ABSTRACT

Massive clusters of galaxies gravitationally shear the images of faint background galaxies. At large impact parameters the shear is weak, but can still be measured, to a reasonable degree of significance, as a statistical anisotropy of the faint galaxy images. We describe techniques for measuring the shear and discuss the interpretation of the shear field. We have applied this analysis to ms1224+007. We find a clear detection of the shear, but, puzzlingly, we find a mass about three times that obtained by application of the virial theorem, and obtain a very large mass-to-light ratio. Similar results have been obtained by other groups, and we discuss their implications.
Giant arcs in clusters provide a very clean probe of the mass distribution in the central parts of clusters (see e.g. the reviews of Soucail and Miralda-Escude in these proceedings). The work we shall describe here follows the pioneering study of A1689 by Tyson, Valdes and Wenk (1990) where they measured for the first time the statistical anisotropy of the background galaxies at much larger radii.

MEASURING THE IMAGE SHEAR

From deep photometry of a field containing a massive cluster we first find the faint galaxies and measure their central second angular moments: 

\[ Q_{ij} = \int d^2\theta W(\theta)\theta_i\theta_j f(\bar{\theta}), \]

where \( f \) is the surface brightness, and form the “polarization”

\[ e_\alpha = \left[ \frac{(Q_{11} - Q_{22})}{(Q_{11} + Q_{22})} \right] \]

The window function \( W(\theta) \) we use is a gaussian with scale length matched to that of the galaxy. In the absence of lensing the \( e_\alpha \) values will scatter around a mean of zero on the \( e_1, e_2 \) plane, but a gravitational shear acting coherently over some angle will displace the distribution. Provided the shear is weak — and we will be measuring shears of typically 10% — the shift in the mean polarization is \( \langle e_\alpha \rangle = P_{sh} s_\alpha \), where the shear \( s_\alpha = \{\phi_{11}, \phi_{22}, 2\phi_{12}\} \) and the surface potential \( \phi \) satisfies \( \nabla^2 \phi = 2\sigma \) with dimensionless surface density \( \sigma = \Sigma_{\text{phys}} / \Sigma_{\text{crit}} \). The effective critical surface density is \( \Sigma_{\text{crit}} = \frac{4\pi G\rho L}{\langle \text{max}(0, 1 - w_g/w_l) \rangle} \) with the average being taken over the distribution of comoving distances \( w_g \) to the faint galaxies. The proportionality constant \( P_{sh} \) — which we call the shear polarizability — depends on the details of the shapes of the galaxies, and can be estimated either for the population as a whole or, as we do, individually for each galaxy.

Each galaxy therefore provides an estimate of the shear along a particular line of sight, though a rather noisy one as there is a substantial scatter in the intrinsic polarizations. These can be averaged together to produce e.g. a smoothed shear map \( s(\bar{\theta}) \) with statistical uncertainty \( \sim \sqrt{\langle s^2 \rangle_{\text{intrinsic}} / N} \) where \( N \) is the number of galaxies being averaged. An interesting feature of cluster studies is that the signal to noise varies rather slowly with radius; at large radii the signal becomes small, but the number of galaxies goes up and these cancel if the surface density varies as \( \Sigma \propto 1/r \) for instance.

A minor complication is that distortion of the images can arise in the telescope and in the earth’s atmosphere. Luckily there are sufficiently many foreground stars to provide a control sample with which to measure the point spread function quite precisely. One could imagine reconvolving the image with a psf designed to recircularise the stars and this would then null out the systematic error in the galaxies. In fact what we do is to calculate for each galaxy a ‘smear polarizability’ \( P_{sm} \) analogous to \( P_{sh} \) which tells us how the polarization shifts in response to smearing by an anisotropic psf \( (P_{sm} \) is essentially a measure of the inverse area of the galaxy), and use this to null out the systematic error. This approach is easier if, as in the data we describe later, the psf anisotropy varies across the chip. An ultimate limit on this technique — which one might eventually hope to apply to very large fields where it is vital to cancel even very tiny systematic effect — may possibly arise due to effects such as...
atmospheric dispersion and chromatic aberrations of the optics which cause the psf to depend on the spectrum of the objects, but with present data such effects are small compared to the statistical error.

