Abstract—Sliding mode observer and event triggering mechanism co-design is the subject of this paper. The synthesis of a sliding mode observer under any event triggering mechanism is given, where the proposed observer can be viewed as a special structure of the Kalman filter. Then, the event triggering condition is updated according to the coverage index value computed with the help of the reachability set representation. A numerical example illustrates the effectiveness of the proposed approach.

Keywords—sliding mode observer; event-based scheduling; co-design; estimation performance; communication design optimization

I. INTRODUCTION

Many estimator design methods have been developed under the assumption that the measurement vector is periodically available. However, sensors may not feed-back the estimator periodically, due to several reasons such as limited network bandwidth and high energy consumption of the radio unit for wireless sensors. In this case, the classic periodic sampling can be replaced by an event-based technique, which insures acceptable estimation performance with limited network resources. Achieving a desired balance between network state estimator quality and sensor to estimator communication rate has led to the use of robust observer methods and in this context sliding mode observer is a very popular tool. The sliding mode observers are proposed with the idea to drive the dynamics of a system to an sliding manifold, that is an integral manifold with finite reaching time [1]. The first sliding mode has been discussed in [2-4]. In addition, some SMO have attractive properties similar to those of the Kalman filter (i.e. noise resilience) [5], but with a simpler implementation [6]. Sometimes this design can be performed by applying an equivalent control method [7, 8], allowing the proposal of robust to noise observers, since the equivalent control is slightly affected by noisy measurements. A key feature in the Utkin observer [9] is the introduction of a switching function in the observer to achieve a sliding mode and also stable error dynamics. For better utilization of shared communication and processing resources or reduction of hardware costs we deal with event-based designs. The area of event-based control and estimation has substantially grown during the last decades [10-12]. In [13], an event-triggered approach was used to trigger the data transmission from a sensor to a remote observer. In [14], a sensor data scheduling problem was considered and a feedback policy to choose the transmission times which provides a trade-off between the communication rate and the estimation error was used. The objective in [15] was the development of predictive triggering mechanisms for event-based state estimation.

The main contribution of this paper is the introduction of an event-based technique and a communication design optimization with a corresponding estimation by adopting a sliding mode from which we obtain high estimation quality with low energy consumption by minimizing the number of sensors.

II. PROBLEM FORMULATION

The event-based sliding mode state estimation process is illustrated in Figure 1.

Consider initially a nominal linear system as described in (1):
framework of the EB state estimation problem is presented.
we call nonlinear EB state estimation or EB state estimation
optimal state estimate of EB sensor data scheduler (3), which
the knowledge of the quantities
feedback communication from the estimator is not needed as
objective is to obtain an estimate of the state
the output error is fed back via a discontinuous switched signal.
error between the observer and the system in a linear fashion,
of the sliding mode observer, instead of feeding back the output

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the measured outputs is independent.
without loss of generality, we

Let \( E_t \triangleq y_t - y_t \), where \( T_t \) denotes the previous time
instant when the measurement of sensor is transmitted defined
the measured outputs is independent.
without loss of generality, we

\[
T_t = \begin{cases} 
T^-_t & \text{if } y_t = 0 \\
T^+_t & \text{if } y_t = 1 , \text{ and } T_0^- = 0
\end{cases} \tag{2}
\]

where \( T^-_t \) denotes the left hand limit of \( T_t \) at the instant \( t \).
The main advantage of using the send-on-delta conditions is that
feedback communication from the estimator is not needed as
the conditions only depend on the sensor measurements. This
fact reduces hardware and energy consumption cost and facilitates
the implantation of event-triggering schemes.
the objective is to obtain an estimate of the state \( x \) based on only
the knowledge of the quantities \( y \) and \( u \). In the simplest form
of the sliding mode observer, instead of feeding back the output
error between the observer and the system in a linear fashion,
the output error is fed back via a discontinuous switched signal.
Furthermore, we consider the following EB sensor data scheduler:

\[
y_t = \begin{cases} 
0, & \text{if } |e_t|_\infty \leq \xi \\
1, & \text{otherwise}
\end{cases} \tag{3}
\]

where \( \xi \leq \infty \) is a predefined threshold and \( e_t \) the output error.

