The propagation of electromagnetic waves in ferrite-dielectric structure with metal shield

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Abstract. The paper deals with the problem of wave propagation in a rectangular waveguide containing a three-layer ferrite-dielectric-ferrite (FDF) structure located near a metal shield. This FDF structure can be used to create low-cost phased array antennas with simple electrical control of the beam position. However, the calculation of the open FDF structure encounters great difficulties. Placing the FDF structure in a rectangular waveguide allows us to obtain a strict solution for same waveguide wave modes including main mode, which have no dependence of the fields on the coordinate directed along the magnetizing field. This is the main mode of the FDF structure, which determines the general properties of the phased array. It is shown that the obtained relations describe the modes of the open FDF waveguide with high accuracy. The dependences of the mode propagation constants on the magnetizing field have been calculated, the structure of the electromagnetic field for the main and some higher order modes has been found, and the optimal parameters of the structure, which provide maximal controllability by magnetizing field, have been determined.

1. Introduction

Today, the task of creation of phased array antennas with optimal technical parameters and low cost remains urgent [1-7]. This is largely due to the upcoming widespread introduction of 5G cellular communication systems [8, 9]. Such systems actively use the millimeter frequency range. This makes the task of creating antennas of this range very relevant. One of the solutions to this problem is the use of electrically controlled ferrite-dielectric-ferrite structures (FDF structures) for the design of integrated phased array antennas (IPA) [2-6, 10, 11]. Such antenna arrays have a simple design that allows them to be manufactured using PCB technology, which ensures low production cost.

FDF structure is an open waveguide operating in multimode regime. Electromagnetic analysis of such a structure is difficult because the waveguide is open (unshielded) and contains magnetized ferrite, which is a non-reciprocal medium. For these reasons, the analysis can be performed only approximately.

It is of interest to find a strict dispersion equation that describes the properties of at least the main wave mode in such a waveguide (the properties of the main wave mode in the FDF structure determine the most important characteristics of the IPA). This will allow a deeper study of the physical properties of the proposed structure, that, in its turn will allow to optimize the antenna.

In paper [11] the method of calculation of parameters of the basic mode of symmetric open FDF structure has been offered, considering this structure in the closed rectangular waveguide. It was shown that at a high value of dielectric permittivity of the dielectric slab in the FDF structure, the
electromagnetic field outside this structure decreases quickly enough, so that the presence of the walls of the waveguide has practically no effect on the basic mode.

In this paper we propose to consider a three-layer FDF structure with metallization nearby one of the ferrite slabs. Namely such structures are used for creation of IPA.

2. Problem statement and solution

So, let us consider the problem of a rectangular waveguide with two ferrite layers and a dielectric slab located between them (figure 1). The external static magnetic field is directed along the $z$ axis (the ferrite slabs are magnetized in the opposite directions).

![Figure 1. Schematic representation of FDF-structure in a rectangular waveguide: $a$ - waveguide width; $a_1$, $g_1$ - $g_3$ - other geometric parameters; $h_{f_1}$, $h_{f_2}$, $h_d$ - widths of ferrite (II, IV) and dielectric (III) slabs respectively; regions I, V - air space, last one having width $s$.](image)

The tensor of magnetic permeability of the ferrite can be written in the form:

$$\tilde{\mu} = \begin{bmatrix} \mu & i\mu_a & 0 \\ -i\mu_a & \mu & 0 \\ 0 & 0 & \mu_0 \end{bmatrix},$$

where $i$ is an imaginary unit, $\mu$, $\mu_a$, $\mu_0$ - components of the magnetic permeability tensor [12].

In work [12, 13] it is shown that only $E_z$ component of an electric field and $H_x$ and $H_y$ components of a magnetic field remain non-zero in the absence of dependence of fields on coordinate $z$ in a rectangular waveguide. In this case, the $E_z$ component in the ferrite satisfies the equation

$$\frac{d^2E_z}{dx^2} + \nu_j^2E_z = 0,$$

where $\nu_j^2 = \frac{k^2}{\varepsilon_f\mu_0 - \beta^2}$, $k$ - wave number of vacuum, $\varepsilon_f$ - permittivity of ferrite, $\beta$ - the desired wave propagation constant in the waveguide.

The components of the magnetic field in the ferrite can be found using the following relations [12]

$$H_x = \frac{1}{k\mu_0} \left( \beta E_z - \frac{\mu_0}{\mu} \frac{dE_z}{dx} \right),$$

$$H_y = \frac{i}{k\mu_0} \left( \beta \frac{\mu_0}{\mu} E_z - \frac{dE_z}{dx} \right),$$

where $\mu_0 = (\mu^2 - \mu_a^2) / \mu$. 

