MULTI-OBJECTIVE AGGREGATE PRODUCTION PLANNING DECISIONS USING TWO-PHASE FUZZY GOAL PROGRAMMING METHOD

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Abstract. In practical aggregate production planning (APP) decisions, the decision maker (DM) must simultaneously handle multiple conflicting goals that govern the use of the constrained resources. This study aims to present a two-phase fuzzy goal programming method to solve multi-objective APP problems with multiple products and multi-time periods. The designed fuzzy multi-objective linear programming model attempts to simultaneously minimize total costs, total carrying and backordering volume, and total rates of changes in labor levels with reference to inventory carrying levels, machine capacity, work-force levels, warehouse space and available budget. An industrial case is used to demonstrate the feasibility of applying the proposed method to real-life APP decisions. The contribution of this study lies in presenting a two-phase fuzzy goal programming methodology to solve multi-objective APP decision problems and provides a systematic decision-making framework that facilitates a DM to interactively adjust the search direction until the preferred efficient compromise solution is obtained.

1. Introduction. The aims of aggregate production planning (APP) decisions are to specify appropriate production levels for each product category to satisfy fluctuating demand over the intermediate planning horizon, often from 2 to 18 months ahead, and to set up overall production policies on regular and overtime production, inventory carrying, subcontracting and backordering levels, and the work-force levels of hiring and layoffs for determining the organizational resources to be used appropriately [33, 36, 40]. In production planning hierarchical levels, APP generally falls between the broad decisions of the long-range strategic planning and the detailed short-range operational planning. Relevant forms of family disaggregation planning, involving master production scheduling, material requirements planning and capacity requirements planning, all depend on APP results in a hierarchical manner.

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Since the famous HMMS decision rule was presented by Holt et al. [13], related issues of APP decisions have attracted considerable attention from both practitioners and academia. Numerous APP techniques including mathematical programming models, algorithms and heuristics have also been presented to solve APP problems [10, 25, 30-31]. When any of traditional APP techniques are used, however, the goals and related parameters are generally assumed to be deterministic/crisp. In practical production systems, model inputs and environmental coefficients, such as market demand, available resources, machine capacity and related operating costs, are normally fuzzy/imprecise because information is incomplete and unobtainable over the intermediate planning horizon. Conventional deterministic APP decision techniques cannot obviously solve all practical APP problems in uncertain environments.

Additionally, the major functional areas in an organization that yield an input to the aggregate plan generally have conflicting goals regarding the use of organization’s resources, and these conflicting goals are required to be solved simultaneously by the decision maker (DM) in the framework of imprecise aspiration levels. These conflicting goals are to minimize total costs, inventory investment, backordering volume, rate of change in labor levels, and/or maximize total profits, customer service levels and utilization of plant equipment and facilities [19, 21, 25, 33, 40-41, 43]. Solutions to optimum fuzzy multi-objective APP problems benefit from assessing the imprecision of the DM’s judgments, such as “annual total costs should be substantially less than or equal to 5 millions,” or “total rate of change in labor levels should be substantially less than or equal to 200 man-hours.”

This study aims to present a two-phase fuzzy goal programming method for solving the multi-objective APP problems with multi-products and multi-time periods. The designed multi-objective linear programming (MOLP) model designed in this study attempts to simultaneously minimize total costs, total carrying and backordering volume, and total rates of changes in labor levels with reference to inventory levels, machine capacity, work-force levels, warehouse space and available budget. The remainder of this study is organized as follows. Section 2 dedicates to a review of the relevant literature. Section 3 describes the problem, details the assumptions and formulates the fuzzy multi-objective APP model. Subsequently, Section 4 develops the fuzzy programming method for solving the fuzzy multi-objective APP problems. Next, an industrial case is used to implement the feasibility of applying the proposed method to a practical APP problem in Section 5. Finally, conclusions are drawn in Section 6.

2. Review of the literature. Conventionally, HMMS linear rule [13] is one of the extensively adopted frameworks employed to solve deterministic APP problems. According to Saad [30] and Nam and Logendran [25], traditional APP models may be classified into five categories: (1) linear decision rule, (2) linear programming (LP) and transportation method, (3) management coefficient approach, (4) simulation, and (5) search decision rule. Shi and Haase [31] designed an APP decision model with multi-goal and multi-capacity-demand levels by using the multi-criteria and multi-constraint level (MC2) linear programming technique. Byrne and Bakir [5] presented a hybrid algorithm that combines mathematical programming and simulation models to solve deterministic APP problems for an entire manufacturing system. That study demonstrated how the analytical model functions in co-operation with the simulation model to yield better results than either approach can achieve
alone. Leung and Chan [19] designed a pre-emptive goal programming model to maximize profits, minimize repairing costs and maximize machine utilization in relation to differential operation constraints, including production capacity, workforce levels, factory location, machine utilization and storage space. Related investigations on deterministic APP decisions include Jain and Palekar [15], Moreno and Montagna [24], Singhvy et al. [32] and Stephen et al. [33].

In real-life production systems, however, environmental coefficients and related parameters are normally fuzzy/imprecise owing to information is incomplete and unobtainable over the planning horizon. To deal with imprecision, Bitran and Yanasse [3] presented a stochastic programming model for solving the multi-time period APP problems having imprecise demand with known probabilistic distribution function in which a distribution bound can be derived by using a deterministic approximation to solve the original APP problems. Feiring [8] focused on an imprecise APP decision of manufacturing resources in order to satisfy stochastic demand with normal probabilistic distribution for a family of products, and to minimize total production and inventory carrying costs over a rolling horizon. Castro et al. [6] developed an optimal production model with stochastic demand, constant defective rate and capacity constraints to analyze the optimum APP problem of a chocolate milk manufacturer. Fleten and Kristoffersen [9] presented a multi-stage mixed-integer stochastic linear programming model to hydropower production planning decisions having uncertain day-ahead market prices. Generally, the main drawbacks of stochastic programming are lack of computational efficiency and inflexible probabilistic doctrines which might not be able to model the real imprecise meaning of DM [17, 45].

