Bi-level Demand Response Game with Information Sharing among Consumers

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Abstract: In this paper, we formulate the demand response problem in smart grid as a bi-level game: a consumer-level noncooperative game and a one-leader-one-follower Stackelberg game between the provider-level and the consumer-level. We prove the existence of a Nash Equilibrium for the noncooperative game and a Stackelberg Equilibrium for the Stackelberg game, focus on the case with information sharing among all consumers, and design distributed algorithms for the supply side and demand side as well as the information platform. Numerical results are provided to illustrate the performance of the proposed algorithms and the effectiveness of information sharing for improving each consumer’s payoff.

Keywords: electric power systems, demand response, game theory, information integration

1. INTRODUCTION

Nowadays, taking advantages of widely deployed smart meters and two-way communication facilities of smart grid, demand response, aiming to change the electricity usage patterns of end-users by the pricing strategies of the supply side, can lead to a significant improvement of grid reliability and efficiency [Siano (2014)]. Among all the pricing programs, real-time pricing (RTP) has been widely considered to be the most efficient, where the electricity price varies with every time interval (usually each hour or each 15 minutes) [Deng et al. (2015)]. In such a system, game theory can be leveraged to model the interactive decision-making process of the power provider and consumer [Saad et al. (2012)].

There have been extensive researches on various demand response game models that characterize the behaviors of the power providers and consumers. For example, Ibars et al. (2010) and Nguyen et al. (2012) studied the demand response game focusing on energy cost minimization and peak-to-average ratio minimization, Mohsenian-Rad et al. (2010) and Deng et al. (2014) studied the noncooperative game among residential consumers, Chen et al. (2010) studied the electricity market from the social perspective, aiming at optimization of the social welfare of the entire system. Wu et al. (2012) focused on PHEV charging and discharging scenarios. Maharjan et al. (2013) and Chai et al. (2014) studied the scenario where multiple power providers interact with multiple consumers.

In spite of the aforementioned works on different demand response game models under different scenarios, there is lack of a unified hierarchical game structure that characterizes the high-level leader-follower relationship between the supply side and demand side, and has flexibility of incorporating different game models within each side. Besides, there is lack of study on comparing different information sharing mechanisms in the game, while information symmetry and sharing are crucial in game theory.

This paper serves as a starting point for addressing the above-mentioned problems, in which we focus on a microgrid system with one power provider and multiple consumers, and model the overall demand response game by a bi-level model, comprising (1) a consumer-level noncooperative game and (2) a one-leader-one-follower Stackelberg game between the provider-level and consumer-level. The structure has good extendability in terms of adding any game on the supply side or changing any game on the demand side, as long as the game satisfies similar good properties. In addition, we have designed an information sharing platform for consumers and studied the case when all consumers share their demand information. Numerical results are provided to illustrate the win-win effectiveness of information sharing among consumers.

The rest of the paper is organized as follows. The system model is described in Section 2. In Section 3, we formulate the demand response problem as a bi-level game and prove the existence of equilibrium. In Section 4, we study the case with information sharing and design distributed algorithms. Numerical results are provided in Section 5 and conclusions are drawn in Section 6.

2. SYSTEM MODEL

2.1 Two-way Communication Infrastructure

Consider a microgrid system of one power provider and N consumers. As shown in Fig. 1, the infrastructure of the system contains two layers. On the power layer,
each user connects to the power provider via the power line. On the communication layer, users connect with each other and the power provider via the local area network (LAN). Through the LAN, the real-time two-way communication between the power provider and each user becomes feasible. In addition, users can share their information with each other via the LAN conveniently.

![Diagram of two-way communication infrastructure](image)

**Fig. 1.** Two-way communication infrastructure.

In smart grid, the computation and optimization is implemented by programmed computers and smart meters. On the supply side, usually the pre-programmed computers are responsible for computing and implementing the real-time pricing strategies. On the demand side, each residential consumer is equipped with a smart meter, which can be pre-programmed to do computation in response to the real-time price, and to take automatic control of all household appliances based on the computation results.

### 2.2 Cost Function for Power Provider

The cost function $C(s)$ models the expense of supplying $s$ unit of energy by the power provider. Demand response accommodates any form of cost functions as long as they satisfy the following two properties [Deng et al. (2015)].

