Research Article

A Supermodular Game Framework for Power Control of Wireless Sensor Networks

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We consider a distributed power control scheme for wireless sensor networks. To derive decentralized solutions that do not require complete information and any cooperation among the users, we formulate this problem as a supermodular game, which each user maximizes its utility function provided transmission rate constraints. Through analyzing the supermodular property of the game, the existence and uniqueness of the Nash equilibrium (NE) are established. Furthermore, we propose a distributed price and power update algorithm (DPPA) to compute the solution of the game which is based on myopic best response. Performance evaluations via numerical simulations verify the existence of the NE and the convergence property of the DPPA algorithm.

1. Introduction

Wireless sensor networks (WSN) have broad application prospects, that is, widely used in health care, military, environmental monitoring, and forecasting, as well as intelligent home, building condition monitoring, and so on. The power control problem with interference management, throughput maximization, or energy efficient target, is an interesting issue and attracts tremendous attention [1–6]. Since different power levels result in a different performance and fundamentally affect many aspects of the operation of the networks, the power control problem becomes a complex and intriguing problem. Since the sensor networks are totally distributed and have no fundamental infrastructures, the traditional centralized power control mechanisms are not feasible. It is well known that game theory is a good tool to handle the distributed problems. Game theory techniques have widely been applied to engineering problems in which the action of one component has impact on and perhaps conflicts with that of any other component [7]. More particularly, supermodular games are interesting since they have several desirable properties, such as they encompass many applied models, have the remarkable property that many solution concepts yield the same predictions and have nice comparative statics properties and behave well under various learning rules [8].

In this paper, we study the power control problem, focus on distributed algorithms with no centralized control using a supermodular game framework, and investigate if any optimality is achievable. Our work is motivated by the following points.

(i) The networks have no central infrastructures; a distributed power control mechanism is desired.

(ii) The game theoretic formulation is a useful tool to design distributed algorithms. And it is amenable to prove the convergence for the distributed algorithm.

(iii) Supermodular game model has several desirable properties.

To formulate the power control design, we take the rate constraints into account and propose a strategic game, where all rational users maximize their own utilities by choosing appropriate power levels. We refer this game to distributed power control game (DPCG). By building the supermodular property of the DPCG, the existence of the Nash equilibrium solution is established. Furthermore, we
present an analysis for users’ myopic best response (MBR) dynamics. Based on the MBR dynamics, a distributed price and power update algorithm (DPPA) is proposed. Then we prove the convergence of the algorithm and show the uniqueness of the NE.

The rest of this paper is organized as follows. In Section 2, the related works are discussed. We depict the system model in Section 3. In Section 4, the proof of the existence of the NE is provided. The proposed pricing mechanism and the distributed convergence algorithm are presented in Section 5. The simulation results are given in Section 6. Finally, Section 7 draws the conclusions.

2. Related Work

There has been a rich literature to study power control in wireless networks. In [9], the authors proposed a joint power and channel resource allocation iterative optimization algorithm. References [1–6] illustrate power allocation problems using traditional game theory and take different factors into account. Evolutionary game theory was used to depict the power problem in [10, 11]. In particular [10] established an evolutionary power mechanisms for W-CDMA and WIM wireless systems. With the analytical limitations of evolutionary game, we can just analyze low-dimension strategy space. Shamik et al. proposed a game theoretic framework for power control in wireless sensor network. They showed the performance differences between continuous levels and discrete levels and gave a framework to find the best transmit power span. However, they did not show the convergence of the power control mechanism. In [12], they used potential game theory and replicator dynamics learning scheme to analyze the distributed power allocation problem in parallel multiple-access channels, and they depicted the sufficient conditions for the unique NE and showed the convergence property. The interaction of several radio devices aiming to obtain wireless connectivity by using a set of base stations was modeled as a noncooperative game in [13]. They showed the existence of the Nash equilibrium of two situations, that is, BS selection and BS sharing. The authors of [14] considered the power control for open-loop overlaid network MIMO systems in a game theoretical perspective. They viewed the problem as a noncooperative game, and the numerical simulations verified the significant performance advantages of the proposed scheme.

