Cepheid Parallaxes and the Hubble Constant

Floor van Leeuwen\textsuperscript{1}, Michael W. Feast\textsuperscript{2}, Patricia A. Whitelock\textsuperscript{2,3,4}, Clifton D. Laney\textsuperscript{3}

\textsuperscript{1} Institute of Astronomy, Madingley Rd, Cambridge, England
\textsuperscript{2} Astronomy Dept., University of Cape Town, Rondebosch, 7701, South Africa
\textsuperscript{3} South African Astronomical Observatory, P.O. Box 9, Observatory, 7935, South Africa
\textsuperscript{4} National Astrophysics and Space Science Programme, Department of Mathematics and Applied Mathematics, University of Cape Town, Rondebosch, 7701, South Africa

1 February 2008

ABSTRACT
Revised Hipparcos parallaxes for classical Cepheids are analysed together with 10 HST-based parallaxes (Benedict et al.). In a reddening-free $V, I$ relation we find that the coefficient of log $P$ is the same within the uncertainties in our Galaxy as in the LMC, contrary to some previous suggestions. Cepheids in the inner region of NGC 4258 with near solar metallicities (Macri et al.) confirm this result. We obtain a zero-point for the reddening-free relation and apply it to the Cepheids in galaxies used by Sandage et al. to calibrate the absolute magnitudes of SNIa and to derive the Hubble constant. We revise their result for $H_0$ from 62 to 70 ± 5 km s$^{-1}$ Mpc$^{-1}$. The Freedman et al. 2001 value is revised from 72 to 76 ± 8 km s$^{-1}$ Mpc$^{-1}$. These results are insensitive to Cepheid metallicity corrections. The Cepheids in the inner region of NGC 4258 yield a modulus of 29.22 ± 0.03 (int.) compared with an maser-based modulus of 29.29 ± 0.15. Distance moduli for the LMC, uncorrected for any metallicity effects, are: 18.52 ± 0.03 from a reddening-free relation in $V, I$; 18.47 ± 0.03 from a period-luminosity relation at $K$; 18.45 ± 0.04 from a period-luminosity-colour relation in $J, K$. Adopting a metallicity correction in $V, I$ from Macri et al. leads to a true LMC modulus of 18.39 ± 0.05.

Key words: astrometry, Cepheids, distance scale, cosmological parameters, Magellanic Clouds, supernovae:general

1 INTRODUCTION
The success of the Hipparcos astrometric satellite in obtaining a large number of absolute stellar parallaxes with much greater accuracy than had previously been possible (ESA 1997), allowed a first estimate to be made of the zero-point of a Cepheid Period-Luminosity (PL) relation directly from Cepheid parallaxes (Feast & Catchpole 1997). Investigations of the Hipparcos data continued after the publication of the catalogue, and led to the identification of a number of repairable problems associated with the reconstruction of the satellite’s attitude (van Leeuwen 2005). This ultimately led to a completely new reduction of the astrometric data (van Leeuwen & Fantino 2005), the results of which are soon to be published (van Leeuwen 2007) and have been incorporated in the current study. The main impact of the reductions is for the brightest stars, where improvements of up to a factor of four in parallax accuracy can be reached. For many of the Cepheids improvements by up to a factor of two have been achieved.

In the present paper we discuss and analyse the revised Hipparcos parallaxes of (classical) Cepheids. Recently, the parallaxes of 10 Cepheids, measured with the Hubble Space Telescope (HST) and corrected to an absolute reference frame using ground-based observations, have been published and discussed (Benedict et al. 2002, Benedict et al. 2007 henceforth FB2007) and we have incorporated these data in our analysis.

Since the discussion of the original Hipparcos data there has been much work on Cepheid photometry, period-luminosity (PL) relations, metallicity effects etc. and we have been able to take advantage of this in the present work. There has also been a great deal of theoretical work on Cepheids. We do not discuss this since our aim has been to obtain empirically-based conclusions. We establish a reddening-free PL relation in $V$ and $I$ (PL($W$)) as well as PL and Period-Luminosity-Colour (PLC) relations in $J$ and $K$. These relations should be of use in a variety of contexts both Galactic and extragalactic and for testing theoretical models. In the present paper we confine ourselves to the implication of our results for Cepheid-based distances of galaxies including the LMC and the estimation of the Hubble constant.
2 DATA

The basic data used here are tabulated in the appendix together with notes on individual stars. The table contains only those stars which had all the data required for the analyses and which are considered to be classical Cepheids.

Table A1 contains:

1. Hipparos number (no).
2. Hipparos parallax (\(\pi\)). Here and throughout in milliarcsec (mas).
3. Hipparos parallax standard error (\(\sigma_\pi\)). In mas.
4. Star name.
5. Logarithm of fundamental period. Where the star is an overtone pulsator the fundamental period, \(P_0\), was derived from the observed period, \(P_1\), using the relation (Alcock et al. 1995, Feast & Catchpole 1997):

\[
P_1/P_0 = 0.716 - 0.027 \log P_1.
\]

Such stars are denoted by ‘O’ in the notes. Here and throughout the periods are in days.

6.7.8. Intensity mean magnitudes \(< B >, < V >, < I >\). The \(I\) magnitudes are on the Cousins system. The \(BV\) photometry used was from Berdnikov (private communication 2005) and is an update of Berdnikov et al. (2000). Where values of \(I\) were not available in this source they were taken from Groenewegen (1999) who transformed other workers’ data to the Cousins system. These stars are marked ‘G’ in the notes.

