Inflation in Supersymmetric Unified Theories

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Abstract

We construct supersymmetric unified models which automatically lead to a period of inflation. The models all involve a U(1) symmetry which does not belong to the MSSM. We consider three different types of models depending on whether this extra U(1) is the subgroup of a non abelian gauge group, is a U(1) factor belonging to the visible sector or is a U(1) factor belonging to the hidden sector. Depending on the structure of the unified theory, on the spontaneous symmetry breaking pattern and on whether we have global or local supersymmetry, inflation may be driven by the non-vanishing vacuum expectation value of a F-term or by that of a D-term. In both scenarios cosmic strings form at the end of inflation, and they have different properties in each model. Both inflation and cosmic strings contribute to the CMBR temperature anisotropies. We show that the strings contribute to the $C_l$'s up to the level of 75%. Hence the contribution from strings to the CMBR and to the density perturbations in the early Universe which lead to structure formation cannot be neglected. We also discuss a very interesting class of models which involve a $U(1)_{B-L}$ gauge symmetry.

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I. INTRODUCTION

Supersymmetric unified theories of the strong, weak and electromagnetic interactions, such as the left-right model $\text{SU}(3)_c \times \text{SU}(2)_L \times \text{SU}(2)_R \times U(1)_{B-L}$ or grand unified theories such as $\text{SO}(10)$ may solve many of the outstanding problems of both the particle physics and the cosmological standard models [1]. In particular, they can solve the question of fermion masses or lead to automatic R-parity conservation, they can explain the matter anti-matter asymmetry of the Universe and provide good dark matter candidates. Such theories beyond the standard model may also be needed to explain the grand desert between the string unification scale $\sim 5 \times 10^{17}$ GeV and the electroweak scale $\sim 10^2$ GeV, if string theory is the theory of quantum gravity needed to explain the state of the Universe at the Planck scale.

In building supersymmetric grand unified models aiming to be consistent with cosmology one is confronted with two main problems. The first problem is that any semi-simple grand unified gauge group which is broken down to the standard model $\text{SU}(3)_c \times \text{SU}(2)_L \times U(1)_Y$ inevitably leads to the formation of topologically stable monopoles, according to the Kibble mechanism [3]. These monopoles, if present today, would dominate the energy density of the universe, and our universe would be very different from what we observe. Even if the grand unified gauge group $G$ is not semi-simple, it may still be confronted with the monopole problem. Topologically stable monopoles form during the spontaneous symmetry breaking (SSB) pattern from $G$ down to the standard model gauge group if the second homotopy group $\pi_2(\text{SU}(3)_c \times \text{SU}(2)_L \times U(1)_Y)$ is non-trivial. For example, unified theories such as the trinification $\text{SU}(3)^3$ or the one based on the Pati-Salam group $\text{SU}(4) \times \text{SU}(2)_L \times \text{SU}(2)_R$ also lead to the formation of topologically stable monopoles making the models inconsistent with observations on their own. Hence some mechanism has to be invoked to get rid of these monopoles. The mechanism which is the most promising is that of inflation. Because inflation not only solves the monopole problem, but also predicts the formation of large scale structures, it predicts anisotropies in the temperature fluctuations of the cosmic microwave background radiation (CMBR), and it solves the horizon problem [2]. The second problem, which is directly related to this first one, is that inflation usually requires very fine tuning of the parameters.

For the monopole problem to be cured, the period of inflation must take place after the phase transition leading to the formation of the unwanted monopoles, and after any other phase transition associated with the formation of other catastrophic defects such as domain walls. Once a scenario for inflation has been chosen, the study of topological defect formation before and after inflation and the various possible SSB patterns from the unified gauge group down the the standard model, together with the requirement that a scenario for baryogenesis occurs after inflation, allows one to select the SSB patterns which are cosmologically interesting. For example, SSB pattern selection from topological defect formation in a supersymmetric SO(10) models can be found in [4].

If inflation comes from the (grand) unified theory (GUT) beyond the standard model which describes the symmetries between particles at $\sim 2 \times 10^{16}$ GeV, it is probably of the hybrid type [5]. This is because hybrid inflationary scenarios occur without fine-tuning and arise in a natural way. By natural we mean that, in the models considered below, no extra field is needed, apart from a scalar field singlet under the considered gauge group. This extra field is probably anyway needed to build the model to force the Higgs field used to
lower the rank of the group to acquire a vacuum expectation value. We point out that potentials which do not require the existence of the scalar singlet can be used to lower the rank of the group, but then inflation cannot arise from this sector since it does not involve any dynamical field. Note that inflation could also come from string theory for example. We do not consider this case here.

There are two main possibilities for the hybrid inflationary scenario to be implemented. The period of inflation can arise from the GUT considered itself, by for example coupling some GUT Higgs fields to a scalar field singlet under the considered gauge group which plays the role of the inflaton, or it can come from another sector, part of the visible sector which must then involve some extra gauge or global symmetry, or from the hidden sector. Inflation assumes that there was a period in the very early universe when the potential, vacuum energy density was dominating the energy density of the universe, so that the cosmic scale factor grew exponentially. Since in supersymmetric theories the scalar potential is basically the sum of the F-terms and D-terms, inflation may be driven by either the non-vanishing vacuum expectation value (vev) of a F-term or by that of a D-term. Almost all inflationary scenarios in supersymmetric theories considered in the literature rely on the non-vanishing vev of a F-term. This is, however, usually a problem in supergravity theories \[6\]. Indeed, for inflation to occur and lead to a nearly scale independent spectrum of density perturbations, as suggested by observations, the potential must be flat in the inflaton direction, and the inflaton must have a very small mass. During inflation, the energy density of the universe is dominated by the vacuum energy density, which is usually close to \( V_0^{1/4} \sim M_{\text{GUT}} \), and all the scalars of the theory acquire a mass \( m^2 = \frac{V_0}{M_{\text{pl}}^2} \). The inflaton mass is then much greater than the Hubble constant \( H \) and the slow roll conditions, which require \( \frac{V'}{V} << H \), where primes denote derivative with respect to the inflaton field, cannot be satisfied. A way out of that problem is to consider special form of the superpotential as well as particular initial conditions \[7\], special form of the Kähler potential \[8\], or extra symmetries such as a global U(1) \[9\]. We do not consider these cases here. Now when \( V_0 \) is the result of a non-vanishing D-term, scalar fields which are uncharged under the unified gauge group, such as the inflaton field, only get masses \( m \ll H \) \[10\]. The slow conditions are satisfied and inflation can take place. Hence in supergravity, hybrid inflationary scenarios driven by the non-vanishing vev of a D-term are favoured.

D-term inflation requires the existence of a Fayet-Illiopoulos term \[12\], which can only exist if the group contains a U(1) factor with \( \text{Tr}Q \neq 0 \) (where Q is the U(1) charge) \[13\]. Hence the Fayet-Illiopoulos D-term of a U(1) gauge symmetry cannot be generated in any order of perturbation theory if this U(1) is at some arbitrary scale unified in a non-abelian gauge group. Thus D-term inflation can arise in the visible sector or in the hidden sector only if there is a U(1) factor with \( \text{Tr}Q \neq 0 \) in the appropriate sector. (Though dynamical generation of a D-term at an intermediate symmetry breaking scale may be possible, we do not consider this as a possible mechanism for inflation here). Hence for a semi-simple gauge group, if inflation arises from the GUT itself, it can only be driven by the non-vanishing vev of a F-term.

The plan of this paper is as follows:

In Sec. II we briefly review the idea of supersymmetric unified theories.

In Sec. III we describe an easy and useful way of building a unified theory of the strong, weak and electromagnetic interactions which automatically lead to a period of false vacuum.
hybrid inflation [3]. ‘Hybrid’ because the inflaton field couples to some Higgs fields which are used to break a U(1) gauge symmetry, and ‘false vacuum’ because during inflation, which occurs for values of the inflaton field much greater than a given critical value, the Higgs fields are trapped at a local minimum of the potential. We consider both F-term inflation with the simplest possible superpotential [6,18] and D-term inflation [16,17]. The models which are discussed are all based on rank greater than five theories. They all involve a U(1) gauge symmetry which does not belong to the minimal supersymmetric standard model (MSSM). We discuss three main class of models, depending on whether this extra U(1) (i) is the subgroup of a non abelian gauge group (ii) is a U(1) factor belonging to the visible sector or (iii) is a U(1) factor belonging to the hidden sector.

