Model-Based Optimization in Induction Heating of Thin-Wall Shells

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Abstract

A mathematical model of induction heating process of paramagnetic thin-wall shells was considered. Electromagnetic processes in the system "inductor heated shell—temperature equalization devices" are modelled by the method of coupled circuits, thermal processes — by the differential-difference method. The model is implemented in MATLAB and is used to optimize the heating process based on partial criteria that are formulated in fuzzy form.

Keywords: fuzzy optimization, control, modeling, induction heating, thin-walled shell

1. Introduction

Induction heating has many advantages: low inertia, energy saving, small size, etc. But it also has disadvantages, the main of which is the irregularity distribution of temperature over the heated object. Therefore, we need to find solutions that will prevent this phenomenon. This work is devoted to the problems of model-oriented design and control in induction heating of thin-wall shells.

The aim of our study was to develop a control system for the induction heating of paramagnetic thin-walled shells to specified conditions. The system should be near optimal for a set of partial criteria formulated in fuzzy form. In our study we find program for a digital controller of a hybrid continuous-discrete control system. The continuous part is the system "inductor — heated shell — additional coils" as non-linear MIMO object. The discrete part is the digital controller. The developed digital controller must ensure that the most general requirements for controlled processes are met, including those that are not clearly defined.

The considered unit is shown in figure 1. The shell is placed inside the induction heater, consisting of an inductor powered by an AC source, and thermal insulation. The heating temperature is controlled by thermocouples at three points spaced along the length of the heated body (signals $y_1, y_2, y_3$). The temperature field at the final stage of the process should be fairly uniform $y_1, y_2, y_3 \in [y_{sp} - \varepsilon, y_{sp} + \varepsilon]$. This requirement is ensured by three control actions $u_1, u_2, u_3$. The main control channel is the power source of the inductor $u_i \in [u_{min}, u_{max}]$. This channel allows you to influence the average temperature of the shell, but it is not able to significantly change the law of temperature distribution along its length. To ensure the necessary uniformity of heating, the inductor is equipped with additional coils as a device for uniformity the temperature along the length of the shell. When opening $(u_i = 0, i = 2, 3)$ the coils do not affect the heating process. When closing the coil $(u_i = 1)$, it causes a counter EMF. This leads to a local decrease in heating power in the coil local area, which can help equalize the temperature in the heated product. As it need temperature equalization devices are activated together or alternately, operating simultaneously with the induction heater.
In many existing works, the study of complex multioperational control systems is based on traditional approaches. The mutual influence of both individual parts of the process and the control subsystems is ignored. The synthesis of stage control algorithms is carried out without taking into account the influence of previous stages on the quality of subsequent ones.

The novelty of the proposed approach lies in the organization of control algorithms on digital controllers in the form of “condition-action” type rule systems. The task of finding rule systems is formulated as a fuzzy optimization problem. An important point is also to take into account when determining the operation algorithms of individual control subsystems not only the requirements for the current stage, but also the possible consequences for subsequent ones.

To solve the formulated problem, it is necessary to have a model for describing the induction heating process with the necessary accuracy. The induction heating process can be divided into two parts: energy transfer from an inductor fed by alternating current of a certain frequency to a heated shell and temperature changing of a shell from initial to required by technology.

2. Modeling of the Induction Heating Processes

2.1 Thermal model

Thermal process in thin shells can be described by the one-dimensional heat equation

\[
\frac{\partial T(x,t)}{\partial t} = \left( \frac{\partial^2 T(x,t)}{\partial x^2} + \frac{\Gamma}{x} \frac{\partial T(x,t)}{\partial x} \right) + P(x,t)
\]

(1)

with boundary conditions

\[
T(x,0) = T_0(x), \quad \frac{\partial T(X_1,t)}{\partial x} = q_{x_1}^0(T), \quad \frac{\partial T(X_2,t)}{\partial x} = q_{x_1}^0(T),
\]

where \( T(x,t) \) — the temperature field of the heated body, depending on time \( t \) and spatial coordinates \( x \in [X_1,X_2] \), \( \Gamma \in [1,2] \) is the form factor of the heated body, \( P(x,t) \) is a power of heat sources, Stefan-Boltzmann border loss

\[
q_{x_1}(T) = k \left( T^{*4}(X_1,t) - T_{cp}^4 \right),
\]

\[
q_{x_2}(T) = k \left( T^{*4}(X_2,t) - T_{cp}^4 \right),
\]

where \( k \) depending on the effective degree of blackness of the heated material and the Stefan-Boltzmann constant, \( T^* \) and \( T_{cp} \) are the absolute temperature of the product and environment.