**Property 1.** Increasing: the cost always increases when the supply amount increases.

$$C(s_1) < C(s_2), \quad \forall s_1 < s_2.$$  

**Property 2.** Strictly convex: the marginal cost always increases when the supply amount increases.

$$\frac{\partial C(s_1)}{s_1} < \frac{\partial C(s_2)}{s_2}, \quad \forall s_1 < s_2.$$  

The piece-wise linear function and quadratic function are two common choices. In this paper, we consider a quadratic cost function [Samadi et al. (2010)]:

$$C(s) = as^2 + bs + c,$$  

where $a > 0$, $b \geq 0$, and $c \geq 0$ are three pre-determined parameters.

### 2.3 Gain Function for Power Consumer

The gain function $G_j(d_j)$ models the power consumer $j$’s satisfaction degree obtained by consuming $d_j$ unit of energy. The gain function should be nondecreasing and concave, i.e., it is increasing before the energy consumption reaches a desired level, and gradually gets saturated when the desired level is satisfied [Deng et al. (2015)].

In this paper, we consider a quadratic gain function [Samadi et al. (2010)]:

$$G_j(d_j) = \begin{cases} \omega_j d_j - \frac{\alpha_j}{2} d_j^2, & 0 \leq d_j < \frac{\omega_j}{\alpha_j} \\ \frac{\omega_j^2}{2\alpha_j}, & d_j \geq \frac{\omega_j}{\alpha_j}. \end{cases}$$  

where $\omega_j > 0$ and $\alpha_j > 0$ are pre-determined parameters, which vary among consumers, and also vary for the same consumer during different times of a day. We can see this gain function corresponds to a linear decreasing marginal benefit:

$$\frac{\partial^2 G_j(d_j)}{\partial d_j^2} < 0,$$  

when $0 \leq d_j < \frac{\omega_j}{\alpha_j}$.

### 3. Bi-Level Game Formulation

#### 3.1 Supply Side: Leader-level

Let $l$ denote the total energy demand from all consumers, i.e.,

$$l = \sum_{j \in M} d_j,$$  

where $M$ denote the set of all consumers. For the power provider, the profit from supplying $s$ unit of electricity at the price of $p$ is calculated as the revenue minus the cost function, i.e.,

$$P_p(s, p) = ps - C(s).$$  

The local optimization problem at the supply side is:

$$\max_{s, p} P_p(s, p)$$  

s.t. \( s \geq l \)

\( p \geq 0. \)

It can be seen from (7) that once the total demand $l$ is fixed, the increasing of the supply amount $s$ will result in increasing of the cost and thus decreasing of the profit. Therefore, the provider tends to stay at the minimum supply amount that matches exactly with the demand $s = l$.

Substituting (9) into (7), we have

$$P_p(s, p) = ps - C(s).$$  

The price is chosen to maximize (10):

$$\frac{\partial P_p(s, p)}{\partial s} = p - \frac{\partial C(s)}{\partial s} = 0 \Rightarrow p = \frac{\partial C(s)}{\partial s} \bigg|_{s=l}. $$  

Substituting (3) into (11), we have

$$p(l) = 2al + b.$$  

The price described in (12) is called the market-clearing price [Chen et al. (2010)], which is the local optimal choice of the power provider under the market-clearing condition.

#### 3.2 Demand Side: Follower-level

Each consumer can respond to the power provider based on only the price information. As shown in Fig. 2, each user only communicates with the power provider to exchange the price and demand information. There is no communication or information sharing among consumers.
In reality, consumers can choose whether to share their demand information with each other or not. In this paper, we focus on the scenario where all consumers share their demand information and all shared information are authentic, as shown in Fig. 3. More complicated situations such as partial population participation or cheating will be considered as potential directions in our future work.

Note that although consumers share demand information, the objective of each individual remains selfish. Each consumer still aims at maximizing his own gain while minimizing his payment. With the rational and selfish assumption, it is still a noncooperative game among consumers. We can utilize noncooperative game theory for building the model.

**Noncooperative Consumer Game** The noncooperative game \( G = \{ \mathcal{M}, \mathcal{D}, \{ P_j(\cdot) \}_{j \in \mathcal{M}} \} \) among consumers consists of three components:

1. Players: \( \mathcal{M} = \{ 1, 2, \ldots, N \} \) is a finite set of players. Each consumer in the system is a player.
2. Strategies: \( \mathcal{D} = \times_{j \in \mathcal{M}} \mathcal{D}_j \) is the strategy space of all players in the game. Each player \( j \in \mathcal{M} \) chooses a demand strategy \( d_j \) from his strategy set \( \mathcal{D}_j \). Let \( d_{-j} = (d_1, \ldots, d_{j-1}, d_{j+1}, \ldots, d_N) \) denote the demand strategies of all consumers but \( j \). We can write \( (d_j, d_{-j}) \) for the overall demand profile \( d \).
3. Payoff functions: the player \( j \)'s payoff is determined by the demand profile \( d \). Each selfish and rational player \( j \in \mathcal{M} \) chooses \( d_j \) according to the other players' strategies \( d_{-j} \) to maximize \( P_j(d_j, d_{-j}) \). For each consumer, the payoff from consuming \( d_j \) unit of energy at the price of \( p \) is calculated as the gain function minus the payment, i.e.,

\[
P_j(d_j, d_{-j}) = G_j(d_j) - p(d_j), \tag{13}
\]

**Definition 1.** Nash Equilibrium (NE) [Fudenberg and Tirole (1991)]: A strategy profile \( d^* = (d^*_j, d^*_j) \) is called NE if and only if \( P_j(d^*_j, d^*_{-j}) \geq P_j(d_j, d^*_{-j}), \forall j \in \mathcal{M}, \forall d_j \in \mathcal{D}_j \).

**Definition 2.** An S-modular game restricts the payoff functions \( \{ P_j(\cdot) \} \) such that for \( \forall j \in \mathcal{M} \) either of the following is satisfied [Neel et al. (2004)]:

\[
\frac{\partial^2 P_j(d)}{\partial d_i \partial d_i} \geq 0, \quad \forall i \neq j \in \mathcal{M} \tag{14a}
\]

\[
\frac{\partial^2 P_j(d)}{\partial d_i \partial d_j} \leq 0, \quad \forall i \neq j \in \mathcal{M}. \tag{14b}
\]

**Lemma 1.** For the S-modular game, when NE exists and is unique, best response can be used to derive the solution converging to NE [Altman and Altman (2003)].

**Theorem 1.** NE of the noncooperative consumer game formulated in this paper exists and is unique.

**Proof 1.** Due to the space limitation, we omit the proof.

**Theorem 2.** NE of the noncooperative consumer game formulated in this paper can be achieved by best response.

**Proof 2.** Due to the space limitation, we omit the proof.

**Best Response** Best response means that at each iteration, each consumer adapts his strategy to the strategies of others to maximize his own payoff. Mathematically, each consumer aims at solving

\[
d^*_j = \arg \max_{d_j \in \mathcal{D}_j} P_j(d_j, d_{-j}), \tag{15}
\]

where \( P_j(\cdot) \) is defined in (13).

### 3.3 Leader-Follower Interaction: Stackelberg Game

In fact, the whole electricity market is a one-leader-multi-follower game, where the power provider leads the game by moving first, i.e., defining the electricity price, and then all consumers make their moves following the price afterwards, i.e., deciding the energy demand, and meanwhile consumers may share information with each other on the follower level.

The main difference of a leader-follower game from a normal game lies in the situation of asymmetry of information. The leader chooses his strategy in advance, and then the follower makes the move accordingly. Therefore, many concepts and strategies in normal games are no longer suitable. For example, the equilibrium in leader-follower games may not satisfy NE conditions.

In this paper, the overall game between the power provider and all consumers is now modeled by a bi-level game, comprising (1) a follower-level noncooperative game and (2) a one-leader-one-follower Stackelberg game.

This model is due to the special structure of the game. On the leader side, the power provider does not care about the detail of how each consumer behaves. The provider’s supply and price are only affected by the aggregate behavior of all consumers, as shown in (9) and (12). Therefore, we can represent the response of the whole follower level by one single follower with strategy \( l = \sum_{j \in \mathcal{M}} d_j \).

Another reason for this model formulation is due to the existing research on Stackelberg games. Stackelberg games are sourced from and have been extensively studied on the one-leader-one-follower case. For multi-leader-multi-follower cases, there exists no special focus on the interactions among followers. Since as long as NE exists in the leader-level game, the game structures and equilibrium theories are well-formed. For more complicated cases where followers interact/cooperate/share information with each other, even for the single leader case, the equilibrium theories are not trivial and have not been well studied yet.
**One-Leader-One-Follower Stackelberg Game** The Stackelberg demand response game $\Theta = \{\Omega, S, \{\Phi^P(\cdot), \Phi^L(\cdot)\}\}$ is modeled as follows:

1. Players: $\Omega = \{P, L\}$. The player set has two players: the player $P$ is the leader, representing the power provider, while the player $L$ is the follower, which is a virtual player representing all consumers.
2. Strategies: two sets of strategies $P$ and $L$. The player $P$’s strategy is the price $p \in \mathcal{P}$, and the player $L$’s strategy is the aggregate demand $l \in \mathcal{L}$.
3. Payoff functions: two sets of payoff functions $\Phi^P(\cdot): \mathcal{P} \times \mathcal{L} \to \mathbb{R}$ and $\Phi^L(\cdot): \mathcal{P} \times \mathcal{L} \to \mathbb{R}^N$, where $\Phi^P(\cdot)$ is described in (10), $\Phi^L(\cdot) = [P_1(\cdot), \ldots, P_N(\cdot)]$ is the payoff vector of all consumers.

**Definition 3.** Best Response Set $\mathcal{R}^L(p)$: The best response $l^*(p) \in \mathcal{R}^L(p)$ of the follower after observing the leader’s strategy $p \in \mathcal{P}(p)$ is $l^*(p) = \sum_{j \in \mathcal{M}} d_j(p)$.

**Definition 4.** Stackelberg Equilibrium (SE): A pair of strategies $(p_S, l_S) \in \mathcal{P} \times \mathcal{L}$ is called SE if $l_S \in \mathcal{R}^L(p_S)$ and $\Phi^P(p, l) \leq \Phi^P(p_S, l_S)$ for every pair $(p, l)$ with $l \in \mathcal{R}^L(p)$ [Bressan (2010)].

**Lemma 2.** If the sets $P$ and $L$ are compact metric spaces, and the payoff functions $\Phi^P(\cdot)$ and $\Phi^L(\cdot)$ are continuous, SE always exists [Bressan (2010)].

**Theorem 3.** SE of the one-leader-one-follower demand response Stackelberg game we formulated in this paper exists.

**Proof 3.** Due to the space limitation, we omit the proof.

### 4. DISTRIBUTED ALGORITHMS WITH INFORMATION SHARING

In this section, we focus on the scenario where all consumers share their demand information with each other. However, in practice, it is obvious that no one would like to share his information if he can obtain the information of others without sharing his own. Therefore the equilibrium is no one obtains any other information. This is the reason why information sharing is not the usual case.

![Demand Information Sharing Platform](image)

To avoid the above-mentioned situation, an information sharing platform can be set up, as shown in Fig. 4, which forces each user to contribute his own demand strategy in order to retrieve the aggregate demand information of others. We derive the game solution for the situation that (1) all consumers participate and (2) everyone contributes his real demand strategy, and design distributed algorithms for the supply side and demand side.

Specifically, we calculate the best response of the virtual follower in the Stackelberg game by achieving NE of the noncooperative game among consumers, and let the leader choose his optimal action accordingly. NE of the noncooperative consumer game is calculated by deriving the best response strategy of each consumer. The above calculation processes repeat until the overall demand response game converges. The convergence condition is that the power provider and all consumers do not revise their strategies any more.

We begin with solving the best response problem for each consumer as formulated in (15), which could be directly solved by letting the first-order derivative equal zero, i.e.,

$$\frac{\partial P_j(d_j, d_{-j})}{\partial d_j} = G'_j(d_j) - 2a(l + d_j) - b = 0.$$

However, this method may not be applicable in practice since both $a$ and $b$ are the power provider’s cost function parameters. It is not realistic that the consumers know them.

Due to the strict concavity of the payoff function, a gradient projection method [Wang and Xiu (2000)] is designed to solve (15), which does not require the functional parameters of the power provider:

$$d_{j+1}^{k+1} = \left[ d_j^k + \gamma \frac{\partial P_j(d_j^k, d_{-j})}{\partial d_j^k} \right]^{\mathcal{D}_j},$$

where

$$d_{j}^k + \gamma \left[ G'_j(d_j^k) - p^k - \frac{\Delta p}{\Delta d_j^k} \right] \right]^{\mathcal{D}_j}.$$  

On receiving each consumer’s demand strategy, the information sharing platform computes the aggregate demand by (6) and shares it with all consumers as well as the power provider. Based on the aggregate demand, the power provider updates his supply and price by (9) and (12) respectively, and broadcasts the updated price to all consumers. On receiving the updated price from the power provider and the aggregate demand from the information sharing platform, each consumer updates his demand strategy by (16). The iteration processes repeat until the game converges, i.e., no one revises his strategy any more. The algorithms for the supply side and demand side as well as the information platform are summarized as Algorithm 1, 2, and 3, respectively.

