Balance functions in a thermal model with resonances

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Abstract

The $\pi^+\pi^-$ balance function in rapidity is computed in a thermal model with resonances. It is found that the correlations from the neutral-resonance decays are important, yielding about a half of the total contribution, which in general consist of resonance and non-resonance parts. The model yields the pionic balance function a few per cent wider that what follows from the recent data for the Au+Au collisions at $\sqrt{s_{NN}} = 130$ GeV.

Key words: ultra-relativistic heavy-ion collisions, particle correlations, charge fluctuations, balance functions

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Microscopic processes of particle production in heavy-ion collisions conserve, in an obvious manner, the electric charge. The charge balance function in rapidity has been proposed as a convenient measure of the resulting correlations between the opposite charges \cite{1,2,3}. Moreover, its features can be used to discriminate between different mechanisms of particle production. In particular, the width of the balance function in rapidity could pin down the time of production of the opposite-charge pair and provide more insight on its subsequent transport in the hadronic environment \cite{2}. In addition, the balance functions are closely related to charge fluctuations \cite{4,5,6,7,8,9,10}, whose study reveals important clues on the evolution of the system formed in the collision.

The balance functions analyzed by the STAR Collaboration \cite{11,12} at RHIC

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are defined by the formula

\[ B(\delta, Y) = \frac{1}{2} \left\{ \frac{\langle N_{+-}(\delta) \rangle - \langle N_{++}(\delta) \rangle}{\langle N_+ \rangle} + \frac{\langle N_{-+}(\delta) \rangle - \langle N_{--}(\delta) \rangle}{\langle N_- \rangle} \right\}. \tag{1} \]

where \( N_{+-}(\delta) \) counts the opposite-charge pairs which satisfy the condition that both members of the pair fall into the rapidity window \( Y \) with the relative rapidity \( |y_2 - y_1| = \delta \), whereas \( N_+ \) is the number of positive particles in the interval \( Y \). Other quantities appearing in Eq. (1) are defined in an analogous way. For sufficiently large rapidity interval \( Y \sim Y_{\text{max}} \), the balance function of all charged hadrons is normalized to unity [2], which is a condition reflecting the overall charge conservation.

The recent measurement of the balance functions by the STAR Collaboration [11] showed that their widths are smaller than expected from models discussed in Ref. [2] and significantly smaller than observed in elementary particle collisions. This issue has been recently addressed by Bia\'ls [13], where the small widths of the balance functions have been explained in the framework of the coalescence model [14]. In this paper we offer an alternative calculation of the width of the \( \pi^+\pi^- \) balance function, based on a thermal model with resonances. A similar calculation has been recently performed by Pratt et al. in the blast-wave model [16,17].

Within our approach, the \( \pi^+\pi^- \) balance function has two contributions, corresponding to two different mechanisms of the creation of an opposite-charge pair. The first one (resonance contribution) is determined by the decays of neutral hadronic resonances. The second one (non-resonance contribution) is related to other possible correlations among the charged particles. We stress that in our terminology the resonance contribution refers only to the decays of neutral resonances which have a \( \pi^+\pi^- \) pair in the final state.

Neutral resonances, having as their decay products a pair of the opposite-charge pions, produce correlations between charges given simply by the kinematics of the decay. In our calculation we explicitly include

\[ K_S, \eta, \eta', \rho^0, \omega, \sigma, \text{ and } f_0. \tag{2} \]

For the scalar-isoscalar channel, involving a wide resonance, we use the formalism with the phase-shifts described in Ref. [18]. The resonance contribution to the balance function is completely defined in the framework of the thermal model. Neutral mesons are present in the final hadronic cocktail in any model of the reaction. Moreover, they have been directly identified in the invariant-mass spectra by the STAR collaboration [19] (see also Ref. [18], where the STAR result is interpreted in the framework of the thermal model used in this
The parameters which must be specified are the percentage of pions originating from the decay of neutral resonances and the momentum distribution for such resonances, here taken from the thermal model of Ref. [15].

A different, non-resonance, mechanism for charge balancing is required when the charged particles are produced directly in the hadronization process, for instance during the elementary collision or the string breakup [1,20]. In this case the details of the process are not known, except for the requirement of the charge conservation. In the spirit of the thermal model with single freeze-out [15], we adopt the late-hadronization hypothesis and assume that the non-resonance charges are balanced locally at the freezeout surface, i.e., the two opposite-charge particles are produced at the same space-time point with thermal velocities (defined in the local rest frame of the fire-cylinder element).

An important point is to note that a neutral resonance ends up (with a given branching ratio) as a $\pi^+\pi^-$ pair, whereas in the non-resonance mechanism of charge balancing a charged pion can be balanced with another charged hadron, not necessarily a pion.