Through discretization by the well-known differential-difference method [1], the thermal process in a thin shell is then described in the form of a state-space model. However, the use of standard programs to optimize induction heating processes seems to be impractical due to their low efficiency under these conditions. To take into account the specifics of the processes and tasks under consideration, the authors developed their own subroutines.

2.2 Electromagnetic model

In [2], the method for calculating electromagnetic fields in non-magnetic and ferromagnetic bodies using coupled circuits was chosen as the base for modeling electromagnetic processes in induction heaters. The calculation of fields in non-magnetic bodies according to [2] is reduced to determining the current \( I_{1N} \) in the inductor and currents \( I_i \), \( i = 1, 2, ..., N - 1 \) in \( N - 1 \) elementary solenoids into the body. To find the currents, a system of equations of the form

\[
[\hat{Z}_i][I_i] = [\hat{U}],
\]

(2)

obtained by applying the 2-nd Kirchhoff law to each solenoid. In (2) the total resistance

\[
[\hat{Z}_i] = [R_i] + j[X_i],
\]

(3)

is the total resistance \( [R_i] \) of the \( i \)-th solenoid, \( X_i \) is the active resistance of the \( i \)-th solenoid, \( X_{ij} = \begin{cases} R_{ij}, & i = k, \\ \alpha L_i, & i \neq j \end{cases} \) is the angular frequency, \( \alpha \) is the form factor of the heated body, Stefan-Boltzmann border loss.
3. Formulation and Solution of the Fuzzy Optimal Control Problem

Consideration of problems of synthesis of systems of rules for the operation of digital controllers of hybrid continuous-discrete control systems has been the subject of a significant number of studies. In [3], it was noted that in relation to a number of applied problems of synthesis (including optimal) of computer control systems, it is possible either to carry out preliminary finite-dimensional parameterization of some elements of the production system that determines the operation algorithm of the desired controller, or to determine the set of elementary functions from which to construct decision. This allows us to solve such problems numerically as finite-dimensional optimization problems. The traditional approach to solving such problems involves formulating them as mathematical programming problems of the form

\[
\min_{z \in \mathbb{Z}} f(z), \quad g_i(z) \geq 0, i = 1, 2, ..., m, \tag{4}
\]

where \( z \in \mathbb{Z} \) is the desired solution, \( f(z) \) is the objective function, \( g_i(z) \) are the constraint.

Problem (4) can be reformulated so that the boundaries that separate acceptable solutions from unacceptable become blurred, and the degrees of acceptability of solutions are represented by fuzzy numbers. For this, the objective function and limitations must be understood in a fuzzy sense [4]. When using the fuzzy logic notation, the fuzzy version of problem (4) can be written as follows:

\[
\min_{z \in \mathbb{Z}} f(z), \quad \tilde{g}_i(z) \geq 0, i = 1, 2, ..., m, \tag{5}
\]

where the wavy line “~” is a symbol of fuzzy operation. Let membership functions \( \mu_0(\tilde{g}_i), \mu_i(\cdot), i = 1, 2, ..., m \) represent degrees of accomplishment of a goal and limitations. The solution of the optimization problem (5) should satisfy, as far as possible, both the goal and the constraints, i.e. maximize the minimum of values \( \mu_i(\cdot), i = 0, 1, ..., m \). So problem (5) of fuzzy mathematical programming can be transformed [5] into a problem \( \max C(z) \), where \( C(z) \) represents the global degree of satisfaction \( z \) with a goal and limitations: \( C(z) = \min \{ v_0, v_1, ..., v_m \} \). Finally, a fuzzy problem (5) takes the form

\[
\max \min_{z \in \mathbb{Z}} \{ v_i \}, i = 0, 1, ..., m, \tag{6}
\]

where membership functions \( \mu_0, \mu_i, i = 1, 2, ..., m \) represent degrees of accomplishment of the goal and constraints, respectively. Further, the problem (6) is solved by numerical methods.

