Uncertainty evaluation for absolute flatness measurement on horizontally aligned Fizeau interferometer

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Abstract. This paper describes and reports uncertainties associated with the absolute flatness measurement for an Ø 300 mm optical flat. Absolute flatness of 3 flats denoted as optical flat A, B and C was determined by conducting a three-flat test method. The measurements were carried out using a Fizeau interferometer. Sources of uncertainty that contributed to the measurement uncertainty were examined. The dominant contributions include off-center alignment during three-flat test measurement, temperature instability and deformation due to gravity. In our experiment, although the alignment was performed crucially, deviation up to 4 nm was observed from 6 sets of three-flat test. Since the optical flat was mounted using a sling type module, deformation due to clamping force can be neglected. Deformation due to gravitational force was investigated by comparing the contour map of the optical flat oriented at 0° and 180°. The maximum difference up to 99 nm was observed which is higher than accuracy of the reference flat (λ/20) itself.

Keywords: flatness, uncertainty, Fizeau interferometer

1. Introduction
The challenge of modern machining industries involves the achievement of high quality and high geometric and dimensional accuracy of work piece [1]. For better part functioning and assembly, part will be made up with good dimensional tolerance as well as with good geometric tolerance. Flatness is a surface form control [2]. Flatness is commonly used on planar surface capable of resting on matting planar surface without any significant rocking. Another important application of flat surface can be found in optics. The mechanism behind the formation of flatness as geometric tolerance is very dynamic, complicated, and process dependent. Several factors that influence the flatness in a CNC milling operations are spindle speed, feed rate, depth of cut and insert type for example.

A perfectly flat surface is defined as having all its elements in the same plane. The flatness tolerance zone consists of the distance between two parallel planes. In order to obtain a reliable assessment of flatness form, an appropriate extraction strategy for obtaining a representative set of points on the work piece is required. Flatness can be measured by various techniques which can be divided into 2 groups, contact type and non-contact type. The contact method can be conducted using dial indicator, height gauge and coordinate measuring machine (CMM) for instance. However neither of these methods can measure flatness more accurately than 2.5 μm [3-7]. Non-contact method is commonly used with lapped parts which is based on the reflection and interference of monochromatic light [8]. A monochromatic
light source and a reference flat are needed for this measuring method. The reference flat, a piece of transparent glass that has itself been lapped and polished on one or both sides, is placed on the lapped surface and plays role as the reference standard. The monochromatic light is then shone down through the glass. The light will pass through glass and reflect off the workpiece. As the light reflects in the gap between the workpiece and the polished surface of the reference flat the light will interfere with itself creating bright and dark fringes. Each fringe represents a change of one half wavelength in the gap width between the reference flat and the workpiece. The light bands display a contour map of the workpiece’s surface and can be readily interpreted for flatness value [9]. The more advanced flatness measuring instrument with the highest accuracy is by using flatness interferometer. Fizeau interferometer is equipped with the He-Ne laser generating a single wavelength illumination and phase shifter. Phases of the interference pattern were determined from phase shifting algorithm and converted to distance between reference flat and the workpiece. The difference between the maximum point and the minimum point is referred to as flatness. The interferometric method of flatness measurement is a form of length measurement in which distances variation between a test surface and a reference surface is measured at many locations simultaneously. The phase differences between the two wavefronts (reflected and transmitted) result in fringe pattern and that is a direct indication of the form of the component being tested with respect to the reference flat [10-11].

Considering that flatness of the reference surface is better than the test surface’s, thus all the differences are counted for the test surface. Therefore, the accuracy of measurement is directly limited by the quality of the reference flat.

In usual flatness test with the reference surface if errors of test surface are similar to those of reference surface, then it is impossible to identify which flat contributes the error. Absolute flatness measurement is an interferometric method to obtain the actual flatness of an optical surface independent of the flatness of reference flat used. In this method, 3 flats in a series of configurations are measured [12-13]. Then the flats are rotated and replaced during measurements in a prescribed order to overcome the error source from the reference flat. The absolute testing is actually beneficial, when we use a tested flat as a reference surface during measurements. The data obtained is extremely accurate and it allows high-accuracy characterization of any optical surfaces.

