A Cosmic Model Parameterizing the Late Universe

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A simple speed-up cosmology model is proposed to account for the dark energy puzzle. We condense contributions from dark energy and curvature term into one effective parameter in order to reduce parameter degeneracies and to find any deviation from flat concordance $\Lambda$CDM model, by considering that the discrimination between dynamical and non-dynamical sources of cosmic acceleration as the best starting point for analyzing dark energy data sets both at present and in future. We also combine recent Type Ia Supernova (SNIa), Cosmic Microwave Background (CMB) and Baryon Oscillation (BAO) to constrain model parameter space. Degeneracies between model parameters are discussed by using both degeneracy diagram and data analysis including high redshift information from Gamma Ray Bursts (GRBs) sample. The analysis results show that our model is consistent with cosmological observations. We try to distinct the curvature effects from the specially scaling dark energy component as parameterized. We study the linear growth of large scale structure, and finally show the effective dark energy equation of state in our model and how the matter component coincidences with the dark energy numerically.

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I. INTRODUCTION

It is now well-established that the expansion of our universe is currently in an accelerating phase, supported by the most direct and robust evidence from the redshift - apparent magnitude measurements of the "cosmic lighthouse" type Ia supernova \cite{1}, and indirect others such as the observations of Cosmic Microwave Background (CMB) by the WMAP satellite \cite{2,3,4,5,6,7,8,9,10}, and large-scale galaxy surveys by 2dF and SDSS \cite{11,12,13,14,15}. Under the assumption that general relativity is valid on cosmological scale, the combined analysis of different observation data sets indicates a spatially-flat universe with about 70\% of the total energy content of the universe today as so called dark energy with effectively negative pressure responsible for the accelerating expansion (see Ref. \cite{16} for reviews on this topic). Among multitudinous candidates of dark energy models, the "simplest" and theoretically attractive one might be the so called vacuum energy, i.e. $\rho_{\Lambda} = \Lambda / 8\pi G$, where $\Lambda$ is the cosmological constant, which has been long considered as a leading candidate and works quite well on explaining observations through out the history of our universe at different scales. But the origin or mechanisms responsible for the cosmic accelerating expansion are not very clear. On the other hand, some authors suggest that maybe there does not exist such mysterious dark component, but instead the observed cosmic acceleration is a signal of our first real lack of understanding of gravitational physics \cite{17} on cosmic scale. An example is the braneworld theory with the extra dimensions compactified or non-compactified

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\cite{18,19,20,21}. Consequently, finding the different cosmological implications to distinguish modified gravity models and dark energy scenario from observations is essentially fundamental to physically understanding of our universe\cite{23}.

Along with the matter (mainly cold dark matter) component and possible curvature term, the mysterious dark energy dominates the fate of our universe (we do not consider the radiation component contribution as it is supposed to be very tiny for the current universe evolution, at least for the present discussion interests). Ironically so far we do not know much to either of them, even full of puzzling to some extends. So any progress or reasonable understanding to each of them is undoubtedly valuable. Specifically, the quest to distinguish between dark energy and modified gravity scenario and further to differentiate cosmological constant and dynamical dark energy models from observations has become the focus of cosmology study since it holds the key to new fundamental physics.

Although we have built up a successful parametrization to describe the properties and evolution of our universe, and in principle distinct dark energy models live at different subspace of fully descriptive multi-dimension parameter space, due to serious degeneracies among different parameters, we cannot get tight enough constraints from observations by global fitting various observational data sets. One way to extract useful information from observation data and get hints for fundamental physics from cosmology study is to reduce the dimension of parameter space (thus reduce the parameter degeneracies) with particular purpose in mind without apparently biased input to model parametrization. Since we have not found any evidence of inconsistency of standard $\Lambda$CDM model, including more parameters which describe detailed properties of each component if the "cosmic pie" will complicate the situation to constrain model parameters.

