Improvement of ore recovery efficiency in a flotation column cell using ultra-sonic enhanced bubbles.

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Abstract. The ore process flotation technique is enhanced by using external ultra-sonic waves. Compared to the classical flotation method, the application of ultrasound to flotation fluids generates micro-bubbles by hydrodynamic cavitation. Flotation performances increase was modelled as a result of increased probabilities of the particle-bubble attachment and reduced detachment probability under sonication. A simplified analytical Navier-Stokes model is used to predict the effect of ultrasonic waves on bubble behavior. If the theory is verified by experimentation, it predicts that the ultrasonic waves would create cavitation micro-bubbles, smaller than the flotation bubble added by the gas sparger. This effect leads to increasing the number of small bubbles in the liquid which promote particle-bubble attachment through coalescence between bubbles and micro-bubbles. The decrease in the radius of the flotation bubbles under external vibration forces has an additional effect by enhancing the bubble-particle collision. Preliminary results performed on a potash ore seem to confirm the theory.

1. Introduction
Froth flotation is considered as the most widely used effective process in environmental engineering for mineral separation [1], wastewater treatment [2], removal hydrocarbon from refinery waters [3], and oil saturated beach sands [4]. Its efficiency comes from the selective ability of separating hydrophobic from hydrophilic materials ([5], [6]). Surfaces of particles are rendered hydrophobic by adding to the aqueous slurry a surfactant (or reagents such as xanthate salts) known as collector. The physical principle of froth flotation is based on the differences in the ability of air bubbles to selectively adhere to specific mineral surfaces in particle/water slurry in a flotation cell (Figure 1) to create the bubble-particle aggregates which move to the surface of slurry and form a stable froth phase. The removal of this froth with hydrophobic particles allows a spatial separation from remaining hydrophilic particles in the cell.

Recent works have investigated froth flotation enhanced by ultrasonic stimulation ([7], [8], [9]). Power ultrasonic waves applied to liquid may reduce the pressure sharply below the saturated vapor pressure, causing the dissolved air to separate out as micro-bubbles, a phenomenon known as hydrodynamic cavitation ([10], [11], [12], [13]). Under ultrasonic oscillation, the shape and geometry of stimulated pre-existing bubbles are modified increasing the attachment probabilities of mineral particles in the froth [9]. Dynamics of bubbles immersed into a non-compressible Newtonian liquid under oscillating pressure fields have been extensively studied these past decades for engineering processes ([14], [15], [16]) such as micro cavitation [10], micro-bubbles as contrast agent in clinical
science [17], ultrasonic enhancement on fine particles flotation [8], and measurement of bubble solids mass loading in cell flotation [18]. If ultrasonic treatments are commonly used to enhance mineral processing flotation recovery [19], little attention on theoretical models has been paid to predict the capture, adhesion, and release probabilities and overall recovery of forced oscillating bubbles. Further interests have been motivated nowadays by the importance of such improvement in a number of applications ranging from cleaning fine coal particles, separating mineral ores, processing waste mining, cleaning waste industrial water or in environmental issues. On the other hand, a number of works have been done on predicting the capture, adhesion and release probability of particles on a single bubble in flotation cells, and a vast literature is now available to describe the particle behavior at the bubble scale ([20], [21], [22], [23]). In addition to these theoretical works at the micro-scale, models for predicting the overall recovery of flotation cells are available since more of a decade. The simplest models rely on first order kinetics type of law ([24], [25]). They both involved cell flotation parameter such as injected gas rate, column diameter and length, percentage of pulp but also collection efficiency through the particle-bubble capture probability.

In this work, we approximate the theoretical oscillating bubble radius time dependent curves by a trigonometric polynomial using results obtained previously ([15], [16]). These approximations are used to predict the particle-bubble ratio against time in order to derive the bubble capture probability as a function of time. This bubble diameter time-dependent function is then use to evaluate the enhancement in flotation recovery due to the application of ultrasonic external fields. It is shown that the size fluctuations of bubbles under ultrasonic wave increase the bubble-particle collision probability, and hence the efficiency of the flotation process.

