Abstract. We calculate the one-loop contributions to the MSSM effective potential when the scalar top fields have non-zero vacuum expectation values and study their impact on charge and colour breaking bounds.
of CCB bounds in phenomenological studies in refs. [5]. A common point in all of these analysis was that only the tree-level potential was studied. It was argued in ref. [3] that a careful choice of renormalisation scale would correctly reproduce the effects of the one-loop minimisation of the potential, both for the “real” minimum as for the CCB one. This led to comparing the value of the potential at the two minima at two different renormalisation scales.

Partial one-loop contributions to the CCB bounds were considered in ref. [6] using renormalisation group arguments. Recently a full one-loop charge breaking effective potential was calculated for the case where the scalar fields \( \tilde{\tau}_L \) and \( \tilde{\tau}_R \), not simply \( H^0_1 \) and \( H^0_2 \), acquire a non-zero vev [7]. At the same time it was argued that one should compare the values of the “real” potential and the CCB one at the same renormalisation scale due to the presence, in the effective potential, of field-independent terms. The minimisation of this one-loop CCB potential showed the Gamberini et al. strategy was not working for the CCB one-loop contributions [8] and raised serious questions about the perturbative believability of bounds derived from this particular CCB direction. This would seem to be closely related to the smallness of the tau Yukawa coupling associated with this pattern of CCB, which generated typical CCB sparticle masses of the order of tens of TeV. CCB associated with a larger Yukawa should not fall victim to the same perturbative problems, and the only Yukawa coupling that satisfies those conditions is the top one 1. In this paper we undertake the calculation of the full one-loop effective potential for the CCB case where the scalar fields \( t_L \) and \( t_R \), \( H^0_1 \) and \( H^0_2 \), acquire non-zero vevs. In section (1) we review the model we will be working in and our conventions. We also discuss our strategy for CCB bounds and review the results of refs. [7, 8]. The CCB mass matrices are introduced in section (2) and the minimisation of the one-loop potential, and its results, is shown in section (3). We conclude with an analysis of the results and their relevance for CCB bounds.

1 Charge and colour breaking in the MSSM

We follow most of the conventions of ref. [9] (except for the sign of the \( \mu \) parameter). Because the Yukawa couplings of the first and second generations are so small compared to those of the third, we set them to zero. The MSSM superpotential is thus given by

\[
W = \lambda_7 H_2 Q t_R + \lambda_6 H_1 Q b_R + \lambda_7 H_1 L \tau_R + \mu H_2 H_1 ,
\]

with \( SU(2) \) doublets \( H_1 = (H^0_1, H^1_1) \), \( H_2 = (H^+_2, H^-_2) \), \( Q = (t_L, b_L) \), \( L = (\nu_L, \tau_L) \) and singlets \( t_R, b_R \) and \( \tau_R \). The MSSM tree-level effective potential is the sum of the \( F \) and \( D \) terms and the soft supersymmetry breaking potential. Its full expression may be found, for instance, in references [7] and [8]. At the renormalisation scale \( M \) the one-loop contributions to the potential are

\[
\Delta V_1 = \sum_\alpha \frac{n_\alpha^0}{64\pi^2} M_\alpha^4 \left( \log \frac{M_\alpha^2}{M^2} - \frac{3}{2} \right) ,
\]

where the \( M_\alpha \) are the (tree-level) masses of each particle of spin \( s_\alpha \) and \( n_\alpha = (-1)^{2s_\alpha} (2s_\alpha + 1) C_\alpha \ Q_\alpha \), \( C_\alpha \) being the number of colour degrees of freedom of each particle and \( Q_\alpha \) counting its particle/anti-particle states. If only \( H^0_1 \) and \( H^0_2 \) acquire vevs \( v_1/\sqrt{2} \), \( v_2/\sqrt{2} \), the negative contributions to the tree-level potential come from the \( -B \mu v_1 v_2 \) term (and usually from the \( n_{H_2}^0 v_2^2 \) term as well). So that CCB occurs extra negative contributions are necessary - these come from terms cubic in the vevs, from the soft and \( F \)-term potentials. There are many possible choices of CCB directions, but one must remember that along with negative trilinear contributions to \( V_0 \) come potentially large positive quadratic and quartic terms as well. In

\footnote{The bottom Yukawa is larger than the tau one, but still of the same order of magnitude.}
We studied in detail the case where the fields $\tilde{t}_L$, $\tilde{t}_R$ had non-zero vevs. This particular direction, being associated with the tau Yukawa coupling, was in principle quite favourable to CCB - not only are $m_L$, $m_t$ usually the smallest of the soft masses but the size of $\lambda_\tau$ should reduce the magnitude of the F-term contributions to the potential. Finally, as vevs at such a CCB minimum should be of the order of $v \sim g_2 A_\tau/\lambda_\tau$, the resulting potential, if negative, ought to be much deeper than the “real” minimum. The CCB direction we propose to study in this article - the scalar fields $\tilde{t}_L$ and $\tilde{t}_R$ acquiring vevs - has none of these advantages: $m_Q$, $m_\ell$ are usually the largest of the soft masses; the top Yukawa being of order 1, the F-term contributions will be large; and the CCB potential, if negative, is not at all guaranteed to be deeper than the “real” one. Nonetheless, as explained in the introduction, we hope this CCB direction is not afflicted by the perturbative problems found in ref. \[8\]. Also, the arguments against top Yukawa CCB rely on an intuitive analysis of the tree-level potential. Such analysis, however, is not possible in the case of the one-loop contributions, which are very complex. Specifying notation, we must remember that $\tilde{t}_L$, $\tilde{t}_R$ are in the 3, 3 representations of $SU(3)$ respectively - each field has therefore three colour degrees of freedom. To limit the size of the $SU(3)$ D-terms it is best to choose both vevs having the same $SU(3)$ index - to simplify calculations we chose the third colour index \[2\]. We emphasize that this choice is not the most general CBB case associated with the top Yukawa, merely the one we expect will produce more interesting minima. So, let us consider that $\{\tilde{t}^{(3)}_L, \tilde{t}^{(3)}_R\}$ have vevs $\{q/\sqrt{2}, t/\sqrt{2}\}$ and $\{H^0_1, H^0_2\}$ have vevs $\{v_1/\sqrt{2}, v_2/\sqrt{2}\}$ as usual. The tree-level potential then becomes

