Measurement uncertainty analysis of field-programmable gate-array-based, real-time signal processing for ultrasound flow imaging

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Abstract. Research in magnetohydrodynamics (MHD) aims to understand the complex interactions of electrically conductive fluids and magnetic fields. A promising approach for investigating complex instationary flow phenomena are lab-scale experiments with low-melting alloys. They require a noninvasive flow instrumentation for opaque liquids with a high spatiotemporal resolution, a low velocity uncertainty and a long measurement duration. Ultrasound Doppler velocimetry can achieve multiplane, multicomponential flow imaging with multiple linear ultrasound arrays. However the average raw data output amounts to $1.2 \text{ GB/s}$ at a frame rate of 33 Hz in a typical configuration for 200 transducers. This usually prevents long-duration measurements when offline signal processing is used.

In this paper, we propose an online signal-processing chain for pulsed-wave Doppler velocimetry that is tailored to the specific requirements of flow imaging for lab-scale experiments. The trade-off between measurement uncertainty and computational complexity is evaluated for different algorithmic variants in relation to the Cramér–Rao bound. By utilizing selected approximations and parameter choices, a prepossessing could be efficiently implemented on a field-programmable gate array (FPGA), enabling a typical reduction of the data bandwidth of $6.5:1$ and online flow visualization. We validated the performance of the signal processing on a test rig, yielding a velocity standard deviation that is a factor of 3 above the theoretical limit despite a low computational complexity.

Potential applications for this signal processing include multihour flow measurements during a crystal-growth process and closed-loop velocity feedback for model experiments.

1 Introduction

Many important industrial processes, such as continuous steel casting and photovoltaic wafer production, involve metal or semiconductor melt flows. The quality of the product and the energy efficiency of the process strongly depends on the flow behavior of the liquid (Müller and Friedrich, 2010; Gardin et al., 1995; Yasuda et al., 2007). A noncontact way of influencing the flow of electrically conductive melts is the application of magnetic fields that introduce Lorentz forces to the fluid. Investigating the interaction of a magnetic field with the flow pattern and optimizing the spatiotemporal structure of the magnetic field for different applications are subjects of ongoing research in magnetohydrodynamics (MHD). Besides numerical simulations, low-temperature, model-scale experiments are important tools for MHD investigations (Eckert et al., 2007b). They often require advanced flow instrumentation for visualizing complex and instationary flows in opaque liquids. A typical set of requirements for MHD research are as follows:

- Noninvasiveness – the influence of the instrumentation to the flow should be negligible (Eckert et al., 2007a).

- Flow imaging capability – the fluid’s velocity should be visualized in multiple planes (2D) with two or three ve-
larity components (2c or 3c) in order to adequately repre-
represent complex flow patterns.

- Spatial resolution – the relevant flow structures have to
be resolved, typically in the range of 10 mm (Timmel
et al., 2011).

- Temporal resolution – fluctuations (typically at
1…5 Hz) have to be resolved in order to capture
instationary flows (Timmel et al., 2011).

- Long measurement duration – flow phenomena on
different timescales should be adequately captured; for in-
stance, rapid spontaneous changes of the flow regime
in a rotating flow (Galindo et al., 2017) or in multhour
model experiments of the semiconductor crystallization
process (Thieme et al., 2017).

- Capability of near-wall measurements – in typical MHD
experiments, the metal melt is contained in a vessel. The
vicinity of the wall is especially important because the
Lorentz force is often concentrated in this region. Con-
trary to, for instance, medical applications, the walls can
be seen as completely stationary in most cases.

- Online capability – conducting long-running MHD ex-
periments requires the ability to examine the data during
the duration of the measurement. Some model experi-
ments in the semiconductor crystallization process even
benefit from an active control of parameters, like mag-
netic field intensity and temperature gradient, based on
the feedback from online velocity data to stabilize the
flow (Thieme et al., 2017).

A measurement system for flow mapping of opaque li-
quids, namely the ultrasound array Doppler velocimetry
(UADV; Nauber et al., 2013a, b), was presented in pre-
vious publications. It extends the pulsed-wave Doppler prin-
Ciple (Takeda, 1986; Baker, 1970) by employing multiple linear
sensor arrays to achieve multiplexing, two-componential
flow imaging. The sensors are designed to achieve a lateral
resolution of ≈ 3 mm in Galinstan (GaInSn). A combination of
spatial- and time-division multiplexing allows one to par-
allelize the scanning process for a planar velocity map; hence
increasing the temporal resolution compared to a strict se-
quential scan. However, online processing of the data for
200 transducer elements simultaneously on 32 channels at a
temporal resolution typically of 33 Hz overburdens PC-based
hardware with 1.2 GBs−1. Therefore, only discontinuous off-
line measurements could be performed with a limited dura-
tion of a few seconds. This severely impedes the usability of
the UADV in the context of MHD experiments and restricts
the investigations into stationary or periodic flows.

