Probing the Weinberg operator at colliders

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Motivated by searches for 0νββ decay in nuclear experiments and collider probes of lepton number violation at dimension $d \gtrsim 7$, we investigate the sensitivity to the $d = 5$ Weinberg operator using the nonresonant signature $pp \rightarrow e^\pm e^\pm j$ at the LHC. We develop a prescription for the operator that is applicable in collisions and decays, and focus on the $e^+e^- = \mu^+\mu^-$ channel, which is beyond the reach of nuclear decays. For a Wilson coefficient $C_5^{\mu\mu} = 1$, scales as heavy as $\Lambda \sim 8.3(11)$ TeV can be probed with $\mathcal{L} = 300$ fb$^{-1}(3$ ab$^{-1})$. This translates to an effective $\mu\mu$ Majorana mass of $|m_{\mu\mu}| \sim 7.3(5.4)$ GeV and establishes a road map for testing the Weinberg operator at accelerators.

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I. INTRODUCTION

Among the most pressing mysteries shared in cosmology, nuclear, and high-energy physics is whether neutrinos are their own antiparticles [1,2]. This importance follows from Majorana neutrinos being necessary ingredients for standard leptogenesis, grand unification, as well as new gauge symmetries. Discovering that neutrinos are Majorana particles would indicate that lepton number (LN) symmetries are not conserved below the electroweak (EW) scale and demonstrate the existence of a mass-generating mechanism beyond those responsible for chiral and EW symmetry breaking (EWSB).

Motivated by this, broad, complementary approaches are taken to explore the nature of neutrinos [3–10]. A foremost probe is the search for the neutrinoless $\beta\beta$ process (0νββ) in decays of nuclei. This is characterized by the transition $(A, Z) \rightarrow (A, Z + 2)$ and the appearance of two same-sign electrons but an absence of neutrinos in the final state.

While no discovery has been confirmed, and assuming that the decay is mediated solely by the light neutrinos observed in nature, searches place upper limits of 79–180 meV at 90% confidence level (C.L.) [11] on the so-called effective $\beta\beta$ Majorana mass, given by [12]

$$|m_{ee}| = \left| \sum_{k=1}^{3} U_{e k} m_{\nu_k} U_{e k} \right|.$$ (1)

In this definition, $m_{\nu_k}$ are the mass eigenvalues of the three light neutrinos and $U_{e k}$ are the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix elements.

From the perspective that the Standard Model (SM) of particle physics is a low-energy effective field theory (EFT), Majorana neutrino masses, and hence $|m_{ee}|$, can be generated most minimally [13,14] at dimension $d = 5$ from the LN-violating Weinberg operator [15],

$$\mathcal{L}_5 = \frac{C_5^{e\nu} \Phi}{\Lambda} [\Phi \cdot \bar{T}_F^e][L_e \cdot \Phi] + \text{H.c.}$$ (2)

Here, $\Lambda$ is the scale at which the particles responsible for LN violation become relevant degrees of freedom, $C_5^{e\nu}$ is a flavor-dependent Wilson coefficient, $L_\ell^e = (\nu_\ell, e^-)$ is the left-handed (LH) lepton doublet, and $\Phi$ is the SM Higgs doublet, whose vacuum expectation value (VEV) $v = \sqrt{2}\langle \Phi \rangle \approx 246$ GeV generates the quantity...
II. THE STANDARD MODEL AT DIMENSION FIVE

To describe Majorana neutrino masses and the $0\nu\beta\beta$ process from $d = 5$ interactions, we work in the SM effective field theory [27] and extend the SM Lagrangian ($\mathcal{L}_{\text{SM}}$) by gauge-invariant operators of $d > 4$. In the canonical representation [28], the Lagrangian is given by [15]

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_5 + \mathcal{O}(\Lambda^{-2}),$$

where $\mathcal{L}_5$ is defined in Eq. (2). By the power counting of Ref. [28], the Weinberg operator is the only gauge-invariant operator at $d = 5$ in the SM [15,29].

