Anti-Anthropic Solutions to the Cosmic Coincidence Problem

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A cosmological constant fits all current dark energy data, but requires two extreme fine tunings, both of which are currently explained by anthropic arguments. Here we discuss anti-anthropic solutions to one of these problems: the cosmic coincidence problem- that today the dark energy density is nearly equal to the matter density. We replace the ensemble of Universes used in the anthropic solution with an ensemble of tracking scalar fields that do not require fine-tuning. This not only does away with the coincidence problem, but also allows for a Universe that has a very different future than the one currently predicted by a cosmological constant. These models also allow for transient periods of significant scalar field energy (SSFE) over the history of the Universe that can give very different observational signatures as compared with a cosmological constant, and so can be confirmed or disproved in current and upcoming experiments.

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\textbf{INTRODUCTION}

The nature of the dark energy (DE) currently causing the accelerated expansion of the Universe is unknown. A cosmological constant, $\Lambda$, can explain all current data, but requires two extreme fine tunings: the value of $\Lambda$ must be many orders of magnitude lower than typical particle physics scales but not zero, and it must be set to just the right value so that the cosmic acceleration started in the recent past as is observed \textsuperscript{1,2}. The first fine-tuning problem above is called the cosmological constant problem and the second fine-tuning problem is called the cosmic coincidence problem.

The essence of the cosmic coincidence problem is that while radiation and matter densities drop very rapidly and at different rates as the Universe expands, a dark energy density described by a cosmological constant stays constant throughout the entire history of the Universe. Thus there is only one unique time in the long history of the Universe where the DE density and matter density are roughly equal. The cosmic coincidence is that this occurred very recently at around a redshift of $z \approx 0.39$. If this current epoch of cosmic acceleration had started even slightly earlier, the DE dominance would have stopped structure formation, and galaxies, stars, and life on this planet would not exist. If this epoch had been even slightly later, we would not have discovered the current accelerated expansion.

The most popular explanation for this cosmic coincidence is an anthropic argument. Here one imagines a large ensemble of Universes, each with its own value of $\Lambda$. Those Universes with values of $\Lambda$ bigger than the current measured value do not form galaxies, stars, life, etc. and so have no observers in them. If the probability for a Universe with a given value of $\Lambda$ is a strong function of the value of $\Lambda$, with smaller values being disfavored, then the mostly likely Universe that has observers in it will be the Universe with the largest value of $\Lambda$ that can form galaxies, stars, and life. That is our Universe, so we have an explanation for the cosmic coincidence.

The general failure of non-anthropic solutions have led to this being the most favored explanation of the cosmic coincidence problem. However, this solution depends upon many unproved and perhaps unprovable hypotheses, most importantly the existence of a huge ensemble of Universes, of which ours is just one.

A potential problem with anthropic solutions such as this is that, if true, it means we can never derive values of $\Lambda$ from a more fundamental theory and perhaps many other phenomena will never be understood from first principles. This does not mean that the anthropic principle cannot be correct, but it would have been a shame if Niels Bohr had noticed that a small change in the values of the atomic levels in atoms meant life could not exist and had concluded that these values were therefore determined anthropically. He might then have never discovered quantum mechanics.

Thus we are led to consider the anti-anthropic principle.

\textbf{The Anti-Anthropic Principle}

Suppose we demand that there is a non-anthropic solution to the cosmic coincidence problem. Is there a way to solve this fine-tuning problem without invoking an ensemble of Universes? As discussed by Griest\textsuperscript{3}, it is possible to replace the ensemble of Universes with an ensemble of scalar fields which cause episodic periods of accelerated expansion, or as discussed by Dodelson, Kaplinghat, & Stewart\textsuperscript{4}, with a scalar field with a complicated potential that has a similar effect.
Thus if there were many periods of cosmic acceleration, and they were spread out over cosmic time, it would not be a coincidence that we are currently experiencing such a period of acceleration. In fact, if an ensemble of scalar fields exist, each of which has a tendency to result in a period of significant scalar field energy (SSFE), and these periods are spread across cosmic time, then even if these periods of SSFE don’t always give rise to accelerated expansion, one can call this a non-anthropic solution to the cosmic coincidence problem.

