New, Efficient and Clean Strategies to Explore
CP Violation Through Neutral $B$ Decays

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Abstract

We point out that decays of the kind $B_d \rightarrow D_{\pm}K_{S(L)}$ and $B_s \rightarrow D_{\pm}\eta^{(')}, D_{\pm}\phi, ...$, where $D_+$ and $D_-$ denote the CP-even and CP-odd eigenstates of the neutral $D$-meson system, respectively, provide very efficient, theoretically clean determinations of the angle $\gamma$ of the unitarity triangle. In this new strategy, we use the $B_0^q\bar{B}_0^{\bar{q}}$ ($q \in \{d, s\}$) mixing phase $\phi_q$ as an input, and employ only “untagged” and mixing-induced CP-violating observables, which satisfy a very simple relation, allowing us to determine $\tan \gamma$. Using a plausible dynamical assumption, $\gamma$ can be fixed in an essentially unambiguous manner. The corresponding formalism can also be applied to $B_d \rightarrow D_{\pm}\pi^0, D_{\pm}\rho^0, ...$ and $B_s \rightarrow D_{\pm}K_{S(L)}$ decays. Although these modes appear less attractive for the extraction of $\gamma$, they provide interesting determinations of $\sin \phi_q$. In comparison with the conventional $B_d \rightarrow J/\psi K_{S(L)}$ and $B_s \rightarrow J/\psi \phi$ methods, these extractions do not suffer from any penguin uncertainties, and are theoretically cleaner by one order of magnitude.
1 Introduction

After the discovery of CP violation in the $B$-meson system by the BaBar (SLAC) and Belle (KEK) collaborations \cite{1}, the exploration of CP violation is now entering another exciting stage, where the central target is the unitarity triangle of the Cabibbo–Kobayashi–Maskawa (CKM) matrix, with its three angles $\alpha$, $\beta$ and $\gamma$. The goal is to overconstrain this triangle as much as possible, thereby performing a stringent test of the Kobayashi–Maskawa \cite{2} mechanism of CP violation (for a detailed review, see \cite{3}).

Using the “gold-plated” mode $B_d \to J/\psi K_S$ and similar channels, the efforts at the $B$ factories have already led to the determination of $\sin \phi_d$ with an impressive accuracy, where $\phi_d$ is the CP-violating weak $B_0^d$–$\bar{B}_0^d$ mixing phase, which equals $2\beta$ in the Standard Model. The present world average is given by $\sin \phi_d = 0.734 \pm 0.054 \pm 0.054$ \cite{4}, implying

$$\phi_d = \left(47^{+5}_{-4}\right)^\circ \vee \left(133^{+4}_{-5}\right)^\circ.$$ \hfill (1)

Here the former solution would be in perfect agreement with the “indirect” range implied by the CKM fits, $40^\circ < \phi_d < 60^\circ$ \cite{5}, whereas the latter would correspond to new physics. Measuring the sign of $\cos \phi_d$, both solutions can be distinguished. There are several strategies on the market to accomplish this important task \cite{6}. In the $B \to J/\psi K$ system, $\text{sgn}(\cos \phi_d)$ can be extracted from the time-dependent angular distribution of the decay products of $B_d \to J/\psi \to \ell^+\ell^- K^* \to \pi^0 K_S$, if the sign of a hadronic parameter $\cos \delta_f$, involving a strong phase $\delta_f$, is fixed through factorization \cite{7, 8}. This analysis is already in progress at the $B$ factories \cite{9}.

A key element in the testing of the Standard-Model description of CP violation is the determination of the angle $\gamma$ of the unitarity triangle. In this context, $B \to \pi K, \pi\pi$ modes are receiving a lot of attention (for recent reviews, see \cite{10}). The theoretical accuracy of these approaches is mainly limited by non-factorizable $SU(3)$-breaking corrections. On the other hand, there are also certain classes of pure “tree” decays, allowing – at least in principle – theoretically clean determinations of $\gamma$ (see, for instance, \cite{11}–\cite{23}). Unfortunately, the practical implementation of these approaches is challenging, and the extraction of $\gamma$ usually suffers from multiple discrete ambiguities, reducing the power to search for possible signals of new physics significantly.

The focus of the present paper is given by particularly interesting colour-suppressed neutral $B_q$-meson decays ($q \in \{d, s\}$), which receive no contributions from penguin-like topologies. In Section 2 we shall first turn to decays of the kind $B_0^d \to D_\pm K_{S(L)}$ and $B_0^s \to D_{\pm \eta(\prime)} D_{\pm \phi}, ...$, which originate from $b \to c\bar{c} s$, $c\bar{u} s$ quark-level processes, and point out that they provide very efficient, theoretically clean determinations of $\gamma$ that are essentially free from discrete ambiguities (for alternative approaches using such modes to extract weak phases, see \cite{11} \cite{18} \cite{20}). As usual, $D_+$ and $D_-$ are the CP-even and CP-odd eigenstates of the neutral $D$-meson system, respectively. As we will see in Section 3, the corresponding formalism can also be applied straightforwardly to decays of the kind $B_0^d \to D_\pm \pi^0, D_{\pm \rho(\prime)}, ...$ and $B_0^s \to D_{\pm \eta K_{S(L)}}$, which arise from $b \to c\bar{u} d, \bar{c} u d$ quark-level transitions. Since the extraction of $\gamma$ relies on certain interference effects, which are doubly Cabibbo-suppressed in these modes, they appear not as attractive as the
$B_d^0 \to D_\pm K_{S(L)}$ and $B_s^0 \to D_\pm \eta^{(')}$, $D_\pm \phi$, ... channels. However, they provide interesting extractions of $\sin \phi_q$, where $\phi_q$ is the $B_q^0 \to \bar{B}_q^0$ mixing phase. In comparison with the conventional $B_d \to J/\psi K_{S(L)}$ and $B_s \to J/\psi \phi$ methods to determine these quantities, these new strategies do not suffer from any penguin uncertainties, and are theoretically cleaner by one order of magnitude. Finally, we summarize our conclusions in Section 4.

