Numerical computation on the scattering sound field distribution of rigid sphere in shallow water waveguide

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Abstract. Numerical computational model about the scattering fields of rigid sphere in shallow water waveguide is accomplished using boundary element method and normal mode model. A kind of modified CHIEF boundary element method is used to solve the non-uniqueness problem at characteristic frequency for exterior sound field computation. The green function for shallow water waveguide acoustic wave propagation is modified based on the normal mode model. The acoustic pressure in the far field yields Kirchhoff Helmholtz equation and the scattering sound field distribution of rigid sphere in a Pekeris waveguide is achieved. Also the scattering sound field distribution of rigid sphere in the free field is calculated. The results indicate that the waveguide influences the sound propagation, and it need to pay attention during the test.

1. Introduction
The theory and the numerical method for acoustic radiation and scattering problem in ocean waveguide are not in the phase of maturation. Most studies were done on the structure radiation or scattering problem separately from sound propagation in the waveguide. The former problem deals with the ideal and uniform medium, and neglects the influence of the sea surface and the bottom regularly. The latter takes the structure as a point source to calculate the sound propagation in the waveguide. However, both need unified approach and they are a whole.

To solve this problem, many scholars have made explorations on the scattering field computation in the ocean waveguide. For example, F.Ingenito made use of normal modes and plane-wave scattering function and got the acoustic field scattered by a rigid sphere in an isovelocity fluid layer overlying a horizontally stratified medium[1]. Collins and Werby used the parabolic equation technique to solve the Helmholtz equation in the waveguide, and their method had been tested against analytical solutions for rigid spheres in 3D, showing good agreement at all scattering angles [2]. Perkins extended the normal mode model applying to the range dependent condition making use of the adiabatic approximation theory [3]. Purnima Ratilal and Makris established a unified model of the scattering and reverberation [4]. Belibassakis and Athanassoulis developed an improved coupled-mode method, based on an enhanced local-mode series for the representation of the acoustic field which can treat propagation and scattering problems in the ocean waveguide [5].

In this paper, boundary element method is combined with normal mode modal to solve the problem of rigid sphere scattering in shallow water waveguide governed by the Helmholtz equation [6].
2. Numerical methods

2.1 CHIEF boundary element method

The boundary integral equation (1):

\[ \sigma_p(X) = \int_S \left( p(Y) \frac{\partial G(X,Y)}{\partial n} - G(X,Y) \frac{\partial p(Y)}{\partial n} \right) ds \]  

(1)

For the direct boundary element method, there may be some trouble on the singular integration, near singular integration and the resonant solutions at characteristic frequency.

The exterior acoustic problem exist the resonant solutions at characteristic frequency. When the number of the boundary element is n, it has to select m interior points in order to overcome the resonant solutions[7].

So these interior points meet the equation (2).

\[ \int_S \left( p(Y) \frac{\partial G(X,Y)}{\partial n} - G(X,Y) \frac{\partial p(Y)}{\partial n} \right) ds = 0 \]  

(2)

When the equations are discretized and then combined with the surface boundary element coefficient matrix, the over determined equations are formed as equation (3) and equation (4).

\[ Ax = b \]  

(3)

\[ A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mm} \\ a_{(m+1)1} & a_{(m+1)2} & \cdots & a_{(m+1)m} \\ a_{(m+2)1} & a_{(m+2)2} & \cdots & a_{(m+2)m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{(m+n-1)1} & a_{(m+n-1)2} & \cdots & a_{(m+n-1)m} \end{bmatrix} \]  

(4)

If there is unique solutions, the over determined equations has to use the singular value decomposition method to calculate the matrix pseudo inverse. The SVD algorithm affects the computing efficiency, so we improve the traditional CHIEF method in this paper. The improved CHIEF method has to select only one interior point where is the structure heart. After discretized the row matrix is added to every row of the boundary element coefficient matrix.

\[ A = \begin{bmatrix} a_{11} + b_1 & a_{12} + b_2 & \cdots & a_{1m} + b_m \\ a_{21} + b_1 & a_{22} + b_2 & \cdots & a_{2m} + b_m \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} + b_1 & a_{m2} + b_2 & \cdots & a_{mm} + b_m \end{bmatrix} \]  

(5)

Equation (5) remains square matrix and it can avoid low efficiency. And the improved CHIEF method has no obvious increase of the calculate quantity and it’s easy to implement.

