Universal Atom Interferometer Simulator - Elastic Scattering Processes

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ABSTRACT

In this article, we introduce a universal simulator covering all regimes of matter wave light-pulse elastic scattering. Applied to atom interferometry as a study case, this simulator solves the atom-light diffraction problem in the elastic case i.e. when the internal state of the atoms remains unchanged. Taking this perspective, the light-pulse beam splitting is interpreted as a space- and time-dependent external potential. In a shift from the usual approach based on a system of momentum-space ordinary differential equations, our position-space treatment is flexible and scales favourably for realistic cases where the light fields have an arbitrary complex spatial behaviour rather than being mere plane waves. Moreover, the numerical package we developed is effortlessly extended to the problem class of trapped and interacting geometries, which have no simple formulation in the usual framework of momentum-space ordinary differential equations. We check the validity of our model by revisiting several case studies relevant to the precision atom interferometry community. We retrieve analytical solutions when they exist and extend the analysis to more complex parameter ranges in a cross-regime fashion. The flexibility of the approach, the insight it gives, its numerical scalability and accuracy make it an exquisite tool to design, understand and quantitatively analyse metrology-oriented matter-wave interferometry experiments.

Introduction and motivation

The commonly used approach for treating light-pulse beam-splitter and mirror dynamics in matter-wave systems consists in solving a system of ordinary differential equations (ODE) with explicit couplings between the relevant momentum states. This formulation starts by identifying the relevant diffraction processes and extracting their corresponding coupling terms in the ODE\textsuperscript{1,2}. In the elastic scattering case, each pair of light plane waves can drive a set of two-photon transitions from one momentum class $m$ to the next neighboring orders $m \pm 2$. The presence of multiple couplings allows for higher order transitions and the system is simplified by choosing a cutoff omitting small transition strengths. This ODE approach works well for simple cases leading to analytical solutions in the deep Bragg and Raman-Nath regimes\textsuperscript{1,2}. Using a perturbative treatment, it was generalised to the intermediate, so-called quasi-Bragg regime\textsuperscript{1}. A numerical solution in this regime has been extended in the case of a finite momentum width\textsuperscript{4}. Realistically distorted light beams, however, sharply increase the number of plane wave states and their couplings required for an accurate description. The formulation of the ODE becomes increasingly large and inflexible, with a set of coupling terms for each relevant pair of light plane waves. In a different approach, Siemß et al.\textsuperscript{5} developed an analytic theory for Bragg atom interferometry based on the adiabatic theorem for quasi-Bragg pulses.

Here, we take a different approach and solve the system in its partial differential equation (PDE) formulation following the Schrödinger equation. This can have several advantages in terms of ease of formulation and implementation, flexibility and numerical efficiency for a broad range of cases. Indeed, this treatment is valid for different types of beam splitters (Bloch, Raman-Nath, deep Bragg and any regime in between) and pulse arrangements. Combining successive light-pulse beam-splitters naturally promoted our solver to a cross-regime or universal atom interferometry simulator that could cope with a wide range of non-ideal effects such as light spatial distortions or atomic interactions, yet being free of commonly-made approximations incompatible with a metrological use.

The position-space representation seems underutilised in the treatment of atom interferometry problems in favor of the momentum-space description although several early attempts of using it were reported for specific cases\textsuperscript{5-10}. In this paper, we show the unique insights this approach can deliver, its great numerical precision and scalability and we illustrate our study with relevant examples from the precision atom interferometry field.
Theoretical model

Light-pulse beam splitting as an external potential

We start with a semi-classical model of Bragg diffraction, where a two-level atom is interacting with a classical light field\(^1,2\). This light field consists of a pair of two counter-propagating laser beams realised by a retro-reflection mirror setup for example. Assuming that the detuning of the laser light \(\Delta\) is much larger than the natural line width of the atom, one may perform the adiabatic elimination of the excited state. This yields an effective Schrödinger equation for the lower-energy atomic state \(\psi(x,t)\) with an external potential proportional to the intensity of the electric field

\[
\textit{i} \hbar \partial_t \psi(x,t) = \left( -\frac{\hbar^2}{2M} \frac{\partial^2}{\partial x^2} + 2\hbar \Omega \cos^2(kx) \right) \psi(x,t)
\]  

with the two-photon Rabi frequency \(\Omega = \Omega_0^2 / 2\Delta\) and wave vector \(k = 2\pi / \lambda\) in a simplified 1D geometry along the x-direction.

For the present study, we consider a \(^{87}\text{Rb}\) atom that is addressed at the D2 transition with \(\lambda = 780\ \text{nm}\) resulting in a recoil frequency and velocity\(^3\) of \(\omega_r = \hbar k^2 / 2m = 2\pi \cdot 3.8\ \text{kHz}\) and \(v_r = \hbar k / m = 5.9\ \text{mm/s}\) respectively.

In the context of realistic precision atom interferometric setups, it is necessary to include Rabi frequencies \(\Omega(x,t)\) and wave vectors \(k(x,t)\) which are space- and time-dependent. This allows to account for important experimental ingredients such as the Doppler detuning or the beam shapes including wavefront curvatures\(^12-14\) and Gouy phases\(^15-18\). Moreover, this generalisation allows to effortlessly include the superposition of more than two laser fields interacting with the atoms as in the promising case of double-Bragg diffraction\(^19-21\) and to model complex atom-light interaction processes where spurious light reflections or other experimental imperfections are present\(^22\).

Atom interferometer geometries

The light-pulse representation presented in the previous section is the elementary component necessary to generate arbitrary geometries of matter-wave interferometers operating in the elastic diffraction limit. Indeed, since the atom-light interaction in this regime conserves the internal state of the atomic system, a scalar Schrödinger equation is needed to describe the physics of the problem in contrast to the model adopted in reference\(^9\).

