Entanglement in the anisotropic Heisenberg XYZ model with different Dzyaloshinskii-Moriya interaction and inhomogeneous magnetic field

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We investigate the entanglement in a two-qubit Heisenberg XYZ system with different Dzyaloshinskii-Moriya(DM) interaction and inhomogeneous magnetic field. It is found that the control parameters ($D_x$, $B_x$ and $b_x$) are remarkably different with the common control parameters ($D_z$, $B_z$ and $b_z$) in the entanglement and the critical temperature, and these x-component parameters can increase the entanglement and the critical temperature more efficiently. Furthermore, we show the properties of these x-component parameters for the control of entanglement. In the ground state, increasing $D_x$ (spin-orbit coupling parameter) can decrease the critical value $b_x$, and increase the entanglement in the revival region, and adjusting some parameters (increasing $b_x$ and $J$, decreasing $B_x$ and $\Delta$) can decrease the critical value $D_{xc}$ to enlarge the revival region. In the thermal state, increasing $D_x$ can increase the revival region and the entanglement in the revival region (for $T$ or $b_x$), and enhance the critical value $B_{xc}$ to make the region of high entanglement larger. Also, the entanglement and the revival region will increase with the decrease of $B_x$ (uniform magnetic field). In addition, small $b_x$ (nonuniform magnetic field) has some similar properties to $D_x$, and with the increase of $b_x$ the entanglement also has a revival phenomenon, so that the entanglement can exist at higher temperature for larger $b_x$.

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I. INTRODUCTION

Entanglement is one of the most fascinating features of quantum mechanics and it provides a fundamental resource in quantum information processing [1]. As a simple system, Heisenberg model is an ideal candidate for the generation and the manipulation of entangled states. This model has been used to simulate many physical systems, such as nuclear spins [2], quantum dots [3], superconductor [4] and optical lattices [5], and the Heisenberg interaction alone can be used for quantum computation by suitable coding [6]. In recent years, the Heisenberg model, including Ising model [7], XY model [8], XXX model [9], XXZ model [10] and XYZ model [11,12], have been intensively studied, especially in thermal entanglement and quantum phase transition. Very recently, F. Kheirandish et al. and Z. N. Gurkan et al. [12] discussed the Heisenberg XYZ model with DM interaction (arising from spin-orbit coupling) and magnetic field. However, almost all the directions of spin-orbit coupling and the external magnetic field are fixed in the z-axis in the above papers. The external magnetic field along other directions has never been taken into account. This motivates us to think about what different phenomena will appear if we choose the parameters along different directions in the generalized Heisenberg XYZ model. To explore this, here we choose x-axis as the direction of spin-orbit coupling and the external magnetic field in the Heisenberg XYZ model.

In this paper, we discuss the differences between the Heisenberg XYZ models with parameters in different directions, and then analyze the influences of these parameters on the ground-state entanglement and thermal entanglement. We find that x-component DM interaction and inhomogeneous magnetic field are more efficient control parameters, and more entanglement and higher critical temperature can be obtained by adjusting the value of these parameters. In order to provide a detailed analytical and numerical analysis, here we take concurrence as a measure of entanglement [13]. The concurrence $C$ ranges from 0 to 1, $C = 0$ and $C = 1$ indicate the vanishing entanglement and the maximal entanglement respectively. For a mixed state $\rho$, the concurrence of the state is $C(\rho) = \max\{2\lambda_{\text{max}} - \sum_{i=1}^{4} \lambda_i, 0\}$, where $\lambda_i$s are the positive square roots of the eigenvalues of the matrix $R = \rho(\sigma^y \otimes \sigma^y)\rho^*(\sigma^y \otimes \sigma^y)$, and the asterisk denotes the complex conjugate. If the state of a system is a pure state, i.e., $\rho = |\Psi\rangle\langle\Psi|$, $|\Psi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$, the concurrence will be reduced to $C(|\Psi\rangle) = 2|ad - bc|$.

Our paper is organized as follows. In Sec. II, we compare the DM coupling parameters along different directions. In Sec. III, we compare the external magnetic fields (including uniform and nonuniform magnetic field) along different directions. In Sec. IV, we analyze the entanglement of generalized Heisenberg XYZ model with x-component DM interaction and inhomogeneous magnetic field. Finally, in Sec. V a discussion concludes the paper.

