A novel effect in a granular material under swirling motion of the container is presented. At low packing densities the material rotates in the same direction as the swirling motion of the container (rotation). At higher densities the cluster of granular material rotates in opposite direction (reptation). The change of the direction of the motion of the cluster takes place at a critical packing density while the diffusion coefficient changes significantly. The measured critical density of the packing is in good agreement with results obtained by molecular dynamics simulation.

The fascination of the flow behavior of granular material like sand is due to the fact that under certain circumstances it behaves like a fluid – an hour glass is the most popular example. Its solid-like behavior is obvious as well: A pile of sand is stable and it can undergo plastic deformation, but, unlike a fluid, it does not dissolve under gravity. The exotic behavior of granular matter and the sometimes unexpected observations have stimulated many scientists to focus their scientific interest to sand-like materials (see e.g. [3]). For a recent review see [3].

We present a novel effect which demonstrates to a certain amount both the similarity and the dissimilarity between granular material and fluid. The experiment is extremely robust, easily visualizable, and can be performed even as a kitchen table experiment using a bunch of marbles and a pot. The basic idea is to bring the pot in a swirling motion, the motion one frequently uses to stir up the bouquet of a glass of wine. If the pot contains only one or a few spheres, they will follow the rotation of the swirl in a similar fashion as the wine in the glass does. When one successively increases the number of spheres the angular velocity of the cluster decreases with increasing density. At a certain critical density an intriguing effect appears: The angular velocity becomes negative, i.e. the cluster rotates in the opposite direction. Now the collective behavior is more reminiscent of a pancake rotated by the swirling motion of a frying pan. The first regime of motion we call “rotation”, the second one “reptation”.

In this paper we present the first qualitative and quantitative description of this crossover effect. The experiments are accompanied by a two-dimensional molecular dynamics simulation. The numerical results are qualitatively and quantitatively in good agreement with the experimental findings.

The experimental setup is shown in fig. 1. A Petri dish of inner diameter 9.0 cm is mounted on a table performing a circular vibration, i.e. each point of the table moves along a circular line during one oscillation period. For the experiments presented here, its frequency is chosen to be 2.5 s⁻¹ and the amplitude of the swirling motion is 1.27 cm. A camera is fixed on the oscillating table, thus the analysis can be done in a comoving frame by visual inspection of the image displayed on a monitor. In between the Petri dish and the camera there is a glass plate with a marker which is fixed in the laboratory frame. Thus the position of the marker relative to the dish indicates the direction of the momentary acceleration. The Petri dish is partially filled with a monolayer of ceramic spheres (density 1.74g/cm³) of diameter 1.223 ± 0.035cm. The number of particles ranges between 1 and 42, where 42 corresponds to the closest packing of spheres. We want to focus here only on the density dependence of the effect. The dependence on the amplitude, the frequency, the material properties etc. will be discussed in more detail in a forthcoming paper.

The system described above was simulated using two-dimensional molecular dynamics. We applied the soft particle Ansatz by Cundall and Strack including interaction between colliding particles in normal and tangential direction. Two colliding particles i and j feel the force

\[ \vec{F}_{ij} = F^N_{ij} \vec{n} + F^T_{ij} \vec{t} \]  

(1)

with

\[ F^N_{ij} = Y \cdot (R_i + R_j - |\vec{r}_i - \vec{r}_j|) - m_{ij}^{\text{eff}} \cdot \gamma_N \cdot (\vec{\dot{r}}_i - \vec{\dot{r}}_j) \cdot \vec{n} \]  

(2)

\[ F^T_{ij} = \text{sign}(v_{ij}^{\text{rel}}) \cdot \min \left( m_{ij}^{\text{eff}} \gamma_T |v_{ij}^{\text{rel}}|, \mu |F^N_{ij}| \right) \]  

(3)

\[ v_{ij}^{\text{rel}} = (\vec{r}_i - \vec{r}_j) \cdot \vec{t} + R_i \cdot \Omega_i + R_j \cdot \Omega_j \]  

