Spin-transfer torque induced spin waves in antiferromagnetic insulators

Matthew W Daniels\textsuperscript{1,5}, Wei Guo\textsuperscript{3,2}, G Malcolm Stocks\textsuperscript{3}, Di Xiao\textsuperscript{1} and Jiang Xiao\textsuperscript{1,4}

\textsuperscript{1} Department of Physics, Carnegie Mellon University, Pittsburgh, PA 15213, USA
\textsuperscript{2} Department of Physics and State Key Laboratory of Surface Physics, Fudan University, Shanghai 200433, People’s Republic of China
\textsuperscript{3} Materials Science and Technology Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831, USA
\textsuperscript{4} Collaborative Innovation Center of Advanced Microstructures, Fudan University, Shanghai, 200433, People’s Republic of China
\textsuperscript{5} These authors contributed equally

E-mail: dixiao@cmu.edu and xiaojiang@fudan.edu.cn

Keywords: spintronics, antiferromagnetism, magnonics, magnon, antiferromagnet, surface, spin-transfer torque

Abstract

We explore the possibility of exciting spin waves in insulating antiferromagnetic films by injecting spin current at the surface. We analyze both magnetically compensated and uncompensated interfaces. We find that the spin current induced spin-transfer torque can excite spin waves in insulating antiferromagnetic materials and that the chirality of the excited spin wave is determined by the polarization of the injected spin current. Furthermore, the presence of magnetic surface anisotropy can greatly increase the accessibility of these excitations.

1. Introduction

The generation, control, and detection of pure spin current carried by spin waves is the subject of magnonics, [1–3] an emerging subfield of spintronics [4–7]. Typically studied in magnetic insulators, magnonics represents a promising platform for next-generation energy-efficient device architectures because spin wave transmission does not induce Joule heating.

So far the research efforts in magnonics have focused on ferromagnetic or effectively ferromagnetic insulators [1–3]. On the other hand, antiferromagnetic insulators are abundant in nature. Compared to ferromagnets, antiferromagnets (AFM) present several technological advantages. Their lack of stray field simplifies the issue of packing multiple nanoscale magnonic devices in close proximity. The resonant frequency of spin waves in AFM is typically THz, making faster dynamics and previously unused communications frequencies more readily available. Finally, in AFMs with easy-axis anisotropy, the spin wave modes are doubly degenerate with opposite chirality, offering a new degree of freedom for information encoding and processing. These advantages, together with recent experimental progress [8–11], make AFMs an attractive candidate for magnonics [12–14].

To realize a magnonic circuit in an AFM, the first priority is understanding the generation of spin waves. Optical excitation methods exist [15], but one would generally prefer an electrical procedure. Recently, Cheng et al showed that the spin-transfer torque (STT) [16–19] can be used to excite magnon modes in easy-axis AFMs [20]; the critical current required \textit{a priori} to excite bulk modes in a typical AFM, though, is too large to be realistically produced by electronic means. One possible way to counter this problem is to excite surface spin waves [21]. It has been shown in ferromagnets that surface anisotropy can significantly lower the spin wave frequency and hence the critical current needed to excite the spin waves [22]. Our present concern is addressing whether a similar reduction in the critical current may be found for surface waves in AFM, bringing electrical excitation down to a more realistic regime.

In this article, we address this possibility of lowering the effective excitation threshold for AFM spin waves by considering the surface spin wave modes of AFMs. Our prediction relies on the fact that surface atoms in certain AFMs can have an effective exchange energy significantly lower than the bulk value [22–24]. One would then...
expect that the resonant frequency of spin waves localized to these exchange-reduced atoms will have lower excitation thresholds and be easier to excite. We compute the surface spin wave spectra of antiferromagnetic insulators with both magnetically compensated and uncompensated surfaces, and show that certain surface modes are, as expected, lower in energy. We then include a contribution from STT due to a polarized spin current injected at the surface. We find that this STT is sufficient to excite the surface spin waves in low-surface-exchange systems, which demonstrates a step toward making AFM magnonics more easily realizable in experiment. We also verify that the sign of the STT determines the handedness of the chiral AFM spin wave, as predicted [20, 25].

This paper is organized as follows: in section 2, we briefly review the eigenstructure of spin waves in antiferromagnetic systems by appealing to a simple two-spin system. In section 3, we review solutions of the LLG equations on an infinite cubic AFM lattice, and then present new results on AFM spin wave spectra for a semi-infinite system with an interface. Based on previous work [22], we expect that surface effects, by their role in modifying the magnons’ activation threshold, will play an important part in the experimental realization of spin wave modes. In particular, we explore variations of the exchange coupling on the surfaces with both compensated and uncompensated net magnetizations. In section 4, we offer concluding remarks and an application for experimental methods.

