SymNMF-Net for The Symmetric NMF Problem

Mingjie Li\(^1\), Hao Kong\(^1\), Zhouchen Lin\(^1\)
\(^1\)Peking University

Abstract

Recently, many works have demonstrated that Symmetric Non-negative Matrix Factorization (SymNMF) enjoys a great superiority for various clustering tasks. Although the state-of-the-art algorithms for SymNMF perform well on synthetic data, they cannot consistently obtain satisfactory results with desirable properties and may fail on real-world tasks like clustering. Considering the flexibility and strong representation ability of the neural network, in this paper, we propose a neural network called SymNMF-Net for the Symmetric NMF problem to overcome the shortcomings of traditional optimization algorithms. Each block of SymNMF-Net is a differentiable architecture with an inversion layer, a linear layer and ReLU, which are inspired by a traditional update scheme for SymNMF. We show that the inference of each block corresponds to a single iteration of the optimization. Furthermore, we analyze the constraints of the inversion layer to ensure the output stability of the network to a certain extent. Empirical results on real-world datasets demonstrate the superiority of our SymNMF-Net and confirm the sufficiency of our theoretical analysis.

1 Introductions

Non-negative Matrix Factorization (NMF) plays an important role in feature extraction task due to its powerful linear representation ability (Guillamet and Vitià, 2002; He et al., 2011; Kang et al., 2014; Kuang et al., 2015; Shahnaz et al., 2006; Zhu et al., 2018). The Classical NMF problem can be formulated as the following:

\[
\begin{align*}
\min_{\mathbf{A}, \mathbf{B}} & \quad \frac{1}{2} \| \mathbf{X} - \mathbf{A} \mathbf{B}^\top \|_F^2 \\
\text{s.t.} & \quad \mathbf{A} \geq 0, \mathbf{B} \geq 0
\end{align*}
\]

(1)

where \( \mathbf{X} \) is a data matrix, \( \mathbf{A} \) and \( \mathbf{B} \) are non-negative matrices, \( \mathbf{A} \in \mathbb{R}^{m \times r} \) and \( \mathbf{B} \in \mathbb{R}^{n \times r} \) with \( r \ll \min\{m, n\} \) to optimize the following:

\[
\min_{\mathbf{A} \geq 0, \mathbf{B} \geq 0} \frac{1}{2} \| \mathbf{X} - \mathbf{A} \mathbf{B}^\top \|_F^2.
\]

In some specific tasks such as image or text clustering, \( \mathbf{X} \in \mathbb{R}^{n \times n} \) can be chosen as a symmetric relational matrix (He et al., 2011; Zhu et al., 2018). Accordingly, Eq. (1) can be reformulated as the following:

\[
\min_{\mathbf{A} \in \mathbb{R}^{m \times r}, \mathbf{B} \in \mathbb{R}^{n \times r}} \frac{1}{2} \| \mathbf{X} - \mathbf{A} \mathbf{B}^\top \|_F^2, \quad \text{s.t.} \quad \mathbf{A} \geq 0, \mathbf{B} \geq 0
\]

(2)

where \( \mathbf{U} \mathbf{U}^\top \) corresponds to a symmetric matrix with rank \( r \). Many related works (Ding et al., 2005; He et al., 2011; Kuang et al., 2015) have analyzed the superiority of SymNMF over other methods in solving clustering problems. In particular, Ding et al. (2005) systematically prove that SymNMF is equivalent to the kernel \( k \)-means and Laplacian-based spectral clustering.