II. THE COMPARISON BETWEEN THE DM COUPLING PARAMETERS ALONG DIFFERENT DIRECTIONS

A. DM coupling parameter along z-axis

The Hamiltonian $H$ for a two-qubit anisotropic Heisenberg XYZ model with z-component DM coupling and external
magnetic field is

\[
H = J_x \sigma^x_1 \sigma^x_2 + J_y \sigma^y_1 \sigma^y_2 + J_z \sigma^z_1 \sigma^z_2 + D_z (\sigma^z_1 \sigma^z_2 - \sigma^z_1 \sigma^z_2) + (B_z + b_z) \sigma^z_1 + (B_z - b_z) \sigma^z_2,
\]

where \(J_i (i = x, y, z)\) are the real coupling coefficients, \(D_z\) is the z-component DM coupling parameter, \(B_z\) (uniform external magnetic field) and \(b_z\) (nonuniform external magnetic field) are the z-component magnetic field parameters, \(\sigma^i (i = x, y, z)\) are the Pauli matrices. The coupling constants \(J_i > 0\) corresponds to the antiferromagnetic case, and \(J_i < 0\) corresponds to the ferromagnetic case. This model can be reduced to some special Heisenberg models by changing \(J_i\). Parameters \(J_x, D_z, B_z\) and \(b_z\) are dimensionless.

Using the similar process to [12], we can get the eigenstates of \(H\):

1. \(|\phi_{1,2}\rangle = \sin \theta_{1,2}|00\rangle + \cos \theta_{1,2}|11\rangle\),
2. \(|\phi_{3,4}\rangle = \sin \theta_{3,4}|01\rangle + \chi \cos \theta_{3,4}|10\rangle\),

with corresponding eigenvalues:

1. \(E_{1,2} = J_z \pm w_1\),
2. \(E_{3,4} = -J_z \pm w_2\),

where \(w_1 = \sqrt{4B_z^2 + (J_x - J_y)^2}, w_2 = \sqrt{4B_z^2 + (J_x + J_y)^2}\), and \(\chi = (J_y - J_x)/(2B_z)\).

The critical temperature above which the entanglement vanishes. These results are in accord with those in Ref. [12].

\[B_z = b_z = 0\]. It shows that the concurrence will decrease with increasing temperature \(T\) and increase with increasing \(D_z\) for a certain temperature. The critical temperature \(T_c\) is dependent on \(D_z\). Increasing \(D_z\) can increase the critical temperature above which the entanglement vanishes. These results are in accord with those in Ref. [12].

B. DM coupling parameter along x-axis

The Hamiltonian \(H'\) for a two-qubit anisotropic Heisenberg XYZ model with x-component DM coupling and external magnetic field is

\[
H' = J_x \sigma^x_1 \sigma^x_2 + J_y \sigma^y_1 \sigma^y_2 + J_z \sigma^z_1 \sigma^z_2 + D_z (\sigma^z_1 \sigma^z_2 - \sigma^z_1 \sigma^z_2) + (B_x + b_x) \sigma^x_1 + (B_x - b_x) \sigma^x_2,
\]

similarly, where \(J_i (i = x, y, z)\) are the real coupling coefficients, \(D_z\) is the x-component DM coupling parameter, \(B_x\) (uniform external magnetic field) and \(b_x\) (nonuniform external magnetic field) are the x-component magnetic field parameters, and \(\sigma^i (i = x, y, z)\) are the Pauli matrices. The coupling constants \(J_i > 0\) corresponds to the antiferromagnetic case, and \(J_i < 0\) corresponds to the ferromagnetic case. This model can be reduced to some special Heisenberg models by changing \(J_i\). Here, all the parameters are dimensionless, and we introduce the mean coupling parameter \(J\) and the partial anisotropy parameter \(\Delta\) in the YZ-plane, where \(J = J_z + J_x\) and \(\Delta = J_y + J_x\).

In the standard basis \(|00\rangle, |01\rangle, |10\rangle, |11\rangle\), the Hamiltonian \(H'\) can be rewritten as:

\[
H' = \begin{pmatrix}
J_z & G_2 & G_3 & J_x - J_y \\
G_4 & -J_x & J_x + J_y & G_1 \\
J_x - J_y & J_x + J_y & -J_z & G_4 \\
J_z & G_4 & G_1 & J_x - J_y \\
\end{pmatrix},
\]

where \(G_{1,2} = iD_x + B_x \pm b_x, G_{3,4} = -iD_x + B_x \pm b_x\). By calculating, we can obtain the eigenstates of \(H'\):

\[
|\psi_{1,2}\rangle = \frac{1}{\sqrt{2}} (|\sin \varphi_{1,2}|00\rangle + |\cos \varphi_{1,2}|01\rangle + |\cos \varphi_{1,2}|10\rangle + |\sin \varphi_{1,2}|11\rangle),
\]

