New modelling techniques for dependability. Case study for a mechanical process

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Abstract: Dependability - a measure of a system's availability, reliability, its maintainability and safety systems, is an integral part of the integrity of engineering design. From a functional perspective, dependability assumes satisfying some of the required functions and implies the correct definition of them. Thus, in the case of complex equipment, multi-variable functions can be generated whose variables are dependent on different states and operating modes. In recent years, automated continuous design techniques have been developed through intelligent computational methodologies to determine the dependability of systems. These techniques also introduce Artificial Intelligence (AI) in integrity engineering design and offer an overview and modelling in designing for reliability, availability, maintainability and system security. This paper combines quantitative research, mathematical and algorithmic modelling with evolutionary techniques. The combination of the four research techniques is done in a coherent application validated on a mechanical device and applicable to a wide range of technical systems. More specifically, the work proposes a comparative study of mathematical - linear and nonlinear models for correlation and optimization of maintainability and reliability values in relation to production and quality policies. These models will be complemented with specific AI techniques: neural and genetic algorithms that validate and optimize mathematical models. It identifies the metrics for the evaluation of the dependent variables (reliability, maintainability) and an independent variables (production and quality), these variables are observed over a given time horizon, their mathematical, neural and genetic modelling is performed to achieve correlations, optimization functions and model validation. The study proves to be a practical and applicative tool in the analysis of system integrity in general and operational safety, especially with a large generalization capacity on systems in many technical fields.

1. Introduction
The character of processes optimization is influenced to a great extent by their complex peculiarities, which implies the utilization of a big number of parameters that act independent or dependent at the same time. For this reason, the mostly used modelling are those of statistic type that can determine distribution for different parameters based of some practically observed values. Yet, the statistical analysis methods cannot solve a whole series of problems of technical and economical processes, due to their complexity from both static (technological) and dynamic (time evolution) standpoint. Accordingly, in parallel with statistic methods, other optimization methods have been developed, such as heuristic algorithms, exact optimization methods, evolutonal algorithms, neuronal networks, neuro-fuzzy algorithms.
The optimization of a technological process implies the initial realization of a mathematical model. When this cannot be determined precisely, either due to the big number of variables, or to incertitude elements characteristic to technological processes, one defines the optimization problem without searching for a mathematical model, and one applies approximate - heuristic or and evolutorial algorithms [1][2][3][4].

2. Material and methods
In order to solve an optimization problem in operational management, it is necessary to solve the following stages:
- **Definition of the target optimization problem.** Discrete optimization problems from the field of operational research can have an absolute or a relative character, depending on the implementation level of the utilized mathematical model. For example, if the mathematical model includes all the technical and economic aspects of a system, one can speak of an absolute optimum (in the case that such an optimum exists); otherwise, optimization leads to a relative optimum. At this stage, one establishes the objective of the economic problem, and then one identifies the input and output data specific to the economic problem and to relations between them.

*The target of this work is to optimize the reliability and maintenance functions of a mechanic system within the production and quality strategies system.*

- **Identify the technical system elements and choose the modeling techniques.** The models for technical processes simulations are of two types [5] [6]: procedural models which present a set of procedures (operations) to be executed for a certain technical system condition and mathematical models, which include one or more mathematical relations that interrelate the process parameters.

- **Data organization and gathering.** Follow up the behavior in exploitation of the production equipment represents one of the information sources for the estimation of the operational maintainability and reliability indices [7][8], and for the optimization of their maintenance. Recording and processing of the information generated by equipment accidental shutdown infer the creation of failure history. At first sight, the process of failure history creation can be considered as costly and time consumer, because it implies gathering an amount of data enough to allows a statistic processing. Yet, failure history can represent a working tool necessary and useful for the production system.