All the models which are discussed in this paper lead to the formation of cosmic strings at the end of inflation. The formation of cosmic strings at the end of inflation is discussed in Sec.IV A. Note that cosmic strings formation at the end of inflation in a somewhat different context has been discussed before [19]. In Sec.IV B, we determine the spectral index of density perturbations coming from inflation and the inflationary scale. We also determine the relative contributions from inflation and cosmic strings to the CMBR.

In Sec.V, we discuss in details the three main classes of models to which the construction described in Sec.III applies. We illustrate each case with an interesting class of unified models: theories beyond the standard model which contain a U(1)B−L gauge symmetry.

In Sec.VI we summarise our main results and conclude.

II. THE SETUP

In studying supersymmetric unified theories, the picture we have in mind is that the very early universe underwent a series of phase transitions associated with the SSB pattern:

\[ G \times \text{SUSY} \xrightarrow{M_{\text{GUT}}} \ldots \rightarrow H \times \text{SUSY} \rightarrow \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y (\times \text{SUSY}) \]

\[ \xrightarrow{M_Z} \text{SU}(3)_c \times \text{U}(1)_Q \]  

(1)

where G is the (grand) unified gauge group (not necessarily semi-simple) and SUSY stands for supersymmetry. The unified scale is \( M_{\text{GUT}} \sim 2 \times 10^{16} \) GeV, the electroweak scale is \( M_Z \sim 100 \) GeV and supersymmetry is broken at \( \sim 10^3 \) GeV. Then there also may or may not be an underlying theory such as superstrings valid at higher energies. If this theory, which must be a theory of quantum gravity, is superstring theory, then the grand unified gauge coupling constant \( g_G \) would run up to the string scale \( M_{\text{string}} \sim 5 \times 10^{17} \) GeV where it would then unify with the gauge coupling of the hidden gauge group and with the gravitational coupling.

If G is assumed to be recovered from superstring compactification schemes, there will be some additional symmetries in both the visible sector and the hidden sector (though not necessarily in the visible sector). In this paper, we suppose the existence of a U(1) symmetry, namely U(1)x, which does not belong to the minimal supersymmetric standard model and can either be a subgroup of G or a U(1) factor belonging to the visible sector or a U(1) factor belonging to the hidden sector in the appropriate case. If G is assumed to be recovered from superstring compactification schemes, then there is a good reason for a U(1) factor to be standing there, since extra U(1) factors normally appear in effective field
theories arising from strings. The low energy spectrum will also be constrained, in particular some high dimensional Higgs representations of the unified theory $G$ may not be allowed \[14\], such as the 126 dimensional Higgs representation in the case of SO(10) \[15\]. This will influence the choice of the grand unified gauge group, the SSB pattern and the choice of the inflationary scenario.

But supersymmetric unified theories can also be thought as standing there on their own. This gives more freedom in choosing the SSB pattern and the Higgs representations to use to implement it, the inflationary scenario and the way of getting some desirable phenomenological effects such as neutrino masses and R parity conservation.

**III. BUILDING A MODEL**

We consider here an easy and useful way of building a supersymmetric unified model which gives rise to a false vacuum hybrid inflationary scenario without fine tuning, which is independent of the process of supersymmetry breaking at low energy, and independent of the form of the supersymmetry breaking parameters.

**A. Assumptions**

We suppose the existence of a $U(1)_x$ gauge symmetry which does not belong to the MSSM and spontaneously breaks at a scale $M_x$. The $U(1)_x$ is related to the inflationary scenario: the inflaton field couples with a pair of Higgs field $\Phi_x$ and $\overline{\Phi}_x$ which are used to break $U(1)_x$. The inflationary scenario may be either of the F-term hybrid type with the simplest possible superpotential \[6,18\] or D-term hybrid type \[16,17\], depending on whether the supersymmetry is global or local, on the SSB pattern and on the origin of $U(1)_x$. We consider three distinct classes of models, depending on whether the $U(1)_x$ (i) is a subgroup of the considered grand unified gauge group $G$, and the SSB given by:

\[
G \times \text{SUSY} \xrightarrow{M_{GUT}} \cdots \to (H \supseteq U(1)_{x}) \times \text{SUSY} \xrightarrow{M_x} (K \nsubseteq U(1)_{x}) \times \text{SUSY} \to \cdots \\
\to \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y \times \text{SUSY} \xrightarrow{M_x} \text{SU}(3)_c \times \text{U}(1)_Q, \tag{2}
\]

(ii) is an extra $U(1)$ symmetry belonging to the visible sector without being a subgroup of a non abelian gauge group, and the SSB is given by:

\[
G \times U(1)_x \times \text{SUSY} \xrightarrow{M_{GUT}} \cdots \to H \times U(1)_x \times \text{SUSY} \\
\xrightarrow{M_x} \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y (\times \text{SUSY}) \\
\xrightarrow{M_x} \text{SU}(3)_c \times \text{U}(1)_Q 
\tag{3}
\]

(iii) is an extra $U(1)$ symmetry belonging to the hidden sector which is not the subgroup of a non abelian gauge group, and the SSB then given by:

\[
G \times [U(1)_x]_{\text{hidden}} \times \text{SUSY} \xrightarrow{M_{GUT}} \cdots H \times [U(1)_x]_{\text{hidden}} \times \text{SUSY} \\
\xrightarrow{M_x} \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y (\times \text{SUSY}) \\
\xrightarrow{M_x} \text{SU}(3)_c \times \text{U}(1)_Q. \tag{4}
\]

In each case, the scale $M_x$ will be constraint by the four year COBE DMR data, see Sec.IV B.
B. The superpotential

The superpotential which will implement the full SSB pattern from $G \times U(1)_x$ down to $SU(3) \times U(1)_Q$ and gives rise to a period of inflation is constructed by considering different sectors, and by adding the superpotentials describing each sector.

The different sectors are:

- The inflaton sector, which breaks the $U(1)_x$ gauge symmetry mentioned above, it is described by $W_{infl}$.
- The GUT sector which implements the SSB of $G$ down to the standard model gauge group (apart from that of $U(1)_x$ when applicable), it is described by $W_{GUT}$.
- The electroweak sector which breaks $SU(3)_c \times SU(2)_L \times U(1)_Y$ down to $SU(3)_c \times U(1)_Q$ and is described by $W_{ew}$.
- And a hidden sector (which also includes the $U(1)_x$ when appropriate) which is described by $W'_{hidden}$. Note that in supergravity, by hidden sector we mean a sector of the theory that couples to the observable sector of quarks, leptons, gauge fields, higgses and their supersymmetric partners only through gravitational interactions. Since globally supersymmetric theories do not include gravity, it probably does not really make sense to talk about hidden sector in that case. Hence in globally supersymmetric theories we distinguish the visible sector from the hidden sector by assuming that the $\Phi_x$ and $\overline{\Phi}_x$ Higgs fields used to break $U(1)_x$ carry G quantum numbers or not and if the particle contain of the MSSM carries $U(1)_x$ quantum numbers or not.

Note that some of these sectors will still be linked for doublet-triplet splitting purpose or the avoidance of unwanted massless pseudo-Goldstone bosons. If they are linked, it must be done in such a way that it does not destabilise the required vevs. Additional link terms (i.e. not actually necessary to implement the SSB pattern nor the inflationary scenario), must be included in $W_{GUT}$ and $W_{ew}$.

The resulting superpotential is then written as

$$W_{tot} = W_{GUT}(A's) + W_{infl}(S, \Phi_x, \overline{\Phi}_x) + W_{ew}(H_1, H_2) + W'_{hidden}(Z's)$$

(5)

where the $A$’s, $H_1$ and $H_2$ are Higgs superfields with the bosonic components in appropriate representations of $G$. In this paper, we will use the same notation for the chiral superfields and their bosonic components if there is no risk of confusion. $H_1$, and $H_2$ must contain SU(2) doublets. $S$ is a scalar field singlet under $G$ which plays the role of the inflaton. $\Phi_x$ and $\overline{\Phi}_x$ are Higgs superfields used to break the $U(1)_x$ gauge symmetry. They have opposite $x$ charges. They may transform non-trivially under $G$ where appropriate, see Sec.\[4]. $Z'$s are Higgs superfields which belong to the hidden sector and interact only gravitationally with the other fields.

