Viscous gravitational aether and the cosmological constant problem

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Abstract

Recently a notion of gravitational aether is advocated to solve the cosmological constant problem. Through the modification of the source of gravity one finds that the effective Newton’s constant is source dependent so as to provide a simple but consistent way to decouple gravity from the vacuum energy. However, in the original paper the ratio of the effective Newton’s constants for pressureless dust and radiation has an upper bound which is 0.75. In this paper we propose a scheme to lose this bound by introducing a bulk viscosity for the gravitational aether, and expect this improvement will provide more space for matching predictions from this theoretical program with observational constraints.

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I. Introduction

The cosmological constant problem is one of the most important and deepest problem in theoretical physics. In the seminal review [1] Weinberg points out that in order to solve this problem completely one need understand the following two puzzles. One is why the observed value of the cosmological constant is so small comparing with the theoretical value of the vacuum energy predicted by quantum field theory, and the second one is why the matter density and the dark energy density now observed have the same order, which is also dubbed as the coincidence problem. Recently Smolin elaborates the first puzzle into two questions, with one being qualitative and the other quantitative [2]. Namely, why is the cosmological constant not of the order of Planck mass? and why is the observed value is $10^{-120} M_p^2$ after some cancelation from symmetry breaking.

Recently Afshordi proposed an interesting program to solve the first cosmological constant problem by introducing a novel notion of gravitational aether [3]. Following the thermodynamical description of general relativity by Jacobson [4], he conjectured that the usual source term in Einstein’s equation can be modified as $T_{ab} - \frac{1}{4} g_{ab} T$. Then to preserve the successful predictions of general relativity and the conservation of the total energy momentum tensor, or the Bianchi identity, an additional term $T'_{ab}$ is needed on the right hand side of the equation, representing the existence of some sort of gravitational aether. One remarkable result derived from this modification is that the vacuum energy can be decoupled at appropriate scale. Previously another approach to degravitate the vacuum energy has been discussed in [5]. However, when degravitating the vacuum energy from the source, the framework of gravitational aether at the same time puts an upper bound on the ratio of two effective Newton’s constant for pressureless dust and radiation, which is $\frac{G_N}{G_R} \leq 0.75$. In the original paper [3] by Afshordi, it is argued that this ratio would be preferred by the current observation data, for instance from the combination of $Ly - \alpha$ forest and CMB observation [6]. Nevertheless, we think this constraint is far from robust, and as the author noticed, this is a controversial result if comparing it with the result from Big Bang Nucleosynthesis [7], which is given as $\frac{G_N}{G_R} = 0.97 \pm 0.09$ [8]. In particular the latter one is a widely admitted and used quantity by most theoretical physicists and cosmological community. Furthermore, in this framework it is possible to match the data only under the condition that the state parameter of the gravitational aether is very large, namely $\omega' \gg 1$, indicating it is forced to be a sort of
incompressible fluid. Though under some consideration a large value of the state parameter is desirable, we view this as another limitation on the nature of the gravitational aether. Therefore in this paper we intend to loose the constraint on the ratio of effective Newton’s constants in this framework, and expect this improvement can provide more space for further tests so as to keep the consistency of this proposed framework with the observational data.

Our key idea to improve this framework is to generalize the fluid characteristic of the gravitational aether. Since it is still beyond the experimental tests, treating the gravitational aether as a perfect fluid is only an assumption for simplicity. In this paper we propose that the gravitational aether may be treated as a fluid with viscosity. It turns out that this treatment will greatly change the form of the effective Newton’s constant, and thus provide a mechanism to adjust the ratio of the Newton’s constants for matter and radiation.

We organize our paper as follows. In next section we briefly review the key ideas and results for an ideal gravitational aether and see how it can degravitate the vacuum energy from gravity. Then we present our scheme to treat the aether as a viscous fluid in section three, focusing on the possibility of adjusting the ratio of the effective Newton’s constants. In section four we demonstrate that this strategy can be extended to the case with multi-fluids. As a summary, we discuss some open questions and further possible improvement in section five.

