Detection of anisotropic particles in levitated optomechanics

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We discuss the detection of an anisotropic particle trapped by an elliptically polarized focused Gaussian laser beam. We obtain the full rotational and translational dynamics, as well as, the measured photo-current in a general-dyne detection. As an example, we discuss a toy model of homodyne detection, which captures the main features typically found in experimental setups.

I. INTRODUCTION

Nanoparticles in optical traps are becoming increasingly interesting as they hold the promise of exploring quantum features at novel scales. Typical nanoparticles of mass $10^{-21}$ - $10^{-18}$ kg will push the classical-quantum boundary of exploration into the mesoscopic regime, improving by several orders on the mass $10^{-23}$ kg, which is the most massive object to have been shown to exhibit quantum interference [1]. Consequently, such systems can be used to test the superposition principle [2], as well as, for the detection of small forces [3–7].

The most direct approach to reach the quantum regime is to cool the system to the ground state in high vacuum [8–14]. This endeavour, which has proven to be non-trivial, has lead to a detailed analysis of the forces involved, namely light-matter interaction and gas collisions [9], as well as gravity [15]. The nanoparticle is often a small homogeneous sphere, which can be modelled as a polarized point particle in a harmonic trap, leading to a distinct harmonic motion for each of the three translational degrees of freedom.

However, it has been recently shown that a non-spherical nanoparticle, of a prefabricated shape, leads to interesting rotational [16–21] and librational motion [17]. Furthermore, these investigations have sparked the discussion of some novel ideas in levitated optomechanics, namely force-sensing using spinning objects [22–25], reaching the ground state of librational motion [26], and the generation of quantum superpositions of such rotational degrees of freedom [27]. Such anisotropic objects have three translational, as well as, three rotational degrees of freedom, where the latter ones, are commonly known as the rigid rotor. These have been studied extensively in both classical [28, 29] and quantum mechanics [30]. However, only recently has the investigation of the rotational degrees been extended to open quantum systems [31–36].

To realise such novel experiments, it is imperative to gain a detailed understanding of the rich dynamics a nanoparticle can exhibit: these motions can only be extracted through measurement. It is thus necessary to consider, not only the system dynamics, but also the detection method, i.e. the measurement apparatus, to give a complete description of an experiment. This can be already important for classical systems, where a measurement using a physical procedure will generally perturb a small system, but the two become even more intertwined in the quantum case, where each measurement will change the system and thus also its subsequent evolution. Moreover, when the system has several degrees of freedom, extracting the motion of a particular degree of freedom becomes a non-trivial exercise: the majority of the detection schemes rely on scattering from the trapped particle which invariably carries information on translational, rotational and librational motions, first coupled in a complicated motion, and then mapped into a scalar signal at the detector.

In this paper, building on the previous work, we investigate the rotational and translational (ro-translational) motion of such systems, namely that of an anisotropic polarized particle in an optical trap. We will consider light-matter interactions, namely the quantum analogue of the gradient, scattering forces and torques. Specifically, we will discuss the case of an elliptically polarized Gaussian beam, from which one can also recover the linear and circular polarizations as limiting cases. In addition, we consider, particle-gas collisions, modelled by extending the Caldeira-Leggett model to ro-translations.

The purpose of this work is twofold. The first goal is to give a detailed description of the rotational and translational motion under continuous monitoring. The second is to obtain the formula for the photo-current in a general dyne detection. This will open the door for the application of state estimation and manipulation techniques in ro-translational optomechanics already developed for other quantum systems [37, 38].

This paper is structured as follows. In Sec. II we describe the optomechanical system subject to light-matter interactions and gas collisions. In Sec. III we then obtain the quantum dynamics with and without laser monitoring.

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In Sec. IIIB we discuss the general dyne detection. In addition, we consider a toy model of homodyne detection, which captures the main features of typical experimental setups with mirrors and lenses. We write the conclusions in Sec. IV.

II. DESCRIPTION OF THE SYSTEM

A. Experimental setup

We consider the experimental setup of an optically levitated particle (see Fig. 1(a)). This experimental setup is generic enough to capture both cavity as well as free space experiments in levitated optomechanics. In a nutshell, a laser light is used to create an intense focal region inside a trapping chamber (vacuum chamber): once the particle is trapped at the focus, it will Rayleigh scatter light, which is collected and directed towards a detector. In this section we briefly introduce its main features using notions from classical electromagnetism mechanics. We discuss in detail their quantum counter-parts in the following sections.

We first discuss light-matter interactions. The incoming tightly focused light beam with a Gaussian profile creates an optical trap, which traps a nanoparticle near its focus point. This corresponds classically to the gradient force and torque. Moreover, the incoming light beam carries also linear and angular momentum. The linear momentum creates a radiation pressure scattering force which displaces the particle along the z axis, while the angular momentum carried by the photons is transferred to the particle, which starts to rotate, i.e. spinning or libration.

We next discuss collisions with the surrounding gas, which is a source of friction. Specifically, the gas of particles acts as a bath for the translational and rotational motions. In the simplest case we expect the particle to eventually reach an out of equilibrium steady state with the surrounding gas: the laser continuously transfers energy to the
particle, which is then dissipated into the gas. This results in a specific variance of the translational and librational degrees of freedom, or, in the case of spinning, an asymptotic angular frequency.

Both photon scattering and gas collisions are a source of diffusion: each random collision, either with a photon or with a gas particle, makes the particle recoil. Loosely speaking, the net effect of these collisions is a stochastic trajectory of the particle state (monitoring by the environment with unit efficiency). In addition, the interaction with photons, as well as with gas particles, couples the rotational and translational motion: only in some limiting cases the motions decouple.

