Resummation of Large Corrections in Inclusive $B$ Meson Decays

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Abstract

We show that the resummation technique developed recently for heavy-light systems and the conventional approach based on the factorization of Wilson loops are equivalent. The two methods lead to the same results of the resummation of large corrections in inclusive $B$ meson decays.
1. Introduction

Perturbative QCD (PQCD) including Sudakov effects [1, 2] has been proposed to be an appropriate approach to exclusive heavy meson decays in the maximal recoil region of final-state hadrons [3, 4, 5, 6]. The progress is attributed to the resummation of large radiative corrections in heavy-light systems such as a $B$ meson containing a light valence quark, through which important higher-order and higher-power contributions are included into factorization theorems. Recently, the resummation technique was extended to the inclusive semileptonic decay $B \to X_u \ell \nu$ [7], whose behavior near the high end of charged lepton energy is crucial to the extraction of the Cabibbo-Kobayashi-Maskawa matrix elements $|V_{ub}|$ in the standard model and to the detection of new physics. It has been observed that large perturbative corrections appear at the end point of the spectrum [8, 9, 10], which must be organized to all orders in order to have a reliable PQCD analysis.

The above approach to the resummation of large corrections in inclusive processes differs from the conventional one. In the study of deep inelastic scatterings and Drell-Yan productions [11], large soft corrections are factorized systematically into path-ordered exponentials, or Wilson lines [12, 13, 14] along the classical trajectories of quarks. Korchemsky and Sterman [15] applied this Wilson-loop formalism to the study of end-point singularities in inclusive $B$ meson decays. The Wilson lines are absorbed into a heavy quark distribution function $S$, and give its evolution governed by renormalization group (RG) equations. The emitted light quark produces a jet of collinear particles at the end point due to its large energy and small invariant mass [15].

In this letter we shall show that the resummation technique is equivalent to the Wilson-loop formalism, if relevant evolution scales are chosen properly. The two methods give the same resummation results for the inclusive decays $B \to X_s \gamma$ and $B \to X_u \ell \nu$ up to next-to-leading logarithms. Therefore, this
work provides a non-trivial check on the two different approaches.

2. The Radiative Decay $B \rightarrow X_s \gamma$

We apply the resummation technique to the inclusive radiative decay $B \rightarrow X_s \gamma$, which occurs through the transition $b \rightarrow s \gamma$ described by an effective Hamiltonian [10]. The basic factorization of this process is shown in fig. 1a, where the bulb represents the $b$ quark distribution function $S(k)$, $k$ being the momentum carried by light partons in the $B$ meson, $k^2 \approx 0$. $k$ has a plus component $k^+$ and small transverse components $k_T$, which serve as the infrared cutoff of loop integrals for radiative corrections. The momentum of the $b$ quark is then $P - k$, $P = m_B/\sqrt{2}(1, 1, 0)$ being the $B$ meson momentum in terms of light-cone components, which satisfies the on-shell condition $(P - k)^2 \approx m_b^2$. Here $m_B$ and $m_b$ are the $B$ meson mass and the $b$ quark mass, respectively. The $b$ quark decays into a real photon of momentum $q$ and a $s$ quark of momentum $P_s$, which is regarded as being light. Assuming that the nonvanishing component of $q$ is $q^+$, we have $P_s = P - k - q = (P^+ - k^+ - q^+, P^-, -k_T)$.

