CONSTRAINTS ON THE SIZE OF EXTRA DIMENSIONS FROM THE ORBITAL EVOLUTION OF BLACK-HOLE X-RAY BINARIES

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Abstract

One of the plausible unification schemes in physics considers the observable universe to be a four-dimensional surface (the “brane”) embedded in a higher dimensional curved spacetime (the “bulk”). In such braneworld gravity models with infinitely large extra dimensions, black holes evaporate fast through the emission of the additional gravitational degrees of freedom, resulting in lifetimes of stellar-mass black holes that are significantly smaller than the Hubble time. We show that the predicted evaporation rate leads to a change in the orbital period of X-ray binaries harboring black holes that is observable with current instruments. We obtain an upper limit on the rate of change of the orbital period of the binary A0620−00 and use it to constrain the asymptotic curvature radius of the extra dimension to a value comparable to the one obtained by table-top experiments. Furthermore, we argue that any measurement of a period increase for low-mass X-ray binaries with a high mass ratio is evidence for new physics beyond general relativity and the standard model.

Key words: black hole physics – gravitation – stars: individual (A0620-00) – X-rays: binaries – X-rays: stars

1. INTRODUCTION

In the search for the unified theory of all forces, an essential ingredient is the solution of the so-called hierarchy problem. The fundamental scale of gravity, the Planck mass, exceeds the electroweak scale by 16 orders of magnitude. In order to resolve this discrepancy, Arkani-Hamed et al. (1998) suggested that gravity is allowed to propagate in more than three spatial dimensions and is hence “diluted” in our universe. This leads to modifications of gravity at distances that are smaller than those probed by experiments. Indeed, Newton’s inverse square law has been tested down to the submillimeter range (Kapner et al. 2007; Geraci et al. 2008), hence verifying that our space is three dimensional at macroscopic scales. Any modification of gravity involving extra dimensions therefore has to ensure that additional space dimensions only effect our world at distances that are smaller than those experimental limits.

Braneworld gravity offers a solution to this problem in the form of two different scenarios. One approach (Arkani-Hamed et al. 1998) is to compactify $n$ extra dimensions at scales smaller than those set by experiment. The fundamental Planck mass can be pushed down to the electroweak scale of about 1 TeV, provided the extra dimensions are large enough. For $n$ extra dimensions, the limit is $R < 10^{20} n^{-1/7}$ cm. For $n \geq 2$, extra dimensions would have a submillimeter size, which is just at the limit up to which the inverse square law has been verified. This model can also be embedded in String Theory (Antoniadis et al. 1998). However, it cannot be tested in astrophysics, because those length scales are well below astronomical distances.

A second scenario (Randall & Sundrum 1999) is based on a different idea. The four-dimensional brane with all standard model particles is embedded in an infinite five-dimensional anti-de Sitter space. Deviations from the inverse square law, however, only manifest at distances smaller than the asymptotic curvature radius $L$ of the bulk because the latter is filled with a negative cosmological constant. This setup has dramatic implications for astrophysical black holes.

No stable solutions for black holes on the brane have been found to date. Numerical integration of the classical bulk equations governing the evolution of black holes in the RS2 scenario that are localized on the brane indicated that black holes are unstable and hence lose energy in the extra dimension (Tanaka 2003). Based on the AdS/CFT correspondence, Tanaka (2003) suggested that stable black holes may not exist on the brane at all. Applying the AdS/CFT correspondence to AdS braneworld models, Emparan et al. (2002) conjectured that black holes localized on the brane that are solutions of the classical bulk equations in $AdS_{D+4}$ with the brane boundary conditions correspond to quantum-corrected black holes in $D$ dimensions. Black holes can then evaporate through the emission of a large number of conformal field theory (CFT) modes with a lifetime given by (Emparan et al. 2003; see, however, Fitzpatrick et al. 2006)

$$\tau \sim 1.2 \times 10^2 \left(\frac{M}{M_\odot}\right)^3 \left(\frac{1 \text{ mm}}{L}\right)^2 \text{yr},$$

which is only of the order of a hundred thousand years for black holes with a mass $M$ of a few solar masses and an asymptotic curvature $L$ in the submillimeter range. Therefore, astrophysical black holes can radiate away most of their mass at cosmologically relevant timescales. This property has been used to constrain $L$ from a kinematic limit on the age of the black hole XTE J1118+480 (Psaltis 2007), yielding $L < 80 \mu m$, as well as to give a possible explanation of the unusual observed black hole mass function (Postnov & Cherepashchuk 2003).

