Unambiguous Tracking Technique Based on Shape Code for BOC Signals

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ABSTRACT The multiple peaks characteristic of binary offset carrier (BOC) autocorrelation function (ACF) makes ambiguity easy to be generated. There are some methods to eliminate ambiguity, but for higher-order BOC Signal, it will sacrifice signal energy or cannot maintain narrow correlation. This paper studies these problems. According to the idea of GRASS algorithm shape code, this paper proposes an unambiguous tracking algorithm, Sub Cross-correlation Shift Technique (SCST), which is suitable for BOC(m,n) signals. The key to this algorithm is to generate a new cross-correlation function between the BOC signal and the PRN code based on the shape code. The new cross-correlation function is linearly combined with the autocorrelation function of the BOC signal to remove sub-peak interference and achieve high-accuracy tracking. The phase discrimination function is given, and the effectiveness of the tracking algorithm is analyzed theoretically. The disadvantage of this method is that it needs multilevel storage, which will bring extra resource consumption to the receiver. For comparison, Unit Correlation, ASPeCT, GRASS, and the algorithm proposed by Yan are proposed. Experiments show that SCST can completely remove the side peak, and the phase discriminator output has only two main peaks, which successfully eliminates the false lock point. The multipath error envelope has only one and the smallest area. In terms of code tracking accuracy, for a BOC(10,5) signal with a received signal-to-noise ratio (SNR) of $-28$ dB, SCST is less than 45.8%, 67.5%, and 12.2% of the Unit Correlation, GRASS, and Yan proposed algorithms.

INDEX TERMS BOC, tracking, PRN code, unambiguous.

I. INTRODUCTION

In recent years, with the development of radio technology, radar, communications and Global Navigation Satellite System (GNSS) have been vigorously developed. The number of mobile devices such as smartphones, wearables, and autonomous driving technologies has increased dramatically [1]–[3]. These devices are inseparable from GNSS. The GNSS is widely used to provide users with high-precision navigation, positioning and timing services on a global, 24×7, real-time basis [4]. However, with the increase of satellite navigation signals, the problems such as mutual interference between signals, overcrowding of frequency band and incompatibility between systems become more and more serious. A new modulation method, binary offset carrier (BOC) modulation, is proposed. This modulation method has a higher code tracking accuracy, and its autocorrelation function (ACF) has the characteristics of narrow main peak and spectrum splitting, which can well solve the problems faced now. BOC Signal not only has the characteristic of the narrow main peak but also has the multi-peak characteristic of the correlation function, which makes the synchronization processing of the BOC signal has serious ambiguity. If the sub-peak is locked, it will result in mis-acquisition and mis-locking of BOC signal, which has a significant impact on the positioning accuracy. The minimum error of BOC(10,5) is 14.6 m, and the minimum error of BOC(14,2) is 10.5 m, which is intolerable to the navigation system [5]. Due to the narrow autocorrelation main peak, the BOC signal has better code tracking performance and anti-multipath performance than the binary phase-shift keying (BPSK) signal with the same code rate, and the multipath error is also the main source...
of BOC signal positioning error [6]–[8]. Therefore, how to eliminate the ambiguity of the BOC signal and improve its anti-multipath performance has become a research hotspot in the field of navigation.

The BOC signal has many advantages, but the multi-peak characteristic of its autocorrelation function makes the acquisition and tracking more difficult. At present, there are many outstanding research results at home and abroad. For example, (1) single sideband algorithm [9]–[11]: The idea of this method is to treat the BOC signal as a combination of multiple BPSK-like signals, process each BPSK-like signal separately and then do incoherent accumulation. This type of method eliminates the sub-peak, but it sacrifices the advantage of the narrow correlation peak of the BOC signal. The sideband techniques [9], BPSK-LIKE techniques [10] and Modified Sideband (MSB) method [11] are typical examples of this category. Furthermore, filtering and dual sideband processing increase the implementation complexity and cause correlation loss due to the mismatch between the received signal (BOC) and the local signal (shifted BPSK) [12]. (2) Ambiguity avoidance detection method. These methods, such as the Bump-jump (BJ) method [13], the double estimation technique (DET) method [14], [15], and the method of coherent combining using a Dual sidebands Double Phase Estimator (DDPE) [16]. These methods detect the occurrence of error acquisition or mis-locking during BOC signal synchronization by adding correlators. This type of method requires extended detection and recovery times, and it is only suitable for strong signal conditions [5]. (3) Side-peak Cancellation (SC) technology: The basic idea of this type of method is to use local auxiliary signals to achieve the elimination or suppression of sub-peaks through a specific combination algorithm, and to retain or generate a new single main peak [17]. For example, the Autocorrelation Side-Peak Cancellation Technique (ASPeCT) [18] uses the local pseudo code as the auxiliary code to track the BOC signal explicitly. Still, it is only applicable to the sinusoidal phase BOC(n,n). Weighted discriminators [19] have the same limitation as ASPeCT. Sub-carrier phase cancellation (SCPC) [20] uses the local BOC code modulated by two orthogonal subcarriers as an auxiliary code, which applies to any BOC signal but sacrifices high precision characteristics. General Removing Ambiguity via Side-peak Suppression (GRASS) [21] defines the shape code vector to generate the auxiliary code, which has poor tracking performance. Two local step-shaped modulated signals are designed in Pseudo correlation function based Unambiguous Delay Lock Loop (PUDLL) [22] to remove the ambiguity of BOC signals. Still, with the increase of BOC order, the code tracking performance decreases rapidly. Yan et al. (2015a; 2015b) proposed two techniques for the unambiguous application of high-order BOC signals [23], [24]. For the poor performance of processing second-order BOC signals, the final cross-correlation function still has two small sub-peaks. The Unit Correlation method [25] is similar to PUDLL. Two auxiliary signals are designed to remove the ambiguity of the BOCs signal.

