Spin-orbit excitations of quantum wells

A. Ambrosetti¹, J. M. Escartín², E. Lipparini³,⁴ and F. Pederiva³,⁴

¹ Dipartimento di Fisica “G. Galilei”, Università di Padova - Via Marzolo 8, Padova, Italy, EU
² Dept. ECM, Facultat de Física, and IN² UB - Universitat de Barcelona - Diagonal 647, 08028 Barcelona, Spain, EU
³ Dipartimento di Fisica, University of Trento - via Sommarive 14, I-38050, Povo, Trento, Italy, EU
⁴ INFN, Gruppo Collegato di Trento - Trento, Italy, EU

received 19 October 2010; accepted in final form 18 March 2011
published online 18 April 2011

PACS 73.21.Fg – Quantum wells
PACS 71.10.Ca – Electron gas, Fermi gas

Abstract – Spin-orbit effects on the photoabsorption of a quantum well are discussed by means of a sum rules approach. We show that while the strength of the excitation is zero when the spin-orbit coupling is neglected, the inclusion of the spin-orbit interaction gives rise to a nonzero strength and mean excitation energy in the far-infrared region. A simple expression for these quantities up to the second order in the Rashba interaction is derived. Modifications of the results due to the Dresselhaus spin-orbit term are discussed. The effect of two-body Coulomb interaction is then studied by means of a Quantum Monte Carlo calculation, showing that electron-electron correlations induce only a small deviation from the independent particle model result.

Introduction. – Quantum wells, i.e. strongly confined electrons at a semiconducting heterostructure, are intrinsically very complex systems, in which the local arrangement of the electronic structure can in principle give rise to a large variety of interesting subtle phenomena. However, the very strong confinement and the weak potential felt by the electrons allowed for successfully using simplified Hamiltonians in describing the phenomenology of these systems. The most celebrated one is the two-dimensional electron gas (2DEG) Hamiltonian in the effective mass and dielectric constant approximation, and its numerous variants [1]. At the semiconductor junction, the effective potential is never symmetric with respect to the junction plane. This implies that electrons feel the effect of an electric field transverse to the plane. In the frame of effective Hamiltonians, this fact is accounted for by the inclusion of the so-called Rashba term [2], which simply describes the interaction between the spin of an electron moving in the plane, and the self-induced magnetic field. For \( N \) electrons the 2DEG Hamiltonian reads

\[
H_{2\text{DEG}} = \sum_{i=1}^{N} \left( \frac{p_i^2}{2m^*} \right) + \sum_{i<j}^{N} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|},
\]

where \( \epsilon \) and \( m^* \) are the dielectric constant and effective electron mass in the semiconductor, respectively. The Rashba interaction [2],

\[
V_{\text{Rashba}} = \frac{\lambda}{\hbar} \sum_{i=1}^{N} [p_i^y \sigma_i^x - p_i^x \sigma_i^y],
\]

where \( p_i \) is the momentum of the \( i \)-th electron and \( \sigma_i^x \) and \( \sigma_i^y \) are the Pauli matrices acting over the spin of particle \( i \), has a strength \( \lambda \) proportional to the modulus of the transverse electric field. Given the formal analogy, the Rashba term is usually reported as a spin-orbit (SO) interaction.

The most interesting fact about SO interaction is that it links spin and charge dynamics, hence opening the possibility of spin control by means of electrical fields. It affects the charge transport [3–9] and the far-infrared absorption in heterostructures [10–14] and can lead to spin inversion [15–19] and variations of the electric spin precession in a magnetic field [20–23]. It also induces linear magneto-optical Kerr or Faraday effects then allowing the measurement of the magnetization of materials by means of laser pulse excitation [24]. In this sense, the Rashba SO contribution has received special attention since it was shown that its intensity can be externally tuned via the application of a gate voltage [25,26]. In view of technical applications, it is very important to have a clean way to measure the initial strength of the Rashba SO interaction in the sample, in order to quantitatively
control the induced effects in the physical processes of interest.

In this work we show that this measurement can be achieved by far-infrared spectroscopy in the absence of external magnetic fields. If the SO term is absent, then the light absorption is zero. On the contrary, when the SO interaction is present, photoabsorption occurs with a strength and at an energy in the far-infrared domain which are proportional to the SO strength. The actual experiment would provide, in addition, a very stringent test on the applicability of the effective model Hamiltonian to real systems.