2.1. **Programmed experiment method to solve the problems of optimum (mathematical model)**
Taking into account technical processes’ multitude, variety and complexity, the methods of modeling through experimental programming and the optimization problems’ types are very different. For this reason, we shall only focus on problems of extreme, which implies the determination of extreme values of the objective function in terms of the variation of the factors of influence \( x_1, x_2, ..., x_n \)

\[ y = f(x_1, x_2, x_3, ..., x_n) \]

In this way, the **programmed experiment method** simplifies the determination of the objective function by reducing the number of experiments. The algorithm is as follows:
- initiate the initial value of the objective function (let this be the solution \( S_1 \))
- generate 4 experimental points around the solution \( S_1 \) in order to determine the gradient direction – the direction of modification of the parameter \( y \) in terms of \( x_1, x_2 \); thus, one determines a linear equation \( y = a_0 + a_1x_1 + a_2x_2 \)
- generate some experiments on gradient direction and retain an extreme value;
- around this extreme value, generate 4 experimental points and determine a new linear equation \( y = b_0 + b_1x_1 + b_2x_2 \) on the direction of optimization gradient of the objective function in terms of the two parameters \( x_1 \) and \( x_2 \);
- generate some experiments on the direction of the last equation until one obtains the optimum value of the objective function.

Even if the method of the programmed experiment does not guarantee the optimality, it is quite useful, due to the quality of the solutions obtained through small number of experiments that implies reduced calculus and time resources.
2.1.1. Choose the variables and objective function

For each variable, one identifies 4 levels: basic “0” (start point), superior “+1”, which represents a value = level 0 + the interval of variable variation, inferior “-1”, which represents a value = level 0 - the interval of variable variation and some level “+1.4”, which represents a certain value different from the values from levels 0, +1, -1.

The utilization of nonlinear models is useful, because most of times the linear models cannot determine the optimum solution, as gradient directions cannot refine enough the solutions zone. These (nonlinear models) introduce additional searches within the neighboring zone of the local extreme points. Most of times, the nonlinear equation is a second order polynomial of the form:

\[ y = b_0 + \sum_{i=1}^{k} b_i x_i + \sum_{i=1}^{k} b_{ij} x_i x_j + \sum_{i=1}^{k} b_i x_i^2 \]  \hspace{1cm} (1)

where \( k \) is the number of factors.

2.1.2. Experimental compound rotatable central model

This model (chosen in this work) allows for a uniform information distribution within the multifactor space. In this modelling, a special importance belongs to the choice of experiences number in the centre of the experiment, \( n_0 \), because these determine the distribution character within the solution space. One can ensure a uniform programming of the experiments, which implies their equal distribution in relation with the centre. In this way, one determines the number of experiences \( N = 2^k + 2k + n_0 \), where \( k \) is the number of factors.

The coefficients of the equation (1) are determined through the least squares method.

2.2. Neural technique

Neural modelling technique is a bio-inspired technique in which, following a learning process, a neural structure can provide values for the output variables of a system for new values of the input variables. In this paper we will use a neural network with supervised learning because we have known values for output variables (MTBF and MTTR) for input patterns (production, quality).

2.3. Genetic algorithm

Genetic algorithms are useful tools in designing methods for discrete optimization issues. Their character, searching in the whole solution space, is searched for optimization problems whose functions are strongly nonlinear, thus avoiding the algorithm stopping in a local optimum. In this paper I used the genetic algorithms for determining the coefficients of the regression equation.

3. Determination of operating regime through statistic methods

Because the quantitative indices concerning the reliability follow statistic distribution laws, their statistical analysis becomes necessary [8], [9]. Thus, the reliable operating time follows an exponential distribution, entering the normal operating regime can be validated by applying the statistic test \( H_i^2 \), and the reliability function is defined statistically [10]. One tests the hypothesis, according to which the distribution function of the reliable operating time is exponential, and then one estimates the confidence intervals and the inferior unilateral limit of the mean operating time until the occurrence of the first failure. It is worth mentioning that confidence intervals’ testing and estimation have been performed based on recorded data, considering that data number is sufficient to consider the volume sampling big enough, such that to provide a reliable interval level for a statistic reliability of 95%. One has taken also into account that at this machine have been ensured the conditions for operating in continuous regime, without shutdowns due to organizing measures, or production downtimes. As the result of processing the data from table 1, one obtains a sequence of numbers that represent operating times until the failures appear, that can be set in order as follows:
Table 1. Reliable operating time (s).

