Invited Comment

Propagation of a probe pulse inside a Bose–Einstein condensate under conditions of electromagnetically induced transparency

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Abstract

We obtain a partial differential equation for a pulse travelling inside a Bose–Einstein condensate under conditions of electromagnetically induced transparency. The equation is valid for a weak probe pulse. We solve the equation for the case of a three-level BEC in $\Lambda$ configuration with one of its ground state spatial profiles initially constant. The solution characterizes, in detail, the effect that the evolution of the condensate wave function has on pulse propagation, including the process of stopping and releasing it.

Keywords: effects of atomic coherence on propagation, electromagnetically induced transparency, Bose–Einstein condensate

1. Introduction

A medium composed of three-level atoms in $\Lambda$ configuration (two ground states, 1 and 2, dipole-coupled to an excited state 0, see figure 1(a)) can be rendered transparent using a technique known as electromagnetically induced transparency (EIT). We will call the field mode coupling states 2 and 0 the probe field and the field mode coupling states 1 and 0 the control field. In the usual EIT configuration the probe field is weak compared with the control field, which, in turn, is in resonance with the transition frequency between states 1 and 0. We denote by $\delta$ the detuning of the probe field from resonance with respect to the transition frequency between states 2 and 0. The range of frequency detunings, $\delta$, where absorption of the probe field by the medium is negligible is known as the transparency window; its size is proportional to the control field. The appearance of this transparency window using the control field is known as EIT; see figure 1(b) for an example of the typical absorption spectrum of a probe field interacting with atoms showing EIT. In [1, 2] the fundamentals of EIT are explained in detail. The probe field inside a medium composed of several three-level atoms exhibiting EIT propagates without distortion with a velocity that is a function of the control field [3, 4]; this can be used to slow down the probe velocity as much as desired [5, 6].

For an atom, the atomic wave function has a dependence on its position; when dealing with pulse propagation inside a medium composed of atoms, we can ignore this dependence when the pulse size is much larger than the variance of the atomic position; in this case only the expectation value of the atomic position enters the description and we say that we treat the atoms as point-like. The theory developed in [3, 4] treats the atoms as point-like. When the atoms that compose the medium are in a Bose–Einstein condensate, they can no longer be treated as point-like. In this case the condensate wave function together with its dynamics have a strong effect on the probe field propagation, as is shown through numerical simulations in [7–9]. To observe this effect it is necessary that the time during which the pulse interacts with the medium is larger than the time scale characterizing the condensate wave function dynamics.
Since a Bose–Einstein condensate has a well defined spatial phase relation, the system evolution does not necessarily decohere the information carried by the pulse, opening the possibility of using slow light and BEC to manipulate optical information [9, 10].

The main question we address in this paper is: how does the evolution of the condensate wave function modify the propagating pulse? We address it in a form where analytical results can be obtained. To solve this question we have to solve the Gross–Pitaevskii equations, describing the condensate evolution, together with the Maxwell equations, describing the field propagation. We obtain a partial differential equation for the pulse propagation inside a Bose–Einstein condensate with atoms in $\Lambda$ configuration. This equation is much simpler than the set of equations that are usually used to find the pulse propagation and the condensate wave function. For some simple cases it can be solved analytically, showing explicitly how the different elements—probe pulse, control field and wave functions—interact and evolve. The equation is obtained using a weak probe and approximations similar to those made in [11], but taking into account the condensate wave function. For a condensate wave function with a spatial profile that is initially constant, we write the solution in integral form. The evolution for the field looks similar to the evolution of the wave function for atoms in state 1, but with its potential being described in a reference system that moves with the pulse; we use this similarity for analyzing the pulse propagation in the medium.

A general overview of a three-level Bose–Einstein condensate interacting with two modes of the field is presented in section 2, where known approximations are used to express the model in a simplified way. In section 3 we obtain the propagation equation for the case of the probe pulse. In section 4 we study how the pulse propagates inside the condensate showing EIT and discuss some physical implications derived from the analytic solution. Concluding remarks are given in section 5.

