MOMENT ANALYSIS, MULTIPLICITY DISTRIBUTIONS AND CORRELATIONS IN HIGH ENERGY PROCESSES:
NUCLEUS-NUCLEUS COLLISIONS

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Abstract

Cumulant oscillations, or $H_q$ moment oscillations, appear if the KNO multiparticle distribution decreases at large $z$, $z \equiv n/ < n >$, faster than the exponential, $\exp(-Dz^\mu)$, with $\mu > 1$. In nucleus-nucleus interactions this behaviour is related to the limitation in the average number of elementary central collisions (or average number of strings centrally produced), due to the finite number of nucleons involved. Colour deconfinement, via percolating string fusion, will drastically decrease the fraction of centrally produced strings and increase the cut-off parameter $\mu$: Moment oscillations will be displaced to smaller $q$ and the width of the KNO distribution and forward-backward particle correlations will become smaller.

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In the framework of perturbative QCD it was theoretically predicted, sometime ago, that the factorial cumulants $K_q$ (or, equivalently, the moments $H_q \equiv K_q/F_q$, $F_q$ being the factorial moments) of the multiparticle distribution should present oscillations in sign as a function of $q$ [1]. The predictions turned out to be confirmed by experiment in $e^+e^-$ annihilations [2], in $p\bar{p}$ interactions [3] and in hadron-nucleus and nucleus-nucleus collisions [4]. It is now clear that such oscillations are present in all known high energy processes.

Recently, in [5], it was shown that a necessary condition for a particle distribution $P(n, <n>)$, $<n>$ being the average multiplicity, with positive two particle correlation, $K_2 > 0$, to have oscillations in $K_q$ is to exist, asymptotically, a KNO distribution $\Psi(z)$, [6], of the form:

$$<n > P(n, <n>) \rightarrow <n> \rightarrow \infty z^n <n> = \text{const.}$$

$$\Psi(z) \rightarrow z > 1 \exp[-Dz^\mu] ,$$

with $z \equiv n/ <n>$ being the scaling variable, $D$ and $\mu$ positive parameters, with

$$\mu > 1.$$

Most of the existing popular parametrizations (of the Negative Binomial Distribution family) do not satisfy (1) and (2). They behave exponentially at large $z$, $\mu = 1$, and, not surprisingly, they do not show oscillations in $q$: $K_q > 0$ for all values of $q$.

The importance of the large $z$ behaviour of the distributions to generate the oscillations can be easily seen from the expression relating $K_q$ to $F_q$ ($F_q \equiv <n(n-1)...(n-q+1)> / <n>^q$) and $K_p$, $p < q$:

$$K_q = F_q - \sum_{r_1,r_2,...} P^q_{r_1,r_2,...} K_{r_1}K_{r_2}...,$$

with $r_1 + r_2 + ... = q$, $r_i \geq 1$, $r_{i+1} \geq r_i$ and $r_1 < q$. The $P^q_{r_1,r_2,...}$ are the positive combinatorial factors associated to the partition of an integer $q$ into integers $r_1, r_2, ...$. The meaning of (3) is straightforward. To obtain the $q$ particle cumulant $K_q$ one has to subtract from the $q$ particle factorial moment $F_q$ (integrated $q$ particle inclusive density) all the $q-1, q-2,...$ particle cumulants (integrated inclusive particle correlations) in all clustering combinations. We have, in particular (note that we are using normalized moments, $K_1 = F_1 = 1$),

$$K_2 = F_2 - 1$$

$$K_3 = F_3 - 3K_2 - 1$$

$$K_4 = F_4 - 4K_3 - 3K_2^2 - 6K_2 - 1$$

$$K_5 = F_5 - 5K_4 - 10K_3K_2 - 10K_3 - 15K_2^2 - 10K_2 - 1.$$
If the distribution if of exponential type the $F_q$ grow very fast with $q$, essentially as $q!$, and all the terms in (3) and (4) remain positive ($K_2$ is assumed positive). If at large $z$ the distribution decreases faster than the exponential, $\mu > 1$, the $F_q$ grow slower and at some value of $q$, $K_q$ start becoming negative. But this negative $K_q$ enter in the future steps, $q + 1, q + 2, \ldots$ giving positive contributions to the right hand side of (3) and (4). At some stage the $K_q$ become again positive, etc.: the oscillations start.

It can be argued that the required supression at large $z$ is not really physical, in the sense that at finite energy and limited experimental acceptance, distributions are always cut at large $z$ [7]. In $e^+e^-$ annihilations there are theoretical reasons to belive that the oscillations are physical [1]. We shall argue here that in nucleus-nucleus collisions, at least, the oscillations have also a physical origin.

