Halometry from Astrometry

Ken Van Tilburg (NYU & IAS)

arXiv:1804.01991

with Anna-Maria Taki, Neal Weiner (NYU)

New Probes for Physics Beyond the Standard Model
KITP, April 9, 2018
time-domain, astrometric, weak gravitational lensing

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Dark Matter
Not-So-Dark Matter
Dark Matter Density Fluctuations

[Image: Diagram of dark matter density fluctuations with three red cubes overlayed on a complex network of interconnected regions.]

[MPA]
Dark Matter Density Fluctuations
Dark Matter Density Fluctuations

[Axel Mellinger]
Dark Matter Density Fluctuations

[Image: Dark matter density fluctuations in the universe, with dark matter clumps and the Milky Way galaxy at the center.]
Astrometric Weak Gravitational Lensing

\[ \Delta \theta_{il} = - \left( 1 - \frac{D_l}{D_i} \right) \frac{4G_N M(b_{il})}{b_{il}} \hat{b}_{il} \approx 4 \, \mu\text{as} \]

\[ \left[ \frac{M(b_{il}^{\text{typical}})}{10^6 \, M_\odot} \right] \left[ \frac{10 \, \text{kpc}}{b_{il}^{\text{typical}}} \right] \]
Astrometric Weak Gravitational Lensing

\[ \Delta \theta_{il} = - \left(1 - \frac{D_l}{D_i}\right) \frac{4G_N M(b_{il})}{b_{il}} \hat{b}_{il} \approx 4 \text{ \mu as} \left[ \frac{M(b_{il}^{\text{typical}})}{10^6 \ M_\odot} \right] \left[ \frac{10 \text{ kpc}}{b_{il}^{\text{typical}}} \right] \]

\[ \theta_i \text{ unknown a priori} \]

\[ \lim_{b_{il} \to 0} M(b_{il}) = 0 \]

for finite density and size of lens,
Astrometric Weak Gravitational Lensing

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\[ \theta_i \text{ unknown a priori, but:} \]

\[ \mathbf{v}_{il} \equiv \mathbf{b}_{il} = \mathbf{v}_l - \left(1 - \frac{D_l}{D_i}\right) \mathbf{v}_\odot - \frac{D_l}{D_i} \mathbf{v}_i \]

\( \Rightarrow \) time-domain \( \Delta \dot{\theta}_{il}, \Delta \ddot{\theta}_{il}, \ldots \)

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\( \theta_i \) unknown a priori, but:

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\( \Rightarrow \) time-domain \( \Delta \dot{\theta}_{il}, \Delta \ddot{\theta}_{il}, \ldots \)

for finite density and size of lens,

\[ \lim_{b_{il} \to 0} M(b_{il}) = 0 \]

\( \Rightarrow \) many sources behind 1 lens
Numerical Estimates

**Time-domain effects:**

\[
\Delta \dot{\theta}_{il} \sim \frac{4G_N M(b_{il})v_{il}}{b_{il}^2} \sim 10^{-3} \text{ mas} y^{-1} \left( \frac{M(b_{il})}{10^6 M_\odot} \right) \left( \frac{10^2 \text{ pc}}{b_{il}} \right)^2
\]

\[
\Delta \ddot{\theta}_{il} \sim \frac{4G_N M(b_{il})v_{il}^2}{b_{il}^3} \sim 4 \times 10^{-3} \text{ mas} y^{-2} \left( \frac{M(b_{il})}{M_\odot} \right) \left( \frac{10^{-2} \text{ pc}}{b_{il}} \right)^3
\]

