Performance analysis of SPAD-based optical wireless communication with OFDM

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In recent years, there has been a growing interest in the use of a single-photon avalanche diode (SPAD) in optical wireless communication (OWC). The SPAD operates in the Geiger mode and can act as a photon counting receiver obviating the need for a transimpedance amplifier. Although a SPAD receiver can provide higher sensitivity compared to traditional linear photodetectors, it suffers from dead-time-induced nonlinearity. To improve the data rates of SPAD-based OWC systems, optical orthogonal frequency division multiplexing (OFDM) can be employed. This paper provides a comprehensive theoretical analysis of the SPAD-based OWC systems using direct-current-biased optical OFDM signaling considering the effects of signal clipping, SPAD nonlinearity, and signal-dependent shot noise. An equivalent additive Gaussian noise channel model is proposed to describe the performance of the SPAD-based OFDM system. The statistics of the proposed channel model and the analytical expressions of the signal-to-noise ratio and bit error rate are derived in closed forms. By means of extensive numerical results, the impact of the unique receiver nonlinearity on the system performance is investigated. The results demonstrate new insights into different optical power regimes of reliable operation for SPAD-based OFDM systems even well beyond the SPAD saturation level.

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1. INTRODUCTION

Due to the scarcity of the radio frequency (RF) spectrum, optical wireless communication (OWC) has been continuously gaining interest in both industry and scientific communities in recent years. Compared to traditional RF wireless communication, the potential advantages of OWC mainly include high data rates, excellent security levels, and license-free spectra [1]. However, the performance of OWC can be strongly degraded by the occasional outages caused by the received optical power fluctuations. For example, for terrestrial free-space optical (FSO) communication, the power fluctuation can be introduced by atmospheric turbulence, misalignment, and adverse weather conditions (e.g., fog and haze) [2], while for indoor visible light communication (VLC) the power fluctuation mainly results from user mobility and random orientation [3]. One effective way of mitigating the effect of outages induced by power fluctuation is employing highly sensitive photon counting receivers. Typical photon counting receivers include photo-multiplier tubes (PMTs) and single-photon avalanche diodes (SPADs). PMTs can offer high sensitivity and gain. However, they are usually bulky, expensive, and require high voltage to operate [4]. SPADs can provide similar gain as PMTs. Compared to PMTs, SPADs have the important advantages of low cost, low operation voltage, and reduced weight, which make them very attractive in OWC [5]. The photon counting capability of a SPAD is achieved by biasing the traditional linear photodiode (PD) above the breakdown voltage so that it operates in the Geiger mode. After detecting each photon arrival, an avalanche process can be initiated to produce a striking current pulse, which leads to a huge receiver gain on the order of $10^6$ [6]. Compared to the commonly used photodetectors, e.g., PIN PD and avalanche photodiode (APD), a SPAD obviates the need for a transimpedance amplifier and has the advantage of higher sensitivity, which can greatly extend the operation regime of the received optical power close to the quantum limit [7]. There are two main types of SPADs, i.e., passive quenching (PQ) and active quenching (AQ) SPADs. For PQ SPADs, no active circuitry is required, and the avalanche currents are quenched by simply flowing through the quenching resistors [8], whereas, for AQ SPADs, active circuitry is adopted to sense the avalanche pulse and then force the quenching and reset the SPAD after an accurate hold-off time [9]. The SPAD has various use cases in OWC systems. It can be adopted in FSO systems to maintain the availability of FSO links in deep fading events [2]. For indoor VLC, the SPAD can be employed to design highly sensitive wide field-of-view (FoV) receivers to support user mobility [10,11] and dimming control [12]. SPAD can also be used in underwater
Different from the systems with linear receivers, when a SPAD corresponding information rate is further analyzed in [30]. To improve the performance of SPAD-based receivers, a detection scheme that uses not only the photon count information but also the photon arrival information is proposed in [17]. A SPAD-based OWC system with joint pre-distortion and noise normalization functionality is proposed to mitigate the SPAD nonlinear distortion [18]. The utilization of a radial basis function neural network-based demodulation method in SPAD-based OWC systems is also investigated [19]. Although most of the prior SPAD-based OWC works focused on the simple on-off keying (OOK) modulation [9,20,21], very recently optical orthogonal frequency division multiplexing (OFDM) has also been employed in SPAD-based systems to improve the spectral efficiency (SE) [19,22–24]. In particular, using the proposed simplified Volterra nonlinear equalizer, a data rate of 1.35 Gbps has been achieved [22]. More recently, in [23], in addition to the utilization of the nonlinear equalization, by further employing the peak-to-average power ratio (PAPR) optimization and adaptive bit and energy loading, an experimental data rate of 5 Gbps using a commercial SPAD receiver has been reported, which is significantly higher than the data rates achieved with OOK [20,25]. Despite a few experimental works, to the best of our knowledge, a complete theoretical performance analysis of SPAD-based OFDM OWC systems in the presence of nonlinear effects at both the transmitter and receiver is still missing. The objective of this paper is to fill this research gap.

The performance analysis of OFDM-based OWC systems with traditional linear photodetectors has been widely investigated. For instance, in [26], the impact of clipping noise on the performance of OWC systems with OFDM is discussed. Note that the time-domain OFDM signal is approximately normal distributed with high PAPR, but practical light sources have limited dynamic ranges. Therefore, signal clipping should be employed to fit the signal into limited dynamic ranges. A detailed comparison among three optical OFDM variations, i.e., direct-current-biased optical OFDM (DCO-OFDM), asymmetrically clipped optical OFDM (ACO-OFDM), and asymmetrically clipped DC biased optical OFDM (ADO-OFDM) is presented in [27]. Some works have also been conducted to investigate the performance of optical OFDM in the presence of LED nonlinearity. For example, in [28], the nonlinear response of the LED is described based on the physical mechanisms in the quantum well, and an efficient pre-distorter is further proposed to overcome the nonlinearity. In [29], a set of polynomials is employed to model the effects of a general nonlinear distortion in OWC systems, and the corresponding information rate is further analyzed in [30]. Different from the systems with linear receivers, when a SPAD is employed in OWC systems, the unique receiver nonlinearity induced by the SPAD dead time has to be taken into account. In addition, being a photon counting receiver, the dominant noise factor of a SPAD receiver is the signal-dependent shot noise rather than the signal-independent thermal noise, which is also distinct from its linear counterparts. As a result, the performance analysis of traditional OFDM with a linear PD cannot be directly applied to OFDM with a SPAD receiver.

This work provides a complete framework for the performance analysis of DCO-OFDM-based OWC systems with SPAD receivers considering practical nonlinear effects at both the transmitter (i.e., signal clipping) and receiver (i.e., dead-time-induced nonlinearity) sides. The main contributions of this paper are summarized as follows:

- The statistics of the proposed channel are studied analytically and are given in closed forms. An equivalent additive Gaussian noise channel model is then defined to characterize the performance of the SPAD-based OFDM OWC systems.
- The analytical expression of the system signal-to-noise ratio (SNR) is derived, which consists of the signal-to-distortion-noise ratio (SDNR) and signal-to-shot-noise ratio (SSNR). Based on the derived SNR expression, the closed form of the bit error rate (BER) performance is achieved.
- Extensive numerical results demonstrate that the SPAD nonlinearity has significant impacts on the SNR, BER, and SE of the considered system. The accuracy of the derived analytical derivations is validated by comparing the analytical and Monte Carlo simulation results. The presented results provide new insights into the different regimes of reliable operation of SPAD receivers showing that high-speed transmission is achievable even at high-power regimes beyond the saturation of SPAD receivers.
- The trade-off between the received signal power and nonlinear distortion indicates that the signal clipping at the transmitter can be optimized. Therefore, the optimization of the signal clipping is further investigated to effectively improve the performance.