Although several investigations on the measurement un-
certainty of Doppler velocity estimation methods for laser-
based instrumentation (Fischer et al., 2010), for flow-rate
measurements in a pipe (Furuichi, 2013), and for blood-flow
measurements in the human body (Lovstakken et al., 2007)
have been performed, no comprehensive measurement un-
certainty budget in the context of instrumenting an MHD ex-
periment has been presented to the knowledge of the authors.

This paper provides a signal-processing chain that is tai-
lored to the specific requirements of MHD model experi-
ments and shows a real-time implementation using a field-
programmable gate array (FPGA). It enables the UADV sys-
tem to perform long-duration measurements with high frame
rates and online flow visualization. Furthermore, we eval-
uate the measurement uncertainty of the whole UADV sys-
tem in the context of MHD experiments and present an un-
certainty budget according to the methodology proposed by
the “Guide to the expression of uncertainty in measurement”
(GUM; JCGM, 2008) for a typical configuration.

2 Pulsed-wave ultrasound Doppler velocimetry
2.1 Measurement principle

In pulsed-wave ultrasound Doppler velocimetry (PW–UDV),
short bursts are emitted periodically with a pulse repeti-
tion frequency fPR (Baker, 1970). The emission times tN =
nN/fPR span the so-called slow-time axis ts, with nN =
0…NEPP being the bursts number. The emitted bursts usu-
ally consist of Nperiods periods of a sinusoidal wave, with the
frequency f0. As the bursts travel through the fluid, scatter-
ing particles reflect a fraction of the signal back to the ul-
trasound transceiver. The received echo signal z(tf, ts) is ac-
quired, starting from the emission time along the fast-time
axis tf. Figure 1 depicts an example of the echo signal for a
single moving scattering particle.

The movement of a scattering particle leads to a phase shift
of the echo signal between multiple burst emissions (Kasai
et al., 1985). The mean phase shift per time unit, expressed
as mean frequency fd, is related to the velocity v for a given
speed of sound c by the following:
\[
v = -\frac{1}{2} \frac{f_d}{f_{xs}} c, \tag{1}
\]

with fxs denoting the mean frequency of the received signal
burst and c ≫ v. The mean phase shift per time unit fd can
be interpreted as a Doppler frequency shift fd (Kasai et al.,
1985); hence the name Doppler velocimetry.

The time since the burst emission tf corresponds to the dis-
cance d between the scattering particle and the transducer ac-
\[
d = \frac{1}{2} tf c. \tag{2}
\]

This allows a spatially resolved flow measurement along the
axis of the transducer, given that the scattering particles fol-
low the motion of the fluid with negligible slip. The axial res-
olution can be estimated with the following (Jensen, 1996):
\[
\Delta d = \frac{1}{2} N_{\text{periods}} c \frac{c}{f_0}. \tag{3}
\]
the frequency $f_{s}$, which is a result of the transducer geometry, temporal resolution $\Delta t$ is determined through the following:

$$\Delta t = \frac{N_{EPP}}{f_{PR}}. \quad (4)$$

### 2.2 Ultrasound array Doppler velocimeter

The ultrasound array Doppler velocimeter (UADV) is a modular research platform developed at the Laboratory of Measurement and Sensor System Technique (MST) for flow imaging in opaque liquids with PW–UDV. It is flexible and especially well suited for instrumenting a wide range of experiments in the field of MHD. The hardware of the UADV consists of individually configurable modules driving 25 ultrasound transducers each. It can be scaled to support up to 200 transducers in various configurations; for instance, in four linear arrays which can be individually parameterized regarding ultrasound frequency, pulse shape and length, and pulse-repetition frequency (Nauber et al., 2016; Büttner et al., 2013).

A module of the UADV consists of an arbitrary function generator and a power amplifier for generating parameterizable burst signals which are routed through a programmable switching matrix and a transmit/receive switch to the transducers. The received echo signals are amplified with a parallel-processing chain, very large data bandwidths have to be processed. This can be achieved by utilizing the parallel-processing capability of a field-programmable gate array (FPGA). Especially narrowband algorithms are very suitable for FPGA-based implementations, due to their low computational complexity (Alam and Parker, 2003; Loupas et al., 1995a). Therefore, this paper focuses on investigating the most common narrowband velocity estimator by Kasai et al. (1985) and the extensions proposed by Loupas et al. (1995b).

