Ultraviolet completion of pseudo-Nambu-Goldstone dark matter with a hidden U(1) gauge symmetry

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We propose an ultraviolet completion model for pseudo-Nambu-Goldstone dark matter with a hidden U(1) gauge symmetry. Compared to previous studies, this setup is simpler, introducing less interactions. Dark matter scattering off nucleons is highly suppressed by the ultraviolet scale and direct detection constraints can be easily evaded. The kinetic mixing between the hidden U(1) and the U(1)$_Y$ gauge fields would lead to dark matter decays. We find that the current bound on the dark matter lifetime implies that the ultraviolet scale should be higher than $10^{10}$ GeV. The phenomenological constraints from the 125 GeV Higgs measurements, the dark matter relic density, and indirect detection of dark matter annihilation are also investigated.

CONTENTS

I. Introduction 2

II. Model 3
   A. Lagrangian 3
   B. Interactions 6

III. Phenomenology 9
   A. WIMP-nucleon Scattering 10
   B. WIMP Lifetime 11
   C. Higgs Physics 13
   D. WIMP Annihilation 14

IV. Parameter Scan 15

V. Conclusions and Discussions 18

Acknowledgments 18

References 19

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I. INTRODUCTION

The cosmological abundance of dark matter (DM) can be naturally explained by neutral weakly interacting massive particles (WIMPs) which are thermally produced in the plasma and subsequently freeze out as the Universe expands [1–3]. The crucial ingredient in this argument is the weak interaction strength of WIMP annihilation into standard model (SM) particles at the freeze-out epoch, implying that WIMP dark matter is quite promising to be probed in current direct detection experiments. Although the direct detection sensitivity has been tremendously improved in the recent two decades, no robust signal has been found, suggesting severe constraints on the WIMP-nucleon scattering cross section [4, 5]. This situation makes the WIMP paradigm questionable.

Nonetheless, an annihilation cross section of weak strength does not necessarily result in a WIMP-nucleon scattering cross section of the same strength. It is possible to greatly suppress the scattering process in direct detection without affecting the annihilation processes at the freeze-out epoch. An elegant way to realize it is to assume that the WIMP is a pseudo-Nambu-Goldstone boson (pNGB) whose interactions are momentum-suppressed [6–32]. Since direct detection experiments basically operate at zero momentum transfer, the WIMP-nucleon scattering cross section totally vanishes at tree level [6], evading the direct detection constraints.

The original pNGB DM model [6] introduces a complex scalar $S$, which is a SM gauge singlet. The Lagrangian respects a U(1) global symmetry $S \rightarrow e^{i\alpha}S$, except for a quadratic term $\mu^2_S(S^2 + S^\dagger_2^2)/4$, which softly breaks the U(1) symmetry into a $Z_2$ symmetry. After the U(1) spontaneous breaking, the imaginary part of $S$ becomes a stable pNGB, which has a mass $\mu_S$ and acts as the WIMP with a vanishing tree-level WIMP-nucleon scattering amplitude. Such a soft breaking term is ad hoc. Other soft breaking terms, such as a trilinear term $\propto S^3 + S^\dagger_3^3$, would spoil the vanishing scattering amplitude. Therefore, it demands an appropriate ultraviolet (UV) completion to realize only this quadratic soft breaking term [6].

A possible UV completion is to gauge the U(1) symmetry with $B - L$ charges [17, 18]. Such a U(1)$_{B-L}$ gauge symmetry would be free from gauge anomalies if three right-handed neutrinos are introduced. Consequently, the WIMP-nucleon amplitude would not exactly vanish at tree level, but be suppressed by a UV scale, i.e., the breaking scale of the U(1)$_{B-L}$ gauge symmetry. In addition, the pNGB WIMP becomes unstable, and the constraint on its lifetime leads to a UV scale typically exceeding $O(10^{11}–10^{13})$ GeV [17, 18]. Such a high scale UV completion can be embedded into a grand unified theory [22, 23].

In this work, we would like to decrease the UV scale, because a lower scale may be easier to be probed in future indirect detection and collider experiments. For this purpose, we assume the pNGB WIMP arising from a hidden U(1)$_X$ gauge symmetry, where all the SM fields do not carry U(1)$_X$ charges. The gauge anomalies are canceled without introducing right-handed neutrinos, so less fields are involved in this setup. Since the U(1)$_X$ gauge boson does not couple to SM fermions via any U(1)$_X$ gauge interaction, the interactions inducing WIMP decays only come from the kinetic mixing between the U(1)$_X$ and U(1)$_Y$ gauge fields, relieving the lifetime constraint on the UV scale.
This paper is organized as follows. In Section II, we construct a UV-complete model for pNGB WIMP by extending the SM by a hidden $U(1)_X$ gauge symmetry and two $U(1)_X$-charged scalar fields. In Section III, we discuss the phenomenology of this model regarding the WIMP-nucleon scattering in direct detection experiments, the WIMP lifetime, the related Higgs physics, and the WIMP annihilation relevant to the relic abundance and indirect detection experiments. Section IV presents results from a random scan in the parameter space. Section V gives a summary of the paper.

II. MODEL

We extend the SM with a $U(1)_X$ gauge symmetry accompanied with two complex scalar fields $S$ and $\Phi$, which are SM gauge singlets but carry $U(1)_X$ charges $q_S = 1$ and $q_\Phi = 2$, respectively. All the SM fields do not carry $U(1)_X$ charges. We assume that $S$ and $\Phi$ develop nonzero vacuum expectation values (VEVs) $v_S$ and $v_\Phi$ with a hierarchy $v_S \ll v_\Phi$. Thus, $v_\Phi$ represents a UV scale that breaks the $U(1)_X$ gauge symmetry into an approximate $U(1)_X$ global symmetry. Beneath the lower scale $v_S$, the $U(1)_X$ global symmetry is spontaneously broken, resulting in a pNGB WIMP.

A. Lagrangian

The SU(2)$_L \times U(1)_Y \times U(1)_X$ gauge-invariant Lagrangian involving $S$, $\Phi$, the SM Higgs doublet $H$, the $U(1)_X$ gauge field $X^\mu$, and the $U(1)_Y$ gauge field $B^\mu$ reads

$$\mathcal{L} \supset (D^\mu H)\dagger (D_\mu H) + (D^\mu S)\dagger (D_\mu S) + (D^\mu \Phi)\dagger (D_\mu \Phi) - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} - \frac{1}{4} X^{\mu\nu} X_{\mu\nu} - \frac{s_\epsilon}{2} B^{\mu\nu} X_{\mu\nu} + \mu^2 H^2 + \mu_S^2 |S|^2 + \mu_\Phi^2 |\Phi|^2 - \frac{\lambda_H}{2} |H|^4 - \frac{\lambda_S}{2} |S|^4 - \frac{\lambda_\Phi}{2} |\Phi|^4 - \lambda_H S |H|^2 |\Phi|^2 - \lambda_S S |S|^2 |\Phi|^2 + \frac{1}{\sqrt{2}} (\mu_S \Phi^\dagger S^2 + H.c.).$$

The covariant derivatives of the scalars are $D_\mu H = (\partial_\mu - i g' B_\mu / 2 - ig W^{\mu}_{\mu} \sigma^a / 2) H$, $D_\mu S = (\partial_\mu - iq g_X X_\mu) S$, and $D_\mu \Phi = (\partial_\mu - iq g_X X_\mu) \Phi$, where $g_X$ denotes the $U(1)_X$ gauge coupling. The field strengths of $B^{\mu\nu}$ and $X^{\mu\nu}$ are defined as $B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$ and $X_{\mu\nu} \equiv \partial_\mu X_\nu - \partial_\nu X_\mu$. The $B^{\mu\nu} X_{\mu\nu}$ term implies a kinetic mixing between $B^{\mu\nu}$ and $X^{\mu\nu}$ with a mixing parameter $s_\epsilon \equiv \sin \epsilon \in (-1, 1)$.