$$V_0 = \frac{\lambda_\tau^2}{4} \left[ v_2^2 (q^2 + t^2) + q^2 t^2 \right] - \frac{\lambda_\tau}{\sqrt{2}} (A_t v_2 - \mu v_1) q t + \frac{1}{2} (m^2_1 v^2_1 + m^2_2 v^2_2 + m^2_Q q^2)
+ m^2_1 t^2) - B \mu v_1 v_2 + \frac{g^2}{32} \left( v^2_2 - v^2_1 + \frac{1}{3} q^2 - \frac{4}{3} t^2 \right)^2
+ \frac{g^2}{24} (v^2_2 - v^2_1 - q^2)^2,$$

with $m^2_1 = m^2_{H_1} + \mu^2$ and $m^2_2 = m^2_{H_2} + \mu^2$. This potential being a 4-variable function analytical studies of CBB are possible only in simplified cases. The usual strategy \[2\] considers only the tree-level value of the potential and the tree-level derived vevs: the latter simplification is based on ref. \[3\] where it was showed that for the “real” MSSM potential, by choosing a renormalisation scale $M$ of the order of the largest mass present in $\Delta V_1$, the tree-level vevs were a good approximation to the one-loop ones - that is to say, in this range of $M$ the one-loop contributions to the minimisation conditions, given by

$$\sum_{\alpha} \frac{n_\alpha}{32 \pi^2} M^2 \frac{\partial M^4_{\alpha}}{\partial v_i} \left( \log \frac{M^2_{\alpha}}{M^2} - 1 \right),$$

are not significative. Notice, however, that this does not necessarily imply that the contributions $\Delta V_1$ will be negligible. In fact, even in the usual MSSM calculations with an appropriate mass scale, the value of the one-loop potential is positive whereas the tree-level one is negative. This is not a problem since the value of the potential, CCB bounds excluded, has no bearing on the phenomenological aspects of the model. But it shows that even if an adequate choice of scale may render insignificant the derivatives of the one-loop contributions to the potential, that same choice does not imply that $\Delta V_1$ itself is negligible. Since the typical mass is usually different in the “real” and CCB potentials this leads to both being compared at different renormalisation scales. It was this point that led us to argue against this procedure in ref. \[7\], based on the fact that the sum $V_0 + \Delta V_1$ is not renormalisation group (RG) invariant. Instead, the complete RG

\[2\] Identical results would be obtained for the colour indices (1) or (2).
The invariant effective potential is given by [10]

$$V(M, \lambda_i, \phi_j) = \Omega(M, \lambda_i) + V_0(\lambda_i, \phi_j) + h \Delta V_1(M, \lambda_i, \phi_j) + O(h^2),$$

where $\lambda_i$ stands for the couplings and masses of the theory and $\phi_j$ for its fields. The field-independent function $\Omega$, implicitly or explicitly, depends on the renormalisation scale $M$. The only difference between the CCB and “real” potentials being the different set of values some of the fields $\phi_j$ have, $\Omega$ is the same in both cases - so, if we compare $V^{MSSM}$ and $V^{CCB}$ at different scales the contributions from $\Omega$ will not be considered correctly. But comparing potentials for the same value of $M$ means that at least in one case the one-loop contributions to the vevs will have to be included. By consistency, if $h$ contributions to the vevs are being taken, $h$ contributions to $V$, that is, $\Delta V_1$, must also be considered. In ref. [8] we found an even stronger argument for studying one-loop CCB potentials and their minimisation: eq. (5) implies that $d(V_0 + \Delta V_1)^{CCB}/dM = d(V_0 + \Delta V_1)^{MSSM}/dM$, up to two-loop effects. In other words, the two potentials must run with the renormalisation scale parallel to one another. In [8] we found that the CCB vevs were so large that the one-loop CCB effective potential was not RG invariant. Perturbation theory had broken down and the typically small two-loop effects had become quite large. As a consequence, the condition $V^{CCB} < V^{MSSM}$ was renormalisation scale dependent and as such bounds thereof derived not reliable. We expect this will not happen for the top-associated CCB direction, as the typical vevs should be smaller.

In short, our CCB strategy will be to calculate the one-loop CCB potential, obtain from its minimisation the one-loop vevs $\{v_1, v_2, q, t\}$ and compare it to the one-loop minimised “real” one-loop potential, at the same renormalisation scale. For comparison, we will also perform tree-level minimisations of the potentials (both MSSM and CCB) and compare their values - even though, as follows from the discussion above, this tree-level procedure is misleading and may induce errors. For the one-loop calculations we will need the CCB masses contributing to $\Delta V_1$. The complication that arises, as in ref. [7], is the fact that the vevs $q$ and $t$ cause the mixing between charged/neutrals and coloured/colourless fields in the theory. For instance, the trilinear terms in the soft potential cause mixing between $\{H_1^0, H_2^0, \tilde{t}^3_L, \tilde{t}^3_R\}$ - and further mixing between them arises from the F and D-terms. Likewise, similar mixing occurs between $\{H_1^-, H_2^+, \tilde{b}^{(3)}_L, \tilde{b}^{(3)}_R\}$. The charged, pseudoscalar and CP-even Higgses will therefore have $4 \times 4$ mass matrices. Because colour symmetry has been broken, particles carrying colour indices $\{(1), (2)\}$ will have different masses from those with colour index (3) - the $\{(1), (2)\}$ squarks have mass matrices very similar to those of the non-CCB case. The existence of vevs carrying colour degrees of freedom gives mass and electric charge to four gluons, three others remaining massless. The eighth gluon remains neutral but becomes also massive by mixing with the $B_\mu$ and $W_\mu^3$ fields. For the fermions the mixing is even more extensive: the charginos become a mixture of charged $SU(2)$ gauginos, the fermionic partners of the charged Higgses and the (3) component of the bottom quark. The case of the neutralinos is even more complex: a $7 \times 7$ mass matrix originates from the mix of neutral $U(1)$ and $SU(2)$ gauginos, fermionic partners of the neutral Higgses, (3) components of the top quark and the eighth gluino. The gauge interaction term between quarks, scalar quarks and gluinos, $-i g_3 q_i^\dagger \lambda_i \tilde{G}^i/\sqrt{2}$, also causes mixing between the $\{(1), (2)\}$ quark components and the $\tilde{G}^{1,\ldots,7}$ gluinos - the results are two $4 \times 4$ identical mass matrices. The mass of the gluinos $\tilde{G}^{1,\ldots,3}$ remains unchanged, $M_3$. We present the CCB mass matrices in the next section.
2 Mass matrices

