Schedule Sequence Design for Broadcast in Multi-channel Ad Hoc Networks

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Abstract—We consider a single-hop ad hoc network in which each node aims to broadcast packets to its neighboring nodes by using multiple slotted, TDD collision channels. There is no cooperation among the nodes. To ensure successful broadcast, we propose to pre-assign each node a periodic sequence to schedule transmissions and receptions at each time slot. These sequences are referred to as schedule sequences. Since each node starts its transmission schedule independently, there exist relative time offsets among the schedule sequences they use. Our objective is to design schedule sequences such that each node can transmit at least one packet to each of its neighbors successfully within a common period, no matter what the time offsets are. The sequence period should be designed as short as possible. In this paper, we analyze the lower bound on sequence period, and propose a sequence construction method by which the period can achieve the same order as the lower bound.

We also consider the random scheme in which each node transmits or receives on a channel at each time slot with a pre-determined probability. The frame length and broadcast completion time under different schemes are compared by numerical studies.

I. INTRODUCTION

A. Overall scenario

We consider a medium access control (MAC) problem for a wireless single-hop ad hoc network, in which each node always has a stream of packets to broadcast to its neighboring nodes. The nodes are within a common hearing range. This broadcast scenario is common. For example, in a sensor network, each sensor node is required to collect data such as temperature and humidity observed by itself and other neighboring nodes for further processing [1]–[3]. Another example comes from vehicular ad hoc networks (VANETs), in which each vehicle broadcasts safety messages such as its speed and location information to its neighboring vehicles, in order to avoid collisions among the vehicles [4]–[6].

The overall objective of the MAC design is to ensure that any node can successfully receive broadcast packets from all other nodes within a short time duration. This is in line with the goal of ultra-reliable low latency communications (URLLC) in the fifth generation (5G) networks [7]. Lots of scheduling algorithms for single-channel environments have been proposed in the literature [8]–[10]. In this paper, we mainly focus on the multi-channel case. Specifically, we assume that the broadcast packets are transmitted over multiple slotted, time division duplex (TDD), equal-bandwidth collision channels. The TDD assumption indicates that each node at any time slot can either receive or transmit a packet over a channel but not both. A broadcast at a given channel is successfully received if during the whole transmission duration it is free from conflicts with other transmissions on the channel and the intended receiver is tuned to receive packets at the same channel. Compared with a single-channel system, the use of multiple channels has two main influences. On one hand, it enables the possibility of concurrent successful transmissions among multiple node pairs, and thus may expedite successful all-to-all broadcasts. On the other hand, for any node pair, the transmitter and the receiver should be matched to the same channel before data transmission. This matching process, referred to as rendezvous [11]–[13], may result in longer delay, especially when their schedules are not under centralized control. Therefore, the problem that whether using more channels is beneficial for decreasing delay compared with using a single channel is nontrivial.

For broadcast in multi-channel ad hoc networks without centralized controller, most of the existing MAC schemes rely on coordination among the nodes, which usually requires control message exchange on a dedicated control channel [14]–[16]. For example, MCB proposed in [14] follows a split phase approach, that is, each node periodically switches between the control channel and one of six service channels. Before broadcasting data, each node should select a service channel for data transmission and should announce this information to other nodes through the control channel. However, this control channel would be a bottleneck when traffic is heavy, and the overhead for frequent control message exchange would be high especially when the data packets are short packets [17].

In this paper, we aim at devising multi-channel MAC schemes without centralized controller and negotiation among the nodes. For such a system, accurate time synchronization among the nodes is challenging to achieve. Therefore, it is desirable to devise asynchronous MAC schemes. Without time synchronization, each node starts its transmission mechanism independently. It follows that the time difference between the start point of a node and the system-wide reference point $t = 0$ may vary from node to node. We refer to this time

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difference as the time offset of a node. The values of time offsets are unknown and remain unchanging during the whole communication session.

To the best of our knowledge, this is the first work that focuses on all-to-all broadcast in a multi-channel single-hop ad hoc network without synchronization and coordination. We mainly consider deterministic schemes, and use random schemes as reference baseline. We regard deterministic schemes as sequence schemes in which each node is pre-assigned a transmission and reception schedule in the format of a schedule sequence [10], [18], [19]. At each time slot, each node reads out its current sequence value, and then conducts corresponding action (transmitting or receiving on a particular channel) according to that value. An appropriately designed schedule sequence set can guarantee successful broadcasts within a common sequence period, for all possible time offsets. In the random schemes, each node at each time slot transmits or receives on a channel with a fixed probability. In both of the sequence schemes and random schemes, each node transmits or receives independently without cooperating with other nodes.

B. Performance metrics

The main goal in this paper is to devise MAC schemes to provide a hard guarantee on broadcast delay, for an asynchronous multi-channel network. The metrics for broadcast delay are frame length and broadcast completion time, which are defined as follows.

1) Frame length: In the system we investigate, each node is required to transmit a sequence of packets to all other nodes. To ensure reliable communication, each node may need to transmit a packet for multiple times. We define the consecutive sequence of time slots in which the same packet is considered for transmission by a node as a frame, in both of the sequence schemes and the random schemes, as shown in Figure 1

In a sequence scheme, each packet is transmitted according to a periodic schedule sequence. Our sequence design goal is to ensure that each node has one or more successful broadcasts per frame to each other node. So for sequence-based schemes, the frame length is equal to the sequence period. For random schemes, the frame length represents the number of trials a node attempts to transmit a given packet, such that the probability that a successful broadcast can be achieved within a frame is close to 1. (Here, the definition of closeness is determined according to QoS requirements motivated by URLLC standards [7], since it it not possible to attain 100% certainty for random schemes.) The frame length upper bounds the broadcast delay for all possible time offsets, so heuristically it should be minimized.

2) Broadcast completion time: The completion time is also a common metric for delay, which is usually referred to as group delay for broadcast in the single channel model [9], [10]. It is defined as the time duration starting from $t = 0$ until each node has transmitted at least one packet to each other node successfully. The completion time varies with time offsets. The comparisons for broadcast completion time under different schemes are mainly conducted by numerical studies.

![Fig. 1](image_url) Each node transmits each packet in its packet stream for a frame duration. The arrows represent the time slots in which a packet is transmitted. In the sequence scheme, each packet is transmitted according to the periodic schedule sequence. In the random scheme, each packet is transmitted random number of times within a frame.

C. Related work

1) Broadcast with a single channel: The design of schedule sequences for asynchronous broadcast was first studied for VANETs in [10]. Various methods for assigning the sequences to vehicles have been discussed in [10], [20], [21]. Note that the schedule sequences proposed in [10] are only applicable for the single channel model and can be represented by binary protocol sequences with the User-Irrepressibility (UI) property, which have been extensively investigated in the literature (please see [22]–[26] and references therein). The symbol value “1” or “0” in binary protocol sequences corresponds to transmitting or receiving on the single channel. The UI property signifies that for all possible time offsets, each protocol sequence has at least one “1” which does not collide with “1”s from other sequences. Protocol sequences with the UI property can be constructed from conflict-avoiding codes [27]. In this paper, we extend the analysis of protocol sequences to more general schedule sequences that are required for modeling multiple channel systems. Using multiple channels can increase throughput, however, sequence design also becomes more difficult due to the rendezvous process.

In [10], the comparison between the proposed sequence scheme and the optimized random scheme in terms of broadcast completion time has been conducted. The comparison result is that the sequence scheme can achieve shorter broadcast completion time than the random scheme.

2) Unicast with multiple channels: Sequence design for another common information exchange pattern, unicast, in asynchronous multi-channel system was investigated in [19]. The difference between unicast and broadcast lies in the contents of the exchanged packets. In all-to-all broadcast, the packets transmitted from one node to other nodes are the same. As a contrast, in all-to-all unicast, the packets transmitted from one node to each of the other nodes are individual. That is, given $K$ nodes within the same hearing range, then for the broadcast model, the total number of packets that should be transmitted successfully by the $K$ nodes within a frame is $K$, while for the unicast model, this value should be $K(K - 1)$. The optimal transmitting and receiving probabilities for unicast under the random schemes are also analyzed in [19]. The simulation results in [19] show that under both sequence scheme and the optimized random scheme, the unicast completion time decreases when the number of available channels
Our major contributions are listed as follows.

D. Main contributions

To the best of our knowledge, this is the first study that considers MAC schemes for asynchronous all-to-all broadcast by multiple channels without coordination among the nodes. Our major contributions are listed as follows.

1) The following results are obtained for the sequence scheme. Given $K$ nodes and $M$ available channels, we derive a lower bound on the shortest common sequence period, and propose a sequence design method based on the Chinese Remainder Theorem (CRT) correspondence. Under some general technical assumptions, the sequence period under our proposed construction has the same order as the lower bound, and can achieve an asymptotic reduction in the order of $\frac{1}{M}$.

2) We analyze random schemes for benchmark against the sequence schemes. We derive optimal transmitting and receiving probabilities for two random schemes.

3) Frame length and broadcast completion time under different schemes are compared by theoretical analysis and numerical studies.

The rest of this paper is organized as follows. After describing the system model in Section II, we present preliminary information in Section III to prepare for subsequent discussions. Then we analyze the lower bound on sequence period in Section IV and propose a sequence construction method in Section V. In Section VI, we present results on the sequence period under our proposed construction method under even group division. In Section VII, we analyze two random schemes. Comparisons on frame length and broadcast completion time under different schemes are shown in Section VIII. Finally, we conclude the paper in Section IX.

II. Problem Formulation

We consider a single-hop ad hoc network consisting of $K$ nodes that are all within a common hearing range. Each node should broadcast packets to each other node at least once within a sequence period, under the sequence scheme. For notation simplicity in this paper, given a positive integer $n$, we use $[n]$ to denote the set $\{1, 2, \ldots, n\}$, and $\mathbb{Z}_n$ to denote the cyclic group $\{0, 1, \ldots, n-1\}$, with addition (resp. subtraction) modulo $n$ denoted by $\oplus_n$ (resp. $\ominus_n$). We denote the $i$-th node by $N_i$, for $i \in [K]$. There are $M$ frequency channels available. Since the bandwidth is a scarce resource in general, we only consider the case where $M \leq K$ in this paper.