**Light source statistics:**

\[
\# \text{ of lensed sources} \sim \Sigma_0 \frac{r_s^2}{D_l^2} \sim 10^4 \left( \frac{\Sigma_0}{10^8} \right) \left( \frac{r_s}{10^2 \text{ pc}} \right)^2 \left( \frac{10 \text{ kpc}}{D_l} \right)^2
\]

\[
\left\langle \min_{i,l} b_{il} \right\rangle \approx \sqrt{\frac{M_l}{\pi \rho_l D_i N_0}} \approx 2 \times 10^{-5} \text{ pc} \sqrt{\left[ \frac{5 \text{ kpc}}{D_i} \right] \left[ \frac{10^{-2} \rho_{DM,\odot}}{\rho_l} \right] \left[ \frac{M_l}{M_\odot} \right] \left[ \frac{10^9}{N_0} \right]}
\]
Signal Observables
Signal Observables

Outlier acceleration $O_\alpha$
Signal Observables

Outlier acceleration $O_{\alpha}$

Mono-blip $B_{\text{mono}}$
Signal Observables

Outlier acceleration $O_{\alpha}$

Mono–blip $B_{\text{mono}}$

Multi–blip $B_{\text{multi}}$
Signal Observables

- Outlier acceleration $O_\alpha$
- Mono–blip $B_{\text{mono}}$
- Multi–blip $B_{\text{multi}}$
- Velocity template $\mathcal{T}_{\mu}$
Multi-Source Observables

blips:

\[ \mathcal{B} [x_l(t)] = \sum_{i \in \square} \frac{1}{\sigma^2_{\delta \theta, i}} \sum_n \delta \theta_i(t_n) \cdot \Delta \theta_{il} [x_l(t_n)] \]

template velocity:

\[ \mathcal{T}_\mu [\theta_t, \beta_t, \hat{v}_t] = \sum_i \frac{\dot{\theta}_i}{\sigma^2_{\mu, i}} \cdot \mu_t \left[ \frac{\beta_{it}}{\beta_t}, \hat{\beta}_{it}, \hat{v}_t \right] \]

correlated acceleration:

\[ C_\alpha [\beta_-, \beta_+, \delta] = \frac{1}{2} \sum_{i \neq j} \frac{\ddot{\theta}_i \cdot \ddot{\theta}_j}{\sigma^2_{\alpha, i} \sigma^2_{\alpha, j}} \frac{B[\beta_{ij}; \beta_-, \beta_+]}{\beta^\delta_{ij}} \]
Precision Astrometry

Space-based optical surveys

current:  *Gaia*

future:  *Theia*

Ground-based radio surveys

current:  *VLA*

future:  *SKA*

per-epoch precision  $\sigma_{\delta \theta} \sim 10 \, \mu \text{as}$

full-sky catalogs  $N_0 \gtrsim 10^9$
Stochastic Noise

\[ \sigma \approx \mu \left( \frac{\mu}{\sigma} \right) \]

\[ \sigma_{\delta \theta}[\mu \text{as}] \]
\[ \sigma_{\mu}[\mu \text{as/y}] \]
\[ \sigma_{\alpha}[\mu \text{as/y}^2] \]
Compact Objects

- mono-blips
- multi-blips
- outlier acceleration
- outlier velocity

$\log_{10}[\rho/\rho_c]$

$\log_{10}[M/I M_\odot]$
NFW Subhalos

correlated acceleration
template velocity
correlated velocity

\begin{align*}
\alpha \mu &= \text{correlated acceleration} \\
\alpha \mu &= \text{template velocity} \\
\mu &= \text{correlated velocity}
\end{align*}
NFW Subhalos

correlated acceleration

template velocity

correlated velocity

\[
\frac{r_s}{\sigma_{v_i}} < \tau
\]

\[
\rho / \rho_\odot
\]

\[
\mu
\]

\[
\alpha
\]

\[
\sigma
\]

\[
\tau
\]

\[
C_\alpha
\]

\[
C_\mu
\]

\[
T_\mu
\]

\[
\text{correlated acceleration}
\]

\[
\text{template velocity}
\]

\[
\text{correlated velocity}
\]

\[
\log_{10}[M_s / M_\odot pc^{-3}]
\]

\[
\log_{10}[M_s / M_\odot]
\]
Primordial Power Spectrum

- Correlated velocity
- Template velocity
- Correlated acceleration

\[ \mathcal{P}_R(k) \]

\[ \mathcal{P}_R(k) \propto \frac{1}{\mu^2} \]