The objective of this article is to find or approximate an
optimal state estimate of EB sensor data scheduler (3), which
we call nonlinear EB state estimation or EB state estimation
problem interchangeably. In the next section, a sliding mode
framework of the EB state estimation problem is presented.

III. EVENT BASED SLIDING MODE OBSERVER

A. Similarity Transformation

Consider the transformation \( x = T_c^{-1}x_1 \) associated with the
invertible matrix:

\[
T_c = \begin{bmatrix} N & T_c^T \end{bmatrix}
\]

where \( N_c \in \mathbb{R}^{n_x \times (n_x - n_y)} \) is a full rank column matrix
composed with the null space basis of \( C \). By substitution we
obtain:

\[
\dot{x}_1(t) = \bar{A}x_1(t) + \bar{B}u(t) \quad \dot{y}_1(t) = \bar{C}x_1(t)
\]

where:
\[
\bar{A} = T_c A T_c^{-1}, \bar{B} = T_c B, \text{ and } \bar{C} = C T_c^{-1} = \begin{bmatrix} 0 & I_p \end{bmatrix}
\]

An observer for (1) was proposed as follows [9]:

\[
\dot{\hat{x}}_1 = A\hat{x}_1 + Bu + G_nv \quad \dot{\hat{y}}_1 = C\hat{x}_1
\]

where \((\hat{x}, \hat{y})\) are the estimates of \( x, y \) and \( v \) is a discontinuous
injection term. \( e_1 = \hat{x}_1 - x_1 \) and \( \bar{e}_y(t) = \bar{y}_1 - \bar{y}_2 \) are defined
as the state estimation and output estimation errors respectively. The term \( v \) is defined component-wise as:

\[
v_i = p\text{sign}(\bar{e}_{y, i}), i = 1, \ldots, p \tag{8}
\]

where \( p \) is a positive scalar and \( \bar{e}_{y, i} \) represents the \( i \)-th component
of \( \bar{e}_y \). The term \( v \) is designed to be discontinuous with respect to
the sliding surface \( S = \{x \in \mathbb{R}^n : Cx = 0\} \) to force the trajectories
of \( \epsilon(t) \) into \( S \) in finite time. Assuming, without loss of
energy, that the system is already in the coordinate associated with (6), then the gain \( G_n \) has the following structure:

\[
G_n = \begin{bmatrix} L & -I_p \end{bmatrix}
\]

where \( L \in \mathbb{R}^{(n_x - p) \times p} \) represents the design freedom, following
the definition of \( \epsilon(t) \) and (1). The error system is given by:

\[
\dot{\epsilon}_i(t) = A\epsilon_i(t) + G_nv_i \tag{10}
\]

From the structure of the output distribution matrix \( C \) in (6)
the state estimation error can be partitioned as \( e = \text{col} \{e_1, \bar{e}_y\} \), where \( e_1 \in \mathbb{R}^{n_x-p} \). Consequently the error system from (10) can be written in the form:

\[
\dot{e}_{1i}(t) = A_{21}e_{1i}(t) + A_{12}\bar{e}_y(t) + Lv \tag{11}
\]

\[
\dot{\bar{e}}_y(t) = A_{21}e_{1i}(t) + A_{12}e_{y}(t) - v \tag{12}
\]

Furthermore, (12) can be written component-wise as:

\[
\dot{e}_{y,i}(t) = A_{21i}e_{1i}(t) + A_{22i}\bar{e}_y(t) - p\text{sign}(\bar{e}_{y,i}) \tag{13}
\]

where \( A_{21i} \) and \( A_{22i} \) represent the \( i \)-th rows of \( A_{21} \) and \( A_{22} \)
respectively. To develop the conditions under which the sliding
will take place, the reachability condition will be tested.