FIG. 1: (Color online) (a) Thermal concurrence versus \(T\) and \(D_z\) without the external magnetic field. (b) Thermal concurrence versus \(T\) and \(D_z\) without the external magnetic field. Here \(J_x = 1, J_y = 0.5, J_z = 0.2\).
In Fig. 1(b), we plot the concurrence in Fig. 1(b), where $J_x = 1, J_y = 0.5, J_z = 0.2$ and $B_{x,z} = b_{x,z} = 0$. It can be seen that for the same parameters $\varphi_3, 4 = \arctan(\frac{2\sqrt{b_1^2 + b_2^2}}{J_y - J_z \pm w_1^2})$. In the above standard basis, the density matrix $\rho(T)$ has the following form:

$$\rho'(T) = \begin{pmatrix} U_1 & Q_1^* & Q_2^* & U_2 \\ Q_1 & V_1 & V_2 & Q_2 \\ Q_2^* & V_1 & V_2^* & Q_1^* \\ U_2^* & Q_2^* & Q_1^* & U_1 \end{pmatrix},$$

(10)

where

$$U_{1,2} = \frac{1}{2Z}(e^{-\frac{J_y + w_1'}{T}} \sin^2 \varphi_1 + e^{-\frac{J_y - w_1'}{T}} \sin^2 \varphi_2 \pm e^{-\frac{J_z - w_1'}{T}} \sin^2 \varphi_3 \pm e^{-\frac{J_z + w_1'}{T}} \sin^2 \varphi_4),$$

$$V_{1,2} = \frac{1}{2Z}(e^{-\frac{J_y + w_1'}{T}} \cos^2 \varphi_1 + e^{-\frac{J_y - w_1'}{T}} \cos^2 \varphi_2 \pm e^{-\frac{J_z - w_1'}{T}} \cos^2 \varphi_3 \pm e^{-\frac{J_z + w_1'}{T}} \cos^2 \varphi_4),$$

$$Q_{1,2} = \frac{1}{2Z}(e^{-\frac{J_y + w_1'}{T}} \sin \varphi_1 \cos \varphi_1 + e^{-\frac{J_y - w_1'}{T}} \sin \varphi_2 \cos \varphi_2 \pm e^{-\frac{J_z - w_1'}{T}} \chi \sin \varphi_3 \cos \varphi_3 \pm e^{-\frac{J_z + w_1'}{T}} \chi \sin \varphi_4 \cos \varphi_4).$$

Then the positive square roots of the eigenvalues of the matrix $R'$ can be obtained

$$\lambda_{1,2} = \frac{e^{\frac{\omega_1'}{Z}}}{Z} \sqrt{2w_2^2 \cosh(2 \omega_1'/T) - \frac{8b_2^2}{w_2^2} \sin^2 \omega_1'/T} \pm \sqrt{\frac{4D_2^2 + (J_y + J_z)^2}{w_2^2}} \left[ \sin^2 \left( \frac{2w_2^2}{T} \right) - \frac{16b_2^2}{w_2^4} \sin^4 \left( \frac{\omega_1'}{T} \right) \right]^{\frac{1}{2}},$$

(11a)

$$\lambda_{3,4} = \frac{e^{\frac{\omega_1'}{Z}}}{Z} \sqrt{2w_2^2 \cosh(2 \omega_1'/T) - \frac{8b_2^2}{w_2^2} \sin^2 \omega_1'/T} \pm \sqrt{(J_y - J_z)^2} \left[ \sin^2 \left( \frac{2w_2^2}{T} \right) - \frac{16b_2^2}{w_2^4} \sin^4 \left( \frac{\omega_1'}{T} \right) \right]^{\frac{1}{2}},$$

(11b)

where $Z' = 2[e^{\frac{\omega_1'}{Z}} \cosh(\frac{\omega_1'}{T}) + e^{\frac{\omega_1'}{Z}} \cosh(\frac{\omega_1'}{T})]$. Thus, according to (13), the corresponding concurrence of this system can be expressed as:

$$C' = \max\{|\lambda_1' - \lambda_3'| - |\lambda_2' - \lambda_4'|, 0\},$$

(12)

