Baryon Density Correlations in the Quark Plasma

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As part of an ongoing effort to characterize the high temperature phase of QCD, we measure the quark baryon density in the vicinity of a fixed test quark and compare it with similar measurements at low temperature and at the crossover temperature. Such an observable has also been studied by the Vienna group. We find an extremely weak correlation at high temperature, suggesting that small color singlet clusters are unimportant in the thermal ensemble. We also find that at $T = 0.75T_c$ the induced quark number shows a surprisingly large component attributable to baryonic screening. A simulation of a simple flux tube model produces results that suggest a plausible scenario: As the crossover temperature is approached from below, baryonic states proliferate. Above the crossover temperature the mean size of color singlet clusters grows explosively, resulting in an effective electrostatic deconfinement.

1. MOTIVATION

Numerical simulations of the quark plasma have suggested seemingly contradictory models. While bulk thermodynamic quantities, such as the energy density and baryon susceptibility, yield values consistent with a nearly free gas of quarks and gluons, measurements of screening propagators, particularly, measurements of the wave functions of exchanged objects, reported in Lattice '91, are consistent with the confinement of color singlets. Indeed, simulations and analytic work in the pure glue sector have demonstrated that space-like Wilson loops obey an area law in the high temperature phase, a signature of confinement.

One resolution of this seeming paradox describes the quark plasma as an ensemble of color singlet clusters of various sizes. Bulk thermodynamic quantities, such as the energy density, would receive contributions from all clusters, whereas long-range screening would be controlled by the lightest clusters. How large is the typical color singlet cluster? What is the typical spatial extent and quark and antiquark content? To answer this question, it is necessary to seek observables that have not hitherto been studied in this context. Thus, we measured the distribution of induced quark charge (baryon number) in the vicinity of a fixed test quark, at low and high temperature, and at the crossover temperature. This observable has also been studied by the Vienna group. At low temperature we expect that, as a result of confinement, a dynamical antiquark or, less often, a pair of quarks, screens the test charge at short distance. Thus, the induced dynamical quark number density should be large and negative close to the test charge. If screening is entirely due to a single antiquark, we should observe that the total induced quark number $Q$ is $-1$. By contrast, if color singlet clusters are large both in size or in the number of quarks and antiquarks,
we would expect only a weak correlation and a small value of $Q$.

2. QUARK NUMBER DENSITY

The construction of the local quark number density starts with the introduction of a baryon chemical potential in the standard way, but with a spatial dependence. Such a definition assures that the total baryon charge so defined is exactly conserved on the lattice. Differentiating the thermodynamic potential with respect to the local chemical potential yields the local quark number density. In the staggered fermion formalism, the quark number density (including all flavors) in the presence of a fixed quark in the ensemble is given by the correlation:

$$\rho_q(r) = -(N_f/2) \frac{\langle \text{Im} P_{\text{stat}}(0) \text{Im} P_{\text{dyn}}(r) \rangle_U}{\langle \text{Re} P_{\text{stat}}(0) \rangle_U},$$

where the Polyakov loop is given by the color trace (including the staggered Dirac phase factors $\eta(r,t)$):

$$P_{\text{stat}}(r) = \text{Tr}_c \left[ \prod_{t=0}^{N_t} \eta(r,t) \prod t U(r,t) \right],$$

and the dynamical quark charge density is given in terms of the fermion propagator $M_{r,r'}^{-1}$:

$$P_{\text{dyn}}(r) = \eta(r,0) \text{Tr}_c \left[ M_{r,1}^{-1}(r,0) U(r,0) \right].$$

An alternative dynamical density operator averages over all time slices.

3. RESULTS OF SIMULATIONS

Simulations were carried out at fixed $\beta = 5.445$ and quark mass $m = 0.025$ for two flavors of staggered fermions on lattices of size $16^3 \times N_t$, where $N_t = 8, 6, 4$. This choice of lattice parameters corresponds to the crossover temperature at $N_t = 6$. Thus, the simulations are done at three temperatures $T = 0.75T_c$, $T = T_c$, and $T = 1.5T_c$, respectively, at the same lattice scale. Spectroscopic simulations at the same temperature allow us to set the scale, viz. $T_c = 145$ MeV and $a = 0.227$ fm. The quark number density was obtained using a random source technique, and the correlation convolution was constructed with the aid of a Fourier transform.

