Bias in Absolute Magnitude Determination from Parallaxes

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1 INTRODUCTION

The general methods for dealing with bias in the analysis of astronomical (or other) data were set out by Eddington (1913, 1940, see also Dyson 1926). Although these are quite straightforward, the current literature shows that there is some uncertainty and misunderstanding in these matters. The present paper attempts to clarify the situation in the case of the derivation of absolute magnitudes from parallaxes.

Since the Hipparcos catalogue (Perryman et al. 1997) became available there has been increased use of parallaxes for absolute magnitude determination and future missions such as GAIA will lead to further work in this area. It is therefore important that there should be agreement on the question of bias correction.

If objects are selected for analysis on the basis of their observed parallaxes ($\pi$) or weighted (including selection or rejection) by the ratio of $\pi$ to its standard error, $\sigma_\pi$, then the result is subject to bias (Lutz & Kelker 1973). On the other hand, provided there is no selection or weighting according to $\pi$ or $\pi/\sigma_\pi$ and if the relative absolute magnitudes of the objects in a group are known then the method of reduced parallaxes can be used to convert relative values to absolute. In the next section the biases that can occur in this method are set out. Following this, the question of the determination of the absolute magnitudes of individual objects from their parallaxes will be considered.

2 THE METHOD OF REDUCED PARALLAXES

There have been a number of discussions of bias applicable to the method of reduced parallaxes but none dealing entirely satisfactorily with the situation likely to arise in practice. For instance Turon & Crézé (1977) consider only the case when all the objects have identical absolute magnitudes (i.e. there is no dispersion in absolute magnitude) and the discussion of Ljunggren & Oja is not completely relevant to the present purpose and seems to confuse the intrinsic dispersion in absolute magnitude with the uncertainty in the mean absolute magnitude derived from parallaxes. It should be noted that they, and the analysis of Malmquist (1920) to which they refer, consider only the case when all the objects of the relevant class down to a certain apparent magnitude are measured for parallax in the area of sky considered.

In the case of recent discussions of the Hipparcos parallaxes of Cepheids (e.g. Feast & Catchpole 1997 (=FC), Oudmaijer et al. 1998, Groenewegen & Oudmaijer 2000) various views on bias corrections have been put forward, none entirely satisfactory. In that particular case it is now generally agreed that any correction is negligibly small (FC, Groenewegen & Oudmaijer 2000, Lanoix et al. 1999) and this is confirmed by Monte Carlo simulations (Pont 1999). However occasions may well arise when these corrections are significant.

In the method of reduced parallaxes it is assumed that the objects in the group under discussion have not been selected by their measured parallaxes ($\pi$) or the ratio, $\pi/\sigma_\pi$. Consider first the case of objects with a mean absolute magnitude derived from parallaxes. The bias in the case of the derivation of absolute magnitudes of individual objects is also considered.

\[ 10^{0.2M} = \sum 0.01\pi 10^{0.2m_o} \cdot p / \sum p \]  \hspace{1cm} (1)

where the parallax is measured in milliarcsec, $m_o$ is the absorption free apparent magnitude and $p$ is the weight given by:

\[ (0.01\sigma_T 10^{0.2m_o})^2 = 1/p \]  \hspace{1cm} (2)

and where $\sigma_T$ is given by:

\[ \sigma_T^2 = \sigma_\pi^2 + b^2\pi^2 \sigma_{M_o}^2 (\sigma_{m_o}^2 + \sigma_{\pi L}^2). \]  \hspace{1cm} (3)

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Here, 
\[ b = 0.2 \log_{10} 10 = 0.4605, \]
\[ \pi_\text{M} \] is the photometric parallax of an object computed from \((m_\text{o} - M_\text{o})\), \(\sigma_{m_\text{o}}\) is the standard error of the reddening corrected apparent magnitude.

Equation 3 is from Koen & Laney (1998).

Equation 1 gives an unbiased estimate of \(10^{\pi_\text{M}a} \) only if \(\sigma_{M_\text{o}}\) is zero.

