Old high-redshift galaxies and primordial density fluctuation spectra

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ABSTRACT
We have discovered a population of extremely red galaxies at $z \simeq 1.5$ which have apparent stellar ages of $\gtrsim 3$ Gyr, based on detailed spectroscopy in the rest-frame ultraviolet. In order for galaxies to have existed at the high collapse redshifts indicated by these ages, there must be a minimum level of power in the density fluctuation spectrum on galaxy scales. This paper compares the required power with that inferred from other high-redshift populations: damped Lyman-\alpha absorbers and Lyman-limit galaxies at $z \simeq 3.2$. If the collapse redshifts for the old red galaxies are in the range $z_c \simeq 6-8$, there is general agreement between the various tracers on the required inhomogeneity on 1-Mpc scales. This level of small-scale power requires the Lyman-limit galaxies to be approximately $\nu \simeq 3.0$ fluctuations, implying a very large bias parameter $b \gtrsim 6$. If the collapse redshifts of the red galaxies are indeed in the range $z_c = 6-8$ required for power spectrum consistency, their implied ages at $z \simeq 1.5$ are between 3 and 3.8 Gyr for essentially any model universe of current age 14 Gyr. The age of these objects as deduced from gravitational collapse thus provides independent support for the ages estimated from their stellar populations. Such early-forming galaxies are rare, and their contribution to the cosmological stellar density is consistent with an extrapolation to higher redshifts of the star-formation rate measured at $z < 5$; there is no evidence for a general era of spheroid formation at extreme redshifts.

Key words: galaxies: clustering – cosmology: theory – large-scale structure of Universe.

1 INTRODUCTION

It is widely believed that the sequence of cosmological structure formation was hierarchical, originating in a density power spectrum with increasing fluctuations on small scales. The large-wavelength portion of this spectrum is accessible to observation today through studies of galaxy clustering in the linear and quasilinear regimes. However, nonlinear evolution has effectively erased any information on the initial spectrum for wavelengths below about 1 Mpc. The most sensitive way of measuring the spectrum on smaller scales is via the abundances of high-redshift objects; the amplitude of fluctuations on scales of individual galaxies governs the redshift at which these objects first undergo gravitational collapse.

The aim of this paper is to apply these arguments about the small-scale spectrum to a particularly interesting class of galaxy which we have recently discovered. It has long been apparent that a significant fraction of the optical identifications of 1-mJy radio galaxies are red and inactive (Windhorst, Kron & Koo 1984; Kron, Koo & Windhorst 1985). More recently, we have obtained the deep absorption-line spectroscopy needed in order to prove that these colours result from a well-evolved stellar population. The minimum age of the stars can be inferred robustly from spectral breaks, and gives ages of 3.5 Gyr for 53W091 at $z = 1.55$ (Dunlop et al. 1996; Spinrad et al. 1997), and 4.0 Gyr for 53W069 at $z = 1.43$ (Dunlop 1998; Dey et al. 1998). Such ages push the formation era for these galaxies back to extremely high redshifts, and it is of interest to ask what level of small-scale power is needed in order to allow this early formation. However, the dating of stellar populations rests on complex modelling, and so it is desirable to have an independent way of checking whether these high collapse redshifts are correct. We have carried out such a test, using the fact that the abundances of early-forming galaxies are sensitive to the amplitude of the small-scale power spectrum. Requiring a level of small-scale power consistent with that implied by other high-redshift objects predicts a collapse redshift for our red galaxies. From this, we can predict an age which can then be compared with the age results obtained by analyzing stellar populations.
We shall adopt a standard framework for interpreting the abundances of high-redshift objects in terms of structure-formation models, as outlined by Efstathiou & Rees (1988). Under the assumption that the growth of structure proceeds as a gravitational hierarchy with Gaussian primordial statistics, the abundance of objects of a given mass is related directly to the rms density fluctuations on that mass scale. In Section 2, we summarize the necessary elements of this Press-Schechter theory. It will be important to achieve a consistent picture in this analysis between these observations of high-redshift density fluctuations and the fluctuation spectrum at the present deduced from galaxy clustering; Section 3 summarizes our knowledge of the large-scale spectrum. We then assemble the data on masses and abundance of high-redshift galaxies in Section 4, summarizing both our own results and those of other classes of high-redshift objects. The implied density fluctuation spectrum is then discussed in Section 5, where we note that these results require a high level of bias for rare high-redshift galaxies. Finally, we return in Section 6 to the question of stellar ages in our red radio galaxies, in the light of the collapse redshifts implied by the constraints on small-scale density fluctuations.

### 2 PRESS-SCHECHTER APPARATUS

The formalism of Press & Schechter (1974) gives a way of calculating the fraction \( F_c \) of the mass in the universe which has collapsed into objects more massive than some limit \( M \):

\[
F_c(\geq M, z) = 1 - \text{erf} \left( \frac{\delta_c}{\sqrt{2}\sigma(M)} \right).
\]

(1)

Here, \( \sigma(M) \) is the rms fractional density contrast obtained by filtering the linear-theory density field on the required scale. In practice, this filtering is usually performed with a spherical ‘top hat’ filter of radius \( R \), with a corresponding mass \( 4\pi\rho_b R^3/3 \), where \( \rho_b \) is the background density. The number \( \delta_c \) is the linear-theory critical value, which for a ‘top-hat’ overdensity undergoing spherical collapse is \( 1.686 - \) virtually independent of \( \Omega \). This form describes numerical simulations very well (see e.g. Ma & Bertschinger 1994). The main assumption is that the density field obeys Gaussian statistics, which is true in most inflationary models. Given some estimate of \( F_c \), the number \( \sigma(R) \) can then be inferred. Note that for rare objects this is a pleasingly robust process: a large error in \( F_c \) will give only a small error in \( \sigma(R) \), because the abundance is exponentially sensitive to \( \sigma \).