2. Macrospin model

To provide a conceptual account of the mechanism underlying STT-generated spin waves in AFM materials, we present a minimal model which includes the important physical terms without the complications of a spatially extended lattice. This so-called ‘macrospin model’ has been considered by many in the past, and we include it here not as new work but as a pedagogical tool to illustrate the mechanism by which STT determines AFM spin wave handedness. This idea is extended to our core result on a semi-infinite lattice in section 3.

In the macrospin model, the magnetization on the two sublattices are modeled as two macrospins [26], \( m_+ \) and \( m_- \), which are coupled by a constant Heisenberg-type exchange interaction \( J_H \). They are additionally subject to Gilbert damping \( \alpha \) and spin transfer torque \( \omega_s/\gamma \); the latter is due to an injected spin current polarized along the \( \hat{z} \) direction. Both macrospins experience a uniaxial anisotropy \( \omega_A \) in the \( \hat{z} \) direction [27]. We also allow for an external magnetic field \( H_0 \hat{z} \) along this axis. The setup yields an equation of motion

\[
\dot{m}_\pm = -m_\pm \times H_{\text{eff}}^\pm + \alpha m_\pm \times m_\mp + \omega_s m_\mp \times (\hat{z} \times m_\pm) \\
= -m_\pm \times \left[ -\omega_H m_\mp + \left( \omega_A m_{\pm z} + \omega_H \right) \hat{z} \right] \\
+ \alpha m_\pm \times m_\mp + \omega_s m_\mp \times (\hat{z} \times m_\pm),
\]

where \( \omega_H = \gamma H_0 \). The effective field term \( H_{\text{eff}}^\pm \) is the negative derivative \( -\nabla m \) \( H \) of the Hamiltonian

\[
H = \omega_H m_\pm \cdot \hat{m} - \omega_H \left( m_{\pm z}^2 + m_{\pm z}^2 \right) - \frac{\omega_A}{2} \left( m_{\pm z}^2 + m_{\pm z}^2 \right) = \frac{\omega_s}{2} \left( m_{\pm z}^2 + m_{\pm z}^2 \right)
\]

where the damping term is added phenomenologically [28], and the STT term—where \( \omega_s \) is linear in the applied spin voltage—is due to [20]; we partially rederive it for the reader in appendix A.

In the small angle approximation, we demand that the deviation \( \theta = 1 + O(\theta^2) \) and \( m_{\pm x} \propto \theta + O(\theta^3) \). Now the \( \hat{z} \)-component of equation (1) vanishes to order \( O(\theta^2) \), and the problem is reduced to two effective dimensions in the \( xy \)-plane. We can exchange these two dimensions for a simple complex one by defining the transverse magnetization \( u \equiv m_x + im_y \) and rewriting equation (1) in terms of this new variable. We then employ a spin wave ansatz \( u_\pm = e^{i\omega t} \) which allows us to solve for the modes that satisfy equation (1). In the small-angle approximation, these eigenfrequencies of precession are

\[
\omega_\pm = \pm \omega_0 - i \alpha \left( \omega_f + \omega_A \right) \left( 1 \mp \frac{\omega_A}{\omega_f} \right).
\]

The resonant frequency in the absence of damping and STT is \( \omega_0 = \sqrt{\omega_A (\omega_A + 2\omega_f)} \). In AFM, two degenerate modes with opposite chirality appear in equation (3). This equation highlights the essential competition between STT and precessional damping: when the applied spin current is sufficiently strong, the dissipative term (second term) in equation (3) changes sign and selectively excites one of the \( \omega_\pm \) modes depending on the sign of \( \omega_s \).

Therefore, spin waves with different chirality can be selectively excited according to the spin current polarization. This behavior is different from STT-induced FM dynamics, for which only one polarization of spin current can excite FM spin waves since FM magnons can only accept spin antiparallel to the local magnetization. We plot a solution of these equations in figure 1. We see on the left of the figure that pumping a positive \( \hat{S}_c \) current into the
system via the STT mechanism described above preferentially selects a mode with positive net magnetization \( m_z = +1 \).

In appendix B, we carry this derivation a bit further to demonstrate explicitly the degree of magnetization carried by the macrospin model in one of its chiral eigenstates. The interested reader may refer there for more detail.

3. Lattice calculation

To consider a more realistic system than that of section 2, we now extend the Heisenberg-type Hamiltonian (2) to a simple cubic lattice as in [29]:

\[
H = \sum_{\langle i,j \rangle} \omega_{ij} \mathbf{m}_i \cdot \mathbf{m}_j - \sum_j \left( \omega_{H} + \frac{\omega_{P}}{2} m_{j,z} \right) m_{j,z},
\]

where the subscripts \( i \) and \( j \) are lattice sites and the first sum is taken over nearest neighbors.

