Lifetime of respiratory saliva droplets

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Respiratory transmission by airborne droplets is the leading mechanism by which SARS-CoV-2 spreads. Motivated by the pressing need to understand this spread, we formulate a model of the dynamics of respiratory saliva droplets, quantifying the relative contribution of key factors to the airborne lifetime of virus-carrying droplets. We consider a simple momentum balance to calculate the spread of respiratory droplets, accounting for their size evolution as they undergo vaporisation via mass and energy balances. The model shows how an increase in relative humidity leads to larger descent rates, which shorten the lifetime of droplets and therefore reduce the risk of transmission. Emulating indoor air turbulence using a stochastic process, we numerically calculate probability distributions for the lifetime of droplets, showing how an increase in the air turbulent velocity significantly enhances the range of lifetimes. The distributions reveal non-negligible probabilities for very long lifetimes, which potentially escalate the risk of transmission. The droplet model may be coupled with more detailed models for the air flow to study the lifetime and spread of saliva droplets in specific spaces.

Various viral diseases, including SARS-CoV-2, spread through respiratory transmission [1]. A leading precautionary measure to mitigate respiratory transmission is maintaining a ‘safe distance’, reflecting a priori knowledge on the spread of droplets and their time airborne. However, despite intense efforts to study the lifetime of saliva droplets [2,5] and alleviate the risk of transmission [6–13], our understanding of the dynamics of airborne saliva droplets – which dictates their spread and airborne lifetime – remains limited.

Droplets expelled from the human body span a wide range of sizes, from less than a micron to hundreds of microns in diameter [14,15]. The vast majority of expelled droplets satisfy $Re = |v - u| R / \nu_a \ll 1$, where $Re$ is the Reynolds number, $v$ and $u$ the droplet terminal velocity. Accordingly, the droplet terminal velocity downwards is $v_t \propto R^2$. This shows that small droplets, rather than large ones, remain airborne for longer and therefore pose the greater threat for virus transmission. Moreover, small droplets ($<10 \mu m$) typically account for more than 95% of the total number of droplets expelled in a single action, e.g. breath, speech or cough [17]. However, a droplet’s potential to infect a new host successfully depends heavily on its viral content [16], which is proportional to the droplet volume. Consequently, intermediate size droplets – small enough to remain airborne longer than several seconds yet large enough to carry significant viral loads – are most likely to effectively transmit viruses.

Once droplets leave the body they vaporise according to the vapour-pressure balance with the air that surrounds them, possibly after a short period of growth [17]. Saliva droplets do not completely vaporise due to non-volatile substances, resulting in droplets saturating to a small finite size as they reach equilibrium with the surrounding air [18,19]. During the final stage of vaporisation, salts in human saliva can precipitate, leading to a change in the droplet spherical shape [8]. However, recent experiments by Lieber et al. [20] demonstrated that saliva droplets retain a near spherical shape upon equilibrium, even when precipitation occurs; this finding is employed in the analysis below. The infectivity of SARS-CoV-2 in airborne droplets with varying concentration of salts remains, to the best of our knowledge, undetermined. In this work, we therefore consider saliva droplets to be potentially infectious throughout their airborne time.

RESULTS

We describe the dynamics of small airborne particles ($Re \ll 1$) using the aerosol limit of the Maxey–Riley model [21]

\begin{align}
\frac{dx}{dt} &= v, \\
\frac{dv}{dt} &= -6\pi R \rho_a (v - u) - mg,
\end{align}

where $x$ is the droplet position, $m = 4\pi R^3 \rho / 3$ is its mass, with $\rho$ the droplet density, $\mu_a$ is the air dynamic viscosity, and $g$ is the gravitational acceleration. Eq. (2) is obtained from the complete Maxey–Riley model by neglecting all terms proportional to the air-to-liquid density ratio $\vartheta_\rho = \rho_a / \rho \ll 1$. Thus we treat droplets as liquid particles, small enough to be considered rigid (and therefore spherical) and heavy enough that $\vartheta_\rho \approx 10^{-3} \ll 1$.

