Study of Fluid Flow Movement by Using Self Potential Data

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Abstract. The Self-Potential (SP) method is passive method in geophysics which works based on the natural presence of an electric field on the surface due to anomalies below the surface. SP value on surface can be generated by fluid flow through rock pores or fractures. We study fluid flow movement in subsurface using the velocity value of fluid flow derived from SP values measured on surface. For that purpose, we carried out mathematical modelling, connecting the Helmholtz-Smoluchovsky’s electrokinetic potential gradient equation with Darcy’s law. The velocity of fluid flow depends on the intrinsic permeability of rocks, electrokinetic potential gradient and electrohydrolic conductivity constant. We tested derived velocity of fluid flow on the SP data from a pilot project test site. Study results show that fluid flow in vertical direction can be identified from SP data at locations where there are significant changes of positive and negative SP values. Fluid flows from a high SP value to a low SP value and this flow is opposite the positive SP gradient. The SP value at study site lie in the range -80mV to -160mV, whereas the value of the water flow velocity lie in the range 0.08 cm/s - 0.21 cm/s.

1. Introduction
There is a lot of environmental pollution in the form of fluids that flows near the ground surface. Such fluid flows cannot be seen directly from surface. The Self-Potential (SP) method is passive method in geophysics which is suitable to be applied in identifying those fluid flows. This technique works based on the natural presence of an electric field on the surface due to anomalies below the surface. SP value on surface can be generated by fluid flows through rock pores or fractures [1]. The SP method is popularly applied by researchers because it is a cheap and efficient method for describing subsurface conditions. This method has many applications, ranging from its use to detect underground caves [1], over hydrological studies [2] to studies in volcanic environments to investigate multiscale fluid flow [3]. Success in our laboratory examination to study fluid flow velocity determination motivated us to apply the results to a pilot project scale. In this study we investigated the movement of lateral fluid flow using the SP values measured at the surface in the study area.
2. Data and methods

2.1. the Helmholtz-Smoluchovsky’s electrokinetic potential gradient equation
Helmholtz developed the theoretical basis of the streaming potential, stating that streaming potentials are present when a conduction current balances the convection current caused by the flow of positive ions as seen in figure 1 [1]. Due to the difference in pressure, fluids will flow in the rock pores and attract positive charges, resulting in a convection current.

![Streaming current in a capillary](image1.png)

**Figure 1.** Streaming current in a capillary [1]

The convection current $I_{\text{CONV}}$ carried by the flow along the pore is represented by the equation [4]

$$I_{\text{CONV}} = \frac{\varepsilon \zeta}{\mu} \Delta P.$$  

where $\varepsilon$ is the dielectric constant of the fluid, $\zeta$ is the zeta potential, $\mu$ is the viscosity of the fluid and $\Delta P$ is the pressure difference.

The conduction current $I_{\text{COND}}$ for each unit area caused by the convection current is formulated by Ohm’s law [5]:

$$I_{\text{COND}} = -\sigma \Delta V$$  

Where $\sigma$ is the conductivity of the fluid and $\Delta V$ is the difference of streaming potential.

At steady state the convection current $I_{\text{CONV}}$ equals the conduction current $I_{\text{COND}}$ and this relation leads to the Helmholtz-Smoluchowski equation [1]

$$\Delta V = \frac{\varepsilon \zeta}{\mu \sigma} \Delta P.$$  

(3)

Substituting $P = \rho gh$ from Bernoulli’s equation, where $\rho$ is the fluid density; $g$ is the earth acceleration of gravity and $h$ is the height of fluid level, into equation (3) results in

$$\Delta V = \frac{\varepsilon \zeta}{\mu \sigma} \rho g h$$  

(4)

In gradient form, equation (4) becomes

$$\nabla V = \frac{\varepsilon \zeta \rho g}{\mu \sigma} \nabla h.$$  

(5)

Equation (5) is also called the Helmholtz-Smoluchovsky’s electrokinetic potential gradient equation.

