Governor: a Reference Generator for Nonlinear Model Predictive Control in Legged Robots

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Abstract—Model Predictive Control (MPC) approaches are widely used in robotics, since they allow to compute updated trajectories while the robot is moving. They generally require heuristic references for the tracking terms and proper tuning of parameters of the cost function in order to obtain good performance. When for example, a legged robot has to react to disturbances from the environment (e.g., recover after a push) or track a certain goal with statically unstable gaits, the effectiveness of the algorithm can degrade. In this work we propose a novel optimization-based Reference Generator, named Governor, which exploits a Linear Inverted Pendulum model to compute reference trajectories for the Center of Mass, while taking into account the possible under-actuation of a gait (e.g. in a trot). The obtained trajectories are used as references for the cost function of the Nonlinear MPC presented in our previous work [1]. We also present a formulation that can guarantee a certain response time to reach a goal, without the need to tune the weights of the cost terms. In addition, foothold locations are corrected to drive the robot towards the goal. We demonstrate the effectiveness of our approach both in simulations and experiments in different scenarios with the Aliengo robot.

I. INTRODUCTION

A. Related Work

Legged robots are becoming popular nowadays, thanks to their ability to operate on irregular and complex terrains. Early developed methods involve the use of heuristic approaches, e.g. [2]. They demonstrated good performance and succeeded in hardware experiments, but are tailored to specific motions and scenarios. More recently, Trajectory Optimization (TO) techniques [3, 4, 5] were introduced, since constraints and cost functions can ensure dynamic feasibility and desired performance. In particular, Model Predictive Control (MPC) approaches can compensate for uncertainties, disturbances and changes in the environment, by computing a new trajectory online, while the robot moves. In our previous work [1] we presented a Nonlinear Model Predictive Control (NMPC) formulation, which runs at 25 Hz and allows the Hydraulically actuated Quadruped (HyQ) [6] robot to perform omni-directional motions and detect a pallet and step on it with improved leg mobility. Minniti et al. [7] integrated Control Lyapunov Functions into their MPC to guarantee stability when the robot has to interact with unknown objects. Hong et al. [8] presented an NMPC implementation with a set of different gaits, while Amatucci et al. [9] exploited a Monte Carlo Tree Search to optimize also for the contact schedule. Both works, however, have only been tested in simulation. Jeon et al. [10] presented a MPC landing controller, exploiting a trained neural network that outputs initial guesses for trajectories. High frequency re-planning allows the robot to comply with external pushes [11], but it makes difficult to track a specific reference Cartesian position or to return, after a push, to the original one. In fact, the MPC implementation is “transparent”, i.e. the new trajectory starts from the actual position, therefore an additional effort is required to drive the robot back to the initial path. A common practice is to have user-defined values as a reference for the MPC [12], but in these approaches the user would have to manually change the reference velocity in order to compensate for the deviation due to a push. Another possible solution would be to add in parallel to the MPC a simple Cartesian Proportional-Derivative (PD) control action to attract the Center of Mass (CoM) to the reference position [13]. However, this control strategy has some notable limitations, in particular: 1) the added wrench does not consider the wrench produced by the MPC layer (hence, there will be a fight between the two controllers with the risk to violate feasibility), 2) the PD is not aware of the hybrid dynamics of the legged robot, i.e. the intermittent contacts and the (possible) under-actuation. Moreover, foothold locations play an important role, since their coherence with the CoM can improve the stability when the robots has to “react” to an external disturbance. The third drawback for PD approaches, thus, is that they cannot change feet trajectories, which is crucial to deal with lateral pushes. For example, Barasuol et al. [14] presented a Push Recovery module which modifies the footholds to counteract the disturbance and track the position with the PD. As explained, the drawback of this approach is that the computed locations of the feet give no guarantee that
the resulting wrench can be generated by the robot. Another possibility is to track a fixed goal in the cost function. This would solve the issue number 1 of the PD controller, because the constant position is embedded in the cost function of the MPC, hence the “conflict” will be dealt with at the cost level resulting in feasible contact forces. However, this solution suffers from the fact that it is able to apply only a limited resistive force before the legs lose their control authority (e.g. when the CoM goes out of the support polygon), and therefore is of restricted applicability. Also in this case, footholds are not generally designed to be consistent with the resulting CoM trajectory, since they are computed with simple heuristics.

