Design and manufacturing of spiral bevel gears using CNC milling machines

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Abstract. Gears are some of the most important elements used in machinery. Their purpose is to transmit motion and torque from one shaft to another. When shafts intersect, bevel gears provide the most efficient mean of transmitting power. There are two main forms of bevel gears, straight bevel gears and spiral bevel gears. This paper is the first of a sequence of papers concerning design and manufacturing of spiral bevel gears using computer numerical control (CNC) milling machines. It is an introduction to their designing features, showing the standards that recommend the geometrical dimensions of gear tooth and a theoretical approach to the traditionally used cutting methods of spiral gears, Gleason, Oerlikon and Klingelnberg methods. Finally, it is presented five-axis machining tools that can be used for cutting of spiral bevel gears.

1. Introduction
Gears are some of the most important elements used in machinery. There are few mechanical devices that do not have to transmit power and motion between rotating shafts. Gears not only do this satisfactorily but can also do so with uniform motion and reliability. They used to increase or decrease the speed of rotation, to reverse the direction of rotation, to move rotational motion to a different axis or to keep the rotation of two axes synchronized. Gear designs are standardized in accordance with size and shape which provides widespread interchangeability.

A complete information on design of bevel gearing is provided by the American Gear Manufacturers Association (ANSI/AGMA standards) and the International Organization for Standardization (ISO 23509:2006 standard). It should be noted that only some essential geometrical dimensions of gear set are mentioned below and not all the design considerations that recommended by these standards. For many decades, information on bevel gear geometry has been developed and published by the gear machine manufacturers. It is clear that the specific formulas for their respective geometries were developed for the mechanical generation methods of their particular machines and tools. In many cases, these formulas could not be used in general for all bevel gear types. This situation changed with the introduction of universal, multiaxis CNC machines, which in principle are able to produce nearly all types of gearing [2-4].
2. Design of spiral gears and cutting methods

To understand the spiral bevel gear tooth geometry, one might observe the case of straight bevel gears. Bevel gears have tapered elements because they are generated and operate, in theory, on the surface of a sphere. Pitch diameters of mating bevel gears belong to frusta of cones, as shown in Figure 1.

![Figure 1. Pitch cone frusta.](image1)

In the full development on the surface of a sphere, a pair of mesh bevel gears are in conjugate engagement as shown in Figure 2 [1], [6]. In Figure 3.a and Figure 3.b, bevel gear terminology is presented, as is referred in ANSI/AGMA/ISO standards [3], [6].

![Figure 2. Pitch cones and the development sphere.](image2)

The pitch angle of a bevel gear is the angle of the pitch cone. It is measure of the amount of taper of the gear. The pitch angles in a set of bevel gears are defined by lines meeting at the cone center.

The root and face angles are defined by lines that do not hit the cone center (or apex.). The outside cone of one gear is parallel to the root cone of its mate. The circular pitch and the pitch diameter of bevel gears are calculated the same as for spur gears. Bevel tooth design involves some consideration of tooth taper because the amount of taper affects the final tooth proportions and the size and the shape of the blank [3-6]. Basic types of tapers are the following, Figure 5 and Figure 6.

![Figure 3. Bevel gear terminology.](image3)
Let $z_1$ and $z_2$ be the pinion and the gear tooth numbers; shaft angle $\Sigma$; and pitch cone angles $\delta_1$ and $\delta_2$; then:

\[
\tan \delta_1 = \frac{\sin \Sigma}{\frac{z_2}{z_1} + \cos \Sigma} \\
\tan \delta_2 = \frac{\sin \Sigma}{\frac{z_1}{z_2} + \cos \Sigma}
\]

(1)

Generally, shaft angle $\Sigma = 90^\circ$ is most used. But other angles can be used, too. When $\Sigma = 90^\circ$,

\[
\delta_1 = \tan^{-1}\left(\frac{z_2}{z_1}\right) \\
\delta_2 = \tan^{-1}\left(\frac{z_1}{z_2}\right)
\]

(2)

**Figure 4.** The pitch cone angle of bevel gear.

**Figure 5.** Bevel gear tooth tapers.

**Figure 6.** Bevel gear depthwise tapers.

Depth taper refers to the change in tooth depth along the face measured perpendicular to the pitch cone. It directly affects the blank of the tooth through its effect on the dedendum angle, which is used in the calculation of the face angle of the mating member.

