Stabilizing dilaton and baryogenesis

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Abstract

Entropy production by the dilaton decay is studied in the model where the dilaton acquires potential via gaugino condensation in the hidden gauge group. Its effect on the Affleck-Dine baryogenesis is investigated with and without non-renormalizable terms in the potential. It is shown that the baryon asymmetry produced by this mechanism with the higher-dimensional terms is diluted by the dilaton decay and can be regulated to the observed value.

I. INTRODUCTION

Many gauge-singlet scalar fields arise in the effective four-dimensional supergravity which could be derived from string theories. Among them the dilaton $S$ has a flat potential in all orders in perturbation theory [1]. Therefore some non-perturbative effects are expected to generate the potential whose minimum corresponds to the vacuum expectation value (VEV). The most promising mechanism of the dilaton stabilization and the supersymmetry breaking is the gaugino condensation in the hidden gauge sector [2–5].

In cosmological consideration, even if the dilaton acquires potential through such non-perturbative effects, there are some difficulties to relax the dilaton to the correct minimum, as pointed out by Brustein and Steinhardt [6]. Since the potential generated by multiple
gaugino condensations is very steep in the small field-value region, the dilaton would have large kinetic energy and overshoot the potential maximum to run away to infinity.

As a possible way to overcome this problem, Barreiro et al. [7] pointed out that the dilaton could slowly roll down to its minimum with a little kinetic energy due to large Hubble friction if the background fluid dominated the cosmic energy density. As a result, the dilaton could be trapped and would oscillate around the minimum.

When the dilaton decays, however, the remaining energy density is transformed to radiation. One may worry about a huge entropy production by the decay, because it could dilute initial baryon asymmetry [8]. Therefore in order to obtain the observed baryon asymmetry, as required by e.g. nucleosynthesis, it is necessary to produce a larger asymmetry than the observed one at the outset.

The attractive mechanism to produce large baryon asymmetry in supersymmetric models was proposed by Affleck and Dine [9]. However, the baryon-to-entropy ratio produced by this mechanism, $n_b/s$, is usually too large. So if we take into account the entropy production after the baryogenesis, we can expect that the additional entropy may dilute the excessive baryon asymmetry to the observed value, as pointed out in e.g. [10,11].

In this paper, we investigate whether the dilution by the dilaton decay can regulate the large baryon asymmetry produced by the Affleck-Dine mechanism to the observed value. The paper is organized as follows. In §II and §III, we describe the potential and the dynamics of the dilaton. Then, in §IV we estimate the baryon asymmetry generated by the Affleck-Dine mechanism taking into account dilution by the dilaton decay. Section V is devoted to the conclusion. We take units with $8\pi G = 1$.

II. DILATON POTENTIAL

We consider the potential of the dilaton non-perturbatively induced by multiple gaugino condensates. In string models, the tree level Kähler potential is given by

$$K = -\ln(S + S^*) - 3\ln(T + T^* - |\Phi|^2),$$

where $S$ is the dilaton, $T$ is the modulus and $\Phi$ represents some chiral matter fields [12].

The effective superpotential of the dilaton [2] generated by the gaugino condensation is given by

$$W = \sum_a \Lambda_a(T)e^{-\alpha_a S}.$$  \hspace{1cm} (2)

Here, for the $SU(N_a)$ gauge group and the chiral matter in $M_a(N_a + \bar{N}_a)$ “quark” representations, $\alpha_a$ and $\Lambda_a$ are given by

$$\alpha_a = \frac{8\pi^2}{N_a - \frac{M_a^2}{3}},$$

$$\Lambda_a = -\frac{N_a - \frac{M_a^3}{3\eta^6(T)}}{\eta^6(T)}(32\pi^2 e)^{3(N_a-M_a/3)/(3N_a-M_a)}(\frac{M_a}{3})^{M_a/(3N_a-M_a)}.$$  \hspace{1cm} (4)
Then the modulus $T$ have a potential minimum due to the presence of the Dedekind function $\eta(T)$ [5]. Hereafter we assume the stabilization of the modulus $T$ at the potential minimum and concentrate on the evolution of the real part of the dilaton field.

Since at least two condensates are required to form the potential minimum, we consider a model with two condensates. Then the indices of the gauge group are $a = 1, 2,$ and we take $10 < \alpha_1 < \alpha_2$.

The potential $V$ for scalar components in supergravity is given by

$$V = e^K \left[ (K^{-1})^i_j D_i W (D_j W)^* - 3|W|^2 \right],$$

where

$$D_i W = \partial W / \partial \Phi^i + \partial K / \partial \Phi^i W,$$

$K^{i,j} = \partial^2 K / \partial \Phi^i \partial \Phi^j$, the inverse $(K^{-1})^{i,j}$ is defined by $(K^{-1})^{i,j} K^{j,k} = \delta^i_k$ and $i = S, T, \Phi$. In the region, $\alpha \text{Re} S \gg 1$, the potential Eq. (5) can be rewritten as

$$V(S) \simeq e^K (K^{-1})^S_S |\partial_S W|^2$$

$$= (S + S^*) |\alpha_1 \Lambda_1|^2 e^{-\alpha_1 (S + S^*)} \left[ 1 + \frac{\alpha_2 \Lambda_2}{\alpha_1 \Lambda_1} e^{-(\alpha_2 - \alpha_1)S} \right]^2.$$ (7)

In this potential the imaginary part of $S$ has a minimum at

$$\text{Im} S_{\text{min}} = \frac{(2n + 1)\pi}{\alpha_2 - \alpha_1},$$

where $n$ is an integer. So we assume $\text{Im} S = \text{Im} S_{\text{min}}$ and concentrate on the behavior of $\text{Re} S$ hereafter. Then we find that the potential minimum, $\text{Re} S_{\text{min}}$, is given by

$$\text{Re} S_{\text{min}} = \frac{1}{\alpha_2 - \alpha_1} \ln \left( \frac{\alpha_2 \Lambda_2}{\alpha_1 \Lambda_1} \right).$$

We assume $\text{Re} S_{\text{min}} \simeq 2$ to reproduce a phenomenologically viable value of the gauge coupling constant of grand unified theory [5]. The negative vacuum energy at the minimum of the potential is assumed to be canceled by some mechanism such as a vacuum expectation value of three form field strength. Although we consider a model with two gaugino condensates, our following estimations would be almost unchanged in a single condensate model with non-perturbative Kähler corrections [13,14], because the evolution of the dilaton is determined only by the slope of the potential in the region $S \ll S_{\text{min}}$ and its mass.

