Preparing a (quantum) belief system

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Abstract

In this paper we investigate the potential for persuasion linked to the quantum indeterminacy of beliefs. We first formulate the persuasion problem in the context of quantum-like beliefs. We provide an economic example of belief manipulation that illustrates the setting. We next establish a theoretical result showing that in the absence of constraints on measurements, any belief state can be obtained as the result of a suitable sequence of measurements. We finally discuss the practical significance of our result in the context of persuasion.

Keywords: belief, quantum-like, persuasion, measurement

1 Introduction

The theory of persuasion was initiated by Kamenica and Gentskow [11] and further developed in a variety of directions. The subject matter of the theory of persuasion is the use of an information structure (or measurement) that generates new information in order to modify a person’s state of beliefs with the intent of making her act in a specific way. The question of interest is how much can a person, call him Sender, influence another one, call her Receiver, by selecting a suitable measurement and revealing its outcome. An example is in lobbying. A pharmaceutical company commissions to a scientific laboratory a specific study of a drug impact, the result of which is delivered to the politician. The question of interest from a persuasion point of view is what kind of study best serves the company’s interest.

Receiver’s decision to act depends on her beliefs about the world. In [11] and related works the beliefs are given as a probability distribution over a set of states of the world. A central assumption is that uncertainty is formulated in the standard classical framework. As a consequence the updating of Receiver’s beliefs follows Bayes’ rule.

However as amply documented the functioning of the mind is more complex and often people do not follow Bayes rule. Cognitive sciences propose alternatives to Bayesianism. One avenue of research within cognitive sciences appeals to the formalism of quantum mechanics. A main reason is that QM has properties that reminds of the paradoxical phenomena exhibited in human cognition. Quantum cognition has been successful in explaining a wide variety of behavioral phenomena such as disjunction

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effect, cognitive dissonance or preference reversal (see [10], [3]). Moreover there exists by now a fully
developed decision theory relying on the principle of quantum cognition (see [5, 6]). Therefore in the
following we shall use the Hilbert space model to represent the belief of an individual and capture the
impact of new information on those beliefs. Clearly, the mind is likely to be even more complex than a
quantum system but our view is that the quantum cognitive approach already delivers interesting new
insights in particular with respect to persuasion.

In quantum cognition, the system of interest is the decision-maker’s mental representation of the
world. It is represented by a cognitive state. This representation of the world is modelled as a quantum-
like system so the decision relevant uncertainty is of non-classical (quantum) nature. As argued in
(8) this modeling approach allows capturing widespread cognitive difficulties that people exhibit when
constructing a mental representation of a ‘complex’ alternative (cf [4]). The key quantum property
that we use is that some characteristics (cf. properties) of a complex mental object may be "Bohr
complementary" that is incompatible in the decision-maker’s mind: they cannot have a definite value
simultaneously. A central implication is that measurements (new information) modifies the cognitive
state in a non-Bayesian well-defined manner.

It turns out that persuasion - measurement operations aimed at moving a cognitive system into a
specific state - has a close counter-part in Physics (see, for example, [9]). When doing experiments in
Quantum Mechanics, one often needs to know the state of the particles before performing operations
on them in order to be able to draw conclusions. In order to determine that state, physicists ‘prepare’
particles in a definite quantum state, e.g. by means of filtering: for instance they measure the spin of a
set of particles and keep for further operation those that are in state + while throwing away the others.
By doing so they have effectively created particles with the spin property +. The term preparation is
rather broad - it covers any kind of operations that affects the state of a single system or the composition
of a set of systems. One such operation is measurement. A von Neumann direct measurement prepares
a system in the (pure) state that obtains as the result of the measurement. Generally, a von Neumann
measurement modifies a quantum state according to the von Neumann-Lüders postulate.

