Logarithmic $O(\alpha_s^3)$ contributions to the DIS Heavy Flavor Wilson Coefficients at $Q^2 \gg m^2$

Isabella Bierenbaum$^*$
Instituto de Física Corpuscular, CSIC-Universitat de València, Apartado de Correos 22085, E–46071 Valencia, Spain
E-mail: Isabella.Bierenbaum@ific.uv.es

Johannes Blümlein$^{†‡}$
Deutsches Elektronen–Synchrotron, DESY, Platanenallee 6, D–15638 Zeuthen, Germany
E-mail: Johannes.Bluemlein@desy.de

Sebastian Klein$§$
Institut für theoretische Teilchenphysik und Kosmologie, RWTH Aachen University, D–52056 Aachen, Germany
E-mail: sklein@physik.rwth-aachen.de

The logarithmic contributions to the massive twist-2 operator matrix elements for deep-inelastic scattering are calculated to $O(\alpha_s^3)$ for general values of the Mellin variable $N$.

XVIII International Workshop on Deep-Inelastic Scattering and Related Subjects, DIS 2010
April 19-23, 2010
Firenze, Italy

$^*$Supported in part by Generalitat Valenciana Grant No. PROMETEO/2008/69.
$^†$Speaker.
$^‡$Supported in part by SFB-TR/9 and EU TMR network HEPTOOLS.
$§$Supported in part by SFB-TR/9.
1. Introduction

The heavy flavor contributions to deep-inelastic scattering (DIS) are rather large in the small $x$ region. With the current DIS data a precision of $\sim 1\%$ is reached [1] for $F_2(x, Q^2)$. This requires to describe the heavy flavor corrections to 3-loop order, to perform a consistent next-to-next-to leading order (NNLO) analysis, to measure the strong coupling constant $\alpha_s(M_Z^2)$ and the unpolarized twist-2 parton distribution functions at highest precision possible, cf. [2]. In the region $Q^2/m^2 \geq 10$ one may compute all contributions except the power suppressed terms, $\propto (m^2/Q^2)^k, k \geq 1$ using the factorization theorem given in Ref. [3]. Here, the massive Wilson coefficients factorize into massive operator matrix elements (OMEs), $A_{ij}$, and the massless Wilson coefficients, which are known to $O(\alpha_s^3)$ [4]. The $O(\alpha_s^3)$ Mellin moments of the massive operator matrix elements up to $N = 10...14$, depending on the process, were computed in [5]. The calculation was performed relating the moments of the massive operator matrix elements to massive tadpoles and using MATAD [4]. In Ref. [5] also the complete renormalization for a single massive quark has been derived. Different other contributions, needed in the renormalization process, were computed at general values of $N$ in Refs. [6]. For the structure function $F_L(x, Q^2)$ the asymptotic corrections to $O(\alpha_s^3)$ are known for general values of $N$ [8]. They are, however, only applicable at scales $Q^2/m^2 \geq 800$. For transversity the matrix elements were computed for general $N$ at 2-loop order and a series of moments at 3-loop order in [9]. Very recently, the general $N$ results at $O(\alpha_s^3)$ for $F_2(x, Q^2)$ for the color coefficients $\propto n_f$ have been completed [10, 11]. These computations use modern summation technologies encoded in the package SİGMA [12] and the results can be expressed in terms of nested harmonic sums [13]. From the single pole terms in the massive computations of Refs. [5, 6, 11, 14] the corresponding contributions to the 3–loop anomalous dimensions were derived, either for the respective moments [14], or at general values of $N$ [15].

In this note we report on the logarithmic $O(\alpha_s^3)$ contributions to the massive operator matrix elements, cf. also [17]. They are known for general values of $N$ and depend on the 3-loop anomalous dimensions and massive OMEs up to $O(\alpha_s^4)$.

2. The heavy flavor Wilson coefficients in the asymptotic region

The heavy flavor correction to the structure function $F_2(x, Q^2)$ with $n_f$ massless and one heavy flavor reads, [5]:

$$F_Q^{(2, L)}(x, n_f, Q^2, m^2) = \sum_{k=1}^{n_f} e_k^2 \left\{ L_{q, (2, L)}^{\text{NS}} \left( x, n_f + 1, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2} \right) \otimes \left[ f_k(x, \mu^2, n_f) + f_{\bar{k}}(x, \mu^2, n_f) \right] + \frac{1}{n_f} L_{q, (2, L)}^{\text{PS}} \left( x, n_f + 1, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2} \right) \otimes \left[ \Sigma(x, \mu^2, n_f) + L_{g, (2, L)}^S \left( x, n_f + 1, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2} \right) \otimes G(x, \mu^2, n_f) \right] \right\}$$

$$+ e_k^2 \left[ H_{q, (2, L)}^{\text{PS}} \left( x, n_f + 1, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2} \right) \otimes \left[ \Sigma(x, \mu^2, n_f) + H_{g, (2, L)}^S \left( x, n_f + 1, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2} \right) \otimes G(x, \mu^2, n_f) \right] \right],$$

