Gauge Coupling Unification in GUT with Anomalous \( U(1) \) Symmetry

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We show that in the framework of grand unified theory (GUT) with anomalous \( U(1)_A \) gauge symmetry, the success of the gauge coupling unification in the minimal \( SU(5) \) GUT is naturally explained, even if the mass spectrum of superheavy fields does not respect \( SU(5) \) symmetry. Because the unification scale for most realizations of the theory becomes smaller than the usual GUT scale, it suggests that the present level of experiments is close to that sufficient to observe proton decay via dimension 6 operators, \( p \to e + \pi \).

INTRODUCTION

As is well known, if a reasonable SUSY breaking scale is assumed, the three gauge coupling constants meet at the usual GUT scale \( \Lambda_G \sim 2 \times 10^{16} \text{ GeV} \), in the minimal supersymmetric standard model (MSSM). This is a very significant result, and it is sometimes regarded as evidence supporting the validity of the existing supersymmetric grand unified theory (SUSY-GUT). However, generically in the SUSY-GUT scenario, there exist color triplet partners of the MSSM Higgs, whose presence results in too rapid proton decay via dimension 5 operators. In many GUTs, suppression of this proton decay is incompatible with the success of gauge coupling unification \( \text{[1, 2, 3]} \). Of course, it is possible to manipulate these effects in such a way to specifically realize both suppression of proton decay and gauge coupling unification - for example, with the theories adjusted so that proton decay is appropriately suppressed, scales below \( \Lambda_G \) can be tuned to realize gauge coupling unification there - but it is obviously desirable to construct a theory in which these effects are realized in a more natural manner. However, if we wish for them to emerge naturally within any of the existing theories, there are few possibilities. Most existing solutions realize the MSSM below \( \Lambda_G \) and include suppression or complete prohibition of proton decay \( \text{[4, 5, 6, 7, 8]} \). However, recently another type of solution has been proposed \( \text{[9]} \) in the context of GUT with anomalous \( U(1) \) gauge symmetry \( \text{[10]} \), whose anomaly is cancelled by the Green-Schwarz mechanism \( \text{[11]} \). It is surprising that, with this solution, gauge coupling unification is realized without fine tuning of the theory, even though the unified gauge group \( G (= SO(10) \text{ or } E_6) \) has a higher rank than \( SU(5) \) (implying that there are several gauge symmetry breaking scales), and the mass spectrum of superheavy fields does not respect \( SU(5) \) symmetry. This unification is made possible because the mass spectrum of superheavy fields and the symmetry breaking scales are determined by anomalous \( U(1)_A \) charges, and most of the charges cancel and as a result do not appear in the relations for gauge coupling unification.

In this letter, we give a group theoretical argument that accounts for the cancellation of the charges within the solution proposed in Ref. \( \text{[3]} \). With the understanding provided by this argument, it becomes obvious that any GUTs with anomalous \( U(1)_A \) gauge symmetry can naturally explain the success of gauge coupling unification in the minimal \( SU(5) \) GUT if the following conditions are satisfied:

1. The unification group \( G \) is simple.
2. The vacuum expectation values (VEVs) of GUT gauge singlet operators (\( G \)-singlets) \( O_i \) with anomalous \( U(1)_A \) charges \( o_i \) are given by
   \[
   \langle O_i \rangle \sim \begin{cases} 
   \lambda^{-o_i} & o_i \leq 0 \\
   0 & o_i > 0 
   \end{cases}
   \tag{1}
   \]
3. Below a certain scale, MSSM is realized.

Here \( \lambda \ll 1 \) is the ratio of the cutoff scale \( \Lambda \) and the VEV of the Froggatt-Nielsen (FN) field \( \Theta \), whose anomalous \( U(1)_A \) charge is normalized to \( -1 \text{ [12]} \). Throughout this letter, we denote all the superfields and chiral operators by uppercase letters and their anomalous \( U(1)_A \) charges by the corresponding lowercase letters. In most situations, we use units in which \( \Lambda = 1 \). In the argument we give, the concept of the “effective charge” is very important, and in fact when it can be defined consistently, gauge coupling unification is naturally realized. We show that such a definition can be given when the VEV structure satisfies \( \text{[11]} \). We thus find that gauge coupling unification emerges naturally in a whole class of GUT models with anomalous \( U(1)_A \) symmetry if their VEV structure satisfies \( \text{[11]} \). In fact, this class includes both our models considered in Ref. \( \text{[13]} \) and those proposed previously in Ref. \( \text{[4, 5, 6, 7, 8]} \). Moreover, some of the above conditions can be weakened. For example, even when the gauge group is non-simple, gauge coupling unification is realized if the charge assignment respects \( SU(5) \) symmetry.