Total masses are of course ill-defined both for real astronomical objects and clumps of particles in simulations; a better quantity to use is the velocity dispersion. Virial equilibrium for a halo of mass \( M \) and proper radius \( r \) demands a circular orbital velocity of

\[
V_c^2 = \frac{GM}{r}.
\]

(2)

For a spherically collapsed object this velocity can be converted directly into a Lagrangian comoving radius which contains the mass of the object within the virialization radius (e.g. White, Efstathiou & Frenk 1993)

\[
R/ h^{-1}\text{Mpc} = \frac{2^{1/2}[V_c/100\text{ km s}^{-1}]^2}{\Omega_m^{1/2}(1+z_c)^{1/2}f_c^{1/6}}.
\]

(3)

\( h \equiv H_0/100 \text{ km s}^{-1} \text{Mpc}^{-1} \). Here, \( z_c \) is the redshift of virialization; \( \Omega_m \) is the present value of the matter density parameter; \( f_c \) is the density contrast at virialization of the newly-collapsed object relative to the background, which is adequately approximated by

\[
f_c = \frac{178}{\Omega_m^{0.4}(z_c)},
\]

(4)

with only a slight sensitivity to whether \( \Lambda \) is non-zero (Eke, Cole & Frenk 1996).

For isothermal-sphere haloes, the velocity dispersion is

\[
\sigma_v = V_c/\sqrt{2}.
\]

(5)

Given a formation redshift of interest, and a velocity dispersion, there is then a direct route to the Lagrangian radius from which the proto-object collapsed. It is sometimes argued that the observed stellar velocity dispersion should be a little ‘cooler’ than that of the dark-matter halo hosting the galaxy (by a factor \( \sqrt{3}/2 \) for a \( \epsilon^{-1} \) stellar density profile in an isothermal sphere, for example). However, this assumes that the dark matter totally dominates the gravity, whereas real galaxies are baryon dominated in the centre. Any velocity correction is therefore likely to be small in practice, and we ignore the effect.

The Press-Schechter collapsed fraction can now be converted to a differential number density of objects, \( n(M) \), using

\[
M = \rho_b \frac{dF_c}{dM}.
\]

(6)

In practice, however, one is more likely to measure an integrated number density \( N \) of objects which lie above above some mass threshold, in which case

\[
F_c \sim \frac{M N}{\rho_b} = N \frac{4\pi R^3}{3}.
\]

(7)

This just says that the collapsed fraction of the mass is the fraction of the volume contained in the Lagrangian spheres around each object. As argued above, even quite large uncertainties in \( F_c \) can have little effect on the implied value of \( \sigma(R) \). This allows us to neglect the uncertain constant of proportionality in the above relation, which is

\[
F_c = \frac{\epsilon + 2}{\epsilon} \frac{M N}{\rho_b},
\]

(8)

for \( F_c \propto M^{-\epsilon} \); similarly, the uncertainties in estimating the appropriate value of \( R \) from the observed circular velocities are often unimportant. Strictly, the observations are lower limits: we must make at least sufficient collapsed objects to site the galaxies under study, but some objects of this mass may give rise to galaxies of a different type. However, the results are highly robust to substantial changes in the assumed abundance, so we shall treat them as measurements.

The number densities require a cosmological model, and we quote figures assuming \( \Omega = 1 \). For specific calculations, we scale to other values of \( \Omega \) using

\[
n D^2 \, dr = n_1 D^2_1 \, dr_1,
\]

(9)

where \( dr \) is increment of comoving distance and \( D \) is angular-diameter distance \( (D = R_0 S_h(r)/[1 + z]; \text{see the Appendix}) \). The scaling of \( F_c \) with model is different, because the inferred density of baryonic material depends on
the element of radial distance only:

\[ F_c \, dr = F_{c1} \, dr_1. \]  

(10)

3 POWER SPECTRA FROM GALAXY CLUSTERING

The small-scale \( \sigma(R) \) data have to be related to the present-day observations of large-scale fluctuations in order to make a consistent picture. The present linear fluctuation spectrum is known independent of uncertainties about bias for \( R \geq 3 \, h^{-1} \text{Mpc} \) (Peacock 1997). We summarize here the results of these studies.