9.10.11. Intensity mean magnitudes \(< J >, < H >, < K >\) on the SAAO system (Carter 1990). Where possible these are from multiple observations. They are mainly from the papers by Laney & Stobie (1992, 1993, 1994) and previously unpublished SAAO data. The individual observations on which these latter intensity means depend will be published separately. In a limited number of cases intensity means are from Groenewegen’s (1999) tabulation. All these stars are indicated by “L” in the notes. Where no source is specified the mean magnitude has been estimated from the single 2MASS measurement transformed to the SAAO system (Carpenter 2001) and corrected for phase by the procedure outlined by Soszyński et al. (2005). Since in many cases the phases used are old (GCVS) the accuracy of these corrections can be poor and this is taken into account in the later work.

12. \(ET\) is the value of \(E(B - V)\) given by Tammann et al. (2003). If this is not available then this column contains \(E \times 0.951\) (see Tammann et al. 2003) and is indicated by “F” in the notes.

13. \(\Delta\) is the full \(V\) amplitude of the star.

14. \(EF\) is the value of \(E(B - V)\), as estimated by Fernie et al. (1995) and given in the DDO data base as \(FE1\). If this is not available \(FE2\) is given.

15. Notes using the following symbols:

\(O?\), possible overtone, but considered a fundamental pulsator.

\(B, B?\), binary or possible binary.

\(VB\), visual binary.

\(SB\), spectroscopic binary.

\(SB2\), spectroscopic binary with both spectra observed.

The data on the binaries are mainly from the Szabados data base (see Szabados 2003). In cases of specific stars, additional references are in the notes. Many of the binaries listed by Szabados are, or possibly are, single line spectroscopic binaries. In the case of the original Hipparos data the effects of binaries on the zero-point was discussed in Feast (1998) both as regards the photometry and the astrometry. Where the photometry may have been affected by a companion this is mentioned in the notes and if thought appreciable the star was omitted from the analysis. \(DM\), double mode pulsator; the fundamental period is listed.

16. The type of astrometric solution using the following codes:

5: standard 5-parameter solution for single stars.

25: standard 5-parameter solution for a variable double star.

65: standard 5-parameter solution for the photo-centre of a variable double star.

105: standard 5-parameter solution for the secondary in a resolved binary with a variable component (a difficult solution).

7: single star with time-dependent proper motion (accelerated solution, which may indicate long-term orbital motion).

3: variability induced mover, a probably spurious indication of duplicity depending on the phase of the lightcurve.

1: stochastic solution, too much unexplained noise left in the data, generally unreliable, and an indication of short-term orbital motion.

Initial tests showed that with the Hipparos parallaxes and listed photometry, the following stars lay \(5\sigma\) or more from any reasonable PL relation: \(Y\) Sgr, V350 Sgr, GQ Ori, SY Nor, HL Pup. The Hipparos data for all of these stars were omitted from the analysis, although only \(Y\) Sgr would have sufficient weight to make a significant contribution to the solutions discussed. In some cases erroneous classification may be the cause of the discrepancy. The reason for the significant discrepancy in the case of \(Y\) Sgr is not fully understood since the star has an HST parallax (FB2007) in good accord with other Cepheid data. In the combined solutions discussed below we use only the HST parallax for \(Y\) Sgr.

Leaving aside \(Y\) Sgr there are nine Cepheids in common with the HST parallax work. These parallaxes are listed with the Hipparos results in Table 1 and the weighted means are also shown. In the solutions below we use these weighted means unless otherwise indicated.

\[
\begin{array}{lllll}
\text{Hipp} & \text{Name} & \pi_{Hipp} & \pi_{HST} & \pi_{mean} \\
47854 & \ell\ Car & 2.06 \pm 0.27 & 2.01 \pm 0.20 & 2.03 \pm 0.16 \\
34088 & \zeta\ Gem & 2.71 \pm 0.17 & 2.78 \pm 0.18 & 2.74 \pm 0.12 \\
26069 & \beta\ Dor & 3.64 \pm 0.28 & 3.14 \pm 0.16 & 3.26 \pm 0.14 \\
88567 & W\ Sgr & 2.59 \pm 0.75 & 2.28 \pm 0.20 & 2.30 \pm 0.19 \\
87072 & X\ Sgr & 3.39 \pm 0.21 & 3.00 \pm 0.18 & 3.17 \pm 0.14 \\
110991 & \delta\ Cep & 3.81 \pm 0.20 & 3.66 \pm 0.15 & 3.71 \pm 0.12 \\
93124 & FF\ Aql & 2.05 \pm 0.34 & 2.81 \pm 0.18 & 2.64 \pm 0.16 \\
102949 & T\ Vul & 2.31 \pm 0.29 & 1.90 \pm 0.23 & 2.06 \pm 0.22 \\
30827 & RT\ Aur & -0.23 \pm 1.01 & 2.40 \pm 0.19 & 2.31 \pm 0.19 \\
89968 & Y\ Sgr & 3.73 \pm 0.32 & 2.13 \pm 0.29 \\
\end{array}
\]
3 METHODS USING $V/I$ PHOTOMETRY

The existence of a Cepheid PL relation goes back of course to Leavitt (1908, 1912). It is known that the PL($V$) relation in the LMC has significant width and that this is greatly reduced, probably to within the observational errors, by the use of a period-luminosity-colour (PLC) relation (e.g. Martin et al. 1979). However, the value of the coefficient of the colour term and whether it varies with period has been a matter of uncertainty and debate.