We consider a group-based channel allocation method, called Assignment $T$. The $K$ nodes are divided into $M$ groups, denoted by $G_1, G_2, \ldots, G_M$. The group division satisfies $\cup_{m=1}^{M} G_m = \{N_1, N_2, \ldots, N_K\}$, and $G_m \cap G_n = \emptyset$, for $m, n \in [M]$ and $m \neq n$. The group size of $G_m$ is denoted by $|G_m|$. We remark that the groups may have different sizes, and a group could be an empty set. If $G_m$ is empty, then channel $m$ would not be used. Among the $M$ groups, we assume $G_1, G_2, \ldots, G_W$ are non-empty, $1 \leq W \leq M$, and denote the smallest (resp. largest) non-zero group size by $k$ (resp. $\ell$), i.e.,

$$k = \min\{|G_1|, |G_2|, \ldots, |G_W|\},$$

$$\ell = \max\{|G_1|, |G_2|, \ldots, |G_W|\}.$$

The values of $W, k, \ell$ depend on how the groups are divided. Especially, we define an even group division, in which the division of the $W$ non-empty groups is as even as possible, that is, $k = \lfloor K/W \rfloor$, $\ell = \lceil K/W \rceil$. Under a given group division, for $m = 1, 2, \ldots, W$, the nodes in group $G_m$ are allowed to transmit on channel $m$ only, but are able to receive packets from any of the $W$ channels.

All time slots are assumed to be of equal duration. Without loss of generality, we normalize the slot duration to 1. We represent a periodic sequence with period $L$ by a sequence of finite length $L$. The schedule sequence of period $L$ assigned to node $N_i$ is denoted by

$$s_i := [s_i(0) \ s_i(1) \ \ldots \ s_i(L-1)],$$

for $i = 1, 2, \ldots, K$. For node $N_i \in G_m$, where $i \in [K]$ and $m \in [W]$, we denote the action to transmit on channel $m$ by the symbol $T_m$, and the action to receive on channel $r$ by the symbol $R_r$, for any $r \in [W]$, then the entries in $s_i$ are chosen from the set

$$\{T_m\} \cup \{R_1, R_2, \ldots, R_W\}.$$

For $i \in [K]$, node $N_i$ has a time offset, denoted by $\tau_i$, which is defined as the time difference between the system-wide reference point $t = 0$ and the starting point of node $N_i$. To facilitate discussions, we assume that all nodes start their schedules no later than $t = 0$ and that the slot boundaries of the nodes are aligned. As a result, the time offsets of the nodes are non-negative integers. Considering that the sequences have a common period $L$, we assume that $\tau_i \in \mathbb{Z}_L$. We let $\tau = (\tau_1, \tau_2, \ldots, \tau_K) \in \mathbb{Z}_L^K$ denote an instance of time offsets of the $K$ nodes. For $N_i \in G_m$ with $\tau_i$, we denote the cyclic shift of $s_i$ by $\tau_i$ by

$$s_i^\tau := [s_i(\tau_i) \ s_i(1+\tau_i) \ \ldots \ s_i((L-1)+\tau_i)].$$

If $s_i(t+\tau_i) = T_m$, node $N_i$ sends out a packet on channel $m$ at the time slot $t$. If $s_i(t+\tau_i) = R_r$, $r \in [W]$, node $N_i$ listens to channel $r$ in time slot $t$ and see if any packet can be received. If multiple nodes transmit on the same channel simultaneously, then a collision occurs and no packets transmitted on this channel at this time slot can be successfully decoded. For $m \in [W]$, if there is only one node transmitting on channel $m$ and multiple nodes are receiving from channel $m$ in the same time slot, the transmitted packet is regarded as successfully received by all the nodes that are listening to channel $m$.

The sequence design should ensure successful transmission between any two nodes within a period, regardless of the time offsets. Specifically, the design of schedule sequences is subject to the following requirements.

1) (Intra-group communication) For any $m \in [W]$, if $G_m$ has size $|G_m| \geq 2$, then for any $N_i, N_j \in G_m$ with $i \neq j$
and for any \( \tau \in \mathbb{Z}_L^\tau \), there exists a time index \( t \in \mathbb{Z}_L \)
\begin{align*}
\begin{cases}
    s_i(t \oplus_L \tau_i) = T_m; \\
    s_j(t \oplus_L \tau_j) = R_m; \\
    s_x(t \oplus_L \tau_x) \neq T_m, \text{ for all } N_x \in G_m \setminus \{N_i, N_j\}.
\end{cases}
\end{align*}

(1)

2) (Inter-group communication) For any two distinct group indices \( m, n \in [W] \), for any \( N_i \in G_m \) and \( N_j \in G_n \), and for any \( \tau \in \mathbb{Z}_L^\tau \), there exists a time index \( t \in \mathbb{Z}_L \)
\begin{align*}
\begin{cases}
    s_i(t \oplus_L \tau_i) = T_m; \\
    s_j(t \oplus_L \tau_j) = R_m; \\
    s_x(t \oplus_L \tau_x) \neq T_m, \text{ for all } N_x \in G_m \setminus \{N_i\}.
\end{cases}
\end{align*}

(2)

Given \( K \) nodes and \( M \) channels, a set of sequences \( \{s_i : i \in [K]\} \) of length \( L \) is called an \((M, K, L)\)-schedule sequence set if there exists a positive integer \( W \leq M \) and a partition of \([N_1, N_2, \ldots, N_K]\) into \( W \) non-empty groups \( G_1, G_2, \ldots, G_W \), so that

(i) For each \( i \in [K] \), if \( N_i \in G_m, m \in [W] \), the entries of sequence \( s_i \) are drawn from \( \{T_m\} \cup \{R_1, R_2, \ldots, R_W\} \);

(ii) The conditions in (1) and (2) are satisfied.

Example 1: For 3 nodes \( K = 3 \) and 2 channels \( M = 2 \) under Assignment T, we let \( W = 2 \) and \( G_1 = \{N_1, N_2\}, G_2 = \{N_3\} \). Then the entries in \( s_1, s_2 \) are drawn from \( \{T_1, R_1, R_2\} \) and the entries in \( s_3 \) are drawn from \( \{T_2, R_1\} \). Here is a \((2, 3, 12)\)-schedule sequence set of length \( L = 12 \):

\[
\begin{align*}
    s_1 &= [T_1, T_1, T_1, T_1, T_1, T_1, T_1, T_1, R_1, R_1, R_2, R_2]; \\
    s_2 &= [T_1, R_1, T_1, R_1, T_1, R_1, T_1, R_1, T_1, T_2, R_2, R_1]; \\
    s_3 &= [T_2, R_1, T_2, R_1, T_2, R_1, T_2, R_1, T_2, R_1, T_1].
\end{align*}
\]

We can check that for all \( \tau \in \mathbb{Z}_L^\tau \), the conditions in (1) and (2) are satisfied. For example, if \( \tau = (3, 7, 10) \), the shifted sequences \( s_1^3, s_2^7, s_3^{10} \) are as follows:

\[
\begin{align*}
    s_1^3 &= [T_1, T_1, T_1, R_1, R_1, R_1, R_2, R_2, R_2, T_1, T_1]; \\
    s_2^7 &= [R_2, T_1, T_1, R_1, R_1, R_1, T_1, T_1, R_2, T_1, R_1; \\
    s_3^{10} &= [R_1, R_1, T_1, T_2, R_1, T_1, T_2, R_1, T_1, T_2, R_1].
\end{align*}
\]

We take \( N_1 \) for instance in this case. It transmits to \( N_2 \) successfully at \( t = 2 \) since \( s_1^3(2) = T_1 \) and \( s_2^7(2) = R_1 \), and transmits to \( N_3 \) successfully at \( t = 0 \) since \( s_1^3(0) = T_1, s_2^7(0) \neq T_1 \) and \( s_3^{10}(0) = R_1 \).

In later sections, we will analyze lower bound on \( L \) and propose construction method for \((M, K, L)\)-schedule sequence set. To facilitate reading, we list the notation introduced in this section in Table I

### Table I

| Notation | Definition |
|----------|------------|
| \( K \) | The total number of nodes |
| \( M \) | The total number of available channels |
| \( G_m \) | The \( m \)-th group, \( m \in [M] \) |
| \( W \) | The number of non-empty groups, \( 1 \leq W \leq M \) |
| \( k \) | The smallest non-empty group size |
| \( L \) | The largest non-empty group size |
| \( t \) | The period of a periodic sequence set |
| \( N_i \) | The \( i \)-th node, \( i \in [K] \) |
| \( \tau \) | The time offset of node \( N_i, \tau \in \mathbb{Z}_L^\tau \) |
| \( s_i \) | The combination of \( \tau_i \), \( \tau \in \mathbb{Z}_L^\tau \) |
| \( s_i^\tau \) | The cyclic shift of \( s_i \) by \( \tau_i \) |

### A. Hamming cross-correlation

We introduce the definition and a basic property of the Hamming cross-correlation of two binary sequences.

**Definition 1**: For two binary sequences \( s_1 := [s_1(0) s_1(1) \ldots s_1(L-1)] \) and \( s_2 := [s_2(0) s_2(1) \ldots s_2(L-1)] \) with common period \( L \) and relative time offset \( \tau \), the Hamming cross-correlation function is defined by

\[
H_{1,2}(\tau) = \sum_{t=0}^{L-1} s_1(t)s_2(t \oplus_L \tau).
\]

When \( s_1 = s_2 \), \( H_{1,2}(\tau) \) is called the Hamming autocorrelation of \( s_1 \).

**Definition 2**: The Hamming weight of a periodic binary sequence is defined as the number of “1”s in a period.

**Lemma 1**: Below illustrates a relationship between the Hamming cross-correlation and the Hamming weights of two binary sequences.

**Lemma 3**: For two binary sequences \( s_1, s_2 \) with common period \( L \) and with Hamming weights \( w_1, w_2 \), respectively, the sum of their Hamming cross-correlation, taken over relative time offset \( \tau \) ranging from 0 to \( L - 1 \), satisfies

\[
\sum_{\tau=0}^{L-1} H_{1,2}(\tau) = w_1 w_2.
\]

### B. CRT correspondence

We remind readers of the Chinese Remainder Theorem (CRT) correspondence, since our proposed construction method for \((M, K, L)\)-schedule sequence set in Section IV is based on it.

**Definition 4**: For \( p \) and \( q \) that are relatively prime, the CRT correspondence is a bijective mapping between \( \mathbb{Z}_{pq} \) and \( \mathbb{Z}_p \times \mathbb{Z}_q \) defined by

\[
\Phi_{p,q}(t) := (t \bmod p, t \bmod q).
\]

By the CRT correspondence, a sequence of length \( L = pq \) can be obtained from a \( p \times q \) array with the \((t \bmod p, t \bmod q)\)-th entry in the array being mapped to the \(t\)-th entry in the sequence, for \( t \in \mathbb{Z}_L \). Cyclically shifting the sequence by \( \tau \), where \( \tau \in \mathbb{Z}_L \), is equivalent to row-wise and column-wise shifting its array representation by \( \tau \bmod p \) and \( \tau \bmod q \), respectively.
C. User-Irrepressible sequences

User-Irrepressible (UI) sequences can be directly employed for broadcast in the single-channel model. They will also be used in our proposed construction method for \((M, K, L)\)-schedule sequence set.