2. Flotation column in a nutshell
A froth flotation cell consists typically of a collection zone (or pulp zone) located below the feed particle point, and a cleaning zone, (or froth zone), above the particle feed point [26] (Figure 2).

The collection zone is fed back-stream by the pulp pushed by the bubble swarm created by a gas sparger at the bottom of the flotation cell. Hydrophobic particles collide with, attach to bubbles and rise to the froth zone. The other remaining particles not attached to the bubbles fall at the bottom of the cell. In the cleaning zone, a downward liquid flow prevents hydraulic entrainment of fine particles into the concentrate [26].

2.1. Key parameters of a flotation cell
Several parameters govern the functioning of a flotation cell including the: (i) type of floated mineral; (ii) superficial air rate \( J_s \), measured as the volumetric air flow rate per unit cross-section (typical values between 1 and 3 cm.s\(^{-1}\) which flows minerals of interest to the froth zone; (iii) bubble size which is generally function of the air flow rate generated by the sparger at the bottom of the column [5], collection rate which is optimum for bubble size of about 1-2mm; (iv) gas holdup which can affect significantly the particle residence time and collection of minerals. It depends on gas rate, bubble size, slurry rate, slurry density, and bubble loading; (v) wash water rate \( J_w \) which increases the froth stability, allows a deep froth layer to develop, and prevents from hydraulic entrainment of non-desirable minerals; (vi) froth depth which is around 0.5 to 1.5 m. A shallow froth depth is used against hydraulic entrainment, while a deep froth is common to select hydrophobic minerals [27]; and (vii) column height. Yianatos [26] gives the following typical values for superficial gas rate: 1-3 cm.s\(^{-1}\), superficial wash water rate: 0.3-0.5 cm.s\(^{-1}\), superficial bias rate: 0.1-0.2 cm.s\(^{-1}\), froth depth: 100 cm, average bubble size: 0.05-0.2 cm, and height/diameter ratio: 10/1. Others parameters are also important and characterize the flotation process including: the overall \( k \) and collection zone \( k_c \) flotation rate constants, the floatability factor \( F \), the collection efficiency \( E_c \), the Peclet number \( P_e \) \( (P_e = (v_p L)/(2D)) \), where \( v_p \) is the superficial velocity of solids, \( L \) the length of the collection zone, and \( D \) the dispersion coefficient of the solid phase in the column.
2.2. Flotation kinetics
The principle of the froth flotation is based on the hydrophobic property of some mineral particles. After colliding with a bubble, a hydrophobic particle has the probability $P_a$ to attach to the bubble and to be transported to the froth zone (Figure 3). Based on the sub-process presentation of a flotation process, the probability $P$ of a given particle being collected to the froth zone is given by:

$$P = P_c P_a (1 - P_d)$$  (1)

where $P_c$ is the probability of collision with a bubble, $P_a$ the probability of attachment, and $P_d$ the probability of detachment from the bubble. Several expressions have been proposed to calculate $P_c$, $P_a$, and $P_d$ depending on the hydrodynamic conditions [9] (Figure 3). The collision angle $\theta_c$ depends weakly on the particle size, but strongly on the particle density and the bubble Reynolds number $Re_b$. Flotation is generally modeled as a first-order rate process with respect to the number of particles and the number of bubble-particle aggregates [28].

