Hierarch from Baryogenesis

Leonardo Senatore*

Center for Theoretical Physics,
Massachusetts Institute of Technology, Cambridge, MA 02139, USA

Abstract

We study a recently proposed mechanism to solve the hierarchy problem in the context of the landscape, where the solution of the hierarchy problem is connected to the requirement of having baryons in our universe via Electroweak Baryogenesis. The phase transition is triggered by the fermion condensation of a new gauge sector which becomes strong at a scale $\Lambda$ determined by dimensional transmutation, and it is mediated to the standard model by a new singlet field. In a “friendly” neighborhood of the landscape, where only the relevant operators are “scanned” among the vacua, baryogenesis is effective only if the higgs mass $m_h$ is comparable to this low scale $\Lambda$, forcing $m_h \sim \Lambda$, and solving the hierarchy problem. A new CP violating phase is needed coupling the new singlet and the higgs field to new matter fields. We study the constraints on this model given by baryogenesis and by the electron electric dipole moment (EDM), and we briefly comment on gauge coupling unification and on dark matter relic abundance. We find that next generation experiments on the EDM will be sensitive to essentially the entire viable region of the parameter space, so that absence of a signal would effectively rule out the model.

1 Introduction

The most problematic characteristic of the Standard Model (SM) seems to be the smallness of its superrenormalizable couplings, the higgs mass $m_h^2$, and the cosmological constant $\Lambda_c$, with respect to the apparent cutoff of the theory, the Planck mass, $M_{pl}$. These give rise respectively to the hierarchy, and cosmological constant problems. According to the naturalness hypothesis, the smallness of these operators should be understood in terms of some dynamical mechanism, and this has been a driving motivation in the last two decades in the high energy theory community.

In the last few years, several things have changed.

Concerning the hierarchy problem, experimental investigation has shown that already many of the theoretically most attractive possibilities for the stability of the weak scale, such as supersymmetry, begins to be disfavored, or present at least some fine tuning issues [1,2].

Concerning the cosmological constant problem, there was the hope that some symmetry connected to quantum gravity would have forced the cosmological constant to be zero. However, first,
cosmological observations have shown evidence for a non zero cosmological constant in our present universe \cite{3, 4}; second, from the string theory point of view, two main things have occurred: on the one hand, consistent solutions with a non zero cosmological constant have been found \cite{5, 6, 7, 8, 9}, and, on the other hand, it has been becoming more and more clear that string theory appear to have a huge landscape of vacua, each one with different low energy parameters \cite{10, 12}.

If the landscape is revealed to be real, it would force a big change in the way physics has to be done, and some deep questions may find a complete new answer. In particular, it is conceivable that some characteristics of our low energy theory are just accidental consequences of the vacua in which we happen to be. This is a very big step from the kind of physics we are used to, and its consequences have been explored recently in \cite{11}. There, it is shown that the presence of a landscape offers a new set of tools to address old questions regarding the low energy effective theory of our universe. On one hand, there are statistical arguments, according to which we might explore the statistically favored characteristics for a vacuum in the landscape \cite{12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22}. On the other hand, there are also some selection rules due to anthropic reasoning, which we might have to impose on the vacuum, in order for an observer to be present in it. Pioneering in this, is Weinberg’s “structure principle” \cite{23}, which predicts the right order of magnitude for the cosmological constant starting from the requirement that structures should have had time to form in our universe in order for life to be present in it.

At this point, it is necessary to speak about predictivity for theories formulated with a lot of different vacua, as it occurs in the landscape. There are two important points that greatly affect the predictivity of the theory. First, we must know how the characteristic of the low energy theory change as we scan among the vacua. Second, we must know also how the probability of populate such a vacuum changes among the vacua, also considering the influence of the cosmological evolution. These are very deep questions whose answer in general requires a full knowledge of the UV theory, and we do not address them here. However, there is still something we can do. In fact, as it was pointed out in \cite{11}, theories with a large number of vacua can be described in an effective field theory approach. In such a study, it was shown that it is very natural for parameters not to effectively scan in the landscape, unless their average value in the landscape is null. So, it is reasonable to assume that some parameters in the landscape do scan, and some do not. Among the parameters of the theory, the relevant operators have a particular strong impact on the infrared properties of the theory. But exactly because of this, if they are the only ones to scan, their value can be fixed by environmental arguments. This is however true only if the marginal couplings do not scan. For this reason, a neighborhood of the landscape in which only the relevant parameters are scanning, is called a “friendly neighborhood”, because it allows to fix the value of the relevant couplings with environmental reasoning, while the marginal parameters do not scan, and so they are analyzed with the usual instruments of physics, judging them on the basis of their simplicity and of their correlations. We do not know if the true string theory landscape have these properties. However, there is some phenomenological motivation to expect that this could well be. In fact, in the physics connected to the standard model, we have been able to successfully address question concerning the marginal parameters with dynamical reasoning, deducing some striking features as gauge coupling unification, chiral symmetry breaking, or the weakly interacting dark matter, while, on the contrary, we have been having big troubles concerning the problems connected to the relevant coupling in the standard model, the cosmological constant, and the higgs scale, both of which arise large fine tuning issues. In this paper, we want to address the hierarchy problem of the electroweak scale assuming we are living in such a ”friendly neighborhood” of the landscape. Because of this, we concentrate a little more on the predictivity of a phenomenological model based on this. As it is clear, predictivity in the landscape is very much enhanced if there is an infrared fragile feature \cite{11}, which must be realized in the universe in order for it not to be lethal, and which is however difficult to realize in
the scanned landscape. Exactly the fact that a necessary fragile feature is difficult to realize gives a lot of constraints on the possible vacua in which we should be, or, in other words, on the parameters of the theory, making then the model predictive. An example of a fragile feature is the presence of structures in the universe. According to Weinberg’s argument [23], if we require the fact that in the universe there should be structures, than the value of the cosmological constant is very much tighten to a value close to the observed one. The presence of structures can then also be used to anthropically require the lightness of the higgs field in the case dark matter particles receive mass from electroweak symmetry breaking [11, 24].

In this paper, we concentrate on another fragile feature which we consider necessary for the development of any sort of possible life in the universe, and which is a necessary completion on Weinberg’s structure principle. In the absence of baryons, the dark matter would just form virilized structures, and not clumped structures, which instead are necessary for the development of life. We can construct a model where baryons are explicitly a fragile feature. Then, we can connect the solutions of the problem of baryons to the hierarchy problem with a simple mechanism which imposes a low energy electroweak scale in order for baryons to be present in the universe. This is the line of thought implemented in [11], which is the model on which we focus in this paper. As we will explain in detail in the next section, the mechanism is based on the fact that baryogenesis is possible in the model only if the electroweak scale is close to a hierarchical small scale. The hierarchical small scale is naturally introduced setting it equal to the scale at which a new gauge sector of the theory becomes strong through dimensional transmutation. This mechanism naturally provides a small scale. In order to successfully implement baryogenesis, we will also need to add some more CP violation. This will force the presence of new particles and new couplings. The model becomes also predictive: the lightest of these particle will naturally be a weakly interacting particle at the weak scale, and so it will be a very good candidate for dark matter; further, the new CP violating terms will lead to the prediction of a strong electron electric dipole.

The main purpose of this paper is to investigate in detail the mechanism in which baryogenesis occurs in this model, and the consequent predictions of the model. In particular, we will find that next generation experiments of the electron Electric Dipole Moment (EDM), together with the turning on of LHC, are going to explore the entire viable region of the parameter space, constraining it to such a peculiar region of the parameter space, so that absence of a signal would result in ruling out the model.

The paper is organized as follows: in sec. II, we explain in detail the model; in sec. III we study the amount of baryons produced; in sec. IV we determine the electron EDM; in sec. V we briefly comment on Dark Matter and Gauge Coupling Unification; and in sec. VI, we draw our conclusions.

2 Hierarchy from Baryogenesis

In this section, following [11], we show how we can connect the electroweak scale to a scale exponentially smaller than the cutoff.

As the presence of baryons is a necessary condition for the formation of clumped structures, it is naturally to require that the vacuum in which we are should allow for the formation of a net baryon number in the universe. This is not a trivial requirement. In fact, in the early hot phase of the universe, before the electroweak phase transition, baryon number violating interactions through weak sphalerons were unsuppressed, with a rate given approximately by [25]:

\[ \Gamma_{ws} = 6k\alpha_w^5T \]  

(1)

where \( k \approx 20 \), with the consequence of erasing all previously generated baryon asymmetry (if
no other macroscopic charges are present). However, at the electroweak phase transition, all the Sakharov’s necessary conditions for generating a baryon asymmetry are satisfied, and so it is possible in principle to generate a net baryon number at the electroweak phase transition, through a process known in the literature as electroweak baryogenesis (see [27, 28] for two nice reviews). However, it is known that the SM electroweak phase transition can not, if unmodified, generate the right amount of baryons for two separate reasons. On one hand, CP violating interactions are insufficient [29], and, on the other hand, the phase transition is not enough first order for preventing weak sphalerons to be active also after the phase transition [30]. Since this last point is very important for our discussion, let us review it in detail.

Let us suppose in some early phase of the universe we have produced some initial baryon number \( B \) and lepton number \( L \), and no other macroscopic charge. In particular \( B - L = 0 \). The quantum number \( B + L \) is anomalous, and the equation for the abundance of particles carrying \( B + L \) charge is given by:

\[
\frac{d}{dt} \left( \frac{n_{B+L}}{s} \right) = -\Gamma \left( \frac{n_{B+L}}{s} \right)
\]

where \( n_{B+L} \) is the number of baryons and leptons per unit volume, \( s \) is the entropy density \( \sim T^3 \), and \( \Gamma \) is related to the sphaleron rate [27], \( \Gamma \sim \frac{13}{2} N_f T^3 e^{-\left(\frac{4\pi}{g} v(T)^3 \sqrt{2} \right)} \), where \( v(T) \) is the temperature dependent higgs vev (\( v(T_0) = 246 \) GeV as measured in our vacuum), \( N_f \) is the number of fermionic families, and \( g \) is the SU(2) weak coupling. The reason why reactions destroying baryons are faster than reactions creating them is due to the fact that the relative reaction rate goes as:

\[
\frac{\Gamma_+}{\Gamma_-} \sim e^{\Delta f}
\]

where \( \Delta f \) is the difference in free energy, and \( \Gamma_\pm \) are the sphaleron rates in the two directions. Now, it is easy to see that the free energy grows with the chemical potential \( \mu_B \), which then grows with the number of baryons \( n_B \). In the limit of small difference, we then get eq. (2).

