Periods of magnetic field variations in the Ap star γ Equulei (HD 201601)

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ABSTRACT
This paper presents a series of 95 new measurements of the longitudinal (effective) magnetic field $B_e$ of the Ap star γ Equ (HD 201601). Observations were obtained at the coudé focus of the 1-m reflector at the Special Astrophysical Observatory (SAO RAS) in Russia over a time period of 4190 days (more than 11 years). We compiled a long record of $B_e$ points, adding our measurements to all published data. The time series of magnetic data consists of 395 $B_e$ points extending for 24488 days, i.e. over 67 years. Various methods of period determination were examined for the case in which the length of the observed time series is rather short and amounts only to 69 percent of the period. We argue that the fitting of a sine wave to the observed $B_e$ points by least squares yields the most reliable period in the case of γ Equ. Therefore, the best period for long-term magnetic variations of γ Equ, and hence the rotational period, is $P_{rot} = 35462.5 ± 1149$ days = 97.16 ± 3.15 years.

Key words: Stars: magnetic fields – Stars: fundamental parameters – Stars: individual (HD 201601)

1 INTRODUCTION
Ap star γ Equ (HD 201601; HR 8097) is an apparently bright object ($V = 4.7$ mag) with a strong global magnetic field. Its longitudinal component $B_e$ (effective magnetic field) exhibits very slow variations plus significant noise in the range from ≈1000 G to −1600 G. The total time period over which the magnetic field $B_e$ data have been recorded exceeds 60 years.

The long-term variability of the longitudinal magnetic field of γ Equ most likely is due to the very slow rotation of the star. That feature has been investigated in many papers [see Bonsack & Pilachowski (1974); Leroy et al. (1994); Bychkov & Shtol (1997); Scholz et al. (1997); Bychkov et al. (2006); Savanov et al. (2014)]. After smoothing, all available $B_e$ points accumulated in published sources plotted against time are arranged as part of a sine wave with a long period. Bychkov et al. (2006) determined the period of long-term magnetic variations as $P_{mag} = 91.1 ± 3.6$ years. However, Savanov et al. (2014) determined this period as $P_{mag} = 93 ± 3$ years after adding their new $B_e$ measurements.

Such a discrepancy between periods $P_{mag}$ in both papers is probably the result of the application of different numerical methods in the particular case when an unknown period of $B_e$ variations is longer than the length of the observed time series. We believe that the most commonly used methods of period determination fail in such a case and yield inaccurate results.

Our paper aims to examine the systematic errors introduced by various commonly used methods of period determination in the particular case of the Ap star γ Equ. We aim to reliably determine the period of long-term $B_e$ variations, as well as to search for a possibly more rapid variability of this star.

2 OBSERVATIONAL DATA
The longitudinal magnetic field $B_e$ of γ Equ was routinely measured at the Special Astrophysical Observatory (SAO RAS) in Russia over many years. Observations were carried out at the coudé focus of the 1-m reflector. During the observational period (4190 days ≈ 11 years) we acquired a total of 95 $B_e$ measurements for this object. Those observations are shown in Table 2 of Appendix A.

2.1 Details of our observations
The magnetic measurements of Gamma Equ were derived from the Zeeman spectra, obtained in the coudé focus of the 1-m telescope of SAO RAS. The instrumentation and data reduction procedures were described in Bychkov (2008). In
our case, we analysed high-quality spectral data \((R = 45000, \text{CCD}, S/N \approx 100)\) and processed them using the MIDAS software. The method for obtaining values of the longitudinal magnetic field is standard and includes the following steps:

1. The selection of lines suitable for measurement was carried out taking into account the physical properties and chemical composition of the star, calculating the synthetic spectrum with the STARSP code (Tsymbal 1996) using the VALD database (Kupka et al. 1999).

2. in the observed spectral line profiles, corresponding to the left (LCP) and right (RCP) circular polarizations, we fitted a gaussian using the method of least squares. Lines with defective registration due to the impact of cosmic rays, etc. were discarded. The value of line splitting under the effect of the magnetic field was determined by the magnitude of the shift between the centres of the fitted Gaussians.

