On Two-Pair Two-Way Relay Channel with an Intermittently Available Relay

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Abstract—When multiple users share the same resource for physical layer cooperation such as relay terminals in their vicinities, this shared resource may not be always available for every user, and it is critical for transmitting terminals to know whether other users have access to that common resource in order to better utilize it. Failing to learn this critical piece of information may cause severe issues in the design of such cooperative systems. In this paper, we address this problem by investigating a two-pair two-way relay channel with an intermittently available relay. In the model, each pair of users need to exchange their messages within their own pair via the shared relay. The shared relay, however, is only intermittently available for the users to access. The accessing activities of different pairs of users are governed by independent Bernoulli random processes. Our main contribution is the characterization of the capacity region to within a bounded gap in a symmetric setting, for both delayed and instantaneous state information at transmitters. An interesting observation is that the bottleneck for information flow is the quality of state information (delayed or instantaneous) available at the relay, not those at the end users. To the best of our knowledge, our work is the first result regarding how the shared intermittent relay should cooperate with multiple pairs of users in such a two-way cooperative network.

I. INTRODUCTION

Physical layer cooperation has been proposed as a promising approach to increase spectral efficiency, where additional resources are dedicated for cooperation, such as relay terminals in the vicinity. Such resources for cooperation could be shared by many users. One of the envisioned scenarios for physical layer cooperation is multi-pair two-way communication via a relay, where multiple pairs of users exchange their messages within their own pairs, with the help of a relay. The shared resource for cooperation in this scenario is the relay shared by multiple pairs of users. The simplest information theoretic model for studying this problem is the two-way relay channel without user-to-user connections. There has been a great deal of works focusing on (multi-pair) two-way relay channels, such as [1], [2]. A conventional assumption in these works is that, the relay is always available for the users to access, so that they can exchange data via the relay all the time.

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In practice, however, the opportunity of cooperation may not always exist, mainly because the management and allocation of resources for cooperation (such as relay terminals in their vicinities) lies beyond the physical layer. When multiple users share the same cooperation resource, it may severely impact the design of such cooperative systems if transmitters cannot timely learn how heavily the common resource is currently being utilized. In the context of multi-pair two-way communication, the issue becomes relevant especially when the spectral activity such as the frequency hopping sequence and/or the frequency coding pattern of a communication link is unknown to a relay which is installed by a third party [3] but shared by multiple pairs of users. Hence, it is of fundamental interest to characterize the capacity of such systems, under various levels of state information availability of other pairs’ accessing activities.

In this paper, we take a first step towards this direction by investigating a two-pair two-way relay channel where the two pairs get to access the relay intermittently, under various settings of temporal availability of activity state information at transmitters. The availability of accessing the relay is governed by two independent Bernoulli i.i.d. processes, one for each pair. The terminals can either have delayed information about the activity states, or instantaneous state information. See Figure 1 for an illustration of the channel model.

Fig. 1. Two-Pair Two-Way Relay Channel with an Intermittent Relay

Our main contribution is the characterization of the capacity region to within a bounded gap in a symmetric setup, both under the delayed state information setting and the instantaneous state information setting. We show that the two-pair two-way relay channel can be decomposed into the uplink and the downlink part, and the approximate capacity region is charac-
terized as the intersection of the uplink outer bound region and the downlink outer bound region. The decomposition principle can be viewed as an extension of that in the multi-pair two-way relay channel with a static relay [2]. An interesting observation is hence that the bottleneck for information flow within the system is the quality of state information available at the relay. Towards establishing the achievability of the bound-gap result, for the downlink phase with delayed state information, we have developed a novel scheme that takes care of unequal received signal-to-noise ratios. The scheme complements that in [4] where equal received SNRs are assumed.

We obtain key insights from the binary expansion model [5] for this problem to develop our scheme, where the main novelty is two-fold. First, since the state is not known instantaneously at the relay (transmitter), a lattice-based dirty paper coding (DPC) is employed instead of conventional DPC based on Gaussian random codes. Second, to take care of the unequal received SNRs, instead of quantizing the erased sequences into a single codeword like [4], we propose a successive refinement framework so that stronger receiver can have higher resolution into the quantized signal.

