Optomechanical sideband cooling of a thin membrane within a cavity

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\textbf{Abstract.} We present an experimental study of dynamical back-action cooling of the fundamental vibrational mode of a thin semitransparent membrane placed within a high-finesse optical cavity. We study how the radiation–pressure interaction modifies the mechanical response of the vibrational mode, and the experimental results are in agreement with a Langevin equation description of the coupled dynamics. The experiments are carried out in the resolved sideband regime, and we have observed cooling by a factor of \(\approx 350\). We have also observed the mechanical frequency shift associated with the quadratic term in the expansion of the cavity mode frequency versus the effective membrane position, which is typically negligible in other cavity optomechanical devices.

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1. Introduction

The preparation and manipulation of macroscopic mechanical systems in the quantum regime has drawn much interest over the last decade [1–4]. Mechanical resonators provide unique opportunities in a disparate field of applications, such as the detection of forces [5], displacements and masses [6] at the ultimate limits imposed by the Heisenberg principle, the realization of quantum information architectures where they can act as a universal quantum bus [7] and for fundamental tests of quantum theory [8, 9]. An important prerequisite for operating in the quantum regime is to reduce as much as possible the thermal noise effect, and this can be obtained by cooling the mechanical resonator close to its quantum ground state. Important results have recently been achieved on this. A GHz frequency piezomechanical oscillator has been cooled to the quantum regime with conventional cryogenics and then probed by a superconducting qubit [10]. In contrast, cooling schemes based on dynamical back-action caused by the parametric coupling with an optical or microwave cavity [11] can be applied to a wider class of nanomechanical and micromechanical resonators which, due to the lower resonance frequency, are practically impossible to cool using only cryogenic techniques. In particular, it has been theoretically shown that dynamical back-action cooling allows one to reach the quantum ground state of a mechanical mode [12–15] in the resolved sideband regime where the cavity linewidth is much smaller than the mechanical frequency. In this limit and if the cavity is resonant with the anti-Stokes sideband of the driving laser, effective phonon emission into the cavity is enhanced and phonon absorption is suppressed, yielding a large net laser cooling rate. After first demonstrations [16–21], sideband cooling has recently been employed to reach a phonon occupancy $n_{\text{eff}} < 1$ for an aluminum membrane capacitively coupled to a cryogenic microwave cavity [22] and for an integrated optical and mechanical nanoscale resonator in a photonic crystal structure [23].

Dynamical back-action cooling can be equivalently described as the consequence of a modification of the mechanical susceptibility of the resonator caused by optomechanical interactions. In fact, the back-action of a detuned cavity modifies both the frequency (the so-called ‘optical spring effect’ [19, 24]) and the damping of the mechanical resonator. When the resonator is overdamped due to back-action, its susceptibility at resonance gets strongly suppressed. As a consequence, the resonator becomes less sensitive to thermal noise, which leads to cooling. Studies of the modification of the mechanical response as a function of the
cavity detuning have been carried out in Fabry–Perot (FP) cavities with a movable micro-
mirror [17, 20], and in the well-resolved sideband regime alone in silica toroidal optomechanical
systems [25]. Here we perform a detailed study of dynamical back-action cooling and of
radiation–pressure modifications of the mechanical properties in a membrane-in-the-middle (MIM)
system formed by a vibrating thin silicon nitride (Si$_3$N$_4$) semitransparent membrane
with high mechanical quality factor, placed within a high-finesse cavity. Such a scheme has
been introduced in [26] and has been studied in [27–29] (see also [30] for a similar scheme
with a GaAs membrane cooled via an optical absorption process and [31] that studies a scheme
based on a nanomechanical scattering element within a cavity). We find that the experimental
results are in excellent agreement with the theoretical prediction of [32] that included also the
effect of membrane absorption and of an additional frequency shift caused by the presence
of quadratic terms in the effective membrane position $\hat{q}$, which are typically negligible in
most optomechanical systems. We measure the effective temperature of the cooled vibrational
mode in three different ways, obtaining consistent results, in agreement with the theoretical
expectations. In particular we demonstrate resolved sideband cooling by a factor of $\approx 350$
starting from room temperature.

The paper is organized as follows. Section 2 describes the experimental setup and section 3
adapts the theoretical description of [32] to the present experimental conditions. Section 4
illustrates the experimental results and their matching with the theory predictions on the
mechanical frequency shift, the effective damping and the effective temperature. Section 5
presents the concluding remarks.