Figure 1 summarizes our preliminary results for the induced quark number density at these three temperatures. Particularly striking is the dramatic decrease at high temperature. The total induced quark number $Q$ is also indicated in the figure, normalized to one for a single quark. At the high temperature point the total induced charge is nearly two orders of magnitude smaller than the charge at low temperature. Thus, we see no evidence for small color singlet clusters in the high temperature plasma. At low temperature we expect that the test charge is attached to a color singlet cluster. A single antiquark would contribute $-1$ to the total induced charge, and a pair of quarks, $+2$. Evidently, already at a temperature of $0.75T_c$, there is a significant baryonic screening component.
4. FLUX TUBE MODEL

Some years ago Patel[11] proposed a flux tube version of the three-state three-dimensional Potts model to explain the mechanism of the deconfining phase transition in QCD. In this model, each site $r$ of a cubic lattice holds either a quark, anti-quark, or none at all, and each link $\ell_{r,\mu}$, a triplet or antitriplet flux, or none at all. Flux is conserved modulo 3. The hamiltonian is given in terms of the quark mass $m$ and the string link energy $\sigma$ by

$$H = \sum_{r,\mu} \sigma |\ell_{r,\mu}| + \sum_r m|n_r|.$$ 

Patel proposed using this model as a paradigm for the QCD phase transition. It has some intriguing features. At low temperature, only small color singlet clusters may occur in the Gibbs ensemble. As the temperature is increased, clusters of increasing size populate the ensemble. Eventually clusters connect to fill the entire spatial volume. For heavy quark masses this phenomenon corresponds to a first-order deconfinement phase transition. For light quarks, as seen in Fig. 2, simulations show that cluster growth is explosive, eventually filling the entire spatial volume.

Here a cluster (color singlet) is defined as a set of sites connected by flux tubes. Shown is the mean fraction of the total volume occupied by the cluster connected to the origin for a $10^3$ lattice with $m = \sigma = 1$. The vertical bar indicates the approximate crossover $T_c$. (Curiously, despite the pervasive growth of the mean cluster size, there is no evident accompanying “percolation” phase transition at this quark mass.) Thus, the addition of a single test quark at high temperature in such an ensemble produces an insignificant perturbation, thereby accounting for the extremely weak correlation seen at high temperature in the QCD simulation. We have an effective electrostatic deconfinement without a phase transition. Thus, our results for the induced charge in QCD may find an explanation within this picture.

The total induced charge $Q$ in the flux tube model can also be observed. Here, too the value is surprisingly less than $-1$ in magnitude at $T = 0.78 T_c$. From Fig. 3 we find that $Q = -0.67(9)$ at the same parameter set and lattice size as Fig. 2. A visual examination of the lattices as in Fig. 4 shows why. In this periodic lattice the

Figure 2. Cluster size vs inverse temperature for the flux tube model

Figure 3. Induced charge vs inverse temperature for the flux tube model.
targons, antiquarks. This typical lattice contains one baryon, one antibaryon, and nine mesons; by contrast a naive application of Boltzmann statistics to only the lowest lying meson and baryon in this model would have predicted fewer than one baryon per hundred mesons. The baryons in Fig. 4 are not the lowest lying states. We note that the density of baryonic states grows with mass more rapidly than that of mesonic states in this model. Thus, baryonic clusters proliferate as the temperature rises through $T_c$, permitting more frequent baryonic screening of a test charge.

To be sure the flux tube model omits many features of QCD. It lacks dynamics, describing only electrostatics. Completely omitted are the important magnetic interactions that give rise to confinement in spacelike propagation. It would be useful to find an elaboration of the model more closely relevant to QCD. Nonetheless, it is highly suggestive both for further exploration of QCD and for the phenomenology of the quark plasma.

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