Notice that \( e_{y,i} = y_i - \bar{y}_i \) and \( \bar{e}_{y,i} = y_{i1} - \bar{y}_i \). Then
\( e_{y,i} - \bar{e}_{y,i} = y_i - y_{i1} \), and from (3) we get:

\[
|e_{y,i}| - \xi \leq |\bar{e}_{y,i}|
\]

(14)

From (13) and (14) we obtain:

\[
e_{y,i}\dot{e}_{y,i} = e_{y,i}(A_{21i}e_{1i} + A_{22i}\bar{e}_y(t)) - p|\bar{e}_{y,i}| \tag{15}
\]

\[
\leq e_{y,i}(A_{21i}e_{1i} + A_{22i}\bar{e}_y(t)) - p|\bar{e}_{y,i}| + \rho_k \tag{16}
\]

\[
<-|\bar{e}_{y,i}|(p - [(A_{21i}e_{1i} + A_{22i}\bar{e}_y(t)) - \rho_k]|/|\bar{e}_{y,i}|)\tag{17}
\]

Provided that the scalar \( p \) is chosen such (large enough)
that:

\[
p > [(A_{21i}e_{1i} + A_{22i}\bar{e}_y(t)) - \rho_k]/|\bar{e}_{y,i}| + n \tag{18}
\]

where the scalar \( n \in \mathbb{R}^+ \). Then:

\[
e_{y,i}\dot{e}_{y,i} < -n|e_{y,i}| \tag{19}
\]

This is the eta-reachability condition and implies that \( e_{y,i} \)
will converge to zero in finite time. When every component of


and the network bandwidth use as a constrained optimization threshold network utilization. Below we consider the choice of the trade-off between communication rate and estimation quality.

During sliding mode \( e_y(t) = \hat{e}_y(t) = 0 \), and the error system defined by (11) and (12) can be written in a collapsed form as:

\[
\dot{e}_1(t) = (A_{11}e_1(t) + Lv_{eq}) \quad (20)
\]

\[
0 = (A_{21}e_1(t) + v_{eq}) \quad (21)
\]

where \( v_{eq} \) is the so-called equivalent output error injection that is required to maintain the sliding motion. This is the natural analogue of the equivalent control. Substituting \( v_{eq} \) from (18) and (19) yields the following expression for the reduced-order sliding motion:

\[
\dot{e}_1(t) = (A_{11} + LA_{21})e_2(t) \quad (22)
\]

This represents the reduced-order motion of order \( n - p \) that governs the sliding mode dynamics. It can be shown that if \( (A, C) \) is observable then \( (A_{11}A_{21}) \) is also observable, and a matrix \( L \) can always be chosen to ensure that the reduced-order motion in (20) is stable.

### B. Communication Rate and Threshold Design

The average sensor communication rate is defined as:

\[
\gamma = \limsup_{T \to +\infty} \left( \frac{1}{T+1} \int_{t=0}^{T} y_i(t) \, dt \right) \quad (23)
\]

The average rate \( \gamma \) gives a general description of the network utilization. Below we consider the choice of the threshold \( \zeta \) issue, given in (3), in order to achieve a desirable trade-off between communication rate and estimation quality.

### C. Communication Design Optimization

In this section, the tradeoff between the estimation quality and the network bandwidth use as a constrained optimization problem is described. For this aim, the following cost functions \( f_1 \) and \( f_2 \) are defined:

\[
f_1(\zeta) = \gamma \quad (24)
\]

This function is related to number of the transmitted sensor measurement packets.

\[
f_2(\zeta) = \lim_{T \to +\infty} \sqrt{\frac{1}{T+1} \sum_{t=0}^{T} (e_y(t))^2} \, dt \quad (25)
\]

where \( f_2 \) is an estimation error RMS performance index.

Finally, both functions can be used to formulate a general performance index as following:

\[
f(\zeta) = Qf_1(\zeta) + Rf_2(\zeta) \quad (26)
\]

where, \( Q \) and \( R \) are weighting matrices used to penalize the number of transmitted packets and the estimation quality respectively. Thus the co-design problem can be formulated as follows:

\[
\zeta^* = \arg\min f(\zeta) \quad (27)
\]

### IV. Simulation results

In this section, the effectiveness of the proposed co-design approach is illustrated with a numerical example. For simulation purposes, let us consider a nominal linear system described by (1) and modeled by:

\[
A = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1, 1] \quad (28)
\]

which represents a simple harmonic oscillator. For simplicity, we assume \( u(t) = 0, D = 0, \) and \( \rho = 1, Q = 5, R = 1. \) The system’s initial state is \( x_0 = [0.5; -0.8]^T \), and the observer has zero initial condition. Figure 2 shows the simulation diagram using Matlab/Simulink version R15.a. using a variable step ode45, Dormand Prince solver with a processor Intel(R) Core(TM) i5-8250U CPU.

