Direct-writing of complex liquid crystal patterns

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Abstract: We report on a direct-write system for patterning of arbitrary, high-quality, continuous liquid crystal (LC) alignment patterns. The system uses a focused UV laser and XY scanning stages to expose a photoalignment layer, which then aligns a subsequent LC layer. We intentionally arrange for multiple overlapping exposures of the photoalignment material by a scanned Gaussian beam, often with a plurality of polarizations and intensities, in order to promote continuous and precise LC alignment. This type of exposure protocol has not been well investigated, and sometimes results in unexpected LC responses. Ultimately, this enables us to create continuous alignment patterns with feature sizes smaller than the recording beam. We describe the system design along with a thorough mathematical system description, starting from the direct-write system inputs and ending with the estimated alignment of the LC. We fabricate a number of test patterns to validate our system model, then design and fabricate a number of interesting well-known elements, including a \( q \)-plate and polarization grating.

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OCIS codes: (260.5430) Polarization; (160.3710) Liquid crystals; (310.010) Thin films; (310.6805) Theory and design.

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#206501 - $15.00 USD  Received 14 Feb 2014; revised 29 Apr 2014; accepted 2 May 2014; published 16 May 2014
(C) 2014 OSA 19 May 2014 | Vol. 22, No. 10 | DOI:10.1364/OE.22.012691 | OPTICS EXPRESS  12691
1. Introduction

Liquid crystals (LCs) are most prominently used in liquid crystals displays (LCDs), where they function as the active elements used to control pixel brightness. In these cases, the alignment of the LC is simple, being homogeneous over relatively large areas. However a number of other applications require complex LC alignment patterns for correct operation. Examples of such elements are polarization gratings (PGs) [1], q-plates [2], beam shaping elements [3], phase plates [4], and LC actuators [5].

With the rise of photo alignment materials [6, 7], the alignment patterns required for such elements has become easier to realize using polarization holography [8]. However, while polarization holography is quite suitable for periodic patterns such as the PG, it does not solve the problem of easily aligning more arbitrary patterns.

In recent years, various photoalignment techniques have been developed to efficiently record specific patterns. These include scanning a cylindrical beam while changing the polarization to record a PG [9], scanning a cylindrical beam in an azimuthal pattern while changing the polarization to record a q-plate [10, 11], using a reconfigurable spatial light modulator (SLM) to record arbitrary patterns including a lens array [12], and using a point-by-point direct-write system with a polarization modulator to record a complex beam-shaping element [3]. It is notable that this last kind of direct-write system is actually quite old, as the first reported direct-write system with polarization modulation dates back to 1976 [13].

In this paper, we present, analyze, and demonstrate a direct-write system capable of creating arbitrary and continuous LC patterns (in-plane orientation angle profiles) with high quality and resolution. The system design and operation is discussed in Section 2. In Section 3, we develop a flexible system description starting with the direct-write scan inputs and ending with the LC response. Although we use the system with photoalignment materials and LCs, the system description is applicable to any material set sensitive to linear polarization. In Section 4, we...
fabricate a few patterns to test the accuracy of our system description and its ability to predict the LC response. In Section 5, we use the parameters obtained from Section 4 to study some interesting patterns, including adjacent scan lines to explore the continuous nature of the LC alignment, intersecting scan lines to study non-intuitive LC responses, and a highest quality $q$-plate and PG.

All of our results indicate that this approach is by far the most flexible and diverse LC patterning approach, allowing the creation of arbitrary continuous LC patterns. In addition, the approach can be easily scaled for large areas and volume manufacturing, where multiple high resolution LC patterns can be created on the same substrate.

An important distinction of the work reported here is the intentional exposure of the photoalignment material to multiple, and usually different, polarizations and intensities of UV light. While LPP photoresponse has been studied extensively in the past [14, 15, 16], including the class of materials we employ, almost all prior work has focused on the effect of polarization, incident angle, wavelength, fluence, and temperature from a single exposure. Except for a few special cases [14], very little is reported on the photoresponse when LPP is exposed multiple times with different polarization parameters and fluences. In addition, this work studies highly inhomogeneous exposure patterns which posses feature sizes tens of $\mu$m or less and approach the scale of the thickness of LC films. Combined with the prior mentioned multiple exposures, this creates a scenario which, to the authors knowledge, has never been studied before.