After EWSB, the Higgs field can be expanded about its VEV, which in the unitary gauge reads $\sqrt{2}\Phi \approx v + h$, where $h$ is the Higgs boson. The resulting Lagrangian is

$$\mathcal{L}_5 = -\frac{C_5^{\ell\ell'}}{2\Lambda} h\bar{\nu_{\ell'}}\nu_{\ell} - \frac{C_5^{\ell\ell'}}{\Lambda} h\bar{\nu_{\ell}}\nu_{\ell'} - \frac{C_5^{\ell\ell'}}{2\Lambda} v^2 \bar{\nu_{\ell}}\nu_{\ell'} + \text{H.c.}$$

Here, $C_5^{\ell\ell'}$ is defined in the flavor basis. The minus signs above originate from the SU(2)$_L$-invariant product $\Phi \cdot \mathcal{T} = \Phi^j \varepsilon_{ij} \mathcal{L}^j$, with $\varepsilon_{12} = 1$. While the first two terms in Eq. (5) signify double- and single-Higgs couplings to neutrinos of flavors $\ell \ell'$, the third term generates the $3 \times 3$ LH Majorana mass matrix $m_{\nu\ell}$, as defined in Eq. (3). After rotating $m_{\nu\ell}$ into the mass basis, the resulting eigenvalues parametrize the three neutrino mass eigenstates $m_{\nu_k}$ that describe neutrino oscillation data.

We make no assumption on the structure of $C_5^{\ell\ell'}$. It is therefore possible under this framework that one neutrino is massless, as allowed by data [30]; that all masses scale as $m_{\nu_3} \sim \mathcal{O}(m_{\nu\ell})$, indicating minor fine-tuning; or that $m_{\nu_3} \ll m_{\nu\ell}$, indicating strong cancellations among the $m_{\nu\ell}$ elements. As nuclear searches are only sensitive to $|m_{\nu\ell}|$, the latter possibilities remain underexplored.

III. THE 0$\nu$ββ PROCESS AT DIMENSION FIVE

A goal of this work is to estimate the sensitivity of the Large Hadron Collider (LHC) and the high-luminosity LHC (HL-LHC) to the $0\nu\beta\beta$ process and hence the Weinberg operator. When simulating the Weinberg operator at the LHC, difficulties arise if working in the neutrinos’ mass eigenbasis. There, $d = 5$ vertices are proportional to $m_{\nu_k}$, which are unknown and small on LHC scales, and to $U_{ek}$, which carry unknown phases. So while the transition in Fig. 1 may proceed through a nontrivial incoherent sum of intermediate states, individual contributions may be too small for practical computations.

We propose a solution to this complication by working in the neutrino flavor basis and treating the mass term in Eq. (5) as a “two-point vertex.” From this perspective, the Weinberg operator in Fig. 1 couples one massless, LH neutrino of momentum $p$ and flavor $\ell$ with the conjugate of a second neutrino of momentum $p$ and flavor $\ell'$. After contracting Dirac matrices, the LN-violating $(\bar{\nu}_p \nu_{\ell'}^c)$ current in Fig. 1 reduces to the ratio of $m_{\ell\ell'}$ and the squared virtuality $p^2$. Explicitly, its graph simplifies to

$$m_{\ell\ell'} = C_5^{\ell\ell'} \frac{v^2}{\Lambda}.$$
vL(p)\nu_bar^e_R(-p) = \frac{i\gamma^\mu}{p^2} - \frac{iC_\alpha^\nu\nu^2}{\Lambda} \frac{p^\mu}{p^2} = \frac{im_{\ell^e}\ell^e}{p^2}.

Up to corrections of \mathcal{O}(m_{\ell^e}/p^2), which are assumed small, one can identify the rightmost ratio as the right-handed (RH) helicity state of an intermediate fermion with mass \(m_{\ell^e}\) and momentum \(p\). That is, one can write

$$\gamma^\mu P_L \frac{i(p + m_{\ell^e})}{p^2 - m_{\ell^e}^2} \nu_R = \gamma^\mu P_L \frac{im_{\ell^e}}{p^2 - m_{\ell^e}^2} P_L \nu_L \rho^\mu$$

$$= \gamma^\mu P_L \frac{im_{\ell^e}}{p^2} P_L \nu^\mu \times \left[ 1 + \mathcal{O}\left( \frac{m_{\ell^e}^2}{p^2} \right) \right].$$

where \(P_{R/L} = \frac{1}{2}(1 \pm \gamma^5)\) are the usual chiral projection operators in four-component notation and recover the same ratio at leading power of the expansion. Intuitively, this identification follows from the inversion of helicity in \(\nu\) violating currents as discussed in Refs. [23,31–36].

