Unification of Couplings and the Dynamical Breakdown of the Electroweak Symmetry

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Abstract

I discuss the properties of the minimal supersymmetric extension of the standard model, in the case in which there is a dynamical breakdown of the electroweak symmetry induced by the formation of condensates of the third generation of quarks and their supersymmetric partners. The top quark and Higgs mass predictions derived within this scheme are essentially equivalent to those ones obtained from the requirement of bottom-tau Yukawa coupling unification in a supersymmetric grand unified scenario, if the compositeness scale is identified with the grand unification scale. I give an explanation of this interesting result, for which the relevance of the infrared quasi fixed point on the top quark Yukawa coupling is emphasized.
1. The Top Condensate Model

The increasing lower bound on the top mass has open the window for a top quark heavy enough to induce the formation of a condensate, which catalyzes the electroweak symmetry breakdown at low energies. In fact, in analogy to what happens in the Nambu Jona Lasinio (NJL) model, a strong Yukawa coupling could be the signature of a dynamical mechanism for the electroweak symmetry breaking which relies only on the observed quark and leptons of the standard model, and in which the Higgs field appears as a $t-\bar{t}$ bound state. The basic mechanism for the physical realization of this idea was first proposed by Nambu, by making an analogy between the spontaneous breakdown of the electroweak symmetry in the Standard Model and the BCS mechanism in condensed matter theories. Several authors analysed the physical consequences of such a scenario and a detailed field theoretical analysis was first done by Bardeen, Hill and Lindner.

They started with a gauged, $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant, NJL model,

$$\mathcal{L} = \mathcal{L}_K^\psi + \mathcal{L}_{YM} + G \left( \bar{\psi}_L^c i t_R^c \right) \left( \bar{t}_R^d \psi_L^d \right)$$

where $\mathcal{L}_K^\psi$ and $\mathcal{L}_{YM}$ are the kinetic terms for the fermion and Yang Mills fields respectively, $\psi_L^c = (t\ b)_L$, $t$ and $b$ are the bottom and top quark fermion fields and the indices $c$ and $d$ indicate a sum over color degrees of freedom. In this first simplified formulation only the top quark acquires mass. The masses for the other fermion fields, however, may be generated by introducing the corresponding Yukawa couplings between the fermions and the scalar composite field. If $G > 0$ the interactions are attractive, and for $G > G_c$, the local chiral symmetry of the theory is broken through a top condensate, $\langle \bar{t}t \rangle \neq 0$. In the scaling region, a composite scalar doublet,

$$H = G \bar{t}_R \psi_L$$

appears in the spectrum of the theory. The quantum numbers of these composite fields are exactly equal to those ones of the elementary Higgs field in the standard model, and hence, for $G > G_c$ the $SU(2)_L \times U(1)_Y$ local symmetry is broken to $U(1)_{em}$. Three massless Goldstone bosons, associated with the breakdown of the gauge symmetry are
induced, giving masses to the electroweak gauge bosons through the usual Higgs mechanism. In addition, a physical, electrically neutral scalar field appears in the spectrum of the theory. In general, the low energy spectrum is completely equivalent to the Standard Model one, although the reduction in the number of free parameters of the theory increases its predictability. In fact, as I shall discuss below, for a given effective cutoff scale $\Lambda$, sharp predictions for the scalar Higgs and top masses can be derived within this context.

1.1 Large $N_C$ Analysis.

The dynamical properties of the gauged NJL model can only be explored by using nonperturbative methods. A systemathical analysis can be done, for example, by solving the self consistent Schwinger Dyson equations of the theory in the large $N_C$ approximation, where $N_C$ is the number of colors. The critical four Fermi coupling may be estimated by solving the self consistent equation for the top quark mass. I shall first study the model in the so called bubble approximation, that is the large $N_C$ limit, for vanishing $SU(3)_C$ gauge coupling value. The dynamical effects due to the inclusion of the $SU(3)_C$ interactions will be discussed below. In the bubble approximation, the self consistent equation for the top quark mass reads

$$M_t = \frac{G N_C}{8\pi^2} \left( \Lambda^2 - M_t^2 \log \left( \frac{\Lambda^2}{M_t^2} \right) \right) M_t.$$  \hspace{1cm} (3)

Hence, for a nontrivial solution of the gap equation, $M_t \neq 0$, the top quark mass is given by

$$M_t^2 \log \left( \frac{\Lambda^2}{M_t^2} \right) = \Lambda^2 - \frac{8\pi^2}{N_C G}.$$  \hspace{1cm} (4)

Observe that, since the left hand side is positive a nontrivial solution only exists if $G > G_c$, with $G_c = 8\pi^2/N_C \Lambda^2$. Since the logarithmic factor is only a slowly varying function of $M_t$, the natural scale for the fermion mass in the broken phase would be just the cutoff scale. A large hierarchy between the cutoff scale and the fermion mass scale requires a very precise fine tuning of the four Fermi coupling to its critical value. This is nothing but the usual fine tuning problem of the standard model.
The relation between this model and the standard Higgs Yukawa model becomes apparent if I rewrite the Lagrangian density in an equivalent form, by introducing an auxiliary scalar doublet $H$

\[ \mathcal{L} = \mathcal{L}_K^{\psi} + \mathcal{L}_{YM} + \bar{\psi}_L^b t_R^b H + H^\dagger t_R^b \psi_L^b - M_0^2 H^\dagger H. \]  