2. System design

A high level schematic of the system is shown in Fig. 1. The optical train begins with a continuous wave 325 nm HeCd laser. The laser then passes through an active polarization modulator. Since we are using a photoalignment material which only responds to linear polarization, we are only interested in changing the linear polarization angle of the beam. Thus, possible polarization modulators include a rotating HWP or a variable retarder combined with a quarter wave plate (QWP) (when configured properly, these two elements rotate the polarization angle as the retardation is changed). Our system uses the later, with a KD*P Pockels cell by ConOptics as the variable retarder. After this, the laser passes through a spatial filter and a collimating lens (not shown in figure). Last, the beam is focused to the substrate with a 40X objective lens. The minimum beam waist [17] we have achieved with this setup is $\sim 1 \mu$m.

The substrate itself is coated with a linear photopolymerizable polymer (LPP) and is placed on two stages for XY translation. The stages are from Newport (ILS200LM) and have 10 nm resolution. Since even the smallest beam waist is 100 times this resolution, the resolution and accuracy of the XY stages is neglected in the analysis. The entire system is controlled via a Newport XPS controller.

After exposure, the substrate is removed and a liquid crystal polymer (LCP) layer is spin cast
3. System description

3.1. Approach

Starting with the inputs to the scanning system, we would like to be able to predict the LC response. By LC response, we specifically mean the in-plane LC optical axis orientation (or nematic director) and the quality of LC alignment. We define quality of alignment as the degree to which the LC is aligned in-plane; LC that is aligned out of plane (i.e., poorly aligned) is characterized by point defects, line defects, and a reduced birefringence.

Finding the LC response is not trivial, even for relatively simple alignment patterns. Sophisticated mechanics-based models such as elastic continuum theory have been developed to analyze specific LC patterns. This model is most commonly applied to study the response of uniformly aligned LCs under applied voltages, but has also been applied to PGs [18] to gain insight into the physical limits of PG fabrication. However, using elastic continuum theory for patterns more complex than the PGs or other simple periodic structures [19] present serious difficulty when approached from either an analytic or numeric perspective. Thus, our approach here is to develop a new model tailored specifically for the problem of LC on complexly patterned LPP, based on fundamental, observed interactions between an LPP exposure and the resulting LC response.

One option to do this would be to a) find the electromagnetic output of the system, b) find the physical response of the LPP to the electromagnetic output, and then c) find the physical response of the LC to the LPP. However, finding the response of the LPP is difficult to do and cumbersome to describe. Instead, we a) find the electromagnetic output, b) adjust it based on assumed properties of the LPP resulting in what we call the adjusted electromagnetic output, and then c) find the LCs physical response from the adjusted electromagnetic response. A high-level view of this process is shown in Fig. 2.

![Diagram](https://example.com/diagram.png)
The electromagnetic output of the system is fully described by $S(r, t)$, a Stokes vector with spatial and temporal dependence. The adjusted electromagnetic output is fully described by $S_r$, a time-independent Stokes vector with spatial dependence. The response of the LC is thus predicted based on the Stokes vector $S_r$. This approach allows us to avoid unneeded complications and also to predict the response of the LC using familiar metrics such as fluence and degree of linear polarization (DOLP).

3.2. System inputs and electromagnetic output

We begin with the independent inputs to the system: the beam intensity profile $I_b(r)$, the XY scan path $r_b(t)$, and the instantaneous exposure polarization $\psi_b(t)$. We will only consider linear polarization, such that $\psi_b(t)$ is the orientation of the linear polarization. However, the model can easily be used to describe systems with elliptically polarized beams as well. Next we find the instantaneous exposure intensity over the entire scan space:

$$I(r, t) = I_b(r - r_b(t))$$

Then we find the instantaneous exposure Stokes vector over the entire scan space, which is the electromagnetic output:

$$S(r, t) = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = I(r, t) \begin{bmatrix} 1 \\ \cos 2\psi(t) \\ \sin 2\psi(t) \\ 0 \end{bmatrix}$$

3.3. Adjusted electromagnetic output

We would like to adjust $S(r, t)$ using some transformation function $H$ that embodies the relevant properties of the LPP and removes the time-dependence. While it may not be valid for all LPPs, our experimental work with at least some supports the assumption that $H$ is a simple average over the entire exposure:

$$S(r) = H(S(r, t)) = \frac{1}{\tau} \int_0^\tau S(r, t) dt$$

where $\tau$ is the duration of the entire exposure.

