Modal analysis of the rotating shell structure based on Absolute Nodal Coordinate Formulation

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Abstract. In recent years, flexible structures composed of cables, plates and shells, such as rotors and solar sails that rotate around a fixed axis have been widely used in aerospace, energy and other industrial fields. Traditional structural dynamics or multi-flexible dynamics modeling methods based on structural small deformation assumptions cannot accurately describe the nonlinear dynamics of flexible structures undergoing large-scale rotation. The Absolute Nodal Coordinate Formulation (ANCF) can accurately describe the flexible multi-body system with both large rigid motions and large deformation. Such flexible structures, while undergoing a wide range of rotational motions, are accompanied by structural vibrations that are excited by unbalanced centrifugal forces, aerodynamic forces, etc., resulting in very complex dynamic behaviors. Therefore, it is of great scientific significance and application value to dynamically model and predict the dynamic response of such flexible structures with a wide range of rotational motions. In this paper, the cylindrical shell element of ANCF proposed by Liu et al. which can realize efficient calculation is briefly introduced. The analytical expressions of elastic force and Jacobin matrix of this shell element are given. Modal analysis of shell structures modeled by ANCF cylindrical shell element and rotating around a fixed axis is studied. The effect of curvature and angular velocities on the modal of shell structures is validated.

1. Introduction

Flexible multi-body systems, especially flexible structures for high-speed rotations, are widely used in aerospace, mechanical, biological, such as solar sails, helicopter rotors and wind power blades [1]. The flexible structures for high-speed rotations are mostly composed of beams, plates and shells. In the high-speed rotation state, the deformation due to the rotation of this structure will be coupled with its elastic deformation, leading to complex nonlinear dynamic behaviors. Kane [2] firstly found the dynamic stiffening effect of beams.

Many researchers have studied the effects of high-speed rotation on the structure dynamics in the early research, where the rotating structure is simplified to a beam. However, this simplification loses its accuracy in the case of low aspect ratio of the structure. In this case, the structure should be modeled by using plate element or shell element. Some researchers studied the modal characteristics of the rotating structures by using finite element techniques. Dokainish et al. [3] analyzed the modal characteristics of a rotating plate by using one of these techniques. Ramamurti et al. [4] analyzed the modal characteristics of twisted rotating plates by using one of these techniques. These finite element techniques get some success in some fields by using classical finite element method. But these theory would lose their accuracies in the systems with both large deformations and overall motions [6]. In this
case, Zhao et al. [7] studied the modal analysis of a rotating thin plate based on ANCF and validated the effects of rotating angular velocity on the natural frequencies.

The ANCF initially proposed by Shabana [8]. The flexible multi-body systems with both large rigid motions and large deformations could be exactly described by using this method. Different from classical finite elements, nodal coordinates in ANCF consist of the global position and its gradients. This choice leads to constant mass matrix and zero centrifugal and Coriolis inertia forces. ANCF is considered to be an important progress of the studies on flexible multi-body dynamics [9, 10] and has been widely used in different fields, such as the dynamics of mechanisms with clearance joints [11], the deployable dynamics of solar sails [12], and the contact dynamics of elastic thin beams [13].

As for plate/shell of ANCF, Shabana et al. [14] firstly proposed a plate element that can accurately describe the rigid motion of flexible plates. Mikkola et al. [15] proposed a fully parameterized plate element with four-node and 48 DOFs which can be used to deal with some flexible multi-body systems. However, this shell element has poor convergence when analyzing the flexible multi-body systems with thin shell structures. Dmitrochenko et al. [16] proposed a new plate element with 36 DOFs which can improve calculation efficiency. Gerstmayr et al. [17] derived a new analytical expression of the elastic force based on the first type of Piola-Kirchhoff stress tensor. Liu et al. [18] proposed a new cylindrical shell element of gradient-deficient ANCF and derived the analytical expression of the elastic force and the Jacobin matrix of ANCF shell element by using the second type of Piola-Kirchhoff stress tensor. This element can be used to modal some parallelogram thin shell structures by introducing the element angle between the two sides of the element into the strain formula.

