Research on operational effectiveness evaluation based on information entropy method

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Abstract. In the face of the future systematic warfare pattern and the development demand of argumentation and demonstration of equipment system in China, it is of great significance to carry out the research of equipment system in order to realize the transformation from single platform technology advantage to combat system capability advantage. Equipment system integrates many factors, such as combat mission requirements, equipment operational capability and weapon operational allocation. In order to construct and optimize equipment system efficiently, it is often necessary to rapidly and dynamically evaluate the operational effectiveness of equipment system. Therefore, the scientific and effective evaluation method of operational effectiveness of equipment system has important reference value for optimizing the performance of weapons and equipment and choosing the direction of equipment pre-research.

1. Introduction

With the state attaching great importance to the construction of the system, many achievements have been made in the research on the contribution rate of equipment system in the military field [1-7]. Su Yaofeng [8], National Defense Information Institute, based on the basic idea of ADC method, establishes an index system for evaluating operational effectiveness, and determines the operational effectiveness of command information system by evaluating the effectiveness and reliability of the system. Liu Huayun [9], 28 Research Institute of China Electronic Science and Technology Group, and others, have staged flight characteristics missile system. Based on the traditional ADC effectiveness evaluation method, the reliability of missile weapon system is analyzed in stages. The credibility of missile flight in the initial, middle and later stages is decomposed into the product of credibility in different flight stages. A phased ADC system effectiveness evaluation method is proposed. Cheng Hao [10] of the Artillery College of the Chinese People's Liberation Army and others, according to the way of weapon operation, applied SEA method, combined with Lanchester equation, synthetically studied the system, environment, mission and other elements of the long-range rocket launcher system, established the SEA model for evaluating the effectiveness of the long-range rocket launcher system, and two values for performance measurement and mission measurement. Based on complex network theory, Zhang Chunhua [11] et al. of National University of Defense Science and Technology extracted the core operational activities in the information-based combat system, established the evaluation model of combat equipment system capability, and proposed a combat system based on combat ring. Effectiveness evaluation method.

Combining with the current research results of weapon and equipment system, it is found that the research of operational effectiveness evaluation of weapon and equipment system lies in the
development of modeling and evaluation methods of weapon and equipment system. However, all the above studies are static evaluation methods, without considering the factors such as cooperation and integration between combat equipment. In order to effectively express the emergence and non-linearity of the system, a dynamic and confrontational method for evaluating the operational effectiveness of the equipment system is proposed based on the information entropy method.

2. Study on the concept of information entropy

As the basic unit of measuring information quantity, information entropy measures the uncertainty of the system by measuring the amount of information contained in a probability distribution. Its essence reflects the uncertainty degree of the state of the system. From the perspective of information theory, it is a good method to measure and study the operational effectiveness of equipment system by using information entropy. It breaks through the traditional static weight measurement and evaluation model, and makes a new dynamic quantitative expression of combat capability evaluation.

The principle of applying information entropy method to combat capability evaluation of equipment system is to decompose equipment system into several combat equipment. There are many information flows between equipment and equipment. There are many factors affecting the effectiveness of each equipment. The less uncertainty these factors satisfy mission requirements in combat, the less uncertainty the system is. The higher the efficiency is, the uncertainty of the influencing factors can be measured by the degree of membership to meet the mission requirements. The greater the degree of membership, the more satisfied the operational requirements.

Assuming that a certain equipment has n factors affecting operational effectiveness, for a certain factor, the membership degree is 1 when it fully meets the operational requirements, and the membership degree is 0 when it is completely unsatisfactory. If \( R_i \) is used to express the membership degree of the influencing factors satisfying the combat mission (\( R_i \in [0,1] \)), then the uncertainty brought by the influencing factors to the combat effectiveness of the equipment can be measured by the self-confidence information quantity \( \ln R_i \), and the weight of each influencing factor is \( w_i \), then the weighted self-confidence information quantity of the combat equipment is recorded as \( I_s \). Then:

\[
I_s = -\sum_{i=1}^{n} w_i \ln R_i
\]

3. OODA combat network model

In order to study the weapon equipment system, it is necessary to describe and model its structure scientifically and accurately. In this paper, the OODA combat network model of the U.S. Army is used to describe and model the weapon and equipment architecture. The combat process is modeled as a dynamic cyclic process [12] consisting of observation, judgment, decision-making and action. The basic structure of the OODA model is shown in Figure 1.

