Analysis of the approximate solutions of HII region expansion problem

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Abstract. The properties of interstellar gas produce significant effect on dynamic phenomena which occur at large heliocentric distances. HII region is a vivid example of the objects with extremely high radiation, where atoms of hydrogen are heated and ionized by ultraviolet radiation. This article deals with the HII region expansion. To study this process researches need effective methods that can provide information about quantitative and qualitative characteristics of the motion of the interstellar medium. In this article Cherny method is applied to a spherically symmetric case and Kompaneets method is used in the two-dimensional axisymmetric case. The solutions obtained were compared to numerical calculations. The result of comparing shows good accuracy of the analytical solution. This method can be applied to solve real problems for specific HII regions. It is also shown that, as in the two-dimensional problem of a strong explosion, the effect of an ionization-shock front exiting the atmosphere takes place.

1. Introduction

Phenomenon of HII region expansion is examined by many scientists who use various astrophysical methods and numerical simulation. The problem stems from the fact that observations conducted by researches provide an opportunity to estimate the age of the star. Density and velocity distribution inside HII regions can give information about existence or absence of stellar wind. Furthermore, the process of HII region expansion is considered to affect the process of star formation. Accumulation of dust and gas in the layer behind the shock wave during the process of HII region expansion is accounted for by one of the possible processes which trigger star formation. The clumps which are formed in this layer under influence of gravitation can potentially give birth to new stars. The process of star birth from the galactic gas is one of the fundamental problems of astrophysics.

Conditions in which regions HII are formed, for example, the numbers of flux of ionizing photons, temperature of the star vary significantly. This fact accounts for the difficulty of studying dynamics of HII regions. Therefore, approximate models of the expansion of the HII regions are of great interest [1–7].

In this paper the problem of HII region expansion is solved on the base of Cherny method [8] and Kompaneets method [9]. Instead of the law of energy conservation the assumption of fulfillment of ionization balance condition is used [10]. This method allows us to consider the motion of a gas in an inhomogeneous medium and in a non-stationary radiation.
2. Process of HII region expansion

According to existing models of star expansion, increasing the temperature of stars to 30–40 thousand degrees occurs in rather small period of time. For sufficiently heated stars we need not take their gravity and stellar winds into account.

In this study the motion of gas influenced by a hot star will be examined. An approximate temperature of the star is 10 thousand degrees Kelvin. The temperature is determined by the amount of energy absorbed and emitted by the unit of gas volume. Energy is absorbed during processes of photoionization of hydrogen atoms and is emitted during photo recombination processes.

Consider in detail the process of HII region expansion: when a massive star is suddenly “turned on” in a homogeneous environment, an ionization front will expand so rapidly into the surrounding medium that the ionized gas has no time to react dynamically. Then, the high pressure of the ionized gas will drive the expansion of the HII region into the neutral medium. The slowly expanding ionization front is then preceded by a shock front compressing the neutral medium.

2.1. System of equations

Consider the system of gasdynamic equations within the model [10].

Assuming that the bulk of the gas involved in the motion is concentrated in a thin layer of radius $r_s$ with mass $M_s$, the equation of motion can be represented as

$$\frac{d}{dt} \left( \frac{2M_s}{\gamma + 1} \right) = 4\pi r_s^2 \rho_i \left( 2r_s \right)^2$$

$$M_s = 4\pi \int_0^r \rho_0(r) r^2 dr, \quad \frac{dr}{dt}, \quad a_s^2 = \frac{2kT_e}{m_{H^+}}$$

where $\rho_i$ and $\rho_0(r)$ are densities of fully ionized gas and neutral gas respectively, $\gamma$ is adiabatic exponent of neutral gas.

According to the model plasma temperature $T_e = \text{const}$. This is due to the fact that in the HII regions the inflow and outflow of heat to gas due to radiation processes are so great that the hydrodynamic movement has little effect on the change in the temperature of the medium.

An additional condition that allows to determine the unknown functions $r_s$ and $\rho_i$ from equations (1), (2) is the approximate equality of the number of photoionization under the influence of the star radiation $Q(t)$ and the number of photorecombinations occurring in the ionized gas:

$$Q(t) = \rho_i^2 \frac{\alpha_{H^+} (T_e)}{m_{H^+}} \int_0^r \frac{r^2}{\pi} (2r)^2 dr,$$

where $\alpha_{H^+}$ is photorecombination coefficient, $m_{H^+}$ is mass of hydrogen atom. Since $Q(t)$ is assumed to be known, the system of equations (1)–(3) is complete.