A typical narrowband signal-processing chain is shown in Fig. 2. In this fully digital realization, the slow time $t_{s}$ is sampled for each burst $n_{b}$ by $t_{s} = n_{b} f_{PR}$ and the fast time $t_{f}$ is sampled with a frequency $f_{s}$ as follows:

$$z_{raw}(k, n_{b}) = z(t_{f} = k / f_{s}, t_{s} = n_{b} f_{PR}), \quad k = 0, 1, \ldots K, \quad n_{b} = 0, 1, \ldots N_{EPP}. \quad (5)$$

The signals are then bandpass filtered to reduce noise contributions outside of the bandwidth of the transmitted ultrasound signal. A quadrature demodulation is performed, consisting of a Hilbert transform and a subsequent down sampling. Static echoes are removed through a clutter reduction filter (CRF) and the velocities are estimated by an autocorrelation.

### 3.2 Quadrature demodulation

In order to meet the assumptions of the narrowband signal processing and to reduce the influence of noise, a bandpass
filtering is performed as follows:

\[ z'(k, n_b) = \sum_{n=0}^{N_{	ext{periods}}} c_i \cdot z'_{\text{raw}}(k-n, n_b), \]  
(6)

with the filter coefficients \( c_i \). In order to maximize the SNR for signals with additive white Gaussian noise, a matched filter is used (Turin, 1960) as follows:

\[ c_i = s_{tx}(N_{tx} - i), \quad i \in [1, N_{tx}], \]  
(7)

with the transmitted signal \( s_{tx} \) with \( N_{tx} \) samples.

The result of the quadrature demodulation is a complex signal \( h'_{\text{unfilt}}(k/n_{sub}, n_b) \) in the baseband, which can be sampled at a lower rate (reduction by a factor of \( n_{sub} \)) than the raw signal, as follows:

\[ h'_{\text{unfilt}}(k/n_{sub}, n_b) = z'(k, n_b) + j \cdot \hat{z}'(k, n_b), \]  
(8)

with \( N_b = 0, 1, \ldots N_{\text{EPP}}, k = 0, 1, \ldots K \) and the Hilbert transform signal \( \hat{z}'(k, n_b) \) (with 90° phase shift with respect to \( z' \)).

3.3 Clutter-reduction filtering

A common problem of ultrasound Doppler flow measurements is distinguishing between static echoes originating from the walls (the so-called clutter) and echoes originating from scattering particles. Multiple reflections from the transmitted burst inside the wall superimpose the signal from scatter particles in the vicinity of the wall. For this problem, a multitude of signal-processing methods were proposed, most of them based on digital filters (finite impulse response (FIR) or infinite impulse response (IIR) filters) with various initialization techniques (Lee et al., 2009). With these methods, the clutter is distinguished from the particle echoes by a velocity close to zero, respectively, by a Doppler frequency shift close to zero. Because filtering will influence the spectrum of the signal, a bias may be introduced to the subsequent velocity estimation, depending on the frequency cutoff. For typical MHD experimental setups, the wall can be assumed to be completely stationary (in contrast to, e.g., medical applications where clutter is often constituted by slowly moving tissue; cf. Jensen, 1996); therefore, a steep cutoff at a frequency of zero is desirable. The simplest and computationally most efficient approach is to filter the constant component of the demodulated IQ signal by subtracting its mean value, which is the equivalent of applying a very narrow-band, high-pass filter (Thomas and Hall, 1994; Jensen, 1996; Torp, 1997; Bjaerum et al., 2002) as follows:

\[ h'(k/n_{sub}, n_b) = h'_{\text{unfilt}}(k/n_{sub}, n_b) - \frac{1}{N_{\text{EPP}}} \sum_{n_{sub}=0}^{N_{\text{EPP}}-1} h'_{\text{unfilt}}(k/n_{sub}, n_b). \]  
(9)

As the filter is noncausal, all \( N_{\text{EPP}} \) samples have to be acquired before the result can be computed.

3.4 One-dimensional autocorrelation algorithm

A widely used approach for velocity estimation is the autocorrelation method proposed by Kasai et al. (1985), which operates solely in the domain of IQ-demodulated echo signals and therefore can be implemented very efficiently (Alam and Parker, 2003). It uses the properties of the signals’ discrete autocorrelation function as follows:

\[ R'(\Delta k, \Delta n_b) = \sum_{m=0}^{K} h'(m, n) \cdot h'^*(m + \Delta k, n + \Delta n_b), \]  
(10)

where its values at a lag of 1 relate to the center of mass of the signal’s power density spectrum through the
Wiener–Khinchin theorem. As shown by Kasai et al. (1985) and Jensen (1996), the mean Doppler shift \( f_d \) can be approximated through evaluating the autocorrelation function at a lag of \( \Delta n_b = 1 \) slow-time samples as follows:

\[
f_d \approx 1/T_{PR} \arg(R(\Delta k = 0, \Delta n_b = 1)). \tag{11}\]

