Periodicity and area spectrum of black holes

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Abstract The recent speculation of Maggiore that the periodicity of a black hole may be the origin of the area quantization law is confirmed. We exclusively utilize the period of motion of an outgoing wave, which is shown to be related to the vibrational frequency of the perturbed black hole, to quantize the horizon areas of a Schwarzschild black hole and a Kerr black hole. It is shown that the equally spaced area spectrum for both cases takes the same form and the spacing is the same as that obtained through the quasinormal mode frequencies. Particularly, for a Kerr black hole, the small angular momentum assumption, which is necessary from the perspective of quasinormal mode, is not employed as the general area spacing is reproduced.

Keywords area spectrum; periodicity; black hole

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1 Introduction

In recent several years, investigation on quasinormal modes of black holes has attracted more and more attention of astrophysicists and theoretical physicists. It is believed that quasinormal modes of black holes can provide not only a way to find black holes by detecting their fingerprints [1] but also a tool to check the complex quantum gravity theory [2]. Actually, the adiabatic invariant stems from the analogy with classical harmonic oscillator. It is found that the action integral of the form \( \oint pdq \) for a quasiperiodic system is an adiabatic invariant in analytical mechanics. For a one-dimensional harmonic oscillator with Hamiltonian \( H = \frac{p^2}{2m} + \frac{m\omega^2}{2}q^2 \), the adiabatic invariant was shown to be \( A = E/\omega \) [7]. As the classical vibrational frequency \( \omega_c \) and system energy \( E \) are treated as the quasinormal mode frequencies \( \omega \) and black hole mass \( M \) in large \( n \) limit, one can get Eq. (2) immediately. In 2007, Maggiore [3] elucidated that the treatment of Hod and Kunstatter should be reexamined. He proved that the proper frequency of the equivalent harmonic oscillator, which is interpreted as the quasinormal mode frequency, contains contributions of both the real part \( \omega_R \) and imaginary part \( \omega_I \) in high damping limit. More importantly, he found that the imaginary part rather than the real part is dominant for the highly excited quasinormal modes. Therefore, one should use the imaginary part of quasinormal mode frequencies to study the area spectrum of

\[ 8\pi M\omega_n = \ln 3 + 2\pi i(n + \frac{1}{2}) + O(n^{-1/2}), \]

where \( \omega \) and \( M \) are quasinormal mode frequency and black hole mass, respectively. Actually, Kunstatter’s idea stems from the analogy with classical harmonic oscillator. It is found that the action integral of the form \( \oint pdq \) for a quasiperiodic system is an adiabatic invariant in analytical mechanics. For a one-dimensional harmonic oscillator with Hamiltonian \( H = \frac{p^2}{2m} + \frac{m\omega^2}{2}q^2 \), the adiabatic invariant was shown to be \( A = E/\omega \) [7]. As the classical vibrational frequency \( \omega_c \) and system energy \( E \) are treated as the quasinormal mode frequencies \( \omega \) and black hole mass \( M \) in large \( n \) limit, one can get Eq. (2) immediately. In 2007, Maggiore [3] elucidated that the treatment of Hod and Kunstatter should be reexamined. He proved that the proper frequency of the equivalent harmonic oscillator, which is interpreted as the quasinormal mode frequency, contains contributions of both the real part \( \omega_R \) and imaginary part \( \omega_I \) in high damping limit. More importantly, he found that the imaginary part rather than the real part is dominant for the highly excited quasinormal modes. Therefore, one should use the imaginary part of quasinormal mode frequencies to study the area spectrum of
black holes. For the case of Schwarzschild black hole, the quantized horizon area was shown to be $$\Delta A = 8\pi l_p^2$$, which is obviously different from the one obtained by Hod and Kunstatter. Another important speculation in Maggiore’s work is that the periodicity of a black hole in Euclidean time may be the origin of the area quantization. In fact, this inference can be got from Eq.(1) directly. Considering the contributions of real part and imaginary part of quasinormal modes, the transition frequency in large $$n$$ limit can be expressed as $$\omega = \omega_n - \omega_{n-1} = 2\pi/T_{BH}$$. It is well known that for any background space time with a horizon in Kruskal coordinates, the period with respect to Euclidean time takes the form as $$T = 2\pi/\kappa_h$$ [9]. Here $$\kappa_h$$ is the surface gravity that relates to the temperature of the black hole with the relation $$T_{BH} = (h\kappa_h)/2\pi$$. Hence one can see immediately that the transition frequency is related to the period of the black hole in Euclidean time.