The parameter $\mu_S \Phi$ can be made real and positive by redefining the phase of $S$, and thus we adopt $\mu_S \Phi > 0$ hereafter. The scalar fields can be decomposed as

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}, \quad S = \frac{1}{\sqrt{2}} (v_S + s + i \eta_S), \quad \Phi = \frac{1}{\sqrt{2}} (v_\Phi + \phi + i \eta_\Phi),$$

where the SM Higgs field is expressed in the unitary gauge and $v = 246.22$ GeV. When mini-
mizing the scalar potential, three stationary point conditions are obtained as
\[ \mu_H^2 = \frac{1}{2}(\lambda_H v^2 + \lambda_H S v_S^2 + \lambda_H \Phi v_\Phi^2), \]
\[ \mu_S^2 = \frac{1}{2}(\lambda_S v_S^2 + \lambda_H S v^2 + \lambda_S \Phi v_\Phi^2) - \mu_S \Phi v_\Phi, \]
\[ \mu_\Phi^2 = \frac{1}{2}(\lambda_\Phi v_\Phi^2 + \lambda_H \Phi v^2 + \lambda_S \Phi v_S^2) - \frac{\mu_S \Phi v_\Phi^2}{2 v_\Phi}. \]  
(3)

The \( v_\Phi \) contribution to the \( \Phi^\dagger S^2 \) term leads to
\[ L_{\text{soft}} = \frac{\mu_S^2}{4} (S^2 + S^2), \]  
(4)

with \( \mu_S^2 = 2 \mu_S \Phi v_\Phi \). This is the quadratic term directly introduced in the original pNGB DM model \([6]\) to softly break the \( U(1)_X \) global symmetry. In the limit \( v_\Phi \to \infty \) and \( \mu_S \Phi \to 0 \) with finite \( \mu_S^2 \), the original model is recovered. For a finite \( v_\Phi \), there should be some phenomenological deviations from the original model, which will be explored below.

After the scalar fields obtain the nonzero VEVs, the mass terms for the \( CP \)-even scalars \( (h, s, \phi) \) and the \( CP \)-odd scalars \( (\eta_S, \eta_\Phi) \) become
\[ L_{\text{mass}} \supset -\frac{1}{2} \begin{pmatrix} h & s & \phi \end{pmatrix} M_E^2 \begin{pmatrix} h \\ s \\ \phi \end{pmatrix} - \frac{1}{2} \begin{pmatrix} \eta_S & \eta_\Phi \end{pmatrix} M_O^2 \begin{pmatrix} \eta_S \\ \eta_\Phi \end{pmatrix}, \]
(5)

where the mass-squared matrices are given by [17]
\[ M_E^2 = \begin{pmatrix} \lambda_H v^2 & \lambda_H S v_S & \lambda_H \Phi v_\Phi \\ \lambda_H S v_S & \lambda_S v_S^2 & \lambda_S \Phi v_\Phi - \mu_S \Phi v_S \\ \lambda_H \Phi v_\Phi & \lambda_S \Phi v_S v_\Phi - \mu_S \Phi v_S & \lambda_\Phi v_\Phi^2 + \frac{\mu_S \Phi v_\Phi^2}{2 v_\Phi} \end{pmatrix}, \]  
(6)
\[ M_O^2 = \mu_S \Phi \begin{pmatrix} 2 v_\Phi & -v_S \\ -v_S & v_S^2/2 v_\Phi \end{pmatrix}. \]
(7)

The two matrices can be diagonalized by two real orthogonal matrices \( U \) and \( V \):
\[ U^T M_E^2 U = \text{diag}(m_{h_1}^2, m_{h_2}^2, m_{h_3}^2), \quad V^T M_O^2 V = \text{diag}(m_\chi^2, 0). \]
(8)

\( V \) can be explicitly expressed as
\[ V = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}, \]
(9)
with a rotation angle $\beta$ satisfying
\[
\sin \beta = \frac{v_S}{\sqrt{v_S^2 + 4v_{\Phi}^2}}.
\] (10)

We have adopted the shorthand notations for the trigonometric functions, $s_\beta \equiv \sin \beta$, $c_\beta \equiv \cos \beta$, and $t_\beta \equiv \tan \beta$. Such notations are used throughout the paper.

The relations between the interaction and mass bases are
\[
\begin{pmatrix} h \\ s \\ \phi \end{pmatrix} = U \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}, \quad \begin{pmatrix} \eta_S \\ \eta_\Phi \end{pmatrix} = V \begin{pmatrix} \chi \\ \tilde{\chi} \end{pmatrix}.
\] (11)

We define $h_1$ to be the SM-like Higgs boson with $m_{h_1} = 125.10 \pm 0.14$ GeV [33], whose dominant component should be $h$, i.e., $|U_{11}| > |U_{21}|, |U_{31}|$. Similarly, the exotic Higgs bosons $h_2$ and $h_3$ with masses $m_{h_2}$ and $m_{h_3}$ are defined as $s$-like and $\phi$-like, respectively. We further use positive $U_{11}, U_{22}$, and $U_{33}$ to fix the signs of the $U$ matrix elements. $\tilde{\chi}$ is the massless Nambu-Goldstone boson associated with the spontaneous breaking of the $U(1)_X$ gauge symmetry, while $\chi$ is a pNGB WIMP with a mass squared of
\[
m^2_\chi = \frac{\mu_S \Phi}{2v_\Phi} (v_S^2 + 4v_{\Phi}^2),
\] (12)
serving as a DM candidate. The typical range for $m_{\chi}$ would be $\mathcal{O}(\text{GeV})$$-\mathcal{O}(\text{TeV})$.

If $\mu_{S\Phi} = 0$, the Lagrangian (1) respects two distinct $U(1)$ global symmetries, one with $S \rightarrow e^{\text{i} \alpha_1} S$ and the other one with $\Phi \rightarrow e^{\text{i} \alpha_2} \Phi$. Consequently, both $\eta_S$ and $\eta_\Phi$ are massless Nambu-Goldstone bosons according to the Goldstone theorem [34, 35]. Nonetheless, the existence of the $\mu_{S\Phi}$ term merges the two $U(1)$ symmetries into the $U(1)_X$ global symmetry with $q_S = 1$ and $q_\Phi = 2$. As a result, only $\tilde{\chi}$ remains massless, while $\chi$ obtains a mass proportional to $\sqrt{\mu_{S\Phi}}$.