Fuzzy set theory, was presented by Zadeh [46], has provided an appropriate methodology to deal quantitatively with decision problems that are formulated as mathematical programming models with imprecise parameters. Zimmermann [47] first introduced fuzzy set theory into an ordinary LP problem with fuzzy goal and constraints. Following the fuzzy decision-making concept of Bellman and Zadeh [2], it confirmed that an equivalent ordinary single-goal LP problem exists. Subsequently, fuzzy set theory and Zimmermann’s fuzzy programming technique have been developed into several fuzzy optimization methods to solve APP problems. Wang and Fang [39] proposed a genetics-based inexact technique to imitate the human decision procedure for fuzzy APP problems. Instead of locating one exact optimal solution, that technique yielded a family of inexact solution with an acceptance levels by adopting a mutation operator to move along a weighted gradient direction. Tang et al. [36] focused on fuzzy linear programming method to modeling multi-product APP problems with fuzzy demand and fuzzy capacities for minimizing the total costs involving quadratic production costs and linear inventory holding costs. Fung et al. [10] developed a fuzzy multi-product APP decision model to cater to different scenarios under various decision-making preferences by applying integrated parametric programming method, best balance and interactive techniques. That model can also effectively enhance the ability of an aggregate plan to provide feasible disaggregate plans under varying circumstances with fuzzy demands and fuzzy capacities. Wang and Liang [42] designed an interactive possibility linear programming model to solve multi-product and multi-time period APP problems with imprecise cost coefficients and imprecise constraints for minimizing total costs. Vasant [38] developed a fuzzy linear programming technique with a modified S-curve membership function to solve fuzzy mix product selection
problem where a performance measure was defined to identify the decision for high level of profit with high degree of satisfaction. Related studies include Hsu and Wang [14], Petrovic et al. [27] and Tang et al. [35].

In practical situations, the many functional areas in an organization that yield an input to the APP decisions generally have conflicting goals regarding the use of organization’s resources, and these conflicting goals are required to be solved simultaneously by the DM in the framework of imprecise aspiration levels. Zimmermann [48] first extended his fuzzy linear programming technique to an ordinary MOLP problem. Regarding the applications of fuzzy mathematical programming to solve multi-objective APP problems, Wang and Fang [40] developed a fuzzy MOLP method to solve imprecise APP problems with multiple goals where the price, unit cost/time coefficients, work-force levels and market demand were fuzzy in nature over the production planning horizon. Wang and Liang [31] developed an interactive fuzzy multi-objective APP decision model with piecewise linear membership function to specify fuzzy objective functions. That model can yield an efficient compromise solution and the DM’s overall levels of satisfaction. Aliev et al. [1] developed a fuzzy-genetic approach to solve aggregate production-distribution planning problems, in which the interactive solution procedure was developed based on the fuzzy integrated model, which allowed sound trade-offs between maximizing profit and fillrate. Liang [21] introduced possibilistic linear programming to multi-product and multi-time period APP decisions with imprecise goals and unit cost coefficients having triangular distributions, in which the imprecise APP model was designed to minimize total production costs and changes in work-force levels. More recently, Torabi and Hassini [37] proposed a multi-objective possibilistic mixed integer linear programming model for integrating procurement, production and planning considering various conflicting objectives simultaneously as well as the fuzzy nature of some critical parameters such as market demands, unit cost/time coefficients and relevant capacity levels.

Generally, these fuzzy APP optimization techniques described above neglect the time value of money for relevant operating cost categories, available budget constraint and conditions of insufficient resources; thus, they are unrealistic in real-life applications. Particularly, although it had been indicated that the minimum operator adopted extensively by the existing fuzzy APP techniques has some good properties, the optimal solution yielded using the minimum operator may not be an efficient solution [7, 11, 18, 20, 26].

3. Problem formulation.

3.1. Problem description, assumptions and notation. The fuzzy multiple products and multiple time periods APP problem with multiple goals examined in this study can be described as follows. Assume that an industrial company manufactures \(N\) types of products to fulfill market demand over the planning horizon \(T\). The APP decision involves determining the most effective means of satisfying market demand by adjusting output rates, hiring and layoffs, inventory carrying levels, overtime work, subcontracting, backordering and other controllable variables. This study focuses on developing a two-phase fuzzy goal programming method to yield the optimum APP plan for meeting imprecise market demand in a fuzzy environment. The expected goals of this APP decision are to simultaneously minimize total costs, total carrying and backordering volume, and total rate of change in work-force levels.
The formulated fuzzy programming model is based on the following assumptions.

1. All of the objective functions are fuzzy with imprecise aspiration levels.
2. The linear membership functions are specified for fuzzy goals, and the minimum operator and the weighted average operator are sequentially used to aggregate fuzzy sets.
3. All of the objective functions and constraints are linear equations.
4. The escalating factors in each of the operating costs categories are certain over the next $T$ planning horizon.
5. The values of related unit cost/time coefficients in the objectives and constraints functions are certain over the planning horizon.
6. The market demand over a particular period can be either satisfied or backordered, but the backorder must be fulfilled in the next period.
7. Actual labor levels, machine capacity, warehouse space and total budget in each period cannot exceed their respective maximum levels.