In fact, horizon area of black holes is quantized in units of $$l_p^2$$ was proposed firstly by Bekenstein [10,11]. He found that the horizon area of a non-extremal black hole is adiabatic invariant classically. According to the Ehrenfest principle, any classical adiabatic invariant corresponds to a quantum entity with discrete spectrum. Bekenstein conjectured that the horizon area of a non-extremal quantum black hole has a discrete eigenvalue spectrum. Based on Christodouloulou’s point particle model [33], Bekenstein found that the smallest possible increase in horizon area of a non-extremal black hole is $$\Delta A = 8\pi l_p^2$$. Obviously, Maggiore’s result [8] is consistent with it.

Following the work of Maggiore, the area spectrum of a lot of black holes has been investigated. Exploiting models proposed by Hod and Kunstatter, Vagenas [14] gave the area spectrum of a Kerr black hole. It was found that there is a logarithmic term when horizon area of black holes can be reproduced directly with the proposal of Hod. In Sec.2, the area spectrum of the Schwarzschild black hole is given with the proposal of Hod. We get the concrete value of vibration frequency by equaling the motion period of outgoing wave to the period of gravity system in the form of Euclidean time. Particles’ motion in this periodic gravity system also has a period, which has been shown to be the inverse Hawking temperature [9]. Therefore the concrete formulism of frequency of outgoing wave can be given by the inverse Hawking temperature. In this case, the area spectrum of black holes can be reproduced directly with the proposal of Hod.

Now, we intend to give the area spectrum of black holes without using the quasinormal mode frequencies. In fact, there have been some similar ideas before. In Ref.[30], it was found that area spectrum of black holes can be obtained by computing the average squared energy of the outgoing wave in the view of quantum tunneling. In Ref.[31], Ropotenko stated that quantization of the angular momentum component with commutation relation in quantum mechanics can be used to quantize the horizon area of black holes. Recently, Majhi and Vagenas [32] elucidated that an adiabatic invariant quantity, $$\int p_i dq_i$$, can also be used to quantize the horizon area. Here we would like to employ the periodicity of outgoing wave to obtain area spectrum of black holes. For a perturbed black hole, the outgoing wave performs periodic motion outside the horizon and the period of motion is related to the frequency of outgoing wave. It is well known that the gravity system in Kruskal coordinates is periodic with respect to Euclidean time. Particles’ motion in this periodic gravity system also has a period, which has been shown to be the inverse Hawking temperature [9]. Therefore the concrete formulism of frequency of outgoing wave can be given by the inverse Hawking temperature. In this case, the area spectrum of black holes can be reproduced directly with the proposal of Hod.

In Sec.2, the area spectrum of the Schwarzschild black hole is given with the proposal of Hod. We get the concrete value of vibration frequency by equaling the motion period of outgoing wave to the period of gravity system in the form of Euclidean time. In Sec.3, our idea is extended to a Kerr black hole. The general area spacing $$8\pi l_p^2$$ is obtained without any assumptions. Finally, some conclusion and a discussion is given in Sec.4.

## 2 Area spectrum of a Schwarzschild black hole

The line element of a Schwarzschild black hole is

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2d\theta^2 + r^2 \sin^2 \theta d\varphi^2,$$

(3)

where

$$f(r) = 1 - \frac{2M}{r},$$

(4)

The location of the black hole horizon, namely $$r_h$$, is determined by $$f(r_h) = 0$$. The Hawking temperature of this spacetime is $$T_{BH} = \frac{h\kappa_h}{2\pi} = \frac{h}{2\pi M}$$. Substituting Eq.(3) into the Klein–Gordon equation

$$\mu^{\mu\nu}\partial_{\mu} \partial_{\nu} \Phi - \frac{m^2}{h^2} \Phi = 0,$$

(5)