According to the above discussion, the $\pi^+\pi^-$ balance function can be constructed as a sum of the two terms

$$B(\delta, Y) = B_R(\delta, Y) + B_{NR}(\delta, Y).$$

The resonance contribution $B_R(\delta, Y)$ is obtained directly from the expressions describing the phase-space of the pions emitted in a decay. One may use here one of the results obtained in Ref. [13], i.e., the fact that the balance functions calculated in a neutral-cluster model do not depend on correlations between the clusters but they are determined solely by the single-particle distribution of the clusters and the two-particle distribution of pions in a decay of a single cluster. Replacing the neutral clusters of Ref. [13] by the neutral resonances we immediately obtain the two-particle rapidity distribution of the $\pi^+\pi^-$ pairs coming from the decay of a neutral resonance

$$\frac{dN_{R}^{\pi^+\pi^-}}{dy_1 dy_2} = \int d\Sigma \int d^2p_1 \frac{d^2p_2}{p^2} C_\pi \left( \frac{dN_R}{dy_1 dy_2} \right) \rho_{R \rightarrow \pi^+\pi^-} (p, p_1, p_2),$$

where $C_\pi$ indicates symbolically the presence of kinematic cuts for the pions. In a thermal approach the momentum distribution of the resonance $R$ is obtained from the Cooper-Frye formula

$$\frac{dN_R}{dy_1 dy_2} = \int d\Sigma(x) \cdot p f_R (p \cdot u(x)),$$

where $f_R$ is the phase-space distribution function of the resonance, $u^\mu$ is the hydrodynamic flow velocity at freeze-out defined by the constant value of the
invariant time $\tau$, and $d\Sigma_\mu$ describes a three-dimensional element of the freeze-out hypersurface. According to Ref. [15], we use the expansion model where $u^\mu = x^\mu/\tau$ and

$$d\Sigma_\mu(x) = d\Sigma(x) u_\mu(x), \quad (6)$$

with $d\Sigma(x) = \tau^3 \cosh(\alpha_\perp) \sinh(\alpha_\perp) \, d\alpha_\parallel \, d\alpha_\perp \, d\phi$. The quantities $\alpha_\parallel$ and $\alpha_\perp$ denote the longitudinal and transverse rapidity of the fluid element, respectively, and $\phi$ is the azimuthal angle [15].

The two-particle pion momentum distribution in a two-body ($\pi^+\pi^-$) resonance decay may be expressed by the Dirac $\delta$ function

$$\rho_{R\rightarrow\pi^+\pi^-} = \frac{b_{\pi\pi}}{N_2} \delta^{(4)}(p - p_1 - p_2), \quad (7)$$

with the normalization constant

$$N_2 = \int \frac{d^3p_1}{E_1} \frac{d^3p_2}{E_2} \delta^{(4)}(p - p_1 - p_2), \quad (8)$$

and the branching ratio $b_{\pi\pi}$.

Expressions (4)-(8) may be easily generalized to the case of three-body decays where the charged $\pi^+\pi^-$ pair is produced together with an extra neutral pion. With the typical technical assumption of a constant transition matrix element we have

$$\rho_{R\rightarrow\pi^+\pi^-\pi^0} = \frac{b_{\pi\pi\pi}}{N_3} \int \frac{d^3p_3}{E_3} \delta^{(4)}(p - p_1 - p_2 - p_3), \quad (9)$$

where the normalization constant is given by the formula

$$N_3 = \int \frac{d^3p_1}{E_1} \frac{d^3p_2}{E_2} \frac{d^3p_3}{E_3} \delta^{(4)}(p - p_1 - p_2 - p_3). \quad (10)$$

The non-resonance correlations arise from pions emitted directly from the fire-cylinder or those produced in the decays of resonances other than (2). In contrast to the correlations between the resonance pions (fully determined by the model), the correlations between the non-resonance pions are not specified by the model and we have to propose their form. We assume that the creation of an opposite-charge pair occurs locally in the fire-cylinder. It means that the two charges have the same longitudinal and transverse collective velocity, however their relative momentum is determined by the local thermal momenta.
of both particles \[2,21\]. This process is described by the following two-particle rapidity distribution for correlated pairs

\[
\frac{dN_{\pi^+\pi^-}}{dy_1dy_2} = A \int d^2p_1^+ d^2p_2^+ C_\pi \times \int d\Sigma(x) p_1 \cdot u(x) f_{NR}^\pi (p_1 \cdot u(x)) p_2 \cdot u(x) f_{NR}^\pi (p_2 \cdot u(x)). \tag{11}
\]

Here \(f_{NR}^\pi\) is the phase-space distribution function of the non-resonance pions and the normalization constant \(A\) in Eq. (11) is obtained from the condition

\[
\int dy_2 \left( \frac{dN_{\pi^+\pi^-}}{dy_1dy_2} \right) = \frac{dN_{\pi^+\pi^-}}{dy_1}. \tag{12}
\]

