Data on the Neutron–Neutron Scattering Length from the nd-Breakup Reaction at \(E_n = 8\) MeV and \(E_n = 11\) MeV

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Abstract—A kinematically complete experiment devoted to studying the \(nd\)-breakup reaction at energies of 8 and 11 MeV via detecting all three secondary particles was performed. The \(1S_0\) neutron–neutron \((nn)\) scattering-length values of \(a_{nn} = -19.8 \pm 0.4\) and \(-19.0 \pm 0.5\) fm at, respectively, \(E_n = 8\) and \(11\) MeV were obtained from a comparison of the experimental dependence of the yield of the \(nd\)-breakup reaction on the relative energy of the \(nn\) pair with the results of a simulation. An analysis of the values obtained for the \(nn\) scattering length, together with data from other experiments, confirms the hypothesis that three-nucleon forces affect the parameters of \(nn\) interaction that are extracted from reactions involving few-nucleon systems and gives a new asymptotic \(nn\) scattering length, \(a_{nn} = -16.1 \pm 0.1\) fm.

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1. INTRODUCTION

It is well known that breaking of charge symmetry of nuclear forces [that is, the difference of nuclear neutron–neutron \((nn)\) and proton–proton \((pp)\) interactions] is a small effect, which, according to modern concepts, is due to the difference of \(u\)- and \(d\)-quark masses, charges, and magnetic moments. In determining the degree of charge-symmetry breaking (CSB), special role belongs to studying low-energy properties of nucleon–nucleon \((NN)\) interaction in the singlet spin state, such as the scattering lengths and energies of a virtual \(1S_0\) level. The singlet \(s\) state of two nucleons is nearly bound owing to the existence of a virtual level at an energy \(E_{NN}\) close to zero. The respective scattering lengths for \(nn\) and \(pp\) interactions \((a_{nn} \text{ and } a_{pp}, \text{ respectively})\) are large in magnitude and are rather highly sensitive to moderately small distinctions between the \(nn\) and \(pp\) potentials. In order to estimate quantitatively the degree of CSB—we denote it by \(\Delta a_{CSB} = a_{pp} - a_{nn}\)—it is therefore necessary to know accurate values of these quantities.

The value of the \(pp\) scattering length was determined to a high precision in experiments devoted to studying free \(pp\) scattering. Its presently adopted value is \(a_{pp} = -17.3 \pm 0.4\) fm [1]. The uncertainty in it stems primarily from the model-dependent procedure for the exclusion of the electromagnetic component of \(pp\) interaction.

Since a direct experiment on \(nn\) scattering could not be implemented thus far, the \(nn\) scattering length is extracted from data on reactions involving two final-state neutrons. Among them, the \(nd\)- and \(dd\)-breakup reactions are used most frequently to determine the \(nn\) scattering length. However, the data obtained on the \(nn\) scattering length in such reactions at different laboratories \(\{\text{TUNL} [2, 3], \text{BONN} [4, 5], \text{TUNL-BONN} [6, 7], \text{and Institute for Nuclear Research (INR, Russian Academy of Sciences) [8–10]}\}\) after 1998 differ significantly.

It was assumed in [11] that a significant spread of the values of \(a_{nn}\) may be associated with a rather strong effect of three-nucleon \((3N)\) forces.

The dibaryon model of nuclear forces [12, 13] predicts a rather strong \(3N\) interaction between the dineutron singlet and the third particle because of scalar-meson exchange. The effect of this interaction on low-energy parameters of \(nn\) interaction may depend on the velocity at which the fragments fly apart.

The velocity at which breakup fragments fly apart can be calculated on the basis of the kinematics of the two-body reaction \(n + ^2\text{H} \rightarrow (nn) + p\) or \(d + ^2\text{H} \rightarrow (nn) + ^2\text{He}\). At a fixed time interval \(t\) (the choice of a specific value of \(t\) is immaterial because of obvious scaling), the parameter \(R\) (the distance to which the fragments move away from each other within the time \(t\)) depends on the relative velocity of the fragments—that is, on the final-state energy and masses of the
fragments. Since the velocity at which the fragments move apart is different in different experiments performed at different projectile energies, the parameter $R$ will also take different values. The larger the value of $R$, the higher the velocity at which the fragments move apart and, hence, the shorter the time it takes for them to leave the region where $3N$ forces are operative, with the result that the effect of $3N$ forces on the parameters of $nn$ interaction that are extracted from experimental data then becomes weaker.

