ION PICKUP BY FINITE AMPLITUDE PARALLEL PROPAGATING ALFVÉN WAVES

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ABSTRACT

Two scenarios of possible ion heating due to finite amplitude parallel propagating Alfvén waves in the solar atmosphere are investigated using a 1D test particle approach. 1. A finite amplitude Alfvén wave is instantly introduced into a plasma (or equivalently, new ions are instantly created). 2. New ions are constantly created. In both scenarios, ions will be picked up by the Alfvén wave. In case 1, the wave scatters ions in the transverse direction leading to a randomization (or heating) process. This process is complete when a phase shift of ±π in the ion gyrospeed is produced between particles with characteristic parallel thermal speed and particles with zero parallel speed. This corresponds to ±π/2 in the ion gyrospeed is produced between particles with characteristic parallel thermal speed and particles with zero parallel speed. A ring velocity distribution can be produced for a large wave amplitude. The process yields a mass-proportional heating in the transverse direction, a temperature anisotropy and a bulk flow along the background magnetic field. In case 2, continuous ion creation represents a continuing phase shift in the ion gyrospeed leading to heating. New particles are picked up by the Alfvén wave within one ion gyroperiod. It is speculated that the mechanism may operate in the chromosphere and active regions where transient events may generate finite amplitude Alfvén waves. To appear in ApJ Letters May 10, 2007.

Subject headings: Waves; Sun: chromosphere; Sun: corona

1. INTRODUCTION

It has been widely speculated that the energy that heats the corona comes from the convective flows in the photosphere. The energy is somehow transported into the coronal part through the magnetic field. It is natural to think that Alfvén waves channel the energy to the corona. Indeed, these waves have been observed in the solar atmosphere (Ulrich 1996), and are ubiquitous in the extended corona — the fast solar wind (Smith et al. 1995). However Alfvén waves are difficult to dissipate in collisionless plasmas. Hence, non-linear processes have been assumed to cascade the wave energy from low to high frequencies where wave dissipation is readily possible (Hollweg 1986, Li and Habbal 2003).

When Alfvén waves propagate in a partially ionized plasma, neutral-ion collisions produce a channel for the wave dissipation (De Pontieu & Haerendel 1998; De Pontieu et al. 2001; Leake et al. 2005). Ions and neutrals can be collisionally coupled. A slippage between the ion and neutral populations leads to the wave dissipation. If we consider neutrals are constantly ionized at the chromosphere, we will show that these newly created He+ ions will be picked-up by the wave and will be energized.

In this Letter, a new scenario, ion pickup by an Alfvén wave, is explored. The pickup process can lead to the heating of ions, and it must also dissipate the wave. In section 2, we discuss the ion pickup process by an instantly introduced wave (or identically, the ions are created instantly). In section 3, the pickup of continuously created ions by an Alfvén wave is investigated. Finally, in section 4 we discuss possible applications of the ion pickup process in the solar atmosphere.

2. ION PICKUP BY ALFVÉN WAVES

Consider a parallel propagating monochromatic dispersionless Alfvén wave with angular frequency ω and wavenumber k, and ω = kvA(νA is the Alfvén speed). The wave electromagnetic field δBw and δEw are

\[
\begin{align*}
\delta B_w &= B_k [\cos \phi_k i_x - \sin \phi_k i_y], \\
\delta E_w &= -v_A B_0/k_0 \times \delta B_w.
\end{align*}
\]

i_x and i_y are unit vectors, B_0 the background magnetic field, and ϕ_k = k(ν_A t - z) denotes the wave phase. The motion of a particle is described by

\[
m_j \frac{dv}{dt} = e_j [δE_w + v × (B_0 i_z + δB_w)],
\]

