Viscous dark energy and generalized second law of thermodynamics

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We examine the validity of the generalized second law of thermodynamics in a non-flat universe in the presence of viscous dark energy. At first we assume that the universe filled only with viscous dark energy. Then, we extend our study to the case where there is an interaction between viscous dark energy and pressureless dark matter. We examine the time evolution of the total entropy, including the entropy associated with the apparent horizon and the entropy of the viscous dark energy inside the apparent horizon. Our study show that the generalized second law of thermodynamics is always protected in a universe filled with interacting viscous dark energy and dark matter in a region enclosed by the apparent horizon. Finally, we show that the the generalized second law of thermodynamics is fulfilled for a universe filled with interacting viscous dark energy and dark matter in the sense that we take into account the Casimir effect.

I. INTRODUCTION

One of the most important problems of modern cosmology is the so-called dark energy (DE) puzzle. The type Ia supernova observations suggest that the universe is dominated by DE with negative pressure which provides the dynamical mechanism for the accelerating expansion of the universe \([1]\). This acceleration implies that if Einstein’s theory of gravity is reliable on cosmological scales, then our universe is dominated by a mysterious form of energy. This unknown energy component possesses some strange features, for example it is not clustered on large length scales and its pressure must be negative so that can drive the current acceleration of the universe. Since the fundamental theory of nature that could explain the microscopic physics of DE is unknown at present, phenomenologists take delight in constructing various models based on its macroscopic behavior. The dynamical nature of dark energy, at least in an effective level, can originate from various fields, such is a canonical scalar field (quintessence) \([2]\), a phantom field, that is a scalar field with a negative sign of the kinetic term \([3]\), or the combination of quintessence and phantom

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in a unified model named quintom [4].

The cosmological models with non-viscous cosmic fluid has been studied widely in the literature. Early treatises on viscous cosmology are given in [5]. The viscous entropy production in the early universe and viscous fluids on the Randall-Sundrum branes have been studied respectively in [6]. A special branch of viscous cosmology is to investigate how the bulk viscosity can influence the future singularity, commonly called the Big Rip, when the fluid is in the phantom state corresponding to $w_D < -1$. A lot of works have been done in this direction [7–9]. In particular, it was first pointed out in [7] that the presence of a bulk viscosity proportional to the Hubble expansion $H$ can cause the fluid to pass from the quintessence region into the phantom region and thereby inevitably lead to a future singularity.

Since the discovery of black hole thermodynamics in 1970, physicists have been speculated on the thermodynamics of the cosmological models in an accelerated expanding universe [10–17]. Related to the present work, the first and the second laws of thermodynamics in a flat universe were investigated for time independent and time dependent EoS [18]. For the case of a constant EoS, the first law is valid for the apparent horizon (Hubble horizon) and it does not hold for the event horizon as systems IR cut-off. When the EoS is assumed to be time dependent, using a holographic model of dark energy in flat space, the same result is gained; the event horizon, in contrast to the apparent horizon, does not satisfy the first law. Also, while the event horizon does not respect the second law, it hold for the universe enclosed by the apparent horizon.

In this paper we study the validity of the generalized second law of thermodynamics for a viscous dark energy in a universe enveloped by the apparent horizon. Recently, it was shown that for an accelerating universe the apparent horizon is a physical boundary from the thermodynamical point of view [19–22]. In particular, it was argued that for an accelerating universe inside the event horizon the generalized second law does not satisfy, while the accelerating universe enveloped by the apparent horizon satisfies the generalized second law of thermodynamics [19]. Therefore, the event horizon in an accelerating universe might not be a physical boundary from the thermodynamical point of view. Then we extend our study to the case where there is an interaction between viscous dark energy and pressureless dark matter. Most discussions on dark energy rely on the assumption that it evolves independently of dark matter. Given the unknown nature of both dark energy and dark matter there is nothing in principle against their mutual interaction and it seems very special that these two major components in the universe are entirely independent. Indeed, this possibility has received a lot of attention recently [23–26] and in particular, it has been shown that the coupling can alleviate the coincidence problem [27].
This paper is organized as follows. In section II we examine the generalized second law of thermodynamics in a universe filled only with viscous dark energy. In section III we extend our study to the case where there is an interaction term between viscous dark energy and pressureless dark matter. In section IV we study the Casimir effect in viscous dark energy. The last section is devoted to conclusions.

