Inelastic Black Hole Production and Large Extra Dimensions

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Black hole production in elementary particle collisions is among the most promising probes of large extra spacetime dimensions. Studies of black holes at particle colliders have assumed that all of the incoming energy is captured in the resulting black hole. We incorporate the inelasticity inherent in such processes and determine the prospects for discovering black holes in colliders and cosmic ray experiments, employing a dynamical model of Hawking evolution. At the Large Hadron Collider, inelasticity reduces rates by factors of $10^3$ to $10^6$ in the accessible parameter space, moderating, but not eliminating, hopes for black hole discovery. At the Pierre Auger Observatories, rates are suppressed by a factor of 10. We evaluate the impact of cosmic ray observations on collider prospects.

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When the Large Hadron Collider (LHC) experiences “first light” later this decade, some of this “light” may be the Hawking radiation of microscopic black holes (BHs) [2, 4, 5]. Likewise, ultrahigh energy cosmic rays that are continuously bombarding the Earth may be producing several BHs per minute in the upper atmosphere [4], as well as in the Antarctic ice cap [3].

The production of microscopic BHs is possible if large extra spacetime dimensions exist. In the simplest scenarios, spacetime is a direct product of the apparent 4-dimensional world, where standard model (SM) fields and gravity propagate, and a flat spatial $n$-dimensional torus (of common linear size $2\pi r_c$), where only gravity propagates [4]. In such a spacetime, gravity is modified at distances below $r_c$ and becomes strong at the energy scale $M_D \equiv |M_{Pl}^2/(8\pi n^2)|^{1/(2+n)}$, where $M_{Pl} \sim 10^{19}$ GeV is the 4-dimensional Planck scale. Gravitational collapse may therefore be triggered in collisions of elementary particles with center-of-mass energy somewhat above $M_D$ [7] at small impact parameters. Astrophysical constraints require $M_D \gg 10$ TeV for $n = 2, 3$ and $M_D \gtrsim 4$ TeV for $n = 4, 5$. For $n \geq 5$, however, $M_D$ may be as low as a TeV [3, 10], and so BH production is possible in $pp$ collisions at the LHC.

Up to now, studies [3, 11] of BH production at particle colliders have taken the mass trapped inside the BH’s apparent horizon, $M_{BH}$, to be identical to the incoming parton energy $\sqrt{s}$. This is a poor approximation: even in head-on collisions at impact parameter $b = 0$, a significant fraction of the incoming energy may escape in gravitational waves. Recently, Yoshino and Nambu have quantified the inelasticity $y \equiv M_{BH}/\sqrt{s}$ as a function of $n$ and $b$ [12]. Although Yoshino and Nambu have only determined lower bounds on $y$, their results may still be taken as reasonable estimates of the effects of inelasticity. In this work we include inelasticity and determine its effect on BH discovery prospects at the LHC.

Including inelasticity, the BH cross section at a hadron collider with center-of-mass energy $\sqrt{s}$ is

$$\sigma^{pp}(s, x_{min}, n, M_D) \equiv \int_0^1 2z dz \int_0^1 \frac{dz}{\sqrt{s}} \frac{du}{v} \int_v^1 dv \times F(n) \pi r_s^2(\mu, n, M_D) \sum_{ij} f_i(v, Q) f_j(u/v, Q),$$

where $z = b/b_{max}$, $F(n)$ is the form factor of Ref. [12],

$$r_s(\mu, n, M_D) = k(n) M_D^{-1} \left[ \sqrt{\mu s/M_D} \right]^{1/(1+n)},$$

where

$$k(n) \equiv \left[ 2^n \sqrt{\pi} (3 + n/2) \right]^{1/(1+n)},$$

is the Schwarzschild radius of the apparent horizon [13], $i, j$ label parton species, $f_i, f_j$ are parton distribution functions [14] with momentum transfer $Q \approx r_s^{-1}$ [15], and $x_{min} = M_{BH}^\text{min}/M_D$, where $M_{BH}^\text{min}$ is the smallest BH mass for which we trust the semi-classical calculation.

The parameter $x_{min}$ plays an important role in interpreting the results derived below. Validity of the semi-classical calculation requires satisfaction of at least three criteria [2]. First, $S_0$, the initial entropy of the produced BH, should be large enough to ensure a well-defined thermodynamic description [16]. Second, the BH’s lifetime $\tau$ should be large compared to its inverse mass so that the black hole behaves like a well-defined resonance. Third, the BH’s mass must be large compared to the scale of the 3-brane tension $T_3$ so that the brane does not significantly perturb the BH metric. Quantitative measures of these three criteria are given in Fig. 4 assuming $T_3 = \sqrt{8\pi}/(2\pi)^6 M_D^3$ for 6 toroidally-compactified dimensions [17]. We find it reasonable to conclude that all three criteria are satisfied for $x_{min} \approx 3$, but not necessarily for lower $x_{min}$. In string theory, as the BH mass decreases toward $M_D$, there is a continuous transition, at least in energy and string coupling, to string ball production [18].
FIG. 1: Quantitative measures of the validity of the semi-classical analysis of BH production for \( n = 6 \). (See text.)