VACUUM DETERMINATION AND MASS SPECTRUM OF SUPERHEAVY FIELDS

In order to examine the gauge coupling unification conditions, we have to know how the VEVs of Higgs fields
and the mass spectrum of superheavy fields are determined by the anomalous $U(1)_A$ charges.

First, let us recall the vacuum structure of theories with anomalous $U(1)_A$ gauge symmetry in which generic interactions, namely all the interactions allowed by the symmetry, are introduced. Generally, in vacua in which the FN mechanism acts, the VEVs of G-singlets $O_i$, which are fixed by $F$-flatness conditions, are as given in Eq. (1). The VEVs of Higgs fields can be evaluated by using those relations for the VEVs of the G-singlets. As an example, let us study $SO(10)$ GUT. Consider an adjoint Higgs field $A(45)$ and a pair of spinor Higgs fields $C(16)$ and $C(\overline{16})$, so that $SO(10)$ is broken into the gauge group of the standard model (SM) $G_{SM}$. Then, the scale of the VEV of $A(45)$ is given approximately by $\langle A \rangle \sim \lambda^{-a}$, because the G-singlet tr $A^2$ must have a VEV equal to $\lambda^{-2a}$. The scale of the VEV of the complex Higgs $C$ and $\bar{C}$ behaves as $\langle CC \rangle \sim \lambda^{-(c+\tilde{c})}$, and the $D$-flatness condition requires $|\langle C \rangle| = |\langle \bar{C} \rangle| \sim \lambda^{-\frac{1}{2}(c+\tilde{c})}$. Note that the VEVs of G-singlet fields can differ from the expected values for G-singlets, that is $\lambda^{-c}$ and $\lambda^{-\tilde{c}}$.

Next, we examine how to determine the mass spectrum of superheavy fields. The mass term of the vector-like fields $X$ and $\bar{X}$ can be written as

$$\lambda^{x+\tilde{x}} X \bar{X}, \tag{2}$$

where $x + \tilde{x} \geq 0$. It is obvious that if $x + \tilde{x} < 0$, this mass term is forbidden by the symmetry if the vacuum structure satisfies (1) (the SUSY zero mechanism). Note that by causing the VEVs to satisfy Eq. (1), the higher-dimensional terms $\lambda^{x+\tilde{x}+\alpha} X \bar{X} O_i$ also induce the masses of $X$ and $\bar{X}$ that are of the same order as that given in Eq. (2). This is one of the most important features of theories with anomalous $U(1)_A$ symmetry. However, G-non-singlet fields $(C)$ have generally VEVs that differ from the expected values for G-singlets (i.e. $\lambda^{-c}$). For this reason, the masses resulting from the VEVs of G-non-singlet fields are generally different from those obtained by simple sum of the charges, given in Eq. (2). For example, by introducing the fields $\psi(16)$ and $\bar{T}(10)$, from the interaction $\lambda^{\psi+\bar{T}} \psi \bar{T} C$, we can obtain the mass of $\tilde{5}_\psi$ and $5_T$ of $SU(5)$ as $\lambda^{\psi+\bar{T}+c} \langle C \rangle \sim \lambda^{\psi+\bar{T}+\frac{1}{2}(c-\tilde{c})}$, which is generally not equal to the simply expected value, $\lambda^{\psi+\bar{T}}$. In such cases, the charges appearing in the mass matrices of superheavy fields can be replaced by effective charges, which are defined so that the relations

$$\langle C \rangle \sim \lambda^{-\tilde{c}}, \langle \bar{C} \rangle \sim \lambda^{-\tilde{c}} \tag{3}$$

are satisfied. Here, the effective charges $\tilde{c}$ and $\tilde{\tilde{c}}$ are related as $\tilde{c} = c + \Delta c = \tilde{\tilde{c}} = \tilde{c} - \Delta c = \frac{1}{2}(c + \tilde{c})$, with the discrepancy between these and the original charges given by $\Delta c \equiv \frac{1}{2}(\tilde{c} - c)$. This discrepancy can be understood as the effect of an additional $U(1)_V$ in the decomposition $SO(10) \rightarrow SU(5) \times U(1)_V$. The VEVs $|\langle C \rangle| = |\langle \bar{C} \rangle|$ that break $U(1)_V$ become the source of the new hierarchical structure (10). The unit of this new hierarchy is given by $\lambda^{\Delta c} (\equiv \lambda^{\Delta \tilde{c}})$. (Here, we normalize the $U(1)_V$ charge of $(C)$ to 1.) We can include the effect of the new hierarchical source by defining the effective charges $f$ for other fields $F$, using $U(1)_V$ charges $v_f$ as

