Optimal power and energy sizing of compressed air energy storage in distribution network using multiparametric programming

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Abstract. As a new type of mechanical energy storage, compressed air energy storage (CAES) has attracted wide attention in recent years. This paper studies the optimal sizing problem of CAES in power distribution network (PDN). CAES plays a role in shaving the peak and filling the valley at demand side and thus reduces the operation cost of PDN to purchase electricity from the main grid. A multi-parametric linear programming (MP-LP) model is formulated where the power and energy capacities of CAES are regarded as parameters; the parameterized optimal value function (OVF) provides a graphical tool to describe the impact on operation cost of CAES configuration, which helps determine the planning strategy in a visual manner. The visualized results reveal not only the optimal solution, but also some useful information, like the sensitivity of operation cost to parameters. Case studies conducted on the IEEE 33-bus distribution system verifies the effectiveness of the proposed method.

1. Introduction
Currently, the penetration of renewable energy generation in power distribution network (PDN) is continuously growing, which brings in great volatility and fluctuation; also, the load level rises and becomes unpredictable. These factors challenge the operation of PDN, from both safe and economic perspective. One remedy is to deploy energy storage, which can improve operating flexibility, shave the peak and fill the valley [1]. Battery is the energy storage technology with the highest maturity, whose cost, however, is still high due to the limited cycle times [2]. Compressed air energy storage (CAES), a newly-developing mechanical energy storage, has attracted wide attention in recent years. CAES has series of appealing advantages, such as long lifetime and the capability of heat-cool-electricity tri-generation [3]; also, CAES can provide reactive power support because there is a generator in the discharging side [4].

In distribution network, energy storage mainly plays a role in reducing the operation cost (or increasing the profits), promoting the integration of renewable energy and improving the system flexibility [5] on condition that a proper sizing strategy is employed. In [6], the optimal placement and sizing of distributed battery storage in low voltage grids is investigated; the fuel cost of generators and investment cost of battery are considered; a Benders decomposition technique is employed to partition
a long-term planning horizon into a series of sequential sub-periods and thus effectively reduce the computational complexity to a manageable level. In wind power-rich distribution networks, optimal sizing and control of energy storage is studied to reduce curtailment from renewable distributed generation [7]; a two-stage framework is proposed where the first stage determines the sizes of storage, and the second stage minimizes the renewable power curtailment and facilitates the adequate use of storage with a high resolution of timescale. In [8], a bi-level optimization model for energy storage sizing is developed for networked microgrids considering reliability and resilience enhancement; The upper-level model optimizes the energy storage sizing problem; the lower-level problem is aimed at operation optimization for profit maximization under multiple operating scenarios.

Above work generally relies on a specified optimization problem to determine the optimal sizing strategy. Although such a mode can give out a strategy, it cannot provide more useful information, such as the sensitivity of operation cost to sizing strategies. Besides, we intend to characterize the impact of storage capacity on the operating cost, which provides reference for CAES planning in a visual manner.

To this end, this work employs the scheme of multiparametric programming [9] to investigate the optimal sizing of energy storage in distribution network. CAES is chosen as the target type of energy storage. The power and energy capacities are regarded as parameters; the results of multiparametric programming visually reveals the relation between parameters and the operation cost of distribution network during the planning horizon. Visualized results can not only determine the optimal strategies, but also show information about sensitivity and some potential indicator, such as the optimal charging time (the ratio of energy capacity and discharging power capacity). The multiparametric programming-base method for energy storage sizing presents advantages in result display, which can be extended to other applications in practice.

The rest of this paper is organized as follows: Section 2 introduces the basic models of distribution power flow and CAES; the solution methodology based on multiparametric programming is proposed in Section 3; case studies are conducted in Section 4, followed by the conclusions drawn in Section 5.

2. Basic models
This section introduces the basic models of the optimal sizing problem. The linearized branch flow model (BFM) is utilized to describe the power flow in the radial PDN. The model of CAES consists of the electric part and thermal part, which reflects the operating characteristics of electric system and thermal system of CAES.