In this work the control of paramagnetic thin-walled shells induction heating is considered. The control task with a fuzzy purpose and limitations is formulated as follows. It is necessary in the time \( t_F \rightarrow \min \) to ensure the transfer of the model of the control object (1) – (4) to the area

\[
\max_{x \in [x_1, x_2]} \{ f(x) \} \leq \varepsilon, \quad T_F(x) = (T(x) - T_{cp}) \leq \varepsilon
\]

under the additional condition \( \max_{i \in [0, t_F]} (T_D(x)) \leq 2 \varepsilon \), where

\[
T_D = \max_{x \in [x_1, x_2]} (T(x,t) - \min_{x \in [x_1, x_2]} (T(x,t))
\]

Membership functions of fuzzy sets are determined programmatically as S-shaped membership functions (for \( t_F \) and \( T_D \)) and triangular-shaped membership function for \( T_F \):

\[
\mu_0(t_F) = \begin{cases} 
1, & t_F \leq 4 \\
1 - (t_F - 4)^2 / 32, & 4 \leq t_F \leq 8 \\
0, & 12 \leq t_F 
\end{cases}
\]

\[
\mu_1(t_F) = \begin{cases} 
0, & \rho_1 \leq 890 \\
(\rho_2 - 890) / 50, & 890 \leq \rho_1 \leq 940 \\
(990 - \rho_1) / 50, & 940 \leq \rho_1 \leq 990 \\
0, & 990 \leq \rho_1 
\end{cases}
\]

\[
\rho_1 = \max_{i \in [0, t_F]} \{ T_F(x) \}:
\]

We will find the fuzzy solution from relation (6) for a system of rules for digital program controller with a unit operating frequency (ones per model time unit) in the following form “if \( i \in \{i, i+1\} \) then \( u(t) = u_i \), \( u(t) = u_j \), \( u(t) = u_k \)”, \( i, j, k = 0, 1, 2, ..., t_F / t_s - 1 \) where \( t_F = 6, t_s = 0.6, u_1(t), u_2(t), u_3(t) \) are respectively the inductor power control signal \( u_i \in [u_{\min}, u_{\max}] \) and control signals \( u_2(t) \in [0,1], u_3(t) \in [0,1] \) of additional coils as temperature equalization devices along the shell length.

The problem of determining the fuzzy optimal control is reduced to the problem of integer programming. The parameter \( z_j(t_j), i = 1, 2, 3, j = 0, 1, 2, ..., t_f / t_s - 1 \) at each of the 10 intervals of control constancy is the number of one of four possible options for the distribution of volumetric power density of internal heat sources. Option \( z_j = 1 \) corresponds to a combination \( u_1 = u_{\max}, u_2 = 0, u_3 = 0 \), option \( z_j = 2 \) — to a combination \( u_1 = u_{\max}, u_2 = 1, u_3 = 0 \), option \( z_j = 3 \) — to a combination of \( u_1 = u_{\max}, u_2 = 0, u_3 = 1 \), option \( z_j = 4 \) — to a combination of \( u_1 = u_{\max}, u_2 = 1, u_3 = 1 \). These combinations lead to the following four options for power distribution along the length of the shell (figure 2). By brute
force method we find one of $4^{10} = 1048576$ options. This mode of the form $z = [1,1,1,1,1,1,2,3,1]$ ensures the quality of heating at a level $\min(\mu_i) = 0.8750$, $i = 1, 2, 3$.

![Figure 2](image1.png)

**Figure 2.** Four possible distributions of volumetric power density of internal heat sources.

The results are presented in figures 3, 4 where $y_1 = T(X_1)$, $y_2 = T(X_1 + X_2)/2$, $y_3 = T(X_2)$, $t_F = 6$, $t_e = 0.6$. The degree of satisfaction of the obtained solution to the given conditions is 0.875.

![Figure 3](image2.png)

**Figure 3.** Temperature in a fuzzy optimal problem.

![Figure 4](image3.png)

**Figure 4.** Control actions in a fuzzy optimal problem.

4. Conclusion

This paper demonstrates the possibility of using fuzzy logic in the synthesis of regulators under the conditions of fuzzy formulated requirements for the intermediate and final state of a nonlinear, multidimensional object in the form of a multiply connected process chain. This approach seems very promising. On its basis it is possible to automate the entire process of designing control systems from setting the task to modeling their work, including software testing, creating technical devices that independently generate the algorithms of their control systems and, if necessary, modernizing them.

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