We work in axial gauge $n \cdot A = 0$ for the analysis below, $n$ being a gauge vector. In the end-point region with $q^+ \rightarrow P^+$, the $s$ quark has a large minus component $P^-$ but a small invariant mass $P_s^2$. Hence, photon vertex corrections and self-energy corrections to the $s$ quark in fig. 1b give double logarithms from the overlap of collinear and soft divergences. In the leading regions with loop momentum $l$ parallel to $P_s$ and with soft $l$, the integrand associated with the photon vertex correction has the partial expression

$$\frac{(P - k + l + m_b)\gamma_\lambda(P + m_B)}{[(P - k + l)^2 - m_b^2]} \approx \frac{2P_\lambda - \gamma_\lambda(P - m_b) + l\gamma_\lambda(P + m_B)}{2P \cdot l}, \quad (1)$$

where the factor $(\not{P} + m_B)$ is the matrix structure of $S$, and those terms
proportional to $k$ have been neglected. The second term in the numerator of eq. (1) is suppressed by power $(m_B - m_b)/m_B$. The third term is negligible for soft $l$. For $l/P_s$, it can be easily shown that the trace of $\gamma$-matrices associated with fig. 1a, which contains the third term, vanishes.

Therefore, in the leading regions of $l$ radiated gluons are detached from the $b$ quark and collected by an eikonal line described by the Feynman rule $P_\lambda/P \cdot l$. The factor $1/P \cdot l$ is associated with the eikonal propagator, and the numerator $P_\lambda$ is assigned to the vertex, where a gluon attaches the eikonal line. This observation is consistent with the flavor symmetry in the heavy quark effective theory [17]. We then absorb this type of radiative corrections into a jet function $J(P_s)$, which is now factorized out of the process. This absorption is reasonable because the soft divergences in fig. 1b cancel asymptotically as explained later, and the double logarithmic corrections are mainly collinear.

Self-energy corrections to the $b$ quark and loop corrections with real gluons connecting the two $b$ quarks in fig. 1c contain only single soft logarithms, and are grouped into the distribution function, which is supposed to be dominated by soft dynamics. Remaining important corrections from hard gluons are absorbed into a hard scattering amplitude $H$. At last, the factorization formula for the spectrum of the decay $B \to X_s \gamma$ is written as

$$\frac{d\Gamma_\gamma}{dE_\gamma} = \int dk^+d^2k_T S(k, \mu)H(P - k, \mu) J(P_s, \mu),$$

which is described graphically in fig. 2. Here $E_\gamma = q^+ / \sqrt{2}$ is the photon energy, and $\mu$ is the factorization and renormalization scale.

The Wilson-loop formalism for the decay $B \to X_s \gamma$ [15] is summarized in fig. 3. Soft gluons radiated from the $b$ quark are collected by the eikonal lines along the classical trajectories of the $b$ and $s$ quarks, which form a Wilson loop, and are absorbed into $S$. Since the $s$ quark is almost on-shell in the end-point region, the Wilson loop introduces extra collinear singularities into the distribution function, which are not consistent with the dominant
soft dynamics in the $B$ meson. Note that the light $s$ quark is not replaced by an eikonal line in our treatment. Hence, both $S$ and $J$ in [13] contain double logarithms, which are treated by RG methods. However, due to the presence of double logarithms, relevant anomalous dimensions involved in RG equations must be accompanied by large logarithms [12], such that perturbative calculation is not reliable.

For the purpose of demonstrating the equivalence of our approach to the Wilson-loop one, it is convenient to choose a time-like gauge vector $n = P$, instead of a space-like $n$ as in [4, 5, 7], under which radiated gluons do not attach the eikonal lines associated with the $b$ quarks. This choice of $n$ does not bring new pinch singularities into $S$.

We drop the intrinsic $k_T$ dependence in $S$, and Fourier transform eq. (2) into $b$ space, $b$ being a conjugate variable to $k_T$,

$$\frac{d\Gamma_\gamma}{dE_\gamma} = \int dk^+ \frac{d^2b}{(2\pi)^2} S(k^+, \mu) \tilde{H}(P^+ - k^+, \mathbf{b}, \mu) \tilde{J}(P_s^+, P_s^-, \mathbf{b}, \mu).$$

(3)

It has been shown that integrands for soft corrections are proportional to $1 - e^{l_T \cdot b}$ [1, 7], which vanish in the asymptotic region with $b \to 0$ as stated before.

