Design and optimization of a Holweck pump via linear kinetic theory

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Abstract. The Holweck pump is widely used in the vacuum pumping industry. It can be a self standing apparatus or it can be part of a more advanced pumping system. It is composed by an inner rotating cylinder (rotor) and an outer stationary cylinder (stator). One of them, has spiral guided grooves resulting to a gas motion from the high towards the low vacuum port. Vacuum pumps may be simulated by the DSMC method but due to the involved high computational cost in many cases manufacturers commonly resort to empirical formulas and experimental data. Recently a computationally efficient simulation of the Holweck pump via linear kinetic theory has been proposed by Sharipov et al [1]. Neglecting curvature and end effects the gas flow configuration through the helicoidal channels is decomposed into four basic flows. They correspond to pressure and boundary driven flows through a grooved channel and through a long channel with a T shape cross section. Although the formulation and the methodology are explained in detail, results are very limited and more important they are presented in a normalized way which does not provide the needed information about the pump performance in terms of the involved geometrical and flow parameters. In the present work the four basic flows are solved numerically based on the linearized BGK model equation subjected to diffuse boundary conditions. The results obtained are combined in order to create a database of the flow characteristics for a large spectrum of the rarefaction parameter and various geometrical configurations. Based on this database the performance characteristics which are critical in the design of the Holweck pump are computed and the design parameters such as the angle of the pump and the rotational speed, are optimized. This modeling may be extended to other vacuum pumps.

1. Introduction
The choice of the equipment that is used for the creation and maintenance of vacuum conditions depend on various parameters such as the required pressure, the throughput and the available time for the process. In many cases, the use of a single vacuum pump is not enough and a combination of pumps is needed. It is common to have a first stage where rough vacuum conditions are created and a second one for the achievement of the desired pressure.

The optimization of the design and the operational parameters of the pumps has lead to the development of numerical tools for the simulation of the flow in the pump. In many approaches the Navier-Stokes equations have been used with the corresponding slip boundary conditions. This method is well tested but its range of applicability is limited to rough vacuum conditions. This is due to the fact that in lower pressures the assumption of the continuum medium collapses and the recovered results are not reliable [2].
Another method that can be used for the simulation of flow systems in high vacuum conditions is the mesoscopic approach with either stochastic or deterministic tools. The first one is the Direct Simulation Monte Carlo method (DSMC) [3] where computational molecules move, reflect from solid boundaries and collide to each other so as to statistically mimic the behavior of real molecules. Each model particle in the simulation represents a large number of real molecules in the physical system. The methodology is stochastic in nature since several modules in the algorithm, such as intermolecular collisions, are modeled in a probabilistic manner using random numbers. The state of the system is defined by the position and velocity vectors of the model particles. The most important drawback of DSMC is that it is appropriate for relatively high Mach numbers since the statistical noise can be significant.

The deterministic approach is based on the solution of the Boltzmann equation or the corresponding kinetic equations [4, 5]. The main unknown is the distribution function, while the macroscopic quantities can be recovered as its moments. The most common and computationally efficient method is by discretizing the kinetic equation in the molecular velocity space by the discrete velocity method (DVM) and by a finite differencing scheme in the physical space. This approach is superior to the DSMC method when linearized flows are tackled.

In the present paper the simulation of the Holweck pump based on linear kinetic theory is presented. This is a vacuum pump that is used either as a single apparatus or as a first stage for a pumping system. Simulations of the Howeck pump have been carried out especially in the last years. Most of them are using either the DSMC [6, 7] or the Navier-Stokes [8, 9] equation in order to recover the results and refer to various geometries. Recently, the deterministic approach has been implemented [1] but the details of the pump dimensions and the function characteristics are not provided. The methodology proposed in this paper is followed here and more specifically the discrete velocity method (DVM) is applied to solve the BGK kinetic equation, which is valid for the case under investigation since there are no significant temperature variations. In addition, the equations are linearized, something that is justified by the fact that the length of the pump channels is much bigger than the characteristic length of the cross section and the local forcing term is relatively small. Results are presented for the mass flow rates and the characteristic curves, while a preliminary parametric study is performed for the optimization of the design parameters.