The rest of this paper is organized as follows. Section 2 introduces the SPAD-based OWC system with OFDM. Section 3 presents the theoretical analysis of such a system. The results and discussion are presented in Section 4. Finally, we conclude this paper in Section 5.

2. SPAD-BASED OFDM SYSTEM

2.1. Optical OFDM Transmission

Figure 1 presents the block diagram of an OFDM OWC communication system using a SPAD-based optical receiver. Note that, in this work, we aim to investigate the application of the SPAD in high-speed OWC systems; hence, DCO-OFDM signaling is employed due to its high SE and implementation simplicity [26,31]. It is worth noting that the proposed framework can also be extended to the optical OFDM variants designed for other objectives, e.g., ACO-OFDM for high power efficiency. At the transmitter, the input bit stream is transformed into a complex symbol stream by the M-quadrature amplitude modulation (QAM) modulator, where $M$ denotes the constellation size. The symbol stream is then...
serial-to-parallel (S/P) converted to form vectors suitable for inverse fast Fourier transform (IFFT) operation. Considering a $K$-point fast Fourier transform (FFT) operation, the subcarriers carrying information bits populate the first half of the OFDM frame, leaving the 0th and $K/2$th subcarriers unused [30]. Therefore, the number of information carrying subcarriers is $K' = K/2 - 1$. Hermitian symmetry is applied to the rest of the OFDM frame in order to obtain real-valued symbols after the IFFT operation. Denote the symbol associated to the $k$th information carrying subcarrier as $X[k]$ with $k = 1, \ldots, K'$. Then, the time-domain signal $x[n]$ can be obtained after the IFFT as

$$x[n] = \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} X[k] e^{\frac{2\pi nk}{K}}. \quad (1)$$

According to the central limit theorem (CLT), the amplitude of $x[n]$ is approximately zero-mean Gaussian distributed as long as $K$ is relatively large [29]. Considering the uniform power allocation over the subcarriers, to ensure that $x[n]$ is with unit variance, the variance of $X[k]$ is set to $\sigma_k^2 = K/(K-2)$ [26].

Considering that the PAPR of the generated signal $x[n]$ is relatively high, whereas practical light sources are with limited dynamic ranges, $x[n]$ has to be properly clipped. The clipped signal can be expressed as

$$x_c[n] = \begin{cases} \kappa_t, & \text{if } x[n] \geq \kappa_t, \\ x[n], & \text{if } \kappa_b < x[n] < \kappa_t, \\ \kappa_b, & \text{if } x[n] \leq \kappa_b, \end{cases} \quad (2)$$

where $\kappa_t$ and $\kappa_b$ denote the normalized clipping levels from the top and bottom, respectively. After scaling, adding a DC bias and digital-to-analog conversion (DAC), which consists of zero-order-hold operation and a low-pass filter, the resultant electrical signal is used to drive the light source. In effect, the optical power of the $n$th time-domain OFDM sample emitted from the source is given by [26]

$$x_c[n] = \delta x_c[n] + P_{\text{bias}}, \quad (3)$$

where $\delta$ denotes the scaling factor and $P_{\text{bias}}$ is the DC bias value. Since the light source can only send unipolar signals, $x_c[n]$ should be nonnegative by choosing proper DC bias. Denoting the minimal and maximal optical power of the light source as $P_{\text{min}}$ and $P_{\text{max}}$, respectively, the following equations should be satisfied: $\delta \kappa_b + P_{\text{bias}} = P_{\text{min}}$ and $\delta \kappa_t + P_{\text{bias}} = P_{\text{max}}$, which leads to

$$\delta = \frac{P_{\text{max}} - P_{\text{min}}}{\kappa_t - \kappa_b}, \quad P_{\text{bias}} = \frac{P_{\text{min}} \kappa_t - P_{\text{max}} \kappa_b}{\kappa_t - \kappa_b}. \quad (4)$$

The average transmit optical power can be expressed as [26]

$$\bar{P}_{\text{Tx}} = \delta (f(\kappa_b) - f(\kappa_t) + \kappa_t Q(\kappa_t) + \kappa_b Q(-\kappa_b)) + P_{\text{bias}}, \quad (5)$$

where

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right). \quad (6)$$

is the probability density function (PDF) of a standard Gaussian distribution and $Q(\cdot)$ denotes the Q-function. Note that, if a symmetric clipping is employed, i.e., $\kappa_t = -\kappa_b = \kappa$, one has $\delta = (P_{\text{max}} - P_{\text{min}})/2\kappa$, $P_{\text{bias}} = (P_{\text{max}} + P_{\text{min}})/2$, and $\bar{P}_{\text{Tx}} = P_{\text{bias}}$. In this work, the bandwidths of the light source and DAC are assumed to be larger than the signaling spectrum; therefore, a flat fading channel is approximately experienced, and the intersymbol interference and the induced overshoot-and-ramp of the waveform beyond the clipping levels are negligible [26,29]. In addition, we focus on the impact of the nonlinear distortions induced by the signal clipping and SPAD receiver on the performance of OFDM-based OWC systems. The nonlinearity of the light source is assumed to be negligible. This is a reasonable assumption when the DC bias and the input AC signal of the light source are carefully selected to ensure that the source operates in its linear dynamic range.

### B. SPAD Receivers

A SPAD is an APD that is biased beyond reverse breakdown voltage in the so-called "Geiger" region. In this mode of operation, a SPAD triggers a large amount of electron–hole pair generations for each detected photon. In other words, a SPAD emits a very large current by receiving a single photon and, thus, can essentially be modeled as a single photon counter. The photodetection process of an ideal photon counter can be modeled using Poisson statistics, which describe the shot noise effects [32]. However, the performance of practical SPAD-based receivers depends on non-ideal effects such as dead time, photon detection efficiency (PDE), dark count rate (DCR), afterpulsing, and crosstalk. In particular, due to the existence of dead time, significant nonlinearity is introduced, which causes the deviation of the detected photon count of SPAD receivers from the Poisson distribution [18]. In order to mitigate the dead time effect and, hence, improve the photon counting capability, an array of SPADs whose output is the superposition of the photon counts of the individual SPADs
is commonly utilized [17,33]. With the array design, when some SPADs are triggered and turn into the dead time, other SPADs might be still active for photon detection. Therefore, the dead-time-induced effects can be mitigated. This is similar to the adoption of receiver diversity in fading channels. The practical circuitry designs of SPAD receivers can be found in many prior works [8,9,34].

Assuming that the channel loss is $\xi$, the average received signal optical power is given by $\bar{P}_R = \xi P_T$. The received optical power when the $n$th OFDM symbol is transmitted can be expressed as $P_R[n] = \xi x[n]$. To the SPAD receiver, it corresponds to an incident photon rate of [4]

$$\lambda_a[n] = \left( \frac{\gamma_{\text{PDE}} x[n]}{E_{\text{ph}}} + \delta_{\text{DCR}} + \delta_B \right) (1 + \varphi_{\text{AP}} + \varphi_{\text{CT}}),$$

(7)

where $\gamma_{\text{PDE}}$ denotes the PDE of the SPAD; $E_{\text{ph}}$ is the photon energy; $\delta_B$ represents the background photon rate; and $\delta_{\text{DCR}}$, $\varphi_{\text{AP}}$, and $\varphi_{\text{CT}}$ denote the DCR of the array, the probabilities of afterpulsing, and crosstalk, respectively. Note that the dark count is defined as the avalanche triggered by the presence of intrinsic carriers inside the active region of the PD. The crosstalk means an avalanche SPAD might trigger an avalanche in a neighboring SPAD in the array. Afterpulsing is defined as the re-trigger of a SPAD after an avalanche due to the trapped carriers. These are all important specifications of SPAD receivers [6]. The photon rate $\delta_B$ equals to $\gamma_{\text{PDE}} P_B/E_{\text{ph}}$, where $P_B$ denotes the power of the background light received by the array receiver. By denoting

$$\begin{cases}
C_a = \gamma_{\text{PDE}} \xi (1 + \varphi_{\text{AP}} + \varphi_{\text{CT}}) / E_{\text{ph}}, \\
C_n = (\delta_{\text{DCR}} + \delta_B) (1 + \varphi_{\text{AP}} + \varphi_{\text{CT}}),
\end{cases}$$

(8)

$\lambda_a[n]$ can be simplified as

$$\lambda_a[n] = C_n x[n] + C_n. \quad \text{(9)}$$

Note that, in this work, precise time synchronization between the transmitter and receiver is assumed, which is a common assumption employed in the performance analysis works [4,26,29,35]. In practical implementations, efficient low-complexity synchronization for OFDM-based systems can be achieved by adopting various techniques such as the transmission of a dedicated pilot sequence [36], the employment of a specially designed training block [37], and the utilization of some blind synchronization algorithms [38].