$\tilde{J}$ depends on $P_s$ and $n$ through the ratio $(P_s \cdot n)^2/n^2$ due to the scale invariance in $n$ of the gluon propagator in axial gauge,

$$N^{\mu\nu}(l) = \frac{-i}{l^2} \left( g^{\mu\nu} - \frac{n^\mu n^\nu + n^\nu l^\mu}{n \cdot l} + n^2 \frac{l^\mu l^\nu}{(n \cdot l)^2} \right),$$

(4)

To sum up the double logarithms in $\tilde{J}$, we consider the derivative $d\tilde{J}/d\ln P_s^-$ [4, 7], $P_s^-$ being regarded as a variable from now on. Replacing $d/d\ln P_s^-$ by $d/dn$ using a chain rule, we have

$$\frac{d}{d\ln P_s^-} \tilde{J} = -n^2 \frac{v \cdot n}{v \cdot n} \frac{d}{dn_\alpha} \tilde{J}$$

(5)

with the vector $v = (0, 1, 0)$. $d/dn_\alpha$ operates only on a gluon propagator,
and gives \[6\]

\[
\frac{d}{dn_\alpha} N^{\mu \nu} = -\frac{1}{l \cdot n} \left( N^{\mu \alpha \nu} + N^{\nu \alpha \mu} \right).
\]

Combining eqs. (5) and (6), we find that a gluon vertex, after contracted with \(l\) that locates at both ends of the differentiated gluon line, has been replaced by a new vertex, described by the Feynman rule \(\frac{n^2}{v_n} l \cdot n\). This new vertex will be represented by a square. Adding together all diagrams with different differentiated gluon lines and using the Ward identity [1, 18], this square moves to the outermost ends of \(\bar{\mathcal{J}}\).

The gluon momentum \(l\) flowing through the square vertex does not lead to collinear divergences because of the denominator \(n \cdot l\). The leading regions of \(l\) are then soft and ultraviolet, in which the subdiagram containing the square can be factorized. We then derive a differential equation,

\[
\frac{d}{d \ln P_s^{-1}} \bar{\mathcal{J}} = 2 [\mathcal{K}(\alpha_s(\mu)) + \mathcal{G}(\alpha_s(\mu))] \bar{\mathcal{J}},
\]

as shown in fig. 4a, where the eikonal lines disappear due to the choice \(n = P\). The factor 2 counts the two ends of \(\bar{\mathcal{J}}\). The functions \(\mathcal{K}\) and \(\mathcal{G}\) collect the soft and ultraviolet divergences in the subdiagram, respectively. Lowest-order diagrams of \(\mathcal{K}\) are exhibited in fig. 4b, and those of \(\mathcal{G}\) are in fig. 4c.

A straightforward calculation gives \(\mathcal{K} = \text{fig. 4b} - \delta \mathcal{K}\) and \(\mathcal{G} = \text{fig. 4c} - \delta \mathcal{G}\), \(\delta \mathcal{K}\) and \(\delta \mathcal{G}\) being the corresponding counterterms. Since \(\mathcal{K}\) contains only single soft logarithms and \(\mathcal{G}\) contains only single ultraviolet logarithms, they can be treated by RG methods:

\[
\mu \frac{d}{d \mu} \mathcal{K} = -\lambda_{\mathcal{K}} = -\mu \frac{d}{d \mu} \mathcal{G}.
\]

\(\lambda_{\mathcal{K}} = \mu \frac{d \delta \mathcal{K}}{d \mu}\) is the anomalous dimension of \(\mathcal{K}\), whose expression up to two loops is given by [1, 4]

\[
\lambda_{\mathcal{K}} = \frac{\alpha_s}{\pi} C_F + \left( \frac{\alpha_s}{\pi} \right)^2 C_F \left[ C_A \left( \frac{67}{36} - \frac{\pi^2}{12} \right) - \frac{5}{18} n_f \right]
\]
with \( n_f \) the number of quark flavors, and \( C_F = 4/3 \) and \( C_A = 3 \) the color factors. \( \lambda_K \) is independent of the gauge vector \( n \), and thus eq. (9) is identical to that derived from a space-like \( n \) in [7].