Point width taper refers to the change in the width formed by a V-shaped cutting tool of nominal pressure angle and is the taper of primary consideration for production. The width of the slot at its narrowest point determines the point width of the cutting tool and limits the edge radius that can be placed on the cutter blade. The point width taper depends upon the lengthwise curvature and the dedendum angle. In spiral bevel and hypoid gears, the amount by which the root line is tilted is further dependent upon a number of geometric characteristics including the cutter radius.

Space width taper refers to the change in the space width along the face. It is generally measured in the pitch plane. Thickness taper refers to the change in the tooth thickness along the face. It is generally measured in the pitch plane.
Standard depth pertains to the configuration where the depth changes in proportion to the cone distance at any particular section of the tooth. If the root line of such a tooth is extended, it intersects the axis at the pitch cone apex, as illustrated in Figure 6.

Uniform depth is the configuration where the tooth depth remains constant along the face width regardless of cutter radius. In this case, the root line is parallel to an element of the face cone (Figure 6).

Duplex depth taper represents a tilt of the root line such that the slot width is constant while maintaining the proper space width taper. The point width taper is zero on both members. For a given design, a large cutter radius increases the sum of the dedendum angles, while a small cutter radius decreases the sum of the dedendum angles.

Tilted root line taper is an intermediate one in which the root line is tilted about the mean point. In this case, the slot width of the gear member is constant along the tooth length and any point width taper is on the pinion member [3-7].

The most complex form of bevel gear is the spiral bevel. These gears have curved oblique teeth which contact each other gradually and smoothly from one end to the other. Well-designed spiral bevels have two or more teeth in contact at all times. The overlapping tooth action transmits motion more smoothly and quietly than with straight bevel gears.

A difficulty that is encountered in the development of spiral bevel gears is due to the fact that the final geometry of spiral bevel teeth is obtained on grinding machines. The complex surfaces of the teeth depend on a considerable number of independent machine settings.

Spiral angle and pressure angle are two design parameters that help determine the shape of a spiral bevel gear teeth. The teeth of spiral bevel gears are curved and oblique. This curved shape is positioned at an angle to a pitch cone element. The angle at the center of the face width shows the spiral angle (Figure 7). The most common spiral angle in use is 35° [1-7].

A spiral bevel gear can be a right-hand spiral bevel gear or a left-hand spiral gear. The more common pressure angles for spiral bevel gears are 16° and 20°, with 20° to be almost standard. In regards to the selection of a pressure angle, a lower pressure angle increases the transverse contact ratio, a benefit which results in increased bending strength, while also increasing the risk of undercut that is a major concern. Lower pressure angles also help to reduce the axial and separating forces and increase the toplands and slot widths. These factors help to strengthen gear teeth. [6][7]

The following table, Table 1, is presenting the formulas that are used to calculate the blank and tooth dimensions of spiral bevel gears according to ANSI/AGMA/ISO standards [3],[4],[7].

In gear designing is very important to analyze the magnitude and the direction of the forces acting upon the gear teeth. Spiral bevel gears have convex and concave sides. Depending on which surface the force is acting on, the direction and the magnitude changes [4],[7].

Two common production methods of spiral bevel gears in gear industry are face-milling and face-hobbing, both of them have similar cutting movements. The two different manufacturing methods are shown in Figures 9 and 10. During the tooth generation process, in the face-milling technique, the cutting wheel interacts with one tooth space and is then indexed to the next location (the cradle axis and the work axis roll together in a timed relationship). The process continues until all tooth spaces are

![Figure 7. Spiral angle.](image-url)
Table 1. Straight and spiral bevel gear formulas acc. to ANSI/AGMA-2005-D03.