This differential equation can be integrated to give:

\[
\left( \frac{n_{B+L}}{s} \right)_{\text{final}} \simeq \left( \frac{n_{B+L}}{s} \right)_{\text{initial}} \exp \left( -\frac{N_f \sqrt{2} m_{pl}}{g_s v(T_c)} e^{-\left(\frac{4\pi}{g} v(T_c) \sqrt{2} T_c \right)} \right)
\]

where \( T_c \) is the critical density, and where \( g_s \) is the number of effective degrees of freedom \( \sim 55 \).

If in the universe there is no macroscopic lepton number, than clearly at present time we would have no baryons left, unless:

\[
\frac{N_f \sqrt{2} m_{pl}}{g_s v(T_c)} e^{-\left(\frac{4\pi}{g} v(T_c) \sqrt{2} T_c \right)} \lesssim 1
\]

which roughly implies the constraint:

\[
\frac{v(T_c)}{T_c} \gtrsim 1
\]

This is the so called ”baryons wash out”, and the origin of the requirement that the electroweak phase transition should be strongly first order.

Note that this is the same condition we would get if we required sphaleron interactions not to be in thermal equilibrium at the end of the phase transition:

\[
\frac{\Gamma}{T_c^3} < H
\]
implies roughly:
\[ T_c e^{-\frac{4\pi v(T_c)}{92\sqrt{2} T_c}} < \frac{T_c^2}{M_{pl}} \]  
(8)
and so
\[ \frac{v(T_c)}{T_c} \gtrsim 1 \]  
(9)
Also note that the requirement for the sphalerons not to be in thermal equilibrium already just after the phase transition is necessary, as otherwise we would have a baryon symmetric universe, which leads to a far too small residual relic density of baryons.

Later on, when we shall study electroweak baryogenesis, we shall get a number for the baryon number. In order to get then the baryon number at, let us say, Big Bang Nucleosynthesis (BBN), we need to multiply that number by the factor in eq. (8). In the next sections, we shall consider the requirement in eq. (6) fulfilled, and we will consider completely negligible the wash out from the sphalerons coming from eq. (4).

Now, let us go back to the higgs phase transition, and let us assume that in the neighborhood of the landscape in which we are, all the high energy mechanisms for producing baryons have been shut down. This is easy to imagine if, for example, the reheating temperature is smaller than the GUT scale. We are then left with the only mechanism of electroweak baryogenesis. From the former discussion, it appears clear that there should be some physics beyond the SM to help to make the phase transition strong enough.

We may achieve this by coupling the higgs field to a singlet \( S \), with the potential equal to:
\[
V = \lambda S^4 + \lambda_h (h^\dagger h - \tilde{\lambda} S^2)^2 + \frac{1}{2} m_S^2 S^2 + m_h^2 h^\dagger h
\]  
(10)
where we have assumed a symmetry \( S \rightarrow -S \). We can couple this field to two fermions \( \Psi, \Psi^c \), which are charged under a non-Abelian gauge group through the interaction
\[ k_S S \Psi \Psi^c \]  
(11)
In order to preserve the symmetry \( S \rightarrow -S \), we give the fields \( \Psi, \Psi^c \) charge \( i \). We can then assume that this sector undergoes confinement and chiral symmetry breaking at its QCD scale determined by dimensional transmutation
\[ < \Psi \Psi^c > \sim \Lambda^3 \]  
(12)
which is naturally exponentially smaller than the cutoff of the theory. We assume that this phase transition is first order, so that departure from thermal equilibrium is guaranteed.

Now, following our discussion in the introduction, suppose that, scanning in the landscape, the only parameters which are effectively scanned are the relevant couplings \( m_h \) and \( m_S \). If then we must have baryons in our universe so that clumped structures can form, than we need to be in the vacuum in which these two parameters allow for a strong enough first order phase transition in the electroweak sector. So, we have to require that this phase transition triggers the electroweak phase transition. It is clear that this can only be if it triggers a phase transition in the \( S \) field, which is possible then only if \( m_S \) is of the order of \( \Lambda \). Finally, the phase transition in \( S \) can trigger a strong first order phase transition in the higgs field only if again \( m_h \sim m_S \sim \Lambda \) (for a more detailed discussion, see next subsection). So, summarizing, we see that, the requirement of having baryons in the universe forces the higgs mass to be exponentially smaller than the cutoff, solving in this way the hierarchy problem.

In order to produce baryons, we still need to improve the CP violating interactions, that in the SM are not strong enough. We can minimally extend the introduced model to include a singlet \( s \) and...
2 SU(2) doublets $\Psi_{\pm}$, with hypercharge $\pm 1/2$ (notice that they have the same quantum numbers as higgsinos in the Minimal Supersymmetric Standard Model (MSSM)), with the following Yukawa couplings:

$$kSss + k'Ss\Psi + gh\Psi + g'h\Psi - s$$

There is then a reparametrization invariant CP violating phase:

$$\theta = \arg(kk'g^*g'')$$

The mass terms for this new fields can be prohibited giving proper charges to the fields $s$, $\Psi_{\pm}$. This implies that, since at the electroweak phase transition these new fermions get a mass of order of the electroweak scale, and also because of the fact that the lightest fermion is stable, we actually have a nice candidate for dark matter. This last point is a connection between the higgs mass, which, up to this point, we have just assured to be exponentially smaller than the cutoff, and the weak scale, but this connection will come out quite naturally later. So, if this model happens to describe our universe, what we should see at LHC should be the higgs, the two new singlets $S$ and $s$, and the two new doublets $\Psi_{\pm}$.

Since the model is particularly minimal, it is interesting to explore the possibility for generating the baryon number of the universe in more detail. Before doing so, however, let us see in more detail how the vevs of the higgs and of the singlet $S$ are changed by the phase transition.

### 2.1 Phase Transition more in detail

Before going on, here we show more in detail how the requirement of having a strong first order phase transition leads to have $m_S \sim m_h \sim \Lambda$.

In unitary gauge, the equations to minimize the potential are (from here on, we mean by $S$ also the vev of the field $S$; the meaning will be clear by the contest):

$$4\lambda S^3 - 4\lambda_h\bar{\lambda}S\left(\frac{v^2}{2} - \bar{\lambda}S^2\right) + m_S^2 S + k_S\Lambda^3 \Theta(T_c - T) = 0$$

$$2\lambda_h\left(\frac{v^2}{2} - \bar{\lambda}S^2\right)\frac{v}{\sqrt{2}} + 2m_h^2 \frac{v}{\sqrt{2}} = 0$$

where $T_c$ is the critical temperature. The first equation comes from the derivative with respect to $S$, and we will refer to it as $S$ equation, while for the other we will use the name $h$ equation.

We first consider the case $m_S^2 > 0$ and $m_h^2 > 0$. Before the phase transition, we have the minimum at the symmetric vacua:

$$S = 0, \ v = 0, \text{ for } T > T_c$$

For $T < T_c$, the minimum conditions change, and we can not solve them analytically. We can nevertheless draw some important conclusions. Let us consider the minimum equation for $S$, which is the only equation which changes. Let us first consider the case $m_S \gg \Lambda$. In this case, we can consistently neglect the cubic terms in the $S$ equation, to get:

$$S \simeq -\frac{k_S\Lambda^3}{(m_S^2 - 2\lambda\lambda_h v^2)}$$

Then, if $m_S^2 \gg 2\lambda\lambda_h v^2$, we have:

$$S \simeq -\frac{k_S\Lambda^3}{m_S^2}$$
while the other solutions are still unphysical, as:

\[ \frac{v^2}{2} \simeq -m_h^2 \frac{\lambda}{\lambda_h} + \tilde{\lambda} \frac{(k_S^2 \Lambda^6)}{m_S^2} < 0 \] (21)

If \( m_S^2 \ll 2\tilde{\lambda} \lambda_h v^2 \), then:

\[ S \simeq -\frac{k_S \Lambda^3}{2\lambda_h v^2} \] (22)

For the higgs, the non null solution is:

\[ v^2 \simeq -m_h^2 \frac{\lambda}{\lambda_h} + \tilde{\lambda} \left( \frac{k_S^2 \Lambda^6}{4\lambda^2 \lambda_h^2 v^4} \right) \] (23)

which has some relevant effect for the electroweak phase transition only if \( m_h \sim \Lambda \). But in that case \( v \sim \Lambda \), and from \( m_S^2 \ll 2\tilde{\lambda} \lambda_h v^2 \) we get that \( m_S^2 \ll \Lambda^2 \), in contradiction with our initial assumption.

Then, if \( \frac{m^2 - 2\tilde{\lambda} \lambda_h v^2}{m_S^2} \ll 1 \), in the \( S \) equation, we can consistently consider just the cubic term, to get:

\[ S \simeq -\frac{k_S^{1/3} \Lambda}{4^{1/3}(\lambda + \lambda \lambda_h)^{1/3}} \] (24)

then, the non null solution for \( v^2 \) becomes:

\[ \frac{v^2}{2} \simeq -m_h^2 \frac{\lambda}{\lambda_h} + \tilde{\lambda} \left( \frac{k_S^{2/3} \Lambda^2}{4^{2/3}(\lambda + \lambda \lambda_h)(\lambda + \lambda \lambda_h)} \right) \] (25)

which has some relevant effect only if \( m_h^2 \ll \Lambda^2 \). However, in this case the condition \( \frac{m^2 - 2\tilde{\lambda} \lambda_h v^2}{m_S^2} \ll 1 \) would imply \( \frac{m_S}{\Lambda} \ll 1 \), again in contradiction with our assumptions.

So, in order to have some effect on the higgs phase transition, we are left with the only possibility of having \( m_S \ll \Lambda \).

