Dynamical instability and its implications for planetary system architecture

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ABSTRACT
We examine the effects that dynamical instability has on shaping the orbital properties of exoplanetary systems. Using N-body simulations of non-EMS (Equal Mutual Separation), multiplanet systems we find that the lower limit of the instability time-scale \( t \) is determined by the minimal mutual separation \( K_{\text{min}} \) in units of the mutual Hill radius. Planetary systems showing instability generally include planet pairs with period ratio \(<1.33\). Our final period ratio distribution of all adjacent planet pairs shows dip-peak structures near first-order mean motion resonances similar to those observed in the Kepler planetary data. Then we compare the probability density function (PDF) of the de-biased Kepler period ratios with those in our simulations and find a lack of planet pairs with period ratio \( >2.1 \) in the observations – possibly caused either by inward migration before the dissipation of the disc or by planet pairs not forming with period ratios \( >2.1 \) with the same frequency they do with smaller period ratios. By comparing the PDF of the period ratio between simulation and observation, we obtain an upper limit of 0.03 on the scale parameter of the Rayleigh distributed eccentricities when the gas disc dissipated. Finally, our results suggest that a viable definition for a ‘packed’ or ‘compact’ planetary system be one that has at least one planet pair with a period ratio less than 1.33. This criterion would imply that 4 per cent of the Kepler systems (or 6 per cent of the systems with more than two planets) are compact.

Key words: methods: numerical – planets and satellites: dynamical evolution and stability.

1 INTRODUCTION
About 40 per cent of planets discovered by the Kepler spacecraft are in multiplanet systems,† some of which have small orbital period ratios between neighbouring planets. The observed period ratios between adjacent pairs (Fig. 1) show that most of the period ratios are smaller than three, and there is a pile-up of period ratios around the 3:2 and 2:1 mean motion resonances (MMRs). The existence of planet pairs near first-order MMRs is often ascribed to disc migration (Snellgrove, Papaloizou & Nelson 2001; Lee & Peale 2002; Lee & Thommes 2009; Wang & Ji 2014). However, we might expect the overabundance to be larger if disc migration is common, though Pan & Schlichting (2017) suggested that resonance capture is more difficult for smaller planets in a disc. Additionally, more planet pairs are observed on the far side of MMRs rather than being symmetrically distributed around them (Lissauer et al. 2011; Fabrycky et al. 2014). Numerous mechanisms have been proposed to explain the asymmetrical period ratio distribution around MMRs including dissipative resonant repulsion (Lithwick & Wu 2012; Batygin & Morbidelli 2013), stochastic and smooth migration (Rein 2012), interactions between the planets and the planetesimal disc (Chatterjee & Ford 2015), in situ growth of planets (Petrovich, Malhotra & Tremaine 2013), and planet–planet interactions (Pu & Wu 2015).

Most of the Kepler planetary systems are perceived as being quite compact, often containing multiple planets with orbital periods shorter than Mercury. However, since the dynamics of the systems are generally scale invariant [dictated primarily by orbital period ratios rather than the orbital periods themselves (Rice, Rasio & Steffen 2018)] the term ‘compact’ is ambiguous. For example, compared with the physical size of the orbits of Kepler planets, the planets in our Solar system are relatively far apart. However, they have similar period ratios – the quantity that is more fundamental – to those observed in Kepler planet pairs (shown in Fig. 1). Thus, either the Kepler planetary systems are
The period ratio between adjacent planets is determined by

\[ a_{i+1} - a_i = K \Delta H, \]  

(1)

and

\[ \Delta H = \frac{(a_i + a_{i+1}) (m_i + m_{i+1})^{1/3}}{2}, \]  

(2)

where \( \Delta H \) is the mutual Hill radius, \( a_i \) is the semimajor axis of the \( i \)-th planet, \( m \) is the planetary mass, and \( K \) is a numerical spacing parameter. In this way, the period ratio between adjacent planets is

\[ \frac{P_{i+1}}{P_i} \approx 1 + \frac{3}{2} K \left( \frac{m_i + m_{i+1}}{3M} \right)^{1/3}. \]  

(3)

For EMS planetary systems, \( K \) is constant within one system. This quantity is a key factor in determining the stability timescale \( \tau \) of EMS planetary systems, where \( \log \tau \propto K \) [see Pu & Wu (2015) for a review]. Another measure of the compactness of a system is the orbital period ratios between the planets. Even with differences in planetary masses, it is clear that planet pairs with smaller period ratios are more compact and can be more strongly perturbed throughout their dynamical history than those with larger period ratios. We see in Fig. 1 that there is an obvious decrease of planet pairs towards small period ratios (<1.5), which may be caused, at least in part, by dynamical instability – a conjecture we investigate here.

