Assessment of enclosing structure unsteady-state moisture behavior using moisture potential theory

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Abstract. The logistics of the moisture movement in the building wall is very important. We need to know the movement of moisture from one point to another. Two known moisture transport equations are analyzed. The first one is based on vapor-state moisture movement in sorption wetting zone, and the second one – on combined transfer of vapor and liquid moisture. Their strengths and shortcomings are described. Analytical formula for moisture potential $F$ developed by V.G. Gagarin and V.V. Kozlov is given. This formula allows taking into account moisture transport simultaneously in sorption and ultrasorption wetting zones in a consistent way. Derivation of a new moisture transfer equation based on moisture potential $F$ is given. Benefits of such approach are introduced. Boundary conditions for moisture transport problem are formulated, problem solution by finite difference method using an explicit difference scheme is given. The proposed method is applied to single-layer enclosing structure made of aerated concrete blocks. It is demonstrated that illustrated method takes into account wetting inertia, and allows moisture distribution determination in all enclosing structure sections at any time. Results can be presented as moisture distribution along enclosing structure thickness, or as enclosing structure average moisture variation during a year. It is highlighted that calculation using the proposed method does not require values of separate moisture transfer potentials.

1. Introduction
A huge set of researches is devoted to the problem of enclosing structure moisture behavior. This set includes researches for moisture diffusion coefficient measurement, experiments in moisture regime determination in laboratory and natural conditions [1–10]. Applied researches for moisture regime affect on thermal protection of buildings, their lifetime, and on human health are also of importance [12–16].

Development of calculation methods for enclosing structure moisture behavior is one of research trends [17]. At first, moisture regime was thought of as vapor-liquid transfer in enclosing structure thickness under the action of partial pressure gradient. The well-known “consistent wetting method” is one example, which moisture transfer differential is written as [18–20]:

\begin{equation}
\gamma_0 \cdot \frac{\xi}{E(t)} \cdot \frac{\partial \varepsilon(w,t)}{\partial \tau} = \frac{\partial}{\partial x} \left( \mu \frac{\partial \varepsilon(w,t)}{\partial x} \right).
\end{equation}
where \( e \) – water vapor partial pressure, Pa; \( E \) – saturated water vapor pressure, Pa; \( \mu \) – vapor permeability coefficient, \( \text{kg/(m} \cdot \text{s} \cdot \text{Pa}) \); \( \gamma_0 \) – dry material density, \( \text{kg/m}^3 \); \( \xi \) – relative vapor capacity, \( \text{kg/kg} \); \( w \) – material moisture content, mass percentage (1 kg/kg = 100 mass percentage); \( \tau \) – time, \( \text{s} \); \( x \) – coordinate, \( \text{m} \); \( t \) – temperature, \( \text{C} \).

The methods based on combined action of several transfer potentials have been developed later. For example, moisture transfer can be assessed by cumulative action of water vapor partial pressure gradient and liquid moisture partial pressure gradient [20]:

\[
\gamma_0 \frac{\partial w}{\partial \tau} = \frac{\partial}{\partial x} \left( \beta (w) \frac{\partial w}{\partial x} \right) + \frac{\partial}{\partial x} \left( \mu \frac{\partial e(w,t)}{\partial x} \right),
\]

(2)

where \( \beta \) – moisture conductivity coefficient, \( \text{kg/(m} \cdot \text{s} \cdot \text{kg/kg}) \).

There are also methods based on moisture potential theory. They allow replacement of several separate transfer potentials by a united moisture potential. One of the most appropriate moisture potential is the moisture potential \( F \) developed by V.G. Gagarin and V.V. Kozlov. The principal distinguishing feature of the moisture potential \( F \) is the need to carry out three experiments: vapor permeability, sorption and static moisture conductivity – for potential determination. Moreover, the potential itself is an analytically representable function [20]:

\[
F(w,t) = E_i(t) \cdot \varphi(w) + \frac{1}{\mu} \int_0^w \beta(\zeta) d\zeta.
\]

(3)

where \( F \) – moisture potential, Pa.

Moisture potential (3) take into account liquid and vapor moisture movement in a consistent way. Total flow density of vapor and liquid moisture is equal to:

\[
g = -\mu \frac{\partial F}{\partial x}.
\]

(4)

where \( g \) – total flow density of vapor and liquid moisture, \( \text{kg/(m}^2 \cdot \text{s}) \).

2. The problem
The problem is to develop unsteady-state moisture regime calculation method based on moisture potential \( F \).