In the Randall–Sundrum model, RS2, the gravitational potential at distances close to $L$ takes the form (Randall & Sundrum 1999)

$$V(r) \approx -G \frac{m_1 M_2}{r} \left(1 + \frac{L^2}{r^2}\right).$$

Adelberger et al. (2007) report a $1 \sigma$ upper limit on $L$ of $11 \mu m$. A $3 \sigma$ constraint has not been computed yet, but it should be significantly larger and comparable to the 95% confidence upper bound of $44 \mu m$ for the size of one compact extra dimension.
We then obtain the rate of black hole evaporation into the higher dimensional bulk. In Section 2, we systematically caused by magnetic braking and the evolution of the companion star (see e.g., Verbunt 1993). In Section 2, we systematically derive the rate of change of the orbital period involving all three effects as well as nonconservative mass transfer. We present the results in Section 3, where we identify systems with predominant black hole evaporation. In Section 4, we focus on the black hole binary A0620−90 in particular and obtain an upper limit of $L < 161 \mu m$, which is already comparable to the limit from table-top experiments. In the final section (Section 5), we discuss the potential of this binary as well as of other sources to constrain $L$ down to a few microns.

2. ORBITAL EVOLUTION OF A BLACK HOLE BINARY IN BRANEWORLD GRAVITY

In this section, we derive the rate of change of the orbital period of a binary system that harbors a black hole following closely the works of Will & Zaglauer (1989) and Psaltis (2008). In our treatment, we also include the effect of CFT emission of the black hole in the extra dimension.

For a black hole of mass $m_1$ with a companion star of mass $m_2$ on a circular orbit, the rate of change of the orbital angular momentum, $J \equiv \mu \sqrt{Gm_1}$, is

$$\dot{J} = \frac{1}{J} \frac{\partial J}{\partial m_1} \dot{m}_1 + \frac{1}{J} \frac{\partial J}{\partial m_2} \dot{m}_2 + \frac{1}{\dot{J}} \frac{\partial J}{\partial \dot{a}} \dot{a} = \left(1 - \frac{1}{2} \frac{m_1}{m_1 + m_2}\right) \frac{\dot{m}_1}{m_1} + \left(1 - \frac{1}{2} \frac{m_2}{m_1 + m_2}\right) \frac{\dot{m}_2}{m_2} + \frac{1}{\dot{J}} \frac{\dot{a}}{2 \dot{a}},$$

where $m \equiv m_1 + m_2$, $\mu \equiv m_1 m_2 / m$, and $a$ is the semimajor axis. We set $m_1 = q m_2$ and $m_1 = - \beta m_2 - M$, where $M$ is the rate of black hole evaporation into the higher dimensional bulk. We then obtain

$$\dot{J} = \left(1 - \frac{\beta}{q} - \frac{1}{2} \frac{1}{1 + q}\right) \frac{\dot{m}_2}{m_2} - \left(1 + \frac{q}{2} \frac{1}{1 + q}\right) \frac{\dot{M}}{m_1} + \frac{1}{2 \dot{a}}.$$  

Angular momentum may be lost because of mass loss from the system or because of the effect of magnetic braking. This leads to

$$\dot{J} = j_\omega (1 - \beta) \frac{1}{q} \frac{\dot{m}_2}{m_2} + \frac{\dot{J}_{mb}}{J},$$

where $j_\omega$ is the specific angular momentum carried away by the stellar wind in units of $2 \pi a^2 / P$, $P$ is the orbital period, and $\dot{J}_{mb}/J$ is the rate of angular momentum loss due to magnetic braking. Following Rappaport et al. (1983) we estimate the corresponding torque by the empirical expression

$$\tau_{mb} \equiv \dot{J}_{mb} \simeq -3.8 \times 10^{-30} m_2 R_0^3 \left(\frac{R_2}{R_0}\right)^{\gamma} \alpha^3 \text{ dyn cm.}$$

Here, $\alpha$ is the angular frequency of the secondary, $\gamma$ is a parameter that characterizes the strength of the magnetic braking, and $R_2$ is the radius of the stellar Roche lobe which is assumed to be filled at all times (Eggleton 1983),

$$R_2 = \frac{0.49 q^{2/3}}{0.6 q^{2/3} + \ln(1 + q^{1/3})} a.$$  

Using the expression for the orbital period

$$\frac{P}{2 \pi} = \frac{m}{m_1 m_2} J^3 G^{-2},$$

we can evaluate $\dot{J}_{mb}/J$ as

$$\frac{\dot{J}_{mb}}{J} = C \left(\frac{m_1 + m_2}{m_1}\right)^2 \left[\frac{0.49 q^{2/3}}{0.6 q^{2/3} + \ln(1 + q^{1/3})}\right]^{\gamma} \times \left[\sqrt{G(m_1 + m_2)} / \beta \right]^{2(\gamma - 5)},$$

where

$$C \equiv -3.8 \times 10^{-30} R_0^{4 - \gamma}.$$  

Additionally, from the period in Equation (8) together with our expressions for $m_1$ and $m_2$ we find

$$\frac{P}{\dot{P}} = \frac{3 \dot{a}}{2 \dot{a}} - \frac{1 - \beta m_2 - M + \dot{m}_2}{m_1 + m_2} = \frac{1}{2} \frac{1 - \beta m_2 - M + \dot{m}_2}{m_1 + m_2} + \frac{1}{2} \frac{M}{m_1} + \frac{3 \dot{a}}{2 \dot{a}}.$$  

