Threshold Resonant Structure of the $^{232}$Th Neutron-Induced Fission Cross Section

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Abstract. The structures observed in the sub-threshold neutron-induced fission of $^{232}$Th were investigated employing a recent developed model. Theoretical single-particle excitations of a phenomenological two-humped barrier are determined by solving a system of coupled differential equations for the motion along the optimal fission path. A rather good agreement with experimental data was obtained using a small number of independent parameters. It is predicted that the structure at 1.4 and 1.6 MeV is mainly dominated by spin 3/2 partial cross-section with small admixture of spin 1/2, while the structure at 1.7 MeV is given by a large partial cross section of spin 5/2.

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1. Introduction

The measured neutron-induced fission cross-section behavior of nuclei in the thorium region represented a challenge for nuclear physicists concerning the shape of the potential energy surface. The experimental data suggested the existence of a triple humped barrier. The neutron-induced cross-sections of $^{230,232}$Th exhibit multiple fine structures [1, 2, 3] superimposed on a gross structure of the threshold cross-section. If the fine structure is interpreted as a serie of rotational states constructed on a $\beta$-vibrational state produced in some well of the deformation energy, it is straightforward to postulate the existence of a triple-humped barrier. The spacing between the members of the band is so small that it is consistent only with a prolate deformation that reaches the vicinity of the second-barrier top. The analysis of Ref. [1] indicates that an intermediate state nucleus must exist at a deformation considerably larger that the normal value. A ternary minimum obtained theoretically in the potential energy surface of $^{210}$Po [4] made this hypothesis credible. Therefore, a shallow minimum was assumed at this deformation to create a new $\beta$-vibrational state. Angular distribution analysis [5, 6] confirmed the existence of the triple well. Up to now, the assumption of a triple-humped barrier seems to be the best interpretation for the fine structure of intermediate cross-section resonances [7].

On the other hand, our analysis explores a different way to consider the cross-section resonant structure phenomenon by quantifying the dynamical single-particle effects associated to vibrational resonances produced in the second well [8]. Our exploratory investigation showed that the $^{230}$Th neutron-induced fission threshold resonant structure can be explained [9] by rearrangements of single-particle orbitals on the way from the initial configuration of the compound nucleus up to scission. This resonant structure depends also on the dynamics of the process.

Sect. 2 will provide a general description of the formalism intended for the evaluation of single-particle excitations, while results concerning the intermediate structure of the fission cross-section will be extensively presented in Sect. 3. Comments are made in Sect. 4.

2. Single-particle excitations

In most usual theoretical treatments of nuclear fission, the whole nuclear system is characterized by some collective coordinates associated with some degrees of freedom that determine approximately the behavior of many other intrinsic variables. The basic ingredient in such an analysis is a shape parametrization that depends on several macroscopic degrees of freedom. The generalized coordinates associated to these degrees of freedom vary in time leading to a split of the nuclear system in two separated fragments. A microscopic potential must be constructed, to be consistent with this nuclear shape parametrization. The three important degrees of freedom encountered in fission, that is, elongation, necking and mass-asymmetry, must be taken into account.
By solving the Schrödinger equation for a reasonable mean field potential associated to the nuclear shape parametrization, the single-particle energies are determined. In the case of odd-nucleon systems, the potential barrier must be increased with an excitation associated to the unpaired nucleon. The amount of which the barrier is increased can be estimated within the specialization energy \[10\]. This quantity can be interpreted as the excess of the energy of the unpaired nucleon with a given spin over the energy of the same spin nucleon state of lowest energy.

In the present work, an axial-symmetric nuclear parametrization is obtained by smoothly joining two intersected spheres of different radii \(R_1\) and \(R_2\) with a neck surface generated by the rotation of a circle of radius \(R_3\) around the symmetry axis, as displayed in Fig. 1. The surface equation is given in cylindrical coordinates:

\[
\rho_s(z) = \begin{cases} 
\sqrt{R_1^2 - (z - z_1)^2}, & z \leq z_{c1} \\
\rho_3 - s \sqrt{R_3^2 - (z - z_3)^2}, & z_{c1} < z < z_{c2} \\
\sqrt{R_2^2 - (z - z_2)^2}, & z_{c2} \leq z,
\end{cases} \tag{1}
\]

where \(z_{c1}\) and \(z_{c2}\) define the region of the necking. The meaning of the geometrical symbols that depends on the shape parametrization can be understood inspecting Fig. 1. This parametrization allows to characterize a single nucleus or two separated nuclei. Throughout the paper, the subscripts 0, 1, and 2 indicate the parent, the heavy and light fragments, respectively. If \(S=1\), the shapes are necked in the median surface characterizing scission shapes and if \(S=-1\) the shapes are swollen characterizing the ground-state and saddle points. The macroscopic parameters used in the following are denoted \(R = z_2 - z_1\) (elongation), \(C = S/R_3\) (necking) and \(\eta = R_1/R_2\) (mass-asymmetry). For large distances between the two nascent fragments, the configuration given by two separated spheres is reached.