We can say that $H$ is a valid transformation from $S(r, t)$ to $S(r)$ if and only if all $S(r, t)$ with the same $S(r)$ produce the same response in the LC. Figure 3 shows some different $S(r, t)$ with the same $S(r)$ for our chosen $H$, which may provide insight into what $H$ implies physically. We interpret Fig. 3(a) as implying that the LPP is non-causal, an assumption we believe is valid if the exposure fluence is low enough, keeping the LPP away from saturation. Figure 3(a) also implies that it is the average alignment of the LPP that matters, not the individual exposures. We interpret Fig 3(b) as implying that orthogonal exposure orientations are equivalent to an unpolarized input, *i.e.* they cancel each other out and have no influence on the alignment of the LC. Our $H$ also implies that the LPP responds at the point $r$ only to incidence fluence delivered to the point $r$ (*i.e.*, neighboring regions do not affect each other), and also that the LPP’s response does not change with incident power.

We will transform $S(r)$ into three terms which we call the key parameters that allow for easy interpretation of the final LPP alignment. They are the total fluence $F(r)$, the average polarization orientation $\psi_{avg}(r)$, and the average DOLP $D_{avg}(r)$. The total fluence is found as:

$$F(r) = S_0(r) \tau$$
Fig. 3. Graphical representation of some properties of our chosen $H$, which takes the average of all input Stokes vectors. Each diagram is a 2D cross-section of the Poincare sphere (since we are only concerned with linear polarization), and the ring in each diagram represents a DOLP of 1.  a) Non-causality and orientation averaging. b) Depolarization by superposition of orthogonal polarizations.

The average polarization orientation is easily found from $S_1(r)$ and $S_2(r)$ using the atan2 function (the unwrapped arctangent):

$$\psi_{avg}(r) = \text{atan2}(S_2(r), S_1(r))$$

$$\text{atan2}(y, x) = 2\tan^{-1} \frac{y}{\sqrt{x^2 + y^2}}$$

The average DOLP goes from 0 to 1 and the calculation is straightforward:

$$D_{avg}(r) = \frac{\sqrt{S_1(r)^2 + S_2(r)^2}}{S_0(r)}$$

3.4. LC response

From the key parameters we find what we define as the LC anchoring vector $A(r)$. The angle of the vector is the in-plane angle we expect the LC to align to. The magnitude of the vector, which varies from 0 to 1, represents our confidence that the LC will align well to $A(r)$. To elaborate, if $|A(r)|$ at some $r$ is a high value such as 0.99, it means we have high confidence that the LC will align to $A(r)$ and that the LC will be well aligned, i.e., without scattering or defects. If $|A(r)|$ at some $r$ is a low value such as 0.01, it means that we do not have confidence that the LC will align to $A(r)$ or that the LC will be well aligned. When LC is not well aligned, it usually means that the azimuthal angle is random and will result in a defect. We emphasize that $|A(r)|$ is our prediction confidence, and when our prediction confidence is low it means we do not know how the LC will respond. Thus, when $|A(r)|$ is low, the LC may be well aligned to $A(r)$, well aligned to some other angle, poorly aligned to some angle, or have no apparent alignment at all depending on a multitude of variables which this model does not account for.

In general, $A(r)$ is found using some function $T$ which itself is a function of the key parameters:
\[ A(\mathbf{r}) = T(\psi_{\text{avg}}(\mathbf{r}), F(\mathbf{r}), D_{\text{avg}}(\mathbf{r})) \]  

(7)

We make the assumption that the angle and magnitude information of \( T \) are independent, which simplifies the function considerably. A further assumption is that the magnitude of \( T \) has the form of two multiplied threshold functions. The hyperbolic tangent is a convenient threshold function to use, and combined with a simple assumption for the angle information, \( A(\mathbf{r}) \) is found as follows:

\[
\begin{align*}
\angle A(\mathbf{r}) &= \psi_{\text{avg}}(\mathbf{r}) \\
|A(\mathbf{r})| &= T_F(F)T_D(D_{\text{avg}})
\end{align*}
\]

(8a)

\[
T_F(F) = \frac{1}{2} \left( \tanh \left( \frac{F(\mathbf{r}) - F_{TH}}{w_{TH}^F} \right) + 1 \right)
\]

(8c)

\[
T_D(D_{\text{avg}}) = \frac{1}{2} \left( \tanh \left( \frac{D(\mathbf{r}) - D_{TH}}{w_{TH}^D} \right) + 1 \right)
\]

(8d)

where \( F_{TH} \) and \( D_{TH} \) are the threshold values and \( w_{TH}^F \) and \( w_{TH}^D \) are the widths of the threshold functions. The widths specify the distance from the threshold value where the function is equal to 0.5 ± 0.33. Examples of the functions \( T_F \) and \( T_D \) are shown in Fig. 4. We use this form of \( T \) for the remainder of this paper.

