Revisiting Gaussian mixture critics in off-policy reinforcement learning: a sample-based approach

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Actor-critic algorithms that make use of distributional policy evaluation have frequently been shown to outperform their non-distributional counterparts on many challenging control tasks. Examples of this behavior include the D4PG and DMPO algorithms as compared to DDPG and MPO, respectively [Barth-Maron et al., 2018; Hoffman et al., 2020]. However, both agents rely on the C51 critic for value estimation. One major drawback of the C51 approach is its requirement of prior knowledge about the minimum and maximum values a policy can attain as well as the number of bins used, which fixes the resolution of the distributional estimate. While the DeepMind control suite of tasks utilizes standardized rewards and episode lengths, thus enabling the entire suite to be solved with a single setting of these hyperparameters, this is often not the case. This paper revisits a natural alternative that removes this requirement, namely a mixture of Gaussians, and a simple sample-based loss function to train it in an off-policy regime. We empirically evaluate its performance on a broad range of continuous control tasks and demonstrate that it eliminates the need for these distributional hyperparameters and achieves state-of-the-art performance on a variety of challenging tasks (e.g. the humanoid, dog, quadruped, and manipulator domains). Finally we provide an implementation in the Acme agent repository.

1. Introduction

The field of Reinforcement Learning (RL) formalizes the study and design of agents which interact with their environment and make observations [Sutton and Barto, 1998]. A typical RL agent is one which optimizes its behavior in order to maximize its expected return—i.e. the expected (discounted) sum of future rewards provided by the environment. However, for many algorithms a crucial prerequisite to this optimization process is the ability to predict the agent's expected return. As a result a key building block of such agents is their value function, i.e. a learned function which maps from the current state of the environment to the return expected under future interaction. This function can be used in an iterative fashion by first improving its predictions and using the updated values to improve the agent's behavior; this process can be repeated until the agent's behavior converges.

Classically, value-based RL algorithms rely on a function which makes point estimate predictions of the expected return. While algorithms making use of more complex predictions or higher moments do exist [Morimura et al., 2012; Prashanth and Ghavamzadeh, 2013; Tamar et al., 2016], they are far from the norm. Recently, though, the work of [Bellemare et al., 2017a] has revived interest in this domain by proposing a mechanism to estimate a distribution over values rather just a single scalar value. By making use of this distributional approach to value estimation Bellemare et al. was able to achieve what was at the time, state-of-the-art performance on the Atari 2600 benchmark suite. These techniques were later extended to continuous action-space Actor-Critic algorithms in the form of D4PG and DMPO [Barth-Maron et al., 2018; Hoffman et al., 2020] achieving state-of-the-art performance in many continuous control tasks.

By design the distribution over values has an expectation equivalent to the standard value function, a key point that allows these methods to easily integrate with many mechanisms for policy improvement. However, the precise form of this distribution plays a key role in how it is updated based on incoming reward observations. In the case of the methods presented thus far this distribution is represented by a categorical distribution defined at a regular grid of atoms over the space of expected returns—i.e. the real line. This use of a discrete distribution over the space of returns has two potential downsides. First, returns are inherently continuous and as a result there is
some loss of representation due to the use of this discretized distribution. Although at first glance this may seem problematic it is still possible to increase the number of atoms in this representation. While such an approach does increase the complexity of the algorithm, it does so only linearly since the returns that must be predicted are a scalar quantity. More importantly, however, the categorical distribution in question has bounded support that must be known ahead of time.

While it is possible to learn the support of the return distribution [see e.g. Dabney et al., 2018a] an alternative is to make use of a distribution which is naturally able to shift its support. In this work we revisit an alternative distributional value function parameterized by a mixture of Gaussians (MoG) and the simple sample-based approach to optimize this object, which was first proposed in [Barth-Maron et al., 2018, appendix A–C]. Compared to its categorical counterpart, the MoG parameterization has received relatively little scrutiny, a fact we attribute primarily to previously inferior results. To address this we also provide a number of experiments to examine the performance of our approach when used within a modern off-policy actor-critic algorithm. Our results show that this technique is able to achieve state-of-the-art performance on a number of hard continuous control tasks. Finally, we provide some preliminary analysis (Sections 5 and 6.1) to help understand what made this implementation successful compared to prior attempts [Barth-Maron et al., 2018].

