Study on Rub-impact Faults in Two Discs of Double Span Rotor System

Yue-gang LUO\textsuperscript{a,*}, Hong-wen ZHI\textsuperscript{b}, Yan-qiu LI\textsuperscript{c} and Bin WU\textsuperscript{d}

Dalian Nationalities University, Dalian 116600, China

\textsuperscript{a}luoyg@dlnu.edu.cn
\textsuperscript{b}hwzhvip@163.com
\textsuperscript{c}1396911958@qq.com
\textsuperscript{d}wb@dlnu.edu.cn

*Corresponding author

Keywords: Double-span rotor system, Rub-impact fault, Improved rubbing force, Nonlinearity

Abstract. A new model of improved friction is proposed considering the bearing oil film force and nonlinear rubbing force and based on dual-span rotor system. The nonlinear dynamic equations of the two-span rotor system under the influence of nonlinear oil film force and rubbing force are established. The dynamic characteristics of the double-disk rub-impact fault of the double-span rotor system were analyzed by numerical simulation method. Through the contrast analysis of the linear friction model and the improved model of the impact force, it is found that the dynamic response of the improved model is more complex and more realistic to reflect the rubbing fault.

Introduction

In the fault of large rotating machinery, the rubbing fault is an early study and plays an important role in the stable operation of the rotor system. When the rubbing fault occurs, the dynamic response of the system exhibits a wealth of non-linear characteristics. At present, the normal contact force model used in the study of rub-impact faults mainly includes spring-damping model [1], Hertz contact model [2] and nonlinear damping model [3]. The commonly used tangential contact model mainly has the Coulomb friction model [4], tangential stiffness model [5]. Wang et al [6] established the nonlinear rub-impact model based on the Jeffcott rotor according to the Hertz contact theory and the linear model and the Hertz model are compared and the dynamic response of the rub-impact under different stiffness is studied. Cao et al [7] Taking Jeffcott rotor as an example, this paper proposes a rubbing model based on hysteresis frictional force, and analyzes the influence of different factors on the hysteresis frictional force model by means of numerical simulation analysis. The model of friction was compared. Luo et al [8] put forward a new improved rub-impact model and the nonlinear dynamics formula of the rotor system was set up considering the seal fluid force and oil-film force. The dynamic characteristics of the rub-impact rotor system were analyzed by numerical simulation method. It shows that the improved rub-impact model can more describe the characteristics of rub-impact fault from comparing with the linear rub-impact. Li et al. [9] established a nonlinear dynamic model of elastic support double-cross-collision and oil-film failure 20-DOF rotor system and carried out numerical analysis, and compared with the rigid support, the results show that the system has obvious Chaotic characteristics.)
Yuan et al. [10] simplified the rotor engine system to the double rotor-gate coupling dynamic system, and derived the fourth-order partial differential equations of the continuum of the double rotor multi-disc system. The four-Kutta method simulates the rubbing fault response of the double rotor system, and analyzes the influence of the rotational speed on the fault response of the system. Zhang et al [11] established a double-disc rotor-bearing system in the axial rubbing, radial rubbing and two rubbing under the common impact of the finite element method of continuous model, taking into account the rotation effect and shear effect Beam element, the nonlinear dynamic behavior of the rotor system under different rubbing conditions is numerically simulated, and the influence of speed and unbalance on the system is studied. In this paper the nonlinear dynamic equations of the two-span rotor system under the influence of nonlinear oil film force and rubbing force are established. The dynamic characteristics of the double-disc rub-impact fault of the double-span rotor system are analyzed by numerical simulation method.

Modal of Two Discs of Double Span Rotor System and Equations

Figure 1 shows the four sliding bearing support double-span elastic rotor-bearing system and the impact force model, the middle of the coupling connected to the system quality equivalent to the center of the disc and the bearing. Ignore torsional vibration and gyro moment and only consider the system lateral vibration. Both ends of the rotor with symmetrical structure of the sliding bearing support, O1, O3, O4, O6 are the geometric center of the pad, O2c, O3c are the rotor geometric center, m1, m3, m4, m6 are the mass of the rotor at the bearing, m2, m5 are the equivalent mass at the disc. Between the disc and the bearing is a massless elastic shaft, k1 and k2 are the elastic shaft stiffness, k3 is the coupling stiffness, k_c is to turn the static pieces of rubbing stiffness.

