Temperature dependent anisotropy and elastic effects in ferromagnetic nanowire arrays

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Abstract

Temperature dependent Ferromagnetic Resonance measurements performed as a function of diameter on Nickel nanowire arrays reveal several interesting features in these systems. With diameter decrease from 100 nm to 15 nm, a transition induced by surface anisotropy increase is observed at 50 nm in easy axis orientation from parallel to perpendicular with respect to individual nanowire geometric axis. Analysis of resonance field \(H_{\text{res}}\) temperature variation (between liquid Helium and room temperature) reveals underlying strong magneto-elastic effects in small and large diameter nanowire arrays with potential applications in recording and spintronic.

Key words: Ferromagnetic resonance, Magnetic properties of nanostructures, Nanowires
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1 Introduction

Ferromagnetic nanowires possess interesting properties that might be exploited in current or future spintronic devices such as race-track type magnetic non-volatile memory called MRAM (based on transverse domain-wall dynamics [1,2]) and magnetic logic devices [3,4,5]. They might also be used in particular (dissipation-less) magnonic devices [6] extending microwave components to the nanoscale regime [7].

Mermin-Wagner [8] theorem forbids (Heisenberg-type) magnetism in low-dimensional systems (for a dimension \(\leq 2\)) with short-range interactions.

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Thus, ferromagnetic nanowires being a quasi-one dimensional system displaying magnetic properties represent an interesting system from the fundamental point of view.

In addition to their interest in fundamental magnetism, ferromagnetic nanowires have many applications in microwave components such as circulators [7], superconducting single-photon GHz detectors and counters [9], information storage (as recording media and read-write recording devices), Quantum transport (Giant magnetoresistance [10]) as well as in Quantum computing and Telecommunication devices.

They are simpler than nanotubes since their physical properties do not depend on chirality and they can be grown with a variety of methods [11]: Molecular Beam Epitaxy, Electrochemical methods (Template synthesis, Anodic Alumina filters), Chemical solution techniques (Self-assembly, Sol-Gel, emulsions...) and can be grown with a tunable number of monolayers and length [12].

Ordered arrays of nanowires may be of paramount importance in areas such as high-density patterned media information recording an example of which is the Quantum Magnetic Disk [13]. They might be also of interest in novel high-frequency communication or signal-processing devices based on the exploitation of spin-waves (in magnonic crystals made of magnetic superlattices or multilayers) [6] to transfer and process information or spin-currents with no dissipative Joule effect.

In this work, Nickel ferromagnetic nanowire arrays (FNA) are fabricated with an electrochemical deposition method [14] similar to the one used by Kartopu et al. [15]. During deposition, the time dependence of electrical current is carefully monitored and recorded since the current intensity profile versus time [11] reveals underlying growth mechanisms (coverage of pore walls, filling of pore interior by growth, growth at the outbound pore end, hemispherical cap growth over nanowire tips, percolative growth outside pores and formation of a 2D film) allowing us to finely tune the nanowire growth process and control it.

After structural and magnetic characterization [16], angle dependent FMR measurements in the X-band (9.4 GHz) are performed to extract the effective anisotropy field $H_{\text{eff}}$ versus angle while varying temperature from liquid Helium (4.2 K) to room temperature and changing nanowire diameter from 15 nm to 100 nm. The length of the Nickel FNA is 6 $\mu$m for all diameters and the average interwire distance is 350 nm.

We find that surface anisotropy (for small diameter) and underlying magnetoelastic effects play an important role (for all diameter) in those systems that might be exploited in novel storage devices, magnonic and spintronic compo-
Fig. 1. Magnetization $M$, applied field $H$ and corresponding angles $\theta$, $\phi$, $\theta_H$, $\phi_H$ they make with the nanowire axis that can be considered as an ellipsoid-shaped single domain with characteristic lengths $a = d/2$ and $c$ with $d$ the diameter. When the aspect ratio $c/a$ is large enough the ellipsoid becomes an infinite cylinder.

This work is organized as follows: In section 2, FMR measurements are presented and later analyzed in section 3. We conclude the work in section 4 and Appendix A details the FMR angular fitting procedure, Appendix B covers dipolar effects while Appendix C deals with transverse single domain issue.

2 FMR Resonance field versus field angle and temperature

Individual wires inside the array are aligned parallel to each other within a deviation of a few degrees. They are characterized by a cylindrical shape with a typical variation in diameter of less than 5% with a low-surface roughness and a typical length of 6 microns.

FMR experiments are performed with the microwave pumping field $h_{rf}$ operating at 9.4 GHz with a DC bias field $H$ making a variable angle $\theta_H$ with the nanowire axis.