(5)

In the above, $M_0^2 = 1/G$ and $H$ can be eliminated through a Gaussian integration, or equivalently by its replacement through its equation of motion $H = G \bar{t}_R \psi_L$. At this level, the scalar field $H$ is a static field, with no independent dynamics. The physical picture changes, however, once the quantum fluctuations of the fermion fields are taken into account. In the bubble approximation, for example, the scalar fields propagate through fermion “bubbles”. The propagator of the scalar field $H$, $D^{-1}(p)$, may be obtained by computing the bubble function with external momentum $p$ through the relation

\[ D^{-1}(p) = \frac{1}{G} + B(p) \]  

(6)

where $B(p)$ is the bubble function. A nonvanishing kinetic term for the unrenormalized scalar field $H$ is induced, together with a correction to its physical mass. The function $B(p)$ is quadratically divergent, but the quadratical divergences are cancelled once the gap equation is taken into account. Observe that the fermion mass is nothing but the vacuum expectation value of the electrically neutral, CP even component of the scalar field, $<H^0>$, and hence, once the gap equation is fulfilled, the quadratical divergences of the scalar propagator are automatically cancelled.

The propagator of the neutral scalar field $H^0$ may be explicitly computed, giving

\[ D^{-1}(p) = \frac{N_C}{8\pi^2} \left( p^2 - 4M_t^2 \right) \log \left( \frac{\Lambda^2}{M_t^2} \right) + \chi(p^2), \]  

(7)

where for $p^2 = \mathcal{O}(M_t^2)$ and $\Lambda \gg M_t$, the function $\chi(p^2)$ is negligible. Hence, the neutral scalar field propagator has a pole at

\[ M_{H^0} = 2M_t. \]  

(8)

I would like to emphasize that the above prediction is obtained in the context of the bubble approximation where important effects, like the QCD corrections, are neglected.
Although these corrections do not change the qualitative physical picture, they have an important incidence on the quantitative relations between physical couplings and masses.

1.2 Effective Lagrangian Analysis.

For $\Lambda \gg M_t$, the values of the relevant quantities are dominated by large logarithms, and all physical results may be reproduced by doing an effective field theory analysis. I start with the Lagrangian density

$$\mathcal{L}(\Lambda) = \mathcal{L}_K + \mathcal{L}_Y + \bar{\psi}_L t_R^b H + H \bar{t}_R^b \psi_L^b - M_0^2 H \dagger H,$$

which characterizes the interactions at the large energy scale $\Lambda$. The effective theory at the low energy scale $\mu$ may be obtained by integrating out the short distance fermion effects, which in this context is equivalent to consider the quadratic and large logarithmic corrections induced by the fermion loops. The effective low energy Lagrangian reads,

$$\mathcal{L}(\mu) = \mathcal{L}_K^\psi + \mathcal{L}_Y^\psi + \bar{\psi}_L^b t_R^b H + H^\dagger \bar{t}_R^b \psi_L^b$$

$$+ Z_H |\mathcal{D}_\mu H|^2 - \frac{\lambda_0}{2} \left( H \dagger H \right)^2 - (M_0^2 + \Delta M^2) H \dagger H,$$

where

$$Z_H = \frac{N_C}{(4\pi)^2} \log \left( \frac{\Lambda^2}{\mu^2} \right); \quad \lambda_0 = 2Z_H,$$

while $\Delta M^2 \approx -1/G_c$. The values of the wave function renormalization constant and of the quartic couplings are normalized so that the effective Lagrangian coincides with Eq. (9) at $\mu = \Lambda$. This leads to the following boundary conditions,

$$Z_H(\mu \to \Lambda) = 0, \quad \lambda_0(\mu \to \Lambda) = 0,$$

which are called the compositeness conditions.

The Lagrangian can be rewritten in a more conventional way by normalizing the field $H$ so that it has a canonical kinetic term, $H \to Z_H^{1/2} H$. In terms of the renormalized field, it reads,

$$\mathcal{L}(\mu) = \mathcal{L}_K^\psi + \mathcal{L}_Y^\psi + h_t \bar{\psi}_L^b t_R^b H + h_t H^\dagger \bar{t}_R^b \psi_L^b$$

$$+ |\mathcal{D}_\mu H|^2 - \frac{\lambda}{2} \left( H \dagger H \right)^2 - m_H^2 H \dagger H.$$
where the renormalized couplings $h_t = Z_t^{-1/2}$ and $\lambda = Z_H^{-2} \lambda_0$. The compositeness conditions imply the divergence of the renormalized couplings when $\mu \to \Lambda$.