Let \( u_\perp = v_x + i v_y, v_\parallel = v_z, \) and δB_w = B_k e^{-iϕ_k}, we have

\[
\frac{du_\perp}{dt} + i Ω_0 u_\perp = i (v_\parallel - v_A) Ω_k e^{-iϕ_k},
\]

\[
\frac{dv_\parallel}{dt} = -\text{Im}(u_\perp Ω_k e^{iϕ_k}),
\]

where Ω_0 = e_j B_0/m_j, Ω_k = e_j B_k/m_j, and j refers physical quantities of ion species j. As a first order approximation, \( v_\parallel \approx v_\parallel(0) \) is a constant, where \( v_\parallel(0) \) is the initial ion parallel velocity. The approximation is valid when \( Ω_k/Ω_0 = B_k/B_0 \) is small and the wave frequency is low so \| k(v_\parallel - v_A) \| ≫ 1. With the initial condition \( u_\perp = u_\perp(0) \) and \( z = z(0) \), the solution of Eq. 3 for a low beta plasma is (Wu et al. 1997, Lu et al. 2006)

\[
u_\perp = \left[ u_\perp(0) + \frac{B_k}{B_0} e^{i\frac{π}{2} k z(0)} \right] e^{-iΩ_0 t} - v_A \frac{B_k}{B_0} e^{-iφ_k[v_A t - z]},
\]

where \( z = z(0) + v_\parallel(0) t \) and we have adopted approximations Ω_0 - k[v_A - v_\parallel(0)] ≈ Ω_0, v_A - v_\parallel(0) ≈ v_A. The last term of \( u_\perp \) is the perturbed ion velocity of an
Alfvén wave. The first term is due to the gyromotion of the particle and the modification of the gyromotion due to the wave. Note, if \( u_{\perp}(0) = -v_A(B_k/B_0)e^{-ik_z(0)} \) and \( v_{th}(0) = 0 \), the particle is surfing on the wave, there is no gyromotion.

Now let us consider an ensemble of newly created particles with a Maxwellian distribution. The particles’ average parallel speed is zero but the transverse component is \( u_{\perp,f} \)

\[
u_{\perp}(0) = u_{\perp,r}(0) + u_{\perp,f}(0) = u_{\perp,r}(0) - \alpha v_A \frac{B_k}{B_0} e^{ik_z},
\]

where \( \alpha \), a constant, describes the degree that the newly created particles are settled in the wave field. If \( \alpha = 1 \), the newly created particles are already settled. \( u_{\perp,r} \) denotes the random perpendicular thermal speed. After time \( t \) particles with different initial position \( z(0) = z - v_{th}(0)t \) will arrive at \( z \). The average transverse velocity at \( z \) is

\[
U_{\perp} = -v_A \frac{B_k}{B_0} e^{-ik(v_A t - z)} + \frac{1 - \alpha}{\sqrt{\pi}v_{th}} \int_{-\infty}^{\infty} \frac{B_k}{B_0} e^{-ikz - v_{th}(0)t} e^{-\Omega t^2} e^{\left(-\frac{v_{th}(0)}{m_0}\right)^2} dv_{th}(0),
\]

where \( A_k = \frac{1}{\sqrt{2}} \int_{-\infty}^{\infty} \cos(kv_A t,x) e^{-x^2} dx = e^{-\frac{x^2}{2}} \),

\[
v_{th} = \left(\frac{2k_B T_{j0}}{m_j}\right)^{1/2}, \quad \text{and} \quad T_{j0} = \text{the initial temperature of species } j.
\]

Eq. (7) illustrates the pickup of ions by the wave. Note \( A_k \approx 0 \) when \( kv_A t \geq \pi \). Hence, regardless of the value of \( \alpha \), these particles will be picked up by the Alfvén wave: eventually \( U_{\perp} \) is determined by the wave only. Subtracting (7) from (5), one finds the random perpendicular velocity at \( z \):

\[
u_{\perp} - U_{\perp} = u_{\perp,r}(0) e^{-\Omega t} + (1 - \alpha) v_A \frac{B_k}{B_0} e^{ikz - v_{th}(0)t} e^{-\Omega t} e^{ikz} - \Omega t.
\]

Eq. (8) describes the particles’ gyrospeed in the frame of \( U_{\perp} \). When \( \alpha = 0 \) (initially the newly created particles have zero flow speed), particles will be strongly scattered in the phase space. The second term on the right hand side is crucial for our understanding the heating in the pickup process. In the phase space frame of \( U_{\perp} \), a randomization process is complete when particles with characteristic speed \( v_{th}(0) = \pm v_{th} \) moving from \( z(0) = z - v_{th}(0)t \) to \( z \) are scattered in the perpendicular direction and a phase shift \( \pm \pi \) in the gyrospeed relative to particles with \( v_{th}(0) = 0 \) is produced. This translates to \( kv_A t, k \approx \pi \). For a finite amplitude Alfvén wave, if \( v_A B_k/B_0 > v_{th} \), the distribution function will be a ring. Hence particles are strongly heated. On the other hand, if \( \alpha = 1 \), initially particles are already “picked up” by (surfing on) the wave. Only the initial random motion survives, and there is no heating. One notices that if \( v_{th} = 0, A_k = 1 \), there is also no heating. Hence, the heating is a warm plasma effect. With \( U_{\perp} \), the perpendicular temperature can be found

