The approach based on paradigm of self-organized criticality proposed for experimental investigation and theoretical modelling of software evolution. The dynamics of modifications studied for three free, open source programs Mozilla, FreeBSD and Emacs using the data from version control systems. Scaling laws typical for the self-organization criticality found. The model of software evolution presenting the natural selection principle is proposed. The results of numerical and analytical investigation of the model are presented. They are in a good agreement with the data collected for the real-world software.

The basic self-organization mechanisms of complex systems in the Nature are intensively studied last years. The proposed in the pioneering paper of P.Bak, C.Thang and K.Wiesenfeld paradigm of self-organized criticality (SOC) appeared to be most fruitful here. The SOC dynamics is characterized by avalanche-like changes of the system state with power law statistics of the avalanche growth. The main feature of the SOC regime is that it is an attractor of the system dynamics approached without any fine tuning of control parameters.

Studies of the fossil records have shown that the biological evolution is a strong non-equilibrium process with long periods of stasis interrupted by avalanches of large changes in biosphere. This is a main point of the punctuated equilibrium paradigm of biological evolution suggested by E.Gould and H.Eldredg. Detailed quantitative analysis of paleontological dates revealed the scaling power laws in distributions of avalanches in extinction and creations of species. Therefore the biological evolution can be considered as a kind of SOC dynamics. This has been demonstrated by P.Bak and K.Sneppen in the proposed model of Darwinian selection in ecosystem.

The development of computer science and engineering created the "virtual biosphere" with specific evolution laws of "virtual species" – computer programs. In this paper we propose an approach to the studies of software evolution in the framework of the SOC conception.

"Life" of large computer program is a perfect example of evolutionary process in complex system. During its creation the program often undergoes multiple internal reorganizations. New devices and platforms supported, new features added, system tuning performed, erroneous code corrected, huge number of cosmetic changes going on during the development of any program. Despite of the fact that the first papers on software evolution study are now decades old, the universal mechanisms of computer program evolution are unclear. The most of existing in this region research methodologies are based on assumption that estimations of possible changes in a program can be obtained without taking into account underlying dynamical laws creating this system. In the multitude of papers the authors propose statistical methods predicting the number of defects in a program using of some kind of metrics describing complexity, size, volume etc.

From our point of view the main disadvantage of such approach is that even the best in the world static metric which forecasts a number of improvements to be done in computer program correspond to a given specification, becomes useless if the specification changes in time essentially. Our approach can be considered as an elaboration of a prototype for dynamical metrics based on the use of characteristics of SOC universality class of the system.

There is a lot of phenomenological work has been done on software evolution. Lehman’s laws suggest that as system grows in size, it becomes increasingly difficult to add new code unless explicit steps are taken to reorganize the overall design. There were some systems examined both at system level and within the top-level subsystems. It has been noted that subsystems can behave quite differently from the system as whole. Good metaphors such as "code decay" has been proposed to describe the continuous process that makes the software more brittle over time. Thus the software evolution has many similar features with the evolution of biological species, and one can expect that evolution of large computer program presents some class of universality of the SOC dynamics.

To study software evolution processes it is necessary to have information about the state of the system in different moments of time. The usual sources of such data are various versions or releases of a product. Unfortunately, the number of releases rarely exceeds a couple of tens. This fact significantly decreases our possibility to study the evolution of program. The better sources of information about changes in computer programs are version control systems. One of them is Concurrent Versions System (CVS). It keeps information about changes happened in short time intervals.

Using the CVS in our work, we studied the histories of three software projects: Mozilla web browser, FreeBSD Operating System and Gnu Emacs text editor. For each of these projects we analyzed only files written in the basic for the project language.
and the exponents are the following:

- Free-BSD: $\mu_a = -1.44 \pm 0.02$, $\mu_d = -1.48 \pm 0.02$
- Mozilla: $\mu_a = -1.43 \pm 0.02$, $\mu_d = -1.47 \pm 0.02$
- Emacs: $\mu_a = -1.39 \pm 0.03$, $\mu_d = -1.49 \pm 0.04$

These scaling laws can be considered as a manifestation of the SOC in the evolution of software.