One nice feature of such an anti-anthropic solution is that it makes several predictions, some of which may be testable:

(i) The dark energy is not a cosmological constant. The current period of accelerated expansion is temporary and might finish; the DE equation of state parameter, $w_\phi$, is not equal to -1, and it is changing with time.

(ii) There were other periods of SSFE in the past and there could be more in the future. These may or may not have caused periods of accelerated expansion, but, as discussed below, these periods can still cause measurable changes in the expansion history of the Universe. It then becomes an experimental question to limit or detect these periods.

(iii) The sum of all the changing scalar field energies may eventually approach zero; that is, the minimum of the total potential of all these fields may be zero, implying that the cosmological constant is actually zero. Thus the solution to the cosmological constant problem (as separate from the cosmic coincidence problem) may be reduced to the older and easier problem of finding some symmetry or reason that sets it exactly to zero.

These issues were discussed earlier by Dodelson, Kaplinghat, & Steward[4], and by Griest[5], but the examples given by these authors required fine-tuning. In particular, the toy models of Griest suffered a severe flaw. In order to have periods of significant scalar field energy (SSFE), the values of the parameters in the scalar fields had to be finely tuned, and also the initial values of the scalar fields themselves had to be extremely finely tuned. Thus one solved a fine-tuning problem by an ensemble of fine-tuned scalar fields. Thus the models proposed by Griest were not really a solution to the cosmic coincidence problem.

In this paper we attempt to address and correct this flaw. We replace the ensemble of monomial scalar field potentials used by Griest with an ensemble of brane-world inspired tracking scalar fields. These have the advantage of having attractor-like solutions that exist independent of the initial values of the scalar fields[6, 7]. In addition, the form of these potentials require that the values of all parameters are of order unity in Planck units and don’t need to be finely-tuned to high precision. The values of these parameters do have to be set to give the current value of the dark energy density, and to avoid conflicts with current cosmological measurements, but this can be viewed as a measurement of the parameter values rather than a fine-tuning. We show some examples of such scalar field ensembles that give SSFE over periods of interest in the Early Universe, but still agree with current experimental measurements. These models make significantly different predictions about the past and future of the Universe than the simple cosmological constant model.

Note that several recent experimental results make an ensemble of scalar fields more aesthetically acceptable. The discovery of cosmic microwave background (CMB) anisotropies points strongly towards an epoch of cosmic inflation in the early universe, most likely caused by an inflaton scalar field. We note that the brane-world inspired tracking potentials similar to the ones we explore here might also make a acceptable inflaton field[8, 9]. recent discussion of hilltop vs monomial potentials notwithstanding[10, 11]. Also, the Higgs Boson mass of around 126 GeV[12, 13] points to a somewhat finely-tuned scalar field sector in the Standard Model.

Thus, perhaps we should abandon our Occam’s razor proclivities and accept that scalar fields seem to be part of modern physics and thus may also be part of the solution to the DE problem.

Experimental Constraints on Multiple Epochs of SSFE

While the idea of many periods of accelerated expansion is appealing, there are severe experimental constraints on such periods. A change in the expansion history causes a change in the relationship between distance and redshift and also changes the growth rate of structure. It also can change the relative ratio of dark energy to radiation and/or matter at different epochs.

The earliest constraint comes from Big Bang Nucleosynthesis (BBN). By requiring that the deuterium to hydrogen ratio be within measured bounds during BBN, Yahiro, et al.[2] find that $\rho_{DE}/\rho_{rad} < 0.02$ between $10^8 < z < 10^3$, where $\rho_{DE}$ is the energy density of DE and $\rho_{rad}$ is the energy density of radiation. Thus there cannot be a period of SSFE during this epoch, but there can periods of SSFE before and after.