In order to find deviation of dark energy equation of state...
parameter \( w \) from \(-1 \) (e.g., evidence of non-cosmological constant dark energy), the assumption of a flat universe is widely accepted in the literature with claims that curvature is negligible from inflation predictions and with emphasis on combined analysis results with prior assumption \( w = -1 \). On the other hand, typically one looks for evidence of dynamical dark energy in the absence of spatial curvature to get better constraints (for an exception, see [24]). It has been concluded in [25] that the non-curvature assumption can induce critically large errors in reconstructing the dark energy equation of state even if the true cosmic curvature is on subpercent level. These claims motivate us proposing a parameterized dark component term to mimic the effective contributions from either dark energy or curvature term plus the dark energy (It is also possible that the parameterized term we postulate may be from a fundamental theory or reasonably modified gravity model we are seeking), besides the conventional matter term.

In the first step, it is reasonable to introduce only one parameter which stands for any kind of deviation from standard cosmology model. In some limit case, it should be reduced to the simple four dimensional (4D) \( \Lambda \)CDM cosmology. The constraint on this parameter from observations should provide insightful hints to further explore fundamental physics.

In the next section, we propose a simple cosmic parametrization for the current universe, a parameterized model for the later evolution of our universe. In section 3 we give various cosmic probes to this model, with comparison to the DGP model Universe[18] and the concordance model with a cosmological constant, i.e., the \( \Lambda \)CDM model, with the hope to locate new features to this new model. Then in section 4, we discuss the new degeneracies between the parameters we introduced and dark matter content. The possible constraints from high redshift observations are also discussed. The last section devotes discussions and conclusions for the general framework studies to this present model.

\[ H^2 = M a^{-3} - ka^{-2} + Ra^{-4} + \frac{A}{3} \]

where the subscript 0 indicating today’s value, curvature fraction \( \Omega_k = k/H_0^2 \), similarly to matter component fraction \( \Omega_m \), cosmological constant (Dark Energy) contribution \( \Omega_\Lambda \) and the radiation part \( \Omega_r = R/H_0^2 \) can be negligible today when compared to the mainly dark components.

In the year 2000, G. Dvali, G. Gabadadze and M. Porrati proposed a new model that can mimic the 4D Newton potential (with same scaling) in short ranges while it makes a 5D gravity model (the corresponding potential scaling differently from the conventional 4D Newton potential) in long ranges. It turns out interesting to compare this ScM to the DGP model, an extended ScM that in the simplest flat geometry case an additional term \( H_c \) contributes a cosmic scale related effect that deviates the common framework at large distances and we hope to know when it functions. The Friedmann equation for Hubble expansion in the DGP model (we take the self-accelerating branch solution with the plus sign in front of the root term) reads as

\[ H^2 - k/a(t)/(H^2 - k/a(t))^{1/2}/r_c = \rho/3 \]

where the Hubble parameter or expansion rate \( H^2 = (\dot{a}/a)^2 \). In the DGP model, gravity is trapped on a four-dimensional (4D) brane world at short distances, but is able to propagate into a higher-dimensional space at large distances. For the convenient comparasions we take its flat geometrical form

\[ H^2 = (\dot{a}/a)^2 = \rho/3 + H_c^2 \]

where the effective term \( H_c^2 = H/r_c \) that we treat as a parameter to be fitted in this work and the cross-over length scale defined by Planck mass over a 5D scale \( r_c = M_p/2M_5 \).

