Event-based secure consensus of multiple AUVs under DoS attacks

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Abstract In this paper, the event-based triggering method is adopted to investigate the secure consensus issue of multiple autonomous underwater vehicles (AUVs) under denial-of-service (DoS) attacks. DoS attack is a form of time-sequence-based cyber attack, which can destroy the normal service of the control target or network. First, based on an event-triggered mechanism, a novel secure control protocol is proposed. Second, the upper bounds of attack duration and attack frequency are given to ensure that multiple AUVs under DoS attacks can reach consensus. Third, an event-triggered mechanism with exponential variables is developed to avoid the continuous update of the controller, thereby reducing the burdens of communication and calculation. Zeno behavior can be strictly ruled out for each AUV under this triggering mechanism. Finally, the simulation results illustrate the feasibility of the proposed scheme.

Keywords AUVs · Event-triggered mechanism · DoS attacks · Consensus

1 Introduction

Recently, collaborative control of multi-agent systems has become a hot research field because of its wide application in various fields, such as the formation of robots [1,2], unmanned vehicles [3,4], and surface ships [5,6]. As we all know, in the numerous studies of cooperative control, the consensus of multiple AUVs is one of the basic problems. The main goal of multiple AUV consensus is to design control protocols based on the local information of the neighbors of the AUV to ensure that the states of all AUVs finally reach consensus [6–8]. Due to the complex underwater environment, it is relatively difficult to share information among multiple AUVs, consensus control becomes a huge challenge.

Consensus of multiple AUVs is a hot topic in both practical and theoretical research. In [6,7,9], the consensus of multiple AUVs under fully actuated and underactuated was discussed. In [8,10–13], the trajectory tracking issue of AUVs with disturbances under different conditions was studied by the sliding mode control method. An impulse network method was developed to study the formation problem of multiple AUVs in fixed or switched communication topologies in [14,15]. In [16], a coordinated controller was proposed based on the hybrid control theory, which enabled the controller to switch freely. To cope with the complex marine environment and achieve the purpose of long-term navigation, energy saving is a hot topic of research and the multiple AUVs control under the event-triggered mechanism is necessary.

It is worth noting that in reality, each AUV usually consists of some specific modules with limited energy,
such as processor modules, communication modules, and drive modules. The event-triggered control strategy is an effective energy-saving control method, which can reduce the communication and calculation burden caused by continuous communication among multiple AUVs and frequent updates of controllers [17,18]. Therefore, this method can save energy while ensuring control performance. The mathematical model of the AUV can be regarded as a special second-order nonlinear system. Some efforts are made to second-order systems under the triggering mechanism [19–23], but there are few studies about AUVs under the triggering mechanism. In [24,25], the trajectory tracking problem of fully actuated and underactuated AUVs under the event-triggered mechanism was investigated.

The multi-agent systems can be regarded as fragile network systems, which are extremely vulnerable to cyber attacks. DoS attacks are a major form of cyber attacks. DoS attacks can affect or even destroy the normal services and communications of the target network. The design of the secure control protocol is an important issue in the network control systems. To eliminate the impact of DoS attacks on network systems, some secure control protocols were developed in [26–33]. Driven by [27,28], the leader-follower multi-agent systems under DoS attacks were discussed in [34]. The distributed consensus controller was designed in [35] under the framework of linear systems. In [36–39], the consensus of linear multi-agent systems under DoS attack based on event-triggered mechanism was studied. DoS attack frequency and attack duration can be obtained so that the positions of multiple AUVs can reach consensus. Third, an event-triggered mechanism including exponential variables is adopted, which is not involved in [29,37]. Under this triggering mechanism, a larger interevent interval can be obtained to avoid Zeno behavior. Finally, the simulation result shows the effectiveness of the algorithm.

The rest of this paper is summarized as follows. Section 2 provides some preliminaries and the problem statement. In Sect. 3, event-based secure consensus of multiple AUVs under DoS attacks is investigated. In Sect. 4, a simulation example is provided to demonstrate the merits and validity of the obtained results. In Sect. 5, the conclusion and future work are presented.

Notation: $R^n$ is the $N$-dimensional column vectors. $\|\cdot\|$ is the Euclidean norm. $1_N(0_N)$ is a vector with entries being 1(0). $I_N$ represents the identity matrix and the dimension is $M$. $\lambda_{\text{max}}(H)(\lambda_{\text{min}}(H))$ is introduced to represent the maximum (minimum) eigenvalues of the matrix $H$. $\text{diag}()$ denotes a diagonal matrix.

2 Preliminaries and problems definition

2.1 Communication topology

The topology among N agents can be described as a graph $G = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ consisting of a set of vertices $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$, a set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $\mathcal{A} = (a_{ij})_{N\times N}$ with nonnegative entries. The directed edge $e_{ij}$ is denoted by a pair of vertices $(v_i, v_j)$, which means $v_j$ can obtain information from $v_i$, the neighbor set of the $v_i$ is denoted by $\mathcal{N}_i = \{v_j \in \mathcal{V} | ((v_i, v_j)) \in \mathcal{E}\}$ and $e_{ij} \in \mathcal{E}$ if and only if $a_{ij} > 0; a_{ij} = 0$ otherwise. The Laplacian matrix $L = (l_{ij})_{N\times N}$ is defined as $l_{ii} = \sum_{j=1, j \neq i}^{N} a_{ij}$, $l_{ij} = -a_{ij}$ for $i \neq j$. For an undirected graph, it follows that $a_{ij} = a_{ji}$, $\forall i, j \in (1, 2, \ldots, N)$. If there is a reachable path between any two agents in $G$, then $G$ is considered to be connected.