**INTERPRETATION OF THE SHEAR**

Assuming we have a map of the shear \( s(\vec{\theta}) \) how do we infer from this the surface density \( \sigma \)? First, there is no local relation between the shear and \( \sigma \). Physically, this reflects the fact that a constant density sheet lens does not produce shear. As one might expect, however, there is a local relation between the angular gradients of \( s \) and of \( \sigma \):

\[
\frac{\partial \sigma}{\partial x} = \frac{1}{2} \left[ \frac{\partial s_1}{\partial x} + \frac{\partial s_2}{\partial y} \right] \\
\frac{\partial \sigma}{\partial y} = \frac{1}{2} \left[ \frac{\partial s_2}{\partial x} - \frac{\partial s_1}{\partial y} \right]
\]

This means that in principle one can construct any differential measurement of the surface density; for instance one can calculate the surface density difference between two points simply by integrating \( \nabla \sigma \) along some line:

\[
\delta \sigma_{12} = \sigma(\vec{\theta}_1) - \sigma(\vec{\theta}_2) = \int_{\theta_1}^{\theta_2} \dd l \cdot \nabla \sigma.
\]

The arbitrariness of the line chosen reflects the inherent non-uniqueness of any \( \sigma \) determination method. This arises essentially because one has two inputs \( s_1, s_2 \) from which we only want to recover the single scalar function \( \sigma \). In the example here one could estimate \( \delta \sigma_{12} \) by averaging over any combination of paths from \( \theta_1 \) to \( \theta_2 \) — and one would presumably try to find some optimum weighted combination of these — and one could also use loop integrals in some way to provide a check on the quality of the data, though how best to do this has yet to be worked out in any systematic way.

The ambiguity in the baseline surface density is something of a problem, particularly when the data coverage is limited. The ambiguity can be resolved to some extent by studying clusters with giant arcs, though this involves some uncertain interpolation of the surface density from the radii where the arcs lie out to the radii where the weak shear analysis can be safely applied. An alternative is to try and measure the perturbation of the background galaxy counts \( n(m) \) (Broadhurst, Peacock and Taylor, 1994) to determine the surface density, though this is quite difficult, as we show below. We have developed a number of tests derived from the relation above between \( \partial \sigma / \partial \theta_i \) and \( \partial s_\alpha / \partial \theta_i \):

**Mass Imaging**

Kaiser and Squires (1992, hereafter KS93) provide an algorithm to reconstruct a smoothed 2-dimensional mass image. In the present context, this can be viewed as an average over all radial paths from infinity (or some large radius where the surface density and the shear can be assumed small) to the point in question. This becomes a two dimensional integral \( \hat{\sigma} = \langle \int \dd l \cdot \nabla \sigma \rangle_{\text{radial paths}} \rightarrow \int \dd^2 \theta \ldots \) which in turn can be replaced by a sum over the background galaxies (provided these are sufficiently dense on the sky).

An important practical limitation of this method comes from boundary terms introduced when the data are finite. Near the centre of the mass reconstruction this causes a constant negative shift which can be expressed in terms of the surface density and shear on the boundary (for a circular field the shift is the mean of \( \sigma \) plus half the mean tangential shear around the boundary). Further out nearer the edge of the field one finds a spurious negative trough;
the lack of observed shear beyond the field effectively introduces an integral constraint which
forces the total mass in the lens to be zero. A nice feature of the method is that while the es-
timator is a convolution of the observed shears with an extended kernel, the noise has a white
spectrum and, provided we average over a scale containing several galaxies, should be quite
accurately gaussian. This makes assigning significance to features in the mass reconstructions
fairly straightforward.

