The $\Sigma_c$ and $\Lambda_c$ magnetic moment from QCD spectral Sum Rules

Shi-lin Zhu,$^1$ W-Y. P. Hwang,$^{2,3}$ and Ze-sen Yang$^1$

$^1$Department of Physics, Peking University, Beijing, 100871, China
$^2$Department of Physics, National Taiwan University, Taipei, Taiwan 10764
$^3$Center for Theoretical Physics, Laboratory for Nuclear Science and
Department of Physics, Massachusetts Institute of Technology, Cambridge,
Massachusetts 02139

March 26, 2022

Abstract

The QCD spectral sum rules in the presence of the external electromagnetic field $F_{\mu\nu}$ is used to calculate the magnetic moment of $\Sigma_c$ and $\Lambda_c$. Our result is $\mu_{\Sigma^{++}_c} = (2.1 \pm 0.3)\mu_N$, $\mu_{\Sigma^{+}_c} = (0.6 \pm 0.1)\mu_N$, $\mu_{\Sigma^{0}_c} = (-1.6 \pm 0.2)\mu_N$ and $\mu_{\Lambda_c} = (0.15 \pm 0.05)\mu_N$.

Keywords: heavy baryon, magnetic moment, QCD sum rule
PACS: 13.40.Em, 14.20.Lq, 14.20.Mr, 12.38.Lg

QCD spectral sum rules (QSSR) are successful in extracting the masses and coupling constants of low-lying mesons and baryons. In this approach the nonperturbative effects are taken into account through various condensates in the QCD vacuum. As shown in $^2$, $^3$ the light baryon masses are determined by the chiral symmetry breaking quark
condensate. In the infinite heavy quark mass limit the QSSR was first used to evaluate the heavy baryon mass [4]. The QSSR with finite heavy quark mass are treated in [5, 6]. Recently the QSSR is employed in the framework of the heavy quark effective theory [7, 8, 9, 10, 11].

The baryon magnetic moment is another important static quantity as the baryon mass. Ioffe and Smilga [12], independently, Balitsky and Yung [13], extracted the nucleon magnetic moment treating the electromagnetic field $F_{\mu\nu}$ as an external field in the QSSR approach. They found that the nucleon magnetic moment is essentially related to the quark condensate and three susceptibilities $\chi, \kappa$ and $\xi$. Later on the octet baryon magnetic moments [15] were obtained in a similar manner. In this work we shall employ the same approach to calculate the magnetic moments of the $\Sigma_c$ and $\Lambda_c$.

We shall consider the two-point correlator $\Pi_{\Sigma_c}(p)$ in the presence of an external electromagnetic field $F_{\alpha\beta}$.

$$\Pi_{\Sigma_c}(p) = \int d^4x \langle 0|T\{\eta_{\Sigma_c}(x), \bar{\eta}_{\Sigma_c}(0)\}|0\rangle F_{\alpha\beta} e^{ip\cdot x}$$

$$= \Pi_0(p) + \Pi_{\mu\nu}(p) F_{\mu\nu} + \cdots \quad (1)$$

where $\Pi_0(p)$ is the polarization operator without the external field $F_{\alpha\beta}$. The $\eta_{\Sigma_c}$ in Eq. (1) is the interpolating current with $\Sigma_c$ quantum numbers.

$$\eta_{\Sigma_c}(x) = \epsilon^{abc}\{[u^a T(x) Cc^b(x)]u^c(x) - [u^a T(x) C\gamma_5 c^b(x)]\gamma_5 u^c(x)\}, \quad (2)$$

and

$$\langle 0|\eta_{\Sigma_c}(0)|\Sigma_c\rangle = \lambda_{\Sigma_c} \nu_{\Sigma_c}(p), \quad (3)$$

where $u^a(x)$ and $c^b(x)$ is the up and charm quark field, $\lambda_{\Sigma_c}$ is the overlap amplitude of the interpolating current with the baryon state, and the $\nu_{\Sigma_c}$ is the Dirac spinor of the heavy quark.

