The term “Space Manifold Dynamics” (SMD) is used to describe the applications of Dynamical Systems methods to spacecraft mission analysis and design. Since the late 1980’s, the application of tools coming from the general field of Dynamical Systems has gone from a mathematical curiosity in the space community to become a serious methodology for the design and operation of real space missions. Missions such as Gaia, Genesis, GRAIL, Herschel, MAP, Plank, and many others, are all using Dynamical Systems concepts for their design.

The Space Manifold Dynamics approach to mission analysis problems allows the analysis of the natural dynamics of the problem in a systematic and efficient way, and can be used to solve questions such as: the description of the phase space in a large vicinity of the collinear Lagrangian points, the analytical computation of libration point orbits (LPO) using Lindstedt-Poincaré methods, the design of optimal station-keeping strategies for LPOs, the determination of low-energy and interplanetary transfers, the computation of transfers between libration point orbits, or the design of eclipse avoidance strategies; in all the cases fitting the required mission constraints.

In this paper some of the main tools of the Dynamical Systems theory used in Astrodynamics are presented, as well as their application to some particular problems of
1. Introduction

For the design of space missions to libration point orbits, the Circular Restricted Three–Body Problem (CRTBP) is the natural and simplest model to start with. Dynamical Systems theory has been extensively used in the study of the CRTBP, for instance to get a detailed analysis of the dynamics in the vicinity of its equilibrium points, where some of the most dynamical complications occur. Its qualitative and quantitative procedures allow us to obtain an accurate picture of the evolution of the states of the system. Next we briefly introduce and discuss the main features of the problem.

The CRTBP describes the motion of a massless particle under the gravitational influence of two point masses $m_1$ and $m_2$, called primaries, in circular motion around their common center of mass. It is usual to consider a synodic reference system, with origin at the center of mass and rotating with the same angular velocity than the primaries, so that they are fixed in this system. The CRTBP has a Hamiltonian structure, with Hamiltonian function $H$, that in terms of the synodic position $(x, y, z)$ and momentum $(p_x, p_y, p_z)$ of the massless particle is given by

$$H = \frac{1}{2}\left(p_x^2 + p_y^2 + p_z^2\right) - xp_y + yp_x - \frac{1-\mu}{r_1} - \frac{\mu}{r_2},$$

where $\mu = m_2/(m_1 + m_2)$, and $r_1$ and $r_2$ the distances from the massless particle to both primaries. The constant value of the Hamiltonian over each solution, $h$, is called the energy of the orbit.

In the synodical reference system there exist five equilibrium (or libration) points (see Figure 1). Three of them, the collinear ones, are on the line joining the primaries and are usually denoted by $L_1$, $L_2$ and $L_3$, where $L_1$ is between the two primaries, $L_2$ is at the left-hand side of the small one (which is assumed to be on the negative $x$-axis), and $L_3$ is at the right-hand side of the big one (on the positive $x$-axis). The last two equilibrium points, $L_4$ and $L_5$, called triangular points, form equilateral triangles with the primaries. Around the triangular equilibrium points, there are large regions with good stability properties that could be used as parking regions at which almost no station keeping is needed.

From a dynamical point of view, the collinear libration points behave as the product of two centers by a saddle. According to Lyapunov’s center theorem, each equilibrium point gives rise to two one-parametric families of periodic orbits, spanning a 2D manifold tangent at the equilibrium point to the real and imaginary parts of the eigenvectors with eigenvalues $\pm (\sqrt{-1}) \omega_1$. These two families are known as the planar and vertical Lyapunov family, respectively, of periodic orbits.
When we consider all the energy levels, the center × center part gives rise to four-dimensional central manifolds around these equilibria. Among the solutions in the central manifold, the quasi-periodic Lissajous orbits are those associated with two-dimensional tori. For a fixed energy level, these solutions can be viewed as families of quasi-periodic solutions that “connect” the planar and the vertical Lyapunov orbit at the same energy level (see Figure 2, left).