3.5. Physical interpretation of \( H \) and \( T \)

Since the material transfer functions we have introduced (\( H, T \)) are atypical, a few comments may be appropriate to provide a physical interpretation of their meaning. The function \( H \) mainly represents the material response of the LPP to one or more exposure processes. While it is true that a photoinduced molecular reaction of the LPP occurs (e.g., isomerization or preferential chemical bonding [14, 15]), we have chosen to abstract away the precise reorientation mechanism because it is not needed for the first-order analysis presented here. Note that this phenomenological representation allows the treatment of different classes of LPP photoresponse (e.g., trans-cis or photocrosslinkable) with the same framework. The function \( T \) mainly represents the LC alignment behavior, in response to an LPP layer exposed with a given fluence, DOLP, and orientation angle. Well-known metrics, such as the LC elastic constants and the Frank free energy density, are accounted for in this function, although in a phenomenological way. Depending on the complexity and sophistication of the functions \( H \) and \( T \), a wide range of physical phenomena occurring in LPP and LC may be accounted for with this model.
Fig. 5. a) Polarizing optical microscope setup used to capture the images in this paper. The Full-Wave Plate (FWP) is used to distinguish between 0° and 90° polarized light. b) The full-color colormap for differently aligned liquid crystals, acquired when using the FWP. c) The grayscale colormap acquired without the FWP (0° and 90° are indistinguishable). Orientation angle profiles of films were measured by matching the film’s color with b) using the least squares algorithm.

4. System description validation

We have done a series of experiments to test the accuracy of our system description in predicting the LC response. The experiments validate the primary assumptions of \( H \) (non-causality, alignment to average exposure angle, and depolarization by orthogonal polarizations) and the basic form of \( T \) (a threshold fluence function multiplied by a threshold DOLP function). Where applicable, all experiments were simulated using a numerical implementation of our system description with a discrete spatial grid and discrete time steps.

The films were fabricated by spin coating LPP (Rolic ROP-108) at 1500 rpm for 30 s, followed by heating at 115°C for 1 min. This LPP layer was then exposed using the scanning system as previously described. After this, two layers of an LC mixture (3:1 ratio of Merck RMS03-001C and PGMEA) were spun at 1500 rpm for 60 s. Each LC layer was then photopolymerized with illumination of 10 mW/cm² for 1 min in a nitrogen environment. The resulting film thickness was about 1.5 µm (200 nm LPP + 1.3 µm LCP). This thickness was chosen for the colors produced when viewed through a polarizing microscope.

All images were captured using the setup shown in Fig. 5. No post-processing was done to the images, and the alignment of the films was determined by least squares error color matching to the colormap in Fig. 5(b). The light source for the microscope was a white-light lamp. Different wavelengths experience different retardation for a given thickness and birefringence of an LC film. Because of this, the polarization state of the light after any LC film will in general be elliptical and will vary by wavelength. The exception is when the LC optical axis is parallel to or orthogonal to the input polarization, in which case no wavelength experiences any retardation and the polarization output is identical to the input. By inserting a full waveplate oriented at 45° (compared to 25° of the input), we cause the light to experience chromatic retardation even in these cases, making them distinguishable. In combination with an analyzing polarizer that is not parallel to or orthogonal to the input polarizer, the final result is that each possible LC orientation angle (0° to 180° due to rotational symmetry) produces a different transmitting color as shown in Fig. 5(b).

In our tests, neither the power of the beam nor the scanning speed affected the LC response (as reflected by their absence in our model). However, some practical details about these pa-
Fig. 6. The first validation experiment, designed to test the effect of fluence on LC alignment. Pattern a) was created by scanning adjacent, uniform vertical lines and varying the fluence in the horizontal dimension. Pattern c) is the reverse of pattern a), and pattern b) is the superposition of a) and c). From this experiment we determined a fluence threshold to be used in subsequent simulations.

Parameters are provided for the benefit of the reader. In our scanning system, the ideal maximum single-direction scanning speed is 200 mm/s, the maximum beam power is 17 mW, and the minimum beam power is in the nW range. For a given beam width, there are many combinations of beam power and scanning speed that result in the same delivered fluence and identical LC response. For a given pattern, the speed is usually chosen first (the fastest speed possible), and then the needed beam power is calculated. In practice, the scanning speed is limited by the desired fidelity of the scan pattern and digital control limitations. For simple patterns such as the PG, the scan speed may be as high as 150 mm/s; for complex patterns such as the $q$-plate, the scan speed may be as low as 100 nm/s near the center of the pattern.

