The problem spreading acoustic waves in a porous environment with air bubbles on por walls

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Abstract. The propagation of acoustic waves in a porous medium with air bubbles on the pore walls is numerically studied. Based on the equations of mechanics of multiphase media, a dispersion relation is obtained that describes the dependence of the complex wave vector on frequency, on the basis of which the dependence of the phase velocity and attenuation coefficient on frequency for “fast” and “slow” waves is studied. The calculation results make it possible to evaluate the effect of gas bubbles on the propagation of sound waves in a porous medium with bubbles on the pore walls. The study was carried out with the financial support of the Russian Federal Property Fund in the framework of the scientific project No. 19-31-60015.

1. Introduction
The study of the propagation of pressure waves in saturated porous media is associated not only with solving practical problems of underwater acoustics, seismic, object protection, non-destructive testing, etc., but also for understanding the fundamental principles of wave processes in such media. A review of the work on the dynamics of waves in porous media is presented in [1-6].

The dynamics of shock waves in a porous medium consisting of sand particles bonded with epoxy resin was experimentally and theoretically studied in [7]. Three cases were investigated in the work: the pores are completely saturated with water; the pores are filled with air and pores contain a mixture of water and air bubbles. It turned out that the shock tube can be used for research reflection of high-frequency components of a shock wave from a porous medium. Good agreement is obtained between theoretical and experimental data in the case of saturation of the porous medium with water. In the case of a partially saturated porous medium, a satisfactory agreement is obtained.

The propagation of longitudinal waves in a porous medium saturated with a liquid with gas bubbles was studied in [8]. In the linear approximation, the dependences of the attenuation coefficient and the phase velocity of the Frenkel - Biot waves of the first and second kind on the frequency are constructed. It is shown that in the vicinity of the resonant frequency of the bubbles, the longitudinal Frenkel - Biot waves change their character. A wave of the first kind is transformed from a fast wave at low frequencies to a slow wave at high. The dispersion curve of a wave of the second kind consists of two branches - the “low-frequency” branch, whose vibrations have classical properties, and the “high-frequency” branch, which is a weakly damped high-speed mode. An expression is obtained for the “cutoff frequency” that defines the beginning of the “high-frequency” branch.

Dispersion curve of a wave of the second kind. In [9-10], wave processes in a porous medium saturated with a bubble liquid were numerically studied. The studies took into account the nonlinearity of bubble oscillations in gas equations and Rayleigh - Lamb. The change in the velocity and attenuation coefficient of deformation and filtration waves as a function of the properties of a porous
medium and the parameters of a bubble fluid is studied. The passage of a stepwise wave is considered profile from a liquid into such a porous medium. Study results show good quality agreement with experimental data of other authors.

In reservoir fluids, gas is often present. For example, a gas-liquid mixture in a porous medium is formed during acid treatments of low-permeability zones, with water-gas treatment of formations, etc. Therefore, it seems relevant to take into account the presence of gas bubbles in the study of wave processes in porous media saturated with a bubble fluid.

2. Basic equations
When describing the propagation of acoustic waves in a porous medium saturated with a bubble mixture, we will accept the following assumptions: the wavelengths of the waves under consideration are much larger than the pore sizes, the radius of the bubbles, and the distance between the bubbles; the velocities of the liquid and gas during the passage of the wave are equal. We take the average pore radius and the characteristic size of gas bubbles (Figure 1).

We write the macroscopic linearized mass equations for the liquid, gas in the pores and the skeleton of the porous medium in the two-velocity approximation and the number of bubbles [1]:

\[
\frac{d\rho_l^0}{dt} + \rho_l^0 \frac{dv_l}{dx} = 0, \quad \frac{d\rho_g^0}{dt} + \rho_g^0 \frac{dv_g}{dx} = 0, \quad \frac{d\rho_s^0}{dt} + \rho_s^0 \frac{dv_s}{dx} = 0
\]

\[
\frac{d\rho_l}{dt} + \frac{d\rho_g^0}{dt} = 0, \quad \frac{d\rho_s}{dt} = 0, \quad \frac{d\rho_g}{dt} = 0
\]

(1)

Here \( \rho_l^0, \rho_g^0, v_l, v_g, n_l, n_g \) – volume average and phase average density, velocity, pressure, volumetric contents and the number of bubbles per unit volume. An additional subscript corresponds to the initial undisturbed state.