For example, a Mach-Zehnder-like interferometer geometry can be generated by a succession of \(\pi/2 - \pi - \pi/2\) Bragg pulses (beam-splitter, mirror, beam-splitter pulses) of order \(n\) separated by a free drift time of \(T\) between each pair of pulses. In the case of Gaussian temporal pulses, this leads to a time-dependent Rabi frequency

\[
\Omega(t) = \Omega_{bs} e^{-(t - T)^2 / 2\tau_{bs}^2} + \Omega_m e^{-(t - T)^2 / 2\tau_m^2} + \Omega_{bs} e^{-(t - T)^2 / 2\tau_{bs}^2},
\]

where \(\Omega_{bs}, \tau_{bs}\) and \(\Omega_m, \tau_m\) are the peak Rabi frequencies and their respective durations associated to the beam-splitter and mirror pulses, respectively. We numerically solve the corresponding time-dependent Schrödinger equation using the split-operator method\(^23\) to propagate the atomic wave packets along the two arms. The populations in the two output ports \(|+\rangle = |\hbar k\rangle\) and \(|-\rangle = |2\pi \hbar k\rangle\) are evaluated after the last recombination pulse waiting for a time of flight \(\tau_{\text{ToF}}\) long enough such that the atomic wave packets spatially separate. They are obtained by the integration

\[
P_{\text{annnormalised}} = \int_{\pm} \text{d}x |\psi(x, \tau_{\text{ToF}})|^2,
\]

where the integration domains extend over a space interval with non-vanishing probability density of the states \(|\pm\rangle\). These probabilities are further normalised to account for the loss of atoms to other parasitic momentum classes

\[
P_{\pm} = \frac{P_{\text{annnormalised}}}{P_{\text{annnormalised}} + P_{\text{annnormalised}}},
\]

Using Feynman’s path integral approach, the resulting phase shift between the two arms can be decomposed as\(^24,25\)

\[
\Delta \phi = \Delta \phi_{\text{propagation}} + \Delta \phi_{\text{laser}} + \Delta \phi_{\text{separation}}.
\]

The propagation phase is calculated by evaluating the classical action along the trajectories of the wave packets centers. The laser phase corresponds to the accumulated phase imprinted by the light pulses at the atom-light interaction position and time. Finally, the separation phase is different from zero if the final wave packets are not overlapping at the time of the final beam splitter, \(t = 2T\).
Figure 1. Probability density after a Raman-Nath pulse with \( \Omega = 50 \omega_r \), \( \tau = 1 \mu s \) and a rectangular temporal profile as implemented in [26]. This shall create a beam splitter of roughly 50% in \(|0\hbar k\rangle\) and 25% in each of the \(|\pm 2\hbar k\rangle\) momentum states with an added time of flight of \( \tau_{ToF} = 20 \text{ ms} \) to clearly separate the wavepackets in position space. The left and right panels show the position- and momentum-space probability density. The initial momentum width of the Gaussian wavepacket is chosen to be \( \sigma_p = 0.01 \hbar k \). Numerical results of this work (continuous blue lines) agree well with the analytical solution of the Raman-Nath regime (green dots, momentum space) given by the Bessel functions of the first kind.

To extract the relative phase \( \Delta \phi \) between the two conjugate ports and the contrast \( C \), one can scan a laser phase \( \phi_0 \in [0, 2\pi] \) at the last beam splitter and evaluate the populations\(^1\) varying as

\[
P_{\pm} = \frac{1}{2} (1 \pm C \cos(\Delta \phi + n\phi_0)).
\]

The resulting fringe pattern is then fitted with \( \Delta \phi \) and \( C \) as fit parameters. This method, analogous to experimental procedures, allows to determine the relative phase modulo \( 2\pi \).

**Results**

**Raman-Nath Beam Splitter**

The Raman-Nath regime, characterised by a spatially symmetric beam splitting, is the limit of elastic diffraction for very short interaction times of \( \tau \ll \frac{1}{\sqrt{2\Omega \omega_r}} \). The dynamics of the system can, in this case, be analytically captured following references [1,2]

\[
|g_n(t)|^2 = J_n^2(\Omega r),
\]

where \( g_n(t) \) describes the amplitude of the momentum state \(|2n\hbar k\rangle\) and \( J_n \) the Bessel functions of the first kind. Such experiments are at the heart of investigations such as the one reported in reference [26] where a Raman-Nath beam splitter was used to initialise a three-path contrast interferometer offering the possibility to measure the recoil frequency \( \omega_r \).

To demonstrate the validity of our position-space approach, we contrast our results to the analytical ones obtained adopting the parameters of reference [26]. Fig. 1 shows the outcome of a symmetric Raman-Nath beam splitter targeting the preparation of three momentum states: 50% into \(|0\hbar k\rangle\) and 25% in each of the \(|\pm 2\hbar k\rangle\) momentum classes. As a feature of our solver, we directly observe the losses to higher momentum states (\( p = \pm 4\hbar k \) and \( p = \pm 6\hbar k \)) due to the finite pulse fidelity. An excellent agreement is found with the analytical predictions (green filled circles) of the populations of the momentum states.

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Figure 2. Space-time diagram of the probability density $|\psi(x,t)|^2$ of a $2\hbar k$-Bragg Mach-Zehnder interferometer. The initial momentum width is chosen to be $\sigma_p = 0.1 \hbar k$ and the splitter and mirror Gaussian pulses have peak Rabi frequencies of $\Omega = 1.0573 \omega_r$ with pulse lengths of $\tau_s = 25 \mu s$ and $\tau_m = 50 \mu s$, respectively. The separation time between the pulses is $T = 10 \text{ ms}$ with a final time of flight after the exit beam splitter of $\tau_{\text{ToF}} = 20 \text{ ms}$. Due to the velocity selectivity of the Bragg pulses, several trajectories can be observed after each pulse. \textit{Inset:} Additional insight into the dynamics of the mirror pulse of the upper arm. The peak amplitude of the Gaussian pulse is reached at $t = 11 \text{ ms}$. The interference fringes of the density plot indicate the overlap between the atoms in momentum class $p = 2 \hbar k$ and the atoms lost due to velocity selectivity remaining at $p = 0 \hbar k$.

Bragg-diffraction Mach-Zehnder interferometers

To simulate a Mach-Zehnder atom interferometer based on Bragg diffraction, we consider a pair of two counter-propagating laser beams with a relative frequency detuning $\Delta \omega = \omega_1 - \omega_2 = 2nkv_r$ and a phase jump $\phi_0 \in [0, 2\pi]$. This gives rise to the following running optical lattice

$$V_{\text{Bragg}}(x,t) = 2\hbar \Omega(t) \cos^2(k(x - nv_r t) + \frac{\phi_0}{2}).$$ (8)

For sufficiently long atom-light interaction times, i.e. in the quasi- and deep-Bragg regimes\textsuperscript{2,3,27,28}, the driven Bragg order $n$ with momentum transfer $\Delta p = 2n\hbar k$ is determined by the relative frequency detuning $\Delta \omega$ of the two laser beams. The relative velocity between the initially prepared atom and the optical lattice is $v = nv_r$. In the rest frame of the optical lattice, the atom has a momentum $p = -n\hbar k$. The difference of kinetic energy between the initial ($p = -n\hbar k$) and target state ($p = +n\hbar k$) is vanishing and therefore this transition is energetically allowed and leads to a $\Delta p = n\hbar k - (-n\hbar k) = 2n\hbar k$ momentum transfer.