Note that if we use the general splitting of $W_{tot}$ into the two main sectors which are the visible and the hidden sector [12]:

$$W_{tot} = W_{visible} + W_{hidden},$$

(6)
we have $W_{\text{visible}} = W_{\text{GUT}} + W_{\text{inf}} + W_{\text{ew}}$, $W_{\text{hidden}} = W'_{\text{hidden}}$ in cases (i) and (ii) and $W_{\text{visible}} = W_{\text{GUT}} + W_{\text{ew}}$, $W_{\text{hidden}} = W_{\text{inf}} + W'_{\text{hidden}}$ in case (iii).

The superpotential in the inflaton sector will look different whether inflation is driven by the non-vanishing vev of a F-term or by that of a D-term. Extra symmetries must be imposed for each particular case. The choice for F-term or D-term inflation is model dependent, see Sec. V.

In this paper, we consider the simplest superpotentials which lead to a period of F-term and D-term inflation respectively, and do not involve any non-renormalisable term, but generalisation to inflationary scenarios with more complicated superpotentials in the inflaton sector is straightforward.

In the case of F-term inflation, the superpotential in the inflaton sector is chosen to be [6,18]:

$$W_{\text{infl}}(S, \Phi_x, \overline{\Phi}_x) = \alpha S \Phi_x \overline{\Phi}_x - \mu^2 S. \quad (7)$$

The $\Phi_x$ and $\overline{\Phi}_x$ are Higgs fields used to break $U(1)_x$ in appropriate representations. $S$ is a scalar superfield singlet under $G$ which plays the role of the inflaton field. $\alpha$ and $\mu$ are two positive constants, and the ratio $\frac{\mu}{\sqrt{\alpha}}$ sets the $U(1)_x$ symmetry breaking scale $M_x$. The superpotential given by Eq.(7) is the most general potential consistent with a continuous $R$ symmetry under which $S \rightarrow e^{i\gamma} S$, $\Phi_x \rightarrow e^{i\gamma} \Phi_x$, $\overline{\Phi}_x \rightarrow e^{i\gamma} \overline{\Phi}_x$ and $W \rightarrow e^{i\gamma} W$. Note that this superpotential with the assumption of minimal Kähler potential in supergravity is viable [1].

If inflation comes from the non-vanishing vev of a D-term, this can only be the case when the $U(1)_x$ gauge symmetry is not the subgroup of a non-abelian gauge group, the superpotential in the inflaton sector is [14,17]:

$$W_{\text{infl}}(S, \Phi_x, \overline{\Phi}_x) = \alpha S \Phi_x \overline{\Phi}_x \quad (8)$$

$\Phi_x$ and $\overline{\Phi}_x$ are Higgs fields with $x$-charges $+Q$ and $-Q$ respectively, which break $U(1)_x$ when acquiring vevs. In the following, we will set $Q = 1$. Note that the qualitative results given in this paper will be unchanged for higher charged Higgs fields. $S$ is a scalar superfield singlet under $G$ which plays the role of the inflaton, and $\alpha$ is a positive constant. The potential given by Eq.(8) is the most general potential consistent with a continuous $R$ symmetry under which $S \rightarrow e^{i\gamma} S$, $\Phi_x \rightarrow e^{i\gamma} \Phi_x$, $\overline{\Phi}_x \rightarrow e^{i\gamma} \overline{\Phi}_x$ and $W \rightarrow e^{i\gamma} W$. There is also a $Z_3$ symmetry under which the fields transform as $\chi \rightarrow e^{i\frac{2\pi}{3}} \chi$, for $\chi = S, \Phi_x$ and $\overline{\Phi}_x$. The existence of a Fayet-Iliopoulos D-term for the $U(1)_x$ symmetry is assumed. It sets the $U(1)_x$ symmetry breaking scale as well as the inflationary scale, see Sec. IV B.

C. The scalar potential

1. Globally supersymmetric case

In globally supersymmetric theories, the scalar potential $V_{\text{tot}}$ which can be derived from the superpotential given in Eq.(3) is [1]:

$$V_{\text{tot}} = \sum_i |F_i|^2 + \frac{1}{2} \sum_{\alpha} |D_{\alpha}|^2 + V_{\text{soft}} \quad (9)$$
where \( i \) runs from 1 to \( N \), where \( N \) is the number of chiral superfields in \( W_{\text{tot}} \). \( V_{\text{soft}} \) contains all the soft terms generated by supersymmetry breaking at low energy. The F and D-terms are respectively given by:

\[
F_i = \frac{\partial W}{\partial h_i} \tag{10}
\]

and

\[
D^\alpha = g \sum_{a,b} h^*_a [T^a_{\beta a} h^b + \xi_\alpha] \tag{11}
\]

where the \( h_i \) are the scalar components of the chiral superfields included in the superpotential \( W_{\text{tot}} \) and \( T^a_\alpha \) are the generators of \( G(\times U(1)) \). \( \xi_\alpha \) is a Fayet-Illiopoulos term which can only exist when \( T^a_\alpha \) is the generator of the U(1) factor. The conditions for unbroken global supersymmetry is that all the F and D-terms equal zero.

From Eqs. (5), (9), (10) and (5), we find that the full scalar potential can be written as:

\[
V_{\text{tot}} = V_{\text{GUT}} + V_{\text{infl}} + V_{\text{ew}} + V_{\text{soft}}. \tag{12}
\]

\( V_{\text{infl}} \) has a global minimum such that the U(1)_x symmetry is broken down to unity (or eventually a discrete Z_n). \( V_{\text{GUT}} + V_{\text{infl}} \) (when U(1)_x is a subgroup of G) has a global minimum such that the gauge group G is broken down to SU(3)_c \times SU(2)_L \times U(1)_Y and supersymmetry is unbroken. Note that when G is a semi-simple gauge group and U(1)_x is a subgroup of G, the \( \Phi_x \) and \( \Phi_x^* \), which transform non-trivially under G, break U(1)_x as well some other generators of G. If some generators of G are broken in both the inflaton and the GUT sector, this will result in the existence of pseudo-Goldstone bosons which may be undesirable. The problem can be cured by linking the two sectors. As mentioned above, see Sec.III.B, this must be done in such a way that it does not destabilise the required vevs. This is not always easy \[20\]. \( V_{\text{GUT}} + V_{\text{infl}} + V_{\text{ew}} \) has a global minimum such that G is broken down to SU(3)_c \times U(1)_Q. The last two terms in Eq.(12) should not affect the behaviour of the fields in the GUT and in the inflationary sector at high energies. Indeed, as pointed out by Dvali \[10\], in the very early universe, at temperatures of order \( 10^{15} - 10^{16} \) GeV, the fields do not know whether supersymmetry will be broken at lower energy or not. Hence, the dynamics of the scenario should be independent of the soft masses for the supersymmetric particles at low energy.

We know briefly discuss the scalar potentials for both F-term and D-term inflation, and the dynamics of the fields in the inflaton sector. They have been discussed before \[18,16\]. Discussion concerning the evolution of all Higgs fields can be found in Sec.V for each appropriate model.