Volatile droplets lose mass while vaporising and hence their radius varies. To account for this variation, we include a mass balance equation which depends on the droplet temperature. This temperature is in turn gov-

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erned by energy balance, leading to the system

\[
\frac{dm}{dt} = -\frac{\overline{h}_m A M_w}{R} \left( \frac{p}{T} - \frac{p_a}{T_a} \right),
\]

(3)

\[
mc_p \frac{dT}{dt} = H \frac{dm}{dt} - \overline{h} h A (T - T_a),
\]

(4)

where \( A = 4\pi R^2 \) is the droplet surface area, \( p \) and \( T \) are the droplet vapour pressure and temperature, \( p_a \) and \( T_a \) their counterparts for air, \( R \) is the universal gas constant, \( M_w \) is water’s molecular weight, \( c_p \) is the droplet isobaric heat capacity, \( H \) is the heat of vapourisation, and \( \overline{h}_m \) and \( \overline{h} h \) are the convective mass and heat transfer coefficients, respectively [22, 23]. In [30–34] we assume that convection dominates over diffusion between the droplet and air. Next, we non-dimensionalise variables by letting

\[
t = \tau \hat{t}, \ v = U \hat{v}, \ u = U \hat{u}, \ x = \tau U \hat{x},
\]

(5)

\[
R = R_0 \hat{R}, \ g = \frac{U}{\tau} \hat{g}, \ T = T_a \hat{T} \text{ and } p = P \hat{p},
\]

where the hat denotes dimensionless quantities, \( U \) is a characteristic velocity, \( R_0 \equiv R(t = 0) \) is the droplet initial radius, \( \tau = 2 R_0^2 / (9 \nu a \partial \rho) \) is the Stokes time scale, and \( P \) is the atmospheric pressure. To simplify our model, we assume that \( Pr = Sc = \lambda \), where \( Pr = \mu_a c_{p,a} / k_a \) is the Prandtl number with \( c_{p,a} \) and \( k_a \) the air isobaric heat capacity and thermal conductivity, and \( Sc = \nu_a / D \) is the Schmidt number with \( D \) the molecular diffusion coefficient for air-water vapour mixture. This assumption is not strictly necessary for the numerical calculations that follow, however it greatly assists in drawing physical insight from the model with only a marginal sacrificed in accuracy (see table I for typical values of \( Pr \) and \( Sc \)). Building upon the scaling theory for convection over a sphere [22], the mass and heat convection dimensionless parameters – the Sherwood and Nusselt numbers, \( Sh(Re, Sc) = \overline{h}_m R / D \) and \( Nu(Re, Pr) = \overline{h} h R / k_a \), respectively – are identical for \( Pr = Sc = \lambda \) and therefore we write \( Sh = Nu = F \). Finally, we write the model in dimensionless form, omitting all hats for convenience and recalling that \( dm = \rho AdR \) to obtain

\[
\frac{dx}{dt} = v,
\]

(6a)

\[
\frac{dv}{dt} = -\left( \frac{v - u}{\theta} + g \right),
\]

(6b)

\[
\frac{dS}{dt} = -\frac{4 \partial \Phi(S, \theta)}{9 \lambda} F(Re, \lambda),
\]

(6c)

\[
\frac{d\theta}{dt} = -\frac{2 [H_d \partial \Phi(S, \theta) + \partial \theta]}{3 \lambda S} F(Re, \lambda),
\]

(6d)

where \( S \equiv R^2, \ \theta \equiv T - 1, \ \partial \Phi = M_w / M_a \approx 0.62 \) and \( \partial_S = c_{p,a} / c_{p,d} \approx 0.24 \) are the ratios between water and air molecular mass and heat capacity, respectively, \( H_d = H / (c_p T_a) \) is the droplet heat capacity parameter, and \( \Phi \) is the scaled vapour pressure difference between the droplet and air. The latter is expressed through the Clausius-Clapeyron relation, in the form

\[
\Phi = \exp \left[ -H_v \left( \frac{1}{\theta + 1} - \frac{1}{T_b} \right) + \frac{\eta}{S^{1/2}} \right] / (\theta + 1) - \phi \exp \left[ -H_v \left( 1 - \frac{1}{T_b} \right) \right],
\]

(7)

where \( \phi \) is the air relative humidity, \( H_v = HM_w / (RT_a) \) is the vapour latent heat parameter, and \( T_b \) is the water scaled boiling temperature. The term \( \eta / S^{1/2} \) is the addition of capillary evaporation [24], stemming from the droplet curvature, where \( \eta = 2 \gamma_c \partial \rho \partial \Phi / (P R_0) \) is the scaled surface tension with \( \gamma_c \) the water-air surface tension. The term \( \beta S^{-3/2} \), with \( \beta \) a constant, is the correction to the droplet boiling temperature due the increasing concentration of non-volatile substances in its composition [25]. Here we assume that saliva may be modelled as a dilute solution throughout its vaporisation, such that the deterioration of mass from the droplet is expressed only through a change in volume – which is proportional to \( S^{3/2} \) – while the droplet density remains that of water, independent of the (small) concentration of non volatiles. The dimensionless constant \( \beta \) depends on the saliva reference composition; its value is determined below using experimental measurements of saliva droplet vaporisation.

