Dilaton–Axion Symmetry

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Abstract

The heterotic string compactified on a six-torus is described by a low-energy effective action consisting of N=4 supergravity coupled to N=4 super Yang-Mills, a theory that was studied in detail many years ago. By explicitly carrying out the dimensional reduction of the massless fields, we obtain the bosonic sector of this theory. In the Abelian case the action is written with manifest global $O(6,6+n)$ symmetry. A duality transformation that replaces the antisymmetric tensor field by an axion brings it to a form in which the axion and dilaton parametrize an $SL(2,R)/SO(2)$ coset, and the equations of motion have $SL(2,R)$ symmetry. This symmetry, which combines Peccei–Quinn translations with Montonen–Olive duality transformations, has been exploited in several recent papers to construct black hole solutions carrying both electric and magnetic charge. Our purpose is to explore whether, as various authors have conjectured, an $SL(2,Z)$ subgroup could be an exact symmetry of the full quantum string theory. If true, this would be of fundamental importance, since this group transforms the dilaton nonlinearly and can relate weak and strong coupling.

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1. Introduction

The unexpected appearance of noncompact global symmetries was one of the most intriguing discoveries to emerge from the study of supergravity theories in the 1970’s. The first appearance of a noncompact symmetry was the discovery of a global $SU(1,1)$ invariance in an appropriate formulation of $N = 4$, $D = 4$ supergravity.\(^1\)

The qualification “appropriate formulation” refers to the fact that duality transformations allow $n$-forms to be recast as $(D - n - 2)$-forms in $D$ dimensions ($d\tilde{A} = *dA$), interchanging the role of Bianchi identities and equations of motion. Only after appropriate transformations is the full noncompact symmetry exhibited. In the $SU(1,1)$ theory there are two scalar fields, nowadays called the “dilaton” and the “axion”, which parametrize the coset space $SU(1,1)/U(1)$.

We shall also focus on theories with half the maximum possible supersymmetry ($N = 1$ in $D = 10$ or $N = 4$ in $D = 4$), as these are most relevant to heterotic string theory. Ref. 2 showed that 10-dimensional $N = 1$ supergravity, dimensionally reduced to $D \geq 4$ dimensions (by dropping the dependence of the fields on $d = 10 - D$ dimensions), has global $O(d,d)$ symmetry. Moreover, when the original $N = 1$, $D = 10$ theory has $n$ Abelian vector supermultiplets, in addition to the supergravity multiplet, the global symmetry of the dimensionally reduced theory becomes extended to $O(d,d+n)$. The coupling of $N = 1$, $D = 10$ supergravity to vector supermultiplets require the inclusion of a Chern–Simons term ($H = dB - \omega_3$) in order to achieve supersymmetry. This was shown in the Abelian case by Bergshoeff et al.\(^3\) and in the non-Abelian case by Chapline and Manton.\(^4\)

In this paper we will focus on the bosonic sector, which can be formulated in any dimension. In section 2 we show that dimensional reduction from $D + d$ dimensions to $D$ dimensions gives rise to a theory with global $O(d,d)$ symmetry when there are no vector fields in $D + d$ dimensions. Adding $n$ Abelian vector fields in $D + d$ dimensions gives rise to a dimensionally reduced theory with $O(d,d+n)$ symmetry, provided that the Chern–Simons term (described above) is included. This construction has been discussed in detail elsewhere\(^5\) and is summarized here to present the theory that is
studied in the subsequent sections.

In section 3 we specialize to the $D = 4$ case and review the duality transformation that replaces the antisymmetric two-form field by the axion and gives rise to global $SL(2, R)$ (or $SU(1, 1)$) symmetry of the equations of motion. The symmetry is not present in the action for those terms involving the vector gauge fields. In section 4 we explore whether this symmetry could be generally valid in the heterotic string theory compactified to four dimensions, or whether it is a special feature of the low-energy effective action. The question is both subtle and profound, because the symmetry gives a nonlinear transformation of the the dilaton, whose expectation value gives the coupling constant (loop expansion parameter). Thus, even if the symmetry is exact, one should not expect to find it order-by-order in perturbation theory. By the same token, the question is certainly of fundamental importance, since such a symmetry is potentially a powerful tool for obtaining non-perturbative information about the theory. We examine the classical string equations of motion in the presence of appropriate background fields and demonstrate that the linearly realized subgroup of $SL(2, R)$ is a symmetry, but the full group is not. However, this is all the symmetry that should appear at this order, so the question remains open.

2. Noncompact Global Symmetry from Dimensional Reduction

In the 1970’s it was noted that noncompact global symmetries are a generic feature of supergravity theories containing scalar fields. One of the useful techniques that was exploited in these studies was the method of “dimensional reduction.” In its simplest form, this consists of considering a theory in a spacetime $M \times K$, where $M$ has $D$ dimensions and $K$ has $d$ dimensions, and supposing that the fields are independent of the coordinates $y^\alpha$ of $K$. For this to be a consistent procedure it is necessary that $K$-independent solutions be able to solve the classical field equations. Then one speaks of “spontaneous compactification” (at least when $K$ is compact). In a gravity theory this implies that $K$ is flat, a torus for example. Of course, in recent times more interesting possibilities, such as Calabi–Yau spaces, have received a great deal of attention. In such a case, the analog of dropping $y$ dependence is to truncate
all fields to their zero modes on $K$.