1We thank Prof. Elias C. Vagenas for pointing out that the period of motion of outgoing wave has the same value as that of gravity system in Kruskal coordinates with respect to the Euclidean time.
and adopting the wave equation ansatz $\Phi = \frac{1}{4\pi c^2} R_\omega (r, t) Y_{\ell m}(\theta, \phi)$ for the scalar field, we can get the solution of wave function. On the other hand, we can also obtain the wave function by the Hamilton–Jacobi equation. It is well known that in keeping the items only in order of $\hbar$ the Klein–Gordon equation after inserting Eq.(7) into it and adopting the wave equation ansatz $\psi$ is a periodic function with period $\pi$. In fact, the Hamilton–Jacobi equation can be obtained from the Klein–Gordon equation after inserting Eq.(12) into it and keeping the items only in order of $\hbar$.\footnote{In fact, the Hamilton–Jacobi equation can be obtained from the Klein–Gordon equation after inserting Eq.(12) into it and keeping the items only in order of $\hbar$.\cite{33,35}}

$$g^{\mu\nu}\partial_\mu S_\Lambda S + m^2 = 0,$$  
where the action $S$ and the wave function $\Phi$ have the relation

$$\Phi = \exp\left(\frac{i}{\hbar} S(t, r, \theta, \phi)\right).$$  

(7)

Now, we will concentrate on using the Hamilton–Jacobi equation to find the wave function. For the spherically symmetric Schwarzschild black hole, the action $S$ can be decomposed as \footnote{This value is used to investigate Hawking tunneling radiation usually \cite{33} under the relation $I' = \exp[-2ImS]$, where $I'$ is the tunneling probability, because only this part contributes to the imaginary part of action.}

$$S(t, r, \theta, \phi) = -Et + W(r) + J(\theta, \phi),$$  
where $E = -\partial S/\partial t$ stands for the energy of emitted particles observed at infinity. Near the horizon, $J$ vanishes and $W$ can be solved as \footnote{Based on the Temperature Green Function \cite{9}, it has been shown that this period is the geometric origin of Hawking thermal radiation.}

$$W(r) = \frac{inE}{\rho f(r_h)},$$  

(9)

where we only consider the outgoing wave near the horizon. In this case, it is obvious that the wave function $\Phi$ outside the horizon can be expressed as the form

$$\Phi = \exp\left(\frac{i}{\hbar} Et\right)\psi(r_h),$$  

(10)

where $\psi(r_h) = \exp\left[-\frac{nE}{\rho f(r_h)}\right]$. From Eq.(10), we can see that $\Phi$ is a periodic function with period

$$T = \frac{2\pi}{E}.$$  

(11)

Taking into account the relation $E = \hbar \omega$, we find

$$T = \frac{2\pi}{\omega}.$$  

(12)

Apparently, the frequency is related to the period of the outgoing wave near the horizon. According to Hod's speculation, the change of horizon area of the Schwarzschild black hole takes the form $\Delta A = 32\pi \hbar M \omega$, where $\omega$ is the frequency of the perturbed black hole. Here, $\omega$ can be regarded as the frequency of outgoing wave, so the change of the horizon area can be expressed as

$$\Delta A = 32\pi \hbar M \times \frac{2\pi}{T},$$  

(13)

where we have used Eq.(12). It is well known that in Kruskal coordinates, the gravity system is a periodic system with respect to the Euclidean time. Particles moving in this background hence also have a period $T$ which should be the same as that of the periodic gravity system, namely \footnote{In Ref.32, this period has been used to study the action of particle motion to find area spectrum of black holes. Here we will also resort to this skill. Substituting Eq.(14) into Eq.(13), we can get immediately $\Delta A = 8\pi l_p^2$. Evidently, the area spectrum of a Schwarzschild black hole is quantized with spacing $8\pi l_p^2$. This is consistent with that obtained by Maggiore \cite{8} from the viewpoint of quasinormal mode.}

$$T = \frac{2\pi}{\kappa_h} = \frac{\hbar}{T_{BH}}.$$  

(14)

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### 3 Area spectrum of a Kerr black hole