Restricting to this, consistently, we can neglect the linear term in the \( S \) equation, to have:

\[ 4\lambda S^3 - 4\lambda_h \tilde{\lambda} S \left( \frac{v^2}{2} - \tilde{\lambda} S^2 \right) = -k_S \Lambda^3 \] (26)

\[ 2\lambda_h \left( \frac{v^2}{2} - \tilde{\lambda} S^2 \right) = -2m_h^2 \] (27)

which implies the following equation for \( S \):

\[ 4\lambda S^3 + 4\tilde{\lambda} m_h^2 S = -k_S \Lambda^3 \] (28)

For \( m_h \gg \Lambda \), we have:

\[ S = -\frac{k_S \Lambda^3}{4\lambda m_h^2} \] (29)

and the non null solution for \( v \) is still not physical:

\[ \frac{v^2}{2} = -m_h^2 \frac{\lambda}{\lambda_h} + \tilde{\lambda} \left( \frac{k_S \Lambda^3}{4\lambda m_h^2} \right)^2 < 0 \] (30)
So, finally, we have that in order to have a strong first order phase transition triggered by the new sector, we are forced to have $m_h \ll \Lambda$, which is what we wanted to show. In detail, for $m_h \ll \Lambda$, we implement the following phase transition

$$S: 0 \rightarrow -\frac{k_S^{1/3} \Lambda}{(4\lambda)^{1/3}} \quad (31)$$

$$v: 0 \rightarrow 2^{1/3} \frac{\tilde{\lambda}^{1/2} k_S^{1/3}}{\lambda^{1/3}} \Lambda$$

Now, the final step is to show that, not only $m_h \ll \Lambda$ and $m_S \ll \Lambda$, but that actually $m_h \sim \Lambda$ and $m_S \sim \Lambda$. The argument for this is that, scanning in the landscape, it will be generically much more difficult to encounter light scalar masses, as they are fine tuned. So, in the anthropically allowed range, our parameters shall most probably be in the upper part of the allowed range. So, we conclude that this model predicts $m_h \sim m_S \sim \Lambda$, as we wanted to show.

This phase transition must satisfy the requirement that the sphalerons are ineffective ($\frac{v(T_c)}{T_c} \gtrsim 1$), after the phase transition, which occurs at, roughly, the critical temperature $T_c \sim \Lambda$. So:

$$\frac{v(T_c)}{T_c} \simeq \frac{\tilde{\lambda}^{1/2} (2k_S)^{1/3}}{\lambda^{1/3}} \gtrsim 1 \quad (32)$$

which can clearly be satisfied for some choices of the couplings. In order to better understand the natural values of the ratio $v(T_c)/T_c$ in terms of the scalar couplings, it is worth to notice that the coupling $\tilde{\lambda}$ appears in the lagrangian as always multiplied by $\lambda_h$. So, the coupling which naturally tends to be equal to the other ones in the lagrangian is $\lambda_e \equiv \lambda_h \tilde{\lambda}$. With this redefinition, we get the constraint:

$$\frac{v(T_c)}{T_c} \simeq \left( \frac{\lambda_e}{\lambda_h} \right)^{1/2} \left( \frac{2k_S}{\lambda} \right)^{1/3} \gtrsim 1 \quad (33)$$

which can clearly be satisfied with some choice of the parameters. It is also worth to write the ratio of the vevs of $S$ and $h$ after the phase transition:

$$\frac{S}{v} = \left( \frac{\lambda_h}{\lambda_e} \right)^{1/2} \quad (34)$$

We can also analyze the case in which $m_h^2 < 0$. In this case, the electroweak phase transition would occur before the actual strong sector phase transition. However, since we know that in the SM the phase transition is neither enough first order, nor enough CP violating, we still need, in order to have baryon formation, to require that the strong sector phase transition triggers a phase transition in the higgs sector. It is then easy to see that all the former discussion still applies with tiny changes, and we get the same condition $m_S^2 \sim m_h^2 \sim \Lambda^2$. There is a further check to make, though, which is due to the fact that, in order for baryogenesis to occur, we need an unsuppressed sphaleron rate in the exterior of the bubble. This translates in the requirement, for $m_h^2 < 0$:

$$\frac{|m_h|}{\Lambda} \ll 1 \quad (35)$$

which can be satisfied in some vacua.

In the next of this paper, we will always assume that these conditions are satisfied, and the sphalerons are suppressed in the broken phase.

Now, we are ready to treat baryogenesis in detail.
3 Electroweak Baryogenesis

During the electroweak phase transition, we have all the necessary conditions to fulfill baryogenesis \cite{27, 28}. We have departure from thermal equilibrium because of the phase transition; we have CP and C violation because of the CP and C violating interactions; and finally we have baryon violation because of the unsuppressed sphaleron rate. The sphaleron rate per unit time per unit volume is in fact unsuppressed in the unbroken phase (see eq. (1)).

There are various different effects that contribute to the final production of baryons. For example, CP violation can be due just to some CP violating Yukawa coupling in the mass matrix, or it can be mainly due to the fact that, in the presence of the wall, the mass matrix is diagonalized with space-time dependent rotation matrixes, which induce CP violation. Further, CP violation can be accounted for by some time dependent effective coupling in some interaction terms. Baryon number production as well can be treated differently. In the contest of electroweak baryogenesis which we are dealing with, there are mainly two ways of approaching the problem, one in which the baryon production occurs locally where the CP violation is taking place, so called local baryogenesis, or one in which it occurs well in the exterior of the bubble, in the unbroken phase, in the so called non local baryogenesis. At the current status of the art, it appears that the non local baryogenesis is the dominant effect, at least for not very large velocity of the wall $v_w$, which however is believed to be not large because the interactions with the plasma tend to slow down the wall considerably \cite{31}, and we concentrate on this case (see Appendix A for a brief treatment of baryogenesis in the fast wall approximation).

In order to compute the produced baryon abundance, we follow a semiclassical method developed in \cite{32}. A method based on the quantum Boltzmann equation and the closed time path integral formalism was developed in \cite{33}, making a part of the method more precise. However, the corrections given by applying this procedure to our case can be expected to be in general not very important once compared to the uncertainties associated to our poor knowledge of certain parameters of the electroweak phase transition, as it will become clear later, which make the computed final baryon abundance reliable only to approximately one order of magnitude \cite{32}. Further, it is nice to note that our general conclusions will be quite robust under our estimated uncertainty in the computation of the baryon abundance. For these reasons, the method in \cite{32} represents for us the right mixture between accuracy and simplicity which is in the scope of our paper.

Since the method is quite contorted, let us see immediately where the basic ingredients for baryogenesis are. Departure from thermal equilibrium obviously occurs because of the crossing of the wall. C violation occurs because of the $V - A$ nature of the interactions. CP violation occurs because the CP violating phases in the mass matrix are rotated away at two different points by two different unitary matrix such that $U(x)^\dagger U(x + dx) \neq 1$. There could be other sources of CP violation, but, in our case, this is the dominant one. Baryon production occurs instead well in the exterior of the wall, in the so called non-local baryogenesis approach, where the weak sphaleron rate is unsuppressed.

Let us anticipate the general logic. The calculation is naturally split into two part. In the first part, we compute the sources for CP violating charges which are due to the CP violating interactions of the particles with the incoming wall. This calculation will be done restricting ourself to the vicinity of the wall, and solving a set of coupled Dirac-Majorana equations to determine the transmission and reflection coefficients of the particles in the thermal bath when they hit the wall, which are different for particles and antiparticles. In the wall rest frame, the wall is perceived as a space-time dependent mass term. This will give rise to a CP violating current. The non null divergence of this current will be the input of the second part of the calculation. In this second part, we shall move to a larger scale, and describe the plasma in a fluid approximation, where we shall study effective
diffusion equations. The key observation is that, once the charges have diffused in the unbroken phase, thermal equilibrium of the sphalerons will force a net baryon number. In fact, in the presence of SU(2) not neutral charges, the equilibrium value of the baryon number is not zero:

\[(B + L)_{eq} = \sum_i c_i Q_i\]  

where \(c_i\) are coefficients which depends on the different charges. Once produced, the baryon number will diffuse back in the broken phase, where, due to the suppression of the sphaleron rate, it will be practically conserved up to the present epoch. This will end our calculation.

In the next two subsections, we proceed to the two parts of the outlined calculation.

### 3.1 CP violation sources

We begin by computing the source for the CP violating charges, following [32]. We restrict to the region very close the the wall, so that the wall can be considered flat, and we can approximately consider the problem as one dimensional. We consider a set of particles with mass matrix \(M(z)\) where \(z\) is the coordinate transverse to the wall, moving in the rest frame of the wall, with energy-momentum \(E, \vec{k}\). Taken \(z_0\) as the last scattering point, these particles will propagate freely for a mean free time \(\tau\), when they will rescatter at the point \(z_0 + \tau v\), where \(v\) is the velocity perpendicular to the wall, \(k_{tr}/E\). Now, because of the space dependent CP violating mass matrix, these particles will effectively scatter, and the probability of being transmitted and reflected will be different for particles and antiparticles. This will create a current for some charges, whose divergence will then be the source term in the diffusion equations we shall deal with in the next section. The effect will be particularly large for charges which are explicitly violated by the presence of the mass matrix, and we shall restrict to them.