Explicit formulas for dimensional reduction were given in a 1979 paper by Joël Scherk and me$^6$ and subsequently explored by Cremmer.$^7$ The main purpose of ref. [6] was to introduce a “generalized” method of dimensional reduction, but here we will stick to the simplest case in which the fields are taken to be independent of the $K$ coordinates.$^*$. The notation is as follows: Local coordinates of $M$ are $x^\mu$ ($\mu = 0, 1, \ldots, D-1$) and local coordinates of $K$ are $y^\alpha$ ($\alpha = 1, \ldots, d$). The tangent space Lorentz metric has signature ($- + \ldots +$). All fields in $D+d$ dimensions are written with hats on the fields and the indices ($\hat{\phi}$, $\hat{g}_{\mu\nu}$, etc.). Quantities without hats are reserved for $D$ dimensions. Thus, for example, the Einstein action on $M\times K$ (with a dilaton field $\hat{\phi}$) is

$$S_{\hat{g}} = \int_{M} dx \int_{K} dy \sqrt{-\hat{g}} e^{-\hat{\phi}} \left[ \hat{R}(\hat{g}) + \hat{g}^{\hat{\mu}\hat{\nu}} \partial_{\hat{\mu}} \hat{\phi} \partial_{\hat{\nu}} \hat{\phi} \right]$$

(1)

If $K$ is assumed to be a torus we can choose the coordinates $y^\alpha$ to be periodic with unit periods, so that $\int_{K} dy = 1$. The radii and angles that characterize the torus are then encoded in the metric tensor. In terms of a $(D+d)$-dimensional vielbein, we can use local Lorentz invariance to choose a triangular parametrization

$$\hat{e}_{\hat{r}}^\mu = \begin{pmatrix} e^r_{\mu} & A^{(1)}_{\mu\beta} E^\alpha_{\beta} \\ 0 & E^\alpha_{\alpha} \end{pmatrix}.$$  

(2)

The “internal” metric is $G_{\alpha\beta} = E^a_{\alpha} \delta_{ab} E^b_{\beta}$ and the “spacetime” metric is $g_{\mu\nu} = e^r_{\mu} \eta_{rs} e^s_{\nu}$. As usual, $G^{\alpha\beta}$ and $g^{\mu\nu}$ represent inverses. In terms of these quantities the complete $(D+d)$-dimensional metric is

$$\hat{g}_{\hat{\mu}\hat{\nu}} = \begin{pmatrix} g_{\mu\nu} + A^{(1)}_{\mu\gamma} G^{\gamma\delta} A^{(1)}_{\nu\delta} \\ A^{(1)}_{\nu\alpha} \\ A^{(1)}_{\nu\alpha} \\ \frac{1}{G_{\alpha\beta}} \end{pmatrix}.$$  

(3)

A convenient property of this parametrization is that $\sqrt{-\hat{g}} = \sqrt{-g}\sqrt{\det G}$. If all fields

$^*$ For a discussion of the application of the generalized method to supersymmetry breaking in string theory see ref. [8].
are assumed to be \( y \) independent, one finds the \( D \)-dimensional action

\[
S_\hat{g} = \int_M dx \sqrt{-g} \ e^{-\phi} \left\{ R + g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi \right. \\
+ \frac{1}{4} g^{\mu \nu} \partial_\mu G_{\alpha \beta} \partial_\nu G^{\alpha \beta} - \frac{1}{4} g^{\mu \rho} g^{\nu \lambda} G_{\alpha \beta} F_{\mu \nu}^{(1) \alpha} F_{\rho \lambda}^{(1) \beta} \left. \right\},
\]

(4)

where we have introduced a shifted dilaton field\(^{9,10} \phi = \hat{\phi} - \frac{1}{2} \log \det G_{\alpha \beta} \) and \( F_{\mu \nu}^{(1) \alpha} = \partial_\mu A_{\nu}^{(1) \alpha} - \partial_\nu A_{\mu}^{(1) \alpha} \).

Another field that is of interest in string theory is a second-rank antisymmetric tensor \( \hat{B}_{\hat{\mu} \hat{\nu}} \) with field strength \( \hat{H}_{\hat{\mu} \hat{\nu} \hat{\rho}} = \partial_\hat{\mu} \hat{B}_{\hat{\nu} \hat{\rho}} + \text{cyc. perm.} \). The Chern–Simons terms that appear in superstring theory are not present here, since we are not yet including \((D + d)\)-dimensional vector fields. The Lorentz Chern–Simons term\(^{11} \) is higher order in derivatives than we are considering. The action for the \( \hat{B} \) term is

\[
S_{\hat{B}} = -\frac{1}{12} \int_M dx \int_K dy \sqrt{-\hat{g}} \ e^{-\hat{\phi}} \hat{g}^{\hat{\mu} \hat{\nu} \hat{\rho}} \hat{g}^{\hat{\rho} \hat{\nu} \hat{\rho}} \hat{H}_{\hat{\mu} \hat{\nu} \hat{\rho}} \hat{H}_{\hat{\mu} \hat{\nu} \hat{\rho}}.
\]

(5)

Again dropping \( y \) dependence, one finds that

\[
S_{\hat{B}} = -\int_M dx \sqrt{-g} \ e^{-\phi} \left\{ \frac{1}{4} H_{\mu \alpha \beta} H^{\mu \alpha \beta} + \frac{1}{4} H_{\mu \nu \alpha} H^{\mu \nu \alpha} + \frac{1}{12} H_{\mu \nu \rho} H^{\mu \nu \rho} \right\},
\]

(6)

where \( H_{\mu \alpha \beta} = \partial_\mu B_{\alpha \beta} \) and \( H_{\mu \nu \alpha} = F_{\mu \nu \alpha}^{(2)} - B_{\alpha \beta} F_{\mu \nu}^{(1) \beta} \). Also, \( \hat{B}_{\alpha \beta} = B_{\alpha \beta} \), \( F_{\mu \nu \alpha}^{(2)} = \partial_\mu A_{\nu \alpha}^{(2)} - \partial_\nu A_{\mu \alpha}^{(2)} \), and \( A_{\mu \alpha}^{(2)} = \hat{B}_{\mu \alpha} + B_{\alpha \beta} A_{\mu \beta}^{(1) \alpha} \). The gauge transformations of the vector fields are simply \( \delta A_{\mu}^{(1) \alpha} = \partial_\mu \Lambda_{(1) \alpha} \) and \( \delta A_{\mu \alpha}^{(2)} = \partial_\mu \Lambda_{\alpha}^{(2)} \), under which \( H_{\mu \nu \alpha} \) is invariant. Also,

\[
H_{\mu \nu \rho} = \partial_\mu B_{\nu \rho} - \frac{1}{2} (A_{\mu}^{(1) \alpha} F_{\nu \rho \alpha}^{(2)} + A_{\mu \alpha}^{(2)} F_{\nu \rho}^{(1) \alpha}) + \text{cyc. perm.},
\]

(7)

where

\[
B_{\mu \nu} = \hat{B}_{\mu \nu} + \frac{1}{2} A_{\mu}^{(1) \alpha} A_{\nu \alpha}^{(2)} - \frac{1}{2} A_{\nu}^{(1) \alpha} A_{\mu \alpha}^{(2)} - A_{\mu}^{(1) \alpha} B_{\alpha \beta} A_{\nu}^{(1) \beta}.
\]

(8)

In this case gauge invariance of eq. (7) requires that under the \( \Lambda^{(1)} \) and \( \Lambda^{(2)} \) transformations \( \delta B_{\mu \nu} = \frac{1}{2} (\Lambda^{(1) \alpha} F_{\mu \nu \alpha}^{(2)} + \Lambda_{\alpha}^{(2)} F_{\mu \nu}^{(1) \alpha}) \). The extra terms in \( H_{\mu \nu \rho} \) have
arisen as a consequence of the dimensional reduction, are Abelian Chern–Simons terms.