In addition to solving Eq. (2) Projected Gradient Descent (PGD) directly, Kuang et al. (2015) introduce an auxiliary variable \( \mathbf{V} \) as another factor matrix, and add \( \frac{\lambda}{2} \| \mathbf{W} - \mathbf{V} \|_F^2 \) to the objective function as a penalty term, formulated as follow:

\[
\begin{align*}
\min_{\mathbf{U} \in \mathbb{R}^{m \times r}, \mathbf{V} \in \mathbb{R}^{n \times r}} & \quad \frac{1}{2} \| \mathbf{X} - \mathbf{U} \mathbf{V}^\top \|_F^2 + \frac{\lambda}{2} \| \mathbf{W} - \mathbf{V} \|_F^2 \\
\text{s.t.} & \quad \mathbf{U} \geq 0, \mathbf{V} \geq 0
\end{align*}
\]

(3)

The most successful algorithms based on the above formulation are the Symmetric Alternative Non-negative Least Square (SymANLS) and the Symmetric Hierarchical Least Square (SymHALS) which are proposed by Zhu et al. (2018).

Although the classical approaches can reach to some local minima quickly, they can randomly approximate the non-negative factorization with respect to the initialization. Some special attributes are not considered in the algorithm which makes it hard to produce the results with desired properties. On this account, the classical algorithms often fail on some real-world tasks such as clustering.

Due to the above defects for the classical algorithm, we deem that the design of a practical factorization algorithm should be data-driven or target-driven. Thus, the factor matrices can extract more effective intrinsic information from data, which may lead to better and more consistent performance. Deep Neural Networks (DNNs) have now achieved great success in many machine learning tasks. On the one hand, back-propagation makes the training process driven by an objective function which is usually designed by specific requirements. On the other hand, training DNN with gradient descent or its variants can ensure the network reach its local minima with proper learning rate even the structure is
non-convex in most cases especially with perturbed gradient descent method (Jin et al., 2017). Inspired by these observations, we want to use a DNN to construct a data-driven architecture for the SymNMF problem. In general, it is quite difficult for DNN to solve many traditional mathematical models directly, such as SymNMF, because the constraints of these problems are hard to be structured as commonly used neural layers or formulated as a part of loss functions.

In this paper, inspired by a straightforward update scheme for problem (3), we propose a new network structure to solve the SymNMF problem. Each block contains three layers sequentially: the inversion layer, linear layer, and ReLU layer. Moreover, in order to ensure the output stability of the inversion layer, we analyze the sufficient lower bound for the penalty parameter λ, which is also a parameter in our network. In summary, our main contributions include:

- We parameterize the optimization iteration of a numerical updating scheme and design SymNMF-Net for solving the SymNMF problem (3). Each block of SymNMF-Net is a differentiable architecture with an inversion layer, a linear layer and ReLU.
- We analyze the lower bound for the learnable parameter λ to ensure the output stability of the inversion layer. We also show that our SymNMF-Net shares some critical points with traditional methods under certain conditions.
- We conduct experiments on numerical comparison and graph clustering to validate the effectiveness of our SymNMF-Net and our theoretical bound for λ.

1.1 Related Work

Differentiable programming, or learning-based optimization, has achieved great success in many tasks. For example, Peng et al. (2018) combine K-means with neural network for clustering. Many works (Chen et al., 2018; Liu et al., 2019; Sprechmann et al., 2015) solve the LASSO problem via a recurrent neural network (RNN). Zhou et al. (2018) combine K-means with neural network for clustering. Also show that our SymNMF-Net shares some critical points with traditional methods under certain conditions. As we can see, the algorithm only involves matrix inversion, multiplication and addition, which means that the forward propagation for a SymNMF-Block is only $O(n^2)$. The details of the forward and backward propagation for our SymNet-Net are listed in the supplementary.

Algorithm 1 Initialization of the SymNMF-Net $\{P_i\}$.

Require: Input a random guess $U_0$ for the factorization, the data matrix $X \in \mathbb{R}^{n \times n}$, the weights of the SymNMF-Net $\{P_i \in \mathbb{R}^{r \times r}\}$, and layer $K$.