We now realise beam splitters and mirrors by finding the right combination of peak Rabi frequency and interaction time ($\Omega, \tau$), either by numerical population optimization or analytically, when we work in the deep Bragg regime. Recent advances by Siemß \textit{et al.}\textsuperscript{5} generalise this to the quasi-Bragg regime in an analytical description of Bragg pulses based on the adiabatic theorem. For the pulses used in this paper, the two approaches give the same result for the optimized Rabi frequencies and pulse durations.

In Fig. 2, we simulate a Mach-Zehnder geometry and illustrate the diffraction outcome by showing a space-time diagram of the density distribution $|\psi(x,t)|^2$. For the parameters chosen here, a clear feature of the dynamics is the appearance of additional atomic channels after the mirror pulse, which can be attributed to the velocity selectivity arising from a pulse with a finite duration characterised by $\tau$. The finite velocity acceptance can, indeed, be estimated over the Fourier width $\sigma_f$ of the
applied pulse as
\[ \mathcal{F}(\Omega e^{\frac{i t}{2\pi}}) = \sqrt{2\pi \tau^2 \Omega^2 e^{-2(i \omega t)^2}}, \]
with \( \sigma_f = 1/(2\pi \tau) \) and \( f \) being the frequency variable. This yields the velocity acceptance
\[ \sigma_v^{\text{pulse}} = \frac{1}{8 \omega_c \tau} v_r = 0.11 v_r. \]

With an initial velocity width of the atomic probability distribution of \( \sigma_v^{\text{atom}} = 0.1 v_r \), it is clear that velocity components with \(|v| = \sigma_v^{\text{pulse}}\) will have a much smaller excitation probability than the components at the center of the cloud, which leads to the characteristic double well densities of the parasitic trajectories.

With momenta \( p_{\text{upper}} = 0 \hbar k \) and \( p_{\text{lower}} = 2\hbar k \), both parasitic trajectories still fulfill the resonance condition with the final Bragg beam splitter, which leads to the emergence of ten trajectories after the exit beam splitter. For a measurement in position space, it is now important that a sufficiently long time of flight \( \tau_{\text{ToF}} \) is applied such that the ports of the Mach-Zehnder interferometer do not overlap with the parasitic ports and bias the relative phase measurement. For large densities, the parasitic trajectories at the Mach-Zehnder ports should not overlap since this may already lead to density interaction phase shifts \( H_{\text{int}} \sim |\psi(x,t)|^2 \). To circumvent these problems it is important to choose \( \sigma_v^{\text{pulse}} \gg \sigma_v^{\text{atom}} \). An example of state-of-the-art experiments\(^{22}\) with delta-kick collimated BEC sources\(^{30-35}\) uses \( \sigma_v^{\text{atom}} = 0.03 v_r \ll 0.14 v_r = \sigma_v^{\text{pulse}} \), for strongly suppressed parasitic trajectories due to velocity selectivity.

Implementing high-order Bragg diffraction is a natural avenue to increase the momentum separation of an atom interferometer, and therefore its sensitivity. In Fig. 3, we run our solver to observe the population distribution across the different ports of a Mach-Zehnder configuration with Bragg orders up to \( n = 3 \). This is done in a straightforward way by scanning the laser phase \( \phi_0 \). We fit the data points corresponding to the population in the fast port \( 2\hbar k \) for the different Bragg orders according to Eq. (6) and observe a clear sinusoidal signal of the simulated fringes, as expected. The resulting contrasts and phase shifts are directly found by our theory model and numerical solver which include the ideal phase shifts commonly found\(^{24,25}\) and go beyond to comprise several non-ideal effects as (i) finite momentum widths, (ii) finite pulse timings and (iii) multi-port Bragg diffraction\(^{36,37}\) and the resulting diffraction phase. The natural occurrence of these effects and the possibility to quantify them are a native feature of our simulator.

**Symmetric double-Bragg geometry**

Scalable and symmetric atom interferometers based on double-Bragg diffraction were theoretically studied\(^{21}\) and experimentally demonstrated\(^{20}\). This dual-lattice geometry has particular advantages, including an increased sensitivity due to the doubled scale factor compared to single-Bragg diffraction, as well as an intrinsic suppression of noise and certain systematic uncertainties due to the symmetric configuration\(^{20}\). Combining this technique with subsequent Bloch oscillations applied to the two interferometer arms, led to reaching momentum separations of thousands of photon recoils as it was recently shown in reference [22].

In double-Bragg diffraction schemes, two counter-propagating optical lattices are implemented such that the recoil is simultaneously transferred in opposite directions, leading to a beam splitter momentum separation of \( \Delta p = 4\hbar k \).\(^{20,21}\) To extend our simulator to this important class of interferometers, we merely have to add a term to the external potential
\[ V_{\text{Double Bragg}}(x,t) = 2\hbar \Omega(t) \left( \cos^2(k(x-nv_f t)) + \cos^2(k(x+nv_f t)) \right). \]

The procedures of realising a desired \( 4\hbar k \) momentum transfer as well as mirror or splitter pulses are identical to the case of single-Bragg diffraction. A simple scan of the Rabi frequency and pulse timings was enough to obtain a full double-Bragg interferometer as shown in Fig. 4. The different resulting paths are illustrated in this space-time diagram of the density distribution \( |\psi(x,t)|^2 \). Similarly to the single-Bragg Mach-Zehnder interferometer we observe additional parasitic interferometers due to the finite velocity filter of the Bragg pulses after the mirror pulse of the interferometer. Due to a finite fidelity of the initial beam splitter, some atoms remain in the \( |0\hbar k \rangle \) port and recombine at the last beam splitter with the trajectories of the interferometer. In a metrological study, these effects are highly important to quantify. Our simulator gives access to all the quantitative details of such a realisation in a straightforward fashion.

