On the habitability of the OGLE-2006-BLG-109L planetary system

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\textbf{ABSTRACT}

We investigate the dynamics of putative Earth-mass planets in the habitable zone (HZ) of the extrasolar planetary system OGLE-2006-BLG-109L, a close analog of the Solar system. Our work is inspired by work of Malhotra and Minton (2008). Using the linear Laplace–Lagrange theory, they identified a strong secular resonance that may excite large eccentricity of orbits in the HZ. However, due to uncertain or unconstrained orbital parameters, the sub-system of Jupiters may be found in dynamically active region of the phase space spanned by low-order mean-motion resonances. To generalize this secular model, we construct a semi-analytical averaging method in terms of the restricted problem. The secular orbits of large planets are approximated by numerically averaged osculating elements. They are used to calculate the mean orbits of terrestrial planets by means of a high-order analytic secular theory developed in our previous works. We found regions in the parameter space of the problem in which stable, quasi-circular orbits in the HZ are permitted. The excitation of eccentricity in the HZ strongly depends on the apsidal angle of jovian orbits. For some combinations of that angle, eccentricities and semi-major axes consistent with the observations, a terrestrial planet may survive in low eccentric orbits. We also study the effect of post-Newtonian gravity correction on the innermost secular resonance.

\textbf{Key words:} celestial mechanics – secular dynamics – relativistic effects – analytical methods – extrasolar planetary systems

1 INTRODUCTION

Recently, Gaudi et al. (2008) announced a discovery of two-planet extrasolar system hosted by the OGLE-2006-BLG-109L star of \( \sim 0.5 \, \text{M}_\odot \). Its jovian companions of \( \sim 0.71 \) and \( \sim 0.27 \) Jupiter masses are in orbits of 2.3 and 4.6 au, resembling a scaled copy of the Jupiter-Saturn pair. This similarity leads to a natural question, whether additional planets can exist in this system, in particular Earth-like planets in the habitable zone (HZ). As for now, the answer can be speculative, because the OGLE-2006-BLG-109L planetary system has been detected during an observed micro-lensing event and hence the observational data are not reproducible. In addition, the faint and distant parent star cannot yet be observed by other planet-hunting techniques, like the Doppler spectroscopy or astrometry, to eventually detect the presence of an Earth-like planet. Moreover, some orbital parameters of the jovian planets are either uncertain or undetermined at all. The orbital architecture of the system, including putative terrestrial planets, and its stability, may be investigated only indirectly by numerical simulations.

In this work, we focus on the dynamical structure of the HZ. The HZ is defined implicitly through a requirement of the liquid state of water (see, e.g., Kasting et al. 1993; Hinse et al. 2008; Malhotra & Minton 2008). That condition is satisfied if the terrestrial orbit, parameterized by the semi-major axis \( a_0 \) and eccentricity \( e_0 \), is confined to the annulus of \( (r_p, r_a) \equiv (0.25, 0.36) \, \text{au} \) [following Hinse et al. (2008)], the terrestrial planet in the HZ will be called with Latin word Tellus from hereafter and we will always refer to its orbital elements with index “0”. It means that the pericenter and apocenter distances of habitable orbit must be limited through the following conditions:

\[
a_0(1-e_0) > r_p \quad \text{and} \quad a_0(1+e_0) < r_a, \tag{1}
\]

providing very long time-scale of stable motion, well enough to develop and support life (see, e.g., Hinse et al. 2008 for details). The term \textit{stable} or \textit{HZ-stable}, regarding the terrestrial orbit, will be understood as “confined to the HZ, as defined above, during the secular time scale counted in tens of Myrs”. Apparently, the HZ of the OGLE-2006-BLG-109L system is separated from the resonant influence of the primaries. The ratio of orbital periods corresponding to \( a_0 \in (0.25, 0.36) \, \text{au} \) is \( \sim 20 \). Therefore, no strong mean motion resonances (MMRs) can perturb putative terrestrial orbits. However, even in the absence of the MMRs, such orbits may be still disturbed by the long-term, secular interactions with jovian companions. Indeed, Malhotra & Minton (2008) identified two secular resonances \( (v_{1,2}) \) in the inner part of the OGLE-2006-BLG-109L system related to the fundamental secular frequencies...
2 DYNAMICS OF THE OGLE-2006-BLG-109L SYSTEM