Conventionally, the redshift is defined by \( z = 1/a - 1, \) thus \( a^{-1} = 1 + z \). Compared with the expression of \( H^2 \) for the power-law \( \Lambda \)CDM model, we have known that from this 4D cosmological model with cosmological constant, the cosmological observation data analysis can be nicely accommodated/explained with curvature contribution near zero, so named as the concordant model. While the global data fitting successful we are still left with the curiosity that whether the cosmic curvature term is really zero or it can be effectively described by the accumulated effects from the cosmic un-known dark components.[22]

The reduction to the \( \Lambda \)CDM model can be also realized in a more economic form as parameterized below

\[ H(z)^2 = H_0[(\Omega_m) a^{-3} + (1 - \Omega_m) a^{B - 2}]. \]

where \( B \) is a parameter to be determined by data fittings and obviously \( B = 2 \) corresponds to the \( \Lambda \)CDM model, we call the term including \( B \) parameter effective dark energy (EDE) in the following. We can compare it with the \( \Lambda \)CDM model in the flat geometry where the Hubble parameter \( \bar{H}(z) \) is

\[ \bar{H}(z) = H_0[\Omega_m(1 + z)^3 + 1 - \Omega_m]. \]
Thus, the general case with all possible components we understand so far reads as

\[ H(z)^2 = \beta(z)^2 - k(1+z)^2 + R(1+z)^4. \]  

(7)

Of course we can encode relevant physics in the parameter \( \beta \), but in the 4D Universe with cosmological constant, each arbitrary parameter and term separately possesses concrete physics meanings compared with the 5D DGP model. So we employ various cosmological tests to see what physics the parameter \( \beta \) may stand for, the effective effects from both curvature and dark energy, or curvature term only with \( \beta = 0 \) or dark energy alone in the flat spacetime geometry. Finally, we want to ask how well can we distinguish the curvature effects from the dark energy component?

Among dark energy candidate models, among which the modified gravity or decaying cosmology term models can effectively describe possibly dark matter interacting with dark energy \([26, 27]\), to which we also expect this parameterized model can help. The detailed discussion on this topic is beyond the scope of this paper.

We also note that there is a long standing issue on breaking degeneracies between curvature and dynamical dark energy model parameters. For example, CMB lensing information can effectively help to break such degeneracy \([28, 29, 30, 31, 32]\) with only already planned ground-based CMB polarization power spectrum measurements. But the results depend on two strong assumptions, one is that the ground-based CMB survey will be able to remove foregrounds and systematics at a level sufficient to enable few percent level measurements of the lensing B-mode polarization power, another one is that the neutrino masses are fixed by oscillation measurements and a theoretical assumption about the neutrino mass hierarchy \([32]\). So even with ideal future measurements on CMB lensing, our new parametrization still has its advantages.

### III. Observational Constraints

In this section, we study the cosmological constraints on our model parameter space. There are several methods which have been used or proposed to constrain cosmological parameters in the literature, e.g., Type Ia supernovae, CMB, linear power spectrum and higher order statistics of large scale structure \([33]\), Lyman-alpha forest \([34]\), Alcock-Paczynski (AP) effect \([35]\), weak/strong gravitational lensing \([36]\), Gamma ray bursts/ultra-compact radio sources as standard candles/rulers \([37, 38, 39, 40]\), X-ray cluster baryon fraction versus redshift test \([41]\), Hubble parameter measurements on different redshift \([42]\), cluster counting \([43]\), and so on. In principle, in order to get self-consistent parameter constraints, one should do a global fitting on whole cosmological parameter space with properly chosen observational data sets. However, global fitting is time/CPU consuming and it is hard to analyze the degeneracies on parameter spaces.

In this paper, in the first setup to look at our model parameter space and to analyze the parameter degeneracies clearly, we use recent SNe Ia gold sample \([44]\) and SNLS data \([45]\), and combine with information from WMAP three year data and SDSS analysis results in our explorations. The SNLS sample consists of 44 nearby \((0.015<z<0.125)\) objects assembled from the literature, and 73 distant SNIa \((0.15<z<1.00)\) discovered and carefully followed during the first year of SNLS group \([45]\). For the cosmological fits, two of the SNLS data points were excluded because they are outliers in the Hubble diagram. For the SNIa data, the distance modulus is defined as

\[ M - m = 5 \log d_L + 25. \]  

(8)

Here \( d_L \) is the luminosity distance in units of Mpc which is written as

\[ d_L = \frac{1+z}{\sqrt{|\Omega_k|}} S \left( \int_0^{z_c} \frac{dz'}{H(z')/H_0} \right) \]  

(9)

where \( S \) is defined as \( S(x) = \sin(x) \) for a closed universe, \( S(x) = \sinh(x) \) for an open universe and simply \( S(x) = x \) with non-curvature universe.

