Behavior of compressed concrete in a glass fiber-reinforced shell

A L Krishan¹, E P Chernysova², M A Astafyeva¹

¹Department of Building Design and Constructions, Nosov Magnitogorsk State Technical University, 38, Lenin Ave., Magnitogorsk 455000, Russia
²Department of Design, Nosov Magnitogorsk State Technical University, 38, Lenin Ave., Magnitogorsk 455000, Russia

E-mail: ep.chernyshova@gmail.com

Abstract. The paper presents a theoretical study of the structural resistance of short compressed concrete elements in a glass fiber-reinforced shell. The methodology is based on a nonlinear strain model of how this element reacts to incremental loading. What makes computing such structures difficult is the need to account for the continuously changing lateral shell pressure on the concrete core. Lateral pressure keeps increasing due to changes in the concrete-core and glass fiber-reinforced shell lateral-strain coefficients, causing greater stress in the material. This paper is the first to propose formulas to account for such changes and measure lateral pressure for any load on such a structure. It also outlines practical guidelines on computing the strength of short compressed concrete elements in a glass fiber-reinforced shell.

1. Introduction
Concrete-filled steel tubes (CFST) have been well-studied and are now widely used in construction [1-5]. Their best application is the overloaded compressed elements of the bearing frames in buildings and structures. Researchers say a rational approach could also make great use of concrete-filled glass fiber-reinforced tubes (CFGT) [6-9]. Those retain the basic technological, structural, and economic advantages of compressed CFST elements. However, they also feature great corrosion resistance.

What limits the use of CFGT structures is the lack of any standards or guidelines on modeling them. As of today, the strength of compressed CFGT elements is found either empirically or by finite-element analysis [10,11]. Such calculations ignore the changes in the concrete-core and glass fiber-reinforced shell lateral-strain coefficients, which cause greater stress in the material. It is primarily these changes that cause the lateral pressure on the concrete to grow gradually; the pressure is what determines the strength and the deformability of the bulk-compressed concrete core and of the whole structure. The goal hereof is to develop a method to calculate the strength of short compressed CFGT elements, which shall take into account the particularities of their structural resistance.

2. Proposed Calculation Method
The proposed calculation method is based on strain calculations. It is essentially as follows. Consider an element with the compressive load $N$, having design or random eccentricity $e_o$, applied to the edges, see Figure 1. The concrete core is considered a transversely-isotropic material. Two planes of different properties can be distinguished in the core:
Figure 1. Calculating the compressed concrete-filled tubes.

Figure 2. N- $N_{u}$ dependencies after attaining:
(1) maximum load; (2) specified strain.

- ZOR with an origin (point O) in the geometric center of the cross-section, the longitudinal axis $Z$ of the calculated element, and the axis $R$ perpendicular to $Z$ and directed along the longitudinal-axis circle radius;
- ROR, which is perpendicular to the longitudinal axis $Z$.

Since fiberglass has a relatively low modulus of elasticity, the axial stresses $\sigma_{pz}$ in the shell are low and neglected (added to the margin of safety). As the compressed element is deformed by loading, circumferential tensile stresses $\sigma_{pr}$ emerge in the shell, which give rise to the lateral concrete-core compression of value $\sigma_{cr}$.

The concrete core is pre-divided into small sections of area $A_{cj}$; stresses within each area are averaged. Each longitudinal rebar (if any) of the cross-section area $A_{sk}$ is linked to a selected origin. Then the axial strain of the most compressed fiber $\varepsilon_{cz,max}$ of the element is increased in increments. The Bernoulli hypothesis is used to plot the strains in the cross-section of the out-of-center compressed element so that the internal forces in the concrete core and longitudinal reinforcements, and the external load forces be in equilibrium. Find the stresses $\sigma_{cj}$ and $\sigma_{sk}$ to verify the conditions for the equilibrium of strains in the center of each smaller concrete section as well as in the rebar centers of gravity. The equilibrium conditions are as follows:

$$N \cdot \varepsilon_{0} = \sum_{j} \sigma_{cj} A_{cj} Z_{cj} + \sum_{k} \sigma_{sk} A_{sk} Z_{sk}, \quad (1)$$

$$N = \sum_{j} \sigma_{cj} A_{cj} + \sum_{k} \sigma_{sk} A_{sk}, \quad (2)$$

where $Z_{cj}$ are the coordinates of the center of gravity $j$-th concrete section; $Z_{sk}$ are the coordinates of the center of gravity $k$-th rebar.

As soon as the left side equals the right side in the equations (1) and (2), record the compressive external-load force and continue to increase the strain $\varepsilon_{cz,max}$. Continue until the compressive strength reaches the maximum value $N_u$ or the axial strain reaches the maximum permissible value $\varepsilon_{cz,u}$ as specified by the analyst, see Figure 2.