The uncertainty of measurement is associated with the result of a measurement that characterizes the dispersion of the values that could reasonably be attributed to the measurand [14]. Toshiyuki et al. demonstrated uncertainty evaluation for three-flat test measurement performed on the vertically aligned Fizeau interferometer. Sources of uncertainty are repeatability, reproducibility, temperature effect, deformation due to gravity and uncertainty of algorithm in three-flat test. The main sources of measurement uncertainty are the environment and gravitational force.

When measuring flatness of optics, surface of an unknown is measured comparatively to the reference surface. In order to determine the absolute flatness error of the unknown face, three-flat test method is need to be conducted. There has been a research reported the uncertainty estimation of the flatness measurement on the vertically aligned Fizeau interferometer. In order to increase accuracy of flatness measurement on horizontally aligned Fizeau Interferometer, uncertainty contribution is examined here.

In this paper, we explored the amount of deformation of the optics mounted horizontally. The three-flat test measurement of optical flat, 300 mm in diameter, on the horizontally aligned Fizeau interferometer was conducted. The optical flat were mounted on the sling type mounting in order to avoid deformation due to clamping force. The uncertainty contributed from environmental effect and deformation due to gravitational force were investigated.

2. Experiment setup
2.1. Instrument
The flatness interferometer used is manufactured by 4D Technology. This system can measure flatness of workpiece diameter up to 300 mm. The system equipped with the stabilized He-Ne laser, illuminating light at 632.8 nm, and phase shifter. Interference fringes were captured by the CCD camera with
resolution of 1200 pixel × 1200 pixel. The system were placed on the anti-vibration table. Figure 1 illustrates schematic diagram of the horizontally aligned flatness interferometer system.

Figure 1. Schematic diagram of the flatness interferometer.

Phases (\(\phi\)) of the interference fringes were determined using 9-steps phase shifting algorithm designed by Phillion D.W. [15]. Phase difference was then converted to distance in unit of nanometer creating a surface topography of the optic under-tested. Flatness value was determined from the difference between the maximum point (peak) and the minimum point (valley). Figure 2 shows optical flat rested on a sling and mounted on the tip/tilt stage. The sling is made of aluminum, 610 mm in length and 9.4 mm in width.

Figure 2. Optical mounting system.

Optical flat A, B and C are 300 mm in diameter with thickness of 50.8 mm. Their weight are approximately 10 kg. The experiments were employed in the environmental controlled laboratory. The environmental condition is 20 ± 1°C for temperature and 50 ± 10 % RH for relative humidity.

2.2. Three-flat test method
Three-flat test method helps eliminates form errors of the reference flat. It is ideal for calibrating reference flats because it gives an absolute calibration. There are various methods for absolute flat testing. Also, several algorithms have been developed to determine absolute surface error [16-21].

The experimental procedure consists of 4 sets of measurement [22-23]. Figure 3 demonstrates configurations of optical flat setup for four measurement require for the three-flat test method. The optical flat to be measured in are labeled as A, B and C. The first set denoted as measurement I, consists of flat B acting as transmission flat and flat A as a reference flat. The second set denoted as measurement II, consists of flat B acting as transmission flat and flat A is rotate 90 degree as a reference flat. The third
set denoted as measurement III, consists of flat C acting as transmission flat and flat A as a reference flat. The last set denoted as measurement IV, consists of flat C acting as transmission flat and flat B as a reference flat.