For instance, to minimize the action integral \[11\] it is very difficult to treat the three independent generalized coordinates in the same time. Some simplifying assumptions must be introduced. As mentioned also in Ref. \[12\], microscopic approaches to fission \[13, 14\] established that the second saddle point is asymmetrical with a value compatible with the observed mass ratio. In the same time, in the region of the second barrier, the mass-asymmetry component of the inertia tensor is very large \[15\]. So, the variations of the mass-asymmetry coordinate are hindered in this region. On another hand, for elongation smaller than that of the second well deformation, the mass-asymmetry component of the inertia is much lower. Therefore, the mass-asymmetry coordinate can be modified without enhancing too much the value of the action integral. Moreover, the deformation energy is less sensitive to variations of the mass-asymmetry coordinate in the region of compact shapes. As in Ref. \[15\], this observation allows us to reduce the number of parameters in order to rend our problem tractable. Therefore, the evolution of the mass asymmetry generalized coordinate will be a priori fixed in the following. It is assumed that the ratio \(R_1/R_2\) varies linearly from unity (first barrier top) to the value associated with the final mass partition (second barrier top). The mass asymmetry in the outer barrier region is deduced by considering that the volume occupied by the light
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Figure 1. Nuclear shape parametrization. $z_1$, $z_2$, and $z_3$ are the positions of the centers of circles of radii $R_1$, $R_2$ characterizing the two nascent fragments, and of $R_3$ determining the neck, respectively. If $s = 1$, the shape is necked, otherwise the shape is swollen in the median surface. The distance between the two centers $z_1$ and $z_2$ determines the elongation $R$.

The deformation energy of the nuclear system is the sum between the liquid drop energy and the shell effects, including pairing corrections. The macroscopic energy is obtained in the framework of the Yukawa-plus-exponential model extended for binary systems with different charge densities [16]. The Strutinsky prescriptions [17] were computed on the basis of the Superasymmetric Two Center Shell Model (STCSM) [18, 19]. For one of the most probable partition $^{233}$Th $\rightarrow ^{98}$Sr $+^{135}$Te, the deformation energy as function of $C$ and $R$ is plotted in Fig. 2.

The theoretical study of binary disintegration processes is limited by the difficulties encountered in the calculation of single-particle levels for very deformed one-center potentials. On one hand, central oscillator potentials are not able to describe in a correct manner the shapes for the passage of one nucleus to two separated nuclei without including a large number of multipole deformation parameters and, on the other hand, for very large prolate deformations the sum of single-particle energies reaches an infinite value, as evidenced within the deformed oscillator model. These difficulties are surpassed by considering that the mean field is generated by nucleons moving in a double center potential. This kind of models allows to describe scission configurations within a small number of degrees of freedom. A more realistic version of the two-center shell model was
realized recently \cite{19} and it is used to generate the single-particle energy evolutions from the ground-state up to the formation of two separated fragments. The nuclear shape parametrization being characterized by an axial symmetry, the good quantum numbers are the projection of the spin $\Omega$. As in the Nilsson model, the single-particle energies depend on two interaction constants $\kappa$ and $\eta$, related to the spin-orbit operator and to the squared orbital momentum correction, respectively. These constants are determined in order to reproduce the ground-state properties \cite{20}.

In order to determine the single-particle excitations, it is not sufficient to have a model for the intrinsic nuclear levels, but is necessary to perform a full calculation of the trajectory of the decaying system in the configuration space. The shape of the fission barrier can be obtained if the trajectory of the nuclear system in our three-dimensional configuration space is obtained, starting with the ground-state of the compound nucleus and reaching the exit from the second barrier. This trajectory can be obtained by minimizing numerically the action functional that gives the quantum penetrability:

$$
P = \exp \left\{ -\frac{2}{\hbar} \int_{R_i}^{R_f} \sqrt{2V(R, C, \eta)M \left( R, C, \eta, \frac{\partial C}{\partial R}, \frac{\partial \eta}{\partial R} \right)} \, dR \right\}
$$

(2)

in the semi-classical Wentzel-Kramers-Brillouin approximation \cite{17}. The two turning points $R_i$ and $R_f$ denote the elongations that characterize the ground-state and the exit point of the barrier, respectively. Here $V(R, C, \eta)$ is the deformation energy and $M(R, C, \eta, \frac{\partial C}{\partial R}, \frac{\partial \eta}{\partial R})$ is the effective mass along the trajectory. The inertia is computed in the frame of the Werner-Wheeler approximation \cite{21}, that means, the flow of the fluid is idealized as non-rotational, non-viscous and hydrodynamic. Using the minimal action principle, in general, the nuclear system does not follow a path characterized by minimal values of the deformation energy, so that the trajectory does not interpolate barrier saddle points values.