Using

$$\frac{\dot{q}}{\dot{a}} = \frac{\beta + q \dot{m}_2}{q m_2} - \frac{M}{m_1},$$

we obtain the rate of change of the radius of the companion as

$$\frac{R_2}{R_2} = \frac{\dot{a}}{a} + \frac{2 \beta + q}{3} \frac{1 - \frac{0.6 + 0.5 q^{1/3} (1 + q^{-1/3})^{-1}}{0.6 + q^{2/3} \ln(1 + q^{-1/3})} \dot{m}_2}{m_2} + \frac{2}{3} \frac{1 - \frac{0.6 + 0.5 q^{1/3} (1 + q^{-1/3})^{-1}}{0.6 + q^{2/3} \ln(1 + q^{-1/3})} \dot{M}}{m_1}.$$  

The third effect that dictates the change of the orbital period in a binary system is the evolution of the companion star. As the secondary leaves the main sequence and starts to burn helium, it expands rapidly. Following Webbink et al. (1983) and Verbunt (1993) we estimate the rate of change of the radius of a star leaving the main sequence as

$$\frac{\dot{R}_2}{R_2}_{ev} = (c_1 + 2 c_2 y + 3 c_3 y^3) \frac{M_c}{M_\odot}.$$  

Here, $M_c$ is the core mass of the companion, $y \equiv \ln(M_c/0.25 M_\odot)$, and $c_1$, $c_2$, and $c_3$ are constants that depend on the composition of the core. The core mass changes in time according to (Verbunt 1993)

$$\dot{M}_c \simeq 1.37 \times 10^{-11} \left(\frac{L_2}{L_\odot}\right) M_\odot \text{ yr}^{-1}.$$  

In this expression, $L_2$ is the luminosity of the companion, which is determined by the core mass (Webbink et al. 1983) according to the empirical relation

$$\ln \left(\frac{L_2}{L_\odot}\right) = a_0 + a_1 y + a_2 y^2 + a_3 y^3,$$  

where $a_0 = 6.05$, $a_1 = 3.97$, $a_2 = -6.10$, and $a_3 = 2.87$.
with $a_0$, $a_1$, $a_2$, and $a_3$ constants depending on the core composition. Combining Equations (14)–(16) leads to

$$
\frac{R_e}{R_2} \simeq 1.37 \times 10^{-11} \times 4^{a_1} (c_1 + 2c_2 y + 3c_3 y^2) \\
\times e^{a_0 + a_2 y^2 + a_3 y^3} \left( \frac{M_\ast}{M_\odot} \right)^{a_1 - 1} \text{yr}^{-1}.
$$

(17)

We now define the adiabatic index for the companion star as

$$
\xi_{\text{ad}} = \frac{d \ln R_2}{d \ln m_2}
$$

(18)

and obtain

$$
\frac{\dot{a}}{a} = \left[ \xi_{\text{ad}} - \frac{2}{3} \frac{\beta + q}{q} \left( 1 - \frac{0.6 + 0.5q^{1/3}(1 + q^{-1/3})^{-1}}{0.6 + q^{2/3} \ln(1 + q^{-1/3})} \right) \right] \frac{m_2}{m_1} - \left( \frac{R_e}{R_2} \right)_{ev}.
$$

(19)

In the following, we will estimate the value of $\xi_{\text{ad}}$ from the expressions for the stellar radius $R$ and mass $m_2$ given by Kalogera & Webbink (1996).