**Algorithm 1** : Executed by the power provider.

1. Initialization.
2. repeat
3. Receive demand $d_j^k$ from all consumers $j \in \mathcal{M}$.
4. Update supply $s^k$ by (9).
5. Update price $p^k$ by (12).
6. Broadcast updated price $p^k$ to all consumers.
7. until price does not change

### 5. NUMERICAL RESULTS

In this section, we provide numerical examples to evaluate the performance of the proposed bi-level demand response game with information sharing among consumers. For ease of illustration, we consider a simple case for a microgrid system with one power provider and three consumers. It can be extended to more users, with similar results.
The simulation parameters for the power provider is $a = 0.1$, $b = 0.5$, and $c = 0$, i.e., the supply cost function is $C(s) = 0.1s^2 + 0.5s$. For the parameters of the consumer gain functions, we first fix $\alpha$ and vary $\omega$, then fix $\omega$ and vary $\alpha$ to show different cases. The iteration step size is fixed at $\gamma = 0.5$. All initial values of the price and demand are set to be random and it is verified through tests that the initial point does not affect the convergence point of the game.

In Fig. 5, performance comparison details between the two scenarios are provided. One is without information sharing among consumers, i.e., each consumer responds to the power provider based on only the price information. The solution to this case is denoted by solution 1 (no). The other is with information sharing among consumers, i.e., each consumer responds to the power provider based on the price information and the aggregate demand information of others. The solution to this case is denoted by solution 2 (share).

Fig. 6 shows more simulation results when $\alpha$ varies from 0.4 to 0.6, and $\omega$ varies from 2.5 to 3.5. In the bar chart, solution 1 (no) is denoted by green bars, while red bars for solution 2 (share). The performance comparison metrics include each user’s energy demand, each user’s payoff, all users’ average payoff, power provider’s profit, and social welfare.

Both figures illustrate that each consumer’s payoff is improved by information sharing, while the power provider’s profit drops and the social welfare slightly decreases. When comparing solution 2 (share) with solution 1 (no), we can see that the information at the supply side remains the same, while each consumer at the demand side gains more information by information sharing. From each consumer’s perspective, by contributing his own demand strategy, he can retrieve the aggregate demand information of others.
Algorithm 2: Executed by each consumer $j \in M$.

1: Initialization.
2: repeat
3: Receive price $p^k$ from power provider.
4: Receive aggregate demand $l^k$ from information platform.
5: if $k > 1$
6: Record price change $\Delta p = p^k - p^{k-1}$.
7: Record aggregate demand change $\Delta l = l^k - l^{k-1}$.
8: Compute $\Delta p/\Delta l$.
9: else
10: Set $\Delta p/\Delta l = 0$.
11: end
12: Update demand $d_{j}^{k+1}$ by (16).
13: Communicate the updated demand $d_{j}^{k+1}$ to power provider.
14: Communicate the updated demand $d_{j}^{k+1}$ to information platform.
15: until demand does not change

Algorithm 3: Executed by the information platform.

1: Initialization.
2: repeat
3: Receive demand $d_{j}^{k}$ from all consumers $j \in M$.
4: Compute aggregate demand $l^k$ by (6).
5: Broadcast aggregate demand $l^k$ to all consumers.
6: until aggregate demand does not change

and increase his own payoff. It is actually a win-win situation for all consumers since every user’s payoff is improved. Therefore, we can conclude that information symmetry and sharing are crucial in game theory and are directly related to solution performance.

6. CONCLUSION AND FUTURE WORK

In this paper, we have studied a bi-level game model for the demand response problem with one power provider and multiple consumers. The bi-level game consists of (1) a consumer-level noncooperative game and (2) a one-leader-one-follower Stackelberg game between the provider-level and consumer-level. An information sharing platform is designed for the scenario where all consumers share their demand information with each other. We have proved the existence of equilibrium for both games and proposed distributed algorithms for the supply side and demand side as well as the information platform. Numerical results are presented to illustrate the performance of the proposed algorithms and the effectiveness of information sharing for improving every user’s payoff.

Serving as a starting point, the bi-level game model and information sharing mechanism analyzed in this paper can be further extended from several aspects. The scalability and convergence rate of the proposed approach will be studied. The equilibrium proof under more general cost and gain functions will also be interesting. In addition, we will explore the scenarios with multiple power providers and consumers, and compare different information sharing mechanisms such as partial population participation or cheating, both numerically and theoretically.

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