Effect of micropolar fluids on squeeze film lubrication between curved annular plates

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Abstract: The theoretical examination is made in this paper to learn the effect of micropolar fluid lubricants on curved annular bearings by allowing the squeezing action. The general modified Reynolds equation of squeeze film curved annular plates with micropolar fluids is derived with general film thickness. The closed form result of this equation is arrived. From the outcome it is scrutinized that, the effect of micropolar fluid is to enhance the pressure, the load capacity and to lengthen the squeeze film time as estimated to the corresponding Newtonian case.

1. Introduction
The study of Micropolar fluids plays on vital role in industries such as cooling of extrusion of polymers, colloidal and adjournment solution and solidification of liquid crystal. For studying all these troubles, classical Navier-Stokes theory is insufficient. The first person proposed the theory of micropolar fluids is Eringen [1]. Using the micropolar fluid theory several investigations are carried out to analysis of numerous bearing structures, such as journal bearings [7], squeeze film bearings [8-9] have observed some utilizes of micropolar fluids over the Newtonian fluids, for example load capacity exceeds and the time exceeds of approach for squeeze film bearings. Naduvinamani and Siddanagouda [10] theoretically proved that it provides an enhanced load carrying capacity and decayed coefficient of resistance as estimated to the corresponding Newtonian case. In the reference of complex and step slider bearings, the properties of micropolar lubricant were demonstrated by Maiti [2] and a hypothetical investigation of a micropolar fluid in porous pivoted slider bearings lubrication by Agarwal et al. [4], it is recognized that, the load capacity is superior than the Newtonian fluid. Isa and Zaheeruddin [3] presented the lubrication of one-dimensional journal bearings and establish that the load capacity exceeds as the micropolar parameter exceeds and decreases as the step height exceeds. Khonsari and Brewe [5] analysed that the action of micropolar fluids in finite journal bearings lubrication and it is proved that finite Reynolds equation for micropolar fluids is evaluated by using central finite difference system and it is exposed that even if the frictional force connected with micropolar fluid is in normally privileged than that of a Newtonian fluid, the friction co-efficient of micropolar fluids inclined to be poorer than that of Newtonian. The discrepancy in viscosity with temperature in journal bearing lubricant with additives with the micropolar theory was studied by Sinha et.al. [6] and they recognized that, the additives in lubricant intensify the temperature in journal bearing.

Squeeze film mechanisms of curved plates is observed in many applications artificial joints, gears, bio-lubrication and machine tools. Analyses of squeeze film action mainly focused on the lubricated
condition on faces of disc clutches. Relating to squeeze film study many researchers have been accessible, Hays [11], Gould [12], Gupta and Vora [13], Christensen [14] according to their results, the load capacity enhanced with decaying the values of film height.

The aim of this paper is to widen the earlier examination of squeeze films between curved annular plates deliberate by Gupta and Vora [13] and Bujurke et al. [15] to include micropolar fluids on squeeze film lubrication between curved annular plates. The general modified Reynolds equation of squeeze film curved annular plates with micropolar fluids is derived with general film thickness. Utilizing the continuity equation and the basic equation of motion applied to expect the squeezing motion behavior. The closed form result of this equation is attained. From the outcome it is examine that, the effect of micropolar fluid is to enhance the pressure, the load capacity and to lengthen the squeeze film time as estimated to the corresponding Newtonian case.

2. Mathematical Formulation
The physical configuration of squeeze film geometry is displayed in figure 1. Where the inward radius \( b \) and the outward radius \( a \), in which the upper plate is imminent the lower plate with a squeezing velocity \( \frac{-dh}{dt} \).

