Unsteady nano fluid flow through magnetic porous sphere under the influence of mixed convection

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Abstract. This paper considers about the nano fluid flow through porous sphere under the influence of mixed convection problem. Dimensional equations are further constructed by the continuity, momentum, and energy equations. The dimensional equations that have been obtained is further converted into non-dimensional equations by using non-dimensional variables. These non-dimensional equations are transformed into similarity equation by using stream function. These similarity equations further resolved numerically by using the Keller-Box Scheme to analyze the effect of both mixed convection and magnetic toward velocity and its temperature profiles. We obtain that the velocity distribution and the temperature decrease when the magnetic parameter’s value increases, and the velocity distributions and the temperature increase when the mixed convection’s value increase.

1. Introduction
Based on shear stress and shear rate, fluid is dividing into two types: Newtonian and Non-Newtonian fluids [1]. Nano fluids is one of the Newtonian fluids. Nano fluid is a mixture of basic fluids with nanoparticles. In this research, the basic fluid that we use is water and the nanoparticle is Cu. Due to its heat transfer ability, nano fluids is used in many major industries, for example in energy supply, electronic, textile, and in the industries of transportation and paper [2]. Fluids have been widely used in research on Magnetohydrodynamics. Magnetohydrodynamics (MHD) is a fluid flow movement study that can conduct electricity and is influenced by magnetic fields. The basic concept of MHD is that the magnetic field can induce an electric current in a moving conductive fluid, which in turn creates forces on the fluid and also alters the magnetic field itself [3]. So, the flow of MHD is an important research in the engineering and industry.

In this research, the nano fluids is considered move upwards and going through a magnetic porous sphere. With the influence of this magnetic porous sphere to the nano fluid flow, this nano fluid can be consider has magnetohydrodynamics characteristics. This unsteady nano fluid flow is not only influenced by magnetic field from the magnetic porous sphere, but also influenced by mixed convection. Mixed convection is a combination of free convection and force convection [4]. In this research, we develope the governing equation of unsteady nano fluid flow through magnetic porous sphere under the influence of mixed convection that is derived from mass, momentum, and energy conservation. This research investigates the velocity and temperature of unsteady nano fluid flow through a magnetic porous sphere under the influence of mixed convection in front of the lower stagnation point \( x \approx 0 \).
2. Numerical Methods
We consider the nano fluid’s laminar flow move vertically through a magnetic porous sphere with velocity \( U_\infty \) and surrounding temperature \( T_\infty \). Consider the unsteady nano fluid flow through a magnetic porous sphere with mixed convection. In this research, the nano fluid that we use for simulation is Cu-water. The illustrates of the physical model and the coordinate system of the porous sphere show in Figure 1.

![Figure 1](image_url)

Figure 1. (a) Physical Model and (b) Coordinate System

The governing equations are developed from mass, momentum, and energy conservation, as follow. Continuity Equation:

\[
\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0
\]  

(1)

Momentum Equation in \( x \) and \( y \) axis:

\[
\rho_n f \left( \frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu_n f \left( \frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2} \right) + \sigma B_0^2 \bar{u} + \frac{\mu_n f}{K} \bar{u} - \rho \beta (\bar{T} - T_\infty) g_y
\]  

(2)

\[
\rho_n f \left( \frac{\partial \bar{v}}{\partial t} + \bar{u} \frac{\partial \bar{v}}{\partial x} + \bar{v} \frac{\partial \bar{v}}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu_n f \left( \frac{\partial^2 \bar{v}}{\partial x^2} + \frac{\partial^2 \bar{v}}{\partial y^2} \right) + \sigma B_0^2 \bar{v} + \frac{\mu_n f}{K} \bar{v} - \rho \beta (\bar{T} - T_\infty) g_x
\]  

(3)

Energy Equation:

\[
\frac{\partial \bar{T}}{\partial t} + \bar{u} \frac{\partial \bar{T}}{\partial x} + \bar{v} \frac{\partial \bar{T}}{\partial y} = \alpha_n f \left( \frac{\partial^2 \bar{T}}{\partial x^2} + \frac{\partial^2 \bar{T}}{\partial y^2} \right)
\]  

(4)

With the free flow velocity \( \bar{u}_e = \frac{3}{2} U_\infty \) \( \sin x \) , and respect to the boundary conditions as follow:

\[ \bar{t} = 0 : \quad \bar{u} = \bar{v} = 0, \bar{T} = T_\infty \text{ for all } \bar{x}, \bar{y} \]

\[ \bar{t} \geq 0 : \quad \bar{u} = \bar{v} = 0, \bar{T} = T_w \text{ at } \bar{y} = 0 \]

\[ \bar{u} = \bar{u}_e(x), \bar{T} = T_\infty \text{ while } \bar{y} \to \infty \]