Overall, the primary contribution of this work is an effective combination of a mixture-of-Gaussians value distribution with the MPO algorithm for policy optimization that we found particularly effective for continuous control domains. Our results in 5 closely examines the reasons why this approach is successful, and may prove useful for further development of distributional algorithms. Finally, while we did find this algorithm to be fairly robust, we did find it to be somewhat sensitive to the choice of initial scale of the Gaussian components; see 6.1.

1.1. Related work

The approach described in this paper is most related to two particularly relevant lines of work. The first line is that of [Bellemare et al., 2017a], whose work renewed recent interest in distributional RL as a whole as well as presenting both both initial theoretical analysis and impressive empirical Atari results. The critic introduced by Bellemare et al. outputs a mixture of fixed delta masses, an approach that has influenced much of the following distributional RL work and that we will refer to as a categorical critic in this report (or CS1 when using 51 atoms as prescribed by Bellemare et al.). The second related line of work is that of Barth-Maron et al. [2018] and Hoffman et al. [2020], who have successfully combined categorical critic with the DDPG and MPO policy optimization approaches, respectively, in order to improve on the state-of-the-art in continuous control.

While the works of Bellemare et al. [2017a] and Barth-Maron et al. [2018] focus on categorical value distributions, the latter work also describes and experiments with a mixture-of-Gaussians distribution. This earlier work, however, found the MoG approach to underperform compared with that of the categorical distribution and left its use relatively unexplored. Much earlier work of Morimura et al. [2012] uses a single Gaussian for its return distribution and minimizes the KL divergence in a similar way to this paper. However this earlier work also uses much simpler policies which do not make use of modern deep neural networks and as a result are not directly comparable. More recently, the works of Nam et al. [2021] and Choi et al. [2019] have returned the the question of a mixture-of-Gaussians distribution. The closest of these to our work is the approach of SR(λ) presented in [Nam et al., 2021] which differs in two primary ways. The first is in the choice of the divergence metric between distributions; unlike our approach SR(λ) makes use of the Cramér metric rather than the KL divergence. The second, and perhaps bigger difference, is the choice of underlying policy optimization metric in which SR(λ) makes use of PPO (whereas we use MPO)—as a result this work need only estimate the value function V and is somewhat more on-policy than the approach described in this work. Although interesting, we leave an in-depth comparison to SR(λ) for future work. Similarly the MOG-Q algorithm introduced in [Choi et al., 2019] differs greatest in its choice of policy optimization algorithm—DQN in this case. The use of DQN however leads to a greater difference in its loss function and requires the authors to introduce their JTD loss. Additionally this approach makes MOG-Q less applicable to the continuous control examples focused on by this work.

In terms of relevant theoretical results, Lemma 3 of [Bellemare et al., 2017a] shows that the distributional Bellman operator is a $\gamma$-contraction in the supremum-Wasserstein metric. In this report, however, we do not benefit from Lemma 3, as we do not minimize the appropriate error metric. This should be considered in future work, perhaps capitalizing on the Cramér distance and Proposition 2 from [Rowland et al., 2018], which denotes the result to a $\sqrt{f}$-contraction in the supremum-Cramér metric. Since in the actor-critic setting we have separate policy evaluation and improvement subroutines, the purpose of the former is simply to get an estimate of $Q^*_x$, therefore
we are not interested in the distributional Bellman optimality operator from [Bellemare et al., 2017a] and we are not affected by the negative results in Lemma 4 and Theorem 1 therein, which proves that the optimality operator is not a contraction.

There are also several works that are left out of the scope of this report which we should nevertheless mention as additional comparisons and potential future work. For instance recent quantile-based work by Dabney et al. [2018a,b], which also removes the hyperparameter requirements of C51; these are more complex algorithms than the one studied in this manuscript and a full comparison is left for future work. Concurrently to the present work, Nguyen et al. [2021] have explored similar sample-based approaches by leveraging the maximum mean discrepancy [MMD; Gretton et al., 2012]; this may be a promising alternative although it brings with it a kernel function as a hyperparameter. Finally, somewhat related albeit non-distributional is the PopArt approach of van Hasselt et al. [2016], which applies an adaptive standardization transformation to the value targets; a similar approach may be useful to remove the vmin/vmax hyperparameters, although the fixed number of delta masses of C51 would remain and would bound the resolution of values that can be considered.