The larger the BOC modulation order, the worse the code tracking performance. In Sharp Reconstruction (RS) [26] algorithm, the relation result is folded by fractal reconstruction to eliminate ambiguity, but the acquisition accuracy is declined caused by its wide main lobe.

This paper proposes a Sub Cross-correlation Shift Technology (SCST) unambiguity tracking algorithm. This method draws on the idea of defining the shape code vector by GRASS [21] to generate a new local auxiliary code, delays and linearly combines the obtained cross-correlation function, and multiplies the new correlation function by the BOC signal autocorrelation function to get a combined correlation function without sub-peak. The phase discrimination function is given, and the code tracking accuracy is theoretically analyzed. This paper also analyzes the unambiguous performance, anti-multipath performance, tracking error, and so on. Compared with the tracking method proposed by Unit Correlation, ASPeCT, GRASS and Yan et al. (2015a), the simulation results show that the proposed method can completely remove the ambiguity problem and significantly improve the tracking accuracy and anti-multipath ability. There is only one multipath error envelope for SCST, and the envelope area is minimal compared to several other methods. In terms of code tracking accuracy, for a BOC(10,5) signal with a received signal-to-noise ratio (SNR) of -28 dB, SCST is less than 45.8%, 67.5%, and 12.2% of the Unit Correlation, GRASS, and Yan proposed algorithms.

The structure of this paper is as follows. Section 2 gives the BOC signal model and analyzes the ambiguity problem, defining the BOC shape code. In the third section, a new local auxiliary signal is given, and an algorithm for solving the ambiguity problem is proposed. The tracking scheme is theoretically analyzed, and the phase discrimination function is provided. In the fourth section, the tracking performance of the algorithm is analyzed from four aspects: correlation function, phase discrimination, anti-multipath and anti-noise. The final section concludes.

II. BOC MODULATION SIGNAL CHARACTERISTICS AND SHAPE CODE

A. BOC MODULATION SIGNAL MODEL

The BOC modulation process can be described as follows. The navigation signal is first combined with the Pseudo-random noise (PRN) code. The baseband signal is obtained by the sub-carrier modulation. Finally, the carrier modulates the baseband combined code so that the navigation signal is successfully modulated to achieve the BOC navigation signal. Figure 1 shows the BOC modulation signal generation process.

In general, the BOC signal is expressed as BOC(m,n), where m represents the ratio of the subcarrier rate to the reference frequency, i.e., $f_c = m \times 1.023MHz$. n represents the reference frequency at which the PRN code rate is n times, that is, $f_c = n \times 1.023MHz$. The reference frequency
The BOC modulation signal generation process.

FIGURE 1. The BOC modulation signal generation process.

is \( f_0 = 1.023 \text{MHz} \), and \( m > n \), the modulation order is \( N = 2m/n \). The mathematical expression according to the definition of the BOC modulation signal is

\[
S(t) = \sqrt{P} c(t) d(t) \text{sign} \left( \cos(2\pi ft + \theta) \right) \tag{1}
\]

where: \( P \) is signal power; \( c(t) \) is PRN code; \( d(t) \) is navigation message; the carrier is cosine modulation; main carrier frequency is \( f \); PRN code \( c(t) \) and subcarrier \( \text{sign}(t) \) can be expressed as follows:

\[
c(t) = \sum_{i=-\infty}^{\infty} C_i P_{TC} (t - iT_C) \tag{2}
\]

\( [C_i] \) represents the symbol value of the \( i \)-th code chip, \( C_i \in (-1, 1) \). \( T_C \) is a PRN code chip duration. \( P_{TC}(t) \) is a rectangular pulse signal with a duration of \( T_C \) and an amplitude value of 1. The subcarriers are further divided into sine subcarriers and cosine subcarriers, which are generated by taking the sign of the sine and cosine function. \( \text{sign} \) represents a symbolic function.

\[
\text{sc}(t) = \begin{cases} 
\text{sign}(\sin(2\pi f_s t)), & \text{sine modulation} \\
\text{sign}(\cos(2\pi f_s t)), & \text{cosine modulation} 
\end{cases} \tag{3}
\]

Subcarriers can also be expressed in mathematical formulas as:

\[
\text{sc}(t) = \sum_{j=0}^{N-1} d_j P_{TSC} (t - jT_{SC}) \tag{4}
\]

