The effects of the massless $O(\alpha_s^2)$, $O(\alpha\alpha_s)$, $O(\alpha^2)$ QCD and QED corrections and of the massive contributions to $\Gamma(H^0 \to b\bar{b})$

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Abstract

We consider in detail various theoretical uncertainties of the perturbative predictions for the decay width of $H^0 \to b\bar{b}$ process in the region $50 \text{ GeV} < M_H \leq 2M_W$. We calculate the order $O(\alpha_s^2)$-contributions to the expression for $\Gamma_{Hb\bar{b}}$ through the pole quark mass and demonstrate that they are important for the elimination of the numerical difference between the corresponding expression and the one through the running $b$-quark mass. The order $O(\alpha\alpha_s)$ and $O(\alpha^2)$ massless and order $O(m_b^2/M_H^2)$ massive corrections to $\Gamma_{Hb\bar{b}}$ are also calculated. The importance of the latter contributions for modeling of the threshold effects is demonstrated. The troubles with identifying of the 4 recent L3 events $e^+e^- \to l^+l^-\gamma\gamma$ with the decay of a Standard Higgs boson are discussed.
Among the most intriguing modern theoretical problems are the investigations of the properties of still nondiscovered Higgs particles. The special attention is nowadays paid to the searches of a Standard Model Higgs boson. Various aspects of "Higgs hunting" were discussed in detail from the theoretical \[1\],\[2\] and the phenomenological \[2\],\[3\], \[4\] points of view.

The current lower bound on a Standard Model Higgs boson mass is \( M_H > 52 \text{ GeV} \) at the 95\% confidence level (see e.g. \[3\]). This lower bound came from the analysis of the LEP data with taking into account of the theoretical expression for \( H^0 \rightarrow q\bar{q} \) decay rate including the results of the calculations of the order \( O(\alpha_s) \) QCD corrections \[3\]. One of the main decay channels of a Higgs boson in the intermediate mass range \( 50 \text{ GeV} < M_H < 2M_W \) is the decay to the \( b\bar{b} \) final states with the coupling constant being proportional to the \( b \)-quark mass. In this mass range the theoretical uncertainties of \( \Gamma_{Hb\bar{b}} = \Gamma(H^0 \rightarrow b\bar{b}) \) are closely related to the theoretical uncertainties of the branching ratio of the process \( H^0 \rightarrow \gamma\gamma \) which is known as one of the most typical reactions for the searches of not too heavy Higgs bosons.

Careful studies of various kinds of theoretical predictions for the cross-sections and branching ratios of a Higgs particles became important in view of the appearance of the announced by L3 collaboration \[4\] yet non-explained recent LEP events in the reaction \( e^+e^- \rightarrow l^+l^-\gamma\gamma \) (one \( e^+e^-\gamma\gamma \) and three \( \mu^+\mu^-\gamma\gamma \) events) with the invariant mass of the photons close to 60 GeV (!?) \[5\]. Moreover the analysis of the QCD uncertainties for \( \Gamma_{Hb\bar{b}} \), \( Br(H^0 \rightarrow \gamma\gamma) \) and for other branching ratios can be useful in the planned searches of a Higgs particles at the future colliders, namely LEP2, LHC, SSC.

The massless QCD corrections to \( \Gamma(H^0 \rightarrow \text{hadrons}) \) were considered at the next-to-next-to-leading order (NNLO) using the concept of the running \( b \)-quark mass \( m_b(M_H) \) \[5\],\[4\] with taking into account of the 3-loop NNLO corrections to the QCD \( \beta \)-function \[10\] and to the anomalous mass dimension function \[11\]. In the process of the consideration of the phenomenological consequences of the results of ref.\[4\] it was noticed further on that taking into account of the variation of the running \( b \)-quark mass from the \( b \)-quark mass-shell (namely from the \( m_b(m_b) \)-value) to the \( M_H \)-scale (namely to the \( m_b(M_H) \)-value) in the leading order (LO) of perturbation theory results in the negative corrections, which diminish the value of the corresponding Born expression for \( \Gamma_{Hb\bar{b}} \) by over 50\% \[3\].

However, the running mass is not the unique way of defining mass parameters. Indeed, in total analogy with the physical mass of electron in QCD one can also define the pole quark mass (for the general consideration of the corresponding renormalization group equations see ref.\[12\]). The definition of the pole quark mass is commonly used for heavy quarks, namely for \( c \)- and \( b \)-quarks. The expression for the decay width \( H^0 \rightarrow q\bar{q} \) \( (H^0 \rightarrow l^-l^+) \) has been explicitly calculated in terms of the pole quark masses \( m_c \) and \( m_b \) in refs.\[3\], \[13\] at the \( O(\alpha_s) \) \( (O(\alpha)) \)-level. The presented in ref.\[2\] numerical studies of these results did not reveal the effect of the 50\% reduction of the Born approximation. Therefore, it is important to understand the origin of the observed in ref.\[2\] puzzle of the differences between various parametrizations of the QCD results for \( \Gamma_{Hb\bar{b}} \) in the experimentally interesting region of \( M_H \).
values.

In this work using the 2-loop relation between the running and the pole quark masses [14] and the results of refs.[9] we calculate the expression for \( \Gamma_{Hb\bar{b}} \) at the \( \alpha_s^2 \)-level in terms of the pole quark mass in two different forms. The first one will contain the \( \ln(M_H^2/m_b^2) \)-contributions explicitly, while in the second one they will be summed up through the renormalization group (RG) technique. Note, that the second parametrization is closely related to the one through the running quark mass.

Our results demonstrate that for the RG-nonimproved expression for \( \Gamma_{Hb\bar{b}} \) the calculated by us \( \alpha_s^2 \)-contribution produce the negative correction which is responsible for the elimination of the numerical difference between various parametrizations of the QCD results for \( \Gamma_{Hb\bar{b}} \).

