The Influence of Tracking Rate on Helioseismic Flow Inferences

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Abstract. Traditionally, most local helioseismic studies of subsurface flows have removed the large signal due to the Sun’s rotation by tracking the analysis region across the solar surface. In order to work in a uniformly rotating reference frame, the ring-analysis pipeline of the recently launched Helioseismic and Magnetic Imager (HMI) will track all analysis regions at the solid-body Carrington rate. To test this tracking scheme, we compare flow determinations resulting from two different tracking schemes. In one scheme we use the HMI pipeline implementation which tracks at the Carrington rotation rate. In the other, the tiles are tracked at the local differential surface rotation rate as measured by Snodgrass (1984). We observe systematic differences between the flows obtained by the two schemes even after transforming them to a common frame (Snodgrass frame), with the zonal flows measured in the Carrington frame being faster by 5-20 m/s.

1. Introduction
When measuring convective flows through local helioseismology, one must deal with the fact that the sun’s rotational velocity is often an order of magnitude larger than the flows being sampled (typical flow speeds for supergranulation are 300 m/s, compared to the equatorial rotational velocity of 2000 m/s). Therefore, in order to measure the more sedate convective flows, ring-analysis procedures have usually tracked data cubes at the local surface differential rotation rate of the plasma (Haber et al. 2002, Corbard et al. 2003). In practice, the Snodgrass rate (Snodgrass 1984) appropriate for the center of the analysis region has been used in order to remove as much of the rotational signature as possible and to keep the measurement procedure in a slow, linear regime.

For consistency in region labelling, to allow for tracking over long time intervals, and to avoid a shearing reference frame, the designers of the HMI ring-analysis pipeline have opted to track at the uniform Carrington rotation rate. If the ring-analysis flow-measurement procedure were purely linear, the choice of reference frame would be immaterial. However, large velocities can Doppler shift power out of the predefined frequency windows used to perform the analysis, thereby causing a potential nonlinearity in the measurement procedure. In order to test the proposed tracking scheme for HMI, in this paper we analyze the same data using both tracking...
2. The Data

We have analyzed Dopplergram data from HMI consisting of a multitude of tracked data cubes each corresponding to a square region, 5.12° on a side, that has been tracked for 9.6 hr. Each cube lies at a different location on the solar disk, and the regions fill the solar disk with roughly equal density. The regions overlap by roughly 50% and, depending on the solar B angle, there are either 2727 or 2748 separate regions. Data from 18 separate 9.6 hr periods, spanning 6 days, have been analyzed and the results averaged together. Two different tracking schemes have been employed on the same HMI images. One set of data cubes has been tracked at the Carrington rate $\Omega_C/2\pi = 456.03$ nHz, and the other at the Snodgrass rate $\Omega_S/2\pi = (451.426 - 54.77 \sin^2 \theta - 80.17 \sin^4 \theta)$ nHz, where $\theta$ is the latitude of the center of the region. Each data cube is apodized in space and time and Fourier transformed to form a 3-D power spectrum that is a function of cyclic frequency $\nu$ and two horizontal wavenumbers, $k_x$ and $k_y$, corresponding to the prograde, zonal direction (the $x$-direction) and northward (the $y$-direction).

3. Mode Fitting

Parameters for the $f$ and $p$ modes have been extracted from each power spectra by a maximum-likelihood fitting procedure with a fitting function given by,

$$P(\nu, k_x, k_y) = \frac{A \Gamma/2}{[(\nu - \nu_0 - U_x k_x - U_y k_y)^2 + (\Gamma/2)^2]} + \frac{b}{k^3},$$

