Application of Transition probability based on random graph and Markov chain for the Monitoring of Complex Network

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Abstract. In recent years, the method of graph theory has been used in the process of solving many practical problems, especially the transition probability of random graphs. Because it can be used to solve the problem of numerical variables with random phenomena, it is also practical. In the middle, the Markov chain provides a transformation and functional relationship from a set of discrete vectors to the state space, and is often used to describe the transition from one initial state to another end state in various state spaces. The process and the transition process are random. The research in this paper is based on the transition probability of the random graph and the Markov chain to establish a mathematical model, so that complex networks can be monitored, including the ability to predict the long-term steady state of complex networks. The required transition time, the quantitative relationship between the number of monitoring points and the reliability and sensitivity of the network were obtained.

1. Introduction
In the world of mathematics, graph theory belongs to the field of discrete mathematics and includes discrete mathematics. Graph theory itself belongs to applied mathematics, many mathematicians have studied graph theory separately and have made great contributions to the development of graph theory. Graph theory is developed around the two most important elements of graphics. The first element is a point, which usually represents a variety of different objects. The other element is a connection point. The connection with the point, the connection is usually used to express the relationship between things. The relationships contained in the links they make up, and the different relationships of various things can be represented by link segments [1, 2]. In addition to the traditional academic research and research fields of science and engineering, graph theory can also be used in sports and other entertainment and life activities, such as how to skillfully complete a sports event schedule in the shortest time. Researchers such as Li [3] ingeniously applied the coloring theory in map theory to the specific feature design of time sequencing and arrangement problems, thus compiling the IS method schedule, and the method through a large number of practical applications shows that the IS method is used for arrangement the schedule of large-scale competitions is efficient and effective. In many disciplines such as natural sciences, chip design, artificial intelligence, machine learning, and other branches of mathematics, these and other practical problems all involve the use of graph theory. It can be said that the use of graph theory can help people solve these problems more intuitively.

In the field of priority search [4-7], both depth-first search and breadth-first search are graph search algorithms. They are similar but different, and they are often used in different places in applications. The main difference between them is the breadth or depth of their walking [8]. The process includes network embedding, rather than manually extracting features [9, 10]. Random graphs are accompanied by random processes. In the process of using random graphs, both knowledge of graph theory and
probability theory are needed. This theory was proposed by Paul Eldersch and Alfred Rainey in a series of papers [11, 12].

If the Markov chain can be divided into several states, past data can form a sequence of these states in time. Then starting from one of the states, after any transition, a certain set of possible states will inevitably appear. This process is called transition probability. Transition probability is an important concept in Markov chain [13-15]. A most simplified Markov process includes one time, and it is the transition probability that governs the transformation of the Markov process. The matrix composed of several transition probabilities is called the transition probability matrix. A characteristic of the transition probability matrix is that the sum of the elements in each row is 1.

In recent years, the method of graph theory has been used in the process of solving many practical problems, especially the transition probability of random graphs. Because it can be used to solve the problem of numerical variables with random phenomena, it is also practical. There is also a large amount of information with uncertainty, which is increasingly being widely used, such as Tang [16] using Markov chain state transition probability matrix to predict market changes; Zhou [17] using the transition probability in Markov model to evaluate pharmaeconomics. Luo [13] using dynamic Markov transition probability for multi-model tracking of maneuvering targets; Zuo [18] based on the transition probability model to predict and analyze the needs of elderly long-term care. Among the above studies, the Markov chain provides a transformation and functional relationship from a set of discrete vectors to the state space, and is often used to describe various state spaces from one initial state to another end state. The conversion process is random.

2. Mathematical Model

(1) The known information involved in the mathematical model

Complex network under study; point set of long-term state of complex network \( NV^L \subseteq NV \); point set of short-term state of complex network \( NV^S \subseteq NV \); weight of all connections \( W \); all possible connections and their corresponding directions \( \vec{N}E \).

(2) Information to be solved in the mathematical model

The directed graph corresponding to the complex network \( NG = < NV, \vec{N}E, W > \) ; establish a dynamic random graph of a complex network: For a random process \( \{S(t), t \in T\} \), if for any positive integer \( n \) and the transition probability \( P_{ij}(n) = P\{S_{n+1} = j | S_n = i\} \) of the random graph corresponding to each step of the state change of the complex network \( P\{S(t_1) = s_1, \ldots, S(t_n) = s_n\} > 0 \); ordered Markov chain \( \{S_n, n \in T\} \); monitor the complex network whether it exceeds a certain threshold for \( \{S_n, n \in T\} \); predict the long-term steady state of complex networks \( \lim_{n \to \infty} S_n, n \in T \); calculate the transition time required for a complex network to transition from a non-equilibrium state to an equilibrium state; determine the quantitative relationship between the number of monitoring points and the reliability and sensitivity of the network