1: for $k = 1$ to $K$ do
2: Initialize $P_i = (X + \lambda I_n)U_{i-1}$.
3: Compute $U_i = (X + \lambda I_n)U_{i-1}(U_{i-1}^T + \lambda I_r)^{-1}$.
4: Compute $U_i = \max(U_i, 0)$.
5: end for

Ensure: The initialization of the SymNMF-Net $\{P_i\}$.

Motivated by such insights, we unfold the above algorithm into a neural network by adding an inversion layer into the network. Inputting a factor matrix $U$, the output inversion layer can be formulated as $(U^T U + \lambda I_r)^{-1}$. We multiply the input factor matrix with its transpose and then add a weighted identity matrix $\lambda I$ to circumvent the ill-posedness of $U^T U$. Finally, we use a matrix inversion to get the output of the inversion layer (the left part in Fig. 2.1). Moreover, considering the condition number of the inversion layer:

$$
\text{cond}((U^T U + \lambda I_r)^{-1}) = \frac{\sigma_1(U) + \lambda}{\sigma_2(U) + \lambda} \leq \frac{\sigma_1^2(U)}{\lambda}.
$$

One can see that, a proper $\lambda$ can make the output of the inversion layer stable even if the input matrix $U$ is degenerated. Meanwhile, the spectral norm of $(U^T U + \lambda I_r)^{-1}$ satisfies $\|((U^T U + \lambda I_r)^{-1})\|_2 \leq 1/\lambda$, which enables our inversion layer to regularize the scale of input and output factor matrices of SymNMF-Net explicitly. Furthermore, we make the inversion layer learnable. Moreover, we analyze the lower bound for $\lambda$ (illustrated in Section 3) in order to make the network converge steadily.

Finally, we replace the project operation $\max\{\cdot\}$ onto the nonnegative orthant with a Rectified Linear Unit (ReLU) layer and parameterize the rest of the factors $(X + \lambda I_n)U_{i-1}$ as linear layers with weights $P_k$ respectively. In summary, the construction of the SymNMF-Block is shown in Fig. 2.1.
Just as other differential architectures, parameters $P_i$ needs to be not far from the original parameter in (4). For this account, we use classical algorithms to initial the linear layers shown in Alg. 1. By stacking the SymNMF blocks (Fig. 2.1) and then training the parameter $\{P_i\}$ and $\lambda$ using gradient descent method with the regression loss ($\|X - U_{\text{out}}U_{\text{out}}^T\|_F$), the network can finally output an accurate factorization $U_{\text{out}}$ for the data matrix $(X)$ with a random input $U_0$.

Comparing with the state-of-the-art algorithms of problem (3), our method can directly optimize the original objective function of the NMF problem instead of its relaxed form, which may lead to better results. Since a pair of solutions of problem (2) are not always the solutions for problem (3), and SymNMF is a non-convex problem, the search space of the local optimum sets for SymNMF-Net is much larger than those of traditional algorithms (e.g., SymANLS and SymHALS), which means that we may find better solutions than them on different tasks.

Our method can not only be considered as a data-driven algorithm, but also can be referred as a task-driven method. If the prior knowledge can be formulated on the target matrix, it needs to satisfy.

In our following analysis and experiments, we call the learnable parameter $P_i$ is $\delta$-bounded if the equations (5) are satisfied.

In the remainder of this section, the lower bound of $\lambda$ is provided in order to achieve the above conditions.

3.1 Notations

Before our analysis, we list some important notations. First, we summarize the classical scheme as iterating the following function:

$$
\tilde{U}_i = (X + \lambda I_n)\tilde{U}_{i-1} - \gamma \tilde{U}_{i-1}^T \tilde{U}_{i-1} + \lambda I_r)^{-1}
$$

And we use $\tilde{U}_i$ to represent the output of the classical algorithm. Meanwhile, the outputs of corresponding SymNMF-Net block are denoted as $U_i$. Then we use $T_{U_i}$ to represent the mappings of the classical algorithm for the $i$-th iteration. We use $F_{U}$ to denote all possible mappings of SymNMF-Net layer (or the inference step), which can evolve to correspondingly $T_{U_i}$, respectively. Finally, following the former settings, we use $X$ to denote the input matrix, and let $P_i$ and $\lambda$ to be learnable parameters for the $i$-th SymNMF-Net block.

