Bayesian Promised Persuasion: Dynamic Forward-Looking Multiagent Delegation with Informational Burning

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Abstract

This work studies a dynamic mechanism design problem in which a principal delegates decision makings to a group of privately-informed agents without the monetary transfer or burning. We consider that the principal privately possesses complete knowledge about the state transitions and study how she can use her private observation to support the incentive compatibility of the delegation via informational burning, a process we refer to as the looking-forward persuasion. The delegation mechanism is formulated in which the agents form belief hierarchies due to the persuasion and play a dynamic Bayesian game. We propose a novel randomized mechanism, known as Bayesian promised delegation (BPD), in which the periodic incentive compatibility is guaranteed by persuasions and promises of future delegations. We show that the BPD can achieve the same optimal social welfare as the original mechanism in stationary Markov perfect Bayesian equilibria. A revelation-principle-like design regime is established to show that the persuasion with belief hierarchies can be fully characterized by correlating the randomization of the agents’ local BPD mechanisms with the persuasion as a direct recommendation of the future promises.

1 Introduction

Building efficient rational multi-agent system is an important research agenda in Artificial Intelligence. In many application domains, a system designer (principal, she) aims to optimize the performance (e.g., social welfare or revenue) of a system in which multiple self-interested agents actively behave. The principal can use mechanism design approaches to determine how agents should strategically interact with each other. She can also influence the agents’ decision makings by restricting their discretion (delegation) or limiting their knowledge about the system (persuasion).

The key component of the system design problem is to provide incentives that align the interests of participants. Information asymmetry between the principal and the agents, however, imposes challenges for the incentive design. Mechanism design literature studies two forms of information asymmetry: unobserved preferences and unknown actions which leads to adverse selection (see, e.g., [Myerson, 1981; Lobel and Leme, 2019]) and moral hazard (see, e.g., [Bohren, 2019; Khalili et al., 2017]), respectively.

Monetary value has been long explored in designing incentive compatible (IC) mechanisms. There are two forms of monetary value: transfer and burning. Monetary transfer is also referred to as payment that is paid by one party to the other. For example, in auctions, the winning bidder has to pay for the items to the auctioneer. Hence, the money reduced from the bidder directly benefits the auctioneer. Monetary burning, on the other hand, captures the setting when decisions are costly to one party but with no direct benefits to the other. For example, the actions in delegation problems may incur a certain amount of wasteful expenditure (i.e., money burned) such as consumption of natural resources that may reduce the welfare of both the agents and the principal. However, in certain scenarios, monetary value may be improper. For example, it is infeasible for organizations to use monetary incentive to efficiently allocate internal resources (e.g., vehicles, conference rooms). In some cases, using monetary value may be controversial (e.g., medical or humanitarian-aid resource allocations) or constrained (incentive mechanism may violates the financial constraints.)

Information also plays an essential role in aligning the agents’ behaviors with the principal’s desire. Bayesian persuasion ([Kamenica and Gentzkow, 2011]) or information design studies how a principal can incentivize agents to behave in her favor through strategic information provision. Instead of using monetary transfer or burning, we consider a process known as informational burning to dynamically support the IC of the delegation design. In particular, we consider an adverse selection problem in a dynamic environment and study incentive compatible (IC) delegation mechanisms, in which a socially maximizing principal repeatedly delegates the decision makings to the agents by choosing an action menu, knowing that each agent will privately observe a payoff-relevant type and then take an action from this menu. The dynamic of the environment is driven by the transitions of the state of the world (state) in a Markovian fashion where agents’ current actions have persistence on future states. The Markovian transition kernel of the state is parameterized by
a variable known as shock. We assume that only the principal observes the shock and then controls what and how each agent gets to know about the shock—information burned—in each period to influence the agents’ decisions in each period. In addition to the delegation rule, the principal also commits to a signaling rule profile that performs Bayesian persuasion to periodically inform the agents about the shock by sending signals in each period. The agents make current decisions by considering the future expected payoffs. As a result, the principal’s delegation design must dynamically capture the signals to periodically inform the agents about the shock by sending a variable known as a signaling rule profile that performs Bayesian persuasion to periodically inform the agents about the shock by sending signals in each period. The agents make current decisions by considering the future expected payoffs. As a result, the principal’s delegation design must dynamically capture the signals to periodically inform the agents about the shock by sending a variable known as 

Since the seminal work of [Kamenica and Gentzkow, 2011], there is a growing literature on Bayesian persuasion and information design (see, e.g., [Rayo and Segal, 2010; Ely, 2017; Bergemann and Morris, 2019; Hahn et al., 2020; Babichenko et al.; Celli et al., 2020; Mathevet et al., 2020; Zhang and Zhu, 2021a]). This paper is closely related to the works studying direct models of persuasion (or information design) by establishing design regimes similar to the Revelation Principle of the mechanism design. In single-agent cases, [Kamenica and Gentzkow; 2011] has shown that for every optimal persuasion model, there is a direct information structure that directly recommends the action to the receiver. [Bergemann and Morris, 2016] has considered multi-agent cases and shown that persuasion for a mixed-strategy Bayes Nash equilibrium has an equivalent Bayes correlated equilibrium model that performs direct action recommendation.

2 Model and Problem Formulation

Conventions. For any measurable set \( Y \), \( \Delta(Y) \) denotes the set of probability measures over \( Y \). Any function defined on a measurable set is assumed to be measurable. We use tilde to indicate the random variables; i.e., \( \tilde{y} \) is a realization of the random variable \( y \). For a given expectation operator \( E[\cdot] \), \( E[F(\tilde{y}, w)] \) is the expectation taken over \( Y \) while a realization \( w \) is given, for any measurable function \( F \). Notation summary, proofs and remarks are provided in the online supplementary document.

