Inflation and Holography in String Theory

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Abstract

The encoding of an inflating patch of space-time in terms of a dual theory is discussed. Following Bousso’s interpretation of the holographic principle, we find that those are generically described not by states in the dual theory but by density matrices. We try to implement this idea on simple deformations of the AdS/CFT examples, and an argument is given as to why inflation is so elusive to string theory.
1 Introduction

The idea that the area of a surface in space-time bounds the number of degrees of freedom in the space-like region inside it has been entertained for some time now [1]. The quintessential example of it is the semi-classical black hole, whose entropy scales as its area and is in some sense the most entropic system one can imagine [2].

Some realizations of this have arisen in string theory, particularly in the AdS/CFT correspondence. Although there the theory constructed is dual to gravity, one can nonetheless estimate that the number of degrees of freedom inside a bounded region is proportional to the area of its boundary [3]. The theory thus constructed - the CFT - can then be sensibly described as living “on the boundary of space-time.”

This holographic description of space was then generalized by Boussou [4], who succeeded in giving a covariant set of rules to associate regions of space-time with the degrees of freedom living on a surface bounding it. One of the strengths of this approach is the transparency of the statistical mechanical interpretation of the entropy, since the rules are independent of the “arrow of time” one chooses. Of particular interest for this construction are the horizons, null surfaces where the boundary of an observable patch of space-time lies. As such, the holographic description of points on either side of the horizon can be located at distant regions of space (see below).

This work addresses the problem of horizon formation in the the supergravity theories that arise from string theory. The motivation seems rather clear, given the amount of information gathered recently from the interplay between supergravity and string theory. More prosaically, this situation seems to combine both ingredients necessary [5]: i) Kaluza-Klein compactifications can generate exponential potentials that are on the verge of inducing inflation, and ii) Even though the antisymmetric tensor fields of supergravity obey the strong energy condition in 10 (or 11) dimensions, this may not be so after compactification.

The paper is organized as follows. The second section reviews Boussou’s construction - and terminology - of preferred screens and the context of formation of horizons. The third and fourth sections deal with supergravity compactifications. it finishes with a discussion about the results and interpretation.


\section{Holographic Screens}

Loosely speaking, holography is the statement that the area of a surface bounds the number of degrees of freedom in its interior. Of course, this idea has a serious drawback that there is no covariant notion of “interior region” when one is dealing with a \( n - 2 \) hypersurface embedded in a \( n \) dimensional space-time. This notion can be recovered \cite{footnote} by considering collapsing null light rays emanating from the surface, the \textit{light-sheets}. This gives then a covariant definition of the interior region described by its boundary degrees of freedom.

By following the geodesic generators of the light-sheet back in the direction of non-negative expansion, one can \textit{project} the degrees of freedom into a larger, possibly infinite, surface \( B \), the \textit{screen}. Generically, the expansion will change sign at \( B \), but, if the expansion vanishes everywhere in \( B \) it will be called a \textit{preferred screen}. Preferred screens are conjectured to be those in which the holographic bound is saturated.

One can then foliate the whole space-time into null-surfaces, and project their degrees of freedom onto screens. The family of screens contains a finite number of degrees of freedom per Planck area. By an abuse of language we will also call these hypersurfaces preferred screens. Examples of preferred screens include the cosmological horizon of an inflationary universe and the conformal boundary of an anti-de Sitter (AdS) space-time.

Consider for instance the maximally extended Schwarzchild-AdS space (Fig. 1): there are two preferred screens, at the conformal boundaries of space-time. By null projection, one can encode all the information inside the black-hole into either boundary. For instance, a state localized inside the horizon of the black hole, at for instance a point \( P \) can be described by the set of degrees of freedom in the boundary as a state in the Hilbert space \( \mathcal{H}_1 \) living at the boundary preferred screen. This view is compatible with the AdS/CFT duality, which states that the (conformal) field theory describing perturbations of AdS space lives at the conformal boundary. Note however that the local measurements on \( P \) do depend on information coming from both regions near the spatial infinities.