In this paper, a rotating shell structure is established with the purpose of modal analysis, based on the cylindrical shell element model deduced by Liu et al. [18]. The effects of curvatures and angular velocities on the shell structures are analyzed. The paper is organized as follows. In Sect. 2, the new shell element is introduced to ensure the completeness of the paper. In Sect. 3, the modal analysis formulation of the rotating shell structure is made by using the external force equivalent method presented by Karmakar et al. [19]. In Sect. 4, two numerical examples are provided to examine the validation of shell modes and to analyze the effects of curvature and angular velocity on the thin shell mode. Some conclusions are made in Sect. 5.

2. A new shell element of ANCF

2.1 Mid-surface strain energy

The shell element proposed by Liu et al. was introduced here to research more complicated thin structures based on the plate element studies [16]. As shown in Figure 1, the global position of the arbitrary point \( P(\xi, \eta) \) based on ANCF can be expressed by

\[
r_0(\xi, \eta) = S(\xi, \eta)e_0
\]

In the above equation, the subscript ‘0’ indicates the initially curved configuration; \( e_0 = [ (e_0)_A^T (e_0)_B^T (e_0)_C^T (e_0)_D^T ]^T \) is the nodal coordinates; \( 0 \leq \xi \leq 1 \) and \( 0 \leq \eta \leq 1 \) are the local coordinates of the element on the mid-surface; \( S \) is the shape function matrix, which can be expressed by

\[
S = [ S_1 I_3 \quad S_2 I_3 \quad \cdots \quad S_3 I_3 ]
\]

where \( I_3 \) is a 3x3 identity matrix; The components of the shape function matrix \( S \) can be expressed by
\[
\begin{aligned}
S_1 &= -\xi(\xi - 1)(\eta - 1)(2\eta^2 - \eta + 2\xi^2 - \xi) \\
S_2 &= -a\xi(\xi - 1)^2(\eta - 1) \\
S_3 &= -b\eta(\eta - 1)^2(\xi - 1) \\
S_4 &= (\eta - 1)(\xi)(2\eta^2 - \eta - 3\xi + 2\xi^2) \\
S_5 &= -a\xi^2(\xi - 1)(\eta - 1) \\
S_6 &= b\xi\eta(\eta - 1)^2 \\
S_7 &= -\xi\eta(1 - 3\xi - 3\eta + 2\eta^2 + 2\xi^2) \\
S_8 &= a\xi^2\eta(\xi - 1) \\
S_9 &= b\eta^2\xi(\eta - 1) \\
S_{10} &= \eta(\xi - 1)(2\xi^2 - \xi - 3\eta - 2\eta^2) \\
S_{11} &= a\xi^2\eta(\xi - 1)^2 \\
S_{12} &= -b\eta^2(\xi - 1)(\eta - 1)
\end{aligned}
\tag{3}
\]

The above shape function satisfies the definition of ANCF finite elements [20].

As shown in Figure 1, a local coordinate frame \((g_0)_1 - (g_0)_2 - n_0\) and a local Cartesian coordinate frame \((e_0)_1 - (e_0)_2 - (e_0)_3\) are established at the arbitrary point \(P(\xi,\eta)\) in initially curved configuration. The local coordinate frame \((g_0)_1 - (g_0)_2 - n_0\) can be expressed by

\[
\begin{align*}
(g_0)_1 &= (r_0)_z = \frac{\partial r_0}{\partial \xi} \\
(g_0)_2 &= (r_0)_\eta = \frac{\partial r_0}{\partial \eta} \\
n_0 &= \frac{(g_0)_1 \times (g_0)_2}{|(g_0)_1 \times (g_0)_2|}
\end{align*}
\tag{4}
\]

**Figure 1.** Mid-surface and nodes of a shell element of ANCF. [18]
The local Cartesian coordinate frame \( \mathbf{e}_i - \mathbf{e}_2 - \mathbf{e}_3 \) can be expressed by

\[
\begin{align*}
\mathbf{e}_i & = \frac{\mathbf{g}_i}{|\mathbf{g}_i|} \\
\mathbf{e}_2 & = \mathbf{e}_3 \times \mathbf{e}_1 \\
\mathbf{e}_3 & = \mathbf{n}_0
\end{align*}
\]

(5)

The local coordinate frame \( \mathbf{g}_1 - \mathbf{g}_2 - \mathbf{n} \) and the local Cartesian coordinate frame \( \mathbf{e}_1 - \mathbf{e}_2 - \mathbf{e}_3 \) in the current configuration can also be expressed by the similar theory.