In recent times there has been considerable discussion on the possibility that some PL relations might be non-linear. Thus, it has been suggested that the PL($V$) relation in the LMC is non-linear (Sandage et al. 2004, Ngeow et al. 2005). However, it is not entirely clear whether this effect is real or, for instance, due to systematic errors in the adopted reddening varying with period. In any case, there seems to be good evidence (Ngeow & Kanbur 2005) that in the LMC, a “reddening-free” relation (Madore 1976) such as,

$$M_W = \alpha \log P + \beta(V - I) + \gamma,$$  \hspace{1cm} (2)

is linear. Here the coefficient, $\beta$, is an adopted ratio of total to selective extinction ($A_V/(A_V - A_I)$). We have restricted our discussion at optical wavelengths to a relation of this form because of this and because of its importance in extragalactic work.

In a very detailed analysis of the data available to them, Sandage et al. (2004) have suggested that the slopes of PL relations in the optical, and hence the log $P$ coefficient in equation 2, differ between the LMC and our Galaxy, presumably due to metallicity effects. In the case of the Galaxy the slopes were derived by combining Cepheid distances derived from Baade-Wesselink (pulsation parallax) type analyses with those from Cepheids in clusters with distances from main-sequence fitting. Their results have remained controversial for the following reasons. Gieren et al. (2005) showed that the Baade-Wesselink type distances of LMC Cepheids were a function of period if a conventional “$p$” factor was used to convert observed radial velocities to pulsational velocities of the stars. Changing “$p$” to remove this effect brings the LMC and Galaxy PL slopes into agreement within the uncertainties. As regards the ‘cluster’ distances, these partly depend, especially at longer periods, not on clusters but on stellar associations. The distances of these associations remain somewhat uncertain and their inclusion can affect any derived PL slope (as can be inferred from the early work of Feast & Walker 1987, their table 3). In view of these various uncertainties, one of our aims has been to derive the PL($W$) slope in our Galaxy.

Two PL($W$) relations are of particular importance for extragalactic applications.

(1) The relation adopted by Freedman et al. (2001) in their HST key project on the Cepheid calibration of the Hubble Constant ($H_0$),

$$M_W = -3.255 \log P + 2.45(V - I) - 2.724.$$  \hspace{1cm} (3)

The log $P$ coefficient was derived from OGLE LMC data (Udalski et al. 1999) and the colour coefficient from their adopted reddening law. The equation as given is applicable to local Galactic Cepheids. The zero-point is derived from their adopted LMC modulus and metallicity correction.

(2) The relations for Galactic Cepheids used by Sandage et al. (2006 henceforth S2006) in their work on a Cepheid-based $H_0$ are equivalent to:

$$M_W = -3.746 \log P + 2.563(V - I) - 2.213.$$  \hspace{1cm} (4)

The basis on which this equation was derived was discussed above.

A third reddening-free relation is of relevance. This was derived (FB2007) from a “cleaned” selection of 581 LMC OGLE Cepheids and adopting the Freedman et al. reddening law

$$M_W = -3.29(\pm 0.01) \log P + 2.45(V - I) - (\text{Mod} - 15.94(\pm 0.01)),$$  \hspace{1cm} (5)

where $\text{Mod}$ is the distance modulus of the LMC and no metallicity correction has been applied.

After making corrections for reddening, Udalski et al. (1999) derived a true PLC relation for LMC Cepheids in the OGLE data base, which can be written as,

$$V_0 = -3.25(\pm 0.02) \log P + 2.41(\pm 0.03)(V - I) + 15.88(\pm 0.02).$$  \hspace{1cm} (6)

Comparison of equations 5 and 6 shows that the coefficients of the slopes and colour terms are nearly the same in the two equations. Thus, we are justified (at least in the LMC) in considering a reddening-free relation as also very close to a true PLC relation. This is important for two reasons. First, as discussed above, we expect a PLC relation to be very narrow. Secondly, because Cepheid overtone pulsators are in general bluer than those of the same fundamental period, they lie systematically above fundamental pulsators in a PL($V$) plot. However, they lie together with the fundamental pulsators in a PLC plot (see, e.g. Beaulieu et al. 1995). This is important if we wish to include overtone pulsators in a Cepheid calibration. We test this result in the case of Galactic Cepheids below.

In analysing the data we have proceeded in two ways:

(1) For a limited number of Cepheids the percentage errors in the parallaxes are sufficiently small that individual values of $M_W$ can be directly combined to derive PL relations; Cepheids selected in this way require a Lutz-Kelker type bias correction (Lutz & Kelker 1973). We have scaled these corrections to be on the same system as that adopted by FB2007. We are primarily interested in using this subset of stars to obtain an estimate of $\alpha$ in equation 2.

(2) We combine the main body of data using the method of reduced parallaxes (e.g. Feast 2002) and fixed values of $\alpha$ to obtain the zero-point, $\gamma$, in equation 2.

4 THE SLOPE $\alpha$ OF THE GALACTIC PL($W$) RELATION

Table 2 lists the subset of Cepheids used in this section. The table gives, Hipparcos number, name, the parallax and its standard error from the combined Hipparcos and HST data, the absolute magnitude, $M_W$, (without Lutz-Kelker correction) and its standard error ($= 2.17\sigma_s/\pi$), from this parallax and the adopted photometry together with $\beta = \ldots$
Figure 1. $M_W$ (with Lutz-Kelker correction) plotted against log $P$ for the 14 stars in Table 2. The line is the relation finally adopted which has $\alpha = -3.29$, $\beta = 2.45$ and $\gamma = -2.58$.

