Changes in the radius of a nucleon in interaction with another nucleon

G. Kälbermann

Rothberg School for Overseas Students and Racah Institute of Physics
Hebrew University, 91904 Jerusalem, Israel

and

L.L. Frankfurt and J.M. Eisenberg

School of Physics and Astronomy
Raymond and Beverly Sackler Faculty of Exact Sciences
Tel Aviv University, 69978 Tel Aviv, Israel

Abstract:- We consider a two-nucleon system described by two different skyrmion models that provide attraction for the central $NN$ potential. One of these models is based on the product ansatz and the other on dilaton coupling. Within these models we ask the question, To what degree does the nucleon swell or shrink when the internucleon separation distance is appropriate to attraction or repulsion? We find typically swelling of 3 to 4 percent for central attraction of some 40 to 50 MeV.

February, 1994.
A major goal of nuclear physics in the years since the establishment of quantum chromodynamics (QCD) as the preferred theory of strong interactions has been to study the mutual influences between nucleon substructure and the behavior of complex nuclei. Well over ten years ago, it was suggested [1] that there might be appreciable changes of the nucleon radius when the nucleon is in a nuclear medium. This issue was then explored in a number of papers [2-7]; the situation as of 1988 is reviewed in ref. [8]. Early arguments [2-5] were in part based on the expectation that \( r M = r_0 M_0 \), where \( r \) and \( M \) are the nucleon radius and mass within the nuclear medium, and \( r_0 \) and \( M_0 \) are these same two quantities for a free nucleon. However, there are many possible sources for the “effective” features in the effective mass \( M \), ranging from nonrelativistic many-body physics to issues of relativistic mean fields, while the changes of the nucleon radius in the hadronic medium are expected to arise mainly from the polarization of the particle. Put in other terms, the issue of the nucleon effective mass in the medium is likely to relate most strongly to its behavior as a quasiparticle, that is to say, to the dressing that is supplied by its interaction with the other nucleons around it, independent of questions of nucleon substructure; the effective mass, for example, is quite naturally a long-range phenomenon in the nucleus. On the other hand, we tend to view changes in nucleon radius as mainly a result of responses of the internal nucleon constituents to their immediate surroundings, and thus as a short-range effect. Further, the empirical limits on the changes of the nucleon radius in the nucleus suggest [8] that \( |r/r_0 - 1| \leq 0.04 \), roughly, while the nucleon effective mass may differ by a few times this amount according to many estimates of \( M/M_0 \), so that it is difficult to accept the simple view that \( r \sim 1/M \).

Since the understanding of the \( NN \) force obtained from models intended to reflect QCD physics also suggests that the central attraction in that force derives from polarization phenomena, it has become increasingly interesting to attempt to link those two polarization features directly [6-8]. The approach to the \( NN \) problem based on skyrmions lends itself particularly readily to such studies. (The use of skyrmions for nuclear problems has by now been surveyed by nearly all the practitioners in the field; we note here the reviews [9-13] that are particularly close to the issues raised in this paper.) There one finds—much as in the application of the nonrelativistic quark model to the \( NN \) force problem [14]—that it is crucial to include the polarization of each individual nucleon in order to get an attractive central potential of medium range. This is perhaps best handled by allowing for
the distortion of the full two-skyrmion solution as the two baryons approach each other [12,13], but very similar effects may also be dealt with by the explicit admixture of nucleon excitations such as the $\Delta(1232)$ and $N(1440)$ as the internucleon separation distance $R$ is decreased [11]. Some skyrmion studies have indeed been made of the changes in nucleon radius as $R$ is varied [15,16], but these were carried out while the search for a fuller understanding of possible sources of attraction in the skyrmion approach was at its peak. As a result, the models in question in fact contained no attraction, and therefore merely confirmed that when skyrmions are mainly engaged in repelling each other they also cause each other to shrink.* As was early pointed out [6-8], there are very general arguments that imply that an attractive potential will be accompanied by nucleon swelling, and it is the purpose of this paper to study this at a quantitative level using skyrmions.