Definition 5: \([24]\) Consider a set of \(K\) binary sequences each of which is of length \(L\). We cyclically shift the \(i\)-th sequence by a time offset \(\tau_i \in \mathbb{Z}_L\), for \(i \in [K]\), and stack these shifted sequences into a \(K \times L\) matrix \(M\). If \(M\) always contains a \(K \times K\) permutation matrix for all possible \(\tau_1, \tau_2, \ldots, \tau_K \in \mathbb{Z}_L\), then this sequence set is a \((K, L)\)-UI sequence set.

By Definition 5, a \((K, L)\)-UI sequence set is equivalent with a \((1, K, L)\)-schedule sequence set. There are a variety of construction methods for UI sequence sets in the literature \(\cite{10, 24, 29}\). It is well known that for a set of \(K\) binary sequences of length \(L\), if the Hamming weight of each sequence is no less than \(K\), and the Hamming cross-correlation between any two of them is no more than 1 for any time offsets, that is,

\[
H_{i,j}(\tau_i \ominus \tau_j) \leq 1, \quad \text{for} \ i, j \in [K], i \neq j, \tau_i, \tau_j \in \mathbb{Z}_L,
\]

then this sequence set is a \((K, L)\)-UI sequence set.

For any given \(K\), we can obtain a \((K, L)\)-UI sequence set that satisfies (4) by the following construction, which is based on the CRT correspondence \(\cite{3}\).

Definition 6: CRT-UI construction \(\cite{24}\): Given \(K\), let \(w \geq K\), \(p\) be a prime and \(p \equiv w\ mod \ q\), be a number coprime with \(p\) and \(q \geq 2w\). For generators \(g \in [K]\), construct a set of \(K\) sequences \(\{s_g = [s_g(0) \ldots s_g(L-1)] : g \in [K]\}\) with common Hamming weight \(w\) and common period \(L = pq\) as follows: for \(t \in \mathbb{Z}_L\),

\[
s_g(t) = \begin{cases} 1 & \text{if } \Phi_{p,q}(t) = (ug \mod p, u \mod q), \text{ for } u \in \mathbb{Z}_w, \\ 0 & \text{otherwise.} \end{cases}
\]

For any prime \(K\), the shortest period \(L\) of a \((1, K, L)\)-schedule sequence set \(((K, L)\)-UI sequence set) obtained by the CRT-UI construction is

\[
L = K(2K - 1).
\]

The period \(L\) in (6) is obtained by letting \(w = K\), \(p = K\) and \(q = 2K - 1\). For general \(K\) which may not be a prime, we can obtain the following equation on this shortest \(L\) by Bertrand's postulate,

\[
L \leq 2K(2K - 1).
\]

The sequences obtained by the CRT-UI construction have the following Hamming auto-correlation property:

Lemma 7: \([18]\) For \(q \in [p - 1], d \in \mathbb{Z}_w\) and \(\tau \in \mathbb{Z}_L\),

\[
H_{g,g}(\tau) = \begin{cases} w - d & \text{if } \Phi_{p,q}(\tau) = \pm(g, 1)d, \\ 0 & \text{otherwise.} \end{cases}
\]

Example 2: Given \(K = 3\), we design three sequences by the CRT-UI construction with \(w = 3\), \(p = 3\), \(q = 5\), generators \(g = 1, 2, 3\) and length \(L = pq = 15\) as follows,

\[
s_1 = [1 1 1 0 0 0 0 0 0 0 0 0 0 0 0];
\]

\[
s_2 = [1 0 0 0 0 0 1 0 0 0 1 0 0 0];
\]

\[
s_3 = [1 0 0 0 0 0 1 0 0 0 0 1 0 0 0].
\]

Since the CRT-UI construction is based on the CRT correspondence, these sequences can be obtained from the following three \(3 \times 5\) arrays, respectively:

\[
s_1 : \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad s_2 : \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad s_3 : \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}
\]

We take the sequence \(s_2\) for instance. In its array representation, “1”s are located at the positions \(\{2u \ mod \ p, u \ mod \ q\} : u = 0, 1, 2\} = \{(0, 0), (2, 1), (1, 1)\}. Since \(\Phi_{p,q}(0) = (0, 0), \Phi_{p,q}(11) = (2, 1), \Phi_{p,q}(7) = (1, 2)\), thus “1”s appear in \(s_2\) when \(t = 0, 11, 7\). When \(\tau_2 = 7\), the shifted sequence \(s_2^7\) is as follows,

\[
s_2^7 : \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}
\]

We can check that, for any \(\tau_1, \tau_2, \tau_3 \in \mathbb{Z}_{15}\), the Hamming cross-correlation of any two of \(s_1^7, s_2^7, s_3^7\) is 0 or 1. Thus \(s_1, s_2, s_3\) form a (3, 15)-UI sequence set.

D. A result for a recursive sequence

In order to establish lower bound on sequence period \(L\), we need a technical lemma concerning a real valued sequence, which is defined recursively as follows.

Lemma 8: Define a recursive sequence \((b_r)_{r=1}^\infty\) by

\[
\begin{align*}
\{b_1 \geq C, \\
\{b_r = b_{r-1} - \left[\frac{b_{r-1} - b_1 \mu}{L}\right], \text{ for } r \geq 2, \end{align*}
\]

where \(C\) is a positive integer and \(\mu\) is a real number that satisfies \(\mu \geq 1\). If \(b_C \geq 1\), then we have

\[
L \geq \left[\frac{8C^2 \mu}{9}\right].
\]

Please refer to Appendix A for the proof of Lemma 8.

IV. LOWER BOUND ON PERIOD L

Given a partition of \(\{N_1, N_2, \ldots, N_K\}\) into \(W\) non-empty groups \(G_1, G_2, \ldots, G_W\) with the smallest group size \(k, W \leq M\), we define \(\Omega(W, k, M, K)\) as the smallest length \(L\) such that an \((M, K, L)\)-schedule sequence set exists.

For a sequence set \(\{s_i : i \in [K]\}\), we denote the number of transmitting symbols in node \(N_i\)’s sequence \(s_i\) by \(\alpha_i\), and the
number of receiving symbols $R_r$’s in $s_i$ by $\beta_r^i$, for $i \in [K]$, $r \in [W]$. It is obvious that
\[ L = \alpha_i + \sum_{r=1}^{W} \beta_r^i, \text{ for any } i \in [K]. \] (10)

From (10), we can derive that there must exist an $i \in [K]$ and an $r \in [W]$ such that
\[ \beta_r^i \leq L/W. \]

Next we consider the nodes in group $G_r$, and denote the node with the smallest number of transmitting symbols $T_r$’s among nodes in $G_r$ by $N_{r_1}$. If this sequence set is an $(M, K, L)$-schedule sequence set, then node $N_1$ can be guaranteed to receive a collision-free packet from node $N_{r_1}$ successfully within a period $L$. This means that at least one $T_r$ in $s_{r_1}$ can match with an $R_r$ in $s_1$, without colliding with $T_r$’s from other nodes in $G_r$. Considering that $|G_r| \geq k$ and that node $N_1$ may also in $G_r$, there should be at least $k-2$ competitors in $G_r$ besides $N_1$ and $N_{r_1}$ that would transmit on channel $r$. To facilitate subsequent discussions, we reduce these schedule sequences to binary sequences in the following way. For sequence $s_{r_1}$, we replace the transmitting symbols $T_r$’s by “1”s, and replace the other symbols by “0”s. This newly obtained binary sequence is denoted by $e_1$. For sequence $s_1$, we replace all the non-$R_r$ symbols by “1”s and replace $R_r$’s by “0”s. This newly obtained sequence is denoted by $e_2$ for the $(k-2)$ sequences corresponding to the $(k-2)$ potential competitors, we replace $T_r$’s by “1”s and other symbols by “0”s. These $(k-2)$ newly obtained sequences are denoted by $e_3, e_4, \ldots, e_k$. The number of “1”s in $e_1$ is denoted by $a_1$, and the number of “1”s in $e_j$ is denoted by $w_j$, for $j \in \{2, 3, \ldots, k\}$.

We have
\[ w_2 = L - \beta_1^i \geq L - L/W, w_3, \ldots, w_k \geq a_1. \]

In the following, we analyze the lower bound on $\Omega(W, k, M, K)$ by using the blocking algorithm [24], in which we fix $e_1$ and cyclically shift $e_2, e_3, \ldots, e_k$ to collide as many “1”s in $e_1$ as possible.

**Blocking algorithm**

**Inputs:** A set of $k$ binary sequences $e_1, e_2, \ldots, e_k$ with common period $L$.

1. Set $j = 2$.
2. Choose a time offset $\tau_j \in [L]$ for $e_j$ such that $w_j$ “1”s in $e_j$ and $a_{j-1}$ “1”s in $e_1$ collide for the most number of times, that is, the Hamming cross-correlation between $e_1$ and $e_j$ with relative time offset $\tau_j$, $H_1,j(\tau_j)$, is maximal.
3. Set the colliding “1”s in $e_1$ after colliding with $e_j$, $a_j = a_{j-1} - \max_{\tau_j \in [L]} H_1,j(\tau_j)$.
4. If $j < k$, increase $j$ by one and go back to Step 2.
5. Output $a_1, a_2, \ldots, a_k$ and stop.

By the blocking algorithm, the values of $a_1, a_2, \ldots, a_k$ are non-negative. If $a_k = 0$, that is, none of the “1”s in $e_1$ can match with a “0” in $e_2^2$, without colliding with “1”s in $e_j$’s, for $j \in \{3, 4, \ldots, k\}$, then correspondingly none of the $T_r$’s in $s_{r_1}$ can match with an $R_r$ in $s_1^i$ without colliding with the $(k-2)$ potential competitors. Thus if the sequence set $\{s_i : i \in [K]\}$ is an $(M, K, L)$-schedule sequence set, then we must have $a_k \geq 1$. Next we will analyze necessary condition for $a_k \geq 1$.

There exists a relation among $a_1, a_2, \ldots, a_k$ which is summarized by the following lemma.