\[
T_{\perp} = \frac{m_j}{2k_B v_{th} \sqrt{\pi}} \int_{-\infty}^{\infty} |u_{\perp} - U_{\perp}|^2 e^{-\left(\frac{v_{th}(0)}{m_0}\right)^2} dv_{th}(0)
\]

\[
= T_{j0} \left[ 1 + \frac{m_j B_k^2}{m_0 \beta_j B_0^2} (1 - A_k^2) \right],
\]

where \( \beta_j = 8\pi n_j k_B T_{j0}/B_0^2 \) is the plasma beta of species \( j \). The heating is obviously mass-proportional.

Substituting Eq. (9) into Eq. (11), and letting \( v_A - v_{th}(0) \approx v_A \) and \( \Omega_0 - k(v_A - v_{th}(0)) \approx \Omega_0 \), one finds

\[
v_{th} = v_{th}(0) + v_A(1 - \alpha) \frac{B_k^2}{B_0^2} (1 - \cos^2(\Omega_0 t - kv_A t - k v_{th}(0)t)),
\]

(10)

Using the same procedure to obtain \( U_{\perp} \) and \( T_{\perp} \), we find the average parallel velocity and temperature:

\[
U_{\parallel} = v_A \frac{B_k^2}{B_0^2} (1 - \alpha)(1 - A_k \cos(\Omega_0 t - kv_A t)),
\]

\[
T_{\parallel} = T_{j0} \left[ 1 + \frac{2m_j B_k^2(1 - \alpha)^2}{m_0 \beta_j B_0^2} (C_k - A_k^2) \right]
\]

\[
\cos^2(\Omega_0 t - kv_A t) + (1 - C_k) \sin^2(\Omega_0 t - kv_A t)),
\]

(12)

where \( C_k = \int_{-\infty}^{\infty} \cos^2(kv_A t,x) e^{-2x^2} dx = 0.5 + 0.5 e^{-k^2 v_{th}^2} \). When \( t \to \infty \), \( A_k = 0 \), and \( C_k = 0.5 \). We then obtain asymptotic values:

\[
u_{\perp} - U_{\perp} = -v_A \frac{B_k}{B_0} e^{-ik(v_A t - z)}, \quad u_{\parallel} = v_A(1 - \alpha) \frac{B_k^2}{B_0^2}
\]

\[
T_{\perp} = T_{j0} \left[ 1 + \frac{m_j B_k^2(1 - \alpha)^2}{m_0 \beta_j B_0^2} \right].
\]

(14)

Subtracting (11) from (10), when \( t \to \infty \), one finds

\[
v_{th} - U_{\parallel} = v_{th}(0) + v_A(1 - \alpha) \frac{B_k^2}{B_0^2} \cos(\Omega_0 t - kv_A t - k v_{th}(0)t).
\]

It is possible that the second term on the right hand side is larger than the initial random thermal velocity \( v_{th}(0) \). In this case, the velocity distribution plotted in the \( v_x - v_y \) plane may be a ring as well (see Fig.2).