On the other hand, it is usual for screens to form during gravitational collapse. Usually, however, for normal and anti-trapped regions the projection into past screens is not altered, and then the details of the collapse can still be encoded into the screen at the boundary. This can be illustrated by following the past null projection of the
Figure 1: The Penrose diagram for the maximally extended Schwarzschild-AdS solution. The thicker vertical lines represent the holographic screens. Holography tells us that a state $P$ inside the black hole can be immersed into either Hilbert space $\mathcal{H}_1$ or $\mathcal{H}_2$.

point $P$ in Fig. 1. For instance, the problem of gravitational collapse in AdS was studied in several instances in the context of AdS/CFT, and corresponds in the dual theory to the evolution of excited states into one or more states in a thermal ensemble $\mathcal{H}_1$. The entropy of the black hole shows as the degeneracy of the thermal ensemble corresponding to it.

But ultimately the details of the collapsing region depend heavily on the model studied. One can cook up models in which a cosmological type of horizon is formed inside the region under collapse, which, in its extreme view, can create causally disconnected patches of space-time $\mathcal{H}_1$. Fig. 2. Our goal is a little more modest: while the uniqueness of the compactifications of 11-dimensional supergravity prevents the creation of “child-universes,” we would still hope that anti-trapped surfaces could be created in a meta-stable state of the theory. Thus, while ultimately the scenario from Fig. 2 would decay into the Schwarzschild-AdS black hole, it is also clear that not all states inside the bubble would allow for a unique description at spatial infinity. For instance, region $B$ would allow for a spatial boundary description by ultimately decaying into AdS space, but the degrees of freedom in region $A$ are projected onto
Figure 2: The birth of a child-universe in AdS space. A domain wall separates the expanding region (to the left) to the vacuum Schwarzchild solution (right). The holographic screens are represented in thicker lines.

the null screen. The past projection cannot be used since in this scenario the past directed null geodesics are not complete [10].

The puzzle relies on this simple fact: by considering a bubble of these states into the usual AdS vacuum of supergravity compactifications, one might create an anti-trapped surface inside a trapped one. As the system evolves, the meta-stable state would decay and form a singularity. So future projection of the degrees of freedom inside the black hole are forbidden. But now degrees of freedom inside the bubble and behind the horizon cannot be projected out of the black-hole either, like the region B in Fig. 2. If this construct can be implemented, the description of the black-hole in the boundary theory would be by means of a density matrix, and not by a wave-function. The density matrix would encode the fact that the degrees of freedom representing the meta-stable state are not all (fore)seen by an observer at spatial infinity.

This interpretation can perhaps be seen more clearly from the Kruskal extension of the Schwarzchild-AdS solution (Fig. 1.) A state localized in a region corresponding to a point $P$ inside the black hole region can be encoded in either screen, $\mathcal{H}_1$ or $\mathcal{H}_2$. That means that in spite of the fact that the dual theory, say in $\mathcal{H}_1$, is still able to determine the state in $P$ uniquely using its Hilbert space, the evolution of that state will depend also on information coming from the other leaf of the universe, which is encoded by Hilbert spaces in the other screen, such as $\mathcal{H}_2$. In other words, the total
Hilbert space of space-time is not nicely separated into the sum of two spaces, one for each screen. Any dual description of the evolution of the state in $P$ would be incomplete since the theory is missing part of the Hilbert space. The dual theory then sees the state as a density matrix.

3 Deformations of the background

Let us consider first the D1-D5 system. One starts from the low energy effective Lagrangian of type IIB

$$S_{IIB} = \frac{1}{g_s^2 l_s^8} \int d^{10} x \sqrt{-g} \left( R - \frac{1}{2} \nabla_a \Phi \nabla^a \Phi - \frac{1}{24} e^\Phi H_{abc} H^{abc} \right) + \ldots$$

(1)

with an ellipsis for all fields not turned on here.