Izidoro et al. (2017) studied the influence of dynamical instability on the period ratio distribution of multiplanet systems starting from compact resonant chains. But in this paper we conduct numerical simulations on non-EMS planetary systems with uniformly distributed initial period ratios which we then evolve to determine the role that instability plays in shaping the final period ratio distribution. By comparing the final distribution to the observed distribution, we should gain insight not only into the effects of dynamical instability, but also into the planet formation process generally. That is, at least a portion of the difference between our simulations and the observations must be a consequence of the formation process itself, independent of the system’s subsequent dynamical evolution.

We describe our simulation techniques and the initial conditions in Section 2. In Section 3, we analyse the factors that influence the dynamical stability of multiplanet systems. The consequences that instability has on the period ratio distribution and a comparison of the probability density function (PDF) between our simulations and the de-biased Kepler observations are presented in Section 4. Finally, our conclusions are outlined in Section 5.
3 THE STABILITY CRITERIA IN MULTIPLANET SYSTEMS

The stability of planetary systems containing more than two planets is more challenging and less well understood than two-planet systems. Here, we study the stability criteria for both two-planet systems and systems with more than two planets. The relationship between period ratio and planetary mass \((m_1 + m_2)/m_0\) of planet pairs in all four kinds of planetary systems after a long time is shown in Fig. 2. We find that the stable planet pairs have period ratios either smaller than 1.05 or larger than 1.1. The two groups of planet pairs remain stable via different mechanisms, which are discussed in the following sections.

For planet pairs with period ratios larger than 1.1, two different criteria are often invoked to determine their stability, either the resonance overlap criteria (Wisdom 1980; Deck et al. 2013) or the Hill stability criteria (Gladman 1993). We find that both criteria are reasonable approximations to the stability cut-off, but that the resonance overlap criteria performs better (it is strictly obeyed in our simulations for period ratios larger than 1.1). For period ratios smaller than 1.05, the systems are stable if they are in the 1:1 MMR.

3.1 Planet pairs in the 1:1 MMR

After an integration time of \(3.65 \times 10^7 t_0\), some planet pairs with period ratios near 1 remain because they are protected by the 1:1 MMR. Co-orbital configurations have been studied extensively, especially in planet–satellite systems (Dermott & Murray 1981a, b; Yoder et al. 1983; Tabachnik & Evans 2000; Christou & Asher 2011). These insights are also applied to the problem where a terrestrial planet co-orbits with a gas giant (Dvorak et al. 2004; Erdi & Sándor 2005; Beaugé et al. 2007). More general problems such as two comparable planets in 1:1 resonance have also been studied (Laughlin & Chambers 2002; Nauenberg 2002).

Of the stable, co-orbital planetary systems, planet pairs with initial differences of mean longitude far from 180° and period ratios very close to 1 evolve in tadpole orbits (shown in the left-hand panel of Fig. 3), while planet pairs with period ratios slightly farther from 1 have horseshoe orbits (shown in the right-hand panel of Fig. 3). The fraction of tadpole orbits among all co-orbital configurations is about 25 per cent. (Recall that all of these co-orbital systems were generated randomly from our distributions of initial parameter values).

Planet pairs in systems with more than two planets account for 87 per cent of all co-orbital configurations. Hence, co-orbital planets are also likely to be stable in multiplanet (\(N > 2\)) systems. We check and find that planet pairs survived 1:1 MMR generally have period ratios <1.03. For planetary systems with more than two planets, the period ratio between the co-orbital pair and their closest companion should be larger than 1.33 to ensure the stability of the co-orbital pair. The resonant angle \(\phi = \lambda_2 - \lambda_1\) of planet pairs in tadpole orbits (where 1 and 2 represent the two planets in the resonance) oscillates within a small range and one planet never crosses the \(L_3\) Lagrange point of the other. For planet pairs in horseshoe orbits, the resonant angles oscillate over a large range >180° of values – where one planet crosses the \(L_3, L_4,\) and \(L_5\) Lagrange points of the other planet.

The co-orbital configuration of the two-planet case can be stable for as long as \(3.65 \times 10^7 t_0\) (possibly longer), for both the tadpole and the horseshoe orbits. Tabachnik & Evans (2000) showed that the Earth tadpole can be stable for as long as \(10^9\) yr, while horseshoe orbits are generally considered less stable than tadpole orbits (Dermott & Murray 1981b). Laughlin & Chambers (2002) suggested that the horseshoe configuration can be stable for a long time if \((m_1 + m_2)/m_0 \leq 2 \times 10^{-4}\), which is the case for our simulations. Although co-orbital planets were not found by \(Kepler\) (Janson 2013), Ford & Gaudi (2006) and Leleu et al. (2017) proposed a method to detect them by combining transit and radial velocity measurements. This method may have different detection sensitivities that may enable their discoveries in the future. Nevertheless, if such planet pairs were common, they would likely have been detected by \(Kepler\)– especially in high signal-to-noise cases. There are a few planet candidate systems that appear to have small period ratios such as KOI-284, KOI-521, and KOI-2248. However, these systems show signs of being false positives, or (as in the case of KOI-284) false multis – where the signal is actually from two separate planetary systems in a stellar binary (Lissauer et al. 2014). Thus, we find it unlikely that co-orbital planet pairs are a common byproduct of planet formation.