Cebe et al. [15] optimize for both quantities, i.e., the velocities and the GRFs that are sent to the robot, so it requires more computational time required. In addition, the MPCs usually employ references for contact forces that come from crude heuristic computations (i.e., dividing the gravity weight along the legs based on static conditions). These values (i.e., purely vertical forces) are often not feasible with respect to the motion of the base. Undoubtedly, accurately tuning of the weights for the different cost terms is a tedious task and having more physically meaningful references can be a preferable solution [16]. To address this problem, Bjelonic et al. [17] use the result of an offline TO as cost terms for their MPC.

In this work, we propose a novel optimization-based reference generator layer (named Governor) that supplies a NMPC with a reference CoM trajectories and Ground Reaction Forces (GRFs) that are suitable to the task of the robot. The novelty of our approach is that the trajectories are computed online (differently from [17]), solving an optimization problem that takes into account the future robot behaviour, and the intermittent contact schedule to generate the references. In particular, we are interested in considering the intrinsic under-actuation of a trot gait when only two feet are in contact with the ground, even though the scheme can be generalized to different gaits. Moreover, the optimization structure of the Governor allows us to impose additional features, such as a desired time in a scenario in which the robot has to reach a fixed goal or recover its position automatically from big pushes. Another advantage of our algorithm is that it is able to affect joint CoM trajectories and footholds, even though the latter are not directly included in the optimization variables.

A possible drawback of such cascade optimization setting is the computational effort, which can result in a reduction of the NMPC frequency. Aiming to find a compromise between accuracy and computational efficiency, we employ models of different complexity using the simplest in the Governor (Linear Inverted Pendulum (LIP) [18]) and a more complex one (the Single Rigid Bidy Dynamics (SRBD) [19]) for the NMPC. The latter, in fact, computes the state trajectories and GRFs that are then sent to the robot, so it requires more accuracy. Thanks to the optimal references, we can avoid the use of a full dynamics model as in [20].

B. Proposed Approach and Contribution

In this work we introduce the Governor, an optimization-based reference generator which endows the cost function of a NMPC [1] with physically informed trajectories to be tracked. We integrated the Governor with our previous works, Fig 1. To summarize, the contributions of the paper are:

- the presentation of a novel Governor that generates reference CoM trajectories to drive the robot to accomplish a task (eventually in a user-defined time interval), taking into account the under-actuation of the statically unstable gaits, like the trot. Foothold locations are heuristically computed to be coherent with the CoM motion, and optimized GRFs are obtained in order to follow those trajectories. The formulation is lightweight enough to maintain the re-planning frequency of 25 Hz of the NMPC.
- simulations and experiments to demonstrate the effectiveness of the proposed approach in three different scenarios: a) straight motion, b) fixed lateral goal and c) recovery after a push. We also compared in simulation our algorithm with a state-of-the-art approach (NMPC + PD action) for the scenario (c).
- as an additional minor contribution we extended the approach of [1] to different dynamic gaits (i.e. trot and pace) and to a different robot platform (Aliengo of Unitree) 1

C. Outline

The paper is organized as follows: Sec. II gives an overview of our planning framework, highlighting the main features of the Governor. Sec. III describes the optimization problem with the LIP model and how it is used to compute CoM position velocity and GRFs references. Simulations and experiments with our Aliengo robot are illustrated in Sec. IV. Finally, we draw the conclusions in Sec. V.

II. LOCOMOTION FRAMEWORK DESCRIPTION

A. Governor algorithm

Figure 2 describes the scheme of our entire locomotion framework. The user decides the leg sequence and the values of both linear $v_{lim}^m \in \mathbb{R}^2$ and heading $\omega_{lim}^m \in \mathbb{R}$ velocities, that the robot should follow. The velocities can be changed during the motion and the NMPC will immediately react accordingly. In this work we use two modules already presented in [1]: the gait scheduler and the robocentric stepping (also used in [21]).