3. The intensity of the effective magnetic field for each line was computed using the well-known relation (Mathys 1991):

\[
\lambda_R - \lambda_L = 2g_{\text{eff}} \cdot 4.671 \times 10^{-13} \lambda^2 B_{\text{eff}},
\]

where the wavelength \(\lambda\) is expressed in Å, and the field strength is in G. The value of \(B_e\) was computed as the average over all the lines, used for measurements. Assuming that the scatter of \(B_e\) values due to inaccurate measurements and other reasons is described by a normal law of distribution, the probable error \(\sigma(B_e)\) was computed. Instrumental effects are taken into account according to Bychkov et al. (2000). During observations, we regularly registered the magnetic standards, \(\alpha^2\) CVn and 53 Cam, and the standards of the null field \(\alpha\) CMi, \(\alpha\) Boo, the Moon, etc., as described by Bychkov et al. (2006). Results obtained for the standards are in good agreement with the literature data.

2.2 Details of the literature data

The full record of \(B_e\) points analysed in this paper includes 372 CCD measurements obtained from Zeeman splitting of metal lines, which were collected from published papers. We list below references to the sources of previous data and the method of observation. Fig. 1 shows the distribution of \(B_e\) points obtained by different methods in the time period when magnetic measurements were done (JD).

This is a very inhomogeneous set of observations certainly with underestimated errors and possible offsets between various observers. The data set includes:

- Photographic estimates of \(B_e\) taken from Babcock (1958), Bonsack & Pilachowski (1974), Scholz (1975; 1979), Zverko et al. (1989) and Bychkov & Shtol (1997).
- CCD estimates (using CCD arrays and a more refined selection of split lines). Open squares: measurements taken from Mathys (1991; 1994), Mathys & Hubrig (1997), Hildeman, Scholz & Lehmann (2000), Leone & Kurtz (2003), Leone (2007) and Bychkov, Bychkova & Madej (2006) (filled squares).
- LSD points (Least Squares Deconvolution method) are marked in Fig. 1 with asterisks. Source: Wade (2015).

Method (1) was used during the longest time period and covers phases of the positive \(B_e\) maximum on the magnetic phase curve of \(\gamma\) Equi.

Method (2) required the use of echelle spectrometers to pack many spectral orders on a CCD matrix of relatively small size. In fact CCD techniques use achromatic analysers of circular polarization that operate in the range \(\approx 2000\) Å (Babcock analyzed the interval of only \(300 - 350\) Å). CCD analyzes higher number of spectral lines. The identification of lines is more accurate when using VALD and model atmospheres. In fact, this is just a more advanced version of method (1).

Method (3) is the most precise method. However, observations of the longitudinal magnetic field of \(\gamma\) Equ are located in a relatively short time interval; they do not cover the minimum of the phase curve but are located at the rising part of the curve.

The principal point is that the local \(B_e\) extrema were obtained mostly using different methods. The \(B_e\) maximum was observed by the photographic method and the minimum by the CCD method (and LSD to a lesser extent). Differences and possible offsets produced by these methods had an impact on the accuracy in the determination of the period.

Note, that all of the \(B_e\) estimates of Bychkov et al. (2006) and of this work were obtained at the same telescope and spectrometer and processed with the same software. As can be seen from Fig. 1 these estimates cover the minimum at the negative part of the magnetic phase curve.

2.3 Details of the LSD observations

We appended to our time series 23 unpublished \(B_e\) points obtained by the LSD method (Wade 2015). These observations contributed to our period determination with the same weight as other \(B_e\) values.

These measurements were obtained from ESPaDOnS Stokes V spectra acquired between September 2004 and July 2010 as part of instrumental commissioning and systematic monitoring of the instrumental crosstalk. The spectra were reduced using Libre-Esprit (Donati et al. 1997) and LSD.
profiles were extracted using a line mask corresponding to an appropriate temperature and metallicity. Longitudinal field measurements were extracted from each LSD profile using the first moment method. Additional details concerning the reduction and analysis will be communicated in a forthcoming publication.

2.4 Results

The final $B_e$ time series extended for 24488 days (or over 67 years) starting from the first measurement of the magnetic field on 9 October 1946 by Babcock (1958). That time period amounts to 69 percent of the expected long-term period of $\gamma$ Equ.

Belonging, as it does, to a rather late Ap spectral subclass, this star is a very good object for studying of global magnetic fields since it shows a large number of extremely narrow, sharp metal lines.

