Study on the Computation Method for the Generalized Force and Mass of Flexible Aerial Rudder in Space Vehicle

Guo Peng-fei, Wu Jian-hui, Zhang Wei-qiang, Zhang Yong and Yang Yang
China Academy of Launch Vehicle Technology, Beijing 100076, China

Corresponding author’s e-mail address: gpfmail@126.com

Abstract. Aerial rudders are significant actuators in the spacecraft control systems and are responsible for the stability and control of different motions such as pitching, yawing and rolling around mass center. With the development of new generation of space vehicles, high speed flight demonstration, light and thin aerial rudders with large span-chord ratio have been widely used. In the design of control strategies in pitching, yawing and rolling channels and control feedback loop parameters, the influence caused by aerial rudder elastic deformation should be considered particularly. In this paper, elastic motion equations of the spacecraft flexible rudder are studied firstly. Then, according to the modal vibration features, aerodynamic force derivative distribution characteristics and generalized force expressions, the rudder station division and computation method of generalized rudder aerodynamic force derivatives are proposed in the pitching, yawing and rolling channels based on the computational fluid dynamics simulation. By comparison the results of the proposed method with these of the traditional computation methods, the accuracy and validity of the proposed method are verified which can provide considerable references and basis in the control design of flexible rudder in the spacecraft.

1. Introduction
The motion of spacecraft during flight is the coupling of the overall rigid movement and the elastic oscillation. Under the loadings including aerodynamic force, power system thrust and servo force, the rigid motion of the spacecraft could lead to structural oscillation which will also affect overall rigid motion simultaneously [1]. Therefore, the spacecraft considering structural elasticity challenges the traditional flight control which is based on rigid body dynamics, and makes the control of real spacecrafts more complicated.

Traditional spacecrafts represented by launch vehicles and missile weapons hold symmetric aerodynamic shapes and structures with relatively large rudder stiffness. The corresponding pitching, yawing and rolling channels which are affected by servo mechanism, control force and aerodynamic force are coupled in a low degree. Thus, tri-channel control system with decoupled strategy is employed generally [2]. However, with the increasing weight, thrust and slenderness ratio of rocket and missile as well as extensive applications of bundled propellants, the new generation spacecraft presents the characteristics of concentrated low frequency mode, strong coupling vibrations and complex local deformations. In this case, Wu et al [3] established elastic vibration equations of spacecrafts with spatial mode based on the finite element method, and demonstrated the influence of lateral, longitudinal, torsional vibrations and booster local mode on the attitude control system.

Aerial rudders are significant actuators in the spacecraft control systems and are responsible for the
stability and control of different motions such as pitching, yawing and rolling around mass center, which are the vital basis to ensure the predetermined trajectory flight [3-4]. In traditional space vehicle design, aerial rudders hold large structural stiffness and connection stiffness and thus can be regarded as rigid body in the control system [5-7]. With the development of new generation of space vehicles represented by USA X-37B and X-51 high speed flight demonstration, light and thin aerial rudders with large span-chord ratio have been widely used. These dimension changes cause a dramatic decline of rudder stiffness comparing with those in traditional launch vehicles and missile weapons, even make local modal frequency of the rudders lower than the bending modal frequency of the structure in directions of pitching and yawing. Besides, these changes increase the difficulty in the design of control systems which guarantee space vehicle stability against attitude divergence and out of control. So in the design of control strategies in pitching, yawing and rolling channels and control feedback loop parameters, the influence caused by aerial rudder elastic deformation should be considered particularly.

In this paper, rudder station division and computation method of generalized rudder aerodynamic force derivatives are proposed according to the modal vibration features, aerodynamic force derivative distribution characteristics and generalized force expressions. By comparing with the results of traditional computation method, the accuracy and validity of the proposed method are verified which can provide considerable references and basis in the control design of the flexible rudder.

2. Elastic motion equation and corresponding parameters

Elastic vibration modal shapes of the spacecraft structure mainly include motions in longitudinal, pitching, yawing and rolling directions with pitching axis perpendicular to trajectory plane and yawing axis in the trajectory plane perpendicular to structural longitudinal axis. When elastic vibrations in pitching and yawing directions are analyzed as shown in Figure 1, computational fluid method and finite element method should be employed to obtain characteristics of the aerodynamic distribution and each order modal shape respectively.