2.3. Ultrasonic enhancements of the flotation process
Compared to classical flotation, ultrasonic stimulated flotation provides many advantages: it acts directly on the pulp by creating a large amount of cavitation tiny bubbles in the collection zone, with diameter from microns down to nano sizes [29], [30]. From energy consideration, it has been shown that micro cavitation bubbles occur preferentially at the surface of hydrophobic particles by rupture of the liquid-solid interface, making hydrodynamic cavitation a selective process with positive effect on flotation efficiency [12]. Nano bubbles created by hydrodynamic cavitation have the following effects [29]: (i) favor particles aggregation by tiny bubble bridging increasing the collision probability with air bubbles of the enlarged particle aggregate; (ii) increase the particle-bubble attachment probability
Figure 3. Schematic view of bubble-particle collision and attachment in flotation:

\( v_p \) = down-stream particle velocity; \( v_b \) = up-stream bubble velocity; \( d_p \) = particle diameter; \( d_b \) = bubble diameter; \( \Theta_c \) = collision (or critical) angle depends on the particle density and the bubble Reynolds number; beyond this angle, liquid pushes particles away and the particle cannot be attached to the bubble surface (after [21])

through coalescence of flotation bubble with tiny bubble frosted on particle surfaces; (iii) increase the contact angle and attachment force between particles and bubbles thanks to tiny bubbles frosted on particle surfaces; (iv) clean the particles surfaces through collapse of tiny bubbles by removing slime coatings, oxidation films [8], [30]; (v) decrease the reagent consumption [8], [30]. Negative effects on the functioning of the flotation cell can be also noticed when using ultrasound treatment [30] including

(i) high energy consumption limiting its use in industry; (ii) temperature increase of both bubbles and liquid; (iii) increase of the reagent solubility, and (iv) induced destabilization of the mineral aggregates by splitting effects.

3. Governing equations for bubble evolution under ultrasonic waves

Hydrodynamics of single bubble immersed in a liquid subjected to ultrasonic waves can be described by the Navier-Stokes equations applied to the gas inside the bubble, initially at rest, and to the incompressible liquid adjacent to the bubble wall. The Navier-Stokes equations can be solved analytically in spherical coordinates [15] or numerically using a radial formulation [16].

3.1. Solving Navier-Stokes equations

A bubble initially at rest with mean radius \( R_0 \) is immersed in an unbounded incompressible liquid submitted to external pressure driven periodically. Neglecting gravitational forces, the Navier-Stokes equations (or mass and momentum conservation equations) describes the flow field in the incompressible liquid with density \( \rho \) surrounding the bubble [16]:

\[
\nabla \cdot \mathbf{u} = 0; \quad \rho \left( \frac{\partial \mathbf{u}}{\partial t} \right) = -\nabla P + \nabla \cdot \mathbf{\tau}
\]

(2)

where \( \frac{Df}{Dt} = \frac{\partial f}{\partial t} + \mathbf{u} \cdot \nabla f \) is the particle derivative of function \( f \), \( t \) the time, \( \mathbf{u} \) the velocity field, \( \mathbf{\tau} \) the deviatoric part of the total stress tensor \( \mathbf{\pi} = -\mathbf{P}(I) + \mathbf{\tau} \), and \( P \) the pressure of the liquid. The shape of the bubble-liquid interface is described by the kinematic equation:

\[
\frac{DR_b}{Dt} = \mathbf{u}|_{r=R_b}
\]

(3)

where \( \mathbf{R}_b(\theta, t) = R_b(\theta, t) \mathbf{e}_r \) is the position vector. The above set of equations can be solved either using (i) an analytical solution in spherical coordinates assuming an ideal gas in the bubble [14].

3.2. Analytical solution to the Navier-Stokes equations for a spherical bubble

Assuming bubbles being spherical, Eq. (3) can be simplified into \( \partial \ (r^2 u(r,t)) = 0 \) using spherical coordinates, with solution for the radial velocity:
\[ u(r, t) = \frac{1}{r^2} \Phi(t) \]  

(4)

where \( \Phi(t) \) is a function of time determined by the boundary conditions. Assuming no mass transfer across the bubble boundary, and using a local coordinate system attached to the bubble, the wall velocity is equal to the change rate of its radius \( u(R_b, t) = \dot{R}_b \) (where \( \dot{R}_b \) is the derivation of \( R_b \) against time \( t \)), thus \( \Phi(t) = R_b^2(t) \dot{R}_b \), and consequently Eq.(4) becomes:

\[ u(r, t) = \left( \frac{R_b(t)}{r} \right)^2 \dot{R}_b \]  