FIG. 2: The number of BHs produced at the LHC with inelasticity included \( (N, \) solid \) and neglected \( (N^{i=1}, \) dashed \) for integrated luminosity \( 1 \text{ ab}^{-1} \) and \( n = 6 \) extra dimensions.

In many string models, the string ball cross section lies well above the semi-classical BH cross section, perhaps justifying extrapolation to \( x_{\text{min}} \approx 1 \) \([10]\).

The number of BHs produced at the LHC with and without inelasticity is given in Fig. 2 in the \( (x_{\text{min}}, M_D) \) plane, assuming a cumulative integrated luminosity of \( 1 \text{ ab}^{-1} \) over the life of the collider. Inelasticity suppresses event rates by factors of \( 10^3 \) to \( 10^6 \) in the region of parameter space where more than 1 inelastic BH event is expected. The effect is large because the LHC is energy-limited for BH production, and inelasticity effectively suppresses the available energy to below BH production threshold in much of parameter space.

Inelasticity also affects event rates for BH-mediated showers at cosmic ray facilities. These showers are initiated by very high energy neutrinos in the atmosphere, and the observable event rate is a function of BH cross section, exposure, and the incoming neutrino flux \([10]\). In this case, inelasticity affects not only the BH production cross section, but also the exposure, which is a function of shower energy. We have presented the effect of inelasticity on existing cosmic ray data elsewhere \([10]\). Following that analysis, we show event rates for a future cosmic ray experiment, the Pierre Auger Observatory (PAO), in Fig. 3 assuming a cosmogenic flux, again with and without inelasticity. Cosmic ray experiments are flux-, not energy-, limited. Consequently, the effect of inelasticity is much less severe than at the LHC, typically reducing event rates by an order of magnitude.

We turn now to a detailed evaluation of BH discovery prospects at the LHC. Following Dimopoulos and Landsberg \([3]\), we consider the signal of events with total multiplicity \( N \geq 4 \) and at least one \( e^\pm/\gamma \) with energy > 100 GeV. To implement these cuts, we first determine average multiplicities \( \langle N \rangle \) for the various particle species, incorporating evolution effects during Hawking radiation.

The average total emission rate for particle species \( i \) is

\[
\frac{d\langle N \rangle}{dt} = \frac{1}{2\pi} \left( \sum c_i g_i \Gamma_i \right) \zeta(3) \Gamma(3) r^2 T^3,
\]

where \( c_i \) is the number of internal degrees of freedom of particle species \( i \), \( g_i = 1 \) (3/4) for bosons (fermions),

\[
\Gamma_i = \frac{1}{4\pi r^2} \int \frac{\sigma_i(\omega) \omega^2 d\omega}{e^{\omega/T} \pm 1} \left[ \int \frac{\omega^2 d\omega}{e^{\omega/T} \pm 1} \right]^{-1},
\]

where \( \sigma_i \) is the greybody absorption area due to the backscattering of part of the outgoing radiation of frequency \( \omega \) into the BH \([20]\), and \( r \) and \( T \) are the instantaneous Schwarzschild radius and Hawking temperature.
The rate of change of the BH mass is

\[ \frac{dM}{dt} = \frac{1}{2\pi} \left( \sum c_i f_i \Phi_i \right) \zeta(3) \Gamma(4) \epsilon^2 T^4, \]  

(7)

where \( f_i = 1 \) \((7/8)\) for bosons (fermions) and

\[ \Phi_i = \frac{1}{4\pi^2} \int \frac{d\omega}{e^{\omega/T} + 1} \left[ \int \frac{\omega^4 d\omega}{e^{\omega/T} + 1} \right]^{-1}. \]  

(8)

Without the absorption correction, \( \Gamma_i = \Phi_i = 1 \). With it, these greybody parameters are given in Table I.