After the spontaneous breaking of the $SU(2)_L \times U(1)_Y \times U(1)_X$ gauge symmetry, the gauge fields obtain the mass terms
\[
\mathcal{L}_{\text{mass}} \supset m^2_W W^{\mu - \mu} W^+_\mu + \frac{1}{2} \begin{pmatrix} B^\mu & W^{3,\mu} & X^\mu \end{pmatrix} M^2_N \begin{pmatrix} B_\mu \\ W^3_\mu \\ X_\mu \end{pmatrix},
\] (13)
with $m^2_W = g^2 v^2 / 4$ and
\[
M^2_N = \begin{pmatrix}
g^2 v^2 / 4 & -gg' v^2 / 4 & 0 \\
-gg' v^2 / 4 & g^2 v^2 / 4 & 0 \\
0 & 0 & g^2 (v_S^2 + 4v_{\Phi}^2)
\end{pmatrix}.
\] (14)

Considering both such a $B_\mu W^3_\mu$ mass mixing and the $B_\mu X_\mu$ kinetic mixing, the physical neutral
gauge fields \((A_\mu, Z_\mu, Z'_\mu)\) can be obtained through a linear transformation \([36, 37]\)

\[
\begin{pmatrix}
B_\mu \\
W^3_\mu \\
X_\mu
\end{pmatrix}
= K
\begin{pmatrix}
A_\mu \\
Z_\mu \\
Z'_\mu
\end{pmatrix}
\] (15)

with

\[
K = \begin{pmatrix}
\hat{c}_W & -\hat{s}_Wc_\xi - t_\varepsilon s_\xi & \hat{s}_W s_\xi - t_\varepsilon c_\xi \\
\hat{s}_W & \hat{c}_W c_\xi & -\hat{c}_W s_\xi \\
0 & s_\xi/c_\varepsilon & c_\xi/c_\varepsilon
\end{pmatrix}.
\] (16)

Here we denote \(\hat{s}_W \equiv \sin \hat{\theta}_W\) and \(\hat{c}_W \equiv \cos \hat{\theta}_W\), where \(\hat{\theta}_W \equiv \tan^{-1}(g'/g)\) is the weak mixing angle. \(\varepsilon\) is the angle related to the kinetic mixing, and \(\xi\) is a rotation angle determined by the equation \([36]\)

\[
t_2\varepsilon = \frac{s_2\hat{s}_Wv^2(g^2 + g'^2)}{c_\varepsilon^2v^2(1 - \hat{s}_W^2t_\varepsilon^2)} - 4g_X^2(v_S^2 + 4v_\Phi^2).
\] (17)

The gauge fields \((A_\mu, Z_\mu, Z'_\mu)\) have canonical kinetic terms as well as diagonalized mass terms. The corresponding masses for the photon, \(Z\), and \(Z'\) bosons are given by \(m_A = 0\) and \([38]\)

\[
m_Z^2 = \frac{v^2}{4}(g^2 + g'^2)(1 + \hat{s}_W t_\varepsilon), \quad m_{Z'}^2 = \frac{g_X^2(v_S^2 + 4v_\Phi^2)}{c_\varepsilon^2(1 + \hat{s}_W t_\varepsilon)},
\] (18)

respectively. Define \(r \equiv m_{Z'}^2/m_Z^2\), and we can further derive \([37]\)

\[
t_\varepsilon = \frac{2\hat{s}_W t_\varepsilon}{1 - r} \left[1 + \sqrt{1 - r \left(\frac{2\hat{s}_W t_\varepsilon}{1 - r}\right)^2}\right]^{-1}.
\] (19)

\(r \gg 1\) would lead to

\[
t_\varepsilon \simeq -\frac{\hat{s}_W t_\varepsilon}{r} + \mathcal{O}(r^{-2}).
\] (20)

If there is no kinetic mixing between \(B_\mu\) and \(X_\mu\), we have \(\varepsilon = \xi = 0\), \(m_Z^2 = (g^2 + g'^2)v^2/4\), and \(m_{Z'}^2 = g_X^2(v_S^2 + 4v_\Phi^2)\).

**B. Interactions**

In the basis of the mass eigenstates \(\chi\) and \(h_i\), the scalar trilinear couplings can be expressed as

\[
\mathcal{L}_{\text{tri}} = -\frac{1}{2} \sum_{i=1}^{3} g_{hi\chi}h_i\chi^2 - \sum_{i,j,k=1}^{3} g_{h_i h_j h_k} h_i h_j h_k,
\] (21)
where

\[
g_{h_1\chi^2} = (\lambda H S c_3^2 + \lambda H s_3^2) v U_{11} + (\lambda_S v S c_2^2 + \lambda_S v S s_2^2 + 2 \mu_S \beta c_3) U_{12}
\]
\[
+ [\lambda_H v \beta s_3^2 + (\lambda_S v \beta + \mu_S \beta) c_3] U_{13},
\]
(22)

\[
g_{h_1 h_2} h_k = \frac{1}{2} (\lambda H v U_{11} + \lambda_H v S U_{21} + \lambda H v \Phi U_{31}) U_{1j} U_{1k}
\]
\[
+ \frac{1}{2} [\lambda H S v U_{11} + \lambda_S v S U_{21} + (\lambda_S v \beta - \mu_S \beta) U_{31}] U_{2j} U_{2k}
\]
\[
+ \frac{1}{2} (\lambda H \Phi v U_{11} + \lambda_S \Phi v S U_{21} + \lambda \Phi v \Phi U_{31}) U_{3j} U_{3k}.
\]
(23)

The Yukawa couplings become

\[
\mathcal{L}_{h_1 f f} = - \sum_{f} \sum_{i=1}^{3} \frac{m_f}{v} U_{i1} h_{1ij} f,
\]
(24)

where \( f \) denotes the SM fermions.

The neutral current interactions arising from the SU(2)_L \times U(1)_Y \times U(1)_X gauge symmetry are given by

\[
\mathcal{L}_{NC} = B_{\mu} j_{\mu}^Y + W_{\mu} j_{\mu}^3 + X_{\mu} j_{\mu}^X,
\]
(25)

with

\[
\begin{align*}
    j_{\mu}^Y &= \sum_f g'(Y_{fL} \bar{f} L \gamma_{\mu} f_L + Y_{fR} \bar{f} R \gamma_{\mu} f_R), \\
    j_{\mu}^3 &= \sum_f g T_f^3 \bar{f} L \gamma_{\mu} f_L, \\
    j_{\mu}^X &= i g_X (\overleftrightarrow{\partial_{\mu} S} + 2 \Phi \overleftrightarrow{\partial_{\mu} \Phi}),
\end{align*}
\]
(26-28)

where \( T_f^3 \) is the third component of the weak isospin for a SM fermion \( f \), and \( Y_{fL} \) and \( Y_{fR} \) are the weak hypercharges for left- and right-handed fermions. If we define \( \hat{A}_\mu \equiv \hat{c}_W B_\mu + \hat{s}_W W_\mu^3 \) and \( \hat{Z}_\mu \equiv -\hat{s}_W B_\mu + \hat{c}_W W_\mu^3 \), the neutral current interactions can be expressed in a familiar form

\[
\mathcal{L}_{NC} = \hat{A}_\mu j_{\mu}^{EM} + \hat{Z}_\mu j_{\mu}^{Z} + X_{\mu} j_{\mu}^{X},
\]
(29)

with

\[
\begin{align*}
    j_{\mu}^{EM} &= \sum_f Q_f e \bar{f} \gamma_{\mu} f, \\
    j_{\mu}^{Z} &= \frac{g}{2\hat{c}_W} \sum_f f_{\gamma_{\mu}} (T_f^3 - 2Q_f \hat{s}_W - T_f^1 \gamma_5)f,
\end{align*}
\]
(30-31)

where \( Q_f \) is the electric charge of \( f \) in units of \( e = g g' / \sqrt{g^2 + g'^2} \).
The relation between \((\hat{A}_\mu, \hat{Z}_\mu, X_\mu)\) and the mass basis \((A_\mu, Z_\mu, Z'_\mu)\) is

\[
\begin{pmatrix}
\hat{A}_\mu \\
\hat{Z}_\mu \\
X_\mu
\end{pmatrix} = R
\begin{pmatrix}
A_\mu \\
Z_\mu \\
Z'_\mu
\end{pmatrix},
\]

