Quantum simulation—the use of one quantum system to simulate a less controllable one—may provide an understanding of the many quantum systems which cannot be modelled using classical computers. Considerable progress in control and manipulation has been achieved for various quantum systems, but one of the remaining challenges is the implementation of scalable devices. In this regard, individual ions trapped in separate tunable potential wells are promising. Here we implement the basic features of this approach and demonstrate deterministic tuning of the Coulomb interaction between two ions, independently controlling their local wells. The scheme is suitable for emulating a range of spin–spin interactions, but to characterize the performance of our set-up we select one that entangles the internal states of the two ions with a fidelity of 0.82(1) (the digit in parentheses shows the standard error of the mean). Extension of this building block to a two-dimensional network, which is possible using ion-trap microfabrication processes, may provide a new quantum simulator architecture with broad flexibility in designing and scaling the arrangement of ions and their mutual interactions. To perform useful quantum simulations, including those of condensed-matter phenomena such as the fractional quantum Hall effect, an array of tens of ions might be sufficient.

The use of effective spin–spin interactions between ions in separate potential wells is a key feature of proposals for simulation with two-dimensional systems of quantum spins with arbitrary conformations and versatile couplings. In addition, these effective spin–spin interactions may enable logic operations to be performed in a multi-zone quantum information processor without the need to bring the quantum bits (qubits) into the same trapping potential well. Such coupling might also prove useful for metrology and sensing. For example, the spin–spin interaction can be tuned from strong to weak by controlling the individual trapping wells. We excite ‘carrier’ transitions and demonstrate deterministic tuning of the Coulomb interaction with a fidelity of 0.82(1) and a stretch mode (ωcom, qcom = (−1/√2, 1/√2)) with motional quanta shared between the two ions.

These shared quantized degrees of freedom can simulate spin–spin interactions, just as for two-qubit quantum logic gates with ions in the same harmonic well; but, unlike in the latter case, the strength of the spin–spin interaction can be tuned from strong to weak by controlling the individual trapping wells. We define the energy eigenstates of the pseudo-spin-1/2 systems as |ψ⟩, corresponding to internal states of the ions, separated by hω0 (h, Planck’s constant divided by 2π), and the number states of the normal modes as |nstr⟩ and |ncom⟩. We excite ‘carrier’ transitions and the stretch mode |nstr, ncom⟩ = (nstr, ncom) with a uniform oscillating field at the transition frequency ω0 and with phase φ. Simultaneously, a single ‘red-sideband’ excitation at frequency ω0 − 2ω and phase φ, between the sideband frequencies for the stretch and centre-of-mass modes, excites both the |nstr, ncom⟩ = (nstr, ncom) transition and the |nstr, ncom⟩ = (nstr, ncom − 1) transition. These excitations emulate an effective spin–spin interaction (Methods)

\[
\hat{H}_{\text{eff}} = \hbar \kappa \sigma_{1}^{x} \sigma_{1}^{y} + \hbar \kappa \sigma_{2}^{x} \sigma_{2}^{y},
\]

where k is the coupling strength and \(\sigma_{i}^{x/y} \) are the Pauli spin-1/2 operators of the respective ions. We can emulate antiferromagnetic (k > 0) and ferromagnetic (k < 0) interactions by our choice of the ion spacing or the detunings \(\delta_{r} \) and \(\delta_{c} \) of the normal modes relative to the sideband drive (Methods). Under the simultaneous carrier and red-sideband drive, the spins become periodically entangled and disentangled with the motion. Starting with a product state |Ψ⟩, spins and motion are disentangled into a product state at \(T_{2} = 2\pi/\Omega_{ex} \) (j > 0 integer), but the spins acquire phases that depend on the ions’ motion in phase space during the off-resonance excitation. These phases simulate the spin–spin interaction. We benchmark our implementation of the spin–spin interaction by starting from the well-defined product state |Ψ⟩ = |jj⟩, effectively evolving it under an antiferromagnetic (k > 0) interaction for time \(T_{2} = \pi/4\kappa \) with \(\phi = 0\), and comparing the resulting state with the maximally entangled state |Ψ⟩ = exp \[−i\pi/4 \sigma_{1}^{x} \sigma_{2}^{x} \] |Ψ⟩ = 1/2 (|jj⟩ − |jj⟩) that would be produced under ideal conditions (Methods).