3.2 Stability of planet pairs with period ratio > 1.1

We now turn from planets in the 1:1 MMR to pairs in multiplanet systems that have larger period ratios. Previous works (Chambers et al. 1996; Zhou et al. 2007; Smith & Lissauer 2009; Funk et al. 2010; Pu & Wu 2015; Morrison & Kratter 2016; Obertas et al. 2017) have shown that the mutual separation in units of mutual Hill radius,
$K$, is one indicator of the instability time-scale of EMS planetary systems. Gladman (1993) showed that for two-planet systems, the minimal $K$ required to remain stable is $\sim 3.5$. Fig. 4 shows the initial and final $K$ distributions for systems with two or more planets. The initial values of $K$ are distributed between 0 and 40. After $3.65 \times 10^7 t_0$, however, $K$ of the remaining pairs are either very close to 0 or larger than the predicted stability cut-off of 3.5. For EMS planetary systems, numerical simulation results in Obertas et al. (2017) show that five-planet systems can survive at least $10^9 t_0$ for $K \geq 8.5$. The criterion $K > 3.5$ between each planet pair alone cannot ensure the stability of the multiplanet $(N > 2)$ systems. For non-EMS planetary systems, we investigate whether $K$ between each planet pair, or some other statistic derived from $K$, best characterizes the stability of the multiplanet systems in the following sections.

### 3.2.1 Factors that determine the stability in multiple planet systems

We consider three statistics derived from $K$: the minimum $K$ in a system ($K_{\text{min}}$), the harmonic mean value of $K$ ($K_{\text{hmn}}$), and the arithmetic mean value of $K$ ($K_{\text{avg}}$). Generally, the minimum mutual separation ($K_{\text{min}}$) represents the local compactness of the planetary system, with the other two means gradually transitioning between local compactness and global compactness (the harmonic mean is the smallest of the three Pythagorean means and the arithmetic mean is the largest). The stable rates of planetary systems at different $K_{\text{min}}$, $K_{\text{hmn}}$, and $K_{\text{avg}}$ are shown in Fig. 5. Here, our measure of the stability of a planetary system is whether or not the planetary orbits remain near their initial values throughout the integration. That is, $|P_f - P_i| < 0.01P_i$, where $P_i$ and $P_f$ represent the initial and final orbital period, respectively.

We see that the stable rates increase with all three statistics $K_{\text{min}}$, $K_{\text{hmn}}$, and $K_{\text{avg}}$. Once $K_{\text{min}}$, $K_{\text{hmn}}$, or $K_{\text{avg}}$ exceeds a particular critical value (noted as $K_{\text{min, crit}}$, $K_{\text{hmn, crit}}$, and $K_{\text{avg, crit}}$, respectively), the stability rates are 100 per cent, meaning that the planetary system is stable for at least $3.65 \times 10^7 t_0$. The critical values for the three statistics of $K$ are shown in Table 1. For the two-planet systems, the critical values of the $K$’s are all near four with uncertainties of 0.4 – slightly larger than the traditional 3.5. Part of the reason for this larger cut-off may be that we have very few samples of planetary systems around 3.5, and our stability criteria is quite restrictive. Increasing the number of planets within one system increases the critical values of $K_{\text{min}}$, $K_{\text{hmn}}$, and $K_{\text{avg}}$.

### 3.2.2 Lower limit of instability time-scale determined by $K_{\text{min}}$

In this section, we calculate the instability time-scale when a first close encounter occurs in our simulations. Fig. 6 compares our results to the results from EMS systems in Chambers et al. (1996), Obertas et al. (2017), and Rice et al. (2018). We can see that the instability time-scales for these systems have a large scatter, even at the same $K_{\text{min}}$, $K_{\text{hmn}}$, or $K_{\text{avg}}$. However, the lower limit of the instability time-scale at different $K_{\text{min}}$ is consistent with the value calculated in EMS systems. At $K_{\text{min}} > 2$, we can determine a lower bound on the stability time-scale for the system. When using $K_{\text{hmn}}$ and $K_{\text{avg}}$, the estimated instability time-scale no longer yields a good lower bound on the measured time-scale, especially for $10 < K_{\text{avg}} < 20$ where the instability time-scale varies between 1 and $3.65 \times 10^7 t_0$. However, we can estimate the upper bound of the instability time-scale with $K_{\text{min}}$ or $K_{\text{avg}}$. A combination of $K_{\text{min}}$ and $K_{\text{hmn}}$ (or $K_{\text{avg}}$) would yield the variation in instability time-scale. Pu & Wu (2015) also conduct numerical simulations on non-EMS...
The instability time-scales of the three groups are shown in Fig. 7. The instability time-scale is well determined by $K_{\text{min},\text{crit}}$, as shown in Obertas et al. (2017). We consider three scenarios. Group 1: we adopt EMS systems where the planet systems are sculpted by their dynamical evolution. Here, we study the variation of instability time-scale of one planetary system can be roughly determined with $K_{\text{min}}$ and $K_{\text{hmn}}$ (or $K_{\text{avg}}$). Nevertheless, with the ability of determining the lower limit of the instability time-scale, $K_{\text{min}}$ performs better than $K_{\text{hmn}}$ and $K_{\text{avg}}$ as a stability criteria in combination with the analysis in previous paragraphs.