Successful models that attempt to explain hadron-hadron, h-h, hadron-nucleus, h-A, and nucleus-nucleus, A-B, collisions, making use, or not, of basic information from $e^+e^-$ annihilations, are multiple scattering models. We shall take as reference the Dual Parton Model (DPM), [8], but most of our results do not depend on detailed features of a particular model.

In any multiple scattering model the distribution $P(n)$ of produced particles is the result of the superposition of the contributions from elementary inelastic collisions. At each elementary collision particles (via string formation, for instance) are emitted with a given distribution. There is a certain probability of $\nu$ elementary collisions to occur. In general, we can write

$$P(n) = \sum_{\nu=1}^{n} \sum_{n_1,\ldots,n_\nu} \phi(\nu)p(n_1)p(n_2)\ldots p(n_\nu)$$

with $n_1 + n_2 + \cdots + n_\nu = n$, $p(n_i)$ being the particle distribution from the $i$-th elementary collision, $\phi(\nu)$ the probability distribution for $\nu$ elementary collisions. The parameter $\nu$ represents, as well, the number of intermediate produced objects: pairs of strings, in DPM. In (3) we have assumed that the formed strings emit independently and that fluctuations in the size of the strings are negligeable, or can be reabsorbed in the distribution $\phi(\nu)$.

Let us now introduce the generating function $G(z)$,

$$G(z) = \sum_{n=0}^{\infty} (1 + z)^nP(n)$$

with $G(0) = 1$, such that

$$F_q = \frac{1}{<n>_q} \frac{d^qG(z)}{dz^q} \bigg|_{z=0},$$

and

$$K_q = \frac{1}{<n>_q} \frac{d^q\log G(z)}{dz^q} \bigg|_{z=0}.$$
By combining (5) and (6) we obtain

\[ G(z) = \sum_{\nu=1}^{\infty} \varphi(\nu) g(z)^\nu, \]  

(9)

where \( g(z) \) is the generating function for the elementary process. Knowing the elementary generating function \( g(z) \) (from \( e^+e^- \), string model, Poisson approximation, etc) and the elementary collision distribution \( \varphi(\nu) \) (from multiple scattering combinatorics, impact parameter integrations, etc) the full generating function \( G(z) \) can be constructed and the moments computed, (9), (7) and (8). A summary of results is contained in Table 1.

For the average multiplicity \( < n > \) and the normalized KNO dispersion \( D/ < n > \), where \( D^2 = < n^2 > - < n >^2 \) we have,

\[ < n > = < \nu >\bar{n} \]  

(10)

and

\[ \frac{D^2}{< n >^2} = \frac{< \nu^2 > - < \nu >^2}{< \nu >^2} + \frac{1}{< \nu >} \frac{d^2}{\bar{n}^2}, \]  

(11)

where \( \bar{n} \) and \( d \) are the average multiplicity and the dispersion of the elementary process, respectively.

Concerning relation (11) an important observation can be made. If fluctuations in the effective number of strings were negligible, i.e., \( (< \nu^2 > - < \nu >^2)/< \nu >^2 \approx 0 \), the second term in the right hand side of (11) should dominate. The KNO dispersion, as \( < \nu > \ll 1 \), should then be smaller than the normalized dispersion of the elementary process (say, \( e^+e^- \) or Poisson distribution). This is against experiment: \( D^2/ < n >^2 \) increases with the complexity of the systems involved, from 0.09 in \( e^+e^- \) annihilations [9], to 0.25 - 0.30 in \( pp \) collisions [10], to larger values in h-A processes, and to 0.8 - 1 in A-B collisions [11, 12]. The conclusion is that in (11) the first term in the right hand side, counting the fluctuations in the effective number of strings, is dominant. This is particularly true in nucleus-nucleus collisions where, for large A and B, \( < \nu > \approx 10^2 - 10^3 \), thus making the second term in the right hand side of (11) completely negligible. What we have shown for \( D/ < n > \) applies to all the cumulants \( K_q \) (see Table 1). At least in nucleus-nucleus collisions the KNO particle distribution function \( < n > P(n, < n >) \) must be very close to the KNO string distribution function \( < \nu > \varphi(\nu, < \nu >) \), with scaling variable \( z = n/ < n > \approx \nu/ < \nu > \).

It should by now be clear that oscillations in the factorial cumulants \( K_q \) (or in the \( H_q \) moments), at least in nucleus-nucleus collisions, have nothing to do with the elementary interactions and cannot be related in a simple way to some perturbative QCD calculations (which may apply to the elementary process). We shall try next to explain the origin of the oscillations in A-B collisions.
In nucleus-nucleus A-B collisions, if instead of measuring the unconstrained multiplicity distributions \(P(n)\) one measures the distribution at a fixed impact parameter \(b\) (in particular in very central collisions, at \(b = 0\)) the obtained distribution is totally different. While in the unconstrained situation the KNO distribution is wide \((D/ < n > \approx 1)\) and roughly independent of the nuclei, the central KNO distribution is very narrow \((D/ < n > \ll 1)\) and strongly dependent on the nuclei involved, specially on the lightest nucleus \([11]\). These differences can be easily understood from \([11]\).