Given the user-defined leg sequence, the gait scheduler returns the gait status (either swing or stance) $\delta_{N,k} \in \mathbb{R}$ of each leg $i$ and for each time instant $k$ in a horizon $N_{g}$, re-conciliating it with the real condition of the robot, e.g. early or late touchdown. Horizon $N_{g}$ corresponds to the maximum response time of the Governor. Note that $N_{g}$ used in the Governor can be different from the one $N$ used in the NMPC, with $N \geq N_{g}$. We use the symbol $\delta \in \mathbb{R}^{4 \times N_{g}}$ to refer to the entire sequence of gait status. The robocentric stepping module, instead, has the important task to compute foothold locations for each leg over the entire horizon. Again, we use the symbol $p_{f} \in \mathbb{R}^{12 \times N_{g}}$ to denote these quantities for the whole horizon. The key idea of robocentric stepping is that the touch-down points are computed with respect to the hip position and they are offset, with respect to that, depending on the CoM.

1https://www.unitree.com/products/aliengo/
reference velocity. Applying the robocentric stepping with the velocities computed by the Governor allows us to obtain the coherence between CoM and footholds. The variables $p_t$ and $\delta$ are also used as parameters in the SRBD model of the NMPC. If the error between the goal and average CoM position is lower than a threshold (see Sec. II-B), the Governor does not perform optimizations and user-defined velocities (heuristic references) are used as references also for the NMPC. We refer to this condition as inactive Governor. Instead, when the Governor is active, the sequences $p_t$ and $\delta$ are the input parameters to a first stage optimization that employs the LIP model (Sec. III-A). Here we compute the optimal X-Y CoM trajectory $p_{\text{e,ref}}^c$, $v_{\text{e,ref}}^c$ to reach the goal $p_{\text{c,goal}}^c$ where dynamic stability is satisfied (i.e., ZMP always inside the support polygon). However, since the foothold locations are computed the first time with the simple heuristic approach, we improve them by updating the velocity used by the robocentric stepping. This will result in a new set of footholds that will be used as inputs for a second optimization. This process iterates until a stop condition is reached, e.g. maximum number of iterations or difference between two consecutive solutions below the defined threshold.

The CoM velocity trajectory $v_{\text{e,ref}}^c$ computed by the LIP model optimization is then used as velocity reference for the NMPC. A Quadratic Program (QP)-based mapping (Sec. III-B) computes the reference for the GRFs $u_{\text{e,ref}}^c$ for the NMPC. Finally, the output of the NMPC are the CoM trajectories (position, orientation, linear and angular velocities) $x_{\text{e,ref}}^c \in \mathbb{R}^{12 \times (N + 1)}$ and GRFs $u_{\text{e,ref}}^c \in \mathbb{R}^{12 \times N}$ (prediction horizon $N = 2 \text{s}$, 50 knots) that will be sent to the Controller block. The Controller is composed of a 250 Hz Whole-Body Control (WBC) [22] and a 1 kHz PD-Joint controller. They generate the torques references $\tau_{\text{e,ref}}^c$ for the low level controller of the robot. The State Estimator module [23] provides the actual values $x_{\text{e,act}}^c$ of the robot at a frequency of 300 Hz.

B. Goal Setting and activation of the Governor

As already explained in Sec. I, the robot cannot follow a user-defined velocity in case of non-idealities or pushes. For this reason, we define the goal $p_{\text{e,goal}}^c$ as the position that the robot would have reached if it had followed the user commanded velocities $v_{\text{e,usr}}^c \in \mathbb{R}^2$. The goal is updated to provide recent values at each iteration of the NMPC. It is initialized with $p_{\text{e,act}}^c + N_s v_{\text{e,usr}}^c T_s$ at the beginning of an experiment and at each iteration it is incremented by $v_{\text{e,usr}}^c T_s$. Variable $T_s$ is the Governor sampling time and it is equal to $1/f_s = 40 \text{ ms}$ where $f_s = 25 \text{ Hz}$ is the planning loop frequency. As an alternative, the user can decide a fixed goal, either for one coordinate or both. To activate the Governor in a meaningful way (e.g., no activation with the normal sway of the robot) we consider the average $p_c$ of the X-Y CoM position during the last gait cycle. To the average position, we add the offset due to the desired motion in the horizon $N_g$ and compare it with the goal. We compute the error as: $e = p_{\text{c,goal}}^c - \left( \bar{p}_c + \int_0^{N_g T_s} v_{\text{e,usr}}^c dt \right)$. When its norm $||e||$ goes beyond a threshold the Governor is said active and optimization blocks are executed. The Algorithm I illustrates the pseudo-code of the different computation phases of the Governor.

