Research on Dielectric Loss Angle of Capacitive Equipment Based on Blackman Window Improved FFT

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Abstract. According to the fact that the dielectric loss angle of capacitive equipment is mainly determined by the phase difference between its terminal voltage and leakage current, cutting short the voltage and current signals by Blackman window, and correcting the spectrum of voltage and current signals by three-spectral line interpolation method and 7-order polynomial fitting technique, a simple and practical frequency correction formula and a formula for calculating dielectric loss angle based on improved FFT are given. The results of simulation and measurement show that the spectrum correction method presented in this paper is effective under harmonic interference and frequency fluctuation of power grid, and the measurement results of dielectric loss angle delta are accurate and stable.

1. Introduction
In power system, if the insulator of high voltage electrical equipment is damped, deteriorated, dirty by stains or discharged in insulation, etc., the development of local defects can lead to the increase of dielectric loss value until the phenomenon of insulation breakdown or partial discharge occurs, which directly threatens the safe operation of the whole power system. It is an important content of preventive test and on-line insulation monitoring to monitor the insulation performance of capacitive equipment by measuring the change of dielectric angle delta to ensure the stability and safety of equipment operation in power system.

2. Digital measurement principle of dielectric loss angle

2.1. Medium angle of capacitive equipment
A high voltage capacitive type equipment and equivalent circuit of its insulation characteristics are shown in figs. 1 (a) and (b).

![Figure 1. A high voltage capacitive type equipment and equivalent circuit of its insulation characteristics.](image-url)
Let the terminal voltage of capacitive devices be $\hat{U}$, the leakage current be $\hat{I}$. From the circuit theory, we can see that:

$$\hat{U} = U e^{i\varphi},$$
$$\hat{I} = I e^{i\varphi}.\quad (1)$$

Solution:

$$\varphi = \varphi_u + \arctan\left(\frac{2\pi fC}{1/R}\right)\quad (2)$$

The tangent $\tan \delta$ of dielectric loss angle $\delta$ of equipment is a factor used to measure insulation performance of insulating medium. It is defined as

$$\tan \delta = \frac{\text{active power}}{\text{reactive power}} = \frac{U^2 / R}{U^2(2\pi fC)} = \frac{1}{2\pi fRC}\quad (3)$$

Derived from formulas (2) and (3)

$$\delta = \frac{\pi}{2} (\varphi - \varphi_u)\quad (4)$$

That is to say, the magnitude of dielectric loss angle $\delta$ depends on the phase difference between current and voltage.

When the insulation of high voltage capacitive equipment is damped, deteriorated or dirty by stains and other defects, it can lead to the increase of dielectric loss value and the decrease of insulation resistance $R$, and the increase of dielectric loss angle $\delta$. From formula (3), it can be seen that the dielectric loss angle of high voltage capacitive equipment changes slightly when the operating frequency fluctuates. According to formula (4), the change of dielectric loss angle $\delta$ can be monitored by real-time monitoring the terminal voltage and current signals, their phase difference $\varphi - \varphi_u$ can be analyzed and calculated, thus the insulation performance of high voltage capacitive equipment can be monitored.

2.2. Principle of digital measurement of dielectric loss angle

The digital technology is applied to the measurement of dielectric loss factor $\tan \delta$. Firstly, the voltage and leakage current signals of the tested equipment are collected by sensor technology. Then these are converted into discrete signals by analogy-to-digital conversion device and frequency spectrum analysis is utilized by FFT. So, the phase difference between the voltage and current signals is calculated to obtain the specific value of dielectric loss angle $\delta$.

Let the voltage be the sinusoidal form of power frequency $f_0$: $u(t) = U_m \cos(2\pi f_0 t + \varphi_u)$. The voltage signal is sampled by sampling frequency $f_s$ and truncated by window function $w(n)$. The discrete voltage signal with $M$ sample points is obtained:

$$u(n) = U_m \cos(2\pi f_s n / f_s + \varphi_u) w(n)\quad n = 0, 1, \ldots, M - 1$$