When inflation is driven by the non-vanishing vev of a F-term, the superpotential in the inflaton sector is given by Eq.(5), and the Fayet-Iliopoulos D-term for U(1)_x vanishes. The scalar potential in the inflaton sector is then given by:

\[
V_{\text{infl}}^F = \alpha^2 |S|^2 (|\Phi_x|^2 + |\Phi_x^*|^2) + |\alpha \Phi_0 - \mu|^2. \tag{13}
\]

\( V_{\text{infl}}^F \) has a unique supersymmetric minimum corresponding to \( \langle |\Phi_0| \rangle = \langle |\Phi_x^*| \rangle = \frac{\mu}{\sqrt{\alpha}} \) and \( S = 0 \). It also has a local minimum for \( |S| > \frac{\mu}{\sqrt{\alpha}} = S_c \), at \( \langle |\Phi_x| \rangle = \langle |\Phi_x^*| \rangle = 0 \). \( \frac{\mu}{\sqrt{\alpha}} \) sets the U(1)_x...
symmetry breaking scale $M_x$. Let’s assume chaotic initial conditions. The potential is very flat in the $|S|$ direction, and the $\Phi_x$ and $\overline{\Phi}_x$ fields settle down to the local minimum of the potential, $\Phi_x = \overline{\Phi}_x = 0$. The universe is dominated by a non vanishing vacuum energy density, $V_0^\frac{1}{4} = \mu$, inflation starts, and supersymmetry is broken. There are therefore some quantum corrections to the effective potential [18]:

$$V_{eff}^F = \mu^4 \left(1 + \frac{\alpha^2}{16\pi^2} \ln \frac{\alpha^2|S|^2}{\Lambda^2}\right)$$

(14)

where $\Lambda$ is a renormalisation constant. These corrections help the inflaton field to slowly roll down the potential. When $|S|$ falls below $S_c$ and the $\Phi_x$ and $\overline{\Phi}_x$ fields quickly settle down to the global minimum of the potential, spontaneously breaking $U(1)_x$. Supersymmetry is restored.

For D-term inflation, the superpotential in the inflation sector is given by Eq.(8), and the existence of a Fayet-Iliopoulos D-term for the $U(1)_x$ gauge symmetry, namely $\xi_x$, is assumed. It will set the $U(1)_x$ symmetry breaking scale $M_x$ as well as the inflationary scale, see Sec.IV B. The scalar potential in the inflaton sector is now given by [16,17]:

$$V_{infl}^D = \alpha^2|S|^2(|\Phi_x|^2 + |\overline{\Phi}_x|^2) + \alpha^2|\Phi_x\overline{\Phi}_x|^2 + \frac{g^2}{2}(|\overline{\Phi}_x|^2 - |\Phi_x|^2 + \xi_x)^2.$$

(15)

To summarise, $V_{infl}$ has a unique supersymmetric minimum where

$$\langle S \rangle = \langle \Phi \rangle = 0 \text{ and } \langle |\Phi| \rangle = \xi_x^\frac{1}{2}.$$

(16)

It also has a local minimum for $|S| > S_c = \frac{g}{\xi_x^\frac{1}{2}}$ at $\langle \Phi_x \rangle = \langle \overline{\Phi}_x \rangle = 0$, and $V = \frac{g^2}{2}\xi_x^2$. Hence setting chaotic initial conditions, $|S|$ is initially much greater than $S_c$ and the $\Phi_x$ and $\overline{\Phi}_x$ fields settle down the local minimum. Inflation can take place and supersymmetry is broken. This leads to a one loop correction to the potential (13) which is then given by [16,17]:

$$V_{eff}^D = \frac{g^2}{2}\xi_x^\frac{1}{2}(1 + \frac{g^2}{16\pi^2} \ln \frac{\alpha^2|S|^2}{\Lambda^2})$$

(17)

where $\Lambda$ is a renormalisation scale. This loop correction drives the inflaton slowly down the potential, independently of the process of supersymmetry breaking at low energy. The slow roll conditions are satisfied for $|S|^2 \gg \frac{M_d^2g^2}{8\pi^2} = S_{end}$. When $|S|$ falls below $S_{end}$ inflation ends. When $|S|$ falls below $S_c$, the $\Phi$ and $\overline{\Phi}$ fields quickly reach the global minimum of the potential. What is important is that the scenario does not depend on the process of supersymmetry breaking at low energy. The value of the Fayet-Iliopoulos constant $\xi_x$ will be determined for successful inflation.

2. Locally supersymmetric case

In supergravity theories, the full superpotential which implements the SSB pattern, the inflationary scenario and the breaking of susy can also be written as in Eq.(5). Note that the superpotential in each sector will in general be different from the one used in the global case.
The scalar potential in supergravity theories is given by \([1]\):

\[
V = e^{\frac{K}{M^2}}(K^{-1})_i^j F_i F^j - 3 \frac{|W|^2}{M^2} + \frac{g^2}{2} \text{Re}[f_{\alpha\beta}] D^\alpha D^\beta
\]

(18)

where \(M = \frac{M_{pl}}{\sqrt{8\pi}}\) is the reduced Planck mass. \(K(H,\overline{H})\) is the Kähler function, here \(H\) denotes any Higgs superfield in the appropriate representation, or a scalar superfield, \(W(H)\) is the superpotential, and \(f(H)\) is the gauge kinetic function. Upper (lower) indexes \((i,j)\) denote derivatives with respect to the scalar components of the corresponding chiral superfields \(h_i \) (\(h_i^*\)). The F-terms in supergravity theories differ from those in the globally supersymmetric case and are given by

\[
F^i = \frac{\partial W}{\partial h_i} + \frac{\partial K}{\partial h_i} \frac{W}{M}
\]

(19)

The supergravity D-terms are

\[
D^\alpha = K^i(T^\alpha)^j h_j + \xi_\alpha
\]

(20)

where \(T^\alpha\) are the generators of the appropriate gauge group in the corresponding representation and \(\xi_\alpha\) is a Fayet-Illiopoulos term which can only exist when \(T^\alpha\) is the generator an abelian group with \(TrQ \neq 0\). It is easy to check that when \(M \rightarrow \infty\), the global supersymmetric potential is recovered.

The conditions for unbroken supergravity is that the supergravity F and D terms vanish. Hence in locally supersymmetric unified theories the superpotential in each sector will in general be different from the one used in the global case. The second reason for these superpotentials to be different is that most of the superpotentials used to build globally supersymmetric GUTs do not vanish and consequently lead to a non-vanishing cosmological constant. To have get a vanishing cosmological constant the superpotential in each sector should vanish. For example, Higgs superpotential used to build globally supersymmetric GUTs usually satisfies \(\langle W_{GUT} \rangle \sim M^3_{GUT}\) which would give rise to an enormous negative cosmological constant, inconsistent with observations. The cosmological constant problem can be cured, for example, by adding a term to the superpotential such that \(\langle W_{GUT} \rangle = 0\).

In what follows, we assume minimal kinetic terms \(f^{\alpha\beta} = 1\) and minimal Kähler potential \(K = \sum_i H_i^* H_i\). We also assume that \(\langle W_{GUT}(A) \rangle = 0\) and \(\langle W_{infl}(S, \Phi_x, \overline{\Phi}_x) \rangle = 0\), for \(A, S, \Phi_x\) and \(\overline{\Phi}_x\) acquiring vev such that G is broken down to \(SU(3)_c \times SU(2)_L \times U(1)_Y\) and \(U(1)_x\) is broken down to unity (note that in case (i) \(U(1)_x\) is a subgroup of the grand unified gauge group G). We also assume that the \(\overline{\Phi}_x\) is initially set to zero.

Whereas in globally supersymmetric GUTs both F-term and D-term inflation can occur, in supergravity GUTs D-term inflation is strongly favoured, see Sec.\([4]\). This requires the \(U(1)_x\) gauge symmetry to be a U(1) factor with a non-vanishing Fayet-Illiopoulos D-term. We consider this case here.

The GUT symmetry breaking at high energy and the inflationary scenarios are independent of the supersymmetry breaking mechanism at low energy, as was justified earlier. With the assumption of minimal supergravity, the scalar potential at high energies is:

\[
V = e^{\frac{K}{M^2}}[F_{GUT} F_{GUT}^* + F_{infl} F_{infl}^* - 3 \frac{(W_{GUT} + W_{infl})^2}{M^2}] + \frac{g^2}{2} D_{GUT} D_{GUT} + \frac{g^2}{2} D_{infl} D_{infl}
\]

(21)
where $F_{\text{GUT}}$, $F_{\text{infl}}$, $D_{\text{GUT}}$ and $D_{\text{infl}}$ are the sum of the F and D-terms for the fields in the GUT and the inflation sector respectively. Recall also that the $A, S, \Phi_x$ and $\overline{\Phi}_x$ fields must satisfy the vanishing conditions of the F and D-terms for unbroken supergravity and hence $\langle F_{\text{GUT}} \rangle = 0$, $\langle F_{\text{infl}} \rangle = 0$, $\langle D_{\text{GUT}} \rangle = 0$ and $\langle D_{\text{infl}} \rangle = 0$. With the above choice (tuning) of the parameters, the cosmological constant at high energy vanishes before and after the inflationary period.

