Local Rule-Based Explanations of Black Box Decision Systems

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ABSTRACT
The recent years have witnessed the rise of accurate but obscure decision systems which hide the logic of their internal decision processes to the users. The lack of explanations for the decisions of black box systems is a key ethical issue, and a limitation to the adoption of machine learning components in socially sensitive and safety-critical contexts. In this paper we focus on the problem of black box outcome explanation, i.e., explaining the reasons of the decision taken on a specific instance. We propose LORE, an agnostic method able to provide interpretable and faithful explanations. LORE first learns a local interpretable predictor on a synthetic neighborhood generated by a genetic algorithm. Then it derives from the logic of the local interpretable predictor a meaningful explanation consisting of: a decision rule, which explains the reasons of the decision; and a set of counterfactual rules, suggesting the changes in the instance’s features that lead to a different outcome. Wide experiments show that LORE outperforms existing methods and baselines both in the quality of explanations and in the accuracy in mimicking the black box.

CCS CONCEPTS
• Information systems → Decision support systems; Data mining; Data analytics;

KEYWORDS
Explanation, Decision Systems, Rules

1 INTRODUCTION
Popular magazines and newspapers are full of commentaries about algorithms taking critical decisions that heavily impact on our life and society, from granting a loan to finding a job or driving our car. The worry is not only due to the increasing automation of decision making, but mostly to the fact that the algorithms are opaque and their logic unexplained. The main cause for this lack of transparency is that often the algorithm itself has not been directly coded by a human but it has been generated from data through machine learning. Machine learning allows building predictive models which map user features into a class (outcome or decision), obtained by generalizing from a training set of examples. This learning process is made possible by the digital records of past decisions and classification outcomes, typically provided by human experts and decision makers. The process of inferring a classification model from examples cannot be controlled step by step because the size of training data and the complexity of the learned model are too big for humans. This is how we got trapped in a paradoxical situation in which, on one side, the legislator defines new regulations requiring that automated decisions should be explained to affected people\(^1\) while, on the other side, even more sophisticated and obscure algorithms for decision making are generated [16, 37].

The lack of transparency in algorithms generated through machine learning grants to them the power to perpetuate or reinforce forms of injustice by learning bad habits from the data. In fact, if the training data contains a number of biased decision records, or misleading classification examples due to data collection mistakes or artifacts, it is likely that the resulting algorithm inherits the biases and recommends discriminatory or simply wrong decisions\(^2\) [6, 7]. The inability of obtaining an explanation for what one considers a biased decision is a profound drawback of learning from big data, limiting social acceptance and trust on its adoption in many sensitive contexts. Starting from [29] a rich literature has been flourishing on discrimination discovery and avoidance. Some of the ideas developed in that context can be reinterpreted for addressing the more general problem of explaining the logic driving a decision taken by an obscure algorithm, which is precisely the problem tackled in this paper.

In particular, in this paper we address the problem of explaining the decision outcome taken by an obscure algorithm by providing “meaningful explanations of the logic involved” when automated decision making takes place, as prescribed by the GDPR. The decision system can be obscure because based on a deep learning approach, or because of inaccessibility of the source code, or other reasons. We perform our research under some specific assumptions. First, we assume that an explanation is interesting for a user if it clarifies why a specific decision pertaining that user has been made, i.e., we aim for local explanations, not general, global, descriptions of how the overall system works [17]. Second, we assume that the vehicle for offering explanations should be as close as possible to the language of reasoning, that is logic. Thus, we are also assuming that the user can understand elementary logic rules. Finally, we assume that the black box decision system can be queried as many times as necessary, to probe its decision behavior to the scope of reconstructing its logic; this is certainly the case in a legal argumentation in court, or in an industrial setting where a company wants to stress-test a machine learning component of a manufactured product, to minimize risk of failures and consequent industrial liability. On the other hand, we make no assumptions on the specific algorithms used in the obscure classifier: we aim at an agnostic

\(^1\)We refer here to the so-called “right to explanation” established in the European General Data Protection Regulation (GDPR), entering into force in May 2018.

\(^2\)www.propublica.org/article/machine-bias-risk-assessments-in-criminal-sentencing
We critically compare local versus global explanations, rule-based explanations with the highest coverage and respecting a precision threshold. Thus, the user is given not only a specific example of how to obtain actionable recourse (e.g., how to improve application for getting a benefit), but also an abstract characterization of its neighborhood instances with reversed black box outcome.

To the best of our knowledge, in the literature there is no work proposing a black box agnostic method for local decision explanation based on both decision and counterfactual rules.
3 PROBLEM AND EXPLANATIONS

Let us start recalling basic notation on classification of tabular data. Afterwards, we define the black box outcome explanation problem, and the notion of explanation for which we propose a solution.

Classification, black boxes, and interpretable predictors. A predictor or classifier, is a function \( b : X(m) \rightarrow Y \) which maps data instances (tuples) \( x \) from a feature space \( X(m) \) with \( m \) input features to a decision \( y \) in a target space \( Y \). We write \( b(x) = y \) to denote the decision \( y \) predicted by \( b \) and \( b(X) = Y \) as a shorthand for \( \{ b(x) \mid x \in X \} = Y \). We restrict here to binary decisions. An instance \( x \) consists of a set of \( m \) attribute-value pairs \( (a_i,v_i) \), where \( a_i \) is a feature (or attribute) and \( v_i \) is a value from the domain of \( a_i \). The domain of a feature can be continuous or categorical. A predictor can be a machine learning model, a domain-expert rule-based system, or any combination of algorithmic and human knowledge processing. We assume that a predictor is available as a software function that can be queried at will. In the following, we denote by \( b \) a black box predictor, whose internals are either unknown to the observer or they are known but uninterpretable by humans. Examples include neural networks, SVMs, ensemble classifiers, or a composition of data mining, legacy software, and hard-coded expert systems. Instead, we denote with \( c \) an interpretable predictor, whose internal processing yielding a decision \( c(x) = y \) can be given a symbolic interpretation understandable by a human. Examples of such predictors include rule-based classifiers, decision trees, decision sets, and rational functions.

