter slicing of the AdS factor in the metric.

Type IIB equations of motion can be solved analytically in this case. We find

$$ds^2_{10} = \rho^2 (-dt^2 + e^{2Ht}d\bar{x}^2) + \frac{L^2d\rho^2}{L^2H^2 + \rho^2} + L^2ds^2_{T^1,1},$$  \hspace{1cm} (2.6)$$

where $(ds^{}_{T^1,1})^2$ is the standard metric on $T^1,1 = (SU(2) \times SU(2))/U(1)$ and

$$L^4 = 4\pi g_s N(\alpha') \frac{27}{16},$$  \hspace{1cm} (2.7)$$

with $N$ being the number of D3 branes. The metric (2.6) is supported by the following 5-form flux

$$F_5 = \mathcal{F}_5 + \ast \mathcal{F}_5, \quad \mathcal{F}_5 = -L^4d\text{vol}_{T^1,1}.$$  \hspace{1cm} (2.8)$$

Above solution is related by a coordinate transformation to the extremal ($H = 0$) D3 brane solution. Indeed, first introduce

$$\tau = \frac{1}{H}e^{-Ht}.$$  \hspace{1cm} (2.9)$$

Then the change of variables that do the job is

$$r = \frac{\rho}{H\tau} = \rho e^{Ht},$$  \hspace{1cm} (2.10)$$

$$d\tilde{t} = -\frac{\sqrt{L^2H^2 + \rho^2}}{\rho}d\tau - \frac{L^2H^2\tau}{\sqrt{L^2H^2 + \rho^2}} \frac{d\rho}{\rho^2},$$  \hspace{1cm} (2.11)$$

where $r$ and $\tilde{t}$ are the radial and the time coordinates of the $H = 0$ solution. Note that in (2.11) $d^2\tilde{t} = 0$, so this equation can indeed be integrated

$$\tilde{t} = \frac{\sqrt{L^2H^2 + \rho^2}e^{-Ht}}{H\rho}.$$  \hspace{1cm} (2.12)$$

From the coordinate transformations (2.10), (2.12) we see that de Sitter slicing of $AdS_5$, as in (2.6), covers “half” ($\tilde{t} \geq 0$ region) of its Poincare patch. It is easy to see that this slicing can be obtained from the analytical continuation (along with some scaling limits) of the Euclidean $AdS$ in the “hyperboloid” parametrization\footnote{This is a Euclidean $AdS_{d+1}$ parametrization where the constant radial slice is $S^d$.}. Really, for the $AdS_{d+1}$ the metric in this parametrization is given by

$$ds^2_{AdS_{d+1}} = \sinh^2 \rho \left(dS^d\right)^2 + d\rho^2 = \sinh^2 \rho \left(d\tau^2 + \sin^2 \tau (dS^{d-1})^2 \right) + d\rho^2 .$$  \hspace{1cm} (2.13)$$
Now Wick rotation of $\tau \rightarrow i\tau$ and the “decompactification” limit of $S^{d-1}$, $(dS^{d-1})^2 \rightarrow (dR^{d-1})^2$, along with $\tau \gg 1$ give

$$ds_{AdS_{d+1}}^2 = \sinh^2 \rho \left( -d\tau^2 + e^{2\tau} (dR^{d-1})^2 \right) + d\rho^2$$

$$= r^2 \left( -d\tau^2 + e^{2\tau} (dR^{d-1})^2 \right) + \frac{dr^2}{1+r^2},$$

where $r \equiv \sinh \rho$. Thus, coordinate transformations (2.10), (2.12) must be represented by the (corresponding scaling limit of the) Wick rotation of local coordinate transformations relating Poincare and “hyperboloid” parametrizations of the Euclidean $AdS$.

In the rest of this section we address the question when does the $H \neq 0$ deformation of a given gauge-gravity duality is related to the original ($H = 0$) correspondence by some change of variables, as in the case above. We will argue that this is so only when the gauge theory in the duality correspondence is conformal. Let

$$(ds_{10E}^0)^2 = (c_1^0)^2 (-dt^2 + d\bar{x}^2) + (c_2^0)^2 d\rho^2 + (d\mu_{9-d}^0)^2$$

be a supergravity metric in the original$^{10}$ gauge-gravity correspondence, and

$$(ds_{10E}^H)^2 = (c_1)^2 (-d\tau^2 + e^{2H\tau} d\bar{x}^2) + (c_2)^2 d\rho^2 + (d\mu_{9-d})^2$$

is the metric corresponding to its $H \neq 0$ deformation. We want to know when (2.16) is related by some coordinate reparametrization to (2.15). Replacing $\tau \rightarrow \frac{1}{H} e^{-H\tau}$ in (2.16) we get

$$(ds_{10E}^H)^2 = \frac{c_1^2}{H^2 \tau^2} (-d\tau^2 + d\bar{x}^2) + c_2^2 d\rho^2 + (d\mu_{9-d})^2.$$  

