Comparison Between Robust and Stochastic Optimisation for Long-term Reservoir Management Under Uncertainty

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Abstract Long-term reservoir management often uses bounds on the reservoir level, between which the operator can work. However, these bounds are not always kept up-to-date with the latest knowledge about the reservoir drainage area, and thus become obsolete. The main difficulty with bounds computation is to correctly take into account the high uncertainty about the inflow to the reservoir. We propose a methodology to derive minimum bounds while providing formal guarantees about the quality of the obtained solutions. The uncertainty is embedded using either stochastic or robust programming; we derived linear models, which ensures good performance to compute the solutions.

Keywords Long-term reservoir management · Rule curve · Stochastic optimisation · Robust optimisation

1 Introduction

Drinking-water production is a vital need, and tap water is a basic service that may not be interrupted. This water may come from multiple sources, such as groundwater or dams (surface water). Utility managers cannot afford lacking water to inject in their distribution system, and this is one reason that impelled them to build large dams and reservoirs. Those serve also multiple other purposes, such as hydropower [10] or flood control [12].

These reservoirs must deliver the expected level of service with a very low probability of failure. A common management technique is to use predefined rules, i.e. rule curves, indicating upper and lower bounds on the level of the reservoir at any period of time throughout the year [14]. Modern operation

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systems usually prefer real-time control [23], with no predefined rules, but this is harder to implement than traditional rule curves.

Rule-curve-based management is expected to take full benefit of the potential of the reservoirs. However, the rule curves must be periodically updated; otherwise, they become less reliable over time and may eventually fail to provide the expected service. The quality degradation of these operational rules is mostly due to external evolution—be they in the required amount of water or in climate change.

In this context, formal optimality guarantees of the rule curve are an asset for the operator: if there is any lack of water, the operator is able to prove that every possible action was taken to prevent this situation. To this end, mathematical optimisation can be used to derive the rule curves; with a well-defined and reproducible computational framework, the rule curves can also be regularly updated, and thus remain relevant, even under external evolutions. Most current optimisation techniques [21] are based on evolutionary computations [3] (mainly genetic algorithms [14], sometimes coupled with simulation [25]; newcomers like harmony search are gaining traction in the community [5]). In contrast to them, mathematical optimisation [20,29] can provide global-optimality guarantees while having acceptable computational time requirements (under some assumptions, such as convexity), which this article demonstrates in the case of long-term rule curves.

Whatever the optimisation method, it must take into account the inherent uncertainty in the input data (namely, the inflow). In mathematical optimisation, there are two well-known paradigms to handle uncertainty: stochastic and robust optimisation.

1. The first is the most common one [28], and is also known as explicit stochastic optimisation [13]. It implies that the model directly uses probabilistic information (usually, by sampling the probability density function, i.e. using inflow scenarios for each river). The objective function minimises some risk measure [24], often the expected value of the deterministic function. However, in our case, the maximum function makes more sense than the expectation (see Section 3.1.2 for a full discussion).

2. Recently, another approach has been explored: robust optimisation [22], whose roots lie in mathematical optimisation communities [7]. It considers the uncertain values as pertaining to a so-called uncertainty set and it optimises for the worst case within that set; a common choice is to use the confidence intervals around the average value. It can be considered as a kind of implicit stochastic optimisation (ISO) [13], as the actual model is deterministic, albeit working with data that is adapted to the uncertainty (without any need for hedging the solutions to different sets of possible inflow scenarios, as is usually necessary in ISO approaches, like in [30]).

In this paper, we compare these two paradigms based on a real-world case study of a Belgian dam on the river Vesdre, at Eupen. Historically, a very conservative process was used to compute the rule curves, with a single objective: water supply. We discuss to which extent the two considered approaches, ro-
bust and stochastic optimisation, may contribute to give more freedom to the operator for other purposes, such as hydropower.

The article is structured as follows. After a brief description of the case study in Section 2, the optimisation models corresponding to each uncertainty paradigm are detailed in Section 3. The results of the optimisation process are discussed in Section 4. Finally, conclusions are drawn in Section 5 with further directions to improve this work.

