Solving the Container Stowage Problem (CSP) using Particle Swarm Optimization (PSO)

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Abstract. Container Stowage Problem (CSP) is a problem of containers arrangement into ships by considering rules such as: total weight, weight of one stack, destination, equilibrium, and placement of containers on vessel. Container stowage problem is combinatorial problem and hard to solve with enumeration technique. It is an NP-Hard Problem. Therefore, to find a solution, metaheuristics is preferred. The objective of solving the problem is to minimize the amount of shifting such that the unloading time is minimized. Particle Swarm Optimization (PSO) is proposed to solve the problem. The implementation of PSO is combined with some steps which are stack position change rules, stack changes based on destination, and stack changes based on the weight type of the stacks (light, medium, and heavy). The proposed method was applied on five different cases. The results were compared to Bee Swarm Optimization (BSO) and heuristics method. PSO provided mean of 0.87% gap and time gap of 60 second. While BSO provided mean of 2.98% gap and 459.6 second to the heuristics.

1. Introduction
Sea transportation is an important mode in international trade. According to the Maritime Transport Review released by the United Nations Conference on Trade and Development at 2012, about 80% of global trade is conducted through sea. Among different ship types, about 62% used containers. Since the containerization era, containers have helped to smooth the flow of goods [1]. With the rapidly growing use of containers, container stowage problem becomes a complex problem faced daily by all shipping lines. Therefore, it requires proper container containment planning. But, good planning is not easy as it depends on the experience of the planner [2]. In the container arrangement into the vessel, we must pay attention to three rules. First rule is the location of the container. The location must be in accordance with the type and size of the container. The second rule, is the heavier container should be placed under a lighter container. The third is the balance of the ship. The arrangement of thousands of containers that have dozens of destinations with regard to the size, type, weight and equilibrium of ships, raises many possibilities that are difficult to solve (combinatorial problems). Out of many combinations, there are some solutions which are infeasible because it must be re-unloaded as much as possible so container can get into the ship. Therefore, the heuristic approach to solve container arrangement problem that makes the number of shifting and unloading time minimal is required [3].

Research on CSP have been done with various of methods, such as genetic algorithm [4], bi-level genetic algorithm [5], heuristics [6], ant colony optimization [1], tabu search [8], constraint programming and integer programming [9], survey method and classification [10] integer linear
programming [11], hybrid metaheuristic and local search heuristics [10]. The algorithm used in this paper is particle swarm optimization (PSO). The reason is aside from its simplicity, PSO has not been applied for container stowage problem (CSP) with considering the five factors: type, size, weight, purpose, and balance.

2. Container Stowage Problem (CSP)
Each container location on the vessel has a position index or location information: position of bay, row, and tier. The bay represents a segment from the front to the rear of the ship. While the row indicates a segment from the center to the outside of the ship if the ship is seen from above. Tier represents a segment from the bottom to the top if the ship is viewed from the side. The description of bay, row, and tier can be seen in figure 1.

![Figure 1. Description of Bay, Row, and Tier.](image)

Figure 1 presents each location based on bay, row, and tier indices. The container size of 20 is placed on the odd bays, i.e. 01, 03, 05, 07, 09 and so on. While, container size of 40 is placed on even bays or two odd bays, i.e. 04 = 03 + 05, 08 = 07 + 09 and so on [6].

3. Model Container Stowage Problem (CSP)
Many research on CSP use various methods and models. One of the models that considers the five factors in the stowage process (the size, type, weight, and purpose of the container, as well as, the ship's balance) was the model that proposed by Ambrosino et al. [6]. This basic model has also been used in Putamawa and Santosa [3]. This model is used in this research. The basic model is:

\[
\begin{align*}
\text{Min } L & = \sum_i \sum_c t_{ic} x_{ic} \\
\sum_i \sum_c x_{ic} & = m \\
\sum_c x_{ic} & \leq 1 \forall c \\
\sum_c x_{ic} & \leq 1 \forall l \\
\sum_i \sum_c w_c x_{ic} & \leq Q \\
\sum_c \mathbb{E} x_{ijkc} & = 0 \forall i \in E, j, k \\
\sum_c \mathbb{E} x_{ijkc} & = 0 \forall i \in O, j, k \\
\sum_c \mathbb{E} x_{ij+k} + \sum_c \mathbb{E} x_{ij+k} & \leq 1 \forall i \in E, j, k \\
\sum_c \mathbb{E} x_{ijk-1} + \sum_c \mathbb{E} x_{ij+k} & \leq 1 \forall i \in E, j, k \\
\sum_c \mathbb{E} x_{ij+k} + \sum_c \mathbb{E} x_{ij+k} & \leq 1 \forall i \in E, j, k = 1, ..., |k| - 1 \\
\sum_c \mathbb{E} x_{ijk-1} + \sum_c \mathbb{E} x_{ij+k} & \leq 1 \forall i \in E, j, k = 1, ..., |k| - 1 \\
\sum_c \mathbb{E} w_{ic} x_{ijkc} & \leq MT \forall i, j \\
\sum_c \mathbb{E} w_{ic} x_{ijk} & \leq MF \forall i, j \\
\sum_c \mathbb{E} x_{ijkc} - w_{ie} x_{ij+k} & \geq 0 \forall i, j, k = 1, ..., |k| - 1 \\
\sum_c \mathbb{E} (d_{ie} x_{ij+k} - d_{ie} x_{ij+k}) & \geq 0 \forall i, j, k = 1, ..., |k| - 1 \\
- Q_2 & \leq \sum_i \sum_c \sum_{w_c} x_{ijkc} - \sum_i \sum_{d_c} w_c x_{ijkc} \leq Q_2 \\
- Q_1 & \leq \sum_i \sum_c \sum_{w_c} x_{ijkc} - \sum_i \sum_{d_c} w_c x_{ijkc} \leq Q_1 \\
x_{ic} & \in \{0, 1\} \forall i, c
\end{align*}
\]

40 bay's
The objective function is to minimize the total unloading time (L), with the decision variable $x_{lc}$ being the container $c$ at location $l$, equals to 1 if the container $c$ is at location $l$ and equals 0 if not. $x_{lc} = x_{ijk}$. Other variables used in the mathematical model are:

- $l$: The location of the 1st placement ... n shown by $i$ = bay, $j$ = row, and $k$ = tier ($l = 1, 2, \ldots n$).
- $c$: 1st container ... m.
- $t_{lc}$: Time needed for unloading container $c$ location $l$.
- $M$: Number of all containers.
- $w_c$: The weight of the container $c$.
- $Q$: Total capacity.
- $Q1$: Tolerance of anterior-posterior balance.
- $Q2$: Left-right balance tolerance.
- $E$: Bay is even. $O$ = Bay odd. $T$ = Container with size 20 feet.
- $F$: Container with size 40 feet.
- $MT$: Weight limit of one level container 20 feet.
- $MF$: Weight limit of one level container 40 feet.
- $K$: Tier ship.
- $d_c$: Destination of container.

The description of each equation is as follows. Eq. (1) objective function, minimizing the total time of unloading. Eq. (2) the total $l$ location of the occupied container should be equal to the number of containers. Eq. (3) one container is placed in only one location. Eq. (4) one location can only be occupied one container. E. (5) the total weight of the container transported shall not exceed the capacity of the ship (Q). Eq. (6) the 20 feet (T) container will not be placed on even bay (E). In accordance with the arrangement rules based on container size is 20 feet container placed on odd bay. Eq. (7) the 40 feet (F) container will not be placed in the odd bay (O). Eq. (8) and eq. (9) container 20 and 40 feet container cannot be placed simultaneously on odd bays and successive even bays. Eq. (10) and eq. (11) container 20 feet cannot be placed on top of container 40 feet. Eq. (12) the total weight of the 20 feet container pile on one tier shall not exceed the MT limit. Eq. (13) the total weight of the 40 feet container pile on one tier should not exceed the MF limit. Eq. (14) the heavier container cannot be placed above a lighter container. Eq. (15) container with closer destination is placed on top of the container with the longest destination. Eq. (16) the total difference of the weight of the container on the anterior part with the posterior part shall not exceed the tolerance Q2. Eq. (17) the total difference of the weight of the container on the left side with the right side shall not exceed the tolerance Q1. In accordance with the rules based on the balance of the vessel is the total weight difference of the container between the axis portion of the vessel forward and the rear axis of the ship does not exceed the specified limit. Finally, Eq. (18) the decision variable is a binary (0,1).