According to the discovery paper (Gaudi et al. 2008), the orbits of jovian companions of OGLE-2006-BLG-109L are determined with significant uncertainties, i.e., $a_c = 2.3 \pm 0.2$ au, $a_b = 4.6 \pm 0.5$ au for the inner and outer planet, respectively; $e_b$ is unconstrained at all, and $e_c = 0.11 \pm 0.07$. To keep our work consistent with a parameterization of the analytical theory in (Migaszewski & Goździewski 2008), we interpret these orbital parameters as the canonical, geometric elements related to the Poincaré coordinates (see, e.g., Morbidelli 2002, Ferraz-Mello et al. 2006). From the “practical” point of view, these elements are not significantly different from the common, astrocentric Keplerian elements. The longitudes of pericenters ($\theta_b, \theta_c$), and mean anomalies are undetermined. The inclination of OGLE-2006-BLG-109Lc has been estimated from observations as $i \sim 59^\circ$, and also masses of companions are constrained within $\sim 10\%$ error. It is reasonable to assume that the whole system is coplanar. Still, significant errors of the semi-major axes leave room for a few qualitatively different orbital configurations which can be studied with the help of 2D dynamical maps. We select and vary two Keplerian elements, and other orbital parameters are fixed. For each point of the parameter plane, representing an initial configuration of the system, we compute the MEGNO signature (Cincotta & Simó 2000) with the symplectic algorithm (Cincotta & Simó 2000). A relatively short integration time required by MEGNO and good sensitivity of the indicator to chaotic motions make it possible to investigate large volumes of the phase-space with high resolution, and to detect efficiently unstable configurations. For instance, the dynamical maps in the $(a_b, e_b)$-plane, illustrated in the left-hand panels of Fig. 1 have the resolution of $640 \times 400$ points and each point in these maps represents a configuration integrated over $\sim 0.4$ Myr. The orbital parameters of the nominal configuration marked with crossed circle in the map are taken from (Malhotra & Minton 2008), and we choose $\Delta\theta_{b,c}$, following a concept of the so called represented plane of initial conditions (Michtchenko & Malhotra 2004). That $(e_b, \cos \Delta\theta_{b,c}, e_c)$-plane, $\mathcal{S}$ from hereafter, is defined through fixed $\Delta\theta_{b,c} = 0$ (the right-hand half-plane, for initially aligned orbits) and for $\Delta\theta_{b,c} = \pi$ (orbits are anti-aligned). For these particular values of $\Delta\theta_{b,c}$, the eccentricities in two-planet configuration reach maximal or minimal values at the same time with an accord to the conservation of the total angular momentum and the condition of $\partial \Delta\theta_{c} / \partial \Delta\theta_{b,c} = 0$ ($\mathcal{H}_c$ is for the secular Hamiltonian of the three-body problem). It can be shown (Michtchenko & Malhotra 2004) that all phase trajectories of non-resonant two-planet system must intersect the $\mathcal{S}$-plane. Hence, the global dynamics of the system may be described conveniently in terms of initial conditions selected in the $\mathcal{S}$-plane. The $\mathcal{S}$-plane is particularly useful for investigating equilibria of the secular system. For a reference, other examples of MEGNO maps in the $\mathcal{S}$-plane are shown in the right-hand panels of Figs. 1 and 2.

Clearly, the pair of Jupiters resides in dynamically active region of the phase space which is spanned by a few low-order MMRs. Allowing for a variation of $a_b$ within the 0.2 au error, the OGLE-2006-BLG-109L system can be found in the neighborhood of the 3c:1b MMR or 5c:2b MMR. Because the error of $a_b$ is $\sim 0.5$ au, even 2c:1b MMR configurations are permitted. Simultaneously, as the dynamical maps indicate, the border of stable zone strongly depends on initial eccentricities. Therefore, to determine the secular evolution of orbits in the HZ, stable configurations of the giant planets must be chosen with care. To illustrate that issue, we select a few representative configurations (still, consistent with the observational uncertainties) and compute their dynamical...
maps. Orbital parameters of these models, in terms of parameter tuples \((a_b, a_c, e_b, e_c, \Delta \omega_{b,c})\), are given in Table 1 for the nominal configuration investigated in Malhotra & Minton (2008), with apsides aligned and model \(I_0\) with apsides anti-aligned; model \(I_\pi\) with moderate eccentricities, close to the best fit elements quoted in Gaudi et al. (2008); models \(III_b\) and \(III_c\), for compact systems with moderate eccentricities; models \(V_0\) for hierarchical configuration of the Jupiters, and model \(V_\pi\) involving Jupiters in the 2c:1b MMR. Dynamical maps for models \(III_b, c\) are shown in the left-hand panel of Fig. 2 and for model \(V_0\) in two remaining panels.

