What Knowledge can be Transferred Between Network Reconstruction and Community Detection?

Kai Wu\(^a\), Chao Wang\(^a,\)^*, Junyuan Chen\(^b\), Jing Liu\(^b,\)^*

\(^a\)School of Artificial Intelligence, Xidian University, Xi’an 710071, China
\(^b\)Guangzhou Institute of Technology, Xidian University, Guangzhou 510555, China

Abstract: This paper focuses on inferring network structure and community structure from the dynamics of the nonlinear and complex dynamical systems, which is prominent in many fields. Many methods have been proposed to solely address these two problems, but none of them consider explicit shareable knowledge across these two tasks. Inspired by the fact that a more precise network structure may promote the accuracy of community discovery and the better communities may promote the performance of network reconstruction (NR), this paper develops an evolutionary multitasking framework to make full use of explicit shareable knowledge among these two tasks to improve their performance; we refer to this framework as EMTNRCD. In EMTNRCD, we first establish these two tasks as a multitasking NR and community detection (CD) problem where one mission is to reconstruct network structure from dynamics and the other is to discover communities from dynamics. In the process of EMTNRCD, the NR task explicitly transfers several better network structures for the CD task and the CD task explicitly transfers a better community structure to assist the NR task, which improves the reconstruction accuracy of the NR task and the community division quality of the CD task. Moreover, to transfer knowledge from the study of the NR task to the CD task, EMTNRCD models the study of CD from dynamics as the problem of finding communities in the dynamic network and then decides whether to conduct knowledge transfer across tasks. This paper also designs a test suite for multitasking NR and CD problems (MTNRCDPs) to verify the performance of EMTNRCD. The experimental results have demonstrated that joint NR with CD has a synergistic effect, where the network structure used to inform the existence of communities is also inherently employed to improve the reconstruction accuracy, which, in turn, can better demonstrate the discovering of the community structure.

Keywords: Evolutionary multitasking optimization, knowledge transfer, network reconstruction, community detection.

1. Introduction

Complex networks are a useful model for understanding complex dynamics and playing an essential role in controlling and synchronizing complex dynamical systems [1], [2], which has attracted attention in many fields including engineering, computer, physical, biological, and social sciences. Moreover, uncovering communities in networks offers coarse-graining the complex relations between entities, which provides a more interpretable summary of a complex system. However, in many complex systems, the network structure between entities
and the complex systems’ nodal dynamics is unknown and unobservable. Instead, we may observe interdependent signals from the nodes, such as time series, which may be employed to infer these relationships. In most cases, only interdependent signals from the nodes are available, such as time series, which can be used to infer the network structure and community structure. Thus, this paper focuses on two tasks: one for reconstructing network structure from dynamics; another one for discovering communities from dynamics, which have become a central challenge in contemporary network science and engineering.

To address the challenge of the first task, a wide range of network reconstruction (NR) methods has been developed to reconstruct network structure from dynamics, which are divided into model-free methods and model-based methods. Model-free methods obtain the presence and strength of a link among nodes by measuring the dependence from their dynamics in terms of correlations[3], maximum entropy distributions [4], mutual information [5], random forest [6], and ensemble method [7]. Model-based methods provide prior knowledge of the dynamics and interactions and then employ this knowledge to infer the network structure, such as compressed sensing [8]-[12], evolutionary algorithm (EA) [13]-[20], and online learning [21]. Current methods for detecting communities when the network structure is unobservable typically involve a complicated process that is highly sensitive to specific design decisions and parameter selection. To address the challenge of the second task, most methods consist of two steps: first, select the NR methods to assess the similarity or the network structure of any pair of factors in a complex system described as above; Second, convert the resemblance to a dense weighted network or a binary network [22]. After determining the underlying network, the community detection (CD) methods are employed to uncover clusters of the network, for example, by maximizing the modularity [23]-[24] or using the map equation [22], [25], EA [26]-[35], deep learning [36], and nonnegative matrix factorization [37]. These approaches rely on the assumption that the network edges have been accurately observed [39]. Unlike the above methods, Hoffmann et al. [40] employed a Bayesian hierarchical model to discover the community structure from multivariate time-series data that bypasses the NR. However, neither of these approaches attempt to perform the NR task together with the CD task, and none of them considers knowledge transfer among these two tasks, which may promote the accuracy of the two
studies. We also find that a more precise network structure may promote the accuracy of community discovery and the better communities may promote the performance of the NR task. Thus, can joint optimization of the NR task and the CD task obtain better results?

To answer the above guess, this paper develops an evolutionary multitasking NR and CD framework to use the information of the network structure and community structure obtained from dynamics to improve the accuracy of the NR task and the CD task, termed as EMTNRCD. EMTNRCD is inspired by evolutionary multitasking optimization (EMTO) [41], a new paradigm for solving multiple optimization tasks by taking advantage of the parallelism mechanism of the human brain and evolutionary algorithm. In recent years, owing to its powerful search capability and easy scalability, EMTO has been successfully applied to overcome many practical challenges [42]-[57]. These studies show that transferring useful knowledge across tasks can improve the method’s convergence characteristics that solve each task solely [58]-[62]. Thus, to share knowledge across the NR task and the CD task, we first establish a multitasking model, where the community structure obtained from the CD task is explicitly transferred to improve the performance of the NR task, and the better network structures acquired from the NR task are explicitly transferred to enhance the performance of the CD task from dynamics. However, these existing EMT methods cannot be employed to handle this problem due to none of the EA-based CD methods can handle the task of CD from dynamics. Moreover, the gene coding of the NR task is continuous and the gene coding of the CD task is discrete. The existing EMT methods cannot solve this problem directly. Thus, we need to design a new EMT framework to handle this problem, which is our main contribution to the EMT community. Due to the difficulties of CD from dynamics, current EA-based CD methods cannot directly handle this problem due to the absence of objective function. Thus, we design a preprocessing stage to obtain the initial network structure before conducting the CD task. Moreover, in traditional CD tasks, communities are discovered from the given network. In our framework, we transfer the new network structure obtained from the NR task to the CD task, which leads to the fact that the CD task is different from discovering communities from the static network. To overcome this issue, we first model this process as a dynamic CD problem that needs to infer communities from a time-varying network.

To validate the performance of EMTNRCD, we design a test suite by employing
evolutionary game (EG) [63], [64] and resistor network (RN) [8], [14] models taking place in four real-world networks. Moreover, two state-of-the-art MOEAs for the NR task and a state-of-the-art population-based CD algorithm of the dynamic network are embedded in EMTNRCD. As shown in the experimental results, joining these two tasks has a synergistic effect, whereby the discovering of communities significantly increases the reconstruction accuracy, which in turn improves the performance of the CD task, when compared to performing these tasks in isolation. The highlights of the proposed EMTNRCD are summarized as follows:

1) At the problem level, we propose the joint optimization of the NR task and the CD task from the dynamics for the first time. This evolutionary multitasking framework is inspired by the phenomenon that a more precise network structure can improve the accuracy of community discovery and the better communities can promote the performance of the NR task [70]. In EMTNRCD, unlike existing methods that cope with the CD task and the NR task alone, we first establish a multitasking NR and CD problems (MTNRCDPs), where the NR task and the CD task are optimized simultaneously and each lesson may promote the performance by explicitly transferring useful knowledge, such as the network structure and communities, among them.

2) Given the difficulties encountered in establishing the EMTNRCD framework, current EMT methods cannot handle MTNRCDPs directly. Thus, we design the EMTNRCD framework, which is our main contribution to the EMT community at the algorithm level. We design a pre-optimization stage to obtain the initial network structure and initial community structure, which is regarded as the cornerstone of explicit knowledge transfer. Moreover, the CD task with knowledge transfer is modeled as a dynamic CD problem that needs to detect communities in a dynamic network obtained from the NR task. To assist the NR task, we design two local search strategies to utilize the inter-community and intra-community structural information transferred from the CD task, respectively.

We organize the rest of this paper as follows. The existing EA-based NR methods, EA-based CD methods, and evolutionary multi-objective multitasking optimization (EMMTO)
are reviewed in Section 2. Section 3 gives an introduction to the designed MTNRCDPs. The
details of EMTNRCD are described in Section 4. Section 5 presents the experimental results
on the designed test suite to illustrate the effectiveness of our methods. Finally, Section 6
summarizes the work in this paper and discusses the potential directions for future works.