2.2 AUV model

Consider that a multi-agent system consists of $N$ AUVs. The AUV obtains position information from its neighbors to determine position and orientation through the sensors. Suppose that the attitudes of AUVs

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Event-based secure consensus of multiple AUVs can be described as [40]

\[
\dot{\eta}_i = J_i(\theta_i)\eta_i,
\]

\[
M_i \dot{v}_i = -C_i(v_i) v_i - D_i(v_i) v_i - g_i(\theta_i) + \tau_i,
\]

where \(i \in \{1, 2, \ldots, N\}\) is the lable of the AUVs, \(\eta_i = [x_i, y_i, z_i]^T \in R^3\) is the position vector of the \(i^{th}\) AUV in the inertial reference frame, \(\theta_i = [\theta_i, \phi_i, \psi_i]^T \in R^3\) is the attitude vector and \(\theta, \phi, \psi\) are the Euler angles (roll, pitch, yaw) of the \(i^{th}\) AUV in the inertial reference frame. \(J_i(\theta_i)\) represents the kinematic transformation matrix. \(v_i = [u_i, q_i, w_i] \in R^3\) is the velocity vector and \(u_i, q_i, w_i\) are the linear velocities (surge, sway, heave) of the \(i^{th}\) AUV in the body-fixed reference frame. \(M_i\) denotes the inertia matrix. \(C_i(v_i)\) is the coriolis and centripetal matrix. \(D_i(v_i)\) is the hydrodynamic drag matrix. \(g_i(\theta_i) \in R^3\) is the vector of restoring forces (gravity and buoyancy). \(\tau_i \in R^3\) is the control input. For brevity, \(J_i = J_i(\theta_i), C_i = C_i(v_i), D_i = D_i(v_i), g_i = g_i(\theta_i)\). The kinematic transformation matrices is as follows:

\[
J_i = \begin{pmatrix}
c_{\theta_i}c_{\phi_i} & -s_{\phi_i}c_{\theta_i} & c_{\phi_i}s_{\theta_i} + c_{\theta_i}s_{\psi_i}
c_{\phi_i}c_{\theta_i} & c_{\theta_i}c_{\phi_i} & -c_{\phi_i}s_{\psi_i} - s_{\theta_i}s_{\psi_i}
-s_{\phi_i} & 0 & c_{\phi_i}s_{\theta_i} + s_{\theta_i}s_{\psi_i}
c_{\phi_i}s_{\theta_i} & -s_{\phi_i}s_{\theta_i} & c_{\phi_i}c_{\theta_i}
\end{pmatrix},
\]

for an angle \(\alpha \in R\), the symbol \(s_{\alpha}\) and \(c_{\alpha}\) denote \(\sin \alpha\) and \(\cos \alpha\).

\[
C_i = \begin{pmatrix}
0 & 0 & -m_{12}q_i \\
0 & m_{11}u_i & 0 \\
m_{12}q_i & -m_{11}u_i & 0
\end{pmatrix},
\]

\[
M_i = \text{diag} \{m_{11}, m_{12}, m_{13}\} , D_i = \text{diag} \{d_{11}, d_{12}, d_{13}\},
\]

\[
m_{11} = m - X_{ix} - m_{12} = -Y_{iq} - m_{13} = m - Z_{iw},
\]

\[
d_{11} = -X_{iu} - X_{iu}, d_{12} = -Y_{iq} - Y_{iq} |q_i|, d_{13} = -Z_{iw} - Z_{iw} |w_i|,
\]

\[
g_i = \begin{pmatrix}
(F_i - H_i)s_{\phi_i}, -(F_i - H_i)c_{\phi_i}c_{\theta_i}
\end{pmatrix}^T ,
\]

where \(F_i\) and \(H_i\) denote the gravitational and buoyancy forces. Note that \(J_i J_i^T = I_3\) and \(x^T D_i(x)x > 0, \forall x \neq 0, x \in R^3\).

### 2.3 DoS attack model

DoS attack [28]–[39]: DoS attacks, as a common form of cyber attacks, can disrupt the information transmission among agents. Under the DoS attacks, the target systems cannot provide normal services. If the length and frequency of the attack are not limited, the stability of the entire system will be undermined. Thus, assuming that the energy of DoS attacks is limited, the systems can enter the recovery phase after each attack to increase energy. Therefore, the entire time can be divided into attack areas and communication areas. As shown in Fig. 1, the red areas represent the communication areas. Note that the event-triggered mechanism exists in this area. The blue areas represent the attack areas, in which the controller is not available.

Suppose \(h_n n \in N\) is a DoS attack sequence, where \(h_n\) represents the instant of the \(n\)th attack. For \(\Delta_n\), the \(n\)th attack interval is \(H_n = [h_n, h_n + \Delta_n]\), in which \(h_{n+1} > h_n + \Delta_n\). For \(t \geq t_0\), similar to [39], the sets of time instants for the attack areas and the communication areas are depicted as follows:

\[
\Lambda_a(t, t_0) = \bigcup_{n \in N} H_n \cap [t, t_0)
\]

\[
\Lambda_s(t, t_0) = [t, t_0] \setminus \Lambda_a(t, t_0)
\]

**Definition 1** (Attack Frequency [36], [38]): Define \(N_a(t, t_0)\) as the number of DoS attacks over \([t, t_0]\). The attack frequency can be depicted as follows:

\[
F_a(t, t_0) = \frac{N_a(t, t_0)}{t - t_0}.
\]

**Definition 2** (Attack Duration Rate [36], [38]): For \(t > t_0 > 0\), \(A_a(t, t_0)\) denotes the total attack interval in \([t, t_0]\). The attack duration rate can be depicted as follows:

\[
rate = \frac{\Lambda_a(t, t_0)}{t - t_0}.
\]

**Remark 1** It is worth mentioning that there are many papers on the multi-agent system under DoS attacks, the control input is directly set to zero or takes the control value of the most recent successful communication moment, such as [34] and [30]. If a multi-agent system is repeatedly attacked by DoS and the duration of the attack is not limited, the local agent cannot receive the neighbor’s data in time. Therefore, all data transfers may fail all the time. This may make the system unstable. This kind of attack requires a continuous supply of energy, so it is impractical in the real world. From a practical point of view, Definition 1 and Definition 2 are general and quite mild, as described in the existing references [36] and [38].

