Vacuum structure and high-energy scattering*

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This short review deals with the manifestations of the vacuum structure of non-Abelian gauge theories in high-energy scattering. Specifically, it concentrates on instanton-induced hard scattering processes, both in the electroweak gauge theory and in QCD. Soft scattering processes in QCD and their connection to models of semi-hard vacuum fluctuations are also briefly discussed.

1. INTRODUCTION

Non-Abelian gauge theories like QCD are known to possess a rich vacuum structure. Notably, there are topologically non-trivial vacuum fluctuations of the gauge fields, whose simplest examples are instantons. Instantons have been argued to play an important role in various long-distance aspects of QCD, such as giving a possible solution to the axial $U(1)$ problem or being at work in $SU(n_f)$ chiral symmetry breaking. Recently, also a number of QCD lattice studies elucidated the topological structure and the instanton content of the vacuum.

Another prominent feature of the QCD vacuum is its apparent dual superconductivity. The latter has been recognized since long as an explanation of confinement and is also intensively studied on the lattice. These investigations seem to indicate that the fundamental dual excitations are monopoles or vortices, while instantons might be composite objects of those.

The QCD vacuum structure may be studied on the lattice, starting from first principles. Alternatively, low-energy phenomenology can be exploited to learn about the vacuum, at the expense of introducing model assumptions as for example in the instanton liquid model or in the model of the stochastic vacuum, which incorporates the dual superconductor picture of the vacuum.

In this short review, I shall emphasize that manifestations of the vacuum structure of non-Abelian gauge theories can also be searched for in high-energy scattering. I shall concentrate in Sect. 2 on instanton-induced hard scattering processes in the electroweak gauge theory (Quantum Flavor Dynamics (QFD)) and in deep-inelastic scattering (DIS) in QCD, which are calculable from first principles within instanton-perturbation theory. In Sect. 3 I shall discuss briefly soft scattering processes in QCD.

2. HARD SCATTERING PROCESSES INDUCED BY INSTANTONS

Instantons ($I$) are minima of the Euclidean action, localized in space and Euclidean time, with unit topological charge $Q=1$. In Minkowski space-time, they describe tunneling transitions between classically degenerate, topologically inequivalent vacua, differing in their winding number by one unit, $\Delta N_{CS} = Q = 1$. The corresponding energy barrier, under which the instantons tunnel, is inversely proportional to the gauge coupling $\alpha_g$ and the effective $I$-size $\rho_{\text{eff}}$, $M_{\text{sp}} \sim \pi/(\alpha_g \rho_{\text{eff}})$, and of order $\pi M_W/\alpha_W \sim 10$ TeV in QFD and $Q$ in hard scattering in QCD, where $Q \gtrsim 10$ GeV is a large momentum transfer e.g. in DIS.

2.1. Instanton-induced processes

Due to axial anomalies, $I$-induced hard scattering processes are always associated with anomalous fermion-number violation, in particular baryon plus lepton number violation, $\Delta (B+L) = -12 Q$, in the case of QFD, chirality violation, $\Delta Q_5 = 2 n_f Q$, in the case of
QCD. Generically, $I$-induced total cross-sections for hard parton scattering processes are given in terms of an integral over the instanton-antinstanton ($I\bar{I}$) collective coordinates (sizes $\rho, \bar{\rho}$, $I\bar{I}$ distance $R$, color orientation $U$) \[24,11,25\] (see also \[11\] and \[23\])

$$\sigma_{I\bar{I}p_1p_2}^{(I)} \sim \int d^4R \int_0^\infty d\rho \int_0^\infty d\bar{\rho} D(\rho)D(\bar{\rho}) \int d\mu e^{-\frac{4\pi}{\rho} \Omega(U, \frac{\rho^2}{\bar{\rho}^2})}$$

Here, the basic blocks arising in $I$-perturbation theory are the $I$-size distribution $D(\rho)$ and the function $\Omega$, which takes into account the exponential of gauge boson production \[8\] and can be identified with the $I\bar{I}$-interaction defined via the valley method \[24,11,23\].

![Figure 1. QCD $I$-induced process in DIS.](image)

The $I$-size distribution $D(\rho)$ is known in $I$-perturbation theory, $\alpha_s(\mu) \ln(\rho\mu) \ll 1$, up to two-loop renormalization group invariance \[20\]. At one-loop, it reads

$$D(\rho) = \frac{d}{\rho^3} \left( \frac{2\pi}{\alpha_s(\mu)} \right)^2 N_c (\rho \mu)^{\beta_0} S^{(I)} e^{-\frac{2\pi}{\alpha_s(\mu)} S^{(I)}}$$

with $\beta_0 = 11 N_c/3 - 2 n_f/3 - 1/6 n_s$ the first coefficient in the $\beta$ function,

$$S^{(I)} = \begin{cases} 1 & \text{QCD} \\ 1 + M_W^2 \rho^2/2 & \text{QFD} \end{cases}$$

the $I$-action, $\mu$ the renormalization scale, and $d$ a scheme-dependent constant. The quite different $\rho$ dependence of the size distribution (2) for QCD and QFD has important consequences for the predictivity: whereas large-size instantons, $\rho \geq M_W^{-1}$, are exponentially suppressed in QCD (cf. (3)) and thus the relevant contributions to the size integrals in (1) arise consistently from the perturbative region ($\alpha_W(\rho^{-1}) \ll 1$) even if both initial partons are on-shell, $p_i^2 \approx 0$, in QCD the power-law behavior of the size distribution, $\sim \rho^{\beta_0-5}$, generically causes the dominant contributions to (1) to originate from large $\rho \sim \Lambda^{-1} = \alpha_s(\rho^{-1}) \sim 1$ and thus often spoils the applicability of $I$-perturbation theory. In DIS (cf. Fig. 1), however, one parton, $p_1 = q'$ say, carries a space-like virtuality $Q'^2 = -p_1^2 > 0$, such that the contribution of large instantons to the integrals is suppressed by an exponential factor in (1), $e^{-Q'^2(\rho+\bar{\rho})}$, and $I$-perturbation theory becomes exploitable, i.e. predictable, for sufficiently large $Q'$ (16). In this connection it is quite welcome that lattice data on the quenched ($n_f = 0$) QCD vacuum \[27\] can be exploited to infer the region of validity of $I$-perturbation theory for $D(\rho)$ \[7,19\]: As illustrated in Fig. 3 (left), there is very good agreement for $\rho \lesssim 0.35$ fm.

