Microwave Experiments Simulating Quantum Search and Directed Transport in Artificial Graphene

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A series of quantum search algorithms have been proposed recently providing an algebraic speedup compared to classical search algorithms from $N$ to $\sqrt{N}$, where $N$ is the number of items in the search space. In particular, devising searches on regular lattices has become popular in extending Grover’s original algorithm to spatial searching. Working in a tight-binding setup, it could be demonstrated, theoretically, that a search is possible in the physically relevant dimensions 2 and 3 if the lattice spectrum possesses Dirac points. We present here a proof of principle experiment implementing wave search algorithms and directed wave transport in a graphene lattice arrangement. The idea is based on bringing localized search states into resonance with an extended lattice state in an energy region of low spectral density—namely, at or near the Dirac point. The experiment is implemented using classical waves in a microwave setup containing weakly coupled dielectric resonators placed in a honeycomb arrangement, i.e., artificial graphene. Furthermore, we investigate the scaling behavior experimentally using linear chains.

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Introduction.—Currently, one of the most fruitful branches of quantum information is the field of quantum search algorithms. It started with Grover’s work [1] describing a search algorithm for unstructured databases, which has been implemented experimentally in NMR [2,3] and in optical experiments [4]. More recently, spatial quantum search algorithms have been proposed based on the quantum walk mechanism [5,6]. All of these algorithms can achieve up to quadratic speedup compared to the corresponding classical search. For quantum searches on generic $d$-dimensional lattices, certain restrictions have been observed, however, depending on whether the underlying quantum walk is discrete [7] or continuous [8]. While effective search algorithms for discrete walks on square lattices have been reported for $d \geq 2$ [9,10], continuous-time quantum search algorithms on the same lattice show speedup compared to the classical search only for $d \geq 4$ [11]. Experimental implementations of discrete quantum walks need time stepping mechanisms such as laser pulses [12–17]. By switching to a continuous-time evolution based, for example, on tight-binding coupling between sites, one can avoid time discretization in an experiment. It has been shown in Ref. [18] that continuous-time quantum search in 2D is indeed possible when performed near the Dirac point in graphene or, more generally, for lattices with a cone structure in the dispersion relation [19], i.e., a linear growth of the density of states (DOS). This effect adds a new dimension to the material properties of graphene [20,21] with potential applications in sensing and detection as well as directed charge carrier transport. This may provide new ways of channeling intensity and information across lattices and between distinct sites, such as for single-molecule sensing, as described in Refs. [22,23]. In this Letter, we present the first proof-of-principle experiment for a continuous 2D search in a tight-binding setup based on a microwave experiment using artificial graphene, as discussed in Refs. [24–26]. As was already noted by Grover and co-workers, “quantum” searching is often a pure wave phenomenon based on interference alone and can thus also be implemented using the single particle Schrödinger equation [27] or classical waves such as coupled harmonic oscillators [28,29] or wave optics [30]. A wave search amongst $N$ sites will take place in a full $N$-dimensional state space, while a full quantum search can be implemented with only $\log N$ particles. The $\sqrt{N}$ speedup is independent of this resource compression issue. In the following, we recapitulate briefly the theory of quantum searching on graphene and describe the experimental setup. We then demonstrate both searching and directed transport in graphenelike lattice structures. The $\sqrt{N}$ scaling behavior with a number of sites will be demonstrated experimentally using linear chains.

Theoretical background.—All quantum search algorithms starting from Grover’s search on an unstructured...
entries in the database are represented by the extended state $|j\rangle$ and the avoided crossing is controlled by the overlap integral $|\langle j|i\rangle|^2$. In the quantum state transfer setup described in Ref. [32], one thus creates artificial graphene [24–26]. The top plate holds a loop antenna coupling via the magnetic field into the TE$_{1}$ mode which can be positioned arbitrarily in the $xy$ plane above each resonator. A kink antenna is placed at a fixed position. For details on the experimental setup and its relations to a tight-binding model, refer to Ref. [26].

In particular, the reflection of the movable loop antenna at the center of the resonator is proportional to the intensity of the eigenmodes $|\langle j|\psi_n\rangle|^2$, where $n$ labels the resonator. These reflection measurements determine the LDOS and, by integrating over $n$, the DOS as well (for details, refer to Ref. [26]). A transmission measurement from the fixed antenna 1 positioned at $r_0$ to the movable antenna 2 positioned at $r_n$ yields amplitudes as well as phases. We conduct the experiment in the frequency domain, but the results can be converted from the frequency to the time domain without any loss of information. The DOS for the artificial graphene flake close to the Dirac point is shown in Fig. 1. One can clearly identify two isolated states, which are extended lattice states. The lower one is denoted by $\nu_l$ and the upper one by $\nu_u$. We will use the frequencies of these states near the Dirac point as working points for our search algorithm. The boundary has been chosen to contain the on-site energy—tuned to an extended state close to the Dirac point.