We are interested in both magnetically compensated (left) and uncompensated (right) surfaces, as depicted in figure 2. We take g- and a-type AFM as prototypical textures for these two cases. g-type AFMs are widely studied; some examples are \( \text{La}_2\text{CuO}_4 \) [30, 31] and \( \text{BiFeO}_3 \) [32]. On the other hand, there are quite a few a-type AFMs, mostly in layered magnetic compounds such as \( \text{RbMnCl}_3 \) [33], and certain transition metal trichalcogenides [34]. Even for AFMs with cubic symmetry, depending on the cut angle, the surface can be uncompensated as well, such as (111) \( \text{NiO} \) [35] or (111) \( \text{BiFeO}_3 \).

We take the lattice constant as \( a = 1 \) so that the wavevector is dimensionless. The derivation from section 2 is repeated for the Hamiltonian in equation (4) to derive an effective on-site magnetic field. Knowledge of this field determines the LLG equation for \( \mathbf{m}_j \), namely:
\[ m_j = -m_j \times \left[ \sum_{\langle i,j \rangle} \omega_i m_i - \left( \omega_H - \omega_j m_{j,z} \right) \hat{z} \right] + \alpha m_j \times m_j. \] (5)

The exchange coefficients \( \omega_{ij} \) will be uniformly constant \( \omega_{ij} = \omega_j \) for the \( g \)-type system where all nearest neighbors are the same, though for the \( a \)-type system we will need to distinguish \( \omega_j = \omega_{1j} < 0 \) and \( \omega_j = \omega_{2j} > 0 \) for the coupling between inter- and intra-plane (respectively AFM-like and FM-like) neighbors.

By assuming a small precession of \( m \) about its easy-axis, the \( \hat{z} \)-component of the LLG equation (5) can be neglected to first order. We then rewrite the equation of motion in terms of the transverse magnetization \( u \equiv m_{x,u} + im_{y,u} \) as in section 2. Translational symmetry in time and the \( yz \)-plane validates the plane wave ansatz

\[ u_{(q,s)} = \mu^\pm_{q,s} e^{i(qs - \omega t)}. \] (6)

where \( j \) is the layer index in the \( \hat{z} \)-direction and \( q \) is the wave vector in \( yz \)-plane. We substitute this equality into the transverse magnetization equation. From now on we will use \( k \) to refer to a 3D wavevector and \( q \) will be \( k \)'s restriction in the \( yz \)-plane.

With these modifications, the LLG equation (5) is rewritten as a recurrence relation among different layers

\[ S\psi_j + N_+ \psi_{j+1} + N_- \psi_{j-1} = 0 \] (7)

with \( \psi_j = (\mu^\pm_{q,j})^T \). The square matrices \( S, N_+, \) and \( N_- \) can be computed directly from considering the coefficients in equation (5). For \( g \)-type,

\[ S^{(g)}(q) = \begin{pmatrix} \omega - \omega_H - \Omega & -\omega_q^{(g)} \\ -\omega_q^{(g)} & \omega - \omega_H + \Omega \end{pmatrix}, \]

\[ N^{(g)}_+ = N^{(g)}_- = \begin{pmatrix} 0 & -\omega_j \\ \omega_j & 0 \end{pmatrix}, \] (8a, b)

with \( \Omega = 6\omega_j + \omega_{1j} - i\omega_j \) and \( \omega_q^{(g)} = 2\omega_j (\cos q_x + \cos q_y) \). For \( a \)-type,

\[ S^{(a)}(q) = \begin{pmatrix} \omega - \omega_H - \Omega_q & -\omega_{1,j} \\ -\omega_{1,j} & \omega - \omega_H + \Omega_q \end{pmatrix}, \]

\[ N^{(a)}_+ = \begin{pmatrix} 0 & -\omega_{1,j} \\ \omega_{1,j} & 0 \end{pmatrix}, \]

\[ N^{(a)}_- = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \] (9a, b)

with \( \Omega_q = 2\omega_{1,j} + \omega_{1,j}' - i\omega_j \) and \( \omega_{1,j}' = 2\omega_j (2 - \cos q_x - \cos q_y) \).

### 3.1. Bulk calculation

For comparison with our results for a semi-infinite lattice in section 3.2, we pause to reproduce the bulk spin wave spectrum this formalism. The reader may refer to [29], or any condensed matter textbook, for a more complete discussion of the g-type spectral calculation. The \( a \)-type calculation is similar except that the primitive lattice vectors differ.