![Figure 2. Illustration of the agreement of lattice data \[27\] for the $I$-size distribution (left) and the $I\bar{I}$-distance distribution (right) with the predictions from $I$-perturbation theory for $\rho \lesssim 0.35$ fm and $R/\rho \gtrsim 1.05$, respectively \[7,19\].](image)

As far as the $I\bar{I}$-interaction $\Omega$ is concerned, it is seen from simple Fourier correspondences in (1), e.g. $R^2 \sim 1/(p_1+p_2)^2 \equiv s'/s$, that at high center-of-mass (cm) energies $\sqrt{s'}$ small $I\bar{I}$-distances are probed, i.e. $\rho\bar{\rho}/R^2 \sim s'/Q^2 \equiv 1/x'-1$ (DIS) or $s'/M_W^2$ (QFD). Thus, in order to make a reliable prediction of $I$-induced hard scattering at high energies, we need to know the interaction for small distances. Again, for QCD one may exploit lattice data on the $I\bar{I}$-distance distribution \[27\] to infer the region of validity of the description of...
the $I\bar{I}$-interaction by its exact expression given in the valley method [11,23]: one finds good agreement for $R/\rho \gtrsim 1.0 \div 1.05$ [14,18] (cf. Fig. 2). In this case, however, there are remaining theoretical ambiguities: The integrations over $\rho, \bar{\rho}$ in the $I\bar{I}$-distance distribution imply significant contributions also from larger instantons with $0.35 \, \text{fm} < \rho, \bar{\rho} < \sim 0.6 \, \text{fm}$, outside the strict region of perturbation theory. It is not excluded that the valley interaction reliably describes the interactions of small-size instantons, $\rho \sim \bar{\rho} \ll \langle \rho \rangle \approx 0.5 \, \text{fm}$, at smaller $R/\rho$, say $R/\rho > 0.5$. 

2.2. QCD-instantons at HERA

With my colleague Fridger Schrempp we have conducted a long-term research program at DESY to work out the theory and phenomenology of hard QCD $I$-induced processes in DIS at HERA [14,15,16,17,18,19]. Meanwhile, the first dedicated search by the H1 collaboration has been published [29]. Several observables characterising the hadronic final state of QCD $I$-induced events were exploited to identify a potentially $I$-enriched domain. The results obtained are intriguing but non-conclusive. While an excess of events with $I$-like topology over the expectation of the standard DIS background is observed, which, moreover, is compatible with the $I$-signal, it can not be claimed to be significant given the uncertainty of the Monte Carlo simulations of the standard DIS background. The data do not exclude the cross-section predicted by $I$-perturbation theory for quite small $I$-sizes $\rho \lesssim 0.2 \, \text{fm}$ and $(R/\rho)_* > 0.9$.

2.3. QFD-instantons at VLHC?

In the early '90s the possibility of observable QFD $I$-effects at cm energies $\gg 10 \, \text{TeV}$ was quite intensively investigated [8,9,10,11,12,13]. But despite considerable theoretical efforts the actual size of the cross-sections in the relevant energy regime was never established. In view of the similarity between QFD and hard QCD $I$-induced processes in DIS and of the recent information about the latter both from the lattice and from experiment, it seems worthwhile to reconsider the subject [18]. Figure 4 represents a state-of-the-art evaluation of the QFD $I$-induced quark-quark cross-section, which neglects Higgs production [11]. It demonstrates that the cross-section is unobservably small in the conservative fiducial region corresponding to $(R/\rho)_* > 1$. However, it becomes of observable size if the $I\bar{I}$-attraction remains valid also for slightly smaller values of $(R/\rho)_* > 0.7$ at $\sqrt{s} \approx 4\pi M_W/\alpha_W = 30 \, \text{TeV}$. This opens up exciting opportunities at future colliders [9] such as VLHC [30] or at cosmic ray facilities and neutrino telescopes [12].
3. SOFT HIGH-ENERGY PROCESSES

One of the main challenges of QCD is to understand the high-energy, but small momentum transfer cross-sections of hadrons from first principles. In this domain, perturbation theory is not applicable and direct lattice simulations are not possible until now. There have been recently several attempts to face this challenge [31]. Most of them are based on the eikonal representation of the amplitudes in terms of correlators of Wilson lines or loops [32], which thereafter are evaluated using different model assumptions. The most advanced attempt is certainly the one of the Heidelberg group [33] based on the model of the stochastic vacuum [7], which has been already successfully applied to a variety of soft high-energy processes (e.g. [34]). Less developed but rapidly evolving is the understanding of a possible connection of semi-hard instantons to diffraction [35]. There exists the intriguing possibility that larger-size instantons build up diffractive scattering, with the marked $I$-size scale $\langle \rho \rangle \approx 0.5$ fm (Fig. 2 (left)) being reflected in the conspicuous “geometrization” of soft QCD [36].

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