3. System Model

3.1. Link Capacity. The links between nodes are modeled by a set \( i = \{1, 2, \ldots, L\} \). The channels are assumed to be additive white Gaussian noise (AWGN) channel with noise power spectral density \( N_0 \) over the bandwidth of operation \( B \). The channel gain from link \( i \)'s transmitter to the link \( j \)'s receiver is denoted by \( G_{ij} \). The interference from link \( i \)'s transmitter to link \( j \)'s receiver is denoted as \( G_{ij}P_i \). Then, the total interference and noise power at the link \( j \)'s receiver is as follows:

\[
t_j = \sum_{i \neq j} G_{ij}P_i + N_0B, \tag{1}
\]

Therefore, the instantaneous signal to interference plus noise ratio (SINR) of node \( j \) is

\[
\gamma_j = \frac{G_{jj}P_j}{t_j}. \tag{2}
\]

Denote the link capacity of link \( j \) by \( R_j \); we can get the link capacity that can be supported over link \( j \) as follows:

\[
R_j = B \log \left( 1 + \gamma_j \right). \tag{3}
\]

To guarantee the communication quality, we assume that

\[
R_j \geq R_j^*, \tag{4}
\]

where \( R_j^* \) is the rate constraint of link \( j \), that is, the minimum transmission rate required by each link.

3.2. Distributed Power Control Game. The power control game is modeled as a strategic game, in which the players are the links and the payoff is the difference between their transmission gain and the power consumption cost. To maximize its own utility, each player chooses transmission power with a given constraint on the minimum achievable information rate to compete against others. The strategy of each player is one-dimension subset of \( R \). The strategy of player \( i \) is denoted by \( p_i \), and the strategy of player \( i \)'s opponents is defined as \( p_{-i} = (p_{i_1}, \ldots, p_{i_{-i}}, p_{i_{+i}}, \ldots, p_{i_L}) \). Then the game has the following structure:

\[
\theta = \{i, \{P_i\}_{i \in i^c}, \{ti(p_i)\}_{i \in i^c}\}, \tag{5}
\]

where \( i = \{1, 2, \ldots, L\} \) is the set of links and \( p_i \) is the power allocation strategy of player \( i \), which is defined as

\[
p_i = \{p_i \in P_i : R_i(p_i, p_{-i}) \geq R_i^*\}, \tag{6}
\]

where \( R_i(p_i, p_{-i}) = B \log(1 + (G_{ii}P_i/i)) \). As defined above, \( R_i^* \) is the transmission rate constraint. We assume that \( R_i^* \geq B \) for all links. \( P_i \) denotes the action set for player \( i \) as follows

\[
P_i = [P_i_{\min}, P_i_{\max}], \tag{7}
\]

where \( P_i_{\min} = 0 \) means that user \( i \) decides not to transmit. We denote the vector \( R^* = (R_i^*)_{i \in i^c} \) by the rate constraint profile. The payoff function of user \( i \) is defined as follows:

\[
u_i(p_i) = B \log(\gamma_i) - c_i \gamma_i, \tag{8}
\]

where \( c_i \) is the pricing parameter for user \( i \). We use \( \log(\gamma_i) \) to replace \( \log(\gamma_i + 1) \) due to the following reasons: in the high SINR regime, logarithmic utility approximates the Shannon capacity \( B \log(1 + \gamma_i) \) and for low SINR, a user's rate is approximately linear in SINR, and so this utility is proportional to the logarithm of the rate. Each user's feasible strategies depend not only on their own action, but also on their opponents'.

Given the power allocation of the others \( p_{-i} \), the optimal strategy for the \( i \)th user is the solution of the following maximization problem:

\[
\max_{\tilde{p}_i} u_i(\tilde{p}_i, p_{-i}), \quad i \in i, \quad \text{subject to } \tilde{p}_i \in P_i. \tag{9}
\]
We refer to the game \( \vartheta \) with maximization problem (9) by distributed power control game (DPCG). The strategy space of the game is \( P = P_1 \times \cdots \times P_2 \).

Now we define the Nash equilibrium of this game. An NE of the DPCG game \( \vartheta \) is defined as follows.

**Definition 1.** Consider an \( L \)-player game, each player maximizing its individual cost function \( u_i : P_i \rightarrow R_+ \), subject to coupled inequality constraint \( R^*_i - R_i(p_i, p_{-i}) \leq 0 \), a vector \( p^* = (p^*_1, p^*_2, \ldots, p^*_L) \) is called an NE solution of DPCG \( \vartheta \) if every given \( p^*_i \) is as follows:

\[
  u_i(p^*_i) \geq u_i(p_i), \quad \forall p_i \in P_i, \forall i \in I. \tag{10}
\]

In the following sections, we will discuss whether the NE solution exists. If it exists, is it unique and how to reach it in a totally distributed way?

