We discuss the flavor structure of soft supersymmetry breaking parameters in 5-dimensional orbifold field theories in which $N=1$ supersymmetry is broken by the Scherk-Schwarz boundary condition and hierarchical 4-dimensional Yukawa couplings are obtained by quasi-localizing the matter fields in extra dimension. In such theories, the resulting soft scalar masses and trilinear scalar couplings at the compactification scale are highly flavor-dependent, but appropriately suppressed in correlation with Yukawa couplings. Those flavor violations can give interesting phenomenological consequences at low energies as well as constrain the mechanism of Yukawa coupling generation.

## I. INTRODUCTION

Supersymmetry (SUSY) is one of the prime candidates for new physics beyond the standard model [1]. An important issue in supersymmetric theories is to understand how SUSY is broken in low energy world. It has been known that theories with compact extra dimension provide an attractive way to break SUSY, imposing non-trivial boundary conditions on the field variables. This mechanism which has been proposed originally by Scherk and Schwarz (SS) [2] can be interpreted as a spontaneous SUSY breaking induced by the auxiliary component of higher dimensional supergravity (SUGRA) multiplet [3]. Extra dimension can provide also an attractive mechanism to generate hierarchical Yukawa couplings [4]. The quark and lepton fields can be quasi-localized in extra dimension in a natural manner, and then their 4-dimensional (4D) Yukawa couplings are determined by the wavefunction overlap factor $e^{-M\pi R}$ where $M$ is a combination of mass parameters in higher dimensional theory and $R$ is the length of extra dimension. This allows that hierarchical Yukawa couplings are obtained from fundamental mass parameters having the same order of magnitude. In this talk, we discuss the flavor structure of soft SUSY breaking parameters induced by the SS boundary condition imposed on the matter fields which are quasi-localized to generate hierarchical Yukawa couplings [2, 3].

## II. YUKA W A COUPLINGS AND SOFT PARAMETERS OF QUASI-LOCALIZED MATTER FIELDS

To proceed, let us consider a generic 5D gauge theory coupled to the minimal 5D SUGRA on $S^1/Z_2$. The action of the model is given by [5]

$$
\int d^5x \sqrt{-G} \left[ \left( \frac{1}{2} R + \frac{1}{2} \bar{\Psi}^I_M A^{MNP} D_N \Psi_{iP} \right) - \frac{3}{4} C_{MN} C^{MN} \right] + \frac{1}{4} g_5^2 \left( F^{MN} F_{MN} \right)
+ \frac{1}{2} D_M \phi^a D^M \phi^a + \frac{i}{2} \bar{\lambda}^{\alpha} \gamma^M D_M \lambda^\alpha
+ |D_M h^i_I|^2 + i \bar{\Psi}^I_M \gamma^M D_M \Psi_{iP} + i \tilde{m}_I \epsilon(y) \bar{\Psi}^I_M \Psi_{iP} \right] \tag{1}
$$

where $R$ is the Ricci scalar of the 5D metric $G_{MN}$, $\Psi^I_M$ ($i=1,2$) are the symplectic Majorana gravitinos, $C_{MN} = \partial_M B_N - \partial_N B_M$ is the graviphoton field strength, and $y$ is the 5th coordinate with a fundamental range $0 \leq y \leq \pi$. Here $\phi^a$, $A^a_M$ and $\lambda^a$ are 5D scalar, vector and symplectic Majorana spinors constituting a 5D vector multiplet, $h^i_I$ and $\Psi_I$ are 5D scalar and Dirac spinor constituting the $I$-th hypermultiplet with kink mass $\tilde{m}_I \epsilon(y)$. The kink masses are related to the gauging of graviphotons as indicated by the following covariant derivatives of hypermultiplets:

$$
D_M h^i_I = (\partial_M + i \tilde{m}_I \epsilon(y) B_M) h^i_I + ..., \\
D_M \Psi_I = (\partial_M + i \tilde{m}_I \epsilon(y) B_M) \Psi_I + ..., 
$$

where the ellipses stand for other gauge interactions. Although not required within 5D SUGRA, it is not unreasonable to assume that the kink masses are quantized in an appropriate unit, which will be adopted here.