2. Statement of the problem
The Holweck pump consists of two coaxial cylinders. One of them is stationary and the other one is rotating, while the inner cylinder has helicoidal grooves printed on it. The rotation causes a flow that can result to a pressure difference between the two ends of the cylinder. The pressure at the high vacuum end is \( P_h \) while the corresponding pressure at the fore vacuum end is symbolized as \( P_f \).

![Figure 1. Typical design of the Holweck pump and basic dimensions of the inner cylinder.](image-url)
The flow is fully three-dimensional, but a simplification can be achieved if the effect of the curvature of the cylinder is neglected as well as the end effects at the inlet and outlet of the pump. This approach can be justified by the fact that the ratio of the characteristic length of the grooves and the radius of the inner cylinder is less than 5% while the length of the channels is big compared to the characteristic length of its cross section.

This approach gives the opportunity to have the solution of the whole flow field by integrating the partial solutions of every cross section of the channel. On the other hand, if the equations are non-dimensionalized by the local forcing term, then only one cross section has to be solved, since the flow can be assumed as a fully developed 3d flow in a duct. This procedure simplifies the solution and decreased drastically the computational time.

The exact geometry of the pump is shown in Figs. 1 and 2 and Table 1. The geometry of the pump is identical to the one used in [7] and it corresponds to a typical design of the Holweck pump. It is obvious that the same approach can be easily applied to any other configuration.

In the present work, this configuration is kept constant except of the angle $\theta$ of the channels. This is one of the parameters (the other one is the rotational speed) which are optimized by using the methodology presented in the next paragraphs.

| Table 1. Basic parameters of the Holweck pump under investigation. |
|---------------------------------------------------------------|
| Parameter                      | Value                  |
| Inner Diameter ($D_{in}$)     | 168.6 mm               |
| Outer Diameter ($D_{out}$)     | 170.4 mm               |
| Pump Length                    | 132.0 mm               |
| Number of Grooves              | 7                      |
| Rotational speed (n)           | 1200-4800 rpm          |
| Angle ($\theta$)               | $4^0 - 24^0$           |
| Gas molecular mass             | 4.002602 (He)          |
| Boltzmann constant             | 8.314462 $k_B$        |

![Figure 2. Cross section A-A’ of the grooves with dimensions and the coordinate system with its origin](image)
3. Formulation
As it has been already stated, the method followed in the present work is the simulation of the flow by the BGK kinetic equation, which can be deduced from the Boltzmann equation if the collision term is replaced by the BGK model. Then the equation takes the form

$$\xi \cdot \frac{\partial f}{\partial r} = \frac{P}{\mu} (f - f^{eq})$$

(1)

where $\xi$ is the microscopic velocity, $P$ the pressure, $r$ the position vector and $f = f(r, \xi)$ the distribution function. Finally, $f^{eq} = f^{eq}(r, \xi)$ represents the local Maxwellian which is

$$f^{eq} = n(r) \left[ \frac{m}{2\pi k T(r)} \right]^{\frac{3}{2}} \left\{ \frac{m(\xi - \bar{u}(r))^2}{2k T(r)} \right\}$$

(2)

Here $m$ is the molecular mass, $k$ is the Boltzmann constant, $n(r)$ is the number density, $u(r)$ is the macroscopic velocity and $T(r)$ is the temperature. These quantities can be calculated as moments of the distribution function

$$n(r) = \int f(r, \xi) d\xi \quad \bar{u}(r) = \int \xi f(r, \xi) d\xi \quad T(r) = \frac{m}{3n(r)k} \int (\xi - \bar{u}(r))^2 f(r, \xi) d\xi$$