II. THE GRAVITATIONAL AETHER AS A PERFECT FLUID

In this section we briefly review the approach proposed in [3] which is supposed to solve the first cosmological constant problem. In this framework Einstein equation in a spacetime with contents of common matter and gravitational aether is modified as

\[
\frac{1}{8\pi G'} G_{\mu\nu} = T_{\mu\nu} - \frac{1}{4} T g_{\mu\nu} + T'_{\mu\nu},
\]  

where \( T_{\mu\nu} \) is the ordinary energy-momentum tensor of matter and \( T \) is its trace, while \( T'_{\mu\nu} \) is the energy-momentum tensor of gravitational aether. Adding such a term is to preserve the divergenceless of the right hand side of Eq.(1), which means

\[
\nabla_\mu T'^{\mu\nu} - \frac{1}{4} g^{\mu\nu} \nabla_\mu T + \nabla_\mu T'^{\mu\nu} = 0.
\]
Since for the usual matter we have $\nabla_\mu T^{\mu \nu} = 0$, the Bianchi identity (2) leads to

$$\frac{1}{4} g^{\mu \nu} \nabla_\mu T = \nabla_\mu T^{\mu \nu},$$

which is a dynamical equation controlling the relations between the matter and the gravitational aether. Consider the homogeneity of spacetime we may have

$$\frac{1}{4} g^{00} \partial_t T = \partial_\mu T^{\mu 0} + \Gamma^\mu_{\rho \mu} T^{\rho 0} + \Gamma^0_{\rho \mu} T^{\rho \mu}.$$  

(4)

In the flat FRW metric with the signature $(-+++)$, we consider the matter ingredient composed of an ideal fluid with a constant state parameter $\omega$, thus $T = -(1 - 3\omega)\rho$. If one assumes that the gravitational aether is also a simple perfect fluid with an energy-momentum tensor as

$$T^{\mu \nu} = p'[(1 + \omega'^{-1})u_\mu u_\nu + g_{\mu \nu}],$$

(5)

then from Eq. (4) we can derive

$$\frac{dp'}{dt} + 3(1 + \omega')Hp' = -\frac{3\omega'}{4}(1 - 3\omega)(1 + \omega)H\rho,$$

(6)

where we have used $\rho = \rho_0 a^{-3(1+\omega)}$ and $H = \dot{a}/a$. Equation (6) plays a key role to degravitate the vacuum energy from gravity. To understand this we write down the cosmological equation from the modified Einstein equation (1),

$$H^2 + \frac{k}{a^2} = \frac{8\pi G_{\text{eff}}}{3} \rho,$$

(7)

with an effective Newton’s constant

$$G_{\text{eff}} = (1 + \frac{T}{4\rho} + \frac{\rho'}{\rho})G'.$$

(8)

Substituting the solution to Eq. (6) into above formula one finds that the effective Newton’s constant becomes

$$G_{\text{eff}} = \frac{3}{4}(1 + \omega)(\frac{\omega'}{\omega'} - \frac{1}{3})G',$$

(9)

where it is required that $\omega' > \omega$ and $\omega' > \frac{1}{3}$ in order to obtain the inhomogeneous solution. First of all, we notice that the effective Newton’s constant depends only on the state parameters of the gravitational aether and matter. In particular the energy density of aether $\rho'$ plays no role in the final version of the modified cosmological equation. One remarkable result which can be read from Eq. (9) is that the effective Newton’s constant would approach
to zero whenever $\omega \to -1$, indicating that the zero-point energy is decoupled from spacetime geometry, known as degravitation. However, the price this mechanism has to pay is that the ratio of the effective Newton’s constants are bounded for various contents of matter. Specifically, for the cases of pressureless dust era and radiation era, one finds that

$$
\frac{G_N}{G_R} = \frac{G_{\text{eff}}|_{\omega=0}}{G_{\text{eff}}|_{\omega=\frac{1}{3}}} = \frac{3(\omega' - \frac{1}{3})}{4\omega'} = \frac{3}{4} - \frac{1}{4}\omega',
$$

(10)

which means the ratio of these two effective Newton’s constant has a maximal value which is 0.75, as the state parameter $\omega'$ goes to infinity. This is a very strong constraint, although in some circumstances this upper bound still falls in a region allowed by recent observations and data analysis.

