Probabilistic Learning in Junior High School: Investigation of Student Probabilistic Thinking Levels

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Abstract. This paper was to investigate level on students’ probabilistic thinking. Probabilistic thinking level is level of probabilistic thinking. Probabilistic thinking is thinking about probabilistic or uncertainty matter in probability material. The research’s subject was students in grade 8th Junior High School students. The main instrument is a researcher and a supporting instrument is probabilistic thinking skills test and interview guidelines. Data was analyzed using triangulation method. The results showed that the level of students probabilistic thinking before obtaining a teaching opportunity at the level of subjective and transitional. After the students’ learning level probabilistic thinking is changing. Based on the results of research there are some students who have in 8th grade level probabilistic thinking numerically highest of levels. Level of students’ probabilistic thinking can be used as a reference to make a learning material and strategy.

1. Introduction

Mathematic learning is a complex learning process. Mathematic learning is aimed to train how students can act with logical, rational, critical, accurate, efficient, and effective thinking. It is important to relate students’ thinking process in mathematic learning, in which thinking is a mental activity to solve the problems, to make decision, or to fulfill a desire to understand [15]. Furthermore, learning process must pay attention on how to teach mathematic and to train thinking ability. Thinking ability is quite needed in the past, present and future time. Human being must be able to predict what will happen in the future situation. It can be revealed with probabilistic matter or something which is equal to probability.

Probability is the mathematical way to deal with the problems of uncertainty [13]. It means that probability is used to estimate an event, for example; whether if it is cloudy, it will rain or not, how is the possibility to pass the school entrance exam, and how is the possibility to win in a match, etc. A same assumption also states that probability is a concept of increasing importance in scientific thought [7]. Therefore, it is clear that probability is essential in daily life.

A probability situation is a situation involving uncertainty. Student who thinks in an uncertainty situation is called probabilistic thinking. Probabilistic thinking is used to describe students’ thinking in responding various probabilistic problems [17]. Students’ thinking in responding various probabilistic problems has different level which is called as level of probabilistic thinking. Level of probabilistic thinking is a hierarchy of thinking level in probabilistic matter. Level of students’ probabilistic thinking can be used to face phenomenon in the future.
Based on some researches, there are some aspects that influence probabilistic knowledge and students’ probabilistic thinking. Those are culture, language, belief, daily and school experience (game on probability theory) [3, 4, 16]. Therefore, in curriculum of National Conference of Teachers in Mathematics (NCTM), it is started since kindergarten. It means that probability is basic material to be learned.

Based on the explanation above, it is important to conduct investigation related to the level of students’ probabilistic in junior high school. Each level of probabilistic thinking has different indicators according to its levels. In learning process, it is hoped that teachers can create an ability to arrange learning strategy in teaching probability material.

2. Method

The research was conducted by taking samples from junior high school students of grade 8 with the age of 13-15. The research was conducted to 237 students in 8 classes at the second semester of 2015/2016 academic year. The survey of the students chose two classes randomly. The random way was taken because there was no superior or low class among the 8 classes. Therefore, all classes have no difference. The result of the research is a description of level of students’ probabilistic thinking in solving probability problem. Therefore, this research belongs to a descriptive research using qualitative approach.

Subjects of the research were students of grade 8 that have been chosen by some considerations, such as 1). Students of grade 8 had had enough learning experience and probability material in the previous class. Therefore, it is hoped that they can finish the questions on probability material. 2). Subjects were not chosen randomly. It was chosen by considering ability in communication to make sure that the level of probabilistic thinking ran well. 3). Grouping subjects were based on the early identification of the level of students’ probabilistic thinking. Technique of taking subjects in this research used purposive sampling. The researcher chose one student from each level of probabilistic thinking randomly. They are expected to be able in getting information related to the level of probabilistic thinking.

Instrument used in this research was test and interview guidance. Technique of collecting valid data uses questionnaire of validation. Furthermore, test was conducted to know the reliability. Reliability test was conducted by using formula of Cronbach’s alpha [5, 2]

Collecting data was conducted by interview based on task. Subjects were given questions of probabilistic thinking ability. Next, there would be a deep interview to know the level of students’ probabilistic thinking on each indicator. The data of interview would be arranged into interview transcript. Furthermore, the analysis was conducted to know the level of students’ probabilistic thinking.

Data validation used triangulation method. There were two strategies, checking the belief degree of research discovery using technique of collecting data and checking the belief degree of data resources using same methods [14]. Triangulation method was conducted by comparing the data of test and interview to obtain the level data of students’ probabilistic thinking.

3. Result and Discussion

3.1. Probability Learning

Probability is an essential material and can be interpreted descriptively by words. Probability is interpreted by using some words; such as never, impossible, unlikely, probably, certain, and so on, but how they are used in probability may be different from the real-life use of these words [12]. Probability applied in everyday situations, in games, in data processing, in insurance, in economics, in natural sciences [13]. Based on this matter, one of the essential aspects in teachers’ attention in teaching is probability matter with two approaches; the objective and the subjective approach [6, 13]. According to objective approach, it can generally use and follow the direction of probabilistic books.