The physical Higgs and top quark masses $M_{H_0}$ and $M_t$, (which in the absence of gauge couplings coincide with the running ones, $m_{H_0}$ and $m_t$, respectively) are given by the on shell relations

$$m_t = h_t(m_t) \, v, \quad m_{H_0}^2 = 2 \lambda(m_0^0) \, v^2$$

where $v \simeq 175$ GeV, is the vacuum expectation value of the renormalized field. Since in the bubble approximation the relation $\lambda(\mu)/(2h_t^2(\mu)) = 1$ is fulfilled, ignoring the small scale dependence, which is of the order of other ignored higher order effects, the relation $m_0^0 = 2m_t$ is recovered.

1.3 Improved Renormalization Group Analysis

The results of the last section can be improved by including the electromagnetic and weak gauge interactions, together with the dynamically generated scalar effects. This can be done by including nonleading order in $1/N$ effects in the self consistent equations for the scalar Higgs and top quark self energies. When the compositeness scale is much larger than the weak scale, the value of the relevant coupling is well determined by computing the leading logarithmic corrections. Hence, the results of this approximation can be reproduced by considering the full one loop renormalization group equations of the standard model\(^4\), while using the compositeness conditions discussed in the previous subsection, as an ultraviolet boundary condition at the compositeness scale $\Lambda$,

$$16\pi^2 \frac{dh_t}{dt} = \left((N_C + \frac{3}{2}) \, h_t^2 - (N_C - 1)g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{12}g_1^2\right) h_t$$

$$16\pi^2 \frac{dg_i}{dt} = \beta_i g_i^3$$

$$16\pi^2 \frac{d\lambda}{dt} = 12 \left(\lambda^2 + (h_t^2 - A)\lambda + B - h_t^4\right)$$

where $A = g_1^2/4 + 3g_2^2/4$, $B = g_1^4/4 + g_1^2g_2^2/8 + 3g_2^4/16$, $\beta_1 = -41/6$, $\beta_2 = 19/6$ and $\beta_3 = 7$. Of course, the perturbative one loop renormalization group equations may not be reliably used to determine the evolution of the top quark Yukawa coupling at scales
close to the compositeness scale $\Lambda$. However, the action of the infrared quasi-fixed point makes the top quark mass predictions very insensitive to the precise high value of the top quark Yukawa coupling at the scale $\Lambda$. In fact, for a compositeness scale of the order of $\Lambda = 10^{10} - 10^{19}$ GeV, the top quark Yukawa coupling is strongly focussed to a small set of infrared values, with corresponding running top quark masses of the order of 230 GeV. A slight variation, of less than 1% (2%) of the top quark mass value is obtained by setting $h_t^2(\Lambda)/4\pi = 1$, for a compositeness scale $\Lambda \geq 10^{16}$ GeV ($\Lambda \geq 10^{10}$ GeV).

**Table 1.** Predictions for the top quark mass, $m_t$, and the Higgs mass, $m_h$, in different approximations.

| $\Lambda (GeV)$ | $10^{19}$ | $10^{15}$ | $10^{11}$ | $10^9$ |
|-----------------|-----------|-----------|-----------|------|
| $m_t$ (GeV)     |           |           |           |      |
| Bubble Sum      | 144       | 165       | 200       | 277  |
| Planar QCD      | 245       | 262       | 288       | 349  |
| Full RG Eq.     | 218       | 229       | 248       | 293  |
| $m_h$ (GeV)     |           |           |           |      |
| Full RG Eq.     | 239       | 256       | 285       | 354  |

The quartic coupling is also attracted to its infrared quasi fixed point value, which as can be seen from Eq.(15), gives a relation between the top quark Yukawa coupling and the quartic coupling, which translates into a mass ratio $m_{H^0}/m_t \approx 1.1$. The numerical values for the top quark and Higgs masses obtained in the different approximations and for different values of the compositeness scale are shown in Table 1.

The value for the top quark (Higgs) mass obtained by using the full one loop renormalization group equations are stable under variations of the compositeness scale $\Lambda$. It follows from Table 1 that, starting from $\Lambda \simeq 10^{19}$ GeV, this mass value varies less than a 15% (25%) under a variation of the compositeness scale of eight orders of magnitude. In general, for $\Lambda \leq 10^{19}$ GeV,

$$m_t > 210 GeV. \quad (16)$$

Quite generally, for a given effective cutoff scale $\Lambda$, the triviality bound on the top quark may be defined as the value of $m_t$ which is obtained assuming that the top quark
Yukawa coupling becomes strong at scales of the order of $\Lambda$. Since in the dynamical scheme under consideration the renormalized coupling diverge at the compositeness scale, the top quark mass obtained within the top condensate model is consistent with the renormalization group trajectories associated with the triviality bounds on this quantity, for an effective cutoff scale equal to the compositeness scale. The presence of the infrared quasi fixed point, makes the value of this bound very insensitive to the exact large value of the top quark Yukawa coupling at the effective cutoff scale$^{5,6}$. Thus, the values of the top quark mass derived above define the triviality bounds on $m_t$ and may be interpreted as the maximum allowed value of this quantity in any theory in which no new physics appear up to scales of order $\Lambda$.

