Testing Neutrino Mass Matrices with Approximate $L_e - L_\mu - L_\tau$ Symmetry

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Abstract

As neutrino experiments are starting to probe the detailed structure of the neutrino mass matrix, we present sumrules relating its matrix elements for a class of models with approximate $L_e - L_\mu - L_\tau$ symmetry and the observables in neutrino oscillation experiments. We show that regardless of how the above symmetry is broken (whether in the neutrino sector or the charged lepton sector), as long as the breaking terms are small, there is a lower bound on the solar neutrino mixing angle, $\sin^2 2\theta_\odot$, correlated with the solar mass difference square, $\Delta m^2_\odot$, or the mixing parameter, $U_{e3}$. We also discuss models where such patterns can arise.

I. INTRODUCTION

With the evidences for neutrino mass becoming stronger, we have reached a stage where it is becoming possible to contemplate probing the detailed structure of the neutrino mass matrix. The initial evidences for neutrino mixing from solar and the atmospheric neutrino data have now been confirmed by several experiments. From the recent results of Super-Kamiokande and SNO experiments, it now appears that the choice for neutrino mixings and therefore the possible profiles for the neutrino mass matrix within the three neutrino framework have been considerably narrowed. There are also evidences for new structures in neutrino masses from the LSND and Heidelberg-Moscow neutrinoless double beta decay.

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decay experiments [4]. These latter results need confirmation. However, if they are included in the neutrino mass analysis, they have a profound effect on the nature of new physics: for example, confirmation of the LSND results would imply that there are four neutrinos rather than three, the fourth one being a “sterile” neutrino. Similarly, if the present results of the Heidelberg-Moscow $\beta\beta_0\nu$ are confirmed [4], then for the first time, we would have at hand a piece of information that oscillation experiments are unable to provide, i.e., the overall scale of the neutrino masses. In this paper, we will take the conservative approach and consider the implications of only the solar and the atmospheric neutrino results within a three neutrino framework. In this case, the mixing pattern is very nearly fixed by the solar and atmospheric data along with important constraints coming from CHOOZ-PALOVVERDE experiments [5].

The general Majorana mass matrix for three neutrinos has nine parameters: three mass eigenvalues, $(m_1, m_2, m_3)$, three mixing angles and three phases. Of the three phases, only one is observable in oscillation experiments. The final goal of neutrino physics is to determine all these parameters, assuming there are only three neutrinos (as we do in this paper).

To fit the atmospheric and solar neutrino experiments, we will make the following choice of mixing angles, which we will incorporate into our analysis, i.e.

(i) mixing between $\nu_\mu$ and $\nu_\tau$ is nearly maximal and

(ii) there is large mixing between $\nu_e$ and the other active neutrinos.

The next piece of information necessary for our analysis is the mass pattern for neutrinos. The data at the moment does not choose between

(i) hierarchical, i.e. $m_1 \ll m_2 \ll m_3$,

(ii) inverted, i.e. $m_1 \simeq -m_2 \gg m_3$ or

(iii) degenerate, i.e. $m_1 \simeq m_2 \simeq m_3$ patterns.

We will choose for our discussion the inverted pattern since that corresponds to a very elegant leptonic symmetry, $L_e - L_\mu - L_\tau$ [6]. Models with this symmetry have been extensively discussed in literature [6]. We will here follow the spirit of the paper [7], where two plausible models for approximate $L_e - L_\mu - L_\tau$ symmetry were considered:

(i) where the symmetry was broken by Planck scale effects and

(ii) where the charged lepton mass matrix was the source of leptonic symmetry breaking.

The first case leads to the so-called low solution for the solar neutrino problem and predicts that KAMLAND experiment currently running [8] should not see a signal. It also predicts that the parameter $U_{e3} = 0$. The second case, for large $tan\beta$ SUSY models leads to the LMA solution to the solar neutrino problem and has a correlation between the $U_{e3}$ and $sin^22\theta_\odot$. In this note we present a detailed phenomenological analysis of the most general class of such models.

