Application of the Mohr-Coulomb model for simulating the biomass compaction process

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Abstract. Presently, there are many different available material models. Some are used for modeling loose materials such as soil and sand. Such models could be usable for modeling the behavior of fragmented biomaterials e.g. sawdust. The subject of the present article is the M-C model, one of the models for describing the characteristics of plasticity in a material. The article presents the method of determining the material constants used in this model, including the cohesion coefficient and angle of internal friction. The results and types of experimental examinations necessary for the purpose of the implemented task are also presented herein. The study was carried out for the oak and pine sawdust mix.

1. Introduction
Sustainable sources of energy become more common and popular. This is caused by the reduced investment expenses caused by lower cost of equipment as well as the policies by national governments and the European Union which very often establish subsidy programs for equipment to obtain energy from renewable sources which include, among others, biomass. The term biomass is understood as various types of natural (organic) materials which can be processed into energy. This includes both organic waste from agricultural production (including materials of animal and plant origin), forestry and related industry branches, including fishing and aquaculture, as well as biogases and biodegradable fractions of industrial and communal waste which include sawdust as well as all types of energy crops [1].

In practical terms, the majority of the biomass is reprocessed before it can be used. This process typically entails its compaction to pellet or briquette form [2–5]. Such a process calls requires using briquetting machines or equipment for breaking down and briquetting of material with suitable efficiency [6–8] which depends on the employed technology, structure and characteristics of the compacted material [9, 10]. The degree of fragmentation of the material for compaction is also relevant [9].

Among the many techniques available, a very common technological process used in compaction utilizes the piston method. It is used for biomass compaction in metallurgical, pharmaceutical and automotive industry. The compaction and comminution process (by cutting) itself became the subject of interest for many researchers [11–18].

The compaction process technology itself is not complex. What is problematic is the determination of its energy consumption as well as the dependencies between the technological parameters of the process and the characteristics of the resulting briquette. These dependencies are described in a function which will have different form for different materials. For this article, a sawdust mix comprising two different types of wood were analyzed. These are oak and pine wood. Before examination, the material was subject to seasoning to achieve moisture content of approx. 10 %.
The analyzed/examined material is wood with prominent anisotropic characteristics. However, in circumstances where the material becomes fragmented, these characteristics diminish. This results from the fact that fragmentation destroys the inner structure/composition of the material which is responsible for the anisotropy. Consequently, further considerations will treat the sawdust as an anisotropic material with different characteristics to the unfragmented material. As a result, it is required to determine the baseline characteristics of the material experimentally.

2. Mohr–Coulomb model
The Mohr-Coulomb model of plasticity is most frequently employed by design engineers and scientists for simulating the behavior of sand in solving geotechnical engineering problems. Its popularity in part stems from simplicity. The Mohr-Coulomb criterion of failure is easy to understand and its construction model is available in nearly all software for carrying out analyses utilizing the finite element method. As such, the model can be easily employed for analyzing practical issues using known strength parameters. The Mohr-Coulomb plasticity model is also widely employed in research, because it allows to determine, depending on effective hydrostatic pressure, the compression strength and non-elastic strain of granular materials such as soil, grains and powders. This behavior is characteristics for the media in which the mechanics is dominated by frictional forces. The linear perfect plasticity model of Mohr-Coulomb (M-C) is one of the most frequently used constitutive models sensitive to compaction pressure, able to model such behavior in an idealized representation [19].

The Mohr-Coulomb model of plasticity was implemented for numerical calculations in numerous software, including Abaqus. The implemented model [20]:

- may be used for describing a material with classical Mohr-Coulomb criterion of failure,
- allows to model isotropic material reinforcement,
- uses a smooth flow potential that has a hyperbolic shape in the meridional stress plane and a piecewise elliptic shape in the deviatoric stress plane,
- can be used with the Rankine surface (tension cutoff) to limit load carrying capacity near the tensile region,
- is used together with the linear-elastic material model,
- can be used to simulate the behavior of material under static/monotone load.

The yield surface is a composite of two different criteria: a shear criterion, known as the Mohr-Coulomb surface, and an optional tension cutoff criterion, modeled using the Rankine surface.

![Figure 1. Mohr-Coulomb yield model [20.]](image-url)
The Mohr-Coulomb criterion assumes that yield occurs when the shear stress on any point in a material reaches a value that depends linearly on the normal stress in the same plane. So the failure is controlled by the maximum shear stress and that this failure shear stress depends on the normal stress. The Mohr-Coulomb model is based on plotting Mohr's circle for states of stress at yield in the plane of the maximum and minimum principal stresses. The yield line is the best straight line that touches these Mohr's circles (figure 1) [20].

Hence, the M-C model may be described with the following expression [20]:

\[ \tau = c - \sigma \tan \phi \]  

(1)

where \( \tau \) is the shear stress, \( \sigma \) is the normal stress (negative in compression), \( c \) is the cohesion of the material, and \( \phi \) is the material angle of friction.

