A simple macro-model of COVID-19 with special reference to India

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Abstract
Motivated by the prevailing severe situation in India, we extend the SIR(S) model of infectious diseases to incorporate demand dynamics and its interaction with COVID-19 spread. We argue that, on one hand, the spread of COVID-19 creates panic among consumers and firms and negatively affects economic activity. On the other hand, economic activity intensifies the spread of the infection. Initially assuming that recovered individuals do not develop antibodies and become susceptible again, we capture the interaction between economic activity and the spread of the disease in a two-dimensional dynamical system. We show that a large fiscal expansion combined with measures to boost community health and improve the health sector's capacity to provide critical care can simultaneously improve the economy and control the spread of the disease. Finally, assuming that only a fraction of recovered individuals become susceptible to contracting the diseases again, we obtain richer dynamics in a three-dimensional dynamical system. This paper also highlights the important role of infection rates and the recovery rate in determining the uniqueness and the stability properties of steady state.

KEYWORDS
animal spirits, consumer confidence, COVID-19, effective demand, fiscal policy, public health

JEL CLASSIFICATION
E12; E62; H51
1 | INTRODUCTION

This paper extends the Susceptible, Infected, and Recovered (SIR(S))\(^1\) model of infectious diseases by introducing economic activity in order to understand the current severe situation and health and economic crisis in India’s context. The interaction between the pandemic and economic activity and the economic consequence of the COVID-19 pandemic is missing in the traditional epidemiological models. Although COVID-19 vaccines have come out, there are still ambiguities about their effectiveness, and many countries, including India, are experiencing a second or a third wave of the pandemic. Most governments continue to rely on non-medicinal policy interventions such as lockdowns, restrictions on several economic activities. These policies have a severe negative effect on economic activity. Moreover, the pandemic itself creates panic among economic agents. Therefore, this paper aims to explain the consequences of the interaction between the COVID-19 pandemic and economic activity and prescribes some policies to control the pandemic and improve the economic situation.

India witnessed a rapid increase in COVID-19 infections in March 2020. The Government of India responded by imposing the strictest country-wide lockdown in the entire world on March 23, which continued till May before being relaxed in a phased manner from June onward. Nonetheless, compared to its neighboring countries, India’s performance in controlling the pandemic has not been very satisfactory. Figure 1a presents the total number of cases since they first reported their 50th case while Figure 1b represents the ratio of death to cumulative number of confirmed cases in India and its neighbor countries.\(^2\) Immense differences exist within the states as well. Estimates of $R_0$ for India during the initial stage of the pandemic by Singh and Adhikari (2020), Bhola et al. (2020) and Ranjan (2020) are between 1.5 and 2.6. However, by September, India’s $R_0$ was below 1, and in December, it was 0.86, although there were still a few states with $R_0$ greater than 1.\(^3\)

The sudden imposition of lockdown caused a breakdown in the supply chain of the economy. The urban poor and migrant workers in the cities were left with neither work nor food and, with long-distance buses and trains becoming unavailable, millions turned toward their native places on foot. No satisfactory government measures were taken for these migrant workers—not even proper food distribution was arranged although sufficient food grains were available with the center.\(^4\)

The Indian economy was already going through a slow down even before the COVID-19 pandemic. Unemployment rates were at unprecedented high levels as per the Periodic Labour Force Survey of 2017–2018. The MSMEs (micro, small, and medium enterprises) are one of the most important sectors of the economy and contribute about 30% of the country’s gross domestic product.

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\(^{1}\)The very last \(S\) indicates that there is a possibility that the recovered individuals may reenter the susceptible group again. See Hirsch et al. (2004) for the basic SIR(S) model. Also see Kermack and McKendrick (1927), Baraun (1983, pp. 456–463), Murray (2002, pp. 315–327).

\(^{2}\)Basu and Srivastava (2020a, 2020b, 2020c) analyze the relative performance of major South Asian nations in addressing the public health as well as economic repercussion of the COVID-19 pandemic. We follow the same procedure as Basu and Srivastava (2020a) for constructing Figure 1a.b.

\(^{3}\)Note that the reproduction number ($R_0$) or the number of people getting infected by an already infected person on average is an important parameter to understand the pandemic scenario. When this number ($R_0$) drops below 1, it means the number of recoveries now outnumber the new infections. By December 18, 2020, $R_0$ dropped to 0.86 in India. Most of the states with the maximum number of active cases had an $R_0$ value less than 1 (source: https://www.deccanherald.com/national/covid-19-r-value-under-one-almost-all-over-india-best-week-since-pandemic-began-929303.html). Among all the states, Maharashtra has the highest $R_0$ value, 0.97, to date (source: http://pracriti.iitd.ac.in/; retrieved on 12 March, 2021).

\(^{4}\)The total stock of food grains in the Central Pool was 56.939 million metric ton (MMT) out of which 24.7 MMT was wheat and 32.239 MMT was rice. These were substantially higher than the food grain stocking norms of 21.040 MMT for the quarter beginning from April 1, 2020 (source: Department of Food and Public Distribution, (2020a, 2020b)).
**FIGURE 1** (a) Logarithm of the total confirmed cases of COVID-19 till March 9, 2021. (b) Ratio of death to total confirmed cases (in percent) of COVID-19 till March 9, 2021. Source: Computed using Our World in Data website
(GDP), 40%–45% of exports, and employ about 30% of the labor force (about 114 million people). About 63 million unincorporated MSMEs are engaged in the non-agricultural sector, most of which are micro-enterprises in the informal sector. The MSMEs were already incurring business losses and liquidity problems much before the lockdown owing to insufficient credit, delays in payments and unsold goods. Despite the announcement of a few measures by the government “a survey of 5,000 MSMEs by the All India Manufacturers Organisation (AIMO) revealed that 71% of the enterprises could not pay salaries to their workers for March 2020 due to the lockdown.” Another issue plaguing the Indian economy throughout this decade is a continuous deterioration in the gross investment rate in Indian industries from 2010 to 2011.

The story is not satisfactory in the health sector, either. For the last two decades, the total health expenditure never exceeded beyond 4.26% of GDP. It constituted merely 3.53% of GDP in 2017. Out-of-pocket expenditure in 2017 was 62.4% of the total health expenditure and 2.2% of GDP. Government expenditure on health in 2017 was only 0.96% of GDP. The doctor–patient ratio in India is much lower (0.8:1,000) than the WHO recommendation of 1:1,000. On the other hand, merely 0.7 hospital-beds are available per 1,000 population.

The twin-damages in the form of demonetization and implementation of GST (Goods and Services Tax) have already harmed the economy (Patnaik, 2019). The health situation during the pandemic is also complicated by the fact that while public health in India is the responsibility of states, the central government has not managed to compensate the states for revenue loss due to GST implementation.

One of the simplest model for infectious diseases, like measles, malaria, chickenpox, mumps, smallpox, rubella, influenza, is the SIR model. In this model, the entire population \( (N) \) is divided into three groups which are mutually exclusive to each other. These are (a) the susceptible class \( (S) \) which consists of those individuals who are still not infected, but can catch the disease and become infected; (b) the infected individuals \( (I) \) who have the disease and can transmit it; and (c) the recovered individuals \( (R) \) who was infected by the disease and have recovered. The SIR(S) model as mentioned in Hirsch et al. (2004, p. 238) is as follows. The rate of transmission of the disease depends on the number of encounters between susceptible \( (S) \) and infected \( (I) \) individuals. However, the recovered individuals also can be returned to the class \( S \) at a rate proportional to the population of recovered individuals \( (R) \). On the other hand, a fraction \( (\nu) \) of the infected individuals is recovered. Therefore, the change in \( S, I \) and \( R \) can be written as

\[
\begin{align*}
\dot{S} &= - \beta SI + \mu R \\
\dot{I} &= \beta SI - \nu I \\
\dot{R} &= \nu I - \mu R
\end{align*}
\]
where $\beta$, $\mu$, and $\nu$ are all positive constants. $\beta$ is the infection rate. However, the interaction between the pandemic and economic activity is missing in this kind of model. We extend this model to explore the consequences of the interaction between economic activity and infection. Although this paper is motivated by Indian experience, this analysis applies to other countries as well. The analytical approach is divided into two parts: the first part where once a person recovers from the disease immediately enters the pool of susceptible individuals, and the second part, where only a fraction of the recovered individuals enter the pool of susceptible individuals. Economic activity is a major site for the spread of infection as infected persons may present themselves as asymptomatic and continue to participate in an economic activity where they interact with susceptible persons. To account for this, assume that the number of infections not only increases because of the interaction of infected and susceptible individuals but also with the increase in output. At the same time, infection also creates panic among consumers and forces them to cut down their spending on non-essential commodities and similarly dampens the investors' animal spirits. Hence, we also assume that the economic activity is negatively affected by the number of infected individuals. The model we study can produce both pandemic-free as well as pandemic equilibria under parametric restrictions. We find that fiscal expansion increases the level of output as well as the number of infected individuals. On the other hand, a fiscal expansion along with some probable measures such as adequate isolation, adequate medical care, public health vigilance and control, mass-scale testing and segregating the infected individuals from others, large-scale use of personal protective equipment (PPE) can reduce the number of infected individuals and improve the economic situation.

A richer dynamics is obtained when we assume that instead of all the recovered individuals only a fraction of them become susceptible to contracting the infection again. Here too, the model can produce endemic equilibrium under certain parametric restrictions. We show that, ceteris paribus, a relatively low infection rate due to non-economic activity ($\beta$), a weaker rate of infection because of economic activity ($\theta$), and a sufficiently strong recovery rate ($\nu$) lead to the existence of an economically meaningful unique stable steady state. On the other hand, relatively high values of $\beta$ and $\theta$, and a low recovery rate may lead to the existence of multiple equilibria, one of which is unstable. We show that a rise in the recovery rate from the disease raises the equilibrium level of output, whereas the level of infected individuals falls. We show that a proper policy-mix of a fiscal expansion and measures that ensure a lower infection rate and a higher recovery rate can increase the steady-state level of output and a fall in the number of infected individuals.