2 $B_d \to D_\pm K_{S(L)}$ and $B_s \to D_\pm \eta^{(')}$, $D_\pm \phi$, ...

In our analysis, we shall neglect $D^0 - \bar{D}^0$ mixing and CP violation in $D$ decays, which are tiny effects in the Standard Model; should they be enhanced through new physics, they could be taken into account with the help of experimental $D$-decay studies [24]. We may then write

$$|D_\pm \rangle = \frac{1}{\sqrt{2}} \left[ |D^0 \rangle \pm e^{i\phi_{CP}(D)} |\bar{D}^0 \rangle \right],$$

where

$$(CP)|D^0 \rangle = e^{i\phi_{CP}(D)} |\bar{D}^0 \rangle, \quad (CP)|\bar{D}^0 \rangle = e^{-i\phi_{CP}(D)} |D^0 \rangle.$$  (3)

In order to simplify the following discussion, let us denote the $B_d^0 \to D_\pm K_{S(L)}$ and $B_s^0 \to D_\pm \eta^{(')}$, $D_\pm \phi$, ... decays generically by $B_q^0 \to D_\pm f_s$, where the label $s$ reminds us that we are dealing with $\bar{b} \to \bar{s}$ transitions, and $f_s$ is a CP eigenstate, satisfying

$$(CP)|f_s \rangle = \eta_{CP}^s |f_s \rangle.$$  (4)

2.1 Untagged Observables and a New Bound on $\gamma$

The most straightforward observable we may consider is the “untagged” rate

$$\langle \Gamma(B_q(t) \to D_\pm f_s) \rangle \equiv \Gamma(B_q^0(t) \to D_\pm f_s) + \Gamma(\bar{B}_q(t) \to D_\pm f_s),$$

where $\Gamma(B_q^0(t) \to D_\pm f_s)$ and $\Gamma(\bar{B}_q(t) \to D_\pm f_s)$ denote the time-dependent decay rates for initially, i.e. at time $t = 0$, present $B_q^0$ and $\bar{B}_q$ states, respectively. It takes the following form [3]:

$$\langle \Gamma(B_q(t) \to D_\pm f_s) \rangle = \left[ \Gamma(B_q^0 \to D_\pm f_s) + \Gamma(\bar{B}_q \to D_\pm f_s) \right] \times [\cosh(\Delta \Gamma_q t/2) - A_{\Delta \Gamma}(B_q \to D_\pm f_s) \sinh(\Delta \Gamma_q t/2)] e^{-\Gamma_q t},$$

where $\Delta \Gamma_q \equiv \Gamma^{(q)}_H - \Gamma^{(q)}_L$ is the decay width difference of the $B_q$ mass eigenstates $B_q^H$ ("heavy") and $B_q^L$ ("light"), and $\Gamma_q \equiv (\Gamma^{(q)}_H + \Gamma^{(q)}_L)/2$ is their average decay width. In the case of the $B_d$-meson system, the width difference is negligibly small, so that the time evolution of (6) is essentially given by the well-known exponential $e^{-\Gamma_q t}$. On the other hand, $\Delta \Gamma_s$ may be as large as $O(-10\%)$ (for a recent review, see [25]). Strategies employing $\Delta \Gamma_s$ to extract $\gamma$ were proposed in [26]. Since we do not have to rely on a sizeable value of $\Delta \Gamma_q$ and the observable $A_{\Delta \Gamma}(B_q \to D_\pm f_s)$ in the following analysis, we
shall not consider these quantities in further detail; the effects of $\Delta \Gamma_q$ could be taken into account straightforwardly.

In the new strategy to determine $\gamma$ discussed below, we apply (6) only to extract the "unevolved", i.e. time-independent, untagged rates

$$\langle \Gamma(B_q \rightarrow D_{\pm} f_s) \rangle \equiv \Gamma(B_q^0 \rightarrow D_{\pm} f_s) + \Gamma(B_q^\mp \rightarrow D_{\pm} f_s),$$

which allow us to determine the following asymmetry:

$$\Gamma_{+-}^f \equiv \frac{\langle \Gamma(B_q \rightarrow D_+ f_s) \rangle - \langle \Gamma(B_q \rightarrow D_- f_s) \rangle}{\langle \Gamma(B_q \rightarrow D_+ f_s) \rangle + \langle \Gamma(B_q \rightarrow D_- f_s) \rangle}.$$

This quantity will play a key role below. Using (2), we obtain

$$A(B_q^0 \rightarrow D_{\pm} f_s) = \frac{1}{\sqrt{2}} \left[ A(B_q^0 \rightarrow D^0 f_s) \pm e^{-i\phi_{CP}(D)} A(B_q^0 \rightarrow \overline{D^0} f_s) \right]$$

If we follow [3], and employ an appropriate low-energy effective Hamiltonian to deal with the $A(B_q \rightarrow D f_s)$ decay amplitudes, we eventually arrive at

$$\Gamma_{+-}^f = \frac{2x_f e^{i\delta_{fs}} \cos \theta_f \cos \gamma}{1 + x_f^2},$$

