Emergent spin-liquid-like state in the bond-disordered breathing pyrochlore

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Abstract

A breathing pyrochlore system is predicted to host a variety of quantum spin liquids. However, perturbations beyond nearest-neighbor Heisenberg interaction are an obstacle to identifying such exotic states. Here, we utilize a bond-alternating disorder to tune a magnetic ground state in the Cr-based breathing pyrochlore. By combining thermodynamic and magnetic resonance techniques, we provide experimental signatures of a spin-liquid-like state in LiGa₁₋ₓInₓCr₄O₈ (x=0.2), namely, a nearly T²-dependent magnetic specific heat and a persistent spin dynamics by muon spin relaxation (μSR). Moreover, ⁷Li NMR, ZF-μSR, and ESR unveil the dichotomic nature of both temporal and thermal spin fluctuations: slowly fluctuating tetramer singlets at high temperatures and a fast fluctuating spin-liquid-like state at low temperatures. Our results suggest that a bond disorder in the breathing pyrochlore offers a new route to achieve an unexplored state of matter.
Introduction

Frustrated quantum magnets provide a fertile ground to discover exotic quantum and topological phenomena, ranging from quantum spin liquids to magnetic monopoles in spin ice to Majorana anyons in the Kitaev honeycomb lattice. A prominent instance is a pyrochlore lattice, which is a three-dimensional network of corner-sharing tetrahedra. The pyrochlore Heisenberg antiferromagnet is predicted to harbor a spin-liquid ground state in the absence of further neighbor interactions. In real materials, the macroscopic ground-state degeneracy can be lifted by a range of perturbations, including spin-lattice couplings, additional magnetic interactions, and quenched disorders. The lifting of the highly degenerate ground states gives rise to emergent phenomena such as zero-energy modes, quantum spin ice, quantum spin liquid, and moment fragmentations.

By the same token, bond alternation as a perturbation to the pyrochlore system leads to a new structural motif called a breathing pyrochlore lattice. Its spin Hamiltonian is described as $\mathcal{H} = J \sum_{i \in A} \mathbf{S}_i \cdot \mathbf{S}_j + J' \sum_{i \in B} \mathbf{S}_i \cdot \mathbf{S}_j$, where $J$ and $J'$ are the nearest-neighbor exchange interactions for the A and B tetrahedra of two alternating sizes, respectively. In the breathing pyrochlore, the breathing anisotropy $B_t = J'/J$ gauges a degree of the bond alternation. Remarkably, the breathing pyrochlore can host Weyl magnons in an antiferromagnetically ordered phase, as well as a fracton spin liquid in the presence of Dzyaloshinskii-Moriya interactions not to mention a Coulomb spin liquid.

On the material side, the Cr-based breathing pyrochlore LiACr$_4$O$_8$ ($A = \text{Ga}^{3+}$, $\text{In}^{3+}$; space group $\bar{F}43m$) is a particularly interesting example because $B_t$ can be tuned by varying the composition of Li and $A$. LiGaCr$_4$O$_8$, having the breathing anisotropy $B_t = 0.6$, behaves much like a uniform pyrochlore. On the other hand, LiInCr$_4$O$_8$ has the small $B_t = 0.1$, in which the A tetrahedra are weakly coupled through inter-tetrahedral interaction $J'$. Despite the large difference in $B_t$, both the LiACr$_4$O$_8$ compounds show commonly two-stage symmetry breaking: a magnetostructural phase transition at $T_S = 16$ (17) K with subsequent antiferromagnetic ordering at $T_M = 13$ (14) K for $A = \text{In (Ga)}$. The lasting magnetic orders suggest that the bond alternation alone is not sufficient to repress the effect of spin-lattice coupling that induces a cubic-to-tetragonal transition. In this situation, bond randomness can counteract the detrimental impact of the magnetoelastic transition. This raises the exciting prospect of tracing intrinsic spin dynamics of the breathing pyrochlore lattice to a putative spin-liquid ground state, not being interrupted by long-range magnetic order.
In this work, we investigate spin dynamics and low-energy excitations of the bond-disordered breathing pyrochlore LiGa$_{0.2}$In$_{0.8}$Cr$_4$O$_8$ by combining multiple magnetic resonances with thermodynamic techniques. We observe a thermal and temporal dichotomy between a coupled tetramer singlet and a spin-liquid-like state. Spin fluctuations probed on the different time scales give evidence that the emergent spin-liquid state is pertinent to a correlated pyrochlore lattice with dimensional reduction.

