Drell-Yan cross-sections with fiducial cuts: impact of linear power corrections and $q_T$-resummation in PDF determination

Simone Amoroso, Ludovica Aperio Bella, Maarten Boonekamp, Stefano Camarda, Alexander Glazov, Alessandro Guida, Renat Sadykov, and Yulia Yermolchyk

(Dated: September 28, 2022)

Measurement at Hadron colliders of neutral- and charged-current Drell-Yan production provide essential constraints in the determination of parton distribution functions. Experimentally, they have reached percent level precision, challenging the accuracy of the theoretical predictions. In this work we benchmark the novel implementation in DYTurbo of linear fiducial power corrections in the $q_T$-subtraction formalism at NLO and NNLO in QCD. We illustrate how the inclusion of linear fiducial power corrections impacts predictions for precise $W$ and $Z$ measurements from the LHC and affects their description by modern global parton distribution functions. The further inclusion of $q_T$-resummation corrections in the theoretical predictions leads to a better modelling of the lepton $p_T$ distribution and we study how these improve the description of the data.

I. INTRODUCTION

The Drell-Yan (DY) process consists of lepton pair production, through the creation of a vector boson, either $\gamma^*/Z$ or $W^\pm$, in hadron-hadron high energy collisions. Measurements of this process help to probe the parton distribution functions (PDF) giving insight on the $u$- and $d$-valence quarks PDFs and the sea/light-quark decomposition.

To extract precise PDFs, the level of accuracy reached by the experiments in DY measurements needs to be matched by the precision of the theoretical predictions. The fully differential cross section considering leptonic decay is known up to next-to-next-to-leading-order (NNLO) in Quantum Chromodynamics (QCD) [1–3] and at next-to-leading-order (NLO) in Electroweak (EW) couplings [4–6]. More recently the N3LO QCD calculations have also been performed [7–11]. It has been noticed [12] that the results from different NNLO QCD codes differ between each other by an amount that can reach the percent level, much higher than the expected numerical differences. The disagreement is understood to be related to the presence of fiducial cuts applied to the final state leptons and the different subtraction schemes adopted in the calculations [13, 14]. Some cut configurations lead to a linear $q_T$ dependence of the cross section and hence induce a bias in non-local subtraction calculations [15–18]. The $q_T$ dependent term is referred to as fiducial power correction (FPC). As a solution to this problem, it has been shown that including a $q_T$ recoil prescription, the nominal accuracy of non-local subtraction codes is recovered [13, 19, 20]. The fixed order results, regardless of the subtraction scheme used, present anyway some instabilities due to the sensitivity to enhanced $q_T$ logarithms at small $q_T$ [21]. In order to obtain a physical result, these logarithms need to be resummed to all orders [22–25].

In this proceeding the effects of fiducial cuts and $q_T$ resummation on DY $q_T$-inclusive cross section calculations are explored. As a benchmark scenario, we consider the ATLAS $W$ and $Z$ cross section measurement at 7 TeV [26]. These experimental data are very precise and offer important constraints on the PDF determination. The predictions are evaluated with DYTurbo [15], a versatile program for fast DY calculations. The code implements a non-local $q_T$ subtraction method and easily allows the user to include the $q_T$ recoil prescription [19], or effects from $q_T$-resummation [27]. The calculations are combined with NLO EW corrections computed with the ReneSANCE code [28].

The document is organized as follows: in section II the simulation setup for the predictions is described, next, in section III, the results are presented with a particular focus on the effects related to the FPC. In section IV a quantitative comparison with the ATLAS data is carried out. The data and the predictions are then used in section V for a PDF profiling study. Finally, in section VI, conclusions and future perspective are discussed.
II. SIMULATION SETUP

Predictions are calculated with DYTurbo using the $G_F$ electroweak scheme: $G_F$, $m_W$, $m_Z$ are the input values. The Standard Model input parameters are set to the following values

$$G_F = 1.1663787 \times 10^{-5} \text{ GeV}^{-2}, \quad m_Z = 91.1876 \text{ GeV},$$
$$m_W = 80.385 \text{ GeV}, \quad \Gamma_Z = 2.4950 \text{ GeV},$$
$$\mu_W = 2.091 \text{ GeV}.$$ (1)