**Method.** – In the following we will consider a quantum well modeled by N conduction electrons laterally confined in the $xy$ plane, at surface density $\rho$, and interacting via the Hamiltonian $H = H_{2DEG} + V_{\text{Rashba}}$. To simplify the expressions we prefer to take into account additional Dresselhaus SO contributions at the end of the paper. We make use of effective atomic units in order to simplify the notation, defining $\hbar = e^2/\epsilon = m^* = 1$. The length unit therefore is the effective Bohr radius $a_0^* = a_0 e m_e/\hbar^2$, and the energy unit the effective Hartree $E_\text{h}^* = E_\text{h} m^*/(m_e e^2)$ ($E_\text{h} = e^4 m_e/((4\pi\epsilon_0\hbar)^2) \approx 27.211$ eV).

The single-particle part of Hamiltonian (1) and (2) can be solved analytically [2] to give the following solutions for single-particle energies and wave functions:

$$
e_k = \frac{k^2}{2m} + \lambda|k|,
\varphi = \frac{1}{\sqrt{2S}} e^{ikr} \left( \begin{array}{c} \cos \psi \\ \sin \psi \end{array} \right),$$

(3)

where $\psi = \arctan \frac{k}{\xi}$ and $S$ is the surface area. Note that the wave function (3) is an eigenstate of a component of the spin which is perpendicular to the direction of $k$. In the following we will address these states as +chirality and -chirality single-particle states, respectively. For a given chemical potential $\mu > 0$, the two branches of the excitation energy $e_{k+}$ are filled up to the Fermi momenta $k_{F+}^\sigma$ and $k_{F-}^\sigma$, where

$$k_{F}^\pm = \sqrt{2\pi \rho (1 \mp \xi)}, \quad \xi = \sqrt{\frac{\beta}{\pi \rho}} \left( 1 - \frac{\lambda^2}{4 \pi \rho} \right),$$

(4)

and the expression for $\xi$ is exact at the order $\lambda^3$ (see, for example, ref. [1]).

**Interaction Hamiltonian and excitation operators.** We now turn to analyze the transitions induced in the quantum well by the interaction with a linearly polarized (along $y$) electromagnetic wave propagating along the $x$-direction, i.e., in the plane of motion of the electrons. The corresponding vector potential is $A(t) = A \sin \omega t$, with $\omega = \omega_t - qx$. The interaction Hamiltonian $\mathbf{J} \cdot \mathbf{A}/c + g^* \mu_B \mathbf{S} \cdot (\nabla \times \mathbf{A})$, where $\mathbf{J} = e\mathbf{v}$ and $g^*$ is the effective gyro-magnetic factor, reads

$$H_{\text{int}} = \sum_i \left( \frac{A}{c} v_y^i \sin \theta - \frac{1}{2} g^* \mu_B q A \sigma^i \cos \theta \right),$$

(5)

and the velocity operator $v^y$ is defined as $v^y \equiv -i[y, H] = p^y + i \lambda \sigma^y$.

This interaction Hamiltonian yields three excitation operators: $\sum \rho_n^p$, $\lambda \sum \sigma_i^z$ and $\sum \sigma_i^x$ to be used for calculating the photoabsorption cross-section in perturbation theory. The first one does not excite the electrons of the system since it commutes with the quantum well Hamiltonian. We are then left with the excitations induced in the target by the operators

$$S^z = \frac{1}{2} \sum_i \sigma_i^z, \quad S^x = \frac{1}{2} \sum_i \sigma_i^x.$$  

(6)

For time-reversal–invariant systems, like the ones considered here for which the expectation value of $S_z$ on the ground state $|0\rangle$: $\langle 0|S^z|0\rangle$ is zero, it is possible to show [27] that the strengths and mean excitations energies corresponding to the operators (6) are the same. Hence in the following we will study the excitations induced in the system by only one of the two operators, e.g., by $S_z$.