Related work: Two-way relay channel with a static relay has been extensively studied. For the single-pair two-way relay channel, [1] characterized the capacity region to within 1/2 bin with compute-and-forward [6] and cut-set based outer bound. [2] extended the result to the two-pair two-way relay channel, using insights from the binary-expansion model [5]. However, when the relay is intermittently available, there has been very few results regarding how the shared relay should cooperate with multiple pairs of users. Related works that address intermittence in wireless networks were focused on bursty interference networks. [7] characterized the generalized degrees of freedom of a bursty interference channel with delayed state information and channel output feedback, while [8] [9] studied the degrees of freedom of binary fading interference channels with instantaneous or delayed state information. However, the intermittent availability of cooperation resources has not been investigated widely.

II. PROBLEM FORMULATION

A. Channel Model

In the system, there are two pairs of end user terminals, pair 1: (A1, B1) and pair 2: (A2, B2), and one relay terminal R. Each terminal can listen and transmit simultaneously, and the blocklength is N. End user Ui in pair i (U = A, B, i = 1, 2) would like to deliver its message Wui to the other end user in pair i. The encoding constraints depend on the state information assumption and are detailed in Section II-B.

The two-pair two-way Gaussian relay channel with an intermittent relay is depicted in Figure 1 and defined as follows. The transmitted signals of the five terminals are XA1, XB1, XA2, XB2, XR ∈ C respectively, each of which is subject to unit power constraint, and the received signals are

\[ Y_A[i] = h_{AR}S_i[t]X_A[i] + Z_A[i], \quad i = 1, 2, \]
\[ Y_B[i] = h_{BR}S_i[t]X_R[i] + Z_B[i], \quad i = 1, 2, \]

where the independent additive noises at the five terminals ZA1, ZB1, ZA2, ZB2, ZR are CN(0, 1) i.i.d. over time. \{Si[t]\} denotes the random process that governs the accessing activity of the two users in pair i, for i = 1, 2. \{S1[t]\} and \{S2[t]\} are independent Bernoulli p processes, i.i.d. over time 1. We denote the signal-to-noise ratios as follows: for i = 1, 2,

\[ \text{SNR}_{RA} := |h_{RA}|^2, \quad \text{SNR}_{RB} := |h_{RB}|^2, \]
\[ \text{SNR}_{AR} := |h_{AR}|^2, \quad \text{SNR}_{BR} := |h_{BR}|^2. \]

Note that we focus the fast fading scenario where a codeword can span over different activity states. This assumption makes our uplink model (2) fundamentally different to the random access channel in [10]. In [10], the slow fading scenario was studied when encoding over different states was prohibited.

B. Activity State Information

We consider two scenarios in this paper regarding how the accessing activity state processes \{Si[t]\} and \{S2[t]\} are known to the five terminals, in terms of how the state information helps in encoding.

1) Delayed State Information:

- For end users: for user Ui in pair i (U = A, B, i = 1, 2),
  \[ X_U[i,t] \subseteq \{ W_{ui}, Y_{U1}^{-1}, S_{11}^{-1}, S_{21}^{-1} \}. \]
- For the relay: \[ X_R[t] \subseteq \{ Y_{R1}^{-1}, S_{11}^{-1}, S_{21}^{-1} \}. \]

2) Instantaneous State Information:

- For end users: for user Ui in pair i (U = A, B, i = 1, 2),
  \[ X_U[i,t] \subseteq \{ W_{ui}, Y_{U1}^{-1}, S_{11}, S_{21} \}. \]
- For the relay: \[ X_R[t] \subseteq \{ Y_{R1}^{-1}, S_{11}, S_{21} \}. \]

The capacity region \( \mathcal{C} \) depends on the available activity state information. We take the following notation to denote the capacity region under certain setting of activity state information: \( \mathcal{C}(u, r) \), where the first argument \( u \in \{d, i\} \) denotes that the end users have delayed state information (d) or instantaneous state information (i), while the second argument \( r \in \{d, i\} \) denotes the type of the available activity state information at the relay terminal.