We first sketch some of the relevant formalism, beginning with the Skyrme model using the product ansatz. The skyrmion is here described by the lagrangian

$$\mathcal{L} = -\frac{F_\pi^2}{16}\text{tr}(L_\mu L^\mu) + \frac{1}{32e^2}\text{tr}[L_\mu, L_\nu]^2$$

$$+ \frac{\gamma}{8e^2}\text{tr}(L_\mu L_\nu)^2 - \frac{\epsilon^2}{8}(\text{tr}B^\mu)^2,$$

with

$$L_\mu \equiv U^\dagger \partial_\mu U$$

and

$$B^\mu \equiv -\frac{\epsilon^{\mu\alpha\beta\gamma}}{24\pi^2}\text{tr}(L_\alpha L_\beta L_\gamma),$$

where $U(\vec{r}, t)$ is the unitary SU(2) chiral field, $F_\pi$ is the pion decay constant (with experimental value 186 MeV), $e$ is the Skyrme parameter, and $\gamma$ and $\epsilon$ are coefficients of additional attractive and repulsive terms, respectively [11]. (We note that these two terms are necessary in order to obtain medium-range attraction in the NN potential in the present approach.) The static solution for the $B = 1$ case is the well-known hedgehog

$$U(\vec{r}) = \exp[i\vec{r} \cdot \hat{r} F(r)],$$

where $F(r)$ is the profile function or chiral angle. For the $B = 2$ soliton, we use the product ansatz [11]

$$U_{B=2} = A(t)U(\vec{r} - \vec{r}_1)A^\dagger(t) B(t)U(\vec{r} - \vec{r}_2)B^\dagger(t),$$

* There also exists an interesting skyrmion study [17] of radius changes in an infinite hadronic medium.
where $\vec{r}_1$ and $\vec{r}_2$ are the baryon locations, and $A(t)$ and $B(t)$ are time-dependent rotations in SU(2). Introducing eq. (5) into the Hamiltonian derived from eq. (1) and subtracting the one-body energies, we find the two-body potential.

Attraction for the $NN$ system is achieved within the product ansatz (5) by carrying out a further variational calculation [11] in which the baryon resonances $\Delta(1232)$ and $N^* = N(1440)$ are admixed by

$$|\tilde{NW}(R)\rangle = \alpha(R)|NN(R)\rangle + \beta(R)|N\Delta(R)\rangle + \gamma(R)|\Delta N(R)\rangle + \epsilon(R)|NN^*(R)\rangle + \zeta(R)|N\Delta^*(R)\rangle + \eta(R)|N^* N(R)\rangle + \theta(R)|N^* N^*(R)\rangle \cdots.$$  

(The ellipsis refers to the possible inclusion of, say, the $\Delta(1670)$, whose contribution proves to be negligible here.) That is to say, at each internucleon separation distance $R$ we minimize the energy for the two-nucleon system with respect to the coefficients $\alpha(R)$ through $\theta(R)$, etc., after we have obtained this energy from projected states for the nucleon and the admixed baryon resonances. The Roper $N(1440)$ is handled as a vibrating breathing mode of the nucleon (see [11] and references therein for details). Changes in the radius of the nucleon while in interaction with a second nucleon in this description are studied by taking the second nucleon as a spectator and evaluating the root-mean-square radius for the first.

We now turn to the Skyrme model involving coupling to a dilaton. It was early realized [18] that such an approach allows for the incorporation of the QCD trace anomaly. Subsequently such models were studied for their possible advantages in obtaining medium-range central attraction [19,20]. The introduction of a new length scale through the dilaton has the effect of inducing a sharper edge for the skyrmion, which now exists with bag-like support within the dilaton. This seems an appealing and natural way to cut off the long tail of skyrmion repulsion, generated by the same mechanism that stabilizes the skyrmion, and thus to enhance attraction at medium ranges. We have thus found this model to be interesting in terms of its predictions for nucleon radius change as well.

Towards such a study we take the usual [18-20] lagrangian for a skyrmion coupled with a dilaton,

$$\mathcal{L} = \mathcal{L}_{\text{sym}} - V(\sigma) = e^{2\sigma}\left[\frac{1}{2}\Gamma^2_0 \partial\mu \sigma \partial^\mu \sigma - \frac{F^2}{16} \text{tr}(L_\mu L^\mu)\right] + \frac{1}{32e^2} \text{tr}[L_\mu, L_\nu]^2 - V(\sigma).$$  

(7)
Here $\mathcal{L}_{\text{sym}}$ preserves scale invariance and $V(\sigma)$ is determined [18] from the trace anomaly to be

$$V(\sigma) = \frac{C_G}{4} [1 + e^{4\sigma} (4\sigma - 1)],$$

and there are two new constants $\Gamma_0$ and $C_G$. The product ansatz of eq. (5) is now augmented by an assumption of additivity for the dilaton $\sigma$-field,

$$\sigma_{B=2} = \sigma_1 + \sigma_2.$$  

Once again the $NN$ potential is identified by subtracting the energy for the $B = 2$ system with separation $R \to \infty$ from that with finite separation $R$.