2.1. Power flow model of PDN
In general, the topology of PDN is radial, as shown in Figure 1. Here we use the well-known linearized BFM [10] to describe the power flow, which can be recursively constructed as follows

\[ P_{ij,t}^l + P_{j,t}^n = \sum_{k \in \Omega(j)} P_{jk,t}^l, \forall j \in \mathbb{B}, \ t \in T \]  

(1a)

\[ Q_{ij,t}^l + Q_{j,t}^n = \sum_{k \in \Omega(j)} Q_{jk,t}^l, \forall j \in \mathbb{B}, \ t \in T \]  

(1b)

\[ V_{j,t} = V_{i,t} - 2(r_{ij}^l P_{ij,t}^l + x_{ij}^l Q_{ij,t}^l), \forall l \in \mathbb{L}, \ t \in T \]  

(1c)

\[-P_{ij,t}^m \leq P_{ij,t}^l \leq P_{ij,t}^m, \forall l \in \mathbb{L}, \ t \in T \]  

(1d)

\[-Q_{ij,t}^m \leq Q_{ij,t}^l \leq Q_{ij,t}^m, \forall l \in \mathbb{L}, \ t \in T \]  

(1f)

\[ V_{i,t}^n \leq V_{j,t} \leq V_{i,t}^n, \forall j \in \mathbb{B}, \ t \in T \]  

(1g)

where \( \mathbb{B}/\mathbb{L} \) is the set consisting of all the buses/lines in PDN; \( \Omega(j) \) is the set of lines whose head node is bus \( j \). \( P_{ij,t}^l/Q_{ij,t}^l \) is the active/reactive power flow in line \( l \) in period \( t \) whose head/tail node is bus \( i/ \) bus \( j \); \( P_{j,t}^n/Q_{j,t}^n \) is the active/reactive net injection at bus \( j \) in period \( t \), depending on the production of renewable energy generation and energy storage, as well as load level, if there is any; \( r_{ij}^l/x_{ij}^l \) is the resistance/reactance in line \( l \); \( V_{i,t} \) is the squared voltage magnitude in period \( t \) at bus \( j \) and \( V_{1,t} \) at the slack bus is set to 1. With these notations above, (1a) and (1b) represent the nodal
power balance; (1c) describes the voltage drop in transmission line. (1d) - (1f) impose limits on the upper and lower bounds of decision variables.

In the linearized BFM (1a)- (1g), the loss terms are ignored, which are included in the intact BFM. In fact, it is necessary to consider the loss terms in the short-term economic dispatch problems because of the relatively high ratio of $r/x$. However, this paper focuses on the long-term sizing of CAES and thus the loss can be neglected. Besides, in the mathematic sense, the relaxed BFM with loss terms is supposed to be a second-order cone model, which is not readily compatible to the multiparametric programming based methodology to be used. Therefore, we resort to the linearized BFM in (1).

2.2. Model of CAES

There are many types of CAES technologies. Specifically, in this paper we consider the advanced adiabatic compressed air energy storage (AA-CAES), which gives up the gas-fire supplement; instead, the heat from compression process is stored in heat storage and will be reused to heat the air to improve its working ability to generate electricity during expansion.

The structure of AA-CAES is drawn in Figure 2, which is mainly composed of the compressing subsystem, expanding subsystem, air storage and heat storage subsystem. For compressing subsystem, the power from external grid drives the electric motor and compressor to compress air into air storage. The electric power consumption can be calculated by

$$P_{f} = \frac{k R m n_f}{\eta_m \eta_c (k-1)} \left[ \sum_{c=1}^{N_c} T_{c,n_c}^{in} (\beta_{c,n_c}^{k-1} - 1) \right], \forall t \in T$$

where $k/R_m$ is the adiabatic exponent/ideal gas constant of air; $\eta_m/\eta_c$ is the efficiency of motor/compressor; $m_{c,n_c}^f$ is the mass flow rate in compressor; $N_c$ is the number of compression stage; $T_{c,n_c}^{in}$ is the inlet temperature of $n_c$-th compressor, whose compressing ratio is $\beta_{c,n_c}$. Please note that in an existing AA-CAES system, only $P_{f}$ is the independent decision variable; $m_{c,n_c}^f$ is related to $P_{f}^e$, and other parameters are constant. So (2) is linear.

