Comment on “Measurement of the $Q^2$ Dependence of the Deuteron Spin Structure Function $g_1$ and its Moments at Low $Q^2$ with CLAS”

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We argue that the recently published CLAS results on the deuteron spin polarizability $\gamma_0$ [Adhikari et al., Phys. Rev. Lett. 120, 062501 (2018)], as well as their comparisons with chiral perturbation theory ($\chi$PT), are misleading. In reality, the deuteron polarizability is larger by 4 orders of magnitude, as demonstrated here by a novel calculation in pionless EFT. The CLAS paper, on the other hand, presents only a tiny correction to it, based on a partial evaluation of a sum rule for $\gamma_0$. The sum rule is assumed to have the same form as for the nucleon; we argue it does not. Moreover, their “test of $\chi$PT” tacitly involves assumptions which, as we demonstrate, may not be valid at the claimed accuracy.

In a recent Letter [1], the CLAS Collaboration presents “the first precise measurement of the $Q^2$ evolution of $\Gamma_4^d$ and of the spin polarizability $\gamma_0$ on the deuteron”, shown in their Figs. 2 and 3, respectively. The results in Fig. 3 are in units of $10^{-4}$ fm$^4$ (cf. their arXiv version). This is 4 orders of magnitude smaller than our estimate, shown here in Fig. 1, based on extending the pionless-EFT calculation [2] to finite $Q^2$.

What Fig. 3 of [1] really represents is a small contribution to the deuteron polarizability, obtained by a partial evaluation of the sum rule in their Eq. (3). The upper limit of integration therein is the lowest inelastic threshold — in this case, the threshold of the deuteron breakup ($\gamma'd \rightarrow pn$). In the evaluation, however, the upper limit of integration is set at a higher energy scale, in the vicinity of the pion-production threshold. The large low-energy contribution is omitted, without stating what the remaining quantity is. The same critique concerns the results for $\Gamma_4^d$, albeit the omitted low-energy contribution therein has a much smaller impact because of the different energy weighting.

We stress that Fig. 1 is an EFT prediction for the deuteron $\gamma_0$, and not the “$\chi$PT results” presented in [1]. The latter are obtained from the single-nucleon calculations [4, 5] assuming the deuteron polarizability is given by the isoscalar nucleon polarizability. It is an approximation, which, among other effects, neglects the breakup channel. Even though the integral is taken from the pion threshold, where the breakup channel is less important, its effect is not necessarily negligible. We have evaluated it at $Q^2 = 0$, using the helicity-difference cross sections of Arenhövel et al. [7], with the result of roughly: $-0.7 \times 10^{-4}$ fm$^4$.

Furthermore, Eq. (3) of [1] is only correct for a spin-1/2 target, such as the nucleon. The deuteron has a different sum rule [6]. For $Q^2 = 0$, it reads:

$$- \frac{\alpha}{4M_T^2}(\gamma + \zeta)^2 + 2\gamma_0 = \frac{1}{4\pi^2} \int_0^{\infty} dv \frac{\sigma_0(v) - \sigma(v)}{v^3}$$

$$= \lim_{Q^2\to0} \frac{16\alpha M^2}{Q^6} \int_0^{\infty} dx x^2 g_{TT}(x,Q^2),$$

with $g_{TT} \equiv g_1 - (2Mx/Q)^2 g_2$. The rhs (either in terms of the helicity-difference cross section of total photoabsorption or, equivalently, in terms of the spin structure functions $g_1$ and $g_2$) is the same. The lhs is different. The first term, given by the anomalous magnetic moment $\gamma$ and the anomalous quadrupole moment $\zeta$, is absent for the spin-1/2 case. Using the empirical values for the deuteron: $\gamma_0 \simeq -0.143$, $\zeta_0 \simeq 13.5$, $M_d \simeq 1.8756$ GeV, this term amounts to: $-0.4 \times 10^{-4}$ fm$^4$. This would be negligible as a correction to the deuteron polarizability, but not for the small contribution to $\gamma_0$ studied in [1]. The second term has a conventional factor of 2, adopted here to compare directly with the literature (e.g., [2, 3]).

To conclude, Ref. [1] concerns with only a tiny contribution to the deuteron polarizability $\gamma_0$, rather than the polarizability itself. Besides the wrong “semantics”, we identify two missing contributions which are potentially important: 1) the breakup channel, affecting the theory curves, in the much discussed comparison with $\chi$PT; 2) the deuteron electromagnetic moments (in general, form factors) in the sum rule of Eq. (1).

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FIG. 1. Deuteron polarizability $\gamma_0$ calculated in leading-order pionless EFT. The data point at $Q^2 = 0$ is from [3].
[1] K. P. Adhikari et al. [CLAS Collaboration], Phys. Rev. Lett. 120, no. 6, 062501 (2018).
[2] X. d. Ji and Y. c. Li, Phys. Lett. B 591, 76 (2004).
[3] M. W. Ahmed et al., Phys. Rev. C 77, 044005 (2008).
[4] V. Bernard, E. Epelbaum, H. Krebs and U.-G. Meißner, Phys. Rev. D 87, 054032 (2013).
[5] V. Lensky, J. M. Alarcón and V. Pascalutsa, Phys. Rev. C 90, no. 5, 055202 (2014).
[6] F. Hagelstein, “Sum rules for electromagnetic moments and polarizabilities of spin-1 particles in massive Yang-Mills QED”, Master thesis (University of Mainz, 2014) doi:10.13140/RG.2.2.35903.82088.
[7] H. Arenhövel, A. Fix and M. Schwamb, Phys. Rev. Lett. 93, 202301 (2004).