There are also strong constraints coming from the CMB. Early constraints[8, 9] have recently been updated by Linder and Smith[10] who find significant changes in the CMB anisotropy power spectrum caused by even very short periods of accelerated expansion. If a period of accelerated expansion happens during, or soon after, recombination then peaks in the CMB power spectrum are shifted to lower values of multipole moment, $l$, because the angular diameter distance to the last scattering surface decreases. In addition, extra decay of the gravitational potential gives an additional Integrated Sachs-Wolfe (ISW) bump[11]. Comparison with the measured
power spectrum rules out any period of accelerated expansion after recombination \((z \approx 1100)\).

Even a period of accelerated expansion earlier than recombination can have important effects on the CMB power spectrum, since the sound horizon at decoupling is decreased leading to a shift in the power spectrum peaks to higher \(l\). In summary, Linder & Smith find that no period of accelerated expansion can occur after \(z \approx 10^5\). At higher redshifts, they find no constraints from the CMB, so we have only the BBN constraint above.

Accelerated expansion occurs whenever \(w_\text{tot} < -1/3\), where \(w_\text{tot}\) is defined in Equation 5. The limits above do not apply directly when there is SSFE which does not cause accelerated expansion. However, it is beyond the scope of this paper to calculate how much DE can exist at various epochs with \(z < 10^5\) without causing measurable changes to the CMB power spectrum. Due to the extreme precision of recent CMB measurements, we suspect that even fairly small amounts of scalar field DE will cause measurable changes and be excluded. Therefore, we only consider cases where the DE has a very small fraction of the total energy density for \(z < 10^5\).

We also must require compatibility with recent measurements of the dark energy equation of state. There are many results from recent experiments, but most apply only to a \(w\) whose value is constant in time. However, the supernova legacy survey (SNLS3) recently reported results[18] from combining supernova measurements with WMAP7, plus the SLOAN survey data release 7, plus Hubble constant measurements, and found \(w = -0.909 \pm 0.196\) and \(w_a = -0.984 \pm 1.09\), where \(w(a) = w + w_a (1 - a)\) is allowed to vary linearly with the scale factor. We can then require to around 1-sigma that our value of \(w\) be less than \(-0.7\), and our value of \(w_a\) be between \(-2.08 < w_a < 0.11\).

**EXAMPLE MODELS**

Here we consider tracking models that don’t require much fine-tuning. There have been many such models suggested and we will not review these here. We will only consider two models from the class of brane-world inspired models discussed by Dvali & Tye[10]. As a first example we consider a potential first discussed in detail by Albrecht and Skordis [20, 21].

\[
V(\phi)_{AS1} = V_0 [ (\phi - B)^2 + A ] e^{-\lambda \phi}, \quad (1)
\]

where \(A, B\) and \(\lambda\) are all of order unity in Planck units. We will refer to this model as AS1. In our anti-anthropic examples, because we want several epochs of SSFE, we introduce two more fields of identical form but with different values of the parameters:

\[
V(\phi_1, \phi_2, \phi_3) = V(\phi_1)_{AS1} + V(\phi_2)_{AS1} + V(\phi_3)_{AS1}. \quad (2)
\]

Our second example is another potential studied by Skordis and Albrecht [21],

\[
V(\phi)_{AS2} = \frac{C}{(\phi - B)^2 + A} e^{-\lambda \phi}, \quad (3)
\]

where, as before, the parameters \(A, B, C, D\) and \(\lambda\) are all of order unity in Planck units. We will refer to this model as AS2. By once again including two additional scalar fields with potentials of identical form but different parameter values we introduce additional periods of increased scalar field energy density,

\[
V(\phi_1, \phi_2, \phi_3) = V(\phi_1)_{AS2} + V(\phi_2)_{AS2} + V(\phi_3)_{AS2}. \quad (4)
\]

For both these example models, we solve the coupled Friedmann-Robertson-Walker (FRW) and scalar field equations in the standard way, making the obvious generalization for three independent scalar fields instead of the usual single field. See for example, equations 1 through 7 of Skordis & Albrecht[21]. We choose parameters so that we have a transient accelerated expansion today that gives the measured values of Dark Energy and Dark Matter energy density, as well as a value of \(w_\phi\) within current limits. We also demand that the calculated distances to the Baryon Acoustic Oscillation (BAO) peaks agree with the measured values for three different redshift ranges [11]. In each case, we also choose parameters to give two earlier periods of SSFE consistent with BBN and CMB constraints.