III. MAIN RESULTS

In this paper, we focus on the symmetric case where \( \text{SNR}_{R0} = \text{SNR}_{R1}, \text{SNR}_{U0,R} = \text{SNR}_{U1,R} \) for U = A, B and i = 1, 2. We focus on characterizing the symmetric rate tuple \( (R_1, R_2) \), where \( R_A = R_B = R \) for i = 1, 2. Without loss of generality, we assume that \( \text{SNR}_{R1} \geq \text{SNR}_{R0}. \)

To present our main result, let us begin with some definitions useful in characterizing the approximate capacity regions.

Notations:

- Define \( \mathcal{C}(x) := \log(1 + x) \) (logarithm is of base 2).

In general, the states may be correlated across time and thus allowing us to predict future and improve the throughput. However, discussing the benefit of predicting the future is beyond the scope of this paper, and thus as [7] [8], we impose i.i.d assumptions on states.
For a \( \mathcal{R} \subseteq \mathbb{R}^2 \), define the pointwise minus operator \( \ominus \) as follows: \( \mathcal{R} \ominus (a, b) := \{(x - a, y - b) : (x, y) \in \mathcal{R} \} \).

**Uplink Rate Regions:** Let \( \mathcal{A}^\text{up} \) be the collection of \((R_1, R_2) \geq 0\) satisfying
\[
\frac{R_1}{p} + \frac{R_2}{p+2} \leq C \left( \frac{\text{SNR}_{R_1}}{p} \right),
\]
\[
\frac{R_1}{p} \leq C \left( \frac{\text{SNR}_{R_2}}{p} \right),
\]
\[
\frac{R_1}{p} + \frac{R_2}{p+2} \leq C \left( \frac{\text{SNR}_{R_1}}{p} \right) + C \left( \frac{\text{SNR}_{R_2}}{p} \right) + pC \left( \frac{\text{SNR}_{R_1} + \text{SNR}_{R_2} + 2\sqrt{\text{SNR}_{R_1} \text{SNR}_{R_2}}}{p} \right).
\]

Let \( \mathcal{A}^\text{up} \) be the collection of \((R_1, R_2) \geq 0\) satisfying (3) – (4) with SNR's replaced by \( \mathcal{SNR} \), and \( \mathcal{A}^\text{up} \) be \( \mathcal{A}^\text{up} \) with SNR's replaced by \( \mathcal{SNR} \).

**Downlink Rate Regions:** Let \( \mathcal{A}^\text{down} \) be the collection of \((R_1, R_2) \geq 0\) satisfying
\[
\frac{R_1}{p} \leq C \left( \frac{\text{SNR}_{R}}{p} \right),
\]
\[
\frac{R_1}{p} + \frac{R_2}{p+2} \leq C \left( \frac{\text{SNR}_{R}}{p} \right) + C \left( \frac{\text{SNR}_{R}}{p} \right) + pC \left( \frac{\text{SNR}_{R} + \text{SNR}_{R} + 2\sqrt{\text{SNR}_{R} \text{SNR}_{R}}}{p} \right).
\]

Our main result is summarized in the following theorem.

**Theorem 3.1** (Capacity Region to within a Bounded Gap): For capacity region \( \mathcal{C} \) (u, r), we have inner and outer bounds
\[
\mathcal{C} \geq \mathcal{A}^\text{in} \cap \mathcal{A}^\text{in} \text{ (u, r) \in \{d, i\} }^2,
\]
\[
\mathcal{C} \leq \mathcal{A}^\text{in} \cap \mathcal{A}^\text{in} \text{ (u, r) \in \{d, i\} }^2.
\]

Since for all \( (u, r) \in \{d, i\} \), \( \mathcal{A}^\text{in} \) and \( \mathcal{A}^\text{in} \) are within a bounded gap, and so are \( \mathcal{A}^\text{out} \) and \( \mathcal{A}^\text{in} \), we have characterized the capacity region to within a bounded gap.

