Chiral tunneling in single layer graphene with Rashba spin-orbit coupling: spin currents

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We study 1D barrier penetration of 2D massless Dirac electrons in single layer graphene in the presence of uniform Rashba spin-orbit coupling, thereby exploring the role Klein paradox in graphene spintronics. Spin density and spin-current density are calculated (in addition to transmission), and shown to be remarkably different from those predicted in bulk single layer graphene. In particular, they are rather sensitive to the strength of the spin-orbit coupling and the height of the potential barrier, and have a non-trivial space dependence (associated with spin torque). Such a system may serve as a graphene based spintronic device without the use of an external magnetic field or magnetic materials.

Introduction: Shortly after the discovery of graphene\cite{1}, numerous novel physical phenomena were exposed in its electronic properties\cite{2,3}. One of these, the occurrence of chiral tunneling and the Klein paradox\cite{4} in single layer graphene (SLG), was reported in a seminal paper\cite{5}. This work was further expanded in Refs.\cite{6,7}. It was shown that, due to chirality near a Dirac point, electrons execute unimpeded transmission for energies below the potential barrier. This phenomenon is related to the absence of back-scattering for electron-impurity scattering in carbon nanotubes\cite{8}. Several additional extensions have been reported in Refs.\cite{9-11}. In parallel, investigation of the role of electron spin in graphene led to the emergence of a new field: graphene spintronics\cite{12-55}. Here we focus on chiral tunneling in the presence of uniform Rashba spin-orbit coupling (RSOC). Thereby, the roles of Klein paradox\cite{5} and RSOC in SLG are combined. The main motivation is to reveal novel aspects in graphene spintronics. In earlier relevant work\cite{33,39}, the focus was mainly on the effect of RSOC on the spin resolved transmission coefficients. Here our objective is to analyze spin related observables, especially spin density, spin current density and spin torque.

RSOC is time reversal invariant, and can be controlled by an externally applied uniform electric field \( \mathbf{E} = E_0 \hat{z} \) perpendicular to the SLG lying in the \( x-y \) plane, as in the Rashba model for the two-dimensional electron gas\cite{56}. In our analysis, the electric field acts over the whole SLG plane (application of the electric field only within the barrier region is experimentally more difficult control\cite{39}). We hope this study will motivate the fabrication of graphene based spintronic devices that do not rely on the use of an external magnetic field or magnetic materials.

T achieve our goal, the problem of 1D barrier penetration of 2D massless Dirac electrons in SLG in the presence of uniform RSOC is formulated and solved. Transmission, spin density, spin current density (related to spin torques\cite{57}) are calculated and analyzed (analytical expressions for some of these quantities are given). The calculated spin observables have properties that are remarkably different from those predicted in bulk SLG\cite{47} (i.e., in the absence of a 1D potential so that the Klein paradox does not play a role). In particular, some symmetry relations are broken, and the spin current is space dependent so that there is a finite spin torque. But most importantly, the response of the spin densities to the RSOC strength is substantial even for small RSOC coupling (the strength of Rashba splitting caused by a strong electric field in SLG reported in Ref.\cite{44} is a fraction of meV). Therefore, we expect the present system to be a good candidate for advancing graphene spintronics.

Formalism: We formulate the problem of massless 2D Dirac electrons in SLG lying in the \( x-y \) plane that are scattered from a 1D rectangular potential barrier of \( u(x) = u_0 \Theta(x)(d - x) \) and subject to a uniform electric field \( \mathbf{E} = E_0 \hat{z} \). The (Fermi) scattering energy \( \epsilon \) and the height potential \( u_0 \) satisfy the inequality \( u_0 \ll \epsilon \ll 0 \) (the condition for the emergence of the Klein paradox).

Our treatment is carried out within the continuum formulation near one of the Dirac points, say \( \mathbf{K}' \). Since the transverse wave number \( k_0 \) is conserved, the wave function can be factorized: \( \Psi(x,y) = e^{i \mathbf{k}_0 \cdot \mathbf{x}} \psi(x) \). Recall that, in addition to the isospin \( \sigma \) encoding the two-lattice structure of graphene, there is now a real spin, \( \tau \). Hence, the wave function \( \psi(x) \) is a four component vector in \( \sigma \otimes \tau \) (spin\otimes isospin) space. \( \psi(x) \) has dimensions of \( 1/\sqrt{\mathcal{A}} \) where \( \mathcal{A} \) is some relevant area. Hereafter we take \( \mathcal{A} = 1 \text{ (nm)}^2 \), and omit this factor when no confusion arises. The Hamiltonian for the scattering problem is given by