3.2 Proximity Condition

Definition 1. For any fixed $\tilde{U}_i$, we call that the $i$-th block of SymNMF-Net is $\gamma$-proximal with respect to the classical algorithm $T$ if the following equations hold:

$$
\sup_{U_i \in B(U_i, \epsilon)} \|F_{U_i}(\tilde{U}_i) - T_{U_i}(\tilde{U}_i)\|_F \leq C\gamma,
$$

where $C$ is a constant, and $B(E, \gamma) = \{D|D - E|_2 \leq \gamma\}$.

In the following, we will prove that with certain lower bounds for $\lambda$, the block of SymNMF-Net satisfies the Proximity Condition.

Theorem 1. Suppose that the input matrix $\|X\|_2 = B$, $\max \{\|\tilde{U}_i\|_F, \|U_i\|_F\} \leq a$, and the learnable parameter $P_i$ of each block are $\epsilon$-bounded. If $\lambda$ obeys the following equation:

$$
\lambda > a^2 + 4a\epsilon,
$$

then the block is $\epsilon$-proximal with the following $C$:

$$
C = \frac{4(B + \lambda)a^2}{(\lambda - a^2)^2} + \frac{(B + \lambda)a}{\lambda - a^2}.
$$

Due to limited space, for the proof of this Theorem, please refer to our Supplementary Materials. Note that since we only need to make the output stable, $\epsilon$ here need not to be too small. Moreover, empirical observations show that a small $\epsilon$ is not good for training the network, because the network will perform almost equivalent to the original update scheme (4).
3.3 Sufficiency Conditions

The objective function for the traditional algorithm is:

\[ f_1(W, V) = \frac{1}{2} \|X - WV^\top\|_F^2 + \frac{\lambda}{2} \|W - V\|_F^2 + \delta_+(W) + \delta_+(V), \]

(6)

where \(\delta_+(\cdot)\) denotes a delta function in order to penalize the negative elements. Here, \(\delta_+(x)\) is equal to 0 if \(x \geq 0\), and +\(\infty\) otherwise. Eq. (6) is the relaxed objective function for SymNMF. But in our network, we use the original objective function as the loss function:

\[ f_2(U) = \frac{1}{4} \|X - UU^\top\|_F^2 + \delta_+(U). \]

(7)

The minimizers of Eq. (7) are not always the solutions of Eq. (6) for different \(\lambda\). Fortunately, Zhu et al. (2018) give a lemma to prove that the solutions of the algorithms for problem (3) are the solutions of ours with proper \(\lambda\). Combining with Theorem 1, we can obtain the following theorem to ensure the proximality and sufficiency conditions simultaneously.

**Proposition 1.** Suppose that the input matrix \(\|X\|_2 = B\), \(\max_i \{\|\tilde{U}_i\|_F, \|U_i\|_F\} \leq a\), and the parameters \(P_i\) of SymNMF-Net are -\(e\)-bounded. If \(\lambda\) satisfies the following inequality:

\[ \lambda > \max \{a^2 + 4ae, \frac{1}{2}(\|X\|_F + \|X - U_0U_0^\top\|_F)\} \]

then SymNMF-Net satisfies the proximality and sufficiency conditions simultaneously:

In brief, if \(\lambda\) stays in the above region stated in Prop. 1, the stability of the SymNMF-Net’s output is ensured. With the theoretical results of the perturbed gradient descent method Jin et al. (2017), our network will finally converge to the local minima of the SymNMF problem.

4 Numerical Experiments

In this section, we conduct empirical results to verify the numerical convergence of SymNMF-Net, the sufficiency of lower bound for \(\lambda\), and the superior performance of the factorization on image and text clustering tasks.