$$\tilde{f} = f + v_f \Delta_{V}. \tag{4}$$

It is obvious that this definition of the effective charges does not change the VEV relation in Eq. (1), because the G-singlets $O_i$ have vanishing $U(1)_V$ charges. Note that the effective charges respect $SU(5)$ symmetry, because $U(1)_V$ respects this symmetry. The extension of the concept of effective charges to a more general situation is straightforward. If there are several Higgs fields that break $U(1)_V$, the unit of the new hierarchy can be defined by the Higgs fields with the largest VEVs. In the case that there are several $U(1)_k$ with the GUT gauge group $G \rightarrow SU(5) \times \prod_k U(1)_k$, the unit of the new hierarchy, $\lambda^{\Delta_{k}}$, can be defined for each $U(1)_k$ from the Higgs fields with the largest VEVs that break $U(1)_k$. Also, by defining the effective charges as $\tilde{x}_i \equiv x_i + \sum_k v_{x_k} \Delta_{k}$ for the superheavy fields $X_i$ with $U(1)_k$ charges $v_{x_k}$, their masses are easily evaluated as

$$\lambda^{x_i + \tilde{x}_i}, \tag{5}$$

unless the mass terms are forbidden by some mechanism, such as the SUSY zero mechanism. Therefore, the determinants of the mass matrices $M_I$ of superheavy fields, which appear in the expressions of the gauge coupling flows, are written $\det M_I = \lambda^{\Sigma_i x_i}$, where $I$ is the index for the SM irreducible representations. Note that $\det M$ can be calculated using the simple sum of the effective charges of the massive fields. The ratio of the determinants for each pair of SM multiplets $I$ and $I'$ (contained in each $SU(5)$ multiplet), $\frac{\det M_I}{\det M_{I'}}$, appears in the relations for gauge coupling unification. Because the effective charges respect $SU(5)$ symmetry, the contributions of the $SU(5)$ multiplet whose $I$ and $I'$ components are both massive cancel. Hence, only the effective charges of massive modes whose $SU(5)$ partners are massless contribute. This can be reinterpreted as meaning that only the effective charges of the massless modes appear in the ratios, that is,

$$\frac{\det M_I}{\det M_{I'}} = \frac{1/\lambda^{\Sigma_i x_i}}{1/\lambda^{\Sigma_i x_i'}}, \tag{6}$$

where $i$ runs over the massless modes.

**GAUGE COUPLING UNIFICATION**

Now, we carry out an analysis based on the renormalization group equations (RGEs) up to one loop. Here, we
consider the most general situation, in which the GUT symmetry $G$ is successively broken into $G_{SM}$ as

$$G(\equiv H_0) \xrightarrow{\lambda_1} H_1 \xrightarrow{\lambda_2} \ldots \xrightarrow{\lambda_N} G_{SM}(\equiv H_N). \quad (7)$$

First, the conditions for the gauge coupling unification are given by $\alpha_3(\Lambda) = \alpha_2(\Lambda) = \frac{3}{5} \alpha_Y(\Lambda) \equiv \alpha_1(\Lambda)$, and the gauge couplings at the cutoff scale $\Lambda$ are given by

$$\alpha_a^{-1}(\Lambda) = \alpha_a^{-1}(MSB) + \frac{1}{2\pi} \left( b_a \ln \left( \frac{MSB}{\Lambda} \right) \right. + \sum_i \Delta b_{ai} \ln \left( \frac{m_{ai}}{\Lambda} \right) \left. + \sum_n \Delta_{an} \ln \left( \frac{\Lambda_n}{\Lambda} \right) \right), \quad (8)$$

where $a = 1, 2, 3$, $MSB$ is the SUSY breaking scale, $(b_1, b_2, b_3) = (33/5, 1, -3)$ are the renormalization group coefficients of MSSM, $\Delta b_{ai}$ are the corrections to the coefficients caused by the massive fields with masses $m_{ai}$, and the last term is the correction due to the enhancement of the gauge symmetry above each symmetry breaking scale $\Lambda_n$:

$$\Delta_{an} = -3 T_a \left[ H_{n-1}/H_n \right] + T_a \left[ N_{Gn} \right] = -2 T_a \left[ N_{Gn} \right]. \quad (9)$$

Here, $NG_n$ denotes the NG modes that are absorbed through the Higgs mechanism at the scale $\Lambda_n$, and $T_a$ are the Dynkin indices, defined as $\text{Tr}(T_A T_B) = T[R] \delta_{AB}$, where $T_A$ are the generators in the $R$ representation. The second equality in Eq. (9) is derived from the relation $T_a \left[ H_{n-1}/H_n \right] = T_a \left[ N_{Gn} \right]$.