Table 3. Determinations of the slope, $\alpha$, in equation 2.

| No. | Sample                      | $\alpha$      | $\beta$ |
|-----|-----------------------------|---------------|---------|
| 1   | 10 HST stars + HST phot.    | -3.335 $\pm$ 0.172 | 2.45 |
| 2   | 10 HST stars + New phot.    | -3.473 $\pm$ 0.183 | 2.45 |
| 3   | 10 stars HST + Hipp         | -3.328 $\pm$ 0.188 | 2.45 |
| 4   | (3) + Polaris (11 stars)    | -3.285 $\pm$ 0.169 | 2.45 |
| 5   | weight $> 10$ (13 stars)    | -3.273 $\pm$ 0.155 | 2.45 |
| 6   | weight $> 8$ (14 stars)     | -3.288 $\pm$ 0.151 | 2.45 |
| 7   | (6) omitting I Car (13 stars)| -3.265 $\pm$ 0.230 | 2.45 |
| 8   | LMC (OGLE)                  | -3.29 $\pm$ 0.01 | 2.45 |
| 9   | Freedman (adopted)          | -3.26         | 2.45 |
|     |                             |               |         |

(a) 10 HST stars + New phot.  
11 10 HST stars + Hipp       
12 (11) + Polaris (11 stars) 
13 weight $> 10$ (13 stars)  
14 weight $> 8$ (14 stars)   
15 Sandage (adopted)

(b) 2.45. Also given are the Lutz-Kelker corrections applicable in this case and the log of the fundamental period. Fig. 1 shows $M_W$ with Lutz-Kelker corrections applied plotted against log $P$ and with our finally adopted relation ($\alpha = -3.29$, $\beta = 2.45$ and $\gamma = -2.58$) shown. The residuals in this case are also listed in Table 2.

Weighted least square fits to equation 2 were made to various subsets of the data and the slopes ($\alpha$) derived are listed in Table 3. This is in two parts: Table 3(a) adopts $\beta = 2.45$ (as in Freedman et al. 2001) and Table 3(b) adopts $\beta = 2.523$ (as in S2006) (see section 3).

From Table 3(a) we draw the following conclusions:
1. Slight changes in the adopted photometry affect the value of the slope. However, this is not the main source of uncertainty.
2. Adding the overtone Polaris at its fundamental period does not change the slope appreciably (see also the next section).
3. Leaving out I Car does not affect the slope appreciably. This is important since I Car is by far the longest period star in this sample and therefore, when included, has a major effect on the slope determined.
4. Our best determinations (solutions 6 and 7 of Table 3) give values of $\alpha$ close to that determined for LMC stars from the OGLE data (as shown in the table). There is still significant uncertainty in our value of the Galactic slope. However, within the uncertainties it agrees with that determined for the LMC.

From Table 3(b) we draw the following conclusions:
1. Using the “Sandage” colour coefficient ($\beta$) we get slopes which are not significantly different from those in Table 3(a).
2. Our slopes, especially the higher weight ones, are distinctly different from that adopted by S2006, and in view of the uncertainties surrounding this latter result (see section 3) we suggest it should be replaced by our best value. Nevertheless, in the next section we give zero-points derived using a value of $\alpha = -3.75$.

The main conclusion of this section is that within current uncertainties the value of $\alpha$ is the same in the Galaxy as in the LMC and our final results will be based on this assumption.

Table 4. The zero-point, $\gamma$, of equation 2 with fixed $\alpha$ and $\beta$.

| No. | Stars | $\alpha$ | $\beta$ | $\gamma$ | Notes        |
|-----|-------|----------|---------|----------|--------------|
|     |       |          |         |          |              |
| (a) |       |          |         |          |              |
| 1   | 14    | -3.255   | 2.45    | -2.606   | $0.022$      |
| 2   | 10    | -3.255   | 2.45    | -2.568   | $0.036$      |
| 3   | 14    | -3.29    | 2.24    | -2.579   | $0.022$      |
| 4   | 14    | -2.75    | 2.523   | -2.264   | $0.028$      |
|     |       |          |         |          |              |
| (b) |       |          |         |          |              |
| 5   | 240   | -3.255   | 2.45    | -2.604   | $0.030$      |
| 6   | 240   | -3.29    | 2.45    | -2.576   | $0.030$      |
| 7   | 239   | -3.29    | 2.45    | -2.558   | $0.044$      |
| 8   | 213   | -3.29    | 2.45    | -2.563   | $0.046$      |
| 9   | 240   | -3.75    | 2.523   | -2.263   | $0.090$      |

5 THE ZERO-POINT $\gamma$ OF THE PL($W$) RELATION

Table 4 gives results of the determinations of the zero-point, $\gamma$ in equation 2 using fixed values of $\alpha$ and $\beta$. In Table 4(a) are the values of $\gamma$ obtained directly from the 14 stars in Table 2 with Lutz-Kelker corrections applied. In Table 4(b) we give the values of $\gamma$ derived by the method of reduced parallaxes (e.g. Feast 2002) to our bulk sample. Points to note are:
1. Values of $\gamma$ in Table 4(a) agree closely with the corresponding values in Table 4(b).
2. In Table 4(b), leaving out the high weight overtone pulsator, Polaris, makes no significant difference. Nor does leaving out all the known overtone pulsators. This is consistent with the discussion in section 3 which noted that the reddening-free relation in the LMC was very closely the same
3. In carrying out the reduced parallax solutions we have assumed that the uncertainties are dominated by the errors in the parallaxes. That is, we have neglected the second term in equation 3 of Feast (2002). However, if we supposed that 
\[ \sigma_m = \sigma_M = 0.14, \]
which we believe would be a gross over-estimate, then the value of \( \gamma \) in Table 4(b) (solution 6) would only change from \(-2.576\) to \(-2.554\).