Black Box Outcome Explanation. Given a black box predictor \( b \) and an instance \( x \), the black box outcome explanation problem consists in providing an explanation \( e \) for the decision \( b(x) = y \). We approach the problem by learning an interpretable predictor \( c \) that reproduces and accurately mimics the local behavior of the black box. An explanation of the decision is then derived from \( c \). By local, we mean focusing on the behavior of the black box in the neighborhood of the specific instance \( x \), without aiming at providing a single description of the logic of the black box for all possible instances. The neighborhood of \( x \) is not given, but rather it has to be generated as part of the explanation process. However, we assume that some knowledge is available about the characteristics of the feature space \( X(m) \), in particular the ranges of admissible values for the domains of features and, possibly, the (empirical) distribution of features. Nothing is instead assumed about the process of constructing the black box \( b \). Let us formalize the problem, and the approach based on interpretable models.

Definition 3.1 (Black Box Outcome Explanation). Let \( b \) be a black box, and \( x \) an instance whose decision \( b(x) \) has to be explained. The black box outcome explanation problem consists in finding an explanation \( e \in E \) belonging to a human-interpretable domain \( E \).

Definition 3.2 (Explanation Through Interpretable Models). Let \( c = \xi(b, x) \) be an interpretable predictor derived from the black box \( b \) and the instance \( x \) using some process \( \xi(\cdot, \cdot) \). An explanation \( e \in E \) is obtained through \( c \), if \( e = \varepsilon(c, x) \) for some explanation logic \( \varepsilon(\cdot, \cdot) \) which reasons over \( c \) and \( x \).

One point is still missing: which is a comprehensible domain \( E \) of explanations? We will define an explanation \( e \) as a pair of objects:

\[
e = (r = p \rightarrow y, \Phi)
\]

The first component \( r = p \rightarrow y \) is a decision rule describing the reason for the decision value \( y = c(x) \). The second component \( \Phi \) is a set of counterfactual rules, namely the minimal number of changes in the feature values of \( x \) that would reverse the decision of the predictor. Let us consider as an example the following explanation for a loan request for user \( x = \{(\text{age}=22), (\text{job}=\text{none}), (\text{amount}=10k), (\text{car}=\text{no})\} \):

\[
e = (r = \{\{\text{age} \leq 25, \text{job}=\text{none}, \text{amount}>5k\} \rightarrow \text{deny}, \\
\Phi = \{\{\text{age}>25, \text{amount} \leq 5k\} \rightarrow \text{grant}, \}
\]

Here, the decision \( \text{deny} \) is due to the age lower than 25, the absence of job and an amount greater than 5k (see component \( r \)). For changing the decision instead it is required either an age higher than 25 and a smaller amount, or owning a clerk job and a car (see component \( \Phi \)). Details are provided in the rest of the section.

In a decision rule (simply, a rule) \( r \) of the form \( p \rightarrow y \), the decision \( y \) is the consequence of the rule, while the premise \( p \) is a boolean condition on feature values. We assume that \( p \) is the conjunction of split conditions \( s \) of the form \( \{ a \in [v_1, v_2]\} \), where \( a \) is a feature and \( v_1, v_2 \) are values in the domain of \( a \) extended with \( ^1 \pm \infty \). An instance \( x \) satisfies \( r \), or \( r \) covers \( x \), if the boolean condition \( p \) evaluates to true for \( x \), i.e., if \( s(x) \) is true for every \( s \in p \). For example, the rule \( \{\text{age} \leq 25, \text{job}=\text{none}\} \rightarrow \text{deny} \) is satisfied by \( x_0 = \{(\text{age}=22), (\text{job}=\text{none})\} \) and not satisfied by \( x_1 = \{(\text{age}=22), (\text{job}=\text{clerk})\} \). We say that \( r \) is consistent with \( c \), if \( c(x) = y \) for every instance \( x \) that satisfies \( r \). Consistency means that the rule specifies some conditions for which the predictor makes a specific decision. When the instance \( x \) for which we have to explain the decision satisfies \( r \), the rule \( p \rightarrow y \) represents a motivation for taking a decision value, i.e., \( p \) locally explains why \( b \) returned \( y \).

