QED radiative correction to spin-density matrix elements in exclusive vector meson production

I. Akushevich and P. Kuzhir

National Center of Particle and High Energy Physics, Bogdanovich str. 153, 220040 Minsk, Belarus

Abstract

QED radiative effects are considered in the case of measurement of spin-density matrix elements of diffractive $\rho$-meson electroproduction. Large radiative correction for $r_{00}^5$ is found in the kinematics of collider experiments at HERA.

The extension of the kinematical region of lepton-nucleon deep inelastic scattering to the domain of the diffractive processes provides hadronic nature of the photon to be studied along with the nucleon structure. Particular emphasis has been placed on the case of exclusive vector meson production

$$e(k_1) + p(p) \rightarrow e(k_2) + p(p') + \rho(p_{V}), \quad \rho \rightarrow \pi^+(p_+) + \pi^-(p_-).$$

The reason is the process (1) can be viewed as an off-diagonal Compton scattering analytically continued in the virtuality of the photon $\gamma^*$ to the vector meson mass $\gamma^* p \rightarrow V p$ and gives access to the whole set of the corresponding helicity amplitudes.

The process (1) is analyzed experimentally by means of spin-density matrix elements. When measured, they give an indication of vector meson internal constituents motion and its spin-angular structure. The angular distribution of unpolarized vector meson decay is parameterized by fifteen matrix elements $r_{ij}^{\alpha}, r_{ij}^{\alpha\beta}$. For a long time it was believed, that their behavior complies with the s-channel helicity conservation (SCHC) hypothesis, which means that the helicity of the virtual photon is conserved in the s-channel process $\gamma^* p \rightarrow \rho p$. In this case ten matrix elements (which corresponds to the case when photon and vector meson have different helicities) are equal to zero. But in the recent measurements $r_{00}^5$ has been observed to be non zero [1–3], what has been considered as an indication to SCHC violation.
The procedure of the experimental data analysis is based on the correlation of the lepton scattering, vector meson production and decay planes, which are affected by the radiative corrections (RC). Hence it is topical to look at whether the measured $r^\alpha_0$ can, at least partly, be the result that RC coming from non-observed QED effects and real photon emission was underestimated. In any case in order to make the data processing of the corresponding experiments [1–3] to be consistent, RC should be taken into account.

Following the analysis of K. Schilling and G. Wolf [4] one reconstructs $r^\alpha_{ij}$, $r^\alpha\beta_{ij}$ through vector meson decay angular distribution $W(\cos\theta, \phi, \Phi)$ and the weight coefficients $F_{ij}(\cos\theta, \phi, \Phi)$ (see Appendix C of [4]), the observed matrix elements $r^{obs}_{ij}$ can be written as:

$$r^{obs}_{ij} = \frac{\int_0^{2\pi} d\Phi \int_{-1}^{+1} d\cos\theta \int_0^{2\pi} d\phi \ W(\cos\theta, \phi, \Phi)F_{ij}(\cos\theta, \phi, \Phi)(1 + \delta)}{\int_0^{2\pi} d\Phi \int_{-1}^{+1} d\cos\theta \int_0^{2\pi} d\phi \ W(\cos\theta, \phi, \Phi)(1 + \delta)}.$$  (2)

Here $\Phi$ is the angle between the lepton scattering plane and $\rho$–production plane, $\phi$ is the angle between $\rho$–decay and production plane, $\theta$ is the polar angle of the direction of flight of the positive decay pion. $\delta$ is RC obtained as the ratio of next to the lowest order cross section of the process (1) to the Born cross section.

The RC in the case that vector meson in the final state is considered as a stable particle was calculated in [5] and can be presented in the form

$$\delta = e^{\delta_{inf}} (1 + \delta_{VR} + \delta_{vac}) + \frac{\sigma_F}{\sigma_0}. $$  (3)

Recall that $\sigma_0$ is the Born cross section, $\delta_{vac}$ comes from the effects of vacuum polarization by leptons and hadrons, the sum of $\delta_{VR}$ and $\delta_{inf}$ originates from contributions of vertex function and soft photon emission (the exponent is due to the multiple soft photon radiation), and $\sigma_F$ appears for the hard photon emission.