We adopt \(-2.58\) (solution 6, Table 4) as the value for \( \alpha \) to use with \( \alpha = -3.29 \) and \( \beta = 2.45 \) in equation 2.

### 6 THE HUBBLE CONSTANT

An important use of Cepheids is as a basis for the determination of the distances of galaxies and from that the estimation of a value of \( H_0 \). This can then be compared with the value derived in other ways (e.g. from the microwave background) which require the adoption of a general cosmological model; thus providing a test of the model. Sandage and his co-workers have recently completed a major programme of reanalysing HST data of Cepheids in galaxies in which supernovae have been observed (see S2006, Saha et al. 2006 and earlier papers in the series). In their summary paper they use the Cepheid data to derive distance moduli to 10 normal SNIs. From these they derive the maximum SNs brightness (as defined by them). They then use this as a zero-point for a determination of \( H_0 \). We have re-determined the distance moduli of these galaxies using equation 2 with our adopted coefficients from section 5 viz. \( \alpha = -3.29 \), \( \beta = +2.45 \), and \( \gamma = -2.58 \). We use the same selection of Cepheids as used by S2006 and adopt the corrected apparent SN magnitudes in Table 2 of that paper. We derive SNIs absolute magnitudes equivalent to those in Table 3 of S2006 and like them derive weighted means. Table 5 gives the weighted mean absolute magnitudes of S2006 and the corresponding values of \( H_0 \) which this implies in their work.

Table 5 also contains the equivalent weighted mean absolute magnitudes derived from our estimates of the moduli. Evidently the difference between our absolute magnitude and theirs implies a change in \( H_0 \) and this is given in the table (in units of km s\(^{-1}\) Mpc\(^{-1}\)). The table contains the results of two estimates we have made for the moduli. In one case we have applied no metallicity correction. In the other we have applied a metallicity correction based on the “Sakai” values of [O/H] in table 1 of S2006. These abundances are in the \( T_e \) system and Macri et al. (2006) find from their work on NGC 4258 that in this system a PL(\( W \)) relation such as we have used requires a correction of \(-0.49\) mag/dex. There is considerable uncertainty in the size of the required metallicity correction. Fortunately, as Table 5 shows, the result we obtain is quite insensitive to the correction used. This is due to the mean metallicity of the S2006 galaxies being close to that of the local Cepheids. Only a large, non-linear metallicity correction would change this conclusion.

Thus our best value of \( H_0 \) based on our PL(\( W \)) relation but with all other data and assumptions as in S2006 is 69.6. S2006 obtained 62.3\( \pm \)5. Our improved Cepheid results would in principle reduce the uncertainty to \( \sim \pm 2 \), but to be conservative we keep it the same. Our revised value (70\( \pm \)5) is clearly compatible with the value of 73\( \pm \)3 found from the WMAP data and a \( \Lambda \)CDM model (Spergel et al. 2006).

There are a large number of other determinations of \( H_0 \), some of them depending on a Cepheid scale. The most widely quoted is that of Freedman et al. (2001) who obtained \( H_0 = 72 \pm 8 \). Since that paper was published there has been much work on the refinement of the basic HST data, on galaxy metallicities and on the various large scale distance indicators used by these workers. However, the fact that the values of \( \alpha \) and \( \beta \) in the PL(\( W \)) relation they use are close to ours, and that the mean metallicity of their sample, weighted according to the contribution of an indicator to \( H_0 \) is close to solar, means that a satisfactory estimate of the effect of our work on theirs can be made by comparing PL(\( W \)) zero-points. Table 4 solution 5 shows that with their \( \alpha \) and \( \beta \) we find \( \gamma = -2.604 \) whereas they used \(-2.724 \) at the metallicity of Galactic Cepheids (see equation 3 above). The difference (0.12 mag) implies an increase of their \( H_0 \) to 76\( \pm \)8, where to be conservative we keep the error the same, though the discussion of Freedman et al. together with our new results would in principle reduce this to \( \sim \pm 6 \). Again this revised \( H_0 \) is quite compatible with the WMAP result.
van Leeuwen, Feast, Whitelock, Laney

Table 5. SNIa absolute magnitudes and $H_0$.

|         | $M_B$ | $M_V$ | $M_I$ |
|---------|-------|-------|-------|
|         | $H_0$ (B) | $H_0$ (V) | $H_0$ (I) |
| S2006   | $-19.49 \pm 0.04$ | $-19.46 \pm 0.04$ | $-19.22 \pm 0.05$ |
|         | $62.4 \pm 1.2$   | $62.5 \pm 1.2$   | $62.1 \pm 1.4$   |
|         | $62.3 \pm 1.3 \text{ (int.)}$ |
| Revised (no metal cor.) | $-19.26 \pm 0.05$ | $-19.22 \pm 0.05$ | $-18.98 \pm 0.07$ |
|         | $69.4 \pm 1.6$   | $69.8 \pm 1.6$   | $69.4 \pm 2.3$   |
|         | 69.5            |
| Revised (with metal cor.) | $-19.26$ | $-19.22$ | $-18.97$ |
|         | 69.4            | 69.8            | 69.7            |
|         | 69.6            |

7 THE DISTANCE MODULUS OF THE LMC USING $VI$ PHOTOMETRY

Combining the LMC PL($W$) relation (equation 5) with our derived value of $\gamma \approx (2.58)$ gives directly the true modulus of the LMC uncorrected for metallicity effects. We thus find a modulus of $18.52 \pm 0.03$. Adopting the results of Andrievsky et al. (2002) and Sakai et al. (2004), as discussed by S2006 the LMC Cepheids are metal deficient by $\Delta[O/H] = 0.26$ on the "$T_e$" abundance scale. As already noted Macri et al. (2006) found a Cepheid metallicity effect, applicable to our PL($W$) results, of $-0.49(\pm0.15) \text{ mag/dex}$. Applying this leads to a metallicity corrected LMC modulus of $18.39 \pm 0.05$.

