Comparing and forecasting performances in different events of athletics using a probabilistic model

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Abstract

Though athletics statistics are abundant, it is a difficult task to quantitatively compare performances from different events of track, field, and road running in a meaningful way. There are several commonly-used methods, but each has its limitations. Some methods, for example, are valid only for running events, or are unable to compare men’s performances to women’s, while others are based largely on world records and are thus unsuitable for comparing world records to one other. The most versatile and widely-used statistic is a set of scoring tables compiled by the IAAF, which are updated and published every few years. Unfortunately, these methods are not fully disclosed. In this paper, we propose a straight-forward, objective, model-based algorithm for assigning scores to athletic performances for the express purpose of comparing marks between different events. Specifically, the main score we propose is based on the expected number of athletes who perform better than a given mark within a calendar year. Computing this naturally interpretable statistic requires only a list of the top performances in each event and is not overly dependent on a small number of marks, such as the world records. We found that this statistic could predict the quality of future performances better than the IAAF scoring tables, and is thus better suited for comparing performances from different events. In addition, the probabilistic
model used to generate the performance scores allows for multiple interpretations which can be adapted for various purposes, such as calculating the expected top mark in a given event or calculating the probability of a world record being broken within a certain time period. In this paper, we give the details of the model and the scores, a comparison with the IAAF scoring tables, and a demonstration of how we can calculate expectations of what might happen in the coming Olympic year. Our conclusion is that a probabilistic model such as the one presented here is a more informative and more versatile choice than the standard methods for comparing athletic performances.

1 Introduction

Quantitatively comparing performances from different athletic events and specifying how much more impressive one performance is than another are not simple tasks. There are a few good models that are valid for running events, particularly longer distances, namely those by McMillan (2011), Cameron (1998), Riegel (1977), and Daniels and Gilbert (1979). These models rely on physiological measurements such as speed and running economy to compare performances at different race distances, either for men or for women, but not between them.

Purdy Points (Gardner and Purdy, 1970) have long been used to compare marks from different events in both track and field, but these scores are based mainly on the world records of each event at a particular date in the past, which leads to two main disadvantages: (1) it is impossible to compare world records to each other if the model is based on them, and (2) basing the model on such a small data set leads to much uncertainty and variation in the scores as the records and model evolve over time. In other words, if a particular world record is “weak” in some sense, Purdy points will likely unfairly assign a higher score to performances in that event when compared to others.

Currently, the most popular method for comparing performances across all events in track and field as well as road running is to consult the IAAF scoring tables (Spiriev and Spiriev, 2011). These tables are updated every few years using methods that are not fully disclosed, with the last two updates occurring in 2008 and 2011. The IAAF is the main official governing body for international athletics, and they also publish the official scoring tables for “combined events competitions” such as the heptathlon and decathlon. These “combined events” consist of seven women’s and ten men’s events, respectively, and which are contested at most major international athletics competitions, and the winner is declared to be the competitor with the highest point total from all of the events. These combined events scoring tables were intended to assign a similar amount of points to performances that are “similar in quality and difficulty” (International Association of
All point values $P$ in these tables can be calculated using a formula of the form $P = a(M - b)^c$, where $M$ is the measured performance (use $M = -T$ for running times $T$, where a lower performance is better) and $a$, $b$, and $c$ are constants estimated by undisclosed methods. The combined events tables are not the same as the general IAAF scoring tables, but it may be deduced that both sets of tables are produced using similar methods. Which data are used and how exactly the constants are estimated is not clear.

In this publication, we introduce a method of scoring athletic performances based on the idea that a good performance is a rare or improbable performance. Two very common reasons why one might think that an athletic performance is good are:

1. A performance is good if few athletes improve upon it, or
2. A performance is good if it is close to or improves upon the [previous] best performance.

The first reason is important because it puts emphasis on what has actually happened. In other words, if an athlete is in the top ten in the world in her event, she is likely better than an athlete who is ranked 50th or 100th. On the other hand, the second reason is important because it focuses more on what is possible. Sometimes in sport, a revolution occurs, whether in training, technique, equipment, or facilities, and performances improve dramatically. Certain events in history cause people to re-think what they thought was good—Bob Beamon’s 1968 Olympic long jump in Mexico City, Paula Radcliffe’s 2003 London Marathon, and more recently Usain Bolt’s 2009 World Championship 100m run in Berlin come to mind. In some of these cases, but not in others, what we once thought was unthinkable becomes commonplace. In 1996, many people thought that Michael Johnson’s 200m world record would last an eternity—it was revolutionary—but now it is only fourth on the all-time list. The men’s marathon record has dropped tremendously in recent years, carried in part by Haile Gebreselassie and Paul Tergat, who accomplished the same feat for the 10,000m run in the 1990s. The point is only that a superb, dominating performance might be one of the greatest feats ever witnessed, but it also might be an inevitability. Usain Bolt’s 9.58s mark in the Berlin 100m dash in 2009 is certainly impressive, but we saw three men running 9.72s or faster in the 100m dash in 2008, all under the world record from 2007; so how impressive was 9.58s really? Is it a statistical outlier, or is it the expected result of a general increase in performance level which by chance had not yet produced the outstanding performance that was bound to happen? These are some questions this paper was intended to answer.
The methods introduced here utilize a large amount of historical data to estimate directly the improbability of athletic performances. Using a data set consisting of the top \( n \) performances of all time—where \( n \) is generally well over 100 and can be different for each event—we estimate a log-normal distribution for each event, allowing us to calculate directly both the probability that a specific mark is exceeded as well as the expected number of such performances within a given time period. We use this model to predict the number and quality of top performances in the subsequent years, for data up until the year 2000 and also 2008, and we show that our scoring tables based on data prior to 2008 correlate more highly with actual data than do the 2008 IAAF scoring tables. Lastly, we look ahead to the coming year and the 2012 Olympic Games in London, and we determine which world records are most in danger of being broken and which are most likely to last a while longer.

2 Methods

In general, we estimate a log-normal distribution for each athletic event \( k \) using a list of the best \( n_k \) marks from that event. Equivalently, we assume that the natural logarithms of performances from each event are normally distributed. We use this second formulation throughout this paper.

A list of best marks represents only one tail of the distribution, and so for simplicity we convert marks so that we perform all calculations on the lower tail. For running events, a lower time is better, and thus we take only the natural logarithm of the times, in seconds, before fitting a normal distribution to the data. For throwing and jumping events, a higher mark is better, so we assume that the inverse (negative) of the natural logarithm is normally distributed. This does not cause any adverse consequences as long as we again take the inverse before converting back to an actual mark, typically in centimeters (cm).

Figure 1 illustrates how a normal distribution can be fit to a list of top [log-]performances, represented by a histogram. Since we are working exclusively with the tail of the distribution, the parameters must be estimated from the shape of the tail.