There is however an important difference between photon scattering and gas collisions. Suppose that the characteristic length of the optically levitated particle is \( r_x \), denote the photon wavelength by \( \lambda \), and the wavelength associated to a gas particle by \( \lambda_g \approx \frac{2\pi h}{m_g k_b T} \) where \( T \) is the gas temperature, \( m_g \) is the mass of a gas particle, and \( k_b \) is Boltzman’s constant. For photon scattering we are in the long wavelength limit, while for gas collisions, for temperatures above \( ~1 \text{mK} \), we are in the short wavelength limit, i.e. \( \lambda_g < r_x < \lambda \). Thus we will model the optically levitated particle in two different ways: on the one hand, for photon scattering, we can approximate it as an anisotropic particle with six degrees of freedom, while, on the other hand, for gas collisions, we will model it initially as a many-body system. However, under some simplifying assumption, e.g. rigid body, the latter will also reduce to the anisotropic particle model with six degrees of freedom.

B. Free Hamiltonian

We model the optically levitated system as an anisotropic polarized particle with six degrees of freedom, i.e. three translational and three rotational. We denote the position and momentum operators by \( \hat{r} = (\hat{x}, \hat{y}, \hat{z})^T \) and \( \hat{p} = (\hat{p}_x, \hat{p}_y, \hat{p}_z)^T \), respectively, the angle operator by \( \hat{\phi} = (\hat{\phi}_x, \hat{\phi}_y, \hat{\phi}_z)^T \), where the three operators denote the quantized Euler angles in the \( z-y'-z'' \) convention, and the corresponding (angle) momentum operator by \( \hat{\pi} = (\hat{\pi}_x, \hat{\pi}_y, \hat{\pi}_z)^T \).

We consider the free Hamiltonian for translational and rotational degrees of freedom:

\[
\hat{H}_\text{free} = \frac{\hat{p}^T \hat{p}}{2M} + \frac{\hat{\pi}^T \hat{M}^{-1} \hat{\pi}}{2},
\]

where \( \hat{M} \) is the moment of inertia tensor in the principal axis (the body frame), \( \hat{\pi} = \hat{M}^{-1} \hat{\phi} \) is the angular momentum, \( \hat{\phi} \) is the matrix that maps \( \hat{\phi} \) to the angular frequency \( \hat{\omega} \) in the laboratory frame, i.e. \( \hat{\omega} = \hat{M}(\hat{\phi})\hat{\phi} \) (see Fig. 1(b)).

C. Light-matter coupling

The total electric field \( \hat{E} \) induces a dipole proportional to \( \propto \hat{\chi} \hat{E} \), where \( \hat{\chi} \) is the susceptibility tensor of the trapped particle, and we suppose that this induced field is coupled with \( \hat{E} \) by the usual dielectric coupling, i.e. \( \propto \hat{E}^\dagger \hat{\chi} \hat{E} \). Specifically, we start from the following interaction Hamiltonian:

\[
\hat{H}_\text{int} = -\frac{1}{2} \epsilon_0 \hat{E}^\dagger \hat{\chi} \hat{E} \hat{F}^\dagger \hat{F} \hat{E}^\dagger \hat{E},
\]

where \( \hat{E}(\hat{r}) \) is the total electric field, \( \epsilon_0 \) is the electric permittivity of free space, and \( \hat{\chi} = \text{diag}(\chi_1, \chi_2, \chi_3) \) is the electric susceptibility tensor in the body frame, i.e. \( \hat{\chi} = \hat{F} \hat{\chi} \hat{F}^\dagger \). We assume that \( \chi_j \) are \( \mathbb{R} \)-valued, i.e. we consider only photon scattering, neglecting absorption and emission.

The total electric field is given by

\[
\hat{E} = \hat{E}_d + \hat{E}_f
\]

where \( \hat{E}_d \) is the field that generates the optical trap and \( \hat{E}_f \) denotes the free electromagnetic field. Specifically, we consider

\[
\hat{E}_d = i E_0 (\epsilon_d \hat{u} \hat{a} - \epsilon_d^* \hat{a}^\dagger \hat{a}^\dagger),
\]

where \( E_0 \) is the electric field of the optical field.
where \( E_0 \) is the amplitude of the field, \( \mathbf{e}_d \) is the polarization vector, \( \hat{a}(\mathbf{r}) \) is the mode function, and \( \hat{a}^\dagger \) (\( \hat{a}^\dagger \)) the corresponding annihilation (creation) operator. In the following we will however limit the discussion to the case of a coherent beam and make the replacement \( \hat{a} \to a \), with \( a \), on the right hand-side of the arrow, a \( \mathbb{C} \)-number. Moreover, we will consider the case of elliptical polarization

\[
\mathbf{e}_d = \frac{1}{\sqrt{b_x^2 + b_y^2}} (b_x, i b_y, 0) ^\top,
\]

where \( b_x \), \( b_y \) are \( \mathbb{R} \)-valued, and \( \mathbf{e}_d^* \mathbf{e}_d = 1 \). More generally, in particular going beyond the paraxial approximation, one could consider also the case \( \mathbf{e}_d = \mathbf{e}_d(\mathbf{r}) \).

