Magnetization suppression of Type-II Superconductors by external alternating magnetic field

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The effect of suppression of static magnetization of an anisotropic hard superconductor by alternating magnetic field is analyzed theoretically. The magnetic moment suppression dynamics is described with respect to the magnetization loop of the superconductor. It is found that in some cases the magnetic moment varies nonmonotonically with the growth in amplitude h of the alternating field. Effect of transition, induced by h(t), of superconductor form paramagnetic into the diamagnetic state is considered. The amplitude of alternating magnetic field h(ωt) for which the complete suppression of the magnetization occurs is calculated as the function of anisotropy parameter δ and orientation angle ϑ with respect to the crystallographic axes of the sample.

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I. INTRODUCTION

For many years the physics of the vortex matter in the superconductors attracts the attention of researchers. This is due to collective properties of the vortices reveal variety of reach and very interesting phenomena which involve different phases and phase transitions Ref. [1,2,3,4,5], magnetic instabilities Ref. [6,7,8,9] mesoscopic and fluctuation effects and others. In this paper one example of the instability effects, namely the problem of magnetic moment suppression by external alternating magnetic field, is considered in some details. Usually the static and quasi-static electromagnetic properties of hard superconductors is treated in terms of critical state model at first suggested by Bean Ref. [10]. This model describes of magnetic induction B distribution inside the superconductors. In the simplest possible case, when superconductor represents rectangular slab with x-axis directed perpendicular to it planes and z-axes along the magnetic field direction, the Bean equation has following form ∂B/∂x = ±4πJc/c, here Jc is critical current density. Critical state equation can be easily understood by analyzing the forces balance which acts on a single vortex. In accordance to the model, the Lorentz force, which acts on the vortex, is compensated by the pinning forces between the vortex and various defects in the crystalline lattice. Later critical state model was generalized for more complicated situations in several directions.

First generalization was performed by the authors of Ref. [11] in order to include quasi-stationary processes. In the most of the cases, for simple geometries, direction of the critical current vector Jc is uniquely determined. But situation is some what more complicated when external magnetic field has several spatial components or when it varies in time. For these time dependent cases usually critical state model is written as curlB = (4πJc/c)E/E, where E is electric field. This model assumes that varying in time magnetic flux produces electric field, superconductor becomes in the resistive state and the direction of electric current coincide with the direction of E similar to normal metal. It is also important to answer the question about the direction of critical current density in that regions of superconductor where there is no electric field. At this point rises one of the most important electrodynamic properties of hard superconductors. The magnetic state of the superconducting sample, at some specific moment of the time, is described not only by critical model equations with some boundary conditions but also depends on prehistory - how this state was prepared. In order to understand this statement let’s consider example of static magnetization. Assume that we have superconducting slab placed in the external magnetic field with the quasi-stationary varied amplitude. Because magnetic field changes in time very slowly we can assume that in each instance superconducting sample is in the critical state with the critical current density Jc(B) which is determined by the value of magnetic induction at this moment of the time. Direction of this current coincides with the direction of electric field E which emerges from slowly varying external magnetic field H. This picture is valid while amplitude of H changes. As soon as magnetic field stop changes the electric field disappears but critical current density persists its direction which was set by the electric field at last instance of its existence. Exactly this dissipationless current determines final magnetization of the sample which is usually called static.

It is also important to point out that critical state model equations are significantly nonlinear and this nonlinearity is peculiar only for superconductors and has no equivalents in other nonlinear medias. This specific nonlinearity leads to several very interesting effects [12] one of which is static magnetization suppression. The essence of this effect is the following. Let a plane superconducting sample cooled in zero magnetic field be placed into an external magnetic field H > Hc1 (here Hc1 is the lower critical magnetic field) which is parallel to the superconductor surface. Pinning leads to the emergence of a nonuniform distribution of the magnetic induction and, accordingly, a static magnetization, in the sample. If an alternating magnetic field h(t) = h cos(ωt) is applied to the magnetized sample in a direction parallel to the sample surface and perpendicular to a constant field...
H, then magnetization M of the sample decreases. Direct measurements\(^{22}\) show that everywhere in the sample where the alternating magnetic field penetrates, the flow of nondissipative currents becomes impossible. The current that previously screened field H and contributed to the magnetization of superconductor disappear. If the amplitude is sufficiently large \(h \sim H_p = 2\pi J_c(H)/cd\) (the alternating field is practically penetrate through the entire sample) magnetization is suppressed.