Here $\gamma$ is the usual angle of the unitarity triangle, and

$$x_f e^{i\delta_{fs}} \equiv R_b e^{-i\phi_{CP}(D)} \left[ \frac{\langle f_s \overline{D^0} | \mathcal{O}_s^1 C_1(\mu) + \mathcal{O}_s^2 C_2(\mu) | B_q^0 \rangle}{\langle f_s D^0 | \mathcal{O}_s^1 C_1(\mu) + \mathcal{O}_s^2 C_2(\mu) | B_q^0 \rangle} \right]$$

denotes a hadronic parameter, which involves the current–current operators

$$\mathcal{O}_s^1 = (\overline{s}_\alpha c_\beta)_{V-A} (\overline{u}_\beta b_\alpha)_{V-A}, \quad \mathcal{O}_s^2 = (\overline{s}_\alpha c_\alpha)_{V-A} (\overline{u}_\beta b_\beta)_{V-A},$$

with their Wilson coefficients $C_{1,2}(\mu)$, and the CKM factor [27]

$$R_b \equiv \left( 1 - \frac{\lambda^2}{2} \right) \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right| = 0.39 \pm 0.04,$$

where $\lambda \equiv |V_{us}| = 0.22$ is the usual Wolfenstein parameter [28].

If we apply the factorization approach to calculate [12] and perform appropriate CP transformations by taking [3] into account, we observe that both the convention-dependent phase $\phi_{CP}(D)$ and the factorized hadronic matrix elements cancel, and arrive at

$$x_f e^{i\delta_{fs}} \bigg|_{\text{fact}} = -R_b \approx -0.4.$$
Note that \( x_f e^{i\delta f_s} \) is governed by a ratio of hadronic matrix elements, which are related to each other through an interchange of all up and charm quarks, hence having a similar structure, and that \( \delta f_s \) measures the relative strong phase between them. Consequently, we expect that the deviation of \( \delta f_s \) from the trivial value of 180° is moderate, even if the individual hadronic matrix elements entering \(|\Gamma(B_q \to D_{\pm}f_s)|\) should deviate sizeably from the factorization case, as advocated in [29]. In particular, the assumption
\[
\cos \delta f_s < 0, \tag{16}
\]
which is satisfied for the whole range of 90° < \( \delta f_s < 270° \), appears very plausible.

An important advantage of the observable \( \Gamma_{+-}^{f_s} \) is that it does not depend on the overall normalization of the untagged \( \langle \Gamma(B_q \to D_{\pm}f_s) \rangle \) rates. Interestingly, already \( \Gamma_{+-}^{f_s} \) provides valuable information on \( \gamma \). Using (11), we obtain
\[
|\cos \gamma| \geq |\Gamma_{+-}^{f_s}|, \tag{17}
\]
which can easily be converted into bounds on \( \gamma \) (for alternative constraints on \( \gamma \) that could be obtained from other pure tree decays involving \( D_{\pm} \) states, see [17, 23]). Note that \( \Gamma_{+-}^{f_s} \) satisfies, by definition, the relation 0 ≤ |\( \Gamma_{+-}^{f_s} \)| ≤ 1. Moreover, we have
\[
\text{sgn}(\Gamma_{+-}^{f_s}) = \text{sgn}(\cos \delta f_s) \text{sgn}(\cos \gamma). \tag{18}
\]
Using now (16), i.e. \( \text{sgn}(\cos \delta f_s) = -1 \), we may fix the sign of \( \cos \gamma \) through the sign of \( \Gamma_{+-}^{f_s} \) as follows:
\[
\text{sgn}(\cos \gamma) = -\text{sgn}(\Gamma_{+-}^{f_s}). \tag{19}
\]
We shall come back to this interesting feature below.

### 2.2 Tagged Observables and Extraction of \( \gamma \)

The final goal is not just to constrain \( \gamma \), but to determine this angle. To this end, we use the following time-dependent rate asymmetry [3]:
\[
\frac{\Gamma(B^0_q(t) \to D_{\pm}f_s) - \Gamma(\overline{B}^0_q(t) \to D_{\pm}f_s)}{\Gamma(B^0_q(t) \to D_{\pm}f_s) + \Gamma(\overline{B}^0_q(t) \to D_{\pm}f_s)} \tag{20}
\]
\[
= \left[ A_{\text{dir}}^{\text{CP}}(B_q \to D_{\pm}f_s) \cos(\Delta M_q t) + A_{\text{mix}}^{\text{CP}}(B_q \to D_{\pm}f_s) \sin(\Delta M_q t) \right] \cosh(\Delta \Gamma_q t/2) - A_{\Delta \Gamma}(B_q \to D_{\pm}f_s) \sinh(\Delta \Gamma_q t/2),
\]
where \( \Delta M_q \equiv M_{H_q}^{(q)} - M_{L_q}^{(q)} > 0 \) denotes the mass difference between the \( B_q \) mass eigenstates. Let us note that \( A_{\Delta \Gamma}(B_q \to D_{\pm}f_s) \), which could be extracted from the untagged rate (3) in the presence of a sizeable \( \Delta \Gamma_q \), is not independent from \( A_{\text{dir}}^{\text{CP}}(B_q \to D_{\pm}f_s) \) and \( A_{\text{mix}}^{\text{CP}}(B_q \to D_{\pm}f_s) \), satisfying
\[
\left[ A_{\text{dir}}^{\text{CP}}(B_q \to D_{\pm}f_s) \right]^2 + \left[ A_{\text{mix}}^{\text{CP}}(B_q \to D_{\pm}f_s) \right]^2 + \left[ A_{\Delta \Gamma}(B_q \to D_{\pm}f_s) \right]^2 = 1. \tag{21}
\]
As discussed in [3], the “direct” CP-violating observables $A_{CP}^{\text{dir}}$ originate from interference effects between different decay amplitudes. In the case of $B_q^0 \rightarrow D_{\pm} f_s$ transitions, these are provided by the $B_q^0 \rightarrow D^0 f_s$ and $B_q^0 \rightarrow D^0 f_s$ decay paths, yielding
\[ C_{\pm}^{f_s} \equiv A_{CP}^{\text{dir}}(B_q \rightarrow D_{\pm} f_s) = \mp \left[ \frac{2 x_{f_s} \sin \delta_{f_s} \sin \gamma}{1 \pm 2 x_{f_s} \cos \delta_{f_s} \cos \gamma + x_{f_s}^2} \right]. \] (22)