One of the important notion being used in description of the SOC dynamic is the avalanche. The SOC process can be presented as a consequence of meta-stable states interrupted by the avalanche-like changes in the system. For evolution of computer program the close analog of the avalanche is a set of changes going on from version to version. We see that the avalanche statistic in evolution of software is described by power functions with nontrivial exponents. The universality of SOC dynamical mechanisms allows one to hope that a simple ”holistic” model can be constructed for its quantitative description

To realize this idea for software evolution modeling we use the following assumptions. The specific of software changes is that one programmer can not modify a program at different points simultaneously (at least using a traditional development tools). The point of changes is characterized as ”weakest” one in the program text: a programmer has some subjective estimation of parts of a program and makes changes in place which is estimated as extremely non-satisfactory. If the change is made on some point, corresponding changes must be made in some other places, i.e. in the program there is a coordination structure of its elements. We suppose that changes in the program can’t make its size less that some minimal one.

We formulate the model presenting this conception as follows. Computer program as a system constitutes a sequence of elements – lines of code. At time point $t$ the $i-$th line is characterized by a number $b_i(t)$, $0 < b_i(t) < 1$ representing its ”fitness” in the program text or a barrier in respect to change in future stages of evolution. The state of the system of $N$ elements is fully given by the set of barriers $B(t) = \{b_i(t), i = 1, 2, ..., N\}$. The evolution of the program is described in our model as a sequence of $B(t)$ for discrete time points $t = 0, 1, 2, ...$. The coordination structure of program is presented by a network of its elements, where each element-node is conformed with its nears neighbors. The node having minimal barrier is defined as weakest unit of the system. At each time point $t$ we define the set $W(t)$ containing weakest unit with all its neighbors. We call $W(t)$ the weakest spoil at time $t$.

Dynamics in the model is defined in the following way. The initial number of nodes $N(0)$ and the minimal possible number of nodes $K$ are supposed to be given. The initial values of barriers $b_i(0)$ are chosen at random. The state $B(t)$ at time point $t$ transforms into state $B(t+1)$ as follows. If the number of nodes $N(t)$ in the system is more than $K$ two kinds of changes are possible. With probability $\alpha$, the weakest unit is deleted from the system or with probability $1 - \alpha$ a new neighbor node to the weakest unit is inserted into the system. After that
the barriers of all nodes from weakest spoil \( W(t) \) are set random. So if \( N(t) > K \), the size of the system decreases or increases by one for one time step. If \( N(t) = K \), then deletion is impossible and the above described insertion is made.

Our model is a modification of well known Simple Model of Biological Evolution suggested by Bak and Sneppen \cite{23, 24, 25}, and its essential specific is that the number of system elements varies in time. In our study we have considered two versions of the model: with 1-dimensional (1D) and random neighbor (RN) coordination structure. In the 1D case the nodes are organized into 1D lattice with periodic boundary condition, and each node has two neighbors. In RN model, there is no fixed coordination structure in the system, and at each time step \( k \) random nodes are chosen as neighbors of weakest unit. We have considered the case \( k = 1 \) only.

An avalanche as the elementary process of complex behavior of non-equilibrium dynamical system can be defined in different ways. Usually in the model of SOC dynamics the \( \lambda \)- and transient avalanches are considered \cite{3, 21, 22}. In studies of our model we were interested mostly in transient avalanches. They can be defined as follows. Let at the time moment \( t_0 \) the minimal barrier has the value \( f_0 \). The sequence of \( S \) time steps during which the minimal barrier does not exceed \( f_0 \): \( b_{\min}(t) < f_0, t_0 < t < t_0 + S \) is called transient avalanche or just avalanche if it finishes at the time point \( t_0 + S \) when the value of minimal barrier becomes larger than \( f_0 \): \( b_{\min}(t + S) > f_0 \). Distribution \( P(S) \) of avalanche temporal duration and distribution \( P(R) \) of avalanche spatial volume are important characteristics of the type of dynamics. For our model it is reasonable to consider two values as characteristics of volume of changes produced in the system by avalanche. One of them is the number \( A \) of new elements appeared in the system at the end of avalanche. Other is the number \( D \) of elements disappeared from the system at the end of the avalanche. In dynamic of our model we studied mostly the distributions \( P(S), P(A), P(D) \) of temporal and spatial characteristics of avalanches.