The main uncertainty in this result is due to the uncertainty in the metallicity correction to our PL($W$) relation (note that corrections at other wavelengths would not necessarily be the same). It has even been recently suggested that the effect may be negligible (Rizzi et al. 2007). This would be somewhat remarkable since it has been long known (Gascoigne & Kron 1965 and much further work) that the intrinsic colours of LMC Cepheids differed from those of Galactic Cepheids of the same period. This was shown by Laney & Stobie (1986) to be due to a combination of changes in atmospheric blanketing together with a real shift of the instability strip in temperature. Fortunately the metallicity correction problem is not important for the work on $H_0$ discussed in the previous section. The use of the LMC modulus, however determined, will remain an uncertain basis for an extragalactic scale based on Cepheids pending further work on the metallicity effect.

8 THE CEPHEID-BASED DISTANCE OF THE MASER-HOST GALAXY NGC 4258

NGC 4258 is of special interest because a distance has been determined (Herrnstein et al. 1999) based on the motions of $H_2O$ masers around the central black hole. Macri et al. (2006) have obtained HST photometry of Cepheids in this galaxy. Here we concentrate on Cepheids in their inner field which has a metallicity close to solar and therefore is immune to the problem of metallicity corrections when combined with our Galactic calibration. Some discussion of the Cepheid-based distance of this galaxy was made in connection with the HST Cepheid parallax work (FB2007).

We first consider the coefficient $\alpha$ of equation 2. Taking the 69 Cepheids which pass the selection criteria of Macri et al.2 We find the following:

1. For our adopted value of $\beta = 2.45$ we find $\alpha = -3.18 \pm 0.13$.
2. For the value of $\beta = 2.523$ favoured by S2006 we find $\alpha = -3.19 \pm 0.13$.

These values are less than 1$\sigma$ from our adopted slope of $-3.29$, and 4.3$\sigma$ different from that favoured by S2006 for Galactic Cepheids ($-3.75$). We take this as further evidence that the slope of the PL($W$) relation in the LMC applies also to Cepheids of approximately Galactic composition. Macri et al. reach a similar conclusion by a different method.

Adopting $\alpha = -3.29$, $\beta = 2.45$ and $\gamma = 2.58$, from section 5, for equation 2 we find from the data of Macri et al. (2006) for their inner region, a distance modulus of 29.22$\pm$0.03 for NGC 4258. The standard error takes into account the internal scatter in the NGC 4258 Cepheid data and the uncertainty in the adopted $\gamma$. The currently available maser-based distance modulus is 29.29$\pm$0.15 (Herrnstein et al. 1999) and is thus in good agreement with the parallax-based Cepheid modulus. The referee has suggested that the Cepheid modulus may be slightly underestimated due to the possible effects of blending on the NGC 4258 Cepheids and an even closer agreement with the maser distance might be obtained if this could be taken into account.

Improvements in the maser-based modulus are expected (Macri et al. 2006, Argon et al. 2007) and these should allow a more stringent comparison with the Cepheids.

9 METHODS USING $JK$ PHOTOMETRY

In the present section our aim is to establish zero points for PL and PLC relations in the near infrared. The data used were outlined in section 2 and listed in Table A1. In the case of the important overtone pulsator, Polaris, the 2MASS observation is heavily saturated and thus has a very large uncertainty and there appears to be no other ground-based near infrared photometry on a system which can be reliably transformed to the SAAO system. There is, however, extensive DIRBE data (Smith et al. 2004). This has been transformed to the SAAO system as follows. There are eleven SAAO $JHKL$ standard stars in the DIRBE catalogue with $J < 1.51$ and $K < 1.00$ and no indication of confusion in the
Using these as calibrators, together with Procyon which has \( J = -0.40 \) and \(-0.65 \) on the SAAO system (transformed from Glass (1974) and Bouchet et al. (1991)) leads to \( J = 0.98 \) and \( K = 0.60 \) for Polaris.

In view of the \( V,I \) results of the present paper and those at \( V,I \) and \( K \) in FB2007, we confine ourselves to determining the zero-points of PL and PLC relations of forms established in the LMC. The most extensive data set in \( J \) and \( K \) for LMC Cepheids, based on multiple observations, is that of Persson et al. (2004). We adopt their PL and PLC relations. Transformed (Carter 1990) onto the SAAO system these are:

\[
M_K = -3.258 \log P + \gamma_1 \tag{7}
\]

and

\[
M_K = -3.457 \log P + 1.894(J - K)_0 + \gamma_2. \tag{8}
\]

We have carried through the calculations with reddening corrections according to both the reddening law derived by Laney & Stobie (1993) and that of Cardelli, Clayton & Mathis (1989) and taking values of \( E(B - V) \) from either Fernie et al. (1995) or Tammann et al. (2003) as outlined in section 2. The differences in the results obtained were all very small and much smaller than the standard errors of the quantities sought, so we list only one of them (Tammann et al. redenings, Laney & Stobie reddening law).

