Constraining the nuclear symmetry-energy at super-density

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The nuclear symmetry-energy has broad implications in both nuclear physics and astrophysics. Due to hard work of many people, the nuclear symmetry-energy around saturation density has been roughly constrained. However, the nuclear symmetry-energy at super-density is still in chaos. By considering both the effects of the nucleon-nucleon short-rang correlations and the isospin-dependent in-medium inelastic baryon-baryon scattering cross sections in the transport model, two unrelated experimental measurements are simultaneously analyzed. A soft symmetry-energy at super-density is first consistently obtained by the double comparison of the symmetry-energy sensitive observables.

PACS numbers: 25.70.-z, 21.65.Cd, 21.65.Mn, 21.65.Ef

I. INTRODUCTION

The symmetry-energy, which describes the single nucleonic energy of nuclei or nuclear matter changes as one replaces protons in a system with neutrons. Besides its impacts in nuclear physics [1, 2], in a density range of 0.1 ~ 10 times nuclear saturation density, the symmetry-energy determines the birth of neutron stars and supernova neutrinos [3], a range of neutron star properties such as cooling rates, the thickness of the crust, the mass-radius relationship, and the moment of inertia [4–7]. The nuclear symmetry-energy also plays crucial role in the evolution of core-collapse supernovae [8] and astrophysical r-process nucleosynthesis [9]. Thus the better we can constrain the symmetry-energy in laboratory measurements, the more we can learn from astroobservations.

To constrain the symmetry-energy in broad density region, besides the studies in astrophysics [10, 11], many terrestrial experiments are being carried out or planned using a wide variety of advanced new facilities, such as the Facility for Rare Isotope Beams (FRIB) in the US, or the Radioactive Isotope Beam Facility (RIBF) in Japan. To unscramble symmetry-energy related experimental data, various isospin-dependent transport models are frequently used to probe the symmetry-energy below and above saturation density [1, 2]. With these efforts, the nuclear symmetry-energy and its slope around saturation density of nuclear matter from 28 analysis of terrestrial nuclear laboratory experiments and astrophysical observations have been roughly pinned down [12], while recent interpretations of experimental measurements by different groups made the symmetry-energy at super-density fall into chaos [13].

Recently, the high-momentum transfer measurements have shown that nucleons in nucleus can form pairs with large relative momentum and small center-of-mass momentum [14–16]. This phenomenon was explained by the short-rang nucleon-nucleon tensor interaction [16, 17]. Such nucleon-nucleon short-range correlations (SRC) in nucleus leads to a high-momentum tail (HMT) in the single-nucleon momentum distribution [18, 19]. More interestingly, in the HMT of nucleon momentum distribution, nucleonic component is evidently isospin-dependent. The number of n-p SRC pairs are about 18 times that of the p-p and n-n SRC pairs [20]. And in neutron-rich nucleus, protons have a greater probability than neutrons to have momentum greater than the nuclear Fermi momentum [21]. Therefore, for a nuclear system, the total kinetic energy caused by the SRC of unlike nucleons can be in principle roughly equal or greater than that due to the ideal Fermi movement.

Unfortunately, effects of the above isospin-dependent SRC was seldom taken into account in most of the currently used isospin-dependent transport models, while the latter have been frequently used to unscramble the symmetry-energy related experimental data [22–27]. In this study, by considering the effects of the isospin-dependent SRC and the important but often-overlooked in-medium baryon-baryon inelastic cross sections in the isospin-dependent transport model, two unrelated experimental measurements are simultaneously analyzed. We first obtained consistent constraints on the symmetry-energy at super-density.

II. THE BUU TRANSPORT MODEL

For our purpose, we use the newly updated isospin-dependent Boltzmann-Uehling-Uhlenbeck (BUU) transport model. In the present version, nucleon-density distribution in initial colliding nuclei is calculated by the Skyrme-Hartree-Fock with Skyrme M* force parameters [28]. Considering the n-p SRC in nucleus [20, 21, 22, 30], the isospin-dependent nucleon momentum distribution can be given by [31]

\[
 n_{\text{proton(neutron)}}(p) = \begin{cases} 
 C_{1, \text{proton(neutron)}} \cdot \frac{A}{22(A^2/N^2)} \cdot \frac{C_2}{p^2}, & p_f \leq p < \lambda p_f; \\
 0, & p > \lambda p_f. 
\end{cases} \tag{1}
\]

\(n(p)\) is nucleon momentum distribution in initial colliding nucleus, \(p_f\) is the nuclear Fermi momentum. \(\lambda = p_{\text{max}}/p_f\), is the factor of maximum momentum of nucleon relative to the nuclear Fermi momentum, i.e., the so-called high-momentum cutoff parameter. We in this study let \(\lambda = 2\) [31]. \(A, Z, N\) are the nuclear mass number, proton number and neutron number, respectively.
energy with different γ parameters.