(3)

while the shear stress tensor can be calculated as

$$P_{i,j}(r) = m \int (\xi_i - u_i)(\xi_j - u_j) f(r, \xi) d\xi \quad i, j = x, y, z$$

(4)

The basic parameter of the flow is the Knudsen number which determines the rarefaction of the flow. In this work, the rarefaction parameter $\delta$ is used, which is proportional to the inverse Knudsen number and is defined as

$$\delta = \frac{P_0 D_h}{\mu_0 u_m} \sim \frac{1}{Kn}, \quad u_m = \sqrt{\frac{2k_B T_0}{m}}$$

(5)

where $P$ is the local pressure, $D_h$ the hydraulic diameter, $\mu_0$ the viscosity at the reference pressure $T_0$ and $u_m$ is the most probable molecular velocity. Since the length of the channels is big enough compared to $D_h$ and the speed of the outer cylinder is much smaller than $u_m$, the kinetic equation can be linearized, non-dimensionalized and solved numerically. On the other hand, taking into account the fact that the flow is linear, a decomposition can be applied and solve four subproblems: Longitudinal Poiseuille and Couette flow and Transversal Poiseuille and Couette flow. Then the results can be combined properly in order to have the full solution. This approach which has been proposed by Sharipov [1], has been proved efficient and is followed in the present work. Applying this procedure simplifies more the numerical solution of the flow since instead of a 3-d problem one has to solve four 2-d problems. In addition, it gives flexibility to the solution because parameters such as the angle of the channel and the velocity of the cylinder are taken in to account only when the results of the subproblems are combined. So, for a given cross section, a data base of the subproblems results can be created and an optimization with respect to the angle or the velocity can be achieved.

For all the four problems, the non-dimensional parameter in the physical space is the hydraulic diameter of the channel cross section

$$D_h = 4 \frac{A}{Pr} = 2 \frac{d \times l + a \times (b + d)}{(b + d + l + a)}$$

(6)
while the velocity vectors are non-dimensionalized as $c = \xi / u_m$. In the following paragraphs, the formulation for each of the four subproblems and the procedure for the results combination are presented. It is noted that the following approach is included here for completeness, eventhough someone can find it in [1]. In addition, here the parameter for the non-dimensionalization of length is the hydraulic diameter instead of the height of the groove and for the two transverse flows the no penetration boundary condition is used while in [1] it is not clear which is the boundary condition.

3.1. Longitudinal Poiseuille flow

The present flow is caused by a pressure difference along the $z$-axis of the channel. The distribution function is linearized as

$$f(r, \xi) = f^{eq}(c) \left[1 + (h(x, y, c) - z)X_z\right]$$

$$X_z = \frac{D_h dP}{P} \ll 1$$

and by taking into account that the flow is considered fully developed and the assumption that the density and the temperature over the whole cross section remain constant, the kinetic equation takes the form

$$c_x \frac{\partial h}{\partial x} + c_y \frac{\partial h}{\partial y} = \delta(2c_zu_z - h) + c_z$$

In addition, the fact that the distribution function $h$ does not change along the $z$-axis, allows us to eliminate one of the microscopic velocity’s components. If the projected distribution function is defined as

$$\Phi(x, y, c_x, c_y) = \frac{1}{\sqrt{\pi}} \int h(x, y, c)e^{-c_x^2}dc_x,$$

the kinetic equation then takes the form

$$c_x \frac{\partial \Phi}{\partial x} + c_y \frac{\partial \Phi}{\partial y} = \delta(u_z - \Phi) + \frac{1}{2}$$

which is solved in order to recover the macroscopic quantities. Diffuse boundary conditions are used for all the solid boundaries and periodic boundary conditions at $-\frac{b + d}{2D}$ and $\frac{b + d}{2D}$. It is noted that this is also the treatment for the boundary conditions at the longitudinal Couette flow.