### 4. Existence of the NE

Herein, we prove that the DPCG game has the NE solution by establishing the supermodular property for it. By verifying the Hessian matrix of the game, we prove the uniqueness of the NE. Following [8], we have the following.

**Definition 2 (supermodular game).** The strategic form game \( \langle I; S; (u_i) \rangle \) is a supermodular game if for all \( i \) one has the following:

1. \( S_i \) is a compact subset of \( R \);
2. \( u_i \) is upper semicontinuous in \( s_i \), continuous in \( s_{-i} \);
3. \( u_i \) has increasing differences in \( (s_i; s_{-i}) \).

With the above definition, we can get the following.

**Theorem 3.** The game \( \vartheta \) defined by (5)-(9) is supermodular and admits a compact sublattice NE solution.

**Proof.** Apparently, the strategy set \( P_i \) of user \( i \) is a sublattice of \( R \); that is, for all \( p_i, p'_i \in P_i \), max\( (p_i, p'_i) \) \( \in P_i \) and min\( (p_i, p'_i) \) \( \in P_i \). For the game DPCG's utility,

\[
  u_i(p_i) = B \log \left( \sum_{j \neq i} G_{ij} p_j + N_i B \right) - c_i p_i. \tag{11}
\]

Let \( f(y_i) = B \log(y_i) \); then we have

\[
  \frac{\gamma_j f''(y_i)}{f'(y_i)} = 1 \geq 1. \tag{12}
\]

For all \( j \neq i \),

\[
  \frac{\partial^2 u_i}{\partial p_i \partial p_j} (p_i, p_{-i}) = -\frac{\gamma_j G_{ii} p_i f''(y_i)}{p_i^2 G_{ii}} \left[ y_j f''(y_i) + f'(y_i) \right]. \tag{13}
\]

Obviously, \( (\partial^2 u_i/\partial p_i \partial p_j)(p_i, p_{-i}) \geq 0 \). It indicates that the utility function of the DPCG meets increasing differences. According to Definition 2, we conclude that the DPCG is a supermodular game. Based on Theorem 1 in [15], the set of the NEs of game \( \vartheta \) is a nonempty and compact sublattice, and it admits an NE solution.

### 5. Distributed Convergence Algorithm and Uniqueness of the NE

#### 5.1. Pricing Mechanism

In the above section, we did not depict the pricing parameter precisely. Herein, we define the price charged to other users for generating interference to user \( i \) as follows:

\[
  \pi_i(p_i, p_{-i}) = -\frac{\partial u_i(p_i, p_{-i})}{\partial p_i}, \tag{14}
\]

where \( I_i = \sum_{j \neq i} p_j G_{ji} \) is the total interference that is created by opponents received by user \( i \). Obviously, \( \pi_i(p_i, p_{-i}) \) is always nonnegative and represents user \( i \)'s marginal increase in utility per unit and decrease in total opponents' interference. User \( i \) therefore maximizes the difference between its gain minus its cost to the other users in the network due to the interference it creates. The cost is its transmit power times a weighted sum of other users' prices, where the weights equal the channel gains between user \( i \)'s transmitter and the other links' receivers. Then the maximization problem becomes that each user \( i \) specifies a power level \( p_i \in P_i \) to maximize the following utility function:

\[
  u_i(p_i, p_{-i}) = B \log(y_i) - p_i \sum_{j \neq i} \pi_j G_{ij}. \tag{15}
\]

#### 5.2. Distributed Update Algorithm

In Section 4, the DPCG game has been shown to admit an NE. Now we focus on distributed algorithm to compute the NE solutions. In particular, we employ asynchronous myopic best response (MBR) update rules; that is, the users update their strategies according to their best response assuming that other player's strategies are fixed. Given \( p_{-i} \) at the user \( i \)'s update time epoch, we can express the MBR updates as follows:

\[
  \bar{p}_i = \arg \max_{p_i \in P_i} u_i(p_i) = \arg \max_{p_i \in P_i} \left[ B \log \left( \frac{G_{ii} p_i}{I_i} \right) - p_i \sum_{j \neq i} \pi_j G_{ij} \right]. \tag{16}
\]