We consider a discrete-time infinite horizon problem in a dynamic environment, where there is one principal (she) and a finite number of agents (he). Time is indexed by \( t = 0, 1, 2, \ldots \) and agents are indexed by \( i \in \mathcal{N} \equiv [n] \) with \( 1 \leq n < \infty \). The principal repeatedly delegates decision makings to the agents. In each period \( t \), each agent \( i \) takes an action \( a_{ij} \in A_{ij} \subseteq A \), where \( A \) is a compact set of actions and \( A_{ij} \subseteq A \) is an action menu determined by the principal in period \( t \). The dynamic of the environment is characterized by three notions of information. At the beginning of each period \( t \), a state \( s_t \in S \) is drawn. Based on \( s_t \), each agent \( i \) observes his type \( \theta_{ij} \in \Theta \) which is drawn according to the distribution \( d_{\Theta}^{s_t}(\cdot|s_t) \in \Delta(\Theta_i) \), for all \( i \in \mathcal{N} \). Both the state \( s_t \) and the type \( \theta_{ij} \) are the payoff-relevant information of each agent \( i \) in each period \( t \). In addition, the environment generates a shock, \( x_t \in X \), which is drawn according to the distribution \( d^X(\cdot|x_t) \in \Delta(X) \). We consider that \( S, \Theta, \) and \( X \) are compact sets of states, types, and shocks, respectively.

The state of the environment evolves in a Markovian fashion. Formally, let \( h_t = (s_0, a_{00}, x_0, \ldots, s_{t-1}, a_{t-1}, x_{t-1}) \) denote the history up to period \( t \). The state in period \( t \) depends on history \( h_t \) only through the state of period \( t - 1 \). If the state in period \( t - 1 \) is \( s_{t-1} \), the agents take joint action \( a_{t-1} = (a_{ij|t-1})_{i \in \mathcal{N}, j} \), and \( x_{t-1} \) is the shock in period \( t - 1 \), then, period-\( t \) state is distributed as \( s_t \sim \kappa(\cdot|s_{t-1}, a_{t-1}, x_{t-1}) \), where the function \( \kappa: S \times A \times X \rightarrow \Delta(S) \) is the transition kernel with \( A = \prod_{i \in \mathcal{N}} A_{ij} \). The initial distribution of the state is given as \( \kappa_0(\cdot) \in \Delta(S) \).
We assume that the environment is two-sided information-asymmetric. Specifically, the state $x_i$ is publicly observed by the principal and all the agents. However, the type $\theta_i$, is the private information of each agent $i$ that is unobserved by the principal and other agents while the shock $x_i$ is only observed by the principal. On the one hand, the principal has imperfect information about the agents’ decision makings of choosing actions because she does not know the agents’ type profile, $\theta_i = (\theta_{ij})_{j \in N^i}$. On the other hand, each agent $i$ has imperfect information about other agents’ decision makings because he does not know others’ types, $\theta_{-i}$; additionally, he has uncertainty regarding the distribution of the next-period state due to the unobservability of the current shock, $x_i$. By knowing the shock, the principal designs how and what each agent should know about the shocks in addition to the action menu. Specifically, the principal informs each agent $i$ about the shock $x_i$ by privately sending agent $i$ a signal $\omega_i \in \Omega_i$ where $\Omega_i$ is a compact set of signals, for $i \in N$. We refer to the tuple $\mathcal{E} \equiv (\mathcal{S}, \mathcal{A}, \mathcal{X}, \Theta, \mathcal{D}, \langle \mathcal{d}_{ij} \rangle_{i \in N, j})$ with the aforementioned two-sided information asymmetry as the environment model.

### 2.1 Dynamic Delegation Mechanism

We consider that the principal designs stationary delegation mechanisms in the dynamic environment. Hence, we suppress the time index for the elements in the environment models, unless otherwise stated. Specifically, the principal uses a menu function $\lambda_i(\cdot): \Theta_i \mapsto \mathcal{A}$ to determine an action menu $A_i$ for each agent $i$ when the current state is $s$. That is,

$$A_i = \{a_i \in \mathcal{A}: a_i = \lambda_i(\theta), \theta_i \in \Theta_i\}. \quad (1)$$

Let $\Lambda_i$, denote a compact set of menu functions the principal can choose from for each agent $i$, for $i \in N$. We restrict attention to contingent delegation setting, in which each menu function $\lambda_i$ is generated according to a stationary mixed-strategy delegation rule, $\sigma_i: S \mapsto \Delta(\Lambda_i)$, such that $\sigma_i(\lambda_i | s) \in [0, 1]$ gives the probability of specifying a menu function $\lambda_i$ for agent $i$ when the state is $s$. Here, the action menu $A_i$ specified for each agent $i$ depends on the current state $s$ only through the randomized generation of the menu function $\lambda_i$ by $\sigma_i(\cdot | s)$.

Additionally, the principal uses a stationary mixed-strategy signaling rule $\phi_i: S \times X \mapsto \Delta(\Omega_i)$ to select a signal $\omega_i \in \Omega_i$ for each agent $i$. The signaling rule profile performs Bayesian persuasion for the agents. In this work, we restrict attention to Markovian mechanisms in which each menu function and each signaling rule only take into consideration the current relevant information and are independent of histories. We refer to such delegation mechanism with looking-forward persuasion as an LFD mechanism, denoted by $D \equiv \langle \sigma, \phi \rangle$, where $\sigma = (\sigma_i)_{i \in N^i}$ and $\phi = (\phi_i)_{i \in N^i}$.