The line element of a Kerr black hole is

$$ds^2 = -(1 - \frac{2Mr}{\rho^2})dt^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2d\theta^2 + [(r^2 + a^2)\sin^2\theta + \frac{2Mar^2\sin^4\theta}{\rho^2}]d\phi^2 - \frac{4Mar^2\sin^2\theta}{\rho^2}dt\,d\phi,$$  

(16)

where $\Delta = r^2 - 2Mr + a^2$, $\rho^2 = r^2 + a^2\cos^2\theta$. The outer(event) horizon and inner horizon of the Kerr space time can be expressed as $r_{\pm} = M \pm (M^2 - a^2)^{1/2}$, where the parameters $M$ and $a = J/M$ represent the mass, and the angular momentum per unit mass, respectively.

The Hawking temperature and horizon area are

$$T_{BH} = \frac{\kappa_h}{2\pi} = \frac{\sqrt{M^4 - J^2}}{4\pi M (M^2 + \sqrt{M^4 - J^2})},$$  

(17)

$$A = \int (r_h^4 + a^2)\sin^2\theta\,d\theta\,d\phi = 8\pi \left(M^2 + \sqrt{M^4 - J^2}\right).$$  

(18)

For a Kerr black hole, there is an ergosphere between the outer horizon and infinite redshift surface. To avoid the dragging effect, usually one should perform the so-called dragging coordinate transformation, where the dragging angular velocity at the event horizon is defined as

$$\Omega_h = \frac{J}{2M (M^2 + \sqrt{M^4 - J^2})}.$$  

(19)

In this case, Eq.(16) takes the form as

$$ds^2 = -\frac{\rho^2\Delta}{(r^2 + a^2)}dr^2 + \frac{\Delta}{\rho^2}\frac{\rho^2}{\Delta}dr^2 + \rho^2d\theta^2 - F(r)dt^2 + \frac{1}{G(r)}dr^2 + H(r)d\phi^2.$$  

(20)\footnote{In Ref.32, this period has been used to study the action of particle motion to find area spectrum of black holes. Here we will also resort to this skill. Substituting Eq.(14) into Eq.(13), we can get immediately $\Delta A = 8\pi l_p^2$. Evidently, the area spectrum of a Schwarzschild black hole is quantized with spacing $8\pi l_p^2$. This is consistent with that obtained by Maggiore \cite{8} from the viewpoint of quasinormal mode.}
In the dragging coordinate frame, the action $S$ can be decomposed as [36]

$$S(t, r, \theta) = -(E - m\Omega_h)t + W(r) + \Theta(\theta),$$  

(21)

where $E$ is the energy of the emitted particle measured by the observer at the infinity, $m$ denotes the angular quantum number about $\phi$. Incorporating Eqs. (23) and (20), we find near the horizon $\Theta$ vanishes and $W$ can be solved as

$$W(r) = \frac{i\pi(E - m\Omega_h)}{\sqrt{F'(r_h)G'(r_h)}},$$  

(22)

where we also only consider the outgoing wave. So the wave function $\Psi$ can be expressed as the form

$$\Psi = \exp\left[-\frac{i}{\hbar}(E - m\Omega_h)t\right]\psi(r_h).$$  

(23)

Obviously, $\Psi$ is a periodic function with the period

$$T = \frac{2\pi}{(\omega - (m\Omega_h)/\hbar)}.$$  

(24)

Based on Eq. (18), the change of horizon area of a Kerr black hole can be written as

$$\Delta A = 8\pi\left[\frac{2M(\sqrt{M^4 - J^2} + M^2)dM - 2JdJ}{2\sqrt{M^4 - J^2}}\right]$$

$$= 8\pi\left[\frac{dM}{2\pi T_{BH}} - \frac{JdJ}{\sqrt{M^4 - J^2}}\right].$$  

(25)

This treatment is different from that from the viewpoint of quasinormal mode frequency [14], where only the change of the mass $M$ is considered. In Ref. [14], the condition $\omega \gg \omega_R$ implies the imposition $M^2 \gg J$.