As a result, up to corrections of \(\mathcal{O}(m_{\ell^e}/p^2)\), the \((\nu_{\ell^e}\nu_{\ell^e}^\nu)\) current itself can be modeled as an unphysical Majorana neutrino \(N\) with mass \(m_{\ell^e}\) that couples to the \(W\) boson and all charged leptons \(\ell^e\) via the Lagrangian

$$\Delta L = -\frac{g_W}{\sqrt{2}} W^+_{\mu} \sum_{\ell=e}^r \frac{\tau}{N} \gamma^\mu P_L \ell^e - \text{H.c.}$$

Here, \(g_W \approx 0.65\) is the SU(2) weak coupling constant. Up to factors of active-sterile mixing, Eq. (8) is identical to the interaction Lagrangian in the phenomenological type I seesaw model [3,37], and therefore can also be employed in \(\nu\) violating decays of hadrons and leptons.

IV. SIGNAL AND BACKGROUND SIMULATION

To simulate the \(0\nu\beta\beta\) process in LHC collisions using mainstream MC tools, we exploit the above observation that the intermediate \((\nu_{\ell^e}\nu_{\ell^e}^\nu)\) current in Fig. 1 can be modeled as an unphysical Majorana neutrino with mass \(m_{\ell^e} = C_{\ell^e}^\nu v^2/\Lambda\). We implement the Lagrangian of Eq. (4) into the FeynRules software package (version 2.3.36) [38–41] by extending the feynrules implementation of the SM (version 1.4.7) by a single Majorana neutrino \(N\) with mass \(m_N\) and EW boson couplings governed by Eq. (8). We ensure that conventional factors are kept according to Ref. [41]. To account for all \(\ell^e\ell^\pm\ell^\mp\) flavor permutations accessible at LHC energies, we make \(m_N\) an internally calculated quantity that is set by

$$m_N = |C_{\ell^e}^{\nu\nu}|^2 + \frac{C_{\ell^e}^{\nu\nu} + C_{\ell^e}^{\nu\nu} + C_{\ell^e}^{\nu\nu} + C_{\ell^e}^{\nu\nu}}{\Lambda} \frac{v^2}{\Lambda}. \quad (9)$$

Using Refs. [42,43], we extract renormalization and \(R_2\) counterterms up to the first order in the quantum chromodynamic (QCD) coupling \(\alpha_s\). Feynman rules are collected into a set of public universal FeynRules output (UFO) libraries that we call the SMWEINBERG libraries.

With this UFO proton collisions are simulated at next-to-leading order (NLO) in QCD with the event generator MadGraph5_aMC@NLO (version 2.7.1.2) [44–49]. Parton showering (PS) and modeling of nonperturbative phenomena are handled by PYTHIA8 (version 243) [50]. Hadron-level events are passed through DELPHES (version 3.4.2) [51] for the simulation of an ATLAS-like detector. Hadron clustering is handled according to the anti-\(k_t\) algorithm at \(R = 0.4\) [52–54] as implemented in Fastjet [55,56]. We tune our simulation tool chain as in the study on \(W^+W^-\) scattering by Ref. [23], whose methodology we also follow to model SM backgrounds.

V. THE \(d=5, 0\nu\beta\beta\) PROCESS AT THE LHC

In LHC collisions, the \(\nu\) violating \(0\nu\beta\beta\) process occurs through the scattering of two same-sign \(W\) bosons that are sourced from quarks and antiquarks, and exit as two high-\(p_T\) jets. At the hadronic level, the collider signature is given by

$$pp \to jj\ell^\pm\ell^{\mp\pm} + X,$$ \quad (10)

where \(X\) represents the additional hadronic and electromagnetic activity that may exist in the inclusive process.