**Results**

![Fig. 1](image)

**Fig. 1** | Crystal structure, phase diagram, and magnetic properties. a Schematic structure of the uniform breathing pyrochlore LiACr$_4$O$_8$ (A = Ga, In) and the bond-disordered breathing pyrochlore LiGa$_{0.2}$In$_{0.8}$Cr$_4$O$_8$. The inverse red triangle marks the investigated compound LiGa$_{0.2}$In$_{0.8}$Cr$_4$O$_8$. b $T$-$x$ phase diagram of LiGa$_{1-x}$In$_x$Cr$_4$O$_8$. The spinels are a nearly perfect realization of the breathing pyrochlore lattice due to the significant difference between the Li$^+$ and A$^{3+}$ valence states. The Ga-for-In substitution disrupts a uniform arrangement of the two alternating tetrahedra and leads to the rich magnetic phase

**Magnetic phase diagram** As sketched in Fig. 1a, the A-site ordered LiACr$_4$O$_8$ (A = Ga, In) spinels are a nearly perfect realization of the breathing pyrochlore lattice due to the significant difference between the Li$^+$ and A$^{3+}$ valence states. The Ga-for-In substitution disrupts a uniform arrangement of the two alternating tetrahedra and leads to the rich magnetic phase
diagram of LiGa$_{1-x}$In$_x$Cr$_4$O$_8$, as shown in Fig. 1b. The magnetic ordering vanishes for $x>0.1$ or $x<0.95$. A spin-glass phase occupies a wide range of $x=0.1$ - 0.625. For the Ga-rich compound LiGa$_{0.95}$In$_{0.05}$Cr$_4$O$_8$, the $^7$Li NMR and neutron experiments revealed the spin nematic transition at $T_1 = 11$ K, driven by the small bond disorder. In the In-rich range of $x=0.7$ - 0.95, LiGa$_{1-x}$In$_x$Cr$_4$O$_8$ exhibits a spin-gap behavior with the lack of magnetic ordering and spin freezing down to 2 K. However, little is known about an exact ground state. Here, we have chosen the $x=0.8$ compound (marked by the inverse triangle in Fig. 1b) to address this issue.

**Magnetic susceptibilities and magnetization** Figure 1c presents the $T$ dependence of dc magnetic susceptibility $\chi_{dc}(T)$ of LiGa$_{0.8}$In$_{0.2}$Cr$_4$O$_8$. On cooling, $\chi_{dc}(T)$ shows a broad hump at around 70 K, and a subsequent upturn below 10 K. Above 100 K, $\chi_{dc}(T)$ follows the Curie-Weiss law with the Curie-Weiss temperature $\Theta_{CW}$=-386(1) K and the effective magnetic moment $\mu_{eff} = 3.873(4) \mu_B$ (see Supplementary Note 1 and Supplementary Fig. 1) that are consistent with the values in the previous report. The low-$T$ upturn of $\chi_{dc}(T)$ is reminiscent of orphan spins. We attempt to single out the intrinsic $\chi_{intr}(T)$ by fitting the $T<5$ K data to $\chi_{dc}(T) = \chi_{intr}(T) + \chi_{orp}(T)\frac{B\Theta_{orp}}{T}$, where $\chi_{orp}(T) = C_{orp}/(T+\Theta_{orp})$. We obtain $\Theta_{orp} = -2.0(2)$ K, confirming the contribution of weakly interacting orphan spins. The orphan spin concentration is estimated to be 0.6% of the Cr$^{3+}$ spins, smaller than the value obtained from the high-field magnetization data (see below). After the subtraction of $\chi_{orp}(T)$ from $\chi_{dc}(T)$, we can identify a nearly constant $\chi_{intr}(T<5$ K), indicative of the presence of abundant low-energy in-gap states.

Exhibited in Fig. 1d is the real component of the ac magnetic susceptibility $\chi_{ac}(T, B)$ as a function of $T$ and $B$ at a fixed frequency $\nu=100$ Hz. The low-$T$ upturn of $\chi_{ac}(T, B)$ is systematically suppressed with increasing $B$. Remarkably, $\chi_{ac}(T, B=2$ T) becomes nearly $T$ independent below $T^*=10$ K. This is attributed to the Zeeman splitting of the orphan spins that weakly interact with an energy scale of $|\Theta_{orp}| \approx 2$ K. In addition, $\chi_{ac}(T)$ shows no apparent frequency dependence, ruling out the formation of spin freezing or spin glass state (see Supplementary Note 2 and Supplementary Fig. 2).