The motivation for exploring such symmetries of leptonic sector are manyfold. Historically, symmetries have played a guiding role in the progress of particle physics as exemplified by symmetries such as isospin, SU(3), color, electroweak symmetries etc. Secondly, if the existence of leptonic symmetries is established, that would make the leptons completely different from quarks and ideas such as quark-lepton unification would not be indicated in physics beyond the standard model. This will have implications for the nature of possible grand
unification theories e.g. with leptonic symmetries, it will be hard to envision unification such as SO(10); instead unification schemes based on [SU(3)]³ may be preferred.

This note focusses on three issues: (A) first, the numerical analysis of the most general Majorana neutrino mass matrix consistent with weakly broken $L_e - L_\mu - L_\tau$ symmetry in a basis where charged lepton mass matrix is diagonal and (B) a second case, where the symmetry is exact in the neutrino mass matrix but broken in the charged lepton masses as in ref. [7] and (C) theoretical scenarios that make definite predictions for the pattern of $L_e - L_\mu - L_\tau$ breaking in the neutrino mass matrix. We derive sumrules involving the parameters of the mass matrix and the observables $\sin^2\theta_\odot, \Delta m^2_\odot, A$ and the mixing parameter $U_{e3}$ for case (A) and find strong correlation between these parameters. In particular we find that $\sin^2\theta_\odot$ in this case is necessarily more than 0.95. This makes it possible to confront these models with the next generation neutrino oscillation experiments. We then discuss case (B) which will be one way to proceed in this framework if the value of $\sin^2\theta_\odot$ determined by KAMLAND and future solar neutrino experiments turns out to be smaller than 0.95. In this case, we observe a correlation between $U_{e3}$ and $\sin^2\theta_\odot$. For the specific breaking pattern in the charged lepton sector described in [7], we find a somewhat relaxed lower bound of $\sin^2\theta_\odot \geq 0.80$ for $U_{e3} \leq 0.22$. Case (C) presents an example of a model, where $U_{e3} = 0$ is a natural consequence without contradicting any known results. We also apply our sumrules to discuss the effective three neutrino models derived from $3+1$ [8] models for LSND [3] based on the same symmetry [11] using the seesaw mechanism [11]. Depending on whether these models are ruled out or confirmed, it can provide important information on the direction of new physics beyond the standard model.

II. REVIEW OF OSCILLATION PHENOMENOLOGY FOR THREE NEUTRINOS

Before discussing the phenomenology of the model of interest, we repeat a few basic formulae well known in the literature in order to set the notation. We express the flavor (or weak) eigenstates in terms of the mass eigenstates as follows:

$$\nu_\alpha = U_{\alpha i} \nu_i$$

where we will use $\alpha, \beta$ for the flavor index and $i, j$ for the mass eigenstates index. $U_{\alpha i}$ are element of the mixing matrix which diagonalizes the mass matrix of the neutrinos. The basic formula for the transition probability between two weak eigenstates is given by

$$|\langle \nu_\beta | \nu_\alpha \rangle|^2 = \sum_{i,j} A_{\alpha i}^* A_{\beta j} \frac{\Delta m^2_{ij}}{2E}$$

$$A_{\alpha i}^* A_{\beta j} = U_{\alpha i} U_{\alpha j}^* U_{\beta j} U_{\beta i}$$

$$\Delta m^2_{ij} = m_{i}^2 - m_{j}^2.$$ It is easy to see that $(A_{\alpha \beta}^*)^* = A_{\beta \alpha}^*$

Experimental data suggest (i) two distinct mass scales that characterize the solar and the atmospheric neutrino oscillations: $D3 = \Delta m^2_{31}$ and $D2 = \Delta m^2_{21}$ with $|D3| > |D2|$ and (ii) $U_{e3}$ very small. In this approximation, the analysis of the two oscillations can be separated effectively into $2 \times 2$ oscillations.