3. Materials and methods

3.1. Derivation of moisture transfer equation
Let’s use known ratio between moisture gradient in time and moisture flow gradient in coordinate [20]:

\[
\gamma_0 \frac{\partial w}{\partial \tau} = -\frac{\partial g}{\partial x}.
\]

(5)

Let’s express moisture time derivative in terms of moisture potential and temperature:

\[
\gamma_0 \frac{\partial w}{\partial \tau} = \gamma_0 \left( \frac{\partial F(w,t)}{\partial F(w,t)} \cdot \frac{\partial F(w,t)}{\partial \tau} + \gamma_0 \frac{\partial w}{\partial \tau} \frac{\partial t}{\partial \tau} \right).
\]

(6)

In equation (6) let’s disregard influence of moisture temperature derivative:

\[
\gamma_0 \frac{\partial w}{\partial \tau} = \gamma_0 \frac{\partial F(w,t)}{\partial F(w,t)} \frac{\partial F(w,t)}{\partial \tau}.
\]

(7)
By substituting (4) and (7) in (5) we obtain:

\[ \gamma_0 \cdot \frac{\partial w}{\partial F(w,t)} \cdot \frac{\partial F(w,t)}{\partial \tau} = \frac{\partial}{\partial x} \left( \mu \cdot \frac{\partial F(w,t)}{\partial x} \right). \]  

(8)

Let's use analytic expression (3) to determine moisture derivative with respect to moisture potential:

\[ \frac{\partial w}{\partial F(w,t)} = \frac{1}{\mu} \beta(w) + \frac{\partial \phi(w)}{\partial w} E_i(t)^{-1}. \]  

(9)

By substituting (9) in (8) we obtain differential moisture transfer equation:

\[ \gamma_0 \cdot \frac{1}{\mu} \beta(w) + \frac{\partial \phi(w)}{\partial w} E_i(t)^{-1} \cdot \frac{\partial F(w,t)}{\partial \tau} = \frac{\partial}{\partial x} \left( \mu \cdot \frac{\partial F(w,t)}{\partial x} \right). \]  

(10)

Let's introduce new value \( \xi_r \) for equation (10) reduction:

\[ \xi_r(w,t) = (\frac{\partial \phi(w)}{\partial w} + \frac{1}{\mu \cdot E_i(t)} \beta(w))^{-1}. \]  

(11)

where \( \xi_r \) – relative potential capacity, kg/kg.

With regard for (11) the equation (10) changes to:

\[ \gamma_0 \cdot \xi_r(w,t) \cdot \frac{\partial F(w,t)}{E_i(t)} = \frac{\partial}{\partial x} \left( \mu \cdot \frac{\partial F(w,t)}{\partial x} \right). \]  

(12)

The equation (12) combines equation (1) expression simplicity, but it takes into account movement both in sorption and ultrasorption wetting zones similar to the equation (2). Relative potential capacity \( \xi_r \) is the complete analog of «relative vapor capacity» \( \xi_o \) in equation (1), but \( \xi_r \) value can be determined both for sorption and ultrasorption wetting zones.

Disregarding vapor permeability coefficient variation within enclosing structure material, we obtain an equation for a single-layer enclosing structure:

\[ \frac{\partial F(w,t)}{\partial \tau} = \frac{\mu}{\gamma_0 \cdot \xi_r(w,t) \cdot E_i(t)} \cdot \frac{\partial^2 F(w,t)}{\partial x^2}. \]  

(13)

Let's introduce new value – «heat-humidity characteristic coefficient» \( \kappa_r \):

\[ \kappa_r(w,t) = \frac{\mu}{\gamma_0 \cdot \xi_r(w,t)}. \]  

(14)

where \( \kappa_r \) – heat-humidity characteristic coefficient, \( m^2/(s \cdot Pa) \).

By substituting (14) in (13) we obtain more compact moisture transfer equation:

\[ \frac{\partial F(w,t)}{\partial \tau} = \kappa_r(w,t) \cdot E_i(t) \cdot \frac{\partial^2 F(w,t)}{\partial x^2}. \]  

(15)

3.2. Boundary conditions

Third-kind boundary conditions are set for enclosing structure edges:
\[-\mu \frac{\partial F}{\partial x} \bigg|_{x=0} = \frac{1}{R_{m,ext}} (F_{ext} - F_1),\]

where \( R_{m,ext} \) – resistance to moisture exchange between outer air and enclosing structure surface, \((m^2 \cdot s \cdot Pa) / kg\); \( F_{ext} \) – outer air moisture potential, \( Pa \); \( F_1 \) – material moisture potential in enclosing structure section in contact with outer air, \( Pa \).

\[\mu \frac{\partial F}{\partial x} \bigg|_{x=t} = \frac{1}{R_{m,in}} (F_m - F_N).\]

where \( R_{m,in} \) – resistance to moisture exchange between inner air and enclosing structure surface, \((m^2 \cdot s \cdot Pa) / kg\); \( F_m \) – inner air moisture potential, \( Pa \); \( F_N \) – material moisture potential in enclosing structure section in contact with inner air, \( Pa \).