4. Particle swarm optimization (PSO)

Particle Swarm Optimization (PSO) is developed based on the behavior of the swarm (bird or fish). The individual movement is influenced by its individual (cognitive) and social (swarm) movements [13]. Each individual (particle) is characterized by velocity and position. Updating the velocity ($v$) and the position ($x$) of each particle based on its initial position, initial velocity, Gbest, and Pbest is done according the following formula:

$$V_{t+1} = V_t + c_1 r_1(Pbest - X_t) + c_2 r_2(Gbest - X_t) \quad (19)$$
$$X_{t+1} = X_t + V_t \quad (20)$$

where Pbest is the best position of each particle among iterations, Gbest is the best of Pbest. PSO is suitable to solve continuous problem, because the value of position and velocity of all of particles are continuous. The disadvantage of PSO is the possibility to be trapped at local optimal. The solution can be trapped at local optimal because of the updating solution only based on the best solution from
previous iteration and the algorithm does not have a mechanism to jump out from the local optimal. The modification by changing the updating mechanism (based on some best solution) can reduce the possibility to be trapped at local optimal. To apply PSO to CSP, it is necessary to customize or modify algorithms as illustrations in figure 2.

![Flowchart](image)

**Figure 2.** Flowchart of solving the CSP using PSO.

5. **Validation**
Validation is done by comparing the results of experiment on small data with those of enumeration. The comparison of the results of both are as shown in table 1.

| Method                  | Enumeration | PSO |
|-------------------------|-------------|-----|
| Penalty                 | 0           | 0   |
| Time **Unloading**      | 366         | 366 |
| Objective function      | 366         | 366 |

Based on the results, particle swarm optimization and enumeration algorithms provided the same performance. Therefore, the algorithm developed can be said to be valid, so that research can be done next step.

6. **Experiment**
The experiments were done on some data sets. The data used are secondary data sets from those in Putamawa and Santosa [3]. The numbers of of bay = 14, row = 4, tier = 5. The description of the data is shown in table 2.
Table 2. Data Criteria for Each Case

| Case | Total Container | Size 20’ | 40’ | Light (10 - 15) | Medium (20 - 25) | Heavy (30 - 35) | Destination |
|------|----------------|---------|-----|----------------|-----------------|----------------|-------------|
| 1    | 100            | 62      | 38  | 45             | 30              | 25             | 47          |
| 2    | 120            | 75      | 45  | 50             | 44              | 26             | 55          |
| 3    | 130            | 90      | 40  | 56             | 46              | 28             | 55          |
| 8    | 140            | 95      | 45  | 60             | 50              | 30             | 65          |
| 9    | 140            | 95      | 45  | 60             | 50              | 30             | 50          |

7. Analysis of performance algorithm

Particle swarm optimization (PSO) for container stowage problem (CSP) has been combined to solve CSP. CSP is the problem of arranging container into ship which is difficult to solve because it has many constraints and where constraints can be conflicted each other. To overcome this issue, the PSO added a penalty to the objective function. The penalty is used to control the solution in finding the optimal solution. Container position is determined by random number, and steps added to arrange the container based on the weight of the container. It is possible to change the position of the container based on the coordinates of the position on the ship. Every position that has not been full until the upper level is possible to be placed a new container obtained from the coordinates of other positions with the least pile position. Then sorted is done by the container destination so that the container with the closer destination is always placed above the container with the further destination. Experiments were conducted on five different cases. The five cases were applied to the PSO, and the PSO modifications were compared with the BSO modifications in the previous study.

