The Cosmological Origins of Nonlinear Electrodynamics

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Abstract—We present a mechanism that allows any nonlinear theory of electrodynamics to be described as a consequence of a coupling of the electromagnetic field to gravity in the presence of a vacuum represented by the cosmological constant. We emphasize gravity’s exclusive role of catalysis.

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1. INTRODUCTION

Recently [1, 2], one of us has examined how a specific self-interaction of a fermionic field, in the Heisenberg description of a dynamical model of elementary particles, can be related to cosmology. To be more specific, it was proved that the nonlinearity of Heisenberg’s dynamics can be interpreted as a consequence of the existence of a cosmological constant $\Lambda$ through the influence of a gravitational process. Such a method has also been the main tool to display a gravitational mechanism in describing the origin of masses of all particles, as shown in [3, 4].

The presence of $\Lambda$ is to be treated, according to Einstein’s general relativity, as a global property of the universe. In the modern framework $\Lambda$ can be alternatively interpreted as some sort of influence of the vacuum configuration. For our purposes here either one of these interpretations is allowed, once the mechanism which we will present does not distinguish between the interpretations. Nevertheless, just to be specific, we will follow Einstein and Mach and interpret $\Lambda$ as the way the rest-of-the-universe affects any material body or energy of any kind.

In the present work we apply this same idea to analyze the effects of $\Lambda$ on the electromagnetic field. We will show that any nonlinear theory of electrodynamics can be interpreted as a consequence of the gravitational interaction of the electromagnetic field $F_{\mu\nu}$ with the rest of the Universe. Let us point out from the very beginning that the function of gravity in this process is just to act as a catalyst, that is an intermediary agent between the cosmic vacuum and the electromagnetic field. Our starting point concerns a generalization of Mach’s principle [5] and a statement that not only inertia depends on the homogeneous distribution of the energy-momentum throughout the whole space, but other properties of bodies of any nature and any distribution of energy may also be influenced by $\Lambda$.

We follow here a similar procedure and use this Extended Mach’s Principle [2, 6] to analyze the action of the cosmic energy-momentum distribution represented by

$$ T_{\mu\nu} = \Lambda g_{\mu\nu} $$

and its induction of nonlinear processes. In the present paper we consider the case of nonlinear electrodynamics.

In short, we shall prove that a non-minimal coupling of $F_{\mu\nu}$ to gravity in the realm of general relativity generates all forms of self-interaction of the electromagnetic field. We shall see that gravity acts only to promote a catalysis that allows an interaction of the electromagnetic field and the rest of the Universe through $\Lambda$.

2. THE SCENARIO

The minimal coupling principle states that in the framework of general relativity the dynamics of the metric and the Maxwell theory of electrodynamics is provided by the action

$$ S = \int \sqrt{-g} \left( -\frac{c^2}{4} F + \frac{1}{2\kappa} R \right) d^4x, \quad (1) $$

where

$$ F = F_{\mu\nu} F^{\mu\nu}. $$

In the case of a nonminimal coupling, we set the general form of the Lagrangian as

$$ L = -\frac{c^2}{4} F + \frac{1}{2\kappa} R + \frac{1}{\kappa} \Lambda + \frac{1}{\kappa} V(F) R + L_{CT}. \quad (2) $$
From now on we set $c = 1$ for the velocity of light. Note that $V(F)$ is dimensionless and represents a nonminimal coupling with gravity; the counter-term $L_{CT}$, which depends only on $F$ and its first derivatives, is introduced in order to eliminate in the equation of motion of the electromagnetic field the presence of higher-order derivatives. The most general form of this counter-term is

$$L_{CT} = H(F)\partial_{\mu}F\partial^{\mu}F. \quad (3)$$

To accomplish this task and to avoid terms of second-order derivatives in the dynamics of $F^{\mu\nu}$, the function $H$ must depend on $V$, and if we set $V = 0$, then $H$ vanishes. This dynamics represents the electromagnetic field coupled nonminimally with gravity. Although at the very beginning it seems that there is no direct influence of $\Lambda$ on the electromagnetic field, we shall see that due to the nonminimal coupling such an influence will appear and, more than that, it will be really responsible for the rising of nonlinearities of the electromagnetic field, even if we take the limit of vanishing gravity. We restrict our analysis here to the case in which the nonlinearity appears only in functions of the invariant $F$.

Independent variation of the electromagnetic potential $A_{\mu}$ and $g_{\mu\nu}$ yields

$$F^{\mu\nu}\partial_{\nu} = -\frac{4}{\kappa}[RV'F_{\mu}]_{,\nu} - 4[H'F_{\chi}F_{\chi}F^{\mu\nu}]_{,\nu} + 8[H'F_{\chi}F_{\chi}F^{\mu\nu}]_{,\nu} = 0, \quad (4)$$

where $V' = dV/dF$ represents a derivative with respect to $F$. For the metric we obtain the equation

$$\frac{1}{\kappa}(1 + 2V)\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) = -E_{\mu\nu} + \frac{\Lambda}{\kappa}g_{\mu\nu} - \left(\frac{4}{\kappa}RV' + 4H'F_{\chi}F_{\chi} - 8H\Box F\right)F_{\mu\alpha}F_{\nu}^{\alpha} + 2\Box V g_{\mu\nu} - 2V_{\mu,\nu} - 2HF_{\mu}F_{\nu}, \quad (5)$$

where

$$E_{\mu\nu} = F_{\mu\alpha}F_{\nu}^{\alpha} + \frac{1}{4}Fg_{\mu\nu},$$

and $F_{\mu} = \partial_{\mu}F$. Once $V = V(F)$, it follows $\Box V = V'\Box F + V''F_{\alpha}F^{\alpha}$. The trace of Eq. (5) yields

$$R(1 + 2V - 4FV') = -4\Lambda - \Box F(6V' + 8HF) - F_{\mu}F^{\mu}(6V'' + 2\kappa H + 4\kappa H'F), \quad (6)$$

or

$$R = \frac{-1}{1 + 2V - 4FV'} \left(4\Lambda + \Box F(6V' + 8HF) + F_{\mu}F^{\mu}(6V'' + 2\kappa H + 4\kappa H'F)\right). \quad (7)$$

