Cosmic Archaeology with Gravitational Waves from Cosmic Strings

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Cosmic strings are generic cosmological predictions of many extensions of the Standard Model of particle physics, such as a $U(1)'$ symmetry breaking phase transition in the early universe or remnants of superstring theory. Unlike other topological defects, cosmic strings can reach a scaling regime that maintains a small fixed fraction of the total energy density of the universe from a very early epoch until today. If present, they will oscillate and generate gravitational waves with a frequency spectrum that imprints the dominant sources of total cosmic energy density throughout the history of the universe. We demonstrate that current and future gravitational wave detectors, such as LIGO and LISA, could be capable of measuring the frequency spectrum of gravitational waves from cosmic strings and discerning the energy composition of the universe at times well before primordial nucleosynthesis and the cosmic microwave background where standard cosmology has yet to be tested. This work establishes a benchmark case that gravitational waves may provide an unprecedented, powerful tool for probing the evolutionary history of the very early universe.

INTRODUCTION

Gravitational waves (GW), vibrations of spacetime itself proposed by Einstein in 1916, were recently observed directly for the first time by the LIGO collaboration [1]. The source of these signals were black hole binaries, and more recently a neutron star binary [2]. Future measurements of such astrophysical events by the LIGO [3] and Virgo [4] detectors and the proposed LISA [5], BBO, and DECIGO [6] detectors will usher in a new era of observational astronomy and a much better understanding of the largest compact objects in the universe.

Gravitational waves may also provide a unique test of fundamental microphysics and early universe cosmology [7–10]. For example, primordial inflation [11], cosmic strings [12, 13] and cosmological first-order phase transitions [14] are all expected to create GWs. While the GWs from inflation are generally below the sensitivity of current and planned future detectors [15–17], GWs from cosmic strings or phase transitions can produce observable signals. In many cases, the potential sensitivity of GW detectors to these phenomena extends well beyond the reach of other foreseeable laboratory and cosmological tests. In this Letter we focus on the GWs from cosmic strings and demonstrate their unique potential for exploring cosmological history.

Cosmic strings are stable one-dimensional objects characterized by a tension $\mu$. They arise in superstring theory as fundamental or $(p,q)$ strings [18, 19]. They can also emerge as vortex-like solutions of field theory [20], such as configurations that wrap one or more times at spatial infinity in theories with a spontaneously broken $U(1)$, in which case the tension is related to the symmetry breaking scale by $\mu \sim \Lambda^2$ [21]. In the cosmological setting, cosmic strings form a network of horizon-length long strings together with a collection of closed string loops [12, 13]. Such a network would distort the cosmic microwave background (CMB), and current observations limit $G\mu < 1.1 \times 10^{-7}$, where $G$ is Newton’s constant [22].

Gravitational radiation is a key part of the evolution of a cosmic string network [12, 13]. Long strings intercommute to create closed string loops. These loops then oscillate, emitting energy in the form of gravitational waves until they decay away [23, 24]. Together, the processes of intercommutation, oscillation, and emission allow the string network to shed energy efficiently and prevent it from dominating the energy density of the universe. Instead, a cosmic string network is expected to reach a scaling regime in which it tracks the total energy density with fraction on the order of $G\mu$ [27, 29]. The scaling property of cosmic strings leads to a stochastic background of gravitational radiation built up from GW emission over the history of the string network [23, 30].

In this Letter we show that the frequency spectrum of GWs from cosmic strings can be used to look back in time and test the evolutionary history of the universe. For each frequency band of the background observed today, the emission was dominated by strings in a particu-
lar era of the early universe \([30]\). The standard thermal picture for the evolution of the cosmos is primordial inflation followed by reheating to a high temperature, and a subsequent long period in which the expansion of the universe is driven by a dominant energy density of radiation until the more recent transitions to matter and then dark energy domination. Evidence for this standard cosmology comes primarily from observations of the CMB \([21]\) and the successful predictions of Big Bang Nucleosynthesis (BBN), corresponding to cosmic temperatures below \(T \approx 5\) MeV \([32]\). Measurements of the GW frequency spectrum from cosmic strings by current and planned detectors could test the standard cosmology at even earlier times and possibly reveal deviations from it.