Absolute flatness testing is an interferometric method for obtaining the actual surface error of flat object, independent of flatness error of the reference flat used. This method, one measures three flats in a series of configurations, rotating and replacing flats between measurements in a prescribed order to eliminate the reference surface’s flatness as an error source. The analysis of measurement results was done by using even and odd functions. The function of even-odd, odd-even, even-even, and odd-odd are illustrated as in (1) - (3). With rotation is required in the test method, horizontally mounted flat experience gravitational effect differently between each measurement series. It should be noted that when the measurement sequence as shown in figure 3, the optical flat was measured with an upward and a downward posture. The gravitational effect is needed to be examined as it directly affects accuracy of the absolute measurement result.

\[
A = A_{ee} + A_{oe} + A_{oo, 2oddθ} \\
B = B_{ee} + B_{oe} + B_{oo, 2oddθ} \\
C = C_{ee} + C_{oe} + C_{oo, 2oddθ}
\]  \hspace{1cm} (1)

\[
A = A_{ee} + A_{oe} + A_{oo, 2oddθ} \\
B = B_{ee} + B_{oe} + B_{oo, 2oddθ} \\
C = C_{ee} + C_{oe} + C_{oo, 2oddθ}
\]  \hspace{1cm} (2)

\[
A = A_{ee} + A_{oe} + A_{oo, 2oddθ} \\
B = B_{ee} + B_{oe} + B_{oo, 2oddθ} \\
C = C_{ee} + C_{oe} + C_{oo, 2oddθ}
\]  \hspace{1cm} (3)

The even-odd, and odd-even function of absolute flatness can be analysis by three-flat test method using 2oddθ from four configurations [12].

**Figure 3.** Configurations of four required for three-flat test method.
3. Results and discussion

3.1. Three-flat test measurement

Figure 4 illustrates surface contour maps obtained from measurement I, II, III and IV. Each measurement is obtained from averaging 20 repeated measurements. The three-flat test measurements were reproduced 6 times.

![Figure 4](image)

**Figure 4.** The measurement results obtained from (a) measurement I, (b) measurement II, (c) measurement III and (d) measurement IV.

It should be emphasized that such results are not absolute form error of optical flat but combined error of 3 optical flats. The absolute form error of optical flat A, B and C were calculate according to as in (1) - (3) yielding surface contour of optical flat A, B and C as shown in figure 5.

![Figure 5](image)

**Figure 5.** The contour map of optical flat (a) A, (b) B and (c) C.

| Flat | PV (nm) | \( \sigma_{\text{rms}} \) (nm) |
|------|--------|------------------|
| A    | 61.92  | 3.06             |
| B    | 71.61  | 4.24             |
| C    | 393.01 | 7.11             |

**Table 1.** PV value of optical flat A, B and C

The averaged peak-to-valley (PV) and standard deviation of optical flat A, B and C obtained from 6 measurements were determined and were summarized in table 1.
3.2. Uncertainty evaluation
The uncertainty was evaluated in accordance with the JCGM 100:2008 (Evaluation of measurement data - Guide to the expression of uncertainty in measurement) [24]. The measurement uncertainty can be determined by combining all sources of uncertainty as described as in (4)

\[ u_c^2(x) = c_1^2 u^2(x_1) + c_2^2 u^2(x_2) + \ldots + c_n^2 (x_n) \]  

where \( u(x) \) is combined standard uncertainty, \( u(x_i) \) is standard uncertainty of component \( i \) and \( c_i \) is sensitivity coefficient of component \( i \). Flatness of optical flat was calculated from the difference between the maximum height \( (x_{\text{max}}) \) and the minimum height \( (x_{\text{min}}) \) of the flat’s topography.

\[ PV = x_{\text{max}} - x_{\text{min}} \]  

The surface height can be determined from phase \( (\phi) \) as in (6) when \( \lambda \) is wavelength of light source.

\[ x = \frac{\phi}{4\pi} \lambda \]  

Phase value was analyzed from 9-step phase shifting algorithm according to as in (7) when \( I_i \) is intensity of light detected by the CCD camera and \( i \) is the step order [25].

\[ \phi = \tan^{-1} \left( \frac{-4I_2 + 12I_4 - 12I_6 + 4I_8}{I_1 - 8I_5 + 14I_7 - 8I_9 + I_9} \right) \]  