Second generalization of critical state model is related to the fact of high anisotropy of the high-\(T_c\) superconductors which will sufficiently important in this work. In the anisotropic model current critical density becomes a tensor \(J_{cij}\) and straightforward generalization of model equations gives \((\text{curl} B)_i = (4\pi/c)J_{cij}E_j/E\). Diagonal components of the \(J_{cij}\) tensor are significantly different, for example for YBaCuO the critical current density in the \(ab\) plane is much greater than in \(c\) direction.

In this work the theoretical analysis of static magnetization suppression in the anisotropic case under influence of alternating magnetic field \(h(t) = h\cos(\omega t)\) is performed. It is known that magnetization curve \(M(H)\) of HTSC reveals hysteresis and it turns out that character of magnetization suppression is very sensitive to the position of current magnetization in the magnetization loop (this reflection of the sensitivity to the magnetic prehistory described above). It is found that in total there are nine regions in the magnetization loop where suppression scenario is qualitatively different and dynamics of magnetic moment suppression with growth of the amplitude \(h\) is described in each of these regions.

II. MAIN EQUATIONS AND GEOMETRY OF THE PROBLEM

Consider an infinite plane-parallel superconducting plate of thickness \(d\) placed in external constant magnetic field \(H\) and alternating magnetic field \(h(t) = h\cos(\omega t)\) which are mutually perpendicular and parallel to the plate surface. It is assumed that all fields and currents depend on only one spatial coordinate \(x\) directed along the normal to the plate. The origin \(x = 0\) is located in the origin of the sample. In this geometry the equations of generalized critical state model for magnetic induction \(B\), written in the components, take form

\[
\frac{\partial B_x}{\partial x} = -\frac{4\pi}{c} J_{cy}(B_y, B_z) \cos(\varphi(x)),
\]

\[
\frac{\partial B_y}{\partial x} = \frac{4\pi}{c} J_{cz}(B_y, B_z) \sin(\varphi(x)),
\]

here \(\varphi(x)\) - angle between the electric field vector \(E\) and \(y\) axis. The spatially averaged component of the magnetization \(\mathcal{M}\) along the direction of the external magnetic field \(H\), which we will denote as \(M_H\), is given by following formula

\[
M_H = \frac{1}{4\pi} \left[ \frac{1}{d} \int_{-d/2}^{d/2} (B_x(x) \cos \varphi + B_y(x) \sin \varphi) dx - H \right]
\]

Eqs. (1) and (2) should be accompanied by the Maxwell equations for electric field

\[
\frac{\partial E_z}{\partial x} = \frac{1}{c} \frac{\partial h_p}{\partial t}, \quad \frac{\partial E_y}{\partial x} = -\frac{1}{c} \frac{\partial h_z}{\partial t}
\]

and boundary conditions

\[
B_z\left(\pm \frac{d}{2}\right) = H \cos \vartheta, \quad B_y\left(\pm \frac{d}{2}\right) = H \sin \vartheta.
\]

It is clear that exact integration of Eqs. (1) is very complicated problem due to specific nonlinearity of equations. For the sake of simplifying of the calculations, the dependence of the components \(J_c(B(x))\) of the critical current density on the \(x\) coordinate, caused by the nonuniformity of the magnetic induction distribution, will be neglected and assumed that \(J_c(B(x)) = J_c(H)\). In addition, the \(x\) - coordinate dependent angle \(\varphi(x)\) between the electric field and the \(y\) axis will be replaced by the angle \(\pi/2 + \vartheta\) between the external alternating magnetic field and the \(z\) axis (as in the anisotropic situation). It turns out that these simplifications have no qualitative influence on the results but make it possible to perform analytical calculations completely.