Previously, several studies have considered reversal modes by domain nucleation and propagation (see for instance Henry et al. [17] for an extensive discussion of the statistical determination of reversal processes and distribution functions of domain nucleation and propagation fields). Moreover, Ferré et al. [18] and Hertel [19] showed the existence of domains with micromagnetic simulations). We do not consider domain nucleation and propagation in this work and rather concentrate on transverse single domain case as explained in
Thus, the angular dependence of $H_{\text{res}}$ in the uniform mode is obtained by considering an ellipsoid with energy $E$ comprised of a small second-order effective uniaxial anisotropy $K_1$ and shape demagnetization energy. Their sum is the effective anisotropy energy $E_A$ to which we add a Zeeman term $E_Z$ due to the external field $H$:

$$E = E_A + E_Z = (K_1 + \pi M^2 S) \sin^2 \theta - M_S H \sin \theta \sin \theta_H \cos(\phi - \phi_H) + \cos \theta \cos \theta_H$$

(1)

$\theta$ is the angle the magnetization makes with the nanowire axis (see fig. 1). The resonance frequency $\omega_r$ is obtained from the Smit-Beljers [21] formula that can be derived from the Landau-Lifshitz equation of motion with a damping term $\alpha$. Calculating the angular second derivatives of the total energy:

$$\left[ \frac{\omega_r}{\gamma} \right]^2 = \frac{(1 + \alpha^2)}{\sin^2 \theta} \left[ \frac{\partial^2 E}{\partial \theta^2} \frac{\partial^2 E}{\partial \phi^2} - \left[ \frac{\partial^2 E}{\partial \theta \partial \phi} \right]^2 \right]$$

(2)

This provides a relationship between the effective anisotropy field $H_{\text{eff}}$ and the external field $H$ at the resonance frequency.

Theoretically, the effective anisotropy field $H_{\text{eff}}$ can be obtained from the vectorial functional derivative of the energy $E_A$ (eq. 1) with respect to magnetization $H_{\text{eff}} = -\frac{\delta E_A}{\delta M}$ that becomes in the uniform case the gradient with respect to the magnetization components $H_{\text{eff}} = -\frac{\partial E_A}{\partial M}$.

The frequency-field dispersion relation obtained from the Smit-Beljers equation is:

$$\frac{\omega}{\gamma} = \sqrt{(1 + \alpha^2)[H_{\text{eff}} \cos 2\theta + H \cos(\theta - \theta_H)]} \times \sqrt{[H_{\text{eff}} \cos^2 \theta + H \cos(\theta - \theta_H)]}$$

(3)

where $H_{\text{eff}} = \frac{2K_{\text{eff}}}{M_S}$. At the resonance frequency $\omega = \omega_r$, $\theta = \theta_H$ and the applied field $H = H_{\text{res}}$ in the saturated case. In the unsaturated case the magnetization angle $\theta \neq \theta_H$ and one determines it directly from energy minimization.

Calling at equilibrium, magnetization orientation $\theta_0$ (taking $\phi = \phi_H$) we de-
termine it by evaluating the derivative \( \left( \frac{\partial E}{\partial \theta} \right)_{\theta_0} = 0 \). We get:

\[
K_{\text{eff}} \sin 2\theta_0 = M_S H \sin (\theta_H - \theta_0)
\]  

\[(4)\]

where \( K_{\text{eff}} = K_1 + \pi M_S^2 \).

Equations 4 and 3 are used simultaneously to determine the resonance field \( H_{\text{res}} \) versus angle \( \theta_H \) at any temperature (see Appendix A).

### 2.1 General analysis of experimental results

We add to the previously defined anisotropy energy \( E_A \), the dipolar interaction among nanowires yielding \( H_{\text{eff}} \) as the sum of the demagnetization field \( H_{\text{dem}} = 2\pi M_S \), the dipolar interaction field \( H_{\text{dip}} \) and the magnetocrystalline anisotropy field \( H_K \):

\[
H_{\text{eff}} = H_{\text{dem}} + H_{\text{dip}} + H_K = 2\pi M_S H_{\text{dip}} + \frac{2K_1}{M_S}
\]  

\[(5)\]

The dipolar field depends on porosity \( P \) (filling factor) in a way such that the demagnetization and dipolar fields (see Appendix B) are written as a single term \( 2\pi M_S(1 - 3P) \).

Experimentally, the resonance field \( H_{\text{res}} \) peaks [22] at \( \omega_r/\gamma \), hence it is possible to extract the effective anisotropy field \( H_{\text{eff}} \) through the use of eq. 3.

From the measured resonance field \( H_{\text{res}} \) versus field angle \( \theta_H \) the \( g \)-factor, saturation magnetization \( M_S \) and cubic anisotropy constant \( K_1 \) are determined with a least-squares fitting method (see Appendix A).

This yields the following table 1 containing fitting parameters \( K_1 \) and \( M_S \) (Anisotropy and saturation magnetization) versus diameter.

From table 1, one infers that as the diameter increases the Ni bulk values are steadily approached which is a good test of the FMR fit.