(32)

where

\[
R = \begin{pmatrix}
\hat{c}_W & \hat{s}_W & 0 \\
-\hat{s}_W & \hat{c}_W & 0 \\
0 & 0 & 1
\end{pmatrix} K = \begin{pmatrix}
1 & -\hat{c}_W t_\xi s_\xi & -\hat{c}_W t_\xi c_\xi \\
0 & \hat{s}_W t_\xi s_\xi + c_\xi & \hat{s}_W t_\xi c_\xi - s_\xi \\
0 & s_\xi/c_\xi & c_\xi/c_\xi
\end{pmatrix}.
\]

(33)

Therefore, the neutral current interactions in the mass basis are \([37, 39]\)

\[
\mathcal{L}_{NC} = A_\mu j_\mu^{EM} + Z_\mu j_\mu^{Z} + Z'_\mu j_\mu^{Z'},
\]

(34)

with

\[
j_\mu^{Z} = \sum_f \bar{f} \gamma^\mu (g_{V, Z}^f - g_{A, Z}^f \gamma^5) f + \frac{s_\xi}{c_\xi} j_\mu^{X},
\]

(35)

\[
j_\mu^{Z'} = \sum_f \bar{f} \gamma^\mu (g_{V, Z'}^f - g_{A, Z'}^f \gamma^5) f + \frac{c_\xi}{c_\xi} j_\mu^{X},
\]

(36)

where

\[
g_{V, Z}^f = \frac{g}{2\hat{c}_W} (\hat{s}_W t_\xi s_\xi + c_\xi)(T_f^3 - 2Q_f \hat{s}_W^2) - Q_f \hat{c}_W t_\xi c_\xi,
\]

(37)

\[
g_{A, Z}^f = \frac{g}{2\hat{c}_W} (\hat{s}_W t_\xi s_\xi + c_\xi)T_f^3,
\]

(38)

\[
g_{V, Z'}^f = \frac{g}{2\hat{c}_W} (\hat{s}_W t_\xi c_\xi - s_\xi)(T_f^3 - 2Q_f \hat{s}_W^2) - Q_f \hat{c}_W t_\xi c_\xi,
\]

(39)

\[
g_{A, Z'}^f = \frac{g}{2\hat{c}_W} (\hat{s}_W t_\xi c_\xi - s_\xi)T_f^3.
\]

(40)

Note that the electromagnetic current interactions \(A_\mu j_\mu^{EM}\) remain in the SM form. For \(\varepsilon = 0\), we have \(A_\mu = \hat{A}_\mu\), \(Z_\mu = \hat{Z}_\mu\), and \(Z'_\mu = X_\mu\), and the \(Z\) couplings to the fermions are the same as in the SM, while the \(Z'\) boson only couples to \(j_\mu^{X}\). The existence of the kinetic mixing makes \(Z\) couple to \(j_\mu^{X}\) and \(Z'\) couple to the SM fermions.

The \(Z-\chi-h_i\) and \(Z'-\chi-h_i\) couplings from the neutral current interactions \(Z_\mu j_\mu^{Z} + Z'_\mu j_\mu^{Z'}\) are

\[
\mathcal{L}_{\chi h_i} = \sum_{i=1}^{3} (g_{Z\chi h_i} Z_\mu \chi \frac{\theta^\mu}{\sqrt{2}} h_i + g_{Z'\chi h_i} Z'_\mu \chi \frac{\theta^\mu}{\sqrt{2}} h_i),
\]

(41)

where

\[
g_{Z\chi h_i} = \frac{g X s_\xi}{c_\xi} (c_\beta U_{2i} - 2s_\beta U_{3i}),
\]

(42)
\[ g_{Z^{c} h} = \frac{g_{X} c_{z}}{c_{z}} (c_{\beta} U_{21} - 2 s_{\beta} U_{31}). \]

These couplings break the $Z_2$ symmetry $\chi \rightarrow -\chi$, inducing decay processes of the pNGB WIMP $\chi$. In order to be a viable DM candidate, $\chi$ should have a sufficiently long lifetime.

The best measured electroweak quantities are the fine-structure constant $\alpha(m_Z)$ in the $\overline{\text{MS}}$ scheme, the Fermi constant $G_F$, and the $Z$ boson pole mass $m_Z$. From these quantities, it is conventional to define the “physical” weak mixing parameters $s_W^2$ and $c_W^2 \equiv 1 - s_W^2$ via the tree-level SM relation [36, 40]

\[ s_W^2 c_W^2 = \frac{\pi \alpha}{\sqrt{2} G_F m_Z^2}. \]

Then the hatted parameters $\hat{s}_W^2$ and $\hat{c}_W^2$ can be determined by [38]

\[ s_W^2 c_W^2 (1 + \hat{s}_W t_\xi t_\varepsilon) = \hat{s}_W^2 \hat{c}_W^2. \]

The values of $g$ and $g'$ are settled by $g = e/s_W$ and $g' = e/c_W$ with $e = \sqrt{4\pi\alpha}$.

There are 10 free parameters in the model, which are chosen to be

\[ v_S, v_\Phi, m_\chi, m_{h_2}, m_{h_3}, m_{Z'}, \lambda_{HS}, \lambda_{H\Phi}, \lambda_{S\Phi}, s_\varepsilon. \]

For a UV completion of the original pNGB DM model, we are particularly interested in the parameter regions with $v_\Phi \gg v_S \sim v$. This implies a mass hierarchy of $m_{Z'} \sim m_{h_3} \gg m_{h_2} \sim m_{h_1}$, leading to $r = m_{Z'}^2/m_Z^2 \gg 1$ and hence $|\xi| \ll |\varepsilon|$. Therefore, the value of $\hat{s}_W$ would be very close to $s_W$ for any value of $s_\varepsilon$.

The kinetic mixing contributes to the electroweak oblique parameters $S$, $T$, and $U$ [41, 42] at tree level. For a small $\varepsilon$, we have

\[ \xi \approx \frac{s_W \varepsilon}{1 - r}, \]

and [43]

\[ S \approx \frac{4(c_W^2 - r)s_W^2 c_W^2 \varepsilon^2}{\alpha(1 - r)^2}, \quad T \approx -\frac{r s_W^2 \varepsilon^2}{\alpha(1 - r)^2}, \quad U \approx \frac{4 s_W^2 c_W^2 \varepsilon^2}{\alpha(1 - r)^2}. \]

Because $S$, $T$, and $U$ are all highly suppressed by $r$, we expect that the constraint on the oblique parameters from the global fit of electroweak precision measurements [44] would not constrain our interested parameter regions.

### III. PHENOMENOLOGY

In this section, we discuss the phenomenological consequences of the model.
A. WIMP-nucleon Scattering

In the original pNGB DM model with a directly introduced soft breaking parameter $\mu_S^2$, the WIMP-nucleon scattering amplitude exactly vanishes at tree level in the zero momentum transfer limit [6]. Our UV completion gives $\mu_S^2$ a dynamical origin, but inevitably introduces the $\chi$-quark coupling, leading to a nonvanishing $\chi$-nucleon scattering amplitude. Nonetheless, we expect that the amplitude is significantly suppressed by a high UV scale $v_\Phi$, since $v_\Phi \to \infty$ limit recovers the original model.