Having in mind the assumption imposed for the variation of the mass-asymmetry, the action integral must be minimized in a two-dimensional space spanned by $C$ and $R$. The first turning point $R_i$ is fixed but the second $R_f$ lies on the equipotential line that characterizes the exit from the outer barrier, that is $R_f$ is a function of $C$. A simple numerical method is used to find the paths characterized by different values of $R_f$, associated with local minimums. For that purpose, the function $C = f(R)$ is approximated with a spline function of $n$ variables $C_j (j = 1, n)$ in fixed mesh points $R_j$ located in the interval $[R_i, R_f]$ along the elongation axis. A numerical expression for the WKB functional \cite{2} that depends only on the parameters $C_j$ is obtained. This expression is minimized numerically. For every value of $R_f$ a local minimum is obtained. The best values are retained. The trajectory is displayed on Fig. 2. This dynamical trajectory starts from the ground-state, reaches the region of the second well and the slope changes suddenly to penetrate the outer barrier. Between the first and second well, the macroscopic coordinate $C$ is less than 0, that is the shapes are swollen in the median region. Penetrating the second well, the shapes become necked. The theoretical potential barrier obtained along the minimal action path is plotted in Fig. 3. The
Deformation energy in MeV for the partition $^{233}$Th$\rightarrow^{98}$Sr$+^{135}$Te. $C$ represents the curvature of the neck and $R$ the distance between the centers of the fragments. Positive values of $C$ characterize necked-in shapes. The mass-asymmetry is varied linearly with $R$ from a value $\eta(R \approx 5 \text{ fm})=0$ (close to the ground-state of the compound nucleus) to the final value $\eta = A_1/A_2$ (in the vicinity of the top of the second barrier). The step between two equipotential lines is 1 MeV. Several values of the deformation energy are marked on the plot. The dynamic trajectory is represented with a thick line that starts in the first well, penetrates the first barrier, attains the second well and finally tunnels the second barrier towards scission.

height of the outer barrier is very large, therefore some corrections are required in order to obtain realistic values of the fission cross-section. This is the main reason that leads to use a phenomenological barrier in calculating the cross section. The first well is located at approximately $R=5.5$ fm and identifies the fundamental state. In Fig. 4 the nuclear shapes of the extreme values of the barrier are displayed.

Using the STCSM the neutron diagram is computed along the minimal action trajectory, as displayed in Fig. 5. Up to $R \approx 5.5$ fm the nuclear system is considered reflection symmetric. From the ground-state (located at approximately $R=5.5$ MeV) up to scission, the system loses the reflection symmetry to reach the final partition $^{233}$Th$\rightarrow^{98}$Sr$+^{135}$Te. In these circumstances, the parity is no longer a good quantum number, the levels being characterized only by the spin projection $\Omega$ as good quantum numbers. The Nilsson coefficients of the orbital momentum operators ($\kappa=0.063$ and $\eta=0.8$) were determined to reproduce as better as possible the experimental sequence of the first excited levels in $^{233}$Th. The first single-particle excited states are retrieved: an $\frac{1}{2}^+$ state (fundamental level) emerging from $2g_{9/2}$ followed by a $\frac{5}{2}^+$ one.

To determine the cross section, several single-particle levels are selected that lie as
close as possible to the Fermi energy region. These levels give the major contribution in the strength of the fission channel due to their low excitation energy and the large amount of macroscopic kinetic energy available for fission. Concerning the $\Omega=1/2$ workspace, 8 selected levels, $E_1$ up $E_8$ are extracted separately in the left panel of Fig. 6 as an example. The last occupied level is denoted $E_F$. The diabatic levels of the subspace $\Omega = 3/2$ are displayed in the right panel of the same figure. In the following, for simplicity, the discussion will be restricted only for the subspace $\Omega = \frac{1}{2}$.
Figure 5. Neutron level scheme as function of the elongation. At elongation zero, the shape parametrization gives a spherical nucleus and the spectroscopic notations are available. For low values of the deformations, the system behaves as a Nilsson level scheme. Asymptotically ($R \to \infty$) the two diagram of the two formed fragments are superimposed. In the adiabatic representation, the last occupied level is displayed with a thick dashed line. The 8 selected levels with $\Omega=1/2$ are represented with full thick line, the 5 levels with $\Omega=3/2$ are plotted with dot-dash thick lines, the 4 times $\Omega=5/2$ and 3 times $7/2$ levels are marked with dotted lines (smaller distance between points for $\Omega=5/2$). The ground-state of the compound nucleus is indicated with an arrow.

For $\Omega = \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$, the same procedure as in the case of $\Omega = \frac{1}{2}$ will be used.

A first behavior can be noticed. The nucleon located on the adiabatic level emerging from $E_1$ reaches a very unfavorable energy configuration after the scission. In the fundamental state, this unpaired nucleon is located on the fundamental level but arrives, in the adiabatic representation, at several MeV under the last occupied level (the $\epsilon_3$ diabatic level). So, if the nucleon is initially on the ground-state, it must follow a diabatic energy path to arrive in a most favorable energy configuration, that is close to the last occupied level (in one of the diabatic states $\epsilon_6$, $\epsilon_5$, $\epsilon_7$ or $\epsilon_2$). So, adiabatically, the fission strength for states with spin $1/2$ is not favored. This effect is a direct consequence of the rearrangement of low spin orbitals during the disintegration. The number of levels with $\Omega = \frac{1}{2}$ in the two nascent fragments that are under the energy of the last occupied level is always larger that the same number in the compound nucleus. So, $\Omega=1/2$ orbitals with larger energies of the parent must decrease in energy to fill
Figure 6. (a) The 8 selected levels with $\Omega=1/2$. The last occupied level in the adiabatic representation is denoted $E_F$ and is represented with a dashed line. The avoided level crossing regions are numbered and the diabatic levels $\epsilon_i$ identified. In the ground-state configuration, the $\epsilon_1$ level (emerging from $E_1$) is superimposed to $E_F$. At $R \approx 20$ fm, $E_F$ is located between $\epsilon_6$ and $\epsilon_5$, while the adiabatic level emerging from $E_1$ dropped to $\epsilon_3$. (b) As in plot (a) for the 5 levels with $\Omega=3/2$. With thin dot-dashed lines the 4 $\Omega=5/2$ adiabatic levels are also displayed.

