Expansion of Filter Design from GHz to THz with Metamaterial Hexagonal Split Ring Resonator

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Abstract. Metamaterial application of hexagonal structures with two hexagonal ring-centered, hexagonal ring resonators (SRR-H) for frequency filter has greatly developed but in microwave frequency range. This paper proposes both higher frequency filter and energy. The SRR-H is designed and computed in the 300GHz - 300THz with outer and inner radii of SRR-H respectively for 0.5μm and 0.36μm to 0.35μm and 0.21μm. The simulation results show that the SRR-H size has an effect on the resonance frequency with the largest refractive index occurring at low frequency with the refractive index peak of -6.0242 corresponding to 9.765THz for SRR-H structure of outer and inner radius 0.4μm and 0.26μm. Resonance frequencies tend to shift at high frequencies when the geometrical size of the structure is reduced. This SRR-H filter can be used in THz frequency because the width of the resonance frequency occurs in SRR-H structure.

1. Introduction
Over the last two decades, electromagnetic metamaterials have grown rapidly with unique electromagnetic effects such as invisibility cloaking [1,2], negative refractive index [3,4] and super resolution [5]. Generally, the material is designed and consists of a resonator ring separator (SRR) and placed on the substrate [6]. The most common SRR modifications made are round and square SRR. Other basic structures have also been investigated such as the structure of the letters 'S' [6,7], omega [8], chiral [9] and helix [10]. The advantages of SRR-H compared to other structures have a high absorptivity of up to 99% [11]. Hexagonal SRR is also designed for cloaking purposes, negative refractive index and super resolution. The hexagonal structure places more effective and can be used for wide applications [7]. Negative refractive index investigation was first performed on microscale [3] and now it develops in Terahertz frequency response [12], near-infrared, and visible light [13-15]. Metamaterial in far-infrared or THz region is considered as a slight development of its frequency region in electromagnetic (EM) spectrum, but it is very attractive for a wide range of applications [16,17]. This non-ionizing radiation can easily penetrates materials such as plastics, fabrics and paper [18]. The size of SRR-H can be made small (in order of μm) because the THz radiation has high energy and frequency so it is necessary to expand the SRR-H metamaterial design.

Metamaterials also have extensive applications such as sub-wavelength imaging, electronic components and optical applications, antennas, absorber in solar cells, invisibility technologies, detectors and biosensors [19]. Based on the above studies, this article aims to design and operate the hexagonal resonator ring structure (SRR-H) to increase the higher frequency and energy, as to investigate the SRR and analyze the transmission function and wave propagation reflection. The
frequency used is typically at 300GHz - 300THz. These frequencies have taken into consideration to the requirements of optical metamaterials, where SRR-H is observed at various sizes using HFSS™ software.

2. Literature Review
The main characteristics of optical metamaterials, particularly optical metamaterials, are the EM properties of materials derived from their structural forms, and not the compositions of their constituents [20]. The size of the optical metamaterial is less than half the wavelength coming to avoid the diffraction effect [6]. Modified from the Nicolson-Ross-Weir (NRW) method has been done by Ziolkowski [21]. The NRW equation is used extensively to calculate the permittivity and the complex permeability of the waveguide by measuring the S-parameter. As the waves propagate toward the material-air boundary, a part of the waves will be reflected and transmitted to the boundary plane. If the length of the waveguide is \( t_m \), then the Equation of the S11 reflection function and the transmission function S21 is as follows [6].

\[
S_{11} = \Gamma_{\text{total}} = \frac{\Gamma_1 (1 - z^2)}{1 - \Gamma_1^2 z^2} \quad (1)
\]

\[
S_{21} = T_{\text{total}} = \frac{z (1 - \Gamma_1^2)}{1 - \Gamma_1^2 z^2} \quad (2)
\]

where \( z = e^{-2j\theta} \) represents the wave impedance and \( \theta = k t_m \) is wave phase, so the parameter quantity and difference of \( S_{21} \) and \( S_{11} \) can be expressed,

\[
V_1 = S_{21} + S_{11} \quad (3)
\]

\[
V_2 = S_{21} - S_{11} \quad (4)
\]

Assume \( |\Gamma_1| \leq 1 \), so that it is obtained the relations as follows,

\[
k = \frac{2}{j t_m} \frac{(1 - V_1)(1 + \Gamma_1)}{1 - \Gamma_1 V_1} \quad (5)
\]

\[
\mu_r = \frac{j k t_m}{2} \frac{1 + V_2}{1 - \Gamma_1 V_1} \quad (6)
\]

\[
\varepsilon_r = \mu_r + j \frac{2 S_{11}}{k t_m} \quad (7)
\]

\[
n = \sqrt{\varepsilon_r \mu_r} \quad (8)
\]

where \( k \) is wave propagation (m\(^{-1}\)), \( \mu_r \) is relative permeability (H.m\(^{-1}\)), \( \varepsilon_r \) is relative permittivity (F.m\(^{-1}\)) and \( n \) is refractive index.