The subcarrier also can be regarded as a combination of multiple rectangular pulses with different delays. In the formula: \( T_{SC} \) is a subcarrier semi-period. \( P_{TSC}(t) \) is a rectangular pulse signal with a duration of \( T_{SC} \) and an amplitude value of 1, \( N \) represents the number of a sub-carrier corresponding to one chip interval in the BOC signal, which is the same as the modulation order. \( d_j \in \{ -1, 1 \} \) \( (j = 0, 1, 2, \cdots, N-1 \)

According to formula (1) (2) (4), the BOC baseband signal mathematical formula can represent:

\[
s_{BOC}(t) = \sum_{i=-\infty}^{\infty} \sum_{j=0}^{N-1} C_{ij} P_{TSC}(t - iT_C - jT_{SC}) \tag{5}
\]

For an ideal spreading code symbol, it has a characteristic of \( E[c_i c_j] = \delta_{ij} \) (or \( E[-1]c_i (-1)c_j] = (-1)^{i+j} \delta_{ij} \)) [27]. Therefore, the correlation function of a signal that is directly spread using a PRN sequence having an ideal correlation characteristic can be expressed as:

\[
R_B(\tau) = E[s_{BOC}(t)s_{BOC}(t+\tau)] = \frac{1}{T_C} \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} E[C_i C_j] \int_0^{T_C} P_{TSC}(t-iT_C) P_{TSC}(t) \times (t + \tau - iT_C) dt = \frac{1}{T_C} \int_0^{T_C} \text{sc}(t) \text{sc}(t+\tau) dt = E[\text{sc}(t) \text{sc}(t+\tau)] \tag{6}
\]

\( \tau \) indicates the code phase delay. According to the characteristics of the rectangular wave autocorrelation function:

\[
\int_{-\infty}^{+\infty} P_{TSC}(t) P_{TSC}(t+\tau) dt = \Lambda_{TSC}(\tau) \tag{7}
\]

\( \Lambda_{TSC} \) is the triangular function with the center at zero, the triangle duration is \( 2T_{SC} \). The autocorrelation function of the BOC signal can be deduced as follows:

\[
R_B(\tau) = E[s_{BOC}(t)s_{BOC}(t+\tau)] = \frac{1}{N} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \Lambda_{TSC}(\tau - (j-i)T_{SC}) \tag{8}
\]

\( d_j \) and \( d_j \) are the same.

FIGURE 2. The comparison of autocorrelation functions.

Figure 2 is a comparison of autocorrelation functions of BPSK, BOC(n,n) and BOC(2n,n). As can be seen from the figures, the larger the modulation order, the greater the number of correlation peaks. BOC (n, n) signal has three peaks, two subpeaks and one main peak. BOC (2n, n) signal has five peaks, four subpeaks and one main peak.

However, for the BOC signal, once the side peak is acquired, the false lock will occur during the subsequent tracking phase [23]. For \( N=2 \), the sub-peak is 6db weaker than the main peak. But for \( N=4 \), the gap between the largest side peak and the main peak is only 2.5 dB. With an increase of \( N \), the difference between the maximum side peak and the main peak decreases, while the false acquisition probability increase [21].
B. BOC SHAPE CODE

d_j \in \{1, -1\} (j = 0, 1, 2, \ldots, N - 1) in the equation (8) is defined as a shape code, and d = [d_0, d_1, \ldots, d_{N-1}]_N composed of multiple shape codes is called the shape code vector. The BOC modulation has two modes: sine subcarrier modulation and cosine subcarrier modulation. The sine BOC (sBOC or BOCs) subcarrier and cosine BOC (cBOC or BOCc) subcarrier can be shown in Figure 3.

\[ \text{FIGURE 3. The BOC shape code.} \]

In FIG. 3, for a subcarrier of a sine modulation BOC(m,n), \( N = 2m/n \), where \( d = [1 \ -1 \ \cdots \ -1]_N \). For example, Sine-BOC(1,1), the shape code vector is \( d = [1 \ -1]_2 \). Similarly, for the cosine modulated BOC(m,n) subcarrier, \( N = 4m/n \), \( d = [1 \ -1 \ -1 \ \cdots \ -1]_N \) at this time. For example, Cosine-BOC(1,1), the shape code vector is \( d = [1 \ -1 \ -1 \ 1]_4 \).

1) FOR BOC(N, N) SIGNAL

For the autocorrelation function of sBOC(1,1) signal, the cBOC(1,1) signal is analyzed identically. And its shape code vectors symmetric about the main diagonal is \( [1 \ 1] \). Further, in the matrix, the triangles composed of \( \sum_{k=0}^{N} d_j d_{j+k} \) on the main diagonal are symmetric about the Y-axis, and the pattern composed of any two shape code vectors symmetric about the main diagonal is also symmetric about the Y-axis. It can, therefore, be inferred that the BOC autocorrelation function is symmetric about the Y-axis.

\[ \text{III. THE ALGORITHM ANALYSIS} \]

A. THE AUXILIARY SIGNAL WAVEFORM DESIGN

In Section 2.2, we define the shape code of the BOC signal. The number of elements in the shape code vector is the same as the modulation order. To obtain a new auxiliary code,
Equation (12) can be changed as shown in Figure 5.