(4)
\[ m_{ij}^{\text{eff}} = \frac{m_i \cdot m_j}{m_i + m_j}. \] (5)

\[ Y = 8 \cdot 10^6 \text{ g cm}^2 \] is the Young modulus, \( \gamma_N = 800 \text{ s}^{-1} \) and \( \gamma_T = 3000 \text{ s}^{-1} \) are the damping coefficients in normal and tangential direction and \( \mu = 0.5 \) is the Coulomb friction coefficient. The unit vectors in normal and tangential direction are given by

\[ \vec{n} = \frac{\vec{r}_i - \vec{r}_j}{|\vec{r}_i - \vec{r}_j|}, \]

\[ \vec{t} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \cdot \frac{\vec{r}_i - \vec{r}_j}{|\vec{r}_i - \vec{r}_j|}. \] (7)

The parameters are chosen to represent the material properties of the ceramic spheres used in the experiment as close as possible. Eq. (4) describes the relative velocity of the surfaces of the particles at the point of contact and eq. (5) gives the effective mass. Eq. (3) takes the Coulomb friction law into account, saying that two particles slide on top of each other if the shear force overcomes \( \mu \) times the normal force. For the integration of Newton’s equations we applied a Gear predictor corrector method of fifth order [6].

The time series of images in fig. 2 visualizes the motion of 26 spheres in a counterclockwise rotating Petri dish at low packing density. They have been taken at time intervals \( T/5 \), with \( T \) being the period of the driving motion of the Petri dish. The evolution of the positions of the white spheres in figs. 2a to 2f indicates that the cluster of spheres also rotates in counterclockwise direction. Figs. 2g to 2l show corresponding to figs. 2a to 2f snapshots from a molecular dynamics simulation using the same parameters as the experiment.

FIG. 1. Experimental setup.

FIG. 2. Temporal behavior of a system of 26 spheres during one counterclockwise cycle of the swirl. The arrow shows the direction of the momentary acceleration. The lower 6 figures show the corresponding molecular dynamics simulation.

FIG. 3. Temporal behavior of a system of 37 spheres. Top: the experiment, bottom: molecular dynamics simulation.

When the number of spheres is increased the rotation...
becomes slower. At a fairly well defined density it stops. Increasing the density then leads to a different kind of motion, which is displayed in figs. 3 for the case of 37 spheres. Now the cluster rotates in the opposite direction. This motion is now better described by the name reptation. Animated sequences of the motion are available via WWW [8].

The experimental snapshots show a white arrow which indicates the direction of the momentary inertial force. It is obtained by drawing a vector from the small white cross through the center of the Petri dish. The small white cross is the marker on the glass plate which is fixed in the laboratory frame as mentioned above. Therefore it rotates in the comoving frame. This procedure shows that the phase shift between the crescent-shaped area and the inertial force is different in the rotation and the reptation mode.

The most striking feature of the motion of the cluster of granular material is the dependence of the direction and the absolute value of its angular velocity on the particle number, i.e. the packing density as presented in the upper part of fig. 3. To obtain the experimental data indicated by the solid circles, the time for a single particle located near the wall for one complete path around the container is measured. The inverse of this time is shown for different numbers of spheres. The effect of the change of the sign of the rotation is thus clearly demonstrated. This method of measurement is not sufficiently well defined for a small number of particles. A particle located at the edge of the cluster does not remain there but has a tendency to migrate inwards, which makes the characterization of the cluster movement by tracing an individual particle meaningless. Therefore in the case of low particle numbers \( N < 23 \) the experimental method breaks down.