We also present the more detailed analysis of the RG-improved (running mass) expression for \( \Gamma_{Hb\bar{b}} \) with taking into account of the effects, neglected in the course of estimates of ref.[3]. Among them are the corrections responsible for the relation between the running mass \( \overline{m}_b(m_b) \) and the pole mass \( m_b \) at the 1-loop and 2-loop levels and the order \( O(m_b^2/M_H^2) \)-corrections to \( \Gamma_{Hb\bar{b}} \). The importance of taking into account of the order \( O(m_b^2/M_H^2) \)-corrections for modeling the threshold effects in the \( H^0 \rightarrow q\bar{q} \) process is demonstrated. Following the lines of ref.[17] we also calculate order \( O(\alpha^2) \) and \( O(\alpha\alpha_s) \) corrections to \( \Gamma_{Hb\bar{b}} \) and demonstrate that they are small.

Our pole mass dependent results support the effect of the 50\% reduction of the value of \( \Gamma_{Hb\bar{b}} \) making it more theoretically substantiated. As to the phenomenology, we discuss the troubles of the relation of the 4 events announced by L3 group [7] with the decay of the standard Higgs boson with the mass \( M_H = 60 \text{ GeV} \).

2. We start from technical considerations. First we remind different QCD expressions for \( \Gamma_{Hb\bar{b}} \) in terms of the pole quark mass which is defined as the position of the pole of the quark propagator. At the Born level of perturbation theory one has

\[
\Gamma_{Hb\bar{b}} = \Gamma(H^0 \rightarrow b\bar{b}) = \Gamma_0^{(b)} \beta^3
\]

where

\[
\Gamma_0^{(b)} = \frac{3\sqrt{2}}{8\pi} G_F M_H m_b^2
\]

\[
\beta = \sqrt{1 - \frac{4m_b^2}{M_H^2}}
\]

The corresponding \( O(\alpha_s) \) leading order (LO) approximation of the decay width reads [3, 13]

\[
\Gamma_{Hb\bar{b}} = \Gamma_0^{(b)} \beta^3 \left[ 1 + \delta(\alpha_s/M_H) \right]
\]

where

\[
\delta = \frac{4}{3} \left[ \frac{A(\beta)}{\beta} + \frac{3 + 34\beta^2 - 13\beta^4}{16\beta^3} \ln \frac{1 + \beta}{1 - \beta} + \frac{3(-1 + 7\beta^2)}{8\beta^2} \right]
\]
with $A(\beta)$ defined as

$$A(\beta) = (1 + \beta^2) \left[ 4L_2\left(\frac{1-\beta}{1+\beta}\right) + 2L_2\left(-\frac{1-\beta}{1+\beta}\right) - 3 \ln \frac{2}{1+\beta} \ln \frac{1+\beta}{1-\beta} - 2 \ln \frac{1+\beta}{1-\beta} \ln \beta \right] - 3\beta \ln \frac{4}{1-\beta^2} - 4\beta \ln \beta$$

(5)

Here $L_2(x) = \int_0^x (dt/t) \ln(1-t)$ is the Spence function [1].

It is well-known that in the limit $m_b^2/M_H^2 \to 0$ (or $\beta \to 1$) the expression for the $\alpha_s$-correction to $\Gamma_{Hb\bar{b}}$ appears to be infinite, namely it contains the logarithmic singularities

$$\Gamma_{Hb\bar{b}} = \Gamma_0^b \left[ 1 + \left(3 - 2\ln(x)\right)\frac{\alpha_s(M_H)}{\pi} \right] - 6m_b^2\frac{M_H^2}{\pi^2} \left[ 1 + \left(\frac{4}{3} - 4\ln(x)\right)\frac{\alpha_s(M_H)}{\pi} \right] + O\left(\frac{m_b^4}{M_H^4}\right)$$

(6)

where $x = M_H^2/m_b^2$. These logarithmic terms are connected to the renormalization of the overall Yukawa coupling, namely to the $b$-quark mass, which appear in the expression for $\Gamma_{Hb\bar{b}}$ at the leading order level. In order to avoid these logarithmic terms one can sum them up by replacing the pole $b$-quark mass by the normalized on the $M_H$-scale running $b$-quark mass $\bar{m}_b(M_H)$. Both the running quark mass and the pole quark mass are commonly used in QCD for the parametrization of the massive-dependent contributions to physical quantities. The relation between these two definitions of quark masses will be discussed in the next Section.

We will now consider the expression for $\Gamma_{Hb\bar{b}}$ in terms of $\bar{m}_b = \bar{m}_b(M_H)$ with taking into account of the order $O(\bar{m}_b^2/M_H^2)$-contributions. Since $\bar{m}_b(M_H)$ is defined through the running coupling constant $\alpha_s(M_H)$ already in the lowest order of perturbation theory, we will identify the Born RG-improved expression with the LO approximation, which can be defined as

$$\Gamma_{Hb\bar{b}} = \Gamma_{0}^{(b)} \frac{\bar{m}_b^2}{m_b^2} \left[ 1 - \frac{4\bar{m}_b^2}{M_H^2} \right]$$

(7)

where in the LO of perturbation theory $\bar{m}_b(M_H) = (\alpha_s(M_H)/\alpha_s(m_b))^{12/23}m_b$. At the NNLO level of perturbation theory the expanded in $O(\bar{m}_b^2/M_H^2)$-terms expression for $\Gamma_{Hb\bar{b}}$ can be presented in the following form

$$\Gamma_{Hb\bar{b}} = \Gamma_{0}^{(b)} \frac{\bar{m}_b^2}{m_b^2} \left[ 1 + \Delta \Gamma_1 + \Delta \Gamma_2 \right] - 6\frac{\bar{m}_b^2}{M_H^2} \left[ 1 + \Delta \Gamma_1^{(m)} + \Delta \Gamma_2^{(m)} \right] + O\left(\frac{\bar{m}_b^4}{M_H^4}\right)$$

(8)