As in the classical context our rational Receiver uses new information to update her beliefs so that
choices based on updated preferences are consistent with ex-ante preferences defined for the condition
(event) that triggered updating. In [6], we learned that a dynamically consistent rational quantum-like
decision-maker updates her beliefs according to the von Neumann-Lüders postulate. We take this result
as starting point to investigate the potential of manipulation of Sender when facing a rational quantum-
like Receiver. Our central result in Theorem 1 is that, in the absence of any constraints on measurements,
there always exists a sequence of direct measurements that secures reaching any target state starting from
any initial state. In terms of persuasion, Sender can always persuade Receiver to believe anything that
favors him. This is in sharp contrast with the classical setting where the expected posterior must be

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1A rational quantum-like decision-maker is a decision-maker who has preferences over items (or actions) that she perceives (represents) as quantum systems. Her preferences over items with uncertain properties (or actions with uncertain consequences), satisfy a number of axioms similar to those in the classical case as we show in [6]. These axioms secure that the preferences can be represented by an expected utility function.
equal to the prior - a property labeled Bayesian plausibility in [11]. Theorem 1 is of course a theoretical result. Achieving the desired belief state may require a sequence of measurements that is not practically feasible. Such measurements may be too costly for Sender to undertake and Receiver could get annoyed. Classical measurements face similar practical constraints. A distinction however is that any number of classical measurements can always be performed as a single measurement while this is generally not true in a quantum situation.

Kamenica and Gentskow motivated their paper by the fact that attempts to manipulate command a sizable share of our resources in the economy. Persuasion is at the heart of advertising, courts hearings, lobbying, financial disclosures and political campaign among other activities. As suggested by Akerlof and Schiller [1], the power of manipulation seems however much larger and more determinant than what the classical approach reveals. Our results suggest that quantum cognition may provide a better model to explain the extent and power of manipulation in society. Other approaches to influence on decision-making that appeal to the quantum formalism have been developed (see e.g. [2]). An important distinction however is that in [2], the authors consider open quantum systems evolving in time. Our research is a step in a broader project which aims at proposing an alternative to the foundations of the functioning of free markets in the spirit of Akerlof and Schiller.

2 The model

We have two players Receiver (her) and Sender (him). We are interested in persuasion aimed at influencing Receiver’s choice over actions with uncertain consequences. The consequences of an action depend on the state of some system (which is often called Nature). Receiver has some a priori representation of the system that is a belief about the state of Nature. Sender can provide new information about the state of Nature by selecting and performing some investigation (measurement). It is assumed that the outcome of the investigation is public (or reported truthfully). The new information triggers a revision of Receiver’s beliefs and as a consequence it affects her choice of action. Sender has an interest in Receiver’s action because his own utility depends on the action chosen by Receiver. Sender’s problem amounts to acting on Receiver’s beliefs so as to maximize his expected utility from Receiver’s decision, i.e. to persuade her to act in ways favorable to him. His means of persuasion are information (signal) structures, we shall also use the term “measurement devices”.

We below provide a brief description of the classical setting and thereafter we develop our argument in the quantum context.

2.1 The classical setting

The classical model of persuasion has been well described in Kamenica and Gentskow (2011), hereafter KG. There is a set $\Omega$ of states of Nature. For the sake of simplicity we shall assume that the set $\Omega$ is finite. Receiver’s belief (or her belief state) is a probability distribution $\beta : \Omega \rightarrow \mathbb{R}$, $\beta(\omega) \geq 0$, $\sum_{\omega \in \Omega} \beta(\omega) = 1$. The set $\Delta(\Omega)$ is the simplex of probability distributions on $\Omega$. 
An action is a function $a : \Omega \to \mathbb{R}$. In a pure belief state $\beta = \omega$ this action gives (to Receiver) utility $a(\omega)$. If our Receiver has a 'mixed' belief $\beta$ she expects to obtain utility $a(\beta) = \sum_\omega a(\omega)\beta(\omega)$.