Combining the equations we derived above, we obtain the rate of change of the orbital period of the binary as

$$
\frac{\dot{P}}{P} = \frac{Q_0}{m_1} + Q_2 \left( \frac{m_1 + m_2}{m_1} \right)^2 \left[ \frac{0.49q^{-2/3}}{0.6q^{-2/3} + \ln(1 + q^{-1/3})} \right]^{1/2} \left[ \frac{\sqrt{G(m_1 + m_2)} P}{2\pi} \right]^{(y-5)/2} + Q_3 (c_1 + 2c_2 y + 3c_3 y^2) \times e^{a_0 + a_2 y^2 + a_3 y^3} \left( \frac{M_\ast}{M_\odot} \right)^{a_1 - 1}.
$$

(20)

In this equation, we have introduced the quantities:

$$
Q_0 \equiv \frac{11 - \beta}{2 + q} + \frac{1 + \frac{1}{2} \frac{q}{1 + q}}{D} + \frac{1}{2} \frac{q}{1 + q} + \frac{3}{2},
$$

$$
Q_2 \equiv \frac{C}{D} \left( \frac{11 - \beta}{2 + q} + \frac{\beta + q}{q} A - \frac{3}{2} \xi_{\text{ad}} \right),
$$

$$
Q_3 \equiv 1.37 \times 10^{-11} \times 4^{a_1} \left[ \frac{11 - \beta}{4D} + \frac{1}{2D} \left( \frac{\beta + q}{q} A - \frac{3}{2} \xi_{\text{ad}} \right) - \frac{3}{2} \right] D, \quad (21)
$$

(22)

$$
A \equiv 1 - \frac{0.6 + 0.5q^{1/3}(1 + q^{-1/3})^{-1}}{0.6 + q^{2/3} \ln(1 + q^{-1/3})},
$$

(23)

and

$$
D \equiv j_w (1 - \beta) \left( \frac{1}{q} + \frac{q}{2} + \frac{11 - \beta - 1}{2} \xi_{\text{ad}} - \frac{2}{3} \frac{\beta + q}{q} A \right).
$$

(24)

For the “evaporation” of the black hole due to the emission of CFT modes, which are the dual description of the infinite dimension, we use (Emparan et al. 2003)

$$
M = 2.8 \times 10^{-3} \left( \frac{M_\odot}{m_1} \right)^2 \left( \frac{L}{1 \text{ mm}} \right)^2 M_\odot \text{yr}^{-1},
$$

(26)

where $L$ is the asymptotic AdS radius of curvature. Defining

$$
Q_1 \equiv 2.8 \times 10^{-3} Q_0 \text{yr}^{-1},
$$

(27)

we arrive at our final equation for the orbital period evolution,

$$
\frac{\dot{P}}{P} = Q_1 \left( \frac{M_\odot}{m_1} \right)^3 \left( \frac{L}{1 \text{ mm}} \right)^2 + Q_2 \left( m_1 + m_2 \right)^2 \frac{0.49q^{-2/3}}{0.6q^{-2/3} + \ln(1 + q^{-1/3})} \frac{\sqrt{G(m_1 + m_2)} P}{2\pi} \right]^{1/2} \left[ \frac{\sqrt{G(m_1 + m_2)} P}{2\pi} \right]^{(y-5)/2} + Q_3 (c_1 + 2c_2 y + 3c_3 y^2) \times e^{a_0 + a_2 y^2 + a_3 y^3} \left( \frac{M_\ast}{M_\odot} \right)^{a_1 - 1}.
$$

(28)

For our analysis it is important that the companion star remains in contact with its Roche lobe. Should the black hole evaporation be so strong that magnetic braking is negligible (as well as stellar evolution), then the Roche lobe of the secondary will grow and the system will eventually get out of contact. This, however, can only occur on timescales of at least $10^8 \text{yr}$, which is much larger than the observational timescales of interest. Studying other implications of the loss of contact for the evolution of the black hole binary is beyond the scope of this paper.

3. RESULTS

In this section, we investigate the potential of the currently known black hole binary systems to constrain the rate of black hole evaporation into higher dimensions. We will start with a general discussion of the various systems and then focus on the system A0620−00 in particular.

First, we investigate which term in Equation (28) dominates the period evolution for a given period $P$, black hole and companion masses $m_1$ and $m_2$, respectively, and curvature radius $L$. Figure 1 shows the orbital periods versus the companion masses of the observed systems (compare Table 1). On the same graph, we plot the curves along which the evaporation term equals the magnetic braking term for $L = 0.1 \mu m$, $L = 1 \mu m$, $L = 10 \mu m$, and $L = 100 \mu m$. The evaporation dominates above the lines, whereas below the lines the magnetic braking dominates. Recent numerical simulations (Yungelson & Lasota 2008) showed that the expression for magnetic braking is actually overestimated for low-mass black hole binaries, which further increases the predominance of the evaporation term. The parameters we used in this figure are $\xi_{\text{ad}} = 0.8$, $\beta = 0$, $j_w = 0$, and $\gamma = 0$, and we set the black hole mass to a nominal value of $10 M_\odot$.

For companion masses $\gtrsim 1 M_\odot$, there exists a maximum period beyond which systems contain companion stars that have evolved past the base of the giant branch (see the curve marked BGB in Figure 1). For these systems, the evolution of the companion star completely dominates the rate of change of the orbital period for any plausible value of the asymptotic curvature $L$.