To further break the parameter degeneracies, it is useful to study the combined constraints with other cosmological observations, we make use of the CMB shift parameter which includes the whole shift information of CMB angular power spectrum. It is defined as

\[ R = \frac{\Omega_m}{\sqrt{|\Omega_k|}} S \left( \int_0^{z_c} \frac{dz'}{H(z')/H_0} \right) \]  

(10)

where \( z_c = 1089 \), the redshift of the epoch of the recombination. The shift parameter is constrained to be \( R = 1.70 \pm 0.03 \) from the three-year WMAP result, CBI and ACBAR \([46]\). The CMB shift parameter contains the main information for the scale of the first acoustic peak in the TT spectrum, and is the most relevant one for constraining dark energy properties as it is not sensitive to different dark energy models. Since we only consider the shift parameter which is determined only by the background evolution for the constraint from CMB, we do not need to include the effect of the fluctuation of dark energy. In this paper, by using shift parameter, we can confine ourselves to considering the effects of the modification of the background evolution alone.

And we also use the information from observation of baryon oscillation acoustic peak which has been detected from the SDSS luminous red galaxy sample \([13]\). The quantity we use to constrain the cosmological parameters in this paper is

### TABLE I: Physics meanings in the Friedmann evolution Eq.

| Functions or constants | Physical meanings | Terms in \( H^2 \) |
|------------------------|-------------------|-------------------|
| \( a \)                | “Expansion velocity” | \( \Omega_m(1+z)^2 \) |
| \( M \)                | Matter (dust)      | \( \Omega_k(1+z)^2 \) |
| \( k \)                | Curvature          | \( \Omega_B(1+z)^2 \) |
| \( R \)                | Radiation          | \( \Omega_r(1+z)^2 \) |
| \( \Lambda \)          | Cosmological constant | \( \Omega_{\Lambda} \) |
defined as

\[
A = \frac{\sqrt{\Omega_m}}{(H(z_1)/H_0)^{1/3}} \left[ \frac{1}{\sqrt{3}a(z)} \int_{z_1}^{z} \frac{dz'}{H(z')/H_0} \right]^{2/3}
\]

where \(z_1 = 0.35\) and \(A\) is measured as \(A = 0.469 \pm 0.017\) [13]. Recently, the new SDSS LRG data were released, and the corresponding power spectrum was analyzed. The BAO peaks are clearly seen in the power spectrum, which, together with the overall shape, put tight constraints on model parameters [14].

For the fitting methodology, we use the standard \(\chi^2\) minimization method. It is well known that parameter estimates depend sensitively on the assumed priors on other parameters. In our study, we choose the allowed range of the Hubble constant \(H_0 = 72 \pm 8\) km s\(^{-1}\) Mpc\(^{-1}\) resulting from the Hubble Space Telescope Key Project with a uniform prior [47], and marginalize over \(H_0\) to get two-dimensional constraints for our parameter space.

Fig.1 shows confidence-level contours on \(\Omega_m - B\) parameter space using the SNIa Gold sample. The black, grey, and light grey region shows the 1, 2, and 3 \(\sigma\) confidence level contours of \(\Omega_m - B\) parameter space respectively with the minimum \(\chi^2 = 158.42\) occurring at \(\Omega_m = 0.46\), and \(B = 4.62\). We note that the best fit point is far from standard concordance cosmology, but the 2 \(\sigma\) confident contour is consistent with standard concordance cosmology. The parameter degeneracy properties between model parameters determine the configurations of constraint contours. We will analyze the degeneracy properties on \(\Omega_m - B\) parameter space in Fig.7. We note that the constraint results depend sensitively on the prior assumptions that one adopts. A strong prior can result in an overestimate on the power of a cosmological probe or make a incorrect constraints on key parameters, bias our judgement on model selection, thus improperly ruling out models. Especially, it is also noted that factitious priors on \(H_0\) can result in strongly biased constraints.