To verify the above analysis, a test particle simulation was conducted by calculating the full dynamics of particles (Eq. 2) in the electromagnetic field of a given Alfvén wave described by Eq. 1. The equations were solved using the Boris algorithm with time step \( \Delta t = 0.01T_p^{-1} \) (\( T_p \) is the proton gyrofrequency) and periodic boundary conditions. Initially, 200000 particles with Maxwellian velocity distribution were evenly distributed in a region with length 1000A_1/(\sqrt{\pi}) (1000 cells). The flow speed of these particles was zero. The average parallel velocity, and the parallel and perpendicular temperatures were obtained by using the following procedure:
we first computed \( U_\parallel = \langle v_z \rangle, \) \( T_\parallel = \frac{m_p}{2k_B} \langle (v_z - \langle v_z \rangle)^2 \rangle, \) and \( T_\perp = \frac{m_p}{2k_B} \langle (v_z - \langle v_z \rangle)^2 + (v_y - \langle v_y \rangle)^2 \rangle \) in every cell (\( \langle \rangle \) denotes an average over a cell). These quantities were then averaged over all cells, so only random motions contribute to the temperatures. Fig. 1 shows the time evolution of (a) the perpendicular temperature \( T_{\perp H_e}/T_{H,e0} \) (here 0 refers to the initial value) and the parallel temperature \( T_{\parallel H_e}/T_{H,e0} \), (b) the temperature anisotropy \( T_{\perp H_e}/T_{\parallel H_e} \), and (c) the average parallel velocity \( U_\parallel/v_A \) of the test particle simulation. The “shaded areas” are due to rapid ion gyrotations. The results are consistent with our analytical predictions. At \( \Omega_p t \approx 12000 \), an asymptotic stage is reached: \( U_\parallel/v_A \approx 0.086, T_{\perp H_e}/T_{H,e0} \approx 34 \) and \( T_{\parallel H_e}/T_{H,e0} \approx 3.9 \) with the temperature anisotropy \( T_{\perp H,e}/T_{\parallel H,e} \approx 8.7 \).

Fig. 2 displays scatter plots of the helium ions in the five cells around \( z = 500v_A\Omega_p^{-1} \) at \( \Omega_p t = 0, 2800, 6600 \) and 11000. Initially, helium ions satisfy the Maxwellian distribution with \( v_{th} = 0.005v_A \) and \( \beta_{H,e} = 0.01 \). Helium ions are dramatically scattered in the transverse direction in the phase space. The ring velocity distributions are nicely shown in Fig. 2. The ring velocity distribution is unstable (Lu and Wang 2005). The velocity distribution may eventually be thermalized by relevant microscopic instabilities of the ring distribution. The bulk acceleration of particles along \( B_0 \) is obvious. This is because in the frame of the Alfvén phase speed, the kinetic energy of particles is conserved. As particles are picked up by the wave, they gain a bulk acceleration along \( B_0 \).

3. PICK-UP OF CONTINUOUSLY CREATED IONS

In a partially ionized plasma, the creation of new ions due to photoionization or collisional ionization is highly random. Consider a region with a low plasma beta and where the amplitude of an Alfvén wave is finite. When ions are created from neutrals, they are subject to rapid gyrotations. The continuous creation of ions represents a continuous phase shift in the ion gyrospeed. This will naturally mix the phase of particles’ gyrospeeds. To explore the pickup process of continuously created ions, a new test particle simulation is conducted. The Alfvén wave and the simulation box are the same as in section 2. However, 1000 particles are deployed every 0.250\( \Omega_p^{-1} \) at \( 0 \leq \Omega_p t \leq 100 \) (one particle per cell). The beta value of the newly created He\(^+1 \) ions is 0.01 and they have zero flow speed. The results of the simulation are shown in Figs. 3 and 4, which are plotted in the same way as Figs. 1 and 2. Note that in the \( v_x - v_y \) plane, because the radius of the ring (Fig. 3) is \( B_k v_A/B_0 \), the origin, where new particles are created around there, is always on the ring. When new He\(^+1 \) ions are continuously created, the heating and scattering process are completed when they have filled the ring of radius \( B_k v_A/B_0 \) in the \( v_x - v_y \) plane. This only takes one gyroperiod of He\(^+1 \) ions, or when \( \Omega_p t = 8\pi \), as shown in Fig. 3 and 4. Hence the heating of these particles is extremely rapid. The ion creation process was terminated at time \( t = 100\Omega_p^{-1} \), however no change is observed as \( \Omega_p t \geq 8\pi \). We believe that the rapid heating due to Alfvén waves reported by Wang et al. (2006) is due to their deployment of particles within one gyroperiod, even though they used many wave modes in their computations.