(5)

The equations above are the dimensional equations. By substituting non-dimensional variables that are given as [5]:

\[
x = \frac{x}{a}, y = Re \frac{y}{a}, u = \frac{u}{U_\infty}, v = Re \frac{v}{U_\infty}, t = \frac{u_\infty t}{a}, p = \frac{\rho}{\rho U_\infty}, r = \frac{r}{a}, \bar{T} = \frac{T - T_\infty}{T_w - T_\infty}
\]  

(6)

where

\[
Re = \frac{u_\infty a}{v}, \quad \nu = \frac{u}{\rho}, \quad \text{and magnetic parameter } M, \text{ mixed convection parameter } \lambda, \text{Gr, Pr, and } \phi \text{ are dimensionless parameter that defined as:}
\]

\[
M = \frac{a \beta \delta^2}{\rho U_\infty}, \quad \lambda = \frac{Gr}{Re^2}, \quad Gr = \frac{\gamma B_0 (T_w - T_\infty) a^3}{\nu^2}, \quad Pr = \frac{\nu_n f}{\alpha_n f}, \quad \phi = \frac{a \mu_n f}{\rho_n f U_\infty K}
\]  

(7)

the dimensional boundary layer equations are transformed into non-dimensional governing equations.

We use the approximation of boundary layer and the stream function, we get the non-dimensional equations as follow:

\[
\frac{\partial u}{\partial \tau} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\nu_n f}{\nu} \frac{\partial^2 u}{\partial y^2} + Mu + \phi u + \lambda T \sin x
\]  

(8)
\[
\frac{\partial r}{\partial t} + u \frac{\partial r}{\partial x} + v \frac{\partial r}{\partial y} = \frac{1}{\rho r} \frac{\partial}{\partial y} \left( \frac{\partial \psi}{\partial y} \right)^2
\]

with the initial and boundary conditions as below
\[ t < 0, u = v = 0, T = 0 \] for all \( x, y \)
\[ t \geq 0, u = v = 0, T = 1 \] for \( y = 0 \)
\[ u = u_e(x), T = 0 \] for \( y \to \infty \)

And then, we substitute the stream function \( \psi \) defined as [5]:
\[ u = \frac{1}{r} \frac{\partial \psi}{\partial y}, \quad v = -\frac{1}{r} \frac{\partial \psi}{\partial x} \]
to equation (7) to (9), we obtain
\[ \frac{1}{r} \frac{\partial \psi}{\partial y} + \frac{1}{r^2} \frac{\partial \psi}{\partial x} \frac{\partial}{\partial x} \left( \frac{\partial \psi}{\partial y} \right)^2 - \frac{1}{r^2} \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial y} = u_e \frac{\partial u_e}{\partial x} + \frac{v_f}{v_f} \frac{1}{r} \frac{\partial \psi}{\partial y} + (M + \phi) \left( \frac{1}{r} \frac{\partial \psi}{\partial y} - u_e \right) + \lambda T \sin x \]

with respect to the following boundary conditions.
\[ t < 0 : \psi = \frac{\partial \psi}{\partial y} = T \] for all \( x, y \)
\[ t \geq 0 : \psi = \frac{\partial \psi}{\partial y} = 0, T = 1 \] for \( y = 0 \)
\[ \frac{\partial \psi}{\partial y} = u_e(x)r(x), T = 0 \] for \( y \to \infty \)

The equation that we use to get the numerical results are the similarity equation. So, by substituting the similarity variable for small time \( t \leq t^* \):
\[ \psi = t^{\frac{1}{2}} u_e(x)r(x)f(x, \eta, t), \quad T = s(x, \eta, t), \quad \eta = \frac{y}{t^{\frac{1}{2}}} \]
and the similarity variable for large time \( t > t^* \):
\[ \psi = t^{\frac{1}{2}} u_e(x)r(x)f(x, \eta, t), \quad T = S(x, \eta, t), \quad Y = y \]
to equation (11) and (12), with the porous sphere’s lower stagnation point \( x \approx 0 \), we obtain:

Small time
\[ -\frac{\eta}{2} \frac{\partial^2 f}{\partial \eta^2} + t \frac{\partial^2 f}{\partial x^2} + 3x \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial \eta^2} - \frac{3t}{2} \frac{\partial^2 f}{\partial x^2} = \frac{3t}{2} + \frac{v_f}{v_f} \frac{\partial^2 f}{\partial y^2} + t(M + \phi) \left( \frac{\partial f}{\partial \eta} - 1 \right) + \frac{2t}{3} \lambda s \]