This autocorrelation computation can be expressed solely by repeatedly multiplying accumulate operations and therefore can be implemented very efficiently. Kasai’s method approximates the center frequency \( f_d \) of the received signal with the frequency of the emitted signal as follows:

\[
f_{rx} \approx f_0. \tag{12}\]

Being based on a phase estimation, the Kasai algorithm is inherently limited in the maximum measurable velocity. Given the 2\( \pi \)-phase ambiguity in Eq. (11), the measurable velocity range resulting from Eq. (1) is (Jensen, 1996) as follows:

\[
v \in [\pm v_{\text{max}}]; \quad v_{\text{max}} = \frac{c}{f_{PR}}. \tag{13}\]

### 3.5 Two-dimensional autocorrelation algorithm

An extension of Kasai’s autocorrelation method is proposed by Loupas et al. to improve its performance in the following two regards (Loupas et al., 1995b):

1. The assumption of an unchanged center frequency of an ultrasound burst throughout emission, propagation inside the fluid and reception is discarded. This allows one to account for the effect of frequency-dependent attenuation, which is present in most relevant fluids. By explicitly estimating the center frequency of the received signal, a systematic velocity error stemming from the relationship in Eq. (1) \( v \propto 1/T_{\text{rx}} \) is avoided.

2. An information loss occurs if only a narrow-band part of a broadband echo signal is processed. Hence a better estimation of the velocity is achieved by including a larger part of the signal spectrum.

Both aspects are addressed by increasing the dimensionality of Kasai’s autocorrelation; instead of just correlating along the slow-time axis, a 2D autocorrelation along the slow- and fast-time axis is performed. An autocorrelation with a lag of one fast-time sample yields the estimate of the center frequency as follows:

\[
f_{rx} \approx f_{rx} \arg\left(\sum_{n} R(\Delta k = 1, \Delta n_b = 0)\right) \approx \frac{1}{2\pi} \frac{f_{rx}}{n_{sub}} \left(2\pi \left[\frac{1}{2} + n_{sub} f_0 / f_s\right] + \arg R'(0, 0)\right) \tag{14}\]

\[
f_{rx} \in \left(-\frac{f_s}{n_{sub}} \left[\frac{1}{2} + n_{sub} f_0 / f_s\right] \pm \frac{f_s}{2 n_{sub}}\right). \tag{15}\]

Furthermore, the estimation of the frequencies \( f_{rx} \) and \( f_{d} \) can be performed using \( M \) samples per gate, as follows:

\[
f_d \approx \frac{1}{2\pi} f_{PR} \arg R'(0, 1). \tag{16}\]

The extension of the Kasai autocorrelation algorithm potentially improves the estimation performance while still preserving a low computational complexity.

### 4 Online-capable, FPGA-based signal-processing implementation

In order to provide online capability, the signal processing depicted in Fig. 2 has been realized on an FPGA (NI PXIe-7965R; National Instruments, Austin, Texas, USA). The FPGA communicates with a host PC through a peripheral component interconnect express (PCIe) bus and has the ability to stream data through direct memory access into the main memory of the PC.

The amplified echo signals \( U_{PP, \text{max}} = 1 \text{ V} \) are digitized through an A/D converter module (NI-5752; National Instruments, Austin, Texas, USA) for \( n_{\text{ch}} = 32 \) channels at an externally provided sampling rate \( 32 \text{ MHz} > f_s > 50 \text{ MHz} \) with a quantization of 12 bit. The raw data rate \( r_{\text{ADC}} \) at this stage is as follows:

\[
r_{\text{ADC}} = n_{\text{ch}} \cdot n_{\text{sampbytes}} \cdot f_{\text{frame}} \cdot N_{\text{EPP}} \cdot N_{\text{SW}} \cdot N_{\text{gates}} \cdot K. \tag{17}\]

Data are processed as signed 16 bit integer (\( n_{\text{sampbytes}} = 2 \text{ B} \)) and, for a typical configuration as listed in Table 1, the data rate amounts to 1.2 GBs\(^{-1}\).

This data bandwidth is hardly suitable for continuous streaming to a storage device over a long duration (> 1 h) with common PC hardware. Therefore, raw data are only briefly retrieved for debugging purposes or for low frame-rate measurements and are otherwise not transferred to the host.