2. The experimental setup

Our MIM setup is schematically described in figure 1. Laser light at $\lambda = 1\,064$ nm is produced
by a Nd:YAG laser (Innolight). After exiting the laser head, the light beam passes through a
half-wave plate (HWP) followed by a polarizing beam splitter (PBS). By rotating the HWP the
optical power can be distributed between a probe and a pump beam, with frequencies $\omega_p$ and $\omega_L$,
respectively. The probe beam power is 100 $\mu$W, while the rest, i.e. about 200 mW, is fed into the
pump beam optical line: at the end only a small fraction of it is used. The frequency detuning
from the probe beam is controlled by two cascaded acousto-optical modulators (AOMs) whose
central operating frequency is 80 MHz. By selecting diffracted beams of the first order but with
opposite signs, detunings from 0 to 40 MHz can be obtained, although only detunings up to
500 kHz have been used. The pump beam intensity is controlled by the modulation amplitude
of the electrical signal used for driving AOM$_2$ (see figure 1). After the AOMs the pump beam
passes through an optical isolator (OFR$_2$) and is mode matched to the FP optical cavity by
means of two lenses (L$_1$ and L$_3$). Before being injected in the FP cavity, the pump beam is then
combined with the probe beam by means of PBS$_2$.

The cavity is $L \approx 93$ mm long and consists of two equal dielectric mirrors, each with a
radius of curvature $R = 10$ cm. The measured value of the empty cavity finesse is $\mathcal{F} \approx 60,000$
and is consistent with the mirror’s nominal reflectivity. Halfway between the mirrors a thin
stoichiometric silicon nitride membrane is mounted on series of piezo-motor driven optical
mounts that control the angular alignment as well as the linear positioning with respect to the
optical axis.

The membrane is a commercial 1 mm $\times$ 1 mm Si$_3$N$_4$ stoichiometric x-ray window
(Norcada) with nominal thickness $L_d = 50$ nm and index of refraction $n_R \approx 2$, supported on
a 200 \mu m Si frame. It has been chosen due to its high mechanical quality factor and very low optical absorption at \( \lambda = 1064 \text{ nm} \) [33]. Its optical properties were also experimentally verified, yielding an intensity reflection coefficient of \( R \approx 0.18 \) and an imaginary part of the index of refraction \( n_I \approx 2 \times 10^{-6} \).

The membrane’s mechanical motion is monitored by the probe beam with frequency \( \omega_p \) that is also used for locking the laser to the FP instantaneous resonant frequency. This is done by polarization multiplexing of the fields on PBS\(_2\) and by observing only the probe light reflected from the cavity by a photodiode PD\(_2\), whose output signal is amplified and fed into a frequency locking loop and a spectrum analyzer where the membrane’s mechanical motion is observed. The locking scheme is a standard Pound–Drever–Hall (PDH) scheme [34, 35], where the sidebands necessary for obtaining the error signal fed into the locking loop are created by modulating directly the laser crystal [36]. Although the pump field is modulated as well, its sidebands are out of resonance and do not contribute to the optomechanical dynamics. In order to avoid the deterioration of the mechanical properties of the membrane and optical properties of the FP cavity, the cavity is mounted inside a vacuum chamber which is evacuated by a turbo-molecular pump down to \( 10^{-5} \text{ mbar} \). Once the base pressure is reached, the pump is switched off and disconnected from the vacuum chamber in order to minimize the mechanical noise. Due to some leaks, the residual pressure inside the chamber reaches values as high as \( 10^{-2} \text{ mbar} \) in about 1 day. This has caused a worsening of the membrane’s mechanical quality factor, which did not reach values of the order of 1 million, as instead reported in [33]. A vibrationless pumping scheme is being prepared for future measurements.
3. A quantum Langevin description

Radiation pressure of the intracavity field excites the membrane vibrational modes, and therefore one has a multimode bosonic system in which mechanical and optical modes interact in a nonlinear way. However, one can adopt a simplified description based on a single-cavity mode interacting with a single mechanical mode [26, 28, 29, 32]. One can restrict to a single-cavity mode if the driving laser populates a given cavity mode only (here a TEM$_{00}$ mode, associated with the annihilation operator $\hat{a}$), and if scattering into other modes is negligible [37]. Moreover, one can consider a single mechanical mode of the membrane (described by dimensionless position $\hat{q}$ and momentum $\hat{p}$ operators, such that $[\hat{q}, \hat{p}] = i$) when the detection bandwidth is chosen so that it includes only a single isolated mechanical resonance with frequency $\Omega_m$.