We use a dimensionless notation for the power spectrum: \( \Delta^2 \) is the contribution to the fractional density variance per unit \( k \). In the convention of Peebles (1980), this is

\[ \Delta^2(k) = \frac{d\sigma^2}{d\ln k} = \frac{V}{(2\pi)^3} 4\pi k^3 |\delta_k|^2 \]  

\[ (V \text{ being a normalization volume}), \text{ and the relation to the correlation function is} \]

\[ \xi(r) = \int \Delta^2(k) \frac{dk}{k} \sin kr. \]  

(12)

Similarly, the variance in fractional density contrast averaged over spheres of radius \( R \) is

\[ \sigma^2(R) = \int \Delta^2(k) \frac{dk}{k} W_k^2, \]  

(13)

where \( W_k = 3(\sin y - y \cos y)/y^3; \ y = kR. \)

The twin problems with galaxy clustering are (i) the power-spectrum measurements are nonlinear, rather than the linear-theory power spectrum required by the Press-Schechter method; (ii) the normalization and even shape of the galaxy spectrum is biased relative to that of the mass. The first problem can be dealt with by calibrating the nonlinear effects using \( N \)-body simulations. The second is more difficult, but soluble in a number of limits. First, the overall normalization can be determined by the Press-Schechter method of Section 2, as applied to rich clusters. This gives a measurement of the rms in spheres of radius \( 8 \, h^{-1} \text{Mpc} \), on which there is general agreement:

\[ \sigma_8 = [0.5 - 0.6] \Omega^{-0.56} \]  

(14)

(Henry & Arnaud 1989; White, Efstathiou & Frenk 1993; Viana & Liddle 1996; Eke, Cole & Frenk 1996). On smaller scales, bias is expected to steepen the galaxy correlations, but this effect operates on the nonlinear-theory power spectrum required by the Press-Schechter method: \( \xi(k) \) is known independent of uncertainties about bias for \( R \geq 3 \, h^{-1} \text{Mpc} \) (Peacock 1997). We summarize here the results of these studies.

The resulting spectrum shape appears to be inconsistent with any variant of pure Cold Dark Matter, and is better described by Mixed Dark Matter with roughly a 30 per cent admixture of light neutrinos (e.g. Klypin et al. 1993; Peacock 1997; Smith et al. 1997). We are now interested in seeing how well this spectrum matches onto the smaller-scale data obtained from abundances of high-redshift galaxies.

4 DATA ON HIGH-REDSHIFT GALAXY ABUNDANCES

In addition to our red mJy galaxies, two classes of high-redshift object have been recently set to constrain the small-scale power spectrum at high redshift.

4.1 Damped Lyman-\( \alpha \) systems

Damped Lyman-\( \alpha \) absorbers are systems with HI column densities greater than \( \sim 2 \times 10^{12} \text{cm}^{-2} \) (Lanzetta et al. 1991). If the fraction of baryons in the virialized dark matter halos equals the global value \( \Omega_B \), then data on these systems can be used to infer the total fraction of matter that has collapsed into bound structures at high redshifts (Ma & Bertschinger 1994, Mo & Miralda-Escudé 1994; Kauffmann & Charlot 1994; Klypin et al. 1995). The highest measurement at \( z \approx 3.2 \) implies \( \Omega_{HI} \approx 0.0025 h^{-1} \) (Lanzetta et al. 1991; Storrie-Lombardi, McMahon & Irwin 1996). We take \( \Omega_{HI} h^2 = 0.02 \) as a compromise between the lower Walker et al. (1991) nucleosynthesis estimate and the more recent estimate of 0.025 from Tytler et al. (1996), giving

\[ F_c = \frac{\Omega_{HI}}{\Omega_{HI}} \approx 0.12 h \]  

(15)

for these systems. In this case alone, an explicit value of \( h \) is required in order to obtain the collapsed fraction; we take \( h = 0.65. \)

The photoionizing background prevents virialized gaseous systems with circular velocities of less than about 50 \( \text{km s}^{-1} \) from cooling efficiently, so that they cannot contract to the high density contrasts characteristic of galaxies (e.g. Efstathiou 1992). We follow Mo & Miralda-Escudé (1994) and use the circular velocity range 50 – 100 \( \text{km s}^{-1} \) (\( \sigma_v = 35 – 70 \text{km s}^{-1} \)) to model the damped Lyman alpha systems. Reinforcing the photoionization argument, detailed hydrodynamic simulations imply that the absorbers are not expected to be associated with very massive dark-matter haloes (Hahnelt, Steinmetz & Rauch 1997). This assumption is consistent with the rather low luminosity galaxies detected in association with the absorbers in a number of cases (Le Brun et al. 1996).

4.2 Lyman-limit galaxies

Steidel et al. (1996) observed star-forming galaxies between \( z = 3 \) and 3.5 by looking for objects with a spectral break redwards of the \( U \) band. Our treatment of these Lyman-limit galaxies is similar to that of Mo & Fukugita (1996), who compared the abundances of these objects to predictions from various models. Steidel et al. give the comoving density of their galaxies as

\[ N(\Omega = 1) \approx 10^{-2.54} \left( h^{-1} \text{Mpc}^{-3} \right). \]  

(16)

This is a high number density, comparable to that of \( L^\ast \) galaxies in the present Universe. The mass of \( L^\ast \) galaxies corresponds to collapse of a Lagrangian region of volume \( \sim 1 \text{Mpc}^3 \), so the collapsed fraction would be a few tenths of a per cent if the Lyman-limit galaxies had these masses.