Let us define the coefficients $G$ as

$$G_1 = v_2^2 - v_1^2 + \frac{1}{3} q^2 - \frac{4}{3} t^2, \quad G_2 = v_2^2 - v_1^2 - q^2, \quad G_3 = q^2 - t^2.$$  \hspace{1cm} (6)

We now list the masses of the MSSM when $q$ and $t$ are non-zero.

- First and second generation sleptons and sneutrinos ($n_1 = n_2 = n_{\tilde{\nu}_e} = 2 \times 2$):

$$M_{\tilde{e}_1}^2 = m_N^2 - \frac{g'^2}{8} G_1 + \frac{g_2^2}{8} G_2 \quad \quad M_{\tilde{\nu}_e}^2 = m_\nu^2 + \frac{g'^2}{4} G_1$$

$$M_{\tilde{e}_2}^2 = m_N^2 - \frac{g'^2}{8} G_1 - \frac{g_2^2}{8} G_2 . \hspace{1cm} (7)$$

- First and second generation squarks, colour indices $\{ 1, 2 \}$ ($n_1 = n_2 = 2 \times 4$, for both up and down type squarks):

$$M_{\tilde{u}_1}^{(1,2)} = m_R^2 + \frac{g'^2}{24} G_1 - \frac{g_2^2}{8} G_2 - \frac{g_3^2}{12} G_3 \quad M_{\tilde{u}_2}^{(1,2)} = m_u^2 - \frac{g'^2}{6} G_1 + \frac{g_2^2}{12} G_3$$

$$M_{\tilde{d}_1}^{(1,2)} = m_R^2 + \frac{g'^2}{24} G_1 + \frac{g_2^2}{8} G_2 - \frac{g_3^2}{12} G_3 \quad M_{\tilde{d}_2}^{(1,2)} = m_d^2 + \frac{g'^2}{12} G_1 + \frac{g_2^2}{12} G_3 . \hspace{1cm} (8)$$

- First and second generation squarks, colour index 3 ($n_1 = n_2 = 2 \times 2$, for both up and down type squarks):

$$M_{\tilde{u}_1}^{(3)} = m_R^2 + \frac{g'^2}{24} G_1 - \frac{g_2^2}{8} G_2 + \frac{g_3^2}{6} G_3 \quad M_{\tilde{u}_2}^{(3)} = m_u^2 - \frac{g'^2}{6} G_1 - \frac{g_3^2}{6} G_3$$

$$M_{\tilde{d}_1}^{(3)} = m_R^2 + \frac{g'^2}{24} G_1 + \frac{g_2^2}{8} G_2 + \frac{g_3^2}{6} G_3 \quad M_{\tilde{d}_2}^{(3)} = m_d^2 + \frac{g'^2}{12} G_1 - \frac{g_3^2}{6} G_3 . \hspace{1cm} (9)$$

- Tau sneutrino and sleptons ($n_{\tilde{\nu}_\tau} = n_1 = n_2 = 2$):

$$M_{\tilde{\nu}_\tau}^2 = m_{\tilde{\nu}}^2 - \frac{g'^2}{8} G_1 - \frac{g_2^2}{8} G_2 , \quad [M_{\tilde{\tau}}^2] = \begin{pmatrix} a_{\tilde{\nu}} & b_{\tilde{\nu}} \\ b_{\tilde{\nu}} & c_{\tilde{\nu}} \end{pmatrix} , \hspace{1cm} (10)$$

with

$$a_{\tilde{\nu}} = m_{\tilde{\nu}}^2 + \frac{\lambda^2}{2} v_1^2 - \frac{g'^2}{8} G_1 + \frac{g_2^2}{8} G_2 \quad \quad b_{\tilde{\nu}} = \frac{\lambda^\tau}{\sqrt{2}} (\mu v_2 - A_\tau v_1)$$

$$c_{\tilde{\nu}} = m_{\tilde{\tau}}^2 + \frac{\lambda^2}{2} v_1^2 + \frac{g'^2}{4} G_1 . \hspace{1cm} (11)$$

- Charged ($n = 6$) and neutral ($n_1 = n_2 = 3$) electroweak gauge bosons:

$$M_W^2 = \frac{1}{4} g_2^2 (v_1^2 + v_2^2 + q^2) \quad , \quad [M_C^2] = \begin{pmatrix} a_C & b_C & d_C \\ b_C & c_C & e_C \\ d_C & e_C & f_C \end{pmatrix} , \hspace{1cm} (12)$$

with

$$a_C = \frac{g_2^2}{4} (v_1^2 + v_2^2 + q^2) \quad \quad b_C = -\frac{g_2^2 q'}{4} \left( v_1^2 + v_2^2 - \frac{1}{3} q^2 \right) .$$

\[ ^3 \text{We considered degenerate first and second generation sparticles, but these results are trivially generalised.} \]
\[ c_{G^0} = \frac{g^2}{4} \left( v_1^2 + v_2^2 + \frac{1}{9} q^2 + \frac{16}{9} t^2 \right) \quad d_{G^0} = -\frac{g_2 g_3}{2\sqrt{3}} q^2 \]
\[ e_{G^0} = -\frac{g_2 g_3}{\sqrt{3}} \left( \frac{1}{6} q^2 + \frac{2}{3} t^2 \right) \quad f_{G^0} = \frac{g_2^2}{3} (q^2 + t^2) . \]  

This matrix has one zero eigenvalue, corresponding to a “photon” resulting from gauge symmetry breaking.  