The channel loss $\zeta$ depends on the specific OWC application. For example, for FSO application, the channel loss is random, which is given by [39]

$$\zeta_{\text{FSO}} = b_f \vartheta \xi = b_f \left[ \text{erf} \left( \frac{\sqrt{\pi} \omega}{2 \sqrt{26} L} \right) \right]^2,$$

(10)

where $\text{erf}(\cdot)$ denotes the error function, $b_f$ refers to the geometric loss induced by diffraction of the Gaussian beam, $b_f$ is the intensity fluctuation caused by atmospheric turbulence, $\omega$ is the receiver aperture diameter, $\theta$ represents the beam divergence angle at the transmitter, and $L$ is the link distance. The intensity fading $b_f$ is gamma-gamma-distributed with the PDF given by [2]

$$F_{b_f}(x) = \frac{2(\rho \beta)(\rho+\beta)/2}{\Gamma(\rho)\Gamma(\beta)} \chi^{(\rho+\beta)/2-1}(K_{\rho-\beta}(2\sqrt{\rho \beta x}),$$

(11)

where $\Gamma(\cdot)$ is the gamma function, $K_{\rho-\beta}(\cdot)$ is the modified Bessel function of the second kind, and the parameters $\rho$ and $\beta$ are given by

$$\rho = \left[ \text{exp} \left( \frac{0.49 \chi^2}{1 + 0.18 \xi^2 + 0.56 \chi^{12/5}} \right) ^{7/6} - 1 \right] ^{-1},$$

$$\beta = \left[ \text{exp} \left( \frac{0.51 \chi^2 (1 + 0.69 \chi^{12/5})^{-5/6}}{1 + 0.9 \xi^2 + 0.62 \chi^{12/5}} \right) ^{5/6} - 1 \right] ^{-1},$$

(12)

respectively, with $\chi^2 = 0.5 C_0^2 k^2/\xi^{11/6}$, $\xi^2 = k_w \omega^2/4 L$, and $k_w = 2\pi/\lambda_w$, where $\lambda_w$ denotes the light wavelength and $C_0$ refers to the turbulence refraction structure parameter. For the application of VLC with the line-of-sight links, the channel loss can be written as [35]

$$\zeta_{\text{VLC}} = \begin{cases}
\frac{\lambda_d}{\xi_{\text{op}}} R_e(v_i) T_r(v_i) G(v_i) \cos(v_i), & 0 \leq v_i \leq \Phi_{\text{FoV}}, \\
0, & v_i > \Phi_{\text{FoV}},
\end{cases}$$

(13)

where $\lambda_d$ refers to the physical area of the detector, $v_i$ is the radiance angle, $v_r$ is the angle of incidence at the receiver, $R_e(v_i)$ denotes the radiant intensity, $T_r(v_i)$ refers to the filter transmission, $G(v_i)$ is the concentrator gain, and $\Phi_{\text{FoV}}$ is the receiver FoV.

According to the different employed quenching circuits, SPAD can be classified into two main types, i.e., AQ and PQ SPADs. AQ SPADs normally have high complexity and cost, and they are usually designed with small array sizes [9], whereas PQ SPADs benefit from simpler circuit design, higher PDE, and low cost and, therefore, are commonly employed in the commercial low-cost photon counting receivers with large array sizes [20]. In this work, we, hence, consider that the employed SPAD receiver is PQ-based. For each PQ SPAD in the array receiver, the average detected photon count during the time-domain OFDM sample duration $T_i$ is given by [2,16]

$$\mu_s(x[n]) = \frac{\lambda_a[n] T_i}{N_i} \text{exp} \left( -\frac{\lambda_a[n] T_d}{N_i} \right),$$

(14)

where $N_i$ denotes the number of SPAD pixels in the array and $T_d$ refers to the dead time. The average detected photon count of the array receiver can be expressed as

$$\mu_s(x[n]) = N_s \mu_s(x[n]) = \lambda_a[n] T_i \text{exp} \left( -\frac{\lambda_a[n] T_d}{N_i} \right).$$

(15)

It is shown in Eq. (15) that, with the increase of the incident photon rate $\lambda_a[n]$, the detected photon count first increases and then decreases. This indicates a nonlinear distortion to the received signal, which results from the paralysis property of the PQ SPAD. The received photon rate that gives the highest detected photon count is $N_i/T_d$, and the corresponding detected photon count is $N_i T_i/\epsilon T_d$, where $\epsilon$ is the Euler’s number [16]. Note that, in the absence of dead time, i.e., $T_d = 0$, the receiver nonlinearity vanishes, which is the case when an ideal photon counting receiver is considered.
The variance of the detected photon count during \( T_s \) for a single PQ SPAD can be written as \([40,41]\)

\[
\sigma^2_s(x[n]) = \mu_s(x[n]) - \mu_s^2(x[n]) \left[ 1 - \left( 1 - \frac{\tau_d}{T_s} \right)^2 \right],
\]

(16)

where it is assumed that the sample duration is larger than the dead time, i.e., \( T_s \geq \tau_d \), which is the operation regime considered in this work. As the detected photon counts of SPADs in the array are independent, the variance of the detected photon count of the array is given by \( \sigma^2(x[n]) = N_s \sigma^2_s(x[n]) \). Invoking the expression of \( \mu_s(x[n]) \) in Eq. (14), \( \sigma^2_s(x[n]) \) can be expressed as

\[
\sigma^2_s(x[n]) = \lambda_s[n] T_s \exp \left( -\frac{\lambda_s[n] \tau_d}{N_s} \right) \left[ \frac{2 \lambda_s[n] \tau_d}{N_s} (2 - \frac{\tau_d}{T_s}) \right].
\]

(17)

In Eq. (17), \( \sigma^2_s(x[n]) \) is the variance of shot noise of the detected photon count, which is a nonlinear function of the signal amplitude. Note that the dead time effect has made this signal-dependent shot noise more complicated than the APD shot noise, whose power is proportional to the signal amplitude \([42]\).

We denote the detected photon count during sample period \( T_s \) after S/P mapping at the receiver as \( y[n] \). When the size of the SPAD array is relatively large, due to the CLT, \( y[n] \) is approximately Gaussian distributed \([5]\) with mean and variance given by Eqs. (15) and (17), respectively. Hence, \( y[n] \) can be rewritten as

\[
y[n] = \mu_s(x[n]) + w_d[n],
\]

(18)

where \( w_d[n] \) represents the signal-dependent shot noise, which is Gaussian distributed with zero mean and signal-dependent variance given by Eq. (17). As shown in Fig. 1, the photon counting signal \( y[n] \) is then converted back to the frequency domain using the FFT operation given by

\[
Y[k] = \frac{1}{\sqrt{K}} \sum_{n=0}^{K-1} y[n] e^{-\frac{2\pi nk}{K}}.
\]

(19)

After the single-tap equalization and parallel-to-serial (P/S) mapping, the received QAM signal can be achieved. The QAM demodulation can then be applied, which results in the recovered bit stream.