The scalar potential now reads:

$$V = e^{\sum_i |A_i|^2 + |S|^2 + |\Phi_x|^2 + |\overline{\Phi}_x|^2} \left[ |\Phi_x \overline{\Phi}_x|^2 (1 + \frac{|S|^4}{M^2}) + |S \Phi_x|^2 (1 + \frac{|S|^4}{M^2}) \right] + |S \Phi_x|^2 (1 + \frac{|S|^4}{M^2}) + \frac{g^2}{2} (|\Phi_x|^2 - |\Phi_x|^2 + \xi^2)^2.$$ (22)

As in the globally supersymmetric case, the potential as a local minimum for $|S| > S_c = \frac{2}{\alpha} \xi^2$ and $\Phi_x = \overline{\Phi}_x = 0$ and a locally supersymmetric one with $S = \overline{\Phi}_x = 0$ and $|\Phi_x| = \xi^2$ and vanishing cosmological constant. The behaviour of the fields is then similar to the globally supersymmetric case.

### IV. CONSTRAINTS FROM COBE DATA

In this section, we determine the scale of inflation and the relative contributions from inflation and cosmic strings to the CMBR. We mainly focus on the case of D-term inflation, where string formation at the end of inflation has not yet been discussed.

#### A. Cosmic strings form at the end of inflation

Recall that cosmic strings are one dimensional topological defects which form according to the Kibble mechanism [3] at the phase transition associated with the SSB of a group $G$ down to a subgroup $H$ of $G$ if the vacuum manifold $G/H$ contains non contractible loops. In other words, cosmic strings form when $G \rightarrow H$ if the first homotopy group $\pi_1(G/H)$ is non trivial. For review on cosmic strings the reader is referred to Refs. [25,26].

The simplest well known example of SSB which leads to the formation of cosmic strings is that of the abelian Higgs model, when a U(1) symmetry breaks down to the identity. Since in both the F-term and D-term inflationary scenarios discussed in this paper, the inflaton field couples to a pair of Higgs fields used to break a U(1) gauge symmetry when acquiring vev at the end of inflation, cosmic strings are expected to form. It is indeed easy to check that the scalar potentials in the inflationary sector both the F-term and D-term inflationary scenarios, given by Eqs.(13) and (15) respectively, have string solutions. For F-term inflation, the scalar potential is given by (13), both $\Phi_x$ and $\overline{\Phi}_x$ acquire a non-vanishing vev, $\langle |\Phi_x| \rangle = \langle |\overline{\Phi}_x| \rangle = \frac{\mu}{\sqrt{\alpha}}$, and the potential is minimised for $\arg(\Phi_x) + \arg(\overline{\Phi}_x) = 0$. The two fields $\Phi_x$ and $\overline{\Phi}_x$ conspire to form the gauge string [22]. Now in the D-term inflationary scenario the scalar potential is given by Eq. (15), only the $\Phi_x$ field acquires a non-vanishing, the string is a Nielsen-Olesen string [23,24] and the Higgs field forming the string is $\Phi_x$. Therefore cosmic strings form at the end of inflation in both the F-term and D-term inflationary scenarios. Whether these
strings are topologically stable is model dependent. They are stable in models (ii) and (iii), and almost always stable in models belonging to the class (i), see Sec. V.

From now on, we focus on the case of D-term inflation. In this case, the masses of the Higgs and gauge fields forming the string are identical. The string mass per-unit-length is then given by [26]:

$$\mu = 2\pi \xi_x.$$  \hspace{1cm} (24)

Hence cosmic strings forming at the end of inflation are very heavy, and we expect them to contribute to the temperature anisotropies in the CMBR as well as to the fluctuations in energy density of the universe which lead to structure formation. These cosmic strings may also have other cosmological consequences as will be discussed in Sec. V. In the following section, we will determine the relative contributions from inflation and cosmic strings to the CMBR.

**B. The predictions**

The temperature fluctuations in the CMBR are proportional to the density perturbations which were produced in the very early Universe and lead to structure formation:

$$\frac{\delta T}{T} = \frac{1}{3} \frac{\delta \rho}{\rho}.$$  \hspace{1cm} (25)

In the case of the F-term and D-term hybrid inflationary scenarios discussed in this paper, cosmic strings form at the end of inflation. Thus we have contributions to \(\delta \rho\) and hence to \(\frac{\delta T}{T}\) from both inflation and cosmic strings in different proportions which we estimate. Whether the strings are topologically stable depends on the specific model which is considered. In the following, we suppose the strings to be topologically stable. We take the normalisation to COBE for inflation from Ref. [23] and for cosmic strings from Refs. [24].

It is usual to expand the temperature anisotropies in spherical harmonics:

$$\frac{\delta T}{T}(\theta, \phi) = \sum_{l,m} a_{lm} Y_{lm}$$  \hspace{1cm} (25)

and then to work with the multipole moments (or angular power spectrum):

$$C_l = \frac{1}{2l+1} \sum_{m=-l}^{m=l} |a_{lm}|^2$$  \hspace{1cm} (26)

which contain all information of the CMB (if it is Gaussian).

The density perturbations (and other quantities) are usually expressed in Fourier space, because the different fluctuation modes are uncorrelated and the solution is greatly simplified:

$$\delta(k) = \int d^3 x \frac{\delta \rho}{\rho}(x) e^{i k \cdot x}.$$  \hspace{1cm} (27)

The spectrum of density perturbations is usually assumed to be a power law:

$$\delta(k) \propto k^{n-1},$$  \hspace{1cm} (28)

as suggested by observations. The exponent \(n\) is called the spectral index.
The spectrum of density perturbations coming from inflation and the spectral index can be calculated analytically using the slow roll parameters. These are given by \[28\]:

\[ \epsilon = \frac{M_{pl}^2}{16\pi} \left( \frac{V'}{V} \right)^2 \quad \text{and} \quad \eta = \frac{M_{pl}^2 V''}{8\pi V} \]  

(29)

where prime denotes derivative with respect to the inflaton field \(|S|\). The conditions for inflation to happen are \[28\]:

\[ \epsilon \ll 1 \quad \text{and} \quad |\eta| \ll 1. \]  

(30)

For D-term inflation, with an effective potential given by Eq.(17), the slow parameters are:

\[ \epsilon = \frac{M_{pl}^2 g^4}{128\pi^2 |S|^2} \quad \text{and} \quad \eta = -\frac{M_{pl}^2 g^2}{64\pi^3 |S|^2}. \]  

(31)

And we see that \(\epsilon\) is always smaller than \(\eta\). Consequently, in this case inflation ends when \(|\eta| = 1\). The value of the inflaton field at the end of inflation is:

\[ S_{\text{end}}^2 = \frac{M_{pl}^2 g^2}{64\pi^3}. \]  

(32)

The spectrum of density perturbations coming from inflation is given by \[28\]:

\[ \delta_H^2(k) = (1 + 4.0\epsilon - 2.1\eta) \frac{512\pi}{75} \frac{V^3}{M_{pl}^6 V''^2} |k=aH| \]  

(33)

where \(k\) is the comoving wave number, \(a\) is the cosmic scale factor and \(H\) is the Hubble parameter. The right-hand side of Eq.(33) must be evaluated at the epoch of horizon exit. The spectral index of density perturbations can also be expressed in terms of the slow parameters:

\[ n = 1 - 6\epsilon + 2\eta \]  

(34)

and can be evaluated at any scale. We also evaluate it at the epoch of horizon exit. The number of e-foldings between two values of the inflaton field \(S_1\) and \(S_2\) is given by

\[ N(|S|) \approx \frac{8\pi}{M_{pl}^2} \int_{S_2}^{S_1} \frac{V}{V'} d|S|. \]  

(35)

Assuming that it takes 60 e-folds for the cosmological scales to leave the horizon, the value of the inflaton field at the epoch of horizon exit is then:

\[ S_{60}^2 = \frac{121g^2}{64\pi^3} M_{pl}^2. \]  