Then, using the fact that in MSSM the three gauge couplings meet at the scale $\Lambda_G \sim 2 \times 10^{16}$ GeV, the relations expressing unification, $\alpha_a(\Lambda) = \alpha_0(\Lambda)$, become

$$(b_a - b_b) \ln(\Lambda_G) + \sum_I (\Delta b_{aI} - \Delta b_{bI}) \ln(\text{det} M_I) + \sum_n (\Delta_{an} - \Delta_{bn}) \ln(\Lambda_n) = 0, \quad (10)$$

where $I$ runs over the SM irreducible representations. Because the sum of $\Delta b_{aI}$ over an $SU(5)$ multiplet is independent of $a$, the second term in Eq. (10) can be written in terms of the ratios of the determinants of the mass matrices in (6), and therefore in terms of the contributions from the massless modes, as mentioned above. In terms of the “effective mass” of massless modes, which is defined as $m_{\text{eff}} = \lambda^2 \bar{y}$, even if $\bar{x} + \bar{y} < 0$, the second term in (10) can be written

$$\sum_{i=\text{massless}} (T_a[i] - T_b[i]) \ln(1/m_{\text{eff}}). \quad (11)$$

These massless modes consist of two types, physical massless modes, such as the MSSM doublet Higgs $(H_u$ and $H_d)$, and unphysical NG modes. From (9), we can see that the contribution of the latter type is cancelled by that of the last term in Eq. (10) if the conditions

$$m_{\text{eff}}^{NG_n} \sim \Lambda_n^{-2} \quad (12)$$

hold. These conditions are satisfied when the vacuum structure satisfies (11), because $m_{\text{eff}}^{NG_n}$ is the coefficient of the bilinear term of the $n$-th NG modes, $\Phi$ and $\tilde{\Phi}$ ($\tilde{\phi} = \tilde{\phi}$), and therefore $m_{\text{eff}}^{NG_n} \sim \lambda^{5\delta}$, and from (13), $\Lambda_n \sim \lambda^{-5\delta}$.

When (12) holds, only the physical massless modes contribute to the conditions for the gauge coupling unification, and they are independent of the details of the Higgs sector, such as the field content and the symmetry breaking pattern. In particular, if all the fields other than those in MSSM become superheavy, only the MSSM doublet Higgs fields $H$ contribute, and we have

$$(b_a - b_b) \ln(\Lambda_G) + (\Delta b_{aH} - \Delta b_{bH}) \ln(1/m_{H_{\text{eff}}}) = 0, \quad (13)$$

for all combinations $(a, b)$. These relations lead to

$$\ln(\Lambda_G) = \ln(m_{H_{\text{eff}}}) = 0, \quad \text{and thus}$$

$$\Lambda \sim \Lambda_G, \quad \tilde{h}_u + \tilde{h}_d \sim 0. \quad (14)$$

The first relation here simply defines the scale of the theory: The cutoff scale $\Lambda$ is taken as the usual GUT scale, $\Lambda_G$. This is also the case in the minimal $SU(5)$ GUT, where the scale at which $SU(5)$ is broken is also taken as $\Lambda_G$. The second relation in (14) corresponds to that for the colored Higgs mass in the minimal $SU(5)$ GUT, because the effective colored Higgs mass is obtained as $m_{H_{\text{eff}}} \sim \lambda^{5\delta} \tilde{h}_d$. Therefore, we have no tuning parameters for the gauge coupling unification other than those in the minimal $SU(5)$ GUT. Note that $(\tilde{h}_u + \tilde{h}_d = 0)$ if we calculate gauge couplings at a low energy scale in the GUT scenario with any cutoff (for example, the Planck scale) and use them as the initial values, the three running gauge couplings calculated in MSSM meet at the cutoff scale. In this way, we can naturally explain the gauge coupling unification in the minimal $SU(5)$ GUT.