2. Supersymmetric Generalization and Unification of Couplings.

In spite of its beauty and simplicity, there are two main problems in the standard formulation of the top condensate model. The first one is that an unnatural fine tuning of the four Fermi coupling is necessary in order to obtain a proper physical spectrum. The second one, is the fact that, even for a compositeness scale of the order of the Planck scale $\Lambda \simeq 10^{19}$ GeV, the running top quark mass turns out to be $m_t \simeq 220$ GeV, a value which could be too large to be consistent with the experimental constraints coming from the $\rho$ parameter measurement. Supersymmetry provides a possible solution to these problems. In a supersymmetric extension of the top condensate model the quadratic divergences disappear, and hence no fine tuning of the four Fermi coupling constants is required$^7$. In addition, the predicted top quark mass values are sensibly lower than in the standard case$^8$.

Most interesting, it has been also recently noticed that the values of the gauge coupling constants measured at LEP are consistent with a supersymmetric grand unified scenario. Indeed, unification of couplings within the Minimal Supersymmetric Standard Model (MSSM) may be achieved if the grand unification scale is of the order of $10^{16}$ GeV and the supersymmetric partners masses, characterized by a common mass scale
$M_{SUSY}$, are of the order of the weak scale$^{9,10}$. It is worth mentioning that the exact value of the supersymmetric threshold scale necessary to achieve unification of gauge coupling constants is strongly dependent on the value of the strong gauge coupling, $\alpha_3(M_Z)$ and the weak mixing angle, $\sin^2 \theta_W(M_Z)$. Moreover, as I shall discuss below, when a splitting of the supersymmetric partner masses is introduced, the effective supersymmetric threshold scale may be far away from the characteristic scale of the supersymmetric mass spectrum$^{11-13}$.

In addition to the unification of gauge couplings, the unification of the bottom quark and tau Yukawa couplings appears naturally in many grand unified scenarios$^{13-17}$. At the one loop level the bottom and tau Yukawa coupling renormalization group equations depend only on the gauge couplings, which are fixed by the unification conditions, and the top quark Yukawa coupling. Hence, the requirement of Yukawa coupling unification, together with the bottom quark and tau mass values, are sufficient to determine the top quark mass as a function of $\tan \beta$, the ratio of vacuum expectation values of the two Higgs doublets present in the theory. This program was recently carried on by several authors. One of the most interesting results is that, for a running bottom quark mass $m_b(M_b) < 4.6$ GeV (which approximately correspond to a physical bottom quark mass $M_b < 5.2$ GeV), the top quark mass predictions are close to its quasi infrared fixed point ones, associated with the triviality bounds on this quantity$^{8,13,14}$. Hence, for these values of the running bottom quark mass the predictions of the grand unified scenario are remarkably close to the ones of the top condensate model with a compositeness scale $\Lambda \simeq M_{GUT}$. One of the purpose of this talk is to explain the origin of this interesting coincidence.

2.1 The Generalized Supersymmetric NJL Model

To describe the dynamics responsible for the top quark multiplet condensation, I shall consider an $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant gauged supersymmetric Nambu-Jona-Lasinio model$^{7,8}$, with explicit soft supersymmetry breaking terms. Written in terms of
the two composite chiral Higgs superfields $H_1$ and $H_2$, the action of the gauged Nambu-Jona-Lasinio model at the scale $\Lambda$ takes the form

$$\Gamma_\Lambda = \int dV \left[ Q e^{2\nu_0} Q + T^C e^{-2\nu_T} \bar{T}^C + B^C e^{-2\nu_B} \bar{B}^C \right] (1 - m_0^2 \theta^2 \bar{\theta}^2)$$

$$+ \int dV \left( \bar{H}_1 e^{2\nu_H} H_1 (1 - M_H^2 \theta^2 \bar{\theta}^2) \right) - \int dS \epsilon_{ij} \left( \mu_0 H^i_1 H^j_2 (1 + B_0 \theta^2) \right)$$

$$- g T_0 H^2_2 Q^i T^C (1 + A_0 \theta^2) + h.c.,$$  \hspace{1cm} (17)

where $Q = \begin{pmatrix} T_B \\ B_C \end{pmatrix}$ is the $SU(2)_L$ doublet of top and bottom quark chiral superfields, $T^C$ ($B^C$) is the $SU(2)_L$ singlet charge conjugate top (bottom) quark chiral multiplet, $m_0$, $M_H^2$, $A_0$ and $B_0$ are soft supersymmetry breaking terms, $dV = d^4 x d\theta^2 d\bar{\theta}^2$ and $dS = d^4 x d\theta^2$. The quark and Higgs multiplets interact with the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge fields via $V_i$. The usual superfield notation has been used. At this level the superfield $H_2$ acts as a Lagrange multiplier, imposing the compositeness condition

$$H_1 = \frac{g T_0}{\mu_0} Q^T T^C. \hspace{1cm} (18)$$

It is then straightforward to show that the Nambu Jona Lasinio model depends only on $\delta = A_0 - B_0$. Observe that the supersymmetric generalization of the four Fermi interaction, $\Gamma_F$, is a $D$ term, which is automatically obtained when the superfield $H_2$ is integrated out,

$$\Gamma_F = \frac{g^2 T_0}{\mu_0^2} \int dV \bar{T}^C Q e^{2\nu_H} QT^C.$$  \hspace{1cm} (19)