The (pseudo-)spin-1/2 system is formed by the \(|2s^{*}S_{1/2}, F = 1, m_{F} = -1⟩ \) and \(|2s^{*}S_{1/2}, F = 2, m_{F} = -2⟩ \) hyperfine ground
states of $^{9}$Be$^{+}$, where $F$ is the total angular momentum and $m_F$ is the component of $F$ along a quantization axis provided by a 1.46(2) mT static magnetic field (Fig. 1). The ions are confined in a cryogenic (trap temperature < 5 K), microfabricated, surface-electrode linear Paul ion trap composed of 10 μm-thick gold electrodes separated by 5 μm gaps, deposited onto a crystalline quartz substrate. An oscillating potential (∼100 V peak at 163 MHz), applied to the radiofrequency electrodes in Fig. 1, provides pseudopotential confinement of the ions in the radial (perpendicular to z) directions at motional frequencies of ∼17 and ∼27 MHz at a distance of approximately 40 μm from the trap surface. Along the trap z axis, a double well is formed by static potentials applied to control electrodes C1–C12. The axial (z) oscillation frequencies $\omega_0$ and $\omega_z$ around the respective minima are typically near 4 MHz. Single-ion heating$^{13}$ is in the range of 100 to 200 quanta per second. This heating is approximately four orders of magnitude larger than that due to our estimate of Johnson noise heating for this apparatus. For two ions spaced 30 μm apart, and in motional resonance ($\delta = 0$), the period required for the ions to exchange their motional energies is $\tau_{\text{ex}} = \pi/2\Omega_{\text{ex}} = 70 \mu s$, compared with an average period of 5–10 ms required to absorb a single motional quantum due to background heating. Fine adjustment of control-electrode potentials (at the 100 μV level) enables individual control of potential-well curvatures to tune the Coulomb interaction between the ions through resonance. Electrode C1 also supports microwave currents (typically of milliampere amplitude) that produce an oscillating magnetic field to drive carrier transitions at the same rate in both ions.

Superimposed $\sigma^-$-polarized laser beams, nearly resonant with the $2s^2S_{1/2} \rightarrow 2p^2P_{3/2}$ and the $2s^2S_{1/2} \rightarrow 2p^2P_{3/2}$ transitions ($\lambda \approx 313 \text{ nm}$) and propagating along the magnetic field direction, are used for optical pumping, Doppler laser cooling and state detection by resonance fluorescence. The positions of the ions, $\mathbf{r}$, are trapped 40 μm from the chip surface; red dots indicate the ion locations, with a 30 μm spacing. Electrode C1 also supports microwave currents at 1.28 GHz to drive carrier transitions on the two ions.

Figure 1 | Microfabricated surface-electrode trap. Microscope image of ion-trap electrodes, showing radio-frequency (RF) and static-potential control electrodes (C1–C12). Dark areas are the 5 μm gaps between electrodes. Ions are trapped 40 μm above the chip surface; red dots indicate the ion locations, with a 30 μm spacing. Electrode C1 also supports microwave currents at 1.28 GHz to drive carrier transitions on the two ions.

Figure 2 | Motional spectroscopy of two coupled ions. a. The red dots connected by black lines indicate separate scans of the red-sideband detuning $\delta_{\text{RSB}}$ from the average mode frequency $\bar{\omega}$ for different values of the difference $\delta$ between the individual well frequencies. The vertical scale is proportional to the sum of the probabilities for each ion to be in $|\uparrow\rangle$. At the centre of the avoided crossing, the normal mode frequency splitting $\Omega_\text{ex}/\pi$ is 12(1) kHz. Each data point represents an average of 200 experiments. Shaded planes are a theoretical prediction for the avoided crossing according to equations (1). b. Resonant ($\delta = 0$) single-quantum motional exchange between two ions, with an exchange time $\tau_{\text{ex}} = 80(2) \mu s$. The vertical scale is proportional to the probability of the laser-addressed ion being in $|\uparrow\rangle$. Each data point represents an average of 500 experiments, and error bars correspond to s.e.m. Dashed lines are included to guide the eye.