**Remark 2** Note that DoS attacks is different from traditional packet dropouts phenomenon. The number of packet dropouts usually belongs to an integer set, and the number of continuous packet dropouts is less
than a small number, while DoS attacks may last for a long period of time.

**Assumption 1** There is a Lipschitz constant $c > 0$, which can make $\| f(a) - f(b) \| \leq c \| a - b \|$, for $a, b \in R$.

**Assumption 2** [5]: For the system (10), Assume that $\dot{v}_i$ are bounded. That is, there is a positive constant $\Gamma_M \in R$ such that $\| \dot{v}_i \| \leq \Gamma_M$.

The purpose of this paper is to design the event-triggered control protocol so that the positions of multiple AUVs under DoS attacks can reach consensus when triggered control protocol is available, i.e., $\tau_i(t) = 0$. Therefore, the kinematic and dynamic model of the $i$th AUV can be rewritten as

$$\begin{align*}
\dot{\xi}_i &= -J_iM^{-1}C_iJ_i^T \xi_i - J_iM^{-1}D_iJ_i^T \xi_i - J_iM^{-1} \mu(\gamma_i(t_k^i)).
\end{align*}$$

According to the definition of DoS attacks, when the system suffers DoS attacks, the control protocol cannot be available, i.e., $\tau_i(t) = 0$. Therefore, the kinematic and dynamic model of the $i$th AUV can be given as

$$\begin{align*}
\dot{\xi}_i &= \xi_i - \xi_j, \\
\xi_i &= -J_iM^{-1}C_iJ_i^T \xi_i - J_iM^{-1}D_iJ_i^T \xi_i - J_iM^{-1}g_i.
\end{align*}$$

A measurement error applicable to the system (11) is defined as

$$e_i = \gamma_i(t_k^i) = [\eta_i^T(t_k^i) - \eta_i^T, \eta_j^T(t_k^i) - \eta_j^T, v_i^T(t_k^i) - v_j^T(t_k^i)].$$

Based on (10) and (12), one has

$$\dot{\beta}_{ij} = \eta_i - \eta_j, \\
\dot{\xi}_i = -J_iM^{-1}C_iJ_i^T \xi_i - J_iM^{-1}D_iJ_i^T \xi_i - J_iM^{-1}g_i.$$
Remark 4 Different from the centralized event-triggered control method utilized in [26] and [29] to deal with attacks, the distributed event-triggered control method is used to study the security consensus of AUVs. In fact, the centralized event-triggered control method wastes network resources, especially when network resources are limited. Under the distributed event-triggered control method, each agent can determine its own triggering moment, which can save network resources.

**Theorem 1** Suppose Assumption 1 holds. Considering system (1) under DoS attacks with the event-triggered mechanism (15) can ensure that the positions of multiple AUVs reach consensus when the following conditions can be satisfied:

\[
F_a(t, t_0) = \frac{N_a(t, t_0)}{t - t_0} \leq \frac{\theta^*}{\ln \mu + (\varepsilon_1 + \varepsilon_2)\Delta},
\]

where \(\varepsilon^* \in (0, \varepsilon_1), \varepsilon \in (\varepsilon_1, \varepsilon_2)\), \(s_{\text{max}}(\omega_{\text{min}}) > 0\), \(\varepsilon_1 = \min\{2(1 - \delta_{\text{max}})/\lambda_{\text{max}}(H), 2s_{\text{min}}/\lambda_{\text{max}}(L)\}\) and \(\varepsilon_2 = \max\{2(s_{\text{max}} + \lambda_{\text{max}}(J(J_i + D_i)J_i^T))/\lambda_{\text{min}}(H), 2\lambda_{\text{max}}(L)/s_{\text{min}}\lambda_{\text{max}}(L)\}\).

**Proof:** When DoS attacks exist, there are two time sequences in the systems: triggering sequence \(\{t^k\}_{k=1}^{\infty}\) and attack sequence \(\{h_n\}_{n=1}^{\infty}\). The update set of agents in the attack areas: \(\mathcal{W} = \{ (i, k) \in \mathcal{V} \times N | t^k_i \in \mathcal{H}_n \}\). After each DoS attack, agent need time \(\Delta_n\) to recover communication. Meanwhile, there is a time delay \(\Delta\) between two attacks and the interval between two attacks is greater than \(\Delta\). \(\Pi_n = [h_n, h_n + \Delta_n + \Delta]\) denotes the \(n\)th interval where the triggering condition (14) cannot work. For any time interval \([t, t_0]\), we have \([t, t_0] = \tilde{A}_x(t, t_0) \cup \tilde{A}_a(t, t_0)\), where \(\tilde{A}_x(t, t_0) = \bigcup \Pi_n \cap [t, t_0]\) and \(\tilde{A}_a(t, t_0) = [t, t_0] \setminus \tilde{A}_a(t, t_0)\).

First, the system without DoS attacks is considered, i.e., the system is in the communication areas \(\tilde{A}_x(t, t_0)\). Choose the Lyapunov function candidate as

\[
V = \frac{\zeta}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} b_{ij} \xi_i^T \xi_j + \frac{1}{2} \sum_{i=1}^{N} \xi_i^T J_i^T J_i \xi_i.
\]

where \(0 < \zeta < 1\) and \(H = (J_i M_i^{-1} J_i^T)^{-1}\).

