Explaining LSND and MiniBooNE using altered neutrino dispersion relations.

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Motivation

- LSND observed a $3.8\sigma$ excess of $\bar{\nu}_e$ events in a pure $\bar{\nu}_\mu$ beam
- MiniBooNE sees excess only in low energy regime $\nu_\mu \rightarrow \nu_e$

- No signal on the antineutrino channel!
- $\Delta m^2_{\odot}$ & $\Delta m^2_{Atm} \Rightarrow 3\Delta m^2$'s!

LSND and MiniBooNE low energy anomaly might hint towards deviations from the usual oscillation mechanism...

- maybe extra dimensions? active-sterile neutrino oscillations?
- CPT- & Lorentz violating terms
Hamiltonian

Neutrino oscillations Hamiltonian (with CPT- & Lorentz violating terms):

\[ h_{\text{eff}} = \text{diag} \left( E + \frac{\Sigma m^2}{4E} \right) + \]

\[ + \begin{pmatrix}
-\frac{\Delta m^2}{4E} \cos 2\theta - C(E) & \frac{\Delta m^2}{4E} \sin 2\theta & B(E) & 0 \\
\frac{\Delta m^2}{4E} \sin 2\theta & -\frac{\Delta m^2}{4E} \cos 2\theta - C(E) & 0 & B(E) \\
B(E) & 0 & -\frac{\Delta m^2}{4E} \cos 2\theta - C(E) & \frac{\Delta m^2}{4E} \sin 2\theta \\
0 & B(E) & \frac{\Delta m^2}{4E} \sin 2\theta & -\frac{\Delta m^2}{4E} \cos 2\theta - C(E)
\end{pmatrix} \]

- Modified dispersion relations
  - A fourth sterile neutrino \(\Rightarrow\) active-sterile neutrino oscillations
    * Hollenberg, Päss, Micu, Weiler; arXiv:0906.0150 [hep-ph]
  - Consider CPT- & Lorentz-violating terms in a 3 neutrino scenario.
    * Kostelecky, Mewes; arXiv:hep-ph/0308300
    * Hollenberg, Päss, Micu; arXiv:0906.5072 [hep-ph].
Active-Sterile Oscillation Probability with Bulk Shortcuts

- **Standard Model** particles confined to the 3+1 brane.
- **Gauge singlet** particles as gravitons or sterile neutrinos may travel off the brane into the bulk!
- Virtual gravitons penetrate the bulk $\rightarrow$ Gauß’s law $\rightarrow$ apparent weak gravity on the brane!

Mechanisms for bulk shortcuts:
- **Self-gravity effects** in the presence of matter localized on the brane $\Rightarrow$ extrinsic brane curvature.
- **Thermal or quantum fluctuations** $\Rightarrow$ brane bending.
- The extra dimension can be asymmetrically warped, i.e. warp factors can shrink the space dimensions $x$ parallel to the brane but leave the time and bulk dimension $t$ and $u$ unaffected

$$ds^2 = dt^2 - \sum_{i=1}^{3} a^2(t)e^{-2k|u|}(dx^i)^2 - du^2,$$
Active-Sterile Oscillation Probability with Bulk Shortcuts

With sterile $\nu$’s traveling in the bulk $\Rightarrow$ Effective Hamiltonian:

$$H'_F = \text{Diag. terms} + \frac{\delta m^2}{4E} \left( \begin{array}{cc} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{array} \right) - E \frac{\epsilon}{2} \left( \begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right)$$

with the shortcut parameter: $\epsilon \simeq (t_{\text{brane}} - t_{\text{bulk}})/t_{\text{brane}} \simeq \delta t/t$.

Resonant energy condition

$$E_{\text{res}} = \sqrt{\frac{\delta m^2 \cos 2\theta}{2\epsilon}}$$

Flavor oscillation probability:

$$P_{as} = \sin^2(2\tilde{\theta}) \sin^2(\delta H L/2)$$

$$\sin^2(2\tilde{\theta}) = \frac{\sin^2(2\theta)}{\sin^2(2\theta) + \cos^2(2\theta) [1 - (E/E_{\text{res}})^2]^2}$$

$$\delta H = \frac{\delta m^2}{2E} \sqrt{\sin^2(2\theta) + \cos^2(2\theta) [1 - (E/E_{\text{res}})^2]^2}$$
Resonance plot

Figure 1: Oscillation amplitude $\sin^2 2\theta$ as a function of the neutrino energy $E_\nu$, for a resonance energy of $E_{\text{res}} = 40\text{MeV}$. The different values correspond to different values for the standard angle $\sin^2 2\theta = 0.2, 0.1, 0.01, 0.001$ (from above).

- Päs, Pakvasa, Weiler, Phys.Rev.D72:095017,2005.
Asymmetrically warped space-times geodesics

→ how do geodesics in the bulk alter the oscillation probabilities?

$$ds^2 = dt^2 - \sum_{i=1}^{3} a^2(t)e^{-2k|u|}(dx^i)^2 - du^2$$

⇔ Geodesic equations ⇒ shortcut parameter ⇒ resonances...
Geodesics

Geodesic: \( u(x) = \pm \frac{1}{2k} \ln \left[ 1 + k^2 x (L - x) \right] \).