4 PERIOD RATIO DISTRIBUTION

Planet pairs with small $K_{\text{min}}$ likely collide with each other or are scattered, and as a consequence, the architecture of multiplanet systems are sculpted by their dynamical evolution. Here, we study the final period ratio distribution of the systems after $3.65 \times 10^7 t_0$. The initial and final period ratio distributions of all planetary systems are shown in the upper panel of Fig. 8.

We can see in that figure that planet pairs with period ratios smaller than 1.05 or larger than 1.1 remain stable. Planet pairs with period ratios near 1 are protected by the 1:1 MMR, as discussed in Section 3.1. The number of stable planetary systems increases with period ratio between 1.1 and 1.33, after which the distribution is

Table 1. Critical values of different statistics of $K$ for planetary systems containing different number of planets (the second, third, and fourth row) and fraction of planetary systems meeting the criteria among all stable planetary systems (the fifth, sixth, and seventh row).

|       | $N = 2$          | $N = 3$          | $N = 4$          | $N = 5$          |
|-------|-----------------|-----------------|-----------------|-----------------|
| $K_{\text{min},\text{crit}}$ | 4.0 ± 0.4       | 6.6 ± 0.3       | 7.1 ± 0.3       | 7.1 ± 0.3       |
| $K_{\text{hmn},\text{crit}}$ | 4.0 ± 0.4       | 6.8 ± 0.4       | 11.6 ± 0.3      | 13.6 ± 0.3      |
| $K_{\text{avg},\text{crit}}$ | 4.0 ± 0.4       | 16.2 ± 0.4      | 20.2 ± 0.3      | 22.9 ± 0.3      |
| $K_{\text{min}} > K_{\text{min},\text{crit}}$ | 99.2% ± 0.5%    | 88.2% ± 1.0%    | 80.7% ± 2.0%    | 75.8% ± 2.7%    |
| $K_{\text{hmn}} > K_{\text{hmn},\text{crit}}$ | 99.2% ± 0.5%    | 97.9% ± 1.3%    | 82.9% ± 2.0%    | 72.5% ± 2.4%    |
| $K_{\text{avg}} > K_{\text{avg},\text{crit}}$ | 99.2% ± 0.5%    | 69.1% ± 2.5%    | 38.9% ± 2.4%    | 15.9% ± 1.5%    |
Dynamical instability implication

Figure 8. The upper panel is the initial (shown in grey) and final (shown in purple) period ratio distribution from our simulations described in Section 2 where the planetary mass are Rayleigh distributed with \(\sigma_m = 6 m_\oplus\). The middle panel is the initial (grey) and final (purple) period ratio distribution from the simulations where the planetary mass are Rayleigh distributed with \(\sigma_m = 30 m_\oplus\). The lower panel is the period ratio distribution of the confirmed Kepler planet pairs. The vertical dashed lines indicate planet pairs near the first-order MMRs and period ratio of 2.17. \(t_0\) is the initial orbital period of the inner most planet.

almost flat. Additionally, we see that there are dips on the near side and peaks on the far side of the first-order MMRs, including 2:1, 3:2, 4:3, 5:4, 6:5, and 7:6. This result is similar to the observed period ratio distribution (lower panel of Fig. 8), except that the width and depth of the gap on the near side of the MMRs are smaller than those in the observation. Also, there is no significant feature at period ratio of 2.17 in the simulation. Period ratio distribution from Pu & Wu (2015) also shows asymmetry features around MMRs, but there is no obvious peaks on the far side of MMRs in their simulations. We investigate how these features were produced in the following paragraphs.

4.1 Period ratio asymmetry near first-order MMR

The behaviour of two accreting planets near the first-order MMRs 2:1 and 3:2 has been studied by Petrovich et al. (2013). They found that the period ratio distribution develops an asymmetric dip-peak structure near the resonance. In our simulations, this feature appears in both two-planet systems and systems with more than two planets, although the planetary mass is fixed during the evolution. We find that planet pairs with initial period ratios near MMR are likely to have final period ratios larger than their initial values.