To identify the dependence of Eq. (10) on the Weinberg operator, we consider the effective \(W\) approximation [57–59] and treat the incoming \(W^+W^-\) pair as perturbative constituents of the proton. In this limit, we find that the \(W^+W^- \rightarrow \ell^\pm\ell^{\mp\pm}\) subprocess is dominated by the scattering of longitudinal \(W\) bosons. After summing over all external helicities, the spin-averaged, parton-level cross section for the \(2 \rightarrow 2\) process is given by

$$\hat{\sigma}(W^+W^- \rightarrow \ell^+\ell^{\mp\pm}) = \frac{2 - \delta_{\ell^e\ell^\nu}}{2\pi\alpha^2} \left| \frac{C_{\ell^e}^{\nu\nu}}{\Lambda} \right|^2 + \mathcal{O}\left( \frac{m_N^2}{M_{WW}^2} \right). \quad (11)$$

This shows that like in nuclear experiments the \(0\nu\beta\beta\) rate at the LHC scales as \(\sigma \sim |m_{\ell^e}/\Lambda|^2\).

Using this scaling behavior, we have checked that setting \(\Lambda \ll 200\) TeV in simulations with the SMWEINBERG UFO will generate unphysical cross sections. This is due to a breakdown of the expansion in Eq. (7), which requires \(v^2/\Lambda\) to be small compared to the virtuality of the internal \((\nu_{\ell^e}\nu_{\ell^e}^\nu)\) current. For the LHC and beyond, physical rates can be obtained by choosing, for example, \(\Lambda = 200\) TeV and using the relationship

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to the cross section, we show in the bottom of Fig. 2 the QCD $K$-factor, defined as the ratio of the NLO and leading order (LO) cross sections. We report that $O(\alpha_s)$ corrections are mild, reaching $K \sim 0.95–1.05$ across $\sqrt{s}$.

To estimate the LHC’s discovery potential of the Weinberg operator, we focus on the $e^+e^- = \mu\mu$ channel with benchmark inputs $C_s^{\ell\ell} = \delta_{\ell\mu} \delta_{\ell\mu}$ and $\Lambda = 200$ TeV (or $m_{\mu\mu} \approx 300$ MeV) and design an analysis inspired by the Run 2 performance of the ATLAS detector [60,61]. We employ particle identification requirements on electrons, muons, and jets that are summarized at the top of Table I. For simplicity, we ignore particles originating from pileup interactions as they would mostly be subtracted with dedicated algorithms in real experiments.

To define our signal enriched region, we demand events to have at least two jets, with the leading pair carrying a large invariant mass, and exactly two muons with the same charge. Events with additional leptons, including hadronically decaying $\tau$ leptons, are vetoed. To further reduce backgrounds, we take into account two qualitative differences between our signal and background processes:

(i) unlike SM processes with the same final state, our signal does not contain outgoing neutrinos. As neutrinos go undetected in LHC experiments, their presence gives rise to missing transverse momentum $E_T^\text{miss}$, which is defined as the $p_T$ recoil against all visible objects. We therefore require that events have a small $E_T^\text{miss}$, in accordance with the detector resolution.

(ii) Due to the lack of QCD color flowing between the two hadrons in Fig. 1, the hadronic activity is much milder than the QCD and $W^{\pm}V$ backgrounds. Following past studies [62–64], we impose an upper limit on the ratio $(H_T/p_T^\mu)^2$, where $H_T$ is the scalar sum of jet $p_T$. To guide our precise cut choice, we plot in Fig. 3 the $(H_T/p_T^\mu)$ distribution for our signal and leading backgrounds after applying all other selection cuts.

At this stage, the leading backgrounds consist of mixed EW-QCD production of $W^{\pm}W^{\pm}jj$, pure EW production of $W^{\pm}W^{\pm}jj$, and the inclusive diboson + jets spectrum $W^{\pm}V + nj$, with $V \in \{\gamma/Z/Z^{*}\}$. We checked that other processes, e.g., $t\bar{t}W^{\pm}$, do not appreciably survive our event selection. We neglect processes that are especially difficult