We find that two sources have relatively large orbital periods and are probably evolved, while the other systems group around
the separatrices only depend weakly on $x_{ad} = 0.8$, $\beta = 0$, $j_w = 0$, and $y = 0$. Note that the separatrices only depend weakly on $m_2$ as long as $y = 0$.

Table 1

| X-Ray Binary | $P$ (d) | $q$ | $m_1(M_\odot)$ |
|--------------|--------|----|----------------|
| GRS1915+105  | 816    | 12 | 14 ± 4         |
| J1118+480    | 4.1    | ~20 | 6.8 ± 0.4     |
| GS2023+338   | 155.3  | 17 ± 1 | 12 ± 2 |
| GS2000+25    | 8.3    | 24 ± 10 | 10 ± 4 |
| H1750-25     | 12.5   | >19 | 6 ± 2          |
| GRS1009-45   | 6.8    | 7 ± 1  | 5.2 ± 0.6     |
| N Mus 91     | 10.4   | 6.8 ± 2 | $6^{+3}_{-2}$ |
| A0620-00     | 7.8    | 17 ± 1b | 10 ± 5 |
| J0422+32     | 5.1    | 9.03±0.7 | 4 ± 1 |
| J1819.3-2525 | 67.6   | 2.31 ± 0.08 | 7.1 ± 0.3 |
| J1655-40     | 62.9   | 2.39 ± 0.15 | 6.6 ± 0.5 |
| 4U1543-47    | 27.0   | 3.6 ± 0.4 | 9.4 ± 1 |

Notes.

a Most data compiled by Charles & Coe (2006).

b Neilsen et al. (2008).

is only ~ 25% below that of a normal K4 dwarf. However, the secondary of A0620−00 is not a normal star, given the extraordinary evolutionary history of this black hole binary system (e.g., de Kool et al. 1987). Nevertheless, for our purposes the secondary functions like a main-sequence star: it is not evolving on a nuclear time scale and the system is kept in contact by magnetic braking (Justham et al. 2006).

Thus, we can neglect the evolution term in Equation (28) and plot the expected rate of change of its orbital period $P$ as a function of the asymptotic AdS curvature radius $L$(see Figure 2).

The parameters for this plot are $\beta = 0$, $j_w = 0$, $y = 0$, and $x_{ad} = 0.8$. We see that for $L \lesssim 20 \mu m$ the magnetic braking dominates and the orbital period derivative is constant because it is independent of $L$. For $L \gtrsim 20 \mu m$ the black hole evaporation dominates and the orbital period derivative increases with increasing AdS curvature as expected. This shows that A0620−00 theoretically allows for a constraint on $L$ as low as $20 \mu m$, assuming that $m_1 = 10 M_\odot$. Since $m_1$ has only been measured to an accuracy of ±50%, the constraint can even be reduced to a few microns. We will return to the question of the black hole mass in the following section.

In order to determine the dependence of the orbital period evolution on the parameters $j_w$, $\beta$, and $y$, we plot the rate of change of the orbital period as a function of one parameter while holding the others constant. In all plots, we set $x_{ad} = 0.8$ (estimated from Kalogera & Webbink 1996) for the companion mass in this system and evaluate the period evolution rate at the current experimental upper limit of the AdS curvature of $L = 44 \mu m$ (Kapner et al. 2007). Figure 3 shows the dependence of the rate of change of the orbital period on the parameters $j_w$, $\beta$, and $y$, respectively. First, we note that for large values of the parameters, the period increases. This behavior is entirely due to the high mass ratio $q = m_1/m_2$ measured for this source; for other sources with substantially smaller mass ratios, the behavior is not monotonic. Furthermore, we find that the rate parameter for this graph are $\mu = 8.0, \beta = 0, j_w = 0$, and $y = 0$. The transition from predominant magnetic braking (constant negative rate) to predominant black hole evaporation (positive and rapidly increasing rate) occurs at $L \approx 20 \mu m$.

Figure 1. Orbital period $P$ of observed black hole binary systems vs. the mass $m_2$ of the companion star. Four separatrices are shown for different values of the asymptotic AdS curvature radius $L$ for a nominal black hole mass of $10 M_\odot$. Below the lines, magnetic braking dominates. Above the lines, black hole evaporation dominates. Binaries above the curve marked BGB contain companions beyond the base of the giant branch; for these systems, the evolution of the companion completely dominates the orbital evolution of the binary. The parameters for this graph are $x_{ad} = 0.8$, $\beta = 0$, $j_w = 0$, and $y = 0$. Note that the separatrices only depend weakly on $m_2$ as long as $y = 0$.