The PRN code is divided into four sub-signal waveforms, for example, tracking the sBOC(10,5) signal, the shape code vector of PRN code is divided into $d' = [1 \ 1 \ 1 \ 1]_N$, and the shape code $d'$ is divided into $d'0, d'1, d'2$ and $d'3$ shape code vectors, as follows:

\[
\begin{align*}
&d'0 = [1 \ 0 \ 0 \ 0]_4 \\
&d'1 = [0 \ 1 \ 0 \ 0]_4 \\
&d'2 = [0 \ 0 \ 1 \ 0]_4 \\
&d'3 = [0 \ 0 \ 0 \ 1]_4
\end{align*}
\]  

(11)

According to the shape code vector in equation (11), the PRN code is divided into four sub-signal waveforms, as shown in Figure 5.

As shown in Figure 5, the new auxiliary codes $c_0(t), c_1(t), c_2(t)$ and $c_3(t)$ are obtained by four shape code vectors modulation PRN code. The mathematical formula can be expressed as

\[
\begin{align*}
c_0(t) &= \sum_{i=-\infty}^{+\infty} \sum_{j=0}^{N-1} C_i d'0 P_{TSC}(t - iT_C - jT_S) \\
c_1(t) &= \sum_{i=-\infty}^{+\infty} \sum_{j=0}^{N-1} C_i d'1 P_{TSC}(t - iT_C - jT_S) \\
c_2(t) &= \sum_{i=-\infty}^{+\infty} \sum_{j=0}^{N-1} C_i d'2 P_{TSC}(t - iT_C - jT_S) \\
c_3(t) &= \sum_{i=-\infty}^{+\infty} \sum_{j=0}^{N-1} C_i d'3 P_{TSC}(t - iT_C - jT_S)
\end{align*}
\]  

(12)

Equation (12) can be changed to

\[
\begin{align*}
c_0(t) &= \sum_{i=-\infty}^{+\infty} \sum_{j=0}^{N-1} C_i d'0 P_{TSC}(t - iT_C - jT_S) \\
c_1(t) &= \sum_{i=-\infty}^{+\infty} \sum_{j=0}^{N-1} C_i d'1 P_{TSC}(t - iT_C - j + 1) T_S \\
c_2(t) &= \sum_{i=-\infty}^{+\infty} \sum_{j=0}^{N-1} C_i d'2 P_{TSC}(t - iT_C - j + 2) T_S \\
c_3(t) &= \sum_{i=-\infty}^{+\infty} \sum_{j=0}^{N-1} C_i d'3 P_{TSC}(t - iT_C - j + 3) T_S
\end{align*}
\]  

(13)

Combined with the formula (8), the cross-correlation function of sBOC(m, n) and PRN code can be expressed as:

\[
R_{B/P}(\tau) = E[s_{BOC}(t) c(t + \tau)]
\]

\[
= \frac{1}{N} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} d_i d'_j \Delta T_{SC} [\tau - jT_S]
\]  

(14)

The correlation function of the sub-signal and the received signal is defined as a sub-correlation function. According to the definition of the sub-correlation function, the cross-correlation function of sBOC(m, n) and PRN code can be expressed as:

\[
R_{B/P} = \sum_{i=0}^{N-1} R_{B/P,i} = \frac{1}{N} \sum H (d, d') \Delta T_{SC} [\tau - kT_S]
\]

(15)

$R_{B/P,i}$ is a sub-correlation function, $k$ is the delay difference between two shape codes. $H$ is the product matrix of the shape code vector $d$ and $d'$, as shown in formula (16). Due to the integrity of the received signal, the obtained sub cross-correlation function is the accumulated value of each row element.

\[
H (d, d') = \begin{bmatrix}
d_0 d'_0 & d_0 d'_1 & d_1 d'_0 & \cdots & d_N d'_0 \\
d_0 d'_1 & d_1 d'_1 & d_2 d'_0 & \cdots & d_N d'_1 \\
d_0 d'_2 & d_1 d'_2 & d_2 d'_1 & \cdots & d_N d'_2 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
d_0 d'_N & d_1 d'_N & d_2 d'_N & \cdots & d_N d'_N
\end{bmatrix}
\]  

(16)

For the sub cross-correlation function $R_{B/P,0}$ of sBOC(10,5), the shape code vectors are $d = [1 \ 1 \ 1 \ 1]_4$ and $d'0 = [1 \ 0 \ 0 \ 0]_4$. Then the product matrix $H$ is

\[
H_0 = \begin{bmatrix}
1 & -1 & 1 & -1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

(17)

Combined with the equation (15), the sub cross-correlation function $R_{B/P,0}$ of sBOC(10,5) is

\[
R_{B/P,0} = d_0 d'_0 \Delta T_{SC} \langle \tau \rangle + d_1 d'_0 \Delta T_{SC} [\tau - T_{SC}] + d_2 d'_0 \Delta T_{SC} [\tau - 2T_{SC}] + d_3 d'_0 \Delta T_{SC} [\tau - 3T_{SC}]
\]

\[
= \Delta T_{SC} \langle \tau \rangle - \Delta T_{SC} [\tau - T_{SC}] + \Delta T_{SC} [\tau - 2T_{SC}] - \Delta T_{SC} [\tau - 3T_{SC}]
\]  

