Determination of matter radius and neutron skin of $^{58}\text{Ni}$ from reaction cross section of proton+$^{58}\text{Ni}$ scattering based on chiral $g$-matrix model

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Background: Using the chiral (Kyushu) $g$-matrix folding model with the densities calculated with Gogny-HFB (GHFB) with the angular momentum projection (AMP), we determined the central values of matter radius and neutron skin from the central values of reaction cross sections $\sigma_R(\text{EXP})$ of p+$^{40,48}\text{Ca}$ and p+$^{208}\text{Pb}$ scattering. As for p+$^{58}\text{Ni}$ scattering, $\sigma_R(\text{EXP})$ are available as a function of incident energy $E_{\text{in}}$.

Aim: Our aim is to determine matter radius $r_m$ and skin radius $r_{\text{skin}}$ for $^{58}\text{Ni}$ from the $\sigma_R(\text{EXP})$ of p+$^{58}\text{Ni}$ scattering by using the Kyushu $g$-matrix folding model with the GHFB+AMP densities.

Results: For p+$^{58}\text{Ni}$ scattering, the Kyushu $g$-matrix folding model with the GHFB+AMP densities reproduces $\sigma_R(\text{EXP})$ in $8.8 \leq E_{\text{in}} \leq 81\text{MeV}$. For $E_{\text{in}} = 81\text{MeV}$, we define the factor $F$ as $F = \sigma_R(\text{EXP})/\sigma_R(\text{AMP}) = 0.9775$. The $F\sigma_R(\text{AMP})$ be much the same as the center values of $\sigma_R(\text{EXP})$ in $8.8 \leq E_{\text{in}} \leq 81\text{MeV}$. We then determine $r_m(\text{EXP})$ from the center values of $\sigma_R(\text{EXP})$, using $\sigma_R(\text{EXP}) = C r_m^2(\text{EXP})$ with $C = r_m^2(\text{AMP})/(F\sigma_R(\text{AMP}))$. The $r_m(\text{EXP})$ thus obtained are averaged over $E_{\text{in}}$. The averaged value is $r_m(\text{EXP}) = 3.697\text{fm}$. Eventually, we obtain $r_{\text{skin}}(\text{EXP}) = 0.023\text{fm}$ from $r_m = 3.697\text{fm}$ and $r_p(\text{EXP}) = 3.685\text{fm}$ of electron scattering.

I. INTRODUCTION AND CONCLUSION

Background: A novel method for measuring nuclear reactions in inverse kinematics with stored beam ions was successfully used to extract the matter radius of $^{58}\text{Ni}$ [1]. The experiment was performed at the experimental heavy-ion storage ring at the GSI facility. Their results determined from the differential cross section for $^{58}\text{Ni}$+$^4\text{He}$ scattering are $r_m(\text{GSI}) = 3.70(7)\text{fm}$, $r_p(\text{GSI}) = 3.68\text{fm}$, $r_n(\text{GSI}) = 3.71(12)$, $r_{\text{skin}}(\text{GSI}) = 0.03(12)\text{fm}$.

Reaction cross section $\sigma_R$ and interaction cross sections $\sigma_I$ are a standard observable to determine a central value of matter radius $r_m$. In fact, we deduced the matter radii $r_m$ for Ne isotopes [2] and for Mg isotopes [3]. One can then evaluate $r_{\text{skin}}$ and $r_n$ from the $r_m$ and the $r_p(\text{EXP})$ [4] of the electron scattering. Eventually, one can determine $r_m$ and $r_{\text{skin}}$ from the central value of $\sigma_R(\text{EXP})$. Recently, we have determined $r_m$ and $r_{\text{skin}}$ for $^{208}\text{Pb}$ [5] and $^{40,48}\text{Ca}$ [6,7], using the chiral (Kyushu) $g$-matrix folding model with the densities calculated with Gogny-D1S-HFB (GHFB) with the angular momentum projection (AMP). As for $^{58}\text{Ni}$, the data on $\sigma_R$ are available for p+$^{58}\text{Ni}$ scattering [8–12].

The $g$-matrix folding model is a standard way of obtaining microscopic optical potential for proton scattering and nucleus-nucleus scattering [13–22]. Applying the folding model with the Melbourne $g$-matrix [16] for $\sigma_I$ for Ne isotopes and $\sigma_R$ for Mg isotopes, we found that $^{31}\text{Ne}$ is a deformed halo nucleus [23], and determined the matter radii $r_m$ for Ne isotopes [2] and for Mg isotopes [3].

Kohno calculated the $g$ matrix for the symmetric nuclear matter, using the Brueckner-Hartree-Fock method with chiral N$^3$LO 2NFs and NNLO 3NFs [24]. He set $c_D = -2.5$ and $c_E = 0.25$ so that the energy per nucleon can become minimum at $\rho = \rho_0$. Toyokawa et al. localized the non-local chiral $g$ matrix into three-range Gaussian forms [21], using the localization method proposed by the Melbourne group [16, 25, 26]. The resulting local $g$ matrix is referred to as “Kyushu $g$-matrix”.