\begin{equation}
\begin{aligned}
h(-i\partial_x, k_0, \lambda) &= \gamma_0 [(-i\partial_x + \lambda (\hat{z} \times \sigma_y)) \tau_x + (k_0 + \lambda (\hat{z} \times \sigma_y)) \lambda y \tau_y] + u(x)
\end{aligned}
\end{equation}

which is a 4x4 matrix first-order differential operator. Here, \( \gamma_0 = 557.107 \text{ meV-nm} \) is the kinetic energy parameter, and \( \lambda \) is the RSOC strength parameter\cite{58} (it is also the inverse spin-orbit length parameter \( \lambda = 1/(\hbar \gamma \times E_0) \)). The products, \( \sigma \tau_x, \sigma \tau_y \), implicitly incorporate a Kronnercker product. Therefore, \( \psi(x) \) is a four component plane-wave spinor, that can be written as,

\begin{equation}
\psi(x) = \begin{cases}
\begin{aligned}
e^{i \epsilon x / 2} \psi(x) \in \mathbb{C}^2, & x \notin [0,d],
\end{aligned}

\end{cases}
\end{equation}

The 4 component vectors \( u(\pm k_0) \) and \( w(\pm q_0) \) in Eqs. (2) above and (3) below are normalized to unity. They satisfy the equations,

\begin{equation}
\begin{aligned}
h_0(-i\partial_x, k_0, \lambda)u(\pm k_0) &= \epsilon u(\pm k_0),
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
h_0(\pm q_0, k_0, \lambda)w(\pm k_0) &= \epsilon w(\pm k_0).
\end{aligned}
\end{equation}

Because RSOC acts in all of space (not only in the barrier), the vectors \( u(\pm k_0) \) cannot be chosen as spin eigenfunctions because \( \psi \) is not conserved. Moreover, Eqs. (3) are not eigenvalue equations. Indeed, assuming fixed transverse wave number \( k_0 \) and potential parameters \( u_0, d \) and RSOC strength \( \lambda \), the wave numbers \( k_0 \) and \( q_0 \) must depend on the (already fixed) scattering energy \( \epsilon \). Equations (3)
are implicit equations for $k_0(\varepsilon), q_0(\varepsilon)$ as well as for $u(\pm k_0(\varepsilon))$ and $w(\pm q_0(\varepsilon))$. For $\varepsilon > 0$, there are two wave numbers in each region that solve these implicit equations, $\pm k_0(\varepsilon)$ for $\varepsilon \not\in [0, d]$, and $\pm q_0(\varepsilon)$ for $x \in [0, d]$ ($n=1,2$). Solution of Eqs. (4) yields

$$k^2_{\pm \varepsilon} = [\varepsilon + (1 - \varepsilon^2) + 2\lambda^2 - k^2]^{1/2},$$

$$q^2_{\pm \varepsilon} = [\varepsilon + (1 - \varepsilon^2) + 2\lambda^2 - k^2_{\pm \varepsilon}]^{1/2},$$

(4) together with the vectors $u_a(\pm k_0(\varepsilon))$ and $w_a(\pm q_0(\varepsilon))$. This leads to a two-channel scattering problem[39].

The wave function corresponding to an incoming wave in channel $n$ ($n=1,2$) in the three regions is,

$$\psi_n(x) = \begin{cases} \begin{pmatrix} k_{n1} & r_{n1} \end{pmatrix} e^{i k_{n2} x}, & x<0, \\ a_{n1} & a_{n2} \end{pmatrix} e^{i q_{n2} x}, & 0<x<d, \\ a_{n3} & a_{n4} \end{pmatrix} e^{i q_{n4} x}, & x>d, \end{cases}$$

where $T$ and $R$ are the transmission and reflection coefficients and $j_L = I_L \otimes \tau$, is the current operator.

Choice of parameters: Our objective is to explore the response of the system to variation of the RSOC strength $\lambda$ (tunable by the electric field), the scattering energy $\varepsilon$ and the potential parameters $u_0, d$ (tunable by gate voltage). For simplicity, we consider forward scattering, $k_0 = 0$. The case $k_0 \neq 0$ will be explored in a future communication. Note that it is experimentally difficult to tune $k_0$ for fixed Fermi energy $\varepsilon$.