To recapitulate the results so far, the dimensionally reduced form of $S = S_\gamma + S_B$ has been written in the form

$$S = \int_M dx \sqrt{-g} \ e^{-\phi} (\mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4) ,$$

where

$$\mathcal{L}_1 = R + g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

$$\mathcal{L}_2 = \frac{1}{4} g^{\mu\nu} (\partial_\mu G_{\alpha\beta} \partial_\nu G^{\alpha\beta} - G^{\gamma\delta} \partial_\mu B_{\alpha\gamma} \partial_\nu B_{\beta\delta})$$

$$\mathcal{L}_3 = \frac{1}{4} g^{\mu\rho} g^{\nu\lambda} (G_{\alpha\beta} F_{\mu\nu}^{(1)\alpha} F_{\rho\lambda}^{(1)\beta} + G^{\alpha\beta} H_{\mu\nu\alpha} H_{\rho\lambda\beta})$$

$$\mathcal{L}_4 = -\frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} .$$

We now claim that there is an $O(d, d)$ global symmetry that leaves each of these four terms separately invariant. The first term ($\mathcal{L}_1$) is trivially invariant since $g_{\mu\nu}$ and $\phi$ are. It should be noted, however, that the individual terms in $\phi = \hat{\phi} - \frac{1}{2} \log \det G_{\alpha\beta}$ are not invariant. To investigate the invariance of $\mathcal{L}_2$ we first rewrite it, using matrix notation, as

$$\mathcal{L}_2 = \frac{1}{4} \text{tr}(\partial_\mu G^{-1} \partial^\mu G + G^{-1} \partial_\mu B G^{-1} \partial^\mu B) .$$

Then we introduce two $2d \times 2d$ matrices, written in $d \times d$ blocks, as follows:¹³

$$\eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad M = \begin{pmatrix} G^{-1} & -G^{-1} B \\ B G^{-1} & G - B G^{-1} B \end{pmatrix}$$

Since $\eta$ has $d$ eigenvalues $+1$ and $d$ eigenvalues $-1$, it is a metric for the group $O(d, d)$ in a basis rotated from the one with a diagonal metric. Next we note that $M \in O(d, d)$, since $M^T \eta M = \eta$. In fact, $M$ is a symmetric $O(d, d)$ matrix, which
implies that \( M^{-1} = \eta M \eta \). It is now a simple exercise to verify that

\[
\mathcal{L}_2 = \frac{1}{8} \mathrm{tr}(\partial_{\mu} M^{-1} \partial^{\mu} M). \tag{13}
\]

Thus \( \mathcal{L}_2 \) is invariant under a global \( O(d, d) \) transformation \( M \rightarrow \Omega M \Omega^T \), where \( \Omega^T \eta \Omega = \eta \). This transformation acts nonlinearly on \( G_{\alpha \beta} \) and \( B_{\alpha \beta} \), which parametrize the coset space \( O(d, d)/O(d) \times O(d) \).

Next we consider the \( \mathcal{L}_3 \) term:

\[
\mathcal{L}_3 = -\frac{1}{4} \left[ F^{(1)\alpha}_{\mu \nu} G_{\alpha \beta} F^{(1)\mu \nu \beta} + (F^{(2)}_{\mu \nu \alpha} - B_{\alpha \gamma} F^{(1)\gamma \mu \nu}) G^{\alpha \beta} (F^{(2)\beta \mu \nu} - B_{\beta \delta} F^{(1)\mu \nu \delta}) \right]
\]

\[
= -\frac{1}{4} F^i_{\mu \nu} (M^{-1})_{ij} F^{j \mu \nu}, \tag{14}
\]

where \( F^i_{\mu \nu} \) is the 2d-component vector of field strengths

\[
F^i_{\mu \nu} = \left( \begin{array}{c} F^{(1)\alpha}_{\mu \nu} \\ F^{(2)}_{\mu \nu \alpha} \end{array} \right) = \partial_{\mu} A^i_{\nu} - \partial_{\nu} A^i_{\mu}. \tag{15}
\]

Thus \( \mathcal{L}_3 \) is manifestly \( O(d, d) \) invariant provided that the vector fields transform according to the vector representation of \( O(d, d) \), i.e., \( A^i_{\mu} \rightarrow \Omega^i_j A^j_{\mu} \). The demonstration of \( O(d, d) \) symmetry is completed by noting that \( \mathcal{L}_4 \) is invariant (if we require that \( B_{\nu \rho} \) is invariant), since \( H_{\mu \nu \rho} \) can be written in the manifestly invariant form

\[
H_{\mu \nu \rho} = \partial_{\mu} B_{\nu \rho} - \frac{1}{2} A^i_{\mu} \eta_{ij} F^j_{\nu \rho} + (\text{cyc. perms.}) \tag{16}
\]