Consider now a set \( \delta \) of split conditions. We denote the update of \( p \) by \( \delta \) as \( p[\delta] = \delta \cup \{ (a \in [v_1, v_2]) \in p \mid \{ a \in [v_1, v_2]\} \in \delta \} \). Intuitively, \( p[\delta] \) is the logical condition \( p \) with ranges for attributes overwritten as stated in \( \delta \), e.g. \( \{\text{age} \leq 25, \text{job}=\text{none}\}[>25] \) is \( \{\text{age} > 25, \text{job}=\text{none}\} \). A counterfactual rule for \( p \) is a rule of the form \( p[\delta] \rightarrow y \), for \( y \neq y \). We call \( \delta \) a counterfactual. Consistency means also for counterfactual rules, meaning that the rule is an instance of the decision logic of \( c \). A counterfactual \( \delta \) describes what features to change and how to change them to get an outcome different from \( y \). Since \( c \) predicts either \( y \) or \( \bar{y} \), if such changes are applied to the given instance \( x \), the predictor \( c \) will return a different decision. Continuing the example before, changing the age feature of \( x_0 \) to any value greater than 25 will change the predicted outcome of \( c \) from \( \text{deny} \) to \( \text{grant} \). An expected property of a consistent counterfactual rule \( p[\delta] \rightarrow \bar{y} \) is that it should be minimal w.r.t. \( x \). Minimality

\[\text{Using } ^1 \pm \infty \text{ we can model with a single notation typical univariate split conditions, such as equality } (a = v) \text{ as } (a \in [v, v]), \text{upper bounds } (a \leq v) \text{ as } (a \in [\infty, v]), \text{strict lower bounds } (a > v) \text{ as } (a \in (v, \infty]) \text{ for a sufficiently small } e. \] However, since our method is parametric to a decision tree induction algorithm, split conditions can also be multivariate, e.g., \( a \leq b + v \) for \( a, b \) features (as in oblique decision trees).
We propose by first generating a set of \( x \) (minimal) motivation for reversing \( n_f \leq Z \) local decision behavior of the black box \( c \), and then extracting from such a set a interpretable predictor.

4 PROPOSED METHOD

\[ \text{Algorithm 1: LORE}(x, b) \]

\begin{itemize}
  \item Input : \( x \) - instance to explain, \( b \) - black box, \( N \) - \# of neighbors
  \item Output: \( e \) - explanation of \( x \)
  \end{itemize}

\begin{verbatim}
1 \ G \leftarrow 10; \ pc \leftarrow 0.5; \ pm \leftarrow 0.2; // init. parameters
2 \ Z_x \leftarrow \text{GeneticNeigh}(x, \text{fitness}_x, b, N/2, G, pc, pm); // generate neigh.
3 \ Z_x \leftarrow \text{GeneticNeigh}(x, \text{fitness}_x, b, N/2, G, pc, pm); // generate neigh.
4 \ Z \leftarrow \text{Merge}(Z); // merge neighborhoods
5 \ c \leftarrow \text{BuildTree}(Z); // build decision tree
6 \ r = (p \rightarrow y) \leftarrow \text{ExtractRule}(c, x); // extract decision rule
7 \ \Phi \leftarrow \text{ExtractCounterfactual}(c, r, x); // extract counterfactuals
\end{verbatim}

is measured w.r.t. the number of split conditions in \( p[\delta] \) not satisfied by \( x \). Formally, we define \( nf(p[\delta], x) = |\{sc \in p[\delta] | \neg sc(x)\}| \) (where \( nf(\cdot, \cdot) \) stands for the number of falsified split conditions), and, when clear from the context, we simply write \( nf \). For example, \( \{\text{age} < 25, \text{job}=\text{clerk}\} \rightarrow \text{grant} \) is a counterfactual with two conditions falsified by \( x \). It is not minimal if the counterfactual \( \{\text{age} > 25, \text{job}=\text{none}\} \rightarrow \tilde{y} \), with only one falsified condition, is consistent for \( c \). In summary, a counterfactual rule \( p[\delta] \rightarrow \tilde{y} \) is a (minimal) motivation for reversing a decision value.

We are now in the position to formally introduce the notion of explanation that we are able to provide.

**Definition 3.3 (Local Explanation).** Let \( x \) be an instance, and \( c(x) = y \) be the decision of \( c \). A local explanation \( e = (r, \Phi) \) is a pair: a decision rule \( r = (p \rightarrow y) \) consistent with \( x \) and satisfied by \( x \); and, a set \( \Phi = \{p[\delta_1] \rightarrow \tilde{y}_1, \ldots, p[\delta_n] \rightarrow \tilde{y}_n\} \) of counterfactual rules for \( p \) consistent with \( c \).

This definition completes the elements of the black box outcome explanation problem. A solution to the problem will then consists of: (i) computing an interpretable predictor \( c \) for a given black box \( b \) and an instance \( x \), i.e., defining the function \( \zeta(\cdot, \cdot) \) according to Definition 3.2; (ii) deriving a local explanation from \( c \) and \( x \), i.e., defining the explanation logic \( \varepsilon(\cdot, \cdot) \) according to Definition 3.2.

4 PROPOSED METHOD

We propose LORE (LOcal Rule-based Explanations, Algorithm 1) as a solution to the black box outcome explanation problem. An interpretable predictor \( c \) is built for a given black box \( b \) and instance \( x \) by first generating a set of \( N \) neighbor instances of \( x \) through a genetic algorithm, and then extracting from such a set a decision tree \( c \). A local explanation, consisting of a single rule \( r \) and a set of counterfactual rules \( \Phi \), is then derived from the structure of \( c \).