**Proof:** Regarding the proof of the converse, we employ cut-set based outer bounds and enhance the downlink channel to a degraded broadcast channel where feedback does not increase the capacity region [11], [12]. Details can be found in the Appendix of the extended version of this paper [13].

Regarding the achievability, here we provide the scheme for the inner bound of \( \mathcal{C} \) (d, i) in (10), the case where end users and relay all have delayed state information. The proofs for the other three combinations in Theorem 3.1 easily follow, and are also provided in the Appendix of [13].

Our scheme consists of two phases: the uplink phase and the downlink phase. In the uplink phase, the relay terminal aims to decode the two XORs of the two pairs of messages \( \Sigma_1 = W_{R_1} \oplus W_{B_1} \), \( \Sigma_2 = W_{R_2} \oplus W_{B_2} \), \( i = 1, 2 \) from its received signal, and store them for later uses. Hence, it can be viewed as a function computation problem over a multiple access channel. In the downlink phase, the relay terminal re-encodes the stored XORs \( \{\Sigma_1, \Sigma_2\} \) and delivers \( \Sigma_2 \) to end users \( \{A_i, B_i\} \) for \( i = 1, 2 \).

Further note that in the symmetric setting, since the rate of the messages \( W_{A_i} \) and \( W_{B_i} \) are both \( R_i \), the rate of the XOR \( \Sigma_i \) is also \( R_i \), for \( i = 1, 2 \). Hence, we are able to establish the inner bound region of achievable \( (R_1, R_2) \) as the intersection of the inner bound region of the uplink phase and that of the downlink phase, denoted by \( \mathcal{A}^\text{in} \) and \( \mathcal{A}^\text{in} \) respectively.

To achieve \( \mathcal{A}^\text{in} \) in the uplink phase, we will use lattice-based compute-and-forward [6]. Casting it as a function computation problem over the four-transmitter multiple access channel, the relay can successfully decode the XORs of messages \( \Sigma_i = W_{R_i} \oplus W_{B_i}, i = 1, 2 \) from its received signal (2) without explicitly decoding the four messages \( \{W_{A_i}, W_{B_i}, W_{A_i}, W_{B_i}\} \), thanks to the linearity of lattice codes. The detailed encoding/decoding steps for our uplink phase are given in Section IV of [13]. Compared with the scheme in [2], our scheme needs to deal with the additional ergodic activity states \( \{S_1(t), S_2(t)\} \) and the delayed state information. Also we adopt joint lattice decoding from [14], which has better performance than the successive lattice decoding in [2].

To finish up, below we give the proof sketches for the downlink phase in Sec. IV. The detailed proofs of both uplink and downlink are given in the Appendix of [13].

**Remark:** If the relay has only delayed state information, the downlink from relay becomes the bottleneck for information flow within the system. One can verify this observation by looking at the sum degree-of-freedoms (DoF). The sum DoF of the uplink region \( \mathcal{A}^\text{in} \) is \( p(2 - p) \), while that of the downlink region \( \mathcal{A}^\text{in} \) is \( \frac{p(2 - p)}{4 - p} \). The latter is strictly smaller than the former.

IV. **Proof Sketch of the Inner Bound \( \mathcal{A}^\text{in} \) in (10) for Downlink with Delayed State Information**

In our symmetric setting, since \( A_i \) and \( B_i \) have the same receiver SNRs and are under the same activity state \( \{S_i(t)\} \), for \( i = 1, 2 \), we can treat the downlink as a broadcast channel (1) where the relay sends \( \Sigma_1 \) to user \( B_1 \) and \( \Sigma_2 \) to user \( B_2 \) respectively, with delayed state information. Compared with [4], which is focused on ergodic Rayleigh fading downlink with equal received SNRs, our downlink (1) has different on/off channel statistics and unequal \( \text{SNR}_{R} \geq \text{SNR}_{R} \). These two differences raise new challenges for obtaining bounded-gap capacity results.

