Dynamical determination of the unification scale
by gauge-mediated supersymmetry breaking

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Abstract

We propose a mechanism for generating the grand-unification (GUT) scale dynamically from the Planck scale. The idea is that the GUT scale is fixed by the vacuum expectation value of a ‘GUT modulus’ field whose potential is exactly flat in the supersymmetric limit. If supersymmetry is broken by gauge mediation, a potential for the GUT modulus is generated at 2 loops, and slopes away from the origin for a wide range of parameters. This potential is stabilized by Planck-suppressed operators in the Kähler potential, and the GUT scale is fixed to be of order $M_*/(4\pi^2)$ (where $M_* \sim 10^{18}$ GeV is the reduced Planck scale) independently of the supersymmetry breaking scale. The cosmology of this scenario is acceptable if there is an epoch of inflation with reheat temperature small compared to the supersymmetry-breaking scale. We construct a realistic GUT that realizes these ideas. The model is based on the gauge group $SU(6)$, and solves the doublet-triplet splitting problem by a sliding singlet mechanism. The GUT sector contains no dimensionful couplings or tuned parameters, and all mass scales other than the Planck scale are generated dynamically. This model can be viewed as a realistic implementation of the inverted hierarchy mechanism.

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I. INTRODUCTION

The study of supersymmetric grand unified theories (SUSY GUT’s) has received renewed impetus in recent years because of the striking agreement between the observed values of the gauge couplings and the predictions of the simplest supersymmetric unified theories [1]. Also, the recent revival of models of gauge-mediated SUSY breaking has opened up new possibilities for the realization of SUSY in nature [2,3,4]. (For a review, see Ref. [5].) The condition that gauge-mediated models can be embedded in a GUT is often imposed, with the motivation that one does not want to give up the successful prediction of gauge coupling unification. In this paper, we explore the possibility that there is a deeper connection between grand unification and gauge-mediated SUSY breaking. We construct a model in which the dynamics that breaks SUSY is also responsible for fixing the GUT scale via an ‘inverted hierarchy’ mechanism [6]. The mechanism is very robust, and the specific model we construct to implement it is quite simple.

The idea is that the GUT scale is determined by minimizing the potential for an almost-flat ‘GUT modulus’ field Φ whose vacuum expectation value (VEV) determines the GUT scale. It is assumed that SUSY is broken by the VEV of a singlet field $X$ below the GUT scale: $F \equiv \langle F_X \rangle \ll \langle \Phi \rangle^2$. The field $X$ has trilinear couplings to charged ‘messenger’ fields, and loop corrections involving gauge bosons give rise to supersymmetry breaking terms in the observable sector. In such models, SUSY breaking is communicated to Φ via 2-loop graphs, which give Φ a logarithmic potential that slopes away from the origin for a wide range of parameters:

$$V_{GMSB}(\Phi) \sim -\frac{F^2}{(4\pi^2)^2} \ln^2 \Phi.$$  (1.1)

If this were the only contribution to the potential, Φ would run away to infinity. However, in the context of supergravity (or string theory), we expect the effective field theory below the Planck scale to contain operators such as

$$L_{\text{eff}} \sim \pm \int d^4\theta \frac{1}{M_*^2} X^\dagger X \Phi^\dagger \Phi = \pm \frac{F^2}{M_*^2} \Phi^\dagger \Phi + \cdots,$$  (1.2)

where $M_* = M_{\text{Planck}}/\sqrt{8\pi} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass. This stabilizes the potential for Φ if the sign is negative, and gives

$$\langle \Phi \rangle \sim \frac{1}{4\pi^2} M_*.$$  (1.3)

This relation is very robust: it is independent of the SUSY breaking scale $F$, and gives the right magnitude for the GUT scale as long as all dimensionless couplings are order one.

The mass of Φ is

$$m_\Phi \sim \frac{F}{M_*},$$  (1.4)

so $m_\Phi$ is small compared to the weak scale precisely when gauge-mediated contributions to SUSY breaking dominate over the supergravity mediated contribution. The interactions of Φ at low energies are given by higher-dimension operators suppressed by powers of
\(\langle \Phi \rangle \sim 10^{16} \text{ GeV}\). This explains why the \(\Phi\) particles cannot be seen directly in laboratory experiments.

We construct a realistic GUT model that realizes these ideas. The model is based on gauge group \(SU(6)\), with the doublet-triplet splitting problem solved using a version of the sliding singlet mechanism \([\text{Ref.}]\) suggested by Barr \([\text{Ref.}]\). The Higgs sector consists of \(35 \oplus 35 \oplus 4 \times (6 \oplus \bar{6})\) plus singlets. To incorporate 3 generations of matter, one needs \(3 \times (15 \oplus 6 \oplus \bar{6})\) and additional \(15 \oplus 15\) Higgs fields. The model is therefore easily perturbative up to the Planck scale. The Higgs superpotential contains only dimensionless couplings, and all mass scales arise from the VEV of the field \(\Phi\), which is fixed dynamically by the mechanism above. We do not specifically address the ‘\(\mu\) problem,’ but this can be solved by simply adding a \(\mu\) term to the model, or by one of the mechanisms previously proposed in the literature \([\text{Ref.}]\).

