Numerical calculation and measurement of low temperature thermal conductivity of polyurethane rigid foam

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Abstract. In the structure of cold compressors, polyurethane rigid foams (PURF) are often used as important thermal insulations for large temperature gradient. Heat transfer in PURF occurs by conduction through the solid skeleton and gases trapped in cells as well as radiation. According to the actual structure and operating conditions of helium cold compressors, a heat transfer hierarchical model has been established, which can predict the apparent thermal conductivity of PURF more simply. Meanwhile the thermal conductivity of a sample PURF has been measured in the physical property measurement system from 4.4K to 300K.

1 Introduction

As a key component of a large cryogenic refrigeration system, the adiabatic effect of the equipment becomes an important factor affecting the performance of helium cold compressors. S Yoshinaga found that the heat transfer through insulating materials accounts for about 50\% of total leakage \cite{1}.

In the structure of the cold compressor, PURF, an ideal insulating material, is divided into two parts by a copper thermal anchor: one part is within the range of 77K to 300K; the other is used between 4.2K and 77K. As a porous material, thermal conductivity of PURF is an important property. By measurement \cite{2-6} and calculation \cite{7-9}, the thermal conductivity can be obtained. Since the cost of the measurement is high, it is simple and efficient to calculate the thermal conductivity of PURF.

2 Calculations

The microstructure observed by a 180 times electronic magnifier shows that it mainly consists of the solid skeleton and gases trapped in cells, shown as Figure 1(a) and (b). Generally speaking, it is acceptable to ignore convection depending on the Rayleigh number. Therefore, there are three ways of heat transfer that occurs in PURF: radiation, conduction of the solid skeleton and conduction of gases. This means that the apparent thermal conductivity is the sum of these parts, shown as equation (1):
\[ k_e = k_{solid} + k_{gas} + k_{radiation} \]  

Where \( k_e \) is the apparent thermal conductivity, \( k_{solid} \) is conduction of the solid skeleton, \( k_{gas} \) is conduction of gases, and \( k_{radiation} \) is radiation.

2.1 Heat transfer hierarchical model

By referring to Yang’s heat transfer hierarchical model, a similar model has been established to analyze and the calculated apparent thermal conductivity has been obtained [10]. Since the closed rate of PURF can reach 95%, it is acceptable to assume that the closed rate is 100%. To simplify the calculation, the effect of \( \text{CO}_2 \) is also ignored because of the low partial pressure.

Because of different condensing temperatures of different gases, these gases exist in different temperature ranges, shown as Figure 2(a). Where, \( T_{WW} \) is the warm wall temperature, \( T_{BC} \) is the blowing agent-condensation temperature, \( T_{AC} \) is the air-condensation temperature, and \( T_{CW} \) is the cold wall temperature. Conduction of the solid skeleton and radiation exist throughout the whole temperature field. When temperature drops from \( T_{WW} \) to \( T_{BC} \), the physical blowing agent gas is condensed, so that there is only the air left in cells. When temperature drops to \( T_{AC} \), the air is in the free molecular conduction region, and conduction of gases is negligible.

2.2 The equivalent solid thermal conductivity \( k_{solid} \)

When PURF is foaming freely, parts of cells are pulled into long strips, and this inhomogeneity phenomenon is improved when PURF is foaming in molds. In this research, PURF is assumed to be homogeneity [9, 11]. Schuetz and Glicksman said that the solid thermal conductivity can be divided into two parts: conductivity of struts and conductivity of walls [12]. Between \( T_{WW} \) and \( T_{CW} \), the equivalent solid thermal conductivity can be calculated by equation (2):
2.3 The equivalent radiation thermal conductivity $k_{\text{radiation}}$

It is common to use diffusion approximate method to study radiation. By this method, the radiant heat flow can be obtained by the Rosseland equation and the equivalent radiation thermal conductivity can be written as the following equation (3):

$$k_{\text{radiation}} = \frac{16\sigma T^3}{K T_{cw} - T_{cw}}$$

Where $K$ is the mean extinction coefficient, $\sigma$ is the Stefan-Boltzmann constant, and $T$ is the temperature.

2.4 The equivalent gas thermal conductivity $k_{\text{gas}}$

As the aging time goes, the blowing agent gas diffuses from the inside to the outside, while the air diffuses oppositely. Because the thermal conductivity of air is greater than that of blowing agent gas, the insulation becomes worse and worse. In order to obtain $k_{\text{gas}}$, $T_{AC}$ needs to be calculated first.

2.4.1 Determination of $T_{AC}$

When the pressure drops to the point where the air is in the free molecular conduction region, the conduction of gases is negligible and this temperature is determined as $T_{AC}$. Knudsen number can be used to determine whether gases are in the free molecular conduction region or not. The calculated $T_{AC}$ is about 57K, which means that gases are always in the continuous conduction region at 77K-300K, shown as Figure 2(b).