The fluid film thickness \( h \) is considered as

\[
h = h_m \exp\left(-\frac{\beta r^2}{a^2}\right), \quad b \leq r \leq a
\]

(1)

Here \( h_m \) denotes the lowest film thickness and \( \beta \) indicates the curved shape parameter, the internal and external radii \( r \) is the radial coordinate. In view of these suspicions, the basic equation of motion and the continuity equation are given by:
\[ \left( \mu + \chi \right) \frac{\partial^2 u}{\partial z^2} + \chi \frac{\partial u}{\partial z} = \frac{\partial p}{\partial r} \quad (2) \]

\[ r \frac{\partial^2 w_1}{\partial z^2} - \chi \frac{\partial u}{\partial z} - 2\chi w_1 = 0 \quad (3) \]

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( ru + \frac{\partial w}{\partial z} \right) = 0 \quad (4) \]

The boundary conditions are given by:

At the higher surface \( z = h \)

\[ u = 0, \ w_1 = 0, \ w = \frac{dh}{dt} \quad (5a) \]

At the lower surface \( z = 0 \)

\[ u = 0, \ w_1 = 0, \ w = 0 \quad (5b) \]

By using the above boundary conditions, the solution of equation (2) and (3) is obtained as

\[ u = p' \left[ \frac{z^2}{2} - \frac{N^2 h}{m} \left( \cosh mz - 1 \right) \right] \]

\[ + \frac{D_1}{2(1-N^2)} \left[ \frac{z}{N} \left( \cosh mz - \frac{1}{(cosh mz - 1)} \right) \right] \]

And \( w_1 = \frac{D_1}{2(1-N^2)} \left( \cosh mz - 1 \right) + \frac{\sinh mz}{\sinh mh} \left[ \frac{hp'}{2\mu} - \frac{D_1}{2(1-N^2)} \left( \cosh mh - 1 \right) \right] \]

where

\[ m = \left( \frac{4\mu \chi}{r(2\mu + \chi)} \right)^{\frac{1}{2}}, \quad N = \left( \frac{\chi}{2\mu + \chi} \right)^{\frac{1}{2}}, \quad D_1 = \frac{-1}{2(1-N^2)} \left( \frac{hp'}{\mu} \right), \quad l = \left( \frac{r}{4\mu} \right)^{\frac{1}{2}} \]

Substituting \( u \) in the continuity equation (4) and integrating we get,

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( ru + \frac{\partial w}{\partial z} \right) = 12\mu \frac{dh}{dt} \quad (7) \]

Where \( f(N, l, h) = h^3 + 12l^2h - 6Nh^2 \cosh \left( \frac{Nh}{2l} \right) \).

Introducing the following dimensionless variables

\[ r^* = \frac{r}{a}, \ h_m^* = \frac{h_m}{h_{m_0}}, \ l' = \frac{l}{h_{m_0}}, \ h^* = \frac{h}{h_{m_0}} = h_m^* \exp \left( -\beta r^* \right), \ P^* = \frac{p h_{m_0}}{\mu a^3 \left( -dh_m/\partial t \right)} \]

\[ \frac{d}{dr^*} \left( f^* \left( N, l', h^* \right) r^* \frac{dp^*}{dr^*} \right) = -12r^* \quad (8) \]

For the current squeezing issue, the boundary conditions are:

\[ P^* = 0 \text{ at } r^* = \delta = \frac{h'}{a}, \quad (9a) \]

\[ P^* = 0 \text{ at } r^* = 1, \quad (9b) \]

Integrating the equation (8) using the boundary conditions (9a and 9b), the non-dimensional squeeze film pressure is arrived as
\[ P^* = \frac{6\left[g_1(1)g_2(r^*) - g_2(1)g_1(r^*)\right]}{g_2(1)} \]  \hspace{1cm} (10)

Where \( g_1(r^*) = \int_0^r r^* \, dr^* \), \( g_2(r^*) = \int_0^r \frac{1}{r} \, dr^* \)  \hspace{1cm} (11)

The dimensionless load-carrying capacity is given by

\[ W^* = \frac{Wh^2 m_a}{2 \pi \mu a^2 \left(-\frac{dh_m}{dt}\right)} \]

\[ = -6\int_0^r g_1(r^*) r^* \, dr^* + 6\frac{g_1(1)}{g_2(1)}\int_0^r g_2(r^*) r^* \, dr^* \]  \hspace{1cm} (12)