Models with the mechanism described in this paper suffer from a version of the well-known ‘Polonyi’ or ‘moduli’ problem \([\text{Ref.}]\). Because the potential for \(\Phi\) is very flat, and the interaction of \(\Phi\) with other light fields is very weak at low energies, it is not easy to understand why \(\Phi\) is close to its minimum in the early universe. Coherent oscillations of \(\Phi\) about the minimum can dominate the energy density of the universe, giving rise to an early matter-dominated era. Nucleosynthesis is not possible during the \(\Phi\)-dominated era \([\text{Ref.}]\), and the eventual decay of \(\Phi\) does not reheat the universe sufficiently to allow nucleosynthesis. These problems can be avoided by assuming an epoch of inflation with low vacuum energy \([\text{Ref.}]\). If the vacuum energy during inflation and reheating is sufficiently small, \(\Phi\) is underdamped and relaxes to its minimum on a time scale \(1/H\), where \(H\) is the expansion rate during inflation. It is important to take into account the fact that the minimum of the \(\Phi\) potential is shifted from its vacuum value due to supersymmetry breaking effects in the early universe \([\text{Ref.}]\). Nonetheless, inflation suppresses the energy density in \(\Phi\) oscillations and allows for a realistic cosmology. As emphasized in Ref. \([\text{Ref.}]\), the density fluctuations that are the seeds for structure formation can arise at a higher scale, and mild constraints on the low-scale inflation ensure that the fluctuations are not destroyed. This model is therefore compatible with the presently-favored scenario of structure formation seeded by inflation at high energy scales.

Of course, the present proposal is not the only possible mechanism for fixing the GUT scale dynamically. The GUT scale can emerge via dimensional transmutation in weakly-coupled models \([\text{Ref.}]\) or strongly-coupled models \([\text{Ref.}]\). In these models, the value of \(M_{\text{GUT}}\) depends sensitively on the values of dimensionless couplings. It has also been proposed that the potential for a GUT modulus such as we are suggesting can arise from supergravity (or string) effects \([\text{Ref.}]\). Our mechanism differs from these proposals in that the ratio \(M_{\text{GUT}}/M_*\) is a robust prediction that does not depend on details of Planck-scale physics.

This paper is organized as follows. In Section II, we describe the model, compute and minimize the potential for the GUT modulus field, and construct the low-energy effective lagrangian. In Section III we discuss the cosmology of this model. Section IV contains our conclusions.
II. THE MECHANISM

In this Section we describe in detail the mechanism for fixing the GUT scale in the context of a specific model. The main ideas are more general than the model we present, and we will emphasize the features that are important for our mechanism.

A. Higgs Sector

The first requirement for a successful model of the type outlined in the Introduction is a Higgs sector that breaks a GUT group down to $SU(3) \times SU(2) \times U(1)$ at a scale given by the VEV of a ‘GUT modulus’ flat direction $\Phi$. In order to explain the success of gauge coupling unification, we demand that the model incorporate a natural solution to the doublet-triplet splitting problem. Since we are trying to explain the origin of mass scales, we demand that the Higgs sector contain no dimensionful couplings.

We now describe a simple model that satisfies all these requirements. The model has gauge group $SU(6)$ and a Higgs sector consisting of the following charged fields:

$$\Sigma, \Delta \sim 35, \quad H_{1,2}, \quad h_{1,2} \sim 6, \quad \bar{H}_{1,2}, \bar{h}_{1,2} \sim \bar{6}.$$  (2.1)

In addition, there are 8 singlets $\Phi$, $S$, $T_{1,2}$, $U_{1,2}$, and $\bar{U}_{1,2}$. The superpotential for the Higgs sector is

$$W_{\text{Higgs}} = \frac{1}{2}S(tr \Sigma^2 - \Phi^2) + \frac{1}{6}tr \Sigma^3$$
$$+ \sum_{J=1}^{2} T_J(H_JH_J - \frac{1}{2}\Phi^2)$$
$$+ \sum_{J=1}^{2} \left[H_J\Sigma h_J + U_JH_Jh_J + \bar{h}_J\Sigma H_J + \bar{U}_J\bar{h}_JH_J \right]$$
$$+ \frac{1}{2}\Phi tr \Delta^2 + (\bar{H}_1\Delta H_2 - \bar{H}_2\Delta H_1).$$  (2.2)

