Analytical and numerical investigation of high-harmonic and attosecond pulse-train generation: the sliding mirror model

Vineeta Jain¹, H K Malav², K P Maheshwari² and N K Jaiman¹

¹Department of Pure and Applied Physics, University of Kota, Kota
²DST-Project, Vardhaman Mahaveer Open University, Kota, India

Abstract. Pulse-shape dependence of high harmonic generation and attosecond pulse generation as a result of the interaction of a short ultra relativistic laser pulse with a thin layer of overdense plasma for normal and oblique incidence and different polarizations is studied analytically and numerically. The temporal profile effects in the relativistic regime on HHG and optimization of the parameters for most effective attosecond pulse train generation in the transmitted pulse are also investigated.

1. Introduction

Much attention has been paid to aspects of the interaction of intense laser light with solid-density targets in which the plasma is overdense in terms of the critical density[1-2]. In the relativistic range of amplitude of the laser radiation, when intensity is as high as $10^{18}$-$10^{20}$W/cm² the ratio $\frac{eE}{mc}\omega_0$ is greater than 1. At these ultrahigh intensities, electron quiver velocities are close to the velocity of light and the motion of even free electrons becomes highly nonlinear. High-order harmonic generation by reflection of an intense laser beam from an overdense plasma has been observed with very different types of lasers, from nano-second CO₂ laser to picoseconds Nd-glass and femtosecond Ti-Sa laser. High order harmonics attract a great attention due to a wide range of their applications for the diagnostics, the UV and coherent x-ray sources, lithography etc [3-5]. In this paper we investigate analytically and numerically the effect of the temporal profile of the laser pulse when it interacts with a ultra-thin, overdense plasma layer leading to the generation of high-order harmonics.

2. Reduction of the oblique incidence to the normal incidence

Making use of Lorentz transformation equations and following the basic equations and figure (1) of reference [6] one can reduce the oblique incidence of the laser pulse to that of normal incidence. The laser pulse is incident on the vacuum-plasma thin layer at an angle $\theta$ with respect to the normal to the thin layer. In the boosted reference frame the plasma moves in the opposite direction. The electric field of the incident pulse in the laboratory frame has the form:

$$\tilde{E}_0 = E_\phi(t) \left[ -\sin \theta \hat{e}_x + \cos \theta \hat{e}_y \right] + E_s(t) \hat{e}_z, \quad (1)$$

$$E_\phi(t) = E_\phi(\theta, P(t) \cos(\omega_0 t + \psi_0)), \quad (2)$$
\[ E_s(t_s) = E_0 \cos(\omega_0 t_s + \psi_s). \]  

(3)

The pulse-envelopes \( P(t_s) \) for a Gaussian, Lorentzian, and hyperbolic secant pulses are taken to be [7]

\[ P_g(t_s) = \exp\left(-\frac{(1.17t_s)^2}{\tau}\right), \quad P_l(t_s) = \left[1 + \left(\frac{1.29t_s}{\tau}\right)^2\right]^{-1}, \quad \text{and} \quad P_h(t_s) = \text{sech}\left(\frac{1.76t_s}{\tau}\right) \]

respectively, where \( \tau \) is full width at half maximum (FWHM) of pulse intensity. Here \( E_{\rho 0} \) and \( E_{\rho s} \) are the electric-field amplitudes of the \( p \)- and \( s \)-polarized components of the pulse \( E_p \) and \( E_s \), \( t_s = t - \tilde{n}\cdot\tilde{r}/c \), \( n = \cos \theta \tilde{e}_x + \sin \theta \tilde{e}_y \), is the unit vector along propagation direction, \( \tilde{r} = x\tilde{e}_x + y\tilde{e}_y + z\tilde{e}_z \), and \( \psi_p, \psi_s \) are the carrier-envelope phases. The electric field of the incident pulse in the boosted reference frame is

\[ E'_0 = \cos \theta E_p(t'_s, \cos \theta \tilde{e}_y) + E_s(t'_s, \cos \theta \tilde{e}_z) \]

(4)

here \( t'_s = t' - x/c \). In the boosted frame the central frequency becomes \( \omega'_0 = \omega_0 \cos \theta \) and the duration becomes \( \tau' = \tau / \cos \theta \). The number of cycle in a pulse (\( \sim \omega_0 \tau \)) and carrier-envelope phase are Lorentz invariants. In the boosted frame the plasma density becomes \( n' = n / \cos \theta \); the electrons and ions move with the speed \( \tilde{v}_0 = -c \sin \theta \tilde{e}_y \) along the plasma layer, which corresponds to the electron momentum \( \tilde{p}_0 = -mc \tan \theta \tilde{e}_y \). A prime denotes quantities in the boosted reference frame. The pulse in the boosted frame has a form of a plane wave propagating along the \( x \) direction i.e. normal to the plasma layer. The relativistic factor is equal to \( \sec \theta \).