![Figure 1. Double-span rub-impact rotor-bearing system and rubbing force model.](image)

In the case of normal rubbing force model, the current application is Herz contact model, which the rubbing force is nonlinear, the impact is limited to elastic deformation, that form does not include damping. It is in the form of

\[ Q_n = k_c \delta^n \]  

(1)

Where \( k \) and \( n \) are constants associated with the contact material and geometric parameters. Hunt et al proposed a collision force model considering both linear elasticity and linear damping force [11]:

\[ Q_n = (e - \delta_0)k_c + c_c \nu, \quad Q_\tau = fQ_n \]  

(2)

In this form, the constant \( c_c \) is the normal viscous coefficient of the contact collision.

In the case of the tangential rubbing force model, it is generally assumed that the friction between the rotor and the stator coincides with the coulomb friction law when
the rubbing occurs and assumes that the coefficient of friction is constant. However, when the relative speed between the rotor and stator is very high, the friction coefficient between the rotor and stator and the relative speed. Based on the advantages and disadvantages of the above model, this paper presents a new model of the impact force, which also considers the influence of bearing oil film force and nonlinear friction force.

When the collision occurs, the normal rubbing force and tangential friction can be expressed as:

\[
\begin{aligned}
Q_n &= (e - \delta_0)^2 k_c + c_c v \quad (e \geq \delta_0) \\
Q_r &= (f + b v) Q_n
\end{aligned}
\]  

(3)

Where \( f \) is the friction coefficient when considering the effect of velocity, \( v = \sqrt{x^2 + y^2} \) is the relative sliding speed between the rotor and stator, \( b \) is the speed influence coefficient, \( e = \sqrt{x^2 + y^2} \) is the radial displacement of the rotor. When the collision occurs, it’s forward rubbing force and tangential friction force decomposition in the x-y coordinate system can be expressed as:

\[
\begin{pmatrix}
Q_x \\
Q_y
\end{pmatrix} = -\left(\frac{e - \delta_0}{e}\right)^2 k_c + c_c v \begin{pmatrix}
1 \\
(f + b v)
\end{pmatrix} \begin{pmatrix}
x \\
y
\end{pmatrix} \quad (e \geq \delta_0)
\]

(4)

Assume that the radial displacement of the rotor system from left to right ends is \((x_1, y_1), (x_3, y_3), (x_4, y_4), (x_6, y_6)\). The radial displacement at the turntable is \((x_2, y_2), (x_5, y_5)\), the system of dimensionless differential equations is:

\[
\begin{aligned}
\ddot{x}_1 + \dddot{x}_1 + \dddot{y}_1 &= \frac{1}{M_1} f(x_1, y_1) \\
\ddot{y}_1 + \dddot{y}_1 + \dddot{y}_1 &= \frac{1}{M_1} f(x_1, y_1) - G \\
\ddot{x}_2 + \dddot{x}_2 + \dddot{y}_2 &= \frac{1}{M_2} Q(x_2, y_2) + a_1 \cos \tau \\
\ddot{y}_2 + \dddot{y}_2 + \dddot{y}_2 &= \frac{1}{M_2} Q(x_2, y_2) + a_1 \sin \tau - G \\
\ddot{x}_3 + \dddot{x}_3 + \dddot{y}_3 &= \frac{1}{M_3} f(x_3, y_3) \\
\ddot{y}_3 + \dddot{y}_3 + \dddot{y}_3 &= \frac{1}{M_3} f(x_3, y_3) - G \\
\ddot{x}_4 + \dddot{x}_4 + \dddot{y}_4 &= \frac{1}{M_4} f(x_4, y_4) \\
\ddot{y}_4 + \dddot{y}_4 + \dddot{y}_4 &= \frac{1}{M_4} f(x_4, y_4) - G \\
\ddot{x}_5 + \dddot{x}_5 + \dddot{y}_5 &= \frac{1}{M_5} Q(x_5, y_5) + a_1 \cos \tau \\
\ddot{y}_5 + \dddot{y}_5 + \dddot{y}_5 &= \frac{1}{M_5} Q(x_5, y_5) + a_1 \sin \tau - G \\
\ddot{x}_6 + \dddot{x}_6 + \dddot{y}_6 &= \frac{1}{M_6} f(x_6, y_6) \\
\ddot{y}_6 + \dddot{y}_6 + \dddot{y}_6 &= \frac{1}{M_6} f(x_6, y_6) - G
\end{aligned}
\]  

