A Decoupled MPC for Motion Control in Robotino Using a Geometric Approach

Daniel Straßberger, Paolo Mercorelli
Institute of Product and Process Innovation, Leuphana University of Lueneburg, Volgershall 1, D-21339 Lueneburg, Germany Phone: +49-(0)4131-677-5571, Fax: +49-(0)4131-677-5300. E-mail: mercorelli@uni.leuphana.de

Oleg Sergiyenko
Engineering Institute of Autonomous University of Baja California, Blvd. Benito Juárez y Calle de La Normal, s/n, Col. Insurgentes Este, C.P.21280, Mexicali, BC, Mexico, tel./fax. 01(52-686) 566-41-50. E-mail: srgnk@ing.mx.uabc.mx

Abstract. This paper proposes a controller for motion control of the Robotino. The proposed controller considers a functional decoupling control strategy realized using a geometric approach and the invertibility property of the DC-drives with which the Robotino is equipped. Horizontal, Vertical and Angular motions are considered and once the decoupling between these motions is obtained, a Model Predictive Control (MPC) strategy is used in combination with the inverse DC-drive model. Simulation results using real data of Robotino are shown.

1. Introduction
This paper presents a systematic procedure in order to obtain the decoupling controllability between horizontal, vertical and angular motions. Here, the decoupling problem is investigated. To achieve a decoupling effect a feedback control law is needed together with a feed-forward regulator. In the past three decades, research on the geometric approach to dynamic systems theory and control has allowed this approach to become a powerful and a thorough tool for the analysis and synthesis of dynamic systems [1], [2], [3]. Over the same time period, mechanical systems used in industry and developed in research labs have also evolved rapidly. Mobile robotics is a notable case of such evolution. The robotics community has developed sophisticated analysis and control techniques to meet increasing requirements on the control of motions of mechanical systems. These increasing requirements are motivated by higher performance specifications and an increasing number of degrees-of-freedom. References [4] and [5] mark progress in the analysis and synthesis of geometric controller for mechanical systems, and [6] proposes non-interacting force-motion control in robotic manipulation. Reference [7] reports the possibility of parameterising input controlled subspaces to guarantee non-interaction. In [4], a robust decoupling controller using an algebraic state input feedback is presented, while this paper presents a robust decoupling controller using an algebraic output-input feedback. The force/motion control problem has attracted significant attention of scientists over the last decade in the fields of robotic manipulation and mobile robotics. Approaches exploiting input-output decoupling controllers...
are found, for instance, in the work [8]. The geometric approach allows very elegant solutions of control problems. Nevertheless, robustness analysis using a linear geometric control offers answers through rank conditions of matrices that are necessary conditions. These conditions are often not constructive ones. Even though the rank conditions offer simple "on-off" conditions, it is also possible to measure the robustness. In [4], a robust decoupling controller is obtained using a state input feedback controller. The drawback of this approach consists of a wide sensing structure. In fact, the whole state space should be available and sensed. With the approach proposed in this paper, the robust decoupling is obtained with an output feedback from the contact forces and the joint positions. The goal of this paper is to propose a complete constructive procedure for the design of a decoupling controller. In Robotino’s case, a preselecting field law is needed together with a feed-forward regulator for achieving a decoupling between Horizontal, Vertical and Angular motions. The paper is organised in the following way: Section 2 presents a possible model of the Robotino. Section 3 shows the decoupling strategy using the geometric approach. Section 4 is devoted to the derivation of the Model Predictive Control (MPC) strategy. At the end, numerical computer simulations, considering real data of the Robotino, are shown.

**Figure 1.** Robotino in action on the pitch.

| The main nomenclature |
|------------------------|
| **A**: state matrix of the mechanical model |
| **g(θ)**: input field of the mechanical model |
| **T(θ)**: decoupling input partition field |
| **B** = **g(θ)** · **T(θ)**: decoupled input matrix of the mechanical model |
| **B** = **imB**: image of matrix **B** |
| (subspace spanned by the columns of matrix **B**) |
| **min I(A, B)** = \( \sum_{i=0}^{n-1} A^i \text{im}B \): minimum **A**–invariant subspace containing **imB** |
| **C** = **kerC**: kernel of matrix **C** |
| (subspace spanned by the columns of matrix **C**) |