2. Model

We only consider dynamics in the $x$ direction. The Hamiltonian of a three-level Bose–Einstein condensate interacting with two modes of light is given by

$$H = H_0 + H_{\text{int}},$$

with

$$H_0 = -\hbar \omega_2 \int dx \psi_2^\dagger(x) \psi_2(x) - \hbar \omega_1 \int dx \psi_1^\dagger(x) \psi_1(x) + \hbar \sum_j \int dx \left( \frac{\hbar^2}{2M} \frac{\partial^2}{\partial x^2} + V_j(x) \right) \psi_j(x) + \hbar \sum_{j \neq k} \int dx \left( \psi_j^\dagger(x) \psi_k(x) \right) \psi_k(x),$$

where $\psi_1(x), \psi_2(x)$ are bosonic operators that create and destroy a particle, with mass $M$, in the state $j$, with $j = 0, 1, 2$, at position $x$; these obey commutation rules

$$[\psi_j(x), \psi_k^\dagger(x')] = \delta_{ij} \delta(x - x').$$

The functions $V_j(x)$ are the trapping potential for the atoms in state $j$. The constants $u_j$ are potentials due to elastic collisions between particles in the same level, $u_{ij}$ are potentials due to elastic collisions between particles in level $i$ with particles in level $j$. We set the energy of the state $j = 0$ as the zero of the energy scale. The energy of atomic state $j = 1, 2$ is given by $-\hbar \omega_j$ and the interaction Hamiltonian reads

$$H_{\text{int}} = \hbar G(t) \int dx \left( \psi_1^\dagger(x) \psi_1(x) e^{i(k_G x - \omega_G t)} + \text{h.c.} \right) + \hbar g \int dx \mathcal{E}(x, t) \left( \psi_1^\dagger(x) \psi_2(x) + \text{h.c.} \right),$$

where $G(t)$ is proportional to the strength of the continuous control pulse, with wave number $k_G$ and frequency $\omega_G$, $\mathcal{E}(x, t)$ is the incoming probe pulse, assumed to be centered at a frequency $\omega$, and $g$ is the atom-field coupling.

In order to calculate the system evolution we use the Heisenberg representation, where the state vectors are time-independent and the evolution is given by time-dependent operators that obey the Heisenberg equations of motion. The
Heisenberg equations of motion for the bosonic operators are
\[ \frac{d}{dt} \psi_j(x, t) = \frac{1}{i\hbar} \left[ \psi_j(x, t), H \right], \] (5)
substituting the Hamiltonian \( H \) given by equation (1) we get the following differential equations for the bosonic operators
\[ \frac{d}{dt} \psi_0 = \left( \frac{i \hbar}{2M \Delta x^2} \right) \psi_0 \right) - \frac{i \hbar}{V_0} \psi_1 - 2iu_1 \psi_0 \psi_2 - \frac{i \hbar}{V_0} \psi_0 \psi_2 - \frac{i \hbar}{V_0} \psi_0 \psi_2 \]
\[ \frac{d}{dt} \psi_1 = \left( \frac{i \hbar}{2M \Delta x^2} \right) \psi_1 \right) - \frac{i \hbar}{V_0} \psi_1 - 2iu_2 \psi_0 \psi_2 - \frac{i \hbar}{V_0} \psi_0 \psi_2 \]
\[ \frac{d}{dt} \psi_2 = \left( \frac{i \hbar}{2M \Delta x^2} \right) \psi_2 \right) - \frac{i \hbar}{V_0} \psi_1 - 2iu_2 \psi_0 \psi_2 - \frac{i \hbar}{V_0} \psi_0 \psi_2 \]
Here we have omitted the time and space dependence of the operators and variables and we assumed that \( u_j = u_j \).