![Figure 1. Schematic diagram of OODA combat model.](image-url)
effectiveness of equipment system is the result of the synergistic action of various weapons and equipment under the command of combatants. In the process of battle, information system connects all systems and units in equipment system to form a battle network driven by information flow. Weapons and equipment are regarded as network nodes, and information flow between them is regarded as network edge.

4. Operational effectiveness evaluation based on information entropy method

Based on the information entropy method and the OODA combat network model, the nodes in the combat system are modeled. Combining with the information entropy method, the operational edge capability and operational ring effectiveness are calculated. A scientific operational effectiveness evaluation method is proposed.

4.1. Modeling of operational node capability index

The equipment system is divided into four types of nodes: target node (T), reconnaissance and warning node (S), command and control node (D) and fire attack node (I). The capability of each type of node is analyzed quantitatively and the capability index of the node is defined. The hierarchy of nodes is shown in Figure 2.

![Node hierarchy diagram](image1)

Figure 2. Node hierarchy diagram.

![Hierarchical decomposition of capability-indicator for target class nodes](image2)

Figure 3. Hierarchical decomposition of capability-indicator for target class nodes.

As can be seen from Figure 2, the characteristics of four types of nodes are described by 11 combat capabilities. Each combat capability has a high degree of discrimination, which meets the functional requirements of nodes. The guidance, penetration and damage performances of fire attack nodes are effectively highlighted by choosing accurate attack capability, damage capability and penetration capability. This paper takes the target class node as an example to illustrate the modeling process.
Target node is enemy equipment. Its equipment survivability mainly includes anti-reconnaissance ability, the ability to avoid enemy hitting after being detected, and the ability to keep part of its functions running normally after being hit. The basic parameters, anti-destruction ability and defensive ability of the target are summarized. Therefore, the target class nodes are decomposed hierarchically into "capability-index", as shown in Figure 3.

As can be seen from Figure 3, the hierarchical relationship between competence and indicators effectively supports the connotation and function of competence. The characteristics of enemy targets are described scientifically through nine capability indices, such as initial distance, maneuvering speed and stealth coefficient. Defense capability plays an important role in survival rate of target. Early warning time, interception distance and interception probability can effectively reflect enemy's means of resisting threat and ability of controlling threat. These nine operational capability indicators also determine whether the nodes of fire attack equipment and reconnaissance and early warning equipment can successfully reconnaissance, track and effectively attack enemy targets. In different combat scenarios such as anti-aircraft/anti-ship/anti-submarine, the membership function of each node is different. The parameters in the function are obtained by simulation, which is not discussed in this paper.

Based on the above analysis, the target class node T and its attributes are described with the following mathematical expressions:

\[ x_T = (x_1^T, x_2^T, x_3^T, x_4^T, x_5^T, x_6^T, x_7^T, x_8^T, x_9^T) \]  

The following definitions are given for the membership function of the node index. The membership functions of the anti-destruction coefficient \( x_4^T \), the maneuvering speed \( x_2^T \), the warning time \( x_7^T \) and the interception ability \( x_9^T \) are defined as follows:

\[
R_{x_4^T} = \begin{cases} 
1.0 & x_4^T \in (0.8, 1.0) \\
2 * x_4^T - 0.3 & x_4^T \in (0.3, 0.5) \\
x_4^T & x_4^T \in (0, 0.3) 
\end{cases}
\]  

\[
R_{x_2^T} = \begin{cases} 
1.0 & x_2^T \in [36, 50] \\
\frac{1}{40} x_2^T + 0.1 & x_2^T \in [20, 36] \\
0.03 * x_2^T & x_2^T \in [0, 20] 
\end{cases}
\]  

\[
R_{x_7^T} = \begin{cases} 
\frac{-4 * x_7^T + 43}{30} & x_7^T \in [0.7, 1.0] \\
-1.25 * x_7^T + \frac{11}{8} & x_7^T \in [0.3, 0.7] \\
1.0 & x_7^T \in [0, 0.3] 
\end{cases}
\]  