The input PDFs are taken from the NNPDF31_nnlo_as_0118 set [29]. The values of the renormalization and factorization scale $\mu_R$ and $\mu_F$ are set equal to the dilepton invariant mass, $m_{\ell\ell}$. The value of the $q_T$-slicing cut-off is set to $(q_T/m_{\ell\ell})_{\text{cut}} = 0.008$. To include the resummation effects, additional parameters are considered: the resummation scale, $\mu_{\text{Res}}$, is set equal to the dilepton invariant mass. Non-perturbative QCD effects at low $q_T$ are included through a Gaussian form factor in the space of the impact parameter $b$. NLO EW corrections are calculated with the RENESANCE program and using the same EW parameters listed above as input. These include virtual weak corrections, QED initial-state radiation and initial-final interference. The data are corrected for the final state QED radiation effects using PHOTOS [30] and for the $\gamma\gamma \to \ell\ell$ process contributions using the SANC program [31].

The ATLAS measurement implements symmetric cuts on the final state lepton transverse momentum, $p_{T,\ell/\nu} > 25 \text{ GeV}$. The $W$ measurement also applies a cut on the $W$-transverse mass, $m_W > 40 \text{ GeV}$, with $m_T^2 = 2p_{T,\ell}p_{T,\nu}(1-\cos \Delta \phi_{\ell,\nu})$. The $Z$ cross section is measured differentially in the dilepton rapidity $|y_{\ell\ell}|$ in two channels: a central channel with leptons at central pseudorapidity, $|\eta| < 2.5$, and a forward channel in which one of the two leptons is produced at high pseudorapidity, $2.5 < |\eta| < 4.9$. Three different mass bins around the $Z$ resonance peak are considered: $m_{\ell\ell} = [46, 66, 116, 150] \text{ GeV}$. The $W^\pm$ production cross section is measured as a function of the lepton pseudorapidity $|\eta|$. These cut configurations induce, at least in some part of phase space, linear-$q_T$ fiducial power corrections.

The predictions are produced with high statistical accuracy; the relative statistical uncertainty is at the level of fractions of permille, completely negligible with respect to the data uncertainties and the size of the effects considered in this work.

III. NNLO QCD RESULTS

Three different sets of predictions at NNLO QCD are produced: the nominal fixed order one using the $q_T$ subtraction method, the fixed order including the $q_T$ recoil prescription (equivalent to a local subtraction result) and the $q_T$ resummed result (at a formal accuracy of NNLO+NNLL in QCD). The different calculations are shown in the example of the $Z$-peak bin, for the central and forward channels, in Figure 1. In the central channel a shape difference, between the three calculations, of 0.5% is observed. In the forward channel the difference between the fixed order and the resummed prediction is more striking and gets as big as 10% in the first rapidity bin. The difference between the $q_T$ recoil calculation and the resummed one is smaller, still of the level of few percents.

IV. DATA-PREDICTIONS COMPARISON

A quantitative comparison of the predictions with the experimental data is carried out using the xFitter framework [32, 33]. The following $\chi^2$ definition is used:

$$\chi^2(b_{\text{exp}}, b_{\text{th}}) = \sum_{i=1}^{N_{\text{data}}} \left[ D_i - T_i(1 - \sum_k \gamma_{ik}^{\text{th}} b_{k,\text{th}} - \sum_j \gamma_{ij}^{\text{exp}} b_{j,\text{exp}}) \right]^2 \delta_{i,\text{uncor}}^2 T_i^2 + \delta_{i,\text{stat}}^2 D_i T_i$$
$$+ \sum_i \log \frac{\delta_{i,\text{uncor}}^2 T_i^2 + \delta_{i,\text{stat}}^2 D_i T_i}{\delta_{i,\text{uncor}}^2 D_i^2 + \delta_{i,\text{stat}}^2 D_i^2}$$
$$+ \sum_{j=1}^{N_{\text{exp,sys}}} b_{j,\text{exp}}^2 + \sum_{k=1}^{N_{\text{th,sys}}} b_{k,\text{th}}^2.$$ (2)

Both the experimental uncertainties and theoretical uncertainties arising from PDF variation are considered. The correlated uncertainty components are accounted with two sets of nuisance parameters, $b_{\text{exp}}$ and $b_{\text{th}}$. The impact of
FIG. 1. Z boson production cross section at NNLO (with and without implementing a $q_T$ recoil prescription) and NNLO+NNLL. Both the central channel (left) and the forward rapidity channel (right) are reported. Some significant differences between the predictions are observed in both cases.