Following the families of Lyapunov periodic orbits, as the energy $h$ increases, the linear stability of the orbits change and there appear bifurcating orbits where other families of periodic orbits are born. At the first bifurcation orbit of the family of planar Lyapunov orbits, there appear two families of 3-dimensional periodic orbits, symmetric with respect the $y = 0$ plane, that are called Halo orbits (see Figure 2 right).
Due to the hyperbolic character of the collinear equilibrium points, the invariant objects around them inherit the hyperbolicity, at least for values of the energy close to that of each equilibrium. This means that the orbits (periodic and quasi-periodic) in the central manifold are unstable and have a stable and an unstable invariant manifold associated. For the periodic orbits, the invariant manifolds look like 2D tubes filled with trajectories tending forwards (for the unstable) and backwards (for the stable) in time to the corresponding orbit. In the case of the Lissajous orbits, these invariant manifolds increase in one unit their dimension.

The stable invariant manifolds allow an efficient determination of transfer trajectories from the Earth to the libration point orbits of the Sun–Earth system, as well as the emergence of other trajectory and mission options. Furthermore, the intersections between the invariant manifolds give rise to homoclinic or heteroclinic connections that, in principle, allow to construct complicated itineraries between neighborhoods of two equilibrium points.

In connection with the computation of transfer orbits, it often appears in the literature the so called weak stability boundary (WSB), introduced by E. Belbruno after the rescue of the Hiten spacecraft. Although the WSB has not a precise definition, it can be seen as a boundary set in the phase space between stable and unstable motion relative to the second primary. After the work in the last decade of Koon, Gómez and Belbruno, it has been shown that the WSB, as well as its “rescue” role in missions like Hiten, can be completely explained in terms of the invariant hyperbolic manifolds associated to the central manifolds of the $L_1$ and $L_2$ libration points.

2. Spacecraft Missions to Libration Point Orbits

The orbits around the libration points, called libration point orbits, LPO, have unique characteristics suitable for performing different kinds of spacecraft missions. Among the most relevant characteristics, one can mention:

- In the Earth–Sun system, they are easy and inexpensive to reach from Earth.
- In the Earth–Sun system, they provide good observation sites, mainly solar observatories at $L_1$ and astronomy observatories at $L_2$. Near $L_2$ more than half of the entire celestial sphere is available at all times.
- Since the libration orbits around the $L_1$ and $L_2$ points of the Sun–Earth system always remain close to the Earth, at a distance of roughly 1.5 million km, and have a near-constant geometry as seen from the Earth, the communications system is simple.
- The $L_2$ environment of the Sun–Earth system is highly favorable for non-cryogenic missions requiring great thermal stability, suitable for highly precise visible light telescopes.
- The libration orbits around the $L_2$ point of the Earth–Moon system, can be used to establish a permanent communications link between the Earth and the hidden part of the Moon, as was suggested by A.C. Clark in 1950 and Farquhar in 1968.
- The LPO’s can provide ballistic planetary captures, such as for the one used by the
Hiten spacecraft.
- The heteroclinic connections between libration point orbits provide Earth transfer and return trajectories, such as the one used for the Genesis mission or by the Artemis-P1 spacecraft.
- The libration point orbits provide interplanetary transport which can be exploited in the Jovian and Saturn systems to design a low energy cost mission to tour several of their moons (Petit Grand Tour mission).
- Formation flight, with a rigid shape, is possible using libration point orbits.

An example of a mission visiting libration points’ neighborhoods is Genesis, launched in 2001 by NASA to study the solar wind and bringing back a sample to the Earth. The trajectory started travelling to the \( L_1 \) Sun-Earth point, resembled several times a halo orbit, and finally was inserted in a trajectory with a loop around \( L_2 \) before being captured back to Earth (see Figure 3 left). Another example is the trajectory of the Artemis-P1 spacecraft, devoted to study magnetism and how the solar wind flows past the Moon and tries to fill in the vacuum on the other side. This spacecraft follows a heteroclinic connection between orbits around the two Lagrangian points \( L_1 \) and \( L_2 \) of the Earth–Moon system (see Figure 3 right).