Direct dynamical determinations of these masses are still lacking in most cases. Steidel et al. attempt to infer a velocity width by looking at the equivalent width of the C and Si absorption lines. These are saturated lines, and so the equivalent width is sensitive to the velocity dispersion; values in the range

\[ \sigma_v \approx 180 – 320 \text{km s}^{-1} \]  

(17)
are implied. These numbers may measure velocities which are not due to bound material, in which case they would give an upper limit to $V_c/\sqrt{2}$ for the dark halo. A more recent measurement of the velocity width of the H$\alpha$ emission line in one of these objects gives a dispersion of closer to $100 \, \text{km} \, \text{s}^{-1}$ (Pettini, private communication), consistent with the median velocity width for Lyα of $140 \, \text{km} \, \text{s}^{-1}$ measured in similar galaxies in the HDF (Lowenthal et al. 1997). Of course, these figures could underestimate the total velocity dispersion, since they are dominated by emission from the central regions only. For the present, we consider the range of values $\sigma_v = 100$ to $320 \, \text{km} \, \text{s}^{-1}$, and the sensitivity to the assumed velocity will be indicated. In practice, this uncertainty in the velocity does not produce an important uncertainty in the conclusions.

4.3 Red radio galaxies

We have observed two galaxies at $z = 1.43$ and 1.55, over an area $1.68 \times 10^{-3} \, \text{sr}$, so a minimal comoving density is from one galaxy in this redshift range:

$$N(\Omega = 1) \gtrsim 10^{-5.87} \, (h^{-1} \, \text{Mpc})^{-3}.$$ (18)

This figure is comparable to the density of the richest Abell clusters, and is thus in reasonable agreement with the discovery that rich high-redshift clusters appear to contain radio-quiet examples of similarly red galaxies (Dickinson 1995).

Since the velocity dispersions of these galaxies are not observed, they must be inferred indirectly. This is possible because of the known present-day Faber-Jackson relation for ellipticals. For $53W091$, the large-aperture absolute magnitude is

$$M_V(z = 1.55 \mid \Omega = 1) \simeq -21.62 - 5 \log_{10} h$$ (19)

(measured direct in the rest frame). From our Solar-metallicity models, this would be expected to fade by about 0.9 mag. between $z = 1.55$ and the present, for an $\Omega = 1$ model of present age 14 Gyr (note that Bender et al. 1996 have observed a shift in the zero-point of the $M - \sigma_v$ relation out to $z = 0.37$ of about the expected size). If we compare these numbers with the $\sigma_v - M_V$ relation for Coma ($m - M = 34.3$ for $h = 1$) taken from Dressler (1984), this gives velocity dispersions in the range

$$\sigma_v = 222 \text{ to } 292 \, \text{km} \, \text{s}^{-1}.$$ (20)

This is a very reasonable range for a giant elliptical, and we adopt it hereafter. Assuming low-density models would increase these figures by an amount smaller than the above range, so we ignore this additional uncertainty.

We note in passing that these figures also make a prediction for the metallicity:

$$M_{g_2} = 0.32 \text{ to } 0.35,$$ (21)

(Dressler 1984) corresponding to

$$[\text{Fe/H}] = 0.11 \text{ to } 0.39,$$ (22)

or a metallicity of between 1.3 and 2.5 times Solar. Care is needed here, however, because this figure refers to the nuclear metallicity, whereas our spectra are effectively total. Given the metallicity gradients in low-redshift ellipticals, such a slightly super-Solar nuclear metallicity would result in an integrated mean metallicity of Solar at best (e.g. González & Gorgas 1996; Buzzoni 1996). This means that the use of Solar-metallicity models in estimating the age of the stellar populations in these galaxies is consistent.

Having established an abundance and an equivalent circular velocity for these galaxies, our treatment of them will differ in one critical way from the Lyman-α and Lyman-limit galaxies. For these, we take the normal Press-Schechter approach, in which the systems under study are assumed to be newly born. For the Lyman-α and Lyman-limit galaxies, this may not be a bad approximation, since they are evolving rapidly and/or display high levels of star-formation activity. For the radio galaxies, conversely, this would be a very poor assumption, since the evidence is that they existed as discrete systems at redshifts much higher than the $z \simeq 1.5$ where we see them today. Our strategy will therefore be to apply the Press-Schechter machinery at some unknown formation redshift, and see what range of redshift gives a consistent degree of inhomogeneity.

5 THE SMALL-SCALE FLUCTUATION SPECTRUM

5.1 The empirical spectrum

Fig. 1 shows the $\sigma(R)$ data which result from the Press-Schechter analysis, for three cosmologies. The $\sigma(R)$ numbers measured at various high redshifts have been translated to $z = 0$ using the appropriate linear growth law for density perturbations (see Appendix).

The open symbols give the results for the Lyman-limit (largest $R$) and Lyman-α (smallest $R$) systems. The approximately horizontal error bars show the effect of the quoted range of velocity dispersions for a fixed abundance; the vertical errors show the effect of changing the abundance by a factor 2 at fixed velocity dispersion. The locus implied by the red radio galaxies sits in between. The different points show the effects of varying collapse redshift: $z_c = 2, 4, \ldots, 12$ [lowest redshift gives lowest $\sigma(R)$]. Clearly, collapse redshifts of 6–8 are favoured for consistency with the other data on high-redshift galaxies, independent of theoretical preconceptions and independent of the age of these galaxies. This level of power ($\sigma(R) \simeq 2$ for $R \simeq 1 \, h^{-1} \, \text{Mpc}$) is also in very close agreement with the level of power required to produce the observed structure in the Lyman alpha forest (Croft et al. 1997), so there is a good case to be made that the fluctuation spectrum has now been measured in a consistent fashion down to $R \simeq 0.5 \, h^{-1} \, \text{Mpc}$.