### 3.3. Unsteady-state moisture behavior determination using finite difference method

Finite difference method using explicit scheme is applied to solve the proposed moisture transfer equation (15) with boundary conditions (16), (17):

\[
\begin{cases}
F_1^{k+1} = F_1^k + \Delta \tau \frac{k_f^i}{h^2} E_{i1}^k (1 - (1 + \frac{h}{\mu R_{m,ext}}) F_1^k + F_2^k) + \Delta \tau \frac{k_f^{i-1}}{h^2} E_{i1}^k (1 - (1 + \frac{h}{\mu R_{m,ext}}) F_1^k + F_2^k)

F_2^{k+1} = F_2^k + \Delta \tau \frac{k_f^i}{h^2} E_{i2}^k (F_{i1}^k - 2 \cdot F_2^k + F_{i2}^k), i = 2, ..., N - 1, k = 0, 1, ..., \end{cases}
\]

\[
F_N^{k+1} = F_N^k + \Delta \tau \frac{k_f^N}{h^2} E_{IN}^k (F_{IN}^k - 1 - (1 + \frac{h}{\mu R_{m,in}}) F_1^k + \Delta \tau \frac{k_f^{N-1}}{h^2} E_{IN}^k (1 - (1 + \frac{h}{\mu R_{m,in}}) F_1^k + F_2^k)
\]

where \( F_1^k \) – moisture potential in the first section of the enclosing structure at the \( k \)-th time step, \( Pa \); \( F_2^k \) – moisture potential in the second section of the enclosing structure at the \( k \)-th time step, \( Pa \); \( F_1^{k+1} \) – moisture potential in the first section of the enclosing structure at the \( (k+1) \)-th time step, \( Pa \); \( F_{ext} \) – material moisture potential of outer air at the \( k \)-th time step, \( Pa \); \( \kappa_f^i \) – material heat-humidity characteristic coefficient in the first section of the enclosing structure at the \( k \)-th time step, \( m^2/(s \cdot Pa) \); \( E_{i1}^k \) – saturated water vapor pressure in the first section of the enclosing structure at the \( k \)-th time step, \( Pa \); \( F_{i1}^k \) – moisture potential in the \( (i-1) \)-th section of the enclosing structure at the \( k \)-th time step, \( Pa \); \( F_i^k \) – moisture potential in the \( i \)-th section of the enclosing structure at the \( k \)-th time step, \( Pa \); \( \kappa_f^i \) – material heat-humidity characteristic coefficient in the \( i \)-th section of the enclosing structure at the \( k \)-th time step, \( m^2/(s \cdot Pa) \); \( E_{ii}^k \) – saturated water vapor pressure in the \( i \)-th section of the enclosing structure at the \( k \)-th time step, \( Pa \); \( F_{N-1}^k \) – moisture potential in the \( (N-1) \)-th section of the enclosing structure at the \( k \)-th time step, \( Pa \); \( F_{i+k}^{k+1} \) – moisture potential at the \( k \)-th time step of the enclosing structure at the \( k \)-th time step, \( Pa \); \( \Delta \tau \) – time step duration.
the $k$-th time step, $Pa$; $F_N^k$ – moisture potential in the $N$-th section of the enclosing structure at the $k$-th time step, $Pa$; $F_{N+1}^{k+1}$ – moisture potential in the $N$-th section of the enclosing structure at the $(k+1)$-th time step, $Pa$; $F_m^k$ – moisture potential of inner air at the $k$-th time step, $Pa$; $\kappa_{\mu}^k$ – material heat-humidity characteristic coefficient in the $N$-th section of the enclosing structure at the $k$-th time step, $m^2/(s \cdot Pa)$; $E_{\tau,N}^k$ – saturated water vapor pressure in the $N$-th section of the enclosing structure at the $k$-th time step, $Pa$; $h$ – coordinate step, $m$; $\Delta \tau$ – time step, $s$.

4. Results and discussion
The proposed method has been applied for unsteady-state moisture regime calculation of a single-layer enclosing structure made of 0.4 m thick aerated concrete. Construction area: Moscow, air temperature $22°C$ and relative humidity 55% has been maintained constant inside the enclosing structure. Humidity distribution along enclosing structure section during maximum moisture accumulation period (Fig. 1) and average humidity change of enclosing structure during a year (Fig. 2) have been investigated.

As can be seen in figures, the proposed method takes into account wetting process response time, and therefore maximum moisture accumulation falls on the beginning of March. The method also allows determining moisture distribution by enclosing structure sections at any moment of time. Accuracy of calculation results using the proposed method corresponds to calculation results using formula (2).

5. Conclusion
Calculation method for unsteady-state moisture regime based on moisture potential theory $F$ was developed. The proposed method takes into account vapor moisture movement in sorption wetting zone and liquid moisture movement in ultrasorption zone, however moisture transfer equation is formulated as regard to moisture potential $F$. This fact makes calculation easier, as separate transfer potential values are not required.

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