The spin-independent (SI) $\chi$-nucleon scattering is induced by the $\chi$-quark scattering via $t$-channel exchanges of the $CP$-even Higgs bosons $h_1$, $h_2$, and $h_3$. In the zero momentum transfer limit, the tree-level $\chi$-quark scattering amplitude becomes

$$iM = \frac{i m_q}{v} \bar{u}(k_2) u(k_1) \left( \frac{gh_{1\chi^2} U_{11}}{m_{h_1}^2} + \frac{gh_{2\chi^2} U_{12}}{m_{h_2}^2} + \frac{gh_{3\chi^2} U_{13}}{m_{h_3}^2} \right),$$

$$= \frac{i m_q}{v} \bar{u}(k_2) u(k_1) \left( g_{h_1\chi^2} g_{h_2\chi^2} g_{h_3\chi^2} \right) \begin{pmatrix} m_{h_1}^{-2} & m_{h_2}^{-2} & m_{h_3}^{-2} \end{pmatrix} U^T \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad (49)$$

where $u(k_1)$ and $\bar{u}(k_2)$ denote the plane-wave spinor coefficients for the incoming and outgoing quarks $q$ of 4-momenta $k_1$ and $k_2$. It is equivalent [6, 14] to express the amplitude in the interaction basis $(\Phi, \phi, s)$, whose couplings to $\chi$ are given by $G = (g_{h\chi^2} g_{s\chi^2} g_{\phi\chi^2})$ with

$$g_{h\chi^2} = (\lambda_{HSC}^2 + \lambda_{HFS}^2) v, \quad (50)$$

$$g_{s\chi^2} = \lambda_{S\Phi} v_{S} c_{\beta}^2 + \lambda_{S\Phi} v_{S} s_{\beta}^2 + 2 \mu_{S\Phi} s_{\beta} c_{\beta}, \quad (51)$$

$$g_{\phi\chi^2} = \lambda_{\Phi} v_{\Phi} c_{\beta}^2 + (\lambda_{\Phi} v_{\Phi} + \mu_{\Phi}) c_{\beta}. \quad (52)$$

Utilizing $\text{diag}(m_{h_1}^{-2}, m_{h_2}^{-2}, m_{h_3}^{-2}) = U^T (M_\Phi^2)^{-1} U$ and $(g_{h_1\chi^2} g_{h_2\chi^2} g_{h_3\chi^2}) = GU$, we derive

$$iM = \frac{i m_q}{v} \bar{u}(k_2) u(k_1) G (M_\Phi^2)^{-1} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}. \quad (53)$$

Expanding the scattering amplitude in orders of $v_\Phi$ for $v_\Phi \gg v, v_S$, we obtain

$$iM \simeq \frac{i \hat{\lambda} m_q m_{\chi}^2}{2v^2 v_\Phi^4} \bar{u}(k_2) u(k_1) + \mathcal{O}(v_\Phi^{-4}), \quad (54)$$

where

$$\hat{\lambda} = \frac{\lambda_{H\Phi} \lambda_{S\Phi} - \lambda_{\Phi} \lambda_{HS} + 2 \lambda_{H\Phi} \lambda_{S\Phi} - 2 \lambda_{S} \lambda_{H\Phi}}{\lambda_{H} \lambda_{S}^{2} \lambda_{\Phi} + 2 \lambda_{H\Phi} \lambda_{S} \lambda_{S\Phi} - \lambda_{S} \lambda_{H\Phi}^{2} - \lambda_{\Phi} \lambda_{H\Phi}^{2} - \lambda_{H} \lambda_{S\Phi}^{2}}. \quad (55)$$

Thus, the amplitude is suppressed by $v_\Phi^{-2}$, as expected. Based on effective field theory [45], we
derive the resulting SI χ-nucleon scattering cross section

\[ \sigma_{\chi N}^{SI} \simeq \frac{\bar{\lambda}^2 m_N^4 m_\chi^4}{1296\pi (m_N + m_\chi)^2 v_\Phi^4 v_\phi^4} + O(v_\Phi^{-6}), \]  

(56)

which is suppressed by \( v_\phi^{-4} \). Here, \( f_{u,d,s}^N \) are the nucleon form factors for light quarks [46].

In Fig. 1(a), we plot the χ-nucleon scattering cross section \( \sigma_{\chi N}^{SI} \) as functions of \( m_\chi \) for \( v_\Phi = 10^5 \) and \( 10^7 \) GeV, with the other related parameters fixed to be \( v_S = 1 \) TeV, \( m_{h_2} = 300 \) GeV, \( m_{h_3} = 0.1v_\phi \), \( \lambda_{HS} = 0.03 \), and \( \lambda_{H\Phi} = \lambda_{S\Phi} = 0.01 \). For \( m_\chi \gg m_N \), Eq. (56) shows that \( \sigma_{\chi N}^{SI} \) is proportional to \( m_\chi^2 \). Therefore, as \( m_\chi \) increases by one order of magnitudes, \( \sigma_{\chi N}^{SI} \) in Fig. 1(a) increases by two orders of magnitudes. \( v_\Phi = 10^7 \) GeV leads to cross sections smaller that those for \( v_\Phi = 10^5 \) GeV by eight orders of magnitudes, because of \( v_\phi^{-4} \). Note that \( v_\Phi = 10^5 \) GeV results in \( \sigma_{\chi N}^{SI} \) much smaller than the 90% confidence level (C.L.) upper limits from the recent LZ direct detection experiment [5], and even beyond the reach of the future DARWIN experiment with a 200 t·yr exposure [47]. Fig. 1(b) displays \( \sigma_{\chi N}^{SI} \) as functions of \( v_\Phi \) for \( m_\chi = 100 \) GeV and 1 TeV, demonstrating an obvious \( \sigma_{\chi N}^{SI} \propto v_\phi^{-4} \) behavior.

B. WIMP Lifetime

The \( Z'-\chi-h_i \) and \( Z'-\chi-h_i \) couplings (41) induce the decay of the pNGB WIMP \( \chi \). We are particularly interested in the parameter regions with \( m_\chi \ll m_{Z'} \sim m_{h_3} \), where the \( \chi \) decay processes involve \( \chi \rightarrow h_i^{(*)} Z^{(*)} \) and \( \chi \rightarrow h_i^{(*)} Z'^{(*)} \). Depending on the mass spectrum, the \( h_1, h_2, \) and \( Z \) bosons could be either on or off shell, while the \( h_3 \) and \( Z' \) bosons must be off shell. The corresponding Feynman diagrams are depicted in Fig. 2.
Six years of Fermi-LAT γ-ray observations of dwarf galaxies imply a conservative constraint on the WIMP lifetime, $\tau_\chi \gtrsim 10^{27}$ s [48], corresponding to a bound on the total WIMP decay width, $\Gamma_\chi \equiv 1/\tau_\chi \lesssim 6.6 \times 10^{-52}$ GeV. In the $v_\Phi \to \infty$ limit, all the $\chi$ decay channels are forbidden, and $\chi$ becomes stable. Thus, the constraint on the $\chi$ lifetime is expected to give a lower bound on the UV scale $v_\Phi$. Therefore, the total decay width of the pNGB WIMP $\chi$ should be carefully calculated.