Combining Eq. (13) and Eq. (24), we obtain

$$dM = \hbar\omega = m\Omega_h + 2\pi T_{BH}.$$  

(26)

Substituting Eq. (20) into Eq. (25) leads to

$$\Delta A = 8\pi(1 + \frac{\hbar\omega}{2\pi T_{BH}} - \frac{JdJ}{\sqrt{M^4 - J^2}}).$$  

(27)

Simplifying it with Eq. (17) and Eq. (19), we get

$$\Delta A = 8\pi l_p^2.$$  

(28)

Obviously, the equally spaced area spectrum of a Kerr black hole produced here is consistent with that obtained by the viewpoint of quasinormal mode [14]. However, the small angular momentum limit, which is necessary from the perspective of quasinormal mode analysis, is not necessary as the general area gap $8\pi l_p^2$ is obtained.

4 Conclusions and discussions

To conclude, a new scheme to quantize the horizon area of a black hole was proposed. It was found that the period of the gravity system with respect to the Euclidean time can determine the area spectrum of black holes exclusively. Obviously, our result confirmed the speculation of Maggiore that the periodicity of a black hole may be the origin of the area quantization. Here, the quasinormal mode frequency is not used and there is also no confusion on whether the real part or imaginary part is responsible for the area spectrum. It is more convenient and simple. If one wants to study area spectrum of more complicated background space times from the viewpoint of quasinormal mode to confirm the conjecture of Bekenstein that the area spectrum is independent of black hole parameters, there are some difficulties to find the quasinormal modes frequency mathematically which can produce the area spectrum. Even for a Kerr black hole, there are some controversies on the different formalisms [14]. In addition, our treatment is also different from the quantum tunneling method [30]. For the quantum tunneling method, the perturbed frequency is obtained through computing the average squared energy of the outgoing wave. Here, the frequency is determined by the period of motion of outgoing wave. Another important difference in the two methods is the area spacing. For the quantum tunneling method, the spacing of the equally spaced area spectrum is $\Delta A = 4l_p^2$, which is smaller than the general value $8\pi l_p^2$ obviously. The reason for this difference is that the tunneling mechanism in the quantum tunneling method is similar to the Schwinger mechanism [33][37], which will contribute to the quantum of the quantized horizon area [38].

We have found that, for a perturbed black hole, the frequency is determined by the period of the motion of an outgoing wave. It is well known that the gravity system in the form of Euclidean time is periodic. The concrete formalism of period, and further the frequency of outgoing wave, can be given because of the same periodicity. For any space times, we can get the quantized horizon area easily. As examples of application, the area spectrum of a Schwarzschild black hole and a Kerr black hole were studied in this paper, and we found that our results are consistent with those obtained from the viewpoint of quasinormal mode frequencies. We only applied the proposal of Hod to investigate the area spectrum of black holes by virtue of the periodicity of the outgoing wave. It can also be checked using the adiabatic invariant proposed by Kunstatter [6]. For the Kerr black hole, of course, the modified adiabatic invariant should be adopted [14]. The obtained area spectrum
is valid only for large values of $n$ since our calculations are based on the semiclassical approximation. Although the quantum gravity theory has not been found, it is also meaningful to investigate the quantum correction to the area spectrum under some heuristic paradigms [39,40].

According to the quantized area spectrum, we also can get the mass spectrum. From Eq. (15), we find that the mass spectrum takes the form

$$M = \sqrt{\frac{\hbar n}{2}}; \quad n = 1, 2, \cdots$$

(29)

implying the $n \rightarrow n - 1$ transition frequency

$$\omega_0 \equiv \frac{dM}{\hbar} = \frac{1}{4M}.$$  

(30)

Thus quantum jumps larger than the minimal produce emission at all frequencies $\omega = \omega_0 \delta n$ with $\delta n = 1, 2, \cdots$. Bekensten and Mukhanov [41] noted that this simple spectrum can provide a way to make quantum gravity effects detectable because the spacing is inversely proportional to the black hole mass over all scales. For the massive black holes, all the lines in the spectrum are dim and unobservable, which is similar to the semiclassical Hawking temperature. But there exists always a mass regime for which the first few uniformly spaced lines should be detectable under optimum circumstances. It was speculated that the detected mass is the primordial mini-black holes [41]. Thus black hole spectroscopy is also useful to look for the remains of primordial mini-black holes.

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