On the other hand, the position of the local maximum of potential, $\text{Re} S_{\text{max}}$, is given by

$$\text{Re} S_{\text{max}} = \text{Re} S_{\text{min}} + \frac{1}{\alpha_2 - \alpha_1} \ln \left( \frac{\alpha_2}{\alpha_1} \right).$$

(10)

For $S \ll S_{\text{min}}$, the potential (7) can be approximated as

$$V(S) \simeq 2 \text{Re} S \nu_0 e^{-\alpha_2 \text{Re} S},$$

(11)
where $V_0 \simeq |\alpha_2 A_2|^2$. The potential (7) has the minimum at $S_{\text{min}}$ and the local maximum at $S_{\text{max}}(> S_{\text{min}})$. For $S \gtrsim S_{\text{cr}} \equiv S_{\text{min}} - 1/(\alpha_2 - \alpha_1)$, the approximate expression for the potential (11) breaks down. Around the potential minimum $S_{\text{min}}$, the potential (7) becomes

$$V(S) \simeq |\alpha_1 \Lambda_1|^2 e^{-2\alpha_1 S_{\text{min}}} (\alpha_2 - \alpha_1)^2 (\text{Re} S - \text{Re} S_{\text{min}})^2.$$  \hspace*{1cm} (12)

However, one can see from the Kähler potential (1) that the variable $\text{Re} S$ does not have the canonical kinetic term. Therefore we introduce the canonically normalized variable, $\phi$, as

$$\phi \equiv \frac{1}{\sqrt{2}} \ln \text{Re} S.$$ \hspace*{1cm} (13)

### III. STABILIZATION MECHANISM FOR THE DILATON

Here, after reviewing the mechanism for dilaton stabilization proposed by Barreiro et al. [7], we estimate the relic energy density of the dilaton and the amount of the entropy density produced by its decay. We will consider the situation that the universe after inflation contains the dilaton, $\phi$, and a fluid with the equation of state, $p = (\gamma - 1)\rho$, where $\gamma$ is a constant. For example, $\gamma = 4/3$ for radiation or $\gamma = 1$ for non-relativistic matter. The latter includes oscillating inflaton field or/and the Affleck-Dine (AD) condensate $\phi_{AD}$.

In the spatially flat Robertson-Walker space-time,

$$ds^2 = -dt^2 + a(t)^2 dx^2,$$  \hspace*{1cm} (14)

with the scale factor $a(t)$, the Friedmann equations and the field equation for $\phi$ read

$$\dot{H} = -\frac{1}{2}(\rho + p + \dot{\phi}^2),$$  \hspace*{1cm} (15)

$$\ddot{\phi} = -3H \dot{\phi} - \frac{dV(\phi)}{d\phi},$$  \hspace*{1cm} (16)

$$H^2 = \frac{1}{3} \left[ \rho + \frac{1}{2} \dot{\phi}^2 + V(\phi) \right],$$  \hspace*{1cm} (17)

where $H = \dot{a}/a$ is the Hubble parameter and a dot denotes time differentiation. We define the new variables,

$$x \equiv \frac{\dot{\phi}}{\sqrt{6H}}, \quad y \equiv \frac{\sqrt{V(\phi)}}{\sqrt{3H}},$$  \hspace*{1cm} (18)

and the number of $e$-folds $N \equiv \ln(a)$.

Then, the equations of motion can be rewritten as

$$x' = -3x - \sqrt{\frac{3}{2}} \frac{\partial \phi V}{V} y^2 + \frac{3}{2} x \left[ 2x^2 + \gamma(1 - x^2 - y^2) \right],$$  \hspace*{1cm} (19)

$$y' = \sqrt{\frac{3}{2}} \frac{\partial \phi V}{V} x y + \frac{3}{2} y \left[ 2x^2 + \gamma(1 - x^2 - y^2) \right],$$  \hspace*{1cm} (20)

$$H' = -\frac{3}{2} H \left[ 2x^2 + \gamma(1 - x^2 - y^2) \right],$$  \hspace*{1cm} (21)
where the prime denotes a derivative with respect to \( N \). In terms of these variables, the Friedman equation becomes \( x^2 + y^2 + \rho/(3H^2) = 1 \). We see that \( x^2 \) and \( y^2 \) are respectively the ratios of the kinetic and potential energy densities of the dilaton to the total energy density. We consider the case of the universe dominated by the background fluid, so the inequalities, \( x^2, y^2 \ll 1 \), hold. Then Eq. (21) can easily be solved and the solution is

\[
H = H_0 e^{-3\gamma N/2}.
\] (22)

Next we introduce another new variable,

\[
x_s \equiv \frac{d\text{Re}S}{d\phi} x.
\] (23)

Then Eqs. (19) and (20) can be respectively rewritten as

\[
x'_s = -3x_s + \frac{3}{2}\gamma x_s + 2\alpha_2 \sqrt{\frac{3}{2} \left( \frac{d\text{Re}S}{d\phi} \right)}^2 y^2,
\] (24)

\[
y' = -2\alpha_2 \sqrt{\frac{3}{2} x_s y} + \frac{3}{2}\gamma y,
\] (25)

where we have used the relation \(-2\alpha_1 V = \partial V/\partial \text{Re}S\). Now we examine the stationary points in the above equations. From \( y' = 0 \), we find

\[
x_s = \sqrt{\frac{3}{2} \frac{\gamma}{2\alpha_2}}.
\] (26)

On the other hand, from \( x'_s = 0 \) we see

\[
y^2 = \frac{3(2 - \gamma)\gamma}{2(2\alpha_2)^2} \left( \frac{d\text{Re}S}{d\phi} \right)^{-2}.
\] (27)

Except for the factor \((d\text{Re}S/d\phi)^{-2}\), Eqs. (26) and (27) represent the scaling solution for the scalar field with exponential potential [15]. In spite of the presence of the factor \((d\text{Re}S/d\phi)^{-2}\), we can verify that the deviation from the scaling solution is small enough, as already shown in [7]. Thus we can write the solution as

\[
\text{Re}S = \frac{3\gamma}{2\alpha_2} N + \frac{1}{\alpha_2} \ln \left[ \frac{4V_0}{(2-\gamma)\gamma} \left( \frac{2\alpha_2}{3H_0} \right)^2 \right] + \epsilon(N),
\] (28)

where

\[
\epsilon(N) = \frac{2}{\alpha_2} \ln \left\{ \frac{3\gamma}{2\alpha_2} N + \frac{1}{\alpha_2} \ln \left[ \frac{4V_0}{(2-\gamma)\gamma} \left( \frac{2\alpha_2}{3H_0} \right)^2 \right] \right\},
\] (29)

denotes the deviation from the scaling solution; we find \( x'_s = \epsilon(N)'' \ll 1 \).