C. Formal guarantees on response time

In an optimization problem, if we set the tracking of the goal as a running cost (i.e., for all the samples) rather than a terminal cost, the response will depend on the tuning of the weights of the cost itself. In addition, with this approach, we cannot impose a predefined time $T_f$ in which the robot reaches the goal (response interval)\(^3\). For these two reasons a\(^3\)

\(^3\)the case of a fixed goal corresponds to a scenario in which $v_{\text{e,usr}}^c = 0$.\(^4\)

\(^4\)For the sake of simplicity, we assume that the sampling time $T_s$ of the NMPC and the Governor are the same.

\(^5\)The size of the horizon $N_s$ will be linked to the response time $T_f$ of the Governor, related the real-time constraint posed by the re-planning frequency $f_s$. This will depend on the computational power of the machine where the optimization is run.
Algorithm 1 Governor

1: Governor ← INACTIVE  
2: e ← \( p_{c}^{\text{goal}} - p_{c} - \left( \int_{0}^{N_{k} T_{s}} v_{\text{usr}}^{T} \, dt \right) \)  
3: if \( \|e\| > \text{tol} \) then  
4: Governor ACTIVE ← INACTIVE  
5: end if  
6: \( p_{f}, \delta \leftarrow \text{heuristic references} \)  
7: if Governor ACTIVE then  
8: while stop condition is not reached do  
9: \( v_{g} \leftarrow \text{LIP Model} \)  
10: \( p_{f} \leftarrow \text{update considering } v_{g} \)  
11: end while  
12: \( u_{\text{ref}} \leftarrow \text{QP Mapping} \)  
13: else  
14: \( v_{\text{ref}} \leftarrow v_{\text{usr}} \)  
15: \( u_{\text{ref}} \leftarrow \text{gravity mapping} \)  
16: end if  
17: \( p_{c}^{\text{goal}} \leftarrow p_{c}^{\text{goal}} + v_{\text{usr}} T_{s} \)  
18: NMPC

A. LIP Model

One of the features of the Governor is to take into account the intrinsic under-actuated nature of a robot (e.g., while trotting). Therefore, we employ the simplest model that is able to capture this under-actuation: the LIP model [18]. Indeed, this allows to compute the optimal trajectory for the CoM, while imposing a desired behavior for the ZMP. The ZMP is defined as "a point on the ground at which the tangential component of the moment generated by the ground reaction force/moment becomes zero" [25], and its position determines the direction and magnitude of the CoM acceleration. Guaranteeing that the GRFs are such that the resulting ZMP is inside the support polygon ensures that they satisfy also the unilateral constraints [26] (the legs can only push and not pull the ground); therefore they can be effectively realized by a real robot. Since during a two leg stance phase the support polygon boils down to be a line connecting the stance feet, the ZMP will be able to move only on that segment. We are aware that the LIP model presents some assumptions (vertical and angular dynamics are neglected), but as already mentioned, our NMPC uses the SRBD model that will consider these dynamics in the lower optimization stage.

We define the Governor state \( \{x_{c,0}^{g}, \ldots, x_{c,N_{g}}^{g}\} \in \mathbb{R}^{4 \times (N_{g} + 1)} \), with \( x_{c,k}^{g} = [p_{c,k}^{g}, v_{c,k}^{g}]^{T} \in \mathbb{R}^{4} \) the stack of X-Y CoM position and velocity at time \( k \). The control inputs are \( u_{c}^{g} \in \mathbb{R}^{2 \times N_{g}} = \{u_{c,0}^{g}, \ldots, u_{c,N_{g} - 1}^{g}\} \), with \( u_{c,k}^{g} \in \mathbb{R}^{2} \) X-Y position of the ZMP at time \( k \). Foothold locations \( p_{f,k} \) and gait status \( \delta_{k} \) for all the feet at each sample \( k \) are the parameters used to compute the support polygon at each node of the Governor horizon \( N_{g} \). Additional parameters are initial Z CoM position \( p_{c,0}^{\text{act}} \) and \( g \in \mathbb{R} = 9.81 \text{m/s}^{2} \). To obtain and \( x_{c}^{g} \) \( u_{c}^{g} \) we cast the following optimization problem:

\[
\min_{x^{g},u^{g}} \sum_{k=0}^{N_{g}} \| p_{c,k}^{g} - p_{c}^{\text{goal}} \|_{Q_{p}} + \| v_{c,k}^{g} \|_{Q_{v}} + \| u_{c,k}^{g} \|_{Q_{u}} + \| u_{c,k}^{g} \|_{Q_{u}} \quad (a)
\]