### 3 SEMI-ANALYTIC MODEL OF THE HZ

The time-scales of orbital periods of giants and Tellus in the HZ are very different. Moreover, due to very long period of the \(v_1\) resonance, of a few Myrs \(\sim 10^8\) orbital periods of Tellus), the direct integrations of the planetary equations of motion would require incredible amounts of CPU time. Therefore, we consider Tellus placed in the HZ and each planet in the dynamically coupled pair of Jupiters as highly hierarchical system, because the ratio of semi-major axes \(a_0/a_{b,c} \sim 1/10\) is small. That makes it possible to average out the short-term variations of the orbit of Tellus over the mean longitude, and to approximate its secular evolution by means of a 24-order analytical theory in the semi-major axes ratio Migaszewski & Goździewski (2008). Yet the theory cannot be directly applied to jovian orbits, because their elements can be varied in wide ranges, and non-resonant as well as resonant or near-resonant configurations are permitted. The averaging of the whole system, with strongly interacting giant planets cannot be done uniformly (see, for instance Malhotra et al. 1989). Hence, we introduce two simplifications to the problem. At first, we assume that Tellus does not affect the orbital evolution of the giants, so we consider the restricted problem. Then the secular Hamiltonian (per mass unit) may be written as a sum of two terms \(\mathcal{H}_L = \mathcal{H}_0 + \mathcal{H}_c\), for interactions of Tellus with each Jupiter, separately.

\[
\mathcal{H}_c = -\frac{e^2 m_i}{a_i} \left(1 + \sqrt{1 - e_i^2} \sum_{i=2}^{\infty} \left[ \frac{\alpha_{ij}}{1 - e_i} \right] R_{ij}^{0,1}(e_0, e_i, \Delta \omega_{0,i}) \right),
\]

where \(\alpha_{ij} = a_0/a_i\) and \(i = b, c\). The explicit formulae for expansion terms, \(R_{ij}^{0,1}(e_0, e_i, \Delta \omega_{0,i})\) can be found in Migaszewski & Goździewski (2008). The equations of motion implied by \(\mathcal{H}_L\) refer to the mean orbital elements of Tellus and the mean orbital elements of Jupiters as parameters of the model. In the realm of the restricted problem, we can derive the mean orbits of Jupiters and their perturbations by numerical averaging. The full, \(N\)-body equations of motion of these planets are integrated numerically at

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**Table 1.** Orbital models of the OGLE-2006-BLG-109L system considered in this work. Masses of Jovian planets are fixed to 0.71 \(m_b\) and 0.27 \(m_c\) for planets \(b\) and \(c\), respectively, following Gaudi et al. (2008). Mass of the parent star is 0.5 \(m_\odot\). The mean anomalies \(M_{b,c}\) in these models are constrained by the mean longitude \(\Delta \omega_{b,c} = \Delta \omega_{0,b,c} + M_{b,c} = 0\).

| Model | \(a_b\) [au] | \(a_c\) [au] | \(e_b\) | \(e_c\) | \(\Delta \omega_{b,c}\) |
|-------|--------------|--------------|--------|--------|------------------|
| \(I_0\) | 2.30 | 4.60 | 0.03 | 0.05 | 0 |
| \(I_\pi\) | 2.30 | 4.60 | 0.05 | 0.05 | \(\pi\) |
| \(III_b\) | 2.30 | 4.60 | 0.10 | 0.10 | 0 |
| \(III_c\) | 2.40 | 4.35 | 0.10 | 0.10 | 0 |
| \(V_0\) | 2.10 | 5.10 | 0.10 | 0.10 | 0 |
| \(V_\pi\) | 2.50 | 4.10 | 0.45 | 0.10 | 0 |
least over of a few \( v_1 \) periods (~10 Myr). Then the temporal, osculating elements are converted with appropriate constant time step-size to the mean elements through the running average (see Figs. 3a,b). Because we intend to integrate the secular equations of motion of Tellus, the step size of the averaging (~10^7 yr) should be a small part of the apsidal period of its orbit. With this relatively long time step, as compared to the orbital periods, we remove all fast, quasi-periodic variations of the osculating elements. The mean orbits of jovian planets need to be computed only once (because we consider the restricted problem), and then we can efficiently reconstruct secular orbits of arbitrary number of Tellus “clones”, integrating numerically the secular equations of motion induced by \( \mathcal{H}_\infty \). The integration of a single terrestrial orbit over the secular time scale (typically, a few Myrs) is rapid, by a few orders of magnitude shorter than of the full system: the CPU time of the analytical solutions counts in minutes, but to integrate ~100 orbits of Tellus during ~20 Myrs, the MERCURY code [Chambers 1999] integrator RADAU spent a week on four 2GHz AMD/Opteron CPU cores. A test of this approach is illustrated in Fig. 3. According with the classic L-L theory [Murray & Dermott 2000], the central peak of eccentricity \( e_0 \) appears when the particle’s apsidal frequency is close to the forcing planetary frequency. Hence that peak shows the position of the resonance in the parameter space (for instance, at the \( a_0 \)-axis). The position of the eccentricity peak as well as the shape of \( \max e_0(a_0) \) graph are reproduced with great precision in the whole range of \( a_0 \), in spite of extremely large eccentricity attained by terrestrial orbits around \( a_0 \sim 0.40 \) au. Actually, after a few Myrs such orbits may become strongly chaotic. We did similar tests, choosing angle \( \Delta \varpi \equiv \varpi - \varpi_0 = \pi \). Initial conditions are: \( \Delta \varpi_0 = 0, e_0 = 0.01 \), and the mean anomalies of Tellus \( M_0 \) were chosen at random for the numerical integrations.