In our first set of analyses, we fit the model to the data as it would have been at the beginning of 2000, and we test its predictive ability for the subsequent years. Below, we elaborate on exactly how we calculate these predictions and their comparison with actual outcomes.

In the second set of analyses, we fit the model to the data as it would have been at the beginning of 2008, and we test its predictive ability for the following four years. Then we generate a set of scoring tables analogous to the IAAF scoring tables and we compare some predictions that could be made from the tables to
Figure 1: **Illustration of model fit.** Both panels in this figure show a histogram of log-performances for the men’s 400m dash (all data until the present day) as well as the fitted normal distribution curve that is re-scaled to match the histogram. The left panel gives a wider view, while the right panel shows in more detail the area of the graph which contains performances appearing on the list of top marks.

those of the IAAF scoring tables. Granted, the IAAF may not have intended for such specific predictions to be made, but we try to be as fair as possible based on what it might mean for one athletic performance to be “better” than another. We think that, generally, performances that are given equal scores should, in any given year, (1) have approximately the same number of marks exceed them, (2) should have the same chance of being broken, and (3) should have a comparable relative margin (in percent) between itself and the best mark of the year.

We then give the results of a third set of analyses that uses data through October 1st, 2011, including predictions about the numbers of top performances that will occur in the coming years as well as what we expect the top mark to be in each event and the probabilities of new world records being set.

2.1 Data

An ideal data set would consist of a complete list of every performance by an elite athlete in the modern era of athletics. Such a list, as far as we can tell, does not exist. We do have, however, lists of the best performances ever. The lists compiled by [www.alltime-athletics.com](http://www.alltime-athletics.com) (Larsson 2011) include all of the top performances of all time—list lengths ranging from a few hundred to several thousand, depending on the event. We have data for all track and field events contested in the modern Olympic Games for men and women, except the heptathlon and decathlon, plus the marathon, half marathon, one mile run and 3000m run.
We assume that these lists are complete, in the sense that each list is indeed the best $n^*_k$ performances for event $k$, with no missing marks.

For the three time periods we consider—to which we will refer by year, 2000, 2008, and 2012—we do two sets of analyses, one using all data prior to that year, and the other using data from only the prior 5 years.

The performance lists for performances prior to the present day (1 October 2011) have lengths between 215 and 9672, with a median of 1596.5. For the five years prior to the present day, list lengths range from 10 to 4630, with a median of 275. The women’s one mile run is the shortest list, and the second shortest list has 51 entries.

The performance lists for performances prior to 2008 have lengths between 205 and 5547, with a median of 1273. Using five years of data prior to 2008 gives a range of list lengths from 18 to 4235, with a median of 298.5. The list of length 18 belongs to the women’s one mile run, and the next shortest is the women’s shot put, with 38 entries. These are special cases where either the event is rarely contested (one mile run) or has a dearth of recent top performances (shot put). All other lists include at least 68 performances.

The performance lists for performances prior to 2000 have lengths between 63 and 3761, with a median of 790. For the five years prior to 2000, list lengths range from 52 to 1288, with a median of 252.5.

2.2 The model

A normal (or log-normal) distribution takes two parameters: mean $\mu$ and variance $\sigma^2$. Given these parameters, we can calculate the probability $p_a$ that a particular performance in event $k$ exceeds a specified mark $a$ using the formula:

$$p_a = \int_{-\infty}^{a} N(x \mid \mu_k, \sigma^2_k) \, dx$$

(1)

where $a$ is a specified performance (natural logarithm of a mark, inverted for events in which greater marks are better) and $N(x \mid \mu_k, \sigma^2_k)$ is the normal distribution probability density function (pdf). Equation (1) is equivalent to the cumulative distribution function (cdf) of the normal distribution with mean $\mu_k$ and variance $\sigma^2_k$, which we call $F(a \mid \mu_k, \sigma^2_k)$. If we accurately estimate $\mu_k$ and $\sigma^2_k$, then $p_a$ is easy to compute.

We can use $F(a \mid \mu_k, \sigma^2_k)$ to formulate the pdf of a normal distribution truncated at $c_k$ as:
Bayes’ Theorem then gives the un-normalized posterior density for the model parameters:

$$
\ell(\mu_k, \sigma_k^2 | X_k) = \prod_{x \in X_k} \frac{N(x | \mu_k, \sigma_k^2) p(\mu_k) p(\sigma_k^2)}{F(c_k | \mu_k, \sigma_k^2)}
$$

where $X_k$ is the set of performances on the list for event $k$, and $p(\mu_k)$ and $p(\sigma_k^2)$ are the prior probability distributions of $\mu_k$ and $\sigma_k^2$, respectively.

### 2.3 Development of an empirical prior

In general, we would like to use non-informative prior distributions for our model parameters $\mu_k$ and $\sigma_k^2$, but when first fitting our model to the data, it quickly became clear that there was much uncertainty about the total population size $N_k$ for each event $k$. So, we used an empirical Bayes approach to estimate reasonable prior expectations for the $N_k$ in order to reduce this uncertainty.

That is, the posterior densities suggested that when using non- or weakly-informative priors for each event, many $\{\mu_k, \sigma_k^2\}$ pairs were nearly equally likely, and they gave a wide range of values for $N_k$, as calculated according to the following relation:

$$
F(w_k | \mu_k, \sigma_k^2) = \frac{n_k}{N_k}
$$

where, $n_k$ is the [constant] length of the list of best performances for event $k$, and $w_k$ is the worst mark on that list. Equation 4 is inherently true, as it says only that the cumulative density through the region for which we have data—i.e. the tail—is equal to the size of the data set, $n_k$, divided by the size of the largest possible data set, $N_k$.

In order to reduce this uncertainty over the $N_k$ and ensure that the estimated population sizes for different events were similar, we re-parametrized the model, using equation 4, to use $N_k$ as a parameter instead of $\sigma_k^2$. Then, we assume a log-normal prior distribution for the $N_k$, with parameters $\mu_N$ and $\sigma_N^2$, as well as a uniform prior distribution over all real numbers for the $\mu_k$, which is non-informative and improper.