![Elastic deformation of space vehicle structure](image)

**Figure 1.** Elastic deformation of space vehicle structure

In modal analysis, the finite element method is widely adopted and the spacecraft structural elastic vibration equation can be expressed by,

\[ M \ddot{X}(t) + C \dot{X}(t) + KX(t) = F(t) \] (1)

where \( M, C \) and \( K \) are the structural system mass matrix, damping matrix and stiffness matrix respectively; \( X(t), \dot{X}(t) \) and \( \ddot{X}(t) \) are the finite element node displacement, velocity and acceleration vector respectively; \( F(t) \) is the loading vector on finite element node. Mode-superposition method is exploited to solve the equation, and it is assumed that

\[ X(t) = \sum_{i=1}^{n} \Phi_i q_i(t) \] (2)

where \( \Phi_i \) is the \( i \)th-order modal shape of the structural system, then Equation (1) can be decoupled as:
\[ \ddot{q}_i(t) + 2\zeta_i \omega_i \dot{q}_i(t) + \omega_i^2 q_i(t) = \frac{F_i(t)}{M_i} \quad i = 1,2,...,n \]  

where \(q_i(t), \dot{q}_i(t)\) and \(\ddot{q}_i(t)\) are the \(i\)-th order mode displacement, velocity and acceleration in generalized coordinates respectively; \(\omega_i, \zeta_i, F_i(t)\) and \(M_i\) are the \(i\)-th order mode natural frequency, damping coefficient, generalized force and generalized mass of the structural system. \(M_i\) and \(F_i(x,y,z,t)\) can be expressed as:

\[ M_i = \Phi_i^T M \Phi_i = m(x,y,z) \cdot \Phi_i^2(x,y,z) \]  

\[ F_i(x,y,z,t) = \Phi_i^T F(x,y,z,t) = \Phi_i X (x,y,z) F_x(x,y,z,t) + \Phi_i y (x,y,z) F_y(x,y,z,t) + \Phi_i z (x,y,z) F_z(x,y,z,t) \]

where \(F(x,y,z,t)\) and \(\Phi_i(x,y,z)\) are the external excitation and modal shape in corresponding position respectively.

When generalized mass and generalized force are substituted into Equation (3), spacecraft lateral elastic vibration equation can be described by

\[ \ddot{q}_i(t) + 2\zeta_i \omega_i \dot{q}_i(t) + \omega_i^2 q_i(t) = D_{1i} \Delta \dot{\phi} + D_{2i} \Delta \alpha + D_{3i} \Delta \delta + D_{3i}'' \Delta \ddot{\delta} + Q_{iy} \]  

\[ D_{1i} = \frac{180qS_M}{\pi Mt} \sum_{S=1}^{K} C_{\alpha n}^{\alpha} \Phi_i(X)(X_T - X_S)dX \]  

\[ D_{2i} = \frac{180qS_M}{\pi Mt} \sum_{S=1}^{K} C_{\alpha n}^{\alpha} \Phi_i(X)dX \]  

If the stiffness of the aerial rudder is large enough, the modal shape \(\Phi_i(X_K, Y_K, Z_K)\) and the modal shape slope \(\Phi'_i(X_K, Y_K, Z_K)\) at any point on rudder are equal, and then the following can be derived.

\[ D_{3i} = \frac{N_K H_K}{M_i} [J_{in} \Phi_i(X_K) + m_X l_K \Phi'_i(X_K)] \]  

\[ D_{3i}'' = \frac{N_K H_K}{M_i} [m_K l_K \Phi_i(X_K) + J_{K} \Phi'_i(X_K)] \]