**Lemma 9:** By the blocking algorithm, we have
\[ a_j \leq a_{j-1} - \left\lfloor \frac{a_{j-1}w_j}{L} \right\rfloor, \text{ for } j \in \{2, 3, \ldots, k\}. \] (11)

**Proof:** By the blocking algorithm, for $j \in \{2, 3, \ldots, k\}$,
\[ a_j = a_{j-1} - \max_{\tau_j \in [L]} H_1,j(\tau_j). \]

By Lemma 5 the sum of $H_1,j(\tau_j)$ for all $\tau_j \in [L]$ satisfies
\[ \sum_{\tau_j=0}^{L-1} H_1,j(\tau_j) = a_{j-1}w_j. \]

Thus
\[ \max_{\tau_j \in [L]} H_1,j(\tau_j) \geq \left\lfloor \frac{a_{j-1}w_j}{L} \right\rfloor. \]

This completes the proof for Lemma 9.

Based on (11), we define a recursive sequence $(b_j)_{j=2}^\infty$ to make the analysis for $a_k \geq 1$ more tractable.

**Theorem 10:** Define a sequence $(b_j)_{j=2}^\infty$ recursively by
\[ \begin{align*}
\{b_2 & = \left\lfloor \frac{a_1}{W} \right\rfloor, \\
b_j & = b_j-1 - \left\lfloor \frac{b_j-1b_2W\varepsilon}{L} \right\rfloor, \text{ for } j \geq 3,
\end{align*} \]

where $\varepsilon$ is a real number that satisfies $b_2W\varepsilon \leq a_1$. Then, $a_j \leq b_j$, for $j \in \{2, 3, \ldots, k\}$.

**Proof:** We will prove $a_j \leq b_j$ for $j \in \{2, 3, \ldots, k\}$ by mathematical induction. At first, we consider the value of $a_2$. Due to (11) and the fact that $w_2 \geq L - L/W$, we have
\[ a_2 \leq a_1 - \frac{a_1(L - L/W)}{L} \leq a_1 - a_1 \left(1 - \frac{1}{W}\right) = a_1/W. \] (12)

Then we have $a_2 \leq b_2$.

Next we assume $a_j-1 \leq b_j-1$ for $j \in \{3, 4, \ldots, k\}$. We have known that $w_3, \ldots, w_k \geq a_1$. Then by (11), we have that for $j \in \{3, 4, \ldots, k\}$,
\[ a_j \leq a_j-1 - \left\lfloor \frac{a_{j-1}a_1}{L} \right\rfloor. \] (13)

Since $a_1/L \in (0, 1)$, given $a_j-1 \leq b_j-1$, we have
\[ a_j-1 - \left\lfloor \frac{a_{j-1}a_1}{L} \right\rfloor \leq b_j-1 - \left\lfloor \frac{b_j-1a_1}{L} \right\rfloor. \]

Since $b_2W\varepsilon \leq a_1$, we have
\[ a_j-1 - \left\lfloor \frac{a_{j-1}a_1}{L} \right\rfloor \leq b_j-1 - \left\lfloor \frac{b_j-1b_2W\varepsilon}{L} \right\rfloor = b_j. \] (14)

By combining (13) and (14), we obtain that $a_j \leq b_j$, for $j \in \{3, 4, \ldots, k\}$. This completes the proof.

**Theorem 11:** For an $(M, K, L)$-schedule sequence set, we have
\[ L \geq \frac{8(k-1)^2W\varepsilon}{9}, \]

where $\varepsilon = 1 - \frac{1}{k}$. 

Proof: We first show that the value of \( \varepsilon \) can guarantee that
\[
b_2 W \varepsilon \leq a_1. \tag{16}
\]
Given \( a_k \geq 1 \), it follows that \( \max H_{1,j}(\tau_j) \geq 1 \) for \( j \in \{3, 4, \ldots, k\} \). Then we have \( a_2 \geq k - 1 \). On the other hand, we have \( a_2 \leq a_1/W \) by (12). Therefore, we obtain that \( a_1 \geq W(k-1) \). Since \( \varepsilon = 1 - 1/k \), we have
\[
W \varepsilon = W(k-1) \leq a_1,
\]
which by simple manipulation can be rewritten as
\[
\left( \frac{a_1}{W} + 1 \right) W \varepsilon \leq a_1.
\]
Since \( b_2 = [a_1/W] < a_1/W + 1 \), then (16) can hold.

With this \( \varepsilon \), we have \( b_k \geq a_k \geq 1 \), as well as \( b_2 \geq a_2 \geq k-1 \) by Theorem 10. Then (15) follows by plugging \( C = k-1 \) and \( \mu = W \varepsilon \) into Lemma 3.

Remark 1: When the number of transmitting symbols in each sequence is a multiple of \( W \), that is, \( \alpha_i \) is a multiple of \( W \), then \( b_2 = [a_1/W] = a_1/W \). In this case, with \( \varepsilon = 1 \), (16) can be satisfied. Then the lower bound on \( L \) can be improved to
\[
L \geq \left\lceil \frac{8(k-1)^2W}{9} \right\rceil.
\]

We can observe from Theorem 11 that the lower bound (15) is loose when \( k \) is small. Here we provide another lower bound as a supplement.

Theorem 12: For an \((M, K, L)\)-schedule sequence set, if \( k \geq 2 \), we have
\[
L \geq 4W(k-1). \tag{17}
\]
If \( k = 1 \), we have \( L \geq 4(W-1) \).

Proof: The least required period for \( K \) nodes is no less than that for \( K' = Wk \) nodes. We will analyze the lower bound on \( L \) for \( K' \) nodes. Consider the transmission from node \( N_i \) to node \( N_j \), for \( i, j \in [K'], i \neq j \). Assume that \( N_i \in G_m, m \in [W] \). For the case of \( k \geq 2 \), there are at least \((k-2)\) nodes in \( G_m \) that would cause collisions to node \( N_i \). Without loss of generality, we fix \( s_i \) and shift \( s_j \). Denote the number of transmitting symbols \( T_m \)'s in \( s_i \) that overlap with receiving symbols \( R_m \)'s in \( s_j \) by \( H_{i,j}(\tau_j) \). By Lemma 3 the sum of the \( H_{i,j}(\tau_j) \) for all \( \tau_j \in \mathbb{Z}_L \) satisfies
\[
\sum_{j=0}^{L-1} H_{i,j}(\tau_j) = \alpha_i \beta_j^m. \tag{18}
\]
If \( \sum_{j=0}^{L-1} H_{i,j}(\tau_j) < (k-1)L \), then there must exist at least a value of \( \tau_j \in \mathbb{Z}_L \) such that \( H_{i,j}(\tau_j) < k-1 \). This means that with such a \( \tau_j \), the number of \( T_m \)'s in \( s_i \) that overlap with \( R_m \)'s in \( s_j \) is no more than \((k-2)\). However, there must exist a time offset combination of other \((k-2)\) nodes in \( G_m \), such that \((k-2)\) \( T_m \)'s in \( s_i \) are collided. In this case, node \( N_i \) would fail to transmit to node \( N_j \). Thus to ensure successful transmissions among all the transmitter-receiver pairs with all possible \( \tau \), we have
\[
\alpha_i \beta_j^m \geq (k-1)L, \text{ for any } i, j \in [K'], j \neq i, m \in [W]. \tag{19}
\]

Summing (10) up for \( i \in [K'] \) yields
\[
K'L = \sum_{i=1}^{K'} \alpha_i + \sum_{i=1}^{W} \sum_{r=1}^{K'} \beta_i^r. \tag{20}
\]
By plugging (19) into (20), we obtain
\[
K'L \geq \sum_{i=1}^{K'} \alpha_i + W(k-1)\sum_{i=1}^{K'} \frac{1}{\alpha_i}.
\]
Due to the fact that the harmonic mean is no more than the arithmetic mean, we obtain
\[
K'L \geq \sum_{i=1}^{K'} \alpha_i + W(k-1)L\sum_{i=1}^{K'} \frac{1}{\alpha_i}. \tag{21}
\]
Then (17) follows by the fact that the minimum value of the RHS of (21) is \( 2K'\sqrt{W(k-1)L} \).

For the case of \( k = 1 \), we consider the lower bound on \( L \) for \( K' = W \) nodes. In this case, any transmitter-receiver pair should satisfy
\[
\alpha_i \beta_j^m \geq L, \text{ for any } i, j \in [K'], j \neq i, m \in [W]. \tag{22}
\]
Then by the same analysis, we have
\[
K'L \geq \sum_{i=1}^{K'} \alpha_i + (K'-1)L\sum_{i=1}^{K'} \frac{1}{\alpha_i}. \tag{23}
\]
Based on (23), we have
\[
L \geq 4(K'-1) = 4(W-1).
\]
This completes the proof for Theorem 12.

By combining Theorem 11 and Theorem 12, we can conclude that for an \((M, K, L)\)-schedule sequence set, when \( k = 1 \),
\[
\Omega(W, k, M, K) \geq 4(W-1), \tag{24}
\]
and when \( k \geq 2 \),
\[
\Omega(W, k, M, K) \geq \max \left\{ \left\lfloor \frac{8W(k-1)^3}{9k} \right\rfloor, 4W(k-1) \right\}. \tag{25}
\]
Especially, for the even group division case with \( W = M \), \( k = \lceil K/M \rceil \geq 2 \), the lower bound is
\[
\Omega(W, k, M, K) \geq \max \left\{ \left\lfloor \frac{8M(\lceil K/M \rceil - 1)^3}{9\lceil K/M \rceil} \right\rfloor, 4M(\lceil K/M \rceil - 1) \right\}. \tag{26}
\]

V. CONSTRUCTION FOR AN \((M, K, L)\)-SCHEDULE SEQUENCE SET

In this section, we propose a CRT-based construction for \((M, K, L)\)-schedule sequence set. For notation simplicity, this construction is called Construction *. Given a group division with \( W \) non-empty groups \( G_1, G_2, \ldots, G_W \) and the largest group size \( \ell \), we first design a set of \( W\ell \) sequences for the case that each of the \( W \) groups contains exactly \( \ell \) nodes, then we obtain an \((M, K, L)\)-schedule sequence set by randomly picking \( K \) sequences out of them.

As mentioned in Section III-B, an array can be mapped to a sequence via the CRT correspondence (3), if the number of
rows and the number of columns of the array are coprime. In Construction *, to design a schedule sequence $s_i$ for the node $N_i$, $i \in [W\ell]$, we first construct an array $A_i$ consisting of $2W$ rows each of which is defined by a CRT-UI sequence of length $L' = pq$. Under the construction, $2W$ and $L'$ are required to be coprime with each other, so that we can map the array $A_i$ to a one-dimensional sequence $s_i$ of length $L = 2WL'$. Cyclically shifting $s_i$ by $\tau_j$ is equivalent to row-wise and column-wise shifting $A_i$ by corresponding time offsets. The shifted version of $A_i$ is denoted by $A_i^\tau_j$.

