Sum rate analysis of two-unicast wireless energy harvesting system

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Abstract: The authors study the sum rate of a two-unicast wireless energy harvesting system. The system consists of two source nodes, two destination nodes and one energy constrained relay node, which only harvests energy from the signals transmitted by the source nodes for its operation. Each source node transmits the signal to its corresponding destination node via the help of the relay node. The transmission process is divided into two time slots each with the same time duration. In the first time slot, two source nodes transmit the signals to the relay node and the destination nodes, and the relay node uses the power splitting (PS) protocol for harvesting energy. In the second time slot, adopting analogue network coding, the relay node retransmits the mixed signal to the destination nodes using the harvested energy. Considering network coding noise, the analytical expressions of a near optimal adaptive PS factor that leads to a maximal sum rate is derived. Numerical results corroborate the analytical results and show the superior performance of the proposed scheme in comparison with the conventional one.

1 Introduction

Wireless energy harvesting (WEH) technology can make wireless devices collect energy from the radio frequency (RF) to meet their energy demands, so the system based on the WEH has drawn more and more attention in the literature. In a cooperative communication, a WEH relay can help source nodes transmit information to destination nodes and then improve the performance of the system. Nasir et al. [1] studied the performance of the classical three nodes model consisting of one source node, one destination node, and one relay node. Nasir et al. [1] only considered the one-way transmission from the source node to the destination node. The relay node can use power splitting (PS) protocol, time switching protocol or ideal receiver architecture to harvest energy and receive information, and amplify-and-forward (AF) and decode-and-forward (DF) schemes to retransmit the signal, respectively. The performances are evaluated by the outage probability and ergodic capacity. Considering the complexity of deriving the closed-form results, the analytical integral form results are obtained.

Based on [1], Liu et al. [2] analysed the outage probability, throughput, and energy efficiency of a two-way AF WEH system, which adopts multiple access broadcast or time division broadcast protocols. In [1, 2], limited to the integral form of analytical results, the authors observed the impact of the key parameters, such as PS factor, on the performance of the system through the simulations. In [3], the authors assumed the system adopting an adaptive power splitter, which adjusts the PS factor according to the instantaneous channel state information (CSI) and derived optimal PS factor that leads to a maximal end-to-end data rate. Adopting the optimal PS factor, the authors derived the closed-form outage probability and average capacity without considering the direct link. Different from [3] without the direct link, Lee et al. [4] derived the closed-form optimal outage probability based on the statistical CSI when the direct link is available. For achieving the information transmissions between multiple source nodes and multiple destination nodes, the power allocation strategies were proposed for a DF relaying system in [5, 6]. In particular, to avoid the co-channel interference among different pairs, orthogonal channel access schemes were adopted [5, 6], thus leading to an inefficient use of spectrum resources.

In [7–9], an analogue network coding (ANC) protocol, in other words, AF protocol, was used to accomplish information transmissions of a two-unicast system with one shared relay. Each unicast system consists of one source node, one destination node, and one shared relay node. The impacts of overhearing link quality and network coding (NC) noise on the sum rate of the system were studied in [7–9], respectively. In the above works [7–9], the authors do not consider WEH at the relay node.

In this study, the sum rate of a two-unicast system with a WEH relay is studied. The relay node uses an adaptive PS factor to maximise the sum rate of the system. Numerical results investigate the superior performance of the proposed scheme compared to the conventional one and the NC noise impact on the sum rate.