Considering the interception capability of the target, focusing on its defensive means such as missiles, torpedoes, and the impact of the combat environment, the interception probability \( R_{x_9^T} \) is described by the following functions combined with the maneuvering speed \( x_8^T \) of the fire attack-like nodes.

\[
R_{x_9^T} = \begin{cases} 
-0.4 * x_8^T + 1.8 & x_8^T \in [2, 3] \\
-0.1 * x_8^T + 0.9 & x_8^T \in [3, 6] \\
\frac{-3 * x_8^T + 0.75}{40} & x_8^T \in [6, 10] 
\end{cases}
\]  

4.2. Calculation of combat side capability

Based on the uncertainty of operational activities brought by node capability index, the basic idea of modeling operational side is to assume that there are \( n \) capability indexes in a certain capability of node X. If the weights of each capability index of node are \( w_1, w_2, \ldots, w_n \) and membership
function, respectively. Numbers are R1, R2, ⋯, Rn. This paper mainly considers six basic functions among weapon equipment nodes: T→S, S→S, S→D, D→D, D→I, I→T. Among them, T→S and I→T depend on the reconnaissance and influence ability of reconnaissance and early warning nodes and fire strike nodes to target, reflecting the main functions of these two types of nodes, and modeling as functional function relationship; while the other four types of relationships are mainly realized by information acquisition and transmission, as information-oriented function. Relations are modeled. Next, take T→S as an example to describe and model its action relationship.

T→S edge is based on the acquisition ability of reconnaissance and early warning node to target information. It mainly investigates the probability of successful reconnaissance and recognition of target by S node. Generally, S-to-T combat activities are usually a process of discovering, identifying and tracking targets. The reconnaissance probability of T→S edge is measured from three aspects.

For discovery probability Pf, there are

\[ P_f = 1 - (1 - t_n)x_1^5 \]  \hspace{1cm} (7)

In Formula (7), \( t_n \) is the probability that S can find T in the nth detection, and x1 S is the scanning frequency per unit time.

\[ t_n = \frac{k \times P_s}{x_3^5 + P_y} \]  \hspace{1cm} (8)

In Formula (8), Ps is the probabilities of searching for class S equipment by random search, Py is the probability of enemy anti-reconnaissance, K is the parameter of environment adjustment, and x3T is the stealth coefficient of target T.

\[ P_s = w_{x_1^5} \times R_{x_3^5} \times R_{x_4^4} + w_{x_2^2} \times R_{x_5^5} + w_{x_3^3} \times R_{x_6^6} \]  \hspace{1cm} (9)

In Formula (9), R(x3 S), R(x4 S), R(x10 S) and R(x11 S) are subordinate functions of detection accuracy x3 S, identification probability x4 S, detection cost x10S and annual maintenance cost x11 S respectively. Wx(i = 1, 2, 3) is the weight of each index.

\[ P_y = R_{x_2^2} \times R_{x_7^7} \]  \hspace{1cm} (10)

In Formula (10), R(x2T) and R(x7T) are the membership functions of the maneuvering speed x2T and the warning time x7T of the T-type nodes, respectively.

Tracking target probability is defined as:

\[ P_g = (P_s)^{x_1^5} \]  \hspace{1cm} (11)

Recognition probability Pb is the ability index x4S of S.

We use Rf, Rg and Rb to represent the efficiency functions of the ability of S-type nodes to discover, track and recognize T-type nodes, namely:

\[ R_m = 1 - \frac{2}{e^{P_m} + e^{-P_m}}, \, m \in \{ f, \, g, \, b \} \]  \hspace{1cm} (12)

PT→S indicates the probability of T→S edge connection when reconnaissance monitors enemy targets.

\[ P_{T→S} = \exp\left(-\sum_{m\in\{f, \, g, \, b\}} w_m \ln R_m\right) \]  \hspace{1cm} (13)

In Formula (13), wm denotes the weight of detection, tracking and recognition relative to T→S edge reconnaissance and early warning probability.