$N$-point FFT ($N \geq M$) for $u(n)$ is obtained:
Let $f_0=\frac{k_0}{N}$. If it is synchronous sampling, $k_0$ must be an integer. Owing to
\[
\max\{|U(k)|, k=0,1,...,N-1\} = |\sum_{n=0}^{N-1} w(n) n| (k)
\]
And then
\[
U(k_0) = 0.5U_u e^{j\pi M} \sum_{n=0}^{M-1} w(n)
\]
In this case, the phase of signal $u(t)$ can be obtained by searching for the sampling number $k_0$ corresponding to the maximum value of $|U(k)|$:
\[
\varphi_u = \arg[U(k_0)]
\]
In actual measurement, the unknown of $f_0$ will lead to asynchronous sampling, that is, $k_0$ is not an integer. If $k_0= k_a + \Delta k$, where $k_a$ is an integer, then $|\Delta k| \leq 0.5$. So
\[
\max\{|U(k)|, k=0,1,...,N/2\} = |U(k_a)|
\]
And
\[
U(k_a) = 0.5U_u e^{j\pi a} \sum_{n=0}^{M-1} w(n) e^{j\frac{2\pi k_a}{N}} + 0.5U_u e^{j\pi a} \sum_{n=0}^{M-1} w(n) e^{-j\frac{2\pi k_a}{N}} + 0.5U_u e^{j\pi a} \sum_{n=0}^{M-1} w(n) e^{-j\frac{2\pi k_a}{N}}
\]
\[
\arg[U(k_a)] = \varphi_u + \arg[\sum_{n=0}^{M-1} w(n) e^{j\frac{2\pi k_a}{N}} + e^{j\frac{2\pi k_a}{N}} \sum_{n=0}^{M-1} w(n) e^{-j\frac{2\pi k_a}{N}}]
\]
It shows that, the sampling number corresponding to the maximum value of $|U(k)|$ is $k_a$ under the condition of asynchronous sampling, the amplitude angle $\arg[U(k_a)]$ and the phase $\varphi_u$ of signal $u(t)$ are not completely equal. It can be seen from formula (12) that the difference between $\varphi_u$ and $\arg[U(k_a)]$ is mainly determined by the value of $\Delta k$ and the shape of window $w(n)$.

3. Spectrum correction algorithms

3.1. Effect of negative frequency of signal
Because Blackman window has better function of spectrum leakage suppression and anti-harmonic interference, so Blackman window is selected to weigh the signal.

In practical application, the voltage of high voltage capacitive equipment in power system comes from power line and its working frequency is 50Hz. If $f_s=1000$Hz, $N=512$, then $k_a=26$. At this time, the proportion of the maximum amplitude of the negative frequency term in formula (12) can reach $10^{-6}$ magnitude, that is, the influence of the negative frequency of the signal on the spectrum amplitude can be neglected. So, formula (12) near $k_a$ can be simplified as follows:
\[ U(k) = 0.5U_0 e^{i\omega} \sum_{n=0}^{N-1} w(n) e^{-j2\pi \frac{k}{N} n} \]  

(13)

So

\[ \arg[U(k_\alpha)] = \phi_\alpha + \arg\left[ \sum_{n=0}^{N-1} w(n) e^{j2\pi \frac{Mn}{N}} \right] \]  

(14)

Since window \( w(n) \) is a real sequence and satisfies \( w(n) = w(M-1-n) \), there must be

\[ \arg\left[ \sum_{n=0}^{M-1} w(n) e^{j2\pi \frac{Mn}{N}} \right] = -\frac{M-1}{2} \times \frac{2\pi}{N} \Delta k = -\frac{M-1}{N} \pi \Delta k \]  

(15)

Thus, formula (14) can be simplified to

\[ \arg[U(k_\alpha)] = \phi_\alpha - \frac{M-1}{N} \pi \Delta k \]  

(16)

Formula (16) shows that the phase correction of the signal can be obtained as long as the correction \( \Delta k \) of the spectrum sampling sequence number. Thus, the frequency component of the signal is obtained by spectrum analysis of the signal.

\[ f_0 = \frac{f}{N} (k_\alpha + \Delta k) \]  

(17)

The phase of this frequency component is

\[ \phi_\alpha = \arg[U(k_\alpha)] + \frac{M-1}{N} \pi \Delta k \]  

(18)

### 3.2. Correction of signal spectrum

In asynchronous sampling, the frequency \( f_0 \) of fundamental voltage is difficult to locate exactly at the sampling frequency point, that is, \( k_0 = N(f_0/f_s) \) is not an integer. Let \( U(e^{j\omega}) = \text{DTFT}[u(n)] \), \( U(k) = \text{DTF}[u(n)] \). Because \( U(k) = U(e^{j\omega}) \big|_{\omega=2\pi k/N} \), and \( \max \{ |U(k)| \} = |U(k_\alpha)| \), \( k_0 = k_\alpha + \Delta k \), the relationship between \( k_\alpha \) and \( k_0 \) is shown in Figure 2.