Here $\Delta \Gamma_1 (\Delta \Gamma_1^{(m)})$ and $\Delta \Gamma_2 (\Delta \Gamma_2^{(m)})$ are the next-to-leading order (NLO) and NNLO QCD corrections to the ”coefficient functions” of the $H^0 \to b\bar{b}$ decay width $\Gamma_{Hb\bar{b}}$. The general analytic expressions for $\Delta \Gamma_1$ and $\Delta \Gamma_2$ were obtained in ref. [4] in the $\overline{MS}$-scheme with the help of the SCHOOSHIP analytical system [10]. In the case of QCD with $f = 5$ numbers of flavours they have the following numerical form [3]

$$\Delta \Gamma_1 = \frac{17}{3} \frac{\alpha_s(M_H)}{\pi}$$

(9)

4Notice that the expression for $A(\beta)$ in the second work from ref.[13] contained misprint-last two terms of eq.(5) were absent.
\[ \Delta \Gamma_2 = 29.14 \left( \frac{\alpha_s(M_H)}{\pi} \right)^2 \]  

The result for \( \Delta \Gamma_1 \) coincides with the one obtained previously \[17\]. The expression for \( \Delta \Gamma_2 \) was recently confirmed \[18\] using FORM analytical system \[19\].

Let us determine the NLO \( \alpha_s \) correction to the order \( O(m_b^2/M_H^2) \)-contributions using the results of calculations of ref. \[20\]. In the \( \overline{MS} \)-scheme it reads

\[ \Delta \Gamma_1^{(m)} = 5 C_F \frac{\alpha_s(M_H)}{\pi} = (C_F = 4/3) = \frac{20}{3} \frac{\alpha_s(M_H)}{\pi} \]  

The corresponding NNLO \( \alpha_s^2 \)-term is still unknown. In principle it can be obtained after the analysis of the results of the calculations of the 3-loop order \( O(m^2) \)-corrections to the 2-point function of the scalar quark currents in the euclidean region \[21\] with taking into account of the effects of the analytical continuation to the physical region \( (\pi^2 \text{-terms}) \). However, varying this correction between \( \Delta \Gamma_2^{(m)} = 0 \) and \( \Delta \Gamma_2^{(m)} = 40 (\alpha_s(M_H)/\pi)^2 \) we will demonstrate that this term is not very important in our phenomenological studies.

3. Let us now express the corresponding approximation for \( \Gamma_{H\bar{b}b} \) of eq.(8) with the radiative corrections defined by eqs.(9,10,11) through the b-quark pole mass. We will use the obtained in ref.\[14\] 2-loop relation between the normalized on the mass-shell running b-quark mass in the \( \overline{MS} \)-scheme and the pole b-quark mass, namely

\[ m_b^2(m_b) = m_b^2 \left[ 1 - \frac{8}{3} \frac{\alpha_s(m_b)}{\pi} - \left( 26.89 - 2.08 \sum_{f=u}^c \left( 1 - \frac{m_f}{m_b} \right) \left( \frac{\alpha_s(m_b)}{\pi} \right)^2 \right) \right] \]  

In the rough estimates presented in ref.\[3\] the corrections of order \( \alpha_s \) and \( \alpha_s^2 \) in eq.(12) were neglected. However, in view of the sizeable value of \( \alpha_s(m_b) \) it is desirable to take these corrections into account in the detailed studies. Using eq.(12) we get the following 2-loop relation between the \( \overline{MS} \)-scheme running and pole b-quark masses :

\[ m_b^2(M_H) = m_b^2 \left[ 1 - \left( \frac{8}{3} + 2 \ln(x) \right) \frac{\alpha_s(M_H)}{\pi} - \left( 26.89 - 2.08 \sum_{f=u}^c \left( 1 - \frac{m_f}{m_b} \right) \ln^2(x) + \frac{245}{36} \ln^2(x) \right) \left( \frac{\alpha_s(M_H)}{\pi} \right) \right] \]  

The main phenomenological difference between the pole mass \( m_b \) and the running mass \( m_b(M_H) \) is that \( m_b \) is the number (over \( m_b = 4.6 \text{ GeV} \) usually extracted from the theoretical studies of the properties of the b-quark bound states, while \( m_b(s) \) is the asymptotically decreasing function, which depends from the values of the QCD parameters \( m_b \) and \( \Lambda_{\overline{MS}} \) (through the expression for \( \alpha_s \); see eq.(22) defined below).

We will apply now eq.(13) for the determination of the higher order corrections to the expression of \( \Gamma_{H\bar{b}b} \) through the pole mass \( m_b \). The corresponding expression has the following form

\[ \Gamma_{H\bar{b}b} = \Gamma_{0}^{(b)} \left[ 1 + \Delta \Gamma_1 + \Delta \Gamma_2 \right] - 6 \frac{m_b^2}{M_H^2} \left[ 1 + \Delta \Gamma_1^{(m)} + \Delta \Gamma_2^{(m)} \right] + O \left( \frac{m_b^2}{M_H^2} \right) \]  

\[ (14) \]
where

\[ \Delta \Gamma_1 = \left(3 - 2 \ln(x)\right) \frac{\alpha_s(M_H)}{\pi} \] (15)

\[ \Delta \Gamma_2 = \left(4.54 - 2.08 \sum_{f=\text{up}}^c \frac{m_f}{m_b} - 18.14 \ln(x) + 0.08 \ln^2(x)\right) \left(\frac{\alpha_s(M_H)}{\pi}\right)^2 \] (16)

\[ \Delta \Gamma_1^{(m)} = \left(4 - 4 \ln(x)\right) \frac{\alpha_s(M_H)}{\pi} \] (17)

\[ \Delta \Gamma_2^{(m)} = \Delta \Gamma_2 - \left(65.6 + 4.16 \sum_{f=\text{up}}^c \frac{m_f}{m_b} + 29.6 \ln(x) - 4.17 \ln^2(x)\right) \left(\frac{\alpha_s(M_H)}{\pi}\right)^2 \] (18)

The results for \( \Delta \Gamma_1 \) and \( \Delta \Gamma_1^{(m)} \) are in agreement with the ones obtained from the complete LO massive dependence of \( \Gamma_{Hb\bar{b}} \) after expanding it in powers of \( m_b^2/M_H^2 \) (see eq.(15)). The expressions for the order \( \alpha_s^2 \) corrections are new.

