AN ANALYSIS OF 4-QUARK ENERGIES
IN SU(2) LATTICE MONTE CARLO

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March 25, 2022

Abstract

Energies of four-quark systems with the tetrahedral geometry measured by the static quenched SU(2) lattice monte carlo method are analyzed by parametrizing the gluon overlap factor in the form $\exp(-b_s E A + \sqrt{b_s F P})$, where $A$ and $P$ are the area and perimeter defined mainly by the positions of the four quarks, $b_s$ is the string constant in the 2-quark potentials and $E, F$ are constants.

1 Introduction

For the purpose of understanding meson-meson interactions from the QCD, we studied the ground and the 1st excited state energy of 4-quark system measured by the SU(2) lattice Monte-Carlo. The configuration of the four quarks that we consider are large square(LS), rectangular(R), tilted rectangular(TR), linear(L), quadrilateral(Q), non-planar(NP) and tetrahedral(TH).
There are three possible choices of the colour singlet pairs which are denoted by $A(14,23)$, $B(12,34)$ and $C(13,24)$. (We interchange the definition of $A,B$ from [1].) Identifying these bases as $P_1, P_2$ and $P_3$, we obtain the eigen-energies by diagonalizing $3 \times 3$ matrix of

$$W_{ij}^T = <P_i|\hat{T}^T|P_j>$$

where $\hat{T} = \exp(-a\hat{H})$ is the transfer matrix. Using a trial wave function $\psi = \sum_i a_i |P_i>$, the eigenvalue equation

$$W_{ij}^T a_j^T = \lambda^{(T)} W_{ij}^{-1} a_j^T$$

is solved to get $\exp(-aV_2) = \lambda^{(T)}$ for large $T[1]$.

2 The model with the gluon overlap factor

We observed that among the three bases, one base can be treated as a linear combination of the other two, but within a base in the coarse lattice, when the shortest path between a quark and an antiquark is not along any link, there are several possible configurations of links, which are specified by the parity. When there are $m(n)$ parity eigenstates in $A(B)$, we solve;

$$det \begin{pmatrix}
V_A & \cdots & c_{Am} & V_{ABf_{11}} & \cdots & V_{ABf_{1n}} \\
\vdots & \ddots & \vdots & \ddots & \ddots & \vdots \\
c_{Am1} & \cdots & V_A & V_{ABf_{m1}} & \cdots & V_{ABf_{mn}} \\
V_{ABf_{11}^*} & \cdots & V_{ABf_{1m}^*} & V_B & \cdots & c_{B1n} \\
\vdots & \ddots & \vdots & \ddots & \ddots & \vdots \\
V_{ABf_{n1}^*} & \cdots & V_{ABf_{nm}^*} & c_{Bn1} & \cdots & V_B
\end{pmatrix}$$

$$-E \begin{pmatrix}
1 & \cdots & 0 & f_{11}/2 & \cdots & f_{1n}/2 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 1 & f_{m1}/2 & \cdots & f_{mn}/2 \\
f_{11}/2 & \cdots & f_{1m}/2 & 1 & 0 & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
f_{n1}/2 & \cdots & f_{nm}/2 & 0 & 0 & 1
\end{pmatrix} = 0$$

We introduce the gluon overlap factor in the form $f_{ij} = \exp(-b_s E \mathcal{A} + \sqrt{b_s F_{ij} \mathcal{P}_{ij}})$, where $\mathcal{A}$ and $\mathcal{P}$ are the area and the perimeter. We assume that
the length of the zig-zag perimeter has the fractal dimension 2, and so it is fixed from that of the coarse lattice, use the string constant \( b_s \) measured in the simulation of the 2-quark potentials and fit constants \( E \) and \( F_{ij} \)\[2\]. The length \( P_{ij} \) depends on the base but the minimal area \( A \) are chosen to be independent of the base and estimated by the analytical form derived in the regular surface approximation. (The analysis of NP of\[2\] is revised, where the area depended on the bases.) The perimeter dependent terms contain lattice artefacts and the physical quantities are obtained by subtracting the artefacts.

The parameter \( E \) for the area term and \( A_0 \) for the self energy in the Linear configuration are fixed in the previous model\[4\]. In the NP case we fitted three coefficients \( F \) corresponding to the three types of the perimeter lengths and a mixing parameter between the parity eigenstates\[2\]. The details of the revised fitting are in \[4\].

In the case of TH, when the length of the links are all equal\( (r = d) \), the ground state energy is doubly degenerate. When they differ\( (r \neq d) \) we evaluate minimal surface area for \( A-C(\text{regular}_1) \) and for \( A-B(\text{regular}_2) \) and redefine among \( B \) and \( C \), the base \( B \) such that it is connected to \( A \) by the smaller area. We solve the secular equation of \( m=4 \) and \( n=4 \).

\[ \begin{array}{c}
A_1 \quad A_2 \quad A_3 \quad A_4
\end{array} \]

Figure 1: The bases \( A_1, A_2, A_3 \) and \( A_4 \) in the TH configuration.

We explain the degeneracy of the ground state energy for the \( r = d \) cases and the ground and the first excited state eigen-energies of \( r \neq d \) cases by introducing an additional coefficient \( F \) corresponding to the new type of perimeter length and a parameter that specifies the internal excitation of the two quark cluster.

3 Discussion and Conclusions

In an analysis of 4-quark energy of the tetrahedral geometry, Green and Pennanen\[3\] proposed a 6-basis model in which the ground \((A_{1g}, A_{1g})\) and an excited state \((E_u, E_u)\) are considered for each of the configuration A, B and
Table 1: Minimal surface area in regular surface approximation and the sum of two triangles for the TH and the experimental and the fitted binding energy.

| r,d | regular₁ | regular₂ | 2 triangles | $E_0$ | $E_0$(fit) | $E_1$ | $E_1$(fit) |
|-----|----------|----------|-------------|-------|------------|-------|------------|
| 1 2 | 4.316    | 3.180    | 4.899       | -0.043| -0.036     | 0.084 | 0.108      |
| 2 2 | 5.123    | 5.123    | 6.928       | -0.021| -0.023     | -0.021| -0.023     |
| 3 2 | 6.209    | 7.260    | 9.381       | -0.008| -0.011     | 0.148 | 0.139      |
| 2 3 | 10.22    | 8.528    | 12.369      | -0.040| -0.043     | 0.051 | 0.072      |
| 3 3 | 11.53    | 11.53    | 15.588      | -0.028| -0.027     | -0.028| -0.027     |
| 4 3 | 13.11    | 14.71    | 19.209      | -0.007| -0.012     | 0.110 | 0.115      |
| 3 4 | 18.69    | 16.45    | 23.324      | -0.041| -0.050     | -0.032| -0.032     |
| 4 4 | 20.49    | 20.49    | 27.713      | -0.030| -0.031     | -0.030| -0.031     |
| 5 4 | 22.56    | 24.71    | 32.496      | -0.010| -0.014     | 0.089 | 0.093      |

C. Our choice corresponds to $A_{1g}A_{1g}$, $E_uE_u$, $A_{1g}E_u$ and $E_uA_{1g}$ for each of the configuration A and B. Our estimation of the minimal surface is more accurate than their triangular approximation. The relatively large coefficients of $P$ suggests that the lattice artefact is still large.

References

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