Large time
\[ \frac{\partial^2 f}{\partial \eta^2} + \frac{3}{2} \left[ 1 - \frac{(\partial f)^2}{\partial \eta^2} + \frac{(\partial^2 f)}{\partial \eta^2} \right] = \frac{\partial^2 f}{\partial x^2} - (M + \phi) \left( \frac{\partial f}{\partial \eta} - 1 \right) - \frac{2}{3} \lambda s \]

with respect to the following boundary conditions as below
\[ t < 0 : f' = s = 0 \] for all \( x, \eta \)
\[ t \geq 0 : f = f', s = 0, s = 1 \] for \( \eta = 0 \)
\[ f' = 1, s = 0 \] for \( \eta \to \infty \)

Let \( \frac{\partial f}{\partial \eta} = f' \) and \( \frac{\partial s}{\partial \eta} = s' \), so (18) to (19) become:
\[ \frac{v_f}{v_f} \frac{f'''}{f''} + \frac{3}{2} \left[ 1 - (f')^2 + f'f'' \right] + \frac{3}{2} f''' + t(M + \phi)(f' - 1) + \frac{2}{3} t \lambda s = t \frac{\partial u}{\partial t} \]

with respect to the following boundary conditions as below
\[ F = \frac{\partial f}{\partial \eta} = 0, S = 1 \] for \( Y = 0 \)
\[ \frac{\partial F}{\partial \eta} = 1, S = 0 \] for \( Y \to \infty \)
with respect to the following boundary conditions in (16).

According to [6], the density, viscosity, specific heat, and thermal conductivity of nano fluid defined as follows:

\[ \rho_{nf} = (1 - \chi)\rho_f + \chi \rho_s, \]
\[ \mu_{nf} = \left( \frac{\mu}{1 - \chi} \right)^{2.5}, \]
\[ \left( \frac{k_{nf}}{k_f} \right) = \left( \frac{k_f + 2k_f - 2\chi(k_f - k_s)}{(k_f + 2k_f) + \chi(k_f - k_s)} \right). \]

So we get

\[ \frac{v_{nf}}{v_f} = \frac{1}{(1-\chi)^{2.5}(1-\chi) + \chi\left(\frac{2\chi}{\rho_f}\right)}, \quad \text{and} \quad \frac{a_{nf}}{a_f} = \frac{(k_f + 2k_f - 2\chi(k_f - k_s))}{(k_f + 2k_f) + \chi(k_f - k_s)(1-\chi) + \chi\left(\frac{\rho C_p}{\rho C_p}\right)}. \]

We solve numerically the equation (19) and (20) by using Keller-Box method. According to [3], the steps are first, we reduce (19) and (20) to first order equations; and then we do discretization using finite difference method to discretize the first order equations; after that, by using the Newton’s Method we linearize the resulting algebraic equations and write in matrix-vector form. The last step is solving the linear system by using the technique of tridiagonal block elimination.

3. Results and Discussion

In this research, the nano fluid flow through magnetic porous sphere under the influence of mixed convection is investigated numerically by using Keller-Box Scheme. The objective of this research is to investigate the velocity and temperature of nano fluid flow with the variation of magnetic parameter \( M \) and mixed convection parameter \( \lambda \).

The results of the numerical methods are compared to Ismail et. al [7] for \( \lambda = 1, \ \chi = 0, \ \text{Pr} = 0.7 \). The difference between this research and Ismail et. al are that in this research the sphere is a magnetic porous sphere, so the fluid induced by magnetic from the porous sphere, and Ismail et. al using a sphere and the fluid induced by magnetic that is not from the sphere. Figure 2 shows that the comparison of the present results and Ismail et. al. Figure 2 shows the velocity and the temperature of the present research and Ismail et. al is coincide. This coincide means that the results of this research is confirm with Ismail et. al. So we are sure that the numerical results of this research can be use for another parameters and variables that in accordance with the problem in this research.