Let’s ignore for now the internal piece of the metric. Comparing $d\bar{x}^2$ pieces of the metric in (2.17) and (2.13) we see that

$$c_1^0(r) = \frac{c_1(\rho)}{H\tau},$$

so that

$$dr = \frac{[c_1(\rho)]' \tau d\rho - c_1(\rho) d\tau}{H\tau^2 [c_1(\rho)]''}. \quad (2.19)$$

Taking the most general ansatz for $dt$

$$dt = g_1(\rho, \tau) d\tau + g_2(\rho, \tau) d\rho,$$

$^{10}$ That is a gauge theory is formulated in Minkowski space-time.
and matching (2.17) and (2.13) we find
\begin{align}
g_1(\rho, \tau) &= \frac{c_1(\rho) \sqrt{|c_1^0(r)|^2 + \tau^2[c_1^1(r)]^2}}{c_1^0(r) H\tau^2[c_1^0(r)]'}, \\
g_2(\rho, \tau) &= -\frac{[c_2^0(r)]^2 [c_1^0(\rho)]'}{H\tau c_1^0(r)[c_1^0(r)]' \sqrt{|c_2^0(r)|^2 + \tau^2[c_1^0(r)]^2}},
\end{align}

plus we have a constraint
\[ 0 = -[c_2^0(r)]^2 [c_1^0(\rho)]^2 + c_2(\rho)^2 H^2 [c_2^0(r)]^2 + c_2(\rho)^2 H^2 \tau^2 [c_1^0(r)]^2. \]

Since we should be able to integrate (2.20)
\[ d^2t \equiv 0. \]

It turns out, given above expressions we can rewrite (2.24) as
\[ 0 = \frac{d}{dr} \left[ \frac{[c_1^0(r)]'}{c_1^0(r)c_2^0(r)} \right]. \]

Without loss of generality we can assume that in the original duality
\[ c_1^0(r) = r. \]

From (2.25) we find then
\[ c_2^0 = \frac{L}{r}, \]

where \( L \) is some constant. Finally, the only way \( (d\mu_{d-9})^2 \) and \( (d\mu_{d-9})^2 \) could ever match is when they are independent of \( \rho \) and \( r \) correspondingly. Thus we conclude that the metric (2.15) is actually
\[ (ds_{10E}^0) = (ds_{AdS_{d+1}})^2 + (d\mu_{9-d})^2, \]

where the metric on \( \mu_{9-d} \) does not depend on the \( AdS_{d+1} \) radial coordinate. The AdS factor in (2.28) points to the conformal invariance of the dual gauge theory.

Above discussion suggests that for the embedding of a de Sitter space-time in supergravity we should look for deformations of gauge/gravity duality where the gauge theory is not conformal. We will present explicit examples of such deformations in the next two sections.

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\[ ^{11} \] We are assuming that the \( c_0^0 \) warp factor in (2.15) is nontrivial, that is not a constant.

\[ ^{12} \] This fixes an arbitrary choice of a radial coordinate in (2.15).
3. (1,1) LST in accelerating universe and the IR singularity resolution by inflation

In this section we describe $H \neq 0$ deformations of the (1,1) little string theory, realized on the world-volume of NS5 branes in type IIB string theory. The effective infrared description of the LST is in terms of $d = 6 \mathcal{N} = 2$ supersymmetric Yang-Mills theory. As this gauge theory is not conformal, we expect to get a nontrivial embedding of $dS_6$ from its $H \neq 0$ deformation.

In the extremal case, $H = 0$, the supergravity approximation breaks down near the core of the branes. This curvature singularity can be thought of as being generated by the zero modes of the IR free $d = 6$ SYM. Since a Hubble parameter provides an infrared cutoff on the dynamics of the theory, we expect that it should regulate the curvature singularity of the extremal background. We argue that this is indeed so.