In the presence of a condensate of top quark superfields, a dynamical mass for the top quark is generated. Its value may be determined in a self consistent way by using the Schwinger-Dyson equations in the bubble approximation. In its simpler form, for $\delta = 0$, the gap equation reads

$$G^{-1} = \frac{N_C m_0^2}{16\pi^2} \left[ \left( 1 + \frac{m_t^2}{m_0^2} \right) \ln \left( \frac{\Lambda^2}{(m_t^2 + m_0^2)^2} \right) - \frac{2m_t^2}{m_0^2} \ln \left( \frac{\Lambda^2}{m_t^2} \right) \right],$$  \hspace{1cm} (20)

where $G = \frac{g^2 T_0}{\mu_0^2}$. The usual quadratic dependence on $\Lambda$, appearing in the standard top-condensate model has been replaced by a mild quadratic dependence on the soft supersymmetry breaking scale $m_0$. In general, the critical four Fermi coupling is of
the order of the largest soft supersymmetry breaking scale appearing in Eq.(17). Hence, as we discussed above, for a soft supersymmetry breaking scale of the order of a few TeV, no fine tuning of the four Fermi coupling is necessary in this framework.

In general, for $\delta \neq 0$, in the scaling region, in which the four Fermi coupling constant is close to its critical value, a gauge invariant kinetic term for $H_2$ is induced at low energies. Rescaling the field $H_2$, so that it has a canonically normalized kinetic term, its low energy effective action is given by

$$\Gamma_{H_2} = \int dV \bar{H}_2 e^{2\nu_2} H_2 (1 + 2m_0^2 \delta \bar{\theta}^2) - \left[ \int dS e_{ij} \left( \mu H_1^i H_2^j (1 + \delta \bar{\theta}^2) - h_t H_2^i Q^j T^C \right) + h.c. \right] + q.t., (21)$$

where I have defined the renormalized mass, $\mu = \mu_0 / \sqrt{Z_{H_2}}$, and Yukawa coupling, $h_t = gT_0 / \sqrt{Z_{H_2}}$ with the wave function renormalization constant $Z_{H_2} = \frac{g^2_0 N_C}{16\pi^2} \ln \frac{\Lambda^2}{m_0^2}$. In the above, $q.t.$ represent the radiative corrections to the quartic terms which will be analyzed below. Since $\mu_0$ and $gT_0$ have finite values, the above renormalized couplings diverge at the scale $\Lambda$. Observe that, although the cancellation of the supersymmetry breaking term $A(\mu)$ at all scales is only a property of the bubble sum approximation, the relation $A(\mu)|_{\mu \rightarrow \Lambda} = 0$ is a prediction of the model. It is also important to remark that although at high energy scales, the mass parameter associated to $H_2$ is positive due to the supersymmetric contribution proportional to $\mu^2$, $m_2^2 = \mu^2 - 2m_0^2$, at low energies it tends to negative values, inducing, therefore, the breakdown of the $SU(2)_L \times U(1)_Y$ symmetry.

3. Predictions of the SUSY Top Condensate Model

Instead of computing gauge fields corrections and higher order in $1/N_C$ effects, it proves convenient to work with the full renormalization group equations of the supersymmetric standard model. The running top quark mass value is given by $m_t = h_t(m_t)v_2$, where $v_i$ is the vacuum expectation value of the scalar Higgs $H_i$. The low energy value of the top quark Yukawa coupling can be obtained by computing its renormalization group flow, using the supersymmetric renormalization group equations for scales
\(M_{\text{SUSY}} \leq \mu \leq \Lambda\) and those of the Standard Model with one or two Higgs doublets, for \(\mu \leq M_{\text{SUSY}}\), where \(M_{\text{SUSY}}\) is the soft supersymmetry breaking scale\(^8\). At energy scales \(\mu\) close to the compositeness scale \(\Lambda\), the perturbative one loop renormalization group equations can not be used in a reliable way, to determine the evolution of the top Yukawa coupling. However, the same as in the standard case, the action of the infrared quasi-fixed point yields the top quark mass predictions quite insensitive to the precise high value of the top quark Yukawa coupling at the scale \(\Lambda\), or more generally, to the inclusion of higher order operators\(^8,18\). In addition, the running top quark mass is only slightly dependent on the exact value of \(M_{\text{SUSY}}\) and for a fixed compositeness scale, it is well approximated by the functional relation \(m_t \simeq M_T \tan \beta / \sqrt{1 + \tan^2 \beta}\), where \(\tan \beta = v_2 / v_1\). For \(\Lambda = 10^{16}\) GeV and a strong gauge coupling \(\alpha_3(M_Z) \simeq 0.12\), the value of the constant \(M_T\) is approximately given by \(M_T \simeq 195\) GeV.

