Massive Schwinger Model for Small Fermion Mass

J.P. Vary and T.J. Fields
Physics Department, Iowa State University of Science and Technology
Ames, IA 50011

H.J. Pirner
Inst. f. Theoret. Physik, Univ. Heidelberg,
Philosophenweg 19, D-69120 Heidelberg, Germany

Abstract
We examine QED\(_{1+1}\) (the massive Schwinger model) to extend existing perturbation results in \(m_f/m_b\), and compare our results with lattice and DLCQ calculations.

1. Introduction
We start with the standard Lagrangian of QED, and work in a coordinate system that is ‘near’ the light front\(^1,2\):

\[
x^+ = \frac{1}{\sqrt{2}} \left[ \left( 1 + \frac{\epsilon}{L} \right) x^0 + \left( 1 - \frac{\epsilon}{L} \right) x^1 \right]
\]
\[
x^- = \frac{1}{\sqrt{2}} (x^0 - x^1)
\]

We interpret \(x^+\) as our time variable, and \(x^-\) as our spatial variable. We will discretize the problem by putting it in a box \(x^- = [0, L]\) and require periodic boundary conditions. Note that with this choice of coordinate system, the two ends of our box are separated by a spacelike separation \(ds^2 = -2\epsilon L\), and no conflict with causality arises by imposing our boundary conditions. In other words, it allows us to specify our initial conditions on the spacelike surface \(x^+ = 0\) (fig. \[\square\]), while allowing the recovery of the ‘usual’ light front variables by letting \(\epsilon \to 0\) for a fixed \(L\).

\(^1\)Based on a talk presented by J.P. Vary at “Theory of Hadrons and Light-Front QCD”, Polona Zgorzelisko, Poland, August 1994.
In ‘pure’ light front coordinates, the negative momentum states are eliminated because, as the dispersion relation

\[ p_+ = \frac{m^2}{2p_-} \]  

(3)

shows, a negative momentum state is a state of negative energy. In our coordinates, we can no longer neglect the negative momentum states. The dispersion relation now reads,

\[ p_+ = \frac{L}{2\epsilon} \left( -p_- \pm \sqrt{(p_-)^2 + \frac{2m^2\epsilon}{L}} \right) \]  

(4)

showing that negative momentum modes can contribute to positive energy states.

As the axial gauge \( A_- = 0 \) is inconsistent with boundary conditions on this finite interval, we pick the light front Coulomb gauge \( \partial_- A_- = 0 \), which poses no such problems. The need to restrict the problem to the charge zero sector is well known, and we choose a heat-kernal regularization for the problem, which is designed to be gauge invariant.

We can completely diagonalize the Hamiltonian in an appropriately defined set of bosonic operators. After manipulation to this form, we can set \( m_f = 0 \), and in the continuum limit, we obtain standard Schwinger model results:

- Physical states are non–interacting Schwinger bosons
- Mass of Schwinger boson is \( m_b^{(0)} = \frac{g}{\sqrt{\pi}} \).
- \( \theta \) vacuum with condensate

\[ \langle \theta | \bar{\psi} \psi | \theta \rangle = \frac{1}{2\pi} m_b^{(0)} e^{-} \cos \theta \]

where \( e \) is Euler’s constant \((e = 0.5772\ldots)\).
2. Mass Perturbation

The idea is to simply let the quark have a small mass $m_f$, and treat the mass term in the Hamiltonian as a perturbation. Therefore the perturbation will be in dimensionless units of $(m_f/g)$ and hence the region of small mass directly corresponds to the region of strong coupling.

Using our bosonized Hamiltonian, we recover known first order results for the change in the mass of the Schwinger boson\(^1\):

\[
(\delta m_b^2)^{(1)} = -4\pi m_f \langle \theta | \bar{\psi} \psi | \theta \rangle = -2m_fm_b^{(0)} e^c \cos \theta
\]

In second order, we obtain new results. We can write our second order series for $\delta m_b^2$ in diagrammatic form (fig. 2). There are a few important things to note in this expansion

- Each term in the expansion is finite.
- Each row, summed separately, diverges.
- Divergences cancel when all diagrams are summed.
- High Fock states are very important: all the diagrams shown explicitly in figure 2 contribute only 40\% to the total sum.

The non-vanishing contributions of the diagrams from the lower row of the figure is a direct consequence of the presence of negative momentum states in our choice of coordinate system\(^3,4\).

Through this evaluation, we identify an expansion parameter:

\[
\beta = 2e^c \frac{m_f}{m_b^{(0)}} \propto m_f \langle \theta | \bar{\psi} \psi | \theta \rangle
\]

with which we can write the results through second order as follows:

\[
m_b^2 = \left( m_b^{(0)} \right)^2 \left[ 1 + \beta + \frac{A}{2} \beta^2 \right]^2
\]
Figure 3: The mass of the Schwinger boson: Our first and second order results, along with lattice and DLCQ calculations.

with A defined by:

\[ A = - \int_{0}^{\infty} \rho \, d\rho \left[ \{ \sinh 2K_0(\rho) - 2K_0(\rho) \} I_0(\rho) + 1 - \cosh 2K_0(\rho) \} \right]. \]  

(8)

As potentially nasty as this integrand looks (The modified Bessel functions \( K_0(\rho) \) and \( I_0(\rho) \) diverge as \( \rho \to 0 \), and as \( \rho \to \infty \), respectively), it is actually well behaved, and \( A \) can be evaluated numerically to be 0.5339 . . ..

A plot of this second order result, our first order result, a lattice result\(^5\), and a DLCQ result\(^6\) is shown in figure 3. There are several things to notice about the plot.

- Our results through second order match the lattice results for an intermediate range of \( m_f/m_b^{(0)} \), and we extend the lattice results to stronger coupling (lighter mass).
- DLCQ misses the linear contribution to the mass: the leading term in a DLCQ calculation is \( \propto m_f^2 \).

3. Structure Functions

Our structure functions will depend on a parameter \( K \):

\[ K = \frac{p-}{m_b(\epsilon/L)^{2}} \]  

(9)

This parameter fixes our frame, and we are interested in results as \( K \to \infty \). There are two ways we could obtain this limit:
• $p_\pm \to \infty$. This is reminiscent of an infinite momentum frame scheme.

• $\epsilon / L \to 0$. This reduces our coordinates to the usual light front coordinates.

In this way, the parameter $K$ forms an useful bridge between the infinite momentum frame method and light front coordinates.

At the moment, the ‘full’ structure function result (with arbitrary numbers of intermediate bosons) seems to be numerically challenging. Calculations to date have concentrated on the lowest 3-boson contribution to the structure functions. Preliminary results show a significantly asymmetric distribution around $x = 0.5$, whereas DLCQ and wave equation approaches (through $qq\bar{q}\bar{q}$ configurations) show symmetric distributions. Calculations are in progress to obtain the entire summed series.

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