**Gravity gradient cancellation for a combined Bragg and Bloch geometry**

Precision atom interferometry-based inertial sensors are sensitive to higher order terms of the gravitational potential, including gravity gradients. In particular, for atom interferometric tests of Einstein’s equivalence principle (EP), gravity gradients pose a challenge by coupling to the initial conditions, \( i.e. \) position and velocity of the two test isotopes\(^{38}\). A finite initial differential
position or velocity of the two species can, if unaccounted for, mimic a violation of the EP. By considering a gravitational potential of the form

$$V(x) = -mgx - \frac{1}{2}m\Gamma x^2,$$

where $\Gamma = \Gamma_{ex}$ is the gravity gradient in the direction normal to the Earth’s surface, the relative phase of a freely falling interferometer can be calculated as

$$\Delta \phi = k_{eff}|\mathbf{g} - a_{\text{Bragg}}|T^2 + k_{eff}\Gamma(x_0 + v_0T)T^2,$$

with $k_{eff} = 2nk$.

In reference [39], it was shown that introducing a variation of the effective wave vector $\Delta k_{eff} = \Gamma k_{eff}T^2/2$ at the $\pi$ pulse can cancel the additional phase shift due to the gravity gradient. This was experimentally demonstrated in reference [40].

The same principle applies to the gradiometer configuration of left panel in Fig. 5 where the effect of a gravity gradient is compensated by the application of a wave vector correction. This is reminiscent of another experimental cancellation of the gravity gradient phase shifts[41]. In our example, we first consider a set of two Mach-Zehnder interferometers vertically separated by 2 meters, realised with $4\hbar k$ Bragg transitions where the atoms start with the same initial velocities $v_0$. Choosing a Doppler detuning according to $a_{\text{Bragg}} = g$, the gradiometric phase reads

$$\Phi = 4k\Gamma h T^2.$$

By scanning the momentum of the applied $\pi$ pulse, one can compensate the gradiometric phase. This is observed in our simulations at the analytically predicted value of $\Delta k_{eff} = \Gamma k_{eff}T^2/2$ (red dashed curve crossing the zero horizontal line).

It is particularly interesting to use our simulator to find this correction phase in the context of more challenging situations, such as a combined scalable Bragg and Bloch Mach-Zehnder interferometer where analytic solutions are not easily found.
Figure 4. Space-time diagram of a symmetric double-Bragg interferometer. The probability density $|\psi(x,t)|^2$ is plotted for an initial momentum width of $\sigma_p = 0.1 \hbar k$. The timings of the Gaussian splitter and mirror pulses are set to $\tau_{bs} = 25 \mu s$ and $\tau_m = 50 \mu s$, respectively. The corresponding Rabi frequencies are found by optimising the desired population transfer. The first $\frac{\pi}{2}$ pulse corresponds to a $2\hbar k$ transfer in two directions, realised by two counter-propagating optical lattices which results in a $4\hbar k$ separation between the two interferometer arms. The mirror pulse is a $4\hbar k$ Bragg transition with a Rabi frequency of $\Omega = 1.9\omega_k$ such that both arms make a transition from $|\pm 2\hbar k\rangle \rightarrow |\mp 2\hbar k\rangle$. The last recombination pulse now realises a $50:50$ split of the upper trajectory to $|−2\hbar k\rangle$ and $|0\hbar k\rangle$ and the lower trajectory to $|+2\hbar k\rangle$ and $|0\hbar k\rangle$. This leads to a final population of 25% in the $|\pm 2\hbar k\rangle$ ports and 50% in the $|0\hbar k\rangle$ port. The separation time between the pulses is $T = 10$ ms with a final time of flight after the exit beam splitter of $\tau_{ToF} = 20$ ms. Due to velocity selectivity of the Bragg pulses and a non-ideal fidelity of the initial beam splitter pulse, several parasitic interferometers can be observed.

Bloch oscillations can be used to quickly impart a momentum of $p = 2n_{\text{Bloch}}\hbar k$ on the atoms. This adiabatic process can be realised by loading the atoms into a co-moving optical lattice, then accelerating the optical lattice by applying a frequency chirp and finally by unloading the atom from the optical lattice. In our model, this corresponds to the following external potential

$$V_{\text{Bloch}}(x,t) = 2\hbar \Omega(t) \cos^2(k(x-x(t)))$$

$$x(0) = 0$$

$$\dot{x}(0) = 2n v_r$$

$$\ddot{x}(t) = \begin{cases} 0, & 0 < t < t_{al} \\ \frac{2n_{\text{Bloch}} v_r}{\tau_{bo}}, & t_{al} < t < t_{bo} \\ 0, & t_{bo} < t < t_{aul} \end{cases}$$

$$\ddot{\Omega}(t) = \begin{cases} \Omega, & 0 < t < t_{al} \\ \frac{\Omega}{\tau_{al}}, & t_{al} < t < t_{bo} \\ \Omega \left(1 - \frac{t_{bo}}{\tau_{al}}\right), & t_{bo} < t < t_{aul} \end{cases}$$

By ramping up the co-moving optical lattice, the atoms are loaded into the first Bloch band with a quasimomentum $q = 0$. An acceleration of the optical lattice acts as a constant force on the atoms which linearly increases the quasimomentum over time. When the criterion for an adiabatic acceleration of the optical lattice is met, the atoms stay in the first Bloch band and undergo a Bloch oscillation, which can be repeated $n_{\text{Bloch}}$ times leading to a final momentum transfer of $\Delta p = 2n_{\text{Bloch}}\hbar k$.

The $\pi$ pulse correction $\Delta k_{\text{eff}} = \Gamma k_{\text{eff}} T^2 / 2$ is proportional to the space-time area $s_{\text{Bragg}} = \hbar k_{\text{eff}} T^2 / m$ of the underlying...
has numerous advantages such as for atom interferometry where the quantum gas of $N$ atoms is trapped is a quasi-1D guide aligned with the interferometry direction and characterised by a transverse trapping at an angular frequency $\omega_\perp$. These interactions can effectively be reduced in 1D to a magnitude of $g_{1D} = 2\hbar a_s\omega_\perp$, where $a_s = 5.272$ nm is the s-wave scattering length of $^{87}$Rb.

