Analytical and numerical approaches to compute the influence of vertical load on lateral response of single pile

Kaustav Chatterjee i) and Deepankar Choudhury ii)

i) Ph.D. Research Scholar, Department of Civil Engineering, Indian Institute of Technology Bombay, Powai, Mumbai – 400076, India.
ii) Professor, Department of Civil Engineering, Indian Institute of Technology Bombay, Powai, Mumbai – 400076, India. Also, Adjunct Professor, Academy of Scientific and Innovative Research (AcSIR), New Delhi, India.

ABSTRACT

In the present study an analytical procedure is developed, based on finite element approach, to determine the bending moment and lateral deflection response of a free headed single pile with floating tip embedded in both dry and saturated cohesionless soils subjected to combined action of vertical and lateral loadings, using the modulus of subgrade reaction method. Also the numerical results using MIDAS GTS are obtained to validate the proposed analytical solution. It is observed under static conditions, when the vertical load is increased from zero to the ‘ultimate’ pile capacity at 0.03D deflection level (where D is the diameter of the pile), the lateral load carrying capacity of the fixed headed and free headed flexible piles is increased by 43.27% and 26%, respectively embedded in dry dense sand. Similarly for fixed headed piles embedded in dry and saturated loose sand, the bending moment is increased by 25% and 27%, respectively when vertical loads varies from zero to ultimate pile capacity for a constant lateral load of 200kN. Thus the above analysis is useful for practical design purpose to estimate the lateral load carrying capacity of a single pile by knowing the allowable deformation and vertical load acting on the pile.

Keywords: pile, finite element, modulus of subgrade reaction, MIDAS GTS, bending moment, lateral deflection

1 INTRODUCTION

Pile foundations are subjected to vertical loadings transferred from overlying superstructures along with lateral loads due to impact of ships and wave actions (tsunami) in harbour and port structures, seismic force acting on high-rise structures in earthquake prone areas, operating machineries in industries and wind action on tall structures like chimney, skyscrapers and transmission towers. The major criteria of design of pile foundation for resisting lateral loads is not only to compute the ultimate lateral capacity of piles but also to determine the maximum deflection of piles; thereby ensuring that serviceability limits are attained.

It is a common practice among present day geotechnical engineers to initially analyze the pile subjected to vertical load only to compute the bearing capacity and settlement and then the influence of lateral load to investigate the flexural behaviour [Karthigeyan et al. (2006)]. However, this approach is valid only for smaller magnitudes of lateral loads and a combined interaction analysis is essential when magnitude of lateral load varies between 10% to 20% of vertical load, as observed for coastal structures. Several numerical, analytical and experimental techniques are available in literature to analyze the behaviour of piles under the action of lateral and vertical loads. Reese and Matlock (1956), Poulos (1971), Poulos and Davis (1980), Randolph (1981) studied the behaviour of single piles under lateral loading only while Poulos (1968), Randolph and Wroth (1978), Mylonakis and Gazetas (1998) studied the deformation of piles under vertical loading only. Theoretical approaches like modulus of subgrade reaction approach [Phanikanth et al. (2010a, b)] and elastic approach [Phanikanth et al. (2014)] have also been extensively used for predicting pile deflection and bending moment. However, analysis of piles under combined action of vertical and lateral loads is scarce in literature and hence the present study fills the existing research gap.

In the present study, an analytical procedure based on finite element method is proposed to compute the bending and deflection responses of a free headed pile with floating tip embedded in cohesionless soil and subjected to both vertical and lateral loads. The modulus of subgrade reaction approach is adopted here and pile-soil interaction analysis is implemented using a mathematical code written in MATLAB (2012). The proposed analytical results is validated by using numerical analysis through MIDAS GTS (2013) and close agreement between the results are observed.