The free electromagnetic field, which forms a bath, is given by:

\[
\mathbf{E} = \sum_{n=1}^{2} \frac{\hbar}{2 V_0} (\mathbf{e}_{k,n}^\dagger \hat{\alpha}_{k,n} \mathbf{e}^{i k \cdot \mathbf{r}} - \mathbf{e}_{k,n}^\dagger \hat{\alpha}_{k,n} \mathbf{e}^{-i k \cdot \mathbf{r}})
\]

where \( \hat{\alpha}_{k,n} \) (\( \hat{\alpha}_{k,n}^\dagger \)) is the annihilation (creation) operator, \( V_0 \) is the quantization volume, \( \mathbf{e}_{k,n} \) is the polarization vector, \( k \) is the wave-vector, \( n \) denotes the two independent polarizations, \( \omega_k = c k \), and \( k = |k| \). Note that \( \mathbf{e}_{k,n} = \mathbf{e}_{k,n}^\dagger \) and \( \mathbf{e}_{k,n}^\dagger \mathbf{e}_{k,n} = 1 \), where \( \mathbf{n} \) is a unit vector in the direction of \( k \), i.e. \( k = k \mathbf{n} \). For more details about the decomposition in Eq. (6) see Appendix A. In the following we will also use the completeness relation:

\[
\sum_n (\mathbf{e}_{n,n})_i (\mathbf{e}_{n,n}^\dagger)_j = \delta_{ij} - \mathbf{n}_i \mathbf{n}_j.
\]

We consider the usual Hamiltonian contribution of the free electromagnetic field:

\[
H_f = \sum_{k,n} \frac{\hbar}{2 V_0} \mathbf{e}_{k,n}^\dagger \hat{\alpha}_{k,n} \mathbf{e}^{i k \cdot \mathbf{r}}
\]

We now use Eq. (5) in Eq. (2) from which we obtain two main contributions: the term \( \propto \mathbf{E}_f^\dagger \mathbf{F} \mathbf{e}^\dagger \mathbf{E}_d \), which gives rise to the unitary dynamics, and the term \( \propto \mathbf{E}_f^\dagger \mathbf{F} \mathbf{e}^\dagger \mathbf{E}_d \), which gives rise to the non-unitary dynamics, while we neglect \( \propto \mathbf{E}_f^\dagger \mathbf{F} \mathbf{e}^\dagger \mathbf{E}_f \), as we assume that the free-field modes are initially empty. Classically these correspond to the gradient and radiation pressure terms, respectively: we now discuss each of these separately.

1. Gradient terms

We consider the term \( \propto \mathbf{E}_f^\dagger \mathbf{F} \mathbf{e}^\dagger \mathbf{E}_d \), where \( \mathbf{E}_d \) is given in Eq. (4). Specifically, from Eqs. (2), (3), making the rotating wave approximation, we obtain the gradient potential:

\[
\dot{H}_{\text{grad}} = -\epsilon_0 V E_d^2 |\hat{a}|^2 (\mathbf{e}_d^\dagger)^\top \mathbf{F} \mathbf{e}^\dagger \mathbf{F} \mathbf{e}_d \hat{a} \hat{a} \dagger.
\]

We now assume that the field \( \mathbf{E}_d \) is coherent and make the replacement \( \hat{a} \to a \), where \( a \) on the right hand-side denotes a \( \mathbb{C} \)-value. We then use Eq. (5) in Eq. (9) to eventually obtain:

\[
\dot{H}_{\text{grad}} = -\frac{V \epsilon_0 E_d^2 |\hat{a}|^2 |a|^2}{2} (\mathbf{e}_d^\dagger)^\top \mathbf{F} \mathbf{e}^\dagger \mathbf{F} \mathbf{e}_d,
\]

where

\[
(\mathbf{e}_d^\dagger)^\top \mathbf{F} \mathbf{e} = b_x^2 (\chi_1 (\cos(\hat{\alpha}) \cos(\hat{\beta}) \cos(\hat{\gamma}) - \sin(\hat{\alpha}) \sin(\hat{\gamma}))^2 \\
+ \chi_2 (\cos(\hat{\alpha}) \cos(\hat{\beta}) \sin(\hat{\gamma}) + \sin(\hat{\alpha}) \cos(\hat{\gamma}))^2 + \chi_3 \cos^2(\hat{\alpha}) \sin^2(\hat{\beta})) \\
+ b_y^2 (\chi_1 (\sin(\hat{\alpha}) \cos(\hat{\beta}) \cos(\hat{\gamma}) + \cos(\hat{\alpha}) \sin(\hat{\gamma}))^2 \\
+ \chi_2 (\cos(\hat{\alpha}) \cos(\hat{\gamma}) - \sin(\hat{\alpha}) \cos(\hat{\beta}) \sin(\hat{\gamma}))^2 + \chi_3 \sin^2(\hat{\alpha}) \sin^2(\hat{\beta})).
\]
For $b_x = b_y$ we obtain circular polarization, while for $b_x = 0$ or $b_y = 0$ we obtain linear polarization along the $y$ or $x$ axis, respectively.

We now suppose that the beam is approximately Gaussian, focused in a relatively small volume. Specifically, we assume that the transverse region has a cross-section of size $\sigma_L$. In this case we have that

$$\epsilon_0 E_0^2 |u|^2 = \frac{P}{c \sigma_L},$$

where $P$ is the laser power, and $c$ is the speed of light.

For the mode function one usually considers the Gaussian beam:

$$\hat{u}(\hat{r}) = \frac{w_0}{w(\hat{z})} e^{-\frac{x^2 + y^2}{w(\hat{z})^2}} e^{i k \hat{z}},$$

where $w(\hat{z}) = w_0 \sqrt{1 + \left(\frac{\hat{z}}{z_0}\right)^2}$, $w_0$ is the beam waist, $k = \frac{2\pi}{\lambda}$, and $\lambda$ is the laser wavelength. More generally, one can consider an expansion of $|u|^2$ up to a given order $O(|\hat{r}|^n)$, where $n \in \mathbb{N}$, with $\frac{(n+2)(n+1)}{2}$ free parameters:

$$-\frac{VP}{2c \sigma_L} |\hat{u}|^2 = \sum_{k+l+m \leq n} c_{k,l,m} \hat{x}^k \hat{y}^l \hat{z}^m.$$ (14)

