Search for the Radiative Leptonic Decay $B^+ \rightarrow \gamma \ell^+ \nu_\ell$

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We present the results of a search for $B^+ \rightarrow \gamma \ell^+ \nu_\ell$, where $\ell = e, \mu$. We use a sample of 232 million $B \bar{B}$ pairs recorded at the $\Upsilon(4S)$ with the BABAR detector at the PEP-II $B$ Factory. We measure a partial branching fraction $\Delta B$ in a restricted region of phase space that reduces the effect of theoretical uncertainties, requiring the lepton energy to be between 1.875 and 2.850 GeV, the photon energy to be between 0.45 and 2.35 GeV, and the cosine of the angle between the lepton and photon momenta to be less than $-0.36$, with all quantities computed in the $\Upsilon(4S)$ center-of-mass frame. We find $\Delta B(B^+ \rightarrow \gamma \ell^+ \nu_\ell) = (-0.3^{+1.5}_{-1.3} (stat) \pm 0.6 (syst) \pm 0.1 (th)) \times 10^{-6}$, assuming lepton universality. Interpreted as a 90% C.L. Bayesian upper limit, the result corresponds to $1.7 \times 10^{-6}$ for a prior flat in amplitude, and $2.3 \times 10^{-6}$ for a prior flat in branching fraction.

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At tree level, the branching fraction (BF) for radiative leptonic $B$ decays is given by:

$$B(B^+ \rightarrow \gamma \ell^+ \nu_\ell) = \frac{G_F^2 |V_{ub}|^2}{288 \pi^2} f_B^2 \tau_B m_B^5 \left( \frac{Q_\ell - Q_b}{m_B - m_b} \right)^2,$$

(1)

where $m_B$ is the $B^+$ meson mass, $m_b$ is the $\Upsilon(2S)$ $b$ quark mass, $\tau_B$ is the $B^+$ meson lifetime, $f_B$ is the $B$ meson decay constant, $Q_\ell$ is the charge of quark flavor $i$ and $\lambda_B$ is the first inverse moment of the $B$ light-cone distribution amplitude [1, 2], a quantity that enters into theoretical calculations [3] of the BF of hadronic $B$ decays such as $B \rightarrow \pi \pi$, and is typically taken to be of the order of $\Lambda_{QCD}$. Thus, a measurement of $B(B^+ \rightarrow \gamma \ell^+ \nu_\ell)$ can provide a determination of $\lambda_B$ free of hadronic final-state uncertainties. The best current 90% C.L. upper bound on the full BF is $5.2 \times 10^{-5}$ [4], for $B^+ \rightarrow \gamma \mu^+ \mu^- \nu_\mu$.

However, Eq.(1) is based on the assumption that the factorization relation for the vector and axial-vector form factors is valid over the entire phase space. Instead, one can relate, at tree-level, $\lambda_B$ to a partial BF, $\Delta B$, over a restricted region of phase space [4, 5]:

$$\Delta B = \frac{G_F^2 |V_{ub}|^2}{32 \pi^4} f_B^2 \tau_B m_B^5 \left[ a + b L + c L^2 \right],$$

(2)

where $L = (m_B/3)(1/\lambda_B + 1/(2m_b))$, the first term describes the effects of photon radiation from the lepton, the third term the internal photon emission, and second their interference. The constants $a$, $b$, and $c$ can be predicted model-independently using factorization at large photon energy, the kinematic region for our analysis.

We present herein the results of a search for charged $B$ meson decays $B^+ \rightarrow \gamma \ell^+ \nu_\ell$, where $\ell = e, \mu$ (“electron channel”, “muon channel”) [6]. Our measurements are based on a sample of 232 million $B \bar{B}$ pairs recorded with the BaBar detector [6] at the PEP-II asymmetric-energy $e^+e^-$ storage rings, comprising an integrated luminosity of 210.5 fb$^{-1}$ collected at the $\Upsilon(4S)$ resonance (“on-peak”). We also use 21.6 fb$^{-1}$ recorded approximately 40 MeV below the $\Upsilon(4S)$ (“off-peak”).