In addition to the translational symmetries used in the previous section, a bulk lattice possesses an additional translational symmetry in the \( \hat{x} \) direction. Therefore we may take a plane wave solution for the \( x \)-coordinate: \( \psi_j = |\phi(q)| e^{i(qj - \omega t)} \). We can then find the eigenfrequencies of AFM spin waves. For \( g \)-type,

\[ \omega_{q,3}^g = \omega_H \pm \sqrt{\Omega^2 - \omega_k^2}, \] (10)

with \( \omega_k = 2\omega_j (\cos k_x + \cos k_y + \cos k_z) \). For the \( a \)-type lattice, we have an infinite stack of alternating ferromagnetic sheets. We choose \( \hat{x} \) to be the direction normal to any given sheet. Then the bulk dispersion is

\[ \omega_{q,a} = \omega_H \pm \sqrt{\Omega^2_q - 4\omega_{1,q}^2 \cos^2 k_x}. \] (11)

One can verify that in the limit \( \omega_j = 0 \) we recover decoupled 1D AF chains with the expected dispersion \( \pm 2\omega_j \sin k_x \) in the simple isotropic case [36]. Likewise, the \( \omega_{1,j} = 0 \) limit recovers a decoupled 2D ferromagnetic system, with dispersion \( \pm 2\omega_j \cos k_y + \cos k_z - 2 \) in the same simple case.

The spin wave eigenfunctions corresponding to equations (10) and (11) are respectively

\[ \phi_{q,3}^g = \frac{\sqrt{\Omega^2 + (\omega - \omega_H)}}{\sqrt{\Omega^2 + (\omega - \omega_H)}}, \] (12a)
and \( \varphi(\omega) = \begin{pmatrix} e^{i k x} \sqrt{\Delta_1} (\omega - \omega_R) \\ -e^{i k x} \sqrt{\Delta_1} (\omega - \omega_I) \end{pmatrix} \).

The dispersion relations \( (10) \) and \( (11) \) enforce a constraint linking the irreducible representations of time \( (\omega) \) and space \( (k) \) translational symmetries. For any particular \( \omega \) and \( q \), there are only a finite number of \( k \) values in its preimage under the eigenvalue equations \( (10) \) and \( (11) \). In the next section, we will need to consider linear combinations of bulk solutions to satisfy the boundary condition. The dispersion relations above will allow us to consider only a small subset of all conceivable wavenumbers \( k_x \).

### 3.2. The semi-infinite case

We now introduce an interface by terminating the insulator along its \( (100) \) plane and replacing the space \( x < 0 \) with a non-magnetic contact from which spin current can be injected; see figure 2. We will modify the equations of motion to allow for special conditions on the atomic surface layer at \( x = 0 \).

First, an enhanced damping term \( [37] \) is inserted into the LLG equation by taking \( \alpha \mapsto \alpha + \beta \delta \) where \( \beta \) is the enhanced damping parameter for the surface spins. This enhanced damping represents spin loss due to spin pumping from the AFM back into the NM contact. The STT term \( \omega m_j \times (\hat{x} \times m_j) \delta \bar{g}_{\bar{a}, a} \) is likewise included on the atomic surface layer. This is an anti-damping term; formally, there is also a field-like term. This field-like term, however, is proportional to the imaginary part of the spin mixing conductance, which is several orders of magnitude smaller than the real part which gives the scale of the anti-damping contribution \[20\]. Therefore, we keep only the anti-damping term in this calculation. Finally, as a form of surface anisotropy, we allow a modulation of the intralayer exchange coupling represented by the ratio \( \epsilon \equiv \omega_1^\text{surf} / \omega_1^\text{bulk} \) (or \( \omega_0^\text{surf} / \omega_0^\text{bulk} \)). It is known that this type of surface anisotropy can modulate the surface spin wave modes in AFM \[29\]. The variation in the exchange energy at the interface of magnetic materials has been studied by many groups. For instance, numerical studies on NiO(100) interfaces have shown that, depending on the assumptions of the model, surface exchange energy can vary by at least 20% with some groups showing as much as a 50% variation \[23\] from the bulk coupling.

We can write new equations of motion for this semi-infinite system as:

\[
\begin{align*}
(S + B)\psi_0 + N_+\psi_1 &= 0 \quad (j = 0), \\
S\psi_j + N_-\psi_{j+1} + N_+\psi_{j-1} &= 0 \quad (j > 0),
\end{align*}
\]

where, for \( g \)- and \( a \)-types, respectively:

\[
\begin{align*}
B^{(g)} &= \left( (5 - 4\epsilon)\omega j + i\left( \omega j + \beta \omega \right) \sigma_\perp + \omega_0 j (1 - \epsilon) i \sigma_\parallel \right), \\
B^{(a)} &= -\left[ \omega_0 j + \omega_0 j (1 - \epsilon) - i\left( \omega j + \beta \omega \right) \right] \frac{1 - \sigma_\perp}{2},
\end{align*}
\]

and \( \sigma_{\perp, \parallel} \) are the Pauli matrices.