At energies below the soft supersymmetry breaking scale the Higgs potential is given by the general expression\(^8,20\)

\[
V_{\text{eff}} = m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 - m_3^2 \left( H_1^T i \tau_2 H_2 + \text{h.c.} \right) + \frac{\lambda_1}{2} \left( H_1^\dagger H_1 \right)^2 \\
+ \frac{\lambda_2}{2} \left( H_2^\dagger H_2 \right)^2 + \lambda_3 \left( H_1^\dagger H_1 \right) \left( H_2^\dagger H_2 \right) + \lambda_4 \left| H_2^\dagger i \tau_2 H_1^* \right|^2 
\]  
(22)

where the radiative corrections induced by the top quark Yukawa coupling\(^19\) may be computed by solving the corresponding renormalization group equations for the quartic couplings. At the supersymmetry breaking scale the quartic couplings must fulfill the boundary conditions

\[
\lambda_1(M_{\text{SUSY}}) = \lambda_2(M_{\text{SUSY}}) = \frac{g_1^2 + g_2^2}{4}, \quad \lambda_3(M_{\text{SUSY}}) = \frac{g_2^2 - g_1^2}{4}, \\
\lambda_4(M_{\text{SUSY}}) = -\frac{g_2^2}{2}. 
\]  
(23)

If \(m_A^0\), defined as \((m_A^0)^2 = m_1^2 + m_2^2\), is of the order of the weak scale, two light Higgs doublets appear in the low energy spectrum. There are two neutral CP-even scalar states, one neutral CP-odd state and a charged state, whose masses may be obtained from the above effective potential\(^8,20\).
From the minimization of the potential, a lower bound on \( \tan \beta \) may be derived. It can be shown that, under reasonable assumptions, the characteristic values of the ratio of vacuum expectation values \( \tan \beta \geq 1 \). Moreover, for a characteristic soft supersymmetry breaking scale \( M_{SUSY} = 1 - 10 \text{ TeV} \) and a compositeness scale \( \Lambda = 10^{10} - 10^{16} \text{ GeV} \), the top quark mass fulfills the condition \( m_t > 140 \text{ GeV} \). For a given compositeness scale \( \Lambda \) and \( M_{SUSY} \), the top quark mass is only a function of \( \tan \beta \), while the Higgs spectrum depends on \( \tan \beta \) as well as on the value of the mass parameter \( m^0_A \). Assuming \( \tan \beta \geq 1 \), \( \alpha_3(M_Z) \simeq 0.12 \), the characteristic squark mass to be of the order of 1 TeV and a compositeness scale \( \Lambda = 10^{16} \text{ GeV} \) the upper bounds on the lightest Higgs mass within this model are \( m_h \leq 65 \text{ GeV} \) if \( \tan \beta \simeq 1 \) \( (m_t \simeq 140 \text{ GeV}) \) and \( m_h \leq 135 \text{ GeV} \) if the ratio of vacuum expectation values is in the range \( \tan \beta = 5 - 30 \) \( (m_t \simeq 195) \text{ GeV} \).

4. Unification of Couplings, \( \alpha_3(M_Z) \) and \( Y_t(M_{GUT}) \)

As we mentioned in the introduction, for given values of the gauge couplings, the condition of unification of bottom and tau Yukawa couplings allows to determine the value of the top quark mass as a function of \( \tan \beta \). The values of the weak gauge couplings must fulfill very tight experimental constraints. Indeed, working in the modified \( \bar{\text{MS}} \) scheme\(^{21} \), the value of the fine structure constant \( \alpha^{-1}(M_Z) = 127.9 \), while\(^{12} \)

\[
\sin^2 \theta_W(M_Z) = 0.2324 \pm 0.006
\]  

(24)

where the top quark mass value has been left free. The strong gauge coupling value is not so precisely known and, a conservative estimate for this quantity is\(^{12} \)

\[
\alpha_3(M_Z) = 0.12 \pm 0.1,
\]  

(25)

where the upper (lower) range of values are preferred by LEP (deep inelastic scattering) data. Since the top quark mass predictions coming from Yukawa coupling unification depend on the value of \( \alpha_3(M_Z) \), a precise determination of the top quark mass can not be done unless the value of \( \alpha_3(M_Z) \) is known. In the following, for a given supersymmetric spectrum, and a given value of the weak mixing angle, the value of \( \alpha_3(M_Z) \) will be
determined by requiring the unification condition. Within this context, the value of the
strong gauge coupling is given by\textsuperscript{12,13}

\[
\frac{1}{\alpha_3(M_Z)} = \frac{(b_1 - b_3)}{(b_1 - b_2)} \left[ \frac{1}{\alpha_2(M_Z)} + \gamma_2 + \Delta_2 \right] - \frac{(b_2 - b_3)}{(b_1 - b_2)} \left[ \frac{1}{\alpha_1(M_Z)} + \gamma_1 + \Delta_1 \right] - \gamma_3 - \Delta_3 + \Delta^{sth}\left( \frac{1}{\alpha_3(M_Z)} \right),
\]