An information structure (or a measurement device) is a map $\varphi : \Omega \to \Delta(S)$, where $S$ is a set of signals (outcomes) of our measurement device. In a state $\omega \in \Omega$ this device gives (randomized) signal $\varphi(\omega) \in \Delta(S)$. If we write this more carefully such a device is given by a family $(f_s, s \in S)$ of functions $f_s$ on $\Omega$; $f_s(\omega)$ gives the probability of obtaining signal $s$ in state $\omega \in \Omega$. Of course all the functions $f_s$ must be nonnegative and their sum $\sum_s f_s$ must yield the unit function $1_\Omega$ on $\Omega$.

Assume Sender and Receiver hold common prior $\beta = (\beta(\omega), \omega \in \Omega)$\footnote{Common priors is the standard assumption. Allowing for different priors would require distinguishing between Sender respectively Receiver’s belief where necessary. In addition we may need to assume that Sender knows Receiver’s belief which is also a common assumption.}. The probability of receiving a signal $s$ given the prior $\beta$ is equal to $p_s = \sum_\omega \beta(\omega) f_s(\omega)$.

More important for us is that our rational classical Receiver uses Bayes’ rule when she receives signal $s$ to update her beliefs, i.e. to form the posterior $\beta_s \in \Delta(\Omega)$, given as $\beta_s(\omega) = f_s(\omega)\beta(\omega)/p_s$. For Receiver (and for Sender) what is important is the change in the belief from $\beta$ to $\beta_s$ upon receiving signal $s$. Because as she receives signal $s$ she will choose her optimal action for the updated beliefs, $a^*(\beta_s)$, and Sender will receive utility $u(a^*(\beta_s))$. On average when Sender uses such a signal structure (measurement) he receives utility $\sum_s p_s u(a^*(\beta_s))$. And so we can ask which is the best measurement device for Sender? Since $\sum_s p_s \beta_s = \beta$, that is the expected posterior is equal to the prior, a property that KG call Bayesian plausibility, the problem simplifies to finding the signal structure with expected posterior (equal to the prior) that maximizes the (expected) utility of Sender.

### 2.2 The quantum setting

The description of a quantum system starts with the fixation of a Hilbert space $H$ (over the field $\mathbb{R}$ of real numbers or the field $\mathbb{C}$ of complex number). Physicists usually work with the complex field $\mathbb{C}$. We, partly for simplicity, shall work with the real field $\mathbb{R}$, although all goes without changes for the complex case. $(\cdot, \cdot)$ denotes the scalar product in Hilbert space $H$ (in our case a finite dimensional space).

We shall be interested not so much in the Hilbert space $H$ as in operators, that is linear mappings $A : H \to H$. Such an operator $A$ is Hermitian (or symmetric) if $(Ax, y) = (x, Ay)$ for all $x, y \in H$. A Hermitian operator $A$ is non-negative if $(Ax, x) \geq 0$ for any $x \in H$.

The notion of trace will be a central instrument in what follows. The trace $\text{Tr}$ of a matrix can be defined as the sum of its diagonal elements. It is known that the trace does not depend on the choice of basis. With the help of the trace one can introduce the notion of state of a quantum system. It is defined as a non-negative Hermitian operator $B$ with trace equal to 1. This notion replaces the classical concept of probability distribution. The non-negativity of the operator is analogous to the nonnegativity of a probability measure, and the trace 1 to the sum of probabilities which equals 1. The set of states is denoted $\text{St} = \text{St}(H)$. It is a convex compact subset in the space of operators. Extreme points of this set are called pure states and have the following form. Let $x \in H$ be a vector in $H$ of the length 1 (that is $(x, x) = 1$). Define the operator $P_x$ by the formula $P_x(y) = (x, y)x$. Then the operator $P_x$ is a pure...
state, and any pure state has such a form for an appropriate vector $x$.