Three different tensor structures contribute to $\Pi_{\mu\nu}(p)$,

$$\Pi_{\mu\nu}(p) = \Pi(p)(\sigma_{\mu\nu}\hat{p} + \hat{p}\sigma_{\mu\nu}) + \Pi_1(p)i(p_\mu\gamma_\nu - p_\nu\gamma_\mu)\hat{p} + \Pi_2(p)\sigma_{\mu\nu}. \quad (4)$$
As in the original QSSR analysis of nucleon magnetic moment [12], we shall consider the first tensor structure \((\sigma_{\mu\nu}\hat{p} + \hat{p}\sigma_{\mu\nu})\).

The presence of the external field \(F_{\mu\nu}\) will induce three new condensates in the QCD vacuum [12].

\[
\langle 0 | \bar{q} \sigma_{\mu\nu} q | 0 \rangle F_{\mu\nu} = e_c \chi F_{\mu\nu} \langle 0 | \bar{q} q | 0 \rangle,
\]

\[
g_s \langle 0 | \bar{q} \frac{\lambda^n}{2} G_{\mu\nu} q | 0 \rangle F_{\mu\nu} = e_c \kappa F_{\mu\nu} \langle 0 | \bar{q} q | 0 \rangle,
\]

\[
g_s \epsilon^{\mu\nu\lambda\sigma} \langle 0 | \bar{q} \gamma_5 \frac{\lambda^n}{2} G_{\lambda\sigma} q | 0 \rangle F_{\mu\nu} = ie_c \xi F_{\mu\nu} \langle 0 | \bar{q} q | 0 \rangle,
\]

where \(q\) refers to the up and down quark, \(e_c\) is the quark charge. The \(\chi, \kappa\) and \(\xi\) in Eq. (5) are the quark condensate susceptibilities, which have been the subject of various studies [12, 13, 14, 15]. Their values employed by different groups are consistent with each other. We shall adopt the values \(\chi = -4.5 \text{ GeV}^{-2}, \kappa = 0.4, \xi = -0.8\).

At the phenomenological level we have

\[
\text{Im}\Pi(s) = \frac{1}{4} \mu_{\Sigma_c} \lambda_{\Sigma_c}^2 \delta'(s - m_{\Sigma_c}^2) + C \delta(s - m_{\Sigma_c}^2) + \text{Im}\Pi^{\text{pert}}(s) \theta(s - t_{\Sigma_c})
\]

where the first term corresponds to the \(\Sigma_c\) magnetic moment and is of the double pole. The second term comes from the transition \(\Sigma_c \rightarrow\) excited states and is of single-pole. The third term is the usual continuum contribution and \(t_{\Sigma_c}\) is the continuum threshold. The single-pole transition term does not damp out after Borel transform. So it should be explicitly included in the QSSR analysis.

Within an operator product expansion we obtain to lowest order of \(\alpha_s\) and for condensates up to dimension six at the quark level,

\[
\text{Im}\Pi(s) = \text{Im}\Pi^{\text{pert}}(s) + \text{Im}\Pi^{\text{np}}(s),
\]

\[
\text{Im}\Pi^{\text{pert}}(s) = \frac{e_u}{8} s(1 - \frac{m_{\Sigma_c}^2}{s})^3,
\]

Making Borel tranform of \(\Pi(p)\) and transferring the continuum contribution to the
left hand side, we obtain:

\[
M_B^2 f_{\Sigma c}^{ls} \text{Im} \Pi_{\text{pert}}(s) e^{-t_{\Sigma c} M_B^2 ds L^{-\frac{1}{2}}} + \left\{ -\frac{1}{12} \chi e_u a^2 M_B^2 (1 - e^{-\frac{t_{\Sigma c} - m_c^2}{M_B^2}}) L^{-\frac{10}{3}} - \frac{1}{24} e_c a^2 - \frac{1}{36} e_u a^2 (\frac{m_c^2}{M_B^2} + 1) + \frac{1}{36} \chi m_0^2 e_u a^2 (2 \frac{m_c^2}{M_B^2} + 1) L^{-\frac{10}{3}} - \frac{1}{72} \kappa e_u a^2 (2 \frac{m_c^2}{M_B^2} - 1) - \frac{1}{36} \xi e_u a^2 (\frac{m_c^2}{M_B^2} + 1) \right\} e^{-\frac{m_c^2}{M_B^2} L^{-\frac{1}{2}}}
\]