In our figures we show \(\Omega_i\) vs. \(\log_{10}(a)\), where \(a = 1/(1 + z)\) is the scale factor and \(\Omega_i\) are the densities of each component divided by the critical density. We take \(\Omega_\text{tot} = 1\), consistent with observations that our Universe is flat [12], and \(\Omega_\phi\) to be the sum of the scalar field densities divided by the critical density. We also show \(w_\text{tot}\) and \(w_\phi\) vs. \(\log_{10}(a)\), where

\[
w_\text{tot} = \frac{p_r + p_m + p_\phi}{p_r + p_m + p_\phi}, \quad (5)
\]

\(p_i\) and \(\rho_i\) are the pressure and density of radiation, matter, and the scalar fields, and \(w_\phi = \frac{p_\phi}{\rho_\phi}\). The pressure of each scalar field is given by \(p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi)\), while the density is given by \(\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi)\).

Figure 11 shows our first example. Starting at the Planck epoch, \(\log_{10}(a) = -30\), corresponding to \(t = 10^{-43}\) s, we follow the evolution of the radiation, matter, and scalar field energy densities by numerically solving the differential equations that describe each. We start the three scalar fields at \(\phi_i = 55.2\ M_P\), and \(\phi_\phi = 0\), where the dot represents a derivative with respect to time. We see in Figure 11 a few wiggles in the energy density of radiation and the vacuum soon after the Planck epoch as the tracking behavior of the scalar field ensemble sets in. During this tracking regime \(\Omega_\phi\) remains relatively
constant for a long period of time. Then, at around $\log_{10}(a) \approx -16$, the first scalar field starts to dominate and $\Omega_\phi$ rises to 0.63. This is only a transient epoch of SSFE and $\Omega_\phi$ drops low again as the $\phi_1$ domination fades away. There is another very small period of SSFE after BBN as $\phi_2$ begins to dominate and raises $\Omega_\phi$ to 0.28 before fading away. The third epoch of SSFE occurs as $\phi_3$ starts to dominate recently and give rise to our current period of accelerated expansion.

In this particular model $\Omega_\phi = 0.68$ today, matching the results of Planck [22], the Universe continues to accelerate, and $\Omega_\phi$ reaches a maximum value of 0.79 in the future when $a = 1.33$. At this point $\Omega_\phi$ begins decreasing and at $a = 2.02$ we have a second epoch of equality between $\Omega_\phi$ and $\Omega_m$. In this particular model the current epoch of accelerated expansion of the Universe will end and give rise to a secondary epoch of matter domination. This is very different from the predictions of a cosmological constant, which predicts that our current epoch of accelerated expansion will last forever.

We can see this all from another perspective in Figure 2 where we plot $w_\phi$ vs. $\log_{10}(a)$. We again see wiggles soon after the Planck epoch due to the onset of tracking behavior as $w_\phi$ finds its tracking solution and then both $w_\text{tot}$ and $w_\phi$ settle in at $1/3$ during the radiation dominated expansion. At the same time as mentioned above ($\log_{10}(a) \approx -16$) $w_\phi$ changes to nearly -1 as an epoch of SSFE unfolds. Note that $w_\text{tot}$ does not drop below $-1/3$ and thus there is no early period of accelerated expansion. During the second epoch of SSFE, both $w_\phi$ and $w_\text{tot}$ begin to drop, but, again, the Universe does not begin to accelerate. During the third epoch of SSFE $w_\phi$ drops to nearly -1 and $w_\text{tot}$ drops below $-1/3$ giving rise to the current period of accelerated expansion.