It is convenient to write the 5D action in $N=1$ superspace [6]. For the 5D SUGRA multiplet, we keep only the radion superfield

$$
T = R + i B_5 + \theta \Psi_{5R} + \theta^2 F^T,
$$

where $R = \sqrt{G_{55}}$ denotes the radius of the compactified 5-th dimension, and $\Psi_{5R} = \frac{1}{2} (1 + \gamma_5) \Psi_{Z=5}$. For 5D vector multiplets and hypermultiplets, the relevant piece of the action is given by [7]

$$
\int d^5x \left[ \int d^4 \theta \frac{T + T^*}{2} \left( e^{-\tilde{m}_I (T + T^*)} |y| H^*_I H^I \right) \right]
$$

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Flavor Structure of Scherk-Schwarz Supersymmetry Breaking for Quasi-Localized Matter Fields

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We discuss the flavor structure of soft supersymmetry breaking parameters in 5-dimensional orbifold field theories in which $N=1$ supersymmetry is broken by the Scherk-Schwarz boundary condition and hierarchical 4-dimensional Yukawa couplings are obtained by quasi-localizing the matter fields in extra dimension. In such theories, the resulting soft scalar masses and trilinear scalar couplings at the compactification scale are highly flavor-dependent, but appropriately suppressed in correlation with Yukawa couplings. Those flavor violations can give interesting phenomenological consequences at low energies as well as constrain the mechanism of Yukawa coupling generation.
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and

H_I = e^{m_I T \mid y}(h^1_I + \theta \psi_I + \bar{\theta} F_I),

H^\prime_I = e^{-m_I T \mid y}(h^1_I + \theta \psi_I + \bar{\theta} F_I),

where \( \lambda^a = \frac{1}{2}(1 - \gamma_5)\lambda^a \), \( \psi_I = \frac{1}{2}(1 - \gamma_5)\Psi_I \), and \( \bar{\psi} = \frac{1}{2}(1 + \gamma_5)\psi_I \).

As the theory is orbifolded by \( Z_2 : y \to -y \), all 5D fields should have a definite boundary condition under \( Z_2 \). To give a massless 4D zero mode, the vector superfield \( V^a \) is required to be \( Z_2 \)-even, while the hypermultiplet can have any \( Z_2 \)-boundary condition:

\[
Y^a(-y) = Y^a(y),
\]

\[
H_I(-y) = z_I H_I(y),
\]

\[
H^\prime_I(-y) = -z_I H^\prime_I(y),
\]

where \( z_I = \pm 1 \). In the superfield basis of \((H_I, H^\prime_I)\), the wavefunctions of 4D zero modes are \( y \)-independent constant.

However in the original 5D field basis for which the 5D action is given by

\[
\int d^5\theta \left\{ \frac{1}{4} \theta^2 \lambda^a \right\} + \lambda^a, \lambda_{IJK}(\omega^a) \text{ and } \lambda_{IJK}(Z^a) \text{ are generic holomorphic functions of } Z \text{ (at } y = \pi \text{)}, \]

With compact extra dimension, SUSY can be broken by imposing nontrivial boundary conditions on the field variables. Such SUSY breaking has been proposed originally by Scherk and Schwarz [2], and can be interpreted as a spontaneous SUSY breaking induced by the auxiliary component of higher dimensional SUGRA multiplet [3].

In 5D orbifold field theory, the SS boundary condition is given by

\[
\lambda^a(y + 2\pi) = (\epsilon^{2\pi i\omega \cdot \sigma \cdot j} \lambda^a(y),
\]

\[
h^a_j(y + 2\pi) = (\epsilon^{2\pi i\omega \cdot \sigma \cdot j} h^a_j(y),
\]

where \( \lambda^a \) and \( h^a_j \) are the \( SU(2)_R \) doublet gauginos and hypermultiplet scalars, respectively, and \( \omega = (\omega_1, \omega_2, 0) \). It has been noticed that imposing the SS boundary condition is equivalent to turning on the F-component of the radion superfield as

\[
F^T = 2R(\omega_2 - \omega_1).
\]

A small but nonzero value of \( F^T \) can be achieved dynamically when a proper radion stabilization mechanism is introduced [4].