For further convenience lets introduce following dimensionless variables

\[
\xi = \frac{2x}{d}, \quad \mathcal{H} = \frac{H}{H_p}, \quad \mathcal{B} = \frac{B}{H_p}, \quad H_p = \frac{2\pi d J_{cy}}{c},
\]

\[
h = \frac{h}{H_p}, \quad b = \frac{b}{H_p}, \quad \mathcal{M} = \frac{M_H}{H_p}, \quad \delta = \frac{J_{cz}}{J_{cy}}
\]

here capital letters refer to the dc-magnetic field \(H\) and small letters to the ac-magnetic field \(h(t)\).

III. CALCULATION OF MAGNETIC MOMENT SUPPRESSION DYNAMICS

The main idea of calculations is as follows, first of all we have to find static magnetization when the alternating magnetic field is absent. This part of the problem is well developed both theoretically and experimentally and we can refer to Ref.\(^{23}\) where magnetization loop was studied in details. For each point of the magnetization loop there is some specific distribution of magnetic induction which determines magnetization. As the second step we have to solve dynamical problem and describe the penetration of ac-magnetic filed. In this way it will be possible to find the penetration depth \(\xi_h\) and critical amplitude of ac-field \(h_c(\vartheta)\) at which total suppression of magnetization occurs. As soon as problem is two-dimensional it means
that from two penetration-depths of each spatial component of alternating magnetic field we have to choose the highest.

One can easily shows that at all places where the alternating field \( h(t) \) penetrates, the magnetic induction can be presented as a sum of two terms. One of these terms is a constant homogeneous quantity coinciding with vector \( \mathcal{H} \). The second term, which corresponds to the nonhomogeneous magnetic induction distribution of ac-magnetic field \( b_{y,z} \), may be described by the following equations (in analogy with Eqs. (4))

\[
\frac{\partial b_z}{\partial \xi} = \sin \vartheta, \quad \frac{\partial b_y}{\partial \xi} = \delta \cos \vartheta. \tag{6}
\]

These equations hold in the superconductor region where both induction components \( b_z \) and \( b_y \) are present. In the region there component \( b_z \) vanishes, which correspond to \( \varphi(x) = \pi/2 \) or equivalently \( \vartheta = 0 \), and there is only induction component \( b_y \), the distribution of this component is described by the equation

\[
\frac{\partial b_y}{\partial \xi} = \delta. \tag{7}
\]

Solution of Eqs. (6) and Eq. (4) is easy to find

\[
b_z(\xi) = h \sin \vartheta + \sin(\vartheta - 1), \quad \xi_z \leq \xi \leq 1, \tag{8a}
\]

\[
b_y(\xi) = \begin{cases} h \cos \vartheta + \delta \cos(\vartheta - 1) & \xi_z \leq \xi \leq 1, \\ b_y(\xi_z) + \delta(\xi - \xi_z) & \xi_y \leq \xi \leq \xi_z, \end{cases} \tag{8b}
\]

here \( \xi_z \) and \( \xi_y \) are penetration depths of each component of ac-magnetic field. Cusp in the distribution of \( b_y \) component is the result of the fact that coordinate dependence of angle \( \varphi(x) \) was neglected. From the condition \( b_z(\xi_z) = 0 \) one finds the penetration depth for \( z \) component of ac-magnetic field \( \xi_z = 1 - h \) and value of \( b_y \) at that point \( b_y(\xi_z) = (1 - \delta)h \cos \vartheta \). Similarly one can find penetration depth for \( y \)-component \( \xi_y = 1 - h - (1 - \delta)h \cos \vartheta \) and finally from the condition \( b_y(\xi = 0) = 0 \) critical amplitude \( h_c(\vartheta) \) of the alternating magnetic field at which total suppression of magnetization occurs

\[
h_c(\vartheta) = \frac{\delta}{\delta + (1 - \delta) \cos \vartheta}. \tag{9}
\]

Function \( h_c(\vartheta) \) has universal character; it is defined via the anisotropy parameter \( \delta \) of the theory which makes this formula to be interesting from the point of view of experiment. Say by measuring the \( h_c(\vartheta) \) at \( \vartheta = 0 \) one can directly find anisotropy parameter because \( \delta = h_c \) or \( \delta = \chi_r/(2\pi d)(\xi_z) \) in dimension variables.