From as in (5) to as in (7), uncertainty of flatness can be written as following

\[ u^2(PV) = u^2(x_{\text{max}}) + u^2(x_{\text{min}}) + u^2(x_{3-\text{flat}}) \]

\[ = 2\left( c^2_x + u^2(x) \right) + \left( c^2_{x_{3-\text{flat}}} + u^2(x_{3-\text{flat}}) \right) \]

where \( u(x_{3-\text{flat}}) \) is the standard uncertainty due to absolute calibration using 3-flat test method. From as in (4), and as in (6) can be rewritten as

\[ u^2(x) = \left( \frac{\lambda}{4\pi} \right)^2 u^2(\phi) + \left( \frac{\phi}{4\pi} \right)^2 u^2(\lambda) \]  

Thus, as in (8) can be described as

\[ u^2(PV) = 2 \left[ \left( \frac{\lambda}{4\pi} \right)^2 u^2(\phi) + \left( \frac{\phi}{4\pi} \right)^2 u^2(\lambda) \right] + u^2(x_{3-\text{flat}}) \]

3.2.1. Uncertainty due to phase determination, \( u(\phi) \)
Phase is the position of point on waveform. The uncertainty of phase is caused by several factors including nonlinearity of piezoelectric transducer (PZT), background noise, signal drift, sensitivity of
the detector and vibration Phillion D.W. suggested that by analysing phase using as in (7), phase error became insensitive to the detector’s sensitivity and vibration. Phase error is limited to calculation as in (11).

\[
\left(\frac{\delta \theta}{\pi}\right)^2 / 2\pi = 2.261 \times 10^{-4} \left(1 - 0.04 \sin^2 \theta\right)^{1/2}, \text{ wave}
\]

The standard uncertainty of phase determination is considered as rectangular distribution and its value is equal to 0.00082.

3.2.2. Uncertainty due to accuracy of wavelength, \(u(\lambda)\)
Wavelength of laser plays a crucial role in converting. Phase value to height value. Hence, accuracy of height value strongly depends on accuracy of the laser’s wavelength and decimal place of the wavelength value used in calculation. Both errors are considered as rectangular distribution and the standard uncertainty of 0.00577 nm was obtained.

3.2.3. Uncertainty in 3-flat test measurement, \(u(x_{3-\text{flat}})\)
Upon conducting 3-flat test method, the measurement was performed at 4 different configurations as illustrated in figure 3. There are various factors that affect accuracy of this absolute calibration. There are temperature fluctuation \(u(x_{3-\text{flat-1}})\), centering error \(u(x_{3-\text{flat-2}})\) and deformation due to gravity \(u(x_{3-\text{flat-3}})\).

For each measurement, the optical flat were left to be stabilized with the setup and temperature for at least 8 hours. Thus, one set of 3-flat test calibration required approximately 4 days to complete. Therefore, effect of temperature fluctuation to flatness measurement was investigated.

Figure 6 illustrates height variation of the optical flat C at position X1 (-147.75 mm, 0 mm), X2 (-134.53 mm, 0 mm), Y1 (0 mm, -139.29 mm) and Y2 (0 mm, -99.64 mm). The variation was examined after 8 hours stabilization. The height variation of ±32 nm and the normal distribution with maximum experimental standard deviation of 13.55 nm were observed.

![Figure 6. Variation in calculated distance at four different positions of reference flat C.](image-url)

Centering is also strongly effect accuracy of 3-flat test analysis. Alignment with out of center by only 5 nm can cause error in absolute calibration. We examined alignment error by reproducing 3-flat test measurement from 6 set of averaged height value and its experimental standard deviation of optical flat A, B and C at position (75 mm, 75 mm), (-75 mm, 75 mm), (-75 mm, -75 mm) and (75 mm, -75 mm).
denoted as position 1-4, respectively, are summarized in table 2. The maximum standard deviation of 5.28 nm was observed.