The nucleon radius modification is this time evaluated by minimizing the total static energy (without rotational energy),

$$E(X) = \frac{1}{\lambda} E_{-1}(X) + \lambda E_1(X) + \frac{1}{\lambda^3} E_{-3}(X),$$

where $X = \lambda R$, with respect to a change in the length scale $\lambda$. In eq. (10) we identify the various contributions to the static energy according to their behavior under $\vec{r} \to \lambda \vec{r}$, namely,

$$E_{-1}(X) = \int e^{2\sigma} \left[ \frac{\Gamma_0^2}{2} (\nabla \sigma)^2 - \frac{F_\pi^2}{16} \text{tr}(L_i L_i) \right] d\vec{x},$$

$$E_1(X) = \int \frac{1}{32e^2} \text{tr}[L_i, L_j]^2 d\vec{x},$$

and

$$E_{-3}(X) = \int V(\sigma) d\vec{x},$$

where $\vec{x} = \lambda \vec{r}$. The resulting minimum then fixes the preferred scale for the interacting system, and with it a slightly modified two-nucleon potential. That is to say, the scale invariance of $\mathcal{L}_{\text{sym}}$ in the original dilaton-skyrmion lagrangian of eq. (7) has been broken by the additivity ansatz for the dilaton, eq. (9), and our restricted minimization with respect to $\lambda$ then provides an approximate solution to the full $B = 2$ dilaton-skyrmion problem which improves upon the simple product-plus-additivity assumptions of eqs. (5) and (9).

For the usual Skyrme model with hedgehog and product ansaetze of eqs. (1) through (5) we show in fig. 1 the central $NN$ potential and ratio of the interacting-nucleon radius to the free-nucleon radius. The central potential in
this case reaches an attraction of about -11 MeV and the corresponding change in the nucleon radius in interaction is about 4 percent. The dilaton case is shown in fig. 2, and yields considerably more attraction, reaching possibly overly-large values deeper than -40 MeV. Nonetheless, the radius changes are again a rather modest 3 percent or so. In both models, as $R$ becomes very small the two nucleons have large overlap, strong repulsion sets in, the nucleons begin to show shrinking, and the entire use of the product ansatz quickly becomes meaningless. In the case of the dilaton, the additivity assumption, eq. (9), is also bad for small $R$. For large $R$, the skyrmion shows an unrealistically long tail of interaction between the two nucleons, reflected both in the potential $V$ and in the deviation of $r/r_0$ from unity. (Since both of these quantities have been calculated on a rather sparse grid—seen in the nexuses of the straight-line segments—there is a lack of smoothness in the results shown here.)

Both cases show that, as the potential moves from large positive values at small $R$ through zero and on to attraction, the nucleon changes from a shrunken condition to a swollen one (though in neither situation do the changeovers of $V$ and of $r/r_0$ occur at precisely the same separations). There is thus a clear link in two rather different skyrmion descriptions of the two-nucleon system between attraction and nucleon swelling. In both cases the nucleon swelling is quit modest, and well within the limits set by present experiment [8]. Shrinking sets in for $R < 1.5$ fm or so, and it is in this region that quark-gluon degrees of freedom may first be required in the description of hadronic systems (e.g., neutron stars). Since we have calculated only for the $B = 2$ system, there is here little or no question of a link between nucleon swelling or shrinking and an effective mass within a hadronic medium as this is normally understood. To the degree that the skyrmion is a valid description for the range $1 \text{ fm} \leq R \leq 3 \text{ fm}$, our results span the region from large separation, where the effects involve overall hadronic behavior, down to small $R$, where perturbative QCD enters. The changes in nucleon radius for small internucleon separation should be enhanced in measurements of nuclear form factors since these will be sensitive to the third power of the scaling factor for moderate momentum transfer. We note that, since we expect, on the whole, nucleon shrinking for small $R$ and swelling for intermediate $R$, the effects in form factors are likely to show changes as different ranges of momentum of the nucleon within the Fermi sea are probed. (For heavy nuclei, the swelling at intermediate ranges of $R$ may receive partial compensation from the suppression of the pion...
cloud around the bound nucleon [21].) One may hope that with the arrival of new data on high-energy electron scattering, for example from CEBAF, the study of changes in the nucleon radius while the particle is in interaction may serve as an additional tool to unravel the delicate question of mutual nucleon polarization at medium-range separations.