It is convenient to consider the following combinations:
\[ \langle C_{f_s} \rangle_+ \equiv \frac{C_+^{f_s} + C_-^{f_s}}{2} = \frac{x_{f_s}^2 \sin 2\delta_{f_s} \sin 2\gamma}{(1 + x_{f_s}^2)^2 - (2 x_{f_s} \cos \delta_{f_s} \cos \gamma)^2} \] (23)
\[ \langle C_{f_s} \rangle_- \equiv \frac{C_+^{f_s} - C_-^{f_s}}{2} = \left[ \frac{2 x_{f_s} (1 + x_{f_s}^2) \sin \delta_{f_s} \sin \gamma}{(1 + x_{f_s}^2)^2 - (2 x_{f_s} \cos \delta_{f_s} \cos \gamma)^2} \right], \] (24)

which give
\[ - \left[ \frac{\langle C_{f_s} \rangle_+}{\langle C_{f_s} \rangle_-} \right] = \frac{2 x_{f_s} \cos \delta_{f_s} \cos \gamma}{1 + x_{f_s}^2} = \Gamma_{f_s}^{\pm} \] (25)

Consequently, (23) and (24) are not independent from $\Gamma_{f_s}^{\pm}$, and allow us to extract the same information. However, the “untagged” avenue offered by (23) is much more promising for the determination of $\Gamma_{f_s}^{\pm}$ from a practical point of view. Note that the $C_{\pm}^{f_s}$ vanish in the case of $\delta_{f_s} = 180^\circ$, since these observables are proportional to $\sin \delta_{f_s}$.

The phase $\delta_{f_s}$ can be probed nicely through the relation
\[ \tan \delta_{f_s} \tan \gamma = - \left[ 1 - \frac{(\Gamma_{f_s}^{\pm})^2}{\Gamma_{f_s}^{\pm}} \right] \langle C_{f_s} \rangle_- , \] (26)

which follows from the elimination of $x_{f_s}$ in (24) through (11). As we will see below, $\tan \gamma$ can be determined straightforwardly, thereby allowing the extraction of $\tan \delta_{f_s}$, which implies a twofold solution for $\delta_{f_s}$ itself. If we use the plausible assumption (10), we may resolve this ambiguity, and arrive at a single solution for $\delta_{f_s}$, which offers valuable insights into hadronic physics.

Let us now turn to the mixing-induced CP-violating observables, which are associated with the $\sin(\Delta M_q t)$ terms in (20), and originate from interference effects between $B_q - \overline{B}_q$ mixing and decay processes. Following the formalism discussed in [3], we obtain
\[ S_{\pm}^{f_s} \equiv A_{CP}^{\text{mix}}(B_q \rightarrow D_{\pm} f_s) = \pm \eta_{f_s} \left[ \frac{\sin \phi_q \pm 2 x_{f_s} \cos \delta_{f_s} \sin(\phi_q + \gamma) + x_{f_s}^2 \sin(\phi_q + 2\gamma)}{1 \pm 2 x_{f_s} \cos \delta_{f_s} \cos \gamma + x_{f_s}^2} \right] , \] (27)

where the factor
\[ \eta_{f_s} \equiv (-1)^L \eta_{CP}^{f_s} \] (28)
takes into account both the angular momentum $L$ of the $D_{f_s}$ state and the intrinsic CP parity $\eta_{CP}^{f_s}$ of $f_s$ (see (11)), and
\[ \phi_q \equiv 2 \arg (V_{tq}^\ast V_{tb}) \equiv \begin{cases} +2\beta = \mathcal{O}(50^\circ) & (q = d) \\ -2\lambda^2 \eta = \mathcal{O}(-2^\circ) & (q = s) \end{cases} \] (29)
is the CP-violating weak $B_q^0 - B_{ar{q}}^0$ mixing phase; the quantity $\eta$ appearing in the $B_s$ case is another Wolfenstein parameter [28]. In analogy to (23) and (24), it is convenient to consider the following combinations:

$$\langle S_{f_s} \rangle^+ \equiv \frac{S_{f_s}^+ + S_{f_s}^-}{2} = \eta_{f_s} \left[ \frac{2 x_{f_s} \cos \delta_{f_s} \sin \gamma \left\{ \cos \phi_q - x_{f_s}^2 \cos(\phi_q + 2\gamma) \right\}}{(1 + x_{f_s}^2)^2 - (2 x_{f_s} \cos \delta_{f_s} \cos \gamma)^2} \right]$$

(30)

$$\langle S_{f_s} \rangle^- \equiv \frac{S_{f_s}^+ - S_{f_s}^-}{2} = \eta_{f_s} \left[ \frac{\sin \phi_q + x_{f_s}^2 \left\{ \sin \phi_q + (1 + x_{f_s}^2) \sin(\phi_q + 2\gamma) - 4 \cos^2 \delta_{f_s} \cos \gamma \sin(\phi_q + \gamma) \right\}}{(1 + x_{f_s}^2)^2 - (2 x_{f_s} \cos \delta_{f_s} \cos \gamma)^2} \right].$$