- Charged gluons \((n_1 = 4 \times 3):\)

\[ M^2_{G^\pm} = \frac{1}{4} g_3^2 (q^2 + t^2) . \]  

- Top scalars, colour indices \(\{1, 2\} (n_1 = n_2 = 4):\)

\[ [M^2_t^{(1,2)}] = \begin{pmatrix} a_i & b_i \\ b_i & c_i \end{pmatrix} , \]

with

\[ a_i = m_t^2 + \frac{1}{2} \lambda^2 v_2^2 + \frac{g^2}{24} G_1 - \frac{g_2}{8} G_2 + \frac{g_3}{12} (3 q^2 - G_3) \]
\[ b_i = -\frac{\lambda t}{\sqrt{2}} (A_1 v_2 - \mu v_1) + \frac{1}{4} (2 \lambda^2 - g_3^2) q t \]
\[ c_i = m_t^2 + \frac{1}{2} \lambda^2 v_2^2 - \frac{g^2}{6} G_1 + \frac{g_3}{12} (3 t^2 + G_3) . \]  

- Bottom scalars, colour indices \(\{1, 2\} (n_1 = n_2 = 4):\)

\[ [M^2_b^{(1,2)}] = \begin{pmatrix} a_b & b_b \\ b_b & c_b \end{pmatrix} , \]

with

\[ a_b = m_b^2 + \frac{1}{2} \lambda^2 v_1^2 + \frac{g^2}{24} G_1 + \frac{g_2}{8} G_2 - \frac{g_3}{12} G_3 \quad b_b = -\frac{\lambda b}{\sqrt{2}} (A_b v_1 - \mu v_2) \]
\[ c_b = m_b^2 + \frac{1}{2} \lambda^2 v_1^2 + \frac{g^2}{12} G_1 + \frac{g_3}{12} G_3 . \]  

- Charged Higgs (mix between \(H_1^-, H_2^+, \tilde{b}_{L}^{(3)}\) and \(\tilde{b}_{R}^{(3)}; n_{1...4} = 2):\)

\[ [M^2_{H^\pm}] = \begin{pmatrix} a_\pm & b_\pm & d_\pm & e_\pm \\ b_\pm & c_\pm & f_\pm & g_\pm \\ d_\pm & f_\pm & h_\pm & i_\pm \\ e_\pm & g_\pm & i_\pm & j_\pm \end{pmatrix} , \]  

with

\[ a_\pm = m_1^2 + \frac{1}{2} \lambda_b^2 q^2 - \frac{g^2}{8} G_1 + \frac{g_3^2}{8} (v_1^2 + v_2^2 - q^2) \]

\[ \frac{4}{3} \]The symmetry breaking we have chosen leaves intact a \(SU(2) \times U(1)\) gauge group, corresponding to an integer charge quark theory. In this theory four gluons couple directly to the photon and as such possess electric charge. See reference [11] for details.
\[ b_\pm = B \mu + \frac{g_2^2}{4} v_1 v_2 \]
\[ c_\pm = m_2^2 + \frac{1}{2} \lambda_t^2 t^2 + \frac{g_2^2}{8} G_1 + \frac{g_2^2}{8} (v_2^2 + v_1^2 + q^2) \]
\[ d_\pm = \frac{\lambda_t}{\sqrt{2}} \mu t + \frac{1}{4} (g_2^2 - 2 \lambda_b^2) v_1 q \]
\[ e_\pm = \frac{1}{2} \lambda_b \lambda_t v_2 t - \frac{\lambda_b}{\sqrt{2}} A_b q \]
\[ f_\pm = \frac{\lambda_t}{\sqrt{2}} A_t t + \frac{1}{4} (g_2^2 - 2 \lambda_t^2) v_2 q \]
\[ g_\pm = -\frac{\lambda_b}{\sqrt{2}} \mu q + \frac{1}{2} \lambda_b \lambda_t v_1 t \]
\[ h_\pm = m_Q^2 + \frac{1}{2} (\lambda_b^2 v_1^2 + \lambda_t^2 t^2) + \frac{g_2^2}{24} G_1 + \frac{g_2^2}{8} (v_2^2 - v_1^2 + q^2) + \frac{g_2^2}{6} G_3 \]
\[ i_\pm = \lambda_b \sqrt{2} (A_b v_1 - \mu v_2) \]
\[ j_\pm = m_b^2 + \frac{1}{2} \lambda_b^2 (v_1^2 + q^2) + \frac{g_2^2}{12} G_1 - \frac{g_2^2}{6} G_3 \].

(20)

- Pseudo scalars (mix between the imaginary parts of \( H_1^0, H_2^0, \tilde{t}^{(3)}_L \) and \( \tilde{t}^{(3)}_R \); \( n_1 \ldots 4 = 1 \)):

\[
[M_{\tilde{H}_0}^2] = \begin{pmatrix}
  a_H & b_H & d_H & e_H \\
  b_H & c_H & f_H & g_H \\
  d_H & f_H & h_H & i_H \\
  e_H & g_H & i_H & j_H \\
\end{pmatrix},
\]

(21)

with

\[ a_H = m_1^2 - \frac{g_2^2}{8} G_1 - \frac{g_2^2}{8} G_2 \]
\[ b_H = B \mu \]
\[ c_H = m_2^2 + \frac{1}{2} \lambda_t^2 (q^2 + t^2) + \frac{g_2^2}{8} G_1 + \frac{g_2^2}{8} G_2 \]
\[ d_H = \frac{\lambda_t}{\sqrt{2}} \mu t \]
\[ e_H = \frac{\lambda_t}{\sqrt{2}} \mu q \]
\[ f_H = \frac{\lambda_t}{\sqrt{2}} A_t t \]
\[ g_H = \frac{\lambda_t}{\sqrt{2}} A_t q \]
\[ h_H = m_Q^2 + \frac{1}{2} \lambda_t^2 (v_2^2 + t^2) + \frac{g_2^2}{24} G_1 - \frac{g_2^2}{8} G_2 + \frac{g_2^2}{6} G_3 \]
\[ i_H = \frac{\lambda_t}{\sqrt{2}} (A_t v_2 - \mu v_1) \]
\[ j_H = m_b^2 + \frac{1}{2} \lambda_b^2 (v_1^2 + q^2) - \frac{g_2^2}{6} G_1 - \frac{g_2^2}{6} G_3 \].