We now take the bulk eigenvectors \( \varphi_{\pm} \) in equation \( (12) \) for the \( g \)-type as a basis for general solutions to a semi-infinite lattice configuration. By using the bulk dispersion relations, \( \varphi_{\pm} \) can be rewritten in terms of a distinguished eigenvalue \( \omega \) and trigonometric functions of \( k_x \) as in equation \( (12) \). Recall from the conclusion of section 3.1 that for a particular value of \( \omega = \omega(q) \) the irreducible representation \( k_x \) is restricted to the finite set of values \( k_x \in \omega^{-1}_{0,\pm} (\omega(q)) \). We will call these at most four values by \( k_{x_0} \). Since the cosine function is even, we see that two of the \( k_{x_0} \) values are related by a sign change to the other two. As will become clear in sections 3.2.1 and 3.2.2, we demand that \( \mathcal{E}(k_{x_0}) \) be positive so that surface solutions decay into the bulk. Then two allowed values of \( k_{x_0} \) remain, which we call \( k^+ \) and \( k^- \).

We can now consider solutions of the form

\[
\psi_j = \eta_+ \varphi_+ e^{i(k_j - \omega_j)} + \eta_- \varphi_- e^{i(k_j - \omega_j)},
\]

where \( \varphi_{\pm} \) are the bulk eigenvectors corresponding to \( k^\pm \), which are the only allowed wavenumbers \( k_x \) in the preimage of the bulk \( \omega \).

#### 3.2.1. \( g \)-type, with compensated surface

Using equation \( (15) \), the boundary condition equation \( (13a) \) for the compensated \( g \)-type system takes the form

\[
\text{det} \left[ B(\varphi_+, \varphi_-) + N \left( \varphi_+ e^{ik^+} - \varphi_- e^{ik^-} \right) \right] = 0.
\]

The exponentials \( e^{ik^\pm} \) can be determined from solving the eigenvalue equations equation \( (10), (11) \) for \( \cos k_x \), employing the Pythagorean identity to expand Euler’s formula, and demanding solutions \( \mathcal{E}(k_{x_0}) > 0 \) which decay into the bulk. Taken together with equation \( (10) \), this equation can be solved analytically for \( \omega \) when
\[ \alpha = \beta = \omega_s = 0. \] This unperturbed eigenfrequency is then used to calculate constant perturbations—namely the \( i\alpha \omega \) and \( \beta \omega \) terms—so that equation (16) can be evaluated to leading order in the presence of damping and STT with straightforward modifications to its coefficients. The results in the complex eigenfrequencies \( \omega = \omega_r + i\omega_i \) are plotted in figure 3, wherein the bulk modes are plotted as the shaded area and the surface modes are plotted in colored curves.

The dispersion relations of \( \omega_r \) for the surface modes of this system are plotted in figure 3 over a spectrum of surface exchange ratios \( \omega_s \). These surface modes are the same as those calculated in [29]. The spin wave profiles for the surface modes are presented in lower panels of the figure. These figures also reveal that the surface modes in a \( g \)-type AFM can be either acoustic or optical; a detailed discussion of the acoustic/optical transition as a function of \( \omega_s \) is given in [29].

Beyond the dispersion relations, we are also interested in the dissipative behavior of various spin wave modes. Especially of interest are their behavior under the influence a STT due to spin current injection from the NM contact. The second panel of figure 3 shows \( \omega_r \) when there is only intrinsic damping included. In this case there is neither spin pumping or STT, and we plot both the bulk modes (shaded continuum) and the surface modes (colored curves) for different values of the surface anisotropy \( \epsilon \). With the additional NM contact at the surface, the spin pumping into NM from AFM increases the dissipation for the surface spin wave modes, as seen in the third panel in figure 3. Far from the \( \gamma \) point, the effective damping enhancement (in the language of [38]) is \( \Delta \alpha \approx \beta \). This is expected since this regime corresponds to high surface localization, wherein \( \beta \) is effectively just added to \( \alpha \) in the local LLG equations. Introduction of a STT can dramatically decrease the damping of some surface spin waves, especially in the low-\( \epsilon \) regime. The low damping combined with low excitation energy makes these low-\( \epsilon \) modes particularly excitable due to strong surface localization. Strong enough \( \omega_s \) together with low \( \epsilon \) (weakened surface exchange coupling) can cause sign changes in \( \omega_r \) and lead to AFM spin wave excitation, as in figure 3.
the last panel of figure 3. Furthermore, STT distinguishes the two spin wave chiralities by enhancing the damping of one while reducing the other. Precisely which chirality is excited depends on the spin current polarization, so that it is distinctly possible to selectively excite a particular chiral mode.