(18)

Similarly, the other sub cross-correlation function product matrix is as follows:

\[
H_1 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
1 & -1 & 1 & -1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]
As can be seen from Figures 6, each sub cross-correlation function consists of two trigonometric functions. And in Figure 7, each sub cross-correlation function consists of four trigonometric functions. Besides, it can be found that $R_{B/P,0}$ and $R_{B/P,N-1}$ are symmetrical about (0, 0) point, the main peaks of $R_{B/P,0}$ and $R_{B/P,N-1}$ are at the same X-axis coordinate, and the side peaks of $R_{B/P,0}$ are at the zero value of $R_{B/P,N-1}$. Based on this feature, we can construct new correlation functions to eliminate ambiguity. Two peak elimination schemes are given.

**Method I:**

Combine $R_{B/P,0}$ and $R_{B/P,N-1}$ for modulo addition and normal addition. To get a positive main peak, the two results are squared and subtracted. And $R_{B/P,N-1}$ can be obtained by $R_{B/P,0}$ shift $(N-1) T_{SC}$. Therefore, combining correlation functions such as formulas

$$
R_0 = \left(|R_{B/P,0}| + |R_{B/P,N-1}|\right)^2 - \left(R_{B/P,0} + R_{B/P,N-1}\right)^2
= \left(|R_{B/P,0}| + |R_{B/P,0}\left(\tau - (N-1) T_{SC}\right)|\right)^2
- \left(R_{B/P,0} + R_{B/P,0}\left(\tau - (N-1) T_{SC}\right)\right)^2
$$

(20)

Figure 8 is a simulation of the combined function of $N = 4$. It can be found that $R_0$ achieves the purpose of eliminating sub-peaks.

**Method II:**

Combine $R_{B/P,0}$ and $R_{B/P,N-1}$ for modulo subtraction and normal subtraction. To get a positive main peak, the two results are squared and subtracted. And $R_{B/P,N-1}$ can be obtained by $R_{B/P,0}$ shift $(N-1) T_{SC}$. Therefore, combining correlation functions such as formulas

$$
R_1 = \left(R_{B/P,0} - R_{B/P,N-1}\right)^2 - \left(|R_{B/P,0}| - |R_{B/P,N-1}|\right)^2
= \left(R_{B/P,0} - R_{B/P,0}\left(\tau - (N-1) T_{SC}\right)\right)^2
- \left(|R_{B/P,0}| - |R_{B/P,0}\left(\tau - (N-1) T_{SC}\right)\right)^2
$$

(21)

Figure 9 is a simulation of the combined function of $N = 4$. It can be found that $R_1$ achieves the purpose of eliminating sub-peaks.

Figure 10 shows the simulation results. It can be found that the larger the modulation order N, the narrower the main peak width of the correlation function. However, the peak value of the main peak will become smaller.

This problem is caused by the deterioration of the correlation between the auxiliary code and the BOC signal. PUDLL and Unit-Correlation methods are also facing this problem. In order to solve this problem, we can multiply the autocorrelation function of the BOC signal by $R_0$ or $R_1$ to increase the energy of the correlation function. As formula (22)

$$
R_{SCST,0} = |R_B \cdot R_0| \quad \text{or} \quad R_{SCST,1} = |R_B \cdot R_1|
$$

(22)
The BOC IF signal received by the GNSS receiver from the satellite can be expressed as [28]:

\[
    r(t) = \sqrt{2P} \cdot D(t) \cdot S_{BOC}(t - \tau) \cdot \cos(2\pi (f_{IF} + f_D) t + \varphi_0) + n(t) \quad (23)
\]

Among them, \( P \) is the total power of the received signal; \( D(t) \) is the navigation data code; \( S_{BOC}(t) \) is defined in equation (5); \( \tau \) is the code delay of the input signal, \( f_{IF} \) is the intermediate frequency after down-converting the received signal, \( f_D \) is the Doppler frequency of the input signal, \( \varphi_0 \) is the initial phase of the carrier, \( n(t) \) is Band-limited white noise [29], and it has the zero mean and double-sided power spectrum density \( N_0 \).

After down-conversion, the I-branch signal is coherently integrated with the early and late local auxiliary codes to obtain \( I_{E0} \) and \( I_{L0} \). \( I_{E1} \) and \( I_{L1} \) are the outputs of the coherent integral between I-branch signal and early and late local BOC codes. Similarly, the Q-branch signal is also coherently integrated with the early and late local auxiliary codes to obtain \( Q_{E0} \) and \( Q_{L0} \). \( Q_{E1} \) and \( Q_{L1} \) are the outputs of the coherent integral between I-branch signal and early and late local BOC codes.

First, the carrier cancellation operation is performed on the received intermediate frequency (IF) signal. At the same time, the local auxiliary code and the local BOC code are generated in the code loop and are performed early and late delay processing. The received signal is coherently integrated with the local code. Then, the phase discriminator and the numerically controlled oscillator (NCO) are used to adjust the phase of the local code to achieve the unambiguous tracking of the BOC signal.