(31)

An important advantage in comparison with $\langle C_{f_s} \rangle^+$ and $\langle C_{f_s} \rangle^-$ is that $\delta_{f_s}$ now enters only in the form of $\cos \delta_{f_s}$. Although (30) and (31) are complicated expressions, we may use (11) to derive the following, very simple final result:

$$\tan \gamma \cos \phi_q = \left[ \eta_{f_s} \langle S_{f_s} \rangle^+ \right] + [\eta_{f_s} \langle S_{f_s} \rangle^- - \sin \phi_q].$$

(32)

It should be emphasized that this relation is valid exactly. In particular, it does not rely on any assumptions related to factorization or the strong phase $\delta_{f_s}$. Interestingly, the first term in square brackets can be considered as the leading $O(1)$ term, as it is determined from a ratio of observables that are both governed by $x_{f_s} \approx 0.4$. The second term starts to contribute at $O(x_{f_s}^2)$, as can easily be seen from (31).

### 2.3 Resolving Discrete Ambiguities

The simple and completely general relation given in (32) has powerful applications. If we assume that $\phi_q$ will be known unambiguously with the help of [6]–[8] by the time the $B_q \to D_{\pm f_s}$ measurements can be performed in practice, (32) allows us to determine $\tan \gamma$ unambiguously, yielding a twofold solution, $\gamma = \gamma_1 \lor \gamma_2$, where we may choose $\gamma_1 \in [0^\circ, 180^\circ]$ and $\gamma_2 = \gamma_1 + 180^\circ$. If we assume – as is usually done – that $\gamma$ lies between $0^\circ$ and $180^\circ$, we may exclude the $\gamma_2$ solution. The range $[0^\circ, 180^\circ]$ for $\gamma$ is implied by the Standard-Model interpretation of $\varepsilon_K$, which measures the “indirect” CP violation in the neutral kaon system, if we make the very plausible assumption that a certain “bag” parameter, $B_K$, is positive. Let us note that we have also assumed implicitly in (29) that another “bag” parameter $B_{B_q}$, which is the $B_q$-meson counterpart of $B_K$, is positive as well. Indeed, all existing non-perturbative methods give positive values for these parameters. For a discussion of the very unlikely $B_K < 0$, $B_{B_q} < 0$ cases, see [30].

If we assume that only $\sin \phi_q$ has been measured through mixing-induced CP-violating effects, we may use the right-hand side of (32) to determine the sign of the product.
\[ \tan \gamma \cos \phi_q. \] As we have seen above, \( \sin \phi_d \) has already been determined with an impressive experimental accuracy due to the efforts at the \( B \) factories. Since \( \text{sgn}(\cos \gamma) = \text{sgn}(\tan \gamma) \) for \( \gamma \in [0^\circ, 180^\circ] \), we may use (19) to fix the sign of \( \tan \gamma \), and may eventually determine the sign of \( \cos \phi_q = \pm \sqrt{1 - \sin^2 \phi_q} \), thereby resolving the twofold ambiguity arising in the extraction of \( \phi_q \) from \( \sin \phi_q \). Consequently, measuring the untagged asymmetry \( \Gamma_f^{\pi \pi} \) and the mixing-induced observables \( \langle S_{f_+} \rangle_+ \) and \( \langle S_{f_+} \rangle_- \) of \( B_d \to D_{\pm K_S(L)} \) modes, we may fix \( \phi_d \) and \( \gamma \in [0^\circ, 180^\circ] \) in an unambiguous way. Following these lines, we may also distinguish between the two cases of \( \phi_d \) which emerge from a recent analysis of CP violation in \( B_d \to \pi^+\pi^- \) [31].

Since the expectation \( \gamma \in [0^\circ, 180^\circ] \) relies on the Standard-Model interpretation of \( \varepsilon_K \), it may no longer be correct in the presence of new physics. Consequently, it would be very interesting to check whether \( \gamma \) is actually smaller than \( 180^\circ \). If we assume that \( \phi_q \) is known unambiguously, [32] implies the twofold solution \( \gamma = \gamma_1 \lor \gamma_2 \), where \( \gamma_1 \in [0^\circ, 180^\circ] \) and \( \gamma_2 = \gamma_1 + 180^\circ \), as we have seen above. Since \( \cos \gamma_1 \) and \( \cos \gamma_2 \) have opposite signs, we may easily distinguish between these solutions with the help of (19).

### 2.4 Special Cases Related to \( \Gamma_f^{\pi \pi} = 0 \)

Let us, for completeness, also spend a few words on two special cases, where \( \Gamma_f^{\pi \pi} \) vanishes. The first one corresponds to \( \gamma = 90^\circ \lor 270^\circ \), yielding

\[
\eta_f \langle S_{f_+} \rangle_+|_{\gamma = 90^\circ \lor 270^\circ} = \left[ \frac{2 x_{f_\pi} \cos \delta_{f_\pi} \cos \phi_q}{1 + x_{f_\pi}^2} \right] \text{sgn}(\sin \gamma). \quad (33)
\]