(5)

Neglecting the dynamic viscosity of the liquid, and substituting the radial component of the velocity, the radial component of the Navier-Stokes equation applicable to the liquid can be written as:

\[
\frac{1}{\rho} \frac{\partial P_t}{\partial r} + \frac{\partial u}{\partial r} \frac{\partial u}{\partial r} + u(r, t) \frac{\partial u}{\partial r} = 0
\]

\[
= -\frac{1}{\rho} \int_{P_i(R_b, t)}^{P_\infty(t)} dP = \int_{R_b}^{\infty} \left( \frac{R_b}{r^2} \frac{\partial P_t}{\partial r} - \frac{2 R_b^2 \dot{R}_b}{r^5} \right) dr \Rightarrow \frac{P_i(R_b(t)) - P_\infty}{\rho} = R_b \dot{R}_b + \frac{3}{2} \dot{R}_b^2
\]  

(6)

where \( P_i(R_b, t) \) is the liquid pressure on the bubble wall which is equal to the sum of all forces acting on the wall bubble: (i) the gas pressure \( P_g \) in the bubble \( \dot{R}_b \), (ii) the viscous stress \( \sigma_r \), of the liquid; and (iii) the superficial tension \( \tau \). It comes \( P_i = \sigma_r + \tau + P_p \). The radial component of the viscous stress \( \sigma_r \), acting on the bubble wall depends on the liquid cinematic viscosity \( \mu \) [10]:

\[ \sigma_{rr}(r, t) = 2\mu \frac{\partial u}{\partial r} = -4\mu \frac{\dot{R}_b}{R_b(t)} \]  

(7)

The tension pressure \( \tau \) (or Laplace’s pressure) acting on the surface bubble is given by the Laplace’s formula \( \tau(t) = -\frac{2\sigma}{R_b(t)} \) where \( \sigma \) is the surface tension and \( 1/R_b \) the bubble curvature. Assuming an ideal gas polytropic transformation for the gas trapped in the bubble, the gas pressure is thus equal to [9]:

\[ P_g(t) = P_{g_0} \left( \frac{R_b(t)}{R_{b_0}} \right)^{3m} \]  

(8)

with \( P_{g_0} \) the initial gas pressure in the bubble at rest. At \( t = 0, P_i(R_b, 0) = P_0 \) and \( \dot{R}_b = 0 \), the initial pressure in the bubble becomes \( P_{g_0} = \frac{2\sigma}{R_{b_0}} + P_0 \). Thus, the total pressure at the bubble boundary \( P_i \) is given by combining Eqs. (7), and (8):

\[ P_i(R_b, t) = \left( \frac{2\sigma}{R_{b_0}} + P_0 \right) \left( \frac{R_b(t)}{R_{b_0}} \right)^{3m} - 4\mu \frac{\dot{R}_b}{R_b(t)} - \frac{2\sigma}{R_b(t)} \]  

(9)

Other gas models (including ideal gas, Soave-Redlich-Kwong, and Peng-Robinson) can be used to predict the evolution of the internal bubble pressure (see [9]).

### 3.3. Numerical solution to the Navier-Stokes equations for complex bubbles

Assuming adiabatic conditions for the gas in the bubble, and accounting for the elasticity conditions represented by the Deborah number \( D_e = \lambda \omega \) (with \( \lambda \) the polymer relaxation time), several authors including [15], and [16] gives a numerical solution to the bubble set of the Navier-Stokes equations:

Model (I) Kim and Kwak [15] \[
\frac{R_b(\omega t)}{R_{b_0}} = \frac{R}{R_0} \left[ 1 + q \sin(\omega t + \omega \tau_0) \exp(\sin(\omega t + \omega \tau_0)) \right]
\]