We next turn to the high-field magnetization $M(T, B)$ measured up to 58 T at selected temperatures (see Fig. 1e). With increasing field, $M(T=1.8$ K, $B)$ shows an upward convex behavior at low fields and then switches to a concave curvature at high fields. The low-$B$ convex $M(H)$ is associated with the orphan spin contribution $M_{orp}(H)$. To separate $M_{orp}(H)$ from $M(H)$, we fit the $T = 1.8$ K data to a Brillouin function $B_J$. The number of the orphan spins is evaluated to be 1.5% of the Cr$^{3+}$ spins. By subtracting $M_{orp}(H)$ from $M(H)$, we can obtain the intrinsic magnetization $M_{intr}(H)$, as illustrated by the green line in Fig. 1e. $M_{intr}(H)$ increases slowly at low fields and then exhibits a steep, linear increase above 30 T. The linear
extrapolation of the high-field $M_{\text{intr}}(H)$ yields the spin gap $\Delta M/k_B \sim 20$ K. The finite slope of $M_{\text{intr}}(H)$ at low fields is ascribed to the in-gap state, consistent with the $\chi_{\text{ac}}(T)$ and $\chi_{\text{ac}}'(T, B)$ data. At $T=60$ K ($\gg \Delta M/k_B$), $M(H)$ shows a quasilinear increase, characteristic of antiferromagnetically coupled spins.

Fig. 2 | Heat capacity of the breathing pyrochlore LiGa$_{0.2}$In$_{0.8}$Cr$_4$O$_8$. a Temperature dependence of the total heat capacity $C_{\text{tot}}$ in various magnetic fields $B=0$, 1, 3, 6, and 9 T. The blue dashed curve denotes the evaluated lattice contributions using the Debye model. The open pink circles represent the estimated magnetic heat capacity $C_m$. b Temperature and field dependences of the magnetic heat capacity divided by temperature $C_m/T$ after subtracting the lattice and nuclear Schottky contributions on a double log scale. The dashed lines are fits to the power-law behavior $C_m/T \sim T^n$ with $n = 0.68 – 0.82$. The inset shows a comparison between the $C_m/T$ data and the theoretical curve for a $s=3/2$ tetrahedron model. c Temperature dependence of the magnetic entropy $S_m$ under various magnetic fields. The dashed black line corresponds to the theoretical value $R \ln 4 = 11.52$. The inset is a log-log plot of $S_m$ vs. temperature.

Heat capacity To elucidate the nature of low-energy magnetic excitations, we performed heat capacity measurements down to 100 mK in various magnetic fields.

In Fig. 2, we plot the heat capacity of LiGa$_{0.2}$In$_{0.8}$Cr$_4$O$_8$ as a function of temperature. The total specific heat comprises the sum of lattice, magnetic, and nuclear contributions, $C_{\text{tot}} = C_{\text{lat}} + C_m + C_{\text{NS}}$. Here, $C_{\text{lat}}$ represents the lattice heat capacity, $C_m$ is the magnetic heat capacity, and $C_{\text{NS}}$ is the nuclear Schottky contribution. To isolate the magnetic contribution $C_m$ from $C_{\text{tot}}$, we estimate the lattice heat capacity $C_{\text{lat}}$ using the Debye model and the nuclear Schottky contribution $C_{\text{NS}}$ using $C_{\text{NS}}/T \sim T^3$ at low temperatures (see Supplementary Fig. 3a). The Debye model well reproduces the high-$T$ $C_{\text{tot}}$ with the Debye temperature $\Theta_D=669$ K, as shown in Fig. 2a. After subtracting the calculated $C_{\text{lat}}$ and $C_{\text{NS}}$ from $C_{\text{tot}}$, we obtain the magnetic heat capacity $C_m$. As evident from Fig. 2a, $C_m(T)$ shows a broad maximum around 70 K, at which both $\chi_{\text{dc}}(T)$ and $\chi_{\text{ac}}'(T, B)$ indicate the onset of short-range ordering.
From a log-log plot of $C_m/T$ vs. $T$ in Fig. 2b, we observe a hump at around $T = 0.6$ K in addition to the high-$T$ broad hump. The lack of a sharp peak excludes long-range magnetic ordering. On the application of an external magnetic field, the $T = 0.6$ K hump shifts to a higher temperature and is quickly suppressed at $\mu_0H > 3$ T. The $B$-dependence of the low-$T$ hump is attributed to the weakly coupled orphan spins, which become frozen in the small field. In addition, $C_m(T)/T$ displays a moderate $B$ dependence in the $T$ range of $T=0.1 – 10$ K. Overall, the $C_m(T)/T$ data are described by the power-law dependence $C_m/T \sim T^n$. The exponent is determined to be $n = 0.68 – 0.82$ in the $T$ range of $T = 2 – 10$ K and $n = 0.67 – 0.86$ below 1 K. The observed power-law behavior $C_m \sim T^{1.68 – 1.82}$ is close to the quadratic $T$ dependence.