When $m_\chi > m_{h_i} + m_Z$ ($i = 1, 2$), the 2-body partial decay width of $\chi \to h_i Z$ is given by

$$\Gamma(\chi \to h_i Z) = \frac{g_Z^2 m_\chi^3}{16\pi m_Z^2} \lambda^{3/2} \left(1 - \frac{m_{h_i}^2}{m_\chi^2} - \frac{m_Z^2}{m_\chi^2} \right),$$

where the $\lambda$ function is defined as $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$. When $m_\chi > m_{h_i} + 2m_f$ ($i = 1, 2$), we should consider the 3-body decays $\chi \to h_i f \bar{f}$. If $m_{h_i} + 2m_f < m_\chi < m_{h_i} + m_Z$, both the Feynman diagrams mediated by the off-shell $Z$ and $Z'$ bosons contribute to $\chi \to h_i f \bar{f}$. However, once the 2-body decay channels $\chi \to h_i Z$ open, the $\chi \to h_i f \bar{f}$ decay diagrams mediated by the $Z$ bosons should be discarded for avoiding double counting.

When $m_Z + 2m_f < m_\chi < m_{h_i} + m_Z$ ($i = 1, 2$), the 3-body decays $\chi \to Z f \bar{f}$ mediated by $h_i$ should be involved. Since the fermion couplings to $h_i$ are commonly suppressed by $m_f/v$, the dominant contributions to $\chi \to Z f \bar{f}$ come from the heaviest SM fermions $t$, $b$, $\tau$, and $c$. If $2m_W + m_Z < m_\chi < m_{h_2} + m_Z$, the 3-body decays $\chi \to W^+W^-Z$ and $\chi \to ZZZ$ mediated by off-shell $h_i$ bosons may happen. Nonetheless, our calculation shows that their contributions are negligible, compared to $\chi \to Z f \bar{f}$ and $\chi \to h_i f \bar{f}$. If all 2- and 3-body decay channels are kinematically forbidden, the 4-body decays $\chi \to f \bar{f} f \bar{f}$ should be taken into account.

For calculating the 3-body partial decay widths, we derive analytic expressions and perform numerical integrals. Since the Feynman diagrams and integrals for the 4-body decays are too complicated to be dealt with by hand, we utilize the Monte Carlo tool MadGraph5_aMG@NLO [49] to automatically evaluate the 4-body partial decay widths. In the latter approach, FeynRules [50] is used to implement the model.

We fix the parameters as $v_S = 1$ TeV, $m_{h_2} = 300$ GeV, $m_{h_2} = m_{Z'} = 0.1 v_\Phi$, $\lambda_{HS} = 0.03$, $\lambda_{H\Phi} = \lambda_{S\Phi} = 0.01$, and $\sin \varepsilon = 0.1$, and show the total decay width of $\chi$ as functions of $m_\chi$ for $v_\Phi = 10^{10}, 10^{13}$, and $10^{15}$ GeV in Fig. 3(a). The threshold effects at $m_\chi = m_{h_1} \simeq 125$ GeV,
FIG. 3. $\chi$ decay width $\Gamma_\chi$ as functions of $m_\chi$ for $v_\Phi = 10^{10}$, $10^{13}$, and $10^{15}$ GeV (a) and $\chi$ lifetime $\tau_\chi$ as functions of $v_\Phi$ for $m_\chi = 20, 200$, and $2000$ GeV (b). Other parameters are fixed as $v_S = 1$ TeV, $m_{h_2} = 300$ GeV, $m_{h_3} = m_{Z'} = 0.1v_\Phi$, $\lambda_{HS} = 0.03$, $\lambda_{HF} = \lambda_{SF} = 0.01$, and $\sin \varepsilon = 0.1$. The dashed lines denote the conservative constraint $\tau_\chi \gtrsim 10^{27}$ s from Fermi-LAT $\gamma$-ray observations of dwarf galaxies [48].

$m_\chi = m_{h_1} + m_Z \simeq 216$ GeV, and $m_\chi = m_{h_2} + m_Z \simeq 391$ GeV are clearly demonstrated. The red dashed line denotes the Fermi-LAT bound on the WIMP lifetime [48] converted to the total decay width, $\Gamma_\chi \lesssim 6.6 \times 10^{-52}$ GeV. Thus, $m_\chi \gtrsim 25, 140$, and $3900$ GeV are excluded for $v_\Phi = 10^{10}, 10^{13}$, and $10^{15}$ GeV, respectively. In Fig. 3(b), we display the $\chi$ lifetime as functions of $v_\Phi$ for $m_\chi = 20, 200$, and $2000$ GeV, where the Fermi-LAT constraint excludes $v_\Phi \lesssim 4 \times 10^9$, $3 \times 10^{13}$, and $4 \times 10^{15}$ GeV, respectively. Thus, the bound on $v_\Phi$ given by the WIMP lifetime is much more stringent than the bound from direct detection experiments.

C. Higgs Physics

In this model, the properties of the SM-like Higgs boson $h_1$ deviate from the SM prediction. The tree-level $h_1$ couplings to SM particles can be parametrized as

$$
\mathcal{L}_{h_1} = \kappa_W \frac{2m_W^2}{v} h_1 W^+ W^- + \kappa_Z \frac{m_Z^2}{v} h_1 Z^\pm Z^{\mp} - \sum_f \kappa_f \frac{m_f}{v} h_1 \bar{f} f,
$$

where $\kappa_W$, $\kappa_Z$, and $\kappa_f$ are the modifiers to the couplings with $W$, $Z$, and fermions. The SM corresponds to $\kappa_W = \kappa_Z = \kappa_f = 1$, while this model gives

$$
\kappa_W = \kappa_f = U_{11},
$$

$$
\kappa_Z = U_{11} \epsilon_\chi \xi (1 + \hat{s}_W t_e t_\xi) + \frac{\xi^2 g_X^2 v}{\epsilon_\chi m_Z^2} (U_{21} v_S + 4 U_{31} v_\Phi).
$$

In addition, exotic Higgs decay channels may exist. If $m_{h_1} > 2m_\chi$, the invisible decay channel
\[ h_1 \rightarrow \chi \chi \] opens, leading to an invisible decay width

\[
\Gamma(h_1 \rightarrow \chi \chi) = \frac{g_{h_1 \chi}^2}{32\pi m_{h_1}} \sqrt{1 - \frac{4m_{\chi}^2}{m_{h_1}^2}}. \quad (61)
\]

If \( m_{h_1} > m_\chi + m_Z \), there is a semi-invisible decay channel \( h_1 \rightarrow \chi Z \) with a partial decay width

\[
\Gamma(h_1 \rightarrow \chi Z) = \frac{g_{Z h_1 m_{h_1}^3}}{16\pi m_Z^2} \lambda^{3/2} \left( 1, \frac{m_\chi^2}{m_{h_1}^2}, \frac{m_Z^2}{m_{h_1}^2} \right). \quad (62)
\]

If \( m_{h_1} > 2m_{h_2} \), the partial decay width of \( h_1 \rightarrow h_2 h_2 \) is given by

\[
\Gamma(h_1 \rightarrow h_2 h_2) = \frac{(g_{h_1 h_2 h_2} + g_{h_2 h_1 h_2} + g_{h_2 h_2 h_1})^2}{8\pi m_{h_1}} \sqrt{1 - \frac{4m_{h_2}^2}{m_{h_1}^2}}. \quad (63)
\]

We utilize a numerical tool Lilith 2 [51, 52] to constrain the model parameter space with the LHC Higgs measurements based on the Lilith database of version 19.09, including ATLAS and CMS Run 2 data of integrated luminosity 36 fb\(^{-1}\). The important results sensitive to this model come from the measurements of \( h_1 \rightarrow \gamma \gamma \) [53], \( h_1 \rightarrow ZZ \) [54], and \( h_1 \rightarrow W^+W^- \) [55], and the search for invisible Higgs decays [56], and the combined measurements of the Higgs couplings from several channels [57]. For each parameter point, Lilith constructs an approximate likelihood function from the measurements of the Higgs signal strengths. The corresponding p-value larger than 0.05 is required, ensuring that each viable parameter point is consistent with the experimental results at 95% C.L.