(36)

Using Eqs. (31), (34) and (36) we find the value for the spectral index:

\[ n = 0.98 \]  

(37)
where we have used the relation $\frac{\pi^2}{4\xi} = \frac{1}{25}$. For power law inflation, the multipole moments \[ (38) \] are related to spectrum of density perturbations at horizon exit \[ (33) \] by:

$$l(l+1)C^\text{infl}_l = f(n,l)\delta_H^2$$

where $f$ is a function of the spectral index $n$ and of the spherical harmonic $l$ which is given by \[ (28) \]:

$$f(n,l) = \frac{\pi^\frac{\frac{3}{2}}{4}}{\Gamma\left(l + \left(\frac{n}{2}\right)\right)} \frac{\Gamma\left(l + \left(\frac{n-1}{2}\right)\right)}{\Gamma\left(l + \left(\frac{5-n}{2}\right)\right)}.$$  

(39)

For cosmic string scenarios, the multipole moments are proportional to the string mass per unit length $\mu = 2\pi \xi_x$. Numerical simulations give, for $l \leq 20$, \[ (39) \]:

$$l(l+1)C^\text{str}_l \sim 350(G\mu)^2$$

(40)

where $G$ is Newton’s constant.

With both inflation and cosmic strings the multipole moments are given by given by:

$$C^\text{tot}_l = C^\text{infl}_l + C^\text{str}_l.$$  

(41)

The best fitting to COBE data occurs at the fourteenth multipole \[ (23) \]. The normalisation to COBE for inflation yields \[ (23) \]:

$$\delta_H^\text{norm}(n,r) = N^\text{infl} = 1.91 \times 10^{-5} \frac{\exp\left(1.01(1-n)\right)}{\sqrt{1+0.75r}} = 1.94 \times 10^{-5}$$

(42)

for the density perturbations, where $r$ measures the relative importance of gravitational waves and density perturbations to the relevant multipole moment. Using the results of Ref. \[ (23) \], we get $r = 2.7 \times 10^{-3}$, and hence the gravitational wave spectrum can be neglected here. Note that this is a common feature of all hybrid models. The uncertainty in $\delta_H^\text{norm}$ is 9%. Now the normalisation to COBE for cosmic strings yields \[ (24) \]:

$$(G\mu)^\text{norm} = N^\text{str} = 1.05^{+0.35}_{-0.20} \times 10^{-6}$$

(43)

for the string mass per unit length. Combining equations \[ (38) \], \[ (10) \], \[ (11) \], \[ (12) \] and \[ (13) \] leads to a normalisation equation for a mixed scenario with inflation and cosmic strings:

$$1 = \left(\frac{G\mu}{N^\text{str}}\right)^2 + \left(\frac{\delta_H}{N^\text{infl}}\right)^2.$$  

(44)

Using Eqs.(24), \[ (33) \], \[ (17) \] and \[ (36) \], Eq.\[ (14) \] becomes:

$$1 = \left(\frac{\xi}{M^2_{pl}}\right)^2 \left(\frac{4\pi^2}{N^2_{str}} + \frac{\alpha_{60} 256 \pi^2 121}{75 N^2_{infl}}\right)$$

(45)

where $\alpha_{60} = 1 + 4.0\epsilon_{60} - 2.1\eta_{60}$.

From Eq.\[ (45) \], we find that the U(1)$_x$ SSB scale is constraint be:
\[ \xi^2 = 4.7^{+0.5}_{-0.6} \times 10^{15} \text{ GeV}. \] (46)

From Eqs. (17), we find that during inflation the Universe is dominated by the following energy density:

\[ V_0^i \simeq 3.3 \times 10^{15} \text{ GeV}. \] (47)

Finally, from Eq. (45) we find that cosmic strings contribute to the \( C_l \)s at the level of:

\[ 75^{+10}_{-15} \%. \] (48)

Note that it is in the case of D-term inflation that the strings contribute the most to the CBR.

This result shows that the contribution from strings to the CBR temperature anisotropies and therefore to the density perturbations in the early Universe which lead to structure formation is non negligible. Density perturbations due to a mixed scenarios with inflation and cosmic strings should be computed. The effects of both inflation and cosmic strings on the temperature of the CBR should be taken into account.

The results obtained in this section are somewhat similar to those obtained in the case of F-term inflation in both globally supersymmetric theories \cite{21} locally supersymmetric ones \cite{4}.

V. THE MODELS

In this section, we discuss the three classes of models to which the construction discussed in this paper can be applied, see Sec. III. We take models which have an intermediate \( SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L} \) gauge symmetry as illustration. These models are very interesting from both a particle physics and a cosmological point of view. First, they satisfy all the requirement for successful inflation which can arise without fine tuning. They also conserve R-parity automatically, predict both hot and cold dark-matter in the form of a massive neutrino and the lightest-superparticle respectively, and lead to a scenario for baryogenesis via leptogenesis. The minimal theory beyond the standard model which belongs to this class of models is the left-right symmetry \( G = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \) and the minimal grand unified theory is \( SO(10) \).

A. Inflation from the unified theory itself, F-term inflation

We consider here case (i) where the \( U(1)_x \) symmetry is the subgroup of a non abelian gauge group belonging to the visible sector. The general SSB pattern which is discussed is the following:

\[ G \times \text{SUSY} \rightarrow H \times \text{SUSY} \rightarrow K \times \text{SUSY} \rightarrow \ldots \]

\[ \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \times \text{SUSY} \rightarrow SU(3)_c \times U(1)_Q \] (49)
where H is a subgroup of G which contains a U(1)$_x$ gauge symmetry, K is a subgroup of H which does not contain U(1)$_x$. G can be identified with H and/or K with the standard model gauge group SU(3)$_c \times SU(2)_L \times U(1)_Y$, where appropriate.

The U(1)$_x$ symmetry is broken at $M_*$ by a pair of Higgs fields $\Phi_x$ and $\Phi_{\bar{x}}$ in complex conjugate representations of G which transform non-trivially under G. The U(1)$_x$ symmetry is a subgroup of a non-abelian gauge group, and hence the hybrid inflationary scenario must be of the F-term type. The inflaton is a scalar field singlet under G which couples to the $\Phi_x$ and $\Phi_{\bar{x}}$ fields, with a superpotential in the inflaton sector given by Eq.(7). The phase transition $H \rightarrow K$ takes place at the end of inflation, and cosmic strings form, see Sec.[V A]. The intermediate symmetry group H (and K when $K \neq SU(3)_c \times SU(2)_L \times U(1)_Y$) must be chosen such that the symmetry breaking $H \rightarrow K$ (and $K \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$ respectively) do not lead to the formation of monopoles or domain walls, which would make the model in conflict with the standard cosmology (unless this latter symmetry breaking scale be very low).

Let us now turn to models with an intermediate U(1)$_{B-L}$ gauge symmetry. The SSB pattern is now given by:

$$G \times \text{SUSY} \xrightarrow{M_{\text{GUT}}} \rightarrow SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L} \times \text{SUSY} \xrightarrow{M_{B-L}} SU(3)_c \times SU(2)_L \times U(1)_Y \times \text{SUSY} \xrightarrow{M_{\text{GUT}}} SU(3)_c \times U(1)_Q$$

(50)

where the unified gauge group G contains U(1)$_{B-L}$. The minimal models are those where G is identified with SU(3)$_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, and SO(10) for semi-simple gauge groups, which break directly down to SU(3)$_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$. The study of SU(3)$_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$ on its own is also possible. We now couple the inflaton field with the pair of Higgs superfields used to break U(1)$_{B-L}$, namely $\Phi_{B-L}$ and $\Phi_{\bar{B}-L}$.