The shaded region at larger $R$ shows the results deduced from clustering data (Peacock 1997). The $\pm 1\sigma$ confidence region was obtained by an approximation to the fractional error in $\Delta^2(k)$ at $k \simeq 1/R$. It is clear an $\Omega = 1$ universe requires the power spectrum at small scales to be higher than would be expected on the basis of an extrapolation from the large-scale spectrum. Depending on assumptions about the scale-dependence of bias, such a ‘feature’ in the linear spectrum may also be required in order to satisfy the small-scale present-day nonlinear galaxy clustering (Peacock 1997). Conversely, for low-density models, the empirical small-scale spectrum appears to match reasonably smoothly onto the large-scale data.
5.2 Comparison with CDM & MDM

Fig. 1 also compares the empirical data with various physical power spectra. A CDM model (using the transfer function of Bardeen et al. 1986) with shape parameter $\Gamma = \Omega h = 0.25$ is shown as a reference for all models. This has approximately the correct level of small-scale power, but significantly over-predicts intermediate-scale clustering, as discussed in Peacock (1997). The empirical shape is better described by MDM with $\Omega h \simeq 0.4$ and $\Omega_\nu \simeq 0.3$. This is the lowest curve in Fig. 1c, reproduced from the fitting formula of Pogosyan & Starobinsky (1995; see also Ma 1996). However, this curve fails to supply the required small-scale power, by about a factor 3 in $\sigma$; lowering $\Omega_\nu$ to 0.2 still leaves a very large discrepancy. This conclusion is in agreement with e.g. Mo & Miralda-Escudé (1994), Ma & Bertschinger (1994), but conflicts slightly with Klypin et al. (1995), who claimed that the $\Omega_\nu = 0.2$ model was acceptable. This difference arises partly because Klypin et al. adopt a lower value for $\delta_c$ (1.33 as against 1.686 here), and also because they adopt the high normalization of $\sigma_8 = 0.7$; the net effect of these changes is to boost the model relative to the small-scale data by a factor of 1.6, which would allow marginal consistency for the $\Omega_\nu = 0.2$ model. MDM models do allow a higher normalization than the conventional figure of $\sigma_8 = 0.55$, partly because of the very flat small-scale spectrum, and also because of the effects of random neutrino velocities. However, such shifts are at the 10 per cent level (Borgani et al. 1997a, 1997b), and $\sigma_8 = 0.7$ would probably still give a cluster abundance in excess of observation. The consensus of more recent modelling is that even $\Omega_\nu = 0.2$ MDM is deficient in small-scale power (Ma et al. 1997; Gardner et al. 1997).

All the models in Fig. 1 assume $n = 1$; in fact, consistency with the COBE results for this choice of $\sigma_8$ requires a significant tilt for flat models, $n \simeq 0.8 - 0.9$. Over the range of scales probed by large-scale structure, changes in $n$ are largely degenerate with changes in $\Omega h$, but the small-scale power is more sensitive to tilt than to $\Omega h$. Tilting the $\Omega = 1$ models is not attractive, since it increases the tendency for model predictions to lie below the data. However, a tilted low-$\Omega$ flat CDM model would agree moderately well with the data on all scales, with the exception of the ‘bump’ around $R \simeq 30 \ h^{-1} \text{Mpc}$. Testing the reality of this feature will therefore be an important task for future generations of redshift survey.

5.3 Limits on high-redshift clustering

An interesting aspect of these results is that the level of power on 1-Mpc scales is only moderate: $\sigma(1 \ h^{-1} \text{Mpc}) \simeq 2$. At $z \simeq 3$, the corresponding figure would have been much lower, making systems like the Lyman-limit galaxies rather rare. For Gaussian fluctuations, as assumed in the Press-Schechter analysis, such systems will be expected to display spatial correlations which are strongly biased with respect to the underlying mass. The linear bias parameter depends on the rareness of the fluctuation and the rms of the underlying field as

$$b = 1 + \frac{\nu^2 - 1}{\nu \sigma} = 1 + \frac{\nu^2 - 1}{\delta_c}$$

(Kaiser 1984; Cole & Kaiser 1989; Mo & White 1996), where $\nu = \delta_c/\sigma$, and $\sigma^2$ is the fractional mass variance at the
redshift of interest.

In this analysis, \( \delta_c = 1.686 \) is assumed. Variations in this number of order 10 per cent have been suggested by authors who have studied the fit of the Press-Schechter model to numerical data. These changes would merely scale \( b - 1 \) by a small amount; the key parameter is \( \nu \), which is set entirely by the collapsed fraction. For the Lyman-limit galaxies, typical values of this parameter are \( \nu \approx 3 \), and it is clear that very substantial values of bias are expected, as illustrated in Figure 2.

