The operator product expansion does not imply parity doubling of hadrons

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Abstract

We examine whether the fact that QCD is chirally invariant at short distances necessarily leads to the prediction that hadrons form approximate parity doublets, as has been recently put forward by Glozman and collaborators. We show that this is not the case, and we exhibit some of the pitfalls of trying to link the operator product expansion to the hadron spectrum. We illustrate our arguments within a model for scalar and pseudo-scalar mesons used recently by Shifman to argue for parity doubling. We find that, whatever the experimental situation may be, there is no theoretical basis for parity doubling based on the use of the operator product expansion.

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1 Introduction

In recent years there has been a renewed interest in the possibility that “chiral symmetry restoration” takes place for the higher excited hadronic states of a given spin and isospin. For each spin and isospin, resonances occur with both parities, and if chiral symmetry, $SU(n_f)_L \times SU(n_f)_R$, would be linearly realized these resonances would have to organize themselves in equal-mass pairs of opposite parity. Of course, it is well known that chiral symmetry is spontaneously broken, and that there is therefore no symmetry reason that such parity pairing should occur. This observation goes back to the work of Ref. [1], and was recently re-emphasized in the language of effective field theory in Ref. [2].

However, one may ask whether there could be any other dynamical reason that approximately degenerate parity pairs occur in QCD. Loosely speaking, the intuition would seem to be that for highly excited resonances the physics of the vacuum (which breaks chiral symmetry) is less important. Such resonances would then couple less strongly to pions, and possibly organize themselves in approximate parity pairs.

Here we examine one such dynamical scenario. The basic observation is that, since chiral symmetry breaking is a non-perturbative phenomenon, and QCD is asymptotically free, chiral symmetry is restored at high energies, and, mutatis mutandis, for highly excited resonances [4]. To our knowledge, this has been most concretely explored in Ref. [5] in the context of the operator product expansion (OPE).

In order to be precise, we begin with setting the stage for our discussion. First, in order to give an unambiguous meaning to the mass of a resonance, we will consider the limit in which $N_c$, the number of colors, is taken to infinity. Otherwise highly excited resonances merge into the perturbative continuum, due to their finite widths, and their individual identity is lost. In contrast, in the limit of infinite $N_c$, all resonances are stable, and spectral functions are (infinite) sums over delta functions. Second, we need a definition of what it means for chiral symmetry to be restored for highly excited resonances. We consider a tower of resonances of given spin, isospin and parity, with masses $M_{S_n}$, and the corresponding tower with the same spin and isospin, but opposite parity, with masses $M_{P_n}$. For definiteness, we will consider scalar and pseudo-scalar mesons, as was done in Ref. [5]. The integer $n$ labels the successive states in each tower. We will now assume that for each $S$ state with label $n$, a closest $P$ state can be identified, and for the sake of convenience we will use the same label $n$ for this $P$ state. We now define as a measure of chiral symmetry the “spectral overlap” $\rho(n)$ by

$$\rho(n) = \frac{M_{P_n} - M_{S_n}}{M_{S_{n+1}} - M_{S_n}}. \quad (1)$$

The choice of denominator here deserves some comment. Obviously, we want $\rho(n)$ to be dimensionless, but we do not want $\rho(n)$ to go to zero for large $n$ trivially. Therefore, the denominator is taken to be the smallest possible quantity with dimension of mass

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1For a recent discussion of other possibilities, see Ref. [3].
2Our arguments extend easily to the example of vector and axial-vector mesons.
3In other words, we assume that approximate parity doublets can be identified, at least for large enough $n$. 
made out of the masses with “resonance number” approximately equal to \( n \). “Chiral symmetry restoration” for a particular spin and isospin takes place if \( \rho(n) \) goes to zero for large \( n \).