To make the system [6a–6d] complete, an explicit form must be assigned to \( F(Re, \lambda) \). In order to recover the well-known \( D^2 \) law of vapourisation, i.e. that \( S \) decreases linearly with \( t [20], F \) must be constant. This decouples [6c–6d] from the droplet dynamics [6a–6b] and forms a closed system. To determine the value of \( F \), one could invoke Ranz and Marshall’s theoretical result [23] giving \( F = 2 \) for convection over a single sphere in the limit \( Re \to 0 \). Instead, we fit values to \( F \) and \( \beta \) according to experimental measurements of saliva droplet vaporisation.
Stage 1: initial cooling

Droplets expelled from a human body are typically warmer than the surrounding air, i.e. \( \theta (t = 0) > 0 \). The vaporisation process begins with a rapid decrease in \( \theta \) as both vaporisation and convection – both terms \( \Phi \) and \( \vartheta \) in (6d) – respectively – cool the droplet, while its size remains nearly unchanged. As \( \theta \) falls below zero, convection heats the droplet until a balance with vaporisation is reached and its temperature stabilises. The temperature then remains nearly constant throughout the second vaporisation stage, and therefore we denote it by \( \theta_{D2} \). The value of \( \theta_{D2} \) may be computed numerically by setting the right-hand-side (RHS) of (6d) to zero and neglecting the terms \( \beta S^{3/2} \) and \( \eta S^{-1/2} \) in (7), since these only become significant for \( S \ll 1 \), whereas here \( S \approx 1 \).

Stage 2: \( D^2 \) vaporisation

Once a droplet temperature stabilises at \( \theta_{D2} \), the RHS of (6d) becomes nearly constant, giving rise to a linear decrease of \( S \) with time – the well-known \( D^2 \) law [20]. The slope,

\[
\alpha = -\frac{4\vartheta M F \Phi (\theta_{D2})}{9\lambda}, \tag{8}
\]

can be estimated by fitting a straight line to experimental measurements of \( S \) vs. \( t \) in the vaporising stage. Eq. (8) then yields an estimate for the value of the constant \( F \). We use the results of Lieber et al. [20], who recorded the vaporisation of levitating saliva droplets, to evaluate \( F \) by least-squares fitting to the data with \( \phi = 0.534 \) in figure 1 (blue), yielding \( F = 0.707 \). This value is then substituted back to (6c) and used to predict the vaporisation at \( \phi = 0.067 \) (green), showing a good quantitative agreement. This vindicates the use of our model to describe the vaporisation of saliva droplets.

Stage 3: saturation to equilibrium

As water vaporises from a droplet and its size diminishes, the concentration of non-volatiles increases, leading to an increase in the droplet boiling temperature. This process is accounted for by the term \( \beta S^{3/2} \) in (7), recalling that \( S^{3/2} \) is proportional to the droplet volume. As \( S \) decreases, the droplet vapour pressure decreases until, at a finite size \( S_{eq} > 0 \), the droplet and air reach a state of thermodynamic equilibrium and the vaporisation terminates. This equilibrium corresponds to the vanishing the right-hand side of (6c) and (6d), resulting in \( \theta = \Phi = 0 \). This readily provides the means for calculating \( S_{eq} \), by solving \( \Phi (S_{eq}, \theta_{eq} = 0) = 0 \). The limit \( \eta \to 0 \), which neglects the minor effect of surface curvature, gives the

Fig. 1: Time evolution of saliva droplets vaporising in air, expressed as the dimensionless radius squared, \( S(t) = R(t)^2 \). The dots mark experimental measurements from Lieber et al. [20], solid lines are the numerical solution to (6c)-(6d), dashed and dashed dotted lines are the \( D^2 \) linear laws with slopes \( \alpha \) and \( \beta S_{eq} \), respectively. The experimental results for \( \alpha = 0.534 \) (blue) were used to fit the model constants \(- F = 0.707 \) and \( \beta = 8 \cdot 10^{-3} \) – and hence theory and experiment match by construction. These values were then used to predict the time evolution at \( \phi = 0.067 \) (green), showing good quantitative agreement. The results indicate that the droplet size evolution may be approximated as a piecewise function with a linear decay followed by an equilibrium value, \( S_{eq} \).