Strain calculations involve the use of strain curves for the axial direction $\sigma_{cz} - \varepsilon_{cz}$ in a bulk-compressed concrete core. The outline of each such curve is assumed to be curvilinear, ascending to the vertex $f_{cz}, \varepsilon_{cz1}$, then descending to the strains $\varepsilon_{cz2}$, see Figure 3. The accuracy of such calculations...
mainly depends on the reliability of the adopted curves. Papers on the subject [4, 12-13] propose analytical dependencies for describing the strain curves of concrete exposed to a specific lateral pressure. As a rule, engineers use the structure-specific limit-state pressure level. In reality, lateral pressure changes as the load is increasing. Each lateral pressure \( \sigma_{cr} \) at the specific \( i \)-th step of the calculation will have its own strain curve. This fact does complicate the strain strength calculations; however, it cannot be ignored. Given the continuous changes in the lateral pressure, we have a set of curves \( \sigma_{cr}^{(i)} - \varepsilon_{c}^{(i)} \), see Figure 3.

The current state-of-the-art is to use empirical approaches to analyze the curves. Paper [14] presents a detailed analysis thereof. For the case under consideration, the seemingly best method is a slightly updated Mander formula [15], which can be written as follows

\[
\sigma_{cr}^{(i)} = f_{cc}^{(i)} \frac{\lambda^{(i)} \varepsilon_{c}^{(i)}}{\varepsilon_{c0}^{(i)}} \lambda^{(i)} - 1 + \left( \frac{\varepsilon_{c}^{(i)}}{\varepsilon_{c0}^{(i)}} \right)^{\lambda^{(i)}} ,
\]

where \( f_{cc}^{(i)} \) and \( \varepsilon_{c0}^{(i)} \) are the stress and strain at the curve vertex corresponding to the adopted current strain \( \varepsilon_{c}^{(i)} \); \( \lambda^{(i)} \) is the coefficient calculated by the formula

\[
\lambda^{(i)} = \frac{E_{c}}{E_{c} - f_{cc}^{(i)} \varepsilon_{c0}^{(i)}} ,
\]

where \( E_{c} \) is the initial concrete modulus of elasticity.

The limit stress of bulk-compressed concrete, which corresponds to the lateral pressure \( \sigma_{cr} \), can be found by the formula [16]

\[
f_{cc}^{(i)} = f_{e} \left[ 1 + \left( 0.5 \frac{\sigma_{c}}{\overline{\sigma}_{c}} - 2 + \sqrt{\left( \frac{\overline{\sigma}_{c} - 2}{4} \right)^{2} + \overline{\sigma}_{c}} \right) \right] ,
\]

where \( f_{e} \) is the strength of uniaxially compressed concrete; \( \overline{\sigma}_{c} \) is the relative lateral shell pressure on the concrete core \( \overline{\sigma}_{c} = \sigma_{cr}^{(i)} / f_{e} \); \( b \) is a material-specific coefficient to be found experimentally (for heavy concrete, \( b=0.096 \)).

Figure 3. Concrete-core strain curves, incrementally growing longitudinal strain: (1) uniaxial compression; (2), (3) bulk compression at intermediate deformation stages; (4) bulk compression, limit state.
The following formula is proposed for finding the concrete strain magnitude \( \varepsilon_{cc}^{(i)} \) at maximum compressive stress \( f_{cc}^{(i)} \)

\[
\varepsilon_{cc}(i) = \alpha_i \left[ \varepsilon_{c1} \alpha_i - \frac{f_{c1}}{E_c} (\alpha_i - 1) \right],
\]

(6)

where \( \varepsilon_{c1} \) is compressive strain in the concrete at the peak stress \( f_{c1} \); \( \alpha_i \) is coefficient of concrete strength growth caused by the bulk compression \( (\alpha_i = f_{cc}^{(i)}/f_{c1}) \).

The strain of the bulk-compressed concrete at the end of the curve can be found by the formula

\[
\varepsilon_{cc2}(i) = \frac{\varepsilon_{c2}^{(i)}}{\varepsilon_{c1}^{(i)}},
\]

(7)

where \( \varepsilon_{c2} \) is the strain at the end of the uniaxially compressed concrete strain curve.

The formulas (5) and (6) make apparent that the coordinates of the strain curve vertex for bulk-compressed concrete will depend on the original type and class of such concrete as well as on the lateral pressure \( \sigma_{cr} \).

Paper [13] proposes the following formula for finding the lateral pressure that the glass fiber-reinforced shell exerts on the concrete

\[
\sigma_{cr}^{(i)} = \frac{\nu_{zo}^{(i)} - \nu_{p}^{(i)}}{2E_{pt} t} \frac{1 - \nu_{zo}^{(i)} \cdot \varepsilon_{zr}^{(i)}}{\nu_{zo}^{(i)} E_c},
\]

(8)

where \( \nu_{zo}^{(i)} \) is the concrete-core lateral-strain coefficient for the plane \( ZOR \); \( \nu_{p}^{(i)} \) is the lateral-strain coefficient for the circumferential direction of the glass fiber-reinforced shell; \( E_{pt} \) is the modulus of elasticity of the shell; \( d \) and \( t \) are the shell outer diameter and wall thickness; \( \varepsilon_{zr}^{(i)} \) is the coefficient of elasticity for the concrete core.