2. Mechanical and Electrical Model Description

In Fig. 2 a diagram with the representation of the forces in the considered system is shown. If state vector **X(t)** is defined as, \( x(t) \) and \( y(t) \) positions of the center of mass of the system and its velocities, \( \dot{x}(t) \) and \( \dot{y}(t) \), moreover, considering the angular dynamics with its angular position
\( \theta(t) \) and its velocity \( \dot{\theta}(t) \), then the following system is derived.

\[
\begin{align*}
\dot{X}(t) &= AX(t) + g(\theta)F(t) \\
O(t) &= CX(t),
\end{align*}
\]  

(1)

with

\[
X(t) =
\begin{bmatrix}
    x(t) \\
    \dot{x}(t) \\
    y(t) \\
    \dot{y}(t) \\
    \theta(t) \\
    \dot{\theta}(t)
\end{bmatrix},
\]

\[
\dot{X}(t) =
\begin{bmatrix}
    \dot{x}(t) \\
    \ddot{x}(t) \\
    \dot{y}(t) \\
    \ddot{y}(t) \\
    \dot{\theta}(t) \\
    \ddot{\theta}(t)
\end{bmatrix},
\]

(2)

matrix

\[
A =
\begin{bmatrix}
    0 & 1 & 0 & 0 & 0 & 0 \\
    0 & -\frac{k_v}{M} & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 1 & 0 & 0 \\
    0 & 0 & 0 & -\frac{k_v}{M} & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 1 \\
    0 & 0 & 0 & 0 & 0 & -\frac{k_\theta}{J}
\end{bmatrix},
\]  

(3)

which for sake of notation can be written as follows:

\[
A =
\begin{bmatrix}
    A_{x2\times2} & 0_{2\times2} & 0_{2\times2} \\
    0_{2\times2} & A_{y2\times2} & 0_{2\times2} \\
    0_{2\times2} & 0_{2\times2} & A_{\theta2\times2}
\end{bmatrix},
\]  

(4)

where \( A_{x2\times2} \), \( A_{y2\times2} \) and \( A_{\theta2\times2} \) are the minor matrices of \( A \) describing the \( x, y \) and \( \theta \) dynamics of the system. Matrix \( C \) represents the output matrix and can be written in the following way:

\[
C =
\begin{bmatrix}
    1 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 1 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix},
\]  

(5)

Figure 2. Mechanical schematic diagram of Robotino.
where the following notation is assumed:

\[
C_x = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (6)
\]
\[
C_y = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \quad (7)
\]
\[
C_\theta = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (8)
\]

Field \( g(\theta) \) is as follows:

\[
g(\theta) = \begin{bmatrix} 0 & 0 & 0 \\
\frac{\sin(\theta)}{M} & \frac{\sin(\frac{2}{3}\pi + \theta)}{M} & -\frac{\sin(\frac{2}{3}\pi - \theta)}{M} \\
0 & 0 & 0 \\
\frac{\cos(\theta)}{M} & \frac{\cos(\frac{2}{3}\pi + \theta)}{M} & \frac{\cos(\frac{2}{3}\pi - \theta)}{M} \\
a & b & c \end{bmatrix} \quad (9)
\]

where

\[
a = \frac{\sin(\theta) \cdot l \cdot \sin(\theta) + \cos(\theta) \cdot l \cdot \cos(\theta)}{J},
\]
\[
b = \frac{\sin(\frac{2}{3}\pi + \theta) \cdot l \cdot \cos(\frac{2}{3}\pi + \theta) + \cos(\frac{2}{3}\pi + \theta) \cdot l \cdot \sin(\frac{2}{3}\pi + \theta)}{J}
\]

and

\[
c = \frac{-\sin(\frac{2}{3}\pi - \theta) \cdot l \cdot \cos(\frac{2}{3}\pi - \theta) + \cos(\frac{2}{3}\pi - \theta) \cdot l \cdot \sin(\frac{2}{3}\pi - \theta)}{J} \quad (10)
\]

and force input vector signal \( \mathbf{F}(t) = \begin{bmatrix} F_1(t) \\ F_2(t) \\ F_3(t) \end{bmatrix} \) in which \( F_1(t) = K_m i_1(t) / r, \ F_2(t) = K_m i_2(t) / r \)

and \( F_3(t) = K_m i_3(t) / r \), where \( K_m \) represents the motor constant, \( r \) the radius of the wheels and variables \( i_1(t), i_2(t) \) and \( i_3(t) \) are the currents of the three DC-electrical drives. In fact, the Robotino consists of 3 DC electrical drives that power the three omniwheels. The models of these three DC-drive are reported below.