### 2.2 Analysis of FMR results at room temperature

Measured FMR absorption derivative spectra [23] are similar for (50, 80 and 100 nm) diameters but differ from the 15 nm case. Angular \( H_{\text{res}} \) curves versus \( \theta_H \) show a minimum at \( \theta_H = 0^\circ \) for the large diameters (50, 80 and 100 nm) and a minimum at \( \theta_H = 90^\circ \) for the 15 nm case (see Fig. 2).
Table 1

| d (nm) | D (nm) | $K_1$ (erg/cm$^3$) | $M_s$ (emu/cm$^3$) | $H_{eff}$ (Oe) | $H_K$ (Oe) |
|--------|--------|-------------------|-------------------|----------------|-------------|
| 15     | 256    | $-1.909 \times 10^6$ | 988.22           | 2344.58        | -3864.61    |
| 50     | 510    | $-1.621 \times 10^6$ | 451.95           | 2122.17        | -717.52     |
| 80     | 393    | $-2.424 \times 10^5$ | 453.25           | 1778.32        | -1069.56    |
| 100    | 497    | $-8.037 \times 10^4$ | 410.24           | 2185.78        | -391.81     |

Room temperature fitting parameters $K_1$ and $M_S$ with corresponding Nickel nanowire diameter $d$ and average separation $D$. Effective $H_{eff}$ and anisotropy $H_K$ fields are determined with Smit-Beljers. Comparing with bulk Nickel parameters at room temperature: $K_1 = -4.5 \times 10^4$ erg/cm$^3$, $K_2 = 2.3 \times 10^4$ erg/cm$^3$, $M_S = 485$ emu/cm$^3$ we infer that as the diameter increases we get closer to the bulk values as expected with $K_1$ changing by about two orders of magnitude (see Appendix A).

From this angular variation, we infer that 15 nm samples behave differently from larger diameter samples with a transition observed about 50 nm in agreement with the hysteresis loop VSM measurements (as displayed in fig. 3).

Previously Nielsch et al. [24] mentioned a change of behavior in nanowire arrays because of the existence of a coherence diameter in Ni to be $\approx 40$ nm.

The coherent diameter $d_c$ separates coherent (Stoner-Wohlfarth style or homogeneous) from inhomogeneous reversal (reversal by curling) obtained by equating nucleation fields in both cases [25]: $d_c = 2\sqrt{\frac{A}{2\pi N_a M_S}}$ where $A$ is the exchange stiffness constant (for Ni, it is about $26$ $1.5 \times 10^{-6}$ erg/cm).

Approximating the nanowire by an infinitely long cylinder, the demagnetizing factor along the minor axis $N_a = \frac{1}{2}$, thus: $d_c = 2\sqrt{\frac{A}{\pi q M_S}}$ with $q = 1.84$ the first positive zero of the first kind Bessel function $J_1(x)$ derivative $\frac{dJ_1(x)}{dx}$. Consequently Nielsch et al. [24] estimate is recovered. Nevertheless in our case, the change in behavior is probably due to change in the values of anisotropy constant $K_1$ and saturation magnetization $M_S$ with the diameter as seen in table 1.

Surface anisotropy might be responsible for this transition since the FMR fit displayed in fig. 1 enables us to define a diameter dependent anisotropy of the form $K_1 = K_V + \frac{K_S}{d}$ where the volume anisotropy $K_V = -2.89 \times 10^5$ erg/cm$^3$ and the surface anisotropy coefficient $K_S = 3.25 \times 10^6$ erg nm/cm$^3$ with the diameter $d$ expressed in nm. The $1/d$ dependence is akin to thin films where $d$ is the film thickness [27].

The transition has also been observed by Kartopu et al. [15] but left with no explanation. Generally, EA orientation of Nickel FNA is believed to be determined by the interplay of volume and shape anisotropies. This implies two regimes exist: for small diameter $d$ ($\leq 50$ nm) the EA is along the nanowire
Fig. 2. Measured resonance field $H_{\text{res}}$(red error bars) versus angle $\theta_H$ at a frequency of 9.4 GHz and at room temperature. The fit with equations 3 and 4 is the continuous curve (black line) as shown for all nanowire diameters 15, 50, 80 and 100 nm. The angle $\theta_H$ for which $H_{\text{res}}$ is minimal gives the easy axis orientation: it is 90° for 15 nm and 0° for all larger diameters.

axis (Isolated nanowire case) whereas for large diameter $d$ ($\geq 100$ nm) it is perpendicular to it (similarly to the thin film case).

Nonetheless, a third regime occurs when $d$ is very small ($\leq 35$ nm) and surface anisotropy intervenes with the EA orientation perpendicular anew to the nanowire axis. This explains our results for $d \leq 15$ nm as well as Kartopu et al.’s [15] for $d \leq 35$ nm.

Considering the fit parameters displayed in table 1 one may understand the easy axis direction change as follows. In the 15 nm diameter case, the values of $K_1$ and $M_S$ are respectively $-1.909 \times 10^6$ erg/cm$^3$ and 988.22 emu/cm$^3$ yielding the ratio $|K_1|/M_S^2$ as 1.94 meaning that magnetocrystalline anisotropy is more important than shape anisotropy that tends to align the magnetization along the nanowire axis. Moreover, since $K_1 < 0$, the anisotropy is planar and consequently the easy axis is in the plane perpendicular to the nanowire axis.
Fig. 3. (Color on-line) Room temperature VSM measured hysteresis loops $M/M_S$ versus $H$ (in Oersteds). The field $H$ is perpendicular (blue for $\perp$) or parallel (red for //) to the wire axis in the 15, 50, 80 and 100 nm diameter cases. The transition of easy axis orientation from perpendicular ($\perp$) in the 15 nm case to parallel (//) for diameter $\geq 50$ nm is clearly visible in the loops (loop merging at 50 nm and interchange for larger diameters) and confirms what is observed with FMR in fig. 2.