The background described is the usual: there are $Q_1$ D1 branes and $Q_5$ D5 branes, sources for the $H$ field. The solution for the Einstein metric is:

$$ds_E^2 = H_1^{-3/4} H_5^{-1/4} \eta_{\mu\nu} dx^\mu dx^\nu + H_1^{3/4} H_5^{1/4} dx^i dx^i + \frac{H_1^{1/4}}{H_5} dx^m dx^m;$$

$$H_1 = 1 + \frac{r_1^2}{r^2}, \quad r_1^2 = \frac{(2\pi)^2 g_s Q_1 l_s^6}{V_4};$$

$$H_5 = 1 + \frac{r_5^2}{r^2}, \quad r_5^2 = g_s Q_5 l_s^2.$$  

(2)

with $\mu, \nu$ parallel to all branes, $m$ tangent to the D5 branes but transverse to the D1, and $i$ transverse to all branes. To keep the total energy finite, we will compactify the $m$ coordinates on a four torus, with volume $V_4$.

Thus the six dimensional infinite space of the solution (2) has two distinct regions: for $r \to \infty$ is asymptotically flat, whereas in the limit $r \to 0$ it factorizes into $AdS_3 \times S^3$. The transition is at $r \approx r_1, r_5$. By taking the near horizon limit - $r \to 0$, one can single out the $AdS_3 \times S^3$ portion. In this limit, the radius of curvature for both the $AdS_3$ and $S^3$ is of order $Q_1 Q_5$. The size of the torus, however is much smaller, being of order $Q_1/Q_5$. This means that we have an effective six-dimensional supergravity on $AdS_3 \times S^3$ with matter fields arising from the compactification. From the 10-dimensional point of view, we are considering low energy excitations near the branes, which, due to the non-flat background, never make it to the flat region.

As far as the dynamics around this region is concerned, the dilaton and the Kaluza-Klein modes will behave as matter fields in AdS3. We will restrict our attention to fluctuations in the $S^3$ radius and the $T^4$ volume, disregarding higher and stringy...
corrections. This system, however, contains some small compactified dimensions, the torus, and some stringy states may be excited during the evolution of this system. Taking them into account would provide a more thorough analysis of the system, but will not be done here. Writing then the ansatz for the metric:

\[ g_{ab} = \tilde{g}_{ab}^{(3)} \oplus \tilde{h}_{ab}^{(4)} \oplus \delta_{ab}^{(4)} = e^{-6\alpha-8\beta} g_{ab} + e^{2\alpha} h_{ab} + e^{2\beta} \delta_{ab} \]  

with \( h_{ab} \) and \( \delta_{ab} \) constant curvature metrics for \( S^3 \) and \( T^4 \) and \( \alpha \) and \( \beta \) generic functions of the “AdS\(_3\)” coordinates \( \{t,x,y\}\). The constraint of \( Q_1 \) electric and \( Q_5 \) magnetic charges determines \( H \) completely as a function of \( \alpha \) and \( \beta \):

\[ H = \frac{(2\pi l)^2 Q_5}{l^3} e^{-3\alpha} + \frac{(2\pi g s l^4)^2 Q_1 l^4}{V_4} e^{-3\alpha-4\beta} \varepsilon_{S^3} \]

From above one sees the flat direction \( \gamma = 2\beta - \Phi \), having to do with the variation of the fields that leaves the effective 6-dimensional Newton’s constant invariant. Shifting \( \gamma \) amounts then to shifting the minimum of the potential, which in turn can be trivialized by changing \( l \). From above it turns out that it’s also desirable to define \( \varphi = \alpha + \beta \). The independent equations of motion in (5) are derivable from an action:

\[ S_{\text{eff}} = \frac{1}{3G_N} \int d^3x \sqrt{-g} \left( R - 3\nabla_a \varphi \nabla^a \varphi - 4\nabla_a \beta \nabla^a \beta + \frac{6}{l^2} e^{-4\varphi} - \frac{4}{l^2} e^{-6\varphi} \cosh 4\beta \right) \]

plus a term \( \nabla_a \gamma \nabla^a \gamma \) to account for the dilaton equation. One notes in the equations of motions (5) and the Lagrangian above the symmetry \( \beta \rightarrow -\beta \), a manifestation of U-duality along the branes directions. Note that the matter fields don’t necessarily satisfy the strong energy condition.
4 Inflation?