\[
\text{s.t.} \quad \begin{align*}
    x_{c,0}^{g} &= x_{c,0}, \\
    x_{c,k+1}^{g} &= x_{c,k}^{g} + \left[ v_{c,k}^{g} T_{s} + \frac{T_{s}^{2}}{2} g \left( p_{c,k}^{g} - u_{c,k}^{g} \right) \right] \\
    u_{c,k}^{g} &\in S(p_{f,k}, \delta_{k}) & k &\in \mathbb{Z}^{N_{g} - 1}
\end{align*} \quad (b)
\]

The terms (1a) and (1b) of the cost function aims to minimize the distance between CoM position and the goal, the norm of the velocities and the distance of the ZMP from the center of the support polygon \( u_{c}^{g} \) (for robustness purposes). Matrices \( Q_{p}, Q_{v}, Q_{u} \in \mathbb{R}^{2 \times 2} \) are positive definite weighting matrices. The initial condition (1c) is expressed by setting \( x_{c,0}^{g} \) equal to the corresponding values \( x_{c,0}^{\text{act}} \) received from the State Estimator. Equation (1d) corresponds to the discrete CoM dynamics for the LIP model. Equation (1e) imposes that the ZMP always lies inside the support polygon \( S(p_{f,k}, \delta_{k}) \). The optimization problem (1) does not have any guarantee on the response time. As already discussed in Sec. II-C a formulation

III. GOVERNOR OPTIMIZATION

As already mentioned, the task of the Governor is to compute along a horizon \( N_{g} \) the linear (i.e., longitudinal and lateral) CoM velocity reference trajectories \( v_{c}^{g} \) to reach a desired goal/recover from a push in a predefined time \( T_{f} \) and the trajectory of GRFs \( u_{c}^{g} \) to follow it.
with slack variable can be used to impose a predefined instant in which the robot has to reach the target:

\[
\min_{x^s, u^p, s} \sum_{k=0}^{N_g} \|v_{c,k}^g\|^2 + \sum_{k=0}^{N_g-1} \|u_k^g - u_k^{ref}\|^2 \quad \text{(2a)}
\]

\[
\sum_{k=0}^{N_g} \left( s_k \|2Q_{aq} + Q_{s,k} s_k \right) \quad \text{s.t.} \quad x_{c,0}^g = x_{c,x,y}^{act} \quad \text{(2b)}
\]

\[
x_{c,k+1}^g = x_{c,k}^g + \begin{bmatrix} v_{c,k}^g T_s + \frac{\tau_{cg}^2}{2p_{cg}^2} (p_{c,k}^g - u_k^g) \\ \frac{g}{p_{cg}^2} (p_{c,k}^g - u_k^g) T_s \end{bmatrix} \quad \text{(2c)}
\]

\[
u_k^g \in \mathcal{S}(p_{f,k}, \delta_k), \quad k \in \mathbb{I}_{N_g}^{N_g-1} \quad \text{(2d)}
\]

\[
\begin{bmatrix} s_k(0) \\ s_k(1) \end{bmatrix} \geq \begin{bmatrix} p_{c,k}^g - p_{c,k}^{goal} \\ p_{c,y,k}^g - p_{c,y}^{goal} \end{bmatrix}, \quad k \in \mathbb{I}_{N_g}^N \quad \text{(2e)}
\]

\[
s_k(0), s_k(1) \geq 0, \quad k \in \mathbb{I}_{N_g}^{N_g} \quad \text{(2f)}
\]

The slack variable \( s_k \in \mathbb{R}^{2 \times 1} \) is added in the cost term (2b) and thanks to (2f), (2g) it allows to impose that robot CoM coincides with the goal after M samples. Differently from (1) the cost function does not include a position term, since it is encoded into (2b). Matrices \( Q_{s,0} \in \mathbb{R}^{2 \times 2} \) and \( Q_{s,1} \in \mathbb{R}^{1 \times 2} \) are the additional weights for the quadratic and linear term for the slack variables. Equations (2c), (2d), (2e) are the same as (1c), (1d), (1e) respectively.