Another measure of “chiral symmetry restoration” considered in the literature \([6, 7]\) is the “chiral asymmetry”
\[
\chi(n) = \frac{M_{P_n} - M_{S_n}}{M_{P_n} + M_{S_n}}.
\]
(2)
This is not a good measure of chiral symmetry restoration, however, because of the choice of the denominator. We know that the resonance masses will increase with \( n \), and thus this denominator will grow with \( n \). That alone will make \( \chi \) go to zero for large \( n \), as long as the numerator grows less fast — in fact, typically one would envisage defining parity pairs such that the numerator stays finite for large \( n \). Clearly, in order to avoid this problem, one should measure the difference in mass between potential parity partners relative to the overall mass density (per unit of \( n \)), as is done by the quantity \( \rho(n) \). We will therefore not consider \( \chi(n) \) in this paper.

The claim made in Ref. \([5]\) can now be expressed as follows: The slowest pattern of chiral symmetry restoration (for the towers of scalar and pseudo-scalar mesons) allowed by the asymptotic behavior of the OPE is given by
\[
|\rho(n)| \sim \frac{1}{n}, \quad n \to \infty.
\]
(3)
It was assumed that the asymptotic behavior for large \( n \) in both the \( S \) and \( P \) channels is Regge like, \( M_{S_n,P_n}^2 \sim n\Lambda^2 \). If Eq. (3) would be true, then clearly it would be appropriate to conclude that chiral symmetry is restored for highly excited resonances (in this channel). This result was inferred from a superconvergence relation,
\[
\sum_{n} \delta M_n^2 = 0,
\]
(4)
in which \( \delta M_n^2 = M_{S_n}^2 - M_{P_n}^2 \). Superconvergence relations of this type were also assumed to be valid in Refs. \([8, 9]\).

In this note, we show, within the context of the same assumptions (large \( N_c \) and Regge behavior), that these claims do not follow from the OPE.\(^4\) To clarify this point, we will, within the context of the same assumptions, exhibit a model of scalar and pseudo-scalar meson resonances which is consistent with what is known about the OPE, and for which \( \rho(n) \) approaches a non-zero constant for large \( n \).

This paper is organized as follows. We begin with reviewing the argument presented in Ref. \([5]\) in the following section, and we point out why in general it is incorrect. We then remind the reader of the fact that large-\( N_c \) QCD in two dimensions satisfies the assumptions, but violates the claims. It therefore is a good counter example, and our explicit calculation of \( \rho(n) \) for this model invalidates statements about this model in

\(^4\)We have pointed out before that superconvergence relations such as Eq. (4) cannot be derived from the OPE \([12]\). However, in view of the recent claims which keep appearing in the literature, it seems worthwhile revisiting this issue once more.
Ref. [6]. In Sec. 4, we construct a model of scalar and pseudo-scalar resonances in four dimensions which further demonstrates that the claims of Ref. [5] do not have to hold. In our concluding section, we comment on the situation at finite $N_c$.

2 Scalar and pseudo-scalar mesons

Following Ref. [5], we start with the scalar and pseudo-scalar two-point functions

$$\Pi_{S,P}(q^2) = i \int dx \, e^{iqx} \langle 0 | T \left\{ J_{S,P}(x) J_{S,P}^\dagger(0) \right\} | 0 \rangle,$$

with $J_S(x) = \overline{d}(x) u(x)$ and $J_P(x) = \overline{d}(x)i\gamma_5 u(x)$, and consider a once-subtracted dispersion relation for one-half the difference of these two correlation functions:

$$\Pi_{S-P}(q^2) = \frac{q^2}{\pi} \int_0^\infty dt \, \frac{1}{2} (\text{Im} \, \Pi_S(t) - \Pi'_P(t)) + \frac{B^2 f_0^2}{-q^2 - i\epsilon} + \Delta \Pi(0),$$

where the prime indicates that we do not include the pion pole inside the integral, instead writing it as a separate term. We work in the chiral limit, $f_0$ is the pion decay constant in that limit (approximately 87 MeV), and $B = -\langle \bar{\psi} \psi \rangle / f_0^2$. We note that there is no singularity in the integrals at $t = 0$ because the spectral functions $\frac{1}{\pi} \text{Im} \, \Pi_S(t)$ and $\frac{1}{\pi} \text{Im} \, \Pi'_P(t)$ vanish at $t = 0$. The subtraction constant $\Delta \Pi$,

$$\Delta \Pi(0) \equiv \frac{1}{2} (\Pi_S(0) - \Pi'_P(0)) = 16B^2L_8$$

is fixed by chiral symmetry, where $L_8$ is one of the order-$p^4$ couplings in the pion effective theory [10].