The levels located under the Fermi energy of the two fragments. This aspect somewhat hinders the possibility to fission through $\Omega = \frac{1}{2}$ channels. The next step is to study the energy paths followed by the unpaired nucleon in the diagram.

The realistic two-center diagram presented before provides an instrument to study the role of individual orbitals during the disintegration process in a similar way as the study of nucleus-nucleus collisions [22, 23] or the alpha- and cluster-decays [24, 25]. Levels with same quantum numbers associated to some symmetries of the system cannot cross during the disintegration process and exhibit avoided level crossing. In our case, due to the axial-symmetry of the system, the good quantum numbers are the projection of the spin $\Omega$. The point of nearest approach between two levels of same $\Omega$ define an avoided level crossing region. If the internuclear distance varies, the transition probability of a nucleon between two adiabatic levels is strongly enhanced in the avoided level crossing region. This promotion mechanism is generically called the Landau-Zener effect.

Concerning the 8 single-particle adiabatic levels ($E_1, \ldots, E_8$) belonging to the $\Omega=1/2$ workspace, the first step is to find the avoided level crossing regions. The avoided crossing regions can be obtained by plotting the energy differences between these adiabatic levels as in Fig. 7. Each pertinent avoided crossing is identified and numbered. The avoided crossings that have a chance to be located along the diabatic single-particle energy paths emerging from the lower levels $E_1$, $E_2$ and $E_3$ are considered pertinent. Due to their low initial excitation energy, the transitions through these levels carry the major part of the fission strength. That property allows to restrict our calculations only
for an initial condition in which the occupation probability of one of these levels is one. The next step is to determine the probability of realization of each diabatic energy path emerging from these levels. Concerning the $\Omega=3/2$ subspace, the analysis is realized in the same manner, for initial conditions restricted to the first 3 low energy levels.

Assuming an $n$-state approximation, the wave function of the unpaired nucleon can be formally expanded [26] in a basis of $n$ diabatic wave functions $\phi_i(r, R)$ as

$$\Psi(r, R, t) = \sum_i^n c_i(t)\phi_i(r, R) \exp\left(-\frac{i}{\hbar}\int_0^t \epsilon_{ii} dt\right)$$  \hspace{1cm} (3)

where the matrix elements with the diabatic states $\phi$ are abbreviated as follows

$$\epsilon_{ij} = \langle \phi_i | H | \phi_j \rangle$$  \hspace{1cm} (4)

where $H$ is the STCSM Hamiltonian and $c_i$ are amplitudes. Inserting $\Psi$ in the time-dependent Schrödinger equation,

$$\left\langle \phi_i | H - i\hbar \frac{\partial}{\partial t} | \Psi \right\rangle = 0$$  \hspace{1cm} (5)
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Figure 8. The occupation probabilities of the diabatic levels $\epsilon_1, \ldots, \epsilon_8$ as function of the distance between the centers of the fragments. The same numbers as in Fig. 7 are used to identify the avoided level crossing regions. The internuclear velocity is $3.5 \times 10^4 \text{ m/s}$, that leads to a reasonable reaction time (time to penetrate the barrier) of approximately $5 \times 10^{-19} \text{ s}$. This example is constructed for an initial condition $p_{\epsilon_1} = 1$, while $p_{\epsilon_i} = 0$ ($i \neq 1$). The occupation probabilities vary in the avoided level crossing regions.

The following system of coupled equations is obtained:

$$\dot{c}_i = \frac{1}{i\hbar} \sum_{j \neq i} c_j \epsilon_{ij} \exp \left( -\frac{i}{\hbar} \int_0^t (\epsilon_{jj} - \epsilon_{ii}) dt \right)$$