One example of a planet pair in the two-planet system with an initial period ratio of 2.0 is shown in the upper panel of Fig. 9. As its period ratio evolves due to mutual interaction, the pair tends to stay on the far side of the 2:1 MMR. To better describe this property, we define the average difference between the period ratio during the evolution and the initial period ratio as \(P_s = \frac{\sum_{i=1}^n (p_{ri} - p_{r0})}{n}\), where \(n\) represents the number of data that is output during the simulation, \(p_{ri}\) represents the period ratio of the \(i\)-th output from the simulation, and \(p_{r0}\) represents the initial period ratio. If \(P_s > 0\), then the period ratio is more likely to be larger than its initial value. We show \(P_s\) at different period ratios for two-, three-, four-, and five-planet systems in Fig. 10. We find that there are significant peaks of \(P_s\) at period ratios 7:6, 6:5, 5:4, 4:3, 3:2, and 2:1 – especially for the two-planet system. A consequence of this feature is that whenever we measure the period ratio distribution, there is excess probability that period ratios initially on the near side of the MMRs will be seen on the far side.

Petrovich et al. (2013) proposed that the equivalent width of the peaks/dips is proportional to the planetary mass. To verify this conclusion, we choose the same two-planet system shown in the upper panel of Fig. 9 to conduct an additional set of simulations. We slowly increase the total mass of the two planets and calculate the median value of the period ratio \(P_m\) during the evolution. The total mass is randomly split between the two planets. We find that \(P_m\) does increase with the planetary mass, in agreement with their work (see the lower panel of Fig. 9).
They also suggest that planetary mass should be in the range of 20–100 $m_\oplus$ in order to explain the structure near 3:2 and 2:1 MMRs in the Kepler observation. Such masses are much larger than the masses we use in our simulations and are larger than the planetary mass obtained for the typical Kepler system as measured with transit time variations (Hadden & Lithwick 2017). To verify that this dip-peak structure persists over a longer evolution time, we integrate the five-planet systems up to $3.65 \times 10^8 t_0$ for a comparison. We found that the two results are similar as all of the features remain (except for an additional 18 systems that go unstable).

4.1 Varying the planetary mass distribution

As mentioned above, the planetary masses in our simulations are too small to fully explain the observations with this mechanism. In this section, we used Rayleigh distributed planetary masses with $\sigma_m = 30 m_\oplus$. (The average value of planetary mass is increased by a factor of five from the previous section.) The other parameter distributions remain the same. The final period ratio distribution for these simulations is shown in the middle panel of Fig. 8. We see that the widths and depths of the dips near the first-order MMRs are larger than those of the smaller planetary mass with $\sigma_m = 6 m_\oplus$ – especially for the 2:1 and 3:2 MMRs where the gap on the near side is only slightly smaller than the observations.

Additionally, the increase of planetary mass by a factor of five leads to a decrease of $K$ by a factor of 1.7 from the original values, substantially reducing the instability time-scale (particularly for planet pairs with period ratios between 1.1 and 1.5). Since the planetary masses observed by Kepler are rarely this large, the mechanism we present here can only account for a portion of the observed asymmetry in the period ratio distribution near MMR. In addition, we note that the shallower period ratio distribution for small period ratios could be used to constrain the planetary masses observed in Kepler systems – though the constraint from TTV observations is likely more stringent.

4.1.2 Varying the eccentricity distribution

We now consider the effects of larger initial eccentricities and inclinations. In previous sections, the orbits of the planets are nearly circular and co-planar with eccentricities and inclinations $\sim 10^{-3}$. Here, we use eccentricity and inclination distributions of $\sigma_{e,i} = 0.01$ and $\sigma_{e,i} = 0.05$. Again, other parameter distributions remain the same with those described in Section 2. The final period ratio distributions are shown in Fig. 11. The results of the simulations with $\sigma_{e,i} = 0.01$ are similar to those of $\sigma_{e,i} = 10^{-3}$, except that the peak on the far side of the 3:2 MMR is not as strong. However, when $\sigma_{e,i}$ increases to 0.05, peaks and dips near MMRs almost disappear, as shown by Xie (2014) that the asymmetry features around MMRs will become weaker with increasing eccentricity.

Planets with higher eccentricities tend to be more unstable when their period ratios are between 1.1 and 1.7 than in the small eccentricity and inclination cases. This is both because the increased eccentricity yields a higher probability that two planets have close encounters and because the resonance width increases with eccentricity (Deck et al. 2013; Hadden & Lithwick 2018), so resonance overlap is more likely to occur. We compare our simulations to the resonance overlap criteria from Hadden & Lithwick (2018) and find that our results conform to that stability criteria. Thus, distributions of eccentricity and inclination with $\sigma_{e,i} = 0.05$ are too large for planet pairs to produce the observed features. Moreover, the larger eccentricities and inclinations yield a period ratio distribution that is more shallow between 1.1 and 1.5 than the observations (similar to what occurred with larger mass planets from the previous section).

We investigate the constraints that can be placed on the eccentricities from this feature in a later section.

