Calculation of branching fraction and $CP$ violation in $B^- \to D_s^- D^0$ decay

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October 17, 2022

Abstract

The most precise measurement of the $CP$ asymmetry in the decay $B^- \to D_s^- D^0$ has been reported by LHCb collaboration with the value of $(-0.4 \pm 0.5 \pm 0.5)\%$. In this study, the $CP$ violation in the decay $B^- \to D_s^- D^0$ has been calculated under the factorization approach. This decay mode includes current-current tree and penguin diagrams and their amplitudes are considered separately. In each of the tree and penguin amplitudes, the strong and weak phases have been introduced. The $CP$ asymmetry has been calculated in this work to be $(-0.35 \pm 0.03)\%$. Finally, from the sum of the amplitudes, we have calculated the total amplitude and obtained comparable results with experimental value for the branching ratio of $B^- \to D_s^- D^0$ decay.

1 Introduction

In the Standard Model (SM), the primary importance of studying the nonleptonic two-body decays of $B$ mesons is to explore $CP$ violation and flavour parameters. The different weak and strong interaction phases that arise from the interference of several competing amplitudes play an important role in $CP$ violation. The weak complex phases are obtained from the argument of quark mixing matrix elements entering each amplitude vertex, which follow the unitarity triangle relation, $V_{ub}^* V_{uq} + V_{cb}^* V_{cq} + V_{tb}^* V_{tq} = 0$ ($q = d, s$). All measured $CP$ asymmetries are in the SM related through this unitarity condition. The weak phases are introduced in a standard convention as $\phi_1 = -\text{arg}(V_{tq})$ and $\phi_2 = \text{arg}(V_{ub}^*)$, the strong interaction phase differences between tree and penguin amplitudes, $\delta$, is another phase that is necessary for $CP$ violation to occur.

In some charm decays of $B$ mesons, such as the $B^- \to D_s^- D^0$ decay, the decay rates are obtained for combined particles and antiparticles, while such decays are affected $CP$ violation. It is in such cases that the weak and strong phases induce the violation that has occurred for the charge and parity in reality.

The LHCb collaboration has reported the first measurement of $CP$ asymmetry in the $B^- \to$
Table 1: Some experimental results and their averages of $CP$-averaged branching ratio for $B^- \rightarrow D_s^- D^0$ decay (in units of $10^{-3}$).

| Refs.      | $B(B^- \rightarrow D_s^- D^0)$  |
|------------|---------------------------------|
| BABAR      | $13.3 \pm 1.8 \pm 3.2$         |
| Belle      | $9.5 \pm 0.2$                  |
| LHCb       | $8.6 \pm 0.2 \pm 0.4 \pm 1.0$  |
| HFLAV      | $13.3 \pm 3.7$                 |
| PDG(2020) Avg. | $9.0 \pm 0.9$             |

$D_s^- D^0$ decay. The value was measured to be $(-0.4 \pm 0.5 \pm 0.5)\%$. There are also several experimental results and their averages for the $CP$-averaged branching ratio of the $B^- \rightarrow D_s^- D^0$ decay, some of them are shown in Tab. 1.

In this work, we have calculated the $CP$ violation and $CP$-averaged branching ratio for the $B^- \rightarrow D_s^- D^0$ decay. First, we have drawn all the contributions of decay diagrams in accordance with Feynman rules. Using the factorization approaches, the matrix elements of effective Hamiltonian have been evaluated. In these approach, the simple factorization of the elements of the hadron matrix appears as the product of two matrices. One of these matrices arises from the transition between meson $B$ and one of the final mesons (form factor). The other matrix is created by the residual end state due to the vacuum (decay constant). In the studied decay, both matrices are for pseudoscalar (spin 0), so the form factor becomes $\langle B^- \rightarrow D^0 \rangle$, and the decay constant takes form $\langle 0 \rightarrow D_s^- \rangle$. The main purpose of obtaining hadron matrix elements is to estimate the amplitude. Then the decay rate, branching ratio and $CP$ violation are obtained from it. The branching fraction is obtained to be $B(B^- \rightarrow D_s^- D^0) = (9.33 \pm 1.17) \times 10^{-3}$ at $\mu = m_b/2$ scale. This value is well compatible with the value of $B(B^- \rightarrow D_s^- D^0) = (9.00 \pm 0.90) \times 10^{-3}$ that reported by PDG(2020) Avg. [3]. A value, $(-0.35 \pm 0.03)\%$, comparable to the experimental result of LHCb [6], is obtained for $CP$ violation.