We recall that in kagome antiferromagnets, the $T^2$ dependence of $C_m$ is taken as evidence of gapless spinon excitations. On the other hand, the coupling between spin and orbital degrees of freedom is evoked in the pyrochlore lattice. Obviously, the orbital degrees of freedom are not relevant to the Cr-based breathing pyrochlore. A comparison between the $C_m/T$ data and the calculated curve of a $s=3/2$ isolated tetrahedron unveils a large discrepancy for temperatures below 10 K (see the inset of Fig. 2b). Thus, we conclude that the observed quadratic dependence of $C_m$ is associated with emergent low-energy excitations of the bond-disordered breathing pyrochlore. We note that the $C_m/T$ data show a steep decrease and deviation from the power-law behavior below 0.1 K. To diagnose the presence of a spin gap, we try to fit the low-$T$ $C_m$ data to $C_m \sim \exp[-\Delta_{low}/k_BT]$. The extracted $\Delta_{low}/k_B \sim 0.29(1)$ K turns out to be tiny (see Supplementary Note 3 and Supplementary Fig. 3b).

Figure 2c presents the magnetic entropy $S_m(T,B)$, which was calculated by integrating $C_m/T$. $S_m(T,B)$ exhibits the saturation to $R\ln 4 \sim 11.52$ at 300 K, consistent with an $s=3/2$ spin system. $S_m(T,B)$ is gradually released with decreasing temperature below 100 K due to the development of short-range magnetic correlations. Only a tiny fraction of $S_m(T,B)$ remains at 0.1 K, which is less than 0.01 %. The nearly-zero residual entropy excludes any kind of magnetic transitions at low temperatures. Taken together, LiGa$_{0.2}$In$_{0.8}$Cr$_4$O$_8$ features a spin-liquid-like ground state with nearly gapless excitations, which evolves from the high-$T$ coupled tetramer singlet state.
**Fig. 3** Thermal and temporal evolution of spin fluctuations of LiGa$_{0.2}$In$_{0.8}$Cr$_4$O$_8$ measured by $^7$Li NMR, ZF-$\mu$SR, and ESR. 

**a** $^7$Li NMR spectra obtained by integrating spin-echo intensity at $T = 2.7$ and 250 K. **b** ZF-$\mu$SR spectra at selected temperatures. **c** Longitudinal field dependence of the $\mu$SR spectra measured at $T = 25$ mK. The solid lines denote the fit to the data using the sum of two simple exponential functions. **d** Derivative of the ESR absorption spectra measured at various temperatures. Spectra are vertically shifted for clarity. **e** Temperature dependence of $^7$Li spin-lattice relaxation rate $1/T_1$ and spin-spin relaxation rate $1/T_2$. The solid red line denotes the Arrhenius equation, yielding the spin gap of $\Delta/k_B = 35(1)\text{ K}$. The solid black line represents a power-law behavior of $1/T_1$ and $1/T_2$. **f** Muon spin relaxation rate as a function of temperature $\lambda_f(T)$. The solid magenta line represents the power-law fit to the data. **g** Muon spin relaxation as a function of longitudinal field $\lambda_{LF}(H)$. The red solid line represents the fit to the Redfield formula. **h** Temperature dependence of the peak-to-peak ESR linewidth $\Delta H_{pp}(T)$. The solid orange lines indicate a power-law behavior of $\Delta H_{pp}(T)$ in two different temperature regions. The shaded region indicates the change of magnetic correlations at around $T^* = 10\text{ K}$.

**Nuclear magnetic resonance** We now examine temporal and thermal spin fluctuations by combining multiple magnetic resonance techniques that probe spin correlations on different time scales.