At the expansion side, the air with high pressure and temperature is released to drive the turbine and generator to produce electricity. Similarly, the production of expanding process is

$$P_{f} = \frac{\eta_g \eta_e k R m n_f}{(k-1)} \left[ \sum_{c=1}^{N_c} T_{c,n_c}^{in} (1 - \beta_{c,n_c}^{k-1}) \right], \forall t \in T$$

where $\eta_g/\eta_e$ is the efficiency of generator/turbine. Other notations can be understood in analogy to compression side; also, (3) is linear. Then, according to the capacity of CAES, we have

$$0 \leq P_{f}^c \leq P_{c,m}^e, \forall t$$

$$0 \leq P_{f}^e \leq P_{e,m}^e, \forall t$$

where $P_{c,m}^e/P_{e,m}^e$ is the power capacity of compressor/expander.

As a form of energy storage system, we still need to consider the state of charge (SOC) of CAES. In fact, CAES stores air and heat simultaneously; given the fixed efficiency of heat exchanger, both the SOC of air and heat are linearly related to the mass flow rate $m_{c,n_c}^f$ and $m_{c,n_c}^e$. Hence, they are linearly related to $P_{f}^c$ and $P_{f}^e$, according to (2) and (3). Further, researches reveal that the SOC of air and heat
can be seen as a whole [11]. In other words, it is practical to consume all the air and heat at the same time. Based on the fact above, we can use a battery-like expression to describe the SOC of CAES from the perspective of electric energy, i.e.

\[ E_t = E_{t-1} + P_c \eta_m \eta_c - P_e / (\eta_p \eta_e), \forall t \in \mathbb{T} \]  
(6)

\[ \beta E^m \leq E_t \leq E^m, \forall t \in \mathbb{T} \]  
(7)

where \( E_t \) is the level of SOC in period \( t \); (6) describes the change of SOC. \( E^m \) is the energy capacity of CAES; \( \beta \) determines the lower bound of SOC.

### 2.3. Economic operation of PDN

Based on the power flow model of PDN in Section 2.1 and the model of CAES in Section 2.2, we can establish the economic operation model of PDN. With the aim to minimize the cost of purchasing electricity from the main grid, we have

\[
\begin{align*}
\min_{\sigma} & \sum_{t \in \mathbb{T}} \pi_t P_{t}^s \\
\text{s. t.} & \quad (1) - (7)
\end{align*}
\]  
(8)

where \( P_t^s \) is the power injection at slack bus from main grid; \( \pi_t \) is the electricity price; \( \Delta \) is the time duration represent one hour. \( \sigma \) is the coefficient to convert the optimization horizon to the lifespan of CAES. The CASE plays a role in smoothing the load curve; it can store an amount of electric energy in the periods with low electricity price and load level; and it can discharge when the demand and electricity price goes up. Due to the linearity of constraints, (8) is a linear program (LP).

### 3. Solution methodology

From the model of CAES (2)-(7), the critical sizing parameters of CAES includes the compressing power capacity \( P^{cm} \), expanding power capacity \( P^{em} \) and energy capacity \( E^m \). Specifically, we define the parameter vector \( \theta = [P^{cm}, P^{em}, E^m]^T \) and enclose the remaining decision variables into vector \( x \); consequently, LP (8) can be equivalently written as

\[
\begin{align*}
\min c^T x \\
\text{s. t. } & \quad Ax \leq b + F\theta; \lambda
\end{align*}
\]  
(9)

where \( A, b, F, c \) are constant coefficients defined by (1)-(7); \( \lambda \) is the vector of dual variables. The equality in (1)-(7) can be replaced by a couple of opposite inequalities. (9) is in the form of multiparametric linear program (MP-LP). Consider the dual problem of LP (9):

\[
\begin{align*}
\nu(\theta) &= \max \lambda^T (b + F\theta) \\
\text{s. t. } & \quad A^T \lambda = c, \lambda \leq 0
\end{align*}
\]  
(10)

where \( \nu(\theta) \) is the multivariate optimal value function (OVF) over \( \theta \). Given different \( \theta \), the optimal solution of (10) \( \lambda^* \) changes; so the OVF is piecewise linear. All the optimal solution comes from the set \( \text{vert}(\Lambda) \) where \( \Lambda = \{ \lambda | A^T \lambda = c, \lambda \leq 0 \} \). Maximization operator determines the convexity of \( \nu(\theta) \). Basically, the optimal sizing problem intends to find an optimal choice of parameter \( \theta \) under a given budget, so the parameter set is described as below:

\[
\Theta = \{ \theta | S\theta \leq H, \theta \geq 0 \}
\]  
(11)

where \( S \) is the cost vector related the devices in compressing subsystem, expanding subsystem and air/heat storage subsystem; \( H \) is the total investment budget. Therefore, the optimal sizing problem comes down to the solution of convex piecewise linear \( \nu(\theta) \) within parameter set \( \Theta \).