**Fig. 2.** Simulation diagram

In Table I the simulation results for different arbitrary values of \( \zeta \) are summarized along with the optimal one for giving priority to the minimisation of energy consumption.

**TABLE I.** OPTIMIZATION OF ENERGY CONSUMPTION

| Case no | Different values of \( \zeta \) with priority to energy minimization | Number of transmissions | Objective function |
|---------|-------------------------------------------------|-------------------------|-------------------|
| 1       | 0.1, 0.0801                                      | 115                     | 0.0806            |
| 2       | 0.50, 0.1059                                     | 22                      | 0.1060            |
| 3       | 1.50, 0.3139                                     | 3                       | 0.3139            |
| Optimal | 0.9409, 0.2291                                   | 9                       | 0.2586            |

Figures 3-6 show the system output \( y_i \), the transmitted output \( y_{\text{transmitted}} \) and the output estimation \( y_{\text{estimation}} \) for different values of \( \zeta \). Figure 6 is the optimal case. Excellent tracking of the output occurs after approximately 1.3s and the output estimation error \( e_y \) after approximately 1.0s becomes zero, and the transmitted outputs are minimized. This is indicative of a high estimation quality with minimization of energy consumption. If we give priority to estimation rather than energy minimization, then \( Q = 1, R = 5. \) Table II exhibits the results of simulation for different values of \( \zeta \).
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Fig. 3. 1st case: The output $y_t$, the transmitted output $y_{t,k}$, the output estimation error $e_y$, and the output estimation $y_{estim}$

Fig. 4. 2nd case: The output $y_t$, the transmitted output $y_{t,k}$, the output estimation error $e_y$, and the output estimation $y_{estim}$

Fig. 5. 3rd case: The output $y_t$, the transmitted output $y_{t,k}$, the output estimation error $e_y$, and the output estimation $y_{estim}$

Fig. 6. Optimal case: The output $y_t$, the transmitted output $y_{t,k}$, the output estimation error $e_y$, and the output estimation $y_{estim}$

Figures 7-10 plot the system output $y_t$, the transmitted output $y_{t,k}$, and the output estimation $y_{estim}$ for different values of $\zeta$. Figure 10 shows the optimal case which clearly demonstrates a high estimation quality always with minimization of the transmitted output $y_{t,k}$.

| Case no | Different values of $\zeta$ with priority to estimation | rms($e_y$) | Number of transmissions | Objective function |
|---------|--------------------------------------------------------|------------|-------------------------|--------------------|
| 1       | 0.20                                                   | 0.0801     | 57                      | 0.4064             |
| 2       | 1.00                                                   | 0.2585     | 10                      | 1.2926             |
| 3       | 1.50                                                   | 0.3139     | 3                       | 1.5693             |
| Optimal | 0.70                                                   | 0.1564     | 14                      | 0.7821             |

TABLE II. OPTIMIZATION OF ESTIMATION QUALITY
Fig. 10. Optimal case: The output $y_{t}$, the transmitted output $y_{tx}$, the output estimation error $e_{y}$, and the output estimation $y_{est}$.

V. CONCLUSIONS

Because of the verified benefits in terms of reduced energy consumption and better resource utilization compared to traditional designs, event-triggered estimation methods can be widely adopted in industrial applications. For instance, in achieving optimal state estimation over a shared network we use a sliding mode observer associated with an event triggering mechanism. It was shown that by the introduction of an event based technique and the communication design optimization with a corresponding estimator by adopting sliding mode, reduction of the estimation error and of the sensor transmission can be achieved. The simulation results illustrate the effectiveness of the proposed sliding mode observer in terms of decreased estimation error and reduced sensor transmitted information.

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