\[= \frac{1}{4} (2\pi)^4 \lambda_{\Sigma c}^2 e^{-\frac{m_c^2}{M_B^2}} \mu_{\Sigma c} (1 + CM_B^2), \]

where \(a = -(2\pi)^2 \langle 0|\bar{q}q|0 \rangle = 0.55\text{GeV}^3\), \(a m_0^2 = (2\pi)^2 g_s \langle 0|\bar{q}\sigma \cdot G q|0 \rangle\), \(m_0^2 = 0.8\text{GeV}^2\), \(q = u, d\), \(L = \frac{\ln(10 M_B)}{\ln(5)}\). \(C\) is the unknown constant to be determined from the sum rule, which parametrizes the transition contribution. We have checked that in the chiral limit \(m_c \to 0\), our result reproduces the sum rule for nucleon magnetic moment \[12\].

For the charm quark mass we use \(m_c = 1.47 \pm 0.1\text{GeV}\). We adopt the estimated continuum threshold and overlapping amplitude in the QSSR analysis \[5, 6, 9, 10, 11, 16\]. \(t_{\Sigma c} = 10\text{GeV}^2\) and \((2\pi)^4 \lambda_{\Sigma c}^2 = 0.8 \pm 0.1\text{GeV}^6\). For the \(\Sigma_c\) mass we may either use the predictions in the QSSR analysis or the experimental value \[22\], \(m_{\Sigma_c} = 2.455\text{GeV}\).

We may further improve the numerical analysis by taking into account of the renormalization group evolutions of the sum rule \[9\] through the anomalous dimensions of the condensates and currents. The working interval of the Borel mass \(M_B^2\) for the sum rule \[9\] is \(1.7\text{GeV}^2 \leq M_B^2 \leq 2.5\text{GeV}^2\) where both the continuum contribution and power corrections are controllable \[4, 5\]. Moving the factor \(\frac{1}{4} (2\pi)^4 \lambda_{\Sigma c}^2 e^{-\frac{m_c^2}{M_B^2}}\) on the right hand side to the left and fitting the new sum rules with a straight line approximation we may extract the \(\Sigma_c\) magnetic moment. We show the Borel mass dependence of the new sum rule and the fitting straight line in Fig. 1 for the continuum threshold \(t_{\Sigma c} = 10\text{GeV}^2\).

It can be seen in Fig. 1 that the nondiagonal transition contribution is important though it is not dominant in the working interval of the Borel mass \(1.7\text{GeV}^2 \leq M_B^2 \leq 2.5\text{GeV}^2\). The sum rule is insensitive to the susceptibilities \(\kappa\) and \(\xi\) due to their small values. Their contributions are less than 5%. When \(\chi\) varies from \(-4.5\text{GeV}^{-2}\) to \(-3.5\text{GeV}^{-2}\) or to \(-5.5\text{GeV}^{-2}\), the sum rules change within 10%.
Our final result is \( \mu_{\Sigma^{++}} = (5.4 \pm 0.5) \frac{e}{2m_{\Sigma^{++}}} \), where \( \frac{e}{2m_{\Sigma^{++}}} \) is a natural unit in QSSR analysis of the baryon magnetic moment. By replacing \( e_u \) in (9) with \( e_u + e_d \) or \( e_d \) we arrive at the magnetic moments for the other \( \Sigma_c \) multiplets, \( \mu_{\Sigma^+} = (0.6 \pm 0.1) \frac{e}{2m_{\Sigma^+}} \) and \( \mu_{\Sigma^0} = (-4.2 \pm 0.4) \frac{e}{2m_{\Sigma^0}} \). In unit of nuclear magneton \( \mu_{\Sigma^{++}} = (2.1 \pm 0.3) \mu_N \), \( \mu_{\Sigma^+} = (0.23 \pm 0.03) \mu_N \) and \( \mu_{\Sigma^0} = (-1.6 \pm 0.2) \mu_N \).