2. Related Work

2.1. EA-based NR Methods

The NR task aims to reconstruct the links between each pair of entities. Various
EA-based methods have been proposed to overcome NR problems. Here, we briefly
introduce several traditional methods to show our motivation for EMTNRCD. These
methods share the same inference pattern in handling the task of NR from dynamics. First,
the inference model, such as fuzzy cognitive maps [65]-[69], S-system [15], [17], [19],
recurrent neural network [20], is employed to model the observed data obtained from
dynamics. Then the EA is used to optimized the parameters of the inference model. Finally,
the designed inference model is used to obtain the network structure of the complex system.
The core of these methods is to develop more accurate models and high-performance
optimizers. Several methods are proposed to reconstruct the network structure based on the
known complex behavior [13], [14]. However, these methods don’t consider the influence of
community structure on the NR task, which may improve the NR task’s performance,
especially for the high-dimensional NR problems. The most related work (CEMO-NR)
proposed in [70] employed community structure information to aid the NR task by
decomposing the original problem into several low-dimensional subproblems. However,
CEMO-NR’s aim didn’t consider the CD task and didn’t consider transfer useful knowledge
to improve the performance of the CD task, which is conducted alone. Moreover, both tasks
were not optimized simultaneously. However, in EMTNRCD, the idea of transferring the
communities for the CD task is inspired by CEMO-NR [70].

2.2. EA-based CD Methods

Many EA-based methods have been proposed to discover the community structure from
the different networks [35], such as large-scale networks [26], [33], dynamic networks [27], [28], attributed networks [32], and signed social networks [34]. Moreover, several works are proposed to handle the more difficult task of discovering overlapping communities [30]-[32]. However, they are not the ability to discovering communities from dynamics. These works assume that the exact network structure has been obtained before performing the CD. Our proposed EMTNRCD is the first EA-based framework to overcome this challenge.

In general, to infer communities from dynamics, most methods need to perform the following two steps: First, one NR method is chosen to reconstruct network structure from dynamics. Second, the EA-based CD methods are employed to detect clusters of the network obtained by the first step. This process is described in Fig. 1. This type of approach relies on the assumption that the network edges have been accurately observed. Meanwhile, the error suffered by the CD will enlarge the error of the whole process. Unlike the above methods, Hoffmann et al. [40] proposed a Bayesian hierarchical model to directly discover communities from time series without considering the process of the NR. However, neither of these approaches attempt to perform the NR together with the CD, and none of them considers knowledge transfer among these two tasks, which may promote the accuracy of the CD from dynamics.

By using the characteristics of the NR task and CD task, the proposed EMTNRCD employs the community structure obtained by the CD task to improve reconstruction accuracy and operates the better network obtained by the NR task to find a better community structure.

2.3. EMMTO

The NR task and the CD task are modeled as MMTO problems in this paper. Therefore, we review the existing methods for EMMTO to demonstrate the effectiveness of our proposal. Inspired by the multifactorial inheritance between organisms and the parallelism of population-based search, a multiobjective multifactorial evolutionary algorithm (MO-MFEA) was proposed in [42] for optimizing multiple tasks simultaneously in a single population. In MO-MFEA, the chromosome is encoded in a unified search space to guarantee population-based search efficiency. Then each task is viewed as a factor of the chromosome in the population. We give several definitions of this factor as follows:
1) **Factorial Cost** $f^j_i$: The objective function value of chromosome $i$ on task $j$.

2) **Factorial Rank** $r^j_i$: The rank index of chromosome $i$ on the ascending factor cost list corresponding to task $j$.

3) **Skill Factor** $\tau_i$: $\tau_i = \text{argmin}\{r_j^i\}$ indicates the task associated with chromosome $i$.

According to the skill factor, MO-MFEA implicitly classifies the population into $K$ groups that are dedicated to different tasks. The chromosome carried knowledge is transferred across tasks by assortative mating and selective imitation during the search process. MO-MFEA designed a fixed-parameter $rmp$ to control the knowledge transfer simply, which does not consider the relationship between tasks. Recently, the theoretical foundations of MO-MFEA reported in [43] show harmful (negative) inter-task interactions are related to $rmp$. Therefore, Bali et al. [43] further proposed MO-MFEA-II to overcome this issue. In this method, a transfer parameter matrix RMP was maintained by online transfer parameter estimation to control the degree of knowledge. The optimal mixture of probabilistic models was designed adaptively to capture useful common knowledge across tasks, which ensures harmful inter-task interactions were reduced.

Besides, some methods with adaptive knowledge transfer capabilities have also attracted great attention. Feng et al. [58] proposed an explicit EMMTO algorithm (EMEA) in which each task is assigned an independent evolutionary solver. Then a denoising autoencoder is used to learn optimal linear mappings between tasks through two populations sampled from two different search spaces. In EMEA, a new explicit knowledge transfer operator is designed to maintain biases of multiple search mechanisms. More recently, Lin et al. [45] proposed a novel EMMTO algorithm based on incremental Naive Bayes classifiers to finding useful knowledge (solutions) during the multitasking search. Moreover, a randomized mapping is proposed to improve the exploration of transferred solutions in the tasks’ search space.

However, none of them is suitable for MTNRCDPs. In traditional CD tasks, communities are discovered from the given network. Current EA-based CD methods cannot directly handle the task of CD from dynamics due to the absence of objective function. Moreover, the coding of the NR task is continuous and the coding of the CD task is discrete. Current EMTO methods cannot be applied to handle this problem. Thus, we need to design a new EMT framework to cope with MTNRCDPs. To deal with this problem, we design a preprocessing
stage to obtain the initial network structure before conducting the CD task. Moreover, we transfer the new network structure obtained from the NR task to the CD task and it is a dynamic process that is different from discovering communities from the static network. To overcome this issue, we first model this process as a dynamic CD problem that needs to infer communities from a time-varying network. It is our contribution to the EMT community. Moreover, in terms of MTNRCDPs, there is no other EMT algorithm available here.

3. Multitasking Network Reconstruction and Community Detection Problems

This section introduces the background knowledge on MMTO problems, NR problems (NRPs), and CD problems (CDPs). Then we discuss how to model NRPs and CDPs as MTNRCDPs.

3.1. MMTO Problems

Generally, for the \( K \) minimization tasks, the MMTO problem can be mathematically formulated as follows:

\[
\begin{align*}
\min_{x_i} F_i(x_i) &= \left( f_{i1}^1(x_i), f_{i2}^2(x_i), \ldots, f_{im_i}^{m_i}(x_i) \right), i = 1, 2, \ldots, K \\
\text{s.t. } x_i &= [x_i^1, x_i^2, \ldots, x_i^{m_i}] \in D_i, i = 1, 2, \ldots, K
\end{align*}
\]

where \( F_i(x_i) \) is the \( i \)-th multiobjective optimization task and \( D_i \) is the search space for optimization task \( i \). \( n_i \) and \( m_i \) are the number of objective functions and the dimensionality of \( x_i \) in the \( k \)-th task, respectively. Suppose that \( x_i^{(1)} \) and \( x_i^{(2)} \) be two solutions for the \( i \)-th multi-objective optimization task, \( x_i^{(1)} \) is said to Pareto dominate \( x_i^{(2)} \), if and only if \( f_i^j(x_i^{(1)}) \leq f_i^j(x_i^{(2)}) \) (\( \forall j \in \{1, 2, \ldots, m_i\} \)) and there exists at least one objective \( f_i^k \) (\( k \in \{1, 2, \ldots, m_i\} \)) satisfying \( f_i^k(x_i^{(1)}) < f_i^k(x_i^{(2)}) \). \( x_i^* \) is known to Pareto optimal if there no \( x_i \) such that \( x_i \) dominates \( x_i^* \). The collection of all Pareto optimal solutions is named the Pareto-optimal set (PS), and the projection of the PS in the objective space is called the Pareto-optimal front (PF). In summary, the goal of MMTO is to find a set of solutions approximating the PF for each multiobjective optimization task, which should be as close as possible to the PF and distributed evenly and widely over the PF [43].

By exploiting the potential similarities between multiple tasks, MMTO aims to improve
the performance of each multiobjective optimization task in terms of convergence and
diversity. In general, the key to designing the EMMTO algorithm lies in the following two
issues [45]: For each task, 1) how to select practical transferred knowledge from other tasks?
and 2) how to make full use of the shared knowledge from other tasks?