Figure 2. Rate of change of the orbital period $P$ of the binary system A0620−00 vs. the asymptotic curvature radius $L$ in the extra dimension. The parameters are $\xi_{ad} = 0.8$, $\beta = 0$, $j_w = 0$, and $y = 0$. The transition from predominant magnetic braking (constant negative rate) to predominant black hole evaporation (positive and rapidly increasing rate) occurs at $L \approx 20 \mu m$.

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we fitted for the three parameters $t$, $\beta$, and $\gamma$, for $L = 44 \mu m$ and $\xi_{ad} = 0.8$. On varying one parameter, the others are held constant at the respective values $j_w = 0$, $\beta = 0$, and $\gamma = 0$.

of change of the orbital period is the smallest when $j_w = 0$ (no angular momentum loss due to stellar wind), $\beta = 0$ (no accretion), and $\gamma = 0$. We choose these values for the respective parameters in the following discussion, where we are aiming to calculate a lower limit on the expected rate of change of the orbital period.

4. DATA

In this section, we use previously published measurements of the orbital period of A0620−00 to set an upper limit on the size $L$ of the asymptotic curvature in the extra dimension. First, we determine that limit using the best-fit black hole mass of $m_1 = 10 M_\odot$, and then we proceed with an analysis of $L$ for different black hole masses.

The orbital period of A0620−00 ($P = 0.32$ days) has been measured several times during the past two decades. A convenient orbital phase reference is the time of maximum radial velocity $T_0$. Four measured values of $T_0$, which span 22 years, are given in Table 2. These times and the individual determinations of the orbital period $P$ uniquely determine the cycle number $n$. The table also gives the calculated times of maximum velocity based on a simple ephemeris with a constant orbital period and referenced to the most precise and recent determination of $T_0$ (see footnote “a” of Table 2). The differences between the observed and calculated times are smaller than their corresponding uncertainties, and thus there is no evidence for any change in the orbital period during the past 22 years. Note that with modern telescopes and instrumentation one can routinely achieve a precision of several seconds in $T_0$ (see the last entry in the table) in $\sim 10$ hr of radial velocity observations of a source like A0620−00 (Neilsen et al. 2008).

Of interest to us is a secure limit on the rate of change of the orbital period. A constant rate of change of the period will result in a quadratic variation in $T_0$ (e.g., Kelley et al. 1983). The time of the $n$th value of $T_0$ is then given by

$$t_n = t_0 + Pn + \frac{1}{2} P \dot{P} n^2,$$

where $P$ and $\dot{P}$ are the orbital period and its derivative, respectively, at time $t_0$; $n$ is the orbital cycle number. Following the standard procedure and using the IDL routine curvefit, we fitted for the three parameters $t_0$, $P$ and $\dot{P}$ using the four observed values of $T_0$ given in Table 2 (Figure 4). The fit yields $P = (-1.66 \pm 2.64) \times 10^{-11}$ s/s. Thus, using a $3\sigma$ upper limit, the period derivative is constrained within the interval $-9.58 \times 10^{-11}$ s/s $< \dot{P} < 6.26 \times 10^{-11}$ s/s. Next, we plot the rate of change of the orbital period versus the AdS curvature for the values of the parameters $\beta$, $j_w$, and $\gamma$ that lead to the lowest limit of the rate of change of the orbital period for a given value of the asymptotic curvature radius $L$ (Figure 5). For $L \gtrsim 20 \mu m$, the rate of change of the orbital period is smallest for the set of parameters $j_w = 0$, $\beta = 0$, and $\gamma = 0$, while for $L \lesssim 20 \mu m$, the parameters $j_w = 1$, $\beta = 0$, and $\gamma = 0$ minimize the orbital period evolution. In a narrow intermediate region, the corresponding set of parameters is $\beta = 1$, and $\gamma = 0$, while $j_w$ is arbitrary. Since the measured upper limit on the period derivative in Equation (29) marks the largest time change of the orbital period, the intersection point of this line with the graph of the smallest rate of change of the orbital period places an upper limit of $L \lesssim 161 \mu m$ (Figure 5).