Similarly we can extract the magnetic moments of \( \Lambda_c \) with the following interpolating current.

\[
\eta_{\Lambda_c}(x) = e^{abc}[u^aT_0(x)C\gamma_5u^b(x)]c^c(x)
\]

(10)

The final sum rule reads as follows:

\[
\begin{align*}
\frac{3e}{16}M_B^2 & \int_{m_c^2}^{t_{\Lambda_c}} \left[ \frac{m_c^2}{2} \left( 1 - \frac{m_c^2}{s^2} \right) - \frac{2}{3} \left( 1 - \frac{m_c^2}{s^2} \right) e^{-\frac{s}{M_B^2}} ds \right] L - \frac{5}{4} \\
& + \left\{ - \frac{e}{24} a^2 + \frac{e_u+e_d}{144} a^2 \left( 1 - \frac{m_c^2}{M_B^2} \right) \right. \\
& \left. + \frac{e_u+e_d}{576} \chi m_0^2 a^2 L - \frac{10}{9} + \frac{e_u+e_d}{48} \kappa a^2 \right\} e^{-\frac{m_c^2}{M_B^2} L} \\
& = \frac{1}{4} (2\pi)^4 \lambda_{\Lambda_c}^2 e^{-\frac{m_c^2}{M_B^2}} \mu_{\Lambda_c} (1 + CM_B^2),
\end{align*}
\]

(11)

With the parameters \( t_{\Lambda_c} = 10 \text{GeV}^2, (2\pi)^4 \lambda_{\Lambda_c}^2 = 1.0 \pm 0.2 \text{GeV}^6 \) and \( m_{\Lambda_c} = 2.285 \text{GeV} \), we get \( \mu_{\Lambda_c} = (0.15 \pm 0.05) \mu_N \). The Borel dependence of \( \mu_{\Lambda_c} \) and the fitting line is shown in Fig. 2. As in the analysis of the \( \Sigma_c \) magnetic moment, the straight line approximation is good.

It is not difficult to extend the same analysis to extract the magnetic moments of \( \Sigma_b \) and \( \Lambda_b \). Yet the overlapping amplitudes \( \lambda_{\Sigma_b} \) and \( \lambda_{\Lambda_b} \) determined in the QSSR approach with finite bottom quark mass have large errors. So we do not tend to present numerical results here.

In summary, we have calculated the magnetic moment of \( \Sigma_c \) and \( \Lambda_c \) using the external field method in the QCD sum rules. There are no experimental data for the heavy baryon magnetic moments. Yet naive predictions have been made in the phenomenological models such as nonrelativistic quark model (NRQM) \cite{17, 18}, bag model \cite{19} and the Skyrme model \cite{20, 21}. Especially in the quark model the heavy baryon magnetic moments have
a rather simple form. \( \mu_{\Lambda_c} = \mu_c \), \( \mu_{\Sigma_c^+} = \frac{8}{9} \mu_p - \frac{1}{3} \mu_c \), \( \mu_{\Sigma_c^0} = \frac{2}{9} \mu_p - \frac{1}{3} \mu_c \) and \( \mu_{\Sigma_c^0} = -\frac{4}{9} \mu_p - \frac{1}{3} \mu_c \).