**Figure 5.** Gravity gradient cancellation in the case of a combined Bragg-Bloch gradiometer scheme. a) Schematic of the Bragg-Bloch interferometer geometry with a baseline of 2 m. This configuration allows to independently imprint momenta of $n\hbar k$ Bragg and $n_{\text{Bloch}}\hbar k$. The Bragg mirror pulse is momentum-adapted to cancel the gravity gradient phase. b) Gradiometric phase for a $4\hbar k$ (red dots) Bragg momentum transfer and a $2\hbar k$ Bragg + $2\hbar k$ Bloch Mach-Zehnder interferometer (blue dots). For both interferometers, the Gaussian pulse lengths of the splitting and mirror pulses are $\tau_{\text{spl}} = 25 \mu s$ and $\tau_{m} = 50 \mu s$, respectively. The initial momentum width of the atomic sample is $\sigma_p = 0.01 \hbar k$. The corresponding Rabi frequencies for the higher order Bragg transitions were found by optimising for an ideal $50:50$ population splitting of the $\Psi$ pulse, which leads to a Rabi frequency of $\Omega_{\text{eff}} = 3.7 \omega_\perp$. For the $(2 + 2)\hbar k$ Bragg+Bloch geometry, the Bloch sequence is implemented with an adiabatic loading time of $\tau_{\text{al}} = 0.5$ ms, a frequency chirp time of $\tau_{\text{rc}} = 0.5$ ms during which the momentum transfer occurs and an adiabatic unloading time of $\tau_{\text{aul}} = 0.5$ ms. The Rabi frequency of the Bloch lattice is $\Omega = 4 \omega_\perp$. For the $4\hbar k$ Bragg geometry we find a vanishing gradiometer phase at $\Delta k_{\text{eff}} = \frac{\Omega}{2\hbar k_{\text{eff}} T^2}$ which agrees with the analytical calculation. For the shell $(2 + 2)\hbar k$ Bragg+Bloch geometry we find a phase shift of $\Phi = -3$ mrad at $\Delta k_{\text{eff}} = \frac{\Omega}{2\hbar k_{\text{eff}} T^2}$ due to the nontrivial correction of the space-time area of the $(2 + 2)\hbar k$ Bragg+Bloch compared to the $4\hbar k$ Bragg geometry. The dashed lines are a guide to the eye.

The 2nhk Mach-Zehnder geometry and does not compensate the gravity gradient effects in the Bloch case. Analysing the space-time area $A_{\text{Bragg+Bloch}}$ immediately shows a non-trivial correction compared to $A_{\text{Bragg}}$. The suitable momentum compensation factor is, however, found using our solver at the crossing of the dashed blue line and the vertical zero limit ($\Delta k_{\text{eff}} = 0.932 \Delta k_{\text{Bragg}}^{\text{Bragg}}$). This straightforward implementation of our toolbox in a rather complex arrangement is promising for an extensive use of this solver to design, interpret or propose advanced experimental schemes.

**Trapped interferometry of an interacting BEC**

Employing Bose-Einstein condensate (BEC) sources for atom interferometry has numerous advantages such as the possibility to start with very narrow momentum widths $\sigma_p$, which enables high fidelities of the interferometry pulses. For interacting atomic ensembles, it is necessary to take into account the scattering properties of the particles. The Schrödinger equation is not anymore sufficient to describe the system dynamics and the ODE approach becomes rather complex to use as shown in the section on scalability and numerics. We rather generalise our position-space approach and consider a trapped BEC atom interferometer including two-body scattering interactions described in a mean-field framework. The corresponding Gross-Pitaevskii equation reads

$$i\hbar \partial_t \psi(x,t) = \left( -\frac{\hbar^2}{2M} \frac{\partial^2}{\partial x^2} + 2\hbar \Omega(t) \cos^2(k(x - mv(t)) + g_{1D}N|\psi(x,t)|^2) \right) \psi(x,t),$$  \hspace{1cm} \text{(20)}

where the quantum gas of $N$ atoms is trapped is a quasi-1D guide aligned with the interferometry direction and characterised by a transverse trapping at an angular frequency $\omega_\perp$. These interactions can effectively be reduced in 1D to a magnitude of $g_{1D} = 2\hbar a_s \omega_\perp$ where $a_s = 5.272$ nm is the s-wave scattering length of $^{87}$Rb.
All atom interferometer considerations mentioned earlier, like the Bragg resonance conditions, construction of interferometer geometries, the implementation of Doppler detunings, phase calculations and population measurements are also valid in this case without any extra theoretical effort. The non-linear Gross-Pitaevskii equation is solved following the split-operator method as in the Schrödinger case.

If the atom interferometer is perfectly symmetric in the two directions of the matterwave guide, no phase shift should occur. In realistic situations, however, the finite fidelity of the beam splitters creates an imbalance \( \delta N \) of the particle numbers between the two interferometer arms. The phase shift in this case can be related to the differential chemical potential by

\[
\Delta \phi_{MF} = \frac{1}{\hbar} \int_0^{2T} dt \left( \mu_{arm1} - \mu_{arm2} \right).
\]

Following the treatment in [46], we introduce \( \delta N \neq 0 \) and analyse the dephasing by assuming a uniform BEC density which gives

\[
\mu_{arm1/arm2} = \left( \frac{N}{2} \pm \frac{\delta N}{2} \right) \frac{g_{1D}}{2R_{TF}},
\]

where \( R_{TF} \) is the initial Thomas-Fermi radius of the BEC. The + and − signs refer here to the arms 1 and 2, respectively. Using Eq. (21) we find a phase shift of

\[
\Delta \phi_{MF} = \frac{2T}{\hbar} \left( \frac{\sqrt{m}}{2\sqrt{3}} g_{1D} \omega_k \right)^{2/3} \frac{\delta N}{N^{1/3}}.
\]

In Fig. 6, we plot this mean-field shift as a function of the atom number imbalance in the two cases of the numerical solution of the Gross-Pitaevskii equation and with the analytical model using the uniform density approximation. We observe an excellent agreement with a maximal phase shift of \( 2.1 \) mrad at an imbalance of \( \delta N = 7\% \). It is worth noting that the dephasing is accompanied by a loss of contrast consistent with previous theoretical studies. We performed a numerical optimisation to find the maximal particle number \( N \) up to which we find a contrast of \( C > 99\% \), which is \( N \leq 6 \cdot 10^4 \) in this case.