The results from five cases showed that PSO still cannot find the optimal solution. It can be seen in table 3 that shows the gap of PSO is still greater than those of BSO modification. Some steps are added to improve the performance of PSO.

\[ \text{Gap} = \frac{L_{PSO} - L_{BSO}}{L_{BSO}} \]  \hspace{1cm} (21)

\[ \% \text{Gap} = \frac{(L_{PSO} - L_{BSO})}{L_{BSO}} \times 100 \]  \hspace{1cm} (22)

Gap is the difference between the two algorithms, % gap is the percentage of the gap. To calculate gap and % gap we can use formulas (21) and (22). \( L \) = Average times unloading.

In table 3. The largest mean objective function is given by BSO. While, the results of PSO in cases 1, 2, 4, and 5 are similar to the solution obtained by the heuristic method. In case 3, PSO gives smaller result than the heuristic method.

8. Conclusion

We have proposed modified PSO to solve Container Stowage Problem. The validation process of PSO is done by comparing the results with those of enumeration technique. The experiment on small problems showed the same results. This validated the proposed algorithm. To apply PSO on CSP we added steps that changes the position of the container based on the number of tiers, sorting the container stack based on the destination container, and sorting the container stack based on the type of container weight. By adding these three steps, it guarantees that two constraints are not violated. This allows the penalty value to be smaller and the value of the objective function to be minimum. The algorithm has been carried out in five cases. The comparison between PSO and BSO provided the Gap of -2.01 percent. It means that PSO gives a smaller value of 2.01 percent or 433.86 seconds lower than the result of modified BSO. Then the comparison of PSO with heuristics provide 0.87 percent Gap. This means that PSO gives greater value of 0.87 percent or 60 seconds greater than heuristic.
For further research, it is suggested to consider not only to minimize unloading time but also loading time. Also, consider to include more than two types of container sizes.

### Table 3. Comparison of Algorithms

| Comparison       | Case | Mean | %GAP | Best | GAP | Comparison       | Case | Mean | %GAP | Best | GAP |
|------------------|------|------|------|------|-----|------------------|------|------|------|------|-----|
| PSO Vs BSO       | 1    | 14629.60 | 0.97 | 14624 | 135.2 | PSO Modifikasi Vs Heuristik | 1    | 14057.80 | 1.47 | 14004 | 150.0 |
| PSO Vs BSO       | 2    | 17294.20 | 0.42 | 17279 | 57.5  | PSO Modifikasi Vs Heuristik | 2    | 16847.00 | 1.59 | 16740 | 156.0 |
|                  | 3    | 18879.30 | 2.03 | 18866 | 362.0 |                  | 3    | 18021.00 | -0.41 | 17910 | -186.0 |
|                  | 4    | 20304.30 | 2.25 | 20297 | 440.0 |                  | 4    | 19628.00 | 0.97 | 19530 | 90.0  |
|                  | 5    | 20308.90 | 2.35 | 20304 | 462.0 |                  | 5    | 19619.00 | 0.77 | 19560 | 90.0  |
| PSO Modifikasi Vs BSO Modifikasi | 1 | 14057.80 | -2.97 | 14004 | -484.8 | | 1 | 14488.80 | 4.58 | 14436 | 582.0 |
|                  | 2    | 16847.00 | -2.17 | 16740 | -481.5 | PSO Modifikasi Vs Heuristik | 2    | 17221.50 | 3.84 | 17190 | 606.0 |
|                  | 3    | 18021.00 | -2.61 | 17910 | -594.0 |                  | 3    | 18504.00 | 2.25 | 18456 | 360.0 |
|                  | 4    | 19628.00 | -1.15 | 19530 | -327.0 |                  | 4    | 19857.00 | 2.15 | 19818 | 378.0 |
|                  | 5    | 19619.00 | -1.12 | 19560 | -282.0 |                  | 5    | 19875.00 | 2.08 | 19842 | 372.0 |

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