In order to test the hypothesis that the extracted parameters of $nn$ interaction depend on the relative distance between the $nn$ pair and the third particle, we decided to perform additional investigations of the $nd$-breakup reaction for various predicted values of the parameter $R$. We assume that the investigation of the $nd$-breakup reaction at the low energies of $E_n = 8$ MeV ($R = 2.94$ fm) and $E_n = 11$ MeV ($R = 3.8$ fm) should lead to a stronger effect of $3N$ forces on the extracted parameters of $nn$ interaction and, accordingly, to $a_{nn}$ values that are larger in magnitude than those at neutron energies in the range of $E_n = 13–25$ MeV in the known experiments performed by the TUNL and BÖNN groups [2–7] and devoted to studying the $nd$-breakup reaction.

2. IMPLEMENTATION OF THE EXPERIMENT

A kinematically complete experiment devoted to studying the reaction $n + ^2$H $\rightarrow n + n + p$ at the energies of $E_n = 8 \pm 1$ MeV and $E_n = 11 \pm 1$ MeV was performed at the RADEX neutron channel of Institute for Nuclear Research (INR, Russian Academy of Sciences). In order to determine the energy of the virtual $nn$ state, $E_{nn}$, and the $nn$ scattering length $a_{nn}$ at this energy, it is necessary to detect two neutrons in coincidence that are emitted within a narrow cone of angles with respect to the direction of motion of their center of mass and to measure the energy of each neutron, $E_1$ and $E_2$, and the angle $\Theta$ between their momenta.

A schematic view of the experimental setup used is shown in Fig. 1. A beamstop for 209-MeV protons at the linear accelerator of INR was employed as a neutron source. Neutrons produced in a tungsten target 60 mm thick were collimated at an angle of $0^\circ$ over a length of 12 m in order to form a beam approximately 50 mm in diameter at a measuring deuterium target.

A C$_6$D$_6$ scintillator (EJ315) was used both as a deuterium target and as a detector of secondary protons. Secondary neutrons were detected by a hodoscope consisting of seven detectors. The central hodoscope detector was arranged at an angle of $40^\circ$ with respect to the neutron-beam axis at a distance of 150 cm from the deuterium target. The remaining six detectors were located on a circle in the plane orthogonal to the direction from the target to the central detector. The opening angles between the central detector and external detectors and between

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**Fig. 1.** Schematic view of the experimental setup at the RADEX channel of Institute for Nuclear Research (INR, Russian Academy of Sciences): (1) tungsten neutron-generating target, (2) collimator, (3) active C$_6$D$_6$ target simultaneously playing the role of a detector, and (4) hodoscope of neutron scintillation detectors.
all neighboring external detectors were $5^\circ$. The neutron energy was determined on the basis of the time of flight of the neutrons to the detector, the time signal from the active scintillation target being used as a starting signal for the time-of-flight system.

The signals from a secondary proton and all detectors of the neutron hodoscope came to the inputs of the CAEN-DT5742 digital signal processor, whose small step of time development made it possible to use it for a time analysis. The recording of oscillograms of these signals was triggered by the actuation of the internal discriminator by a signal from the active detector target. Digitized signals were logged into buffer memory and were transferred to the main computer as soon as the buffer was filled.

Information was processed in an off-line mode. The processing amounted to determining the pulse heights and areas, specifying the times of the appearance of signals in the detectors, and performing a numerical analysis of the pulse shape with the aim of discriminating neutron events from events initiated by photons. Selection of coincidence events involving a proton and two neutrons in neighboring detectors of the neutron hodoscope with an opening angle of $5^\circ$ was performed.

The energy spectrum of neutrons from the RADEX channel, which were incident to the target, is broad and includes all energies up to the limiting one that is equal to the proton-beam energy. A simultaneous detection of all three final-state particles (proton and two neutrons) makes it possible to reconstruct the energy of the primary neutron in the reaction $n + ^2H \rightarrow n + n + p$ for each detected event—that is, to sort all events in this energy.

3. SIMULATION

The $nn$ final-state interaction (FSI) manifests itself in the form of a maximum in the distribution of the reaction yield versus the relative energy of two neutrons,

$$\varepsilon = \frac{1}{2}(E_1 + E_2 - 2\sqrt{E_1E_2}\cos\Delta\Theta).$$

The shape of this distribution is sensitive to the $nn$ scattering length, $a_{nn}$. The distribution in question is frequently described by the Migdal–Watson formula

$$F_{MW} = A\frac{\sqrt{\varepsilon}}{\varepsilon + E_{nn}},$$

where $E_{nn}$ is the absolute value of the energy of the $1S_0$ virtual state of the $nn$ system (it can be obtained from a comparison of the experimental distribution with the results of the simulation for various values of $E_{nn}$), $\sqrt{\varepsilon}$ is a phase-space factor, and $A$ is a normalization factor.

