Phase coherent control in electron-argon scattering in a bichromatic laser field

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We study the elastic scattering of atomic argon by a electron in the presence of a bichromatic laser field. The numerical calculation is done in the first Born approximation (FBA) for a simple screening electric potential. With the help of numerical results we explore the dependence of the differential cross sections (DCS) on the relative phase \(\varphi\) between the two components of the radiation field and discuss the influence of the number of photons exchanged on the phase-dependence effect. Moreover, we also discuss the numerical results of the DCS for different scattering angles and impact energies.

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Multiphoton free-free transitions (MFFT) have attracted much attention since the pioneering papers by Bunkin and Fedorov [1] and Kroll and Watson [2], and a great deal of work has been devoted to them [3, 4, 5, 6, 7]. As the experimental technology improved, and a great deal of work has been devoted to them [8, 9, 10, 11, 12, 13, 14, 15, 16, 17], thus the investigation on the phase coherent control of elastic electron-atom collisions in a multicolor laser field becomes a very active research domain. In most of the theoretical work, the laser radiation is treated as a classical radiation filed with a single frequency \(\omega\), or some narrow band multi-mode approximation has been employed, yielding better agreement with the experiments by Weingartner [18]. Describing a laser beam by a monochromatic classical background field relies on the argument that in a laser beam the density of radiation quanta is so large that the depletion of this beam by emitting or absorbing quanta from it is negligible. If the laser frequency \(\omega\) and intensity \(I\) are sufficiently low so that the excitations of atomic transitions can be neglected, the atomic target can be described by a short range potential \(V(r)\) and the scattering can be treated in the first Born-approximation, as was done by Bunkin and Fedorov [1]. In this letter, we will investigate the relative phase \(\varphi\) dependence of free-free transitions for the electron-argon collisions in the presence of bichromatic laser field.

During a laser-assisted electron-impact scattering process \(l\) photons may be exchanged with the laser field. In our work, we consider the free-free transition in argon atom in the presence of a bichromatic laser field, accompanied by transfer of \(l\) photons in the first Born approximation \((l > 0\) for emission and \(l < 0\) for absorption). The laser field is treated classically as an electromagnetic field [19] which is a superposition of two components of frequencies \(\omega\) and \(2\omega\). The bichromatic laser field is described as \(E(t) = E_0[\sin \omega t + \sin(2\omega t + \varphi)]\), where \(E_0\) is the electric field amplitude vector and the relative phase \(\varphi\) can be arbitrarily changed. Atomic units \(\hbar = m = e = 1\) are used throughout.

In case the laser is much weak compared with the internal field of an atom (ion), the dressing effect in atom can be neglected, and the target atom is described by a screening potential [20, 21, 22]

\[
V(r) = -\frac{Z}{r} \sum_{i=1}^{2} A_i \exp(-\alpha_i r),
\]

where \(r\) denotes the position of the electron with respect to the nucleus, and \(Z\) is the nuclear charge number. For argon, \(A_1 = 2.1912, A_2 = -2.8252, \alpha_1 = 5.5470, \alpha_2 = 4.5687\).

The scattering matrix for the laser-assisted free-free transition in the first Born approximation reads

\[
S_{fi}^{(1)} = -i \int_{-\infty}^{\infty} dt \int d\mathbf{r} \chi_{\mathbf{k}_f,i}(\mathbf{r}, t) V(\mathbf{r}) \chi_{\mathbf{k}_i}(\mathbf{r}, t),
\]

where \(\chi_{\mathbf{k}_f,i}\) and \(\chi_{\mathbf{k}_i}\) are the initial and the final states of the electron, described by the Volkov wave function

\[
\chi_{\mathbf{k}_f,i}(\mathbf{r}, t) = \exp(i \mathbf{k}_{f,i} \cdot \mathbf{r}) \exp \left[ -i E_{f,i} t - \frac{i}{\omega^2} \mathbf{k}_{f,i} \cdot \mathbf{E}_0 \sin \omega t \right] \times \exp \left[ -\frac{i}{4\omega^2} \mathbf{k}_{f,i} \cdot \mathbf{E}_0 \sin(2\omega t + \varphi) \right],
\]

where \(\mathbf{k}_{i,f}\) are the wave vectors of electron in the initial and final states, and \(E_{i,f}\) is the corresponding kinetic energies.