The $^7$Li NMR spectra and the spin-lattice (spin-spin) relaxation rate $1/T_1$ ($1/T_2$) are plotted as a function of temperature in Fig. 3a and 3e. At high temperatures, the NMR spectrum exhibits a single sharp line with no quadrupolar splitting, similarly observed in LiInCr$_4$O$_8$. As the temperature is lowered, the NMR spectra show a continuous broadening and a shift to higher magnetic fields, emulating the development of magnetic correlations and internal fields down to $T = 2.7\text{ K}$. We observe neither a structured broadening of the NMR spectrum nor a sharp peak in $1/T_1$, further confirming the absence of long-range magnetic ordering and structural phase transition.
On cooling, $1/T_1$ displays an exponential-like decrease down to 10 K, and then a subsequent upturn to a power-law growth. The activation behavior of $1/T_1$ in the $T$ range of 25 – 100 K is well fitted with the Arrhenius equation $1/T_1 \sim \exp(-\Delta/k_B T)$ (see the semi-log plot of $1/T_1$ vs. $1/T$ in the inset of Fig. 3e). We obtain the spin gap $\Delta/k_B = 35(1)$ K, somewhat larger than $\Delta_{\text{m}}/k_B \approx 20$ K extracted from $M_{\text{int}}(H)$. Generally, the spin gap $\Delta/k_B$ obtained from $1/T_1$ is larger than the true spin gap $\Delta_{\text{min}}(q)$ occurring at a specific point $q$.\textsuperscript{29–31} This is because $1/T_1$ maps out the $q$-average of the dynamical spin susceptibility, $1/T_1 \approx T \Sigma_q A^2(q)\chi''(q,\omega_0)$. Here, $A(q)$ is the form factor of hyperfine interactions, and $\omega_0$ is the nuclear Larmor frequency.

The salient feature is that $1/T_1$ follows a power law $1/T_1 \sim T^{-n}$ with the exponent of $n = -0.46(3)$ below $T^* = 10$ K, indicative of the development of highly correlated states. The drastically distinct behavior of $1/T_1$ through $T^* = 10$ K suggests the switching of a dominant relaxation mechanism: a high-$T$ activated vs. a low-$T$ critical slowing-down. The spin-spin relaxation rate $1/T_2$ shows a weak power-law decrease $1/T_2 \sim T^{0.07(4)}$ as $T \to 0$. A nearly $T$-independent $1/T_2$ means that spin dynamics is in the fast fluctuating exchange-narrowed limit.

**Muon spin relaxation ($\mu$SR)** To discriminate between static and dynamic magnetism, we performed $\mu$SR experiments for $\text{LiGa}_{0.2}\text{In}_{0.8}\text{Cr}_4\text{O}_8$. In Fig. 3b, we present ZF-$\mu$SR spectra of $\text{LiGa}_{0.2}\text{In}_{0.8}\text{Cr}_4\text{O}_8$ at selected temperatures. At high temperatures, the ZF-$\mu$SR spectra show a Gaussian-like shape and gradually change to an exponential-like form. With decreasing temperature, the muon spin polarization rapidly relaxes in the initial time interval ($t = 0 - 2 \mu$s). We observe neither oscillating muon signal down to $T = 25$ mK nor a recovery to 1/3 of the initial polarization, evidencing the formation of a dynamically fluctuating state.\textsuperscript{32}

For quantitative analysis, we fitted the ZF-$\mu$SR spectra to the sum of a stretched exponential function and an exponential function, $P_{\text{(t)}} = a_{\text{fast}}\exp[-(\lambda_{\text{fast}}t)\beta] + a_{\text{slow}}\exp[-(\lambda_{\text{slow}}t)]$. Here, $a_{\text{fast}}$ ($a_{\text{slow}}$) is the fraction of the fast (slow) relaxation component, and $\lambda_{\text{fast}}$ ($\lambda_{\text{slow}}$) is the muon spin relaxation rate for the fast (slow) relaxing component. We summarize the obtained parameters in Fig. 3f and Supplementary Information (see Supplementary Fig. 4). As the temperature decreases, the slow relaxation component suddenly appears below $T^* = 10$ K, at which the $^7\text{Li}$ $1/T_1$ shows an upturn to the power law. Furthermore, the slow muon spin relaxation rate $\lambda_{\text{slow}}(T)$ shows a $T$-independent behavior ($\lambda_{\text{slow}} \approx 0.042$ $\mu$s$^{-1}$) below 1 K.

Figure 3f is a log-log plot of the fast relaxation rate vs. $T$. As the temperature is lowered, $\lambda_{\text{fast}}(T)$ exhibits a weakly increasing $T$-dependence down to 10 K, and then a power-law increase with the exponent of $n = -0.71(6)$. We note that the steep increment of $\lambda_{\text{fast}}(T)$ concurs with the leveling-off of $\chi_{\text{int}}(T)$ in Fig. 1c and the upturn of $1/T_1$ in Fig. 3e. Upon further cooling toward $T = 25$ mK, $\lambda_{\text{fast}}(T)$ flattens out below 2 K, entering a persistent-spin-dynamical regime. Such a leveling-off of the muon spin relaxation rate is often observed in an assortment of quantum spin liquid candidates.\textsuperscript{33–35} With decreasing temperature, the stretched exponent $\beta$ gradually decreases from $\beta = 2$ (a Gaussian-like decay) to $\beta = 1$ (an exponential decay) below $T^* = 10$ K (see
Supplementary Fig. 4b). The low-$T$ exponential relaxation suggests the persisting spin fluctuations to $T = 25$ mK without forming frozen moments.