### 3. THEORETICAL ANALYSIS OF SPAD-BASED OFDM

In the considered SPAD OFDM system, two nonlinear distortions exist. The first is the clipping-induced distortion due to the limited dynamic range of the light source as presented in Eq. (2). Such distortion also exists in standard OFDM systems with linear photodetectors \([26]\). The second is the additional SPAD-induced distortion given in Eq. (15). In this work, we combine these two nonlinear distortions and investigate the performance of the SPAD OFDM system under the combined nonlinear distortion. By substituting Eqs. (2), (3), and (9) into Eq. (15), the combined nonlinear distortion of the transmitted signal \( x[n] \) can be expressed as

\[
\begin{align*}
\mu_s(x[n]) &= \begin{cases} 
\psi_1 \kappa_t + \psi_2 & \text{if } x[n] \geq \kappa_t, \\
\psi_1 x[n] + \psi_2 & \text{if } \kappa_b < x[n] < \kappa_t, \\
\psi_1 \kappa_b + \psi_2 & \text{if } x[n] \leq \kappa_b,
\end{cases} \\
&= \begin{cases} 
\psi_1 \kappa_t + \psi_2 & \text{if } x[n] \geq \kappa_t, \\
\psi_1 x[n] + \psi_2 & \text{if } \kappa_b < x[n] < \kappa_t, \\
\psi_1 \kappa_b + \psi_2 & \text{if } x[n] \leq \kappa_b,
\end{cases}
\end{align*}
\]

(20)

where

\[
\psi_1 = C_1 \delta, \quad \text{and} \quad \psi_2 = C_1 \rho_{bias} + C_n.
\]

(21)

The nonlinear distortion in an OFDM-based system can be described by a gain factor \( \alpha \) and an additional distortion-induced noise \( (w_d[n]) \), both of which can be explained and quantified by the Bussgang theorem \([26,29]\). According to the Bussgang theorem, we have

\[
\mu_s(x[n]) = \alpha x[n] + w_d[n],
\]

(22)

where

\[
\mathbb{E}[y[n]w_d[n]] = 0
\]

(23)

holds. Note that \( \mathbb{E} \{ \cdot \} \) denotes the expectation operation. In the following discussion, both \( \alpha \) and variance of \( w_d[n] \) will be derived.

**Proposition 1**: The gain factor \( \alpha \) can be expressed as Eq. (24):

\[
\alpha = \frac{\psi_1 \tau_d T_s}{\sqrt{2 \pi N_s}} \left\{ \exp \left[ -\frac{\tau_d^2}{2 \psi_1^2} - \frac{\tau_d}{\psi_1 \kappa_t + \psi_2} \right] \right\} \left[ 1 + \frac{\psi_1^2 \tau_d^2}{N_a^2} - \frac{\psi_2 \tau_d}{N_a} \right]
\]

(24)

**Proof**: Invoking Eqs. (22) and (23), the gain factor \( \alpha \) can be written as

\[
\alpha = \mathbb{E}[\mu_s(x)] / \mathbb{E}[x^2] = \mathbb{E}[\mu_s^2(x)],
\]

(25)

where the sample index \( n \) is dropped for simplicity. After some mathematical manipulations, one can get

\[
\alpha = \frac{(\psi_1 \kappa_t + \psi_2) T_s}{\sqrt{2 \pi}} \left[ e^{-\frac{(\psi_1 \kappa_t + \psi_2) \tau_d}{N_a}} - \frac{1}{2} \psi_t^2 + W \right],
\]

(26)

where the term \( W \) is given by

\[
W = \int_{\kappa_b}^{\kappa_t} x \mu_s(x) f(x) dx.
\]

(27)
By substituting Eq. (20) into Eq. (27), \( W \) can be calculated as
\[
W = \frac{T \psi_1}{2\pi} \int_{-\infty}^{\infty} x^2 \exp \left[ -\left( \frac{(\psi_1 x + \psi_2) \tau_d}{N_s} - \frac{1}{2} x^2 \right) \right] dx + \frac{T \psi_2}{2\pi} \int_{-\infty}^{\infty} x \exp \left[ -\left( \frac{(\psi_1 x + \psi_2) \tau_d}{N_s} - \frac{1}{2} x^2 \right) \right] dx . \tag{28}
\]

The integral \( W_1 \) and \( W_2 \) can be solved analytically as given by Eq. (29):
\[
W_1 = e^{-\frac{\psi_2^2 \tau_d^2}{2N_s}} \left\{ \sqrt{\frac{\pi}{2}} \left( \frac{1 + \psi_1^2 \tau_d^2}{N_s^2} \right) \left[ \text{erf} \left( \frac{\kappa_b + \psi_1 \tau_d}{\sqrt{2}} \right) - \text{erf} \left( \frac{\kappa_b + \psi_1 \tau_d}{\sqrt{2}} \right) \right] + g(\kappa_b) - g(\kappa) \right\} .
\]
\[
W_2 = e^{-\frac{\psi_1^2 \tau_d^2}{2N_s}} \left[ \sqrt{2\pi} \left( \frac{\psi_1 \tau_d}{N_s} \right) f(\kappa) + \sqrt{2\pi} \left( \frac{\psi_1 \tau_d}{N_s} \right) \text{erfc} \left( \frac{\kappa_b + \psi_1 \tau_d}{\sqrt{2}} \right) \right] - \sqrt{2\pi} \left( \frac{\psi_1 \tau_d}{N_s} \right) \text{erfc} \left( \frac{\kappa_b + \psi_1 \tau_d}{\sqrt{2}} \right) . \tag{29}
\]

where
\[
g(\kappa) = \sqrt{\frac{\pi}{2}} \left( \kappa - \frac{\psi_1 \tau_d}{N_s} \right) f(\kappa + \frac{\psi_1 \tau_d}{N_s}) . \tag{30}
\]

\( \text{erfc}(\cdot) \) denotes the complementary error function, and \( f(\cdot) \) is defined in Eq. (6). By substituting Eqs. (29) and (28) into Eq. (26), the analytical expression of \( \alpha \) can be obtained as presented in Eq. (24).

We can consider a special case when \( \tau_d = 0 \), which refers to the linear ideal photon counting receiver. Substituting \( \tau_d = 0 \) into Eq. (24) leads to a gain factor of
\[
\alpha_{\text{id}} = \psi_1 T \left[ Q(\kappa_b) - Q(\kappa) \right] . \tag{31}
\]

This result is in line with the derived gain factor of OFDM systems with traditional linear receivers [26,43].

According to Eqs. (22) and (23), the variance of the distortion induced noise, denoted as \( \sigma_{\text{id}}^2 \), can be calculated as
\[
\sigma_{\text{id}}^2 = \mathbb{E}\{\mu_\alpha^2(x)\} - \mathbb{E}^2\{\mu_\alpha(x)\} - \alpha^2 . \tag{32}
\]

Therefore, in order to calculate \( \sigma_{\text{id}}^2 \), the first two moments of \( \mu_\alpha(x) \) should be investigated.