Table 2. Height of optical flat at position 1, 2, 3 and 4

| Flat | Position | X̄ (nm) | σ_{x-1} (nm) |
|------|----------|---------|---------------|
| A    | 1        | 2.04    | 3.84          |
|      | 2        | -0.63   | 2.00          |
|      | 3        | 7.20    | 4.61          |
|      | 4        | 4.71    | 2.97          |
| B    | 1        | -2.94   | 2.15          |
|      | 2        | 1.80    | 4.05          |
|      | 3        | -3.60   | 1.84          |
|      | 4        | 0.29    | 2.38          |
| C    | 1        | -28.22  | 3.68          |
|      | 2        | -54.26  | 1.59          |
|      | 3        | 16.68   | 5.28          |
|      | 4        | 15.15   | 3.54          |

3-flat test method requires rotation of the optical flat. Unlike vertically aligned flatness interferometer, the deformation due to gravity is non uniform throughout the measurement for the horizontally aligned system. Deformation due to gravitational force was investigated by comparing the profile of optical flat C along y-axis with those of the optical flat C after 180° rotation and the deviation. The maximum difference of 99 nm was observed. Figure 7 illustrates result the measured of cross section profile along y-axis of the reference flat C aligned at 0° and 180°. According in to the guideline [26], the deformation of optical flat with the theoretically perfect edge support was calculated to be approximately 90 nm which is in good agreement with our experimental result.

![Figure 7](image_url)

**Figure 7.** Cross section profile along y-axis of the reference flat C aligned at 0° and 180° and the difference between the two profiles.

As the result, the standard uncertainty in 3-flat test measurement became 57.22 nm. Uncertainty budget of flatness value is summarized in table 3. The expanded uncertainty of PV value at the confidence level of approximately 95% is 115 nm.
### Table 3. Uncertainty budget

| Symbol                | Value   | Unit | Standard uncertainty | Probability distribution | Sensitivity coefficient | Uncertainty contribution | Unit   |
|-----------------------|---------|------|-----------------------|--------------------------|-------------------------|-------------------------|--------|
| $u(\lambda_1)$       | 6.33E-06| nm   | 3.65E-06              | Rectangular              | 0.056                   | 2.06E-07                | nm     |
| $u(\lambda_2)$       | 0.01    | nm   | 5.77E-03              | Rectangular              | 0.056                   | 3.25E-04                | nm     |
| $u(\phi)$            | 1.42E-03| rad  | 8.20E-04              | Rectangular              | 50.357                  | 4.13E-02                | nm     |
| $u(x_i)$              |         |      |                       | Normal                   | 4.13E-02                |                         | nm     |
| $u(x_{i_{\text{flat1}}})$ | 13.55  | nm   | 1.51                  | Normal                   | 1                       | 1.51E+00                | nm     |
| $u(x_{i_{\text{flat2}}})$ | 5.28   | nm   | 2.16                  | Normal                   | 1                       | 2.16E+00                | nm     |
| $u(x_{i_{\text{flat3}}})$ | 99     | nm   | 57.16                 | Rectangular              | 1                       | 5.72E+01                | nm     |
| $u(x_{x_{\text{flat}}})$ |         |      |                       | Normal                   | 5.72E+01                |                         | nm     |
| $u(PV)$               |         |      |                       | Normal                   | 57.22                   |                         | nm     |
| $U_{95\%}(PV)$        |         |      |                       | Normal                   | 115                     |                         | nm     |

### 4. Conclusions
We found that the major source of uncertainty is the environmental fluctuation and deformation due to the gravitational force. Optical flats were absolute calibrated using 3-flat test method performing on the horizontally aligned Fizeau interferometer. Absolute flatness error of optical flat A, B and C were deformed and the PV value are 61.92 nm, 71.61 nm and 393.01 nm respectively. The measurement uncertainty was evaluated and the expanded uncertainty of 115 nm at 95% confidence was obtained. The major source of uncertainty is deformation of the optical flat due to gravitational force. In order to reduce the measurement uncertainty to be within accuracy of the reference flat ($\lambda/20$), deformation due to gravity shall be corrected.

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