By explicitly including cavity driving by the pump laser with frequency $\omega_L$ and input power $P$, the system Hamiltonian reads
\[
\hat{H} = \frac{\hbar \Omega_m}{2} (\hat{p}^2 + \hat{q}^2) + \hbar \omega(\hat{q})\hat{a}^\dagger \hat{a} + i \hbar E (\hat{a}^\dagger e^{i\Omega_1 t} - \hat{a} e^{-i\Omega_1 t}),
\]
where $E = \sqrt{2P/\hbar \omega_0}$, with $\omega_0$ the coupling rate through the input mirror. The optomechanical interaction is described by the position-dependent optical frequency
\[
\omega(\hat{q}) = \omega_0 + \text{Re}[\delta \omega(\hat{z}_0(\hat{q}))],
\]
where $\omega_0$ is the cavity mode frequency in the absence of the membrane, and $\text{Re}[\delta \omega(\hat{z}_0(\hat{q}))]$ is the frequency shift caused by the insertion of the membrane. This shift depends on the membrane position along the cavity axis $\hat{z}_0(\hat{q})$, which in turn depends on the coordinate $\hat{q}$, as one can see by writing $\hat{z}_0(\hat{q}) = \hat{z}_0 + x_0 \Theta \hat{q}$, where $\hat{z}_0$ is the membrane center-of-mass position along the cavity axis, $\Theta$ is the transverse overlap integral between the optical mode and the vibrational mode [32], and $x_0 = \sqrt{\hbar/m \Omega_m}$, with $m$ the effective mass of the mechanical mode. The parameter $x_0$ is the natural unit length of the problem and corresponds to the width of the zero-point position fluctuations of the mechanical resonator multiplied by $\sqrt{2}$.

The system is, in general, affected by fluctuation–dissipation processes: the mechanical mode undergoes a viscous force with damping rate $\gamma_m$ and a Brownian stochastic force with zero mean value $\xi(t)$. We operate at room temperature $T$, and the correlation function of this Brownian noise is well approximated by the following [38–40]:
\[
\langle \xi(t) \xi(t') \rangle \approx \gamma_m \left( \frac{2n + 1}{\Omega_m} + \frac{\delta'(t - t')}{\Omega_m} \right),
\]
where $k_B$ is the Boltzmann constant, $n = [\exp(\hbar \Omega_m/k_B T) - 1]^{-1} \approx k_B T/\hbar \Omega_m$ is the mean thermal phonon number at temperature $T$ and $\delta'(t - t')$ denotes the derivative of the Dirac delta.

The cavity mode loses photons through the input mirror with decay rate $\kappa_0$, through the back mirror with decay rate $\kappa_2$, and also due to optical absorption of the membrane with decay rate $\kappa_1(\hat{q}) \equiv |\text{Im}[\delta \omega(\hat{z}_0(\hat{q}))]|$, which is nonzero owing to the small imaginary part of the refractive index, $n_1$ [32]. Optical absorption of the membrane depends on the mechanical position operator $\hat{q}$, and represents therefore a nonlinear dissipative process affecting both the optical and the mechanical modes. Each decay channel is associated with a vacuum optical input noise $\hat{a}^\dagger_j(t)$, $j = 0, 1, 2$, with correlation functions [39]
\[
\langle \hat{a}_j^\dagger(t) \hat{a}_j^\dagger(t') \rangle = \delta_{kj} \delta(t - t').
\]
Adding the above damping and noise terms to the Heisenberg equations of motion derived from the Hamiltonian of equation (1), one obtains the following set of nonlinear quantum Langevin equations (QLE) that, in the frame rotating at the pump laser frequency $\omega_L$, read

$$\dot{\hat{q}} = \Omega_m \hat{p},$$

$$\dot{\hat{p}} = -\Omega_m\hat{q} - \gamma_m\hat{p} - \partial_q \omega(\hat{q})\hat{a}^\dagger \hat{a} + \hat{\xi} - i \frac{\partial_q \kappa_1(\hat{q})}{\sqrt{2\kappa_1(\hat{q})}} (\hat{a}^\dagger \hat{a}_1^{in} - \hat{\alpha}_1^{in,\dagger}),$$