For the corner point of the outer bound region where (7) and (5) intersect, achieving it to within a bounded gap can be simply done by Gaussian superposition coding. Thus we focus...
on the other corner point where (6) and (7) intersect:

\[
R_1 = p \left( C(SNR_{1R}) - C(SNR_{2R}) \right) + \frac{p(2-p)}{3-p} C(SNR_{2R}), \quad (12)
\]

\[
R_2 = \frac{p(2-p)}{3-p} C(SNR_{3R}). \quad (13)
\]

Our scheme to achieve \((R_1 - \Delta_1, R_2 - \Delta_2)\) with \(R_1, R_2\) taken from (12)(13) is a non-trivial extension of the binary in the binary erasure broadcast channel [11], where \((\Delta_1, \Delta_2)\) are given in (8)(9). To obtain insights, we start with a binary-expansion model [5] for this problem as follows.

A. Insights from Binary-Expansion Model

![Diagram](image)

Fig. 2. Example for achieving corner point (14) with \((n_{1R}, n_{2R}) = (3, 2)\) for the binary expansion downlink with delayed state information.

In this subsection, we employ a binary expansion model corresponding to the downlink phase (1) to obtain insights. In this model, the transmitted and received signals are binary vectors in \(F_2^q\), where \(F_2\) denotes the binary field \([0, 1]\). The received signals are \(Y_{B_i}[t] = H_{B_i} R S_{i}[t] X_R[t], i = 1, 2,\) where additions are modulo-two component-wise. Channel transfer matrices are defined as follows: for \(i = 1, 2, H_{B_i} := S_0^{Q_i}\) where \(q = \max_{i=1,2} (n_{iR})\) and \(S \in F_2^{q} \times q\) is the shift matrix defined in [5]. The corner point corresponds to (12) and (13) in this model is

\[
(R_1, R_2) = \left( p(n_{1R} - n_{2R}) + \frac{p(2-p)}{3-p} n_{2R}, p(2-p) n_{2R} \right). \quad (14)
\]

To achieve this point, the relay uses a three-phase coding scheme extending that in [11]. In Phases I and II (each with block length \(T\)), the relay sends bits intended for \(B_1\) and \(B_2\), using the top \(n_{1R}\) and \(n_{2R}\) levels respectively. In addition, in Phase II the relay also uses the bottom \((n_{1R} - n_{2R})\) levels to deliver additional bits to \(B_1\). Hence, \(B_1\) and \(B_2\) receive roughly \(T p(n_{1R} + n_{1R} - n_{2R})\) and \(T p_{12R}\) desired bits in Phase I and II respectively.

In Phase I, there will be roughly \(T p(1-p)n_{1R}\) bits which are erased at \(B_1\) but erroneously sent to \(B_2\). These bits can be used as side-information. We denote this length-\(T p(1-p)\) sequence of \(n_{1R}\)-level binary vector by \(X^e\). Note that the bottom \((n_{1R} - n_{2R})\) levels will lie below the noise level at \(B_2\) and will NOT appear in this binary expansion model. Similarly in Phase II, there will be such a length \(T p(1-p)\) sequence of \(n_{2R}\)-level binary vector intended for \(B_2\) but only received by \(B_1\). We denote it by \(X^d\). We aim to recycle these bits in Phase III.

The block length of Phase III is roughly \(T p(1-p)\). In Phase III, the relay makes use of delayed state information to form \(X^e\) and \(X^d\). Then it sends out \(X^e \oplus X^d\) from the MSB level as depicted on the rightmost of Figure 2. Hence the bottom \((n_{1R} - n_{2R})\) levels consists of bits in the bottom levels of \(X^e\) only. With side information received in Phase I and II, each receiver can decode the desired bits from the received XORs. In total the numbers of bits recycled in this phase are \(T p(1-p)n_{1R}\) and \(T p(1-p)n_{2R}\) at \(B_1\) and \(B_2\) respectively.

