On the $^{11}$Li two-cluster model for calculations of the $^{11}$Li+p elastic scattering at low and high energies

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Abstract. The two-cluster model is utilized to the $^{11}$Li nucleus consisting of the $^{9}$Li and $h=2n$ clusters for calculations of the microscopic optical potentials and the $^{11}$Li+p scattering cross sections. The obtained density of $^{11}$Li is compared to densities known from the p+$^{11}$Li scattering data at $T_{lab}=700$ MeV fitted using the Glauber theory. Effect of 2n-halo and the $^{9}$Li core contribution to the cross sections is analysed.

1. Motivation
In [1], cross sections of the $^{11}$Li+p elastic scattering at low energies were successfully explained with a help of the microscopic OP [2] where the Large scale shell model (LSSM) [3] density of $^{11}$Li was employed (see dashed line in the left panel of Fig. 1). Also, the two-cluster model developed in [4] to the $^6$He nucleus, was utilized in [1] to estimate the $^{11}$Li breakup and the momentum distributions of fragments.

On the other hand, the phenomenological density of $^{11}$Li (denoted 0.2GG+0.8GO in [5], see solid line in the left panel of Fig. 1) was obtained in [5] by fitting the Glauber theory to experimental data on the $^{11}$Li+p elastic scattering at high energy about 700 MeV. It seems interesting to check (i) if this density can explain experimental data at low energies; (ii) if the two-cluster model density is applicable in both cases of low and high energies.

Below we construct the one-particle $^{11}$Li density inherent to the dynamic two-cluster model (D2C) of $^{11}$Li consisting of the $^9$Li core and $h=2n$ halo bounded with energy about 0.3 MeV. The parameters of clusters are varied to get the $^{11}$Li-density being close to the 0.2GG+0.8GO-form, and the effect is studied of the halo and core contributions to the $^{11}$Li structure. Then we calculate the respective microscopic OPs to reproduce experimental data on the $^{11}$Li+p elastic scattering at both low and high energies.

2. One-particle density in two-cluster model of $^{11}$Li
Relative motion of two clusters in $^{11}$Li is described by the two-node s-state wave function $\Phi(R)$ which is a solution of the Schrödinger equation with WS-potential, that reproduces the respective binding energy and rms-radius of the ($^{9}$Li+$h$) system (this fit was done in [1]). Then the $^{11}$Li one-particle density can be calculated as folding integral of the sum of $^{9}$Li- and $h$-densities $\rho_c$ and $\rho_h$ with the density probability $\rho_m = |\Phi(R)|^2$ obtained in [1]:

$$\rho_m(r) = \int d^3R \left[ \rho_h(r - \frac{2}{11}R) + \rho_c(r - \frac{9}{11}R) \right] \cdot |\Phi(R)|^2.$$  \hspace{1cm} (1)

The halo-density we take in the form of the symmetrized Fermi function (SF):

$$\rho_h(r) = \rho_0 \cdot \sinh \left( R/a \right) \cdot \left[ \cosh \left( R/a \right) + \cosh \left( r/a \right) \right]^{-1}, \quad R^2 = [5R_h^2 - 7(\pi a)^2] / 3.$$ \hspace{1cm} (2)
The halo effect on the $^{11}\text{Li}$ density is shown in Fig. 1, right panel. For the halo density we put $a = 0.5$ and vary rms-radius $\bar{R}_h = 2 \div 6$ fm while the $^9\text{Li}$ density is taken in the LSSM form. Respective rms-radii of halo $\bar{R}_h$ and of whole $^{11}\text{Li}$ nucleus $\bar{R}_m$ are given in the Table 1. It is seen that when $\bar{R}_h$ is varied from 2 to 6 fm then the $\bar{R}_m$ radius changes only 30%. The last line in the Table 1 shows the “bestfit” parameters of our D2C density, and the respective curve is given in the left panel of Fig. 2 (dashed) in comparison with the 0.2GG+0.8GO-density (solid).