We would, ideally, optimize the parameters $\mu_N$ and $\sigma_N^2$ of the prior for $N_k$, as suggested by [MacKay](1999), iteratively as we fit the model, but since the model is fit independently for each event and because calculation takes a considerable
amount of time, we are not able to use many iterations. We chose to approxi-
mate two such iterations, where in the first iteration we fit all models using a very 
weakly-informative prior for \( N_k \) (i.e. \( \mu_N = 10,000 \) and \( \sigma^2_N = e^{20} \)), and then, in 
the second iteration, we re-fit the models with updated parameters \( \mu_N \) and \( \sigma^2_N \), 
which were optimized based on point estimates for the \( N_k \). Specifically, we calcu-
late from the first-iteration posterior distributions, for each \( k \), the expected value 
of \( N_k \), \( E[N_k] \), and then using these estimates to update \( \mu_N \) and \( \sigma^2_N \) according to the 
following:

\[
\mu_N = \text{median}(\{\log(E[N_k]) : \text{for all } k\}) \tag{5}
\]
\[
\sigma^2_N = \min_{K \subset \{\text{all } k\}} \text{var}(\{\log(E[N_k]) : k \in K\}) \tag{6}
\]

where the subset \( K \) comprises 75% of the set of all events. Thus, both prior 
distribution parameters \( \mu_N \) and \( \sigma^2_N \) are robust to some outlying \( N_k \), which we en-
countered in a few cases, particularly in events for which we have little data, as 
well as with data from the sprints, high jump, and pole vault, because those data 
are more discrete than others, as many competitors share the same mark. We 
chose the value 75% somewhat arbitrarily, but it ensures that most of the data are 
used while allowing for inaccurate values due, for example, to small or highly 
discrete data sets. Updating the prior distribution for the \( N_k \) only once in this 
manner gives a compromise between non-informative and fully optimized priors, 
while improving convergence and sharing some information between models for 
different events.

While we do not expect the population sizes from different events to be identical—
there are many reasons why there could be more participants or performances in 
one event than another—we do not expect them to be vastly different, either. For 
example, there are more marathon times posted each year than in any other event, 
though admittedly most are not elite times. Also, the one mile run and the 1500m 
run are very similar in distance, yet each year there are far more 1500m races than 
mile races. Sprinters tend to run more races each year than long distance runners, 
as well. On the other hand, we expect the population sizes to be relatively simi-
lar, perhaps within an order of magnitude of each other, simply because—among 
other reasons—awards, medals, and championships are generally identical in na-
ture and quantity for most events, and identical incentive leads us to believe that 
population sizes would be approximately equal. We have tried to address this in 
choosing our prior distributions.

### 2.4 Fitting the model to the data

To fit the model (3) for each event, we use Markov chain Monte Carlo (MCMC) 
methods as implemented in the \textit{mcmc} package \cite{Geyer2010} of the \textit{R} program-
ming language (R Development Core Team, 2008), which is a version of the Metropolis-Hastings algorithm (Hastings, 1970). We use a “burn in” period of 1,000 steps, after which we test the sample acceptance rate, requiring it to be between 0.2 and 0.4 (we found that this range generally gives good convergence), and if unacceptable we re-do the burn-in with an adjusted sample step size. This process is automated. Following burn-in, we use a subsequent 1,000 batches of 50 steps each with 10 random parameter initializations to determine the joint distribution of $\mu_k$ and $\sigma_k^2$—and/or $N_k$—for each $k$.

Convergence of the MCMC sampling was assessed visually using various plots as well as using the multivariate diagnostic of Gelman and Rubin (1992) as implemented in the coda (Plummer, Best, Cowles, and Vines, 2006) package in R (R Development Core Team, 2008).

### 2.5 Some meaningful statistics

The value of $p_a$ as calculated in equation 1 can be interpreted as the probability that in a given performance a specified member of the total elite athlete population for the given event performs better than the mark $a$. This is a natural measure of performance quality, but it is not easy to test its accuracy using real data. Therefore, in this section we give some other statistics based on the model that may be better at describing the performances we witness during an athletic season. They are based on the ideas stated in the introduction to this paper, that we can measure the rarity—and quality—of a performance by the number of marks that improve upon it or by comparing it with a reference performance. Unless stated otherwise, the statistics below are estimated using 1000 samples of the parameter values.

#### 2.5.1 Expected number of performances improving upon a specified mark

If we fit the model to $t_m$ years of data, then for any point estimates of $\mu_k$, $\sigma_k$, and $N_k$ (and hence the cdf $F(a \mid \mu_k, \sigma_k^2)$) for each event $k$, the expected number of performances during one calendar year that are better than $a$ is:

$$A_k(a \mid \mu_k, N_k) = \frac{N_k}{t_m} F(a \mid \mu_k, N_k)$$

(7)

using the re-parametrized version of the cdf function $F$ (with $\mu_k$ and $N_k$ as given parameters instead of $\mu_k$ and $\sigma_k^2$). We can use our previously-obtained samples from the posterior distributions of the parameters to efficiently find the posterior expected value $\hat{n}_k(a)$ of $A_k(a \mid \mu_k, N_k)$:

$$\hat{n}_k(a) = \int \int A_k(a \mid \mu_k, N_k) p(\mu_k, N_k \mid X_k) \, d\mu_k \, dN_k$$

(8)
This expected number of marks can be compared with data from future athletics seasons (i.e. data not included when fitting the models).

2.5.2 Probability of a record being broken

If we fit the model to $t_m$ years of data, then for any point estimates of $\mu_k$, $\sigma_k^2$, and $N_k$ for each event $k$, the probability that the best performance over $t_f$ calendar years is better than a performance $a$ is:

$$B_k(a \mid \mu_k, N_k) = 1 - \left[1 - F(a \mid \mu_k, N_k)\right]^{t_f N_k \over t_m}$$  \hspace{1cm} (9)

We can compute the posterior expectation of $B_k(a \mid \mu_k, N_k)$ as we did in equation 8:

$$\hat{\lambda}_k(a) = \int \int B_k(a \mid \mu_k, N_k) p(\mu_k, N_k \mid X_k) \, d\mu_k \, dN_k$$  \hspace{1cm} (10)

This estimated probability $\hat{\lambda}_k(a)$ of a mark $a$ being broken by anyone during the given year can be useful for comparing the very best performances—as we do in the Results section—but is less suitable for comparing lesser marks. This is because the probability of a lesser mark being broken in the course of a year is very high, and quickly approaches 1 as the quality of the mark $a$ decreases.

2.5.3 Expected best performance

Equation [10] gives the estimated probability that a particular mark will be broken in a given calendar year. In other words, it is the estimated cdf of the best performance for the year. Therefore, the probability density of the best performance $y_1$ during that year is the derivative of $\hat{\lambda}_k(a)$ from equation 10 and the expected best performance is:

$$\hat{\gamma}_1 = \int_{-\infty}^{\infty} y_1 \left( d \over dy_1 \hat{\lambda}_k(y_1) \right) dy_1$$  \hspace{1cm} (11)

The quantity $\hat{\gamma}_1$ is the expectation of an order statistic on normally distributed data, for which there is no closed-form expression. Furthermore, we have calculated the values of the function $\hat{\lambda}_k(a)$ using numerical integration over the posterior parameter distributions, so the calculation of $\hat{\gamma}_1$ is not straightforward. However, the high-density region of the derivative of $\hat{\lambda}_k(y_1)$—i.e. the pdf of the year’s best performance—is unimodal and in a predictable location, namely close to other years’ best performances. Thus, to calculate $\hat{\gamma}_1$, we first estimate the derivative of $\hat{\lambda}_k(y_1)$ by estimating $\hat{\lambda}_k(y_1)$ for a large number of values of $y_1$ (using samples $\{\mu_k, N_k\}$ from the parameter posterior distributions) and calculating the estimated differentials $\Delta \hat{\lambda}_k(y_1)$ between adjacent values of $y_1$. Then, we use the esti-
mates $\Delta \hat{p}_k(y_1)/\Delta y_1$ in place of the derivative to perform the integral in equation 11 numerically. Because the density function for $y_1$—the derivative of $\hat{p}_k(y_1)$—is unimodal and has high density only in a predictable location, this numerical integration is quick, easy, and accurate.