Then, one can obtain that

\[
\dot{V} = \zeta \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} b_{ij} (\xi_i - \xi_j) + \sum_{i=1}^{N} \xi_i^T J_i^T \xi_i - \sum_{i=1}^{N} \xi_i^T J_i M_i^{-1} D_i J_i^T \xi_i - \sum_{i=1}^{N} \xi_i^T J_i M_i^{-1} \xi_i - \sum_{i=1}^{N} \xi_i^T H J_i M_i^{-1} g_i
\]

\[
+ \sum_{i=1}^{N} \xi_i^T H J_i M_i^{-1} (\mu (\gamma) - \mu_1 (\gamma)) + \sum_{i=1}^{N} \xi_i^T H J_i M_i^{-1} \xi_i - \sum_{i=1}^{N} \xi_i^T H J_i M_i^{-1} g_i + \sum_{i=1}^{N} \xi_i^T H J_i M_i^{-1} \xi_i - \sum_{i=1}^{N} \xi_i^T \bar{J}_i J_i^T \xi_i - \sum_{i=1}^{N} \xi_i^T J_i^T J_i \xi_i - \sum_{i=1}^{N} \xi_i^T H J_i M_i^{-1} g_i
\]
According to Assumption 1, suppose the Lipschitz constant is $\ell$. Then, we have $\|\mu(y_t + e_t) - \mu_i(y_t)\| \leq \ell\|e_t\|$.

Thus,

$$
\dot{V} \leq -(1 - \zeta) \sum_{i=1}^{N} \xi_i^T \xi_i - (1 - \zeta) \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} \beta_{ij}^T \beta_{ij} - \sum_{i=1}^{N} \xi_i^T J_i C_i J_i^T \xi_i
$$

$$
- \sum_{i=1}^{N} \xi_i^T J_i D_i J_i^T \xi_i + \sum_{i=1}^{N} \xi_i^T J_i C_i J_i^T \xi_i
$$

$$
+ \sum_{i=1}^{N} \xi_i^T J_i D_i J_i^T \xi_i - \sum_{i=1}^{N} \xi_i^T \xi_i + \sum_{i=1}^{N} \xi_i^T J_i \|e_t\|
$$

$$
\leq -(2 - \zeta - \rho_i) \sum_{i=1}^{N} \xi_i^T \xi_i - (1 - \zeta) \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} \beta_{ij}^T \beta_{ij} + \sum_{i=1}^{N} \rho_i \|e_t\|^2
$$

$$
- \zeta) \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} \beta_{ij}^T \beta_{ij} + \sum_{i=1}^{N} \rho_i \|e_t\|^2
$$

$$
= \delta_i \sum_{i=1}^{N} ||\xi_i||^2 - t \sum_{i=1}^{N} \sum_{j=1}^{N} ||\beta_{ij}||^2 + \sum_{i=1}^{N} \rho_i \|e_t\|^2
$$

where $\delta_i = 2 - \zeta - \rho_i > 0$, $t_i = (1 - \zeta) \|a_{ij}\| > 0$ and $\rho_i = \|J_i\| \|\xi_i\|$. Based on the event-triggered mechanism (15), it can be obtained that

$$
\|e_t\|^2 \leq \frac{\delta_i}{\rho_i} (t_i \sum_{j=1}^{N} \|\beta_{ij}\|^2 + k_i e^{-d_i t}).
$$

From the inequations (20) and (21), one has

$$
\dot{V} \leq \sum_{i=1}^{N} [-\delta_i ||\xi_i||^2 - (1 - \sigma_i) t_i \sum_{j=1}^{N} \|\beta_{ij}\|^2 + k_i e^{-d_i t}]
$$

$$
\leq -\varepsilon_1 \sum_{i=1}^{N} ||\xi_i||^2 - \varepsilon_1 \sum_{i=1}^{N} \sum_{j=1}^{N} \|\beta_{ij}\|^2 + \sum_{i=1}^{N} k_i e^{-d_i t}.
$$

where $\varepsilon_1 = \min\{2\delta_{\min}/\lambda_{\max}(H), 2(1 - \sigma_{\max})\delta_{\min}/\lambda_{\max}(L)\}$.

Consider the following Lyapunov function candidate:

$$
W = V + \sum_{i=1}^{N} \frac{1}{d_i} k_i e^{-d_i t}.
$$

According to (22) leads to

$$
\dot{W} \leq -\varepsilon_1 \sum_{i=1}^{N} ||\xi_i||^2
$$

$$
-\varepsilon_1 \sum_{i=1}^{N} \sum_{j=1}^{N} \|\beta_{ij}\|^2.
$$

For $t \in [0, +\infty)$, $W$ is nonincreasing. Since $W \geq 0$, $\lim_{t \to \infty} W$ exists and is finite. According to the Cauchy’s convergence criterion, for every $\kappa > 0$, there exists a constant $\tilde{T}$, such that $W(\tilde{T}_1) - W(\tilde{T}_2) < \kappa$, if $\tilde{T}_2 > \tilde{T}_1 > \tilde{T}$. Assume that the triggering instants are sorted as $\tau_i^{k_1} < \tau_i^{k_2} < \cdots < \tau_i^{k'}$ during ($\tilde{T}_1, \tilde{T}_2$). Then, according to (24), one obtains that

$$
\varepsilon_1 \int_{\tilde{T}_1}^{\tilde{T}_2} \left( \sum_{i=1}^{N} ||\xi_i(t)||^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} \|\beta_{ij}(t)\|^2 \right) dt
$$

$$
\leq -\int_{\tilde{T}_1}^{\tilde{T}_2} \dot{W}(t) dt
$$

$$
= -\int_{\tilde{T}_1}^{\tau_i^{k_1}} \dot{W}(t) dt - \int_{\tau_i^{k_1}}^{\tau_i^{k_2}} \dot{W}(t) dt - \cdots
$$

$$
- \int_{\tau_i^{k'}} \dot{W}(t) dt
$$

$$
= W(\tilde{T}_1) - W(\tau_i^{k_1}) + \cdots + W(\tau_i^{k'}) - W(\tilde{T}_2)
$$