We assume that both the principal and the agents share the same discount factor denoted by $\delta \in [0, 1)$. In every period $t$, each agent $i$ chooses an action $a_{it}$ from the menu $A_i$ (uniquely determined by $\lambda_i$) to maximize his period-$t$ expected pay-off including period-$t$ one-stage utility and discounted sum of total utility of the future starting from period $t+1$. The objective of the principal is to maximize the ex-ante social welfare, i.e., the ex-ante expected discounted sum of utilities of all the agents, by designing an incentive compatible LFD mechanism to restrict agents’ discretion (through delegation) and their additional information about the unobserved shocks (through persuasion).

### 2.2 Belief Model

In this work, we consider that each agent is a Bayesian player. That is, each agent forms beliefs about the unobserved shock and his opponents’ types and signals. We assume that the generation of each agent $i$’s type is from a move by Nature (i.e., a Harsanyi’s type [Harsanyi, 1967]) with $d^\theta \equiv \langle d^\theta_i \rangle_{i \in N}$ as common priors. That is, a type $\theta_i$ is a full description of agent $i$’s beliefs about the data of the game, beliefs about the beliefs of the his opponents about the data of the game and about his own beliefs, etc. (see, e.g., [Zamir, 2020]). Hence, the information asymmetry raised from the private types is information incompleteness.

However, the information asymmetry due to the unobservability of the shock and other agents’ signals is information incompleteness, which unavoidably induces agents’ interactive reasoning about the beliefs. Specifically, since the principal’s $\phi$ is publicly known, each agent $i$ forms a posterior belief $\mu_i(\cdot | s, \omega_i) \in \Delta(X \times \Omega_i)$ about the shock $x$ and other agents’ signals $\omega_{-i}$, using $\phi$ and $d^\theta$ according to the Bayes’ law. Since each agent $i$’s period-$t$’s choice of action takes into consideration of the future, the posterior belief $\mu_i$ is payoff-relevant. As other agents’ actions $a_{-i}$ are payoff-relevant to agent $i$, so are their posterior beliefs $\mu_{-i}$. As a result, each agent $i$ needs to form beliefs about other agents’ posterior beliefs. For the same reason, each agent $i$ has to form a belief about other agents’ beliefs of his beliefs of $\mu_{-i}$, and so on. Therefore, this information incompleteness leads to a belief hierarchy for each agent in each period.

Formally, a belief hierarchy is an infinite sequence of beliefs, $b_{i1} \equiv \langle b_{i2}, b_{i3}, \ldots, b_{ik}, \ldots \rangle$. Here, $b_{i1} = \max_{\theta_i} \mu_i(\cdot | s, \omega_i) \in \Delta(X)$ is the first-order belief about the unobserved shock $x$, given the state $s$ and the signal $\omega_i$. Since every agent $j \neq i$ has a first-order belief $b_{j1}$, agent $i$ uses $(s, \omega_i)$ to form a belief about $b_{j1}$. Hence, each agent $i$ forms a second-order belief $b_{i2} \in \Delta(X \times \Delta(X)^{|\omega_i|})$ about $b_{j1}$. For the same reason, the $k$-th-order belief $b_{ik} \in \Delta(X \times \Delta(X)^{|\omega_i|})$ about $b_{j1}$.

To ensure that every agent’s belief is coherent ([Brandenburger and Dekel, 1993]) if $b_{ik}$, for any $k > 1$, coincides with all beliefs of lower order, $\{b_{ik'}\}_{k' = 1}^{k-1}$, i.e., $b_{ik-1} = \max_{\theta_i} b_{ik}$ for all $i \in N, t \geq 0, k > 1$. In a coherent belief hierarchy, any event in the space of
the $k$th-order beliefs, $B_i^{[k]}$, must have the same marginal probability in every $k'$th-order beliefs, for all $k' > k$. Let $B_i^{[k]} \subseteq B_i^{[k]}$ denote the set of coherent belief hierarchies of order $k$ of agent $i$, for all $i \in \mathcal{N}$, $k \geq 1$, with $B_i^{[1]} = B_i^{[1]} \supseteq B_i^{[2]}$. From the concherecy, the projection of $B_i^{[k+1]}$ on $B_i^{[k]}$ is $B_i^{[k]}$ (see, [Zamir, 2020]). [Brandenburger and Dekel, 1993] has shown that there exists a homomorphism $\Gamma_i : (s) : \Omega \rightarrow \Delta(X \times B_{-i})$ such that $\Gamma_i(x, b_{-i}, s_i) \in [0, 1]$ gives agent $i$ a probability of an event that the shock is $x$ and other agents’ (coherent) belief hierarchies are $b_{-i}$, when the state is $s$ and agent $i$’s belief hierarchy is $b_i$. We assume that $\Gamma_i(\cdot) = (\Gamma_i^X(\cdot | b_i; s_i) = \arg\max_{x \in \mathcal{N}} \Gamma_i^X(\cdot | b_i; s_i))$ is given for every $s \in S$, $i \in \mathcal{N}$, and it is publicly known. Given the state $s$ and the belief hierarchy $b_i$, we denote the transition kernel perceived by agent $i$ by $\kappa_i(s, b_i; a_i) = \int_s \kappa_i(s', s, a_i) \Gamma_i^X(\cdot | b_i; s_i)$. We assume that (i) for all $(s, b_i) \in S \times B_i$ and all $a \in \mathcal{A}_i$, $\kappa_i(s, b_i, a)$ is absolutely continuous and (ii) for all $(s, b_i) \in S \times B_i$, the mapping $a \rightarrow \kappa_i(s, b_i, a)$ is norm-continuous.