$$\dot{\hat{a}} = -i [\omega(\hat{q}) - \omega_L] \hat{a} - \kappa_1(\hat{q}) \hat{a} + E + \sqrt{2\kappa_0} \hat{a}_0^{in} + \sqrt{2\kappa_1(\hat{q})} \hat{a}_1^{in} + \sqrt{2\kappa_2} \hat{a}_2^{in},$$

where $\partial_q$ denotes the derivative with respect to $\hat{q}$, and $\kappa_1(\hat{q}) = \kappa_0 + \kappa_1(\hat{q}) + \kappa_2$ is the total cavity decay rate.

Equations (5)–(7) illustrate the peculiar aspects of the MIM scheme with respect to the paradigm optomechanical system represented by a FP cavity with a highly reflecting movable micro-mirror, which satisfactorily applies to a large number of optomechanical devices [41]. In the latter scheme, $\omega(\hat{q}) = \omega_0(1 - x_0 \hat{q}/L)$ and the optical absorption can be usually neglected $[\kappa_1(\hat{q}) \approx 0]$, and therefore the nonlinearities in $\hat{q}$ appearing in equations (5)–(7) are absent. Our experimental results will show that the quadratic term in the power expansion of $\omega(\hat{q})$ has appreciable effects on the optically induced mechanical frequency shift. In contrast, we will see that membrane absorption can be neglected also in our case, due to the very low value of $n_1$ of the stoichiometric Si$_3$N$_4$ membrane employed here.

### 3.1. Linearized quantum Langevin equations

Our experiment is carried out in the usual ‘linearized’ regime characterized by an intense stationary intracavity field with amplitude $\alpha_s$ ($|\alpha_s| \gg 1$), easily achievable with moderate pump power due to the large cavity finesse $F$. The vibrational mode is correspondingly deformed, with a new stationary position $q_s$, satisfying, together with $\alpha_s$, the coupled nonlinear conditions

$$q_s = -\frac{\partial_q \omega(q_s)|\alpha_s|^2}{\Omega_m},$$

$$|\alpha_s|^2 = \frac{E^2}{\kappa_1(q_s)^2 + (\omega_L - \omega(q_s))^2},$$

which may show optical bistability [42–44].

When the system is stable, the relevant dynamics concern the fluctuations of the cavity and mechanical modes around the classical steady-state described by equations (8) and (9). Rewriting each Heisenberg operator of equations (5)–(7) as the classical steady-state value plus an additional fluctuation operator with zero mean value, and neglecting all the nonlinear terms in the equations, one obtains the following linearized QLE for the fluctuations [32]:

$$\delta \dot{\hat{q}} = \Omega_m \delta \hat{p},$$

$$\delta \dot{\hat{p}} = -[\Omega_m + \frac{\gamma_m}{\sqrt{2}} |\alpha_s|^2] \delta \hat{q} - \gamma_m \delta \hat{p} + \frac{G}{\sqrt{2}} (\delta \hat{a} + \delta \hat{a}^\dagger) + \hat{\xi} + i \frac{\Gamma}{2\kappa_1(q_s)} (\hat{a}_1^{in} - \hat{a}_1^{in,\dagger}),$$

$$\delta \dot{\hat{a}} = -[\kappa_1(q_s) + i \Delta] \delta \hat{a} + \frac{i G - \Gamma}{\sqrt{2}} \delta \hat{q} + \sqrt{2\kappa_0} \hat{a}_0^{in} + \sqrt{2\kappa_1(q_s)} \hat{a}_1^{in} + \sqrt{2\kappa_2} \hat{a}_2^{in},$$

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We have redefined the phase reference of the cavity field so that $\alpha_s$ is real and positive, and we have defined the effective detuning $\Delta = \omega(q_s) - \omega_L$ and $\Gamma = \sqrt{2} \partial_q \kappa_1(q_s) \alpha_s$. The linearized QLE (10)–(12) show that the mechanical and cavity mode fluctuations are coupled by the effective optomechanical coupling

$$G = - \sqrt{2} \partial_q \omega(q_s) \alpha_s = -2 \left( \frac{\partial \omega}{\partial z_0} \right) \Theta \sqrt{\frac{P \kappa_0}{m \Omega_m \omega_L \left[ \kappa_1(q_s)^2 + \Delta^2 \right]}}$$

(13)

which can be enhanced by increasing the intracavity amplitude $\alpha_s$. Furthermore, $G$ can be fine tuned in the MIM system by shifting the membrane along the cavity axis, thereby changing $\partial \omega / \partial z_0$.