In the case of a PL relation we expect the overtones, at their fundamental periods, to be brighter than fundamental pulsators of the same period, because of a temperature difference, and this is clear from the PL(\( K \)) relations in the LMC by, e.g. Groenewegen (2000). We must therefore treat fundamental pulsators and overtones (at their fundamental periods) separately in discussing equation 7. Note that because of the relation between fundamental and first overtone periods (equation 1) the PL relation when transformed from the fundamental to the observed, overtone, periods will have a somewhat different slope, as found by Groenewegen. Using the method of reduced parallaxes as outlined earlier we obtain the results listed in Table 6 where Polaris is treated separately. Using the whole sample or only those with well covered light curves makes no significant difference to the results.

The difference in \( \gamma_1 \) between Polaris (at its fundamental period) and the mean of the fundamental pulsators (\(-0.07 \pm 0.06 \) mag) is not significant. However, it in fact agrees closely with the difference between overtones (at their fundamental periods) and fundamental pulsators expected (\(-0.08 \) mag) at the period of Polaris from the LMC data of Groenewegen (2000).

Results for the PLC zero-point are given in Table 7.

We expect for the PLC relation that overtones at their fundamental periods should follow the same relation as fundamental pulsators. The table shows that Polaris is not \(+0.05 \pm 0.06 \) mag fainter than the mean of the fundamental pulsators, not significantly different from zero.

## 10 THE LMC MODULUS FROM \( J \) AND \( K \)

The LMC relations of Persson et al. (2004) converted to the SAAO system using the relations of Carter (1990) are:

\[
K_0 = -3.258 \log P + 16.048 \tag{9}
\]

and

\[
K_0 = -3.457 \log P + 1.894(J - K)_0 + 15.402. \tag{10}
\]

Also Groenewegen (2000) obtained for LMC overtone pulsators in the 2MASS system:

\[
K_0 = -3.381 \log P_1 + 15.533. \tag{11}
\]

The zero-point, \( \gamma_1 \), for fundamental pulsators in Table 6 together with equation 9 gives an LMC modulus of 18.45 \pm 0.05. Polaris, at its observed (overtone) period with \( K = 0.58 \) (the value from section 9 converted to the 2MASS system using Carpenter (2001)) and equation 11, gives 18.49 \pm 0.04 (taking into account the uncertainty in Groenewegen’s result. A straight mean of these two values (18.47 \pm 0.03) is our best estimated of the PL(\( K \)) modulus of the LMC corrected for abundance effects. This is in agreement with 18.45 \pm 0.04 derived by FB2007. A weighted mean of the last three entries in Table 7 leads to \( \gamma_2 = -3.05 \pm 0.02. \) Together with equation 10 and an estimate of its uncertainty leads to an infrared PLC modulus of 18.45 \pm 0.04 again without any metallicity correction.

These values may be compared with those found from \( V,I \) in section 7 which were 18.52 \pm 0.03 uncorrected for metallicity effects and 18.39 \pm 0.05 with a metallicity correction from Macri et al. (2006). These results suggest that any metallicity corrections to the infrared relations will be small.

## 11 CONCLUSIONS

Our main conclusions based on the combined revised Hipparcos and HST data are the following:

1. The coefficient of the \( \log P \) term in a reddening free \( V,I \) relation is found to be the same, within the uncertainties, in

3 Groenewegen’s relation for LMC fundamental pulsators is in good agreement with that of Persson et al. (2004). This latter is in the LCO system which is very close to that of 2MASS. The PL slopes given by Ita et al. (2004) (LCO system) do not agree well with Groenewegen or Persson et al. This may be connected with the inclusion of LMC Cepheids with \( \log P < 0.4 \) in the Ita et al. sample (Y. Ita private communication)
our Galaxy and in the LMC.
2. This result is supported by an analysis of the data of Macri et al. (2006) for Cepheids in the inner region of NGC 4258.
3. Our reddening-free \( V,I \) relation applied to the Cepheids in the inner region of NGC 4258 leads to a modulus of 29.22 \( \pm 0.03 \) in agreement (but of higher accuracy than) the maser-based distance (29.29 \( \pm 0.15 \)).
4. Our revised Cepheid \( V,I \) calibration leads to a revision of the Cepheid-based distances to the 10 galaxies on which S2006 base their SNi calibration and Hubble constant. We revise their results from \( H_0 = 62 \) to 70 \( \pm 5 \) km s\(^{-1}\) Mpc\(^{-1}\) This result is immune to metallicity effects on the Cepheid scale as long as these are linear. The Freeman et al. (2001) result is revised from 72 to \( H_0 = 76 \) \( \pm 8 \) km s\(^{-1}\) Mpc\(^{-1}\). Both these results are consistent with the recent WMAP value (\( H_0 = 73 \) \( \pm 3 \) km s\(^{-1}\) Mpc\(^{-1}\)).
5. The zero-points of Galactic PL(\( K \)) and PLC(\( J,K \)) are derived.
6. Applying our various relations to the LMC we find the following distance moduli, uncorrected for metallicity effects: 18.52 \( \pm 0.03 \) from a reddening-free \( V,I \) relation; 18.47 \( \pm 0.03 \) from a PL(\( K \)) relation; 18.45 \( \pm 0.04 \) from a PLC(\( J,K \)) relation.
7. Applying a metallicity correction derived by Macri et al. (2006) to our LMC modulus leads to a new zero-point of Galactic PL(\( K \)) modulus of 18.39 \( \pm 0.05 \).