**Proposition 2:** The average of the received distorted signal \( \mathbb{E}\{\mu_\alpha(x)\} \) can be expressed as Eq. (33):
\[
\mathbb{E}\{\mu_\alpha(x)\} = (\psi_1 \kappa_b + \psi_2) T e^{-\frac{\psi_1 \kappa_b + \psi_2 \tau_d}{N_s}} Q(-\kappa_b) + (\psi_1 \kappa_b + \psi_2) T e^{-\frac{\psi_1 \kappa_b + \psi_2 \tau_d}{N_s}} Q(\kappa_b) + T \int_{-\infty}^{\infty} e^{-\frac{\psi_1 \tau_d}{N_s}} \left[ e^{-\frac{\psi_1 \tau_d}{N_s}} - e^{-\frac{\psi_1 \tau_d \tau_1}{N_s}} \right] dx + T \int_{-\infty}^{\infty} \left( \frac{\tau_d \psi_1^2}{N_s} - \psi_2 \right) e^{-\frac{\psi_1 \tau_d}{N_s} + \frac{\psi_1^2 \tau_d^2}{2N_s}} \left[ Q(\kappa_b + \psi_1 \tau_d) - Q(\kappa_b + \psi_1 \tau_d \tau_1) \right] . \tag{33}
\]

**Proof:** The expectation of \( \mu_\alpha(x) \) is given by
\[
\mathbb{E}\{\mu_\alpha(x)\} = (\psi_1 \kappa_b + \psi_2) T e^{-\frac{\psi_1 \kappa_b + \psi_2 \tau_d}{N_s}} Q(-\kappa_b) + (\psi_1 \kappa_b + \psi_2) T e^{-\frac{\psi_1 \kappa_b + \psi_2 \tau_d}{N_s}} Q(\kappa_b) + \int_{-\infty}^{\infty} \mu_\alpha(x) f(x) dx . \tag{34}
\]

After some mathematical manipulations, the integral \( M \) can be expressed as
\[
M = \frac{T \psi_1}{\sqrt{2\pi}} W_1 + \frac{T \psi_2}{\sqrt{2\pi}} e^{-\frac{\psi_1^2 \tau_d^2}{2N_s}} \int_{-\infty}^{\kappa_b} \left[ \text{erf} \left( \frac{\kappa_b + \psi_1 \tau_d}{\sqrt{2}} \right) - \text{erf} \left( \frac{\kappa_b + \psi_1 \tau_d}{\sqrt{2}} \right) \right] dx . \tag{35}
\]

The term \( M_1 \) can be solved analytically as
\[
M_1 = \sqrt{\frac{\pi}{2}} e^{-\frac{\psi_1^2 \tau_d^2}{2N_s}} \left[ \text{erf} \left( \frac{\kappa_b + \psi_1 \tau_d}{\sqrt{2}} \right) - \text{erf} \left( \frac{\kappa_b + \psi_1 \tau_d}{\sqrt{2}} \right) \right] . \tag{36}
\]

Therefore, by substituting Eqs. (36) and (35) into Eq. (34) and after some manipulations, the first moment of the received distorted signal can be written as Eq. (33).

**Proposition 3:** The second moment of the received distorted signal, i.e., \( \mathbb{E}\{\mu_\alpha^2(x)\} \) can be calculated as Eq. (37):
\[
\mathbb{E}\{\mu_\alpha^2(x)\} = \frac{T^2}{\sqrt{2\pi}} e^{-\frac{2\psi_1 \kappa_b + 2\psi_2 \tau_d}{N_s}} \left[ \psi_1^2 Q(\kappa_b) + 2\psi_1 \psi_2 \right] + \psi_1^2 \kappa_b - \frac{2\psi_1 \tau_d}{N_s} + 2\psi_1 \psi_2 \times e^{-\frac{\psi_1^2 \tau_d^2}{2N_s}} + \sqrt{2\pi} (\psi_1 \kappa_b + \psi_2) Q(-\kappa_b) - T^2 e^{-\frac{2\psi_1 \kappa_b + 2\psi_2 \tau_d}{N_s}} \left[ \psi_1^2 Q(\kappa_b) + 2\psi_1 \psi_2 \right] + \sqrt{2\pi} (\psi_1 \kappa_b + \psi_2) Q(-\kappa_b) \times e^{-\frac{\psi_1^2 \tau_d^2}{2N_s}} + \sqrt{2\pi} (\psi_1 \kappa_b + \psi_2) Q(\kappa_b) + \sqrt{2\pi} (\psi_1 \kappa_b + \psi_2) Q(\kappa_b) + \sqrt{2\pi} (\psi_1 \kappa_b + \psi_2) Q(\kappa_b) + \sqrt{2\pi} (\psi_1 \kappa_b + \psi_2) Q(\kappa_b) \times \left[ Q(\kappa_b + \psi_1 \tau_d) - Q(\kappa_b + \psi_1 \tau_d) \right] . \tag{37}
\]
Proof: The second moment of $\mu_a(x)$ is given by

$$
\mathbb{E}(\mu^2_a(x)) = \int_{-\infty}^{\infty} \mu^2_a(x) f(x) \, dx,
$$
which can be further calculated as

$$
\mathbb{E}(\mu^2_a(x)) = (\psi_1, + \psi_2)^2 T^2 e^{-\frac{2(x+\psi_1\psi_2\frac{\kappa}{N})}{N^2}} Q(\kappa) \\
+ (\psi_1, + \psi_2)^2 T^2 e^{-\frac{2(x+\psi_1\psi_2\frac{\kappa}{N})}{N^2}} Q(-\kappa) + \mathcal{U},
$$

(38)

where the term $\mathcal{U} = \int_{-\infty}^{\infty} \mu^2_a(x) f(x) \, dx$. After some mathematical calculations, $\mathcal{U}$ can be written as

$$
\mathcal{U} = \frac{T^2}{\sqrt{2\pi}} e^{-\frac{\psi_1^2 t^2}{N^2} + \frac{\psi_2^2 t^2}{N^2}} (\psi_1 U_1 + 2\psi_1 \psi_2 U_2 + \psi_2^2 U_3),
$$

(39)

where

$$
U_1 = \int_{-\infty}^{\infty} x^2 e^{-\frac{1}{2}(x+\frac{\psi_1 t}{N})^2} \, dx = e^{-\frac{\psi_1^2 t^2}{N^2}} \mathcal{W}_1',
$$

(40)

$$
U_2 = \int_{-\infty}^{\infty} x^2 e^{-\frac{1}{2}(x+\frac{\psi_2 t}{N})^2} \, dx = e^{-\frac{\psi_2^2 t^2}{N^2}} \mathcal{W}_2',
$$

(41)

$$
U_3 = \int_{-\infty}^{\infty} e^{-\frac{1}{2}(x+\frac{\psi_1 \psi_2 t}{N})^2} \, dx
$$

$$
= \sqrt{2\pi} \left[ Q\left(\kappa - \frac{2\psi_1 t}{N}\right) - Q\left(\kappa + \frac{2\psi_1 t}{N}\right) \right].
$$

(42)

Note that $\mathcal{W}_1'$ and $\mathcal{W}_2'$ are the same as $\mathcal{W}_1$ and $\mathcal{W}_2$ presented in Eq. (29) but with replacing $\psi_1$ by $\psi_2$. Finally, by plugging Eq. (39) into Eq. (38), the second moment of $\mu_a(x)$ can be calculated, as shown in Eq. (37).

By substituting Eqs. (24), (33), and (37) into Eq. (32), the analytical expression for the variance of the distortion-induced noise $\sigma^2_{w_d}$ can be achieved. For the ideal photon counting receiver, by substituting $t_d = 0$ into Eq. (32), this distortion-induced noise turns out to be only introduced by the signal clipping as

$$
\sigma^2_{w_d} = \psi_1^2 T^2 \left\{ Q(\kappa_b) - Q(\kappa_a) + \kappa_b f(\kappa_b) - \kappa_a f(\kappa_a) \right\}
$$

$$
+ 2\psi_1 T^2 \left\{ \kappa_b^2 Q(-\kappa_b) + \kappa_a^2 Q(-\kappa_a) - Q(\kappa_b) - Q(\kappa_a) \right\}
$$

$$
- \psi_1^2 T^2 \left[ f(\kappa_b) - f(\kappa_a) + \kappa_b Q(-\kappa_b) + \kappa_a Q(-\kappa_a) \right].
$$

(43)

This result is again in line with the derived clipping noise when the linear receiver is employed [26,43].