2. Background and notation

We adopt the standard formalization of reinforcement learning (RL) as a Markov decision process (MDP) and much of the notation from Rowland et al. [2018]. In particular we denote with $\mathcal{S}$ and $\mathcal{A}$ the spaces of states and actions respectively; a transition distribution $p(R_t, S_{t+1} | S_t, A_t)$ over immediate rewards and next states conditioned on a given state-action pair; a discount factor $\gamma \in [0, 1)$; and we denote an agent’s policy with $\pi(A_t | S_t)$. A trajectory of states, actions, and rewards will be denoted with $(S_t, A_t, R_t)_{t \geq 0}$ where we have used capital letters to emphasize the fact that these are random variables. To generate a trajectory the environment starts from an initial state $s_0 \in \mathcal{S}$ and for every subsequent timestep $t \geq 0$, an agent produces an action $A_t \sim \pi(\cdot | S_t = s_t)$ sampled from its policy. The environment then produces an immediate reward and next state according to the transition distribution $(r_t, s_{t+1}) \sim p(\cdot, \cdot | S_t = s_t, A_t = a_t)$. We can then define the random variable $Z^{(s,a)}_t$ representing the sum of $\gamma$-discounted rewards, conditioned on an initial state and action $(s,a)$:

$$Z^{(s,a)}_t = \sum_{t=0}^{\infty} \gamma^t R_t \quad S_0 = s, A_0 = a,$$

where the implicit dependence on the policy $\pi$ is indicated as a subscript on $Z$. Hereafter we refer to this quantity as the return of policy $\pi$. Classical off-policy evaluation methods are interested in evaluating a policy’s expected return given an arbitrary initial state-action pair $(s,a)$, a quantity known as the state-action value function $Q_\pi(s,a) = \mathbb{E}Z^{(s,a)}_\pi$ where the expectation is taken with respect to the trajectory distribution induced by the policy $\pi$ and the unknown transition distribution $p$. In distributional RL, we are interested in the distribution of the conditional random variable $Z^{(s,a)}_\pi$ rather than only its first moment $Q_\pi(s,a)$. Therefore, in a slight abuse of notation, we will also let $Z^{(s,a)}_\pi$ denote the probability density function of the eponymous random variable as the symbol’s meaning should always be clear from context.

In this report, we focus on off-policy actor-critic algorithms. These are algorithms that combine a policy $\pi$ with a critic $Q_\pi$, alternating between (a) training the critic to evaluate the current policy, i.e., policy evaluation, and (b) training the policy to shift its probability mass towards higher quality actions (according to the critic), i.e., policy improvement. Since this study is focused on distributional policy evaluation, for most of this report we fix the policy improvement algorithm to be maximum a posteriori policy optimization (MPO; Abdolmaleki et al., 2018a,b), although we show in Figure 9 that our findings generalize to deterministic policy gradient algorithms [Silver et al., 2014; Barth-Maron et al., 2018]. In both settings the policy improvement steps target the expectation of the distributional critic estimate, i.e. $Q_\pi = \mathbb{E}Z_\pi$ as produced by the policy evaluation step.

In order to discuss policy evaluation we must first introduce the distributional Bellman operator. Following [Bellemare et al., 2017a] for an arbitrary scalar random variable $Z$ which can be conditioned on state-action pairs, the operator in question can be written as

$$(\mathcal{T}^\pi Z)^{(s,a)} = \mathbb{E}_{r,s' \sim p} \left[ \mathbb{E}_{a' \sim \pi} \left[ r + \gamma Z^{(s',a')} \right] \mid s,a \right].$$

With this operator in hand, and analogously to non-distributional Q-learning, the distribution function $Z_\pi$ satisfies the distributional Bellman equation, i.e. that the distribution is a fixed point under the distributional Bellman
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Figure 1 | Diagram depicting the three different kinds of critics considered in this report.

operator. For any state and action this can be written as

$$Z_{\pi}^{(s,a)} = (T^\pi Z_{\pi})^{(s,a)}.$$  

(3)

This can then be used to perform policy evaluation by starting from an arbitrary estimate $Z$ and using the Bellman operator to move this estimate towards its fixed point.

3. Implementation

The algorithm described in this work can be summarized as a combination of the following key ideas:

- **Parameterize the distributions.** We use a mixture of Gaussians (MoG) to parameterize the distributions $Z_{\pi}$ because it is very expressive, it can be sampled from, and has a differentiable log-probability density function. It also has infinite support, obviating the need for projections onto a fixed support, unlike its categorical counterparts [Bellemare et al., 2017a; Barth-Maron et al., 2018].