The momentum equations for a liquid containing gas bubbles and for the entire system:

\[
\left( \alpha_0 p_{l0}^0 + \alpha_g a_0 p_{g0}^0 \right) \frac{dv_l}{dt} = \left( \alpha_{g0} + \alpha_{l0} \right) \frac{dp_l}{dx} - F_r, \quad F = F_m + F_\mu + F_B
\]

\[
F_m = \frac{1}{2} \eta_m \alpha_s \left( \alpha_{l0} p_{l0}^0 + \alpha_g a_0 p_{g0}^0 \right) \left( \frac{dv_l}{dt} \right)^2
\]

\[
F_\mu = \frac{9}{2} \eta_p \left( \alpha_{l0} + \alpha_{g0} \right) \alpha_{s0} a_{s0} \left( v_l - v_g \right)^2
\]

\[
F_B = 6 \eta_B \left( \alpha_{l0} + \alpha_{g0} \right) \alpha_{s0} a_{s0} \left( \frac{\pi \rho_{l0}^0 a_{l0}}{\rho_{g0}^0} \right)^{1/2} \int_{-\infty}^{t} \frac{d}{dt} \left( v_l - v_g \right) \left( t - \tau \right)^{1/2} dt
\]

\[
\left( \alpha_{l0} p_{l0}^0 + \alpha_g a_0 p_{g0}^0 \right) \frac{dv_l}{dt} + \alpha_{s0} p_{s0}^0 \frac{dv_s}{dt} = \frac{d\rho_l^0}{dx} - \frac{dp_l}{dx}
\]

(2)

Here \( p_l \) – fluid pressure, \( \alpha_{s0}, \alpha_{l0}, \alpha_{g0} \) – volumetric content of solid, liquid and gas phases, respectively, \( \sigma_s \) – reduced voltage in the skeleton; \( F_m \) – force of attached masses caused by inertial interaction of phases, \( F_\mu \) – Stokes viscous friction analog, \( F_B \) – analogue of the Basset force, which
manifests itself at high frequencies due to the non-stationary state of the viscous boundary layer near the boundary with the solid phase, $\mu_f$ – fluid viscosity.

For the solid and liquid phases, the linear equation of state in the acoustic approximation is accepted; for the skeleton of the porous medium, we accept the Maxwell model [5,6]:

$$\frac{\rho_s^0}{\rho_s} = 1 + \beta_p (p_s - p_{s0})$$
$$\frac{\rho_l^0}{\rho_l} = 1 + \beta_l (p_l - p_{l0})$$

$$p_s = \rho_s \frac{\alpha_s^0}{\alpha_{s0}}$$
$$\alpha_s \frac{d \alpha_s}{dt} = \frac{1}{E_s} \frac{d \sigma_s}{dt} + \frac{d \alpha_s}{dt}$$

(3)

We assume that the gas in the bubbles is compressed adiabatically:

$$\frac{p_g}{p_{g0}} = \left(\frac{b_0}{b_k}\right)^{3\gamma}$$

(4)

$$\frac{p_g}{p_{g0}} = \left(\frac{p_g}{p_{g0}}\right)^\gamma$$

(5)

Here $\beta_j$ – phase compressibility, $p_g$ – pressure in the gas phase.

The additional subscript (0) defines the parameters corresponding to the unperturbed state, and the parameters without the index express small perturbations of the parameters from the equilibrium value; superscript (0) corresponds to the true value of the parameter.

For volumetric contents $\alpha_j$ the following kinematic relation is valid:

$$\alpha_{g0} + \alpha_{l0} + \alpha_{s0} = 1$$

(6)

Where $j = g, l, s$.

The change in the radius of the bubble obeys the Rayleigh – Lamb equation for the bubble in a porous medium [1].

$$\rho_{l0} \left( b \frac{dw_b}{dt} + \frac{3}{2} w_b^2 \right) = \rho_g - \rho_l - 4 \mu_l \left( \frac{w_b}{b} \right)^2 \left( 1 + \frac{1}{4} \eta_l \left( \frac{b}{w_b} \right)^2 \right)$$

$$w_g = \frac{db}{dt}, w_l = w_R + w_s, w_a = \frac{p_g - p_l}{\rho_{l0} C_l (\phi^{0.8})^{1.3}}$$

(7)

Where $C_l$ – the speed of sound in a liquid, $\phi_{g0}$ – volume fraction of gas in the bubble fluid.