PBH

CMB $\gamma/\mu$

CMB $TT$

Ly-$\alpha$

$\log_{10}[k/\text{Mpc}^{-1}]$
Planet Nine

Terrestrial Planets

Mercury
Venus
Earth
Mars
Planet Nine

Terrestrial Planets

- Mercury
- Venus
- Earth
- Mars
Planet Nine

reflected sunlight: \( P \propto \frac{M^{2/3}}{R^4} \)

\[
\Delta \theta_{il} = \frac{4GM_l}{b_{il}^*} \approx 0.024 \mu\text{as} \left[ \frac{M_l}{M_\oplus} \right] \left[ \frac{\text{AU}}{b_{il}} \right]
\]

\[
N_0^B \approx \frac{4\Sigma_0 \text{AU}^2}{R_l^2} \approx 400 \left[ \frac{\Sigma_0}{10^8} \right] \left[ \frac{1000 \text{AU}}{R_l} \right]^2
\]

\[
\text{SNR}_{B_l}^{\text{multi}} \approx \frac{4GNM_l}{R_l} \frac{\sqrt{f_{\text{rep}}\tau}}{\sigma_{\delta \theta, \text{eff}}} \sqrt{2\pi \Sigma_0 \ln \left[ \frac{4\Sigma_0 \text{AU}^2}{R_l^2} \right]}
\]
Planet Nine

Multi-blip searches for outer Solar System planets

\[ R[\text{AU}] \]

\[ M[M_\oplus] \]

\[ \Sigma_0 = 5 \times 10^{10}, \sigma_{\delta \theta \text{ eff}} = 5 \mu\text{as} \]

\[ \Sigma_0 = 10^9, \sigma_{\delta \theta \text{ eff}} = 5 \mu\text{as} \]

\[ \Sigma_0 = 5 \times 10^9, \sigma_{\delta \theta \text{ eff}} = 100 \mu\text{as} \]

\[ \Sigma_0 = 10^8, \sigma_{\delta \theta \text{ eff}} = 100 \mu\text{as} \]
| Observable                  | Systematic                          | Discrimination                                      |
|-----------------------------|-------------------------------------|-----------------------------------------------------|
| outlier velocities          | hypervelocity stars                 | nearby proper motions                                |
|                             |                                     | line-of-sight velocity                               |
| outlier accelerations       | wide binary companions              | disk vs Magellanic Clouds                           |
| blips                       | baryonic lensing                    | disk vs Magellanic Clouds                           |
| template velocities         | rotational velocity                 | mass determination                                  |
|                             | streams                             | follow-up optical studies                           |
| correlated velocities       | rotational velocity                 | template checks                                      |
|                             | streams                             | extra-galactic sources                              |
| correlated accelerations    | MW disk + halo attraction           | angular scaling                                      |
|                             | local anomalies                     | extra-galactic sources                              |
|                             |                                     | line-of-sight distance                               |
|                             |                                     | angular scaling                                      |
Future Outlook

modern astrometric surveys: precision + statistics

time-domain, astrometric, weak gravitational lensing

halometry = measure motions and spectrum of dark objects in Galactic Halo
Backup
Universality Classes

\[ \rho(r) = \frac{2^{3-\gamma} \rho_s}{\left(\frac{r}{r_s}\right)^\gamma \left(1 + \frac{r}{r_s}\right)^{3-\gamma}} \]

\[ \Delta \dot{\theta}_{il} \sim \frac{4G_N M_s v_{il}^2}{b_{il}^3} \min \left[ 1, \left( \frac{b_{il}}{r_s} \right)^{\min[2,3-\gamma]} \right] \]

\[ \Delta \ddot{\theta}_{il} \sim \frac{4G_N M_s v_{il}^2}{b_{il}^2} \min \left[ 1, \left( \frac{b_{il}}{r_s} \right)^{\min[2,3-\gamma]} \right] \]