$$
= W(\tilde{T}_1) - W(\tilde{T}_2) < \kappa
$$

which means that $\lim_{t \to \infty} \int_{0}^{t} \left( \sum_{i=1}^{N} \|\xi_i(t)||^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} \|\beta_{ij}(t)\|^2 \right) dt$ exists, based on the Cauchy’s convergence criterion. By assumption 2 and (10), we know that $||\xi_i||$ and $||\dot{\xi}_i||$ are bounded, which indicates that $\int_{\tilde{T}_1}^{\tilde{T}_2} \sum_{i=1}^{N} \|\xi_i(t)||^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} \|\beta_{ij}(t)||^2 dt$ is twice differentiable in each interval $[\tau_i^{k_1}, \tau_i^{k_2})$. Then, we have

$$
\sup_{t \in [\tau_i^{k_1}, \tau_i^{k_2}]} \left( \sum_{i=1}^{N} ||\xi_i^T(t)\dot{\xi}_i(t)|| + \sum_{i=1}^{N} \sum_{j=1}^{N} \|\beta_{ij}(t)\| \dot{\beta}_{ij}(t) \right) \leq +\infty.
$$

The generalized Barbalat’s lemma [23] is applied to the function $\int_{0}^{t} \left( \sum_{i=1}^{N} \|\xi_i(t)||^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} \|\beta_{ij}(t)||^2 \right) dt$. Then, it can be concluded that $\lim_{t \to \infty} \left( \sum_{i=1}^{N} \|\xi_i(t)||^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} \|\beta_{ij}(t)||^2 \right) = 0$. This shows that multiple AUVs can achieve position consensus, i.e., $\beta_{ij} = \eta_i - \eta_j = 0$, $\forall i, j \in \mathcal{V}, v_i = 0, \forall i \in \mathcal{V}$. Based on (23) and (24), the following formula holds:

$$
W \leq -\varepsilon_1 \dot{W}.
$$
Next, the system with DoS attacks is considered, i.e., the system is in the attack areas $A_a(t, t_0)$. Choose the same Lyapunov function as (18). Then, one has

$$
\dot{V} \leq \sum_{i=1}^{N} \xi_i^T \sum_{j=1}^{N} a_{ij} \beta_{ij} + \sum_{i=1}^{N} \xi_i^T J_i (C_i + D_i) J_i^T \xi_i
$$

where $\xi_i = \max \{2(\zeta_{\max} + \lambda_{\max} (J_i (C_i + D_i) J_i^T)) / \lambda_{\min} (H), 2\lambda_{\max} (L) / \zeta_{\min} \min (L) \}$. For $t \in [h_{n-1} + \Delta_n - 1, h_n]$ and $t \in [h_n, h_n + \Delta_n + \Delta_s]$, we define $W(t) = V_a(t)$ and $V(t) = V_b(t)$, respectively. Inspired by [24], we have

$$
V(t) \leq \begin{cases} 
& e^{\epsilon_1 (t-h_{n-1}-\Delta_n-1)} V_a(t) (h_{n-1} + \Delta_n-1) \\
& e^{\epsilon_2 (t-h_n)} V_b(t) (h_n) 
\end{cases}
$$

When $t \in [h_{n-1} + \Delta_n-1, h_n)$, one has

$$
V(t) \leq e^{\epsilon_1 (t-h_{n-1}-\Delta_n-1)} V_a(t) (h_{n-1} + \Delta_n-1) \leq \mu e^{\epsilon_1 (t-h_{n-1}-\Delta_n-1)} V_b(t) (h_n-1 + \Delta_n-1) \leq \ldots \leq \mu^{n+1} e^{\epsilon_1 (\tilde{A}_s, t_0)} e^{\epsilon_2 |\tilde{A}_s, t_0|} V_b(t_0).
$$

When $t \in [h_n, h_n + \Delta_n + \Delta_s]$, one has

$$
V(t) \leq e^{\epsilon_2 (t-h_n)} V_b(t) (h_n) \leq \mu e^{\epsilon_2 (t-h_n)} V_a(t) (h_n) \leq \ldots \leq \mu^{n+1} e^{\epsilon_1 (\tilde{A}_s, t_0)} e^{\epsilon_2 (\tilde{A}_s, t_0)} V_b(t_0).
$$

where $\mu = \max \{\lambda_{\max} (L), \lambda_{\max} (H)\}$.

Based on Definition 1 of attack frequency, we can get the number of the communication areas and the attack areas as $N_a(t, t_0) = n$ and $N_a(t, t_0) = n + 1$, respectively. For $[t_0, t]$, note that $|\tilde{A}_s(t, t_0)| = t - t_0 - |\tilde{A}_s(t, t_0)|$ and $|\tilde{A}_s(t, t_0)| \leq |\tilde{A}_s(t, t_0)| + (1 + N_a(t, t_0)) A_s$. From (30) and (31), one has

$$
V(t) \leq \mu^{n+1} e^{\epsilon_1 (\tilde{A}_s, t_0)} e^{\epsilon_2 (\tilde{A}_s, t_0)} V_b(t_0).
$$

Since $\mathcal{F}_a(t, t_0) = N_a(t, t_0) \leq \frac{\epsilon_1^*}{\ln \mu + (\epsilon_1 + \epsilon_2) \Delta_s}$ and rate

$$
V(t) \leq e^{\epsilon_1 + \epsilon_2) \Delta_s} e^{\epsilon_1 + \epsilon_2 + \epsilon_3} V(t_0) \leq e^{\epsilon_1 + \epsilon_2) \Delta_s} e^{\epsilon_1 + \epsilon_2 + \epsilon_3} V(t_0) \leq e^{\epsilon_1 + \epsilon_2) \Delta_s} e^{\epsilon_1 + \epsilon_2 + \epsilon_3} V(t_0) \leq 0.
$$

From (31), it can be seen from the above formula that multiple AUVs can achieve position consensus, i.e., $\eta_i - \eta_j \to 0$, $\forall i, j \in \mathcal{V}$, $v_j \to 0$, $\forall i \in \mathcal{V}$.

