MAJORANA NEUTRINO TRANSITION MAGNETIC MOMENTS IN LEFT-RIGHT SYMMETRIC MODELS

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Transition magnetic moments of Majorana neutrinos are discussed in the frame of the most natural version of the LR model (with left- and right-handed triplets and a bidoublet in the Higgs sector). We show that their largest values could be at most $6 \cdot 10^{-11} \mu_B$ from diagrams with $W_L$ in the loop. This could happen for specific models where (i) neutrino-charged lepton mixing is maximal and (ii) $\kappa_1 \simeq \kappa_2$ (VEVs for neutral Higgs fields in the bidoublet $\phi$ are equal). Contributions from diagrams with charged Higgses in the loop are smaller than those in the SM with right-handed neutrinos.

1 Introduction

1.1 Bounds on $\nu$-magnetic moments

The existence of a nonzero neutrino magnetic moment is a theoretically interesting issue in neutrino physics which is even strengthened by the first indication that neutrinos are massive particles. Whether it is also an experimentally relevant quantity depends obviously on its magnitude.

Measurements of the $\nu_\tau e^- \rightarrow \nu_\tau e^-$ (with $\nu_\tau = \bar{\nu}_e, \nu_e, \nu_\mu$) and the $\nu_\tau e^- \rightarrow \nu_\tau e^-$ cross sections give the following limits:

\begin{align*}
\mu_{\nu_\tau} & \leq 1.8 \cdot 10^{-10} \mu_B, \\
\mu_{\nu_e} & \leq 7.4 \cdot 10^{-10} \mu_B, \\
\mu_{\nu_\mu} & \leq 5.4 \cdot 10^{-7} \mu_B.
\end{align*}

There are also astrophysical bounds in addition to direct laboratory limits given above. In particular red giant luminosity and helium stars cooling by neutrinos emission impose $\nu$-magnetic moments smaller than $10^{-12}$ $\mu_B$. However, this limit is not as reliable as the terrestrial. We expect that values in the range $\mu \sim 10^{-10} - 10^{-12} \mu_B$ could have practical implications for the Sun, Supernova and/or neutron star physics.

Unfortunately, the Standard Model (SM) with its massless neutrinos and sole left-handed currents leaves no space for a nonvanishing neutrino magnetic moment. It is easily understandable. Since it arises from the operator $\sigma_{\mu\nu} q^\mu$ and as

$$\bar{\Psi}_f \sigma_{\mu\nu} \Psi_i = \bar{\Psi}_f L \sigma_{\mu\nu} \Psi_i R + \bar{\Psi}_f R \sigma_{\mu\nu} \Psi_i L,$$

we can see that there is a chirality change which makes it necessary to have both left and right-handed particle states.

The easiest way to avoid this shortcoming of the SM is to add right-handed singlets with additional mass terms. The latter also change chirality.

As it was found such a theory yields

$$\mu_{\nu_e\nu_\tau} = \frac{3eG_F}{16\sqrt{2}\pi^2} (m_\alpha + m_\beta) \sum_{l=e,\mu,\tau} \text{Im} \left( U_{\beta l}^\dagger U_{l\alpha} \right) \left( \frac{m_l}{M_W} \right)^2$$

$$\simeq 1.6 \times 10^{-19} \left( \frac{m_{\nu_e} + m_{\nu_\tau}}{1\text{eV}} \right) \times \sum_{l=e,\mu,\tau} \text{Im} \left( U_{\beta l}^\dagger U_{l\alpha} \right) \left( \frac{m_l}{M_W} \right)^2. \tag{5}$$

Recent Superkamiokande as well as solar neutrino results and cosmological arguments suggest that neutrino masses are much smaller than present terrestrial bounds. Since obviously

$$\sum_{l=e,\mu,\tau} \text{Im} \left( U_{\beta l}^\dagger U_{l\alpha} \right) \left( \frac{m_l}{M_W} \right)^2 < 10^{-4}$$

we can safely assume that in the very best of the cases $\mu_{\nu_e(\nu_\tau)} \leq 10^{-16} \mu_B$ for $m_{\nu_e}$ of order of a few MeV.