3. Case Study

The case analysis used in this research is based on the mathematical model established above, and applied to the transition probability of the random graph and the Markov chain, so that the water system composed of 10 lakes and the passenger flow connected between them can be analyzed. The evaluation of a complex network includes the ability to predict the long-term steady state of the complex network, the transition time required for the transition of the complex network from non-equilibrium to equilibrium, and the quantitative relationship between the number of monitoring points and the reliability and sensitivity of the network. The known information involved in the case analysis includes: the complex network under study, the network includes a total of 10 nodes, as shown in Figure 1, the directed relationship diagram of a complex network with ten nodes involved in the example analysis, assuming that there is a directional connection between any two nodes, representing
A water system consisting of ten lakes and rivers connecting the ten lakes; the point set of the long-term state of the complex network, including nodes 1-10; point set of short-term state of complex network, node 1-10; all possible connection relationships and their corresponding directions, the network includes a total of 10 nodes (Figure 1). The directed relationship diagram of a complex network with ten nodes involved in the example analysis. There is a directional connection between them, which represents a water system composed of ten lakes and the rivers connecting the ten lakes. The direction of the arrow in the figure is the direction of the corresponding connection relationship.
After running the model, the results obtained are as follows, covering almost all the content that we hope to solve above: According to the magnitude and weight of the mutual influence between the nodes in the dynamic process, a complex network DRG of ten nodes with negligible mutual influence (stage 1) can be obtained, as shown in Figure 2; DRG of a complex network with ten nodes in a subgraph (stage 2); DRG without ten nodes with relative subgraphs that is not completely interconnected (stage 3); there is no complex network DRG with ten nodes with relative subgraphs that are not completely interconnected (stage 4); A DRG with ten nodes of the graph (stage 5); DRG with ten nodes without relative subgraphs that are not completely interconnected (stage 6); DRG with ten nodes without relative subgraphs that are not completely interrelated (stage 7); DRG with three nodes (stage 8); DRG with ten nodes without relative subgraphs that are not completely interrelated (stage 9); there is no fully interrelated complex network DRG with ten nodes without relative subgraphs (stage 10); there is no fully interrelated complex network with ten nodes without relative subgraphs (stage 11); the complex network DRG (stage 12) is shown in Figure 13 without completely interrelated and ten nodes without relative subgraphs.
The directed vector diagram of the interaction of the dynamic complex network obtained after the model is run is shown in Figure 17 to Figure 38. It can be seen from the figure that the interaction mechanism between complex network nodes is relatively complicated at the beginning, but as time goes by, the mutual influence between complex network nodes gradually weakens and slows down. After 20 time periods, three relatively independent directed subgraphs are formed: 1-->7, 6-->2-->3, 4-->10, and the remaining three nodes, Node 5, node 8, and node 9 have almost no mutual influence with the above seven nodes. After a time period of 21, the mutual influence of 10 nodes is almost nonexistent, and the directed graph becomes relatively independent of each other. 10 isolated points that are not connected by a directed relationship vector can be regarded as a long-term steady state.

The observation of the state of the complex network includes the observation of the network nodes and the evaluation of the mutual influence between the nodes in the complex network. With the passage of time, the mutual influence of the network nodes on each node itself has gradually decreased. The number is equal to the sum of the number of nodes and the number of directed connections between nodes whose influence is greater than the reliability and sensitivity of the network, so as time goes by, the number of monitoring points will gradually decrease, from the initial need to monitor ten nodes and their connections In the end, the relationship path between nodes only needs to monitor ten nodes.
4. Conclusion
The research in this article is based on the application of graph theory and used it to solve practical problems. The required transition time, the quantitative relationship between the number of monitoring points and the reliability and sensitivity of the network were obtained. The established model was used to evaluate the complex network of water systems composed of 10 lakes and the rivers connected between them. After running the model, the conclusions are as follows:

1. According to the magnitude and weight of the mutual influence between nodes in the dynamic process, different relationship network diagrams can be made: a complex network DRG of ten nodes with negligible mutual influence, and ten nodes with relative subgraphs that are locally interrelated. A DRG of a complex network of three nodes, a DRG that does not have ten nodes with relative subgraphs that are not completely interrelated, and DRG that contains all possible connections.

2. From the directional vector diagram of the interaction of the dynamic complex network, the interaction mechanism between the complex network nodes is more complicated at the beginning, but as time goes by, the complex network nodes and the mutual influence between them gradually weakened and slowed down. After a certain number of time periods, a number of relatively independent directed subgraphs and a number of isolated points are formed. If the mutual influence of nodes is almost non-existent, the directed graph becomes relatively independent of each other, including non-directed Several isolated points connected by relation vector.

3. The observation of the state of the complex network includes the observation of the network nodes and the evaluation of the mutual influence between nodes in the complex network. With the passage of time, the mutual influence of the network nodes on each node itself has gradually decreased. The number of points is equal to the sum of the number of nodes and the number of directed connections between nodes whose influence is greater than the reliability and sensitivity of the network. Therefore, as time goes by, the number of monitoring points will gradually decrease. In the end, the relationship path between nodes only needs to monitor the node itself.

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