### Scalability and numerics

#### Numerical accuracy and precision

To gain a better understanding of the numerical accuracy of the simulations, we plot in Fig. 7 the dependency of the phase shift \( \Delta \phi \) on the momentum width of the atomic sample \( \sigma_p \) for a \( 2\hbar k \) Bragg interferometer. We clearly observe a relative phase larger than one microradian for large momentum widths around \( \sigma_p = 0.1 \hbar k \). A physical explanation of this dephasing is gained looking at Fig. 2, which corresponds to one run of the phase scan in Fig. 7 for \( \sigma_p = 0.1 \hbar k \). It is clearly visible that after the time of flight of \( \tau_{TF} = 2T \), there is no clear separation between the parasitic trajectories and the main ports of the Mach-Zehnder interferometer, which leads to interference patterns between them. This implies that for momentum widths \( \sigma_p \in [0.07 \hbar k, 0.1 \hbar k] \), the integration borders used to extract the populations \( P_x \) mix the contributions of different interferometers. We choose the integration borders by setting up a symmetric interval around the peak value of each of the ports, ensuring a minimal influence of the parasitic atoms on the interferometric ports. This explains the phase fluctuations above one microradian in the interval \( \sigma_p \in [0.07 \hbar k, 0.1 \hbar k] \).

For momentum widths \( \sigma_p \in [0.005 \hbar k, 0.06 \hbar k] \) where the number of particles in parasitic interferometers is dramatically lower, we observe a maximal variation of the relative phase of approximately 0.7 μrad. These results indicate that our simulator reaches a relative phase accuracy on the microradian level at least. It is worth mentioning, that the numerical parameters chosen to reach this performance are very accessible on modestly powerful desktop computers. The computation took \( \tau_{CPU/time} = 12.7 \) s on an Intel Xeon X5670 processor using four cores (2.93 GHz, 12 MB last level cache). Modeling precision atom interferometry problems with this method is therefore practical, flexible and highly accurate approach. Using improved resolutions in position and time or higher order operator splitting schemes leads to even smaller levels of residual phase shifts and therefore better numerical precision and accuracy.

#### Numerical convergence

To analyse the numerical convergence of the split-operator method applied to the previously presented systems, we simulate a scalable trapped \((2 + 2)\hbar k\) Bragg-Bloch transfer for different time and space grids. We consider one Bragg \( \pi \) pulse and one Bloch oscillation and quantify the necessary resolution and grid sizes that can be derived from these results.

Fig. 8 and Fig. 9 show the probability to find an atom in the \( 4\hbar k \) momentum state \( P(4\hbar k) \) after an interaction time \( t \). First we drive a Bragg transition to transfer the atoms from the momentum state \( p = 0\hbar k \) to the state \( p = 2\hbar k \) and then one Bloch...
sequence to accelerate the atoms from $p = 2\hbar k$ to $p = 4\hbar k$. The first step of the Bloch sequence consists of an adiabatic loading of the atoms into the first Bloch state of the lattice for $\tau_{al} = 0.5$ ms. This state already has a nonzero $4\hbar k$ momentum component which explains the first jump of the probability $P(4\hbar k)$ at $t \approx 1.5$ ms. The second step of the Bloch sequence is the acceleration of the Bloch lattice to $4v_f$ where the atoms undergo one Bloch oscillation which is driven at $\tau_{bo} = 0.5$ ms indicated by the second jump of the probability $P(4\hbar k)$. The last step of the Bloch sequence is the adiabatic unloading which converts the atoms of the first Bloch state into the $4\hbar k$ momentum state which can be seen by the last jump at $t \approx 2.5$ ms.

From Fig. 8 and Fig. 9, one can extract the critical time and position steps to be $dt \approx 1$ µs and $dx \approx 0.06 \lambda$. These findings can be put into perspective by relating them to the natural time, position, energy and momentum resolutions as well as grid sizes determined from the physical quantities to be resolved.

The fast Fourier transform (FFT) efficiently switches between momentum and position representations to apply kinetic and potential propagators. The corresponding position and momentum grids are defined by the number of grid points $N_{grid}$ and the total size of the position grid $\Delta x$ as

$$dx = \frac{\Delta x}{N_{grid} - 1}, \quad dp = \frac{2\pi\hbar}{\Delta x} \quad \text{and} \quad \Delta p = \frac{2\pi\hbar}{dx},$$

(24)

where $dp$ and $dx$ are the steps in momentum and position, respectively, and $\Delta p$ the total size of the momentum grid.

To resolve a finite momentum width of the atomic cloud we are restricted to

$$dp \ll \hbar k \leftrightarrow \Delta x \gg \lambda,$$

(25)

which sets a bound to the size of the position grid. Finally, $\Delta x$ has to be chosen according to the maximal separation of the atomic clouds $\Delta x_{sep}$. With this we find

$$\Delta x \gtrsim \Delta x_{sep} \gg \lambda.$$

(26)
Figure 7. Phase shift offset of a $2\hbar k$ Mach-Zehnder interferometer as a function of the initial momentum width of an atomic sample. We evaluate the phase shift for pulse lengths of $\tau_{bs} = 25 \mu s$ and $\tau_{m} = 50 \mu s$ (blue dots) and for $\tau_{bs} = 50 \mu s$ and $\tau_{m} = 100 \mu s$ (red dots), using peak Rabi frequencies of $\Omega_{25\mu s} = 1.0573 \omega_r$ and $\Omega_{50\mu s} = 0.52865 \omega_r$. The dashed lines are a guide to the eye. The longer pulses lead to a better velocity selectivity. For an initial momentum width smaller than $\sigma_p = 0.07 \hbar k$, the residual phase shift of the interferometer reaches a level below one microradian. For momentum widths larger than this limit, the effect of spurious atom interferometers becomes relevant and is responsible for the large phases. The numerical simulations were performed with 65536 grid points, an interaction time step of $dt_{int} = 1 \mu s$ and a free evolution time step of $dt_{free} = 10 \mu s$, leading to a computational time of $\tau_{CPUtime} = 12.69$ s on four cores of an Intel Xeon X5670 processor with 2.93 GHz frequency and 12 MB of cache.

To include all momentum orders necessary to simulate the considered atom interferometric sequences, we are naturally bound by

$$\Delta p = \frac{2\pi \hbar}{dx} \Leftrightarrow \frac{\Delta p}{\hbar k} = \frac{\lambda}{dx}$$

(27)

Hence, we find that

$$\frac{\Delta p}{\hbar k} = \frac{\lambda}{dx} \gg 1 \Leftrightarrow dx \ll \lambda,$$

(28)

which is the natural condition imposed by the necessity of resolving the atomic dynamics in the optical lattice nodes and anti-nodes of the Bragg and Bloch beams.

Fig. 9 shows the limits given by Eq. (28). Choosing $dx = 0.236 \lambda$ leads to a maximal computed momentum of $\pm 2.1 \hbar k$, which results in the impossibility to find probabilities at $4\hbar k$. Imposing that the position step is roughly one order of magnitude smaller than the wavelength ($dx \lesssim 0.06 \lambda$) results in a reasonable momentum truncation and in the convergence of the numerical routine.