In Construction *, the $\ell$ nodes in each group are associated with a set of $\ell$ CRT-UI sequences $u_1, u_2, \ldots, u_\ell$. Specifically, if node $N_i$ is the $n$-th node in group $G_m, m \in [W], n \in [\ell]$, then each row in its array $A_i$ is defined by the CRT-UI sequence $u_n$: the positions of transmitting symbols $T_{m,n}$ in each row are determined by “1”s of $u_n$. For two nodes $N_i$ and $N_j$, if node $N_i$ is the $n$-th node in $G_m$, and node $N_j$ is the $n$-th node in $G_m$, then all rows in their arrays $A_i$ and $A_j$ are defined by $u_n$. If $\tau_i = \tau_j$, then the transmitting symbols $T_{m,n}$ in $A_i^\tau_j$ would exactly overlap with the transmitting symbols $T_{m,n}$ in $A_j^\tau_j$, indicating that the transmissions between $N_i$ and $N_j$ would fail. To prevent the occurrence of this case, we pre-assign rows in $A_i$ and $A_j$ with different time offsets. The effect of the pre-assigned time offsets, which is based on the auto-correlation property of CRT-UI sequences, will be explained in detail in the proof for Theorem 3. Here we use a simple example to illustrate the intuitive effect.

**Example 3:** Given a binary sequence $s = [1 1 1 0 0]$ with length $L = 5$, we construct two $2 \times 5$ arrays $A_1$ and $A_2$ based on $s$. In each array, the first row is exactly $s$ itself, while the second row is a shifted version of $s$ with a pre-assigned time offset. We set the time offsets as 1 and 2 for the two arrays. Then the obtained arrays $A_1$ and $A_2$ are as follows,

$$A_1 = \begin{bmatrix} s \\ s' \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} s \\ s'' \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

We can check that no matter how we construct a column-wise and row-wise version of the two arrays, if the first rows in the two shifted arrays are exactly the same, then the second rows must be different, and vice versa. Thus for $A_1$, even if all the “1”s in the first or second row are colliding with “1”s in $A_2$, there is at least one “1” in the other row that can survive without collisions and can successfully match with a “0” in $A_2$.

For $K$ nodes and $M$ available channels with group division parameters $W$ and $\ell$, the detailed steps of Construction * are as follows.

**Construction * **

1) Construct a set of $\ell$ sequences $u_1, u_2, \ldots, u_\ell$ by the CRT-UI Construction, with generators $g = 1, 2, \ldots, \ell$, Hamming weight $w = \ell + 1$, $p$ being the smallest prime that satisfies $p \geq \max \{w, 2W - 2\}$, $q$ being the smallest integer that is coprime with $p$ and $2W$, and satisfies $q \geq 2w - 1$. The common period of these CRT-UI sequences is $L' = pq$.

2) For the node $N_i, i \in [W\ell]$, which is the $n$-th node in group $G_m, m \in [W], n \in [\ell]$, we define $u_n(T_m, R_r)$ as the sequence obtained from $u_n$ by replacing “1”s and “0”s with $T_m$’s and $R_r$’s, respectively. Define $u_{n\delta}(T_m, R_r)$ as the sequence obtained by cyclically shifting $u_n(T_m, R_r)$ by a pre-assigned time offset $\delta$, where $\delta$ is defined as the unique integer in $\mathbb{Z}_{L'}$ that satisfies

$$\Phi_{p,q}(\delta) = (m - 1, 0).$$

Stack $u_n(T_m, R_r), u_{n\delta}(T_m, R_r)$ together for $r = 1, 2, \ldots, W$ to form a $2W \times L'$ array $A_i$ as follows,

$$A_i = \begin{bmatrix} u_n(T_m, R_1) \\ u_{n\delta}(T_m, R_1) \\ \vdots \\ u_n(T_m, R_W) \\ u_{n\delta}(T_m, R_W) \end{bmatrix}.$$

3) Schedule sequence $s_i = [s_i(0) s_i(1) \ldots s_i(L - 1)]$ of length $L = 2WL'$ is obtained from $A_i$ via the following CRT correspondence:

$$s_i(t) = A_i(t \mod 2W, t \mod L'), \ t \in \mathbb{Z}_{L'}.$$

4) Randomly pick $K$ sequences out of the $W\ell$ sequences to form an $(M, K, L)$-schedule sequence set.

**Example 4:** For 4 nodes ($K = 4$) and 2 channels ($M = 2$), we let $W = 2$ and $\ell = 2$, specifically, $G_1 = \{N_1, N_2\}, G_2 = \{N_3, N_4\}$. Under this group division, we construct 4 schedule sequences $s_1, s_2, s_3, s_4$ according to Construction *.

At first, we design a set of two CRT-UI sequences $u_1, u_2$ with $w = 3, p = 3, q = 5$ and generators 1, 2 as follows,

$$u_1 = [1 1 1 0 0 0 0 0 0 0 0 0 0 0 0];$$
$$u_2 = [1 0 0 0 0 0 1 0 0 0 0 0 0 0 0].$$

For nodes $N_1, N_2 \in G_1$, $\Phi_{p,q}(\delta_1) = (0, 0)$, thus $\delta_1 = 0$. For nodes $N_3, N_4 \in G_2$, $\Phi_{p,q}(\delta_2) = (1, 0)$, thus $\delta_2 = 10$. For each node, we construct a $4 \times 15$ array as shown in [27]. The symbols $T_1, T_2, R_1$ and $R_2$ are displayed in different colors in order to facilitate easier reading. Schedule sequence $s_i (i \in [4])$ of length $L = 60$ is obtained from $A_i$ by the mapping: $s_i(t) = A_i(t \mod 4, t \mod 15)$.

**Theorem 13:** A sequence set $\{s_i; i \in [K]\}$ obtained by Construction * is an $(M, K, L)$-schedule sequence set.

**Proof:** We consider the transmission from $N_i$ to $N_j$, for $i, j \in [K], i \neq j$. Assume that $N_i$ is the $n$-th node in group $G_m$, and $N_j$ is the $y$-th node in group $G_x$, for $m \in [W], n \in [\ell]$. Since the transmission is successful only when $N_j$ receives on channel $m$ and at the same time $N_i$ transmits on channel $m$ without colliding with other nodes, thus we need to consider the positions of $R_m$’s in $s_i^\tau$, or equivalently, $A_j^\tau$. By Construction *, there are two rows containing $R_m$’s in $A_j$: row $(2m - 1)$ which is $u_y(T_x, R_m)$ and row $(2m - 1)$ which is $u_y(T_x, R_m)$. After row-wise and column-wise shifting, there are still only two rows containing $R_m$’s in $A_j^\tau$. The row indices of these two rows are denoted by $\eta_1$ and $\eta_2$. 

where $\eta_1, \eta_2 \in \mathbb{Z}_{2W}$. Row $\eta_1$ and row $\eta_2$ in $A_i^\tau$ and $A_j^\tau$ are shown as follows,

$A_i^\tau = \begin{bmatrix}
\eta_1(T_1, R_1) \\
\eta_1(T_2, R_2) \\
\vdots \\
\eta_1(T_m, R_m)
\end{bmatrix}$

$A_j^\tau = \begin{bmatrix}
\eta_2(T_1, R_1) \\
\eta_2(T_2, R_2) \\
\vdots \\
\eta_2(T_m, R_m)
\end{bmatrix}$

Note that in row $\eta_1$ and row $\eta_2$ of $A_i^\tau$, $R_m$ positions are defined by the CRT-UI sequence $u_k$ with time offsets $\phi_1$ and $\phi_2$ respectively. The values of $\phi_1, \phi_2$ depend on $\tau_j$, and satisfy $|\phi_1 - \phi_2| = \delta_x$. In row $\eta_1$ and row $\eta_2$ of $A_i^\tau$, $T_m$ positions are defined by the CRT-UI sequence $u_k$, with time offsets $\theta_1$ and $\theta_2$ respectively. The values of $\theta_1, \theta_2$ depend on $\tau_i$, and satisfy $|\theta_1 - \theta_2| = \delta_m$. The symbol $R_m$ in $A_i^\tau$ indicates a receiving symbol and the exact channel number is immaterial for the discussion.

We denote the number of collision-free $T_m$’s in row $\eta_1$ (resp. $\eta_2$) of $A_i^\tau$ that overlap with an $R_m$, instead of a $T_j$, in row $\eta_1$ (resp. $\eta_2$) of $A_j^\tau$ by $N_{\eta_1}^\tau$ (resp. $N_{\eta_2}^\tau$), denote the number of $T_m$’s in row $\eta_1$ (resp. $\eta_2$) of $A_i^\tau$ that overlap with a $T_j$ in row $\eta_1$ (resp. $\eta_2$) of $A_j^\tau$ by $N_{\eta_1}^\tau$ (resp. $N_{\eta_2}^\tau$); and denote the number of $T_m$’s in row $\eta_1$ (resp. $\eta_2$) of $A_i^\tau$ that collide with $T_m$’s in other arrays of $G_m$ by $N_{\eta_1}^c$ (resp. $N_{\eta_2}^c$).

It is obvious that

$$N_{\eta_1}^\tau \geq w - N_{\eta_1}^t - N_{\eta_1}^c, \quad N_{\eta_2}^\tau \geq w - N_{\eta_2}^t - N_{\eta_2}^c,$$

(28)

where $w = \ell + 1$.

Next we verify whether the following condition can be satisfied: for all possible $\tau$, in row $\eta_1$ or row $\eta_2$ of $A_i^\tau$, there is at least one collision-free $T_m$ that overlaps with an $R_m$ in $A_j^\tau$. That is, $\max(N_{\eta_1}^\tau, N_{\eta_2}^\tau) \geq 1$ for all possible $\tau$. To verify this, we need to check the following three cases. In each case, we have $N_{\eta_1}^\tau, N_{\eta_2}^\tau \leq \ell - 1$ for all possible $\tau$. This is because by Construction $\star$, rows of the arrays in $G_m$ are determined by $|G_m|$ CRT-UI sequences from $u_1, u_2, \ldots, u_{2r}$, which have the property that the Hamming correlation of any pair of them is at most 1, no matter how we cyclically shift them.

1) Both the transmitter $N_i$ and the receiver $N_j$ come from the same group, i.e., $m = x, n = y$. In this case, $N_{\eta_1}^t + N_{\eta_2}^t \leq \ell - 1, N_{\eta_1}^c + N_{\eta_2}^c \leq \ell - 1$. Then by (28), we have $N_{\eta_1}^\tau, N_{\eta_2}^\tau \geq 2$.

