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1. Editor’s note

This festive issue concludes the civilian year 2008 with details on a special issue of Topology and its Applications dedicated to SPM, and with a quite large list of research announcements.

Have a fruitful 2009,

Boaz Tsaban, tsaban@math.biu.ac.il
2. PROCEEDINGS OF THE THIRD WORKSHOP ON COVERINGS, SELECTIONS AND GAMES IN TOPOLOGY

Special Issue 156 of *Topology and its Applications*. Guest editor: Ljubiša D.R. Kočinac.

From the preface: “The Third Workshop on Coverings, Selections and Games in Topology was held in Vrnjačka Banja, Serbia, from April 25 to April 29, 2007, and organized by Faculty of Sciences and Mathematics, University of Niš, and Technical Faculty in Čačak, University of Kragujevac.

“The previous two workshops under the same name were held in Lecce, Italy (June 27–29, 2002 and December 19–22, 2005). The main theme of this workshop was Selection Principles Theory and variety of its relations with other fields of Topology and Mathematics (game theory, Ramsey theory, set theory, combinatorics, function spaces, hyperspaces, uniform spaces, topological algebras, analysis, dimension theory, lattice theory, Boolean algebras). This special issue of Topology and its Applications contains 19 papers presented at the meeting and evaluated following the usual editorial procedure of the journal.”

Contents and links to abstracts and full texts:

(1) Ljubiša D.R. Kočinac, *Preface*, p. 1.  
http://dx.doi.org/10.1016/j.topol.2008.05.022

(2) Liljana Babinkostova, *Selective screenability in topological groups*, pp. 2–9.  
http://dx.doi.org/10.1016/j.topol.2008.02.014

(3) Taras Banakh, Lyubomyr Zdomskyy, *Separation properties between the σ-compactness and Hurewicz property*, pp. 10–15.  
http://dx.doi.org/10.1016/j.topol.2007.12.017

(4) Pavle V.M. Blagojevic, Aleksandra S. Dimitrijevic Blagojevic, John McCleary, *Equilateral triangles on a Jordan curve and a generalization of a theorem of Dold*, pp. 16–23.  
http://dx.doi.org/10.1016/j.topol.2008.04.008

(5) Lev Bukovský, *On $wQN_*$ and $wQN^k$ spaces*, pp. 24–27.  
http://dx.doi.org/10.1016/j.topol.2007.10.010

(6) Giuseppe Di Maio, Ljubiša D.R. Kočinac, *Statistical convergence in topology*, pp. 28–45.  
http://dx.doi.org/10.1016/j.topol.2008.01.015

(7) Dragan Djurcic, Ljubiša D.R. Kocinac, Mališa R. Žižović, *Classes of sequences of real numbers, games and selection properties*, pp. 46–55.  
http://dx.doi.org/10.1016/j.topol.2008.02.013

(8) Jakub Duda, Boaz Tsaban, *Null sets and games in Banach spaces*, pp. 56–60.  
http://dx.doi.org/10.1016/j.topol.2008.04.009
(9) Vitaly V. Fedorchuk, Evgenij V. Osipov *Certain classes of weakly infinite-dimensional spaces and topological games*, pp. 61–69. http://dx.doi.org/10.1016/j.topol.2007.11.007

(10) Dimitris N. Georgiou, Stavros D. Iliadis, *On the greatest splitting topology*, pp. 70–75. http://dx.doi.org/10.1016/j.topol.2007.11.008

(11) Stavros D. Iliadis, *Universal elements in some classes of mappings and classes of G-spaces*, pp. 76–82. http://dx.doi.org/10.1016/j.topol.2008.04.010

(12) Maria Joita, *On frames in Hilbert modules over pro-C*-algebras*, pp. 83–92. http://dx.doi.org/10.1016/j.topol.2007.12.015

(13) Marion Scheepers, *Rothberger’s property in all finite powers*, pp. 93–103. http://dx.doi.org/10.1016/j.topol.2007.10.011

(14) Dušan Repovš, Boaz Tsaban, Lyubomyr Zdomskyy, *Continuous selections and σ-spaces*, pp. 104–109. http://dx.doi.org/10.1016/j.topol.2008.03.025

(15) Andrzej Kucharski, Szymon Plewik, *Inverse systems and I-favorable spaces*, pp. 110–116. http://dx.doi.org/10.1016/j.topol.2007.12.016