(22)
• Higgs scalars (mix between the real parts of $H_1^0$, $H_2^0$, $\tilde{t}_L^{(3)}$ and $\tilde{t}_R^{(3)}$; $n_{1...4} = 1$):

$$[M_{H_0}^2] = \begin{pmatrix}
a_H & b_H & d_H & e_H \\
b_H & c_H & f_H & g_H \\
d_H & f_H & h_H & i_H \\
e_H & g_H & i_H & j_H
\end{pmatrix}, \tag{23}$$

with

$$a_H = m_1^2 - \frac{g'^2}{8} (G_1 - 2 v_1^2) - \frac{g_2^2}{8} (G_2 - 2 v_1^2)$$

$$b_H = -B \mu - \frac{1}{4} (g'^2 + g_2^2) v_1 v_2$$

$$c_H = m_2^2 + \frac{1}{2} \lambda_t^2 (q^2 + t^2) + \frac{g'^2}{8} (G_1 + 2 v_2^2) + \frac{g_2^2}{8} (G_2 + 2 v_2^2)$$

$$d_H = \frac{\lambda_t}{\sqrt{2}} \mu t + \frac{1}{12} (3 g_2^2 - g'^2) v_1 q$$

$$e_H = \frac{\lambda_t}{\sqrt{2}} \mu q + \frac{1}{3} g'^2 v_1 t$$

$$f_H = \lambda_t^2 v_2 q - \frac{\lambda_t}{\sqrt{2}} A_1 t + \frac{1}{12} (g'^2 - 3 g_2^2) v_2 q$$

$$g_H = -\frac{\lambda_t}{\sqrt{2}} A_1 q + \frac{1}{3} (3 \lambda_t^2 - g'^2) v_2 t$$

$$h_H = m_Q^2 + \frac{1}{2} \lambda_t^2 (v_2^2 + t^2) + \frac{g'^2}{24} \left(G_1 + \frac{2}{3} q^2 \right) - \frac{g_2^2}{8} (G_2 - 2 q^2) + \frac{g_3^2}{6} (G_3 + 2 q^2)$$

$$i_H = -\frac{\lambda_t}{\sqrt{2}} (A_t v_2 - \mu v_1) + \frac{1}{9} (9 \lambda_t^2 - g'^2 - 3 g_2^2) q t$$

$$j_H = m_t^2 + \frac{1}{2} \lambda_t^2 (v_2^2 + q^2) - \frac{g'^2}{6} \left(G_1 + \frac{8}{3} t^2 \right) - \frac{g_3^2}{6} (G_3 - 2 t^2). \tag{24}$$

• Tau lepton and bottom quark, colour indices $\{1, 2\}$ ($n_\tau = -4$, $n_b = -2 \times 4$), $M_\tau = \lambda_\tau v_1/\sqrt{2}$, $M_b = \lambda_b v_1/\sqrt{2}$.

• Charginos (mix between the fermionic partners of the charged Higgses $\tilde{H}_1^\pm$, $\tilde{H}_2^\pm$, the SU(2) gauginos $\Psi^+$, $\Psi^-$ and the colour index 3 components of the bottom quark, $b_L^{(3)}$ and $b_R^{(3)}$; $n_{1...6} = -2$):

$$[M_{\chi^\pm}] = \begin{pmatrix}
0 & 0 & M_2 & -\frac{g_2}{\sqrt{2}} v_2 & 0 & 0 \\
0 & 0 & \frac{g_2}{\sqrt{2}} v_2 & -\mu & \frac{\lambda_b}{\sqrt{2}} t & 0 \\
0 & 0 & M_2 & \frac{g_2}{\sqrt{2}} v_2 & 0 & 0 \\
-\frac{g_2}{\sqrt{2}} v_1 & -\mu & 0 & 0 & 0 & -\frac{\lambda_b}{\sqrt{2}} q \\
0 & \frac{\lambda_b}{\sqrt{2}} t & \frac{g_2}{\sqrt{2}} q & 0 & 0 & \frac{\lambda_b}{\sqrt{2}} v_1 \\
0 & 0 & 0 & -\frac{\lambda_b}{\sqrt{2}} q & \frac{\lambda_b}{\sqrt{2}} v_1 & 0
\end{pmatrix}. \tag{25}$$

• Neutralinos (mix between the fermionic partners of the neutral Higgses $\tilde{H}_1^0$, $\tilde{H}_2^0$, the U(1) and SU(2) gauginos $\tilde{B}$, $\tilde{W}_3$, the colour index 3 components of the top quark, $t_L^{(3)}$, $t_R^{(3)}$ and
the eighth gluino $\tilde{G}^8; n_{1..7} = -2$:

$$[M_{\chi^0}] = \begin{pmatrix}
    M_1 & 0 & -\frac{q'}{2} v_1 & \frac{q'}{2} v_2 & -\frac{q}{6} q & \frac{2q'}{3} t & 0 \\
    0 & M_2 & \frac{q}{2} v_1 & -\frac{q}{2} v_2 & \frac{q}{2} q & 0 & 0 \\
    -\frac{q'}{2} v_1 & \frac{q'}{2} v_1 & 0 & -\mu & 0 & 0 & 0 \\
    \frac{q'}{2} v_2 & -\frac{q}{2} v_2 & -\mu & 0 & -\frac{\lambda}{\sqrt{2}} t & -\frac{\lambda}{\sqrt{2}} q & 0 \\
    -\frac{q'}{2} q & \frac{q}{2} q & 0 & -\frac{\lambda}{\sqrt{2}} t & 0 & \frac{\lambda}{\sqrt{2}} v_2 & -\frac{g_3}{\sqrt{2}} q \\
    \frac{2q'}{3} t & 0 & 0 & -\frac{\lambda}{\sqrt{2}} q & \frac{\lambda}{\sqrt{2}} v_2 & 0 & \frac{g_3}{3} t \\
    0 & 0 & 0 & 0 & -\frac{g_3}{3} q & \frac{g_3}{3} t & M_3
\end{pmatrix}$$ (26)