A weakness of the method is that it does not appear to make full use of the redundancy in
the data described above. We have developed an alternative method which creates the most
probable density field compatible with the data, under the prior assumption of gaussian noise
with user specified colour and amplitude. This should incorporate the extra information.
The results appear quite similar in shape to those obtained with the KS93 method, but there
is a bias in amplitude in the reconstructed $\sigma(\vec{\theta})$ which is hard to calibrate.

**Laplacian Map**

An alternative is to construct a map of the laplacian of $\sigma$:

$$\nabla^2 \sigma = D_\alpha s_\alpha$$

where $D_\alpha \equiv \{\partial^2/\partial x^2 - \partial^2/\partial y^2, 2\partial^2/\partial x\partial y\}$. As we want a smoothed map of the laplacian we implement this as a filter in fourier space, the $D_\alpha$ operator becoming an algebraic function
which multiplies the smoothing filter transfer function. This relation is local, so we don’t
need to worry about boundary terms. This method is probably going to be most useful
for attempting to find clusters blindly in future large survey fields; a cluster will appear as
a negative dip. A nice feature of this method is that one can also use the redundancy in
the input data to calculate a map of $\nabla \times \nabla \sigma$ — which is simply obtained by applying the
‘rotation’ $s_1 \rightarrow s_2, s_2 \rightarrow -s_1$ to the data before applying the $D_\alpha$ operator — and which
should of course be zero. This provides a useful check on the consistency of the data.

**Aperture Massometry**

It is possible to put a rigorous lower bound on the mass contained within a circular
aperture. This method uses the mean tangential shear around a circular path of radius $\theta$:

$$\langle s_T \rangle = \int \frac{d\phi}{2\pi} [s_1 \cos 2\phi + s_2 \sin 2\phi] = -\frac{d\sigma}{d\ln \theta}$$

where $\sigma$ is the mean surface density within the circle and the second equality (which is trivial
for a circularly symmetric lens) follows in the general case from the 2-dimensional version of
Gauss’ law. It is then easy to show that the statistic

$$\zeta(\theta_1, \theta_2) \equiv (1 - \theta_1^2/\theta_2^2)^{-1} \int_{\theta_1}^{\theta_2} d\ln \theta \langle s_T \rangle = \sigma(< \theta_1) - \sigma(\theta_1 < \theta < \theta_2)$$

where the last symbol represents the mean surface density in the annulus $\theta_1 < \theta < \theta_2$. Since
this is necessarily non-negative $\zeta$ provides a lower bound on $\sigma(< \theta_1)$. The $\int d\ln \theta \langle s_T \rangle$ can
again be replaced by an area integral and thereby by a discrete sum over galaxies and, as
with the massmap determination, the error analysis is straightforward.

An interesting feature of this analysis is that it uses only data which lie in the control
annulus, so provided a sufficiently large field, this can be placed outside of most of the cluster
light, greatly simplifying the problem of distinguishing background galaxies from cluster
members.

Unless the ratio of inner and outer radii $\theta_2/\theta_1$ is made very large, in which case we will
only be able to estimate $\overline{\sigma}$ in a very small region, the systematic underestimation of $\overline{\sigma}$ can
be quite substantial. For a power law surface density profile $\sigma \propto \theta^{-\gamma}$ and with $\theta_2 = a\theta_1$,
$\zeta/\overline{\sigma} = (a^2 - a^{2-\gamma})/(a^2 - 1)$, so for $\gamma = 1$, $\zeta$ underestimates the true $\overline{\sigma}$ by 33% and 25% for
$a = 2, 3$ respectively.

As with the laplacian map, it is possible to obtain a check on the consistency of the
data by ‘rotating’ the inputs, $s_1 \rightarrow s_2$, $s_2 \rightarrow -s_1$ which should result in $\zeta = 0$ within the
statistical error.