With the identification [3], SUSY breaking soft parameters induced by the SS boundary condition can be most easily computed by constructing the effective action of 4D matter superfields and the radion superfield in \( N = 1 \) superspace. Let \( V^a \) denote the massless 4D vector superfield originating from \( V^a \), and \( Q_I \) to be the massless 4D chiral superfield originating from \( H_I \) (\( z_I = 1 \)) or \( H^\prime_I \) (\( z_I = -1 \)). Their 4D effective action can be written as

\[
\int d^4\theta Y_{IJ} Q_I Q_J + \int d^2\theta \left( \frac{1}{4} f_a W^a W^a + \tilde{Y}_{IJK} Q_I Q_J Q_K \right),
\]

where \( Y_{IJ} \) are hermitian wave function coefficients, \( f_a \) are holomorphic gauge kinetic functions, and \( \tilde{Y}_{IJK} \) are holomorphic Yukawa couplings. Using [2] and [4], it is straightforward to find

\[
Y_{IJ} = Y_I \delta_{IJ} + L_{IJ} (Z, Z^a),

f_a = \frac{2\pi}{g_0^a} T + \omega_a(Z) + \omega_a(Z'),

\tilde{y}_{IJK} = \lambda_{IJK}(Z) + \frac{\lambda_{IJK}(Z^a)}{e^{(m_I + m_J + m_K)\pi T}},
\]

where

\[
Y_I = \frac{\Lambda}{m_I} \left( 1 - e^{-m_I \pi(T + T^*)} \right).
\]
for the cutoff scale \( \Lambda \) of 5D orbifold field theory. Note that 5D SUSY enforces that the Yukawa couplings of \( Q_I \) originate entirely from the brane action (4).

A naive dimensional analysis in the large radius limit \( R \gg 1/\Lambda \) suggests that

\[
\frac{g_{5a}^2}{\Lambda} = \mathcal{O}(\pi R),
\]

\[
L_{1I}, L_{1J}^I = \mathcal{O}(1),
\]

\[
\omega_a, \omega_a' = \mathcal{O}(1/\pi R \Lambda)
\]

\[
\lambda_{IJK}, \lambda_{IJK}' = \mathcal{O}(\sqrt{\pi R \Lambda}).
\]

This shows that the brane gauge kinetic functions \( \omega_a, \omega_a' \) are suppressed by \( 1/\pi R \Lambda \) compared to the bulk gauge kinetic functions. If \( |m_I| < 1/R \), the matter zero mode \( Q_I \) is equally spread over the 5-th dimension, the brane wavefunction coefficients \( L_{1I}, L_{1J}^I \) are similarly suppressed by \( 1/\pi R \Lambda \) compared to the bulk wavefunction coefficients \( Y_I \). On the other hand, if \( |m_I| > 1/R \), so \( Q_I \) is quasi-localized, the brane wavefunction coefficients appear to be less suppressed since \( L_{1I}/Y_I, L_{1J}/Y_I = \mathcal{O}(m_I/\Lambda) \). In the following, we will ignore all the brane wavefunction coefficients and brane gauge kinetic functions under the assumption that \( |m_I|/\Lambda \) are small enough. In fact, a phenomenologically favored parameter region is given by \( \frac{\pi R}{\Lambda} \approx 10^{-2} - 10^{-3} \),

\[
\frac{m_I}{\Lambda} = \mathcal{O}(10^{-1} - 10^{-2}),
\]

which would justify our assumption.

Let \( y_{IJK} \), \( M_a \), \( m_{aI}^2 \), and \( A_{IJK} \) denote the Yukawa couplings, gaugino masses, soft scalar masses, trilinear scalar couplings, respectively, for the canonically normalized matter superfields. \( Q_I = \phi^I + \theta \phi^I + \theta^2 \phi^I \) and gauginos \( \lambda^a \) which are renormalized at the compactification scale \( M_{KK} \):

\[
\frac{1}{2} y_{IJK} \phi_I \psi_J \psi_K - \frac{1}{2} M_a \lambda^a \lambda^a - \frac{1}{2} m_{aI}^2 \phi^I \phi^I - \frac{1}{6} A_{IJK} \phi^I \phi^J \phi^K + \text{h.c.}
\]