4. DISCUSSION FOR POTENTIAL APPLICATIONS

The Alfvén wave pickup of newly created ions may have applications at the top of the chromosphere or lower transition region. Let’s consider the region with an ion cyclotron frequency much higher than Coulomb collision frequencies, low plasma beta and finite amplitude Alfvén waves. For neutral helium, when wave frequencies are higher than the ion neutral collision frequencies, the perturbed ion and neutral velocity will have a considerable difference between their phases and amplitudes (De Pontieu & Haerendel 1998). In coronal funnels, the plasma beta at the lower transition region is much smaller than unity (Li 2003). For finite amplitude waves, the pickup process described above may operate and heat newly created He\(^+1 \) ions. Due to collisions, the temperature is not expected to reach the value given by (14). Instead, the energy will be transferred to protons and electrons via Coulomb coupling. Obviously, the thermal energy of He\(^+1 \) gained through the pickup process will be passed to He\(^{+2} \) later on when He\(^{+2} \) are created. As the ionization continues to the transition region (Hansteen et al. 1997), the pickup process will contribute to the plasma heating there.

The pickup of instantly created ions by an Alfvén wave or the pickup of ions by an Alfvén wave instantly introduced into a plasma is quite efficient even for low frequency waves in collisionless plasmas. The pickup process is complete when \( k v_{th} t = \pi \) or \( t = \frac{2\pi}{k v_{th}} t_A \), where \( t_A \) is the period of the wave. Transverse oscillations of active region coronal loops that suddenly appear with periods of 2–33 minutes have been frequently observed by TRACE (Nakariakov et al. 1999; Aschwanden et al. 2002). The oscillations usually just last several wave periods. The relative amplitude \( B_k/B_0 \) of these waves can reach 0.05 (Aschwanden et al. 2002). For typically observed 10\(^8 \) K coronal loop with density of 10\(^6 \) cm\(^{-3} \) and magnetic field of 30 G, the plasma beta value is 0.00385.
Fig. 2.— Top panel: Scatter plots of helium He$^{+1}$ velocity in the velocity plane perpendicular to $v_p$. Bottom panel: scatter plots of helium He$^{+1}$ velocity in the plane parallel to the background magnetic field. Relevant parameters are those in Fig.1.

Due to the pickup process, the perpendicular temperature of helium ions and oxygen ions may be increased to 3.6 and 11.4 times of their original values, respectively.

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Fig. 3.— The time history of (a) $T_{\perp He}/T_{He0}$ and $T_{\parallel He}/T_{He0}$, (b) $T_{\perp He}/T_{\parallel He}$, and (c) $U_{\parallel}/v_A$. Here $k_{vA}/\Omega_p = 0.25\pi/125 \approx 0.0063$, $\beta_{He} = 0.01$, $B^2_2/B_0^2 = 0.09$. The He$^{+1}$ ions are steadily and uniformly created between $z = 0$ and 1000$v_A\Omega_p^{-1}$ at $0 < \Omega_p t < 100$.

REFERENCES

Aschwanden, M.J., De Pontieu, B., Schrijver, C.K., & Title, A.M., 2002, Sol Phys., 206, 99.
De Pontieu, B., and Haerendel, G., 1998, A&A, 338, 729.
De Pontieu, B., Martens, P.C.H, and Hudson, H.S., 2001, ApJ, 558, 859.
Hansteen, V. H., Leer, E., & Holzer, T. E. 1997, ApJ, 482, 498
Hollweg, J. V. 1986, J. Geophys. Res., 91, 4111
Leake, J.E., Arber, T.D., and Khodachenko, M.L., 2005, A&A, 442, 1091.

Li, X., 2003, A&A, 406, 345.
Li, X., and Habbal, S.R., 2003, ApJ, 598, L125.
Lu, Q. M., and Wang, S., 2005, Geophys. Res. Lett., 32, L03111.
Lu, Q. M., Wu, C. S., and Wang, S., 2006, ApJ, 638, 1169
Nakariakov, V., Ofman, L., Deluca, E.E., Roberts, N., Davilla, J.M., 1999, Science, 285, 862.
Smith, E.J., Balogh A., Neugebauer, M., & McComas, D., 1995, Geophys. Res. Lett., 22, 3381.
Ulrich, R.K., 1996, ApJ, 465, 436.
Fig. 4.— Top panel: Scatter plots of helium He\(^+1\) velocity in the velocity plane perpendicular to \(v_y\). Bottom panel: scatter plots of helium He\(^+1\) velocity in the plane parallel to the background magnetic field. Relevant parameters are those in Fig.3.

Wang, C. B., Wu, C. S., & Yoon, P. H., 2006, Phys. Rev. Lett., 96, 125001

Wu, C. S., Yoon, P. H., & Chao, J. K., 1997, Phys. Plasmas, 4, 856