2 System model

As shown in Fig. 1 [7], the two-unicast WEH system consists of two source nodes, denoted by $S_1$ and $S_2$, one shared WEH relay node, denoted by $R$, and two destination nodes, denoted by $D_1$ and $D_2$. Since the direct links from $S_1$ to $D_1$ and from $S_2$ to $D_2$ do not exist [3, 5–7], the information transmission from $S_1$ and $S_2$ to their respective destination nodes $D_1$ and $D_2$ relies on the help of $R$. Note that in Fig. 1, $D_1$ and $D_2$ are in the coverage area of $S_2$ and $S_1$ [5, 6], respectively. All channels are subject to independently distributed Rayleigh fading and $E[|h|^2] = d_r^{-m}$, $E[|f|^2] = d_i^{-m}$, $E[|k|^2] = d_k^{-m}$, $E[|p|^2] = d_p^{-m}$ and $E[|q|^2] = d_q^{-m}$, where $h$, $g$, $i$, $k$, $l$, $p$, and $q$ denote the channel coefficients of the corresponding links $S_1 \to R$, $S_2 \to R$, $R \to D_1$, $R \to D_2$, $S_1 \to D_2$ and $S_2 \to D_1$, $d_r$, $d_i$, $d_k$, $d_l$, $d_p$, and $d_q$ denote the distances of the corresponding links and $m$ denotes the path loss exponent. $d_r$ denotes the distance between two sources or destinations. The channels are assumed to be quasi-static, which means that the channel gains remain constant during each transmission and may vary in different transmissions [3]. If adopting the conventional scheme, the two-unicast system can be regarded as two independent classical models, denoted by $(S_1 \to R \to D_1)$ and $(S_2 \to R \to D_2)$, and each accomplishes the information.
transmission using AF mode and PS protocol within two time slots [1]. So, every whole transmission frame needs four time slots [5, 6]. If the relay node adopts the ANC protocol, every whole transmission frame only needs two time slots [7–9]. The details of the ANC protocol will be described in the next section. It is assumed that the perfect CSI can be obtained at R, D1 and D2 through the channel estimation before the transmission [1–6]. All nodes are equipped with a single antenna and are operating in the half-duplex mode. R is equipped with only one power splitter. To simplify the mathematical analysis, it is assumed that $d_{1} = d_{o}$, $d_{2} = d_{o}$ and $d_{s} = d_{o}$ [7].

### 3 Sum rate analysis

For the symmetry of the proposed system, D2 is used as an example to analyse the signal transmission process. In the first time slot $T/2$, using the same power $P_{s}$, S1 and S2 simultaneously transmit their respective signals. The received signals for information processing at D2 and R can be expressed as

$$y_{2s}(t) = \sqrt{P_{s}}p_{s}(t) + n_{t}(t), \quad (1)$$

$$y_{t}(t) = \sqrt{1 - \rho}(\sqrt{P_{s}}h_{s}(t) + \sqrt{P_{r}}g_{s}(t)) + n_{t}(t). \quad (2)$$

where $s(t)$ and $s(t)$ are both the unit power signals sent by S1 and S2, respectively. $n_{t}(t)$ and $n_{t}(t)$ are the baseband additive white Gaussian noises (AWGNs) with the same variances $\sigma^{2} = \sigma_{s}^{2} = \sigma_{t}^{2}$ at D1 and R [3–5]. $n_{t}(t)$ consists of the received baseband AWGN introduced by the antenna and the AWGN due to the RF band to baseband conversion [3–5]. The received power at R is proportionally split into two parts, $\rho$ and $1 - \rho$, for harvesting energy and processing information, respectively. $0 < \rho < 1$ denotes the adaptive PS factor. So, the harvested energy at R can be written as $E_{2} = \eta P_{r} (|r|^{2} + |t|^{2})$, and the transmitted power by R in the second time slot can be derived as $P_{t} = (E_{2}/T/2) = \eta P_{s} (|r|^{2} + |t|^{2})$, where $\eta \in (0, 1)$ denotes the energy conversion efficiency.

The received signal at D2 in the second time slot $T/2$ can be written as

$$y_{2s}(t) = \sqrt{P_{s}}G_{2}l(t) + n_{t}(t) = \sqrt{P_{s}}G_{2}l(t) + \sqrt{P_{s}}G_{2}l(t) + \sqrt{P_{s}}G_{2}l(t) + \sqrt{P_{s}}G_{2}l(t) + n_{t}(t). \quad (3)$$

where

$$G = \frac{1}{\sqrt{(1 - \rho)P_{s}(|r|^{2} + |t|^{2})}} \approx \frac{1}{\sqrt{(1 - \rho)P_{s}(|r|^{2} + |t|^{2})}}$$

is the power normalisation factor and step (a) is for the high signal-to-noise ratio (SNR) approximation [1, 2]. In (3), the part $\sqrt{P_{s}}G_{2}l(t)$ is an undesired signal for D2. Comparing (1) and (3), since D2 knows the CSI, it can use the signal $y_{2s}(t)$ to subtract the signal $\gamma_{s}(t) = \sqrt{P_{s}}G_{2}l(t)/\gamma_{s}(t)$ and obtain the expected signal $s(t)$. The final obtained signal can be expressed as

$$\gamma_{2s}(t) = \sqrt{P_{s}}G_{2}l(t) \approx \sqrt{P_{s}}G_{2}l(t) \approx \frac{1}{\sqrt{(1 - \rho)P_{s}(|r|^{2} + |t|^{2})}}n_{t}(t). \quad (4)$$

The second part at the right of the equal sign in (4) is named NC noise [8, 10]. Substituting the expressions of $G$ and $P_{s}$ and after some mathematical manipulations, the SNR at D2 is written as

$$\gamma_{2s}(t) = \frac{\gamma_{s}(t)P_{s}(1 - \rho)|r|^{2}|t|^{2}}{\eta P_{r}(|r|^{2} + (1 - \rho)|t|^{2}) + \eta P_{s}(1 - \rho)|t|^{2}}. \quad (5)$$

where $\gamma = P_{r}/\sigma^{2}$ is no faded SNR. In (5), the third part of the denominator is the power introduced by the NC noise and will degrade the received SNR.