(2.1)

with boundaries for the Mellin integral $[x(1+4m^2/Q^2), 1]$, and $e_i$ the quark charges. Here the different Wilson coefficients are denoted by $L_i, H_i$ in case the photon couples to a light (L) or the
heavy (H) quark. For \( Q^2 \gg m^2 \) they can be expressed in terms of the massive OMEs \( A_{ij} \) and the massless Wilson coefficients \( C_j \). To \( O(a_s^3) \) they read (\( a_s = \alpha_s/(4\pi) \))

\[
\begin{align*}
L_{q,(2L)}^{NS}(n_f+1) &= a_s^2 \left[ A_{gqQ,q,(2L)}^{(2),NS}(n_f+1) \delta_2 + \tilde{C}^{(2),NS}_{gqQ,q,(2L)}(n_f) \right] \\
&\quad + a_s^3 \left[ A_{gqQ,q,(2L)}^{(3),NS}(n_f+1) \delta_2 + A_{gqQ,q,(2L)}^{(2),NS}(n_f+1) C_{gqQ,q,(2L)}^{(1),NS}(n_f+1) + \tilde{C}^{(3),NS}_{gqQ,q,(2L)}(n_f) \right] \\
L_{\bar{g},(2L)}^{PS}(n_f+1) &= a_s^2 \left[ A_{gqQ,q,(2L)}^{(3),PS}(n_f+1) \delta_2 + A_{gqQ,q,(2L)}^{(2),PS}(n_f) C_{gqQ,q,(2L)}^{(1),PS}(n_f+1) + \tilde{C}^{(3),PS}_{gqQ,q,(2L)}(n_f) \right] \\
&\quad + a_s^3 \left[ A_{gqQ,q,(2L)}^{(3),PS}(n_f+1) \delta_2 + A_{gqQ,q,(2L)}^{(2),PS}(n_f+1) C_{gqQ,q,(2L)}^{(1),PS}(n_f+1) + \tilde{C}^{(3),PS}_{gqQ,q,(2L)}(n_f) \right],
\end{align*}
\]

\[
H_{q,(2L)}^{PS}(n_f+1) = a_s^2 \left[ A_{gqQ,q,(2L)}^{(1),PS}(n_f+1) \delta_2 + \tilde{C}^{(1),PS}_{gqQ,q,(2L)}(n_f+1) \right] + a_s^3 \left[ A_{gqQ,q,(2L)}^{(3),PS}(n_f+1) \delta_2 + A_{gqQ,q,(2L)}^{(2),PS}(n_f+1) C_{gqQ,q,(2L)}^{(1),PS}(n_f+1) + \tilde{C}^{(3),PS}_{gqQ,q,(2L)}(n_f+1) \right],
\]

\[
H_{\bar{g},(2L)}^{PS}(n_f+1) = a_s \left[ A_{gqQ,q,(2L)}^{(1),PS}(n_f+1) \delta_2 + \tilde{C}^{(1),PS}_{gqQ,q,(2L)}(n_f+1) \right] + a_s^2 \left[ A_{gqQ,q,(2L)}^{(2),PS}(n_f+1) \delta_2 + A_{gqQ,q,(2L)}^{(1),PS}(n_f+1) C_{gqQ,q,(2L)}^{(1),PS}(n_f+1) + \tilde{C}^{(3),PS}_{gqQ,q,(2L)}(n_f+1) \right] + a_s^3 \left[ A_{gqQ,q,(2L)}^{(3),PS}(n_f+1) \delta_2 + A_{gqQ,q,(2L)}^{(2),PS}(n_f+1) C_{gqQ,q,(2L)}^{(1),PS}(n_f+1) + \tilde{C}^{(3),PS}_{gqQ,q,(2L)}(n_f+1) \right],
\]