Let now consider the dynamics of suppression of the static magnetic moment \( \mathcal{M} \) of the sample by an orthogonal alternating magnetic field. As has been noted above, the suppression effect essentially depends on the magnetic prehistory of the sample, i.e. on the position of the starting point on the magnetization loop. Below will be considered the most simple but nevertheless practically interesting case of the magnetic prehistory, when the external magnetic field monotonically increased up to certain maximal value \( \mathcal{H}_m \), such that \( 1 \ll \mathcal{H}_m \ll \mathcal{H}_z \), and then decreased back to zero. We will distinguish between two cases namely direct magnetization, when \( \mathcal{H} \) monotonically increased \( \mathcal{H} \in [0, \mathcal{H}_m] \) and reverse magnetization when magnetic field \( \mathcal{H} \) gradually decreased from its maximum value to zero \( \mathcal{H} \in [\mathcal{H}_m, 0] \). It turns out that there are three distinct regions for the direct magnetization and six for the reverse where suppression occurs qualitatively and quantitatively different all these regions will be discussed. For each of these regions the dependence of the magnetization on the amplitude of alternating magnetic field \( \mathcal{M}(h) \) will be found.

### A. Direct Magnetization

For the direct magnetization there are three distinct regions \( \mathcal{H} \leq h_c(\vartheta) \), \( h_c(\vartheta) \leq \mathcal{H} \leq 1 \) and \( 1 \leq \mathcal{H} \leq \mathcal{H}_m \). The calculations of \( \mathcal{M}(h) \) are straightforward but cumbersome so that the only final result will presented and discussed.

**A.1 Range:** \( 0 \leq \mathcal{H} \leq h_c(\vartheta) \)

\[
\mathcal{M}(h) = \begin{cases} \frac{2\mathcal{H}^2}{8\pi} & \mathcal{H} < h_c(\vartheta), \\ \frac{\mathcal{H}^2(1-\delta)\sin^2\vartheta}{8\pi h_c(\vartheta)} & \mathcal{H} \geq h_c(\vartheta), \end{cases}
\]

**A.2 Range:** \( h_c(\vartheta) \leq \mathcal{H} \leq 1 \)

\[
\mathcal{M}(h) = \begin{cases} \left(1 - \frac{\mathcal{H}^2}{8\pi h_c(\vartheta)} + \frac{\mathcal{H} \sin \vartheta}{8\pi h_c(\vartheta)} + \frac{\mathcal{H} \sin \vartheta}{8\pi h_c(\vartheta)} - \frac{\mathcal{H}}{4\pi}ight) & \mathcal{H} < h_c(\vartheta), \\ \left(1 - \frac{h}{h_c(\vartheta)}\right) + \frac{\mathcal{H}^2}{8\pi} \frac{\delta \sin \vartheta}{8\pi h_c(\vartheta)} & \mathcal{H} \geq h_c(\vartheta). \end{cases}
\]