2) The transmitter $N_i$ and the receiver $N_j$ come from different groups and $n \neq y$. In this case, $N_{\eta_1}^t, N_{\eta_2}^t \leq 1$. This is because the values of $N_{\eta_1}^c, N_{\eta_2}^c$ are determined by the Hamming cross-correlation of $u_n$ and $u_y$, which is no more than 1 for all possible $\phi_1, \phi_2$ and $\phi_1, \phi_2$. Then, by (28) and the fact that $N_{\eta_1}^c, N_{\eta_2}^c \leq \ell - 1$, we have $N_{\eta_1}^\tau, N_{\eta_2}^\tau \geq 1$.

3) The transmitter $N_i$ and the receiver $N_j$ come from different groups and $n = y$. In this case, the values of $N_{\eta_1}^t, N_{\eta_2}^t$ are determined by the auto-correlation of $u_n$. By the following Proposition $14$, we have min$(N_{\eta_1}^t, N_{\eta_2}^t) \leq 1$ for all possible $\theta_1, \theta_2$ and $\phi_1, \phi_2$. Then by (28) and the fact that $N_{\eta_1}^c, N_{\eta_2}^c \leq \ell - 1$, we have max$(N_{\eta_1}^\tau, N_{\eta_2}^\tau) \geq 1$.

In summary, max$(N_{\eta_1}^\tau, N_{\eta_2}^\tau) \geq 1$ for all possible $\tau$. That is, the sequence set $\{s_i : i \in [K]\}$ obtained by Construction $\star$ can guarantee at least one collision-free transmission from $N_i$ to $N_j$, for any $i, j \in [K], i \neq j$, and for all possible $\tau$. Therefore, it is an $(M, K, L)$-schedule sequence set.

**Proposition 14:** If $m \neq x, n = y$, then min$(N_{\eta_1}^t, N_{\eta_2}^t) \leq 1$ for all possible $\theta_1, \theta_2, \phi_1, \phi_2$.

**Proof:** By Construction $\star$, $|\theta_1 - \theta_2| = \delta_m, |\phi_1 - \phi_2| = \delta_x$.

Denote the time offset between row $\eta_1$ in $A_i^\tau$ and row $\eta_1$ in $A_j^\tau$ by $\tau_{i,j,1}, \tau_{i,j,1} = \theta_1 - \phi_1$. Denote the time offset between row $\eta_2$ in $A_i^\tau$ and row $\eta_2$ in $A_j^\tau$ by $\tau_{i,j,2}, \tau_{i,j,2} = \theta_2 - \phi_2$.

It follows that

$$\tau_{i,j,2} = \tau_{i,j,1} \pm (\delta_m \pm \delta_x).$$

(29)

Next we first analyze the value of $N_{\eta_1}^\tau$ which equals the Hamming auto-correlation between $u_n$ and $u_{n-j,1}, H_{n,n}(\tau_{i,j,1}),$ in the following two cases.

1) $\Phi_{p,q}(\tau_{i,j,1}) \neq (n,1)a$, for $a \in \{0, \pm 1, \ldots, \pm (w - 2)\}$. In this case, $N_{\eta_1}^\tau = 0$ or 1 since we have $H_{n,n}(\tau_{i,j,1}) = 0$ or 1 by Lemma $7$. 


2) \( \Phi_{p,q}(\tau_{i,j,1}) = (n,1)a, \) for \( a \in \{0, \pm 1, \pm 2, \ldots, \pm (w-2)\}. \) In this case, \( N_{n_2}^0 = H_{n,n}(\tau_{i,j,1}) \geq 2. \) Then we analyze the value of \( N_{n_2}^0, \) which equals the Hamming auto-correlation between \( u_n \) and \( \hat{u}_{n,i,j,n} H_{n,n}(\tau_{i,j,2}). \) We prove \( N_{n_2}^0 \leq 1 \) by contradiction as follows. Assume that \( H_{n,n}(\tau_{i,j,2}) \geq 2. \) It implies that \( \Phi_{p,q}(\tau_{i,j,2}) = (n,1)b, \) for \( b \in \{0, \pm 1, \pm 2, \ldots, \pm (w-2)\}. \) Let \( c = a - b, \ c \in \{0, \pm 1, \pm 2, \ldots, \pm 2(w-2)\}. \) Then we discuss whether this assumption can hold in the following cases indicated by (29):

a) \( \tau_{i,j,2} = \tau_{i,j,1} - (\delta_m - \delta_x). \) In this case,

\[
\Phi_{p,q}(\delta_m - \delta_x) = \Phi_{p,q}(\tau_{i,j,1} - \tau_{i,j,2})
\]

\[(nc \mod p, c \mod q). \]

By Construction *, \( \Phi_{p,q}(\delta_m) = (m-1,0), \) \( \Phi_{p,q}(\delta_x) = (x-1,0), \) then we have \( \Phi_{p,q}(\delta_m - \delta_x) = ((m-x) \mod p, 0). \) Thus

\[
m - x \equiv nc \mod p, \quad 0 \equiv c \mod q.
\]

Since \( q \geq 2w - 1 > |c|, \) then (31) implies \( c = 0. \) However, (30) cannot hold with \( c = 0 \) because \( m-x \in \{ \pm 1, \pm 2, \ldots, \pm (W-1) \} \) and \( p \geq 2W - 2. \)

b) \( \tau_{i,j,2} = \tau_{i,j,1} + (\delta_m + \delta_x). \) In this case,

\[
\Phi_{p,q}(\delta_m + \delta_x) = (nc \mod p, c \mod q).
\]

By Construction *, \( \Phi_{p,q}(\delta_m + \delta_x) = ((m+x-2) \mod p, 0), \) thus

\[
m + x - 2 \equiv nc \mod p, \quad 0 \equiv c \mod q.
\]

Again, (33) implies \( c = 0. \) However, (32) cannot hold with \( c = 0 \) since \( m+x-2 \in \{ 1, 2, \ldots, 2W-3 \} \) and \( p \geq 2W - 2. \)

c) \( \tau_{i,j,2} = \tau_{i,j,1} + (\delta_m + \delta_x). \) By the same analysis for case (a) and (b), we can also derive that the assumption that \( H_{n,n}(\tau_{i,j,2}) \geq 2 \) cannot hold.

Therefore, we conclude that if \( N_{n_2}^0 \geq 2, \) then \( N_{n_2}^0 \leq 1. \)

This completes the proof for Proposition 14.

VI. DISCUSSION ON PERIOD L UNDER EVEN GROUP DIVISION 

In this section, we discuss the sequence period obtained by Construction * under even group division. Given \( K \) and \( M, \) we consider the optimal value of \( W \) that can minimize period \( L. \) We propose the following algorithm: Define

\[
M' = \left\lfloor \sqrt{\frac{K}{2} + \frac{9}{16} + \frac{3}{4}} \right\rfloor.
\]

if \( M > M', \) then let \( W = M', \) otherwise let \( W = M. \)

We provide an intuitive argument for this algorithm as follows. By Construction *, \( w = \ell + 1 = \left\lfloor \frac{K}{W} \right\rfloor + 1, \) \( p \) is the smallest prime that satisfies \( p \geq \max\{w, 2W - 2\}. \) To simplify the following discussion, we assume \( \left\lfloor \frac{K}{W} \right\rfloor \approx \frac{K}{W}. \)

1) \( M \leq M'. \) In this case, \( W \leq M', \) then \( w \geq 2W - 2, \) thus \( p \geq w. \) When \( w \) is a prime, and \( (2w - 1) \) is coprime with \( 2W \) and \( w, \) we set \( p = w \) and \( q = 2w - 1, \) then

\[
L = 2Wpq = \frac{4K^2}{W} + 6K + 2W.
\]

By (35), \( L \) is a decreasing function of \( W \) when \( K \) is fixed and \( W \leq M'. \) Therefore, in the case of \( M \leq M', \) \( L \) is minimized when \( W = M. \)

2) \( M > M'. \) We consider \( W \geq M'. \) In this case, \( p \geq 2W - 2. \) When \( (2w - 1) \) is a prime, and \( (2w - 1) \) is coprime with \( 2W \) and \( (2W - 1), \) we set \( p = 2W - 1 \) and \( q = 2w - 1, \) then

\[
L = 2Wpq = 4W^2 + 2W(4K - 1) - 4K.
\]

We can see that \( L \) in (36) is an increasing function of \( W \) when \( K \) is fixed. Therefore, in the case of \( M > M', \) \( L \) is minimized when \( W = M'. \)

For general \( K \) and \( M < M', \) under even division with \( W = M, \) we have the following result for \( L \) obtained by Construction *.

Proposition 15: Under even group division with \( W = M \leq M', \) there exists a schedule sequence set by Construction * with sequence period

\[
L \leq 2M \left( 2 \left\lceil \frac{K}{M} \right\rceil + 2 \right) \left( 4 \left\lceil \frac{K}{M} \right\rceil + 2 \right).
\]

Proof: By Construction *, \( w = \ell + 1 = \left\lfloor \frac{K}{M} \right\rfloor + 1 \geq 2W - 2, \) \( p \) is the smallest prime that satisfies \( p \geq w. \) By Bertrand’s postulate, we have \( p < 2w. \) We set \( q \) as the smallest prime that satisfies \( q \geq 2w - 1. \) It is obvious that such a \( q \) is coprime with \( 2M \) and \( p. \) Again by Bertrand’s postulate, we have \( q \leq 4w - 2. \) Thus, the obtained period \( L = 2Mpq \) satisfies (37).

By comparing (37) with the lower bound (26) obtained in Section IV, we can observe that under even group division with \( W = M \leq M', \) the period under Construction * can achieve the same order in \( K \) and \( M \) as the lower bound, that is, \( O(K^2/M). \) To illustrate the gap between the period under Construction * and the lower bound, we list their ratios for some \( K \) and \( M \) in Table II. For example, when \( K = 70 \) and \( M = 4, \) the shortest period by Construction * is \( L = 5624, \) and the lower bound by (26) is \( L \geq 857, \) then the ratio between them is \( 5624/857 \approx 6.56. \) We will try to reduce the gap in future work.