![Figure 3. Left: Trajectory of the Genesis spacecraft. Right: Trajectory of the Artemis-P1 spacecraft following a heteroclinic connection in the Earth-Moon system. (From NASA’s official web page).](image)

Many more missions (past, current or future) use the above mentioned properties. Among the most relevant ones we can mention: ISEE-3 (1978), WIND (1994), SOHO (1996), ACE (1997), Herschel (2008), Plank (2008), Chang’e 2 (2010), GRAIL (2011), GAIA (2012), DARWIN, Constellation X, LISA Pathfinder, SAFIR, TPF, Triana, JWST (previously known as NGST), ...

### 2.1. LPO In Lunar and Exploration Missions

In the past few years there has been a renewed interest in the exploration of the Moon and, in particular, in its far side. Among the current missions to the Moon there is the previously mentioned Artemis, an extended mission of a constellation of five spacecrafts, two of which were moved into a lunar orbit, and GRAIL that will produce a high-resolution map of the Moon’s gravitational field. GRAIL is composed by two small probes orbiting the Moon, which made use of a low-energy lunar transfer via the
Sun-Earth Lagrange point $L_1$ in order to reduce the fuel requirements and to slow down the velocity at lunar arrival.

Furthermore, the possibility of performing a temporary ballistic capture allowed us to keep the 40N engines available to the spacecraft low-cost bus: such a moderate thrust would have not allowed us to perform the classical one-shot Lunar Orbit Insertion (LOI) maneuver foreseen by a Hohmann-like transfer; thus the space manifold dynamics transfer removes the “single-point failure” character of the classical LOI.

Space manifold dynamics tools are currently used to design lunar missions, such as the preceding ones, with a significant energy ($\Delta v$) saving factor with respect to classical two-body problem approach. Departing from the Earth, it is possible to perform a ballistic capture in an elliptic orbit around the Moon using the manifolds associated to some particular libration point orbits.

The resulting transfer has an important saving at the lunar injection maneuver (up to 40% in missions like LunarSat, but with an additional mission duration. It must be also said that this gain vanishes when a low-altitude circular orbit (such as those used for manned missions, remote sensing or gravimetry) must be eventually achieved. Another example using these tools is a study of how to launch three small spacecraft on-board the same launch vehicle and send them to different orbits around the Moon with no significant difference in their $\Delta v$ budgets (Marson et al, 2010).

It is known that the design of interplanetary transfers from the Earth to the planets can be optimised, from the energy point of view, by incorporating lunar swing-byes at the departure from the Earth sphere of influence (see Figure 4). Those transfers can also incorporate trajectory paths through the WSB region and in this way save up to 150 kg of propellant to missions like Mars Express, but again with the penalty of a larger transfer duration.

The use of libration point dynamics has been also considered in the design to inner planet capture missions, like Bepi Colombo to Mercury, Venus Express to Venus and Mars Express to Mars. In this case, the energy saving is low, but the mission design is highly flexible compared with the classical patched conics approach. In particular, the use of classical procedures imposes a given argument of pericenter and right ascension of ascending node of the resulting planetary orbit, while the use of LPO techniques give practically a full freedom to select above parameters, with a not too large penalty in the mission duration. From a scientific point of view, the capacity to choose freely the orbital plane orientation gives an extraordinary increase in the final outcome of the mission.

A similar conclusion can be obtained for the application of SMD techniques to the outer planet capture (Jupiter, Saturn, Uranus, Neptune). However, if a tour of giant planet natural moons (Jupiter tour) is designed, the use of SMD techniques gives again an important energy saving factor in addition to the high flexibility.
2.2. Mission Design around Libration Points

The mission design of satellite flying orbits around libration points includes the consideration of the following aspects:

1. **Definition of a nominal trajectory**: the first step is the selection of the environment (the two-body system: Earth-Sun, Earth-Moon), the libration point (collinear L1, L2, L3 or triangular points L4 or L5) and the type of trajectory (Halo, Lissajous,...)
2. **Transfer trajectories** to the selected nominal orbit from initial launch conditions or parking orbits.
3. **Launch window** calculations taking into account the main mission constraints imposed for scientific or technical reasons.
4. **Navigation** of transfer and nominal trajectories: computation of the required trajectory correction maneuvers to correct launch injection dispersion, orbit determination errors and maneuvers mechanisation errors.
5. **Orbit maintenance**: strategies to keep the spacecraft in a neighborhood of the selected nominal path.
6. **Formation flying** techniques: new astronomy missions to LPO imposes the formation flying of several probes to implement interferometric techniques, the design of the formation architecture, the deployment, the tight control and the collision avoidance techniques must be defined.
7. **Eclipse avoidance**: most of the missions flying LPO orbits must avoid eclipses in order to continue nominal operations.
8. **Transfer between libration point orbits**: in some cases there is a need to transfer the
probe from one initial LPO orbit to another larger or smaller amplitude trajectory.

The dynamical systems approach provides solutions to all the above items as will be shown in the sections that follow.

Bibliography

Alessi – E.M. (2010) The Role and Usage of Libration Point Orbits in the Earth–Moon System. PhD thesis, Universitat de Barcelona. [Study of transfers from either Moon and Earth to a nominal libration point orbit of the collinear equilibrium points \( L_{1,2} \) of the Earth-Moon. The work includes also the study of some tracks leading to collisional events with the Moon.]

Alessi E.M., Gómez G., and Masdemont J.J. (2009) Leaving the moon by means of invariant manifolds of libration point orbits. Communications Nonlinear Science Numerical Simulations, 14, 4153–4167. [Computation of resque trajectories the leave the surface of the Moon and belong to the stable manifolds associated with the central manifold of the Lagrangian points \( L_{1,2} \) of the Earth-Moon system.]

Baoyin H. and McInnes C.R. (2006) Trajectories to and from the Lagrange points and the primary body surfaces. Journal Guidance, Control and Dynamics, 29, 998–1003 [Investigation of ballistic trajectories to and from the surfaces of the primaries in the planar circular restricted three body problem and both the collinear equilibrium points and the planar Lyapunov orbits associated.]

Beichman C., Gómez G., Lo M.W., Masdemont J.J., and Romans L. (2004). Searching for life with the terrestrial planet finder: Lagrange point options for a formation flying interferometer. Advances in Space Research, 34, 637–644. [Description of the mission design for TPF, assuming a distributed spacecraft concept using formation flight around both a halo orbit about the Sun–Earth \( L_2 \) as well as a heliocentric orbit.]

Belbruno E. (1987). Lunar capture orbits, a method for constructing earth-moon trajectories and the lunar gas mission. In AIAA paper No. 87-1054. [Description of a method to construct a trajectory from the Earth to the Moon which utilizes the existence of lunar capture orbits and the “weak stability boundary”.]

Belbruno E. and Miler J.K. (1990). A ballistic lunar capture trajectory for the Japanese spacecraft Hiten. Technical Report IOM 312/90. 4–1371, JPL.. [Description of the transfer trajectory from the Earth to the Moon used for the resque of the Japanese spacecraft Hiten.]

Belbruno E., Gidea M., and Topputo F. (2010). Weak stability boundary and invariant manifolds. SIAM Journal on Applied Dynamical Systems, 9 (3): 1061–1089. [Definition of the weak stability boundary in the context of the planar circular restricted three-body problem and its identification as the set of points that, for some energy range, have zero radial velocity and lie on the stable manifolds of the Lyapunov orbits about the libration points \( L_1 \) and \( L_2 \).]

Belló M., Gómez G., and Masdemont J.J. (2010). Invariant manifolds, lagrangian trajectories and space mission design. In Ettore Perozzi and editors Sylvio Ferraz-Mello, editors, Space Manifold Dynamics. Novel Spaceways for Science and Exploration. Springer. [Survey paper about the dynamics around the collinear libration points of the restricted three body problem and the mission design of spacecraft moving.
in the vicinity of those points.]

Breakwell J.V., Kamel A.A. and Ratner M.J. (1974). Station-keeping for a translunar communication station. Celestial Mechanics, 10 (3): 357–373. [Pioneering article on the station keeping for a halo orbit about the $L_2$ point of the Earth-Moon system.]