In the 50 nm case, the corresponding values are respectively $-1.621 \times 10^5$ erg/cm$^3$ and 451.95 emu/cm$^3$ yielding a ratio $|K_1|/M_S$ of 0.79. This means magnetocrystalline anisotropy is less important than shape anisotropy that tends to align the magnetization along the nanowire axis.

In fig. 4 the theoretical behavior at room temperature of the resonance frequency $\omega_r/\gamma$ versus field is shown. The intersection of the FMR measurement frequency line at 9.4 GHz we infer that in both diameter cases, we have a low field mode for $\theta_H = 0^\circ$ and a high field mode at $\theta_H = 90^\circ$ as observed in fig. 2. In the latter the low-field mode is around 1000 Oe for the 15 nm diameter and 1500 Oe for the 100 nm case. In contrast, fig. 4 indicates smaller values (250 Oe for the 15 nm diameter and 500 Oe for the 100 nm case) stemming from averaging effects arising from a dispersion of array anisotropy or geometry.
Fig. 4. (Color on-line) Room temperature theoretical resonance frequency $\omega$ as a function of field for the 15 nm (left) and 100 nm (right) diameter with angles: $\theta_H = 0^\circ$ and $\theta_H = 90^\circ$. The horizontal line is the FMR measurement frequency of 9.4 GHz. The low-field quarter of a circle shaped curve is the unsaturated case where the equilibrium magnetization angle is different from $\theta_H$. It can be expressed as $\omega = \sqrt{H_{eff}^2 - H^2}$. In the 15 nm, we have a high field mode for $\theta_H = 0^\circ$ and a low field mode at $\theta_H = 90^\circ$ as shown in fig. 2. It is exactly the opposite for diameter $d > 15$ nm as observed in fig. 2.

2.3 Analysis of FMR results versus temperature

Extending the above approach to the temperature dependent case by explicitly expressing $M_S$ and $K_1$ as $M_S(T)$ and $K_1(T)$ (taken as the first cubic anisotropy constant [28]), such that the temperature dependent effective field writes:

$$H_{eff}(T) = 2\pi M_S(T)(1 - 3P) + \frac{2K_1(T)}{M_S(T)}$$  \hspace{1cm} (6)

The temperature variation of the resonance field $H_{res}(T)$ is obtained from the Smit-Beljers equation 3 (neglecting $\alpha$) after relating it to the effective field $H_{eff}$.

When $H$ is applied along or perpendicularly to the wire axis, we get:

$$H_{res}(T) = \left| \frac{\omega_r}{\gamma} + H_{eff}(T) \right|; \quad \theta_H = 0^\circ$$

$$H_{res}(T) = \frac{1}{2} \left| H_{eff}(T) - \sqrt{H_{eff}^2(T) + 4\left(\frac{\omega_r}{\gamma}\right)^2} \right|; \quad \theta_H = 90^\circ$$  \hspace{1cm} (7)

Performing the analysis of measured FMR lineshapes versus field, temperature and angle (fig. 5 displays the FMR lineshapes as a function of field for
Fig. 5. (Color on-line) Measured FMR absorption lineshapes as a function of field for temperature varying from 4.2 K to room temperature for the 100 nm nanowire array. The field angle is $\theta_H = 0^\circ$ (top) and $\theta_H = 90^\circ$ (bottom).

Temperature varying from 4.2 K up to 300 K for two field angles $\theta_H = 0^\circ$ and $\theta_H = 90^\circ$ in the 100 nm case) entails extracting $H_{res}(T)$ (as displayed in fig. 6) and analyze its behavior with theoretical models.

In Fig. 6 theory and experiment of the temperature dependence of $H_{res}(T)$ are displayed. The disagreement observed indicates that additional temperature dependent anisotropies might intervene in the behavior of $H_{res}(T)$.

Noticing that in all cases $H_{res}(T)$ decreases with temperature a possible source of discrepancy might originate from the difference between thermal expansion coefficients of the metallic nanowire $\alpha_{Ni}$ with respect to the supporting dielectric material (DM) $\alpha_{DM}$. Thus, magneto-elastic effects might play an im-
Fig. 6. (Color on-line) Experimental (crosses) and theoretical (continuous lines) resonance field $H_{res}$ as a function of $T \in [4.2K, 300K]$ for the 15 nm (left) and 100 nm (right) diameter with both angles: $\theta_H = 0^\circ$ (red crosses for experiment and green lines for theory) and $\theta_H = 90^\circ$ (blue crosses for experiment and magenta lines for theory). Magneto-elastic terms are not considered in anisotropy. Larger values of $H_{res}$ occur for $\theta_H = 0^\circ$ in the 15 nm case and for $\theta_H = 90^\circ$ in the 100 nm case.