Remark 5 Based on Theorem 1, it can be seen that the upper bound of length rate and frequency of DoS attacks are given. Once the length rate or frequency of the DoS attacks is greater than the upper bound, the control algorithm is invalid, which means that as long as the duration of the DoS attacks is too long, the consensus can be destroyed. This is in line with the actual situation.

Theorem 2 Zeno behavior can be ruled out for each AUV in the communication area $A_s(t, t_0)$ under the proposed event-triggered protocol (9) if the following conditions are satisfied:

$$
t_{k+1}^i - t_k^i \geq \tau_i \geq \frac{1}{\ell} \ln \frac{\ell}{\sqrt{\frac{\sigma_i}{\rho_i}}} \frac{\sqrt{\sigma_i k_i}}{\rho_i} e^{-d_i t} + 1
$$

where $\rho_i = \max \{\rho_i\}, i = 1, 2, \ldots, N$.

Proof: For $t \in A_s(t, t_0)$, Zeno phenomenon can be excluded for any AUV under the triggering mechanism (15). From the Lyapunov function $V$, it can
be obtained that $\|\eta_{i}^{T}, \eta_{j}^{T}, v_{i}^{T}, v_{j}^{T}\| \leq \omega_{i} = \sqrt{\text{max}\lambda(L)\lambda(H)\text{min}\lambda(L)\lambda(H)\omega_{i}^{2}}$, and $\|\eta_{i}^{T}(t_{0})_{i}, \eta_{j}^{T}(t_{0})_{j}, v_{i}^{T}(t_{0})_{i}, v_{j}^{T}(t_{0})_{j}\| \leq \tilde{\omega}_{i}, \tilde{\omega}_{j} > 0$. According to the property of the triangle inequality, then, one has

$$d \|e_{i}\| / dt \leq ||J_{i}^{T}v_{i}, J_{j}^{T}v_{j}, [-M^{-1}C_{j}J_{j}^{T}k_{i}] \quad \leq ||J_{i}^{T}v_{i}, J_{j}^{T}v_{j}, [-M^{-1}C_{j}J_{j}^{T}k_{i}] \quad \leq \ell \|e_{i}\| + \ell \|e_{i}\| (37)$$

where $\sigma_{i} = \ell_{1}\omega_{i} > 0$.

By contradiction, we prove that each AUV can exclude Zeno behavior. Suppose Zeno behavior for the $i^{th}$ AUV exists, we define a finite value $\hat{\xi}_{i} > 0$, which has the following relationship with the triggering time series $t_{i}^{k}$, where $k = (0, 1, \ldots, \infty)$ and $t_{i}^{k} \leq \hat{\xi}_{i}$, then $\lim_{k \rightarrow \infty} t_{i}^{k} = \hat{\xi}_{i}$. Based on the definition of the finite sequence, there is $\kappa_{i} > 0$, so $\Delta_{i} - \kappa_{i} < t_{i}^{k} \leq \hat{\xi}_{i}$ can be established, when $k \geq l_{i}$ and $l_{i} > 0$.

For $t \in [t_{i}^{k}, t_{i}^{k+1})$, based on (37), we have

$$\|e_{i}\| \leq \sigma_{i} \left[ e^{\ell(t-t_{i})} - 1 \right] (38)$$

From the event-triggered function (15), when the measurement error $\|e_{i}\|^{2}$ goes from 0 to $\sigma_{i}^{2} = \sigma_{j}^{2} = \sum_{j=1}^{N} \sigma_{j}^{2}$, the interevent interval can be calculated. Then, based on (38), a lower interevent interval $\tau_{i}$ can be given from the following equation:

$$\frac{\sigma_{i}}{\ell} [e^{\ell\tau_{i}} - 1] = \sqrt{\frac{\sigma_{i}^{2}}{\rho_{i}} k_{i} e^{-\ell t}} (39)$$

where $\rho_{i}$ is the $\hat{\rho}_{i} = \max \{\rho_{i}\}$, $i = (1, 2, \ldots, N)$. According to equation (39), we have

$$\kappa_{i} = \frac{1}{2 \ell_{1}} \ln \left[ \frac{\ell_{1}}{\rho_{i}} \sqrt{\frac{\sigma_{i}}{\rho_{i}}} k_{i} e^{-\ell t_{i}} + 1 \right] (40)$$

where $\kappa_{i} > 0$ for $k \geq l_{i}$. By solution $\tau_{i}$ of the equation (36), it can be obtained that $\tau_{i} \geq 2\kappa_{i}$, which means that $t_{i}^{k+1} \geq t_{i}^{k} + \tau_{i} > \hat{\xi}_{i}$. The above result contradicts hypothesis $\hat{\xi}_{i} - \kappa_{i} < t_{i}^{k+1} \leq \hat{\xi}_{i}$. The above analysis shows that the Zeno behavior does not exist for each AUV.

4 Simulation example

In this section, a simulation example is given to illustrate the effectiveness of the proposed algorithms. The communication topology is shown in Fig. 2. It can be seen from the figure that the topology is connected and contains four AUVs. The parameters of the $i^{th}$ AUV dynamics model are given in Table 1.