SymNMF can be used for graph clustering by factorizing the similarity matrix \(X\) of the data (Kuang et al., 2015). In this part, we use the graph clustering methods on different image datasets and text datasets to evaluate the performance of SymANLS, SymHALS and SymNMF-Net.

**Loss Functions:** According to the conclusions by Xie et al. (2019) that the learnable parameters would better lie near the original optimization algorithm, we add regularizers for \(P\) to the loss function:

\[ \text{Loss} = \sum_{i=1}^{L} (\|X - U_i U_i^\top\|_F^2 + \beta \|P_i - (X + \lambda I_n)U_{i-1}\|_F^2), \]

where \(L\) represents the layer number of SymNMF-Net.

**Table 1:** The final relative error for different models on ORL and COIL-20.

| Model       | ORL  | COIL-20 |
|-------------|------|---------|
| SymANLS     | 0.2669 | 0.8405 |
| SymHALS     | 0.2723 | 0.8501 |
| SymNMF-Net  | 0.2659 | 0.8390 |

4.1 Numerical Analysis

**Convergence Evaluation** Firstly, we conduct experiments on ORL and COIL datasets to compare the numerical error among SymANLS, SymHALS, and our SymNMF-Net. We evaluate the performance with the relative error of the output \(U\) and the input matrix \(X\):

\[ \text{(Relative Error)} \quad E = \frac{\|X - UU^\top\|_F^2}{\|X\|_F^2}. \]

We use the similarity matrix of COIL-20 and ORL dataset to compare the convergence property of different models (shown in Figure 4.1). Our SymNMF-Net achieves superior performance than SymHALS and the convergence speed is comparable with SymANLS on ORL dataset. Meanwhile, our model converges much faster than both SymANLS and SymHALS on COIL-20. Furthermore, the relative error of our model is better than others (Table 1). In general, the empirical results confirmed that our model can achieve better performance than classical algorithms under numerical evaluation.

**Sparse output with \(\ell_1\) regularizer** Since the SymNMF problem is a non-convex problem, there are a variety of local minima with different characteristics like sparsity. Never-
theless, classical algorithms are hard to involve certain constraints. Consequently, they are difficult to obtain results with demanding attributes. In contrast, our SymNMF-Net can obtain different results with demanding properties by modifying the loss functions.

![Figure 4.2: SymNMF-Net with different $\lambda$ on COIL-20 dataset.](image)

For instance, our SymNMF-Net can generate more sparse factor matrix than the classical products by adding $\ell_1$ norm of the outputs to the objective function. We compare our model with the state-of-the-art algorithms on COIL-20, ORL and evaluate the sparse factor (CF) of the outputs, which is a widely used indicator to measure the sparsity. The formula of SF is illustrated as follow:

$$SF(x) := \frac{\# \{i, |x_i| > T\}}{\# \{x\}}$$

where $T$ is the sparse threshold which we set to be $0.01 \ast \text{mean}(x)$. All the methods produce the results with similar relative error but different in sparseness shown in Figure 4.3. As we expected, the decomposition from our model is the most sparse one among the three methods, which claimed our motivation for constructing an architecture which can handle non-negative matrix factorization with different expectations on the output.

**The effect of $\lambda$** To study the effects of the parameter $\lambda$ and validate the analyses in Section 3, we show the relative error versus iteration for different $\lambda$ on COIL-20 in Figure 4.2. The original setting is that we initialize the learnable parameter $\lambda$ with $\|X\|_F (\approx 11.2)$, and ensure it obeys our lower bound during the training process by a projection. For the other cases, we fix the $\lambda$ as listed in the legends in the course of the training process. As we can see from Figure 4.2, the network does not converge if $\lambda$ is small, which validates the sufficiency of the conditions in Theorem 1 that $\lambda$ should have a lower bound.

### 4.2 Image Clustering

In the following two sections, we use the SymNMF based graph clustering methods on datasets to evaluate the performance of the numerical algorithms and our SymNMF-Net.