The superpotential is constructed by considering different sectors and by adding each sector as in Eq.(3). The effective scalar potential can then be written as in Eq.(12). The different sectors are the following:

- The inflationary sector, which is described by:

$$W_{\text{infl}}(S, \Phi_{B-L}, \Phi_{\bar{B}-L}) = \alpha S \Phi_{B-L} \Phi_{\bar{B}-L} - \mu^2 S$$

(51)

where $\Phi_{B-L}$ and $\Phi_{\bar{B}-L}$ are Higgs fields used to break $B - L$, with opposite $B - L$ charges. They transform non-trivially under G. The components of $\Phi_{B-L}$ and $\Phi_{\bar{B}-L}$ which acquire a vev transform as gauge singlet under the standard model gauge group. For SO(10) for example, $\Phi_{B-L}$ and $\Phi_{\bar{B}-L}$ can be a pair of 16 + $\bar{16}$ dimensional Higgs representation or 126 + 126 dimensional one. $S$ is a chiral superfield whose bosonic component is a singlet under G which plays the role of the inflaton. $\alpha$ and $\mu$ are two positive constants. $W_{\text{infl}}$ gives rise to a period of false vacuum hybrid inflation and U(1)$_{B-L}$ breaks spontaneously at the end of inflation with the fields $S$, $\Phi_{B-L}$ and $\Phi_{\bar{B}-L}$ acquiring vevs: $S = 0$, $\langle |\Phi_{B-L}| \rangle = \langle \Phi_{\bar{B}-L} \rangle = \frac{\mu}{\sqrt{\alpha}}$.

- The GUT sector, which is described by $W_{\text{GUT}}(A_i)$, $i = 1...n$. The n Higgs fields $A$ are in various representations of G. $V_{\text{GUT}}$ must have a global minimum such that $G$ is broken down to SU(3)$_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$ with the A fields acquiring
An example of $W_{GUT}$ in the case of SO(10) can be found in Refs. \cite{20,21}. In the case of SO(10), $W_{GUT}$ will involve Higgs superfields in the adjoint representation, and possibly in the 54 dimensional representation.

- The electroweak sector, which is described by $W_{ew}(H_1, H_2)$, breaks $SU(3)_c \times SU(2)_L \times U(1)_Y$ down to $SU(3)_c \times U(1)_Q$. Hence the Higgs fields $H$ and $H'$ are in complex representations of $G$ which must contain $SU(2)$ doublets. Extra coupling with the Higgs superfields of the GUT sector may be needed to solve the doublet-triplet splitting problem; see for example Ref. \cite{20}.

Note that the inflationary and GUT sectors may have to be linked in order to avoid any unwanted light pseudo-Goldstone particles. This can be done in a number of ways, for example by introducing extra Higgs fields chosen in such a way that they do not affect the vevs of the other Higgs, or by introducing non-renormalisable couplings between the $A$ and the $\Phi_{B-L}$ fields. This will not affect the dynamics of the model discussed below. Furthermore, as mentioned above, the electroweak sector will usually be facing the second hierarchy problem. And hence $W_{ew}$ also involves a coupling with one of the $A$ fields and $H_1$ and $H_2$ such that $SU(2)$ triplets get a large mass while the doublets remain light.

The scalar potential \cite{12} is given by:

$$V_{tot} = V_{GUT}(A_i) + V_{infl} + V_{ew}(H_1, H_2) + V_{soft}$$

The usual scenario then applies \cite{18}. We impose chaotic initial conditions. The potential is very flat in the $|S|$ direction. The $A$ fields acquire a vacuum expectation value at a scale $\sim 2 \times 10^{16}$ GeV, the vanishing conditions for the F and D term must be satisfied, and $G$ breaks down to $SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$. Topologically stable monopoles form.

Initially $|S| >> S_c = \frac{\mu}{\sqrt{\alpha}}$, value of the inflaton field for which $V_{tot}$ as a local minimum in the $\Phi_{B-L}$ and $\overline{\Phi}_{B-L}$ directions at $\Phi_{B-L} = \overline{\Phi}_{B-L} = 0$. So the $\Phi_{B-L}$ and $\overline{\Phi}_{B-L}$ fields settle down to this local minimum. There is a non-vanishing $F_S$ and hence a non-vanishing vacuum energy density, $V_{eff}^\frac{1}{4} = \mu$, the slow roll conditions are satisfied and inflation starts. There is a non-vanishing vacuum energy density and supersymmetry is broken. Quantum correction to the effective potential can be taken into account and play a crucial role in pulling the inflation field down the potential \cite{13}. When $|S|$ falls below $S_c$, the slow roll conditions are violated, and inflation stops. The unwanted monopoles have been inflated away. At the end of inflation $SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$ breaks down to $SU(3)_c \times SU(2)_L \times U(1)_Y$ and $B - L$ cosmic strings form. Baryogenesis via leptogenesis takes place at the end of inflation \cite{27}. In this type of scenario, the $B - L$ breaking scale is constraint by COBE to be $M_{B-L} \sim 5 \times 10^{15}$ GeV, see Sec. IV B. Also density perturbations in the early universe are due to a mixed scenario with inflation and cosmic strings, with a bigger contribution from inflation, see Ref. \cite{21,27} and Sec. IV B.

**B. Inflation from an extra U(1), D-term inflation**

If the $U(1)_x$ symmetry is not the subgroup of a non abelian gauge group, it can arise from both the non-vanishing vev of a F-term or that of a D-term, in particular for globally supersymmetric models. However, whereas in globally supersymmetric GUTs both F-term
and D-term inflation can occur, in supergravity GUTs D-term inflation is strongly favoured, see Refs. [10] and Sec.I. This however requires the presence of U(1) gauge symmetry with $TrQ \neq 0$. We therefore consider here only the case of inflation driven by the non-vanishing vev of a D-term. We assume the existence of a Fayet-Iliopoulos D-term for $U(1)_x$.

The inflationary sector is now described by Eq.(8). The only main change from F-term models comes from the superpotential in the inflationary sector. Hence switching from D-term to F-term models in globally supersymmetric theories is straightforward.

1. case (ii): The extra $U(1)$ belongs to the visible sector

The general SSB pattern which is assumed here is the following:

$$G \times U(1)_x \times \text{SUSY} \xrightarrow{M_{\text{GUT}}} \cdots \xrightarrow{M_x} H \times U(1)_x \times \text{SUSY}$$

$$\xrightarrow{M_{\text{GUT}}} \text{SU}(3)_c \times \text{SU}(2)_L \times U(1)_Y (\times \text{SUSY})$$

$$\xrightarrow{M_x} \text{SU}(3)_c \times U(1)_Q$$  \hspace{1cm} (52)

where $\text{rank}(H) = 4$. $H$ is a subgroup of $G$ and $\text{SU}(3)_c \times \text{SU}(2)_L \times U(1)_Y$ is a subgroup of $H \times U(1)_x$. The scale $M_{\text{GUT}}$ is model depend but expected to be around $2 \times 10^{16}$ GeV (it should be calculated by solving renormalisation group equations for each specific model). The scale $M_{\text{GUT}}$ is constraint by the scale $M_x$ which is fixed by COBE.

The $U(1)_x$ symmetry is broken by a pair of Higgs superfields $\Phi_x$ and $\overline{\Phi}_x$ which are charged under $G$. The $\Phi_x$ and $\overline{\Phi}_x$ fields may transform non trivially under $G$. The existence of a Fayet-Iliopoulos D-term associated $U(1)_x$ is assumed. It sets the $U(1)_x$ symmetry breaking scale. The value of the Fayet-Iliopoulos term sets the $U(1)_x$ symmetry breaking scale and is constraint by COBE data to be $\sim 5 \times 10^{15}$ GeV.

The phase transition associated with the spontaneous symmetry breaking of $H \times U(1)_x \xrightarrow{M_x} \text{SU}(3)_c \times \text{SU}(2)_L \times U(1)_Y$ takes place at the end of inflation and leads to the formation of cosmic strings with $U(1)_x$ magnetic flux. Fermions and bosons belonging to the visible sector may have non-vanishing $x$ charge. This may have some cosmological effects. For example, surrounding $x$-charged particles are subjected to the Aharonov-Bohm effect [30]. If fermions couple with the Higgs field $\Phi_x$ and acquire a mass at $M_x$, there are fermion zero modes in the core of the strings. If these fermions are charged, the strings may carry very large currents [31].