Shown in Fig. 3c and 3g are the LF-$\mu$SR spectra and the LF dependence of $\lambda_\delta(H_{LF})$ measured at $T = 25$ mK. Remarkably, the LF-$\mu$SR spectra exhibit substantial relaxation even at $H_{LF} = 1$ T, lending further support to the dynamic ground state. In the Redfield model $\lambda_{LF}(H) = 2\gamma_\mu^2 (H_{loc}^2) v / (v^2 + \gamma_\mu^2 H^2)$, the LF dependence of $\lambda_\delta(H_{LF})$ is related to the fluctuation frequency $v$, and the fluctuating time-averaged local field $\langle H_{loc}^2 \rangle$. Here, $\gamma_\mu$ is the muon gyromagnetic ratio. Fitting to the $\lambda_\delta(H_{LF})$ data yields $v = 1.6(1)$ GHz and $H_{loc} = 296(11)$ G (see Fig. 3g). We note that the fluctuation frequency is faster than the value reported thus far for spin-liquid materials while the magnitude of the local field is rather large, possibly due to a large spin number $s=3/2$.\(^{33,35}\) Taken the $\mu$SR data together, we conclude that the emergent correlated spin state below $T^* = 10$ K has a dynamic nature of a GHz fluctuation frequency.

**Electron spin resonance** To take a close look at spin fluctuations on the GHz time scale, we turn to high-frequency ESR. As shown in Fig. 3d, with decreasing temperature, the ESR spectra broaden continuously down to 2.3 K and shift to a lower field. We observe no impurity signals. All of the ESR spectra are fitted with a single Lorentzian profile, suggesting that they originate solely from the correlated Cr spins. The extracted parameters are plotted as a function of temperature in Fig. 3h and Supplementary Fig. 5. The peak-to-peak linewidth $\Delta H_{pp}$ is associated with the development of spin-spin correlations, while the resonance field $H_{res}$ reflects the build-up of internal magnetic fields.

As the temperature is lowered, $\Delta H_{pp}(T)$ exhibits a critical increase, which is described by a power-law $\Delta H_{pp}(T) \sim T^{-n}$. We can identify a small change of the exponent from $n=0.37(1)$ to $0.30(2)$ through $T^* = 10$ K. This weak anomaly in the exponent is in sharp contrast to the drastically changing character of the magnetic correlations on the MHz time scale.

A critical line broadening in the paramagnetic state is due to the development of local spin correlations below the Curie-Weiss temperature $\Theta_{CW}$ and is generic to frustrated magnets subject to critical spin fluctuations. We note that the observed critical exponent of $n = 0.30 - 0.37$ is much smaller than $n = 0.6 - 0.7$ for the uniform pyrochlore $MCr_2O_4$ (for $M = Zn, Mg, Cd$)\(^{37}\), as shown in Fig. 3h. Compared to the uniform pyrochlore, the reduced exponent in LiGa$_{0.2}$In$_{0.8}$Cr$_4$O$_8$ is interpreted in terms of the dimensional reduction as discussed below. As such, the emergent low-$T$ state bears a resemblance to the ground state inherent to a pyrochlore lattice.
Fig. 4 | Schematics of the emergent magnetic subsystems in a bond-disordered breathing pyrochlore. Each square represents the tetrahedra formed by the Cr spins. The Cr spins are omitted for clarity. The orange and green squares are the alternating A and B tetrahedra for LiInCr₄O₈. The grey square denotes the B' tetrahedra introduced by the Ga-for-In substitution. Due to an energy hierarchy of the tetrahedral \( E_A \approx E_{B'} > E_B \), the breathing pyrochlore is effectively decomposed into two subsystems: (i) coupled tetramer singlets (red shaded circle) and (ii) diluted pyrochlore lattices (blue dashed region).

Discussion

With the aid of various thermodynamic and magnetic resonance techniques (0 Hz – 1 THz), we are able to disclose the temporal and thermal structures of magnetic correlations in the bond-disordered breathing pyrochlore LiGa₁₀₂In₀.₈Cr₄O₈.