The analysis procedure consists of selecting a lepton and photon recoiling against a reconstructed $B$, and identifying signal candidates by reconstructing the neutrino using missing energy and momentum. We use a variety of selection criteria, optimized using Monte Carlo (MC) samples, to discriminate signal from background. We then extract the number of signal events in data using a binned maximum-likelihood (ML) fit.

The backgrounds are divided into three categories: continuum (non-$B \bar{B}$), specific exclusive $b \rightarrow u \ell \nu_\ell$ decays, and “generic $B^+$” decays, defined as a combination of all $B$ hadronic decays, $b \rightarrow c \ell \nu_\ell$ decays, and the remaining inclusive $b \rightarrow u \ell \nu_\ell$ decays. In particular, we study the seven exclusive $b \rightarrow u \ell \nu_\ell$ modes: $B^+ \rightarrow h^0 \ell^+ \nu_\ell$ ($h^0 = \pi^0, \rho^0, \eta, \eta', \omega$) and $B^0 \rightarrow h^- \ell^+ \nu_\ell$ ($h^- = \pi^-, \rho^-$), referred to below, for each $h$, as the “$h$ mode”.

Our signal MC samples were generated using the tree-level model of Ref. [1]. The $\pi^0$ and $\pi^\pm$ mode samples were generated using the form factor parameterization of Ref. [6], with the value of the shape parameter based on lattice QCD results [9]. Light cone sum rule-based form factor models were used to generate samples for the $\rho^0$, $\rho^\pm$, and $\omega$ modes [10], and $\eta$ and $\eta'$ modes [11].

We find an excess of events in the off-peak data compared to continuum MC ($e^+e^- \rightarrow q\bar{q}, \tau^+\tau^-$, and in the muon channel, $\mu^+\mu^-\gamma$), with the excess more pronounced in the electron channel. This is likely to result from unmodeled higher-order QED and hadronic two-photon events. We thus use off-peak data instead of continuum MC to represent continuum background in our analysis.

We take as the signal lepton and photon the highest center-of-mass (CM) energy electron (muon) and the highest CM energy photon candidate in each event. The remaining charged tracks, each assigned a pion mass, and neutral clusters, treated as photons, are assigned to the “recoil $B$” candidate. We reconstruct the recoil $B$ in two ways: we construct an “unscaled” recoil momentum as the sum of the CM 3-momenta of its constituents, and we define a “scaled” recoil momentum in the direction of the unscaled recoil, with its magnitude determined from lattice QCD results [9]. Using either the scaled or unscaled momentum, we reconstruct the 3-momentum of a corresponding scaled or unscaled signal neutrino candidate. The reconstructed neutrino CM energy is calculated as the difference between the CM beam energy and the sum of the lepton and photon candidates’ CM energies.

We optimize a set of selection criteria for the best signal sensitivity at a significance of 3σ using MC samples,
splitting each in half, with one sample used for the optimization and the other used to evaluate its performance.

On the signal side, we require that the electron (muon) have a CM energy between 2.00 and 2.85 (1.875 and 2.775) GeV. We require that the photon have a CM energy between 0.65 and 2.35 (0.45 and 2.35) GeV. We define \( \cos \theta_{\ell\gamma} \) to be the cosine of the angle between the lepton and photon in the CM frame, and require its value to be less than \(-0.42 \) (\(-0.36\)). We require 
\[-1.10 < \cos \theta_{\ell\gamma} < 1.10(1.00) \]
where \( \cos \theta_{\ell\gamma} \) is the cosine of the angle between the signal \( B \) and the lepton-photon combination \( Y \) in the CM frame \([12]\), computed from the known \( B \) mass, the beam energy, and the 3-momenta of the signal lepton and photon.