II. GSL AND VISCIOUS DARK ENERGY

We start from a homogenous and isotropic Friedmann-Robertson-Walker (FRW) universe which is described by the line element

$$ds^2 = h_{\mu\nu}dx^\mu dx^\nu + \tilde{r}^2(d\theta^2 + \sin^2 \theta d\phi^2),$$

(1)

where $\tilde{r} = a(t)r$, $x^0 = t$, $x^1 = r$, the two dimensional metric $h_{\mu\nu} = \text{diag} \left( -1, \frac{a^2}{1 - kr^2} \right)$. Here $k$ denotes the curvature of space with $k = 0, 1, -1$ corresponding to open, flat, and closed universes, respectively. A closed universe with a small positive curvature ($\Omega_k \approx 0.01$) is compatible with observations [28]. The dynamical apparent horizon, a marginally trapped surface with vanishing expansion, is determined by the relation $h^{\mu\nu}\partial_\mu \tilde{r}\partial_\nu \tilde{r} = 0$, which implies that the vector $\nabla \tilde{r}$ is null on the apparent horizon surface. The apparent horizon was argued as a causal horizon for a dynamical spacetime and is associated with gravitational entropy and surface gravity [29, 30]. For the FRW universe the apparent horizon radius reads

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + k/a^2}}.$$

(2)

The Friedmann equation for a non-flat universe filled with viscous dark energy takes the form (we neglect the dark matter)

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho_D,$$

(3)

where $\rho_D$ is the energy density of dark energy inside apparent horizon. In an isotropic and homogeneous FRW universe, the dissipative effects arise due to the presence of bulk viscosity in cosmic fluids. The theory of bulk viscosity was initially investigated by Eckart [31] and later on pursued by Landau and Lifshitz [32]. Dark energy with bulk viscosity has a peculiar property to cause accelerated expansion of phantom type in the late evolution of the universe [7, 8]. It can also alleviate several cosmological puzzles like age problem, coincidence problem and phantom crossing. The energy-momentum tensor of the viscous fluid is

$$T_{\mu\nu} = \rho_D u_\mu u_\nu + \tilde{p}_D (g_{\mu\nu} + u_\mu u_\nu),$$

(4)
where $u_\mu$ is the four-velocity vector and

$$\tilde{p}_D = p_D - 3H\xi,$$

(5)

is the effective pressure of dark energy and $\xi$ is the bulk viscosity coefficient. We require $\xi > 0$ to get positive entropy production in conformity with second law of thermodynamics [33]. The energy conservation equation is

$$\dot{\rho}_D + 3H(\rho_D + \tilde{p}_D) = 0,$$

(6)

which can be written

$$\dot{\rho}_D + 3H\rho_D(1 + w_D) = 9H^2\xi,$$

(7)

where $w_D = p_D/\rho_D$ is the equation of state parameter of viscous dark energy. In terms of the apparent horizon radius, we can rewrite the Friedmann equation as

$$\frac{1}{\tilde{r}_A^2} = \frac{8\pi G}{3} \rho_D.$$  

(8)

The associated surface gravity on the apparent horizon can be defined as

$$\kappa = \frac{1}{\sqrt{-h}} \partial_a \left( \sqrt{-h} h^{ab} \partial_b \tilde{r} \right).$$

(9)

Then one can easily show that the surface gravity at the apparent horizon of FRW universe can be written as

$$\kappa = -\frac{1}{\tilde{r}_A} \left( 1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} \right).$$

(10)