2 Case study

In this study, we consider the Eupen dam (also called the Vesdre dam), which lies on the river Vesdre in Eastern Belgium. The reservoir is fed by its natural drainage area (6920 ha) and by another river (the Helle), from which a diversion tunnel was built. This diversion increases the effective catchment area to 10,595 ha. In normal-operation mode, the tunnel is open and only a minimum environmental flow remains in the river Helle. If the reservoir level reaches its maximum level, the tunnel can be closed, so that the whole discharge of the river Helle is conveyed through its natural riverbed. The dam height is 50 m and the maximum storage capacity is $25 \text{ hm}^3$. Currently, the main purpose of the Vesdre reservoir is to provide drinking water to its surroundings. It can also be used to produce hydropower (2.6 MW) and to help controlling floods and low flows.

For now, its operation uses empirical rules, which are functions of the measured discharges and the weather forecasts, e.g. in order to prevent flooding at some points in the hydraulic network. Drinking-water availability is guaranteed through a minimum rule curve: the reservoir level may not drop below a certain threshold (which varies over the year). This curve was computed once, in the 1980s, using a very conservative process. A worst-case scenario was derived from historical data: each month in this scenario was taken as the driest one in history; then, the lowest acceptable level was determined to ensure the expected-water supply guarantee. In the following, we analyse how stochastic and robust optimisation may contribute to revise this rule curve, by making use of today’s enhanced computational power and more recent data.

We use as main input data the characteristics of the catchment, reservoir, and dam (see Table 3 in appendix), as well as time series of recorded natural inflow discharge to the reservoir and flow rates in the Helle river during 24 calendar years (1992-2015).

3 Methodology

We develop optimisation models to compute the minimum acceptable level in a reservoir to ensure that drinking water availability will not be exposed. More specifically, starting from the beginning of a drought, the available drinking
water storage must be sufficient to enable water supply during a predefined period of time. In the considered case study, the risk level is set such that water production for two years is ensured, but the methodology does not depend on the specific value of this parameter.

3.1 Uncertain reservoir models

Remark 1 In the following mathematical notations, the optimisation variables are indicated by a **bold** font. In other words, the value for these symbols is the result of the optimisation process, and are not fixed beforehand.

3.1.1 Deterministic model

The real-world decision variable is the reservoir level, but it is not directly used in the model: we use the volume of water instead, which is in one-to-one mapping to the reservoir level. This variable is denoted by storage, at time \( t \) (m\(^3\)). The optimisation problem consists mainly of one equation, the mass balance over a time step [28,4,22]:

\[
\text{storage}_{t+1} = \text{storage}_t - \text{output}_t + \text{input}_t, \quad \forall t,
\]

where the inputs correspond to the inflowing rivers, and the outputs to the various dam purposes (drinking water, hydropower, etc.).

The inputs are composed of tributary rivers (the set tributaries) and of diverted rivers (the set diverted):

\[
\text{input}_t = \sum_{r \in \text{tributaries}} \text{flow}_{t,r} + \sum_{r \in \text{diverted}} \text{diverted}_{t,r}, \quad \forall t.
\]

For the considered case, the output from the reservoir is the sum of the drinking water consumption, the minimum environmental flow in the river, and the release that is not useful to fulfil the reservoir's purposes:

\[
\text{output}_t = \underbrace{\text{drinkingWater}_t + \text{environmentalFlow}_t}_{\text{constant (i.e. not fixed by the optimisation process)}} + \text{release}_t, \quad \forall t.
\]

In this equation, several terms are fixed by the operator before the optimisation (drinking water and environmental flow requirements); the only actual decision variables are the releases. Other losses could be taken into account, such as evaporation, but they are negligible in the considered area [17].

The reservoir level may not drop below a minimum threshold, defined such that the operator can still extract water from the reservoir for water supply (it corresponds to the position of water outlets, and not to any risk level). Similarly, it has a maximum level that is fixed beforehand to keep a safety margin in case of floods. These constraints can be expressed as:

\[
\text{minStorage} \leq \text{storage}_t \leq \text{maxStorage}, \quad \forall t.
\]
Likewise, the contributions of the diverted rivers can be decided up to some level, as long as two constraints are respected: a minimum environmental flow must remain in the diverted river (and thus cannot flow into the reservoir), while the maximum discharge capacity through the diversion pipe (due to its dimensions) constitutes an upper bound.

\[
\begin{align*}
diverted_{t,r} & \leq \text{maxDischarge}_r, \quad \forall t, \forall r \in \text{diverted}. \\
diverted_{t,r} & \leq \text{flow}_{t,r} - \text{environmentalFlow}_r, \quad \forall t, \forall r \in \text{diverted}.
\end{align*}
\]

Also, the release from the dam release, has an upper bound, related to the hydraulic capacity of the hydropower plant and the bottom outlets.

\[
\text{release}_t \leq \text{penstockHydropower} + \text{bottomOutlet}, \quad \forall t.
\]

Due to the relative position between the spillway crest and the maximum allowable reservoir level, the spillway is not taken into account in this model.