We studied numerically the 1D and RN versions of the model for \( \alpha = \frac{1}{2} \). The initial size of the system was 8000 elements. The experiment went on until one million of avalanches were registered. We got the following results. The \( P(S), P(A), P(D) \) distributions can be sufficiently approximated by the power functions \( P(S) \sim S^\gamma, P(A) \sim A^\alpha, P(D) \sim D^\beta \) with exponents \( \tau = -1.358 \pm 0.005, \mu_a = 1.45 \pm 0.01, \mu_d = -1.47 \pm 0.02 \) for the 1D model and \( \tau = -1.901 \pm 0.008, \mu_a = 1.98 \pm 0.01, \mu_d = -2.10 \pm 0.02 \) for the RN model.

For the RN model it is possible to obtain analytical description in the framework of master equation formalism. To do it one can use the method of construction of master equation proposed for analysis of the SOC dynamic of random neighbor version of Bak-Sneppen model \cite{22}. If we denote \( P_{n,N}(t) \) the probability that at time point \( t \) there are \( N \) nodes in the system, and \( n \) of ones have barriers less than \( \lambda \), where \( 0 < \lambda < 1 \), then the dynamical rules of RN model result in the following master equation

\[
P_{n,N}(t + 1) = (\alpha + \beta \delta_{N,K+1})P_{n,N}(t) + \beta P_{n,N}(t)
\]

where \( \beta = 1 - \alpha \), and in terms of \( \mu = 1 - \lambda, \rho_{n,N} = (n - 1)/(N - 1), \sigma_{n,N} = 1 - \rho_{n,N} \) the quantities \( P_{n,N}(t) \) and \( P_{n,N}(t) \) can be presented by the following relations:

\[
P_{n,N}(t) = A_n^a P_{n+2,N-1}(t) + B_n^a P_{n+1,N-1}(t) + C_n^a P_{n,N-1}(t) + D_n^a P_{n-1,N-1}(t) + E_n^a P_{n-2,N-1}(t) + f_{n-2,N-1}(t) + (\mu \alpha \delta_{n,0} + 3 \lambda \mu^2 \delta_{n,1} + 3 \lambda^2 \mu \delta_{n,2} + \lambda^3 \delta_{n,3}) P_{n-1,N}(t),
\]

\[
P_{n,N}^d(t) = A_n^d P_{n+2,N+1}(t) + B_n^d P_{n+1,N+1}(t) + C_n^d P_{n,N+1}(t) + D_n^d P_{n-1,N+1}(t) + E_n^d P_{n-2,N+1}(t) + f_{n-2,N+1}(t) + (\mu \delta_{n,0} + \lambda \delta_{n,1}) P_{n,N+1}(t),
\]

\[
A_{n,N} = \mu^3 \rho_{n,N}, \quad B_{n,N} = 3 \mu^2 \rho_{n,N} + \mu^3 \sigma_{n,N}, \quad C_{n,N} = 3 \mu \rho_{n,N} + \mu^3 \rho_{n,N}, \quad D_{n,N} = 3 \lambda^2 \rho_{n,N} + \lambda^3 \rho_{n,N}, \quad E_{n,N} = \lambda^3 \sigma_{n,N}, \quad A_{n,N}^d = \mu \rho_{n,N}, \quad B_{n,N}^d = \mu \sigma_{n,N} + \lambda \rho_{n,N}, \quad C_{n,N}^d = \lambda \rho_{n,N}.
\]

Here, the coefficients \( A_{n,N}^a, B_{n,N}^a, C_{n,N}^a, D_{n,N}^a, E_{n,N}^a, A_{n,N}^d, B_{n,N}^d, C_{n,N}^d, \) are defined in the last two lines for \( 0 < n \leq N \). For \( n \leq 0 \) and \( n > N \) they assumed to be zero. The master equation (1) enables one to find \( P_{n,N}(t) \) for \( t > 0 \), if initial values \( P_{n,N}(0) \) are given. Basing on this equation one can obtain analytical results for characteristics of dynamics in RN model. With that end in view it is convenient to use the formalism of generating function appeared to be very effective for construction of exact solution for master equations of RN version of Bak-Sneppen model \cite{22, 24, 25, 26}. Dynamic of RN model is more complex than one of Bak-Sneppen model, and solution of master equation for RN model for software evolution appears to be not easy problem. Here, we present only the exact result for \( P_N(t) = \sum_{n=0}^{N} P_{n,N}(t) \) being the probability that the system has \( n \) element with barriers less then \( \lambda \) at time point \( t \). Let us denote \( N(y, u) \) the generating function for probabilities \( P_N(t) \):

\[
N(y, u) = \sum_{N=K, t=0}^{\infty} P_N(t) y^{-K} u^t.
\]

