Faraday rotation and primordial magnetic fields constraints on ultraviolet Lorentz violation with spacetime torsion

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Abstract

Recently Kahliaishvili et al (2006) presented a unified treatment for ultraviolet Lorentz violation (LV) testing through electromagnetic wave propagation in magnetised plasmas, based on dispersion and rotation measured data. Based on the fact discovered recently by Kostelecky et al (2008), that LV may place constraints on spacetime torsion, in this paper it is shown that on the limit of very low frequency torsion waves, it is possible to constraint torsion from Faraday rotation and CMB on a similar fashion as Minkowski spacetime plus torsion. Here the Maxwells modified equations are obtained by a perturbative method introduced by de Sabbata and Gasperini (1981). Torsion is constraint to $Q_{CMB} \approx 10^{-18} \text{GeV}$ which is not so stringent as the $10^{-31} \text{GeV}$ obtained by Kostelecky et al. However, Gamma Ray Bursts (GBRs) may lead to the more string value obtined by Kostelecky et al. Another interesting constraint on torsion is shown to be placed by galactic dynamo seed magnetic fields. For torsion effects be compatible with the galactic dynamo seeds one obtains a torsion constraint of $10^{-33} \text{GeV}$ which is two orders of magnitude more stringent that the above Kostelecky et al limit.

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1 Introduction

As recently pointed out by Gamboa et al [1] the cornerstone of cosmology is the cosmological principle which is based on the fact that the spacetime is Lorentz invariant. Nevertheless, physical problems such as that primordial magnetic fields (PMF) [2], matter antimatter asymmetry and dark energy (matter) lead us to rethink Lorentz invariant Einsteinian relativity dogma. One of the ways to address this issue has been recently discovered by Kostelecky et al [3], which considers that LV could be associated to another alternative gravity theory called torsion theory [4]. An important issue at this point is to stress that here as in most string inspired Kalb Rammond theory, torsion propagates in vacuum instead of being considered as a contact interaction as in Einstein-Cartan gravitation [5]. In this paper we shall be concerned with some examples where LV not only is present in spacetime but it can be used to place limits in spacetime torsion by its manifestations on dynamo effects and GBRs and cosmic microwave background (CMB) with an interesting analogy of electromagnetic (EM) waves in magnetised plasmas [6]. Faraday rotation, which is so important in measuring magnetic fields [7] is used to place constraints on LV torsion vector. An important issue is that here one adopts perturbative approach to quantum electrodynamics (QED) effects and not the non-perturbative ones used by Enqvist et al [8]. The idea of using this EW analogy in GRB ultraviolet LV has been used by Kahniatishvilly et al [9] in Minkowski spacetime without torsion. One of the main differences between their results and ours, is that to EM waves frequency a torsion wave frequency is summed up, and only in the very low frequency torsion waves, they approach. In this first paper one takes in account this low frequency limit and left the high frequency torsion wave limit to a future work. One also consider here a linearised approach to dispersive and rotations measures. The paper is organised as follows: In section II the de Sabbata-Gasperini formulation of the Riemann-Cartan (RC) Maxwells vacuum electrodynamics, with photon-torsion semi-minimal coupling is reviewed. In section III the EM waves analogy in magnetised plasma is considered and the CMB torsion limit is placed in the low frequency torsion wave limit. In section IV galactic magnetic dynamo seeds are used to place limits on LV through torsion modes. In section V conclusions and discussions are presented.
2 Perturbative QED in Minkowski torsioned spacetime

Throughout the paper second order effects on torsion shall be neglected in the electrodynamics including LV terms due to the three-dimensional torsion vector $Q$. In this section we consider a simple cosmological application concerning the electrodynamics in vacuum QED spacetime background. Perturbative approach to electrodynamics leads to the following set of equations

\[ \partial_i F^{ji} = 4\pi j^i + \frac{2\alpha}{3\pi} \epsilon^{jklm} Q_l F_{kj} \]  
\[ \partial_i [F_{jk}] = 0 \]

where $\partial_i$ is the partial derivative, and $\alpha$ is the e.m fine structure constant, while $F^{ij} = \partial^i A^j - \partial^j A^i$ is the electromagnetic field tensor non-minimally coupled to torsion gravity. Here $A^i$ is the electromagnetic vector potential and $(i, k = 0, 1, 2, 3)$ and $Q_l$ represents the torsion four-vector. In three-dimensional notation the above Maxwell's generalised equations read

\[ \nabla \cdot E = 4\pi \rho + \frac{4\alpha}{3\pi} Q \cdot B \]  
\[ \nabla \cdot B = 0 \]
\[ \nabla \times E = -\frac{\partial B}{\partial t} \]
\[ \nabla \times B = \frac{4\alpha}{3\pi} E \times Q + \frac{\partial E}{\partial t} \]

After some algebraic manipulation on these generalised Maxwell equations one obtains the EM wave equations

\[ \nabla^2 E - \frac{\partial^2 E}{\partial t^2} + \frac{4\alpha}{3\pi} [Q \times \frac{\partial E}{\partial t} - E \times \frac{\partial Q}{\partial t}] = 0 \]  
\[ \nabla^2 B - \frac{\partial^2 B}{\partial t^2} - \frac{16\alpha}{3} \rho Q - \left(\frac{4\alpha}{3\pi}\right)^2 Q(B) = 0 \]

