Study on route selection and collision avoidance of UUV based on fuzzy control

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Abstract: The route selection and collision avoidance performance of UUV was studied based on fuzzy control theory. Firstly, a six-degree-of-freedom motion experiment is simulated for the submarine, and a fuzzy controller is designed according to the motion parameters obtained from the experiment. The fuzzy control system includes the heading control and depth control, and uses the fuzzy BK triangle sub product algorithm to design the next instantaneous safety course, so as to achieve the anti-collision function. In the numerical simulation, the performance of the controller is evaluated by setting a single point obstacle and a multi-point obstacle, respectively, for both the heading control and the collision avoidance path planning. The results show that: The fuzzy control system has better function of route selection, can determine the best course in time, and can pass the obstacle on the course with enough safe distance, which verifies the feasibility of this method.

1. Introduction
With the deepening of marine environmental scientific research and the development of marine resources, the economic and military application value of UUV as a new type of unmanned aircraft has become increasingly prominent. In order to ensure the safety and maneuverability of UUV at sea, the research on heading control and collision avoidance performance has always been a hot spot in the research field \cite{1-3}. The submarine is constantly affected by disturbances such as wind, waves and ocean currents during its navigation, and its motion is actually a time-varying, high-noise nonlinear system. Traditional heading control and path planning use PID controllers, which are also widely used technologies at present. However, traditional PID controllers have better control performance only under certain models, fixed working conditions and small disturbances. When sea conditions change, the robustness of the PID controller will become worse\cite{4-7}. In order to solve this problem, the best way is to design a nonlinear controller. Therefore, based on fuzzy control theory, this paper designs a fuzzy controller of BK triangular sub product algorithm and validates its performance by simulation.

2. Motion modeling
The motion of UUV is a highly nonlinear coupling mode, and its operating environment is a three-dimensional sea area, which is affected by buoyancy, fluid resistance, additional mass, thruster thrust, damping coefficient, etc.. To accurately describe the motion mode of the UUV, there are many related factors that need to be considered. Therefore, this paper proposes a six-degree-of-freedom motion mode and simulates its motion. Firstly, two different reference coordinate systems need to be
defined. As shown in Figure 1, O-XYZ is the inertial coordinate system, the origin O is selected at any point on the surface of the earth, o-xyz is the carrier coordinate system, and the origin o is located on the center of gravity of the submarine.

Suppose the position coordinates of the submarine are \((X, Y, Z)\), which represent traversing, vertical movement, and undulation respectively. The attitude angle is represented by \(\phi\) (roll), \(\theta\) (pitch), \(\psi\) (yawing). The instantaneous velocity and angular velocity of the submarine are \((u, v, w)\) and \((p, q, r)\). The six-degree-of-freedom motion mode is shown in Table 1. If \((\dot{X}, \dot{Y}, \dot{Z})\) is the linear velocity of the submarine relative to the inertial coordinate system, the linear velocity conversion relationship between the two coordinate systems can be expressed as [8-9]:

\[
\begin{bmatrix}
\dot{X} \\
\dot{Y} \\
\dot{Z}
\end{bmatrix} =
\begin{bmatrix}
\cos \theta \cos \psi & \sin \phi \sin \theta \cos \psi & \sin \phi \sin \psi + \\
-\cos \phi \sin \psi & \cos \phi \sin \theta \cos \psi & \cos \phi \sin \psi \\
\cos \theta \sin \psi & \sin \phi \sin \theta \cos \psi & \cos \phi \sin \theta \sin \psi
\end{bmatrix}\begin{bmatrix}
u \\
w
\end{bmatrix}
\]

(1)

The relationship of angular velocity between the two coordinate systems can be derived as follows:

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} =
\begin{bmatrix}
1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi / \cos \theta & \cos \phi / \cos \theta
\end{bmatrix}\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\]

(2)

After integrating the linear velocity and angular velocity in the above expressions (1) and (2), the position and angular velocity of the submarine relative to the inertial coordinate system can be obtained. The motion of the submarine in the water is a highly nonlinear coupling mode. Considering the effects of rigid body inertia, additional mass, centripetal force, additional mass centripetal force, fluid damping, gravitational buoyancy restoration force, and thrust generated by the propeller, its motion can be decomposed into six-degree-of-freedom motion mode, the expressions of each sub-motion are respectively:

Traversing—the equation of motion for moving in the x-direction:

\[
m\left[ \ddot{u} - v \dot{r} + w \dot{q} - x_G (q^2 + r^2) + y_G (p \dot{q} - \dot{r}) + z_G (pr + \dot{q}) \right] + (W - B) \sin \theta = X
\]

(3)

Vertical movement—the equation of motion for moving in the y-direction:
Undulation—the equation of motion for moving in the z-direction:
\[ m \left[ \dot{\omega}_r + u_r r - w_r p + x_G (pq + \dot{r}) - y_G (p^2 + r^2) + z_G (qr - \dot{p}) \right] - (W - B) \cos \theta \sin \phi = Y \]  
(4)

Roll—the equation of motion for rotating along the x-axis:
\[ I_x \ddot{\phi} + (I_z - I_y) q r + I_{xy} (pr - \dot{q}) - I_{zx} (q^2 - r^2) - I_{xz} (pq + \dot{r}) + m \left[ y_G (\dot{\omega}_r - u_r q + v_r p) - z_G (\dot{u}_r - v_r r + w_r q) \right] - (y_G W - y_B) \cos \theta \cos \phi + (z_G W - z_B) \sin \theta \sin \phi = K \]  
(5)

Pitch—the equation of motion for rotating along the y-axis:
\[ I_y \ddot{\theta} + (I_x - I_z) p r - I_{xz} (qr + \dot{p}) + I_{zx} (p^2 - r^2) - m \left[ z_G (\dot{\omega}_r + u_r r - w_r p) - x_G (\dot{u}_r - v_r r + w_r q) \right] + (x_G W - x_B) \cos \theta \sin \phi + (y_G W - y_B) \sin \theta \sin \phi = K \]  
(6)

Yawing—the equation of motion for rotating along the z-axis:
\[ I_z \ddot{\phi} + (I_y - I_x) p q - I_{yx} (p^2 - q^2) - I_{yz} (pr + \dot{q}) + I_{zy} (qr - \dot{p}) + m \left[ x_G (\dot{\omega}_r + u_r r - w_r p) - y_G (\dot{u}_r - v_r r + w_r q) \right] - (z_G W - z_B) \cos \theta \sin \phi - (y_G W - y_B) \sin \theta \sin \phi = N \]  
(7)

In equations (3)~(8):
- \( m \)—UUV mass;
- \( I_x, I_y, I_z \)—rotational inertia;
- \( u_r, v_r, w_r \)—the velocity components of the UUV in different directions;
- \( x_B, y_B, z_B \)—the distance between the geometric center of the UUV and the center of buoyancy;
- \( x_G, y_G, z_G \)—the distance between the geometric center of the UUV and the center of gravity;
- \( B \)—buoyancy;
- \( W \)—gravity;
- \( X, Y, Z, K, M, N \)—the resultant force and moment acting on the submarine.

The hydrodynamic parameters in the submarine motion model can be obtained through experimental simulation, and the motion simulation experiment is carried out in a pool. The UUV has 4 thrusters, horizontal thrusters T1, T2, vertical thrusters T3, T4. The motion simulation results are numerically calculated according to the modeling equation, and the fluid dynamic parameters are obtained from experiments, where \( X, Y, Z \) represent respectively the forces received in three directions, \( K, M, N \) represent the moments in the three directions. The force (moment) in these six directions is differentiated to the acceleration (angular acceleration) to obtain the corresponding additional mass coefficient. If the velocity (angular velocity) is differentiated, the fluid damping coefficient can be obtained. The initial coordinates of the submarine are \((80, 0, 19)\), the initial heading angle is \(180^\circ\), and the speed \(V=0.5\) knots. The simulated trajectory of the submarine's turning motion without the action of ocean currents is shown in Figure 2. The motion trajectory is decomposed into a six-degree-of-freedom motion mode, as shown in Figure 3. It can be seen from the figure that the submarine performs a two-dimensional movement on the x-y plane, the depth does not change, and there is rotational movement in three sub-directions, where in the z direction is 360 gyrations. The six-degree-of-freedom motion parameters obtained from the experimental results are used to design the fuzzy rule library of the fuzzy controller, and it is determined that the motion simulation conforms to the actual physical law.
Table 1 Six-degree-of-freedom motion mode of the UUV

| DOF | Motion mode  | Velocity or angular velocity | Location or angular |
|-----|--------------|------------------------------|---------------------|
| 1   | traversing  | u                            | X                   |
| 2   | vertical movement | v                  | Y                   |
| 3   | undulation   | w                            | Z                   |
| 4   | roll         | p                            | ϕ                   |
| 5   | pitch        | q                            | θ                   |
| 6   | yawing       | r                            | Ψ                   |