The available set of $B_e$ observations consists of 395 measurements obtained from Zeeman splitting of metal lines and 51 points measured in hydrogen lines. Since values of $B_e$ (met) and $B_e$ (H line) are markedly different, we decided to determine periods using only estimates of $B_e$ (met). Moreover, the time interval over which $B_e$ (H line) values were observed was 13131 days, which is approximately half the time interval for $B_e$ (met) observations, since the latter is equal to 24488 days.

The observed time series is very unevenly filled by estimates. For clarity, we constructed a histogram which shows the distribution of time intervals between neighbouring measurements. Fig. 1 presents such a histogram with a step of 10 days. The horizontal axis in Fig. 1 was restricted to intervals not exceeding 350 days, but intervals that are higher than 500 and 1000 days exist, and there is one interval equal to 2614 days.

![Figure 2. Distribution of time separation between two neighbouring $B_e$ measurements.](image)

3 METHODS OF PERIOD DETERMINATION

There exist many different techniques for period determination [Lafler & Kinman (1965); Jurkevich (1971); Deeming (1975); Lomb (1976); Burki et al. (1978); Renson (1978); Pelt (1975, 1983); Kurtz (1985); Scargle (1982) Terebizh (1992) and many others]. Those methods were transformed to usable computer codes or software packages [e.g. Kurtz’s code (1985) or the Period04 package (Goranskij 1976)]. One of the most serious problems in our case is that a given interval of the time series data is shorter than the expected period. In the actual $B_e$ set it amounts only to $\approx$ percent of the expected long-term period. Naturally the accuracy of the period determination will be affected by unevenly distributed data points of non-uniform accuracy.

We selected the most commonly used methods and codes for the determination of periods:

- 1. The Discrete Fourier Transform (DFT) for unevenly spaced data by Deeming (1975), see also Scargle (1982) and Kurtz (1985). We used here Fortran code kindly provided to us by Dr. Don Kurtz.
- 2. Periodogram analysis (Burnasheva & Gollandskij 1989). This is a modified version of Lomb (1976) and Scargle (1982).
- 3. The least squares method. Determination of the minimum sum of squared deviations from the sine wave yields the best fit. The sum should be computed for a set of trial periods. The range of trial periods and the frequency step are selected jointly with parameters of the analysed time series. The minimum sum indicates the most probable period.

Method (3) was implemented in the form of the FORTRAN code written by Bychkov (1988, unpublished). The advantage of this method is that the user explicitly defines the shape of the phase curve which is a very useful option in particular cases.

Each of the above most commonly used methods for period determination has some limitations of accuracy, especially when the length of the period exceeds the length of time of the measurement set.

We also used a well-known package Period04 in the following tests [Goranskij (1976), who implemented methods of Deeming (1975), Lafler and Kinman (1965) and Jurkevich (1971)]. As indicated above, the specific case of $\gamma$ Equ is highly unusual, since the length of the $B_e$ time series equals 70 percent of the expected period.

4 ANALYSIS OF OBSERVATIONAL DATA

The simplest and most common form of phase curve is a simple harmonic curve

$$B_e(\phi) = B_0 + B_1 \sin(\phi),$$

where

$$\phi = 2\pi \left( \frac{t - T_0}{P} \right).$$

We performed a very simple numerical experiment. First, we estimated the preliminary value of the period $P_{\text{est}}$ by fitting a sine wave to the observed $B_e$ points by least-squares (one degree of freedom). Then, for the fixed best period $P_{\text{est}} = 35394.22$ days, we determined the preliminary
values of the half-amplitude $B_e = 850.78$ G, the average $B_0 = -266.24$ G and the initial epoch $T_0 = 2416365.12$ (JD).

Subsequently, we generated a new group of sine $B_e$ time series using the above preliminary estimates of parameters $B_0, B_1, T_0$ and various trial periods $P_{tr}$. Discrete strengths of the longitudinal magnetic field $B_e$ were determined at the same time points as in the observed time series. We then added a gaussian noise component with errors $\sigma$ in G given in specific source papers.

The trial periods $P_{tr}$ of the simulated $B_e$ time series ranged from 0.5 to 4 $\times$ $P_{test}$. Then, for each $P_{tr}$ we assessed again the period $P_{true}$ using various methods and the relative difference between them. In the ideal case, the trial period $P_{tr}$ should be precisely reproduced.