Define the belief hierarchy distribution (belief distribution) induced by the principal’s signaling rule as, for any $s \in S$,

$$Z(b | s) = \int_{x \in X} \phi_{(s \in X)}(\phi(b, x)) d^X(\cdot | b_i, s)$$

We let $Z_i(b | s) \in \Delta(B_i)$ denote the marginal distribution for each $i \in \mathcal{N}$.

Following [Mathevet et al., 2020], we establish the following three conditions that is sufficient and necessary for the existence of a signaling rule profile that induces a belief distribution $Z_i(\cdot | s) \in \Delta(B_i)$ for all $s \in S$. (i) The mechanism should have a common prior $p(\cdot | s) \in \Delta(X \times B)$ such that, for all $i \in \mathcal{N}$, $x \in S$, $p(x, b_i | s) = \Gamma_i^X(b_i | s, b_i) p(b_i | \cdot)$, where $p(b_i | \cdot) = \int_{s \in S} \kappa_i(s, b_i, a_i) p(b_i | \cdot)$ and (ii) the belief distribution induced by the signaling rule is consistent; i.e., for all $s \in S$, $Z_i(\cdot | s) = \arg\max_{s \in S} p(\cdot | s)$, (iii) The belief distribution induced by the signaling rule is Bayes’ plausible; i.e., for all $s \in S$, $Z_i(\cdot | s) = \int_{s \in S} \Gamma_i^X(\cdot | b_i, s) Z_i(b_i | s) d^X(\cdot | b_i, s_i)$. We use $T = \arg\min_{b_i \in B_i} \text{denote the belief hierarchy induced by the signaling rule profile } \phi$. We refer to $T$ that satisfies (i)-(iii) as regular belief model. Given $T$, we expand the information structure $< \mathcal{K}, \mathcal{S}, S >$ used in delegation rule profile to $< \mathcal{K}, \mathcal{S}, S >$ such that each $\sigma_i(\cdot | s, b_i) \in \Delta(A_i)$ for all $i \in \mathcal{N}$.

### 2.3 Dynamic Bayesian Game

Given any LFD mechanism $< \sigma, \phi >$ with the belief model $T$, the agents play a dynamic Bayesian game, in which each agent $i$ takes an action from the action menu $A_i$ in each period and receives a single-period utility. Define each agent $i$’s utility function as $u_i : S \times \Theta_i \times A_i \rightarrow \mathbb{R}$ such that agent $i$ receives one-period utility $u_i(s, \theta_i, a_i)$ when the state is $s$, his type is $\theta_i$, and the agent takes $a_i$. We denote the underlying dynamic Bayesian game in the mechanism $< \sigma, \phi >$ as a tuple $\mathcal{M}(\sigma, \phi) < \mathcal{N}, S, A, \Theta, \Theta, (u_i)_{i \in \mathcal{N}} >$. We consider that in each period $t$, each agent $i$ uses a pure-strategy Markov policy $\pi_{i,t} : \Theta_i \times B_i \rightarrow A_i$ to select an action $\pi_{i,t}(s_i, \theta_i, b_i, d_i) \in A_i$. We say that that a policy profile $\{\pi_{i,t}\}_{t \geq 0}$ is obedient if agent $i$ selects an action $a_i$ according to his type $\theta_i$ in each period $t$.

We say that that a policy profile $\{\pi_{i,t}\}_{t \geq 0}$ is obedient if agent $i$ selects an action $a_i$ according to his type $\theta_i$ in each period $t$; i.e., $\pi_{i,t}(s_i, \theta_i, b_i, d_i) = \lambda_i(\theta_i)$, for all $s_i \in S$, $\theta_i \in \Theta_i$, $b_i \in B_i$, $t \geq 0$.

Following the revelation principle, it is without loss of generality to focus on mechanisms where agents are obedient. In particular, we consider Markov perfect Bayesian equilibria (MPBE) in obedient policies, in which each agent believes with probability 1 that other agents are obedient. To incentivize the obedience, we impose periodic ex-ante incentive compatibility (PIC) constraints to ensure that under the delegation mechanism, obedience is each agent’s best response to other agents’ obedience.

Due to the Ionescu Tulcea theorem (see, e.g., [Hernández-Lerma and Lasserre, 2012]), the initial distribution $\kappa_0$ on $S$, the transition kernel $\kappa$, conditional probability distributions $d^\Theta, d^X >$, the delegation mechanism role profiles $< \sigma, \phi >$ with the belief model $T$, and the agents’ policy profile $\pi = (\pi_{i,t})_{i \in \mathcal{N}}$ uniquely define a probability measure $P_{\pi}^\sigma$ on $(S \times \Theta \times X \times B)^{\infty}_{t=0}$. In addition, given any $(s_i, \theta_i, b_i, a_i)$, each agent $i$ perceives a unique probability measure $P_{\pi}^\sigma(\cdot | s_i, \theta_i, b_i, a_i)$ on $\Theta_i \times B_i \times (S \times \Theta \times X \times B)^{\infty}_{t=0}$. The expectation operators with respect to $P_{\pi}^\sigma$ and $P_{\pi}^\sigma(\cdot | s_i, \theta_i, b_i, a_i)$ are denoted by $E_{\pi}^\sigma$ and $E_{\pi}^\sigma(\cdot | s_i, \theta_i, b_i, a_i)$, respectively. Then, we define the ex-ante expected payoff to-go (payoff to-go) of agent $i$ as:

$$J_i^\sigma(\pi_{i,t}) = J_i(\sigma, \phi, a_i, b_i, d_i, t; \pi) = \arg\max_{\pi \in \Pi_i} \left[ 1 - \delta \right] E_{\pi}^\sigma \left[ u_i(s_i, \theta_i, a_i, b_i, d_i, t) \right]$$