In Fig.2, we show the constraint results from combining CMB shift parameter with BAO from large scale structure of galaxies on different cosmological models. The black, grey, and light grey regions show the 1, 2, and 3 \(\sigma\) confidence level contours of EDE, \(\Lambda\)CDM and DGP model parameter spaces, respectively, with the minimum \(\chi^2\) occurring at \(\Omega_m = 0.31\), and \(B = 1.10\) for EDE model. It has been clearly shown in the figure that even combining information from CMB and BAO, which gives tight constraints on both \(\Omega_M - \Omega_\Lambda\) parameter space in \(\Lambda\)CDM model (middle sub-figure) and \(\Omega_M - \Omega_\Lambda\) (bottom sub-figure) parameter space in DGP model, cannot constrain our model parameter space tightly. The reason is that in \(\Omega_m - B\) parameter space, CMB shift parameter and BAO factor show similar degeneracy properties and thus cannot break the ‘banana’ shape of constraint contours. Fortunately, we find that the constraint contours on \(\Omega_m - B\) parameter space by using SNIa are almost perpendicular to contours from CMB+BAO constraints as two sets of ‘mirror bananas’ (see figure 3 for combined results). That means that in our model luminosity-distance measurements from SNIa contributes considerably to the cosmological constraints comparing with \(\Lambda\)CDM model and DGP model due to different degeneracy properties shown on each parameter space.

In Fig.3, we plot the results of the combined analysis of Riess SNIa data + BAO + CMB. Again, the black, grey, and light grey region shows the 1, 2, and 3 \(\sigma\) confidence level contours on \(\Omega_m - B\) parameter space respectively with the minimum \(\chi^2 = 164.09\) occurring at \(\Omega_m = 0.27\), and \(B = 1.83\) for EDE model. We show clearly that combining two constraint ‘bananas’ from SNIa/BAO+CMB can get tight constraints on both the matter content \(\Omega_m\) and parameter \(B\) without significant degeneracy direction. On the other hand, the two sets of constraints from SNIa/\(\Lambda\)CDM+BAO are largely consistent with each other, indicating the feasibility of our \(\Omega_m - B\) parametrization as a successful way to parameterize our later universe. Considering the best fit values of \(\Omega_m\) and \(\Omega_\Lambda\), however, there exist some differences between the constraint results from SNIa and \(\Lambda\)CDM+BAO. Such discrepancies have also appeared in data analysis on other cosmology models. These might imply the existence of some systematics for cosmological observations we have used here and/or potential inconsistencies which deserve further investigations.

Fig.4 shows the confidence contours of the combined analysis on EDE model \(\Omega_m - B\) parameter space combining CMB and BAO constraints with SNLS data instead of Riess gold data. The minimum \(\chi^2\) locates at \(\Omega_m = 0.26\), and \(B = 1.90\). The constraints from combining SNLS SNIa data with CMB+BAO are less restrictive than combining Riess gold data, but more consistent with standard flat concordance model. We note that the difference between two best fit parameter values is due to difference between Riess gold data and SNLS data. SNLS data gives the minimum \(\chi^2\) = 110.97 occurring at \(\Omega_m = 0.30\), and \(B = 2.35\), much more consistent with concordance model than result from Riess gold data (see figure 1). In addition to cosmological constraints from kinetic dis-
FIG. 2: The black, grey, and light grey regions in top, middle and bottom figures show the 1, 2, and 3 σ confidence level contours of Ω_m – B parameter space respectively, on combining CMB shift parameter from WMAP three years data and BAO from SDSS.