The associated temperature on the apparent horizon can be defined as

$$T_h = \frac{|\kappa|}{2\pi} = \frac{1}{2\pi\tilde{r}_A} \left( 1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} \right).$$

(11)

where $\frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} < 1$ ensures that the temperature is positive. Recently the connection between temperature on the apparent horizon and the Hawking radiation has been observed in [34]. Hawking radiation is an important quantum phenomenon of black hole, which is closely related to the existence of event horizon of black hole. The cosmological event horizon of de Sitter space has the Hawking radiation with thermal spectrum as well. Using the tunneling approach proposed by Parikh and Wilczek, the authors of [34] showed that there is indeed a Hawking radiation with a
finite temperature, for locally defined apparent horizon of the FRW universe with any spatial cur-
vature. This gives more solid physical implication of the temperature associated with the apparent
horizon. The entropy associated to the apparent horizon is

\[ S_h = \frac{A}{4G} = \frac{\pi \tilde{r}_A^2}{G}. \] (12)

where \( A = 4\pi \tilde{r}_A^2 \) is the area of the apparent horizon. Differentiating Eq. (8) with respect to the
cosmic time and using Eq. (7) we get

\[ \dot{\tilde{r}}_A = 4\pi GH \tilde{r}_A^3 [\rho_D(1 + w_D) - 3H\xi]. \] (13)

Let us now turn to find out \( T_h \dot{S}_h \):

\[ T_h \dot{S}_h = \frac{1}{2\pi \tilde{r}_A} \left( 1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} \right) \frac{d}{dt} \left( \frac{\pi \tilde{r}_A^2}{G} \right). \] (14)

After some simplification and using Eq. (13) we get

\[ T_h \dot{S}_h = 4\pi H \tilde{r}_A^3 [\rho_D(1 + w_D) - 3H\xi] \left( 1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} \right). \] (15)

As we argued above the term \( \left( 1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} \right) \) is positive to ensure \( T_h > 0 \), however, in an accelerating
universe the equation of state parameter of dark energy may satisfy \( w_D < -1 + 3H\xi/\rho_D \). This
indicates that the second law of thermodynamics, \( \dot{S}_h \geq 0 \), does not hold on the apparent horizon.
Then the question arises, “will the generalized second law of thermodynamics, \( \dot{S}_h + \dot{S}_D \geq 0 \), can
be satisfied in a region enclosed by the apparent horizon?” The entropy of the viscous dark energy
inside the apparent horizon, \( S_D \), can be related to its energy \( E_D = \rho_D V \) and its pressure \( \tilde{p}_D \) by the Gibbs equation (11)

\[ T_D dS_D = d(\rho_D V) + \tilde{p}_D dV = V d\rho_D + (\rho_D + p_D - 3H\xi) dV, \] (16)

where \( T_D \) and is the temperature of the viscous dark energy and \( V = \frac{4\pi \tilde{r}_A^3}{3} \) is the volume enveloped
by the apparent horizon. We also limit ourselves to the assumption that the thermal system
bounded by the apparent horizon remains in equilibrium so that the temperature of the system must
be uniform and the same as the temperature of its boundary. This requires that the temperature
\( T_D \) of the viscous dark energy inside the apparent horizon should be in equilibrium with the
temperature \( T_h \) associated with the apparent horizon, so we have \( T_D = T_h \). This expression holds
in the local equilibrium hypothesis. If the temperature of the fluid differs much from that of the
horizon, there will be spontaneous heat flow between the horizon and the fluid and the local
equilibrium hypothesis will no longer hold. This is also at variance with the FRW geometry. In
general, when we consider the thermal equilibrium state of the universe, the temperature of
the universe is associated with the apparent horizon. Therefore from the Gibbs equation (16) we can
obtain
\[ T_h \dot{S}_D = 4\pi \tilde{r}_A^2 [\rho_D(1 + w_D) - 3H\xi] \dot{\tilde{r}}_A - 4\pi H\tilde{r}_A^3 [\rho_D(1 + w_D) - 3H\xi]. \]  
(17)