Under the first approximation, the survival probability is, for atmospheric neutrino,
\[ P_{\mu\mu} = 1 - 4(A_{\mu\mu}^{13} + A_{\mu\mu}^{23}) \sin^2 \left( \frac{D_3}{4p} t \right) \]  

for solar neutrino

\[ P_{ee} = 1 - 2(A_{ee}^{13} + A_{ee}^{23}) - 4(A_{ee}^{12}) \sin^2 \left( \frac{D_2}{4p} t \right) \]

\[ = (1 - 2(A_{ee}^{13} + A_{ee}^{23}))(1 - \frac{4(A_{ee}^{12})}{1 - 2(A_{ee}^{13} + A_{ee}^{23})} \sin^2 \left( \frac{D_2}{4p} t \right)) \]

\[ A_{ee}^{23} \text{ and } A_{ee}^{13} \text{ are both proportional to } U_{e3}^2 \text{ and so the oscillation amplitude of atmospheric } \nu_e \text{ is suppressed. Note that the analysis becomes effectively that of two neutrino oscillation.} \]

From the above equation, we can read out the mixing angle

\[ \sin^2 2\theta_A = 4(A_{\mu\mu}^{13} + A_{\mu\mu}^{23}) = 4U_{\mu3}^2(1 - U_{\mu3}^2) \]

\[ \sin^2 2\theta_\odot = \frac{4(A_{ee}^{12})}{1 - 2(A_{ee}^{13} + A_{ee}^{23})} = \frac{4U_{e3}^2 U_{\mu3}^2}{1 - 2U_{e3}^2(1 - U_{e3}^2)} \]

Note that the above formulae for the survival probabilities remain valid even in the presence of complex phases in the mixing matrices. We will use these formulae in deriving the effective mixing angles for the atmospheric and solar neutrino oscillations \[12\].

III. APPROXIMATE \( L_E - L_\mu - L_\tau \) SYMMETRY FOR LEPTONS

In order to explore the possible existence of symmetries in the lepton sector of physics beyond the standard model, we consider two cases in this section. In the first one we allow the most general small symmetry breaking terms in the neutrino mass matrix keeping the charged lepton sector completely symmetry conserving and in the second case, we keep the neutrino sector symmetry conserving and allow small breakings in the charged lepton sector. In the subsequent section we discuss physics scenarios where these two cases can arise.

A. Symmetry breaking in the neutrino sector and sumrules for mass matrix elements

In this section, we will consider the neutrino mass matrix for the case with weakly broken \( L_e - L_\mu - L_\tau \) symmetry in the neutrino sector. We will assume the dominant terms to respect \( L_e - L_\mu - L_\tau \) symmetry with the symmetry breaking terms to be less than one third of the symmetry preserving ones and work in the perturbative approximation of these parameters. We find sumrules that relate the symmetry breaking parameters to the observables in neutrino oscillation experiments. One can then use these to test for the possible existence of the leptonic symmetry \( L_e - L_\mu - L_\tau \).

To proceed with the derivation of the sumrules, we write down the neutrino mass matrix in the exact symmetry limit i. e.

\[ M^{(0)}_\nu = m \begin{pmatrix} 0 & \sin \theta & \cos \theta \\ \sin \theta & 0 & 0 \\ \cos \theta & 0 & 0 \end{pmatrix} \]
We add to it the symmetry breaking matrix of the following general form:

\[
\Delta M_\nu = m \begin{pmatrix} z & 0 & 0 \\ 0 & y & d \\ 0 & d & x \end{pmatrix}.
\]  

(7)

The full neutrino mass matrix is \( M_\nu = M_\nu^{(0)} + \Delta M_\nu \). The charged lepton mass matrix can be cast into a diagonal form in this basis in the symmetry limit by redefining the parameters \( m \) and angle \( \theta \). In this example we do not assume any symmetry breaking terms in the charged lepton sector.

In the perturbative approximation, we find the following sumrules involving the neutrino observables and the elements of the neutrino mass matrix. The two obvious relations are

\[
\sin^2 2\theta_A = \sin^2 2\theta + O(\delta^2) \\
D_3 = \Delta m^2_A = -m^2 + 2\Delta m^2_\odot + O(\delta^2)
\]

(8)

The nontrivial relations that also hold for this model are:

\[
\sin^2 2\theta_\odot = 1 - \left(\frac{\Delta m^2_\odot}{4\Delta m^2_A} - z\right)^2 + O(\delta^3) \\
D_2 \equiv \frac{\Delta m^2_\odot}{\Delta m^2_A} = 2(z + \vec{v} \cdot \vec{x}) + O(\delta^2) \\
U_{e3} = \vec{A} \cdot (\vec{v} \times \vec{x}) + O(\delta^3)
\]