4.1. Fluence threshold

We wish to know if the use of the fluence threshold function $T_F$ in Eq. (8) accurately reflects the LC response when the LPP is exposed to various fluence levels; that is to say, if the LC aligns poorly below and well above some fluence value. To test this, we scanned three large areas using many adjacent stripes with a beam of width $\sim 12 \mu m$. An image of the LCP response and corresponding exposure fluence is shown in Fig. 6. The scan patterns were uniform in the vertical dimension, and varied slowly in the horizontal dimension. In Fig. 6(a) and 6(c), the fluence varied from low (50 nJ/cm$^2$) to high (50 mJ/cm$^2$), and high to low respectively. Figure 6(b) is a combination of Fig. 6(a) and 6(c) written on top of each other in the same location.

From Fig. 6(a) and Fig. 6(c) we see that poor alignment occurs at lower fluence and good alignment occurs at higher fluence. Figure 6(b) has higher fluence everywhere and the alignment is also good everywhere. In addition, since $\psi_{\text{avg}} = 0^\circ$ everywhere, $D_{\text{avg}} = 1$ everywhere according to Eq. (6). Therefore, we can confidently attribute the decrease in alignment quality to a decrease in fluence, and conclude that the use of $T_F$ in Eq. (8) is reasonable. Based on these results, we identify the threshold fluence as 19 mJ/cm$^2$.

We note that the LCP aligns poorly for very low fluence (<1 mJ/cm$^2$), then aligns somewhat well with low fluence(1-10 mJ/cm$^2$), then aligns poorly again with higher fluence (10-19 mJ/cm$^2$), then finally aligns well for even higher fluence (>19 mJ/cm$^2$). These effects can be seen in Fig. 6 and are highly consistent and reproducible but lie outside the scope of this paper. The fluence threshold function we use treats all fluence values less than the threshold as
resulting in poor alignment, which is an adequate approximation for our purposes.

4.2. DOLP threshold

Next, we address whether or not the DOLP threshold function \( T_D \) in Eq. (8) accurately reflects the LC response when the LPP is exposed to various DOLP levels. If our assumptions about \( H \) and \( T \) are correct, then a low DOLP should result in poor alignment and a high DOLP should result in good alignment. To test this, we scanned three large areas again using many adjacent stripes with a beam of width \( \sim 12 \mu m \). An image of the LCP response and corresponding DOLP and average polarization is shown in Fig. 7. The scan patterns were uniform in the vertical dimension, and varied slowly in the horizontal dimension. In Fig. 7(a) and 7(c), the fluence is constant but \( \psi \) varied linearly from 0° to 90°. However, importantly, the sign of the rotation of \( \psi \) is opposite for Fig. 7(a) and 7(c). Figure 7(b) is a combination of Fig. 7(a) and 7(c) written on top of each other in the same location, except they were each half fluence so that the total fluence would be the same as the other patterns.

For each pattern, the beam power, scan speed, and spacing between adjacent scan lines were chosen such that the average fluence delivered was 50 mJ/cm\(^2\). For example, for patterns 7(a) and 7(c), the power of the Gaussian beam was set to 25 \( \mu W \), the beam was scanned at 10 mm/s, and the lines were spaced 5 \( \mu m \) apart.

In Fig. 7(b), \( D_{avg} \) varies from 1 at the far left (both patterns have \( \psi = 0^\circ \)) to 0 at the middle (the difference between \( \psi \) of the two patterns is 90°) and back to 1 at the far right (both patterns have \( \psi = 90^\circ \)). Of the three patterns in Fig. 7, poor alignment only occurs near the center of Fig. 7(b) where the DOLP is lowest. Since the fluence is constant, we can attribute the decrease in alignment quality to a decrease in DOLP, and conclude that the use of \( T_D \) in Eq. (8) is reasonable. Based on these results, we identify the DOLP threshold as 32.1%.