Figure 1 depicts time evolution of \( \text{Re}S \) as a function of \( N \) for various initial values of the dilaton field amplitude. As is seen there if the dilaton relaxes to the scaling solution before reaching \( S_{\text{min}} \), its energy is small enough to prevent overshooting. Attractor behavior in a
different situation has been studied in [16]. The Hubble parameter at $S = S_{\text{min}}$ is estimated as

$$H_{\text{min}} \simeq \frac{2\alpha_2}{3} \sqrt{\frac{2V_0}{(2 - \gamma)\gamma}} e^{-\alpha_2 S_{\text{min}}} \equiv t_{\text{min}}^{-1},$$  

from Eqs. (22) and (28). On the other hand, as is seen from Eq. (7), the mass of the dilaton in vacuum is given by

$$m_\phi \simeq 2(\alpha_2 - \alpha_1)\sqrt{V_0} e^{-\alpha_2 S_{\text{min}}}. \quad (31)$$

We therefore find $m_\phi \simeq H_{\text{min}}$, so the dilaton begins to oscillate immediately when it approaches the potential minimum. Since the mass of the gravitino is given by $m_{3/2} \simeq \Lambda_2 e^{-\alpha_2 S_{\text{min}}}$, the mass of the dilaton is

$$m_\phi \simeq \alpha_1^2 m_{3/2} \simeq 10^2 \text{TeV} \left(\frac{m_{3/2}}{1\, \text{TeV}}\right) \left(\frac{\alpha_1}{10}\right)^2. \quad (32)$$

When the dilaton $S$ approaches the critical point $S_{c\tau}$, the single exponential approximation (11) breaks down and the scaling behavior terminates. Then the energy density of the dilaton is estimated as

$$\rho_{\phi}^{in} = \frac{1}{2} \dot{\phi}^2 + V \bigg|_{t_{\text{min}}} = 3H^2 (x^2 + y^2) \bigg|_{t_{\text{min}}}$$

$$= \frac{3}{2} \left(\frac{\gamma}{2\alpha_1}\right)^2 \rho \left(\frac{d\Re S}{d\phi}\right)^{-2} \left(1 + \frac{2 - \gamma}{2\gamma}\right) \bigg|_{t_{\text{min}}}$$

$$\simeq 10^{-3} \rho \left(\frac{10}{\alpha_1}\right)^2 \left(\frac{2}{\Re S}\right)^2 \text{ for } \gamma = 4/3. \quad (33)$$

Indeed, the numerical calculation gives a close value $\rho_{\phi} \simeq 10^{-4} \rho$ at the beginning of the oscillation. After that, the dilaton begins to oscillate and the energy density decreases as $a(t)^{-3}$ until it decays.

In Fig. 2, we present the energy density of the dilaton in the universe filled by the $\gamma = 4/3$ background fluid at the beginning of the oscillation regime for various model parameters, $\alpha_1$ and $\alpha_2$. The values along the contour lines represent the energy density $\rho_{\phi}$ in the unit of $10^{-4} \rho$. The case with $\gamma = 1$ is depicted in Fig. 3, where we find smaller energy density of the dilaton by a factor of $\sim 3$. These figures are drawn in the two-parameter space, though the potential contains four parameters as seen from Eq. (7). The other two have been fixed by setting $m_{3/2} = 1$ TeV and $S_{\text{min}} = 2$ [5].

The decay of the dilaton produces huge entropy. If we assume that the background fluid is radiation with $\gamma = 4/3$, at the time $t = t_{\text{min}}$ its temperature is $T \sim 10^{11}$ GeV and the entropy density is

$$s = \frac{4\pi^2}{90} g_s T^3, \quad (34)$$
where \( g^* \sim 10^2 \) is the effective number of relativistic degrees of freedom. By using Eq. (33), we find that the entropy density increases by the factor

\[
\Delta = \left. \frac{T}{T_D} \frac{\rho_\phi}{\rho} \right|_{t_{min}} \simeq 10^9 \left( \frac{T}{10^{11}\text{GeV}} \right) \left( \frac{10^{-2}\text{GeV}}{T_D} \right) \left( \frac{\rho_\phi/\rho}{H_{min}} \right),
\]

when the dilaton decays at

\[
H = \Gamma_D \simeq m_\phi^3 \simeq 10^{-21}\text{GeV} \left( \frac{m_\phi}{10^2\text{TeV}} \right)^3,
\]

where

\[
T_D \simeq m_\phi^{3/2} \simeq 10^{-1}\text{GeV} \left( \frac{m_\phi}{10^2\text{TeV}} \right)^{3/2},
\]

is the reheating temperature after decay of the dilaton.

IV. AFFLECK-DINE BARYOGENESIS

The Affleck-Dine mechanism is an efficient mechanism of baryogenesis in supersymmetric models [9,17]. In fact, it is too efficient and the produced baryon asymmetry, \( n_b/s \), is in general too large. However, additional entropy release by the dilaton decay may significantly dilute the baryon asymmetry [10,11] and we examine this possibility here.

It is known that the Q-ball formation occurs for many Affleck-Dine flat directions [17–19]. Whether Q-balls form or not depends on the shape of radiative correction to the flat direction [19,20]. In this paper we estimate the baryon asymmetry providing that the Affleck-Dine field does not lead the Q-ball formation. For instance, one example is a flat direction with large mixtures of stops in the case of light gaugino masses, another is \( H_uL \)-direction [20].

A. Original Affleck-Dine mechanism

First, we investigate the originally proposed Affleck-Dine mechanism with a flat potential up to \( \phi_{AD} \sim 1 \) [9]. We consider the situation that there are the dilaton and the Affleck-Dine condensate in radiation dominated universe. As mentioned above, at the moment \( H = m_\phi \), the dilaton begins to oscillate with the initial energy density \( \rho_\phi \simeq 10^{-4}\rho_\gamma \), where \( \rho_\gamma \) is the energy density of the background radiation. Then, on the other hand, the AD condensate is expected to take a large expectation value, \( \phi_{AD} \sim 1 \), above which its potential blows up exponentially.

