Study of multi-level atomic systems with the application of magnetic field

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Abstract. The complexity of multiple energy levels associated with each atomic system determines the various processes related to light- matter interactions. It is necessary to understand the influence of different levels in a given atomic system. In this work we focus on multi-level atomic schemes with the application of magnetic field. We analyze the different EIT windows which appears in the presence of moderately high magnetic field (~ 10 G) strength.

1. Introduction

Multiple energy levels in a given atomic system influences the processes related to light-matter interactions [1]. Interaction between magnetic and optical fields in an atomic medium results in interesting phenomena related to non-linear magneto-optical rotation [2]. In this work we include the optical fields which can enhance the polarization rotation in an atomic medium. We analyze the effect of different magnetic fields in the presence of additional optical fields. It has been understood that there should be a significant improvement in the birefringence effects with the application of magnetic and optical fields [3]. Optical pumping mechanism exhibits an unequal population distribution for different ground state Zeeman sublevels. This type of anisotropic system shows birefringent effects for the incoming linearly polarized probe beam. The plane of polarization for the incident linearly polarized beam is rotated after passing through the atomic medium with the applied fields. This is similar to the Faraday effect [4]. The additional optical fields produces number of EIT subsystems. This has been studied for the $^{87}$Rb and $^{85}$Rb atoms with the external magnetic field. Density matrix formalism has been adopted to write the equations for multiple energy levels associated with the rubidium atomic system.

2. Theoretical Formalism

$^{87}$Rb three-level $\Lambda$ system is depicted in figure 1. A circularly polarized pump beam induces birefringence effects while a linearly polarized probe beam detects the anisotropy in an atomic medium. $\Omega_2$ and $\Omega_1$ are the Rabi frequencies for the probe and control laser beams. $\Delta_1$ and $\Delta_2$ are the probe and control laser detuning frequencies. Zeeman shifts in the ground and excited states are denoted as $\Delta g$ and $\Delta e$ respectively. Zeeman shift = $g \mu_B m_B B \hbar$.

The density matrix elements are calculated from the Liouville equation.

\[
\frac{d\rho}{dt} = \frac{i}{\hbar} [\rho, H] - \Gamma \rho \tag{1}
\]
The Hamiltonian for the given atomic configuration is
\[
H = \hbar \sum_{i=1}^{3} \omega_{a_i} |a_i \rangle \langle a_i| + \sum_{j=1}^{b} \omega_{b_j} |b_j \rangle \langle b_j| + \\
\sum_{k=1}^{c} \omega_{c_k} |c_k \rangle \langle c_k| + \hbar \sum_{l=1}^{d} \Omega_{l} |a \rangle \langle c| e^{i\omega_{l}t} + |c \rangle \langle a| e^{-i\omega_{l}t}| + \\
\frac{\hbar}{2} \left[ \sum_{m=0}^{n} \Omega_{m} |b \rangle \langle c| e^{i\omega_{m}t} + |c \rangle \langle b| e^{-i\omega_{m}t}\right]
\] (2)

Rabi frequency, \( \Omega = -\frac{\mu E}{\hbar} \) (3) \( \mu \)
is the dipole moment for the appropriate atomic transition and \( E \) is the electric field. The probe Rabi frequencies are very much smaller than the strong pump frequencies.

According to the steady state condition we can assume that
\[
\sum_{m} \rho_{ma} = 1; \sum_{m} \rho_{mb} = \sum_{m} \rho_{mc} = 0
\] (4)

Under the weak probe conditions, we can incorporate electric dipole and rotating wave approximations. For a three-level system we can obtain the density matrix equations. The steady state solution for the non diagonal element of a density matrix \( \rho_{cab} \) as
\[
\rho_{cab} = \frac{i\Omega_{c} \rho_{cab}}{2} + \frac{\Omega_{c}^{2}}{4} \left[ i((\Delta_{a} - \Delta_{c}) - \Gamma_{cab}) + \frac{i((\Delta_{a} - \Delta_{c}) - (\Delta_{c} + \Delta_{d}) - \Gamma_{b1a})}{4} \right]
\] (5)

Where \( \Omega_{c} \) is the Rabi frequency for the probe laser and \( \Omega_{a} \) is the Rabi frequency for the control laser. \( \Delta_{1} \) and \( \Delta_{2} \) are the detuning terms. After applying the magnetic field, there are Zeeman shift terms in the ground state \( \Delta_{g} \) and the excited state \( \Delta_{e} \) are included in the following equations.

\[
\rho_{c1a1} = \frac{i\Omega_{c} \rho_{c1a1}}{2} + \frac{\Omega_{c}^{2}}{4} \left[ i((\Delta_{1} + 2\Delta_{g} - \Delta_{e}) - \Gamma_{c1a1}) + \frac{i((\Delta_{1} - \Delta_{e}) - (\Delta_{e} + \Delta_{g}) - \Gamma_{b2a1})}{4} \right]
\]

\[
\rho_{c2a2} = \frac{i\Omega_{c} \rho_{c2a2}}{2} + \frac{\Omega_{c}^{2}}{4} \left[ i((\Delta_{1} + \Delta_{g} - \Gamma_{c2a2}) + \frac{i((\Delta_{1} - \Delta_{e}) - \Gamma_{b3a2})}{4} \right]
\]

\[
\rho_{c3a3} = \frac{i\Omega_{c} \rho_{c3a3}}{2} + \frac{\Omega_{c}^{2}}{4} \left[ i((\Delta_{1} + \Delta_{g} - \Gamma_{c3a3}) + \frac{i((\Delta_{1} + \Delta_{g}) - (\Delta_{e} - \Delta_{g}) - \Gamma_{b4a3})}{4} \right]
\]
Here $\Gamma$’s are decay constants. $\Omega$ is the Rabi frequency, it is calculated for different atomic transitions by considering the corresponding laser intensity.