4.3. Alignment to average polarization angle

Lastly, we wish to confirm that the alignment angle of the LC is equal to the average exposure angle of the LPP, as assumed in Eq. (8). The experiment just discussed for testing \( T_D \) serves this purpose as well. In Fig. 7(a) and Fig 7(c) we see that the alignment angle varies linearly along with the average polarization angle. In Fig 7(b), \( \psi_{avg} \) forms two discrete domains, being \( \psi_{avg} = 0^\circ \) to the left of the middle and \( \psi_{avg} = 90^\circ \) to the right of the middle. The alignment angle of the LCP in the fabricated film is remarkably close to \( \psi_{avg} \). We suspect that differences in the results can be attributed to very small errors in the polarization purity and accuracy of the scanning system, which prevent two discrete domains from being formed and instead induce an averaging from one angle to the other near the center.

5. Analysis of selected scan patterns

We will now analyze a few interesting scan patterns. For the remainder of this paper, we will use \( F_{TH} = 19 \text{ mJ/cm}^2, D_{TH} = 32.1\%, w^{TH}_F = F_{TH}/4, \) and \( w^{TH}_D = D_{TH}/4 \). The threshold functions with these values are shown in Fig. 4.

5.1. Adjacent scan lines

We consider a pattern that comprises three vertical lines spaced a distance \( d \) apart, where the lines have \( \psi_{b1} = \psi_0 - 50^\circ \), \( \psi_{b2} = \psi_0 \), and \( \psi_{b3} = \psi_0 + 50^\circ \) respectively. Since our system is assumed to be non-causal, we can describe the pattern as three lines being scanned simultaneously as follows:
Fig. 7. The second validation experiment, designed to test 1) the effect of DOLP on LC alignment quality, and 2) if the LC aligns to the average exposure angle of the LPP. Pattern a) was created by scanning adjacent, uniform vertical lines and varying the polarization angle in the horizontal dimension from 0° to 90°. Pattern c) is the same as a), except the polarization varies from 0° to -90°. Pattern b) is the superposition of a) and c) with half fluence each (equivalent total fluence). From this experiment we determine a DOLP threshold to be used in subsequent simulations and confirm the aligning properties of the LC.

\[ I_b(r) = I_0 \exp \left( -2 \frac{x^2 + y^2}{w_0^2} \right) \]  
\[ r_{b1}(t) = \tilde{x} d + \tilde{y} t \]  
\[ r_{b2}(t) = t \tilde{y} \]  
\[ r_{b3}(t) = - \tilde{x} d + \tilde{y} t \]

where \( w_0 = 23 \, \mu m \), \( \psi_0 = -20° \), and \( d = 18 \, \mu m \), 36 \( \mu m \), and 72 \( \mu m \). Using these input parameters, we numerically solve for the total fluence and expected LC orientation angle in the x dimension. Shown in Fig. 8 are images of films fabricated using these parameters, the fluence of each scan line, the resulting total fluence, and the predicted and measured LCP orientation angles. There is a marginal degree of error in the orientation angle measurement (e.g., at position 0 in Fig. 8(a)) for two primary reasons. First, the used measurement method can only measure angles in coarse 10° steps. Second, the LC film in this region had slightly different characteristics (e.g., thickness) than the rest of the film, resulting in slightly different transmitting colors.

We make three important observations. First, when the scans are farther apart (Fig. 8(a) and 8(b)), the orientation angle plateaus at \( \psi_0 \) near the second scan line (\( x = 0 \)). Second, when the lines are very close (Fig. 8(c)), the presence of the second scan line cannot be detected, as the orientation angle varies smoothly from from \( \psi_0 - 50° \) to \( \psi_0 + 50° \). Third, when the scans are overlapping to any degree (Fig. 8(b) and 8(c)) the orientation angle does not have discrete boundaries, but rather continuously varies from the orientation of one scan line to the next.
Fig. 8. Adjacent, uniform scan lines with polarization orientations of $\psi_0 - 50^\circ$, $\psi_0$, and $\psi_0 + 50^\circ$. The distance between the scan lines decreases from a) to c), and we observe an averaging of the LC orientation angle. By sufficiently overlapping neighboring scan lines, we can create continuously varying orientation profiles. The LC orientation angles are only plotted where the fluence is $\geq 0$; in the other regions, the alignment confidence is very low, the LC becomes randomly aligned, and $\psi_{\text{avg}}$ becomes undefined.

is quite amazing, though not altogether surprising.

Therefore, by controlling the spacing and polarization of adjacent scan paths, we can for the first time create arbitrary, continuously varying orientation profiles, subject to some limitations by the size of the recording beam and material response parameters. This feature is incredibly useful, allowing a discrete scan to create a continuous profile, and is a large incentive to use a direct-write system to pattern LC elements. We also point out that the orientation plateau in Fig. 8(b) is smaller than the beam width; hence creation of features smaller than the beam is also possible with this system.