We introduce \(J_{\pm}\) as the average current resulting from particles moving towards the positive and the negative \(z\) direction, between \(z_0\) and \(z_0 + \Delta\), where \(\Delta = \tau v\), and \(\tau\) is the coherence time of the particles due to interactions with the plasma [32]. \(J_{\pm}\) are the CP violating currents associated with each layer of thickness \(\Delta\). \(J_{\pm}\) receives, for example, contributions from particles originating from the thermal bath at \(z_0\) with velocity \(v\), and propagating until \(z_0 + \Delta\), as well as from particles originating at \(z_0 + \Delta\) with velocity \(-v\), and being reflected back at \(z_0 + \Delta\). The formula for \(J_{\pm}\) is given by:

\[J_{\pm} = \left(\text{Tr} \left( \rho_{z_0} \left( T^\dagger QT - \bar{T}^\dagger Q\bar{T} \right) \right) - \text{Tr} \left( \rho_{z_0+\Delta} \left( \bar{R}^\dagger \bar{Q} \bar{R} - \bar{T}^\dagger \bar{Q}\bar{T} \right) \right) \right) (1, 0, 0, \bar{v}) \]

\[J_{-} = \left(\text{Tr} \left( \rho_{z_0} \left( \bar{R}^\dagger \bar{Q} \bar{R} - \bar{T}^\dagger \bar{Q}\bar{T} \right) \right) - \text{Tr} \left( \rho_{z_0+\Delta} \left( \bar{T}^\dagger \bar{Q}\bar{T} - \bar{T}^\dagger \bar{Q}\bar{T} \right) \right) \right) (1, 0, 0, -v)\]

where \(R(\bar{R})\) and \(T(\bar{T})\) are reflection and transmission matrices of particles (antiparticles) produced at \(z_0\) with probability \(\rho_{z_0}\), evolving towards positive \(z\); while \(\bar{T}\) and \(\bar{R}\) are the correspondent quantities for particles produced at \(z_0 + \Delta\) with probability \(\rho_{z_0+\Delta}\) and evolving towards negative \(z\); \(v\) is the group velocity perpendicular to the wall at the point \(z_0\), and \(\bar{v}\) is the same quantity at \(z_0 + \Delta\); \(Q\) is the operator correspondent to the chosen charge; and the trace is taken over all the relevant degrees of freedom and averaged over location \(z_0\) within a layer of thickness \(\Delta\). When boosted in the plasma rest frame, these currents will become the building block to construct the CP violating source for the charges that, diffusing into the unbroken phase, will let the production of baryon number possible.

Now, consider a small volume of the plasma, in the plasma rest frame. As the wall crosses it, it leaves a current density equal to \((J_+ + J_-)^{\mu}_{\text{plasma}}\) every time interval \(\tau\), where the subscript
plasma refers to the quantity boosted in the plasma frame. So, at a time \( t \), the total current density accumulated will be given by:

\[
s^\mu = \int_{t-\tau_R}^t dt' \frac{1}{\tau} (J_+(\vec{x}, t') + J_-(\vec{x}, t'))_{\text{plasma}}^{\mu}
\]

where \( \tau_R \) is the relaxation time due to plasma interaction. From this, the rate of change of the charge \( Q \) per unit time is given by:

\[
\gamma_Q(\vec{x}, t) = \partial_\mu s^\mu = \frac{1}{\tau} (J_+(\vec{x}, t) + J_-(\vec{x}, t))_{\text{plasma}}^0 - \frac{1}{\tau} (J_+(\vec{x}, t - \tau_R) + J_-(\vec{x}, t - \tau_R))_{\text{plasma}}^0 - \int_{t-\tau_R}^t dz \partial_z (J_+ + J_-)_{\text{plasma}}^z
\]

Since in the SM CP violation is very small, and already proved to be not enough to account for the baryon number of the universe, we can clearly concentrate in the sector of the neutral particles of the new theory, where the mass matrix is given by:

\[
M(z) = \begin{pmatrix}
    k S(z) & \frac{g v}{\sqrt{2}} & \frac{g' v}{\sqrt{2}} \\
    \frac{g v}{\sqrt{2}} & 0 & k' S(z) \\
    \frac{g' v}{\sqrt{2}} & k' S(z) & 0
\end{pmatrix}
\]

The transmission and reflection coefficients can be found by solving the free coupled Dirac-Majorana equations for these particles with the mass matrix given in \( M(z) \). We can solve this by a method developed in [32], in a perturbative expansion in mass insertion. As it is explained in [32 34], the small parameter in the expansion is \( m \Delta \), where \( m \) is the typical mass of the particles, in the case \( w \Delta < 1 \), or \( m/w \) in the case \( w \Delta > 1 \). In both cases, the expansion parameter is smaller than one. This can be understood noticing the analogy of our system with the scattering off a diffracting medium with a step potential of order \( m \). In that case, reflection and transmission are comparable (and this is the only case in which we produce a net CP violating charge) only if the wave packet penetrates coherently over a distance of order \( 1/m \), and has few oscillation over that distance. Suppression of the reflection occurs both if \( m \Delta \ll 1 \) and if \( m/w \ll 1 \). In the first case, this is because only a layer of thickness \( \Delta \) contributes to the coherent reconstruction of the reflected wave, while in the second case, because fast oscillations tend to attenuate the reconstruction of the reflected wave. Up to sixth order in the mass insertion, we get:

\[
T = 1 - \int_0^\Delta dz_1 \int_0^{z_1} dz_2 M_2 M_1^* e^{2iw(z_1 - z_2)} + \int_0^\Delta dz_1 \int_0^{z_1} dz_2 \int_0^{z_2} dz_3 \int_0^{z_3} dz_4 M_4 M_3^* M_2 M_1^* e^{2iw(z_1 - z_2 + z_3 - z_4)} + \int_0^\Delta dz_1 \int_0^{z_1} dz_2 \int_0^{z_2} dz_3 \int_0^{z_3} dz_4 \int_0^{z_4} dz_5 \int_0^{z_5} dz_6 M_6 M_5 M_4 M_3^* M_2 M_1^* e^{2iw(z_1 - z_2 + z_3 - z_4 + z_5 - z_6)} + \ldots
\]
\[
\tilde{T} = 1 - \int_0^\Delta dz_1 \int_{z_1}^\Delta dz_2 M_2^* M_1 e^{-2iw(z_1-z_2)} + \\
+ \int_0^\Delta dz_1 \int_{z_1}^\Delta dz_2 \int_0^{z_2} dz_3 \int_{z_3}^\Delta dz_4 M_4^* M_3 M_2^* M_1 e^{-2iw(z_1-z_2+z_3-z_4)} + \\
- \int_0^\Delta dz_1 \int_{z_1}^\Delta dz_2 \int_0^{z_2} dz_3 \int_{z_3}^\Delta dz_4 \int_0^{z_4} dz_5 \int_{z_5}^\Delta dz_6 M_6^* M_5 M_4 M_3 M_2^* M_1 e^{-2iw(z_1-z_2+z_3-z_4+z_5-z_6)} + \\
+ \ldots
\]

\[
R = - \int_0^\Delta dz_1 M_1^* e^{2iw(z_1)} + \\
+ \int_0^\Delta dz_1 \int_{z_1}^{z_2} dz_2 \int_0^{z_2} dz_3 M_3^* M_2 M_1^* e^{2iw(z_1-z_2+z_3)} + \\
- \int_0^\Delta dz_1 \int_{z_1}^{z_2} dz_2 \int_0^{z_2} dz_3 \int_{z_3}^{z_2} dz_4 \int_0^{z_4} dz_5 M_5^* M_4 M_3^* M_2^* M_1 e^{2iw(z_1-z_2+z_3-z_4+z_5)} + \\
+ \ldots
\]

\[
\tilde{R} = \int_0^\Delta dz_1 M_1 e^{-2iw(z_1)} + \\
- \int_0^\Delta dz_1 \int_{z_1}^{z_2} dz_2 \int_0^{z_2} dz_3 M_3^* M_2 M_1 e^{-2iw(z_1-z_2+z_3)} + \\
+ \int_0^\Delta dz_1 \int_{z_1}^{z_2} dz_2 \int_0^{z_2} dz_3 \int_{z_3}^{z_2} dz_4 \int_0^{z_4} dz_5 M_5 M_4^* M_3 M_2^* M_1 e^{-2iw(z_1-z_2+z_3-z_4+z_5)} + \\
+ \ldots
\]

where \( M_i = M(z_i) \). The analogous quantities for the antiparticles are obtained replacing \( M \to M^* \) in all the former formulas.

We also need to have the density matrices \( \rho_{z_0} \) and \( \rho_{z_0+\Delta} \). We can choose these densities as describing thermal equilibrium densities in eigenstates of the unbroken phase:

\[
\rho_{z_0} = \text{Diag} \left( n_s (E, \bar{v}), n_{\psi_+} (E, \bar{v}), n_{\psi_-} (E, \bar{v}) \right)
\]

where \( n(E, v) \) is the Fermi-Dirac distribution, boosted in the wall frame:

\[
n = \frac{1}{e^{\frac{E-w(E-v_\text{w}k_B)}{T_c}} + 1}
\]

and \( \rho_{z_0+\Delta} = \rho_{z_0} (\bar{v} \to -v) \). The motivation of this is that particles are produced in interaction eigenstates which differ from mass eigenstates by a unitary rotation; ignoring this, amounts at ignoring small corrections of order \( (M(z)/T_c)^2 \). The choice of thermal distribution is particularly good in the small velocity \( v_w \) regime, in which we have restricted, where the non-thermal contribution is of order \( v_w^2 \), and it induces corrections of order \( v_w^3 \) in the final baryon density [32].

Finally, we have to consider the charges which can play a role in generating the baryon number. When choosing such charges, one has to consider that the most important charges are those which are approximately conserved in the unbroken phase, as these are the ones which can efficiently diffuse in the unbroken phase, and induce a large generation of baryon number. Keeping this in mind, it is easy to see that the only relevant charge in our model is the “higgs number” charge, which in
the same basis in which we expressed the mass matrix, for the new CP violating sector particles, is given by:

\[ Q_h = \text{Diag}(0, 0, 1, -1) \]  

(47)

The name “higgs number” just comes from the fact that the fields \( \Psi_\pm \) have the same quantum numbers as higgsinos in the MSSM. Now, we can substitute in the formulas for \( J_+ \) and \( J_- \). In order to keep some analytical expression, we decide to do a derivative expansion later the reason of this. For the moment, we get:

\[ (1, 0, 0, 0) \times \]

(48)

\[ +4 \int_0^{\Delta} dz_1 \int_0^{z_1} dz_2 \int_0^{z_2} dz_3 \int_0^{z_3} dz_4 \int_0^{z_4} dz_5 \int_0^{z_5} d\tau \sin(2w(z_1 - z_2 + z_3 - z_4 + z_5 - \tau)) \]

\[ \text{Im}(\rho_{z_0} Q_h M_6^* M_4^* M_2^* M_1^* + \rho_{z_0} Q_h M_1^* M_6^* M_4^* M_2^* M_1^* + \rho_{z_0} Q_h M_1^* M_6^* M_4^* M_3^* M_2^*) \]

\[ J_- = (1, 0, 0, -v) \times \]

(49)

\[ -4 \int_0^{\Delta} dz_1 \int_0^{z_1} dz_2 \int_0^{z_2} dz_3 \int_0^{z_3} dz_4 \int_0^{z_4} dz_5 \int_0^{z_5} d\tau \sin(2w(z_1 - z_2 + z_3 - z_4 + z_5 - \tau)) \]

\[ \text{Im}(\rho_{z_0+\Delta} Q_h M_6^* M_4^* M_2^* M_1^* + \rho_{z_0+\Delta} Q_h M_1^* M_6^* M_4^* M_2^* M_1^* + \rho_{z_0+\Delta} Q_h M_1^* M_6^* M_4^* M_3^* M_2^*) \]

where the \( z \) dependence in each mass matrix \( M_i \) is to be understood at linearized level.

