Bell’s Inequality, Quantum Measurement and Einstein Realism:
A Unified Perspective

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Abstract

The logical foundations of Bell’s inequality are reexamined. We argue that the form of the reality condition that underpins Bell’s inequality comes from the requirement of solving the quantum measurement problem. Hence any violation of Bell’s inequality necessarily implies nonlocality because of the measurement problem. The differences in the implications of deterministic and stochastic formulations of Bell’s inequality are highlighted. The reality condition used in Bell’s inequality is shown to be a generalisation of Einstein’s later form of realism.
I. INTRODUCTION

It is well known that Bell’s inequality (BI) is violated in quantum mechanics (QM) when the two spatially separated systems are in an entangled or nonfactorisable state\[1\]. Similarly, two systems in an entangled state underlie the measurement problem, known as ”the most important puzzle in the interpretation of quantum mechanics ” \[2\]. This is the problem of associating an element of ”reality”, such as a ”complete state”, with a definite outcome of an individual measurement. The significant relationship between BI and the measurement problem becomes apparent when it is noted that the derivations of BI are based upon conditions of ”local realism” involving the concept of a ”complete” or ”real” state.

In section 2 of this paper we argue that the ”realism” problem of measurement provides a more physical motivation for adopting a ”complete state” formulation of ”realism” than has been given previously. Such a formulation of reality when coupled with a locality condition leads to BI, this is discussed in sections III and IV.

Bell’s original argument justifying the reality condition that he used was derived from the Einstein-Podolsky-Rosen (EPR) argument\[3\] expressed in terms of Bohm’s example (EPRB). It assumed perfect quantum mechanical correlations between the outcomes in the two wings in addition to the reality of the individual outcomes. This assumption is not needed in the Clauser, Horne, Shimony and Holt derivation \[5\] nor in the argument given here. As emphasised by Shimony\[5\], in order that the empirical validity of local realism can be tested independently of QM, no quantum mechanical outcomes that arise from the use of the EPRB state, such as the input of perfect correlations, should be used to derive BI. Bell had used this input to justify the existence of the functions $A(a, \lambda), B(b, \lambda)$ by following the EPR argument. This is logically permissible for showing an inconsistency between QM and local realism but not for empirically refuting local realism by testing the validity of BI. A further point against the use of perfect correlations is that the input, viz. the equality of magnitudes of $A(a, \lambda)$ and $B(a, \lambda)$ with opposite signs, cannot be empirically verified as an exact equality because of the inevitable imprecision involved in taking measurements along perfectly opposed directions.

However, any derivation of BI which avoids the input of perfect correlations cannot get off the ground unless it assumes a more complete state specification (eg by a variable $\lambda$) than the wave function $\psi$ of QM. In the recent review by Shimony \[5\], this point is stressed : ”It
should be emphasized that the price of abstaining from a quantum mechanical assumption is that the definiteness of the functions A and B must be ‘postulated’, rather than ’derived’ as in the argument of EPR, which Bell was following”.

In view of the importance of BI in quantum nonlocality discussions, its use as a measure of entanglement, and its role in quantum communications it is important to have a logical foundation of BI that is as cogent as possible. We think that the following form of argument provides this.

We also clarify the differences between deterministic and stochastic formulations of BI through the nature of nonlocality implied by the QM violation of BI. Finally, in section V, we point out the difference between the forms of realism used in the EPR argument and that used in Einstein’s later version, which was given most concisely in a letter to Besso.

We begin by stating the crux of the quantum measurement problem and sharpen the usual discussion by emphasising the gap between what is required and what is obtained in the standard theory concerning the nature of the final ensemble of states. Then the measurement problem is related to the form of ”realism” which is required to address it.

II. THE QUANTUM MEASUREMENT PROBLEM

The measurement problem comes from the well known fact that quantum mechanics predicts only a probability distribution of the values that would be obtained by measuring a physical quantity on an ensemble of systems which are all prepared identically in a state represented by a wave function. Hence QM predictions can only be verified by identical measurements performed on a homogeneous ensemble of systems, all corresponding to a common wave function. After such measurements have been performed, the final ensemble, comprising systems and apparatuses, needs to be such that probabilities (relative frequencies) of various outcomes can be determined. However, these can only be determined if each outcome is observationally distinct from the others. This requires that the post-measurement ensemble be heterogeneous, that is, the post-measurement ensemble is a mixture of distinguishable apparatus states with distinct apparatus states corresponding to distinct outcomes. The acuteness of this requirement is further emphasized by the following argument.

