Scaling behaviour of a scalar field model of dark matter halos

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ABSTRACT
Galactic dark matter is modelled by a scalar field. In particular, it is shown that an analytically solvable toy model with a non–linear self–interaction potential $U(\Phi)$ leads to dark halo models which have the form of quasi–isothermal spheres. We argue that these fit better the observed rotation curves of galaxies than the centrally cusped halos of standard cold dark matter. The scalar field model predicts a proportionality between the central densities of the dark halos and the inverse of their core radii. We test this prediction successfully against a set of rotation curves of low surface brightness galaxies and nearby bright galaxies.

Key words: galaxies: kinematics and dynamics

1 INTRODUCTION
The evidence for dark matter is overwhelming, although its nature is not clear. Most promising seems at present the concept of cold dark matter. However, this is flawed on galactic scales. One of its major difficulties is that models of cold dark matter halos of galaxies show inner density cusps, $\rho \propto r^{-\alpha}$, with $\alpha$ in the range 0.5 to 1.5 (Navarro, Frenk & White 1997, hereafter referred to as NFW, Moore et al. 1999), which are not consistent with observed rotation curves of galaxies (de Blok, McGaugh, & Rubin 2001, de Blok & Bosma 2002, hereafter referred to as dBMGR&dBB) much better than cusped density profiles. Besides this interesting density profile, MSP’s model predicts a scaling law between the central densities of the dark halos and their core radii which we investigate here. For this purpose we recapitulate briefly the field theoretical approach in the next section and discuss then the predicted scaling law using decompositions of the rotation curves of a set of nearby bright and low surface brightness galaxies.

2 SCALAR FIELD WITH A $\Phi^6$ SELF–INTERACTION POTENTIAL
MSP chose ad hoc for the scalar field an analytically solvable toy model (Mielke 1978) with a $\Phi^6$ type self–interaction potential,

$U(\Phi) = m^2 |\Phi|^2 \left(1 - \chi |\Phi|^4\right), \quad \chi |\Phi|^4 \leq 1$, \hspace{1cm} (1)

where $m$ is a tiny ‘bare’ mass of the scalar field and $\chi$ a coupling constant. Both characterize the hypothetical scalar field and are thought of as constants of nature. The self–interaction in the radial Klein–Gordon equation takes the form $dU(P)/dP^2 = m^2 - 3m^2 \chi P^4$, where $P = \Phi e^{-imt}$. For a spherically symmetric configuration the corresponding non–linear Klein–Gordon equation simplifies to an Emden type equation

$P'' + \frac{2}{x}P' + 3xP^5 = 0$, \hspace{1cm} (2)

familiar from the theory of gaseous spheres. It has the completely regular exact solution

$P(r) = \pm \chi^{-1/4} \sqrt{\frac{A}{1 + A^2 x^2}}$, \hspace{1cm} (3)

where the dimensionless radial coordinate $x = mr$ has been introduced and $A = \sqrt{P^2(0)}$ is related to the central value.
The solution depends essentially on the non-linear coupling parameter $\chi$, since the limit $\chi \to 0$ would be singular. This feature is rather characteristic for soliton solutions. In the following we restrict ourselves to the range $A \leq 1$ for which the potential $U(|\Phi|)$ remains positive.

The canonical energy–momentum tensor of a relativistic spherically symmetric scalar field is diagonal, i.e. $T^\mu_\nu(\Phi) = \text{diag}(\rho, -p_r, -p_\perp, -p_\perp)$ with

$$\rho = \frac{1}{2} \left( m^2 P^2 + P^2 + U \right),$$

$$p_r = \rho - U,$$

$$p_\perp = p_\perp - P^2,$$

where the prime indicates a radial derivative. The form (4) is familiar from perfect fluids, except that the radial and tangential pressures generated by the scalar field are in general different, i.e. $p_r \neq p_\perp$. The scalar field proposed here is not interacting by self-gravity but exerts a gravitational force. From (4) MSP find in flat spacetime the energy–density

$$\rho = \frac{m^2}{2} \left[ 2P^2 + P^2 - \chi P^6 \right] = \frac{Am^2}{\sqrt{1+(A^2 x^2)^2}} \left[ 1 + \frac{A^4 x^2 - A^2}{2(1+A^2 x^2)^2} \right].$$

The leading term of the Newtonian type mass concentration (5) is exactly the density law of the quasi–isothermal sphere

$$\rho(r) \simeq \frac{\rho_0 r_c^2}{r_c^2 + r^2}.$$ (6)

At large radii the density falls of like $\rho \propto r^{-2}$ which corresponds to an asymptotically flat rotation curve. Comparing with the MSP model, the central density of the quasi–isothermal sphere is given by $\rho_0 \simeq Am^2/\sqrt{A}$ and the core radius is $r_c \simeq 1/mA$. This implies a scaling law for the dark halos of the form

$$\rho_0 \simeq \frac{m}{\sqrt{A} r_c} \propto \frac{1}{r_c},$$ (7)

where $A$, which may vary from halo to halo, cancels out.

Finally we note that this non-linearly coupled scalar field exerts the radial and tangential pressures

$$p_r = \frac{m^2}{2} \left[ \chi P^6 + P^2 \right] = \frac{A^3 m^2}{2\sqrt{1+A^2 x^2})^2} \simeq \frac{A^3 m^2}{2\sqrt{A}},$$

$$p_\perp = \frac{m^2}{2} \left[ \chi P^6 - P^2 \right] = \frac{A^3 m^2(1-A^2 x^2)}{2\sqrt{1+A^2 x^2})^3} \simeq \frac{A^3 m^2}{2\sqrt{A}},$$

respectively. Thus, at the center the pressure is isotropic, $p_r(0) = p_\perp(0)$, whereas asymptotically at infinite radius

$$p_r, -p_\perp \rightarrow \frac{m^2}{2\sqrt{A} x^2}.$$ (9)

### 3 ASTRONOMICAL TESTS

We have tested the theoretical model predictions against rotation curve data of a set of low surface brightness galaxies taken from dBMGR&dBB. The authors have measured high–resolution rotation curves of in total 54 galaxies. For about half of them surface photometry is available. For these galaxies the authors do not provide only kinematical data, but have also constructed dynamical models of the galaxies.