(16) Masami Sakai, *Function spaces with a countable cs*-network at a point*, pp. 117–123. http://dx.doi.org/10.1016/j.topol.2007.10.012

(17) Mila Mršević, Milena Jelić, *Selection principles in hyperspaces with generalized Vietoris topologies*, pp. 124–129. http://dx.doi.org/10.1016/j.topol.2008.01.016

(18) Heike Mildenberger, *Cardinal characteristics for Menger-bounded subgroups*, pp. 130–137. http://dx.doi.org/10.1016/j.topol.2008.04.011

(19) Tomasz Weiss, *A note on unbounded strongly measure zero subgroups of the BaerSpecker group*, pp. 138–141. http://dx.doi.org/10.1016/j.topol.2008.03.026

(20) Sophia Zafiridou, *Dendrites with a countable closure of the set of end points*, pp. 142–149. http://dx.doi.org/10.1016/j.topol.2008.04.012

3. Research announcements

3.1. The commutant of $L(H)$ in its ultrapower may or may not be trivial. Kirchberg asked in 2004 whether the commutant of $L(H)$ in its (norm) ultrapower is trivial. Assuming the Continuum Hypothesis, we prove that the answer depends on the choice of the ultrafilter.

http://arxiv.org/abs/0808.3763
3.2. **Partitions of trees and ACA'**. We show that a version of Ramsey's theorem for trees for arbitrary exponents is equivalent to the subsystem ACA' of reverse mathematics.

http://arxiv.org/abs/0809.2267

**Bernard A. Anderson, Jeffry L. Hirst**

3.3. **Products of straight spaces**. A metric space $X$ is straight if for each finite cover of $X$ by closed sets, and for each real valued function $f$ on $X$, if $f$ is uniformly continuous on each set of the cover, then $f$ is uniformly continuous on the whole of $X$. A locally connected space is straight iff it is uniformly locally connected (ULC). It is easily seen that ULC spaces are stable under finite products. On the other hand the product of two straight spaces is not necessarily straight. We prove that the product $X \times Y$ of two metric spaces is straight if and only if both $X$ and $Y$ are straight and one of the following conditions holds:

(a) both $X$ and $Y$ are precompact;
(b) both $X$ and $Y$ are locally connected;
(c) one of the spaces is both precompact and locally connected.

In particular, when $X$ satisfies (c), the product $X \times Z$ is straight for every straight space $Z$.

Finally, we characterize when infinite products of metric spaces are ULC and we completely solve the problem of straightness of infinite products of ULC spaces.

http://arxiv.org/abs/0809.5080

**Alessandro Berarducci, Dikran Dikranjan, Jan Pelant**

3.4. **Each second countable abelian group is a subgroup of a second countable divisible group**. It is shown that each pseudonorm defined on a subgroup $H$ of an abelian group $G$ can be extended to a pseudonorm on $G$ such that the densities of the obtained pseudometrizable topological groups coincide. We derive from this that any Hausdorff $\omega$-bounded group topology on $H$ can be extended to a Hausdorff $\omega$-bounded group topology on $G$. In its turn this result implies that each separable metrizable abelian group $H$ is a subgroup of a separable metrizable divisible group $G$. This result essentially relies on the Axiom of Choice and is not true under the Axiom of Determinacy (which contradicts to the Axiom of Choice but implies the Countable Axiom of Choice).

http://arxiv.org/abs/0810.3030

**T. Banakh, L. Zdomskyy**

3.5. **A dichotomy for Borel functions**. The dichotomy discovered by Solecki states that any Baire class 1 function is either $\sigma$-continuous or ”includes” the Pawlikowski function $P$. The aim of this paper is to give an argument which is simpler
than the original proof of Solecki and gives a stronger statement: a dichotomy for all Borel functions.

http://arxiv.org/abs/0810.1391
Marcin Sabok

3.6. More Results on Regular Ultrafilters in ZFC. We prove, in ZFC alone, some new results on regularity and decomposability of ultrafilters. We also list some problems, and furnish applications to topological spaces and to extended logics.