Put \( x = (m_\text{o} - M_\text{o}) \). This will differ from the true distance modulus by \(\epsilon\) (say), due to observational errors in \(m_\text{o}\) and intrinsic dispersion in \(M_\text{o}\). It is then evident that equation 1 yields an estimate of:

\[ 10^{\pi_\text{M}a} = 10^{\pi_\text{M}a + \epsilon} = e^{b_\text{M}o + \epsilon b}. \]  

Consider objects all of the same \(m_\text{o}\) (and \(x\)). From Malmquist (1920) or more compactly, from Feast (1972) where some of the present nomenclature and sign convention is used, we find:

\[ e^\sigma_o = e^{\sigma_o^2 / 2} v(x - \sigma_o^2) / v(x), \]  

(5)

where:

\[ \sigma_o^2 = \sigma_{m_o}^2 + \sigma_{M_o}^2, \]  

(6)

and \(v(x)\) is the frequency distribution of \(x\) which would have been found if a complete survey had been made. Note that this is the case whether or not the objects under consideration actually form a complete survey.

Evidently at any \(m_\text{o}\) an unbiased estimate of \(10^{\pi_\text{M}a}\) is obtained by multiplying the r.h.s. of equation 1 by the reciprocal of the r.h.s. of equation 5. Summed over all \(m_\text{o}\), as in a practical case, the final result will depend on \(v(x)\) which in general will depend on \(x\). Furthermore if one were analysing parallaxes over a significant galactic volume (as may well be possible with GAIA data), \(v(x)\) might not be the same in all heliocentric directions. In such cases the problem would benefit from Monte Carlo simulations (Pont 1999, Sandage & Saha 2002). However if the underlying density distribution is constant then the r.h.s. of equation 5 becomes \(10^{-2.5 \log^2 \sigma_x^2} \) (see for instance Feast (1972) equation 9). Thus provided \(\sigma_x\) is constant for the objects studied, this is simply a constant bias factor.

The relation between natural and logarithmic quantities shows that if \(\sigma_x\) is the standard error of the mean value of \(M_\text{o}\) derived as above, then:

\[ M_\text{o} = 5 \log(10^{\pi_\text{M}a}) - 0.23 \sigma_x^2. \]  

(7)

Thus for the case of a constant underlying space density the best estimate of \(M_\text{o}\) is given by:

\[ M_\text{o} = 5 \log(10^{\pi_\text{M}a}) + 1.15 \sigma_x^2 - 0.23 \sigma_x^2. \]  

(8)

Note that the coefficient of \(\sigma_x^2\) in this equation is different from that in the conversion between \(M_\text{o}\) and \(m_\text{o}\), the mean value for objects of a given \(m_\text{o}\) (=1.38). This latter factor is the well-known Malmquist (1920) bias, first given explicitly by Eddington (1914).

In the case of the Hipparcos parallaxes of Cepheids discussed by FC, the bias terms in equation 8 are negligibly small. The total bias correction is 0.010mag and this would change the PL zero-point from –1.43 to –1.42. The very small bias correction is due to the method of analysis adopted by FC. This was to reduce the intrinsic scatter about the PL relation by the combined use of a PL and PC relation for reddening correction and the determination of relative absolute magnitudes. If this is not done it might be necessary to consider whether Cepheids are distributed uniformly throughout a strip in the PL plane, at least at long periods. In that case, and assuming \(\sigma_{m_o}\) is small and a constant space density distribution, equation 5 above would need to be replaced by:

\[ e^\sigma_o = 3 \sinh(2b \Delta) / 2 \sinh(3b \Delta) \]  

(9)

where \(\Delta\) is the halfwidth of the strip in magnitudes. If one adopts \(2 \Delta \sim 0.7\) mag at \(V\) from the longer period LMC Cepheids (Caldwell & Coulson 1986), the correction factor would be \(\sim +0.05\) mag.

The above considers the case when the objects have a mean absolute magnitude per unit volume of \(M_\text{o}\) with a gaussian scatter \(\sigma_{M_\text{o}}\). In a number of cases the relative absolute magnitudes of the objects concerned are known through the measurement of some auxiliary quantity, \(y\), and a relation of the form:

\[ M_\text{o} = A y + B \]  

(10)

where \(A\) is a known constant. Two separate cases then arise. If the measuring error in \(y\), \(\sigma_y\), introduces an error in \(M_\text{o}\) which is negligibly small compared with the intrinsic dispersion in \(M_\text{o}\) at a given \(y\), then the results given above apply in this case with obvious modifications to equation 1 (see Feast 1987). If this is not the case, then the quantities \(v(x)\) etc. have to be replaced by \(P(x)\) etc., where \(P(x)\) is the distribution in \(x\) of the objects actually under discussion (i.e. taking into account that only a fraction, \(f(x)\), of all the objects may have been observed at a given \(x\) (see Feast 1972). This procedure essentially involves using the inverse solution of equation 10. In the case of the Hipparcos Cepheids, the quantity \(y\) is the period and this is very accurately known, so that the earlier discussion holds in this case.