This diagram shows how the predicted bias parameter varies with the assumed circular velocity, for a number density of galaxies fixed at the level observed by Steidel et al. (1996). The sensitivity to cosmological parameter is only moderate; at \( V_c = 200 \, \text{km s}^{-1} \), we have \( b \approx 4.6, 5.5, 5.8 \) for the open, flat and critical models respectively. These numbers scale approximately as \( V_c^{-0.4} \), and \( b \) is within 20 per cent of 6 for most plausible parameter combinations. Strictly, the bias values determined here are upper limits, since the numbers of collapsed haloes of this circular velocity could in principle greatly exceed the numbers of observed Lyman-limit galaxies. However, the undercounting would have to be substantial: increasing the collapsed fraction by a factor 10 reduces the implied bias by a factor of about 2. A substantial bias seems difficult to avoid, as has been pointed out in the context of CDM models by Baugh, Cole & Frenk (1997).

We now compare these calculations to the recent detection by Steidel et al. (1997) of strong clustering in the population of Lyman-limit galaxies at \( z \approx 3 \). The evidence takes the form of a redshift histogram binned at \( \Delta z = 0.04 \) resolution over a field \( 8.7' \times 17.6' \) in extent. For \( \Omega = 1 \) and \( z = 3 \), this probes the density field using a cell with dimensions

\[
\text{cell} = 15.4 \times 7.6 \times 15.0 \, [h^{-1} \text{Mpc}]^3.
\]

\( (24) \)

Conveniently, this has a volume equivalent to a sphere of radius \( 7.5 \, h^{-1} \text{Mpc} \), so it is easy to measure the bias directly by reference to the known value of \( \sigma_8 \). Since the degree of bias is large, redshift-space distortions from coherent infall are small; the cell is also large enough that the distortions of small-scale random velocities at the few hundred km s\(^{-1} \) level are also small. Using the model of equation (11) of Peacock (1997) for the anisotropic redshift-space power spectrum and integrating over the exact anisotropic window function, we confirm that the above simple volume argument should be accurate to a few per cent for reasonable power spectra:

\[
\sigma_{\text{cell}} \approx b(z = 3) \sigma_{r,5}(z = 3),
\]

where we define the bias factor at this scale. The results of Mo & White (1996) suggest that the scale-dependence of bias should be weak.

In order to estimate \( \sigma_{\text{cell}} \), we have made simulations of synthetic redshift histograms, using the method of Poisson-sampled lognormal realizations described by Broadhurst, Taylor & Peacock (1995). We use a \( \chi^2 \) statistic to quantify the nonuniformity of the redshift histogram, and find that \( \sigma_{\text{cell}} \approx 0.9 \) is required in order for the field of Steidel et al. (1997) to be typical. It is then straightforward to obtain the bias parameter since, for a present-day correlation function \( \xi(r) \propto r^{-1.8} \),

\[
\sigma_{r,5}(z = 3) = \sigma_8 \times [8/7.5]^{1.8/2} \times 1/4 \approx 0.146,
\]

implying

\[
b(z = 3 \mid \Omega = 1) \approx 0.9/0.146 \approx 6.2.
\]

Steidel et al. (1997) use a rather different analysis which concentrates on the highest peak alone, and obtain a minimum bias of 6, with a preferred value of 8. They use the Eke et al. (1996) value of \( \sigma_8 = 0.52 \), which is on the low side of the published range of estimates. Using \( \sigma_8 = 0.55 \) would lower their preferred \( b \) to 7.6, which is satisfyingly close to our estimate. Note that, with both these methods, it is much easier to rule out a low value of \( b \) than a high one; given a single field, it is possible that a relatively ‘quiet’ region of space has been sampled, and that much larger spikes remain to be found elsewhere. Henceforth, we assume that the Steidel et al. (1997) field is typical, since there is evidence that other fields have a similar appearance (Steidel, private communication).

Having arrived at a figure for bias if \( \Omega = 1 \), it is easy to translate to other models, since \( \sigma_{\text{cell}} \) is observed, independent of cosmology. For low \( \Omega \) models, the cell volume will increase by a factor \( [D^2dr]/[D^2dr]_1 \); comparing with present-day fluctuations on this larger scale will tend to increase the bias. However, for low \( \Omega \), two other effects increase the predicted density fluctuation at \( z = 3 \); the cluster constraint increases the present-day fluctuation by a factor \( \Omega^{-0.56} \) and the growth between redshift 3 and the present will be less than a factor of 4. Using the Appendix to calculate these corrections, we get

\[
\frac{b(3 \mid \Omega = 0.3)}{b(3 \mid \Omega = 1)} = \begin{cases} 0.42 \text{ (open)} \\ 0.60 \text{ (flat)} \end{cases},
\]

\( (28) \)

which suggests an approximate scaling as \( b \propto \Omega^{0.72} \) (open) or \( \Omega^{0.42} \) (flat). Multiplying the \( \Omega = 1 \) figure of 6.2 by these
factors gives bias values of 2.6 (Ω = 0.3 open) or 3.7 (Ω = 0.3 flat). The significance of this observation is thus to provide the first convincing proof for the reality of galaxy bias: for Ω ≃ 0.3, bias is not required in the present universe, but we now see that b > 1 is needed at z = 3 for all reasonable values of Ω.