http://arxiv.org/abs/0810.5587
Paolo Lipparini

3.7. Minimal pseudocompact group topologies on free abelian groups. A Hausdorff topological group \( G \) is minimal if every continuous isomorphism \( f : G \to H \) between \( G \) and a Hausdorff topological group \( H \) is open. Significantly strengthening a 1981 result of Stoyanov we prove the following theorem: For every infinite minimal group \( G \) there exists a sequence \( \{ \sigma_n : n \in \mathbb{N} \} \) of cardinals such that

\[
w(G) = \sup \{ \sigma_n : n \in \mathbb{N} \} \quad \text{and} \quad \sup \{ 2^{\sigma_n} : n \in \mathbb{N} \} \leq |G| \leq 2^{w(G)},
\]

where \( w(G) \) is the weight of \( G \). If \( G \) is an infinite minimal abelian group, then either \( |G| = 2^\sigma \) for some cardinal \( \sigma \), or \( w(G) = \min \{ \sigma : |G| \leq 2^\sigma \} \). Moreover, we show that the equality \( |G| = 2^{w(G)} \) holds whenever \( \text{cf}(w(G)) > \omega \).

For a cardinal \( \kappa \) we denote by \( F_\kappa \) the free abelian group with \( \kappa \) many generators. If \( F_\kappa \) admits a pseudocompact group topology, then \( \kappa \geq c \), where \( c \) is the cardinality of the continuum. We show that the existence of a minimal pseudocompact group topology on \( F_\kappa \) is equivalent to the Lusin’s Hypothesis \( 2^{\omega_1} = c \). For \( \kappa > c \), we prove that \( F_\kappa \) admits a (zero-dimensional) minimal pseudocompact group topology if and only if \( F_\kappa \) has both a minimal group topology and a pseudocompact group topology. If \( \kappa > c \), then \( F_\kappa \) admits a connected minimal pseudocompact group topology of weight \( \sigma \) if and only if \( \kappa = 2^\sigma \). Finally, we establish that no infinite torsion-free abelian group can be equipped with a locally connected minimal group topology.

http://arxiv.org/abs/0811.0914
Dikran Dikranjan, Anna Giordano Bruno, Dmitri Shakhmatov

3.8. Weakly infinite dimensional subsets of \( \mathbb{R}^N \). The Continuum Hypothesis implies an Erdős-Sierpiński like duality between the ideal of first category subsets of \( \mathbb{R}^N \), and the ideal of countable dimensional subsets of \( \mathbb{R}^N \). The algebraic sum of a Hurewicz subset - a dimension theoretic analogue of Sierpinski sets and Lusin sets - of \( \mathbb{R}^N \) with any strongly countable dimensional subset of \( \mathbb{R}^N \) has first category.

http://arxiv.org/abs/0811.3661
Liljana Babinkostova and Marion Scheepers
3.9. Selections, Extensions and Collectionwise Normality. We demonstrate that the classical Michael’s selection theorem for l.s.c. mappings with a collection-wise normal domain can be reduced only to compact-valued mappings modulo the Dowker’s extension theorem for such spaces. The technique developed to achieve this result is applied to construct selections for set-valued mappings whose point images are in completely metrizable absolute retracts.

http://arxiv.org/abs/0811.3945
Valentin Gutev and Narcisse Roland Loufouma Makala

3.10. Openly factorizable spaces and compact extensions of topological semigroups. We prove that the semigroup operation of a topological semigroup $S$ extends to a continuous semigroup operation on its the Stone-Čech compactification $\beta S$ provided $S$ is a pseudocompact openly factorizable space, which means that each map $f : S \to Y$ to a second countable space $Y$ can be written as the composition $f = g \circ p$ of an open map $p : X \to Z$ onto a second countable space $Z$ and a map $g : Z \to Y$. We present a spectral characterization of openly factorizable spaces and establish some properties of such spaces.

http://arxiv.org/abs/0811.4272
Taras Banakh, Svetlana Dimitrova

3.11. Embedding the bicyclic semigroup into countably compact topological semigroups. We study algebraic and topological properties of topological semigroups containing a copy of the bicyclic semigroup $C(p, q)$. We prove that each topological semigroup $S$ with pseudocompact square contains no dense copy of $C(p, q)$. On the other hand, we construct a consistent example of a Tychonov countably compact semigroup containing a copy of $C(p, q)$.