- Gluinos (mix between the colour index $\{1, 2\}$ components of the top quark and the gluinos $\tilde{G}^{4..7}; n_{1..4} = -2 \times 2^5$):

$$[M_G] = \begin{pmatrix}
    0 & \frac{\lambda}{\sqrt{2}} v_2 & -\frac{g_3}{\sqrt{2}} q & 0 \\
    \frac{\lambda}{\sqrt{2}} v_2 & 0 & 0 & \frac{g_3}{3} t \\
    -\frac{g_3}{\sqrt{2}} q & 0 & 0 & M_3 \\
    0 & \frac{g_3}{\sqrt{2}} t & M_3 & 0
\end{pmatrix}$$ (27)

The remaining gluinos ($\tilde{G}^{1..3}$) contribute to the one-loop potential with mass $M_3$ and are affected by a factor $n = -2$ each. At the tree-level minimum the matrices $[15]$ and $[10]$ have each a zero eigenvalue, the matrix $[21]$ has two. Counting the multiplicities of each particle this corresponds to a total of eight Goldstone bosons, corresponding to eight gauge bosons that acquire mass (the $Z^0$, the $W^\pm$ and five of the gluons). We checked these mass matrices by computing $Str M^2 = \sum_\alpha n_\alpha M_\alpha^2$ - because supersymmetry is broken in a “soft” way this quantity should be field-independent (that is, it should be independent of the value of the vevs), and it is simple to verify that condition is met.

## 3 Minimisation of the one-loop CCB potential

With the mass particles for the whole sparticle spectrum obtained it is a simple task to obtain the derivatives of eq. [4] and perform the one-loop minimisation of the CCB potential as was done in ref. [8]. Here we will use a different method to minimise the potential: we will not bother with its derivatives and instead look directly at the values of $V_0 + \Delta V_1$ as function of $(v_1, v_2, q, t)$ by means of a modified simulated annealing algorithm incorporating Q-sampling of the phase space $[13]$. The code for the application of this algorithm was developed by J.M. Pacheco. The reason for choosing this method is purely a practical one: the algorithm is extremely efficient and the computation time necessary for a scan of the MSSM parameter space drastically reduced. We checked the code by comparing its results with those of a MSSM calculation using the potential’s derivatives, for a vast region of parameters. We also compared its results with those obtained using the numerical minimisation tools of the MATLAB package. All these approaches produced the same results.

\footnote{\textsuperscript{3}l_{t_R}^{(1)}, l_R^{(1)} \textsuperscript{3} mix with $\tilde{G}^+ = (\tilde{G}^5 + i\tilde{G}^4)/\sqrt{2}, \tilde{G}^- = (\tilde{G}^5 - i\tilde{G}^4)/\sqrt{2}$. An identical mixing occurs between $t^{(2)}, \tilde{G}^6$ and $\tilde{G}^7$, producing degenerate gluino masses - thus, the extra factor of 2 in the coefficients $n$.}
In this work we will mostly study the MSSM with universal input soft parameters. At the gauge unification scale we choose common values \( A_G, M_G \) and \( m_G \) for the \( A \) parameters, the soft gaugino and scalar masses. We took \(-4 \leq A_G \leq 4 \text{ TeV} \) and \( 20 \leq M_G, m_G \leq 900 \text{ GeV} \). Further, we have taken \( 2.5 \leq \tan \beta \leq 10.5 \) and considered both possible signs for the \( \mu \) parameter. This selection of parameter space is by no means an exhaustive one (we left out high values of \( \tan \beta \), for instance) but is already very extensive and should provide a good idea of the importance of the CCB bounds derived from our one-loop potential. We follow the “top-bottom” approach outlined in ref. \(^9\): at the weak scale \( M_Z \) we input \( M_Z = 91.19 \text{ GeV}, M_t = 167.2 \text{ GeV}, M_b = 2.95 \text{ GeV}, M_r = 1.75 \text{ GeV} \) (these are running fermion masses, not the pole ones), \( \alpha_1 = 0.01667, \alpha_2 = 0.032, \alpha_3 = 0.1 \) and the value of \( \tan \beta \). Because the gauge and Yukawa \( \beta \)-functions do not depend on the soft parameters \(^6\), we can run these parameters up in the energy scale until we find the gauge unification scale, defined as the value \( M_U \) for which the couplings \( \alpha_1 \) and \( \alpha_2 \) meet. At that point we input the values of the soft parameters, chosen as explained above, and run the whole theory to a scale \( M_C = \text{max}(M_Z, M_G, g_3 A_G/\lambda_t) \) - this scale is a good estimate of the heaviest masses in the theory, thereby reducing, in principle, the size of the logarithmic contributions to the potential. At that scale we use the one-loop MSSM minimisation conditions (see, for instance, ref. \(^9\)) and determine - if possible - the parameters \( \mu \) and \( B \). We then calculate the sparticles’ masses \(^7\) and use recent experimental bounds \(^{14}\) to reject those “points” already in contradiction with observational evidence. We end up with over 39000 “points” of parameter space. We also studied a small non-universal parameter space where we took the universal “points” obtained earlier and set the \( m_\mu \) soft parameter to negative values (of the order of, at the most, \((100 \text{ GeV})^2\)), as this situation would seem one of the likeliest to result in CCB. This situation clearly requires non-universality, as universal values of the soft masses are unlikely to result in negative \( m_\mu^2 \) at the weak scale, due to the form of the \( \beta \)-function for this soft parameter. With the new value of \( m_\mu^2 \) we minimised the (new) potential and, again, used experimental sparticles’ mass bounds to reject “points” in disagreement with observations. Our non-universal parameter space ended up with about 4000 “points”.

