Microwave power dissipation in a microwave plasma filament

V G Brovkin¹, P V Vedenin¹, I Ch Mashek²

¹Joint Institute for High Temperatures, Russian Academy of Sciences, Izhorskaya 13
Bldg 2, Moscow 125412, Russia
²Saint-Petersburg State University, Saint-Petersburg, Russia
brovkin47@mail.ru

Abstract. A plasma channel (filament) formed in the electromagnetic field of microwave (MW) radiation shows promise for its application for off-body non-electrode modification of a gas flow (plasma aerodynamics) and in the plasma assisted combustion process. In these problems, the channel is considered as an object that converts the energy of electromagnetic radiation into gas heating. To estimate the total power absorbed in the plasma from above, simple formula is obtained. Numerical calculations in air in the pressure range \(P = 20-100\) Torr based on a simplified 2D model using a fairly complete system of plasma-chemical reactions have shown that the total absorbed power is close to its estimate from above when the plasma channel develops.

1. Introduction

Recently, there has been a significant interest in the problems of the impact of MW discharge on the flow at high speeds (the problem of flow control) [1,2] and plasma assisted combustion [3, 4]. For these applications, the most important characteristic is the energy stored in the filament.

A plasma channel is formed as a result of the development of a freely localized (away from surfaces) non-equilibrium MW discharge in the electric field of a linearly polarized wave beam (or beams) at high pressure under the condition \(\nu >> \omega\) (\(\omega\) is the electromagnetic wave angular frequency). The results of experiments [5-10] show that the plasma channel is an almost ellipsoid stretched along the electric field of incident radiation. The maximum length of the filament visible in the photos is in the range of values \((0.5 \div 0.9) \lambda\), where \(\lambda\) is the wavelength. The channel transverse dimensions are significantly smaller than the wavelength.

Three main phases of the plasma channel formation in the above breakdown electric field are revealed both experimentally [5, 9] and theoretically [11, 12]. (i) The development of an electron avalanche in an incident electric field (initial phase). (ii) Elongation of the channel-streamer in mutually opposite directions in parallel with the incident electric field to a length comparable to the wavelength of the incident radiation. (iii) At the quasi-stationary phase, the dimensions of the plasma region change very slightly, the plasma density, the electric field inside a filament as well as the absorbed power reach quasi-stationary levels. At the initial stage, the absorption of MW energy is close to zero. The streamer phase is very short. The plasma channel accumulates its energy at the quasi-stationary phase.
The density of the power dissipated in the channel is proportional to the plasma conductivity. The amplitude of the electric field decreases with increasing conductivity. Therefore, it is natural to assume that at each moment of time the total absorbed power has a maximum value that depends on the spatial distributions of the plasma density and the electric field.

In this report, we have obtained a simple formula for estimating the absorbed power in a MW plasma channel from above. For the applications mentioned above, it is important to know how much the absorbed power differs from its maximum value. Therefore, we carried out a numerical study of the dynamics of Joule’s losses during channel formation in the above breakdown electric field of linearly polarized MW beam. In air in the pressure range \( P = 20-100 \) Torr, a comparison of the results of calculations and estimates showed that they are very close to each other. Based on the results obtained, as well as experimental data on the evolution of radiation scattered by a plasma dipole, the possibility of an “instantaneous” estimate of the energy absorbed during a MW pulse is shown.

2. Model

A thin plasma filament is in a high-frequency electric field, \( \Re\{E_0(r) \exp(-i\omega t)\} \). The variations in its scales and the average for the period, \( T = \frac{2\pi}{\omega} \), electron density are insignificant for a cycle \( T \). The plasma averaged over a period is quasineutral. Uncompensated high-frequency space charges of opposite signs arise during the oscillatory motion of electrons relative to the immobile ion background. The expression for the complex amplitude of the high-frequency (the superscript “h”) electron charge density, which follows from the continuity equation, is

\[
\rho_r^h = -\frac{i\omega j_r^h}{\omega} = -\varepsilon_0 E \varepsilon \propto \nabla N_e,
\]