2.5.4 Proposed formula for performance scoring

We propose a formula for scoring that is analogous to the IAAF scoring tables. For this, we choose to define the quality of an elite performance mainly using $\hat{n}_k(a)$ above, i.e. the expected number of performances exceeding a given reference mark. That is, two elite-level marks may be considered equal if we expect them to be exceeded by the same number of individual performances during a calendar year. The statistic $\hat{n}_k(a)$ is itself valid only for the highest levels of competition—those represented on the lists of top performances that we have—but we would like our scoring formula to be valid for most events also at sub-elite levels. To do this, we took a particular value for $\hat{n}_k(a)$—we chose 0.125 because it was close to most of the current world records—and we defined the corresponding mark $a_0$ to be equal to 1300 points, which is approximately equivalent to most world records on the IAAF scoring tables. We then define the score $S_a$ of any mark $a$ to be

$$S_a = \begin{cases} 
1300\log_2(a_0) + 1 - \log_2(a) & \text{for times} \\
1300\log_2(a_0) - 1 + \log_2(a) & \text{for distances}
\end{cases}$$

(12)

A problem that we encountered here is that a good mark in the one mile run is far more rare than than a comparable mark in the 1500m run, since the mile is run less often. Because the training and ability to run the two events are practically identical, we can assume that the athletes are interchangeable, and so, to remedy the discrepancy between the population sizes $N_k$ for the two events, we set the population size $N_k$ for the mile equal to that of the population size for the 1500m, for both men and women. This is a somewhat arbitrary choice, but the mile is not contested at the major championships and is thus rather dissimilar to the other events; rather than throwing it out entirely, we found that borrowing the $N_k$ from the 1500m run produced satisfactory results.

2.6 Correlation with future performances

For each of the above-mentioned statistics, we would like to compare our predictions with those of other scoring methods. However, the other scoring methods give only a relative score, and no predictions. Thus, to compare our methods to the others, we must use a relative measure. Given a list of performances, one for
each athletic event, we assign scores to each mark and then calculate the Pearson correlation coefficient between the scores and some future outcome, either the number of better performances for each event or the improvement in performance over some reference mark. For the purposes of comparing with the IAAF scoring tables, we define “improvement in performance” of a new mark $a_{new}$ over an old mark $a_{old}$ to be $-\log(a_{new}/a_{old})$. This gives a measure of the relative improvement, which could be negative if the new mark is worse than the old mark. As above, we use the inverse of this score for events in which a higher mark is better. The expected relative improvement is another estimate of the quality of a given performance. Below, we use as reference performances $a_{old}$ the 10th, 25th, 50th, and 100th best all-time performances prior to the analysis year (2000, 2008, or 2012).

For example, for the year 2000 analysis, we calculate the expected best performance $x_1$ over the next two years (2000-2001) and we let this be $a_{new}$ while the 10th, 25th, 50th, and 100th best performances prior to 2000 are each used as $a_{old}$. This gives four different versions of the expected improvement score for each athletic event for each analysis year, for which we can then calculate a Pearson correlation with actual performances in those subsequent years. If an $a_{old}$ for a particular event is weaker than that of other events, we expect to see a larger improvement in subsequent years, and likewise a smaller improvement for stronger reference performances $a_{old}$.

Below, we list many such correlations for our scoring methods, and we compare them with correlations for the IAAF scoring tables.

### 3 Results

In this section, we give three sets of results: one for data preceding 2000, which we compare with later performances; one for data preceding 2008, which we compare with later performances as well as to the 2008 IAAF scoring tables; and one for data up to the present day (1 October 2011), which we use to make predictions for the coming years.

#### 3.1 Convergence

For the three time periods, 2000, 2008, and 2012, and for each of these using all prior data and then only five years of data (thus, six cases in total), the MCMC sampling converged usually without using the empirical prior on the total population size. The slowest convergence in general occurred when using five years of data prior to 2008. Only 37 out of 48 events had Gelman-Rubin diagnostic statistics less than 1.1. When using the empirical prior, the Gelman-Rubin diagnostic
was less than 1.1 for every event in every case, and in each case was less than 1.05 for at least 43 of the 48 events.

Population sizes varied between the events, and the use of the empirical prior on $N_k$ improved convergence and moderated unreasonable population sizes. For example, for all data preceding 2008, the median population size was 19,028, and the robust standard deviation (using 75% of the events) of $\log(N_k)$ was 2.93. The smallest (unrestricted) estimated population size was 510.4 for the women’s mile run, and the largest was $2.71 \times 10^{16}$ for men’s pole vault. Large population sizes such as that of the men’s pole vault are clearly too large, and thus using the empirical prior makes intuitive sense as well as improves convergence. The estimated population size for men’s pole vault when using the empirical prior was still 38.0 million ($3.8 \times 10^{7}$), and that of the women’s mile run was 1309.9, so some flexibility in the choice of population sizes was preserved.

A set of selected posterior expectations of parameter values are shown in table 1. Fans of track and field will notice that the marks $e^{\mu_k}$ are rather mediocre for elite athletes, and those events with larger estimated population sizes have less impressive values for $e^{\mu_k}$, which makes sense intuitively. Assuming that the very best athletes are always participating in their respective events, a larger population size indicates that there are more less-talented athletes participating and making the average performance weaker.

### 3.2 Predictions made prior to 2000

We used data from before 2000 to predict both the number of performances exceeding and the expected improvement over four different reference marks in each event, namely the 10th, 25th, 50th, and 100th best ever marks in each event at the end of 1999. The Pearson correlations of our predictions with the actual outcomes in the subsequent 12 years can be seen in tables 2 and 3.

We can see in table 2 that the predicted number of better performances correlates much more highly with the actual outcomes when we used only the previous five years of data. In fact, the predictions using all data had very poor correlation (Pearson) with the actual outcomes, but the same is not true of the predicted performance improvement. The predicted improvements were significantly correlated with the actual improvements both when we used all data and when we used only the previous five years of data, though the latter still gives better results. We suspect that that the total number of athletes participating in the various events has changed more dramatically over time than has the quality of the very best performers, making our predictions of best performances—and the associated improvement score over the reference marks—more accurate than our predictions of numbers of athletes exceeding the same reference mark.