To solve this system, the internuclear velocity $\dot{R}$, the diabatic energies and the interaction matrix elements must be known. Apart the relative velocity, the other ingredients are supplied by the STCSM. The diabatic states are constructed by using spline interpolations in the level crossing regions. The interaction matrix elements $\epsilon_{ij}$ between the diabatic states is a measure of the difference between adiabatic and diabatic energies. The occupation probability of each adiabatic level as function of $R$ is now obtained $p_{\epsilon_i} = |c_i|^2$. For the unpaired neutron initially located in the fundamental state $E_1$, the system (6) is solved within the boundary condition $c_1 = 1$ and $c_i = 0$ for $i \neq 1$. The occupation probabilities of each diabatic level plotted in Fig. 8 are represented in Fig. 8. Within the selected levels and avoided level crossings, 40 different energy paths of the unpaired neutron can be obtained as indicated in Table I. Here, an approximation is made by considering that the points of the avoided level crossings 1 and 2 form a single avoided level region. Otherwise, the number of paths gets doubled. Each path
represents an excitation of the nuclear system. The probability of each path can be estimated. For example, it can be deduced from Fig. 9 that the path $\epsilon_1 - 2 - \epsilon_1 - 3 - \epsilon_4$ carries about 0.5 of the probability. The line between letters and digits connects diabatic levels and avoided level crossing regions. A strong mixing is produced in the region 4, that leads us to conclude that the paths $\epsilon_1 - 2 - \epsilon_1 - 3 - \epsilon_4 - 4 - \epsilon_3$ (no. 1 in table 1) and $\epsilon_1 - 2 - \epsilon_1 - 3 - \epsilon_4 - 4 - \epsilon_4 - 7$ carry each of them about 0.25 probability. Finally, it can be considered that the path $\epsilon_1 - 2 - \epsilon_1 - 3 - \epsilon_4 - 4 - \epsilon_4 - 7 - \epsilon_1$ (no. 2) has about 0.05 probability of realization while $\epsilon_1 - 2 - \epsilon_1 - 3 - \epsilon_4 - 4 - \epsilon_4 - 7 - \epsilon_4$ (no. 3) remains with 0.2. The other probabilities are estimated in the same manner. The same procedure is repeated for the case when the unpaired nucleon is initially located on the other selected levels.

The excitations of the barriers due to one diabatic path $k$ is given by the specialization energy. Considering that the fundamental barrier corresponds to the nucleon at the Fermi energy, the excitation $E_x$ as function of $R$ is

$$E_{xk}(R) = \sqrt{(\epsilon_k(R) - \lambda(R))^2 + \Delta^2(R) - \Delta(R)}$$

in the frame of the superfluid model. Here, $\epsilon_k$ is the single-particle energy of the path $k$, $\lambda$ is the Fermi energy and $\Delta$ the gap. These excitations are added to the fundamental barrier. These quantities have the same meaning as the so-called transition bandheads found in the literature.
3. Cross-section

The partial fission cross section $\sigma_f$ for a spin $J$ of the compound nucleus and excitation energy $E^*$ is obtained within a statistical principle:

$$\sigma_f(J, E^*) = \sigma_c(J, E^*) \frac{\Gamma_f(J, E^*)}{\Gamma_n(J, E^*) + \Gamma_\gamma(J, E^*) + \Gamma_T(J, E^*)}$$

where the ratio in the right-hand side is the probability that the system decays through fission. It is given by a ratio between energy widths for fission (subscript $f$), neutron emission (subscript $n$) and $\gamma$-de-excitation (subscript $\gamma$). The subscript $T$ addresses the total transmission in the fission channel including absorption in the second well. The neutron transmission was computed for a squared complex potential \[27\] in order to evaluate the compound nucleus cross section. To determine the participation of different $\Omega$ excitations in the fission channel for a given spin $J$ of the compound nucleus, a unfolding procedure in term of Clebsh-Gordon coefficients is used

$$\Gamma_f(J, E^*) = \frac{1}{2\pi\rho(J, E^*, A)} \sum_{L=0}^{L_M} \sum_{\Omega} \frac{<J\Omega|\Omega_0>^2}{C}$$

$$\times \int_{E^* - E_L}^{E^*} T_f(E, L, \Omega) \rho(\Omega, E^* - E - E_L) dE$$

where a normalization coefficient is used:

$$C = \sum_{L=0}^{L_M} \sum_{\Omega} <J\Omega|\Omega_0>^2$$

and the condition $J = L + \Omega$ is imposed. Here $\rho$ represents the density of states, $E_L$ is the rotation energy in the fundamental state of the compound nucleus with an angular momentum $L$ and $L_M$ is the maximum orbital momentum taken into consideration. This formula can be obtained easily by simplifying the model underlined in Ref. \[8\], that is, neglecting the additional collective excitations as gamma, sloshing or bending vibrations. Analog formulas can be obtained for the $\gamma$ and neutron energy widths as detailed in Ref \[8\].

In the fission channel, the spin $\Omega$ density of states can be shared as function of the excitation energy between a discrete component and a continuum one:

$$\rho(J, E) = \begin{cases} \sum_i \delta(E - \epsilon_{\Omega,i}), & E < E_0 \\ \rho_{CG}(\Omega, E), & E \geq E_0 \end{cases}$$

where $\rho_{CG}$ is the statistical Gilbert and Cameron approximation and $\epsilon_{\Omega,i}$ ($i = 1, n$) are the set of diabatic single particle energies that are taken into consideration for a spin projection $\Omega$. So that, the transmission in the fission channel can be decomposed as follows:

$$\int_{E^* - E_L}^{E^*} T_f(E, L, \Omega) \rho(\Omega, E^* - E - E_L) dE$$

$$= \sum_i T_f(E^* - E_L - \epsilon_{\Omega,i})$$

$$+ \int_{E^* - E_L - E_0}^{E^*} T_f(E, L, \Omega) \rho(\Omega, E^* - E - E_L) dE$$

The sum over $i$ takes into account all the transmissions for diabatic levels with spin $J = \Omega + L$ located in the energy interval $[0, E_0]$. In this context, the transmission
$T_f(E^* - E_L - \epsilon_{\Omega,i})$ means a weighted sum of the transmissions of all available diabatic energy paths emerging from the level $\epsilon_{\Omega,i}$.