Using the Gaussian mode in Eq. (13), expanding up to order $n = 4$, we obtain the following terms: the $\hat{x}$ and $\hat{y}$ harmonic contributions ($c_{2,0,0} = c_{0,2,0} \propto \frac{2}{w_0}$), the $\hat{z}$ harmonic contribution ($c_{0,0,2} \propto \frac{1}{z_0}$), the $\hat{x}$ and $\hat{y}$ quartic terms ($c_{4,0,0} = c_{0,4,0} \propto \frac{2}{w_0^2}$), the $\hat{z}$ quartic term ($c_{0,0,4} \propto \frac{1}{z_0^2}$), and a coupling between $\hat{x}$ and $\hat{y}$ ($c_{2,2,0} \propto \frac{1}{w_0^2}$). To limit the number of free parameters one can also consider a slightly modified Gaussian case with asymmetry between $\hat{x}$ and $\hat{y}$ ($c_{2,0,0} \neq c_{0,2,0}$, $c_{4,0,0} \neq c_{0,4,0}$), and with nonzero couplings between $\hat{x}$ and $\hat{z}$ ($c_{2,0,2}$), and between $\hat{y}$ and $\hat{z}$ ($c_{0,2,2}$). In this case the mode function is given by:

$$\hat{u}(\hat{r}) = \frac{w_0}{w(\hat{z})} \exp \left( -\frac{a_1 \hat{x}^2 + a_2 \hat{y}^2}{w(\hat{z})^2} \right) e^{i k \hat{z}},$$

where $a_1$, $a_2$ are two adimensional parameters that quantify the asymmetry between $x$ and $y$.

2. Scattering terms

We consider the term $\hat{E} \hat{\chi} \hat{F} \hat{\chi} \hat{F}^\dagger \hat{E}_d$, where $\hat{E}_d$ and $\hat{E}_f$ are given in Eqs. (4) and (6), respectively. This term, after tracing out the free field degrees of freedom gives a decoherence term $[39]$. Specifically, from Eqs. (2), (3) we obtain the interaction Hamiltonian

$$\hat{H}_{\text{scattering}} = \sum_{k,\nu,\mu} \hat{B}_{k,\nu,\mu} \hat{S}_{k,\nu,\mu},$$

where

$$\hat{B}_{k,\nu,\mu} = \begin{cases} \hat{a}_{k,\nu}, & \text{for } \mu = 0, \\ \hat{a}_{k,\nu}^\dagger, & \text{for } \mu = 1, \end{cases}$$

are the bath operators, and

$$\hat{S}_{k,\nu,\mu} = \begin{cases} -i \frac{\hbar \omega_k}{2 \gamma_\nu \epsilon_0} e^{i k \hat{r}} c_{k,\nu}^\dagger (c_0 V_\nu \hat{F} \hat{\chi} \hat{F}^\dagger \hat{E}_d), & \text{for } \mu = 0, \\ i \frac{\hbar \omega_k}{2 \gamma_\nu \epsilon_0} e^{-i k \hat{r}} c_{k,\nu} (c_0 V_\nu \hat{F} \hat{\chi} \hat{F}^\dagger \hat{E}_d), & \text{for } \mu = 1, \end{cases}$$

(18)
are the system operators. We now make the formal replacement
\[
\sum_{k,\nu} \to \frac{V_0}{\pi(2\pi)^3} \int d\mathbf{k}
\]
and assume a zero temperature bath (corresponding to an initially empty bath):
\[
\langle \hat{a}_{k,\nu} \hat{a}^\dagger_{k',\nu'} \rangle = \langle \hat{a}^\dagger_{k,\nu} \hat{a}_{k',\nu'} \rangle = \langle \hat{a}^\dagger_{k,\nu} \hat{a}^\dagger_{k',\nu'} \rangle = 0,
\]
(19)
\[
\langle \hat{a}_{k,\nu} \hat{a}^\dagger_{k',\nu'} \rangle = \delta^{(3)}(k-k') \delta_{\nu,\nu'}.
\]
(20)
In the Born Markov approximation, assuming the particle degrees of freedom are not evolving during photon scattering (we assume that the incoming and scattered wavelengths are the same, i.e. Rayleigh scattering), making the rotating wave approximation, supposing that the field \(\hat{E}_d\) is coherent (we make the replacement \(\hat{a} \to a\), where \(a\) on the right hand-side denotes a \(\mathbb{C}\)-value), using Eq. (12) and Eqs. (16)-(20), we eventually obtain the Lindblad dissipator:
\[
\mathcal{L}_{\text{scattering}}[\cdot] = \gamma_s \sum_{\nu} \int d\mathbf{n} \left( \hat{A}_{n,\nu} \cdot \hat{A}_{n,\nu}^\dagger - \frac{1}{2} \left\{ \hat{A}_{n,\nu}^\dagger \hat{A}_{n,\nu}, \cdot \right\} \right),
\]
(21)
where
\[
\hat{A}_{n,\nu} = (\epsilon_{kn,\nu}^* \hat{F} \hat{F}^\dagger \epsilon_d) \hat{u}_{\mathbf{c}} + i \hbar \mathbf{k} \cdot \hat{r}_n
\]
(22)
and
\[
\gamma_s = \frac{\tilde{\sigma}_R}{\sigma_L} \frac{P}{\hbar \omega L}
\]
(23)
is the scattering rate. \(\mathbf{n}\) denotes the unit vector and \(\tilde{\sigma}_R = \frac{\pi^3 V_0^2}{\lambda^4}\) is an effective cross-section area. For the case of an isotropic polarized point particle, Eq. (21) reduces to the dissipator considered in [40, 41]: in particular, we also re-obtain the Rayleigh cross-section \(\sigma_R = \frac{24 \pi^3 V_0^2}{\lambda^4} \epsilon_R - \frac{1}{\epsilon_R + 2}\), where \(\epsilon_R\) is the dielectric function, by combing the factors contained in \(\tilde{\sigma}_R\) and \(\chi\). The case of linear rotors with linearly polarized light, and the case of arbitrary rotors with unpolarized light has been discussed in [32, 36] and [31], respectively.

