The Kozai-Lidov Mechanism in Binary Star Systems and the Stabilizing Effect of Binary Stars

Zhifeng Chen

1 York School, Monterey, CA 93940
issae8007@outlook.com

Abstract—This study used Python simulations to compare the orbits of planets in single-planet binary systems and two-planet binary systems with equivalent single star systems to examine the differences of the Kozai-Lidov mechanism in three-body single star systems and four-body binary star systems. The Python program simulated the forces between the stars and the planets, and produced inclination graphs and orbital diagrams. In the single-planet scenario, no Kozai-Lidov effect is observed as the additional star did not affect the planet’s inclination, but the orbit is considerably rotated. In the two-planet scenario, the Kozai-Lidov oscillation period decreases from 650 years to 370 years, and the maximum inclination decreases from 0.690 radians to 0.280 radians, indicating that the binary star system acts against the Kozai-Lidov Mechanism. Additional trials with modified initial configurations of the two-planet binary star system confirm the universality of the suppressive effect in varying initial conditions in binary star systems.

1. Introduction

The Kozai-Lidov Mechanism is a gravitational phenomenon that describes the gravitational influence of an inclined perturber body on an inner binary, which often consists of a body orbiting a central object. Specifically, it states that the perturber and the orbiting body will torque each other and exchange angular momentum, resulting in the orbit of the less massive object periodically trade eccentricity for inclination at time scales orders of magnitudes larger than their orbital periods [Lidov, 1962]. It is most commonly used to describe long term orbital variations in three-body systems. Understanding the Kozai-Lidov Mechanism has allowed scientists to more accurately simulate mergers and formation of different astronomical objects, such as black hole binaries [Valtonen, 2006; Di Matteo et al. 2005], type 1a supernovae [Naoz, 2016], and hot Jupiters [Knutson et al. 2014; Ngo et al. 2015]. Even in our solar system, the Kozai-Lidov Mechanism helps scientists to understand the orbital dynamics of a variety of objects, such as planet pairs, binaries of asteroids, moons, and satellites, which forms a triple body system with the planet or the sun [Polishook and Brosch, 2006; Margot et al., 2015]. However, the effect becomes less predictable and more chaotic when the planets orbit around more than one central object. Binary star systems are one of the examples. Similar to single star systems, all of the bodies in binary star systems revolve around their shared center of mass. However, binary star systems differ from single star systems in the sense that planets are pulled towards to point that is different from the barycenter, as the center of the combined gravitational attraction is at a different point than the barycenter since one of the stars is almost always closer to the planet and the magnitude of the gravitational force follows the inverse square law. Additionally, the gravitational effect of the perturber planet would further interfere with the orbit, resulting in a chaotic orbit that can only be analyzed through computer simulations. This
study uses Python simulations to calculate the orbits of planets in simple binary star systems to compare the effects of the Kozai-Lidov mechanism in binary star systems with single star systems.

2. Methods

2.1. Setup
In this study, two different systems were simulated: a two-planet system and a one-planet system. In each of the two systems, there were a binary star scenario and a single star scenario. The latter was included to isolate the effects of the binary stars. The simulation is run at an interval of 0.1 days. Results run at different time intervals show that time intervals shorter than 0.1 days have no improvement in the accuracy and reliability of the data; while time intervals larger than 0.1 days show less accurate results. The single star is treated as stationary, while the orbital period of the binary stars is set to be 7.5 Earth days. The orbital period is assumed to be arbitrary since this study does not focus on the orbital period of the stars. The central star in the single star system has the mass of the two binary stars combined. To keep the simulation in a practical time interval, the positions of the binary stars were calculated through sinusoidal functions, instead of force and acceleration calculations. Additionally, in single star scenarios, the stars were treated as stationary, which should not affect the simulation since the planets are far less massive than the central star. After observing the stabilizing effect of the binary star, additional trials were conducted to examine the effect in different relative positions. In these trials, the time interval remained at 0.1 days, and the duration decreased to 200 years. Rotations of 45 and 90 degrees were added to the sinusoidal functions modeling the positions of the binary stars to simulate different initial arrangements of the binary star system. All parameters about the stars and the planets are listed in Table 1.