4. Effect of radiation pressure on the membrane vibrational mode

We observe the motion of the vibrational mode by detecting the noise spectrum of the phase $\phi(\omega)$ of the resonant weak probe field reflected by the cavity by means of the PDH technique. This phase is related to the effective position $\hat{q}$ by

$$\phi(\omega) = \frac{G_p}{\kappa_1(q_s) - i \omega} \delta \hat{q}(\omega) + s(\omega),$$

(14)

where

$$G_p = -2 \left( \frac{\partial \omega}{\partial z_0} \right) \Theta \sqrt{\frac{P \kappa_p}{m \Omega_m \omega_p \kappa_1(q_s)^2}}$$

(15)

is the effective optomechanical coupling of the resonant probe beam, analogous to the coupling of the driving field of equation (13), but with the corresponding input power $P_p$, coupling rate $\kappa_p$ and overlap integral $\Theta_p$. $s(\omega)$ denotes the detection noise, essentially given by shot noise, which at resonance is uncorrelated with $\delta \hat{q}$, implying that the phase noise spectrum is given by

$$S_\phi(\omega) = \left[ \frac{G_p^2}{\kappa_1(q_s)^2 + \omega^2} \right] S_{\hat{q}}(\omega) + S_s,$$

(16)

where $S_{\hat{q}}(\omega)$ is the spectrum of the dimensionless position $\hat{q}$, and $S_s$ is the (typically flat) shot noise spectrum. After calibration in $m^2 \text{Hz}^{-1}$ (and using $\hat{x} \equiv x_0 \hat{q}$), the detected spectrum can be written as

$$S_{\hat{x}}^{\text{det}}(\omega) = x_0^2 S_{\hat{q}}(\omega) + \left[ \frac{\kappa_1(q_s)^2 + \omega^2}{G_p^2} \right] x_0^2 S_s.$$

(17)

The explicit expression for $S_{\hat{q}}(\omega)$ is obtained by solving the linearized QLE (10)–(12) in the frequency domain [14, 32, 45] and reads

$$S_{\hat{q}}(\omega) = |\chi_{\text{eff}}(\omega)|^2 \left[ S_{\text{th}}(\omega) + S_{\text{rp}}(\omega) + S_{\text{abs}}(\omega) \right],$$

(18)

where

$$S_{\text{th}}(\omega) = \frac{\gamma_m \omega}{\Omega_m} \coth \left( \frac{\hbar \omega}{2 k_B T} \right) \approx \frac{2 \gamma_m k_B T}{\hbar \Omega_m}$$

(19)
is the thermal noise spectrum,

\[ S_{th}(\omega) = \frac{G^2\kappa_L(q_s)[\Delta^2 + \kappa_L^2(q_s) + \omega^2]}{[\kappa_L^2(q_s) + (\omega - \Delta)^2][\kappa_L^2(q_s) + (\omega + \Delta)^2]} \]

(20)

is the radiation–pressure noise spectrum due to the intense pump beam at frequency \( \omega_L \), and

\[ S_{abs}(\omega) = \frac{\Gamma^2 G}{4\kappa_L(q_s)} \left[ \frac{\Gamma G \Delta^2 + \kappa_L^2(q_s) - \omega^2}{[\kappa_L^2(q_s) + (\omega - \Delta)^2][\kappa_L^2(q_s) + (\omega + \Delta)^2]} \right] \]

(21)

is the additional noise spectrum associated with membrane absorption [32]. It is worth noting that the position (phase) noise is dominated only by the intense pump field \( (G) \), and that the probe field \( (G_p) \) is only involved in the detection. The main effect of the optomechanical interaction on the membrane vibrational mode is the modification of its mechanical susceptibility (see equation (18)), which becomes

\[ \chi_{\text{eff}}(\omega) = \frac{\Omega_m}{\Omega_m^2 - \omega^2 - i\omega\gamma_m - \frac{G\Omega_m^2[\Gamma\kappa_L(q_s)-i\omega]}{[\kappa_L(q_s)-i\omega]^2 + \Delta^2}}, \]

(22)

where

\[ \hat{\Omega}_{\text{m}}^2 = \Omega_m^2 + h\Omega_m, \quad h = \frac{\partial^2 \omega(q_s)\alpha_s^2}{\partial q_s^2} = (\partial^2 \omega/\partial q_s^2)\kappa_0^2 \theta^2 |\alpha_s|^2, \]

(23)

and \( \hat{\Omega}_m \) is the mechanical frequency modified by the second-order contribution to the expansion of \( \omega(\hat{q}) \).