Droplet vaporisation

The vaporisation process of saliva droplets may be divided into three distinct stages: initial cooling, \( D^2 \) vaporisation, and saturation to equilibrium. Figure 1 shows the size evolution of saliva droplets vaporising in air, where the two last stages are clearly depicted – the linear decrease marking the \( D^2 \) vaporisation stage and the saturation to a constant value corresponding to the last stage. The initial cooling stage typically lasts less than a second and is therefore difficult to observe in figure 1. Below we describe the process in each of these stages, and derive analytic approximations that are key for the analysis that follows.
Droplet dynamics

We now turn our focus to the dynamics of respiratory saliva droplets, governed by (6a)-(6d) with $S(t)$ given by the solution to (6c), (6d). We concentrate our analysis on the vertical motion of droplets as it determines the lifetime – the time for a droplet released from a height $z_0 > 0$ to reach the ground – which is a key metric for assessing the risk of respiratory infection. Longer lifetimes indicate that indoor spaces can remain infectious for extended periods of time, and generally correspond to larger horizontal spread of droplets. Accordingly, $z, v, u$ and $g$ are hereinafter scalar quantities representing vertical components, i.e. aligned with gravity. We begin our analysis by considering the simplest case of $u = 0$, corresponding to a room of completely quiescent air.

Quiescent air

The case $u = 0$ provides a benchmark result for free-falling droplets, from which valuable physical insight can be drawn. By employing the piecewise form (10) of $S$, we derive an analytic solution for $z(t)$. The explicit solution to $z(t)$ is tedious and difficult to interpret (see supplementary information note 1); however, after a droplet reaches its equilibrium size this expression reduces to

$$z(t) \approx z_0 + v_0 - \frac{g}{2|\alpha|} - gS_{eq}(t - t_{eq}), \quad t > t_{eq}, \quad (11)$$

with $z_0$ and $v_0$ the droplet initial height and vertical velocity, respectively. In deriving (11) we considered the case $|\alpha| \ll 1$ and $S_{eq}^2 \ll 1$, which holds throughout the realisable range for $T_a$ and $\phi$ (see supplementary information note 1 for the complete derivation). The first and second terms on the RHS of (11), $z_0 + v_0$, denote the droplet initial conditions, where $|v_0| \ll z_0$ is obtained for most realisable conditions. For simplicity, all the results for quiescent air are calculated with $v_0 = 0$. The third term represents the altitude decrease by time $t_{eq}$, inversely proportional to the $D^2$ evaporation rate, $|\alpha|$, which decreases nearly linearly with $\phi$ (see supplementary information note 2). This indicates that an increase in relative humidity increases the altitude drop during the evaporation stage, which shortens the droplet lifetime. The fourth term recovers the expected linear descent at terminal velocity $v_t = -gS_{eq}$, proportional to $S_{eq}$ that increases with an increase in $\phi$. This demonstrates how an increase in $\phi$ translates to faster descent rates that lead to shorter droplet lifetimes. By setting $z(t) = 0$ in (11) and substituting $t_{eq} = (1 - S_{eq})/|\alpha|$, one easily derives an analytic approximation for a droplet
lifetime in quiescent air,

\[ t_f \approx \frac{1}{S_{eq}} \left( \frac{z_0 + v_0}{g} - \frac{1 - 2S_{eq}^2}{2|\alpha|} \right). \tag{12} \]

Figure 2 (blue curve, left vertical axis) shows the lifetime of two respiratory droplets \( R_0 = 20, 30 \mu m \) in quiescent air as a function of relative humidity, calculated both analytically through (12) as well as numerically by solving (6a)–(6d) with \( u = 0 \) (solid and dashed curves, respectively). The curves clearly show that (i) a droplet lifetime monotonically decreases with an increase in \( \phi \) and that (ii) the analytic approximation (12) closely follows the numerical result, verifying the use of (10) to approximate \( S(t) \) as well as the asymptotic approximations \( |\alpha| \ll 1 \) and \( S_{eq}^2 \ll 1 \).