The combination results of correlation functions in different \( N \).

**FIGURE 10.** The combination results of correlation functions in different \( N \).

The combination results of correlation functions for method2.

**FIGURE 9.** The combination results of correlation functions for method2.

**FIGURE 11.** Code tracking loop based on the sub cross-correlation shift technique.

**FIGURE 11.** Code tracking loop based on the sub cross-correlation shift technique.
In the formula, the subscripts $E$ and $L$ respectively indicate the Early and Late branches, and other branches are similarly represented. $T_{coh}$ is the coherent integration time. $\Delta \tau$ and $\Delta \varphi$ respectively represent the estimation error of the code phase and the initial phase of the carrier, $d_s = 2\delta T_C$ is the delay spacing between the early and late correlators. In digital signal processing and communication theory, the normalized $\sin^2$ function is usually defined as $\sin(\pi x)/\pi x$. All $n$ are noise terms obeying the Gaussian distribution [24], [29].

Using Early-Minus-Late-Power (EMLP) phase discriminator, as shown in Figure 11, the final result of the discriminator output $D$ is as follows:

For method I:

$$D_{SCST, 0} (\Delta \tau) = \left( \begin{array}{c} I_{E1}(\Delta \tau) \cdot (|I_{E0}(\Delta \tau) + I_{E0}(\Delta \tau - (N - 1)T_{SC})|^2) \\ -|I_{E0}(\Delta \tau) + I_{E0}(\Delta \tau - (N - 1)T_{SC})|^2 \\ + Q_{E1}(\Delta \tau) \cdot (|Q_{E0}(\Delta \tau) + Q_{E0}(\Delta \tau - (N - 1)T_{SC})|^2) \\ -|Q_{E0}(\Delta \tau) + Q_{E0}(\Delta \tau - (N - 1)T_{SC})|^2 \end{array} \right)$$

Or for Method II:

$$D_{SCST, 1} (\Delta \tau) = \left( \begin{array}{c} I_{L1}(\Delta \tau) \cdot (|I_{L0}(\Delta \tau) - I_{L0}(\Delta \tau - (N - 1)T_{SC})|^2) \\ -|I_{L0}(\Delta \tau) - I_{L0}(\Delta \tau - (N - 1)T_{SC})|^2 \\ + Q_{L1}(\Delta \tau) \cdot (|Q_{L0}(\Delta \tau) - Q_{L0}(\Delta \tau - (N - 1)T_{SC})|^2) \\ -|Q_{L0}(\Delta \tau) - Q_{L0}(\Delta \tau - (N - 1)T_{SC})|^2 \end{array} \right)$$

$I_{E1}$ and $I_{L1}$ are the in-phase correlator outputs of the local BOCs E and L branches, respectively. $I_{E0}$ and $I_{L0}$ are the in-phase correlator outputs of the auxiliary code E and L branches. Similarly, $Q_{E1}$ and $Q_{L1}$ are the quadrature-phase correlator outputs of the local BOCs E and L branches, respectively. $Q_{E0}$ and $Q_{L0}$ are the quadrature-phase correlator outputs of the auxiliary code E and L branches. The joint distribution function of the outputs at $\Delta \tau = 0$ is

$$(I_{E1}, I_{E0}, I_{L1}, I_{L0})^T \sim N (\mu \cos (\Delta \varphi), \omega)$$

$$(Q_{E1}, Q_{E0}, Q_{L1}, Q_{L0})^T \sim N (\mu \sin (\Delta \varphi), \omega)$$

With

$$\mu = \sqrt{2P} \cdot \sin (\pi f_0 T_{coh}) \cdot [R_B (-d_s/2) R_B (d_s/2)]^T$$

$$R_{B/P0} (-d_s/2) R_{B/P0} (d_s/2)^T$$

$$R_B (d_s) R_B (\omega) R_{B/P0} (-d_s) R_{B/P0} (\omega)$$

$$R_{B/P0} (d_s) R_{B/P0} (\omega) R_{P0} (d_s) R_{P0} (\omega)$$

$$= \frac{N_0}{T_{coh}} \begin{bmatrix} R_B (0) & R_B (d_s) & R_{B/P0} (0) & R_{B/P0} (d_s) \\ R_B (d_s) & R_B (0) & R_{B/P0} (-d_s) & R_{B/P0} (0) \\ R_{B/P0} (0) & R_{B/P0} (-d_s) & R_P (0) & R_P (d_s) \\ R_{B/P0} (d_s) & R_{B/P0} (0) & R_P (d_s) & R_P (0) \end{bmatrix}$$

$R_{P0} (\Delta \tau)$ is the ACF of the auxiliary signal. The distribution function of the outputs $Q_{E1}, Q_{E0}, Q_{L1}, Q_{L0}$ at $\Delta \tau = 0$ is

$$N (0, \omega)$$

Figure 12 shows the phase discrimination curves of the proposed method for BOC(10,5). The linear pull-in range is from $-d_s/2$ to $d_s/2$. Delay spacing is 0.05TC. Compared with the classic EMLP phase discriminator, the proposed method has only two peaks and successfully removes the mis-lock point. The code tracking error is discussed in reference [19], and the following formula is given.