The amplitude of the AD field and its energy density remains constant while the Hubble parameter is larger than \( m_{AD} \), where \( m_{AD} \) is the mass of the AD condensate. One should keep in mind, however, that though the energy density, \( \rho_{AD} \), remains constant the baryon number density decreases as \( 1/a^3 \) if baryonic charge is conserved. So if baryon charge is accumulated in “kinetic” motion of the phase of \( \phi_{AD} \) it would decrease as \( 1/a^3 \). If, on the other hand, the AD-field is frozen at the slope of not spherically symmetric potential then baryonic charge of the AD-field is not conserved and after \( H < m_{AD} \) both radial and angular degrees of freedom would be “defrosted” and baryonic charge may be large.
As we noted above, when \( H \simeq m_{3/2} \simeq m_{AD} \), the AD field begins to oscillate. Its energy density at that moment becomes comparable to that of the radiation, while the energy density of the dilaton is estimated as

\[
\rho_\phi |_{H=m_{3/2}} = \left( \frac{m_\phi}{m_{3/2}} \right)^{1/2} \frac{\rho_\phi}{\rho_\gamma} |_{H=m_\phi} \rho_\gamma |_{H=m_{3/2}} \simeq 10^{-3} \rho_\gamma |_{H=m_{3/2}},
\]

for the initial energy density \( \rho_\phi = 10^{-4} \rho_\gamma \) and \( m_\phi \simeq 10^2 m_{AD} \). After that the universe becomes dominated by the oscillating AD condensate and enters into approximately matter dominated regime (see below).

The energy density of the condensate and its baryon number density are given respectively by the expressions:

\[
\rho_{AD} = m_{AD}^2 \phi_{AD}^2, \quad n_b = \kappa m_{AD} \phi_{AD}^2,
\]

where \( \kappa = n_b/n_{AD} < 1 \) is a numerical coefficient and \( n_{AD} \) is the number density of the AD field.

The rate of evaporation of the condensate, given by the decay width of the AD field into fermions, \( \Gamma_{AD} = C m_{AD} \) with \( C = 0.1 - 0.01 \), is quite large. When the Hubble parameter becomes smaller than \( \Gamma_{AD} \), thermal equilibrium would be established rather soon. However, the condensate would evaporate very slowly and disappear much later [21]. The low evaporation rate is related to a large baryonic charge and relatively small energy density of the condensate. Below we will find the temperature and the moment of the condensate evaporation repeating the arguments of ref. [21]. Let us assume that the condensate evaporated immediately when \( H = \Gamma_{AD} \) producing plasma of relativistic particles with temperature \( T_{AD} \) and chemical potential \( \mu_{AD} \). The temperature can be estimated as \( T_{AD} \simeq \rho_{AD}^{1/4} \) and since \( T_{AD} \gg m_{AD} \) the chemical potential is given by

\[
\mu_{AD} \simeq \frac{n_b}{T_{AD}^2} = \kappa \phi_{AD} \gg m_{AD},
\]

if \( \kappa \) is not very small. On the other hand, chemical potential of bosons cannot exceed their mass. It means that instantaneous evaporation of the condensate is impossible. The process of evaporation proceeds rather slowly with an almost constant temperature of the created relativistic plasma. During the process of evaporation the energy density of the latter was small in comparison with the energy density of the condensate, except for the final stage when the condensate disappeared.

The cosmological baryon number density and energy densities are given by the equilibrium expressions:

\[
\frac{n_{b,\text{tot}}}{T^3} = \frac{2 N_f N_c B_q}{6 \pi^2} \left( \xi_q^3 + \pi^2 \xi_q \right) + \frac{1}{2 \pi^2} \int_0^\infty d\eta \eta^2 \left[ \frac{1}{\exp(\epsilon - \xi) - 1} - \frac{1}{\exp(\epsilon + \xi) - 1} \right] + B_c \quad (41)
\]

\[
\frac{\rho_{\text{tot}}}{T^4} = \frac{\pi^2 g_*}{30} + \frac{7 2 N_f (N_c + 1) \pi^2}{8} \left[ 1 + \frac{30}{7} \left( \frac{\xi_q}{\pi} \right)^2 + \frac{15}{7} \left( \frac{\xi_q}{\pi} \right)^4 \right] \left[ 1 - \frac{30}{7} \left( \frac{\xi_q}{\pi} \right)^2 \right] + \frac{1}{2 \pi^2} \int_0^\infty d\eta \eta^2 \epsilon \left[ \frac{1}{\exp(\epsilon - \xi) - 1} + \frac{1}{\exp(\epsilon + \xi) - 1} \right] + \rho_c
\]

\( (42) \)
where $\eta \equiv p/T$ is dimensionless momenta, $\epsilon \equiv \sqrt{\eta^2 + m_{AD}^2/T^2}$, $\xi \equiv \mu_{AD}/T$ and $\xi_q \equiv \mu_q/T$ are dimensionless chemical potential of the AD field and quarks, respectively. $N_f = 6$ and $N_c = 3$ are the numbers of flavors and colors and factor 2 came from counting spin states, $B_q = 1/3$ is the baryonic charge of quarks while the baryonic charge of the AD field is assumed to be 1, and $B_c$ and $\rho_c$ are baryon number density and energy density of the condensate normalized to $T^3$ and $T^4$, respectively. The first term in $\rho_{tot}$ includes energy density of light particles with zero charge asymmetry and $g_*$ is the number of their species. The second term includes the contribution from leptons with the same chemical potential as quarks - it is given by $(N_c + 1)$.

For definiteness, let us assume that the AD field decays into the channel $\phi_{AD} \to 3q + l$ and taking into account that the sum of baryonic and leptonic charges is conserved$^1$, so that $B - L = 0$, we find

$$\mu_q = \mu_l = \frac{\mu_{AD}}{4}. \quad (43)$$

Before complete evaporation of the condensate, the chemical potential of AD-field remains constant and equal to its maximum allowed value $m_{AD}$. Thus the only unknowns in these expressions are the temperature and the amplitude of the field in the condensate. According to Eq.(39), from Eqs. (41) and (42) we obtain

$$\frac{B_c}{\rho_c} = \kappa \frac{T}{m_{AD}}. \quad (44)$$

The same relation was true for the initial values of $n_{b,tot}/T^3$ and $\rho_{tot}/T^4$. Assuming that the ratio $n_{b,tot}/\rho_{tot}$ remains the same during almost all process of evaporation, though it is not exactly so, we can exclude $B_c, n_{b,tot}, \rho_c$ and $\rho_{tot}$ from expressions (41) and (42) and find one equation that permits to calculate the plasma temperature in presence of evaporating condensate as a function of the baryonic charge fraction in the initial condensate, $\kappa$. We find

$$m_{AD}/T \simeq 20, \quad \text{for} \quad \kappa = 1, \quad (45)$$

$$m_{AD}/T \simeq 2, \quad \text{for} \quad \kappa = 0.1. \quad (46)$$

Exact solution of the problem demands much more complicated study of the evolution of energy density according to the equation $\dot{\rho} = -3H(\rho + P)$, while the evolution of baryonic charge density is determined by the conservation of baryonic charge which is assumed to be true at the stage under consideration and thus $n_{b,tot} \propto a^{-3}$. The temperature of plasma found in this way would not be much different from the approximate expressions presented above.