This effective susceptibility can be read as the susceptibility of an oscillator with effective resonance frequency and damping rate, respectively, given by

\[ \Omega_m^{\text{eff}}(\omega) = \sqrt{\hat{\Omega}_{\text{m}}^2 - \frac{G\Omega_m^2[\Gamma\kappa_L(q_s) - \omega^2 + \Delta^2]}{[\kappa_L^2(q_s) + (\omega - \Delta)^2][\kappa_L^2(q_s) + (\omega + \Delta)^2]}} \]

(24)

\[ \gamma_m^{\text{eff}}(\omega) = \gamma_m + \frac{G\Omega_m^2[2\Gamma\Delta\kappa_L(q_s) - \Gamma\kappa_L(q_s) + \omega^2 - \Delta^2]}{[\kappa_L^2(q_s) + (\omega - \Delta)^2][\kappa_L^2(q_s) + (\omega + \Delta)^2]}. \]

(25)

The present experiment is carried out in the weak-coupling regime \( G \ll \Omega_m \), and consequently the effective susceptibility \( \chi_{\text{eff}}(\omega) \), even if appreciably modified, remains peaked approximately around \( \Omega_m \). Therefore, the effective mechanical frequency and damping modified by the optomechanical interaction can be satisfactorily estimated by evaluating equations (24) and (25) at \( \omega = \Omega_m \).

The modification of the mechanical response due to the interaction with the driven cavity mode is clearly visible in figure 2, where the calibrated position spectrum is shown at different positive values of the detuning of the driving field \( \Delta \): the mechanical resonance peak shifts and broadens with varying \( \Delta \). We note that just outside the resonance peak, the displacement sensitivity of our optomechanical detection system is equal to \( \sqrt{S_x} \approx 2.4 \times 10^{-15} \text{ m}(\sqrt{\text{Hz}})^{-1} \approx 40\sqrt{S_{x,\text{SQL}}}(\Omega_m) \), where \( S_{x,\text{SQL}}(\Omega_m) = 2hQ/m\Omega_m^2 \) is the standard quantum limit (SQL) for a displacement spectral measurement at its peak value at resonance. This value is not far from the value \( \sqrt{S_x} = 1.9 \times 10^{-16} \text{ m}(\sqrt{\text{Hz}})^{-1} \) recently achieved in a Michelson–Sagnac interferometer also based on a SiN membrane [46].

We have studied in detail the effective mechanical frequency \( \Omega_m^{\text{eff}} \) and the effective mechanical damping \( \gamma_m^{\text{eff}} \) as a function of the detuning \( \Delta \), which is shown in figures 3 and 4.
Figure 2. Calibrated position noise spectrum around the resonance associated with the fundamental vibrational mode of the membrane, with bare mechanical frequency $\Omega_m/2\pi = 356.6$ kHz, mass $m = 45$ ng, quality factor $Q = 24000$, at different values of the detuning $\Delta/2\pi = (30, 60, 180, 280, 320, 340, 355, 380, 410, 600)$ kHz, from the lower to the upper curve. The membrane is fixed at 10 nm distance from a field node and the driving input power is $P = 670 \mu$W. The best-fit curve yields a negligible value for the membrane absorption-related parameter $\Gamma, |\Gamma| \approx 4 \times 10^{-8} \Omega_m$, which is consistent with the value $n_1 \approx 2 \times 10^{-6}$ estimated in [47] and reasonable for stoichiometric Si$_3$N$_4$ membranes [29]. The best-fit curve also yields a small, but non-negligible, value for the parameter $h$ related to the mechanical frequency shift caused by the second-order term of the expansion of the position-dependent cavity frequency $\omega(\hat{q})$, $h = 10^{-5} \Omega_m$ (around $\Delta = \Omega_m$).
Figure 3. Scaled effective mechanical damping $\gamma_m^{\text{eff}}/\gamma_m$ versus the scaled cavity-driving detuning $\Delta/\Omega_m$, in the region of positive detunings corresponding to a driving red-detuned with respect to the cavity mode. Dots are the experimentally measured values, while the red full line refers to the prediction of equation (25). See figure 2 and text for the other parameter values.