**SymNMF for Clustering:** Putting all the images or texts to be clustered into a data matrix $M$ whose rows are vectorized images or texts, we can construct the similarity matrix $X$ following the procedures in previous works (Kuang et al., 2015; Zhu et al., 2018). After running our SymNMF-Net or traditional algorithms on the similarity matrix $X$, the non-negative approximation $U$ can be obtained, then the label of the $i$-th sample can be obtained by:

$$l(M_i) = \arg \max_j \tilde{U}_{ij}.$$ 

The experiments are conducted on the following image datasets for face, instances or digits:

**ORL:** The dataset has 400 gray-scale images for 40 distinct persons, the size of which is $92 \times 112$. Each person has ten different photos taken from different angles and emotions.\(^3\)

**COIL-20:** It consists of 1,440 images of 20 objects. Each instance is a $128 \times 128$ gray-scale image.\(^4\)

**MNIST:** MNIST is a classical handwritten digits dataset containing 60,000 $32 \times 32$ gray-scale images for training and 10,000 images for testing. Following the setting of the previous work (Zhu et al., 2018), we use 3,147 images in the training set for 10 classes (denoted as MNIST$_1$) and 3,147 images in the test set which belong to 3 digits (denoted as MNIST$_2$).\(^5\)

Comparing the clustering accuracy (shown in Table 2 and Table 3) we can see that, our SymNMF-Net performs much better than all other state-of-the-art graph clustering methods on different evaluation methods for clustering.

Furthermore, the performances of SymNMF-Net comparing with SymHALS and SymANLS on MNIST$_1$ and other datasets (shown in Table 2) we can see that, our SymNMF-Net performs much better than all other state-of-the-art graph clustering methods on different evaluation methods for clustering.

| Method       | ORL   | COIL-20 | MNIST$_1$ | MNIST$_2$ |
|--------------|-------|---------|-----------|-----------|
| SymANLS      | 80.75 | 79.79   | 64.77     | 85.89     |
| SymHALS      | 75.70 | 58.54   | 66.57     | 86.08     |
| ADMM         | 76.50 | 69.03   | 58.03     | 87.13     |
| SymNewton    | 76.25 | 74.72   | 59.90     | 85.89     |
| PGD          | 77.00 | 72.43   | 64.75     | 87.10     |
| SymNMF-Net   | 81.50 | 80.13   | 70.64     | 98.98     |

Table 2: Summary of image clustering accuracy (%) for different algorithms on different image datasets.

---

3\[^3^\]http://www.cl.cam.ac.uk/research/dtg/attarchive/facedatabase.html

4\[^4^\]http://www.cs.columbia.edu/CAVE/software/softlib/coil-20.php

5\[^5^\]http://yann.lecun.com/exdb/mnist/
MNIST2 are shown in Figures 4.4 and 4.5, respectively. From these two figures, one can see that our SymNMF-Net beat them with a large margin on both datasets in terms of both metrics cluster purity and NMI. Note that the inference of our SymNMF-Net can be regarded as an alternating algorithm for SymNMF (3) with 5 iterations, therefore we draw the curves for the clustering purity and NMI versus iterations for SymNMF-Net, SymANLS and SymHALS, which are shown in Figure 4.4 and Figure 4.5. As can be seen that, the alternating algorithm represented by our network enjoys a huge advantages for speed and performance on clustering.

In addition, from the curves of clustering purity and NMI w.r.t. training iterations in Figure 4.5 we can see that, our SymNMF-Net is still able to overperform SymANLS and SymHALS for clustering purity and NMI even with less iterations.

### 4.3 Text Clustering

In addition to image clustering, we also finish the experiments on text data.