\footnotetext{1}{One may wonder if this assumption is inappropriate because the baryon asymmetry created in this channel would be washed out by anomalous electroweak processes [22]. As will be seen later, however, we can avoid this difficulty because in most cases of our interest the AD condensate evaporates at a lower temperature when these anomalous processes are no longer effective.}
Using the above-calculated plasma temperature (46) we find that at the moment of condensate evaporation (when $\mu_{AD} = m_{AD}$) the cosmological energy and baryon number densities of the created relativistic plasma are given by

$$\rho_p \simeq 1000 T^4, \quad (47)$$

$$n_b \approx 50 T^3, \quad (48)$$

for $\kappa = 1$, and

$$\rho_p \approx 70 T^4, \quad (49)$$

$$n_b \approx 1.75 T^3, \quad (50)$$

for $\kappa = 0.1$.

We see that a large baryon asymmetry prevents from fast condensate evaporation, though the interaction rate could be much larger than the expansion rate. From Eq. (48) or (50) and the baryon number conservation

$$n_b = \kappa m_{AD} \left( \frac{a_{AD}}{a(t)} \right)^3 \left( \phi_{AD}|_{H=m_{AD}} \right)^2, \quad (51)$$

we find

$$\left( \frac{a_{ev}}{a_{AD}} \right)^3 = \kappa \frac{m_{AD}}{n_b} \left( \phi_{AD}|_{H=m_{AD}} \right)^2 \quad (52)$$

$$\simeq 160 \left( \frac{\phi_{AD}|_{H=m_{AD}}}{m_{AD}} \right)^2 \simeq 10^{33}, \quad \text{for} \quad \kappa = 1, \quad (53)$$

$$\simeq 0.46 \left( \frac{\phi_{AD}|_{H=m_{AD}}}{m_{AD}} \right)^2 \simeq 3 \times 10^{30}, \quad \text{for} \quad \kappa = 0.1, \quad (54)$$

where $a_{AD}$ and $a_{ev}$ are the value of the scale factor at the moment $H = m_{AD}$ and that at the evaporation of the AD field, respectively. Then at the evaporation the Hubble parameter and the baryon-to-entropy ratio are respectively given by

$$H_{ev} = (\rho_p/3)^{1/2} \simeq \begin{cases} 2 \times 10^{-14} \text{ GeV} & \text{for} \quad \kappa = 1, \\ 5 \times 10^{-13} \text{ GeV} & \text{for} \quad \kappa = 0.1 \end{cases} \quad (55)$$

$$\left. \frac{n_b}{s} \right|_{ev} \simeq 1, \quad \text{for} \quad \kappa = 1, \quad (56)$$

$$\left. \frac{n_b}{s} \right|_{ev} \simeq 0.04, \quad \text{for} \quad \kappa = 0.1, \quad (57)$$

from Eqs. (47), (48), (49) and (50).

Now we have to calculate the ratio of the baryon asymmetry to the entropy of the plasma after thermalization of the products of dilaton decay. Initially, at the moment of evaporation of AD-condensate the energy density of the dilaton is roughly $10^{-3}$ with respect to the energy density of plasma. The latter is dominated by chemical potential $\mu = m_{AD} > T$. When the universe expanded by the factor $\rho_{AD}/\rho_{\phi}|_{ev} \equiv a_{eq}/a_{ev} \simeq 10^3$ the dilaton starts to dominate.
and the relativistic expansion regime turns into matter dominated one at \( a = a_{\text{eq}} \). To the moment of the dilaton decay the energy density of the dilaton becomes larger than the energy density of the plasma formed by the evaporation of the AD-condensate by the factor \( a_d/a_{\text{eq}} = (H_{\text{eq}}/H_d)^{2/3} \), where \( a_d \) and \( H_d \) are the scale factor and the Hubble parameter at the time of the dilaton decay. Keeping in mind that \( H_{\text{eq}} = (a_{\text{eq}}/a_{\text{ev}})^2 H_{\text{ev}} = 10^{-6} H_{\text{ev}} \), we obtain the dilution factor by the dilaton decay

\[
\Delta = \left( \frac{\rho_\phi}{\rho_{AD}} \right)^{3/4} = \left( \frac{H_{\text{ev}} a_{\text{ev}}^2}{H_d a_{\text{eq}}^2} \right)^{1/2} = \left. \frac{\rho_\phi}{\rho_{AD}} \right|_\text{ev} \left( \frac{H_{\text{ev}}}{H_d} \right)^{1/2}
\]

Thus for \( \kappa = 1 \) the dilution factor is only 14, while for \( \kappa = 0.1 \) it is 70.

Finally we find that the baryon asymmetry after the dilaton decay is given by

\[
\frac{n_b}{s} = \left. \frac{n_b}{s} \right|_\text{ev} \Delta \simeq 0.1 - 0.001.
\]

Thus in this model the dilution of originally produced asymmetry from the decay of AD field is not sufficient.

### B. Affleck-Dine mechanism with non-renormalizable potential

As we have seen, additional entropy production due to the dilaton decay is too small to dilute the baryon asymmetry generated in the original Affleck-Dine scenario. Therefore it is necessary to suppress the generated baryon asymmetry. The presence of the non-renormalizable terms can reduce the expectation value of the AD field during inflation. As a result, the magnitude of the baryon asymmetry can be suppressed. Hence we introduce the following non-renormalizable term in the superpotential to lift the Affleck-Dine flat direction as a cure to regulate the baryon asymmetry [23,24].

\[
W = \frac{\lambda}{nM^{n-3}} \phi_{AD}^n,
\]

where \( M \) is some large mass scale.

The potential for the AD field in the inflaton-dominated stage reads

\[
V(\phi_{AD}) = -c_1 H^2 |\phi_{AD}|^2 + \left( \frac{c_2 \lambda H \phi_{AD}^n}{nM^{n-3}} + \text{H.C.} \right) + |\lambda|^2 |\phi_{AD}|^{2n-2} M^{2n-6},
\]

where \( c_1 \) and \( c_2 \) are constants of order unity. The first and the second terms are soft terms which arise from the supersymmetry breaking effect due to the vacuum energy of the inflaton. The minimum of the potential is reached at

\[
|\phi_{AD}| \sim \left( \frac{HM^{n-3}}{\lambda} \right)^{1/(n-2)}.
\]

During inflation, the AD field takes the expectation value \( |\phi_{AD}| \simeq (H_{\text{inf}}M^{n-3}/\lambda)^{1/(n-2)} \), where \( H_{\text{inf}} \) is the Hubble parameter during inflation. After inflation, it also traces the
instantaneous minimum, Eq. (62), until the potential is modified and the field becomes unstable there. The AD field starts oscillation when its effective mass becomes larger than the Hubble parameter. There are three possible contributions to trigger this oscillation: the low energy supersymmetry breaking terms, the thermal mass term from a one-loop effect [25], and the thermal effect at the two-loop order [26].