We take the following ansatz for the metric of LST holographic dual in the inflationary patch of the $dS_6$

$$(ds_{10E})^2 = c_1^2 (-dt^2 + e^{2Ht}d\bar{x}^2) + c_2^2 d\rho^2 + \frac{c_3^2}{4} (g_1^2 + g_2^2 + g_3^2),$$

(3.1)

where $c_i = c_i(\rho)$, and $g_i$ are the $SU(2)$ left-invariant one-forms

$$g_1 = \cos \phi \ d\theta + \sin \phi \sin \theta \ d\psi,$$

(3.2)

$$g_2 = \sin \phi \ d\theta - \cos \phi \sin \theta \ d\psi,$$

$$g_3 = d\phi + \cos \theta \ d\psi.$$

We assume the dilaton $\Phi \equiv \ln g_s$ to be a function of $\rho$ only and the same NS-NS 3-form fluxes as in the extremal case

$$H_3 = n \ g_1 \wedge g_2 \wedge g_3,$$

(3.3)

where $n$ is related to the number of NS5 branes. Solving type IIB supergravity equations we get

$$0 = \left[ \frac{g_1 c_1^6 c_3^3}{g_s c_2} \right]' + \frac{32n^2 c_1^6 c_2}{c_3^3 g_s},$$

(3.4)

$$0 = \left[ \frac{c_1 c_1^5 c_3^3}{c_2} \right]' - \frac{c_1^4 c_2}{g_s c_3^3} \left( 5H^2 g_s c_3^6 + 8n^2 c_1^2 \right),$$

(3.5)

$$0 = \left[ \frac{c_3 c_3^6 c_1^6}{c_2} \right]' - \frac{2c_1^6 c_2}{g_s c_3^3} \left( g_s c_3^4 - 12n^2 \right).$$

(3.6)
along with the first order constraint
\[
0 = \frac{12g_s^2c_3^4}{c_1^4} [c_3c_1]' [c_1^5c_3]'' - c_3^6c_1^2 (g_s')^2 + 4g_s^2c_2^2 (16n^2c_1^2 - 3g_s c_3^4 [c_2^2 + 5H^2c_3^2]) .
\] (3.7)

It is consistent with (3.4) - (3.7) to choose an ansatz for the warp factors \(c_i\) similar to the extremal NS5 brane solution
\[
c_1 = fg_s^{-1/4}, \quad c_2 = c_3 = 2n^{1/2}g_s^{-1/4}.
\] (3.8)

We will end up with the following equations for \(f, g_s\)
\[
0 = \left[ \frac{g_s f^6}{g_s^3} \right]' + \frac{2f^6}{g_s^2},
\] (3.9)
\[
0 = \left[ \frac{(g_s/f^4)'}{g_s^3} \right]' + \frac{2f^4(40H^2n + f^2)}{g_s^2},
\] (3.10)
along with a first order constraint
\[
0 = g_s^2(60H^2n + 2f^2) - (15g_s^2[f']^2 - 12g_s^4f'g_sf + 2[g_s^4][f^2]) .
\] (3.11)

Though we can not solve (3.9) - (3.11) analytically, it is straightforward to exhibit a smooth solution. Really, a smooth solution as \(\rho \to 0\) is
\[
g_s = g_0 \left( 1 - \frac{1}{7}\rho^2 + \frac{5}{378}\rho^4 + O(\rho^6) \right),
\] (3.12)
\[
f = 2Hn^{1/2} \left( \rho - \frac{1}{63}\rho^3 + \frac{1}{1470}\rho^5 + O(\rho^7) \right),
\] (3.13)
where \(g_0\) is an integration constant related to the string coupling. As \(\rho \to \infty\) we rather find\(^\text{13}\)
\[
g_s \to g_0 \rho^{3/4} e^{-\rho} , \quad f \to Hn^{1/2} \sqrt{20\rho}.
\] (3.14)

Note that curvature of (3.1) can be maintained arbitrarily small by taking \(g_0\) small. Thus the \(H \neq 0\) deformation indeed regulates the strong curvature region of the extremal NS5 brane background. On the other hand, from (3.14) we see that turning on a Hubble parameter induces a logarithmic correction to the asymptotically linear dilaton background.

\(^{13}\) These asymptotics can also be verified by numerical integration.
of the extremal NS5 branes. This should be contrasted with the finite energy density regularization of this geometry where one still recovers asymptotically linear dilaton [10].