CT14nnlo 68%CL

| Dataset                                      | NNLO $q_T$-subtr. | NNLO recoil $q_T$-subtr. | NNLO+NNLL $q_T$-subtr. |
|----------------------------------------------|-------------------|--------------------------|------------------------|
| $W^+$ lepton rapidity                        | 9.4/11            | 8.8/11                   | 8.8/11                 |
| $W^-$ lepton rapidity                        | 8.2/11            | 8.7/11                   | 8.2/11                 |
| Low mass, Z rapidity                         | 11/6              | 7.2/6                    | 7.5/6                  |
| Mass peak, central Z rapidity                | 15/12             | 10/12                    | 7.7/12                 |
| Mass peak, forward Z rapidity                | 9.6/9             | 5.3/9                    | 6.4/9                  |
| High mass, central Z rapidity                | 6.0/6             | 6.5/6                    | 5.8/6                  |
| High mass, forward Z rapidity                | 5.2/6             | 5.6/6                    | 5.3/6                  |
| Correlated $\chi^2$                          | 40                | 40                       | 31                     |
| Log penalty $\chi^2$                         | -4.33             | -3.39                    | -4.20                  |
| Total $\chi^2$/dof                          | 99/61             | 88/61                    | 77/61                  |
| $\chi^2$ p-value                            | 0.00              | 0.01                     | 0.08                   |

TABLE I. Results of the comparison of the ATLAS data [26] with the predictions. The CT14nnlo 68%CL PDF is used. An improvement in the $\chi^2$ agreement is observed when including resummation effects in the predictions.

The $\chi^2$, at its minimum, provides a test of the compatibility between the data and the predictions. The penalty term for determining the nuisance parameters is given by the last line in equation 2, this is referred to as correlated $\chi^2$ component. The first line in the definition is instead quoted in the results as the data set $\chi^2$ component. Finally, the second line, the log penalty term, is a small bias correction term.

Predictions for different PDFs are obtained using APPLgrids [34] generated at NLO QCD with MCFM [16, 35, 36]. The NNLO QCD accuracy is obtained through NNLO k-factors ($kF$) calculated using the DYTurbo predictions described in section II. The $kF$ are combined multiplicatively with NLO EW $kF$ calculated with the ReneSANCE program.

As a first test, the CT14nnlo PDF set [37] is used in the comparison. The $\chi^2$ results are reported in Table I. The three sets of predictions introduced in section III are used for the study. A reduction of $\sim$ 10 points in the total $\chi^2$ when using a theory that include a $q_T$ recoil prescription is observed. A further improvement of additional $\sim$ 10 points is obtained when considering the $q_T$ resummation effects. The trend of the results is in line with the theoretical expectation of section I. The study is extended testing other PDF sets. The total $\chi^2$ are reported in Table II. In all the cases a similar reduction of the total $\chi^2$, of about 20 – 30 when including the $q_T$-resummation, is observed.
TABLE II. Total $\chi^2$ for the comparison between the predictions and the ATLAS data [26], using different PDF sets. The three different theory definitions are tested. The first half of the table considers PDFs that did not include ATLAS 7 TeV $W$ and $Z$ data, while in the second half are PDFs that used these data in their determination.

| PDF set          | Total $\chi^2$ (ndf=61) |
|------------------|--------------------------|
|                  | NNLO $q_T$ subtr. | NNLO $q_T$-subtr. | NNLO+NNLL $q_T$-subtr. |
| CT10nnlo68%CL    | 100 | 85 | 76 |
| CT14nnlo68%CL    | 99  | 88 | 77 |
| CT18NNLO68%CL    | 102 | 90 | 79 |
| MMHT14nnlo68%CL  | 124 | 99 | 94 |
| NNPDF30nnlo      | 139 | 133| 111 |
| ABMP16.5_NNLO    | 124 | 106| 92 |
| HERAI PDF        | 199 | 201| 160 |
| CT18ANNLO68      | 96  | 84 | 74 |
| MSHT20nnlo       | 111 | 87 | 79 |
| NNPDF31          | 91  | 84 | 71 |
| NNPDF40nnlo      | 89  | 83 | 69 |