with \( \delta_2 = 0(1) \) for \( F_L(F_2) \) and \( \hat{f}(n_f) = f(n_f+1) - f(n_f), \hat{f}(n_f) = f(n_f)/n_f \). The massive OMEs depend on the ratio \( m^2/\mu^2 \), while the scale ratio of the massless Wilson coefficients is \( \mu^2/Q^2 \). The latter are pure functions of the momentum fraction \( z \), or the Mellin variable \( N \), if one sets \( \mu^2 = Q^2 \). The massive OMEs obey then the general structure

\[
A_{ij}^{(2)} \left( \frac{m^2}{Q^2} \right) = a_{ij}^{(2)} \ln^2 \left( \frac{m^2}{Q^2} \right) + a_{ij}^{(1)} \ln \left( \frac{m^2}{Q^2} \right) + a_{ij}^{(0)}.
\]

3. The matrix element \( A_{Q_S}^{(3)}(N) \)

In the following we present, as an example, the logarithmic expansion coefficients of Eq. (2.3), \( a_{Q_S}^{(3),k}, k \geq 1 \), for the massive OME \( A_{Q_S}^{(3)}(N) \) in the \( \overline{\text{MS}} \)-scheme. They are given by:

\[
a_{Q_S}^{(3),3} = \frac{8(N^2+N+2)}{9N(N+1)(N+2)} T_F n_f \left[ C_F \left( \frac{P_1}{(N-1)N^2(N+1)^2(N+2)} - 4S_1 \right) + C_A \left( 4S_1 - \frac{8(N^2+N+1)}{(N-1)N(N+1)(N+2)} \right) \right] - 8T_F^2 + C_A^2 \left( \frac{(11N^4 + 22N^3 - 59N^2 - 70N - 48)S_1}{(N-1)N(N+1)(N+2)} \right).
\]
\begin{equation}
-12S_1^2 + \frac{2(N^2 + N + 1)(11N^4 + 22N^3 - 35N^2 - 46N - 24)}{(N - 1)^2N^2(N + 1)^2(N + 2)^2} + C_A T_F \left( -\frac{56(N^2 + N + 1)}{(N - 1)N(N + 1)(N + 2)} \right) + C_A T_F \left( -\frac{56(N^2 + N + 1)}{(N - 1)N(N + 1)(N + 2)} \right)
\end{equation}

\begin{equation}
\alpha_{Qg}^{(3),1} = \frac{n_f}{2} T_F^2 n_f \left( C_F \left( -\frac{4(N^2 + N + 2)}{3N(N + 1)(N + 2)} \left( S_2^1 + S_2^2 \right) + \frac{8(5N^3 + 8N^2 + 19N + 6)}{9N^2(N + 1)(N + 2)} S_1 \right) + \frac{8(5N^4 + 20N^3 + 47N^2 + 58N + 20)}{9N(N + 1)^2(N + 2)^3} S_1 \right) + \frac{8(5N^4 + 20N^3 - N^2 - 14N + 20)}{9N(N + 1)^2(N + 2)^2} S_1 \right) + \frac{8(5N^4 + 20N^3 - N^2 - 14N + 20)}{9N(N + 1)^2(N + 2)^2} S_1 \right) + \frac{8(5N^4 + 20N^3 + 47N^2 + 58N + 20)}{9N(N + 1)^2(N + 2)^3} S_1 \right) \right)
\end{equation}

\begin{equation}
\alpha_{Qg}^{(3),2} = 4T_F^2 n_f \left( C_F \left( -\frac{4(N^2 + N + 2)}{3N(N + 1)(N + 2)} \left( S_2^1 + S_2^2 \right) + \frac{8(5N^3 + 8N^2 + 19N + 6)}{9N^2(N + 1)(N + 2)} S_1 \right) + \frac{8(5N^4 + 20N^3 + 47N^2 + 58N + 20)}{9N(N + 1)^2(N + 2)^3} S_1 \right) + \frac{8(5N^4 + 20N^3 - N^2 - 14N + 20)}{9N(N + 1)^2(N + 2)^2} S_1 \right) + \frac{8(5N^4 + 20N^3 + 47N^2 + 58N + 20)}{9N(N + 1)^2(N + 2)^3} S_1 \right) \right)
\end{equation}