**Experiments on artificial graphene.**—The experimental microwave setup is shown in the inset of Fig. 1. A metallic plate supports ceramic cylinders of height $h = 5$ mm and radius $r = 4$ mm which have a high index of refraction ($n \approx 6$), thus acting as resonators for a transverse electric resonance, called TE$_{1}$, at $\nu_0 \approx 6.65$ GHz. The system is closed from above by a metallic top plate at a distance of $h_p = 16$ mm (not shown). We form a “graphene flake” by positioning 216 resonators in a hexagonal lattice (see inset of Fig. 1), thus creating artificial graphene [24–26].

![DOS vs Frequency Graph](image)

**FIG. 1** (color online). (Inset) A photograph of the artificial graphene flake, including the supporting metallic plate and the perturber resonators at the boundary (white arrows). The graph shows the DOS of the unperturbed flake (for details, see Ref. [26]), i.e., without a single resonator and dimer attached. The resonance frequencies of the single resonator $\nu_s$ and the dimer $\nu_{\text{dim}}$ are marked by the dashed vertical lines.
Spatial quantum search can also be used for communication and “quantum state transfer” [18,32,39] by perturbing the lattice with two equivalent resonators whose eigenfrequencies are both tuned to a single eigenfrequency of the unperturbed lattice. This scenario can also be interpreted as a search starting from one resonator and finding the other similar to the search algorithm presented in Refs. [29,34]. The perturbers are attached at two different

FIG. 2 (color online). (a) and (c): The initial lattice state at \( t = 0 \) for two different initial frequency ranges is shown (a) for the range around the lower lattice state \( \nu_L \) including the dimer frequency \( \nu_{dim} \) and (c) the range around \( \nu_S \), including \( \nu_u \). (b) and (d): The illuminated perturber state at the search time \( t = T_{dim} \) and \( T_s \), respectively. The resonators are indicated by the black circles and the color code corresponds to the intensity \( P(r,t) \) (dark red: high probability and white: low probability). The color code is rescaled to the maximal value. See the Supplemental Material [38] for a video showing the full dynamics.
positions to the lattice and interact only via the lattice state. Launching an initial pulse at $r_0$ will illuminate both resonators equally, but with reduced brightness. If we, however, prepare the initial state in one of the perturbing resonators and let the system evolve thereafter, the pulse will actually travel from the initial resonator via the lattice to the second resonator. The evolution is presented in Fig. 3. This time the upper resonators [Fig. 3(a)] is illuminated initially; we then go to a state living both in the lattice and on the two perturbing resonators [Fig. 3(b)]. Then the other resonator lights up [Fig. 3(c)] and (nearly) the entire amplitude is transferred from one resonator to the other. This opens the way towards directed signal transfer and control in graphene. However, graphene cannot yet be manipulated on a single atom level, as would be necessary for making use of the effects described in this Letter; our results may guide future research efforts in this direction.

**Experiment on linear chains.**—We demonstrate the $\sqrt{N}$ scaling behavior of the search time in our experimental setup on a quasi-one-dimensional system, a linear chain. Searching is possible here only for small $N$'s, as the distance between neighboring resonances scales like $N^{-2}$ and the number of resonances will flood the avoided crossing eventually for $N > N_{\text{cut}}$ [18]. A photograph of the unperturbed chain with $N = 11$ resonators is depicted in Fig. 4(a), that is, well below the cutoff $N_{\text{cut}} = 27$ for the setup shown here. The reflection spectrum $1 - |S_{11}|^2$ of the unperturbed chain containing 11 resonances is shown in Fig. 4(b) (black line, central spectrum). The central

[FIG. 4 (color online). (a) Experimental setup of the linear chain with $N = 11$ (without a perturber). The top plate and antenna are not shown. The wave function corresponding to the central resonance is superimposed. (b) Reflection measurement of a single resonator (red line, bottom), a regular chain with 11 resonators (black line, center), and a single resonator attached to the chain (blue line, top). The dotted lines mark the frequency range used for the Fourier transform (baselines shifted).]

[FIG. 5 (color online). (a) The intensity $P(r, t)$ of the chain integrated over the 11 resonators (black line) and of the perturber (red line) is shown. The signals are normalized such that the initial state in the chain at $t = 0$ is 1. The corresponding intensity distributions for specific times are shown as insets. See the Supplemental Material [38] for a video showing the full dynamics. (b) The beating time as a function of $N'$; square root dependence; dashed (blue) line: linear increase. The error bars indicate the time difference between different recurrences.]

**Conclusion.**—We have demonstrated a first proof-of-principle experiment of a continuous quantum wave search
on an (artificial) graphene lattice. The search is facilitated by bringing a lattice state into resonance with a localized perturber state at an avoided crossing. Apart from searching perturber states, one can also use this scheme to address particular sites using a frequency scan, thus initiate a switching behavior, and to transfer signals between sites without knowing their positions. The experiment is limited mainly by losses and absorption. Reducing the resonance widths further, i.e., obtaining a better quality factor $Q$, can be realized using coupled supraconducting cavities (quality factors of about $Q \approx 10^8$ are possible).

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