In Fig. 1(b), we plot the concurrence $C'$ as a function of $T$ and $D_x$, where $J_x = 1, J_y = 0.5, J_z = 0.2, B_{x,z} = b_{x,z} = 0$. It shows the similar results to those in the last subsection, but if we compare the two figures in Fig. 1 in detail, we can easily find some differences between them, i.e., there is less disentanglement region in Fig. 1(b) than in Fig. 1(a), and increasing x-component parameter $D_x$ can make the entanglement increase more rapidly, for example, when $T = 6$ the concurrence increases more rapidly in Fig. 1(b) than in Fig. 1(a).
the concurrence decreases with increasing $T$ for $B_z = 1$ (Fig. 3(b)) and $b_z = 1.5$ (Fig. 4(b)).

**B. External magnetic field along x-axis**

In this case, we use the model in Eq. (6). According Eq. (12), we analyze similarly the variation of the entanglement and the roles of the external magnetic field along x-axis, by fixing other parameters.

In Fig. 3(a), by comparing with $|B_z|$, we can see that, though $|B_z|$ has some similar properties to $|B_z|$, $|B_z|$ has a larger critical value and a more remarkable revival. In Fig. 4(a), we find that the entanglement decreases more slowly with the increase of nonuniform magnetic field $|b_z|$ than $|b_z|$, and $|b_z|$ has more entanglement when $|b_z| = |b_z|$. Also, in Fig. 3(b), we find that $B_z$ has more entanglement than $B_z$ for a certain temperature when $B_z = B_z$. Fig. 4(b) shows that, for the same $b_z$ and $b_z$, $b_z$ has a higher critical temperature and more entanglement (for a certain temperature).

It is evident that for some fixed parameters, using x-component magnetic field can increase the entanglement more efficiently than z-component magnetic field. So we can change the direction of the external magnetic field to obtain a more efficient control parameter.

**IV. HEISENBERG XYZ MODEL WITH X-COMPONENT DM INTERACTION AND INHOMOGENEOUS MAGNETIC FIELD**

According to Secs. II and III, we know that x-component DM interaction and inhomogeneous magnetic field are more efficient control parameters of entanglement. So in this section, we will analyze the properties of parameters of the model (Eq. (6)) in detail.

**A. Ground state entanglement**

When $T = 0$, this system is in its ground state. It is easy to find that the ground-state energy is equal to

$$E'_2 = J_x - w'_1, \quad if \quad J_x < \frac{1}{2}(w'_1 - w'_2), \quad (13a)$$

$$E'_4 = -J_x - w'_2, \quad if \quad J_x > \frac{1}{2}(w'_1 - w'_2), \quad (13b)$$

and for $E'_2$ and $E'_4$, the ground state is $|\psi_2\rangle$ and $|\psi_4\rangle$ respectively. At the critical point $J_x = \frac{1}{2}(w'_1 - w'_2)$ (i.e. $E'_2 = E'_4$), the ground state is $|G\rangle = \frac{1}{\sqrt{2}}(|\psi_2\rangle + |\psi_4\rangle)$ (an equal superposition of $|\psi_2\rangle$ and $|\psi_4\rangle$). Thus, the concurrence of ground state can be expressed as

**III. THE COMPARISON BETWEEN THE EXTERNAL MAGNETIC FIELDS ALONG DIFFERENT DIRECTIONS**

**A. External magnetic field along z-axis**

Here, we use the same model expressed in Eq. (1). According to Eq. (4), we can write the expression of concurrence, and then analyze the variation of the entanglement and the roles of the external magnetic field along z-axis, by fixing other parameters.

In Fig. 3(a), the thermal concurrence as a function of the uniform magnetic field is plotted, where $T = 0.1$. It is shown that, for small $|B_z|$, the concurrence has a high entanglement and does not vary with the variation of $|B_z|$, and with increasing $|B_z|$, the entanglement drops to zero suddenly at the critical value of $|B_z|$, and then undergoes a revival before decreasing to zero. In Fig. 4(a), the thermal concurrence is plotted versus the nonuniform magnetic field with $T = 0.2$. It is evident that increasing $|b_z|$ will decrease the entanglement. In Fig. 3(b) and Fig. 4(b), the concurrence as a function of the temperature $T$ is plotted for the uniform magnetic field and the nonuniform magnetic field, respectively. It is shown that

**FIG. 3:** (Color online) (a) Thermal concurrence versus uniform magnetic field $B_z$ (red dotted line), $B_z$ (blue solid line), where $T = 0.1$. (b) Thermal concurrence versus temperature $T$ for $B_z = 1$ (red dotted line) and $B_z = 1$ (blue solid line). Here $J_z = 1, J_y = 0.8, J_z = 0.2$, and $D_{z,x} = b_{z,x} = 0$.