| Lower limit | Upper limit | Absolute frequency |
|-------------|-------------|--------------------|
| 37500       | 327300      | 54840              | 1362900         |
| 5400        | 86400       | 784800             | 406500          |
| 225000      | 129600      | 320340             | 0               |
| 37800       | 34500       | 0                  | 151200          |
| 3480        | 6720        | 127920             | 233940          |
| 318960      | 40020       | 144000             | 0               |
| 412440      | 174900      | 0                  | 727500          |
| 143460      | 27600       | 0                  | 273300          |
| 50100       | 34800       | 3720               | 302880          |
| 13860       | 12900       | 123300             | 238200          |
| 72060       | 28680       | 307200             | 239100          |
| 0           | 114300      | 204900             | 451080          |
| 29520       | 0           | 222900             | 433200          |
| 15960       | 0           | 45420              | 207240          |
| 35580       | 239220      | 89400              | 207600          |
| 219420      | 57300       | 85980              | 305100          |
| 17760       | 385800      | 60300              | 84600           |
| 71460       | 697500      | 89340              | 108000          |
| 27780       | 975660      | 153600             | 38580           |
| 123060      | 27540       | 158400             | 67860           |

Table 2. Reliable operating time – orderly ascending (min).

| Lower limit | Upper limit | Absolute frequency |
|-------------|-------------|--------------------|
| 0           | 478         | 1440               | 3715            | 12125          |
| 0           | 492         | 1489               | 3750            | 13080          |
| 0           | 575         | 1490               | 3899            | 16261          |
| 0           | 580         | 1800               | 3970            | 22715          |
| 0           | 593         | 1905               | 3985            | 22715          |
| 0           | 625         | 2051               | 3987            | 22715          |
| 0           | 630         | 2055               | 4555            | 22715          |
| 0           | 643         | 2132               | 5048            | 22715          |
| 58          | 667         | 2160               | 5085            | 22715          |
| 62          | 757         | 2391               | 5120            | 22715          |
| 90          | 835         | 2400               | 5316            | 22715          |
| 112         | 914         | 2520               | 5339            | 22715          |
| 215         | 955         | 2560               | 5455            | 22715          |
| 231         | 1005        | 2640               | 6430            | 22715          |
| 266         | 1131        | 2915               | 6775            | 22715          |
| 296         | 1191        | 3415               | 6874            | 22715          |
| 459         | 1201        | 3454               | 7220            | 22715          |
| 460         | 1410        | 3460               | 7518            | 22715          |
| 463         | 1433        | 3657               | 11625           | 22715          |

The appearance of null elements in the above table derives from the fact that while repairing a failure, appeared another failure that needs to be repaired.

The operating times until the first failure appears are presented in increasing order in the table 2. By eliminating the null times (failures appeared when the machine does not work) and the last value, considering it as a deviation from machine normal operation, one finds a series of 71 numbers, such that it is sufficient to divide the series in 7 classes of values $k = 7). According to these values, one considered the width of group interval $\Delta t$ as: $\Delta t = (16261 - 58) / 7 = 2315 \text{ (min)}$

Table 3. Descriptive statistic for probability density function.

| Class | Lower limit | Upper limit | Absolute frequency | Relative frequency | Interval centre | Theoretical points |
|-------|-------------|-------------|--------------------|--------------------|-----------------|------------------|
| 1     | 58          | 2373        | 39                 | 0.55               | 1215            | 0.52             |
| 2     | 2373        | 4687        | 17                 | 0.24               | 3530            | 0.24             |
| 3     | 4687        | 7002        | 9                  | 0.13               | 5845            | 0.11             |
| 4     | 7002        | 9317        | 2                  | 0.03               | 8160            | 0.05             |
| 5     | 9317        | 11632       | 1                  | 0.01               | 10474           | 0.02             |
| 6     | 11632       | 13946       | 2                  | 0.03               | 12789           | 0.01             |
| 7     | 13946       | 16261       | 1                  | 0.01               | 15104           | 0.00             |
| Total |             |             | 71                 |                    |                 |                  |

The reliability of the operation is ordered in ascending order.
Figure 1. Probability density function – experimental and theoretical points.