V. PDF PROFILING

The values of the nuisance parameters, $b_{th}$, at the $\chi^2$ minimum, equation 2, are used to obtain an optimized version of the central PDF $f'_0$

$$f'_0 = f_0 + \sum_k b_{k,th}^{\min} \left( \frac{f^+_k - f^-_k}{2} + b_{k,th}^{\min} \frac{f^+_k + f^-_k - 2f_0}{2} \right).$$

Here $f_0$ is the original central PDF and $f_k^\pm$ are the eigenvector up/down variation sets. Furthermore the updated PDFs have reduced uncertainties. The profiling procedure is used to test the impact of new data set on existing PDF sets.

Here the CT14nnlo 68% CL PDF set is used for the profiling; this set does not include the ATLAS $W$ and $Z$ 7 TeV data. The different NNLO(+NNLL) QCD predictions introduced in section III are used and the differences in the outcome of the profiling are examined. In Figure 2 the profiling results for two relevant quantities that are constrained by DY data are shown: the ratio $R_s = \frac{x(s + \bar{s})}{(\bar{u} + \bar{d})}$ and the gluon PDF. The significant impact of these data sets, as was already observed in [26], is clearly visible. A difference between the profiled PDFs when using the different NNLO QCD calculations is also noticeable. A general observation is that the profiled PDFs using the resummed predictions are somewhat closer to the one using the fixed order $q_T$ subtraction.

VI. CONCLUSION

In this work the effects of fiducial cuts in the accuracy of theoretical predictions for DY cross sections has been investigated. Predictions for the ATLAS $W$ and $Z$ 7 TeV cross section have been produced with the DYTurbo program. The NNLO (with and without lepton $q_T$-recoil) and NNLO+NNLL in QCD accuracy, plus NLO EW corrections, have been studied. A quantitative comparison with data, including PDF uncertainties, shows a better agreement when the $q_T$ resummation effects are taken into account in the predictions. A PDF profiling procedure has been carried out to estimate the impact of using different theory definitions in PDF determinations. This exercise shows small, but noticeable differences. Further direction for investigation will be to study the effect of fiducial cuts on other measurement phase spaces can be investigated, and the impact on the PDF determination can be evaluated through
FIG. 2. Profiling results for the $R_s$ quantity (left) and the gluon PDF ratio to the CT14 set (right). The CT14 PDF set and the profiled PDF using the ATLAS data and different NNLO QCD definitions are shown.

a PDF fit to DY data.

[1] K. Melnikov and F. Petriello, “$W$ boson production cross section at the large hadron collider with $O(\alpha_s^2)$ corrections,” Physical Review Letters 96 no. 23, (Jun, 2006). https://doi.org/10.1103/PhysRevLett.96.231803.

[2] K. Melnikov and F. Petriello, “Electroweak gauge boson production at hadron colliders through $O(\alpha_s^2)$,” Physical Review D 74 no. 11, (Dec, 2006). https://doi.org/10.1103/PhysRevD.74.114017.

[3] S. Catani, L. Cieri, G. Ferrera, D. de Florian, and M. Grazzini, “Vector boson production at hadron colliders: A fully exclusive QCD calculation at next-to-next-to-leading order,” Physical Review Letters 103 no. 8, (Aug, 2009). http://dx.doi.org/10.1103/PhysRevLett.103.082001.

[4] S. Dittmaier and M. Krämer, “Electroweak radiative corrections to $W$-boson production at hadron colliders,” Physical Review D 74 no. 11, (Dec, 2006). https://doi.org/10.1103/PhysRevD.74.114017.

[5] U. Baur and D. Wackeroth, “Electroweak radiative corrections to $pp/\bar{p}p \to W^\pm \to \ell^\pm \nu$ beyond the pole approximation,” Physical Review D 70 no. 7, (Oct, 2004). https://doi.org/10.1103/PhysRevD.70.073015.

[6] V. A. Zykunov, “Radiative corrections to the Drell-Yan process at large dilepton invariant masses,” Phys. Atom. Nucl. 69 (2006) 1522.

[7] C. Duhr, F. Dulat, and B. Mistlberger, “Charged current Drell-Yan production at N3LO,” Journal of High Energy Physics 2020 no. 11, (Nov, 2020). https://doi.org/10.1007/JHEP11(2020)29143.