\begin{equation}
\alpha_{Qg}^{(3),3} = \frac{1}{2} \frac{n_f}{2} T_F^2 \left( C_F \left( -\frac{4(N^2 + N + 2)}{3N(N + 1)(N + 2)} \left( S_2^1 + S_2^2 \right) + \frac{8(5N^3 + 8N^2 + 19N + 6)}{9N^2(N + 1)(N + 2)} S_1 \right) + \frac{8(5N^4 + 20N^3 + 47N^2 + 58N + 20)}{9N(N + 1)^2(N + 2)^3} S_1 \right) + \frac{8(5N^4 + 20N^3 - N^2 - 14N + 20)}{9N(N + 1)^2(N + 2)^2} S_1 \right) + \frac{8(5N^4 + 20N^3 + 47N^2 + 58N + 20)}{9N(N + 1)^2(N + 2)^3} S_1 \right) + \frac{8(5N^4 + 20N^3 - N^2 - 14N + 20)}{9N(N + 1)^2(N + 2)^2} S_1 \right) \right)
\end{equation}
Logarithmic $O(\alpha_s^2)$ contributions

\[
\begin{align*}
&+ \frac{4(N^4 - N^3 - 20N^2 - 10N - 4)S_1}{3N^2(N+1)^2(N+2)} + \frac{2P_{15}}{3(N-1)N^3(N+1)^2(N+2)^2} + \frac{4P_{16}S_2}{3(N-1)N^3(N+1)^3(N+2)^2} \\
&+ C_A \left( \frac{4(N^2 + N + 2)}{9N(N+1)(N+2)} \left( S_1^3 + 9S_2S_1 + 6S_{-3} + 12S_{-2}S_1 + 8S_3 + 12S_{-2,1} \right) - \frac{4P_{27}S_1}{5N(N+1)(N+2)^2} \right) \\
&- \frac{4(N^3 + 8N^2 + 11N + 2)S_1^2}{3(N+1)^2(N+2)^2} + \frac{4P_{18}}{3(N-1)N^4(N+1)^4(N+2)^4} - \frac{4P_{19}S_2}{3(N-1)N^2(N+1)^2(N+2)^2} \\
&+ \frac{16(N^2 - N - 4)S_{-2}}{3(N+1)^2(N+2)^2} + 2C_A^2 T_F \left( \frac{8(N^2 + N + 2)}{3N(N+1)(N+2)} \left( 12S_{-2,1}S_1 - S_1^3 - 9S_2S_1^2 - 8S_3S_1 - 6S_{-3}S_1 \right) - 12S_{-2}S_1^2 \right) - \frac{2P_{20}S_1^3}{9(N-1)N^2(N+1)^2(N+2)^2} + \frac{2P_{21}S_1^2}{3(N-1)N^2(N+1)^2(N+2)^3} \right) \\
&- \frac{2P_{22}S_1}{3(N-1)N^4(N+1)^4(N+2)^4} - \frac{(N-1)N^2(N+1)^2(N+2)^2}{3(N-1)N^3(N+1)^3(N+2)^5} - \frac{2P_{23}S_1^2}{2P_{24}} \\
&+ \frac{4(N^2 + N + 2)(11N^4 + 22N^3 - 35N^2 - 46N - 24)}{9(N-1)N^2(N+1)^2(N+2)^2} \left( 6S_{-2,1} - 4S_3 - 3S_{-3} \right) - \frac{8P_{25}S_2S_1}{3(N-1)N^2(N+1)(N+2)^2} \\
&+ \frac{2(11N^4 + 22N^3 - 35N^2 - 46N - 24)P_{19}S_2}{3(N-1)^2N^3(N+1)^3(N+2)^5} + 4C_A T_F^2 \left( \frac{8(N^2 + N + 2)}{3N(N+1)(N+2)} \left( S_1^3 + 8S_3 - 12S_{-2,1} \right) + \frac{8P_{19}S_2}{3(N-1)N^2(N+1)(N+2)^3} \right) \\
&+ \frac{9S_2S_1 + 12S_{-2}S_1 + 6S_{-3}}{3N^3(N+1)(N+2)^2} - \frac{8(N^3 + 8N^2 + 11N + 2)S_1^2}{3(N-1)N^2(N+1)^2(N+2)^3} + \frac{8P_{19}S_2}{3(N-1)N^2(N+1)(N+2)^3} \right) \\
&+ \frac{2P_{26}}{27(N-1)N^4(N+1)^4(N+2)^4} + \frac{32(N^2 - N - 4)S_{-2}}{3(N+1)^2(N+2)^2} + \frac{4P_{27}S_1}{27N(N+1)^2(N+2)^3} + 2C_A^2 T_F \left( \right) \\
&+ \frac{8(N^2 + N + 2)}{3N(N+1)(N+2)} \left( 4S_1S_1 - S_1^3 - 3S_2S_1^2 \right) + \frac{2(3N^4 + 42N^3 + 107N^2 + 92N + 28)S_1}{3N^2(N+1)^2(N+2)} \\
&+ \frac{2P_{29}S_1^2}{N^3(N+1)^2(N+2)^2} + \frac{2P_{30}S_1}{N^4(N+1)^4(N+2)^4} + \frac{2P_{31}S_1}{N^4(N+1)^4(N+2)^4} \right) + \frac{2(7N^4 + 74N^3 + 79N^2 - 12N - 4)S_{-2}S_1}{N^3(N+1)^2(N+2)^2} \\
&- \frac{8(N^2 + N + 2)(3N^2 + 3N + 2)S_1}{3N^2(N+1)^2(N+2)} - \frac{2(3N^2 + 3N + 2)(N^4 + 17N^3 + 17N^2 - 5N - 2)S_2}{N^3(N+1)^3(N+2)} \\
&- \frac{P_{20}}{N^5(N+1)^3(N+2)} + 4C_F T_F^2 \left( \frac{8(N^2 + N + 2)}{9N(N+1)(N+2)} \left( 4S_1 - S_1^3 - 3S_2S_1 \right) \right) \\
&+ \frac{P_{27}S_1}{3(N-1)N^6(N+1)^5(N+2)^5} + \frac{8(N^4 - N^3 - 20N^2 - 10N - 4)S_1}{3N^2(N+1)^2(N+2)} + 8(N^4 + 17N^3 + 17N^2 - 5N - 2)S_2 \\
&+ \frac{8(3N+2)S_1^2}{3N^3(N+2)} + 2C_F T_F \left( \frac{8(N^2 + N + 2)}{3N(N+1)(N+2)} \left( 2S_1^3 + 12S_2S_1^2 + 4S_1^3 - 12S_{-2,1}S_1 + 6S_{-3}S_1 \right) \\
&+ 12S_{-2}S_1^2 \right) + \frac{4P_{29}S_1^3}{9(N-1)N^2(N+1)^2(N+2)^2} - \frac{4P_{30}S_2}{3(N-1)N^3(N+1)^3(N+2)^3} + \frac{P_{36}}{N^3(N+1)^3(N+2)^2} \\
&- \frac{8P_{37}S_{-2}S_1}{N^2(N+1)^2(N+2)^2} + \frac{4P_{38}S_{-2}}{4P_{39}S_{-2}S_1} + \frac{4P_{34}S_1}{4P_{35}S_{-2}S_1} + \frac{P_{36}}{N^3(N+1)^3(N+2)^2} \\
&- \frac{8(N^2 - N - 4)(3N^2 + 3N + 2)S_{-2}}{N^3(N+1)^3(N+2)^2} - \frac{2P_{38}S_2}{3(N-1)N^4(N+1)^4(N+2)^2}
\end{align*}
\]
Logarithmic $O(\alpha_s^3)$ contributions