The typical time scales we need to consider are set on one hand by the velocities of the optical lattice beams and of atomic cloud speed and on the other side by the duration of the atom-light interaction $\tau$. The beams as well as the atomic cloud move with velocities which are proportional to the recoil velocity $v_r$. Given that we want to drive Bragg processes of the order of $n$, we find the following bound on the time step $dt$

$$dt \ll \frac{\lambda}{nv_r} \approx \frac{100 \mu s}{n}.$$  

(29)
Figure 8. Numerical convergence of a simulated \((2+2)\hbar k\) Bragg+Bloch beam splitter in a waveguide. We analyse the numerical convergence behaviour when changing the time step \(d t\) of the numerical simulation using the third-order split-operator method. The transverse trapping frequency of the waveguide is \(2\pi \times 50\) Hz and the initial trap frequency in which the BEC is condensed is set to \(2\pi \times 1\) Hz. The effective 1D scattering length of the atoms is \(a_{\text{eff}} = 10^{-2}a_s\) with \(a_s = 5.2\) nm and a number of atoms of \(N = 6 \cdot 10^4\). The first Gaussian Bragg pulse has a peak Rabi frequency of \(\Omega = 1.0573\omega_r\) with pulse length of \(\tau = 50\) \(\mu s\). The following Bloch sequence is implemented with an adiabatic loading time of \(\tau_{\text{al}} = 0.5\) ms, a frequency chirp time of \(\tau_{\text{bo}} = 0.5\) ms during which the momentum transfer occurs and an adiabatic unloading time of \(\tau_{\text{aul}} = 0.5\) ms. The peak Rabi frequency of the Bloch lattice is \(\Omega = 4\omega_r\). At a spatial step of \(d x = 30\) nm, the simulation converges well for a time step of \(d t \approx 1\) \(\mu s\).

The typical duration of a pulse in the quasi-Bragg regime is strongly depending on the initial atomic momentum width that is being considered. Here, we assume a lower bound of \(\tau = 10\) \(\mu s\), which leads to \(d t < \tau\). This bound can be seen in Fig. 8 where the numerical convergence is achieved when we choose a time step \(d t \lesssim 4\) \(\mu s\).

It is worth noting that this time step is only necessary during the atom-light interaction. One can simulate the free evolution between the pulses with a much larger time step (without external- and interaction potentials a single step suffices) or using scaling techniques\(^{33,52-55}\).

**Time complexity analysis**

In this section, we compare the time complexity behaviour of the commonly-used method of treating the beam splitter and mirror dynamics given by the ODE approach to the PDE formulation presented in this paper, based on a position-space approach to the Schrödinger equation. To assess the time complexity of the ODE treatment, we re-derive it from the Schrödinger equation

\[ i\hbar \partial_t \psi(x,t) = \left( -\frac{\hbar^2}{2M} \frac{\partial^2}{\partial x^2} + 2\hbar\Omega \cos^2(kx) \right) \psi(x,t). \]  

We decompose the wave function in a momentum state basis as done in references\(^{2-4}\)

\[ \psi(x,t) = \sum_{m,\delta} g_{m+\delta}(t) e^{i\hbar (m+\delta) px}, \]  

where \(m\) denotes the momentum orders considered and \(\delta\) the discrete representation of momenta in the interval \([k_m - k/2, k_m + k/2]\) which captures the finite momentum width of the atoms around each momentum class \(k_m\). Making the two exponential
terms appear in $\cos^2(kv)$, one obtains

$$i\hbar g_{m+n}(t) = \hbar ((m+\delta)^2\omega_r + \Omega) g_{m+n}(t) + \frac{\hbar\Omega}{2} (g_{m+n+2}(t) + g_{m+n-2}(t)),$$

which is a set of $N_{eq}$ coupled ordinary differential equations. This number $N_{eq}$ of equations to solve is equal to $N_m N_\delta$, set by the truncation condition restricting the solution space to $N_m$ momentum classes each discretised in $N_\delta$ sub-components. Using standard solvers for such systems like Runge-Kutta, multistep or the Bulirsch-Stoer methods, we generally need to evaluate the right hand side of the system of equations over several iterations. With $N_{eq}$ differential equations, where each one has only two coupling terms, one finds a time complexity of $O(N_{eq})$.

In a next step, the coupling terms are calculated for more general potentials with time- and space-dependent Rabi frequencies $\Omega(x,t)$ and wave vectors $k(x,t)$. For this purpose, the momentum-space representation of the Schrödinger equation is more appropriate and can be written for the Fourier transform of the atomic wave function $g(p,t)$

$$i\hbar g(p,t) = \frac{p^2}{2m} g(p,t) + V(p,t) * g(p,t),$$

where

$$V(p,t) * g(p,t) := \int dx \frac{e^{-iwx}}{\sqrt{2\pi\hbar}} V(x,t)\psi(x,t).$$

Expressing the wave function in momentum space gives

$$V(p) * g(p,t) = \frac{1}{2\pi\hbar} \int dp' \int dx \frac{e^{i[\frac{p}{\hbar},x]}}{x+p} V(x,t) g(p',t).$$

Discretising $p \rightarrow (m+\delta)\hbar k$ and $p' \rightarrow (l+\gamma)\hbar k$, one finds

$$V(p) * g(p,t) \approx \frac{1}{2\pi\hbar} \sum_{l,\gamma} F((m+\delta)\hbar k, (l+\gamma)\hbar k, t) g_{l+\gamma}(t).$$

Figure 9. Numerical convergence of a simulated $(2+2)\hbar k$ Bragg+Bloch beam splitter in a waveguide. We analyse the numerical convergence behaviour when changing the position step $dx$ of the numerical simulation using the third-order split-operator method. The same parameters as Fig. 8 are used. At a time step of $dt = 10^{-6}$ s, the simulation converges well for a position step of $dx \approx 0.06\lambda$. 