Suppose the distribution function of the operating time until the failure is the exponential,

\[ F(t) = 1 - e^{-\lambda t} \]  

where we denoted with \( \lambda \) the failure intensity, that was estimated with the relation \( \lambda = \frac{1}{\bar{t}} \) where \( \bar{t} \) represents the average of the reliable operation times. Accordingly, \( F(t) = 1 - e^{-0.00034t} \).

In order to check up the hypothesis according to which the distribution function of the reliable operation time up to failures in exponential, we have determined the variation coefficient:

\[ V = \frac{\sigma}{\bar{t}} = \sqrt{\frac{\sum (t_i - \bar{t})^2}{n-1}} = N_{\text{c}}^{-1} = 1.09(3) \]

where \( n= \) total numbers of shutdowns.

Because \( V \) is approximately equal to unit, it follows that the experimental data satisfy the exponential distribution law. Within this law, the probability density function is expressed by the relations:

\[ f(t)_{\text{exp}} = \frac{n_{ci}}{N_c} \text{, } i = 1, \ldots, k \text{ si } f(t)_{\text{theor}} = \lambda e^{-\lambda t} \]  

where: \( n_{ci} \) – number of failures per the interval, \( N_c \) – number of total failures, \( \lambda \) – failures intensity, \( k \) – number of intervals.

Let \( f(t) \) = differential function or density of reliability distribution in the case of the exponential law. and \( R_{\text{theor}}(t) = e^{-\lambda t} \) the function of theoretical reliability distribution.

The reliability function in the case of exponential distribution law is: \( R(t) = e^{-\frac{1}{\bar{t}}} \) and its experimental and theoretical values are calculated and summarized in table 4, by applying the following relations:

\[ D_{\text{exp}}(t) = \sum_{i=1}^{\text{int}} f(t_i), D(t) = \text{integral function}, R_{\text{exp}}(t) = 1 - D_{\text{exp}}(t), R_{\text{theor}}(t) = e^{-\lambda t}, D_{\text{theor}}(t) = 1 - R_{\text{theor}}(t) \]

| Class | Interval | Interval center | \( n_{ci} \) | \( f(t)_{\text{exp}} \) | \( f(t)_{\text{theor}} \) | \( D_{\text{exp}}(t) \) | \( R_{\text{exp}}(t) \) | \( R_{\text{theor}}(t) \) | \( D_{\text{theor}}(t) \) |
|-------|----------|----------------|------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1     | 58       | 2373           | 1215       | 0.55          | 0.52          | 0.55          | 0.45          | 0.66          | 0.34          |
| 2     | 2373     | 4687           | 3530       | 0.24          | 0.24          | 0.79          | 0.21          | 0.30          | 0.70          |
Figure 2. The functions of reliability and failures distribution (theoretical and experimental).

In figure 2, one can notice that the theoretical distributions are “followed” almost exactly by the experimental values.

4. Determination of the mathematical model
Table 5 summarizes the quantitative indices according to the section Data organization and gathering.

Table 5. Determination of quantitative indices concerning the system reliability.