(We will discuss fermion masses below.) This superpotential is invariant under the $Z_2$ symmetry

$$H_1 \leftrightarrow H_2, \quad \bar{H}_1 \leftrightarrow \bar{H}_2, \quad h_1 \leftrightarrow h_2, \quad \bar{h}_1 \leftrightarrow \bar{h}_2, \quad U_1 \leftrightarrow U_2, \quad \bar{U}_1 \leftrightarrow \bar{U}_2, \quad \Delta \leftrightarrow -\Delta.$$  (2.3)

with all other fields invariant. Because the superpotential contains only dimension-3 terms, it is also invariant under a $U(1)_R$ symmetry under which all fields have charge $\frac{2}{3}$. Eq. (2.2) is not the most general superpotential allowed by symmetries (e.g. a $S^3$ term is allowed). However, we do not consider this to be problematic since perturbative non-renormalization theorems and their non-perturbative generalizations \cite{17} allow the superpotential to naturally be ‘non-generic.’

The freedom to rescale the fields has been used to set all Yukawa couplings to 1. The information about the strength of the superpotential couplings is therefore contained in the normalization of the kinetic terms.
This potential has supersymmetric minima with

\[ \langle \Sigma \rangle = \frac{\langle \Phi \rangle}{\sqrt{6}} \left( \begin{array}{cc} 1_3 & 0 \\ 0 & -1_3 \end{array} \right), \quad \langle H_{1,2} \rangle = \frac{1}{\sqrt{2}} \langle \mathcal{H}_{1,2} \rangle = \left( \begin{array}{c} 0 \\ \vdots \\ 0 \end{array} \right), \quad \langle U_{1,2} \rangle = \frac{\langle \Phi \rangle}{\sqrt{6}}. \] (2.4)

and all other VEV’s vanishing. This breaks \( SU(6) \) down to \( SU(3) \times SU(2) \times U(1) \). In the supersymmetric limit, there is a flat direction along which \( \Phi \) changes, and \( \langle \Phi \rangle \) is undetermined at this stage. We therefore refer to the field \( \Phi \) as the ‘GUT modulus.’ We require \( \langle \Phi \rangle \sim 10^{16} \) GeV for a successful model.

The roles of the various terms in Eq. (2.2) are not hard to understand. The terms of the form \( S(\Phi^2 - \text{tr} \Sigma^2) \) and \( T(\mathcal{H}H - \frac{1}{2} \Phi^2) \) force \( \langle \Sigma \rangle \sim \langle H \rangle \sim \langle \Phi \rangle \). The terms \( \mathcal{H} \Sigma h + U \mathcal{H} h \) and \( h \Sigma \mathcal{H} + \bar{U} h \mathcal{H} \) force

\[ \langle \Sigma + U_{1,2} 1_6 \rangle = \langle \bar{U}_{1,2} 1_6 \rangle = \frac{2\langle \Phi \rangle}{\sqrt{6}} \left( \begin{array}{cc} 1_3 & 0 \\ 0 & 0 \end{array} \right), \] (2.5)

which gives mass terms to the Higgs triplets while leaving the doublets massless. This is a version of the sliding singlet mechanism \(^1\) due to Barr \(^2\). Finally, the terms involving \( \Delta \) are needed to give mass to two doublet components of the fields \( H_{1,2} \), \( \mathcal{H}_{1,2} \). (In the model of Ref. \(^3\), one doublet gets a mass from a higher-dimension operator, potentially upsetting gauge coupling unification.)

A nontrivial feature of the superpotential Eq. (2.2) is that the VEV of \( \Phi \) is not fixed in the supersymmetric limit. This is ensured by coupling \( \Phi \) only to fields that have zero VEV in the desired vacuum. This requirement is nontrivial for the term \( S(\Phi^2 - \text{tr} \Sigma^2) \), since the equation of motion for the diagonal components of \( \langle \Sigma \rangle \) is a quadratic equation, and \( \langle S \rangle \) is proportional to the sum of the roots. We have checked explicitly that the only massless fields correspond to the flat direction and a pair of doublets

\[ h^\alpha \equiv \frac{1}{\sqrt{2}} (h_1 - h_2)^\alpha, \quad \bar{h}_\alpha \equiv \frac{1}{\sqrt{2}} (\bar{h}_1 - \bar{h}_2)_\alpha, \quad \alpha = 4, 5. \] (2.6)

These are the features we need for our mechanism to fix the GUT scale dynamically.