As a matter of fact, it is pretty difficult to obtain the exact expression of \( \nu(\theta) \) due to the large cardinality of \( \text{vert}(\Lambda) \), as well as the degeneracy in LP [9], especially when the dimension of \( \theta \) is high; also, for a long-term sizing problem, that is unnecessary. In view of this, we propose the following algorithm to calculate the approximate expression of \( \nu(\theta) \), as well as the parameter subset (i.e. critical region, CR) corresponding to the relevant piece of \( \nu(\theta) \).

1) Sample \( \theta \) within \( \Theta \) and constitute \( \Theta' = \{ \theta_1, ..., \theta_N \} \).
2) Solve (10) with \( \forall \theta_i \in \Theta' \); save the optimal solution \( \lambda_i^* \).
3) Calculate the intercept \( \lambda_i^T b \) and slope \( \lambda_i^T F \) of the piece of OVF corresponding to \( \theta_i \).
4) Report the piecewise linear OVF \( \nu(\theta) = \max_{\theta_i} \nu_i(\theta) = \max_{i} \lambda_i^T b + \lambda_i^T F\theta, \theta \in \Theta \).
5) Report the CR \( \Theta_i = \Theta \cap \{ \theta | \lambda_i^T b + \lambda_i^T F \theta \geq \lambda_j^T b + \lambda_j^T F \theta, j \in \{1, \ldots, N\} - \{i\} \} \).

Apparently, the more samples in \( \Theta' \), the more accurate the approximation of \( v(\theta) \). This approximation guarantees a lower bound of the accurate one. An illustrative example of the OVF and CR is depicted in Figure 3. When the expression of OVF and CR is obtained, the relation between parameter (sizing strategy) and optimal value (operation cost) is clear.

4. Case studies

The case studies are conducted on the modified IEEE 33-bus distribution system, which is drawn in Figure 4. A wind turbine with 1.5MW rated power is located at bus-18; and the CAES under consideration is at bus-6. The investment cost of CAES, \( s_1, s_2 \) and \( s_3 \) are set to 5 \( \times \) 10\(^5\)$/MW, 5 \times 10^5$/MW and 2 \( \times \) 10\(^5\)$/MWh. Four days (one day a season) are chosen as representative days and thus the time horizon is 96h. The lifespan of CAES is set to 40 years [3], so \( \sigma = \frac{365 + 40}{4} = 3650 \). The price data, load data, wind production, as well as other system parameters are given in [12].

![Figure 3. Illustration of CR and piecewise linear OVF.](image)

![Figure 4. Modified IEEE 33-bus distribution system.](image)

Basically, there definitely is an optimal solution \( \theta^* \) within parameter set \( \Theta \), which can be derived by solving (9) where \( \theta \) is treated as decision variables herein. As a result, \( \theta^* = [0.48MWh, 0.45MWh, 7.67MWh] \). Then, note that in (9), \( \theta \) has the dimension of 3, including the power capacity of compression side and expansion side and the energy capacity, which is not convenient for visualization. To this end, we make specific assumptions in cases to cut the dimension of parameter down to 2 for visualization. Accordingly, the parameter set \( \Theta \) is changed with minor modifications.

4.1. Case 1

In this case, we fix \( E^m = 6MWh \); the budget \( H = 10^6 \). In this way, we can investigate the relation between optimal value \( v(\theta) \), charging capacity and discharging capacity in a visual manner. In the first step of our approximation algorithm in Section 3, 66 points of \( \theta \) are sampled, leading to the optimal value function \( v(\theta) \) with 41 linear pieces, as well as 41 corresponding CR, which is the projection of \( v(\theta) \) on parameter space. \( v(\theta) \) is visually depicted in Figure 5; some finding are given as below:

1) Without CAES, the operation cost of such a distribution network is $3.137 \times 10^7$ in time scale of 40 years, i.e. the lifespan of CAES.