For clarity, we express the deviation $R$ of the re-estimated period from the trial value in percent, $R = (P_{true} - P_{tr})/P_{true} \times 100\%$. The results of this simulation are shown in Fig.3 and Table I which shows deviations $R$ as a function of the number of the trial periods contained in the specific $B_e$ time series.

For the length of the $B_e$ time series less than two expected periods, the most reliable results are provided by method (3) (deviations $R$ close to zero). On the contrary, if the $B_e$ time series is much longer than $P_{true}$, all cited methods are reliable (deviations $R$ converge to zero).

The results of the period determination by the various methods are also shown in Fig.3. Black full asterisks denote the preferred method (3); gray line and open squares – method (1) (Scargle); blue line and open circles – method (2) (program by Kurtz); red line and blank diamonds – code Period04; green line and open triangles – code by Goransky (Lafleur-Kinman method); pink line and open triangles – code by Goransky (Deeming method).

Fig.3 clearly shows that methods (1) and (2) yield reliable results only when the length of time of a test series is not less than two or three periods. Method (3) is most independent of the duration of the time period. Of course, this simple test gives only a rough idea of the performance of these methods when they are applied to a specific time series. In fact, those features are useful only to solve a specific problem of secular variability of the effective magnetic field of mCp star $\gamma$ Equ.

Similar testing of various methods of period determination for simulated $B_e$ time series of length similar to the $\gamma$ Equ, both equally and unevenly spaced, gave similar results. Note, that this is a rare case, when the observed $B_e$ time series is short, since the length of the time series is less than the period of variations and this naturally affects the final result.

We finally accepted the value of the magnetic period obtained with method (3), $P = 35462.5$ days (97.16 years) – as the most correct. It is evident that using the very popular method (1) for this specific time series caused a reduction in the period of approximately 10 percent – $P = 32051.69$ days (87.81 years), which is too low.

### Table 1. Deviations of the re-estimated period $P_{true}$ from the trial value $P_{tr}$ (in percent) as a function of the number of trial periods covered by the $B_e$ time series (24487.3 days); see column 2.

| P   | $dt/P$ | R1 | R2 | R3 | Per04 |
|-----|--------|----|----|----|-------|
| 48974.60 | .50   | -39.90 | -24.36 | .12 | -41.18 |
| 40812.20 | .60   | -23.65 | -9.23 | .01 | -25.00 |
| 35462.50 | .69   | -9.56 | 4.47  | -.03 | -7.93  |
| 34981.90 | .70   | -8.15 | 5.90  | -.04 | -6.67  |
| 30069.10 | .80   | -24.88 | -5.06  | -.13 | -23.81 |
| 27208.10 | .90   | -4.05 | -9.12  | -.05 | -5.19  |
| 24487.30 | 1.00  | 10.58 | 9.82  | .02 | 11.11  |
| 22261.20 | 1.10  | 8.59  | -3.55  | .03 | 10.00  |
| 20406.10 | 1.20  | -8.83 | 5.44  | .06 | -7.69  |
| 18836.40 | 1.30  | 26.14 | 31.28  | .05 | 23.81  |
| 17490.90 | 1.40  | 21.64 | 8.24  | .03 | 21.83  |
| 16324.90 | 1.50  | -2.63 | 2.85  | .04 | -3.23  |
| 14404.30 | 1.70  | -4.55 | 3.17  | .00 | -5.56  |
| 12243.70 | 2.00  | 1.53  | 11.26  | 2.56 | .51    |
| 9794.90  | 2.50  | -1.07 | -6.91  | .41 | .41    |
| 8162.40  | 3.00  | .97   | .91    | 1.70 | .34    |

![Figure 3](image-url)
non-gaussian scatter around the best-fitted sine wave. Such a scatter also can be enhanced by a rapid variability of the magnetic field, comparable to rapid oscillations, for example, with periods in the range of 6 to 30 minutes. See papers by Bychkov (1988), Leone & Kurtz (2003), Savanov et al. (2003), Hubrig et al. (2004) and Bychkov et al. (2005a).

6 COMPARISON WITH THE MOST PRECISE MEASUREMENTS

In the period and phase curve determination, we used 23 recent measurements of the longitudinal magnetic field $B_e$ of $\gamma$ Equ measured in metal lines with the precise LSD method (Wade 2015). Fig. 6 shows our homogeneous $B_e$ measurements collected in Table 1 (blue circles with error bars) and the phase curve in JD just after the $B_e$ minimum. LSD measurements are concentrated in the area of red empty squares close to the phase curve.