**Theorem 4.** As the other player's strategies are fixed, the users' MBR is single-valued. Moreover, the user's MBR dynamics can be given as follows:

\[
  \bar{p}^{(t+1)}_i = \max \left[ \min \left[ \frac{e^{R_i / B}}{G_{ii} I_i}, p_{i_{\text{max}}}, p_{i_{\text{min}}} \right] \right]. \tag{17}
\]

**Proof.** The MBR updates (16) are as follows:

\[
  \max_{p_i \in P_i} \left[ B \log \left( \frac{G_{ii} p_i}{I_i} \right) - p_i \sum_{j \neq i} \pi_j G_{ij} \right], \tag{18}
\]

subject to

\[
  p_{i_{\text{min}}} \leq p_i \leq p_{i_{\text{max}}}, \tag{19}
\]

\[
  g(p_i) = R^* - B \log \left( 1 + \frac{G_{ii} p_i}{I_i} \right) \leq 0.
\]
The Lagrangian function associated with problem (18) can be given as follows:
\[
\Psi(\lambda_i, p_i) = \log\left(\frac{G_{ii}p_i}{t_i}\right) - p_i\sum_{j \neq i} \frac{G_{ij}p_j}{t_i} - \lambda_{i1}(p_i - p_{i,\text{min}}) - \lambda_{i2}(p_i - p_{i,\text{max}}) - \lambda_{i3}\left(R^* - B \log\left(\frac{1 + \frac{G_{ii}p_i}{t_i}}{N_0B}\right)\right),
\]
where \(\lambda_{i1}, \lambda_{i2}, \lambda_{i3}\) are the Lagrange multipliers on link \(i\). The Karush-Kuhn-Tucker (KKT) conditions for user \(i\) are given by
\[
\begin{align*}
\frac{B}{p_i} - \sum_{j \neq i} G_{ij}\frac{p_j}{p_i} - \lambda_{i1} - \lambda_{i2} + \frac{B}{p_i}\lambda_{i3} &= 0, \\
\lambda_{i1}(p_i - p_{i,\text{min}}) &= 0, \\
\lambda_{i2}(p_i - p_{i,\text{max}}) &= 0, \\
\lambda_{i3}\left(R^* - B \log\left(\frac{1 + \frac{G_{ii}p_i}{t_i}}{N_0B}\right)\right) &= 0.
\end{align*}
\]
Consider these two situations:

1. \(\lambda_{i1} = 0, \lambda_{i2} = 0, \lambda_{i3} = 0, \sum_{j \neq i} G_{ij}\frac{p_j}{p_i} = \frac{B}{\sum_{j \neq i} \pi_j G_{ij}}\)
2. \(\lambda_{i1} = 0, \lambda_{i2} = 0, \lambda_{i3} = 1, p_i^{(t+1)} = e^{R^*/B}G_{ii}\).

In situation 1, all the constraints are not active and we cannot get a desirable solution. It is necessary to let the constraints to be active, and thus we can easily get the solution of problem (18) as follows:
\[
\begin{align*}
\lambda_{i1} &= 0, \\
\lambda_{i2} &= 0, \\
\lambda_{i3} &= 1,
\end{align*}
\]
Based on the above analysis, the MBR dynamics can be simplified as (17), and thus the proof is completed.

From these expressions, we can conclude that to implement the update process, each user only needs to know the following information:

(i) its own utility \(u_i\), the current SINR \(y_i\) (this can be obtained over observations, that is, the SINR before current update time epoch), and its own channel gain \(G_{ii}\);
(ii) the channel gains \(G_{ij}\) for \(j \in L\) and \(j \neq i\);
(iii) the price profile \(\pi\).

These pieces of information are not difficult to get due to the SINR \(y_i\) and channel game \(G_{ij}\) can be measured at the receiver and fed back to the transmitter. Also measuring the adjacent channel gains \(G_{ij}\) can be accomplished by letting each receiver periodically broadcast a control beacon as we assumed channel reciprocity. The price information can also be broadcast through this control beacon.

Based on Theorem 4, the detailed implementation of the asynchronous distributed power control algorithm can be shown in Algorithm 1.

Note that the power and price need not to update simultaneously. And for each user, the two update processed also need not to be at the same time. We referes to this distributed power control algorithm as DPPA.

5.3. Uniqueness of the NE

**Theorem 5.** The power control game DPCG \(G\) has a unique Nash equilibrium.

**Proof.** The Hessian matrix of each user, \(H(p) = \nabla u(p)\), consists of diagonal elements as follows:
\[
H_{ii}(p) = \frac{1}{p_i^2},
\]
for all \(i \in L\), and off-diagonal elements as follows:
\[
H_{ij}(p) = 0
\]
for all \(j \neq i\). Then, it is easy to verify that \(H(p)\) is positive definite as desired. It follows that it has a unique global optimum, which is the only solution to the KKT conditions.

