Effective degrees of freedom of the Quark-Gluon Plasma

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Abstract

The effective degrees of freedom of the Quark-Gluon Plasma are studied in the temperature range \( \sim 1 - 2 T_c \). Employing lattice results for the pressure and the energy density, we constrain the quasiparticle chiral invariant mass to be of order 200 MeV and the effective number of bosonic resonant states to be at most of order \( \sim 10 \). The chiral mass and the effective number of bosonic degrees of freedom decrease with increasing temperature and at \( T \sim 2 T_c \) only quark and gluon quasiparticles survive. Some remarks regarding the role of the gluon condensation and the baryon number-strangeness correlation are also presented.

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Quantum Chromodynamics (QCD) predicts that at extremely high temperatures matter consists of a gas of weakly interacting quarks and gluons, the Quark-Gluon Plasma (QGP). However at moderate temperatures \( T = 1 - 2 T_c \), \( (T_c = 170 \text{ MeV deconfinement temperature}) \) it is less clear what the dynamical phase is.

The experimental data obtained at the Relativistic-Heavy-Ion-Collider (RHIC), with the measurement of the \( p_t \) spectra and the related indications on the radial and elliptic flow (see [1] for a review), clearly suggest that at moderate temperatures the produced system is in a strongly interacting phase (sQGP) and there are remnant of the confining interaction up to temperatures \( \sim 2 T_c \), in agreement with lattice (lQCD) results. Actually lattice calculations of the pressure and energy density of the system do not reach the Stefan-Boltzmann values for a weakly interacting quark-gluon plasma even at very large temperatures \( \sim 5 T_c \) [2, 3].

This surprising picture calls for understanding the relevant degrees of freedoms to describe such a phase and, in this respect, several models have been proposed, where deconfinement and chiral symmetry restoration occur at a lower temperature than the \( q\bar{q} \) dissociation temperature and “resonance” states may play an important dynamical role [4, 5, 6, 8, 9].

However, recent lattice and phenomenological analyses [10] have shown that the emerging degrees of freedom are quark and gluon quasiparticles and this result has to be compatible with the survival of light \( \bar{q}q \) states [2, 11] as obtained by the analysis of mesonic spectral functions above \( T_c \).

In this letter we address the previous puzzling aspects by performing a phenomenological analysis of the pressure and of the energy density of the system taking into account the presence of quark and gluon quasiparticles as well as of \( \bar{q}q \) states in the temperature range \( 1 - 2 T_c \).

Let us consider a system of quark, antiquark, gluons and correlated particle states.

The gluonic sector contains quasiparticle contributions as well as non perturbative condensation effects. The ratio, \( c_e(T) \), between the chromo-electric condensate evaluated at finite temperature and at \( T = 0 \) strongly decreases for temperature \( T > 1.2 T_c \), whereas the same ratio for the magnetic condensate is \( \sim 1 \) up to larger temperatures [12]. Since at \( T = 0 \) the chromo-electric and chromo-magnetic parts are equal, one can write, in the considered temper-
ature range, the gluon condensate as
\[ < \frac{\alpha_s}{\pi} G^a_{\mu \nu} G^{a\mu\nu} >_{T=\infty} = \frac{1}{2} < \frac{\alpha_s}{\pi} G^a_{\mu \nu} G^{a\mu\nu} >_0 [1+c_q(T)] \]
(1)
where the ratio \( c_q(T) \) can be approximated by the unquenched data of Ref. [12] and we take \( < \frac{\alpha_s}{\pi} G^2 >_T \approx 0.01 \text{ GeV}^4 \), consistently with QCD sum rules [13].

The gluon condensate, i.e. a macroscopically populated state with zero momentum, does not essentially contribute to the pressure but is crucial for the evaluation of the energy density [14]. On the other hand, the gluonic sector contains also gluon quasiparticles which contribute to the pressure and to the energy density and that we shall treat in a phenomenological way (see below).

Concerning the fermionic sector, we assume that the number of quark/antiquark degrees of freedom is fixed, \( D_q = D_{\bar{q}} = 18 \). Recently some dispersion relations of the general form
\[ \omega_q(k) = \sqrt{k^2 + m^2 + \Sigma_R} \]
have been proposed in [6, 15] where \( m \) and the self energy \( \Sigma_R \) have been evaluated taking into account the interaction of the quasiparticles with the medium. For the relevant momenta, of the order of the thermal momentum, we consider that \( m/k \) is small [6] and treat the chiral invariant term \( \Sigma_R \) as a constant parameter \( M \) by using the dispersion relation
\[ \omega_q(k) = \omega_q(k) = k + M. \]
(3)
Therefore, at this level of approximation, we neglect the dynamical phenomena related to frequencies with \( \omega/T \ll 1 \), such as viscosity.