We can substitute the results in eq.(41), to get an expression for the higgs charge source. In reality, if we take the relaxation time large enough, and if we keep performing a derivative expansion, only the first term in eq.(41) is relevant. At first order in the wall velocity, we get:

\[ \gamma_Q = \frac{\gamma_Q \nu w}{\tau} \left[ e^{E_i / T_c} \frac{E_i}{(1 + e^{E_i / T_c})^2 T_c} \right] \]

(50)
where $J$ is a CP violating invariant:

$$J = \frac{1}{4}gg'kk'(g^2 - g'^2)\sin(\theta)$$

and $f_i(w\Delta)$ is:

$$f_q(w\Delta) = \frac{v(z_0)^3S(z_0)}{24T_c^4}((S'(z_0)(9 - 24w^2\Delta^2 - 14w^4\Delta^4)v(z_0) - 9v'(z_0)S(z_0)$$

$$-2w^2\Delta(11v'(z_0)\Delta + 6w^2\Delta^2) +$$

$$3(5 + 2w^2\Delta^2)v(z_0))S(z_0) + 6\cos(6w\Delta)S(z_0))v(z_0) + v'(z_0)S(z_0)) +$$

$$3\cos(2\Delta\Delta)S'(z_0)(1 + 6w^2\Delta^2)v(z_0) - 3v'(z_0)S(z_0) + 2w^2\Delta(3v'(z_0)\Delta + 4v(z_0))S(z_0))$$

$$+2w(2S'(z_0)\Delta(-3 + 14w^2\Delta^2)v(z_0) + 5v'(z_0)\Delta(3 + 7w^2\Delta^2) +$$

$$3(1 + 7w^2\Delta^2)v(z_0))S(z_0))\sin(2\Delta)3w(16S'(z_0)\Delta v(z_0)$$

$$+(-13v'(z_0)\Delta + v(z_0))S(z_0))\sin(4\Delta) + 6w\Delta(-S'(z_0)v(z_0) + v'(z_0)S(z_0))\sin(6\Delta)))$$

$$f_s(w\Delta) = \frac{v(z_0)^3S(z_0)}{48T_c^4}(S'(z_0)(15 + 12w^2\Delta^2 - 28w^4\Delta^4)v(z_0)$$

$$-15v'(z_0)S(z_0) - 4w^2\Delta(v'(z_0)\Delta(48 + 11w^2\Delta^2) +$$

$$3(5 + 2w^2\Delta^2)v(z_0))S(z_0) + 12\cos(6w\Delta)S'(z_0)v(z_0) - v'(z_0)S(z_0)) + 3\cos(4\Delta\Delta)(S'(z_0)$$

$$(1 - 12w^2\Delta^2)v(z_0) - v'(z_0)S(z_0) + 4w^2\Delta(6v'(z_0)\Delta + v(z_0))S(z_0)0 +$$

$$6\cos(2\Delta\Delta)S'(z_0)(-5 + 14w^2\Delta^2)v(z_0) + 5v'(z_0)S(z_0) + 2w^2\Delta(5v'(z_0)\Delta +$$

$$4v(z_0))S(z_0)) + 12w(S'(z_0)\Delta(-7 + 6w^2\Delta^2)v(z_0) + (v(z_0) +$$

$$\Delta(5v'(z_0)(2 + 3w^2\Delta^2) + 7w^2\Delta v(z_0))S(z_0))\sin(2\Delta)$$

$$-6w(-9S'(z_0)\Delta v(z_0) + 12v'(z_0)\Delta + v(z_0))S(z_0))\sin(4\Delta) + 12w\Delta(S'(z_0)v(z_0)$$

$$-v'(z_0)S(z_0))\sin(6\Delta)))$$

where $z_0$ here is a typical point in the middle of the wall. We can estimate the coherence time as

$$\tau \sim (g_wT_c)^{-1},$$

and so we can take, roughly, the typical interval which is valid also in the MSSM $^{32}$: $15 \leq \tau T_c \leq 35$, and in the approximation of making the particles massless, which is still consistent with our approximations, we can integrate numerically the integral, and fit it linearly in $\tau$ (which is a good approximation in the range of interest), to obtain:

$$\gamma Q(\bar{x}, t) \simeq 24v_w\gamma_w \left( gg'kk'(g^2 - g'^2)\sin(\theta) \right) \times$$

$$\left( 1 + \frac{1}{8}(\tau T_c - 25) \right) \frac{v^3S}{T_c} \frac{T_c}{T_c} \left( S'v - v'S \right)$$

Even though the reparametrization invariant CP violating phase $\theta$ requires only 4 Yukawa couplings to be present in the theory, it is somewhat puzzling that the leading effect appears at sixth order in the couplings. The reason is as follows. In the case $g = g'$, there is an approximate $Z_2$ symmetry which exchanges the $\Psi_\pm$ fields. This is only an approximate symmetry, because $\Psi_\pm$ differ by their hypercharge. However, the hypercharge is considered a subleading effect here, and never comes into play in our computation, and so we do not see the breaking of this symmetry. The $Z_2$ symmetry has, in the basis we have been using up to now, the matrix form:

$$Z_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$
We clearly see that:

\[ [M(z), Z_2] = 0, \ [\rho(z), Z_2] = 0, \ \{Q, Z_2\} = 0 \]  \hspace{1cm} (55)

which then implies that:

\[ Tr\left(\rho_{z0}TQ^T\right) = 0 \]  \hspace{1cm} (56)

and similar for the others terms in \(J_+\) and \(J_-\). So, in the limit in which \(g = g'\), the baryon asymmetry should vanish. Then, since each particle needs to have an even number of mass insertions to be transmitted or reflected, and since the CP violation requires at least 4 mass insertion, we finally obtain that the coupling dependence in the CP violating source must be of the form present in eq \(54\). From this discussion, it is clear that in the case \(g = g'\) the baryon production will be heavily suppressed, and, from what we will see later, it will be clear that the couplings in this case will have to be so large to necessarily hit a Landau pole at very low energies, making the model badly defined. We shall neglect this degeneracy of the couplings for the next of the paper.

Now, we are ready to begin the second part of the computation.

### 3.2 Diffusion Equations

Here, we begin the second part of the calculation, still following \[32\]. We turn to analyze the system at a larger scale, and approximate it to a fluid. We then study the evolution of the CP violating charges due to the presence of the sources and of the diffusion effects in the plasma. To this purpose, we shall write a set of coupled differential equations which include the effects of diffusion, particle number changing reactions, and CP violating source terms, and we shall solve them to find the various densities. We shall be interested in the evolution of particles which carry some charges which are approximately conserved in the unbroken phase. Near thermal equilibrium, which is a good approximation for small velocities, we can approximate the number density as:

\[ n_i = k_i \mu_i T_c^2 / 6 \]  \hspace{1cm} (57)

where \(\mu_i\) is the chemical potential, and \(k_i\) is a statistical factor which is equal to 2 for each bosonic degree of freedom, and 1 for each fermionic.

The system of differential equations simplifies a lot if we neglect all couplings except the gauge couplings, the top quark Yukawa coupling, and the Yukawa couplings in the new CP violating sector. From the beginning, we take the interactions mediated from these last ones to be fast with respect to the typical timescale of the fluid. We include the effect of strong sphaleron, but neglect the one of the weak sphalerons until almost the end of the computation. This allows us to forget about leptons. We need only to keep track of the following populations: the top left doublet: \(Q = (t_L + b_L)\), the right top: \(T = t_R\), the Higgs particle plus our new fields \(\Psi_{\pm}; H = h_0 + \Psi_+ + \Psi_-\) Strong sphalerons will be basically the only process to generate the right bottom quarks \(B = b_R\), and the quarks of the first two generations \(Q_{(1,2)}L, U_R, C_R, S_R, D_R\). This implies that all these abundance can be expressed in terms of the one of \(B\):

\[ Q_{1L} = Q_{2L} = -2U_R = -2D_R = -2S_R = -2C_R = -2B = 2(Q + T) \]  \hspace{1cm} (58)

The rate of top Yukawa interaction, Higgs violating process, and axial top number violation are indicated as \(\Gamma_y, \Gamma_h, \Gamma_m\), respectively. We take all the quarks to have the same diffusion equation, and the same for the higgs and the \(\Psi_s\). The charge abundances are then described by the following
set of differential equations:

\[
\begin{align*}
\dot{Q} &= D_q\nabla^2 Q - \Gamma_y (Q/k_Q - H/k_H - T/k_T) - \Gamma_m (Q/k_Q - T/k_T) \\
&\quad - 6\Gamma_{ss} (2Q/k_Q - T/k_T + 9(Q + T)/k_B) \\
\dot{T} &= D_q\nabla^2 T - \Gamma_y (-Q/k_Q + H/k_H + T/k_T) \\
&\quad - \Gamma_m (Q/k_Q + T/k_T) + 3\Gamma_{ss} (2Q/k_Q - T/k_T + 9(Q + T)/k_B) \\
\dot{H} &= D_h\nabla^2 H - \Gamma_y (-Q/k_Q + T/k_T + H/k_H) - \Gamma_h H/k_h + \gamma_Q
\end{align*}
\]