The exponential growth of scales was proposed about 20 years ago \cite{1} to explain the asymptotic flatness and the horizon problem of the recent universe. Along with the idea, a simple system was proposed where an unstable excited state of a scalar field would drive the exponential or polynomial growth of spatial scales. The idea behind is that, if the spatial scales grow too fast \cite{6} (faster than the proper time), two nearby observers won’t be able to see each other, if the expansion lasts for enough time. A cosmological horizon would then be formed.

As usual, one slices up space-time into homogeneous surfaces of constant time, whose flow is represented by a vector field \( \xi^a = \nabla^a t \). Orbits of \( \xi^a \) are geodesics (\( \xi^a \nabla_b \xi^b = 0 \)), and \( \xi^a \xi_a \) is normalized to \(-1\). This allows us to interpret \( \nabla_a \xi_b = \hat{h}_{ab} \) as the time derivative of the spatial metric \( \mathcal{L}_\xi h_{ab} = \dot{h}_{ab} \). Because space is assumed to be homogeneous, the spatial metric \( h_{ab} \) has constant curvature, and the only allowed evolution is a dilation \( \nabla_a \xi_b = \dot{h}_{ab} = \theta h_{ab} \). The dynamical system (5) will then inflate if there is a critical point for which the expansion factor \( \theta \) is positive. The equation for \( \theta \) is then just the Raychaudhuri equation \cite{11}:

\[
\dot{\theta} = -\frac{1}{2} \theta^2 - \xi^a R_{ab} \xi^b = -\frac{1}{2} \theta^2 - 3 \dot{\varphi}^2 - 4 \beta^2 - \frac{2}{l^2} \left( 3 e^{-4 \varphi} - 2 e^{-6 \varphi} \cosh 4 \beta \right) \quad (7)
\]

Using these coordinates, the equations for \( \varphi \) and \( \beta \) are

\[
\ddot{\varphi} = -\theta \dot{\varphi} - \frac{4}{l^2} e^{-4 \varphi} + \frac{2}{l^2} e^{-6 \varphi} \cosh 4 \beta \\
\ddot{\beta} = -\theta \dot{\beta} - \frac{2}{l^2} e^{-6 \varphi} \sinh 4 \beta \quad (8)
\]

If the system is now placed at a point where \( 4 \beta \gg 2 \varphi \sim 0 \), the \( \beta \) term will dominate in the potential. After a reparametrization \( \frac{d\tau}{dt} = e^{-3 \varphi + 2 \beta} \), the system can be written as:

\[
\tilde{\theta}' \approx -\frac{1}{2} \tilde{\theta}^2 + 3 \varphi' \tilde{\theta} - 2 \beta' \tilde{\theta} - 3 \varphi'^2 - 4 \beta'^2 + \frac{2}{l^2} \\
\varphi'' \approx 3 \varphi'^2 - 2 \beta' \varphi' - \tilde{\theta} \varphi' + \frac{2}{l^2} \\
\beta'' \approx 3 \beta' \varphi' - 2 \beta'^2 - \tilde{\theta} \beta' - \frac{1}{l^2} \quad (9)
\]

where \( \tilde{\theta} = \theta e^{3 \varphi - 2 \beta} \) and the primes denote differentiation with respect to \( \tau \).

As it turns out the system above does have a critical point:

\[
\tilde{\theta} = 4 \sqrt{2} \frac{l}{l}, \quad \beta' = -\frac{1}{2 \sqrt{2 l}}, \quad \varphi' = \frac{1}{\sqrt{2 l}} \quad (10)
\]

\footnote{See, for instance, Appendix E in \cite{1}}
With the initial conditions, $\beta(0) = \beta_0$ and $\phi(0) = 0$, the approximate solution for $t \ll le^{2\beta_0}$ is:

$$
\begin{align*}
\theta &= \frac{4\sqrt{2}e^{2\beta_0}}{1 + 2\sqrt{2}e^{2\beta_0}t} \\
\beta &= \beta_0 - \frac{1}{8}\ln\left(1 + 2\sqrt{2}e^{2\beta_0}\frac{t}{2}\right) \\
\varphi &= \frac{1}{4}\ln\left(1 + 2\sqrt{2}e^{2\beta_0}\frac{t}{2}\right).
\end{align*}
$$

(11)

As $t$ approaches $le^{2\beta_0}$ the number of e-foldings $\frac{1}{2} \int \theta dt$ grows as $4\beta_0$. Observers following the flow of time $\xi^a$ will then see an expansionary phase for an arbitrarily long time. After some number of e-foldings, however, the system will relax to its unique ground state: AdS$_3$.