http://doi.org/10.3208/jgssp.IND-11
2 PRESENT ANALYTICAL AND NUMERICAL APPROACH FOR ANALYSIS OF SINGLE PILE

A vertical circular pile having diameter $d$, length $l$, and flexural stiffness $EI$ is embedded in soil medium and subjected to a compressive vertical load having magnitude $P$. According to Hetenyi (1955), the governing differential equation for a vertically and laterally loaded pile is given as:

$$EI \frac{d^4y}{dx^4} + P \frac{d^2y}{dx^2} + k_h y = 0$$  \hspace{1cm} (1)

where, $k_h = \eta_k x$, $\eta_k$ being the coefficient of horizontal subgrade reaction in kN/m$^3$ and $y$ is the lateral deflection of the pile at depth $x$. The solution of the above differential equation is given by:

$$y(x) = c_1 e^{\frac{\beta x}{h}} \cos \frac{\alpha x}{h} + c_2 e^{\frac{-\beta x}{h}} \cos \frac{\alpha x}{h} + c_3 e^{\frac{\beta x}{h}} \sin \frac{\alpha x}{h} + c_4 e^{\frac{-\beta x}{h}} \sin \frac{\alpha x}{h}$$  \hspace{1cm} (2)

where, $c_1, c_2, c_3$, and $c_4$ are the integrating constants. A non-linear soil-pile interaction analysis is done in the present study using a mathematical code written in MATLAB (2012) and pile deflection and bending moment for different combinations of vertical and lateral loadings are computed.

Similarly, numerical analysis is implemented using MIDAS GTS (2013) where-in a single pile is modeled via 3-dimensional elastic elements. The soil is modeled as eight nodded brick element and fine meshing is done for both soil and pile in the present analysis. Initially the pile elements are generated at one meter above the ground level and then it is inserted into the soil block. The first half of model is created and then, due to its symmetrical nature, it is reflected to obtain the full model, as shown in Figure 1. A conventional Mohr – Coulomb failure criterion is used for modeling the soil block while an elastic model is chosen for the single pile in the present study. The base of the influence zone is fixed in all the directions and sides are laterally restrained against movement.

2.1 Validation of the present method under combined loadings in static analyses

The numerical model developed in MIDAS GTS (2013) and analytical model based on finite element technique is validated with the field test results of Karasev et al. (1977) and numerical analyses results of Karthikeyan et al. (2006), wherein a 3m long and 600mm diameter concrete test pile is installed in a very stiff sandy loam underlain by a sandy clay layer. The pile is subjected to a constant vertical load of 400kN and horizontal loads, varying from 0kN to 1100kN, are applied. It is observed from Figure 2 that the present results are in good agreement and compares well with the available solutions in literature.
Poisson’s ratio ($\mu$) 0.2 and unit weight ($\gamma$) 25kN/m$^3$ as per Indian Standard code IS 456 (2000). The length of the pile is varied from 5.0m to 15.0m to capture effects of both short and long piles while the diameter of the pile is kept as 600mm.

Table 1. Soil properties and soil-pile interface properties considered in the present study for static analysis [after Terzaghi (1955), IS 2911 Part1: Section 4 (1984), Bowels (1997) and Timoshenko and Goodier (2002)].

| Properties                              | Types of Soil |
|-----------------------------------------|---------------|
|                                         | Dense Sand    | Loose Sand |
| Young’s Modulus $[E]$ (MPa)             | 70            | 18          |
| Poisson’s Ratio $[\mu]$ (dry)          | 0.35          | 0.3         |
| Poisson’s Ratio $[\mu]$ (saturated)    | 0.45          | 0.39        |
| Relative Density [%]                    | 80            | 40          |
| Unit Weight $[\gamma_{dry}]$ (kN/m$^3$) | 18.05         | 14          |
| Cohesion $[c_s]$ (kPa)                  | 0             | 0           |
| Friction Angle $[\delta]$ (°)          | 40            | 30          |
| Normal stiffness $[k_n]$ (MN/m)         | 53.57         | 11.87       |
| Shear stiffness $[k_s]$ (MN/m)          | 3.65          | 0.91        |
| Friction angle $[\delta]$ (°)          | 40            | 30          |
| Modulus of Subgrade Reaction: dry $[\eta]$ (kN/m$^3$) | 20000         | 2600        |
| Modulus of Subgrade Reaction: submerged $[\eta]$ (kN/m$^3$) | 12500         | 1500        |