Note that the \( U(1)_R \) symmetry is spontaneously broken, and so the model as written has an \( R \) axion with a decay constant of order \( M_{\text{GUT}} \). However, the \( U(1)_R \) symmetry may be explicitly broken by dimension-5 terms in the effective Kähler potential that couple the GUT modulus to the field \( X \) that is responsible for SUSY breaking:

\[ \delta \mathcal{L}_{\text{eff}} = \int d^4 \theta \left[ \frac{a_1}{M_*} X^\dagger \Phi^2 + \frac{a_2}{M_*} X^\dagger X \Phi + \text{h.c.} \right]. \] (2.7)

The term proportional to \( a_1 \) gives the \( R \) axion a mass of order \( m_\Phi \sim F/M_* \) if \( a_1 \sim 1 \). The term proportional to \( a_2 \) gives a large shift to the vacuum found above unless \( a_2 \ll g^2/(4\pi^2) \).

It is natural to have \( a_2 \ll a_1 \) because the terms have different charges under global (discrete) symmetries.\(^3\) Therefore, the \( R \) axion mass can be anywhere below \( m_\Phi \) depending on the pattern of breaking of global symmetries at the Planck scale.

\(^1\)We have checked that there is a discrete \( R \) symmetry that forbids \( a_2 \) under which the superpotential Eq. (2.2) (and Eqs. (2.9) and (2.20) below) is invariant.
B. Fermion Masses

Although we view the model above mainly as an illustration, it is worth noting that fermion masses can be incorporated without significantly complicating the Higgs sector. We add the fields

\[ N_j \sim 15, \quad \bar{P}_{1j}, \bar{P}_{2j} \sim \bar{6}, \quad Y \sim 15, \quad \bar{Y} \sim \bar{15}, \tag{2.8} \]

where \( j = 1, 2, 3 \) is a generation index. (Note that there must be two \( \bar{6} \)'s for each \( 15 \) to satisfy \( SU(6) \) anomaly cancellation.) The additional superpotential terms required are

\[
W_{\text{fermion}} = \lambda^{jk} N_j (\bar{P}_{1k} \bar{H}_1 + \bar{P}_{2k} \bar{H}_2) \\
+ y_d^{jk} N_j (\bar{P}_{1k} \bar{h}_1 + \bar{P}_{2k} \bar{h}_2) \\
+ y_u^{jk} N_j N_k Y + \Phi \bar{Y} Y + \bar{Y} (H_1 h_1 - H_2 h_2). \tag{2.9} \]

The discrete symmetry Eq. (2.3) is extended to the additional fields via

\[ N \mapsto -iN, \quad \bar{P}_1 \mapsto i\bar{P}_2, \quad \bar{P}_2 \mapsto i\bar{P}_1, \quad Y \mapsto -Y, \quad \bar{Y} \mapsto -\bar{Y}. \tag{2.10} \]

(The symmetry is therefore \( Z_4 \) rather than \( Z_2 \).) We have scaled the fields \( Y \) and \( \bar{Y} \) to set some of the superpotential couplings to 1.

The roles of the terms in Eq. (2.9) are as follows. The \( N (\bar{P}_1 \bar{H}_1 + \bar{P}_2 \bar{H}_2) \) term gives a mass of order \( \langle \Phi \rangle \) to the unwanted components of the \( 15 \) and two \( \bar{6} \)'s, leaving 3 generations of quarks and leptons massless. The remaining terms give rise to the fermion Yukawa couplings. The up-type Yukawa couplings arise when the massive fields \( Y \) and \( \bar{Y} \) are integrated out. It is not hard to see that adding these terms to the superpotential preserves the vacuum described above.

Like most SUSY GUT’s, the present model has a potential problem with proton decay from the effective dimension-5 operator

\[
W_{\text{eff}} \sim \frac{y_u^{jk} y_d^{\ell m}}{\langle \Phi \rangle} Q_j Q_k Q_{\ell} L_m \tag{2.11} \]

arising from exchange of heavy color triplet Higgs fields.

As written, this model embodies unsuccessful \( SU(5) \) GUT relations among fermion masses, namely the equality of lepton and down-type quark masses (up to radiative corrections). As is well-known, this actually works well for the third generation but not for the first two generations. This can be remedied by assuming that there are higher-dimension operators such as

\[
\delta W_{\text{fermion}} = \frac{c^{jk}}{M_s} N_j \Sigma (\bar{P}_{1k} \bar{h}_1 - \bar{P}_{2k} \bar{h}_2), \tag{2.12} \]

whose contribution to the Yukawa couplings can be important for the light fermions.
Contributions to the effective potential of $\Phi$ in gauge-mediated models. Here $q$ denotes a messenger field, $H$ denotes a superheavy Higgs field, and $G$ denotes a superheavy gauge boson field. The dependence on $\Phi$ enters because the masses of $H$ and $G$ are proportional to $\langle \Phi \rangle$. The $H$ contribution drives $\Phi \to \infty$, while the $G$ contribution drives $\Phi \to 0$.