When any agent $i$’s policy is obedient, we omit it in the notation of the payoff-to-go function (e.g., $J_i^\sigma(\lambda_i(\theta_i), s_i, \theta_i, b_i, d_i, t; \pi)$ and $J_i^\sigma(\lambda_i(\theta_i), s_i, \theta_i, b_i, d_i, t; \pi)$ if $\pi$ and $\pi_{-i}$ are obedient, respectively). Then, we define the PIC constraints as follows: for all $i \in \mathcal{N}$, $t \geq 0$, $s_i \in S$, $\theta_i \in \Theta_i$, $b_i \in B_i$, $a_i \in A_i$, $J_i^\sigma(\lambda_i(\theta_i), s_i, \theta_i, b_i, d_i, t; \pi) \geq J_i^\sigma(\lambda_i(\theta_i), s_i, \theta_i, b_i, d_i, t; \pi)$ (PIC$_i$).

We say that an LFD mechanism $< \sigma, \phi >$ is PIC if $< \sigma, \phi >$ satisfies (PIC$_i$) for all $i \in \mathcal{N}$, $t \geq 0$. If the principal implements such $< \sigma, \phi >$, then each agent $i$, believing with probability 1 that others are obedient, being obedient in every period is a MPBE. As is standard, when there are multiple equilibria, tie-breaking rule is in the principal’s favor.

Under the mechanism $< \sigma, \phi >$ with $T$, each agent $i$’s ex-ante expected payoff is defined as $J_i(\sigma, \phi, \pi) = E_{\pi}^\sigma \left[ \sum t=0 \delta^t u_i(s_i, \theta_i, a_i, \pi) \right]$. When $\pi$ is obedient, we write $J_i(\sigma, \phi, \pi) = J_i(\sigma, \phi, a_i, b_i, d_i, t; \pi)$. We refer to the vector $(J_i(\sigma, \phi, \pi))_{i \in \mathcal{N}}$ as the principal’s target of the mechanism. We assume that the every target is bounded for all $i \in \mathcal{N}$. We define the set of attainable targets of the principal as follows:

$$V \equiv \left\{ v \in \mathbb{R}^\mathcal{N} : v_i = J_i(\sigma, \phi, \pi), i \in \mathcal{N}, \text{ for a PIC } < \sigma, \phi > \right\}.$$
Hence, for any vector \( v = (v_i)_{i \in \mathcal{A}} \in \mathcal{G} \), there exists a PIC LFD mechanism \( < \sigma, \phi > \) under which each agent \( i \) is incentivized to adopt the obedient policy and obtains ex-ante expected payoff as \( v_i \), for all \( i \in \mathcal{N} \). Since the principal aims to maximize the ex-ante social welfare which is the ex-ante expected discounted sum of all the agents’ utilities, an optimal PIC delegation mechanism \( < \sigma, \phi > \) has a corresponding \( v^* \in \mathcal{V} \) such that \( \sum_{i \in \mathcal{A}} J_i(\sigma^*, \phi^*) = \sum_{i \in \mathcal{A}} v^*_i = \max \sum_{v \in \mathcal{G}} \sum_{i \in \mathcal{A}} v_i \).

### 3 Dynamic Bayesian Promised Delegation Model

In this section, we propose the model of Bayesian promised delegation (BPD) mechanism in the same environment \( \mathcal{G} \). A BPD is a randomized mechanism consisting of a signaling rule profile \( \phi^i = (\phi^i_j)_{j \in \mathcal{A}} \) and a randomization rules profile \( \psi^i = (\psi^i_j)_{j \in \mathcal{A}} \). Similar to \( \phi \) in the original delegation mechanism, the principal uses each \( \phi^i_j \) to inform each agent \( i \) about the realized shock in period \( t \). As shown in Sec. 2.2, the profile \( \phi^i \) induces a belief model \( T^i = \langle p, Z, \Gamma, B \rangle \). We first define attainable state value. Then, we introduce the notion of stage rules and formally define the BPD model. Finally, we establish a revelation principle to obtain a direct BPD mechanism in which the signaling profile is fully characterized by the direct randomization rule profile.