LSD measurements alone essentially form an almost horizontal line and then suggest that real period of $\gamma$ Equ is much longer than $\approx 100$ years, in contrast to our current determination (Wade 2015). We note, that such a hypothesis should be verified by further LSD observations performed using the same instrument, since the current LSD data cover only a small period of time and such a low rise in $B_e$ tells us little about the long term behaviour of the LSD longitudinal magnetic field.

We believe, that precise LSD measurements of the longitudinal magnetic field (those obtained from the splitting of metal lines) are consistent with the new phase curve and our other conclusions.

Additionally, Fig. 6 shows the run of all available $B_e$ measurements for $\gamma$ Equ, which were obtained from hydrogen lines (51 points). Red dots denote recent DAO dimaPol determinations of $B_e$ in $\gamma$ Equ [Bohlender (2015); see Monin et al. (2012) for a description of that technique]. Note, that the $B_e$ values obtained from hydrogen lines exhibit an offset compared to those from metal lines after the minimum.

We did not use the latter $B_e$ points for period determination because of that offset, which is a fact that has been well-known from a long time. Hydrogen is uniformly distributed on the surface of stars and metals apparently are not. Minor deviations from the apparently uniform distribution of HI can be observed only at the He-rich and He-weak stars (Kudryavtsev & Romanyuk 2012).

Moreover, the time interval covered by the hydrogen line estimates is only 13131 days (37 percent of the period) and the corresponding $B_e$ time series contains only 51 points.

7 SUMMARY

In this paper, we present a series of 95 new measurements of the longitudinal magnetic field $B_e$ of Ap star $\gamma$ Equ = HD 201601, which were obtained over 4190 days (more than 11 years) at the Special Astrophysical Observatory. We also compiled and analysed all the published and available unpublished LSD $B_e$ data for this star. A useful subset of magnetic data consists of 395 $B_e$ measurements obtained in Zeeman split metal lines over 24488 days (more than 67 years).

We examined the reliability of the most frequently applied methods for period determination in the rare case, in which the length of the observed $B_e$ time series is lower than an unknown period (in the case of $\gamma$ Equ it equals $\approx 69$ percent of the period $P_{\text{mag}}$). In such a case the most reliable technique is the least squares method of fitting a sine wave to the actual $B_e$ time series. However, such a conclusion applies only to the particular record of $B_e$ observations of $\gamma$ Equ analysed in this paper.

(i) 1. In the uncommon case of a relatively short time series of $B_e$ measurements which is shorter than an unknown
period, the fitting of a sine wave to the time series is the only reliable method for determination of the magnetic (rotational) period. It is possible, that such a conclusion is restricted to the specific set of $B_\text{e}$ measurements of $\gamma$ Equ.

(ii) 2. We determined that the period of secular variations $P = 35462.5$ days (97.16 years) and is longer than the previously accepted value.

(iii) 3. We present a series of 23 precise LSD measurements of its longitudinal magnetic field (Wade 2015) which confirms the accuracy of our measurements.

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REFERENCES

Babcock, H.W. 1958, Ap&SS, 3, 141
Bohlender, D. 2015, private communication
Bonsack, W.K., Pilachowski, C.A. 1974, ApJ, 190, 327
Burki, G., Maeder, A., Rulener, F. 1978, A&A, 65, 363
Burnasheva, B.A., Gollandskij, O.P. 1989, BCrAO, 77, 177
Bychkov, V.D., 1988, Proc. Int. Meet. on ”Physics and Evolution of Stars”, held in Nizny Arkhyz, October 12-17, 1987. Edited by Yu.V. Glagolevskij. Leningrad: Nauka, 197
Bychkov, V.D., 2008, Astrophysical Bulletin, 63, 83

Figure 6. All of the available $B_\text{e}$ measurements obtained from hydrogen lines plotted against JD. Red points denote recent measurements of the $B_\text{e}$ field obtained using the DAO dimaPol method (Bohlender 2015). The solid line is our final $B_\text{e}$ phase derived from metal lines. Note the apparent offset of $B_\text{e}$ values measured in hydrogen lines.
Table 2. Magnetic field measurements, $B_e$, of the Ap star $\gamma$ Equ, obtained with the 1-m telescope at SAO RAS.