The Kyushu $g$-matrix folding model is successful in reproducing $\sigma_R$ and differential cross sections $d\sigma/dQ$ for $^4\text{He}$ scattering in $E_{\text{lab}} = 30 \sim 200\text{MeV}$ per nucleon [21]. The success is true for proton scattering at $E_{\text{lab}} = 65\text{MeV}$ [19].

Proton and neutron densities used in the folding model: In Ref. [22], GHFB and GHFB+AMP reproduce the one-neutron separation energy $S_1$ and the two-neutron separation energy $S_2$ in $^{41–58}\text{Ca}$ [27,29]. We found, with $S_1$ and $S_2$, that $^{64}\text{Ca}$
is an even-dripline nucleus and $^{59}$Ca is an odd-dripline nucleus. Our results are consistent with the data [2] in $^{40-58}$Ca for the binding energy $E_B$. This means that the proton and neutron densities are good.

**Aim:** Our aim is to determine matter radius $r_m$ and skin radius $r_{skin}$ for $^{58}$Ni from the $\sigma_R(\text{EXP})$ of $p+^{58}$Ni scattering by using the Kyushu $g$-matrix folding model with the GHFB+AMP densities.

**Results:** For $p+^{58}$Ni scattering, the Kyushu $g$-matrix folding model with the GHFB+AMP densities reproduces $\sigma_R(\text{EXP})$ in $8.8 \leq E_{in} \leq 81$ MeV. As a fine-tuning, for $E_{in} = 81$ MeV, we define the factor $F$ as $F = \sigma_R(\text{EXP})/\sigma_R(\text{AMP}) = 0.9775$. The $F \sigma_R(\text{AMP})$ is much the same as the center values of $\sigma_R(\text{EXP})$ in $8.8 \leq E_{in} \leq 81$ MeV. We then determine $r_m(\text{EXP})$ from the center values of $\sigma_R(\text{EXP})$, using $\sigma_R(\text{EXP}) = C r_m^2(\text{EXP})$ with $C = \sigma_R(\text{AMP})/(F \sigma_R(\text{AMP}))$. The $r_m(\text{EXP})$ thus obtained are averaged over $E_{in}$. The averaged value is $r_m(\text{EXP}) = 3.697$ fm. Eventually, we obtain $r_{skin}(\text{EXP}) = 0.023$ fm from $r_m = 3.697$ fm and $r_p(\text{PCNP}) = 3.685$ fm of electron scattering.

**Conclusion:** Our conclusion is that the central value of $r_m(\text{EXP})$ is $3.697$ fm and that of $r_{skin}(\text{EXP})$ is $0.023$ fm. Our results are close to with those shown in Ref. [1].

**II. MODEL**

Our model is the Kyushu $g$-matrix folding model [21] with densities calculated with GHFB+AMP [22]. The folding model itself is clearly shown in Ref. [18]. The Kyushu $g$-matrix is constructed from chiral interaction with the cutoff 550 MeV.

**III. RESULTS**

Figure 1 shows reaction cross sections $\sigma_R$ as a function of incident energy $E_{in}$ for $p+^{58}$Ni scattering. In 2-$\sigma$ level, the Kyushu $g$-matrix folding model with the GHFB+AMP densities (closed circles) reproduces $\sigma_R(\text{EXP})$ [8-12] in $8.8 \leq E_{in} \leq 81$ MeV; note that the data has high accuracy of 2.7 %.

Now, we introduce the fine-tuning factor $F$. We consider $E_{in} = 81$ MeV, since total cross section of nucleon-nucleon scattering is smallest in $8.8 \leq E_{in} \leq 81$ MeV. For $E_{in} = 81$ MeV, we define the factor $F$ as $F = \sigma_R(\text{EXP})/\sigma_R(\text{AMP}) = 0.9775$. The $F \sigma_R(\text{AMP})$ (open circles) are much the same as the center values of $\sigma_R(\text{EXP})$ in $8.8 \leq E_{in} \leq 81$ MeV. We then determine $r_m(\text{EXP})$ from the center values of $\sigma_R(\text{EXP})$, using $\sigma_R(\text{EXP}) = C r_m^2(\text{EXP})$ with $C = \sigma_R(\text{AMP})/(F \sigma_R(\text{AMP}))$. The $r_m(\text{EXP})$ thus obtained are averaged over $E_{in}$. The averaged value is $r_m(\text{EXP}) = 3.697$ fm. We then obtain $r_{skin}(\text{EXP}) = 0.023$ fm. The $r_p(\text{PCNP}) = 3.685$ fm of electron scattering. Our results agree with $r_m(\text{GSI}) = 3.707$(7) fm, $r_p(\text{GSI}) = 3.685$, $r_n(\text{GSI}) = 3.71(12)$, and $r_{skin}(\text{GSI}) = 0.03(12)$ fm.

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