(5)
In the form, \( \xi_i = \frac{c_i}{m_i \omega} \), \( c_1 \), \( c_3 \), \( c_4 \) and \( c_6 \) are the damping coefficient of the rotor at the bearing, \( c_2 \) and \( c_5 \) are the rotor disc damping coefficient.

\[ \eta_{i1} = \frac{k_1}{m_i \omega^2}, \quad \eta_{i2} = \frac{k_2}{m_i \omega^2}, \quad \eta_{i3} = \frac{k_3}{m_i \omega^2}, \quad M_i = \frac{c_i \omega^2}{\Delta_i}, \quad G = \frac{g}{c \omega^2}, \quad x_i = \frac{X_i}{c}, \quad y_i = \frac{Y_i}{c} \]

is a dimensionless displacement relative to the bearing gap \( c \). \( f_x = \frac{F}{\Delta}, f_y = \frac{F_y}{\Delta} \) are the dimensionless nonlinear oil film force component, specific expressions see the literature [12]. \( \Delta_i \) is the number of Sommerfeld corrections, \( \Delta_i = \frac{\mu \omega R L}{m g} \left( \frac{R}{c} \right)^2 \left( \frac{L}{2R} \right)^2 \). \( \mu \) is the viscosity of the lubricant, \( \omega \) is the excitation frequency, \( R \) is the bearing radius, \( L \) is the bearing length. \( \tau \) is a dimensionless time, \( \tau = \omega \), \( u_1 \) and \( u_2 \) are the rotor dimensionless eccentricity, \( u_i = \frac{e_i}{c} \). \( \mathbf{Q}_i(x_2, y_2), \mathbf{Q}_i(x_2, y_2), \mathbf{Q}_i(x_5, y_5), \mathbf{Q}_i(x_5, y_5) \) is the rotor system rubbing force in the \( x, y \) direction of the component, as shown in formula (4).

**Analysis of Numerical Simulation Results**

Because of the strong nonlinearity of the system, this paper uses the numerical simulation method to analyze the dynamic characteristics of the system. The system parameters are: \( k_c = 3.5 \times 10^8 \text{N/m} \), \( c_c = 1500 \text{N} \cdot \text{s/m} \), \( f = 0.1 \), \( b = 0.02 \), \( q = 3 \), \( r = 2 \). The first order critical speed of rotor system is 882.5 rad/s.

Figure 2 shows the bifurcation diagram of the rotor system with the rotational speed under the action of two kinds of rubbing force. It can be seen from the figure that the linear rubbing force model is a quasi-periodic motion around the critical speed and the speed of the quasi-periodic motion, interval is very small, and the improved frictional force model is used as chaotic motion around the critical speed and two independent chaotic islands on the bifurcation diagram. The instability speed of the model is 750 rad/s by using the linear rubbing force model and the simulated rotational force model is 710 rad/s. In the super-first order critical velocity region, the motion of the periodic two-cycle and the periodic one is changed to the quasi-periodic motion at the second-order critical speed and the rotational speed range of the quasi-periodic motion is small. The latter is in the super-order critical velocity region to the cycle of decreasing the way into a cyclical movement, in the second-order critical speed left and right to quasi-periodic motion, do the cyclical movement of the speed range is larger.