Below, the lengths $x, y, d, \ldots$ are given in nm, and energies $\varepsilon, u_0 \lambda$ as well as the wave numbers $k_0, k_1, q_0, q_1$ (introduced above) are given in (nm)$^{-1}$. [1 (nm)$^{-1}$ corresponds to 659.107 meV]. The size of $\lambda$ is dictated by experiments on Rashba spin-splitting in SLG. In Ref. [44], it is shown that $\lambda$ is in the order of fraction of 1 meV. Here we let $0 < \lambda < 0.001$ (0.659 nm)$^{-1}$, and vary barrier height in the range 0 < $u_0 < 200$ meV, and keep the barrier width fixed at $d = 200$ nm. Changing $d$ mainly affects the oscillation frequency of the wave function in the barrier region 0 < $x < d$, and has only small effects on the transmission coefficient and spin observables that are measured outside the barrier. The choice of other parameters is determined as follows: First, to fulfill the Klein paradox, it is required that $u_0 > \varepsilon > 0$ (the Fermi energy outside the barrier is in the conduction band and inside the barrier it is in the valence band). Second, we require that outside the barrier, both wave numbers $k_0$ be real [see Eq. (4)]. For $k_0 = 0$, this implies $\varepsilon > 2\lambda$. Yet, it is expected that the interesting physics occurs for $\varepsilon$ close to $\lambda$. The reason is that the RSOC partially lifts the spin degeneracy, and at the Dirac point the energy of the upper spin level is of order $\lambda$. The role of electron spin is relevant when the scattering energy is close to the two spin split levels at energies 0 and $\pm \lambda$ (see Fig. 8(b) in Ref. [33]).

Consider now the wave numbers ($q_0(\varepsilon)$ within the barrier, Eq. (4). In the absence of RSOC, there is only one wave number $q_0$ [5, 7], and the Klein paradox occurs when $q_0$ is real. In the presence of RSOC, it is sufficient that at least one of the wave numbers is real. For $u_0 > \varepsilon > 0$ and $k_0 = 0$ this implies the inequality $(u_0 - \varepsilon)^2 + 2\lambda(u_0 - \varepsilon) > 0$. These relations are highlighted in Fig. 1(a).

Transmission above and below the barrier: For $u_0 = 0$ the channels are not coupled, and the amplitudes $t_{n1}, r_{n1}$ are obtained analytically. The transmission coefficient is,

$$T(u_0, \varepsilon; \lambda, k_0 = 0) =$$

$$\begin{aligned}
&\sum_{n=1}^{2} \left| t_{n1} \right|^2 \\
&+ \left| r_{n1} \right|^2.
\end{aligned}$$

Note that $\lim_{u_0 \to 0} T(u_0, \varepsilon; \lambda, k_0 = 0) = 2$ (the transmission is transparent). It is interesting to check what happens if one of the squared wave numbers, say $q_{n2}$, is negative under the barrier. Strictly speaking, the second term $n = 2$ in Eq. (7) remains real and non-negative. Indeed, the denominator is real and negative because $\cos(q_{n2}d) = \cos(|q_{n2}|d) > 1$. Practically, however, for $|q_{n2}|d \gg 1$ the denominator is so large that the contribution of this term to the transmission is negligible, and channel 2 is virtually closed. However, the Klein paradox is still manifest even if only one channel is open. That happens if $u_0 < \varepsilon < \varepsilon + 2\lambda$, (the transmission coefficient is then bounded by 1). A single channel scattering occurs either above the barrier (no Klein tunneling) $0 < u_0 < \varepsilon$ [blue curve in Fig. 1(a)] or below the barrier $\varepsilon < u_0 < \varepsilon + 2\lambda$ ("partial" Klein tunneling, red curve in Fig. 1(a)]. The separate contributions of each channel to the transmission are shown in Fig. 1(b). On the other hand, if both channels are open (for $\varepsilon + 2\lambda < u_0$, $q_{n2} > 0$), the transmission jumps above 1, and for large $u_0$, it oscillates slightly below its maximal value $T = 2$. This scenario is shown in Fig. 1(c). Recall that for $\lambda = 0$ the transmission is unimpaired, $T(u_0, k_0 = 0) = 2$ (identically). Therefore, a pertinent experiment for $\lambda \neq 0$ should be an excellent probe of the strength of RSOC in SLG.