Previous work in supergravity\textsuperscript{14,15} and superstring theory\textsuperscript{16} suggests that if we add \( n \) Abelian \( U(1) \) gauge fields to the original \((D + d)\)-dimensional theory, that \( O(d, d + n) \) symmetry should result from dimensional reduction to \( D \) dimensions. The additional term to be added to the action is

\[
S_{\hat{A}} = -\frac{1}{4} \int_{M} dx \int_{\mathcal{K}} dy \sqrt{-\hat{g}} \, e^{-\phi} \hat{g}^{\hat{\mu} \hat{\nu}} \hat{g}^{\hat{\rho} \hat{\lambda}} \delta_{IJ} \hat{F}^I_{\hat{\mu} \hat{\nu}} \hat{F}^J_{\hat{\rho} \hat{\lambda}}, \tag{17}
\]

where \( \hat{F}^I_{\hat{\mu} \hat{\nu}} = \partial_{\hat{\mu}} \hat{A}^I_{\hat{\nu}} - \partial_{\hat{\nu}} \hat{A}^I_{\hat{\mu}} \) and the index \( I \) takes the values \( I = 1, 2, \cdots, n \). The most important point to note is that the original \((D + d)\)-dimensional theory should
have $O(n)$ symmetry with $M_{IJ} = \eta_{IJ} = \delta_{IJ}$. Looking at the various pieces of the Lagrangian, we see that $L_1$ has the usual form, $L_2 = 0$, and $L_3$ gives $S_\hat{A}$. The crucial observation concerns $L_4$, which is built from the square of $\hat{H}_\mu \hat{H}_\nu$. This contains the Chern–Simons term (for the $U(1)$ gauge fields), a feature that is clearly crucial for the symmetries we wish to implement.

The dimensional reduction of $S_\hat{B}$ is unchanged from before. For the vectors we obtain

$$S_\hat{A} = -\frac{1}{4} \int dx \sqrt{-g} \, e^{-\phi} \left\{ F^{I}_\mu F^{I\mu} + 2 F^{I}_\mu F^{I\mu} \right\} , \quad (18)$$

where we define

$$A^{(3)I}_\mu = \hat{A}_\mu - a^I_\alpha A^{(1)\alpha}_\mu$$

$$F^{(3)I}_\mu = \partial_\mu A^{(3)I} - \partial_\nu A^{(3)I}$$

$$a^I_\alpha = \hat{A}_I$$

$$F^{(1)I}_\mu = F^{(3)I}_\mu + F^{(1)I}_\mu a^I_\alpha$$

$$F^{(3)I}_\mu = \partial_\mu a^I_\alpha . \quad (19)$$

The reduction of $S_\hat{B}$ is still given by eq. (6), but including the Chern–Simons term gives

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} - \frac{1}{2} A^I_\mu \eta_{ij} F^j_{\nu\rho} + \text{cyc. perms.} , \quad (20)$$

where we have used the definitions

$$A^{(2)}_{\mu\alpha} = \hat{B}_{\mu\alpha} + B_{\alpha\beta} A^{(1)\beta}_\mu + \frac{1}{2} a^I_\alpha A^{(3)I}_\mu \quad (21)$$

$$C_{\alpha\beta} = \frac{1}{2} a^I_\alpha a^I_\beta + B_{\alpha\beta} . \quad (22)$$

and $A^{(3)I}_\mu A^{(1)\alpha}_\nu a^I_\alpha - A^{(3)I}_\mu A^{(1)\alpha}_\nu a^I_\alpha$ should be added to the definition of $B_{\mu\nu}$ in eq. (8). We have introduced a $(2d + n)$-component vectors $\mathcal{A}^I_\mu = (A^{(1)\alpha}_\mu, A^{(2)}_{\mu\alpha}, A^{(3)I}_\mu)$ and
\[ F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu. \] The \( O(d, d+n) \) metric \( \eta \), written in blocks, now takes the form

\[ \eta = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \] (23)

With these definitions, \( H_{\mu\nu\rho} \) has manifest \( O(d, d+n) \) symmetry.

Now we can combine all terms that are quadratic in field strengths in the form

\[ \mathcal{L}_3 = -\frac{1}{4} F^i_{\mu\nu} (M^{-1})_{ij} F^{j\mu\nu}. \] (24)

Contributions come from \( S_\tilde{g} \) (as before), from \( \frac{1}{4} F^I_{\mu\nu} F^{I\mu\nu} \), and from \( \frac{1}{4} H_{\mu\nu\alpha} H^{\mu\nu\alpha} \). Altogether, we read off the result

\[ M^{-1} = \begin{pmatrix} G + C^T G^{-1} C + a^T a & -C^T G^{-1} & C^T G^{-1} a^T + a^T \\ -G^{-1} C & G^{-1} & -G^{-1} a^T \\ a G^{-1} C + a & -a G^{-1} & 1 + a G^{-1} a^T \end{pmatrix} \] (25)

Since \( M^{-1} \eta M^{-1} \eta = 1 \), \( M^{-1} \) and \( M = \eta M^{-1} \eta \) are symmetric \( O(d, d+n) \) matrices.

The last remaining check of \( O(d, d+n) \) symmetry is to verify that we recover \( \mathcal{L}_2 = \frac{1}{8} \text{tr}(\partial_\mu M^{-1} \partial^\mu M) \), with the matrix \( M \) given above. Relevant contributions come from \( S_{\tilde{g}} \), \( -\frac{1}{2} (F^I_{\mu\alpha})^2 \), and \( -\frac{1}{4} (H_{\mu\alpha\beta})^2 \). The calculation is a bit tedious, but the desired result is obtained.

3. \( SL(2, R) \) Symmetry in Four Dimensions

In four-dimensional supersymmetric models arising from string theory, it is well known that the dilaton and axion belong to the same chiral supermultiplet and can be described by a coset construction based on \( SL(2, R) \). The same coset construction turns out to be true for the class of models under consideration here, even though no supersymmetry is assumed (just as with the Chern–Simons terms). As we will see, the symmetry is realized on the equations of motion, but not on the action.
This symmetry first appeared in the “$SU(4)$ formulation” of N=4 supergravity, and it was extended to include the coupling to $n$ Abelian vector supermultiplets by de Roo. He showed that the global $O(6, 6 + n)$ symmetry is present off shell (i.e., as a symmetry of the action), whereas the $SU(1, 1)$ symmetry is only present on shell (i.e., as a symmetry of the equations of motion). The extension to non-Abelian Yang–Mills supermultiplets has also been investigated. These authors found that the non-Abelian gauge interactions cause the $SU(1, 1)$ symmetry to be broken, even on shell. The present analysis confirms these results for the bosonic portions of the theories. From the supergravity studies, we know that there are no surprises when the fermions are added. In the next section we will discuss whether higher-order string effects destroy the symmetry in the Abelian case, or (more optimistically) whether they could restore it in the non-Abelian case.