Let us discuss the angular dependence of RC (3). We remind that the angle $\Phi$ is described by the vector meson momentum and nonmeasured vector $\vec{q}$ (which is determined by measured momenta of initial and scattered leptons). It is clear that the radiation of unobserved real photon changes the vector $\vec{q}$ into $\vec{q} - \vec{k}$ ($\vec{k}$ is the real photon momentum) leading in fact to reorientation of production and scattering planes and, therefore the hard photon contribution $\sigma_F$ to RC can be significantly dependent on $\Phi$. The quantities $\delta_{VR}$ and $\delta_{inf}$ depend only weakly on $\Phi$. We note that this dependence has kinematical origin. It means that if the integration region over photon variable is divided into soft and hard
parts\(^1\), the splitting parameter could be chosen in such a way that mentioned dependence would be completely reduced.

Strictly speaking, \(\delta\) was found in [5] as the ratio of four-fold cross sections \(d^4\sigma/dxdydtd\Phi\). It can be shown, however, that in our case, when \(\rho\)-meson decays into \(\pi^+\pi^-\) and RC is denoted by a ratio of seven-fold cross sections, the results of [5] can be applied unchanged. All one has to do is to show that momenta of \(\pi^\pm\) mesons are appear in the correction (3) only as \(p^+ + p^- = p_V\), but not separately. In this case there would be no any scalar products of the four momenta of pions which can produce dependence on \(\cos\theta\) and \(\phi\).

Really, \(\delta_{\text{vac}}\) and infrared finite part of vertex function contributed to \(\delta_{VR}\) are determined by \(Q^2\) only. Other contributions to \(\delta'\)s in (3) being the result of the infrared divergence cancellation can depend along with the kinematical variables \(Q^2, W^2, t\) also on scalar products of vector \(\Lambda\) (\(\Lambda = p' + k = k_1 - k_2 - p_+ - p_-\) is the four-momentum of the system of unobserved particles): \(\Lambda k_1, \Lambda k_2, \Lambda^2\), and therefore only on \(\Phi\). The real photon phase space is also specified by \(\Lambda\); but not \(p^+\) or \(p^-\). And at last if the natural assumption [7] has been done that all structure functions excepting \(\sigma_L, \sigma_T\) vanish in the hadronic tensor, the hard photon contribution \(\sigma^F\) to RC is also found to be free of \(\cos\theta, \phi\)-dependence. As a result, RC depends only on \(\Phi\) and is consequently reduced to one calculated in [5].

Then by definition (2) the QED corrections \((\Delta r = r_{\text{obs}} - r_{\text{Born}})\) to the matrix elements are

\[
\begin{align*}
\Delta r_{00}^{04} &= -\epsilon I_2 r_{00}^1 + a I_1 r_{00}^5, \\
\Delta \text{Re} r_{10}^{04} &= -\epsilon I_2 \text{Re} r_{10}^1 + a I_1 \text{Re} r_{10}^5, \\
\Delta r_{1-1}^{04} &= -\epsilon I_2 r_{1-1}^1 + a I_1 r_{1-1}^5, \\
\Delta r_{10}^1 &= \frac{1}{\epsilon} [-2I_2 r_{00}^{04} + \epsilon I_4 r_{00}^1 - a(I_1 + I_3)r_{00}^5], \\
\Delta r_{11}^1 &= \frac{1}{\epsilon} [I_2 (r_{00}^{04} - 1) + \epsilon I_4 r_{11}^1 - a(I_1 + I_3)r_{11}^5], \\
\Delta \text{Re} r_{10}^1 &= \frac{1}{\epsilon} [-2I_2 \text{Re} r_{10}^{04} + \epsilon I_4 \text{Re} r_{10}^1 - a(I_1 + I_3)\text{Re} r_{10}^5], \\
\Delta r_{1-1}^1 &= \frac{1}{\epsilon} [-2I_2 r_{1-1}^{04} + \epsilon I_4 r_{1-1}^1 - a(I_1 + I_3)r_{1-1}^5].
\end{align*}
\]