By plugging Eq. (22) into Eq. (18), the SPAD output $y[n]$ can be rewritten as

$$
y[n] = \alpha x[n] + w_d[n] + w_s[n].
$$

(44)

The analytical expressions of both $\alpha$ and $\sigma^2_{w_d}$ have been derived as shown in Propositions 1 to 3. After applying FFT operation, the received time-domain signal $y[n]$ is converted back to the frequency domain, which can be expressed as

$$
Y[k] = \alpha X[k] + W_0[k] + W_d[k],
$$

(45)

where $W_d[k]$ and $W_s[k]$ denote the FFT of $w_d[n]$ and $w_s[n]$, respectively. According to the CLT, both $W_d[k]$ and $W_d[k]$ are zero-mean Gaussian distributed noise terms when the number of subcarriers is sufficiently large [29,30]. Therefore, the received signal in the frequency domain is given by the transmitted signal multiplied by a gain factor plus additive Gaussian noise induced by nonlinear distortion and shot noise. For the distortion-induced noise $w_d[n]$, since it is uncorrelated with the signal according to the Bussgang theorem, the frequency-domain noise is also uncorrelated with the signal; thus, its variance is equal to the time-domain noise variance [29]. Therefore, one has $\sigma^2_{w_d} = \sigma_w^2$, where $\sigma_w^2$ denotes the variance of $W_d[k]$. Different from the distortion-induced noise, the shot noise $w_s[n]$ is signal dependent. In the following proposition, we derive the shot noise variance in the frequency domain, denoted as $\sigma_{w_s}^2$.

**Proposition 4:** The analytical expression of the frequency-domain shot noise variance $\sigma_{w_s}^2$ is given by Eq. (46):

$$
\sigma_{w_s}^2 = \sigma_w^2 \left[ Q\left(\kappa_b + \frac{\psi_1 t_d}{N_0}\right) - Q\left(\kappa_b + \frac{\psi_1 t_d}{N_0}\right) \right],
$$

$$
+ \sqrt{2\pi} \left[ \psi_1^2 T^2 e^{-\frac{\psi_1^2 t^2}{N_0}} - \psi_1^2 T^2 e^{-\frac{\psi_1^2 t^2}{N_0}} \right]
$$

$$
+ \sqrt{2\pi} \left[ \psi_1^2 T^2 e^{-\frac{\psi_1^2 t^2}{N_0}} - \psi_1^2 T^2 e^{-\frac{\psi_1^2 t^2}{N_0}} \right],
$$

(46)

$$
\sigma^2_{w_s} = \mathbb{E} [W_s[k]^2] = \frac{1}{K} \sum_{n=0}^{K-1} \sum_{m=0}^{K-1} \mathbb{E} \{w_s[n] w_s[m]\} e^{\frac{2\pi i n m}{K}} + \frac{2\pi i n k}{K},
$$

$$
= \frac{1}{K} \sum_{n=0}^{K-1} \mathbb{E} [w_s[n]^2] - \mathbb{E}^2 \{w_s[n]\}.
$$

(47)

Considering that $w_s[n]$ is with zero mean, Eq. (47) can be simplified as
\[ \sigma_{w_i}^2 = \frac{1}{K} \sum_{n=0}^{K-1} \sigma^2_n(x[n]) \simeq \mathbb{E} \{ \sigma^2_n(x[n]) \} \tag{48} \]

where the approximation is accurate for relatively large FFT size \( K \). Equation (48) suggests that the variance of the shot noise in the frequency domain is equal to the average of the signal-dependent shot noise variance in the time domain and, hence, is signal independent. The average of the shot noise in the time domain can also be written as

\[ \mathbb{E} \{ \sigma^2_n(x) \} = \int_{-\infty}^{+\infty} f(x) \sigma^2_n(x) dx. \tag{49} \]

where again the sample index \( n \) is dropped for simplicity and \( f(x) \) is defined in Eq. (6). Note that the expression of \( \sigma^2_n(x) \) can be achieved by substituting Eqs. (2), (3), and (9) into Eq. (17).

Equation (49) can be solved as

\[ \sigma_{w_i}^2 = \sigma^2_a(\kappa_b) Q(-\kappa_b) + \sigma^2_a(\kappa_i) Q(\kappa_i) + \int_{\kappa_b}^{\kappa_i} f(x) \sigma^2_n(x) dx. \tag{50} \]

Now let us derive the analytical expression of the term \( S \). According to Eq. (17), the conditional variance \( \sigma^2_n(x) \) when \( x \in [\kappa_b, \kappa_i] \) is given by

\[ \sigma^2_n(x) = (\psi_1 x + \psi_2) T_e \left( \frac{(\psi_1 + \psi_2) \tau_d}{N_0} \right) \]

\[ - \left( \psi_1 x + \psi_2 \right)^2 T_e T_x \left( 2 - \frac{\tau_d}{T_x} \right) e^{-\frac{2(\psi_1 + \psi_2) \tau_d}{N_0}}. \tag{51} \]

Substituting Eq. (51) into \( S \) results in

\[ S = \int_{\kappa_b}^{\kappa_i} (\psi_1 x + \psi_2) T_e \left( \frac{(\psi_1 + \psi_2) \tau_d}{N_0} \right) f(x) dx \]

\[ - \int_{\kappa_b}^{\kappa_i} \left( \psi_1 x + \psi_2 \right)^2 T_e T_x \left( 2 - \frac{\tau_d}{T_x} \right) e^{-\frac{2(\psi_1 + \psi_2) \tau_d}{N_0}} f(x) dx. \tag{52} \]

The first integral in Eq. (52) can be solved analytically as

\[ S_1 = \frac{\psi_1 T_e W_2}{\sqrt{2\pi}} \left. 2^{\frac{-\psi_1^2 \tau_d^2}{2N_0}} + \frac{\psi_1^2 \tau_d^2}{2N_0} \right] \]

\[ \times \left[ Q \left( \kappa_b + \frac{\psi_1 \tau_d}{N_0} \right) - Q \left( \kappa_i + \frac{\psi_1 \tau_d}{N_0} \right) \right]. \tag{53} \]

The second integral can be calculated as

\[ S_2 = \left( 2 - \frac{\tau_d}{T_x} \right) \frac{\tau_d}{N_0 T_x} U, \tag{54} \]

where the term \( U \) is given by Eq. (39). Therefore, by substituting Eqs. (52), (53), and (54) into Eq. (50), the expression of \( \sigma_{w_i}^2 \) can be achieved, which is summarized as Eq. (46).

Again, for the special case of the ideal photon counting receiver, the variance of the shot noise in the frequency domain can be simplified as

\[ \sigma_{w_i, \text{ad}}^2 = (\psi_1 \kappa_b + \psi_2) T_e - \psi_1 \kappa_b Q(\kappa_b) T_e \]

\[ + \psi_1 \kappa_i Q(\kappa_i) T_e + \psi_1 T_e f(\kappa_b) - \psi_1 T_e f(\kappa_i). \tag{55} \]

Proposition 4 implies that, for OFDM transmission, due to the averaging effect of the FFT operation, the time-domain signal-dependent shot noise turns out to be signal-independent noise in the frequency domain. Therefore, both noise terms in the received signal shown in Eq. (45) are uncorrelated with the signal, making it a standard additive Gaussian noise channel model. Note that, unlike the original OWC channel model, the equivalent additive Gaussian channel model for the OFDM SPAD-based OWC system in Eq. (45) is not constrained by the nonnegativity condition of the intensity modulation. Therefore, considering symbol-by-symbol detection, the achievable SE of the system can be upper bounded by

\[ S_{\text{upper}} = \log_2(1 + \gamma), \tag{56} \]

where the SNR of the received signal can be written as

\[ \gamma = \frac{\alpha^2 K}{\sigma_{w_i}^2 + \sigma_{w_i, \text{ad}}^2} = \frac{1}{\gamma_d + \gamma_s}. \tag{57} \]