The strength, duration, and beginning of each SSFE is set by our choice of the parameters in the AS1 potential. For example, the $A$ parameter determines the height of each epoch of SSFE, the $B$ parameter determines when each epoch occurs, and the $\lambda$ parameter effects the height and location of each peak. As the value of lambda becomes larger, the peaks of each SSFE become smaller and get pushed further out into the future.

We next check whether the current accelerated ex-
expansion period predicted by our example looks enough like a cosmological constant to satisfy current observations. For this model we find today that \( w = -0.94 \) and \( w_a = -0.2 \), within the 1-sigma constraints from SNLS3 mentioned above.

As our second example we calculate predictions for a particular choice of parameters for the AS2 model. Figure 3 shows the evolution of the \( \Omega_i \) and Figure 4 shows the evolution of the \( w_i \). Even though the potential is quite different we see fairly similar results as for AS1. Thus one cannot say that one potential form is greatly favored over another, and we expect that there are other forms for the potential that would work equally well, or better. In Figure 3 we again see wiggles in the fractional energy density of radiation and the vacuum as the tracking behavior of the vacuum sets in soon after the Planck epoch. This is followed by two periods of early SSFE and a final period of SSFE that continues today. In this example, the DE dominance does continue into the future, so this set of potentials will approach a cosmological constant model.

Figure 4 shows a similar evolution in the \( w_i \) as for AS1. We find today that this model has \( w_0 = -0.9833 \), and \( w_a = 0.15 \). The \( w_a \) parameter is just outside the 1-sigma Sullivan, et al. contour, but this is to be expected because during the current epoch \( w \) is oscillating as it drops towards \( w = -1 \) due to the oscillations \( \phi_3 \) is undergoing as it gets stuck in the potential well.

Finally, in Figures 5 and 6 we use the AS2 potential again and show results for an epoch of SSFE after \( \log_{10}(a) \approx -5 \) where Linder and Smith’s CMB constraints are in full force. The \( w_i \) vs. \( \log_{10}(a) \) plot shows that \( w_{tot} \) never gets even close to \(-1/3\), thus there is no period of accelerated expansion and Linder and Smith’s constraint is therefore not violated. However, the \( \Omega_i \) plot shows the density of scalar field energy is noticeable and thus may result in CMB, or large scale structure power spectra, that are in conflict with observations. Addressing this question is beyond the scope of this work, but this example shows that even without periods of accelerated expansion one can have a cosmology that differs from a cosmological constant in meaningful ways.
predicted distances match the distances measured by the WiggleZ team.

FIG. 5. Transient increases in the fractional energy densities of the AS2 model with $\lambda_1 = \lambda_2 = \lambda_3 = 14$, $B_1 = 16$, $B_2 = 19.25$, $B_3 = 20.18$, $A_1 = 0.004$, $A_2 = 0.01$, $A_3 = 0.001$, $D_1 = D_2 = D_3 = 0.1$, and $C_1 = C_2 = C_3 = 1$.

FIG. 6. $w_\phi$ and $w_{tot}$ for the AS2 model with the same choice of parameters as shown in Figure 5.

For any cosmological model to be seriously considered it must satisfy an ever increasing set of observational measurements. We will not attempt a careful check of all these recent results since we intend our models as examples, not as proposals for the actual cosmology of the Universe. However, we will now check our examples against the recent WiggleZ measurements of the distance to the Baryon Acoustic Oscillation (BAO) peaks [11]. We view these measurements as a sort of proxy for the many recent experimental results.

The BAO distances can be calculated according to the formula [11],

$$D_V(z) = \left[ \frac{z}{H(z)} \left( \int_0^z \frac{dz'}{H(z')} \right)^2 \right]^{1/2},$$

(6)

where $z$ is the redshift and $H(z)$ is the Hubble parameter.