Canalias E. and Masdemont J.J. (2004). Eclipse avoidance for Lissajous orbits using invariant manifolds. In International Astronautical Federation - 55th International Astronautical Congress 2004, volume 1, pages 536–546. [Development of a complete methodology for the eclipse avoidance on Lissajous orbits using the geometry of the phase space around the collinear libration points of the restricted circular three body problem.]

Canalias E., Cobos J., and Masdemont J.J. (2003). Impulsive transfers between Lissajous libration point orbits. Journal of the Astronautical Sciences, 51 (4): 361–390. [Study of the transfer between two Lissajous orbits around the same collinear equilibrium point of the restricted circular three body problem.]

Conley C. (1968). Low energy transit orbits in the restricted three-body problem. SIAM Journal on Applied Mathematics, 16 (4): 732–746. [Seminal work about the dynamics about the libration points $L_1$ and $L_2$ of the restricted circular three body problem with especial attention to the orbits that make a transit, through the “neck” of Hill’s region, from one primary to the other. A scheme low-energy Earth–Moon orbits is outlined.]

Eckart P. (2000). Lunarsat, the european lunar micro-orbiter mission - a project status report. In Space 2000. In S. Johnson, K. Chua, R. Galloway, and P. Richter, editors. [Summary of results of the Phase B Study of the micro–spacecraft LunarSat (an educational and outreach project) which is planned to be sent into an orbit around the Moon.]

Eissmont N., Dunham D., Sho-Chiang J., and Farquhar R.W. (1991). Lunar Swingby as a Tool for Halo–Orbit Optimization in Relict–2 Project. In Third International Symposium on Spacecraft Flight Dynamics, ESA SP–326. [Mission analysis of the transfer of the Relict–2 spacecraft using a swingby with the Moon.]

Farquhar R.W. (1967). Far Libration Point of Mercury. Astronautics & Aeronautics, 5 (8): 4.

Farquhar R.W. (1968). The Control and Use of Libration Point Satellites. Technical report, Technical Report TR R346, Stanford University Report SUDAAR–350. Reprinted as NASA, 1970. [Pioneering study about the station keeping of the unstable collinear libration point $L_1$ and $L_2$ of the Earth–Moon system as well as its possible use for spacecraft missions.]

Farquhar R.W., Muhonen D.P., Newman C.R., and Heuberger H.S. (1980). Trajectories and orbital maneuvers for the first libration-point satellite. Journal of Guidance and Control, 3 (6): 549–554. [Maneuver strategy for halo orbit insertion and station keeping (prior to the completion of the first halo orbit) of the ISEE-3 spacecraft.]

Farrés A. and Jorba A. (2008). A dynamical system approach for the station keeping of a solar sail. Journal of the Astronautical Sciences, 56 (2): 199–230. [Design and analysis of a station keeping strategy for a solar sail in the Sun–Earth system in the vicinity of the unstable equilibrium points of the system. The strategy uses the knowledge of the invariant manifolds and how they change when the sail orientation is varied.]

García F. and Gómez G. (2007). A note on weak stability boundaries. Celestial Mechanics and Dynamical Astronomy, 97 (2): 87–100. [Analysis of algorithmic definition of the weak stability boundary and of the role of the invariant manifolds associated to the central manifolds of the collinear libration points as boundary of the weak stability region.]

Gómez G. and Mondelo J.M. (2001). The dynamics around the collinear equilibrium points of the RTBP. Physica D., 157 (4): 283–321. [Analysis of an extended neighborhood of the collinear equilibrium points of the restricted three body problem using numerical tools for the determination of periodic orbits and invariant 2D tori.]

Gómez G., Jorba A., Masdemont J.J., and Simó C. (1991). A dynamical systems approach for the analysis of the soho mission. In European Space Agency., editor, In Proceedings Third International Symposium on Spacecraft Flight Dynamics, pages 449–454. [Summary of the results obtained in the analysis of the SOHO mission using techniques which are usual in the study of dynamical systems. The aspects considered include the determination of the nominal orbit, the control strategy and the transfer.]