Model (II) Foteinopoulou and Laso [12] \[
\frac{R_b(\omega t)}{R_{b_0}} = \frac{R}{R_0} \left[ D_e \left[ 1 + q \left( D_e \right) \cos(\omega t + \omega \tau_0(D_e)) \right] \right]
\]

(10)

where the radius equilibrium bubble \( R_{b_0} = 500-700 \) _mm, \( \omega = 2\pi f \) angular frequency with \( f \) the ultrasonic frequency (around 28.5 kHz), external pressure at \( P_s = 0.142 \) MPa, the normalized amplitude \( R/R_0 = 1 \), \( q = 0.623, \omega \tau_0 = -0.374, k = 1.114 \) and \( \tau_0 \) is a delay term describing the time necessary for a bubble
to change its shape under a compressive wave stress. In the model (II), the coefficients $R/R_0(D_e)$, $\rho(D_e)$ and $\omega \tau_0(D_e)$ are functions of the elasticity conditions represented by the Deborah number $D_e$. These two expressions can be generalized into [9]:

$$\frac{R_R(\omega t)}{R_R(0)} = \frac{R}{R_0}f(\psi, \nu, \mu, \omega \tau_0)(\omega t)$$

$$f(\psi, \nu, \mu, \omega \tau_0)(\omega t) = 1 + \psi \cos \left( \omega t + \omega \tau_0 + \frac{\nu}{2} \right) \left[ 1 - \mu [1 - \exp(\sin(\omega t + \omega \tau_0))] \right]$$ (11)

where the coefficients $(R/R_0, \psi, \omega \tau_0, \nu = \mu = 1)$ can be constant as in model (I), or function of $D_e$ $(R/R_0(D_e), \rho(D_e), \omega \tau_0(D_e), \nu = \mu = 0)$ as in model (II). The tuning parameter is always observed to be less than 1: $\psi < 1$ (value $\psi = 1$ may lead to singularity). $R/R_0(D_e)$ and $\omega \tau_0(D_e)$ are fitted by:

$$\frac{R_R(D_e)}{R_R(0)} = R_R(0) + a_1 D_e + a_2 D_e^2 + \omega \tau_0(D_e) = \omega \tau_0(0) + a_1' D_e + a_2' D_e^2$$ (12)

with the fitted parameters equal to: $R_R(0) = 1.009; a_1 = 0.0135; a_2 = -0.0007; \omega \tau_0(0) = 0.088; a_1' = 0.0315; a_2' = -0.002; \omega \tau_0(0) = 0.211; a_1' = 0.723; a_2' = 0.316$ (with $R^2 = 0.98$) accounts for the inside bubble pressure response to an external pressure solicitation. These models will be used in the followings to estimate the effects of ultrasound on various parameters of a flotation cell.

4. Impact of ultrasound on governing equations for particle flotation

Modeling of flotation columns is a major step for optimizing recovery ([31], [32], [33]). Micro- and macro-scale models have been often used to predict the flotation cell kinetics. Macro-scale systems such as kinetic models have received most attention in the literature [34]. The column is modeled as a mass transfer unit using a kinetic model that is then used to predict the large-scale performance flotation system, provided the kinetics and hydrodynamics of the cell are well defined [24]. These models can then be used for developing control strategies in industrial applications.