Putting everything together, we achieve \(R_1 = \frac{p(2-p)}{3-p} n_{2R}\) and \(R_1 = p(n_{1R} - n_{2R}) + \frac{p(2-p)}{3-p} n_{2R}\).

B. Proof Sketch of Bounded-gap Achievement to the Corner Point (12) (13) of the Outer Bound Region

Extending to the Gaussian case, we face the following two challenges. First, in Gaussian channel, we are sending complex symbols instead of bits and there will be additive Gaussian noise. Second, we need to incorporate superposition coding into Phase II of Fig. 2, while \(B_1\) may not be able to decode and cancel the higher-layer codeword since the erasure state process at \(B_1\) and \(B_2\) are different. Note that only signals of \(B_2\) in Phase II have recycling from Phase III.

We solve the first challenge by resending erased symbols instead of bits in the third phase. To do this, the relay will quantize the sum sequence formed by the erased symbols, \(X^e + X^d\), and then send out the quantization indices. Based on the insight learned in the binary expansion model, we know that the resolution of reconstruction must be different: \(B_1\) requires higher resolution than \(B_2\) since \(X^e\) goes deeper in the bit levels. Hence, instead of directly quantizing into a single quantization index, we employ successive refinement source coding [15] so that \(B_2\) is able to get a higher resolution in reconstruction. Again gaining insights from the binary expansion model, since the number of layers used by \(B_1\) is \(n_{1R}\) for \(i = 1, 2\) in Phase I and II respectively, the MSE of the reconstruction at \(B_1\) should be inverse proportional to \(SNR_{1R}: i = 1, 2\).

For the second challenge, we aim to solve it using dirty paper coding (DPC). However, the conventional DPC requires fully known channel information \(S_{i}(t) h_{B_iR}\) at the transmitter [16]. In our case, the current on/off state \(S_{i}(t)\) is unknown at the relay. Hence, we propose new one-dimensional (symbol-based) lattice strategy to solve this problem.

Our scheme is summarized as follows

**Phase I:** By using random Gaussian codebook, relay sends coded symbols \(X_R[t], t = 1, \ldots, T\) from the codeword representing message for user \(B_1\).

**Phase II:** Relay sends \(X_R[t] = X_{2R}[t] + X_{1R}[t], t = T + 1, \ldots, 2T\), where \(X_{2R}[t]\) are coded symbols for user \(B_2\) and \(X_{1R}[t] = (C_{1R}[t] - w \cdot h_{B_1R} X_{2R}[t] - d[t]) \mod L\) (15) where \(C_{1R}[t]\) is coded symbol for user \(B_1\). \(d[t]\) is the independent dither. For a real number \(x, x \mod L = x - Q(x)\) with \(Q(x)\) being the nearest multiple of \(L\) to \(x\).