If the impact parameter is fixed the fluctuation in the number of strings is drastically reduced, \((< \nu^2 > - < \nu >^2)/ < \nu >^2 \rightarrow 0\), and the second term in the right hand side of \([11]\) dominates:

\[
\frac{D^2}{< n >^2} \bigg|_{\text{central}} = \frac{1}{< \nu >^2} \frac{d^2}{\bar{n}^2},
\]

where \(< \nu >_c\) is the average number of central elementary collisions. One sees that the KNO distribution has to be very narrow \((d/\bar{n}\) is almost Poisson-like and \(< \nu >_c\) may be very large), and the width depends on \(< \nu >_c\). The value of \(< \nu >_c\) increases with the atomic weight of the nuclei, being limited by the atomic weight of lightest nucleus. Independently of A and B one roughly has \([11]\)

\[
< \nu > / < \nu >_c \approx 1/4
\]  

A good way of parameterising multiplicity distributions, from \(e^+e^-\) annihilations to A-B collisions, is by using the generalized gamma function for the asymptotic KNO function,

\[
\Psi(z) = \frac{\mu}{\Gamma(\kappa)} \left[ \frac{\Gamma(\kappa + 1/\mu)}{\Gamma(\kappa)} \right]^{\kappa \mu} z^{\kappa \mu - 1} \exp \left( - \left( \frac{\Gamma(\kappa + 1/\mu)}{\Gamma(\kappa)} z \right)^\mu \right),
\]

with

\[
< z^q > = \frac{\Gamma(\kappa + q/\mu)}{\Gamma(\kappa + 1/\mu)^q} \Gamma(\kappa)^{q-1},
\]

and, in particular, \(< z^0 > = < z^1 > = 1\). The parameters \(\kappa\) and \(\mu\) have to be fixed.

We shall now impose the constraint that the large \(n\) behaviour of the central collision particle distribution must be the same as the behaviour of the unconstrained distribution:

\[
P(n, < n >) \xrightarrow{n \to \infty} P(n, < n >)_{\text{central}}
\]
If the central distribution is also parametrized as (14), with parameters $\mu_c$ and $\kappa_c$, it is clear, from (16), that $\mu_c = \mu$. However, $\kappa_c$ is very different from $\kappa$, as the central distribution is very narrow. This means, see (15) for $q = 2$, $<z^2> \simeq 1$ or $\kappa_c \gg 1$. We can finally impose (16) in a stronger way, see (14):

$$\frac{\Gamma(\kappa + 1/\mu)}{\Gamma(\kappa)} = \frac{<\nu>}{<\nu>_c}.$$  \hfill (17)

In order to have estimates for $\kappa$ and $\mu$ we introduce the additional relations, see (15),

$$\frac{\Gamma(\kappa + 2/\mu)}{\Gamma(\kappa + 1/\mu)} \frac{\Gamma(\kappa)}{\Gamma(\kappa + 1/\mu)} = \frac{D^2}{<n>^2 + 1},$$  \hfill (18)

and make use of experimental information on $D/ <n>$. Reasonable values for $\kappa$ and $\mu$ in agreement with (17) and (18), $<\nu> / <\nu>_c \simeq 1/4$ and $D/ <n> \simeq 0.9$, are:

$$\kappa \approx 0.1$$

and

$$\mu \approx 5$$  \hfill (19)

A few remarks can now be made:

1) As, from (19), $\kappa \mu < 1$, the KNO function does not turn to zero as $z \to 0$, in agreement with experimental data on multiplicity and transverse energy distributions, [12, 13];

2) The parameter $\mu$ is very large, which means that the particle distribution is drastically cut at large $n$, in agreement with multiple scattering models [11, 12, 13, 14] and data;

3) As $\mu$ is large, oscillations occur in the cumulants $K_q$.

In Fig.1 we show a plot of the KNO distribution which agrees with experimental data. In Fig.2 we present $H_q$ as a function of $q$. In both cases $\kappa$ and $\mu$ were fixed at the values (19).

Let us next suppose that the road to the formation of the quark-gluon plasma in nucleus-nucleus collisions is by string fusion [15], mainly occuring in central collisions, where the density of strings is higher. The net result is that the large $n$ tail of the multiplicity distribution is further cut at large $n$, or, in other words, $\mu$ is further increased. This can directly be seen in (17): an increase of $<\nu> / <\nu>_c$, due to string fusion, is translated in an increase in $\mu$.