4.1 Neighborhood Generation

The goal of this phase is to identify a set of instances \( Z \), with feature characteristics close to the ones of \( x \), that is able to reproduce the local decision behavior of the black box \( b \). Since the objective is to learn a predictor, the neighborhood should be flexible enough to include instances with both decision values, namely \( Z = Z_+ \cup Z_- \).

\[ \text{Algorithm 2: GeneticNeigh}(x, \text{fitness}, b, N, G, pc, pm) \]

\begin{itemize}
  \item Input : \( x \) - instance to explain, \( b \) - black box, \( \text{fitness} \) - fitness function, \( N \) - population size, \( G \) - \# of generations, \( pc \) - crossover probability, \( pm \) - mutation probability
  \item Output: \( Z \) - neighbors of \( x \)
  \end{itemize}

\begin{verbatim}
1 \ P_0 \leftarrow \{x|V 1 \ldots N\}; \ i \leftarrow 0; // population init.
2 \ evaluate(P_0, \text{fitness}, b); // evaluate population
3 \ while \ i < G \ do
4 \ \ \ P_i \leftarrow select(P_i); // select sub-population
5 \ \ \ P'_i \leftarrow \text{crossover}(P_{i+1}, pc); // mix records
6 \ \ \ P'_i \leftarrow \text{mutate}(P'_i, pm); // perform mutations
7 \ \ \ evaluate(P'_i, \text{fitness}, b); // evaluate population
8 \ \ \ P_{i+1} \leftarrow P'_i; \ i \leftarrow i + 1 // update population
9 \ Z \leftarrow P_i \text{ return } Z;
\end{verbatim}

where instances \( Z \) are such that \( b(z) = b(x) \), and instances \( z \in Z_\pm \) are such that \( b(z) \neq b(x) \). In Algorithm 1, we extract balanced subsets \( Z_+ \) and \( Z_- \) (lines 2–3), and then put \( Z = Z_+ \cup Z_- \) (line 4). This task differs from approaches to instance selection [28], based on genetic algorithms [36] (also specialized for decision trees [40]), in that their objective is to select a subset of instances from an available training set. In our case, instead we cannot assume that the training set used to train \( b \) is available, or not even that \( b \) is a supervised machine learning predictor for which a training set exists. Our task is instead similar to instance generation in the field of active learning [14], also including evolutionary approaches [10].

We adopt an approach based on a genetic algorithm which generates \( z \in Z_\pm \) by maximizing the following fitness functions:

\[
\text{fitness}_+^z(z) = I_{b(x)=b(z)} + (1 - d(x, z)) - I_{z=x} \\
\text{fitness}_-^z(z) = I_{b(x)\neq b(z)} + (1 - d(x, z)) - I_{z=x}
\]

where \( d : X^m \rightarrow [0, 1] \) is a distance function, \( I_{true} = 1 \), and \( I_{false} = 0 \). The first fitness function looks for instances \( z \) similar to \( x \) (term \( 1 - d(x, z) \)), but not equal to \( x \) (term \( I_{z=x} \)) for which the black box \( b \) produces the same outcome as \( x \) (term \( I_{b(x)=b(z)} \)). The second one leads to the generation of instances \( z \) similar to \( x \), but not equal to it, for which \( b \) returns a different decision. Intuitively, for an instance \( z_0 \) such that \( b(x) \neq b(z_0) \) and \( x \neq z_0 \), it turns out \( \text{fitness}_-^z(z_0) < 1 \). For any instance \( z_0 \) such that \( b(x) = b(z_0) \), instead, we have \( \text{fitness}_-^z(z_0) \geq 1 \). Finally, \( \text{fitness}_+^z(x) = 1 \). Thus, maximization of \( \text{fitness}_+^z(\cdot) \) occurs necessarily for instances different from \( x \) and whose prediction is equal to \( b(x) \).

Like neural networks, genetic algorithms [20] are based on the biological metaphor of evolution. They have three distinct aspects. (i) The potential solutions of the problem are encoded into representations that support the variation and selection operations. In our case these representations, generally called chromosomes, correspond to instances in the feature space \( X^m \). (ii) A fitness function evaluates which chromosomes are the “best life forms”, that is, most appropriate for the result. These are then favored in survival and reproduction, thus shaping the next generation according to the fitness function. In our case, these instances correspond to those similar to \( x \), according to distance \( d(\cdot, \cdot) \), and with the same/different outcome returned by the black box \( b \) depending on the fitness function \( \text{fitness}_+^x \) or \( \text{fitness}_-^x \). (iii) Mating (called crossover) and mutation
produce a new generation of chromosomes by recombining features of their parents. The final generation of chromosomes, according to a stopping criterion, is the one that best fits the solution.

Algorithm 2 generates the neighborhoods $Z_\pm$ and $Z_\pm$ of $x$ by instantiating the evolutionary approach of [4]. Using the terminology of [10], it is an instance of generational genetic algorithms for evolutionary prototype generation. However, prototypes are a condensed subset of a training set that enable optimization in predictor learning. We aim instead at generating new instances that separate well the decision boundary of the black box $b$. The usage of classifiers within fitness functions of genetic algorithms can be found in [5, 8, 12, 41]. However, the classifier they use is always the one for which the population must be selected or generated for and not another one (the black box) like in our case. Algorithm 2 uses a uniform distribution of the instance $x$ to explain. Then it enters the evolution loop that begins by selection of the $P_{i+1}$ population having the highest fitness score. After that, the crossover operator is applied on a proportion of $P_{i+1}$, according to the $p_c$ probability, the resulting and the untouched individuals are placed in $P_{i+1}^{\rho}$. We use a two-point crossover which selects two parents and two crossover features at random, and then swap the crossover feature values of the parents (see Figure 1). Thereafter, a proportion of $P_{i+1}^{\rho}$, determined by $p_m$, is mutated and placed in $P_{i+1}$. The unmutated individuals are also added to $P_{i+1}$. Mutation consists of replacing feature values at random according to the empirical distribution of a feature (see Figure 2). Individuals in $P_{i+1}$ are evaluated according to the fitness function, and the evolution loop continues until $G$ generations are completed. Finally, the best individuals according to the fitness function are returned. Algorithm 2 is run twice, once using the fitness function $\text{fitness}^i$ to derive neighborhood instances $Z_\pm$ with the same decision as $x$, and once using $\text{fitness}^b$ to derive neighborhood instances $Z_\pm$ with different decision as $x$. Finally, we set $Z = Z_\pm \cup Z_\pm$.