Figure 4. Mechanical frequency shift $\Omega_m^{\text{eff}} - \Omega_m$ versus the scaled cavity-driving detuning $\Delta/\Omega_m$, in the region of positive detunings corresponding to a driving red detuned with respect to the cavity mode. Dots are the experimentally measured values and the red full line refers to the prediction of equation (24). The membrane is shifted by $z_0 = 10$ nm along the cavity axis with respect to a field node. See figure 2 and text for the other parameter values.

This fact suggests that it should be possible to observe directly the frequency shift caused by the nonzero second-order derivative $\partial^2\omega/\partial z_0^2$, which is absent in FP cavities with a vibrating micromirror. Equation (24) suggests that such an effect should be visible around optical resonance $\Delta \approx 0$, where the frequency shift caused by the radiation–pressure interaction (the second fractional term in equation (24)) is negligible. This is confirmed by the data shown in figure 5, where the mechanical frequency shift $\Omega_m^{\text{eff}} - \Omega_m$ is plotted for a membrane position $z_0$.
Figure 5. Mechanical frequency shift $\Omega_{m}^{\text{eff}} - \Omega_{m}$ versus the membrane position along the cavity axis, from a node to an antinode of the cavity field, at the optical resonance $\Delta = 0$. Dots refer to the measured values and the continuous red line refers to the prediction of equation (24). The other parameter values are the same as in figure 2 except that $P = 76 \mu W$. The oscillatory behavior and the change of sign of the frequency shift are due to the similar behavior of the second-order derivative (see equation (23)). The narrow interval where the data depart from theory prediction corresponds to the presence of an avoided crossing between the driven TEM$_{00}$ mode and a higher-order transverse mode (see also text).

Measuring the position spectrum allows us to deduce also the effective mean thermal phonon number $n_{\text{eff}}$, i.e. the effective temperature $T_{\text{eff}}$, of the vibrational mode, as one can see from the general relation [12–15]

$$\hbar \Omega_{m}(n_{\text{eff}} + \frac{1}{2}) \equiv \frac{\hbar \Omega_{m}}{2} [\langle \delta \hat{q}^2 \rangle + \langle \delta \hat{p}^2 \rangle].$$

(26)

However, as shown in [14, 15], energy equipartition $\langle \delta \hat{p}^2 \rangle \approx \langle \delta \hat{q}^2 \rangle$ holds in a large parameter regime implying that

$$\langle \delta \hat{q}^2 \rangle \approx n_{\text{eff}} + \frac{1}{2} \equiv \frac{k_{B} T_{\text{eff}}}{\hbar \Omega_{m}}.$$

(28)
where the latter definition can be applied far from the quantum regime \( n_{\text{eff}} \approx 1 \), pertaining to our experimental situation. Therefore, \( T_{\text{eff}} \) can be obtained evaluating the area below the mechanical resonance peak after subtraction of the flat noise floor due to detection noise (see equation (17)).

There are two further ways of inferring \( T_{\text{eff}} \) from the measured position noise spectrum. On the one hand, \( T_{\text{eff}} \) can be deduced from \( \gamma_{\text{eff}} / \gamma_{m} \) of figure 3: from a thermodynamical point of view, due to radiation–pressure cooling, the vibrational mode passes from a thermal environment characterized by a Langevin force with strength proportional to \( \gamma_{m} T \) to an effective one with a Langevin force of the same strength, which is however proportional to \( \gamma_{\text{eff}} m T_{\text{eff}} \), so that \[ 49, 50 \] \[
T_{\text{eff}} = T \frac{\gamma_{m}}{\gamma_{\text{eff}}}. \quad (29)
\]

On the other hand, one can use the height of the resonant peak of the calibrated spectrum \( S_{q} (\omega) \). The extremely small value of \( \Gamma \), and the fact that our experiment is carried out at room temperature \( T = 295 \) K and in the weak-coupling limit \( G \approx -0.01 \Omega_{m} \) at \( \Delta = \Omega_{m} \) in figures 3 and 4, implies that both the radiation–pressure contribution \( S_{\text{rp}} (\omega) \) and the absorption contribution \( S_{\text{abs}} (\omega) \) are negligible with respect to that of thermal noise. Therefore one can safely assume that

\[
S_{q} (\omega) \approx \frac{2 \gamma_{m} k_{B} T}{h \Omega_{m}} \left| \chi_{\text{eff}} (\omega) \right|^{2}. \quad (30)
\]