**Phase III:** Let the erased symbols sent to the wrong receiver in Phase I and 2 be \(X^e\) and \(X^d\) respectively. Relay first quantizes
the length $T_p(1 - p)$ sequence $X^I_1 + X^I_2$ using successive refinement into indexes $i_c$ and $i_r$, where $i_c$ is the common index which will be decoded for both $B_1$ and $B_2$ while $i_r$ is the refinement index which will be decoded only at $B_1$. Gaussian superposition channel coding with length $T_p(1 - p)$ is adopted to transmit $(i_c, i_r)$. Now, user $B_2$ can know the noisy reconstruction $X^I_1 + X^I_2 + Z_{D_2}$ with MSE $D_{2}$, by decoding $i_c$. With proper power allocation, $B_1$ (better channel) knows the reconstruction $X^I_1 + X^{II}_1 + Z_{D_1}$ by successively decoding $i_c$ and $i_r$, where reconstruction error $Z_{D_1}$ has smaller MSE $D_1$ than that of $Z_{D_2}$. For this two-receiver source-channel coding, the rates for common index $i_c$ and refinement index $i_r$ are chosen as
\[ \log \left(1 + \frac{2}{\gamma} \right) \text{ and } \log \left(1 + \frac{2}{\gamma} \right) - \log \left(1 + \frac{2}{\gamma} \right) \] respectively. To ensure successful channel decoding at receivers, we need to carefully choosing the power allocation of the superposition channel coding, as well as $D_1$ and $D_2$ in (16). Let the power allocation for indexes $i_c$ and $i_r$ be $SNR_c$ and $SNR_r$ respectively. We choose $SNR_r = 1/\text{SNR}_{2R}, SNR_c = 1 - SNR_r$. (Here we only provide the proof when $SNR_{2R} \geq 2$, since the bounded-gap result for $SNR_{2R} < 2$ is trivial.) For $B_2$ to correctly decode $i_c$, from (16) and the lengths of channel and source codes, we need to choose
\[ D_2 = \frac{1}{SNR_{2R} - 1} \] (17)
For receiver $B_1$, to decode both $i_c$ and $i_r$, we choose
\[ D_1 = \frac{1}{SNR_{2R} + SNR_R} \] (18)
Note that $D_1$ is inverse proportional to $SNR_{2R}, i = 1, 2$, consistent with the insights from the binary expansion model.

Now receivers $B_1, i = 1, 2$ can obtain reconstructions of erased symbols $X^I_1 + X^{II}_1$ with $D_1$ in (18) and $D_2$ in (17) respectively. With side-information $X^I_1$ (noisy) from Phase I, $B_2$ can combine $T_p(1 - p)$ reconstructed symbols $X^{II}_1 + Z_{D_2} - Z_{B_2}$ and the $T_p$ un-erased symbols received in Phase II to decode XOR $\Sigma_2$. Then (13) is achievable with bounded gap $\Delta_2$. To see this, we can first upper-bound the MSE distortion $D_2$ in (17) as
\[ D_2 \leq \frac{1}{SNR_{2R}} \] (19)
By choosing the power allocation of $X_{1R}$ and $X_{2R}$ in Phase II be $1/\text{SNR}_{2R}$ and $1 - 1/\text{SNR}_{2R}$ respectively, together with the independence of these two signals, we have the following achievable rate for user $B_2$
\[ R_2 \geq \frac{1}{3 - p} \left[ (p(1 - p)c \left( \frac{1}{SNR_{2R}} + \frac{1}{SNR_c} \right) \right] + p \left( \frac{1}{SNR_{2R}} + \frac{1}{SNR_r} \right) \] (20)
where (19) is applied to obtain the first term in the RHS of (20). Then bounded gap result can be obtained from (20).

Now we show that for user $B_1$, rate $R_1 - \Delta_1$, with $R_1$ in (12), is achievable. Following similar procedure as $B_2$ aforementioned, by combining the erased symbols with the un-erased symbols received in Phase I, the following rate is achievable to decode XOR $\Sigma_1$,
\[ \frac{p(2 - p)c(SNR_{1R}) - p(1 - p)}{3 - p} \log (3) \] (21)
where the following inequality from (18) is used
\[ D_1 \leq \frac{2}{SNR_{1R}} \]
Moreover, user $B_1$ can decode additional messages by forming the following channel from the un-erased symbols in Phase II,
\[ [C_{1R}[t] + E_L(t)] \bmod L, \]
where $E_L(t) = (w_{B_1} + 1)(X_{1R}[t] + u_{B_2}[t])$. The channel (22) is a modulo-$L$ channel with length $T_p$ and power $L^2/12 = 1/\text{SNR}_{2R}$, then rate
\[ \frac{p}{3 - p} \left( \log \left( \frac{SNR_{1R}}{SNR_{2R}} \right) - \log \left(2\pi e/12 \right) \right) \] (23)
is achievable. By summing (23) and (21), our achievable rate for user $B_1$ has bounded gap to (12).

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