The ground water table is considered at the ground surface level for saturated cases and the same is simulated in the numerical model. The lateral displacements and bending moments of piles are plotted against lateral load for different vertical loads ($V=0$, $V=0.25V_{ult}$, $V=0.5V_{ult}$, $V=0.75V_{ult}$ and $V=V_{ult}$) expressed in terms of ultimate axial capacity of piles ($V_{ult}$). The ultimate load carrying capacity ($V_{ult}$) of a single pile embedded in different types of soil is calculated as per Indian design code IS 2911 Part1: Section 4 (1984). The analysis in the lateral direction is implemented by using load control instead of displacement control, so that various lateral loads are applied at pile top and corresponding displacements along pile depth are observed.

4 RESULTS AND DISCUSSIONS

4.1 Influence of vertical loading on pile deflection

Figure 3 shows the variation of lateral load on pile with lateral displacement for different magnitudes of vertical loading and it is observed as the magnitude of lateral load is increased from 150kN to 900kN, keeping $V=0$, for a fixed headed pile embedded in dry loose sand, the lateral deflection increases from 12mm to 195mm. However when vertical load is applied at the pile head varying from $V=0.25V_{ult}$ to $V=V_{ult}$, the lateral displacement of the pile is considerably reduced from 170mm to 132mm, at a constant lateral loading of 900kN. Hence for short rigid fixed headed piles embedded in dry and saturated loose sand, the lateral load carrying capacity increases by 33.3% (450kN to 600kN) and 20% (250kN to 300kN) at a deflection level of 0.1D (=60mm) [D being the diameter of the pile], when vertical loading is increased from 0 to $V_{ult}$. Similarly, for long flexible fixed headed and free headed piles embedded in dry dense sand, the lateral load carrying capacity is increased by 43.27% (698kN to 1000kN) and 26% (400kN to 502kN), respectively when vertical load is increased from 0 to $V_{ult}$ at a deflection level of 0.03D (=18mm). Thus the above analysis is useful to estimate the lateral load carrying capacity of a pile if the allowable deformation and vertical load acting on the pile is known.

4.2 Influence of vertical loading on pile bending moment

The variation of bending moment with depth for fixed headed piles embedded in dry loose sand is shown in Figure 4, where an increase of 25% in bending moment is observed when vertical load is increased from 0 to $V_{ult}$, while under saturated conditions the increase is 27% when subjected to a lateral load of 200kN. A similar variation is observed for free headed and fixed headed piles embedded in dry dense sand, where the bending moment is increased by 42% and 14%, respectively subjected to a lateral load of 300kN. It is observed from the above analysis that with an increase in vertical loads the magnitude of bending moment increases, irrespective of the soil type and the point of maximum bending moment occurs almost at the same depth irrespective of the magnitude of the vertical load acting on the pile top. This clearly occurs because of the increasing moment arising from the vertical load acting with the increasing lateral deflection (the so-called P-delta effect).

4.3 Percentage improvement in lateral capacity of pile (PIC)

The Percentage Improvement in Lateral capacity (PIC) [Karthigeyan et al. (2006)] has been defined to measure the influence of vertical loads on the lateral response of piles in various soils considered in the
present study. It is expressed as:

$$PIC = \frac{LCWV - LCNV}{LCNV} \times 100\% \quad (3)$$

where, LCWV and LCNV denotes the Lateral load Capacity With Vertical load and Lateral load Capacity under pure lateral load [Karthigeyan et al. (2006)]. It is observed that for saturated loose sand, the PIC values increase to 23% at 0.1D deflection level and reduce to 20% for vertical loads beyond 0.75V\text{lult}, while under dry conditions the increase is from 5% to 33%. In case of free headed pile in saturated dense sand and at a deflection level of 0.05D, the PIC value rises from 5% to 30%, while under dry conditions the increase in PIC value is from 6% to 32%. However at lower deflection level of 0.03D, the increase is from 3% to 26% and 8% to 21%, respectively under dry and saturated dense sand conditions. Hence it is observed from the above tables that the lateral capacities of pile in dense sand improve considerably under the influence of vertical loading.