The infinitesimal arc segment \( d\mathbf{r}_0 \) denotes the displacement between the point \( P(\xi, \eta) \) and the point \( N(\xi + d\xi, \eta + d\eta) \) which is very close to \( P(\xi, \eta) \) on the mid-surface in initially curved configuration. It can be expressed by the two local coordinate respectively as

\[
d\mathbf{r}_0 = (\mathbf{g}_0)_i \, d\xi = (\mathbf{e}_0)_i \, dx^i
\]

(6)

where \( \chi' = \xi, \chi'' = \eta \), \( x'^1 = x_0, x'^2 = y_0 \). Thus, the relationship between the two local coordinate frames \( (\mathbf{g}_0)_1 - (\mathbf{g}_0)_2 - \mathbf{n}_0 \) and \( (\mathbf{e}_0)_1 - (\mathbf{e}_0)_2 - (\mathbf{e}_0)_3 \) can be expressed as [21]

\[
(\mathbf{g}_0)_i = \beta^i_j (\mathbf{e}_0)_j
\]

(7)

where \( \beta^i_j = (\mathbf{g}_0)_i (\mathbf{e}_0)_j \) is the transformation coefficient. Therefore, the contravariance components \( d\chi'^i \) can be expressed by

\[
d\chi'^i = \beta^i_k dx^k
\]

(8)

where \( \beta^i_k \) is the contravariance transformation coefficient. For example, the transformation between the two local coordinate frames \( \mathbf{g}_1 - \mathbf{g}_2 - \mathbf{n} \) and \( \mathbf{e}_1 - \mathbf{e}_2 - \mathbf{e}_3 \) can be expressed by

\[
\begin{bmatrix}
\frac{d\xi}{d\eta} \\
-\cot(\theta)
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{a} - \frac{\cot(\theta)}{a} \\
\frac{0}{b}
\end{bmatrix}
\begin{bmatrix}
dx_0 \\
dy_0
\end{bmatrix}
= T dX_0
\]

(9)

where \( a \) is the length of element; \( b \) is the width of element; \( \theta \) represents the angle between the two base vectors \( (\mathbf{g}_0)_1 \) and \( (\mathbf{g}_0)_2 \); \( T \) is the transformation matrix of the two vectors \( [d\xi, d\eta]^T \) and \( [dx_0, dy_0]^T \).

According to the continuum mechanics, the Green-Lagrange strain can be calculated by

\[
\left( \frac{ds}{d\xi} \right)^2 - \left( \frac{ds}{d\eta} \right)^2 = 2dXxdX
\]

(10)

In the above equation, the square of the infinitesimal arc segment \( ds_0 \) and \( ds \) can be expressed as

\[
\begin{align*}
(ds_0)^2 & = d\mathbf{r}_0 \cdot d\mathbf{r}_0 = ((\mathbf{g}_0)_1, d\xi + (\mathbf{g}_0)_2, d\eta) \cdot ((\mathbf{g}_0)_1, d\xi + (\mathbf{g}_0)_2, d\eta) \\
& = [d\xi, d\eta] [g_0]_{11} [g_0]_{12} [g_0]_{22} [d\xi, d\eta]
\end{align*}
\]

(11)
\[ (ds)^2 = dr \cdot dr = \left[ \frac{d\xi}{d\eta} \right] \begin{bmatrix} g_{11} & g_{12} \\ g_{12} & g_{22} \end{bmatrix} \begin{bmatrix} d\xi \\ d\eta \end{bmatrix} \]  

where \( (g_0)_\alpha = (g_0)_\alpha \cdot (g_0)_\beta \cdot \, \, (g_0)_\alpha = g_{\alpha\alpha}, \, \, g_{\alpha\beta} = (\alpha=1,2 \, \text{and} \, \beta=1,2). \) Considering equations (9-12), the matrix form of the Green-Lagrange strain on the mid-surface can be expressed as

\[ \varepsilon_{\text{mid}}^n = \frac{1}{2} T^T \begin{bmatrix} g_{11} & g_{12} \\ g_{12} & g_{22} \end{bmatrix} - \begin{bmatrix} (g_0)_11 & (g_0)_12 \\ (g_0)_21 & (g_0)_22 \end{bmatrix} T = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{12} & \varepsilon_{22} \end{bmatrix} \]  