#### 3.1 Attainable State Value

In the LFD mechanism \( < \sigma, \phi > \), each agent \( i \) periodically decides whether his choice of action \( a_{it} \) can maximize his current payoff to-go, \( J_i(\sigma, \phi)(a_{it}, s, \theta, a_{i1:t}, \pi_{1:t}) \) which is composed of current expected one-period utility \( \bar{u}_i(s, \theta, a_{it}, a_{i1:t}) = \mathbb{E}^\theta \mathbb{E}_{\pi_{1:t}} [u_i(s, \theta, a_{it}, a_{i1:t}, \lambda_{i-1}(\theta_{i-1})) | s_t] \) and a continuing value, \( c_i(\sigma, \phi)(s, a_{i1:t}, a_{i1}, \pi_{1:t+1}) \equiv (1 - \delta) \mathbb{E}^\theta \mathbb{E}_{\pi_{1:t+1}} \left[ \sum_{s_t} \delta \bar{u}_i(s_t, \theta_t, a_{it}, a_{i1:t}) | s_t, a_{i1:t}, a_{i1} \right] \), where \( \pi_{1:t+1} = (\pi_{1:t})_{i \in \mathcal{N}} \). When \( \pi_{1:t+1} \) is obedient, we write \( c_i(\sigma, \phi)(s, a_{i1:t}, a_{i1}, \lambda_{i}(\theta_i)) = c_i(\sigma, \phi)(s, a_{i1:t}, a_{i1}, \pi_{1:t+1}) \).

**Lemma 1.** Fix an LFD mechanism \( < \sigma, \phi > \) with belief model \( T \). Then, the delegation mechanism is PIC if and only if, for all \( i \in \mathcal{N}, s \in S, \theta, \theta' \in \Theta_i, b_i \in B_i \),

\[
(1 - \delta) \bar{u}_i(s, \theta, a_{i1:t}, \lambda_{i}(\theta'_i)) + c_i(\sigma, \phi)(s, b_i, \lambda_{i}(\theta'_i)) \geq \bar{u}_i(s, \theta, a_{i1:t}, \lambda_{i}(\theta'_i)) + c_i(\sigma, \phi)(s, b_i, \lambda_{i}(\theta'_i)).
\]

Lemma 1 establishes a one-shot deviation principle which directly follows the subgame perfection of Markov perfect Bayesian equilibria and we omit the proof here. Although the delegation mechanism is stationary, each agent’s deviation from obedience can be arbitrarily nonstationary. That is, in each period \( t \), every agent \( i \) can use current policy \( \pi_{it} \) and plan future policies \( \pi_{t+1} \). Lemma 1 implies that if the delegation mechanism is incentive compatible when each agent might deviate from obedience only in current period, then it is also incentive compatible for agents’ arbitrary deviations.

Define the state value function \( g^i_\sigma(\psi)(s) = \mathbb{E}^\sigma \mathbb{E}^\psi \left[ \sum_{j \neq i} \delta \bar{u}_j(s, \theta_j, a_{ij}, \lambda_{ij}(\theta_j)) | s \right] \), assuming that agents are obedient. It is straightforward to obtain that in any PIC LFD mechanism \( < \sigma, \phi > \), the state value function \( g^i_\sigma(\psi) \) is uniquely defined by the recursion:

\[
g^i_\sigma(\psi)(s) = \mathbb{E}^\sigma \mathbb{E}^\psi \left[ (1 - \delta) \bar{u}_i(s, \theta_i, a_{i1:t}, \lambda_{i}(\theta_i)) + \delta \int g^i_\sigma(\psi)(s') \mathbb{P}(ds' | s, b_i, \lambda(\theta)) | s \right].
\]

Let \( g^i(\cdot) = (g^i(\cdot)) \in \mathbb{R}^\mathcal{G} \) denote the vector of agents’ state value functions. Then we define the set of attainable state value functions:

\[
G = \left\{ g_i(\cdot) = g^i(\cdot), i \in \mathcal{N}, \text{ for a PIC } < \sigma, \phi > \right\}.
\]

**Corollary 1.** For every attainable target \( v \in \mathcal{V} \), there is a \( g \in G \) such that \( v_i = \mathbb{E}^{\sigma, \phi}[g_i(\delta)], \) for all \( i \in \mathcal{N} \).

#### 3.2 Stage Rules and BPD Model

Lemma 1 implies that the expected next-period obedient payoff-to-go, characterized by \( g^i_\sigma(\psi) \), constitutes a sufficient statistic for determining whether an LFD mechanism is incentive compatible in the current period. The key design principle for our BPD mechanisms is to allow the principal to provide a promise of the future delegations in addition to the action menus. Define a pure-strategy stage menu function profile, denoted by \( \lambda^i = (\lambda^i_k)_{k \in \mathcal{K}} \in \Lambda_i \), and a stage promise rule, denoted by \( \rho = (\rho_i)_{i \in \mathcal{N}} \in \mathbb{P} = \prod_{i \in \mathcal{N}} \mathbb{P}_i \) where \( \mathbb{P}_i = \{ \rho : S \rightarrow \mathbb{R} \} \) is a compact set of promise rules for agent \( i \), for all \( i \in \mathcal{N} \). Here, each stage menu function \( \lambda^i : \Theta_i \rightarrow A_i \) defines an action menu \( A_i \), in the same way as the menu function \( \lambda_i \) in (1). When the current period state is \( s \), agent \( i \)'s belief hierarchy is \( b_i \), each \( \rho_i : S \rightarrow \mathbb{R} \) specifies a promise function such that we obtain the expected promised value of the next period for agent \( i \), \( L_i(s, b_i, a; \rho_i) \equiv \int \rho_i(s, b_i, a) \mathbb{P}(ds' | s, b_i, a) \).

Formally, a BPD mechanism model is defined by \( \mathbb{BPD}(\phi^i, \psi, P) \equiv \langle \phi^i, \psi, T^i, P, \Lambda \rangle \). The principal uses the signaling rule profile \( \phi^i \) to send signals to the agents such that a belief hierarchy profile \( \psi \) is formed. Each randomization rule \( \psi^i(\cdot | s, b_i) \) specifies \( \Delta(\Lambda_i \times P_i) \) the probability of privately generating a pair of stage rules, \( \langle \lambda^i, \rho_i \rangle \), for agent \( i \) when the state is \( s \) and the agent’s belief hierarchy is \( b_i \).