We use the Skyrme-type parametrization for the mean field, which reads

$$U(\rho) = A(\rho/\rho_0) + B(\rho/\rho_0)^\sigma. \quad (2)$$

Where σ = 1.3, A = -232 MeV accounts for attractive and B = 179 MeV accounts for repulsive. With these choices, the ground-state compressibility coefficient of nuclear matter K = 230 MeV \[33\]. Considering the n-p SRC, we let the kinetic symmetry-energy be -6.71 MeV \[34\], and the symmetry-potential becomes \[35\]

$$U_{\text{sym}}(\rho, \delta) = 38.31(\rho/\rho_0)^\gamma \times [\pm 2\delta + (\gamma - 1)\delta^2], \quad (3)$$

where δ = (ρn - ρp)/ρ is the isospin asymmetry of the medium. In the above, we let the value of the symmetry-energy at saturation density be 31.6 MeV \[36, 37\]. And the corresponding symmetry-energy becomes \[38\]

$$E_{\text{sym}} = -6.71(\rho/\rho_0)^{2/3} + 38.31(\rho/\rho_0)^\gamma. \quad (4)$$

Shown in Fig. 1 is the density dependent symmetry-energy with different γ parameters. Due to the neutron-proton SRC, this form of γ-dependent symmetry-energy has a negative kinetic symmetry-energy around saturation density. This point is quite different from the frequently used un-correlated Fermi Gas kinetic symmetry-energy 12.5(ρ/ρ_0)^{2/3} in most transport models \[39\].

In this model, the in-medium baryon-baryon (BB) elastic cross sections are factorized as the product of a medium correction factor and the free baryon-baryon scattering cross sections \[39\], i.e.,

$$\sigma_{\text{medium}}^{BB, \text{elastic}} = (\frac{1}{3} + \frac{2}{3}e^{-u/0.54568}) \times (1 \pm 0.85\delta) \times \sigma_{\text{free}}^{BB, \text{elastic}}. \quad (5)$$

To unscramble the FOPI pion data, the inelastic baryon-baryon scattering cross section becomes very important. Because pion’s production and absorption are in fact mainly determined by the inelastic baryon-baryon scattering cross section. If one use π^-/π^+ ratio to probe the symmetry-energy, the isospin-dependence of the inelastic baryon-baryon scattering cross section also has to be taken into account. In this model, for the important but often-overlooked inelastic baryon-baryon scattering cross sections, we use the form similar with that in Ref. \[40\] but add an isospin-dependent factor 1 ± 0.85δ by extrapolation, i.e.,

$$\sigma_{\text{medium}}^{BB, \text{inelastic}} = (e^{-1.3u}) \times (1 \pm 0.85\delta) \times \sigma_{\text{free}}^{BB, \text{inelastic}}. \quad (6)$$

u = ρ/ρ_0 is the relative density. In Eqs. (5) and (6), for neutron-neutron (proton-proton) scattering whether they are in the initial or the final states, the above isospin dependent factor is 1 + 0.85δ(1 − 0.85δ), otherwise the isospin dependent factor is 1. The treatments related to π production and absorption are also similar with that in Ref. \[40\]. With the above BUU transport model, both π^- and π^+ multiplicities agree well with the FOPI pion production data.

### III. RESULTS AND DISCUSSIONS

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![FIG. 1: (Color online) Density dependent symmetry-energy with different γ parameters.](image1)

![FIG. 2: (Color online) Effects of the symmetry-energy and the HMT in colliding nucleus on the kinetic energy distribution of the n/p ratio of free nucleons.](image2)
Before we constrain the symmetry-energy, it is instructive to first check both the effects of the HMT in colliding nucleus and the effects of the symmetry-energy on the frequently used observable free n/p ratio. Shown in Fig. 2 is the kinetic energy distribution of the n/p ratio of free nucleons with or without the symmetry-energy (γ = 0.3 as example) and the HMT. As expected, the nuclear symmetry-energy causes larger free n/p ratio than that without considering the symmetry-energy. With the HMT, however, the ratio of free n/p becomes smaller at higher kinetic energies. This is understandable since neutron and proton are correlated together above the Fermi momentum [21], the n/p ratio at high energies should trend towards 1. Without the symmetry-energy (i.e., soft Equation of State of nuclear matter), the longer collision time among nucleons in heavy-ion collisions causes the effects of the HMT on the free n/p are partly cancelled out. Thus with the symmetry-energy or the stiffer symmetry-energy, effects of the HMT on the free n/p ratio are larger than that without considering the symmetry-energy. From Fig. 2, we can also see that the effects of the HMT on the free n/p ratio at lower energies are smaller than that at higher energies.

![Graph showing the kinetic energy distribution of the n/p ratio](image)

**FIG. 3:** (Color online) Elliptic flow ratios of neutrons and protons $V_2^n/V_2^p$ in Au + Au collision at 400 MeV per nucleon incident beam energy calculated by the present BUU model with different symmetry-energy’s stiffness parameter γ. The shadow region is the experimental FOPI-LAND data [20].