The microscopic model used to compute the theoretical barrier is subject to some limitations as described in Ref. [19]. It is not possible to obtain pertinent values of the heights of the barriers. In this circumstances, it is necessary to use a phenomenological barrier. A phenomenological barrier is conventionally simulated as a function of a dimensionless parameter $\beta$, that characterizes a deformation, within three smoothed joined parabolas [28]. In our work, an imaginary component of the potential is added between the turning points of the second well, in order to simulate the damping due to gamma and neutron emission. The additional excitations are considered as specialization energies and are added to the phenomenological barrier. This operation is achieved in the simplest possible way, by realizing a linear interpolation based on a correspondence between the elongation $R$ and the dimensionless parameter $\beta$ in some points. The correspondence was chosen for the two minimums, the two heights and the exit point. The hybrid model emerges. New barriers are constructed as displayed in Fig. 9. When only the collective rotations are taken into account, the heights of the barriers and that of the second well are modified with a quantity

$$\Delta E_L = \frac{L(L + 2\Omega + 1)\hbar^2}{2I_j} - E_L$$

(13)

where $I_j$ is the moment of inertia, $j$ labels one of the two heights or the second well. The decoupling parameter is neglected. The moment of inertia is computed simply with the formula $I_j = \mu R_j^2$ where $\mu$ is the reduced mass and $R_j$ is the theoretical elongation obtained at the extreme point $j$. The quantity

$$E_L = \frac{L(L + 2\Omega + 1)\hbar^2}{2I_0}$$

(14)

addresses the fundamental state of the compound nucleus. The previous formulas represents an improvement of the formalism found in Ref. [8].

A large number of excited states are obtained that are characterized by the projection $\Omega$ and the angular momentum $L$. The transmission is calculated numerically by approximating the shape of the excited barrier within 500 constant potential steps using the numerical recipe found in Ref. [29]. A search of the heights and of the widths of the phenomenological barrier is realized in order to reproduce as well as possible the experimental fission cross-section threshold structure. A behavior that agree satisfactory with the experimental data is obtained. The heights of the inner phenomenological barrier, the second well and the outer barrier are 6.81, 4.83 and 6.61 MeV, respectively. In the same order, the widths are 1.2, 0.4 and 1.1 MeV. The theoretical cross-section is represented in Fig. 10 and compared with experimental data and evaluations.

The evaluations succeeded to reproduce better the experimental data. In general many parameters are taken into account to evaluate a cross-section in terms of Bohr-channels. For example, in evaluations phenomenological level densities functions appropriately matched to the available experimental structure data at low excitation
energies are used. Multiplication factors are also applied to level density functions to account for enhancements in the fission transition state densities at each fission barrier. It is a common practice to describe the cross-section as the sum of excitations for discrete levels constructed to fit the resonance. In other words, the evaluation takes into account many other parameters to fit the experimental data apart the heights and the widths of the phenomenological barrier. In the work presented in this paper, no adjustments are made to improve the agreement, the simulations being based only on the phenomenological barrier parameters and the internuclear velocity.

Our simulations evidence an oscillatory behavior of the cross-section close to 1.4 MeV. This aspect is in agreement with the experimental data given in Ref. [32]. The experimental data combined with theoretical arguments estimate a ratio 2:1 between the partial cross section of spin 3/2 and 1/2, respectively. The model shows that the partial cross section for the spin 3/2 is responsible for the oscillations of the cross section at these energies. Experimentally, the peak at 1.6 MeV is explained entirely by a partial cross section of spin 3/2 with a small 5/2 component. In our plot a strong 3/2 component is present with small admixture of 1/2 and 5/2 partial cross sections. A discrepancy is obtained for the 1.7 MeV structure. The experiment evidences the existence of a mixing between 3/2 and 5/2 components while our model predicts a large 5/2 partial cross section followed by the 3/2 and 1/2 components.

In Fig. [11] the cross-section is plotted on an extended scale. It can be observed that the theoretical results exhibits an oscillatory behavior in the low energy region, up to 1.2 MeV, around the smooth variation of the experimental data. In the panel (b), the transmissions computed for the barriers with different calculated excitations are
The oscillatory behavior is due to a large number of resonances associated to the different excited barriers.

4. Summary and discussion

The scope of the present work is to understand the mechanism for the formation of the fission cross section structure and of the high number of resonances by appealing essentially to dynamical single-particle effects associated to $\beta$-vibration in the second well. The number of free parameters is kept as minimal as possible (six parameters that characterize the phenomenological barrier and one parameter for the internuclear velocity) to show evidence of the physics of the problem.