We proceed now by writing the field operator as the sum of its expectation value plus fluctuations \[ \psi_j = \psi_j(x, t) \mathbb{1} + \delta \psi_j, \] (7)
where \( \mathbb{1} \) is the identity operator and \( \psi_j(x, t) = \langle \psi_j \rangle \) (\( \langle \cdots \rangle \) indicates the quantum expectation value) is known as the mean field. The Gross–Pitaevskii equations are partial differential equations where the fluctuations, \( \delta \psi_j \), are not taken into account. Their solutions give an approximation to the evolution of the mean field, \( \psi_j(x, t) \), whose squared norm gives the condensate spatial profile of the atoms in level \( j \). The Gross–Pitaevskii equations are valid when the mean field dominates the fluctuations, this is the case when the interactions between atoms are weak and the system temperature is well below the transition temperature for Bose condensation.

Using equation (7) in (6) and neglecting fluctuations, we obtain a set of coupled Gross–Pitaevskii equations, they look similar to equation (6), but with functions \( \psi_j(x, t) \) instead of operators \( \hat{\psi}_j(x, t) \).

We assume that the control field is in resonance with the dipole transition \( \gamma \rightarrow 0 \) of atoms at rest, \( \omega_1 = \omega_2 \). The dominant frequency of the probe field is \( \bar{\omega} \), assumed to be in resonance with the dipole transition \( 2 \rightarrow 0 \) of atoms at rest, \( \bar{\omega} = \omega_2 \). In order to eliminate rapidly rotating terms we make the following transformations: \( \psi_j \rightarrow e^{i\bar{\omega}t} \psi_j \), \( \psi_0 \rightarrow e^{i\omega_1 t} \psi_0 \), \( \psi_0 \rightarrow e^{-i\omega_2 t} \psi_0 \), and \( \bar{\omega} \rightarrow e^{-i\bar{\omega}t} \). All the atoms are initially in state 2, i.e. \( \psi_j(x, t = 0) = \psi_j(x, t = 0) = 0 \). We use the weak probe approximation, \( G \gg g \bar{\omega} \), and assume that the wave functions \( \psi_j \) can be expanded in powers of \( \bar{\omega} \): \( g \bar{\omega} \psi_j \) (8)

The expansion limits the validity of our results to the case where the Rabi frequency of the probe field is much weaker than the the Rabi frequency of the control field, this is the standard approximation in EIT based slow light physics.

To zeroth order we obtain
\[ \frac{d}{dt} \psi_j^{(0)} = \left( \frac{i \hbar}{2M \Delta x^2} - \frac{i \hbar}{V_0} - 2iu_2 \psi_j^{(0)} \psi_0^{(0)} \right) \psi_j^{(0)}, \] (9)
and \( \psi_j^{(0)} = \psi_j^{(0)} = 0 \).
At first order we get
\[ \frac{d}{dt} \psi_j^{(1)} = \left( \frac{i \hbar}{2M \Delta x^2} - \frac{i \hbar}{V_0} - 2iu_2 \psi_j^{(0)} \psi_0^{(0)} - i\Delta - \frac{\gamma}{2} \right) \psi_j^{(1)}, \] (10)
\[ \frac{d}{dt} \psi_0^{(1)} = \left( \frac{i \hbar}{2M \Delta x^2} - \frac{i \hbar}{V_0} - 2iu_2 \psi_j^{(1)} \psi_0^{(0)} - i\Delta - \frac{\gamma}{2} \right) \psi_0^{(1)}, \] (11)
where \( \Delta = \omega_1 - \omega_2 \). We inserted spontaneous decay from state 0, at rate \( \gamma \), phenomenologically in equation (10). Since the probe pulse is weak, the population of level 1 of the condensate is small, this is the reason that justify keeping only terms up to first order in \( \bar{\omega} \) in equation (8).