The formulas (2) – (4) describe the fields of region II and IV (ferrite slabs). The analogous relations can be written for region III (dielectric), and for regions I, V (air). The difference is only in the changing of material parameters. So, for the dielectric instead of the value \( v_{\eta}^2 \) will be written as 
\[ v_{\eta}^2 = k^2 e_d - \beta^2 \] 
where \( e_d \) — dielectric constant of region III. In the air areas I and V instead \( v_{\eta}^2 \) will be \( v_0^2 = k^2 - \beta^2 \). Similarly, the relations (3) and (4) will change in these areas.

It can be written down the solution of equation (2) for all five regions and apply boundary conditions to them: the equality of the \( E_z \) component on the side walls to zero and the continuity of the field components \( (E_z \text{ and } H_j) \) on the interface between the regions.

Taking into account the above, we can write the fields on the waveguide areas I-V (see figure 1):

I. 
\[ E_z = A \sin(v_\eta x), \]

II. 
\[ E_z = B \sin[v_\eta (x - a_1)] + C \cos[v_\eta (x - a_1)] \]

III. 
\[ E_z = D \sin[v_\eta (x - a_1 - h_{j1})] + E \cos[v_\eta (x - a_1 - h_{j1})] \]

IV. 
\[ E_z = F \sin[v_\eta (x - a_1 - h_{j1} - h_{j2})] + G \cos[v_\eta (x - a_1 - h_{j1} - h_{j2})] \]

V. 
\[ E_z = I \sin(v_0 (a - x)) \]

Here the boundary conditions on the side walls are already taken into account.

As a result, we obtain a homogeneous system of linear equations of the eighth order with unknown coefficients \( A, B, C, D, E, F, G \) and \( I \). A non-trivial solution exists only if this system determinant \( \hat{Z} \) is equal to zero:

\[ \det \hat{Z} = 0 \]

So, we have obtained a dispersion equation that allows us to find the constant propagation of the modes of a three-layer waveguide, and, first of all, the main mode. In this case, the resulting equation is strict, i.e. the accuracy of the solution will be determined only by the accuracy of the procedure for calculating its roots.

Non-zero elements of the matrix \( \hat{Z} \) are given below:

\[ z_{11} = \sin(v_\eta a_1), \quad z_{12} = -1, \]
\[ z_{21} = -v_\eta \cos(v_\eta a_1), \quad z_{22} = v_\eta \mu_1 \mu_1, \quad z_{23} = -\beta \mu_1 \mu_1, \]
\[ z_{32} = \sin(v_\eta h_{j1}), \quad z_{33} = \cos(v_\eta h_{j1}), \quad z_{34} = -1, \]
\[ z_{41} = \frac{1}{\mu_1} \left[ \frac{\beta \mu_1}{\mu} \cos(v_\eta h_{j1}) - v_\eta \sin(v_\eta h_{j1}) \right], \quad z_{42} = v_\eta, \]
\[ z_{43} = \frac{1}{\mu_1} \left[ \frac{\beta \mu_1}{\mu} \cos(v_\eta h_{j1}) - v_\eta \sin(v_\eta h_{j1}) \right], \quad z_{44} = 1, \]
\[ z_{45} = \sin(v_\eta h_{j2}), \quad z_{55} = \cos(v_\eta h_{j2}), \quad z_{56} = -1, \]
\[ z_{57} = v_\eta \sin(v_\eta h_{j2}), \quad z_{65} = -v_\eta \sin(v_\eta h_{j2}), \quad z_{66} = v_\eta \mu_1 \mu_1, \quad z_{67} = \beta \mu_1 \mu_1, \]
\[ z_{68} = \sin(v_\eta h_{j2}), \quad z_{76} = \cos(v_\eta h_{j2}), \quad z_{77} = -\sin(v_\eta h_{j2}), \quad z_{78} = 1, \]
\[ z_{86} = \frac{1}{\mu_1} \left[ \frac{\beta \mu_1}{\mu} \sin(v_\eta h_{j2}) - v_\eta \cos(v_\eta h_{j2}) \right], \quad z_{87} = -v_\eta \cos(v_\eta h_{j2}), \quad z_{88} = -v_\eta \sin(v_\eta h_{j2}), \quad z_{99} = -v_\eta \cos(v_\eta h_{j2}). \]
3. Study of waveguide modes of FDF structure with a shield

By solving the dispersion equation (10) it is possible to obtain the dependence of the propagation constant on the non-diagonal element of the tensor $\hat{\mu}$ (i.e., in fact, on the magnetizing field), as well as on other parameters. Further, using equations (2-4), we can find the structure of the fields of each mode. In the article [14] a combination of parameters of the FDF structure, which made it possible to create a workable IPA, has been proposed and experimentally tested. However, no conclusions could be made about the optimality of such a combination of parameters. Now, having an analytical solution for the main mode of the FDF waveguide, such a study is possible.