A key to implementing spin–spin interactions with ions in separate trapping zones is being able to tune the well frequencies precisely enough to control the eigenfrequencies and eigenmodes (equations (1)) near the avoided crossing. In Fig. 2a, we characterize this avoided crossing. For these experiments, the ions are separated by 27(2) μm. They are laser-cooled nearly to their motional ground states (mean motional mode occupation, $n_{\text{atm}} = 0.1$), optically pumped to the $|\uparrow\rangle$ state and then rotated into the $|\uparrow\rangle$ state with a microwave carrier $\pi$-pulse. Fine adjustments are made to control electrodes C2 and C12 to tune the harmonic confinement of the two trapping zones, stepping the system through the avoided crossing. At each step, after cooling and optical pumping, we implement the Raman red-sideband drive and scan its detuning $\delta_{\text{RSB}}$ with respect to $\bar{\omega}$. If the sideband excitation frequency is equal to $\omega_0 - \omega_{\text{ex}}$ or $\omega_0 - \omega_{\text{com}}$, then the spin of one or both ions can flip to $|\downarrow\rangle$ while absorbing quanta of motion, and a peak in the resonance fluorescence counts is observed. The spectral resolution is set by the duration of the square-pulse sideband excitation (120 μs). At the centre of the avoided crossing, the splitting of the mode frequencies is $2\Omega_{\text{ex}} = 2\pi \times 12(1) \text{ kHz}$. In Fig. 2b, we show data that demonstrate single-phonon exchange between the two ions. With the trapping zones tuned to resonance ($\delta = 0$), both modes are cooled to near the motional ground state and the ions...
are prepared in $|\uparrow \uparrow\rangle$. In this experiment, the two Raman beams are tightly focused onto only one of the ions and are used to add a single phonon to that ion (and flip its spin) with a π-pulse on the red sideband of its local frequency in a duration short compared with $\tau_{\text{ex}}$. In this limit, after the pulse, the resulting motional state is an equal superposition of both modes, and the phonon energy is therefore exchanged between the ions with a period $2\tau_{\text{ex}}$ (ref. 16). To monitor the exchange, the same Raman interaction is applied again after a variable delay $\tau$. This can flip the spin and remove the quantum of motion only if the motion resides solely in the addressed ion after a particular delay. The level of fluorescence is proportional to the probability of this spin flip. From this, we determine an exchange time of $\tau_{\text{ex}} = 80(2) \mu$s, consistent with an ion spacing of 30(2) μm for this experiment. The reduction in contrast for longer delays is caused mainly by fluctuations and drifts of the trapping potential. We estimate that $\delta/2\pi$ drifted by approximately 500 Hz (a significant fraction of $\Omega_{\text{ex}}/2\pi$) during the 2–3 minutes required for the 20,000 experiments that provided the data for Fig. 2b.