Is there any plausible mechanism to improve this situation? This is what we intend to do in next section.

III. VISCOUS GRAVITATIONAL AETHER

Our main idea is to generalize the fluid characteristic of gravitational aether, which is treated as a perfect fluid in previous section. However this is just an assumption. If the aether has a bulk viscosity, then its energy-momentum tensor is modified as

$$
T'_\mu\nu = [(\tilde{p}' + \rho')u_\mu u_\nu + \tilde{p}'g_{\mu\nu}],
$$

(11)

where the relation between the pressure and the energy density of the fluid is generalized to

$$
\tilde{p}' = \omega'\rho' + \zeta \theta,
$$

(12)

where $\theta$ is the scalar expansion which is proportional to the Hubble parameter in the context of viscous cosmology, usually set as $\theta = 3H$ in comoving coordinates[9, 10, 11]. $\zeta$ is the bulk viscosity which in general is a function of energy densities of multi-fluids as well as their state parameters, and its specific forms can be found in literature, for instance [10]. Here we take one of them, setting $\zeta = \alpha H$ where $\alpha$ may be a function of state parameters $\omega$ and $\omega'$. Then, with the use of the modified Friedmann equation we can rewrite the pressure as $\tilde{p}' = p' + 3\alpha H^2 \equiv p' + \tau \rho$, where $\tau$ is independent of energy densities, but only a function of $\omega$ and $\omega'$ which should be chosen. In this paper we consider one simple form with $\tau = \beta \omega' \omega(1 + \omega)$ for the viscosity of the gravitational aether, where $\beta$ is just a dimensionless constant. As a result, the pressure has the following form
\[ \tilde{p}' = p' + \beta \omega'(1 + \omega)p. \]  

(13)

Now, with the generalized energy-momentum tensor we turn to Eq.(6) and find it is changed into

\[ \frac{d\rho'}{dt} + 3H[\rho' + \omega'\rho' + \beta \omega' \omega (p + \rho)] = \frac{1}{4}(1 - 3\omega)\frac{d\rho}{dt}. \]  

(14)

When both state parameters \( \omega \) and \( \omega' \) are constants, we obtain a solution to this equation as

\[ \frac{\rho'}{\rho} = (\frac{1}{4} - \frac{3}{4} \omega + \beta \omega')\frac{1 + \omega}{\omega - \omega'}. \]  

(15)

We notice that the ratio between the energy densities of matter and aether keep constant. In contrast to the case of perfect aether, we find the ratio is not vanishing as \( \omega' \to \infty \), but approaching to \( -\beta \omega(1 + \omega) \). Since the modification of energy-momentum tensor will not affect the relation between \( G_{\text{eff}} \) and \( G' \), substituting Eq.(15) into Eq.(8) we obtain the effective Newton’s constant as

\[ G_{\text{eff}} = \frac{1 + \omega}{\omega' - \omega} \left[ \frac{3}{4}(\omega' - \frac{1}{3}) - \beta \omega' \right] G'. \]  

(16)

This is the main result in our paper, and several remarks are given as follows.

1. First of all, viscous gravitational aether does provide a similar mechanism to degravi-
tate the vacuum energy from gravity, as \( \omega \to -1, G_{\text{eff}} \to 0 \).

2. In this case the ratio of the effective Newton’s constants for pressureless dust and radiation changes to the form

\[ R = \frac{G_N}{G_R} = \frac{3(\omega' - \frac{1}{3})}{4\omega'} \left[ 1 + \frac{\beta}{3} \frac{\omega'}{\frac{3}{4}(\omega' - \frac{1}{3}) - \frac{1}{3} \beta \omega'} \right]. \]  

(17)

Obviously when \( \beta \to 0 \), we go back to the case of ideal aether, i.e. \( R = \frac{3}{4} - \frac{1}{4\omega'} \leq \frac{3}{4} \).