| Stops | Shutdown time [min] | Operational time | Pieces/h | Pieces/week | Ra [microM] | MTBF | MTTR |
|-------|---------------------|------------------|----------|-------------|-------------|------|------|
| w1    | 5                   | 76               | 10004    | 65          | 10838       | 0.011| 2001 | 15   |
| w2    | 3                   | 401              | 9679     | 75          | 12099       | 0.123| 3226 | 134  |
| w3    | 1                   | 125              | 9955     | 105         | 17421       | 0.234| 9955 | 125  |
| w4    | 2                   | 37               | 10043    | 88          | 14730       | 0.055| 5022 | 19   |
| w5    | 2                   | 155              | 9925     | 85          | 14060       | 0.063| 4963 | 78   |
| w6    | 1                   | 45               | 10035    | 108         | 18063       | 0.244| 10035| 45   |
| w7    | 5                   | 78               | 10002    | 67          | 11169       | 0.013| 2000 | 16   |
| w8    | 1                   | 35               | 10045    | 105         | 17579       | 0.231| 10045| 35   |
| w9    | 0                   | 0                | 10080    | 114         | 19152       | 0.321| 10080| 0    |
| w10   | 2                   | 625              | 9455     | 108         | 17019       | 0.212| 4728 | 313  |
| w11   | 6                   | 231              | 9849     | 66          | 10834       | 0.012| 1642 | 39   |
| w12   | 3                   | 80               | 10000    | 77          | 12833       | 0.131| 3333 | 27   |
| w13   | 5                   | 470              | 9610     | 81          | 12974       | 0.015| 1922 | 94   |

Definition of the experimental matrix
The independent parameters of the considered technological process are:
- \( x_1 \) – real values of the honing machine production
- \( x_2 \) – quality represented through roughness values measured on honing machine parts [\( \mu \)]
Table 6 presents synthetically the coded and real values of the independent parameters.
Table 6. The real and coded values of independent parameters.

| Code | Independent param. |
|------|---------------------|
| -1,414 | x1 – Production (P) |
| -1 | 10834 |
| 0 | 11169 |
| +1 | 14060 |
| +1,414 | 17579 |
| x2 – Quality (Q) | 0.321 |
| | 0.244 |
| | 0.123 |
| | 0.055 |
| | 0.011 |

Table 7 presents the experimental matrix, based on the variation levels of the two independent parameters, with 13 experimental variants [13].

Table 7. The experimental matrix.

| Experimental variants | Independent variables | MTBF |
|-----------------------|-----------------------|------|
|                       | Code | Real | Code | Real |       |
| 1.                    | -1   | 11169 | -1   | 0.244 | 10045 |
| 2.                    | 1    | 17579 | -1   | 0.244 | 9955  |
| 3.                    | -1   | 11169 | 1    | 0.055 | 3226  |
| 4.                    | 1    | 17579 | 1    | 0.055 | 10035 |
| 5.                    | -1.414 | 10834 | 0    | 0.123 | 1922  |
| 6.                    | 1.414 | 19152 | 0    | 0.123 | 10080 |
| 7.                    | 0    | 14060 | -1.414 | 0.321 | 3333  |
| 8.                    | 0    | 14060 | 1.414 | 0.011 | 5022  |
| 9.                    | 0    | 14060 | 0    | 0.123 | 4963  |
| 10.                   | 0    | 14060 | 0    | 0.123 | 2001  |
| 11.                   | 0    | 14060 | 0    | 0.123 | 4728  |
| 12.                   | 0    | 14060 | 0    | 0.123 | 1642  |
| 13.                   | 0    | 14060 | 0    | 0.123 | 1642  |

Value and significance of the regression equation coefficients
The regression equation coefficients determined through the least squares method are written in table 8. After eliminating the insignificant factors, the final form of the regression equation becomes:

\[ y = 3068 + 2281.8x_1 + 2272.5x_1^2 + 1361x_2^2 + 1724.8x_1x_2 \]

Table 8. Regression equation coefficients.

| Coefficient | Statistics \( t_c \) | Statistics \( t_{\text{tab}}(\alpha, \nu) \) | Significance |
|-------------|----------------------|---------------------------------|--------------|
| \( b_0 \)   | 3068.0               | 4.202                           | significant  |
| \( b_1 \)   | 2281.8               | 3.9536                          | significant  |
| \( b_2 \)   | -543.8               | 0.9423                          | insignificant |
| \( b_{11} \)| 2272.5               | 3.671                           | significant  |
| \( b_{32} \)| 1361.0               | 2.1986                          | significant  |
| \( b_{12} \)| 1724.8               | 2.11                            | significant  |

Statistical validation of the model
Statistical validation of the model is performed through two criteria: Fisher (a) and (b).