The observed rotation curves are modeled as

$$v_c^2(R) = v^2_{\text{bulge}}(R) + v^2_{\text{disc}}(R) + v^2_{\text{gas}}(R) + v^2_{\text{halo}}(R),$$ (10)

where $v_{\text{bulge}}$, $v_{\text{disc}}$, $v_{\text{gas}}$, and $v_{\text{halo}}$ denote the contributions due to the bulge, the stellar disc, the interstellar gas, and the dark halo, respectively. The radial variations of $v_{\text{bulge}}(R)$, $v_{\text{disc}}(R)$, and $v_{\text{gas}}(R)$ were derived from the observations, while the normalizations by the mass–to–light ratios were left as free parameters of the fits of the mass models to the data. Fits of the form (10) to observed rotation are notoriously ambiguous. Thus, dBMGR&dBB provide for each galaxy several models, one with zero bulge and disc mass, one model with a ‘reasonable’ mass–to–light ratio of the bulge and the disc, and finally a ‘maximum–disc’ model with bulge and disc masses at the maximum allowed by the data. Furthermore these authors try for each galaxy two types of dark halo models. One is the cusped NFW density law and the second is the quasi–isothermal sphere. While varying the disc contribution to the observed rotation curve leads to fits of the same quality, dBMGR&dBB find that the quasi–isothermal sphere models of the dark halos give significantly better fits to the data than the cusped NFW density law. Thus the scalar field model presented here is in this aspect even superior to the cold dark matter model in its present form.

Prada et al. (2003) have attempted using SDSS data on satellite galaxies of isolated host galaxies to probe on 100 kpc scale the outer halo mass distributions. They find that the line–of–sight velocity dispersions of the satellites follow closely the radially declining velocity dispersion profile of halo particles in a NFW halo. This implies an outer mass density distribution of the form $\rho \propto r^{-3}$ which is at variance with the prediction of the quasi–isothermal sphere (6). In the cold dark matter model the system of satellite galaxies is assembled during the same accretion processes as the dark halo, and Prada et al. (2003) assume consistently for the satellites the same distribution function in phase space as for the halo particles. In the scalar field model, however, the dark halo provides for the baryonic matter simply a Newtonian force field. The distribution function of satellite galaxies in phase space is thus not specified and can be modelled according to the observations, even if the potential trough of the quasi–isothermal sphere has a shallower profile than a NFW halo.

Next we examine the predicted scaling relation (7). A relation of this type was found empirically by Salucci & Burkert (2000), although this was based on the universal rotation curve model of Persic, Salucci & Stel (1996) and the, also empirically derived, halo density profile of Burkert (1995). This density law resembles the quasi–isothermal sphere in that it has also a homogeneous core. In Fig. 1 we show central densities versus the inverses of the core radii of quasi–isothermal dark halo models constructed by dBMGR&dBB assuming for the discs mass–to–light ratios consistent with current population synthesis models. There is despite some scatter a clear correlation between $\rho_0$ and $r_c^{-1}$ over several orders of magnitude, $\log(\rho_0) \propto (1.46 \pm 0.55) \log(r_c^{-1})$. Thus the dark halo model data seem to confirm statistically the scaling relation (7). The scatter in the correlation diagram is probably due to the near degeneracy of the fits to the observed rotation curves, even if the
Figure 1. Central densities versus the inverses of the core radii of dark halos of low surface brightness galaxies derived from modelling their rotation curves. The halo models are constructed assuming ‘realistic’ mass–to–light ratios for the discs. The solid line is the predicted $\rho_0 \propto r_c^{-1}$ relation.

Figure 2. Same as Fig. 1, but the halo models are constructed assuming ‘maximum discs’. Open symbols: low surface brightness galaxies, filled symbols: nearby bright galaxies.

We believe, however, that this has not changed the general trend. Using arguments of the density wave theory of galactic spiral structure, one of us has pointed out, judging from the implied internal dynamics of the galactic discs, that the discs might be near to maximum (Fuchs 2002, 2003a, b). This would imply for the discs of some galaxies mass–to–light ratios which are significantly higher than in current population synthesis models of the discs of low surface brightness galaxies. These can be modified, though, to yield higher mass–to–light ratios (Lee et al., in preparation). Therefore we show in Fig. 2 the central densities versus the inverses of the core radii of the dark halo models, if they are constructed assuming ‘maximum discs’. Although the dark halo parameters are shifted to other values, the linear correlation persists. With $\log(\rho_0) \propto (1.08 \pm 0.39) \log(r_c^{-1})$ the dark halo models fit nearly ideally to the predicted scaling relation (7). Included into Fig. 2 are also ‘maximum disc’ dark halo parameters of nearby bright galaxies (Fuchs 1999, 2003a, Fried & Fuchs, in preparation) which fit also well to the scaling law. We conclude from this discussion that the astronomical tests are rather encouraging for the scalar field model of dark halos presented here. What is missing at present, is of course a cosmological setting of the scalar field model. We hope to address this and, in particular, the question of large scale structure formation, which is on the other hand successfully described by cold dark matter cosmology, in the future.

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