- **Neural network parameterization.** We use a neural network, denoted $Z_{\theta}$, to approximate $Z_{\pi}^{(s,a)}$ and, as is common, we also use a target network $Z_{\bar{\theta}}$ to estimate bootstrap targets to stabilize Q-learning. In this work, the target network is periodically updated but we do not anticipate any issues with using incremental target updates.

- **Distributional loss.** A distributional loss (distance or divergence) $d(Z, Z')$ is necessary to to minimize the distributional Bellman error which we can write as $d(T^\pi Z_{\pi}, Z_{\theta})$, i.e. the difference between the return distribution before and after applying the distributional Bellman operator. In practice we use the cross-entropy loss $H$, which although it does not guarantee a fixed point this loss works well in practice; we leave the consideration of other losses for future work.

- **Sample-based approximation.** We use Monte Carlo approximation for all integrals required to compute the distributional Bellman operator (2), creating a fully sample-based empirical critic loss.

Combining all of these components, we arrive at the following expression for our distributional loss:

$$L(\theta) = -E_{s,a} H \left[T^\pi Z_{\theta}^{(s,a)}, Z_{\theta}^{(s,a)} \right] = -E_{s,a} \left[ E_{r,s'} E_{a'-\pi} \left[ r + \gamma Z_{\theta}^{(s',a')} \right], Z_{\theta}^{(s,a)} \right]$$  

(4)

where the expectation over $(s, a)$ is taken over a replay buffer as usual in lieu of the stationary distribution of the policy $\pi$. As we do not have access to the environment’s transition kernel or reward function, for every $(s, a)$ we naturally only have the observed subsequent $(r, s')$ as samples available for Monte Carlo integration. Omitting the indices on the $B$ minibatch samples from replay, we obtain the following loss

$$= \frac{1}{B} \sum_{s,a,r,s'} H \left[ E_{a'-\pi} \left[ r + \gamma Z_{\theta}^{(s',a')} \right], Z_{\theta}^{(s,a)} \right] = \frac{1}{B} \sum_{s,a,r,s'} \ell(\theta; s, a, r, s'),$$  

(5)
We have implicitly defined a per-transition loss function $\ell$ as the expected cross-entropy Bellman error over (unobserved) next actions according to the policy $\pi$. Therefore we have two more integrals left to approximate, the outer one over the real line representing possible returns, this is the one hidden by the cross-entropy symbol $H$; and the inner one over the policy we are bootstrapping with respect to. Again we estimate these via Monte Carlo, generating sampled returns by transforming samples from $Z_\theta^{(s,a)}$ to get the final expression for our empirical critic loss:

$$
\ell(\theta; s, a, r, s') \approx -\frac{1}{NM} \sum_{i,j=1}^{M} \log Z_\theta^{(s,a)} \left( r + yz^{(i,j)} \right), \quad \text{where} \quad \begin{cases} z^{(i,j)} \sim Z_\theta^{(s',a'(j))}, \\ a'(j) \sim \pi(\cdot | s'). \end{cases}
$$

In practice we often bootstrap with respect to the greedy policy such that $M = 1$ and $a'$ is the mode of the stochastic policy $\pi(\cdot | s')$. Notice that the only requirements for this approximate loss are that (i) we can sample realizations of the random variable $Z_\theta$ and (ii) we can compute differentiable log-probabilities. This means that while in this report we focus on a mixture-of-Gaussians parameterization of the distributional Q-function, this loss can be used more broadly.

4. Experiments

We compared the Mixture of Gaussian critic with 5 components to two other baselines: the C51 critic and the non-distributional critic. In all cases we fixed the policy optimizer to MPO with an identical configuration. All three critics shared the exact same network architecture and hyperparameters: batch sizes, learning rates, etc. Note that this meant adding a 51-unit layer to the non-distributional critic for a fair comparison. Please refer to the released agent source code for the default hyperparameters used.

On 25 control suite environments the MoG critic achieves similar results on simpler tasks when compared to C51 as shown in Figure 2. However we observed much better performance for more difficult tasks, shown in Figure 3 and Figure 4. Note that use of C51 in this setting already provides a strong baseline as shown by Hoffman et al. [2020] under the name of DMPO. In comparison, the MoG variant overall achieves similar or better performance across the board, while not having the limitations of C51, e.g. requiring the knowledge of minimum and maximum Q-values. Additionally the exact same comparison with DDPG instead of MPO revealed the same trends shown in Figure 9, confirming that the gains are agnostic to the policy optimization approach.