Using the fact that at resonance \( \omega = \Omega_{m} \), the mechanical susceptibility becomes \( \left| \chi_{\text{eff}} \right|^{2} = \left( \Omega_{m} / \Omega_{m}^{\text{eff}} \gamma_{m} \right)^{2} \), and recalling that \( \gamma_{m} T = \gamma_{\text{eff}} T_{\text{eff}} \), equation (30) yields

\[
S_{q} (\omega_{m}) = \frac{2 k_{B} \Omega_{m}}{h \gamma_{m} \left( \Omega_{m}^{\text{eff}} \right)^{2}} T^{p} \left( \frac{T_{\text{eff}}^{p}}{2} \right)^{2}, \quad (31)
\]

providing the definition of a `peak' temperature \( T_{\text{eff}}^{p} \).

The three different estimates for the effective vibrational mode temperature, \( T_{\text{eff}}^{\gamma} \), \( T_{\text{eff}}^{p} \), and the one associated with equation (28), \( T_{\text{area}}^{\text{eff}} \), are plotted in figure 6 versus the scaled cavity-driving detuning \( \Delta / \Omega_{m} \), for the same set of parameters of figures 2–4. The three different sets of temperature values agree with each other and with the theoretical prediction. A subset of these data around the resonant condition \( \Delta / \Omega_{m} = 1 \) is then compared with the corresponding data with a larger optomechanical coupling in figure 7. The upper curve refers to the small coupling regime of figure 6, while the lower curve, showing better cooling, is obtained in a different experimental condition, with the membrane at 15 nm distance from a field node (larger \( \partial \omega / \partial z_{0} \)) and a driving input power \( P = 1.60 \) mW, which corresponds to the larger coupling \( G \approx -0.031 \Omega_{m} \) (at \( \Delta = \Omega_{m} \)). In the second case, the vibrational mode is cooled down by a factor of \( \approx 350 \). Moreover, the three different estimates of the effective temperature are again consistent between them and with the theoretical prediction (full line in figure 7).

5. Concluding remarks

We have performed an experimental study of the optomechanical device formed by a vibrating 50 nm thick Si$_{3}$N$_{4}$ membrane placed within a high-finesse optical FP cavity at room temperature. We have studied in particular how the radiation–pressure interaction with the driven cavity mode modifies the mechanical susceptibility of the vibrational mode due to the
Figure 6. The figure shows the three different estimates of the vibrational mode temperature, $T_\gamma$, $T_p$ and $T_{\text{area}}$ versus the scaled cavity-driving detuning $\Delta/\Omega_m$. The parameters of the fitting curve are as in figure 2. Circles correspond to $T_\gamma$, diamonds to $T_p$, and squares to $T_{\text{area}}$.

Figure 7. The figure shows the three different estimates of the vibrational mode temperature, $T_\gamma$, $T_p$ and $T_{\text{area}}$ versus the scaled cavity-driving detuning $\Delta/\Omega_m$, around resonance with the red sideband of the cavity mode frequency, $\Delta/\Omega_m = 1$. The upper curve refers to the small coupling regime of figure 2, while the lower curve refers to a different situation with larger coupling ($G \approx -0.031\Omega_m$ at $\Delta = \Omega_m$), obtained with the membrane at 15 nm distance from a field node, and larger driving input power $P = 1.60\,\text{mW}$. Circles correspond to $T_\gamma$, diamonds to $T_p$, and squares to $T_{\text{area}}$.

back-action of the detuned optical field. The measured mechanical frequency shift and the modified mechanical damping as a function of detuning and optomechanical coupling are well reproduced by a Langevin equation description of the system [32]. The increase of damping is equivalent to cooling the vibrational mode, and we demonstrate a decrease of the effective temperature by a factor of $\approx 350$. We also observe a mechanical frequency shift which is not
associated with the standard optical spring effect, but that can be explained only taking into account the quadratic term in the parametric dependence of the cavity frequency on the effective membrane position. This second-order term, which is usually negligible in most optomechanical devices, cannot be neglected in a proper description of the present MIM setup.

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