Note that the relation $\tilde{h}_u + \tilde{h}_d \sim 0$ does not imply $\tilde{h}_u + \tilde{h}_d = 0$, because there is an ambiguity involving $O(1)$ coefficients, and we have used only one loop RGEs. Using the VEV $(A)$, which breaks $SU(5)$ symmetry, the contribution to the mass of $X$ and $\bar{X}$ from higher-dimensional interactions, $\lambda^{x+y} \bar{X} A^n X$, is of the same order as that from the mass term $\lambda^{x+y} \bar{X} X$, because $\langle A \rangle \sim \lambda^{-a}$. Therefore the $O(1)$ coefficients do not respect $SU(5)$ symmetry. This contrasts with the usual situation, in which the contribution from higher-dimensional interactions is suppressed. The fact that the $O(1)$ coefficients do not respect $SU(5)$ symmetry allows a non-zero value of $\tilde{h}_u + \tilde{h}_d$. This is important, because it is necessary for $\tilde{h}_u + \tilde{h}_d$ to be negative in order to suppress proton decay via dimension 5 operators, the order of whose coefficients is determined by their effective charges and is independent of their origin (e.g. colored Higgs exchange or new physics around the cutoff scale). The suppression requires the effective colored Higgs mass $m_{H_{\text{eff}}} \sim \lambda^{5\delta} \tilde{h}_d \Lambda > O(10^{18}$ GeV), and therefore $\tilde{h}_u + \tilde{h}_d \leq -3$ is needed. Note that $m_{H_{\text{eff}}} > \Lambda$, but the physical masses of the colored Higgs are smaller than $\Lambda$. 


DISCUSSION AND SUMMARY

The argument given here is strongly dependent on the vacuum structure 11, which is naturally realized in the GUT scenario with anomalous 12 U(1)_A gauge symmetry 13. Our results can also be applied to previously studied GUT scenarios with anomalous U(1)_A symmetry 14, 15, distinct from that considered here, if all the VEVs of the G-singlets satisfy the VEV relations 16, even in the case that there are some flat directions.

We have shown that in the more general framework of GUT with anomalous U(1)_A gauge symmetry than in Ref. 13, the success of gauge coupling unification in the minimal SU(5) GUT is naturally explained. Usually, if we adopt a simple group whose rank is higher than that of the standard gauge group (for example, SO(10), E_6, SU(6), etc.), gauge coupling unification can always be realized by tuning the additional degrees of freedom related with the several scales of Higgs VEVs. However, we have shown that in the framework, all the charges of the Higgs fields, except that of the MSSM doublet Higgs, are cancelled in the relations for gauge coupling unification 17, and therefore we have no tuning parameters for the gauge coupling unification other than those in the minimal SU(5) GUT. We have elucidated the conditions for this cancellation with a group theoretical proof.

In the GUT scenario with anomalous U(1)_A symmetry, the unification scale is given by \( \Lambda_A \sim \lambda^{-a} \Lambda \), where \( a \leq 0 \) is the charge of the Higgs field whose VEV breaks the usual SU(5) gauge symmetry. Because the realization of gauge coupling unification requires the cutoff scale to be taken as the usual GUT scale, \( \Lambda_G \sim 2 \times 10^{16} \text{GeV} \), the negative charge \( a \) leads to a unification scale smaller than \( \Lambda_G \). Therefore, proton decay via dimension 6 operators can be enhanced, although that via the dimension 5 is suppressed when \( h_u + h_d \leq -3 \). If we choose \( a = -1 \) and \( \lambda \sim 0.22 \) as typical values \( 19 \), the proton lifetime can be roughly estimated, using a formula in Ref. 11 and a recent result provided by a lattice calculation for the hadron matrix element parameter \( \alpha \), as

\[
\tau(p \rightarrow e^+\pi^0) \sim 5 \times 10^{33} \left( \frac{\Lambda_A}{5 \times 10^{15} \text{GeV}} \right)^4 \left( \frac{0.015 \text{GeV}^3}{\alpha} \right)^2 \text{ yrs.}
\]

This value is near the present experimental limit 17. Thus, our study in this letter gives a strong motivation to search the proton decay \( p \rightarrow e^+\pi^0 \) in future experiments.

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18. We can use non-anomalous U(1) symmetry instead of anomalous U(1) symmetry if 3 conditions in the Introduction are satisfied. However, since we don’t know such models, we adopt anomalous U(1) symmetry in this letter.
19. Note that the slowest proton decay via dimension 6 operators is obtained when \( a = 0 \), and the value must be the same as that in the usual GUT scenario. However, when \( a = 0 \), generally terms of the form \( \int d^8A' W_\alpha W^\alpha \) are allowed, where \( W_\alpha \) is a SUSY field strength. This makes it impossible to realize natural gauge coupling unification. The most natural way to forbid these terms is to choose \( a \) to be negative, which leads to a shorter proton lifetime.