The signal-processing steps that perform an IQ demodulation (bandpass filtering, Hilbert transform and down sampling) are significantly reduced in their computational complexity by fixing the ratio of the sampling frequency \( f_s \) to the ultrasound center frequency \( f_0 \) at \( f_{rx} / f_s = 1/4 \). The matched filter can be realized for a sinusoidal transmit signal at \( f_{tx} \)
Table 1. Overview of the parameters of the signal processing.

| Parameters               | Value |
|--------------------------|-------|
| Number of channels       | $N_{\text{ch}} = 32$ |
| Number of bytes per sample | $N_{\text{sambytes}} = 2\text{B}$ |
| Number of gates          | $N_{\text{gates}} = 51$ |
| Subsampling factor       | $N_{\text{sub}} = 13$ |
| Multiplexing steps       | $N_{\text{sw}} = 6$ |
| Number of emissions      | $N_{\text{app}} = 50$ |

with $N_{\text{periods}}$ periods, assuming $f_{\text{fs}} \approx f_{\text{Rx}}$ with only trivial filter coefficients $c_i$, as follows:

$$c_i = \begin{cases} 
1 & i = 2 + 4n \\
0 & i = 1 + 4n, i = 3 + 4n ; \ n \in [0, N_{\text{periods}}]. \\
-1 & i = 4n 
\end{cases}$$

(18)

This allows one to implement the filtering without multiplication operations, only negations and additions are needed.

To provide a low computational complexity approximation of the Hilbert transform for a narrowband case, a fixed time delay can be employed (Kantz et al., 2012) as follows:

$$\hat{\gamma}(k, n_b) \approx \gamma(k - 1, n_b),$$

(19)

where $f_{\text{fs}}/f_{\text{Rx}} = 1/4$. The signal processing up to this point contains just the summation, negation and storage primitives and therefore can be implemented on an FPGA with modest resources. The data rate $r_{\text{IQ}}$ at this stage for a typical configuration is given by the following:

$$r_{\text{IQ}} = r_{\text{ADC}} \cdot 2 \cdot 1/n_{\text{sub}}.$$

(20)

Through the data reduction of $6.5:1$, the data rate at this stage is $r_{\text{IQ}} = 185\text{MBs}^{-1}$ for a typical configuration, as listed in Table 1. A continuous data streaming to a storage device can be sustained for a long duration at this rate.

5 Performance evaluation of narrow-band signal-processing algorithms

5.1 Theoretical limit of measurement uncertainty

In order to characterize the performance of a signal-processing algorithm, it is not only helpful to have relative data compared to other algorithms but also to relate it to a fundamental limit of attainable precision. This absolute limit of uncertainty can be provided by means of the estimation theory using the Cramér–Rao bound (CRB; Radhakrishna Rao, 1945; Cramér, 1946). Given a suitable signal model, the CRB represents the lowest possible variance for estimating a parameter from the signal with an unbiased estimator. In the following, a simple signal model for a discrete time idealized ultrasound echo is described and a derivation of the CRB for velocity estimation is given.

A simple approximation of the ultrasound echo signal realizations $x(k, n_b, \theta, \sigma_n^2)$ consists of a sinusoidal signal $s(k, n_b, \theta)$ superimposed with additive white Gaussian noise $n(k, n_b, \sigma_n^2)$ sparsely and periodically sampled in the fast-($k$) and slow-time ($n_b$) axis as follows:

$$x(k, n_b, \theta, \sigma_n^2) = s(k, n_b, \theta) + n(k, n_b, \sigma_n^2),$$

(21)

with

$$s(k, n_b, \theta) = A \cos \left(2\pi f_0 + \varphi_0 + \frac{k}{f_{\text{Rx}}} + \varphi_0 \right).$$

(22)

and $A$ being the amplitude of the scattering particles’ echo, $\varphi_0$ a constant phase, and $n(k, n_b, \sigma_n^2)$ Gaussian white noise, with a variance $\sigma_n^2$ and zero mean.

The unknown quantities are as follows:

$$\theta = \begin{pmatrix} A \\ f_0 \end{pmatrix}.$$

(23)

The CRB provides the lower boundary for the variance of an estimator $\hat{\theta}_i$ according to the inequality, as follows:

$$\text{var}(\hat{\theta}_i) \geq \text{CRB}(\hat{\theta}_i) = \left[ I^{-1}(\theta) \right]_{ii},$$

(24)

with $I(\theta)$ being the Fisher information matrix, as follows:

$$[I(\theta)]_{ij} = -\mathbb{E} \left[ \frac{\partial^2 \ln(p(x, \theta))}{\partial \theta_i \partial \theta_j} \right].$$

(25)