From the above analysis it appears that given the Hubble parameter $H$, and for a fixed number of NS5 branes, there is a one parameter family of the LST de Sitter deformations, characterized by $g_0$. Furthermore, it is $g_0$ and not $H$ that controls curvature of the geometry (3.1). This is suprising, as LST does not have any continuous coupling constant. Also, physically, we expect that supergravity approximation describing deformed LST should break down for sufficiently small $H$ (in string units), as this theory should still be weakly coupled at low energies. This suggests that $g_0$ can not be a free parameter. In what follows we argue that this is indeed so. We find that

$$g_0 \sim 1/H^4,$$  \hspace{1cm} (3.15)

so that small $g_0$ (necessary for the validity of the supergravity description) corresponds to a large Hubble parameter in string units, and thus the full picture is consistent with the general lore for the absence of a dual supergravity description to a weakly coupled gauge theory. Before we proceed with an argument for (3.15) we would like to mention that a somewhat similar phenomenon occurs in the near extremal deformation of the NS5 branes [10]. Really, the near extremal deformation of LST is characterized by a single parameter, namely the energy density $\mu$. On the other hand, its holographic dual naively has two parameters: $r_0$ (the location of the black five-branes horizon) and $g_h$ (the value of the string coupling at the horizon). It turns out that by a simple change of a radial coordinate the background geometry of the near extremal NS5 branes can be shown to depend only on a combination $r_0^2/g_h^2$, which can be further identified with the energy density above the extremality in string units $\mu$ [10]. In our case, though we described a two-parameter family $\{g_0, H\}$ of the regular solutions of (3.9) -(3.11), $H$ dependence of the geometry can also be eliminated by redefining the time coordinate $t \rightarrow \tau \equiv 1/He^{-Ht}$. This is not very illuminating, as in doing so we are changing the reference energy scale from the LST perspective. Rather, we continue measuring all energies in string units. To relate $g_0$ and $H$ we study propagation of a minimally coupled scalar in the background (3.1) and on the NS5 brane probe. Specifically, consider a massless scalar $\chi$ minimally coupled to

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14 Classically, the temperature of the LST is independent of the energy density.
the Einstein metric (3.1) with zero angular momentum on $S^3$. The corresponding wave equation is

$$0 = -\partial_t \left[ e^{5Ht} \partial_t \chi \right] + e^{3Ht} \partial_i^2 \chi + \frac{e^{5Ht} g_s^2(\rho)}{4nf^4(\rho)} \partial_\rho \left[ f^6(\rho) \partial_\rho \chi \right] ,$$  (3.16)

where $i$ denotes the spatial directions of the NS5 brane. The last term in (3.16) can be interpreted as a $\rho$-dependent mass term operator on the LST space-time. Using (3.9)-(3.11) we can explicitly factor out $\{g_0, H\}$ dependence of this operator

$$\frac{e^{5Ht} g_s^2(\rho)}{4nf^4(\rho)} \partial_\rho \left[ f^6(\rho) \partial_\rho \cdots \right] \equiv e^{5Ht} H^2 O(\rho)[\cdots] .$$  (3.17)

Assuming the factorized dependence of $\chi$ on $\rho$

$$\chi(t, \bar{x}; \rho) = \kappa(\rho) \tilde{\chi}(t, \bar{x}) ,$$  (3.18)

we get from (3.16)

$$0 = -\partial_t \left[ e^{5Ht} \partial_t \tilde{\chi} \right] + e^{3Ht} \partial_i^2 \tilde{\chi} + e^{5Ht} H^2 \lambda(\rho) \tilde{\chi} ,$$  (3.19)

where $\lambda(\rho) \equiv 1/\kappa(\rho) O(\rho)[\kappa(\rho)]$. As in the original gauge-gravity correspondence of Maldacena we would like to interpret $\rho$ as a (measured in string units) holographic RG scale. Thus the dynamics of $\tilde{\chi}$ should be qualitatively similar to the dynamics of the generically massive scalar $\eta$ propagating along a probe NS5 brane oriented along $\{t, \bar{x}\}$, and sitting at a fixed radial coordinate $\rho$. With a scalar $\eta$ minimally coupled to the induced Einstein frame metric on the probe we find it’s wave equation to be

$$0 = -\partial_t \left[ e^{5Ht} \partial_t \eta \right] + e^{3Ht} \partial_i^2 \eta + e^{5Ht} m^2 g_s^{-1/2}(\rho) \eta ,$$  (3.20)