This research was supported in part (G.K. and J.M.E.) by the Israel Science Foundation, in part (L.L.F.) by the U.S.-Israel Binational Science Foundation, and in part (J.M.E.) by the Yuval Ne’eman Chair in Theoretical Nuclear Physics at Tel Aviv University. Parts of it were carried out while J.M.E. was a guest at the Institute for Theoretical Physics of the University of Frankfurt, and he would like to thank Professor Walter Greiner there for his very kind hospitality.

References:

1. J.V. Noble, Phys. Rev. Lett. 46 (1981) 412 and Phys. Lett. B 178 (1986) 285.

2. L.S. Celenza, A. Rosenthal, and C.M. Shakin, Phys. Rev. Lett. 53 (1984) 892 and Phys. Rev. C 31 (1985) 232.

3. P.J. Mulders, Phys. Rev. Lett. 54 (1985) 2560 and Nucl. Phys. A 459 (1986) 525.

4. T.D. Cohen, J.W. Van Oerden, and A. Picklesimer, Phys. Rev. Lett. 59 (1987) 1267.

5. M.K. Banerjee, Phys. Rev. C 45 (1992) 1359.

6. L. Frankfurt and M. Strikman, Nucl. Phys. B 250 (1985) 123.

7. M. Oka and R.D. Amado, Phys. Rev. C 35 (1987) 1586.

8. L. Frankfurt and M. Strikman, Phys. Repts. 160 (1988) 235.

9. I. Zahed and G.E. Brown, Phys. Repts. 142 (1986) 1.

10. G. Holzwarth and B. Schwesinger, Repts. Prog. Phys. 49 (1986) 825.
11. J.M. Eisenberg and G. Kälbermann, Prog. Part. Nucl. Phys. 22 (1989) 1.

12. T.S. Walhout and J. Wambach, Int. J. Mod. Phys. E 1 (1992) 665.

13. M. Oka and A. Hosaka, Ann. Rev. Nucl. Part. Sci. 42 (1992) 333.

14. K. Maltman and N. Isgur, Phys. Rev. D 29 (1984) 952.

15. M. Oka, K.F. Liu, and H. Yu, Phys. Rev. D 34 (1986) 1575.

16. G. Kälbermann and J.M. Eisenberg, Phys. Lett. B 188 (1987) 311.

17. I.N. Mishustin, Sov. Phys. JETP 71 (1990) 21 [Zh. Eksp. Teor. Fiz. 98 (1990) 41].

18. J. Schechter, Phys. Rev. D 21 (1980) 3393;
   H. Gomm, P. Jain, R. Johnson, and J. Schechter, Phys. Rev. D 33 (1986) 801.

19. H. Yabu, B. Schwesinger, and G. Holzwarth, Phys. Lett. B 224 (1989) 25.

20. K. Tsushima and D.O. Riska, Nucl. Phys. A560 (1993) 985.

21. B.L. Friman, V.R. Pandharipande, and R.B. Wiringa, Phys. Rev. Lett. 51 (1983) 763.
Figure captions:-

Fig. 1. The $NN$ potential and ratio of the radius for the interacting nucleon to that of the free nucleon $r/r_0$ for the usual skyrmion with product ansatz and baryon-resonance admixtures, eqs. (1) through (6). The parameters in eq. (1) are taken to have the values $F_\pi = 130$ MeV, $e = 20$, $\gamma = 0.50$, $\epsilon = 2.58$, and $m_\pi = 139$ MeV, known from earlier studies [11] to yield a fair amount of attraction in the central potential. They produce the masses $M_N = 998$ MeV, $M_\Delta = 1211$ MeV, and $M_{N^*} = 1270$ MeV for the nucleon, $\Delta$, and Roper.

Fig. 2. The $NN$ potential and ratio of the radius for the interacting nucleon to that of the free nucleon $r/r_0$ for the skyrmion coupled to a dilaton, eqs. (7) through (11). The parameters used were $\Gamma_0 = 306$ MeV, $C_G = (121$ MeV$)^4$, and $F_\pi = 186$ MeV, which produce $M_N = 1335$ MeV. No attempt was made here to search for more realistic parameters since our interest was only in the link between attraction/repulsion and swelling/shrinking.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9402344v1
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9402344v1
Figure 2

$r/r_0$

$R [fm]$

$V [MeV]$

$R [fm]$