We then find (in the unit with \( \Lambda = 1 \)) \( [5, 6] \)

\[
y_{IJK} = \frac{1}{\sqrt{Y_{IJK}}} \left( \lambda^i_{IJK} + \frac{\lambda^i_{IJK}}{e^{(m_I + m_J + m_K) \pi R}} \right),
\]

\[
m_{aI}^2 = \delta_{IJ} \left( \frac{2 \pi m_I R}{e^{m_I \pi R/2} - e^{-m_I \pi R/2}} \right) \left( \frac{2 \pi m_I R}{e^{m_I \pi R/2} - e^{-m_I \pi R/2}} \right) \frac{F^T}{2 R},
\]

\[
A_{IJK} \frac{y_{IJK}}{y_{IJK}} = - F^T \left( \frac{\partial}{\partial T} \ln \left( \frac{\lambda^i_{IJK} + \lambda^i_{IJK} e^{-(q_I + q_J + q_K) \pi R}}{Y_{IJK}} \right) \right),
\]

\[
M_a = \frac{F^T}{2 R}.
\]

As we have noticed, \( Q_I \) with \( m_I < 0 \) is quasi-localized at \( y = \pi \) with an exponentially small wavefunction \( (e^{m_I \pi R}) \) at \( y = 0 \), while \( Q_I \) with \( m_I > 0 \) is quasi-localized at \( y = 0 \). As a result, in the case that Yukawa couplings originate from \( y = 0 \), the quark/lepton superfields with \( m_I < 0 \) would have (exponentially) small canonical Yukawa couplings, while the quark/lepton superfields with \( m_I > 0 \) can have Yukawa couplings of order unity. Then one can obtain hierarchical Yukawa couplings with an appropriate set of (quantized) kink masses having the same order of magnitude. The results of [5, 6] show that the trilinear \( A \)-coefficients \( A_{IJK} \) at \( M_{KK} \) induced by the SS boundary condition are essentially of the order of \( y_{IJK} M_a \), however the ratios \( A_{IJK}/y_{IJK} \) are not universal. Such non-universal \( A_{IJK}/y_{IJK} \) can lead to interesting flavor violations of \( LR/RL \)-type at the weak scale. The squark/slepton masses at \( M_{KK} \) induced by the SS boundary condition are not universal also, but they are (exponentially) suppressed for quasi-localized quark/lepton superfields. Still those non-universal squark/slepton masses at \( M_{KK} \) can cause interesting flavor-violations of \( LL/RR \)-type at the weak scale.

### III. A SIMPLE MODEL

The results of [5, 6] show that the soft parameters at the compactification scale induced by the SS boundary condition for quasi-localized quark/lepton superfields are highly flavor dependent. The resulting flavor-violations are suppressed in parallel to the suppressed Yukawa couplings, however still they can give interesting flavor-violating processes at the weak scale. To be more concrete, let us consider the case that the Yukawa couplings come from the brane action at \( y = 0 \) and the Higgs superfields are brane fields confined at \( y = 0 \). In this case, the Yukawa couplings and soft parameters at \( M_{KK} \) can be written as [5, 6]

\[
y_{IJK} = \frac{\lambda_{IJK} \ln(1/e)}{\pi R} \sqrt{\frac{N_I N_J}{(1 - \epsilon^{2N})}} \
\]

\[
M_a = \frac{F^T}{2 R},
\]

\[
A_{IJK} = 2 y_{IJK} \ln(1/e) \left( \frac{N_I}{\epsilon^{2N_J} - 1} + \frac{N_J}{\epsilon^{2N_I} - 1} \right) \frac{F^T}{2 R},
\]

\[
m_{aI}^2 = \delta_{IJ} \left( \frac{2 \ln(1/e)}{\epsilon^{N_I} - \epsilon^{N_J}} \right) \left( \frac{2 \ln(1/e)}{\epsilon^{N_J} - \epsilon^{N_I}} \right) \frac{F^T}{2 R},
\]

\[
N_I = \frac{m_I \pi R}{\ln(1/e)}
\]
quark/lepton superfield. Here we will assume that $m_I$ are quantized in such a way that $N_I$ are integers.