Therefore, the strain energy on the mid-surface can be expressed as

\[ U_{\text{mid}} = \frac{1}{2} \int_{V_0} (\varepsilon_{\text{mid}}^n)^T E(\varepsilon_{\text{mid}}^n) dV_0 \]  

In the above equation, \( E \) is the elastic tensor which can be express by

\[ E = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1-\nu \end{bmatrix} \]  

When calculating the integration over the element, Mikkola and Shabana [15] used the volume \( V \) instead of the volume \( V_0 \) in initially curved configuration. The relationship between the two volumes can be expressed by

\[ dV_0 = \left[(g_0)_1 \times (g_0)_2\right] dV = D dV \]  

where \( V \) is the volume of the master element defined in the local coordinate frame. For example, combined with equation (14), the strain energy on the mid-surface can be further expressed as

\[ U_{\text{mid}} = \frac{1}{2} Dh \int_{S} (\varepsilon_{\text{mid}}^n)^T E(\varepsilon_{\text{mid}}^n) d\xi d\eta \]  

2.2 Bending strain energy

As shown in Figure 2, the global position of an arbitrary point \( P(\xi, \eta) \) on the outer surface of the shell element can be expressed by

\[ r = r(\xi, \eta) + zn(\xi, \eta) \]  

where \( n \) is a unit normal to the mid-surface.

Two vectors of the local coordinate frame \( g_1^z \) and \( g_2^z \) defined at the point \( P(\xi, \eta) \) can be expressed by

\[ \begin{cases} g_1^z = \frac{\partial r}{\partial \xi} + z \frac{\partial n}{\partial \xi} \\ g_2^z = \frac{\partial r}{\partial \eta} + z \frac{\partial n}{\partial \eta} \end{cases} \]  

Similarly to the derivation of the Green-Lagrange strain on the mid-surface, the total strain can be express as

\[ \varepsilon = \varepsilon_{\text{mid}} + \varepsilon^z = \frac{1}{2} T^T \begin{bmatrix} g_{11}^z & g_{12}^z \\ g_{12}^z & g_{22}^z \end{bmatrix} - \begin{bmatrix} (g_0)_11^z & (g_0)_12^z \\ (g_0)_21^z & (g_0)_22^z \end{bmatrix} T \]  

where \( (g_0)_\alpha = (g_0)_\alpha \cdot (g_0)_\beta \cdot \, \, (g_0)_\alpha = g_{\alpha\alpha}, \, \, g_{\alpha\beta} = (\alpha=1,2 \, \text{and} \, \beta=1,2). \)

Substituting equations (13, 19) into equation (20) and ignoring the higher-order terms of \( z \), the bending strain \( \varepsilon^z \) can be expressed by
where
\[
\kappa = \begin{bmatrix}
\kappa_{11} & \kappa_{12} \\
\kappa_{21} & \kappa_{22}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial^2 \mathbf{r}}{\partial \zeta^2} \cdot \mathbf{n} & \frac{\partial^2 \mathbf{r}}{\partial \zeta \partial \eta} \\
\frac{\partial^2 \mathbf{r}}{\partial \zeta \partial \eta} & \frac{\partial^2 \mathbf{r}}{\partial \eta^2} \cdot \mathbf{n}
\end{bmatrix}
\]
\[
\kappa_0 = \begin{bmatrix}
(k_0)_{11} & (k_0)_{12} \\
(k_0)_{21} & (k_0)_{22}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial^2 \mathbf{r}_0}{\partial \zeta^2} \cdot \mathbf{n}_0 & \frac{\partial^2 \mathbf{r}_0}{\partial \zeta \partial \eta} \\
\frac{\partial^2 \mathbf{r}_0}{\partial \zeta \partial \eta} & \frac{\partial^2 \mathbf{r}_0}{\partial \eta^2} \cdot \mathbf{n}_0
\end{bmatrix}
\]

Thus, the bending strain energy can be expressed by
\[
U^b = \frac{1}{2} \int_{V_0} (\varepsilon^b)^T \mathbf{E} \varepsilon^b \, dV_0
\]  