The dimensionless velocity $u_z$ and stress tensor $\Pi_{yz}$ are given as

$$u_z(x, y) = \int \int \Phi e^{-c_x^2 - c_y^2}dc_xdc_y$$

$$P_{yz}(x, y) = \int \int c_y \Phi e^{-c_x^2 - c_y^2}dc_xdc_y$$

The dimensional velocity and stress tensor are

$${\hat u}_z = -u_zu_mX_z$$

$${\hat P}_{yz} = -2P_{yz}PX_z$$

Finally the dimensionless mass flow rate and the reduced drag coefficient on the outer cylinder surface is given as

$$G_z^P = \int_A u_z dx dy$$

$$\Pi_z^P = \int_{\frac{b+d}{2\pi}}^{\frac{b+d}{2\pi}} P_{yz} dx$$

and the dimensional flow rate is

$$\dot M_z^P = \rho u_m D_h^2 X_z G_z^P$$
3.2. Longitudinal Couette flow

When the flow due to the motion of the upper plate in the z-direction is considered, the distribution function is linearized as follows

\[ f(r, \xi) = f_{\text{eq}}(c) \left[ 1 + h(x, y, c) \frac{U_z}{u_m} \right] \]  

in this case the kinetic equation becomes

\[ c_x \frac{\partial h}{\partial x} + c_y \frac{\partial h}{\partial y} = \delta(2c_z u_z - h) \]  

again, \( c_z \) is eliminated by projecting the kinetic equation on the velocity space

\[ c_x \frac{\partial \Phi}{\partial x} + c_y \frac{\partial \Phi}{\partial y} = \delta(u_z - \Phi) \]  

The dimensionless macroscopic quantities are given by Eq.(11). The dimensional velocity and stress tensor are

\[ u_z = u_z U_z \quad \hat{p}_{yz} = -2P_{yz} \frac{U_z}{u_m} \]  

The dimensionless mass flow rate and the reduced drag coefficient on the outer cylinder surface are

\[ G_z^C = \int_A u_z dxdy \quad \Pi_z^C = \int_{x_{\text{b-d}}}^{x_{\text{b+n}}} P_{yz} dx \]  

while the dimensional mass flow is

\[ \dot{M}_z^C = \rho u_m D_h^2 \frac{U_z}{u_m} G_z^C \]  

It has to be noted that by using the Onsager-Casimir theory the following relation is recovered

\[ G_z^C = 2 \Pi_z^P \]  

3.3. Transversal Poiseuille flow

When the flow is caused by a pressure difference along the x direction, the first difference is that there are two components of the velocity i.e. \( u_x \) and \( u_y \). In addition, the density variations can not be neglected. On the contrary, the temperature perturbations are small and the flow can be assumed isothermal [10]. Finally the distribution function is linearized as

\[ f(r, \xi) = f_{\text{eq}}(c) \left[ 1 + (h(x, y, c) - x)X_x \right] \]  

the new form of the kinetic equation after the projection is

\[ c_x \frac{\partial h}{\partial x} + c_y \frac{\partial h}{\partial y} = \delta(\rho + 2(c_x u_x + c_y u_y) - h) + c_x \]  

Again, using the fact that the distribution function \( h \) does not change along the z-axis, allows us to eliminate one of the microscopic velocity’s components. The projected distribution function is defined as

\[ \Phi(x, y, c_x, c_y) = \frac{1}{\sqrt{\pi}} \int h(x, y, c)e^{-c^2} dc_z \]
and the kinetic equation then takes the form

\[
  c_x \frac{\partial \Phi}{\partial x} + c_y \frac{\partial \Phi}{\partial y} = \delta (\rho + 2 c_x u_x + 2 c_y u_y - \Phi) + c_x \tag{25}
\]

The existence of density variations makes the use of the typical Maxwell boundary conditions inappropriate for the flow under investigation and the no-penetration boundary condition is used. According to it, on the walls a new parameter \( \rho_w \) is calculated in order to satisfy the equilibrium of the momentum on the wall in the vertical direction. This is a parameter without a physical meaning, but it allows us to ensure that no momentum is crossing the solid boundaries. The exact expressions for the estimation of \( \rho_w \) can be found in [11].