In order to understand the relative value of the sizeable \( \ln(M_H^2/m_b^2) \)-terms in the above presented parametrization of \( \Gamma_{Hb\bar{b}} \) we will sum them back to the running b-quark mass \( \overline{m}_b \) by solving the corresponding RG equation

\[ \frac{\overline{m}_b^2(M_H)}{\overline{m}_b^2(m_b)} = exp\left[-2 \int_{\alpha_s(m_b)}^{\alpha_s(M_H)} \frac{\gamma_m(x)}{\beta(x)} dx\right] \] (19)

with taking into account of the 3-loop approximations of the QCD \( \beta \)-function [10] and of the anomalous mass dimension function \( \gamma_m(\alpha_s) \) [11]. Using then eq.(12) we express \( \overline{m}_b(M_H) \) through the pole quark mass \( m_b \) as

\[ \overline{m}_b^2(M_H) = m_b^2 \Phi_b(\alpha_s(M_H), \alpha_s(m_b)) \] (20)

The NNLO approximation of the \( \Phi_c \)-function in the \( \overline{MS} \)-scheme is determined by the considerations of refs.[1,2] and read

\[ \Phi_b = \left(\frac{\alpha_s(M_H)}{\alpha_s(m_b)}\right)^{\frac{24}{3}} \left[1 + 2.34 \frac{\alpha_s(M_H)}{\pi} + 4.37 \left(\frac{\alpha_s(M_H)}{\pi}\right)^2\right] \times \] (21)

\[ \left[1 - 2.67 \frac{\alpha_s(m_b)}{\pi} - \left(18.58 + 2.08 \sum_{f=\text{up}}^c \frac{m_f}{m_b}\right) \left(\frac{\alpha_s(m_b)}{\pi}\right)^2\right] \]

where we will use the following NNLO approximation of \( \alpha_s \) in the \( \overline{MS} \)-scheme

\[ \frac{\alpha_s(s)}{\pi} = \frac{1}{\beta_1 L_s} - \frac{\beta_2 \ln L_s}{\beta_1^2 L_s} + \frac{1}{\beta_2^3 L_s} (\beta_2^2 \ln^2 L_s - \beta_2^2 \ln L_s + \beta_1 \beta_3 - \beta_2) \] (22)

for \( f = 5 \) numbers of flavours with \( L_s = \ln(s/\Lambda_{\overline{MS}}^2) \), \( \beta_1 = 23/12 \), \( \beta_2 = 29/12 \) and \( \beta_3 = 9769/3456 \) [10].

Substituting now eq.(20) and eq.(21) into eq.(7) and eq.(8) we get the LO RG-improved expression for \( \Gamma_{Hb\bar{b}} \) through the b-quark pole mass

\[ \Gamma_{Hb\bar{b}} = \Gamma_0^{(b)} \left(\frac{\alpha_s(M_H)}{\alpha_s(m_b)}\right)^{\frac{24}{3}} \left(1 - \frac{4 m_b^2}{M_H^2} \left(\frac{\alpha_s(M_H)}{\alpha_s(m_b)}\right)^{\frac{24}{3}}\right)^{\frac{3}{2}} \] (23)
and the corresponding NNLO expression

\[ \Gamma_{Hb\bar{b}} = \Gamma_0^{(b)} \left( \frac{\alpha_s(M_H)}{\alpha_s(m_b)} \right)^{2\delta} \left[ 1 + \Delta \tilde{\Gamma}_1 + \Delta \tilde{\Gamma}_2 \right] - 6 \frac{m_b^2}{M_H^2} \left( \frac{\alpha_s(M_H)}{\alpha_s(m_b)} \right)^{2\delta} \left[ 1 + \Delta \tilde{\Gamma}_1^{(m)} + \Delta \tilde{\Gamma}_2^{(m)} \right] + O \left( \frac{m_b^4}{M_H^4} \right) \]

where

\[ \Delta \tilde{\Gamma}_1 = 8.01 \frac{\alpha_s(M_H)}{\pi} - 5.01 \frac{\alpha_s(m_b)}{\pi} \]

\[ \Delta \tilde{\Gamma}_2 = 46.8 \left( \frac{\alpha_s(M_H)}{\pi} \right)^2 - (11.2 + 2.08 \sum_{f=u}^c m_f) \left( \frac{\alpha_s(m_b)}{\pi} \right)^2 - 40.13 \frac{\alpha_s(M_H)}{\pi} \frac{\alpha_s(m_b)}{\pi} \]

\[ \Delta \tilde{\Gamma}_1^{(m)} = 11.35 \frac{\alpha_s(M_H)}{\pi} - 10.02 \frac{\alpha_s(m_b)}{\pi} \]

\[ \Delta \tilde{\Gamma}_2^{(m)} = \Delta \Gamma_2^{(m)} + 45.4 \left( \frac{\alpha_s(M_H)}{\pi} \right)^2 + (2.7 - 4.16 \sum_{f=u}^c m_f) \left( \frac{\alpha_s(m_b)}{\pi} \right)^2 - 113.7 \frac{\alpha_s(M_H)}{\pi} \frac{\alpha_s(m_b)}{\pi} \]

For the estimate of the numerical contribution of the \( \Delta \tilde{\Gamma}_2^{(m)} \)-term we will take \( \Delta \tilde{\Gamma}_2^{(m)} = 0 \) and \( \Delta \Gamma_2^{(m)} = 40 (\alpha_s(M_H)/\pi)^2 \).