We take now again the examples of models where $U(1)_x$ is identified with $U(1)_{B-L}$. The scale $M_x$ is identified with the $B - L$ breaking scale $M_{B-L}$. The existence of a Fayet-Iliopoulos term for $U(1)_{B-L}$ is now assumed, and hence $U(1)_{B-L}$ cannot be embedded in a larger gauge group such as SO(10) here. The simplest model is given by the SSB pattern:

$$\text{SU}(3)_c \times \text{SU}(2)_L \times U(1)_R \times U(1)_{B-L} \times \text{SUSY}$$

$$\xrightarrow{M_{B-L}} \text{SU}(3)_c \times \text{SU}(2)_L \times U(1)_Y (\times \text{SUSY})$$

$$\xrightarrow{M_x} \text{SU}(3)_c \times U(1)_Q$$  \hspace{1cm} (53)

and the next to simplest one is given by the SSB pattern
SU(3)_c × SU(2)_L × SU(2)_R × U(1)_{B-L} × SUSY
\xrightarrow{M_{\text{GUT}}} SU(3)_c × SU(2)_L × U(1)_R × U(1)_{B-L} × SUSY
\xrightarrow{M_{\text{B-L}}} SU(3)_c × SU(2)_L × U(1)_Y (×SUSY)
\xrightarrow{M_{\text{F}}SU(3)_c × U(1)_Q. \quad (54)

Note that according to [32] the Left-Right and B − L breaking scale must be different.

The superpotential which implements the full spontaneous symmetry breaking pattern from G down to SU(3)_c × U(1)_Q and gives rise to a period of inflation can again be constructed by considering different sectors, and by adding the superpotentials describing each sector. We consider the Left-Right model (54). The inflationary sector is now described by Eq.(8), with the Φ_x and Φ_x fields identified with Φ_{B-L} and Φ_{B-L}. The existence of a Fayet-Iliopoulos D-term associated with the U(1)_{B-L} symmetry is assumed. The scalar potential in the inflaton sector is given by Eq.(15).

The scenario is the following. We impose chaotic initial conditions. The initial value for the inflaton field is much greater than its critical value S_c, see Sec.III. Since the potential is flat in the |S| direction, the potential can be minimised for fixed |S| and the fields settle down to their local minimum. The A fields acquire a non-vanishing vev and SU(3)_c × SU(2)_L × SU(2)_R × U(1)_{B-L}G breaks down to SU(3)_c × SU(2)_L × U(1)_R × U(1)_{B-L}. Topologically stable monopoles form. The Φ_{B-L} and Φ_{B-L} fields settle to zero. Once the Φ_{B-L} and Φ_{B-L} are trapped in the local minimum of potential, and inflationary period starts. Supersymmetry is broken and the one loop correction to the potential helps the inflaton field |S| to slowly roll down the potential. The slow roll conditions are satisfied for |S|^2 >> \frac{9}{64\pi^4}M_{pl}^2 = S_{\text{end}}^2. When |S| falls below S_{\text{end}} inflation ends. The symmetry breaking SU(3)_c × SU(2)_L × U(1)_R × U(1)_{B-L} \xrightarrow{M_{\text{B-L}}} SU(3)_c × SU(2)_L × U(1)_Y takes place when |S| falls below S_c. The unwanted monopoles have been inflated away, and B − L cosmic strings form. Baryogenesis via leptogenesis takes place [27]. In this scenario, the B − L breaking scale is constraint by COBE to be M_{B-L} \sim 5 \times 10^{15} \text{ GeV.}

2. case(iii): The extra U(1) belongs to the hidden sector

The general SSB pattern is now given by:

\begin{align*}
G × [U(1)_x] × SUSY \xrightarrow{M_{\text{GUT}}} \ldots H × [U(1)_x] × SUSY \\
\xrightarrow{M_x} SU(3)_c × SU(2)_L × U(1)_Y (×SUSY) \\
\xrightarrow{M_{\text{F}}} SU(3)_c × U(1)_Q \quad (55)
\end{align*}

where the U(1)_x symmetry now belongs to the hidden sector and breaks down to unity. H is a subgroup of G. The inflaton field is coupled with a pair of Higgs fields Φ_x and Φ_x used to break U(1)_x. Effective field theories arising from strings usually involve a certain number of U(1) factors. One of these U(1)s could be the U(1)_x considered here. Note that it cannot be an anomalous U(1) since the Fayet-Iliopoulos term which sets the scale of inflation must be ξ_x \sim 5 \times 10^{15} \text{ GeV as constrained by COBE, see Sec.IV.B, whereas for an anomalous...}
U(1) the Fayet-Iliopoulos term can be calculated by the Green-Schwarz mechanism, giving
\[ \xi_{GS} = \frac{\text{Tr}(Q_x) g^2 M_x^2}{192\pi^2} \gg \xi_{COBE}. \]

All particles belonging to the visible sector are uncharged under this U(1) symmetry. And the S, \( \Phi_x \) and \( \overline{\Phi}_x \) are uncharged under G. The U(1) and the visible sector may only interact gravitationally. The scale \( M_{GUT} \) is model dependent but expected to be \( \sim 2 \times 10^{16} \) GeV (should be calculated by solving renormalisation group equations). The scale \( M_x \) is constraint by COBE to be \( \sim 5 \times 10^{15} \) GeV. The scale \( M_{GUT} \) is independent of the scale \( M_x \).

The phase transition associated with the spontaneous symmetry breaking of U(1) down to unity takes place at the end of inflation and leads to the formation of cosmic strings with U(1) magnetic flux. These strings interact with elementary particles in the visible sector only gravitationally. They can only be superconducting if there are fermions in the hidden sector acquiring mass with the field \( \Phi_x \) acquiring a vev.

Let us now turn again back to our example, with a SSB pattern similar to that given by Eq. (49), but the existence a U(1) gauge symmetry in the hidden sector is now assumed. We now couple the inflaton field with the Higgs field used to break U(1) rather than to \( \Phi_{B-L} \). COBE sets the U(1) symmetry breaking scale at \( \sim 5 \times 10^{15} \) GeV and there is now considerable freedom in choosing the \( B - L \) breaking scale and the Higgs potential in the \( B - L \) sector. Note that in the case of models where U(1) is the subgroup of a non abelian gauge group, topological stable monopoles form when G breaks down to \( SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L} \), and hence the \( M_{GUT} \) scale must be greater than \( M_x \).

VI. CONCLUSIONS

False vacuum hybrid inflation emerges naturally in most supersymmetric unified theories of the strong, weak and electromagnetic interactions with rank greater or equal to five. Inflation can a priori be driven by the non-vanishing vev of a F-term or that of a D-term. However, supergravity models favour D-term inflation. On the other hand, D-term inflation requires the existence of a Fayet-Iliopoulos D-term which can only exist if the theory has a U(1) factor, and hence D-term inflation cannot arise from semi-simple gauge groups.

The models we have considered in this paper include a period of false vacuum hybrid inflation with cosmic strings forming at the end of inflation. The scenario for large scale structure formation implied by the models is a mixed scenario for inflation and cosmic strings. In Sec. IV we made a good estimate to the relative contributions from inflation and cosmic strings to the CMBR at the centre of the COBE data was made. We found that cosmic strings contribute in this type of scenario at the level of 75% to the \( C_i \)'s. Hence their contribution is non negligible and should be taken into account when calculating the power spectrum or density perturbations in the early Universe which lead to structure formation.

Note that when cosmic strings form in the hidden sector they must interact only gravitationally with particles in the visible sector.

There is a particularly interesting class of models which fits in our discussion, those are models which involve an intermediate \( SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L} \) gauge symmetry. Hybrid inflation can arise in this type of scenario by coupling the inflaton field with the pair of Higgs fields used to break U(1)_{B-L}. In that case \( B - L \) cosmic strings form at the end of inflation and baryogenesis via leptogenesis occurs at the end of inflation. The \( B - L \) breaking scale is also constrained by COBE to be \( \sim 5 \times 10^{15} \) GeV. If inflation comes form a hidden
sector, there is then considerable freedom for choosing the $B - L$ breaking scale. If another scale is involved in the model, say $M_{GUT}$, which leads to the SSB $G \supset U(1)_R(U(1)_{B-L})$ down to $SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$, $M_{GUT}$ is then constrained to be greater than $\sim 5 \times 10^{15}$ GeV.

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