The DoS attack sequence is shown in Figs. 3, 4, 5 under the event-triggered control protocol. In the simulation example, the total number of DOS attacks is five. The DOS attacks launch at 7, 18, 30, 48, 58 and the DOS attacks end at 12, 21, 38, 51, 64. The total length of the attacks is 25. The attack duration rate is $rate = 0.36 < \frac{\varepsilon_{1} - \varepsilon_{2}}{\varepsilon_{1} + \varepsilon_{2}} = 0.378$ and the attack fre-

| Table 2: Triggering numbers for AUVs under different triggerig mechanisms |

| AUV | Case A | Case B | Case C |
|-----|--------|--------|--------|
| 1   | 29     | 62     | 116    |
| 2   | 26     | 59     | 108    |
| 3   | 28     | 61     | 112    |
| 4   | 26     | 66     | 121    |

Table 1: Parameters of $i^{th}$ AUV model

| Parameter | Value |
|-----------|-------|
| $m$       | 100kg |
| $F_{i}$   | 1148N |
| $H_{i}$   | 1108N |
| $X_{iu}$  | -120kg/s |
| $X_{iu}$  | -75.4kg |
| $X_{iu}$  | -90kg/m |
| $Y_{iq}$  | -90kg/s |
| $Y_{iq}$  | -40.8kg |
| $Y_{iq}$  | -90kg/m |
| $Z_{iw}$  | -150kg/s |
| $Z_{iw}$  | -40.8kg |
| $Z_{iw}$  | -120kg/m |
Fig. 3 State trajectories of the AUVs in x direction

Fig. 4 State trajectories of the AUVs in y direction

Fig. 5 State trajectories of the AUVs in z direction
Fig. 6 Controller states for AUVs in x direction

Fig. 7 Controller states for AUVs in y direction

Fig. 8 Controller states for AUVs in z direction
Fig. 9  Triggering instants for AUVs in x direction (Case: A)

Fig. 10  Triggering instants for AUVs in x direction (Case: B)

Fig. 11  Triggering instants for AUVs in x direction (Case: C)
frequency is $F(t, I_0) = \frac{e^{\tau_i}}{\ln(\mu + (\varepsilon_1 + \varepsilon_2)\Delta_t)} = 0.0016$, which satisfy the condition of Theorem 1.

The simulation parameters are selected as follows: $\omega_i = 0.6$, $\xi_i = 0.3$, $\zeta_i = 0.5$, $\rho_i = 1$, $\delta_i = 0.5$, $k_i = 0.25$, and $d_i = 0.5$. The initial states are chosen as $\eta_i(0) = [-2, -2, -2]^T$, $\eta_2(0) = [-1, -1, -1]^T$, $\eta_3(0) = [1, 1, 1]^T$, $\eta_4(0) = [2, 2, 2]^T$, $v_1(0) = [-2, -2, -2]^T$, $v_2(0) = [-1, -1, -1]^T$, $v_3(0) = [1, 1, 1]^T$, $v_4(0) = [2, 2, 2]^T$, and $\Theta_i = \left[\frac{\pi}{5}, -\frac{\pi}{10}, \frac{\pi}{12}\right]^T$, where $i = 1, 2, 3, 4$. Figures 3, 4, 5 show that the states of four AUVs are asymptotical consensus in the $x$, $y$, and $z$ directions. The controller states are demonstrated in Figs. 6, 7, 8. The total triggering numbers for AUVs under the proposed control algorithm are shown in Fig. 7.

For the sake of brevity of the paper, we only give the simulation results in the $x$ direction. In order to verify the advantage of the triggering mechanism (15) (case A) proposed in this paper, we have given the simulation results of the event-triggered mechanisms used in [38] (case B) and [31] (case C), respectively, when the same parameters are adopted. In case B, the threshold is composed of state-related variables and a constant value. In case C, the threshold is a constant value. Figure 7 shows the triggering instants under different cases. For the convenience of comparison, Table 2 shows the triggering numbers under different triggering mechanisms. From Table 2, we can see that the event-triggered mechanism proposed in this paper can effectively reduce the triggering numbers.

5 Conclusion

In this paper, we study the secure consensus of multiple AUVs with DoS attacks by adopting event-triggered mechanism method. Based on the DoS attack characteristics adopted in this paper, we know that the attacks are unknown and occur irregularly. Compared with the existing results on the consensus problem of DoS attacks, we propose a novel triggering condition. Under the proposed triggering conditions, the attack frequency and attack duration of DoS attacks are discussed. As long as these conditions are satisfied, the consensus problem can be solved. In future work, we will try to study the consensus of multiple AUVs in the case of directed topology.

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References

1. Lindemann, L., Dimarogonas, D.: Barrier function based collaborative control of multiple robots under signal temporal logic tasks. IEEE Trans. Control. Net. Syst. 7(4), 1916–1928 (2020)
2. Nuno, E., Loria, A., Hernández, T.: Distributed consensus-formation of force-controlled nonholonomic robots with time-varying delays. Automatica 120, 109–114 (2020)
3. Peng, Z., He, L., Huang, C.: Adaptive time-varying formation tracking control of unmanned aerial vehicles with quantized input. ISA Trans. 85, 76–83 (2019)
4. Gu, N., Wang, D., Peng, Z.: Distributed containment maneuvering of uncertain under-actuated unmanned surface vehicles guided by multiple virtual leaders with a formation. Ocean Eng. 187, 105996 (2019)
5. Cui, R., Ge, S., How, R., Choo, Y.: Leaderless and leader-follower cooperative control of multiple marine surface vehicles with unknown dynamics. Nonlinear Dyn. 74(1), 95–106 (2013)
6. Chen, S., Ho, D.: Consensus control for multiple AUVs under imperfect information caused by communication faults. Inf. Sci. 370, 565–577 (2016)
7. He, S., Wang, M., Dai, S.: Leader-follower formation control of USVs with prescribed performance and collision avoidance. IEEE Trans. Ind. Inf. 15(1), 572–581 (2019)
8. Yan, Y., Yu, S.H.: Sliding mode tracking control of autonomous underwater vehicles with the effect of quantization. Ocean Eng. 151, 322–328 (2018)
9. Cui, R., Ge, S., How, R., Choo, Y.: Leader-follower formation control of underactuated autonomous underwater vehicles. Ocean Eng. 37(17), 1491–1502 (2010)
10. Elmokadem, T., Zribi, M., Youcefoumi, M.: Trajectory tracking sliding mode control of underactuated AUVs. Nonlinear Dyn. 84(2), 1079–1091 (2016)
11. Elmokadem, T., Zribi, M., Youcefoumi, M.: Terminal sliding mode control for the trajectory tracking of underactuated Autonomous Underwater Vehicles. Ocean Eng. 219, 613–625 (2017)
12. Joe, H., Kim, M., Yu, S.: Second-order sliding-mode controller for autonomous underwater vehicle in the presence of unknown disturbances. Nonlinear Dyn. 78(1), 183–196 (2014)
13. Li, Y., Wei, C., Wu, Q.: Study of 3 dimension trajectory tracking of underactuated autonomous underwater vehicle. Ocean Eng. 105, 270–274 (2015)
14. Hu, Z., Mab, C., Zhang, L., Halme, A., Hayat, T.: Formation control of impulsive networked autonomous underwater vehicles under fixed and switching topologies. Neurocomputing 147, 291–298 (2015)
15. Yuan, C., Licht, S., He, H.: Formation learning control of multiple autonomous underwater vehicles with heterogeneous dynamics. Ocean Eng. 201, 613–625 (2017)
16. Yin, S., Yang, H., Kaynak, O.: Coordination task triggered formation control algorithm for multiple marine vessels. IEEE Trans. Ind. Electron. 64(6), 4984–4993 (2016)