Fig 2 Turning motion trajectory

Fig 3 A six-degree-of-freedom motion diagram of turning motion

3. Fuzzy controller design

3.1. Fuzzy control block diagram

The fuzzy control system of the submarine is divided into two parts: heading control and depth control. The heading control block diagram is shown in Figure 4(a). The input is the error of the instantaneous heading angle and the instantaneous yaw speed. The input value is mapped to the interval of the membership function through the scale factors G1 and G2. Then the fuzzification method is used to fuzzify the input value and the fuzzy information is obtained referring to the fuzzy rule library for logical judgment. Finally the thrust values of the output thrusters T1 and T2 are defuzzified. The output value needs to pass the limiter to limit the maximum thrust and the maximum negative thrust within the allowable range. The depth control block diagram is shown in Figure 4(b). The fuzzy basic structure is the same as the heading control, but the inputs are the depth error and the instantaneous fluctuation speed, and the corresponding scale factors are G3 and G4. Finally, it is defuzzified to output the thrust values of the thrusters T3 and T4.
3.2. Fuzzy methods and control rules

In order to make the clarity values correspond to the fuzzy rules described by the language and perform approximate inferences, they must be turned into fuzzy quantities. Fuzzy processing is the process of mapping these input clear values into fuzzy subsets, and finding the degree of membership of each fuzzy subset. This paper uses Gaussian membership function to convert the input clear value set into a fuzzy set. The nonlinear characteristic of Gaussian type can effectively eliminate errors caused by clutter.

Fuzzy control rules are the core of the fuzzy controller, and its generation rules adopt the form of If-Then to express the relationship between input and output. The two controllers in this study are in the form of dual input and single output. This fuzzy rule library is designed based on the results of the simulation experiment of submarine pool motion. The fuzzy rule tables for heading control and depth control are shown in Table 2 and Table 3, where X1 is the heading error input; X2 is the pan angular velocity input; X3 is the depth error input; X4 is the undulating speed input.

The membership function is used to define the membership degrees of five fuzzy semantics, with different definitions for different input values. When carrying out obstacle avoidance control, it is necessary to require the fuzzy rule library to achieve a certain degree of controllability before being modified, and it cannot completely rely on the self-adjusting training ability. Therefore, high-resolution and low-resolution membership functions are established, as shown in the figure. In order to enable the input value to be mapped to the membership function input interval to achieve fuzzification, the four scale factors of fuzzy control must be obtained according to the maximum value of the input and the maximum value between the membership function. In this study, $G_1 = 0.05$, $G_2 = 0.18$, $G_3 = 1.0$, $G_4 = 25$.

| X1  | X2  | NB | NM | ZO | PM | PB |
|-----|-----|----|----|----|----|----|
| NB  | -0.5| 0  | 1  | 1  | 1  | 1  |
| NM  | -0.5| 0  | 0.5| 0.5| 1  |    |
| ZO  | -0.5| -0.5| 0  | 0.5| 0.5| 1  |
| PM  | -1  | -0.5| -0.5| 0  | 0.5|    |
| PB  | -1  | -1  | -1  | 0  | 0.5|    |
| X3 | NB | NM | ZO | PM | PB |
|----|----|----|----|----|----|
| NB | 1  | 1  | 1  | -0.3 | -1 |
| NM | 1  | 1  | 0.5 | -0.5 | -1 |
| ZO | 1  | 0.8| 0  | -0.8 | -1 |
| PM | 1  | 0.5| -0.5| -1   | -1 |
| PB | 1  | 0.3| -1  | -1   | -1 |