![Figure 2. The relationship between \( k_\alpha \) and \( k_0 \) (\( k_0 < k_\alpha, \Delta k < 0 \)).](image)

For the relationship between \( k_\alpha \) and \( k_0 \) shown in Figure 2, the relative amplitude difference parameter is defined

\[ a = \frac{\left| U(k_\alpha + 1) \right| - |U(k_\alpha - 1)|}{|U(k_\alpha)|} \times 10 \]  

(19)

Formula (13) is substituted and sorted out
\[ \alpha = \frac{\left| \sum_{n=0}^{M-1} w(n)e^{j2\pi \Delta k N n} \right|}{\left| \sum_{n=0}^{M-1} w(n)e^{j2\pi n} \right|} 	imes 10 \]

(20)

Obviously, when \( N \) and \( M \) are determined, the magnitude of relative amplitude difference \( \alpha \) is related to the value of \( \Delta k \) and the shape of Blackman window \( w(n) \), but not to the value of \( k_w \). The relationship between \( \Delta k \) and \( \alpha \) by 7-order polynomial approximation is

\[ \Delta k = a_1 \alpha^7 + a_2 \alpha^6 + a_3 \alpha^5 + a_4 \alpha^4 + a_5 \alpha^3 + a_6 \alpha^2 + a_7 \alpha + a_8 \]

(21)

Because of the even symmetry of \( w(n) \), the coefficient \( a_2 = a_4 = a_6 = a_8 = 0 \) in formula (21). For \( \Delta k \in [-0.5, 0.5] \), the relative amplitude difference \( \alpha \) of each \( \Delta k \) is calculated. The other fitting coefficients in derivation (21) are as follows:

\[ a_1 = -3.3247 \times 10^{-8}, \quad a_3 = 1.5980 \times 10^{-7}, \quad a_5 = -7.7768 \times 10^{-5}, \quad a_7 = 7.8425 \times 10^{-2} \]

(22)

Therefore, after searching for the maximum value of \( |U(k)| \) corresponding to the sampling sequence number is \( k_w \), the relative amplitude difference \( \alpha \) is calculated by formula (19), and then the correction value \( \Delta k \) of spectrum sampling sequence number is calculated by formula (21), so the frequency measurement value of signal \( u(t) \) can be calculated by formula (17), and the initial phase measurement value of frequency component \( f_0 \) of signal \( u(t) \) can be obtained by formula (18).

4. Digital measurement algorithm of dielectric loss angle based on improved FFT

The terminal voltage \( u(t) \) and leakage current \( i(t) \) of the equipment to be measured are processed by anti-aliasing filtering and analogy-to-digital conversion to form digital signals \( u'(n) \) and \( i'(n) \). After weighting \( u'(n) \) and \( i'(n) \) with Blackman window \( w(n) \), \( N \)-point FFT is done. The relative amplitude difference \( \alpha \) of the spectrum of voltage and current signal is calculated by formula (19), and the correction \( \Delta k \) of the spectrum sampling sequence number is obtained by formula (21). The dielectric loss angle \( \delta \) of the equipment to be measured can be calculated from formula (4) and formula (18) as:

\[ \delta = \frac{\pi}{2} \cdot \frac{\arg[U(k_w)] + M - 1}{N} - \frac{\pi \times \Delta k}{N} - \frac{\arg[U(k_w)] + M - 1}{N} \times \Delta k \]

(23)

5. Research on simulation experiment

For the capacitive device shown in Fig. 1, the equivalent circuit parameters are \( R=1.2795\times10^3\text{M} \Omega \), \( C=591.02\text{pF} \). When \( f_0=50\text{Hz} \), the theoretical dielectric loss angle is \( \delta=4.2093\times10^{-3} \text{rad} \). Considering that the actual grid voltage must contain some odd harmonic components, and the amplitude of the harmonic is not more than 10%, the voltage signal of the equipment is shown in (24):

\[ u(t) = \sum_{l=1}^{k} U_l \cos(2\pi f_l t + \phi_{\text{u}_l}) \]

(24)

In the formula, the value of \( U_l \) is the amplitude of the \( l \)-th harmonic component, which is shown in Table 1. And the value of \( \phi_{\text{u}_l} \) is the initial phase angle of the \( l \)-th harmonic component, which is randomly selected.