We begin by setting $D = 4$ and performing a Weyl rescaling that brings the Einstein term to canonical form (let $g_{\mu\nu} = e^\phi g'_{\mu\nu}$ and drop the prime). Then the action becomes

$$S^{(4)} = \int_M dx \sqrt{-g} \left\{ R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \mathcal{L}_2 + e^{-\phi} \mathcal{L}_3 + e^{-2\phi} \mathcal{L}_4 \right\}, \quad (26)$$

with $\mathcal{L}_2$, $\mathcal{L}_3$, and $\mathcal{L}_4$ as defined in the preceding section.

The next step is to perform a duality transformation, which replaces the field $B_{\mu\nu}$ by a scalar field $\chi$. This is achieved by first forming the $B_{\mu\nu}$ equation of motion

$$\partial_\mu (\sqrt{-g} e^{-2\phi} H^{\mu\nu\rho}) = 0 , \quad (27)$$

and solving it by setting

$$\sqrt{-g} e^{-2\phi} H^{\mu\nu\rho} = \gamma \epsilon^{\mu\nu\rho\lambda} \partial_\lambda \chi , \quad (28)$$

where $\chi$ is the “axion” and $\gamma$ is a constant to be fixed later. In the language of
differential forms,

\[ H = \gamma e^{2\phi} * d\chi \tag{29} \]

or, using \( H = dB - \frac{1}{2} \eta_{ij} A^i A_j \),

\[ dB = \frac{1}{2} \eta_{ij} A_i A^j + \gamma e^{2\phi} * d\chi . \tag{30} \]

The Bianchi identity \( (d^2 B = 0) \) now turns into the \( \chi \) field equation

\[ \frac{1}{2} \eta_{ij} F_i A^j + \gamma d(e^{2\phi} * d\chi) = 0 , \tag{31} \]

or, in terms of components, (choosing a convenient value for \( \gamma \))

\[ \partial_\mu (e^{2\phi} \sqrt{-g} g^{\mu\nu} \partial_\nu \chi) - \frac{1}{8} \eta_{ij} \epsilon^{\mu\nu\rho\lambda} F_i A^i \chi A_j \rho \lambda = 0 . \tag{32} \]

This is an equation of motion if we replace the \( L_4 \) term in \( S^{(4)} \) by

\[ S_\chi = - \int_M dx \sqrt{-g} \left( \frac{1}{2} e^{2\phi} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi + \frac{1}{4} \chi F \cdot \tilde{F} \right) , \tag{33} \]

where

\[ F \cdot \tilde{F} \equiv \frac{1}{2\sqrt{-g}} \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} \eta_{ij} F_{i j} \rho \lambda . \tag{34} \]

Let us now regroup the terms in the dual action in the following way:

\[ \tilde{S}^{(4)} = \int_M dx \sqrt{-g} (R + L_2) + S_D + S_F , \tag{35} \]

where

\[ S_D = -\frac{1}{2} \int_M dx \sqrt{-g} (\partial_\mu \phi \partial_\nu \phi + e^{2\phi} \partial_\mu \chi \partial_\nu \chi) \]
\[ S_F = -\frac{1}{4} \int_M dx \sqrt{-g} \left( e^{-\phi} F^2 + \chi F \cdot \tilde{F} \right) \]
\[ F^2 \equiv g^{\mu\rho} g^{\nu\lambda} F_{\mu\nu} (M^{-1})_{ij} \chi A_j ^{i j} \rho \lambda . \tag{36} \]

The claim now is that \( S_D \) is given by a \( SL(2, R)/SO(2) \) coset construction. Start-
ing with the $SL(2, R)$ matrix

$$
T = \begin{pmatrix}
e^{-\phi/2} & 0 \\
e^{\phi/2} & e^{\phi/2}
\end{pmatrix},
$$

(37)

the idea is that under a global $SL(2, R)$ transformation $M$ and a local $SO(2)$ transform-
tion $A$, $T \rightarrow ATM^T$. For a given $M$, one can always choose $A$ to preserve the
triangular form of $T$. We compute the symmetric $SL(2, R)$ “metric”

$$
S = T^T T = \begin{pmatrix}
e^{-\phi} + e^{\phi} \chi^2 & e^{\phi} \chi \\
e^{\phi} \chi & e^{\phi}
\end{pmatrix}.
$$

(38)

Then one finds that

$$
\text{tr}(\partial_\mu S \partial_\nu S^{-1}) = -2(\partial_\mu \phi \partial_\nu \phi + e^{2\phi} \partial_\mu \chi \partial_\nu \chi). 
$$

(39)

Therefore we learn that

$$
S_D = \frac{1}{4} \int_M dx \sqrt{-g} \ g^{\mu\nu} \text{tr}(\partial_\mu S \partial_\nu S^{-1}),
$$

(40)

showing that $\phi$ and $\chi$ parametrize the coset $SL(2, R)/SO(2)$.

Another way of describing the $SL(2, R)$ symmetry of the dilaton and axion kinetic
terms is to introduce a complex modular parameter

$$
\tau = \chi + ie^{-\phi},
$$

(41)

which has the nice property that under a linear fractional transformation

$$
\tau \rightarrow \frac{a\tau + b}{c\tau + d}
$$

(42)

the combination

$$
g^{\mu\nu} \partial_\mu \tau \partial_\nu \bar{\tau} \left(\text{Im } \tau\right)^2 = g^{\mu\nu}(\partial_\mu \phi \partial_\nu \phi + e^{2\phi} \partial_\mu \chi \partial_\nu \chi)
$$

(43)

is invariant. (In $N = 1$ supersymmetry models one often introduces a chiral superfield
\[ S, \text{ whose bosonic part is } i\tau. \] It follows that

\[ S_D = -\frac{1}{2} \int_M dx \sqrt{-g} \frac{g^{\mu\nu} \partial_\mu \tau \partial_\nu \bar{\tau}}{(\text{Im } \tau)^2}. \tag{44} \]

Let us now turn to the last remaining piece of the theory, namely \( S_F \), the terms that depend on the gauge fields. This part of the story is of particular interest, because the \( SL(2, R) \) transformations give rise to an electric-magnetic duality rotation. This fact has been exploited in a number of recent works, which construct black hole solutions with both electric and magnetic charge by applying \( SL(2, R) \) transformations to known solutions with electric or magnetic charge only.\(^{18,19,20}\) (Note that there are no fields in the theory that carry electric or magnetic charge. Indeed, it is not known how to maintain \( SL(2, R) \) symmetry when such fields are added. Still, one can construct charged black hole solutions.)