The dimensionless response time is given by

\[ i^* = \frac{Wh^2 m_a}{\pi \mu a^2} - t \]

\[ = \frac{1}{6} \int_0^r g_2(1) \left[ g_2(1) \int_0^r g_1(r^*) r^* \, dr^* - g_1(1) \int_0^r g_2(r^*) r^* \, dr^* \right] \, dh^* \]  \hspace{1cm} (13)

3. Results and discussion

Utilizing the continuity equation and the basic equation of motion, the general modified Reynolds equation of squeeze film lubrication between curved annular plates with microporous fluids is derived with general film thickness. According to the present study, some special cases in the literature can be obtained from specific values of the parameters \( i^* \), \( N \), \( \delta \) and \( \beta \).

3.1 Non-dimensional pressure distribution

Figure 2 illustrates the deviation of non-dimensional pressure \( P^* \) with \( r^* \) for dissimilar values of \( N \) with \( \beta = 0.2, i^* = 0.3, \delta = 0.2 \) and \( h^*_m = 0.6 \) and it is demonstrate that \( P^* \) enhanced for expanding the values of \( N \). The deviation of non-dimensional film pressure \( P^* \) as a function of \( r^* \) with \( N = 0.3, i^* = 0.3, \delta = 0.2 \) and \( h^*_m = 0.6 \) for dissimilar values of \( \beta \) is depicted in figure 3 and it is exhibit that \( P^* \) increases for raising the values of \( \beta \). The effect of \( i^* \) on the deviation of non-dimensional pressure \( P^* \) as a function of \( r^* \) with \( N = 0.3, \beta = 0.2, \delta = 0.2 \) and \( h^*_m = 0.6 \) is depicted in figure 4 and it is displays that \( P^* \) increases for increasing values of \( i^* \).

3.2 Non-dimensional load carrying capacity

Figure 5 represents the variation of non-dimensional load carrying capacity \( W^* \) as a function of \( \beta \) with \( \delta = 0.2, i^* = 0.3 \) and \( h^*_m = 0.6 \) for dissimilar values of \( N \) and it is exhibit that load capacity enhanced with raising the values of \( N \). Figure 6 illustrate that deviation of non-dimensional load carrying capacity \( W^* \) as a component of \( \beta \) with \( N = 0.3, i^* = 0.3 \) and \( h^*_m = 0.6 \) for dissimilar values of radius ratio \( \delta \) and it is proves that larger values of the load-carrying capacity are attained for smaller radius ratio \( \delta \).

3.3 Non-dimensional squeeze film time

Figure 7 describes the deviation of non-dimensional time \( i^* \) as a component of \( h^*_m \) with \( \beta = 0.2, i^* = 0.3, \delta = 0.2 \) with distinct values of \( N \) and it is examine that raising the values of \( N \) the response time
increases. Figure 8 represents that deviation of non-dimensional time $t'$ as a component of $h_m'$ with $N = 0.3, l' = 0.3, \delta = 0.2$ for dissimilar values of $\beta$ and it proves that response time $t'$ increases for raising the values of $\beta$. 

![Figure 2: Dimensionless film pressure $P'$ versus $r'$ with $\beta = 0.2$, $\delta = 0.2$, $l' = 0.3$ and $h_m' = 0.6$ for different values of $N$.](image1)

![Figure 3: Dimensionless film pressure $P'$ versus $r'$ with $N = 0.3$, $l' = 0.3$, $\delta = 0.2$, $\beta$ for different values of $\beta$.](image2)

![Figure 4: Dimensionless film pressure $P'$ versus $r'$ with $\beta = 0.2$, $\delta = 0.2$, $N = 0.3$ and $h_m' = 0.6$ for different values of $l'$.](image3)

![Figure 5: Dimensionless load-carrying capacity $W'$ versus $\beta$ with $\delta = 0.2$, $l' = 0.3$ and $h_m' = 0.6$ for different values of $N$.](image4)
4. Conclusions
In this paper, effect of micropolar fluids on squeeze film lubrication between curved annular bearings is studied. It is analysed that, the effect of micropolar fluid is to enhance the pressure, the load capacity and to lengthen the squeeze film time as estimated to the corresponding Newtonian case.