The Ioinescu-Tulcea theorem implies that the initial distribution \( k_0 \) on \( S \), the transition kernel \( \kappa \), conditional probability distributions \( < d\phi^i, d\chi^i > \), the profiles \( \phi^i, \psi \), and the induced belief model \( T^i \) uniquely define a probability measure over \( (\mathcal{X} \times \Theta \times \Theta \times \mathcal{X})^\omega \). We use \( \mathbb{E}^\mathcal{X}[\cdot] \) and \( \mathbb{E}^\Theta[\cdot] \), respectively, denote the ex-termin (for any specific conditions) and the ex-ante expectation operators. Define the one-stage ex-post expected payoff of agent \( i \) as \( R_i(s, \theta_i, b_i, a; \rho_i) \equiv (1 - \delta) \bar{u}_i(s, \theta_i, a) + \delta L_i(s, b_i, a; \rho_i) \). Given BPD(\( \phi^i, \psi, P \)), define the one-stage ex-termin expected payoff of agent \( i \) as \( \mathbb{E}^\mathcal{X}[R_i(s, \theta_i, b_i, a; \rho_i)] \equiv \mathbb{E}^\mathcal{X}[\mathbb{E}^\Theta[R_i(s, \theta_i, b_i, a; \rho_i)] | s, b_i] \).

**Definition 1 (Bayesian Incentive Compatibility).** A mechanism \( \mathbb{BPD}(\phi^i, \psi, P) \) is Bayesian incentive compatible (BIC) if for all \( i \in \mathcal{N}, s \in S, \theta_i \in \Theta_i, b_i \in B_i, \lambda^i_j \in \Lambda_i, \) and \( \rho_i \in P \) with \( \psi(\lambda^i_j, \rho_i | s, b_i) > 0, a'_i \in A_i, \)

\[
R_i(s, \theta_i, b_i, \lambda^i_j(\theta_i); \rho_i | \psi_{-i}) \geq \tilde{R}_i(s, \theta_i, b_i, a'_i; \rho_i | \psi_{-i}),
\]

(\( \text{BIC} \))
while each agent beliefs with probability 1 that other agents are obedient.

The inequality displayed in (BIC) is referred to as the BIC constraint for agent $i$. It ensures that agent $i$, after observing $(s, \theta_i)$ and the stage rules $(\lambda_i, \rho_i)$ and forming $b_i$ (induced by a signal sent by $\phi_i^\kappa_i$), finds that it is better off for him to be obedient while believing with probability 1 that others are obedient. Given the obedience, the profile $z(\cdot) = \{z_i(\cdot)\}_{i \in \mathcal{N}}$ where each $z_i(\cdot|s, b_i) = \int \text{arg}_{\hat{\theta}_i} \psi(\lambda_i(\cdot), z_i(\cdot|s, b_i) \rho_i(\cdot|\theta_i)|s)$ constitutes a Bayes Nash equilibrium.

We also require the model BPD[$\phi^i$, $\psi$, $P$] to satisfy the following Bayesian promise keeping (BK) constraints: for all $i \in \mathcal{N}$, $s \in S$, $g_i \in P$, $g_i(s) = \mathbb{E}_\psi \left[ R_i(s, \hat{\theta}_i, \tilde{b}_i, \tilde{\lambda}_i(\hat{\theta}_i); \tilde{\rho}_i|\psi(\cdot) \right] s$. (BKG[$g_i$])

Let BK[$g$] = $\{BKG[g_i]\}_{i \in \mathcal{N}}$. Specifically, BK[$g$] requires the principal to promise a current state-value function profile $g$ by promising future state-value function profile through the randomization rule $\psi$. We refer to a BPD mechanism satisfying BIC and BK[$g$] as BIC-BK[$g$] mechanism. Based on the constraints BIC and BK[$g$], we define the following set: for any compact subset $P \subseteq \mathbb{R}$,

$$H[G] = \left\{ g \in P \mid \exists \text{BPD}[\phi^i, \psi, \tilde{P}], \text{s.t., BIC,BK}[g_i]\right\}. \quad (5)$$

**Proposition 1.** The set of attainable state-value functions, $G$, satisfies $H[G] = G$.

Proposition 1 implies that set of attainable state value functions of the RD mechanisms is a “fixed point” of $H[\cdot]$ and for every $g \in G$ there is a BIC and BK BPD[$\phi^i$, $\psi$, $\tilde{P}$] that can achieve the same $g$. According to Corollary 1, we conclude that any social welfare that can be achieved by a RD mechanism can also be achieved by some BPD mechanism.

### 3.3 Correlated Bayesian Promised Delegation

In this section, we introduce the direct BPD (DPD) mechanisms in which the signal set coincides with the set of promise rules; i.e., $\Omega_i = P_i$, for all $i \in \mathcal{N}$. Formally, a DPD mechanism is defined by the model DPD[$\eta$, $P$] $\equiv \eta, P, A >$, in which $\eta : S \times X \rightarrow \Delta(\Lambda \times P)$ is a direct randomization rule. Note that $\eta$ is not a profile. In each period, the principal generates a profile of stage rule pairs $(\lambda_i, \rho_i)$ according to $\eta(\cdot|s,x)$ based on the state $s$ and the shock $x$. Each $\lambda_i, \rho_i$ is privately sent to each agent $i$. Based on $\lambda_i, \rho_i$ and his type $\theta_i$, agent $i$ chooses an action from the menu defined by $\lambda_i$.