Using this result in the equation of the electromagnetic field (4) gives

$$XF^{\mu\nu}_{,\nu} + \Omega F^{\mu\nu}F_{\nu} = 0, \quad (8)$$

where

$$X = 1 - \frac{4}{\kappa}RV' + 4H'F_{\alpha}F_{\alpha} + 8H\Box F.$$

Using the equation for the trace with the scalar curvature, we obtain

$$X \equiv 1 + \frac{16\Lambda V'}{\kappa(1 + 2V - 4FV')} + F_{\alpha}F^{\alpha}\left(4H' + \frac{24V'V'' + 8\kappa HV' + 16\kappa FHF'V'}{(1 + 2V - 4FV')^2}\right) + \Box F(8H + \frac{24(V')^2 + 32FH}{(1 + 2V - 4FV')}), \quad (9)$$

and

$$\Omega F_{\mu} = -\frac{4}{\kappa}(RV')_{,\nu} + 4(H'F_{\alpha}F_{\alpha})_{,\nu} + 8(\Box F)_{,\nu}. \quad (10)$$

At this stage it is worth selecting, among all possible candidates of $V$ and $H$, particular ones that make the factor on the gradient of $F$ and $\Box F$ disappear from the expressions of $X$ and $\Omega$. This provides four conditions involving the functions $V$ and $H$. However, a direct calculation shows that these four equations are related and provide only one condition for the dependence of $V$ and $H$. Indeed, the factor on $X$ multiplying $\Box F$ gives

$$H + 3\frac{(V')^2}{\kappa(1 + 2V)} = 0. \quad (11)$$

and the other condition on $X$, that is, the factor multiplying $F_{\mu}F^{\mu}$, yields

$$H' + \frac{2HV'}{1 + 2V} + \frac{6V''}{\kappa(1 + 2V)} = 0. \quad (12)$$

By a simple inspection we recognize that the second equation is nothing but the derivative of Eq. (11). Then $X$ reduces to

$$X = 1 + \frac{16\Lambda V'}{\kappa}\frac{V'}{1 + 2V - 4FV'}. \quad (13)$$

A beautiful result comes next: the above condition (11) also eliminates all unpleasant terms containing the gradients and $\Box F$ appearing in $\Omega$. Indeed, we have

$$\Omega = \frac{d}{dF}X,$$

which allows us to rewrite Eq. (4) for the electromagnetic field in the form

$$LF_{\mu}^{\mu}_{,\nu} + LFF_{\mu}F^{\mu}F_{\nu} = 0. \quad (14)$$
where $L_F = dL/dF \equiv L'$. Thus as a result of the gravitational interaction the electromagnetic field obeys a nonlinear equation of motion characterized by the Lagrangian $L(F)$. Indeed, from the identification

$$X = L_F$$

in Eq. (8) and using the connection between $V$ and $H$ given by Eq. (11), it follows that the nonlinear electrodynamics can be obtained by integration of the expression

$$\frac{4V'}{1 + 2V} = \frac{L' - 1}{F(L' - 1) + 4\Lambda/\kappa}. \quad (15)$$

Setting

$$Z(F) \equiv \frac{L' - 1}{F(L' - 1) + 4\Lambda/\kappa},$$

the form of $V$ is given by

$$2V = e^\Delta - 1, \quad (16)$$

where

$$\Delta = \frac{1}{2} \int ZdF.$$

Thus, given any nonlinear Lagrangian $L(F)$, we determine the values of $V$ and $H$ through which the nonminimal interaction with gravity induces such nonlinearities. In the other way round, specification of the dependence of $V(F)$ provides the form of the equivalent nonlinear Lagrangian.

Let us point out that this is possible due to the influence of the rest of the Universe represented by the cosmological constant on the electromagnetic field. If $\Lambda$ vanishes, then the self-interaction disappears, and we are left with Maxwell’s linear dynamics. Indeed, from Eq. (13), when $\Lambda = 0$, then $X = 1$ yielding Maxwell’s linear dynamics.

3. FINAL REMARKS

In this work we have analyzed the influence of all the material content of the universe on the electromagnetic field described by a homogeneous energy distribution $T_{\mu\nu} = \Lambda g_{\mu\nu}$. If $\Lambda$ vanishes, the dynamics of the field is independent of the global properties of the Universe, and it reduces to the Maxwell equations. In the second case, the rest of the Universe induces a non-linear dynamics yielding the equation of motion (14).

Such a scenario is a particular example of generalization of Mach’s principle setting a global mechanism by means of which the influence of the global structure of the universe appears in any physical theory. We would like to emphasize gravity’s catalysis role in the generation of nonlinearities of the electromagnetic field.

Although we have analyzed the case in which the electromagnetic Lagrangian depends only on the invariant $F$, the generalization to the case in which one introduces the dependence on $G = F_{\mu\nu}F^{\mu\nu}$ follows along the same lines. Finally, let us recall that the term “rest of the Universe” that we used concerns the environment of any body $A$, that is the whole domain of influence on $A$ of the remaining bodies in the Universe. Note that it is completely irrelevant—for the gravitational mechanism employed in this paper—if the parameter $\Lambda$ has a classical global origin identified with the cosmological constant introduced by Einstein or a local quantum one identified with the vacuum of quantum fields.

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