This considered by us parametrization of \( \Gamma_{Hb\bar{b}} \) is closely related to the one through the running \( b \)-quark mass \( m_b \) (see eq.(8)). The difference results from the truncation of the corresponding perturbative series at the NNLO level after the substitution of eqs.(20), (21) into eq.(8). We think that this parametrization is more convenient for the study of the numerical difference between LO, NLO and NNLO RG-improved parametrizations of \( \Gamma_{Hb\bar{b}} \) (for the detailed discussion see Sec.4) and for the analysis of the theoretical uncertainties of the phenomenological estimates of ref.[3].

4. In order to understand the relative value of different QCD corrections to \( \Gamma_{Hb\bar{b}} \) discussed and derived in Secs.2,3 we plot at Figs.1-4 the expression for the ratio \( R_{Hb\bar{b}} = \Gamma_{Hb\bar{b}}/\Gamma_0^{(b)} \) (\( \Gamma_0^{(b)} = 3\sqrt{2/(8\pi)}G_F M_H m_b^2 \)) as the functions of \( M_H \) for the values of \( \Lambda_{MS}^{(5)} = 150 \) MeV and \( m_b = 4.8 \) GeV without and with order \( m_b^2/M_H^2 \)-corrections.

Fig.1 corresponds to the massless expression of \( R_{Hb\bar{b}} \) with \( \Gamma_{Hb\bar{b}} \) defined through the pole quark mass \( m_b \) via eq.(14) with (a) \( \Delta \Gamma_1 = 0, \Delta \Gamma_2 = 0 \); (b) \( \Delta \Gamma_1 = \text{(eq.(15))} \), \( \Delta \Gamma_2 = 0 \) and (c) \( \Delta \Gamma_1 = \text{(eq.(15))} \), \( \Delta \Gamma_2 = \text{(eq.(16))} \).

At Fig.2 the same ratio is expressed using the RG-improved massless formula of for \( \Gamma_{Hb\bar{b}} \) (see eq.(24)) with (a) \( \Delta \tilde{\Gamma}_1 = 0 \), \( \Delta \tilde{\Gamma}_2 = 0 \); (b) \( \Delta \tilde{\Gamma}_1 = \text{(eq.(25))} \), \( \Delta \tilde{\Gamma}_2 = 0 \) and (c) \( \Delta \tilde{\Gamma}_1 = \text{(eq.(25))} \), \( \Delta \tilde{\Gamma}_2 = \text{(eq.(26))} \) and the QCD coupling constant \( \alpha_s \) defined by the LO, NLO and NNLO approximations of eq.(22) correspondingly.

Fig.3 shows the dependence of \( R_{Hb\bar{b}} \) from \( M_H \) in the case of the pole quark mass parametrization of \( \Gamma_{Hb\bar{b}} \) with taking into account of the \( O(m_b^2/M_H^2) \)-corrections. The solid curve (a) displays the complete massive dependence of the Born approximation for \( R_{Hb\bar{b}} \) with \( \Gamma_{Hb\bar{b}} \) defined by eq.(1); the dashed curve (a) corresponds to the expanded in \( m_b^2/M_H^2 \)-term LO expression for \( \Gamma_{Hb\bar{b}} \) (see eq.(14) with \( \Delta \Gamma_1 = 0 \), \( \Delta \Gamma_2 = 0 \), \( \Delta \Gamma_1^{(m)} = 0 \) and \( \Delta \Gamma_2^{(m)} = 0 \). The dashed-dotted curve (b) demonstrates the dependence from \( M_H \) of the complete LO
expression for $R_{Hb\bar{b}}$ with $\Gamma_{Hb\bar{b}}$ defined by eqs.(3)-(5)); the dotted curve (b) corresponds to the LO approximation of $\Gamma_{Hb\bar{b}}$ (see eq.(14)) with $\Delta \Gamma_1=(eq.(15))$, $\Delta \Gamma_2=0$, $\Delta \Gamma_1^{(m)}=(eq.(17))$, $\Delta \Gamma_2^{(m)}=0$. The curves (c) compare the NLO behavior of the expanded expression for $R_{Hb\bar{b}}$ with $\Gamma_{Hb\bar{b}}$ defined by eqs.(14)-(18) with $\Delta \tilde{\Gamma}_1^{(m)}=0$, $\Delta \tilde{\Gamma}_2^{(m)}=0$, $\Delta \tilde{\Gamma}_1^{(m)}(m)=0$, $\Delta \tilde{\Gamma}_2^{(m)}=0$ and the LO approximation of $\alpha_s$. The dotted curve (b) gives the understanding of the behavior of the NLO RG-improved approximant for $R_{Hb\bar{b}}$ with $\Gamma_{Hb\bar{b}}$ defined by eq.(24) with $\Delta \tilde{\Gamma}_1^{(m)}=(eq.(25))$, $\Delta \tilde{\Gamma}_2^{(m)}=0$, $\Delta \tilde{\Gamma}_1^{(m)}(m)=0$, $\Delta \tilde{\Gamma}_2^{(m)}=0$ and the NLO approximation of $\alpha_s$. Finally the curves (c) demonstrate the behavior of the NNLO RG-improved massive approximation of $R_{Hb\bar{b}}$ defined by eqs.(24)-(28) with $\Delta \tilde{\Gamma}_2^{(m)}=0$ (solid curve) and $\Delta \tilde{\Gamma}_2^{(m)}=40(\alpha_s(M_H)/\pi)^2$ (dashed curve).