### D. WIMP Annihilation

The relic abundance of the pNGB WIMP \( \chi \) is essentially determined by \( \chi \chi \) annihilation cross section at the freeze-out epoch, \( \langle \sigma_{\text{ann}} v \rangle_{\text{FO}} \). Potential \( \chi \chi \) annihilation channels include \( f \bar{f} \), \( W^+W^- \), \( ZZ \), and \( h_i h_j \) (\( i, j = 1, 2 \)). The related Feynman diagrams are enormous. We make use of a MadGraph5_aMG@NLO plugin MadDM [58] to automatically generate and calculate all tree-level annihilation diagrams, and to solve the Boltzmann equation for predicting the relic abundance \( \Omega_{\chi h}^2 \).

\( \chi \chi \) annihilation would still occur at the present day, inducing potential \( \gamma \)-ray signals in indirect detection experiments. A combined search for such \( \gamma \)-ray signals in the dwarf galaxies from the Fermi-LAT space experiment and the MAGIC Cherenkov telescopes [59] have given important constraints on the DM annihilation cross section. We further utilize MadGraph5_aMG@NLO [49] to evaluate the \( \chi \chi \) annihilation cross section at a typical average WIMP velocity \( 2 \times 10^{-3} \) for dwarf galaxies, \( \langle \sigma_{\text{ann}} v \rangle_D \). Thus, the Fermi-MAGIC result can be used to constrain the model.
IV. PARAMETER SCAN

We perform a random scan in the following parameter ranges,

\[ 10 \text{ GeV} < v_S, m_{h_2}, m_\chi < 10^4 \text{ GeV}, \quad 10^9 \text{ GeV} < v_\Phi < 10^{17} \text{ GeV}, \]
\[ 10^8 \text{ GeV} < m_{h_3}, m_{Z'} < 10^{16} \text{ GeV}, \quad 0.01 < |s_\varepsilon| < 0.9, \]
\[ 10^{-3} < |\lambda_{HS}|, |\lambda_{H\Phi}|, |\lambda_{S\Phi}| < 1. \]

The induced couplings \( \lambda_H, \lambda_S, \lambda_\Phi, \) and \( g_X \) are further required to range from \( 10^{-3} \) to 1. We select the parameter points that satisfy the phenomenological requirements below.

- The WIMP lifetime satisfies the Fermi-LAT bound \( \tau_\chi \gtrsim 10^{27} \text{ s} \) [48].

- The signal strengths of the 125 GeV Higgs boson \( h_1 \) are consistent with the Higgs measurements after LHC Run 2 at 95\% C.L. according to the Lilith calculation.

- The predicted WIMP relic abundance \( \Omega_h \chi^2 \) lies within the 3\( \sigma \) range of the Planck measurement \( \Omega_{DM} h^2 = 0.1200 \pm 0.0012 \) [60].

We project the selected parameter points onto the \( v_S-m_{h_2}, v_\Phi-m_{h_3}, v_\Phi-m_{Z'}, \) and \( m_{Z'}-|\xi| \) planes in Figs. 4(a), 4(b), 4(c), and 4(d), with color axes corresponding to \( \lambda_S, \lambda_\Phi, g_X, \) and \( |s_\varepsilon| \), respectively. Since we focus on the parameter region with \( v_\Phi \gg v, v_S, m_\chi \), the mass-squared matrix (6) implies \( m_{h_3}^2 \simeq \lambda_\Phi v_\Phi^2 \), which is clearly shown in Fig. 4(b). More precisely, this plot demonstrates that \( m_{h_3} \) is proportional to \( v_\Phi \) and positively correlated to \( \lambda_\Phi \). For \( |\lambda_{HS}| \ll 1 \), Eq. (6) leads to \( m_{h_2}^2 \simeq \lambda_S v_S^2 \). Nonetheless, such positive correlations of \( m_{h_2} \) to \( v_S \) and \( \lambda_S \) do not totally manifest in Fig. 4(a). The exceptions should be due to large \( |\lambda_{HS}| \).

According to Eqs. (18) and (20), \( v_\Phi, m_{Z'} \gg v, v_S \) means that \( m_{Z'} \simeq 2g_X v_\Phi/c_\varepsilon \) and \( t_\varepsilon \simeq -s_W t_\varepsilon m_{h_2}^2/m_{Z'}^2 \). Fig. 4(c) illustrates the positive correlations of \( m_{Z'} \) to \( v_\Phi \) and \( g_X \), while Fig. 4(d) displays the negative (positive) correlation of \( |\xi| \) to \( m_{Z'} (|s_\varepsilon|) \). From Figs. 4(b) and 4(c), we find that the lower limit of the UV scale \( v_\Phi \) is down to \( \sim 10^{10} \text{ GeV} \), given by the Fermi-LAT constraint on the WIMP lifetime.

In Fig. 5(a), we demonstrate the selected parameter points in the \( \Gamma_{h_1}-(1-\kappa_Z) \) plane, with colors denoting \( 1-U_{11} \). For \( v_\Phi, m_{Z'} \gg v, v_S \), Eq. (60) becomes \( \kappa_Z \simeq U_{11} = \kappa_W = \kappa_f \), and thus the color axis is basically identical to the vertical axis in Fig. 5(a). There is an obvious curve constituted by parameter points in this plot, indicating the positive correlation between the \( h_1 \) total decay width \( \Gamma_{h_1} \) and \( \kappa_Z \) (or \( U_{11} \), equivalently). The exceptional parameter points have larger \( \Gamma_{h_1} \), which are contributed by the exotic Higgs decays \( h_1 \to \chi\chi, h_1 \to \chi Z, \) and \( h_1 \to h_2h_2 \). We can see that the constraints from current LHC Higgs measurements give \( 1-\kappa_Z \gtrsim 0.1 \) (or \( U_{11} \gtrsim 0.9 \)) and 3.3 MeV \( \lesssim \Gamma_{h_1} \lesssim 5 \text{ MeV} \).

Future experiments at the planning Higgs factories, such as CEPC [61], FCC-ee [62], and ILC [63], would be rather sensitive to \( \kappa_Z, \kappa_W, \kappa_b, \) and \( \Gamma_{h_1} \). For instance, the 1\( \sigma \) precision of the CEPC measurements on these quantities are estimated to be \( \delta\kappa_Z = 0.25\%, \delta\kappa_W = 1.4\%, \delta\kappa_b = 1.3\%, \) and \( \delta\Gamma_{h_1} = 2.8\% \) [61]. Figure 5(a) also shows the expected 95\% C.L. coverage.
of the CEPC experiment on $\kappa_Z$ and $\Gamma_{h_1}$, and a large fraction of the selected parameter points would be properly tested.