Dividing Eq. (3) by Eq. (7) and integrating, we obtain a compact expression for the average multiplicity [21]

\[ \langle N \rangle = \frac{4\pi \rho k(n)}{2 + n} \left[ \frac{M_{BH}}{M_D} \right]^{\frac{2 + n}{n}} = \rho S_0, \]  

(9)

where

\[ \rho = \frac{\sum c_i g_i \Gamma_i \zeta(3) \Gamma(3)}{\sum c_i f_i \Phi_i \zeta(4) \Gamma(4)}, \]  

(10)

and

\[ S_0 = \left( \frac{1 + n}{2 + n} \right) \frac{M_{BH}}{T_{BH}} \]  

(11)

is the initial value of the entropy in terms of the initial BH mass and Hawking temperature \( T_{BH} \). The average multiplicity for any subset of states \( \{s\} \) is \( \langle N_{\{s\}} \rangle = B_{\{s\}} \langle N \rangle \), where the branching fraction is

\[ B_{\{s\}} = \frac{\sum_{i \in \{s\}} c_i g_i \Gamma_i}{\sum_{i} c_i g_i \Gamma_i}. \]  

(12)

For \( n = 6 \), using the the parameters given in Table I we find \( \langle N \rangle = 0.30 M/T \) and \( \langle N_{e\gamma} \rangle = 0.052 \langle N \rangle = 0.016 M/T \). Note that \( \langle N \rangle \) is a factor of 3 smaller than the entropy \( S_0 \), and somewhat smaller than \( \langle N \rangle \) is therefore

\[ P(N_{e\gamma} = 0) P(N = N_{e\gamma} \geq 4) = e^{-\langle N_{e\gamma} \rangle} e^{-\langle N - N_{e\gamma} \rangle} \sum_{i \geq 4} \frac{(N - N_{e\gamma})^i}{i!}. \]  

(13)

Finally, the probability of \( N_{e\gamma} \geq 1 \) and \( N \geq 4 \) is

\[ e^{-\langle N \rangle} \sum_{i \geq 4} \frac{\langle N \rangle^i}{i!} - e^{-\langle N \rangle} \sum_{i \geq 4} \frac{(N - N_{e\gamma})^i}{i!} = \left( 1 - e^{-\langle N \rangle} \sum_{i = 0}^{3} \frac{\langle N \rangle^i}{i!} \right) e^{-\langle N_{e\gamma} \rangle} \left( 1 - e^{-\langle N - N_{e\gamma} \rangle} \sum_{i = 0}^{3} \frac{(N - N_{e\gamma})^i}{i!} \right). \]  

(14)

The SM background masking such events is dominated by \( Z(e^+e^-) + \text{jets} \) and \( \gamma + \text{jets} \). We adopt the background rates estimated using PYTHIA in Ref. [3], and require a 5\( \sigma \) excess for discovery [22]. The resulting reach is shown in Fig. 4.

Since the PAO will begin operation before the LHC, it is of interest to see what cosmic ray observations might imply for collider prospects. In Fig. 4 we have superimposed the region of parameter space excluded if no BH events above background are found at the PAO. Assuming 1 ab\(^{-1}\) luminosity for the LHC and a background of up to 2 SM neutrinos and 10 hadronic events at the PAO, we find that the PAO can limit the discovery reach of LHC to a triangular region in the \((x_{\text{min}}, M_D)\) plane, ranging from 2.2 to 4.0 TeV for \( x_{\text{min}} = 1 \) and only from 1.4 to 1.8 TeV at the favored value \( x_{\text{min}} = 3 \). At \( x_{\text{min}} = 3 \) the \( M_D \)-sensitivity of the PAO is reduced with respect to our previous estimate [23] by a factor of 1.6 as a result of inelasticity.

To summarize, we have analyzed the impact on event rates of inelasticity in BH production. The effects of inelasticity are considerable, suppressing event rates at the LHC by factors of \( 10^3 \) to \( 10^6 \) in the semi-classical regime. Our dynamical treatment of Hawking evolution also reduces event multiplicities compared to previous estimates. In spite of the enormous suppression, BH dis-

| particle’s spin | \( c_i \) | \( \Gamma_i \) | \( \Phi_i \) |
|-----------------|--------|--------|--------|
| 0               | 1      | 0.80   | 0.80   |
| \( \frac{3}{2} \) | 90     | 0.66   | 0.62   |
| 1               | 27     | 0.60   | 0.67   |

TABLE I: Degrees of freedom of particle species and greybody parameters as defined in Eqs. 4 and 5.
FIG. 4: The discovery reaches for the LHC (solid) for 3 different integrated luminosities and \( n = 6 \) extra dimensions. Also shown is the region of parameter space which can be excluded at 95% CL if no neutrino showers mediated by BHs are observed in 5 years at the PAO. The shaded (cross-hatched) region assumes 2 SM neutrino + 0 (10) hadronic background events.

dovery at the LHC is still possible. We recall also that the trapped mass estimate in Ref. \[12\] is a lower bound on the BH mass; a change in the radiation profile, especially at large impact parameters, could considerably raise event rates. Finally, we have also shown that non-observation of an excess of deeply-penetrating showers at the PAO would significantly restrict the parameter space for BH discovery at the LHC.

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