From (1) we obtain the following equation for \( N(y, u) \):

\[
N(y, u) = y u \mathcal{N}(y, 0) + u \beta y^2 - 1) \mathcal{N}(0, u).
\]
It describes 1-dimensional discrete diffusion with reflection and can be solved by methods, used in [23, 24, 25, 26]. The result has the form
\[ N(y, u) = \frac{yN(y, u) - u\alpha \tau N(\tau, 0)(y^2 - 1)}{(u - \tau)(y - u(\alpha^2 + \beta))} \]
where \( \tau = (1 - \sqrt{1 - 4u^2\alpha\beta})/2u\alpha \) is the analytical in the point \( u = 0 \) solution of equation \( \tau = u(\alpha^2 + \beta) = 0 \).
Mean value \( n(t) = K + \sum N P_N(t)N \) of the system element number at time point \( t \) has the following asymptotic for large \( t \): \( n(t) = (2\alpha - 1)t \) for \( \alpha > 1/2 \), \( n(t) \equiv \sqrt{T/\pi} \) for \( \alpha = 1/2 \), and if we denote \( p_{ev} \) \( (p_{od}) \) the probability that the initial number \( N(0) \) of nodes is even (odd), then
\[ n(t) \approx K + [1 + (-1)^t + K(1 - 2\alpha)^2(p_{ev} - p_{od})]/[2(1 - 2\alpha)] \]
for \( \alpha < 1/2 \). The corrections to the leading terms of asymptotic are of the form: \( n(0) - \frac{2\alpha^2}{\sqrt{\pi}} + f(t)g_l(t) \), for \( \alpha > 1/2 \), \( t^{-1/2}g_l(t) \) for \( \alpha = 1/2 \) and \( f(t)g_l(t) \), for \( \alpha < 1/2 \). Here \( f(t) = [4\alpha\beta]^{1/2}/t^{-3/2} \) and \( g_l(t), i = 1, 2, 3 \) are bounded for large \( t \), i.e., there are constants \( T, M \) that \( g_l(t) < M \), if \( t > T \). Since \( 4\alpha\beta < 1 \) for \( \alpha \neq 1/2 \), the function \( f(t) \) decreases exponentially fast for large \( t \).

The asymptotic behavior of \( n(t) \) demonstrates the dynamical phase transition at the point \( \alpha = 1/2 \). For \( \alpha < 1/2 \), the volume of system remains finite. For \( \alpha > 1/2 \) it can became as large as one likes. At the point \( \alpha = 1/2 \), the dynamics of the system is critical one.

In above formulated model we tried to present elementary mechanisms of software changes. They are made by programmer locally in the place where these changes most of all needed. But a program changed in one place often must be changed in other places in some way connected to the first one. For example, in order to change the number of arguments of subroutine call, one needs to change not only the line containing the call operator but the definition of the subroutine either. This would lead to some subsequent changes of all the calls to the subroutine in all the program. If one adds the line in which some data read from a disk one should add some lines to check whether the data have been read successfully, and this in turn can require some change in the list of the modules included which in turn can cause a name conflict which in turn can cause other changes, etc. Thus, the avalanche-like processes seems to be natural for modifications of programs. Avalanche ends up when all the parts of the program code are more or less satisfy some subjective and implicit criteria of programmer. Naively speaking, the program as whole becomes "a little bit better". In the model it can be presented as a process terminating when the value of minimal barrier becomes greater than initial one. This was the point why we studied transient avalanches of self organization period and not the \( \lambda \)-avalanches of the stationary mode. The obtained statistical characteristics of avalanches make it possible to conclude that SOC is the dominating dynamical regime in evolution of free software. Our results demonstrate that the natural selection can create this type of "punctuated equilibrium" of such complex "virtual beings" in info-sphere. We believe that in the framework of proposed approach the modern methods of investigation of the SOC dynamics can appear to be very effective for studies of basic problem of software evolution. Our results could be seen also as a theoretical prerequisite for the development of new tools and methods for advanced measures of software quality engineering.

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