**Figure 2.** Dynamical MEGNO maps for two putative configurations of the OGLE-2006-BLG-109L system. The left-hand, and the middle panels are for the \( S \)-plane (see the text for details), the right-hand panel is for the \((a_0, e_0)\)-plane. Orbital elements of tested configurations are marked with crossed circle and labeled. The left hand panel is for model III, the middle- and the right-hand panels are for the 2c:1b MMR of Jupiters (model \( V_0 \)).

**Figure 3.** The semi-analytic theory of Tellus explained in graphic form. Panel a). A part of \( e_c(t) \)-curve (dots, the full three-body numerical integration), and the averaged eccentricity of the outer planet (red, filled circles). Panel b). The mean eccentricity of the outer planet after averaging. Panel c). A comparison of models of the \( v_1 \) secular resonance derived with the help of the semi-analytical theory (smooth, black curve) and with the direct numerical integration of the restricted four-body problem (open circles). Orbital parameters of the jovian sub-system are written in the box (model \( V_0 \)). Orbits with largest \( e_0 \sim 0.8–0.9 \) are chaotic and that explains significant deviations of the numerical values from the semi-analytic solution. The red double curve marks geometric boundary of the HZ implied by Eq. 1. Initial conditions are: \( \Delta \varpi_0 = 0, e_0 = 0.01 \), and the mean anomalies of Tellus \( M_0 \) were chosen at random for the numerical integrations.
(Eq. 1) implied by the orbital annulus of the HZ. Its geometrical borders are marked with red solid curves in Fig. 4, also the relevant range of semi-major axes, providing secularly low-eccentric HZ-orbits for initial $\Delta \nu_{b,c} = 0$, are marked by projections of green areas under the max $e_0$ curve onto the $a_0$-axis. Panels of Fig. 4a,b are for the configuration studied by Malhotra & Minton (2008), i.e., $e_b = 0.03$, $e_c = 0.05$, and $a_0 = 2.3$ au, $a_c = 4.6$ au. Panel of Fig. 4c is for aligned orbits (our model I), while Fig. 4d is for anti-aligned orbits (our model I). We can observe significant differences between these plots, in spite of that initial eccentricities and semi-major axes are the same, and only initial relative orbital phase of jovian planets (in fact, $\Delta \nu_{b,c}$ has changed). That is quite a different conclusion than could be derived in the framework of the L-L theory. For $\Delta \nu_{b,c} = 0$ a large part of the HZ is rendered unstable by the secular resonance, nevertheless, for the anti-aligned configuration, most particles would survive in the HZ, and max $e_0(a_0)$ would be relatively small. Moreover, the instability generated by the secular resonance (with the centre at approximately 0.29 au) is much stronger when initial eccentricities of Jupiters are relatively small. Moreover, the instability generated by the second resonance (with the centre at approximately 0.58 au) is much stronger when initial eccentricities of Jupiters are relatively small. Moreover, the instability generated by the second resonance (with the centre at approximately 0.58 au) is much stronger when initial eccentricities of Jupiters are relatively small.

According with the definition of the HZ, the maximum range of $a_0$ is $\max \Delta a_0 \sim 0.11$ au. To measure a fraction of HZ-stable orbits as a function of $a_0$, with respect to different configuration of jovian planets, we define the linear coefficient of habitability:

$$f_{HZ} = \frac{\Delta a_0}{\max \Delta a_0}, \quad f_{HZ} \in [0,1],$$

where $\Delta a_0$ is the range of initial semi-major axis implying orbits entirely confined to the HZ during the secular time-scale of a few Myrs. If the whole HZ is rendered unstable by the $v_1$ resonance (or other perturbation) then $f_{HZ} = 0$, and if all orbits remain within the HZ annulus, we have $f_{HZ} = 1$. For instance, for the nominal system in Fig. 4, $f_{HZ} \sim 0.4$. The next configuration illustrated in Fig. 4 has $f_{HZ} \sim 0.95$; in this case, the very narrow $v_1$ resonance affects a small part of the HZ. We can observe in Fig. 4 that the position and the width of $v_1$ depends on assumed orbital parameters of the jovian system, hence $f_{HZ} \equiv f_{HZ}(e_b, e_c, \nu_{b,c}, \Delta \nu_{b,c})$, also parameterized by mass of large bodies $m_b, m_c, m_c$. In general, $f_{HZ}$ depends also on initial $e_0$ and the relative phase of Tellus and a jovian planet, e.g., $\Delta \nu_{b,c}$.