In Fig.1 and 2 we also show the KNO multiplicity distribution and moments $H_q$ as function of $q$ when the ratio $<\nu> / <\nu>_c$ is increased from roughly $1/4$ to $1/2$. The oscillations tend to start earlier.
The same kind of effect occurs if one is triggering on a heavy particle like the $J/\Psi$. The n-tail of the multiplicity distribution is further cut and therefore the oscillations tend to start earlier, as it is seen in Fig.2 where it is plotted $H_q$ for the multiplicity distribution in S-U collisions at $\sqrt{s} = 19.4$ GeV when a $J/\Psi$ is triggered. Also, it is shown the result when Drell-Yan pairs are triggered instead of $J/\Psi$.

We look now to the forward-backward correlations. If one fixes the number of particles forward, and studies the distribution in the backward hemisphere, one approximately has

$$< n_B > = a + bn_F ,$$

(20)

with

$$b \equiv D_{FB}^2 / D_F^2 ,$$

(21)

$$D_{FB}^2 \equiv < n_B n_F > - < n_B > < n_F > ,$$

(22)

and $D_F^2$ being the variance in the forward hemisphere. Within the spirit of our approximation of keeping only fluctuations in the number of elementary collisions, we assume that particles are produced symmetrically in center of mass rapidity, see [18], to obtain

$$D_{FB}^2 = \frac{< \nu^2 > - < \nu >^2}{< \nu >^2} < \nu_B > < \nu_F >$$

(23)

and, see (11),

$$D_F^2 = \left[ \frac{< \nu^2 > - < \nu >^2}{< \nu >^2} + \frac{2}{< \nu >^2} \frac{d^2}{n^2} \right] < n_F > < n_F > ,$$

(24)

where the factor 2 in the second term in the right hand side of (24) corresponds to particles being independently emitted in rapidity in each elementary collision.

It is clear that in an unconstrained nucleus-nucleus, A-A, collision, from (20), (21) and (22), the forward-backward correlation parameter $b$ is essentially 1. If we trigger in a central collision, $< \nu^2 > - < \nu >^2 / < \nu >^2 \to 0$, as seen before, and the parameter $b$ drops to zero. For instance, in the case of percolating fusion [14], as $< \nu > / < \nu >_{c} \simeq 1/2$, if one triggers on forward events with

$$n_F \gtrsim 2 < n_F >$$

(25)

one should already obtain $b \simeq 0$. This agrees with the results of [20], but as we have here strong fusion the net effect is more spectacular.

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**Figure captions**

**Fig.1.** KNO negative particle distributions in heavy nucleus-nucleus collisions. Continuous curve: present energy situation, $\kappa = 0.1$, $\mu = 5$ in eq. (4). Dashed curve: the same distribution for percolating string fusion, with $\kappa = 0.1$, $\mu = 10$, corresponding to about 50% reduction in the density of central strings.

**Fig.2.** The moments $H_q \equiv K_q/F_q$ for $q = 2, 3, 4, 5, 6, 7$ in the case of nucleus-nucleus collisions without fusion ($\mu = 5$, black circles), and with fusion ($\mu = 10$, open circles). Also represented with crosses (squares) the $H_q$ deduced from the multiplicity distributions of S-U collisions at $\sqrt{s} = 19.4$ GeV when a $J/\Psi$ (a Drell-Yan pair) is triggered.
Table 1: The factorial cumulants $K_q$ for $q = 2, 3$ and 4, obtained from (9) and (8). When the fluctuations in the number of strings are negligible the cumulants are the cumulants resulting from $< \nu >$ independent sources: $K_q = \kappa_q / < \nu >^q$. When string fluctuations dominate, as in heavy nuclei collisions, only the terms inside square brackets are important. In that limit, the factorial cumulants $K_q$ are given by the cumulants of the string distribution. In that same limit the factorial moments $F_q$ are given by the moments $< \nu^q > / < \nu >^q$ of the string distribution. For definition of moments see, for instance, [16].

| $q$ | $K_q$ |
|-----|-------|
| 2   | $\left[ <\nu^2> - <\nu>^2 \right] / <\nu>^2 + \frac{\kappa_2}{<\nu>} + \kappa_3 / <\nu>^2$ |
| 3   | $\left[ <(\nu-<\nu>)^3> / <\nu>^3 \right] + 3 <\nu^2> - <\nu>^2 \frac{\kappa_2}{<\nu>^2} / <\nu> + \frac{\kappa_3}{<\nu>^2}$ |
| 4   | $\left[ <(\nu-<\nu>)^4> / <\nu>^4 \right] - 3 <\nu^2> - <\nu>^2 \frac{\kappa_2}{<\nu>^2} / <\nu> + 3 <\nu^2> - <\nu>^2 \left( \frac{\kappa_2}{<\nu>} \right)^2 + 4 <\nu^2> - <\nu>^2 \frac{\kappa_3}{<\nu>^2} / <\nu>^2 + \frac{\kappa_4}{<\nu>^3}$ |