Whereas the Mohr's circle allows us to derive the following [20]:

\[ \tau = s \cos \phi \]  

(2)

\[ \sigma = \sigma_m + s \sin \phi \]  

(3)

Substituting \( \tau \) and \( \sigma \), and multiplying both sides of the expression by \( \cos \phi \) (where \( \phi \) is the angle of friction), and making the appropriate reductions, we derive the following formula [20]:

\[ s + \sigma_m \sin \phi - c \cos \phi = 0 \]  

(4)

where

\[ s = \frac{1}{2}(\sigma_1 - \sigma_3) \]  

(5)

is equal to half of the difference between the highest main stress \( \sigma_1 \) and the lowest main stress value \( \sigma_3 \) (and therefore is the maximum shearing stress).

\[ \sigma_m = \frac{1}{2}(\sigma_1 + \sigma_3) \]  

(6)

is the mean value of the maximum and minimum main stress value, whereas \( \phi \) stands for the angle of friction. The friction angle determines the shape of the plasticity area at the deviatoric plane (figure 2). The angle of friction assumes values in the range from 0° to 90°. In situations where \( \phi = 0° \), the Mohr-Coulomb model is simplified to the Tresca model with perfectly hexagonal shape of the plasticity area in the deviatoric cross-section. Whereas in situations where the angle of friction is equal to \( \phi = 90° \), the M-C model is simplified to the Rankine model with triangular shape of the cross-section at the deviatoric plane [20].

![Figure 2. Mohr-Coulomb surfaces in deviatoric planes [20].](image-url)
3. Determining the values of material constants for the Mohr-Coulomb model

Utilizing the Mohr-Coulomb model for numerical calculations in Abaqus software necessitates the determination of two parameter values: the cohesion coefficient and the angle of internal friction. These parameters can be calculated from expressions (1) to (3). However, doing so requires to first determine, through experimental study, the values of shear stress and main stress.

![Figure 3. General view of the testing station for studying shear stress and coefficient of friction.](image)

The study used a mix of oak and pine sawdust mix in a weight ratio of 1:1. The average moisture value was determined with a scale-dryer and was equal to approximately 10 %.

The determination of all the M-C parameters called for performing at least two different experimental studies. The first experiment entailed the measurement of shear force. To this end, the testing station depicted on figure 3 was employed.

It comprises of two hydraulic servomotors: horizontal and vertical. The vertical servomotor is used to compress the sample with suitable compression force. Whereas the vertical servomotor is used to generate the necessary force for the shearing process. The testing station allows to measure the pressure in the operating liquid and allows to identify the force value affecting the sample, and consequently to determine the shear stress. The examination was carried out for two different values of compaction pressure and the obtained results are presented in table 1.

| Shear stress (MPa) | Axial stress (MPa) | Radial stress (MPa) |
|-------------------|-------------------|--------------------|
| 1.37              | 10                | 1.84               |
| 4.81              | 20                | 3.3                |
| 9.74              | 50                | 5.09               |
| 9.54              | 75                | 8.09               |
| 10.850            | 100               | 8.84               |
| 12.67             | 150               | 8.71               |

In order to determine the material constants for the M-C model, the main stress values also need to be identified. To this end, a specialized testing station designed for installation on a strength machine. It
allows to carry out a tri-axial compression test and through measuring force values, to determine the stress values acting both along the vertical axis and radial stress (figure 4). In the examination, sawdust was compressed with sufficient axial force to induce a compressive stress $\sigma_z$ with values, sequentially, 10 MPa, 20 MPa, 50 MPa, 75 MPa, 100 MPa and 150 MPa. For every stress value, the test was repeated three times. The measurements allowed to determine the change in the Poisson's ratio as a function of relative density.

The carried out examination enabled to determine the necessary parameters of the M-C model – the cohesion coefficient and the angle of internal friction (table 2). These values were determined for different degrees of material compaction.

### Table 2. Breakdown of the M-C model parameter values depending on the value of compaction pressure.

| Compaction pressure (MPa) | Cohesion coefficient (MPa) | Angle of internal friction ($^\circ$) |
|---------------------------|----------------------------|-------------------------------------|
| 10                        | 28.77                      | 70.37                               |
| 20                        | 31.11                      | 54.87                               |
| 50                        | 109.76                     | 64.38                               |
| 75                        | 257.71                     | 73.45                               |
| 100                       | 416.44                     | 76.28                               |
| 150                       | 826.92                     | 79.65                               |

### 4. Conclusion

Presently, many different models for describing various characteristics of materials are available. Many of them are only utilized in a single area of study, e.g. for modeling the behavior of soil and rocks, whereas they are likely to be employed successfully in analyzing of other materials, e.g. biomass such as sawdust.

The sole existing problem with employing such models is the determination of material constants. Unfortunately, such studies call for specialized testing equipment, sometimes designed and built for the particular purpose of carrying out such measurements.

The article presents the method of determining the material constant values for the M-C model. They were determined for a sawdust mix of oak and pine. The results of this work will allow to utilize the
M-C model of plasticity in the analysis and numerical simulation of issues related to mechanical compaction of fragmented wooden waste. These simulations will be the basis for formulating the design assumptions for designing modern, efficient and economical equipment for material agglomeration.

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