| Item | Pinion | Both pinion and gear | Gear |
|------|--------|----------------------|------|
| Pitch diameter (metric) | \( d = \frac{D}{N} \) | \( D = \frac{d}{N} \) | |
| Pitch angle | \( \gamma = \arctan \left( \frac{\sin \Sigma}{\frac{N}{2} \cos \Sigma} \right) \) | \( \Gamma = \Sigma - \gamma \) | |
| Outer cone distance | \( A_o = 0.5D \) | \( A_o = A_o - 0.5F \) | |
| Mean cone distance | \( A_m = A_m - 0.5F \) | \( k_1 \) (see table 4 / ANSI/AGMA 2005-D03) | |
| Depth factor | \( k_1 \) (see table 4 / ANSI/AGMA 2005-D03) | \( c = k_1 \) | |
| Mean working depth | \( h = \frac{k_1}{k_1} \left( \frac{A_m}{A_o} \right) \cos \psi \) | \( h = \frac{k_1}{k_1} m_w \left( \frac{A_m}{A_o} \right) \cos \psi \) | |
| Clearance factor | \( k_2 \) (see table 7 / ANSI/AGMA 2005-D03) | \( c = k_1 \) | |
| Clearance | \( h_m = h + c \) | \( h_m = h - c \) | |
| Mean whole depth | \( m_{w1} = \frac{N \cos \psi}{\frac{N}{2} \cos \Sigma} \) | \( c_1 \) (see table 5 / ANSI/AGMA 2005-D03) | |
| Equivalent 90° ratio | \( c_1 \) (see table 5 / ANSI/AGMA 2005-D03) | \( \frac{p_w}{p_d} = \frac{a_p}{a_d} \left( \frac{A_m}{A_o} \right) \) | |
| Mean addendum factor | \( s_{ap} = \frac{h}{s_p} - h_a \) | \( s_{ap} = c_1 \frac{h}{s_p} \) | |
| Mean addendum | \( s_{gp} = h_m - a_p \) | \( h_m = h_a - s_{ap} \) | |
| Sum of addendum angles | \( \Sigma_1 \) (see table 6 / ANSI/AGMA 2005-D03) | \( \Sigma_2 \) (see table 6 / ANSI/AGMA 2005-D03) | |
| Dedendum angle | \( \theta_0 = \gamma + \delta_p \) | \( \delta_0 \) (see table 7 / ANSI/AGMA 2005-D03) | |
| Face angle | \( \theta_0 = \gamma + \delta_p \) | \( \Gamma_0 = \Gamma + \delta_p \) | |
| Root angle | \( \gamma = \gamma - \delta_p \) | \( \Gamma = \Gamma - \delta_p \) | |
| Outer addendum | \( a_{ap} = a_p + 0.5F \tan \delta_p \) | \( s_{apg} = s_{ap} + 0.5F \tan \delta_p \) | |
| Outer working depth | \( h_{ap} = \frac{d_p}{p_d} \tan \delta_p \) | \( h_{apg} = \frac{d_p}{p_d} \tan \delta_p \) | |
| Outer whole depth | \( h = s_{ap} + h_{ap} \) | \( h = h_a - s_{ap} \) | |
| Outside diameter | \( d_a = d + 2a_{ap} \cos \gamma \) | \( D_p = D + 2a_{ap} \cos \Gamma \) | |
| Pitch cone apex to crown | \( x_a = A_p \cos \gamma - a_{ap} \sin \gamma \) | \( X_a = A_p \cos \Gamma - a_{ap} \sin \Gamma \) | |
| Mean diametral pitch | \( \frac{P_{dm}}{P_{pm}} = \left( \frac{A_m}{A_o} \right) \) | \( \frac{P_{dm}}{P_{pm}} = \frac{N}{P_{bm}} \) | |
| Mean pitch diameter | \( d_m = \frac{d_p}{P_{dm}} \) | \( D_m = \frac{D}{P_{dm}} \) | |
| Thickness factor | \( k_2 \) (see figure 21 / ANSI/AGMA 2005-D03) | \( k_2 \) (see figure 21 / ANSI/AGMA 2005-D03) | |
| Mean normal circular thickness theoretical without backlash | \( t_m = \frac{P_{bm} \cos \psi - T_m}{N} \) | \( T_m = 0.5F \cos \psi - \left( s_{ap} - s_{ap} \right) \cos \phi \) | |
| Outer normal backlash allowance | \( B \) (see table 8 / ANSI/AGMA 2005-D03) | \( B \) (see table 8 / ANSI/AGMA 2005-D03) | |
| Outer spiral angle (face milling) | \( \sin \phi_0 = \frac{2a_m \cos \psi - A_m^2 + A_o^2}{2a_m \cos \lambda} \) | \( A_p = \frac{N \sin \Gamma}{r_c} \) | |
| Outer spiral angle (face hobbing) | \( \sin \phi = \frac{2a_m \cos \psi - A_m^2 + A_o^2}{2a_m \cos \lambda} \) | \( \sin \phi = \frac{A_p \cos \psi}{r_c} \) | |
| Mean normal chordal thickness | \( s_{nch} = s_{nch} \left( \frac{r_p^2}{2a_m^2} \right) - 0.5F \left( \frac{A_o}{A_p \cos \psi} \right) \) | \( T_m = T_m - \left( \frac{r_p^2}{2a_m^2} \right) - 0.5F \left( \frac{A_o}{A_p \cos \psi} \right) \) | |
| Mean chordal addendum | \( a_{ap} = a_p + 0.25 \frac{2 \cos \gamma}{d_a} \) | \( a_{ap} = a_p + 0.25 \frac{2 \cos \gamma}{d_a} \) | |
finish cut to the required depth. In the face-hobbing technique individual cutting blades interact with different tooth spaces (the cradle axis, work axis and cutter axis roll together in a timed relationship).