For benchmarking the spin–spin interaction, the laser beams for fluorescence detection, Doppler cooling and stimulated Raman transitions are made to spatially overlap both ions with equal intensity. The ion spacing (approximately 27 μm here) is adjusted to an integer number of half-wavelengths of the difference wavevector of the two Raman laser fields, by a technique described elsewhere$^{27}$, such that $\cos(2\delta) \approx 1$. The wells are tuned to resonance ($\delta = 0$) with adjustments to control electrodes C2 and C12. The ions are first Doppler-cooled, then Raman sideband-cooled to near the ground state on both normal modes, and finally optically pumped into the $|\downarrow \downarrow\rangle$ state. The spin–spin interaction is implemented by simultaneously applying a relatively strong resonant microwave carrier excitation (Rabi frequency, $\Omega_z = 2\pi \times 23.1(2)$ kHz) and an optical sideband excitation at $(\omega_0 - \nu)$ (Rabi frequency, $\eta \Omega_\nu = 2\pi \times 2.4(2)$ kHz). The exchange frequency satisfies $2\Omega_{\text{ex}} = 2\pi \times 13(1)$ kHz, so that $\kappa = 2\pi \times 446(13)$ Hz. In the middle of the coupling period, we shift the phases $\phi_a$ and $\phi_0$ of both driving fields by 180° relative to their phases during the first half of the coupling period. These phase reversals suppress the dependence of the final state on the carrier Rabi frequency and reduce sensitivity of the spin–spin interaction to drifts in the detuning and the coupling time (Methods). At the end of the coupling period, fluorescence detection and subsequent fitting of the photon-count histograms to those for the three possible outcomes (two ions bright, $|\downarrow \downarrow\rangle$; one ion bright, $|\uparrow \downarrow\rangle$ or $|\downarrow \uparrow\rangle$; or both ions dark, $|\downarrow \downarrow\rangle$) yield the respective probabilities $P_0$, $P_1$, and $P_2$.

Evolution of these probabilities as functions of the coupling duration is shown in Fig. 3a. Near 300 μs, $P_0$ and $P_2$ are approximately equal ($P_2 + P_0 = 0.91(2)$) and $P_1$ has reached a minimum. To show that the resulting state is entangled, in a subsequent experiment we stop the evolution at 300 μs, apply a carrier π/2–pulse of variable phase $\phi_0$, and determine the parity $\tilde{P}_1 = P_2 + P_0 - P_1$ as a function of $\phi_0$. These data are shown in Fig. 3b together with a fit to $A\cos(2\phi_0 + \phi_0) + B$. The fitted probabilities and the contrast $A = 0.73(2)$ imply a state fidelity$^{25} F = \langle \Psi_\rho | \rho_\chi | \Psi_\rho \rangle = (P_0 + P_2 + A)/2 = 0.82(1)$, where the density matrix $\rho_\chi$ describes the experimentally produced state (Methods). From simulations and independent measurements, we estimate the leading contributions to the observed infidelity as follows: drift and fluctuations of the trapping potentials (including 'anomalous' motional heating) contribute $\sim 0.08$; spontaneous emission due to off-resonance excitation by Raman laser beams contributes $\sim 0.02$; Raman laser beam intensity fluctuations contribute $\sim 0.03$; and state preparation and detection errors contribute $\sim 0.03$.

For scalable implementations of lattices of interacting spins, the quality and ease of tuning of the spin–spin interaction must be improved; however, there are no apparent fundamental barriers to this. Trap potential fluctuations in our experiments appear to be dominated by changes in surface charging and work functions rather than changes in externally applied control potentials. It should be possible to suppress these fluctuations by improving the surface quality of the electrodes$^{26}$, reducing the amount of nearby dielectric materials and minimizing the exposure of the electrodes to ultraviolet light through better beam shaping. Laser intensity and pointing noise can be reduced by passive or active stabilization of the beams with respect to the ions (or both), or potentially avoided entirely by using microwave gradient fields for the sideband interactions$^{22}$. The microfabrication techniques used to construct the trap are scalable to larger arrays of trapped ions, thus potentially enabling informative ‘analogue’ quantum simulations without requiring arbitrarily precise quantum control. Theoretical work to quantify the common belief that many observables of interest in analogue quantum simulations are sufficiently robust is ongoing$^{28}$ (Methods). Initial indications are that the proposed technical improvements may be sufficient. A three-by-three lattice is sufficient to simulate quantum Hall physics, and with six-by-six lattices fractional Hall effects and other intriguing solid-state phenomena become accessible$^{29,30}$. Even for these modest numbers of spins, modelling of quantum interactions with conventional computers is challenging; this difficulty may be overcome with quantum simulations.

Online Content Methods, along with any additional Extended Data display items and Source Data, are available in the online version of the paper; references unique to these sections appear only in the online paper.

Received 5 February; accepted 2 June 2014.