3. Structure Design of SRR-H
The SRR-H structure is designed and operated using HFSS [6]. The SRR-H cell unit consists of two concentric aluminum rings with inner radius, \( R_1 \) and the outer radius, \( R_2 \) as shown in Table 1 and Figure 1. SRR-H is set on the FR4-Epoxy substrate and simulated from 300GHz to 300THz to describe frequency filters. The boundary conditions are symmetrically set perfect electrical and magnetic conductors. Its mechanism works by launching the EM source on SRR-H which will be measured by S11 and S21 parameters. The values of S11 and S21 are then calculated using the NRW Equation to obtain refractive index values and resonance frequencies. Table 1 shows the design of SRR-H size in which the substrate has axial dimension with thickness of \( t_s = 0.09 \mu m \), ring thickness tc
\[ \varepsilon = 0.03 \mu m \] and gap \( c_1 \) and \( c_2 \) are set constant i.e., \( 0.07 \mu m \) and \( 0.04 \mu m \), and ring width is constant, \( l = 0.1 \mu m \).

**Table 1. SRR-H parameter input**

| Size | \( R_1 \) (\( \mu m \)) | \( R_2 \) (\( \mu m \)) | \( a \) (\( \mu m \)) | \( b \) (\( \mu m \)) |
|------|-----------------|-----------------|------------|------------|
| r1   | 0.36            | 0.5             | 1          | 1          |
| r2   | 0.31            | 0.45            | 0.9        | 0.9        |
| r3   | 0.26            | 0.4             | 0.8        | 0.8        |
| r4   | 0.21            | 0.35            | 0.7        | 0.7        |

**Figure 1.** Structure of SRR-H. (a) front view. (b) side view.

4. Results and Discussion

Figure 2 and 3 depict the effects of frequency corresponding to energy involved to wave propagation. Geometrical structure changes cause resonance frequency shifts as shown in Figure 3. Resonance frequencies tend to shift larger with structure size. At first resonance frequency (\( f_{01} \)) all the structural variations have the same resonant frequency, but at second (\( f_{02} \)) and third (\( f_{03} \)) resonance frequencies have a uniform resonance frequency shift. This indicates that at lower frequency, the permittivity oscillates over the higher energy but it cannot work at much higher frequency since the change of refractive indices is affected by the geometry structure of both radius gaps. Either real or imaginary parts, the period oscillations are nearly similar but, the expansion of attenuation over the frequency is more affected in imaginary part. Real part having the oscillation is more dominant rather than the attenuation intensity.

In Figure 2, the red line, S11 is the reflection line. The purple line, S21 is the transmission line. The S11 value has several peaks where the filter of waves occurs. It is the same as S21 but the peak tends to exist at low frequencies compared to S11. This is due to EM waves are more easily filtered at lower frequencies rather than at high frequencies. As can be seen in Y1 peak, the filter goes down when the frequency increases. Transmission waves can maintain at high value when reflection waves tend to
decrease. The sum of S11 and S21 is unstable due to the absorption effect and the medium of the structure [22]. It is not surprising when the peak of transmission is fluctuated and cannot be predicted along the frequencies. EM propagating to the ring structure is not only trapped by the ring gap but also protected by the hexagonal structure which is not resonant to the geometry, so that the filters can operate well at the reflection regimes in S11.

Figure 2. Aluminum SRR-H as a function of frequency for ring radius of 0.5 μm (variant r1).

Tabel 2. Resonant frequency of SRR-H

| Size | f01 (THz) | Im(n)   | f02 (THz) | Im(n)   | f03 (THz) | Im(n)   |
|------|-----------|---------|-----------|---------|-----------|---------|
| r1   | 9.255     | -5.0204 | 75.54     | -1.3849 | 135.78    | -1.0224 |
| r2   | 9.765     | -5.3335 | 84.42     | -1.4272 | 150.165   | -1.0298 |
| r3   | 9.765     | -6.0242 | 95.145    | -1.4621 | 168.225   | -1.0264 |
| r4   | 11.775    | -5.709  | 109.395   | -1.5313 | 191.34    | -1.0213 |

Figure 3. Effect of geometry size of SRR-H over resonance frequency. (a) Real part of refractive indices (b) Imaginary part of refractive indices.

The SRR-H structure size variation provides a negative refractive index over a wide frequency range which is useful for extensive filter applications, invisibility technologies or for other EM applications. The other results are depicted in Figure 3 and Table 2 where the size variation has a peak trend, but on
the peak r4 variation in f02 and f03 is smaller than f01. The peak shift of r1, r2, r3 and r4 is nearly periodic, and by increasing the frequency, the peaks of wave are widened and weak until reach zero.

In Figure 4, the frequency difference from the large to small ring radius moves from $\Delta f = \sim 100$THz to $\Delta f = \sim 60$THz which explains that f03 can maintain high frequencies. The largest refractive index values occur at low frequencies with peak refractive index of -6.0242 obtained at a frequency of 9.765THz for r3 structure. The outer ring condition cannot support f01 significantly compared to f02 and f03 corresponding to enhancement of its radius respectively, so that the frequency decay is much dominant for f03.

![Figure 4. Resonance frequency shift over outer ring radius.](image)

5. Conclusion
The structure of two central hexagonal rings, SRR-H has been designed and operated for high energy and frequency range. There are resonance frequencies that occur at intervals of 300GHz - 50THz, 50THz - 125THz and 125THz - 200THz. The metamaterial size of SRR-H gives the resonant frequency tends to expand when the size of the structure is reduced. The refractive index of SRR-H metamaterials has a resonance response on the Re(n) and imaginary Im(n) structure. This simulation has successfully investigated the hexagonal ring resonator characteristics to filter waves at high frequency.

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