5. Detailed analysis of the MoG distribution

In this section we wish to deconstruct and understand what about this distributional loss is providing such an improvement in performance over its non-distributional alternative. In order to study more closely the cross-entropy loss, in this section we consider a distributional critic consisting of a single Gaussian component $Z_\theta^{(s,a)} = N(\mu_\theta(s,a), \sigma_\theta^2(s,a))$. In this special case, the cross-entropy between the target distribution and the online network’s distributional estimate can be expressed analytically. In an effort to lighten the notation we will consider a single element of the minibatch of transitions $(s, a, r, s')$ and a single sampled action $a'$ with which we can write

\[\ell(\theta; s, a, r, s') \approx -\frac{1}{M} \sum_{i=1}^{M} \log \left( N(\mu_\theta(s,a), \sigma_\theta^2(s,a)) \right) \left( r + yz^{(i)} \right), \quad \text{where} \quad \begin{cases} z^{(i)} \sim N(\mu_\theta(s',a'), \sigma_\theta^2(s',a')), \\ a' \sim \pi(\cdot | s'). \end{cases}\]
Figure 2 | Benchmark runs on control suite tasks, showing median and quartiles across 10 seeds. Comparing MPO with a C51 (red) and a sample-based MoG (green) critic. On most tasks, the final performance of the MoG matches that of the state-of-the-art C51, which corresponds the DMPO from [Hoffman et al., 2020]. On the most challenging tasks in the humanoid and manipulator domains, the MoG critic provides a significant performance boost.
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Figure 3 | Benchmark runs on a selection of control suite tasks, showing median and quartiles across 10 seeds. Comparing MPO policy improvement with various policy evaluation approaches. The mixture of Gaussian evaluation trained with the sample-based method consistently outperforms the alternatives.

Figure 4 | Benchmark runs on more challenging control suite tasks, showing median and quartiles across 10 seeds. Comparing MPO policy improvement with various policy evaluation approaches. The mixture of Gaussian evaluation trained with the sample-based method consistently outperforms the alternatives.
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Figure 5 | Diagrams of how neural models are used in this work. (a) The original model outputting a single Gaussian. (b–d) Models used to test Hypotheses 1 to 3, respectively. Hypotheses 2 and 3 use a different loss for their $Q_\theta$ heads.

the distributional loss as

$$ \ell(\theta) = H \left[ r + \gamma Z_{\theta}^{(s',a')} - Z_{\theta}^{(s,a)} \right] \approx \frac{(\mu_\theta - \hat{\mu}_\theta)^2}{\sigma_{\theta}^2} + \frac{\hat{\sigma}_{\theta}^2}{\sigma_{\theta}^2} + \ln \sigma_{\theta}^2 $$

where we have dropped the dependence of the Gaussian components on the state-action pair inputs. We have also introduced $\hat{\mu}_\theta$ and $\hat{\sigma}_{\theta}^2$ as the transformed mean and variance of the target distribution. This is possible due to the fact that a linear transformation of a Gaussian remains Gaussian, however note that their exact values can be treated as constants (due to their independence of $\theta$). We can then further simplify the expression as

$$ \ell(\theta) = \frac{(Q_\theta - \hat{Q}_\theta)^2}{\sigma_{\theta}^2} + \frac{\hat{\sigma}_{\theta}^2}{\sigma_{\theta}^2} + \ln \sigma_{\theta}^2 $$

where we've made use of the fact that the first moments of both distributions correspond to their associated state-action values, e.g. $Q_\theta(s,a) = \mu_\theta$. Finally, we further decouple $Q_\theta$ into a final linear layer (with weights $w$) and a feature vector output by the penultimate layer, denoted $\phi_\theta(s,a)$, which yields

$$ \ell(\theta) = \frac{(w^T \phi_\theta(s,a) - \hat{Q}_\theta)^2}{\sigma_{\theta}^2} + \frac{\hat{\sigma}_{\theta}^2}{\sigma_{\theta}^2} + \ln \sigma_{\theta}^2 $$