17. Yi, X., Liu, K., Dimarogonas, D.V., Johansson, K.: Dynamic event-triggered and self-triggered control for multi-agent systems. IEEE Trans. Autom. Control 64(8), 3300–3307 (2019)

18. Zhao, M., Peng, C., He, W., Song, Y.: Event-triggered communication for leader-following consensus of second-Order multiagent systems. IEEE Trans. Cybern. 48(6), 1888–1897 (2018)

19. Li, H., Liao, X., Huang, T., Zhu, W.: Event-triggering sampling based leader-following consensus in second-order multi-agent systems. IEEE Trans. Autom. Control. 60(7), 1998–2003 (2015)

20. Yang, Y., Li, Y., Yue, D., Yue, W.: Adaptive event-triggered consensus control of a class of second-order nonlinear multiagent systems. IEEE Trans. Cybern. 50(12), 5010–5020 (2020)

21. Nair, R., Behera, L., Kumar, S.: Event-triggered finite-time integral sliding mode controller for consensus-based formation of multirobot systems with disturbances. IEEE Trans. Control. Syst. Tech. 27(1), 39–47 (2017)

22. Xu, C., Wu, B.L., Cao, X.B., Zhang, Y.C.: Distributed adaptive event-triggered control for attitude synchronization of multiple spacecraft. Nonlinear Dyn. 95(4), 2625–2638 (2019)

23. Su, Y., Huang, J.: Stability of a class of linear switching systems with applications to two consensus problems. IEEE Trans. Autom. Control. 57(6), 1420–1430 (2012)

24. Jiao, J., Wang, G.: Event triggered trajectory tracking control approach for fully actuated surface vessel. Neurocomputing 182, 267–273 (2016)

25. Jiao, J., Wang, G.: Event driven tracking control algorithm for marine vessel based on backstepping method. Neurocomputing 207, 669–675 (2016)

26. Cetinkaya, A., Ishii, H., Hayakawa, T.: Event-triggered control over unreliable networks subject to jamming attacks. In 54th IEEE conference on decision and control (CDC). IEEE, pp. 4818–4823 (2015)

27. Persis, C.D., Tesi, P.: Input-to-state stabilizing control under denial of-service. IEEE Trans. Autom. Control. 60(11), 2930–2944 (2015)

28. Zhang, D., Liu, L., Feng, G.: Consensus of heterogeneous linear multiagent systems subject to aperiodic sampled-data and DoS attack. IEEE Trans. Cybern. 49(4), 1501–1511 (2018)

29. Zhang, H., Cheng, P., Shi, L., Chen, J.: Optimal Denial-of-Service attack scheduling with energy constraint. IEEE Trans. Autom. Control. 60(11), 3023–3028 (2015)

30. Hu, S., Yue, D., Xie, X.: Resilient event-triggered controller synthesis of networked control systems under periodic DoS jamming attacks. IEEE Trans. Cybern. 49(12), 4271–4281 (2018)

31. Liu, Y.: Secure control of networked switched systems with random dos attacks via event-triggered approach. Int. J. Control. Autom. 18(10), 2572–2579 (2020)

32. Dong, T., Gong, Y.: Leader-following secure consensus for second-order multi-agent systems with nonlinear dynamics and event-triggered control strategy under DoS attack. Neurocomputing 416, 95–102 (2020)

33. Lu, A., Yang, G.: Input-to-state stabilizing control for cyberphysical systems with multiple transmission channels under denial of service. IEEE Trans. Autom. Control. 63(6), 1813–1820 (2017)

34. Lu, A.Y., Yang, G.H.: Distributed consensus control for multi-agent systems under denial-of-service. Inf. Sci. 439, 95–107 (2018)

35. Zhang, D., Feng, G.: A new switched system approach to leader-follower consensus of heterogeneous linear multiagent systems with DoS attack. IEEE Trans. Syst. Man Cybern. 51, 1258–1266 (2021)

36. Xu, Y., Fang, M., Wu, Z.G.: Input-based event-triggering consensus of multiagent systems under Denial-of-Service attacks. IEEE Trans. Syst. Man Cybern. Syst. 50(4), 1455–1464 (2020)

37. Dolk, V.S., Tesi, P., Persis, C.D., Heemels, W.P.M.H.: Event-triggered control systems under denial-of-service attacks. IEEE Trans. Control. Netw. Syst. 4(1), 93–105 (2017)

38. Feng, Z., Hu, G.: Secure cooperative event-triggered control of linear multiagent systems under DoS attacks. IEEE Trans Control Syst. Tech. 28(3), 741–752 (2020)

39. Ma, Y., Che, W., Deng, C.: Observer-based event-triggered containment control for MASs under DoS attacks. IEEE Trans. Cybern. (2021). https://doi.org/10.1109/TCYB.2021.3104178

40. Fossen, T.: Guidance and Control of Ocean Vehicles. Wiley, New York (1994)

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