5.4. Convergence of the Distributed Algorithm. We characterize the convergence of the distributed algorithm in this section. We consider it in a game theoretic framework. Each user \(i\) specifies a power \(p_i\) and a price \(\pi_i\) to maximize its utility function (15). It is easy to notice that the best response for each user is to choose a large enough price to force all other users transmit at \(p_{\text{min}}\) since there is no penalty for users to announce a high price. This is not a desirable result from the system's perspective. To improve this situation, we consider an externally procedure to determine the price parameter. Let each user split to two fictitious players, and we consider the following External Power-Price (EPP) control game as follows:
\[
G_{EPP} = \left[\text{EW} \cup \text{EC}, \left\{p_i^{\text{EW}}, p_i^{\text{EC}}\right\}, \left\{s_i^{\text{EW}}, s_i^{\text{EC}}\right\}\right],
\]
where the players are from the union of set $EW$ and set $EC$, which are both copies of $L$. $EW$ is a fictitious power player set; each player $i \in EW$ chooses a power $p_i$ from the strategy set $P_i^{EW} = P_i$ and receives the following payoff:

$$ u_i^{EW}(p_i; p_{-i}, \pi_{-i}) = \log (\gamma_i) - p_i \sum_{j \neq i} \pi_j G_{ij}. $$

(26)

EC is a fictitious price player set; each player $i \in EC$ chooses a price $\pi_i$ from the strategy set $P_i^{EC} = [0, \pi_i]$ and receives the following payoff:

$$ u_i^{EC}(\pi_i; p) = -(\pi_i - C_i(p))^2. $$

(27)

Here, $\pi_i = \sup_j C_i(p)$, which could be infinite for some utility functions. The players in the $G_{EPP}$ are selfish and maximize their own payoff function.

In $G_{EPP}$ the players’ best responses are given by $B_i^{EW}(p_{-i}, \pi_{-i}) = W_i(p_{-i}, \pi_i)$ for $i \in EW$ and by $B_i^{EC}(p) = C_i(p)$ for $i \in EC$, where $W_i$ and $C_i$ are the update rules for the distributed algorithm. In other words, the distributed algorithm can be interpreted as if the players in $G_{EPP}$ employ asynchronous myopic best response (MBR) updates; that is, the players update their strategies according to their best responses to the given other players’ strategies. It is known that the set of fixed points of MBR updates is the same as the set of the NEs of a game.

**Proposition 6.** $G_{EPP}$ is supermodular in the transformed strategies $(p, -n)$.

**Proof.** The proof of the proposition is straightforward. For the player in $EW$, the utility function is as follows:

$$ u_i^{EW}(p_i; p_{-i}, \pi_{-i}) = \log (\gamma_i) - p_i \sum_{j \neq i} \pi_j h_{ij}. $$

(28)

Let $\pi'_j = -\pi_j$; then, we get

$$ u_i^{EW}(p_i; p_{-i}, \pi_{-i}) = \log (\gamma_i) + p_i \sum_{j \neq i} \pi'_j h_{ij}, $$

\[ \frac{\partial^2 u_i^{EW}(p_i; p_{-i}, \pi_{-i})}{\partial p_i \partial p_j} = \frac{\partial^2 u_i^{EW}(p_i; p_{-i}, \pi_{-i})}{\partial p_i \partial p_j} = 0 \geq 0, \]

\[ \frac{\partial^2 u_i^{EW}(p_i; p_{-i}, \pi_{-i})}{\partial p_i \partial \pi_j} = \frac{1}{p_i} + \sum_{i \neq j} h_{ij} > 0, \]

\[ \frac{\partial^2 u_i^{EW}(p_i; p_{-i}, \pi_{-i})}{\partial \pi'_j \partial p_i} = \sum_{i \neq j} h_{ij} > 0, \]

\[ \frac{\partial^2 u_i^{EW}(p_i; p_{-i}, \pi_{-i})}{\partial p_j \partial \pi'_{i}} = \frac{\partial^2 u_i^{EW}(p_i; p_{-i}, \pi_{-i})}{\partial \pi'_j \partial p_i} = 0 \geq 0. \]

(29)