3. Spherically symmetric case

To solve spherically symmetric problem Cherny method [8] with taking into account particular qualities of gas interaction with radiation will be used.

Cherny method is widely used, for example, in the problem of point explosion. According to Cherny method, it is assumed that most of the mass of gas compressed by the shock front is in a thin layer in comparison with the radius of the shock wave (figure 1). The thin layer of compressed gas is formed between IF and IS.
Unlike this method, where total energy is presumed to be constant, in this particular problem approximate equality between emitted and absorbed energy is used. Only a small fraction energy, which is absorbed of the gas, is supposed to be transformed into gas motion. In other words, the temperature of ionized gas does not depend on the state of motion and is considered to be constant.

The problem can be stated as follows: in the initial time $t = 0$ in static neutral medium with given distributions of density $\rho$ and temperature $T$ a point source flashes with effective temperature equal to $10^{-1}$–$10^{3}$ K. Under the influence of photons emitted by the source, the surrounding neutral gas will be heated and ionized. The system of equations for determination the trajectory of the shock wave is represented by equations (1)–(3). It should be pointed out that using these equations allows to solve the problem of HII region expansion numerically with arbitrary $Q(t)$ and $\rho_0(r)$.

In the special case, when $Q(t) = Q_0 = \text{const}$, $\rho_0 = \text{const}$ and $\gamma = 5/3$, the following result was obtained (with initial conditions $r(0) = r_\text{st}$, where $r_\text{st}$ is Stromgren radius)

$$\frac{r}{r_\text{st}} = \left(1 + \frac{7}{3} \frac{a_\alpha}{r_\text{st}} t\right)^{4/7}. \quad (4)$$

This result was compared to the results obtained by numerical calculations and some similarities were found.

The comparison illustrated on figure 2 shows good accuracy of the analytical solution obtained. According to the study [11], where the star marks the stage of this HII region evolution approximate age of the HII region RCW120.

4. Application of the Kompaneets method

Consider the axisymmetric case of HII region expansion, as it was discovered by researchers that its shape somewhat deviates from a spherically symmetric. In the following part of the paper the case when parameters of gas depends on (in cylindrical coordinate system) two coordinates $r$ and $z$ will be investigated (figure 3).
To solve this problem the modified Kompaneets method [9] was used. These two methods have the following differences: in the modified method instead of constant energy the approximate equality between emitted and absorbed energy was used, the definition of pressure also varies.

The problem can be reduced to solving the following differential equation:

$$
\left( \frac{\partial r}{\partial y} \right)^2 = \left( 1 + \frac{\partial r}{\partial z} \right)^2 \exp \left( \frac{z}{z_0} \right),
$$

where \( z_0 \) is equivalent thickness of the atmosphere and

$$
y = \int_0^{QT} \left( \frac{Q T}{\alpha_n \pi V} \right)^{1/4} \left( \frac{k}{2 \rho_0} \right)^{1/2} dt.
$$

Using the method of solving such equations that was suggested by Kompaneets [9], the following solution can be found:

$$
r = 2z_0 \arccos \left[ \frac{1}{2} \exp \left( \frac{z}{z_0} \right) \left( 1 - x^2 + \exp \left( -\frac{z}{z_0} \right) \right) \right]
$$

where \( x = y / 2z_0 \).

According to this solution in finite time \( \tau \) radius of the HII region reaches infinity. The time when the shock front leaves the atmosphere is

$$
\tau = z_0 \left( \frac{16 \rho_0 \pi}{k T (\gamma + 1)} \right)^{1/4} \left( \frac{\alpha_n z_0^3}{Q} \right)^{1/4} \int_0^{\Omega(x)} dx,
$$

where

$$
\Omega(x) = \int_{\arcsin(-1)}^{\arcsin(1)} \arccos^2 \left[ \frac{1}{2} e^{u^2} (1 - x^2 + e^u) \right] du.
$$

### 4.1. Analysis of the solution

To analyze the obtained solution some characteristic values were found. The maximum radius of the shock front can be found from the expressions for up and down points of the shock front. Index 1 marks the down point and index 2 marks the up point.

$$
\exp \left( -\frac{z_{1,2}}{z_0} \right) = x \pm 1,
$$

then

$$
e^{-z_{1,2}/z_0} = 1 - x^2, r_m = 2z_0 \arcsin x.
$$
Velocities of up and down points of the shock front were found. Their equations of motion are

\[ z_1 = -2z_0 \ln(1-x), \quad z_2 = -2z_0 \ln(1+x). \]  