The similar dependence of the RG-improved results of eqs.(23),(24) are depicted at Fig.4. The solid curve (a) corresponds to the RG-improved non-expanded LO massive dependent expression for $R_{Hb\bar{b}}$ where $\Gamma_{Hb\bar{b}}$ is defined by eq.(23) with the LO approximation of $\alpha_s$; the dashed curve (a) demonstrates the behavior of the expanded in masses LO RG-improved expression for $R_{Hb\bar{b}}$ with $\Gamma_{Hb\bar{b}}$ defined by eq.(24) with $\Delta \tilde{\Gamma}_1^{(m)}=0$, $\Delta \tilde{\Gamma}_2^{(m)}=0$, $\Delta \tilde{\Gamma}_1^{(m)}(m)=0$, $\Delta \tilde{\Gamma}_2^{(m)}=0$ and the LO approximation of $\alpha_s$. The dotted curve (b) gives the understanding of the behavior of the NLO RG-improved approximant for $R_{Hb\bar{b}}$ with $\Gamma_{Hb\bar{b}}$ defined by eq.(24) with $\Delta \tilde{\Gamma}_1^{(m)}=(eq.(25))$, $\Delta \tilde{\Gamma}_2^{(m)}=0$, $\Delta \tilde{\Gamma}_1^{(m)}(m)=0$, $\Delta \tilde{\Gamma}_2^{(m)}=0$ and the NLO approximation of $\alpha_s$. Finally the curves (c) demonstrate the behavior of the NNLO RG-improved massive approximation of $R_{Hb\bar{b}}$ defined by eqs.(24)-(28) with $\Delta \tilde{\Gamma}_2^{(m)}=0$ (solid curve) and $\Delta \tilde{\Gamma}_2^{(m)}=40(\alpha_s(M_H)/\pi)^2$ (dashed curve).

Let us now discuss our understanding of the behavior of the different approximations of $R_{Hb\bar{b}}$ for $\Lambda^{(5)}_{MS} = 150$ MeV.

1. For the pole mass parametrization of eq.(3) and eq.(14) taking into account of the LO and NLO QCD corrections far above threshold region decreases the value of $R_{Hb\bar{b}}$ by over 40% and 10% correspondingly in both massless and massive-dependent cases (see Figs.1,3). At the NLO level the total numerical reduction is therefore of over 50%.

2. For the RG-improved parametrization the similar pattern can be observed already for the LO Born results of eq.(23),(24) (see Figs.2,4). This welcomed feature, which was already noticed in ref.[3] starting from the results of ref.[2], is related to the effect of variation of the running b-quark mass from the on-shell scale (namely from $m_b(m_b)$) to the $M_H$ scale (i.e. to $m_b(M_H)$) and demonstrate the importance of application of the RG-formalism in the phenomenological studies.

3. Figs.2,4 show that, on the contrary to the pole mass approach, in the RG-approach the LO approximation does not differ significantly from the NLO and the NNLO results. This is the one more welcomed feature of the RG-analysis, which demonstrates that the theoretical uncertainties of the RG-improved Born approximation are significantly smaller of the uncertainties of the Born expression for $\Gamma_{Hb\bar{b}}$ through the pole quark mass. Notice, however, that far above the threshold region the NLO corrections are smaller (sometimes they are even zero!) then the negative NNLO ones, which decrease the LO and NLO approximations by 2% to 4% [5]. This pattern might indicate the

\[ \text{On the contrary to the considerations of [23, 24] we expect the manifestation of the similar behavior of the RG-improved massive-dependent contributions to $Z^0 \rightarrow b\bar{b}$ decay.} \]
possible problems of applicability of the NNLO perturbative approximation of $\Gamma_{Hb\bar{b}}$ (see eq.(24)) and of the related 2-loop mass relation of eq.(14) already discussed from this point of view in the original work on the subject [14].

4. One of our main results is that the calculated by us $\alpha_s^2$-corrections to the pole-mass parametrization of both "massless" and the order $m_b^2/M_H^2$ contributions to $\Gamma_{Hb\bar{b}}$ (see eqs.(14,16,18)) decrease the difference between the pole-mass and RG-improved expressions for $R_{Hb\bar{b}}$ (compare Figs.1,3 with Figs.2,4). Indeed, the 15% difference of the order $O(\alpha_s)$ approximations shrinks to over 4% at the $O(\alpha_s^2)$-level. Therefore, the observed in ref.[2] puzzle of the differences between RG-non-improved and RG-improved parametrizations of $\Gamma_{Hb\bar{b}}$ can be resolved after taking into account of these effects. We have checked that the similar situation also holds in the case of the vector and axial contributions to $Z^0 \rightarrow b\bar{b}$ decay, which were calculated at the NNLO in ref.[23] and ref.[24] correspondingly. We hope to return to the more detailed consideration of this topic in future.

5. The comparison of Figs.1,3 and Figs.2,4 demonstrate that starting from $M_H \geq 60 \text{ GeV}$ the order $O(m_b^2/M_H^2)$-contributions can be safely neglected both in the pole-mass and RG-improved approaches. However, beyond this region these corrections are very important for modeling the threshold behavior of the corresponding approximants.

6. Indeed, the comparison of two curves (a) and (b) of Fig.3 and of two curves (a) of Fig.4 shows that near the threshold region the Born and the LO massive dependence of the pole-mass expression for $R_{Hb\bar{b}}$ (see eqs.(3,4,5)) and the Born LO RG-improved massive dependence (see eq.(23)) are nicely approximated by the corresponding expanded formulae with taking into account of the leading order $m_b^2/M_H^2$ and $\pi_b^2/M_H^2$-terms. In view of this excellent agreement we can hope that this might be the feature, which do not depend neither from the order of perturbation theory, nor from the concrete process. This observation joins the results of the more rigorous considerations of the non-expanded and expanded expressions of the 2-loop self-energy diagrams with different masses [25] in the understanding of the necessity of taking into account of the massive corrections in the problems connected with the phenomenological studies of the threshold effects in different processes, including $e^+e^- \rightarrow \text{hadrons}$ and $Z^0 \rightarrow \text{hadrons}$, which are now under consideration [26]. This observation is another new result of our work.

7. The comparison of the threshold behavior of the pole-mass and RG-improved massive-dependent expressions for $R_{Hb\bar{b}}$ with different values of the unknown at present order $O(\alpha_s^2m_b^2/M_H^2)$ term $\Delta \Gamma_2^{(m)}$ (compare solid curves (c) at Figs.3,4 with $\Delta \Gamma_2^{(m)} = 0$ and the corresponding dashed curves (c) with $\Delta \Gamma_2^{(m)}=40(\alpha_s(M_H)/\pi)^2$) demonstrate that this term is not very important in modeling the threshold behavior of the corresponding
2-loop approximants. However, in view of the discussions of the previous Subsection 4.6 it might be of theoretical interest to calculate this term explicitly.