Consequently, if we employ again (16) and assume that the sign of \( \cos \phi_q \) will be known by the time the \( B_q \to D_{\pm f_\pi} \) measurements can be performed in practice, this observable allows us to determine the sign of \( \sin \gamma \), thereby distinguishing between \( \gamma = 90^\circ \) and \( 270^\circ \). Concerning the extraction of \( \delta_{f_\pi} \) (see (26)), we may now use the ratio of

\[
\langle C_{f_\pi} \rangle_+|_{\gamma = 90^\circ \lor 270^\circ} = -\left[ \frac{2 x_{f_\pi} \sin \delta_{f_\pi}}{1 + x_{f_\pi}^2} \right] \text{sgn}(\sin \gamma) \quad (34)
\]

and (33) to determine \(- \tan \delta_{f_\pi} / \cos \phi_q \). In the unlikely case of \( \delta_{f_\pi} = 90^\circ \lor 270^\circ \), which is the second possibility yielding \( \Gamma_f^{\pi \pi} = 0 \), \( \langle S_{f_\pi} \rangle_+ \) would vanish as well. However, we could then employ

\[
\eta_f \langle S_{f_\pi} \rangle_-|_{\delta_{f_\pi} = 90^\circ \lor 270^\circ} = \sin \phi_q + \frac{2 x_{f_\pi}^2}{1 + x_{f_\pi}^2} \sin \gamma \cos(\phi_q + \gamma) \quad (35)
\]

and

\[
\langle C_{f_\pi} \rangle_-|_{\delta_{f_\pi} = 90^\circ \lor 270^\circ} = -\left[ \frac{2 x_{f_\pi} \sin \gamma}{1 + x_{f_\pi}^2} \right] \text{sgn}(\sin \delta_{f_\pi}) \quad (36)
\]

to determine \( \gamma \) and \( x_{f_\pi} \). Unfortunately, we would then have to struggle with discrete ambiguities, and would have to rely on the experimental resolution of terms entering at the \( x_{f_\pi}^2 \) level. Note that the value of \( x_{f_\pi} \) could differ dramatically from \( R_b \approx 0.4 \) in the case of \( \delta_{f_\pi} = 90^\circ \lor 270^\circ \).
2.5 Remarks on $B_s \to D_\pm \eta^{(')}$, $D_\pm \phi$, ... 

Considering decays of the kind $B_s \to D_\pm \eta^{(')}$, $D_\pm \phi$, ..., we may implement the strategy discussed above in the $B_s$-meson system. These transitions are not accessible at the asymmetric $e^+e^-$ $B$ factories operating at the $\Upsilon(4S)$ resonance, i.e. they cannot be measured by the BaBar and Belle collaborations, but may be studied at hadronic $B$ experiments, in particular at LHCb (CERN) and BTeV (Fermilab). Since $\phi_s$ is negligibly small in the Standard Model, yielding $\sin \phi_s = 0$ and $\cos \phi_s = +1$, (32) simplifies even further in this interesting case:

$$\tan \gamma|_{B_s}^{\text{SM}} = \frac{\eta_{f_s} \langle S_{f_s} \rangle_+}{\Gamma_{f_s}^{+,-}} + \eta_{f_s} \langle S_{f_s} \rangle_-.$$  \hspace{1cm} (37)

As is well known, $\phi_s$ is a sensitive probe for new-physics contributions to $B^0_s - \overline{B}^0_s$ mixing, which may lead to a sizeable value of $\phi_s$. In addition to discrepancies between the values for $\gamma$ extracted from (37) on the one hand, and its $B_d$ counterpart on the other hand, such scenarios may also be probed through the search for CP violation in $B_s \to J/\psi \phi$ modes. If we use these channels (or those proposed below) to fix $\phi_s$, we may extract the “true” value of $\gamma$, and may resolve the discrete ambiguities as discussed above.

In this game, it is also useful to combine information provided by the $B_d$ and $B_s$ systems. For example, an interesting case arises if actually no CP violation should be found in $B_s \to J/\psi \phi$, thereby indicating $\sin \phi_s = 0$. Then the question arises whether $\cos \phi_s$ is positive, as in the Standard Model, or negative, as in the case of new physics. If we determine $\tan \gamma$ unambiguously through $B_d \to D_\pm K_{S(L)}$ modes as discussed above, we may use, for instance, $B_s \to D_\pm \phi$ to fix $\cos \phi_s$ with the help of (32); for the extraction of the sign of $\cos \phi_s$, it is of course sufficient to use only information on the sign of $\tan \gamma$ and the right-hand side of (32) (alternative approaches to determine $\phi_s$ unambiguously can be found in [8]).

2.6 Numerical Examples

In order to illustrate the strategies proposed in this paper in further detail, we consider $x_{f_s} = 0.4$ and $\delta_{f_s} = 180^\circ$, which corresponds to (15), and give in Table 1 the results for $\Gamma_{f_s}^{+,-}$ and the relevant mixing-induced observables for various values of $\phi_q$ and $\gamma$. Since the strong phase $\delta_{f_s}$ enters in these observables only through $\cos \delta_{f_s}$, they are rather robust with respect to deviations of $\delta_{f_s}$ from $180^\circ$. Playing with the numerical examples given in Table 1, it is an easy exercise to see how the strategy proposed above and the resolution of the discrete ambiguities work in practice. Moreover, it is interesting to observe that the first term in square brackets on the right-hand side of (32) is actually the leading one, and that the second term, which arises at the $x_{f_s}^2$ level, plays only a minor rôle. Let us also note that (17) implies $-70^\circ \leq \gamma \leq 70^\circ$ and $110^\circ \leq \gamma \leq 250^\circ$ for $\Gamma_{f_s}^{+,-} = -0.345$ and $+0.345$, respectively, where we have also taken (19) into account.