Assumption 1 relates to the fuzziness of the objective functions in practical APP problems, and incorporates the variations in the DM judgments regarding the solutions of fuzzy optimization problems in a framework of imprecise aspiration levels. Assumption 2 is made to specify the fuzzy objective functions with the simplified linear membership functions and to convert the fuzzy MOLP problem into an equivalent ordinary LP form that can be solved efficiently with the simplex method [16, 48-49]. Assumptions 3-5 indicate that the linearity and certainty properties must be technically satisfied as a standard LP form. Assumption 6 concerns the portion of market demand that must be satisfied during any period, whereas the rest of the market demand can be backordered [21, 40]. Assumption 7 represents the limitations on the maximum available resources in the normal business operations.

The following notation is used.

**Index sets**

$n$ index for product type, for all $n = 1, 2, ..., N$

$t$ index for planning time period, for all $t = 1, 2, ..., T$

$g$ index for objective, for all $g = 1, 2, ..., K$

**Objective functions**

$z_1$ total costs

$z_2$ total carrying and backordering volume

$z_3$ total rate of change in labor levels

**Decision Variables**

$Q_{nt}$ regular production volume for $n$th product in period $t$

$O_{nt}$ overtime production volume for $n$th product in period $t$

$S_{nt}$ subcontracting volume for $n$th product in period $t$

$I_{nt}$ inventory level for $n$th product in period $t$

$B_{nt}$ backordering volume for $n$th product in period $t$

**Parameters**

$D_{nt}$ market demand for $n$th product in period $t$

$a_{n1}$ regular production cost per unit for $n$th product in period 1

$e_a$ escalating factor for regular production cost

$b_{n1}$ overtime production cost per unit for $n$th product in period 1

$e_b$ escalating factor for overtime production cost

$c_{n1}$ subcontracting cost per unit for $n$th product in period 1

$e_c$ escalating factor for subcontracting cost

$d_{n1}$ inventory carrying cost per unit for $n$th product in period 1
3.2. Fuzzy multi-objective linear programming model.

3.2.1. Objective functions. The proposed model selected multiple fuzzy objectives for solving the APP decision problems based on a literature review and by considering industrial situations. In real-life situations, most practical APP decisions minimized total costs, total carrying and backordering volume and total rate of change in labor levels. Particularly, these goals are normally fuzzy owing to incomplete and unavailable information over the intermediate planning horizon [21, 23, 35, 41, 43]. Additionally, most industrial APP decisions must consider related operating costs, inventory level, available capacity and resources, market demand, product life cycle, employment law, and other factors, to minimize total costs. Accordingly, the formulation of the practical APP problem generally considered three goals simultaneously, as follows.

Minimize total costs

$$\text{Min } Z_1 \cong \sum_{n=1}^{N} \sum_{t=1}^{T} \left[ a_{n1}Q_{nt}(1 + e_d)^t + b_{n1}O_{nt}(1 + e_b)^t + c_{n1}S_{nt}(1 + e_c)^t + d_{n1}I_{nt}(1 + e_d)^t + e_{n1}B_{nt}(1 + e_c)^t \right] + \sum_{t=1}^{T} (k_1H_t + m_1F_t)(1 + e_f)^t$$

Minimize total carrying and backordering volume

$$\text{Min } Z_2 \cong \sum_{n=1}^{N} \sum_{t=1}^{T} (I_{nt} + B_{nt})$$

Minimize total rate of change in labor levels

$$\text{Min } Z_3 \cong \sum_{t=1}^{T} (H_t - F_t)$$

The symbol $\cong$ is the fuzzified version of $=$ and refers to the fuzzification of the aspiration levels. In real-world APP decision problems, equations (1) to (3) are normally fuzzy and incorporate the variations in the DM’s judgments relating to the solutions of the fuzzy optimization problem, and these conflicting goals are
required to be optimized simultaneously by the DM in the framework of imprecise aspiration levels. Escalating factors in equation (1) are introduced to represent the time value of money in relation to relevant operating cost categories. The adjustment of cash flows to a common time basis is necessary when determining the time value of money. Accordingly, the index $t$ should be replaced by 1 in the subscripts of the related cost categories.

3.2.2. Constrains.

Constraints on carrying inventory

$$I_{nt-1} - B_{nt-1} + Q_{nt} + O_{nt} + S_{nt} - I_{nt} + B_{nt} = \hat{D}_{nt} \quad \forall n, \forall t$$  \hspace{1cm} (4)

Constraints on labor levels

$$\sum_{n=1}^{N} l_{nt-1}(Q_{nt-1} + O_{nt-1}) + H_t - F_t = \sum_{n=1}^{N} l_{nt}(Q_{nt} + O_{nt}) \quad \forall t$$  \hspace{1cm} (5)

$$\sum_{n=1}^{N} l_{nt}(Q_{nt} + O_{nt}) \leq \hat{W}_{t \text{max}} \quad \forall t$$  \hspace{1cm} (6)

Constraints on machine capacity

$$\sum_{n=1}^{N} r_{nt}(Q_{nt} + O_{nt}) \leq \hat{M}_{t \text{max}} \quad \forall t$$  \hspace{1cm} (7)

Constraints on warehouse space

$$\sum_{n=1}^{N} v_{nt}I_{nt} \leq V_{t \text{max}} \quad \forall t$$  \hspace{1cm} (8)

Constraints on total budget

$$z_1 \leq Z$$  \hspace{1cm} (9)

Non-negativity constraints on decision variables

$$Q_{nt}, O_{nt}, S_{nt}, I_{nt}, B_{nt}, H_t, F_t \geq 0 \quad \forall n, \forall t$$  \hspace{1cm} (10)

In real-world situations, the market demand can never be forecasted precisely owing to the dynamic nature of demand and supply, and the sum of regular production, overtime, inventory carrying levels, and subcontracting and backordering levels come from various sources essentially should equal the market demand, as equation (4). equations (5) and (7) represent the limitations of actual labor levels and machine capacity for each source in each period. The available resources in right-hand sides of equations (6) and (7) are normally imprecise, owing to the uncertainty of the demand and supply of the labor forces, worker skills, public policy and other factors.
4. Solution methodology.