In Figure 7 we show grey solid bands representing the 1-sigma observed distances to the BAO peaks in three different redshift intervals as measured by the WiggleZ team [11], as well as our calculations of these distances in the AS1 and AS2 examples. We see that our calculated

FIG. 7. The calculated fiducial distances of these example models match the BAO distances observed by the WiggleZ team [11]. The bottom grey band is the BAO peak at $z = 0.44$, the middle grey band is the BAO peak at $z = 0.60$, and the top grey band is the BAO peak at $z = 0.73$.

There may be other observations that more strongly constrain our example models, but if we can satisfy the SNLS3 and WiggleZ constraints it is likely we can satisfy these other constraints by small adjustments to our model parameters.

CONCLUSIONS

In this paper we solved the combined FRW/scalar field equations for two quintessence models with multiple scalar fields designed so that they gave several periods of significant scalar field energy (SSFE). Such an ensemble of scalar fields can replace the anthropic cosmological constant model which uses an ensemble of Universes as a solution to the cosmic coincidence fine-tuning problem. We find that there are a wide class of such models available including models which exhibit tracking behavior implying that no fine-tuning of initial conditions is needed. Such models can give different predictions from the simple anthropic cosmological constant model and therefore can be tested for experimentally in current and future experiments.

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[1] Carroll, Sean M. "The cosmological constant." arXiv preprint astro-ph/0004075 (2000).
[2] Padmanabhan, Thanu. "Cosmological constant-the weight of the vacuum." Physics Reports 380.5 (2003): 235-320.
[3] Griest, K., Phys. Rev. D, 66. 123501 (2002).
[4] Dodelson, S., Kaplinghat, M., & Stewart, E. Phys. Rev. Lett., 80, 1582 (1998).
[5] Steinhardt, Paul J., Limin Wang, and Ivaylo Zlatev. "Cosmological tracking solutions." Physical Review D 59.12 (1999): 123504.
[6] Zlatev, Ivaylo, Limin Wang, and Paul J. Steinhardt. "Quintessence, cosmic coincidence, and the cosmological constant." Physical Review Letters 82.5 (1999): 896.
[7] Yahiro, M., Mathews, G.J., Ichikik, T., Kajino, T., & Orito, M., Phys. Rev. D 65, 063502 (2002).
[8] Hu, W., Scott, D, Sugiyama, N., & White, M., Phys. Rev. D 52, 5498 (1995).
[9] Bean, R., Hansen, S.H., & Melchiorri, A, Phys. Rev. D 64, 103508 (1002).
[10] Linder, E.V., & Smith, T.L., JCAP, 04, 001 (2011)
[11] Blake, C., et al., Mon. Not. Roy. Astr. Soc., 425, 405 (2012).
[12] Hinshaw, G., et al., "Nine-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: cosmological parameter results." arXiv preprint arXiv:1212.5226 (2012).
[13] Bento, M. C., R. Gonzalez Felipe, and N. M. C. Santos. "Brane assisted quintessential inflation with transient acceleration." Physical Review D 77.12 (2008): 123512.
[14] Ijjas, A., Steinhardt, P.J., & Loeb, A., Phys. Lett. B, 723, 261 (2013).
[15] Kallosh, R. & Linde, A., [arXiv:1306.3211v2 (2013).
[16] Aad, G., et al, Phys Lett. B, 716,1 (2012).
[17] Chatrchyan, S., et a., Phys. Lett. B, 716, 30 (2013).
[18] Sullivan, M., et al., Astro. Phys. J., 737, 102 (2011).
[19] Dvali, G. & Tye, H.S.H., Phys. Lett. B, 450, 72 (1999).
[20] Albrecht, Andreas, and Constantinos Skordis. "Phenomenology of a realistic accelerating universe using only Planck-scale physics." arXiv preprint astro-ph/9908085 (1999).
[21] Skordis, Constantinos, and Andreas Albrecht. "Planck-scale quintessence and the physics of structure formation." Physical Review D 66.4 (2002): 043523.
[22] Collaboration, Planck, et al. "Planck 2013 results. I. Overview of products and scientific results." arXiv preprint arXiv:1303.5082 (2013).