Information about the space slices can be gathered by decomposing the curvature:

$$3R = 2R + \frac{3}{2}\theta^2 + 2\dot{\theta}$$

and taking the trace in (5). Using the equation of motion for $\dot{\theta}$ (7) one arrives at the Hamiltonian constraint:

$$2R = -\frac{1}{2}\theta^2 + 3\dot{\varphi}^2 + 4\dot{\beta}^2 - \frac{1}{l^2}(6e^{-4\varphi} - 4e^{-6\varphi} \cosh 4\beta).$$

(12)

At the critical point, and at early stages $t \ll le^{2\beta_0}$, we have:

$$2R \simeq -\frac{12}{l^2} e^{-6\varphi+4\beta} = -\frac{12}{l^2} \frac{e^{4\beta_0}}{\left(1 + 2\sqrt{2}e^{2\beta_0}t\right)^2}.$$  

(13)

so, although the system initially has a large spatial curvature, it actually “opens up” $2R \rightarrow 0$ as a result of the expansion. The spatial curvature is, however, negative definite, and a cosmological horizon would still not be formed. In fact, the geometry for intermediate times is not unlike the radiation-filled hyperbolic FRW universe. Despite having non-trivial exponential potentials for the scale factors of $S^3$ and $T^4$, the conditions for inflation are not met.

The resulting 10-dimensional geometry is computed by reversing the definition of $\varphi$ and the conformal transformations relating the six-dimensional and the ten-dimensional metrics:

$$ds^2_{(10)} = -dt^2 + K\nu^2 t_{ij}dx^i dx^j + \ldots$$

(14)

with $t_{ij}$, $(i, j = 1, 2)$ a metric of constant negative curvature, $K$ a constant depending on the initial conditions and the metric of the sphere and the torus were omitted. Note that the ten dimensional causal structure is the same as the three-dimensional one, due to the symmetry of the internal space and the fact that the two metrics are related by a conformal transformation.
5 Other Examples

5.1 AdS$_5 \times S^5$

Much the same way the D1-D5 system gives rise to the AdS$_3 \times S^3 \times T^4$ space, the near-horizon geometry of a number of D3 branes is AdS$_5 \times S^5$. We turn now to study deformations around this background. The situation here is \textit{a priori} a bit different from the previous example in which there are no small dimensions: the radius of the five-sphere is the same as the radius of AdS$_5$. There is then some justification in considering perturbations of the radii of each direct summand of space-time. The system is expected to behave more “gravitationally” than “stringy” at the moderate energies we are dealing with.

One starts from the Ansatz:

$$10g_{ab} = \tilde{g}^{(5)}_{xy} \oplus \tilde{h}^{(5)}_{a_1b_1} = e^{-\frac{10\alpha}{3}} g_{xy} \oplus e^{2\alpha} h_{a_1b_1}$$

The 3-branes act as electric and magnetic sources for the $F_5$ field:

$$\int_{S^5} 10^* F_5 = N = \int_{S^5} F_5$$

By the condition of duality (see, for instance, \[\text{[8]}\]). Then

$$F_5 = N(e^{-5\alpha}\tilde{\varepsilon}_{S^5} + ie^{-5\alpha}\tilde{\varepsilon}_{\tilde{g}})$$

with $\tilde{\varepsilon}$ being the volume form associated with the full metric (before the scaling factors have been extracted). The stress-energy is then:

$$T_{xy} = -\frac{2}{3l^2} e^{-10\alpha}\tilde{g}_{xy}$$

$$T_{a_1b_1} = +\frac{2}{3l^2} e^{-10\alpha}\tilde{h}_{a_1b_1},$$

with $l$ defined so that $\alpha = 0$ correspond to the vacuum of the theory. Then $l$ has some factors of the coupling and the number of 3-branes. The equations of motion are:

$$R_{xy} = \frac{40}{3} \nabla_x \alpha \nabla_y \alpha - \frac{5}{3} \nabla^2 \alpha g_{xy} - \frac{2}{3l^2} e^{-\frac{40}{3}\alpha} g_{xy}$$

and

$$\nabla^2 \alpha = \frac{2}{3l^2} e^{-\frac{22}{3}\alpha} - \frac{2}{3l^2} e^{-\frac{40}{3}\alpha}.$$
Defining the unit, future directed, time vector $\xi^a$, one can write the Raychaudhuri equation for the system:

$$\dot{\theta} = - \frac{1}{4} \theta^2 - \xi^a R_{ab} \xi^b = - \frac{1}{4} \theta^2 - \frac{40}{3} \alpha^2 - \frac{2}{3} l^2 \left( \frac{5}{3} e^{-\frac{22}{3} \alpha} - \frac{2}{3} e^{-\frac{40}{3} \alpha} \right), \quad (22)$$

where $\theta = \nabla_a \xi^a$ is the expansion factor, related to the Hubble factor by $H = \frac{\dot{a}}{a} = \frac{1}{4} \theta$. Dots mean differentiation with respect to proper time, which happens to be the affine parameter of integral curves of $\xi^a$. The equation for $\alpha$ is

$$- \ddot{\alpha} - \theta \dot{\alpha} = \frac{2}{3} l^2 e^{-\frac{22}{3} \alpha} - \frac{2}{3} l^2 e^{-\frac{40}{3} \alpha}, \quad (23)$$

on the supposition that spatial derivatives are small, and hence space is homogeneous.

The stable critical point of the system above, parametrized with $\frac{d\tau}{dt} = e^{-\frac{20}{3} \alpha}$, for $\alpha \ll 0$ is:

$$\tilde{\theta} = \theta e^{-\frac{20}{3} \alpha} = \frac{8}{3} \sqrt{\frac{10}{3}} \alpha^{\prime}$$

$$\alpha^{\prime} = \frac{d\alpha}{d\tau} = \frac{1}{\sqrt{30}} \quad (24)$$

Then $\frac{a(t)}{a_0} = \exp \left( \frac{1}{4} \int \theta \right) \sim t$, the evolution is FRW-like and no cosmological horizon is formed.

### 5.2 AdS$_7 \times$ S$^4$

This space arises as the near horizon limit of the eleven dimensional supergravity (11-SUGRA) solution for a system of M5 branes\footnote{See, for instance, Chapter 6 in \cite{[13]} for a review.}. The AdS/CFT conjecture here is quite interesting since neither the string theory on this backgroung (the “little string theory”) neither its dual conformal field theory (the $A_{N-1}$ (2, 0) 6-dimensional SCFT) are known enough to fully test the results predicted. As stressed in the second section, knowing whether horizons form in excitations of the theory is then important if one is to encode the information inside AdS$_7$ in its boundary, by Bousso’s prescription.

The M5 branes are magnetic sources for the 11-SUGRA’s antisymmetric 4-tensor field:

$$\int_{S^4} F_4 = N. \quad (25)$$

Note that, with this solution \cite{[14]}, the Chern-Simons term vanishes and doesn’t contribute to the equations of motion.
So the energy-momentum tensor, written in terms of the eleven dimensional metric
\[ \tilde{g}_{ab}^{(11)} = \tilde{g}_{xy}^{(7)} \oplus \tilde{h}_{a_1b_1}^{(4)} = e^{2\psi} g_{xy}^{(7)} \oplus e^{2\alpha} h_{a_1b_1}^{(4)} \]
is:

\[ \tilde{T}_{xy} = \frac{1}{4} N^2 e^{-8\alpha} \tilde{g}_{xy} \]
\[ \tilde{T}_{a_1b_1} = \frac{1}{4} N^2 e^{-8\alpha} \tilde{h}_{a_1b_1}. \]  