2.3 Elastic force and its Jacobin matrix
The mass matrix of the shell element can be expressed by
\[
\mathbf{M} = \rho \int_{V_0} \mathbf{S}^T \mathbf{S} \, dV_0
\]  

The elastic force is the partial derivative of the strain energy to the generalized coordinates based on continuum mechanics. Thus, the elastic force on the mid-surface can be expressed as
\[
\mathbf{Q}^{\text{mid}} = \left( \frac{\partial U^{\text{mid}}}{\partial \varepsilon} \right)^T = \int_{V_0} \left( \frac{\partial (\varepsilon^{\text{mid}})}{\partial \varepsilon} \right)^T \mathbf{E} (\varepsilon^{\text{mid}}) \, dV_0
\]  

According to equations (21-24), the bending strain energy can be further expressed by
\[
U^b = \frac{1}{2} \int_{V_0} (\mathbf{Q} - \mathbf{Q}_0)^T \mathbf{E}^T (\mathbf{Q} - \mathbf{Q}_0) \, dV_0
\]  

where \( \mathbf{Q} = -z [\kappa_{11} \quad 2\kappa_{12}]^T \), \( \mathbf{Q}_0 = -z [(k_0)_{11} \quad (k_0)_{12} \quad 2(k_0)_{12}]^T \). \( \mathbf{E}^T = \mathbf{H}^T \mathbf{E} \mathbf{H} \), in which
\[
H = \begin{bmatrix}
\frac{1}{a^2} & 0 & 0 \\
\frac{\cot^2 \theta}{a^2} & \frac{\csc^2 \theta}{b^2} & -\frac{\cot \theta \csc \theta}{ab} \\
\frac{2\cot \theta}{a^2} & 0 & \frac{\csc \theta}{ab}
\end{bmatrix}
\] (28)

Considering equations (27, 28), the bending elastic force can be express as
\[
Q^e = \left( \frac{\partial U^e}{\partial \mathbf{e}} \right)^T = \int_V \left( \frac{\partial \Omega}{\partial \mathbf{e}} \right)^T \mathbf{E}^e \left( \Omega - \Omega_0 \right) dV_0
\] (29)

The Jacobin matrix of elastic forces can be express as
\[
\mathbf{J}^{\text{mid}} = \frac{\partial Q^{\text{mid}}}{\partial \mathbf{e}} = \int_{V_0} \frac{\partial}{\partial \mathbf{e}} \left( \frac{\partial \mathbf{e}^{\text{mid}}}{\partial \mathbf{e}} \right)^T \mathbf{E} \left( \mathbf{e}^{\text{mid}} \right) dV_0 + \int_{V_0} \left( \frac{\partial \mathbf{e}^{\text{mid}}}{\partial \mathbf{e}} \right) \mathbf{E}^e \left( \frac{\partial \Omega}{\partial \mathbf{e}} \right) dV_0
\] (30)

\[
\mathbf{J}^e = \frac{\partial Q^e}{\partial \mathbf{e}} = \int_{V_0} \frac{\partial}{\partial \mathbf{e}} \left( \frac{\partial \Omega}{\partial \mathbf{e}} \right)^T \mathbf{E}^e \left( \Omega - \Omega_0 \right) dV_0 + \int_{V_0} \left( \frac{\partial \Omega}{\partial \mathbf{e}} \right)^T \mathbf{E}^e \left( \frac{\partial \Omega}{\partial \mathbf{e}} \right) dV_0
\] (31)

3. Modal analysis of a rotating shell

A modal analysis theory of rotating shell structures is presented in this section. The equivalent transformation method presented by Karmakar et al. [19] is introduced for modal analysis. As shown in figure 3, the rotating shell is treated as a stationary shell by add a distributive force \( \mathbf{F} \). The magnitude of the distributive force is equal to the centrifugal force.