\section*{TABLE I. Particle identification and signal region definitions.}

| Particle identification cuts                                                                 |
|---------------------------------------------------------------------------------------------|
| $p_T^{(\ell\ell)} > 10(10)[25]$ GeV, Anti-$k_T(R = 0.4)$                                     |
| $|\eta^{(\ell\ell)}| < 2.5(2.7)[4.5]$                                                        |
| Signal region cuts                                                                          |
| $p_T^{D(jj)} > 27(10)$ GeV, $n_\ell = 2, n_j \geq 2, n_\ell = n_{had} = 0$, $Q_{\mu\ell} \times Q_{\mu\ell} = 1$, $M(j_1, j_2) > 700$ GeV |
| $E_T^\text{miss} < 30$ GeV, $(H_T/p_T^{\mu}) < 1.6$                                        |
| Changes to identification and signal cuts at $\sqrt{s} = 100$ TeV                           |
| $|\eta^{(\ell\ell)}| < 4.0(4.0)[5.5]$, $M(j_1, j_2) > 1$ TeV                                |
| $E_T^\text{miss} < 20$ GeV, $(H_T/p_T^{\mu}) < 0.6$                                       |
to simulate from MC methods alone. This includes when muons are assigned the wrong charge during event reconstruction. While subdominant for dimuon final states, muons are assigned the wrong charge during event reconstruction. This includes when muons are assigned the wrong charge during event reconstruction. We then solve this equality for \( \sigma \) to simulate from MC methods alone. This includes when muons are assigned the wrong charge during event reconstruction. While subdominant for dimuon final states, muons are assigned the wrong charge during event reconstruction.

**VI. SENSITIVITY TO THE WEINBERG OPERATOR**

To quantify any excess of events, we apply a Poisson-counting likelihood with a background rate uncertainty that is constrained by an auxiliary Poisson measurement \[68,69\]. Assuming a \( \delta_b = 20\% \) systematic uncertainty in the background, we estimate the sensitivity at 95\% C.L. to \( |C_S^{\mu\mu}|/\Lambda \propto m_{\mu\mu} \) by fixing our signal significance to \( \Lambda = 200 \text{ TeV} \) to scales below

\[
\Lambda/|C_S^{\mu\mu}| \lesssim 8.3(11) \text{ TeV.} \tag{14}
\]

These translate into effective \( \mu\mu \) Majorana masses of

\[
|m_{\mu\mu}| \gtrsim 7.3(5.4) \text{ GeV.} \tag{15}
\]

With an outlook to potential successors of the HL-LHC \([1,2]\), we estimate the sensitivity of a \( \sqrt{s} = 100 \text{ TeV} \) proton collider. We employ our LHC analysis but with changes listed at the bottom of Table I. We set \( \delta_b = 5\% \) to account for improved detector resolution and control region modeling. For \( \mathcal{L} = 50 \text{ ab}^{-1} \), we find sensitivity to \( \Lambda/|C_S^{\mu\mu}| \lesssim 48 \text{ TeV at 95\% C.L.} \) Our precise choice of cuts is for illustration and optimization should be investigated. This is especially relevant as we neglect an \( O(30\%) \) statistical uncertainty on our \( W^\pm W^\pm \) simulation despite starting from \( 10^7 \) NLO + PS events.

As described above, treating the Weinberg operator as an unphysical Majorana fermion is applicable to LN-violating decays of mesons, so long as the expansion in Eq. (7) is satisfied. Using Refs. \([70–72]\), we update the limits and projections on \( m_{\mu\mu} \) from \( B^{\pm \to \pi^\pm \mu^+ \mu^-} \) and \( K^{\pm \to \pi^\pm \mu^+ \mu^-} \) decays. We find that LHCb with \( \mathcal{L} = 300 \text{ fb}^{-1} \) can only probe \( \Lambda/|C_S^{\mu\mu}| \lesssim 9 \text{ MeV} \) while NA-62 has excluded with its 2017 data set \([72]\), and allowed values by best fits to neutrino oscillation data \([73]\).

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possibly charged lepton flavor. Motivated by the flavor limitations of nuclear decay experiments, we have investigated the LHC’s sensitivity to the Weinberg operator using the $W^\pm W^\pm \rightarrow \ell_i^\pm \ell_j^\pm$ process, which permits muon- and tau-flavored final states. We find sensitivity that exceeds representative searches at $B$- and $K$-meson factories and establishes a complementarity across accelerator facilities in the search for the Weinberg operator.