From the Appendix, \( \psi = -\frac{4}{5} \alpha \), and then one can readily write the Einstein equations:

\[ R_{xy} = \frac{36}{5} \nabla_x \alpha \nabla_y \alpha - \frac{4}{5} \nabla^2 \alpha \ g_{xy} - \frac{1}{l^2} e^{-\frac{48}{5} \alpha} \ g_{xy} \]  
\( \text{and} \)

\[ \nabla^2 \alpha = \frac{2}{l^2} e^{-\frac{18}{5} \alpha} - \frac{2}{l^2} e^{-\frac{48}{5} \alpha} \]  

The dynamical system for \( \alpha \ll 0 \) is approximated by:

\[ \tilde{\theta}' - \frac{24}{5} \tilde{\theta} \alpha' \tilde{\theta} + \frac{1}{6} \tilde{\theta}^2 + \frac{2}{5} e^{-\frac{48}{5} \alpha} \alpha^2 = \frac{3}{l^2} \]
\[ \alpha'' - \frac{24}{5} \alpha' \alpha' + \tilde{\theta} \alpha' = \frac{2}{l^2} \]  

with \( \tilde{\theta} = \theta e^{\frac{24}{5} \alpha} \) and \( \alpha' = \frac{d\alpha}{d\tau} = \frac{d\alpha}{dt} e^{-\frac{48}{5} \alpha} \). The system has a critical point at:

\[ \tilde{\theta} = \frac{24\sqrt{3}}{5l}, \ \alpha' = \frac{1}{2\sqrt{3}l} \]  

with \( a(t) \approx t \).

6 Discussion

The perspective of disjoint holographic screens seems both interesting and mysterious. For asymptotically AdS spaces it would mean that the description we have of a gravitational state, like a black hole, in terms of the boundary theory is at best in terms of a density matrix. Bousso [4] circumvented the problem by distinguishing between the spatial-infinity conformal boundary (dual) theory and the holographic one. The dual theory would only know about those patches of space-time whose degrees of freedom could be projected to spatial infinity, whereas the holographic theory would assign Hilbert spaces to each point of each screen and give correlation functions between them with no further distinction. One would then like to embed this construction in string theory, and possibly give it a more precise account.
However, it seems that solutions with disjoint screens cannot be found as distortions of the simple compactifications of string/M-theory in spaces with some symmetry. The reason lies on the construction of those reduced systems, which, despite coming from string sources, are weakly coupled enough to allow for a supergravity description of the matter fields. The constraints of 11 dimensional supergravity still hold in the reduced system. One can see that from the fact that the 11-dimensional metric is related to the reduced metric by a conformal transformation. A horizon formed by inflation in the latter would also mean a horizon in the former, which violates the (11 dimensional) strong energy condition. If string theory does have such solutions and holography is expected to hold, perhaps by taking the near-horizon limit one is constraining the set of initial conditions allowed by the system. The dual boundary theory would not be able to describe the set of conditions which would form a screen in the bulk.

So it seems that a further knowledge of string effects at moderate energies is necessary. A good laboratory for this might again be the D1-D5 system, with its small compactified dimensions. Combinations of massive fields and exponential potentials could prove to be just the ingredients necessary to spawn inflation [5]. Another way to proceed is to look at backgrounds with less supersymmetry. Intersecting branes are an example, where the matter fields do not arise from the gravity multiplet. On the other hand, by having less supersymmetry one allows for non-perturbative effects [15], which may generate potentials for moduli which do not have a direct geometrical interpretation in terms of ten or eleven dimensional supergravity. More work is indeed necessary.

As this paper was in its final phase of preparation I became aware of another work by Hellerman et al. [16] which arrived at similar conclusions.