3. Propagation equation for the probe pulse
In the slowly varying approximation, the equation for the probe electromagnetic field becomes \[ \left( \frac{\partial}{\partial t} + c \frac{\partial}{\partial x} \right) E = i\gamma e^{-\bar{\omega} x} \left( \psi_j^{(0)}(x, t) \psi_0(x, t) \right), \] (13)
with \( \bar{E} = E e^{ikx} \) being the slowly varying part of the electromagnetic field with \( k_x = k(\bar{\omega}) \). To first order in \( \bar{\omega} \), the dipole operator in the previous equation is \( \left( \psi_j^{(0)}(x, t) \psi_0(x, t) \right) \approx \left( \psi_j^{(0)}(x, t) \psi_j^{(0)}(x, t) \right) \).
Equation (11) implies
\[ \psi_0^{(1)} = \frac{i\omega_j^{(0)}}{G} \left( \frac{d}{dt} - \frac{i \hbar}{2M \Delta x^2} + \frac{i \hbar}{V_0} \right) \psi_0^{(1)} + i\omega_2 \psi_j^{(0)} \psi_0^{(0)} \right), \] (14)

We assume that \( \psi_0^{(1)} \) reaches its stationary value very quickly compared with the time scale evolution of the other wave functions. Note that in the stationary regime the system reaches the EIT regime and the population of state 0 is negligible. These two considerations allow us to write equation (10) as
\[ \psi_j^{(1)} \approx \frac{g \bar{\omega} e^{ikx}}{G} e^{-\bar{\omega} x} \psi_j^{(0)}, \] (15)
this expression is equivalent to equation (27) in [11]. We insert equation (15) into equation (14) and denote the condensate wave function of level 2, when there is no field, as

\[ \psi_2^{(0)} = \alpha(x, t), \]

obtaining

\[
\{ \psi_2^{(0)}(x, t) \psi_0^{(1)}(x, t) \} = -i g e^{i k x} e^{i \alpha x} \frac{\partial}{\partial t} E(x, t) \alpha(x, t) + \frac{g e^{i k x} e^{i \alpha x}}{G(t)} \frac{\hbar^2}{2 M} \frac{\partial^2}{\partial x^2} e^{i(k_x - k_F)x} \alpha(x, t) E(x, t) + \frac{g e^{i k x} e^{i \alpha x}}{G^2(t)} (\alpha(x, t))^2 V_1 + \frac{g e^{i k x} e^{i \alpha x}}{G^2(t)} (\alpha(x, t)) \hat{\mu}_1 |\alpha(x, t)|^4 G(t). \tag{16}\]

Substituting in the propagation equation we obtain

\[
\left\{ \begin{array}{l}
\left( \frac{\partial}{\partial t} + c \frac{\partial}{\partial x} \right) E(x, t) \\
= -g e^{i k x} e^{i \alpha x} \frac{\partial}{\partial t} \left( \frac{\partial}{\partial t} \frac{\alpha(x, t)}{G(t)} \right) \\
- \frac{i \hbar g}{2 M G(t)} \left[ -k_G - k_F \right] E(x, t) \alpha(x, t) \\
- i 2 (k_G - k_F) \frac{\partial}{\partial x} E(x, t) \alpha(x, t) \\
+ \frac{\partial^2}{\partial x^2} E(x, t) \alpha(x, t) \\
+ i E(x, t) \alpha(x, t) \right] V_1 \\
+ \frac{i E(x, t) \mu_1 |\alpha(x, t)|^4 G(t)}{G(t)} \end{array} \right. \tag{17}\]

which is one of our main results. The function \( \alpha(x, t) = \psi_2^{(0)}(x, t) \) is given by the solution of equation (9) and does not depend on the field or the condensate wave functions of the other atomic levels. In the following section we take advantage of this to obtain an analytic expression for the solution in the particular case of an initially spatially uniform condensate. This solution exemplifies how the different terms on the right side of equation (17) contribute to the probe pulse evolution.