(9)

where \( \vec{v} = (\cos^2 \theta, \sin^2 \theta, \sqrt{2}\sin \theta \cos \theta) \), \( \vec{x} = (x, y, \sqrt{2}d) \) and \( \vec{A} = \frac{1}{\sqrt{2}}(1, 1, 0) \). \( \delta \) in Eq. (8) and (9) represents the small parameters in the mass matrix. These equations represent one of the main results of this paper. Below we study their implications. Finally, there is the relation \( \langle m \rangle_{\beta\beta} = z \). This is an exact relation with no \( O(\delta^n) \) corrections. Note that in the first of the two equations in Eq. (9), if we set \( z = 0 \), we get a relation which is similar to the one that holds in the Zee model [13] but apparently valid for a wider class of models.

Experimental data requires \( \frac{\Delta m^2_\odot}{\Delta m^2_A} < 0.1 \), which implies that our model predicts \( \sin^2 2\theta_\odot > 0.95 \) for \( |z| \leq 0.2 \). Combining this with the mean value for the \( \Delta m^2_{\text{Atmos}} \), we predict that the effective neutrino mass in the neutrinoless double beta decay in this case is given by \( \langle m \rangle_{\beta\beta} \simeq 0.009 \) eV. The detailed correlation between \( \sin^2 2\theta_\odot \) and \( \Delta m^2_\odot \) is given in Fig. 1. It includes also the second order perturbation effects not present in Eq.(9). In Fig. 2, we present the predictions for \( U_{e3} \) in this model. We see that \( U_{e3} \) is pretty much allowed up to its present limit. Therefore, this model can be tested once the KAMLAND experiment is complete assuming that it confirms the LMA solution to the solar neutrino problem.

An immediate implication of our result above is that if \( \sin^2 2\theta_\odot \) is found to be smaller than the above value 0.95, then either \( L_e - L_\mu - L_\tau \) symmetry has to be very badly broken or there must be contributions to symmetry breaking from the charged lepton sector.
FIG. 1. The figure shows the predictions for $\sin^2 2\theta_\odot$ as a function $\Delta m^2_\odot / \Delta m^2_A$ for different values for the symmetry breaking parameters. The left most line corresponds to $z = -0.2$ and the right most (only partially visible) to $z = 0.16$. The thickness of the individual lines reflect the higher order contributions.

B. Effect of symmetry breaking in the charged lepton sector

We now include the charged lepton sector and following the suggestion of Ref. [7], consider the charged lepton mass matrix of the form

\[
M_e = \begin{pmatrix} 0 & 0 & a \\ 0 & b & 0 \\ a' & 0 & 1 \end{pmatrix} \frac{m_\tau}{q}
\]

\[q = \sqrt{1 + a^2 + a'^2}\]  \hspace{1cm} (10)

We work in the charged lepton mass eigenstate basis. In this basis, the neutrino mass matrix can be rotated from its original mass matrix by $V_L$

\[M_\nu \rightarrow V_L^\dagger M_\nu V_L\]  \hspace{1cm} (11)

where $V_L$ is defined by $V_L^\dagger (M_e M_e^\dagger) V_L$ is diagonal. The charged lepton masses require

\[
\frac{a^2 a'^2}{q^2} \simeq (\frac{m_e}{m_\tau})^2 \simeq 7.7 \times 10^{-8}
\]

\[
\frac{b^2}{q^2} \simeq (\frac{m_\mu}{m_\tau})^2 \simeq 3.4 \times 10^{-3}\]  \hspace{1cm} (12)
we need $a$ to be relatively big to be able to generate the large mixing angle solution to the solar neutrino problem; $a'$ is then has to be small and we approximate it to be zero. In the charged lepton mass eigenstate basis, the neutrino mass matrix is found to be

$$
M_\nu = \begin{pmatrix}
\frac{-2a \cos \theta}{1+a^2} & \sin \theta & \frac{(1-a^2) \cos \theta}{1+a^2} \\
\sin \theta & \frac{-2a \sin \theta}{\sqrt{1+a^2}} & \frac{a \sin \theta}{\sqrt{1+a^2}} \\
\frac{\sqrt{1+a^2}}{(1-a^2) \cos \theta} & \frac{a \sin \theta}{\sqrt{1+a^2}} & \frac{-2a \cos \theta}{1+a^2}
\end{pmatrix} \ m
$$