\[
\begin{bmatrix}
\frac{d i_1(t)}{dt} \\
\frac{d i_2(t)}{dt} \\
\frac{d i_3(t)}{dt} \\
\end{bmatrix} = \begin{bmatrix}
\frac{-R}{L} & -\frac{K_m}{L} & 0 & 0 & 0 & 0 \\
\frac{K_m}{J} & -\frac{K}{J} & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{R}{L} & -\frac{K_m}{L} & 0 & 0 \\
0 & 0 & \frac{K_m}{J} & -\frac{K}{J} & 0 & 0 \\
0 & 0 & 0 & -\frac{R}{L} & -\frac{K_m}{L} & -\frac{K}{J} \\
0 & 0 & 0 & 0 & -\frac{K_m}{J} & -\frac{K}{J} \\
\end{bmatrix} \begin{bmatrix}
i_1(t) \\
i_2(t) \\
i_3(t) \\
\omega_1(t) \\
\omega_2(t) \\
\omega_3(t) \\
\end{bmatrix} + \begin{bmatrix}
\frac{1}{\tau} & 0 & 0 \\
0 & \frac{1}{\tau} & 0 \\
0 & 0 & \frac{1}{\tau} \\
\end{bmatrix} \begin{bmatrix}
U_{inp1}(t) \\
U_{inp2}(t) \\
U_{inp3}(t) \\
\end{bmatrix}, \quad (11)
\]

where \( U_{inp1}(t), U_{inp2}(t) \) and \( U_{inp3}(t) \) represent the input voltage of the DC-drive, \( L \) is the inductance, \( R \) is the resistance, \( J \) represents the motor inertia factor, \( K_m \) is the motor moment and finally \( K \) can be seen as the friction factor. This model is important in realising the simulation and is basis for the inverted drive used in the control strategy (see Fig. 3) but is otherwise not explicitly used in the MPC.
3. Design of a Decoupling Controller

This section describes the design of a decoupling controller with respect to the x-y and \( \theta \) motions above defined. A geometric approach is used in this analysis. The earliest geometric approaches to decoupling control were due to Basile and Marro ([9], [1]) and to Wonham and Morse ([10], [11], and [3]). Since the geometric relations depend on the rotational position \( \theta \), the forces actuated by the three DC drives needed to move in a certain direction may be difficult to obtain.

**Definition 1** A control law for the dynamic system (1) is decoupling with respect to the regulated outputs \( x(t), y(t), \) and \( \theta(t) \), if there exist partitions \( F_x(t), F_y(t), \) and \( F_\theta(t) \) of the input vector \( F(t) \) such that for zero initial conditions, each input \( F_{1,\gamma}(t) \) (with all other inputs, identically zero) only affects the corresponding output \( x(t), y(t), \) or \( \theta(t) \).

It is to be shown that there exists a decoupling and stabilizing state feedback field \( D(\theta) \), along with three input partition fields \( T_x(\theta), T_y(\theta), \) and \( T_\theta(\theta) \) such that, for the dynamic triples

\[
\begin{align*}
(C_x, A + g(\theta)D(\theta), g(\theta)F_1(t)), \\
(C_y, A + g(\theta)D(\theta), g(\theta)F_2(t)), \\
(C_\theta, A + g(\theta)D(\theta), g(\theta)F_3(t)),
\end{align*}
\]  

it holds the following conditions:

\[
\mathcal{R}_x(\theta) = \min \mathcal{I} \left( A + g(\theta)D(\theta), g(\theta)T_x(\theta) \right) \subset C_x \cap C_\theta \quad \forall \theta,
\]

and

\[
C_x \mathcal{R}_x(\theta) = \text{im}(C_x), \quad \forall \theta.
\]

\[
\mathcal{R}_y(\theta) = \min \mathcal{I} \left( A + g(\theta)D(\theta), g(\theta)T_y(\theta) \right) \subset C_y \cap C_\theta \quad \forall \theta,
\]

and

\[
C_y \mathcal{R}_y(\theta) = \text{im}(C_y), \quad \forall \theta.
\]

\[
\mathcal{R}_\theta(\theta) = \min \mathcal{I} \left( A + g(\theta)D(\theta), g(\theta)T_\theta(\theta) \right) \subset C_x \cap C_y \quad \forall \theta,
\]

and

\[
C_\theta \mathcal{R}_\theta(\theta) = \text{im}(C_\theta), \quad \forall \theta.
\]