**A.3 Range:** \( 1 \leq \mathcal{H} \leq \mathcal{H}_m \)

\[
\mathcal{M}(h) = \begin{cases} \frac{1}{8\pi} \left(\cos^2 \vartheta + \frac{\delta \sin^2 \vartheta}{8\pi h_c(\vartheta)}\right)(1 - \frac{h}{h_c(\vartheta)})^2 & \mathcal{H} < h_c(\vartheta), \\ \frac{1}{8\pi} \left(\cos^2 \vartheta + \frac{\delta \sin^2 \vartheta}{8\pi h_c(\vartheta)}\right)(1 - \frac{h}{h_c(\vartheta)})^2 & \mathcal{H} \geq h_c(\vartheta). \end{cases}
\]
\( H \in [0, h_c(\vartheta)] \) we can distinguish three steps in the evolution of the magnetic induction distribution, and subsequently magnetization \( \mathcal{M} \), in the sample. Each of these steps occurs at some specific range of amplitudes of alternating magnetic field which is described by inequalities in the formulas Eqs. (11a) and (11b). Static magnetization is negative which means that sample is in the diamagnetic state. With the gradual increase of \( h \) the static magnetization is smoothly vanishes. This happens because ac-field penetrates dipper inside the sample, magnetic induction in that region becomes homogeneous and doesn’t contribute to the magnetization. Full suppression occurs at the critical field Eq. (9).

B. Reverse Magnetization

Suppression of \( \mathcal{M} \) for the reverse magnetization is qualitatively different from that of direct magnetization. For all magnetic filed ranges in the magnetization loop (only with exception for the last one) there is characteristic cusp in the distribution of the magnetic induction. This cusp is the result of redistribution the magnetic induction after lowering amplitude of \( H \) below the \( H_m \) and there is peculiar feature of critical state model. Precisely this cusp is responsible for some new effects. This happens because now static magnetization has two terms one of which is negative but another is positive. Depending on the value of \( H \) and \( h \) one of this terms wins such that magnetization may be either positive (paramagnetic) or negative (diamagnetic) and paramagnetic-diamagnetic transition is possible.

B.1 Range: \( H_m - \delta < H < H_m \)

\[
\begin{align*}
\left\{ \begin{array}{l}
0 < h < h_c(\vartheta)(H_m - H), \\
\mathcal{M}(h) = -\frac{1}{8\pi}(\cos^2 \vartheta + \delta \sin^2 \vartheta)\left(1 - \frac{h^2}{2h_c^2(\vartheta)}\right) + \\
\quad + \frac{H}{4\pi} \left(1 - \frac{h}{2h_c(\vartheta)}\right) - \frac{(H_m - H)^2}{16\pi \delta} (\delta \cos^2 \vartheta + \sin^2 \vartheta),
\end{array} \right. \\
\text{B.2 Range: } H_m - 2\delta/(1 + \delta) < H \leq H_m - \delta. \quad \text{In the interval of the amplitudes } 0 < h < h_c(\vartheta)(H_m - H) \text{ magnetization is defined by the formula Eq. (11a) and in the interval } h_c(\vartheta)(H_m - H) < h < h_c(\vartheta)(2 - \delta^{-1}(H_m - H)) \text{ by the formula Eq. (11b). For the final interval we have}
\end{align*}
\]

\[
\begin{align}
\mathcal{M}(h) &= -\frac{1}{8\pi}(\cos^2 \vartheta - \delta \sin^2 \vartheta)\left(1 - \frac{h}{h_c(\vartheta)}\right)^2. \quad (12)
\end{align}
\]

Here dynamical picture of magnetic moment suppression is the same as in B.1.

B.3 Range: \( H_m - 2\delta < H \leq H_m - 2\delta/(1 + \delta) \). In the amplitude interval \( 0 < h < h_c(\vartheta)(2 - \delta^{-1}(H_m - H)) \) the \( \mathcal{M}(h) \) is described by the formula Eq. (11a) and for the next interval we have

\[
\begin{align}
\mathcal{M}(h) &= \frac{1}{4\pi}(H_m - H)\cos \vartheta\left(1 - \frac{h}{2h_c(\vartheta)}\right) - \frac{(H_m - H)^2 \cos^2 \vartheta}{16\pi \delta} \\
&\quad + \frac{\sin^2 \vartheta}{8\pi} \left(1 - \frac{h}{h_c(\vartheta)}\right)^2 \left[-\cos^2 \vartheta\left(1 - \frac{h}{2h_c(\vartheta)}\right)\right]. \quad (13)
\end{align}
\]

And finally for the interval \( h_c(\vartheta)(H_m - H) \leq h \leq h_c(\vartheta) \) is described by Eq. (12).