4.1. Effect of ultrasound on the flotation rate constant

Flotation performance depends on the ability of bubbles to collect suspended particles and carry them to the surface, where a layer of froth can be removed over a launder [23] (Figure 2). The basic collection mechanism consists of collision and attachment of particles on the interfacial gas bubble surface. Gas (usually air or nitrogen) is injected at the base of the column to transport the floatable solids to the overflow. The volumetric gas flow-rate per unit cross-section, $J_g$, (also referred as superficial air rate [26]), is commonly used to describe bubble column operations. Gas dispersion parameters comprising bubble size ($d_b$), gas hold-up ($\epsilon_b$), and derived parameter such as the bubble surface area flux ($S_b = 6J_g/d_b$) also control flotation performances [33]. Assuming the overall ($k$) and column ($k_c$) flotation rate constants being equal ($k_c = k$), they can be expressed as a function of the collection efficiency ($E_i$), bubble size ($d_b$) and superficial gas velocity ($J_g$) [24]:

$$k_c = \frac{3}{2} \frac{I_{F_E k}}{d_b} = \frac{S_b E_k}{4} = F S_b \propto d_b^n; \quad n \in ] - 1, -1[ \{ n = 3, -1.5 \} \text{ according to [35]}$$ (13)

where $F$ is the floatability factor, which includes the contribution of particle size and hydrophobicity. When pilot and industrial scale experiments are not available at similar gas rates and bubble sizes, it is suggested [24] using a proportional fractal relationship between the bubble size ($d_b$) and the rate constant ($k_c$) established by [25] and [35]. The collection efficiency ($E_i$) can be estimated using the collision-particle probability models developed at the micro-scale [25].

Under ultrasound, the bubbles in flotation cell oscillate changing radius size and shapes according to the driving frequency and amplitude of the ultrasound as shown in Eq. (11). From Eq. (13), the column rate constant fluctuates against time $t$ under the ultrasonic oscillation according to:
\[ \frac{k_c(\omega t)}{k_c_0} = \left( \frac{d_p(\omega t)}{d_{b_0}} \right)^n = f^n(\omega t) ; \quad n \in -1, -1 \{ \text{or } n \in -3, -1.5 \text{ according to [35]} \} \]  

where \( k_c_0 \) is the column rate constant rate without ultrasonic treatment, the other parameters being kept the same, \( f(\omega t) \) is the function describing the radius-time bubble oscillations under ultrasonic solicitation. Eq. (14) shows that the rate constant \( k_c \) increases when the bubble radius \( d_b \) decreases under ultrasound compression. Given the fact that the oscillation time scale is very small compared to the flotation time-scale, we calculate the average value for \( k_c(\omega t)/k_c \) over six periods for evaluating the overall recovery \( R_c \) (see next paragraph).

4.2. Effect of ultrasound on the column recovery

Assuming a non-ideal plug flow reactor with axial dispersion for the column so that the flotation process obeys first order kinetics, the final collection zone overall recovery \( R_c \) is predicted by [36], [24]:

\[ R_c = 1 - \frac{4a \exp \left( \frac{2J_g}{J_g + 1} \right)}{1 + a \exp \left( \frac{2J_g}{J_g + 1} \right)} ; \quad a = \left( 1 + 4 \frac{d_c}{v_p} \right)^{1/2} \]  

where \( P_c = v_p L/D \) is the dimensionless Péclet axial dispersion number with \( v_p \) the superficial velocity of solid particles, \( L \) the collection zone length of the column, \( D \) the dispersion coefficient of the solid phase in the column, \( k_c \) the overall and collection zone first order constant rate, respectively. Using an axial dispersion model, the dispersion coefficient \( D \) of the solid phase in large columns can be predicted as a function of the column diameter \( d_c \), superficial gas rate \( J_g \) and the % of solid in the pulp \( S \) according to the following semi-empirical model [24] \( D = a_1 d_c^2 \exp^{-b_1 S} \), with \( a_1 = 6.7 \times 10^{-2} \) and \( b_1 = 0.025 \). A simplified expression for \( R_c \) is proposed by [25] as:

\[ \frac{R_c}{R_{\infty}} = 1 - \frac{4a \exp \left( \frac{2J_g}{J_g + 1} \right)}{1 + a \exp \left( \frac{2J_g}{J_g + 1} \right)} \left[ \frac{100}{9(10k_c \tau + 12)} - \frac{12}{(k_c \tau + 12)^2} - \frac{10}{9(k_c \tau + 12)} \right] \]  