We note that a charge can be produced in the fire-cylinder directly as a pion, a resonance decaying eventually into a charged pion, or as a different charged hadron. The last case is not registered in the \(\pi^+\pi^-\) balance function, and leads to a reduction of its norm. The first two cases can be effectively taken into account by combining in the distribution \(f_{NR}^\pi\) the direct pions and the charged pions originating from the decays of the resonances other than \(2\). This has a softening effect on the pion spectra and leads to a slight reduction of the width of the balance function as compared to the case with direct pions only. For both the resonance and the non-resonance production of correlated opposite-charge pairs we assume that after the freeze-out the momentum distribution of pions is not modified by rescattering. This picture is consistent with the single-freezout thermal model we use.

Knowing the two-particle distribution functions (4) and (11), we calculate the balance function of the resonance pions from the expression

\[
B_R(\delta) = \frac{1}{N_\pi} \sum_R \int dy_1 dy_2 C_\pi \left( \frac{dN_{\pi^+\pi^-}}{dy_1dy_2} \right) \delta(|y_2 - y_1| - \delta), \tag{13}
\]

where the sum over \(R\) includes the resonances \(2\). Similarly, for the non-resonance contribution (marked by the tilde here, which indicates the neglect of the balancing from other hadrons) we obtain

\[
\tilde{B}_{NR}(\delta) = \frac{1}{N_\pi} \int dy_1 dy_2 C_\pi \left( \frac{dN_{\pi^+\pi^-}}{dy_1dy_2} \right) \delta(|y_2 - y_1| - \delta). \tag{14}
\]

In the above formulas \(N_\pi = (N_{\pi^+} + N_{\pi^-})/2\) is one half of the total number of
Fig. 1. Contributions to the balance function from the neutral resonances (solid line) and from the non-resonance pions (dashed line), plotted as a function of the rapidity difference of the two pions. The geometric parameters of the model satisfy $\rho_{\text{max}}/\tau = 0.9$, which corresponds to $\langle \beta_\perp \rangle = 0.5$.

charged pions observed in the acceptance window of rapidity,

$$
N_{\pi} = \int dy_1 d^2 p_1^\perp \left( \frac{dN_{\pi}}{dy_1 d^2 p_1^\perp} \right) C_{\pi}.
$$  \hspace{1cm} (15)

We note that $N_{\pi}$ is the sum of the number of non-resonance pions $N_{NR}^\pi$ and the number of resonance pions $N_{R}^\pi$. As a consequence, in the absence of cuts, the two contributions to the pion balance functions are normalized to the relative weight of the corresponding pion-pair production processes. More explicitly, $\int_{Y_{\text{max}}}^0 d\delta B_R(\delta) = N_{R}^\pi/N_{\pi}$, and $\int_{Y_{\text{max}}}^0 d\delta \tilde{B}_{NR}(\delta) = N_{NR}^\pi/N_{\pi}$. Taking into account the fact that some of the thermal pions are balanced by other charged hadrons, the final expression for the pion balance function is

$$
B(\delta) = B_R(\delta) + \frac{N_{NR}^\pi}{N_{\text{charged}} - N_{R}^\pi} \tilde{B}_{NR}(\delta),
$$  \hspace{1cm} (16)

where $N_{\text{charged}} = (N_+ + N_-)/2$ is a half of the total multiplicity of the charged (positive plus negative) particles. The factor $N_{NR}^\pi/(N_{\text{charged}} - N_{R}^\pi)$ is the ratio of the non-resonance pions to all charged particles, with the exclusion of the pions coming from resonances, which have already been accounted for in $B_R(\delta)$. With the parameters of the single freezeout thermal model [15] one finds $N_{NR}^\pi/(N_{\text{charged}} - N_{R}^\pi) = 0.68$.

Our model results for the two (resonance and non-resonance) contributions to the balance function are shown in Fig. 1. One can observe that the balance
function for pions originating from the neutral-resonance decays is very similar to that obtained for the non-resonance contribution. In general, the thermal pion balance function [2,3,21] is known to be wider than the experimental balance function measured by the STAR collaboration [11]. The narrowing of the balance function may be achieved by imposing the strong transverse flow, however, this effect is not sufficient to explain the data, unless unreasonably low values of the temperature are chosen. The non-resonance pions in our calculation give a relatively wide balance function, since we use the high freeze-out temperature of 165 MeV and an average transverse velocity \( \langle \beta_\perp \rangle = 0.5 \).