We will seek a solution to the system of equations in the form of damped traveling waves [1]:

$$\rho_0^0, v_j, p_j, a_j \approx A_j \exp[i(Kx - \omega t)], K = k + i\delta$$

(8)

After solving equations (1) - (7), the dispersion relation is obtained. Due to the bulkiness, this ratio is not given. Based on this relation, the phase velocity and the attenuation decrement of linear waves are calculated.

### 3. Calculation results

In the calculations, the phase parameters are taken at ambient temperature 300 K. For air: $p_{g0} = 10^5$ Pa, $\gamma = 1.4$, $\rho_{g0}^0 = 1.17$ kg/m$^3$, $\mu_g = 1.86 \times 10^{-5}$ Pa·s. For sandstone: $\rho_{s0}^0 = 2560$ kg/m$^3$, $\mu_s = 10^8$ Pa·s, $E_s = 3.7 \times 10^{10}$ Pa·s. For water: $\rho_{l0}^0 = 1000$ kg/m$^3$, $\mu_l = 10^3$ Pa·s, $C_l = 1500$ m/s.
In figure 2 shows the attenuation coefficient $\delta$ and phase velocity $C_p$ from frequency $\omega$. Characteristic sizes: $a_0 = 10^{-3} \text{ m}$, $b_0 = 10^{-4} \text{ m}$, $\alpha_{s_0} = 0.3$, $\alpha_{g_0} = 0.69$, $\alpha_{g_0} = 0.01$.

Figure 2. Dependences of the attenuation coefficient $\delta$ and phase velocity $C_p$ “fast” (dashed lines) and “slow” (solid lines) waves of frequency $\omega$ for a sandstone system with air bubbles - liquid. Characteristic system parameters: $a_0 = 10^{-3} \text{ m}$, $b_0 = 10^{-4} \text{ m}$, $\alpha_{s_0} = 0.69$, $\alpha_{g_0} = 0.01$.

From figure 2 shows that there is a resonant frequency $\omega = 2 \times 10^5 \text{s}^{-1}$, at which there is a sharp jump in the speed of the “slow” wave. Also, worth noting is another frequency $\omega > 6 \times 10^5 \text{s}^{-1}$, at which the speed of the “slow” wave becomes greater than the speed of the “fast” wave.

In the frequency range $2 \times 10^5 \text{s}^{-1} < \omega < 2 \times 10^6 \text{s}^{-1}$ the speed of the “fast” wave is constant and equal to 3000 m/s.

In figure 3 shows the attenuation coefficient $\delta$ and phase velocity $C_p$ from frequency $\omega$. Characteristic dimensions: $a_0 = 10^{-3} \text{ m}$, $b_0 = 5 \times 10^{-5} \text{ m}$, $\alpha_{s_0} = 0.3$, $\alpha_{g_0} = 0.69$, $\alpha_{g_0} = 0.01$. 
Figure 3. Dependences of the attenuation coefficient $\delta$ and phase velocity $C_p$ “fast” (dashed lines) and “slow” (solid lines) waves of frequency $\omega$ for a sandstone system with air bubbles - liquid. Characteristic system parameters: $a_0 = 10^{-3}$ m, $b_0 = 10^{-5}$ m, $\alpha_{d0} = 0.3$, $\alpha_{l0} = 0.64$, $\alpha_{g0} = 0.05$.

Figure 3 shows that with an increase in the volumetric content of gas bubbles and a decrease in the volumetric content of the liquid, a decrease in the attenuation coefficient of the slow wave is observed.

In figure 4 shows the attenuation coefficient $\delta$ and phase velocity $C_p$ from frequency $\omega$. Characteristic dimensions: $a_0 = 10^{-3}$ m, $b_0 = 5 \cdot 10^{-5}$ m, $\alpha_{d0} = 0.3$, $\alpha_{l0} = 0.69$, $\alpha_{g0} = 0.01$.

It can be seen from figure 4 that in the case of the radius of gas bubbles $b_0 = 5 \cdot 10^{-5}$ m resonant frequency is $\omega = 10^5$ s$^{-1}$, moreover, at this frequency the speed of the "slow" wave is greater than the speed of the "fast" wave. It is also worth noting that at high frequencies the speed of the "slow" wave is lower than the case of the radius of the bubble in the pore space $b_0 = 10^{-5}$ m. Moreover, also with $b_0 = 5 \cdot 10^{-5}$ m the maximum value of the "fast" wave is lower.

4. Conclusion
The propagation of acoustic waves in an internal medium with air bubbles on pore walls is theoretically studied, taking into account interphase force interactions. The calculation results make it possible to evaluate the effect of gas bubbles on the level of sound wave propagation.

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