$$\sigma_{te}^2 = \frac{2B_L (1 - 0.5B_L T_{coh}) T_{coh} \sigma_v^2}{K_v^2}$$

where $B_L$ is the code loop filter bandwidth, $\sigma_v$ is the discriminator output standard deviation, and $K_v$ is the discriminator gain [5]. Calculating the differential coefficient in the position of zero point, and discriminator gain can be derived as:

$$K_v = \left. \frac{dD_{SCST}}{d \Delta \tau} \right|_{\Delta \tau = 0}$$

$$= \left. \frac{PT_{coh}^2 \left( R_{SCST}^2 (\Delta \tau - \delta T_C) - R_{SCST}^2 (\Delta \tau + \delta T_C) \right) \right|_{\Delta \tau = 0}$$

$$= -4PT_{coh}^2 \left( 1 - \frac{4m}{n} \right) \left( \frac{4m}{n} \right)$$

In Equation (31), $m$ and $n$ are coefficients in BOC(m,n) defined in Section 2.1. Similarly, in Eq. (27), the error function caused by thermal noise in the
discriminator is:

\[ n(t) = \sqrt{\nu \cdot P_{coh}} \left( R_{SCST}(\Delta \tau - \delta \cdot T_C) \left( n_{i,i}^E + n_{i,N-1}^E \right) + R_{SCST}(\Delta \tau + \delta \cdot T_C) \left( n_{i,i}^L + n_{i,N-1}^L \right) \right) \]

\[ + \left( n_{i,i}^E + n_{i,N-1}^E \right)^2 + \left( n_{i,i}^E + n_{Q,Q-1}^E \right)^2 \]

\[ - \left( n_{i,i}^L + n_{i,N-1}^L \right)^2 - \left( n_{i,i}^L + n_{Q,Q-1}^L \right)^2 \]  

(32)

\( n_{i,i}, n_{i,N-1}, q_{i,i} \) and \( q_{N,N-1} \) are Gaussian noise. So, we can get the variance of thermal noise:

\[ \sigma_N^2 = E \left[ \frac{1}{n(t) \cdot n(t + \tau)} \right] \]

\[ = 4 \cdot P \cdot T_c^3 \cdot N_0 R_{SCST}^2 (\Delta \tau + \delta T_C) \]

\[ \times \left[ R_{SCST}(\Delta \tau) - R_{SCST}(\Delta \tau + 2\delta T_C) \right] \]

\[ + N_0 T_{coh}^2 \left[ R_{SCST}^2(\Delta \tau) - R_{SCST}^2(\Delta \tau + 2\delta T_C) \right] \]  

(33)

Substituting Eq. (31) and Eq. (33) into Eq. (30), assuming the delay spacing equals \( d_s = 2\delta T_C \), we can derive the code variance of the proposed method as:

\[ \sigma_{c,t}^2 = \frac{2B_L (1 - 0.5B_L T_{coh}) T_{coh} \sigma_N^2}{K_2} \]

\[ = \frac{B_L \cdot d_s}{2 \cdot \frac{4m}{\pi} \cdot C / N_0} \left( 1 + \frac{1}{\left( 1 - \frac{d_s^2}{4m^2} \cdot \frac{4m}{N_0} \cdot T_{coh} \right)} \right) \]  

(34)

IV. PERFORMANCE SIMULATION AND ANALYSIS

Based on the Matlab platform, the intermediate frequency of the input signal is 4.092MHz, and the sampling rate is set to 40.92MHz. The code phase offset is in the 0-th, the Doppler is set to 2000 Hz. The number of experimental iterations was set to 2000. In order to better reflect the performance of the SCST method, BOCs(1,1) and BOCs(10,5) common BOC signals were selected to evaluate its performance. For comparison, the Unit Correlation method, the ASPeCT method, the GRASS method, and the tracking method of Yan et al. (Yan 2015 a) were proposed. When the modulation order \( N=2 \), ASPeCT is the same as GRASS, and ASPeCT is a special case of GRASS.

A. DISAMBIGUATION PERFORMANCE ANALYSIS

Figures 13 and 14 show the normalized two-dimensional correlation functions for the five methods. When \( N=2 \), the main peak span of the SCST method is 20 sampling points (half-chip point), which is the same as the Unit Correlation, ASPeCT and GRASS, and better than the 26 sampling points of the Yan2015 method. For the correlation peak maximum, SCST is the same as the Unit Correlation, ASPeCT, and GRASS, which is 42.9% higher than the Yan2015 method. However, the SCST and Unit Correlation methods completely removed the effects of the sub-peaks. ASPeCT and GRASS still have two small sub-peaks, and Yan2015 has four small sub-peaks. When \( N=4 \), the SCST and Unit Correlation methods can completely remove the sub-peaks, but the Unit Correlation method sacrifices the main peak energy. ASPeCT and GRASS have more sub-peaks, which are easy to cause ambiguity problems. Compared with \( N=2 \), the Yan2015 method has a better ability to suppress the side peaks, but still sacrifices the main peak energy.