There are already a few papers that extend the canonical SIR model to understand the economic consequences of COVID-19. Eichenbaum et al. (2020) investigate the interaction between economic decisions (such as purchasing consumption goods and working) and epidemics. As they argue, the epidemic generates both the supply and the demand effects on economic activity, which in turn can generate a large and persistent recession. As the economic agents are atomistic, they do not internalize the impact of their actions on the infection and death rates of other agents, and therefore the competitive equilibrium of their model economy is not Pareto optimal. Bethune and Korinek (2020) analyze the trade-off between economic costs and epidemiological control and the externalities that arise when social and economic interactions transmit infectious diseases like COVID-19. Based on a SIS model, they argue that the individually rational susceptible agents rationally reduce the level of their economic activity to reduce the risk of infection. However, the rational infected agents, recognizing that they have nothing to lose from further social interaction, do not internalize the social cost they impose on others. While the decentralized SIS economy converges to a steady state in which the disease is endemic, the social planner as it internalizes the infection externalities, by inducing infected agents to reduce their economic activity lowers the spread of the disease. Extending the analysis into a SIRS model, they find similar kind of result. Considering a planner who wants to control a pandemic's fatalities and minimize the output costs of the lockdown, Alvarez et al. (2020) analyze the optimal
lockdown policy in a SIR model. However, unlike this paper, the abovementioned papers do not adequately focus on the interaction between demand dynamics and the spread of COVID-19.9

Constructing a two-equation dynamical system of COVID-19 positivity rate and the unemployment rate, Barbieri-Góes and Gallo (2021) capture the evolution of COVID-19 and its effects on the economy. They get both endemic-free and endemic equilibria, whereas the endemic equilibrium, as they suggest, is more realistic. In the absence of extensive vaccination, the endemic equilibrium causes pandemic-driven waves in the unemployment rate. Barbieri-Góes and Gallo (2021) then simulate the unemployment shock on the overall level of economic activity and find that the shock causes negative effect on the output level. Moreover, in the absence of adequate stimulus policies, the emergence of cyclical downswings may determine an L-shaped recession in the medium run. They find that the stimulus policies helped prevent fluctuations in the labor market caused by the pandemic waves. However, more unemployment benefits and significant policy intervention are required for boosting the aggregate demand.

Considering an epidemiological Susceptible, Exposed, Infectious, and Recovered (SEIR) model, Flaschel et al. (2021) analyze the interaction between epidemiological dynamics and the dynamics of economic activity in a demand-constrained economy. In their model, the contact rate and the probability of exposure to the disease depend positively on economic activity. On the other hand, a rise in infection decreases household spending, which in turn reduces aggregate demand. Flaschel et al. (2021) analyze the model using numerical methods and find that physical distancing without rigid restrictions on economic activity cannot reduce the infections significantly. Instead, lockdowns are more effective in reducing the infections, although at the cost of sharp but temporary recessions. On the other hand, laissez-faire leads to sharp fluctuations in both demand and infections before herd immunity is achieved. Flaschel et al. (2021) argue that employment protection and unemployment benefits are pivotal, particularly if strict public health measures affecting economic activity are endorsed.

Closest to the approach used in this paper is that of Razmi (2020). Incorporating macroeconomic considerations in a Susceptible-Infected (SI) model of infectious diseases, Razmi (2020) analyzes the interaction between the economics and epidemiology. His focus is only on employed workers. Each period, a fraction of the new workers entering the workforce is infected while the rest become susceptible. The output is demand determined, and interaction between the susceptible individuals and the economic activity leads to the occurrence of two different cases. In both these cases, a unique stable steady-state exists. Once the consumer (and investment) sentiment is allowed to depend on the state of viral spread, the instability may arise in the system. Then, by introducing a ceiling on employment, he incorporates the supply-side constraint.

Our model, however, has a broader scope than Razmi (2020) since we divide the entire population into groups of susceptible and infected individuals. Furthermore, we consider a nonlinear dynamics that allows the existence of multiple equilibria and opens the possibility of instability in the system even when the confidence of the consumers and the investors' animal spirit are not affected by the state of viral spread (we discuss this in Appendix A.1). Finally, unlike Razmi (2020), we convert the

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9Using a classical growth model with induced technical change, Michl and Tavani (2020) study the long-run consequences of temporary shocks such as COVID-19. However, they do not employ the SIR(S) framework. It is also worth noting that there is a recent special issue of the *Journal of Mathematical Economics* (Vol. 93) on “[T]he economics of epidemics and emerging diseases.” There are 19 articles organized in five thematic sections on this issue. The first section discusses the interactions among economics, epidemiology, and mathematics. The second section is related to the microeconomic approach on the role that individual actions and decisions play in transmitting the disease. The third and fourth sections seek the optimal response policy. The last section is about the quantitative and applied macroeconomic explorations on wealth distribution, the value of economic assets, and environmental issues.
SI(S) model into a SIR(S) model and obtain a richer dynamics. On the other hand, unlike our analytical model, Barbieri-Góes and Gallo (2021) and Flaschel et al. (2021) focus on numerical methods. Similar to our analysis, both Barbieri-Góes and Gallo (2021) and Flaschel et al. (2021) prescribe for significant policy intervention for boosting the aggregate demand in the economy.

The rest of the paper is as follows. Section 2 sets up the basic model where we assume that there is no (or negligible number of) recovered people and the economy is purely demand-constrained. However, as the number of infected individuals rises, people start to panic. Therefore the consumers’ behavior and the animal spirits of investors are affected by the state of infection. In Section 3, we extend the model by incorporating a positive number of recovered people (a fraction of whom again returns to the susceptible class). Section 4 offers some concluding remarks.

2 | THE MODEL

In the canonical SIR model, the entire population \((N)\) is divided into three mutually exclusive groups—the susceptible class \((S)\), the infected individuals \((I)\), and the recovered individuals \((R)\).

In many diseases such as measles and smallpox, once an infected person recovers, he/she does not contract the infection again. There are, however, other diseases such as malaria and tuberculosis in which a recovered person can get infected again. So far, there is no concrete evidence that once COVID-19 infects a person, he/she will not be infected in future. To begin our analysis, we will assume that once a person is recovered, immediately enters the pool of susceptible individuals, that is, \(\dot{R} = 0\) always holds (see Figure 2a). This assumption helps us to operate with a two-dimensional system. We assume the total population \(N (= S + I)\) is constant.\(^{10}\) So rather than the SIR(S) model, the model presented in this section is more of a SI(S) model.\(^{11}\)

In the standard SI(S) model, the rate of change in the infected number of persons is \(\dot{I} = \beta SI - \nu I\). This equation says that the number of infected persons at any point in time increases by \(\beta SI\) because of interaction between susceptible and infected individuals and decreases by \(\nu I\) because a fraction of

\(^{10}\)As a recovered person immediately joins the group of susceptible individuals, we are not separately categorizing it.

\(^{11}\)Along with susceptible, infected, and recovered individuals, few epidemiological models includes death as a separate compartment, that is, instead of SI(S) or SIR(S) models, these models are of SIRD type. As Figure 1b suggests, death rate due to the pandemic is approximately 1.5% in India. Therefore, to make the model as simple as possible, we assume away the possibility of death in our model. However, the introduction of death in our model will be an interesting extension.
infected persons recover. Since our objective is to understand how the dynamics of the economic activity interacts with the dynamics of the spread of the diseases, we modify this equation to

$$\dot{I} = \beta SI - \nu I + \theta YI$$  \hspace{1cm} (2.1)$$

where $\beta$, $\theta$, and $\nu$ are positive constants. $\beta$ and $\theta$ are the infection rates.\(^{12}\) We are also assuming that the number of infected individuals at any point in time, increases because of interaction between susceptible and infected persons. However, we distinguish between economic and non-economic interactions and their implications for change in the number of infected persons. Economic activity is a major site for the spread of infection as infected persons may present themselves as asymptomatic (sometimes unknowingly) and continue to participate in an economic activity where they interact with susceptible persons. To account for this effect of economic activity, we assume $I$ increases because of the interaction among workers in the workplace where some of the workers are infected. Since economic activity is not the only trigger for interactions amongst infected and susceptible persons, we also include the term $\beta SI$ in the right-hand side of Equation (2.1) but interpret it as the increase in $I$ irrespective of economic activity. When it comes to decreasing the number of infected persons at any point in time, we interpret the fraction of infected persons who recover, $\nu$, as the health sector’s effectiveness in providing timely aid and care to infected persons. The rate of increase in the number of infected persons may also slow down because of stricter adherence to social distancing norms, diligent contact tracing and large-scale testing. The success of such policies ultimately boils down to a fall in $\beta$ and $\theta$.

As $S + I = N$ and $N$ is constant, $\dot{I} = -S$ must hold. Hence the discussion of the dynamics of $I$ is sufficient here. Inserting $S = N - I$ into Equation (2.1) and after some rearrangement we get

$$\dot{I} = \left( (\beta N - \nu) - \beta I + \theta Y \right) I$$  \hspace{1cm} (2.2)$$

Now let us focus on the goods market. There is excess supply of labor and no depreciation of capital in the economy. The production function is of Leontief type i.e.,

$$Y = \min \{ \gamma_L L, \gamma_K K \} = L; \gamma_L = \frac{Y^P}{K}, \gamma_K = Y^P \quad (2.3)$$

where $Y$ is the real income or level of output, $L$ is the total amount of labor employed, $K$ is the existing capital stock, and $Y^P$ is the potential output level. Therefore, the actual output is below the potential output level. In other words, the economy operates at the low rate of capacity utilization. For simplicity we assume the labor productivity, $\gamma_L$ to unity. Therefore, the output ($Y$) at any given point is equal to the labor employed ($L$). Note that $L \leq N$.