\[
- \frac{8(N^2+N+2)(29N^4+58N^3-41N^2-70N-48)S_3}{9(N-1)N^2(N+1)^2(N+2)^2}.
\]

(3.3)

Here, $S_3 \equiv S_3(N)$, $P_k$ denote some polynomials in $N$, cf. [17]; and $\Gamma_i^{(2)}$ are the 3-loop anomalous dimensions. The expansion coefficients given above depend on harmonic sums up weight $w = 3$. Numerical studies show, that within the kinematic region of HERA the constant terms to (2.3) are as important as the logarithmic contributions. Further details will be given in [17].

References

[1] F. D. Aaron et al. [ H1 and ZEUS Collaborations ], JHEP 1001 (2010) 109. [arXiv:0911.0884 [hep-ex]].

[2] S. Alekhin, J. Blümlein and S. Moch, arXiv:1007.3657 [hep-ph];
S. Alekhin, J. Blümlein, S. Klein and S. Moch, Phys. Rev. D 81 (2010) 014032 [arXiv:0908.2766 [hep-ph]];
J. Blümlein, H. Böttcher and A. Guflanti, Nucl. Phys. B 774 (2007) 182 [arXiv:hep-ph/0607200];
Nucl. Phys. Proc. Suppl. 135 (2004) 152 [arXiv:hep-ph/0407089];
M. Glück, E. Reya and C. Schuck, Nucl. Phys. B 754 (2006) 178 [arXiv:hep-ph/0604116];
P. Jimenez-Delgado and E. Reya, Phys. Rev. D 79 (2009) 074023 [arXiv:0810.4274 [hep-ph]];
A. D. Martin, W. J. Stirling, R. S. Thorne and G. Watt, Eur. Phys. J. C 63 (2009) 189
[arXiv:0901.0002 [hep-ph]].