D. Gas collisions

To account for the interaction with the gas of particles we suppose that the optically levitated particle is a many-body rigid system composed of \(n\) particles. Specifically, we model the effect of gas collisions on this system using the dissipative Caldeira-Leggett master equation [42, 43]:
\[
\mathcal{L}_{\text{collisional}}[\hat{\rho}] = \frac{i\gamma_c}{2\hbar} \sum_{j=1}^n \left[ \hat{\mathbf{r}}_n \cdot \hat{\mathbf{p}}_n + (\hat{\mathbf{r}}_n \cdot \hat{\mathbf{p}}_n)^\dagger, \hat{\rho} \right] + \frac{4mk_b T \gamma_c}{\hbar^2} \sum_{j=1}^n \left( \hat{\mathbf{L}}_n \cdot \hat{\mathbf{r}}_n \hat{\mathbf{L}}_n^\dagger \hat{\rho} - \frac{1}{2} \left\{ \hat{\mathbf{L}}_n \cdot \hat{\mathbf{L}}_n^\dagger, \hat{\rho} \right\} \right)
\]
(24)
where \(\hat{\mathbf{r}}_n\) and \(\hat{\mathbf{p}}_n\) are the position and momentum operators of particle \(n\), respectively, \(m\) is the mass of a single particle, \(\gamma_c\) is the collision rate (assumed for simplicity the same for each particle), \(k_b\) is Boltzmann constant, \(T\) is the temperature of the gas, and
\[
\hat{\mathbf{L}}_n = \hat{\mathbf{r}}_n + \frac{i\hbar}{4mk_b T} \hat{\mathbf{p}}_n
\]
(25)
We now change to the center-of-mass (c.m.) coordinates:
\[
\hat{\mathbf{r}}_j = \hat{\mathbf{r}} + \hat{\mathbf{r}}_j,
\]
(26)
\[
\hat{\mathbf{p}}_j = \frac{m}{M} \hat{\mathbf{p}} + \hat{\mathbf{p}}_j
\]
(27)
where $\hat{r}$, $\hat{p}$, $\hat{r}_n$, $\hat{p}_n$ are the c.m. position, c.m. momentum, relative position of $n$-th particle, relative momentum of $n$-th particle, operators, respectively, and $M = n m$ is the total mass. We now use Eqs. (26), (27), and the relations $\sum_{j=1}^n \hat{r}_j = 0$, $\sum_{j=1}^n \hat{p}_j = 0$, to decouple c.m. and relative degrees of freedom in Eq. (24):

$$L_{\text{collisional}}[\cdot] = L_{\text{collisional}}^{(t)}[\cdot] + L_{\text{collisional}}^{(r)}[\cdot],$$

where $L_{\text{collisional}}^{(t)}[\cdot]$ and $L_{\text{collisional}}^{(r)}[\cdot]$ denote the dissipator on translations and, as discussed below, rotations, respectively. Specifically, we find the following dissipator for translations:

$$L_{\text{collisional}}^{(t)}[\hat{\rho}] = \frac{i\gamma_c}{2\hbar} \left[ \hat{r} \cdot \hat{p} + (\hat{r} \cdot \hat{p})^\dagger, \hat{\rho} \right] + \frac{4Mk_bT}{\hbar^2} \gamma_c \left( \hat{L} \cdot \hat{\rho} \hat{L}^\dagger - \frac{1}{2} \left\{ \hat{L} \cdot \hat{L}^\dagger, \hat{\rho} \right\} \right),$$

where $\hat{L} = \hat{r} + \frac{i\hbar}{4Mk_bT} \hat{p}$. Under the assumption of a rigid body we eventually find the following dissipator for rotations:

$$L_{\text{collisional}}^{(r)}[\hat{\rho}] = \frac{4mk_bT}{\hbar^2} \gamma_c \sum_{\zeta=1}^3 \hat{D}_\zeta \left( \hat{C}_\zeta \cdot \hat{\rho} \hat{C}_\zeta^\dagger - \frac{1}{2} \left\{ \hat{C}_\zeta^\dagger \cdot \hat{C}_\zeta, \hat{\rho} \right\} \right),$$

where

$$\hat{C}_\zeta = F e_\zeta - \frac{i\hbar}{4k_bT} \hat{F} L_\zeta I^{-1} \hat{F}^\dagger (M^T)^{-1} \hat{\pi},$$

$e_\zeta$ is the unit vector along the $\zeta$-axis, $L_\zeta$ is the generator of rotations about the $\zeta$-axis, and

$$\hat{D}_\zeta = \left( \frac{1}{2} \text{tr} I - I_\zeta \right).$$

For later convenience, we also define the operators:

$$\hat{L}_j = \frac{i\sqrt{4Mk_bT}}{\hbar} \hat{L} \cdot e_j,$$

$$\hat{C}_{\zeta,j} = \frac{i\sqrt{4k_bT}}{\hbar} \hat{D}_\zeta \hat{C}_\zeta \cdot e_j.$$

E. Non-inertial terms

For completeness we also include the non-inertial term, which arises in Earth-bound laboratories. Specifically, we consider the following contribution to the Hamiltonian:

$$\hat{H}_{\text{ni}} = -M g \hat{x},$$

where $M$ is the total mass, and $g$ is the gravitational acceleration. Although the contribution from this term is typically much smaller than from light-matter interactions and gas collisions, it can become relevant in certain experimental settings [15] [16] [27].