Table 4 gives the Pearson correlation of the predicted probabilities of a world
\( \mu_k - 2\sigma_k \), (2) the mean, and (3) mean plus two standard deviations, as well as (4) the posterior expectation of the total population size. Running times are given in hours:minutes:seconds, where applicable, distances and heights are given in meters, and population sizes are the number of performances in the five-year period 2007-2011.

| Event           | \( e^{\mu_k - 2\sigma_k} \) | \( e^{\mu_k} \) | \( e^{\mu_k + 2\sigma_k} \) | \( E[N_k] \) |
|-----------------|-------------------------------|-----------------|-------------------------------|--------------|
| mens100m        | 10.55                         | 11.28           | 12.05                         | 1371048      |
| mens200m        | 21.76                         | 23.66           | 25.72                         | 1543952      |
| mens1500m       | 3:32.22                       | 3:38.78         | 3:45.55                       | 2469         |
| mensMarathon    | 2:05:55.76                    | 2:11:13.44      | 2:16:44.49                    | 1284         |
| mensHJ          | 2.00                          | 2.11            | 2.22                          | 695184       |
| mensLJ          | 6.30                          | 6.96            | 7.68                          | 326672       |
| womens100m      | 11.33                         | 12.01           | 12.73                         | 83707        |
| womens200m      | 23.32                         | 24.75           | 26.27                         | 146548       |
| womens1500m     | 3:59.52                       | 4:05.33         | 4:11.29                       | 625          |
| womensMarathon  | 2:22:38.28                    | 2:29:23.24      | 2:36:27.37                    | 823          |
| womensHJ        | 1.69                          | 1.82            | 1.96                          | 11229        |
| womensLJ        | 5.53                          | 6.00            | 6.51                          | 174252       |

Table 2: Correlations, 2000 number of better performances. Given in the table are the Pearson correlation coefficients between the predicted and actual number of performances exceeding a reference mark, based on the year 2000. The reference marks (the columns) are the 10th, 25th, 50th, and 100th best prior mark in each event.

| years          | using all prior data | using 5 years of prior data |
|----------------|----------------------|-----------------------------|
|                | 10th  | 25th  | 50th  | 100th | 10th  | 25th  | 50th  | 100th |
| 2000-2001      | -0.185 | -0.139 | -0.118 | 0.090 | 0.226 | 0.414 | 0.498 | 0.612 |
| 2000-2003      | -0.198 | -0.095 | -0.100 | 0.062 | 0.163 | 0.380 | 0.463 | 0.581 |
| 2000-2005      | -0.175 | -0.082 | -0.094 | 0.050 | 0.139 | 0.352 | 0.423 | 0.572 |
| 2000-2007      | -0.164 | -0.082 | -0.096 | 0.049 | 0.124 | 0.331 | 0.397 | 0.554 |
| 2000-2009      | -0.161 | -0.085 | -0.097 | 0.051 | 0.117 | 0.323 | 0.388 | 0.548 |
| 2000-2011      | -0.158 | -0.085 | -0.097 | 0.049 | 0.116 | 0.319 | 0.382 | 0.552 |

 record being set with the actual outcome (1 for a world record, 0 for none) over a given time period. Again, there is significant correlation between the predictions and the outcomes, and the predictions based on five years of data were generally
Table 3: **Correlations, 2000 performance improvement.** Given in the table are the Pearson correlation coefficients between the predicted and actual performance improvement over the reference mark, based on the year 2000. The reference marks (the columns) are the 10th, 25th, 50th, and 100th best prior mark in each event.

Better than those based on all data. Also, the correlations generally increased when more years were considered; this is likely due to the rarity of records, whereby the calculated probability of a world record occurring in the next 12 years will be more accurate than the probability for only one or two years. Based on only five years of data, we achieved Pearson correlation coefficients of approximately 0.7 for time periods of length 6-12 years.

| years     | using all prior data | using 5 years of prior data |
|-----------|----------------------|-----------------------------|
|           | 10th  | 25th  | 50th  | 100th | 10th  | 25th  | 50th  | 100th |
| 2000-2001 | -0.274| 0.343 | 0.583 | 0.562 | 0.765| 0.832 | 0.877 | 0.864 |
| 2000-2003 | 0.049 | 0.484 | 0.641 | 0.609 | 0.696| 0.783 | 0.837 | 0.839 |
| 2000-2005 | 0.176 | 0.512 | 0.644 | 0.607 | 0.674| 0.769 | 0.822 | 0.807 |
| 2000-2007 | 0.248 | 0.538 | 0.659 | 0.624 | 0.699| 0.785 | 0.834 | 0.824 |
| 2000-2009 | 0.261 | 0.519 | 0.637 | 0.592 | 0.698| 0.777 | 0.824 | 0.808 |
| 2000-2011 | 0.302 | 0.545 | 0.656 | 0.617 | 0.731| 0.804 | 0.845 | 0.835 |

Table 4: **Correlations, 2000 world records.** Given are the Pearson correlation coefficients between the predicted probability of a world record being set and the actual occurrence (vector of zeros and ones), based on the year 2000.

| years     | all data | 5 years |
|-----------|----------|---------|
| 2000-2001 | 0.144    | 0.324   |
| 2000-2003 | 0.289    | 0.443   |
| 2000-2005 | 0.467    | 0.712   |
| 2000-2007 | 0.442    | 0.706   |
| 2000-2009 | 0.383    | 0.675   |
| 2000-2011 | 0.344    | 0.678   |

3.3 **Predictions made prior to 2008**

In general, the predictions we made based on data prior to 2008 were much better than those from 2000. This could be due to a number of factors, such as the much larger data set, the increased modernization of training and competition, or the likely decrease in the use of performance-enhancing drugs. However, the predictions made using only five years of prior data were again considerably better than
those using all prior data. In fact, our predictions of both the number of performances exceeding and the relative improvement over the 100th best performances of all time have Pearson correlations greater than 0.83 with the actual outcomes in the 2008 athletics season as well as for all seasons through 2011. Tables 5 and 6 show the details of the correlation coefficients.