Taking into account the discussed above QCD uncertainties and the current uncertainties of the value of the parameter $\Lambda^{(5)}_{\overline{MS}}$ (for the detailed discussions see ref. [27]) we obtain the following theoretical estimate for $\Gamma_{Hb\bar{b}}$ in the intermediate region of $M_H$ values $50 \text{ GeV} < M_H < 160 \text{ GeV}$

$$\Gamma_{Hb\bar{b}} = \left(0.55 \div 0.45\right) \frac{3\sqrt{2}}{8\pi} G_F M_H m_{b}^2$$

This result contain less theoretical uncertainties then the estimat es of ref.[3]. Even more refined estimates can be obtained from Figs.1-4 using the concrete values of $M_H$-mass.

It should be mentioned, that for $\Lambda^{(5)}_{\overline{MS}}=250 \text{ MeV}$ the relative value of the NLO corrections to the RG-improved expression of $\Gamma_{Hb\bar{b}}$ in the typical region of $M_H$ values is slightly different then in the case of $\Lambda^{(5)}_{\overline{MS}}=150 \text{ MeV}$, being identically zero for $M_H \approx 65 \text{ GeV}$ [28].

Note, that in our considerations of the order $O(\alpha_s^2)$ QCD massive-dependent corrections to $\Gamma_{Hb\bar{b}}$ we did not take into account the possible contributions of the diagrams of Fig.5 with the b-quark and t-quark virtual loops. These diagrams arise from the 3-loop ones of Fig.6 after the unitarity cut (a) (another cut (b) gives the contribution to $H^0 \rightarrow gg$ subprocess). In the limit $M_H >> 2m_q$ (namely in the case of the neglecting of the masses of the quarks propagating in the internal loops) these diagrams are zero identically. However, they can give some massive-dependent corrections, not studied in ref. [21]. Indeed, as was shown in ref. [29], the calculation of the contribution of the similar diagrams to the axial channel of $Z^0 \rightarrow b\bar{b}$ decay results in the appearance of the sizeable $\alpha_s^2 \ln(m_t/M_Z)$-terms. In our case we also expect the appearance of the massive correction of the similar origin (namely with the top-mass dependence). We think that this, not yet calculated contribution, will not change significantly our result of eq.(29).

In the region $50 \text{ GeV} < M_H < 160 \text{ GeV}$ the result (29) for $\Gamma_{Hb\bar{b}}$ is also not affected by the 1-loop electroweak (EW) corrections, calculated in ref. [13]. Indeed, for the analyzed values of a Higgs mass these negative EW corrections are negligible (less then $-1\%$) [13].

They start to play the significant role in the region $M_H > 250 \text{ GeV}$ [13], not considered in our work. Another source of theoretical uncertainties in the $\Gamma_{Hb\bar{b}}$-expression comes from the higher order EW-corrections (which due to the smallness of the 1-loop EW effects in the analyzed region of $M_H$-values should be small) and from the mixed EW-QCD contributions. Following the lines of ref. [15] and using the results of [9] we can calculate the part of these corrections, namely the contributions to $\Gamma_{Hb\bar{b}}$ of the QED corrections of order $O(\alpha^2)$ and $O(\alpha\alpha_s)$.

The QED corrected expression for $\Gamma_{Hb\bar{b}}$ reads

$$\Gamma_{Hb\bar{b}} = \Gamma^{QCD}_{Hb\bar{b}} + \Gamma^{QED}_{Hb\bar{b}}$$

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\[\text{\textsuperscript{6}}\text{We are far away from extracting from this observation any theoretical consequences}\]
where the QCD contributions were discussed in detail in Secs. 2, 3, 4. The QED contribution $\Gamma_{Hb\bar{b}}$ can be presented as

$$\Gamma_{Hb\bar{b}} = \Gamma_0 \left( 1 + \frac{m_b^2}{M_H^2} \right) \Delta \Gamma_{1,QED} + \Delta \Gamma_{2,QED} - \frac{6 m_b^2}{M_H^2} \Delta \Gamma_{1,QED}^{(m)} + O \left( \frac{m_b^4}{M_H^4} \right)$$

(31)

where $m_b = m_b(M_H)$ is defined with taking into account of the QED corrections to the anomalous mass dimension function $\gamma_m$. The expressions for the QED corrections to the "coefficient functions" have the following form

$$\Delta \Gamma_{1,QED} = \frac{17}{4} Q_b^2 \alpha(M_H) = 0.47 \frac{\alpha(M_H)}{\pi}$$

(32)

$$\Delta \Gamma_{2,QED} = \left[ \left( \frac{691}{64} - \frac{9}{4} \zeta(3) - \frac{3\pi^2}{8} \right) Q_b^4 - \left( \frac{65}{16} - \frac{163}{16} \zeta(3) - \frac{\pi^2}{12} \right) Q_b^2 \left( 3 \sum_{j=u}^b Q_j^2 + N \right) \right] \left( \frac{\alpha(M_H)}{\pi} \right)^2$$

$$+ \left( \frac{691}{24} - 6 \zeta(3) - \pi^2 \right) Q_b^2 \alpha(M_H) \frac{\alpha_s(M_H)}{\pi}$$

(33)

$$\Delta \Gamma_{1,QED}^{(m)} = 5 Q_b^2 \alpha(M_H) \frac{\alpha_s(M_H)}{\pi} = 0.56 \frac{\alpha(M_H)}{\pi}$$

(34)

where $Q_j$ are the charges of u,d,s,c and b-quarks, N is the number of leptons (N=3) and $\alpha(M_H)$ is the QED running coupling constant in the $\overline{MS}$-scheme. One can see that the higher order QED contributions, as calculated by us, are very small.