### B. QP Mapping

The Governor computes CoM trajectories which must be followed by the NMPC, but the latter requires also reference GRFs \( u^{ref} \in \mathbb{R}^{2 \times N} \). Since the output of the optimization problem (2) is a trajectory for the ZMP \( u^g \), we need to map this into a set of consistent GRFs, thus moving from a bi-dimensional to the higher dimensional space of contact forces. For this reason, we define the set of the indices of the legs in contact with the ground by \( \mathcal{C} \). A parametric QP is solved to find the vector of GRFs \( u^{qp} \in \mathbb{R}^{4M} \) which corresponds to the ZMP location \( u^g \). If a foot is in swing phase, its GRFs are set 0 by default.

\[
u_i^{ref} = \begin{cases} u_i^{qp} & i \in \mathcal{C} \\ 0_{4 \times 1} & i \not\in \mathcal{C} \end{cases} \quad \text{(3)}
\]

where \( i \) is the leg index. Note that we need to solve a QP for each sample \( k \in \text{horizon} \) in order to avoid overloading the notation, we do not specify the subscript \( k \) in the following quantities. The parameters of the model are foothold locations \( p_{fi} \), the X-Y components of the CoM position \( p_{cg}^g \) computed by the Governor, the initial Z CoM position \( p_{cz}^{act} \) and the mass of the robot \( m \in \mathbb{R} \). Thus, for every \( k \) we solve:

\[
\min_{u^{qp}} \left\| u_{i,y}^{qp} \right\|^2 + \sum_{i \in \mathcal{C}} \| S_x(p_{f,i} - [p_{cg}^{i}, p_{cz}^{act}]) u_i^{ref} \|^2 \quad \text{(4a)}
\]

\[
s.t. \quad u_{i,y}^{g} = \frac{\sum_{i \in \mathcal{C}} P_{i,x,y} u_{i,y}^{qp}}{\sum_{i \in \mathcal{C}} u_{i,y}^{qp}}, \quad i \in \mathcal{C} \quad \text{(4b)}
\]

First term of (4a) is the regularization term on the X-Y components of the GRFs, with \( Q_t \in \mathbb{R}^{2 \times 2} \) as weighting matrix, while the second one minimizes the angular momentum rate, and it is weighted by \( Q_t \in \mathbb{R}^{3 \times 3} \). Variable \( S_x \) represents the skew-symmetric matrix associated to the cross product. Equation (4b) is the definition of ZMP as the center of pressure, (4e) represents the gravity compensation. Equation (4d) guarantees that the X-Y CoM acceleration computed with the LIP model in the Governor (left term) coincides with the one of the SRBD used in NMPC. It is worth to highlight that when the robot is on two legs, the ZMP lies on a line, and so it can only move in a 1D manifold. During that phase, imposing (4b) for the X component is enough to guarantee that also the Y coordinate of the ZMP respects the same constraint. Along the horizon \( N_g \), each problem is independent from the others, so they can be solved in parallel. In this way the computation effort remains low (a couple of ms to solve a QP problem with linear constraints) and therefore the Governor can be integrated in our high-frequency NMPC scheme.

### IV. SIMULATION AND EXPERIMENTAL RESULTS

The Governor endows the NMPC planner with the capability to recover from disturbances and to avoid drifting in the face of non-idealities. To show its effectiveness, we tailored three template scenarios: (a) motion along a straight line, (b) reaching a fixed goal, and (c) recover from external pushes. We performed the simulations and experiments on the 22 kg quadruped robot AlienGo of Unitree.

We consider trot as a template gait for our experiments because of its inherently unstable nature. Indeed, any asymmetry in the real robot can make it drift when setting a pure forward velocity. The quadruped is thus not able to follow a straight line. The trot parameters are cycle time = 1 s, duty factor = duration of stance phase / cycle time = 0.65. We used the following values for the weighting matrices: \( Q_s = \text{diag}(200, 300) \), \( Q_a = \text{diag}(100, 350) \), \( Q_t = \text{diag}(100, 100) \), \( Q_l = \text{diag}(1, 1, 1) \). Without any lack of generalization, slack variables are considered only for the Y component, i.e. \( Q_{s,q} = \text{diag}(0, 1000) \) and \( Q_{s,1} = (0, 1000) \). We used HPIPM [27] solver integrated in acados [28] library to find the solutions of the problem (2). The problem (4), instead, is solved using eiquadprog [29].

In this section we will call reference the output of the Governor, desired the output of the NMPC, and actual the real values measured by the State Estimator.