Fig 5 Gaussian membership function curve of high and low resolution type

3.3. Clarification

After fuzzy logic judgment, the output is a fuzzy set, but because it is a comprehensive conclusion drawn by multiple fuzzy rules, its membership functions are mostly segmented and irregular. The purpose of clarification is to equate them to a clear value and map it to a representative value. The output values of the fuzzy controller in this study are all single-point outputs, so the weighted average method is used for clarification. Let $u^*$ be the clear value finally output by the fuzzy control system, $u_j$ be the fuzzy value, and $\bar{u}_j$ be the single-point output value in the fuzzy rule library, then there is:\[u^* = \frac{\sum_{j=1}^{25} u_j \bar{u}_j}{\sum_{j=1}^{25} \bar{u}_j}\] (9)

The final output value $u^*$ must be multiplied by a scale factor to fit its operating area. The larger the scale factor, the higher the sensitivity, but it may be too large to cause the error to fail to converge. In addition, after multiplying by the scale factor, a limiter still needs to be added to limit the output value within the actual thrust range of the actual thruster. The scale factor of the heading controller is set to $k_1 = 6$, and the scale factor of the depth controller is set to $k_2 = 2.9$. The performance of the controller is simulated and analyzed. The initial heading angle is 180°, the initial depth is 2m, and the controller wants to follow the target at 150 degrees and 5m in depth. The obtained high- and low-resolution Gaussian membership function control planes are shown in the figure 6(a)(b) respectively. As shown in figure 6(a)(b), the z-axis in the figure represents the height of the output.
value and the output has a larger nonlinear characteristic. There are more areas on the depth control surface with smaller output values, which can reach the target depth faster and have better control performance.

![Gaussian membership function of high and low resolution type](image)

**Fig 6** Gaussian membership function of high and low resolution type

4. Route selection and collision avoidance performance simulation calculation

The collision avoidance navigation function is regarded as one of the most important functions of the UUV. Because the submarine cannot obtain complete and definite information when operating underwater, this paper adopts the fuzzy BK triangle sub-product method (BK-products of fuzzy relation), a heuristic search technology for obstacle avoidance, based on a fuzzy controller, used to select a safe path so that the submarine can avoid obstacles in time\(^\text{[12-13]}\). The fuzzy BK triangle sub-product is used to describe the interactive relationship between the degree of obstacle danger and the efficiency of going to the waypoint. First, \( R \) is defined as the binary relationship between \( X \) and \( Y \), \( X \) contains the subset \( X_i \), \( Y \) contains the subset \( Y_j \), and \( i \) and \( j \) represents the number of items in this set, and each item in this binary relationship is represented by \( R_{ij} \). In the same way, \( S \) is defined as the binary relationship between \( X \) and \( Y \). The degree of the relationship is represented by the ambiguity, which is between 0 and 1. The relationship between \( R \) and \( S \) is expressed as:

\[
(R \circ S)_{ij} = \frac{1}{N} \sum_{j=1}^{N} (R_{ij} \rightarrow S_{jk})
\]  

(10)

The principle of the fuzzy controller using the BK triangle sub-product algorithm is shown in Figure 7. The submarine route selection and collision avoidance must first detect obstacles from the originally planned path, and then re-plan the path according to the obstacle position. The efficiency of this path should be considered when choosing the route, and it should not keep turning to cause a large deviation from the original course because of avoiding obstacles. Obstacle detection relies on the properties of sonar, and then the sonar detection range is divided into multiple areas from left to right, representing multiple selectable headings. The area in the center represents the current heading. If there is an obstacle in the detection range, the submarine should choose the most efficient heading without obstacles, and the BK triangle sub-product completes the logical judgment during the control process. The sonar detection model is shown in Figure 8. The heading to be selected is represented by \( S_i \) to \( S_j \). \( S = \{s_1, s_2, \cdots, s_j\} \) is represented by the set \( S \), and the direction pointed by the arrow is the current heading. \( P \) is used to indicate the degree of safety and the efficiency of going to the path point, and the set \( P \) is expressed as \( P = \{p_1, p_2, \cdots, p_j\} \). After defining \( P \) and \( S \), the relationship matrix \( R \) can be obtained, where \( r_{ij} \) is the fuzzy relationship information transformed by the membership function.
The calculation result of the above-mentioned fuzzy relationship is for heading judgment. If you encounter wide obstacles, you need to deal with it separately. That is, when obstacles are detected in all the candidate headings, the best heading cannot be found at this time. In the process of object collision avoidance, you must first determine whether it is a wide obstacle. If it is a wide obstacle, perform a floating motion until the obstacle disappears and continue to float for a given time to reach the safe range, and then perform depth control according to the reference depth.