Table 1 Star and planet parameters in each scenario

| Scenario               | Parameters | Objects                                      | Mass          | Semimajor Radius | Initial Inclination |
|------------------------|------------|----------------------------------------------|----------------|------------------|--------------------|
| a) Single Star Single Planet |
|                        | Parameters | Mass, Semimajor Radius, Initial Inclination  |                |                  |                    |
| Primary Star           | Mass       | 1.2×10^{31} kg                              | N/A            | N/A              |                    |
| Planet                 | Mass       | 6×10^{27} kg                                | 1.4 AU         | 0.37 Rad         |                    |
| b) Binary Star Single Planet |
|                        | Parameters | Mass, Semimajor Radius, Initial Inclination  |                |                  |                    |
| Primary Star           | Mass       | 9×10^{30} kg                                | 0.03 AU        | N/A              |                    |
| Secondary Star         | Mass       | 3×10^{30} kg                                | 0.1 AU         | N/A              |                    |
| Planet                 | Mass       | 6×10^{27} kg                                | 1.4 AU         | 0.37 Rad         |                    |
### c) Single Star Two Planets

| Parameters | Mass     | Semimajor Radius | Initial Inclination |
|------------|----------|------------------|--------------------|
| Objects    |          |                  |                    |
| Primary Star | 9×10^{30} kg | N/A              | N/A                |
| Planet     | 6×10^{24} kg | 1.0 AU           | 0 Rad              |
| Perturber  | 6×10^{27} kg | 1.4 AU           | 0.37 Rad           |

### d) Binary Star Two Planets

| Parameters | Mass     | Semimajor Radius | Initial Inclination |
|------------|----------|------------------|--------------------|
| Objects    |          |                  |                    |
| Primary Star | 9×10^{30} kg | 0.03 AU         | N/A                |
| Secondary Star | 3×10^{30} kg | 0.1 AU         | N/A                |
| Planet     | 6×10^{24} kg | 1.0 AU           | 0 Rad              |
| Perturber  | 6×10^{27} kg | 1.4 AU           | 0.37 Rad           |

#### 2.2. Calculation Method

This study was conducted through computer simulations based on Python 3.7; graphs were produced with Matplotlib 3.3.1. The program created arrays for the planets x, y, and z coordinates. The coordinate system is centered at the single star in single star scenarios or the barycenter in binary star scenarios, and the binary stars and the subject planet orbit on the x, y plane. The program is based on Newton’s Law of Universal Gravitation, which indicates that the gravitational force between the central object with mass \( M \) and the orbiting planet with mass \( m \), separated by a distance \( r \), is

\[
F_G = G \frac{M m}{r^2}
\]  

(1)

where \( G \) is the gravitational constant.

The acceleration of the planets due to this force, as expressed by Newton’s Second Law of Motion, is,

\[
a = \frac{F_G}{m}
\]  

(2)

equating equations (1) and (2) and solving for the acceleration of the planet obtains

\[
a = \frac{G M}{r^2}
\]  

(3)

which could be broken down into components as

\[
a_i = \frac{-a r_i}{r}
\]  

(4)

where \( a_i \) is the subject planet’s acceleration on a specific axis and \( r_i \) is the distance between the two objects on the same axis.
The program calculates the planet’s net acceleration in each direction by adding the acceleration in the corresponding direction due to each object. For instance, in the two planet binary star scenario, the net acceleration on the x-axis is,

\[ \sum a_x = a_{M_x} + a_{S_x} + a_{p_x} \]  

(5)

where \( a_{M_x} \), \( a_{S_x} \), and \( a_{p_x} \) are acceleration on the x-axis of the subject planet due to the primary star, secondary star, and the perturber, respectively.

From the acceleration, we obtain the x-coordinate of the subject planet after the calculation interval

\[ v_x = v_{0x} + a_x (\Delta t) \]  

(6)

\[ r_x = r_{0x} + (\Delta t) \frac{v_x + v_{0x}}{2} \]  

(7)

where \( \Delta t \) is the calculation interval, 0.1 days; and \( v_{0x} \), and \( r_{0x} \) are the current velocity and position at the x-axis.

These positions were then concatenated to the position arrays.

The program also calculated the inclination \( I \) of the planets’ orbits on the apsis using an absolute value inverse tangent function,

\[ I = \tan^{-1} \left( \frac{r_z}{\sqrt{r_x^2 + r_y^2}} \right) \]  

(8)

where \( r_z \) is the distance between the planet and the barycenter or the single star along the z-axis, and \( \sqrt{r_x^2 + r_y^2} \) is the distance between the two points on the orbital plane.

The coordinate and inclination arrays were then plotted on graphs using Matplotlib. The orbital diagram and the inclination diagrams of the subject planet were graphed.