We set out to determine the impact of the one-loop contributions on CCB bounds and as such we endeavored to compare results coming from tree-level and one-loop minimisations of the potential. The tree-level minimisation of the MSSM potential is performed analytically in the standard way, see for example \(^9\). So, both at tree-level and one-loop, we compute the value of the MSSM potential, \( V_{\text{MSSM}} \), and perform the numerical minimisation of the CCB one, obtaining the vevs \( \{v_1, v_2, q, t\} \). With these we calculate the value of \( V_{\text{CCB}} \) and compare it with \( V_{\text{MSSM}} \). Finally, a word on thresholds: we follow the procedure of ref. \(^{17}\), using the full MSSM \( \beta \)-functions from \( M_Z \) to \( M_U \), and choosing the input parameters at \( M_Z \) such that the threshold contributions are automatically taken into account - this is an effective procedure, but shown to produce good results. And for our purposes - determining if CCB occurs or not - this degree of precision should be more than adequate. Being based on a Monte Carlo method the algorithm depends on the initial conditions used, so several runs of the program were necessary. We only accepted those extrema with all CCB squared masses positive, except the expected four zero eigenvalues corresponding to the Goldstone bosons. We remember that when performing a one-loop minimisation of the MSSM potential it is usual to find negative squared masses in the Higgs sector \(^9\). They correspond to the Goldstone bosons which have zero masses for a tree-level minimisation. Since we do a one-loop minimisation and compute their masses using tree-level matrices, negatives do occur sometimes. However, the absolute value of these negative squared masses is very small when compared to the other masses in the theory and thus, based on \(^{15}\), they can be safely set to zero.

\(^6\)Except indirectly, in the form of particle thresholds.
\(^7\)With the tree-level mass matrices except for the neutral CP-even Higgses, for which full one-loop expressions were used.
The results of this scan of the MSSM parameter space may be resumed as follows:

- No unbound-from-below (UFB) directions were found for the one-loop minimisations. They appear frequently, however, for the tree-level minimisations. It is easy to see from equation (3), for instance, that with $q$ and $t$ constant and $v_1 \approx v_2 \to \infty$ we have a potential tree-level UFB direction. These directions are characterized by vevs assuming arbitrarily large values, causing the potential to become arbitrarily negative. No such thing happened in the one-loop minimisation of the potential, which confirms the expectations of the authors of [4]: they had assumed that the UFB directions they had found at tree-level might not be present if a one-loop minimisation was performed. We confirmed that that is the case, and that the one-loop contributions to the potential stabilize the vevs (which is also in agreement with the conclusions of [3]).

- For the tree-level minimisations, the value of the CCB potential was found to be always deeper than the MSSM one. For many of these minima, however, the values of $q$ and $t$ found were practically zero. What happens is that the tree-level MSSM potential is found for very specific values of the vevs (namely such that $v_1^2 + v_2^2 = (246 \text{ GeV})^2$ and $\tan \beta = v_2/v_1$), but that is not the only possible MSSM minimum - the tree-level MSSM minimisation conditions consist of two coupled cubic equations and as such can have as much as nine different solutions. What we find is that, for the CCB case, we let the four vevs “roam freely” as we minimise numerically the potential and, as such, many solutions with $q \approx t \approx 0$ are found. However, those solutions have values of $v_1$ and $v_2$ which are not phenomenologically acceptable (giving as they do wrong values for the gauge bosons’ masses, for instance). These alternative minima, which preserve the MSSM gauge symmetry, are actually deeper than the “standard” minimum. But again we must remember that a comparison of tree-level potentials with tree-level derived vevs is inherently flawed, and we only undertook it to show the radical differences with the one-loop case.

- The MSSM potential is always deeper than the CCB one at one-loop, both for the universal and non-universal parameter spaces. There are now no “alternative MSSM minima” as in the tree-level case, and the reason is easy to understand: while a given combination of supersymmetric parameters might correspond to several possible combinations of vevs as minimisation solutions at tree-level, it is almost impossible that that happens for the one-loop potential, given its complexity.

The comparison between these two final points reveals how different a one-loop procedure can be from the tree-level one. But it also requires some explanation: what is it about the one-loop CCB contributions that raises the value of the potential, always above its MSSM value? First, we have already seen the importance of a one-loop minimisation to stabilise the values of the vevs (avoiding UFB directions and preventing the appearance of “alternative MSSM minima”). Secondly, the consequences of the different gauge symmetry breakings become apparent only at the level of the one-loop potential, when we consider the different spectra of masses thereof resulting. In section 2 we showed that a very significant difference between the MSSM and the CCB case is the mass of the gluons: all zero for the MSSM, five of them gaining mass for CCB. For CCB vevs $q$ and $t$ of the same order of the electroweak vevs $v_1$ and $v_2$ (as we expect them to be, if not even higher), we expect the gluons to have masses considerably larger than $M_Z$ or $M_W$ since, from eq. (14), their masses are proportional to the strong coupling constant. As these are bosonic masses, their contributions to $\Delta V_1$, from eq. (2), should be positive. Assuming a CCB minimum exists, then, its gluon one-loop contribution to the potential is expected to be large and positive, whereas for the same SUSY parameters the similar contribution to the MSSM potential is zero. This is a possible explanation, but the one-loop contributions are very complex and the real reason may be lying with other terms.
Ultimately, all we can conclude from these results is that we have a “numerical demonstration”, for this vast parameter space, of the MSSM minimum being deeper than any possible CCB minimum. This follows similar conclusions reached in ref. [8], where no acceptable CCB minima associated with the tau Yukawa coupling were found. Also, recent work [19] demonstrated that for two Higgs doublet models (2HDM) at tree-level, if a minimum preserving $U(1)_{em}$ and CP symmetries exists, it is a global minimum. In such models it is impossible to have tunneling for charge or CP breaking vacua. Of course, in this work we are dealing with charge and colour breaking, in a model that is far more complex than the 2HDM. We are also studying one-loop minima, not tree-level ones. But these different elements may be suggesting that the occurrence of CCB is not as easy to occur as a tree-level analysis in the MSSM leads us to believe.