### A. Simulation

The four leg stance phase in a walking trot is the only moment of the gait where the robot has full control authority and is able to track the reference velocities. Neglecting this fact would lead to failure or unpredictably longer response times. In our algorithm, the Governor already takes into account...
the under-actuation, enabling us to successfully deal with this issue. The simulations of the scenario (a) are only reported in the accompanying video.‡

The Governor also allows the robot to reach a goal which is different from the initial position and then to follow user velocity commands, or eventually reach another specified fixed target. Figure 4 refers to the simulation of the scenario (b) in which the robot has to go to a lateral target (-0.2 m on the Y coordinate) in a predefined time ($T_f = 4.8$ s) and then keep it while walking. Since X-Y directions are decoupled in the LIP model, the X component of the goal $p_{c,x}^{\text{goal}}$ is updated at each iteration of the NMPC according to Algorithm 1 and continuously tracked. The top plot demonstrates that the Governor is able to compute reference CoM trajectories (yellow line) that accomplish the task. The actual CoM position (red line) matches with the references, due to the fact that the reference velocities computed by the Governor are nicely tracked by the NMPC (respectively yellow and blue line, bottom part of Fig. 4). Figure 5, instead, shows the reference $u^k$ and actual X-Y components of the ZMP locations for the same simulation (computed by (4b)). As it can be seen, the robot is able to track the reference values for the entire cycle, for both X-Y coordinates. In this simulation we decided to keep the Governor always active to show how the algorithm is able to track (once the target has been reached) a zero user-defined lateral velocity $v_{\text{usr}}^{c,y}$.

Figure 6 reports scenario (b) with the same goal (-0.2 m) but different response interval ($T_f = 3$ s). Due to the smaller interval, the response is more aggressive and presents an overshoot which is then recovered within the 3 s interval. The task is achieved by simply modifying the response interval $T_f$, no tuning of the weights of the cost function is required. Shaded areas highlight the response interval.

As already mentioned in Sec. I, a possible way to track a goal is to compute a feed-forward wrench $w_{f,y} \in \mathbb{R}$ which depends on the error between the actual state and the goal, i.e. $w_{f,y} = K_p(p_{c,y}^{\text{goal}} - p_{c,y}^{\text{act}}) + K_d(v_{c,y}^{\text{sur}} - v_{c,y}^{\text{act}})$. To validate our Governor, we implemented this approach and tested it in a simulation of the scenario (c). The user velocity $v_{\text{usr}}^{c,y}$ is zero, so the task is to come back to the initial Y position. For the first 5 seconds of the motion the robot is pushed with a force of 15 N in the Y direction. In Fig. 7 we report the actual Y CoM position $p_{c,y}^{\text{act}}$ in three different cases. The purple line represents the case in which the values of proportional and derivative gain matrix are chosen such that the system has a critically damped response and the robot is able to perform a straight motion. The result is $K_p = 170 N/m$ and $K_d = 2\sqrt{mK_p} = 122$ Ns/m for a total robot mass $m = 22$ kg. As it can be noticed in the plot, once the disturbance is removed, the robot slowly moves towards the goal, but it is not able to follow it once it has been reached. In fact, the robot keeps drifting, moving away from the initial position. Green line, instead, represents the same scenario in which $K_p$ and $K_d$ are chosen such that a response time of 4.8 s is guaranteed (response time $= 4/\sqrt{K_p m}$, $K_p = 15$ N/m and $K_d = 36$ Ns/m). It is evident that the CoM is far to reach the goal in the predefined time. We can conclude thus that the

‡https://www.dropbox.com/s/zn7hc5crepl41fu/ral22_final.webm?dl=0
approach with a PD wrench results in a slow response, with a steady state error with respect to the goal. In addition, tuning the parameters does not allow the user to obtain the predefined behaviour. The result of our approach, instead, is reported with light blue line. During the push, it ensures the stability of the robot. Once the disturbance has been removed, the Governor computes a velocity trajectory that brings the robot towards the goal.

\[ \text{Fig. 7: Simulation, scenario (c): comparison of the actual Y CoM positions between the PD wrench approach (purple and green lines) and our Governor (light blue line).} \]

The goal is to come back to the initial position after a 15 N lateral push of 5 s. For the purple line, \( K_p \) and \( K_d \) are computed such that the system should have a critically damped response and good performance in a straight forward motion. For the green line, instead, the values are chosen to impose a response time of 4.8 s. In both cases the robot is not able to converge to the goal. With the Governor (light blue line) the robot recovers its initial position after the push, without any steady-state error.