The distributive force \( \mathbf{F} \) can be obtained by the principle of virtual displacement, and can be expressed as
\[
\mathbf{F}^T \delta \mathbf{e} = \int [\rho \ddot{x} \omega^2 \quad \rho \ddot{z} \omega^2] \mathbf{e} dV_0
\]

\[
= \rho \omega^2 \int [\ddot{x} \quad \ddot{z}] \mathbf{d}V_0 \delta \mathbf{e}
\]

\[
= \rho \omega^2 \int \mathbf{e}^T S^T S \mathbf{d}V_0 \delta \mathbf{e}
\] (32)

In the above equation, \( \omega \) is the rotating angular velocity; \( \ddot{x} \) and \( \ddot{z} \) is the quantities on the x axis and z axis of the distance between the integral point and the rotating axis, respectively; \( \mathbf{S} \) is the shape function matrix; The components of the first and third rows in \( \mathbf{S} \) are equal to the components of the first and third rows in \( \mathbf{S} \), while the other components in \( \mathbf{S} \) are all equal to zero. According to equation (32) the distributive force can be expressed by

![Figure 3. Centrifugal force equivalent diagram.](image-url)
\[ F = \rho \omega^2 \int S^T \dot{S} dV dV' \]  
(33)

In order to analyze the vibration of the equivalent system, a linearization is performed [22] for the model obtained based on the ANCF. The perturbation form [7] of the system dynamic equations can be expressed by

\[ M \delta \ddot{e} + K_T \delta e = 0 \]  
(34)

where \( M \) is the mass matrix of the system; \( K_T \) is the tangential stiffness matrix of the system at the static equilibrium configuration which can be expressed by

\[ K_T = \frac{\partial Q}{\partial \dot{e}} \frac{\partial F}{\partial e} \]  
(35)

The static equilibrium configuration can be obtained from the following static equilibrium equation

\[ Q - F = 0 \]  
(36)

Assume the solution of equation (34) as

\[ \delta e = \Lambda e^{i \omega t} \]  
(37)

Substituting equation (37) into equation (34) leads to a generalized eigenvalue problem

\[ (K_T - \omega^2 M) \Lambda = 0 \]  
(38)

The natural frequency and the corresponding mode shape can be determined from the above equation.

4. Numerical results and discussion

4.1 Static bending of parallelogram plate

As shown in Figure 4, the effectiveness of the thin shell element was verified by a plate bending example. Two concentrated force \( F/2 \) was applied to points B and C of the plate, respectively. The specific parameters of the plate are as follows: \( L=0.5 \) m, \( W=0.15 \) m, \( h=0.001 \) m, \( E=2.07 \times 10^{11} \) Pa, \( v=0.3 \), \( F=20 \) N, and the sine and cosine of the angle \( \theta \) between AB and BC are \( \sin(\theta)=0.8 \), \( \cos(\theta)=0.6 \), respectively. The parameter settings are the same as Document [18].

| Element number | X-displacement (m) | Y-displacement (m) | Z-displacement (m) |
|----------------|--------------------|--------------------|--------------------|
| present        |                    |                    |                    |
| 3x10           | -8.622x10^{-2}     | 1.490x10^{-2}      | 2.574x10^{-1}      |
| 6x20           | -9.264x10^{-2}     | 1.788x10^{-2}      | 2.682x10^{-1}      |
| 15x50          | -9.313x10^{-2}     | 1.144x10^{-2}      | 2.691x10^{-1}      |
| 3x10           | -8.599x10^{-2}     | 1.498x10^{-2}      | 2.572x10^{-1}      |
| Liu [18]       |                    |                    |                    |
| 6x20           | -9.251x10^{-2}     | 1.184x10^{-2}      | 2.681x10^{-1}      |
| 15x50          | -9.308x10^{-2}     | 1.146x10^{-2}      | 2.691x10^{-1}      |

Figure 4. Initial configuration of parallelogram plate bending.
Table 1 lists the displacement of point C on the plate with the different number of elements which is obtained by ANCF. The results are good agreement with the results obtained by Liu [18].

4.2 Modal analysis of a rotating cylindrical shell
This subsection presents a numerical example of a rotating cylindrical shell with different rotating speed and curvature to validate the influence of rotating speed and curvature on the natural frequency and the corresponding mode shape of the rotating shell. The cylindrical shell rotating around the cylinder generatrix as shown in Figure 5. The length $L$ and the width $W$ of the cylindrical shell are all set to 1 m, and the thickness $h$ is set to 0.01 m. The Young’s modulus $E$ is set to $2.07 \times 10^{11}$ Pa and the poisson ration $\nu=0.3$. The density $\rho$ of the cylindrical shell is 7800 kg/m$^3$.