**TDT-2:** The TDT-2 corpus contains data collected during the first half of 1998 from newswires, radio programs and television programs. We use its largest 30 categories for the experiments, which contain 9,394 documents in total. 6

As can be seen from Table 4, no matter using clustering accuracy, purity, or NMI as metrics for evaluation, the performance of SymNMF-Net is much better than other comparative methods on both TDT-2.

Moreover, there is an important phenomenon from the two sections that, SymANLS performs better than SymHALS on image clustering whereas worse than SymHALS on text clustering. This mainly because these two methods are not sta-

---

### Table 3: Summary of image clustering Purity and Normalized Mutual Information (NMI) for SymNMF-Net (ours), SymANLS and SymHALS on ORL and COIL.

| Method    | ORL NMI | ORL Purity | COIL-20 NMI | COIL-20 Purity |
|-----------|---------|------------|-------------|---------------|
| SymANLS   | 0.884   | 0.805      | 0.849       | 0.804         |
| SymHALS   | 0.862   | 0.763      | 0.664       | 0.538         |
| SymNMF-Net| 0.904   | 0.835      | 0.896       | 0.853         |

---

### Table 4: Summary of graph clustering accuracy (%), normalized mutual information(NMI), and clustering purity of SymNMF-Net (ours), SymANLS, and SymHALS on TDT-2.

| Method    | ACC | NMI | Purity |
|-----------|-----|-----|--------|
| SymANLS   | 75.36 | 0.746 | 0.794 |
| SymHALS   | 81.31 | 0.755 | 0.833 |
| SymNMF-Net| 82.34 | 0.807 | 0.859 |

---

### Figure 4.4: The curves of (a) clustering purity and (b) NMI with respect to iterations/blocks of SymNMF-Net, SymANLS and SymHALS on MNIST1.

---

### Figure 4.5: The curves of (a) clustering purity and (b) NMI with respect to training or running iterations of SymNMF-Net, SymANLS and SymHALS on MNIST2.