Since the upper limit on the asymptotic curvature radius depends strongly on the black hole mass, which has only been measured to an accuracy of $\pm 50\%$, we plot in Figure 6 the upper limit $L_{\text{max}}$ versus the black hole mass for different limits on the period change. For the upper curve, we used the current limit
Table 2

| Orbital Cycle | \( T_0 \) in JD | \( T_0(n) \) in JD | \( T_0 - T_0(n) \) | Reference |
|---------------|------------------|-------------------|------------------|-----------|
| n             | Observed         | Computed          | (s)              |           |
| 0             | 2, 446, 082.7481 ± 0.0008 | 2, 446, 082.7483 | 17 ± 69         | 1         |
| 6764          | 2, 448, 267.6155 ± 0.0002 | 2, 448, 267.6154 | 8 ± 17          | 2         |
| 20,321        | 2, 452, 646.7173 ± 0.0005 | 2, 452, 646.7170 | 26 ± 43         | 3         |
| 24,773        | 2, 454, 084.77560 ± 0.00005 | 2, 454, 084.77560 | 0 ± 4           | 4         |

Notes.

a \( T_0(n) = JD 2,454,084.77560 - (24773 - n) \times 0.32301406. 

b References: (1) McClintock & Remillard (1986); (2) Orosz et al. (1994); (3) Shahbaz et al. (2004); (4) Neilsen et al. (2008).

c \( T_0 \) corrected to date of observation on 1991 January 11 using ephemeris in reference (1).

Figure 5. Rate of change of the orbital period \( P \) of the binary system A0620−00 as a function of the asymptotic AdS curvature radius \( L \) for a black hole mass of 10 \( M_\odot \). For positive values of the period derivative, this line represents the lower limit among all possible values of the period evolution. The 3σ error bars of the observed orbital period derivative are shown as horizontal lines. The intersection point of the upper limit on \( \dot{P}/P \) with the lower limit curve marks our constraint on the asymptotic curvature in the bulk of \( L = 161 \mu m \) (vertical line).

Figure 6. Upper limit on the asymptotic AdS curvature radius \( L \) as a function of the black hole mass \( m_1 \) of A0620−00 using the current lower limit on the orbital period evolution (upper curve) and on 10% of that limit (lower curve). The dashed line shows the current upper limit on \( L \) from table-top experiments.

There are ample opportunities to substantially improve the above limit on \( L \) using our method. For example, because the uncertainty in \( \dot{P} \) depends quadratically on \( n \), even a single future observation of A0620−00 that extends the 22 year baseline by just five years (i.e., \( n = 30423 \)) with a precision of 4 s would reduce the error in \( \dot{P} \) by a factor of six. Furthermore, independent limits of comparable quality on \( P \) and \( L \) could be obtained by monitoring the ephemerides of several other black hole binary systems (e.g., GRS 1124−683, XTE J1118+480, and 4U 1543−47; Remillard & McClintock 2006).

5. DISCUSSION

For a binary system consisting of a black hole and a companion star we derived the rate of change of the orbital period in the RS2 braneworld gravity model incorporating the emission of CFT modes in the extra dimension. Magnetic braking and the evolution of the companion star can also change the orbital period, but they are negligible if the secondary is a main-sequence star and if the asymptotic AdS curvature radius \( L \) is large enough so that the evaporation dominates. Measuring the rate of change of the orbital period then allows us to constrain the asymptotic AdS curvature radius.

We analyzed in detail the binary system A0620−00, which is a good candidate for such a constraint for both theoretical and observational reasons. The evaporation term dominates the change of the orbital period as long as the asymptotic curvature radius \( L \) is at least \( \sim 20 \mu m \) large. Measurements of the orbital period over the last 20 years allow for a constraint of
$L < 161 \mu$m assuming a black hole mass of 10 $M_\odot$. Refining the measurement of the mass of the black hole and of the rate of change of the orbital period can further improve the constraint on the asymptotic curvature radius. As an example, we showed that improving the measurement of the rate of change of the orbital period by one order of magnitude will constrain the AdS curvature radius to a value smaller than the current experimental limit $L = 44 \mu$m (Kapner et al. 2007), provided the black hole mass is measured not to exceed 9 $M_\odot$.

Considering the other known black hole X-ray binaries, we see from Figure 1 that there are more systems that we can use in constraining the asymptotic curvature radius in the extra dimension. The requirement of unevolved secondaries rules out some of them, but several sources have the potential of a constraint on the curvature radius down to a few microns.

A very exciting aspect of our method is that it not only allows us to constrain the AdS radius $L$, but also to potentially measure it—thereby giving evidence for new physics beyond general relativity and the standard model—provided that the rate of change of the orbital period is measured to be positive. This is due to the fact that, for a black hole binary with a sufficiently high mass ratio ($q = m_1/m_2$), magnetic braking can only shorten the orbital period. The sign of the magnetic braking term in Equation (28) and hence whether this effect leads to a positive and a negative change of the orbital period, is determined exclusively by the prefactor $Q_2$. This factor depends only on the mass ratio $q$ and on the parameters $\beta$ and $j_\omega$ assuming a fixed adiabatic index. In Figure 8, we plot the magnetic braking term for various combinations of the parameters $\beta$ and $j_\omega$, and we set the black hole mass and the orbital period to the nominal values $m_1 = 10$ $M_\odot$ and $P = 7.8$ hr, respectively, as well as $\gamma = 0$. The mass of the primary, the orbital period, and the parameter $\gamma$ only effect the magnitude of the magnetic braking term but not its sign. For all curves we set $\xi_{ad} = 0.8$. We see that the magnetic braking term is negative for $q > 5.5$ for any value of the parameters $\beta$, $j_\omega$, and $\gamma$.