3. Sliding-mirror model

We consider the limit of dense plasma, in which the displacement of the reflecting electron layer in the direction perpendicular to the target surface is negligibly small. In this limit the plasma density is so high that the charge separation electric field suppresses the electron-layer motion inside and outside the target. The electron layer can move only along the target surface. Under this assumption of the so called “sliding mirror” model we investigate numerically the electromagnetic field evolution inside the plasma. The crucial parameter of the laser-thin-plasma slab interaction is the normalized plasma density:

\[ \varepsilon_p = \frac{\int n(x) dx}{2 \omega_0 c n_{cr} \lambda_0}, \]

(5)

here \( \omega_p \) is the plasma frequency, \( n \) is the electron density, \( n_{cr} \) is the critical electron density. For the constant-density plasma of thickness \( l, \varepsilon_p = (\pi n l)/(n_{cr} \lambda_0) \approx a_0 \). The parameter \( \varepsilon_p \sim a_0 \) or larger is important from the point of view of attosecond pulse generation.

4. High harmonic generation & attosecond pulse train generation

The electric fields of the reflected and transmitted pulses are expressed in boosted frame of reference as [8]

\[ E_s(x,t') = \frac{\varepsilon_p}{\cos \theta} \left[ \frac{\tilde{p}_0^*}{\left| \tilde{p}_0^* \right|^2} - \frac{\tilde{p}_0^* + \tilde{A}'(0,t'+x)}{\sqrt{1+\left| \tilde{p}_0^* + \tilde{A}'(0,t'+x) \right|^2}} \right], \quad x < 0, \]

(6)
\begin{align}
E'_i(x,t') = E'_0(x,t') - \frac{e_p}{\cos \theta} \left[ \frac{\hat{p}'_0}{\sqrt{1 + |\hat{p}'_0|^2}} - \frac{\hat{p}'_0 + \tilde{A}'(0,t'-x)}{\sqrt{1 + |\hat{p}'_0 + \tilde{A}'(0,t'-x)|^2}} \right], \quad x > 0, \tag{7}
\end{align}

where \( \hat{p}'_0 \) is normalized by \( mc \) and the normalized vector potential inside the plasma layer \( \tilde{A}'(0,t'+x) \) satisfy the following ordinary differential equation

\begin{align}
\frac{d\tilde{A}'(0,t')}{dt'} = \frac{e_p}{\cos \theta} \left[ \frac{\hat{p}'_0}{\sqrt{1 + |\hat{p}'_0|^2}} - \frac{\hat{p}'_0 + \tilde{A}'(0,t')}{\sqrt{1 + |\hat{p}'_0 + \tilde{A}'(0,t')|^2}} \right] = -\hat{E}'_0(0,t') \tag{8}
\end{align}

5. Results and discussions:

We solve the system of coupled Maxwell-Lorentz equations of the incoming and scattered radiation numerically by using GEAR method. Numerical results for HHG and attosecond pulse train are obtained for Gaussian, Lorentzian and hyperbolic secant laser pulses with identical full width at half maxima of intensity.

![Spectrum for Gaussian pulse shape](a)

Figure 1. Reflected pulse spectra for p-polarized incident double cycle Gaussian pulse \((\lambda = 800 \text{ nm})\) for parameters \(a_0 = 10\), \(\epsilon_p = 5\), \(\omega_0 \tau = 4\pi\), \(\psi_p = 0\) and \(\theta = 30^\circ\)

The left-hand side of equation (8) represents the current density of the sliding mirror and is responsible for HHG. The right hand side is the driving force. We have chosen the driving force to be pulse shape dependent. We consider the case of p-polarized few-cycle intense laser pulse incident on a thin overdense plasma slab. The transmitted electric field resembles the trains of attosecond pulses. Besides the dependence of the pulse train on the angle of incidence, carrier
envelope phase, plasma density, number of cycles in a pulse, intensity of the incident pulse, it also depends on the pulse-shape of the incident laser-pulse. In order to qualify an attosecond pulse train with high peak intensity and shorter pulse duration we define a merit function \( S = \frac{I_{\text{max}}}{\tau_{\text{FWHM}}} \), where \( I_{\text{max}} \) is the maximum intensity and \( \tau_{\text{FWHM}} \) is full width at half maxima of intensity of the main peak of the pulse train. The parameters that govern the nonlinear interaction of the laser pulse with the thin plasma are \( a_0 \), density parameter \( \varepsilon_p \), carrier envelope phase (CEP), angle of incidence, and number of cycles in the pulse. Figure 1 shows the reflected pulse spectra showing harmonics of \( p \)-polarized incident double cycle Gaussian pulse \( (\lambda = 800 \text{ nm}) \). Figure 2 shows the variation of intensity of the attosecond pulse train generated by the \( p \)-polarized single cycle incident pulse for optimized parameters as a function of normalized time. Results of attosecond pulse train generation are found to depend on the temporal profile of the pulse.

\[
\begin{align*}
\text{Train of Attosecond pulses} \\
\text{Gaussian} & \quad \text{Lorentzian} & \quad \text{H. Secant}
\end{align*}
\]

**Figure 2.** Variation of intensity in the attosecond pulse train generated by the optimized parameters of the \( p \)-polarized single cycle incident pulse as a function of normalized time. The optimized parameters are \( a_0 = 10 \), \( \varphi_p = 0.0 \), and \( \theta = 40^\circ \), \( \varepsilon_p = 7.7 \) for Gaussian, \( \varepsilon_p = 7.6 \) for Lorentzian, and \( \varepsilon_p = 8.1 \) for hyperbolic secant pulse shape.

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