[8] X. Chen, T. Gehrmann, N. Glover, A. Huss, T.-Z. Yang, and H. X. Zhu, “Dilepton rapidity distribution in Drell-Yan production to third order in QCD,” Physical Review Letters 128 no. 5, (Feb, 2022). https://doi.org/10.1103/PhysRevLett.128.052001.

[9] C. Duhr and B. Mistlberger, “Lepton-pair production at hadron colliders at N3LO in QCD,” Journal of High Energy Physics 2022 no. 3, (Mar, 2022). https://doi.org/10.1007/JHEP03(2022)2916.

[10] X. Chen, T. Gehrmann, N. Glover, A. Huss, P. F. Monni, E. Re, L. Rottoli, and P. Torrielli, “Third-order fiducial predictions for Drell-Yan production at the LHC,” Physical Review Letters 128 no. 25, (Jun, 2022). https://doi.org/10.1103/PhysRevLett.128.252001.

[11] T. Neumann and J. Campbell, “Fiducial Drell-Yan production at the LHC improved by transverse-momentum resummation at N^4LL+N^4LO,” 2022. https://arxiv.org/abs/2207.07056.

[12] S. Alekhin, A. Kardos, S. Moch, and Z. Trócsányi, “Precision studies for Drell–Yan processes at NNLO,” The European Physical Journal C 81 no. 7, (Jul, 2021). http://dx.doi.org/10.1140/epjc/s10052-021-09361-9.

[13] M. A. Ebert, J. K. L. Michel, I. W. Stewart, and F. J. Tackmann, “Drell-yan $q_T$ resummation of fiducial power corrections at N3LL,” Journal of High Energy Physics 2021 no. 4, (Apr, 2021). https://doi.org/10.1007/JHEP04(2021)219.
Y. Dokshitzer, D. D’yakonov, and S. Troyan, “On the transverse momentum distribution of massive lepton pairs,” *Physical Review Letters* **127** no. 7, (Aug, 2021) .

G. P. Salam and E. Slade, “Cuts for two-body decays at colliders,” *Journal of High Energy Physics* no. 6, (Jun, 2022) .

https://doi.org/10.1140/epjc/s10052-022-10510-x.

L. Buonocore, S. Kallweit, L. Rottoli, and M. Wiesemann, “Linear power corrections for two-body kinematics in the $q_T$ subtraction formalism,” 2021. [arXiv:2111.13661](https://arxiv.org/abs/2111.13661).

G. Parisi and R. Petronzio, “Small Transverse Momentum Distributions in Hard Processes,” *Nucl. Phys. B* **254** (1979) 427–440.

J. Collins, D. E. Soper, and G. Sterman, “Transverse momentum distribution in Drell-Yan pair and $W$ and $Z$ boson production,” *Nuclear Physics B* **250** no. 1, (1985) 199–224.

https://www.sciencedirect.com/science/article/pii/0550321385904791.

S. Camparda, L. Cieri, D. de Florian, G. Ferrera, and M. Grazzini, “Universality of transverse-momentum resummation and leptonic decay,” *Journal of High Energy Physics* **2015** no. 12, (Dec, 2015) 1–47.

https://doi.org/10.1007%2Fjhep12%282015%29047.

S. Camparda, L. Cieri, and G. Ferrera, “Fiducial perturbative power corrections within the $q_T$ subtraction formalism,” *The European Physical Journal C* **82** no. 6, (Jun, 2022) .

https://doi.org/10.1140/epjc/s10052-022-10510-x.

G. P. Salam and E. Slade, “Cuts for two-body decays at colliders,” *Journal of High Energy Physics* **2021** no. 11, (Nov, 2021) .

https://doi.org/10.1007/JHEP11(2021)220.

Y. Dokshitzer, D. D’yakonov, and S. Troyan, “On the transverse momentum distribution of massive lepton pairs,” *Physics Letters B* **79** no. 3, (1978) 269–272. 

https://www.sciencedirect.com/science/article/pii/037026937890240X.

G. Parisi and R. Petronzio, “Small Transverse Momentum Distributions in Hard Processes,” *Nucl. Phys. B* **154** (1979) 427–440.