![Figure 2](image)

**Figure 2.** (a) Velocity and (b) Temperature

The results about the velocity and temperature of the unsteady nano fluid flow at various value of magnetic and mixed convection parameter are illustrated in Figure 3 and Figure 4. These results have been made at fixed values of \( \phi = 1, \text{Pr} = 1, \) and \( \chi = 0.1 \).
The variation of magnetic parameter that use in this research is $M = 0, 1.3, 1.8, 2$, and $2.3$. Figure (3a) shows that the increases of the velocity is at $f' = 0$ to $f' = 1$. Furthermore, the velocity of unsteady nano fluid flow decreases when magnetic parameter increases. This is caused by the Lorentz force where $M$ is proportional to magnetic field $B_0$. Figure (3b) shows that the decreases of the temperature is at $s = 1$ to $s \approx 0$. Furthermore, the temperature of unsteady nano fluid flow decreases when the magnetic parameter increases. This is caused by the magnetic field $B_0$ that makes the density increases so the heat transfer decreases, and temperature decreases.

The variation value of mixed convection parameter that use in this research is $\lambda = 0.5, 0.7, 1.5,$ and $2$. Figure (4a) shows that the increases of the velocity of unsteady nano fluid flow is at $f' = 0$ to $f' = 1$. Furthermore, the velocity of unsteady nano fluid flow increases when mixed convection parameter increases. This caused by the location of the heater is near the stagnation point so the density decreases, and the velocity increases. Figure (4b) shows that decreases of the temperature of unsteady nano fluid is at $s = 1$ to $s \approx 0$. Furthermore, the temperature of unsteady nano fluid flow increases when the mixed convection parameter increase. This caused by the Grashof number increases along with $\lambda$, and it causes the viscosity decreases and temperature increases.

In conclusion, the equations as follow:

\[
\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial y} = 0
\]
\[
\begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} & = -\frac{\partial p}{\partial x} + \frac{\nu_f}{\nu} \frac{\partial^2 u}{\partial y^2} + Mu + \phi u + \lambda T \sin x \\
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} & = \frac{1}{Pr} \frac{\alpha_f}{\alpha} \frac{\partial^2 T}{\partial y^2}
\end{align*}
\]

with the initial and boundary conditions as below:

\[t < 0 : u = v = 0, T = 0 \text{ for all } x,y\]
\[t \geq 0 : u = v = 0, T = 1 \text{ for } y = 0\]
\[u = u_e(x), T = 0 \text{ for } y \to \infty\]

can be used to solve the problem of unsteady nano fluid flow through magnetic porous sphere under the influence of mixed convection. From our numerical results, we declare that the velocity and the temperature distributions of the unsteady nano fluid flow decrease when the value of magnetic parameter increases, and the velocity and the temperature distributions increase when the value of mixed convection increases.

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5. References

[1] Fox, R.W., McDonald, A.T., and Pritchard, P.J., (2011), Introduction to Fluid Mechanics, 8th Edition, Jonh Wiley and Sons, United States of America.

[2] Widodo, B., Arif, D.K., Aryany, D., Asiyah, N., Widjajati, F.A., and Kamiran, (2017), The Effect of Magnetohydrodynamic Nano Fluid Flow Through Porous Cylinder, AIP Conference Proceeding. 1867, 020069-1-020069-10. Published by The American Institute of Physics.

[3] Widodo, B., Khalimah, D.A., Zainal, F.D.S. and Imron, C., (2016), The Effect of Prandtl Number and Magnetic Parameters on unsteady magnetohydrodynamic Forced Convection Boundary Layer Flow of a Viscous Fluid Past A Sphere, International Journal of Advances in Science Engineering and Technology ISSN: 2321-9009, Vol 4, Issue 1, Page 75-78.

[4] Widodo, B., Siswono, G.O. and Imron, C., (2015), Viscoelastic Fluid Flow With The Presence Of Magnetic Field Past A Porous Circular Cylinder, International Journal of Mechanical And Production Engineering ISSN: 2320-2092. Vol 3 Issue 8, page 123-126.

[5] Widodo, B., Anggriani, I., Imron, C., (2016), The Characterization Of Boundary Layer Flow In The Magnetohydrodynamic Micropolar Fluid Past A Solid Sphere, International Journal of Advances in Science Engineering and Technology, ISSN:2321-9009.

[6] Kandasamy, R., Zailani, N. A. B., Jaafar, F. N. B., (2017), Impact of Nanoparticle Volume Fraction on Squeezed MHD Water Based Cu, Al2O3, and SWCNTs Flow Over A Porous Sensor Surface, Journal Physics and Mathematics.

[7] Ismail, M.A., Mohammad, N.F., and Shafie, S., (2017), Separation Time Analysis of Transient Magnetohydrodynamic Mixed Convection Flow of Nanofluid At Lower Stagnation Point Past A Sphere, Malaysian Journal of Fundamental and Applied Sciences Vol. 13, No. 3 (2017) 151-154.