We identify the characteristic temperature \( T^* = 10 \text{ K} \), below which a highly correlated spin state emerges from high-\( T \) tetramer singlet fluctuations. The emergent low-energy state features nearly \( T \)-squared \( C_m \) as well as critical spin fluctuations over a wide time scale of \( 10^{-12} \sim 10^{-4} \) sec, as evident from a power-law \( 1/T_1 \sim T^{-0.46(3)}, \lambda_c(T) \sim T^{-0.71(6)}, \) and \( \Delta H_{pp}(T) \sim T^{-0.30} \). We recall that the energy scale of \( T^* = 10 \text{ K} \) is smaller than the spin gap of \( \Delta/k_B = 20 \sim 35 \text{ K} \), deduced from \( M(H) \) and \( 1/T_1 \). Yet, it is bigger than a few kelvins of weakly correlated orphan spins. The latter contribution is quenched by applying the external field of \( \mu_0 H > 3 \text{ T} \). As such, the low-\( T \) correlated state does not stem from extrinsic defect spins. Rather, it pertains to the in-gap magnetic state created by the bond randomness in a breathing pyrochlore. Here, we stress that
there is no hint for random singlets whose fingerprints are scalings of thermodynamic quantities, namely, \( \chi(T) \sim T^{1-a} \), \( M(H) \sim T^a \), and \( C_m(T) \sim T^a \) with \( 0 < a < 1 \). Therefore, the low-\( T \) correlated state is distinct from a bond-disorder induced random singlet.

We next turn to the temporal spin dynamics. The thermodynamic data reveal the spin gap decorated with the in-gap state. On the slow time scale \((1 - 10^4 \text{ Hz})\), the NMR data demonstrate a dichotomic spin dynamics, namely, high-\( T \) singlet correlations and low-\( T \) critical correlations. On the fast time scale \((10^4 - 10^{12} \text{ Hz})\), the \( \mu \text{SR} \) and ESR data give no signature of the tetramer singlet correlations. Instead, highly correlated spins prevail over the whole measured temperature range and show a spin-liquid behavior. This behavior means that the singlet fluctuations average out beyond the time scale of the NMR technique at low temperatures.

Another perspective is that \( \text{LiGa}_{0.2}\text{In}_{0.8}\text{Cr}_4\text{O}_8 \) comprises two spin subsystems whose temporal magnetic correlations differ from each other. As the tetramer singlet forms an entangled state of four spins, its spin fluctuations are slower than the spin fluctuations of the rest subsystem that are governed by two-spin correlations. In the \( \text{GHz} \) time window, the high-\( T \) critical spin dynamics persists to the low-\( T \) state. More specifically, the power-law dependence of \( \Delta H_{\text{pp}}(T) \), which is an ESR signature of the pyrochlore-like spin dynamics, is observed down to \( T=2.3 \text{ K} \). This gives indirect evidence that the low-\( T \) in-gap state is linked to a pyrochlore-like subsystem.

Lastly, we discuss the origin of the dichotomic magnetic correlations emerging from the bond-alternating disorder in \( \text{LiGa}_{0.2}\text{In}_{0.8}\text{Cr}_4\text{O}_8 \). As sketched in Fig. 4, the Ga-for-In substitution introduces the B’ tetrahedra randomly to the alternating A and B tetrahedra. Consequently, the perturbed system entails two different networks consisting of (i) the alternating A and B tetrahedra with \( B_t=0.1 \) and (ii) the alternating A and B’ tetrahedra with \( B_t=0.6 \). Neglecting a small fraction of the alternating B and B’ tetrahedra with \( B_t=0.17 \), the large difference in \( B_t \) brings about two subsystems: (i) coupled tetramer singlets and (ii) diluted pyrochlore lattices. The coupled tetramer singlets are responsible for the high-\( T \) and slow spin fluctuations. The diluted pyrochlore subsystem weakly bonded to the tetramer singlets contributes to the emergence of the low-\( T \) spin-liquid-like state. \( \text{LiInCr}_4\text{O}_8 \) is in the vicinity of a critical point and is proximate to a Coulomb phase.\(^{28,39}\) However, the magnetoelastic coupling does not allow for exploring this exotic phase. Nonetheless, the bond disorder offers an efficient way to overcome restrictions imposed by the magnetostructural transition. As a by-product of quenching symmetry breaking, we obtain a \textit{finite-size} breathing pyrochlore lattice that dictates a low-energy and fast spin behavior. As such, the low-\( T \) state of \( \text{LiGa}_{0.2}\text{In}_{0.8}\text{Cr}_4\text{O}_8 \) may realize a diluted Coulomb spin liquid, deserving further theoretical and experimental investigations.