Moreover, the CEPC project could also constrain the invisible Higgs decay branching ratio $\mathrm{BR}_{\text{inv}}$ down to 0.3% at 95% C.L. [61]. In this model, we have nonzero $\mathrm{BR}_{\text{inv}} = \Gamma(h_1 \to \chi\chi)/\Gamma_{h_1}$ for $m_\chi < m_{h_1}/2$. Figure 5(b) displays the selected parameter points projected onto the $m_\chi$-$\mathrm{BR}_{\text{inv}}$ plane. Current LHC data allow the parameter points with $\mathrm{BR}_{\text{inv}} \lesssim 14\%$, while the CEPC experiment could probe most of the parameter points with $m_\chi < m_{h_1}/2$.

In Fig. 6(a), the selected parameter points are presented in the $\Omega_{\chi}h^2\langle\sigma_{\text{ann}}v\rangle_{\text{FO}}$ plane, with a color axis indicating the pNGB WIMP mass $m_\chi$ and colored regions corresponding to the 1σ, 2σ, and 3σ ranges of the relic abundance $\Omega_{\text{DM}}h^2 = 0.1200 \pm 0.0012$ measured by the Planck experiment [60]. The majority of the parameter points gather around the standard annihilation cross section $\langle\sigma_{\text{ann}}v\rangle_{\text{FO}} \sim 2 \times 10^{-26} \text{ cm}^3/\text{s}$. The rest points with nonstandard freeze-out annihilation cross sections should arise from resonance or threshold effects of specific annihilation channels that leads to velocity-dependent cross sections [64].
FIG. 5. Selected parameter points projected onto the $\Gamma_{h_1} - (1 - \kappa_Z)$ (a) and $m_\chi$-BR$^{-\text{inv}}$ (b) planes, with color axes corresponding to $1 - U_{11}$ and $\Gamma_{h_1}$, respectively. The green regions denote the expected coverage of the future CEPC experiment at 95% C.L. [61].

FIG. 6. Selected parameter points projected onto the $\Omega_{h} h^2 - \langle \sigma_{\text{ann}} v \rangle_{\text{FO}}$ (a) and $m_\chi - \langle \sigma_{\text{ann}} v \rangle_{\text{D}}$ (b) planes, with color axes corresponding to $m_\chi$ and $\langle \sigma_{\text{ann}} v \rangle_{\text{D}} / \langle \sigma_{\text{ann}} v \rangle_{\text{FO}}$, respectively. The blue dot-dashed line and the colored regions in the left panel indicate the central value and the 1$\sigma$, 2$\sigma$, and 3$\sigma$ ranges of the Planck measured relic abundance $\Omega_{\text{DM}} h^2 = 0.1200 \pm 0.0012$ [60]. In the right panel, the green dashed line denotes the upper limits from the Fermi-MAGIC $\gamma$-ray observations of dwarf galaxies at 95% C.L. [59], while the blue dotted line corresponds to $m_\chi = m_{h_1}/2$.

The selected parameter points are further shown in the $m_\chi - \langle \sigma_{\text{ann}} v \rangle_{\text{D}}$ plane in Fig. 6(b), where the color axis denotes the ratio of $\langle \sigma_{\text{ann}} v \rangle_{\text{D}}$ to $\langle \sigma_{\text{ann}} v \rangle_{\text{FO}}$. $\langle \sigma_{\text{ann}} v \rangle_{\text{D}} \simeq \langle \sigma_{\text{ann}} v \rangle_{\text{FO}}$ means that $\chi\chi$ annihilation is $s$-wave dominated, corresponding to the standard case. The velocity dependence induced by the resonance or threshold effects would make $\langle \sigma_{\text{ann}} v \rangle_{\text{D}}$ different from $\langle \sigma_{\text{ann}} v \rangle_{\text{FO}}$. It is obvious that the parameter points with $\langle \sigma_{\text{ann}} v \rangle_{\text{D}} \neq \langle \sigma_{\text{ann}} v \rangle_{\text{FO}}$ around the $m_\chi = m_{h_1}/2$ line in Fig. 6(b) are caused by the $h_1$ resonance effect, while the $h_2$ resonance effect leads to $\langle \sigma_{\text{ann}} v \rangle_{\text{D}} \neq \langle \sigma_{\text{ann}} v \rangle_{\text{FO}}$ for some of the rest parameter points. The green dashed line in
Fig. 6(b) indicates the 95% C.L. upper limits of $\langle \sigma_{\text{ann}} v \rangle_D$ from the Fermi-MAGIC observations of dwarf galaxies assuming a $b\bar{b}$ annihilation channel. These limits can be approximately used to constrain the model. We find that only a small fraction of the selected parameter points have been excluded.

V. CONCLUSIONS AND DISCUSSIONS

In this paper, we have constructed a UV-complete model for pNGB dark matter with a hidden U(1)$_X$ gauge symmetry. Two complex scalar fields $S$ and $\Phi$ carrying U(1)$_X$ charges of 1 unit and 2 units are introduced. The development of the $\Phi$ VEV $v_\Phi$ at a high scale breaks the U(1)$_X$ gauge symmetry into an approximate U(1)$_X$ global symmetry, which is softly broken by the $\mu_{S\Phi}$ term, leading to the desired pNGB WIMP DM setup. As a result, the tree-level WIMP-nucleon scattering is suppressed by the UV scale $v_\Phi$. We have found a scaling relation $\sigma_{\chi N}^{SI} \propto v_\Phi^{-4}$, and hence $v_\Phi \gtrsim 10^{10}$ is high enough to escape direct detection.

Compared to the UV-completion with the U(1)$_{B-L}$ gauge symmetry \cite{17, 18}, our model do not need to introduce right-handed neutrinos for anomaly cancellation. Moreover, since the SM fermions do not carry U(1)$_X$ charges, the interactions leading to WIMP decays are reduced. Specifically, the interactions that induce WIMP decays only originate from the kinetic mixing between the U(1)$_X$ and U(1)$_Y$ gauge fields. This would relative relie the WIMP lifetime constraint on the UV scale $v_\Phi$.

A random scan in the parameter space has been carried out to obtain the parameter points satisfying the phenomenological constraints from the WIMP lifetime, the 125 GeV Higgs measurements, the observed DM relic abundance, and indirect detection of WIMP annihilation. We have found that the WIMP lifetime bound from the Fermi-LAT $\gamma$-ray observations has set a lower limit on the UV scale, $v_\Phi \gtrsim 10^{10}$ GeV, which is indeed looser than $v_\Phi \gtrsim O(10^{11}-10^{13})$ GeV in the U(1)$_{B-L}$ case estimated in Refs. \cite{17, 18}. The parameter points satisfying current LHC Higgs measurements have $U_{11} \gtrsim 0.9$, $3.3 \text{ MeV} \lesssim \Gamma_{h_1} \lesssim 5 \text{ MeV}$, and BR$_{\text{inv}} \lesssim 14\%$. A large fraction of these parameter points could be properly tested by future Higgs factories.

Additional constraints on this model come from direct searches for the $h_2$ boson at the 13 TeV LHC from decay channels such as $h_2 \to ZZ$ \cite{65, 66}, $h_2 \to W^+W^-$ \cite{67, 68}, $h_2 \to t\bar{t}$ \cite{69}, and $h_2 \to h_1h_1$ \cite{70}. By reinterpreting these constraints, some parameter points remaining in our scan may have been excluded. Nonetheless, the $h_2$ couplings to the $W$ and $Z$ bosons and to the top quark are highly suppressed by the mixing parameter $U_{12}$. Thus, we expect most of the parameter points are still available.

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