![Graph showing numerical convergence of a simulated Bragg+Bloch beam splitter in a waveguide.](image-url)
We analyse now the time complexity class for the PDE approach, using the split-operator method written as

\[ \nu \]

where \( \nu \) which yields the necessary momentum couplings for an arbitrary potential \( V(x,t) \). In the worst case, the sum in Eq. (37) runs over \( N_{eq} \) nonzero entries \( (N_{eq} N_p = N_{eq} N_{grid}) \) which leads to a time complexity of \( O(N_{eq}^2) \). This, however, is an extreme example that contrasts with commonly operated precision interferometric experiments since it would correspond to white light with speckle noise. Realistic scenarios rather involve time-dependent potentials with a smaller number of momentum couplings, i.e. \( N_{eq} \gg \#coupling\ terms \gg 2 \). To evaluate the momentum couplings, it is necessary to calculate the integral \( F(p,p',t) \) at each time step using the FFT, which leads to a final time complexity class for solving the ODE of \( O(N_{eq} \log N_{eq}) \).

The next important generalisation aims to include the effect of the two-body collisions analysed in the mean-field approach. In every realistic case where the light field is allowed to be spatially inhomogeneous, the amount of couplings increases and it is preferable to employ the PDE approach with a scaling of \( O(N_{grid} \log N_{grid}) \), independent of any further complexity to be modeled.

The time complexity analysis is summarised in Table 1. It shows that the standard ODE approach is only better suited in the case of ideal light plane waves. In every realistic case where the light field is allowed to be spatially inhomogeneous, the amount of couplings increases and it is preferable to employ the PDE approach with a scaling of \( O(N_{grid} \log N_{grid}) \), independent of any further complexity to be modeled.

Table 1. Comparison of the different time complexity classes of the commonly-used ODE treatment with the position-space approach developed in this work (PDE-based). Including more and more realistic features of the atom-light system leads to an ODE time complexity unfavorably scaling. The PDE formulation, however, routinely scales with \( O(N_{grid} \log N_{grid}) \).

| Feature                           | Numerical Operations ODEs | Numerical Operations PDE |
|----------------------------------|---------------------------|--------------------------|
| Infinitely sharp momentum widths (\( \delta \rightarrow 1 \)) | \( O(N_{eq}) \) | -- |
| Finite momentum width and ideal light potential | \( O(N_{grid}) \) | \( O(N_{grid} \log N_{grid}) \) |
| Inhomogeneous light potential     | \( O(N_{eq} \log N_{eq}) \rightarrow O(N_{grid}^2) \) | \( O(N_{grid} \log N_{grid}) \) |
| Mean-field interaction           | \( O(N_{grid}^2) \) | \( O(N_{grid} \log N_{grid}) \) |

where \( l \) and \( \gamma \) span the same indices ensembles as \( m \) and \( \delta \). The new equations to solve read

\[
 i\hbar g_{m+\delta}(t) \approx \frac{(m+\delta)\hbar k}{2m} g_{m+\delta}(t) + \frac{1}{2\pi\hbar} \sum_{f} F((m+\delta)\hbar k, (l+\gamma)\hbar k, t) g_{l+\gamma}(t), \tag{37}
\]

which yields the necessary momentum couplings for an arbitrary potential \( V(x,t) \). In the worst case, the sum in Eq. (37) runs over \( N_{eq} \) nonzero entries \( (N_{eq} N_p = N_{eq} N_{grid}) \) which leads to a time complexity of \( O(N_{eq}^2) \). This, however, is an extreme example that contrasts with commonly operated precision interferometric experiments since it would correspond to white light with speckle noise. Realistic scenarios rather involve time-dependent potentials with a smaller number of momentum couplings, i.e. \( N_{eq} \gg \#coupling\ terms \gg 2 \). To evaluate the momentum couplings, it is necessary to calculate the integral \( F(p,p',t) \) at each time step using the FFT, which leads to a final time complexity class for solving the ODE of \( O(N_{eq} \log N_{eq}) \).

The next important generalisation aims to include the effect of the two-body collisions analysed in the mean-field approximation, i.e. \( H_{int} = g_{1D}|\psi(x,t)|^2 \). In this case, the equation describing the dynamics of the system and the couplings can be written as

\[
 i\hbar \dot{g}_{m+\delta}(t) = \hbar \left( (m+\delta)^2 \omega_r + \Omega \right) g_{m+\delta}(t) + \frac{\hbar \Omega}{2} \left( g_{m+\delta+2}(t) + g_{m+\delta-2}(t) \right) + g_{1D} \left( \sum_{\ell,\gamma} g_{\ell+\gamma}(t) g_{2\ell-\gamma}(t) \right) g_{m+\delta}(t), \tag{38}
\]

where \( v \) and \( o \) are running indices over the same values than \( n \) and \( \gamma \). One ends up with \( N_{eq} \) differential equations where each has more than \( N_{eq}^2 \) coupling terms, and finds a time complexity class of \( O(N_{eq}^2) \).

We analyse now the time complexity class for the PDE approach, using the split-operator method. Based on the application of the FFT, it is known that the complexity class of this method is scaling as \( O(N_{grid} \log N_{grid}) \), where \( N_{grid} \) is the number of grid points in the position or momentum representations. Since the discretisation of the problem for the ODE and PDE (Schrödinger equation) approaches is roughly the same \( (N_{eq} \approx N_{grid}) \), a direct comparison between the two treatments is possible.

The time complexity analysis is summarised in Table 1. It shows that the standard ODE approach is only better suited in the case of ideal light plane waves. In every realistic case where the light field is allowed to be spatially inhomogeneous, the amount of couplings increases and it is preferable to employ the PDE approach with a scaling of \( O(N_{grid} \log N_{grid}) \), independent of any further complexity to be modeled.

**Conclusion**

In this paper, we have shown that the position-space representation of light-pulse beam splitters is quite powerful for tackling realistic beam profiles in interaction with cold atom ensembles. It was successfully applied across several relevant regimes, geometries and applications. We showed its particular fitness in treating metrologically-relevant investigations based on atomic sensors. Its high numerical precision and scalability makes it a flexible tool of choice to design or interpret atom interferometric measurements without having to change the theoretical framework for every beam geometry, dimensionality, pulse length or atomic ensemble property. We anticipate the possibility to generalise this method to Raman or 1-photon transitions if we account for the internal state degree of freedom change during the diffraction.

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Author contributions statement

F.F. implemented the numerical model, performed all numerical simulations, and prepared the figures. J.-N.S. and H.A. helped with the interpretation of the results. H.A. and N.G. designed the research goals and directions. E.M.R. and K.H. contributed to scientific discussions. F.F. and N.G. wrote the manuscript. S.S. critically reviewed the manuscript. All authors reviewed the results and the paper and approved the final version of the manuscript.

Additional information

Competing interests: The authors declare no competing interests.