Here,

\[
\min \mathcal{I}(A, \text{im}(g(\theta))) = \sum_{i=0}^{n-1} A^i \text{im}(g(\theta))
\]

is a minimum \( A \)-invariant subspace containing \( \text{im}(g(\theta)) \) \( \forall \theta \). Moreover, the partition fields \( T_x(\theta), T_y(\theta) \) and \( T_\theta(\theta) \) satisfy the following relationships:

\[
\begin{align*}
\text{im}(g(\theta) \cdot T_x(\theta)) &= \text{im}(g(\theta)) \cap \mathcal{R}_x(\theta), \\
\text{im}(g(\theta) \cdot T_y(\theta)) &= \text{im}(g(\theta)) \cap \mathcal{R}_y(\theta), \\
\text{im}(g(\theta) \cdot T_\theta(\theta)) &= \text{im}(g(\theta)) \cap \mathcal{R}_\theta(\theta).
\end{align*}
\]

The stabilizing field \( D(\theta) \) is such that:

\[
(A + g(\theta)D(\theta))\mathcal{R}_x(\theta) \subseteq \mathcal{R}_x(\theta),
\]
Considering matrix $A$ already decoupled and intrinsically stable, this implies that field $D(\theta) = 0 \ \forall \theta$. Considering $\mathbf{T}(\theta) = [\mathbf{T}_x(\theta), \mathbf{T}_y(\theta), \mathbf{T}_\theta(\theta), \mathbf{T}_c(\theta)]$, where $\mathbf{T}_c(\theta)$ is defined in a complementary fashion and it is straightforward to show that matrix $\mathbf{T}_c = 0$. In particular, matrix $\mathbf{T}_c$ represents the complementary field partition to the subspaces of $x$-position, $y$-position and angular position of the Robotino. The Robotino motion is described using just three variables, therefore partitions $\mathbf{T}_x(\theta), \mathbf{T}_y(\theta), \mathbf{T}_\theta(\theta)$ complete the transformation and thus $\mathbf{T}_c = 0$.

According to relation (9), it is straightforward to observe that the following three equations hold $\forall \theta$:

$$\text{im}\mathbf{T}(\theta) = \text{im}[\mathbf{T}_x(\theta), \mathbf{T}_y(\theta), \mathbf{T}_\theta(\theta)] = \text{im}\mathbf{T}_x(\theta) \oplus \text{im}\mathbf{T}_y(\theta) \oplus \text{im}\mathbf{T}_\theta(\theta).$$

Considering the output matrices (6), (7) and (8) corresponding to $x$-position, $y$-position and angular position their respective kernels are as follows:

$$\mathbf{C}_x = \text{im} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}, \quad \mathbf{C}_y = \text{im} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}, \quad \mathbf{C}_\theta = \text{im} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{24}$$

Field $g(\theta)$ is a function of $\theta$ without singularities if $\theta \neq \pi/2 + k\pi$ with $k \in \mathbb{N}$. For sake of brevity the following field is calculated considering just $\theta = 0$:

$$\begin{bmatrix} g(\theta) \end{bmatrix}^\dagger = \begin{bmatrix} 0 & 0 & 0 & a(\theta) & 0 & b(\theta) \\ 0 & c(\theta) & 0 & -a(\theta) & 0 & b(\theta) \\ 0 & -c(\theta) & 0 & -a(\theta) & 0 & b(\theta) \end{bmatrix}, \tag{28}$$

where with $\begin{bmatrix} g(\theta) \end{bmatrix}^\dagger$ the pseudo inverse of field $g(\theta)$ is indicated. Functions $a(\theta)$, $b(\theta)$ and $c(\theta)$ are functions of the variable $\theta$ with $\theta \approx 0$. The following calculation is obtained for $\theta \neq \pi/2 + k\pi$ with $k \in \mathbb{N}$:

$$\begin{align*}
\mathbf{C}_x \cap \mathbf{C}_y &= \text{im} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, & \mathbf{C}_\theta \cap \mathbf{C}_y &= \text{im} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, & \mathbf{C}_\theta \cap \mathbf{C}_x &= \text{im} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{29} \end{align*}$$
The following calculation allow to get the required fields for the decoupling of the mechanical system:

\[ T_\theta(\theta) = (g(\theta))^T \cdot \text{im}(g(\theta)) \cap C_x \cap C_y, \]  
\[ T_x(\theta) = (g(\theta))^T \cdot \text{im}(g(\theta)) \cap C_\theta \cap C_y, \]  
\[ T_y(\theta) = (g(\theta))^T \cdot \text{im}(g(\theta)) \cap C_\theta \cap C_x. \]