In order to reduce background from neutral hadrons, we require the lateral moment \([13]\) of the electromagnetic calorimeter energy distribution of the signal photon candidate to be less than 0.55 for both channels. The polar angle of the photon candidate in the laboratory frame is required to be between 0.326 and 2.443 rad for both channels. We pair the candidate with every other neutral constituent and the CM beam energy to be between 0.85 (1.00) and unscaled neutrino polar angle in the laboratory frame.

To reduce continuum background, we require the ratio of the second to zeroth Fox-Wolfram moment \([14]\) of all charged tracks and neutral clusters to be less than 0.5, and the absolute value of the cosine of the angle between the CM thrust axes of the recoil \( B \) and the lepton-photon system be less than 0.98 (0.86). We use a Fisher discriminant, \( F \equiv a_2 L_0 + a_3 L_2 \), calculated from the momentum-weighted zeroth and second Legendre moments, \( L_0 \) and \( L_2 \), of the recoil \( B \) about the lepton-photon CM thrust axis, with coefficients \( a_2 \) and \( a_3 \) equal to 0.43 and \(-1.86 \) (0.008 and \(-1.590\)), respectively. \( F \) is required to be greater than 1.50 (0.310).

In the electron channel, we veto two-photon events via the charge-angle correlation of the signal lepton arising from the initial state. For a positively (negatively) charged signal electron, we require the cosine of its CM polar angle to be between \(-0.74 \) and 0.78 (\(-0.94 \) and 0.70). In the muon channel, we require this variable to be between \(-1.00 \) and 0.78 for both charges. These criteria were optimized on a loosely-selected sample of events, where the off-peak data are used for the continuum, and the MC for the signal and other backgrounds.

We also reject two-photon events using a parameterized combination of the missing CM momentum in the beam direction and the invariant mass of the hypothetical two-photon system. For the muon channel, the entire observed event is taken as the two-photon system, while for the electron channel, the signal electron is assumed to be from the initial state, and so is excluded from the two-photon system. The selection criteria were adjusted to preserve a 94% efficiency for signal for both channels.

After applying our selection criteria, we use the two-dimensional distribution of \( \Delta E_P \), the difference between the scaled neutrino candidate’s CM energy and the magnitude of its 3-momentum, and \( m_{ES} \), the invariant mass of the recoil \( B \), calculated from its unscaled CM 3-momentum and the CM beam energy, as inputs to the ML fit. These distributions provide distinct signatures for signal, \( B \) background, and continuum, with the signal distribution shown in Fig. 1. The signal (S) and three sideband (B1, B2, B3) regions were selected to maximize separation of signal from \( B \) background and continuum background.

We extract signal events by fitting on-peak data for the contributions of signal and background, while allowing the predicted shapes of signal and background to vary within statistical uncertainties. The scale of signal and generic \( B \) contributions are allowed to vary, while the scale of off-peak data is fixed using the on-peak/off-peak luminosity ratio. For the seven semileptonic (SL) modes, we fit for three of the BF modes and relate the other four to them as follows: The \( \pi^\pm \) and \( \rho^\pm \) mode BFs are obtained from BABAR measurements \([12]\), and the \( \eta \) mode BF is obtained from CLEO \([12]\). The charged and neutral \( \pi \) and \( \rho \) modes are related by the lifetime ratio, \( \tau_{B^\pm} / \tau_{B^0} = 1.071 \pm 0.009 \) \([16]\), and an isospin factor of 2. The \( \omega \) mode BF is taken as equal to the \( \rho^0 \) mode BF. We take the ratio of the \( \eta \) to \( \eta' \) mode BFs to be 2.057 \pm 0.020 \([17]\).

We maximize a likelihood function consisting of the product of four Poisson probability distribution functions (PDFs), modeling the total counts in each of the four regions, three Gaussian PDFs for the BFs of the three SL modes, and 40 Poisson PDFs for the 4-region shapes of the various samples. All of the shapes are obtained from MC, except for continuum, where off-peak data are used, introducing a larger statistical uncertainty.