We also investigated the influence of the general relativity (GR) correction on the secular dynamics of Tellus. In our recent paper (Migaszewski & Goździewski 2009), we have shown that this apparently subtle correction to the Newtonian gravity may affect the secular dynamics significantly, and they can be particularly important for small and close-in planetary companions. With the semi-analytic model of the jovian orbits, we could repeat the calculations of max $e_0$, adding the GR correction to $\mathcal{H}_{sec}$. The results derived with these corrections (the relativistic model) and with the Newtonian interactions only (the classic model) are compared in Fig. 5. For the nominal configuration, we observe only a small shift of the eccentricity peak (of $\sim 0.01$ au). The maximal eccentricity is not affected significantly. Indeed, the GR corrections become
important when the rate of the apsidal motion, which it causes to change, is similar to the effect of the Newtonian point-mass interactions. In the region of HZ, the GR induced apsidal advance has the period of a few Myrs, while the Newtonian period is ~ 30,000 yr only. Hence, we may conclude that the GR effects are not really important for the stable motion of Tellus in the HZ, and we do not consider them anymore.

4 SIZE OF THE HZ IN THE PARAMETRIC SPACE

The one-dimensional parametric survey of the HZ with respect to \( a_0 \) (Fig. 4) can be generalized to the second dimension of the initial eccentricity \( e_0 \). Similarly to \( f_{HZ} \), we define the planar coefficient of habitability:

\[
s_{HZ} \equiv s_{HZ}(a_0, e_0, e_1, e_2, \Delta \delta_{b,c}) = \frac{\Sigma_0}{\max \Sigma_0}, \quad s_{HZ} \in [0, 1],
\]

where \( \max \Sigma_0 \) is the area of initial conditions in the given parameter plane consistent with the geometrical boundaries of the HZ, while \( \Sigma_0 \) measures the area of initial conditions in that plane, implying orbits confined to the HZ during the secular time-scale. In contrary to \( f_{HZ} \), the planar coefficient of habitability does not depend on \( e_0 \). Fixing \( \Delta \delta_{b,c} = 0, \pi \), we follow again the concept of the representative plane of initial conditions.

The results for the same configurations of Jupiters as in Fig. 4 are illustrated in Fig. 5. Because the stable zone is bordered by orbits of Tellus initially aligned or anti-aligned with the apsidal line of planet c (or, alternatively, with planet b), both these limiting cases can be conveniently shown in one \( (a_0, e_0 \cos \Delta \delta_{b,c}) \)-plane. The negative values on the y-axis tell us that \( \Delta \delta_{b,c} = \pi \), positive values mean that \( \Delta \delta_{b,c} = 0 \). The borders of the zone with HZ-stable orbits are marked with green solid curves. The geometric boundary of max \( \Sigma_0 \) is marked with thick dotted curves. The stable zone has rather complex shape, and, depending on orbits of the jovian sub-system, we can derive quite unexpected conclusions. For their apsides aligned (Fig. 5a), there is still significant area of habitable orbits for initial \( e_0 \neq 0 \). Curiously, if the jovian orbits are initially anti-aligned, the HZ is almost wholly preserved (Fig. 5b). In the next instance (panel c), the HZ is rendered unstable, while in all remaining cases (Fig. 5d,e,f), there are HZ-stable orbits up to \( e_0 \sim 0.15 \).

Yet the tests illustrated in Figs. 4 and 5 still provide limited information on the HZ because we analyzed only isolated configurations of Jupiters. To obtain better insight into the dependence of the HZ on different orbital setups of Jupiters, we extend the survey to computation of \( s_{HZ} \) in two-dimensional parameter planes of the jovian sub-system.

4.1 The HZ in the representative plane

The features of the \( S \)-plane make it particularly convenient to analyze \( s_{HZ} \) for fixed semi-major axes of the jovian sub-system and its dependence on the eccentricities. The results are illustrated in the left-hand panel of Fig. 7 which shows color-coded values of \( s_{HZ} \) in the \( S \)-plane constructed for the nominal system \( (a_0 = 2.3 \text{ au}, a_c = 4.6 \text{ au}, \text{ our model I}) \). It can be regarded as non-resonant or near-resonant (see Fig. 7). In such a case, in the regime of moderate eccentricities, the apsidal angle \( \Delta \delta_{b,c} \) circulates or librates around 0 or \( \pi \) (these regions are dotted in Fig. 7). For a reference, the red, thick curves mark approximate positions of stable stationary solutions of the non-resonant secular model of the jovian sub-system, calculated with the help of a high-order analytic theory (Migaszewski & Gozdiewski 2008). The curve in the positive half-plane of \( S \) is for the so called mode I equilibria, characterized by librations of \( \Delta \delta_{b,c} \), 0 for close phase trajectories; mode II curve (in the negative half-plane of \( S \)) is surrounded by orbits with secular angle \( \Delta \delta_{b,c} \) librating around \( \pi \) (see Michtchenko & Malhotra 2004 for details). The right-hand panel of Fig. 7 is for the phase diagram constructed for a fixed angular momentum in the \( (e_5 \cos \Delta \delta_{b,c}, e_b \sin \Delta \delta_{b,c}) \)-plane, illustrating the equilibria and neighboring phase trajectories, including the nominal configuration marked with open circles. White regions in the left-hand panel of Fig. 7 indicate eccentricities and \( \Delta \delta_{b,c} \) for which the HZ is strongly affected by the \( v_1 \) resonance, hence \( s_{HZ} = 0 \). Black regions are for \( s_{HZ} \sim 1 \). Before integrating secular orbits of Tellus, we must average out orbits of primaries, hence we have also “an occasion” to eliminate unstable configurations (disrupted during \( \sim 10 \) Myrs, \( \langle Y \rangle > 10 \)). They are marked with gray crosses. Nominal eccentricities of the Jupiters are marked with filled circles (labeled with \( I_0 \) for \( \Delta \delta_{b,c} = 0 \), and \( I_0 \) for \( \Delta \delta_{b,c} = \pi \)). For the right-hand half-plane, \( s_{HZ} \sim 0.5 \), and for the left-hand half-plane of \( S, s_{HZ} \sim 0.95 \). Of course, it must be in accord with the results of two-dimensional surveys (see Fig. 5) performed for the same initial conditions.