Adding all 3 T-Fields together we get a new field \( T(\theta) \):

\[ T(\theta) = T_x(\theta) + T_y(\theta) + T_\theta(\theta). \]

Field \( T(\theta) \) can be seen as a preselecting field and the following product realises the mechanical decoupling:

\[ B = \text{im}(g(\theta)T(\theta)) = \text{im} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \]

in which matrix \( B \) can be seen as a resulting input matrix.

4. Solving Three Decoupled Linear Position MPC Optimization Problems for the Mechanical System

If matrix (3) is observed, it is possible to notice that this matrix is a block matrix and thus represents, as already observed, an internally decoupled system. Considering that together with the preselecting matrix \( T_\theta \) above calculated, it is possible to consider a centralized decoupled model predictive controller as follows. Considering \( k = nT_s \) and \( k + 1 = (n + 1)T_s \), where \( T_s \) represents the sampling time and \( n \in \mathbb{N} \), then:

\[ u_{\text{mpc}}(k) = u_{\text{mpc}}(k - 1) + \Delta u_{\text{mpc}}(k), \]

then the following expression can be obtained:

\[ x(k + 1) = A(\cdot)_k x(k) + B(\cdot)_k \left( \Delta u_{\text{mpc}}(k) + u_{\text{mpc}}(k - 1) \right), \]

\[ y(k) = H(\cdot)_k x(k), \]

where: \( A(\cdot)_k \) represents the discretisation of each block matrix of definition (4) of matrix \( A \), matrix \( B(\cdot)_k \) represents the same way the discretisation of each block matrix of definition (34) of matrix \( B \), \( x(k) \) is the corresponding discrete part of the state variable and matrix \( H(\cdot)_k \) is the output matrix which determines the position. A general MPC approach with control increments is considered. Using the recursive relation in (35) with a prediction horizon of four the following equation can be obtained:

\[ Y(k) = G_{\text{mpc}} x(k) + F_{1\text{mpc}}(k) \Delta U_{\text{mpc}}(k) + F_{2\text{mpc}} u_{\text{mpc}}(k - 1), \]

where:

\[ Y(k) = \begin{bmatrix} \dot{y}(k + 1/k) \\ \dot{y}(k + 2/k) \\ \dot{y}(k + 3/k) \\ \dot{y}(k + 4/k) \end{bmatrix}, \]

\[ \Delta U_{\text{mpc}}(k) = \begin{bmatrix} \Delta u_{\text{mpc}}(k) \\ \Delta u_{\text{mpc}}(k + 1) \\ \Delta u_{\text{mpc}}(k + 2) \\ \Delta u_{\text{mpc}}(k + 3) \end{bmatrix}. \]
and matrices $G_p$, $F_{1p}$ and $F_{2p}$ is given here below:

$$G_p = \begin{bmatrix} H_k A_{(-)_k}^1 \\ H_k A_{(-)_k}^2 \\ H_k A_{(-)_k}^3 \\ H_k A_{(-)_k}^4 \end{bmatrix},$$

(39)

$$F_{1p} = \begin{bmatrix} H_k B_{(-)_k} \\ H_k A_{(-)_k} B_{(-)_k} \\ H_k \sum_{i=1}^{2}(A_{(-)_k}^i + I) B_{(-)_k} \\ H_k \sum_{i=1}^{3}(A_{(-)_k}^i + I) B_{(-)_k} \end{bmatrix},$$

(40)

$$F_{2p} = \begin{bmatrix} H_k B_{(-)_k} \\ H_k (A_{(-)_k} + I) B_{(-)_k} \\ H_k \sum_{i=1}^{2}(A_{(-)_k}^i + I) B_{(-)_k} \\ H_k \sum_{i=1}^{3}(A_{(-)_k}^i + I) B_{(-)_k} \end{bmatrix},$$

(41)

If the following performance criterion is assumed,

$$J = \frac{1}{2} \sum_{j=1}^{N} \left( y_d(k+j) - \hat{y}(k+j) \right)^T Q_p \left( y_d(k+j) - \hat{y}(k+j) \right) + \sum_{j=1}^{N} \left( \Delta u_{mpc}(k+j) \right)^T R_p \Delta u_{mpc}(k+j),$$