(26)

where

\[
\Delta^{sth} = \frac{19}{28\pi} \ln \left( \frac{T_{SUSY}}{M_Z} \right)
\]

(27)
is the contribution to $1/\alpha_3(M_Z)$ due to the inclusion of the supersymmetric threshold corrections at the one loop level, $\gamma_i$ includes the two loop corrections to the value of $1/\alpha_i(M_Z)$, $\Delta_i$ are correction constants which allow to transform the gauge couplings from the minimal $\overline{MS}$ scheme to the dimensional reduction scheme, $\overline{DR}$, more appropriate for supersymmetric theories, and $b_i$ are the supersymmetric beta function coefficients associated to the gauge coupling $\alpha_i$. The effective supersymmetric threshold scale is defined as that one which would produce the same threshold corrections to the value of $\alpha_3(M_Z)$ in the case in which all the supersymmetric particles were degenerate in mass.

In order to study the dependence of $T_{SUSY}$ on the different sparticle mass scales of the theory, we define $m_{\tilde{q}}, m_{\tilde{g}}, m_{\tilde{l}}, m_{\tilde{W}}, m_{\tilde{H}}$ and $m_H$ as the characteristic masses of the squarks, gluinos, sleptons, electroweak gauginos, Higgsinos and the heavy Higgs doublet, respectively. Assuming different values for all these mass scales, I derive an expression for the effective supersymmetric threshold $T_{SUSY}$ which is given by\textsuperscript{13}

\[
T_{SUSY} = m_{\tilde{H}} \left( \frac{m_{\tilde{W}}}{m_{\tilde{g}}} \right)^{28/19} \left[ \left( \frac{m_{l}}{m_{\tilde{q}}} \right)^{3/19} \left( \frac{m_{H}}{m_{\tilde{H}}} \right)^{3/19} \left( \frac{m_{\tilde{W}}}{m_{\tilde{H}}} \right)^{4/19} \right].
\]

(28)
The above relation holds whenever all particles considered above have a mass $m_\eta > M_Z$. If, instead, any of the sparticles or the heavy Higgs boson has a mass $m_\eta < M_Z$, it should be replaced by $M_Z$ for the purpose of computing the supersymmetric threshold corrections to $1/\alpha_3(M_Z)$. In the following, unless otherwise specified, I shall assume that all sparticles and the heavy Higgs doublet acquire masses above $M_Z$. From Eq.(28), it follows that, for fixed mass values of the uncolored sparticles, that is sleptons, Higgsinos
and the weak gauginos, together with the heavy Higgs doublet, the value of $T_{SUSY}$ decreases for larger mass values of the colored sparticles - squarks and gluinos. Moreover, $T_{SUSY}$ depends strongly on the first two factors in Eq.(28), while it is only slightly dependent on the expression inside the squared brackets. This is most surprising, since it implies that $T_{SUSY}$ has only a slight dependence on the squark, slepton and heavy Higgs masses and a very strong dependence on the overall Higgsino mass, as well as on the ratio of masses of the gauginos associated with the electroweak and strong interactions. The mild dependence of the supersymmetric threshold corrections on the squark and slepton mass scales is in agreement with a similar observation made in the context of the minimal supersymmetric SU(5) model$^{10}$. In Table 2, I show the predictions for the strong gauge coupling, for different values of $\sin^2 \theta_W(M_Z)$ and the supersymmetric threshold scale.

Table 2. Dependence of $\alpha_3(M_Z)$ on $\sin^2 \theta_W(M_Z)$ and $T_{SUSY}$, in the framework of gauge and bottom - tau Yukawa coupling unification, for $m_b(M_b) = 4.3$ GeV.

| $\sin^2 \theta_W(M_Z)$ | $\alpha_3(M_Z)$ for $T_{SUSY} = 1$ TeV | $\alpha_3(M_Z)$ for $T_{SUSY} = 100$ GeV |
|------------------------|--------------------------------------|-----------------------------------|
| 0.2335                 | 0.111                                | 0.118                             |
| 0.2324                 | 0.115                                | 0.122                             |
| 0.2315                 | 0.118                                | 0.126                             |

The issue of unification of Yukawa couplings have been recently analyzed in some detail$^{13−17}$. It was shown that$^{13}$, for a given value of the running bottom quark mass and the weak mixing angle, the top quark Yukawa coupling at the grand unification scale depends strongly only on the value of $\alpha_3(M_Z)$. One of the most interesting results of this analysis is that, for a running bottom quark mass $m_b(M_b) = 4.3$ GeV, which approximately correspond to a physical mass $M_b = 4.9$ GeV, the value of the top quark Yukawa coupling at the grand unification scale must be much larger than the gauge couplings$^{13}$. Moreover, the larger the value of $\alpha_3(M_Z)$, the larger the value of $Y_t(M_{GUT})$ becomes. In Table 3, I present the predicted values of the top quark Yukawa coupling at the grand unification scale for $\sin^2 \theta_W = 0.2324$, $m_b(M_b) = 4.3$ GeV and different values of the effective supersymmetric threshold scale $T_{SUSY}$. 