Each Poisson PDF that models the total count in one

\[ \Delta E_P \text{ vs. } m_{ES} \text{ signal MC, using a color scale to represent relative contents of each bin.} \]
of the four fit regions has a measured value obtained from the on-peak data count, and an expected value based on the fitted contributions of signal and background, including fitted variations of the shapes. For the seven SL modes (where three of the fitted BFs are independent), the variances of the three Gaussian likelihoods are obtained from the published statistical and experimental systematic uncertainties, combined in quadrature. In all, there are 47 PDFs, and 45 free parameters.

We fit for the partial BF $\Delta B$ for the kinematic region with lepton CM energy between 1.875 and 2.850 GeV, photon CM energy between 0.45 and 2.35 GeV, and $\cos \theta_F$, less than $-0.36$ — the union of the electron and muon channel regions. We perform three fits: separate electron and muon channel fits, and a joint fit in which the signal and three SL BFs are constrained to be equal for the two channels. For each fit, errors on the fitted signal BF are obtained by finding the two values at which the signal BF likelihood decreased by a factor of $e^{-1/2}$.

Table I shows the results from the joint fit.

| TABLE I: Comparison of fit results and experimental observations for the joint fit to the muon and electron channels. For each of the four fit regions, the individual fitted contributions of continuum (cont.), $B\bar{B}$ background, and signal are shown, along with their total. The on-peak and off-peak (scaled to the integrated on-peak luminosity) observations are shown for comparison, in indented rows, and are not included in the “Total fit” value shown. |
|-----------------------------|\hline
|                            | Muon channel |                |                |
|                            | S            | B1            | B2            |
| Fit cont.                  | 20.0±11.8    | 116.3±14.7    | 42.6±12.8     |
| Off-peak                   | 23.0±16.2    | 158.1±40.8    | 17.4±12.3     |
| Fit $B\bar{B}$             | 50.1±8.5     | 61.0±9.9      | 9.8±28.6      |
| Fit signal                 | $-5.2±13.8$  | $-1.3±3.4$    | $-0.4±1.0$    |
| Total fit                  | 74.0±8.1     | 176.0±12.4    | 103.9±9.8     |
| On-peak                    | 73.0±8.5     | 170.0±13.0    | 111.0±10.5    |
|                            | 498.0±22.3   | 500.0±22.1    |                |
|                            | 176.0±12.4   | 111.0±10.5    | 498.0±22.3    |
|                            | 500.0±22.1   |                |                |
|                            |                |                |                |
|                            | Electron channel |                |
|                            | S            | B1            | B2            |
| Fit cont.                  | 55.4±20.5    | 181.1±16.2    | 48.9±14.1     |
| Off-peak                   | 41.4±20.7    | 239.7±48.9    | 79.0±27.9     |
| Fit $B\bar{B}$             | 69.2±8.5     | 59.2±8.5      | 140.1±15.5    |
| Fit signal                 | $-8.4±22.3$  | $-1.5±3.9$    | $-1.2±3.3$    |
| Total fit                  | 116.2±10.3   | 238.7±14.5    | 187.7±12.5    |
| On-peak                    | 119.0±10.9   | 231.0±15.2    | 176.0±13.3    |
|                            | 764.0±27.6   |                |                |

Table II shows all systematic uncertainties on $\Delta B$ except for theoretical uncertainties on the signal model, which are shown in Table III.

The theoretical uncertainty within the kinematic region of $\Delta B$ is conservatively estimated by evaluating the change in efficiency when the model of Ref. 1 is modified by setting the axial vector form factor equal to zero.

The results for $\Delta B$ are given in Tables III and IV. We determine 90% C.L. Bayesian upper limits by integrating the signal BF likelihood with two different priors, both of which take values of 0 for negative values of the signal BF: a prior flat in the BF (“flat BF prior”), and a prior flat in the square root of the BF (“flat amplitude prior”), equivalent to assuming a flat prior for $|V_{ub}|$ or $f_B$.