\hspace{8cm} 8
Table 1: The relevant observables in the case of \( x_{f_d} = 0.4 \) and \( \delta_{f_s} = 180^\circ \) for various values of \( (\phi_q, \gamma) \). The columns “1st term” and “2nd term” refer to the 1st and 2nd terms in square brackets on the right-hand side of (32), as discussed in the text.

| \( (\phi_q, \gamma) \) | \( \Gamma_{+}^{J_{+}} \) | \( \eta_{f_d} S_{J_{+}}^{f_d} \) | \( \eta_{f_s} S_{J_{+}}^{f_s} \) | \( \eta_{f_d} \langle S_{f_d} \rangle_{+} \) | \( \eta_{f_s} \langle S_{f_s} \rangle_{-} \) | 1st term | 2nd term |
|-----------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| \( (47^\circ, 60^\circ) \) | -0.345 | +0.003 | -0.982 | -0.490 | +0.493 | +1.420 | -0.239 |
| \( (133^\circ, 120^\circ) \) | +0.345 | +0.982 | -0.003 | +0.490 | +0.493 | +1.420 | -0.239 |
| \( (47^\circ, 240^\circ) \) | +0.345 | +0.982 | -0.003 | +0.490 | +0.493 | +1.420 | -0.239 |
| \( (0^\circ, 60^\circ) \) | -0.345 | -0.729 | -0.533 | -0.631 | -0.098 | +1.830 | -0.098 |
| \( (180^\circ, 60^\circ) \) | -0.345 | +0.729 | +0.533 | +0.631 | +0.098 | -1.830 | +0.098 |
| \( (0^\circ, 240^\circ) \) | +0.345 | +0.533 | +0.729 | +0.631 | -0.098 | +1.830 | -0.098 |

3 \( B_d \rightarrow D_{\pm} \pi^0, D_{\pm} \rho^0, \ldots \) and \( B_s \rightarrow D_{\pm} K_{S(L)} \)

Making straightforward replacements of variables, the formalism developed above can also be applied to decays of the kind \( B_d^{0} \rightarrow D_{\pm} \pi^0, D_{\pm} \rho^0, \ldots \) and \( B_s^{0} \rightarrow D_{\pm} K_{S(L)} \), which originate from \( \bar{b} \rightarrow \bar{u}d \bar{c} \), \( \bar{u}d \bar{d} \) quark-level transitions. To this end, we have only to substitute \( x_{f_d} e^{i\delta_{f_d}} \) through

\[
x_{f_d} e^{i\delta_{f_d}} = - \left( \frac{\lambda^2 R_b}{1 - \lambda^2} \right) e^{-i\phi_{CP}(D)} \frac{\langle f_d \bar{D}^0 | O_4^{d} C_1^d(\mu) + O_2^{d} C_2^d(\mu) | B_q^0 \rangle}{\langle f_d D^0 | O_4^{f_d} C_1^{f_d}(\mu) + O_2^{f_d} C_2^{f_d}(\mu) | B_q^0 \rangle},
\]

yielding

\[
x_{f_d} e^{i\delta_{f_d}} \bigg|_{\text{fact}} = - \left( \frac{\lambda^2 R_b}{1 - \lambda^2} \right) \approx 0.02.
\]

Note that the tiny value of \( x_{f_d} \) in (39) reflects that the interference effects between the \( \bar{B}_q^0 \rightarrow \bar{D}^0 f_d \) and \( B_q^0 \rightarrow D^0 f_d \) decay paths are doubly Cabibbo-suppressed, in contrast to the \( B_q \rightarrow D f_s \) case. Consequently, since \( \Gamma_{+}^{f_d} \) and \( \langle S_{f_d} \rangle_{+} \) are proportional to \( x_{f_d} \), these observables are now strongly suppressed, taking values of at most a few per cent, and the strategy to determine \( \gamma \) proposed above does not appear to be very attractive in this case. However, \( \langle S_{f_d} \rangle_{-} \) provides an interesting extraction of \( \sin \phi_q \) through

\[
\eta_{f_d} \langle S_{f_d} \rangle_{-} = \sin \phi_q + \mathcal{O}(x_{f_d}^2) = \sin \phi_q + \mathcal{O}(4 \times 10^{-4}).
\]

Note that the individual mixing-induced CP asymmetries \( S_{J_{+}}^{f_d} \) and \( S_{J_{-}}^{f_d} \) are affected by terms of \( \mathcal{O}(x_{f_d}) = \mathcal{O}(2 \times 10^{-2}) \), as can be seen in (27). In the conventional strategies to determine \( \sin \phi_d \) and \( \sin \phi_s \) through \( B_d \rightarrow J/\psi K_{S(L)} \) and \( B_s \rightarrow J/\psi \phi \), respectively, unknown penguin contributions limit the theoretical accuracy of the extracted value of \( \sin \phi_q \) through terms of \( \mathcal{O}(R_b \lambda^3) = \mathcal{O}(4 \times 10^{-3}) \). Consequently, the simple determinations of this quantity with the help of (40) are theoretically cleaner by one order of magnitude. For approaches to control the penguin uncertainties in \( B_d \rightarrow J/\psi K_{S(L)} \)
and $B_s \to J/\psi \phi$ through flavour-symmetry arguments, see [34]. These issues may become important in the era of the LHC.

 Needless to note, it will be very challenging to achieve such a tremendous accuracy in practice. To this end, we must also keep an eye on CP violation in $B^0_q - \bar{B}^0_q$ mixing, which enters at the $10^{-4}$ level in the Standard Model and can be probed through “wrong-charge” lepton asymmetries [3]; these effects may be enhanced sizeably through new physics. As we have noted above, a similar comment applies to $D^0 - \bar{D}^0$ mixing and CP violation in $D$ decays. Moreover, also indirect CP violation in the neutral kaon system has to be taken into account if $K_S$ or $K_L$ are involved. However, these effects do not lead to serious problems and can be included through the corresponding experimental data.