| JD2400000.+ | $B_e$ | $\sigma_B_e$ | JD2400000.+ | $B_e$ | $\sigma_B_e$ |
|-------------|-------|--------------|-------------|-------|--------------|
| 53274.38    | -1041.| 35           | 54490.16    | -1203.| 24           |
| 53274.40    | -1041.| 38           | 54779.25    | -1005.| 29           |
| 53275.43    | -995. | 55           | 54780.14    | -1052.| 29           |
| 53277.43    | -1042.| 44           | 54780.18    | -1053.| 27           |
| 53278.41    | -1043.| 38           | 54781.14    | -916. | 20           |
| 53278.43    | -1042.| 41           | 54782.15    | -974. | 22           |
| 53279.37    | -1064.| 37           | 54783.17    | -1092.| 25           |
| 53279.39    | -1064.| 37           | 55081.32    | -1020.| 37           |
| 53346.25    | -1025.| 38           | 55081.36    | -1020.| 38           |
| 53511.47    | -1078.| 43           | 55082.31    | -1004.| 37           |
| 53511.51    | -1079.| 51           | 55082.34    | -1005.| 38           |
| 53513.52    | -1268.| 48           | 55083.28    | -971. | 38           |
| 53514.53    | -827. | 36           | 55083.31    | -971. | 34           |
| 53528.35    | -1235.| 44           | 55348.48    | -1037.| 38           |
| 53629.27    | -902. | 160          | 55448.50    | -1038.| 39           |
| 53629.29    | -973. | 99           | 55351.52    | -889. | 69           |
| 53629.31    | -1001.| 61           | 55494.17    | -815. | 34           |
| 53629.34    | -1028.| 58           | 55494.20    | -814. | 35           |
| 53629.36    | -851. | 63           | 55495.18    | -879. | 45           |
| 53629.38    | -974. | 60           | 55495.21    | -880. | 56           |
| 53629.40    | -1023.| 54           | 55496.18    | -1009.| 39           |
| 53629.43    | -1065.| 67           | 55496.21    | -1010.| 39           |
| 53629.45    | -1040.| 87           | 55881.27    | -894. | 34           |
| 53629.47    | -937. | 118          | 55881.30    | -895. | 33           |
| 53630.25    | -1002.| 40           | 55911.12    | -1051.| 99           |
| 53630.27    | -1002.| 57           | 56021.55    | -907. | 26           |
| 53632.36    | -1159.| 33           | 56021.61    | -905. | 25           |
| 53634.30    | -1084.| 52           | 56058.53    | -1036.| 49           |
| 53635.27    | -1102.| 51           | 56058.55    | -1036.| 45           |
| 53635.29    | -1102.| 49           | 56114.37    | -862. | 37           |
| 53636.25    | -912. | 51           | 56114.49    | -862. | 32           |
| 53637.23    | -1133.| 32           | 56228.26    | -946. | 25           |
| 53637.25    | -997. | 29           | 56228.29    | -946. | 30           |
| 53638.21    | -1091.| 35           | 56229.20    | -1035.| 29           |
| 53638.23    | -1090.| 31           | 56229.23    | -1035.| 33           |
| 53666.17    | -1104.| 32           | 56262.12    | -931. | 25           |
| 53666.19    | -1104.| 38           | 56262.14    | -931. | 28           |
| 53667.18    | -1141.| 32           | 56263.11    | -827. | 28           |
| 53667.21    | -1140.| 36           | 56263.14    | -827. | 30           |
| 53668.19    | -1086.| 28           | 56264.12    | -946. | 39           |
| 53718.12    | -1232.| 32           | 56264.14    | -945. | 36           |
| 53719.11    | -1133.| 55           | 56500.40    | -843. | 38           |
| 53721.12    | -1153.| 48           | 56500.43    | -804. | 41           |
| 54373.19    | -1024.| 27           | 56587.18    | -712. | 57           |
| 54373.22    | -1023.| 31           | 56587.21    | -698. | 51           |
| 54374.20    | -1153.| 29           | 56590.15    | -755. | 48           |
| 54374.23    | -1153.| 49           | 56590.18    | -757. | 51           |
| 54430.13    | -1204.| 25           |             |       |              |

8 APPENDIX A