This discouraging conclusion shows that we need more sophisticated models than the SM alone (with right-handed neutrinos) to get experimentally viable magnetic moments.

1.2 Models with large neutrino magnetic moments

Plenty of models with large $\nu$-magnetic moments have emerged in literature during past decades. Those that remain interesting from the phenomenological point of view can be subdivided into two categories with reference to our problem (i) renormalizable ones (charged scalars...
in the SM, left-right symmetric models,...) and (ii) finite ones (MSSM, supersymmetric left-right model,...).

In the latter class there is a direct connection between the mass of the neutrino and its magnetic moment which requires special treatment (see the case of MSSM in [b]). The situation turns up to be much simpler in the former class where corrections to the mass are divergent. Then renormalization makes it a free parameter. As the magnetic moment contribution is always finite we can safely consider it alone.

So, how to make the magnetic moment contribution larger than in the SM?

First we need to generate a term containing no external neutrino mass. To one loop order this can only be done by left-right transition in vertices which is possible by adding

1. charged Higgs scalars, whose Yukawa couplings contain left-right transition from neutrinos to charged leptons,

and/or

2. right currents

Both of these are present in the left-right symmetric models, on which we shall concentrate in the present article. Although many estimations have been done by other authors, too rough treatments led to quite contrary conclusions [c][d]. Here we attempt to clarify the situation by extending the calculation to the Higgs sector and using more phenomenological arguments.

2 \(\nu\)-transition magnetic moments in Left-Right symmetric models

We concentrate on the popular version of the L-R symmetric model with Higgs bidoublet \(\phi\) and two Higgs triplets \(\Delta_{L,R}\). In such a model neutrinos have a Majorana character meaning only transition magnetic moments are allowed. Other Higgs sectors may lead to Dirac neutrinos, however the magnetic moment is very small [e].

Until now only diagrams with gauge bosons in the loop (see Fig.1) have been considered in the literature, with the dominant \(W_1\) gauge boson contribution.

However, as the charged Higgs - leptons (\(H^\pm l\nu\)) coupling is proportional to heavy neutrino masses, even if \(H^\pm\) masses are very large the contribution of diagrams with exchanged Higgs particles to \(\mu_\nu\) seems to be interesting.

The contribution to the \(\mu_{\alpha\beta}\) can be described by the following diagrams: (1) with gauge bosons \(W_{1,2}\) exchange (Fig.1), (2) with charged Higgs bosons \(H_{1,2}\) exchange (Fig.2), (3) with both \(W_{1,2}\) and \(H_{1,2}\) exchange (Fig.3).

\(\text{The calculation has been performed in the unitary gauge.}\)
2.1 Diagrams with gauge bosons in the loop

As $M_{W_2} \gg M_{W_1}$ the diagrams with $W_1$ exchange dominate. Their contribution to the $\mu_{\alpha\beta}$ can be classified into two categories:

1. Diagrams with neutrino mass insertion on the external neutrino legs, and

2. A diagram with charged lepton mass insertion on the internal lepton leg.

Diagrams from class (1) are proportional to the sum of neutrino masses $m_\alpha + m_\beta$ and are the same as in Eq.(5). The diagram (2) is proportional to the $W_L - W_R$ mixing angle $\xi$, namely

$$\mu_{\alpha\nu_b} \simeq \sqrt{\frac{2G_F}{\pi^2}} \sin \xi \cos \xi \cdot m_\epsilon \sum_{\alpha=\epsilon,\mu,\tau} m_\alpha Im\left( [K_R]_{\alpha\alpha}(K^\dagger_L)_{\alpha\beta} + (K_L)_{\alpha\alpha}(K^\dagger_R)_{\alpha\beta} \right) \mu_B. \quad (6)$$