where $m$ is a constant mass of $\eta$. Extracting the $g_0$ dependence of the last term in (3.20) and comparing with (3.19) we are led to the identification (3.15)

$$g_0^{-1/2} \sim H^2 .$$

We believe that above arguments relating $g_0$ and $H$ are qualitatively correct, and apparently lead to the expected physical picture. It is important to find a more precise understanding of this relation, or in other words the map between the supergravity parameters $\{g_0, H\}$ and their LST dual. This will likely require an understanding of how to measure energies in non-static, asymptotically non-flat supergravity backgrounds. Finally, it is well known that string propagation in the throat geometry of the near extremal NS5 branes corresponds to an exact conformal field theory. It would be interesting to see whether there is a CFT description of the (1,1) LST in the de Sitter background presented here.

\[15\] See, for example, \[11\].
4. de Sitter deformations of the KT/KS backgrounds

Our aim in this section will be to explore dS embedding in the supergravity in the context of the corresponding deformation of the KT model \([6]\). Here, the conformal invariance of the gauge theory on the D3 branes at a conical singularity \([4]\) is broken by adding fractional D3 branes \([12]\). We also comment on the de Sitter deformation of the KS background \([5]\).

We will start with the same ansatz as in \([6]\) and simply replace 1+3 “longitudinal” directions by the Robertson-Walker metric with flat spacelike hypersurfaces

\[
(dM_4^H)^2 = -dt^2 + e^{2H}d\bar{x}^2.
\]  

As we show\(^{16}\), there will be a direct relation to the KT model on \(S^4\) considered in \([3]\).

As in \([6]\) we will impose the requirement that the background has abelian symmetry associated with the \(U(1)\) fiber of \(T^{1,1}\) as we will consider a phase where chiral symmetry is restored\(^{17}\). Our general ansatz for a 10-d Einstein-frame metric will involve 3 functions \(y, z\) and \(w\) of radial coordinate \(u\) \(^{18}\)

\[
ds_{10E}^2 = e^{2y}(dM_4^H)^2 + e^{-2y}[e^{10y}du^2 + e^{2y}(dM_5)^2].
\]  

Here \(M_5\) is a deformation of the \(T^{1,1}\) metric

\[
(dM_5)^2 = e^{-8w}e^2\psi + e^{2w}(e_{\theta_1}^2 + e_{\phi_1}^2 + e_{\theta_2}^2 + e_{\phi_2}^2),
\]  

\[
e_{\psi} = \frac{1}{3}(d\psi + \cos\theta_1d\phi_1 + \cos\theta_2d\phi_2), \quad e_{\theta_i} = \frac{1}{\sqrt{6}}d\theta_i, \quad e_{\phi_i} = \frac{1}{\sqrt{6}}\sin\theta_i d\phi_i.
\]  

As for the matter fields, we will assume that the dilaton \(\Phi\) may depend on \(u\), and our ansatz for the \(p\)-form fields will be exactly as in the extremal KT case \([6]\) and in \([13,3]\):

\[
F_3 = Pe_{\psi} \wedge (e_{\theta_1} \wedge e_{\phi_1} - e_{\theta_2} \wedge e_{\phi_2}), \quad B_2 = f(u)(e_{\theta_1} \wedge e_{\phi_1} - e_{\theta_2} \wedge e_{\phi_2}).
\]  

\(^{16}\) I would like to thank Arkady Tseytlin for pointing this out.

\(^{17}\) Much like in the case of the LST on the de Sitter space-time, we expect the Hubble scale to realize an IR cutoff on the gauge theory dynamics. Thus for sufficiently high \(H\) (which we take to be the case in this section) we expect restoration of the chiral symmetry in the dual gauge theory.