First we consider the case that the low energy supersymmetry breaking terms are most important and that these thermal effects are negligible. When \( H \sim m_{3/2} / 2 \), the low energy supersymmetry breaking terms appear and the potential for the AD field becomes

\[
V(\phi_{AD}) = m_{AD}^2 |\phi_{AD}|^2 + \left( \frac{Am_{3/2} \phi_{AD}^n}{nM^{n-3}} + \text{H.C.} \right) + |\lambda|^2 \frac{|\phi_{AD}|^{2n-2}}{M^{2n-6}},
\]

(63)

where \( A \) is a constant of order unity, and the ratio of the energy density of the AD field \( \rho_{AD} \) to that of the inflaton \( \rho_I \) is given by

\[
\frac{\rho_{AD}}{\rho_I} \simeq \left( \frac{m_{3/2} M^{-n-3}}{\lambda} \right)^{2/(n-2)},
\]

(64)

up to numerical coefficients depending on \( c_1, c_2 \) and \( A \). For example, the typical value for \( n = 4 \) is \( \rho_{AD}/\rho_I \simeq 10^{-16} (M/\lambda) \).

Let us first assume that the inflaton decayed at \( H = m_{AD} (\simeq m_{3/2}) \) with the decay rate \( \Gamma_I = m_{AD} \) and after that the universe was dominated by radiation. Note that the corresponding reheating temperature is \( T_R \simeq \sqrt{\Gamma_I} \simeq \sqrt{m_{AD}} \simeq 10^{10} \text{ GeV} \). Then the evaporation of the AD condensate into relativistic plasma would be different from the evaporation into cold plasma considered in the previous subsection. Due to interaction with plasma the products of the evaporation acquire much larger temperature than in the case of the evaporation into vacuum. Since the energy density of the condensate is negligible in comparison with the total energy density of the plasma, the temperature of the latter drops in the usual way, \( T \propto 1/a \), in contrast to the previously considered case when \( T = \text{const} \). Since the temperature of the plasma is high, \( T \gg m_{AD} \), the baryon number density is \( n_b = B_c T^3 + C_B T^2 \mu \) where \( C_B \sim 1 \) is a constant coefficient, \( \mu \leq m_{AD} \) is the value of the chemical potential, and we have neglected terms of the order of \( \mu^3 \). Since \( n_b \propto a^{-3} \), the ratio of \( a_{ev} \) to \( a_{AD} \) is

\[
\frac{a_{ev}}{a_{AD}} = \frac{n_b|_{H=m_{AD}}}{m_{AD} T_R^2}
\]

(65)

\[
\simeq 10 \left( \frac{m_{AD}}{1 \text{ TeV}} \right) \left( \frac{10^{10} \text{ GeV}}{T_R} \right)^2 \left( \frac{M}{\lambda} \right) \quad \text{for} \quad n = 4,
\]

(66)

(compare to Eq. (52)). Here we took for the initial value of the baryonic charge density \( n_b|_{H=m_{AD}} = \kappa m_{AD} \phi_{AD}^2 \) with \( \kappa \sim 1 \). For \( n = 4 \), \( a_{ev}/a_{AD} \) becomes of order 10, and we find that the condensate would evaporate soon.

To be more precise, however, we must take into account that the interaction rate of the condensate is \( \Gamma_{AD} = (0.1 - 0.01) m_{AD} \) and the evaporation cannot start before \( H = \Gamma_{AD} \). At that moment the plasma temperature would be smaller by the factor \( (m_{AD}/\Gamma_{AD})^2 = 10^2 - 10^4 \) and the baryon number density of the condensate would be smaller by \( (m_{AD}/\Gamma_{AD})^6 \). Correspondingly the red-shift of the end of evaporation should be shifted
by a factor $(m_{AD}/\Gamma_{AD})^2$ with respect to the beginning of evaporation and it means that it would remain the same with respect to the initial moment $H = m_{AD}$.

As we have already noted, for $n = 4$ the condensate decays quickly and the baryon number density produced in the decay is diluted by the plasma created by the inflaton decay as

$$\frac{n_b}{s} \simeq \frac{T_R \rho_{AD}}{m_{AD} \rho_I} = 10^{-9} \left( \frac{T_R}{10^{10}\text{GeV}} \right) \left( \frac{1\text{TeV}}{m_{AD}} \right) \left( \frac{M}{\lambda} \right).$$  \hspace{1cm} (67)

where $T_R$ is the reheating temperature of the inflaton and we used the estimate of Eq. (64). The result does not depend upon the moment of the decay of AD-condensate, since its energy density remains sub-dominant. If an additional dilution by the dilaton and the early oscillation by a thermal effect [25] are operative, the baryon asymmetry become even smaller than the observed one and the $n = 4$ model cannot explain the observed baryon asymmetry. Hence we must consider the flat direction with $n > 4$.

Hereafter, we study the AD fields with $n > 4$ including the thermal effect. For the AD fields with $n > 4$, the relevant thermal effect comes from the running of the gauge coupling constant [26] rather than the thermal plasma effect [25]. The potential for the AD field in the inflaton-dominated stage reads

$$V(\phi_{AD}) = (-c_1 H^2 + m_{AD}^2)|\phi_{AD}|^2 + \alpha T^4 \ln \left( \frac{\sqrt{\phi_{AD}}}{T^2} \right) + \left( \frac{c_2 \lambda H \phi_{AD}}{n M^{n-3}} + \frac{A m_{3/2} \phi_{AD}^n}{n M^{n-3}} + \text{H.C.} \right) + |\lambda|^2 \frac{|\phi_{AD}|^{2n-2}}{M^{2n-6}},$$  \hspace{1cm} (68)

where the second term is the thermal effect at two loop order which is pointed out in [26] and $\alpha$ denotes the gauge coupling.