Note also that for a different geometry, e.g., the one corresponding to transitions induced in the system by the interaction with a left-circular polarized electromagnetic wave propagating along the $z$-direction perpendicular to the plane of motion of the electrons, the relevant excitation operators are

$$S^\pm = \frac{1}{\sqrt{2}} \sum_i \sigma_i^\pm,$$  

(7)

where $\sigma^\pm = \sigma_x \pm i \sigma_y$. Once again, for time-reversal–invariant systems, the strengths and mean excitations energies corresponding to the operators (7) coincide with the ones induced by the operators (6). Photoabsorption experiments attempting to measure the strength $\lambda$ of the SO interaction could then be performed with electromagnetic waves propagating in a direction either parallel or perpendicular to the quantum well.

**Sum rules.** The strength and mean excitation energy of the transitions induced in the quantum well by the operator $S_z$ can be studied in a convenient way by means of the moments $m_p$ of the dynamic form factor $S(S^2, \omega)$ (which is defined as $\sum_{n\neq 0} |\langle n|S_z|0\rangle|^2 \delta(\omega - \omega_{n0})$):

$$m_p = \int_0^\infty d\omega \omega^p S(S^2, \omega) = \sum_{n\neq 0} \omega_{n0}^p |\langle 0|S^2|n\rangle|^2,$$  

(8)

where $\omega_{n0} = E_n - E_0$ is the excitation energy, $p$ is an integer and $|0\rangle$ and $|n\rangle$ are the ground and excited states of $H = H_{2DEG} + V_{\text{Rashba}}$, respectively.

By using the completeness relation $\sum_{n} |n\rangle\langle n| = 1$ and the equation $H|n\rangle = E_n|n\rangle$, it is possible to write the moments (8) as the mean values on the ground state of commutators $[\cdot]$ and anticommutators $\{\cdot\}$ of the excitation operator $S^2$ and of the Hamiltonian $H$.

For time-reversal–invariant systems, using the property
\( \langle S^z \rangle = 0 \), the following sum rules are derived for the operator \( S^z \) [1]:

\[
m_1 = \frac{1}{2} \langle 0 | [S^z, [H, S^z]] | 0 \rangle ,
\]

\[
m_0 = \frac{1}{2} \langle 0 | [S^z, S^z] | 0 \rangle ,
\]

\[
m_{-1} = \frac{1}{2} \langle 0 | [H, X], X] | 0 \rangle .
\]

In deriving the last sum rule we used the operator \( X \), which is solution of the equation

\[
[H, X] = S^z .
\]

Making use of the definition (8) and employing the newly defined operator \( X \), one has

\[
m_{-1} = \sum_{n \neq 0} \frac{|\langle 0 | S^z | n \rangle|^2}{E_n - E_0} = \langle 0 | [H, X], X | 0 \rangle
\]

from which the third of (9) is easily derived. The moments \( m_p \) can be used to define different mean excitation energies, namely

\[
E_{1,-1} = \sqrt{m_1/m_{-1}}
\]

and

\[
E_{1,0} = m_1/m_0,
\]

which satisfy the relations

\[
E_{1,0} \geq E_{1,-1} .
\]

These energies strictly coincide only when the whole strength is exhausted by a single state. However, in the presence of collective states, they can be used to study the spreading of the excitation strength [27] and to bind the energy of the collective modes.

**Results.** – Hereafter we report the analytic results obtained for the independent particle model, and the numerical results for the interacting system computed by means of the Quantum Monte Carlo method.

**Independent particle model.** – In the independent particle picture and to the lower order in \( \lambda \), the moments \( m_1, m_0, \) and \( m_{-1} \) can be computed analytically. The expressions are

\[
m_1 = - \frac{1}{2} \langle 0 | V_{Rashba} | 0 \rangle = N \lambda \left( 1 - \frac{1}{3} \frac{\lambda^2}{\pi \rho} \right) ,
\]

\[
m_0 = N \lambda \left( 1 - \frac{1}{4} \frac{\lambda^2}{\pi \rho} \right) ,
\]

\[
m_{-1} = \frac{S}{8 \pi} .
\]

The corresponding mean excitation energies (13) and (12) become

\[
E_{1,-1} = \sqrt{\frac{8 \pi \rho \lambda}{\lambda^2} \left( 1 - \frac{1}{6} \frac{\lambda^4}{\pi \rho} \right)} ,
\]

\[
E_{1,0} = \sqrt{\frac{8 \pi \rho \lambda}{\lambda^2} \left( 1 - \frac{1}{12} \frac{\lambda^4}{\pi \rho} \right)} .
\]

Some comments are in order here. The total strength \( m_0 \) and the energies (16) vanish for \( \lambda \rightarrow 0 \). As anticipated in the introduction, light absorption in the system only occurs in the presence of SO interaction and the excitation strength and energy are linear in \( \lambda \) at the main order. This fact characterizes these excitations as a genuine effect of the well asymmetry, or of the presence of a SO interaction in the system.