General Hermitian operators play the role of classical random variables on $\Omega$. In fact for any Hermitian operator $A$ and state $B$ we can define the ‘expected value’ of $A$ in state $B$ as $\text{Tr}(AB)$. In [6] it is shown that a decision/action subject to non-classical uncertainty that is an action whose consequence depends on the uncertain state of a quantum system, can be expressed as a Hermitian operator. The expected utility of action $a$ represented by operator $A$ in belief state $B$ is expressed as $\text{Tr}(AB)$ and this number linearly depends on $B$. In this way, Receiver’s preferences over actions are determined by her belief state $B$ and actions are understood as affine functions on $\text{St}$.

When it comes to measurement devices (or information structures), we shall focus on a limited subset of devices referred to as direct (or von Neumann) measurements. These simple devices are sufficient for the purpose of the present paper. Such a device is given by a family $(P_s, s \in S)$ of projectors with the property $\sum_s P_s = E$, where $E$ is the identity operator on $H$. (A projector is an Hermitian operator such that $P^2 = P$. The set $S$ again is understood as the set of signals of the device.) The probability $p_s$ to obtain a signal-outcome $s$ (in a state $B \in \text{St}$) is equal to $\text{Tr}(P_sB)$. And the posterior belief-state is $B_s = P_sBP_s/p_s$ (it is easy to check that it is a state). Here all is standard and simple especially if projector $P_s$ is one-dimensional (that is a pure state); in this case the posterior $B_s$ is equal to $P_s$. If we repeat the measurement we obtain the same outcome $s$ and the state does not change. This type of measurement is repeatable or "first kind". The only thing that must be underlined is that the expected posterior $B^{\text{ex}} = \sum_s p_s B_s = \sum_s P_sBP_s$ is generally different from the prior $B$ as is illustrated in the example below. This contrasts with the classical case and as a consequence the issue of optimality of measurements for Sender cannot be addressed straightforwardly. Instead, our objective in this paper is confined to establishing what can be achieved with a sequence of measurements.

Quantum measurements be they direct or more general are characterized by a few distinguishing features. Most importantly, the performance of a measurement impacts on the state of the system. As a first consequence and in contrast with the classical case, beliefs do not converge toward complete information about ‘a true state’ of the system. In the quantum context, the belief state may be pure, i.e. represent maximal (rather than complete) information and yet change upon the reception of new information. An expression of this is that measurements may be incompatible. As a consequence and in contrast with the classical case, a sequence of direct measurements cannot generally be merged into a single direct measurement. Therefore, in this paper we opted for the following approach. On the one hand we limit ourselves to direct measurements while on the other hand allow for sequences of direct measurements.

A second consequence of the impact of measurements on the state is the so called ‘decoherence’ which turns out to be of great value in our context. Assume that we perform a measurement (for instance a direct one given by the family of projectors $P_s$) but do not learn the result. Such a measurement can be simply ignored in the classical context. In the quantum context such a ‘blind’ measurement induces nevertheless a change in the posterior $B' = \sum_s P_sBP_s$. As we shall see such blind measurements (clearly useless in a classical context) provide a powerful mean of changing the beliefs of Receiver. They also have a meaningful interpretation in the cognitive quantum context.
3 An illustrative example

Before introducing our central result we wish to provide an example of quantum persuasion in a very common context, i.e. when a seller wants to persuade a potential buyer to purchase an item. So in that situation we identify Sender with the seller and Receiver with the consumer.

More specifically, our consumer is considering the purchase of a second hand smartphone at price 30 euros of uncertain value to her, it depends on its technical quality which may be standard or excellent. She holds subjective beliefs about the probability that the smartphone is excellent. Based on those beliefs, she assigns an expected utility value to the smartphone which determines her decision to buy or not the item.

Let $H$ be a two-dimensional Hilbert space with an orthonormal basis $(e_1, e_2)$ and let $(P_1, P_2)$ be the corresponding projectors. Following [6], the utility of the smartphone is expressed as operator $A$ which gives a utility equal to 100 in state $|e_1\rangle$ (the smartphone is Excellent) and 0 in $|e_2\rangle$ (the smartphone is Standard); in matrix form this utility can be written as $A = \begin{pmatrix} 100 & 0 \\ 0 & 0 \end{pmatrix}$. Assume further that the utility when not buying the smartphone is $u = 30$ (she keeps the money). The consumer’s (Receiver) decision is $d \in \{Y, N\}$ accept or refuse to buy. The seller (Sender) receives utility 10 when selling the phone at price 30 whatever its quality and 0 otherwise.