Figure 3 shows an example of neighborhood generation for a black box consisting of a random forest model and a bi-dimensional feature space. The figure contrasts uniform random generation around a specific instance $x$ (stared) to our genetic approach. The latter yields a neighborhood that is denser in the boundary region of the predictor. The density of generated instances will be a key factor in extracting local (interpretable) predictors.

**Distance Function.** A key element in the definition of the fitness functions is the distance $d(x, z)$. We account for the presence of mixed types of features by a weighted sum of simple matching coefficient for categorical features, and of the normalized Euclidean distance for continuous features. Formally, assuming $h$ categorical features and $m - h$ continuous ones, we use:

$$d(x, z) = \frac{h}{m} \cdot \text{SimpleMatch}(x, z) + \frac{m - h}{m} \cdot \text{NormEuclid}(x, z).$$

However, it can be done as follows. Consider the path from the leaf of $p$ to the leaf of $q$. When moving from a child to a father node, we retract the split condition. E.g., $a \leq c_2$ is retracted from $\{a \in [c_1, c_2]\}$ by adding $a \in [c_1, +\infty)$ to $\delta$. When moving from a father node to a child, we add the split condition to $\delta$. Empirical results with different distance functions are reported in Section 5.3.

### 4.2 Local Rule-Based Classifier and Explanation Extraction

Given the neighborhood $Z$ of $x$, the second step is to build an interpretable predictor $c$ trained on the instances $z \in Z$ labeled with the black box decision $b(z)$. Such a predictor is intended to mimic the behavior of $b$ locally in the $Z$ neighborhood. Also, $c$ must be interpretable, so that an explanation for $x$ (decision rule and counterfactuals) can be extracted from it. The LORE approach considers decision tree classifiers due to the following reasons: (i) decision rules can naturally be derived from a root-leaf path in a decision tree; and, (ii) counterfactuals can be extracted by symbolic reasoning over a decision tree. For a decision tree $c$, we derive an explanation $e = \langle r, \Phi \rangle$ as follows. The decision rule $r = (p \rightarrow y)$ is formed by including in $p$ the split conditions on the path from the root to the leaf node that is satisfied by the instance $x$, and setting $y = c(x)$. By construction, the rule $r$ is consistent with $c$ and satisfied by $x$. Consider now the counterfactual rules in $\Phi$. Algorithm 3 looks for all paths in the decision tree $c$ leading to a decision $\hat{q} \neq y$. Fix one of such paths, and let $q$ be the conjunction of split conditions in it. Again by construction, $q \rightarrow \hat{q}$ is a counterfactual rule consistent with $c$. Notice that the counterfactual $q$ for which $q = p(\delta)$ has not to be explicitly computed — this is a benefit of using decision trees. Among all such $q$’s, only the ones with minimum number



**Algorithm 3: ExtractCounterfactuals**(c, r, x)

**Input**: c - decision tree, r - rule (p → q), x - instance to explain  
**Output**: Φ - set of counterfactual rules for p

```plaintext
1 Q ← GetPathsWithDifferentLabel(c, y); // get paths with label y ≠ y
2 Φ ← ∅, min ← +∞; // init. counterfactual set
3 for q ∈ Q do
   4   q_len ← nf(q, x) = |{sc ∈ q | ¬sc(x)}|
   5   if q_len < min then Φ ← {q → ˆy}; min ← q_len;
   6   else if q_len = min then Φ ← Φ ∪ {q → ˆy};
4 end
8 return Φ;
```

**Figure 4: Example decision tree.**

of split conditions sc not satisfied by x (line 4 of Algorithm 3) are kept in Φ. As an example, consider the decision tree in Figure 4, and the instance x = {(age, 22), (job, clerk), (income, 800)} for which the decision deny (e.g., of a loan) has to be explained. The path followed by x is the leftmost one in the tree. The decision rule extracted from the path is {age ≤ 25, job=clerk, income ≤ 900} → deny. There are four paths leading to the opposite decision: q1 = {age ≤ 25, job=clerk, income > 900}, q2 = {17 < age ≤ 25, job = other}, q3 = {age > 25, income ≤ 1500, job = other}, and q4 = {age > 25, income > 1500}. It turns out: nf(q1, x) = 1, nf(q2, x) = 1, nf(q3, x) = 2, and nf(q4, x) = 2. Thus, Φ = {q1→deny, q2→deny}.

As a further output, LORE computes a counterfactual instance starting from a counterfactual rule q → ˆy and x. Among all possible instances that satisfy q, we choose the one that minimally changes attributes from x according to q. This is done by looking at the split conditions falsified by x: {sc ∈ q | ¬sc(x)} and modifying the lower/upper bound in sc that is closer to the value in x. As an example, for the above path q1, the counterfactual instance of x is {(age, 22), (job, clerk), (income, 900 + ε)}, and for q2 is {(age, 22), (job, other), (income, 800)}.