where \( j_r^h = \sigma^h E \) is the complex amplitude of the high-frequency electron current density, \( \sigma^h = \sigma / (1 - i\theta) \) is the high-frequency complex electrical conductivity, \( \sigma = \frac{e^2 N_e}{m v} \), \( N_e \) is the average for the period electron density, \( \theta = \frac{\omega}{c}, \varepsilon = 1 + i\sigma^r \) is the complex permittivity of the plasma, \( \sigma^r \); \( \sigma^c \) = \( \frac{\sigma}{\omega \varepsilon_0} \); \( \varepsilon_0 \) is the permittivity of free space. It follows from Exp. (1) that the excess space charge emerges only if plasma is non-uniform.

When considering the interaction of a MW beam with a thin plasma filament, it is convenient to present the complex amplitude of the total high-frequency electric field in the integral form

\[
E(r, t) = E_0(r) + E_p(r, t),
\]

where \( E_p = E_r + E_j \) is the plasma response electric field

\[
E_p = -\frac{k}{4\pi \varepsilon_0} \int dV \varepsilon (\kappa) \nabla \rho_r (r', t),
\]

\[
E_j(r) = \frac{i\omega \mu_0 k}{4\pi} \int dV \varepsilon (\kappa) j_r (r', t)
\]

\[
k = \frac{\omega^2 c}{\varepsilon^r} c
\]

where \( c \) is the speed of light, \( \mu_0 \) is permeability of free space, \( G = \exp(ik)/k, k = k |r - r'| \).

Let the plasma channel is extended along the \( z \) axis in parallel with the incident electric field \( E_0 = (0, 0, E_0) \). The coordinate origin is in its center. It is assumed that (i) the electron density distribution is symmetric with respect to the plane passing through the center perpendicular to its axis, (ii) the electric field in the center has only \( z \)-component \( E_c = (0, 0, E_c) \). Everywhere below, the subscript “c” will denote the quantity referring to the center.

The central idea of our model, based on numerical simulations results from [11, 13–15] is that

\[
\int dV \sigma \cdot E_0 \approx E_c \int dV \sigma^c, \sqrt{dV} \sum E_0 \sim |E_0|^2 \sqrt{dV} Re \sigma^c \approx 0, \sum \int dV j_{x,y}(r) \cdot (\ldots) \approx 0.
\]

if the electrical conductivity in the central region is near its maximum. Since the maximum values of the electric field amplitude are reached at the tips of a channel [11, 13–15] where \( \sigma(r) \rightarrow 0 \), the contribution
of these volumes to the integrals is negligible. It should be noted that this integral approach has allowed to adequately describe the development of both a single plasma channel [12], a chain and chains of plasma filaments [16].

In this approximation, the expression for the electric field in the center of a thin plasma channel whose length is less than the wavelength can be written as [10]

\[
E_c \approx \omega_0 \frac{k_c}{\gamma_v} \psi_c, \tag{5}
\]

where \( \sigma_c \) is the normalized high-frequency complex electrical conductivity in the center of the channel,

\[
\psi_c = -\frac{k_e}{4\pi} \int dV f(r, t) G(\kappa) \left[ 1 + i\kappa^{-1} - \kappa^{-2} - \left( \frac{k_r}{\kappa} \right)^2 \right] \left( 1 + 3i\kappa^{-1} - 3\kappa^{-2} \right).
\]

\( f = \sigma / \sigma_c \). Here \( \kappa = kr \).

Since the length of the plasma channel is less than the wavelength of the incident radiation, we used the following representation for the function \( \psi_c \) (obtained by expanding the function \( G(\kappa) \) in series in powers of the argument \( \kappa \)):

\[
\psi_c = n_x - k^2 \sum_{m=0}^{\infty} \left( \frac{ik}{m(m+2)} \right)^m n_x = \frac{1}{4\pi} \int dV \left( -\frac{df(r)}{dr} \right)^2 r^2.
\]

Here \( n_x \) is the depolarization factor in the \( z \)-direction. It should be emphasized that the amplitude of the electric field in the center depends integrally on the distribution of electrical conductivity. The integral dependence means that taking into account the details of this distribution can not lead to a noticeable effect on the magnitude of the amplitude.