There are two \( P_s^- \)-dependent scales involved in \( \tilde{J} \): one is \( P_s^- \) itself, which serves as the lower limit of \( \mu \), and the other comes from the invariant \( P \cdot P_s \approx P^+ P_s^- \). Solving eq. (8), we have

\[
K + G = K(\alpha_s(P^+_s)) + G\left(\alpha_s\left(\sqrt{P^+ P^-_s}\right)\right) + \int_{P^-_s}^{\sqrt{P^+ P^-_s}} \frac{d\mu}{\mu} \lambda_K(\alpha_s(\mu)),
\]

\[
\approx \int_{P^-_s}^{\sqrt{P^+ P^-_s}} \frac{d\mu}{\mu} \lambda_K(\alpha_s(\mu)). \tag{10}
\]

In the second expression we have neglected the initial condition of \( K + G \).

Substituting eq. (10) into (7), we derive

\[
\tilde{J}(P^+_s, P^-_s, b, \mu) = \exp\left[-2 \int_{1/b}^{P^-_s} \frac{dP^-_s}{P^-_s} \int_{P^-_s}^{\sqrt{P^+ P^-_s}} \frac{d\bar{\mu}}{\bar{\mu}} \lambda_K(\alpha_s(\bar{\mu}))\right] \tilde{J}(P^+_s, b, \mu), \tag{11}
\]

where we have taken the infrared cutoff \( 1/b \) (conjugate to \( k_T \)) as the lower bound of \( P^-_s \) and the largest scale \( P^- \) involved in the process as the upper bound. The four scales appearing in eq. (11) satisfy the ordering \( P^- > \sqrt{P^+ P^-_s} > P^-_s > 1/b > \Lambda_{\text{QCD}} \), which justifies the neglect of the initial condition of \( K + G \).

The function \( \tilde{J}(P^+_s, b, \mu) \) obeys the RG equation,

\[
\mathcal{D} \tilde{J}(P^+_s, b, \mu) = -2\lambda_J \tilde{J}(P^+_s, b, \mu), \tag{12}
\]

with \( \mathcal{D} = \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \) and \( \lambda_J = -\alpha_s/\pi \) the quark anomalous dimension in axial gauge. The invariant mass of the jet, \( P_s^2 \approx 2P^+_s P^- \approx 2P^-/b \), if assuming that the small \( P^+_s \) is of the same order as \( 1/b \), is a natural initial scale of the evolution. The solution to eq. (12) is then given by

\[
\tilde{J}(P^+_s, b, \mu) = \exp\left[-\int_{P^-/b}^{\mu^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \lambda_J(\alpha_s(\bar{\mu}))\right] \tilde{J}(P^+_s, b, \sqrt{P^-/b}). \tag{13}
\]
To sum up next-to-leading logarithms, we have to organize the single soft logarithms in $S$. The RG equation for $S$ is written as

$$\mathcal{D} S(k^+, \mu) = -\lambda_S S(k^+, \mu),$$

where $\lambda_S = -(\alpha_s/\pi)C_F$ is the anomalous dimension derived from fig. 1c with the eikonal approximation. We allow $S$ to evolve to the infrared cutoff $1/b$, and obtain

$$S(k^+, \mu) = \exp \left[ -\int_{1/b}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \lambda_S(\alpha_s(\bar{\mu})) \right] S(k^+, 1/b).$$

The initial condition $S(k^+, 1/b)$ has a nonperturbative origin arising from soft QCD dynamics in the $B$ meson.