The very concept of objective definiteness of a measurement outcome means that it is inter-subjective and it occurs independently of whether or not it is observed. In other words,
the actualisation of an outcome takes place due to an interaction between a system and an apparatus, with no necessary participation by an external observer. Once actualised or recorded, an outcome is “out there” so that it can be inspected at any later instant by any observer. The actualisation of an individual outcome is therefore associated with an “objectively real” change in the apparatus state that is induced by a measurement process. However, unless the final apparatus states are distinguishable, they cannot correspond to the actualisation of different outcomes. In other words, the definiteness of an individual outcome requires that any particular apparatus state is distinguishable from all other apparatus states. Hence the final ensemble needs to be heterogeneous in an objectively real sense. It is precisely this requirement that is not satisfied by standard QM description of a measurement process.

If both system and apparatus are described quantum mechanically and the time evolution due to a measurement interaction is governed by a Schroedinger equation, the final composite state is inevitably a pure entangled state. The following is a generic feature of all QM models of measurement. Suppose a system is initially in a state given by the wave function \( \psi = a\psi_1 + b\psi_2 \), that is a superposition of, say, two eigenstates \( \psi_1 \) and \( \psi_2 \) of a dynamical variable which is being measured. Then after an interaction with a suitable measuring device, the final system-apparatus state is given by the wave function

\[
\Psi = a\psi_1\phi_1 + b\psi_2\phi_2, \tag{2.1}
\]

where \( \phi_1 \) and \( \phi_2 \) are states of the apparatus which can be distinguished by some macroscopic means. Note that equation (2.1) implies that no element of the system-apparatus ensemble is assigned a separate state because all elements of the ensemble have a common wave function. Hence in the standard formulation of QM the ensemble is homogeneous, implying that its members are indistinguishable. This is clearly inconsistent with the fact that in any one run of a measurement, a definite value of the apparatus variable is obtained which corresponds to either \( \phi_1 \) or \( \phi_2 \). The question of how to resolve this inconsistency given a wave function of the form (2.1) is the quantum measurement problem [6]. Unless a resolution is obtained, it is a logical non sequitur to ascribe physical significance to the computed statistical frequencies or probabilities of various outcomes because the occurrence and the definiteness of an individual outcome is not ensured within the standard framework of QM.

When the Schroedinger equation is kept unmodified, there have been two general approaches in the literature to the problem: (a) those that use the idea of decoherence being
induced by an interaction of the apparatus with the environment, (b) those in which the state of an individual member of the ensemble is more completely specified.

We shall argue below that decoherence alone is not sufficient to ensure that in a single run the apparatus is left in a state characterised by a definite outcome. In other words, the decoherence approach does not actually produce a heterogeneous post-measurement ensemble of the type that is necessary to solve the measurement problem. Therefore, provided one does not assume that there is a special role for conscious awareness by which outcomes are perceived, the remaining option for tackling the measurement problem while the quantum dynamics is unmodified is (b). Then solving the measurement problem gives a justification of the postulate that there is a more complete state specification.

Let us now examine the essentials of the decoherence approach. A measuring apparatus is categorised as a system whose interaction with its environment (modelled quantum mechanically) is necessarily nonnegligible in contrast to the measured system whose interaction with the environment can be neglected. Therefore a measurement process is described quantum mechanically by a tripartite entanglement involving system, apparatus and environment (environment-induced decoherence models \([7]\)). Then due to orthogonality between the states of the environment which are coupled to system-apparatus states, the reduced density operator of the system-apparatus subsystem is diagonal (i.e., an effective decoherence occurs between system-apparatus states). This is then taken to imply that a pure entangled state at the end of a measurement behaves like a mixture of system-apparatus states (this approach to the measurement problem was first suggested by Heisenberg \([8]\)).