## 2 Branching fraction and $CP$ asymmetry in $B^- \rightarrow D_s^- D^0$ decay

In this section, we calculate the $CP$ asymmetry and the branching ratio for comparison with experimentally measured results. In the study of $CP$ violation in $B$ decays it turned out to be useful to make a classification of $CP$ violating effects that is more transparent than the division into the indirect and direct $CP$ violation. Generally, complex phases may enter the particle-antiparticle mixing and in the decay processes through the complex elements of CKM matrix. As the phases in mixing and decay are convention dependent, the $CP$ violating effects depend only on the differences of these phases. Three types of $CP$ violation are: $CP$ violation in mixing, $CP$ violation in decay, $CP$ violation in the interference of mixing and decay [7].

In this study, the $CP$ violation in decay may occur for $B^- \rightarrow D_s^- D^0$ decay. $CP$ violation in decay may be zero due to very close values of particle and antiparticle decay rates. In order for this symmetry not to be zero, two different contributions in the amplitude with the strong ($\delta_i$) and weak ($\phi_i$) phases are needed. These could be for instance two tree diagrams, two
penguin diagrams or just a diagram of each of them. Considering the process of \( B^- \to D_s^- D^0 \) decay (and \( B^+ \to D_s^+ \bar{D}^0 \) decay), this decay mode can proceed through two different elementary amplitudes \( A_1 \) and \( A_2 \), this means that the decay can proceed by two different paths: tree (\( A_1 \)) and penguin (\( A_2 \)) diagrams, for the total decay amplitude we can write [8]:

\[
A(B^- \to D_s^- D^0) = |A_1| e^{i\delta_1} e^{i\phi_1} + |A_2| e^{i\delta_2} e^{i\phi_2},
\]

where \(|A_1|\) and \(|A_2|\) represent \(|A_1(B^- \to D_s^- D^0)|\) and \(|A_2(B^- \to D_s^- D^0)|\). To obtain the antiparticle amplitude (\( B^+ \to D_s^+ \bar{D}^0 \)), in the Eq. [1], the weak phases (\( \phi_i \)) becomes its complex conjugate and the strong phases (\( \delta_i \)) remain unchanged.

For the decay examined in this study, \( B^- \to D_s^- D^0 \) decay, the Feynman’s main diagrams are the tree level diagram that have the largest contribution in the amplitude and the penguin level diagram, which is significantly smaller than the tree level diagram. The Feynman diagrams of \( B^- \to D_s^- D^0 \) decay are shown in Fig. [1] and then the decay amplitude reads

\[
A(B^- \to D_s^- D^0) = \frac{iG_F}{\sqrt{2}} f_{D_s} F_0^{B \to D}(m_{D_s}^2) \left( V_{cb} V_{ts}^* a_1 - V_{tb} V_{ts}^* (a_4 + a_{10} + \xi (a_6 + a_8)) \right),
\]

where the tree and penguin level amplitudes are as follows, respectively

\[
A_1(B^- \to D_s^- D^0) = \frac{iG_F}{\sqrt{2}} f_{D_s} F_0^{B \to D}(m_{D_s}^2) V_{cb} V_{cs}^* a_1,
\]

and

\[
A_2(B^- \to D_s^- D^0) = \frac{iG_F}{\sqrt{2}} f_{D_s} F_0^{B \to D}(m_{D_s}^2) V_{tb} V_{ts}^* (a_4 + a_{10} + \xi (a_6 + a_8)),
\]

\( \xi \) depends on properties of the final-state mesons involved and is defined as [9]

\[
\xi = \frac{2m_{D_s}^2}{(m_b - m_c)(m_c + m_s)}
\]