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APPENDIX: TECHNICAL DETAILS ON METHODOLOGY

In this appendix, we provide additional discussions and details of our methodology.

In a generic gauge, the Higgs field in terms of the EW Goldstone bosons $G^{\pm,0}$ is $\sqrt{2} \Phi = ( -i \sqrt{2} G^+, v + h + i G^0 )^T$. Explicit contraction of $SU(2)_L$ indices then gives

$$L_\ell \cdot \Phi = L_i^j e_{ij} \Phi^j = \frac{1}{\sqrt{2}} \nu_\ell (v + h + i G^0) + i \ell^G^+,$$  
(A1)

$$\Phi \cdot L_\ell^T = -i G^+ \ell^T - \frac{1}{\sqrt{2}} (v + h + i G^0) \ell^T.$$  
(A2)

This allows us to express the full Weinberg operator as

$$L_\ell \cdot \Phi = -\frac{C_5^G}{2\Lambda} (v^2 + 2vh + hh) \ell^T \nu_\ell,$$  
(A3)

$$-i \frac{C^G}{\sqrt{2} \Lambda} G^+ (v + h)(\ell^T \nu_\ell + \ell^T \nu^T_\ell),$$  
(A4)

$$-i \frac{C^G}{\Lambda} G^0 (v + h) \ell^T \nu_\ell.$$  
(A5)

Here and below, the Hermitian conjugate is understood to apply to the full expression, not simply the final line.

With this, the interaction Lagrangian by which we extend the SM Lagrangian in the smWeinberg UFO is

$${\Delta L} = -\frac{g_W}{\sqrt{2}} W_\mu^\ell \sum_\ell \bar{N}_\ell \gamma^\mu P_L \ell^\nu,$$  
(A8)

$$-\frac{g_W}{2 \cos \theta_W} Z_\mu \sum_\ell \bar{N}_\ell \gamma^\mu P_L \ell^\nu,$$  
(A9)

$$-\frac{g_W m_N}{2 m_W} \left( 1 + \frac{g_W}{4 m_W} h \right) \sum_\ell \bar{N}_\ell P_L \ell^\nu,$$  
(A10)

$$-\frac{g_W m_N}{2 \sqrt{2} m_W} G^+ \left( 1 + \frac{g_W}{2 m_W} h \right) \sum_\ell \bar{N}_\ell P_L \ell^\nu,$$  
(A11)

$$+ \frac{g_W m_N}{8 m_W^2} G^0 \left( 1 + \frac{g_W}{2 m_W} h \right) \sum_\ell \bar{N}_\ell P_L \ell^\nu,$$  
(A12)

$$+ \frac{g_W m_N}{4 \sqrt{2} m_W^2} G^0 G^+ \left( \sum_\ell \bar{N}_\ell P_L \ell^\nu + \sum_\ell \bar{N}_\ell P_L \ell^\nu \right),$$  
(A13)

$$+ \frac{g_W m_N}{4 \sqrt{2} m_W^2} G^0 G^+ \left( \sum_\ell \bar{N}_\ell P_L \ell^\nu + \sum_\ell \bar{N}_\ell P_L \ell^\nu \right) + \text{H.c.}$$  
(A14)

To further understand the identification in Eq. (7), we recall that the fermions in the LN-violating ($\ell^T \nu_\ell \nu^T_\ell$) current in Fig. 1 experience an additional parity inversion beyond the standard $SU(2)_L$ chiral couplings [31,32]. In terms of Feynman rules [33,34], this manifests as a chiral inversion of the $(W^\ell \nu_\ell)$ vertex, i.e., $\gamma^\ell P_L \rightarrow \gamma^\ell P_R$. In the absence of additional new physics, this ensures [35,36] the presence of the $P_{R/L}$ projection operators that envelope the $(\nu_\ell \nu^T_\ell)$ current in Eq. (6), and hence that the $(\nu_\ell \nu^T_\ell)$ current propagates in the RH helicity state.