3.2.2. a-type, with uncompensated surface
For the uncompensated surface in an a-type AFM insulator, there is effectively only one $k_x$ which satisfies both the bulk eigenfrequency equations and the reality condition $\Im(k_x) > 0$ for any given $\omega$. The reasoning follows: first, the orientation of the unit cell is necessarily different in the a-type system, so that the coupling to the next unit cell along the $x$-direction requires a factor of $e^{2i k}$ rather than just $e^{i k}$ in the a-type analog to equation (15); second, solving equation (11) for $k_x$ gives a family of four solutions—namely $k, \pi + k$, and $\pi - k$—but as we mentioned in section 3.2, only one of $k$ and $-k$ will have a positive imaginary part, and they furthermore will each appear identical to their $\pi$-shifted partners when expressed in the form $e^{i k}$. This simplifies the form of the boundary condition equation (13a), as well as the a-type analog of equation (16). A procedure similar to that employed in the previous section is used to solve the unperturbed and then perturbed versions of this equation.

The spin wave dispersion $\omega^e$ for an a-type AFM is different from that for g-type AFM; this is evident in the left panel of figure 4. However, the surface anisotropy still modifies the surface spin wave modes. Typical surface mode profiles are shown below the dispersion plots. In the absorption spectra (right three panels of figure 4), the spin pumping (third panel) enhances the dissipation for both chiralities (again at $\Delta \alpha \approx \beta$) while STT reduces the dissipation for one chirality and enhances the other. These results coincide with the outcomes of section 3.2.1 for the g-type configuration, again distinguishing spin wave chiralities and demonstrating that a non-zero $\omega$ in the a-type system can cause a change in sign of the absorption spectrum, and can consequently excite spin wave modes.

Figure 4. Frequency dispersion for the a-type semi-infinite system with $(\omega_q, \omega_s) = (0.5, 1)$. From top left to top right: $\omega_q$ and then $\omega_q / \alpha \omega$; $\beta = \omega_s = 0$; $\beta = \alpha / 4$ but $\omega_s = 0$; and finally, $\omega_s = \alpha$ while $\beta = \alpha / 4$. In all cases, we use $\alpha = 0.01$. The gray regions indicate the bulk spectrum. The horizontal axes measure $\omega_q \equiv \omega_q$, so that $\omega_q = 0$ corresponds to the $\Gamma$ point in the surface Brillouin zone. An array of spin wave profiles plotting the magnitudes of $\psi^e_j = \psi^o_j \sqrt{e^{2i k}}$ at $\omega_q = 0.3$ is shown below.
4. Conclusions

In this article, we have calculated the spin wave spectrum of STT-induced AFM surface excitations. In particular, we found that surface spin wave modes modulated by surface anisotropy are particularly easy to excite compared to bulk modes, implying a lowering of the critical current required to perform magnonic operations in AFM insulators.

The efficiency of spin pumping processes in AFMs is known to be comparable to their ferromagnetic cousins [20]. However, because AFMs have a much stronger exchange coupling, an \textit{a priori} estimate of the threshold current for exciting AFM surface spin waves is two to three orders of magnitude higher than in ferromagnet insulators. Nevertheless, the critical current for exciting a ferromagnetic magnon current was found in [39] to be two to three orders of magnitude lower than the expectation initially accorded to YIG’s resonant frequency. If the same unforeseen reduction occurs in AFM, then the critical current would be on the order \( I_c \approx 10^8 \text{A cm}^{-2} \), which is within experimental feasibility. Our contribution is to take a first step in investigating this potential reduction in the critical barrier. One may of course seek materials with appropriate exchange or anisotropy energies in accordance with equation (3) in order to lower the barrier; we find that seeking materials with low surface exchange coupling reduces the threshold further.

Our work also takes a first step toward developing new experimental techniques for investigating AFM. Because it is relatively straightforward to generate a spin current and measure spin waves, STT-based methods could provide a new tool for probing and controlling AF materials. In particular, parameters such as damping, anisotropy, or surface exchange coupling could be inferred by retrofitting experimental data to models like those we present here. Since this data would be obtained by purely electrical means via a polarized spin current, it could be considerably easier to collect than neutron scattering results. Such a method could be a powerful complement to current experimental procedures, but is intractable without an understanding of the spin wave response to surface STT akin to that which we have outlined above. In any case, such a scheme would require considerable refinement to what we have presented in this article; one would want to keep higher order terms, introduce another thin-film boundary, and break translational invariance along the surface. Treating non-single-crystal AFMs would introduce even more complication. We leave these details to future research, noting here only that continual improvement of our understanding of AFM spin waves should begin to open new routes to experimental investigation on the topic.

Acknowledgments

This work was supported by the National Science Foundation, Office of Emerging Frontiers in Research and Innovation EFRI-1433496 (MWD and DX), the US Department of Energy, Office of Basic Energy Sciences, Materials Sciences and Engineering Division (GMS), and by the special funds for the Major State Basic Research Project of China (grants No. 2014CB921600, No. 2011CB925601) and the National Natural Science Foundation of China (grant No. 91121002) (WG and JX).

Appendix A. STT on AFM

In section 2 we presented an extended LLG equation of motion which included a STT term induced by a \( \hat{z} \)-polarized spin current: \( \tau_n = \omega_s \hat{m}_{\pm} \times (\hat{m} \times \mathbf{v}) \). This form of term is plausible phenomenologically, but in this appendix we provide a more rigorous derivation of its physical content.