Figure 3 shows the Poincare cross-section, the axis trajectory and the spectrum of the system when the rotational force is \( \omega = 860 \text{rad/s} \). From the perspective of the Poincare section, the rotor system using the linear rubbing force model is the quasi-periodic motion. The rotor system with the improved rubbing force model is chaotic motion. The former axis trajectory is a regular annular shape, The latter is irregular ring-shaped; from the spectrum point of view, the linear collision force system mainly to 1/3 frequency-based continuous spectrum, the frequency amplitude is greater than the frequency, in addition to 1/4, 4/5 two very small amplitude of the frequency, to improve the impact force model mainly 1/4, 1/2, 3/4 frequency showed continuous spectrum, and 1/2 frequency is greater than the rotor frequency.
Conclusions

(1) Based on the advantages and disadvantages of various models, a new model of improved friction is proposed, and a nonlinear dynamic equation of the double-span rotor system under the influence of nonlinear oil film force and rub-impact force is established.

(2) When the linear rubbing force model is adopted, the rotor system is quasi-periodic motion around the critical speed, and the rotor system is used as the chaotic motion around the critical speed, and both of them have the quasi-periodic motion after the second-order critical speed. The range of the quasi-periodic motion of the improved rubbing force model is larger.

(3) The axis of the model is improved and irregular, and the spectrum is mainly composed of 1/4, 1/2, 3/4 frequency. The rotor system of the friction model is more complex and more responsive to the real rotor rubbing fault.
Acknowledgments
This research was financially supported by the Fundamental Research Funds for the Central Universities (No. DC201502010202) and the Foundation of Liaoning Education Department, China (No. L2014545).

References
[1] L. S. Ding, J. Z. Hong, Research on contact collision dynamics of flexible multi-body system, Journal of Shanghai Jiaotong University, Vol. 37(2003), pp. 1927-1930.
[2] S. H. Yang, J. D. Zheng, T. S. Zheng, et al, Rotor rubbing model based on Hertz contact theory, Journal of Applied Mechanics, Vol. 20(2003), pp. 61-64.
[3] K. H. Hunt, F. R. Crossley, Coefficient of restitution interpreted as damping in vibroimpact, Journal of Applied mechanics, Vol. 42(1975), pp. 440-445.
[4] F. C. Xu, Vibration characteristics of radial gas flake bearing-rotor system based on coulomb friction effect, Harbin: Harbin Institute of Technology, 2013.
[5] X.P. Li, X. Ju, G.H. Zhao et al, Fractal prediction model of tangential contact stiffness of joint surface considering friction factors and its simulation analysis, Journal of Tribology, Vol. 33(2013), pp. 463-468.
[6] T. Wang, X. J. Fu, Research on dynamic response of rub-impact rotor based on Hertz contact model, Turbine Technology, Vol. 51(2009), pp. 39-41.
[7] D. Q. Cao, Y. Yang, D. Y. Wang et al, Research on rubbing response of rotor system based on hysteresis rubbing force model, Aircraft Engine, Vol. 40(2014), pp. 1-9.
[8] Y. G. Luo, B. Wu, H. W. Zhi, et al, Dynamics behaviours of labyrinth seal-sliding bearing-rotor system based on improved rub-impact force model, Journal of Dalian Mizu University, Vol. 18(2016), pp. 33-37.
[9] D. Li, H. Q. Yun, L. M. Wu, Nonlinear properties of rotor system with elastic bearing two-span friction, Vibration, Testing and Diagnosis, Vol. 29(2009), pp. 414-418.
[10] H. Q. Yuan, W. He, Q. K. Han, Rubbing fault analysis of double rotor-engine coupling system of engine, Journal of Aeronautical Dynamics, Vol. 26(2011), pp. 2401-2408.
[11] Y. Zhang, W. M. Wang, J. F. Yao, Analysis of nonlinear dynamic characteristics of axial-radial rub-impact of double-disc rotor system, Vibration and Shock, Vol. 31(2012), pp. 141-145.
[12] W. Zhang, X. Xu, Modeling of nonlinear oil-film force acting on a journal with unsteady motion and nonlinear instability analysis under the mode, International Journal of Nonlinear Sciences and Numerical Simulation, Vol. 1(2000), pp. 179-186.