The mathematical model is statistically adequate, the two conditions being satisfied:

\( F_c = 3.0695, \quad F_{\text{tab}}(0.05;4;4) = 6.39, \quad \text{condition}\ F_c < F_{\text{tab}} \) is satisfied.

\( F_c = 16.4, \quad F_{\text{tab}}(0.05;12;12) = 2.69, \quad \text{condition}\ F_c > F_{\text{tab}} \) is satisfied.
The presence of the quadratic terms from the mathematical model means that the reliability depends on production and quality, the difference being given by the absence of the coefficient $x_2^2$, which implies a bigger influence of the production on reliability, as compared to that of the quality. The presence of the term $x_1x_2$ leads to nonlinearity of the response $F$ in term of the P/Q plane.

With the view to convert the regression equation to the standard form, the coordinates of the new centre are: $x_{1n} = -0.6610$ and $x_{2n} = 0.4188$, with a minimum computed value of $2.3139e+003$. The angle of axes rotation is $\alpha = 31.0725^\circ$, with the new coordinates plane $(Z_1, Z_2)$ has the form:

$$Y - 2313.9 = 2972.1*Z_1^2 + 841.3352*Z_2^2$$

The coefficients B11 and B22 have the same signs, therefore the surface has the form of a parabola rotated by $31.0725^\circ$ in the plane $(Z_1, Z_2)$.

**Figure 3.** The response surface $MTBF=f(P, Q)$.

**Figure 4.** The response surface $MTTR=f_2(P, Q)$.

### Mathematical model for defining the maintenance in terms of P/Q plane

The definition of the experimental matrix preserves the hypotheses considered in previous section. Table 9 presents the maintainability values measured by measuring the parameter MTTR; $x_1$ codifies the independent parameter “Production”, $x_2$ codifies the independent parameter “Quality”, and Mm represents the measured values of the parameter “Maintainability” for the considered process.

| $x_1$ | -1  | 1   | -1  | 1   | -1.414 | 1.414 | 0    | 0    | 0    | 0    | 0    | 0    |
|-------|-----|------|-----|-----|--------|-------|------|------|------|------|------|------|
| $x_2$ | -1  | -1   | 1   | 1   | 0      | 0     | -1.414| 1.414| 0    | 0    | 0    | 0    |
| Mm    | 0   | 15.2 | 125.0| 133.7| 15.6   | 26.7  | 18.5 | 312.5| 77.5 | 45   | 35   | 38.5 | 94   |

**Table 9.** The measured values of the parameter maintainability.

### Value and significance of regression equation coefficients

Table 10 presents the coefficients of the model whose mathematical expression is given in relation 1, determined with the least squares method, as well as their significance.

| Coefficient | Statistics $t_c$ | Statistics $t_{tab}(\alpha, \nu)$ | Significance |
|-------------|------------------|----------------------------------|--------------|
| $b_0$       | 58.0225          | 4.9441                           | significant  |
| $b_1$       | 4.9494           | 0.5335                           | insignificant|
| $b_2$       | 82.4020          | 8.8816                           | significant  |
| $b_{11}$    | -24.6449         | 2.4766                           | significant  |
| $b_{22}$    | 47.5083          | 4.7742                           | significant  |
| $b_{12}$    | -1.6250          | 0.1238                           | insignificant|

After eliminating the insignificant factors, the final form of the regression equation becomes:
\[ y = 58.0225 + 82.402 x_2 - 24.6449 x_1^2 + 47.5083 x_2^2 \]

**Statistical validation of the model**

The mathematical model is statistically adequate, the two conditions being satisfied:

a) \( F_c = 2.52588 \), \( \text{F}_{\text{tab}(0.05;4;4)} = 6.39 \), the condition \( F_c < \text{F}_{\text{tab}} \) is satisfied.

b) \( F_c = 10.6975 \); \( \text{F}_{\text{tab}(0.05;12;12)} = 2.69 \); the condition \( F_c > \text{F}_{\text{tab}} \) is satisfied.