Similarly, the player’s utility in $FC$ is as follows:

$$ u_i^{FC}(\pi'_i; p) = -(-\pi'_i - C_i(p))^2, $$

\[ \frac{\partial^2 u_i^{FC}(\pi'_i; p)}{\partial p_i \partial p_j} = \frac{\partial^2 u_i^{FC}(\pi'_i; p)}{\partial p_i \partial p_j} = 0 \geq 0, \]

\[ \frac{\partial^2 u_i^{FC}(\pi'_i; p)}{\partial p_i \partial \pi_j} = \frac{\partial^2 u_i^{FC}(\pi'_i; p)}{\partial \pi'_i \partial p_j} = 0 \geq 0, \]

\[ \frac{\partial^2 u_i^{FC}(\pi'_i; p)}{\partial \pi'_j \partial p_i} = \frac{\partial^2 u_i^{FC}(\pi'_i; p)}{\partial \pi'_j \partial p_i} = 0. \]

(30)

Then the $G_{EPP}$ is supermodular game, and the proof is completed.

With Proposition 6, we can conclude that the fictitious game $G_{EPP}$ is a supermodular game. We can also conclude that the game holds the following properties.

(i) The NE set of the game is a nonempty and compact sublattice, and there exists a component-wise smallest and largest NE. (Followed by Lemmas 4.2.1 and 4.2.2 in [16].)

(ii) If each user starts from feasible strategy and uses MBR update rule, the strategy profile will eventually locate in the set bounded by the smallest NE and the largest NE. And if the NE is unique then the MBR update rule globally converge to the NE (followed by Theorem 8 in [17]).

In this case, Theorem 1 in [15] can again be used to characterize the structure of $G_{EPP}$ as well as the convergence of the DPPA. Hence, the fixpoint set of the distributed algorithm is a singleton set containing only the global optimum point. Therefore, the distributed algorithm globally converges to this point. Together with Theorem 5, we conclude the following.

**Proposition 7.** The DPPA algorithm is globally convergent to the unique NE of the DPCG.

### 6. Numerical Results

We provide some numerical results to verify the performance of the distributed update algorithm. The simple sensor network example considered here consists of four pairs of links. The maximum power value $p_{max}$, power $p_{min}$, for all $i \in L$, band width $B$, and noise power spectral density $N_0$ are chosen as 100 mW, 0 mW, 0.1, and 100 Mbps, respectively. The channel gains of links are determined by independent exponential random variables, where the expected value of the gain value of the gain matrix is

$$ E[G_{ij}] = \begin{bmatrix} 1 & 0.05 & 0.03 & 0.02 \\ 0.06 & 1 & 0.04 & 0.04 \\ 0.06 & 0.04 & 1 & 0.05 \\ 0.03 & 0.04 & 0.05 & 1 \end{bmatrix}. $$

(31)
We evaluate the convergence properties of the proposed distributed update algorithm with the same and heterogeneous rate constraints, respectively. Figure 1 illustrates the situation that the four users have the same rate requirement, which means $R_1^* = R_2^* = R_3^* = R_4^* = 17$ Mbps. We see that each of them converges to a reasonable power level after iterations. The power values are different due to the various channel gains.

The results in Figure 2 are nearly the same but with different rate constraints for users, that is, $R_1^* = 17$, $R_2^* = 16$, $R_3^* = 17$, $R_4^* = 18$ Mbps. Note that although user 3 in Figures 1 and 2 has the same rate requirement, the power values assigned to it are not equal because of the different network scenario.

The utility of each user is depicted in Figure 3. In the same rate requirement situation, we can see that the payoff of the users converges to a stable state, and the system reaches the NE state after iterations. Similarly, the utility of the users with different rate target is shown in Figure 4. We can conclude that after the system reaches the NE, the users cannot get extra profit by changing their strategies unilaterally.

7. Conclusion

In this paper, we use game theory to address the issue of power control in wireless sensor networks. We formulate
the distributed power control game as a noncooperative game, where each user maximizes its own utility by choosing a power level from the feasible area. The existence and uniqueness of the NE are established by building the game’s supermodular properties. A distributed power and price update algorithm is proposed which is based on MBR update rules. By correlating to another supermodular game, we show that the proposed distributed update algorithm converges to the unique NE. Numerical experiments have been done to evaluate the algorithm. The results indicate that after relatively little iteration, the game converges to the NE, and the users cannot get extra profit by changing their strategies unilaterally.

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