\(^{18}\) For \(H = 0\) this metric can be brought into a more familiar form \(ds_{10E}^2 = h^{-1/2}(r)(dM_4^{H=0})^2 + h^{1/2}(r)[dr^2 + r^2d\bar{x}_5^2]\), where \(h = e^{-4z}, \quad r = e^{y+w}, \quad e^{10y}du^2 = dr^2\). When \(w = 0\) and \(e^4v = r^4 = \frac{1}{4u}\), the transverse 6-d space is the standard conifold with \(M_5 = T^{1,1}\). Small \(u\) thus corresponds to large distances in 5-d and vice versa. In the \(AdS_5\) region large \(u\) is near the origin of \(AdS_5\) space, while \(u = 0\) is its boundary.
\( F_5 = \mathcal{F} + *\mathcal{F} \), \( \mathcal{F} = K(u) e_\psi \wedge e_{\theta_1} \wedge e_{\phi_1} \wedge e_{\theta_2} \wedge e_{\phi_2} \), \( K(u) = Q + 2Pf(u) \), (4.5)

where, as in [4], the expression for \( K \) follows from the Bianchi identity for the 5-form. The constants \( Q \) and \( P \) are proportional to the numbers of standard and fractional D3-branes.

We could now directly derive the corresponding system of type IIB supergravity equations of motion describing the radial evolution of the five unknown functions of \( u: y, z, w, K, \Phi \). A better approach is to notice that the background we consider here could be obtained from the KT model on \( S^4 \) discussed in [3]. Really, the only difference of our case with the \( S^4 \) compactification of [3] is the replacement of the “longitudinal” directions (4.1) with (\( dS^4 \))^2

\[
(dM_4^H)^2 \to (dM_4)^2 \equiv (dS^4)^2 = d\alpha^2 + \sin^2 \alpha [d\beta^2 + \sin^2 \beta (d\gamma^2 + \sin^2 \gamma d\delta^2)] .
\] (4.6)

Now, Wick rotation of (4.6) \( \alpha \to i\alpha \) and the scaling limit on \( S^3 \) parameterized by \( \beta, \gamma, \delta \) \((dS^3)^2 \to d\bar{x}^2\) along with \( \alpha \gg 1 \) gives

\[
(dS^4)^2 \to -d\alpha^2 + e^{2\alpha}d\bar{x}^2 ,
\] (4.7)

which is precisely (4.1) with \( H = 1 \). Thus, the resulting equations\(^{19}\) are just the straightforward modification of (4.7)-(4.12) of [3]

\[
10y'' - 8e^{8y}(6e^{-2w} - e^{-12w}) - 30H^2e^{10y-4z} + \Phi'' = 0 ,
\] (4.8)

\[
10w'' - 12e^{8y}(e^{-2w} - e^{-12w}) - \Phi'' = 0 ,
\] (4.9)

\[
\Phi'' + e^{-\Phi+4z-4y-4w}(\frac{K'^2}{4P^2} - e^{2\Phi+8y+8w}P^2) = 0 ,
\] (4.10)

\[
4z'' - K^2e^{8z} - e^{-\Phi+4z-4y-4w}(\frac{K'^2}{4P^2} + e^{2\Phi+8y+8w}P^2) - 12H^2e^{10y-4z} = 0 ,
\] (4.11)

\[
(e^{-\Phi+4z-4y-4w}K')' - 2P^2Ke^{8z} = 0 ,
\] (4.12)

with the first order constraint

\[
5y'^2 - 2z'^2 - 5w'^2 - \frac{1}{8}\Phi'^2 - \frac{1}{4}e^{-\Phi+4z-4y-4w}\frac{K'^2}{4P^2}
\]

\[- 3H^2e^{10y-4z} - e^{8y}(6e^{-2w} - e^{-12w}) + \frac{1}{4}e^{\Phi+4z+4y+4w}P^2 + \frac{1}{8}e^{8z}K^2 = 0 .
\] (4.13)

\(^{19}\) We also obtained these equations directly in the background (4.2).
Lacking the exact solution of the above system, it was nonetheless demonstrated in [3] the existence of a smooth interpolation (in radial coordinate only) between (i) a non-singular short-distance region where the 10-d background is approximately $AdS_5 \times T^{1,1}$ written in the coordinates where the $u = \text{const}$ slice is $S^4$, and (ii) a long-distance region where the 10-d background approaches the KT solution. This was shown by starting with the short-distance ($u = \infty$ or $\rho = 0$) region, i.e. $AdS_5 \times T^{1,1}$ space (with the radius determined by the effective charge $K_*$) and demonstrating that by doing perturbation theory in the small parameter $\frac{P^2}{K_*} \ll 1$ one can match it onto the KT asymptotics at large distances ($u \to 0$ or $\rho \to \infty$). The crucial point was that $O(\frac{P^2}{K_*})$ perturbations were regular at small distances. One can literally repeat this analysis of [3] in our case to argue that for large enough Hubble parameter $H$ the naked singularity of the KT geometry will be resolved. Here the short distance region is a direct product of approximately a de Sitter slicing of $AdS_5$ (as in section 2) and a $T^{1,1}$ coset.