To calculate the form factor \( F_0 \) we take the form [10]

\[
f(q^2) = \frac{f(0)}{1 - \sigma_1 q^2/m_{B_s}^2 + \sigma_2 q^4/m_{B_s}^4},
\]

here \( q^2 = P_{B^-}^2 - P_{D^0}^2 = P_{D_s^+}^2 \). The value of the parameter \( m_P \) (pole mass), which is equal to the lowest resonance mass, is fixed to its physical value for proper choice of the quark-model parameters and for the reliability of the calculations. With this description the \( m_P \) is the \( m_{B_s} \) for \( B \to D \) transition. The values of \( f(0), \sigma_1 \) and \( \sigma_2 \) are as follows [10]

\[
F_0^{B \to D} : \quad f(0) = 0.67, \quad \sigma_1 = 0.78, \quad \sigma_2 = 0.
\]

The quantities \( a_i \) (\( i = 1, \ldots, 10 \)) are the following combinations of the effective Wilson coefficients

\[
a_{2i-1} = c_{2i-1} + \frac{1}{3} c_{2i}, \quad a_{2i} = c_{2i} + \frac{1}{3} c_{2i-1}, \quad i = 1, 2, 3, 4, 5.
\]
The Wilson coefficients, \( c_i \), in the effective weak Hamiltonian have been reliably evaluated by the next-to-leading logarithmic order. To proceed, we use the following numerical values at three different choices of \( \mu \) scale, which have been obtained in the NDR scheme and are shown in Tab. 2. The meson and quark masses and decay constants needed in our calculations are taken as (in units of MeV) \( m_B^c = 6274.9 \pm 0.8 \), \( m_{D^-} = 1968.35 \pm 0.07 \), \( m_{B^\pm} = 5279.34 \pm 0.12 \), \( m_{D^0} = 1864.84 \pm 0.05 \), \( m_b = 4180^{+40}_{-30} \), \( m_s = 93^{+11}_{-5} \), \( m_c = 1270 \pm 20 \), \( f_{D_s} = 241 \pm 3 \) [12].

(9)

The decay rates corresponding to the \( \mathcal{A}(B^- \to D_s^- D^0) \) and \( \mathcal{A}(B^+ \to D_s^+ D^0) \) amplitudes can be then written as [13]

\[
\Gamma(B^- \to D_s^- D^0) = \left| A_1 \right|^2 e^{i(\delta_1 + \phi_1)} + \left| A_2 \right|^2 e^{i(\delta_2 + \phi_2)}^2,
\]

\[
\Gamma(B^+ \to D_s^+ D^0) = \left| A_1 \right|^2 e^{i(\delta_1 - \phi_1)} + \left| A_2 \right|^2 e^{i(\delta_2 - \phi_2)}^2.
\]

(10)

The branching fraction for the \( B^- \to D_s^- D^0 \) decay is given by

\[
\mathcal{B}(B^- \to D_s^- D^0) = \frac{\Gamma(B^- \to D_s^- D^0)}{\Gamma_{tot}},
\]

(11)

where the \( \Gamma_{tot} \) for charged \( B \) meson is \( (4.02 \pm 0.01) \times 10^{-13} \) GeV. By dividing the difference between these two decay rates by their sum, the \( CP \) asymmetry in decay rates is given by [14]

\[
\mathcal{A}_{CP} = \frac{\mathcal{B}(B^- \to D_s^- D^0) - \mathcal{B}(B^+ \to D_s^+ D^0)}{\mathcal{B}(B^- \to D_s^- D^0) + \mathcal{B}(B^+ \to D_s^+ D^0)}
\]

\[
= \frac{2|A_2/A_1|\sin(\delta_1 - \delta_2)\sin(\phi_1 - \phi_2)}{1 + |A_2/A_1|^2 + 2|A_2/A_1|\cos(\delta_1 - \delta_2)\cos(\phi_1 - \phi_2)}.
\]

(12)

It should be noted that the numerical value of the difference between the tree and penguin amplitudes, \( \mathcal{A}_1(B^- \to D_s^- D^0) - \mathcal{A}_2(B^- \to D_s^- D^0) \), is obtained as a complex number. The

| NLO | \( \mu = m_b/2 \) | \( \mu = m_b \) | \( \mu = 2m_b \) |
|-----|----------------|----------------|----------------|
| \( c_1 \) | 1.137 | 1.081 | 1.045 |
| \( c_2 \) | -0.295 | -0.190 | -0.113 |
| \( c_3 \) | 0.021 | 0.014 | 0.009 |
| \( c_4 \) | -0.051 | -0.036 | -0.025 |
| \( c_5 \) | 0.010 | 0.009 | 0.007 |
| \( c_6 \) | -0.065 | -0.042 | -0.027 |
| \( c_7/\alpha \) | -0.024 | -0.011 | 0.011 |
| \( c_8/\alpha \) | 0.096 | 0.060 | 0.039 |
| \( c_9/\alpha \) | -1.325 | -1.254 | -1.195 |
| \( c_{10}/\alpha \) | 0.331 | 0.223 | 0.144 |