The presence of the quadratic terms from the mathematical model means that the maintainability depends nonlinearly on production and quality. The absence of \( x_1 \) element means that the weight of “quality” parameter in the value of maintainability parameter is bigger than the weight of “production” parameter (figure 3).

In order to convert the regression equation to standard form, the coordinates of the new systems are \( x_{\text{new}} =0 \) and \( x_{\text{new}} = -0.8672 \), with a computed value of 23.8893. The axes rotation angle is \( \alpha = 0^\circ \).

The canonical equation for the analysed parameter in the new coordinate system \((Z_1, Z_2)\) has the form:

\[ Y \cdot 23.8893 = 47.15 * Z_1^2 - 27.3221 * Z_2^2 \]

The response surface has the same shape as that for reliability, with the difference that the value of \( Y \) parameter increases when moving along the direction \( Z_d(Q) \) (figure 4).

**5. Neural modeling**

For this type of correlation, a neural network with 2 internal layers, a feed-forward network with flexible neurons (2 ... 20) with sigmoidal transfer function and linear output neurons was used. The network drive algorithm is Levenberg-Marquardt backpropagation [11], [12].

To increase the generalization of the neural network, the method of delaying the stopping moment of the training step was used. Thus, the set of vectors consisting of 23 drive paths with Production, Quality (network inputs) and MTBF and MTTR - network outputs is divided into 3 sets: the first set contains the vectors of training (the cardinal of the set is fixed to 70% of the plurality of vectors) - these vectors are used to modify the weights and activation thresholds of the network, the second contains the validation vectors (15%) used to validate network generality and stop the training before a super-matching - validation error is monitored over the entire network learning period; thus it decreases in the first part of the training process and then has a tendency to increase; when the validation error increases for a set number of iterations then the drive process is stopped. The last set of patterns, comprising 15% of the entrainment vectors, is used to test the generality of the network - this set is not used in the learning phase, its role being to compare other neuronal network behavior with other pattern patterns. The neural network was created following the assumptions provided in the previous section and was implemented in the Matlab R2008b (Neural Network Toolbox) [13][14].

The neural network sets are:

\[ \text{P} = \{10838, 2099, 17421, 14730, 14060, 18063, 11169, 17579, 19152, 17019, 10834, 12833, 12974, 11461, 16875, 13972, 16118, 14786, 14422, 8234, 10175, 8295, 14426 \} \]

\[ \text{Q} = \{0.011, 0.123, 0.234, 0.055, 0.063, 0.244, 0.013, 0.231, 0.321, 0.212, 0.012, 0.131, 0.015, 0.142, 0.221, 0.18, 0.254, 0.244, 0.021, 0.012, 0.016, 0.011, 0.192 \} \]

\[ \text{MTBF} = \{2001, 3226, 9955, 5022, 4963, 10035, 2000, 10045, 10080, 4728, 1642, 3333, 1922, 2456, 10025, 4990, 9868, 9857, 4973, 817, 816, 854, 4975 \} \]

\[ \text{MTTR} = \{15.2, 133.7, 125.0, 18.5, 77.5, 45, 15.6, 35, 0, 312.5, 38.5, 26.7, 94.0, 64.0, 55.0, 50.0, 212.0, 223.0, 67.0, 99.7, 100.2, 62.7, 65.5 \} \]

Since the 10-neuron network led to a satisfactory regression coefficient \( R = 0.93172 \) (figure 6) and \( R = 0.98323 \) (figure 7), it was decided to keep the structure of the neural network and its training.
The performance of the neural network in the training phase is shown in Figure 5. After 10 iterations, the values of the internal structures of the network stabilized, with optimal performance at iteration 4. The training process is stopped when the validation error does not show a decrease for 3 iterations. Figure 5 shows training, validation, and test errors. From this figure, we can conclude that the results of the training process are good because the final average square error is small, and the three graphs have a similar pattern after the 10th stage of training.