**C. Fixing the GUT Modulus**

In the class of models we are considering the VEV of the GUT modulus field is undetermined in the SUSY limit, but SUSY breaking will ultimately lift all flat directions and determine the GUT scale. If supersymmetry breaking is broken in a hidden sector and communicated to the observable sector by supergravity, the leading contribution to the potential for $\Phi$ is expected to arise from effective Kähler terms of the form

$$\mathcal{L}_{\text{eff}} = \int d^4 \theta \left[ \frac{c}{M_*^2} X^\dagger X \Phi^\dagger \Phi + \mathcal{O}(1/M_*^2) \right] = \frac{c|F|^2}{M_*^2} \Phi^\dagger \Phi + \cdots, \quad (2.13)$$

where $X$ is a field whose VEV breaks SUSY:

$$F \equiv \langle F_X \rangle \ll \langle X \rangle^2. \quad (2.14)$$

The potential Eq. (2.13) drives $\Phi$ either to the origin or to infinity depending on the sign of $c$, so this does not give a mechanism to determine the GUT scale. If $\Phi$ is driven to infinity, new physics presumably enters for $\langle \Phi \rangle \gtrsim M_*$, but it is not easy to see how this could lead to $\langle \Phi \rangle \sim 10^{-2} M_*$.\(^2\)

However, we now describe a natural mechanism to fix the GUT modulus at the correct value in theories of gauge-mediated supersymmetry breaking. In these models, the leading

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\(^2\)Eq. (2.13) is expected to be the leading correction to the Kähler potential as long as all field strengths are small compared to $M_*$ in models where the only new physics (compactified dimensions, excited string modes, etc.) arise at or above the scale $M_*$. If there are additional scales or light fields coupling the ‘hidden’ and observable sectors, then there may be a stable minimum for the GUT modulus below $M_*$. 

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contribution to the potential for $\Phi$ comes from the 2-loop graphs shown in Fig. 1. We assume that $X$ is coupled to charged messenger fields $q, \bar{q}$ via

$$W_{\text{mess}} = \lambda X \bar{q} q.$$  \hspace{1cm} (2.15)

Using the techniques of Ref. [18] this contribution to the effective potential can be computed by solving 1-loop renormalization group equations analytically continued into superspace. (For earlier calculations, see Ref. [19].) The result is

$$|\Phi| \frac{\partial V}{\partial |\Phi|} \simeq \sum_q \frac{|\lambda|^2|F|^2}{4\pi^2} \left[ \gamma_q(|\Phi|) - \gamma_{q,\text{eff}}(|\Phi|) \right] \ln \frac{|\Phi|}{|\lambda X|},$$ \hspace{1cm} (2.16)

where the sum is over all components of the messenger field $q$, and

$$\gamma_q(\mu) \equiv \mu \frac{d \ln Z_q}{d\mu}$$  \hspace{1cm} (2.17)

is the messenger anomalous dimension. Here, $\gamma_q$ ($\gamma_{q,\text{eff}}$) refers to the anomalous dimension in the theory above (below) the GUT threshold evaluated at $\mu = |\Phi|$. In Eq. (2.16) we have not summed logs of $|\lambda X|/|\Phi|$, and we have identified the GUT threshold with $\Phi$ (rather than Yukawa couplings times $\Phi$). These effects can be easily included, but they are not important at the level of accuracy we are working.

Because the potential is proportional to the difference of messenger anomalous dimensions above and below the scale $\Phi$, it can be viewed as arising from fields that get massive at the scale $\Phi$. This makes the sign easy to understand. Gauge loops give a positive contribution to $\gamma_q$, while matter loops give a negative contribution. The potential will therefore slope away from the origin provided that the contribution from matter loops dominates, a condition that is easily satisfied (see below).

For sufficiently large values of $\Phi$, the supergravity contribution Eq. (2.13) becomes important. We assume $c < 0$, which corresponds to a positive value for the supergravity-induced mass for $Y$. This stabilizes the runaway behavior of the potential and fixes $\langle \Phi \rangle$. Neglecting couplings and group theory factors, $\gamma_q \sim 1/(4\pi^2)$, and the minimum is

$$\langle \Phi \rangle \sim \frac{M_*}{4\pi^2} \left( \ln \frac{\langle |\Phi| \rangle}{\langle |X| \rangle} \right)^{1/2}.$$  \hspace{1cm} (2.18)

Since $M_*/(4\pi^2) \simeq 6 \times 10^{16}$ GeV, this nicely explains the ‘observed’ value $\langle \Phi \rangle \simeq 3 \times 10^{16}$ GeV. It is remarkable that the result is independent of $F$, and depends only logarithmically on $\langle X \rangle$.