Since the differential decay rate is independent of $\mu$, the RG equation of the hard scattering $\tilde{H}$ is written as

$$\mathcal{D} \tilde{H}(\mu) = (2\lambda_J + \lambda_S) \tilde{H}(\mu),$$

which is solved to give

$$\tilde{H}(\mu) = \exp \left[ -\int_{\mu^2}^{(P^-)^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \lambda_J(\alpha_s(\bar{\mu})) - \int_{\mu}^{P^-} \frac{d\bar{\mu}}{\bar{\mu}} \lambda_S(\alpha_s(\bar{\mu})) \right] \tilde{H}(P^-).$$

We have chosen the largest scale $P^-$ as the upper bound of the integral.

Combining eqs. (14)-(17), we derive the differential decay rate

$$\frac{d\Gamma_\gamma}{dE_\gamma} = \int dk^+ \frac{d^2b}{(2\pi)^2} S(k^+, 1/b) \tilde{H}(P^+ - k^+, b, P^-) \tilde{J}(P^+, b, \sqrt{P^-/b})$$

$$\times \exp(-s_\gamma(P^-, 1/b)),$$

with the Sudakov exponent

$$s_\gamma = 2 \int_{1/b}^{P^-} \frac{dP^-}{P^+} \int_{P^-}^{\sqrt{P^+P^+_s}} \frac{d\bar{\mu}}{\bar{\mu}} \lambda_K(\alpha_s(\bar{\mu}))$$

$$+ \int_{1/b}^{P^-} \frac{d\bar{\mu}}{\bar{\mu}} \lambda_S(\alpha_s(\bar{\mu})) + \int_{P^-/b}^{(P^-)^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \lambda_J(\alpha_s(\bar{\mu})).$$
Employing the variable changes $P_s^- = yP^-$ in the first term, $\bar{\mu} = yP^-$ in the second term and $\bar{\mu}^2 = yP^{-2}$ in the last term, eq. (19) reduces to

$$s_\gamma = \int_1^{1/bP^+} \frac{dy}{y} \left[ 2 \int_{yP^+} \frac{d\bar{\mu}}{\bar{\mu}} \lambda_K(\alpha_s(\bar{\mu})) + \lambda_S(\alpha_s(yP^-)) + \lambda_J(\alpha_s(\sqrt{y}P^-)) \right].$$

(20)

The corresponding expression for the exponent in (15) is quoted below:

$$s_N = \int_{n_0/N}^1 \frac{dy}{y} \left[ 2 \int_{yM}^{\sqrt{y}M} \frac{dk_t}{k_t} \Gamma_{\text{cusp}}(\alpha_s(k_t)) + \Gamma(\alpha_s(yM)) + \gamma(\alpha_s(\sqrt{y}M)) \right],$$

(21)

where $s_N$ describes the evolution of the $N$-th moment of the differential decay rate, with $M$ the $b$ quark mass, the constant $n_0 = e^{-\gamma_E}$, $\gamma_E$ being the Euler constant, and $\Gamma_{\text{cusp}} = \lambda_K$, $\Gamma = -\alpha_s(\pi)/\pi C_F$ and $\gamma = -\alpha_s(\pi)$ the anomalous dimensions. The two-loop expressions for $\Gamma_{\text{cusp}}$ and $\Gamma$ can be found in [12, 19].

Comparing eq. (20) to (21), we easily identify the equality of the anomalous dimensions:

$$\lambda_K = \Gamma_{\text{cusp}}, \quad \lambda_S = \Gamma, \quad \lambda_J = \gamma.$$

(22)

The two expressions $s_\gamma$ and $s_N$ are basically the same except the lower bounds of $y$. The scale $1/b$ comes from the inclusion of transverse degrees of freedom in our formalism, and the $N$ dependence is due to the moment analysis in [14].