For instance, if one includes the environment in the entanglement described by (2.1), then by tracing over the environment states, the reduced density operator \(\rho_{SA}\) pertaining to system and apparatus states is given by

\[
\rho_{SA} = |a|^2 |\psi_1\rangle \langle \psi_1| \otimes |\phi_1\rangle \langle \phi_1| + |b|^2 |\psi_2\rangle \langle \psi_2| \otimes |\phi_2\rangle \langle \phi_2|.
\]

(2.2)

It is important to note that although \(\rho_{SA}\) is diagonal it is a sum of two terms each of which describes a different individual outcome after a measurement. Thus the fact that the reduced density operator of the joint system-apparatus system is diagonal (after tracing over the environment states) implies only the non-observability of interference between apparatus states corresponding to various outcomes (these correspond to different diagonal elements of the reduced density operator). The vanishing of the off-diagonal elements of the reduced
density operator is merely a statement about the coherence property of the system-apparatus states. There is no ingredient in the formalism that ensures that in a single run of a measurement an individual apparatus is left in a state characterised by a definite outcome. The question of how one particular diagonal element of the reduced density operator is picked to be the outcome of a single measurement is not addressed.

Moreover, by simply rewriting the tripartite entangled state of system, apparatus and environment in a different basis, it is possible to interpret the same reduced density operator of system-apparatus as corresponding to a different probabilistic mixture of superpositions of system-apparatus states resulting from the same measurement interaction. Thus the decoherence approach has an additional ambiguity in failing to uniquely specify how a particular mixture, out of the myriad of possible mixtures, emerges for a given measurement interaction. Therefore there is a problem of uniqueness of the interpretation of the outcomes of a given measurement interaction even after the environment states have been traced over because a relevant eigenbasis for a given measurement interaction cannot be uniquely identified. This ambiguity cannot be resolved unless a particular basis is chosen or preferred by some criterion for determining the set of outcomes obtained in a specific measurement. This point has been stressed, for example, by Penrose in his debate with Hawking [18].

To sum up, although by considering an interaction with the environment it is possible to explain why the joint system-apparatus pure state behaves with respect to the interference properties as if it were a mixture of states, the crucial requirement that the system-apparatus ensemble after the measurements be heterogeneous is not fulfilled.

There are a number of models/interpretations of quantum mechanics: the statistical interpretation [9], many worlds [10], consistent histories [11] and their many variants (see for example [12]), all of which basically invoke the idea of decoherence, either explicitly or implicitly, in tackling the measurement problem. Hence the criticism above applies to all of these approaches [13].

In approaches of type (b) given an ensemble that is described by a wave function the state of an individual member of the ensemble is specified in a more complete way than that provided by the wave function. Details of how such a specification can be provided (whether by introducing suitable ontological variables supplementing a wave function a la Bohm’s model [14], or by using open subsets of states close to the wave function’s pure state [19]) are not relevant to our subsequent discussion. It is enough to consider the possibility of a
specification in terms of what may be called a complete state which is postulated to provide a complete description of the state of an individual system independent of any measurement. Then the heterogeneity of the post-measurement ensemble emerges because the measurement process is treated on the same footing as any other physical interaction and no cut is made between a system and an apparatus. Different outcomes will be characterised by different labellings of the complete state. It is this idea of a complete state that underlies what we call Bell realism.

In general, this form of realism requires that a complete state description of an individual observed system or an apparatus is one that is sufficiently complete to causally connect to an individual outcome. The argument starts by noting that a definite outcome must be associated with a postmeasurement complete state of the apparatus. This postmeasurement apparatus state must then be coupled to a postmeasurement complete state of the observed system. These postmeasurement states and their coupling should emerge from premeasurement complete states of the apparatus and system through the measurement process. Therefore the definite outcome of a measurement is causally related to the complete state that the observed system was in before the measurement. In essence, the reality of the complete state of the apparatus after the measurement requires that the observed system was in a complete state before the measurement.