Even in the case of Mira variables, the percentage uncertainty in the periods probably makes a much smaller contribution to the uncertainty in \(M_\text{o}\) than the intrinsic dispersion. Thus equation 8 can be applied to the discussion of the Hipparcos parallaxes of these stars by Whitelock & Feast (2000). These workers found an infrared PL zero-point of \(+0.84 \pm 0.14\). Using their data one obtains from equation 8 a correction of \(\sim 0.02\) mag, making the zero-point \(+0.86 \pm 0.14\).

There is another source of bias that should be discussed. In an analysis it is necessary to work with \(m_\text{o}\), the absorption free absolute magnitude. Evidently if absorption is present, objects of a given \(m_\text{o}\) will have a range of uncorrected apparent magnitudes and selection according to apparent magnitude will affect the bias correction. In general absorption increases with distance. Thus if the fraction of objects of a given apparent magnitude observed decreases with increasing apparent magnitude, this will have the effect of reducing the fraction of objects of a given \(m\) whose true distance modulus is greater than \((m_\text{o} - M_\text{o})\). Thus absorption, if it affects the bias, is likely to do so by reducing the number of objects in the sample whose luminosities are greater than the average at a given \(m_\text{o}\). This means for instance that, if anything, the coefficient of the second term of the r.h.s of equation 8 will need decreasing. Thus the value of \(M_\text{o}\) produced from an unchanged equation 8 would be too faint.
3 OBSERVATIONS OF INDIVIDUAL OBJECTS

Provided there is no systematic error in the measuring process, an observed parallax is not in itself biased. This was, in fact, clearly recognized by Lutz & Kelker (1973) who point out that the distribution of measured parallaxes about the true parallax is expected to be gaussian. So provided the objects concerned are not chosen or weighted by their measured $\pi$ or by $\sigma_{\pi}$, an analysis is not subject to bias of the Lutz-Kelker type. This has been stressed recently by e.g. Whitelock & Feast (2000) and Groenewegen & Oudemaijer (2000).

It is of interest to consider the case when only one object of a class is measured, as in the important Hubble Space telescope (HST) observations of the parallaxes of RR Lyrae and $\delta$ Cephei (Benedict et al. 2002a, Benedict et al. 2002b). These stars were presumably chosen as bright members of their class and their HST parallaxes subsequently measured. The absolute magnitudes derived from the parallaxes are therefore not subject to Lutz-Kelker bias. However, since the method of determining an absolute magnitude from a single object is effectively the method of reduced parallaxes applied to one member alone of the class, the bias corrections outlined above will be required, i.e. for the case of an adopted uniform space distribution of the objects of the class, equation 8, if an estimate of $M_\odot$ is required. If the best estimate of the absolute magnitude of the individual object itself is wanted then, obviously, the term in $\sigma_{\pi}$ in equation 8 is not required.

In the case of $\delta$ Cephei (Benedict et al. 2002b) and adopting their parallax, apparent magnitude and reddening, and with $\sigma_{M_\odot} = 0.21$ (Caldwell & Coulson 1986), $\sigma_{\pi} = 0.1$ and assuming $\sigma_{\pi}$ can be neglected one obtains an estimate of $M_\odot$ of $-3.41 \pm 0.10$ and a zero-point of the PL relation adopted by FC of $-1.36 \pm 0.10$. Alternatively one can use the PL and PC relations together for the reasons discussed above to obtain the reddening and absolute magnitude. This gives $M_\odot = -3.37 \pm 0.10$ and a PL zero-point of $-1.32 \pm 0.10$ which may be compared with the FC zero-point (corrected as above) of $-1.42 \pm 0.12$ (where the standard error of PC has been increase from 0.10 for reasons given in Feast 1999).