At the lowest order in \( \lambda \) the two energies (16) coincide, and the higher-order corrections are very small (of order \( \lambda^3 \)). As a consequence the excitation strength is practically concentrated in a single peak. This peak is measurable since it lies in the far infrared region. In fact taking, as an example, a GaAs quantum well for which \( \epsilon = 12.4 \) and \( m^* = 0.067m_e \), yielding \( E_h^* = 11.86 \text{ meV} \) and \( a_0^* = 97.93 \text{ Å} \), one gets for \( E_{1,0} \) a value ranging from 2 to 6 meV, for densities in the range (2–13) \( \times 10^9 \text{ cm}^{-2} \) and \( \lambda \simeq 10^{-9} \text{ eV cm} \simeq 0.1 \text{ a.u.} \) [20, 25, 28, 29].

In order to study the modifications of the above results due to the presence of the Dresselhaus SO interaction [30]

\[
V_{Dres} = \frac{\gamma}{\hbar} \sum_{i=1}^{N} [p_i^x \sigma_i^x - p_i^y \sigma_i^y] ,
\]

we computed the mean excitation energy \( E_{1,0} \) including both Rashba and Dresselhaus SO interactions. An explicit evaluation of the moments (9) would require an integration over all states contained within the Fermi surface. Unfortunately, the inclusion of both SO couplings causes a strong distortion of the Fermi surface making a derivation of analytical formulas for \( E_{1,0} \) complicated. A numerical integration could however be performed, leading to the results shown in fig. 2. These will be commented in the final part of the manuscript. We want to stress that the photoabsorption strength is exhausted by the SO mode only in the absence of transverse external magnetic fields. Otherwise, the strength is almost completely absorbed by the cyclotronic mode which is affected by the SO interaction at order \( \lambda^2 \). We also stress that, since the excitation operator \( S_z \) commutes with the Coulomb interaction, the result \( m_1 = - \frac{1}{2} \langle 0 | V_{Rashba} | 0 \rangle \) for the energy-weighted sum rule is exact for the total Hamiltonian (1) and (2). Deviations from the result (15) for \( m_1 \) are due to two-body correlations induced in the ground state by the Coulomb interaction. Finally we note that result (15) for \( m_{-1} \), valid at all orders in \( \lambda \), is exact for the independent particle model where the Coulomb interaction is neglected.

**Monte Carlo calculations.** – In order to evaluate the effects of the two-body Coulomb interaction on the strength \( m_0 \) and the energy \( E_{1,0} \) we have performed a quantum Monte Carlo (QMC) calculation. Our recent extension of the Diffusion Monte Carlo (DMC) [31], in fact, allows for the projection over the ground state of the system corresponding to the full Hamiltonian (1) and (2). DMC is a very accurate method to solve the Schrödinger equation (for details see ref. [32]), based on imaginary time projection. Given an initial wave function

\[
27004-p3
\]
a function of the SO strength $\lambda$, the number of + chirality ($N_+$) and − chirality ($N_-$) states used in the trial wave function are reported.

| $\lambda$ | $N_-$ | $N_+$ | $m_0$       | $m_1$       | $E_{av}$  |
|---------|------|------|-------------|-------------|-----------|
| 0.1     | 33   | 25   | 2.05(3)     | 0.561(3)    | 0.274(4)  |
| 0.2     | 37   | 21   | 4.20(7)     | 2.250(6)    | 0.54(1)   |
| 0.5     | 48   | 10   | 11.1(5)     | 13.19(1)    | 1.19(5)   |

where $V_{\text{Coul}}$ is the Coulomb potential. The factor including $V_{\text{Rashba}}$ contains an explicit dependence on the spin coordinates of the electrons. This fact demands a substantial modification of the standard algorithm, which must now take care of the propagation of the spin degrees of freedom together with the space coordinates. We should mention that, as in all applications of standard DMC to many Fermion systems, one must introduce an artificial constraint (in this case a fixed-phase constraint) in order to avoid the notorious sign problem. The details of the algorithm and further discussions can be found in ref. [31].