Important role in these systems. Moreover, the difference between $\alpha_{Ni}$ and $\alpha_{DM}$ might be strengthened further with chemical bonding or interaction effects between nanowires and the supporting material.

In order to account for the effect of temperature effect, let us start from the definition of effective anisotropy constant comprising magneto crystalline, shape and dipolar contributions $K_{eff} = K_1 + \pi M^2 S (1 - 3 P)$ and extend it in a way such that it becomes function of temperature.

Taking account of surface and magneto-elastic contributions, yields:

$$K_{eff}(T) = K_1(T) + \pi M_S^2(T)(1 - 3P) + K_{me}(T) + \frac{K_S}{d}$$  \hspace{1cm} (8)

$K_1(T)$ and $K_{me}(T)$ are respectively the magneto crystalline [29] and magnetoelastic [30]. In the following, temperature dependence for all anisotropies is determined except $K_S$.

Regarding saturation magnetization $M_S(T)$ we use Kuzmin [31] parameterization, i.e.:

$$M_S(T) = M_S(0)[1 - sx^{3/2} - (1 - s)x^p]^{1/3}$$  \hspace{1cm} (9)

with $x = T/T_c, M_S(0) = 57.6\text{emu/g}$, $p = 5/2$, $s = 0.15$ and the Curie temperature [31] $T_c = 628K$. 

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The temperature dependence of the magnetocrystalline coefficient $K_1(T)$ follows the Carr model [28], i.e.:

$$K_1(T) = K_0(1 - \delta x) \left[ \frac{M_S(T)}{M_S(0)} \right]^{10} \quad (10)$$

with $\delta = 1.74$ and $K_0 = -4.5 \times 10^6$ erg/cm$^3$ Ni first anisotropy constant at $T = 0K$ which is increased by one order of magnitude with respect to its room temperature [28] value.

Magnetooelastic anisotropy $K_{me}(T)$ is estimated along the following: $K_{me}(T) = \frac{3}{2} \lambda_S \sigma$ with $\sigma = E_Y \varepsilon(T)$ where $\sigma$ is stress, $E_Y$ is Young modulus and $\varepsilon(T)$ the temperature dependent deformation.

Deformation $\varepsilon(T)$ is estimated from the difference of dilation coefficients between Nickel and the DM: $\varepsilon(T) = (\alpha_{Ni} - \alpha_{DM}) \Delta T$ with $\Delta T$ the difference between fabrication (300 K) and measurement temperatures. $\alpha_{Ni}$, $\alpha_{DM}$ are respectively the thermal expansion coefficients of Nickel and of the DM. Their actual values are: $\alpha_{Ni} = 13 \times 10^{-6}$/K and in the case Alumina (Al$_2$O$_3$) is the DM, $\alpha_{DM} = 8.1 \times 10^{-6}$/K.

$\lambda_S$ is Nickel saturation isotropic magnetostriiction constant [32] equal to $-34.10^{-6}$. Young modulus of bulk Nickel is about $2.05 \times 10^{12}$ dyn.cm$^{-2}$ yielding a temperature dependent magnetoelastic anisotropy varying as: $K_{me}(T) \approx 512 \Delta T$. This result being approximate, we use an expression of the form $K_{me}(T) \approx a \Delta T$ and searched for the best value of $a$ that might explain all results for all diameters and field angles. We found the best value as $1500$ erg/K cm$^3$ as displayed in fig. 7.

Theoretical and experimental values of the resonance field $H_{res}$ versus temperature are compared in fig. 7 after accounting for the magneto-elastic anisotropy $K_{me}(T)$ contribution in the form $K_{me}(T) \approx 1500 \Delta T$. The agreement as a function of temperature for both nanowire diameters and both angles ($\theta_H = 0^\circ$ and $\theta_H = 90^\circ$) of the field is a strong indication of the presence of magneto-elastic effects.

**3 Discussion and Conclusion**

We have performed angle and temperature dependent FMR on Ni nanowire arrays with variable diameter and shown with FMR that the easy axis orientation for the 15 nm diameter sample is perpendicular to the wire axis in sharp contrast with the 50 nm, 80 nm and 100 nm samples. Note that we expect
Results obtained from the angular behavior of $H_{res}$ versus $\theta_H$ show that $H_{res}$ is minimum at 90° for the 15 nm sample whereas it is minimum at 0° for the larger diameter samples agree with hysteresis loops obtained from VSM measurements and confirm presence of the transition of easy axis direction from perpendicular at 15 nm to parallel to nanowire axis at 50 nm diameter.

The transition observed at 50 nm is interesting because of several potential applications in race-track MRAM devices. Yan et al. [4] predicted that in Permalloy nanowires of 50 nm and less, moving zero-mass domain walls may attain a velocity of several 100 m/s beating Walker limit obeyed in Permalloy strips with same lateral size. Hence, nanowire cylindrical geometry in contrast to prismatic geometry of stripes bears important consequences on current injection in nanowires that applies Slonczewski type torques [33] on magnetization affecting domain wall motion with reduced Ohmic losses [34].