We begin from equation (6) of [20], which provide the STT on the \( \mathbf{m} \) (magnetization) and \( \mathbf{n} \) (staggered) sublattices due to an applied spin voltage \( \mathbf{V} \),

\[
\tau_n = -\frac{a^3}{e\mathcal{V}} G_n \mathbf{n} \times (\mathbf{m} \times \mathbf{V}), \tag{A.1}
\]

\[
\tau_m = -\frac{a^3}{e\mathcal{V}} G_m \mathbf{n} \times (\mathbf{n} \times \mathbf{V}), \tag{A.2}
\]

where \( \mathcal{V} \) is the volume of the system, \( a \) is the lattice constant, and \( G_n \) is the real part of the spin mixing conductance for an NM \slash{} AFM interface; the corresponding imaginary part of \( G \) is several orders of magnitudes smaller [20] and consequently ignored. By definition, we have \( \mathbf{m}_{\pm} = \mathbf{m} \pm \mathbf{n} \) on the two sublattices from section 2. Thus \( \tau_{\pm} = \tau_{m\pm} + \tau_{m\pm} \) and the use of equations (A.1), (A.2) gives

\[
\tau_{\pm} = -\frac{a^3}{e\mathcal{V}} \mathbf{n} \times (\pm G_{m\pm} \mathbf{m}_{\pm} \times \mathbf{V}). \tag{A.3}
\]
Now, as in the main text, we take the spin voltage to be collinear with the easy-axis \( \hat{z} \): \( V_z = V_z \hat{z} \). Allowing \( n \approx 2m_+ \approx -2m_- \), we have

\[
\tau_\pm = \frac{a^3 V_e}{eV} G_i m_\pm \times (\hat{z} \times m_\pm),
\]

and we define the relevant constant of proportionality as \( \omega_i = (a^3 V_e/eV) G_i \), thus achieving the form exhibited in equation (1).

### Appendix B. Chiral modes in the macrospin model

In section 2, we solved a two-spin system coupled by antiferromagnetic exchange and including relevant terms (anisotropy, STT, etc.) that were needed in the main text. Here, we push that derivation a bit further to provide the reader with some more qualitative details concerning the chiral eigenmodes that emerge from this prototype system.

We can re-express \( m_\pm \) in spherical angular coordinates \( (\theta^\pm, \varphi^\pm) \) and derive a set of exact, coupled ODEs for this system directly from the coupled LLG equations of section 2. \( \theta^\pm \) is taken to be the polar angle between \( m_\pm \) and \( \hat{z} \), and \( \phi^\pm \) is the corresponding azimuthal angle. We find

\[
\begin{align*}
\dot{\theta}^+ &= \omega_\theta \sin \Delta \varphi \sin \theta^+ - (\alpha \varphi^+ + \omega_i) \sin \theta^+ , \\
\dot{\varphi}^+ &= \omega_{\varphi} + \omega_i \sin \theta^* \cos \theta^+ \cos \Delta \varphi \\
&\quad - \omega_i \cos \varphi^+ + \alpha \hat{\theta}^+ \csc \theta^+ , \\
\dot{\theta}^- &= -\omega_\theta \sin \Delta \varphi \sin \theta^- - (\alpha \varphi^- + \omega_i) \sin \theta^- , \\
\dot{\varphi}^- &= \omega_{\varphi} - \omega_i \sin \theta^* \cos \theta^- \cos \Delta \varphi \\
&\quad - \omega_i \cos \varphi^- + \alpha \hat{\theta}^- \csc \theta^- ,
\end{align*}
\]

where \( \Delta \varphi = \varphi^+ - \varphi^- \). This result is analytically exact. Some numerical calculations for these ODEs are depicted in figure 1. Since the exchange energy is locally minimized where \( \varphi^+ - \varphi^- = \Delta \varphi = \pi \), we expect \( \varphi^+ = -\varphi^- \). In the small angle approximation and neglecting \( \hat{\theta}^\pm \) terms, this condition is satisfied when

\[
\frac{\dot{\theta}^+}{\dot{\theta}^-} = \frac{\omega_\theta + \omega_i}{\omega_\theta \cos \Delta \varphi} \pm \sqrt{\left(\frac{\omega_\theta + \omega_i}{\omega_\theta \cos \Delta \varphi}\right)^2 - 1},
\]

where \( \theta^\pm \) are the angles that \( m_\pm \) make with the \( \pm \hat{z} \) axes. Choosing \( \Delta \varphi = \pi \), as energetically expected, recovers the results from [27]. Within the \( \dot{\theta} \approx 0 \) approximation, there is no real solution for \( \hat{\theta}^+ = \hat{\theta}^- \) in the presence of easy-axis anisotropy, and one spin will always dominate the dynamics. Because the spins stay antiparallel, the two chiral modes correspond to a right-handed or left-handed rotation of the \( (+) \)-sublattice, and always carry a net angular momentum [20].