Comparing these bias values with Fig. 2, we see that the observed value of b is quite close to the prediction in the case of Ω = 1 − suggesting that the simplest interpretation of these systems as collapsed rare peaks may well be roughly correct. Indeed, for high circular velocities there is a danger of exceeding the predictions, and it would create something of a difficulty for high-density models if a velocity as high as $V_c \approx 300 \text{ km s}^{-1}$ were to be established as typical of the Lyman-limit galaxies. For low Ω, the ‘observed’ bias is lower than the predictions, so there is no immediate conflict. For a circular velocity of 200 km s$^{-1}$, we would need to say that the collapsed fraction was underestimated by roughly a factor 10 to close the gap in the case of an open universe. This change in collapsed fraction increases the values of σ in Fig. 1 by a factor of about 1.5, increasing the ‘observed’ bias by the same factor. At the same time, this makes ν smaller, reducing the predicted bias by about a factor 2 and producing agreement on a bias factor of between 3 and 4. Such a change in $F_c$ could come about either by postulating that the conversion from velocity to $R$ is systematically in error, or by suggesting that there may be many haloes which are not detected by the Lyman-limit search technique. It is hard to argue that either of these possibilities are completely ruled out. Nevertheless, we have reached the paradoxical conclusion that large-amplitude clustering in the early universe is more naturally understood in an Ω = 1 model, whereas one might have expected the opposite conclusion.

6 AGES AND COLLAPSE REDSHIFTS

We now return to the red radio galaxies, and ask if the collapse redshifts inferred above are consistent with the age data on these objects. First bear in mind that in a hierarchy some of the stars in a galaxy will inevitably form before the epoch of collapse. Indeed, some direct observational evidence for the assembly of galaxies from sub-galactic clumps may now be starting to emerge (Pascarell et al. 1996). At the time of final collapse, the typical stellar age will be some fraction α of the age of the universe at that time:

$$\text{age} = t(z_{\text{obs}}) - t(z_c) + \alpha t(z_c). \quad (29)$$

We can rule out α = 1 (i.e. all stars forming in small sub-units just after the big bang). For present-day ellipticals, the tight colour-magnitude relation only allows an approximate doubling of the mass through mergers since the termination of star formation (Bower et al. 1992). This corresponds to α ≃ 0.3 (Peacock 1991). A non-zero α just corresponds to scaling the collapse redshift as apparent $(1 + z_c) \propto (1 - \alpha)^{-2/3}$. \quad (30)

since $t \propto (1 + z)^{-3/2}$ at high redshifts for all cosmologies. For example, a galaxy which collapsed at $z = 6$ would have an apparent age corresponding to a collapse redshift of 7.9 for α = 0.3.

Converting the ages for the galaxies to an apparent collapse redshift depends on the cosmological model, but particularly on $H_0$. We can circumvent some of this uncertainty by fixing the age of the universe. After all, it is of no interest to ask about formation redshifts in a model with e.g. Ω = 1, $h = 0.7$ when the whole universe then has an age of only 9.5 Gyr. If Ω = 1 is to be tenable then either $h < 0.5$ against all the evidence or there must be an error in the stellar evolution timescale. If the stellar timescales are wrong by a fixed factor, then these two possibilities are degenerate. It therefore makes sense to measure galaxy ages only in units of the age of the universe − or, equivalently, to choose freely an apparent Hubble constant which gives the universe an age comparable to that inferred for globular clusters. In this spirit, Fig. 3 gives apparent ages as a function of effective collapse redshift for models in which the age of the universe is forced to be 14 Gyr (e.g. Jimenez et al. 1996).

This plot shows that the ages of the red radio galaxies are not permitted very much freedom. We have argued for a consistent formation redshift in the range 6 to 8 on abundance grounds, and this clearly predicts an age of close to 3.0 Gyr for Ω = 1, or 3.7 Gyr for low-density models, irrespective of whether Λ is zero. The age-$z_c$ relation is rather flat, and this gives a robust estimate of age once we have some idea of $z_c$ through the abundance arguments. Conversely, it is almost impossible to determine the collapse redshift reliably from the spectral data, since a very high precision would be required both in the age of the galaxy and in the age of the universe.

What conclusions can then be reached about allowed cosmological models? If we take an apparent $z_c = 8$ from the power-spectrum arguments, then the apparent minimum age of > 4 Gyr for 53W069 can very nearly be satisfied in both low-density models (a current age of 14.5 Gyr would be required), but is unattainable for Ω = 1. In the high-density case, a current age of 17.6 Gyr would be required to attain the required age for $z_c = 8$; this requires a Hubble constant of $h = 0.38$. As argued above, this conclusion is highly insensitive to the assumed value of $z_c$. If the true value of $h$ does turn out to be close to 0.5, then it might be argued that Ω = 1 is consistent with the data, given realistic uncertainties. The ages for the low-density models would in this case be large by comparison with the observed radio-galaxy ages. However, the ages obtained by modelling spectra with a single burst can only be lower limits to the true age for the bulk of the stars; we could easily be observing an even older burst which is made bluer by a little recent star formation. A low $h$ measurement would therefore not rule out low-density models.

The main conclusion of this paper is thus that the existence of old radio galaxies at $z = 1.5$ poses two serious difficulties for an Ω = 1 Universe: (i) a consistent picture of structure formation through gravitational instability from Gaussian initial conditions requires a high formation redshift for these objects, leading to an old Universe and, particularly, a very small Hubble constant if the stellar ages of these objects are accepted; (ii) the shape of the power spectrum is complicated, with a large change in power between smoothing scales of 0.5 $h^{-1}$ Mpc and 5 $h^{-1}$ Mpc; no known model predicts a spectrum with this shape. The second difficulty might be avoided through non-Gaussian statistics, but the first would require our age estimates for the radio galax-
ies to be too high by a factor of about 1.5, which we consider implausible. The simple solution to these problems is of course to lower the density parameter, and either open or flat models with $\Omega \approx 0.3$ work quite well. The only counter-argument is that the empirical fluctuation spectrum then predicts a higher degree of bias for Lyman-limit galaxies at $z \approx 3$ than is observed, whereas prediction and observation match well for $\Omega = 1$. However, this problem disappears if the $\sigma(R)$ data are increased by a factor $\lesssim 1.5$; the question of bias therefore does not significantly affect our claim that the empirical small-scale fluctuation spectrum is now measured, once the geometry of the universe is given.