Our result of the \( \Sigma_c \) magnetic moment is in good agreement with the NRQM prediction. In the \( \mu_{\Lambda_c} \) sum rule (11), the contribution from higher-dimension condensates is significant and comparable with the charm quark perturbative contribution numerically, since the light quark perturbative contribution and the induced light quark condensate vanishes. If we turn off all the higher-dimension nonperturbative contribution by setting the quark condensate \( a = 0 \), we arrive at \( \mu_{\Lambda_c} = 0.35 \mu_N \), which is very close to the NRQM prediction! So the higher-dimension condensates lead to the possible deviation from the naive quark model result. It will be very interesting to measure \( \mu_{\Lambda_c} \) experimentally.

This work is supported in part by the National Natural Science Foundation of China and the Doctoral Program of State Education Commission of China. It is also supported in part by the National Science Council of R.O.C. (Taiwan) under the grant NSC84-2112-M002-021Y.

References

[1] M.A.Shifman, A.I.Vaishtein, and V.I.Zakharov, Nucl. Phys. B \textbf{147} (1979) 385, 448.

[2] Y. Yung et al., Phys. Lett. B \textbf{102} (1981) 175; Nucl. Phys. B \textbf{197} (1982) 55.

[3] B. L. Ioffe, Nucl. Phys. B \textbf{188} (1981) 317.

[4] A. Shuryak, Nucl. Phys. B \textbf{198} (1982) 83.

[5] V. M. Belyaev and B. Y. Blok, Z. Phys. C \textbf{30} (1986) 151; B.Y. Blok and V. L. Eletsky, Z. Phys. C \textbf{30} (1986) 229.

[6] E. Bagan et al., Phys. Lett. B \textbf{278} (1992) 367; B \textbf{287} (1992) 176.

[7] A. G. Grozin and O. I. Yakovlev, Phys. Lett. \textbf{285} (1992) 254.

[8] E. Bagan et al., Phys. Lett. B \textbf{301} (1993) 243.
[9] Y. B. Dai et al., Phys. Lett. B 371 (1996) 99.

[10] P. Colangelo et al., Phys. Rev. D 54 (1996) 4622.

[11] S. Groote, J. G. Korner and O. I. Yakovlev, hep-ph/9609468.

[12] B. L. Ioffe and A. V. Smilga, Nucl. Phys. B 232 (1984) 109.

[13] I. I. Balitsky and A. V. Yung, Phys. Lett. B 129 (1983) 328.

[14] V. M. Belyaev and Ya. I. Kogan, Yad. Fiz 40 (1984) 1035. (Sov. J. Nucl. Phys. 40 (1984) 659.)

[15] C. B. Chiu, J. Pasupathy, and S. J. Wilson, Phys. Rev. D 33 (1986) 1961; D 36 (1987) 1451, 1553.

[16] T. Schäfer, E. Shuryak and J. J. M. Verbaarscot, Nucl. Phys. B 412 (1994) 143.

[17] D. B. Lichtenberg, Phys. Rev. D 15 (1977) 345.

[18] G. Karl and J. E. Paton, Phys. Rev. D 30 (1984) 238.

[19] S. K. Bose and L. P. Singh, Phys. Rev. D 22 (1980) 773.

[20] Y. Oh, D.-P. Min, M. Rho and Scoccola, Nucl. Phys. A 534 (1991) 493.

[21] M. Björnberg et al., Nucl. Phys. A 539 (1992) 662.

[22] Particle Data Group, Phys. Rev. D 54 (1996) 55.
Figure Captions

Fig. 1 The Borel mass dependence of the $\Sigma_c^{++}$ magnetic moment for the continuum threshold $t_{\Sigma_c} = 10\text{GeV}^2$. The solid curve is the QCD sum rule prediction for $\mu_{\Sigma_c^{++}}$. The dotted line is a straight-line approximation. The intersect with Y-axis is the $\Sigma_c$ magnetic moment in unit of $\frac{e}{2m_{\Sigma_c}}$.

Fig. 2 The Borel mass dependence of $\mu_{\Lambda_c}$ and the fitting straight line.
Fig. 1

\( \mu \Sigma_c \)

\( M_B^2 \)
\( (1 + C M_B^2) \)