[3] M. Buza, Y. Matiounine, J. Smith, R. Migeron and W. L. van Neerven, Nucl. Phys. B 472 (1996) 611.

[4] J. A. M. Vermaseren, A. Vogt and S. Moch, Nucl. Phys. B 724 (2005) 3.

[5] I. Bierenbaum, J. Blümlein and S. Klein, Nucl. Phys. B 820 (2009) 417.

[6] M. Steinhauser, Comput. Phys. Commun. 134 (2001) 335 [arXiv:hep-ph/0009029].

[7] M. Buza, Y. Matiounine, J. Smith and W. L. van Neerven, Eur. Phys. J. C 1 (1998) 301,
[hep-ph/9612398];
M. Buza, Y. Matiounine, J. Smith and W. L. van Neerven, Nucl. Phys. B485 (1997) 420,
[hep-ph/9608342];
I. Bierenbaum, J. Blümlein and S. Klein, Nucl. Phys. B 780 (2007) 40; [hep-ph/0703285];
Phys. Lett. B 648 (2007) 195, [hep-ph/0702265];
Phys. Lett. B 672 (2009) 401, [hep-ph/0901.0669];
[arXiv:0706.2738 [hep-ph]] and in preparation;
I. Bierenbaum, J. Blümlein, S. Klein and C. Schneider, Nucl. Phys. B 803 (2008) 1,
[hep-ph/0803.0273].

[8] J. Blümlein, A. De Freitas, W. L. van Neerven and S. Klein, Nucl. Phys. B 755 (2006) 272.

[9] J. Blümlein, S. Klein and B. Tödtli, Phys. Rev. D 80 (2009) 094010.

[10] J. Ablinger, J. Blümlein, S. Klein, C. Schneider and F. Wißbrock in preparation;

[11] J. Ablinger, I. Bierenbaum, J. Blümlein, A. Hasselhuhn, S. Klein, C. Schneider and F. Wißbrock,
arXiv:1007.0375 [hep-ph].

[12] C. Schneider, J. Symbolic Comput. 43 (2008) 611, [arXiv:0808.2543v1]; Ann. Comb. 9 (2005) 75; J.
Differ. Equations Appl. 11 (2005) 799; Ann. Comb. (2009) to appear, [arXiv:0808.2596];
Proceedings of the Conference on Motives, Quantum Field Theory, and Pseudodifferential Operators, To appear in
Logarithmic $O(\alpha_s^3)$ contributions

Johannes Blümlein

the Mathematics Clay Proceedings, 2010; Sém. Lothar. Combin. 56 (2007) 1, Article B56b, Habilitationsschrift JKU Linz (2007) and references therein;
J. Ablinger, J. Blümlein, S. Klein and C. Schneider, arXiv:1006.4797 [math-ph].

[13] J. Blümlein and S. Kurth, Phys. Rev. D 60 (1999) 014018 [arXiv:hep-ph/9810241];
J. A. M. Vermaseren, Int. J. Mod. Phys. A 14 (1999) 2037 [arXiv:hep-ph/9806280].

[14] S. A. Larin, T. van Ritbergen and J. A. M. Vermaseren, Nucl. Phys. B 427 (1994) 41;
S. A. Larin, P. Nogueira, T. van Ritbergen and J. A. M. Vermaseren, Nucl. Phys. B 492 (1997) 338;
A. Retey and J. A. M. Vermaseren, Nucl. Phys. B 604 (2001) 281; [hep-ph/0007294];
J. Blümlein and J. A. M. Vermaseren, Phys. Lett. B 606 (2005) 130; [hep-ph/0411111];

[15] J. A. Gracey, Nucl. Phys. B662, 247 (2003) ; Nucl. Phys. B667, 242 (2003) ; JHEP 10, 040 (2006) ;
Phys. Lett. B643, 374 (2006) ;

[16] J. A. Gracey, Phys. Lett. B 322 (1994) 141;
S. Moch, J. A. M. Vermaseren and A. Vogt, Nucl. Phys. B 688 (2004) 101; [hep-ph/0403192].
Nucl. Phys. B 691 (2004) 129; [hep-ph/0404111].

[17] I. Bierenbaum, J. Blümlein, and S. Klein, in preparation.