We now assume that \(\Gamma_y\) and \(\Gamma_{ss}\) are very fast, and we develop the result at \(O(1/\Gamma_y, 1/\Gamma_{ss})\). This allows to algebraically express \(Q\) and \(T\) in terms of \(H\), to get the following relationships:

\[
Q = H \left( \frac{k_Q (9k_T - k_B)}{k_H (k_B + 9k_Q + 9k_T)} \right) \\
T = -H \left( \frac{k_T (2k_B + 9k_Q)}{k_H (k_B + 9k_Q + 9k_T)} \right)
\]

and, substituting back, we find the following effective differential equation for \(H\):

\[
v_w H' = \tilde{D} H'' - \tilde{\Gamma} H + \tilde{\gamma}
\]

where the effective couplings are given by:

\[
\tilde{D} = \frac{D_q (9k_Q k_T - 2k_Q k_B - 2k_B k_T) + D_h k_H (9k_Q + 9k_T + k_B)}{9k_Q k_T - 2k_Q k_B - 2k_B k_T + k_H (9k_Q + 9k_T + k_B)}
\]

\[
\tilde{\gamma} = \gamma_Q \left( \frac{k_H (9k_Q + 9k_T + k_B)}{9k_Q k_T - 2k_Q k_B - 2k_B k_T + k_H (9k_Q + 9k_T + k_B)} \right)
\]

\[
\tilde{\Gamma} = (\Gamma_m + \Gamma_h) \left( \frac{(9k_Q + 9k_T + k_B)}{9k_Q k_T - 2k_Q k_B - 2k_B k_T + k_H (9k_Q + 9k_T + k_B)} \right)
\]

We can estimate the relaxation rates for the higgs number and the axial quark number as \[32\]:

\[
(\Gamma_m + \Gamma_h) \sim \frac{4M_W^2 (T_c, z)}{21g^2 T_c} \lambda_t^2
\]

where \(\lambda_t\) is the top Yukawa coupling, and \(M_W\) is the W boson mass, and, in order to keep analytical control, we approximate the source term and the relaxation term as step functions:

\[
\begin{align*}
\tilde{\gamma} &= \tilde{\gamma}, \quad w > \tilde{z} > 0 \\
\tilde{\gamma} &= 0, \quad \text{otherwise}
\end{align*}
\]
and
\[ \bar{\Gamma} = \tilde{\Gamma}, \bar{z} > 0 \]
\[ \bar{\Gamma} = 0, \bar{z} < 0 \] (69)

For the source term \( \bar{\gamma} \), we can take the averaged value of expression (54). However, due to our lack of knowledge of the details of the profiles of the fields during the phase transition, we can just approximate that expression with:
\[ \tilde{\gamma}_Q = 24v_w\gamma_w (gg'kk' (g^2 - g'^2) \sin(\theta)) \times \]
\[ \frac{1 + \frac{4}{3}(\tau T_c - 25)}{\tau T_c} \frac{v^4 S^2}{T_c W} \] (70)

where we have taken \( S' \sim S/W \) and \( v' \sim v/W \), with \( W \) the wall width, and we have assumed, as expected, that no cancellation is occurring. Here \( S \) and \( v \) are taken to be of the order they are today.

In the approximation that \( \bar{D} \) is constant, and with the boundary conditions given by \( H(\pm \infty) = 0 \), we have an analytical solution in the unbroken phase [32]:
\[ H = A e^{2v_w/D} \] (71)

where, in the limit that \( \bar{D} \tilde{\Gamma} \ll v_w^2 \), which is in general applicable,
\[ A \simeq \frac{\tilde{\gamma}}{\bar{\Gamma}} \left( 1 - e^{-2W\sqrt{\bar{\Gamma}/D}} \right) \] (72)

Note that diffusion of the higgs field, and so of the other charges, in the unbroken phase, occurs for a distance of order \( z \sim \bar{D}/v_w \).

We now turn on the weak sphaleron rate, which is the responsible for the baryon generation. The baryon density follows the following equation of motion:
\[ v_w \rho_B = D_q \rho_B'' - \Theta(-\bar{z})3\Gamma_{ws} n_L(\bar{z}) \] (73)

where we have assumed that the weak sphaleron operates only in the unbroken phase, and where \( n_L \) is the total number density of left fermions. The solution to this equation is given by, at first order in \( v_w \):
\[ \rho_B = -\frac{3\Gamma_{ws}}{v_w} \int_{-\infty}^{0} d\bar{z} n_L(\bar{z}) \] (74)

Now, in the approximation in which all the particles in our theory are light, we have:
\[ k_Q = 6, k_T = 3, k_B = 3, k_H = 8 \] (75)

and in the limit as \( \Gamma_{ss} \to \infty \), the resulting baryon abundance is zero [32]. This means that we have to go to the next order in the strong sphaleron rate expansion. Note also that, with this particle content, using the SM quark and higgs diffusion equation \( D_q \sim 6/T_c, D_h \sim 110/T_c \) [35], we have \( \bar{D} \sim 96/T_c \) and \( D_{\Gamma y}/v_w^2 \gg 2/v_w^2 \), so that the assumption that the Yukawa interaction is fast is self-consistent, since \( \Gamma_y \sim (27/2)\lambda^2 T_c \) [32]. Finally, we take:
\[ \Gamma_{ws} = 6k'_Q \alpha^5 T_c, \Gamma_{ss} = 6k' \frac{8}{3} \alpha^4 T_c \] (76)

where \( k' \) is an order one parameter and \( k \sim 20 \) [25]. To go to next order in the expansion in large \( \Gamma_{ss} \), we write:
\[ Q = H \left( \frac{k_Q(9k_T - k_B)}{k_H(k_B + 9k_Q + 9k_T)} \right) + \delta Q \] (77)
\documentclass{article}
\usepackage{amsmath}
\begin{document}

\[ T = -H \left( \frac{k_T(2k_T + 9k_Q)}{k_H(k_B + 9k_Q + 9k_T)} \right) + \frac{k_T}{k_Q} \delta Q \]  

(78)

Substituting in (60), we get:

\[ \delta Q = \left( \frac{D_qH'' - v_wH'}{\Gamma_{ss}} \right) \left( \frac{-k_B^2k_Q(k_B + 2k_T)}{3k_H(9k_Q + 9k_t + k_B)^2} \right) + O(1/\Gamma_{ss}, 1/\Gamma_y) \]  

(79)

Using \( n_L = Q + Q_{1L} + Q_{2L} = 5Q + 4T \), we get:

\[ n_L = 7\delta Q = -\frac{3}{112} \left( \frac{D_qH'' - v_wH'}{\Gamma_{ss}} \right) \]  

(80)

Substituting the solution for the Higgs field, we finally get:

\[ \frac{\rho_B}{s} = -\left( \frac{9A_{gw}}{112 s \Gamma_{ss}} \right) \left( 1 - \frac{D_q}{D} \right) \]  

(81)

where \( s = (2\pi^2 g_*/45)T_c^3 \approx 55T_c^3 \). Substituting our parameters, we get:

\[ \frac{\rho_B}{s} \simeq 5 \times 10^{-5} \left( \frac{k_{sp}/20}{k_{sp}} \right) v_w\gamma_w \left( 1 + \frac{1}{3}(T_c \tau - 25) \right) \]  

\[ (gg'(g^2 - g'^2)kk'\sin(\theta)) \left( \frac{v}{T_c} \right)^3 \left( \frac{S}{T_c} \right)^2 \]  

(82)

It is worth to make a couple of small comments on the parametric dependence of this expression. The dependence of the coupling terms, and therefore on the vevs of \( h \) and \( S \), was explained in the former section. The wall width \( W \) has simplified away because the exponential in eq.(72) is small and can be Taylor expanded. The presence of \( \Gamma_{ss} \) in the denominator is due to the particular particle content of this model, for which the leading term in the expansion in \( 1/\Gamma_{ss} \) is zero. The factor \( 10^{-5} \) is mainly due to the factor of \( \sim 10^{-2} \) from the entropy density, and the ratio \( 20 \alpha^5_w/\alpha^4_s \), times some other factors coming from the diffusion terms. For typical values of the wall velocity, we can take approximately the SM range: \( v_w \sim 0.05 - 0.3 \), while the mean free time is \( \tau \sim 20/T_c - 30/T_c \). With these values, the produced baryon number ranges in the regime:

\[ \frac{\rho_B}{s} \simeq (2 \times 10^{-7} - 3 \times 10^{-6}) (gg'(g^2 - g'^2)kk'\sin(\theta)) \left( \frac{v}{T_c} \right)^3 \left( \frac{S}{T_c} \right)^2 \]  

(83)

This is the number which has to be equal to the baryon density at BBN:

\[ \left( \frac{\rho_B}{s} \right)_{BBN} = 9 \times 10^{-11} \]  

(84)

For the moment, let us try do draw some preliminary conclusions on what this does imply on the parameters of the model. Clearly, there are still too many parameters which could be varied, so, as a beginning, we can set all the Yukawa couplings in the new CP violating sector to be roughly equal to each other \( k \sim k' \sim g \sim g' \), and \( v/T_c \sim 1 \), we then get:

\[ \frac{\rho_B}{s} \sim (2 \times 10^{-7} - 4 \times 10^{-6}) (g^6 \sin(\theta)) \left( \frac{S}{v} \right)^2 \simeq 10^{-10} \]  

(85)

\end{document}
We then get the following constraint:

\[ g^6 \left( \frac{S}{v} \right)^2 \sin(\theta) \sim 10^{-4} \]  

(86)

It is natural, in this theory, to take the CP violating phase to be of order 1. In this case:

\[ g \sim g' \sim k \sim k' \sim 0.21 \left( \frac{v}{S} \right)^{1/3} \]  

(87)

From this we see that the assumption of considering the interactions mediated by these Yukawa couplings to be fast is justified for a large fraction of the parameter space. These values become lower bounds for the couplings if we allow for the CP violating phase to be smaller than order 1 (see eq. (86)).

## 4 Electric Dipole Moment

The same CP violating phase which is responsible for baryogenesis, induces an electric dipole moment through the 2-loop diagram shown in fig. 1.

The situation here is much different than in the MSSM, where a CP violating phase in general introduces EDM at one loop level, the constraints on which generically force very small CP violating phases in the MSSM. It is instead much more similar to the case of split supersymmetry \[44, 45, 47\], where all the one loop diagrams contributing to the EDM are decoupled. Here the leading diagram is at two loops level, and, as we will soon see, it will induce electron EDM naturally just a little beyond the present constraints, and on the edge of detection by future experiments.