Theoretical excitations and their associated probabilities were determined for a given partition in the isotopic distribution of fragments. These excitations are added to a phenomenological barrier in the framework of the hybrid model. After a suitable search of the parameters of the double humped phenomenological barrier, the cross section is computed. The results give a rather good qualitative agreement with experimental data. It is evidenced that the structure at 1.4 and 1.6 MeV is mainly dominated by spin 3/2
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The partial cross-section with small admixture of spin 1/2, while the structure at 1.7 MeV is given by a large partial cross section of spin 5/2.

In this exploratory analysis, only one partition for the fission fragments is taken into consideration. For other partitions in the same mass region, it is expected that the level scheme changes slightly leading to a small shift in the energy of the resonances. By taking into account several partitions in the same mass-region and folding their yields it is possible to obtain broader resonances as experimentally observed.

In this context, it will be interesting to explore experimentally if the isotopic fragment distribution in the fission process changes in the energetic region covered by a resonance, showing a preference for several mass partitions. If such a phenomenon can be experimental evidenced, that will represent a strong experimental support for our model because the statistical theories don’t include ingredients related to this aspect.

The model can be further improved. Up to now, only the radial coupling was used to explain the intermediate structure of the cross-section. It is possible to have better results by taking into account the Coriolis mixing and the residual interactions by using evolved forms for the system of coupled equations that describes the microscopic motion.

Other models succeed to reproduce better the experimental data using an extensive number of free parameters: 10 variables for the heights and widths of the triple humped phenomenological barrier plus 5 times 16 variables for the transition bandheads constructed on different intrinsic excitations (with a significance of excitations given by single-particle energies). Despite the overall excellent agreement on a very large neutron energy region, this treatment, generally used in evaluations, takes into account a peculiar behavior for the single-particle excitation energies. The levels that characterize the transition bandheads never intersect. The first $\frac{1}{2}^+$ level have practically the same value (having as reference the fundamental state) during the penetration of the barrier. This behavior, as remarked previously, cannot be expected. Moreover, the statistical models consider that the population of each fundamental transition band is essentially one.

The formalism presented in the Sect. 2 indicates that such a behavior is physically not reasonable.

The present investigation shows that the resonant structure of the fission cross section can be explained by the existence of many barriers associated to single-particle excitations. So, it is possible that the complex structure in the fission cross section is due to rearrangement of orbitals and the dynamic of the process, beginning from the initial state of the compound nucleus and terminating at the scission. A large number of different excited barriers are formed leading to a large number of vibrational resonances in the second well. These resonances carry information about the structure of the nucleus at hyperdeformations and the dynamics. The model presented in this work represents an alternative to the actual statistical models and may determine a competitive way to consider the fission process.

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Table 1. Energy paths open for the first $E_1 \Omega=1/2$ level