**FIG. 4:** (Color online) (a) Thermal concurrence versus nonuniform magnetic field $b_z$ (red dotted line), $b_z$ (blue solid line), where $T = 0.2$. (b) Thermal concurrence versus temperature $T$ for $b_z = 1.5$ (red dotted line) and $b_z = 1.5$ (blue solid line). Here $J_z = 0.2, J_y = 0.4, J_z = 1$, and $D_{z,x} = B_{z,x} = 0$. 
\[
C' (T = 0) = \begin{cases} 
\frac{J_y - J_x}{w_1}, & \text{if } J_x < \frac{1}{2} (w_1' - w_2'), \\
\frac{1}{2} \left( \frac{J_x - J_y}{w_1} + \frac{J_x + J_y}{w_2} \right)^2 + \frac{4D^2}{w_2}, & \text{if } J_x = \frac{1}{2} (w_1' - w_2'), \\
\frac{4D^2 + (J_x + J_y)^2}{w_2} \right)^{1/2}, & \text{if } J_x > \frac{1}{2} (w_1' - w_2'). 
\end{cases}
\]

FIG. 5: (Color online) The ground-state concurrence versus \(b_x\) for different \(D_x\). Here \(J = 0.5\), \(\Delta = 0.8\), \(J_x = -1\), and \(B_x = 1\).

In Fig. 5, the ground-state concurrence is plotted as a function of \(b_x\) for different \(D_x\). With increasing \(b_x\), the ground-state concurrence \(C'\) keeps a constant initially, and then varies suddenly at the critical value of \(b_x\) \(b_{xc} = \frac{1}{2} \sqrt{(2J_x - w_1')^2 - (J_y + J_z)^2 - 4D_x^2}\), at which the quantum phase transition occurs. In the region of \(b_x < b_{xc}\), \(C'\) has a revival before decreasing to zero. By increasing \(D_x\), the critical value \(b_{xc}\) will decrease, and thus the revival region will become larger. In addition, there will be more entanglement in the revival region for larger \(D_x\). The ground-state concurrence as a function of \(D_x\) for different parameters is shown in Figs. 6(a)-6(d). These figures show that, with the increasing of \(D_x\), the concurrence keeps a constant initially, which depends on \(J, \Delta\) and \(B_x\), before reaching the critical value \(D_{xc} = \frac{1}{2} \sqrt{(2J_x - w_1')^2 - (J_y + J_z)^2 - 4D_x^2}\), then varies suddenly at \(D_{xc}\), and then undergoes a revival to reach its maximum value \((C' = 1)\) finally in the region of \(D_x > D_{xc}\). The critical point \(D_{xc}\) decreases as \(b_x\) and \(J\) increase, and increases as \(B_x\) and \(\Delta\) increase. Thus, by adjusting the parameters of the system (increasing \(b_x\) and \(J\), decreasing \(B_x\) and \(\Delta\)), we can decrease the critical point \(D_{xc}\) to get larger revival region with more entanglement.

\[\text{B. Thermal entanglement}\]

As the temperature increases, the thermal fluctuation is introduced into the system, thus the entanglement will be changed due to the mix of the ground state and excited states. To see the variation the entanglement, we use the thermal state \(\rho'(T)\) to describe the system state, and the concurrence \(C'\) to measure the entanglement. According to Eq. (12), the concurrence \(C'\) is plotted in Fig. 7 and Fig. 8 by fixing some parameters.

Fig. 7 demonstrates the influence of some parameters on the entanglement and critical temperature. In Fig. 7(A) and 7(a), when \(D_x\) is small, with increasing temperature the concurrence decreases to zero at \(T_{c1}\) (the first critical temperature), and then undergoes a revival before decreasing to zero again at \(T_{c2}\) (the second critical temperature). When \(D_x > D_{xc}\), with increasing temperature the concurrence decreases to zero at \(T_{c2}\). Increasing \(D_x\) can increase the revival region (decreasing \(T_{c1}\) and increasing \(T_{c2}\)) and enhance the entanglement in the revival region, so the entanglement can exist at higher temperature for larger \(D_x\). In Fig. 7(B) and 7(b), we see that decreasing \(B_x\) can enhance the entanglement
and increase the revival region (decreasing $T_{c1}$), so the entanglement has its maximum value when $B_x = 0$ and $T = 0$. Fig. 7(C) and 7(c) show that small $B_z$ has some similar properties to $D_x$. In addition, Fig. 7(C) shows that the concurrence has a revival phenomenon as $b_z$ increases, so the entanglement can also exist at higher temperature for larger $b_z$.

