Hadronic total cross sections, Wilson loop correlators and the QCD spectrum

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Abstract

We show how to obtain rising hadronic total cross sections in QCD, in the framework of the nonperturbative approach to soft high-energy scattering based on Wilson-loop correlators. Total cross sections turn out to be of “Froissart”-type, i.e., the leading energy dependence is of the form $\sigma_{\text{tot}} \sim B \log^2 s$, in agreement with experiments. The observed universality of the prefactor $B$ is obtained rather naturally in this framework. In this case, $B$ is entirely determined by the stable spectrum of QCD, and predicted to be $B_{\text{th}} = 0.22 \text{ mb}$, in fair agreement with experiments.

Keywords: total cross sections, QCD, nonperturbative approach

1. Introduction

Explaining the behaviour of hadronic total cross sections at high energy is a very old problem, which is very rarely attacked within the framework of QCD. Most of the approaches to this problem are based on phenomenological models, which are sometimes QCD-inspired, but a full derivation from first principles of QCD is still lacking.

The observed rise of hadronic total cross sections at high energy is well described by a “Froissart-like” behaviour $\sigma_{\text{tot}} \sim B \log^2 s$ with a universal prefactor $B \approx 0.27 \div 0.28 \text{ mb}$ [1]. This behaviour respects unitarity, as encoded in the Froissart-Lukaszuk-Martin bound [2, 3, 4], $\sigma_{\text{tot}} \leq B_{\text{FLM}} \log^2 \frac{1}{s}$, since $B \ll B_{\text{FLM}} = \pi/m_s^2 \approx 65 \text{ mb}$.

Besides deriving $\sigma_{\text{tot}}$ from the first principles of QCD, one should be able to explain the observed universality of $B$, and also the two orders of magnitude separating $B$ and $B_{\text{FLM}}$. A step in this direction has been made in Ref. [5], where we have derived the asymptotic behaviour of hadronic total cross sections in the framework of the nonperturbative approach to soft high-energy scattering [6, 7, 8], finding indeed a “Froissart-like” behaviour, and a theoretical prediction for $B$ in fair agreement with experiments.

2. Nonperturbative approach to soft high-energy scattering

Understanding the rise of $\sigma_{\text{tot}}$ is part of the problem of soft high-energy scattering, characterised by $|t| \leq 1 \text{ GeV}^2 \ll s$. In this regime perturbation theory is not fully reliable, and a nonperturbative approach is needed. In a nutshell, the nonperturbative approach to elastic meson-meson scattering is as follows [6, 7, 8]:

1. mesons are described as wave packets of transverse colourless dipoles;
2. in the soft high-energy regime, the dipoles travel essentially undisturbed on their classical, almost lightlike trajectories;
3. mesonic amplitudes are obtained from the dipole-dipole ($dd$) amplitudes after folding with the appropriate squared wave functions, $A(s, b) = \langle A^{dd}(s, b; \nu) \rangle$.
Here \(b\) is the impact parameter\(^2\), \(\nu\) denotes collectively the dipole variables (longitudinal momentum fraction, transverse size and orientation), and \(\langle \ldots \rangle\) stands for integration over the dipole variables with the mesonic wave functions. This approach extends also to processes involving baryons, if one adopts for them a quark-antiquark picture [9].

At high energy the \(dd\) amplitude is given by the normalised connected correlator \(C_M\) of the Wilson loops (WL) running along the classical trajectories of the two dipoles, \(A^{(dd)}(s, b; \nu) = -C_M(\chi, b; \nu)\), with \(\chi \approx \log \frac{s}{m^2}\) the hyperbolic angle between the trajectories, and \(m\) the mass of the mesons (taken to be equal for simplicity).

The correlator \(C_M\) is obtained from the correlator \(C_E\) of two Euclidean WL at angle \(\theta\) (see Fig. 1),

\[
C_E(\theta, b; \nu) \equiv \lim_{T \to \infty} \langle W_{C_1} \rangle \langle W_{C_2} \rangle - 1,
\]

through the analytic continuation (AC) [10, 11]

\[
C_M(\chi, b; \nu) = C_E(\theta \rightarrow -i\chi, b; \nu).
\]

A more detailed discussion of the approach can be found in Ref. [5] and references therein. The Euclidean formulation has allowed the study of the relevant WL correlator by means of nonperturbative techniques, which include instantons [12, 13], the model of the stochastic vacuum [14], holography [15, 16, 17, 18, 19], and the lattice [13, 20, 21].

### 3. Large-\(s\) behaviour of \(\sigma_{\text{tot}}\)

The energy dependence of \(\sigma_{\text{tot}}\) is determined by the large-\(s\), large-\(b\) behaviour of the amplitude through the “effective radius” of interaction \(b_c = b_c(s)\), beyond which the amplitude is negligible, as \(\sigma_{\text{tot}} \propto b_c^2\).