Working at $N_c = \infty$, there is an infinite tower of infinitely narrow resonances in each channel, and we thus have that

$$\frac{1}{\pi} \text{Im} \, \Pi_S(t) = 2 \sum_n^\infty F_{S_n}^2 \delta(t - M_{S_n}^2),$$

$$\frac{1}{\pi} \text{Im} \, \Pi'_P(t) = 2 \sum_n^\infty F_{P_n}^2 \delta(t - M_{P_n}^2).$$

Furthermore, we will assume Regge-like behavior asymptotically in $n$ [11], i.e.

$$M_{S_{n,P_n}}^2 = n\Lambda^2 + \ldots, \quad n \to \infty,$$

with $\Lambda \sim 1$ GeV, and we also require that

$$F_{S_{n,P_n}}^2 \sim \Lambda^2 M_{S_{n,P_n}}^2, \quad n \to \infty.$$
in order guarantee the correct parton-model behavior.\textsuperscript{5} Substituting these expressions into Eq. (3), one obtains

\[ \Pi_{S-P}(q^2 = -Q^2) = \Delta \Pi(0) - \frac{B^2 f_0^2}{Q^2} - Q^2 \sum_n \left( \frac{F_{S_n}^2}{M_{S_n}^2 (Q^2 + M_{S_n}^2)} - \frac{F_{P_n}^2}{M_{P_n}^2 (Q^2 + M_{P_n}^2)} \right), \]

which is finite, with the masses and residues of Eqs. (9) and (10).

Next, we define the mass splitting in a parity pair (for convenience we label both the S and P states of the parity pair with the same resonance number \( n \), as already mentioned in the previous section)

\[ \delta M_n^2 = M_{S_n}^2 - M_{P_n}^2, \]

and we assume that \( \delta M_n^2 \) grows less rapidly than \( M_n^2 \equiv (M_{S_n}^2 + M_{P_n}^2) / 2 \) for \( n \to \infty \). Expanding Eq. (11) in \( \delta M_n^2 \) then yields, with \( F_{S_n,P_n}^2 = \kappa \Lambda^2 M_{S_n,P_n}^2 \), where \( \kappa \) is a proportionality constant fixed by the parton-model logarithm,

\[ \Pi_{S-P}(q^2 = -Q^2) = \Delta \Pi(0) - \frac{B^2 f_0^2}{Q^2} + \kappa \Lambda^2 Q^2 \sum_n \frac{\delta M_n^2}{(Q^2 + M_n^2)^2} \left( 1 + O \left( \frac{\delta M_n^2}{M_n^2} \right) \right). \]

This is to be compared with the leading term for large \( Q^2 \) in the OPE for this two-point function:

\[ \Pi_{S-P}(q^2 = -Q^2) \sim \frac{\langle \bar{\psi} \psi \rangle^2}{Q^4}, \quad Q^2 \to \infty \]

up to logarithmic corrections. From the comparison between Eqs. (13) and (14), Ref. [5] concludes that

\[ \sum_n \delta M_n^2 < \infty \]

(and in fact that it vanishes), to avoid \( 1/Q^2 \) terms in the OPE, and, from this, that

\[ \delta M_n^2 \sim (-1)^n \frac{\Lambda^2}{n}, \quad n \to \infty \]

is the slowest possible fall off with \( n \). Equations (16) and (9) then lead to Eq. (3).

Both these conclusions are unfounded. First, Eq. (15) assumes that the sum over \( n \) in Eq. (13) commutes with the expansion in powers of \( 1/Q^2 \), and hence that the sum \( \sum_n \delta M_n^2 \) converges.\textsuperscript{6} In writing Eq. (13) we assumed that the sum in that equation converges, which is indeed the case if \( \delta M_n^2 \) increases less rapidly than \( n \). But clearly this does not imply that \( \sum_n \delta M_n^2 \) converges, much less that it vanishes. With the

\textsuperscript{5} We ignore \( \alpha_s \) corrections to the parton-model logarithm, and, likewise, the fact that \( \Pi_S \) and \( \Pi_P \) have non-trivial anomalous dimensions.