\[
\dot{\rho}_{c,3a1} = \frac{i\Omega_1}{2} \rho_{a1e} + \frac{\Omega_0^2}{4} \frac{i(\Delta_1 - \Delta_2) - \Gamma_{c3a1}}{[i(\Delta_1 - \Delta_2)] - \Gamma_{b4a1}}
\]

\[
\rho_{c,4a2} = \frac{i\Omega_2}{2} \rho_{a2e} + \frac{\Omega_{10}^2}{4} \frac{i(\Delta_1 - \Delta_2) - \Gamma_{c4a2}}{[i(\Delta_1 - (\Delta_2 - 2\Delta_2)] - \Gamma_{b5a2}}
\]

\[
\dot{\rho}_{c,5a3} = \frac{i\Omega_0}{2} \rho_{a3e} + \frac{\Omega_{10}^2}{4} \frac{i(\Delta_1 - 2\Delta_2 + \Delta_2) - \Gamma_{c5a3}}{[i(\Delta_1 - 2\Delta_2 + \Delta_2) - \Gamma_{c5a3}}
\]

The pump field is very much stronger than the probe laser field. In general, this can be studied as a $\Lambda$ type three-level EIT system. A linearly polarized probe beam can be considered as a combination of clockwise and counter clockwise circularly polarized components. There are different three-level subsystems which can be individually solved for multiple Zeeman sublevel systems. There are three $\Lambda$ type EIT systems for $(\sigma_+, \sigma_-)$ polarization scheme in the case of $^{87}\text{Rb}$. Likewise there are two $\Lambda$ type EIT systems and a two-level system in the $(\sigma_+, \sigma_0)$ polarization scheme. The energy level diagrams for these configurations are shown in figures 2 and 3. The mismatch between these number of 3 level and 2 level subsystems in a given polarization configuration $(\sigma_+, \sigma_0)$ and $(\sigma_-, \sigma_0)$ lead to a circular asymmetry. This circular asymmetry stimulates the birefringence in an atomic medium. The Zeeman levels of $F=1, F=2$ and $F'=2$ in $^{87}\text{Rb}$ are denoted as $|a>, |b>, |c>$, where $x=1, 2, 3; y=1, 2, 3, 4, 5$ and $z=1, 2, 3, 4, 5$. 

![Figure 1. Three-level system](image-url)
In $^{85}$Rb, five different $\Lambda$ type EIT subsystems are present for the ($\sigma_+, \sigma_-$) polarization scheme. Likewise, there are four different $\Lambda$ type EIT subsystems and a two-level system in the ($\sigma_+, \sigma_-$) polarization scheme.

Faraday rotation

$$\phi = \frac{\pi I}{2\lambda} \text{Re}(\chi^+ - \chi^-)$$  \hspace{1cm} (7)

3. Results and discussion

EIT windows appear when we apply the magnetic field. When the magnetic field is relatively strong (in comparison with intermediate range) to split the resonances, then the asymmetry saturates. In this case the birefringence effects do not depend on the magnitude of the applied magnetic field. It is shown in figure 4 for the ($\sigma_+, \pi$) control-probe polarization configuration at 10 G magnetic field. It is found that there are three separate resonances for $^{87}$Rb, which is also predicted in the calculation. There are three EIT windows
for the \((\sigma, \sigma_{\pi})\) control-probe polarization configuration, which is denoted as \(\text{Re}(\chi^+_{\pi})\) in figure 4 and two EIT windows for the \((\sigma, \sigma_{\pi})\) control-probe polarization configuration, which is denoted as \(\text{Re}(\chi^-)\). The resultant anisotropy is \(\text{Re}(\chi^+_{\pi} - \chi^-)\), which follows the trend of experimental curve.

![Figure 4. \(^{87}\text{Rb}\) Optical/magnetic field induced rotation at 10G](image)

In the case of \(^{85}\text{Rb}\), similar measurements and calculations are made for the \((\sigma_{\pi}, \pi)\) control-probe polarization configuration at 10G magnetic field. It is noticed that there are five peaks in the experimental curve and the calculated results follow the experimental curve. This is shown in figure 5. There are five EIT windows for the \((\sigma_{\pi}, \sigma_{\pi})\) control-probe polarization configuration, which is denoted as \(\text{Re}(\chi^+_{\pi})\) in figure 5 and four EIT windows for the \((\sigma, \sigma_{\pi})\) control-probe polarization configuration, which is denoted as \(\text{Re}(\chi^-)\).
Figure 5. $^{85}$Rb Optical/magnetic field induced rotation at 10G

4. Conclusions
Rotation in the intermediate field region shows the signature of additional EIT windows. It is also found that the circular asymmetry and linear asymmetry induce birefringence effects, which are observed in the $^{87}\text{Rb}$ and $^{85}\text{Rb}$ atomic systems at intermediate magnetic fields. Three EIT windows for $^{87}\text{Rb}$ and five EIT windows for $^{85}\text{Rb}$ are observed from the calculation which qualitatively agrees with the measurement results.

References
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