Clearly, the region of large \( s_{HZ} \) lies in the neighborhood of stationary mode II. On contrary, the secular resonance in the configurations selected in the positive half-plane (\( \Delta \delta_{b,c} = 0 \)) preserves stable orbits in a very small part of the HZ. This seems a general feature of the system, in fact it can be explained through the fundamental frequencies. Close to the equilibria, \( \Delta \delta_{b,c} \) oscillates around 0, or \( \pi \). To the first significant octupole terms of \( H_{sec} \),

\[
\frac{de_0}{dt} \sim A \sin \Delta \delta_{b,c} + B \sin \Delta \delta_{b,c},
\]

where \( A \) and \( B \) are coefficients dependent on the orbital elements, having the same sign. Hence, the anti-aligned orbits of the giant planets favor small (or much smaller) excitation of \( e_0 \) than the aligned jovian orbits.

The results of the next simulation are shown in Fig. 8. Now, we changed the semi-major axes to \( a_b = 2.5 \text{ au}, a_c = 4.1 \text{ au} \) (our model \( V_0 \), in the 2c:1b MMR). We recall that the middle- and the right-hand panels of Fig. 2 are for the dynamical maps of this configuration in the \( (e_0, e_1) \)-, and \( (a_0, e_3) \)-planes. Figure 8 shows the results in terms of color-coded \( s_{HZ} \) plotted in the right-hand half-plane of \( S \) (there is no equivalent of a stable region in the left-hand half-plane of \( S \)). We compute the \( s_{HZ} \) for jovian orbits stable at least over \( \sim 10 \) Myrs. There is a perfect match of the border of stable zone derived through the long-term integrations, and with the MEGNO map (both maps are plotted in Fig. 8 see also Fig. 5), although the indicator was calculated over \( \sim 0.4 \) Myr “only”. Comparing these two maps, we found a transient zone around \( (a_0 \cos \Delta \delta_{b,c}, e_1) \sim (0.45 \text{ au}, 0.05) \), in which MEGNO already detects chaotic motions, while the system is disrupted after a few Myrs. It confirms a precise calibration of the integration time of \( \langle Y \rangle \).

4.2 The habitable zone in the \( (a_b, a_c) \)-plane

Finally, we computed \( s_{HZ} \) in the \( (a_b, a_c) \)-plane, fixing the reference values of \( e_0 = 0.03, e_1 = 0.05 \) (our model I). For a comparison, we choose two values of \( \Delta \delta_{b,c} = 0, \pi \). The right-hand panel of Fig. 9 is for initially aligned orbits of Jupiters, the left-hand panel is for initially anti-aligned orbits. The position of the nominal system in
Figure 5. A comparison of the maximal eccentricity of Tellus derived with the help of the semi-analytical theory, in the range of \( a_0 \) relevant for the HZ in model \( I \). The initial eccentricity \( e_0 = 0.01 \) and \( \Delta \omega_{0,c} = 0 \). The left-hand panel is for \( \Delta \omega_{0,c} = 0 \), the right-hand panel is for \( \Delta \omega_{0,c} = \pi \). The black curve is for the model with the general relativity corrections, the gray curve is for the classic, Newtonian point-mass model.