The dimensionless velocity \( u_x \) and stress tensor \( P_{xy} \) are given as

\[
  u_x(x,y) = \int \int c_x \Phi e^{-c_x^2 - c_y^2} d\xi d\eta \quad
  u_y(x,y) = \int \int c_y \Phi e^{-c_x^2 - c_y^2} d\xi d\eta \\
  P_{xy}(x,y) = \int \int c_x c_y \Phi e^{-c_x^2 - c_y^2} d\xi d\eta \tag{26}
\]

The dimensionless velocity and stress tensor are

\[
  \hat{u}_x = -u_x u_m X_x \quad \hat{u}_y = -u_y u_m X_x \quad \hat{P}_{xy} = -2 P_{xy} P X_x \tag{27}
\]

Finally the dimensionless mass flow rate and the reduced drag coefficient on the outer cylinder surface is given as

\[
  G^P_x = \frac{1}{ \frac{\partial h}{ \partial y} } u_x \frac{y}{ h } \quad \Pi^P_x = \frac{ \frac{b+d}{2 \pi} }{ \frac{b+d}{2 \pi} } P_{xy} \frac{dy}{ dx } \tag{28}
\]

the dimensional flow rate is

\[
  \dot{M}^P_x = \rho u_m D_h^2 X_x G^P_x \tag{29}
\]

3.4. Transversal Couette flow

The last flow that has to be considered is the one due to a motion of the upper plate in the x-direction. As in the corresponding flow due to pressure gradient, three macroscopic quantities are involved in the kinetic equation ie. \( \rho, u_x \) and \( u_y \). The linearization of the distribution function is

\[
  f(r, \xi) = f^{\text{eq}}(c) \left[ 1 + h(x,y,c) \frac{U_x}{u_m} \right] \tag{30}
\]

and the kinetic equation becomes

\[
  c_x \frac{\partial h}{\partial x} + c_y \frac{\partial h}{\partial y} = \delta (\rho + 2 c_x u_x + 2 c_y u_y - \Phi) \tag{31}
\]

and y using the projection procedure

\[
  c_x \frac{\partial \Phi}{\partial x} + c_y \frac{\partial \Phi}{\partial y} = \delta (\rho + 2 c_x u_x + 2 c_y u_y - \Phi) \tag{32}
\]

Here, the dimensionless macroscopic quantities are given by Eq.(26) and the corresponding dimensional by Eq.(27). The boundary conditions used, are identical with the corresponding
of the Transversal Poiseuille flow. The dimensionless mass flow rate and the reduced drag coefficient on the outer cylinder surface are

\[ G_x^C = \int \frac{b+d}{2d} u_x dy \quad \Pi_x^C = \int \frac{b+d}{2d} P_{xy} dx \]  

(33)

while the dimensional mass flow is

\[ \dot{M}_x^C = \rho u_m D_h^2 \frac{U_x}{u_m} G_x^C \]  

(34)

It has to be noted that by using the Onsager-Casimir theory again for the two transversal flows the following relation is recovered

\[ G_x^C = 2 \Pi_x^D \]  

(35)

4. Numerical scheme

For the four subproblems the kinetic equation has to be solved in order to recover the macroscopic quantities and the dimensionless flow rate. In order to do so in the present work the DVM method is used.

The main idea of the method is that the kinetic equation is solved for a set of discrete microscopic velocity vectors. Then numerical integration is applied in order to recover the moments of the distribution function, which are in fact the macroscopic quantities. The discrete velocities are chosen carefully and most often they are the roots of an orthogonal polynomial, at least as far as the magnitude of the velocities is considered, in order to recover the integrals with the best accuracy for a given number of velocities. Depending on the rarefaction of the flow, different number of velocities is required. In general, the more dense a flow is, the less velocities are required.