To check the generalized second law of thermodynamics, we have to examine the evolution of the
total entropy \( S_h + S_D \). Adding equations (15) and (17), we get
\[ T_h (\dot{S}_h + \dot{S}_D) = 2\pi \tilde{r}_A^2 [\rho_D(1 + w_D) - 3H\xi] \dot{\tilde{r}}_A = \frac{A}{2} [\rho_D(1 + w_D) - 3H\xi] \dot{\tilde{r}}_A. \]  
(18)
where \( A > 0 \) is the area of apparent horizon. Finally, substituting \( \dot{\tilde{r}}_A \) from Eq. (13) into (18) we
reach
\[ T_h (\dot{S}_h + \dot{S}_D) = 2\pi GAH\tilde{r}_A^3 [\rho_D(1 + w_D) - 3H\xi]^2. \]  
(19)
The right hand side of the above equation cannot be negative throughout the history of the universe,
which means that \( \dot{S}_h + \dot{S}_D \geq 0 \) always holds. This indicates that for a universe with spacial
curvature filled with viscous dark energy, the generalized second law of thermodynamics is fulfilled
in a region enclosed by the apparent horizon.

III. GSL AND INTERACTING VISCOUS DARK ENERGY WITH NON-VISCOUS
DARK MATTER

In this section we extend our study to the case where there is an interaction between viscous
dark energy and pressureless dark matter. In this case the Friedmann equation can be written as
\[ H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} (\rho_m + \rho_D), \]  
(20)
where \( \rho_m \) and \( \rho_D \) are the energy density of dark matter and dark energy inside apparent horizon,
respectively. Since we consider the interaction between dark matter and dark energy, \( \rho_m \) and \( \rho_D \)
do not conserve separately, they must rather enter the energy balances
\[ \dot{\rho}_m + 3H\rho_m = Q, \]  
(21)
\[ \dot{\rho}_D + 3H\rho_D(1 + w_D) = 9H^2\xi - Q. \]  
(22)
where \( Q = \Gamma \rho_D \) denotes the interaction between the dark components. We also assume the
interaction term is positive, \( Q > 0 \), which means that there is an energy transfer from the dark
energy to dark matter. In terms of the apparent horizon radius, we can rewrite the Friedmann equation as

$$\frac{1}{\tilde{r}_A^2} = \frac{8\pi G}{3} (\rho_m + \rho_D).$$  \hfill (23)$$

Differentiating Eq. (23) with respect to the cosmic time and using Eqs. (21) and (22) we get

$$\dot{\tilde{r}}_A = 4\pi GH\tilde{r}_A^3 [\rho_D(1 + u + w_D) - 3H\xi].$$  \hfill (24)$$

where $u = \rho_m/\rho_D$ is the ratio of energy densities. Next we turn to calculate $T_h\dot{S}_h$. It is easy to show that

$$T_h\dot{S}_h = 4\pi H\tilde{r}_A^3 [\rho_D(1 + u + w_D) - 3H\xi]\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right).$$  \hfill (25)$$

Again in an accelerating universe the equation of state parameter of dark energy may satisfy the condition $w_D < -1 - u + 3H\xi/\rho_D$. This implies that the second law of thermodynamics, $\dot{S}_h \geq 0$, does not hold on the apparent horizon. Then we examine the validity of the generalized second law, $\dot{S}_h + \dot{S}_m + \dot{S}_D \geq 0$. The entropy of the viscous dark energy plus dark matter inside the apparent horizon, $S = S_m + S_D$, can be related to the total energy $E = (\rho_m + \rho_D)V$ and pressure $\tilde{p}_D$ in the horizon by the Gibbs equation

$$TdS = d[(\rho_m + \rho_D)V] + \tilde{p}_DdV = V(d\rho_m + d\rho_D) + [\rho_D(1 + u + w_D) - 3H\xi]dV,$$  \hfill (26)$$