Assume now that Receivers’ belief (her cognitive state) is a pure (superposed) state represented by vector $b = (1/\sqrt{5}, 2/\sqrt{5})$ or by the corresponding projector $B = \begin{pmatrix} 1/5 & 2/5 \\ 2/5 & 4/5 \end{pmatrix}$ in the basis $(e_1, e_2)$. Upon a measurement of $B$, we would find that she assigns probability 1/5 to the state $|e_1\rangle$ (the smartphone is Excellent) and 4/5 to the state $|e_2\rangle$ (the smartphone is Standard). Receiver’s expected utility for the smartphone in the belief state $B$ is represented by the trace of the product of operators $A$ and $B$:

$$Eu(A; B) = \text{Tr}(AB) = (1/5) \times 100 + (4/5) \times 0 = 20 < 30 = u.$$  

Given belief $B$ Receiver does not want to buy the smartphone so the seller earns 0.

Can Sender persuade Receiver to buy by selecting an appropriate measurement? We next show that he indeed can induce her to buy with probability 1. Consider another property (perspective) of the smartphone that we refer to as Glamour (i.e. whether celebrities have this brand or not). The Glamour property can be measured with direct von Neumann measurement $(Q_1, Q_2)$ with two possible outcomes

Glamour $|G\rangle$ corresponding to projector $Q_1 = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$ and not Glamor $|NG\rangle$ corresponding to

$Q_2 = \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix}$. The Glamour perspective is represented by the basis $(|G\rangle, |NG\rangle)$ of the state space (of the mental representation of the smartphone) which is a 45° rotation of basis $(e_1, e_2)$\footnote{The specific relationship between the two properties i.e., a 45° rotation can be interpreted as follows. In Receiver’s mind (e.g., based on experience) there is no correlation between the true user value of an item and its glamour value.}. This means that the $(|e_1\rangle, |e_2\rangle)$ and $(|G\rangle, |NG\rangle)$ are two properties (perspectives) that are incompatible in the
mind of Receiver. Or equivalently \((P_1, P_2)\) and \((Q_1, Q_2)\) are measurements that do not commute with each other. Receiver can think in terms of either one of the two perspectives but she cannot synthesize (combine in a stable way) pieces of information from the two perspectives. This is illustrated in figure 1.

Assume that Sender brings the discussion to the Glamour perspective and performs the measurement so Receiver learns whether her preferred celebrity has this smartphone. With some probability \(p \approx 0.9\) the new cognitive state is \(B' = Q_1\) and with the complementary probability \(1 - p = 0.1\) it is \(B'' = Q_2\). We note that \(Eu (A; B') = \text{Tr} (AB') = 50 > 30\) and \(Eu (A; B'') = \text{Tr} (AB'') = 50 > 30\). In both cases Receiver is persuaded to buy and Sender gets utility 10.

Interestingly the example also illustrates the non-classical phenomenon of ‘decoherence’. Namely that the mere fact of performing a measurement - without learning the outcome (a blind measurement) - triggers a significant change in beliefs and subsequent action. Here the ‘expected posterior’ \(B^{ex} = pB' + (1 - p)B'' = \begin{pmatrix} 1/2 & 2/5 \\ 2/5 & 1/2 \end{pmatrix} \neq B\) in contrast with classical persuasion which is constrained by Bayesian plausibility meaning that the expected posterior must equal the prior.

In general, it is not possible to persuade Receiver with probability 1 by means of a single measurement. However as Theorem 1 below shows it is theoretically possible to manipulate beliefs with a sequence of appropriate measurements.