where \(q\) is the dynamic pressure; \(S_M\) is the reference area; \(V\) is the flight velocity; \(M_i\) is generalized mass of the \(i\)-th order mode; \(\Phi_i\) and \(\Phi'_i\) are the modal shape and corresponding slope respectively; \(\Delta \dot{\phi}\) is the pitching(or yawing) rotational angular velocity; \(\Delta \alpha\) is the attack angle (or sideslip angle); \(\Delta \delta\) is the angle of aerial rudder rotation; \(\Delta \ddot{\delta}\) is the angular acceleration of aerial rudder rotation; \(Q_{iy}\) is the generalized force corresponding to the disturbing force in the direction of main shaft vibration; \(C_{\alpha n}\) is the coefficient derivative of transverse(in the direction of pitching or yawing) distribution of aerodynamic force; \(F_{in}^{\alpha}\) is the coefficient of the transverse(in the direction of pitching or yawing) aerodynamic force of aerial rudder; \(X_T\) is the longitudinal coordinate of the mass center of space vehicle; \(X_S, X_K\) are the longitudinal coordinates of all rockets(missiles) body stations and connection location of the aerial rudder respectively; \(m_X\) is the longitudinal acceleration; \(m_K\) is the mass of aerial rudder; \(l_K\) is the distance between mass center of the aerial rudder and corresponding spinning shaft; \(J_{K}\) is the moment of inertia of aerial rudder; \(N_K\) is the number of aerial rudders; \(H_K\) is the distribution coefficient of aerial rudder; \(F_{in}^{K}\) are the derivative of the generalized aerodynamic force on rudder.

However, when the stiffness of aerial rudder is small, vibration of the rudder under lateral vibration mode exists different patterns as shown in Figure 2. The modal shape \(\Phi_i(X_K, Y_K, Z_K)\) and the corresponding slope \(\Phi'_i(X_K, Y_K, Z_K)\) at any point on rudder are quite different. When effect of flexibility is considered, rudder generalized aerodynamic force derivative \(F_{in}^{\alpha}\) and generalized mass \(M_i\) in Equations (9) and (10) according to overall spacecraft station computation are not suitable.
any more.

(a) Pattern 1  (b) Pattern 2  (c) Pattern 3

Figure 2. Rudder vibration shape patterns in lateral vibration mode

3. Computation method on generalized force and generalized mass of flexible aerial rudder

According to the expressions of generalized mass in Equation (4) and generalized force in Equation (5), generalized mass of the aerial rudder and generalized aerodynamic force on the rudder can be generated as follows:

\[ M_i = \int \int \int \left[ \Phi_{i,x}^2(x,y,z)m(x,y,z) + \Phi_{i,y}^2(x,y,z)m(x,y,z) + \Phi_{i,z}^2(x,y,z)m(x,y,z) \right] \, dx \, dy \, dz \]  

(11)

\[ F_i = \int \int \int \left[ \Phi_{i,x}F_{x}(x,y,z,t) + \Phi_{i,y}F_{y}(x,y,z,t) + \Phi_{i,z}F_{z}(x,y,z,t) \right] \, dx \, dy \, dz \]  

(12)

Because aerial rudder is a continuous structure, each vibration modal shape is distributed continuously in space, namely, there are equal modal shape perpendicular lines as shown in Figures 3(a)-3(c), which means these modal shapes perpendicular to vertical plane are equal.

(a) Pattern 1  (b) Pattern 2  (c) Pattern 3

Figure 3. Equal vibration shape axis of rudder vibration shape patterns under lateral vibration mode of spacecraft

When the vertical direction of equal vibration shape is considered as the station division direction of flexible rudder, the rudder can be divided into \( s \) stations, then the station mass \( m^{(j)} \) and aerodynamic force \( F^{(j)}, j = 1,2,...,s \) can be calculated. The generalized mas and the generalized aerodynamic force can be defined as follows.

\[ M_i^* = \sum_{j=1}^{s} \left[ (\Phi_{i,x}^{(j)})^2 m^{(j)} + (\Phi_{i,y}^{(j)})^2 m^{(j)} + (\Phi_{i,z}^{(j)})^2 m^{(j)} \right] \]  

(13)

\[ F_i^* = \sum_{j=1}^{s} \left[ \Phi_{i,x}^{(j)}F_{x}^{(j)} + \Phi_{i,y}^{(j)}F_{y}^{(j)} + \Phi_{i,z}^{(j)}F_{z}^{(j)} \right] \]  

(14)