For example, in our $SU(6)$ model, the simplest possibility is to take

$$q \sim 6, \quad \bar{q} \sim \bar{6},$$  \hspace{1cm} (2.19)

and couple the messengers to the adjoint Higgs field $\Delta$ as well as the singlet field $X$ responsible for SUSY breaking:

$$W_{\text{mess}} = \kappa \bar{q} \Delta q + \lambda X q \bar{q}.$$  \hspace{1cm} (2.20)
(Recall that \( \langle \Delta \rangle = 0 \), so this does not generate a superheavy mass for the messengers.) We therefore have
\[
\gamma_q = \frac{C_q}{4\pi^2} g^2 - \frac{C_q}{8\pi^2} |\kappa|^2, \tag{2.21}
\]
where \( C_q = (N^2 - 1)/(2N) \) is the messenger Casimir. Below the scale \( \Phi \), there are contributions to the \( q \) anomalous dimension from the unbroken \( SU(3) \times SU(2) \times U(1) \) gauge fields, and we obtain
\[
\sum_q \left[ \gamma_q(|\langle \Phi \rangle|) - \gamma_q,\text{eff}(|\langle \Phi \rangle|) \right] = \frac{1}{4\pi^2} \left( \frac{23}{2} g^2 - \frac{35}{4} |\kappa|^2 \right). \tag{2.22}
\]
This is naturally negative for \( \kappa \sim 1 \). Assuming that the contribution proportional to \( \kappa \) dominates, we obtain
\[
|\langle \Phi \rangle| \simeq \frac{M_* |\lambda\kappa|}{4\pi^2} \left( \frac{35}{8|c|} \ln \left| \frac{|\langle \Phi \rangle|}{|\lambda\langle X \rangle|} \right| \right)^{1/2}. \tag{2.23}
\]
We see that a realistic GUT scale does in fact emerge for reasonable values for the couplings.

**D. Interactions of the GUT Modulus**

From Eqs. (2.13) and (2.16), we see that the mass of the GUT modulus is
\[
m_\Phi \sim \frac{F}{M_*}, \tag{2.24}
\]
\emph{i.e.} the same magnitude as the SUSY-breaking masses communicated by supergravity. Because we want gauge-mediation to dominate SUSY breaking in the observable sector, we have \( m_\Phi \lesssim 100 \text{ GeV} \). Also, the smallest possible value for \( F \) is of order \( (10 \text{ TeV})^2 \), which gives
\[
10^{-1} \text{ eV} \lesssim m_\Phi \lesssim 100 \text{ GeV}. \tag{2.25}
\]

The existence of such a light particle is not immediately ruled out because it interacts with other light fields only through higher-dimension operators suppressed by powers of \( 1/|\langle \Phi \rangle| \). This follows from the double role of the field \( \Phi \). On the one hand, the VEV of \( \Phi \) sets the scale for all the superheavy masses. On the other hand, if we write
\[
\Phi = \langle \Phi \rangle + \Phi', \tag{2.26}
\]
\( \Phi' \) can only appear in the combination \( \langle \Phi \rangle + \Phi' \) because of the invariance under simultaneous shifts of \( \langle \Phi \rangle \) and \( \Phi' \) that keep \( \Phi \) fixed. \footnote{The precise statement is that it is possible to choose the fields in the low-energy effective theory to have this property.}

\[\]
in the effective theory from the dependence on the GUT threshold. The leading dependence on \( \Phi \) comes from the fact that the dimensionless couplings in the low-energy theory depend logarithmically on \(|\Phi|\) from the renormalization group evolution from the GUT scale. This gives rise to interactions such as

\[
\mathcal{L}_{\text{eff}} \sim \frac{g^2}{4\pi^2} \frac{\Phi'}{\langle \Phi \rangle} F_{\mu\nu}^\mu F_{\mu\nu},
\]

where the factor \( \Phi'/\langle \Phi \rangle \) arises from a difference of anomalous dimensions, as in the calculation of the potential above. The scalar component of the GUT modulus can therefore decay to photons with rate

\[
\Gamma(\phi_\Phi \to \gamma\gamma) \sim \frac{1}{4\pi} \left( \frac{\alpha}{\pi} \right)^2 \frac{m_\Phi^3}{\langle \Phi \rangle^2} \approx \frac{1}{2} \times 10^{45} \text{sec} \left( \frac{\sqrt{F}}{10 \text{ TeV}} \right)^6.
\]

Decay widths to other modes (such as \( e^+e^- \) and \( \bar{\nu}\nu \)) are comparable. The fermion component of \( \Phi \) is expected to be stable, since there is generally no lighter fermion that it can decay into.