Indoor turbulence

Air motion in confined spaces is driven by a variety of factors, including wind and forced ventilation. Since windows, doors and most ventilation inlets/outlets are positioned on rooms’ walls (and not on the ground or ceiling), variations in the vertical air velocity are typically much smaller than in the horizontal directions. To avoid describing the flow in specific conditions, we neglect the vertical velocity mean component – which is expected, for example, to carry droplets from the ventilation inlet to the outlet – and propose a simple model for turbulence in the entire room. We mimic the chaotic nature of the flow using an Ornstein–Uhlenbeck process

\[ du = -\gamma u dt + \sigma dW, \tag{13} \]

in which \( W \) is a Wiener process, \( \gamma = 0.58 \, \text{s}^{-1} \) is fitted using experimental measurements of indoor air velocity [27], and \( \sigma = \sqrt{2\gamma U} \) with \( U \) the root-mean-square (RMS) velocity.

DISCUSSION

We calculate the airborne lifetime of droplets, defined as the time for a droplet released at \( z = 1.5 \, \text{m} \) to reach the ground, by solving (6a)–(6d) and (13) numerically. Figure 3 show trajectories of 5 droplets with \( R_0 = 20 \mu m \) at relative humidity \( \phi = 0.6 \) and 0.8. The random trajectories obtained in turbulent air with RMS velocities (a) \( U = 0.1 \) and (b) \( 0.3 \, \text{m/s} \) (solid blue and orange curves) are compared with the deterministic trajectories obtained for quiescent air (black dashed and dashed-dotted curves). The distinction between the \( \phi = 0.6, 0.8 \) bundles of trajectories (clearly visible in figure 3a) demonstrates that the decrease in droplet lifetime as \( \phi \) is increased, predicted analytically for quiescent air, also holds for \( u \neq 0 \). An increase in air velocity results in greater variability of droplet lifetime – as clearly seen in figure 3b – which stems from the enhanced entrainment of droplets with the turbulent air flow.

The findings above can clearly be noted by observing random droplet trajectories, such as the ones depicted in figure 3. In the context of disease transmission, however, it is essential to quantify the probability for anomalously long lifetimes which can dramatically increase the rate of transmission. Accordingly, we characterise the entire range of lifetimes statistically by calculating 5,000 droplet trajectories and collating their lifetimes into a probability density function (PDF). Figure 4 show these PDFs for saliva droplets with initial radii \( R_0 = 20, 35 \) and \( 50 \mu m \), at relative humidity of 60% (red), 70% (blue), and 80% (green), for (a) \( U = 0.1 \) and (b) \( 0.3 \, \text{m/s} \).
The ambient indoor temperature is 20°C. We emphasise that the horizontal axis is logarithmic, demonstrating the extensive variability in droplet lifetime. The black solid and dashed curves are distributions fitted to the data (discussed below), and the markers on the horizontal axis mark 10.lognormal and log-lognormal distributions fitted to the data, respectively.

For comparison, less than %1 of droplets for the equivalent PDF in figure 4 follow the black solid curves, obtained by fitting lognormal distributions to each data set. These distributions have large kurtosis, i.e. fat 'tails', indicating that the probability for anomalously long lifetime is much larger compared with normal distributions with the same mean and variance. As the air RMS velocity increases (figure 4b), the lifetime variability is significantly enhanced. Recalling that lognormal distributions appear as Gaus- sians on a logarithmic scale, we note that the PDFs for $R_0 = 35 \mu m$ in figure 4b deviate strongly from lognormal distribution, displaying substantially fatter tails. These PDFs are reasonably well fitted by log-lognormal distributions, denoted by dashed black curves.

To understand the remarkable statistics in figure 4 we separate the discussion on the lifetime of large ($R_0 = 50 \mu m$) and small ($R_0 = 20 \mu m$) droplets. The large droplets PDFs at varying $\phi$ are grouped together, indicating that evaporation only weakly affects their dynamics. These droplets reach the ground well before the evaporation terminates, and their motion is approximately ballistic regardless of relative humidity. The PDFs in this case are skewed due to the absorbing boundary condition at $z = 0$, inducing an asymmetry in the effect of air velocity – which fluctuates about a zero mean – on the droplet motion. Indeed, increasing the initial height above the ground allows more time for the air velocity to change direction through the droplet lifetime and entrain it more symmetrically, resulting in convergence towards normal statistics.