3. The Semileptonic Decay $B \rightarrow X_u\ell\nu$

The analysis of the inclusive semileptonic decay $B \rightarrow X_u\ell\nu$ is similar to that of the radiative decay $B \rightarrow X_s\gamma$. Large double logarithmic corrections are also observed near the high end of the charged lepton spectrum. We choose the $B$ meson momentum $P$ and the light parton momentum $k$ as in
Section 2. The lepton momentum $P_\ell$ has only a large plus component $P_\ell^+$. The neutrino carries the momentum $P_\nu$, $P_\nu^2 = 0$. The $u$ quark, emitted by the $b$ quark through the transition $\bar{u}\gamma^\mu b$, has the momentum $P_u = P - k - q$ with the lepton pair momentum $q = P_\ell + P_\nu = (P_\ell^+ + P_\nu^+, P_{\nu T}, P_\nu)$. When $P_\ell^+$ reaches its maximum $P^+$, implying $P_\nu^+ \approx 0$ and $P_{\nu T} \approx 0$, the $u$ quark has a small invariant mass, and moves fast in the minus direction with $P_u^- = P_\ell^+ - P_\nu^-$. The situation is then similar to the end-point region of the radiative decay $B \to X_s \gamma$.

Repeating the resummation procedures in Section 2, we derive the corresponding Sudakov exponent $s_\ell$, which is similar to $s_\gamma$ but with the largest scale $P^-$ replaced by $(1 - x_\nu)P^-$, $x_\nu = P_\nu^-/P^-$. $s_\ell$ is given by

$$
\begin{align*}
\log \frac{dP^-}{P^-} &\int_0^{P^+} \frac{d\mu^2}{\mu^2} \lambda K(\alpha_s(\mu)) \\
&+ \int_0^{(1-x_\nu)P^-} \frac{d\mu^2}{\mu^2} \lambda S(\alpha_s(\mu)) + \int_0^{(1-x_\nu)^2 P^-} \frac{d\mu^2}{\mu^2} \lambda J(\alpha_s(\mu)) \\
&\int_{1/P^-}^{1} dy \left[ 2 \int_{P^-}^{yP^-} \frac{d\mu^2}{\mu^2} \lambda K(\alpha_s(\mu)) + \lambda S(\alpha_s(yP^-)) \right] \\
&+ \int_0^{(1-x_\nu)^2} \frac{dy}{y} \lambda J(\alpha_s(\sqrt{y}P^-)) .
\end{align*}
$$

(23)

Note the change in the $y$-integration limits compared to eq. (20).

4. Conclusion

In this letter we have shown in detail that the resummation technique is indeed equivalent to the conventional Wilson-loop formalism. The two approaches lead to the same resummation results in the end-point region of inclusive $B$ meson decays, except that the large logarithm appears as $\ln bP^-$ in our expression and as $\ln N$ in [15]. This equivalence provides a
justification of our PQCD analysis of $B$ meson decays including Sudakov effects in previous works [4, 3, 7].

We have neglected the initial condition of $K + G$ in demonstrating the equivalence, which in fact contributes to next-to-leading logarithms. However, we believe that physical predictions should be insensitive to the distinction in next-to-leading logarithms. It is known that a substitution of the scale $P^-$ by another one $CP^-$, with $C$ a constant of order 1, is allowed in resummation [1, 18]. This constant $C$ in the integral involving $\lambda_K$ then makes a difference at the level of next-to-leading logarithms. Hence, if the initial condition of $K + G$ is included, predictions will be modified only slightly. Certainly, this issue needs more careful study. Other interesting subjects, such as the gauge dependence of resummation and the application of our technique to deep inelastic scatterings and Drell-Yan productions, will be discussed in a separate work [20].

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Figure Captions

Fig. 1. (a) Basic factorization for the decay $B \rightarrow X_s \gamma$, and (b) and (c) $O(\alpha_s)$ radiative corrections.

Fig. 2. Factorization for the decay $B \rightarrow X_s \gamma$ in our formalism for a general gauge vector $n$.

Fig. 3. Factorization for the decay $B \rightarrow X_s \gamma$ in the Wilson-loop formalism.

Fig. 4. (a) Graphic representation of eq. (7) for a gauge vector $n = P$, and $O(\alpha_s)$ diagrams for (b) the function $KJ$ and for (c) the function $GJ$. 
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