Evidently the agreement is good, as it is with other Cepheid zero-point estimates (see e.g. Feast 2002). Note that, although uncertainty in the reddening is a limiting factor in these determinations of absolute magnitude, it is not important in the use of Cepheids as distance indicators so long as a consistent reddening scale is used for both calibrators and programme stars. This is most easily accomplished using a standard PC relation. The application of the Cepheid scale to derive the distance of the LMC is made somewhat uncertain by the need for rather poorly known metallicity corrections. However, this is not a problem in their use in the HST "key" programme on extragalactic Cepheids (Freedman et al. 2001) since the weighted mean metallicity of these galaxies is close to solar (Feast 2001).

In the case of RR Lyrae, using the adopted parallax, apparent magnitude and reddening from Benedict et al. (2002a) and assuming RR Lyrae variables of the relevant metallicity ([Fe/H] = $-1.39$) uniformly fill an instability strip of width $\sim 0.4$mag, as is the case in globular clusters (see Sandage 1990, especially fig. 16), one obtains as an estimate of the absolute magnitude of RR Lyrae variables an [Fe/H] of $-1.39$, $M_\odot = +0.64 \pm 0.11$. It has of course to be realized that, especially in the case of single objects, the uncertainty in any applied bias corrections is large.

Obviously the same general arguments apply if more than one object of a class is observed. Bias of the Lutz-Kelker type arises when the individual absolute magnitudes of objects derived from parallaxes are combined by weighting them using the observed parallaxes. The effect of this on the weighting is illustrated in a practical case in Feast (1998). In some at least of these cases there will also be selection by apparent magnitude, so that corrections such as that given by equation 8 will apply in addition to the Lutz-Kelker correction.

4 CONCLUSION

The bias corrections to the absolute magnitude scales of Cepheids, RR Lyrae and Miras discussed in this paper are all relatively small. However, with the forthcoming great improvements in parallax measurements expected from GAIA and other space missions, bias corrections of this type will become increasingly significant relative to other sources of uncertainty.

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REFERENCES

Benedict G.F. et al., 2002a, AJ, 123, 473
Benedict G.F. et al., 2002b, astro-ph/0206212
Caldwell J.A.R., Coulson I.M., 1986, MNRAS, 218, 223
Dyson F., 1926, MNRAS, 86, 686
Eddington A.S., 1913, MNRAS, 73, 359
Eddington A.S., 1914, Stellar Motions and the Structure of the Universe, MacMillan, London, p.172
Eddington A.S., 1940, MNRAS, 100, 354
Feast M.W., 1972, Vistas in Astronomy, 13, 207
Feast M.W., 1987, The Observatory, 107, 185
Feast M.W., 1998, Mem. Soc. Ast. It., 69, 31
Feast M.W., 1999, PASP, 111, 775
Feast M.W., 2001, Odessa Pubs., 14, 141
Feast M.W., 2002, astro-ph/0010595
Feast M.W., Catchpole R.M., 1997, MNRAS, 286, L1
Freedman W.L. et al., 2001, ApJ, 553, 47
Groenewegen M.A.T., Oudmaijer R.D., 2000, A&A, 356, 849
Koen C., Laney C.D., 1998, MNRAS, 301, 582
Lanoix P., Faturel G., Garnier R., 1999, MNRAS, 308, 969
Ljunggren B., Oja T., 1965, Arkiv f. Astron., 3, 501
Lutz T.E., Kelker D.H., 1973, PASP, 85, 573
Malqueist K.G., 1920, Lund Medd. Ser. II, no.22
Oudmaijer R.D., Groenewegen M.A.T., Schrijver H., 1998, MNRAS, 294, L55
Perryman M.A.C. et al., The Hipparcos and Tycho Catalogues, ESA SP-1200
Pont F., 1999 in ASP Conf. Ser. 167, Harmonizing the Cosmic Distance Scale in the Post-Hipparcos Era, ed. Egret D, Heck A., San Francisco, p.113
Sandage A., 1990, ApJ, 350, 663
Sandage A., Saha A., 2002, astro-ph/0201381

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Turon Lacarrieu C., Crézé M., 1977, A&A, 56, 273
Whitelock P.A., Feast M.W., 2000, MNRAS, 319, 759