Figure 6. Color coded maximal eccentricity of Tellus in the \((a_0, e_0 \cos \Delta \omega_{0,c})\)-plane; \( \Delta \omega_{0,c} = 0 \) in the positive half-plane, and \( \Delta \omega_{0,c} = \pi \) in the negative half-plane. The thick, dotted curves mark geometric borders of the HZ with accord to condition (Eq. 1 see the text for details). The green solid curves border initial conditions of Tellus implying orbits confined to the HZ during at least \( \sim 10 \) Myrs. Subsequent panels are for the orbital configurations of Jupiters given in Table 1 and labeled accordingly, see also Fig. 4 (models \( I \)–\( V \)).

the parameter space is marked with filled circles. Crosses, along straight lines, indicate unstable solutions detected with the help of MEGNO and, in fact, they are related to the relevant, low-order MMRs, e.g., 5c:2b, 3c:1b and 7c:2b. Clearly, the jovian system would be more “friendly” for inhabitants of Tellus, when the orbits are anti-aligned in the mean, and this result supports a similar conclusion following simulations illustrated in Figs. 3–4. The secular \( \nu_1 \) resonance manifests itself as an upside-down V-like structure (marked with yellow color) in the right-hand panel of Fig. 9. It is located between the 5c:2b and 3c:1b MMRs. Still, even for \( \Delta \omega_{0,c} = 0 \), there is a large region above the 3c:1b MMR line, which is accessible for the HZ-stable orbits of Tellus. These results are in accord with shaded areas illustrated in Fig. 1 of Malhotra & Minton (2008). Here, we found that mode II configurations (characterized by librations of \( \Delta \omega_{0,c} \) around \( \pi \) for the neighboring trajectories), permit stable orbits of Tellus for almost entire observationally determined range of semi-major axes and eccentricities implying dynamically stable configurations of the jovian sub-system.

5 CONCLUSIONS

The detection levels of extrasolar planets reach masses of super-Earths (i.e., Neptune-mass objects). The results of recent RV and transit surveys [Bouchy et al. (2008); Bakos et al. (2009), Udry 2008 (an invited talk in Toruń conference "Extrapol Planets in Multi-body Systems") suggest that such small planets are common.
Figure 7. The left-hand panel. Color coded \( s_{HZ} \) in the \( S \)-plane for \( a_b = 2.3 \) au, \( a_c = 4.6 \) au (model \( I_{0,\pi} \)). Gray crosses indicate unstable configurations of Jupiters in terms of \( \langle Y \rangle > 10 \). Nominal elements for \( \Delta \sigma_{b,c} = 0, \pi \) are labeled with \( I_0 \) and \( I_\pi \), respectively. Dotted areas are for initial conditions implying librations of \( \Delta \sigma_{b,c} \) around 0 (the right-hand half-plane of \( S \)) and around \( \pi \) (the left-hand half-plane of \( S \)). The right-hand panel. The phase diagram calculated for a constant level of the total angular momentum (marked with gray curve in the \( S \)-plane).

Figure 8. Color coded coefficient of habitability, \( s_{HZ} \), illustrated in the \( S \)-plane. Jovian semi-major axes are \( a_b = 2.5 \) au, and \( a_c = 4.1 \) au, \( \Delta \sigma_{b,c} = 0 \) (model \( V_0 \), planets involved in the 2:1b MMR). Gray crosses indicate strongly unstable configurations of the jovian sub-system, with \( \langle Y \rangle > 10 \). The tested configuration (model \( V_0 \), see Table 1) is marked with a circle.

Figure 9. Color coded coefficient of habitability, \( s_{HZ} \), illustrated in the \( (a_b, a_c) \)-plane. The left-hand panel is for \( \Delta \sigma_{b,c} = \pi \), and the right-hand panel is for \( \Delta \sigma_{b,c} = 0 \). Eccentricities of the jovian planets are \( e_b = 0.03 \), \( e_c = 0.05 \) (models \( I_{0,\pi} \)). Gray crosses indicate chaotic configurations related to MMRs (labeled) of the b-c pair, \( \langle Y \rangle > 10 \).

Hence, we expect a growing interest in dynamical studies of extrasolar systems with terrestrial planets. Although we focus on the best analog of the Solar system (Gaudi et al. 2008), in fact, our work concerns a whole class of multi-planet systems involving a hypothetical Earth-like object in the HZ of a low-mass star hosting also large, jovian planets. To investigate the structure of the HZ, we adopted a simple quasi-analytic theory applicable to hierarchical configurations of the terrestrial planet and jovian companions. Then the averaging principle can be applied, and the four-body problem may be further simplified in terms of the restricted model. Our quasi-analytic approach makes it possible to investigate the secular motion in an uniform way. Both the resonant and non-resonant orbits of primary bodies (Jupiter-mass planets) may be averaged out numerically. The secular equations of motion derived from the analytic theory, make it possible to reduce the CPU time by a few orders of magnitude. Hence, the method is useful to review the global features of the secular dynamics. In accord with the assumptions of the averaging principle, the quasi-analytic model can be also easily adopted to study long-term evolution of similar, dynamically scaled configurations, for instance the motion of jovian planets in multiple systems with brown dwarfs or substellar companions.

