Geometry mediated supersymmetry breaking

YUTAKA SAKAMURA

Department of Physics, Korea Advanced Institute of Science and Technology
373-1 Guseong-dong Yuseong-gu, Daejeon 305-701, Korea
E-mail: sakamura@muon.kaist.ac.kr

ABSTRACT

We investigate SUSY breaking mediated through the deformation of the spacetime geometry due to the backreaction of a nontrivial configuration of a bulk scalar field. To illustrate its features, we work with a toy model in which the bulk is four dimensions. Using the superconformal formulation of SUGRA, we provide a systematic method of deriving the 3D effective action expressed by the superfields.

1. Introduction

When we construct brane-world models, the size of the extra dimensions has to be stabilized. One of the main stabilization mechanisms is proposed in Ref. [2], and similar mechanisms have also been studied [3,4]. These mechanisms involve a bulk scalar field that has a nontrivial vacuum configuration. In such a case, the background geometry receives the backreaction of the scalar configuration. However, effects of such backreaction have been neglected in most works. Here, we investigate the SUSY breaking effects mediated through the deformation of the spacetime geometry due to the backreaction [5].

In order to focus on the effects of SUSY breaking through the spacetime geometry, we consider a situation where a scalar field in the hidden sector has a non-BPS configuration. Then, the dominant contribution to SUSY breaking in the visible sector comes from the geometry-mediated effects. In order to understand qualitative features of this type of scenario, we work with a simplified toy model, in which the bulk spacetime is four dimensions and the effective theory is three-dimensional. An interesting example of the stabilization mechanisms is proposed in Ref. [4]. In this article, the authors found a non-BPS solution in the 4D SUGRA, which stabilizes the radius of the extra dimension and simultaneously generates a warped geometry.

In the following, we will assume the existence of a non-BPS solution in 4D SUGRA, and derive the action on that background in terms of 3D superfields. Our method of deriving the action is based on the superconformal formulation of SUGRA [6], but is easy to handle thanks to the superfield formalism.

2. Invariant action

Here, we choose the direction of $y \equiv x_2$ as the extra dimension, and $m, n = 0, 1, 3$ denote the 3D vector indices. The underbarred indices denote the local Lorentz indices.

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We will consider the following gravitational background.

\[ \langle e_\mu^a \rangle = \text{diag}(e^A, e^A, 1, e^A), \quad \langle \psi_\mu^a \rangle = 0, \]
\[ \langle A_m \rangle = 0, \quad \langle A_y \rangle = \frac{\kappa}{2} \Im(\bar{\phi}_h^b \partial_y \phi_h^b), \]  
where \( A(y) \) is a warp factor, \( A_\mu \) is a gauge field of \( U(1)_A \), a constant \( \kappa \equiv 1/M_{\text{pl}} \) is the gravitational coupling, and \( \phi_h^b(y) \) is the background scalar configuration in the hidden sector. On the above background, an superconformal invariant action can be expressed in terms of 3D superfields as follows.

\[ S_{\text{bg}} = \int d^4 x d^2 \theta \left[ 2e^A G_{1I} D^a \bar{\phi}^I D_a \phi^I + 4e^{2A} \Im \left\{ G_I \nabla_y \phi^I + e^{2\theta A} W(\phi) \right\} \right] 
+ \frac{8}{3} e^{2A} \langle A_y \rangle G + \frac{e^{2A}}{2} \text{tr} \left\{ T_1^{-1}(u^g)^2 - T(w^g)^2 \right\}, \]  
(2)

where \( G(\bar{\phi}, \phi) \) is a real function of \( \phi^I \) and \( \bar{\phi}^I \), and \( W(\phi) \) is a holomorphic function of \( \phi^I \).

3D scalar superfields \( \phi^I \) is constructed from 4D chiral multiplets, and the 3D spinor superfields \( u_\alpha^a \) and \( w_\alpha^a \) are constructed from a 4D vector multiplet. Especially, \( w_\alpha^a \) corresponds to the 3D superfield strength. Derivative operators \( D_a \) and \( \nabla_y \) are covariant derivatives for the gauge symmetry. (For their explicit definitions, see Ref. [5].) Furthermore,

\[ T \equiv 1 + e^A \theta \langle A_y \rangle \]  
(3)

is a spurion superfield corresponding to the radion superfield.

3. Action after gauge fixing

In order to obtain the Poincaré SUGRA, we have to remove the redundant superconformal gauge symmetry by imposing the gauge fixing conditions. After the gauge fixing, the invariant action becomes