5 EXPERIMENTS

LORE has been developed in Python\textsuperscript{9}, using, for the genetic neighborhood generation, the deap library [13], and for decision tree induction (the interpretable predictor), the yadt system [32], which is a C4.5 implementation with multi-way splits of categorical attributes. After presenting the experimental setup, we report next: (i) some analyses on the effect of the genetic algorithm parameters for the neighborhood generation; (ii) evidence that the local genetic neighborhood is more effective than a global approach; (iii) a qualitative and quantitative comparison with naive baselines and state of the art competitors.

5.1 Experimental Setup

We ran experiments on three real-world tabular datasets: adult, compas and german\textsuperscript{10}. In each of them, an instance represents attributes of an individual person. All datasets includes both categorical and continuous features.

The adult dataset from UCI Machine Learning Repository, includes 48,842 instances with demographic information like age, workclass, marital-status, race, capital-loss, capital-gain etc. The income divides the population into two classes “<=50K” and “>50K”.

The compas dataset from Propublica contains the features used by the COMPAS algorithm for scoring defendants and their risk (Low, Medium and High), for over 10,000 individuals. We considered two classes “Low-Medium” and “High” risk, and we use the following features: age, sex, race, priors_count, days_b_screening_arrest, length_of_stay, c_charge_degree, is_recid.

In the german dataset from UCI Machine Learning Repository each person of the 1,000 entries is classified as a “good” or “bad” creditor according to attributes like age, sex, checking_account, credit_amount, duration, purpose, etc.

We experimented the following predictors as black boxes: support vector machines with RBF kernel (SVM), random forests with 100 trees (RF), and multi-layer neural networks with ‘lbfgs’ solver (NN). Implementations of the predictors are from the scikit-learn library. Unless differently stated, default parameters were used for both the black boxes and the libraries of LORE. Missing values were replaced by the mean for continuous features and by the mode for categorical ones.

Each dataset was randomly split into train (80% instances), and test (20% instances). The former is used to train black box predictors. The latter, denoted by X, is the set of instances for which the black box decision have to be explained. In the following, for some fixed set of instances, we denote by Y the set of decisions provided by the interpretable predictor c and by ˆY the decisions provided by the black box b on the same set.

We consider the following properties in evaluating the mimic performances of the decision tree c inferred by LORE and of the explanations returned by it against the black box classifier b:

- **fidelity**(Y, ˆY) ∈ [0, 1]. It compares the predictions of c and of the black box b on the instances Z used to train c [11]. It answers the question: how good is c at mimicking b?
- **l-fidelity**(Y, ˆY) ∈ [0, 1]. It compares the predictions of c and b on the instances Zc covered by the decision rule in a local (hence “l-”) explanation for x. It answers the question: how good is the decision rule at mimicking b?
- **cl-fidelity**(Y, ˆY) ∈ [0, 1]. It compares the predictions of c and b on the instances Zc covered by the counterfactual rules in a local explanation for x.
- **hit**(y, ˆy) ∈ [0, 1]. It compares the predictions of c and b on the instance x under analysis. It returns 1 if y = c(x) is equal to ˆy = b(x), and 0 otherwise.
- **c-hit**(y, ˆy) ∈ [0, 1]. It compares the predictions of c and b on a counterfactual instance of x built from counterfactual rules in a local explanation of x.

\textsuperscript{9}The source code and datasets will be available at anonymized url. The experiments were performed on Ubuntu 16.04.1 LTS 64 bit, 32 GB RAM, 3.30GHz Intel Core i7

\textsuperscript{10}https://archive.ics.uci.edu/ml/datasets/adult, https://github.com/propublica/compas-analysis, https://archive.ics.uci.edu/ml/datasets/statlog+(german+credit+data)
We analyze here the impact of the number of generations \( G \) and of population size \( N \) parameters of the genetic neighborhood generation. Bottom plots also report elapsed running times.

![Figure 5: Impact of the number of generations \( G \) and of population size \( N \) parameters of the genetic neighborhood generation. Bottom plots also report elapsed running times.](image)

| Distance | hit    | fidelity | f-fidelity | \(|c|\) | \(|p|\) | nf |
|----------|--------|----------|------------|--------|--------|-----|
| cosine   | .938±.24 | .976±.11 | .936±.24 | 4.4±2.5 | 2.1±1.8 | 1.9±1.0 |
| minmax   | .958±.19 | .965±.15 | .956±.17 | 4.5±2.7 | 2.3±2.3 | 1.8±0.9 |
| neulid   | .966±.17 | .967±.15 | .963±.19 | 4.3±2.6 | 2.2±2.1 | 1.8±1.0 |

Table 1: Comparison of distance measures.

We measure the first three of them by the f1-measure [35]. Aggregated values of f1 and hit/c-hit are reported by averaging them over the the set of test instances \( x \in X \).