At low temperature, we find that $H_{eff}$ increases when temperature is decreased. This may be attributed to the increase of $M_S(T)$, as temperature is decreased, affecting all anisotropy fields ($H_K$, $H_{dem}$, and $H_{dip}$) that depend on $M_S(T)$.

Separating the various contributions to anisotropy by using thermal, frequency, diameter and angular variations we have been able to pinpoint the main contribution to anisotropy at low temperature as stemming from magneto-elastic effects between the nanowire array and the DM supporting it.
Magneto-elastic effects affect directly the thermal variation of \( H_{\text{eff}} \) that increase at low temperature due to the increasing difference between the thermal expansion of the metallic nanowire and the DM. Thermal effects must be analyzed properly for building extremely high density storage devices.

Ordered arrays of nanowires are good candidates for patterned media and may also be used in plasmonic applications such as nano-antenna arrays or nanophotonic waveguides in integrated optics [35]. Recently [36], heat assisted magnetic perpendicular recording using plasmonic aperture nano-antenna has been tested on patterned media in order to process large storage densities starting at 1 Tbits/in\(^2\) and scalable up to 100 Tbits/in\(^2\).

In thermally-assisted perpendicular magnetic recording, a waveguide delivers light to a plasmonic nano-antenna placed just above the disk platter surface creating an intense optical pattern in the near-field region. However, this heats the disk on the nanometer scale (around 25 nm) and consequently temperature dependent magneto-elastic effects analyzed in this work might affect the physics of high-density information writing in these systems.

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4 Appendix A: Angular FMR fitting procedure

We have developed a procedure based on a least squares minimization procedure of the curve \( H_{\text{res}} \) versus \( \theta_H \) to the set of \( n \) experimental measurements \( [x_i, y_i]_{i=1,n} \) where \( x_i = \theta_{H,i} \) and \( y_i = H_{\text{res}}(\beta; x_i) \). \( \beta \) represents the set of parameters to fit \( M_S, K_1 \).

Hence the set of minima equations for the data points are:

\[
\frac{1}{n} \sum_{i=1}^{n} [H_{\text{res}}(\beta; x_i) - y_i]^2 \text{ minimum,} \\
|y_0 - H_{\text{res}}(\beta; x_i = 0^\circ)| \text{ minimum,} \\
|y_{90} - H_{\text{res}}(\beta; x_i = 90^\circ)| \text{ minimum}
\]

where the values \( y_0 \) (resp. \( y_{90} \)) are the experimental values of \( H_{\text{res}} \) corresponding to the angle \( \theta_H = 0^\circ \) (resp. \( \theta_H = 90^\circ \)).
The fitting method is based on the Broyden algorithm, a generalization to higher dimension of the one-dimensional secant method [37] that allows us to determine in a least-squares fashion, the set of unknowns \( g, M_S, K_1 \). Broyden method is selected because it can handle over or under-determined numerical problems and that it works from a singular value decomposition point of view [37]. This means it is able to circumvent singularities and deliver a practical solution to the problem at hand as an optimal set within a minimal distance from the real one. A practical solution might artificially increase some of the fitting values as observed in table 1.

5 Appendix B: Dipolar interaction field

Demagnetization energy is given [38] by \( E_{dem} = 2\pi N_{ij} M_i M_j \) where the demagnetization coefficients are such that \( 0 \leq N_{ij} \leq 1 \) and \( i, j = 1, 2, 3 \). Since the demagnetization field is given by \( \mathbf{H}_{dem} = -\frac{\partial E_{dem}}{\partial \mathbf{M}} \), component \( i \) of \( \mathbf{H}_{dem} \) is \( -4\pi N_{ij} M_j \). We have two limits:

- Single isolated nanowire (for which \( P = 0 \) and \( N_{xx} = 1/2, N_{yy} = 1/2, N_{zz} = 0 \)), \( E_{dem} = \pi(M_x^2 + M_y^2) = \pi(M_S^2 - M_z^2) \) and \( \mathbf{H}_{dem} = 2\pi M_S \cos \theta \mathbf{\hat{z}} \) with \( \mathbf{\hat{z}} \) the unit vector along \( z \).

- Thin film limit (for which \( P = 1 \) and \( N_{xx} = 0, N_{yy} = 0, N_{zz} = 1 \)), \( E_{dem} = 2\pi M_S^2 \) and \( \mathbf{H}_{dem} = -4\pi M_S \cos \theta \mathbf{\hat{z}} \).

When the sample is saturated along \( z \) (nanowire axis) \( M_x = 0, M_y = 0, M_z = M_S \), the demagnetization field in both cases has a single \( z \) component: \( \mathbf{H}_{dem} = 2\pi M_S \) in the single wire case and \( \mathbf{H}_{dem} = -4\pi M_S \) in the thin film case. Making a linear interpolation between these two limits, we get: \( \mathbf{H}_{dem} = 2\pi M_S (1 - 3P) \).