where $T = T_m = T_D$ and $S = S_m + S_D$ are the temperature and the total entropy of the energy and matter content inside the horizon, respectively. Here we assumed that the temperature of both dark components are equal, due to their mutual interaction. We also assume the local equilibrium hypothesis holds, so $T = T_h$. Therefore from the Gibbs equation (26) we obtain

$$T_h(\dot{S}_m + \dot{S}_D) = 4\pi\tilde{r}_A^2 [\rho_D(1 + u + w_D) - 3H\xi]\dot{\tilde{r}}_A - 4\pi H\tilde{r}_A^3 [\rho_D(1 + u + w_D) - 3H\xi].$$  \hfill (27)$$

To check the generalized second law of thermodynamics, we have to examine the evolution of the total entropy $S_h + S_m + S_D$. Adding equations (25) and (27), we get

$$T_h(\dot{S}_h + \dot{S}_m + \dot{S}_D) = 2\pi\tilde{r}_A^2 [\rho_D(1 + u + w_D) - 3H\xi]\dot{\tilde{r}}_A = \frac{A}{2} [\rho_D(1 + u + w_D) - 3H\xi]\dot{\tilde{r}}_A.$$  \hfill (28)$$

Substituting $\dot{\tilde{r}}_A$ from Eq. (24) into (28) we get

$$T_h(\dot{S}_h + \dot{S}_m + \dot{S}_D) = 2\pi GAH\tilde{r}_A^3 [\rho_D(1 + u + w_D) - 3H\xi]^2,$$  \hfill (29)$$

which cannot be negative throughout the history of the universe and hence the general second law of thermodynamics, $\dot{S}_h + \dot{S}_m + \dot{S}_D \geq 0$, is always protected for a universe filled with interacting
viscous dark energy and dark matter in a region enclosed by the apparent horizon. To see the effect on the generalized second law of thermodynamics derived from the interaction $Q$, one can consider the $Q = 0$ in Eqs. (21), (22). After this substitution, our result (29) do not change, so we conclude that the interaction term does not affect on the generalized second law of thermodynamics.

IV. CASIMIR EFFECTS IN VISCOUS COSMOLOGY

In this section we would like to examine the GSL of thermodynamics for an interacting viscous dark energy in the sense that we take into account the Casimir effect. A natural way of dealing with the Casimir effect in a non-flat universe is to relate it to the apparent horizon radius $\tilde{r}_A = 1/\sqrt{H^2 + k/a^2}$. It means effectively that we should put the Casimir energy $E_c$ inversely proportional to the apparent horizon radius. This is consistent with the basic property of the Casimir energy, which states that it is a measure of the stress in the region interior to the “shell” as compared with the unstressed region on the outside. The effect is evidently largest in the beginning of the universe’s evolution, when $\tilde{r}_A$ is small. At late times, when $\tilde{r}_A \to \infty$, the Casimir influence should be expected to fade away. Therefore, we assume the Casimir energy can be written as

$$E_c = \frac{c}{\tilde{r}_A},$$

where $c$ is a constant. We also assume that $c$ is small compared with unity. This is physically reasonable, in view of the conventional feebleness of the Casimir force. The Casimir pressure corresponding to energy (30) is

$$p_c = \frac{-1}{4\pi \tilde{r}_A^2} \frac{\partial E_c}{\partial \tilde{r}_A} = \frac{c}{4\pi \tilde{r}_A^4}.$$  

