ANTI-DE SITTER FRAGMENTATION

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Abstract  Some general aspects of the $AdS_2/CFT_1$ correspondence, previously discussed in hep-th/9812073, are summarized. The majority of this summary is devoted to the question of where the CFT$_1$ should live. Almost as a byproduct, the decay—or fragmentation—of $AdS_2$ is also discussed.

As is hopefully symptomatic of the number of contributions in the Cargese '99 proceedings devoted to this conjecture, the $AdS/CFT$ correspondence has been very fruitful in illuminating aspects of both large $N$ gauge theory, and supergravity on $AdS$ backgrounds. The $AdS_5/CFT_4$ conjecture is both very interesting and widely studied, because it sheds light on four-dimensional gauge theories. The $AdS_3/CFT_2$ conjecture has also been widely studied. Because of the simplifications that result in lower dimension—and because of the amount of knowledge that has accumulated on two-dimensional CFTs—the $AdS_3/CFT_2$ correspondence has been very useful at elucidating information on $AdS_3$ supergravity. For example, a detailed microscopic description of the BTZ black hole has been obtained in this way.

Continuing down in dimension, the $AdS_2/CFT_1$ conjecture is important for the understanding of four and five-dimensional black holes—the near horizon geometry of such black holes is respectively $AdS_2 \times S^2$ and

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1Gong Show talk given at the Cargese '99 ASI “Progress in String Theory and M-Theory”
2The reader may have already noticed a dearth of references in this summary. That is because we are making every effort to acknowledge only our own research; all other citations are completely inadvertent.
AdS$_{D \geq 3}$. One might think that this lowest-dimensional correspondence should be the simplest of all, and the easiest to check. In fact, very little is known about either side of the AdS$_2$/CFT$_1$ correspondence. In this summary, we will discuss some aspects of the AdS$_2$ side of the correspondence. A more complete discussion, and references, can be found in [1]. For later work on this subject, we refer the reader in part to [2, 3, 4, 5, 6, 7] and references therein.

One of the main puzzles in understanding the AdS$_2$/CFT$_1$ correspondence, is the question of where the CFT should “live”. In higher di-

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**Figure 1.1** The Penrose diagram for (a) $AdS_{D \geq 3}$ (each point is an $S^{D-2}$) and (b) $AdS_2$. The solid line is the boundary, the dotted lines delimit the Poincaré patches, and the dashed line indicates the origin ($r = 0$) of the global coordinates (1.1).

**Figure 1.2** Approaching the near-horizon limit of the multi-black hole configuration considered. From afar, the geometry looks like that of a single black hole but as the throat is approached it splits up into several (three depicted) black holes. In the near-horizon limit, only the (four) throats remain.
dimensions, the CFT is usually taken to live on the boundary of the $AdS$ space; however, $AdS_2$ has two boundaries, as shown in figure 1.1. To be precise, in global coordinates, the $AdS_{D \geq 3}$ metric takes the form

$$ds^2 = \cosh^2 r \, dt^2 + dr^2 + \sinh^2 r \, d\Omega_{D-2}^2,$$

where $0 < r < \infty$ and the boundary is the $S^{D-2} \times \mathbb{R}$ at $r = \infty$. If $D = 2$, then the last term of equation (1.1) is absent, $-\infty < r < \infty$, and the boundaries are at $r = \pm \infty$. So, are there two CFTs and if not, then on which boundary does the CFT live?

A strong hint that the CFT lives on only one boundary is given by the following consideration. We can obtain $AdS_2 \times S^2$ from the near-horizon geometry of a four-dimensional extremal Reissner-Nordström black hole. In fact, we can obtain a much richer system by considering the geometry obtained by taking many extremal black holes and allowing them to approach each other as the near-horizon limit is taken—see figure 1.2.

In particular, we examine the system with two black holes with charges $Q_1, Q_2$ and consider the case when $Q_2 \ll Q_1$. Then, we can recover an $AdS_2 \times S^2$ geometry by spherically averaging the small, “test” black hole about the large black hole. More precisely, we obtain an asymptotically $AdS_2 \times S^2$ geometry in this way. In the two-dimensional theory, the second black hole appears as a test particle, and, as depicted in figure 1.3, one of the boundaries—namely the one just inside the horizon of the large black hole—of the spacetime has become singular. Thus, we conclude that the CFT lives only on one boundary.

Finally—and to justify the title of this summary—we note that the analytic continuation of this story to Euclidean signature, is an instanton description of $AdS_2$ fragmentation or splitting. Roughly, the throat at

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$^2$In Poincaré coordinates $(0 < z < \infty, x^\mu \in \mathbb{R}^{D-2,1})$—which cover only a patch of $AdS_D$—the metric is $ds^2 = (dz^2 + \eta_{\mu\nu}^{(D-2,1)} dx^\mu dx^\nu)/z^2$.

$^3$A derivation can be found in [1].
the top of figure 1.2 is the “initial” state—a large $AdS_2 \times S^2$—which then splits into many smaller throats. One might ask whether higher-dimensional $AdS$ spaces can also split. The answer turns out to be negative: the action for the $AdS_2$ instanton is precisely one-half the change in the entropy (so that the rate of black hole splitting is suppressed by the entropy) but for higher-dimensional splitting, there is an infinite factor, related to the fact that the geometry is the near-horizon limit of a noncompact $(D-2)$-brane. Thus, only $AdS_2$ fragments.\(^4\) One might also ask how $AdS_2$ fragmentation manifests itself in the CFT on the boundary. This is still an open question.

Acknowledgments

I thank J. Maldacena and A. Strominger for very fruitful collaboration on this topic, and the organizers of this excellent school for the opportunity to attend. This work was supported by an NSF Graduate Fellowship, an NSERC PGS B Scholarship and DOE grant DE-FGO2-91ER40654.

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\(^4\)A detailed alternative explanation can be found in [8].