III. BELL REALISM

The connection between the measurement problem and Bell realism may now be summed up as follows. Since outcomes are “objectively real” and are inferred from differences between pre- and post-measurement apparatus recordings, the complete state of any individual apparatus must be “objectively real” both at the pre- and post-measurement level. If both the measured system and apparatus are treated on an equal footing in terms of complete states and by viewing a measurement interaction as a physical interaction, “objective reality” must be ascribed to the complete states of the measured system, both before and after the measurement. Then, in such a treatment, the outcome of measuring a dynamical variable is causally related to a complete state of the pre-measured system. A complete state of the system thus gives a description of an individual system which is more complete than that given by a QM wavefunction in the sense that it causally corresponds to an individual
outcome of a measurement.

The physical import of this inference comes from the following \textit{counterfactual} implication of ascribing "objective reality" to the correspondence between an outcome and the complete state of the pre-measured system:

\textbf{Definition 1} \textit{Bell Realism}: If an ensemble of systems is prepared identically in a complete state, a particular outcome of a measured variable will be obtained either with certainty, or with a specific probability.

The above statement is a condition of "realism", henceforth designated as BR, which we call Bell realism. It is \textit{deterministic} or \textit{stochastic}, depending upon whether the correspondence between outcomes and complete states is respectively \textit{one-to-one} or \textit{probabilistic}. There is no locality condition involved in its formulation.

To put it mathematically, Bell realism means "objective definiteness" of the functions of the parameters characterizing pre-measurement complete states; such functions correspond to definite individual outcomes of measurements. It is precisely this form of the reality condition that is used in conjunction with a locality condition, LC, to derive BI. The measurement problem therefore justifies the use of complete states that is basic to this type of derivation of BI. In other words, measurements are characterised as a class of physical process whose complete and self-consistent treatment requires that the QM framework be augmented by introducing the notion of complete states at both the micro and macro levels.

\section*{IV. BELL’S INEQUALITY, DETERMINISTIC AND STOCHASTIC FORMS}

We shall first consider the \textit{deterministic} form of BR in order to make our key point about the nature of nonlocality implied by the QM violation of BI. Subsequently, we shall point out implications of our approach for the stochastic form of BR.

The relevant LC contingent on the deterministic form of BR may be stated as follows:

\textbf{Definition 2} \textit{Locality Condition}: The one-to-one correspondence between an outcome and the complete state of a measured system is independent of what measurement is performed on its spatially separated partner with which it may have interacted in the past but is presently noninteracting.
The conjunction of BR and LC implies that for any individual system prepared in a
putative complete state and for a given dynamical variable, there is a definite result one
“would” get if that variable is measured, irrespective of what is measured on a distant
system. We shall now briefly recapitulate how this leads to BI.

Let \( A_1, A_2 \) and \( B_1, B_2 \) be two pairs of dynamical variables associated with a pair of systems
\( S(A) \) and \( S(B) \). The systems \( S(A) \) and \( S(B) \) have been prepared in an initial complete joint
state, \( FS \), and are now widely separated and non-interacting. We assume that each of the
four quantities, \( A_1, A_2, B_1 \) and \( B_2 \) can take only two values, \(+1\) or \(-1\). Since all the members
of the ensemble of joint systems are assumed to have been prepared in a particular \( FS \), then
BR implies that each of the four quantities will give a particular value when it is measured
on any member of the ensemble. That is, the measured value of of each of the quantities
\( A_1, A_2, B_1 \) and \( B_2 \) is fixed. Then, using LC, the outcomes of the measurements of the joint
quantities \( A_1B_1, A_1B_2, A_2B_1, \) and \( A_2B_2 \) can be either \(+1\) or \(-1\).

For each of the 16 different combinations corresponding to possible choices of \( \pm 1 \) for each
\( A_1, A_2, B_1, B_2 \) the following equality holds

\[ A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2 = \pm 2. \quad (4.1) \]

If, on the other hand, the members of the ensemble of joint systems were prepared in
a number of different \( FS \)'s, each \( FS \) being compatible with the restrictions imposed by the
preparation procedure, then we would have to use BR on each sub-ensemble determined
by a specific \( FS \) to ensure that equation (4.1) holds for the results of measurements on the
systems \( S(A) \) and \( S(B) \) for each sub-ensemble. Then summing the results, given by equation
(4.1) for each \( FS \) specified sub-ensemble, over the collection of \( FS \) specified sub- ensembles
and taking an average, we obtain

\[ |\langle A_1B_1 \rangle + \langle A_1B_2 \rangle + \langle A_2B_1 \rangle - \langle A_2B_2 \rangle| \leq 2. \quad (4.2) \]

Assuming that the randomly chosen samples of particles on which pairs of quantities
such as \( A_1B_1, A_1B_2, \ldots \) are measured are typical of the entire ensemble (the principle of
induction), the averages \( \langle A_1B_1 \rangle, \langle A_1B_2 \rangle, \ldots \) are interpretable as the actual measured values
of these quantities.