The model’s input parameters along with their FeynRules and Les Houches [75] information are summarized in Table II. The syntax used to import the UFO into MadGraph5_aMC@NLO and simulate the process...
where \( q \) is any light quark or antiquark, at NLO is import model SMWeinbergNLO

\[
q_1q_2 \rightarrow q'_1q'_2 \mu^+\mu^-.
\]  
(A15)

As a check of the SM Weinberg UFO, we consider the process \( p p \rightarrow \mu^+ \mu^- j j \) \( \text{QED}=4 \text{ QCD}=0 \)

generate \( p p \rightarrow \mu^- \mu^- j j \) \( \text{QED}=4 \text{ QCD}=0 \)

For SM inputs, we approximate the quark sector by \( n_f = 5 \) massless quarks that do not mix. Values of couplings and masses are set to global averages reported in the 2020 Particle Data Group review [76],

\[
m_t(m_t) = 172.76 \text{ GeV}, \quad m_b = 125.1 \text{ GeV};
\]

\[
M_Z = 91.1876 \text{ GeV}, \quad \alpha_{\text{QED}}(M_Z) = 127.952;
\]

\[
G_F = 1.1663787 \times 10^{-5} \text{ GeV}^{-2}, \quad \alpha_s(M_Z) = 0.118.
\]  
(A16)

We employ the NNPDF3.1 NLO + LUXqed parton distribution function set (lhaid=324900) [77–79], with scale evolution driven by LHAPDF [80], and PDF uncertainties are extracted using the replica method [79,80]. We fix the collinear factorization (\( \mu_f \)), QCD renormalization (\( \mu_r \)), and shower matching (\( \mu_s \)) scales to the default values in Ref. [45]. The uncertainty in choosing \( \mu_f \) and \( \mu_s \) is quantified by scaling their baseline values by factors of 0.5, 1, and 2 to obtain a nine-point uncertainty band.

As a check of the SM Weinberg UFO, we consider the amplitudes for the processes \( W^\pm (p_1^W, \lambda_1^W)W^\pm (p_2^W, \lambda_2^W) \rightarrow \epsilon^\pm (p'_1, \lambda'_1)\epsilon^{\mp} (p'_2, \lambda'_2) \) process. Explicit calculation reveals that in the high-energy limit, i.e., when \( M_{WW}^2 = (p_1^W + p_2^W)^2 \gg m_W^2 \), the \( 2 \rightarrow 2 \) process is dominated by the scattering of two longitudinally polarized \( W^\pm \) bosons. For the \( (\lambda_1^W, \lambda_2^W) = (0, 0) \) helicity configuration, the exact helicity amplitude is

\[
-i\mathcal{M}(W_0^+ W_0^+ \rightarrow \epsilon_R^+ \epsilon_R^{+*}) = -i\mathcal{M}_f + -i\mathcal{M}_w.
\]  
(A17)

\[
-i\mathcal{M}_w = ie^{-i\phi}(\frac{C_S^{\epsilon\ell}(\Lambda)}{\Lambda})(\frac{M_{WW}^3}{t})
\]

\[
\times [1 - 2r_w + \sqrt{1 - 4r_w^2 \cos \theta_1}],
\]  
(A18)

where \( r_w = m_W^2 / M_{WW}^2 \); \( \theta_1 \) and \( \phi_1 \) are, respectively, the polar and azimuthal angles of \( \epsilon(p'_1) \) in the \( (WW) \) frame, and the kinematic invariants are defined by \( t = (p'_1^W - p'_2^W)^2 \) and \( u = (p_1^W - p_2^W)^2 \). Further evaluation of \( t \) and \( u \) results in the somewhat simple expression

\[
\mathcal{M}(W_0^+ W_0^+ \rightarrow \epsilon_R^+ \epsilon_R^{+*}) = e^{-i\phi}(\frac{4C_S^{\epsilon\ell}M_{WW}}{\Lambda}).
\]  
(A20)

The \( J = 0 \) partial wave is subsequently given by

\[
\mathcal{M}(W_0^+ W_0^+ \rightarrow \epsilon_R^+ \epsilon_R^{+*}) = \frac{1}{4\pi} \frac{C_S^{\epsilon\ell}M_{WW}}{\Lambda}.
\]  
(A21)

Since \( s \)-wave perturbative unitarity requires that \( |a_J| < 1 \), one obtains the constraint that

\[
|C_S| M_{WW} < 4\pi \Lambda.
\]  
(A23)