The actual output level is demand-determined and adjusted according to the demand gap with a time lag. Rules and regulations are changing every now and then. Consumption habit of individuals is also changing. On the other hand, reestablishment of the employment networks for the industries is also time-consuming. Therefore, we can expect the actual output is adjusting with a time lag. It can be written as follows:

$$\dot{Y} = \rho [AD - Y]; \quad \rho > 0$$  \hspace{1cm} (2.4)$$

\(^{12}\)While $\theta$ represents the infection rate due to economic activity, infection rate because of non-economic activity is represented by $\beta$.\)
where $\rho$ represents the speed of adjustment parameter. Aggregate demand ($AD$) consists of the consumption demand ($C$), investment demand ($E$), and the government expenditure ($G$). Infection on one hand creates panic among consumers and makes them reluctant of spending on non-essential commodities.\(^{13}\) It dampens the animal spirits of the investors on the other hand. Therefore, we assume $C = \bar{c} + c_Y Y - c_I I$, $E = \bar{e} + e_Y Y - e_I I$, and $G = \bar{G}$. Various kind of fiscal measures has been taken to keep the economy afloat and enable people to retain their jobs and incomes worldwide. While Japan announced a relief package of 21.1% of GDP, in the United States it was 13.3%, in Australia 10.8%, and in Germany 10.7%. Therefore, one can consider an infection-dependent policy $G = \bar{G} + g_I I$, where $g_I > 0$. Here $g_I$ represents responsiveness of government expenditure to a change in infection rate. However, India so far has not announced any significant amount of relief package. Compared to the first quarter of 2019–2020, India's GDP has declined by 23.9% in the first quarter of 2020–2021, whereas in comparison to the second quarter of previous year's GDP, India's GDP has declined by 7.5% in the second quarter of 2020–2021. In contrast with the first quarter of the previous year, there is a fall of 10.3% in public administration, defense, and other services in the first quarter (Q1) of 2020–2021. The same has declined by 12.2% in the second quarter (Q2) of 2020–2021. Government final consumption expenditure has increased from Rs. 4,182.49 billion in Q1 of 2019–2020 to only Rs. 4,866.36 billion in Q1 of 2020–2021. However, it has decreased from Rs. 4,656.43 billion in Q2 of 2019–2020 to Rs. 3,623.68 billion in the second quarter of 2020–2021.\(^{14}\)

Government expenditure in India, instead of being “counter-cyclical” surprisingly has been “pro-cyclical.” Therefore, for simplicity, we assume $g_I = 0$, that is, $G = \bar{G}$. The $AD$ is $C + E + G = a + bY - fI$, where $a = \bar{c} + \bar{e} + \bar{G}$ is the autonomous part of the aggregate demand, $b = c_Y + e_Y$ is the induced part, and $f = c_I + e_I$. Inserting $AD = a + bY - fI$ into Equation (2.4), we get

$$\dot{Y} = \rho \left[ a - (1 - b) Y - fI \right] \quad (2.5)$$

Equations (2.2), and (2.5) yields two steady-state values $(I^*, Y^*) = \left( 0, \frac{a}{1-b} \right)$ and $(I^*, Y^*) = \left( \frac{a}{\theta f + (1-b)\beta}, \frac{a}{\theta f + (1-b)\beta} \right)$. We will call the steady-state $(I^*, Y^*) = \left( 0, \frac{a}{1-b} \right)$ as the pandemic-free equilibrium as there is no infected individuals in this equilibrium. On the other hand, the steady-state $(I^*, Y^*) = \left( \frac{a}{\theta f + (1-b)\beta}, \frac{a}{\theta f + (1-b)\beta} \right)$ will be called as the pandemic equilibrium.

Analysis of the local stability condition of the equilibrium is provided in Appendix A.2. Figure 3 shows the steady states of the model. Equation (2.2) implies either $I = 0$ or $I \neq 0$. When $I = 0$ the vertical line represents the $I = 0$ isocline. On the other hand if $I \neq 0$, the slope of the $I = 0$ isocline is then $\frac{dY}{dI}|_{I=0} = \frac{\beta}{\theta f + (1-b)\beta} > 0$. Therefore, for a positive number of infected individuals we get a positively sloped straight line isocline $I = 0$. On the other hand, slope of the $\dot{Y} = 0$ isocline is $\frac{dY}{dI}|_{Y=0} = -\frac{a}{\theta f + (1-b)\beta} = -\frac{f}{(1-b)} < 0$. Consequently, $\dot{Y} = 0$ isocline is a negatively sloped straight line.

As is shown in Figure 3c,d, we get two meaningful equilibria: one is at point $A$ where $I^* = 0$ and $Y^* = \frac{a}{1-b}$, and the other one is at point $B$ where both $I^*$ and $Y^*$ are positive. Depending on the values of $\beta, \theta, \nu, a$, and $N$ we get following two propositions.

\(^{13}\)In fact, in case of India, a sizable section of the country is unable to spend even on essential commodities as well.

\(^{14}\)Ministry of Statistics & Programme Implementation, retrieved from https://pib.gov.in/PressReleasePage.aspx?PRID=1676486.
Proposition 1 When \( a\theta + (1 - b)(\beta N - \nu) < 0 \) and \( a\beta - f(\beta N - \theta) > 0 \), there exists a unique economically meaningful equilibrium \( A \) (i.e., \( 0, \frac{a}{1 - b} \)) which is a stable steady state. On the other hand, when \( a\theta + (1 - b)(\beta N - \nu) > 0 \) and \( a\beta - f(\beta N - \theta) < 0 \), there exists a unique economically meaningful equilibrium \( A \) (i.e., \( 0, \frac{a}{1 - b} \)) which is a saddle point unstable steady state.

Proof. See Appendix A.3.

Proposition 2 When \( a\theta + (1 - b)(\beta N - \nu) > 0 \), and \( a\beta - f(\beta N - \nu) > 0 \), there exist multiple economically meaningful equilibria \( A \) (i.e., \( 0, \frac{a}{1 - b} \)) and \( B \) (i.e., \( \frac{a\beta + (1 - b)(\beta N - \nu)}{\theta f + (1 - b)\beta}, \frac{a\beta - f(\beta N - \nu)}{\theta f + (1 - b)\beta} \)). \( B \) is locally stable whereas \( A \) is saddle-point unstable.

Proof. See Appendix A.4.
Let us discuss these propositions intuitively. Let us consider Proposition 1 first. Suppose the system is originally at the steady-state A. Suppose there is a sudden rise in the number of infected individuals $I$. In this situation, as $J_{11} = \frac{dJ}{dI} = [a\theta + (1 - b)(\beta N - \nu)] < 0$, $I$ receives a negative feedback from Equation (A.4), and therefore, a rise in $I$ decreases itself. This is a direct stabilizing effect. As $J_{12} = \frac{dJ}{dY} = 0I^* = 0$, there is, however, no indirect effect around the steady state $\left( I^*, Y^* \right) = \left( 0, \frac{a}{1 - b} \right)$.

Consequently, $[a\theta + (1 - b)(\beta N - \nu)] < 0$ ensures the steady state to be locally stable. Hence, ceteris paribus, lower infection rates $\beta$ and $\theta$, and a sufficiently high recovery rate ($\nu$) may ensure stability in the steady-state $A$. In other words, a very low infection rate due to non-economic activity, a weaker rate of infection because of economic activity, and a significantly high recovery rate— all of these are required to get rid of the pandemic crisis which also will be a stable steady state. On the other hand, if there is a very low recovery rate, even if the economy gets rid of the pandemic crisis, the steady state cannot be stable. Similarly, Proposition 2 suggests that even if the recovery rate is high and the infection rate due to non-economic activity is low, as long as the infection rate due to economic activity is significantly high, the economy cannot be able to achieve a stable pandemic-free equilibrium. Instead, a stable pandemic equilibrium can be attained.

Now consider the economically meaningful equilibrium with a positive number of infections and relatively lower output (the pandemic equilibrium), that is, the steady-state $B$ in Proposition 2. A sudden rise in the number of infected individuals $I$ from its equilibrium level $I^*$, in this situation, receives a negative feedback through Equation (A.4). This is a direct stabilizing effect. In a nutshell, either (a) a low recovery rate and a sufficiently high infection rate due to non-economic activity, or (b) a high recovery rate, a low infection rate due to non-economic activity but a significantly high infection rate due to economic activity is associated with this stable negative feedback. On the other hand, a rise in $I$ through Equation (A.6) leads to a fall in $Y$. This, in turn, leads to a fall in $I$ through Equation (A.5). This is the indirect stabilizing effect. As both (the direct as well as the indirect effects) are stabilizing effects, the steady-state $B$ becomes a stable steady state.

As the pandemic is not yet over in India, and as well as in other countries, we are primarily interested in the economically meaningful pandemic equilibrium (steady-state $B$ in Figure 3c,d) and its properties (Figure 4). Figure 4a shows that an increase in government expenditure increases both output and number of infected individuals in the pandemic equilibrium. As $G$ increases, the autonomous part of the aggregate demand rises. Consequently, ceteris paribus, the aggregate demand of the economy rises and it pushes the $\dot{Y} = 0$ isocline upwards. For a given output level $Y$, at the old steady-state $B$ the number of infected individuals are lower than required for satisfying the $I = 0$ isocline. This lower level of $I$ puts upward pressure on the goods market through Equation (A.6). As a result, output level starts rising initially. This rise in output level increases $I$ through Equation (A.5). This rise in $I$ mitigates the initial rise in $Y$. However, the final result is a rise in both $I$ and $Y$. Mathematically, we get $\frac{dI^*}{d\nu} = \frac{\theta}{\theta f + \beta (1 - b)} > 0$ and $\frac{dY^*}{d\nu} = \frac{\beta}{\theta f + \beta (1 - b)} > 0$.

A rise in $\nu$, for a given level of $I$, leads to a rise in $Y$. Therefore the $I = 0$ isocline shifts upward. However, there is no change in the $Y = 0$ isocline.\(^{15}\) As depicted in Figure 4b, for a rise in $\nu$, the system shifts from point $B$ to point $B'$ resulting a fall in the equilibrium level of infected individuals and a rise in output level. The intuition behind this is that as $\nu$ rises, through Equation (2.2), $I$ falls. This fall in $I$ through Equation (A.6) leads to a rise in $Y$. As $Y$ rises, there must be a rise in $I$. This rise in $I$ mitigates the initial fall in $I$. However, the final result is a fall in $I$ and a rise in $Y$. A simple algebra yields $\frac{dI^*}{d\nu} = \frac{- (1 - b)}{\theta f + \beta (1 - b)} < 0$ and $\frac{dY^*}{d\nu} = \frac{f}{\theta f + \beta (1 - b)} > 0$. Thus, the effectiveness of the health sector in providing

\(^{15}\)See Appendix A.5 for analytical explanation behind shift in isoclines due to changes in various parameters.
FIGURE 4  Comparative statics diagram of various parameters. (a) Effect of a rise in \( a \). (b) Effect of a rise in the recovery rate, \( \nu \). (c) Effect of a fall in the infection rate due to non-economic activity, \( \beta \) (when \((\beta N - \nu) > 0\)). (d) Effect of a fall in the infection rate due to economic activity, \( \theta \) (when \((\beta N - \nu) > 0\)). (e) Effect of a fall in the responsiveness of aggregate demand to a change in infection rate, \( f \). (f) A policy-mix
timely aid and care to infected persons increases the recovery rate, which in turn causes the equilibrium level of $I$ to fall and $Y$ to rise.