Thus BI, given by Eq. (4.2), is a testable consequence of BR in conjunction with LC.
Note that the notion of counterfactual definiteness that underlies this derivation of BI is not
merely counterfactual but is crucially linked to the concept of a FS which defines the form of realism that has been used.

Now, it is well known that QM predictions for entangled states violate BI for appropriate choice of observables. In fact, it has been proved that for any nonfactorisable wave function of correlated quantum systems it is always possible to choose observables so that BI is violated by QM predictions [9]. Hence in view of the way we have formulated the conditions BR and LC that give rise to BI and our argument that BR is necessary to tackle the quantum measurement problem, it follows that the incompatibility with LC implied by QM violation of BI means the following:

The correspondence between an outcome and the FS of a system depends on what measurement is done on the system’s spatially separated partner. In other words, the correspondences between outcomes and FS cannot be specified in a mutually independent way for the “parts” of a composite (entangled) quantum system, even if the “parts” may be widely separated. This implies that the very form of “connection” between FS of “parts” depends on the entangled wave function for the state of the “whole” (nonreductionism) - this is what constitutes the essence of quantum nonlocality entailed by Bell’s theorem. How such a “connection” is analysed or whether it actually implies a “mechanical” influence across ordinary space-time depends on the kind of model one uses for characterising FS.

We now turn to consider the stochastic form of BR. It is well known that BI then follows [10] if, in conjunction with BR, one uses the following assumptions concerning the relevant probabilities occurring in the correspondences between outcomes and FS:

(a) The probabilities are bounded between 0 and 1.

(b) The joint probability of outcomes for measurements on spatially separated systems is factorisable with respect to their FS. This means that the joint probability is expressible as a product of individual probabilities of the respective outcomes pertaining to the systems prepared in the respective FS (traditionally, in such derivations, a FS is labelled by the parameter \( \lambda \) - the so-called “hidden variable”; the factorisability is then assumed at the level of “hidden variables”).

(c) A specific probability associated with the correspondence between an outcome and the FS of a measured system is independent of what measurement is performed on its spatially separated partner.

Now, we recall that in the formulation of BR, probabilistic correspondence between an
outcome and a FS physically means the following. If a system is repeatedly prepared in the same FS and the same variable is measured, a particular outcome will be obtained with a fixed relative frequency. This is necessarily bounded between 0 and 1. The assumption (a) is thus justified. Arguments interpreting quantum incompatibility with BI as requiring the use of negative/complex probabilities [11] in stochastic “hidden variable” models are therefore unacceptable.

If BI is viewed as a consequence of stochastic BR, the quantum violation of BI may be attributed to the invalidity of either assumption (b) or assumption (c). Note that assumptions (b) and (c) are mutually independent. While assumption (c) is clearly a locality condition, assumption (b) expresses what may be called the “separability” condition for the probabilities of outcomes pertaining to the respective FS. Violation of assumption (b) would mean that for quantum entangled states that are incompatible with BI, probabilities are nonfactorisable (nonseparable) at the level of FS. Thus, quantum violation of BI in the stochastic case implies either nonlocality or nonseparability. However, such a distinction cannot be made in the deterministic case.

V. RELATION TO EINSTEIN REALISM

At this stage it is instructive to compare Bell realism with the form of realism used in the EPR argument [3]. We recall that the EPR argument is based on associating an element of physical reality with a physical quantity, provided its measured value can be predicted with certainty “without in any way disturbing” the system in question. Evidently, the very formulation of this criterion of realism (EPRR) is contingent upon a locality condition. Therefore it rules out, by fiat, the possibility of a nonlocal realist explanation of quantum phenomena, in particular, of the perfect correlations in the EPR example. However, a counterexample is provided by the Bohmian causal realist model of QM which is manifestly nonlocal [14].