In order to avoid those difficulties in the simulation we used a slightly different measurement method for the rotation velocity drawn in fig. 4 (crosses). The motion of the center of mass is extracted from the numerical data.

\[
f = \left\langle \frac{1}{N} \sum_{i=1}^{N} \frac{\mathbf{\bar{r}}_i^* \times \mathbf{\bar{v}}_i^*}{|\mathbf{\bar{r}}_i^*|^2} \right\rangle_t ,
\]

where \( \langle \ldots \rangle_t \) denotes the time average. \( \mathbf{\bar{r}}_i^* \) and \( \mathbf{\bar{v}}_i^* \) are the relative position and velocity of the \( i \)th particle with respect to the position and velocity of the center of mass:

\[
\mathbf{\bar{r}}_i^* = \mathbf{\bar{r}}_i - \frac{1}{N} \sum_{j=1}^{N} \mathbf{\bar{r}}_j
\]

\[
\mathbf{\bar{v}}_i^* = \mathbf{\bar{v}}_i - \frac{1}{N} \sum_{j=1}^{N} \mathbf{\bar{v}}_j .
\]

\[
f = \frac{1}{N T} \sum_{i=1}^{N} \frac{\mathbf{\bar{r}}_i^* \times \mathbf{\bar{v}}_i^*}{|\mathbf{\bar{r}}_i^*|^2} \]

The most striking feature of the motion of the cluster is the dependence of the direction and the absolute value of its angular velocity on the particle number, i.e. the packing density as presented in the upper part of fig. 3. To obtain the experimental data indicated by the solid circles, the time for a single particle located near the wall for one complete path around the container is measured. The inverse of this time is shown for different numbers of spheres. The effect of the change of the sign of the rotation is thus clearly demonstrated. This method of measurement is not sufficiently well defined for a small number of particles. A particle located at the edge of the cluster does not remain there but has a tendency to migrate inwards, which makes the characterization of the cluster movement by tracing an individual particle meaningless. Therefore in the case of low particle numbers \( N < 23 \) the experimental method breaks down.

The experiment and the simulation agree quantitatively, showing that the rotation velocity decreases with particle number. For a critical number of particles \( N_c = 32 \) we observe the reversal of the direction of the rotation. To obtain the experimental equivalent of eq. (8) requires computer aided image analysis which is currently under construction. In the experiment the rotation velocity is measured from the velocity of particles close to the wall of the Petri dish only, while in the simulation the velocity is calculated due to eq. (6) taking into account data of all particles. Hence the particles close to the center of mass having lower rotation velocity lead to lower values of \( f \) for the results of the simulation (s. fig. 3). One can get a visual impression of the described behavior in [8].

The change of the sense of rotation might be caused by the change of the dynamical structure of the cluster, which depends on the packing density. For high densities one finds that the cluster rolls like a rigid body along the inner wall of the Petri dish, i.e. the neighborhood relations of the particles do not change significantly in time. At lower density the bulk of particles behaves more fluid-like, i.e. the relative positions of the particles change quickly in time. A quantitative measure of this
The effect is given by the diffusion coefficient $D$ shown in the lower part of fig. 4:

$$D = \frac{1}{\pi} \left\langle \frac{d}{dt} \left( \sum_{i=1}^{N} \sum_{j \in U(i) \neq i} (\vec{r}_i - \vec{r}_j)^2 \right) \right\rangle_t.$$  \hspace{1cm} (11)

Here $U$ denotes the surrounding of the $i$th particle defined by

$$j \in U(i) \quad \text{if} \quad |\vec{r}_i - \vec{r}_j| \leq R_U.$$  \hspace{1cm} (12)

In our simulation we used $R_U = 3$ cm. As indicated by fig. 4, the diffusion coefficient changes significantly for $N$ approaching $N_c$.

The agreement between the experimental results and numerical calculation is remarkable, when considering the fact that the theory is a two-dimensional one with idealized boundary conditions and particle interactions. Obviously, this experiment can serve as a test for the quality of theoretical models describing the behavior of granular matter, in a field where robust and reproducible experimental effects are not easily obtained.

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[7] The black and white spheres have been checked to have identical mechanical properties. In test series we did not find effects like segregation which might originate from different bulk or surface properties.
[8] The animated sequences mentioned in this paper can be found via World Wide Web under URL http://summa.physik.hu-berlin.de/~volkhard/swirl.html. The following 6 “movies” from the simulation can be accessed: a) 26 particles recorded by a resting camera, b) camera moving with the dish, c) camera moving with the center of mass of the particles, e-g) the same for 37 particles.