Due to overcrowding, the epidemics build up rapidly that is overcrowding increases the value of $\beta$. On the other hand, a frequent hand wash, proper physical distancing, and isolation lead to a fall in $\beta$. A rise in $\beta$ leads to an unambiguous rise in $I^*$ and a fall in $Y^*$. A simple algebra yields $\frac{dI^*}{d\beta} = \frac{(1-b)(N-I^*)}{\theta + \beta(1-b)^2} > 0$ and $\frac{dY^*}{d\beta} = \frac{(a-fN)-(1-b)Y^*}{[\theta + \beta(1-b)]^2} < 0$.\(^{16}\) Figure 4c represents the case for a fall in $\beta$ when $(\beta N - \nu) > 0$.\(^{17}\)

In our model, $\theta$ represents the infection rate because of economic activity. So the comparative static of a change in $\theta$ deserves some discussion. A proper information among people, a stricter adherence to social distancing norms, diligent contact tracing, mass-scale testing and segregating the infected individuals from others, and large-scale use of PPE (not only by health workers but also workers of other sectors especially where maintaining physical distances is difficult)\(^{18}\) can curb the value of $\theta$ and slow down the change in $I$. The effect of a change in $\theta$ on the equilibrium levels of $I$ and $Y$ are

$$\frac{dI^*}{d\theta} = \frac{(1-b)[a\theta - f_0N - \nu]}{[\theta + \beta(1-b)]^2} > 0$$

and

$$\frac{dY^*}{d\theta} = \frac{-f(a\theta - f_0N - \nu)}{[\theta + \beta(1-b)]^2} < 0.$$  

As $\theta$ rises the $I=0$ isocline pivots downward whereas a fall in $\theta$ leads to an upward pivot in the $I=0$ isocline (the fall in $\theta$ is shown Figure 4d). This is happening because $\frac{d}{d\theta} \left( \frac{dY^*}{d\theta} \right)_{I=0} = \frac{-b}{\theta^2} < 0$.

A fall in $\theta$ shifts the economy from $B$ to $B'$ ensuring a rise in the equilibrium output level and a fall in the number of infected individuals. The intuition behind this is that a fall in $\theta$ through Equation (2.2) lead to a fall in $I$. As $\frac{dY^*}{d\theta} < 0$, this fall in $I$ increases the level of output $Y$. As $Y$ rises, there must be a rise in $I$. This rise in $I$ mitigates the initial fall in $I$. However, the final result is a fall in $I$ and a rise in $Y$.

Furthermore, a policy-mix such as a fiscal expansion and all the measures that lower the value of $\theta$ or $\beta$ and/or higher the value of $\nu$ may lead to a rise in the steady-state value of $Y$ leading $I$ unchanged. A proper policy-mix such as a rise in government expenditure along with all the measures that increase the recovery rate significantly (so that $I=0$ isocline shifts up sufficiently) not only can increase $Y$ but can even decrease the state of infected individuals (see Figure 4f).

### 2.1 Policy prescriptions in Indian context

We have already discussed the health crisis and economic hardship India was/is going through in Section 1. Therefore along with controlling the COVID-19 spread, the government also should focus on reviving the economy. In the context of India, we can infer some policy prescriptions. First of all, more availability of hospital beds, ventilators, more recruitment of nurses, paramedic staff and other health workers on an emergency basis, a time-to-time counselling to COVID-19-infected people as well as health workers, and proper training to the health workers may help to improve the recovery rate. Second, more emphasize on community participation is needed. More community involvement can earn the trust of local people and identify the people’s needs at the local level. It can also actively participate in developing local level planning, which may help prevent the disease through testing, contact tracing, isolation, and quarantine. All the patients coming to the hospitals (irrespective of whether they have COVID-19 symptom or not) must be provided face-masks, hand sanitizers, tissues,

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\(^{16}\)Note that in the steady-state $Y = 0$ must hold. This implies, $\{a - (1-b)Y^* - f^*\} = 0$. As $I^* < N$, therefore $\{a - (1-b)Y^* - f^*\} = 0$ implies $\{(a-fN) - (1-b)Y^*\} < 0$.

\(^{17}\)The same results are also obtained for $(\beta N - \nu) < 0$.

\(^{18}\)Azad and Saratchand (2020a).
etc., so that the transmission of the disease is reduced as much as possible. Third, a mass-scale production of test kits, PPE, masks, sanitizers etc., are required on an emergency basis. The government either can directly produce these (and provide it to all the workers as well as front-line healthcare workers) or provide subsidies to the private sectors that produce these. This production can help mitigate the unemployment rate, improve the economy, and help mitigate the infection rate.\footnote{Azad and Saratchand (2020b) also prescribed this.} Lastly, a large-scale fiscal expenditure and the immediate initiative to pay the State governments their pending GST compensation dues are needed. The government can either borrow the required amount from the Reserve Bank of India, that is, through monetizing the deficit, or collect the fund through wealth taxation.

Recently based on the incidence of the infected individuals, doubling rate, the extent of testing and surveillance feedback, the Indian government has divided the districts into three zones: red, orange and green. They have allowed private and public sectors to be opened in less infected zones with maximum of 33% workers. In other words, there is a ceiling of maximum workers that can be employed. In our model, let us assume that maximum a $\delta$ fraction of the population is allowed to work. In this simple two-dimensional model that will be used as a ceiling in the diagram. As long as $L \leq \delta N$ there is no change in our results (see Figure 5a). However, as Figure 5b depicts, if $L > \delta N$, there will always be an excess demand and as a result the goods market always will be in out of equilibrium.

These comparative static results are encapsulated in Table 1.

3 | SIR(S) MODEL: WHEN ONLY A FRACTION OF RECOVERED INDIVIDUALS ENTERS THE POOL OF SUSCEPTIBLE CLASS

In the last section, we assumed that infected persons upon recovery, immediately become susceptible to further infections. However, this may not be the only scenario as some of the recovered individuals may develop antibody. To account for this possibility, we now assume only a fraction ($\mu$) of the
recovered individuals \((R)\) joins the pool of susceptible individuals (see Figure 2b). The main implication in terms of our formal analysis is that the dynamical system becomes a three-dimensional system as now, along with dynamics of \(I\) and \(Y\), we need to consider the dynamics of either \(S\) or \(R\). The reason why we only need the dynamics of either one of \(S\) or \(R\) is because \(S + I + R = N\) is a constant. In this section, we work with the following dynamical system.

\[
\dot{I} = \beta SI - \nu I + \theta YI \tag{3.1}
\]

\[
\dot{Y} = \rho \left[ a - (1 - b) Y - fI \right] \tag{3.2}
\]

\[
\dot{S} = -\beta SI - \theta YI + \mu R = -\beta SI - \theta YI + \mu (N - S - I) \tag{3.3}
\]

From Section 2, we are already familiar with Equations (3.1) and (3.2). Equation (3.3) suggests that the number of susceptible individuals at any point in time \((\dot{S})\) depends on three things. First, because of the interaction between susceptible and infected individuals, the number of infected individuals at any point in time increases by \(\beta SI\) units, and thereby, \(\dot{S}\) falls by \(\beta SI\) units. Second, it \((\dot{S})\) falls by \(\theta YI\) units because of the interaction among workers in the workplace due to economic activity. Third, as a fraction \((\mu)\) of the recovered individuals \((R)\) joins the pool of susceptible individuals, the number of susceptible individuals at any point in time increases by \(\mu R = \mu(N-S-I)\) units. Therefore, the overall number of susceptible individuals at any point in time changes by \(-\beta SI-\theta YI+\mu R\) units.

In the steady state, \(\dot{I} = \dot{Y} = 0\). From Equations (3.1–3.3), we get two steady-state values \((I^*, Y^*, S^*) = \left( 0, \frac{a}{1-b}, N \right)\) and \((I^*, Y^*, S^*) = \left( \frac{\mu b (1 - b)(\beta N - \nu + \mu \theta)}{\theta (1 - b) (\mu +\nu + \mu \theta)^2} - \frac{a \mu N + (\mu +\nu)(1 - b)(\theta - \nu)}{(1 - b)(\mu +\nu + \mu \theta)} \right)\) \(^{20}\)

The equilibrium level of infected individuals, susceptible individuals, and the equilibrium level of output for the second steady state depend on \(\beta, \nu, \mu, \theta, a, b, f, \) and \(N\). However, the steady state does not depend on the speed of adjustment parameters \(\rho\). \(^{21}\)

\(^{20}\)Note that for the second equilibrium, a positive values of \(I^*, Y^*,\) and \(S^*\) are very much possible. For a detailed calculation, see Appendix A.6.

\(^{21}\)Note that apart from SI(S) and SIR(S) model, there is another possibility where all people develop antibody once they are infected. In that case, \(\mu\) becomes zero. A unique pandemic-free equilibrium with \(I^* = 0, Y^* = \frac{\mu}{1-b},\) and \(S^* + R^* = N\) will be achieved in that scenario. However, according to WHO (source: https://www.who.int/director-general/speeches/detail/who-director-general-s-opening-remarks-at-the-media-briefing-on-covid-19---12-october-2020) “herd immunity” cannot be achieved by exposing people to the virus. Instead, it can be achieved by protecting people from a virus through vaccination. As there is ambiguity about the effectiveness of the recently discovered vaccines, we are not emphasizing “herd immunity” in our analysis.
Analysis of the local stability condition of the equilibrium is provided in Appendix A.7. Depending on the values of $\beta$, $\theta$, $\nu$, $a$, and $N$ we get following two propositions.$^{22}$

**Proposition 3** When $[a\theta + (1 - b)(\beta N - \nu)] < 0$, there exists a unique economically meaningful equilibrium $(0, \frac{a}{1 - b}, N)$ which is stable. On the other hand, when (i) $[a\theta + (1 - b)(\beta N - \nu)] > 0$, and (ii) either $[a\beta (\mu + \nu) - \mu f(\beta N - \nu)] < 0$ or $[\mu fN + (\mu + \nu) ((1 - b)\nu - a\theta)] < 0$, there exists a unique economically meaningful equilibrium $(0, \frac{a}{1 - b}, N)$ which is unstable.