The goal is to determine an enhanced rule curve, i.e. a new lower bound for the reservoir level: the objective function minimises the total stored volume throughout the year, with all periods having the same weight:

\[
\min \sum_t \text{storage}_t.
\]

This way, the solution is the most critical situation while still being feasible from the beginning to the end of the optimisation horizon.

**Remark 2** This objective function is supported by the fact that, if the solver lowers the value for one time step at the expense of another, the variations for these two time steps have the same effect on the objective value. In other words, at the optimality, it is not possible to decrease the total amount of water that is stored throughout the year, and changing the obtained value for any time step forces to reconsider the solution at other time steps.

All constraints detailed above and the objective are linear: this optimisation problem is thus very tractable (large instances can be solved quickly) [27]. However, the model ignores the uncertainty in the inflow (i.e. flow\(_{t,r}\)); it can only consider one inflow scenario, and optimises over that scenario, which is not necessarily representative of basin’s dynamics. The following sections present two approaches to incorporate the uncertainty into the model while keeping it linear.

### 3.1.2 Stochastic model

A first approach considers the inflow as stochastic, the uncertainty being modelled as a series of scenarios [24]; the rule curve is such that the drinking-water requirement is guaranteed, whatever the inflow scenario is. To optimise the rule curve, we simulate each scenario independently; then, the actual solution to the optimisation problem is the upper envelope of the solutions to the individual scenarios (a convex risk measure, in [24]). This risk-averse modelling
ensures that the computed rule curve is conservative enough, based on the information that is known.

Let $\text{storage}_t^s$ denote the solution at time $t$ for scenario $s$, and $\text{ruleStorage}_t$ the actual value for the rule curve at time $t$. The rule curve must be above the minimum level for any scenario (as it is the maximum of all the solutions), which is translated by the constraint $\text{storage}_t^s \leq \text{ruleStorage}_t$. The complete model is thus:

\[
\begin{align*}
\min & \sum_t \text{ruleStorage}_t \\
\text{s.t.} & \quad \text{storage}_t^{s+1} = \text{storage}_t^s - \text{output}_t^s + \text{input}_t^s \\
& \quad \text{input}_t^s = \sum_{r \in \text{tributaries}} \text{flow}_{t,r}^s + \sum_{r \in \text{diverted}} \text{diverted}_{t,r}^s \\
& \quad \text{output}_t^s = \text{drinkingWater}_t + \text{environmentalFlow}_t + \text{release}_t^s \\
& \quad \text{minStorage} \leq \text{storage}_t^s \leq \text{maxStorage} \\
& \quad \text{diverted}_{t,r}^s \leq \text{maxDischarge}_r \\
& \quad \text{ruleStorage}_t \geq 0 \\
& \quad \text{storage}_t^s \geq 0, \quad \text{output}_t^s \geq 0, \\
& \quad \text{input}_t^s \geq 0, \quad \text{release}_t^s \geq 0 \\
& \quad \text{diverted}_{t,r}^s \geq 0
\end{align*}
\]

**Scenario generation.** An interesting point is the way of generating the scenarios. As a property must be guaranteed for two years, each scenario should be two-year long. They can correspond to two successive years in the historical data (“merging”); another way of generating them is to consider all possible pairs of one-year scenarios (“mixing”). The latter technique destroys any kind of inter-year correlation (as opposed to the first one), even though both keep intra-year correlations.

Traditional stochastic models are multistage [20], i.e. they consider a point in time where the operator is allowed to reconsider their decisions (this includes stochastic dynamic programming). However, this formalism would be detrimental to our use case: our goal is to determine a rule curve that does not depend on the actual scenario, thereby giving the operator a sure lower bound for their reservoir management. This case is more pessimistic than what multistage stochastic optimisation allows: our solution must be valid in all cases at all time steps, while a multistage formulation considers one solution per branch in the scenario tree [11]; as a consequence, the present approach is more suited for strategic planning of the management, while multistage models apply for real-time decisions.