4.1. Treatment of the fuzzy constraints. This study assumes the DM has already adopted the triangular fuzzy number to represent the imprecise data in the fuzzy multi-objective APP decision model designed above. The primary advantages of the triangular fuzzy number are the simplicity and flexibility of the fuzzy arithmetic operations [4, 17, 29]. Practically, the DM can construct the distribution of triangular fuzzy number based on the following three prominent data: (1) the most pessimistic value that has a very low likelihood of belonging to the set of available values (membership degree 0 if normalized); (2) the most possible value that definitely belongs to the set of available values (membership degree 1 if normalized); and (3) the most optimistic value that has a very low likelihood of belonging to the set of available values (membership degree 0 if normalized). Figure 1 shows the triangular fuzzy number $\tilde{D}_{nhj} = (D_{mnhj}^m, D_{mnhj}^p, D_{mnhj}^o)$ in equation (4).

![Figure 1. The Distribution of Triangular Fuzzy Number $\tilde{D}_{nt}$](image)

Recalling equation (4) from the original fuzzy MOLP model, consider the situations in which the market demand, $\tilde{D}_{nt}$, is a triangular fuzzy number with the most and least possible values. In the process of defuzzification, this work applies the weighted average method to convert $\tilde{D}_{nt}$ into a crisp number [17, 28, 34]. Moreover, if the minimum acceptable membership level, $\alpha$, is given, the corresponding auxiliary crisp expression of equation (4) can be presented as follows.

$$I_{nt-1} - B_{nt-1} + Q_{nt} + O_{nt} + S_{nt} - I_{nt} + B_{nt} = w_1 D_{nt,\alpha}^p + w_2 D_{nt,\alpha}^m + w_3 D_{nt,\alpha}^o \quad \forall n, \forall t$$

(11)

where, $w_1 + w_2 + w_3 = 1$, $w_1$, $w_2$ and $w_3$ represent the corresponding weights of the most pessimistic, most likely and most optimistic values of the fuzzy demand, respectively. Similarly, the corresponding auxiliary crisp expression of equations (6) and (7) can be presented using the weighted average method, as follows.

$$\sum_{n=1}^{N} l_{nt}(Q_{nt} + O_{nt}) = w_1 W_{t,\text{max,}n}^p + w_2 W_{t,\text{max,}n}^m + w_3 W_{t,\text{max,}n}^o \quad \forall t$$

(12)
\[
\sum_{n=1}^{N} r_{nt}(Q_{nt} + O_{nt}) = w_1 M_{t_{\text{max},a}}^p + w_2 M_{t_{\text{max},a}}^o + w_3 M_{t_{\text{max},a}}^o \quad \forall t \quad (13)
\]

4.2. Solving the fuzzy multi-objective APP problems.

4.2.1. Phase I: the minimum operator method. In phase I, the original fuzzy multi-objective APP problems can be solved using the fuzzy decision-making concept of Bellman and Zadeh [2], together with the fuzzy MOLP technique of Zimmermann [48]. First, the positive ideal solution (PIS) and negative ideal solution (NIS) for each of the fuzzy objective functions can be specified as follows.

\[
z_{g}^{PIS} = \text{Min} z_{g}, \quad z_{g}^{NIS} = \text{Max} z_{g} \quad g = 1, 2, ..., K
\]

Furthermore, the non-increasing continuous linear membership functions are defined for representing fuzzy objective functions involved. The linear membership functions can be specified by requiring the DM to select the goal value interval \([z_{g}^{PIS}, z_{g}^{NIS}]\). Accordingly, the non-increasing continuous linear membership functions for each of the fuzzy objective functions can be expressed as follows. Figure 2 shows the graph of the non-increasing continuous linear membership function.

\[
f_g(z_g) = \begin{cases} 
1 & z_g \leq z_g^{PIS} \\
\frac{z_g^{NIS} - z_g}{z_g^{NIS} - z_g^{PIS}} & z_g^{PIS} < z_g < z_g^{NIS} \\
0 & z_g \geq z_g^{NIS}
\end{cases} 
\]

\[
f_g(z_g)
\]

**Figure 2.** The Non-increasing Continuous Linear Membership Function

In practical applications, the ordinary crisp single-goal LP optimal solution is typically utilized as a starting point of the (PIS, NIS) for specifying the interval of membership degree for each of the fuzzy objective functions in equation (15). Additionally, the minimum operator is used to aggregate fuzzy sets. By introducing the auxiliary variable \(L^{(1)}\), the original fuzzy MOLP problem can be converted into an
equivalent ordinary single-goal LP form. The fuzzy decision-making concept regarding the use of the minimum operator is presented in Appendix A. Consequently, the completed equivalent ordinary LP model is as follows.

\[ \text{Max } L^{(1)} \]
\[ \text{s.t. } 0 \leq L^{(1)} \leq f_g(z_g) \quad \forall g \quad \text{Equations (4) to (13)} \]

where the auxiliary variable \( L^{(1)} \) represents overall DM satisfaction with the obtained goal values.