**Proof.** See Appendix A.8.

Proposition 3 suggests that a weaker infection rate ($\beta$), a weaker value of $\theta$, and a sufficiently strong recovery rate ($\nu$) (so that $D = \beta N - \nu + \theta Y^*$ becomes negative) ensures the steady state $(I^*, Y^*, S^*) = (0, \frac{a}{1 - b}, N)$ to be locally stable. Suppose that the system is originally at a steady state and there is a sudden rise in the number of infected individuals $I$. In this situation, if $D < 0$, $I$ receives a negative feedback from Equation (A.9), and therefore a rise in $I$ decreases $(A.12)$ decreases $N$ (for over seven years) for capital expenditure, transfer payments, and other revenue expenditure, as Bose and Bhanumurthy (2014) found, are 4.8, 0.95, and 0.96, respectively. However, the cumulative multiplier (for over seven years) for capital expenditure, transfer payments, and other revenue expenditure, as Bose and Bhanumurthy (2015), the expenditure multipliers' value lies in between 0.98 and 2.45 in India depending on the type of expenditure. They calculated the values of capital expenditure multiplier, transfer payments multiplier, and other revenue expenditure multiplier as 2.45, 0.98, and 0.99, respectively. However, the cumulative multiplier becomes 0.79. Therefore, $b < 1$ very much holds in India.

**Proposition 4** When $[a\theta + (1 - b)(\beta N - \nu)] > 0$, there exist multiple economically meaningful equilibria $(0, \frac{a}{1 - b}, N)$ and $(\frac{\theta [1 - b (N - \nu)] + a\theta}{(1 - b)\beta (\mu + \nu) + \mu f}, \frac{\theta [1 - b (N - \nu)] + a\theta}{(1 - b)\beta (\mu + \nu) + \mu f}, \frac{\mu fN + (\mu + \nu) (1 - b)\nu - a\theta}{(1 - b)\beta (\mu + \nu) + \mu f})$ while $(0, \frac{a}{1 - b}, N)$ is unstable. $(\frac{\theta [1 - b (N - \nu)] + a\theta}{(1 - b)\beta (\mu + \nu) + \mu f}, \frac{\theta [1 - b (N - \nu)] + a\theta}{(1 - b)\beta (\mu + \nu) + \mu f}, \frac{\mu fN + (\mu + \nu) (1 - b)\nu - a\theta}{(1 - b)\beta (\mu + \nu) + \mu f})$ is locally stable.

**Proof.** See Appendix A.9.

Let us discuss Proposition 4 intuitively. Suppose that the system is initially in a pandemic equilibrium (i.e., where $I^* > 0$, $Y^* > 0$, and $S^* > 0$) and there is a sudden rise in the number of infected individuals $I$. As $J_{11} = 0$, in this situation, $I$ receives no direct feedback through Equation (A.9), and therefore there is no direct stabilizing/destabilizing effect. However, a rise in $I$ through Equation (A.12) decreases $Y$ which in turn through Equation (A.10) leads to a fall in $I$. This is an indirect stabilizing effect. A rise in $I$ through Equation (A.15) decreases $S$ which in turn through Equation (A.11) leads to a fall in $I$. This is the second indirect stabilizing effect. As both the indirect effects are stabilizing effects, the system becomes stable.

Similar to Section 2, for the comparative static analysis, we stick to the pandemic equilibrium (where $I^*$, $Y^*$, and $S^*$ all are positive). Table 2 summarizes the main results of the comparative statics.

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$^{22}$According to Bose and Bhanumurthy (2015), the expenditure multipliers' value lies in between 0.98 and 2.45 in India.
analysis. Here we restrict ourselves to an intuitive discussion of some of these results. We provide all details of these calculations in Appendix A.10.

A rise in $\beta$, the infection rate due to non-economic activity, decreases the equilibrium level of output (and hence employment level) and the number of susceptible individuals $S^*$, whereas $I^*$ rises. Due to the interaction between susceptible and infected individuals, the equilibrium number of infected individuals rises for an increase in infection rate. On the other hand, a rise in $I^*$ negatively influences the aggregated demand through Equation (A.12). Therefore, due to the increase in infection rate, the equilibrium output level falls. This fall in $Y$ through Equation (A.16) leads to an indirect rise in the number of susceptible individuals. On the other hand, the rise in infection rate raises $I^*$ which in turn through Equation (A.15) leads to a fall in the number of susceptible individuals. Finally, a rise in infection rate directly through the first term of the r.h.s. of Equation (3.3) leads to a fall in the number of susceptible individuals. These last two effects, however, dominates the first indirect effect. Therefore, a rise in infection rate causes a fall in the equilibrium number of susceptible individuals. We get a similar kind of result for a rise in the infection rate due to economic activity, $\theta$.

A rise in the recovery rate ($\nu$) although decreases the number of infected individuals and increases the equilibrium level of output, it has an ambiguous impact on the number of susceptible individuals. A rise in recovery rate through Equation (3.1) leads to a fall in the equilibrium number of infected individuals. A fall in $I^*$ on the other hand creates upward pressure to the aggregated demand through Equation (A.12). Therefore, the equilibrium output level rises. This rise in $Y$ indirectly through Equation (A.16) leads to a fall in the number of susceptible individuals. On the other hand, the rise in recovery rate causes a fall in $I^*$ which in turn through Equation (A.15) leads to a rise in the number of susceptible individuals. For a low number of infected individuals (i.e., if $I^* < \frac{(1-b)(\mu+\nu)}{\delta f}$), the later dominates the previous indirect effect (on $S^*$ through a change in $Y^*$), and therefore, a rise in $\nu$ increases $S^*$. On the other hand, if $I^*$ exceeds $\frac{(1-b)(\mu+\nu)}{\delta f}$, the previous indirect effect dominates the later one. Therefore, in that case, a rise in recovery rate causes a fall in the equilibrium number of susceptible individuals.

A rise in government expenditure increases the equilibrium values of $I$ and $Y$, and decreases the equilibrium value of $S$. As government expenditure increases, the autonomous part of the aggregate demand rises. Consequently, *ceteris paribus*, the aggregate demand and hence the output level of the economy rises. This rise in output level leads to an increase in infected individuals through Equation (A.10). As the output level and infected individuals increase, through Equations (A.16) and (A.15) the equilibrium number of susceptible individuals falls.

From the above discussion, we can conclude that an effective health sector in providing timely aid and care to infected persons, a stricter adherence to social distancing norms, diligent contact tracing, and large-scale testing can decrease the level of infected individuals and increase the output level. However, the effect of a fiscal expansion on the equilibrium output level as well as the equilibrium number of infected individuals is positive. Similar to Section 2, a proper policy-mix of fiscal

| Parameter | $I^*$ | $Y^*$ | $S^*$ |
|-----------|-------|-------|-------|
| $\beta$   | +     | −     | −     |
| $\nu$     | −     | +     | +/-   |
| $\theta$  | +     | −     | −     |
| $\alpha$  | +     | +     | −     |
expansion and all the probable measures that ensure low infection rates and a rise in the recovery rate can lead to a rise in the output level and a fall in the level of infected individuals.

4 | CONCLUSION

Considering the economic activity and its interaction with the infectious disease, extended the SIR(S) model of infectious diseases and tried to explain the current severe situation and impending health and economic crisis in India's context. In Section 2, we assumed that a recovered person immediately loses immunity and enters the pool of susceptible individuals. The pervasiveness of the disease creates panic among consumers and forces them to cut down the spending on non-essential commodities. It dampens the animal spirits of the investors. Therefore, the economic activity is affected by the number of infected individuals. We found a very low infection rate due to non-economic activity, a weaker infection rate because of economic activity, and a significantly high recovery rate—all of these are required for achieving a unique stable pandemic-free equilibrium. On the other hand, if there is a very low recovery rate, the unique pandemic-free equilibrium cannot be stable. Moreover, the model we study in Section 2 can produce a stable pandemic equilibrium under parametric restrictions. We found that an expansionary fiscal policy increases the level of output (and hence the labor employment) and the number of infected individuals. On the other hand, a fiscal expansion along with some probable measures such as improvement in the effectiveness of health sector in providing timely aid and care to infected persons, public health vigilance and control, more emphasis on community participation, stricter adherence to social distancing norms, diligent contact tracing and large-scale testing, segregating the infected individuals from others, and large-scale production of PPE, masks, sanitizer, etc., by the government and distribution of these to the common people, can reduce the number of infected individuals and improve the economic condition.

In Section 3, we assumed that instead of all the recovered individuals, only a fraction of them enters the pool of susceptible individuals. We discussed the stability of the system. We showed that when there is a unique economically meaningful pandemic-free steady state, weaker infection rates, and a sufficiently strong recovery rate ensure the steady state to be stable. However, when there exist multiple equilibria, the pandemic-free steady state becomes unstable. On the other hand, under some parametric restrictions, the system achieves a stable pandemic steady state. We showed that a lower infection rate, and a rise in the recovery rate raise the equilibrium level of output and decrease the number of infected individuals. However, the effect of a fiscal expansion on both the equilibrium output level as well as the equilibrium number of infected individuals is positive. Here too, a proper policy-mix of fiscal expansion and all the measures that ensure better community health and a rise in the recovery rate can lead to a rise in the output level and a fall in the level of infected individuals.

The analytical framework in this paper is subject to a few limitations. First, our model focuses on the short run. Analyzing the impact of COVID-19 pandemic on the economy from the long run perspective would be an interesting exercise. Second, we did not focus on the health sector (explicitly) in our analysis. Introduction of a health sector and taking into account its feedback effect on the economy from both the short-run and the long run perspective would also be an interesting extension of the model. These issues are, however, left for future research.