5. The analysed by us corrections do not affect the conclusion that there are the problems with the relation of the announced by L3 collaboration 4 events with the decay of a Standard Higgs boson with the mass $M_H = 60$ GeV. Indeed, the number of $H \to \gamma\gamma$ events at the detector with the acceptance $A$ is

$$N = A \times L \times \sigma_{\text{tot}} \times \text{Br}(H \to \gamma\gamma)$$

(35)

where $L$ is the integrated luminosity and $\sigma_{\text{tot}}$ is the total cross-section of a Higgs production. For the process $e^+e^- \to q\bar{q}, H$ with $M_H = 60$ GeV $\sigma_{\text{tot}}$ with taking into account of the initial state radiation is about 0.4 pb at $Z$-peak.7 This means that for the process $e^+e^- \to l^-l^+, H$ $\sigma_{\text{tot}}$ is about 0.04 pb. For the L3 data the integrated luminosity is 27.1 pb. Since for the $l^-l^+, 2\gamma$ channel the L3 detector has the acceptance $A = 0.85$, one has

$$N < 10^{-3} (!)$$

(36)

Moreover, for $M_H = 60$ GeV one should have the same number of the events for the process $e^+e^- \to H \gamma \to 3\gamma$. Indeed, for $M_H = 60$ GeV $\sigma_{\text{tot}}(e^+e^- \to H \gamma) = \sigma_{\text{tot}}(e^+e^- \to l^-l^+ H)$ (see e.g.3). Notice, that $e^+e^- \to 3\gamma$ L3 data do not contradict QED.

\footnote{The process $e^+e^- \to q\bar{q}, H$ was first considered in ref.31 and then discussed in ref.32.}
Conclusions.

In this work we consider in detail various QCD uncertainties of the perturbative predictions for the decay width of the $H^0 \to b\bar{b}$ process, which is the dominant decay mode of a Higgs boson in the intermediate mass range $50 \text{ GeV} < M_H < 2M_W$. We calculate the $\alpha_s^2$-contributions to the expression for $\Gamma_{Hb\bar{b}}$ through the b-quark pole mass. We present two different parametrizations of the results for the decay width $\Gamma_{Hb\bar{b}}$ through the b-quark pole mass and demonstrate that the calculated by us $\alpha_s^2 \ln^2(M_H/m_b)$ and $\alpha_s^2 \ln(M_H/m_b)$ terms (which are responsible for the variation of the running b-quark mass at the NNLO level) are important since they are decreasing the difference between the corresponding approximation for $\Gamma_{Hb\bar{b}}$ and the one obtained after the summation of these $\ln(M_H/m_b)$-terms using the RG-technique. We also analyse in detail various massive corrections of order $O(m_b^2/M_H^2)$ and demonstrate their importance for the analysis of the threshold contributions to $\Gamma_{Hb\bar{b}}$.

Far above threshold region the analyzed by us QCD corrections are decreasing the value of $\Gamma_{Hb\bar{b}}$ by the factor of over 0.5 (for $\Lambda_{\overline{MS}}(5)=150 \text{ MeV}$) or over 0.45 (for $\Lambda_{\overline{MS}}(5)=250 \text{ MeV}$). The reduction of the value of $\Gamma_{Hb\bar{b}}$ is manifesting itself most obviously for the RG-improved expressions which is connected with the running mass parametrization. This fact demonstrates the importance of using the RG-method in the phenomenological studies.

The discussed QCD contributions to $\Gamma_{Hb\bar{b}}$ are increasing the values of the branching ratios of the decays $H^0 \to l^-l^+$ and $H^0 \to \gamma\gamma$ by the factor of over 2. The consideration of the latter process with taking into account of all existing theoretical information about its decay width [34],[35] can be of interest from the point of view of planning of the experiments aimed to the searches of the signals from a Higgs boson in the intermediate mass range. In future we are going to return to the detailed considerations of the QCD uncertainties for the branching ratios of different decays of a Standard Model Higgs boson.

In the process of our work we became aware of the work [36], where the effects of the NNLO corrections to $\Gamma_{Hb\bar{b}}$ [9] were also considered using the running mass parametrization. The certain physical results of our more detailed studies are in qualitative agreement with the presented in refs.[36] considerations, which however do not touch the analysed by us problems. The proposed by us RG-improved parametrization of the massive dependent effects (see ref.[28] and the discussions above) was also subsequently used in ref.[37] for the analysis of the massive-dependent contributions to the axial $Z$-boson decay rate. This analysis confirm our qualitative expectations. However, it might be also of theoretical interest the further more detailed study of the prescription dependence (or scheme dependence) of the perturbation serious predicions for the massive-dependent contributions to physical quantities, say to $Z \to b\bar{b}$.

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Figure captions

Fig.1: The pole-mass parametrization of the different approximations for $R_{H\bar{b}b} = \Gamma_{H\bar{b}b}/\Gamma_o^{(b)}$ without massive-dependent corrections ($\Lambda_{\overline{MS}}^{(5)} = 150 \text{ MeV}$).

Fig.2: The RG-improved approximation of the different expressions for $R_{H\bar{b}b}$ without massive-dependent corrections ($\Lambda_{\overline{MS}}^{(5)} = 150 \text{ MeV}$).

Fig.3: The pole-mass parametrization of the different approximations of $R_{H\bar{b}b}$ with the massive-dependent corrections ($\Lambda_{\overline{MS}}^{(5)} = 150 \text{ MeV}$).

Fig.4: The RG-improved approximations of the different expressions for $R_{H\bar{b}b}$ with the massive-dependent contributions ($\Lambda_{\overline{MS}}^{(5)} = 150 \text{ MeV}$).

Fig.5: The diagram with the top-quark loop, which was not taken into account in our considerations.

Fig.6: The 3-loop graph which gives different non-analysed by us contributions to $\Gamma(H^0 \rightarrow \text{hadrons})$ after the unitarity cuts.