Table 1. Parameters of 2n-halo in D2C-density

| $\bar{R}_h$ | $R$  | $\bar{R}_m$ | curve | $\bar{R}_h$ | $R$  | $\bar{R}_m$ | curve   |
|------------|------|-------------|-------|------------|------|-------------|---------|
| 6          | 7.365| 3.77        | circles | 3         | 3.04| 3.02        | dash-points |
| 5          | 6    | 3.49        | points  | 2         | 0.95| 2.79        | dashes   |
| 4          | 4.573| 3.24        | triangles | 4.22 | 4.94| 3.29        | $a = 0.48$ best |

3. Cross sections at low energies

Here we apply the D2C densities (1) to calculate the $^{11}\text{Li}+p$ microscopic OP [2]. Then the DWUCK4 code [6] is utilized to compute the respective elastic scattering cross sections. Right panel of Fig. 2 demonstrates a weak effect on them of the halo contribution in the case of low energies. Also, one sees that in spite of the “bestfit” D2C-based density is slightly different to the 0.2GG+0.8GO-form, both cross sections are in good agreement with experiment as well as the curve with $\bar{R}_h = 5$.

In order to analyze the core contribution effect we fix parameters of halo and consider three forms of $^9\text{Li}$ densities, see Fig. 3, left panel. Respective cross sections are presented on the right panel of Fig. 3. When we increase the core radius about 30% then the cross section curves change significantly. So, one can conclude that effect of the $^9\text{Li}$ dimension on the cross sections is strong.

4. Cross sections at high energy

At energy 697 MeV we need to account for relativization. Relativistic momentum $p$ in c.m. system is as follows:

$$p = p_{lab} \cdot m_2 / \sqrt{2E m_2 + m_1^2 + m_2^2} \quad p_{lab} = \sqrt{T_{lab} (T_{lab} + 2m_1)}$$

(3)
Here \( E = T_{\text{lab}} + m_1 \) is the total energy, \( T_{\text{lab}} \) – kinetic energy in laboratory system, \( m_1 \) and \( m_2 \) are the proton mass and the \(^{11}\text{Li} \) mass. The Klein–Gordon–Fock equation at \( E \gg |U| \) is reduced to the form of Schrödinger equation with relativistic momentum \( p \) and relativistic reduced mass \( \bar{\mu} = E \cdot M_A / (E + M_A) \):

\[
(\Delta + p^2)\psi(r) = 2\bar{\mu} V(r)\psi(r), \quad V(r) = U(r) + U_C(r).
\]

The potential consists of the Coulomb potential \( U_C(r) \) and the microscopic OP \( U(r) \).

In Fig. 4 one sees a good agreement of our D2C-based calculations with the high energy data. Fig. 5 shows cross sections calculated using different forms of halo density (left panel) and different forms of the \(^9\text{Li} \) density (right panel). It is seen that at high energy, the effect of halo contribution remains to be weak while effect of the core-density dimension is strong.

5. Summary
The one-particle \(^{11}\text{Li} \) density is constructed within the frame of the dynamic two-cluster model of \(^{11}\text{Li} \) consisting of bounded \(^9\text{Li} \)-core and \( h = 2n \) halo clusters. Microscopic OPs obtained with
Figure 4. $p+^{11}\text{Li}$ elastic scattering at 697 MeV. Left: experimental data and calculations with $0.2\text{GG}+0.8\text{GO}$-density as they done in [5]. Right: our calculations with D2C-density using HEA OP (solid) and folding real and HEA imaginary OP (dashed) compared with the same data but in the other presentation of axes.

Figure 5. Effect of halo and core structure at high energies. Left: the same as in Fig. 2(right) but for 697 MeV. Right: the same as in Fig. 3(right) but for 697 MeV.

such a density can reproduce experimental data on the $^{11}\text{Li}+p$ elastic scattering at both low and high energies. Effect of the halo-density is shown to be weak while effect of the core-density dimension is strong at both low and high energies.

The D2C model makes possible to calculate not only the one-particle density of $^{11}\text{Li}$ and respective OPs but also the $(p+h)$ and $(p+c)$ optical potentials and therefore one can estimate the breakup cross sections and momentum distributions of the $^{11}\text{Li}$ fragments.

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