III. DETECTION FOR RO-TRANSLATIONS

In this section we combine the terms from the previous section and discuss the resulting dynamics. In particular, we consider the unconditional dynamics, i.e. without a detector keeping track of the intensity gathered from the collected scattered photons, and the dynamics conditioned upon the measured intensity in a general dyne detection. We then apply the obtained formulae to construct to a toy model of homodyne detection.
A. Dyne detection

The dynamics of the optically levitated particle is given by:

$$\dot{\rho} = -\frac{i}{\hbar} [\hat{H}_{\text{free}} + \hat{H}_{\text{gradient}} + \hat{H}_{\text{ni}}, \rho] + \mathcal{L}_{\text{scattering}}[\rho] + \mathcal{L}_{\text{collisional}}[\rho]$$  \hspace{1cm} (36)

where $\hat{H}_{\text{free}}$, $\hat{H}_{\text{grad}}$, $\hat{H}_{\text{ni}}$, $\mathcal{L}_{\text{scattering}}[\cdot]$, and $\mathcal{L}_{\text{collisional}}[\cdot]$ are defined in Eqs. (1), (10), (35), (21), and (28), respectively. We will refer to Eq. (1) as the unconditional dynamics, and to the state $\rho$ as the unconditional state.

However, usually one collects part of the scattered light, either with the same laser that keeps the particle suspended, or by a second detection laser. Here we will consider the former case \cite{41}, but the latter case can be analyzed in a similar fashion (see Fig. 1). Anyhow, the detected photo-current (signal) allows to continuously update the description or by a second detection laser. Here we will consider the former case \cite{41}, but the latter case can be analyzed in a similar fashion (see Fig. 1). Anyhow, the detected photo-current (signal) allows to continuously update the description of the system: we will refer to resulting state $\hat{\rho}_C$ as the conditional state. Mathematically we can describe this by considering an unraveling of the photon scattering term $\mathcal{L}_{\text{scattering}}[\hat{\rho}]$ in Eq. (36). The most general diffusive unraveling, also known as the Belavkin equation, is given by (in Itô form) \cite{11, 15}:

$$d\hat{\rho}_C = \gamma_s \sum_{\nu=1}^{2} \int d\mathcal{D}[\hat{A}_{\nu,\nu}]\hat{\rho}_C dt + \frac{\sqrt{\gamma_s}}{2} \sum_{\nu=1}^{2} \int d\mathcal{H}[\hat{A}_{\nu,\nu} dW_{\nu,\nu}]\hat{\rho}_C,$$  \hspace{1cm} (37)

where \cite{46}

$$\mathcal{D}[\hat{K}] = \hat{K} \cdot \hat{K}^\dagger - \frac{1}{2} \left\{ \hat{K}^\dagger \hat{K}, \cdot \right\},$$

$$\mathcal{H}[\hat{K}] = \hat{K} \cdot + \cdot \hat{K}^\dagger - \text{tr}[\hat{K} \cdot + \cdot \hat{K}^\dagger].$$  \hspace{1cm} (38, 39)

and $\hat{K}$ denotes an operator. Note that the first term on the right hand-side of Eq. (37) corresponds to $\mathcal{L}_{\text{scattering}}[\hat{\rho}]$. $W_{\nu,\nu}$ are $\mathbb{C}$-valued, zero mean Wiener processes with correlations:

$$\mathbb{E}[dW_{\nu,\nu} dW^*_{\nu',\nu'}] = dt\delta_{(\nu,\nu), (\nu',\nu')} \delta(2)(n - n'),$$

$$\mathbb{E}[dW_{\nu,\nu} dW_{\nu',\nu}] = dt\Xi_{(\nu,\nu), (\nu',\nu')},$$  \hspace{1cm} (40, 41)

where $\delta(2)$ is positive semi-definite. The photo-currents associated to Eq. (37) are given by:

$$J_{\nu,\nu} dt = \text{tr} \left[ \sum_{\nu' = 1}^{2} \int d\nu' \left( \eta_{(\nu,\nu), (\nu',\nu')} \hat{\rho}_{n',\nu'} + \Xi_{(\nu,\nu), (\nu',\nu')} \hat{\rho}_{n',\nu'} \right) \right] + dW_{\nu,\nu}.$$  \hspace{1cm} (42)

One can also invert Eq. (43) to obtain the expression of $dW_{\nu,\nu}$ as a function of the measured photo-current $J_{\nu,\nu}$, i.e. $dW_{\nu,\nu}(J_{\nu,\nu})$. Inserting $dW_{\nu,\nu}(J_{\nu,\nu})$ in Eq. (37) one then explicitly shows that $\hat{\rho}_C$ is the state conditioned on the photo-currents $J_{\nu,\nu}$. The full conditional dynamics can be obtained by adding the Hamiltonian terms and $\mathcal{L}_{\text{collisional}}[\hat{\rho}] dt$ to the right hand-side of Eq. (37). Discontinuous unravellings, where each photon triggers a discontinuous update of the conditional state, could be treated in a similar way.

In general, the currents $J_{\nu,\nu}$ are $\mathbb{C}$-valued and thus cannot be directly associated to the intensity current measured by a physical detector; these can be reconstructed from the $\mathbb{R}$-valued currents $\text{Re}(J_{\nu,\nu})$ and $\text{Im}(J_{\nu,\nu})$. In the next section we consider the case of homodyne detection, which is a special case of the formalism used in this section, where we obtain explicit expression for the physical photo-currents.
B. Homodyne detection model

In order to discuss a detection model we have to specify the measuring operator(s). In general, the measuring operator will be a functional of the system degrees of freedom as well as of the experimental setting, i.e. $A(\hat{r}, \hat{\phi}; \text{exp. setting})$. For example, only some of the scattered photons are collected by optical elements: these are then recorded by a physical detector, where the detector’s efficiency, orientation, distance, size, and integration time, all affect the measured signal. Here we consider a simplified detector model, completely characterized by the operator $\sqrt{\eta \gamma} \sum_{\nu=1}^{2} \int_{S} dn \hat{A}_{n,\nu}$, where $S$ denotes the surface of a toy detector, $\gamma_{s}$ is defined in Eq. (23), and $\eta$ is the detector’s efficiency (see Fig. 1(b)). In this case, as we show below, the total photo-current is of the form $\sum_{\nu} \int_{S} dn J_{n,\nu}$, where $J_{n,\nu}$ is associated to $A_{n,\nu}$.