As the stress is increasing, the coefficient \( \nu_{zo}^{(i)} \) rises from the initial \( \nu_{zo} = 0.18 \div 0.25 \) to the limit \( \nu_{zo}^{(i)} \), at which the concrete destruction occurs. In case more accurate data is not available, the recommended initial coefficient value is \( \nu_{zo} = 0.2 \).

Paper [17] proposes the following formula for finding the current \( \nu_{zo}^{(i)} \):

\[
\nu_{zo}^{(i)} = \nu_{zo}^{(i)} - (\nu_{zo}^{(i)} - \nu_{zo}) \left( \frac{\varepsilon_{zr}^{(i)} - \nu_{zo}^{(i)}}{1 - \nu_{zo}^{(i)}} \right)^{0.5}.
\]

(9)

Paper [14] proposes the following formula for finding the limit value of the lateral-strain coefficient for concrete cores

\[
\nu_{zo}^{(i)} = \nu_{zo} + (1 - \frac{1}{2 \nu_{zo}^{(i)}}),
\]

(10)

where \( \nu_{zo}^{(i)} \) is the coefficient of elasticity for the concrete-core deformation vertex

\[
\nu_{zo}^{(i)} = \frac{f_{cc}^{(i)}}{E_c \varepsilon_{cc}^{(i)}},
\]

(11)

The current coefficient of elasticity for the concrete core can be found by the formula
\[ \nu_{\text{ez}}^{(i)} = \frac{\sigma_{\text{ez}}^{(i)}}{E_{\text{ez}}e_{\text{ez}}^{(i)}}. \]  \hspace{1cm} (12)

Note that when calculating \( \nu_{\text{ez}}^{(i)} \) and \( \nu_{\text{ez}}^{(i)} \) at this \( i \)-th step, the values \( f_{\text{ez}}^{(i)} \), \( e_{\text{ez}}^{(i)} \) and \( \sigma_{\text{ez}}^{(i)} \) are yet unknown. However, given a sufficiently small increment in the strain \( \Delta e_{\text{ez, max}} \), these values can be taken from the previous step. Numerical analysis shows that the calculation error is insignificant even without refining such values.

A glass fiber-reinforced shell can be conditionally deemed an elastic isotropic material for the calculation purposes. In that case, the lateral-strain coefficients may for any stress be deemed equal to the initial Poisson’s ratio, i.e. \( \nu_{p}^{(i)} = \nu_{p0} \). Stated as such, changes in the lateral pressure will only depend on three parameters, \( e_{\text{ez}}^{(i)} \), \( \nu_{p}^{(i)} \), and \( \nu_{\text{ez}}^{(i)} \).

Longitudinal reinforcement stress \( \sigma_{l} \) must be found in the context of the concrete-core deformations causing deformations in such reinforcements. The relationship between reinforcement stresses and strains is either bilinear or trilinear, depending on the strength [16].

### 3. Discussion

The above dependencies can be used to calculate the strength of compressed CFGT elements. Using the variable coefficients of elasticity and lateral strain enables taking into account the physical nonlinearity of the concrete core and its work. Another important feature of the proposed strain calculation procedure consists in the method for finding the current axial stress \( \sigma_{\text{ez}}^{(i)} \) in the concrete core as a function of the adopted deformation level \( e_{\text{ez}}^{(i)}/e_{\text{ez}}^{(i)} \). Essentially, this necessitates using a set of concrete strain curves rather than a single one. However, it also enables a significantly more accurate calculation of the stresses \( \sigma_{\text{ez}}^{(i)} \), making the entire calculation more accurate.

To validate the proposed method, the research team is planning to run a series of experiments to study the strength of compressed CFGT elements of various geometries and designs.

### 4. Conclusions

This paper proposes a method for calculating the strength of short CFGT elements; the method is based on the nonlinear reinforced-concrete strain model. It takes into account the complex stress state of the structures under analysis, which changes as the axial strains increase. The method considers all the major peculiarities of the CFGT structural resistance under short-term compressive load, while offering minimum reliance on empirical coefficients.

Practical implementation of this method gives engineers an ability to limit the axial strain of elements to a limit specified by the designer. Given the high limit on the deformability of the studied elements, this ability might be of use for operating the bearing frames.

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**Acknowledgments**

This article was prepared by results of implementation of the scientific project within the state task of the Ministry of Education and Science of the Russian Federation No. 7.3379.2017/4.6.