To determine \(b_c\), in Ref. [5] we employed the following strategy. On the Euclidean side, we obtain information on the \(b\)- and \(\theta\)-dependencies of \(C_E\) by inserting between the WL operators (for large enough \(b\)) a complete set of asymptotic states, characterised by their particle content and by the momenta and spin of each particle. After AC this gives us information on the \(s\)- and \(b\)-dependencies. This requires two crucial analyticity assumptions:

1. AC can be performed separately for each term in the sum;
2. WL matrix elements are analytic in \(\theta\).

A few reasonable finiteness assumptions on the WL matrix elements are also made.

Under these assumptions, at large \(\chi, b\), the relevant Minkowskian correlator reads [5]

\[
C_M(\chi, b; \nu) \approx \sum_{\alpha \neq 0} f_\alpha(\nu) \prod_a [w_a(\chi, b)]^{n_a(\alpha)},
\]

where the sum is over (non-vacuum) states \(\alpha\), and \(n_a(\alpha)\) is the number of particles of type \(a\) in state \(\alpha\). Here

\[
w_a(\chi, b) = \frac{e^{i\alpha\chi - \beta m(\rho)}}{\sqrt{2\pi b m(\rho)}}, \quad \rho(\alpha) \equiv \frac{s(\alpha) - 1}{m(\alpha)},
\]

with \((s(\alpha), m(\alpha))\) spin and mass of particles of type \(a\), and \(f_\alpha\) are functions of the dipole variables only. Particles of type \(a\) contribute only for \(b \lesssim r(\alpha)\chi\), and so the effective radius of interaction is given by

\[
b_c(s) = \left(\max_a \rho(\alpha)\right) \log \frac{s}{m^2} \equiv \frac{1}{\mu} \log \frac{s}{m^2}.
\]

Here we assume the maximum to exist and to be positive. If it were zero or negative, \(\sigma_{\text{tot}}\) would be constant or vanishing at high energy. The spectrum is supposed to be free of massless states: in case they were present and with spin at most 1, the maximisation should be performed on the massive spectrum only [5].

Using Eq. 3 we find for \(\sigma_{\text{tot}}\)

\[
\sigma_{\text{tot}} \approx \sum_{s \to \infty} \left(1 - \kappa\right)[b_c(s)]^2 \approx \frac{2\pi}{\mu^2} \left(1 - \kappa \right) \log^2 \frac{s}{m^2},
\]

with \(|\kappa| \leq 1\) due to unitarity [5]. In general \(\kappa\) depends on the colliding hadrons. Analyticity and crossing symmetry [22, 23] requirements show that universality is most naturally achieved if \(\kappa = 0\), corresponding to a vanishing or oscillating correlator as \(\chi \to \infty\) at fixed \(b\).
4. Total cross sections from the hadronic spectrum

The total cross section satisfies the bound [5]

$$\sigma_{\text{tot}} \lesssim \frac{4\pi}{\mu} \log^2 \frac{s}{m^2} = 2B_{\text{th}} \log^2 \frac{s}{m^2}, \quad (7)$$

with $\mu^{-1} = \max_{a} r^{(a)}$ determined from the hadronic spectrum by maximising $r^{(a)}$ over the stable states of QCD in isolation, as electroweak effects have been neglected from the onset. Only states with $s^{(a)} \geq 1$ are considered, including nuclei (see Fig. 2).

Quite surprisingly, this singles out the $\Omega^+$ baryon, which yields $B_{\text{th}} \approx 0.56 \text{ GeV}^{-2}$. The resulting bound on $\sigma_{\text{tot}}$ is much more stringent than the original bound, and the prefactor is of the same order of magnitude of the experimental value $B_{\exp} \approx 0.69 \div 0.73 \text{ GeV}^{-2}$ [1]. Furthermore, our bound is not singular in the chiral limit [27].

If universality is achieved as discussed above in Section 3, then $\sigma_{\text{tot}}$ is entirely determined by the hadronic spectrum, and reads [5]

$$\sigma_{\text{tot}} \simeq \frac{2\pi}{\mu} \log^2 \frac{s}{m^2} = B_{\text{th}} \log^2 \frac{s}{m^2}. \quad (8)$$

This prediction for the prefactor is in fair agreement with $B_{\exp}$, taking into account a systematic error of order 10% on $B_{\exp}$, estimated by comparing the results of different fitting procedures [1, 28, 29, 30]. The same conditions leading to universal total cross sections also give universal, black-disk-like elastic scattering amplitudes [5].

Total cross sections are usually believed to be governed by the gluonic sector of QCD. However, in the quenched theory one finds from the glueball spectrum $B_Q \gtrsim 1.6B_{\exp}$, suggesting large unquenching effects [5].

5. Conclusions

In Ref. [5] we have derived the asymptotic, high-energy behaviour of hadronic total cross sections in the framework of the nonperturbative approach to soft high-energy scattering [6, 7, 8]. We find a “Froissart-like” behaviour $\sigma_{\text{tot}} \sim B \log^2 s$, with $B$ (mainly) determined by the hadronic spectrum (see Eqs. 7 and 8), and in fair agreement with experiments.

Our main results do not depend on the detailed description of the hadrons in terms of partons: adding gluons and sea quarks to the wave functions would lead to more complicated WLs, but since our argument is independent of their detailed form, our conclusions remain unchanged.

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