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APPENDIX A

A.1 When the infection does not have any impact on the aggregate demand

Considering $f = 0$, from Equations (2.2) and (2.5) we get two steady-state values $(I^*, Y^*) = \left(0, \frac{a}{1-b} \right)$ and $(I^*, Y^*) = \left(\frac{a\theta+(1-b)(\beta N-v)}{1-b\theta}, \frac{a}{1-b} \right)$. When $[a\theta + (1-b)(\beta N-v)] < 0$, at point $B$ we get $I^* = \frac{a\theta+(1-b)(\beta N-v)}{1-b\theta} < 0$. Therefore, we get a unique economically meaningful steady-state $A$, i.e., $(I^*, Y^*) = \left(0, \frac{a}{1-b} \right)$ (see Figure A1a). At point $A$, as $I^* = 0$, $J_{12} = 0$ and as $[a\theta + (1-b)(\beta N-v)] < 0$, $J_{11} = \{\beta N-v - 2\beta I^* + \theta Y^*\} = \frac{[a\theta+(1-b)(\beta N-v)]}{(1-b)} < 0$. So, the determinant of the Jacobian matrix

$$\text{Det}(J) = \det(J_{11}J_{22} - J_{12}J_{21}) = J_{11}J_{22} > 0$$

and the trace $\text{tr}(J) = (J_{11} + J_{22}) < 0$. Therefore, $A$ is a stable steady state. On the other hand, when $[a\theta + (1-b)(\beta N-v)] > 0$, $J_{11} = \{\beta N-v + \theta Y^*\} = \frac{[a\theta+(1-b)(\beta N-v)]}{(1-b)} > 0$. So, the determinant of the Jacobian matrix

$$\text{Det}(J) = \det(J_{11}J_{22} - J_{12}J_{21}) = J_{11}J_{22} < 0.$$ Therefore, $A$ is a saddle-point unstable (see Figures A1b and A2). Note that when $[a\theta + (1-b)(\beta N-v)] > 0$, at point $B$ we get $I^* = \frac{a\theta+(1-b)(\beta N-v)}{1-b\theta} > 0$. Therefore, here economically meaningful multiple equilibria $A$ and $B$ exist. At the steady state $B$, $J_{11} = \{\beta N-v - 2\beta I^* + \theta Y^*\} = \frac{[a\theta+(1-b)(\beta N-v)]}{(1-b)} < 0$. Here $f = 0$ and therefore inserting it in Equation (A.6) we get $J_{21} = \frac{\partial Y}{\partial f} = 0$. Therefore at $B$, $\text{Det}(J) = J_{11}J_{22} > 0$ and the trace of the Jacobian matrix $\text{tr}(J) = (J_{11} + J_{22}) < 0$. Hence $B$ is a stable steady state.23

A.2 Analysis of the local stability condition of Section 2

We know the system of differential Equations (2.2) and (2.5) are

$$\dot{I} = \{\beta N - v - \beta I + \theta Y\} I \quad \text{(A.1)}$$

$$\dot{Y} = \rho \left[ a - (1-b)Y - fI \right] \quad \text{(A.2)}$$

To analyze the local stability of the equilibrium, we linearize the system of differential equations (A.1) and (A.2) around the equilibrium and get

---

23Here as $J_{21} = 0$ so, $\text{tr}(J)J_{21} - 4\text{Det}(J) = (J_{11} - J_{22})^2 > 0$ and hence the steady state is a stable node.
FIGURE A1 Steady states. (a) When \((\beta N - \nu) < 0\), and \(\frac{a}{(1-b)} < \frac{-(\delta N - \nu)}{\theta}\). (b) When \((\beta N - \nu) < 0\), but \(\frac{a}{(1-b)} > \frac{-(\delta N - \nu)}{\theta}\).
where the sign structure of the Jacobian matrix \( \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \) is \( \begin{pmatrix} +/ & +/ \\ - & - \end{pmatrix} \) and the elements of the Jacobian matrix \( J \) are given by

\[
\begin{align*}
J_{11} &= \frac{\partial \dot{I}}{\partial I} = (\beta N - \nu) - 2\beta I^* + \theta Y^* \geq 0 \\
J_{12} &= \frac{\partial \dot{I}}{\partial Y} = \theta I^* \geq 0 \\
J_{21} &= \frac{\partial \dot{Y}}{\partial I} = -f \rho < 0 \\
J_{22} &= \frac{\partial \dot{Y}}{\partial Y} = -(1 - b) \rho < 0
\end{align*}
\]
All the above elements are evaluated at the steady-state values. $J_{11}$ represents the effect of an increase in the number of infected individuals on a change in the number of infected individuals themselves. $J_{12}$ shows the effect of a rise in economic activity on the number of infected individuals. As the economic activity rises, more people get in touch with others and maintaining physical distancing becomes difficult. Consequently, more people are infected. $J_{21}$ represents the effect of a rise in $I$ on the change in $Y$. As more number of people are infected, people panic and therefore the confidence of consumers and the animal spirits of the investors fall. This, in turn, ceteris paribus, lowers the aggregate demand and hence the output level/level of income. $J_{22}$ shows the effect of a rise in the level of output on a change in the output level itself. For the Keynesian stability condition to hold, we assume $J_{22} < 0$.

### A.3 | Proof of Proposition 1

**Proof.** When $[a \theta + (1 - b)(\beta N - \nu)] < 0$, at point $B$ we get $I^* = \frac{a \theta + (1 - b) (\beta N - \nu)}{\theta f + (1 - b) b} < 0$. Therefore we get a unique economically meaningful steady state $A$ (see Figure 4a). At point $A$, as $I^* = 0$, $J_{12} = 0$; and as $[a \theta + (1 - b)(\beta N - \nu)] < 0$, $J_{11} = \{ (\beta N - \nu) - 2 \beta I^* + \theta Y^* \} = \frac{[a \theta + (1 - b)(\beta N - \nu)]}{(1 - b)} < 0$. So, the determinant of the Jacobian matrix $\text{Det} (J) = (J_{11} J_{22} - J_{12} J_{21}) = J_{11} J_{22} > 0$ and the trace $\text{tr} (J) = (J_{11} + J_{22}) < 0$. Therefore, $A$ is a stable steady state.\(^{24}\)

On the other hand, when $[a \beta - f(\beta N - \nu)] < 0$, we get $Y^* = \frac{[a \theta - \beta (\nu - \nu)]}{\theta f + (1 - b) b} < 0$ at equilibrium point $B$. Therefore we get a unique economically meaningful steady state $A$ (see Figure 4b). At point $A$, as $I^* = 0$, $J_{12} = 0$; and as $[a \theta + (1 - b)(\beta N - \nu)] > 0$, $J_{11} = \{ (\beta N - \nu) - 2 \beta I^* + \theta Y^* \} = \frac{[a \theta + (1 - b)(\beta N - \nu)]}{(1 - b)} > 0$. So, the determinant of the Jacobian matrix $\text{Det} (J) = (J_{11} J_{22} - J_{12} J_{21}) = J_{11} J_{22} < 0$. Therefore, $A$ is a saddle point unstable steady state.

### A.4 | Proof of Proposition 2

**Proof.** At point $A$, as $I^* = 0$, $J_{12} = 0$ and as $[a \theta + (1 - b)(\beta N - \nu)] > 0$, $J_{11} = \{ (\beta N - \nu) - 2 \beta I^* + \theta Y^* \} = \frac{[a \theta + (1 - b)(\beta N - \nu)]}{(1 - b)} > 0$. So, the determinant of the Jacobian matrix $\text{Det} (J) = (J_{11} J_{22} - J_{12} J_{21}) = J_{11} J_{22} < 0$. Therefore, $A$ is a saddle point unstable (see Figure 4c,d).

At the steady state $B$, $J_{11} = \{ (\beta N - \nu) - 2 \beta I^* + \theta Y^* \} = \frac{- \beta \theta [a \theta + (1 - b)(\beta N - \nu)]}{(1 - b)} < 0$. At point $B$ the slope of the $I = 0$ isocline is greater than the slope of the $Y = 0$ isocline i.e., $-\frac{d I}{d Y} > 0$ which in turn (as $J_{12} > 0$ and $J_{22} < 0$) implies $\text{Det} (J) = (J_{11} J_{22} - J_{12} J_{21}) > 0$. Further, the trace of the Jacobian matrix $\text{tr} (J) = J_{11} + J_{22} < 0$. Hence point $B$ is a stable steady state.\(^{25}\)

### A.5 | Analytical explanation behind shift in isoclines

A rise in $\nu$ decreases the horizontal intercept of the $I = 0$ isocline i.e., $\frac{d}{d Y} (I)_{Y=0} = \frac{d}{d Y} (\frac{\nu}{\beta}) = \frac{-1}{\beta} < 0$ and increase the vertical intercept of the $I = 0$ isocline i.e., $\frac{d}{d V} (I)_{I=0} = \frac{d}{d V} (\frac{\nu}{\theta}) = \frac{1}{\theta} > 0$. However, it does not change the slope of the $I = 0$ isocline i.e., $\frac{d}{d Y} (d I/d Y)_{I=0} = \frac{d}{d Y} (\frac{\beta}{\theta}) = 0$. Also note that a rise in $\nu$, for a given level of $I$, leads to a rise in $Y$ i.e., $\frac{d}{d \nu} (Y)_{I=0} = \frac{d}{d \nu} (\frac{(\beta - (\beta N - \nu))}{\theta}) = \frac{1}{\theta} > 0$.\(^{26}\)

\(^{24}\)Here as $J_{12} = 0$, $\text{tr}(J^2) - 4\text{Det}(J) = (J_{11} - J_{22})^2 > 0$. Hence the steady state is a stable node.