However, when \( \beta \neq 0 \), we find the ratio can be larger than \( \frac{3}{4} \). For instance, let \( \omega' \to \infty \), we have

\[ G_N = G_{\text{eff}}|_{\omega=0} = \frac{3G'}{4}, \quad G_R = G_{\text{eff}}|_{\omega=\frac{1}{3}} = (1 - \frac{4\beta}{9})G'. \]  

(18)

\(^1\) It means that the energy density of the gravitational aether is negative whenever the state parameter of matter \( \omega > 0 \) and \( \beta > 0 \). Similar situation occurs for an ideal aether fluid, where the energy density of the aether has to be negative for \( 0 < \omega < \frac{1}{3} \) and \( \omega' > \omega \).
and

\[
R = \frac{G_N}{G_R} = \frac{3}{4} \frac{1}{1 - \frac{4\beta}{9}}.
\]  

(19)

Therefore the ratio can be larger than \(\frac{3}{4}\) whenever \(\frac{9}{4} > \beta > 0\). In particular, when \(\beta = \frac{9}{16}\), the ratio is exactly equal to one, \(R = 1\). In Figure one we illustrate the variation of the ratio with the state parameter of the gravitational aether when \(\beta\) takes various values.

3. In above discussion we intend to lift the upper bound with an assumption that the aether is a kind of nearly incompressible fluid, namely \(\omega' \gg 1\). As a matter of fact, we may loose this constraint when the gravitational aether has a viscosity. We can choose a team of proper values for \(\beta\) and \(\omega'\) to let \(R = 1\). For instance, if we take \(\omega' = \frac{3}{4} > \omega\), we find the ratio \(R \to 1\) as \(\beta \to \frac{45}{64}\), and the effective Newton’s constants are \(G_N = G_R = \frac{3}{8} G'\). Alternatively, if \(\omega' = 1 > \omega\), then \(R \to 1\) as \(\beta \to \frac{3}{4}\), and the effective Newton’s constants are \(G_N = G_R = \frac{1}{2} G'\). In Figure two we illustrate the relation between \(1/\omega'\) and \(\beta\) when setting \(R = 1\), which is nothing but

\[
\beta = \frac{9}{16} (1 + \frac{1}{\omega'})(1 - \frac{1}{3\omega'}).
\]  

(20)

### IV. GRAVITATIONAL AETHER WITH MULTI-FLUIDS

In previous sections we discussed the effective Newton’s constants could be different for perfect fluids with different state parameters under the condition that the universe is filled with the gravitational aether and a single perfect fluid which is supposed to be dominating in different era of the universe. To make this proposal more practical we need consider the case that the gravitational aether is mixed with multi-fluids including all the ingredients of the universe. Next we demonstrate that the analysis presented in previous sections can be extended to this case indeed. For simplicity we consider the gravitational aether is mixed with two perfect fluids, but all the derivations and results can be straightforwardly generalized to the case with multi-fluids.

Consider two perfect fluids with energy densities and constant state parameters \((\rho_1, \omega_1)\) and \((\rho_2, \omega_2)\), respectively, we obtain the modified Friedmann equation as
FIG. 1: The ratio of the effective Newton’s constants for pressureless dust and radiation.

\[ H^2 + \frac{k}{a^2} = \frac{8\pi G_{1}^{\text{eff}}}{3} \rho_1 + \frac{8\pi G_{2}^{\text{eff}}}{3} \rho_2, \]  

with two effective Newton’s constants which have the form

\[ G_{1}^{\text{eff}} \rho_1 + G_{2}^{\text{eff}} \rho_2 = [(\rho_1 + \frac{T_1}{4}) + (\rho_2 + \frac{T_2}{4}) + \rho']G'. \]  

At first we assume that the gravitational aether is a perfect fluid without viscosity. Then from Eq.(4) we can derive the equation which constrains the evolution of the gravitational aether and the fluids as

\[ \frac{dho'}{dt} + 3H(1 + \omega')\rho' = \frac{1}{4}(1 - 3\omega_1)\frac{d\rho_1}{dt} + \frac{1}{4}(1 - 3\omega_2)\frac{d\rho_2}{dt}. \] 