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Gómez G., Jorba A., Masdemont J.J., and Simó J.J. (1993). Study of the transfer from the earth to a halo orbit around the equilibrium point $L_4$. *Celestial Mechanics and Dynamical Astronomy*, 56 (4): 541–562, 1993. [Study of a transfer strategy from the vicinity of the Earth to a halo orbit around the equilibrium point $L_4$ of the Sun–Earth system. The approach used makes use of the hyperbolic character of the halo orbits under consideration.]

Gómez G., Jorba A., Simó C., and Masdemont J.J. (1998). Study of the transfer between halo orbits. *Acta Astronautica*, 43 (9-10): 493–520. [Two methods of transfer between halo orbits of the same family are developed making use of the geometry of the phase space around these solutions and the Floquet theory of periodic orbits.]

Gómez G., Jorba A., Masdemont J.J., and Simó C. (2001). *Dynamics and Mission Design Near Libration Point Orbits – Volume 3: Advanced Methods for Collinear Points*. World Scientific Publishing Co. Inc. ISBN 981-02-4211-5. [Global description of the orbits near the $L_1$ point of the Sun–Earth system using the reduction to the central manifold. Numerical refinement of libration orbits by means of a multiple shooting procedure. Gravitational effect of the Moon on the orbits of the stable manifold of a halo orbit of the Sun–Earth system suitable for transfers.]

Gómez G., Llibre J., Martínez R., and Simó C. (2001). *Dynamics and Mission Design Near Libration Point Orbits – Volume 1: Fundamentals: The Case of Collinear Libration Points*. World Scientific Publishing Co. Inc., ISBN 981-02-4285-9. [In this book the problem of station keeping is studied for orbits near libration points in the solar system. The main focus is on orbits near halo ones in the (Earth+Moon)-Sun system. Taking as starting point the restricted three-body problem, the motion in the full solar system is considered as a perturbation of this simplified model. All the study is done with enough generality to allow easy application to other primary-secondary systems.]

Gómez G., Koon W.S., Lo M.W., Marsden J.E., Masdemont J.J., and Ross S.D. (2004). Connecting orbits and invariant manifolds in the spatial Restricted Three-Body Problem. *Nonlinearity*, 17 (5): 1571–1606. [The invariant manifold structures of the collinear libration points can be used to construct new spacecraft trajectories, such as a ‘Petit Grand Tour’ of the moons of Jupiter. This work extends the results of previous work to the three-dimensional case. Besides providing a full description of different kinds of libration motions in a large vicinity of these points, this paper numerically demonstrates the existence of heteroclinic connections between pairs of libration orbits, one around the libration point $L_1$ and the other around $L_2$. Since these connections are asymptotic orbits, no maneuver is needed to perform the transfer from one libration point orbit to the other.]

Gómez G., Marcote M., and Masdemont J.J. (2005). Trajectory correction maneuvers in the transfer to libration point orbits. *Acta Astronautica*, 56 (7): 652–669. [Using simple dynamical systems concepts, related with the invariant manifolds of the target orbit, the paper studies the maneuvers to be done by a spacecraft in order to correct the error in the execution of the injection maneuver in the transfer trajectory. The results are compared with those obtained by Serban et al.]

Hiday L.A. and Howell K.C. (1992). Transfers between libration-point orbits in the elliptic restricted problem. In *Advances in the Astronautical Sciences*, 79, 231–250. [A strategy is formulated to design optimal time-fixed transfers between 3-dimensional libration point orbits in the vicinity of the $L_1$ point.]

Howell K.C. and Gordon S.C. (1994). Orbit Determination Error Analysis and a Station–Keeping Strategy for Sun–Earth $L_1$ Libration Point Orbits. *Journal of the Astronautical Sciences*, 42 (2): 207–228. [Development of a station-keeping strategy applicable to libration point orbits using impulsive maneuvers executed at discrete time intervals. The analysis includes some investigation of a number of the problem parameters that affect the overall maneuver costs. Several orbit determination error analysis methods are mentioned and some results are summarized.]