The terms

\[ \gamma_d = \frac{\alpha^2 K}{\sigma_{w_i}^2}, \quad \gamma_s = \frac{\alpha^2 K}{\sigma_{w_i, \text{ad}}^2}, \tag{58} \]

denote the SDNR and SSNR, respectively. The derivation of the analytical expression of the SNR is now complete. The term \( \alpha^2 K \) is used to measure the signal power; therefore, the absolute value of the gain factor, i.e., \(|\alpha|\), can be used to measure the signal power. In addition, the value of the SNR is determined by two factors, i.e., SDNR and SSNR. The BER of the system when \( M \)-QAM is employed can be further written as [26]

\[ P_e = \frac{4(\sqrt{M} - 1)}{\sqrt{M} \log_2(M)} Q \left( \sqrt{\frac{3\gamma}{M - 1}} \right) + 4(\sqrt{M} - 2) \sqrt{\frac{3\gamma}{M - 1}} Q \left( \frac{3\gamma}{M - 1} \right). \tag{59} \]

4. NUMERICAL RESULTS

In this section, the numerical performance analysis of the SPAD-based OFDM system is presented. Unless otherwise mentioned, the parameters used in the simulation are given in Table 1. For simplicity, we assume that symmetric clipping is employed, i.e., \( \kappa_i = - \kappa_b = \kappa \), and \( P_{\text{min}} = 0 \); hence, the average transmitted optical power is \( P_{\text{Tx}} = P_{\text{bias}} = P_{\text{max}} / 2 \). By changing channel loss \( \zeta \), various average optical power at the receiver \( P_{\text{Rx}} = \zeta P_{\text{Tx}} \) can be achieved. The proposed framework in this work is for the performance analysis of the general OFDM-based OWC systems. Therefore, in this section, we focus on the performance of the systems under various received power rather than considering a specific use case. In some
Table 1. Parameter Setting [44,45]

| Symbol        | Definition                      | Value          |
|---------------|---------------------------------|----------------|
| $\lambda_{op}$ | Optical wavelength              | 450 nm         |
| $\eta_{PDE}$  | PDE of the SPAD at $\lambda_{op}$ | 0.35           |
| $N_s$         | Number of SPAD pixels in the array | 8192           |
| $\tau_d$      | Dead time of the SPAD            | 10 ns          |
| $P_B$         | Background light power           | 10 nW          |
| $\tau_{DCR}$  | Dark count rate                  | 0.5 MHz        |
| $\varphi_{AP}$| Afterpulsing probability         | 0.75%          |
| $\varphi_{CT}$| Crosstalk probability            | 2.5%           |
| $P_{max}$     | Maximal transmitted power        | 20 mW          |
| $K$           | Size of the FFT and IFFT         | 1024           |
| $T_s$         | Time-domain sample duration      | 20 ns          |
| $M$           | Modulation order                 | [16, 32]       |

practical OWC use cases, the fluctuation of $\zeta$ might exist due to the slow fading effects induced by atmospheric turbulence, terminal vibrations, random blockages, etc. The proposed framework can work well in such use cases. Specifically, at the beginning of the fading coherence time, the instantaneous channel state information $\zeta$ can be first estimated, and then the proposed analytical results can be used to calculate the performance under this instantaneous $\zeta$ or equivalently $\bar{P}_{Rx}$. Next, the average or outage performance can be easily calculated by considering the statistics of the fading effects.

Figure 2 presents the mean and variance of the SPAD receiver output versus the received optical power, which are calculated based on Eqs. (15) and (17). With the increase of the received optical power, the average value of the detected photon count first increases and then decreases due to the paralysis characteristics of the employed PQ SPAD. The maximal detected photon count of the SPAD receiver is given by $N_s T_s / (\tau_d)$, which is around 6000 in the considered system. For the considered receiver, this photon count is achieved when $\bar{P}_{Rx}$ is around 1 $\mu$W. In the following discussion, we denote this power as the saturation power of the receiver. The regimes with power less and higher than this power are denoted as the low- and high-power regime, respectively. In addition, the variance of the photon count is signal dependent and is less than the mean value, which shows the sub-Poisson property of the SPAD receiver [18]. This is different from the ideal photon counting receiver, i.e., when $\tau_d = 0$, whose mean and variance of the detected photon count are identical and both increase linearly with the received optical power, as also plotted in Fig. 2. Therefore, the dead-time-induced nonlinear effect can strongly reduce the detected photon count and, hence, limit the communication performance.

The gain factor $\alpha$ versus the $\bar{P}_{Rx}$ with various $\kappa$ is demonstrated in Fig. 3. Both simulated and analytical results are presented. The analytical results are calculated through Eq. (24). The simulated results are achieved by dividing the frequency-domain signal $Y[k]$ given in Eq. (44) by the transmitted signal $X[k]$ and then taking the expectation to average out the noise components. It is shown that the analytical results exactly match with the simulation ones, which justifies our analytical derivations. Note that, as mentioned above, $\alpha^2 K/(K - 2)$ refers to the received electrical signal power; hence, a larger $|\alpha|$ indicates higher electrical signal power. It is presented that, initially in Regime 1, with the rise of $\bar{P}_{Rx}$, $\alpha$ increases but then drops in Regime 2 because of the severer SPAD nonlinearity. In particular, when $\bar{P}_{Rx}$ is around the saturation power, $\alpha$ approaches zero since at this point the received optical waveform experiences a significant folding effect distorting the modulated signal. This means that this received power would result in the least undistorted electrical signal power and, hence, the worst performance. Further increase of $\bar{P}_{Rx}$ leads to a negative $\alpha$ in Regime 3. This is because when $\bar{P}_{Rx}$ is beyond the saturation power, the SPAD receiver operates in the high-power regime where any increase of the optical power would cause the reduction of the detected photon count, as presented in Fig. 2. Invoking the definition of $\alpha$ in Eq. (25), such negative correlation between the received power and detected photon count introduces a negative $\alpha$. Note that the increase of $|\alpha|$ with the rise of $\bar{P}_{Rx}$ can be observed in Regime 3 again. This is because, when the average signal power goes to this regime, the majority of the waveform dynamic range moves to a linear part of the SPAD receiver response again where the detected photon count monotonically decreases with $\bar{P}_{Rx}$. However, when $\bar{P}_{Rx}$ further increases to the levels of Regime 4, $\alpha$ starts to tend to zero. In this scenario, the slope reduction of the average photon count versus the received power (see Fig. 2) shrinks the dynamic range of the detected photon count signal and, hence, reduces the electrical signal power. In addition, it is also demonstrated that $\alpha$ also varies with the clipping level $\kappa$ and generally a lower $\kappa$ can provide a higher electrical signal power.

Figure 4 illustrates the SDNR versus $\bar{P}_{Rx}$ with various $\kappa$. Again the match between analytical and simulation results validates our analytical derivations. When $\bar{P}_{Rx}$ is very small, the nonlinearity of the SPAD receiver is negligible; hence, its SDNR converges to that of the ideal linear receiver. However, with the increase of $\bar{P}_{Rx}$, the SDNR of the SPAD receiver deviates from that of the ideal receiver, which remains fixed over $\bar{P}_{Rx}$. This is because, for the ideal receiver, the distortion noise is only introduced by the signal clipping at the transmitter; hence, the SDNR is irrelevant to the received optical power [26,43]. However, a SPAD receiver suffers from the signal distortion from both clipping and receiver nonlinearity. The change of the $\bar{P}_{Rx}$ causes not only the change of the average electrical signal power but also the change of the utilized

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Fig. 2. Mean and variance of the detected photon count of the considered SPAD receiver versus the received optical signal power.
dynamic range of the receiver, which introduces a change in distortion-induced noise variance $\sigma_{WD}^2$. Initially, the rise of $\bar{P}_{Rx}$, although it results in higher $\alpha$ as presented in Fig. 3, introduces much larger $\sigma_{WD}^2$ and causes the overall reduction of the SDNR. When $\bar{P}_{Rx}$ approaches the saturation power, the significant SDNR dip can be observed mainly because of the very small electrical power at this point as mentioned before. The further rise of $\bar{P}_{Rx}$ in turn increases the SDNR, but eventually the SDNR again diminishes because of the reduction of the electrical signal power in the high-power regime. Similar to the ideal receiver, the SPAD receiver also benefits from higher $\kappa$, which can generally provide a higher SDNR. The reason is twofold. First, higher $\kappa$ means less distortion noise introduced by the signal clipping at the transmitter [26, 43]. Secondly, higher $\kappa$ also indicates a smaller scaling factor $\delta$ and, hence, a smaller AC optical signal [see Eq. (3)]. As a result, the received signal occupies a smaller dynamic range, which leads to less distortion caused by receiver nonlinearity.