**ACKNOWLEDGMENTS**

We are very grateful to Dr L. Berdnikov for placing the unpublished revision of his catalogue of Cepheid photometry at our disposal. Dr L. Macri and Dr Y. Ita, very kindly and promptly, answered a number of queries about their work. We thank the referee (Prof. W. Gieren) for his comments.

**REFERENCES**

Alcock C. et al., 1995, AJ, 109, 1653
Andrievsky S.M. et al., 2002, A&A, 381, 32
Antonello E., Poretti E., 1986, A&A, 169, 149
Antonello E., Poretti E., Reduzzi L., 1990, A&A, 236, 138
Argon A.L., Greenhill L.J., Reid M.J., Moran, J.M., Humphreys E.M.L., 2007, [astro-ph/0703396]
Beaulieu J.P. et al., 1996, A&A, 303, 137
Benedict G.F., MacArthur B.E., Feast M.W., Barnes T.G., Harri son T.E., Patterson R.J., Menzies J.W., Bean J.L., Freedman W.L., 2007, AJ, in press [astro-ph/0612465] (FB2007)
Benedict G.F. et al., 2002, AJ, 124, 1695 (FB2002)
Berdnikov L.N., 1992, Sov. Ast. Let., 18, 398
Berdnikov L.N., Dambis A.K., Voziatova M.A., 2000, A&A Sup, 143, 211
Bouchet P., Manfroid J., Schmider F.X., 1991, A&A Sup, 91, 409
Caldwell J.A.R., Coulson I.M., 1987, AJ, 93, 1090
Caldwell J.A.R., Coulson I.M., 1989, MNRAS, 240, 285
Cardelli J.A., Clayton G.C., Mathis J.S., 1989, ApJ, 345, 245
Carpenter J.M., 2001, AJ, 121, 2851
Carter B.S.C., 1990, MNRAS, 242, 1
ESA, 1997, The Hipparcos Catalogue, ESA SP-1200
Evans N.R., 1992a, ApJ, 384, 220
Evans N.R., 1992b, ApJ, 389, 657
Evans N.R., 1994, ApJ, 436, 273
Evans N.R., 1995, ApJ, 445, 393
Evans N.R., Carpenter K.G., Robinson R., Kienzel F., Dekas A.E., 2005, AJ, 130, 789
Feast M.W., 1998, Mem. Soc. Ast. It., 69, 31
Feast M.W., 1999, PASP, 111, 775
Feast M.W., 2002, MNRAS, 337, 1035
Feast M.W., Catchpole R.M., 1997, MNRAS, 286, L1
Feast M.W., Walker A.R., 1987, ARA&A, 25, 345
Fernie J.D., Beattie B., Evans N.R., Soager S., 1995, IBVS, 4148 [http://ddo.astro.utoronto.ca/cephids.html]
Freedman W.L. et al., 2001, ApJ, 553, 47 (WLF 2001)
Gosolique S.C.B., Kron G.E., 1965, MNRAS, 130, 333
Gieren W., Storm J., Barnes T.G., Fouqué P., Pietrzynski G., Kienzel F., 2005, ApJ, 627, 224
Glass I.S., 1974, Mon. Not. Ast. Soc. S.A. 33, 53
Gorynya N.A., Samus N.N., Sachkov M.E., Antipin S.V., Rastorguev A.S., 2000, in: The Impact of Large-Scale Surveys on Pulsating Star Research, (eds.) Szabados, L, Kurtz D.W., ASP Conf. Ser. 203, 242
Groenevewen M.A.T., 1999, A&A Sup, 139, 245
Groenevewen M.A.T., 2000, A&A 363, 901, 2000
Groenevewen M.A.T., Oudmaijer R.D., 2000, A&A, 356, 849
Herrnstein et al., 1999, Nature, 400, 539
Ita Y. et al., 2004, MNRAS, 347, 720
Kienzel F., Moskalik P., Bersier D., Pont, F., 1999, A&A, 341, 818
Kienzel F., Pont F., Moskalik P., Bersier D., 2000, in: The Impact of Large-Scale Surveys on Pulsating Star Research, (eds.) Szabados L., Kurtz D.W., ASP conf. Ser. 203, 239
Kiss L.L., Vinkó J., 2000, MNRAS, 314, 420
Lane C.D., Stobie R.S., 1986, MNRAS, 222, 449
Lane C.D., Stobie R.S., 1992, A&A Sup, 93, 93
Lane C.D., Stobie R.S., 1993, MNRAS, 263, 921
Lane C.D., Stobie R.S., 1994, MNRAS, 266, 441
Leavitt H.S., 1988, Harvard Ann. 60, 87
Leavitt H.S., 1912, Harvard Circ. 173
Luck R.E., Kovtyukh V.V., Andrievsky S.M., 2001, A&A, 373, 589
Lutz T.E., Kelker D.H., 1973, PASP, 85, 573
Macri L.M., Stanek K.Z., Bersier D., Greenhill L.J., Reid M.J., 2006, ApJ, 652, 1133 (LM2006)
Madore B.F., 1976, RGO Bulletins No. 182, 153
Martin W.L., Warren P.R., Feast M.W., 1979, MNRAS, 188, 139
Manetegazza L., Poretti E., 1992, A&A Sup, 93, 93
We thank the referee (Prof. W. Gieren) for his comments.
van Leeuwen F., 2007, Hipparcos: The new reduction of the raw data, Springer, Berlin (in press)
van Leeuwen F., Fantino E., 2005, A&A, 439, 791
Zakrzewski B., Ogłoza W., Moskalik P., 2000, Act. Ast., 50, 387