According to this approach, probability be assigned only to an event that can be repeated. The subjective approach interprets the term probability as the degree of belief rather than as a relative
occurrence; thus, it can be applied to one-time events [6]. Therefore, to teach possibility, it can use those two approaches.

Learning is an adaptive process of our cognition to predict the future on the basis of past experience [1]. Based on that definition, learning process directs to probability learning. Probability learning is a principle that states probability of a response tends to match the probability of the reinforcement [1]. Generally, probability learning is a learning process conducted by students with the activity to interpret all students’ interpretations in responding phenomenon or experiment randomly. There are various learning strategies that can be conducted. One of them is experiment. In the beginning, students can start learning probability by interpreting data and doing simulation and experiment [12].

3.2. Student Probabilistic Thinking Levels

The result of instrument validation research that consists of test and interview has been revealed valid by the validators (lecturers of mathematics and mathematics education, especially on statistic field). The reliability result of the test of probabilistic thinking is 0.43. It shows that the reliability level is medium [8] and the test has been done to obtain the data.

Analysis result on students’ task related to probabilistic problems on probability material consists of sample space, probability of an event, and probability comparison of one-stage, two-stage and three-stage experiment. This paper shows some samples of students’ work and the excerpts of the interview related to probabilistic matter.

On the answer sheets given, students answered the questions related to probabilistic matter in determining the probability of a two-stage experiment. Probabilistic matter used was pairing clothes and pants that would be used. It was known that there were 3 white clothes (P), grey (A), and brown (K), and brown pants (C), and black (H). In the beginning, students arranged all experiment results by pairing clothes and pants. Furthermore, all students answered the possibility of an event easily by using numeric as the Figure 1 described below.

![Figure 1. Photograph Task of Subject 1 (S₁)](image)

Figure 1 shows that subjects can arrange the one-stage experiment result completely with generative strategy. However, subjects did not write sample space from the arranging result of all experiment results in mathematic language. When subjects determined the probability of an event, subjects did not previously write the event or event members. It might be caused that the questions were too easy for subject 1 (S₁).

Several days after conducting post-test on the ability test of probabilistic thinking, interview was conducted to state the validation of thinking level of S₁ students. Below is the data analysis of interview excerpt related to the level of students’ probabilistic thinking.

| P52 | S₁₅₂ | P53 |
|-----|------|-----|
| How is the explanation of this work? | Firstly, we make factor tree (diagram) and paired clothes and pants and looked for clothes and brown pants. The result is two. | Which one? |
The explanation of the subject about the way to determine sample space was making tree diagram or adopting and applying a generative strategy. Subjects arranged all results of the experiment generatively. When the students were asked to explain the probability, students could explain that members of the event are two, and the number of sample space members is 6. Therefore, the probability of the event is 2/6. Thus, subject is capable of applying a generative strategy to make complete listing of the outcomes for two-stage experiment. Subject is able to determine numerical probability in the probability an event for one-stage, two-stage and three-stage experiment.

Another probabilistic matter in probabilistic thinking test was comparing the event probability of two-stage experiments. Another subject work result related to probabilistic matter was shown in Figure 2. In this matter, it would be determined which one has bigger probability in wearing clothes and pants. It was known 3 clothes of white (P), grey (A), and brown (K), and brown clothes (C) and black (H). Mr. Wari did not like any color and combination of the clothes and the pants. Subject 2 wrote sample space of two-stage experiment; black pants (h), clothes P, A, and K. Next, subjects determined the probability of two events; wearing grey clothes, black pants and wearing brown clothes. The event of grey clothes and black pants was given symbol A and the event of brown clothes was not given any symbols. The event probability of A was ¼ and the event probability of brown clothes was 1/5. Then, subject 2 compared 2 event probabilities without answering the questions, in which the question was “which one has bigger probability to be worn”.

Figure 2 shows that subject 2 (S2) cannot arrange the two-stage experiment result using generative strategy completely. Students are capable determining probability of an event in a one-stage using numeric. Subjects 2 (S2) could not determine the probability of an event using numeric numbers. Next, to strengthen the analysis result, the interview was conducted to subject 2 with the interview excerpt below.

**P40:** Pay attention and read to the question first and then understand the meaning of the question.

**RS2.40:** In your opinion, it means that black pants is bigger than brown clothes….

The probability of black pants and grey clothes is ¼ from this (pointing to their work), brown clothes is 1/5. So, it shows big probability when Mr. Wari wears grey clothes and black pants. It has a comparison of ¼ and 1/5.