After evaluating the exact helicity amplitude for each \( (\lambda_1^W, \lambda_2^W) \) permutation, taking their sum, and then taking the high-energy limit, we obtain

\[
\sum_{(\lambda_1^W, \lambda_2^W)} |\mathcal{M}(W^+ W^+ \rightarrow \epsilon^+ \epsilon'^{+*})|^2 = 8(2 - \delta_{\epsilon\ell})\left(\frac{C_S^{\epsilon\ell}M_{WW}}{\Lambda}\right)^2 + O\left(\frac{m_W^2}{M_{WW}^2}\right).
\]  
(A24)

The Kronecker \( \delta \) accounts for the \( 1/2! \) symmetry factor needed for amplitudes with identical final-state particles. This implies a totally differential cross section of

\[
\frac{d\sigma}{d\cos \theta_1 d\phi_1} = \frac{(2 - \delta_{\epsilon\ell})}{8\pi^2 2^2} \frac{C_S^{\epsilon\ell}(\Lambda)^2}{\Lambda^2} + O\left(\frac{m_W^2}{M_{WW}^2}\right).
\]  
(A25)

Integration over the full solid angle recovers Eq. (11).

Using Eq. (11) as a check of the SM Weinberg UFO, we list in Table III the total \( 2 \rightarrow 4 \), hadronic cross section \( \sigma \) for Eq. (10) at NLO in QCD for representative cutoff scales \( \Lambda \), assuming a Wilson coefficient of \( C_S^{\epsilon\ell} = \delta_{\epsilon\ell}\delta_{\epsilon'\ell'} \), for the \( \sqrt{s} = 13 \text{ TeV} \) LHC and proposed experiments at \( \sqrt{s} = 27 \) and 100 TeV. Also listed are the corresponding (unphysical) Majorana neutrino mass \( m_N \), as defined by Eq. (9), the nine-point scale uncertainty \( \delta_{\text{scale}} \), the parton distribution
function uncertainty $\delta_{\text{PDF}}$, and the QCD $K$-factor, which is defined as the ratio $K = \sigma / \sigma^{\text{LO}}$, where $\sigma^{\text{LO}}$ is the LO rate.

For $\Lambda = 10$ TeV and 100 TeV at $\sqrt{s} = 13$ TeV, we observe a cross section scaling of $\sigma(\Lambda = 10 \text{ TeV}) / \sigma(\Lambda = 100 \text{ TeV}) \sim 93$, undershooting the 100x scaling expected from Eq. (11). We attribute this to a breakdown of Eq. (7), which requires the mass $m_N \sim n^2 / \Lambda$ to be small compared to the virtuality of $(\nu_{e} \nu_{\tau})$. At larger $\Lambda$, we find, for example, that $\sigma(\Lambda = 100 \text{ TeV}) / \sigma(\Lambda = 200 \text{ TeV}) \sim 3.93$ and $\sigma(\Lambda = 200 \text{ TeV}) / \sigma(\Lambda = 400 \text{ TeV}) \sim 3.99$, indicating behavior more inline with Eq. (11). We conclude that choices of $\Lambda \gtrsim 200$ TeV generate sufficiently small $m_N$ so that Eq. (7) remains valid for $\sqrt{s} \gtrsim 13$ TeV.

Assuming benchmark signal inputs of $\Lambda = 200$ TeV and $C_5^{\nu \nu} = \delta_{\nu \nu}\delta_{\tau \tau}$, we summarize in Table IV the expected number of signal and background events after all cuts for the LHC (HL-LHC) with $\mathcal{L} = 300 \text{ fb}^{-1}$ (3 $\text{ab}^{-1}$). To quantify the LHC’s sensitivity to the Weinberg operator, we define our signal significance $Z$ as [68,69]

$$Z = \frac{n(n_b - n)}{n(n_b - n)} \sqrt{2 \left[ n \log x - \frac{n^2}{\delta_b^2} \log y \right]}, \quad \text{with} \quad (\text{A26})$$

$$x = \frac{n_b(n_b + \delta_b^2)}{n_b^2 + n_b \delta_b}, \quad \text{and} \quad y = 1 + \frac{\delta_b^2(n - n_b)}{n_b(n_b + \delta_b^2)}, \quad \text{with} \quad (\text{A27})$$

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