| No. | Energy path                                                                 |
|-----|-----------------------------------------------------------------------------|
| 1   | $\epsilon_1 - 2 - \epsilon_1 - 3 - \epsilon_4 - 4 - \epsilon_3$           |
| 2   | $\epsilon_1 - 2 - \epsilon_1 - 3 - \epsilon_4 - 4 - \epsilon_4 - 7 - \epsilon_1$ |
| 3   | $\epsilon_1 - 2 - \epsilon_1 - 3 - \epsilon_4 - 4 - \epsilon_4 - 7 - \epsilon_4$ |
| 4   | $\epsilon_1 - 2 - \epsilon_1 - 3 - \epsilon_1 - 6 - \epsilon_3 - 4 - \epsilon_3$ |
| 5   | $\epsilon_1 - 2 - \epsilon_1 - 3 - \epsilon_1 - 6 - \epsilon_3 - 4 - \epsilon_4 - 7 - \epsilon_1$ |
| 6   | $\epsilon_1 - 2 - \epsilon_1 - 3 - \epsilon_1 - 6 - \epsilon_3 - 4 - \epsilon_4 - 7 - \epsilon_4$ |
| 7   | $\epsilon_1 - 2 - \epsilon_1 - 3 - \epsilon_1 - 6 - \epsilon_1 - 7 - \epsilon_1$ |
| 8   | $\epsilon_1 - 2 - \epsilon_1 - 3 - \epsilon_1 - 6 - \epsilon_1 - 7 - \epsilon_4$ |
| 9   | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_2 - 9 - \epsilon_3 - 6 - \epsilon_3 - 4 - \epsilon_3$ |
| 10  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_2 - 9 - \epsilon_3 - 6 - \epsilon_3 - 4 - \epsilon_4 - 8 - \epsilon_1$ |
| 11  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_2 - 9 - \epsilon_3 - 6 - \epsilon_3 - 4 - \epsilon_4 - 8 - \epsilon_4$ |
| 12  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_2 - 9 - \epsilon_3 - 6 - \epsilon_1 - 8 - \epsilon_1$ |
| 13  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_2 - 9 - \epsilon_3 - 6 - \epsilon_1 - 8 - \epsilon_4$ |
| 14  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_4 - 3 - \epsilon_4 - 4 - \epsilon_3$ |
| 15  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_4 - 3 - \epsilon_4 - 4 - \epsilon_4 - 8 - \epsilon_1$ |
| 16  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_4 - 3 - \epsilon_4 - 4 - \epsilon_4 - 8 - \epsilon_4$ |
| 17  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_4 - 3 - \epsilon_1 - 6 - \epsilon_3 - 4 - \epsilon_3$ |
| 18  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_4 - 3 - \epsilon_1 - 6 - \epsilon_3 - 4 - \epsilon_4 - 8 - \epsilon_1$ |
| 19  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_4 - 3 - \epsilon_1 - 6 - \epsilon_4 - 4 - \epsilon_4 - 8 - \epsilon_4$ |
| 20  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_4 - 3 - \epsilon_1 - 6 - \epsilon_1 - 7 - \epsilon_1$ |
| 21  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_4 - 3 - \epsilon_1 - 6 - \epsilon_1 - 7 - \epsilon_4$ |
| 22  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_2 - 9 - \epsilon_2 - 10 - \epsilon_5 - 11 - \epsilon_6 - 12 - \epsilon_8$ |
| 23  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_2 - 9 - \epsilon_2 - 10 - \epsilon_5 - 11 - \epsilon_6 - 12 - \epsilon_6$ |
| 24  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_2 - 9 - \epsilon_2 - 10 - \epsilon_5 - 11 - \epsilon_5 - 14 - \epsilon_8 - 12 - \epsilon_8$ |
| 25  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_2 - 9 - \epsilon_2 - 10 - \epsilon_5 - 11 - \epsilon_5 - 14 - \epsilon_8 - 12 - \epsilon_6$ |
| 26  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_2 - 9 - \epsilon_2 - 10 - \epsilon_5 - 11 - \epsilon_5 - 14 - \epsilon_5$ |
| 27  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_2 - 9 - \epsilon_2 - 10 - \epsilon_2 - 10 - \epsilon_3 - 11 - \epsilon_6 - 12 - \epsilon_8$ |
| 28  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_2 - 9 - \epsilon_2 - 10 - \epsilon_2 - 10 - \epsilon_2 - 11 - \epsilon_6 - 12 - \epsilon_6$ |
| 29  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_2 - 9 - \epsilon_2 - 10 - \epsilon_2 - 10 - \epsilon_3 - 11 - \epsilon_5 - 14 - \epsilon_8 - 12 - \epsilon_8$ |
| 30  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_2 - 9 - \epsilon_2 - 10 - \epsilon_2 - 10 - \epsilon_2 - 13 - \epsilon_6 - 11 - \epsilon_5 - 14 - \epsilon_8 - 12 - \epsilon_6$ |
| 31  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_2 - 9 - \epsilon_2 - 10 - \epsilon_2 - 10 - \epsilon_2 - 13 - \epsilon_6 - 11 - \epsilon_5 - 14 - \epsilon_5$ |
| 32  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_2 - 9 - \epsilon_2 - 10 - \epsilon_2 - 13 - \epsilon_2 - 15 - \epsilon_7 - 16 - \epsilon_8 - 14 - \epsilon_8 - 12 - \epsilon_8$ |
| 33  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_2 - 9 - \epsilon_2 - 10 - \epsilon_2 - 13 - \epsilon_2 - 15 - \epsilon_7 - 16 - \epsilon_8 - 14 - \epsilon_8 - 12 - \epsilon_6$ |
| 34  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_2 - 9 - \epsilon_2 - 10 - \epsilon_2 - 13 - \epsilon_2 - 15 - \epsilon_7 - 16 - \epsilon_8 - 14 - \epsilon_5$ |
| 35  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_2 - 9 - \epsilon_2 - 10 - \epsilon_2 - 13 - \epsilon_2 - 15 - \epsilon_7 - 16 - \epsilon_7$ |
| 36  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_2 - 9 - \epsilon_2 - 10 - \epsilon_2 - 13 - \epsilon_2 - 15 - \epsilon_2 - 17 - \epsilon_8 - 16 - \epsilon_8 - 14 - \epsilon_8 - 12 - \epsilon_8$ |
| 37  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_2 - 9 - \epsilon_2 - 10 - \epsilon_2 - 13 - \epsilon_2 - 15 - \epsilon_2 - 17 - \epsilon_8 - 16 - \epsilon_8 - 14 - \epsilon_8 - 12 - \epsilon_6$ |
| 38  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_2 - 9 - \epsilon_2 - 10 - \epsilon_2 - 13 - \epsilon_2 - 15 - \epsilon_2 - 17 - \epsilon_8 - 16 - \epsilon_8 - 14 - \epsilon_5$ |
| 39  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_2 - 9 - \epsilon_2 - 10 - \epsilon_2 - 13 - \epsilon_2 - 15 - \epsilon_2 - 17 - \epsilon_8 - 16 - \epsilon_7$ |
| 40  | $\epsilon_1 - 2 - \epsilon_2 - 5 - \epsilon_2 - 9 - \epsilon_2 - 10 - \epsilon_2 - 13 - \epsilon_2 - 15 - \epsilon_2 - 17 - \epsilon_2$ |
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