4.2.2. Phase II: the weighted average operator method. In phase II, the initial solution obtained via the minimum operator method in model (16) is improved by adding the lower bound of satisfaction degrees for each fuzzy objective function, \( L^i_g (g = 1, 2, ..., k) \) as a constraint; and the weighted average operator is then used to aggregate fuzzy sets. By introducing the auxiliary variable \( L^{(2)} \), the model (16) can be converted into an equivalent ordinary LP model, as follows.

\[ \text{Max } L^{(2)} = \sum_{g=1}^{k} w_g L_g \]
\[ \text{s.t. } L^i_g \leq L_g \leq f_g(z_g) \quad \forall g \quad \text{Equations (4) to (13)} \]

where, \( 0 \leq w_g \leq 1 \), \( w_g (g = 1, 2, ..., k) \) is the weight of the \( g \)th objective function chosen by DM.

4.3. Solution procedure.

Step 1: Formulate the fuzzy MOLP model for the multi-objective APP problems using equations (1) to (10).

Step 2: Provide the minimum acceptable membership level, \( \alpha \), and then convert the fuzzy constraints into crisp ones according to equations (11) to (13).

Step 3: Specify the (PIS, NIS) for each of the fuzzy objective function \( z_g (g = 1, 2, ..., k) \), and then define the corresponding linear membership functions, as equations (14) and (15).

Step 4: Introduce the auxiliary variable \( L^{(1)} \), thus enabling aggregation of the original fuzzy MOLP problem into an equivalent ordinary single-goal LP form using the minimum operator method, as model (16).

Step 5: Solve the model (16) to obtain an initial solution.

Step 6: Specify the lower bound of satisfaction degree \( L^i_g \) and the corresponding weight
Table 1. Fuzzy Market Demand Data

| Item     | Period          |
|----------|-----------------|
|          | 1               | 2               | 3               | 4               |
| $\hat{D}_{1t}$ | (1000, 900, 1080) | (3000, 2750, 3200) | (5000, 4600, 5300) | (2000, 1850, 2100) |
| $\hat{D}_{2t}$ | (1000, 900, 1080) | (500, 450, 540) | (3000, 2750, 3200) | (2500, 2300, 2650) |

Table 2. Related Unit Cost/Time Coefficients Data (in Units of US Dollars)

| Product | $a_{n1}$ ($/\text{unit}$) | $b_{n1}$ ($/\text{unit}$) | $c_{n1}$ ($/\text{unit}$) | $d_{n1}$ ($/\text{unit}$) | $e_{n1}$ ($/\text{unit}$) | $l_{nt}$ (hour/\text{unit}) | $r_{nt}$ (hour/\text{unit}) | $v_{nt}$ (ft$^2$/\text{unit}) |
|---------|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|-----------------------------|-----------------------------|-----------------------------|
| 1       | 20                        | 30                        | 25                        | 0.30                      | 40                        | 0.05                        | 0.10                        | 2                           |
| 2       | 10                        | 15                        | 12                        | 0.15                      | 20                        | 0.07                        | 0.08                        | 3                           |

$w_d$ for each fuzzy objective function based on the initial solution in model (16).

Step 7:
Reformulate model (16) into an equivalent ordinary single-goal LP model using the weighted average operator method, as model (17).

Step 8:
Solve the model (17) to generate an improved solution.

5. Implementation.

5.1. Case description. Daya Technologies Corporation was used as a case study to demonstrate the practicality of the developed methodology [21, 41]. Daya is the leading producer of precision machinery and transmission components in Taiwan. Its products are mainly distributed throughout Asia, North America and Europe. The conventional APP strategy used by Daya is to maintain a constant work force level, and fluctuated demands can then be met by using some combination of regular and overtime production, inventory carrying, subcontracting and backordering levels. However, the expected performance was unable to achieve owing to main drawbacks on the traditional experiential method that evaluation comparisons can only be done for specific plans under specific conditions and indication is vague for the resulted APP plan. Alternatively, the DM would apply a fuzzy mathematical programming method to develop a suitable APP plan for the Daya’s ballscrew plant. The model produced by Daya includes two types of standard ballscrew, namely the external recirculation type (product 1) and the internal recirculation type (product 2). The time horizon of APP decision comprises four months, including May, June, July, and August. According to the preliminary production-marketing information, Tables 1 to 3 summarize the relevant market demand, unit operating cost/time coefficients, capacities and warehouse space data used in the Daya case. Notably, the market demand, available labor levels and machine capacity are imprecise numbers with triangular distributions.

Other relevant data are as follows.
(1) Initial carrying inventory in period 1 is 400 units of product 1 and 200 units of product 2, and the end inventory in period 4 is 300 units of product 1 and 200
Table 3. Maximum labor level, machine capacity and warehouse space data

| Period | $W_{t \max}$ | $M_{t \max}$ | $V_{t \max}$ |
|--------|--------------|--------------|--------------|
| 1      | (300, 275, 320) | (400, 360, 430) | 10000        |
| 2      | (300, 275, 320) | (500, 450, 540) | 10000        |
| 3      | (300, 275, 320) | (600, 540, 650) | 10000        |
| 4      | (300, 275, 320) | (600, 540, 650) | 10000        |

Table 4. The (PIS, NIS) for the Fuzzy Objective Functions

| Item                      | LP-1        | LP-2        | LP-3        | (PIS, NIS)          |
|---------------------------|-------------|-------------|-------------|---------------------|
| Objective function        | $\text{Min } Z_1$ | $\text{Min } Z_2$ | $\text{Min } Z_3$ | —                   |
| $Z_1(\text{dollars})$     | 327, 859*   | 331, 496    | 361, 329    | (327, 859, 361, 329) |
| $Z_2(\text{units})$       | 9397        | 8671        | 500*        | (500, 9397)         |
| $Z_3(\text{man-hours})$   | 176         | 42*         | 432         | (42, 432)           |

Note: "*" denotes the optimal value by ordinary single-goal LP model.

units of product 2. The initial backordering volume in period 1 and the end backordering volume in period 4 for two products are zero.

(2) Initial labor level is 300 man-hours. The costs associated with hiring and layoffs are $10 and $2.5 per worker per hour, respectively.