Thus the Casimir energy evolves as $\rho_c \propto \tilde{r}_A^{-4}$. The continuity equation for the Casimir energy takes the form

$$\dot{\rho}_c + 3H \rho_c (1 + w_c) = 0,$$

where $w_c = p_c/\rho_c$ is the equation of state parameter of Casimir energy. Using Eq. (31) as well as relation

$$\rho_c = \frac{E_c}{V} = \frac{3c}{4\pi \tilde{r}_A^4},$$

we have

$$w_c = \frac{p_c}{\rho_c} = \frac{1}{3}.$$
The Friedmann equation now takes the form
\[ H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} (\rho_m + \rho_D + \rho_c), \] (35)
which can be rewritten as
\[ \frac{1}{\tilde{r}_A^2} = \frac{8\pi G}{3} (\rho_m + \rho_D + \rho_c). \] (36)
Differentiating Eq. (36) with respect to the cosmic time and using Eqs. (21), (22), (32) and (34) we find
\[ \dot{\tilde{r}}_A = 4\pi GH \tilde{r}_A^3 \left[ \rho_D (1 + u + \frac{4z}{3} + w_D) - 3H\xi \right], \] (37)
where \( z = \rho_c/\rho_D \). Next we calculate \( T_h \dot{S}_h \). It is a matter of calculation to show
\[ T_h \dot{S}_h = 4\pi H \tilde{r}_A^3 \left[ \rho_D (1 + u + \frac{4z}{3} + w_D) - 3H\xi \right] \left( 1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} \right). \] (38)
From the Gibbs equation for the total energy content of the universe we have
\[ T_h dS = d[(\rho_m + \rho_D + \rho_c)V] + (\tilde{p}_D + p_c)dV \\
= V(d\rho_m + d\rho_D + d\rho_c) + \left[ \rho_D (1 + u + \frac{4z}{3} + w_D) - 3H\xi \right] dV, \] (39)
where \( S = S_m + S_D + S_c \) and we have assumed that the temperature of all the energy content are identical and equal with the apparent horizon temperature \( T_h \). Thus from Eq. (39) we obtain
\[ T_h (\dot{S}_m + \dot{S}_D + \dot{S}_c) = 4\pi \tilde{r}_A^2 \left[ \rho_D (1 + u + \frac{4z}{3} + w_D) - 3H\xi \right] \dot{\tilde{r}}_A \\
- 4\pi H \tilde{r}_A^3 \left[ \rho_D (1 + u + \frac{4z}{3} + w_D) - 3H\xi \right]. \] (40)
Now we are in a position to examine the GSL of thermodynamics. Adding equations (38) and (40), we get
\[ T_h (\dot{S}_h + \dot{S}_m + \dot{S}_D + \dot{S}_c) = 2\pi \tilde{r}_A^2 \left[ \rho_D (1 + u + \frac{4z}{3} + w_D) - 3H\xi \right] \dot{\tilde{r}}_A \\
= \frac{A}{2} \left[ \rho_D (1 + u + \frac{4z}{3} + w_D) - 3H\xi \right] \dot{\tilde{r}}_A. \] (41)
Substituting \( \dot{\tilde{r}}_A \) from Eq. (37) into (41) we reach
\[ T_h (\dot{S}_h + \dot{S}_m + \dot{S}_D + \dot{S}_c) = 2\pi GA H \tilde{r}_A^3 \left[ \rho_D (1 + u + \frac{4z}{3} + w_D) - 3H\xi \right]^2. \] (42)
The right hand side of the above equation cannot be negative throughout the history of the universe, which means that \( \dot{S}_h + \dot{S}_m + \dot{S}_D + \dot{S}_c \geq 0 \) always holds. This indicates that the GSL of thermodynamics is fulfilled for a universe filled with interacting viscous dark energy and dark matter in the sense that we take into account the Casimir effect.
V. CONCLUSIONS

We have investigated the validity of the generalized second law of thermodynamics in a non-flat universe with viscous dark energy. We have examined the total entropy evolution with time, including the derived apparent horizon entropy and the entropy of viscous dark energy inside the apparent horizon. Then, we have extended our study to the case where there is an interaction between viscous dark energy and pressureless dark matter. We have shown that the generalized second law of thermodynamics is always fulfilled for a universe filled with interacting viscous dark energy and dark matter in a region enclosed by the apparent horizon. We have also examined the validity of the GSL of thermodynamics for an interacting viscous dark energy in the sense that we take into account the Casimir effect.

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