5.2. *Intersecting scan lines*

Next we examine a simple pattern that produces some interesting results. The pattern is two intersecting scan lines with polarizations orientated $90^\circ$ apart. We might qualitatively expect that where the lines meet there will not be good alignment, as the LC will have no preferred direction. However, without a quantitative description of the system such as the one we have developed, it is hard to ascertain where exactly well and poor alignment will occur. We can express the scan as follows:
Fig. 9. Two intersecting scan lines with polarization orientations $0^\circ$ and $90^\circ$. The average polarization angle at every location is either $0^\circ$, $90^\circ$, or undefined where the fluence delivered by each line is equal. As a result, discrete domains are produced. a) Simulated $F$, b) $D_{avg}$, c) $|A|$, d) $\psi_{avg}$ using the color bar in Fig. 5(b), e) $A$, created by overlaying c) and d). f) Actual image of fabricated film, color bar in Fig. 5(b).

\begin{align*}
I_0(r) &= I_0 \exp \left(-\frac{2x^2+y^2}{w_0^2}\right) \\
r_{b1}(t) &= \hat{x}t \\
r_{b2}(t) &= \hat{y}t \\
\psi_{b1}(t) &= 0 \\
\psi_{b2}(t) &= \frac{\pi}{2}
\end{align*}

where $w_0 = 23 \, \mu m$. The power was set such that the fluence at the center of each line was 50 mJ/cm$^2$. Using these input parameters, we numerically solved for the key parameters and the final LC response.

All of the relevant parameters are shown in Fig. 9, as well as the actual result of experimentally scanning the pattern. The plot of $F$ (Fig. 9(a)) needs no explanation. The average DOLP $D_{avg}$ (Fig. 9(b)), is very high near the center of each scan line and drops to 0 at points which are equidistant from each scan line. The alignment prediction confidence $|A|$ (Fig. 9(c)), combines $F$ and $D_{avg}$ and the threshold functions. The average polarization angle $\psi_{avg}$ (Fig. 9(d)), is not intuitive at first glance, but the concept here is similar to Fig. 7(b). Since the scan lines have orthogonal polarizations, they cannot be averaged together. Therefore, at any given point one of the scan lines will dominate and solely determine $\psi_{avg}$.

We have combined Fig. 9(c) and Fig. 9(d) giving a complete picture of the predicted result.
Fig. 10. A simulated and fabricated q-plate, which is a kind of azimuthal waveplate. a) Simulated \( F \), b) \( D_{\text{avg}} \), c) \( |A| \), d) \( \psi_{\text{avg}} \) using the color bar in Fig. 5(b), e) \( A \), created by overlaying c) and d), f) Actual image of fabricated film, color bar in Fig. 5(b).

In Fig. 9(e), upon comparing this to the actual result shown in Fig. 9(f), we see that the results match very well, including the X shape in the center. Last, we note that the random line defects that appeared in this pattern are unusual; they did not appear in any of the other patterns fabricated, even though all patterns were created on a single substrate with the same LPP and LCP fabrication steps.

### 5.3. Q-plate

A q-plate is a kind of azimuthal waveplate used to affect the orbital angular momentum of light. High quality q-plates have been fabricated with a variety of material sets including LPP/LC [10, 11, 20]. Of particular interest in q-plate fabrication is the minimum possible size of the defect at the center of the element, also called the singularity defect. We can fabricate q-plates by scanning the beam in a spiral pattern while continually changing \( \psi \). This pattern can be expressed as follows:

\[
I_b(r) = I_0 \exp \left( -\frac{2r^2 + y^2}{w_0^2} \right) \\
r_b(t) = \left( m + \frac{rt}{2\pi} \right) (\hat{x}\cos(2\pi t) + \hat{y}\sin(2\pi t)) \\
\psi_b(t) = qt
\]

where \( w_0 = 1 \ \mu m \), \( q \) is the charge of the q-plate, \( m \) is the minimum radius of the spiral, and \( r \) is the spacing between the rings of the spiral. The power was set such that \( F = 200 \text{ mJ/cm}^2 \).
In practice, the scan pattern will be discretized by some time step as required by the scanning hardware. Here we assume the time discretization of the pattern is sufficiently small, so have neglected it in the above description.