\(^{25}\)Here as $J_{12} > 0$ and $J_{21} < 0$, $\text{tr}(J^2) - 4\text{Det}(J) = (J_{11} - J_{22})^2 + 4J_{12}J_{21} \geq 0$ and hence the steady state can be any of either a stable node or a stable spiral.
A rise in $\beta$ increases the horizontal intercept of the $I = 0$ isocline i.e., $I_{\beta I=0} = \frac{\frac{\nu N - \nu}{\beta}}{\theta} > 0$ and decrease the vertical intercept of the $I = 0$ isocline i.e., $Y_{\beta Y=0} = \frac{\frac{\nu N - \nu}{\theta}}{\beta} < 0$. However, it raises the slope of the $I = 0$ isocline i.e., $\frac{dY}{dI}_{I=0} = \frac{\frac{\nu N - \nu}{\theta}}{\beta} = \frac{N}{\theta} > 0$. Also note that a rise in $\beta$, for a given level of $I$, leads to a fall in $Y$, that is, $\frac{dY}{d\beta}_{I=0} = \frac{\frac{\nu N - \nu}{\theta}}{\beta} = \frac{1-N}{\theta} < 0$.

Note that, for a change in $\theta$, there is no change in the horizontal intercept of the $I = 0$ isocline (as $\frac{dI}{d\theta}_{I=0} = 0$). However, when $(\beta N - \nu) > 0$, for a rise in $\theta$, the vertical intercept rises whereas, for $(\beta N - \nu) < 0$ the vertical intercept falls. Mathematically, $\frac{dY}{d\theta}_{I=0} = \frac{\frac{\nu N - \nu}{\theta}}{\beta} \geq 0$ depending on whether $(\beta N - \nu) \leq 0$. However, for a change in $\theta$, there is no change in the $Y = 0$ isocline.

### A.6 Analytical explanation behind a positive values of $I^*$, $Y^*$, and $S^*$

Note that, as $[(1 - b) \beta (\mu + \nu) + \mu \theta f] > 0$, for $I^*$, $Y^*$, and $S^*$ to be positive, $[(1 - b) (\beta N - \nu) + a \theta] > 0$, $[a \beta (\mu + \nu) - \mu f(\beta N - \nu)] > 0$, and $[\mu \theta f N + (\mu + \nu) ((1 - b) \nu - a \theta)] > 0$ must hold. $[(1 - b) (\beta N - \nu) + \mu f] > 0$ implies $a > \frac{(1-b)(\beta N - \nu)}{\theta} [a \beta (\mu + \nu) - \mu f(\beta N - \nu)] > 0$ implies $a > \frac{\mu f(\beta N - \nu)}{\mu (\mu + \nu)}$, and $[(1 - b) (\mu + \nu) + \mu \theta f] > 0$ implies $a \frac{\mu f(\mu + \nu)(1-b)\nu}{\theta(\mu + \nu)}$. This implies $a < \frac{\mu f(\mu + \nu)}{\theta(\mu + \nu)}$ which in turn implies $N [1-b] \beta (\theta \nu + \mu \theta f] > 0$. As $[(1 - b) \beta (\mu + \nu) + \mu \theta f] > 0$, $\theta(\mu + \nu) > 0$ is satisfied. Similarly, $a > \frac{\mu f(\mu + \nu)}{\theta(\mu + \nu)}$, and $a \frac{\mu f(\mu + \nu)(1-b)\nu}{\theta(\mu + \nu)} < a < \frac{\mu f(\mu + \nu)(1-b)\nu}{\theta(\mu + \nu)}$. This in turn implies $\nu [(1 - b) \beta (\mu + \nu) + \mu \theta f] > 0$ which is also satisfied. Therefore, $I^* > 0, Y^* > 0$, and $S^* > 0$ are possible.

### A.7 Analysis of the local stability condition of Section 3

To analyze the local stability of the equilibrium, we linearize the system of differential Equations (3.1)–(3.3) around the equilibrium and get

$$
\begin{pmatrix}
I \\
Y \\
S
\end{pmatrix}
\approx
\begin{pmatrix}
J_{11} & J_{12} & J_{13} \\
J_{21} & J_{22} & J_{23} \\
J_{31} & J_{32} & J_{33}
\end{pmatrix}
\begin{pmatrix}
I - I^* \\
Y - Y^* \\
S - S^*
\end{pmatrix}
\tag{A.8}
$$

where the elements of the Jacobian matrix $J$ are given by

$$
J_{11} = \frac{\partial I}{\partial I} = (\beta S^* - \nu + \theta Y^*) \geq 0
\tag{A.9}
$$

$$
J_{12} = \frac{\partial I}{\partial Y} = \theta I^* \geq 0
\tag{A.10}
$$

$$
J_{13} = \frac{\partial I}{\partial S} = \beta I^* \geq 0
\tag{A.11}
$$
Equations (A.9)–(A.17) are all evaluated at the steady state. Most elements of the Jacobian matrix have the same interpretation as in Section 2 except a few. $J_{11}$ represents the effect of an increase in the number of infected individuals on a change in the number of infected individuals themselves. As more individuals get infected, *ceteris paribus*, the more is the chance that the susceptible individuals as well as workers also get infected. If there are a large number of susceptible individuals ($S$), large number of employed workers $Y$, and $\beta$ and $\nu$ are high, $(\beta S^* + \theta Y^*)$ outweighs the recovery rate, $\nu$ and consequently Equation (A.9) becomes positive. Otherwise, the opposite happens. However, $J_{11} = 0$ at the pandemic equilibrium (where $I^*$, $Y^*$, $S^*$ - all are positive). $J_{13}$ represents the effect of a rise in the pool of susceptible individuals on the pool of infected people. As $S$ rises, because of the interaction between it and $I$, the number of infected individuals rises. However, when $I^* = 0$, $J_{13}$ also becomes zero. A rise in $I$ affects the change in the pool of susceptible individuals negatively (it is captured by $J_{31}$). As $I$ rises, because of the non-economic interaction between $S$ and $I$, the number of infected individuals rises. $Ceteris paribus$, this leads to a fall in $S$ by $\beta S$ units. Moreover, as the number of infected individuals rises, because of the economic interaction between $I$ and workers, the number of infected individuals rises. This leads to a fall in $S$ by $\theta Y$ units. Furthermore, $I$, due to its indirect effect through $R$, decreases $S$ by $\mu$ units. Together these imply that a rise in $I$ negatively affects $S$ by $(\beta S + \theta Y + \mu)$ units. Given the value of $I$, a rise in economic activity decreases the number of susceptible individuals by $\theta I$ units (it is captured by $J_{32}$). Similarly, $J_{33}$ (the effect of an increase in the number of susceptible individuals on a change in the number of susceptible individuals themselves) is also negative. As $S$ rises, because of the non-economic interaction between $S$ and $I$, the number of infected individuals rises. $Ceteris paribus$, this leads to a fall in $S$ by $\beta I$ units. On the other hand, due to its indirect effect through $R$, $S$ decreases itself by $\mu$ units. These two together imply that a rise in $S$ negatively affects $S$ by $(\beta I^* + \mu)$ units. The characteristic equation of the Jacobian matrix (A.8) is given by

$$J_{21} = \frac{\partial \dot{Y}}{\partial I} = -\rho f < 0$$

(A.12)

$$J_{22} = \frac{\partial \dot{Y}}{\partial Y} = -(1 - b) \rho < 0$$

(A.13)

$$J_{23} = \frac{\partial \dot{Y}}{\partial S} = 0$$

(A.14)

$$J_{31} = \frac{\partial \dot{S}}{\partial I} = -(\beta S^* + \theta Y^* + \mu) < 0$$

(A.15)

$$J_{32} = \frac{\partial \dot{S}}{\partial Y} = -\theta I^* \leq 0$$

(A.16)

$$J_{33} = \frac{\partial \dot{S}}{\partial S} = -(\beta I^* + \mu) < 0$$

(A.17)

\(^{26}\)Note that in our model $Y = L$. 
\[ \lambda^3 + a_1 \lambda^2 + a_2 \lambda + a_3 = 0 \] (A.18)

where \( \lambda \) denotes a characteristic root. Coefficients of Equation (A.18) are:

\[ a_1 = -\text{tr} J = -(J_{11} + J_{22} + J_{33}), \] (A.19)

\[ a_2 = \begin{vmatrix} J_{22} & 0 \\ J_{32} & J_{33} \end{vmatrix} + \begin{vmatrix} J_{11} & J_{13} \\ J_{31} & J_{33} \end{vmatrix} + \begin{vmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{vmatrix} = J_{22}J_{33} + J_{11}J_{33} - J_{13}J_{31} + J_{11}J_{22} - J_{12}J_{21}, \] (A.20)

\[ a_3 = -\text{Det} J = -J_{11}J_{22}J_{33} + J_{12}J_{21}J_{33} - J_{13}J_{21}J_{32} + J_{13}J_{22}J_{31} \] (A.21)

where \(-a_1 = \text{tr} J\) denotes the trace of \( J \); \( a_2 \), the sum of the principal minors' determinants; and \(-a_3 = \text{Det} J\), the determinant of \( J \).

The necessary and sufficient condition for the local stability is that all characteristic roots of the Jacobian matrix must have negative real parts, which, from Routh–Hurwitz condition, is equivalent to \( a_1 > 0, a_2 > 0, a_3 > 0, \) and \( a_1a_2 - a_3 > 0 \).