The $\zeta$ statistic is really a special case of the KS93 method for a particular choice of
smoothing kernel; in this case a ‘compensated top-hat’. In fact, from $\frac{d\overline{\sigma}}{d\ln r} = -\langle s_T \rangle$
one can show that an estimator for the surface density smoothed with an arbitrary circular
window function with zero total weight: $\int d^2 r W(r) = 0$ is

$$\int d^2 r W(r) \sigma(\vec{r}) = 2\pi \int dr \langle s_T \rangle W'(r)$$

where

$$W'(r) = \frac{1}{r} \int_0^r dr' r' W(r') - \frac{r W(r)}{2}$$

which provides a simple way to construct the window function $W'$ for e.g. any mexican hat
type $W(r)$. Note that if $W$ vanishes beyond some radius then so does $W'$, which again is
nice if one is dealing with finite data.

**Dilution of the Counts**

The ambiguity in the baseline surface density is a nuisance. This can be resolved in prin-
ciple by measuring the dilution of the background counts $\delta n(m)$ caused by the amplification
(BPT). Under the weak lens assumption, the perturbation to the counts is just proportional
to the surface density, though with a rather small constant of proportionality since the ampl-
ification bias for faint galaxies is rather small. There are clearly some technical difficulties
in applying this method; here one must look for a perturbation under the lens, so to speak,
rather than around it as in the shear measurement, so one must be careful to correct for the
faint cluster galaxy counts. This can be aided by using colour information, but getting accu-
rate colours is expensive. One would also have to correct for masking of background galaxies
by the high surface brightness parts of foreground galaxies, and perhaps subtle effects in the
image detection and photometry where the extended diffuse light around the cluster galaxies
overlays the background.

With effort these problems can perhaps be overcome, but the following example suggests
that the method will still suffer from rather low signal to noise: If we use the $\zeta$ statistic to
estimate $\sigma$, the statistical uncertainty is $\langle \hat{\sigma} \rangle^{1/2} = (1 - 1/a^2)^{-1} \sqrt{\langle s_1^2 \rangle / (4\pi \bar{\pi} r_2^2)}$ where $r_2 = ar_1$, $\bar{\pi}$ is the surface number density of galaxies, and the rms shear noise (with both instrumental and intrinsic contributions) is $\langle s_1^2 \rangle^{1/2} \simeq 0.43$ for the data described below. From the observed faint galaxy counts, the amplification bias appears to be quite small; $\simeq -0.25$. The amplification is $2\sigma$, so the corresponding estimator is $\hat{\sigma} \simeq -2\delta n(m)/n(m)$ (BPT describe alternatives such as looking for a change in the slope of the counts, but the simple example here illustrates the basic idea and it is hard to imagine that the noise in any other estimator is likely to be significantly different). Assuming a poisson distribution for the background galaxies, the noise in this estimator is then $\langle \hat{\sigma}^2 \rangle^{1/2} = 2/\sqrt{4\pi \bar{\pi} r_2^2}$. Clustering of the background galaxies will inflate this by an uncertain but probably appreciable factor, but the minimal poisson error is already about a factor 8 larger than that for the $\zeta$ statistic, so for the ms1224 data described below, for example, the expected S/N is below unity.

**MS1224**

This cluster was chosen for its high X-ray luminosity, its high redshift ($z = 0.33$) making it possible to survey a $\sim 2h^{-1}$Mpc square field in a reasonable time, and because it was also a target of the CNOC cluster project. The optical spectroscopy studies (Carlberg, Yee and Ellingson, 1994, hereafter CYE) gave a modest velocity dispersion of $\simeq 750$km/s, and apparently consistent with this, a low richness, though as we shall see, the mass found from the lensing appears to be much greater.