In the electrostatic limit (\( \psi_c = n_x \)), when the dimensions of the plasma formation are small compared to the wavelength, the field amplitude in the central region decreases only due to the influence of charges located mainly at the channel ends. With increasing channel length, the role of the electric field \( E_{fs} \) created by the high-frequency longitudinal current increases. This field tends to compensate (\( \psi_c \approx n_x - k^2 \psi_0 \)), \( \psi_0 > 0 \) for the space charge field \( E_{fs} \).

The results of the experiments [5-10] indicate that a plasma channel is nearly ellipsoidal. This allows us to approximate the distribution of the conductivity as follows

\[
\sigma(r, t) = \sigma_0(t) f(t, \xi), \tag{6}
\]

where \( \xi = \sqrt{\left( \frac{x}{X_{vis}} \right)^2 + \left( \frac{y}{Y_{vis}} \right)^2 + \left( \frac{z}{Z_{vis}} \right)^2} \). \( X_{vis}, Y_{vis}, Z_{vis} \) are visible dimensions of a prolate ellipsoid, \( f \) is a function which satisfies the conditions \( f(0) = 1, f(\xi \rightarrow \infty) \rightarrow 0 \).

The ratio of the transverse visible dimensions, \( Y_{vis} / X_{vis} \), is determined primarily by the structure of the MW field in the plasma region. In our experiments \( [9, 10] \) this ratio is approximately equal to \( Y_{vis} / X_{vis} \approx 1.1 \pm 1.2 \). Based on this experimental result as well as the estimation from Ref. 10 we used the azimuthally symmetric approximation with \( \xi = \sqrt{\left( \frac{r}{R_{vis}} \right)^2 + \left( \frac{z}{Z_{vis}} \right)^2} \), where \( R_{vis} = \sqrt{0.5(Y_{vis}^2 + Y_{vis})} \).

In this approximation, the expressions for the depolarization factor \( n_z \) and form-factors \( \psi_m \) take the forms

\[
n_x = \left( \frac{1-e^2}{e} \right) \frac{1}{2\pi} \ln \frac{1+e}{1-e} - 1, \quad e = \sqrt{1 - R_{vis}^2 / Z_{vis}^2}, \quad \psi_m = R_{vis}^2 Z_{vis}^m e^{m-1} \eta_m \eta_{m}. \tag{7}
\]

Within the approximation (4), the expression for the absorbed power averaged over a period, \( W_f = \frac{\int \int |E|^2 Re \sigma}{2} \), takes the form

\[
W_f \approx \frac{|E_c|^2}{2} k \eta \frac{1-c}{1+c} \left( 1 - \sigma_c \right) \frac{\sigma_c}{2} \tag{7}
\]

where \( \sigma_c \) is the electrical conductivity in the center of the channel.
where $S_0 = c E_0^2 E_0 / 2 V_{eff} = \int dV f$ is the effective volume.

Since the amplitude of the electric field in the channel decreases with increasing parameter $\sigma_c$, the function $W_f(\sigma_c(t))$ at each instant of time has a maximum for the value of the normalized conductivity in the center

$$\sigma_c^{(m)}(t) = \sqrt{\frac{1 + \delta^2}{(1m \psi_c(t))^{-1} + (Re \psi_c(t))^2}}.$$  \hfill (8)

The expression for the maximum total absorbed power is

$$W_0 \frac{1}{2(1 + \delta^2)(\chi^2 + \mu^2) + \chi - \mu} f_{max}^{-1},$$  \hfill (9)

where $W_0 = \frac{31^2 S_0}{Btr}$.

$$\chi = \frac{3}{k^2 \psi_1} \left( \frac{1 + \frac{3}{2} \sum_{m=1}^{n} \frac{(-1)^m}{k^2 \psi_1} \frac{(2m+1)!}{(2m+1)!} \frac{\eta_{2m+1}}{\gamma_{1}} \frac{\gamma_{2m+1}}{\gamma_{1}} \right) + \frac{3 n_e}{k^2 \psi_1} \frac{3}{4} \sum_{m=0}^{\infty} \frac{\gamma_{2m+1}}{\gamma_{1}} \frac{\gamma_{2m+1}}{\gamma_{1}}.$$  \hfill (10)

Figure 1 shows the evolution of the cycle-averaged normalized absorbed power, $W_r = \frac{W_f}{W_0}$, as well as the estimate from above, $W_{r, max} = W_{f, max} / W_0$ (dashed line) at air pressures $P = 20, 60, 100$ Torr for $\lambda = 2.3$ cm, $E_{eff} = 1.2 E_{br}$.