The fact that these modes carry a non-zero packet of spin is critical for both the experimental detection and technological application of antiferromagnetic magnonics. It means that these spin waves carry a spin current, even though the two AFM sublattices would naïvely cancel each other’s magnetizations. The association of opposite spin to each chiral mode is what gives rise to the chiral selection mechanism of figures 3 and 4: pumping opposite spin current into the system (via STT) will excite opposite chiral modes.

### References

[1] Chumak A V, Vasyuchka V I, Serga A A and Hillebrands B 2015 Nat. Phys. 11 453
[2] Demokritov S O and Slavin A N 2012 Magnonics: From Fundamentals to Applications (Berlin: Springer)
[3] Serga A A, Chumak A V and Hillebrands B 2010 J. Phys. D.: Appl. Phys. 43 264002
[4] Han W, Kawakami R K, Gmitra M and Fabian I 2014 Nat. Nano 9 794
[5] Pesin D and MacDonald A H 2012 Nat. Mater. 11 499
[6] Lindner J and Robinson J W A 2015 Nat. Phys. 11 307
[7] Pulizzi F 2012 Nat. Mater. 11 367
[8] Bahir S, Alfonson A, Jackeli G, Khalifah G, Matsumoto A, Takayama T, Takagi H, Büchner B and Kataev V 2014 Phys. Rev. B 89 180401
[9] Martí X et al 2014 Nat. Mater. 13 1367
[10] Wang C, Seinige H, Cao G, Zhou J, Soodenough J B and Tsoi M 2014 Phys. Rev. X 4 041034
[11] Wang H, Du C, Hammel P C and Yang F 2014 Phys. Rev. Lett. 113 097202
[12] Tveten E G, Qaiumzadeh A and Brataas A 2014 Phys. Rev. Lett. 112 147204
[13] Tveten E G, Qaiumzadeh A, Tretiakov O A and Brataas A 2013 Phys. Rev. Lett. 110 127208
[14] Kim S K, Tserkovnyak Y and Tchernyshyov O 2014 Phys. Rev. B 90 104406
[15] Dodge J S, Schumacher A B, Bigot J-Y, Chemla D S, Ingle N and Beasley M R 1999 Phys. Rev. Lett. 83 4650
[16] Berger L 1996 Phys. Rev. B 54 9353

[9]
[17] Slonczewski J C 1996 J. Magn. Magn. Mater. 159 L1
[18] Ralph D and Stiles M 2008 J. Magn. Magn. Mater. 320 1190
[19] Stiles M D and Zangwill A 2002 Phys. Rev. B 66 014407
[20] Cheng R, Xiao J, Niu Q and Brataas A 2014 Phys. Rev. Lett. 113 057601
[21] Osborne C F 1971 J. Phys. C: Solid State Phys. 4 2354
[22] Xiao J and Bauer G E W 2012 Phys. Rev. Lett. 108 217204
[23] Ködderitzsch D, Hergert W, Temmerman W M, Szotek Z, Ernst A and Winter H 2002 Phys. Rev. B 66 064434
[24] De Wames R E and Wolfram T 1969 Phys. Rev. 185 752
[25] Gomonay H V and Loktev V M 2010 Phys. Rev. B 81 144427
[26] Xiao J, Zangwill A and Stiles M D 2005 Phys. Rev. B 72 014446
[27] Keffler F and Kittel C 1952 Phys. Rev. 85 329
[28] Roepke G 1971 Theor. Math. Phys. 6 216
[29] Wolfram T and De Wames R E 1969 Phys. Rev. 185 762
[30] Vaknin D, Sinha S K, Moncton D E, Johnston D C, Newsam J M, Safinya C R and King H E 1987 Phys. Rev. Lett. 58 2802
[31] Endoh Y et al 1988 Phys. Rev. B 37 7442
[32] Fischer P, Polomska M, Sosnowska I and Szymanski M 1980 J. Phys. C: Solid State Phys. 13 1931
[33] Alperin H A, Melamud M and Horowitz A 1981 J. Appl. Phys. 52 2225
[34] Sivadas N, Daniels M W, Swendsen R H, Okamoto S and Xiao D 2015 Phys. Rev. B 91 235425
[35] Roth W L 1960 J. Appl. Phys. 31 2000
[36] Kittel C 2005 Introduction to Solid State Physics (New York: Wiley)
[37] Tserkovnyak Y, Brataas A and Bauer G E W 2002 Phys. Rev. Lett. 88 117601
[38] Kapelrud A and Brataas A 2013 Phys. Rev. Lett. 111 097602
[39] Kajiwara Y et al 2010 Nature 464 262