Table 2: Wilson coefficients \( c_i \) in the NDR scheme (\( \alpha = 1/129 \)) [11].
strong phase $\delta_1 - \delta_2$ arises from the ratio of the imaginary part to the real part. In fact, the argument of the difference of two amplitudes gives the strong phase, the estimated value is $\delta_1 - \delta_2 = 89.94^\circ$.

As mentioned in the introduction section, the weak phase $\phi_1$ is calculated from the argument of the complex element of $V_{ts}$ in the CKM matrix, the result is achieved to be $\phi_1 = 1.10^\circ$. Considering that the element $V_{ub}$ in the CKM matrix is a real number and the weak phase $\phi_2$ comes from the argument of this element, so we set $\phi_2 = 0$.

In the Standard Model, the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix is a unitary matrix, in which an expansion is introduced by the small value of $\lambda$. The CKM matrix at order $\lambda^3$ can be parameterized as [15]

$$V = \begin{pmatrix}
1 - 1/2\lambda^2 - 1/8\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda + 1/2A^2\lambda^5[1 - 2(\rho + i\eta)] & 1 - 1/2\lambda^2 - 1/8\lambda^4(1 + 4A^2) & A\lambda^2 \\
A\lambda^3[1 - (1 - 1/2\lambda^2)(\rho + i\eta)] & -A\lambda^2 + 1/2A\lambda^4[1 - 2(\rho + i\eta)] & 1 - 1/2A^2\lambda^4
\end{pmatrix}$$

Recent Particle Data Group (PDG) average values for the Wolfenstein parameters are [5]

$$\lambda = 0.22650 \pm 0.00048, \quad A = 0.790^{+0.017}_{-0.012}, \quad \bar{\rho} = 0.141^{+0.016}_{-0.017}, \quad \bar{\eta} = 0.357 \pm 0.011,$$  (14)

where $\bar{\rho} = \rho(1 - 1/2\lambda^2)$ and $\bar{\eta} = \eta(1 - 1/2\lambda^2)$ [16]. The CKM matrix elements used in this work are obtained by the above calculations as follows (in units of $10^{-3}$)

$$V_{cb} = 40.529, \quad V_{cs} = 973.198,$$

$$V_{tb} = 999.179, \quad V_{ts} = -39.790 - 0.762i.$$  (15)

### 3 Numerical results and conclusion

The numerical results of the $CP$ violation and $CP$-averaged branching ratio for $B^- \rightarrow D_s^- D^0$ is presented in Tab. [3]. In this paper, we have analyzed the decay of $B$ meson into two pseudoscalar mesons. We have drawn Feynman diagrams completely for the $B^- \rightarrow D_s^- D^0$ decay based on the standard model. This decay can violate $CP$ symmetry. In general, $CP$ asymmetry can be calculated from the difference between the particle and the antiparticles decay rates relative to their sum and it becomes non-zero for the mentioned decay. Here we have obtained the $CP$ violation by calculating the amplitude of the current-current tree and penguin diagrams that are considered separately and using the strong ($\delta_i$) and weak ($\phi_i$) phases. The weak and
strong phases have been obtained by complex elements of CKM matrix and from current-current tree and penguin amplitude differences, respectively. We have estimated the $CP$ violation as $A_{CP}(B^- \rightarrow D^-_s D^0) = (-0.35 \pm 0.03)\%$. Also, from the sum of the amplitudes, we have calculated the total amplitude and obtained comparable results with experimental values for the branching ratio as $B(B^- \rightarrow D^-_s D^0) = (9.33 \pm 1.17) \times 10^{-3}$ at $\mu = m_b/2$ scale. Theoretical uncertainties in our calculations are due to the uncertainties in the form factors, decay constants, meson masses and the uncertainties of the input parameters in CKM elements.

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Figure 1: Feynman diagrams contributing to $B^- \rightarrow D_s^- D^0$ decay.