3.1.3 Robust model

The second uncertain model considers the inflow as belonging to an uncertainty set, which we chose to be the confidence interval of the inflow at the corresponding time period based on the historical data. This choice is sometimes called *interval uncertainty* or *Soyster’s model* [8].
The worst inflow to the reservoir can be computed explicitly: it corresponds to the minimum inflow within the confidence interval (i.e. the lower bound). As such, the robust model is very similar to the basic deterministic one that is detailed in Section 3.1.1, except in the way the inflow is chosen: it is no more raw historical data, but rather this worst case, computed with the historical data.

**Confidence intervals.** Multiple ways of computing those confidence intervals (and thus their lower bounds) can be thought of. The simplest approach is probably to separately consider periods of time, and applying a standard statistical technique (e.g., based on t-Student distribution), albeit this has limited physical meaning. Other methods include fitting a dedicated statistical model (such as the one in [2]) and using standard time series analysis procedures (like ARMA [11]).

3.2 Model predictive control (MPC)

Uncertain models as such have one important defect: they guarantee the two-year supply only for the first time step of the solution; for the following ones, the guarantee is limited by the time horizon of the model (i.e. the second time step has a guarantee for two years minus one time step). Using longer scenarios would not solve this issue: for example, with three-year scenarios, the solution for the first time step would be guaranteed for three years, which is more constraining than expected.

To work around this deficiency, we use moving time horizons: the algorithm uses three-year scenarios, but the actual optimisation (performed with an uncertain model, as explained earlier) merely happens on two consecutive years of the scenario; only the solution for the first time step is used to build the final solution (with the two-year guarantees). To construct the complete solution (for one year), this two-year horizon shifts, as depicted in Figure 1.

This technique is called *receding horizon control* [19], and is often used in industrial process control, including beforehand operational-rules computation (which is precisely the case here). It is part of a more generic framework, *model predictive control* (MPC), that has already been used with great success in water resources management, but mostly for real-time control [26,6].

As a consequence, to fully ensure the water supply guarantees, the uncertain models must be solved a high number of times—albeit these are completely independent computations and can be performed in parallel.

Thanks to this algorithm, all obtained solutions are year-to-year continuous: there is no gap in the computed rule curve between the end of a year and the beginning of the next one. The solutions of the basic uncertain models of Section 3.1 do not have this continuity property, which is important for operators to implement the rule curve.
Figure 1: Behaviour of the MPC algorithm. The solution for the first time step $t_1$ has an optimisation horizon limited to the first two years (i.e. ensures the drinking-water guarantee for two years); the one for $t_2$ is computed for a two-year period starting at the second time step, i.e. the optimisation horizon of $t_1$ shifted by one time step.

4 Results

4.1 Outcome of the optimisation

The two described models (stochastic and robust models, both with model predictive control) have been implemented in a Julia package [9], ReservoirManagement.jl (https://github.com/dourouc05/ReservoirManagement.jl) which is freely available on GitHub; it is based on the JuMP mathematical modelling layer [16]. Both models are compared to the existing minimum rule curve in Figure 2. Multiple such curves have been computed for each uncertainty model:

- for the stochastic model: the two scenario generation techniques (merging and mixing);
- for the robust model: multiple confidence intervals (based on a $t$-Student model), starting at 95% and up to 98.5%, with increasing conservativeness (as shown in appendix, Figure 5). Higher confidence levels (99% and beyond) do not allow for any solution: too little water is available in the corresponding scenario.

Overall, the computed rule curves strongly depend on how the uncertainty is handled, i.e. stochastic or robust approach; nonetheless, the curves remain relatively close to the current rule curve; yet, at some time steps, all the proposed models are below the current rule curve. In other words, depending on the way to model the uncertainty, the current rule curve is either too conservative (none of our solutions needs a reservoir level as high as prescribed by the existing rule curve) or marginally unsafe (one model proposes to keep a slightly higher level for about one third of the year).
Figure 2: Comparing the two uncertainty models to the current rule curve: on the one hand, stochastic, with two scenario generation techniques; on the other one, robust.

Figure 3: Closest confidence intervals corresponding to the stochastic solutions.