2.2. Darcy’s law
Darcy’s law acts as the basic law for fluid flows in porous media. It says that the discharge rate is proportional to the gradient in hydraulic head and the hydraulic conductivity. It is represented mathematically by the following equation [6]
\[ q = \frac{Q}{A} = -K \frac{\Delta h}{\Delta L} = -K \frac{dh}{dl} \] (6)

where \( q \) is the discharge rate, \( Q \) is the volume of flow in unit time, \( A \) is the cross-sectional area normal to the flow and \( K \) is the coefficient of permeability. \( \Delta h \) and \( \Delta L \) are the difference in total head and the length of flow in a medium, respectively.

In gradient form, equation (6) is written as

\[ q = -K \nabla h \] (7)

where

\[ K = \frac{k \rho g}{\mu} \] (8)

\( q \) is also known as the discharge velocity \( v \), which is the velocity of fluid flows in porous media. The minus sign in equation (6) means that \( q \) is in the direction of decreasing total head. \( k \) is intrinsic permeability of fluid.

2.3. Velocity of fluid flows

Velocity of fluid flowing through porous media can be determined from the streaming potential by connecting the Helmholtz-Smoluchovsky’s electrokinetic potential gradient equation with Darcy’s law.

First, we define Electrohydraulic conductivity constant \( C \) as the ratio between \( \nabla V \) and \( \nabla h \) [7]

\[ C = \frac{\nabla V}{\nabla h} = \frac{\varepsilon \rho g}{\mu \sigma} . \] (9)

Substituting equation (9) into equation (5) results in

\[ \nabla V = C \nabla h \] (10)

Using \( C \) value from equation (9), equation (7) can be written as

\[ v = q = -K \nabla h = -K \frac{\nabla V}{c} = -\frac{K}{c} \nabla V . \] (11)

Thus, the velocity of fluid flow \( v \) depends on the intrinsic permeability of rocks \( K \), on the electrokinetic potential gradient or streaming potential gradient \( \nabla V \) and electrohydraulic conductivity constant \( C \). Equation (11) states that the velocity flow direction is opposite to the SP gradient direction. Based on this equation, fluid flows in the opposite direction to the SP gradient and flows from locations with high SP value to locations with low SP value.

In three-dimensional Cartesian coordinate system, with origin \( O \) and axis lines \( X \), \( Y \) and \( Z \) and assuming isotropic media, the velocity component of fluid flow in each direction \( v_x \), \( v_y \) as well as \( v_z \) is

\[ v_x = -\frac{K}{c} \frac{\partial V}{\partial x} \approx -\frac{K}{c} \frac{\Delta V}{\Delta X} \] (12)

\[ v_y = -\frac{K}{c} \frac{\partial V}{\partial y} \approx -\frac{K}{c} \frac{\Delta V}{\Delta Y} \] (13)

\[ v_z = -\frac{K}{c} \frac{\partial V}{\partial z} \approx -\frac{K}{c} \frac{\Delta V}{\Delta Z} \] (14)

with \( X \), \( Y \) and \( Z \) are horizontal axis in \( x \) direction, horizontal axis in \( y \) direction perpendicular to \( x \) axis and vertical axis, respectively.

If the fluids flow only in horizontal direction, its total velocity magnitude can be calculated as

\[ |v| = \frac{K}{c} |\nabla V| = \frac{K}{c} \sqrt{\left(\frac{\Delta V}{\Delta X}\right)^2 + \left(\frac{\Delta V}{\Delta Y}\right)^2} . \] (15)
Based on measurement in laboratory using pure water flowing in sand as medium, K and C are correlable each other and have this relation [7]

\[ K \left( \frac{cm}{s} \right) = 0.0262 \ln \left( C \left( mV/cm \right) \right) + 0.1105 \]  \hspace{1cm} (16)

2.4. Data

Pilot project test site of 2400 meters x 1600 meters was used for this study. The topography and morphology of the research area is quite flat with an elevation between 43 - 66 m above sea level. In general, the research area consists of rubber plantations, tall shrubs, swamps, and small forests. In general, the morphology of the study area is a slightly wavy plain, with a maximum slope of 7% which is classified as gently sloping. Overall, the elevation factor does not really affect the measurement results. The lithology in the study area is composed of claystone, siltstone, and tuff sandstone with coal inserted. Water flowing horizontally in subsurface and surface was observed in some parts of the study area. Upflow water was found also near some SP stations.

