SYM Correlators and the Maldacena Conjecture

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We report on progress in evaluating quantum field theories with supersymmetric discrete light-cone quantization (SDLCQ). We compare the method to lattice gauge theory and point out its relevance for lattice calculations. As an exciting application we present a test of the Maldacena conjecture. We test the conjecture by evaluating the correlator of the stress-energy tensor in the strong coupling field theory and comparing to the string theory prediction of its behavior as a function of the distance. Our numerical results support the Maldacena conjecture and are within 10-15% of the predicted results.

1. Introducing the Method: SDLCQ

Supersymmetric Discretized Light-Cone Quantization (SDLCQ) is a discrete, Hamiltonian, manifestly supersymmetric approach to solving quantum field theories. Light-cone coordinates \((x^+, x^-, \vec{x}^\perp)\) are defined as

\[ x^\pm = \left( x^0 \pm x^1 \right)/\sqrt{2}, \]

where \(x^+(x^-)\) plays the role of a time(space) coordinate. Transverse coordinates are treated in the usual way. The conjugate variables are \((P^+)p^\pm\), the (total) longitudinal momentum. The light-cone energy \(P^-\) is the Hamiltonian operator which propagates the system in the light-cone time, and is of utmost importance. In light-cone quantization all individual longitudinal momenta are positive, \(p^+_i \geq 0\). This allows for a convenient discretization of the theory by putting (anti-)periodic boundary conditions on the fields. The momenta are then characterized by an integer \(n_i\), which symbolizes a momentum fraction

\[ p^+_i = \frac{n_i P^+}{K}; \quad n = 1, 2, 3, \ldots, K. \]

Here, \(K \equiv P^+ L/\pi\) is the harmonic resolution and also by construction the maximal number of partons. The continuum limit is reached as \(K \to \infty\).

The framework of DLCQ can be utilized to create a manifestly supersymmetric approach, namely SDLCQ. The key ingredient is the preservation of supersymmetry even at finite cutoff by discretizing first the supercharge \(Q^-\) and then constructing the Hamiltonian via

\[ P^- = \frac{1}{2\sqrt{2}} \{Q^-, Q^-\}. \]

A schematic comparison between the essential properties of SDLCQ and lattice gauge theory is compiled in Table 1. Since the approaches are complementary, results can be tested against each other! Interesting results in this direction have been obtained by Hamiltonian lattice methods \[4\] and in work on supersymmetry on the lattice \[5\].

There is a host of results in SDLCQ in two and three dimensions on correlators \[1\], bound states \[2\], and other topics, including an overview article, Ref. \[3\].

2. Application: Maldacena conjecture

The Maldacena conjecture \[6\] states, *cum grano salis*, that a field theory can be equivalent to a string theory on a special background. The drawback of the exciting perspectives of this conjecture are the problems to verify it. The crucial issue is that we need a matching point where the theories are equivalent. It should have small curvature, so that the supergravity approximation is valid, together with a small coupling allow for the use of perturbation theory. There is no such scenario known. Here the non-perturbative features of SDLCQ come to the rescue.

A variant of the Maldacena conjecture states that two-dimensional \(\mathcal{N} = (8, 8)\) supersymmetric Yang-Mills (SYM) theory should be equivalent to a system of D1 branes in Type IIB string theory.
We will use the correlation function of a gauge invariant operator, namely $T^\mu \nu$, to test this conjecture. The agenda is then clear: we have to compute the form of correlator in supergravity (SUGRA) approximation, and then perform a non-perturbative calculation of the correlator in SDLCQ.

### 2.1. The Correlator from SUGRA

One can compute the two-point correlation function of the stress-energy tensor from string theory using the SUGRA (i.e. small curvature) approximation. The leading non-analytic term in the flux factor yields the correlator

$$\langle O(r)O(0) \rangle = N_c^3/\sqrt{g}^5. \quad (4)$$

As a check we remark that $\mathcal{N}=(8,8)$ SYM$_3$ has conformal fixed points at the ultra-violet and the infra-red with central charges $N_c^2$ and $N_c$, respectively. One expects to deviate from the conformal behavior at $r_{UV}=g\sqrt{N_c}^{-1}$, and $r_{IR}=\sqrt{N_c}g^{-1}$. This yields the following phase diagram:

| UV | SUGRA | IR |
|----|-------|----|
| $N_c^2/r^4$ | $N_c^{3/2}/(g r^5)$ | $N_c/r^4$ |
| 0 | $1/(g\sqrt{N_c})$ | $\frac{\sqrt{g}}{2}$ |