### III. COSMOLOGY

The potential for the field \( \Phi \) is very flat, and the interaction of \( \Phi \) with other light fields is very weak at energies below the GUT scale. This leads to a potential cosmological problem known as the ‘Polonyi’ or ‘moduli’ problem [10]. The problem is to understand why \( \Phi \) is close to the minimum of its potential in the early universe. \( \Phi \) will begin to oscillate about the minimum of its potential when the expansion rate becomes smaller than its mass. The coherent oscillations scale like non-relativistic matter, so \( \Phi \) oscillations will dominate the energy density unless \( \Phi \) is very close to its minimum in the early universe. If this condition is not satisfied, \( \Phi \) oscillations dominate the universe, and the eventual \( \Phi \) decay reheats the universe. The reheat temperature is

\[
T_{\Phi,\text{RH}} \sim \frac{\sqrt{T_{\Phi,\text{RH}}}}{g_{*}^{1/4}(\Phi,\text{RH})} \sim (10 \text{ eV}) \left( \frac{\sqrt{F}}{10^{19} \text{ GeV}} \right)^3.
\]

This is far too low to allow nucleosynthesis after \( \Phi \) decay, and successful nucleosynthesis is impossible during the \( \Phi \)-dominated era [11].

The natural framework for solving this problem is low-scale inflation [12]. During the slow-roll phase of inflation, the \( \Phi \) equation of motion is

\[
\ddot{\Phi} + 3H_{\text{inf}} \dot{\Phi} + \frac{\partial V}{\partial \Phi} = 0,
\]

where \( H_{\text{inf}} \) is the expansion rate of the universe during inflation, and we have neglected the \( \Phi \) decay term. If \( H_{\text{inf}} \lesssim m_\Phi \), the equation for \( \Phi \) is underdamped or critically damped, and \( \Phi \) approaches the minimum of its potential on a time scale \( 1/(3H_{\text{inf}}) \). As long as inflation persists for several \( e \)-folds, \( \Phi \) will be driven close to its minimum. In terms of microscopic parameters, the condition for this to occur is
\[ V_{\text{inf}} \lesssim F^2, \quad (3.3) \]

where \( V_{\text{inf}} \) is the vacuum energy that drives inflation.

If \( V_{\text{inf}} \gg F^2 \), then \( H_{\text{inf}} \gg m_\Phi \) and the evolution of \( \Phi \) is overdamped. \( \Phi \) therefore approaches its minimum on a time scale \( 3H/m_\Phi^2 \gg 1/H \). We will see that this scenario is ruled out even if we accept the enormous number of \( e \)-folds required for \( \Phi \) to relax to its minimum.

The effective potential for \( \Phi \) during inflation is not the same as the vacuum potential. This is important because the flatness of the \( \Phi \) potential is protected by supersymmetry, which is explicitly broken by the finite energy density in the early universe [13]. In general, there will always be a contribution to the effective potential for \( \Phi \) of the form
\[ \Phi \frac{\partial V_{\text{eff}}}{\partial \Phi} \propto \rho, \quad (3.4) \]

where \( \rho \) is the energy density. It will be important for us to have an estimate of the constant of proportionality during the slow-roll phase of inflation. For this we need to know the couplings between the inflaton field \( I \) and the GUT modulus \( \Phi \). The most conservative possible assumption is that \( \Phi \) couples to the inflaton as strongly as to visible fields. For example, if the inflaton couples to superheavy fields at the GUT scale with dimensionless coupling constant \( h \), we obtain
\[ \Phi \frac{\partial V_{\text{eff}}}{\partial \Phi} \sim \frac{h^2}{4\pi^2} V_{\text{inf}}. \quad (3.5) \]

We will use this formula to parameterize the effect of the inflation energy on the \( \Phi \) potential in general models. The inflaton may couple very weakly to \( \Phi \), and it is important to keep in mind that \( h \ll 1 \) is a natural possibility.

Even though \( \Phi \) is not at its vacuum value, it will track the instantaneous minimum of its effective potential as long as the potential is changing sufficiently slowly in time. This will be the case as long as
\[ \frac{\dot{\rho}}{\rho} \ll m_\Phi^2. \quad (3.6) \]

Since \( \dot{\rho} \lesssim H\rho \), this is satisfied provided \( H \ll m_\Phi \). In order for this condition to be satisfied at the end of inflation, we require
\[ V_{\text{inf}} \ll F^2. \quad (3.7) \]

With the suppression of \( \Phi \) oscillations guaranteed by low-scale inflation, the fact that \( \Phi \) has a long lifetime and decays into photons does not cause problems.