\(^1\) In this case we would come to the formulae analogous to ones of traditional approach of Mo and Tsai [6] for deep inelastic scattering.
\( \Delta \text{Im } r_{10}^2 = -I_4 \text{Im } r_{10}^2 + \frac{a}{\epsilon} (I_1 + I_3) \text{Im } r_{10}^6, \)
\( \Delta \text{Im } r_{1-1}^2 = -I_4 \text{Im } r_{1-1}^2 + \frac{a}{\epsilon} (I_1 + I_3) \text{Im } r_{1-1}^6, \)
\( \Delta r_{10}^5 = \frac{1}{a} [2I_1 r_{10}^{04} + aI_2 r_{10}^{05} - \epsilon (I_1 + I_3) r_{10}^1], \)
\( \Delta r_{11}^5 = \frac{1}{a} [I_1 (1 - r_{11}^{04}) + aI_2 r_{11}^5 - \epsilon (I_1 + I_3) r_{11}^1], \)
\( \Delta \text{Re } r_{10}^5 = \frac{1}{a} [I_1 \text{Re } r_{10}^{04} + aI_2 \text{Re } r_{10}^5 - \epsilon (I_1 + I_3) \text{Re } r_{10}^1], \)
\( \Delta r_{1-1}^5 = \frac{1}{a} [I_1 r_{1-1}^{04} + aI_2 r_{1-1}^5 - \epsilon (I_1 + I_3) r_{1-1}^1], \)
\( \Delta \text{Im } r_{10}^6 = -I_2 \text{Im } r_{10}^6 + \frac{\epsilon}{a} (I_1 + I_3) \text{Im } r_{10}^2, \)
\( \Delta \text{Im } r_{1-1}^6 = -I_2 \text{Im } r_{1-1}^6 + \frac{\epsilon}{a} (I_1 + I_3) \text{Im } r_{1-1}^2. \) \hspace{1cm} (4)

Polarization parameter of the virtual photon density matrix \( \epsilon = \frac{1-y^{1-y-y^2/2}}{y} \) is close to 1 at HERA kinematics, \( a = \sqrt{2\epsilon(1+\epsilon)}, \)
\[ I_n = \int_0^{2\pi} \frac{d\Phi}{2\pi} \cos n\Phi \frac{\delta(\Phi)}{1+\delta(\Phi)}, \quad n = 0, \ldots, 4. \] \hspace{1cm} (5)

Thus the absolute radiative correction to spin-density matrix elements is linear in the lowest order of \( r_{ij}^\alpha, r_{ij}^{\alpha,\beta}, \) and the dependence on \( \delta = \delta(\Phi) \) is included in the coefficients \( I_n. \)

It is clear, that Born matrix elements can be easily extracted from formulae (4) without using any model for \( r' \)s. For realistic radiative correction procedure a system of equations (4) can be solved by the simplest and traditional way: to perform the iteration procedure, where extracted matrix element at the \( n \) step is calculated via \( r' \)s estimated at the \( n-1 \) step as
\[ r_{i ext}^{(n)} = r_{obs} - \Delta r(r_{i ext}^{(n-1)}). \] \hspace{1cm} (6)

Note that the value of RC \( \Delta r \) is expected to be small in respect to correction to the cross section. The reason is only contributions of higher harmonics \( I_n \) survive, but the large contribution of \( I_0 \) vanishes. The quantities \( I_{1,2,3,4} \) are shown in Figure 1. \( I_1 \) is of order 1-2\%, \( I_2 \) is less then 1\% while \( I_{3,4} \) are
Fig. 1. The dependence of $I_n, n = 1 \ldots 4$ on $Q^2$ under the kinematical conditions of H1/ZEUS experiments: $\sqrt{s} = 300$ GeV, $W = 75$ GeV.