The strategy provided by (40) is particularly interesting for the $B_s$-meson case. If \( \phi_s = \phi_s^{SM} = -2\lambda^2 \eta \approx -3 \times 10^{-2} \), we may determine $\phi_s$ with the help of $B_s \to D_{\pm}K_{S(L)}$ modes with a theoretical accuracy of $\mathcal{O}(1\%)$. On the other hand, as emphasized in [32], using the time-dependent angular distribution of the $B_s \to J/\psi \phi$ decay products, the theoretical accuracy of $\phi_s$ is limited by penguin contributions to $\mathcal{O}(10\%)$. An accurate determination of $\phi_s$ is very important, since it is essentially given by the Wolfenstein parameter $\eta$ within the Standard Model, measuring the height of the unitarity triangle. An alternative approach to determine this parameter is provided by the rare kaon decay $K_L \to \pi^0 \nu \bar{\nu}$ [34, 27], which would offer an interesting consistency check.

Let us finally note that the $B_d^0 \to D^0 \pi^0$ mode has already been observed by the Belle, CLEO, and BaBar collaborations, with branching ratios at the $3 \times 10^{-4}$ level [35]. Rescaling them by $\lambda^2$, we obtain the estimate $\text{BR}(B_d^0 \to D^0 K_S) = \mathcal{O}(10^{-5})$. Interestingly, the $B_d^0 \to D^0 K_S$ channel has very recently been observed for the first time by the Belle collaboration, with the branching ratio $(5.0^{+1.3}_{-1.2} \pm 0.6) \times 10^{-5}$ [36].

4 Conclusions

In summary, we have shown that $B_d \to D_{\pm}K_{S(L)}$ and $B_s \to D_{\pm} \eta^{(*)}, D_{\pm} \phi, \ldots$ modes provide a very efficient strategy to determine $\gamma$ in a theoretically clean and essentially unambiguous manner. Here we mean by “efficient” that only the “untagged” asymmetry $\Gamma_{\pm}^{f_s}$ and the mixing-induced CP-violating observables $\langle S_{f_s} \rangle_+$ and $\langle S_{f_s} \rangle_-$ of these colour-suppressed decays have to be measured, which satisfy the simple, exact relation (32), allowing us to extract $\tan \gamma$. Interestingly, first information about $\gamma$ can already be obtained from $\Gamma_{\pm}^{f_s}$ with the help of $|\cos \gamma| \geq |\Gamma_{\pm}^{f_s}|$, which can straightforwardly be converted into bounds on $\gamma$.

It is useful to compare this new strategy with other approaches to extract weak phases from decays of the kind $B_d \to DK_S$. In [11], it was pointed out that these modes allow a clean determination of $\sin \phi_d \overset{\text{SM}}{=} \sin 2\beta$ and $\sin(\phi_d + 2\gamma) \overset{\text{SM}}{=} -\sin 2\alpha$, leaving fourfold discrete ambiguities for the values of $\beta$ and $\alpha$. To this end, time-dependent rate measurements of $B_d \to D^0 K_S$ and $B_d \to \bar{D}^0 K_S$ processes are complemented with a complex triangle construction involving the $B_d \to D_{\pm}K_S$ channel. In [18], it was then shown that this method is affected by subtle interference effects between $D^0 \to \pi^+ K^-$
and $\bar{D}^0 \rightarrow \pi^+ K^-$, which are Cabibbo-favoured and doubly Cabibbo-suppressed decay processes, respectively. The approach proposed in [15] to extract $\beta$ and $\gamma$ from time-dependent $B_d \rightarrow D^0 K_S$ and $B_d \rightarrow \bar{D}^0 K_S$ measurements ($B_d \rightarrow D^\pm K_S$ is not used here), taking into account the $D$-decay interference effects, can no longer be implemented in an analytical manner, and leaves a 16-fold discrete ambiguity for the extraction of $\beta$, $\gamma$ and two strong phases. The essentially unambiguous extraction of $\gamma$ through decays of the kind $B_d \rightarrow D^\pm K_S$ proposed above is obviously much simpler and in this sense more efficient. Moreover, it is of advantage not to employ the Standard-Model expression $\phi_d = 2\beta$, since this phase may well be affected by new-physics contributions to $B^0_d - \bar{B}^0_d$ mixing, and can straightforwardly be determined separately.

The formalism developed for $B_d \rightarrow D^\pm K_{S(L)}$ and $B_s \rightarrow D^\pm \eta^\prime, D^\pm \phi, ...$ modes can also be applied to decays of the kind $B_d \rightarrow D^\pm \pi^0, D^\pm \rho^0, ...$ and $B_s \rightarrow D^\pm K_{S(L)}$, where the interference effects between the $\bar{D}^0_q \rightarrow D^0 f_q$ and $\bar{D}^0_{q'} \rightarrow D^0 f_{q'}$ decay paths are doubly Cabibbo-suppressed, thereby reducing the prospects of these modes to determine $\gamma$. However, these channels allow interesting determinations of $\sin \phi_q$. In comparison with the conventional methods provided by $B_d \rightarrow J/\psi K_{S(L)}$ and $B_s \rightarrow J/\psi \phi$, these extractions are theoretically cleaner by one order of magnitude. A more comprehensive analysis of $B_q \rightarrow D f_{s,a}$ modes is given in [37]. There we also have a closer look at the interesting case where the neutral $D$ mesons are observed through decays into CP non-eigenstates. We look forward to detailed experimental studies of the corresponding decays.

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