In binary star scenarios, the positions of the binary stars, which have a period of 7.5 days, are computed through sinusoidal functions below,

\[ s_x = x_0 \cos \left( \frac{2\pi}{7.5} t - \frac{7.5}{\pi/d} \right) \]  

(9)

\[ s_y = x_0 \sin \left( \frac{2\pi}{7.5} t - \frac{7.5}{\pi/d} \right) \]  

(10)

\[ s_z = z_0 \cos \left( \frac{2\pi}{7.5} t - \frac{7.5}{\pi/d} \right) \]  

(11)

where \( s_x, s_y, \) and \( s_z \) are the x, y, and z coordinate of each star; \( x_0 \) and \( z_0 \) are the star’s initial x and z coordinate; \( d \) is the angle in radians at which the initial positions of the binary star is rotated. To more clearly demonstrate the changes of the orbits, only the initial, middle, and final orbit are being shown, which are marked with a solid, dashed, and dotted line, respectively.

3. Results and Discussion

3.1. Single planet systems

As demonstrated in Figure 1, the inclination remains at a constant 0.366 radian in the single star scenario, which is the exact decimal value of the planet’s initial inclination. The result shows that the calculation interval of 0.1 days is adequate for accurately simulating the orbits in such systems. Figure 2 shows a considerable alteration in the planet’s orbit, as the orbit appears wider due to the gradual rotation of its orbit. On the vertical axis, the orbital inclination shows no changes as it stayed at a constant 0.366 radian. This suggests that the binary stars have a larger impact on the planet’s orbit along the stars’ orbital plane compared to the vertical direction. The additional central star causes the orbit of the subject planet to rotate more than fluctuate in inclination.
3.2. Two planet systems:

In both of the scenarios, periodic oscillations of the inner planet’s orbital inclination are shown in Figure 3 and Figure 4. Figure 3 shows that in the single star scenario, the Kozai-Lidov oscillation period is 650 years. The planet starts with a zero inclination, which is expected since it initially orbits along the x-y plane, and peaks at 0.690 radian, which exceeds the inclination of the perturber. Additionally, the planet’s inclination does not fall back to zero, indicating that the perturber’s gravitational attraction is much larger than that of the central star in the vertical direction when the subject planet has low orbital inclinations. Furthermore, the final (dotted) orbit in figure 3 indicates that the orbital velocity of the subject planet is significantly higher, since there are 2 dotted orbits, suggesting that in the same period, the subject planet completed 2 revolutions. Figure 4 shows that the orbital period reduced to 370 years in the binary star scenario, which is 56.9% of the oscillation period in the single star scenario. The peak inclination reaches 0.280 radian, which is 40.5% of the original peak inclination. The decreases in both the period and the angle show that the gravitational pull of the binary stars acts against the Kozai-Lidov Mechanism. The orbital diagrams in Figures 3 and 4 further demonstrate the suppressive effect of the central star on the Kozai-Lidov Mechanism, as the orbit of the subject planet in binary star systems appear thinner, indicating that the orbit is much less rotated and distorted by the perturber in binary star systems. The observation matches the analytical hypothesis that the center of combined gravitational attraction from the planets is always closer to the planet, because gravitational force follows the inverse...
square law while the stars’ center of mass does not. This results in a larger magnitude of the gravitational force, which explains the suppressive force that acts against the Kozai-Lidov Mechanism.

![1 Star 2 Planet Inclination](image1)

a) Inclination

![1 Star 2 Planet x, y](image2)

b) Orbital Diagram

Figure 3 The subject planet in two planet single star system.

![2 Star 2 Planet Inclination](image3)

a) Inclination

![2 Star 2 Planet x, y](image4)

b) Orbital Diagram

Figure 4 The subject planet in two planet binary star system.

3.3. Universality of the Stabilizing Effect

Figure 5 shows the inclination graphs of the subject planet at different initial positions were consistent with the unshifted graph. The general trends in both scenarios indicate a maximum inclination of 0.280 radians and a half period of approximately 180 Earth years, showing no significant differences from the original unshifted scenario. Therefore, the differences in the directions at which the stars pull on the subject planet do not constitute an observable difference. Since these scenarios represent a variety of real-world binary star systems, it is safe to conclude that the suppressive gravitational effect of the additional star is present in most binary star systems.
Figure 5 Inclination of subject planet with rotations of the initial position of the central stars in two planet binary star systems.

4. Conclusion
This study sufficiently concludes that binary stars significantly alter the Kozai-Lidov Mechanism on the subject planet. Single planets revolving around binary stars at an incline will experience no fluctuation in its orbital inclination, while rotations of the planets’ orbits are much more apparent. In multi-planet systems, the additional central star counters the Kozai-Lidov Mechanism by exerting a large suppressive force, decreasing the oscillation period and maximum inclination by 43% and 59%, respectively.

Future studies may include simulating how different factors affect the orbits of the planets in multi-planet binary star systems to find more evidence and quantifiable patterns of the stabilizing effect of binary stars systems or other four body systems.

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