http://arxiv.org/abs/0811.4276
Taras Banakh, Svetlana Dimitrova, Oleg Gutik

3.12. Building suitable sets for locally compact groups by means of continuous selections. If a discrete subset $S$ of a topological group $G$ with the identity 1 generates a dense subgroup of $G$ and $S \cup 1$ is closed in $G$, then $S$ is called a suitable set for $G$. We apply Michael’s selection theorem to offer a direct, self-contained, purely topological proof of the result of Hofmann and Morris on the existence of suitable sets in locally compact groups. Our approach uses only elementary facts from (topological) group theory.

http://arxiv.org/abs/0812.0489
Dmitri Shakhmatov

3.13. The near coherence of filters principle does not imply the filter dichotomy principle.

http://www.ams.org/journal-getitem?pii=S0002-9947-08-04806-X
Heike mildenberger and Saharon Shelah
4. UNSOLVED PROBLEMS FROM EARLIER ISSUES

Issue 1. Is \( \binom{\Omega}{T} = \binom{\Omega}{T} \)?

Issue 2. Is \( U_{\text{fin}}(\mathcal{O}, \Omega) = S_{\text{fin}}(\Gamma, \Omega) \)? And if not, does \( U_{\text{fin}}(\mathcal{O}, \Gamma) \) imply \( S_{\text{fin}}(\Gamma, \Omega) \)?

Issue 4. Does \( S_1(\Omega, \Gamma) \) imply \( U_{\text{fin}}(\Gamma, \Gamma) \)?

Issue 5. Is \( p = p^* \)? (See the definition of \( p^* \) in that issue.)

Issue 6. Does there exist (in ZFC) an uncountable set satisfying \( S_{\text{fin}}(\mathcal{B}, \mathcal{B}) \)?

Issue 8. Does \( X \notin \text{NON}(\mathcal{M}) \) and \( Y \notin \text{D} \) imply that \( X \cup Y \notin \text{COF}(\mathcal{M}) \)?

Issue 9 (CH). Is \( \text{Split}(\Lambda, \Lambda) \) preserved under finite unions?

Issue 10. Is \( \text{cov}(\mathcal{M}) = \text{cof} \)? (See the definition of \( \text{cof} \) in that issue.)

Issue 11. Does \( S_1(\Gamma, \Gamma) \) always contain an element of cardinality \( \mathfrak{b} \)?

Issue 12. Could there be a Baire metric space \( M \) of weight \( \aleph_1 \) and a partition \( \mathcal{U} \) of \( M \) into \( \aleph_1 \) meager sets where for each \( \mathcal{U}' \subset \mathcal{U}, \bigcup \mathcal{U}' \) has the Baire property in \( M \)?

Issue 14. Does there exist (in ZFC) a set of reals \( X \) of cardinality \( \mathfrak{d} \) such that all finite powers of \( X \) have Menger’s property \( S_{\text{fin}}(\mathcal{O}, \mathcal{O}) \)?

Issue 15. Can a Borel non-\( \sigma \)-compact group be generated by a Hurewicz subspace?

Issue 16 (MA). Is there an uncountable \( X \subseteq \mathbb{R} \) satisfying \( S_1(\mathcal{B}_\Omega, \mathcal{B}_\Gamma) \)?

Issue 17 (CH). Is there a totally imperfect \( X \) satisfying \( U_{\text{fin}}(\mathcal{O}, \Gamma) \) that can be mapped continuously onto \( \{0, 1\}^{\omega_1} \)?

Issue 18 (CH). Is there a Hurewicz \( X \) such that \( X^2 \) is Menger but not Hurewicz?

Issue 19. Does the Pytkeev property of \( C_p(X) \) imply that \( X \) has Menger’s property?

Issue 20. Does every hereditarily Hurewicz space satisfy \( S_1(\mathcal{B}_\Gamma, \mathcal{B}_\Gamma) \)?

Issue 21 (CH). Is there a Rothberger-bounded \( G \subseteq \mathbb{Z}^{\omega_1} \) such that \( G^2 \) is not Menger-bounded?

Issue 22. Let \( \mathcal{W} \) be the van der Waerden ideal. Are \( \mathcal{W} \)-ultrafilters closed under products?

Issue 23. Is the \( \delta \)-property equivalent to the \( \gamma \)-property \( \binom{\Omega}{\Gamma} \)?