The structure of the in-medium correlated states as a function of temperature is not easily evaluated. These states may describe \( \bar{q}q \) states as well as more exotic states [6]. Close to \( T_c \), it should be reasonable to consider that the number of correlated state degrees of freedom \( D_b \) is of order 10, which corresponds to the pseudoscalar nonet. However in our analysis we will treat \( D_b \) as a parameter indicating that an effective number of bosonic states is present (see below). In the following we will neglect, as a first approximation, the effect on the thermodynamics quantities of the width of the bosonic states. Therefore we employ the dispersion law \( \omega_b = \sqrt{k^2 + 4M^2} \).

Finally, in the present approach, the interaction of the gluonic sector with fermions and correlated states is described, in a mean field-like treatment, by the effective mass \( M \) and \( D_b \) which we will be evaluated employing unquenched lattice data of pressure and energy density.

Therefore our expressions of pressure and energy density are given by the sum of the contributions of quarks, antiquarks, bosonic pairs and gluons:
\[ P(T) = \sum_{i=q,\bar{q}} D_i \int \frac{d^3k}{(2\pi)^3} \frac{k^2 f_i(k)}{\omega_i(k)} + P_g(T), \]
(4)
\[ \epsilon(T) = \sum_{i=q,\bar{q},b} D_i \int \frac{d^3k}{(2\pi)^3} \omega_i(k) f_i(k) + \epsilon_g(T) + \epsilon_{\text{con}}(T), \]
(5)
where \( f_i(k) = f_q(k) \) is the Fermi distribution law, \( f_b(k) \) is the Bose distribution law, \( P_g(T) \) and \( \epsilon_g(T) \) are respectively the contributions to the pressure and energy density due to gluon quasiparticles and \( \epsilon_{\text{con}}(T) \) is the gluonic condensate given in Eq. (1).

In order to evaluate \( M \) and \( D_b \) we have to perform a simultaneous fit of the data of Refs. [2, 3] as a function of the temperature employing Eqs. (4), (5) and (1).

As suggested by quenched data, the contribution, \( \epsilon_g \), to the energy density is small with respect to the gluon condensate and also the effect of \( P_g \) on the pressure of the whole system (i.e. including fermions and correlated states) is expected to be small [2].

Therefore, as a first step, let us assume that \( P_g = \epsilon_g = 0 \). As we shall see, this approximation gives an upper limit on the effective degrees of freedom above \( T_c \). Moreover, we will numerically study how different values of \( P_g \) and \( \epsilon_g \) may affect our final results.

The result of the combined, \( P(T) - \epsilon(T) \), analysis, for \( P_g = \epsilon_g = 0 \), are shown in the plots of Fig. [1] full (blue) lines.

From the upper panel of Fig. [1] one sees that also when the gluon quasiparticles are turned off,
FIG. 1: Effective number of bosonic degrees of freedom (upper panel) and quasiparticle chiral mass (lower panel) as a function of the temperature for $T = 1.2 - 2 T_c$. Full (blue) lines correspond to $P_b = \epsilon_g = 0$; dashed (red) lines correspond to pressure and energy density evaluated with $m_g = 1.0$ GeV and dotted (green) lines to $m_g = 0.9$ GeV.

i.e. one is artificially increasing the number of correlated pairs, the effective number of bosonic degrees of freedom, in the range $T = 1.2 - 2 T_c$, remains of order 10, suggesting that essentially the states of the mesonic nonet are present.

The chiral invariant mass of the quark quasiparticles, shown in the lower panel of Fig. 1 turns out to be a decreasing function of the temperature suggesting that the mechanism which determines the chiral mass becomes less efficient as the temperature increase. It is interesting to note that the decreasing of the chiral mass as a function of the temperature determined with this approach is in qualitative agreement with the one determined in Ref. [10] with a different method.

Notice that the previously obtained value of $D_b$ has to be considered as an upper limit to the effective number of correlated degrees of freedom. Indeed, if in Eqs. [11] and [15] one switches on gluons, that is if one includes the contributions of the gluon quasiparticles to the pressure and to the energy density, these terms reduce the weight of the fermions and of the correlated bosonic states. To check numerically this effect the same unquenched lattice results have been fitted with $P_g$ and $\epsilon_g$ calculated with different gluon quasiparticle masses $m_g$ and for $D_g = 24$ gluonic degrees of freedom. The results are reported in Fig. 1 which shows that increasing the values of $P_g$ and $\epsilon_g$, i.e. employing different values of $m_g$, $D_b$ and $M$ drastically decrease$^1$.

Therefore our analysis of the unquenched lattice data strongly constraints the number of correlated states and of the quasiparticle masses.