The energy of the virtual level, $E_{nn}$, is related to the $nn$ scattering length, $a_{nn}$, by the equation

$$\frac{1}{a_{nn}} = -\left(\frac{m_nE_{nn}}{\hbar^2}\right)^{1/2} - \frac{1}{2}r_{nn} \frac{m_nE_{nn}}{\hbar^2} + \ldots,$$

where $r_{nn}$ is the effective radius of $nn$ interaction and $m_n$ is the neutron mass.
The energies measured in our experiment for two neutrons at an opening angle of about 5° correspond to the kinematical region where \( nn \) FSI manifests itself most strongly.

By means of codes for a kinematical simulation of reactions leading to the appearance of three particles in the final state [14], a detailed simulation of the reaction \( n + ^2H \rightarrow n + n + p \) was performed in order to determine necessary conditions and parameters of the experimental setup.

The kinematics of the reaction \( n + ^2H \rightarrow n + n + p \) is simulated in two steps. At the first step, one examines the formation of a neutron pair with invariant mass \( M_{nn} = 2m_n + E_{nn} \) in the two-body reaction \( n + ^2H \rightarrow ^2n + p \) and calculates the emission angles and kinetic energies for the proton, \( \Theta_p \) and \( E_p \), and for the center of mass of the \( nn \) pair, \( \Theta_{2n} \) and \( E_{2n} \), in the laboratory frame. The \( \varepsilon \) dependence of the reaction yield is taken into account via the number of simulated events with different \( \varepsilon \) according to the curves calculated by expression (2) at a specific value of the parameter \( E_{nn} \) (see Fig. 2). As a result, one introduces the dependence of the shape of the reaction-yield distribution on the energy of the virtual \( nn \) state (or on the \( nn \) scattering length).

At the second step, one examines the breakup of the \(^2n\) system, \(^2n \rightarrow n_1 + n_2\), and calculates the emission angles (\( \Theta_1 \) and \( \Theta_2 \)) and kinetic energies (\( E_1 \) and \( E_2 \)) of the two neutrons in the laboratory frame, taking into account experimental conditions, including the positions and number of detectors, as well as their angular and energy resolutions. Events where the proton hits the proton detector and where the two neutrons with an opening angle \( \Delta\theta \) hit simultaneously the respective two neutron detectors are selected among the total number of simulated events. For these events, the relative energy \( \varepsilon \) is calculated according to expression (1).

For the yield of the reaction \( n + ^2H \rightarrow n + n + p \) as a function of the relative energy \( \varepsilon \), the simulation described above gives the results corresponding to the experimental conditions at preset values of the primary-neutron energy, the neutron–neutron opening angle, and the energy of the virtual \( nn \) state. In Fig. 3, this dependence is contrasted against the experimental dependence of the yield of the \( nd \)-breakup reaction at \( E_n = 11 \pm 1 \) MeV. One can see that the shapes of the distributions are similar and that the experimental curve in the region of the low-energy peak lies between the simulated curves for \( E_{nn} = 0.06 \) and 0.15 MeV.

We have introduced the shape factor (SF), defining it as the ratio of the sum of events covering the whole peak region at small \( \varepsilon \) between 0 and \( \varepsilon_1 \) to the sum of events over a wide region of \( \varepsilon \) from 0 to \( \varepsilon_2 \).

Figure 4 shows the simulated shape factor SF (in this simulation, we took into account all parameters of the experiment) as a function of the virtual-state energy \( E_{nn} \). In the energy region around the virtual \( nn \) state (\( E_{nn} = 0.06 \)–0.25 MeV), the shape factor changes substantially.

In order to determine the value of \( E_{nn} \), the simulated dependence of SF was compared with SF\( _\text{exp} \) (see Fig. 4). The dashed straight lines in Fig. 4 show the value SF\( _\text{exp} \) within the error interval. At \( E_n = 11 \pm 1 \) MeV, the energy of the virtual \( nn \) state was found to be \( E_{nn} = 100 \pm 5 \) keV. The respective \( nn \) scattering length calculated according to expression (3) at \( r_{nn} = 2.83 \) fm is \( a_{nn} = -19.0 \pm 0.5 \) fm.

Thus, a comparison of the experimental value of SF with its simulated counterpart makes it possible to determine the energy of the virtual \( nn \) state, \( E_{nn} \), and accordingly the scattering length \( a_{nn} \).