3.2. NRPs

A complex network can be modeled as a graph $G = (V, E)$, where the nodes (vertices) in $V$
represent the individuals in the network, and the edges in $E$ represent the relationship
among individuals [71]. The network structure between nodes is defined as an $N \times N$ weight
matrix $X:

$$X = \begin{bmatrix}
x_{11} & \cdots & x_{1N} \\
\vdots & \ddots & \vdots \\
x_{N1} & \cdots & x_{NN}
\end{bmatrix}.$$ \hspace{1cm} (2)

where $x_{ij} \in \{0, 1\}$ represents the connection between nodes $i$ and $j$ and $N$ represents the
number of nodes.

Let $Y$ and $h(X, Y)$ be the observed data and the inference model simulation from the
candidate network structure, respectively. The goal of the NRP is to infer the connections
between each pair of nodes according to $Y$ and $h(X, Y)$. In general, A NRP can be simplified
as follows [70]:

$$\min_{X} F(X) = (h(X, Y), g(X))$$

$s.t. X = \begin{bmatrix}
x_{11} & \cdots & x_{1N} \\
\vdots & \ddots & \vdots \\
x_{N1} & \cdots & x_{NN}
\end{bmatrix} \in \{0, 1\}^{N \times N}.$ \hspace{1cm} (3)

The definition of $h(X, Y)$ and $g(X)$ is determined by the complex network’s inference
model. Next, we introduce two common NRPs, EGNRPs and RNRPs.

EGNRPs. The evolutionary game (EG) model is commonly used to model node-to-node
interactions in various complex systems. In each round of an EG model, two players must
choose a cooperation (Co) strategy or defection (De). Then the payoffs of these players are
determined by the strategy and the payoff matrix of the game. The prisoner’s dilemma games
(PDG) [64] are employed in this paper. Its payoff matrix is denoted as:
\[ P_{\text{EG}} = \begin{pmatrix} 1 & 0 \\ 1.2 & 0 \end{pmatrix}. \] (4)

If both choose different strategies, the defector gets a reward of 1.2, and the cooperator receives a prize of 0. And if both decide to cooperate (or defect), all players get rewards 1 (or 0). Formally, for player \( i \) in round \( t \), its payoff can be expressed as follows:

\[ Y_i(t) = \sum_{j=1}^{N} x_{ij} S_j^T(t) P_{\text{EG}} S_j(t) \] (5)

where \( S_j(t) \) is the strategy of player \( i \) in the \( t \)-th round, and \( T \) represents “transpose”. And \( x_{ij} = 1 \) if players \( i \) and \( j \) are connected and \( x_{ij} = 0 \) otherwise. To maximize their payoff at the next round, players adjust strategies according to their payoff and their neighbors after each round of the game. Fermi rule [63] is adopted to update the strategy in our simulations, which can be expressed as follows:

\[ W(S_i \leftarrow S_j) = \frac{1}{1 + \exp \left( \frac{Y_i - Y_j}{\kappa} \right)} \] (6)

where \( \kappa = 0.1 \). To find the links among players, the EGNRPs can be formulated as:

\[
\min \sum_{i,j} \left[ X_i \cdot Y_i - Y_i \cdot Y_j \right] + \frac{1}{\kappa} \sum_{i=1}^{N} \left\| X_i \right\|_2
\]

\[ U_i = \begin{pmatrix} S_{i1}^T(1)P_{\text{EG}}S_1(1) & \cdots & S_{i1}^T(1)P_{\text{EG}}S_N(1) \\ \vdots & \ddots & \vdots \\ \vdots & \cdots & \vdots \\ S_{iL}^T(L)P_{\text{EG}}S_1(L) & \cdots & S_{iL}^T(L)P_{\text{EG}}S_N(L) \end{pmatrix}, i = 1,2,\ldots,N \] (7)

where \( L \) is the number of rounds. The first goal is to minimize the difference between the real payoff data and the generated payoff data of all players, and the second goal is to ensure the sparsity of the learned EG network.

The numerical simulation of the EG is described as follows [9]:

a) Input an EG network with nodes representing players;

b) Each player chooses a strategy of cooperation or defection;

c) Calculate the payoff of player \( i \) by (5);

d) Update the strategy of player \( i \) by (6);

e) Repeat Step c) to Step d) \( T \) times;

For this dynamic process, we record the strategies and the payoffs of all players in different rounds as observational data.
RNRPs [8]. Resistor network dynamics is a standard circuit system considering current transportation in the resistor. We denoted the resistance of a resistor between nodes \( i \) and \( j \) as \( r_{ij} \). For simplicity, \( r_{ij}=1 \) if \( i \) and \( j \) are directly connected by a resistor and \( r_{ij} = \infty \) otherwise. According to Kirchhoff’s laws at different periods, if the voltages at the nodes and resistances of connections are known, the currents at the nodes can be calculated as follows:

\[
I_i(t) = \sum_{j=1}^{N} \frac{1}{r_{ij}} (V_j(t) - V_i(t)) \tag{8}
\]

where \( I_i \) is the total current at node \( i \) and \( V_i = V^* \sin[(w+\Delta w_i)t] \) is the voltage. In this paper, \( V^*=1 \) is the voltage peak, \( w=10^3 \) is the frequency, and \( \Delta w_i \in [0, 20] \) is the perturbation. We can assume that only the voltages and currents at the nodes are measurable, and the resistor network can be reconstructed as follows:

\[
\min_X F(X) = \left( h(X,Y) = \sum_{i=1}^{N} \| R X_i - Y_i \|_2, g(X) = \| Y \|_1 \right)
\]

\[
\begin{align*}
X_i = \begin{bmatrix} x_{i1} = \frac{1}{r_{i1}}, x_{i2} = \frac{1}{r_{i2}}, \ldots, x_{iN} = \frac{1}{r_{iN}} \end{bmatrix}^T 
\in \{0,1\}^N
\end{align*}
\]

\[ s.t. \quad Y_i = [I_i(1), I_i(2), \ldots, I_i(L)]^T, i = 1, 2, \ldots, N \]

\[ R_i = \begin{bmatrix} V_i(1) - V_i(1) & \ldots & V_i(1) - V_i(N) \\ \vdots & \ddots & \vdots \\ V_i(L) - V_i(1) & \ldots & V_i(L) - V_i(N) \end{bmatrix} \]

where \( L \) is the rounds of the observation data. The first goal is to minimize the difference between the real current data and the generated current data, and the second goal is to ensure the sparsity of the RN.

The numerical simulation of the RN is described as follows:

a) Input an RN network;

b) Each node state is obtained from a random number \( \Delta w_i \in [0, 20] \);

c) Calculate the voltage of node \( i \) by \( V_i = V^* \sin[(w+\Delta w_i)t] \);

d) Calculate the electrical current of the node \( i \) by (8);

e) Repeat Step c) to Step d) \( T \) times;

We record the voltages and the currents at the nodes simultaneously as observational data for this dynamic process.

3.3. CDPs
It is difficult to handle the task of detecting communities from dynamics. We also introduce the traditional CD from network methods, which will be employed in our framework. CD is an important field in complex network research, aiming to divide all network nodes into multiple communities with dense intra-community links and sparse inter-community links [24]. We can model the complex network as a graph $G = (V, E)$, where $V = \{v_1, v_2, \ldots, v_N\}$ is a set of $N$ nodes and $E = \{(v_i, v_j) \mid v_i, v_j \in V, i \neq j\}$ for undirected networks. Let $C = \{C_i \mid C_i \subset V, C_i \neq \emptyset, i = 1, 2, \ldots, S\}$ be a set of $S$ communities obtained from $G$. Thus, it must satisfy the following conditions:

$$\bigcup_{i=1}^{S} C_i = V \quad \forall i \neq j, C_i \neq C_j, i, j \in \{1, 2, \ldots, S\}$$

For CDPs, modularity $Q$ is one of the most well-known functions for evaluating the quality of network partitions, which can be expressed as follows [24]:

$$Q = \sum_{i,j} \left[ \frac{l_{ij} - \left( d_i \right) \left( d_j \right)}{2e} \right]^2$$

where $e$ denotes the total number of edges; $l_i$ and $d_i$ represent the number of edges and the sum of the degree of nodes in the $s$th community, respectively. The larger the $Q$ value, the better the CD results of the network.