The values of the regression curves for the two outputs are shown in Figures 6 and 7:

- **Figure 6.** The regression curve of y1 output.
- **Figure 7.** The regression curve of y2 output.

The response surface for the MTBF parameter based on P/Q values in the neural network usage stage is shown in Figure 8. New test patterns [Production, Quality] were generated with randomly generated data in the value range for production and quality measured in the technical process. We observe the more irregular form compared to the surface obtained in Figure 3 due to the fact that the neural network is not limited by a polynomial function with a certain degree, the exit found by the number of layers, the number of drive patterns, transfer functions, and training thresholds. Figure 9 shows the level curves, the red ones representing the maximum values obtained for the high values of the parameter P, and dark blue values, minimum values obtained for relatively small values of the parameter P on a wide range of Q parameters.
Figure 10 shows the response surface for the MTTR parameter based on P/Q values in the neural network usage phase. The same patterns were recorded for MTTRs. We also observe the more irregular form compared to the surface obtained in figure 4.

![Figure 10](image_url)

**Figure 10.** The response surface MTTR=f(P,Q).

![Figure 11](image_url)

**Figure 11.** MTTR=f(P,Q) - level curves.

From figure 10 we can see that the MTTR parameter reaches large values for a high level of parameter Q over the entire value range of parameter P. The level curves in figure 11 confirm that the dependence of the parameter Q is strong [17].

### 6. Genetic modeling

The results of genetic modeling (using the evolutionary Solver toolbox) are presented in table 11, where \(a_0, b_1, b_2, c_1, c_2, d_1\) are the coefficients of the equation \(y=a_0+b_1x_1+b_2x_1^2+c_1x_1+c_2x_2^2+d_1x_1x_2\). GA settings are: stop criteria, \(\varepsilon=0.000001\), mutation_rate=0.25, crossover_rate=0.5, population_size=300, maximum time without improvement, 30s.

**Table 11.** Results of GA and regression coefficients for GA modelling.

| Production | Quality | Measured MTBF | Computed MTBF | Dif in abs. val | A [%] | Regression coefficients |
|------------|---------|---------------|---------------|----------------|-------|------------------------|
| 10838      | 0.011   | 2001          | 1799          | 202            | 10.1  | d1 0.31                |
| 12099      | 0.123   | 3226          | 3028          | 198            | 6.1   | c2 -2                   |
| 17421      | 0.234   | 9955          | 9092          | 863            | 8.7   | c1 2                    |
| 14730      | 0.055   | 5022          | 5031          | 9              | 0.2   | b2 0                    |
| 14060      | 0.063   | 4963          | 4420          | 543            | 10.9  | b1 -0.6                |
| 18063      | 0.244   | 10035         | 10035         | 0              | 0.0   | a0 1926                 |
| 11169      | 0.013   | 2000          | 2000          | 0              | 0.0   |                        |
| 17579      | 0.231   | 10045         | 9290          | 755            | 7.5   |                        |
| 19152      | 0.321   | 10080         | 12106         | 2026           | 20.1  |                        |
| 10834      | 0.012   | 1642          | 1800          | 158            | 9.6   |                        |
| 12833      | 0.131   | 3333          | 3632          | 299            | 9.0   |                        |

It can be observed that approximation amplitude values by GA are within the range of [0, 20]%, greater than those provided by mathematical modeling [0, 10]%.
7. Conclusions and future research directions
The paper combines three modeling techniques to solve an optimization problem from the operational management applied to a case study - mechanical system. Mathematical modeling provides a second non-linear regression equation, and the response surface is validated by other modeling techniques belonging to the artificial intelligence: genetic algorithms and neural networks. It is worth noting that the optimization problem is not an extreme one, but one that determines two values of Safety in Operation (Reliability and Maintainability) according to production and quality policies. The tool is valuable in terms of results confirmed by different optimization techniques.
A future research direction is to determine extreme values by transforming the correlation problem into a discrete optimization problem of min/max.

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