By differentiation of these expressions and non-dimensionnalisation the following expressions for velocities were found

\[ \hat{V}_1 = \frac{dx}{dt} = 4 \sqrt{\frac{1}{\Omega(x)}} \int_0^1 \frac{1}{1-x} \frac{1}{x} \Omega(x) dx, \quad \hat{V}_2 = \frac{dx}{dt} = 4 \sqrt{\frac{1}{\Omega(x)}} \int_0^1 \frac{1}{1+x} \frac{1}{x} \Omega(x) dx. \]  

where

\[ z_{1,2} = z_0 \hat{z}_{1,2}, \quad t = \tau \hat{t}. \]

Parameters \( z_0 \) and \( \tau \) are characteristic size of the HII region and time when the up point of HII region reaches infinity consequently.

![Figure 4. Non-dimensional velocities of the up and down points of the shock front, \( \hat{V}_1, \hat{V}_2 \) are corresponding to HII region case, \( \hat{V}_1^k, \hat{V}_2^k \) – Kompaneets solution [9].](image)

According to graphics of velocity on figure 4, when time tends to infinity, velocity of the up point of HII region has minimum and then increases, because the density decreases very fast and velocity of the down point tends to zero, as the environment becomes denser.

4.2. Rayleigh–Taylor instability

To establish the characteristic time for Rayleigh–Taylor instability development, the expression for characteristic time in the case of ideal incompressible liquid was used. For this case characteristic time of perturbation growth with wave length \( \lambda_0 = 2\pi / k \) is equal to

\[ \delta = (Wk)^{-1/2}, \]  

where \( W \) is acceleration of the medium and \( k \) is a wave number.

For the HII region characteristic time of perturbation evolution is following

\[ \tau_{RT}^{HII} = \frac{1}{\delta} \left( \frac{\lambda_0}{2\pi} \right)^{1/2} \left( \frac{16\rho_{o}\pi}{kT(\gamma+1)} \right)^{1/2} \left( \frac{\alpha}{Q} \right)^{1/4} \frac{1}{\int_0^1 \sqrt{\Omega(x)} dx}. \]  

The ratio of characteristic time of the shock front up point to leave the atmosphere and characteristic time of perturbation evolution can be expressed as follows
\[ \tau \sim \frac{2z_0 \left( \frac{8z_0^3 \rho_0 \pi}{\lambda E (\gamma^2 - 1)} \right)^{1/2}}{\int_0^\infty \sqrt{\rho(x)} dx} \left( \frac{2\pi z_0}{\lambda_0} \right)^{1/2}. \quad (17) \]

So, if we concentrate on large scale disturbances with wave length \( \lambda_0 \) which is the same order as \( z_0 \), then the characteristic time of perturbation development is proportional to the time when HII region up point leaves the atmosphere.

5. Conclusion
This paper provides analysis of the approximate solutions obtained for the problem of HII region expansion. Characteristic time of the exit of the upper point of the shock wave to infinity was determined. For the solution found by Kompaneets and for the case of HII region, the nature of change in the velocity of the shock wave was determined, and also the possibility of developing the Rayleigh–Taylor instability was investigated. In the approximate solution of HII region expansion problem, inhomogeneity with a scale of the order of the radius of the HII region deserve special attention, because of the largest amount of mass accumulation, which increases the probability of undergoing gravitational compression.

References
[1] Schatzman E, Kahn F D 1995 *IAU Symposium series, Symposium* 2 163
[2] Savedoff M P, Greene J 1955 *Astrophysical J.* 122 477
[3] Mathews W G, O'dell C R 1969 *Ann. Rev. of Astronomy and Astrophys.* 7 67
[4] Icke V 1979 *Astrophys. J.* 234 615
[5] Korycansky D G 1992 *Astrophys. J.* 398 184
[6] Bisbas T G, Haworth T J, Williams R J R, Mackey J, Tremblin P, Raga A C, Geen S 2015 *Monthly Notices of the Royal Astronom. Soc.* 453 1324
[7] Kim J G, Kim W T, Ostriker E C 2016 *Astrophys. J.* 819 137
[8] Chernyi G G 1957 The problem of a point explosion *Dokl. AN SSSR* 112 213
[9] Kompaneets A S 1960 A point explosion in an in homogeneous atmosphere *Dokl. AN SSSR* 130 1001–3
[10] Krasnobaev K V, Kotova V Y, Tagirova R R 2008 Problems of Modern Mechanics: To the 85th Anniversary of Academician G G Chernyi p 190
[11] Zavagno A, Pomares M, Deharveng L, Hosokawa T, Russeil D, Caplan J 2007 *Astronomy & Astrophys.* 472 835