where \( \tau \) the mean residence time determined as the ratio between effective volume of the column (i.e. total volume - gas + froth volume) and volumetric feed flow rate, \( R_{\infty} \) is the recovery at \( t = \infty \) (about \( R_{\infty} = 90\% \)), which depends on the ore mineralogy and flotation chemistry. The final overall recovery \( R_{\infty} \) depends on the collection zone recovery \( R_c \) and on the froth zone recovery \( R_f \) [33], [37]:

\[ R_{fc} = \frac{R_c R_f}{R_c(R_f - 1) + 1} = \frac{1}{(R_f - 1)(1 - R_f)} ; \quad R_f = \alpha \exp \left( -\beta \frac{J_g^{1.16}}{J_g} \right) \quad \alpha = 85\% ; \quad \beta = 5.2 \times 10^{-3} \]  

where \( L_f \) is the froth depth, and \( J_g \) the superficial gas velocity. Finally, the overall recovery \( R_c \) depends on two dimensionless parameters, the collection first order rate constant \( k_c \) and the Péclet number \( P_c \).

At the micro-scale, bubble radius changes impact the constant rate \( k_c \) through the collection efficiency \( E_i \) terms, and thus the overall recovery \( R_c \), assuming the other parameters be kept constant (i.e. \( J_g, D, v_b, R_c \)). As \( k_c \) depends as an inverse power function of the bubble radius \( k_c \propto d_e^{-n}, n \in [-2, -1] \) (see Eq. (14)), when the bubble radius \( d_e \) decreases under ultrasound compression, the constant rate \( k_c \) increases, thus the overall recovery \( R_c \) improves (Figure 4a), and vice versa when \( k_c \) decreases under bubble radius expand, the overall recovery \( R_c \) worsens.

4.3. Effect of ultrasound on particle-bubble capture probability

The probability \( P \) of a particle being collected by a gas bubble during flotation is given by Eq. (4). A generalized expression for the particle-bubble collision and attachment probabilities \( P_c \) and \( P_a \), respectively, is proposed by [22], [38]:

\[ P_c = A \left( \frac{d_p}{d_b} \right)^2 ; \quad P_a = \sin^2 \left[ 2 \arctan \exp \left( \frac{1}{15d_b \left( \frac{d_p}{d_b} + 1 \right)^{1/3}} \right) \right] \quad t_i = \frac{75}{v_b} d_p^{1/2} \]  

where \( d_p, d_b \) are the particle and bubble diameters, and \( A \) varies with the flow regime (i.e. Reynolds number), \( v_b \) the bubble raise velocity, \( t_i \) (in s) the induction time which is a function of the particle size
$d_b$ (in m) and contact angle $\theta$; $R_{eb} = v_b d_b \rho_b / \eta$ is the dimensionless bubble Reynolds number which combines the effect of bubble rise velocity $v_b$, mean bubble diameter $d_b$ and density $\rho_b$ and dynamic viscosity $\eta$ of fluid around of the bubble; $v_p$ is the particle velocity. $R_{eb}$ values are used to describe the flow condition from Newtonian flow: ($R_{eb} \leq 1$), intermediate I: ($1 < R_{eb} < 20$), intermediate II: ($20 < R_{eb} < 400$), and turbulent: ($R_{eb} > 400$). The flow regime dependent coefficient $A$ varies as: Newtonian flow [39] $A = 2/3$; intermediate I [38]: $A = 2/3 (1.178 R_{eb})$; intermediate II [40]: $A = 2/3 (1.1875 R_{eb}(1+249 R_{sb}^{-5.6})$.

The variation in bubble radius under ultrasound impacts the collision probability as $P_c (\omega t) / P_{c0} = f^{a (\omega t)}$ assuming a constant Reynolds number (Figure 4c and d). The attachment probability $P_a (\omega t) / P_{a0}$ is a more complex function of $\omega t$ which can be calculated numerically. Globally, ultrasound has less impact on the collision probability than on the attachment probability; the relationship between the recovery impact and the collision and attachment probability is complex and not direct. An average increase in capture probability not necessary leads to an increase in recovery performance.