To this mass matrix, we must add the renormalization group corrections due to the charged lepton mass, which then induces the solar mass splitting. As was noted in [7], this process of rotation of the charged lepton and the renormalization group corrections lead to a nonzero value for the $U_{e3}$ parameter correlated with the solar mixing angle, $\sin^2 2\theta_\odot$. In Fig. 3, we show the correlation between these two parameters. We note that as $U_{e3}$ reduces, the value of $\sin^2 2\theta_\odot$ increases. For $U_{e3} \leq 0.22$, we get $\sin^2 2\theta_\odot \geq 0.85$. This can therefore be used to test the model. It is important to note that the solar mixing angle $\sin^2 2\theta_\odot$ will be more precisely measured by KAMLAND (down to the level of 0.91) if the LMA solution to the solar neutrino puzzle is correct. There are many experimental projects (such the off axis NUMI beam [14], JHF and the neutrino factory proposals) whose main goal is to measure (or limit) $U_{e3}$ down to the level of a few percent. Clearly this will then very severely constrain or rule out the case where the $L_e - L_\mu - L_\tau$ symmetry is broken in the charged lepton sector.
FIG. 3. This figure shows the correlation between $U_{e3}$ and $\sin^2 2\theta_\odot$ for the model where $L_e - L_\mu - L_\tau$ symmetry is broken by the charged lepton sector. Note that present upper limit of 0.22 corresponds to a minimum value of $\sin^2 2\theta_\odot \geq 0.80$ and $U_{e3} \leq 0.16$ corresponds to $\sin^2 2\theta_\odot \geq 0.90$. The different lines in the figure correspond to various values for the $\sin^2 2\theta_A$ with the rightmost line corresponding to the maximal value of 1. The lines in this figure correspond to the following relation between $U_{e3}$ and $\sin^2 2\theta_\odot$ i.e. $\sin^2 2\theta_\odot = \left[1 - \frac{1 + \cos^2 \theta_A}{\sin^2 \theta_A} U_{e3}^2\right]^2 / (1 - U_{e3}^2)^2$.

**IV. THEORETICAL ORIGIN OF THE MASS MATRICES**

In this section, we will discuss possible theoretical origin of the mass matrices analyzed above in simple gauge models. We will first present a simple extension of the standard model by adding two additional singlet right handed neutrinos, $N_1, N_2$ assigning them $L_e - L_\mu - L_\tau$ quantum numbers of $= 1$ and $-1$ respectively. Denoting the standard model lepton doublets by $\psi_{e,\mu,\tau}$, the $L_e - L_\mu - L_\tau$ symmetry allows the following new couplings to the Lagrangian of the standard model:

$$L' = (a\tilde{\psi}_\tau + b\tilde{\psi}_\mu)HN_2 + \tilde{\psi}_eHN_1 + MN_1^T C^{-1} N_2$$

where $H$ is the Higgs doublet of the standard model; $C^{-1}$ is the Dirac charge conjugation matrix. We add to it the symmetry breaking mass terms for the right handed neutrinos, which are soft terms, i.e.

$$L_B = \epsilon(M_1 N_1^T C^{-1} N_1 + N_2^T C^{-1} N_2) + H.c.$$  

with $\epsilon \ll 1$. These terms break $L_e - L_\mu - L_\tau$ by two units but since they are dimension 3 terms, they are soft and do not induce any new terms into the theory.
It is clear from the resulting mass matrix for the $\nu_L, N$ system that the linear combination $bv_\mu - a\nu_e$ is massless and the atmospheric oscillation angle is given by $\tan\theta_A = b/a$; for $a \sim b$, the $\theta_A$ is maximal. The seesaw mass matrix then takes the following form (in the basis $(\nu_e, \tilde{\nu}_\mu, N_1, N_2)$ with $\tilde{\nu}_\mu \equiv a\nu_\mu + b\tau$):

$$M = \begin{pmatrix}
0 & 0 & m_1 & 0 \\
0 & 0 & 0 & m_2 \\
m_1 & 0 & \epsilon M_1 & M \\
0 & m_2 & M & \epsilon M_2
\end{pmatrix}$$