![Figure 1. The compared calculation results.](attachment:image.png)
As shown in figure 1, according to the calculation results compared with the traditional chief method and analysis solution, it is very useful to remove the resonant solutions at characteristic frequency.

2.2 BEM-Normal model method
Normal modes method has been used for many years in underwater acoustics. The normal modes method calculates an isolated point source in acoustic medium, which is based on separation of variables. The solution is governed by the acoustic Helmholtz equation (6):

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) + \frac{1}{c_p^2 r^2} \frac{\partial^2 p}{\partial \zeta^2} + k_m^2 p = -4\pi \delta(r - r_0) \]  

(6)

The green function formula equation (7) in the ocean waveguide gives [8]:

\[ G = \frac{i}{4\rho_0} \sum_{m=1}^{N} \psi_m(z) \psi_m^*(z) H_0^{(1)}(k_m r) \]  

(7)

When the Kirchhoff Helmholtz equation is solved, the scattering far field distribution of the rigid sphere in the shallow water waveguide can be computed by equation (8).

\[ p(X) = \int \left( p(Y) \frac{\partial G(X, Y)}{\partial n} - G(X, Y) \frac{\partial p(Y)}{\partial n} \right) ds \]  

(8)

3. Model simulating and analysis

3.1 Computational model
A Pekeris waveguide describes as follows: the uniform fluid velocity is 1500 m/s, and the bottom density is 1500 kg/m³, the sound velocity is 1600 m/s. The parameters list in figure 2 in detail.

\[
\begin{align*}
D & = 100 \text{ m} \\
c_s & = 1500 \text{ m/s} \\
\rho_s & = 1000 \text{ kg/m}^3 \\
c_w & = 1600 \text{ m/s} \\
\rho_w & = 1500 \text{ kg/m}^3 \\
a_w & = 0.2\beta/\lambda
\end{align*}
\]

Figure 2. Schematic graph of a Pekeris waveguide.

3.2 Accuracy Verification
The accuracy validation method using an acoustic point source is as follows: Firstly, build a spherical auxiliary surface with radius 1m. Secondly, the pressure on the surface and outside the sphere can compute by point source propagation method such as normal mode method. Then according to the Huygens principle, the wavelet on the auxiliary surface can be taken as secondary source, and the external acoustic field can compute by improved chief BEM algorithm making use of the boundary condition on the auxiliary surface. At last, the results can be compared to verify the algorithm.

In a Pekeris waveguide, we can select field points on a horizontal line 1-500 meters far away from the point source and give the quantification comparison. As shown in figure 3, the results of direct solution method and boundary element method are used respectively, and they are basically the same.
3.3 Simulation results and analysis

The rigid sphere of 1m diameter is located at 50m or 30m depth separately. The source is at 30m depth, where the horizontal distance is 500m far away from the rigid sphere, and the frequency is 1 kHz. The scattering acoustic field with three profiles which is azimuth angles at 0, pi/4, pi/2 are shown. Azimuth 0 stands for the field points (its center is the same as the center of the rigid sphere) locate in the plane of the source and sphere, and azimuth pi/4 stands for the degrees counter clockwise rotation.

The scattering field distributions in free field are shown in figure 4. As shown in figure 5, the scattering field distributions also show some symmetry, because the water is uniform isosonic profile. Compare figure 4 with figure 5, there are significant difference between the scattering field in the free field and in a Pekeris waveguide, and the directivity has changed due to the function of waveguide.

The scattering intensity is failing continuously in all directions. However, the scattering field in a Pekeris waveguide alternates with distance, which reflects the interference effect by the waveguide.
Figure 6. Scattering field distribution in a Pekeris waveguide (sphere is at 30m depth).

Figure 7. Scattering field distribution (the depth of a Pekeris waveguide is 200m).

Figure 6 shows that the scattering field distribution (the rigid sphere is at 30m depth) is not only related to the waveguide properties but also relating to the position of the acoustic source and rigid sphere. When the depth of a Pekeris waveguide is 200m, figure 7 shows that the starting point of coherent distance increases with the increase of the waveguide depth.

4. Conclusion
The traditional boundary element method uses the green function of free field, which is not suitable for the ocean waveguide. In this paper, the green function is established using the normal mode model and the improved algorithm can consider the influence of the sea surface, bottom and sound velocity gradient distribution.

Through the simulation calculation of the rigid sphere in a Pekeris waveguide, it can be seen that the scattering field exists the interference effect clearly. And the scattering field distribution is not only related to the waveguide properties but also related to the position of the acoustic source and rigid sphere.

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