Face-hobbing is a continuously indexing tooth generation process, where all the teeth are cut a little at a time, until all the teeth are finished to the final desired depth. With the face hobbing process, the curve in the lengthwise direction of the tooth is an extended epicycloid and is a function of the relative roll between the workpiece and the cutter [9-11].

Figure 8. When meshing on the concave side of tooth face [4].

Two common production methods of spiral bevel gears in gear industry are face-milling and face-hobbing, both of them have similar cutting movements. The two different manufacturing methods are shown in Figures 9 and 10. During the tooth generation process, in the face-milling technique, the cutting wheel interacts with one tooth space and is then indexed to the next location (the cradle axis and the work axis roll together in a timed relationship). The process continues until all tooth spaces are finish cut to the required depth. In the face-hobbing technique individual cutting blades interact with different tooth spaces (the cradle axis, work axis and cutter axis roll together in a timed relationship). Face-hobbing is a continuously indexing tooth generation process, where all the teeth are cut a little at a time, until all the teeth are finished to the final desired depth. With the face hobbing process, the curve in the lengthwise direction of the tooth is an extended epicycloid and is a function of the relative roll between the workpiece and the cutter [9-11].

There are three major design systems used to produce spiral bevel gear sets and generally bevel gear sets - Gleason, Oerlikon and Klingelnberg. The different cutting methods employed by these three systems produce significantly different tooth forms.

- The Gleason tooth form is arc shaped and has a tooth depth that varies along the length of the tooth.

Figure 9. Face-milling method.

Figure 10. Face-hobbing method.
The Oerlikon and Klingelnberg tooth forms have a constant tooth depth and are designed to provide a beneficial rolling motion of the gear teeth as they mesh.

The Klingelnberg Palloid system, which used hobs instead of cutters, produces a tooth form that follows a true involute curve, whereas the Gleason and Oerlikon designs only approximate an involute curve.

In the following Figures 11-15, are presented the various types of spiral bevel gears and theirs geometrical features. [8] [2] [10] [11]

Figure 11. Gleason geometry type
- Longitudinal curvature of the tooth: arc
- Transverse thickness, height of the tooth and tooth space tapered towards the vertex of the cone.
- Angle of the spiral ranges from 0° (toothing Zero) up to about 45°; normally 35°

Figure 12. Oerlikon geometry type
- Longitudinal curvature of the tooth: epicycloid
- Tooth height constant
- Toothing N: the normal module reaches its maximum value at the center of the tooth and reduces towards the two sides
- Angle of spiral between 30° and 50°
- Toothing G: the spiral angle is from 0° to about 50°

Figure 13. Klingelnberg-Palloid geometry type
- Longitudinal curvature of the tooth: involute to a circle
- Constant tooth height
- Normal pitch and thickness
- The inclination angle is from 35° to 38°
- Slightly faceted surface

Figure 14. Klingelnberg-Zyclo-Palloid geometry type
- Longitudinal curvature of the tooth: epicycloid
- Constant height of the teeth
- Normal module and pitch, according to the angle of the spiral tapered down to almost constant
- Angles of the spiral form from 0° to 45°

Figure 15. Modul-Kurvex geometry type
- Longitudinal curvature of the tooth: arc of a circle
- Tooth height constant or tapered
- Corners of the spiral 25° to about 45°
- Mating gears can be milled by means of the same tool
Due to economic factors, the majority of spiral bevel gears manufactured today are of the Gleason design, and second in majority produced by the Oerlikon method. Klingelnberg Palloid tooth profiles are generated using specially designed spiral hobs, which produce a gear tooth shape that is close to the theoretical ideal. The resulting gears tend to be stronger and more accurate with smoother running tooth forms, but they are generally more expensive to produce, especially in large production runs.