As described in Fahlman et al., 1994 (hereafter FKSW), we took 1 hr total I-band integrations on each of four fields surrounding the cluster centre under excellent seeing conditions. Our software found $\sim 5000$ objects over an area (after allowing for masked regions around bright foreground stars) of about 120 square arcmin, from which we extracted a ‘faint-extended’ subsample of about 2000 galaxies covering a range of about 3 magnitudes to $I=23.4$. As a test, these data were artificially stretched, rebinned onto a $2 \times$ coarser pixel grid and then degraded to simulate the effects of seeing. These synthetic data were then analysed in the same way as the real data. This verified that the analysis software was indeed able to detect the artificial shear, and also allowed us to estimate a small, but critical ‘signal loss factor’ due to seeing and other biases in the analysis. The anisotropy of the psf was measured and the correction applied as described above.

In the real data a clear shear signal was seen: the mass-map has a peak which coincides quite well with the bulk of the smoothed cluster light measured by CYE, and the tangential shear was also clearly seen (as a significance level of about 5-sigma). The $\zeta$-statistic gave $\bar{\sigma}(> 2.7') \geq 0.06 \pm 0.012$. The signal was also seen repeatedly in independent magnitude sub-samples. A mysterious dark peak also appears repeatedly in the mass-map. This is not seen in the cluster light, but as it is only a $\simeq 3$-sigma detection it should not be taken too seriously.

Fig. 1.—Spatial distribution and polarization (or ellipticity) parameters for the ms1224 faint galaxy subsample. The left panel shows the individual galaxies to be fairly uniformly distributed on the sky, though with some holes around bright foreground stars and galaxies. No particularly strong coherent distortion is apparent to the eye. However, in the panel on the
right we have smoothed the polarizations to make a map of the shear, and the characteristic shear pattern is now clearly seen.

While the location of the main mass peak is in nice accord with the optical measurement, the amplitude is not. The mass scale calibration depends on the effective inverse critical surface density which in turn depends on the redshift distribution of the background galaxies. For our brighter galaxies we can measure $\Sigma_{\text{crit}}^{-1}$ directly using the data of Lilly, 93 and Tresse et al., 1993; the much larger CFRS redshift survey has recently been completed, increasing the sample size by an order of magnitude, and seems to reinforce the results obtained from the smaller published samples. For the fainter galaxies we need to extrapolate. Extrapolating the trend at brighter magnitudes suggest a very slow increase in $\Sigma_{\text{crit}}$ of about 30% per magnitude. A similar or even more modest increase is indicated by the slow variation of shear with magnitude limit seen in our studies of ms1224 and now of several other similar clusters.

The upshot of all of this is a lower bound (aside from statistical error, of course) on the mass within a radius of 2.76' (or roughly $0.5h^{-1}\text{Mpc}$ in physical radius) of $M_{\text{lens}} \geq 3.5 \times 10^{14}M_\odot/h$. This can be compared with the virial mass in the same aperture — obtained by taking the total virial mass estimate and multiplying by the fraction of the total cluster light lying within the aperture — of $M_{\text{virial}} = 1.15 \times 10^{14}M_\odot/h$; a factor 3 smaller than $M_{\text{lens}}$. We can also use our photometry (or that of CYE) to obtain a mass to light ratio $(M/L)_V \approx 800h$, much larger than the values typically obtained for nearby optically selected clusters.

Fig. 2.—Contour plot of the projected mass reconstruction superposed on the summed I-band image of the central 7' square region of ms1224.
We have thought long and hard about possible errors or biases in our analysis which might have corrupted our mass estimates. Here is a non-exhaustive list of factors we have considered, though most of these can be safely discounted: 

i) Biases: There are a number of biases: $\zeta < \sigma$; the optical light estimate includes projected material in the centre beam, but does not cover the entire control aperture, and no attempt has been made to remove the cluster galaxies, which will tend to dilute the shear signal. Allowing for these biases would only exacerbate the discrepancy. 

ii) Statistical Uncertainty: The statistical uncertainty in $\sigma$ is about 20%. At the time we submitted FKSW there was a larger statistical uncertainty arising from the rather small size of the redshift surveys, but the larger CFRS survey seems to give similar results. The redshift surveys are, of course, not 100% complete and the missing 10% or so of the galaxies are most probably at higher redshift. However, even if we put these at redshift 2 say, the effect on our mass estimate is small. We found that in order to reconcile the mass estimates would require a median galaxy redshift $\sim 4$, which is clearly unreasonable. 