This total photo-current, which we label as $J$, can be considered as a toy model for the experimental configuration in Fig. 1. Loosely speaking, optical elements, such as a paraboloidal mirror, collect the scattered photons and direct them towards the beam splitter: this conceals, at least partially, the information about the scattering direction $n$ and polarization $\nu$. We denote the annihilation operator for the corresponding collective mode by $\hat{a}_{\text{out}}$, i.e. the annihilation operator of all the photons travelling towards the detector. At the beam splitter the signal from the scattered photons is combined with the local oscillator $a_{\text{LO}}$ (a $\mathbb{C}$-value) from which we obtain the current $J$ (see Fig. 1(c)). Here we are supposing that the local oscillators $(a_{n,\nu})_{\text{LO}}$ for each direction $n$ and polarization $\nu$, can be approximated by a single local oscillator $a_{\text{LO}}$. To obtain a more refined model of detection in this specific experimental situation, or to adapt it to describe a different experimental setup, one would need to take into account the specific details of the experiment and repeat the analysis, e.g. by imposing the specific boundary conditions.

We can now apply the general procedure discussed in the previous Sec. IIIA. Specifically, for each dissipator term $D(A_{n,\nu})$ we have to consider the corresponding noise term $H(A_{n,\nu},dW_{n,\nu})$, where we assume that $W_{n,\nu}$ are $\mathbb{R}$-valued and independent, since they are associated to different modes. Moreover, we assume that each mode is detected with the same efficiency $\eta$. This simplifies Eqs. (40), (41) to $\mathbb{E}[dW_{n,\nu}dW_{n',\nu}^{*}] = \eta dt \delta_{\nu,\nu'} \delta^{(2)}(n - n')$. It is then straightforward to obtain the equation for the conditional state (in Itô form):

$$
d\hat{\rho}_{c} = -\frac{i}{\hbar} [\hat{H}_{\text{free}} + \hat{H}_{\text{gradient}} + \hat{H}_{\text{ni}}, \hat{\rho}_{c}] dt + \gamma_{s} \sum_{\nu=1}^{2} \int dn D[\hat{A}_{n,\nu}]\hat{\rho}_{c} dt + \sqrt{\eta \gamma} \sum_{\nu=1}^{2} \int_{S} dn \hat{A}_{n,\nu}\hat{\rho}_{c} dW$

$$+ \gamma_{c} \sum_{j=1}^{3} \hat{D}[\hat{L}_{j}]\hat{\rho}_{c} dt + \gamma_{c} \sum_{\zeta,j=1}^{3} \hat{D}[\hat{C}_{\zeta,j}]\hat{\rho}_{c} dt.$$  \hspace{1cm} (44)

$W$ is a zero mean, $\mathbb{R}$-valued Wiener process with correlation

$$\mathbb{E}[dW dW] = 2\Omega dt,$$  \hspace{1cm} (45)

where $\Omega = \int_{S} dn$, and the factor 2 reflects the fact that both independent polarizations are detected. Using Eq. (43), summing all the currents, we finally obtain that the state $\hat{\rho}_{c}$ in Eq. (44) is conditioned on the following photo-current:

$$J = \sqrt{\eta \gamma} \text{Tr} \left[ \hat{A}_{n,\nu} + \hat{A}_{n,\nu}^{\dagger} \right] \hat{\rho}_{c} + dW.$$  \hspace{1cm} (46)

We recover Eq. (36) from Eq. (44) either by taking the expectation value $\mathbb{E}[\cdot]$ over the noise realizations, or by considering a completely inefficient detection, i.e. $\eta = 0$.

The above derivation (in the the Schrödinger picture), on the one hand, has the advantage that it clearly shows the effect of photon detection on the nanoparticle, i.e. Eqs. (43) and (46), on the other hand, it does not provide an intuitive picture of the interaction between the photons and the nanoparticle. In particular, the scattered photon and the nanoparticle become entangled during the interaction. This becomes more apparent in Heisenberg picture using the input-output formalism \[47\]. Specifically, we have that:

$$(\hat{a}_{n,\nu})_{\text{out}} = (\hat{a}_{n,\nu})_{\text{in}} + \sqrt{\eta \gamma_{s}} \hat{A}_{n,\nu},$$  \hspace{1cm} (47)

where $(\hat{a}_{n,\nu})_{\text{in}}$ and $(\hat{a}_{n,\nu})_{\text{out}}$ denote the bath operators before and after the interaction with the nanoparticle, respectively (for more details see for example \[37\], \[48\]). Assuming now again homodyne detection, we add a local oscillator to
the output field, i.e. \( (\hat{a}_{n,\nu})_{out} \rightarrow (\hat{a}_{n,\nu})_{out} + (a_{n,\nu})_{LO} \), where \((a_{n,\nu})_{LO}\) is a \(\mathbb{R}\)-valued local oscillator, and we consider:

\[
\hat{I}_{n,\nu} = ((\hat{a}_{n,\nu})_{out} + (a_{n,\nu})_{LO})^\dagger ((\hat{a}_{n,\nu})_{out} + (a_{n,\nu})_{LO}).
\]  (48)

The statistics of the photo-current \(J\) in Eq. (46) can then be recovered by considering (see Fig. [1]):

\[
\hat{J} = \sum_{\nu} \int_S dn \lim_{(a_{n,\nu})_{LO} \to \infty} \left[ \hat{I}_{n,\nu} - (a_{n,\nu})_{LO}^2 \right],
\]  (49)

which is a consequence of the fact that \((\hat{a}_{n,\nu})_{in}\) and \(dW_{n,\nu}\) have the same statistical properties \[37, 47, 48\].