4.2 The probability density function of the period ratios

4.2.1 De-biased period ratios of the Kepler planets

We have shown that (at least a portion of) the asymmetry feature near MMRs can be produced via planetary dynamics originating from a distribution that lacks those features. In this section, we compare the PDF of period ratios between the observed Kepler data and our simulations. As expected, Kepler observations contain geometric bias and pipeline incompleteness (Ragozzine & Holman 2010; Borucki et al. 2011; Lissauer et al. 2011; Ciardi et al. 2013; Steffen & Haw 2015; Brakensiek & Ragozzine 2016; Coughlin et al. 2016). Steffen & Haw (2015) suggest that the influence of pipeline incompleteness, compared to the geometric bias, is the smaller of the two effects so we only consider the geometric bias here.

We have a total of 583 confirmed planet pairs with period ratio $<5$ from the Q1–Q17 DR25 catalogue. To avoid the influence of very long-period planets, which can significantly affect the distribution if not treated correctly, we cut off the sample with $a/R_s < 150$. According to previous studies (Lissauer et al. 2011; Fang & Margot 2012; Tremaine & Dong 2012; Fabrycky et al. 2014), Kepler multiplanet systems are rather flat, so we assume the mutual inclination of planets in a system are Rayleigh distributed with $\sigma \sim 1.5$, similar to Steffen & Haw (2015). We use the CORBITS algorithms from Brakensiek & Ragozzine (2016) to calculate the probability of detecting the outer planet given that the inner planet is detected. The inverse of the probability is adopted as the weight of the planet pair. Finally, we construct a kernel density estimator of the period ratio distribution. For each period ratio, we use a Gaussian distribution with the median value $\mu$ equal to the period ratio $Pr$ and the standard deviation $\sigma$ to be $0.0005Pr$. The total area of the Gaussian distributions is normalized to 1.

The PDF of the observed period ratio and the de-biased period ratio distributions are shown in the lower panel of Fig. 12. After
and 2.17. 3.65 of the period ratio of four-planet systems with an integration time of 3:2 MMR) which are also seen in Fig. 4 of Brakensiek & Ragozzine as significant as the original ones (especially for the peak near the de-biasing, the peaks near 3:2 and 2:1 MMRs persist, but they are not represented relative to what dynamical stability would otherwise allow.

We find that the PDF of our simulation and the observations (the red and yellow curves in the lower panel of Fig. 12) roughly coincide between period ratios of 1.5 and 2.1. For the deficit of planet pairs with period ratios between 1.1 and 1.5, the data show fewer systems than what our simulations suggest could survive. However, given our limited integration time, there may be some residual instabilities that have not had time to manifest. Rather than continuing to integrate all planetary systems to a longer time, we simulate a set of four-planet systems and integrate them to $3.65 \times 10^7 t_0$ (100 times longer than the previous simulations). We choose the four-planet systems for further integration because the shape of the PDF for four-planet systems roughly resembles the shape of the PDF for the whole samples (see upper panel of Fig. 12). The new simulations contain five hundred four-planet systems with the parameter distributions described in Section 2.

The re-normalized PDF of the new simulations is shown as the orange curve in the lower panel of Fig. 12. We see that the shape of the new PDF changes very little when compared to that of the previous simulations (the red curve in the lower panel of Fig. 12). Therefore, we suspect the shape of the PDF at period ratio $>1.5$ in the observation is not entirely due to instability, but may also be influenced by the initial eccentricity distribution, which we will discuss later.

For planet pairs with period ratio $>2.1$, there is an obvious deficit in the observations when compared with the prediction of planet pairs that would otherwise survive given our simulations. Since systems with period ratios this large should be stable for very long time (i.e. longer than the age of the Universe), these results indicate planet pairs do not emerge from the protoplanetary disc with those period ratios to the same degree that they do with smaller period ratios, at least for systems like those observed by Kepler. Thus, whatever formation or dynamical processes are ongoing while the protoplanetary disc is present, the frequency of planet pairs that are produced with period ratios between 2.1 and 3 is 30–50 per cent lower than the frequency of those produced between 1.5 and 2.1.

The sizable fraction of planet pairs that survive in the 1:1 MMR is at odds with the lack of observed planet pairs in those orbits. This discrepancy likely indicates that planets either rarely form or are rarely driven into those configurations – if they did form, a large fraction would have survived. It is possible that such planet pairs have been missed by the transit search algorithms, but the high signal-to-noise ratios of many of the Kepler detections makes this explanation difficult to justify in most cases. (Though, we recommend revisiting the Kepler discoveries with this in mind.)
planet pairs with orbital period ratios 

4.2.3 Eccentricity of multiplanet systems when gas disc dissipates

We showed that the observed period ratios between 1.1 and 1.5 cannot be explained by the effects of instability with initial orbits that are nearly circular. But we see from Fig. 11 that orbital eccentricity drives more planet pairs with small period ratios into instability. While the eccentricities of planets are likely to grow during the dynamical evolution following the dispersion of the gas disc, we can constrain the maximum initial eccentricity by comparing the shape of the PDFs between the observations and our simulations using different values of initial eccentricity. We conduct a set of simulations with the same orbital parameters as those described in Section 2, except for the eccentricity and the inclinations. The integration time is $3.65 \times 10^7$ $t_0$. The results are shown in Fig. 13.