Shortcut parameter: \( \epsilon(L) = 1 - \frac{2}{\beta k L} \text{arcsinh} \frac{kL}{2} \)

Figure 2: The relative difference between the travel time for SM neutrinos and sterile neutrinos. Curves are parametrized by geodesic mode number \( n = 1, 2, 5, 10 \) (from top to bottom)
Oscillation Probability

Path integral weight

\[ A_{as} = \sum_{n=1}^{\infty} \Delta n e^{iS_{cl}(n)} \]

\[
\begin{align*}
\Delta n & = \frac{4nkL}{(4n^2 + (kL)^2)^{3/2}} \left[ \frac{\sqrt{2}\beta E}{\sqrt{\pi}\sigma} e^{-\frac{(\beta E kL)^2}{2\sigma^2(4n^2 + (kL)^2)}} \right] \sin \Theta_{\text{eff}} \sin 2\tilde{\theta}_n \sin \frac{L\delta\tilde{H}_n}{2} \\
\end{align*}
\]

Modes \( n \)

Distribution of initial \( \dot{u}_0 \)

Path-integral weight: \( S_{cl}(n) = m \int d\tau = \left( \frac{m^2L}{\beta E} \right) (1 - \epsilon_n) \)

Distribution of initial \( \dot{u}_0 \) (normalized) \( \mapsto \) Gaussian:

\[ dN_G(p_u) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left\{ \frac{p_u^2}{2\sigma^2} \right\}} dp_u \]

The weight \( \Delta n \):

\[ \Delta n = \frac{dn}{dS} \Delta S \sim \frac{1}{dS/\Delta n} \]

The probability of oscillation:

\[ P_{as} = |A_{as}|^2 = \left| \sum_{n=1}^{\infty} A(n) \right|^2 \]
**Figure 3:** Oscillation probability as a function of the experimental baseline (red and green curves). The green curve presents the phase-averaged oscillation probability, and the sinusoidal blue curve presents the standard 4D vacuum oscillation probability between sterile and active neutrinos. Parameter choices are $\sin^2 2\theta = 0.003$, $k = 5/(10^8 \text{ m})$, $E = 15 \text{ MeV}$, $\Delta m^2 = 64 \text{ eV}^2$, and $\sigma = 100 \text{ eV}$. .

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Summary

★ Bulk shortcuts

• Can arise naturally in extra dimensional theories

• **Shortcut parameter** is **baseline dependent**!

• **Affect neutrino mixings** and imply **new resonances**!

• The resonances depend on the product $LE$ rather than $E$!

• **LSND** data and the **MiniBooNE** null result may be explained.

• It can also help solve the problems with the long baseline experiments.

• Disappearance into sterile neutrinos?

• No explanation of MiniBooNE excess only for neutrinos.
Model II. CPT-\& Lorentz violation

- Schrödinger equation

\[ i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_e^c \\ \nu_\mu^c \end{pmatrix} = h_{\text{eff}} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_e^c \\ \nu_\mu^c \end{pmatrix} \]

- Effective Hamiltonian

\[ h_{\text{eff}} = \text{Diagonal part} + \]
\[ \begin{pmatrix} -\frac{\Delta m^2}{4E} \cos 2\theta - C(E) & \frac{\Delta m^2}{4E} \sin 2\theta & B(E) & 0 \\ \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{4E} \cos 2\theta - C(E) & 0 & B(E) \\ B(E) & 0 & -\frac{\Delta m^2}{4E} \cos 2\theta - C(E) & \frac{\Delta m^2}{4E} \sin 2\theta \\ 0 & B(E) & \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{4E} \cos 2\theta - C(E) \end{pmatrix} \]

- Block diagonalize it so that \( h_{\text{eff}} = U \tilde{h}_{\text{eff}} U^\dagger \) with the unitary matrix:

\[ U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \]
Model II. CPT- & Lorentz violation

- Change of basis

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_e^c \\
\nu_\mu^c
\end{pmatrix}
\rightarrow \frac{1}{\sqrt{2}}
\begin{pmatrix}
1 & 0 & -1 & 0 \\
0 & 1 & 0 & -1 \\
1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_e^c \\
\nu_\mu^c
\end{pmatrix}
= \frac{1}{\sqrt{2}}
\begin{pmatrix}
\nu_e - \nu_e^c \\
\nu_\mu - \nu_\mu^c \\
\nu_e + \nu_e^c \\
\nu_\mu + \nu_\mu^c
\end{pmatrix}
= \begin{pmatrix}
\nu_e^- \\
\nu_\mu^- \\
\nu_e^+ \\
\nu_\mu^+
\end{pmatrix}
\]

- Charge conjugation eigenstates:

\[
C \nu^- = -\nu^-, \\
C \nu^+ = +\nu^+
\]