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Acknowledgements We thank K. McCormick, A. Keith and D. Alcock for comments on the manuscript. This research was funded by the Office of the Director of National Intelligence (ODNI), Intelligence Advanced Research Projects Activity (IARPA), ONR, and the NIST Quantum Information Program. All statements of fact, opinion or conclusions contained herein are those of the authors and should not be construed as representing the official views or policies of IARPA or the ODNI. This work, a submission of NIST, is not subject to US copyright.

Author Contributions A.C.W. and D.L. designed the experiment, developed components of the experimental apparatus, collected data, analysed results and wrote the manuscript. D.L. developed the theory. Y.C. fabricated the ion-trap chip. K.R.B. built components of the apparatus, most notably the cryostat, and participated in the early design phase of the experiment. E.K. assisted with data analysis. D.J.W. participated in the design and analysis of the experiment. All authors discussed the results and the text of the manuscript.

Author Information Reprints and permissions information is available at www.nature.com/reprints. The authors declare no competing financial interests. Readers are welcome to comment on the online version of the paper. Correspondence and requests for materials should be addressed to A.C.W. (andrew.wilson@nist.gov).
Interaction Hamiltonian. The ions are driven resonantly by a pair of laser fields with carrier frequencies \( \omega_{\text{carrier}} \) and Rabi frequencies \( \Omega_{\text{carrier}} \), and phase \( \phi_{\text{carrier}} \). In the interaction picture and rotating-wave approximation, the carrier interaction takes the form

\[
H_{\text{int}} = \hbar \Omega_{\text{carrier}} (\hat{a} \hat{b}^\dagger + \hat{b} \hat{a}^\dagger) e^{i\phi_{\text{carrier}}} + (\hat{a}^\dagger \hat{b} + \hat{b}^\dagger \hat{a}) e^{-i\phi_{\text{carrier}}},
\]

with \( \hat{a} \equiv \hat{a}_{\uparrow} - \hat{a}_{\downarrow} \). Simultaneously, the ions are driven close to the Raman red sidebands of both normal modes by two laser beams (quantities associated with which will be denoted using indices 1 and 2) with difference wavevectors \( \mathbf{k}_1 - \mathbf{k}_2 \), where \( \mathbf{k}_1 = \mathbf{k}_2 + \mathbf{k}_d \). The difference frequency between the carrier field and the driven sideband is denoted \( \Delta \), and the phase \( \phi \) is determined by the relative phase of the two laser fields. The Lamb–Dicke factor \( \eta \) is defined as the ratio of the reduced mass of the ions to their de Broglie wavelength, \( \eta = m_{\text{ion}} \lambda \). The Lamb–Dicke factor is related to the operating depth \( \Omega_{\text{carrier}}^2 / (2 \Delta) \) of the interaction by

\[
\eta = \frac{\Delta}{\Omega_{\text{carrier}}^2 / (2 \Delta)}. \]

The spin–spin interaction. In the limit of a strongly driven carrier, such that \( |\Delta_{\text{carrier}}| > \Omega_{\text{carrier}}^2 / (2 \Delta) \), it is helpful to first transform to an internal-state basis where the bare spin states are dressed by the carrier field. In this dressed frame, the basis states \( |\pm \rangle \) are eigenstates of

\[
\sigma^z_{\text{carrier}} = \cos(\phi_{\text{carrier}}) \sigma^z_{\text{ion}}(\pm) - \sin(\phi_{\text{carrier}}) \sigma^x_{\text{ion}}(\pm),
\]

with \( \sigma^x_{\text{ion}}(\pm) \) and \( \sigma^y_{\text{ion}}(\pm) \) the Pauli matrices for the ion states. In such cases, the motion is displaced around \( \Delta \rangle \) full circles in the respective phase spaces of the two modes by the interaction. Also, because \( \delta_{\text{carrier}} = \Delta \rangle \), the interaction duration can assume only certain values, determined by \( \Delta \rangle > \delta_{\text{carrier}} > \delta_{\text{min}} > 0 \), for the motion to return to its original state:

\[
T = \frac{\pi \Delta}{\sqrt{\delta^2 + \Omega_{\text{carrier}}^2}}.
\]