Kay (1993) provided a formula for the case when the probability density function $p(x, \theta)$ of the signal model $x$, Eq. (22), is a Gaussian joint probability function as follows:

$$[I(\theta)]_{ij} = \frac{1}{\sigma_n^2} \sum_k \sum_{n_b} \frac{\delta s(k, n_b, \theta)}{\delta \theta_i} \frac{\delta s(k, n_b, \theta)}{\delta \theta_j}. $$

(26)

The differentiation of $s(k, n_b, \theta)$ with respect to the unknown quantities is performed analytically, while the matrix inversion was performed numerically using MATLAB (The MathWorks, Inc., Natick, Massachusetts, USA). The resulting CRB for the velocity uncertainty as a function of the signal-to-noise ratio (SNR) is given in Fig. 4e–d. It has a slope of $-20\text{dB/decade}$, which is consistent with the CRB of other Doppler-based signal-processing problems (Fischer et al., 2010; Chan et al., 2012; Demirli and Saniee, 2001).

5.2 UADV measurements on a reference experiment

For an experimental characterization of the measurement performance of the UADV system, a test rig based on the linear translation of a single scattering object is used (Fig. 3).

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compared as follows:

\[ \text{SNR} = -6 \text{,} \ldots, -3, \ldots, 12 \text{ dB} \]

Four algorithmic variants were compared as follows:

- (DEF) – the 1D Kasai velocity estimator without clutter filtering, as described in Sect. 3.4
- (CRF) – the 1D Kasai velocity estimator with a clutter filtering including the estimation of \( f_{tx} \)
- (CRF 2D) – the 2D velocity estimator as described in Sect. 3.5 with clutter filtering but without an estimation of \( f_{tx} \)
- (CRF 2D RF) – the 2D velocity estimator as with clutter filtering including the estimation of \( f_{tx} \)

The parameterization of the experiment and of the algorithms is listed in Table 2.

Table 2. Overview of the ultrasound parameters and the signal-processing algorithms.

| Parameters                      | Values |
|---------------------------------|--------|
| Excitation pulse                | Sinusoidal signal; \( f_0 = 8 \text{ MHz} \) |
| Pulse length                    | \( N_{\text{periods}} = 8 \) |
| Pulse repetition frequency      | \( f_{PR} = 900 \text{ Hz} \) |
| Number of emissions             | \( N_{\text{EPP}} = 50 \) |
| Speed of sound                  | \( \chi_{\text{H}_2\text{O}} = 1480 \text{ ms}^{-1} \) |
| Clutter-to-signal ratio \( \text{CSR}_1 \) | \(-7.3 \text{ dB} \) (near the wall) |
|                                | \(-19.0 \text{ dB} \) (far from the wall) |
| Sampling frequency              | \( f_s = 32 \text{ MHz} \) |
| Velocity set point              | \( v_{ref} = 10 \text{ mms}^{-1} \) |
| Number of repetitions           | \( N = 130 \) |
| (DEF)                           | \( M = 1 \), \( f_{tx} \approx f_0 \) |
| (CRF)                           | \( M = 1 \), \( f_{tx} \approx f_0 \) |
| (CRF 2D)                        | \( M = 3 \), \( f_{tx} \approx f_0 \) |
| (CRF 2D RF)                     | \( M = 3 \), \( f_{tx} \approx f_0 \) |

It consists of a linear stage (41.121.102E; OWIS GmbH, Staufen, Germany) that is mounted over a glass tank with the dimensions of 212 × 81 × 135 mm³. It moves a scattering object (glass fiber with a spherical tip, and diameter of 0.6 mm, mounted in a hollow needle) with a constant velocity through water (\( \theta = 20^\circ \text{C} \); \( c = 1480 \text{ ms}^{-1} \)). The ultrasound sensor array is mounted on the front wall of the tank and therefore insonates through an 8 mm glass wall and a water-based ultrasound couplant.

In order to trace back the measurement results of the UADV to the definitions of the respective units in the SI system, a simultaneous measurement of the relative position and velocity was done with a vibrometer (OFV-503; Polytech, Waldbronn, Germany; displacement decoder DD-900 and velocity decoder VD-09). A retroreflective tape (3M Scotchlite) was attached to the shaft of the scattering object’s mount. For a velocity set point of 10 mms⁻¹, a standard deviation of the velocity \( \sigma_{v,\text{ref,rel}} = 0.178 \% \) was determined for the linear stage–vibrometer combination (for the same averaging time as the UADV system).