3. Results

Figure 1 shows the evolution of the cycle-averaged normalized absorbed power, $W_r = \frac{W_f}{W_0}$, as well as the estimate from above, $W_{r, max} = W_{f, max} / W_0$ (dashed line) at different air pressures for $\lambda = 2.3$ cm, $E_{eff} = \frac{|E_0|}{\sqrt{2(1 + (\frac{E}{V})^2)}} = 1.2 E_{br}$ ($E_{br}$ is the breakdown electric field, $E_{br} / P = 40$ V/cm$^1$·Torr$^1$). In
numerical calculations we used a simplified 2D model coupled with fairly complete system of plasma-chemical reactions [17].

As mentioned in the introduction, there are three main phases of a channel development. At the initial stage, the power tends to zero. In the electrostatic limit (streamer phase), the power proportional to the effective volume \( V_{\text{eff}} \) is still negligible. It can be seen that the streamer phase is short: its typical duration is about a few scales \( \nu_{i,0}^{-1}(E_0) \) [8] (\( \nu_{i,0}^{-1}(E_0) \) is the impact ionization frequency in an incident electric field). The most intense absorption of energy begins at the final quasi-stationary stage of development when the dimensions and plasma conductivity in the central region are close to the limiting values. At this stage, we have \( W_*, W_{\text{max}} \approx 1 \) (\( W, W_{\text{max}} \approx W_0 \)). It also follows from the Figure 1 that the functions \( W(t), W(t)_{\text{max}} \) are close to each other. Calculations carried out in the ranges of values of the discharge parameters \( P = 20 - 130 \) Torr, \( E_{\text{eff}} = (1.1 - 1.3)E_{\text{tr}} \) revealed that (i) the magnitude of the maximum relative deviation, \( \Delta(t) = \left| \frac{1 - W_j(t)}{W_j(t)_{\text{max}}(t)} \right| \), does not exceed 20\% , (ii) as the pressure and amplitude of the electric field increase, the deviation value decreases.

Note that based on this study it possible to “instantly” estimate the energy, \( Q_j \), absorbed during a MW pulse with duration \( t_p \). As follows from the above, the plasma channel accumulates energy at the final quasi-stationary stage. Thus, roughly

\[
Q(t_p - t_{qs})_{j_{\text{max}}},
\]

where \( t_{qs} \) is the quasi-stationary stage start time. This time can be estimated using experimental data on the evolution of radiation scattered by a plasma dipole [9, 10].

The results obtained allowed us to make the following cautious assumption that the developing plasma filament tends to absorb the maximum power depending on the spatial distribution of the plasma density. If this hypothesis is true, then any incorrect interference inside the kinetic block should lead to an increase in the deviation. Therefore, within the framework of this hypothesis, the deviation value can serve as an indicator when choosing a kinetic model. We carried out the calculations by excluding associative ionization reactions

\[
N_2(a^3\Sigma_u^+) + N_2(a^3\Sigma_u^+) \rightarrow e + N_4^+, \quad k_{as} = 5 \times 10^{-11} \text{ cm}^{-3}\text{s}^{-1}
\]

\[
N_2(a^3\Sigma_u^+) + N_2(a^3\Sigma_u^+) \rightarrow e + N_4^+, \quad k_{as} = 2 \times 10^{-10} \text{ cm}^{-3}\text{s}^{-1},
\]

which are still actively discussed in the literature. This simplification of the kinetic model led to a noticeable increase in the deviation (about twice).