Dependence of the Transmission and current on $\lambda$ and $u_0$: We now inspect the transmission coefficient $T$ and the charged current $j(x)$ as a function of potential barrier height parameter $u_0$ and the RSOC strength parameter $\lambda$. Figure 2(a) shows the transmission and current versus $\lambda$ for fixed $u_0$ and Fig. 2(b) shows the transmission and current versus $u_0$ for fixed $\lambda$. The main conclusion from these figures is that in the presence of RSOC, the transmission coefficient is no longer unimpaired. Rather, for fixed $u_0$ and for experimentally relevant interval $0 < \lambda < 0.001$ nm$^{-1}$ (corresponding Rashba spin splitting 0.65 meV) it smoothly decreases as in Fig. 1(a). Moreover, for fixed $\lambda$, considered as function of the barrier height $u_0$ it exposes a beautiful pattern of oscillations below the unitarity upper limit $T = 2$ as in Fig. 2(b). It is of course not surprising that the charge current and the transmission coefficient are highly correlated. Note that the charge current is space-independent, see Eq. (6).

Spin density operators and observables: Spin density and spin current density operators $O^\tau$ are representable as 4 x 4 matrices in $\sigma \otimes \tau$ space. Spin observables are obtained as $O(x) = \psi^\dagger(x)O\psi(x)$ (this is not an expectation value, spin-observables may depend on $x$). The spin density operators $S$ and
FIG. 1. Relations between wave numbers $q_n$, and the transmission $T(u_0; k_0 = 0)$, Eq. (7), as function of $u_0$ for $\varepsilon = 0.002$ (nm)$^{-1}$, $\lambda = 0.001$ (nm)$^{-1}$, $d = 5000$ nm (intentionally exaggerated) through a p-n-p junction. (a) Squared wave-numbers, $q_n^2$ ($n = 1, 2$ for blue, red) defined in Eq. (4). The blue-green-red points mark $u_0 = \varepsilon - 2\lambda < \varepsilon$, $u_0 = \varepsilon$, $u_0 = \varepsilon + 2\lambda > \varepsilon$ respectively. Negative $q_n^2$ implies that channel $n$ is closed (it does not contribute to the transmission). (b) Separate contributions of the two terms in Eq. (7) to the total transmission. (c) The total transmission is the sum of the two contributions shown in (b). Note the symmetry about $u_0 = \varepsilon = 0.004$. In this grazing energy scattering $\varepsilon = u_0$, the transmission almost vanishes. Recall that in the corresponding 1D Schrödinger problem for $\varepsilon = u_0$, the transmission coefficient is $T = \frac{\lambda}{k + (\lambda^2)}$

where $k = \sqrt{\frac{\lambda}{\varepsilon}}$

the spin densities $S(x)$ are given by,

$S = (S_x, S_y, S_z) = \frac{1}{2}\sigma \otimes I_2$,

$S(x) = \sum_{n=1}^{2} \psi_n(x)^2 S_0 \psi_n(x)$,  \hspace{1cm} (8)

where $\psi_n(x)$ is defined in Eq. (5). The unit of spin density used here is $S_0 = \hbar A$.

Of the three spin density observables, two of them vanish, $S_x = S_z = 0$, i.e., there is no polarization along the direction of motion or in a direction perpendicular to the SLG plane. This is consistent with the results of Ref. [47] wherein the spin density distribution is calculated in bulk SLG [for $k_0 = 0$ and $\sin \theta = 0$, see Eq. (5) therein]. Figure 3(a) shows the spin density $S_y(x = 0)$ just outside the left wall of the barrier of the p-n-junction as a function of $\lambda$ [note however, that the spin densities are space-independent, $S_y(x) = S_y(0)$]. The size of the polarization substantially increases for $\lambda > 0.0003$ (nm)$^{-1} \approx 0.20$ meV. Figure 3(b) plots $S_y$ versus $u_0$, and shows a rich oscillatory pattern that decreases near the lower limit $u_0 = \varepsilon + 2\lambda$, below which one channel is closed (see discussion of Fig. 1). Both parts of Fig. 3 substantiate the role of the RSOC strength and the barrier height as useful parameters to control the degree of polarization.

Spin current density operators and observables: The spin-current density operator $J$ (a tensor) and the observed components of the spin current density $J_{ij}(x)$ are defined as

$J = \frac{1}{2} (S \cdot V)$, \hspace{1cm} $J_{ij} = \hat{S}_i V_j + V_i \hat{S}_j$,

$J_{ij}(x) = \sum_{n=1}^{2} \psi_n(x)^2 \hat{S}_i \psi_n(x)$,  \hspace{1cm} (9)

where $S$ is the spin density operator defined in Eq. (8) and $V = \hat{I}_2 \otimes \tau$ is the velocity operator [equal the charge current operator defined in Eq. (6)]. In Eq. (9), $i, j = 1, 2, 3 \equiv x, y, z$ specifies the polarization direction, and $j = 1, 2 \equiv x, y$ specifies the axis along which electrons propagate. The unit of spin current density is $J_0 = S_0 \hbar \gamma / A = 659.107$ meV/um.