To demonstrate the power of GWs from cosmic strings to probe the very early universe, we study the frequency spectrum emitted by a string network in the standard cosmology and in two well-motivated variations. We focus on an ideal Nambu-Goto (NG) cosmic string network and apply the results of recent simulations of string networks to compute the GW spectrum. We show that a combination of current and planned GW detectors with different frequency sensitivities may enable us to reconstruct a timeline of cosmic history well beyond the BBN epoch.

**GW FROM COSMIC STRINGS**

Oscillating closed string loops are typically the dominant source of GWs from a cosmic string network in the scaling regime. The length \(\ell\) of a string loop created by the network at time \(t_i\) evolves according to

\[
\ell = \alpha t_i - \Gamma G \mu (t - t_i) .
\]  

The first term is the initial loop size as a fraction \(\alpha\) of the formation time \(t_i\), i.e., a fraction of the horizon size. Recent cosmic string simulations find that about 10% of the energy released by the long string network goes to \(\alpha \approx 10^{-1}\) large loops, with the remaining 90% going to the kinetic energies of highly-boosted smaller loops \([38, 39]\). The kinetic energy redshifts away and is not transferred to GWs. The second term above describes the shortening of the loop as it emits gravitational radiation, characterized by the dimensionless constant \(\Gamma \approx 50\) \([23, 21, 38, 39]\).

String loops emit GWs from normal mode oscillations at frequencies \(f_{\text{emit}} = 2k/\ell\), \(k \in \mathbb{Z}^+\). After emission, the frequency of the GW redshifts as \(a^{-1}\), where \(a(t)\) is the cosmological scale factor. For a given GW frequency \(f\) observed today from mode \(k\), this implies the emission time \(\hat{t}\) is related to the loop formation time by

\[
f = \frac{a(\hat{t})}{a(t_0)} \frac{2k}{\alpha t_i - \Gamma G \mu (t - t_i)} ,
\]

where \(t_0\) is the current time.

The stochastic GW background depends on the rate of loop production by the cosmic string network. We model this using the velocity-dependent one-scale (VOS) model \([41, 43]\), with a loop-dependent one-scale (VOS) model, with a loop-dependent energy going to large loops discussed earlier \([38, 39]\). The function \(C_{\text{eff}}(t_i)\) depends on the redshift scaling of the dominant energy density \(\rho\) of the universe. When \(\rho\) is dominated by a single source, it scales approximately as

\[
\rho \propto a^{-n} .
\]

For \(n = 3\) (matter domination), 4 (radiation domination), and 6 (kination – to be explained later), the VOS model predicts \(C_{\text{eff}} = 0.41, 5.5, 30\), respectively.

Summing over all harmonic modes, the GW density per unit frequency seen today is

\[
\Omega_{GW}(f) = \frac{f}{\rho_c} \frac{d\rho_{GW}}{df} = \sum_k \Omega_{GW}^{(k)}(f) ,
\]

with

\[
\Omega_{GW}^{(k)}(f) = \frac{1}{\rho_c} \frac{2k (0.1) \Gamma_{\text{g}} G \mu^2}{\alpha(\alpha + \Gamma G \mu)} \times \int_{t_F}^{t_0} dt \frac{C_{\text{eff}}(t_i)}{t_i^4} \left[ \frac{a(\hat{t})}{a(t)} \right]^5 \left[ \frac{a(\hat{t})}{a(t)} \right]^3 \Theta(t_i - t_F) ,
\]

where \(\rho_c = 3H_0^2/8\pi G\) is the critical density, \(\Gamma_{\text{k}} = \Gamma/(3.60 \cdot k^4/3)\) \([38, 39]\), \(t_i\) is obtained by inverting Eq. 2 and \(t_F\) is the formation time of the string network.