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Table 3. $Y_t(M_{GUT})$ predictions, as a function of $\alpha_3(M_Z)$ and $T_{SUSY}$, in the framework of gauge and bottom-tau Yukawa coupling unification.

| $T_{SUSY}$ [GeV] | $\alpha_3(M_Z)$ | $Y_t(M_{GUT})$ |
|------------------|-----------------|----------------|
| $10^3$           | 0.115           | 0.3            |
| 15               | 0.127           | 0.7            |
| 7                | 0.130           | 1.0            |

Observe that, for the above value of the bottom quark mass the requirement of perturbative consistency of the top quark Yukawa sector, $Y_t(M_{GUT}) \leq 1$ is sufficient to constrain on the allowed value for the strong gauge coupling. Indeed, in this case, the obtained upper bound coincides with that one coming from experimental limits on the strong gauge coupling, $\alpha_3(M_Z) \leq 0.13$.

The large value of the top quark Yukawa coupling necessary to achieve unification of bottom and tau Yukawa couplings explains why the top quark mass values predicted from the unification condition quantitatively coincide with those ones obtained within the supersymmetric top condensate model. In fact, for values of $Y_t(M_{GUT}) \geq 0.2$, and a grand unification scale $M_{GUT} = \mathcal{O}(10^{16})$ GeV, the low energy values of the top quark Yukawa coupling are strongly focussed to its quasi infrared fixed point. I illustrate this behaviour in Fig. 1, in which I plot the top quark mass as a function of $\tan \beta$ for different values of the running bottom quark mass in the range $m_b(M_b) = 4.1 - 4.6$ GeV (which approximately correspond to the experimental allowed range for the physical bottom mass range $M_b = 4.7 - 5.2$ GeV), and characteristic values of $\sin^2 \theta_W(M_Z) = 0.2324$ and $\alpha_3(M_Z) = 0.122$.

Observe that, if in the minimal supersymmetric model the physical top quark mass is below $M_t \leq 160$ GeV, as may be inferred from precision measurement analysis, then the running top quark mass $m_t \leq 152$ GeV. As it may be seen from Fig. 1, such relatively low values of the running top quark mass can only be obtained, in the framework of
gauge and bottom quark - tau Yukawa coupling unification, for values of tan β close to one, or for very large values of tan β\textsuperscript{13}. Assuming moderate values of tan β, an upper limit on tan β ≤ 1.3 may be obtained. In addition, this bound implies strong constraints on the Higgs sector of the theory. In fact, if the characteristic squark mass is lower than or of the order of 1 TeV, then the lightest CP even mass will be \( m_b < 80 \text{ GeV} \) and it should be observed at the LEP2 experiment\textsuperscript{13,20}.

\textbf{Fig.1.} The predicted top quark mass as a function of tan β, assuming gauge and also bottom and tau Yukawa coupling unification, for \( m_b(M_b) = 4.6 \text{ GeV} \) (dot-dashed line), \( m_b(M_b) = 4.3 \text{ GeV} \) (solid line) and \( m_b(M_b) = 4.1 \text{ GeV} \) (dashed line).

As illustrated in Table 4, the variation of the running bottom quark mass in the range
\( m_b(M_b) = 4.1 - 4.7 \) GeV, implies a large variation of the top quark Yukawa coupling at the grand unification scale. In fact, for the particular value of \( \alpha_3(M_Z) \) and \( \sin^2 \theta_W(M_Z) \) considered in Fig. 1 and Table 4, and for \( m_b = 4.1 \) GeV, the top quark mass predictions exactly coincide with the ones of the SUSY top condensate models since the top quark Yukawa coupling \( Y_t(M_{GUT}) = h_t^2(M_{GUT})/4\pi \) acquires the maximum allowed value consistent with a perturbative analysis of the theory\(^{13}\).

### Table 4. \( Y_t(M_{GUT}) \) predictions as a function of the running bottom quark mass.

| \( \sin^2 \theta_W(M_Z) = 0.2324 \) | \( \alpha_3(M_Z) \) | \( Y_t(M_{GUT}) \) |
|--------------------------------|------------------|------------------|
| \( m_b(M_b) = 4.6 \) GeV  | 0.122            | 0.2              |
| \( m_b(M_b) = 4.3 \) GeV  | 0.122            | 0.5              |
| \( m_b(M_b) = 4.1 \) GeV  | 0.122            | 1.0              |

It is important to remark that, at the grand unification scale, the characteristic value of the top quark Yukawa coupling is five to ten times the gauge coupling values. This may only be avoided by choosing very large values of \( \tan \beta \), which, in the minimal \( SU(5) \) model, are disfavoured by proton decay constraints\(^{22}\). As a matter of fact, the existence of such large values of the top quark Yukawa coupling at the grand unification scale provides a challenge for model builders. Most interesting, it might provide an interrelation between the unification of couplings and the minimal dynamical breaking of the electroweak symmetry within the Minimal Supersymmetric Standard Model.

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