“soft lens”: \( \gamma < 2 \)  
“hard lens”: \( \gamma \geq 2 \)
Systematics: outliers

accelerations: disk vs Magellanic Clouds:

\[ \ddot{\theta}_{ic} = \frac{G^{1/3}}{D_i} \frac{m_c}{m_i^{2/3}} \left( \frac{2\pi}{T} \right)^{4/3} \]

\[ \approx 0.18 \ \mu\text{as} \ y^{-2} \left[ \frac{10 \ \text{kpc}}{D_i} \right] \left[ \frac{m_c}{10^{-3} \ M_\odot} \right] \left[ \frac{M_\odot}{m_i} \right]^{2/3} \left[ \frac{10 \ y}{T} \right]^{4/3} \]

velocities: nearby proper motions + line-of-sight velocity
Systematics: blips

mono-blips: disk vs Magellanic Clouds
mass determination

multi-blips: follow-up optical studies
Systematics: correlations

accelerations: \[ \alpha_d[D_i, R, z] \approx \frac{G_N \Sigma_{d,0}}{2D_i} e^{-\frac{R}{R_d}} \left[ 1 - e^{-\frac{|z|}{z_d}} \right] \]
\[ \approx 2 \times 10^{-6} \mu \text{as yr}^{-2} \left( \frac{10 \text{ kpc}}{D_i} \right) \left( \frac{e^{-\frac{R}{R_d}}}{e^{-\frac{R_{\odot}}{R_d}}} \right) \left[ 1 - e^{-\frac{|z|}{z_d}} \right] \]

velocities: scale separation + template checks + extra-galactic sources
Look-Elsewhere Effect

$$\text{SNR}^{\text{global}} \simeq \frac{\text{SNR}^{\text{local}}}{\sqrt{1 + \ln N_{\text{trial}}}}$$

$$\sqrt{1 + \ln N_{\text{trial}}}$$

\[
\begin{align*}
&\lesssim 4.7 \quad (O_\mu, O_\alpha) \\
&\approx 5.4 \quad (B^{\text{mono}}) \\
&\approx 4.1 \quad (B^{\text{multi}}) \\
&\lesssim 3.5 \quad (T_\mu) \\
&= 1 \quad (C_\mu, C_\alpha)
\end{align*}
\]
\[
\langle \min_{i,l} b^*_{il} \rangle = \frac{M_l}{\sigma_{vl} \tau \rho_i \Sigma_0 \Delta \Omega} \lesssim \sigma_{vl} \tau
\]

\[
\text{SNR}_{B}^{\text{mono}} \approx \frac{4G_N M_l}{\sigma_{\delta \theta, \text{eff}}} \sqrt{\frac{\pi f_{\text{rep}}}{\sigma_{vl}}} \frac{1}{\langle \min_{i,l} b^*_{il} \rangle}
\]
SNR: mono-blip

\[
\langle \min_{i,l} b_{il}^* \rangle = \frac{M_l}{\sigma_{v_l} \tau \rho_l D_i \Sigma_0 \Delta \Omega} \lesssim \sigma_{v_l} \tau
\]

\[
\text{SNR}_B^{\text{mono}} \approx \frac{4G_N M_l}{\sigma_{\delta \theta, \text{eff}}} \sqrt{\frac{\pi f_{\text{rep}}}{\sigma_{v_l}}} \frac{1}{\langle \min_{i,l} b_{il}^* \rangle}
\]
SNR: velocity template

\[
\text{SNR}_{T\mu} [D_l, v_{il}] = C_1 \left( 1 - \frac{D_l}{D_i} \right) \frac{4G_N M_s v_{il}}{r_s D_l} \sqrt{\frac{\Sigma_0}{\sigma_{\mu,\text{eff}}}}
\]

\[
\left\langle \min_l D_l \right\rangle = \left( \frac{3}{n_l \Delta \Omega} \right)^{1/3} \approx 18 \text{ kpc} \left[ \frac{M_s}{10^7 M_\odot} \frac{1}{n_l \Delta \Omega} \frac{0.01}{\Omega_{\text{sub}}} \right]^{1/3}
\]