B.4 Range: \( H_m - 1 < H \leq H_m - 2\delta \). For the case \( 0 \leq h < h_c(\vartheta)(H_m - H) \) for \( \mathcal{M}(h) \) we have Eq. (13) and for the interval \( h_c(\vartheta)(H_m - H) \leq h \leq h_c(\vartheta) \) magnetization is described by the formula Eq. (12).

As soon as physical processes are similar for the ranges B.3 and B.4 we will discuss them together. Because of relatively big contribution from the positive component of the magnetic moment, sample at the beginning is paramagnetic. Increase in the amplitude of the alternating magnetic field causes decrease in the magnetic moment because at the beginning suppression affects only positive component of the magnetization. At some characteristic amplitude of ac-filed, which is smaller then \( h_c(\vartheta) \), magnetization becomes zero. This situation correspond to the case when contributions from positive and negative components to the total magnetization are equal. For further increase of the amplitude \( h \) sample becomes diamagnetic, magnetic moment reaches its minimal value then growth back and suppress completely at the field \( h_c(\vartheta) \).

B.5 Range: \( H_m - 2 \leq H \leq H_m - 1 \). For the amplitudes \( 0 \leq h < h_c(\vartheta)(2 - (H_m - H)) \) magnetization is described by Eq. (13) and for the final interval \( h_c(\vartheta)(2 - (H_m - H)) \leq h \leq h_c(\vartheta) \) we have new formula for the the magnetic moment \( \mathcal{M}(h) \)

\[
\begin{align}
\mathcal{M}(h) &= \frac{\cos^2 \vartheta + \delta \sin^2 \vartheta}{8\pi} \left(1 - \frac{h}{h_c(\vartheta)}\right)^2. \quad (14)
\end{align}
\]

B.6 Range: \( 0 < H \leq H_m - 2 \). For all possible amplitudes of alternating magnetic field \( 0 \leq h \leq h_c(\vartheta) \) magnetization \( \mathcal{M}(h) \) is described by the Eq. (14).

Behavior of \( \mathcal{M}(h) \) in the ranges B.5 and B.6 is completely different form that considered above. For these ranges there is no cusp in the distribution of magnetic
induction components in the sample and magnetization is always positive for any interval of $h$ so that sample is totally paramagnetic. Growth of the amplitude of alternating field smoothly suppress magnetic moment which nevertheless remains positive.

IV. CONCLUSIONS

Theoretical analysis and recent experiments\textsuperscript{8,9,14} shows that the alternating magnetic field exerts significant influence on the static magnetic properties of anisotropic disordered high-$T_c$ superconductor. Switching on a sufficiently strong field $h(t)$ orthogonal to the static magnetizing field results in the complete suppression of the magnetic moment of the sample. The reason for this suppression is that, at all places where the alternating field penetrates, levelling of the distribution profile of the static magnetic induction is observed. In the other words, in the same spatial region of the sample, the constant and alternating screening currents cannot coexist. In the conditions when the alternating field penetrates into the entire volume of the sample, complete suppression of the static magnetization takes place. The nature of the magnetization suppression, which consists in a local effect of the mutual influence of different components of the critical current density vector, is manifested in the anisotropic situation in a rather peculiar way. Different components of the magnetic field penetrate at different depths, since the are screened by critical current densities components of quite different magnitudes. This is the reason why the magnetization suppression is primarily caused by the alternating filed component deeply penetrating into the sample. As the result, the anisotropy induces a quite interesting effect: to suppress large magnetic moment a small amplitude of the alternating signal is sufficient. In the paper, the dynamics of the magnetization suppression with increase in the amplitude of the alternating filed $h$ is studied in details and results are in agreement with that obtained in the experiments Ref.\textsuperscript{8,9,14}. It was shown that in some cases the dependence of the moment on $h$ is nonmonotonic and, in addition, during the suppression transition of the sample from the paramagnetic state into diamagnetic state sometimes occurs. All results can be interpreted within the framework of a critical state model generalized to the anisotropic case.

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