For our kinematic region, the constants $a$, $b$, and $c$ of Eq.(2) are 0.88, $-3.24$, and 3.25, respectively. Using input values of $f_B = 216 \text{ MeV}$ 15, $|V_{ub}| = 4.31 \times 10^{-3}$ 16, $\tau_B = 1.638 \text{ ps}$ 10, and $m_b = 4.20 \text{ GeV}$ 10,
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TABLE III: Comparison of $\Delta B$ two-sided results for all three fits. All values have been multiplied by $10^6$.

|        | Central values $\times 10^6$ | Statistical uncertainty | Systematic uncertainty | Theoretical uncertainty |
|--------|-----------------------------|-------------------------|------------------------|-------------------------|
| Muon   | $-1.33$                     | +1.74                   | +0.80                  | 0.03                    |
| Electron | $0.11$                       | +1.73                   | +0.61                  | 0.08                    |
| Joint  | $-0.25$                     | +1.33                   | +0.60                  | 0.07                    |

TABLE IV: The 90% Bayesian upper-limits for all three fits, for the two different choices of prior, in terms of $\Delta B$.

|                   | Prior flat in amplitude | Prior flat in BF |
|-------------------|-------------------------|------------------|
| Muon              | $< 1.5 \times 10^{-6}$  | $< 2.1 \times 10^{-6}$ |
| Electron          | $< 2.2 \times 10^{-6}$  | $< 2.8 \times 10^{-6}$ |
| Joint             | $< 1.7 \times 10^{-6}$  | $< 2.3 \times 10^{-6}$ |

our 90% C.L. Bayesian limits on $\Delta B$ correspond to values of $\lambda_B$ of $> 669$ MeV and $> 591$ MeV, for the choice of the flat amplitude and flat BF priors, respectively.

Given a theoretical model, a measurement of $\Delta B$ may be converted into an estimate of the total BF. In the model of Ref. [1], the result of the joint fit corresponds to a BF of $(-0.6^{+3.0}_{-3.1}(\text{stat})^{+2.5}_{-1.2}(\text{syst})) \times 10^{-6}$, and 90% C.L. Bayesian upper limits of $3.8 \times 10^{-6}$ and $5.0 \times 10^{-6}$ for the flat amplitude and flat BF priors, respectively.

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[1] G. P. Korchemsky, D. Pirjol, and T. M. Yan, Phys. Rev. D 61, 114510 (2000).

[2] S. Descotes-Genon and C. T. Sachrajda, Nucl. Phys. B 650, 356 (2003).

[3] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999).

[4] T. E. Browder et al. [CLEO Collaboration], Phys. Rev. D 56, 11 (1997).

[5] The calculation of the partial BF and its dependence on $\lambda_B$, based on Ref. [1], was performed by D. Pirjol.

[6] Charge-conjugate modes are included implicitly.

[7] B. Aubert et al. [BABAR Collaboration], Nucl. Instrum. Meth. A 479, 1 (2002).

[8] D. Becirevic and A. B. Kaidalov, Phys. Lett. B 478, 417 (2000).

[9] M. Okamoto et al., Nucl. Phys. Proc. Suppl. 140, 461 (2005).

[10] P. Ball and R. Zwicky, Phys. Rev. D 71, 014029 (2005).

[11] P. Ball and R. Zwicky, Phys. Rev. D 71, 014015 (2005).

[12] B. Aubert et al. [BABAR Collaboration], Phys. Rev. D 72, 051102 (2005).

[13] A. Drescher et al., Nucl. Instrum. Meth. A 237, 464 (1985).

[14] G. C. Fox and S. Wolfram, Phys. Rev. Lett. 41, 1581 (1978).

[15] S. B. Athar et al. [CLEO Collaboration], Phys. Rev. D 68, 072003 (2003).

[16] W. M. Yao et al. [Particle Data Group], J. Phys. G 33, 1 (2006).

[17] C. S. Kim and Y. D. Yang, Phys. Rev. D 65, 017501 (2002). The uncertainty on this factor was determined in consultation with the authors.

[18] A. Gray et al. [HPQCD Collaboration], Phys. Rev. Lett. 95, 212001 (2005).