FIG. 3: The black, grey, and light grey region shows the 1, 2, and 3 σ confidence level contours of Ω_m – B parameter space respectively on combining the SNIa Gold data, CMB shift parameter, and BAO.

FIG. 4: The black, grey, and light grey region shows the 1, 2, and 3 σ confidence level contours of Ω_m – B parameter space respectively on combining the SNIa SNLS data, CMB shift parameter, and BAO.

The CMB anisotropies and matter power spectrum provide in principle suitable discriminatory tests. These tests require a detailed understanding of the evolution of density perturbations in our model. Fig. 5 shows the linear growth factor G(a) of ΛCDM model (red curve), DGP model (green curve) and our

FIG. 5: Linear growth factors G(a) of ΛCDM model (red curve), DGP model (green curve) and our model (blue curve) as a function of a. The solid line represents our model, while the dashed line represents the ΛCDM model. The dotted line represents the DGP model.
The growth history for a flat universe can be solved as shown in [48]. The growth factor $G(a)$ is defined by solving the following differential equation:

$$
\frac{dG}{d\ln a} + \left( 4 + \frac{1}{2} \frac{d \ln H^2}{d \ln a} \right) G + G^2 + 3 + \frac{1}{2} \frac{d \ln H^2}{d \ln a} - \frac{3}{2} \Omega_m(a) = 0,
$$

where $G = d \ln (\delta/\alpha)/d \ln a$, $H = \dot{a}/a$ is the Hubble parameter. The growth history for a flat universe can be solved as

$$
G(a) = -1 + \left[ a^4 H(a) \right]^{-1} \int_{a_0}^a \frac{da'}{a'} a^4 H(a') \times \left[ \frac{5}{2} - \frac{3}{2} \Omega_m(a') - G^2(a') \right].
$$

For growth during the matter-dominated era, $G$ will be small. A reasonable approximation throughout the growth history even as dark energy comes to dominate has also been shown in [48].

$$
G(a) = -\frac{1}{2} \Omega_m(a) - \frac{1}{4} a^{-5/2} \int_0^a \frac{da'}{a'} a^5 \Omega_m(a').
$$

For any particular model of $H(a)$, or $\Omega_m(a)$ or $\Omega_w(a)$, we can then evaluate the growth history.

The values of model parameters we chose to plot $G(a)$ in Fig.5 correspond to the combined analysis results including CMB, BAO, and Riess gold SNIa data. We can find that our best fit model mimic $\Lambda$CDM linear structure formation quite well both in the early universe and in the late universe. Just for comparison, we also show the linear growth factor for DGP model. The non-linear structure formation in our model is definitely worth to study but it is beyond the scope of this present paper.

In Fig.6, we plot the relative weight of EDE component and dark matter component with respect to total energy contents in our universe versus redshift with best fitting parameter value from combined analysis of SNIa Gold data, CMB shift parameter, and BAO. We can see that the DM-EDE equality time happened at $z \sim 0.7$ which is quite close to the result from fitting to $\Lambda$CDM model.

### IV. PARAMETER DEGENERACY ANALYSIS

In this section, we discuss the new degeneracies on $\Omega_m - B$ parameter space, where new introduced parameter $B$ describes either dark energy or curvature term plus the dark energy. In this section, we use the first year of SNLS data in our analysis instead of Riess gold sample, since SNLS data set has a relatively narrow redshift range with $z < 1$ thus with more clear degenerate features between parameters. We note again that for the cosmological fits, two of the SNLS data points were excluded because they are outliers in the Hubble diagram. We also take the advantage of the recent GRB sample compiled by Schaefer [39] including 69 bursts with properly estimated and corrected redshifts to investigate the cosmological constraints. The redshift of the sample extends to $z = 6.3$ with considerable objects having $z > 1.5$. Upon using these distance modulus from GRBs, we fully aware of the circulation problem associated with GRBs as cosmological probes. In this paper, we only use the GRBs data to study the degeneracy properties on $\Omega_m - B$ parameter space, but not combing to other cosmological observations to constrain parameter space.