Fig. 2. The relative RC $\delta r$ to non-zeroth in accordance with SCHC matrix elements under the kinematical conditions of H1/ZEUS experiments. Practically negligible in the considered kinematical region. It follows that only those $\Delta r$ would be significantly different from zero, which are proportional to non-vanished matrix elements with relatively large coefficient $I_1$.

In order to treat the radiative effects numerically the model [8] reasonably reproduced experimental data has been attracted. We estimate the relative RC $\delta r = \Delta r/r$ for those matrix elements, which should according to SCHC be non-zeroth. We found (see Figure 2) that in kinematics of experiments at HERA $\delta r$ do not exceed 1%. Let us stress, that if SCHC is true, $\delta r_{04}^{04}$ will be identically equal to zero, since it is proportional to zeroth (by SCHC) matrix elements $r_{00}^1, r_{00}^5$.

For the majority of the matrix elements vanishing in the SCHC limit, radiative corrections turn out to be not greater then 1%. However there are two of them, Re $r_{10}^{04}$ and $r_{00}^5$, which RC appears to be substantial (see Figure 3). One can see, that corrections $\Delta \text{Re } r_{10}^{04}$ and $\Delta r_{00}^5$ may reach $\sim 20\%$.

The last result is interesting from point of view of the found SCHC violation: the radiative correction procedure reduces the observed effect.

As an illustration let us follow the origin of the radiative effect within the experimental data processing. $r_{00}^5$ is defined experimentally by fitting of vector meson decay $\Phi$ distribution ($W(\cos \theta, \phi, \Phi)$ integrated over $\cos \theta, \phi$)

$$W(\Phi) \sim 1 - \epsilon \cos 2\Phi (2r_{11}^1 + r_{00}^1) + a \cos \Phi (2r_{11}^5 + r_{00}^5).$$ (7)
Fig. 3. The dependence of Born (dashed line) and radiative corrected (solid line) spin-density matrix elements on $Q^2$ under the kinematical conditions of H1/ZEUS experiments.

Fig. 4. Vector meson decay $\Phi$–distribution: SCHC curve (straight line), experimental curve [2] (solid line) and radiative corrected theoretical curve (dashed line).

Based on SCHC hypothesis this distribution would be flat, what corresponds to zeroth $r_{00}^{r5}$. But this is true only for matrix elements in the lowest order of QED. It can be seen (Figure 4, see also [9]), that the theoretical radiative corrected $\Phi$–distribution deviates from flat, and has the form similar to the experimental distribution. Thus the observed effect comes not only from SCHC violation but due to radiative corrections as well.

It follows that if RC procedure is included in the data processing, it would lead (as one can see from Figures 3 and 4) to reducing of the found effect of SCHC violation almost on 20%.
We are grateful to A.B.Borissov, N.N.Nikolaev, A.S.Proskuryakov, who took their time to read this paper attentively and discuss it with us. Their advices and suggestions were very helpful for us.

References

[1] J. Breitweg et al. [ZEUS Collaboration], Eur. Phys. J. C6 (1999) 603

[2] C. Adloff et al. [H1 Collaboration], hep-ex/9902019.

[3] J. Breitweg et al. [ZEUS Collaboration], hep-ex/9908026.

[4] K.Schilling, G.Wolf, Nucl. Phys. B61 (1973) 381

[5] I. Akushevich, Eur. Phys. J. C8 (1999) 457

[6] L. Mo, Y. Tsai Rev. Mod. Phys. 41 (1969) 205

[7] M. Ryskin, Z.Phys. C 57 (1993) 89

[8] I.P. Ivanov and N.N. Nikolaev, JETP Lett. 69 (1999) 294
   E.V. Kuraev, N.N. Nikolaev and B.G. Zakharov, JETP Lett. 68 (1998) 696
   I.Akushevich, I.Ivanov, N.N.Nikolaev and A.Pronyaev, private communication.

[9] I. Akushevich, hep-ph/9906410 In: Hamburg 1998/1999, "Monte Carlo generators for HERA physics". p.547