Lastly, it is interesting to note that it has been possible to construct a consistent picture which incorporates both the large numbers of star-forming galaxies at $z \lesssim 3$ and the existence of old systems which form en route to the general era of star and galaxy formation. While we cannot rule out such a bimodal history of star formation, the rareness of the red radio galaxies indicates that there is no difficulty with the former picture. We can demonstrate this quantitatively by integrating the total amount of star formation at high redshift. According to Madau et al., the star-formation rate at $z = 4$ is

$$\rho_\star \sim 10^{7.3} h M_\odot \text{Gyr}^{-1} \text{Mpc}^{-3},$$

(31)

declining roughly as $(1+z)^{-4}$. This is probably an underestimate by a factor of at least 3, as indicated by suggestions of dust in the Lyman-limit galaxies (Pettini et al. 1997), and by the prediction of Pei & Fall (1995), based on high-$z$ element abundances. If we scale by a factor 3, and integrate to find the total density in stars produced at $z > 6$, this yields $\rho_\star(z_f > 6) \sim 10^{6.2} M_\odot \text{Mpc}^{-3}$. (32)

Since the mJy galaxies have a density of $10^{-5.87} h^3 \text{Mpc}^{-3}$ and stellar masses of order $10^{11} M_\odot$, there is clearly no conflict with the idea that these galaxies are the first stellar systems of $L^*$ size which form en route to the general era of star and galaxy formation.

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APPENDIX: FORMULAE FOR GENERAL COSMOLOGIES

If a nonzero cosmological constant is allowed, not all of the important cosmological formulae exist as analytical expressions, but in many cases accurate approximation formulae may be used: see Carroll, Press & Turner (1992). For convenience, we summarize the necessary expressions here.

In general, it is necessary to distinguish matter (m) and vacuum (v) contributions to the total density parameter. Both these parameters and the Hubble parameter vary with scale factor $a = 1/(1 + z)$:

$$H[a] = H_0 \sqrt{\Omega_v (1 - a^{-2}) + \Omega_m (a^{-3} - a^{-2}) + a^{-2}}. \quad (33)$$

$$\Omega_m[a] = \frac{\Omega_m}{a + \Omega_m(1 - a) + \Omega_v(a^3 - a)}. \quad (34)$$

$$\Omega_v[a] = \frac{a^3 \Omega_v}{a + \Omega_m(1 - a) + \Omega_v(a^3 - a)}. \quad (35)$$

The age of the universe (at a a given epoch, taking the appropriate redshift-dependent $H \& \Omega$) can be approximated to a few per cent by

$$H t = \frac{2}{3} \left| 1 - f \right|^{-1/2} S_h^{-1} \sqrt{|1 - f|/f}. \quad (36)$$

where $f = 0.7\Omega_m - 0.3\Omega_v + 0.3$ and $S_h$ is sinh if $f < 1$, otherwise sin.

The increment of comoving distance is

$$R_0 dr = \frac{|c/H_0| dz}{\sqrt{\Omega_v + \Omega_m(1 + z)^3(1 - \Omega)(1 + z)^2}}. \quad (37)$$

where $\Omega = \Omega_m + \Omega_v$. This integrates to

$$R_0 S_h(r) = \frac{c}{H_0} \left| 1 - \Omega \right|^{-1/2} \times$$

$$S_h \left[ \int_0^z \frac{|1 - \Omega |^{1/2} dz'}{\sqrt{(1 - \Omega)(1 + z')^2 + \Omega_v + \Omega_m(1 + z')^3}} \right] \quad (38)$$

For the linear growth of density perturbations, there is a density-dependent suppression of the $\Omega = 1$ linear growth law:

$$\sigma(a) \propto a g[\Omega_m(a), \Omega_v(a)]. \quad (39)$$

where a high-accuracy fitting formula is

$$g(\Omega) = \frac{5}{2} \Omega_m \left[ \Omega_{v}^{1/7} - \Omega_v + (1 + \Omega_m/2)(1 + \Omega_v/70) \right]^{-1}. \quad (40)$$

The required growth factor is then

$$\frac{\sigma(z)}{\sigma(0)} = a g[\Omega_m(a), \Omega_v(a)] / g[\Omega_m(0), \Omega_v(0)]. \quad (41)$$

The age of the universe (at a given epoch, taking the appropriate redshift-dependent $H \& \Omega$) can be approximated to a few per cent by

$$H t = \frac{2}{3} \left| 1 - f \right|^{-1/2} S_h^{-1} \sqrt{|1 - f|/f}. \quad (36)$$

where $f = 0.7\Omega_m - 0.3\Omega_v + 0.3$ and $S_h$ is sinh if $f < 1$, otherwise sin.