If the effective mass of the AD field becomes comparable to the Hubble parameter when it is larger than the low energy supersymmetry breaking scale,

$$\alpha \frac{T^4}{|\phi_{AD}|^2} \simeq H^2 \left( > m_{AD}^2 \right),$$  \hspace{1cm} (69)

then AD field undergoes the early oscillation by the thermal effect. During the oscillating inflaton dominated stage $(t < t_{rh})$, the temperature of the plasma behaves as

$$T \simeq T_R \left( \frac{a(t_{rh})}{a(t)} \right)^{3/8} \simeq T_{R}^{1/2} H^{1/4}. \hspace{1cm} (70)$$

From Eqs. (62) and (70), the effective mass term of the AD field is rewritten as

$$\alpha \frac{T^4}{|\phi_{AD}|^2} \simeq \alpha T_R^2 \left( \frac{\lambda}{M^{n-3}} \right)^{2/(n-2)} H^{(n-4)/(n-2)}. \hspace{1cm} (71)$$

By comparing with Eq. (69), when the AD field begins to oscillate at $t \equiv t_{os}$, the Hubble parameter is given by

$$H_{os} \simeq \left( \alpha T_R^2 \right)^{(n-2)/n} \left( \frac{\lambda}{M^{n-3}} \right)^{2/n}, \hspace{1cm} (72)$$
From now on we concentrate on the case $n = 6$, for which

$$H_{os} \simeq 1\text{TeV} \left(\frac{\alpha}{10^{-2}}\right)^{2/3} \left(\frac{T_R}{10^{8}\text{GeV}}\right)^{4/3} \left(\frac{\lambda}{M^3}\right)^{1/3}. \quad (73)$$

Thus we find that the AD field begins to oscillate at $H > \sim m_{AD}$ due to the thermal term if $T_R \gtrsim 10^8$ GeV for $M = 1$, and if $T_R \gtrsim 10^6$ GeV for $M = 10^{-2}$, respectively. During the early oscillation driven by the thermal term, the amplitude of the AD field decreases as

$$|\phi_{AD}(t)| = |\phi_{AD}|_{t_{os}} \left(\frac{a_{os}}{a(t)}\right)^{9/4}, \quad (74)$$

where $a_{os}$ denotes the scale factor at the beginning of the oscillation. The analytic derivation of Eq. (74) is shown in Appendix and we have confirmed this result by the numerical calculation. It also agrees with the analysis in [27]. Using Eqs. (62), (70) and (74), we obtain the ratio of the amplitude of the AD field to temperature of plasma as

$$\frac{|\phi_{AD}|}{T} = \left(\frac{M^3}{T_R^2 \lambda}\right)^{1/4} \left(\frac{a_{os}}{a(t)}\right)^{15/8}. \quad (75)$$

We consider the evaporation rate of the AD field. The condition that the particles coupled to the AD field are light enough to exist as much as radiation reads $h|\phi_{AD}| < T$, where $h$ is the corresponding coupling constant. Following [25] let us adopt the scattering rate of the AD field $\Gamma \sim h^4T$ in this situation as the rate of its evaporation. Estimating $h|\phi_{AD}|/T$ and $\Gamma$ at the reheating time, we find

$$\frac{h|\phi_{AD}|}{T}{|_{t_{rh}}} \simeq 10^{-1} \left(\frac{T_R}{10^{10}\text{GeV}}\right)^{1/3} \left(\frac{h}{10^{-2}}\right) \left(\frac{10^{-2}}{\alpha}\right)^{5/6} \left(\frac{M^3}{\lambda}\right)^{2/3}, \quad (76)$$

and

$$\frac{\Gamma}{H} \simeq \frac{h^4}{T_R} \simeq \left(\frac{10^{10}\text{GeV}}{T_R}\right) \left(\frac{h}{10^{-2}}\right)^4. \quad (77)$$

Eq. (76) shows that the particles coupled to the AD field are thermally excited and populated well before the reheating time for any reheating temperature, $T_R \lesssim 10^6$ GeV, while we find that the AD condensate can evaporate around the typical reheating time from Eq. (77).

The baryon number density for the AD field $\phi_{AD}$ is given as

$$n_b = -iq(\phi_{AD}^* \dot{\phi}_{AD} - \dot{\phi}_{AD}^* \phi_{AD}), \quad (78)$$

where $q$ is a baryonic charge for the AD field.

The baryon number density at $H = H_{os}$ is estimated as

$$n_b|_{t_{os}} = \frac{4q m_{3/2}}{3HM^3} \text{Im}(A\phi_{AD}^6)|_{t_{os}} = \frac{4q \delta m_{3/2}}{3\lambda} \left(\frac{H_{os} M^3}{\lambda}\right)^{1/2}, \quad (79)$$

where $\delta$ is a effective relative CP phase. The baryon-to-entropy ratio at the reheating time is estimated as
\[ \frac{n_b}{s} \bigg|_{t_{\text{rh}}} = \frac{3T_R}{4} \frac{n_b}{\rho_I} \bigg|_{t_{\text{os}}} \frac{q \delta m_{3/2}}{3 \lambda} \frac{T_R}{H_{\text{os}}^2} \left( \frac{H_{\text{os}} M^3}{\lambda^2} \right)^{1/2}. \]  

(80)

Since the dilution factor by the dilaton decay is given by

\[ \Delta = \frac{T_R}{10^4 T_D} \left( \frac{\rho_\phi}{\rho_I \mid_{t_{\text{min}}}} \frac{10^{-4}}{10^{-4}} \right), \]

(81)

from Eqs. (80) and (81), we obtain the final baryon asymmetry,

\[ \frac{n_b}{s} = n_b \bigg|_{t_{\text{rh}}} \frac{1}{\Delta} \approx 10^{-8} q \delta \left( \frac{m_{3/2}}{H_{\text{os}}} \right)^{3/2} \left( \frac{M}{\lambda} \right)^{3/2} \left( \frac{T_D}{10^{-1} \text{GeV}} \right) \left( \frac{1 \text{TeV}}{m_{3/2}} \right)^{1/2} \left( \frac{\rho_\phi}{\rho_I \mid_{t_{\text{min}}}} \frac{10^{-4}}{10^{-4}} \right)^{-1}, \]

(82)

for \( H_{\text{os}} > m_{3/2} \). This result can easily meet the observation if we take, for example, \( H_{\text{os}} \approx 10^2 m_{3/2} \) and other factors to be of order of unity. In this case, from Eq. (77), we find that for the present case the AD condensate can evaporate before the inflaton decay is completed.

On the other hand, in the case the reheating temperature is so low that the thermal effect does not lead the early oscillation, the reduction of the cut-off scale \( M \) can lead to the reasonable baryon asymmetry:

\[ \frac{n_b}{s} \approx 10^{-11} q \delta \left( \frac{M}{10^{-2} \lambda} \right)^{3/2} \left( \frac{T_D}{10^{-1} \text{GeV}} \right) \left( \frac{1 \text{TeV}}{m_{3/2}} \right)^{1/2} \left( \frac{\rho_\phi}{\rho_I \mid_{t_{\text{min}}}} \frac{10^{-4}}{10^{-4}} \right)^{-1}. \]

(83)

This expression implies that the cut-off scale \( M \) should be around the GUT scale \( 10^{-2} \) or \( 10^{16} \text{GeV} \) for inflation models with a low reheating temperature, \( T_R \lesssim 10^6 \text{GeV} \). Furthermore we find that the final baryon asymmetry is independent of the reheating temperature of inflation within the range.