Normalized mutual information (NMI) is a commonly used function to measure the similarity between two community structures. Let $B_1$ and $B_2$ be two partitions of a network and $A$ be a confusion matrix, where element $A_{ij}$ is the number of nodes that belong to both the $i$-th community of $B_1$ and the $j$-th community of $B_2$. Then NMI can be expressed as follows:

$$\text{NMI}(B_1, B_2) = \frac{-2\sum_i \sum_j A_{ij} \log \left( \frac{A_{ij} \cdot N}{A_i \cdot A_j} \right)}{\sum_i A_i \log \left( \frac{A_i}{N} \right) + \sum_j A_j \log \left( \frac{A_j}{N} \right)}$$

where $S_1$ and $S_2$ denote the number of communities in $B_1$ and $B_2$, respectively; $A_i$ and $A_j$ represent the sum of the elements of $A$ in the $i$-th row and $j$-th column, respectively. Generally speaking, the larger the NMI value, the more similar $B_1$ and $B_2$.

3.4. MTNRCDDPs
The NR and CD are two hot topics in the research of complex network systems. The former aims to infer the network structure from the limited observed data, while the latter aims to explore the complex interactions and relationships among nodes from dynamics. As is discussed in Section 2.2, we can generally reconstruct the network structure from dynamics first, and then detect the community structure, as shown in Fig. 1.

![Fig. 1. A general process of the NR and the CD.](image1)

The work proposed in [23] shows that social networks naturally tend to cluster into groups or communities. The nodes in the same group link more densely than nodes outside the group. Inspired by this, we consider the explicit knowledge transfer across the CD task and NR task to improve each task’s performance, which is modeled as an MMTO problem, as shown in Fig. 2. In the optimization process of MTNRCD, the community partition obtained from the CD task can be considered useful knowledge to reconstruct the connection between nodes. Similarly, the network structure obtained from the NR task can also be transferred to assist the CD task. The detailed procedure is showed in Section 4.

![Fig. 2. A process of the multitasking NR and CD.](image2)
4. Proposed Method

We describe the proposed EMTNRCD framework for MNRCDPs in this Section. Specifically, we introduce the outline of EMTNRCD firstly. Then the explicit knowledge transfer from the NR task to the CD task is presented. Next, we also give the optimization process of the NR task, including the knowledge transfer from the CD task to the NR task. Finally, the framework of EMTNRCD is presented.

4.1. Outline of EMTNRCD

The goal of EMTNRCD is to simultaneously reconstruct the network structure and detect the community structure from the given limited observed data by transferring knowledge across the NR task and CD task. The outline of EMTNRCD for MNRCDPs is shown in Fig. 3, which consists of two different optimization stages: a pre-optimization stage and a normal optimization stage. In the pre-optimization stage, we optimize the NR task with a fixed number of function evaluations by employing the general population-based NR algorithm firstly. Then, the initialized network structure is selected from the optimized population. Next, based on the initial network structure, we optimize the CD task with a fixed number of function evaluations by employing the general population-based CD algorithm. After the pre-optimization stage, we obtain the initial network structure and initial community structure, which is employed by the subsequent stages. In the normal optimization stage, the NR task and CD task are optimized simultaneously with explicit knowledge transfer. In the optimization of each generation of the CD task, the network structures obtained from the NR task in the current generation and the previous generation can be considered as the snapshots of a dynamic network at two consecutive time steps, which is viewed as prior knowledge to assist the CD task. Thus, we model this process as the dynamic CD problem and then solve the CD task using the general population-based CD algorithm of a dynamic network. In optimizing each generation of the NR task, the same population-based NR algorithm as the pre-optimization stage is employed. Since the nodes in the same community link more densely than nodes outside the community, the community structure obtained from the CD task is transferred to help reconstruct the links between nodes in the NR task, which is
inspired by the work [70]. Next, we introduce the procedure of explicit knowledge transfer across the NR task and CD task in detail.

Fig. 3. Outline of EMTNRCD.

4.2. Knowledge Transfer from the NR Task to CD Task

When solving the CD task, we usually need to know prior knowledge about the network structure $X$. In the evolutionary process of EMTNRCD, more and more precise network structures are obtained with the development of the optimization of the NR task. Inspired by this, the network structures obtained from the NR task at consecutive generations can be viewed as multiple snapshots of a dynamic network to assist the CD task. To realize the above ideas, the following two key issues need to be resolved: 1) How to obtain the network structure $X^{(t)}$ from optimized population $P_{NR}^{(t)}$ of the NR task in the $t$-th generation? 2) How to take full advantage of the network structure $X^{(t)}$ to assist the CD task?

1) How to obtain the network structure $X^{(t)}$ from optimized population $P_{NR}^{(t)}$ of the NR task in the $t$-th generation?
How to select the network structure $X^{(t)}$ has a significant effect on the performance of EMTNRCD. Unlike single-objective optimization, we cannot directly obtain the best solution from $P_{NR}^{(t)}$. In this paper, we select network structure $X^{(t)}$ by applying the crowding distance metric to the first nondominated front of the population $P_{NR}^{(t)}$ reported in [72], which considers the diversity and convergence of the nondominated set. The pseudo-code of the selection operator is shown in Algorithm 1. Firstly, the fast nondominated sorting method [72] is employed to partition $P_{NR}^{(t)}$ into $L$ nondominated fronts $\{F^1, \ldots, F^L\}$, where $F^1$ is the first nondominated front, and $F^L$ is the last one. Next, the crowding distance [72] of each solution in $F^1$ is calculated. The network structure $X^{(t)}$ in $F^1$ with the largest crowding distance value is selected to assist the CD task. If there are two extreme solutions in $F^1$, we choose one at random.

**Algorithm 1: Selection**

**Input:**

$P_{NR}^{(t)}$: Optimized population of the NR task in the $t$-th generation;

**Output:**

$X^{(t)}$: Network obtained from the NR task in the $t$-th generation;

1. $\{F^1, \ldots, F^L\} \leftarrow$ Fast Nondominated Sorting ($P_{NR}^{(t)}$);
2. count $\leftarrow$ Count the number of solutions in $F^1$;
3. if count $> 2$ then
4. $X^{(t)} \leftarrow \arg\max_{X \in F^1} \text{Crowding Distance}(F^1)$;
5. else
6. $X^{(t)} \leftarrow$ Randomly choose a solution from $F^1$;
7. end if;
8. return $X^{(t)}$;

2) How to take full advantage of the network structure $X^{(t)}$ to assist the CD task?

After obtaining the network structure $X^{(t)}$ from the optimized population $P_{NR}^{(t)}$, we need to consider how to apply the information of network structure $X^{(t)}$ to assist the CD task. The evolution analysis of the community structure of dynamic networks is also one of the current research hotspots in the field of network sciences. In the evolutionary process of EMTNRCD, the network structures $X^{(t)}$ and $X^{(t-1)}$ obtained from the NR task for two consecutive generations can be regarded as the snapshots of a dynamic network $G$ in two consecutive time steps. For convenience, a dynamic network $G$ can be denoted as $G = \{G^{(t)} = (V, E^{(t)}), t=0,$
1, ..., \(T\), where \(G^{(t)}\) represents a snapshot of \(G\) in the \(t\)-th generation of EMTNRCD. Let \(C^{(t)} = \{C_j^{(t)} \mid C_j^{(t)} \subset V, C_j^{(t)} \neq \emptyset, j=1, 2, \ldots, S^{(t)}\}\) be the community structure containing \(S^{(t)}\) communities obtained from \(G^{(t)}\) and \(C_j^{(t)} \cap C_j^{(t)} = \emptyset\) for any \(C_j^{(t)}, C_j^{(t)} \in C\). Then, the CD in dynamic networks aims to find the communities \(C = \{C^{(0)}, C^{(1)}, \ldots, C^{(T)}\}\). To uncover dynamic networks’ evolutionary behavior, a framework, named temporal smoothness, is proposed by Chakrabarti et al. [74], which emphasizes that the network should not shift significantly from one timestep to the next. A cost function \(f^{(t)}\) combining snapshot cost \(f_1^{(t)}\) and temporal cost \(f_2^{(t)}\) at generation \(t\) is described as follows:

\[
f^{(t)}(C^{(t)}) = \begin{cases} f_1^{(t)}(C^{(t)}), & t = 0 \\ \alpha \cdot f_1^{(t)}(C^{(t)}) + (1-\alpha) \cdot f_2^{(t)}(C^{(t)}, C^{(t-1)}), & t = 1, 2, \ldots, T \\ \end{cases}
\]

where \(\alpha \in [0, 1]\) is a balance parameter that controls the trade-off between \(f_1^{(t)}\) and \(f_2^{(t)}\). The first term \(f_1^{(t)}\) evaluates the quality of the community structure at generation \(t\), and the second term \(f_2^{(t)}\) evaluates the similarity of community structures between generation \(t\) and \(t-1\). To automatically obtain the best trade-off between the above two items, Folino et al. [27] transformed the CD in dynamic networks into a multiobjective optimization problem, which can be expressed as follow:

\[
\begin{align*}
\max_{C^{(t)}} & \ f_1^{(t)}(C^{(t)}), t = 0 \\
\max_{C^{(t)}} & \ \left[f_1^{(t)}(C^{(t)}), f_2^{(t)}(C^{(t)}, C^{(t-1)})\right], \quad t = 1, 2, \ldots, T
\end{align*}
\]