3. Manufacturing of spiral bevel gears on five-axis CNC milling machines.

The spiral bevel gears design remains complex since tooth geometry is affected by the manufacturing process. There are different cutter systems to choose from, depending on the method of producing gears and the supplier of the cutter system. Manufacturing of spiral bevel gears has been implemented using dedicated bevel gear cutting machines. Straight bevel gears are produced by a generating machine that moves forward and back a cutting tool in a motion of a shaper. Spiral bevel gears are cut with a generating machine that uses a series of cutting blades mounted on a circular tool holder. The tooth profiling between the cutter and the generating tooth does not require any rotation of the generating gear. The virtual generating gear is formed by the cutter head in a non-generating process is shown in Figure 17. In the following Figure 16, it is presented a conventional bevel gear generator. [8][9]

Special machines are produced by Gleason, Oerlikon or Klingelnberg manufacturers, using technologies developed by them, as described in section 2, make the production of this type of gears quite costly. This high production dependence on existing monopoly of Gleason, Klingelnberg and Oerlinkon methods and its high cost, forced many gear manufacturers start using CNC milling machines for cutting spiral bevel gears. Modern CNC technology has led to a booming five-axis machine industry and many experts eager to find new applications for these machines. These machines typically require only one setup to machine an entire part, saving time and improving accuracy between operations. These advanced machines also have the capability to change the tool axis direction in order to reach machinable areas with shorter, more rigid tools or to reach zones that could not be cut at all with standard methods. [2][10][11][12]

A standard machine has linear motion along X, Y, Z axes. A five-axis machine has two additional axes of rotation. According to the ISO standard, rotary axes have been identified as:

- Rotation around X axis, A axis
- Rotation around Y axis, B axis
- Rotation around Z axis, C axis
Figure 18. Both rotary axes are in the tool. B axis and C axis located in the milling head and tilt the tool into any position. This type of machines can support heavier parts, since the table is always horizontal [12].

Figure 19. Both rotary axes are in the table. B axis tilts and C axis rotates the part, while the milling head can handle only linear motion. These machines offer larger work volumes, since there is no need to compensate for the space taken up by the swiveling spindle [12].

Figure 20. One rotary axis is in the table and the other is in the tool. The table rotates in C axis and B axis tilts the tool [12].

Figure 21. One rotary axis is in the turning spindle and the other axis in the tool. The turning spindle makes the C axis to rotate the part and the B axis controls the tilt of the tool [12].

A 5-axis machine’s specific configuration determines which two of the three rotational axes it utilizes. Some common machine configurations are presented in the following Figures 18-21.

One should be noted is the difference between 5-axis machining and 3+2-axis machining. A 5-axis machining, also called continuous or simultaneous 5-axis machining, involves continuous adjustments of the cutting tool along all five axes to keep the tip optimally perpendicular to the part. On the other hand, the 3+2-axis machining rotates the part into position before the start of each cut and then a standard 3-axis toolpath is run. This makes programming easy since rotary motion only occurs between operations. It’s also worth noting that with the speed advantage of 5-axis machining comes more moving parts since there is no need for stopping and starting between each reorientation of the tool, which leads to increased wear and tear as well as a greater need for part crash detection. This is one of the reasons continuous 5-axis machining is more difficult from a programming standpoint. [12]

The combination of 5-axis milling machine centers and capable computer aided design (CAM) software can be used for the machining of spiral bevel gears. The increased availability of 5-axis milling machine center and the continuous development of modern CAM systems that today support a vast number of complicated machining procedures, offer the opportunity of machining high quality bevel gears even in hardened condition, avoiding the restrictions that govern the traditional bevel gear manufacturing community.
4. Conclusions
The goal of this paper is to show the designing features of spiral bevel gears, as they described by ANSI/AGMA/ISO standards. It also presents the traditionally cutting methods of gears and introduces five-axis CNC machining center that can be used for spiral bevel gear cutting. It constitutes the background knowledge of an attempt to design and manufacture a spiral bevel gear set using a 5-axis CNC milling machine, the progress of which is to be presented in future papers.

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