With using the potential of Eq. (1) and the wave functions of Eq. (3), we obtain

\[
S_{fi}^{(1)} = -2\pi i \sum_{l} T_{f,i}^{(1)}(l) \delta(E_f - E_i + l\omega).
\]
is the Fourier transformation of the potential, and in which

\[ T_{f,i}^{(1)}(l) = B_l(\lambda, \frac{1}{4}\lambda, \varphi) V(k_{f,i}), \]

(5)

where \[ B_l(\lambda, \frac{1}{4}\lambda, \varphi) = \sum_{n=-\infty}^{\infty} J_l(\lambda) J_{n-l}(\lambda) \exp(-in\varphi), \]

(7)

is the generalized bessel function with \( \lambda = (k_f - k_i) \cdot \epsilon_0/\omega^2 \). \( J_n(\lambda) \) is the ordinary bessel function. The DCS for the net exchange of \( l \) photons between the colliding system and the bichromatic laser field is

\[ \frac{d\sigma}{d\Omega} = \frac{1}{(2\pi)^2} \frac{k_f}{k_i} |T_{f,i}^{(1)}(l)|^2. \]

(8)

Where \( T_{f,i}^{(1)}(l) \) is the transition matrix element, resolved with respect to multiphoton exchange processes

\[ T_{f,i}^{(1)}(l) = B_l(\lambda, \frac{1}{4}\lambda, \varphi) V(k_{f,i}), \]

(5)

in which

\[ V(k_{f,i}) = \int d\mathbf{r} e^{-i(k_f - k_i) \cdot \mathbf{r}} V(\mathbf{r}) \]

(6)

is the Fourier transformation of the potential, and

\[ B_l(\lambda, \frac{1}{4}\lambda, \varphi) = \sum_{n=-\infty}^{\infty} J_l(\lambda) J_{n-l}(\lambda) \exp(-in\varphi), \]

(7)

is the generalized bessel function with \( \lambda = (k_f - k_i) \cdot \epsilon_0/\omega^2 \). \( J_n(\lambda) \) is the ordinary bessel function. The DCS for the net exchange of \( l \) photons between the colliding system and the bichromatic laser field is

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(8)

FIG. 1: The \( \varphi \)-dependence of DCS at a scattering angle \( \theta = 13^\circ \) for electron-argon elastic scattering in a CO\(_2\) laser field (\( \hbar\omega = 0.117eV \)) with \( l \) photons exchanged between the scattering system and the bichromatic laser field. The impact energies of the incident electron is \( E_i=9.5eV \), and laser amplitude \( \epsilon_0 = 2.7 \times 10^4Vcm^{-1} \). (a) Results for emission \( (l > 0) \); (b) Results for absorption \( (l < 0) \).

For numerical calculation, we study the electron-argon scattering under the geometry of the experiment of Weingartshofer [18] in which the angle between the polarization vector \( \epsilon_0 \) and the momentum \( p_i \) of the incident electron is \( \psi_0 = 38^\circ \), the momentum \( p_f \) of the scattered electron is in the plane defined by the polarization vector \( \epsilon_0 \) and the momentum \( p_i \) of the incident electron. The bichromatic laser frequencies are respectively \( \omega = 0.117eV \) and its double harmonic, with the field amplitude \( \epsilon_0 = 2.7 \times 10^4Vcm^{-1} \), all taken from Ref. [3].

In Fig.1, we display cross section dependence on for each multiphoton process. Generally speaking, at the field strength considered, only the processes with a few photon exchanged have significant contributions. With the exchanged photon number \( l \) increasing, the DCS decline rapidly. The results for \( l = 0 \) and odd numbers are less sensitive to the variation of \( \varphi \) than \( l = \) even numbers. For the photon emission processes \( (l > 0) \), the DCS at \( \varphi = 180^\circ \) attain the maximum, while results for photon absorption attain minimum \( (l < 0) \). All the results are symmetric about \( \varphi = 180^\circ \).

FIG. 2: The \( \varphi \)-dependence of DCS for \( l = \pm 2 \) at different scattering angles. The impact energy and laser parameters are the same as in Fig.1. (a) Results for emission \( (l = +2) \); (b) Results for absorption \( (l = -2) \).
In summary, we have studied the electron-argon elastic scattering in a bichromatic laser field with employing a simple potential. The dependence of DCS on the relative phase $\varphi$ is investigated at different scattering angles and impact energies. These features may be used in the coherent control in the free-free transition.

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**FIG. 3**: The $\varphi$-dependence of DCS for $l = \pm 2$ at different impact energies. The scattering angle is $\theta = 13^\circ$. The laser parameters are the same as in Fig.1. (a) Results for emission ($l = +2$); (b) Results for absorption ($l = -2$).

Fig. 2 shows the $\varphi$-dependence of DCS for $l = \pm 2$ at the scattering angles $\theta = 8^\circ$, $13^\circ$, $30^\circ$ and $50^\circ$. The result for $\theta = 30^\circ$ is much more sensitive to $\varphi$ than the results of other angles. Thus at this angle, one can more effectively control the dynamics by varying the phase.

Fig. 3 displays the result for $l = \pm 2$ at different impact energies. The tendencies for emission ($l = +2$) and absorption ($l = -2$) are opposite. With the impact energies increasing, the result becomes more and more sensitive to the variation of phase.

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