There is some variation in the redshift distribution from field to field in the $z$-surveys, and while the fluctuations expected for the larger field here are smaller, there is a remote possibility that we hit an extreme statistical fluctuation in $n(z)$ due to large-scale structure. However, it is relevant to note that the counts of galaxies we find agree well with those of e.g. Lilly, Cowie and Gardner (1991). 

iii) Contamination: Our method measures the total projected mass (excess) within our aperture. It is possible (though extremely improbable) that there is another high mass object within the aperture. However, if this lies at a redshift $> 0.33$ then the projected mass-to-light ratio for the combined system only increases, and if it lies at lower redshift it is hard to see why it was not detected. If we had no knowledge at all of the lens redshift, the lowest mass to light ratio is obtained by placing the lens at a somewhat lower redshift, and is only marginally less than our quoted value. We consider the effect of adding mass around the cluster itself below. 

iv) Inhomogeneous Sampling: Our massmap algorithm assumes uniformly sampled data, whereas in fact there are some masked holes which can result in a bias in the shape of real observed signal (though they do not create a bias in the sense of causing spurious detections where there is no real signal). Could this have created the dark-blob? We have explored this with simulations obtained by using the real galaxy positions but by shuffling the shear estimates and adding a known lensing signal, and we find any such bias to be very small. 

v) Weak Shear Assumption: If the shear becomes strong then the assumed linear response of the population mean polarisation will fail. In particular one might worry that gravitational amplification might have increased the mean redshift of the background galaxies behind the cluster. However, the measured shear in ms1224 is in fact very weak: $s \sim 10\%$ or so after correction for seeing; the light in the cluster is compact (more than half the light lies within our central aperture) so if mass traces light at least, the amplification for the galaxies we actually use is very small and in any case all indications are that the median redshift increases very weakly with increasing magnitude. One might worry that even though the mean surface density is low, the dark matter has clumpy substructure giving localised regions of non-linearity which somehow bias the mean shear estimation. This possibility probably deserves detailed study, but one argument against this
is that one often sees very long smooth arcs, suggesting the dark matter in clusters is actually smoothly distributed, and, as emphasised by Tyson (1990), if one makes the dark mass too clumpy this produces a large number of conspicuous arcs which are not seen.  

vi) Cosmological Model: Our calculation of $\Sigma_{\text{crit}}$ assumes a flat, zero cosmological constant model. Changing these assumptions makes only a very slight change to our results.  

vii) Correlated intrinsic ellipticities: Our error analysis assumes that the intrinsic ellipticities are uncorrelated. Flin (1993a,b) finds a tendency for galaxies in physical pairs and triplets to be aligned, but this is a weak statistical effect seen at the $\simeq 2$-sigma level in a sample of $\sim 10^3$ galaxies, and would negligibly effect the noise level in our mass-maps. We are currently analysing some blank fields where we can check for spurious detections. None have been found as yet.  

viii) Signal loss factor: Our simulations, which show that due to seeing etc. we recover only about 70\% of the input shear, assume that the faint galaxies are simply scaled down replicas of their brighter cousins. This may be false, but the sizes of the synthetic and real galaxies at least are quite similar. A better way to establish this calibration factor would be stretch and then degrade deep images from HST, which will shortly become possible. We are confident however, that seeing does not increase the shear. 

None of these loopholes appear very promising and it is therefore hard to escape the conclusion that, in this cluster at least, the mass and the mass-to-light ratio are indeed very large. We have subsequently obtained similarly extensive data on A2218, and smaller fields on A2163 and A2390. These clusters all have arcs with measured redshifts, and while our analysis is still ongoing, we see no indication that the weak shear analysis overestimates the mass as compared with the arcs.
Fig. 2.—Contour plot of the projected mass reconstruction superposed on the photometry in A2218.
WHAT DOES IT MEAN?