Modeling the rotating shell with the ANCF thin shell element introduced in Sect.2. In order to obtain more general numerical simulation results, the dimensionless parameters are presented as: $\Omega = \omega_0 / \omega_b$, $\gamma = \omega \times \omega_b$, $\omega_b = \sqrt{D / (\rho h L^2)}$, $D = (1 - \nu^2) / 12(1 - \nu^2)$, where $\Omega$ is the dimensionless natural frequency; $\omega_b$ is the natural frequency; $\gamma$ is the dimensionless rotating angular velocity; $\omega$ is the rotating angular velocity. This model also modeled by ANSYS with 181 shell element.

Table 2 lists the first six dimensionless natural frequencies in three different curvatures of the thin shell under the dimensionless rotating angular velocity 10. Comparing the results in Table 2, it can be seen that the first dimensionless natural frequencies decrease with the decreases of the curvature, while the next five dimensionless natural frequencies increase with the decrease of the curvature. Furthermore, the results obtained in this work are good agreement with the results obtained by ANSYS.

![Figure 5. Rotating shell schematic diagram.](image-url)

| Mode | Present $\pi$ | ANSYS $\pi$ | Present $2\pi/3$ | ANSYS $2\pi/3$ | Present $\pi/3$ | ANSYS $\pi/3$ |
|------|--------------|-------------|------------------|----------------|----------------|--------------|
| 1    | 4.2761       | 4.2947      | 3.8315           | 3.8396         | 3.5852         | 3.5872       |
| 2    | 7.6077       | 7.5929      | 7.8064           | 7.7956         | 8.2779         | 8.2695       |
| 3    | 13.630       | 13.614      | 16.1206          | 16.109         | 19.732         | 19.775       |
| 4    | 17.477       | 17.361      | 19.9129          | 19.803         | 25.412         | 25.356       |
| 5    | 46.942       | 47.062      | 53.4679          | 53.869         | 58.858         | 59.559       |
| 6    | 48.932       | 48.763      | 56.5469          | 56.669         | 64.199         | 64.693       |
Figure 6 shows the first five dimensionless natural frequencies of the thin shell versus the dimensionless rotating angular velocity with the curvature $\pi/3$. It can be seen clearly from the figure that the results obtained in this work are in agreement with the results obtained by ANSYS. The figure also demonstrates that all dimensionless natural frequencies become larger with the increase of the angular velocity.

![Figure 6. First five natural frequencies versus angular velocity.](image)

(a)  
(b)  
(c)  
(d)  

**Figure 7.** First four order mode shape with the curvature $\pi$ ($\gamma=50$), (a) first order, (b) second order, (c) third order, (d) fourth order.
Figure 8. First four order mode shape with the curvature $\pi/3$ ($\gamma=50$), (a) first order, (b) second order, (c) third order, (d) fourth order.

Figure 7 is the first four order mode shapes obtained by ANCF for the rotating shell with a curvature $\pi$ and a dimensionless angular velocity 50. Figure 8 is the first four order mode shapes obtained by ANCF for the rotating shell with a curvature of $\pi/3$ and a dimensionless angular velocity 50. The white surfaces in the figure 7 and 8 are the shapes of the rotating shell and the colored surfaces are the mode shapes. It can be seen clearly from figure 7 and figure 8 that no mode switching happens in the lowest four mode under different curvature of the thin shell.

5. Conclusions
A cylindrical thin shell element of ANCF proposed by Liu [18] was introduced and derived the elastic forces and Jacobin matrix in detail. The validity and the convergence of the element are demonstrated through the case of static bending of parallelogram plate. The corresponding numerical results are in good agreement with the results obtained by Liu [18]. The modal analysis is studied through the second case. The effect of the angular velocity and the curvature on the modal of the rotating shell is discussed. The results of the rotating thin shell case indicate that the first dimensionless natural frequencies of the thin shell decrease with the decreases of the curvature of the thin shell, while the next five dimensionless natural frequencies of the thin shell increase with the decrease of the curvature of the thin shell. Furthermore, the results obtained in this work are in good agreement with the results obtained by ANSYS. They also demonstrated that all dimensionless natural frequencies become larger with the increase of the angular velocity. However, there is no mode switching happens in the lowest four mode under different curvature of the thin shell.

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