Let $\psi_I = \{q_i, u_i, d_i, \ell_i, e_i\}$ ($i = 1, 2, 3$) denote the known three generations of the left-handed quark-singlets ($q_i$), up-type antiquark-singlets ($u_i$), down-type antiquark singlets ($d_i$), lepton-doublets ($\ell_i$), and anti-lepton singlets ($e_i$). The Yukawa couplings can be written as

$$
\mathcal{L}_{\text{Yukawa}} = y^u_{ij} H_2 q_i u_j + y^d_{ij} H_1 q_i d_j + y^\ell_{ij} H_1 \ell_i e_j
$$

and the squark/sleptons $\phi_j = \{\tilde{q}_i, \tilde{u}_i, \tilde{d}_i, \tilde{\ell}_i, \tilde{e}_i\}$ have the soft SUSY breaking couplings:

$$
\mathcal{L}_{\text{soft}} = - \left( A^u_{ij} H_2 \tilde{q}_i \tilde{u}_j + A^d_{ij} H_1 \tilde{q}_i \tilde{d}_j + A^\ell_{ij} H_1 \tilde{\ell}_i \tilde{e}_j + m_{2(j)}^2 \tilde{q}_i \tilde{q}_i^* + m_{2(j)}^{(\tilde{u})} \tilde{u}_i \tilde{u}_i^* + m_{2(j)}^{(\tilde{d})} \tilde{d}_i \tilde{d}_i^* + m_{2(j)}^{(\tilde{\ell})} \tilde{\ell}_i \tilde{\ell}_i^* + m_{2(j)}^{(\tilde{e})} \tilde{e}_i \tilde{e}_i^* \right).
$$

There can be several different choices of $N_I \equiv N(Q_I)$ which would yield the observed quark/lepton masses and mixing angles [11]. Here we will consider one example:

- $N(q_i) = (-3, -2, 1),$
- $N(u_i) = (-5, -2, 1),$
- $N(d_i) = (-3, -2, -2),$
- $N(\ell_i) = (-5, -2, -1),$
- $N(e_i) = (-2, -2, -1).$

These values of $N(Q_I)$ give the following forms of Yukawa coupling matrices

$$
y^{u}_{ij} = \left( e^8 \lambda^u_{11} \ e^5 \lambda^u_{12} \ e^3 \lambda^u_{13} \right),
$$

$$
y^{d}_{ij} = \left( e^8 \lambda^d_{11} \ e^5 \lambda^d_{12} \ e^3 \lambda^d_{13} \right),
$$

$$
y^{\ell}_{ij} = \left( e^8 \lambda^\ell_{11} \ e^5 \lambda^\ell_{12} \ e^3 \lambda^\ell_{13} \right),
$$

where $\lambda^{u,d,\ell}_{1J}$ are determined by the brane Yukawa couplings $\gamma:\mathcal{O}(\pi R A)$ and $\lambda:\mathcal{O}(\pi R A)$ as

$$
\lambda^{u,d,\ell}_{1J} = \mathcal{O}\left( \frac{\lambda_{1J}\gamma N_1 N_2 \ln(1/\epsilon)}{\pi R A} \right) = \mathcal{O}\left( \frac{m_1 \pi R}{\sqrt{\pi R A}} \right) = \mathcal{O}(1).
$$

The soft parameters renormalized at $M_{KK}$ are determined to be

- $A_{ij}^u = 2 M_{1/2} \ln 5 \left( \begin{array}{ccc} 8 & 5 & 3 \\ 7 & 4 & 2 \\ 5 & 2 & 2 e^2 \end{array} \right),$
- $A_{ij}^d = 2 M_{1/2} \ln 5 \left( \begin{array}{ccc} 6 & 5 & 5 \\ 5 & 4 & 4 \\ 3 & 3 & 2 \end{array} \right),$
- $A_{ij}^\ell = 2 M_{1/2} \ln 5 \left( \begin{array}{ccc} 7 & 7 & 6 \\ 4 & 4 & 3 \\ 3 & 3 & 2 \end{array} \right),$