The diagonalization of this mass matrix leads to the mass matrix of the form discussed in Eq. (6) and (7) with the constraint that $d/x = y/d = m_2/m_1$. This has the implication that $Ue_3 = 0$. One could get nonzero values for $Ue_3$ by further extending the model to include another heavy right handed neutrino.

V. SYMMETRY BREAKING INDUCED BY STERILE NEUTRINO

In this section, we consider an application of our sumrules to a four neutrino model which accommodates the LSND results within the 3+1 scenario [9] and where the mass matrix in the three active neutrino subsector obeys $L_e - L_\mu - L_\tau$ symmetry. The only symmetry breaking terms in this case are due to the mixing of the active neutrinos with the sterile one. The mass matrix in this case is given by:

$$M^{(4)} = \begin{pmatrix}
0 & m_1 & m_2 & \delta_1 \\
m_1 & 0 & 0 & \delta_2 \\
m_2 & 0 & 0 & \delta_3 \\
\delta_1 & \delta_2 & \delta_3 & \Delta
\end{pmatrix}$$

Such patterns are predicted by minimal quark lepton symmetric seesaw models for the sterile neutrino as discussed in [10]. In this mass matrix $\delta_i \ll \Delta$; The diagonalization of this mass matrix can be carried by using the seesaw technique [15] which leads to the effective $3 \times 3$ active neutrino submatrix of the form that now has induced $L_e - L_\mu - L_\tau$ breaking terms.

$$M^{(3)} = \begin{pmatrix}
\frac{-\delta_1^2}{\Delta} & m'_1 & m'_2 \\
-m'_1 & \frac{-\delta_2^2}{\Delta} & -\frac{\delta_2 \delta_3}{\Delta} \\
m'_2 & -\frac{\delta_2 \delta_3}{\Delta} & \frac{-\delta_3^2}{\Delta}
\end{pmatrix}$$

where $m'_{1,2}$ has new small contributions from the diagonalization (terms of the form $\frac{\delta \delta L}{\Delta}$). In order to fit the LSND results, we must have $(\delta_1 \delta_2/\Delta^2) \approx 0.03$. For $\Delta \approx 1\text{eV}$, this implies that roughly $\delta_i \approx 0.17$. Using this along with the sumrules of section 2, we immediately conclude that the model predicts $\Delta m^2 \approx 10^{-3} \text{eV}^2$, which is much too large a value. This simple version of the model is therefore in contradiction with data. If we want to fit the solar neutrino data in the context of such a model, then it would predict a maximum value for the $\nu_\mu - \nu_e$ transition probability at the level of $10^{-4}$. Unfortunately this is below the sensitivity of Mini-Boone [16] for detecting this process.
VI. CONCLUSION

In summary, we have outlined the predictions of the assumption that the neutrino mass matrix obeys a softly broken $L_e - L_\mu - L_\tau$ symmetry either in the neutrino or in the charged lepton sector. These predictions if confirmed will constitute evidence both for an inverted mass hierarchy among neutrinos as well as for the existence of an approximate $L_e - L_\mu - L_\tau$ symmetry, a new symmetry of physics beyond the standard model. It is interesting that these predictions can be tested in the current round of solar neutrino experiments such as KAMLAND and more definitively once future neutrino projects are able to improve the limits on the parameter $U_{e3}$. The presence or absence of such leptonic symmetries will clearly be of fundamental significance for physics beyond the standard model.

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Note added: After this paper was posted in the Archives, it was brought to our attention that the model presented in section (iv) of this paper was also discussed by W. Grimus and L. Lavoura in the paper now cited in Ref. [6]. It was also brought to our attention that mass matrices with inverted hierarchy were analyzed in [17].
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