\[ R = S \times P = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1j} \\ r_{21} & r_{22} & \cdots & r_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ r_{i1} & r_{i2} & \cdots & r_{ij} \end{bmatrix} s_i \]

\[ R^T \]

\[ T = R \triangleleft R^T = \begin{bmatrix} t_{i1} & t_{i2} & \cdots & t_{ii} \\ t_{21} & t_{22} & \cdots & t_{2i} \\ \vdots & \vdots & \ddots & \vdots \\ t_{i1} & t_{i2} & \cdots & t_{ii} \end{bmatrix} s_i \]

\[ S = \max \left\{ t_{i1} \times t_{21} \times \cdots \times t_{i1}, \cdots, t_{ik} \times t_{2k} \times \cdots \times t_{ik} \right\}, k = 1, 2, \cdots, i \]

The fuzzy controller of the BK triangle sub-product algorithm designed above is used to carry out...
the simulation analysis of the route selection and collision avoidance performance of the submarine. The collision avoidance path planning under a single obstacle is shown in Figure 9. The original path of the submarine is a turning motion. When an obstacle appears on the original path, the submarine automatically adjusts its course and successfully avoids the obstacle. The collision avoidance adjustment process is shown in Figure 10. The square obstacle is set at the center point of (35, 10, 19), and the length, width, and height are 4m each. Each grid on the grid point represents a distance of 1m. The submarine moves to (23,11,19), and moves on from this point, the heading angle is 180 degrees and the target point that crosses the obstacle is (43,11,19). The underwater detection resolution is assumed to be 5m in the simulation process, and the detection angle is 100 degrees, the controller will act once every 0.1 seconds. The dashed line represents seven headings to be selected, which are \( S_1, S_2, S_3, S_4, S_5, S_6, S_7 \) from left to right. The heading angles are 45 degrees, 30 degrees, 15 degrees, 0 degrees, -15 degrees, -30 degrees, and -45 degrees. The submarine detected an obstacle at the position (28,11,19), and the heading angle of the detected obstacle are \( S_3, S_4, S_5, S_6 \). Although only a part of the course of \( S_3 \) has detected obstacles, it is still a more dangerous course. Although no obstacles are detected, the course of \( S_7 \) will travel a long distance to reach the target waypoint. Therefore, according to the BK triangle sub-product algorithm, the controller judges that the heading angle should be \( S_2 \) and perform a 30-degree turn. When the submarine sails to the left of the obstacle, the coordinate point is (34,12.7,19), and the controller continuously corrects the course to keep a certain safe distance from the obstacle and continue to move toward the target path point. Finally, when the submarine crosses the obstacle, it stops the BK triangle sub-product operation, and directly advances toward the target path point to complete the selection of the control path.

Fig 9 The collision avoidance path planning under a single obstacle

Fig 10 Collision avoidance adjustment process
From the simulation results, it can be seen that under a single obstacle, the controller can automatically adjust the course and successfully avoid the obstacle. So what is the effect of the controller under multiple obstacles? Therefore, three obstacles are set on the original path to plan the collision avoidance path. The position of the first obstacle remains unchanged and is on the original path, the second obstacle is located to the right of the original path, and the third obstacle is located to the left of the original path, and the volume of the obstacle is relatively large. The simulation results of the collision avoidance path planning are shown in Figure 11. The simulation results show that the planned path is within a reasonable and safe range, which verifies the feasibility of the method. In addition, the yaw angle signal is added to the heading controller. As shown in Figure 12, the adjustment time of the yaw angle step response curve is shorter, and the pulse disturbance signal is added at 12 seconds, which returns to normal in a short time, and there is basically no overshoot phenomenon, the controller has strong robustness.

![Fig 11 The collision avoidance path planning under multiple obstacles](image)

5. Conclusion

Based on the fuzzy control theory, this paper studies the route selection and collision avoidance performance of the UUV, and designs the fuzzy controller of the BK triangle sub-product algorithm. The controller can realize path planning and design under single-point obstacles and multi-point obstacles, automatically adjust the course and successfully avoid obstacles, and the system has strong stability and anti-interference ability, which verifies the method feasibility. Through the research of
this article, it is hoped to promote the engineering application of the optimization of collision avoidance performance of UUV.

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