| \( l \) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|------|---|---|---|---|---|---|---|---|---|
| \( U_l \) | 1 | 0 | 0.1 | 0 | 0.07 | 0 | 0.01 | 0 | 0.03 |

According to circuit theory, the current of capacitive devices is
\[ i(t) = \sum_{l=0}^{L} I_l \cos(2\pi f_0 t + \varphi_l) \]  

(25)

here:

\[ I_l = U_l \sqrt{\frac{1}{R^2 + (2\pi f_0 C)^2}}, \quad \varphi_l = \varphi_{il} + \arg(2\pi f_0 RC) \]  

(26)

For signals \( u(t) \) and \( i(t) \), their frequency spectrum \( U(k) \) and \( I(k) \) are obtained by sampling with \( f_s=1000\text{Hz} \), truncating Blackman window with length \( M=512 \), and FFT with \( N=512 \) points. Moreover, the fluctuation range of the fundamental frequency of the power grid is \( f_0 \in [49.5, 50.5] \text{Hz} \).

The improved FFT algorithm in this paper and direct-FFT are used to measure the fundamental frequency of signals \( u(t) \) when the fundamental frequency of the actual grid voltage varies in the range of 49.5 Hz to 50.5 Hz, respectively. The measurement value of \( f_0 \) and its error are shown in Table 2.

Table 2. The measurements and their errors based on improved FFT

| Theory | Improved FFT | FFT |
|--------|--------------|-----|
| 49.5  | 1.0628e-06  | -6.7188e-01 |
| 49.6  | 1.4201e-06  | -7.7188e-01 |
| 49.7  | 2.1006e-06  | -8.7188e-01 |
| 49.8  | 1.8836e-06  | -9.7188e-01 |
| 49.9  | -1.1037e-06 |  8.8125e-01 |
| 50.0  | -5.8286e-07 |  7.8125e-01 |
| 50.1  | -3.9789e-07 |  6.8125e-01 |
| 50.2  | -4.8110e-07 |  5.8125e-01 |
| 50.3  | -5.5822e-07 |  4.8125e-01 |
| 50.4  | -5.1631e-07 |  3.8125e-01 |
| 50.5  | -3.8631e-07 |  2.8125e-01 |

Table 2 shows that the measurement errors of frequency based on the improved FFT algorithm is less than \( 2.2 \times 10^{-6} \), and the errors based on FFT algorithm is less than \( 9.8 \times 10^{-1} \). This shows that the improved FFT algorithm is more superior.

When the frequency \( f_0 \) of the national standard power grid fluctuates in the range of 49.5 Hz to 50.5 Hz, the improved FFT algorithm is used to correct \( U(k) \) and \( I(k) \) respectively, and the measurement value of dielectric loss angle \( \delta \) and its error can be obtained as shown in Table 3.

Table 3. The measurement value of \( \delta \) and its error based on improved FFT

| Theory     | Measure    | Error       |
|------------|------------|-------------|
| 4.2518e-03 | 4.2499e-03 | -1.9136e-06 |
| 4.2432e-03 | 4.2412e-03 | -2.0152e-06 |
| 4.2347e-03 | 4.2331e-03 | -1.5518e-06 |
| 4.2262e-03 | 4.2256e-03 | -5.3229e-07 |
| 4.2177e-03 | 4.2218e-03 |  4.0648e-06 |
| 4.2093e-03 | 4.2130e-03 |  3.7060e-06 |
| 4.2093e-03 | 4.2037e-03 |  2.8448e-06 |
| 4.1925e-03 | 4.1942e-03 |  1.7130e-06 |
| 4.1842e-03 | 4.1847e-03 |  5.5862e-07 |
| 4.1759e-03 | 4.1754e-03 |  -4.0182e-07 |
| 4.1676e-03 | 4.1666e-03 | -9.9175e-07 |
Table 3 shows that the measurement errors of dielectric loss are less than $4.1 \times 10^{-6}$ when the frequency of power grid varies. This shows that the improved FFT algorithm is effective and stable for the measurement of dielectric loss angle of equipment.

6. Conclusion
Under the condition of asynchronous sampling, the corrected value $\Delta k$ of the spectrum sampling sequence number is estimated by the relative amplitude difference parameter $\alpha$, so as to realize the correction of the signal spectrum, i.e. the improved FFT algorithm. Experiments show that the measurement result of dielectric loss angle of capacitive equipment is accurate and stable by using the improved algorithm presented in this paper under the condition of harmonic interference and frequency fluctuation of power grid.

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