The used in article [14] parameters are given in table 1.

| $ka$ | $h_j / a$ | $h_d / a$ | $\varepsilon_f$ | $\varepsilon_d$ |
|------|-----------|-----------|----------------|----------------|
| 4,82 | 0,097     | 0,042     | 12             | 40             |

Figure 2 shows the dependence of the main mode deceleration $q = \beta / k$ of the symmetric ($h_1=h_2$) FDF waveguide, obtained using the parameters from table 1, on the value of the non-diagonal tensor element $\mu_\alpha$ (the other elements of the tensor are assumed to be 1). Curve 3 corresponds to that obtained in [11, 14]. For the FDF structure shifted relative to the center of the waveguide, its controllability slightly increases, reaching a maximum when a ferrite slab is pressed against the metal shield (curve 1).

The numerical experiment shows that the propagation constant does not change with increasing waveguide size $a$ (while maintaining the other sizes). This is due to the fact that in the location of the left wall of the waveguide field is almost equal to zero. Therefore, the result will not change if the left wall moves to infinity. The wide walls of the waveguide can be removed, as this does not change the boundary conditions. Thus, we obtain a solution for an open FDF structure with a metal shield near one of the ferrite slabs.

The dependence of controllability on the thickness of the dielectric $h_d / a$ has been investigated (figure 3). Controllability has a maximum value at $h_d / a=0.04$. The physical meaning of the presence of the dielectric slab in the FDF structure is that it "draws in" the electromagnetic field, so that the energy propagates through the waveguide inside dielectric slab and close to it, i.e., where the ferrite slabs are located. This ensures good control. If the dielectric is too thick, the field is concentrated in it, and the ferrite has a small amount of electromagnetic field, resulting in drop of the controllability. Too thin dielectric is unable to concentrate the field, a lot of energy is distributed outside the ferrite, and control also falls.
Figure 3. The dependence of the controllability of FDF structure on the thickness of the dielectric slabs

Figure 4. The dependence of the controllability of FDF-waveguide to the thickness of the ferrite slabs

Figure 4 shows the dependence of the controllability of FDF structure on the thickness of the ferrite slabs. It is seen that the controllability has a maximum value at $h_f = 0.097$, and then slowly decreases. This is due to the fact that the ferrite interacts effectively with the electromagnetic wave only in those areas where the polarization of the magnetic field is close to the circular; the value of the field should be large enough. These areas are located near the dielectric slab. When using thick ferrite slabs, areas, where the magnetic field is small, are captured by ferrite.

We now investigate the dependence of controllability on the ferrite thickness $h_{f1}$ and $h_{f2}$ (in figure 5 $h_{f1}$ is being changed at fixed $h_{f2}$, and in figure 6 — $h_{f2}$ at fixed $h_{f1}$). Other parameters are taken from table 1).

Figure 5. Dependence of controllability on the normalized thickness of the left ferrite slab

Figure 6. Dependence of controllability on the normalized thickness of the right ferrite slab

Figures 5 and 6 show that there are optimal values for the thickness of the ferrite slabs, providing maximum control. Moreover, the maximum controllability is achieved with some asymmetry of the FDF structure.
4. Conclusion
The paper strictly solves the problem of wave propagation in a rectangular waveguide containing a three-layer ferrite-dielectric-ferrite structure with a shield located nearby one of the ferrite slabs. The proposed approach allows us to study the modes of electromagnetic waves, which have no dependence on the coordinate directed along the magnetizing field. It is shown that the obtained relations describe the wave modes for an open FDF waveguide with shield disposed near one of the ferrite slabs. Namely the main wave mode in the open FDF structure determines the general properties of integrated phased array antennas, such as the scanning sector and the width of the radiation pattern. As a result, it is not necessary to involve cumbersome numerical methods that take a lot of machine time to study FDF structure. Thus, the calculation of the performance characteristics of the integrated phased antenna arrays is significantly simplified.

The obtained results can be used in the design of IPA.

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