When vibration shape pattern in Figure 3(a) is taken as an example, the rudder vibration shape \( \Phi_1(x,y,z) \) varies along X axis, while has no obvious change along the other two axis. Therefore, the vibration shape of the rudder can be simplified as \( \Phi_1(x,y,z) = \Phi_1(x) \). When this simplification is substituted to Eq.(11) and Eq.(12), and the following can be derived,
\[ M_i = \int \{ m(x) \left[ \phi_{i,x}^2(x) + \phi_{i,z}^2(x) + \phi_{i,y}^2(x) \right] \} dx \]

\[ F_i = \int \left\{ \phi_{i,x}(x) F_X(x) + \phi_{i,y}(x) F_Y(x) + \phi_{i,z}(x) F_Z(x) \right\} dx \]

where

\[ m(x) = \int m(x, y, z) dy dz, \quad F_X(x) = \int F_X(x, y, z) dy dz, \quad F_Y(x) = \int F_Y(x, y, z) dy dz \quad \text{and} \quad F_Z(x) = \int F_Z(x, y, z) dy dz. \]

When the stations of aerial rudder are divided densely, the mass, vibration shape and derivatives of aerodynamic force among successive stations will vary moderately and linearly. It is assumed that the aerodynamic force is \( F_X(x) = Ax + B \), vibration shape is \( \phi_{i,X}(x) = Cx + D \), distance between two stations is \( L \), left and right station masses are \( m_L \) and \( m_R \) respectively, as a result, generalized mass and generalized force can be expressed by

\[ M_{i,X} = \int_0^L (m_L + (m_R - m_L) / L \cdot x) / L \cdot (C x + D)^2 dx \]

\[ F_{i,X} = \int_0^L (A x + B) (C x + D) dx \]

Similarly, the similar results could be obtained in \( Y \) and \( Z \) directions.

When stations are divided in the direction of equal vibration shape vertical line, generalized mass and generalized force can be expressed by

\[ M_{i,X}^* = m_L D^2 + m_R (CL + D)^2 \]

\[ F_{i,X}^* = D \int_0^{L/2} (A x + B) dx + (C L + D) \int_{L/2}^L (A x + B) dx \]

Similarly, the similar results could also be obtained in \( Y \) and \( Z \) direction.

It can be proved that when the stations of flexible aerial rudder are divided in the vertical direction of equal vibration shape and detailed enough, the generalized mass and the generalized force with high accuracy of the flexible rudder can be calculated, where \( M_{i,X} = M_{i,X}^* \), and \( F_{i,X} = F_{i,X}^* \).

4. Simulation case studies

In order to verify the effectiveness of the proposed method, the aerial rudder shown in Figure 4 is taken as an example. The thickness of the aerial rudder is 30mm and the mass is \( m_d \) (kg) and distributes uniformly, the other dimensions are shown in Figure 4. Simulation cases are listed in Table 1.
Table 1. Simulation cases

| Mach Number | height(km) | Flight deviation (°) | Dynamic pressure(N/m²) |
|-------------|------------|----------------------|------------------------|
| 2.5         | 12         | 0                    | 84872                  |
| 2.5         | 12         | 1                    | 84872                  |

The aerodynamic pressure distribution of the aerial rudder can be obtained in conditions of rudder deflection 0 degree and 1 degree respectively by using aerodynamic CFD simulation, and the distribution characteristics of its derivatives of aerodynamic force in different directions can be obtained by subtraction, as shown in Figure 5.

![Figure 5](image_url)

(a) Direction of X axis                   (b) Direction of Y axis

(c) Direction of Z axis

Figure 5. Distribution of certain aerial rudder grid aerodynamic force derivative

The aerial rudder is connected with spacecraft structure through the rudder shaft. However, there can be many different modes for the rudder \((\Phi_i,x(x,y,z),\Phi_i,y(x,y,z) \) and \(\Phi_i,z(x,y,z)\)) under the modes of pitching, yawing and rolling of the spacecraft. When three modes shown in Figure 2 are considered as examples, the mode functions can be assumed as follows:

1. pattern1: \(\Phi_i,x(x,y,z) = 0, \Phi_i,y(x,y,z) = -0.4x + 0.1 \) and \(\Phi_i,z(x,y,z) = 0\);
2. pattern2: \(\Phi_i,x(x,y,z) = 0, \Phi_i,y(x,y,z) = 0.4z \) and \(\Phi_i,z(x,y,z) = 0\);
3. pattern3: \(\Phi_i,x(x,y,z) = 0, \Phi_i,y(x,y,z) = 0.5x - 0.5z - 0.5 \) and \(\Phi_i,z(x,y,z) = 0\).