In Fig. 1 we show the frequency spectrum of GWs \(\Omega_{GW} h^2\) from cosmic strings for \(G \mu = 5 \times 10^{-12}\), \(\alpha = 10^{-1}\), and a standard cosmological history. Also shown are the current sensitivity bands of LIGO \([1, 3, 44]\), and the projected sensitivities of LISA \([45]\), DECIGO, and BBO \([6]\). The upper left solid triangle indicates the current limit from timing measurements by the European Pulsar Timing Array (EPTA) \([46]\), with the expected sensitivity of the future Square Kilometre Array (SKA) \([47]\) shown below. The EPTA limit implies \(G \mu \lesssim 10^{-11}\) giving the strongest current bound on the cosmic string network. Our results are consistent with the recent calculations of Refs. \([48, 49]\).

Fig. 1 illustrates the key relationship between the GW frequency spectrum and the cosmological era at which a given frequency today was produced. At higher frequencies, the result of Eq. 6 is approximately independent of \(\alpha\) and scales as

\[
\Omega_{GW}(f) \propto \begin{cases} f^{5/2} & n > 10/3 \\ f^{-1} & n \leq 10/3 \end{cases}
\]
This yields a flat spectrum for radiation and a $1/f$ spectrum for matter. At lower frequencies, the scaling (for $\alpha \gg \Gamma G\mu$) goes like $f^{3/2}$. The characteristic flatness of the spectrum at higher frequencies from early radiation domination implies that deviations from this scenario would be dramatic.

**TESTS OF NON-STANDARD COSMOLOGIES**

Cosmic string scaling implies that the relic spectrum of GWs today originated from an extended period of evolution of the early universe. As a result, the GW spectrum from cosmic strings can probe deviations from the standard cosmological evolution and identify the specific era in which it occurred. To illustrate this, we focus on two well-motivated non-standard histories. The first is a transient period of radiation domination prior to the standard radiation era, corresponding to $n = 3$ in Eq. [3]. This can arise from a large density of a long-lived massive particle or from the oscillation of a scalar moduli field in a quadratic potential [54]. Matter dominance typically ends during a reheating phase in which the relevant species decays to light Standard Model (SM) particles. The second non-standard cosmology we consider is a period of “kination”, with $n > 4$ in Eq. [4]. This can arise from the oscillation of a scalar field in a non-quadratic potential: for $V(\phi) \propto \phi^N$ one obtains $n = 6N/(N + 2)$, which can occur in quintessence models for dark energy or inflation [51] [52]. For both non-standard scenarios, above, let $t_{\Delta}$ be the time at which the universe transitions to the standard period of radiation domination. The evolution of the energy density of the universe during and after the non-standard phase can be parameterized according to

$$\rho(t) = \begin{cases} \rho_{\text{st}}(t_{\Delta}) \left[ \frac{a(t_{\Delta})}{a(t)} \right]^n ; & t < t_{\Delta} \\ \rho_{\text{st}}(t) ; & t \geq t_{\Delta} \end{cases}$$

where $\rho_{\text{st}}$ is the energy density extrapolated assuming the standard cosmological history, and $n = 6$ ($n = 3$) for early kination (matter) domination. We also define $T_{\Delta}$ as the temperature at time $t_{\Delta}$ when radiation domination resumes. In scenarios with early matter domination, $T_{\Delta}$ coincides with the reheating temperature. For both the matter-dominated and kination scenarios, $T_{\Delta} \gtrsim 5$ MeV is needed for consistency with BBN [55].

In Fig. 2 we illustrate the effect of non-standard cosmologies on the frequency spectrum of GWs from a cosmic string network with $G\mu = 5 \times 10^{-15}$ (blue), $5 \times 10^{-17}$ (orange) and $\alpha = 10^{-1}$ for standard and non-standard cosmological histories. The solid lines show the spectra for the standard evolution while the dashed (dotted) lines show the GW spectra for an early $n = 6$ kination ($n = 3$ matter) era ending at temperature $T_{\Delta} = 5$ MeV. The upper shaded regions indicate the current and future sensitivities of GW detectors and pulsar timing arrays.
putting it out of range of LIGO detection or constraint. However, the turn-over it induces is still potentially observable by future detectors such as LISA, DECIGO and BBO.