The numerical value of $\mu_{\alpha\beta}$ depends on the mixing angle $\xi$ and the $K_R(K_L)$ matrix elements. If we assume that the $K_L$ matrix is diagonal (lepton number conservation), the $K_R$ matrix elements are given by the see-saw mechanism

$$K_R \sim O\left( \frac{<m_D>}{M_N} \right) \quad (7)$$

and $K_R \sim 0.01$ for $<m_D> \sim 1$ GeV and $m_N = 100$ GeV. Taking also that (limits from fits to low energy data

$$M_{W_2} \geq 477 \text{ GeV} \quad \xi \leq 0.031 \text{ rad} \quad (8)$$

we obtain (for $e(\mu)$ transition to $\tau$ neutrino)

$$\mu_{\alpha\beta} \lesssim 6 \cdot 10^{-13} \mu_B \quad (9)$$

This value is at the edge of physical interest. However, (9) is very optimistic. For $M_{W_2} \gg M_{W_1}$ we have

$$\xi \simeq \epsilon \left( \frac{M_{W_1}}{M_{W_2}} \right)^2 \quad (10)$$

From the above and (8) it follows that $\epsilon \simeq 1$ but this value (which is equivalent to $\kappa_1 \simeq 1$ for the bidoublet $\phi$) is very unprobable.

2.2 Diagrams with scalars in the loop

To calculate the diagrams of Fig.(2) and (3) we need couplings of charged Higgs particles to gauge bosons and leptons. To this end we must define the Yukawa and the Higgs sector. The latter is the most general potential with a vanishing VEV for the neutral field of the left-handed triplet $\Delta_L$, $v_L = 0$. All necessary couplings can be found in Eq.(11).

We found that all couplings vanish in the $\epsilon = \frac{2m_e}{m^2_{W_2}} \rightarrow 0$ limit. For $\epsilon \neq 0$ the magnitude of the Higgs diagram contribution to $\mu_{\alpha\beta}$ is also very small. For example diagrams of Fig.(3) for $H_2$ and $W_2$ exchange yield

$$\mu_{ab} \simeq \frac{1}{\sqrt{2} \cdot 4\pi^2} f(\epsilon) \cdot (m^N_a + m^N_b) Im(\bar{K}_R^L)$$

$$\times \frac{m_e}{M^2_{H_2} - M^2_{W_2}} \left( \frac{2M^2_{H_2}}{M^2_{H_2} - M^2_{W_2}} ln \left( \frac{M_{H_2}}{M_{W_2}} \right) - 1 \right) \mu_B. \quad (11)$$

where $f(\epsilon) < 1$ and $f(\epsilon) \rightarrow 0$ for $\epsilon \rightarrow 0$. Taking $M_{W_2(H_2)} = 1(1.6) \text{ TeV}$ we have

$$\mu_{ab} \lesssim f(\epsilon) \cdot 10^{-22} \left( \frac{m^N_a + m^N_b}{eV} \right) \mu_B. \quad (12)$$

Let’s note that the expectation that the Higgs diagrams contain terms proportional to the heavy neutrino masses was incorrect. Only light masses remain after cancellation due to the Majorana nature of neutrinos. Taking into account all diagrams (Figs.1-3) we can see that the dominant contribution to Majorana neutrino transition magnetic moment $\mu_{\alpha\beta}$ is given by $Eq.(6)$.

We also see that $\mu_{\alpha\beta} \neq 0$ if $K_{L(R)}$ matrices are complex which is the case of broken CP symmetry. When CP is conserved the magnetic transition is possible only for neutrinos of opposite CP eigenvalues. For three light neutrinos this leaves two nonzero values at most.

3 Conclusion

We have recalculated the transition magnetic moments of Majorana neutrinos in the Left-Right symmetric model with a bidoublet and two triplets. We found that the contribution of the diagrams with Higgs scalar exchange is very small, smaller than the SM contribution (Eq.5). Only one diagram (with $W_1$ exchange) gives an interesting result which for $\mu_{\nu_e\nu_\mu}$ transition magnetic moment is

$$\mu_{\nu_e\nu_\mu}/\mu_B \simeq 7 \cdot 10^{-12} |(\bar{K}_R)_{e\mu}| \left( \frac{0.5 TeV}{M_{W_2(TeV)}} \right)^2 \epsilon \quad (13)$$

For acceptable values of the $\bar{K}_R$ matrix elements, the mass of the heavy $W_2$ particle and the $\epsilon$ parameter, the value of $\mu_{\nu_e\nu_\mu}$ is however much too small to be interesting from experimental and astrophysical point of view.
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