5.2 Analysis of Neighborhood Generation

We analyze here the impact of the number of generations \( G \) and size of neighborhood \( N \) on the performances of instance generation and on the size complexity of the LORE output. We report only results for \( \text{german} \) dataset, since we get similar results for the other ones. The other parameters of Algorithm 2 (probabilities of crossover \( pc \) and mutation \( pm \)) are set with the default values of 0.5 and 0.2 respectively [4]. Figure 5 shows in the top plots the value of fitness functions and measures of sizes of local classifier \( c \) (decision tree depth), of decision rule (size of the antecedent \( p \)), and of counterfactual rules (number \( nf \) of falsified split conditions). The bottom plots show fidelity (f1-measure) and hit (rate) as well as running times of neighborhood generation. Fixed \( N = 1000 \), after 10 generations, the fitness function converges around the optimal value (top left), fidelity is almost maximized (bottom left), and also the measures of sizes (top right) become stable and small. We then set \( G=10 \) in all other experiments. Figure 5-(bottom right) shows instead that the size \( N \) of the neighborhood instances to be generated is relevant for the hit rate but not for fidelity. By taking into account also the running time (right side scale of the bottom plots), a good trade-off is obtained by setting \( N=1000 \).

5.3 Comparing Distance Functions

A key element of the neighborhood generation is the distance function used by the genetic algorithm. A legitimate question is whether the results of the approach are affected by the choice of the distance function adopted (see Section 4.1). For instance, [38] presents considerable differences in their output of counterfactual instance varying the choice of the distance in their stochastic optimization approach. Table 1 reports basic measures contrasting the normalized Euclidean distance adopted by LORE with cosine and min-max distance on \( \text{german} \) dataset. The table does not highlight any considerable difference. This can be justified by the fact that, following instance generation, there are phases, such as decision tree building, that abstract instances to patterns, resulting in resilience against variability due to the distance function adopted.

### 5.4 Validation of Local Explanations

We now compare our local approach with a global approach, and discuss alternative neighborhood instance generation methods.

**Local vs Global Explanations.** Extracting a predictor from the neighborhood of an instance is a winning strategy, if contrasted to an approach that builds a single predictor from all instances in the test set, i.e., \( Z = X \). In particular, this means that the interpretable predictor will be the same for all instances in the test set. Let compare our approach with such a global approach. Table 2 reports the mean and standard deviation values of hit, fidelity, fairness and tree depth for each dataset aggregating over the results of the various black boxes. While for hit both LORE and global obtain similar high performances, for the other scores LORE considerably over-takes global. In particular, the size and depth of the decision tree of the global approach may lead to explanations (decision rules and counterfactuals) more complex to understand than those returned by the proposed local approach LORE.

| Dataset | Method | hit | fidelity | f-fidelity | tree depth |
|---------|--------|-----|----------|------------|------------|
| adult   | lore   | 912±29 | .959±17 | 892±29 | 4.16±0.21 |
|         | global | 901±28 | .750±00 | 873±27 | 12.00±0.00 |
| compas  | lore   | 942±23 | .992±03 | 937±23 | 4.72±2.15 |
|         | global | 902±29 | .935±00 | 857±29 | 12.00±0.00 |
| german  | lore   | 925±26 | .988±07 | 920±26 | 4.95±2.54 |
|         | global | 880±32 | .571±00 | 824±31 | 6.00±0.00 |

Table 2: Local vs global approach.
Table 3: Comparison of neighborhood generations methods.

| Method | hit fidelity | l-fidelity | c-hit | cl-fidelity |
|--------|--------------|------------|-------|-------------|
| lore   | 0.962 ± 0.19 | 0.993 ± 0.04 | 0.959 ± 0.19 | 0.588 ± 0.42 | 0.756 ± 0.40 |
| crd    | 0.924 ± 0.26 | 0.855 ± 0.23 | 0.894 ± 0.25 | 0.349 ± 0.26 | 0.583 ± 0.48 |
| rnd    | 0.946 ± 0.22 | 0.904 ± 0.15 | 0.920 ± 0.22 | 0.494 ± 0.24 | 0.712 ± 0.40 |
| ris    | 0.916 ± 0.27 | 0.869 ± 0.05 | 0.870 ± 0.26 | 0.501 ± 0.22 | 0.708 ± 0.39 |
| ros    | 0.968 ± 0.17 | 0.965 ± 0.03 | 0.953 ± 0.17 | 0.491 ± 0.22 | 0.733 ± 0.34 |

Table 4: LORE vs LIME: hit scores.

Table 4: Comparison of neighborhood generations methods.

| Dataset | german | compass | adult |
|---------|--------|---------|-------|
| Black Box | lore   | lime    | lore   | lime    |
| RF      | 0.925 ± 0.2 | 0.880 ± 0.3 | 0.941 ± 0.2 | 0.826 ± 0.4 | 0.901 ± 0.3 | 0.824 ± 0.4 |
| NN      | 0.980 ± 1.0 | 1.000 ± 0.0 | 0.987 ± 1.0 | 0.902 ± 0.3 | 0.918 ± 0.0 | 0.998 ± 1.0 |
| SVM     | 1.000 ± 0.0 | 0.966 ± 0.1 | 0.997 ± 0.1 | 0.900 ± 0.3 | 0.985 ± 0.1 | 0.987 ± 0.1 |

5.5 Comparison with the State-of-Art

In this section we compare our approach with the state of the art.

5.5.1 Rules vs Linear Regression for Explanations. We present first a quantitative and qualitative comparison with the linear explanations of LIME\textsuperscript{12} [30]. A first crucial difference is that in LIME, the number of features composing an explanation is an input parameter that must be specified by the user. LORE, instead, automatically provides the user with an explanation including only the features useful to justify the black box decision. This is a clear improvement over LIME. In experiments, unless otherwise stated, we vary the number of features of LIME explanations from two to ten and we consider the performance with the highest score.