---

### 5 Conclusion

In this paper, we propose a deep-learning-based optimization method called SymNMF-Net for solving the SymNMF problem. In particular, each block of our network contains an inversion layer, a linear layer and ReLU. Moreover, we analyze the conditions which can ensure the output stability of each inversion layer. We further show that our SymNMF-Net shares some critical points with traditional methods under our conditions. Besides, the empirical results demonstrate that our model finally evolves to a better data-driven algorithm which converges much faster than the traditional optimization algorithms. Experiments also show that, comparing with other state-of-the-art SymNMF methods on graph clustering, our model consistently leads to a better performance. In this paper, we have proposed an inversion in neural network and designed our SymNMF-Net, a differentiable architecture for SymNMF problem. Experiments have been done to show our model will leads to consistently better performance on model convergence and clustering. Besides, the empirical results demonstrate that our model finally evolves to a better data-driven algorithm which converges much faster than traditional optimization problem.
References
Xiaohan Chen, Jialin Liu, Zhangyang Wang, and Wotao Yin. Theoretical linear convergence of unfolded ISTA and its practical weights and thresholds. In *Advances in Neural Information Processing Systems*, pages 9061–9071, 2018.
G Di Pillo and L Grippo. Exact penalty functions in constrained optimization. *SIAM Journal on Control and Optimization*, 27(6):1333–1360, 1989.
G Di Pillo. Exact penalty methods. *Algorithms for Continuous Optimization*, 434:209–253, 1994.
Chris Ding, Xiaofeng He, and Horst D Simon. On the equivalence of nonnegative matrix factorization and spectral clustering. In *Proceedings of the 2005 SIAM International Conference on Data Mining*, pages 606–610. SIAM, 2005.
Radu-Alexandru Dragoni, Jérôme Bolte, and Alexandre d’Aspremont. Fast gradient methods for symmetric nonnegative matrix factorization. *ArXiv preprint ArXiv:1901.10791*, 2019.
David Guillamet and Jordi Vitrià. Non-negative matrix factorization for face recognition. In *Catalonian Conference on Artificial Intelligence*, pages 336–344. Springer, 2002.
Zhaoshui He, Shengli Xie, Rafal Zdunek, Guoxu Zhou, and Andrzej Cichocki. Symmetric nonnegative matrix factorization: Algorithms and applications to probabilistic clustering. *IEEE Transactions on Neural Networks*, 22(12):2117–2131, 2011.
Chi Jin, Rong Ge, Praneeth Netrapalli, Sham M Kakade, and Michael I Jordan. How to escape saddle points efficiently. In *Proceedings of the 34th International Conference on Machine Learning-Volume 70*, pages 1724–1732. JMLR.org, 2017.
Tae Gyoon Kang, Kisoo Kwon, Jong Won Shin, and Nam Soo Kim. NMF-based target source separation using deep neural network. *IEEE Signal Processing Letters*, 22(2):229–233, 2014.
Diederik P Kingma and Jimmy Ba. Adam: A method for stochastic optimization. In *International Conference on Learning Representations*, 2015.
Da Kuang, Sangwoon Yun, and Haesun Park. SymNMF: nonnegative low-rank approximation of a similarity matrix for graph clustering. *Journal of Global Optimization*, 62(3):545–574, 2015.
Jialu Liu, Chi Wang, Jing Gao, and Jiawei Han. Multi-view clustering via joint nonnegative matrix factorization. In *Proceedings of the 2013 SIAM International Conference on Data Mining*, pages 252–260. SIAM, 2013.
Jialin Liu, Xiaohan Chen, Zhangyang Wang, and Wotao Yin. ALISTA: Analytic weights are as good as learned weights in LISTA. In *International Conference on Learning Representations*, 2019.
Songtao Lu, Mingyi Hong, and Zhengdao Wang. A nonconvex splitting method for symmetric nonnegative matrix factorization: Convergence analysis and optimality. *IEEE Transactions on Signal Processing*, 65(12):3120–3135, 2017.
Adam Paszke, Sam Gross, Francisco Massa, Adam Lerer, James Bradbury, Gregory Chanan, Trevor Killeen, Zeming Lin, Natalia Gimelshein, Luca Antiga, et al. Pytorch: An imperative style, high-performance deep learning library. In *Advances in Neural Information Processing Systems*, pages 8024–8035. Curran Associates, Inc., 2019.
Xi Peng, Ivor W Tsang, Joey Tianyi Zhou, and Hongyuan Zhu. k-meansnet: When k-means meets differentiable programming. *ArXiv preprint ArXiv:1808.07292*, 2018.
Farial Shahbazi, Michael W Berry, V Paul Pauca, and Robert J Plemmons. Document clustering using nonnegative matrix factorization. *Information Processing and Management*, 42(2):373–386, 2006.
Pablo Sprechmann, Alexander M Bronstein, and Guillermo Sapiro. Learning efficient sparse and low rank models. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 37(9):1821–1833, 2015.
Jian Sun, Huibin Li, Zongben Xu, et al. Deep ADMM-Net for compressive sensing mri. In *Advances in Neural Information Processing Systems*, pages 10–18, 2016.
Xingyu Xie, Jianlong Wu, Zhisheng Zhong, Guangcan Liu, and Zhouchen Lin. Differentiable Linearized ADMM. In *Thirty-sixth International Conference on Machine Learning (ICML)*, 2019.
Joey Tianyi Zhou, Kai Di, Jiawei Du, Xi Peng, Hao Yang, Sinno Jialin Pan, Ivor W Tsang, Yong Liu, Zheng Qin, and Rick Siow Mong Goh. Sc2net: Sparse LSTMs for sparse coding. In *Thirty-Second AAAI Conference on Artificial Intelligence*, 2018.
Zhihui Zhu, Xiao Li, Kai Liu, and Qiwei Li. Dropping symmetry for fast symmetric nonnegative matrix factorization. In *Advances in Neural Information Processing Systems*, pages 5154–5164, 2018.