Based on the reported results in [29], [38], modularity \(Q\) in (11) and NMI in (12) are used as the two optimization objectives for finding communities of dynamic networks in this paper. The former evaluates the quality of community structures, while the latter evaluates the similarity of community structures in two consecutive generations. Then we can optimize (14) by the general population-based CD algorithm of the dynamic network to obtain communities \(C = \{C^{(0)}, C^{(1)}, \ldots, C^{(T)}\}\). Based on (14), the CD task of a dynamic network composed of the network structure obtained by two consecutive generations \(t\) and \(i-1\) in this paper can be expressed as follow:

\[
\begin{align*}
\max_{C^{(t)}} & \ Q\left(C^{(t)}\right), NMI\left(C^{(t)}, C^{(t-1)}\right) \\
& \text{s.t.} \ C^{(t)} = \{C_j^{(t)} \mid C_j^{(t)} \subset V, C_j^{(t)} \neq \emptyset, j = 1, 2, \ldots, S^{(t)}\} \\
& \forall C_j^{(t)}, C_j^{(t)} \in C^{(t)}, C_j^{(t)} \cap C_j^{(t)} = \emptyset
\end{align*}
\]
4.3. Knowledge Transfer from the CD Task to NR Task

There are multiple communities in the real complex network, among which the same communities are more closely linked. Inspired by this, the community structure \( C(t) \) from the CD task in the \( t \)-th generation is explicitly transferred to the NR task to take advantage of the property of the community. To realize the above ideas, the knowledge transfer from the CD task to the NR task is proposed as shown in Algorithm 2. Firstly, the solution \( X' \) is obtained from the \( P_{NR}^{(t)} \) by the same selection method as algorithm 1. Then we perform two local searches \( ls_1 \) and \( ls_2 \) on \( X' \) by utilizing the link information within and between communities, respectively. Further, the environment selection procedure that is the same as the procedure of the population-based NR algorithm in the normal optimization stage is performed on the union set \( \{ P_{NR}^{(t)}, P_{in}, P_{out} \} \) to obtain the new population \( P_{NR}^{(t)} \) with \( N_1 \) chromosomes after knowledge transfer. Next, we introduce how to perform two local search operators in detail. Moreover, the example of these two operators can be found in Fig. 4.

![diagram](image-url)

Fig. 4. Outline of two local search operators.
Algorithm 2: Knowledge Transfer\(_{CD\rightarrow NR}\)

**Input:**
- \(N_1\): Population size for the NR task;
- \(P_{NR}^{(t)}\): Population obtained from the normal optimization stage of the NR task in \(t\)-th generation;
- \(C^{(t)}=\{C_j^{(t)} \mid C_j^{(t)} \subset V, C_j^{(t)} \neq \emptyset, j=1, 2, \ldots, S^{(t)}\}\): Network structure obtained from the NR task in the \(t\)-th generation;
- \(\alpha\): Population size for the knowledge transfer from the CD task to NR task;
- \(t_1\): Number of function evaluations for the knowledge transfer from the CD task to NR task;

**Output:**
- \(P_{NR}^{(t)}\): Population obtained from the knowledge transfer from the CD task to NR task;

1. \(X'\leftarrow\)Selection \((P_{NR}^{(t)})\); // Algorithm 1
2. // local search operator \(ls_1\) on \(X'\) by utilizing the structural information within a community.
3. \(C'\leftarrow\)Randomly choose a community from \(C^{(t)}\);
4. \(P_{in}\leftarrow\)Initialization\(_{within}\) \((\alpha, X', C')\);
5. while the used number of function evaluations \(\leq t_1\) do
6. \(O\leftarrow\)Offspring Generation\(_{within}\) \((P_{in})\);
7. \(P_{in}\leftarrow\)Environment Selection \(\{P_{in}, O\}, \alpha\);
8. end while
9. // local search operator \(ls_2\) on \(X'\) by utilizing the structural information between communities.
10. \(P_{out}\leftarrow\)Initialization\(_{between}\) \((\alpha, X', C^{(t)})\);
11. while used number of function evaluations \(\leq t_1\) do
12. \(O\leftarrow\)Offspring Generation\(_{between}\) \((P_{out})\);
13. \(P_{out}\leftarrow\)Environment Selection \(\{P_{out}, O\}, \alpha\);
14. end while
15. \(P_{NR}^{(t)}\leftarrow\)Environment Selection \(\{P_{NR}^{(t)}, P_{in}, P_{out}\}, N_1\);
16. return \(P_{NR}^{(t)}\);

A community \(C'\) is randomly selected from \(C^{(t)}\) and then a local search operator \(ls_1\) (lines 3-8 in Algorithm 2) is proposed by utilizing the links information within a community \(C'\). Firstly, each link relationship within the community \(C'\) in the network \(X'\) is changed with a probability of 0.8 while other link relationships remain unchanged. Then an initial population \(P_{in}\) with \(\alpha\) chromosomes can be obtained by repeating this operator (line 4 in Algorithm 2). In the main loop, \(\alpha\) individuals are generated by repeating the following operation. We randomly select two solutions from the population \(P_{in}\) and then perform the single-point crossover and bitwise mutation operator on the decision variables corresponding
to the link relationship within the community $C'$ (line 6 in Algorithm 2). Finally, the environment selection procedure that is the same as the population-based NR algorithm procedure in the normal optimization stage is used to select solutions that survive to the next generation (line 7 in Algorithm 2).

Besides, a local search operator $ls_2$ (lines 10-14 in Algorithm 2) is proposed by utilizing the links information between communities and its difference from $ls_1$ includes two operations: Initialization and offspring generation. We only need to modify the links information within a community $C'$ used in the above two operators of $ls_1$ to the links information between communities to obtain $ls_2$. In summary, the links information within and between communities, as prior knowledge, is embedded in two local search operators to optimize the NR task.

4.4. Framework of EMTNRCD

The general framework of EMTNRCD, including two optimization stages, is presented in Algorithm 3. In the pre-optimization step of the NR task, a population $P_{NR}^{(0)}$ for the NR task is initialized randomly, and the general population-based NR algorithm $OptimizerNR$ can be employed to pre-optimize the NR task with a fixed number of function evaluations $(1-\lambda)\times TFE_1$ (lines 1-2 in Algorithm 3). Then an initial network structure $X^{(0)}$ is selected from $P_{NR}^{(0)}$ by Algorithm 1, and the NR task on $X^{(0)}$ is pre-optimized by the general population-based CD algorithm $OptimizerCD_{pre}$ with a fixed number of function evaluations $(1-\lambda)\times TFE_2$ (line 3-4 in Algorithm 3). After the pre-optimization stage, the initial network structure $X^{(0)}$ and initial community structure $C^{(0)}$ can be obtained. Next, a normal optimization stage is performed to get more accurate solutions by utilizing the common knowledge across the NR task and CD task. In the main loop, the same algorithm as $OptimizerNR$ is employed to optimize the NR task with a fixed number of function evaluations $N_1$ in the $t$ generation of EMTNRCD (line 8 in Algorithm 3). Then a general population-based CD algorithm of dynamic network $OptimizerCD_{normal}$ is performed to optimize (15) with a fixed number of function evaluations $t_2$ by utilizing the knowledge (network structure) acquired from the NR task for two consecutive generations (lines 10-11 in Algorithm 3). Next, knowledge transfer from the CD task to the NR task is employed to
improve the convergence and diversity of the population $P_{NR}^{(0)}$ (line 12 in Algorithm 3). Finally, the decision-maker has selected the network structure $X^*$ and community structure $C^*$ (lines 17-18 in Algorithm 3).