(3) The expected escalating factor for each of the operating cost categories is fixed to 5% in each period.

(4) The minimum acceptable membership level, $\alpha$, is specified to 0.5 for each fuzzy parameters.

(5) The available budget is $400,000 over the planning horizon.

5.2. Solution procedure for the Daya case. The solution procedure with the proposed method to solve fuzzy multi-objective APP problem for the Daya case is demonstrated as follows. The fuzzy MOLP model is first formulated according to equations (1) to (10). Then, convert the fuzzy constraints into crisp ones according to equations (11) to (13) at $\alpha = 0.5$. This study applies the concept of the most possible values, specifying $w_2 = 4/6$, $w_1 = w_3 = 1/6$ for all fuzzy constraints [17, 34]. Moreover, the original MOLP problem is respectively solved by the ordinary single-goal LP model, and the corresponding (PIS, NIS) for each of the fuzzy objective function can be specified with equation (14). The results are $(z_1^{PIS}, z_1^{NIS}) = (327, 859, 331, 496)$, $(z_2^{PIS}, z_2^{NIS}) = (361, 329)$, $(z_3^{PIS}, z_3^{NIS}) = (500, 9397)$ man-hours. Table 4 lists the optimal solutions obtained by the ordinary LP model and the specifying interval values of (PIS, NIS) for each of the fuzzy objective functions.

Accordingly, the non-increasing continuous linear membership functions for each fuzzy objective function can be defined via equation (15). Moreover, the fuzzy multi-objective APP problem for the Daya case can be transformed into an equivalent ordinary LP form using model (16). LINDO computer software is used to run this ordinary LP model. The resulting initial solutions are $z_1 = 338,955$, $z_2 = 3450$ units, $z_3 = 171$ man-hours, and the overall DM satisfaction is 0.6684. Additionally, the DM specifies $(L_1^L, L_2^L, L_3^L) = (0.75, 0.80, 0.72)$ and $(w_1, w_2, w_3) = (1/2, 1/4, 1/4)$ for the three fuzzy objective functions, and the equivalent ordinary LP form can be
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Table 5. Initial and Improved APP Plan for the Daya Case

| Item          | Initial solutions (phase I) | Improved solutions (phase II) |
|---------------|-----------------------------|-------------------------------|
|               | Period | 1 | 2 | 3 | 4 | Period | 1 | 2 | 3 | 4 |
| Product 1     |        |   |   |   |   |        |   |   |   |   |
| $Q_1t$ (units) | 598    | 3521 | 4467 | 2296 | 598 | 2996 | 4992 | 2296 |        |   |   |   |   |
| $O_1t$ (units) | 0      | 0   | 0   | 0   | 0   | 0     | 0   | 0   | 0   | 0   |
| $S_1t$ (units) | 0      | 0   | 0   | 0   | 0   | 0     | 0   | 0   | 0   | 0   |
| $I_1t$ (units) | 0      | 525 | 0   | 300 | 0   | 0     | 0   | 0   | 0   | 0   |
| $B_1t$ (units) | 0      | 0   | 0   | 0   | 0   | 0     | 0   | 0   | 0   | 0   |
| Product 2     |        |   |   |   |   |        |   |   |   |   |
| $Q_2t$ (units) | 2203   | 115 | —   | 1320 | 1863 | 150  | 1093 | 1320 |        |   |   |   |   |
| $O_2t$ (units) | —      | —   | 1975 | 1376 | 0   | 0     | 0   | 0   | 0   | 0   |
| $S_2t$ (units) | —      | —   | 1975 | 1376 | 0   | 0     | 0   | 0   | 0   | 0   |
| $I_2t$ (units) | 1404   | 1020 | —   | 200 | 1064 | 715  | 0   | 200 | 4       |   |   |   |   |
| $B_2t$ (units) | 0      | 0   | 0   | 0   | 0   | 0     | 0   | 0   | 0   | 0   |
| $H_t$ (man-hours) | 0     | 0   | 39  | 0   | 0   | 0     | 0   | 25  | 0   |
| $F_t$ (man-hours) | 116   | 0   | 0   | 16  | 89   | 0     | 0   | 29  | 0   |
| $L$ and goal values | $L^{(1)}=0.6684$ | $L^{(2)}=0.7602$ |
|                 | $Z_1=338,955$, $Z_2=3450$ (units) | $Z_1=336,226$, $Z_2=2279$ (units) |
|                 | $Z_3=171$ (man-hours) | $Z_3=143$ (man-hours) |

formulated via the model (17). The improved efficient solutions are $z_1 = 336,226$, $z_2 = 143$ man-hours, $z_3 = 2279$ units, and overall degree of DM satisfaction is up to 0.7602. Table 5 lists initial and improved APP plans for the Daya case.

5.3. **Computational analysis.** Several significant implications when applying the fuzzy programming methodology developed in this study to practical APP decisions are as follows. First, the proposed method yields an efficient solution and presents the overall DM satisfaction with the given goal values. From Table 5, it indicates the optimal results by the proposed method are an efficient solution, because of the solutions obtained using the proposed two-phase fuzzy goal programming method are obviously better than that of one-stage minimum operator method. Relevant works verified why the output results obtained using the two-phase fuzzy programming method are always efficient solution for using the minimum operator and the weighted average operator sequentially to aggregate fuzzy sets in the decision-making process. As a result, an improved APP plan can be obtained under an acceptable degree of DM satisfaction [11, 20, 26].