If we rather consider a 2D square lattice of nanowires with parameter \( D \) the average nanowire separation, the porosity is given by: \( P = \frac{\pi a^2}{D^2} \) and we can calculate directly the dipolar energy as follows.

Starting from a single dipole \( \mathbf{p}_0 \) surrounded by an array of dipoles on the lattice \( \mathbf{p}_i \), the interaction energy is:

\[
E_{int} = \sum_i \frac{\mathbf{p}_0 \cdot \mathbf{p}_i}{r^3} - \frac{3(\mathbf{p}_0 \cdot \mathbf{r})(\mathbf{p}_i \cdot \mathbf{r})}{r^5}
\]

(12)

This is equivalent to a field \( \mathbf{H}_{dem} \) acting on the dipole such that: \( E_{int} = -\mathbf{p}_0 \cdot \mathbf{H}_{dem} \).

When \( \mathbf{p}_0 \) is directed along the \( z \) axis, \( \mathbf{p}_0 = (0, 0, 1) \) and \( \mathbf{p}_0 \cdot \mathbf{H}_{dem} = H_{dem,z} \) with \( H_{dem,z} \) the \( z \) component of \( \mathbf{H}_{dem} \).
At any node \((i, j)\) of the lattice \(\mathbf{p}_i = (p_x, p_y, p_z)\), and \(\mathbf{r} = (iD, jD, 0)\), hence \(\mathbf{p}_0 \cdot \mathbf{r} = 0, \mathbf{p}_i \cdot \mathbf{p} = p_z\).

The field \(H_{\text{dem},z}\) is then:

\[
H_{\text{dem},z} = -\sum_{i,j=-\infty}^{+\infty} \frac{p_z}{r^3} = -\sum_{i,j=-\infty}^{+\infty} \frac{p_z}{D^3(i^2 + j^2)^{3/2}} \tag{13}
\]

When all surrounding dipoles are saturated along \(z\), \(p_z = M_S V\) with \(V = \pi a^2 l\) the nanowire volume:

\[
H_{\text{dem},z} = -\frac{M_S V}{D^3} \sum_{i,j=-\infty}^{+\infty} \frac{1}{(i^2 + j^2)^{3/2}} \approx -\frac{4.2 M_S V}{D^3} = -4.2 M_S P \left( \frac{l}{D} \right) \tag{14}
\]

This should be compared to the interpolated dipolar term \(-6\pi M_S P\). The evaluation of \(H_{\text{dem},x}\) the \(x\) component of \(H_{\text{dem}}\) proceeds along the same lines. Taking \(\mathbf{p}_0\) along the \(x\) axis, \(\mathbf{p}_0 = (1, 0, 0)\), \(\mathbf{p} = (p_x, p_y, p_z)\) and \(\mathbf{r} = (iD, jD, 0)\), the dipolar energy writes:

\[
E_{\text{int}} = -\mathbf{p}_0 \cdot \mathbf{H}_{\text{dem}} = -H_{\text{dem},x} = -\sum_{i,j=-\infty}^{+\infty} \frac{r^2 p_x - 3 D^2 (i^2 p_x + i j p_y)}{r^5} \tag{15}
\]

Therefore:

\[
H_{\text{dem},x} = -\sum_{i,j=-\infty}^{+\infty} \frac{p_x (i^2 + j^2) - 3 (i^2 p_x + i j p_y)}{D^3 (i^2 + j^2)^{3/2}} \tag{16}
\]

If all the surrounding dipoles are saturated along \(x\), \(p_x = M_S V, p_y = p_z = 0\), we get:

\[
H_{\text{dem},x} = \frac{M_S V}{D^3} \sum_{i,j=-\infty}^{\infty} \frac{(2i^2 - j^2)}{(i^2 + j^2)^{3/2}} \approx 2.1 \frac{M_S V}{D^3} \tag{17}
\]

Thus we obtain:

\[
H_{\text{dem},x} = 2.1 M_S P \left( \frac{l}{D} \right) = -\frac{1}{2} H_{\text{dem},z} \tag{18}
\]
A simple argument given in Landau-Lifshitz *Electrodynamics of Continuous Media* [38] gives the size below which a single domain (or Stoner-Wohlfarth) behavior is observed in the plane perpendicular to nanowire axis. It is based on the following: When the demagnetization energy $2\pi N_{ij} M_i M_j \sim 2\pi N_c M_s^2$ (where $N_c$ is the demagnetization coefficient along some preferred axis, usually the long one in an ellipsoid- approximated nanowire) is equal to the exchange energy $\frac{A_{ij}}{M_s^2} \frac{\partial M_k}{\partial x_i} \frac{\partial M_k}{\partial x_j} \sim \frac{A}{R_{sd}^2}$ ($A_{ij}$, $i, j, k = 1, 2, 3$ is the exchange stiffness constant along $i, j$ directions).

Considering that $A_{ij} \sim A$ a typical exchange stiffness constant (regardless of $i, j$) results in $R_{sd} \sim \sqrt{\frac{A}{2\pi N_c M_s^2}}$. Exchange energy is the largest contribution to non-uniformity energy due to spatial variation of the magnetization $M$.