**Algorithm 3: Framework of EMTNRCD**

**Input:**
- $Z$: MNRCDPs;
- $N_1$: Population size for the NR task;
- $N_2$: Population size for the CD task;
- $TFE_1$: Number of function evaluations for the NR task;
- $TFE_2$: Number of function evaluations for the CD task;
- $\lambda$, $\alpha$, $t_1$, $t_2$: Key parameters;
- OptimizerNR: Optimizer for the NR task;
- OptimizerCD$_{pre}$: Optimizer for the CD task in pre-optimization stage;
- OptimizerCD$_{normal}$: Optimizer for the CD task in normal optimization stage;

**Output:**
- Result: Learned network structures $X^*$ and community structure $C^*$;

1. $P_{NR}^{(0)} \leftarrow$ Initialization ($N_1$, $Z$);
2. $P_{NR}^{(0)} \leftarrow$ OptimizerNR ($N_1$, $Z$, $P_{NR}^{(0)}$, $(1-\lambda) \times TFE_1$); // pre-optimization stage of the NR task
3. $X^{(0)} \leftarrow$ Selection($P_{NR}^{(0)}$); // Algorithm 1
4. $C^{(0)} \leftarrow$ OptimizerCD$_{pre}$ ($N_2$, $Z$, $(1-\lambda) \times TFE_2$); // pre-optimization stage of the CD task
5. $t \leftarrow 1$;
6. while termination criterion not fulfilled do
7. if used $FE_1 \leq \lambda \times TFE_1$ then
8. $P_{NR}^{(1)} \leftarrow$ OptimizerNR ($N_1$, $Z$, $P_{NR}^{(1)}$, $N_1$); // normal optimization stage of the NR task
9. if used $FE_2 \leq \lambda \times TFE_2$ then
10. $X^{(1)} \leftarrow$ Selection ($P_{NR}^{(1)}$); // Algorithm 1
11. $C^{(1)} \leftarrow$ OptimizerCD$_{normal}$ ($N_2$, $Z$, $X^{(1)}$, $C^{(t-1)}$, $t_2$); // normal optimization stage of the CD task
12. $P_{NR}^{(1)} \leftarrow$ KnowledgeTransferCD$\rightarrow$NR ($N_1$, $P_{NR}^{(1)}$, $C^{(1)}$, $\alpha$, $t_1$); // Algorithm 2
13. end if
14. end if
15. $t \leftarrow t+1$;
16. end while
17. $X^* \leftarrow$ Selection ($P_{NR}^{(1)}$); // Algorithm 1
18. $C^* \leftarrow C^{(1)}$;
19. return $X^*$ and $C^*$.

5. Experiments

This section verifies the effectiveness of our proposed EMTNRCD on multiple
MNRCBDPs. Firstly, we give the experimental settings in Section 5.1. Then, the performance of EMTNRCBD is tested in Section 5.2. Next, we analyze the effect of knowledge transfer of EMTNRCBD in Section 5.3. Finally, in Section 5.4, the parameter analysis is also given. We perform all experiments on a PC with Windows and Intel Core i7-8700 CPU at 3.20 GHz and 16GB RAM.

5.1. Experimental Setup

### TABLE I TEST SUITE OF MNRCBDPS

| ProblemID | Network | N_s-L | Type | D   | N   | I   | S_c |
|-----------|---------|-------|------|-----|-----|-----|-----|
| EG_1      | ZK      | 5-10  | Binary | 1156 | 34  | 78  | 2   |
| EG_2      | polbooks|       |       | 11025| 105 | 441 | 3   |
| EG_3      | football|       |       | 13255| 115 | 613 | 12  |
| EG_4      | dolphin |       |       | 3844 | 62  | 159 | 2   |

| ProblemID | Network | N_s-L | Type | D   | N   | I   | S_c |
|-----------|---------|-------|------|-----|-----|-----|-----|
| EG_5      | ZK      | 20-10 | Binary | 11025| 105 | 441 | 3   |
| EG_6      | polbooks|       |       | 13255| 115 | 613 | 12  |
| EG_7      | football|       |       | 3844 | 62  | 159 | 2   |
| EG_8      | dolphin |       |       | 1156 | 34  | 78  | 2   |

| ProblemID | Network | N_s-L | Type | D   | N   | I   | S_c |
|-----------|---------|-------|------|-----|-----|-----|-----|
| RN_1      | ZK      | 5-10  | Binary | 11025| 105 | 441 | 3   |
| RN_2      | polbooks|       |       | 13255| 115 | 613 | 12  |
| RN_3      | football|       |       | 3844 | 62  | 159 | 2   |
| RN_4      | dolphin |       |       | 1156 | 34  | 78  | 2   |

| ProblemID | Network | N_s-L | Type | D   | N   | I   | S_c |
|-----------|---------|-------|------|-----|-----|-----|-----|
| RN_5      | ZK      | 20-10 | Binary | 11025| 105 | 441 | 3   |
| RN_6      | polbooks|       |       | 13255| 115 | 613 | 12  |
| RN_7      | football|       |       | 3844 | 62  | 159 | 2   |
| RN_8      | dolphin |       |       | 1156 | 34  | 78  | 2   |

**Test Suite of MNRCBDPs.** In this section, four real social networks, football [75], polbooks [76], dolphin [77], and ZK [78] are employed to construct the test suite of MNRCBDPs. The details of those networks are presented in Table I, including the type of variables for the NR task, the number of variables $D$ for the NR task, the number of nodes $N$, the number of links $I$, and the number of communities $S_c$. In terms of $D$, these problems are high-dimensional. In the test suite, two types of response sequences are generated from the given networks by the methods described in Section 3. The first one is the case of 5 response sequences with 10 rounds each ($N_s=5, L=10$). The second one is the case of 20 response sequences with 10 rounds each ($N_s=20, L=10$). In Table I, EG_1–EG_8 and RN_1–RN_8 represent the EG MNRCBDPs and the RN MNRCBDPs, respectively. In general, to see the efficiency of solving simultaneous tasks, different multitasking environments should be created with synergistic tasks and non-synergistic tasks. The CD from dynamics and NR from dynamics are nature synergistic tasks which is the motivation of EMTNRCBD. Thus, we employ the
commonly used networks to construct the test suite and these networks represent most types of networks. In the worst case, one network contains a community. In this case, EMTNRCD can be considered as the single task of reconstructing the network alone. Moreover, it is difficult to create non-synergistic tasks since the more accurate the given network is, the more accurate the detection results are. Also, due to the limited ability of evolutionary algorithms, the performance of the NR decreases with bigger networks as shown in Table III. The core of experiments is to verify whether the proposed knowledge transfer strategy is effective and whether EMTNRCD can handle MTNRCDPs.

**Algorithms.** NSGA-II [72] and SPEA2 [73], two state-of-the-art MOEAs, are embedded in EMTNRCD as OptimizerNR to form EMTNRCD-NSGA-II and EMTNRCD-SPEA2, respectively. Then ECD [38], a state-of-the-art EA of the CD method in the dynamic network, and PNGACD [29], a state-of-the-art EA of the CD method that has the same base search operator as ECD, are considered as OptimizerCDnormal and OptimizerCDpre in EMTNRCD-NSGA-II and EMTNRCD-SPEA2, respectively. Please note that none of the current EMT methods can handle MTNRCDPs as discussed in Section 2.3. Thus, we cannot compare our proposal with the state-of-the-art EMT methods. Moreover, as discussed in Section 2.2, none of the current EA-based methods can handle the task of the CD from dynamics and our proposal is the first work to detect communities from dynamics. Thus, we cannot compare our proposal with the single task of the CD from dynamics. However, in general, we should compare EMTNRCD with the alternative of solving the tasks with a parallel method but not sharing any kind of knowledge. To verify the performance of our proposal in the experiments, a comparison algorithm called NRTOCD is designed, which follows the general process of the NR task and the CD task, as shown in Fig. 1. In NRTOCD, we reconstruct the network structure from the observed data first and then detect the community structure, where any form of knowledge transfer does not occur across these two tasks. NSGA-II and SPEA2 are embedded in NRTOCD as the base optimizer for the NRP to form NRTOCD-NSGA-II and NRTOCD-SPEA2, respectively. For a fair comparison, PNGACD [29], a state-of-the-art EA of CD with the same base search operator as ECD, is considered the base optimizer for the CD task in NRTOCD-NSGA-II and NRTOCD-SPEA2. Moreover, NSGA-II and SPEA-II are employed to handle the NR task alone. In addition to
NRTOCD, we should compare our proposal with the single task of the NR from dynamics. Because the first stage of NRTOCD is the same as the NR task, the results of the NR task in NRTOCD can be used to finish this comparison. All experiments are implemented on the evolutionary multiobjective optimization platform PlatEMO [80].