The spin current density was calculated in bulk SLG in Ref. [47]; it was found that (1) $J_{xx} = J_{yy} = J_{zz} = J_J = 0$, (2) $J_{xy} = -J_{yx}$, and (3) The spin currents are independent on space (see Eq. (5) in Ref. [47]). Below, we show that: (1) In the presence of a 1D potential (where there is no rotational symmetry around the $z$-axis), the symmetry relation $J_{xy} = -J_{yx}$ that is valid in bulk SLG [47] is broken. (2) Although the value of $\lambda$ used in our calculations is much smaller than that used in Ref. [47], the size of spin current densities are both systems is the same. (3) The divergence of the spin current does not vanish, and hence the continuity

FIG. 2. Transmission $T$ (blue) and charge current $j_j$ (red) for the p-n-p junction. The RSOC acts uniformly over the SLG plane (not only within the barrier region). The current was calculated on the right wall of the barrier but its divergence vanishes, as required by Eq. (6), hence it is uniform (space-independent). (a) Transmission and current versus $\lambda$ for $u_0 = 0.3$ (nm)$^{-1}$, $d = 200$ nm, $\varepsilon = 0.004$ (nm)$^{-1}$ and $k_0 = 0$. Thus, whereas for $\lambda = 0$, $T(k_0) = 2$; for $\lambda > 0$, $T(k_0) < 2$ (b) Transmission and current versus $u_0$ for $\lambda = 0.001$ (nm)$^{-1}$ [other parameters are as in (a)].
implies \( S \) (wherein there is no Klein paradox). In Eq. (7) compare them with those obtained in bulk SLG.

The nearly linear increase of \( J_{xy}(0) \) and \( J_{xy}(d) \) with \( \lambda \) is an encouraging feature in the quest for exposing novel and practical aspects graphene spintronics.

To stress the role of the Klein paradox in the present system we re-examine our results and compare them with those obtained in bulk SLG (wherein there is no Klein paradox). In Eq. (7) of Ref. [47], the authors found that in bulk SLG, \( J_{yx} = J_0 \frac{\sin(2\eta)}{2\sqrt{\sin^2\eta}} \) where \( \eta = \sqrt{\frac{\lambda^2}{\alpha^2} \hbar^2} \) is of order unity and \( \phi = \arctan \frac{\alpha}{\lambda} \). Thus, for \( k_y = 0 \), this implies \( J_{yx} \approx J_0 \). As shown in Figs. 4, 5 and 6, in the presence of 1D potential barrier, the spin current density can reach similar values. However, in Ref. [47] the value of \( \lambda \) is about 200 times higher than the one we have used. As shown in Ref. [44], such high values of \( \lambda \) are not achievable on SLG. It is possible to have higher values of RSOC if the SLG is in contact with metals such as Au or Pb. But upon passing a current through such a slab, the electrons will flow through the metal and not through the SLG. Thus, at the Fermi level, the electronic states are metallic.

The spin current densities versus the potential height \( u_0 \) is shown in Fig. 5(b). Recalling the behavior of \( S_x \) as a function of \( u_0 \) shown in Fig. 3(b), and noting the behavior of the spin current densities versus \( u_0 \) in Fig. 5(a), it is clear that spin density and the spin current density are significantly correlated.

Finally, it is interesting to consider the space dependence of the spin current density (recall that the spin density \( \mathbf{S}(x) \) is space-independent. The spin current densities is plotted in the barrier region \( 0 < x < d \), wherein the wave function is a linear combination of four different plane wave spinors [see Eq. (5)]. The fact that \( u_0 \gg \varepsilon, \lambda \) and inspection of Eq. (4) implies the inequality \( |q_{xu}| \gg |k_{xu}| \). As shown in Fig. 6, this leads to an interference pattern of spin density currents that is quite rich.