On the whole, Fig. 7 can be divided into two regions: (i) The region with reversal. In this region, $D_x < D_{xc}, b_x < b_{xc}$, or $B_z > B_{xc}$, where $D_{xc}, b_{xc}$ and $B_{xc}$ are the critical values of the parameters. When $T < T_{c1}$, $\lambda'_{\text{max}} = \lambda'_2$ and thus $C' = \max\{\lambda_3 - \lambda_1 - \lambda_2 - \lambda_4, 0\}$. When $T = T_{c1}$, $\lambda'_3 = \lambda'_1$ and thus $C' = 0$. When $T > T_{c1}$, $\lambda'_{\text{max}} = \lambda'_1$ and thus $C' = \max\{\lambda'_1 - \lambda'_3 - \lambda'_2 - \lambda'_4, 0\}$. With the increase of temperature, the entanglement decreases to zero at $T_{c2}$ ultimately. (ii) The region without reversal. In this region, $D_x > D_{xc}, b_x > b_{xc}$, or $B_z < B_{xc}$. For arbitrary temperature, $\lambda'_{\text{max}} = \lambda'_1$, so the concurrence is $C' = \max\{\lambda'_1 - \lambda'_3 - \lambda'_2 - \lambda'_4, 0\}$. Also, the entanglement vanishes finally at $T_{c2}$ with increasing temperature.

Fig. 8 shows the cross influence of the DM parameter and the magnetic field on entanglement. In Fig. 8(A) and 8(a), when $D_z$ is fixed, with increasing $B_z$ the entanglement is a constant initially, and then dropped to zero suddenly at $B_{xc}$. For $B_z > B_{xc}$, a revival phenomenon occurs, and the entanglement becomes another constant. In addition, as $D_z$ increases, the critical value $B_{xc}$ increases, and the entanglement enhances in the region of $B_x < B_{xc}$. In Fig. 8(B) and 8(b), when $D_x$ is small, with increasing $B_z$ the entanglement is also a constant initially, then dropped to zero suddenly at $B_{xc}$, and then undergoes a revival. Furthermore, increasing $D_z$ can increase the revival region (decreasing $b_{xc}$), and enhance the entanglement in the revival region. When $D_z$ is large enough, with increasing $b_z$ the entanglement decreases from its maximum value to zero.

V. DISCUSSION

We have investigated the entanglement in the generalized two-qubit Heisenberg $XYZ$ system with different DM interaction and inhomogeneous magnetic field. By comparing with the common $x$-component parameters ($D_x, B_z$ and $b_z$), we find that the $x$-component DM interaction and inhomogeneous magnetic field ($D_x, B_x$ and $b_x$) are more efficient control parameters for the increase of entanglement and critical temperature. Furthermore, we analyze the properties of $x$-component parameters for the control of entanglement. In the ground state, increasing $D_x$ can decrease the critical value $B_{xc}$ to increase to revival region, and increase the entanglement in the revival region. The critical value $D_{xc}$ can also be decreased by increasing $b_x$ and $J$, or decreasing $B_x$ and $\Delta$. In the thermal state, for $T > b_z$, increasing $D_x$ can increase the revival region and the entanglement in the revival region, for $B_{xc}$, increasing $D_z$ can increase the critical value $B_{xc}$ to lager the region of high entanglement. In addition, decreasing $B_z$ can also increase the entanglement and the revival region. Small $b_x$ possesses some similar properties to $D_x$, and with the increase of $b_z$, the entanglement also has a revival phenomenon.
so that the entanglement can exist at a higher temperature for larger $b_\tau$. Our results imply that more efficient control parameters can be gotten by changing the direction of parameters, and more entanglement and higher critical temperature can be obtained by adjusting the values of these parameters. Thus, by using the state with more entanglement as the quantum channel, the ultimate fidelity will be increased in quantum communication. In addition, in practice the need for low temperature enhances the complexity and difficulty of constructing a quantum computer, however by increasing the critical temperature the entanglement can exist at a higher temperature, this will reduce the technical difficulties for the realization of a quantum computer.

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