The induced EDM is (see \[38\]):

\[ \frac{dW_f}{e} = \pm \frac{\alpha^2 m_f}{8\pi^2 s_W^4 M_W^2} \sum_{i=1}^{3} \frac{m_{\chi_i} m_{\omega}}{M_W^2} \text{Im} (O^L_i O^R^*) \mathcal{G} \left( r^0_i, r^{\pm}, r_f' \right), \]  

(88)

\[ \mathcal{G} \left( r^0_i, r^{\pm}, r_f' \right) = \int_0^\infty \int_0^1 \frac{d\gamma}{\gamma} \int_0^1 dy \frac{y z (y + z/2)}{(z + R)^3(z + K_i)} \]  

(89)

\[ = \int_0^1 \frac{d\gamma}{\gamma} \int_0^1 dy y \left[ \frac{(R - 3K_i) R + 2(K_i + R)y}{4R(K_i - R)^2} + \frac{K_i(K_i - 2y)}{2(K_i - R)^3} \ln \frac{K_i}{R} \right]. \]  

(90)
with
\[ R = y + (1 - y) r_f' , \quad K_i = \frac{r_i^0}{1 - \gamma} , \quad r_\pm = \frac{m_\omega^2}{M_W^2} , \quad r_i^0 = \frac{m_{\omega_i}^2}{M_W^2} , \quad r_f' = \frac{m_{f'}^2}{M_W^2} . \tag{91} \]

\[ O^R_i = N_{3i}^*, \quad O^L_i = -N_{4i} C_R \tag{92} \]

where \( C_R = e^{-i\theta} \) and \( N^T M_N N = \text{diag}\{m_{\chi_1}, m_{\chi_2}, m_{\chi_3}\} \) with real and positive diagonal elements. The plus (minus) sign on the right-hand side of eq. (88) corresponds to the fermion \( f \) with weak isospin \( \pm \frac{1}{2} \). \( f' \) is its electroweak partner. We mean by \( \omega \) the charged mass eigenstate, and with \( \chi_i, i = 1, 2, 3 \), the neutral mass eigenstates in order of increasing mass.

Diagonalizing the \( 3 \times 3 \) mass matrix numerically, we see that generically the model predicts EDM very close to detection. The induced EDM for some generic parameters are shown in fig. 2.

Figure 2: The predicted EDM in this model. We plot the induced electron EDM as a function of \( k'S \). The solid line represents the induced EDM with \( g = g' = 1/2 \) and \( kS = 100 \text{ GeV} \), while the dashed line represents \( g = g' = 1/10 \) and \( kS = 100 \text{ GeV} \), and we take maximal CP violating phase. The horizontal line represents the present electron EDM constraint \( d_e < 1.7 \times 10^{-27} \text{ e cm} \) at 95\% CL [37].

It is clear then that improvements of the determination of the EDM are going to explore the most interesting region of the parameter space. Ongoing and next generation experiments plan to improve the EDM sensitivity by several orders of magnitude within a few years. For example, DeMille and his Yale group [39] will use the molecule PbO to improve the sensitivity of the electron EDM to \( 10^{-29} \text{ e cm} \) within three years, and possibly to \( 10^{-31} \text{ e cm} \) within five years. Lamoreaux and his Los Alamos group [40] developed a solid state technique that can improve the sensitivity to the electron EDM by \( 10^4 \) to reach \( 10^{-31} \text{ e cm} \). By operating at a lower temperature it is feasible to eventually reach a sensitivity of \( 10^{-35} \text{ e cm} \), an improvement of eight orders of magnitude over the present sensitivity. The time scale for these is uncertain, as it is tied to funding prospects. Semertzidis and his Brookhaven group [41] plan to trap muons in storage rings and increase the sensitivity of their EDM measurement by five orders of magnitude. A new measurement has been presented by the Sussex group [42]. A number of other experiments aim for an improvement in sensitivity by one or two orders of magnitude, and involve nuclear EDMs.
In order to understand the current and future constraints on the model, we can study what is the induced electron EDM, once we have satisfied the constraint from baryogenesis in eq. (83):

$$gg' (g^2 - g'^2) k T_c v^3 S^2 / T_c^2 \sim 10^{-4}$$

The way we proceed is as follows. First, we fix $g' = g / \sqrt{2}$. This is a good representative of the possible ratios between $g$ and $g'$, as it is quite far from the region where the approximate symmetry in the case $g = g'$ suppresses baryogenesis, and $g'$ is not too small to suppress baryogenesis on its own. Later on we shall relax this condition. Having done this, we invert eq. (93), to get an expression for $g$ in terms of the other parameters of the model. Now, the couplings $g, g'$ are expressed in terms of $T_c$, and in terms of $k S$ and $k' S$ which, as it will be useful, in the case of small mixing, can be thought of as respectively the singlet and the doublet mass. We shall impose the constraint on the couplings $g, g', k, k'$ to be less than $\sim 1$, in order for these Yukawa couplings not to hit a Landau pole before the unification scale $\sim 10^{15}$ GeV.

We decide to restrict our analysis to the case in which the possible ratios between the marginal couplings of the same sector of the theory are smaller than 2 orders of magnitude. Here, by same sector of the theory we mean either the CP violating sector, or the sector of the scalar potential and the strong gauge group. The justification of this relies in the fact that we wish to explore the most natural region of the parameter space. For this reason, we expect that there is no large hierarchy between the marginal couplings of the same sector. We think that 2 order of magnitudes is a threshold large enough to delimitate this natural region. However, since marginal couplings are radiatively relatively stable, and large hierarchies among them does not give rise to fine tuning issues, in principle, large hierarchies among the marginal couplings are acceptable. We think that, however, such a hierarchy would require the addition of further structure to the model to justify its presence, and we decide to restrict to the simplest realization of the model. As a consequence of this, the most important restrictions we apply are: $10^{-2} \lesssim k / k' \lesssim 10^2$, and $10^{-2} \lesssim k S / \lambda \lesssim 10^2$. In particular, using eq. (33) and eq. (34), this implies $\lambda T_c \lesssim S \lesssim 5 T_c$, which, using the limit $k, k' \lesssim 1$, implies that the largest of $k S$ and $k' S$ must be smaller than 5 $T_c$.

The upper bound on the CP violating sector particles tells us that there is a small part of the parameter space which we are going to explore, in which the particles responsible for the production of baryons are non relativistic. In that case, we can approximately extend the result found in eq. (83), with the purpose of having order of magnitude estimates, in the following way. From eq. (50), it is clear that baryon production is Boltzmann suppressed if the particles are non relativistic. In that case, the CP violating charge will be generated by the scattering of these particle in the region where the induced mass on the particles is of the order of the critical temperature, as this is the condition of maximum CP violating interaction compatible with not being Boltzmann suppressed. Having observed this, it is easy to approximately extend the result of eq. (83) to the non relativistic case, by taking the vevs of the fields at the a value such that the induced mass is of the order of the critical temperature.

In fig. 3, we show the induced EDM as a function of $k S$, for several values of the critical temperature $T_c$, for $k' S = 100$, on top, and $k' S = 500$, at the bottom, with the couplings $g, g'$ chosen as explained above, in order to fulfil the baryogenesis requirement. Notice that 500 GeV is roughly the limit that LHC will put on SU(2) doublets. We choose the maximum CP violating phase. The horizontal lines represent the present constraint on EDM $d_e < 1.7 \times 10^{-27} e cm$ at 95% CL, and the future expected one $39, 40, 41$ of order $10^{-31} e cm$. A few features are worth to be noted. We see that, for fixed $T_c$, the EDM decreases as we decrease $k S$, for $k S$ light enough. This is due the fact that, reducing $k S$, we reduce both the EDM and the produced quantity of baryons. However, the loss in the production of baryons is compensated with a much smaller, compared to the decrease
in $kS$, increase in the couplings $g, g'$, so that, the baryon abundance can remain constant, while the EDM decreases. For large $kS$, the EDM decreases both because the mass of the particles in the loops becomes heavier and heavier, and also because the mixing becomes more and more suppressed. The maximum is located at the point where $kS \sim k'S$. In that case, in fact, the mass matrix has a diagonal piece roughly proportional to the identity, so, even though the diagonal elements are much larger than the off diagonal ones, mixing is much enhanced, and so is the EDM, which is proportional to the mixing. The lower and upper limit on the value of $kS$, as well as the minimum temperature, are dictated by the restrictions $k'S \lesssim 5 T_c$, $kS \lesssim 5 T_c$, and $k/k' \gtrsim 10^{-2}$. Now, as we decrease the critical temperature, the resulting EDM tends to decrease. In fact, decreasing the temperature much enhances the baryon production. This allows to decrease the couplings $g, g'$, which explains why the induced EDM decreases. We verify that increasing the hierarchy between $g$ and $g'$ does not change the results a lot, as the decrease in baryons production requires to make the other couplings large. Increasing the value of $k'S$ up to the maximum allowed value of 1.1 TeV (since $T_{c\max} \simeq v$) does not produce any relevant change in the result, because this raises the minimum temperature, so that baryogenesis requires larger couplings, which forces the EDM not to decrease relevantly. It is only once the CP violating phase is lowered to less than $10^{-2}$ that a small experimentally unreachable region is opened up around $k'S = 500$ GeV and $kS \lesssim 1$ GeV. The same region is opened also enlarging the allowed hierarchy between the couplings to above $5 \times 10^5$. However, clearly, the presence at the same time of a large hierarchy and of a very small CP phase requires some additional structure on the model to explain the reason for their presence.

The main conclusion we can draw from combining the analysis on baryogenesis and EDM is that, at present, the most natural region of the parameter space is perfectly allowed, however, improvements in the determination of the EDM, are going to explore the entire viable region of the parameter space, so that absence of signal, would result in effectively ruling out the model, at least if no further structure is added.

5 Comments on Dark Matter and Gauge Coupling Unification

The proposed model finds its main motivation in stabilizing the weak scale through the requirement of attaining baryons in our universe. However, further than this, it is clear that the model provides two other interesting phenomenological aspects: gauge coupling unification and dark matter. Here, we just briefly introduce these aspects, and we postpone a more detailed discussion to future work.