4. Summary. Within the scope of the simplifying assumptions relating to a thin channel that develops along the electric field of an electromagnetic wave with linear polarization, a formula is obtained for estimating the total absorbed power from above. To find out how much the total absorbed power \( W_j \) differs from its maximum value \( W_{j_{\text{max}}} \), numerical calculations based on a simplified 2D model using a fairly complete system of plasma-chemical reactions are carried out in air in the pressure range \( P = 20-100 \) Torr. The comparison showed that the maximum relative deviation \( \Delta(t) = \frac{1 - W_j(t)}{W(t)_{j_{\text{max}}}} \) does not exceed 20\%. As the pressure and amplitude of the electric field increase, the deviation value decreases. Based on these results we hypothesized that the forming plasma channel tends to absorb the maximum energy of the incident radiation. Numerical calculations showed that simplification of the kinetic model by eliminating some key ionization processes led to a noticeable increase in the deviation.

It is proposed to use the obtained results for an “instantaneous” estimate of the energy absorbed in a MW pulse based on experimental data on the evolution of radiation scattered by a plasma dipole. This work was supported by RFBR grant 18-08-00707.

References

[1] Kolesnichenko Y, Brovkin V, Azarova O, Grudnitsky V, Lashkov V and Mashek I 2003
Microwave Energy Deposition for Aerodynamic Application AIAA 2003-0361, American Institute of Aeronautics and Astronautics.

[2] Knight D 2015 A short review of microwave and laser discharges for supersonic flow control J. AerospaceLab. 10, AL10-02 (2015).

[3] Starikovskaya S M 2006 Plasma assisted ignition and combustion J. Phys. D: Appl. Phys. 39 R265–R299.

[4] Michael J B, Dogariu A, Shneider M N, and Miles R B 2010 Subcritical microwave coupling to femtosecond and picosecond laser ionization for localized, multipoint ignition of methane/air mixtures J. Appl. Phys. 108, 093408.

[5] Vikharev A L, Gorbachev A M, Kim A V, and Kolysko A L 1992 Sov. J. Plasma Phys. 18 554

[6] Brotvin V, Kolesnichenko Yu 1995 Structure and dynamics of stimulated microwave discharge in wave beams J. Moscow Phys. Soc. 5, 23.

[7] Shishkov A. A.1996 Teplofiz. Vys. Temp. 34, 525.

[8] Kolesnichenko Yu, Brotvin V, Khmara D, et al., 2003 Fine structure of microwave discharge Proc. of the 41st AIAA Aerospace Meeting and Exhibition, Reno, Nevada, AIAA 2003, 362.

[9] Bityurin V, Brotvin V, and Vedenin P 2012 Investigation of the electromagnetic wave scattering dynamics during microwave streamer evolution Tech.Phys. 57, 95.

[10] Bityurin V, Brotvin V, and Vedenin P 2017 Investigation of structured microwave discharge based on data from the scattered radiation J. Phys. D: Appl. Phys. 50, 75201.

[11] Vedenin P, Rozanov N 1994 Initial stage of development of a high-pressure self-sustained microwave discharge in a plain-polarized field: elongation and stopping of a microwave streamer Sov. Phys. JETP 78, 465.

[12] Bityrin V and Vedenin P, 2010 An integral approach to considering the evolution of a microwave streamer Sov. Phys. JETP 111 512.

[13] Gildenburg V B, Guschin I S, Dvinin S A, and A.V. Kim, 1990 Sov. Phys. JETP 70, 645.

[14] Naidis G V 1996 High-frequency streamer dynamics in air Sov. Phys. JETP 82, 694.

[15] Chaudhury B, Boeuf J P, Zhu G Q, and Pascal O 2011 Physics and modelling of microwave streamer at atmospheric pressure J. Appl. Phys. 110, 113306.

[16] Vednin P, and Popov N 2003 Propagation of a high-pressure microwave discharge in prebreakdown fields: formation of plasma structures Sov. Phys. JETP 96, 40.

[17] Brotvin V, and Vedenin P 2018 A semi-analytical model for the study of the evolution of a microwave plasma channel IOP Conf. Series: Journal of Physics: Conf. Series 1112 (2018) 012025.