The improved information entropy is used to measure the operational activity completion degree at the battle side, i.e. the confidence information quantity Is_{T→S} at the T→S side. The calculation formulas are as follows:

\[ I_{S_{T→S}} = -\ln P_{T→S} \]  \hspace{1cm} (14)
4.3. Operational effectiveness calculation

Operational Ring Design Idea: Use pointer to identify the front and rear nodes of the side, select the tail node of one side and the same head node of the other side, such as T1→S2, S2→D1, and S2 is the same, then the two sides are connected head and tail, forming T1→S2→D1. Multilateral links are formed according to different interaction relationships, such as T1→S2→D1→I1→T1. The design method of combat ring is shown in Figure 4.

![Diagram of combat rings](image)

**Figure 4.** Design method of T1 S2 D1 I1 T1 combat ring.

The operational effectiveness of a battle ring is calculated by superposition principle, and each battle side is arranged in the order of T→S→D→I→T. Therefore, the operational effectiveness of a battle ring is used to evaluate the completion degree of a battle mission. The calculation method is as follows:

The confidence information of the nth battle ring is $I_n$, and the calculation formula is as follows:

$$I_n = \sum_{j=1}^{M} I_{S_j}$$  \hspace{1cm} (15)

In Formula (15), $I_{S_j}$ denotes the confidence information of the jth battle side and $M$ denotes the number of battle sides included in the battle mission.

$I_n$ can measure the operational effectiveness $P_n$ of a combat mission, and the formula is as follows:

$$P_n = \exp (-I_n)$$  \hspace{1cm} (16)

In the combat network model, each combat task established for the enemy target T is abstracted as a combat ring. The different combinations of these combat tasks constitute the military action that can be taken against the enemy target T. That is to say, by constructing different combat ring combinations in the combat process, the weapon and equipment system can take different military actions against T. Operations, their operational effectiveness is the comprehensive operational effectiveness of all these operational rings. Then, the greatest operational effectiveness of a military operation against T is the aggregation of operational effectiveness of all these operational tasks. Therefore, by evaluating the effectiveness of the combat ring, we can calculate the maximum operational effectiveness of the weapon and equipment system to take military action against the same target.

In this paper, analogous to the method of calculating the resistance of parallel circuit in physics, the relationship between rings is regarded as the parallel relationship between circuits, and the uncertainty of each battle ring is regarded as the resistance of the circuit. Assuming that our weapon and equipment system can formulate $M$ kinds of combat tasks against enemy target T, we can form $M$ coordination rings. The overall uncertainty of these rings can be defined as:

$$I = \frac{1}{\sum_{n=1}^{M} 1/I_n}$$  \hspace{1cm} (17)

Then, the operational effectiveness $P$ of the military operations constituted by these synergistic rings against the enemy target T can be calculated as follows:

$$P = \exp \left(-\frac{1}{\sum_{n=1}^{M} 1/I_n}\right)$$  \hspace{1cm} (18)
5. Evaluation examples

Space-based information support anti-submarine warfare is an important mode of ocean warfare. It contains complex information flow and is a typical form of information warfare. This paper takes space-based information support missile to attack submarine as an example.

5.1. Network model of antisubmarine warfare based on space-based information

The Red and Blue sides clashed in a certain sea area. The Red side reconnaissance detected the Blue side submarine and sent the information back to the command center by satellite. The command center dispatched fighter planes to launch torpedoes to attack the target. If the Red Square destroys the Blue Square submarine, the combat mission is completed.

Based on the operational scenario, for the blue submarine T1, the Red Square combat network weapons and equipment are reconnaissance and early warning categories: electronic reconnaissance satellite S1, photo reconnaissance satellite S2, ocean surveillance satellite S3, reconnaissance aircraft S4, early warning radar S5 and relay satellite S6; command and control categories: command center D1 and fighter D2; Strike Class: Torpedo I1. Based on the idea of OODA ring modeling, a combat network model is built, as shown in Figure 5.