Second, the DM generally must solve APP problems with multiple fuzzy goals owing to some information being incomplete and unobtainable, and these conflicting goals must be optimized by the DM in the framework of imprecise aspiration levels. The comparisons listed in Tables 4 and 5 shows that the interaction of trade-offs and conflicts among dependent multiple objective functions. Analytical results obtained by implementing Daya case indicate that the proposed method satisfies the requirement for the practical application since it attempts to simultaneously minimize the total costs, total carrying and backordering volume and total rates of changes in work-force levels in a fuzzy environment. Generally, applying fuzzy sets and fuzzy goal programming method to multi-objective decisions may provide more efficient and flexible model formulation and arithmetic operations [4, 10, 35-37, 40, 45].
Moreover, the comparison of initial and improved compromise solutions in Table 5 reveals that the changes in the weight and the minimum satisfaction degree for each of the fuzzy objective function in model (17) influence both goal and L values. If the minimum satisfaction degree for each of the objective functions is improperly given, it will make the solution procedure more complicated. Practically, the derived degree of membership of each fuzzy objective function in model (16) can be taken as its initial minimum satisfaction degree. As a result, the computing amounts of model (17) will be decreased. If the DM increases the minimum satisfaction degree of one fuzzy objective function, it implies that the value of this fuzzy objective function is closer to the optimal value, but it may make other fuzzy objective values far from their optimal values [20]. In real-world situations, the values of the relative weights among multiple objective functions can be adjusted subjectively based on the experience and knowledge of DM.

Additionally, the proposed method exhibits greater computational efficiency and flexibility by respectively adopting the linear membership functions and triangular fuzzy numbers to represent both fuzzy goals and imprecise data for the multi-objective APP decisions [4, 17, 29]. The development of the proposed method is based on the fuzzy decision-making concept of Bellman and Zadeh [2] and fuzzy programming technique of Zimmermann [47-48], in which the fuzzy MOLP problem can be converted into an equivalent ordinary single-goal LP form by the minimum operator the weighted average operator sequentially to aggregate fuzzy goal sets, and is easily solved by the simplex method.

Finally, the optimal solution yielded by using the minimum operator method may not be an efficient solution, and the computing efficiency of the solution is not been assured. Aggregate operators can be roughly classified into three categories - intersection, union and averaging operators. Table 6 lists the comparisons of major types of aggregation operators in the existing literature [16, 22, 41, 44, 49]. Among the various types of aggregate operators, the minimum operator is used most generally for solving the fuzzy mathematical programming problems. However, the main drawback of the minimum operator is its lack of discriminatory power between solutions that strongly differ with respect to the fulfillment of membership to the various constraints [7]. Alternatively, compensatory average operator considers the relative importance of each fuzzy set and has the compensative property so that the result of combination will be medium.

5.4. Comparisons. As indicated in Table 7, the proposed method can yield an efficient solution, compared to the two existing fuzzy programming techniques based on current information for the Daya case. Moreover, Table 8 presents the qualitative comparisons among the proposed methods with those of the representative APP techniques. Several advantages of the proposed method are summarized as follows. First, the APP model designed here considers the time value of money of related operating cost categories. The value of total costs is impacted significantly by the monetary interest and, thus, a DM must consider the time value of money for each cost category when solving the real-life APP problems. Practically, the DM must increase the efficiency of internal management and seek to reduce the cost of capital to reduce the escalating factor. Moreover, the proposed method exhibits greater computational flexibility of the fuzzy arithmetic operations by employing the linear membership functions and triangular fuzzy numbers to represent fuzzy goals and imprecise data, and the fuzzy MOLP model can be converted into an
Table 6. Comparisons of Common Aggregation Operators

| Operator          | Example                                      | Brief description                                                                 |
|-------------------|----------------------------------------------|----------------------------------------------------------------------------------|
| Intersection      | • Minimum                                     | • An aggregation scheme is implemented where fuzzy sets are connected by a logical 'and'. |
| $t$-norms         | • Algebraic product                          | • The result of combination is high if and only if all values are high.          |
|                   | • Bounded sum                                 | • The minimum operator is a greatest $t$-norm.                                   |
|                   | • Drastic intersection                       |                                                                                  |
| Union             | • Maximum                                     | • An aggregation scheme is implemented where fuzzy sets are connected by a logical 'or'. |
| $t$-conorms       | • Algebraic sum                               | • The result of combination is high if some values are high.                     |
|                   | • Bounded difference                         | • The minimum operator is a smallest $t$-conorm.                                  |
|                   | • Drastic union                               |                                                                                  |
| Averaging         | • Mean                                        | • Have the compensative property so that the result of combination will be medium. |
| Compensative      | • Weighted averaging                          | • Consider the relative importance of the fuzzy sets.                            |
|                   | • OWA (Ordered Weighted averaging)            | • The $\gamma$-operator is the convex combination of the min-operator and the max-operator. |
|                   | • DIFWA (Dynamic intuitionistic fuzzy        | • OWA enables a DM to specify linguistically his agenda for aggregating a collection of fuzzy sets. |
|                   | Weighted averaging)                          | • DIFWA is used to solve the dynamic intuitionistic fuzzy multi-attribute decision making problems (DIF-MADM). |
|                   | • UDIFWA (Uncertain dynamic intuitionistic   | • UDIFWA is used to solve the DIF-MADM problems under interval uncertainty at different periods. |
|                   | fuzzy Weighted averaging)                    |                                                                                  |

Table 7. Solution Comparisons

| Item               | Zimmermann [47-48], Wang and Liang [43] | Hannan [12], Wang and Liang [41] | The proposed method |
|--------------------|-----------------------------------------|----------------------------------|---------------------|
| Objective function | Max $L$                                 | Max $L$                          | Max $L$             |
| $L$                | 66.84 %                                 | 66.68 %                          | 76.02 %             |
| $Z_1($$\$$)        | 339,955                                 | 399,011                          | 339,226             |
| $Z_2$(units)        | 3450                                    | 2398                             | 2279                |
| $Z_3$(man − hours)  | 171                                     | 137                              | 142                 |

equivalent ordinary LP form that is easily solved by the simplex method. Additionally, the proposed method provides a systematic fuzzy decision-making framework that the DM adjusts the search direction interactively, until the efficient solution satisfies the DM’s preferences and is considered to be the preferred satisfactory solution. Particularly, the proposed method essentially provides a more efficient way of solving the fuzzy constraints in the fuzzy multi-objective APP decision problems with the simplified weighted average method given by Eqs.(11)-(13). Finally, the computational methodology can easily be extended to any other situations and can handle the realistic APP decisions. The industrial case illustrated here is sufficient to lay a strong foundation on which a DM can formulate additional applications to large scale APP decisions in a fuzzy environment.