For the experiment at \( E_n = 8 \pm 1 \) MeV, the procedure for the simulation and determination of SF\( _\text{exp} \) is similar. The values of \( E_{nn} = 93 \pm 4 \) keV and \( a_{nn} = -19.8 \pm 0.4 \) fm were obtained for, respectively, the energy of the virtual \( nn \) state and the \(^1S_0 \) \( nn \) scattering length.

4. DATA ANALYSIS

Figure 5a shows data on the \( nn \) scattering length that were obtained in the present study and in other studies devoted to the \( nd\)– and \( dd\)-breakup reactions [2–10]. In [11], it was assumed that substantial discrepancies between experimental results from different studies may be explained by the effect of three-nucleon (3N) forces, which depend on the velocity at which the \( nn \) pair and the charged fragment fly apart. The data under analysis can be approximated by a smooth curve representing the dependence of the scattering length on the parameter \( R \) determining the distance to which the fragments move away from each other over a fixed time.

In order to approximate available data on \( a_{nn} \), including the results of the present study, we employed the three-parameter exponential function

\[
a_{nn}(R) = a + b \exp(-R/r_0),
\]

which is shown in Fig. 5b. The parameter \( a \) determines the asymptotic value of \( a_{nn} \) found upon the extrapolation of this curve for \( R \rightarrow \infty \) and should be free from the contribution of 3N forces. The parameters \( a, b, \) and \( r_0 \) can be obtained from a \( \chi^2 \) analysis of the experimental data.

For the experimental data used, we have found a new parameter value of \( a \equiv a_{nn}(\infty) = -16.1 \) ±
Fig. 3. Comparison of the experimental (thick black curve) and simulated dependences of the yield of the \( nd \)-breakup reaction on \( \varepsilon \) at the experimental values of \( E_0 = 11 \pm 1 \) MeV, \( \Theta_{2n} = 40^\circ \), and \( \Delta \Theta = 5^\circ \) for the following values of the virtual-state energy \( E_{nn} \): (1) 0.06, (2) 0.15, and (3) 0.25 MeV. The dashed straight lines show the boundaries of event summation for calculating the shape factor SF: \( \varepsilon_1 = 0.03 \) MeV and \( \varepsilon_2 = 0.4 \) MeV.

Fig. 4. Comparison of \( SF_{\exp} = 0.239 \pm 0.004 \) (dashed straight lines with allowance for the error interval) with the results of the simulation of SF in the reaction \( n + ^2H \rightarrow n + n + p \). The primary-neutron energy is \( E_0 = 11 \pm 1 \) MeV, the \( ^2n \) emission angle is \( \Theta_{2n} = 40^\circ \), and the opening angle for secondary neutrons is \( \Delta \Theta = 5^\circ \).

0.1 fm. We assume that this value of \( a_{nn} \) is not subject to the effect of \( 3N \) forces and is in better agreement with pure nucleon–nucleon interaction. At the same time, the values obtained in the present study for the \( nn \) scattering length (\( a_{nn} = -19.8 \pm 0.4 \) and \( -19.0 \pm 0.5 \) fm at the neutron energies of 8 and 11 MeV, respectively) are large in magnitude and clearly demonstrate the importance of taking into account \( 3N \) forces in determining the parameters of \( nn \) interaction.
5. CONCLUSIONS

A kinematically complete experiment devoted to studying the $nd$-breakup reaction at the neutron energies of 8 and 11 MeV has been performed at the RADEX neutron channel of the Institute for Nuclear Research (INR, Russian Academy of Sciences).

From an analysis of the shape of the reaction yield as a function of the relative energy $\varepsilon$ of two neutrons, we have determined the low-energy parameters of $nn$ interaction—that is, the energy of the virtual $nn$ level and the $nn$ scattering length: $E_{nn} = 93 \pm 4$ keV and $a_{nn} = -19.8 \pm 0.4$ fm at the primary-neutron energy.
of 8 MeV and $E_{nn} = 100 \pm 5$ keV and $a_{nn} = -19.0 \pm 0.5$ fm at the primary-neutron energy of 11 MeV.

An analysis of the resulting values of the $nn$ scattering lengths, together with the data from other experiments that studied the $dd$- and $nd$-breakup reactions, has confirmed the hypothesis that $3\!N$ forces affect the values that are extracted for the parameters of $nn$ interaction from reactions involving few-nucleon systems. The inclusion of this contribution even within a simple three-parameter exponential dependence permits matching experimental data obtained at different laboratories (and in different years) and to obtain a new estimate of the asymptotic $nn$ scattering length, $a_{nn} = -16.1 \pm 0.1$ fm.

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