\textsuperscript{6} We refer to Ref. [11] for a detailed discussion on how the sums in Eq. (8) should be regulated so as to be consistent with chiral symmetry. Obviously, the regulator should be taken to infinity \textit{before} taking \( Q^2 \) — a physical scale — large [12].
superconvergence relation gone, there is also no ground for the second claim, Eq. (16). Asymptotic behavior like that of Eq. (15) may of course be consistent with the absence of $1/Q^2$ terms in the OPE, but it certainly does not follow from it.

Second, we also see immediately that demanding that $\sum_n \delta M_n^2$ be finite leads to inconsistent physics, because it would follow that $\lim_{Q^2 \to \infty} \Pi_{S-P}(Q^2)$ is equal to $\Delta \Pi(0)$. The reason this is inconsistent is that chiral symmetry at low energy predicts that this constant does not vanish according to Eq. (7), while perturbation theory predicts that $\Pi_{S-P}(Q^2)$ should vanish for $Q^2 \to \infty$ (cf. Eq. (14)), expressing the fact that perturbation theory does not know about chiral symmetry breaking.

All these problems get resolved by the simple fact that the assumptions made about the scalar and pseudo-scalar two-point functions (large-$N_c$, Regge behavior and parton-model behavior) can be fully consistent with the OPE, regardless of what happens to $\sum_n \delta M_n^2$. We will demonstrate this by explicit example in Sec. 4.

3 Absence of parity pairs in the ’t Hooft model

Before getting to our example, it is instructive to consider the situation in a simple solvable model, two-dimensional QCD at $N_c = \infty$ [13]. The asymptotic behavior of the meson spectrum in this model is known: at large resonance number, the masses scale like

$$M_n^2 \sim n \Lambda^2, \quad n \to \infty,$$

(17)

and resonances of opposite parity alternate: resonances with odd $n$ have the same parity, which, in turn, is opposite to that of resonances with even $n$.

It follows that the numerator of Eq. (1) is given by

$$M_{n+1} - M_n = M_{n+1}^2 - M_n^2 \sim \frac{\Lambda}{2\sqrt{n}}, \quad n \to \infty,$$

(18)

while the denominator is given by

$$M_{n+2} - M_n \sim \frac{\Lambda}{\sqrt{n}}, \quad n \to \infty.$$

(19)

In the ’t Hooft model, therefore,

$$\lim_{n \to \infty} |\rho(n)| = \frac{1}{2}.$$  

(20)

While mass differences decrease as a function of $n$, there is clearly no way in which parity pairs can be unambiguously identified. This is reflected in the result for the spectral overlap, Eq. (20): the value $1/2$ is the maximum possible value that $\rho(n)$ can have. This means that asymptotically the pseudo-scalar states lie precisely halfway two successive scalar states.\footnote{A larger value would imply that parity pairs have been misidentified.} In this sense, chiral symmetry is maximally broken by all meson resonances in the ’t Hooft model, unlike what is claimed in Refs. [6, 7].
We note that the quantity \( \chi(n) \), defined in Eq. (2), goes like
\[
\chi(n) \sim \frac{1}{4n}, \quad n \to \infty,
\]
simply because the denominator of \( \chi(n) \) goes like \( 2\sqrt{n}\Lambda \). The spectrum gets more crowded with higher \( n \), and thus naturally the pseudo-scalar states get more squeezed between the scalar states. The 't Hooft model thus confirms that \( \chi(n) \) is not a good measure of chiral symmetry restoration.

4 A counter example in four dimensions

Returning to four dimensions, we will now construct an example of a mass spectrum which satisfies all OPE constraints, but which does not satisfy Eqs. (15) and (16). Our model will be that of Sec. 2, with the specific choice
\[
M^2_{S,P,n} = m^2_{S,P} + n\Lambda^2, \quad n = 1, \ldots, \tag{22}
\]
\[
F^2_{S,P,n} = \kappa\Lambda^2 M^2_{S,P,n}, \quad n = 1, \ldots, \tag{22}
\]
where \( \kappa \) is a numerical constant, equal to \( N_c/32\pi^2 \) in large-\( N_c \) QCD, and we choose \( m_S \neq m_P \). In the pseudo-scalar sector, there is an additional massless pion pole, with residue \( 2B^2f_0^2 \) (cf. Eq. (6)).