Howell K.C. and Hiday-Johnston L.A. (1994). Time-free transfers between libration-point orbits in the elliptic restricted problem. *Acta Astronautica*, 32 (4): 245–254. [Using primer vector theory, a strategy is formulated to design optimal time-free impulsive transfers between 3D libration-point orbits in the vicinity of the interior $L_1$ libration point of the Sun–Earth/Moon system.]

Howell K.C. and Pernicka H.J. (1993). Stationkeeping method for libration point trajectories. *Journal of
Guidance, Control, and Dynamics, 16 (1): 151–159. [A method is presented that uses maneuvers executed impulsively at discrete time intervals. The analysis includes some investigation of a number of the problem parameters that affect the overall maneuver costs. Simulations are designed to provide representative station keeping costs for a spacecraft moving in a libration-point trajectory.]

Keeter T.M. (1994). Station–Keeping Strategies for Libration Point Orbits: Target Point and Floquet Mode Approaches. PhD thesis, School of Aeronautics and Astronautics, Purdue University, West Lafayette, Indiana. [Formulation and comparison of two station keeping procedures for libration point orbits: the target point approach and the Floquet mode approach.]

Koon W.S., Lo M.W., Marsden J.E., and Ross S.D. (2008). Dynamical Systems, the Three-Body Problem and Space Mission Design. Marsden Books. [This book considers global solutions to the restricted three-body problem from a geometric point of view. The authors include a wealth of background material, but also bring the reader up to a portion of the research frontier.]

Koon W.S., Lo M.W., Marsden J.E., and Ross S.D. (2001). Low energy transfer to the moon. Celestial Mechanics and Dynamical Astronomy, 81 (1-2): 63–73. [Using the invariant manifold structures of the Lagrange points of the 3-body systems, is possible to construct low energy transfer trajectories from the Earth which execute ballistic capture at the Moon. The techniques used in the design and construction of this trajectory may be applied in many situations.]

Koon W.S., Lo M.W., Marsden J.E., and Ross S.D. (2001). Resonance and capture of Jupiter comets. Celestial Mechanics and Dynamical Astronomy, 81(1-2): 27–38. ISSN 0923-2958. [Study of the libration point invariant manifold structures for $L_4$ and $L_5$ as a starting point for understanding the capture and resonance transition of comets such as Oterma and Gehrels 3. These comets make a rapid transition from heliocentric orbits outside the orbit of Jupiter to heliocentric orbits inside the orbit of Jupiter and vice versa.]

Lo M.W., Williams B.G., Bollman W.E., Han D., Hahn Y., Bell J.L., Hirst E.A., Corwin R.A., Hong P.E., Howell K.C., Barden B.T., and Wilson R. (1998). Genesis mission design. In AAS/AIAA Space Flight Mechanics, Paper No. AIAA 98-4468 [The Genesis spacecraft had to collect solar wind samples from a halo orbit about the Sun-Earth L1 point for two years, returning those samples to Earth in 2003 for on-Earth analysis and examination. This paper is a review of the mission analysis performed for this mission.]

Marson R., Pontani M., Perozzi E., and Teofilatto P. (2010). Using space manifold dynamics to deploy a small satellite constellation around the moon. Celestial Mechanics and Dynamical Astronomy, 106 (2): 117–142. [The aim of this paper is to show how Space Manifold Dynamics can be profitably applied in order to launch three small spacecraft onboard the same launch vehicle and send them to different orbits around the Moon with no significant difference in the Delta-V budgets. Internal manifold transfers are considered to minimize also the transfer time.]

Masdemont J.J. (1991). Estudi i Utilització de Varietats Invariants en Problemes de Mecànica Celeste. PhD thesis, Universitat Politècnica de Catalunya, Barcelona. [Numerical study of the homoclinic and heteroclinic orbits associated to the triangular equilibrium points of the restricted three body problem. Study of the transfer from the vicinity of the Earth to a halo orbit using its stable manifold.]