**Evaluation Metrics.** To evaluate the performance of our proposal, two measurement indices are employed, the Matthews correlation coefficient (MCC) and normalized mutual information (NMI). The MCC is used to measure the accuracy of the reconstructed network structure and the NMI is used to measure the quality of community partitions. In each MNRCDP, all comparison algorithms are run 20 times independently to obtain statistical results, and the Wilcoxon rank-sum test is employed to test the significance of the experimental results.

| Table II Parameter Settings |
|-----------------------------|
| Parameter | Value | Descriptions |
| \( N_1 \) | 100 | The population size for the NR task |
| \( N_2 \) | 100 | The population size for the CD task |
| \( TFE_1 \) | 200000 | The maximum number of function evaluations for the NR task |
| \( TFE_2 \) | 200000 | The maximum number of function evaluations for the CD task |
| \( \lambda \) | 0.5 | The share of the \( TFE \) used for the normal optimization stage in EMTNRCD |
| \( t_1 \) | 1000 | The number of function evaluations for the knowledge transfer from the CD task to NR task in EMTNRCD |
| \( P_c \) | 1 | The probabilities of crossover in NSGAII and SPEA2 |
| \( P_m \) | 1/D | The probabilities of mutation in NSGAII and SPEA2 |
| \( P_{mu} \) | 0.2 | The probabilities of mutation in ECD and PNGACD |
| \( P_{mi} \) | 0.2 | The probabilities of migration in ECD and PNGACD |
| \( P_{mu/mi} \) | 0.5 | The parameter to control the execution of mutation and migration in ECD and PNGACD |

**Parameter Settings.** NSGA-II and SPEA2 for the NR task employ the same single-point crossover and bitwise mutation operators. For the CD task, ECD and PNGACD use the same crossover, mutation, and migration operators in [29], [38]. Both the population size for the NR task and CD task is set to 100 and both the maximum number of function evaluations for the NR task and the CD task is assigned to 200000 in all experiments. In EMTNRCD, \( \lambda \) and \( t_1 \) are set to 0.5 and 1000, respectively. Then the number of time steps for the CD task in EMTNRCD, marked as \( T \), is \( \lceil \lambda \times TFE_1/(N_1+2 \times t_1) \rceil \) and the number of function evaluations for the CD task in each time step in EMTNRCD, marked as \( t_2 \), is \( \lambda \times TFE_2/T \). The Sub-population size \( \alpha \) for the knowledge transfer from the CD task to NR task in EMTNRCD is set to 20. The details of parameter settings are listed in Table II.
5.2. Results and Discussion

Table III lists the mean and standard deviation of MCC (NR Task) and NMI (CD Task) obtained by EMTNRCD-NSGA-II, EMTNRCD-SPEA2, NRTOCD-NSGA-II, NRTOCD-SPEA2, CEMO-NR-NSGAI1, and CEMO-NR-SPEA2 on the problems EG1–EG8 and RN1–RN8 over 20 independent runs, where symbols “−”, “≈”, and “+” imply that the NRTOCD-Alg is significantly worse, similar and better than EMTNRCD-Alg on the Wilcoxon rank-sum test with 95% confidence level, respectively. Then the comparison results of EMTNRCD-Alg and NRTOCD-Alg are analyzed, where Alg is the NSGAI or SPEA2. We mark the best results in boldface.

| Problem | Type of Tasks | NRTODC-NSGA-II | EMTNRCD-NSGA-II | NRTOCD-SPEA2 | EMTNRCD-SPEA2 |
|---------|---------------|----------------|-----------------|--------------|---------------|
| EG1     | NR Task       | 7.87e-01       | 8.66e-01        | 7.98e-01     | 8.37e-01      |
|         | CD Task       | 4.25e-01       | 9.72e-01        | 4.76e-01     | 9.75e-01      |
| EG2     | NR Task       | 5.22e-01       | 5.20e-01        | 5.19e-01     | 5.19e-01      |
|         | CD Task       | 3.03e-01       | 3.21e-01        | 3.10e-01     | 3.27e-01      |
| EG3     | NR Task       | 5.14e-01       | 5.10e-01        | 5.05e-01     | 5.09e-01      |
|         | CD Task       | 2.18e-01       | 5.51e-01        | 1.74e-01     | 5.27e-01      |
| EG4     | NR Task       | 6.77e-01       | 7.41e-01        | 6.10e-01     | 6.73e-01      |
|         | CD Task       | 2.77e-01       | 7.29e-01        | 3.23e-01     | 4.18e-01      |
| EG5     | NR Task       | 8.55e-01       | 9.40e-01        | 8.44e-01     | 9.02e-01      |
|         | CD Task       | 7.57e-01       | 8.57e-01        | 4.12e-01     | 8.79e-01      |
| EG6     | NR Task       | 5.28e-01       | 5.27e-01        | 5.20e-01     | 5.23e-01      |
|         | CD Task       | 3.09e-01       | 3.24e-01        | 3.09e-01     | 3.09e-01      |
| EG7     | NR Task       | 5.16e-01       | 5.17e-01        | 5.10e-01     | 5.07e-01      |
|         | CD Task       | 3.43e-01       | 5.55e-01        | 5.31e-01     | 5.31e-01      |
| EG8     | NR Task       | 7.19e-01       | 8.12e-01        | 6.10e-01     | 7.21e-01      |
|         | CD Task       | 4.80e-01       | 7.21e-01        | 3.17e-01     | 3.86e-01      |
| RN1     | NR Task       | 8.73e-01       | 9.15e-01        | 8.58e-01     | 8.92e-01      |
|         | CD Task       | 2.09e-01       | 7.34e-01        | 3.22e-01     | 5.77e-01      |
| RN2     | NR Task       | 5.05e-01       | 5.11e-01        | 5.15e-01     | 5.09e-01      |
|         | CD Task       | 3.01e-01       | 3.33e-01        | 3.08e-01     | 3.10e-01      |
| RN3     | NR Task       | 5.05e-01       | 4.97e-01        | 5.04e-01     | 5.04e-01      |
|         | CD Task       | 3.66e-01       | 5.30e-01        | 1.83e-01     | 5.14e-01      |
| RN4     | NR Task       | 6.51e-01       | 7.18e-01        | 6.09e-01     | 6.01e-01      |
|         | CD Task       | 1.03e-01       | 2.71e-01        | 6.58e-02     | 2.27e-01      |
| RN5     | NR Task       | 8.88e-01       | 9.53e-01        | 8.93e-01     | 9.31e-01      |
|         | CD Task       | 4.93e-01       | 8.24e-01        | 4.83e-01     | 6.16e-01      |
| RN6     | NR Task       | 5.18e-01       | 5.21e-01        | 5.13e-01     | 5.12e-01      |
|         | CD Task       | 2.95e-01       | 3.29e-01        | 2.84e-01     | 3.19e-01      |
| RN7     | NR Task       | 5.03e-01       | 4.99e-01        | 5.04e-01     | 5.12e-01      |
|         | CD Task       | 1.78e-01       | 5.07e-01        | 1.86e-01     | 5.51e-01      |
| RN8     | NR Task       | 6.94e-01       | 7.69e-01        | 6.19e-01     | 6.88e-01      |
|         | CD Task       | 5.96e-02       | 3.40e-01        | 1.42e-01     | 2.30e-01      |
| −/+     | NR Task       | 10/5           | 11/5           |             |               |
|         | CD Task       | 16/0           | 13/2           |             |               |

According to the comparison results of EMTNRCD-Alg and NRTOCD-Alg on EG1–EG8
and RN₁~RN₈ in Table III, EMTNRCD-Alg has shown excellent performance in terms of MCC (NR Task) and NMI (CD Task) in the test suite. Specifically, compared to the single task of NR from dynamics, EMTNRCD-Alg performs better on 21, ties 10, and loses once out of 32 tasks in terms of MCC (NR Task), which demonstrates that the transferred community information in our method may be useful. EMTNRCD-Alg exceeds the NRTOCD-Alg in 50, ties 11, and loses 3 out of 64 tasks in MCC or NMI. For the problems with ZK and dolphin networks, the performance of EMTNRCD-Alg is better than that of NRTOCD-Alg in all tasks. In EMTNRCD-Alg, useful knowledge is transferred between the NR task and the CD task to improve each task’s performance. Since EMTNRCD-Alg has the same basic evolutionary solver of the NR task and CD task as NRTOCD-Alg, knowledge transfer’s effectiveness in EMTNRCD-Alg is confirmed. For the problems with the polbooks network, EMTNRCD-Alg and NRTOCD-Alg have similar performance in terms of the NR task. Since the community structure in the reconstructed imprecise network may have a large gap with the community structure in the actual network, knowledge transfer may fail in large-scale networks. This case leads to negative transfer. Moreover, it is found that the number of communities is much larger than the real number of communities. The size of transferred communities is too small to offset its consumption of computational resources. From Table III, the quality of both networks and communities, except for Zackary, is not impressive. NMI is very often below 0.5, which means that the approach is not able to recover the true communities, but also the MCC is not high. This phenomenon appears since we don’t design the specified operator for these two tasks, which may improve the performance violently. These experiments aim to verify the effectiveness of the designed knowledge transfer operator and whether EMTNRCD can handle the task of CD and NR from dynamics jointly. We think the experiments have achieved these goals.