If the spin and motional states are in a product state initially, they will be in a product state at \( T \) and any integer multiple of \( T \). The spin-dependent phases acquired during \( T \) simplify to

\[
\Phi_{\text{spin}}(\pm) = \frac{\Omega_{\text{carrier}} \sqrt{\Delta^2 - \delta_{\text{carrier}}^2}}{2T} \left( 1 - e^{i\phi_{\text{carrier}}} \right).
\]

The spin-dependent term is largest if \( \phi = \pi/2 \) with \( i \) integer. This corresponds to the ions being spaced by an integer number of half-wavelengths \( \pi/k \). In the experiment, the separation of the ions is controlled by slight changes in the well curvatures to ensure half-integer wavelength spacing. Also, \( \sin(2\phi_{\text{carrier}}) \) is reduced for \( \phi_{\text{carrier}} > 0 \) and eventually vanishes as the modes decouple in the limit \( \Delta \rangle \gg \delta_{\text{carrier}} \). Therefore, the most efficient spin–spin interactions are implemented for \( \delta_{\text{carrier}} = 0 \). For our experimental conditions and \( \delta_{\text{carrier}} = 0 \), the mode splitting is much smaller than the average mode frequency \( \omega_0 \), so we can approximate \( \eta_{\text{carrier}} = \eta = k \sqrt{\hbar / 2m_0 \omega_0} \), the Lamb–Dicke parameters of the respective normal modes.

The near-resonant terms of the red-sideband Hamiltonian are

\[
H_{\text{red}} = \hbar \Omega_{\text{carrier}} (\hat{a} \hat{d}^\dagger + \hat{d} \hat{a}^\dagger) e^{i\phi_{\text{carrier}}} + (\hat{a}^\dagger \hat{d} + \hat{d}^\dagger \hat{a}) e^{-i\phi_{\text{carrier}}},
\]

with \( \hat{d} \equiv \hat{a}_{\uparrow} + \hat{a}_{\downarrow} \). The phase of the sideband excitations is set to \( \phi_{\text{carrier}} = 0 \). The phase of the red sideband is the phase of the red-sideband interaction at the mean position of the ions.
The histograms $h_q$ recorded at phase $\phi$ are sampled from the mixture $P_{\text{ex}}(\phi) + P_{\text{det}}(\phi + \epsilon)$, where the $q_2$ are the count distributions for zero, one or two bright ions. From this model and the Ramsey data, we can determine $w_0$ so that $\sum w_i (|h_q(\phi)|^2)$ yields $P_\phi(\phi)$. We use a linear least-squares fit, regularizing it to minimize the anticipated variance when inferring $P_\phi$ for the completely mixed state.

Given a probability estimator $\hat{w}$, a recent histogram $h_q(\phi)$, we estimate the experimental variance of the inferred probability $P$ according to $v = \left(\sum w_i(\tilde{h}(\phi) - |h_q(\phi)|^2)^2\right)/(N-1)$. This variance determines the error bars in Fig. 3. For the fidelities and related quantities, the variation in the probability estimators due to the finite statistics of the Ramsey experiments contributes an error comparable to this variance. To determine the overall statistical error in the fidelities, we used non-parametric bootstrap resampling on all contributing histograms with 100 bootstrap resamples to determine error bars for fidelities and contrasts.

The assumed model for the Ramsey experiments makes no assumptions about the shapes or relationships of the count distributions $q_2$. This was important because we found that the $q_2$ exhibit clear deviations from Poissonian distributions. We also coprime $q_2$, the mean number of counts according to $q_2$, and found that $\hat{v} - v_0$ exceeded $v_0$ by about 8% for all the Ramsey scans considered.