4.2 Associating confidence levels

Based on both the stochastic and the robust approaches, we can estimate a confidence level of feasibility for the stochastic solutions, by choosing the robust solution that is the closest to the stochastic curve to analyse within a fixed set of confidence levels (between 95% and 99%). This correspondence is shown in Figure 3. We can say that the merging-scenario-generation technique gives a confidence level of approximately 96.5%, the existing rule curve 97.5%, the mixing scenario generation 98%.
Table 1: Analysis of the time periods that define the optimisation results. There are 22 scenarios, of which 9 are support scenarios.

| Considered period of time for the average | Average rank of support scenarios | Average rank of nonsupport scenarios | Average discharge of support scenarios ($10^6$ m$^3$/day) | Average discharge of nonsupport scenarios ($10^6$ m$^3$/day) |
|------------------------------------------|---------------------------------|-------------------------------------|--------------------------------------------------------|--------------------------------------------------------|
| Year                                     | 12.111                          | 11.077                              | 0.981                                                  | 0.857                                                  |
| Wet season                               | 12.889                          | 10.538                              | 1.338                                                  | 1.097                                                  |
| Dry season                               | 14.222                          | 9.615                               | 0.473                                                  | 0.517                                                  |
| Driest six months                        | 11.444                          | 11.539                              | 0.842                                                  | 0.938                                                  |
| Driest three months                      | 13.778                          | 9.923                               | 0.618                                                  | 0.651                                                  |
| Driest month                             | 14.333                          | 9.538                               | 0.418                                                  | 0.488                                                  |

4.3 Most important scenarios for stochastic optimisation

For the stochastic model (Section 3.1.2), there is not a single scenario that fully defines a given solution, as shown in Figure 4a; instead, a limited number of scenarios have an impact on the rule curve (these scenarios may be called support vectors [15]); the other scenarios have no influence on the solution.

Among them, some are closer to the wettest year, others to the driest one (Figure 4b): a low average discharge throughout the scenario does not imply that it yields the most conservative solution; the distribution of the inflow over the year has a substantial influence on the result. This means that limiting the study to the driest years would not be enough to derive a sufficiently reliable minimum rule curve. In contrast, a more important factor is the driest month (as depicted in Figure 4c), as the support scenarios correspond instead to those containing some of the driest months, and these define the solution for a large period of time. Figures 6a to 6d in appendix present similar results for averaging periods of three or six months, and of the wet and dry seasons. The highest correlation is seen for the driest month, but the three driest months and the dry season are similarly correlated, as indicated in Table 1. The support scenarios are not only made up of dry years, but also a few very wet ones. Also, these results suggest that the month is a very relevant time scale for defining the rule curve.

The main conclusion for the optimisation of this analysis is that the time step for the optimisation of this model should never be longer than one month, otherwise the resulting rule curve may miss some important events. It also gives an easy (but approximate) criterion to discriminate support scenarios from redundant ones, based on the driest month contained in the time series.
(a) The colour of the scenario curve corresponds to the average discharge: the bluer curves indicate a wetter year.

(b) The colours correspond to the average discharge during the driest month: the reddest curve indicates the driest month among all the scenarios.

Figure 4: Impact of the scenarios on the solution for a stochastic solver (here, merging is depicted, without MPC). The optimisation curves (i.e. the result of the optimisation and the scenarios) correspond to a 2-year scenario; each of them is plotted against the original rule curve for the first year. The colour scales are relative: dark blue indicates the wettest scenario, dark-red the driest scenario. A dashed line indicates a support scenario.

4.4 Computational time analysis

As shown in Table 2, the computations are very efficient, while they use the whole set of available river discharges (twenty-three hydrological years). The most time-consuming computations took a little more than half an hour. It corresponds to the MPC algorithm being used with a weekly time step and mixing scenario generation, each iteration having to deal with $23^3 = 12,167$
Table 2: Computational efficiency comparison between the different optimisation models. Historical data spans over 23 years. Preprocessing time (such as reading input files) is not included in this benchmark. These tests are performed on a current high-end laptop (Intel i7-6700HQ, 4 cores, 2.6 GHz, 16 GB of RAM) with the solver Gurobi 6.5.0 and the modelling language Julia 0.4.7 (under Windows 10, 64 bits).

| Model                | Time (seconds) | Number of scenarios |
|----------------------|----------------|---------------------|
| Stochastic (merge)   | 1.8            | $23 - 1 = 22$       |
| Stochastic (mix)     | 2140.2         | $23^3 = 12,167$     |
| Robust (95%)         | 0.1340         | 1                   |
| Robust (99%)         | 0.1346         | 1                   |

scenarios. Hence, both formalisms lead to efficient computation, without specific implementation care, which is a positive aspect in terms of scalability of the method.