\[ S_{\text{vis}} = \int d^4 x d^2 \theta \left[ e^A \sum_i D^a D^a_i \phi^i + e^{2A} \text{Im} \left\{ \frac{1}{2} \sum_i \bar{\phi}^i \nabla_y \phi^i + e^{(\kappa^2/2)} \phi_h^b \right\} P_{\text{vis}}(\phi) \right] 
\frac{4}{3} e^{2A} \langle A_y \rangle \sum_i \bar{\phi}^i \phi^i + \frac{e^{2A}}{2 \kappa^2} \text{tr} \left\{ (u^g)^2 - (w^g)^2 \right\} + \kappa^2 \int d^4 x e^{2A} \left[ \text{Im} \left\{ -2 \sum_i f_{bg}^G \phi^i \bar{f}_I + 6 f_{bg}^\Sigma P_{\text{vis}} - \frac{4 \langle A_y \rangle}{3 \kappa^2} \left( 3P_{\text{vis}} - \sum_i \phi^i \partial P_{\text{vis}} \phi^i \right) \right\} \right] 
- \frac{\langle A_y \rangle}{2 \kappa^2} \text{tr} \left\{ \left( \lambda_1^g \right)^2 + \left( \lambda_2^g \right)^2 + \ldots \right\} + \mathcal{O}(\kappa^4), \]  
(4)

where the scalar \( \phi^i, \lambda_1^g, \) and \( \lambda_2^g \) are the lowest components of \( \phi^i, u^g, \) and \( w^g \), respectively.

Here, we have assumed the minimal Kähler potential, and \( P \) is a superpotential which is related to \( W \) in Eq.(2) as

\[ W = \left( \frac{2}{3} \right)^{3/2} \kappa^3 \left( \phi^\Sigma \right)^3 \left\{ P_{\text{hid}}(\phi^h) + P_{\text{vis}}(\phi^i) \right\}, \]  
(5)
where \( \varphi^\Sigma \) is a compensator superfield, \( \varphi^h \) is a hidden-sector superfield whose lowest component has a nontrivial vacuum configuration, and \( \varphi^i \) are matter superfields in the visible sector. Since we are interested only in the visible sector, we have dropped the fluctuation modes of \( \varphi^h \) in the hidden sector. Note that the action is supersymmetric at \( \mathcal{O}(\kappa^0) \). This is a trivial result. Because all the SUSY breaking effects in the visible sector are induced through the deformation of the geometry, they will vanish in the limit that the gravity is turned off. Thus, SUSY breaking terms appear at \( \mathcal{O}(\kappa^2) \).

4. Summary and comments

We have discussed the effects of SUSY breaking mediated by the deformation of the spacetime geometry due to the backreaction of a bulk scalar field. We derived the action expressed by 3D superfields and the SUSY breaking terms.

Scales introduced in the theory are the 4D Planck mass \( M_{\text{pl}} \), the mass parameters for the matter fields \( m_i \), the characteristic scale of the hidden sector dynamics \( \Lambda \), and the compactification scale \( r^{-1} \). In terms of these scales, the SUSY-breaking scalar masses induced in the visible sector have a form of

\[
m^2_S = \frac{\Lambda^3 m_i}{M^2_{\text{pl}}} \alpha(m_i, \Lambda, r),
\]

where \( \alpha(m_i, \Lambda, r) \) is a dimensionless function expressed by an overlap integral of the mode functions and the functions \( f^h_{bg}, f^G_{bg} \), and \( \langle A_y \rangle \). The gaugino mass is induced by the nonzero \( \langle A_y \rangle \), and roughly estimated as

\[
m_g \sim \frac{\Lambda^3}{M^2_{\text{pl}}}.
\]

Here, we have supposed that \( \arg(\phi^h_{bg}) \) varies with \( \mathcal{O}(1) \) amplitude as \( y \) goes from 0 to \( \pi r \).

Since all the SUSY breaking effects discussed here are suppressed by the Planck mass, our scenario can be considered as a kind of the gravity mediation. However, there are some points that should be noticed. First, from the viewpoint of the effective theory, SUSY breaking discussed here cannot be regarded as a spontaneous breaking because the order parameter of SUSY breaking is roughly of \( \mathcal{O}(\Lambda) \) and is generally higher than the compactification scale \( r^{-1} \), which is the cut-off scale of the 3D effective theory. Second, the induced SUSY breaking scale in the effective theory can be suppressed by the overlap integral of \( f^h_{bg}, f^G_{bg} \), and the mode functions. Third, the gaugino mass can be induced by non-zero \( \langle A_y \rangle \) without introducing a non-minimal gauge kinetic function. This is very similar to the Scherk-Schwarz (SS) SUSY breaking \(^7\) interpreted in the Hosotani basis. However, this breaking is irrelevant to the \( U(1)_R \) twisting since \( U(1)_R \) is a symmetry after the gauge fixing and independent of \( U(1)_A \), which is completely fixed by the gauge fixing condition. In addition, \( \langle A_y \rangle \) is not an input parameter as in the SS breaking, but is determined by the bulk scalar dynamics. Further, the non-zero \( \langle A_y \rangle \) indicates that the
background configuration is non-BPS. Thus, it inevitably leads to SUSY breaking terms that associate with $f_{bg}^2$ and $f_G$; to which the SS breaking does not have any resemblances.

To investigate more phenomenological aspects, we should extend our discussion to 5D SUGRA. Note that the procedure explained here requires only the knowledge of the superconformal formulation of 4D SUGRA and the 3D superfield formalism. The 4D superfield formalism is not necessary. Therefore, the extension to 5D SUGRA only requires the 5D superconformal formulation \cite{8} and the well-known 4D $\mathcal{N} = 1$ superfield formalism. This work is done in Ref. \cite{9}.

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