![Figure 5. Anti-submarine combat network diagram.](image)

From Figure 4, it can be seen that the nodes are closely linked. The submarine can be attacked by multiple combat tasks, and the operational effectiveness of the combat system can be evaluated by calculating the degree of uncertainty in the combat network. For blue submarine T1, red side has four means of reconnaissance, and increases the information exchange between nodes, which effectively guarantees the credibility of information and the reliability of combat process.

| Operational Capability | Operational Capability Indicators | Evaluation Value |
|------------------------|----------------------------------|------------------|
| Basic parameter initial| distance (km)                    | 1200             |
|                        | Manoeuvring speed (knot)         | 15               |
|                        | Stealth coefficient              | 0.7              |
|                        | Damage resistance coefficient    | 0.68             |
| Survivability          | Maintenance support capacity     | 0.71             |
|                        | Risk aversion capacity           | 0.65             |
|                        | Early Warning Time (s)           | 0.3              |
| Defense Capability     | Interception distance (km)       | 200              |
|                        | Interception probability         | 82%              |
5.2. Effectiveness evaluation of antisubmarine operations based on space-based information

Based on the network model of anti-submarine warfare, the index value of each node's capability is evaluated, and the operational efficiency of the system is evaluated scientifically by using the information entropy method to calculate the operational efficiency value of the combat ring, combining with the idea of parallel connection.

(1) Setting of Node Indicators

Considering the complex and changeable operation process, this paper gives the evaluation values of various node capability indices based on theoretical experience value and previous test value. The evaluation results of submarine T1 capability indices are given here, as shown in Table 1.

(2) Operational effectiveness evaluation

According to the operational network constructed, the degree of uncertainty in the operational side is measured by information entropy, and the operational effectiveness of all feasible paths of the operational network is calculated accurately. The results are shown in Table 2.

| Loop | Name of Systematic Operational Loop | Operational effectiveness |
|------|-----------------------------------|--------------------------|
| 1    | T₁→S₁→D₁→I₁→T₁                   | 0.4787                   |
| 2    | T₁→S₁→D₁→D₂→I₁→T₁               | 0.4584                   |
| 3    | T₁→S₂→D₁→I₁→T₁                   | 0.4744                   |
| 4    | T₁→S₂→D₁→D₂→I₁→T₁               | 0.4571                   |
| 5    | T₁→S₃→S₅→D₁→I₁→T₁               | 0.4695                   |
| 6    | T₁→S₃→S₅→D₁→D₂→I₁→T₁            | 0.4556                   |
| 7    | T₁→S₃→S₆→D₁→I₁→T₁               | 0.4784                   |
| 8    | T₁→S₄→S₅→D₁→D₂→I₁→T₁            | 0.4583                   |
| 9    | T₁→S₄→S₅→D₁→I₁→T₁               | 0.4637                   |
| 10   | T₁→S₄→S₆→D₁→D₂→I₁→T₁            | 0.4538                   |
| 11   | T₁→S₄→S₆→D₁→I₁→T₁               | 0.4707                   |
| 12   | T₁→S₄→S₆→D₁→D₂→I₁→T₁            | 0.4559                   |

From Table 2, it can be seen that the operational effectiveness of the 12 loops is 0.4538 at the lowest and 0.4787 at the highest. The difference between them is 5.49%. This indicates that the capability of each loop to accomplish operational tasks is quite different, and it is necessary to improve the performance of the equipment in the loop. Compared with each loop, it is found that the more the loop nodes, the lower the combat effectiveness. Generally, the error caused by the information diversification is large, which increases the uncertainty of the system, but the effectiveness value of the 7th loop is higher than that of the 3rd loop. The research shows that improving the performance of the equipment in the loop can improve the reliability of the system and guarantee the combat responsibility. The efficient completion of business is also in line with the actual situation.

Then, the operational effectiveness P of the system is:

$$P = \exp\left(-\sum_{j=1}^{12} I_{S_j}\right) = 0.9381$$  \hspace{1cm} (19)

The operational effectiveness of the system is calculated to be 0.9381. The results show that the Red Square equipment system can effectively complete the attack mission against the Blue Square submarine. Therefore, the information entropy method can scientifically evaluate the operational effectiveness of the system, effectively find out the weak links in the system, and provide reliable theoretical reference for argumentation of weapons and equipment.
6. Summary
Based on the space-based information of anti-submarine combat scenario, the operational effectiveness of the Red Square system is calculated accurately, which verifies the feasibility of the operational network modeling method and operational effectiveness evaluation method. Therefore, the operational effectiveness evaluation method of the equipment system can find out the weak links of the equipment system, provide important data reference basis for equipment deployment and operational application, and predict the future development direction of weapons and equipment and key research objects.

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