An additional constraint not yet included comes from the total radiation density of GWs, which must not exceed the limits from the CMB and BBN. This translates into the bound\[54, 55\]

\[\int d(\ln f) \Omega_{GW} \lesssim 3.8 \times 10^{-6} . \] (9)

For standard early radiation domination, this places a moderate constraint on \(\Omega_{GW}\) with a logarithmic sensitivity to the highest frequencies created in this era. The bound becomes more severe for \(n > 4\) since the relic GW spectrum now increases with frequency as a power law, Eq. (7). This growth is expected to be cut off at a high frequency that corresponds to the onset of the \(n > 4\) phase or the creation of the string network. In the early \(n = 6\) scenario with \(T_{\Delta} = 5\) MeV, the maximal temperature in this phase is \(T \simeq 4\) GeV (20 GeV) for \(G\mu = 5 \times 10^{-15} (5 \times 10^{-17})\). As can be seen in Fig. 2, the deviation in the GW spectrum due to a non-standard cosmic history occurs at a characteristic frequency \(f_{\Delta}\) that depends on the string network parameters and \(T_{\Delta}\), but is nearly independent of the energy redshift exponent \(n\). In Fig. 3 we show \(f_{\Delta}\) as a function of \(T_{\Delta}\) for several values of \(G\mu\). These curves follow the approximate relation

\[f_{\Delta} \propto T_{\Delta} (G\mu)^{-\frac{2}{5}} \alpha^{-\frac{1}{2}} \] (valid for \(\alpha \gg \Gamma G\mu\)), which we derive in a future work [56]. Roughly speaking, \(T_{\Delta}\) characterizes the formation time of loops that produce the dominant contribution to GWs with frequency \(f_{\Delta}\) today. Figure 3 shows that the frequency range of LIGO could be sensitive to non-standard cosmologies all the way back to \(T_{\Delta} \sim 10^{4}\) GeV for \(G\mu = 10^{-12}\), well beyond the reach of other known probes.

**DISCUSSION**

If cosmic strings are realized in nature, they could provide a unique and powerful tool for probing the history of the early universe. We have demonstrated that the frequency spectrum of GWs emitted by a cosmic string network depends dramatically on the energy content of the universe when they are produced. Current and planned experiments will have the potential to measure such a spectrum and thereby test the evolution of the cosmos at much earlier times than ever before.

The reach of this method for looking back in time depends on the frequency sensitivity of GW detectors and the properties of the cosmic string network. In general, deviations from the standard cosmological evolution at earlier times imprint themselves on the GW spectrum at higher frequencies, as shown in Fig. 3. These considerations provide strong motivation to explore new methods to extend the sensitivity of future GW observatories to higher frequencies beyond LIGO [57, 58].

The frequency spectrum of GWs also depends on the distribution of string loop sizes. Based on cosmic string simulations [38, 39], we have modeled this by assuming that 10% of the energy shed by a scaling string network goes to loops with a fixed initial loop size parameter of \(\alpha = 10^{-1}\). Our simple prescription reproduces the radiation-era loop length distribution found in the simulation of Ref. [37, 38] and is similar to their result for the matter era, but it is also optimistic. Smaller values of \(\alpha\) would push the spectral features of non-standard cosmologies to higher frequencies and potentially outside the range of GW detectors. Let us also mention that we do not know of any simulations of loop formation or scaling during a kination phase, which we have assumed in our calculation.

In addition to the two well-motivated non-standard cosmologies studied in this Letter, there are many other cosmological scenarios that could be probed by the GW spectrum from cosmic strings. Our results could also be modified in theories with more complex cosmic string dynamics such as \((p, q)\) strings with small intercommutation probabilities [59] or in scenarios where strings emit non-gravitational radiation [60–65]. We defer the investigation of such possibilities to a future work [56].

An important further question is whether there are other sources that could mimic the changes in the frequency spectrum from cosmic string GWs due to non-standard cosmologies. While it is difficult to provide a definitive answer, we note that the flat part of the cosmic string GW spectrum is very distinctive suggesting that it would be challenging to reproduce with a single alternative source.

In this Letter we have investigated the effects of non-
standard cosmological histories on the frequency spectrum of stochastic GWs produced by cosmic strings. We have demonstrated that cosmic string GWs could provide an unprecedented window on the very early universe prior to BBN and the CMB. This work may serve as an inspiring benchmark for exploiting the full potential of GW as a new tool for probing particle physics and cosmology beyond the horizon of our current knowledge.

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