### 4 Our central result

In order to formulate our central result we return in more detail to the description of direct measurements. A direct measurement device \(M\) will be given by an orthonormal basis \((e_1, ..., e_n)\), where \(n = \dim H\), and \((e_i, e_j) = \delta_{ij}\), and a collection of signals \((s_1, ..., s_n)\). The vector \(e_i\) defines the one-dimensional projector \(P_i; P_i (x) = (x, e_i) e_i\). How does such a measurement device operate? Upon the reception of signal \(s_i\) the system transits into state \(|e_i\rangle\) (or \(P_i\)) with probability \(p_i = \text{Tr} (BP_i)\). If we represent the initial state \(B\) in matrix form in the basis \((e_1, ..., e_n)\), we have \(p_i = b_{ii}\) where \(b_{ii}\) is the corresponding diagonal element.
of the matrix. This is true when the signal is fully disclosed. If the result is not communicated (a blind measurement) the system transits into the mixed state $\sum p_i P_i$. The impact of blind measurements is a distinguishing feature of the quantum formalism.

A second useful feature of the quantum situation is related to conditional measurements. Assume that we performed a measurement according to the above described device (with the signal set $S$). As we receive signal $s$ we may thereafter perform a new measurement $M_s$ (which can depend on signal $s$). In the classical context this kind of conditionality does not play any role because a compound measurement also is a measurement. But in the quantum context as we shall see further, such a composition is generally not a direct measurement although it is a measurement in the most general meaning. We can iterate that procedure conditioning on the result from the second measurement and so on. As we consider a sequence of measurements, we assume that Receiver updates her beliefs each time she receives a new piece of information in the order of reception.

Our main result is that starting from any initial state it is possible to transit to any state by means of a suitable sequence of conditional measurements. In term of persuasion and beliefs, it means that Sender can always persuade Receiver to believe what is most favorable to him in the sense that Receiver will take the decision (consistent with her preferences) that is most desirable for Sender independently of her prior. Of course, this is a purely theoretical result. In practice, it may not be possible to ‘play’ with Receiver so easily. Measurements are connected with costs, Receiver may be impatient or get tired of all information etc...

**Theorem 1.** For any prior $B$, any target belief state $T$, and any $\varepsilon > 0$, there exists a sequence of direct conditional measurements such that with a probability larger than $1-\varepsilon$ the posterior is equal to $T$.

We understand this result as follows: there exists in principle a rather simple and constructive strategy for Sender to persuade Receiver to believe anything Sender wants her to believe.

We first show that the target state can be taken as pure. In fact the final state $T$ can be represented (by force of the Spectral theorem) as a mixture of orthogonal pure states, $T = \sum q_i P_i$, where $q_i \geq 0$, $\sum q_i = 1$, $P_i$ are projectors on (unit) vector $e_i$. Let $P$ be the projector on the following vector $e = \sum \sqrt{q_i} e_i$, that is $P(x) = (x,e) e$ for $x \in H$. In the basis $(e_1,\ldots,e_n)$ the projector $P$ is given by the matrix $(\sqrt{q_i q_j})$.

We assert that if we perform a blind measurement with basis $(e_1,\ldots,e_n)$, state $P$ transits into state $T$. In fact, the expected posterior is $P^{ex} = \sum_i P_i P P_i = \sum_i q_i P_i = T$.

Hence, in order to arrive at the state $T$ it is sufficient to arrive at the pure state $P$. We next show how to transit into any arbitrary pure state $P$.