The equipment and materials used in SP measurements consist of the Fluke Multimeter, switch box, cable, porous pot, CuSO4 solution, meter, CS60 Garmin GPS receiver, copper electrodes, and other field measurement equipment. Figure 2 shows equipments used for data acquisition.

![Figure 2](image)

**Figure 2.** Equipments used in data acquisition: porous pot with CuSO4 in it (a), Fluke Multimeter as resistivitymeter and its switchbox (b) and rolls of cables (c)

SP measurement was carried out in a gridded system at 100 meters station distance in X direction (longitudinal direction) and 50 meters station distance in Y direction (transverse direction). Totally 1155 measurement stations with 4 geological wells were available for this study. Figure 3 show map of measured SP in the study area, SP profile of line A-A’ and its corresponding resistivity profile in the same line. SP values lie in the range of -160 mV – 110 mV. Resistivity data was acquired using pole-pole configuration at 20 meters electrode spacing and with wells are used for validation.

3. Results and discussion

The velocity of fluid flowing in the study area is calculated by using equations (12), (13) and (15). Assuming \( \frac{K}{C} \) value satisfies equation (16) this relation depends only on the medium properties in the study area and K is calculated as

\[ K_{\text{fluid}} = \frac{\rho_{\text{fluid}} \cdot \mu_{\text{pure water}}}{\mu_{\text{fluid}} \cdot \rho_{\text{pure water}}} \cdot K_{\text{pure water}} \]  \hspace{1cm} (17)

Using properties value for pure water and that of fluid in the study area, K in equation (17) is easily determined.

The velocity distribution of fluid flowing in X-direction, Y-direction as well as total velocity, all in horizontal direction are shown in figures (4) – (6). Water flowing in subsurface was observed in Wells
1 – 4 and showed dominantly in lateral direction as seen also in all figures. The magnitude of water velocity in subsurface lies in the range 0.08 cm/s – 0.21 cm/s. Figure (7) shows the distribution of lateral velocity direction overlying SP map. Water velocity direction is shown by arrow sign and its size denotes its magnitude.

![Figure 3](image3.png)

**Figure 3.** Map of measured SP in the study area (a), SP profile (b) and its corresponding resistivity profile of line A-A’ (c). W1 – W2 are the location of test wells.

![Figure 4](image4.png)

**Figure 4.** Map of fluid velocity in X direction. W1 – W2 are the location of test wells.
Figure 5. Map of fluid velocity in Y direction. W1 – W2 are the location of test wells.

Figure 6. Map of total velocity in lateral direction. W1 – W2 are the location of test wells.

Figure 7. Distribution of lateral total velocity overlying SP map.
As seen also from figure (3), the A-A’ line passes through 4 test wells. Those wells coincide with the SP profile which shows a significant change in positive and negative SP signs. A significant change in value from negative to positive or vice versa in the SP profile indicates a vertical fluid flow, because the fluid contains an electric charge greater than the surrounding medium.

4. Conclusion
Our study concludes following results:
1. The location of water seepage near the surface can be well identified using the SP method. The SP value due to water flowing at the study site ranges from: -80mV - -160mV.
2. DC resistivity method is good to be used as a support method to identify the location of water flows, at which the resistivity value for water at the study site is $<0.5 \ \Omega m$.
3. The formula derived to determine the velocity of the fluid flow works well in modeling the water flow velocity in the study area with the velocity value lie in the range 0.08 cm/s - 0.21 cm/s.
4. A significant change in value from negative to positive or vice versa in the SP profile indicates a vertical fluid flow which is observed in the study area.

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