### A.8 Proof of Proposition 3

**Proof.** For \( I^* \neq 0 [a \theta + (1 - b)(\beta N - \nu)] < 0 \) implies \( I^* = \frac{\mu [1 - \beta (\beta N - \nu)] + \theta \rho}{(1 - b) \beta (\mu + \nu) + \mu \theta} \) < 0. Therefore we get a unique economically meaningful steady state \((0, \frac{a}{1 - b}, N)\). At \((I^*, Y^*, S^*) = (0, \frac{a}{1 - b}, N)\),

\[
\begin{bmatrix}
J_{11} & J_{12} & J_{13} \\
J_{21} & J_{22} & J_{23} \\
J_{31} & J_{32} & J_{33}
\end{bmatrix}
= \begin{bmatrix}
J_{11} & 0 & 0 \\
J_{21} & J_{22} & 0 \\
J_{31} & 0 & J_{33}
\end{bmatrix}.
\]

Therefore,

\[ a_1 = -\text{tr} J = -(J_{11} + J_{22} + J_{33}) \equiv a_1 (\rho) = \{\nu + \mu + \beta (I - S) - \theta Y\} + (1 - b) \rho \]

\[ = \left\{ \nu + \mu - \beta N - \frac{\theta a}{(1 - b)} \right\} + (1 - b) \rho = A + (1 - b) \rho. \] (A.22)

\[
\begin{aligned}
a_2 &= \begin{vmatrix} J_{22} & 0 \\ 0 & J_{33} \end{vmatrix} + \begin{vmatrix} J_{11} & 0 \\ J_{31} & J_{33} \end{vmatrix} + \begin{vmatrix} J_{11} & 0 \\ J_{21} & J_{22} \end{vmatrix} = J_{22}J_{33} + J_{11}J_{33} + J_{11}J_{22} \equiv a_2 (\rho) \\
&= \left\{ \nu + \mu - \beta N - \frac{\theta a}{(1 - b)} \right\} (1 - b) \rho - \mu \left\{ \beta N - \nu + \frac{\theta a}{(1 - b)} \right\} = (1 - b)A \rho - D \quad \text{\(\equiv A > 0\)}
\end{aligned}
\] (A.23)

\[ a_3 = -J_{11}J_{22}J_{33} \equiv a_3 (\rho) = -\mu \left\{ \beta N - \nu + \frac{\theta a}{(1 - b)} \right\} (1 - b) \rho = -D (1 - b) \rho. \] (A.24)
\[ a_1 \alpha_2 - \alpha_3 \equiv \xi(\rho) = (1 - b)^2 \alpha \rho^2 + (1 - b) \lambda \rho - AD > 0. \]

Hence \( a_1 > 0, a_2 > 0, a_3 > 0 \), and \( a_1 \alpha_2 - \alpha_3 > 0 \) i.e., all the conditions are satisfied. Consequently, \( 0, \frac{\alpha}{1 - b}, N \) is a stable steady state.

On the other hand if \( [a \beta (\mu + v) - \mu f(\beta N - v)] < 0 \), we get \( Y^* = \frac{a \beta (\mu + v) - \mu f(\beta N - v)}{(1 - b) \beta (\mu + v) + \mu f} < 0 \). Therefore we get a unique economically meaningful steady state \( \left( 0, \frac{\alpha}{1 - b}, N \right) \). Similarly, when \( [\mu f(\beta N + (\mu + v) ((1 - b) v - a \theta)] < 0 \), we get \( S^* = \frac{\mu f(\beta N + (\mu + v) ((1 - b) v - a \theta)}{(1 - b) \beta (\mu + v) + \mu f} < 0 \). Consequently, we get a unique economically meaningful steady state \( \left( 0, \frac{\alpha}{1 - b}, N \right) \). As in both the cases \( [a \theta + (1 - b) (\beta N - v)] > 0 \), \( D = \left\{ \beta N - v + \frac{\theta \mu}{(1 - b)} \right\} \) becomes positive. As a result \( a_3 \) becomes negative. Consequently, \( 0, \frac{\alpha}{1 - b}, N \) becomes an unstable steady state.

A.9 | Proof of Proposition 4

**Proof.** As \( [a \theta + (1 - b) (\beta N - v)] > 0 \), \( a_3 \) becomes negative and consequently, \( (I^*, Y^*, S^*) = \left( 0, \frac{\alpha}{1 - b}, N \right) \) becomes an unstable steady state. Now let us focus on the second steady state. At

\[
\begin{pmatrix}
\mu(1 - b)(\beta N - v + a \theta) & a \beta (\mu + v) - \mu f(\beta N - v) \\
(1 - b) \beta (\mu + v) + \mu f & \mu f(\beta N + (\mu + v) ((1 - b) v - a \theta)
\end{pmatrix}
\]

we get

\[
\begin{pmatrix}
J_{11} & J_{12} & J_{13} \\
J_{21} & J_{22} & J_{23} \\
J_{31} & J_{32} & J_{33}
\end{pmatrix}
= \begin{pmatrix}
0 & J_{12} & J_{13} \\
J_{21} & J_{22} & 0 \\
J_{31} & J_{32} & J_{33}
\end{pmatrix}.
\]

Therefore,

\[
a_1 = -wJ = -(J_{22} + J_{33}) \equiv a_1(\rho) = (\beta I^* + \mu) + (1 - b) \rho = A + (1 - b) \rho > 0. \tag{A.25}
\]

\[
a_2 = \begin{pmatrix}
J_{22} & 0 \\
J_{32} & J_{33}
\end{pmatrix}
+ \begin{pmatrix}
0 & J_{13} \\
J_{12} & J_{22}
\end{pmatrix}
= J_{22}J_{33} - J_{13}J_{31} - J_{12}J_{21} \equiv a_2(\rho)
= \begin{pmatrix}
(\beta I^* + \mu) (1 - b) \\
\theta fI^*
\end{pmatrix}
\]

\[\equiv A > 0 \]

\[\equiv C > 0 \]

\[
\rho + \beta I^* \{ \beta S^* + \theta Y^* + \mu \} = Cp + D > 0. \tag{A.26}
\]

\[
a_3 = J_{12}J_{21}J_{33} - J_{13}J_{21}J_{32} + J_{13}J_{22}J_{31} \equiv a_3(\rho)
= \begin{pmatrix}
\theta \mu fI^* + \beta I^* \{ \beta S^* + \theta Y^* + \mu \} (1 - b)
\end{pmatrix}
\rho = \left[ \theta \mu fI^* + D (1 - b) \right] \rho > 0. \tag{A.27}
\]

\[\text{As } I = \beta SI - vI + \theta YI = 0 \text{ implies either } I = 0 \text{ or } (\beta S - \nu + \theta Y) = 0. \text{ As here } I > 0, (\beta S - \nu + \theta Y) = 0 \text{ must hold.}\]
\( a_1 a_2 - a_3 \equiv \xi(\rho) = (1 - b)C \rho^2 + \{(1 - b)(\beta f^* + \mu \rho^2 + \beta \theta f^* \rho^2)\} \rho + AD > 0. \)  

(A.28)

At the steady state \((I^*, Y^*, S^*) = \left( \frac{\mu (1 - b) (\mu + \nu) \{ (1 - b) \nu - a \theta \}}{(1 - b) (\mu + \nu) \beta + \mu \theta f^*} \right)\) is a stable steady state. 

\( a_1 > 0, a_2 > 0, a_3 > 0, \) and \( a_1 a_2 - a_3 > 0, \) that is, all the conditions for stability are satisfied. Therefore, 

\((I^*, Y^*, S^*) = \left( \frac{\mu (1 - b) (\mu + \nu) \{ (1 - b) \nu - a \theta \}}{(1 - b) (\mu + \nu) \beta + \mu \theta f^*} \right)\) is a stable steady state. 

### A.10 \quad \textbf{Effects of parametric changes}

We examine the effect of changes in the fiscal policy, the infection rate due to non-economic activity \( (\beta) \) and due to economic activity \( (\theta) \), and the recovery rate \( (\nu) \) on the steady-state values of \( I, Y, \) and \( S \) when all of these \( (I, Y, \) and \( S) \) are positive. 

The impact of a change in \( \beta \):

\[
\frac{dI^*}{d\beta} = \frac{\mu (1 - b) \{ (1 - b) \nu - a \theta \}}{(1 - b) (\mu + \nu) \beta + \mu \theta f^*} > 0
\]

\[
\frac{dY^*}{d\beta} = -\frac{\mu f \{ (1 - b) \nu - a \theta \}}{(1 - b) (\mu + \nu) \beta + \mu \theta f^*} < 0
\]

\[
\frac{dS^*}{d\beta} = -\frac{(1 - b) (\mu + \nu) \{ (1 - b) \nu - a \theta \}}{(1 - b) (\mu + \nu) \beta + \mu \theta f^*} < 0
\]

The impact of a change in \( \nu \):

\[
\frac{dI^*}{d\nu} = -\frac{\mu (1 - b) \{ (1 - b) \beta N + \mu + \nu \} \{ (1 - b) \beta f^* + a \beta \}}{(1 - b) (\mu + \nu) \beta + \mu \theta f^*} < 0
\]

\[
\frac{dY^*}{d\nu} = \frac{\mu f \{ (1 - b) \beta N + \mu + \nu \} \{ (1 - b) \beta f^* + a \beta \}}{(1 - b) (\mu + \nu) \beta + \mu \theta f^*} > 0
\]

\[
\frac{dS^*}{d\nu} = \frac{-\mu \theta f \{ a \theta + (1 - b) (\beta N - \nu) \} (1 - b) (\mu + \nu) \{ (1 - b) (\mu + \nu) \beta + \mu \theta f^* \}}{(1 - b) (\mu + \nu) \beta + \mu \theta f^*}
\]

\[
\frac{dS^*}{d\nu} \geq 0 \quad \text{according to whether} \quad I^* = \frac{\mu \{ (1 - b) (\beta N - \nu) + a \theta \}}{(1 - b) \beta (\mu + \nu) + \mu \theta f^*} \leq \frac{(1 - b) (\mu + \nu)}{\theta f}
\]

The impact of a change in \( \theta \):
\[
\frac{dI^*}{d\theta} = \frac{\mu (1 - b) [a\beta (\mu + v) - \mu f (\beta N - v)]}{[(1 - b)(\mu + v)\beta + \mu \theta f]^2} > 0
\]

\[
\frac{dY^*}{d\theta} = -\frac{\mu f [a\beta (\mu + v) - \mu f (\beta N - v)]}{[(1 - b)(\mu + v)\beta + \mu \theta f]^2} < 0
\]

\[
\frac{dS^*}{d\theta} = -\frac{(1 - b)(\mu + v) [a\beta (\mu + v) - \mu f (\beta N - v)]}{[(1 - b)(\mu + v)\beta + \mu \theta f]^2} < 0
\]

The impact of a change in \(a\):

\[
\frac{dI^*}{da} = \frac{\mu \theta}{[(1 - b)(\mu + v)\beta + \mu \theta f]^2} > 0
\]

\[
\frac{dY^*}{da} = \frac{\beta (\mu + v)}{[(1 - b)(\mu + v)\beta + \mu \theta f]^2} > 0
\]

\[
\frac{dS^*}{da} = -\frac{(\mu + v) \theta}{[(1 - b)(\mu + v)\beta + \mu \theta f]^2} < 0
\]