- \(C\)-eigenstates basis:

\[
i \frac{d}{dt} \begin{pmatrix}
\nu^- \\
\nu^+
\end{pmatrix} = \begin{pmatrix}
h_{\text{eff}}^{C-\text{odd}} & 0 \\
0 & h_{\text{eff}}^{C-\text{even}}
\end{pmatrix}
\begin{pmatrix}
\nu^- \\
\nu^+
\end{pmatrix}
\]
Resonant $C$-odd oscillations

- **$C$-odd effective Hamiltonian**

$$h_{\text{C-odd}}^{\text{eff}} = \text{diag} \left( E + \frac{\Sigma m^2}{4E} \right) + \begin{pmatrix} -\frac{\Delta m^2}{4E} \cos 2\theta - \frac{(b_e+c_{ee})E}{2} & \frac{\Delta m^2}{4E} \sin 2\theta \\ \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{4E} \cos 2\theta - \frac{(b_{\mu}+c_{\mu\mu})E}{2} \end{pmatrix}$$

- **Effective mixing angle**

$$\tan 2\theta_{C\text{-odd}} = \frac{\Delta m^2 \sin 2\theta}{(b_e-b_{\mu}+c_{ee}-c_{\mu\mu})E^2+\Delta m^2 \cos 2\theta}$$

- **Resonant mixing**

$$E_{\text{res}}^{C\text{-odd}} = \sqrt{\frac{\Delta m^2 \cos 2\theta}{b_{\mu}-b_e+c_{\mu\mu}+c_{ee}}}$$

- **Effective mass eigenvalues**

$$m_1^2 = \frac{\Sigma m^2}{2} - \frac{1}{2} \left[ (b_e + b_{\mu} + c_{ee} + c_{\mu\mu})E^2 + \kappa_{C\text{-odd}} \right]$$

$$m_2^2 = \frac{\Sigma m^2}{2} - \frac{1}{2} \left[ (b_e + b_{\mu} + c_{ee} + c_{\mu\mu})E^2 - \kappa_{C\text{-odd}} \right]$$

where

$$\kappa_{C\text{-odd}}^2 = (b_e-b_{\mu}+c_{ee}-c_{\mu\mu})^2 E^4 + 2\Delta m^2 (b_e-b_{\mu}+c_{ee}-c_{\mu\mu}) \cos 2\theta E^2 + (\Delta m^2)^2$$
Resonances

- Effective mixing angle
  \[ \tan 2\theta_{C\text{-even}} = \frac{\Delta m^2 \sin 2\theta}{(b_\mu - b_e + c_{ee} - c_{\mu\mu})E^2 + \Delta m^2 \cos 2\theta} \]

Resonant mixing for states with an energy

\[ E_{C\text{-even}}^{\text{res}} = \sqrt{\frac{\Delta m^2 \cos 2\theta}{b_e - b_\mu + c_{\mu\mu} - c_{ee}}} \]

- Diagonalization connects $C$-eigenstates with mass eigenstates

\[
\begin{pmatrix}
\nu_e^+ \\
\nu_\mu^+
\end{pmatrix} = \begin{pmatrix}
\cos \theta_{C\text{-even}} & \sin \theta_{C\text{-even}} \\
-\sin \theta_{C\text{-even}} & \cos \theta_{C\text{-even}}
\end{pmatrix} \begin{pmatrix}
\nu_3 \\
\nu_4
\end{pmatrix}
\]

- Translation between flavor and mass eigenstates

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_e^c \\
\nu_\mu^c
\end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix}
\cos \theta_{C\text{-odd}} & \sin \theta_{C\text{-odd}} & \cos \theta_{C\text{-even}} & \sin \theta_{C\text{-even}} \\
-\sin \theta_{C\text{-odd}} & \cos \theta_{C\text{-odd}} & -\sin \theta_{C\text{-even}} & \cos \theta_{C\text{-even}} \\
-\cos \theta_{C\text{-odd}} & -\sin \theta_{C\text{-odd}} & \cos \theta_{C\text{-even}} & \sin \theta_{C\text{-even}} \\
\sin \theta_{C\text{-odd}} & -\cos \theta_{C\text{-odd}} & -\sin \theta_{C\text{-even}} & \cos \theta_{C\text{-even}}
\end{pmatrix} \begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3 \\
\nu_4
\end{pmatrix}
\]
Resonances

\( \sin^2 2\theta_{\text{eff}} \)

\( E \text{[eV]} \)

\( \sin^2 2\theta_{\text{eff}} \)

\( E \text{[eV]} \)

Figure 4: Resonance structures between charge conjugation eigenstates. Shown is the sine-squared of the effective mixing angles \( \theta_{C\text{-odd}} \) (blue curve) and \( \theta_{C\text{-even}} \) (red curve).
Summary

- **CPT- and Lorentz-violating terms**
  - Neutrino-antineutrino oscillations become possible!
  - Can explain low energy MiniBooNE data.
  - Generate new resonance peaks which can be at different energies.
  - Resonance peaks can be narrower.
  - No disappearance due to oscillations into sterile $\nu$'s

CPT- and Lorentz-violating effects generate new resonances and can help us understand the LSND & MiniBooNE data!