The de Sitter deformation of the KT model above and of the LST in the previous section are similar in that as one turns off a Hubble parameter (or rather sufficiently lower it), one ends up with a singular geometry. A way to turn on small (vanishingly small) de Sitter deformation is to start with a gauge-gravity correspondence for a confining gauge theory like, say, the KS model [5]. It is straightforward to repeat above analysis for the deformed KS background and obtain a consistent system of equations. We do not present this system here due to its complexity and the fact that we could not find analytical solution. The added difficulty (compare to the extremal KS background) comes from the fact that it is inconsistent (on the level of equations of motion) to demand $H \neq 0$ along with a constant dilaton. A similar phenomenon has been observed in studies of the near extremal deformation of the KS background [14]: it was shown there that a black hole with a regular Schwarzschild horizon in the KS geometry necessarily has nonconstant dilaton. This observation has a simple physical interpretation. In the extremal KS solution the string coupling $g_s$ was an exact modulus of the cascading gauge theory, dual to the sum of the individual gauge couplings

$$\frac{1}{g_s} = 4\pi \frac{g_1^2}{g_1} + 4\pi \frac{g_2^2}{g_2} = \text{const}. \quad (4.14)$$

As both the finite temperature and the Hubble parameter breaks supersymmetry, this modulus is expected to be lifted, thus developing a nontrivial radial dependence in the dual supergravity.

13
5. Concluding remarks

In this paper we presented a simple framework how one can embed an accelerating universe in the supergravity. The idea is to start with a gauge-gravity duality of Maldacena, and consider deformations of this duality where Minkowski background space-time of the gauge theory is replaced with a de Sitter space-time.

We argued that to get nontrivial time-dependent solutions (i.e. unrelated by coordinate reparametrization to a static solution) the starting point for the deformation must be a gravitational dual to a non-conformal gauge theory. We discussed two examples of such deformations: the little string theory and the KT model. In the both cases conformal invariance is broken by considering (adding) NS5 (D5) branes. We argued that the expansion of the background geometry on the gauge theory side serves as an infrared cutoff in the dual supergravity. In particular, for a sufficiently high expansion rate this resolves a naked singularity of the KT solution \[ T_{\text{GH}} = \frac{H}{2\pi} \], analogous to the Hawking temperature of a black hole. The KT deformation discussed here is very similar to the finite temperature deformation of the KT solution considered in \[ [13,15] \]. By comparing a critical expansion rate for the \( H \neq 0 \) deformation of the KT model with the critical temperature for its finite temperature deformation, one should be able to relate the Gibbons-Hawking temperature of the expanding universe with the temperature of the gauge theory in the standard near extremal deformation.

There are several interesting future directions. The vacuum state in an accelerating universe has a nonzero Gibbons-Hawking temperature \( T_{\text{GH}} = H/2\pi \), analogous to the Hawking temperature of a black hole. The KT deformation discussed here is very similar to the finite temperature deformation of the KT solution considered in \[ [13,15] \]. By comparing a critical expansion rate for the \( H \neq 0 \) deformation of the KT model with the critical temperature for its finite temperature deformation, one should be able to relate the Gibbons-Hawking temperature of the expanding universe with the temperature of the gauge theory in the standard near extremal deformation.

Another interesting question is a dynamical stability of the deformed backgrounds discussed here. Since de Sitter deformation breaks supersymmetry one has to worry about potential tachyons. The similarity of this deformation with the near-extremal one suggests that the KT model is likely to be stable, while there could be a tachyon in the LST deformation, in analogy with \[ [11] \]. It would be nice to explicitly verify these conjectures. In the case of the KS deformation, at least for small values of the Hubble parameter, we expect to get a stable nonsupersymmetric background. The argument is identical to the one given in \[ [16] \]: the original supergravity background had a mass gap, and thus a small deformation should not produce a tachyon.

In this paper we only constructed de Sitter backgrounds in supergravity. It is important to understand the spectrum of density fluctuations and the physics of D-brane probes in these geometries.
Recently Giddings, Kachru and Polchinski [17] studied embedding of the KS model in the type IIB string compactifications in the context of moduli stabilization and generation of large hierarchies of physical scales. It would be very interesting to explore de Sitter deformation of these models.

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