To see how the \( SL(2, R) \) symmetry works for \( S_F \), we define

\[ F_{\mu\nu}^\pm = M \eta F_{\mu\nu} \pm i \tilde{F}_{\mu\nu}. \tag{45} \]

Then, using the identity \( F^{\mu\nu}_{\mu\nu} M^{-1} F^\pm_{\mu\nu} = 0 \), we can rewrite \( S_F \) in the form

\[ S_F = -\frac{1}{16i} \int_M dx \sqrt{-g} \left( \tau F^{\mu\nu} M^{-1} F^+_{\mu\nu} - \bar{\tau} F^{-\mu\nu} M^{-1} F^-_{\mu\nu} \right). \tag{46} \]

The \( A_\mu \) equation of motion is

\[ \nabla^\mu \left( \tau F^+_{\mu\nu} - \bar{\tau} F^-_{\mu\nu} \right) = 0 \tag{47} \]

* The \( SU(1, 1) \) formulation of refs. \([15, 17]\) is obtained by the change of variables \( Z = \frac{\tau - i}{\tau + i} \), which maps the upper half plane to the unit disk. In this formulation

\[ S_D = -2 \int_M dx \sqrt{-g} \frac{g^{\mu\nu} \partial_\mu Z \partial_\nu \bar{Z}}{(1 - |Z|^2)^2}. \]
and the Bianchi identity is
\[ \nabla^\mu (F^{+\mu \nu} - F^{-\mu \nu}) = 0. \] (48)

To exhibit \( SL(2, R) \) symmetry it is necessary to have \( A_\mu \) transform at the same time as \( \tau \). The appropriate choice is to require that \( F^{\pm \mu \nu} \) transform as modular forms as follows

\[ F^{+\mu \nu} \rightarrow (c\tau + d)F^{+\mu \nu}, \quad F^{-\mu \nu} \rightarrow (c\bar{\tau} + d)F^{-\mu \nu}. \] (49)

This implies that

\[ \tau F^{+\mu \nu} \rightarrow (a\tau + b)F^{+\mu \nu}, \quad \bar{\tau} F^{-\mu \nu} \rightarrow (a\bar{\tau} + b)F^{-\mu \nu}. \] (50)

Thus the equation of motion (47) and the Bianchi identity (48) transform into linear combinations of one another and are preserved. In particular, the negative of the unit matrix sends \( F^{\pm \mu \nu} \rightarrow -F^{\pm \mu \nu} \). This result is acceptable if we identify the symmetry as \( SL(2, R) \), not just \( PSL(2, R) = SL(2, R)/Z_2 \). Note that \( SL(2, R) \) is not a symmetry of the action. The transformation in (49) is a nonlocal transformation of \( A_\mu \), and such transformations can do strange things to the action. For example, the total derivative \( F \cdot \tilde{F} \) transforms into an expression that is not a total derivative.

To complete the demonstration of \( SL(2, R) \) symmetry one should also examine the equations of motion of the other fields in the theory. Each of them works nicely, as has been amply discussed by previous authors. For example, in forming the Einstein equation one needs to show that the contribution of \( S_F \) to the energy–momentum tensor is \( SL(2, R) \) invariant. After a short calculation one finds that only terms of the structure \( e^{-\phi} F^+ F^- \) survive, and these are invariant since \( e^{-\phi} \rightarrow |c\tau + d|^{-2} e^{-\phi} \).

At this point we can note the problem that arises when one attempts to generalize the discussion to allow non-Abelian gauge field interactions. In this case the divergences in eqs. (47) and (48) become covariant derivatives involving the vector potentials. Since they undergo horrible non-linear transformations, implied by eq. (49) it is quite clear that the Bianchi identity and the equation of motion can no longer be preserved.
To recapitulate, the $SL(2, R)$ symmetry described in this section differs from the noncompact symmetries in section 2 in several respects: 1) It is special to four dimensions. More precisely, it is compatible with Lorentz invariance in four dimensions. (It might be present in higher dimensions.) 2) It is realized in terms of nonlocal field transformations. 3) The coupling strength $<e^\phi>$ is involved nonlinearly in the transformations. 4) It is destroyed by non-Abelian gauge field interactions.

4. Could SL(2,R) Be a Symmetry of Heterotic String Theory?

It is an important question whether $SL(2, R)$, or at least an $SL(2, Z)$ subgroup, is a symmetry of string theory, as has been conjectured by Font et al.\textsuperscript{21} and emphasized once again in the recent work of Sen.\textsuperscript{20} Sen has argued that since the effective field theory admits string-like solutions, whose zero modes correspond to the dynamical degrees of freedom of four-dimensional heterotic strings,\textsuperscript{22} the symmetry of the effective field theory might carry over to the full heterotic string theory. This reasoning is analogous to that introduced long ago by Montonen and Olive.\textsuperscript{23} Indeed the $SL(2, Z)$ transformation $\tau \rightarrow -1/\tau$, evaluated at $\chi = 0$, corresponds to $e^\phi \rightarrow e^{-\phi}$ and hence $\kappa \rightarrow 1/\kappa$, which is Montonen–Olive duality. Indeed, their reasoning is most compelling in the context of N=4 super Yang–Mills theories.\textsuperscript{24} The Montonen–Olive duality transformation exchanges elementary fields with monopoles. The $SL(2, Z)$ symmetry under consideration is precisely that duality combined with Peccei–Quinn symmetry and generalized to the supergravity and string contexts.