4. Slow light inside a condensate

In this section, we study how the pulse probe propagates inside the condensate showing EIT. We will focus on the case where \( \alpha(x, t) = \alpha \exp(i \omega t) \), with \( \alpha \) a constant, is a solution of the Gross-Pitaevsky equation for the state 2 of the condensate. Spatially uniform condensates have been achieved experimentally [14]. The potential, \( V_2 \), that generates this distribution, could be a square well; it can also be a wide harmonic potential and the following discussion is valid in the region near the center of the potential, where there is little change in the density distribution [15]. This case avoids the effect that density change in atoms in level 2 has on pulse propagation; we are interested in the effect that state evolution of atoms in level 1 has on the pulse. This approximation allows us to obtain an analytic expression for the solution that completely characterizes both, the pulse propagation and the atomic state evolution. Defining \( k_i = k_G - k_F, \) equation (17) reads

\[
\left( \frac{\partial}{\partial t} + c \frac{\partial}{\partial x} \right) E(x, t) \\
= -g^2 |\alpha|^2 \frac{\partial}{\partial t} \left( \frac{E(x, t)}{G(t)} \right) + i \mu E(x, t) \frac{G(t)}{G(t)} \\
+ \frac{i g^2 |\alpha|^2}{2 M G(t)} \left[ -k^2 - i 2 k_i \frac{\partial}{\partial x} + \frac{\partial^2}{\partial x^2} \right] E(x, t). \tag{18}\]

To solve this equation we use a reference system that moves with the probe pulse. With substitution

\[
u = -\frac{x}{c} + \int_0^t \frac{G^2(\xi)}{2 M G(t)} d\xi \tag{19}\]

we obtain

\[
E(u, t) = \frac{G(t) e^{i \phi(t)}}{\sqrt{G^2(t) + g^2 |\alpha|^2}} \exp \left( \int_0^t \frac{i}{\hbar} \tilde{H}(\tau) d\tau \right) E(u, t = 0), \tag{20}\]

where

\[
\tilde{H} = \frac{1}{(1 + G^2(t)/g^2 |\alpha|^2)} \times \left\{ \frac{\hbar^2}{2 M} \left( k_i + \frac{i}{\hbar} \frac{\partial}{\partial \tau} \right)^2 + V_1(u, t) \right\} \tag{21}\]

and

\[
\phi(t) = \left( \mu + u_{12} |\alpha|^2 \right) \int_0^t \frac{G^2(\tau)}{G^2(\tau) + g^2 |\alpha|^2} d\tau - t. \tag{22}\]

The solution for the field shown in (20), includes some of the usual characteristic behavior of light traveling through media showing EIT. The pulse velocity depends on the value of \( G(t) \); as \( G(t) \to 0 \) the pulse is stopped and, as can be seen from expression (15), its information is transferred to the atoms [3]. Note that if \( k_i = 0 \) and the pulse is already stopped \( (G(t) = 0) \), the operator \( \tilde{H} \), defined in (21), coincides with the Hamiltonian operator of an atom trapped in the potential \( V_1 \). As shown in relation (15), the wave function of atoms in state 1 is given by the probe field once the pulse is trapped; then, while the pulse is trapped, the atomic wave function for
atoms in state 1 evolves in terms of its own Hamiltonian. As solution (20) shows, the profile of it gets written in the pulse once it is released \((G(t) \rightarrow \infty)\). If \(k_0 \neq 0\), the photons transfer momentum to the atoms that go from state 2 to state 1. This precisely appears in (21) as a displacement of the momentum operator by \(k_0\). This effect has been used to move a pulse by physically moving the atoms in state 1, and then releasing the pulse once the atoms are in a new position [16].