$$
   \approx |M_{1/2}|^2 \left( \begin{array}{ccc} 6 \times 10^{-3} & 0 & 0 \\ 0 & 6 \times 10^{-2} & 0 \\ 0 & 0 & 0.4 \end{array} \right),
$$

$$
   \approx |M_{1/2}|^2 \left( \begin{array}{ccc} 3 \times 10^{-5} & 0 & 0 \\ 0 & 6 \times 10^{-2} & 0 \\ 0 & 0 & 0.4 \end{array} \right),
$$

$$
   \approx |M_{1/2}|^2 \left( \begin{array}{ccc} 6 \times 10^{-3} & 0 & 0 \\ 0 & 6 \times 10^{-2} & 0 \\ 0 & 0 & 0.4 \end{array} \right),
$$

where $M_{1/2} = F^T / 2 R$ denotes the universal gaugino mass at $M_{KK}$.

The Yukawa coupling matrices of [12] produce well the observed quark/lepton masses and also the quark mixing angles for a reasonable range of $\lambda^{u,d,\ell}_{1J}$. The soft parameters of [12] are highly flavor-dependent, but appropriately suppressed in correlation with Yukawa couplings. After taking into account the renormalization group evolution from $M_{KK}$ to the weak scale $M_W$, the flavor-violations from the squark and slepton masses at $M_{KK}$ can pass the known phenomenological constraints for a reasonable range of parameters [6]. (Here we assume $M_{KK}$ is close to the unification scale $2 \times 10^{16}$ GeV.) It is still true that the squark and slepton masses induced by the SS boundary condition at $M_{KK}$ can lead to interesting flavor-changing new physics signals at the weak scale, which may be able to be observed in future experiments. As for the squark A-parameters $A_{ij}^u$ and $A_{ij}^d$, one arrives at a similar conclusion.

However the slepton A-parameter $A_{ij}^\ell$ can yield a too rapid $\mu \to e\gamma$ unless $\lambda_{12,21}$ are appropriately tuned. More explicitly, to satisfy the experimental bound on $Br(\mu \to e\gamma)$, one needs

$$
\lambda_{12} \lesssim 5 \times 10^{-2} \left( \frac{M_{1/2}}{500 \text{ GeV}} \right)^2,
$$
\[ \lambda_{21}^\ell \lesssim 10^{-2} \left( \frac{M_{1/2}}{500 \text{ GeV}} \right)^2. \] 

(14)

This suggests that the holomorphic brane Yukawa couplings of leptons at \( y = 0 \), i.e. \( \lambda_{1JK}^\ell \) in (4), conserve the lepton flavor \( L_e \) or \( L_\mu \), for instance \( L_e - L_\tau \), which would give flavor-diagonal

\[ \lambda_{ij}^\ell = \lambda_i^\ell \delta_{ij}. \]

Still one can achieve large lepton flavor-mixing in the neutrino mass matrix by introducing gauge-singlet bulk neutrinos \( N_i \) which have flavor diagonal Yukawa couplings

\[ \delta(y) \kappa_i H_2 \ell_i N_i \]

at \( y = 0 \) and flavor non-diagonal Majorana mass masses

\[ \delta(y - \pi) M_{ij} N_i N_j \]

at \( y = \pi \). One can then adjust the kink masses of \( N_i \) to make the light neutrino mass matrix induced by the seesaw mechanism to take a form which can explain the observed large neutrino mixings [2].

IV. CONCLUSION

Imposing the Sherk-Schwarz boundary condition is an attractive way to break SUSY in theories with compact extra dimension. It can be interpreted as a spontaneous SUSY breaking by the auxiliary component of higher dimensional SUGRA multiplet. Another attractive possibility associated with extra dimension is the quasi-localization of matter fields which would generate hierarchical Yukawa couplings in a natural manner. In this talk, we discussed the flavor structure of soft SUSY breaking parameters induced by the SS boundary condition for quasi-localized quark/lepton superfields. The resulting squark/slepton masses and trilinear couplings at the compactification scale are highly flavor-dependent, but appropriately suppressed in correlation with Yukawa couplings. Those flavor violations can give interesting phenomenological consequences at low energies as well as constrain the mechanism of Yukawa coupling generation.

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