Using
\[
\lim_{N \to \infty} \sum_{n=1}^{N} \left( \frac{1}{z + n} - \frac{1}{n} \right) = -\psi(z) - \frac{1}{z} - \gamma_E, \tag{23}
\]
in which \( \gamma_E \) is the Euler–Mascheroni constant and \( \psi(z) \) is the digamma function
\[
\psi(z) = \int_0^\infty dt \left( \frac{e^{-t}}{t} - \frac{e^{-zt}}{1 - e^{-t}} \right) = \frac{d}{dz} \log \Gamma(z), \tag{24}
\]
we find that
\[
\Pi_{S-P}(q^2 = -Q^2) = \Delta\Pi(0) - \frac{B^2f_0^2}{Q^2} + \kappa Q^2 \left[ \psi \left( \frac{Q^2 + m^2_S}{\Lambda^2} \right) - \psi \left( \frac{Q^2 + m^2_P}{\Lambda^2} \right) + \frac{\Lambda^2}{Q^2 + m^2_S} - \frac{\Lambda^2}{Q^2 + m^2_P} \right] \tag{25}
\]
\[
= \Delta\Pi(0) + \kappa(m^2_S - m^2_P) - \left( B^2f_0^2 + \frac{1}{2} \kappa(m^4_S - m^4_P + \Lambda^2(m^2_S - m^2_P)) \right) \frac{1}{Q^2}
\]
\[
+ \kappa \left( \frac{1}{3} m^6_S - m^6_P \right) + \frac{1}{2} \Lambda^2(m^4_S - m^4_P) + \frac{1}{6} \Lambda^4(m^2_S - m^2_P) \right) \frac{1}{Q^4} + O \left( \frac{1}{Q^6} \right),
\]
where we used
\[
\psi(z) = \log z - \frac{1}{2z} - \sum_{n=1}^{\infty} \frac{B_{2n}}{2nz^{2n}}, \tag{26}
\]
valid for \( \text{Re} \, z > 0 \) (\( B_{2n} \) are the Bernoulli numbers). Requiring that this be equal to the leading OPE expression, 
\[-6\pi\alpha_S \langle \bar{\psi}\psi \rangle^2 / Q^4,\]
leads to the relations
\[
16B^2 L_s = \kappa (m^2_P - m^2_S), \tag{27}
\]
\[
B^2 f_0^2 = \frac{1}{2} \kappa (m^4_P - m^4_S + \Lambda^2 (m^2_P - m^2_S)),
\]
\[
6\pi\alpha_S \langle \bar{\psi}\psi \rangle^2 = \frac{1}{6} \kappa \left(2(m^6_P - m^6_S) + 3\Lambda^2 (m^4_P - m^4_S) + \Lambda^4 (m^2_P - m^2_S)\right),
\]
where we used Eq. (7). Clearly, our model wants \( m^2_P \) to be larger than \( m^2_S \), which is not inconsistent with what is known about the lowest lying resonances in the scalar and pseudo-scalar channels.

This example demonstrates several facts. First, in this model, \( \delta M^2_n = m^2_S - m^2_P \) for all \( n = 1, \ldots, \infty \), and therefore \( \sum_{n=1}^{\infty} \delta M^2_n \) does not converge. Nevertheless, there is no clash with the OPE. Second, the coefficients of the OPE appearing in Eq. (27) are related to low-energy constants and know about resonances at all scales, and not just about those for asymptotically large \( n \). Clearly, if we would include other low-lying resonances which are not part of the equally-spaced towers, their masses and residues would show up in these sum rules as well. Our example satisfies all the assumptions of Ref. [5], but invalidates the claims. In particular, it is straightforward to see that this model does not show chiral symmetry restoration:
\[
\lim_{n \to \infty} \rho(n) = \min \left\{ \frac{m^2_P - m^2_S}{\Lambda^2}, \frac{m^2_S + \Lambda^2 - m^2_P}{\Lambda^2} \right\}
\tag{28}
\]
(assuming that \( m^2_S < m^2_P < m^2_S + \Lambda^2 \)).