Regarding the OGLE-2006-BLG-109L system, we found that the structure of its HZ strongly depends on orbits of Jupiters, and not only on their semi-major axes and eccentricities, but also on the orbital phase and a libration mode of apsidal angle \( \Delta \sigma_{b,c} \). From the recent theory we know (Michtchenko & Malhotra 2004), that apsidal librations around 0 or \( \pi \) are generic orbital states of non-resonant, two-planet systems. In general, we found that the anti-aligned mode of the jovian orbits favors much larger fraction of stable orbits remaining entirely in the annulus of HZ between or-
habitable radii of 0.25 au and 0.36 au, than are permitted by the aligned configurations. Having in mind unconstrained parameters of the OGLE-2006-BLG-109L system, many scenarios are possible, even assuming that only one terrestrial planet resides in the HZ. The disastrous $\nu_1$ secular resonance can be moved out of the HZ, through perturbations of additional smaller planets (Malhotra & Minton 2008). They have shown that if such planets exist in the system then the secular dynamics of Earth-like planets would be very complex. Here, we also demonstrate that the dynamics are rich in the realm of the restricted four-body problem.

Still, there remains an open question whether the creation of Earth-like bodies is possible in the OGLE-2006-BLG-109L system. We did preliminary simulations (Musieliński et al., in preparation) with the help of MERCURY code (Chambers 1999), setting orbital parameters of the Jovian system consistent with the error bounds (Gaudi et al. 2008). We applied the common model of elastic coagulation of small protoplanets and planetesimals (see, e.g., Raymond 2008). The initial distribution of planetary embryos involved $\sim 50$ Moon-sized objects distributed between 0.2 au and 2 au. Curiously, as a typical outcome from 50 runs of 50 Myrs each, we obtained single 0.3–0.8 Earth-mass planet around 0.3 au and (sometimes) a second object of a similar mass beyond 0.6–0.8 au. The eccentricity distribution in the small sample of runs is basically uniform, with a number of quasi-circular orbits. These results are encouraging but they must be confirmed by new, intensive simulations. Nevertheless, we already found some evidence, that the terrestrial planets may emerge in the HZ of the OGLE-2006-BLG-109L system, and we have got a new motivation to investigate the effects of the $\nu_1$ resonance in this region.

In this work, we applied a few different techniques to study the dynamics of planetary systems. In particular, we apply the fast indicator (the symplectic MEGNO algorithm) to map the phase space of the system and to detect chaotic (unstable) configurations. The numerical integrations of the equations of motion are necessary to calibrate the integration time of the fast indicator and to test the quality of analytic expansions with the help of semi-analytical averaging. To study the structure of the HZ, it is necessary to analyse simultaneously the short-term and the long-term behavior of the system. As we think, our study benefits from the synthesis of analytical and numerical techniques.

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REFERENCES

Bakos G. A., et al., 2009, ApJ (arXiv:0901.0282)  
Bouchy F., et al., 2008, A&A (arXiv:0812.1608)  
Chambers J. E., 1999, MNRAS, 304, 793  
Cincotta P. M., Giordano C. M., Simó C., 2003, Physica D Nonlinear Phenomena, 182, 151  
Cincotta P. M., Simó C., 2000, A&A Supp. Ser., 147, 205  
Ferraz-Mello S., Michtchenko T. A., Beaugé C., 2006, Regular motions in extra-solar planetary systems. Chaotic Worlds: from Order to Disorder in Gravitational N-Body Dynamical Systems, pp 255–
Gaudi et al. B. S., 2008, Science, 319, 927  
Goździewski K., Breiter S., Borczyk W., 2008, MNRAS, 383, 989  
Hinse T. C., Michelsen R., Jørgensen U. G., Goździewski K., Mikkola S., 2008, A&A, 488, 1133  
Innanen K., Mikkola S., Wiegert P., 1998, AJ, 116, 2055  
Kasting J. F., Whitmire D. P., Reynolds R. T., 1993, Icarus, 101, 108  
Malhotra R., Fox K., Murray C. D., Nicholson P. D., 1989, ApJ, 221, 348  
Malhotra R., Minton D. A., 2008, ApJL, 683, L67  
Michtchenko T. A., Malhotra R., 2004, Icarus, 168, 237  
Migaszewski C., Goździewski K., 2008, MNRAS, 388, 789  
Migaszewski C., Goździewski K., 2009, MNRAS, 392, 2  
Morbidelli A., 2002, Modern celestial mechanics: aspects of Solar system dynamics. Taylor & Francis  
Murray C. D., Dermott S. F., 2000, Solar System Dynamics. Cambridge University Press, 2000.  
Namouni F., Murray C. D., 1999, AJ, 117, 2561  
Pilat-Lohinger E., Robutel P., Süli Á., Freistetter F., 2008, Celestial Mechanics and Dynamical Astronomy, 102, 83  
Pilat-Lohinger E., Süli Á., Robutel P., Freistetter F., 2008, ApJ, 681, 1639  
Raymond S. N., 2008, in IAU Symposium 249 of IAU Symposium, Terrestrial planet formation in extra-solar planetary systems, pp 233–250