Table 5: **Correlations, 2008 number of better performances.** Given in the table are the Pearson correlation coefficients between the predicted and actual number of performances exceeding a reference mark, based on the year 2008. The reference marks (the columns) are the 10th, 25th, 50th, and 100th best prior mark in each event.

| year(s) | using all prior data | using 5 years of prior data |
|---------|----------------------|-----------------------------|
|         | 10th  | 25th  | 50th  | 100th | 10th  | 25th  | 50th  | 100th |
| 2008    | 0.322 | 0.330 | 0.294 | 0.193 | 0.561 | 0.765 | 0.774 | 0.841 |
| 2008-2009 | 0.329 | 0.298 | 0.298 | 0.197 | 0.672 | 0.752 | 0.784 | 0.834 |
| 2008-2010 | 0.333 | 0.299 | 0.331 | 0.206 | 0.602 | 0.728 | 0.783 | 0.831 |
| 2008-2011 | 0.321 | 0.308 | 0.345 | 0.210 | 0.605 | 0.751 | 0.806 | 0.847 |

Table 6: **Correlations, 2008 performance improvement.** Given in the table are the Pearson correlation coefficients between the predicted and actual performance improvement over the reference mark, based on the year 2008. The reference marks (the columns) are the 10th, 25th, 50th, and 100th best prior mark in each event.

| year(s) | using all prior data | using 5 years of prior data |
|---------|----------------------|-----------------------------|
|         | 10th  | 25th  | 50th  | 100th | 10th  | 25th  | 50th  | 100th |
| 2008    | 0.188 | 0.129 | 0.249 | 0.286 | 0.825 | 0.821 | 0.830 | 0.835 |
| 2008-2009 | 0.110 | 0.064 | 0.191 | 0.260 | 0.847 | 0.842 | 0.849 | 0.851 |
| 2008-2010 | 0.038 | 0.042 | 0.181 | 0.260 | 0.846 | 0.841 | 0.849 | 0.853 |
| 2008-2011 | 0.030 | 0.068 | 0.214 | 0.298 | 0.837 | 0.836 | 0.847 | 0.853 |

Table 7 shows the Pearson correlation between predicted probabilities of world record being set and the actual outcomes. For the period 2008-2011, the predicted probabilities had a correlation coefficient of 0.48 with the actual outcomes, which is slightly higher than the corresponding correlation coefficient from the four-year period beginning in 2000, as shown in table 4. Thus, our predictions from the beginning of the year 2008 are better in nearly every case than those from the year 2000.
| year(s)     | all data | 5 years |
|------------|----------|---------|
| 2008       | 0.225    | 0.338   |
| 2008-2009  | 0.363    | 0.497   |
| 2008-2010  | 0.278    | 0.491   |
| 2008-2011  | 0.257    | 0.484   |

Table 7: **Correlations, 2008 world records** Given are the Pearson correlation coefficients between the predicted probability of a world record being set and the actual occurrence (vector of zeros and ones), based on the year 2008.

### 3.4 Comparison with IAAF scoring tables

The scoring tables we have constructed based on the model described in this paper are designed to be analogous to the IAAF scoring tables (Spiriev, 2008, Spiriev and Spiriev, 2011), ranging from a score of zero for a relatively poor performance to approximately 1300 points for the current world records. A subset of scores from our tables can be found in table 8; a full table can be found in the supplementary materials. (Note: for the five years preceding 2012, there were only 10 marks in the data set for the women’s one-mile run; though parameter convergence was achieved, the scores assigned were clearly not in line with the women’s 1500m performances. We include the women’s one-mile run in the scoring tables for completeness, but we discourage their use in performance comparison.) Thus, the two sets of tables have both been made mainly to compare elite-level performances, though they both are applicable to the performances of even recreational athletes. In previous sections, we tested the predictive ability of the model and the various statistics we calculate from it; in this section, we do the same tests on the predictive ability of the scoring tables constructed in this paper—using data prior to 2008—and we compare the results to those of the 2008 IAAF scoring tables, which to the best of our knowledge were constructed based on the same available data.

Table 9 gives the Pearson correlations of the reference performance scores (as assigned by the sets of scoring tables to the same reference performances we used in previous analyses) with the number of marks exceeding the reference performances in subsequent years. Similarly, table 10 gives the correlations of the same scores with the relative improvements over the reference performances. Note that these correlations should be negative because a higher score indicates a better performance, which should then see fewer better performances and less improvement in the subsequent years.

The scoring tables constructed in this paper using five years of data (but not those using all data) are more predictive of future performances than the IAAF tables. For example, using the 10th best all-time performance (as of 2008) as a
points | 800 | 900 | 1000 | 1100 | 1200 | 1300 | 1400
---|---|---|---|---|---|---|---
mens100m | 12.50 | 11.85 | 11.24 | 10.66 | 10.10 | 9.58 | 9.08
mens200m | 25.12 | 23.82 | 22.58 | 21.41 | 20.30 | 19.24 | 18.24
mens400m | 56.85 | 53.89 | 51.10 | 48.44 | 45.93 | 43.54 | 41.28
mens800m | 2:11.64 | 2:04.81 | 1:58.33 | 1:52.18 | 1:46.36 | 1:40.84 | 1:35.60
mens1500m | 4:30.89 | 4:16.82 | 4:03.49 | 3:50.84 | 3:38.86 | 3:27.49 | 3:16.72
mens3000m | 9:35.72 | 9:05.83 | 8:37.48 | 8:10.62 | 7:45.14 | 7:20.99 | 6:58.09
mens5000m | 16:26.21 | 15:35 | 14:46.45 | 14:00.43 | 13:16.79 | 12:35.42 | 11:56.20
mens10000m | 33:54.31 | 32:08.69 | 30:28.54 | 28:53.60 | 27:23.59 | 25:58.25 | 24:37.34
mensHalfMarathon | 1:15:34.51 | 1:11:39.07 | 1:07:55.85 | 1:04:22.22 | 1:01:03.58 | 57:53.36 | 54:53.01
mensMarathon | 2:40:02.90 | 2:31:44.29 | 2:23:51.58 | 2:16:23.40 | 2:09:18.50 | 2:02:35.66 | 1:56:13.74
womens100m | 13.80 | 13.09 | 12.41 | 11.76 | 11.15 | 10.57 | 10.02
womens200m | 28.29 | 26.82 | 25.43 | 24.11 | 22.86 | 21.67 | 20.54
womens400m | 1:03.38 | 1:00.09 | 56.97 | 54.01 | 51.21 | 48.55 | 46.03
womens800m | 2:29.47 | 2:21.71 | 2:14.35 | 2:07.37 | 2:00.76 | 1:54.49 | 1:48.55
womens1500m | 7:56.41 | 7:22.33 | 6:50.01 | 6:19.38 | 6:03.34 | 5:22.80 | 5:16.69
womens3000m | 18:13.36 | 17:16.59 | 16:22.77 | 15:31.74 | 14:43.36 | 13:57.49 | 13:14.01
womens5000m | 38:35.22 | 36:35.01 | 34:41.04 | 32:52.98 | 31:10.54 | 29:33.42 | 28:01.34
womens10000m | 1:25:54.31 | 1:21:26.69 | 1:17:12.96 | 1:13:12.40 | 1:09:24.34 | 1:05:48.11 | 1:02:23.12
womensHalfMarathon | 3:01:13.87 | 2:51:49.27 | 2:42:53.98 | 2:34:26.49 | 2:26:25.36 | 2:18:49.20 | 2:11:36.73

Table 8: Subset of scoring tables. A sample of scores from the scoring tables based on our model, using five years of data prior to 2012. Here, we show only running events, but scores for other events can be found in the full table.

