We report an interaction that controls spin squeezing in a collection of spin 1/2 particles. We describe how spin squeezing can be generated and maintained in time. Our scheme can be applied to control the spin squeezing in a Bose condensate with two internal spin states.

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Squeezed spin states (SSS) of atoms have reduced quantum fluctuations that are useful in enhancing sensitivity in precision spectroscopy. Since the early work by Kitagawa and Ueda, there have been several proposals of generating SSS in different configurations. In the original proposal, Kitagawa and Ueda first proposed the model $H_1$ by adding a linear interaction term $\Omega J_x$, where the interaction strength $\Omega$ (assumed positive) can be controlled by an external field. We consider that the system starts from the lowest eigenvector of $J_x$, $|J, m_x = -J\rangle$. Such an initial state is favorable in generating spin squeezing because of the twisting effect due to the nonlinear interaction $2\kappa J_x^2$ along the $z$-axis. In Fig. 1 we show the typical behavior of the squeezing parameter as a function of time. In the case of $\Omega = 0$ (i.e., the $H_1$ model) shown in Fig. 1a, we see that $\xi_s$ reaches a minimum after a characteristic time. However, such a squeezing can only be maintained in a certain time period. As the time increases, the system is less squeezed and eventually becomes unsqueezed.

The key advantage of the model $H_3$ ($\Omega \neq 0$) is the maintenance of squeezing in an extended period of time. This is clearly shown in Fig. 1b-d. We see that $\xi_s$ can be kept below unity in a much longer period of time than that for the $\Omega = 0$ case. Indeed, for the choice of $\Omega$ used in Figs. 1b-d, the system exhibits squeezing most of the time. The interaction strength $\Omega$ affects the minimal $\xi_s$ that the system can reach. Fig. 1c represents a near optimal choice of $\Omega$ for $J = 100$. A closer look at the minimal value of $\xi_s$ indicates that it is less than (i.e., more squeezing) that in Fig. 1a.

![FIG. 1. Typical time dependence of $\xi_s$ for the system starting from the initial state $|J, m_x = -J\rangle$. Here $J = 100$ is used.](image-url)
Another advantage of the interaction model $H_3$ is the maintenance of large coherent (mean) component of the collective spin. This feature is shown in Fig. 2 where the expectation values $\langle J_z \rangle$ are plotted against time for various values of $\Omega$. We remark that for the system starts from $|J, m_x = -J\rangle$, the only nonvanishing spin component is $J_z$ because $\langle J_y \rangle = \langle J_y \rangle = 0$ at all times. In Fig. 2, we see that for a sufficiently large $\Omega$ (for example the $\Omega = 25\kappa$ curve), $\langle J_z \rangle$ changes slightly in the course of time. This is in contrast to the $\Omega = 0$ case in which $\langle J_z \rangle$ vanishes after some time. Since a strong coherent spin component is often needed in increasing the sensitivity of precision measurement (such as in Ramsey spectroscopy\cite{9}), $H_3$ is more desirable in this regard.

![FIG. 2. Time dependence of the expectation value of $J_z$, same parameters as in Fig. 1](image)

Although exact analytic solutions of the nonlinear problem are not available, the squeezing behavior produced by $H_3$ can be understood when $\Omega$ is sufficiently larger than $\kappa$, i.e., $\Omega \gg \kappa$. First we recall that the system is prepared to start from the lowest eigenstate of $J_z$, $|J, m_x = -J\rangle$, i.e.,

$$J_z |J, m_x = -J\rangle = -J |J, m_x = -J\rangle.$$  \hspace{1cm} (5)

Such a state minimizes the energy associated with the interaction $\Omega J_z$. If $\Omega \gg \kappa$, the external field forces the total spin to remain polarized in the $-ve x$-direction because it costs energy to change the spin vector. This explains why a large coherent component of $\langle J_z \rangle$ can be maintained. Now we look at the Heisenberg’s equation of motion of the angular momentum operators in the $y$– and $z$– directions,

$$\dot{J}_y = \Omega J_z,$$  \hspace{1cm} (6)

$$\dot{J}_z = -\Omega J_y + 2\kappa (J_x J_z + J_z J_x).$$  \hspace{1cm} (7)

Based on the fact that $J_z$ remains unchanged approximately, it is justified to make an approximation: Replacing $J_x$ by $-J$. We call such an approximation as a \textit{frozen spin approximation} \cite{9}. In this way, we have

$$\dot{J}_z \approx - (\Omega^2 + 4\kappa \Omega J) J_z,$$  \hspace{1cm} (8)

which permits harmonic solutions,

$$J_z(t) \approx J_z(0) \cos \omega t + \Omega J_y(0) \sin \omega t / \omega$$  \hspace{1cm} (9)

$$J_y(t) \approx -\omega J_z(0) \sin \omega t / \Omega + J_y(0) \cos \omega t,$$  \hspace{1cm} (10)

where the frequency $\omega = \sqrt{\Omega^2 + 4\kappa \Omega J}$ is defined.