In our model, the parameter $B$ represents the deviation from standard flat $\Lambda$CDM concordance model. We can easily find that $B = 2$ corresponding to $\Lambda$CDM model with cosmological constant as dark energy and with flat geometry of our universe, whereas $B > 2$ describes effective positive curvature geometry
of our universe and/or effective dark energy equation of state \( w < -1 \), namely phantom like dark energy, and \( B < 2 \) describes effective negative curvature geometry of our universe and/or effective dark energy equation of state \( w > -1 \), namely quintessence like dark energy. It is well known that there exists significant degeneracies among \( \Omega_c, \Omega_m \) and dark energy equation of state \( w \) parameters. In the first step to explore evidences beyond standard cosmology model, it might be helpful and reasonable to introduce only one parameter which collapses both curvature effect and dark energy effect into this single parameter, and maybe includes other unknown features of new physics beyond flat \( \Lambda \)CDM concordance model, simplify the degeneracy relations, thus make the signal of deviation from flat \( \Lambda \)CDM model easily spotting out.

It is known that the CMB data alone cannot constrain well the dynamics of dark energy. Additional information from large scale structure of galaxies helps to tight on dark energy constraints mostly because they provide a tight limit on \( \Omega_m \), which in turn helps to constrain the properties of dark energy due to breaking the degeneracy between \( \Omega_m \) and the equation of state of dark energy in cosmological observable quantities. Here, we concentrate our study on parameter degeneracies in luminosity/angular diameter distance since it can be clearly and easily understood and it can also give rise to the most direct constraints on dark energy models. With flat universe assumption, the luminosity distance can be written as

\[
d_L = c(1+z) \int_0^z \frac{dz'}{H_0[\Omega_m(1+z')^3 + (1-\Omega_m)(1+z')^3+B]^\frac{1}{2}}. \tag{15}
\]

The degeneracies between \( B, \Omega_m \) and \( H_0 \) are clearly seen in this integral.

In order to see the degeneracy between the parameters \( B \) and \( \Omega_m \), in Fig. 7 we present the degeneracies in luminosity distance on the \( \Omega_m - B \) parameter plane at different redshifts. The different color bands describe the parameter spaces of \( \Omega_m, B \) where given the variation of \( d_L \) is in between \( \pm 1\% \) for \( z = 0.5 \) (black), 0.1(blue),2(yellow), 3(green),6(red),1100(magenta) with respect to a given fiducial model with parameter value given by our best fittings from Ries gold SNIa+CM+m+BAO before. One can find that, the degeneracy between \( \Omega_m \) and \( B \) varies with the redshift, which in turn implies that combining the information of \( d_L \) at different redshifts can indeed helps break such a degeneracy. This is the ideal case for showing the degeneracy between the parameters \( \Omega_m \) and \( B \) for different redshifts, because we fix the nuisance parameter \( H_0 \) instead of marginalizing it as we did in fitting procedure. Figure 8 is the results coming from the data fitting of GRBs (including high redshift information up to \( z \sim 6 \) and SNLS with information from much lower redshift range. We can find that the rotation of degeneracy direction from low redshift to high redshift showing in the plot can be explained by the degeneracy analysis on Fig. 7. The trend of degeneracy rotation in \( \Omega_m - B \) parameter plane is the same for Fig. 7 and 8. In order to constrain the cosmological parameters \( \Omega_m \) and \( B \) well from only distance measurements, one needs distance determinations for a wide range of redshifts. Or instead, one can break the parameter degeneracies by other cosmological observations with different degeneracy properties shown in Fig. 7. For current SNIa data, their redshift range is limited with the highest observed redshift \( \sim 1.7 \) up to now. On the other hand, for the GRB sample used in our analysis, the redshift extends to as high as \( \sim 6.3 \). Due to the different degeneracies at different redshift range, the complementarity of GRBs to SNIa is highly expected with assumption of well controlled systematics of using GRBs as standard candles.