Moreover, we can see from (37) and the achievable length (7) for the single channel case that under even group division with \( W = M \leq M', \) the period under Construction * can achieve an asymptotic reduction by a factor of \( M. \)

TABLE II

| K   | 60   | 70   | 80   | 90   | 100  | 110  | 120  | 130  | 140  | 150  |
|-----|------|------|------|------|------|------|------|------|------|------|
| 2   | 3.23 | 3.26 | 3.28 | 3.30 | 3.14 | 3.13 | 3.55 | 3.45 | 3.72 | 3.12 |
| 3   | 6.18 | 6.97 | 5.23 | 5.39 | 5.96 | 5.16 | 5.46 | 5.99 | 5.26 | 5.63 |
| 4   | 6.45 | 6.56 | 6.15 | 7.34 | 5.01 | 5.22 | 5.99 | 5.09 | 5.26 | 5.63 |
| 5   | 7.12 | 7.06 | 5.98 | 3.79 | 6.18 | 5.78 | 6.3 | 5.75 | 5.29 | 5.23 |
VII. RANDOM SCHEMES

In this section, we analyze the optimal transmitting and receiving probabilities for two random schemes, given \( K \) nodes and \( W \) of \( M \) channels being employed, \( 1 \leq W \leq M \).

A. General random scheme

In the general random scheme, there is no concept of groups. For any node, at a time slot, it transmits on any one of the \( W \) channels with probability \( p_a \) and receives on any one of the \( W \) channels with probability \( q_a \). The values of \( p_a \) and \( q_a \) satisfy \( 0 < p_a < 1 \), \( 0 < q_a < 1 \), and \( W(p_a + q_a) = 1 \). The probability for any node to successfully receive a packet from another node in a time slot is

\[
P_a = Wp_a q_a (1 - p_a)^{K-2} = p_a (1 - W p_a) (1 - p_a)^{K-2}. \quad (38)
\]

B. Assignment T based random scheme

In the Assignment T based random scheme, we divide \( K \) nodes into \( W \) non-empty groups. The \( \{G_m\} \) nodes belong to group \( G_m \) can only transmit on channel \( m \), for \( m \in [W] \). For node \( N_i \in G_m \), \( i \in [K] \), \( m \in [W] \), at a time slot, it transmits on channel \( m \) with probability \( p_b \), receives on channel \( m \) with probability \( q_1 \), and receives on any other channel with probability \( q_2 \). The values of \( p_b, q_1, q_2 \) satisfy \( 0 < p_b < 1 \), \( 0 < q_1 < 1 \), \( 0 < q_2 < 1 \), and

\[
p_b + q_1 + (W-1)q_2 = 1. \quad (39)
\]

The probability for node \( N_i \) to successfully transmit a packet to a node in \( G_m \) in a time slot is

\[
P_a = p_b q_1 (1 - p_b)^{|G_m|-2}. \quad (40)
\]

The probability for node \( N_i \) to successfully transmit a packet to a node in another group in a time slot is

\[
P_b = p_b q_2 (1 - p_b)^{|G_m|-1}. \quad (41)
\]

C. Optimized random scheme

In this section, we try to optimize the two random schemes. At first, for the general random scheme, we find from (38) that for any given \( K \) and \( p_a \), \( P_a \) monotonically decreases as \( W \) increases. Next we consider \( P_a \) (see (40)) and \( P_b \) (see (41)) in the Assignment T based random scheme. To simplify the discussion, we assume \( P_b = P_b \) and \( |G_m| = K/W \) for any \( m \in [W] \). Then by (39), (40) and (41), we have

\[
P_b = p_b (1 - p_b)^{K/W}/(W - p_b). \quad (42)
\]

We can observe from (42) that for any given \( K \) and \( p_b \), \( P_b \) also monotonically decreases as \( W \) increases. Thus for both of the general random scheme and the Assignment T based random scheme, using only one channel, i.e., \( W = 1 \), is optimal for maximizing \( P_a \) or \( P_b \). This implies that the two random schemes cannot efficiently make use of the multi-channel resources.

With only one channel, the two random schemes are equivalent. For each node at each time slot, it transmits on this channel with probability \( p \) and receives on this channel with probability \( 1 - p \). Then the probability for a node to receive from one of its \((K-1)\) neighboring nodes successfully at a time slot, denoted by \( P \), equals \( p (1 - p)^{K-1} \). By taking derivative of \( P \) with respect to \( p \), we obtain that \( P \) attains its maximum value when \( p = 1/K \). Thus the optimal transmitting probability for the two random schemes with one channel is

\[
p^* = 1/K.
\]

The corresponding \( P \) is denoted by \( P^* \),

\[
P^* = \frac{(K-1)^{K-1}}{K^K}. \quad (43)
\]

VIII. COMPARISON BETWEEN SEQUENCE SCHEME AND RANDOM SCHEME

In this section, we compare the frame length and broadcast completion time under our proposed sequence scheme with those under the optimized random scheme. The schedule sequences employed are obtained by Construction * under even group division.

A. Frame length

First, we explain how frame length is defined for random schemes. While it is possible to find sequence schemes that can ensure each node has at least one successful broadcast per frame, it is impossible to provide such guarantee for the random schemes. Using a high value for frame length would strengthen the guarantee but weaken the performance of the random schemes. In order to determine a fair frame length value for the random schemes, we adopt the following argument. We assume that all the nodes start at time \( t = 0 \) without any offset to render the analysis manageable. Let \( X \) be the time required for each of the \( K \) nodes to broadcast a packet to all other nodes at least once, which is also the time required for each node to receive a packet from each other node at least once. Motivated by the least required reliability for URLLC [8], we set the probability \( P(X \leq L_{rand}) \) as 99.999\% . We then set the frame length to be \( L_{rand} \).

Let \( X_i \) be the time required for node \( N_i \) to receive a packet from each other node at least once, for \( i \in [K] \). By definition, we have

\[
X = \max_{i \in [K]} X_i.
\]

To simplify calculation, we follow the assumption in [10], that is, \( X_i \)'s are assumed to be independent for all \( i \in [K] \). Then for any \( \ell \geq 0 \), \( P(X \leq \ell) \) can be obtained from \( P(X_i \leq \ell) \) by

\[
P(X \leq \ell) = \prod_{i=1}^{K} P(X_i \leq \ell). \quad (44)
\]

We analyze \( P(X_i \leq \ell) \) by using results in the coupon collector's problem [30]. We use \( \mathcal{E}_i^j \) to denote the event that at a time slot, the \( j \)-th of the \((K-1)\) neighboring nodes of \( N_i \) successfully transmits a packet to node \( N_i \), for \( j = 1, 2, \ldots, K - 1 \). We use \( \mathcal{E}_i^K \) to denote the event that at a time slot, none of the \((K-1)\) events \( \mathcal{E}_i^1, \mathcal{E}_i^2, \ldots, \mathcal{E}_i^{K-1} \) happens. Then by definition, \( X_i \) is exactly the time slots needed for the \((K-1)\) events \( \mathcal{E}_i^1, \mathcal{E}_i^2, \ldots, \mathcal{E}_i^{K-1} \) to happen at least once. Now consider a coupon collector’s problem: in a container indexed by \( i \), there are \( K \) coupons which are
randomly drawn one by one with replacement. Among these \( K \) coupons, there are \((K - 1)\) different coupons corresponding to events \( \mathcal{E}_1^j, \mathcal{E}_2^j, \ldots, \mathcal{E}_{K-1}^j \), and a null coupon corresponding to the event \( \mathcal{E}_K^j \). Let \( Y_i \) be the time required to get a collection of \((K - 1)\) different coupons. Then \( X_i \) has the same distribution as \( Y_i \). That is, for any \( \ell \geq 0 \),

\[
P(X_i \leq \ell) = P(Y_i \leq \ell).
\] (45)

Let \( p_i = [p(\mathcal{E}_1^i)p(\mathcal{E}_2^i) \ldots p(\mathcal{E}_{K-1}^i)] \), where \( p(\mathcal{E}_j^i) \) denotes the probability that the event \( \mathcal{E}_j^i \) happens at a time slot, which also denotes the probability that coupon \( j \) is drawn at a time slot, for \( j = 0, 1, \ldots, K - 1 \). Since the distribution of \( Y_i \) depends on \( p_i \), we will use the notation \( Y_i(p_i) \). Let \( P = [p_1 \ p_2 \ \ldots \ p_K] \). Since the value of \( X \) depends on \( P \), we will abuse \( X(P) \) and \( X \) when they are clear from the context. We will also abuse \( L_{\text{rand}}(P) \) and \( L_{\text{rand}} \).

Based on (44) and (45), we can calculate \( P(X \leq \ell) \) as follows,

\[
P(X \leq \ell) = \prod_{i=1}^{K} P(Y_i(p_i) \leq \ell).
\] (46)

For \( Y_i(p_i) \), we have found the following result from the literature.

**Lemma 16:** \cite{31} For any given \( i \) and \( \ell \geq 0 \), if \( p(\mathcal{E}_j^i) = (1 - p(\mathcal{E}_j^i))/(K - 1) \) for \( j = 1, \ldots, K - 1 \), then

\[
P(Y_i(p_i) \leq \ell) = 1 - \sum_{i=0}^{K-2} (-1)^{K-2-i} \left( \begin{array}{c} K - 1 \cr i \end{array} \right) \frac{(K - 1 - i)p(\mathcal{E}_0^i) + i}{K - 1} \ell^i.
\]

Next we analyze \( L_{\text{rand}} \) under the optimized random scheme. We have obtained in Section VII-C that in the optimized random scheme, for any node, the probability that another node successfully transmits to it in a time slot equals \( P^* \) (see (43)), that is,

\[
p(\mathcal{E}_1^i) = p(\mathcal{E}_2^i) = \cdots = p(\mathcal{E}_{K-1}^i) = P^*, \quad p(\mathcal{E}_0^i) = 1 - (K - 1)P^*.
\]

for any \( i \in [K] \). Let \( P^* = [p_1^* \ p_2^* \ \ldots \ p_K^*] \), where \( p_i^* = [p(\mathcal{E}_0^i) \ P^* \ \ldots \ P^*] \), for any \( i \in [K] \). Then by Lemma 16 and (46), we obtain that for any \( \ell \geq 0 \),

\[
P(X(P^*) \leq \ell) = \left[ 1 - \sum_{i=0}^{K-2} (-1)^{K-2-i} \left( \begin{array}{c} K - 1 \cr i \end{array} \right) \frac{(K - 1 - i)p(\mathcal{E}_0^i) + i}{K - 1} \ell^i \right]^K.
\] (47)

Based on (47), we can find \( L_{\text{rand}}(P^*) \) to satisfy \( P(X(P^*) \leq L_{\text{rand}}(P^*)) = 99.999\% \). We have listed \( L_{\text{rand}}(P^*) \) for some \( K \) and \( M \) in Table III.