This scenario potentially suffers from a naturalness problem, since terms in the effective theory of the form
\[ \delta \mathcal{L} \sim \int d^4\theta \frac{1}{M_*^2} X^i X^j I^i I^j \quad (3.8) \]

naturally imply \( \partial^2 V/\partial I^2 \gtrsim F^2/M_*^2 \), which contradicts the slow-roll condition \( \partial^2 V/\partial I^2 \ll V_{\text{inf}}/M_*^2 \ll F^2/M_*^2 \). This may be avoided if the inflaton is a pseudo-Nambu-Goldstone
A boson associated with the breaking of an approximate global symmetry. The flatness of the inflaton potential is then unaffected by terms such as Eq. (3.8) that do not violate the global symmetry. Terms that do violate the symmetry may be naturally small. Alternatively, terms such as Eq. (3.8) may be naturally smaller than expected on the basis of dimensional analysis if the inflaton sector and the SUSY breaking sector are close to a limit where they decouple.

If the condition Eq. (3.7) on the inflation energy is not satisfied, then the time-dependent effective potential will cause \( \Phi \) to oscillate with an amplitude of order the difference between its value after inflation and its vacuum value:

\[
\Delta \Phi \sim h^2 \frac{V_{\text{inf}}}{F^2} M_* ,
\]

where we have assumed that the dimensionless couplings \( g, \lambda, \text{ and } \kappa \) (defined above) are of order 1. The \( \Phi \) energy density at the end of inflation is therefore

\[
\frac{\rho_\Phi}{\rho_{\text{inf}}} \sim h^4 \frac{V_{\text{inf}}}{F^2}.
\]

This need not dominate the present energy of the universe if \( h \ll 1 \), i.e. the inflaton is very weakly coupled to \( \Phi \). As an illustration, we consider the case \( V_{\text{inf}} \sim F^2 \). The field \( \Phi \) then begins to oscillate immediately after inflation, and we obtain

\[
\frac{\rho_\Phi}{\rho_0} \sim \left( \frac{g_s^{1/3} T_{\text{RH}}}{T_{\text{EQ}}^{1/3}} \right)^{4/3} \left( \frac{T_{\text{RH}}}{100 \text{ GeV}} \right) ,
\]

since the oscillations of both \( \Phi \) and the inflaton scale like non-relativistic matter during the reheating phase of inflation. We see that having a reheating temperature above the weak scale requires \( h \lesssim 10^{-3} \). Recall that \( h \sim 1 \) is the maximal possible coupling of the inflaton to \( \Phi \), so such a weak coupling is not unnatural.

In this scenario, the \( \Phi \) oscillations are not necessarily suppressed, and we must consider the consequences of the \( \Phi \) decay into photons. For \( F \lesssim 10^8 \text{ GeV} \), the lifetime is longer than the present age of the universe, and \( \Phi \) decays are presently producing photons. Assuming that the photon energy is \( E_\gamma \sim m_\Phi \), the bound on the present energy in \( \Phi \) oscillations is

\[
\frac{\rho_\Phi \Gamma_\Phi}{4\pi H_0} \lesssim \frac{1 \text{ MeV}}{1 \text{ cm}^2 \text{ sec}}.
\]

For the case \( V_{\text{inf}} \sim F^2 \) considered above, this gives the bound

\[
h \lesssim 10^{-2} \left( \frac{T_{\text{RH}}}{100 \text{ GeV}} \right)^{-1/4} \left( \frac{\sqrt{F}}{10^7 \text{ GeV}} \right)^{-3/2}.
\]

We have not separately discussed the interactions of the \( R \) axion in our model. If its mass is of order \( m_\Phi \), then the considerations above apply equally to the real and complex components of \( \Phi \). For axion masses below \( m_\Phi \), the cosmological constraints are more restrictive, but the model is still acceptable for a wide range of parameters. We leave detailed investigation of the cosmology of these models for future work. The considerations above are sufficient to show that the cosmology of this model is acceptable for a wide range of parameters.
IV. CONCLUSIONS

We have constructed a model in which the GUT scale is identified with the vacuum expectation value of a ‘GUT modulus’ field with a potential that is exactly flat in the supersymmetric limit. In the context of gauge-mediated supersymmetry breaking, the potential for the GUT modulus field arises at 2 loops, and pushes the modulus to large values for a wide range of couplings. If this runaway behavior is stabilized by Planck-suppressed operators, the vacuum expectation value of the GUT modulus has the desired magnitude independently of the scale of supersymmetry breaking, as long as all dimensionless couplings are order one. We find this mechanism to be quite compelling. We have constructed an explicit model where this mechanism is embedded in a realistic GUT with no dimensionful parameters and natural doublet-triplet splitting. We have also argued that the GUT modulus poses no problems for cosmology provided there is an epoch of low-scale inflation. This low-scale inflation need not be responsible for structure formation.

This mechanism for fixing the GUT scale described here is very simple, and there is no reason that it cannot occur in other models. For example, it should be possible to construct models based on $SO(10)$, and there is no reason it cannot occur in a string GUT.

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