We see from these simulations that systems with initial eccentricities and inclinations $\sigma_{e,i} \approx 0.03$ are roughly consistent with the observed period ratios between 1.1 and 1.5 while larger values of initial eccentricity do not match the profile of the observed distribution. Thus, the eccentricity and inclination distributions should have typical values $\sigma_{e,i} < 0.03$ when the gas disc dissipates. Xie et al. (2016) proposed $\epsilon = 0.04 \pm 0.04$ for multiplanet systems, which places an upper limit to the initial eccentricities around 0.04.

Our prediction that $\sigma_{e,i} < 0.03$ is consistent with their limit.

5 CONCLUSION

In this paper, we studied non-EMS multiplanet systems to investigate their stability and the evolution of their period ratio distribution. In contrast to previous works, which assume the planets have equal mutual separation after the disc dissipates (Chambers et al. 1996; Zhou et al. 2007; Smith & Lissauer 2009; Obertas et al. 2017), we begin with the premise that the orbital periods between adjacent planet pairs in multiplanet systems are uniformly distributed. Thus, any differences between the observed distribution of period ratios and the results of our simulations are likely due to some physical process other than dynamical instability.

After an evolution time of $3.65 \times 10^7$ $t_0$, we find that surviving planet pairs with orbital period ratios $<1.1$ are protected by the 1:1 MMR (both in two-planet systems and systems with more than two planets). These planets can be stable for $3.65 \times 10^8$ $t_0$ or longer whether in tadpole or horseshoe orbits. Thus, the lack of co-orbital planet pairs in the observations indicates that either such planets are difficult to detect (which seems unlikely), or are rarely produced in planetary systems similar to those seen by Kepler. If there was a viable mechanism to produce a large population of 1:1 MMR planet pairs, many would survive and should be seen.

For planets far from the 1:1 MMR, the lower limits of their stability time-scales determined by $K_{\text{min}}$ are consistent with what is predicted in EMS systems. While planets in our simulations are not of equal mass, the differences between them are within one order of magnitude, our results should be largely unchanged as the Hill radius depends only weakly on planetary mass. Of the statistical quantities we studied to characterize instability time-scales, we find that $K_{\text{min}}$ performs most consistently.

Our period ratio distribution shows a dip-peak asymmetry near first-order MMRs, where more planets are on the far side of the resonance than near side. We find that period ratios that are initially on the near side of these resonances are observed on the far side of the resonance more often due to their orbital evolution. This result may partly explain the observed features near MMR in the Kepler data. (Period ratios farther from the first-order MMRs do not show such asymmetries in their orbital evolution.) This deviation of the period ratio near MMR increases with planetary mass. Petrovich et al. (2013) proposed that in order to explain the observed asymmetric structure, the planetary mass should be in the range of 20–100 $m_{\oplus}$. However, the TTV-determined masses in Hadden & Lithwick (2017) are too small to account for the dip-peak feature of the Kepler systems. We also investigate the influence that eccentricity can have on the period ratio distribution and find that the dip-peak structure depends inversely upon the eccentricity of the planetary orbits – larger eccentricities show smaller asymmetry. A non-zero initial eccentricity distribution with $\sigma_e = 0.05$ is too large to produce the dip-peak structure.

Finally, we compare the PDF of the de-biased period ratio distribution of the Kepler observation to our simulations. We find that the general shape of the period ratio distribution less than $\sim 2.1$ can be explained by dynamical instability of planetary systems with non-circular orbits with initial eccentricities $\lesssim 0.03$. This same eccentricity preserves the asymmetry features near MMR while larger eccentricities simultaneously alters the resulting period ratio distribution removes the asymmetries. (We note, however, that the asymmetries near MMR may not be caused by the mechanism we present here). Local features near MMR and near 2.17 (Steffen & Hwang 2015) may require unique explanations.

We also find an obvious deficit of planet pairs with period ratios $\gtrsim 2.1$ in the Kepler data (the deficit is nearly 50 per cent of what would survive if they were initially present). Thus, we suspect that planet pairs are either not formed as often with these period ratios, or if they are produced, that interactions with the gas disc may drive them to smaller period ratios. For example, it may be that the initial distribution of period ratios is essentially flat, but that $\sim 25$ per cent of the planet pairs eventually converge to period ratios between 1.5 and 2.1 – producing the two-plateaus shown in Fig. 12.