\[
\left\langle \max_l \text{SNR}_{T\mu} \right\rangle \approx \frac{\pi^{1/2} C_1}{2^{1/2} 3^{1/3}} \frac{4G_N M_s \sigma_{v_l}}{r_s} (n_l \Delta \Omega)^{1/3} \sqrt{\frac{\Sigma_0}{\sigma_{\mu,\text{eff}}}}
\approx 0.4 \frac{\Omega_{\text{sub}}^{1/3}}{M_s} \left[ \frac{M_s}{10^7 M_\odot} \right]^{2/3} \left[ \frac{10 \text{ pc}}{r_s} \right] \left[ \frac{N_0}{10^7} \right]^{1/2} \left[ \frac{0.01}{\Delta \Omega} \right]^{1/6} \left[ \frac{200 \mu\text{as y}^{-1}}{\sigma_{\mu,\text{eff}}} \right]
\]
**SNR: correlations**

\[
\text{SNR}_{C_{\mu}} = \sqrt{\frac{\pi}{2}} \sqrt{\Sigma_0^2 \Delta \Omega} \left( n_l r_s^3 \right) \left( \frac{4 G_N M_S v_{il}}{r_s^2 \sigma_{\mu,\text{eff}}} \right)^2 I_1 \left[ \frac{\beta_-}{r_s/D_{\text{max}}}, \frac{\beta_+}{r_s/D_{\text{max}}}, \delta, \epsilon_s \right]
\]

\[
I_1 [z_-, z_+, \delta, \epsilon_s] \equiv \sqrt{\frac{2 - 2\delta}{z^2_r - 2\delta - z^2 \delta}} \int_{z_-}^{z_+} dz z^{1-\delta} \int_0^1 dy \int_{x>zy+\epsilon_s} d^2x \ |G_1[x, \hat{\mathbf{x}}, \hat{\mathbf{v}}]|^2
\]

\[
\text{SNR}_{C_{\alpha}} = \sqrt{\frac{\pi}{2}} \sqrt{\Sigma_0^2 \Delta \Omega} \left( n_l r_s^3 \right) \left( \frac{4 G_N M_S v_{il}^2}{r_s^3 \sigma_{\alpha,\text{eff}}} \right)^2 I_2 \left[ \frac{\beta_-}{r_s/D_i}, \frac{\beta_+}{r_s/D_i}, \delta, \epsilon_s \right]
\]

\[
I_2 [z_-, z_+, \delta, \epsilon_s] \equiv \sqrt{\frac{2 - 2\delta}{z^2_r - 2\delta - z^2 \delta}} \int_{z_-}^{z_+} dz z^{1-\delta} \int_0^1 dy \int_{x>zy+\epsilon_s} d^2x \ |G_2[x, \hat{\mathbf{x}}, \hat{\mathbf{v}}]|^2
\]
SNR: correlations

\[ \delta \equiv \frac{z_+}{r_s} \]

- \[ z_- = 0, z_+ = 10^2, \tau = 5y \]
- \[ z_- = 0, z_+ = 10^4, \tau = 5y \]
- \[ z_- = 0, z_+ = 10^2, \delta = 0.70, \tau = 5y \]
- \[ z_- = 0, z_+ = 10^2, \delta = 0.85, \tau = 5y \]
- \[ \delta = 0.70, r_s = 10^2 \text{pc}, \tau = 5y \]
- \[ \delta = 0.85, r_s = 10^{-1} \text{pc}, \tau = 5y \]

Graphs illustrating the relationship between SNR and various parameters.
Planet Nine
NFW templates

azimuthal average of square template vector

NFW lens angle template: $-\tilde{\beta}_d(\beta_s/\beta_d)G_0[\beta_d/\beta_s]$

NFW lens velocity template: $G_1[\beta_d/\beta_s]$

NFW lens acceleration template: $G_2[\beta_d/\beta_s]$
Tidal stripping