Thus we conclude that any measurement of a positive orbital period derivative in a black hole X-ray binary with a mass ratio $q > 5.5$ (assuming an unevolved companion star) is strong evidence for new gravitational physics and directly measures the asymptotic curvature radius $L$. Since our method is model dependent (RS2), such a measurement, together with the top-down experiments, would even allow for a distinction between the ADD and the RS2 scenario. Current table-top experiments, that probe Newton’s inverse square law in the submillimeter range, are insensitive to the underlying model, so that their results together with a measurement as discussed above would indeed allow for a distinction between ADD and RS2.

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REFERENCES

Adelberger, E. G., Heckel, B. R., Hoedl, S., Hoyle, C. D., Kapner, D. J., & Upadhye, A. 2007, Phys. Rev. Lett., 98, 131104
Antoniadis, I., Arkani-Hamed, N., Dimopoulos, S., & Dvali, G. 1998, Phys. Lett. B, 436, 257
Arkani-Hamed, N., Dimopoulos, S., & Dvali, G. 1998, Phys. Lett. B, 429, 263
Charles, P. A., & Coe, M. J. 2006, in Compact Stellar X-ray Sources, ed. W. H. G. Lewin & M. van der Klis (Cambridge: Cambridge Univ. Press), arXiv:0308020
de Kool, M., van den Heuvel, E. P. J., & Pylyser, E. 1987, A&A, 183, 47
Eggleton, P. P. 1983, ApJ, 268, 368
Emparan, R., Fabbri, A., & Kaloper, N. 2002, JHEP, 0208, 043
Emparan, R., García-Bellido, J., & Kaloper, N. 2003, JHEP, 0301, 079
Fitzpatrick, A. L., Randall, L., & Wiseman, T. 2006, JHEP, 0611, 033
Frank, J., King, A. R., & Raine, D. 2002, Accretion Power in Astrophysics (Cambridge: Cambridge Univ. Press)
Genat, A. H., Smullin, S. J., Wels, D. M., Chiaverini, J., & Kapitulnik, A. 2008, Phys. Rev. D, 78, 022002
González Hernández, J. I., Rebolo, R., Israelian, G., & Casares, J. 2004, ApJ, 609, 988
Justham, S., Rappaport, S., & Podsiadlowski, P. 2006, MNRAS, 366, 1415
Kalogera, V., & Webbink, R. F. 1996, ApJ, 458, 301
Kaper, D. J., Cook, T. S., Adelberger, E. G., Gundlach, J. H., Heckel, B. R., Hoyle, C. D., & Swanson, H. E. 2007, Phys. Rev. Lett., 98, 021101
Kelley, R. L., Rappaport, S., Clark, G. W., & Petro, L. D. 1983, ApJ, 268, 790
Marsh, T. R., Robinson, E. L., & Wood, J. H. 1994, MNRAS, 266, 137
McClintock, J. E., & Remillard, R. A. 1986, ApJ, 308, 110
Neilsen, J., Steeghs, D., & Vrtilek, S. D. 2008, MNRAS, 384, 849
Orosz, J. A., Bailyn, C. D., Remillard, R. A., McClintock, J. E., & Foltz, C. B. 1994, ApJ, 436, 848
Postnov, K. A., & Cherepashchuk, A. M. 2003, Astron. Rep., 80, 1075
Psaltis, D. 2007, Phys. Rev. Lett., 98, 181101
Psaltis, D. 2008, ApJ, 688, 1282
Randall, L., & Sundrum, R. 1999, Phys. Rev. Lett., 83, 4690
Rappaport, S., Verbunt, F., & Joss, P. C. 1983, ApJ, 275, 713
Remillard, R. A., & McClintock, J. E. 2006, ARA&A, 44, 49
Shahbaz, T., Hynes, R. I., Charles, P. A., Zurita, C., Casares, J., Haswell, C. A., Araujo-Betancor, S., & Powell, C. 2004, MNRAS, 345, 31
Tanaka, T. 2003, Prog. Theor. Phys. Suppl., 148, 307
Verbunt, F. 1993, ARA&A, 31, 93
Webbink, R. F., Rappaport, S., & Savonije, G. J. 1983, ApJ, 270, 678
Will, C. M., & Zaglauer, H. W. 1989, ApJ, 346, 366
Yungelson, L., & Lasota, J.-P. 2008, NewAR, 51, 860