(42)

where $x_d(k+j)$, $j = 1, 2, \ldots, N$ is the position reference trajectory and $N$ the prediction horizon, and $Q_p$ and $R_p$ are non-negative definite matrices, then the solution minimizing performance index (42) may be then obtained by solving

$$\frac{\partial J}{\partial \Delta u_{mpc}} = 0.$$  

(43)

A direct computation of the vectorial optimal solution may be obtained with

$$\Delta \hat{U}_{mpc} = (F_{1p}^T Q_p F_{1p} + R_p)^{-1} \left( F_{1p}^T Q_p (Y_{d_p}(k) - G_p x(k) - F_{2p} u_{mpc}(k-1)) \right),$$

(44)

where $Y_{d_p}(k)$ is the desired output column vector. The component at time "k" of the optimal vector of Eq. (44) is to be considered. Further details concerning this MPC can be found in [12].

5. Simulation Results

Fig. 3 shows the general setup used for simulation and validation. Using a centralized MPC-controller with the geometric decoupling approach acting as a pre-control, different trajectories can be tracked. As such the MPC accepts trajectories in horizontal, vertical and angular
direction and attempts to direct the Robotino only in the specified direction. Therefore, it is possible to move the Robotino along the x-axis without a rotation for example. In the case presented here a circular movement was attempted while the angular position remained constantly zero. Figure 4 shows very accurate movements along the desired trajectory which is represented in the tested case by a circle. The circle is realized considering a continuous sine and a cosine functions on the x and y movements. All the time the angular position of the robot remained unchanged at zero and this also has been achieved: As it can be observed in Figure 5, the simulated result is so small that it is considered to be a numerical error in the simulation. Additionally it is vital, that the drives are not forced to output more than they can handle, so voltages and currents would have to remain in their respective boundaries. Figure 5 shows that aside from the transients at the start, the nominal current of 0.9 Ampere was not exceeded. Further testings suggest that this could be avoided with better reference trajectories.

![Figure 3. Block diagram of the control system structure](image)

6. Conclusion
This paper presents a procedure in order to obtain the decoupling controllability between horizontal, vertical and angular motions using an geometric approach. A feedback control law is needed together with a feed-forward regulator to achieve a decoupling effect. Simulation results are shown to indicate the potential of this approach.

References
[1] G. Basile and G. Marro. Controlled and conditioned invariants in linear system theory. Prentice Hall, New Jersey, 1992.
[2] A. Isidori. Nonlinear control Systems: an introduction. Springer, Berlin, 1989.
[3] W.M. Wonham. Linear multivariable control: a geometric approach. Springer, New York, 1979.
[4] P. Mercorelli and D. Prattichizzo. A geometric procedure for robust decoupling control of contact forces in robotic manipulation. Kybernetika, 39(4):433–455, 2003.
[5] D. Prattichizzo and P. Mercorelli. On some geometric control properties of active suspension systems. Kybernetika, 36(5):549–570, 2000.
Figure 4. X-Y-Trajectory of the Robotino

Figure 5. Change in angular position (left). Currents of the three drives acting the three wheels (right)

[6] P. Mercorelli. Invariant subspace for grasping internal forces and non-interacting force-motion control in robotic manipulation. *Kybernetika*, 48(6):12291249, 2012.

[7] P. Mercorelli. Geometric structures for the parameterization of non-interacting dynamics for multi-body mechanisms. *International Journal of Pure and Applied Mathematics-IJPM*, 59(3):257–273, 2010.

[8] Y. Yamamoto and X. Yun. Effect of the dynamic interaction on coordinated control of mobile manipulators. *IEEE Transactions on Robotics and Automation*, 12(5):816–824, 1996.

[9] G. Basile and G. Marro. A state space approach to non-interacting controls. *Ricerche di Automatica*, 1(1):68–77, 1970.

[10] W.M. Wonham and A.S. Morse. Decoupling and pole assignment in linear multivariable systems: a geometric approach. *SIAM J. Control*, 8(1):1–18, 1970.

[11] A.S. Morse and W.M. Wonham. Decoupling and pole assignment by dynamic compensation. *SIAM J. Control*, (1):317–337, 1970.

[12] H. Sunan, T.K. Kiong, and L.T. Heng. *Applied Predictive Control*. Springer-Verlag London, Printed in Great Britain, 2002.