The comparison of in plane to out-of-plane nucleon emission rates, i.e., elliptic flow, can provide the Equation of State information [41]. While this emission rates of neutron compared with that of proton reflect the density-dependent symmetry-energy [20]. Shown in Fig. 3 is the elliptic flow ratios of neutrons and protons $V_2^n/V_2^p$ calculated by the present BUU model with different symmetry-energy’s stiffness parameter γ and compared with the experimental FOPI-LAND data [20]. Since the stiffer symmetry-energy causes more squeezed-out neutrons to emit in the direction perpendicular to the reaction plane [12] and also the n-p SRC in the HMT, the stiffer symmetry-energy causes a smaller average neutron momentum than that of proton in the direction perpendicular to the reaction plane, one sees smaller values of the elliptic flow ratios of neutrons and protons $V_2^n/V_2^p$ with stiffer symmetry-energies. This figure indicates the FOPI-LAND elliptic flow experimental data favors a soft symmetry-energy with γ < 0.3.

![Graph showing the elliptic flow ratios](image)

**FIG. 4:** (Color online) $\pi^-/\pi^+$ ratios in Au + Au collision at 400 MeV per nucleon incident beam energy calculated by the present BUU model with different symmetry-energy’s stiffness parameter γ. The shadow region is the experimental FOPI-LAND data [14].

It is meaningful to make multiple-observable comparisons with the same transport model and the same settings. For this purpose, shown in Fig. 4 is the $\pi^-/\pi^+$ ratios in Au + Au collision at 400 MeV per nucleon incident beam energy calculated by the same BUU model with different symmetry-energy’s stiffness parameter γ. The $\pi^-/\pi^+$ ratio has been proposed to be a potential probe of the symmetry-energy ten years ago [31] and after that many authors demonstrate its relationship to the symmetry-energy. Because the softer symmetry-energy causes more neutron-rich dense matter and $\pi^-$‘s are mainly from neutron-neutron collisions whereas $\pi^+$‘s are mainly from proton-proton collision [43], and also the effects of the HMT decrease the value of $\pi^-/\pi^+$ ratio [31], especially for the stiffer symmetry-energy, it is not surprising that one sees large $\pi^-/\pi^+$ ratio with the soft symmetry-energy while the stiff symmetry-energy corresponds small value of the $\pi^-/\pi^+$ ratio. This figure also indicates the FOPI π experimental data favors a soft symmetry-energy with γ < 0.3.

While one may ask if the physical result here would change when the momentum-dependent nucleon-potential is used, e.g., similar with that in Ref. [31]. In fact, the ratio or difference of the neutron and the
proton elliptic flows is less affected by the momentum-
dependence of the symmetry-potential [46]. The 
momentum-dependence of the symmetry-potential in-
creases the value of the $\pi^-/\pi^+$ ratio at most 10% [47], 
thus the value of $\pi^-/\pi^+$ ratio with $\gamma = 0.3$ studied here 
may just reach the lower limit of the FOPI data error 
band.

Combining the studies of the elliptic flow ratios of neu-
trons and protons and the $\pi^-/\pi^+$ ratio, one obtained 
the symmetry-energy stiffness parameter $\gamma < 0.3$. It in 
fact corresponds a soft symmetry-energy at super-
density. This result seems to be qualitatively consistent 
with the results in Ref. [25], but the latter simulation 
did not consider the effects of the n-p SRC and the in-
medium inelastic baryon-baryon scattering cross section 
as well as its isospin-dependence. While all of them surely 
play crucial roles in unscrambling the FOPI pion data. It is worth noting that the present constraints on the 
nuclear symmetry-energy are just in the density around 
1-2 times saturation density [48]. To probe the nuclear 
symmetry-energy at even higher densities, more great ef-
forts are needed in both nuclear physics and astrophysics, more related experimental and theoretical 
efforts are needed to confirm the equation of state of 
asymmetric matter at super-density.

IV. CONCLUSIONS

Using the FOPI and FOPI-LAND experimental 
measurements, based on the newly updated isospin-
dependent BUU model, the symmetry-energy at super-
density is consistently constrained by the double comp-
parison of the elliptic flow ratios of neutrons and protons and the $\pi^-/\pi^+$ ratio. The study indicates a soft symmetry-
energy at super-density. Since the nuclear symmetry-
energy plays crucial roles in both nuclear physics and 

Acknowledgements

The author thanks M. D. Cozma for providing the 
FOPI-LAND elliptic flow analysis routine. The work was 
carried out at National Supercomputer Center in Tianjin, 
and the calculations were performed on TianHe-1A. The 
work is supported by the National Natural Science Foun-
dation of China under Grant Nos. 11375239, 11435014.
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