2) The deployment of CAES can help reduce the operation cost of distribution network. The optimal choice of CAES parameters is \([p^{cm}, p^{em}] = [0.907, 1.093]MW\) and the cost saving is $2.68 \times 10^6$. So the ratio of income (saving) to capital is 2.68. Actually, the cost saving only accounts for 8.54% of the operation cost without CAES; however, the base value is very large due to the long lifespan of CAES and thus the reduction is considerable.
3) Figure 5 shows that when $P^{cm} = 0$, the operation cost increases with the growth of $P^{em}$ in a narrow range; the reason is that the initial SOC of CAES can be discharged, which is relaxed as a decision variable in our test. When $P^{em} = 0$, no cost can be saved. Nevertheless, it is meaningless to discuss the situation where either $P^{cm}$ or $P^{em}$ is minimal.

4) If we regard $H = 10^6$ $\$ as the capital ceiling that the investor can provide, Figure 5 reveals the optimal solution given different budgets $H \leq H$, which can help decision-making from a global perspective.

5) Every piece of optimal value function has a unique intercept and slope, whose value are $\lambda_i^T b$ and $\lambda_i^T F$, respectively; importantly, the slope provides the information of sensitivity from optimal value to parameters. For example, in P14, the optimal value function is, $v_{14}(\theta) = 29300900 - 596900P^{cm} - 162200P^{em}$, which implies that the unit MW increment of $P^{cm}/P^{em}$ leads to a cost reduction of $596900/162200$. The information of sensitivity, which cannot be given by ordinary optimization, reflects the marginal income of energy storage investment and is of great significance in practice.

Figure 5. Piecewise linear optimal value function in case 1. Figure 6. Piecewise linear optimal value function in case 2.

4.2. Case 2
In this case, we assume $P^{cm} = 1$ MW, which is speculated by the results of case 1; thus, $P^{em}$ and $E^m$ are independent parameters. The budget $H$ is set to $10^6$. The graphical results are shown in Figure 6. Some discussion is given below:

1) The cost without CAES is $3.137 \times 10^7$, same as that in Case 1.

2) When $P^{em} = 0$, there is no capacity to discharge; when $E^m = 0$, there is no space for energy storage, let alone energy shift between periods. Therefore, when either $P^{em}$ or $E^m$ is 0, the operation cost remains at the highest level.

3) The optimal choice of parameter is $[P^{em}, E^m] = [0.259$ MW, $4.353$ MWh], with which the cost is $2.96 \times 10^7$, giving rise to a 5.64% reduction.

4) Figure 6 also provides useful sensitivity information, which can be similarly analyzed as in case 1 and thus omitted.

5) Unlike case 1, the optimal solution of $P^{em}$ and $E^m$ is more conspicuous in a visual manner. As shown in Figure 6, given different investment budgets, the optimal solution moves along with a valley, inspiring the optimal ratio of $E^m/P^{em}$, which represents the rated discharging time of CAES. In this case, the ratio reaches 16.8. Such a ratio implies that from the perspective of economic operation, CAES is expected to provide continuous energy output, which is more helpful for operation cost reduction rather than short-term power support that needs a lower ratio of $E^m/P^{em}$ in general.
5. Conclusions
This paper studies the optimal sizing problem of CAES in distribution network from the perspective of operating economy using multiparametric linear programming (MP-LP). In a visual manner, the results reveal useful information about optimal sizing strategy, sensitivity, optimal rated discharging time, which cannot be found by ordinary optimization methods. Results shows that in the situation where CAES is used to redispatch the energy in distribution network for cost reduction, the ratio of energy capacity to power capacity, both charging and discharging power, is relatively high, which implies that from the perspective of economic operation, CAES is expected to provide continuous energy output or storage, which is more helpful for operation cost reduction rather than short-term power support. Moreover, such a MP-based scheme provides a general and graphical tool for many applications in practical, including but not limited to planning and expansion, techno-economic assessment of specific devices and infrastructures.

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