Let $E^\eta[\cdot]$ and $E^0[\cdot]$ denote the corresponding expectation operators. Given any $(s, x) \in S \times X$, define the belief distribution induced by the signaling rule profile $\phi_i^\kappa$ as: $Z^\kappa(\delta|s, x) = \phi(\{\omega : \Xi(\omega|x, s) = \delta\})$. With a slight abuse of notation, let $R_i(s, \theta_i, x, a; \rho_i) = (1 - \delta) u_i(s, \theta_i, a) + \delta \rho_i(s')\kappa(ds'|s, x, a)$. Then, define $m_i(\theta_i, a; \lambda_i, \rho_i|s, x) = \mathbb{E}_\eta^\kappa \left[ R_i(s, \theta_i, x, a; \tilde{\lambda}_i(\hat{\theta}_i); \tilde{\rho}_i) \right] s$. $v$ attainable target

**Definition 2 (Bayesian Correlated Incentive Compatibility).** A BPD mechanism $\eta$ is Bayesian correlated incentive compatible (BCIC) if for each $i \in \mathcal{N}$, $s \in S$, $\theta_i \in \Theta_i$, $\lambda_i \in \Lambda$, $\rho_i \in P_i$,

$$E^\eta \left[ m_i(\theta_i, \lambda_i(\cdot); \lambda_i, \rho_i|s, x) \right] \geq E^\eta \left[ m_i(\theta_i, d_i^*; \lambda_i, \rho_i|s, x) \right],$$

for all $d_i^* \in A_i$.

The inequality displayed in (6) is referred to as the BCIC constraint for agent $i$. This constraint guarantees that agent $i$, after observing $(s, \theta_i)$ and stage rules $(\lambda_i, \rho_i)$, finds that it is optimal to be obedient when he believes with probability 1 that other agents are obedient. Given the obedience, the distribution $\tilde{z}(\cdot|s, x) = \int \text{arg}_{\hat{\theta}_i} \eta(\lambda_i(\cdot), \rho_i|s, x) \prod_{i \in \mathcal{N}} d_i^\kappa(\theta_i)$ constitute a Bayes correlated equilibrium. Besides the BCIC constraints, we require the mechanism DPD[$\eta$, $P$] $\equiv \eta, P, A >$ to satisfy following Bayesian correlated promise keeping (BCK) constraints: for all $i \in \mathcal{N}$, $s \in S$, $g_i \in P_i$,

$$g_i(s) = \mathbb{E}_\psi^\kappa \left[ R_i(s, \hat{\theta}_i, \tilde{b}_i, \tilde{\lambda}_i(\hat{\theta}_i); \tilde{\rho}_i|\psi(\cdot) \right] s. \quad \text{(BCK[g_i])}$$

Let BCK[$g$] = $\{BCK[g_i]\}_{i \in \mathcal{N}}$. We refer to a DPD mechanism satisfying BIC and BCK[$g$] as BCIC-BCK[$g$] mechanism. We say that a randomization profile $\psi$ induces a direct randomization rule $\eta$ if, for all $s \in S, x \in X, \lambda \in \Lambda, \rho \in G$,

$$\eta(\lambda_i, \rho_i|s, x) = \int \left( \prod_{i \in \mathcal{N}} \psi_i(\lambda_i, \rho_i|s, b_i) \right) Z^\kappa(db|x).$$

**Theorem 1.** For any PIC LFD mechanism $< \sigma, \phi >$ that achieves an attainable $g \in G$, there exists a BCIC-BCK[$g$] DPD mechanism DPD[$\eta$, $P$] that achieves the same $g$ if there exists a BIC-BK[$g$] BPD mechanism BPD[$\phi^i$, $\psi$, $G$] that induces DPD[$\eta$, $G$].

Following Corollary 1, Theorem 1 implies that for every attainable target $v \in V$ that can be achieved by a PIC LFD mechanism $< \sigma, \phi >$, there exists a BCIC DPD mechanism that can achieve $v$ if there exists a BCIC BPD mechanism that induces this DPD mechanism. Therefore, an equivalence of social welfare is obtained. Theorem 1 establishes our version of revelation principle for the looking-forward persuasion. In particular, the persuasion with information structure $< \phi, \Omega >$ which induces belief hierarchies for the agents can be fully characterized by the randomized mechanism DPD in which the randomization of the agents’ individual stage rules are correlated with the recommendation of the future promises as the direct persuasion. As a result, the explicit formulation of belief hierarchies is avoided.

### 4 Conclusion

In this work, we have studied a dynamic delegation mechanism design problem with informational burning that is supported by a looking-forward persuasion. We have proposed a novel randomized mechanism known as Bayesian promised delegation (BPD) model in addition to the signaling rule that jointly generates an action menu and a promise of future to each agent based on the state and agent’s belief hierarchy.
induced by the persuasion. We have shown that by imposing the incentive compatibility and the promise keeping constraints, BPD mechanism can achieve a same social welfare as the original LFD mechanism. A revelation principle is obtained to show that the delegation with informational burning can be fully characterized by a direct BPD in which we avoid explicit formulations of the belief hierarchies by persuading each agent through direct recommendation of the promise. These results contribute as foundations for the algorithmic analysis of the dynamic mechanism design which is our next step.

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