Masdemont J.J. (2005). High-order expansions of invariant manifolds of libration point orbits with applications to mission design. Dynamical Systems, 20 (1): 59–113. [Two methods for computing the stable and unstable manifolds of libration point orbits in series expansions are studied. One procedure is based on the Lindstedt–Poincaré method, and the other in a normal form of the Hamiltonian equations of motion.]

Meyer K.R. and Hall G.R. (1992). Introduction to Hamiltonian dynamical systems and the $N$ -body problem, volume 90 of Applied Mathematical Sciences. Springer-Verlag, New York.. ISBN 0-387-97637-X. [Textbook giving a systematic grounding in the theory of Hamiltonian systems, an introduction to the theory of integrals and reduction. Poincaré’s continuation of periodic solution, normal forms, and applications of KAM theory.]

Perozzi E. and Salvo A.D. (2008). Novel spaceways for reaching the moon: An assessment for exploration. Celestial Mechanics and Dynamical Astronomy, 102 (1-3): 207–218. [Description of a method for evaluating the efficiency of novel spaceways for reaching the Moon if compared to more traditional mission profiles.]

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Serban R., Koon W.S., Lo M.W., Marsden J.E., Petzold L.R., Ross S.D., and Wilson R.S. (2002). Halo orbit mission correction maneuvers using optimal control. *Automatica*, 38 (4): 571–583. [Procedure for the computation of the required trajectory correction maneuvers for a halo orbit space mission to compensate for the launch velocity errors introduced by inaccuracies of the launch vehicle.]

Siegel C.L. and Moser J.K. (1995). *Lectures on Celestial Mechanics*. Classics in Mathematics. Springer. [This is a 1971 update by Jurgen Moser of an earlier text in German based on Carl Siegel’s lectures. With just three chapters, The three–body problem, Periodic solutions and Stability, is the analytic standard on Celestial Mechanics. The text is not easy reading but well worth the effort.]

Simó C., Gómez G., Llibre J., and Martínez R. (1986). Station Keeping of a Quasiperiodic Halo Orbit Using Invariant Manifolds. In European Space Agency, editor, *Second International Symposium on Spacecraft Flight Dynamics*, pages 65–70. [An analysis of the dynamic behavior near a halo orbit is presented. The analysis shows that there is only one strong unstable direction. The control proposed is based on reducing the orbital error with a component in that direction to be zero.]

Simó C., Gómez G., Llibre J., Martínez R., and Rodríquez R. (1987). On the Optimal Station Keeping Control of Halo Orbits. *Acta Astronautica*, 15 (6): 391–397. [The paper presents techniques for computing and controlling a halo orbit. A semi-analytical theory for the halo orbits, that is valid and amenable to any order is introduced. The Floquet modes of the monodromy matrix are used to define a local optimal control procedure.]

Szebehely V. (1967). *Theory of Orbits. The Restricted Problem of Three Bodies*. Academic Press, Inc. [The standard and most complete reference on the restricted three body problem.]

Wiesel W. and Shelton W. (1983). Modal Control of an Unstable Periodic Orbit. *Journal of the Astronautical Sciences*, 31 (1): 63–76. [Floquet theory is applied to the problem of designing a control system for a satellite in an unstable periodic orbit.]

**Biographical Sketches**

**Gerard Gómez** (born in 1952 in Barcelona, Spain) received his Master degree in 1974 at the Universitat de Barcelona and his PhD at the Universitat Autònoma de Barcelona in 1981, under Prof. C. Simó supervision. He is currently professor of Applied Mathematics at the Universitat de Barcelona. His research interest concerns Celestial Mechanics and Astrodynamics, with particular reference to the application of numerical and Dynamical Systems methods to Astronomy and spacecraft mission design.

**Esther Barrabés** (born in 1967 in Barcelona, Spain) received her Master degree in 1990 and her PhD in 2001, under Prof. G. Gómez supervision, both at the Universitat Autònoma de Barcelona. She is currently professor of Applied Mathematics at the Universitat de Girona. Her research interest concerns Celestial Mechanics and Dynamical Systems, with particular reference to the restricted three-body problem and the dynamics around libration points and \( N \) -body problems.