Few comments are now in order. Let us notice that the value of the mass of the quasiparticles that we obtain is smaller than the one obtained in Ref. [16] or in Ref. [17] where $M$ is estimated to be $\sim 3 - 4 T$ from fits of lattice data. This difference essentially relies on the fact that we have considered the dispersion law of Eq. (3) with a chirally invariant mass, whereas in [16] and [17] the quasiparticle dispersion law has been parameterized as $\omega_q = \sqrt{k^2 + M^2}$. Moreover in Ref. [16], in order to reproduce the lattice results, a (small) bag constant has been employed. In our case the contribution of the gluonic condensate and of the mesonic resonant states play a crucial role in determining the correct values of pressure and energy density.

Finally let us comment on the correlation between baryon number and strangeness ($C_{BS}$) as an indication of the effective dynamical degrees of freedom of the system. The analysis of lQCD results performed in [10] indicates that at $1.5 T_c$ the BS correlation is very close to 1.

We can estimate, in an admittedly rough way, the BS correlation as

$$C_{BS} \sim \frac{\frac{1}{2}D_f < n_f >}{\frac{1}{2}D_f < n_f > + \frac{4}{9}D_b < n_b >},$$

(6)

where $< n_b >$ ($< n_f >$) is the number density of

$^1$ The bump in $D_b$ at $T \sim 1.4 T_c$ is due to have considered a thermal independent gluon mass in $P_g$ and $\epsilon_g$. Indeed, in Ref. [16], a minimum in $m_g(T)$ is found at approximately the same value of $T$. 

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bosonic (fermionic) states, the coefficient $D_f/3$ takes into account that in the chiral symmetric limit one third of fermions are strange, whereas the coefficient $4/9D_b$ is an effective way to weight the number of strange boson in $D_b$ according to the meson nonet.

Employing the data of Fig. 1 at $T = 1.5 T_c$ it turns out that the correlation is about 0.95 for $m_g = 1.0$ GeV. Using smaller values of $m_g$ the correlation further increases. Moreover in every case considered at $T = 2.0 T_c$ the correlation is in any cases $\simeq 1$.

In conclusion, according to the present work, the relevant degrees of freedom in QCD, for temperatures above $T_c$ are $q, \bar{q}$, $g$ quasiparticles and bosonic states. The effective number of degrees of freedom associated with the bosonic states is at most of order 10 suggesting that only light non exotic states are present.

For $T \gtrsim 2 T_c$ the contribution of mesonic bound states to pressure and energy density is vanishing small and only quasiparticles are relevant. Gluon condensation and its persistence above $T_c$ is a fundamental ingredient of the energy balance.

The correlation between baryon number and strangeness can be understood by considering the reduction of the effective bosonic degrees of freedom due to the gluon quasiparticle mass. Further investigations are needed to clarify the underlying non perturbative dynamics in terms of resonance scattering, chiral phase fluctuations and instantons.

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[1] P. F. Kolb and U. W. Heinz (2003), nucl-th/0305084.
[2] F. Karsch and E. Laermann (2003), hep-lat/0305025.
[3] F. Karsch, E. Laermann, and A. Peikert, Phys. Lett. B478, 447 (2000), hep-lat/0002003.
[4] T. Hatsuda and T. Kunihiro, Phys. Rev. Lett. 55, 158 (1985).
[5] E. V. Shuryak and I. Zahed, Phys. Rev. D70, 054507 (2004), hep-ph/0403127.
[6] M. Mannarelli and R. Rapp (2005), hep-ph/0505080.
[7] M. Mannarelli and R. Rapp (2005), hep-ph/0509310.
[8] P. Castorina, G. Nardulli, and D. Zappala (2005), hep-ph/0505089.
[9] H. van Hees, V. Greco, and R. Rapp (2005), nucl-th/0508055.
[10] V. Koch, A. Majumder, and J. Randrup (2005), nucl-th/0505052.
[11] M. Asakawa and T. Hatsuda, Nucl. Phys. A721, 869 (2003).
[12] M. D’Elia, A. Di Giacomo, and E. Meggiolaro, Phys. Rev. D67, 114504 (2003), hep-lat/0205018.
[13] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, M. B. Voloshin, and V. I. Zakharov, Nucl. Phys. B237, 525 (1984).
[14] D. Zwanziger, Phys. Rev. Lett. 94, 182301 (2005), hep-ph/0407103.
[15] M. Kitazawa, T. Kunihiro, and Y. Nemoto (2005), hep-ph/0510167.
[16] P. Levai and U. W. Heinz, Phys. Rev. C57, 1879 (1998), hep-ph/9710463.
[17] A. Peshier and W. Cassing, Phys. Rev. Lett. 94, 172301 (2005), hep-ph/0502138.