**B. Experiments**

In this section we present preliminary experiments for the scenarios (a), (b), (c) carried out with the real robot platform. The results are presented in the accompanying video. In these experiments we employed the problem (1) without ensuring guarantees on the response time. The Governor is capable to correct the lateral drifts present on the real robot due to non idealities. The video demonstrates that the NMPC without Governor does not succeed in moving on a straight line in presence of a statically unstable gait as trot. Indeed, the robot suffers lateral and backward (less visible) drifts because of two reasons: 1) the trot being an unstable gait the CoM always diverges between two four leg stance events in opposite directions. Any little asymmetry in the robot results in a cumulative drift in one direction. 2) since Aliengo has c-shaped legs, they create nonzero moments about the pitch axis during a swing.

By enabling the Governor, instead, the robot is able to reach the goal and to prevent the trajectory from drifting. Figure 8 shows the change in the reference lateral velocity (yellow line) done by the Governor. When the average Y position (\( p_{y,x} \), red line) exceeds the bounds (\( p_{y,x}^{\text{bound}} \), dashed lines), the Governor is activated and brings back the CoM close to the goal. A threshold of 1 cm around the constant goal has been chosen for the activation. Once the goal has been reached the Governor automatically switches off and the reference velocity becomes equal to the user one (zero). The continuous switching on and off of the Governor demonstrates the need of having an external module, as the Governor, which corrects the reference trajectories during a trot.

Figure 9 shows the Y position of the robot in the scenario (b) with a fixed goal of -0.15 m. As in simulation, the robot is able to track the reference value, due to the fact that the velocity takes into account the under-actuation of the trot gait. In this case we decided to keep the Governor always active to demonstrate that it is able to work properly also when the target has been reached, compensating drifts as in scenario (a).

\[ \text{Fig. 8: Experiment, scenario (a): Aliengo moving forward with zero user lateral velocity. The peaks in the reference velocity (yellow line) represent the moment in which the average Y position (red line) has passed the threshold (dashed lines) around the initial position and the Governor is activated.} \]

In the last experiment, we show how we can use the Governor to react to external disturbances. As we have already mentioned in the Introduction, the task is not to reject the disturbance, but to cope with it and later to recover from its effect. Figure 10 shows the Y position of the CoM in a real hardware experiment in the scenario (c) when the robot receives two constant pushes. During the push the robot tries to resist to the disturbance and, thanks to the high-frequency re-planning of the NMPC, is able to keep the stability and avoid falling. Once the pushing force is removed, the Governor drives the robot back to the initial position, with a threshold of 1 cm. As in the previous cases, the Governor becomes active when the robot is diverging from the goal.

\[ \text{Fig. 9: Experiment, scenario (b): Aliengo robot reaches the target position of -0.15 m and then keeps moving following the user defined velocity.} \]

\[ \text{Fig. 10: Experiment, scenario (c): CoM Y position of the robot. During the motion the robot has been pushed twice and it automatically comes back to the initial position when the push is removed.} \]
In this work we presented an optimization-based reference generator, named Governor, which deals with the under-actuation of the statically unstable gaits and with external disturbances. It exploits the LIP model to compute feasible reference trajectories that allow the robot to accomplish a tracking task. A QP mapping is used to determine the GRFs which correspond to the ZMP location computed by the LIP model. Velocities and GRFs are used as informative tracking references by a 25 Hz lower-stage NMPC planner introduced in [1]. This results in the absence of conflicting tasks in the cost function, which simplifies the tuning of the cost weights of the NMPC. We validated our approach performing simulations and experiments with the 22 kg quadruped robot Aliengo in three different scenarios: (a) straight forward motion, (b) tracking of a fixed goal, (c) recover after a push. For the last scenario we demonstrated that the naïve solution of adding a Cartesian PD in parallel to the NMPC is not enough to return to the required position. In addition, we presented and validated in simulations a formulation with slack variables that can guarantee to reach the goal in a specified time, without the need to further tune any parameters. Future work involves extending the idea of the Governor also to the heading (i.e. yaw) orientation, and to perform additional experiments with formulation that respects a specified response time.

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