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Appendix: Kaluza-Klein reduction and scale deformations

The most general metric used here has the form:

$$\tilde{g}_{ab}^D = e^{2\psi} g_{xy}^{(p)} \oplus e^{2\alpha} h_{a1}^{(m)} \oplus e^{2\beta} h_{a2}^{(n)}$$

of a 10 or 11 dimensional space compactified on $S^m$ and $T^n$. We will use the axioms of the covariant derivative to relate the Ricci tensor $\tilde{R}_{ab}$ associated with $\tilde{g}_{ab}^D$ with the corresponding $R_{ab}$ obtained as $\psi = \alpha = \beta = 0$. Beginning with the Christoffel symbols:

$$C_{yz}^x = \nabla_y \psi \delta_z^x + \nabla_z \psi \delta_y^x - \nabla_x \psi g_{yz}, \quad (32)$$

$$C_{a1}^{e_1b_1} = -\frac{1}{2} e^{-2\psi} \nabla (e^{2\alpha}) = -e^{-2\psi+2\alpha} \nabla \alpha h_{a1b_1}, \quad (33)$$

$$C_{a1}^{a1b_1} = \frac{1}{2} e^{-2\alpha} \nabla (e^{2\alpha}) \delta_{a1}^{a1} = \nabla \alpha \delta_{a1}^{a1}, \quad (34)$$

with analogous expressions for $\beta$. All indices will from here on be raised with $g_{xy}^{(p)}$.

Proceeding by computing the parcels in the definition of the curvature tensor:

$$C_{a}^{a} = p \nabla_x \psi + m \nabla_x \alpha + n \nabla_x \beta,$$

$$\nabla_b C_{xy}^b = \nabla_z C_{xy}^z = 2 \nabla_x \nabla_y \psi = g_{xy} \nabla^2 \psi,$$

$$C_{xy}^a C_{da} = 2p \nabla_x \nabla_y \psi - pg_{xy} (\nabla \psi)^2 + 2m \nabla (\nabla \psi) \alpha + mg_{xy} \nabla \psi \nabla^2 \alpha +$$

$$+ 2n \nabla (\nabla \psi) \beta - ng_{xy} \nabla \psi \nabla^2 \beta,$$

$$C_{ab}^a C_{ya}^b = C_{zt}^{a_z} C_{yt}^{z} + C_{a1}^{a_1} C_{yt}^{z} a_1 + C_{a2}^{a_2} C_{yt}^{z} a_2$$

$$= (p + 2) \nabla_x \psi \nabla_y \psi - 2g_{xy} (\nabla \psi)^2 + m \nabla_x \alpha \nabla_y \psi + n \nabla_x \beta \nabla_y \psi.$$

Then we obtain

$$\tilde{R}_{xy} = R_{xy} - (p - 2) \nabla_x \nabla_y \psi - m \nabla_x \nabla_y \alpha - n \nabla_x \nabla_y \beta - g_{xy} \nabla^2 \psi +$$

$$(p - 2) \nabla_x \nabla_y \psi - (p - 2) g_{xy} (\nabla \psi)^2 + 2m \nabla (\nabla \psi) \alpha - mg_{xy} \nabla \psi \nabla^2 \alpha +$$

$$2n \nabla (\nabla \psi) \beta - ng_{xy} \nabla \psi \nabla^2 \beta - m \nabla_x \alpha \nabla_y \psi - n \nabla_x \beta \nabla_y \psi,$$

and
\[
\begin{align*}
\tilde{R}_{ab} &= R_{ab} - e^{-2\psi} e^{2\alpha} \left[ \nabla^2 \alpha + ((p-2)\nabla^x \psi + m \nabla^x \alpha + n \nabla^x \beta) \nabla_x \alpha \right] h_{ab}, \\
\tilde{R}_{2b} &= R_{2b} - e^{-2\psi} e^{2\beta} \left[ \nabla^2 \beta + ((p-2)\nabla^x \psi + m \nabla^x \alpha + n \nabla^x \beta) \nabla_x \beta \right] h_{2b}.
\end{align*}
\] (36)

One sees from above that choosing \( \psi = -\frac{m}{p-2} \alpha - \frac{n}{p-2} \beta \) one can not only make the two-derivative terms in the reduced Ricci tensor vanish but also the non-linear terms in the internal curvature. This choice is called the Einstein frame since then the energy-momentum tensor for the extra “matter” fields \( \alpha, \beta \), which can be constructed from the equation for \( \tilde{R}_{xy} \) above via the Einstein equation, will be conserved.

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