Small droplets, on the other hand, are less affected by gravity and do not fall a significant distance, on the average, during their evaporation. The fat tails in their PDFs derive from the nonlinear interplay between evaporation and the drag force acting to entrain droplets to the air flow. At early times, when a droplet vaporises and $S(t)$ decreases, the drag force $(u-v)/S(t)$ in 6b increases non-linearly. The symmetry in $u$ about a zero mean, imposed by the Ornstein-Uhlenbeck process, then leads to an asymmetric effect on the lifetime.

Each of the two effects described above – finite domain for large droplets and evaporation for small droplets – leads to a departure from normal to lognormal distribution. For intermediate size droplets ($R_0 = 35 \mu m$), sufficiently large air velocity can trigger a combined effect that dramatically increases the probability for anomalously long lifetimes, as manifested by the $\phi = 0.7$ PDF in figure 4b. The average lifetime for these relatively large droplets is 45 seconds, however %10 of all droplets are predicted to remain airborne more than 90 seconds. For comparison, less than %1 of droplets for the equivalent PDF in figure 4a remain airborne after 90 seconds. Recalling that the number of viral particles carried by a droplet is proportional to $R_0^3$, such non-negligible probabilities for anomalously long lifetime in large droplets can lead to high infection rates.

Our findings have practical implications: the results throughout suggest that maintaining a high relative hu-

FIG. 4: Probability density function for the lifetime of respiratory saliva droplets released 1.5 m above ground with initial radii of 20 (right), 35 (centre), and 50 $\mu m$ (left). The red, blue, and green histograms correspond to relative humidity of $\phi = 0.6$, 0.7 and 0.8, respectively. Solid and dashed black curves are lognormal and log-lognormal distributions fitted to the data, respectively. Solid and dashed black curves are distributions fitted to the data (discussed below), and the markers on the horizontal axis give the lifetimes for $u = 0$ (colours matching the histograms). As expected, smaller droplets remain airborne longer. An increase in relative humidity slows the vapourisation rate and increases droplets’ equilibrium size, thus accelerating their descent to the ground and shortening their lifetime. An increase in air velocity increases the lifetime variability.
atory transmission by shortening the lifetime of saliva droplets. This may be achieved, for example, by using commercially available humidifiers. The statistical analysis proposes that damping of turbulence in a room can significantly decrease the probability for anomalously long lifetime of droplets, thus further alleviating the risk of transmission. This in turn suggests that indoor ventilation strategies, the aim of which is to discharge droplets with a mean flow of air, should also account for a possibly undesired byproduct of mean flows – the triggering or enhancement of turbulence in the entire room. Droplets can possibly be entrained within vortices rather than flow out of a room, which may significantly increase the risk of respiratory transmission.

**DATA AVAILABILITY**

The data and code used to produce the results in this study are available from the corresponding author on request.

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AUTHOR CONTRIBUTION

A.O. designed the research, derived the results and wrote the manuscript; J.V. contributed to the model and to the manuscript.

COMPETING INTERESTS

The authors declare no competing interests.
NOTE 1: ANALYTIC SOLUTION FOR FREE-FALLING DROPLETS IN QUIESCENT AIR

We derive an analytic solution for the descent of free-falling droplets by solving the system (6) in the main text, with the following simplifications: setting the air velocity to $u = 0$ (quiescent air), and approximating the droplet size evolution $S(t)$ using the piecewise form (10) in the main text. The system then simplifies to

$$\frac{dz}{dt} = v,$$  \hspace{1cm} (S1)

$$\frac{dv}{dt} = -\frac{v}{S(t)} - g,$$  \hspace{1cm} (S2)

with

$$S(t) = \begin{cases} 1 - |\alpha| t, & t < t_{eq} \\ S_{eq}, & t > t_{eq} \end{cases},$$  \hspace{1cm} (S3)

subject to the initial conditions $z(t=0) = z_0$ and $v(t=0) = v_0$. The equations are solved successively, with the solution to (S2) substituted into (S1) and integrated, to obtain