A total of 130 measurement cycles were conducted, consisting of a constant translation away from the front wall of the tank with a velocity set-point \( v_{\text{ref}} = 10 \text{ mms}^{-1} \) and the respective backward motion. Of the continuously obtained UADV measurements, only those that originate from two defined positions near to and far from the wall during the movement away from the ultrasound transducer (Richter Sensor and Transducer Technology, Germany) are selected in the postprocessing. The clutter-to-signal ratio (CSR) is \( \text{CSR}_1 = -7.3 \text{ dB} \) and \( \text{CSR}_2 = -19.0 \text{ dB} \), respectively. To ensure a common time base for vibrometer and UADV measurements, the trigger signal of the UADV is acquired simultaneously with the velocity and position signals. To test the performance under different SNR conditions, white Gaussian noise was added to the raw digitized signals to achieve SNR = -6, -3, ..., 12 dB. Four algorithmic variants were compared as follows:

- (DEF) – the 1D Kasai velocity estimator without clutter filtering, as described in Sect. 3.4
- (CRF) – the 1D Kasai velocity estimator with a clutter filtering according to Sect. 3.3
- (CRF 2D) – the 2D velocity estimator as described in Sect. 3.5 with clutter filtering but without an estimation of \( f_{tx} \)
- (CRF 2D RF) – the 2D velocity estimator as with clutter filtering including the estimation of \( f_{tx} \)
Figure 4. Relative systematic deviation (a, b) and relative standard deviation (c, d) of the velocity versus SNR for reference measurements far from the wall (a, c) and near to the wall (b, d); the relative systematic deviation of (DEF) (b) is outside of the axis, with $\Delta v/v_{\text{ref}} < -42\%$; the error bars denote the 95% confidence interval from 130 measurement cycles.

Figure 5. Example of a flow image of magnetically stirred GaInSn in the central horizontal plane of a cubic vessel. Panel (a) depicts the experimental setup, (b) the mean flow velocity along the $d$ axis, and (c) the standard deviation.
Table 3. Measurement uncertainty budget for typical MHD experiments in liquid GaInSn.

| Quantity | Uncertainty source                                                                 | Type of uncertainty estimation according to GUM                                                                 | Relative standard uncertainty; \( \sigma_{v, rel} \) |
|----------|------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------|--------------------------------------------------|
| \( f_d \); \( f_{tx} \) | Random effects of Doppler frequency estimation, including phase jitter and electrical noise. | Type A estimation from calibration measurements (Fig. 4) for (CRF 2D RF) and an SNR of 5 dB; normal distribution with \( \sigma_{v, rel} = 0.4 \% \). | 0.4 \% |
| \( f_d \); \( f_{tx} \) | Unknown systematic effects of Doppler frequency estimation, including frequency-dependent attenuation of the fluid and drift in the slow-time clock source. | Type B estimation from calibration measurements (Fig. 4) for an SNR of 5 dB; uniformly distributed in the interval \( \Delta v_{rel} = \pm 0.08 \% \). | 0.05 \% |
| \( c \) | Value of the speed of sound of the fluid \( c_{GaInSn} = 2740 \text{ ms}^{-1} \) (given by Morley et al., 2008, without a measurement uncertainty). | Type A estimation based on Proffit and Carome (1962); \( \sigma_{c, rel} = 0.03 \% \). | 0.03 \% |
| \( \Delta c \) | Unknown systematic variations of the speed of sound in the fluid due to temperature changes. | Type B estimation for the sound–speed temperature coefficient of liquid gallium (Proffit and Carome, 1962; Popel et al., 2005); \( \frac{dc}{dT} = -0.3 \text{ m(sK)}^{-1} \) for \( \Delta T = \pm 10 \text{ K} \) rectangular distributed in the interval \( \Delta c_{rel} = \pm 0.11 \% \). | 0.06 \% |
| \( v \) | Influence of the spatial resolution from the finite width of the sound field. | Type B estimation for a beam width \( \Delta x = 3 \text{ mm} \) and typical velocity gradients of MHD experiments estimated from numerical simulation; \( \frac{dv}{dx} = 0.16 \text{ mm}^{-1} \cdot t_{\max} \) (Galindo et al., 2017); \( \Delta v = 1/2 \cdot \frac{dv}{dx} \cdot b \); \( \Delta v_{rel} = \pm 24 \% \). | 13.9 \% |

Total uncertainty \( \sqrt{\sum_i \sigma_{v, rel,i}^2} = 13.9 \% \).

For the given experimental data, the algorithm variant (CRF 2D RF) provides a suitable trade-off between systematic and standard deviation and computational complexity.

6 Measurement uncertainty budget of the UADV in liquid metal

A measurement uncertainty budget according to the GUM (JCGM, 2008) is used to assess the contributions of measurement uncertainty for the UADV system. Based on Eq. (1), the measurand \( v \) is derived from the quantities \( f_d \), \( c \) and \( f_{tx} \). Furthermore, the direct influence of the spatial averaging over the flow within the ultrasound beam width is considered. In Table 3, the uncertainties contribution of these quantities are given for a typical MHD experiment.