Kepler planetary systems are often portrayed as compact since planet pairs typically have small period ratios and orbit close to their host star. However, the criteria for describing a system this way is ill defined. Dynamical processes for planetary orbits are scale invariant, where resonance or other effects occur near certain period ratios regardless of the overall size of the system. Only when some new physical scale enters the description is the invariance broken and the dynamics changed. The results from Rice et al. (2018) indicate that dynamical effects related to instability are not markedly different between systems at 0.1 au (where most Kepler planets are found) and at 1 au where the Solar system terrestrial planets are found – though more work on this issue is warranted.
The only scale where planetary system architecture is seen to change in the observations of Kepler planets is when the inner planet has an orbital period less than a few days (~0.05 au; Steffen & Farr 2013; Steffen & Coughlin 2016). Moreover, period ratios observed in the Solar system are similar to period ratios observed in most Kepler systems. With the exception of the Jupiter/Mars ratio, Solar system period ratios lie between 1.5 and 3 with the majority being less than 2.5. Thus, unless the Solar system is considered to be ‘compact’ there is little to suggest that the typical Kepler planetary system should be so described.

This work shows that instability plays a significant role in sculpting planetary system architectures for period ratios less than 1.33 (see lower panel of Fig. 12). These results suggest that a reasonable criterion for ‘compactness’ could be that for a system to be considered compact, it must contain a planet pair with a period ratio less than this value. For the Kepler multiplanet systems, this criterion would classify roughly 4 per cent of the systems as compact (or roughly 6 per cent of systems containing more than two planets – which may be more representative of multiplanet systems generally).

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APPENDIX A: PERIOD RATIO DE-BIASING

A1 Two kinds of weight calculation

In Section 4.2, we discuss the PDF of the de-biased period ratios. The weight of each period ratio is assumed to be the inverse of the transiting probability of the outer planet given that the inner planet is transiting. This is the method used in Steffen & Hwang (2015). We record the PDF calculated this way as $F_{\text{out}}(\text{in})$.

Another weighting method could be the inverse of the probability when both planets are transiting the host star directly, $F_{\text{out} \& \text{in}}$. It...
is not obvious which of these approaches is correct. The assumption in Steffen & Hwang (2015) is that you would not detect a planet pair if you had not detected the inner planet in that pair—hence their use of $\mathcal{F}_{\text{out}}$. Either way, we show the PDF of the original $\mathcal{F}_{\text{orig}}$ and the two kinds of de-biased period ratios $\mathcal{F}_{\text{out}&\text{in}}$, where the weight is calculated as the inverse of the probability when both planets are transiting the host star. The lower right-hand panel represents the de-biased PDF of period ratios $\mathcal{F}_{\text{out}&\text{in}}$, where the weight is calculated as the inverse of the transiting probability given that the inner planet is transiting. The dashed vertical lines show the period ratios at 1.52, 1.85, 2.04, and 2.17.

Another interesting peak is 2.17. It exists in $\mathcal{F}_{\text{orig}}$ and $\mathcal{F}_{\text{out}&\text{in}}$ for all orbital periods, but for $\mathcal{F}_{\text{out}&\text{in}}$, the peak at 2.17 disappears once we include planet pairs with orbital period $>130 \text{ d}$. We checked the samples with orbital period ratios near 2.17 and find that they mainly constitute of planet pairs with orbital periods between 10 and 20 d. Hence, in the calculation of $\mathcal{F}_{\text{out}&\text{in}}$, the inclusion of planet pairs with long orbital periods increases the weight of other period ratios and simultaneously reduces the weight of the period ratio at 2.17. Thus, it may be that the process that creates the feature at 2.17 is something that occurs only in the innermost parts of the protoplanetary disc.

### A2 Influence of mutual inclination between planet pairs

In Section 4.2, we assumed that the mutual inclination of planets in a system are Rayleigh distributed with $\alpha \sim 1.5^\circ$ (noted as the co-planar case). However, Zhu et al. (2018) proposed that the dispersion of planetary inclinations within a given system is a function of its number of planets, i.e., $\sigma_{i,N} = \sigma_{i,5}(N/5)^{\alpha}$, where $\sigma_{i,5} \approx 0.8^\circ$, $-4 < \alpha < -2$. Based on this inclination distribution function, we recalculate the transiting probability of each planet pair assuming an extreme case where $\alpha = -4$ (noted as the inclined case). The weight of each period ratio is calculated using the method described in Section 4.2. The PDF for the inclined case, the co-planar case and our simulation are shown in Fig. A2. Compared to the co-planar case, the PDF at period ratio $>2$ for the inclined case increases, while the PDF at period ratio $<2$ decreases. It is because that most of planet pairs with period ratio $>2$ are from two-planet systems. If we assume a larger mutual inclination for two-planet systems than planetary systems with higher multiplicity, the weight of period ratio $>2$ will increase, which leads to the increase of PDF at period ratio $>2$. Nevertheless, there is still deficit of planet pairs in the observation at period ratio $>2.1$.