Table 9: Correlations, scoring tables with number of better performances. Shown are the Pearson correlation coefficients between the points assigned by scoring tables and the actual number of better performances, based on the year 2008. The reference marks (the columns) are the 10th, 25th, 50th, and 100th best prior mark in each event. More negative correlations are better.

reference, the scores assigned by the 2008 IAAF tables have a Pearson correlation coefficient of -0.22 with the numbers of better performances from 2008 to 2011, compared to -0.43 for our tables. Likewise, the relative improvements over this same reference performance during the same time period had a correlation coefficient of -0.69 with the IAAF scores and -0.80 with our scores. Our scores were more predictive in all cases that we tested. See tables 9 and 10 for more details.

3.5 Predictions for 2012 and beyond

Heading into 2012, an Olympic year, it is interesting to examine the predictions we might make. Most interesting, we feel, is the probability that a new world
Table 10: **Correlations, scoring tables with performance improvement.**

Shown are the Pearson correlation coefficients between the points assigned by scoring tables and the actual performance improvements over the reference mark, based on the year 2008. The reference marks (the columns) are the 10th, 25th, 50th, and 100th best prior mark in each event. More negative correlations are better.

| year(s)   | IAAF scoring tables | our tables, all data | our tables, 5 years |
|-----------|---------------------|----------------------|---------------------|
|           | 10th 25th 50th 100th | 10th 25th 50th 100th | 10th 25th 50th 100th |
| 2008      | -0.65 -0.66 -0.68 -0.69 | 0.02 -0.06 -0.18 -0.28 | -0.77 -0.77 -0.78 -0.80 |
| 2008-2009 | -0.68 -0.69 -0.71 -0.72 | 0.06 -0.02 -0.14 -0.24 | -0.78 -0.78 -0.79 -0.80 |
| 2008-2010 | -0.68 -0.68 -0.71 -0.71 | 0.07 -0.01 -0.14 -0.24 | -0.79 -0.79 -0.80 -0.81 |
| 2008-2011 | -0.69 -0.69 -0.71 -0.72 | 0.05 -0.04 -0.18 -0.28 | -0.80 -0.80 -0.81 -0.83 |

The probabilities range from less than 1/100,000 for women’s discus to almost certain (0.95) for women’s steeplechase. Most of the world records (26 out of 48) have less than a 10% chance of being broken, a quarter (12) have less than a 1% chance, and only two—women’s steeplechase and men’s 110m hurdles—are likely to get broken. In both of these events, the world record was set recently, in 2008 in both cases, and there are many other recent marks that come close to the record. In particular, there are nine women’s steeplechase performances from the past five years that are within ten seconds of the world record, including the record itself. There are seven marks (including the record) in the men’s 110m hurdles from the past five years that are within 0.05s of the world record. This suggests that in both of these events, with so many recent marks that are close to the record, it is more likely than not that a record will be set in 2012.

On the other end of the spectrum, those records least likely to get broken are some of the older records, with only 6 of the 25 toughest (according to table [11]) records occurring in the past 15 years, whereas 17 of the 25 weakest records have occurred in the past 15 years. In the women’s discus, where the record is least likely to get broken, no one has produced a mark in the top 100 in nearly 20 years. The women’s 1500m run, which has the second toughest record, has seen no time within five seconds of the record in over ten years.

Notably, two events, the one mile run and the 3000m run (non-Olympic events), are contested less frequently than the rest, and therefore the probabilities of their records being broken are lower than if they were contested more often. For instance, the men’s one mile world record is obviously—to any track and field fan—easier for a well-trained athlete to break than the 1500m world record, but the probability of the mile record actually being broken is lower since there are far fewer attempts.
| Event              | WR Mark | Athlete               | Date       | Prob of WR in 2012 |
|--------------------|---------|-----------------------|------------|--------------------|
| womensDisc         | 76.8    | Gabriele Reinsch      | 09.07.1988 | 7.41×10⁻⁶          |
| womens1500m        | 3:50.46 | Qu Yunxia             | 11.09.1993 | 9.24×10⁻³          |
| mensHJ             | 2.45    | Javier Sotomayor      | 27.07.1993 | 7.09×10⁻⁴          |
| womensLJ           | 7.52    | Galina Chistyakova    | 11.06.1988 | 8.56×10⁻⁴          |
| womens3000m        | 8:06.11 | Wang Junxia           | 13.09.1993 | 1.62×10⁻³          |
| mensHammer         | 86.74   | Yuriy Syedikh         | 30.08.1986 | 1.86×10⁻³          |
| womensMarathon     | 2:15:25 | Paula Radcliffe       | 13.04.2003 | 2.52×10⁻³          |
| mens110mH          | 12.87   | Dayron Robles         | 12.06.2008 | 6.62×10⁻¹          |
| mensHalfMarathon   | 1:41:01 | David Rudisha         | 29.08.2010 | 1.61×10⁻¹          |
| womens400mH        | 46.78   | Kevin Young           | 06.06.1992 | 1.62×10⁻²          |
| mens5000m          | 21.34   | Florence Griffith-Joyner | 29.09.1988 | 1.14×10⁻²          |
| mensLJ             | 8.95    | Mike Powell           | 30.08.1991 | 1.49×10⁻²          |
| mens400m           | 47.60   | Marita Koch           | 06.10.1985 | 5.14×10⁻³          |
| mens1 mile         | 3:43.13 | Hicham El Guerrouj    | 07.07.1999 | 5.28×10⁻³          |
| womensShot          | 22.63   | Natalya Lisovskaya    | 07.06.1987 | 8.20×10⁻³          |
| mensPV             | 6.14    | Sergey Bubka          | 31.07.1994 | 9.53×10⁻³          |
| mens200m           | 2:54.29 | United States         | 22.08.1993 | 2.35×10⁻²          |
| mensShot            | 23.12   | Randy Barnes          | 20.05.1990 | 2.86×10⁻²          |
| mensDisc           | 74.08   | Jurgen Schultz        | 06.06.1986 | 3.13×10⁻²          |
| womens100mH        | 12.21   | Yordanka Donkova      | 20.08.1988 | 3.17×10⁻²          |
| mensTJ             | 18.29   | Jonathan Edwards      | 07.08.1995 | 3.84×10⁻²          |
| mens100m           | 10.49   | Florence Griffith-Joynes | 16.07.1988 | 3.92×10⁻³          |
| womens800m         | 1:53.28 | Jarmila Kratochvilova | 26.07.1983 | 2.11×10⁻²          |
| mens400m           | 43.18   | Michael Johnson       | 26.08.1999 | 2.28×10⁻²          |
| mens4x400m         | 2:54.29 | United States         | 22.08.1993 | 2.35×10⁻²          |
| mens800m           | 19.19   | Usain Bolt            | 20.08.2009 | 8.60×10⁻²          |
| mens3000m          | 7:20.67 | Daniel Komen          | 01.09.1996 | 1.08×10⁻¹          |
| womens10000m       | 29:31.78| Wang Junxia           | 08.09.1993 | 1.14×10⁻¹          |
| mens100m           | 9.58    | Usain Bolt            | 16.08.2009 | 1.23×10⁻¹          |
| womensHalfMarathon  | 65.50   | Mary Keitany          | 18.02.2011 | 1.32×10⁻¹          |
| mens4x100m         | 37.04   | Jamaica               | 04.09.2011 | 1.42×10⁻¹          |
| mens4x100m         | 41.37   | German Democratic Republic | 06.10.1985 | 1.43×10⁻²          |
| womens1 mile       | 4:12.56 | Svetlana Masterkova   | 14.08.1996 | 1.51×10⁻²          |
| womens4x400m       | 3:15.17 | Soviet Union          | 01.10.1988 | 1.54×10⁻¹          |
| mens800m           | 1:41.01 | David Rudisha         | 29.08.2010 | 1.61×10⁻¹          |
| mens5000m          | 12:37.35| Kenenisa Bekele       | 31.05.2004 | 1.84×10⁻¹          |
| mens3000mSC        | 7:53.63 | Saif Saeed Shaheen    | 03.09.2004 | 2.35×10⁻¹          |
| womensTJ           | 15.50   | Inessa Kravets        | 10.08.1995 | 2.40×10⁻¹          |
| mensHJ             | 2:09    | Sterka Kostadinova    | 30.08.1987 | 2.91×10⁻²          |
| mens400mH          | 52.34   | Yuliya Pechonkina     | 08.08.2003 | 3.32×10⁻¹          |
| mens10000m         | 26:17.53| Kenenisa Bekele       | 26.08.2005 | 3.82×10⁻¹          |
| mensJav            | 72.28   | Barbora Spotakova     | 13.09.2008 | 3.84×10⁻¹          |
| mensMarathon        | 2:03:38 | Patrick Makau         | 25.09.2011 | 3.91×10⁻¹          |
| mensHalfMarathon    | 58.23   | Zersenay Tadesse      | 21.03.2010 | 3.96×10⁻¹          |
| womensHammer       | 79.42   | Betty Heidler         | 21.05.2011 | 4.72×10⁻¹          |
| womens5000m        | 14:11.15| Tirunesh Dibaba       | 06.06.2008 | 4.76×10⁻¹          |
| mens110mH          | 12.87   | Dayron Robles         | 12.06.2008 | 6.62×10⁻¹          |
| womens3000mSC       | 8:58.81 | Gulnara Galkina       | 17.08.2008 | 9.52×10⁻¹          |