In the present model, the supersymmetry breaking is caused by the F-term of the dilaton. Therefore, when the dilaton decays, it can decay into gravitinos through their mass term and this process could lead to overproduction of the gravitinos. The constraint derived in [28] to avoid the overproduction is \( m_\phi \gtrsim 100 \text{ TeV} \). The mass of the dilaton, (32), in the model considered here is in the allowed region.

V. CONCLUSION

In this paper, we have studied the Affleck-Dine baryogenesis in the framework of the string cosmology. In string models, the dilaton is ubiquitous and does not have any potential perturbatively. We adopted the non-perturbatively induced potential of the dilaton via the gaugino condensation in the hidden gauge sector. Then we set phenomenologically desired values for the gravitino mass and the VEV of the dilaton.

The attractive mechanism to stabilize the dilaton at the desired minimum was proposed by Barreiro \textit{et al.} [7]. They did not estimate the energy density of the oscillating dilaton. It is estimated in the presented paper where we have found \( \rho_\phi \approx 10^{-4} \rho \) at \( H = m_\phi \). This
energy transforms into the radiation after the decay of the dilaton before nucleosynthesis because the mass $m_\phi \simeq 10^2$ TeV is sufficiently high.

We have discussed cosmological baryogenesis in this model. In the above-mentioned cosmological history with the entropy production, the Affleck-Dine baryogenesis might be the only workable mechanism for baryogenesis. We have investigated the Affleck-Dine baryogenesis with and without non-renormalizable terms. We have shown that while the original Affleck-Dine scenario produces too much baryon asymmetry even if there is the dilution by the dilaton decay, the model with $n = 6$ non-renormalizable terms can lead to the appropriate baryon asymmetry.

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APPENDIX

In this appendix, we derive Eq. (74). Although the similar discussion can be found in the literature [29], we derive it for completeness.

We consider the evolution of the AD field after the beginning of oscillation induced by the thermal effect at the two-loop level. Then the AD field obeys the equation of motion

$$\ddot{\phi}_{AD} + 3H\dot{\phi}_{AD} + \alpha \frac{T^4}{\phi_{AD}} = 0.$$  \hspace{1cm} (A1)

By decomposing $\phi_{AD}$ into

$$\phi_{AD} = |\phi_{AD}| e^{i\theta} \equiv \Phi e^{i\theta},$$  \hspace{1cm} (A2)

Eq. (A1) is reduced to the following equations

$$\ddot{\Phi} + 3H\dot{\Phi} - \dot{\theta}^2\Phi + \alpha \frac{T^4}{\Phi} = 0,$$

$$\left(a^3\dot{\theta}^2\Phi^2\right) = 0.$$  \hspace{1cm} (A3) (A4)

The second equation (A4) is interpreted as the conservation of the angular momentum which corresponds to the baryon number density and rewritten as

$$\dot{\theta}\Phi^2 = \dot{\theta}\Phi^2|_{t_{\text{os}}} \left(\frac{a_{\text{os}}}{a(t)}\right)^3 \equiv m\Phi_0^2 \left(\frac{a_{\text{os}}}{a(t)}\right)^3,$$  \hspace{1cm} (A5)

where $\Phi_0$ represents the initial amplitude of the AD field and $m$ means the initial angular velocity of the order of $m_{3/2}$ for $n = 6$. By eliminating $\dot{\theta}$ in Eqs. (A3) and (A4), we obtain

$$\ddot{\Phi} + 3H\dot{\Phi} - m^2 \left(\frac{a_{\text{os}}}{a(t)}\right)^6 \left(\frac{\Phi_0}{\Phi}\right)^4 \Phi + \alpha \frac{T^4}{\Phi} = 0.$$  \hspace{1cm} (A6)
Multiplied by $\dot{\Phi}$ and using $T^4 \propto a^{-3/2}$, Eq. (A6) yields

$$\frac{d}{dt} \left[ \dot{\Phi}^2 + m^2 \left( \frac{a_{\text{os}}}{a(t)} \right)^6 \frac{\Phi_0^4}{\Phi^2} + \alpha T^4 \ln \frac{\Phi^2}{T^2} \right] = -6H \left[ \dot{\Phi}^2 + m^2 \left( \frac{a_{\text{os}}}{a(t)} \right)^6 \frac{\Phi_0^4}{\Phi^2} \right] - \frac{3}{2}H \alpha T^4 \ln \frac{\Phi^2}{T^2} + \frac{3}{4}H \alpha T^4.$$  (A7)

On the other hand, multiplying Eq. (A6) by $\Phi$, we obtain

$$\frac{1}{a^3} (\Phi a^3 \ddot{\Phi}) - \ddot{\Phi}^2 - m^2 \left( \frac{a_{\text{os}}}{a(t)} \right)^6 \frac{\Phi_0^4}{\Phi^2} + \alpha T^4 = 0.$$  (A8)

By taking the time average over the time scale of the cosmic expansion, we obtain the cosmic virial theorem

$$\langle \dot{\Phi}^2 \rangle = \langle \alpha T^4 \rangle,$$  (A9)

where $\langle \ldots \rangle$ denotes the time average. Moreover, since the second term which represents the centrifugal force becomes efficient around only $\Phi \approx 0$, Eq. (A9) could be rewritten as

$$\langle \dot{\Phi}^2 \rangle = \langle \alpha T^4 \rangle.$$  (A10)

From Eqs. (A7) and (A10), we obtain

$$\frac{d}{dt} \left( 1 + \ln \frac{\Phi^2}{T^2} \right) = -\frac{15}{4}H.$$  (A11)

Since we know $T \propto a^{-3/8}$, hence, we find

$$\Phi \propto a^{-9/4},$$  (A12)

for $\Phi \gg T$. 

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FIG. 1. Evolution of Re $S$ as a function of $N$ for various initial conditions with $S_{\text{min}} = 2$ in case of $\gamma = 4/3$. We set the initial values of the velocity and the Hubble expansion rate as $d\text{Re}S/dt|_0 = 0$, and $H_0 = 1$, respectively.
FIG. 2. Energy density of the dilaton at the beginning of the oscillations for various model parameters. The number associated with each contour line represents the value of $\rho_\phi$ normalized by $10^{-4}\rho$. Here we adopt $\gamma = 4/3$.

FIG. 3. Same as Fig. 2 except for $\gamma = 1$. 