By Fourier analyzing the first expression or substituting $\partial_t \rightarrow i\omega$ and $\nabla \rightarrow -ik$ one obtains from expression (7) the following expression

\[ [(\omega^2 - k^2)\delta_{ab} - \frac{4\alpha}{3\pi} i(\omega_1 + \omega)\epsilon_{ac}Q^c]E^b = 0 \]
\[ \frac{4\alpha}{3\pi} Q e_c E_c \] (10)

where \((a, b = 1, 2, 3)\), \(\omega\) is the EM wave frequency while \(\omega_1\) is the torsion wave frequency. Here we also chose the charge density \(\rho = 0\) since we are adopting vacuum QED. The dispersion relation is given by

\[ \omega^2 \mp (\omega_1 + \omega) k Q - k^2 [1 \mp \gamma] = 0 \] (11)

where \(\gamma(k)\) is the photon-spin-sign-dependent term on the LHS of equation (11), to account for the phenomenological LV of an energy-dependent photon speed. Now by considering the analogy to EM waves in a magnetised plasma with an index of refraction of refraction of \(n = \frac{k}{\omega}\), one obtains

\[ n_{L,R} = (\epsilon_1 \pm \epsilon_2)^{\frac{1}{2}} \] (12)

where \(\epsilon\) is the electric permittivity. From the dispersion relation above one obtains the permittivities

\[ \epsilon_1 = \frac{1}{(1 \pm \gamma(k))} \] (13)

\[ \epsilon_2 = -\frac{(\omega_1 + \omega) Q}{(1 \pm \gamma(k))} \approx (\omega + \omega_1) Q \] (14)

From the approximation of low torsion frequency \(\omega_1 << \omega\), this can be dropped in the last expression and torsion vector reduces to the g LV vector used by Khniatishivilly et al. Thus these expressions one obtains the refractive index

\[ n_{L,R} = (1 \pm \omega Q \pm \gamma(k))^{\frac{1}{2}} \] (15)

By making the approximation \(\gamma \ll Q\omega\) the refractive index reduces to

\[ n_{L,R} \approx (1 \pm \omega Q)^{\frac{1}{2}} = \frac{k}{\omega} \] (16)

Now the dispersion measure and rotation measure (RM) of the GRBs depend on the photon travel distance \(\Delta l\) and are expressed as

\[ \Delta t_{L,R} = \Delta l (1 - \frac{\partial k_{L,R}}{\partial \omega}) \] (17)

\[ \Delta \phi = \frac{1}{2} (k_L - k_R) \Delta l \] (18)
where $\phi$ is the polarization plane rotation of the electric field describing the Faraday rotation. These expressions can be written in terms of torsion by

$$\Delta t_{L,R} = \pm \Delta l \omega Q$$

(19)

$$\Delta \phi \approx \frac{1}{2} \omega^2 Q \Delta l$$

(20)

Therefore, when the photon-spin is damped by the torsion wave, the Faraday rotation of $\Delta \phi \approx 10^{-2} \text{rad}$, allows one to estimate torsion as

$$Q_{CMB} \approx 10^{-18} \text{GeV}$$

(21)

thus establishing new limits for LV from torsin distinct from those of Kostelecky et al.

3 Galactic dynamo seeds constraints to LV in spacetime with torsion

Recently a more complicated approach to place constraints on LV from galactic dynamo magnetic seed fields appeared in the literature [3, 4]. Here following the perturbative method above and the magnetic field equation one obtains a much simpler and straightforward method of placing limits on LV from torsion and galactic dynamo seeds. Performing the Fourier spectrum of the magnetic field equation yields

$$[\left(\omega^2 - k^2\right) \delta_{ab} + \frac{16\alpha^2}{9\pi} Q_a Q_b] B^b = 0$$

(22)

which yields the following dispersion relation

$$\omega^2 = k^2 + \frac{16\alpha^2}{9\pi^2} Q^2$$

(23)

Actually $\omega$ coincides with dynamo growth rate $\gamma_0$ with the ansatz

$$B(t) = B_0 e^{\gamma t}$$

(24)
From the dispersion relation one may conclude that in order that torsion may contribute to dynamo action it must be comparable with the large scale coherence which is the inverse of the wave vector $k$, under the law

$$k^2 \approx \frac{16\alpha^2}{9\pi^2}Q^2$$

(25)

As for today coherence scales torsion would be extremely weak and of the order of $Q \approx 10^{-21} cm^{-1}$. This is exactly the estimate obtained by Laemmerzahl [?] on the basis of Earth laboratory Hughes-Drever experiment. It is interesting to note that Kostelecky et al have also obtained LV with table top experiments on Earth lab.

## 4 Discussion and conclusions

The investigation of Faraday rotation has been proved very important in high-energy astronomy of magnetic fields in the universe. Here one uses Faraday rotation to establish limits of the LV in terms of torsion as established by Kostelecky et al with torsion being a constant vector. Actually here torsion vector is not constant though LV is attainable. Methods used here were previously investigated by Kahniatshivilly et al in the context of GRBs in torsionless Minkowski spacetime. Dynamo plasma is obtained from the dispersion relation where torsion can be expressed in terms of the coherence scale of magnetic fields. Quantum effects may be obtained here from perturbative QED instead of non-perturbative primordial magnetic fields obtained by Enqvist.

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