Because the spin current density is space dependent (its divergence does not vanish), the corresponding continuity equation requires the inclusion of spin torque density. In the general case, one considers a 3D material wherein the observable spin current density tensor \( J_{iy}(r) \) depends on the position vector \( r \). One then defines the three vector fields \( \mathbf{J}_i(r) = \langle J_{ix}(r), J_{iy}(r), J_{iz}(r) \rangle \) for a given polarization direction \( i = x, y, z \). Since spin is not conserved in systems with spin-orbit coupling, the spin current density vector field \( \mathbf{J}_i(r) \) has non-zero divergence, and its continuity equation reads:

\[
\frac{\partial}{\partial t} \mathbf{S} + \nabla \cdot \mathbf{J}_i(r) = \mathbf{T}_i(r) \equiv \text{Re}[\psi^\dagger(r)\hat{T}_i\psi(r)]. \tag{10}
\]

Here the scalar \( T_i(r) \) is the spin torque density and \( \hat{T}_i = \frac{\partial}{\partial r}\left[ S_i, H \right] \) is the spin torque density operator, where \( \hat{H} = (p + \sigma \cdot \mathbf{r}) \cdot \tau \) is the Hamiltonian operator and \( \mathbf{n} \) is the unit vector in the direction of the electric field. The volume integral of \( T_i(r) \) sometimes vanishes due to symmetry relations [57], and then there exists a vector field \( \mathbf{P}_i(r) \) such that \( T_i(r) = -\nabla \cdot \mathbf{P}_i(r) \). The continuity equation then becomes \( \frac{\partial}{\partial r} \mathbf{S} + \nabla \cdot (\mathbf{J}_i(r) + \mathbf{P}_i(r)) = 0 \). In the static case, this yields

\[

T_i(r) = \nabla \cdot \mathbf{J}_i(r). \tag{11}
\]
FIG. 5. (a) Spin current densities $J_{xy}(0)$, $J_{yx}(0)$ on the left wall of the barrier as function of $u_0$, for $d = 200$ nm, $\varepsilon = 0.004$ (nm)$^{-1}$, $\lambda = 0.001$ (nm)$^{-1}$ and $k_y = 0$. (b) Spin current densities $J_{xy}(d)$, $J_{yx}(d)$ on the right wall of the barrier as function of $u_0$. The other parameters are as in (a).

FIG. 6. Space dependence of Spin current density (in units of $J_0$) in the barrier region, $0 < x < d=200$ nm, for $\lambda = 0.001$ (nm)$^{-1}$, $\varepsilon = 0.004$ (nm)$^{-1}$, and $u_0 = 0.2853$ (nm)$^{-1}$. (a) $J_{xy}(x)$ and (b) $J_{yx}(x)$.

In our case $r \rightarrow x$ and the two vector fields are $J_1(x) = (0, J_{xy}(x))$ and $J_2(x) = (J_{yx}(x), 0)$. Therefore, the only non-zero spin torque density is $T_2(x) = dJ_{yx}(x)/dx$.

Summary and Conclusion: The Klein paradox in SLG occurs when an electron at Fermi energy $\varepsilon$ tunnels through a 1D potential barrier of height $u_0$ (which can be experimentally controlled by a gate voltage) in the region $u_0 > \varepsilon > 0$. When, in addition, a uniform perpendicular electric field $E = E_0 \hat{z}$ is applied, the role of electron spin enters due to the RSOC. In this work we elucidated the physics when the Klein paradox and RSOC are combined, in order to expose interesting facets of graphene spintronics within a time-reversal invariant formalism. The fact that $u_0$ and $\lambda$ are experimentally controlled makes our analysis readily verifiable.

This combination of Klein paradox and RSOC shows up in the behaviour of transmission [33, 39], and, as demonstrated here, in the pattern of spin observables. As shown in Figs. 1(c), 2(a) and 2(b), the transmission is strongly affected by the presence of RSOC (recall that in the absence of RSOC, the transmission in the forward direction $k_y = 0$ is constant and at its maximum possible $T = 2$). Spin observables are calculated and their behaviour as function of $\lambda$ and $u_0$ and position are traced. The only non-zero spin density is $S_y$ shown in Figs. 3(a) and 3(b) ($S_y$ is independent of $x$). Spin current density is plotted as function of $\lambda$ in Fig. 4, as function of $u_0$ in Fig. 5, and as function for $x$ in Fig. 6. Compared with those found in bulk SLG, they have a much richer pattern and non-trivial space dependence (associated with non-zero spin torque density).

This work is partially motivated by the quest for constructing spintronic devices without the use of an external magnetic field or magnetic materials (in addition to the many references mentioned above, see also Refs. [62-64]). We hope that our results advance this goal. The nearly linear dependence of the spin current density $J_{xy}(d)$ on $\lambda$ is particularly useful in this regard.

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