When the change in the direction of $M$ occurs over distances that are large compared to interatomic distances, non-uniformity energy can be expressed through derivatives of $M$ with respect to spatial coordinates (see Landau-Lifshitz [38] and Brown [39]). Exchange stiffness constant $A_{ij}$ is on the order of Heisenberg exchange energy per unit length $J/a_0$ ($a_0$ is the average nearest neighbour distance in the nanowire material).

![Fig. 8. Single domain radius (nm) for a Nickel prolate (elongated) ellipsoid as a function of the demagnetization coefficient along its axis $N_c$. The Landau approximation $R_{sd} \sim \sqrt{\frac{A}{2\pi N_c M_s^2}}$ is scaled in a way such that it agrees with the exact result when $m \to 0$. The coherent radius, in contrast, increases with $N_c$ and is around 10 nm. The room-temperature Nickel data taken from Kittel [40]) are $M_s = 485$ Gauss; $a_0 = 2.49$ Å. The exchange stiffness constant $A$ is taken as $10^{-6}$ erg/cm. Typically $J \sim 10$ meV and $a_0 \sim 1$ Å, hence we get $A_{ij} \sim 10^{-6}$ erg/cm (see fig. 8). The exchange length $\ell_{ex}$ is defined as $\ell_{ex} = \sqrt{\frac{A}{K}}$ with $A$, the exchange stiffness constant $(A \sim 10^{-6}$ erg/cm). It is obtained from the comparison between the exchange energy $\frac{A_{ij}}{M_s^2} \frac{\partial M_k}{\partial x_i} \frac{\partial M_k}{\partial x_j} \sim \frac{A}{\ell_{ex}^2}$ and the anisotropy energy.
\( \frac{K_{ij}}{M_s^2} M_i M_j \sim K \) with \( K \) representing the anisotropy constant of the material. In soft materials whose anisotropy constant \( K \sim 0 \), one uses the magnetostatic exchange length defined as \( \ell_{ex} = \sqrt{A/M_s^2} \) since \( K \sim 0 \) in soft materials. It is obtained from the comparison between the exchange energy \( \frac{A}{\ell_{ex}} \) and the demagnetization energy \( 2\pi N_s M_s^2 \).

The length \( \ell_{ex} = \sqrt{A/K} \) is in fact on the order of the domain wall thickness, therefore we follow Frei et al. [41] to estimate rigorously the single-domain radius \( R_{sd} \) from a minimization of the energy using Euler variational equations.

The energy density of an infinite cylinder (see fig.1) made of a ferromagnetic material accounting for only exchange terms and Zeeman energy is given in cylindrical coordinates \((r, \varphi, z)\) by:

\[
A[(\frac{\partial M}{\partial r})^2 + (\frac{\partial M}{r \partial \varphi})^2 + (\frac{\partial M}{\partial z})^2] - M.H
\]

The total energy is found from an average over the volume:

\[
E = \frac{1}{\pi a^2} \int_{a_0/2}^a \{A[(\frac{d\theta}{dr})^2 + \frac{\sin^2 \theta}{r^2}] - M_s H \cos(\theta - \phi)\} 2\pi r dr
\]

Minimization of the energy obtained from nulling the variational derivative with respect to the angle \( \theta \) leads to:

\[
-2 \frac{d^2 \theta}{dr^2} - 2 \frac{1}{r} \frac{d\theta}{dr} + 2 \frac{1}{r} \sin(\theta) \cos(\theta) + M_s H \sin(\theta - \phi) = 0
\]

When we consider small values of \( \theta \), the above differential equation becomes a Bessel equation in terms of \( \theta(r) \).

In order to evaluate the average of the different energy terms, we replace \( \theta(r) \) by \( \pi/2 \) in the evaluation of the exchange energy term that becomes \( E_X = \frac{A}{\ell_{ex}} \).

This can be simply averaged over the ellipsoid volume (with short axis \( 2a \) and long axis \( 2c \) see fig. 1) representing the nanowire:

\[
\overline{E_X} = \frac{1}{3} \frac{1}{\pi c a^2} \int_{a_0/2}^a 2\pi r dr \int_0^1 dz \frac{A}{r^2}
\]

\( a_0 \) is a cutoff length given by the nearest-neighbour distance or the lattice parameter of the corresponding crystal (see fig 8). The average magnetostatic...
(demagnetization) energy can be calculated following ref [42] accounting for the curling of the magnetization from the initial state with $M_0$:

\[ E_m = 2\pi N_c M_s^2 \]  

(23)

with $N_c$ (the long axis demagnetization coefficient).

Comparing the average exchange and demagnetization energies gives:

\[ R_{sd} - \sqrt{\frac{3A}{2\pi N_c M_s^2}} \left[ \ln \left( \frac{4R_{sd}}{a_0} \right) - 1 \right] = 0 \]  

(24)

Solving eq. 24 for $R_{sd}$, in the case of Ni, fig. 8 gives the variation of $R_{sd}$ versus $N_c$. From the figure, we infer that for Nickel nanowires $R_{sd}$ is within a few 100 nm range.

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