In order to investigate this question further, let us examine the equations for strings propagating in the presence of background fields satisfying the equations of the previous sections. These equations were derived in ref. [5], and so we simply sketch the derivation here. The $D + d$ string coordinates $X^\mu(\sigma, \tau)$ decompose into two sets $\{X^\mu\}$ and $\{Y^\alpha\}$ where $\mu = 0, 1, \ldots, D - 1$ and $\alpha = 1, 2, \ldots, d$. In order to make contact with the low-energy theory of the preceding sections, we consider $(D + d)$-dimensional massless background fields $\hat{g}_{\mu\nu}$ and $\hat{B}_{\mu\nu}$ that depend only on the
The world sheet action is
\[ S = \frac{1}{2} \int d^2 \sigma (\hat{g}_{\hat{\mu} \hat{\nu}} \eta^{ab} + \hat{B}_{\hat{\mu} \hat{\nu}} \epsilon^{ab}) \partial_a \hat{X}^\hat{\mu} \partial_b \hat{X}^\hat{\nu}. \]  

(51)

Even though this is a bosonic string action, it is also the relevant part of the heterotic string action, as well. Varying \( S \) with respect to \( \hat{X}^\hat{\mu}(\sigma, \tau) \) gives the classical equation of motion for the string
\[
\frac{\delta S}{\delta \hat{X}^\mu} = -\hat{\Gamma}_{\hat{\mu} \hat{\nu} \hat{\rho}} \partial_a \hat{X}^\hat{\nu} \partial_a \hat{X}^\hat{\rho} - \hat{g}_{\hat{\mu} \hat{\nu}} \partial_a \hat{X}^\hat{\nu} \partial_a \hat{X}^\hat{\mu} + \frac{1}{2} \epsilon^{ab} (\partial_a \hat{B}_{\hat{\nu} \hat{\rho}} + \partial_{\hat{\nu}} \hat{B}_{\hat{\rho} \hat{\mu}} + \partial_{\hat{\rho}} \hat{B}_{\hat{\mu} \hat{\nu}}) \partial_a \hat{X}^\hat{\nu} \partial_b \hat{X}^\hat{\rho} = 0 ,
\]

(52)

where
\[
\hat{\Gamma}_{\hat{\mu} \hat{\nu} \hat{\rho}} = \frac{1}{2} (\partial_{\hat{\nu}} \hat{g}_{\hat{\mu} \hat{\rho}} + \partial_{\hat{\rho}} \hat{g}_{\hat{\mu} \hat{\nu}} - \partial_{\hat{\mu}} \hat{g}_{\hat{\nu} \hat{\rho}}).
\]

(53)

To analyze these equations it is convenient to consider \( X^\mu \) and \( Y^\alpha \) separately. Since the \( Y^\alpha \) equation is somewhat simpler we begin with that. Indeed for that case, let us back up and focus on those terms in \( S \) that are \( Y \) dependent. These are
\[
S_Y = \int d^2 \sigma \left\{ \frac{1}{2} (\eta^{ab} G_{\alpha \beta}(X) \partial_a Y^\alpha \partial_b Y^\beta + \epsilon^{ab} B_{\alpha \beta}(X) \partial_a Y^\alpha \partial_b Y^\beta) + \Gamma^a(X) \partial_a Y^\alpha \right\},
\]

(54)

where
\[
\Gamma^a = \eta^{ab} \hat{g}_{\mu a} \partial_b X^\mu - \epsilon^{ab} \hat{B}_{\mu a} \partial_b X^\mu = \eta^{ab} G_{\alpha \beta} A_{(1) \alpha}^{(1) \beta} \partial_b X^\mu - \epsilon^{ab} (A_{(2) \alpha}^{(1) \beta} - B_{\alpha \beta} A_{(1) \alpha}) \partial_b X^\mu.
\]

(55)

Since the backgrounds are independent of \( Y^\alpha \), the Euler–Lagrange equations for the \( Y \) coordinates take the form
\[
\partial_a \left( \frac{\delta S}{\delta \partial_a Y^\alpha} \right) = 0.
\]

(56)

Therefore, locally, we can write
\[
\frac{\delta S}{\delta \partial_a Y^\alpha} = \eta^{ab} \partial_b Y^\beta G_{\alpha \beta} + \epsilon^{ab} \partial_b Y^\beta B_{\alpha \beta} + \Gamma^a = \epsilon^{ab} \partial_b \tilde{Y}_\alpha,
\]

(57)

where \( \tilde{Y}_\alpha \) are the dual coordinates. If we define an enlarged manifold combining the
coordinates $Y^\alpha$ and $\tilde{Y}_\alpha$ such that $\{Z^i\} = \{Y^\alpha, \tilde{Y}_\alpha\}, i = 1, 2, \ldots, 2d$, then we obtain

$$M\eta D_a Z = \epsilon_a^b D_b Z , \quad (58)$$

where

$$(D_a Z)^i = \partial_a Z^i + A^{i\mu}_a \partial_a X^\mu , \quad (59)$$

and $A^{i\mu}_a$ is comprised of $A^{(1)\mu}_a$ and $A^{(2)\mu}_a$, as in section 2. This equation (which appears in ref. [28] for the special case $A_\mu = 0$) has manifest $O(d, d)$ invariance provided the transformation rules $M \rightarrow \Omega M \Omega^T$ and $A_\mu \rightarrow \Omega A_\mu$, obtained in section 2, are supplemented with $Z \rightarrow \Omega Z$.

Even though these equations have continuous $O(d, d)$ invariance, the symmetry is broken to the discrete subgroup $O(d, d, Z)$ by the boundary conditions $Y^\alpha \simeq Y^\alpha + 2\pi$ and $\tilde{Y}_\alpha \simeq \tilde{Y}_\alpha + 2\pi$. The fundamental point is that all geometries related by $O(d, d, Z)$ transformations correspond to the same conformal field theory and are physically equivalent. The moduli space of conformally inequivalent (and hence physically inequivalent) classical solutions is given by the coset space $O(d, d, Z)$ and is parametrized locally by the scalar fields $G_{\alpha\beta}$ and $B_{\alpha\beta}$.