In order to create the smallest possible singularity defect for a given beam width and $q$-plate charge, we used a numerical implementation of our model to simulate and optimize $m$ and $r$ for a charge 4 $q$-plate. The key parameters are shown in Fig. 10 for one of our best designs ($m = 2 \, \mu m$ and $r = 0.2 \, \mu m$), along with an image of the predicted A and the actual result of experimentally scanning the pattern. (We fabricated a small size $q$-plate because the central region is the only particular region of interest and it was easier to simulate; larger sized $q$-plates can easily be fabricated in the same way.)

The singularity defect predicted is about 5 $\mu m$ in diameter. However, the actual film produced a kind of rectangular defect about 2 by 10 $\mu m$ large. This demonstrates a case where our system description is not entirely accurate and more sophisticated $H$ and $T$ functions would be beneficial. Nevertheless, this $q$-plate has a singularity defect comparable to the smallest reported to date [20].

5.4. Polarization grating

The polarization grating (PG) is one of the most useful liquid crystal elements, being used in imaging, beam-steering [1], spectroscopy [21], and many other fields. When light hits an ideal PG, the incident light is diffracted into a single order. The ideal PG has a linear phase profile which corresponds to a linearly varying optical axis. If the optical axis does not vary linearly, then other diffraction orders are produced decreasing the efficiency of the element. Thus, we are primarily interested in producing a perfectly linearly varying optical axis profile. The most
convenient way to do this is to scan lines of constant polarization with some spacing between the lines, just like in Section 5.1 but with many more lines.

Our target PG has a pitch of 50 µm, meaning the optical axis rotates 180° in 50 µm. We must choose the size of the beam to scan and the spacing between the lines. If the beam size is very large, e.g. 90 µm, then every point will be exposed by many polarizations and we expect that the DOLP will be very low, causing poor alignment. Therefore, we expect a smaller beam size will be better and we chose a beam with a width of 15 µm. As for the line spacing, if the spacing is too far apart then space between the lines will not receive enough fluence. Thus, adjacent beams should be overlapping somewhat and we chose a line spacing of 7 µm. We chose a beam power and scan speed such that the average fluence delivered was 200 mJ/cm², well-above the required threshold.

The fabricated film using these parameters is shown in Fig. 11. Below the image of the film we show the ideal and measured optical axis profile, and below that the unwrapped phase profile. The phase of the PG is equal to twice the optical axis orientation [3]. The measured profile looks like a staircase because our colormap only contains 18 samples, causing the measured angle to jump in steps of 10°. Taking this into account, we see that the fabricated film is extremely close to the ideal PG profile. Using a laser, we measured >99.5% of the power going into the +1 and 0 orders. If the retardation was half-wave (which can be achieved relatively easily by optimizing the fabrication), we would expect the +1 order diffraction efficiency to be >99.5% [1, 22, 23]. Using this approach, we can very easily set the pitch of the PG and create large-area PGs that have the exact same pitch over the entire area, a major challenge for traditional PG fabrication methods [24].

6. Conclusion

We have presented a direct-write system for fabrication of complex LC patterns. We developed a complete system description which appears to be very accurate in predicting the alignment angle and alignment quality of LC given a set of input scan parameters. We studied some patterns which highlighted the advantages of this system, including the continuous nature of the LC alignment and the creation of features smaller than the beam size. Using the system, we were able to fabricate a high-quality q-plate with minimal effort. Finally, we created a PG with a near-perfect linear optical axis profile which would result in a diffraction efficiency equivalent to prior best results if the retardation were half-wave.

We note that the system description, as we have presented it, is limited in a number of ways. For example, it does not account for causal effects in the photoalignment material (e.g., saturation), the effect of neighboring LC regions on each other, or the thickness of the LC film. It also does not predict anything regarding the out-of-plane LC orientation angle (i.e., pre-tilt). However, we suspect that by choosing different functions for $H$ and $T$, it is possible to account for more complex alignment responses in the LC such as those just mentioned. The presented formalism is also flexible enough that it can be used with many different photoalignment and liquid crystal materials. In addition, we also envision that the adjusted electromagnetic response of this system may be used as an input to an elastic continuum model to provide an even more accurate prediction of the LC response in very complex situations. To conclude, this work enables for the first time a simple means of fabricating sophisticated LC elements, beneficial to a wide set of applications including displays, holography, imaging, spectroscopy, sensing, beam-steering, beam-shaping, and astronomy.

Acknowledgment

The authors gratefully acknowledge the support of the National Science Foundation (NSF grant ECCS-0955127).