We have evaluated the moments $m_1$ and $m_0$ by explicitly calculating the expectation values of eqs. (9) on the Monte Carlo ground state at $r_s = 1$, corresponding to a density $\rho \simeq 3 \cdot 10^{11} \text{cm}^{-2}$. Calculations were performed using the trial wave function described in [31], using the combination of + chirality and − chirality single-particle states giving the lowest ground-state energy. The obtained results are reported in table 1 and fig. 1 for three values of $\lambda$ expressed in effective atomic units.

In fig. 1 we compare the $\lambda$ dependence of the excitation energy predicted by the independent particle model (with and without corrections at order $\lambda^2$), and by the DMC calculations. As can be noticed, the deviations due to the presence of the electron-electron correlations are rather small. At values of $\lambda$ of order of the estimates for typical quantum wells they are almost negligible, and increase with the strength of the transverse electric field. This implies that the analytic results (15) and (16) can be considered very good estimates for the strength and excitation energies of far infrared absorption of polarized light in the quantum well.

As already mentioned, in fig. 2 we report the average excitation energy $E_{1,0}$ of the independent particle model.

![Fig. 1](Colour on-line) Average excitation energy estimates in $H^*$. The green line and the black dots correspond, respectively, to the independent particle model estimates at order $\lambda$ and $\lambda^3$, respectively. The red squares correspond to the QMC estimates in the presence of Coulomb interaction.

![Fig. 2](Colour on-line) Average excitation energy (in $H^*$) as a function of the strength $\lambda$ of the Rashba term, including the Dresselhaus contribution at different strengths $\gamma$. The results have been obtained within the independent particle model.
including both the Rashba and Dresselhaus SO interactions. From the figure one sees that inclusion of the Dresselhaus term always increases the value of the mean excitation energy which still remains in the far-infrared domain. One also notices that for zero Rashba interaction, the Dresselhaus contribution to $E_{1,0}$ coincides with the one of fig. 1 in which only the Rashba term is included. As discussed earlier, the mean excitation energy is influenced by the the Fermi surface of the system. It can be easily proved that the two SO couplings taken separately lead to the same kind of Fermi surface, and for this reason, give exactly the same result for $E_{1,0}$. Taken both, their contributions, due to the Fermi surface distortion, lead to an enhancement of the mean excitation energy. Finally we comment about possible impurity and subband effects that have been neglected in the present investigation. In sufficiently high mobility carrier systems like, for example, the ones fabricated in ref. [10] impurities influences on far-infrared absorption are strongly suppressed and can be neglected in the present analysis. As far as subbands effects are concerned, the Hamiltonian (1) and (2) is valid only if there is a single subband. In general, due to the confinement in the $z$-direction there exists more than one subband. However for strong confinement the energy gap among subbands is very high and we have estimated that the effect of higher subbands is not very relevant for infrared transitions. The case of SO interaction in multi-subband quantum wells has been considered in refs. [33]. In fact, from such a two band model one can prove that the corrections to the excitation energy contain at least a factor of order $\eta/\omega_g$, where $\eta$ is the intersubband-induced SO coupling and $\omega_g$ is the energy difference between the two bands.

Conclusions. – In summary, we have calculated strength and mean excitation energy of the transitions induced in a two-dimensional quantum well by a linearly polarized electromagnetic wave propagating in the plane of motion of the electrons by means of sum rules techniques. In the absence of SO couplings there is no absorption of light in the system. When the SO is turned on, the predicted strength and mean excitation energy turn out to be proportional to the SO coupling parameters. Calculations have been performed analytically in the independent particle picture which neglects the Coulomb interaction and takes into account the SO potential. The effects of the Coulomb interaction have been accurately tested by numerical Diffusion Monte Carlo simulations including the recently developed extension to the treatment of spin-orbit interactions.

***

We acknowledge useful discussions with S. Gandolfi and L. Mitas. JME acknowledges support from the Spanish FPU program (ME). DMC calculations were performed on the Wigelaf HPC facility of the Physics Department of the University of Trento and on the CINECA HPC under a Computing Project Grant of the University of Trento.