$$z(t) = \begin{cases} z_0 - \frac{g(2 - |\alpha| t)}{2(1 - |\alpha|)} + \frac{g + v_0(1 - |\alpha|)}{1 - |\alpha|^2} \times \\ \left[ 1 - (1 - |\alpha| t) \right]^{\frac{1 + |\alpha|}{|\alpha|}} \\ z_c - gS_{eq}(t - t_{eq}) + S_{eq}(v_c + gS_{eq}) \left[ 1 - \exp \left( -\frac{t - t_{eq}}{S_{eq}} \right) \right], & t > t_{eq} \end{cases},$$  \hspace{1cm} (S4)

where $t_{eq} = (1 - S_{eq}) / |\alpha|$, and

$$z_c = z(t = t_{eq}) = z_0 - \frac{1}{1 - |\alpha|} \times \\
\left[ \frac{g}{2} \left( \frac{1 - S_{eq}^2}{2} \right) + \frac{g + v_0(1 - |\alpha|)}{1 + |\alpha|} \left( 1 - S_{eq} \right) \frac{1 + |\alpha|}{|\alpha|} \right] ;$$  \hspace{1cm} (S5)

$$v_c = v(t = t_{eq}) = v_0 \frac{1}{1 - |\alpha|} - \frac{gS_{eq}}{1 - |\alpha|} \left( 1 - S_{eq} \right) \frac{1 + |\alpha|}{|\alpha|} .$$  \hspace{1cm} (S6)

For $t > t_{eq}$, namely after the vaporisation terminates, (S4) can be greatly simplified. Based on experimental measurements of saliva droplet vaporisation [1], we note that $|\alpha| \sim 10^{-4} - 10^{-2}$ and $S_{eq} \sim 0.02 - 0.2$ for the realisable range of air temperature and humidity. Accordingly, we consider the asymptotic limits $|\alpha| \ll 1$ and $S_{eq}^2 \ll 1$, for which the descent simplifies to

$$z(t) = z_0 + v_0 - \frac{g}{2|\alpha|} - gS_{eq}(t - t_{eq}) , \hspace{1cm} t > t_{eq},$$  \hspace{1cm} (S7)

recovering (11) in the main text.

NOTE 2: DEPENDENCY OF $\alpha$ ON RELATIVE HUMIDITY

We being by rewriting the definition of the $D^2$ vaporisation rate $\alpha$,

$$\alpha = -\frac{4\vartheta_M F \Phi (\theta_{D2})}{9\lambda} .$$  \hspace{1cm} (S8)

Recalling that $\vartheta_M, F$ and $\lambda$ are constants, $\alpha$ varies only according to $\Phi$, which is generally a function of both $\theta$ and $S$ and is given as Eq. (7) in the main text. However, during the first two vaporisation stages the droplet is large enough so that both effects involving $S$ are negligible – namely surface tension $\eta S^{-1/2}$ and composition-driven increase in boiling temperature $\beta S^{-3/2}$ – and hence

$$\Phi (\theta_{D2}) = \exp \left[ -\mathcal{H}_v \left( \frac{1}{\theta_{D2} + 1} - \frac{1}{T_b} \right) \right] / (\theta_{D2} + 1)$$

$$- \phi \exp \left[ -\mathcal{H}_v \left( 1 - \frac{1}{T_b} \right) \right] .$$  \hspace{1cm} (S9)

The temperature $\theta_{D2} \leq 0$ reflects a balance between vaporisation and convection, acting to decrease and increase the droplet temperature, respectively. In practice, droplets cool to within several degrees below the air temperature, which in scaled form translates to very small variations from zero. Accordingly, we expand (S9) as a series near $\theta_{D2} = 0$,

$$\Phi (\theta_{D2}) = (1 - \phi) \exp \left[ -\mathcal{H}_v \left( 1 - \frac{1}{T_b} \right) \right] + O \left( \theta_{D2} \right) ,$$  \hspace{1cm} (S10)

showing that $\Phi (\theta_{D2})$, to leading order, is linearly dependent on relative humidity. In turn, we find that $|\alpha|$ decreases nearly linearly with $\phi$. 

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[1] Lieber, C., Melekidis, S., Koch, R. & Bauer, H. J. Insights into the evaporation characteristics of saliva droplets and aerosols: Levitation experiments and numerical modeling. *J. Aerosol Sci.* **154**, 105760 (2021).