In the present work, 16 values have been used for the magnitude of the velocities and 400 angles, since a polar coordinates system is used for the microscopic velocities space. On the other hand, when the rarefaction parameter is larger than 15, the discrete angles are reduced to 80.

In the physical space, the grid used was uniform with \( \Delta x = \Delta y = 0.1 mm \). For more dense flows (\( \delta > 1 \)) the grid lattice was tripled (\( \Delta x = \Delta y = 1/30 \) ) since it is known that higher values of \( \delta \) require more dense grids. The numerical scheme used for all the four syb-problems, is a typical central-difference scheme but it is applied on the characteristic of the microscopic velocity since it gives more accurate results due to the Lagransian nature of the Boltzmann equation and is described in detail in [11].

5. Overall quantities

For the characterization of a pump, one of the quantities required is the pressure difference created and the corresponding throughput. The most important problem is that the dimensional quantities \( G \) recovered by the numerical solution of the four subproblems depend on the local pressure and on the local rarefaction parameter \( \delta \). This is also the reason for the variance of their values while the mass flow rate has to be constant for every cross section of the pump.

In order to solve this problem the quantity \( G_\eta \) is defined which is related to the mass flow rate as

\[ \dot{M}_\eta = \frac{2P_h}{u_m} D_h^2 G_\eta \]  

(36)

with \( P_h \) being the pressure on the high vacuum chamber. Since all the other quantities are constant, \( G_\eta \) should not be dependent on the position of the cross section.
Application of the mass conservation law in the triangle of Fig.3 gives
\[ \dot{M}_\eta = \dot{M}_x + \dot{M}_z. \] (37)

In addition, from Eqs (14,20) it is deduced that
\[ \dot{M}_z = \dot{M}_z^P + \dot{M}_z^C = \rho u_m D_h^2 \left\{ - G_z^P X_z + G_z^C U_z \right\} \] (38)

and since
\[ X_z = - \frac{D_h}{P} \frac{dP}{d\eta} \sin(\theta), \quad U_z = - U \cos(\theta) \] (39)

it is concluded that
\[ \dot{M}_z = \rho u_m D_h^2 \left\{ \frac{D_h}{P} \frac{dP}{d\eta} \sin(\theta) G_z^P - \frac{U \cos(\theta)}{u_m} G_z^C \right\} \] (40)

Accordingly for the x-direction
\[ \dot{M}_x = \dot{M}_x^P + \dot{M}_x^C = \rho u_m D_h \frac{l_x}{D_h} \left\{ - G_x^P X_x + G_x^C \frac{U_x}{u_m} \right\} \] (41)

with
\[ X_x = \frac{D_h}{P} \frac{dP}{d\eta} \cos(\theta), \quad U_x = U \sin(\theta) \] (42)
and finally
\[
\dot{M}_x = \rho u_m D_h^2 \frac{l_x}{D_h} \left\{ \frac{D_h}{P} \frac{dP}{d\eta} \cos(\theta) G_x^P + \frac{U \sin(\theta)}{u_m} G_x^C \right\}
\] (43)

\(l_x\) is the dimensional length of side \(x\). By substituting Eq. (40) and (43) to Eq. (37) a differential equation for the local pressure is recovered
\[
D_h \frac{dP}{d\eta} = \frac{\sin(\theta)}{G_x^P \sin^2(\theta) + \frac{D_h}{l_x} \cos^2(\theta) G_x^P} \left\{ P \frac{U}{u_m} \cos(\theta) \left[ G_x^C - \frac{l_x}{D_h} G_x^C \right] - G_x P_h \right\}
\] (44)

For a known \(P_h\), Eq. (44) can be solved and the pressure distribution for the whole length of the pump recovered. It has to be noted that on every cross section, the values of the dimensionless flow rates for the local \(\delta\) have to be used. Finally the pumping speed \(S\) and the throughput \(Q\) can be found as
\[
S = u_m D_h^2 G_x N_{gr} \quad Q = S P_h = P_h u_m D_h^2 G_x N_{gr}
\] (45)

where \(N_{gr}\) is the number of grooves.