Table 11: World record probabilities, 2012. Shown is a list of the current world records for all athletic events considered in this paper, sorted by the probability of being broken in 2012.
4 Discussion

This paper has been an attempt to rigorously quantify what it means for an athletic performance to be “good”, and, alternatively, what it means for a performance to be better than another performance, particularly if the two performances are in different events. We use primarily two alternative reasons why an observer of track and field might believe that a performance is good, restated from the introduction:

1. A performance is good if few athletes improve upon it, or
2. A performance is good if it is close to or improves upon the [previous] best performance.

In the introduction, we suggested that the 9.58s 100m dash that Usain Bolt ran in the 2009 World Championships might be one of the greatest athletics feats ever. But, we can see in table [11] that there are many records that are less likely to be broken next year than Usain Bolt’s 9.58s. In fact, his own 200m world record (19.19) is one of them. On the other hand, of the world records that were set since the year 2000 (18 of them), these are the third and fourth least likely to be broken, so perhaps they are so impressive because they are among the best records of recent memory.

In addition to calculating probabilities of world records, we also calculated expected number of performances improving upon a given mark, expected best performances, and a set of scoring tables intended to be analogous to the IAAF scoring tables. Our results, particularly tables [9] and [10], show that our model can predict the levels of future performances with considerable success, and better than the most common method of performance scoring, the IAAF scoring tables. Given a set of performances or records, we can predict which ones will be broken, how many times, and by how much, and these predictions have a Pearson correlation coefficient of over 0.8 in many cases with actual future outcomes. Our scoring tables, which are derived from the expected number of annual performances exceeding a given mark, outperformed the IAAF scoring tables for two different prediction types, each with four sets of reference marks and four time periods, giving 32 cases wherein our predictions correlated more highly in every case.

The keys to the success, we believe, are the large amount of data used in model fitting and the probabilistic approach. Past scoring methods typically have used a fixed number of top performances—in some cases very few—such as the top ten or one hundred within a particular time period; we wanted to avoid this restriction and use all available data to compute actual probabilities. In general, more data is better, though admittedly there were some outlying circumstances in the past when, for example, performance enhancing drugs have been used without
detection, or marks were set under other questionable circumstances. One glaring example of this is the fact that no woman has produced a top-100 mark in the shot put or the discus in the past ten years. Likely because of these questionable performances, we have found that the most accurate way to predict the performances of the next year is by fitting the model only to recent data. Another example of a negative shift in performance is the recent switch to all-women road races, particularly in the marathon. Paula Radcliffe’s marathon world record is one of the best marks in athletics, but it was set with men running alongside the women. It has been ruled (by the IAAF) that mixed-sex races are no longer eligible for women’s records, but it seems that previous marks will be allowed to stand. Though not previously considered cheating, male pacers can help women significantly, and in their absence we have indeed seen a drop in the quality of women’s marathon times, as most major marathons have in the past few years switched to separate men’s and women’s races. These shifts in performance level are a problem we might address in future research. It is reasonable to assume that performance levels improve over time due to improved training and technique, and any large-scale decline is the result of a reduction in the prevalence of performance enhancing drugs or other forms of performance aid or cheating. There are a number of ways we might detect and remove—or otherwise take into account—these questionable performances, possibly using robust statistics or parameter optimization techniques. In addition, other probability distributions might also be considered if they seem to fit the data.

In a more general sense, it would likely help the predictive ability of the model if time were included as a contributing variable. Modeling general performance changes over time would give us further abilities to discuss and describe the history of athletics, such as in detecting or predicting eras of great improvement or change and also in modeling the maturity of an event, in the sense that, for example, the women’s steeplechase isn’t quite mature yet since it has been an Olympic event since only 2008 and its records still fall quite often.

Lastly, the type of analysis demonstrated in this paper need not be limited to athletics. Any standardized competition with a large number of performances that are either normally or log-normally distributed can be modeled in this way. Swimming and rowing come to mind, though those are more dependent on technology than athletics and thus may be more difficult to model. All in all, a probabilistic approach to studying sports performances seems to be a practical and valuable tool in examining the history and predicting the future of sport.
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