Eqs. (9) and (10) are operator solutions under the frozen spin approximation. The time-dependent spin fluctuations are given by,

$$\langle (\Delta J_z(t))^2 \rangle \approx \langle J_z^2(t) \rangle \cos^2 \omega t + \frac{\Omega^2}{\omega^2} \langle J_y^2(t) \rangle \sin^2 \omega t,$$  \hspace{1cm} (11)

$$\langle (\Delta J_y(t))^2 \rangle \approx \langle J_y^2(t) \rangle \cos^2 \omega t + \frac{\omega^2}{\Omega^2} \langle J_z^2(t) \rangle \sin^2 \omega t.$$  \hspace{1cm} (12)

Here the cross terms $\langle J_z(0)J_y(0) \rangle$ and $\langle J_y(0)J_z(0) \rangle$ do not appear because they are identically zero with respect to the initial state (5). Now using the fact that $\omega > \Omega$ and

$$\langle J_z^2(0) \rangle = \langle J_y^2(0) \rangle = J/2,$$  \hspace{1cm} (13)

we find that reduced spin fluctuations occurs in the $z$–direction, i.e., $\langle (\Delta J_z(t))^2 \rangle \leq J/2$. In other words the \textit{system is always squeezed} except at the times $t = n\pi / \omega$. The strongest squeezing occurs at $t = t^* = (2n+1)\pi / 2\omega$ with

$$\langle (\Delta J_z(t))^2 \rangle_{t=t^*} \approx \frac{\Omega^2 J}{2\omega^2}.$$  \hspace{1cm} (14)

This corresponds to the squeezing parameter at $t^*$,

$$\xi_{min} \equiv \xi_s|_{t=t^*} \approx \frac{\Omega}{\omega} < 1.$$  \hspace{1cm} (15)

From the definition of $\omega$ above, we see that the squeezing parameter $\xi_{min}$ is approximately $(4\kappa J / \Omega)^{-1/2}$ when $\kappa J \gg \Omega$. Therefore the system is less squeezed if $\Omega$ is large, but more squeezing can be achieved by increasing the number of particles. We should point out that the frozen spin approximation becomes less valid when $\Omega$ is comparable to $\kappa$. Nevertheless, the approximation captures the essential physical picture. We have compared the approximate analytical results with the exact numerical solutions, we found a good agreement in $\xi_{min}$ and the oscillation frequency $\omega$ as long as $\Omega \gg \kappa$.

In order to determine how optimal squeezing depends on $\Omega$ and particle number $2J$ beyond the frozen spin approximation, we have examined the exact numerical solutions for a wide range of $J$ and $\Omega$. In Fig. 3 we show the values of $\xi_{min}$ that can be attained by our model $H_3$ for different $J$s. These values of $\xi_{min}$ are attained by using optimal $\Omega$ for the corresponding $J$ (see the inset of Fig. 3). For example, we have found that for $J = 500$, $\Omega \approx 10\kappa$ yields an optimal squeezing parameter $\xi_{min} \approx 0.09$. These numerical findings are consistent with the prediction from the frozen spin approximation that strong squeezing exhibits in the domain $\kappa J \gg \Omega \gg \kappa$.
FIG. 3. The minimum value of the squeezing parameter that can be attained for various $J$. The inset shows the optimal $\Omega$ used.

In conclusion, we have discovered how an external field can be applied to control spin fluctuations, which is expected to play an prominent role in the internal dynamics of Bose condensates. Our results indicate that a cooperation of the nonlinear self-interaction $2\kappa J_x^2$ and the external interaction $\Omega J_x$ can generate spin squeezing in an extended period of time. In our scheme the coherent component of the collect spin can be locked in the $x$–direction, and the reduced fluctuations always appear in the $z$–direction. These advantages are not found in the standard model $H_1$. Finally we remark that spinor Bose condensates have an intrinsic $H_1$ type self-interaction among particles. The application of an external magnetic field or Raman fields can be used to prepare the required initial state and to realize the coupling $\Omega J_x$.

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