The pion pairs originating from the decays of neutral resonances lead to a similar balance function. As a result, the sum of the two is a few per cent wider than the experiment. The norms, including the kinematic cuts of \(|\eta_\pi| < 1.3 \) and \( p_\perp^{(1,2)} > 0.1 \) GeV, are \( \int d\delta B_R(\delta) = 0.25 \), and \( \int d\delta B_{NR}(\delta) = 0.29 \), with the total reaching 0.53. The number is less than one, which reflects the presence of cuts, and the mentioned effect of balancing of non-resonance pions by other charged particles.

In Fig. 2 we show our full result for the balance function for four different centrality windows compared to the STAR experimental data. The geometric (expansion) parameters of the thermal model are taken in such a way that their ratio \( \rho_{\text{max}}/\tau \) is fixed and equal to 0.9, for each centrality. The dependence on the \( \tau \) parameter vanishes, since it appears as a factor \( \tau^3 \) in both the numerator and denominator of Eq. (1). In each case the normalization of the balance function has been adjusted arbitrarily, due to our lack of knowledge of the experimental efficiency [12]. The resulting factors are given below the plot labels, and vary from 0.40 to 0.51. The basic conclusion is that the model and the experiment agree quite reasonably for the shape of \( B(\delta) \). In fact, the model agrees best for the most periferic data, where the measured balance function is wider. We note that the dips of the balance functions observed at small values of \( \delta \) are caused by the Bose-Einstein correlations and cannot be explained by our model, unless such correlations are incorporated as an additional effect.

The mean values of \( \delta \) are presented in Table 1 independently for the resonance and non-resonance contributions, and also for the total. For comparison with the experimental values [11], we present the results for the the mean width \( \langle \delta \rangle \) with the exclusion of the region \( \delta < 0.2 \). In addition, in Table 1 we show the results obtained for \( \rho_{\text{max}}/\tau = 0.8 \), which corresponds to a smaller value of the transverse flow.

The obtained model width of the pion balance function is consistent with the STAR collaboration results for the 70 – 96% centrality bin, where \( \langle \delta \rangle = 0.664 \pm 0.029 \). For the central events \( (c = 0 – 10\%) \) the experiment gives \( \langle \delta \rangle = 0.594 \pm 0.019 \), for the mid-central \( (c = 10 – 40\%) \) \( \langle \delta \rangle = 0.622 \pm 0.020 \), and for the mid-periferal \( (c = 40 – 70\%) \) \( \langle \delta \rangle = 0.633 \pm 0.024 \), which is few per cent larger than the model result. The dependence of the width of the balance
Fig. 2. Balance functions for the pions in the thermal model calculated for four different centrality classes and compared to the experimental data of Ref. [11]. The normalization, affected by the detection efficiency, is adjusted a posteriori, as explained in the text. The resulting normalization factor, obtained by a $\chi^2$ fit in the range $0.2 < \delta < 2.2$, is listed near the plot labels.

The function on centrality cannot be reproduced in our approach by varying the transverse flow within limits consistent with the single-particle spectra.

We summarize our main points:

(1) An important source of the correlation between the opposite-charge pions is the decay of neutral resonances. The decay of the neutral resonances gives about half of the pion pairs in the rapidity window considered.

(2) The rest of the pion pairs has been assumed to be emitted locally from the freeze-out surface [2,3,13,21]. The balance function for such pairs is relatively wide since the temperature at the freeze-out surface is large and the average transverse flow is small. Hence, this mechanism of local

| $\rho_{\text{max}}/\tau$ | $\langle \beta_\perp \rangle$ | $\langle \delta \rangle_{\text{res}}$ | $\langle \delta \rangle_{\text{therm}}$ | $\langle \delta \rangle_{\text{tot}}$ |
|--------------------------|-----------------|-----------------|-----------------|-----------------|
| 0.8                      | 0.46            | 0.65 (0.52)     | 0.68 (0.54)     | 0.66 (0.53)     |
| 0.9                      | 0.50            | 0.65 (0.51)     | 0.67 (0.53)     | 0.66 (0.52)     |

Table 1
The widths of the pionic balance functions in rapidity for different geometry/flow parameters. The numbers for $\langle \delta \rangle$ are obtained with the range $0.2 < \delta < 2.6$, while the values in parenthesis are for the range $0 < \delta < 2.6$. 

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charge balancing in the fireball cannot explain the data by itself.

(3) By summing the contribution of the two mechanisms of charge balancing we obtain a pion balance function with the shape similar to the periferic data of Ref. [11].

(4) The normalization of the balance function obtained from the model is significantly larger than the experimental value (a factor of 2 - 2.5) because of the effect of a limited detector efficiency and acceptance that we are not able to take into account. We include only the sharp kinematic cuts in pseudorapidity and transverse momentum to simulate the acceptance window of the STAR experiment.

(5) The limited detector efficiency and acceptance may also affect the width of the balance function.

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