B. PHASE DISCRIMINATION CURVE ANALYSIS

The classical EMLP (Early-Minus-Late-Power) phase detector is often used to generate the phase discrimination curve to evaluate the tracking performance of each method [23]. As shown in FIG. 15 and FIG. 16, it is assumed that the front-end bandwidth is infinite, and the phase discrimination curves of SCST, Unit Correlation, ASPeCT, GRASS, and Yan2015 methods are respectively generated. For the BOC(1,1) signal, compared with the traditional delay DLL, the five algorithms can eliminate the false lock point and stabilize in the interval \([-0.1Tc, +0.1Tc] \). The peaks of SCST are sharper than others. And there are two very small peaks in ASPeCT/GRASS, which may cause the false lock. For the fourth-order BOC(10,5) signal, the phase discrimination curve of the traditional DLL has six false lock points. SCST, Unit Correlation and Yan2015 can remove the mis-locking...
The phase discrimination curve of BOC(1,1).

The phase discrimination curve of BOC(10,5).

FIGURE 15. The phase discrimination curve of BOC(1,1).

FIGURE 16. The phase discrimination curve of BOC(10,5).

The phase detection error output is proportional to the slope of the center zero-crossing phase discrimination curve, and the performance of noise immunity and tracking jitter accuracy of one method is closely related to it.

C. ANTI-MULTIPATH PERFORMANCE ANALYSIS

Multipath Error Envelope (MEE) is a typical indicator for evaluating the multipath performance of tracking loops, reflecting the sensitivity of a code tracking loop to different parameters of multipath signals [30]–[31]. The envelope extreme value (the absolute maximum value of the MEE), the length of the envelope interval (the sum of the abscissa intervals when the MEE takes a non-zero error), and the envelope area (the area enclosed by the MEE) are three measures of anti-multipath performance [32]. The smaller the three indexes, the better the anti-multipath performance.

FIGURE 17. The Multipath error envelopes for BOC(1,1).

FIGURE 18. The Multipath error envelopes for BOC(10,5).

The three methods of ASPeCT, GRASS and Yan2015 have more multipath errors envelopes than traditional EMLP, but the envelope area is small. ASPeCT and GRASS have three envelopes, and Yan2015 has five envelopes. Figure 18 shows a multipath envelope comparison of the five tracking algorithms with the traditional EMLP for the BOC(10,5) signal when the correlator span is set to 0.05Tc. The GRASS main envelope interval length is the shortest, but the number of envelopes is the most compared to other methods. There is still only one envelope for SCST and Unit Correlation methods. Since ASPeCT is only suitable for BOC(n,n), it has seven envelopes and has poor anti-multipath performance. Therefore, the method proposed in this paper has excellent anti-multipath performance.

D. ANTI-NOISE PERFORMANCE ANALYSIS

Thermal noise is another important cause of tracking error, and loop code tracking error is an important indicator to measure the anti-noise performance of tracking methods [33]. Here, the code tracking error is defined as the code phase
offset of the phase discrimination function. Using the Monte Carlo experimental method, the number of experimental iterations was set to 2000. As shown in Figure 19, BOC(1,1) is tracked by several tracking methods, and the code tracking error under different SNR is obtained. SCST has the best performance. The performance of ASPeCT/GRASS and Unit Correlation methods is almost the same. The performance of Yan2015 is not as good as the other four, which still has four sub-peaks at N=2, and the side peak suppression effect is not good. Figure 20 shows the code tracking error at different signal-to-noise ratios obtained by tracking the BOC(10,5) by five tracking methods. ASPeCT performs best, followed by SCST. However, for ASPeCT, its correlation function, multipath error envelope and phase discrimination function have many peaks, which are not suitable for BOC signals above 2nd order. The larger the modulation order is, the worse the correlation of the auxiliary codes of the Unit Correlation method is, and the worse the code tracking error performance is. The side peak suppression ability of the GRASS is not good, there are still side peak interference, and the code tracking error performance is not as good as other methods. For a BOC(10,5) signal with a received signal-to-noise ratio (SNR) of −28 dB, SCST is less than 45.8%, 67.5%, and 12.2% of the Unit Correlation, GRASS, and Yan proposed algorithms.

V. CONCLUSION

The multi-peak characteristic of BOC autocorrelation function is easy to cause ambiguity. This paper studies this problem. Combining the idea of GRASS algorithm shape code, an unambiguous tracking algorithm suitable for BOC(m,n) signals is proposed. In this method, the sub-peak interference is eliminated by the reconstruction of the cross-correlation function between BOC Signal and PRN code. The theory analyzes the effectiveness of the tracking algorithm. The disadvantage is that it requires multi-level storage, which will bring additional resource consumption to the receiver. Compared with the unit, ASPeCT, GRASS and Yan 2015, the SCST performs best in eliminating the side peak performance, phase discrimination curve, anti-multipath performance and code tracking accuracy. This provides a prerequisite for the integration of the new generation of navigation systems with other systems, such as DOA [1]–[3], [42]–[44] and 3D imaging [45] combined with GNSS and radar.

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