6. Conclusion. This study aims to develop a two-phase fuzzy goal programming method for solving the multi-objective APP problems with multiple product and
Table 8. Solution Comparisons

| Factor                  | Masud and Hwang [23] | Bitran and Hwang [3] | Tang et al. [36] | Wang and Fang [40] | The proposed method |
|-------------------------|----------------------|----------------------|------------------|---------------------|---------------------|
| Objective function      | Multiple, linear     | Single, nonlinear    | Single, nonlinear| Multiple, linear    | Multiple, linear    |
| Objective property      | Crisp                | Stochastic           | Fuzzy            | Fuzzy               | Fuzzy               |
| Operator                | Minimum              | Minimum              | Minimum and      | Minimum             | Minimum and average |
| Output solution         | Efficient            | Not guaranteed       | Not guaranteed   | Not guaranteed      | Efficient           |
| Degree of satisfaction  | —                    | —                    | —                | —                   | —                   |
| Market demand           | Crisp                | Stochastic           | Fuzzy            | Fuzzy               | Fuzzy               |
| Product item            | Single item          | Single item          | Product family   | Product family      | Product family      |
| Time value of money     | Not included         | Not included         | Not included     | Not included        | Included            |
| Subcontracting          | Not included         | Not included         | Not included     | Included            | Included            |
| Backordering            | Not included         | Not included         | Not included     | Included            | Included            |
| Labor level             | Not included         | Not included         | Included/fuzzy   | Included/fuzzy      | Included/fuzzy      |
| Machine capacity        | Infinite             | Infinite             | Infinite         | Limited/fuzzy       | Limited/fuzzy       |
| Warehouse space         | Infinite             | Infinite             | Limited          | Limited             | Limited             |
| Revised flexibility     | —                    | Low                  | Medium           | Medium              | High                |

The designed fuzzy multi-objective APP decision model attempts to simultaneously minimize total costs, total carrying and backordering volume, and total rates of changes in labor levels in relation to inventory levels, machine capacity and labor levels, warehouse space and the constraint on available budget. An industrial case is used to demonstrate the feasibility of applying the proposed method to practical multi-objective APP decisions. On the whole, the major contribution of this study lies in presenting a fuzzy goal programming methodology to fuzzy multi-objective APP decisions with multi-products and multi-time periods and provides a systematic decision-making framework that facilitates a DM to interactively adjust the search direction until the preferred efficient solution is obtained. The primary limitations of the proposed method concern the certain assumptions made for related unit cost/time coefficients. Therefore, future researchers may explore the fuzzy properties of related unit cost/time coefficients and model parameters in real-world APP decision problems. Additionally, future studies can adopt the dynamic fuzzy programming models, such as dynamic intuitionistic fuzzy weighted averaging (DIFWA) and uncertain dynamic intuitionistic fuzzy weighted averaging (UDIFWA) operators, or relevant sequential decision making techniques into the multi-objective APP decisions by considering the important weights of time series to make it better suited to practical applications.

Appendix A. Let $X$ be a given set of all possible solutions to a decision problem. A fuzzy goal $G$ is a fuzzy set on $X$ characterized by its membership function

$$
\mu_G : X \rightarrow [0, 1]
$$

(A.1)

A fuzzy constraint $C$ is a fuzzy set on $X$ characterized by its membership function

$$
\mu_C : X \rightarrow [0, 1]
$$

(A.2)

Then, $G$ and $C$ combine to generate a fuzzy decision $D$ on $X$, which is a fuzzy set resulting from intersection of $G$ and $C$, and is characterized by its membership function

$$
L = \mu_D(x) = \mu_G(x) \wedge \mu_C(x) = \text{Min}(\mu_G(x), \mu_C(x))
$$

(A.3)
and the corresponding maximizing decision is defined by

\[ \max L = \max \mu_D(x) = \max \min(\mu_G(x), \mu_C(x)) \quad (A.4) \]

More generally, suppose the fuzzy decision \( D \) results from \( k \) fuzzy goals \( G_1, \ldots, G_k \) and \( m \) constraints \( C_1, \ldots, C_m \). Then the fuzzy decision \( D \) is the intersection of \( G_1, \ldots, G_k \) and \( C_1, \ldots, C_m \), and is characterized by its membership function

\[
L = \mu_D(x) = \mu_{G_1}(x) \land \mu_{G_2}(x) \land \ldots \land \mu_{G_k}(x) \land \mu_{C_1}(x) \land \mu_{C_2}(x) \land \ldots \land \mu_{C_m}(x) \quad (A.5)
\]

and the corresponding maximizing decision is defined by

\[
\max L = \max \mu_D(x) = \max \min(\mu_{G_1}(x), \mu_{G_2}(x), \ldots, \mu_{G_k}(x), \mu_{C_1}(x), \mu_{C_2}(x), \ldots, \mu_{C_m}(x))
\quad (A.6)
\]

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