### 2.2. The Correlator from SDLCQ

To reproduce SUGRA scaling relation, we will calculate the cross-over behavior at $1/g\sqrt{N_c}<r<\sqrt{N}/g$ using SDLCQ. We want to compute correlator

$$F(x^-,x^+) = \langle O(x^-,x^+)O(0,0) \rangle. \quad (5)$$

As a gauge invariant (two-body) operator we take $T^{++}(-K)$. In DLCQ one fixes $P^+ = K \pi/L$. Therefore we Fourier transform the last equation and decompose it into modes. We then continue to Euclidean space by taking distance $r^2 = 2x^+x^-$ to be real. This yields

$$F(r) = \left( \frac{x^-}{x^+} \right)^2 F(x^-,x^+) = \sum_n \left\{ \frac{L}{\pi} \langle n|T^{++}(-K)|0 \rangle \right\}^2 \times \frac{M_n^4}{8\pi^2 K^3} K^4(M_n r).$$

We emphasize that this result is dependent on the harmonic resolution $K$, but involves no other unphysical quantities. We recover the continuum limit by sending $K \to \infty$. The correct small $r$ behavior is retained.

From the numerical perspective the evaluation of the expression for $F(r)$ is straightforward. We need to calculate the mass spectrum by solving eigenvalue problem, i.e. diagonalizing the Hamiltonian,

$$2P^+P^-(K)|\psi_n(K)\rangle = M_n^2(K)|\psi_n(K)\rangle. \quad (7)$$

The problem is the large number of particles in the theory which has the Fock space growing exponentially with the harmonic resolution $K$. The
necessary numerical improvements include writing a C++ code with an efficient data structure, incorporation of the discrete flavor symmetry of the problem, and an increase of numerical efficiency by an improved version of the Lanczos algorithm. The hardware requirements are quite modest. We work with a Linux workstation with a Pentium III processor at 733 MHz and 2 GB RAM. Typical running times for large-scale computations are in the order of a few days.

2.3. Results

The correlator $F(r)$, Eq. (6), is determined by a numerical calculation of the mass spectrum of the $\mathcal{N} = (8,8)$ SYM theory. One problem with the discrete approach is the existence of unphysical states. Additionally, the number of partons in the massless unphysical states is even/odd for $K$ even/odd. Since the correlator, Eq. (6), is only sensitive to two-particle contributions, the resulting curves $F(r)$ are different for even and odd $K$. Furthermore, the unphysical states yield also a $1/r^4$ behavior, but have a wrong and dominant $N_c$ dependence. Therefore we cannot see regular contribution at large $r$. We can, however, take the different behavior of the curves to establish where the approximation breaks down. As a consistency check we note that the approximation breaks down at larger and larger $r$ as the harmonic resolution $K$ grows.

From the discussion of Sec. 2.1 we expect the correlator $F(r)$ to change its behavior from $1/r^4$ to $1/r^5$ as $r$ increases. We should thus approach $dF/dr = -1$ in the continuum limit and would claim success if the derivative flattens at this value before the approximation breaks down. As we see in Fig. 1 the derivative approaches this line, but the approximation breaks down when the curve reaches a value of $dF/dr \approx -0.85$ at $K = 6$. We therefore have evidence that the Maldacena conjecture is correct, although not yet a decisive result.

3. Conclusions and Perspectives

To put things in perspective we state that SDLCQ is a viable way to solve quantum field theories. Results within this framework include spectra, correlators and other properties of two- and three-dimensional YM- and SYM-theories. A test of these results by an independent method, namely lattice gauge theory, is necessary and would be very much appreciated! As an example we showed a test of the Maldacena conjecture. Though the results are not totally conclusive, the values are within 10-15% of results expected from the conjecture. Improvements of the code and the numerics are possible and on the way.

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