To better illustrate the performance of EMTNRCD-Alg, we visualize the network structure and community structure obtained by EMTNRCD-NSGA-II on ZK networks. We mark the different communities with different colors. Fig. 5 shows an intuitive example of the network structure and community structure of the ZK network obtained by EMTNRCD-NSGA-II on problem EG₁. As can be seen from Fig. 5, according to the connection between every two nodes obtained by knowledge transfer from the NR task to the
CD task, EMTNRCD-NSGA-II divides all nodes into two communities, which is consistent with the actual community division as shown in Table I. There are two colors shown in the figure which represent the division made by EMTNRCD-NSGA-II. Our proposal organizes the network in two divisions according to the density of connection between nodes, where closely connected nodes are divided into the same community.

![Illustrative example of the community structure about the ZK network obtained by EMTNRCD-NSGA-II on problem EG1.](image)

**Fig. 5.** Illustrative example of the community structure about the ZK network obtained by EMTNRCD-NSGA-II on problem EG1.

### 5.3. Effect of Knowledge Transfer in EMTNRCD

![Non-dominated solution set before (grey) and after (red) the knowledge transfer from the CD task to NR task obtained by EMTNRCD on the problem EG1.](image)

**Fig. 6.** The non-dominated solution set before (grey) and after (red) the knowledge transfer from the CD task to NR task obtained by EMTNRCD on the problem EG1. (a) The number of function evaluations for the NRP is 102100, (b) The number of function evaluations for the NRP is 198700.

This section further illustrates the effectiveness of the knowledge transfer operators from
the CD task to the NR task and from the NR task to the CD task in EMTNRCD and takes the algorithm EMTNRCD-NSGA-II and the problem EG\textsubscript{1} as an example. Fig. 6 shows the non-dominated solution set before and after the knowledge transfer from the CD task to NR task obtained by EMTNRCD when the number of function evaluations for the NRP is 102100 and 198700, namely, Algorithm 2 is performed once. As can be observed in Fig. 6 (a), in the mid-stage of evolution, compared with the non-dominated solution set (grey) before knowledge transfer from the CD task to NR task, the new non-dominated solution set (red) has better overall performance in terms of convergence and diversity, which illustrates the effectiveness of knowledge transfer from the CD task to NR task. As shown in Fig. 6 (b), in the later stages of evolution, the overall performance of the non-dominated solution set before and after the knowledge transfer is not much different in terms of convergence and diversity. Since the network structure obtained by the NR task has fully utilized the community structure obtained by the CD task, the process of knowledge transfer from the CD task to the NR task in the later stage of evolution is not apparent for improving the non-dominated solution set.

Fig. 7. A curve of the values of NMI over generations, in which the values of NMI are calculated based on the optimal community structures returned by EMTNRCD at each generation on the problem EG\textsubscript{1} and the real community structure.

Besides, Fig. 7 shows a curve of the values of NMI over time steps. NMI values are calculated by (12) based on the optimal community partitions returned by EMTNRCD after each knowledge transfer stage on the problem EG\textsubscript{1} and the real community partition. As shown in Fig. 7, NMI shows an increasing trend over time, which illustrates that knowledge from the NR task to the CD task positively impacts the CD task. We also find that the
negative knowledge may affect the performance shown in the early stage of evolution, but this phenomenon can be eliminated with development. We also find that the transfer process has little impact on the performance of our proposal at the last stage, but the knowledge transfer still uses the computational budget. We can decrease this negative knowledge transfer by reducing the frequency of knowledge transfer.

5.4. Analysis of the Parameter Sensitivity in EMTNRC

This section analyses the effect of two critical parameters in EMTNRC and takes the algorithm EMTNRC-NSGA-II and the problem EG1 as an example. Two key parameters are summarized as follows: 1) \( \lambda \), the share of the TFE used for the normal optimization stage in EMTNRC; 2) \( t_1 \), the number of function evaluations for the knowledge transfer from the CD task to NR task in EMTNRC. To analyze one parameter visually, the values of other parameters are fixed.

Fig. 8 shows the MCC and NMI of EMTNRC versus varying \( \lambda \). The value of \( \lambda \) is set to 0.1, 0.3, 0.5, and 0.7. In Fig. 8(a), the value of MCC increases with \( \lambda \) until \( \lambda > 0.3 \). Then with increasing \( \lambda \), the median value of MCC decreases. Fig. 8 (b) shows that when \( \lambda \) is 0.5, the value of NMI is the most stable in all cases. With the increase of \( \lambda \), the NMI and MCC obtained by EMTNRC first increase and then decrease. This phenomenon appears because the accuracy of initializing the network impacts knowledge transfer in the normal optimization stage. With the increase of \( \lambda \), the TFE used for the normal optimization stage in
EMTNRCD decreases, which results in the inability to fully utilize the knowledge between the NR task and the CD task.

![Fig. 9. (a) MCC of EMTNRCD-NSGA-II versus varying $t_1$; (b) NMI of EMTNRCD-NSGA-II versus varying $t_1$.](image)

Fig. 9 shows the MCC and NMI of EMTNRCD versus varying $t_1$. The value of $t_1$ is set to 200, 400, 600, 800, and 1000. In Fig. 9 (a), with increasing $t_1$, EMTNRCD obtains the greater value of MCC. In Fig. 9 (b), when $t_1$ is 1000, the median value of NMI obtained by EMTNRCD is 1, that is, the community structure in the EG1 problem is completely identified. With increasing $t_1$, the transferred knowledge (network structure) can be more fully utilized to assist the NR task in EMTNRCD.

It can be seen from the observation that these parameters significantly impact the performance of EMTNRCD. These parameters affect the frequency of knowledge transfer and the convergence and diversity of the obtained non-dominated solution. Since different NR and CD problems have unique properties, the collection of parameters in the problem EG1 may not be suitable for the others.

6. Conclusions

This paper answers the question: could the joint optimization of NR and CD tasks effectively improve these two tasks’ accuracy? The answer is Yes, which is ensured by the proposed EMTNRCD. The core of EMTNRCD is to determine what knowledge should be transferred across two tasks. Benefited from the proposed evolutionary multitasking framework, we explicitly transfer the better community structure obtained by the CD task to
improve the performance of the NR task and share the better network structure obtained by
the NR task to enhance the performance of the CD task. Moreover, given the difficulties
encountered in developing the EMTNRCD framework, a preprocessing stage is designed to
overcome EA-based CD methods by obtaining the initial network structure. Besides, we first
model the CD task with explicit knowledge transfer as a dynamic CD problem. The
experimental results on 16 cases show that this strategy is effective and supports the claim
that joining these two tasks has a synergistic effect. The discovery of communities
significantly improves the reconstruction accuracy, which finds a better community structure
to perform these tasks in isolation.

However, several issues need to be solved. In terms of large-scale complex systems, the
NR task’s low accuracy will decrease the CD task’s performance. Thus, more robust NR
methods should be further studied. Besides, in future work, applying our proposal to find
overlapping communities is also a promising research topic.

Acknowledgements

This work was supported in part by the Key Project of Science and Technology
Innovation 2030 supported by the Ministry of Science and Technology of China under Grant
2018AAA0101302 and in part by the General Program of National Natural Science
Foundation of China (NSFC) under Grant 61773300.

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