**P42:** What does it mean?

**RS2.42:** ¼ is bigger. So? Grey clothes and black pants is chosen.
The explanation given by subject 2 (S₂) stated that subject knew the meaning of the question given. Subject 2 (S₂) was not able to arrange the two-stage experiment result completely. However, subject 2 (S₂) is capable of determining numerical probability in the probability of an event for one-stage experiment. Subject 2 (S₂) can list a complete set of outcomes for a two-stage experiment without good understanding on the concept of probability an event in the two-stage experiment.

Based on the analysis result of students’ task and interview, it can be concluded that S₁ can enroll the complete result experiment of one and two levels, use generative energy to enroll the complete result experiment of three levels, determine the probability size using numeric and compare two events of probability even it still experiences the difficulty. It means that S₁ students are on the semi-numeric level. It is same as the indicators of level of semi-numeric probabilistic thinking [17].

The analysis result of subject 2 (S₂) for the material of sample space is subject can arrange the member of the sample space of one-stage experiment, it also sometimes arranges the member of the sample space of two-stage experiment completely. Subject 2 (S₂) can determine the probability of an event from one-stage experiment even it is not too correct. Subjects 2 (S₂) can compare the event probability from one-stage experiment even they still involve subjective reasons. That characteristic indicates that subject 2 (S₂) is on the subjective level. It is appropriate with the level indicator of probabilistic thinking of level 2 (subjective) [9, 10, 17].

In this research, the indicator is used to investigate the level of students’ probabilistic thinking in junior high school. Level indicator of probabilistic thinking is adapted from the material in junior high school in Indonesia. It is chosen because of cultural factors that influence the level of students’ probabilistic thinking. The research shows that teachers do not give high attention in the importance of level of students' probabilistic thinking, especially in the material of probability.

Level of probabilistic thinking shows hierarchical thinking. Level of probabilistic thinking divides the level into subjective, transitional, informal quantitative, and numeric [9, 10]. Meanwhile, level of probabilistic thinking also divided the level into pre-subjective, subjective, transitional, informal quantitative, semi-numeric and numeric [17]. Level indicator of probabilistic thinking can be used as an alternative way to teach the material of probability to the students. It can be used as a criteria to score how far the students’ understanding toward probability concept.

In pre-subjective level, students do not tend to give answers. They just try to answer as they can even it is wrong. In subjective level, students use a thinking way related to subjective reason. In transitional level, it is a transitional period between subjective and quantitative thinking. In informal quantitative level, students use quantitative thinking informally. In semi-numeric level, students start to use intellectual activity numerically, even it is limited. In numeric level, students have taught numerically. It means that students are able to apply and use the concept of probability in probabilistic problem. It can be found that in this research, on the level of informal quantitative, arranging all result of two/ three-stage experiment uses another strategy that is not structural.

In the result of this research, subject has not fulfilled the indicator of numeric level [9, 10]. Numeric level is the highest level of probabilistic thinking. The indicators are shown on the Table 1.

| Construct              | Indicator                                                                 |
|------------------------|---------------------------------------------------------------------------|
| Sample space           | Adopts and applies a generative strategy that enables a complete listing of the outcomes for two or three-stage. |
| Probability of an event| Predicts most/least likely events for single-stage experiment.            |
|                        | Assigns a numerical probability to an event (either a real probability or a form of odds). |
| Probability comparison | Assigns numerical probability measures and compares events.                |
|                        | Incorporates noncontiguous and contiguous outcomes in determining probabilities. |
|                        | Assigns equal numerical probabilities to equally likely events.            |
Based on the result of the level of students’ probabilistic thinking in junior high school, it has been known that they have not been able to achieve the highest level. The highest level of probabilistic thinking is that numeric can be reached by the students if they understand the probability concept from statistic experiment, event, sample space, probability of an event and probability comparison of one-stage, two-stage and three-stage experiment. Besides that, probability concept can also be applied by students in daily life. This case is caused by some factors like students are less preparation in facing the test.

Nevertheless, it needs a learning plan that can optimize the level of students’ probabilistic thinking. Learning probability can be conducted through experiment learning, using contextual problem started by teaching related events and using software or computer to give understanding on probability concept. Learning probability can also be conducted through various representation and model such as table, tree diagram, pipe diagram, Venn diagram, area model, formula and etc [13].

4. Conclusion
This paper we have discussed the investigation findings as follow. Level of students’ probabilistic thinking is the level of students’ thinking in responding probability problem. It is a problem that involves certainty or points an activity or random experiment with various results. The results might happen however the actual result cannot be previously determined. Second, level of students’ probabilistic is started from subjective level to numeric. Level of students’ probabilistic thinking in junior high school has not reached the numeric level yet. It is caused by some factors. One of them is the learning process.

In teaching probability material, it must also focus on the process of students’ probability thinking. The learning strategy is started by doing random experiment, possible and impossible events, and certain event. Experiment must also be conducted to generalize it. If the number of experiment is unlimited, it can use technology to make it attractive in understanding concept. In addition, teachers must also understand the probability concept well.

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