J. Collins, D. E. Soper, and G. Sterman, “Transverse momentum distribution in Drell-Yan pair and $W$ and $Z$ boson production,” *Nuclear Physics B* **250** no. 1, (1985) 199–224.

https://www.sciencedirect.com/science/article/pii/0550321385904791.

S. Camparda, L. Cieri, D. de Florian, G. Ferrera, and M. Grazzini, “Universality of transverse-momentum resummation and hard factors at the NNLO,” *Nuclear Physics B* **881** (Apr, 2014) 414–443.

https://doi.org/10.1016%2Fj.nuclphysb.2014.02.011.

ATLAS collaboration, “Precision measurement and interpretation of inclusive $W^+/-/W$ and $Z/\gamma$ production cross sections at high precision with the ATLAS detector,” *The European Physical Journal C* **77** no. 6, (Jun, 2017) .

https://doi.org/10.1140%2Fepjc%2Fs10052-017-4911-9.

S. Camparda, L. Cieri, and G. Ferrera, “Drell-Yan lepton-pair production: $q_T$ resummation at N$^3$LO accuracy and fiducial cross sections at N$^3$LO,” *Physical Review D* **104** no. 11, (Dec, 2021) .

https://doi.org/10.1103%2FPhysRevD.104.1111503.

S. Bondarenko, Y. Dydyshka, L. Kalinovskaya, R. Sadykov, and V. Yermolchyk, “Hadron-hadron collision mode in RenoSANCe-v1.3.0,” 2022. https://arxiv.org/abs/2207.04332.

R. D. Ball, V. Bertone, S. Carrazza, L. D. Debbio, S. Forte, P. Groth-Merrild, A. Guffanti, N. P. Hartland, Z. Kassabol, J. I. Latorre, E. R. Nocera, J. Rojo, L. Rottoli, E. Slade, and M. Ubiali, “Parton distributions from high-precision collider data,” *The European Physical Journal C* **77** no. 10, (Oct, 2017) .

http://dx.doi.org/10.1140/epjc/s10052-017-6199-5.

N. Davidson, T. Przedzinski, and Z. Was, “PHOTOS interface in C++; technical and physics documentation,” 2015. 

S. Sadykov, A. Arbuzov, D. Bardin, S. Bondarenko, P. Christova, L. Kalinovskaya, V. Kolesnikov, A. Sapronov, and E. Uglov, “SANC system and its applications for LHC,” *Journal of Physics: Conference Series* **523** (Jun, 2014) 012043.

https://doi.org/10.1088%2F1742-6596%2F523%2F1%2F012043.

S. Alekhin et al., “HERAFitter,” *Eur. Phys. J. C* **75** no. 7, (2015) 304, arXiv:1410.4412 [hep-ph].

V. Bertone, M. Botje, D. Britzger, S. Camparda, A. Cooper-Sarkar, F. Giuli, A. Glazov, A. Luszczak, F. Olness, R. Placakyte, V. Radescu, W. Skomlski, and O. Zenaiev, “$x$Fitter 2.0: An open source QCD fit framework,” 2017. [arXiv:1709.01151](https://arxiv.org/abs/1709.01151).

T. Carli, D. Clements, A. Cooper-Sarkar, C. Gwenlan, G. P. Salam, F. Siegert, P. Starovoitov, and M. Sutton, “A posteriori inclusion of parton density functions in NLO QCD final-state calculations at hadron colliders: the APPLGRID project,” *The European Physical Journal C* **66** no. 3–4, (Feb, 2010) 503–524.

http://dx.doi.org/10.1140/epjc/s10052-010-1255-0.

J. M. Campbell, R. K. Ellis, and C. Williams, “Vector boson pair production at the LHC,” *Journal of High Energy Physics* **2011** no. 7, (Jul, 2011) .

http://dx.doi.org/10.1007/JHEP07(2011)018.

J. M. Campbell, R. K. Ellis, and W. T. Giele, “A multi-threaded version of MCFM,” 2015.

S. Dutil, T.-J. Hou, J. Gao, M. Guzzi, J. Huston, P. Nadolsky, J. Pumpkin, C. Schmidt, D. Stump, and C.-P. Yuan, “New parton distribution functions from a global analysis of quantum chromodynamics,” *Physical Review D* **93** no. 3, (Feb, 2016) .

http://dx.doi.org/10.1103/PhysRevD.93.033006.