Quantitative Comparison. Table 4 reports the mean and standard deviation of hit for each black box predictor and dataset. Moreover, Figure 7 details the box plots of fidelity (top) and l-fidelity (bottom). Results show that LORE definitely outperforms LIME under various viewpoints. Regarding the hit score, even when LORE is worse than LIME, it has a score close to 1. For RF black box, instead, LIME performs considerably worse than LORE. The box plots show that, in addition, LORE has better (local) fidelity scores and is more robust than LIME, which, on the contrary, exhibits very high variability in the neighborhood of the instance to explain (i.e., for l-fidelity). This can be tracked back to the genetic instance generation of LORE. Figure 8 reports a multidimensional scaling of the neighborhood of a sample instance x generated by the two approaches.

Qualitative Comparison. We claim that the explanations provided by LORE are more abstract and comprehensible than the ones of LIME. Consider the example in Figure 9. The top part reports a LORE local explanation for an instance x from the german dataset. The central part is a LIME explanation. Weights are associated to the categorical values in the instance x to explain, and to continuous upper/lower bounds where the bounding values are taken from x. Each weight tells the user how much the decision would have changed for different (resp., smaller/greater) values of a specific feature. In the example, the weight 0.11 has the following meaning [30]: if the duration in months had been higher than the value it is for x, the prediction would have been, on average, 0.11 less “0” (or 0.11 more “1”). A not very easy logic to follow when compared to a single decision rule which characterizes the contextual conditions for the decision of the black box. Another major advantage of our notion of explanation consists of the set of counterfactual rules. LIME provides a rough indication of where to look for a different decision: different categorical values or lower/higher continuous values of some feature. LORE’s counterfactual rules provide high-level and minimal-change contexts for reversing the outcome prediction of the black box.

On a qualitative level of comparison, the Anchor approach requires the apriori discretization of continuous features, while the decision rule of LORE benefits of the capabilities of decision tree to split continuous features. Contrast, for instance, the example rules in Figure 9. Moreover, the approach of Anchor does not clearly extend to compute counterfactuals.

Let us compare now the two approaches on a quantitative level. Figure 10 reports the average precision of decision rules, where the

\textsuperscript{12}https://github.com/marcotcr/lime

\textsuperscript{13}https://github.com/marcotcr/anchor
Figure 7: LORE vs LIME: box plots of fidelity and l-fidelity. Numbers on top are the mean values.

Figure 8: Neighborhoods of LORE (left) and LIME (right).

- LORE

\[
\begin{align*}
    r &= \{(\text{credit} \_\text{amount} > 836, \text{housing} = \text{own}, \text{other} \_\text{debtors} = \text{none}, \text{credit} \_\text{history} = \text{critical account}) \rightarrow \text{decision} = 0) \\
    \Phi &= \{(\text{credit} \_\text{amount} \leq 836, \text{housing} = \text{own}, \text{other} \_\text{debtors} = \text{none}, \text{credit} \_\text{history} = \text{all paid back}) \rightarrow \text{decision} = 1), \\
    &((\text{credit} \_\text{amount} > 836, \text{housing} = \text{own}, \text{other} \_\text{debtors} = \text{none}, \text{credit} \_\text{history} = \text{all paid back}) \rightarrow \text{decision} = 1))
\end{align*}
\]

- LIME

\[
\begin{align*}
    a &= \{(\text{credit} \_\text{history} = \text{critical account}, \\
    &\text{duration} \_\text{in} \_\text{month} \in [0, 18.00]) \rightarrow \text{decision} = 0)
\end{align*}
\]

Figure 9: Explanations of LORE, LIME and Anchor.

Table 5: LORE vs Anchor: Jaccard measure of stability.

| Dataset | german | compass | adult |
|---------|--------|---------|-------|
| Black box | lore | anchor | lore | anchor |
| RF | .76 ± .15 | .75 ± .12 | .70 ± .15 | .69 ± .15 |
| NN | .69 ± .18 | .83 ± .13 | .81 ± .12 | .65 ± .16 |
| SVM | .82 ± .16 | .71 ± .16 | .87 ± .14 | .67 ± .13 |

6 CONCLUSION

We have proposed a local black box agnostic explanation approach based on logic rules. LORE builds an interpretable predictor for a given black box and instance to be explained. The local interpretable predictor, a decision tree, is trained on a dense set of artificial instances similar to the one to explain generated by a genetic algorithm. The decision tree enables the extraction of a local explanation, consisting of a single rule for the decision and a set of counterfactual rules for the reversed decision. An ample experimental evaluation of the proposed approach has demonstrated the effectiveness of the genetic neighborhood procedure that leads LORE to outperform the proposals in the state of the art. A number of extensions and additional experiments can be mentioned as future work. First, LORE now works tabular data. An interesting future research direction is to make the method suitable for image and text data, for example by applying a pre-processing step for extracting semantic tags/concepts that may be mapped to a tabular format. Second, another study might be focused on the possibility...
to derive a global description of the black box bottom-up by composing the local explanations and minimizing the size (complexity) of the global description. Third, research lab experiments would be useful for evaluating the human comprehensibility of the provided explanations. Finally, LORE explanations can be used for identifying data and/or algorithmic biases. After the local explanations are retrieved, it would be interesting to develop an approach for deriving an unbiased dataset for safely training the obscure classifier, or to prevent the black box from introducing an algorithmic bias.

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