It is clear from the solution that the pulse is modified as it travels through the medium, and in the process of stopping it; the strength of this modification depends on \(G(t)\) and on the trapping potential for atoms in state 1. The behavior described above is the expected behavior for trapping a pulse in a BEC [7]. The advantage of the analytic expression for the field \((20)\), is that it tells us how the pulse evolve: as if it were a quantum particle with a Hamiltonian given by equation (21). Note that \(\hat{H}\) looks similar to the Hamiltonian operator for atoms in state 1 but multiplied by the factor \(1/(1 + G^2(t)/g^2 \vert \alpha \vert^2)\) and in a reference frame that moves with the pulse; when the control field is zero, \(u = x\) and \(\hat{H}\) coincides with the Hamiltonian of atoms in state 1. Due to these facts we claim that: the way the probe pulse propagation is affected by the evolution of the wave function of atoms in state 1, is given by \(\hat{H}\). With this interpretation, the coefficient factor \(1/(1 + G^2(t)/g^2 \vert \alpha \vert^2)\) in Hamiltonian (21), gives the strength of how the evolution of the atomic wave function affects the propagating pulse.

We will now explore some of the physical insights that solution (20) exhibits.

The field, shown in (20), does not depend on \(u_1\): collisions between atoms in state 1 does not affect the field propagation; in the limit of weak probe field, the number of atoms transferred to state 1 is small and collisions between them are negligible.

The field has a global phase which depends on \(\phi(t)\) (see (20), (22)), which in turn depends on term \(u_{12}\) that characterizes the strength of collisions between atoms in state 2 with atoms in state 1. Pulses that propagate in the medium with different control fields, \(G(t)\), will have different phases; the phase accumulation is maximum when the pulse is stopped, and it is negligible when \(G(t)\) is large. Depending on which parameters are known, making these pulses interfere could be used to extract the value of \(u_{12}, \mu\) or \(g\). If we make interfere a pulse that has been stopped and then released, with a pulse that propagates at constant velocity, we could extract information on the phase advance due to the process of trapping and releasing the pulse.

For intermediate cases \((G(t) \neq 0, \infty)\) the pulse is modified; how it is modified depends on the wave function dynamics of atoms in state 1 and on the coupling between this dynamics and the pulse, \(1/(1 + G^2(t)/g^2 \vert \alpha \vert^2)\). For example, if \(V_1\) is constant inside the region the pulse propagates, the width of the wave function for state 1 expands as a free particle, then the probe pulse width expands as if it were a free particle with mass \(M(1 + G^2(t)/g^2 \vert \alpha \vert^2)\). In this case, not taking into account that the field strength changes as a function of the control field, the evolution of the field, in the process of trapping it, would be similar to the evolution of a free particle wave function whose mass change with time.

The pulse propagation can be manipulated using \(V_1\). For example, if \(V_1 = M\omega^2\alpha^2\), the center of the pulse (in the moving frame) will behave as if it were a harmonic oscillator with mass \(M(1 + G^2(t)/g^2 \vert \alpha \vert^2)\) and frequency \(\omega/(1 + G^2(t)/g^2 \vert \alpha \vert^2)\).

5. Conclusion

When a probe pulse of light propagates through a three-level Bose–Einstein condensate under conditions of EIT, its shape is modified by the evolution of the condensate wave function. The strength of this modification is a function of the control field. This is in contrast with the case where atoms are assumed point-like, where the probe pulse propagates without modification.

We derived a partial differential equation whose solution gives the probe pulse propagation inside a three-level Bose–Einstein condensate under conditions of EIT. We obtained an analytic expression when the initial profile for the atoms in state 2 is constant. The solution shows explicitly how different parameters of the system—trapping potential of the atoms, time dependent control field, scattering lengths, atom-field coupling—affect the pulse propagation and the evolution of the condensate wave function. The form of the solution for the probe propagation, expression (20), is similar to the formal solution of the Schrödinger equation for the wave function of an atom in state 1, but with the Hamiltonian given in a reference frame that moves with the pulse.

The solution allows us to see how to manipulate the pulse by choosing different trapping potentials for atoms in state 1 and different control fields. For example, if the potentials are zero, the width of the probe pulse expands in time as if it were a free quantum particle, but with an expansion rate which is a function of the control field. The propagated pulse acquires a global phase that depends on the control field and the scattering length between atoms in state 1 and 2. This phase could be measured by performing interference experiments with pulses that propagate with different control fields.

We are confident that our results can help in the discussion of manipulating light memories using condensates.

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