We note that gravitational radiation opens another window by providing high redshift information to constrain our model. Observations of the gravitational waves emitted from the coalescence of supermassive black holes with independent determination of redshift through an electromagnetic counterpart can be used as standard sirens to provide an excellent probe of the expansion history of the Universe, especially by high redshift information, which can be used to constrain the dark energy properties[50]. The degeneracy properties of model parameters are the same as by using standard candle, standard ruler or standard siren, as discussed in this paper. Potentially, several well measured standard sirens will be enough to give us tight constraints on dark energy parameters.

In Fig. 9, we plot the effective dark energy equation of state \( w_{\text{DE,eff}} \) with different choices of parameter \( B \). The effective dark energy equation of state is determined purely by the Hubble parameter \( H(z) \) , and there is a general formula that can relate \( H(z) \) and \( w_{\text{DE,eff}} \) as

\[
w_{\text{DE,eff}}(z) = -1 + \frac{1}{3} \frac{d \ln(\delta H^2/H_0^2)}{d \ln(1+z)}, \tag{16}
\]
FIG. 8: 1, 2, and 3 σ confidence level contours on $\Omega_m - B$ parameter space using the GRB sample and SNIa SNLS data. The red-dashed lines are the results from the GRB sample and the blue-dotted lines show the constraints resulting from the SNIa SNLS data.

FIG. 9: Effective dark energy equation of state $w(a)$. The different color curves correspond to different value of $B$, namely $B = 1.5$ (black), $B = 1.7$ (blue), $B = 2.0$ (yellow), $B = 2.3$ (green), $B = 2.5$ (red).

V. CONCLUSION

We have presented a cosmic model parameterizing the late universe which collapses curvature and dark energy effects into one parameter $B$ that may indicate any deviation from standard flat $\Lambda$CDM model and we find that we can not conclude that the cosmic curvature term is constantly zero, instead it may contribute rich phenomenological effects. In order to show the advantages of our parametrization, we study the degeneracy properties between $B$ and $\Omega_m$, emphasizing the contribution from high redshift distance information from GRB or gravitational waves experiments on-going and upcoming. It is well-known that deducing the number of free parameter without significant physics lost is quite important to constrain cosmology models and to find new physics behind.

In this paper we also investigated the DGP cosmological model in the simplest flat geometry case with the extra dimension contribution as an effective "cosmological constant", compared with our parameterized model and the reduction to the power-law $\Lambda$CDM model for the 4D real Universe. We find that the DGP model even in the simplest case is still an interesting candidate for the current cosmic speed-up expansion mechanism at long distances, while we know that in the short ranges the model behaves as 4D conventional gravity. We will exploit the non-compact extra dimension to see its possible existence signatures via cosmic effects in the general DGP model later as a promising model, while we do not intend to discuss the quantum aspects of this model as a basic theory\cite{54}.

As a generalization of the $\Lambda$CDM model with naive cosmological constant as dark energy candidate we has parameterized a curvature like term with new phenomenological features via numerical fittings and show the term explicit physics meanings when we perform the parameter $B$ reduction directly to zero or 2. It may be interesting also to study the general properties of the parameterized term as the matter-energy contents in our Universe continuous equation to see what kind of "matter" it may describe effectively, without specifying the form of the parameter. Besides, the phantom case can be realized too, for example, the equation of state parameter $w = p/\rho < -1$ if we take $B > 2$ and quintessence corresponds to $B < 2$ with $w = p/\rho > -1$ numerically. We think this picture is in conformity with other popular models and enlarges phenomenological dark energy study possibilities to explain the late-time accelerating expansion of our Universe, thus it is worth of further endeavors.

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