## 5 Discussion

In this paper, we have restricted ourselves to \( N_c = \infty \), because in that case all resonances in a certain channel are stable, and their masses are thus well-defined. Within this framework, we have demonstrated that from what we know about the short-distance behavior of QCD (in this limit) nothing can be inferred about parity doubling in the hadron spectrum. The basic reason for the difficulty comes from the following pitfall: even though it is true that perturbation theory tells us that chiral symmetry breaking effects die off at high energy, this is true only in the euclidean regime. The spectrum lies in the Minkowski regime, and perturbation theory clearly fails badly for infinite \( N_c \), predicting a smooth continuum where there is in fact an infinite tower of Dirac delta functions. The lack of a straightforward connection between the two regimes is at the origin of the difficulty. We illustrated the absence of parity doubling in detail in the context of a Regge-like model for scalar and pseudo-scalar mesons which satisfies both the requirements of large \( N_c \) in the Minkowski regime and those of leading-order perturbation theory in the euclidean regime. We see no reason why the situation would be different in channels with different spin and isospin.

A further fact illustrated by our example is that the coefficients of the OPE know about all resonances in a certain channel, and not only about the asymptotic behavior
for large $n$. This is as one would expect from the OPE, which can be used to connect information about hadron spectra as a whole to the behavior of correlation functions deep in the euclidean regime. The fact that this connection must work constrains the possibilities for the spectrum quite generally. For instance, in Sec. 2 we showed that a superconvergence relation of the form of Eq. (4) cannot hold for large $N_c$, because it is in conflict with perturbation theory.

In the model we considered in Sec. 4, potential candidates for parity doublers can still be identified, because the slope in scalar and pseudo-scalar channels is taken to be equal, as was done in Ref. [5]. This leads to a definite prediction for $\rho(n)$. However, we do not really know whether the slope in the scalar and pseudo-scalar channels is equal, and one way to generalize the model is to allow for $\Lambda_S \neq \Lambda_P$. Even in this case, a set of sum rules generalizing Eq. (27) can still be derived, and thus there is no conflict with the OPE. But in this case it clearly makes little sense to try identify potential parity partners, let alone define $\rho(n)$.

Another important generalization is to go to the world of finite $N_c$. Assuming that at finite but large $N_c$ the masses of the model of Sec. 4 pick up decay widths $\Gamma_n$ which scale like $\sim \sqrt{n}/N_c$ [14], it can be shown [14, 15] that as a consequence in this model the spectral function $\frac{1}{\pi} \text{Im} (\Pi_S(t) - \Pi_P(t))$ exhibits a fall off $\sim 1/t^2$, for large $t$.[5] It would however simply be incorrect to interpret this fall off in the spectral function as a sign of chiral symmetry restoration between the scalar and pseudo-scalar channels. It is just the growing of the widths with excitation number which causes this effect, by washing out the delta functions in Eq. (8), and thus increasing the overlap between opposite-parity doublets. The mass spectrum (defined as the location of the real part of the masses) is still given by Eq. (22), and no parity alignment takes place.

While the details of this argument are based on the finite-$N_c$ version of our model, it is generally expected that broadening of resonances toward larger $n$ takes place also in QCD. This smoothens the spectral functions, ameliorating the connection to the perturbative regime, and one expects that the difference of the $S$ and $P$ spectral functions will fall off bounded by some power of $t$ also in the real world. However, our model demonstrates that this does not imply that chiral symmetry restoration has to take place. The slowest possible pattern of chiral symmetry restoration compatible with the OPE is no chiral symmetry restoration at all.

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8For $\Pi_{V-A}$ one finds a fall off $1/t^3$ [15], but a similar analysis gives a fall off $1/t^2$ for $\Pi_{S-P}$.
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