There are really two puzzles here: Why is $M_{\text{tens}} \gg M_{\text{virial}}$ and why is the lensing derived $M/L$ so large. It is quite possible to imagine that the observed velocity dispersion underestimates the mass: after all, the well documented ‘beta-discrepancy’ problem seems to indicate a sizeable scatter in $\sigma^2_{\text{v}}/T_X$ and we might just be seeing a low-beta cluster. Perhaps the cluster consists of two clumps merging along a direction perpendicular to the line of sight and that is why we see a low $\sigma_v$. It is interesting that the conspicuous giant elliptical in the cluster does not lie near the centre of the main cluster concentration, reinforcing the suspicion that we are not dealing with a well relaxed system. This might explain a low virial mass, but would not reduce the $M/L$.

Another possibility is that the galaxies are relaxed, but that they suffer from velocity bias because their scale length is shorter than that of the mass. The key question is what is the mass profile derived from the shear? Unfortunately this is rather difficult to answer as with the current data we only have a 5-sigma detection, so there is considerable noise as well as bias in the mass profile. In addition, one should be wary of dilution of the shear signal by cluster galaxies which will further bias the profile in the centre. Our lower bound on the projected aperture mass applies if the projected mass vanishes in the control annulus: i.e. implicitly assuming a very steep mass profile. The lensing data themselves would be quite compatible with a more extended mass distribution. Under the empty annulus assumption the shear should fall off within the annulus as $s \propto 1/r^2$ whereas in fact it appears to be remarkably flat with radius, suggesting, at face value, a very flat surface density profile indeed. However, we must emphasise that invoking an extended mass profile will increase the mass to light ratio if one self-consistently corrects for the bias in the $\zeta$ statistic. With an isothermal sphere type mass profile, for example, $\sigma$, and therefore $M/L$ within our aperture would increase by about 30%, and the $M/L$ within $1h^{-1}$Mpc would increase even more. Nor is it clear that this type of solution can really explain the low observed velocity dispersion; in the isothermal sphere example, the 1-D velocity dispersion for any population of finite radial extent is $\sigma^2_v = V^2_{\text{rot}}/3 = \theta \sigma(< \theta)/(6\pi) = (940\text{km}/\text{s})^2$, still larger than that observed.

Even our minimal mass gives a surprisingly high mass-to-light ratio (or essentially equivalent, but easier to measure, a very high mass per galaxy). However, the value is not at all out of line with other studies using the same technique: Bonnet et al., 1993 claim the large scale shear around cl0024 requires a mass roughly 3-times larger than the virial mass, and Smail et al., 1994 quote $M/L$ of $\sim 550h$, from their studies of cl1455, cl0016, quite comparable to the value here. What makes these high $M/L$ ratios more surprising is when we allow for the considerable increase in the comoving number density of $\sim L_{\ast}$ galaxies inferred from the faint galaxy redshift surveys. We can clearly measure the excess cluster counts $N_c$ in our central aperture over about a two magnitude range below $L_\ast$, and thereby obtain $(M/N)_{\text{cluster}}$. Similarly, we can readily estimate the comoving number density of field galaxies over the same magnitude range at this redshift and thereby obtain a estimate of $(M/N)$ for a closed universe. An estimate of $\Omega$ then follows if one assumes that the mass-per-galaxy in the cluster...
is representative of the universe as a whole:

\[
\Omega = \frac{(M/N)_{\text{cluster}}}{(M/N)_{\text{universe}}} = \frac{\sigma d\Omega \sqrt{1 + z_c (dn/dz) z_c}}{3 N_c w_l \langle \max(0, 1 - w_l/w_g) \rangle} \approx 1.8
\]

This is very large compared to the typical values found applying the same kind of analysis to low redshift, optically selected clusters. The difference stems roughly equally from the high cluster $M/L$ and from the evolution of the field galaxy population. There is of course considerable uncertainty in this estimate due to the uncertain $n(z)$ for the background galaxies, and, as with any $\Omega$ estimate of this kind, one is really under no obligation to believe that the mass-per-galaxy of this particular cluster, or indeed of clusters in general, is representative.