Note that in the sequence scheme, even though \( L \) obtained from Construction \( \ast \) is asymptotically decreasing with respect to \( W \) when \( W \leq M \), there are some cases where \( L \) with a larger \( W \) is longer than that with a smaller \( W \) due to the irregularity in occurrence of prime numbers. Therefore, given \( M \), we choose the smallest one among \( L \)’s corresponding to \( W = 1, 2, \ldots, M \). We take the case of \( K = 10 \) and \( M = 2 \) for example. By Construction \( \ast \), when \( W = 1, L = 209 \); while when \( W = 2, L = 308 \). Then given \( M = 2 \), we only use one channel and thus \( L = 209 \). We have also listed \( L \) for some \( K \) and \( M \) in Table III in which we have shown \( P(X(P^*) \leq L) \) as well.

### Table III

| \((K, M)\) | \(L_{\text{rand}}(P^*)\) | \(P(X(P^*) \leq L)\) |
|---------|-----------------|-----------------|
| (10, 1) | 209             | 0.9769          |
| (10, 2) | 209             | 0.9769          |
| (15, 1) | 493             | 0.9999          |
| (15, 5) | 462             | 0.9999          |
| (18, 1) | 665             | 0.9999          |
| (18, 2) | 665             | 0.9999          |
| (18, 3) | 546             | 0.9997          |
| (20, 1) | 897             | 0.9999          |
| (20, 2) | 616             | 0.997           |
| (24, 1) | 1123            | 0.999999        |
| (24, 3) | 1123            | 0.99998         |
| (24, 4) | 728             | 0.9944          |

From Table III we can observe that in most cases, the frame length under our proposed sequence scheme is shorter than that under the optimized random scheme, that is, \( L < L_{\text{rand}}(P^*) \). There exist some cases where \( L_{\text{rand}}(P^*) < L \). For example, when \( K = 24, M = 1 \), we have \( L_{\text{rand}}(P^*) = 1130 \), \( L = 1363 \). However, we should note that \( L_{\text{rand}}(P^*) \) just indicates that \( P(X(P^*) \leq L_{\text{rand}}(P^*)) = 99.999\% \), but cannot provide a hard guarantee on broadcast delay due to its probabilistic nature. In this case, even if we set \( L_{\text{rand}} = L = 1363 \), we only have \( P(X(P^*) \leq L_{\text{rand}}) = 99.999\% \), instead of \( P(X(P^*) \leq L_{\text{rand}}) = 1 \).

We can conclude that in terms of frame length, our proposed sequence scheme outperforms the random scheme in two aspects. One is that the sequence scheme can efficiently utilize multi-channel resources to reduce frame length while the random scheme cannot. The other is that the sequence scheme can provide a hard guarantee on delay.

### B. Broadcast completion time

In this section, we consider another performance metric – broadcast completion time. In order to show the relationship between the broadcast completion time and the number of employed channels, \( W \), we let \( W = 1, 2, \ldots, M \) in each scheme. For the general random scheme, we find \( p_a \) to optimize \( P_a \) in \cite{38}. For the Assignment T based random scheme, we assume \( P_a = P_b \) and find \( p_a \) to optimize \( P_b \) in \cite{42}. Since we observe that the optimized \( P_b \) is no less than the optimized \( P_a \) for any given \( K \) and \( W \), we will only compare the Assignment T based random scheme with the sequence scheme.

Fig. 2 shows the probability distribution of the broadcast completion time in 10000 runs for the case where \( K = 18, M = 3 \) and \( W = 1, 2, 3 \) under the sequence scheme and the Assignment T based random scheme. The time offset of each node in each run is randomly generated. We can observe from Fig. 2 that for both sequence scheme and random scheme, using only one channel can achieve shorter broadcast completion
time with higher probability. We have conducted simulations for many other cases and observed the same result. For the random scheme, this is not surprising since we have obtained similar result when we discuss frame length. However, for the sequence scheme, this is an interesting phenomenon and is contradictory to the performance for unicast completion time we considered for unicast in [19]. In [19], we have shown that the sequence scheme can utilize multi-channel resources to decrease sequence period as well as the unicast completion time. The reasons behind may lie in the nature of broadcast and unicast and the tradeoff caused by multiple channels. We will try to explore the cause of this in the future.

IX. CONCLUSION

We investigate schedule sequence design to guarantee successful broadcast in an asynchronous ad hoc network. Previous works on the sequence design for broadcast are mainly developed with a single channel. In this paper, we derive a lower bound on the shortest common period and propose a CRT-based sequence construction method, for the multi-channel model. Under even group division with \( W = M \leq M' \), the period under our proposed construction has the same order as the lower bound. We also achieve an asymptotic reduction in the order of \( M \) compared with the shortest known sequence period for the single channel case.

We also analyze optimal transmitting and receiving probabilities for two random schemes. Comparisons for frame length and broadcast completion time under different schemes are conducted. By comparison, we find that our proposed sequence scheme can ensure successful broadcast within shorter frame length than the optimized random scheme. Moreover, our proposed sequence scheme can decrease the frame length by utilizing multiple channels while the random schemes cannot. However, using more channels would result in longer broadcast completion time, for both sequence scheme and random scheme.

APPENDIX

Proof: The sequence \( \{b_r\}_{r=1}^{\infty} \) is non-negative and monotonically non-increasing. Let \( \lambda = \lceil \mu b_1^2 / L \rceil \), which is the largest difference between two adjacent entries in the sequence \( \{b_r\}_{r=1}^{\infty} \), and for \( j = 1, 2, \ldots, \lambda \), let \( n_i \) be the number of indices \( r \geq 1 \) such that \( b_r - b_{r+1} = i \). We have the following identity

\[
n_1 + 2n_2 + \cdots + \lambda n_\lambda = b_1. \tag{48}
\]

We denote the largest \( b_r \) in \( \{b_r\}_{r=1}^{\infty} \) such that \( b_r - b_{r+1} = i \) by \( b_r^+ \), and denote the smallest such \( b_r \) by \( b_r^- \). The two entries followed by \( b_r^- \) are \( b_r^- + 1 \) and \( b_r^- + 2 \). For \( b_r^- \), we have

\[
b_r^- = b_1 - \sum_{j=i+1}^{\lambda} j n_j.
\]

For \( b_r^- + 1 \) and \( b_r^- + 2 \), we have

\[
\left| b_r^- + 1 - b_r^- \right| = \left| \frac{\mu b_1 b_r^- + 1}{L} \right| \leq i - 1. \tag{49}
\]

The inequality in (49) indicates

\[
b_r^- + 1 \leq \frac{(i - 1)L}{\mu b_1}.
\]

Since \( b_r^+ - b_r^- = n_i \), then we have

\[
i n_i \geq \left( b_1 - \sum_{j=i+1}^{\lambda} j n_j \right) - \frac{(i - 1)L}{\mu b_1},
\]

that is,

\[
\frac{(i - 1)L}{\mu b_1} + \sum_{j=1}^{\lambda} j n_j \geq b_1. \tag{50}
\]

For \( i = 2, 3, \ldots, \lambda \), by dividing both sides of (50) by \( i(i - 1) \), and summing up the resulting inequalities, we have

\[
\sum_{i=2}^{\lambda} \frac{L}{\mu b_1 i} + \sum_{i=2}^{\lambda} \sum_{j=i}^{\lambda} j n_j \geq \sum_{i=2}^{\lambda} \frac{b_1}{i(i - 1)}. \tag{51}
\]

The RHS of (51) is equal to

\[
\sum_{i=2}^{\lambda} \frac{b_1}{i(i - 1)} = b_1 \left( 1 - \frac{1}{\lambda} \right),
\]

and the double summation in (51) is equal to

\[
\sum_{i=2}^{\lambda} \sum_{j=1}^{\lambda} \frac{j n_j}{i(i - 1)} = \sum_{j=2}^{\lambda} j n_j \sum_{i=2}^{\lambda} \frac{1}{i(i - 1)} = \sum_{j=2}^{\lambda} n_j (j - 1).
\]

Therefore we can rewrite (51) as

\[
\frac{L}{\mu b_1} \sum_{i=2}^{\lambda} \frac{1}{i} + \sum_{j=2}^{\lambda} n_j (j - 1) \geq b_1 \left( 1 - \frac{1}{\lambda} \right),
\]

\[
\sum_{j=1}^{\lambda} n_j \leq b_1 \frac{\lambda}{\mu b_1} + \frac{L}{\mu b_1} \sum_{i=2}^{\lambda} \frac{1}{i}.
\]

If \( b_C \geq 1 \), then the number of strictly positive differences between two adjacent entries in \( \{b_r\}_{r=1}^{\infty} \) must be no less than \( C \), that is, \( C \leq \sum_{j=1}^{\lambda} n_j \). Thus, we have

\[
C \leq b_1 \frac{\lambda}{\mu b_1} + \frac{L}{\mu b_1} \sum_{i=2}^{\lambda} \frac{1}{i}. \tag{52}
\]
Note that when $\lambda = 1$, (52) still holds since it is reduced to $C \leq b_1$.

The inequality in (52) can be re-written as

$$C \leq \sqrt{\frac{L}{\mu}} \left( \frac{b_1}{\sqrt{\lambda}} + \frac{\sqrt{T}}{b_1} \sum_{i=2}^{\lambda} \frac{1}{i} \right). \quad (53)$$

Let $z = b_1/\sqrt{T/\mu}$. Then we write (52) as

$$C \leq \sqrt{\frac{L}{\mu}} \left( \frac{z}{\lambda} + \sum_{i=2}^{\lambda} \frac{1}{i} \right),$$

where $\lambda = \lceil z^2 \rceil$, that is, $\sqrt{\lambda - 1} < z \leq \sqrt{\lambda}$. Now we partition $\mathbb{R}^+$ into subintervals $I_d = (\sqrt{d-1}, \sqrt{d})$ for $d = 1, 2, 3, \ldots$, and let $F: \mathbb{R}^+ \rightarrow \mathbb{R}^+$ be a piecewise function defined as

$$F(x) = \frac{x}{d} + \frac{1}{d} \sum_{i=2}^{d} \frac{1}{i}, \quad \text{for } x \in I_d, d = 1, 2, 3, \ldots.$$  

As shown in Figure 3 the function $F(x)$ attains global maximum at $x = \sqrt{2}$, with maximal value $F(\sqrt{2}) = 3/\sqrt{8}$. Thus

$$C \leq \sqrt{\frac{L}{\mu} F(x)} \leq \frac{3}{\sqrt{8}} \sqrt{\frac{L}{\mu}}.$$  

Therefore we can obtain that $L \geq \left[ \frac{8C^2\mu}{9} \right]$.

Fig. 3. The image of $F(x)$.

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