5 CONCLUSIONS

In the present study, numerical analysis using MIDAS GTS (2013) and analytical study based on finite element technique is implemented to study the static response of a single pile in homogenous soil and subjected to both axial and lateral loadings. Based on static analyses, the following conclusions are drawn:

• The vertical load can have a significant impact on the lateral response of a single pile embedded in dense sand because under the action of vertical loads, higher vertical soil stresses and hence higher lateral stresses are generated along the surface of the pile and larger friction force is mobilized along the pile depth.

• The presence of vertical load increases bending moment in piles irrespective of soil type and pile length and the increase is maximum for free headed piles in saturated loose sand.

REFERENCES

1) Bowles, J.E. (1997): Foundation analysis and design, 5th Edition, McGraw-Hill Inc.
2) Hetenyi, M. (1955): Beams on elastic foundation: theory with applications in the fields of civil and mechanical engineering. 4th reprint, Ann Arbor: The University of Michigan Press.
3) IS 2911 Part 1 Section 4 (1984): Indian standard code of practice for design and construction of pile foundations, Bureau of Indian Standards, New Delhi.
4) IS 456 (2000): Indian standard on plain and reinforced concrete-code of practice, Bureau of Indian Standards, New Delhi.
5) Karasev, O.V., Talanov, G.P. and Benda, S.F. (1977): Investigation of the work of single situ-cast piles under different load combinations, Journal of Soil Mechanics and Foundation Engineering, 14(3), 173-177.
6) Karthigeyan, S., Ramakrishna, V.V.G.S.T. and Rajagopal, K. (2006): Influence of vertical load on the lateral response of piles in sand, Computers and Geotechnics, 33, 121-131.
7) MATLAB (2012): Programming, version 7. The Math Works Inc.
8) MIDAS GTS (2013): Geo-Technical analysis System, Korea.
9) Mylonakis, G. and Gazetas, G. (1998): Settlement and additional internal forces of grouped piles in layered soil, Geotechnique, 48(1), 55-72.
10) Phanikanth, V.S. and Choudhury, D. (2014): Single piles in cohesionless soils under lateral loads using elastic continuum approach, Indian Geotechnical Journal, 44(3), 225-233.
11) Phanikanth, V.S., Choudhury, D. and Reddy, G.R. (2010a): Response of single pile under lateral loads in cohesionless soil, Electronic Journal of Geotechnical Engineering, 15(4), 813-830.
12) Phanikanth, V.S., Choudhury, D. and Reddy, G.R. (2010b): Behavior of fixed head single pile in cohesionless soil under lateral loads. Electronic Journal of Geotechnical Engineering, 15(M), 1243-1262.
13) Poulos, H.G. (1968): Analysis of the settlement of pile groups, Geotechnique, 18(4), 449-471.
14) Poulos, H. G. (1971): Behaviour of laterally loaded piles I- single piles, Journal of Soil Mechanics and Foundations Division, ASCE, 97(SM5), 711-731.
15) Poulos, H.G. and Davis, E.H. (1980): Pile foundation analysis and design, Wiley.
16) Randolph, M.F. (1981): The response of flexible piles to lateral loading, Geotechnique, 31(2), 247-259.
17) Randolph, M.F. and Wroth, C.P. (1978): Analysis of deformation of vertically loaded piles, Journal of the Geotechnical Engineering Division, ASCE, 104(12), 1465-1488.
18) Reese, L.C. and Matlock, H. (1956): Non-dimensional solutions for laterally-loaded piles with soil modulus assumed proportional to depth, Proceedings of the 8th Texas Conference on Soil Mechanics and Foundation Engineering, Austin, Texas, 1-41.
19) Terzaghi, K. (1955): Evaluation of coefficients of subgrade reaction, Géotechnique, 4, 297-326.
20) Timoshenko, S.P. and Goodier, J.N. (2002): Theory of elasticity, 3rd Edition, Tata McGraw-Hill Education.