It is important to remark that we did not exhaust the MSMM parameter space. The possibility of dangerous CCB minima occurring may happen in some portion of parameter space not included in our study. Even though the range and number of parameters we chose was very general, this possibility cannot be wholly dismissed. Finally, all our calculations have assumed that the MSSM vacuum is the absolute minimum of the theory. There is however [2] the theoretical possibility that the the CCB minimum is deeper than the real one and the tunneling time between both vacua be superior to the age of the universe. This leads to a relaxation of the CCB bounds usually obtained. In our case we do not need to worry about this possibility since no CCB absolute minima were found.

We hope to have convinced the reader of the importance of full one-loop calculations in estimating CCB bounds. In references [7], [8] and this work we showed that for RG consistency one had to compare the one-loop MSSM and CCB potentials, and that the results of that enterprise gave very different results from tree-level studies. We studied very specific CCB directions and vast sections of the MSSM parameter space, and no CCB minima deeper than the “normal” vacuum were found. Although at this stage we cannot make sweeping generalisations and claim all CCB bounds used in the literature are wrong, these results urge some caution. In reference [5], for example, large sections of parameter space are excluded on CCB grounds, and predictions of supersymmetric masses are affected by it. If indeed, as this work suggests, tree-level CCB bounds are over-estimated, we may be excluding areas of parameter space that can be of experimental interest. A careful re-evaluation of CCB bounds might be in order.

Acknowledgments: My deepest thanks to Jorge Pacheco for providing me with the code for the minimisation algorithm. This work was supported by a fellowship from Fundação para a Ciência e Tecnologia, SFRH/BPD/5575/2001.

References.

[1] J.M. Frére, D.R.T. Jones and S. Raby, Nucl. Phys. B222 (1983) 11.
[2] L. Alvarez-Gaumé, J. Polchinski and M. Wise, Nucl. Phys. B221 (1983) 495;
   J.P. Derendinger and C.A. Savoy Nucl. Phys. B237 307;
   C. Kounnas, A.B. Lahanas, D.V. Nanopoulos and M. Quirós, Nucl. Phys. B236 (1984) 438;
   M. Claudson, L.J. Hall and I. Hinchliffe, Nucl. Phys. B228 (1983) 501;
   M. Drees, M. Glück and K. Grassie, Phys. Lett. B157 (1985) 164;
   J.F. Gunion, H.E. Haber and M. Sher, Nucl. Phys. B331 (1988) 320.
[3] G. Gamberini, G. Ridolfi and F. Zwirner, Nucl. Phys. B331 (1990) 331.
[4] J.A. Casas, A. Lleyda and C. Muñoz, \textit{Nucl. Phys.} \textbf{B471} (1996) 3.

[5] U. Ellwanger and C. Hugonie, \texttt{hep-ph/9811386};
   S. Abel and T. Falk, \textit{Phys. Lett.} \textbf{B444} (1998) 427;
   S. Abel and C. Savoy, \textit{Phys. Lett.} \textbf{B444} (1998) 119;
   S. Abel and B. Allanach, \textit{Phys. Lett.} \textbf{B431} (1998) 339.
   OPAL Collaboration, \textit{Eur. Phys. Jour} \textbf{C7} (1999) 407; \textit{ibid.}, \textit{Eur. Phys. Jour} \textbf{C12} (2000) 567.

[6] H. Baer, M. Brhlik and D. Castaño, \textit{Phys. Rev.} \textbf{D54} (1996) 6944.

[7] P.M. Ferreira, \textit{Phys. Lett.} \textbf{B509} (2001) 120; \textit{Err. Phys. Lett.} \textbf{B518} (2001) 333.

[8] P.M. Ferreira, \textit{Phys. Lett.} \textbf{B512} (2001) 379; \textit{Err. Phys. Lett.} \textbf{B518} (2001) 334.

[9] V. Barger, M.S. Berger and P. Ohmann, \textit{Phys. Rev.} \textbf{D49} (1994) 4908.

[10] C. Ford, D.R.T. Jones, P.W. Stephenson and M.B. Einhorn, \textit{Nucl. Phys.} \textbf{B395} (1993) 17.

[11] P.M. Ferreira, \texttt{hep-ph/0210024}.

[12] J. Rosiek, \textit{Phys. Rev.} \textbf{D41} (1990) 3464.

[13] A.K. Hartmann and H. Rieger, \textit{Optimization algorithms in physics}, Wiley VCH, Berlin 2002.

[14] K. Hagiwara \textit{et al}, \textit{Phys. Rev.} \textbf{D66} (2002) 010001.

[15] P.M. Ferreira, I. Jack and D.R.T. Jones, \textit{Phys. Lett.} \textbf{B387} (1996) 80.

[16] D. Pierce, \texttt{hep-ph/9407202}.

[17] P.H Chankowski, Z. Pluciennik and S. Pokorski, \textit{Nucl. Phys.} \textbf{B439} (1995) 23.

[18] Y. Fujimoto, L. O’Raifeartaigh and G. Parravicini, \textit{Nucl. Phys.} \textbf{B212} (1983) 268.
   E.J. Weinberg and A. Wu, \textit{Phys. Rev.} \textbf{D36} (1987) 2474.

[19] P.M. Ferreira, R. Santos and A. Barroso, \textit{Phys. Lett.} \textbf{B603} (2004) 219.