For the uncertainties of \( f_d \) and \( f_{tx} \), the results of Sect. 5.2 are transferred from the reference experiments in water to typical measurement conditions in low-melting liquid metals. The maximum relative systematic deviation and standard deviation for both investigated CSR and a typical SNR of SNR = 5 dB are used to calculate the equivalent uncertainty of the velocity. The influence of an uncertainty in the fluid’s speed of sound, \( c \), is estimated by the uncertainty of the measurement of this quantity, in the literature and the temperature dependence, assuming a temperature gradient of \( \Delta T = \pm 10 \text{ K} \). The uncertainty arising from spatial averaging through the ultrasound beam characteristics is calculated by assuming a lateral averaging of \( \Delta x = 3 \text{ mm} \) and velocity gradients of numerical simulations of typical MHD experiments (Galindo et al., 2017).

It can be seen that the biggest contribution to the velocity uncertainty of the UADV measurement system for typical MHD settings with \( \sigma_{v, rel} = 13.9 \% \) stems from the spatial averaging over lateral resolution given by the ultrasound beam width of the unfocused transducers. This provides the most promising starting point for further improvements regarding the measurement uncertainty of the UADV system. Furthermore, it justifies the approximations taken for computationally efficiently implementing the signal processing, even though lower uncertainty algorithms exist that approach...
the CRB (Chan et al., 2012) because signal processing is not the limiting factor in the measurement uncertainty budget.

7 Example of liquid metal flow imaging

To demonstrate the capabilities of the ultrasound array Doppler velocimeter (UADV) with the proposed signal processing, it is applied to a simple MHD experiment. A cubic vessel with the dimensions of $67 \times 67 \times 67 \text{mm}^3$ is filled with GaInSn and a 25-element linear transducer array (Richter Sensor and Transducer Technology, Germany) is attached to insonify the central horizontal plane (cf. Fig. 5a). With the application of a horizontally counterclockwise rotating magnet field, a counterclockwise central vortex forms. The UADV measures the velocity component along the axis of the transducers ($d$ axis) with the parameterization given in Table 2 and with $f_{PR} = 200 \text{Hz}$. The resulting planar flow image, using signal-processing variant (CRF 2D RF), is shown in Fig. 5b and c.

8 Conclusions

Experimental research in the field of MHD can benefit from online, noninvasive flow imaging for investigating fundamental phenomena, such as flow instabilities and optimizing industrial processes. We describe an online-capable signal processing for pulsed-wave Doppler velocimetry that is tailored to the specific requirements of lab-scale model experiments. It is based on a 2D autocorrelator, which allows for a reduction of systematic and stochastic errors through explicitly estimating the RF and utilizing multiple samples per gate. We optimized the signal processing for low computational complexity and implemented substantial parts on an FPGA. A typical reduction of the data bandwidth of 6.5 : 1 enables continuous data streaming to PC hardware.

We evaluated the performance of the implemented signal processing in a water test rig with a single scattering object and a reference velocity obtained through a laser vibrometer. Two different clutter signal levels emulate a measurement close to and far from a wall. A velocity standard deviation of $\sigma_v, \text{rel} = 0.4\%$ was found, which is about 3 times the fundamental limit of the uncertainty, the CRB, for velocity estimation. The systematic deviation is $\Delta v_{\text{rel}} = \pm 0.08\%$.

We investigated the measurement uncertainty budget for flow velocity measurements in a typical MHD experimental setup for the low-melting alloy GaInSn. The total measurement uncertainty of $\sigma_v, \text{rel} = 13.9\%$ almost solely stems from the effect of spatial averaging over the lateral resolution for flows with high-velocity gradients. This justifies the approximations taken for lowering the computational complexity of the signal processing.

A measurement uncertainty budget of a typical MHD experiment at laboratory scale suggests improvements towards a better lateral resolution. In the context of flow imaging, this can be provided by the focusing and steering of the ultrasound beam using the phased-array principle.

The presented signal processing enables online, multi-plane flow visualization with the UADV research platform. A long measurement duration (> 1 h), combined with a high frame rate (> 10 Hz), allows one to investigate complex, instationary flows such as instability phenomena in cubes.

Data availability. Research data are available upon request from the authors.

Author contributions. RN implemented the signal processing, designed and conducted the numerical and experimental investigations. LB and JC supervised the research. All authors discussed and proofread the manuscript.

Competing interests. The authors declare that they have no conflict of interest.

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