To conclude, we have investigated the temporal and thermal spin dynamics of a bond-disordered breathing pyrochlore compound. We find that a dichotomy of magnetic correlations
is linked to two emergent subsystems. The observed low-$T$ spin-liquid-like state bears pertinent spin fluctuations to a diluted breathing pyrochlore lattice, which opens up a venue for a fundamentally new state.

**Methods**

**Sample synthesis** Polycrystalline samples of LiGa$_{0.2}$In$_{0.8}$Cr$_4$O$_8$ were synthesized by a solid-state reaction method. Stoichiometric amounts of Li$_2$CO$_3$, Cr$_2$O$_3$, Ga$_2$O$_3$, and In$_2$O$_3$ were mixed in 1:4:0.2:0.8 molar ratio and thoroughly ground in a mortar. The mixture was pelletized and sintered at 800 °C in the air for 12 h. The substance was ground, pressured into pellets, and finally sintered at 1000 °C for 24 h and 1100 °C for 72 h. We checked the quality of the samples by X-ray diffraction measurements.

**Magnetic properties characterization** dc and ac magnetic susceptibilities were measured using a superconducting quantum interference device magnetometer and vibrating sample magnetometer (Quantum Design MPMS and VSM). High-field magnetization experiments were performed at the Dresden High Magnetic Field Laboratory using a pulsed-field magnet (20 ms duration). The magnetic moment was detected by a standard inductive method with a pick-up coil system in the field range of $\mu_0 H = 0 – 60$ T.

**Heat capacity** Heat capacity measurements were carried out in the temperature and field range of $T=0.03 – 300$ K and $B=0 – 9$ T using a commercial set-up of Quantum Design PPMS with a thermal relaxation method. The raw heat capacity data $C_{tot}/T$ follow a $T^3$ power-law behavior at extremely low temperatures, corresponding to a nuclear Schottky contribution (see Supplementary Fig. 3a). After subtracting the nuclear Schottky contribution $C_{NS}/T = AT^{-3}$ from $C_{tot}/T$, we present the magnetic and lattice contributions to the specific heat in the main text.

**Nuclear magnetic resonance** $^7$Li ($I = 3/2$, $\gamma_N = 16.5471$ MHz/T) NMR measurements were conducted at National High Magnetic Field Laboratory (Tallahassee, USA) by using a locally developed NMR spectrometer equipped with a high-homogeneity 17 T sweepable magnet. $^7$Li NMR spectra were recorded by a fast Fourier transform of spin-echo signals while sweeping the field at a fixed frequency $\nu = 198.317$ MHz. The nuclear spin-lattice (spin-spin) relaxation time $T_1$ ($T_2$) was measured by a modified inversion recovery (Hahn pulse) method width $\pi/2 = 1 \mu$s in the temperature range of $T = 2.6 – 250$ K.
**Muon spin relaxation (μSR)** μSR measurements were carried out on the DR spectrometer ($^3\text{He}/^4\text{He}$ dilution refrigerator, $25 \text{ mK} \leq T \leq 10 \text{ K}$) at M15 beamline and the LAMPF spectrometer ($^4\text{He}$ cryostat, $2 < T < 300 \text{ K}$) at M20 beamline in TRIUMF (Vancouver, Canada). The polycrystalline samples of LiGa$_{0.2}$In$_{0.8}$Cr$_4$O$_8$ were wrapped with a silver foil and then attached to the sample holder. After the mounted samples were inserted into the cryostat, μSR spectra were measured in zero-field, longitudinal field (parallel to muon spin direction), and transverse field (perpendicular to muon spin direction).

**Electron spin resonance (ESR)** High-frequency ESR experiments were performed at National High Magnetic Field Laboratory (Tallahassee, USA). The ESR spectra were recorded using a quasioptical heterodyne spectrometer in the temperature range of $T = 2 – 290 \text{ K}$, enabling the detection of a magnetic field derivative of a microwave absorption signal. An external magnetic field was swept with a 12.5 T sweepable superconducting magnet at the fixed frequency $\nu = 240 \text{ GHz}$.

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**Author contributions**
K.-Y.C. designed and conceived the study. S.L., S.-H.D., and K.-Y.C. planned the experiments. S.-H.D. and Y.S.C. synthesized the sample and characterized structural and magnetic properties. W.L. and A.P.R. conducted NMR experiments. S.L. and W.L. performed μSR experiments, and S.L. analyzed the data. S.L. and J.V.T. carried out the high-field ESR measurements. S.L. carried out the heat capacity measurements. S.L. and K.-Y.C. wrote the manuscript with contributions from all authors.

**Additional information**
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