The time ratio between the merging and mixing scenario-generation techniques is about one thousand, while the size of the optimisation problems (measured by the number of variables and constraints) grows by a factor of approximately five hundred: this difference is explained by the fact that the optimisation solver uses algorithms that do not have a linear complexity in the number of variables \[27\]. So far, no specific mitigation has been implemented to lower the computational cost, such as taking advantage of the decoupled nature of the scenarios (optimisation for each scenario can be performed independently from the others).

5 Conclusion

This article compares two paradigms to take uncertainty into account within mathematical optimisation techniques applied to rule-curve derivation: one is similar to many existing tools (stochastic programming), with scenario generation to help deal with limited data; the other one is more synthetic and directly uses confidence levels (robust programming). Our results show that the current operating guidelines can be improved at some points. Moreover, the dam operator can implement it in such a way that rule curves are regularly updated.

Besides those practical issues, the proposed methodology is easy to implement efficiently enough, while being based on sound mathematical principles. Also, the mathematical models used in this article are exploited to their utmost potential: there is no point in pursuing the research to get better solutions to these models, as the global optimality has been reached. Nevertheless, the models could still be improved to get more detailed results. For example, the discharge through the hydropower penstock and the bottom outlets both depend on the hydraulic head; they are currently considered as constants. An-
other weak point is the computation of inflow confidence intervals, which is crude, and could be greatly enhanced.

Furthermore, this approach is directly applicable to any kind of water demand, such as controlling low flows, as long as there is no uncertainty in this demand; otherwise, another stage of uncertainty modelling is needed, applying the same techniques as developed in this article.

The rule curves have been evaluated with feasibility-related criteria, which are the most relevant in this case for the operator. Nevertheless, other evaluation processes could bring more information about the behaviour of each potential policy with respect to the other dam purposes, such as hydropower like in [4].

The approach can also be extended to handle flooding, by defining a maximum rule curve for normal operations: this lets some free space to store the excess water due to flood events. This could be computed, season per season, by analysing the maximum level so that the usual constraints are not violated (as done in Section [4], with the flood event as input. This requires some flood detection algorithm, such as the one presented in [18].

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A Technical characteristics

Table 3: Characteristics of the catchment, reservoir, and dam.

| Characteristic                                      | Value for the Eupen reservoir |
|-----------------------------------------------------|-------------------------------|
| Capacity                                            | $25 \cdot 10^6$ hm$^3$        |
| Dam height                                          | 66 m                          |
| Natural rivers                                      | Vesdre, Getzbach              |
| Natural drainage area                               | 6920 ha                       |
| Diverted river                                      | Helle                         |
| Diverted drainage area                              | 3675 ha                       |
| Reservoir level for drinking water                  | 343 m                         |
| Reservoir level for chosen flood safety margin      | 361 m                         |
| Spillway crest                                      | 362 m                         |

Table 4: Characteristics of the catchment, reservoir, and dam that are used for the optimisation.

| Characteristic                                      | Symbol            | Value for the Eupen reservoir |
|-----------------------------------------------------|-------------------|-------------------------------|
| Minimum stored volume for drinking water            | $\text{minStorage}$ | $2.25 \text{ hm}^3$          |
| Maximum stored volume for flood safety margin       | $\text{maxStorage}$ | $22 \text{ hm}^3$            |
| Required amount of drinking water                   | $\text{drinkingWater}$ | $55,000 \text{ m}^3$/day     |
| Minimum environmental flow from the dam             | $\text{environmentalFlow}$ | $0.22 \text{ m}^3$/s         |
| Maximum discharge through the hydropower penstock   | $\text{penstockHydropower}$ | $4.5 \text{ m}^3$/s          |
| Maximum discharge through the bottom outlets        | $\text{bottomOutlet}$     | $70 \text{ m}^3$/s           |
| Maximum deviation from the river Helle              | $\text{maxDischargeHelle}$ | $20 \text{ m}^3$/s           |
| Minimum environmental flow for river Helle          | $\text{environmentalFlowHelle}$ | $0.05 \text{ m}^3$/s         |
B Impact of the confidence level on the robust solutions

Figure 5: Impact of the confidence level on the solutions.
C Determination of important years to perform the stochastic optimisation

(a) The colours correspond to the average discharge during the driest three-month period.

(b) The colours correspond to the average discharge during the driest six-month period.

(c) The colours correspond to the average discharge during the dry season (May-September).

(d) The colours correspond to the average discharge during the wet season (October-April).

Figure 6: Supplementary analysis (refers to Figures 4a and 4b).