The SSNR versus $\bar{P}_{Rx}$ with various $\kappa$ is demonstrated in Fig. 5. Different from the ideal linear receiver whose SSNR monotonically increases with $\bar{P}_{Rx}$, the change of the SNR over $\bar{P}_{Rx}$ is similar to the SDNR and SSNR discussed above; therefore, a larger $\bar{P}_{Rx}$ does not always mean a higher SNR. It is also illustrated that, for different $\bar{P}_{Rx}$, the optimal clipping level is also different. Generally, smaller $\kappa$ provides a higher SNR when $\bar{P}_{Rx}$ is low, whereas larger $\kappa$ in turn gives a higher SNR when $\bar{P}_{Rx}$ is high. We attribute this to the fact that, under different $\bar{P}_{Rx}$, the effect of either the distortion-induced noise or shot noise would be dominant. For $\bar{P}_{Rx}$ where the former is the dominant factor, the SNR is limited by the SDNR. In this case, higher $\kappa$ can lead to a higher SDNR and, hence, a higher SNR. However, if the shot noise is the dominant factor, the SNR is in turn limited by the SSNR, and less $\kappa$ gives a higher SNR. Therefore, for any given $\bar{P}_{Rx}$, the clipping level can be optimized to achieve the optimal performance. In addition, Fig. 6 also illustrates that the SNR of the SPAD receiver is significantly less than that of the ideal receiver especially when the received optical power is high, which shows the significant impact of the SPAD nonlinearity.

To approach the communication performance limits of the SPAD receiver, in this work, the optimization of signal clipping level $\kappa$ is also investigated through exhaustive search. Figure 7 presents the recorded optimal clipping level and the corresponding optimal SNR versus $\bar{P}_{Rx}$. Both the SPAD receiver and ideal photon counting receiver are considered. For the ideal receiver, with the increase of $\bar{P}_{Rx}$, the nonlinear distortion-induced noise becomes the dominant noise factor. Since the nonlinear distortion is solely introduced by
signal clipping, which can be sufficiently eliminated when $\kappa$ is above 4, the optimal $\kappa$ monotonically increases with $\bar{P}_{\text{Rx}}$ and finally saturates at around 4. On the other hand, for the SPAD receiver, the behavior of optimal $\kappa$ as a function of $\bar{P}_{\text{Rx}}$ is more complicated. This is because of the change of the relative amounts of the SDNR and SSNR over $\bar{P}_{\text{Rx}}$ in the presence of the receiver nonlinearity. It is worth noting that, when $P_{\text{Rx}}$ is relatively high, the optimal $\kappa$ of the SPAD receiver can be significantly beyond 4. In fact, when the clipping level is above this value, the clipping noise already vanishes, and the increase of $\kappa$ is simply for the reduction of the optical signal dynamic range, which can effectively mitigate the nonlinear effect introduced by the receiver. This result indicates that, when the receiver nonlinearity is severe, using a smaller dynamic range of the transmitter is actually beneficial.

Let us further consider the BER performance of the SPAD-based OWC system with OFDM. The BER versus $\bar{P}_{\text{Rx}}$ when 16-QAM and 32-QAM are employed is demonstrated in Figs. 8 and 9, respectively. Note that the square 16-QAM and cross 32-QAM constellations are considered. Both simulated and analytical results are presented, which again match with each other. Considering the two peaks of the SNR with the change of $\bar{P}_{\text{Rx}}$ as shown in Fig. 6, with the increase of $\bar{P}_{\text{Rx}}$, two dips of BER can be observed. The BER dip when the received power is high (around 2 $\mu$W) indicates that reliable communication can even be achieved when the power is beyond the SPAD saturation power, which may be counterintuitive. As mentioned before, this is because, in this scenario, the received waveform goes into another linear regime of the SPAD at high power. Although in this high-power regime the waveform is reversed after the SPAD detection, because higher optical power leads to lower detected photon count as shown in Fig. 2, this reverse of the waveform can be easily corrected at the receiver through zero-forcing equalization before decoding. As a result, a decent performance can still be achieved. In addition, a different $\kappa$ also results in distinct performances. In these figures, the BER performance when the optimal clipping level is employed is also plotted. It is demonstrated that, by choosing the optimal clipping level, the best BER performance over the whole range of $\bar{P}_{\text{Rx}}$ can be achieved, which justifies the effectiveness of employing the optimal clipping level to improve the
performance. By comparing the system utilizing the optimal clipping with those with fixed clipping levels, we can observe that, when $P_{\text{Rx}}$ is low (i.e., $P_{\text{Rx}} < 0.02 \, \mu\text{W}$), its BER is close to the system with $\kappa = 2$. This is because, in this power regime, the optimal clipping level is indeed around 2, as illustrated in Fig. 7. With the increase of $P_{\text{Rx}}$, its performance becomes first closer to the system with $\kappa = 3$ and then $\kappa = 4$, which is also in line with the result presented in Fig. 7. In addition, by increasing the modulation order from 16 to 32, although the SE increases from 4 bits/symbol to 5 bits/symbol, the BER performance is also degraded. This suggests that, in practical implementations, appropriate modulation order should be employed considering the signal SNR and the BER target.

To explore the throughput of the SPAD-based OFDM system, Fig. 10 is plotted, which presents the achievable SE versus $P_{\text{Rx}}$. Note that, to achieve this result, a vector of QAM modulation order [4, 8, 16, 32, 64] is considered. For each $P_{\text{Rx}}$, the BERs when different modulation schemes are employed are calculated according to Eq. (59), and the maximal SE with the BER less than a BER target of $3 \times 10^{-3}$ is recorded. The BER target is selected, which is below the 7% forward error correction limits [46]. It is demonstrated in Fig. 10 that the achievable SE varies with $P_{\text{Rx}}$, and, by employing the optimal clipping level, the highest SE can be achieved over the considered $P_{\text{Rx}}$, as expected. Therefore, for OWC links with highly dynamic channels, the adaptive signal clipping and modulation scheme can be employed to achieve the best throughput over a wide range of the received power. In addition, the upper bound on achievable SE of the considered system with optimal signal clipping, calculated based on Eq. (56), is also plotted in Fig. 10. This upper bound is higher than the SE of the QAM and is the fundamental limit of the investigated system.

5. CONCLUSION

SPAD receivers have a great potential for improving the sensitivity of OWC systems and, hence, combating the outage caused by power fluctuations. OFDM can be used in SPAD-based OWC systems to improve the SE and boost the data rate. In this work, a theoretical performance analysis of the SPAD-based OFDM system is presented. The analytical expressions of both the SNR and BER are derived, which perfectly match Monte Carlo simulation results verifying the accuracy of the analytical derivations. The presented analytical results provide an effective and accurate way to estimate the system performance of practical SPAD-based systems employing OFDM. Through extensive numerical results, the impact of the SPAD nonlinearity on its performance is investigated showing new insights into the best operation regime of PQ SPADs. In addition, the optimization of the signal clipping is further investigated to improve the communication performance. This work paves the way for the future applications of SPAD receivers in general OWC systems, e.g., FSO, VLC, and UWOC, to achieve both high sensitivity and SE. The analytical framework proposed in this work can be utilized by the future works aiming to design more efficient techniques to combat SPAD nonlinearity, such as nonlinear equalizations and pre-distortions.

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