2.4.2 Determination of $T_{BC}$ and air layer $x_2$.

When the partial pressure of blowing agent gas is much smaller than that of air, the influence of blowing agent gas can be ignored and this temperature can be determined as $T_{BC}$. $T_{AC}$, an assumed apparent thermal conductivity based on experiments, will be used as an initial value to calculate the total heat flow. Between $T_{CW}$ and $T_{BC}$, there are mainly three parts that affect the heat transfer: radiation, conduction of the solid skeleton and air. Since the heat flow at this temperature difference is equal to the total heat flow, $T_{BC}$ and $x_2$ can be calculate by several iterations.

2.4.3 Calculation of thermal conductivity of mixed gas layer $x_j$.

The thermal conductivity of mixed gases can be calculated by equations (4):

$$k_{\text{mix}} = \sum_{i=1}^{n} m_i k_i = \sum_{i=1}^{n} (p_i (\sum_{j=1}^{n} p_j)^{-1} k_i)$$

Where $k_{\text{mix}}$ is the thermal conductivity of the mixed gases; $m_i$, $k_i$ and $p_i$ are the molar fraction, the thermal conductivity and the partial pressure of each component. The mixed gases layer $x_j$ is divided into N layers equally, the thermal conductivity of each layer is calculated respectively, and the total
thermal conductivity of mixed gases is obtained by several iterations shown as equation (5). The approximate temperature of the $i_{th}$ layer is $T_i$, and the temperature rise of the $i_{th}$ layer is $\Delta T_i$.

$$q = \frac{N}{\chi_T} \int_{T_i}^{T_i+\Delta T_i} k_{i}(T) \, dT + \int_{T_i}^{T_i+\Delta T_i} k_{g}(T) \, dT + \int_{T_i}^{T_i+\Delta T_i} k_{mix}(T) \, dT + \int_{T_i}^{T_i+\Delta T_i} k_{r}(T) \, dT$$  \hspace{1cm} (5)

The equivalent gas thermal conductivity $k_{gas}$ contains two parts: one is the air in $x_3$, and the other is mixed gases in $x_1$, so $k_{gas}$ is the sum of them.

2.5 Results

When $|k_e-k_0|$ is very small, it is reasonable to say that the calculated results are relatively accurate. Yang’s experiment shows the measurement result of PURF, whose thickness is 25mm, between 77K and 300K [10]. Table 1 shows the results compared with his experiment, and the error is 17%.

| Density   | Blowing agent | Thermal conductivity |
|-----------|---------------|----------------------|
| Experiment | 32kg/m$^3$    | R11                  | 14.5 mW/(mK)         |
| Calculation| 32kg/m$^3$    | R11                  | 17 mW/(mK)          |

The error is mainly caused by four parts: (1) Physical parameters of gases are used by fitting formulas which may not be optimal. (2) In the process of calculating the thermal conductivity of mixed gases, the selection of $T_i$, which is approximately regarded as the temperature of the $i_{th}$ layer, affects the results of calculation. (3) By analysis, several simplifications of the model are acceptable. However, in practice, the apparent thermal conductivity is the comprehensive reflection of these complex phenomena. (4) In Yang’s experiment, factors such as the filling percentage of liquid nitrogen and the evaporation rate of liquid nitrogen will affect the final results.

Between 4.2K and 77K, based on the heat transfer hierarchical model, the apparent thermal conductivity can also be calculated. Instead of mixed gases, there is only the air left. As a result, the calculated apparent thermal conductivity is about 10.2 mW/(mK).

3 Measurement

Based on the steady-state method, Technical Institute of Physics and Chemistry builds a comprehensive physical property test system (PPMS), which is using a refrigerator as a cold source. A sample of PURFs of helium cold compressors, whose density is 136.5kg/m$^3$, is tested. The thermal conductivity from 4.4K to 300K are obtained and shown as Figure 3(a), (b) and (c).

As can be seen from the Figure 3(c), the thermal conductivity varies with the increase of temperature. This change is not linear for two reasons: (1) The proportions of solid conduction, gas conduction and radiation are different in different temperature ranges, so the contribution of each part to the thermal conductivity varies greatly with the change of temperature. (2) The nonlinearity of physical parameters of the solid skeleton and gases trapped in cells is an important factor.
Figure 3 (a) PPMS, (b) a sample of PURFs, (c) the thermal conductivity of PURF.

4 Conclusions

According to the operating conditions of helium cold compressors, a heat transfer hierarchical model of PURF has been established. The calculated apparent thermal conductivity of PURF with R11 as blowing agent is about 17mW/(mK), which meets the design requirements of helium cold compressors. By using this model, the apparent thermal conductivity of PURF at 4.2K-77K is also calculated, and the value is about 10.2 mW/(mK). The thermal conductivity of a sample of PURFs of helium cold compressors has been measured with PPMS, and the measure temperature reaches 4.4K.

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