To obtain a solution to this equation we firstly notice that from the conservation equations of fluids \( \dot{\rho}_1 + 3H(1 + \omega_1)\rho_1 = 0 \) and \( \dot{\rho}_2 + 3H(1 + \omega_2)\rho_2 = 0 \), a relation between the energy densities of two fluids can be obtained

\[ \rho_2 = \frac{\rho_1}{1 + \omega_2}. \]
FIG. 2: The relation between $\beta$ and $1/\omega$ when setting $R = 1$.

Substituting this into equation (23), we can solve for $\rho'$ as a function of the energy density $\rho_1$

$$\rho' = C\rho_1 + D\rho_1^m,$$  \hspace{1cm} (25)

with

$$m = \frac{1 + \omega_2}{1 + \omega_1},$$

$$C = \frac{1 - 3\omega_1}{4} \frac{1 + \omega_1}{\omega_1 - \omega'},$$

$$D = \frac{1 - 3\omega_2}{4} \frac{1 + \omega_2}{\omega_2 - \omega'}.$$ \hspace{1cm} (26)

Now it is straightforward to obtain the effective Newton’s constants for these two fluids by substituting Eq. (25) into Eq. (22)

$$G_1^{eff} = \frac{3}{4}(1 + \omega_1)(\omega' - \frac{1}{3})G',$$

$$G_2^{eff} = \frac{3}{4}(1 + \omega_2)(\omega' - \frac{1}{3})G'.$$ \hspace{1cm} (27)
As a result, we show that the gravitational aether proposal can be generalized to the case with multi-fluids. In particular, the effective Newton’s constants have the same form as that for a single perfect fluid.

When the viscosity of the gravitational aether is taken into account, we propose that the pressure of the aether is modified as

\[ \tilde{p}' = p' + \beta \omega' \left[ \omega_1 (1 + \omega_1) \rho_1 + \omega_2 (1 + \omega_2) \rho_2 \right]. \tag{28} \]

Then with the same algebra we obtain the effective Newton’s constants for these two fluids as

\[
\begin{align*}
G_1^{\text{eff}} &= \frac{1 + \omega_1}{\omega' - \omega_1} \left[ \frac{3}{4} (\omega' - \frac{1}{3}) - \beta \omega' \omega_1 \right] G' \\
G_2^{\text{eff}} &= \frac{1 + \omega_2}{\omega' - \omega_2} \left[ \frac{3}{4} (\omega' - \frac{1}{3}) - \beta \omega' \omega_2 \right] G'.
\end{align*} \tag{29}\]

They have the same form as that for a single fluid. Therefore we conclude that our strategy is general and can be applicable to the case with multi-fluids.

V. DISCUSSION AND CONCLUSIONS

The notion of gravitational aether is introduced to solve the first cosmological constant problem. In this mechanism the effective Newton’s constant is matter dependent such that the vacuum energy can be decoupled from gravity, but the price is that the ratio of effective Newton’s constants for pressureless dust and radiation is bounded, no larger than 0.75. In this paper we proposed that this difficulty may be overcome by generalizing the ideal gravitational aether to the one with viscosity. Moreover, the aether need not be a kind of incompressible fluid.

To obtain such effects we have taken a special form of the viscous term. In particular, it depends on the energy density of ordinary matter rather than that of the gravitational aether, which looks peculiar. It means that the viscosity is not an intrinsic property of the gravitational aether itself, but generated whenever the aether is mixed with matter fluids. Although we may change its form into one proportional to the square of the Hubble parameter through the Friedmann equation, we think this form of viscosity calls for further understanding.
To push the proposal of gravitational aether forward, we need do more, specially in the interpretation of current accelerating expansion of the universe and the proportion of the dark energy in contents of cosmological ingredients\[12\]. Since our strategy can be extended to the case with multi-fluids, the program along this direction is under progress and will be presented elsewhere.

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