Thanks to the two doublets we have inserted in our model, which have the same quantum numbers as higgsinos in the MSSM, gauge coupling unification works much better than in the standard model. As it was shown in [11], gauge coupling unification with the standard model plus higgsinos works roughly as well as the MSSM at two loops level, with the only possible problem being the fact that the unification scale is a bit low at around $\sim 10^{15}$ GeV. The problem from proton decay can however be avoided with some particular model at the GUT scale [11, 43]. We expect that 2-loop gauge coupling unification works quite well also in this model, with only small corrections coming from the presence of the singlet scalar $S$.

Concerning the Dark Matter relic abundance, the lightest of the newly introduced particles is stable, and so it provides a natural candidate for Dark Matter. Estimating the relic abundance is quite complex, as it depends on the composition of the particle, and on its annihilation and coannihilations rate. However, if our newly introduced Yukawa couplings are not very close to their upper limit (of order one), and if the lightest particle is mostly composed of the two doublets, then its relic abundance is very similar to the one of pure higgsino dark matter in split supersymmetry.
Figure 3: Top, the predicted EDM given the constraint from Baryogenesis satisfied, as a function of $kS$ for $k'S = 100$ GeV, $g' = g/\sqrt{2}$, maximum CP violating phase, and $T_c = 200$ GeV (solid line), 100 GeV (long dashed), 25 GeV (short dashed). Bottom, the same for $k'S = 500$ GeV, and $T_c = 200$ GeV (solid line), 150 GeV (long dashed), 110 GeV (short dashed). The horizontal lines represent the present electron EDM constraint $d_e < 1.7 \times 10^{-27} \text{e cm}$ at 95% CL [37], and the expected improvement up to $d_e < 10^{-31} \text{e cm}$ [38, 40, 41].

In this paper, we have addressed the solution to the electroweak hierarchy problem in the context of the landscape, following a recent model proposed in [11]. We have shown that it is possible to connect the electroweak scale to a hierarchically small scale at which a gauge group becomes strong by dimensional transmutation. The assumption is that we are in a "friendly neighborhood" of the landscape in which only the relevant parameters of the

6 Conclusions

In this paper, we have addressed the solution to the electroweak hierarchy problem in the context of the landscape, following a recent model proposed in [11].

We have shown that it is possible to connect the electroweak scale to a hierarchically small scale at which a gauge group becomes strong by dimensional transmutation. The assumption is that we are in a "friendly neighborhood" of the landscape in which only the relevant parameters of the
low energy theory are effectively scanned. We then realize the model in such a way that there is a fragile, though necessary, feature of the universe which needs to be realized in our universe in order to sustain any sort of life. As a natural continuation of Weinberg’s ”structure principle”, the fragile feature is the presence of baryons in the universe, which are a necessary ingredient for the formation of clumped structures. We assume that, in the friendly neighborhood of the landscape in which we should be, baryogenesis is possible only through the mechanism of electroweak baryogenesis. Then, in order to produce a first order phase transition strong enough so that sphalerons do not wash out the produced baryon density, we develop a new mechanism to implement the electroweak phase transition. We introduce a new gauge sector which becomes strong at an exponentially small scale through dimensional transmutation. We couple this new sector to a singlet \( S \) which is then coupled to the higgs field. The electroweak phase transition occurs as the new gauge sector becomes strong, and produces a chiral condensation. This triggers a phase transition for the singlet \( S \), which then triggers the phase transition for the higgs fields. In order to preserve baryon number, we need the phase transition to be strong enough, and this is true only if the higgs mass is comparable to the QCD scale of the strong sector. This solves the hierarchy problem. In order to provide the necessary CP violation, we introduce 2 SU(2) doublets \( \Psi_{\pm} \) with hypercharge \( \pm 1/2 \), and a gauge singlet \( s \).

When we require the model to describe our world, the model leads to falsifiable predictions. The main result of the paper is the computation of the produced baryon number, for which we obtain:

\[
\frac{\rho_B}{s} \approx (2 \times 10^{-7} - 3 \times 10^{-6}) (g g' (g^2 - g'^2) k k' \sin(\theta)) \left( \frac{v}{T_c} \right)^3 \left( \frac{S}{T_c} \right)^2
\]

(94)

The requirement that this baryon abundance should cope with the observed one leads to a lower bound on a combination of the product of the CP violating phase, the new couplings, the \( S \) vev, and the critical temperature \( T_c \).

We infer that Gauge Coupling Unification, and the right amount of Dark Matter relic abundance are easily achieved in this model.

We study in detail the induced electron Electric Dipole Moment (EDM), and we find that, at present, the most natural region of the parameter space of the theory is allowed. However, soon in the future improvement in the EDM experiments will be sensitive the entire viable region of the parameter space of the model, so that absence of a signal would result in practically ruling out the model.

### Acknowledgments

I would like to thank Nima Arkani-Hamed, who inspired me the problem, and without whose constant help I would have not been able to perform this calculation. I would like to thank also Paolo Creminelli, Alberto Nicolis, Toby Wiseman, and Rakhi Mahbubani for interesting conversations.

### Appendix A  Baryogenesis in the Large Velocity Approximation

The method we have used in the main part of this work is based on some approximations that fail in limit of very fast wall speed: \( v_w \sim 1 \). Even though this regime seems to be disfavored by actual computations of the wall speed[31], it is worth to try to estimate the result even in this case. In this appendix we are going to do an approximate computation in the large wall velocity approximation. In this new regime, calculations become much more complicated, as the assumption
of local thermodynamical equilibrium begins to fail, and the baryon number tends to be produced in the region of the wall, in the so called local baryogenesis scenario.

We follow the treatment of [51]. In this approach, the phase transition is treated from an effective field theory point of view. A good way of looking at the high temperature phase of the unbroken phase is imagining it as discretized in a lot of cells, whose side is given by the typical size of the weak sphaleron barrier crossing configuration of the gauge field: \( \zeta \sim (\alpha W T_c)^{-1} \). Concentrating on each cell, we have that the thermal energy in a cell is in general much larger than the energy necessary to create a gauge field oscillation capable of crossing the barrier, and this means that most of the energy is in oscillations with smaller wavelength than \( \zeta \). So, these configurations cross the barrier at energy far above the one of the sphaleron configuration, and so their rate has nothing to do with the sphaleron rate, and this explain why their rate per unit volume is of order \( \zeta^4 \) (In the case \( m_h^2 < 0 \), electroweak symmetry breaking has already occurred outside the bubble, but this discussion still roughly applies). We can parameterize the configuration in one cell with one variable \( \tau \), which depends only on time. Obviously, this is a very rough approximation, but this is at least a beginning. We try to describe the dynamics of the configuration near crossing the barrier, at a maximum of the energy, that we fixe to be at \( \tau = 0 \). Near this point, we can write the following Lagrangian, which is very similar to the one in [51]:

\[
L(\tau, \dot{\tau}) = \frac{c_1}{2\zeta} \dot{\tau}^2 + \frac{c_2}{2\zeta^3} \tau^2 + \frac{c_3}{\zeta} b \dot{\phi} \tau
\]

(95)

where \( c_i \) are dimensionless parameters depending on the different possible barrier crossing trajectories, while \( b \) will be determined shortly. Since the point \( \tau = 0 \) is a maximum of the energy, no odd powers of \( \tau \) can appear. The appearance of the odd power in \( \dot{\tau} \) can be understood because, at one loop level, the CP violating mass matrix of the particles in the CP violating sector induces an operator which contains the term \( \text{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu} \). This can be seen in the following way, in an argument similar to the one shown in [47]. We can imagine to do a chiral rotation on these field of amplitude equal to the CP violating phase. Because of the anomaly, this will induce the operator:

\[
\mathcal{O} = \frac{g_2^2}{16\pi^2} \phi \text{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu}
\]

(96)

where \( g_2 \) is the SU(2) weak coupling, and \( F_{\mu\nu} \) is the SU(2) field strength, and where

\[
\phi = \text{Arg}(\text{Det}(M)) = \text{Arcsin}\left(\frac{kk'S^2\sin(\theta)}{kk'S^2\sin(\theta)^2 + (gg'v^2 + kk'S^2\cos(\theta))^2}\right)
\]

(97)

Promoting the vevs of \( h \) and \( S \) to the actual fields, we find the operator we were looking for. Now, the term \( \text{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu} \) contains a term proportional to the time derivative of the Chern Simons number, and so the operator \( \mathcal{O} \) must contribute to the effective action (95) with a term proportional to \( b \phi \dot{\tau} \), with \( b = \frac{g_2^2}{16\pi^2} \), explaining the reason for the presence of the term in \( \dot{\tau} \)

The equation of motion for \( \tau \) is:

\[
\ddot{\tau} = \frac{c_2}{c_1 \zeta^2} \tau - \frac{c_3}{c_1 16\pi^2} g_2^2 \dot{\phi}
\]

(98)

If the wall is fast enough, we can solve it in the impulse approximation, to get:

\[
\Delta \dot{\tau} = -\frac{c_3}{c_1 16\pi^2} g_2^2 \Delta \phi
\]

(99)

This kick to \( \Delta \dot{\tau} \) makes the distribution of velocities of barrier crossing configurations asymmetric, leading to a production of baryons with respect to antibaryons.
Now, this kick will be very inefficient in changing the distribution of baryons, unless the kick is larger than the typical speed $\tau_0$ a generic configuration would have if it crossed the barrier in the absence of the wall. So, we require: $\Delta \dot{\tau} > \tau_0$. The fraction of configurations which satisfy this requirement is proportional to $\Delta \dot{\tau}$, but is very difficult to estimate. Following [51], we just say that it is equal to $f \Delta \dot{\tau}$, where $f$ is our “ignorance” coefficient. So, we finally get:

$$n_B \sim f \Delta \dot{\tau} \zeta^{-3}$$  \hspace{1cm} (100)

where we have reabsorbed the constants $c_i$ into $f$. We finally get:

$$\frac{\rho_B}{s} \sim f \frac{\alpha_Y^3}{45} \frac{g_s^2}{16\pi^2} \Delta \phi$$  \hspace{1cm} (101)

There are a lot of heavy approximations which suggest that this estimate is very rough [51]: from the value of the coefficient $c_i$, to the approximation of restricting to one degree of freedom, to the impulse approximation in solving the differential equation, and to the estimate of the fraction of configurations influenced by the kick. All these suggests that we should take eq.(101) with $f \sim 1$, as at most an upper limit on the baryon production, as the authors of [51] suggest.

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