The instantaneous received data rate at D2 is defined as [3, 5, 6]

$$R_{1} = \frac{1}{2} \log_{2}(1 + \gamma_{2}). \quad (6)$$

Using the same analytical process, the SNR and the received data rate at D1 can be derived as

$$R_{2} = \frac{1}{2} \log_{2}(1 + \gamma_{2}). \quad (7)$$

The sum rate of the system can be defined as [5, 6]

$$R_{sum} = R_{1} + R_{2}. \quad (8)$$

For maximising the sum rate, we first consider maximising $R_{1}$. By calculating the second-order derivative of $\gamma_{2}$ with respect to $\rho$, we obtain

$$\frac{\partial^{2} \gamma_{2}}{\partial \rho^{2}} = -2\eta^{2}||r||^{2}||t||^{2}||r||^{2}||t||^{2} \left(1 - 3\rho + 3\rho^{2} + (\eta(||r||^{2} - 1)||t||^{2})\right)$$

$$\frac{\partial^{2} \gamma_{2}}{\partial \rho^{2}} = \frac{1}{1 + \sqrt{\eta}||t||} \text{ or } \frac{\partial^{2} \gamma_{2}}{\partial \rho^{2}} = \frac{1}{1 - \sqrt{\eta}||t||}. \quad (9)$$

Since $0 < \rho < 1$, $\eta > 0$ and $|t| > 0$, we select $\rho_{opt} = (1/1 + \sqrt{\eta}||t||)$ as the final solution. Observing the expressions of $\rho_{opt}$, we find an interesting phenomenon that the expressions of $\rho_{opt}$ have almost the same form compared to [3, 10] except increasing a factor $\sqrt{\eta}$ in the denominator. In the same way, we can obtain optimal $\rho_{opt} = (1/1 + \sqrt{\eta}||t||)$ which leads to maximal $\gamma_{2}$. Since we have assumed $d_{i} = d_{o}$, $E(||r||^{2}) = d_{m}$ and $E(||t||^{2}) = d_{m}$, $E(\rho_{opt}) = E(\rho_{opt})$ can be obtained. In a statistical viewpoint, $\rho_{opt} = 1/(1 + \sqrt{\eta}||t||)$ can maximise $R_{1}$ and meanwhile maximise $R_{2}$, which finally lead to the maximal sum rate $R_{sum}$ of the system. So, $\rho_{opt} = 1/(1 + \sqrt{\eta}||t||)$ can be regarded as an approximately optimal solution to the system.

### 4 Numerical results and discussion

In the simulations, the system parameters are set as follows: $m = 2.7$, $\eta = 0.8$, $\sigma^{2} = -70$ dBm. The two-dimensional coordinates of $S_{1}, S_{2}, D_{1}$ and $D_{2}$ are set to $(0, m, 0)$, $(5, m, 5)$, $(5, 5, 5)$ and $(m, 5, m)$, respectively [7, 11].
Fig. 2 Sum rate comparison of the proposed scheme and fixed PS factor scheme with respect to \( P_s/\sigma^2 \) (\( d_7 = 10 \) m)

Fig. 3 Maximal sum rate comparison of the proposed scheme and conventional orthogonal channels scheme with respect to \( d_7 \) (\( P_s = 20 \) dBm)

Fig. 3.1 shows the sum rate comparison of the proposed scheme and the conventional scheme for different values of \( d_7 \). The peak sum rate is achieved when \( d_7 = 6 \) m. The analytical results match well with the simulation results. The gap between the schemes is larger than that of the fixed PS factor scheme over a wide SNR range. The sum rates of both schemes increase with the increasing SNR.

As shown in Fig. 3.1, the analytical and simulation results match very well, and the sum rate of the proposed scheme is nearly twice compared to that of the conventional four time slots scheme particularly when the distance between two source nodes is large.

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9 References

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