How do we understand the high $M/L$’s if they are indeed real? Does the mass-to-light ratio of a cluster decrease with time; implying either that galaxy rich matter falls in later or that somehow galaxy formation is stimulated within the cluster? Or perhaps is the explanation simply that there is really a wide variation in $M/L$’s for clusters; the well studied optically selected clusters at low redshift preferentially seeing the low $M/L$ cases and the arcs selected clusters naturally sampling the higher end. This is attractive, but it is not easy to see why ms1224 should have been biased in this way; the highly speculative possibility that ms1224 got into the EMSS sample by macro-lensing a background AGN will shortly be testable with ROSAT and ASCA spectra.

How could a strong variation in $M/L$ on cluster scales arise? Could it be that in an early stage of explosive galaxy formation the bulk of the gas was disturbed in the manner envisaged by Ostriker and Cowie (1981), resulting in a highly inhomogeneous gas entropy and density distribution? The evolution of the dark-matter clustering would proceed essentially undisturbed, but there will be DM concentrations which happen to lie in regions of high entropy gas where galaxy formation might plausibly have been impeded. An appeal of this idea is that this might also help explain the ‘baryon catastrophe’ problem (White, et al., 1993). The dark-clumps seen in some of the mass reconstructions are certainly of interest in this regard, but the significance of the dark feature in the ms1224 map at least is only marginal.

The results described here and those of Tyson’s group, the Toulouse group and the Durham/Caltech group clearly show that these observations are a practical way to directly map the dark matter in clusters. The high mass-to-light ratios obtained are admittedly somewhat puzzling. The calibration of this method, on which these results rest, is not perfect, but we have argued that it is very hard to make the discrepancy with the virial mass in ms1224 go away and indeed, at face value, the lensing data are quite compatible with an even larger mass. The great strength of this method is that it makes no assumptions regarding the shape, dynamical stability or state of relaxation of the cluster. There are still uncertainties in e.g the redshift distribution of the faint galaxies, which affects the calibration, but these are small and of an ‘engineering’ nature and should be solvable with a combination of HST imaging, ground-based spectroscopy to fainter limits and a much larger sample of weak-shear cluster studies.

These observations are currently limited by detector technology. It would be of great
value to obtain data out to large radii. CFHT has a corrected field $\sim 50'$ across, yet we currently use only use a 7' square 2048$^2$ chip. The 4096$^2$ MOCAM array will speed observations by a factor 4. With a thinned mosaic of this size on a 10m telescope the ms1224 study could be made in about 4-minutes, so it would be quite practical to go much fainter, increasing the number of background galaxies substantially, and thereby boosting the precision of the measurement. A further boost in signal to noise can be obtained by studying clusters at somewhat lower redshift, though to explore the same physical radius becomes more expensive and one also becomes more sensitive to how well correlated psf variations and other systematic effects can be corrected for. With these developments it should be quite feasible to obtain detailed individual mass profiles and shapes for the most massive clusters and also, with large random field surveys, to obtain a mass-selected sample of clusters. Galaxies, groups and poor clusters will be hard to detect individually, but by stacking results it should be possible to determine e.g. the galaxy-mass cross-correlation function directly. Another window of opportunity is to study coherent shear on the scale of superclusters; current observations being right at the level of precision where we expect to see a signal appearing. This will potentially provide a direct measure of the mass power spectrum $P_\rho(k)$, giving a strong test of cosmogonical theories and, combining with COBE type measurements on a similar scale, giving us a handle on the relative contribution of tensor and scalar modes and/or the ionisation history. Finally, a further spin-off from these studies will be quite accurate measurement of the mean relative geometrical distances to faint galaxies as a function of their size and magnitude. Combining these with directly measured redshifts should allow a fundamental test of the cosmological world model.

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