6. Results and Discussion

6.1. Partial solutions

The first step for the recovery of the overall quantities is the creation of a full database including the dimensionless flow rate for the four subproblems described above. The results are presented in Figs. 4 and 5. They cover the range \(0 \leq \delta \leq 200\). All the quantities have been obtained by using the developed kinetic numerical codes. As it can be seen, the dimensionless flow rate for the longitudinal flows is an order of magnitude larger than the corresponding transversal flows. This is due to the chosen geometry and more specifically to the small gap between the two cylinders. The relative small effect of the transversal compared to the longitudinal phenomena is desirable because these flows in fact decrease the efficiency of the pump since these are secondary flows where the gas returns back towards the high vacuum chamber. On the other hand it is obvious that for \(\delta \leq 10\) the Poiseuille and the Couette flow rate are of the same magnitude but for larger values the Poiseuille flow becomes dominant. Of course, the exact contribution of each flow can be seen only when the operational parameters, such as rotational speed and angle of the channels, are defined.
In Fig. 6 the drag coefficient is plotted for the two Couette flows. Again the phenomena of the transversal flow are much smaller than the corresponding of the longitudinal. It has to be mentioned here that the drag coefficient for the Poiseuille flows can be easily deduced if Eqs. (21) and (35) are used.

6.2. Parametric analysis
One of the parameters that are expected to affect the performance of the pump is the angle of the grooved channels with respect to the front surface of the pump (see Fig. 1). The angle in the present work is assumed to be constant for the whole length of the pump. In Fig. 7 the dependence of the exit pressure on the angle of the pump is examined. As it can be seen, the influence of angle is strong and the optimum angle depends on the rarefaction at the inlet of the pump. For $\delta_h = 1$ it seems that the optimum angle is close to $\phi = 12^0$, while for $\delta_h = 0.01$ it is about $\phi = 15^0$. This correlation is expected even though the exact value of the optimal angle can not be anticipated without numerical or experimental results. It can be deduced that a gradual change of the angle along the pump or a multi-stage pump of equal length but different angles for each stage can be much more efficient.

Finally, on Fig. 8 the influence of the rotating cylinder speed is examined. The increase of the rotational speed is increasing the pressure difference which is the expected behavior.
Figure 7. Characteristic curves for various values for angle, \( n = 2400 \), and \( \delta_h = 1 \) (left) and \( \delta_h = 0.01 \) (right).

Figure 8. Characteristic curves for \( \theta = 12^0, \delta_h = 1 \) (left) and \( \theta = 15^0, \delta_h = 10^{-2} \) (right) and various values for \( n \).

7. Concluding remarks
The flow in a Holweck pump has been simulated by solving the linearized Boltzmann-BGK equation using the discrete velocity method in the velocity space and an equi-distributed Cartesian grid in the physical space. A decomposition method has been applied in order to reduce the computational cost. Results containing the dependence of the flow on the rarefaction parameter have been provided while the influence of the grooves’ angle and the rotational speed on the pressure difference produced by the pump has been examined. It has been shown that for the sets of parameters tested, there is an optimum angle where the pump can produce the highest pressure difference for almost the whole range of the pump throughput, while the increase of the rotational speed seems to increase the pressure difference in any case.

Future studies can include a detailed investigation of the dependence of the pump performance on the design parameters such as the grooves’ dimensions, the pump length, the gas type.
In addition the angle of the grooves has to be optimized for various sets of the other design parameters and the overall efficiency of the pump has to be taken into account. Finally the same approach can be modified and applied in other kind of pumps such as the Gaede pump.

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