Suppose that $P$ is a projector on (unit) vector $|e_1\rangle$. We complete it to an orthonormal basis $(e_1,\ldots,e_n)$ and as our main measurement $\mathcal{M}$ we take a non-degenerated (complete) direct measurement in this basis. Whatever the initial state $B$, after the performance of $\mathcal{M}$, the state transits into one of the pure states $|e_1\rangle,\ldots,|e_n\rangle$ and the signal informs us about which one. Assume that the signal is $s_2$ so the system is now in the state $|e_2\rangle$. In that case we construct an auxiliary direct measurement device $\mathcal{M}_2$ with the following orthonormal basis $(e_1 + e_2)/\sqrt{2}, (e_1 - e_2)/\sqrt{2}, e_3,\ldots,e_n$. If we perform $\mathcal{M}_2$ (recalling that the system
is now in $|e_2\rangle$ the system will with equal probability transit into $|(e_1 + e_2)/\sqrt{2}\rangle$ or $|(e_1 - e_2)/\sqrt{2}\rangle$ (we could let $\mathcal{M}_2$ be a blind measurement). Now we once more apply $\mathcal{M}$ and with probability $1/2$ we obtain the desired state $|e_1\rangle$ (and the undesired state $|e_2\rangle$ with the same probability). If we obtain $|e_1\rangle$ we are done. If we obtain $|e_2\rangle$ we again apply $\mathcal{M}_2$ and thereafter $\mathcal{M}$. After $N$ iterations the state will have transited into the target state $|e_1\rangle$ (or $P$) with probability $1 - (1/2)^N$.

Above we consider the case when the first measurement gave outcome $s_2$. A similar procedure secures the desired state whatever the first outcome $s_i$, $i \neq 1$. Instead of $\mathcal{M}_2$ we use $\mathcal{M}_i$ with basis $(e_1 + e_i)/\sqrt{2}$, $(e_1 - e_i)/\sqrt{2}$, $e_3$, $e_{i-1}$, $e_{i+1}$, $e_n$. Which proves the Theorem 1.

In figure 2 we illustrate the conditional measurement scheme used in the proof of Theorem 1 for the case when $n = 3$. $\mathcal{M}_{2,3}$ represent blind measurements.

![Figure 2](image)

The strategy used in the proof of the theorem has two nice features. First it does not require knowing the prior $B$ of Receiver. Second it only appeals to a restricted set of types of measurements, $\mathcal{M}$ and the instrumental $\mathcal{M}_i$ ($i = 2, ..., n$). As earlier mentioned in practice the optimal strategy will have to take into account costs of measurements and constraints on the feasible number of iterations. Therefore, it can be interesting to consider the case when the number of measurements is limited as we do in [7].

5 Discussion

Manipulation is both an old and a very active field of research in social sciences (see e.g., [4]). Recently an approach has received a lot of attention in the field of economics thanks to the seminal paper by Kamenica and Gentskow on Bayesian persuasion. The starting point is that one can manipulate rational people’s behavior by acting upon their beliefs. This is done through the choice of a suitable information structure or measurement that generates new information. This theory is developed in the classical uncertainty setting.

An alternative approach to decision-making under uncertainty uses the quantum formalism to describe (subjective) uncertainty. Relying on the recent success of quantum cognition in explaining behavioral
anomalies, we have investigated the scope of manipulation when a person’s representation of the world, i.e. her beliefs are represented as a quantum-like system.

Quite remarkably, we establish that a person’s beliefs are in theory fully manipulable. For any target state and any initial state there exists a sequence of direct measurements such that the target state is reached with a probability close to one.

In practice, this potential for manipulation is not expected to be realized because measurements are costly, people have limited patience or because constructing the appropriate measurements may not be practically feasible. However, there are situations where it makes perfect sense. We provide a simple example showing that under some circumstances a single measurement in a very common situation can be sufficient to achieve desired behavior with probability one. The example also illustrates the impact and power of blind measurements which captures the idea of changing the focus of a person’s mind without bringing any new information. In our example simply ”diverting” Receiver’s attention to the Glamour perspective of the smartphone (that is performing the corresponding blind measurement) modifies her state of belief so as to make her willing to buy. As noted by Akerlof and Schiller ” just change people’s focus and one can change the decisions they make.” [1, p.173].

Our result suggests that the potential for manipulation of human behavior may indeed be much greater than what is proposed by mainstream economic theory. This finding is in line with recent works in economics that emphasize the role of manipulation in the functioning of markets.
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