According to the different vibration modes of the aerial rudder, the rudder is divided into 10 stations by using three different station division methods as shown in Fig. 3. The average value of the section vibration mode data of each station is taken, and the corresponding generalized mass and generalized force derivative are calculated, and compared with the definitions in Equations (11) and (12) of the generalized mass and generalized force derivative, the results are listed in Tables 2 and 3.

Table 2. Generalized mass with different station divisions in different vibration patterns (kgmm²)

| Shape pattern | Generalized mass definition | Axis1 station division | Axis2 station division | Axis3 station division |
|---------------|-----------------------------|------------------------|------------------------|------------------------|
| 1             | 0.012m_d                    | 0.014m_d               | 0.031m_d               | 0.022m_d               |
| 2             | 0.0066m_d                   | 0.0031m_d              | 0.0070m_d              | 0.0042m_d              |
| 3             | 0.055m_d                    | 0.042m_d               | 0.062m_d               | 0.052m_d               |
Table 3. Generalized force derivative with different station divisions in different vibration patterns (N)

| Shape pattern | Generalized force derivative definition | Axis1 station division | Axis2 station division | Axis3 station division |
|---------------|-----------------------------------------|------------------------|------------------------|------------------------|
| 1             | -25.69                                  | -24.21                 | -41.22                 | -33.75                 |
| 2             | -26.75                                  | -10.92                 | -27.11                 | -17.85                 |
| 3             | -82.19                                  | -65.26                 | -97.23                 | -83.21                 |

It can be seen from Table 3 that for the flexible aerial rudder with obvious local modes, the equal modal shape vertical direction (Axis1 station in pattern 1, Axis2 station in pattern 2, and Axis3 station in pattern 3) should be taken as the direction of station division, and then the generalized mass, generalized force derivative can be calculated which holds higher engineering calculation accuracy.

5. Conclusions

In this paper, influence of the flexible rudder used in the control system design of new generation spacecraft is analysed and the elastic motion equation is studied. Rudder station division and computation method of generalized aerodynamic force are proposed according to the modal vibration features, derivative distribution characteristics of aerodynamic force and generalized force expressions of modal shape so as to determine generalized mass and force of flexible aerial rudder. The research results can provide an important theoretical basis for the wide application of thin and light rudder with large span-chord ratio, and it is also an important technical basis for the realization of high precision and high reliability control of flexible rudder.

References

[1] Pan Z W. The dynamical modeling and modal slope indicate technique of launch vehicle [J]. Science in China, Series E: Technological Sciences, 2009, 39(3): 469—473.
[2] Zhang W D, Liu Y X, Liu H B, Ding X F, Zhang K B. Development trend and prospect of attitude control technologies of launch vehicle. Aerospace Control, 2017, 3(35): 85-89.
[3] Wu Y S, He L S. Attitude control technology of new generation launch vehicles [J]. Journal of Beijing University of Aeronautics and Astronautics, 2009, 35(11): 1294-1297.
[4] Wei C Z, Ju X Z, Zhang L, Cui N G. The modeling and controller design for aeroelastic rocket launcher [J]. Astronautical Systems Engineering Technology, 2017, 1(1): 21-26.
[5] Yin Y Y. Loads design foundation for solid rockets [B]. China Aerospace Publishing House, 2007.
[6] Deng W Y. Research on the methods of bending modes data for launch vehicle [J]. Aerospace Control, 2010, 28(2): 7-11.
[7] Feng X X, Xia G Q, Han X L, Han X M. Analysis of stability for launch vehicle attitude control system [J]. Journal of Dalian University of Technology, 2015, 5(55): 542-547.