It is useful to derive approximate photo-currents for a classical nanoparticle, e.g. for force sensing applications. Specifically, we replace quantum observables \(\hat{O}\) by their corresponding classical observables \(O\), and the commutators with Poisson Brackets, i.e \([\cdot, \cdot] \rightarrow i\hbar\{\cdot, \cdot\}_PB\). In addition, we suppose that \(k \cdot r \ll 1\), i.e. we use \(e^{ik \cdot r} \approx 1 + ik \cdot r + \mathcal{O}((k \cdot r)^2)\), and we write for convenience \(\chi = \chi_0(\mathcal{I} + \Delta \chi)\), where \(\chi_0\) is the susceptibility in the limit of an isotropic particle, and \(\mathcal{I}\) denotes the \(3 \times 3\) identity matrix. Using these expressions we can decompose the expectation value of the photo-current in Eq. (46) in four parts:

\[
E[J] = E[J_0 + J_T + J_R + J_{RT}],
\]  (50)

where \(J_0\) is a constant term, and

\[
J_T = 2k\sqrt{\eta s} \left( \int_S dn Re \left[ i \left( \sum_{\nu=1}^{2} e^{\dagger}_{k n,\nu} \epsilon_d \right) n \right] \cdot r \right)
\]  (51)

\[
J_R = 2\sqrt{\eta s} \left( \int_S dn Re \left[ \sum_{\nu=1}^{2} e^{\dagger}_{k n,\nu} F(\phi) \Delta \chi F(\phi)^\dagger \epsilon_d \right] \right)
\]  (52)

\[
J_{RT} = 2k\sqrt{\eta s} \left( \int_S dn Re \left[ i \left( \sum_{\nu=1}^{2} e^{\dagger}_{k n,\nu} F(\phi) \Delta \chi F(\phi)^\dagger \epsilon_d \right) n \right] \cdot r \right)
\]  (53)

denote the purely translational, purely rotational, and the ro-translational correction currents, respectively. This expressions can be used, as an alternative way of obtaining the conversion between the measured intensity and the nanoparticle position and orientation, given by the position \(r\) and angle \(\phi\) vector, respectively.

### IV. SUMMARY

We have discussed the motion and detection of optically levitated nanoparticles. Specifically, we have considered an anisotropic particle trapped in an elliptically polarized Gaussian beam, and immersed in a bath of gas particles. We have first introduced the dynamics of such systems using notions of classical electromagnetism and mechanics: the resulting ro-translational motion is driven (photon scattering), damped (gas particle collisions), as well as diffusive (photon scattering and gas particle collisions). We have then derived the complete quantum dynamics and discussed in detail the detection of the nanoparticle. Specifically, under the Born-Markov assumption we have obtained the unconditional dynamics and the dynamics conditioned upon a general dyne measurement. We have discussed the relation between the photo-currents, the measuring operators, and the dynamics both in the Schrödinger, as well as in the Heisenberg picture. We have illustrated the use of the general formulae by constructing a toy model of homodyne detection. We have obtained approximate formulae, which could be used to extract the nanoparticle position and orientation from the measured signal.

### ACKNOWLEDGMENTS

We also like to thank A. Bassi, M. Carlesso, and G. Gasbarri for discussions. We wish to thank for research funding, The Leverhulme Trust and the Foundational Questions Institute (FQXi). This project has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement No 766900. We also acknowledge support by the EU COST action QTSpace (CA15220).
Appendix A: Polarization of scattered light

In this section we briefly discuss the decomposition in Eq. (6). Consider a fixed scattering direction $\mathbf{n}$ and the orthogonal plane described by the tensor $\sum_\nu \mathbf{e}_{n,\nu} \otimes \mathbf{e}_{n,\nu}^*$, i.e. the completeness relation in Eq. (7). We consider two orthogonal axis in this plane, which we denote by $x$ and $y$, and the corresponding unit vectors along these axis, which we denote by $\mathbf{e}_x$ and $\mathbf{e}_y$, respectively. Moreover, we require that $\mathbf{e}_x$, $\mathbf{e}_y$ and $\mathbf{n}$ form the directions of a right-handed coordinate system.

In this coordinate system we can consider different decompositions. Particularly simple is the linear decomposition:

$$\sum_\nu \epsilon_{n,\nu} \hat{a}_{k,\nu} = e_x \hat{a}_{k,x} + e_y \hat{a}_{k,y},$$

(A1)

where $\hat{a}_{k,x}$, $\hat{a}_{k,y}$ denote annihilation operators for photons with polarizations along $x$ and $y$, respectively. Alternatively, we can consider the circular decomposition:

$$\sum_\nu \epsilon_{n,\nu} \hat{a}_{k,\nu} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ 0 & 0 \end{pmatrix} \hat{a}_{k,R} + \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ 0 & 0 \end{pmatrix} \hat{a}_{k,L},$$

(A2)

where $\hat{a}_{k,L}$, $\hat{a}_{k,R}$ denote annihilation operators for left and right photons, respectively. Comparing the two expressions in Eqs. (A1) and (A2) we find:

$$\hat{a}_{k,L} = \frac{\hat{a}_{k,x} + i \hat{a}_{k,y}}{\sqrt{2}},$$

(A3)

$$\hat{a}_{k,R} = \frac{\hat{a}_{k,x} - i \hat{a}_{k,y}}{\sqrt{2}}.$$

(A4)

Similarly, one could also consider other decompositions, such as the elliptical, and find the decomposition of corresponding annihilation operators in terms of the annihilation operators for linearly polarized photons.

To fully specify the decomposition in expression in Eq. (6), one would need to apply this procedure for each direction $\mathbf{n}$. However, any decomposition is valid, as physical quantities are independent of the chosen decomposition, and thus the chosen one is a matter of convenience.

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