REFERENCES

[1] Lipparini E., Modern Many-Particle Physics: Atomic Gases Quantum Dots and Quantum Liquids, 2nd edition (World Scientific, Singapore) 2008.
[2] Rashba E. I., Physica E, 20 (2004) 189.
[3] Folk J. A., Patel S. R., Birnbaum K. M., Marcus C. M., Duruţţ C. I. and Harris J. S., Phys. Rev. Lett., 86 (2001) 2102.
[4] Halperin B. I., Stern A., Oreg Y., Cremer J. N. H. J., Folk J. A. and Marcus C. M., Phys. Rev. Lett., 86 (2001) 2106.
[5] Aleiner I. L. and Fal’ko V. I, Phys. Rev. Lett., 87 (2001) 256801.
[6] Moroz A. V. and Barnes C. H. W., Phys. Rev. B, 60 (1999) 014272.
[7] Pershin Y. V., Nesteroff J. A. and Privman V., Phys. Rev. B, 69 (2004) 121306(R).
[8] Serra Ll., Sánchez D. and López R., Phys. Rev. B, 72 (2005) 235309.
[9] Lipparini E., Barranco M., Malet F. and Pi M., Phys. Rev. B, 79 (2009) 115310.
[10] Manger M., Batke E., Hey R., Friedland K. J., Köhler K. and Ganser P., Phys. Rev. B, 63 (2001) 121203(R).
[11] Valín-Rodríguez M., Puente A. and Serra Ll., Phys. Rev. B, 66 (2002) 045317.
[12] Chakraborty T. and Pietiläinen P., Phys. Rev. Lett., 95 (2005) 136603.
[13] Tonello P. and Lipparini E., Phys. Rev. B, 70 (2004) 081201(R).
[14] Usaj G. and Balseiro C. A., Phys. Rev. B, 70 (2004) 041301(R).
[15] Valín-Rodríguez M., Puente A. and Pi M., Phys. Rev. B, 66 (2002) 165302.
[16] Emperador A., Lipparini E. and Pederiva F., Phys. Rev. B, 70 (2004) 125302.
[17] Ashoori R. C., Nature, 379 (1996) 413.
[18] Ciorga M., Sachrajda A. S., Hawrylak P., Gould C., Zawadzki P., Jullian S., Feng Y. and Wasilewski Z., Phys. Rev. B, 61 (2000) R13615.
[19] Yoskoboykinov O., Lee C. P. and Tretyak O., Phys. Rev. B, 63 (2001) 165306.
[20] Valín-Rodríguez M., Puente A. and Serra Ll., Phys. Rev. B, 73 (2006) 125302.
[21] Malet F., Lipparini E., Barranco M. and Pi M., Phys. Rev. B, 73 (2006) 085306.
[22] Murakami S., Nagaosa N. and Zhang S. C., Science, 301 (2003) 1348.
[23] Sinova J., et al., Phys. Rev. Lett., 92 (2004) 126603.
[24] Bigot J. Y., Vomir M. and Beaurepaire E., Nat. Phys., 5 (2009) 515.
[25] Nitta J., Akazaki T., Takayanagi H. and Enoki T., Phys. Rev. Lett., 78 (1997) 1335.
[26] Engels G., Lange J., Schäpers T. and Lüth H., Phys. Rev. B, 55 (1997) R1958; Kohda M., Nihei T. and Nitta J., Physica E, 40 (2008) 1194.
[27] Lipparini E. and Stringari S., Phys. Rep., 175 (1989) 103.
[28] Bychkov Yu. A. and Rashba E. I., J. Phys. C, 17 (1984) 6039.
[29] Vagner I. D., Rozhavsky A. S., Wyder P. and Zyuzin Yu. A., Phys. Rev. Lett., 80 (1998) 2417.
[30] Dresselhaus G., Phys. Rev., 100 (1955) 580.
[31] Ambrosetti A., Pederiva F., Lipparini E. and Gandolfi S., Phys. Rev. B, 80 (2009) 125306.
[32] Foulkes W. M. C., Mitas L., Needs R. J. and Rajagopal G., Rev. Mod. Phys., 73 (2001) 33.
[33] Bernardes E., Schliemann J., Lee M., Egues J. C. and Loss D., Phys. Rev. Lett., 99 (2007) 076603; Calsaverini R. S., Bernardes E., Egues J. C. and Loss D., Phys. Rev. B, 78 (2008) 155313.