4.4. Effect of ultrasound on collision angle

The collision angle $\theta_c$ (Figure 3) depends weakly on particle size $d_p$, but strongly on the particle density and the dimensionless bubble Reynolds number $R_{eb}$ and may be predicted by [21]:

$$\theta_c = \arccos(\delta); \quad x = \frac{3}{2} + \frac{0.91 R_{eb}}{3.236+R_{eb}^{0.514}}; \quad y = \frac{0.375 R_{eb}}{1+0.217 R_{eb}^{0.514}}; \quad c = \frac{v_b}{v_p} \left( \frac{d_p}{d_b} \right)^2; \quad \zeta = \frac{x+y}{2y}; \quad \delta = \sqrt{\zeta^2 + \frac{1}{3}} - \zeta \quad (19)$$
A simplified expression for $\theta_c$ which depends only on the hydrodynamic conditions (i.e. $Re_b$) is [32]:



\[
\begin{align*}
\text{Newtonian} &: \quad \theta_c = 85.0 - 2.50 \log R_{eb}; \\
\text{Intermediate I} &: \quad \theta_c = 85.0 - 12.49 \log R_{eb}; \\
\text{Intermediate II} &: \quad \theta_c = 78.1 - 7.37 \log R_{eb}
\end{align*}
\]

Ultra sound oscillation impacts the parameter $c$ through the velocity and diameter of bubbles, and thus may increase the collision angle $\theta_c$.

These above results demonstrate that improvement on overall flotation recovery $R_c$ under ultrasound is a very complex process. It may be due to: (i) an increase in particle-bubble collision or attachment probabilities by reduction of the bubble radius, but it is not always the case; (ii) the overall recovery improves under ultrasound because the radius oscillation not only impacts the capture probability but also indirectly the constant rate and collision angle. Effects can be compensated or amplified.

5. Conclusions

This theoretical work shows that ultrasound effect on flotation can be predicted both at micro and macro scale, the major impact being the time-variation of bubbles volumes under pressure oscillation. The most direct consequences would be that the bubble ability for capturing particles increases when bubble size diminishes, by an increase in collision and attachment probability. This work also demonstrates a strongly dependency on the flow regime and on the bubble oscillation. If overall flotation recovery is chosen as criteria for measuring the impact of ultrasound, it has been shown that: (i) the recovery efficiency improve due to a reduction of the bubble radius increasing the particle-bubble collision and attachment probabilities by a factor ranging between 0.8 and 2 on average depending on the model used (or both depending on the flow regimes or/and on the model used to describe the radius oscillation); (ii) the improvement in overall flotation recovery under ultrasound is a very complex process involving not only the collision and attachment probabilities, but also the variation in bubble radius and flotation rate constant, effects being exaggerated or compensated depending on the flow regimes or/and on the model used to describe the radius oscillation; (iii) model I give similar results while model II seems to under estimate the bubble radius oscillation.

Several assumptions have been proposed during this theoretical work which might be confirmed or not by experimental work or direct measurement of the bubble oscillation to valid the different suggested models [41]. Non Newtonian regimes such as intermediate turbulent flows with intermediate Reynolds numbers seem to improve the overall flotation recovery. This point must be investigated more in future experimental works. In some flow regimes, it has been found numerically that ultrasound may decrease the overall flotation recovery, a point not fully understood yet but with important consequences in flotation cell design and applications. Finally, in certain conditions bubble may enter in resonance for a given frequency, and this implication has been not investigated here but needs some further theoretical works.

6. Acknowledgements

The work was supported by the Russian Foundation for Basic Research (grant 17-41-590974). The authors would like to express their thanks for the support to Lorraine-Russia Arcus Project.

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