Several effects result in deviations from an ideal Ramsey experiment. We found that there is a phase offset of approximately 5° in the Ramsey scans. We shifted the phase accordingly before determining the probability estimators. This had a statistically negligible effect on inferred probabilities and fidelities. After adjusting for the phase shift, we found no signature of a mismatch between the model and the data. In addition to checking that the dependence of the histograms on the phase was as expected, we considered whether there are more than three count distributions contributing to the Ramsey scans. We found no signature of such an effect. Furthermore, all other histograms, including those used to determine fidelities, could be explained as arising from a mixture of the same three count distributions.

An important effect that need not be apparent from the data is state-preparation error. By simulating Ramsey experiments with state-preparation error and $q_2$, as inferred from the data, we determined that such errors lead to systematic overestimates of fidelities that are well correlated with the state-preparation error. The simulations involved initial states that are mixtures of the basis states. Let $e(\epsilon < 1)$ be the probability that the state in this mixture is not $|\psi\rangle$. For the inferred fidelities, we estimate a systematic increase in fidelity of approximately 1%. The quoted systematic errors are based on a pessimistic upper bound of 0.01 on $\epsilon$. In inferring $P_\phi$ for a single histogram (as required for the plots in Fig. 3), these biases are small enough that the statistical error and were therefore not included in the error bars. We assumed that pulse errors had a statistically small effect on inferred probabilities and fidelities.

**Discussion on robustness of analogue simulations.** Richard Feynman stated that, “with a suitable class of quantum machines you could imitate any quantum system, including the physical world.” For arbitrarily precise quantum simulations, this requires scalable quantum computers that employ error correction, but realizing these computers has proven to be very difficult. An alternative that may circumvent the difficulties is to faithfully map the dynamics of the physical model of interest onto sufficiently controllable quantum systems. This is called “analogue quantum simulation”. Because the overall physical properties of interest are often determined by local observables, the expectation is that the full quantum state need not be arbitrarily precise for useful information to be obtained. For example, although the global many-body state of the simulator is sensitive to a local perturbation, the expectation values of intensive properties can be more robust. It is also noteworthy that many material properties are robust in the presence of naturally occurring imperfections. This suggests that a useful analogue quantum simulator might be significantly easier to construct than a quantum computer, even in the absence of sufficiently precise quantum gates or explicit quantum error-correction strategies needed for fault tolerance.

Although the robustness of analogue quantum simulations is frequently asserted, it is not a simple matter to quantify the effects of experimental imperfections on physical properties of interest. At present, there does not exist a perfect and rigorous way to assess the quality of the results that one can expect from an analogue quantum simulation. Nevertheless, one can seek models and conditions for which the effects of the quantum simulator’s imperfections are expected to be minor and well...
understood. A number of experimental groups, across multiple platforms, are currently pursuing this strategy. An alternative is to seek validation of the results on small systems that can be classically verified before obtaining results on large systems realizing the same model. In addition, validation may come from consistent results on multiple independent simulator platforms. This can eliminate simulator artefacts, as has been suggested in ref. 37.

For many developers of quantum simulators, a common Hamiltonian for testing their setups is the transverse Ising model. Recently, a theoretical investigation into the influence of disorder on the fidelity of quantum simulations of the Ising model was performed. With relatively large spin chains, analogue quantum simulator results are predicted to be usefully robust to random variation in the coupling coefficient up to a few per cent. This high tolerance to coupling imperfections, relative to a comparable universal quantum computation, is achieved because the simulation required that only local observables, rather than the entire simulator state, be robust. Although this work does not account for other technical issues that often limit the performance of experiments, it is nonetheless a useful performance indicator. In relation to our work, it suggests that although further progress on reducing experimental imperfections is probably required, the future technical improvements we propose may be sufficient. It may also be possible to ensure that the experimental imperfections correspond to physically relevant effects in the model under consideration. For example, Lloyd suggested that, “decoherence and thermal effects in the quantum computer can be exploited to mimic decoherence and thermal effects in the system to be simulated,” as was recently demonstrated. To ensure that the platform’s imperfections represent physically relevant interactions between the model and its normal environment, one can sometimes engineer the mapping from the ideal model to the experimental platform. Although we cannot make a general statement on the robustness of analogue quantum simulations, the above discussion is suggestive and many promising examples have been proposed.

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