where the fitting is performed separately for each total horizontal wavenumber $k = (k_x^2 + k_y^2)^{1/2}$ and for each radial order $n$. The fitted parameters are the mode amplitude $A$, the line width $\Gamma$, the rest frequency $\nu_0$, two flow speeds $U_x$ and $U_y$, and a background power level $b$. For the entire set of data cubes, 60812 modes were common to both data sets. Figure 1 shows the dispersion diagram for these fits. Modes ranging in order from the $f$ mode to $p_5$ were obtained.
Figure 2. (a) Difference between the mean zonal flow measured in the two frames \((v_C - v_S)\) and (b) mean meridional flow plotted separately for each frame. The means were computed by averaging over longitude and time, with further averaging over 7.5° bins in latitude and 3 bins in horizontal phase speed performed to increase the ratio of the signal to noise. All zonal flows have been transformed to a common reference frame (Snodgrass) for comparison. The different colors of the curves correspond to averages over \(p\) modes with different horizontal phase speeds, with green corresponding to modes with the shallowest acoustic cavity and violet to those with the deepest. The colors correspond to those used in Figure 1. The zonal flows measured in the Carrington frame are systematically faster than those measured in the Snodgrass frame. The meridional flows measured in the two frames are identical to within their measurement errors. Measurement made in the Carrington frame are shown with solid curves and those made in the Snodgrass frame with dashed curves.

4. Mean Flow Analysis

In order to compare the data obtained by the two different tracking rates, we have transformed the zonal flows \(U_z\) measured in the Carrington frame to the Snodgrass frame, by adding the difference between the surface velocity for the two frame rates. Error weighted means were then computed by averaging all flow speeds in longitude and time. The ratio of signal to noise was further increased by averaging over latitudinal bins that were 7.5° wide and by averaging all modes falling within specified ranges of horizontal phase speed \(2\pi\nu/k\) (see Figure 1). Modes with a common phase speed have acoustic cavities with the same lower turning point; thus, the phase speed binning averages over modes with similar sampling in depth. Furthermore, since the \(f\) modes are surface gravity waves without an acoustic cavity, we have performed a separate analysis for the \(f\) modes and the \(p\) modes. Finally, only modes common to both data sets were included in the means.

5. Results

The mean flow speeds are shown in Figures 2 and 3. The mean zonal flows measured in the Carrington reference frame are systematically faster than those measured in the Snodgrass frame (shown as positive values in Figures 2a and 3a). The difference is small near the equator (less than 5 m/s), but grows toward the poles reaching values of 20 m/s or more. Furthermore, for modes of large harmonic degree \((l > 1000)\) the discrepancy is greatly enhanced preferentially in the northern hemisphere.

The meridional flows measured in the two reference frames are identical to within their measurement errors. The flows are poleward with a speed of 15-20 m/s at mid-latitudes, vanishing flow at the equator, and evidence for reduced flow near the poles. For the \(p\) modes there is a marked lack of dependence on phase speed indicating little variation with depth; however, the flows obtained with \(f\) modes are systematically higher by a factor of roughly 50%, perhaps suggesting a near-surface enhancement of the meridional flow.
Figure 3. Same as Figure 2, except the analysis uses only $f$ modes. Orange corresponds to high degree modes $l > 1000$

6. Discussion

The fact that the mean flows are systematically different depending on the tracking rate is an indication that the mode fitting procedure isn’t linear with respect to tracking. One possible source for this systematic error arises because the fitting procedure is performed over fixed frequency windows and large flow velocities shift the power within this window. For small analysis regions, the $p$-mode power profiles are sufficiently broad that this redistribution of power may move the wings of the target mode ridge out of the fitting domain, and perhaps move the wings of nearby ridges into the domain as well. This causes a systematic mismeasurement of the flow speed. Such problems would be exacerbated by the use of the Carrington rate because the rotation rate of the frame and the local plasma rate have substantial mismatch. Further, this problem is more prominent when analyzing small tiles. The mode profiles for small analysis regions are broadened due to the spatial window function. Thus, a further test would be to repeat the analysis with tiles that are 15° on a side. The disagreement in the zonal velocity should be reduced for such a tile size, if the source is as suggested.

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7. References

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