The $X^\mu$ equation of motion is obtained by considering eq. (52) for the case of $\hat{\mu} = \mu$ and substituting the various definitions given in section 2. After a certain amount of algebra, one finds that the $X^\mu$ equation of motion can be written in the manifestly $O(d, d)$ invariant form

$$\frac{1}{2} D_+ Z (\partial_\mu M^{-1}) D_- Z + \epsilon^{ab} \partial_a X^\nu F_{\mu\nu} \eta D_b Z - \Gamma_{\mu\nu\rho} \partial^\alpha X^\nu \partial_a X^\rho - g_{\mu\nu} \partial^a \partial_a X^\nu + \frac{1}{2} \epsilon^{ab} H_{\mu\nu\rho} \partial_a X^\nu \partial_b X^\rho = 0 . \quad (60)$$

Together with eq. (58), this gives the classical dynamics of strings moving in an arbitrary $X$-dependent background.
Eqs. (58) and (60) continue to hold for the $O(d, d + n)$ generalization, provided that $M, \eta, \text{ and } A_i^\mu$ are defined as in section 2. Also, $Z^i$ now becomes a $(2d + n)$-component vector made by combining $Y^\alpha, \tilde{Y}^\alpha,$ and $Y^I$, where $Y^I$ are $n$ additional internal coordinates. One must require that

$$\partial_- Y^I + A^{(3) I}_\mu \partial_- X^\mu = 0,$$

(61)

as a “gauge invariant” generalization of what we know to be true for the heterotic string with vanishing $A^{(3) I}_\mu$ background fields, viz. that the $Y^I$ are left-moving.

Let us now consider how eqs. (58) and (60) transform under the $SL(2, R)$ transformations introduced in section 3. For this purpose we should specialize to $D = 4$ and eliminate $H_{\mu\nu\rho}$ in favor of the axion field $\chi$ using eq. (28). Also, the space-time metric that appears in eq. (60) is the “string metric” of section 2, and it needs to be Weyl rescaled, as in section 3 ($g^S_{\mu\nu} = e^\phi g_{\mu\nu}$) in order to make contact with the equations of that section. In this way the dilaton field enters the equation. (The usual coupling of the dilaton to the world-sheet curvature is a higher-order effect than is being considered here.)

For symmetry of eq. (58), we need a covariant interpretation of eq. (59). We learned in section 3 that $F^+_\mu = (c \tau + d) F^+_\mu$ under an $SL(2, R)$ transformation. This has no simple solution for $A_\mu$ unless $c = 0$ in which case we have $A_\mu \to d A_\mu$. Therefore eqs. (58) and (59) are covariant for this subgroup ($\tau \to (a \tau + b)/d$) provided that we simultaneously transform $Z \to dZ$. It is straightforward to verify that eq. (60), with the dilaton and axion introduced as described above, is also invariant under the same linear subgroup of $SL(2, R)$.

What should we conclude about the status of $SL(2, R)$ (or $SL(2, Z)$ when we restrict to discrete translations of the axion field) in string theory? Infinitesimal $SL(2, R)$ transformations can be written in the form $\delta \tau = \alpha + \beta \tau + \gamma \tau^2$, and we have found that the $\alpha$ and $\beta$ transformations are okay, but the $\gamma$ one is not. However, the $\gamma$ transformation mixes up different powers of the string loop expansion parameter $e^\phi$, and the equations we are studying are only lowest-order equations. So, it is
still possible that the symmetry could be restored when higher-order corrections are taken into account. I am not very sanguine about this, but the evidence so far is not sufficient to exclude this possibility.

Even if this works, we would still need to understand what happens when non-Abelian gauge fields are present. In heterotic string theory, the Yang–Mills coupling constant \( g^2 \sim \kappa^2 / \alpha' \sim < e^\phi > \). So, the extra gauge field terms are also of higher order, and should not be included in the leading order analysis. Therefore, is is conceivable that they could also be reconciled with the symmetry.

One piece of evidence in support of the conjecture that the \( SL(2, Z) \) symmetry is present in the quantum theory was presented recently by Sen.\(^3\) He showed that, when all normalizations are carefully accounted for, the quantization conditions of electric and magnetic charge for dyons\(^3\) are preserved under by \( SL(2, Z) \) transformations. Electric and magnetic charges play much the same role for the \( SL(2, Z) \) group as momentum and winding modes do for the \( O(d, d + n, Z) \) groups.

5. Conclusion

This work has reviewed the noncompact \( O(d, d) \) group that appears in toroidal compactification of oriented closed bosonic strings, as well as the \( O(d, d + n) \) generalization that is required for the heterotic string. Using methods of dimensional reduction, we showed that these noncompact groups are exact symmetries of the (classical) low-energy effective field theory that is obtained when one truncates the dependence on the internal coordinates \( y^\alpha \) keeping zero modes only. Starting from the two-dimensional sigma model describing string world sheet dynamics in the presence of background fields, we found that the classical string equations of motion also have the full noncompact symmetry, but that in string theory it is broken to the discrete subgroup \( O(d, d + n, Z) \) by the boundary conditions that describe the toroidal topology of the compactified dimensions. These subgroups are, in fact, “discrete gauge symmetries,”\(^3\) which means that they should be quite robust, surviving the plethora of phenomena that typically lead to explicit breaking of global symmetries. (However, they are broken \emph{spontaneously} in general.)
A much deeper question is the status of the axion–dilaton $SL(2, Z)$ symmetry, which combines Peccei–Quinn symmetry with Montonen–Olive duality. If it is an exact symmetry of heterotic string theory (compactified to four dimensions), that is very profound. Unlike the $O(d, d+n)$-type symmetries, it transforms the dilaton field nonlinearly, and therefore has non-perturbative implications, rather like those that have been suggested for five-branes. In particular, an understanding of how it works could give insight into how the dilaton acquires mass and supersymmetry is broken. I think it could even shed light on the question of why the cosmological constant remains zero when supersymmetry is broken by non-perturbative effects. Unlike $O(d, d+n)$-type symmetries, this symmetry is presumably not a discrete gauge symmetry of the heterotic string. If it were, this would make its fundamental status much more convincing, and it could also have phenomenological benefits. Lacking that, it seems more likely that it is broken (at least) by small non-perturbative effects.

Let me conclude with a more optimistic remark. Toroidal compactification and N=4 supersymmetry certainly simplify the analysis, but if the $SL(2, Z)$ symmetry is really fundamental, it should also apply in more realistic situations, such as Calabi–Yau compactification.

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