5. Nomenclature:
\begin{itemize}
  \item a Outer radius of the plate
  \item b Inner radius of the plate
  \item h Film thickness
  \item h_{\text{min}} Minimum film thickness
\end{itemize}
\[ h_m^* \quad \text{Non-dimensional film thickness} \]
\[ l \quad \text{Couple stress parameter} \left( \frac{r}{4\mu} \right)^{1/2} \]
\[ l^* \quad \text{Non-dimensional couple stress parameter} \left( \frac{l}{h_m} \right) \]
\[ N \quad \text{Coupling number} \left( \frac{\chi}{2\mu + \chi} \right)^{1/2} \]
\[ p \quad \text{Pressure in the film region} \]
\[ P^* \quad \text{Non-dimensional fluid film pressure} \]
\[ r, z \quad \text{Radial and axial coordinates} \]
\[ t \quad \text{Time of approach} \]
\[ t^* \quad \text{Non-dimensional time of approach} \]
\[ u, w \quad \text{Velocity components in} \ r \ \text{and} \ z \ \text{directions} \]
\[ W \quad \text{Load carrying capacity} \]
\[ W^* \quad \text{Non-dimensional load carrying capacity} \]

**Greek Symbols:**

\[ \beta \quad \text{Curved shape parameter} \]
\[ \mu \quad \text{Lubricant viscosity} \]

**References**

[1] Eringen A C 1966 Theory of micropolar fluids *J. Math. Mech.* 16 1-18.
Maiti G 1973 Composite and step slider bearings in micropolar fluids *Japanese J. Appl. Physics* 12(7) 1058-64.
[2] Isa M and Zaheeruddin K H 1978 Micropolar fluid lubrication of one-dimensional journal bearings *Wear* 50 211- 20.
Agrawal V K and Bhatt S B 1980 Porous pivoted slider bearings lubricated with a micropolar fluid *Wear* 61 issue 1 1-8.
[3] Khonsari M M and Brewe D E 1989 On the performance of finite journal bearings lubricated with Micropolar fluids *Tri. Trans.* 32 155-60.
[4] Sinha P, Chandan S and Prasad K R 1983 Viscosity variation considering cavitation in a journal bearing lubricant containing additives *Wear* 86(1) 43-56.
[5] Das S, Guha S K and Chattapadhyay A K 2004 Theoretical analysis of stability characteristics of hydrodynamic journal bearings with micropolar fluid *Proc. Inst. Mech. Engrs. J. Eng. Tribology*(part J) 218 no. 1 45-56.
[6] Wang X and Zhu K Q 2006 Numerical analysis of journal bearings lubricated with micropolar fluids including thermal and cavitating effects *Tribol Int* 39 no. 3 227–37.
[7] Naduvinamani B N and Kashinath B 2008 Surface roughness effects on the static and dynamic behavior of squeeze film lubrication of short journal bearings with micropolar fluids *Proc IMechE Part J: J Engineering Tribology* 222 no. 2 121–31.
[8] Naduvinamani N B and Siddanagouda A 2008 Porous inclined stepped composite bearings with micropolar fluid *Tribol. Materials, Surfaces and Interfaces* 4 224-32.
[9] Hays D F 1963 Squeeze films for Rectangular plates *ASME J. Basic Engin.* 88 243-46.
[10] Gould P 1967 Parallel surface Squeeze films *ASME J. Lubric. Technol.* 89 375-80.
[11] Gupta J L and Vora K H 1980 Analysis of Squeeze films between Curved Annular Plates *ASMEJ. Lubric. Technol.* 102 48-50.
[12] Christensen H 1970 Elastohydrodynamic theory of Spherical Bodies in Normal Approach *ASME J. Lubric. Technol.* 92 145-54.
[13] Bujurke N M, Naduvinamani N B and Basti D P 2007 Effect of surface roughness on the squeeze film lubrication between curved annular plates *Industrial Lubrication and Tribology* 59 no. 4 178–85.