Einstein’s own version of the EPR argument, which can be found in his letter to Besso in 1952 [4], rests on a criterion of realism, Einstein realism or ER, that is independent of the locality condition. In it Einstein assumes a one-to-one correspondence between a wave function and the real (complete) state of a system. In order to demonstrate the incompleteness of the wave function description Einstein uses a "reductio ab absurdum"
argument in which, as well as the one-to-one relation, a particular form of the locality condition is assumed.

Take two spatially separated but previously interacting systems $S_1$ and $S_2$ with a joint wave function $\psi_{12}$. Depending on the kind of measurement made on $S_1$ and the particular measurement result, one can subsequently ascribe a certain wave function $\psi_2$ to the other system $S_2$. Assuming the collapse of a wave function, $\psi_2$ can thus take different forms according to the kind of measurement done on $S_1$. Then Einstein applied the locality condition in the form that the complete state of $S_2$ should not be affected by what is done on $S_1$ which is spatially separated from $S_2$ and no longer interacting with it. Hence he inferred that there was no one-to-one correspondence between the complete state of $S_2$ and $S_2$’s wave function $\psi_2$ which varied depending on the measurement performed on $S_1$. This contradiction could only be avoided if the locality condition was dropped which was unacceptable for Einstein. Therefore he inferred that the standard formulation of QM was incomplete.

The weakness of Einstein’s argument is that within the standard framework of QM it can be argued that although $\psi_2$ changed when different measurements were performed on $S_1$, the reduced density matrix of $S_2$ was unaffected and hence the observable statistical properties of $S_2$ were unchanged. Then the reduced density matrix of $S_2$, rather than the wave function, could be used to get a unique correspondence with the complete state.

The realism condition BR underlying BI avoids the above weakness of ER by generalising ER through an objective reality being ascribed to an individual outcome of a measurement. As we have argued earlier, such a reality condition is required to address the measurement problem and this in turn implies that an individual outcome has a causal correspondence with the complete state of a premeasured system. Thus while ER is formulated specifically with reference to the question as to whether a quantum wave function describes a complete state or not, BR is independent of the specifics of how a complete state is described.

Moreover, Einstein’s locality condition is modelled in terms of a complete state being unaffected by distant measurements, while the locality condition (LC) leading to BI pertains to the causal correspondence between a definite outcome and a complete state being unaffected by distant measurements. This form of LC together with BR enables BI to be derived without any reference to QM or to any specifics of the way a real state or complete state is modelled.
VI. CONCLUDING REMARKS

The key elements of this paper are as follows: (i) The central point is that the postulate of realism underlying BI can be justified in a logically cogent way on the basis of the quantum measurement problem, without requiring to either assume it a priori (as is assumed in all the derivations of BI which follow the CHSH type logic) or by following Bell’s original argument using the EPR-type reasoning whose logical weakness we have discussed.

Furthermore, we have pinpointed the precise problem with Einstein’s later version of the EPR argument. We have also clarified in what sense Bell realism can be viewed as a generalisation of Einstein realism, thereby avoiding that problem.

To the best of our knowledge, such a unified perspective on the relationship between Bell’s Theorem, Measurement Problem and Einstein Realism is lacking in the literature. This clarification, we believe, is helpful because it increases our understanding of the logical foundations of BI which is now being so widely used in diverse areas.

(ii) Though the distinction between separability and locality has been discussed earlier, the precise distinction between the implications of the deterministic and stochastic versions of BI in terms of the ”in-principle impossibility of preparing a complete state of an individual system” has not been stressed previously in the way done in this paper.

To sum up, starting from general considerations arising from the quantum measurement problem, we have motivated the Bell realism required to derive BI. Thus the nonlocality inferred from the quantum mechanical violation of BI cannot be avoided if one takes on board the quantum measurement problem. Our analysis also helps to clarify the distinctions between Bell realism and the forms of realism used in the EPR argument or by Einstein in a way that has not been stressed in the literature. However, Bell realism can be either deterministic or stochastic. Accordingly, as we have discussed above, the implications of the quantum violations of deterministic or stochastic BI differ as regards the nature of quantum nonlocality. The significance of these differences needs to be investigated further.

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