Note that in all our experiments thus far, the feature map we refer to as $\phi_\theta(s,a)$ has been shared among all final heads, e.g., those that parameterized the locations, scales, and mixture logits of our MoG distributions. This final expression reveals the similarities and differences between the traditional squared Bellman error (SBE) and the distributional counterpart we consider in this report. The squared Bellman error, in the same notation reads:

$$ \ell_{\text{SBE}}(\theta) = \frac{(Q_\theta(s,a) - \hat{Q}_\theta(s,a))^2}{\sigma_{\theta}^2} = \frac{(\mu_\theta(s,a) - \hat{Q}_\theta(s,a))^2}{\sigma_{\theta}^2} = \left( w^T \phi_\theta(s,a) - \hat{Q}_\theta(s,a) \right)^2 $$

assuming a deterministic critic outputting only a prediction of the mean. Inspecting this expression leads us to three hypotheses as to what may be responsible for the improved performance. For each one that follows below we devise an alteration of the original distributional model to compare it to the non-distributional baseline. The three alterations are depicted along side the original model in Figure 5. Further, empirically we assess each of these hypotheses on a set of four challenging DM control suite tasks.

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\[ \ell(\theta) = \frac{(\mu_\theta - \hat{\mu}_\theta)^2}{\sigma_{\theta}^2} + \frac{\hat{\sigma}_{\theta}^2}{\sigma_{\theta}^2} + \ln \sigma_{\theta}^2 \]

\[ \ell(\theta) = \frac{(Q_\theta - \hat{Q}_\theta)^2}{\sigma_{\theta}^2} + \frac{\hat{\sigma}_{\theta}^2}{\sigma_{\theta}^2} + \ln \sigma_{\theta}^2 \]

\[ \ell(\theta) = \frac{(w^T \phi_\theta(s,a) - \hat{Q}_\theta)^2}{\sigma_{\theta}^2} + \frac{\hat{\sigma}_{\theta}^2}{\sigma_{\theta}^2} + \ln \sigma_{\theta}^2 \]
Figure 6 | Testing our three hypotheses for which elements of the distributional loss are contributing to the success of the sample-based distributional loss, showing median and quartiles across 5 seeds. Combining both the feature learning and the Mahalanobis reweighting recovers the performance of the fully distributional agent.

**Hypothesis 1:** Adaptive Mahalanobis reweighting of losses at each $s \cdot a$ is beneficial. This hypothesis concerns the question of whether the distributional network is able to focus on those transitions that it is relatively certain about because smaller values of $\sigma_\theta$ associates an increased weight (compared to the non-distributional SBE) with those transitions? To test this hypothesis we consider an alteration to the model that uses same loss with entirely independent architectures for $\mu_\theta$ and $\sigma_\theta$, thus controlling for any shared representation learning in order to assess the added value of Mahalanobis reweighting alone.

**Hypothesis 2:** The additional auxiliary loss leads to better representation learning. Since all but the final mean and variance heads of our network are shared between the two pathways, the question is: are the additional $\sigma_\theta$ terms in the loss serving as a grounded auxiliary loss from which to learn better features? To test this hypothesis we create a copy of the $\mu_\theta$ head, denoted $Q_\theta$ in Figure 5 (c), which is trained without the Mahalanobis reweighting and with a stop-gradient preventing backpropagation from this new head to influence representation learning from the joint torso. Importantly, we then use the output of the $Q_\theta$ head for policy optimization; this ensures that the Mahalanobis reweighting only influences the torso’s feature mapping and not the $Q$-learning used for policy optimization.

**Hypothesis 3:** Both are necessary to recover the full benefit of the distributional loss. Finally, is the improved performance of the studied distributional loss due to both the adaptive reweighting and the improved feature learning? To test this final hypothesis we take the previous model and scale the $Q_\theta$ head’s loss by $\sigma_\theta$ without propagating any additional gradients down the torso. Once again, the only signal training the torso’s feature mapping is the studied distributional loss, and the only signal training the policy is $Q_\theta$, which is now able to adaptively reweight $(s, a)$ examples during training.

Our results, presented in Figure 6, show quite convincingly that both the effect on feature learning and the adaptive reweighting of instances are responsible for the improved performance, as evidenced by the blue Hypothesis 3 curve recovering the performance of the single Gaussian baseline in black. In contrast, while the feature learning alone (green) is helpful in three out of four tasks, the reweighting alone (red) can have a catastrophic effect on learning performance.
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6. Sensitivity analysis

6.1. Hyperparameter Sensitivity

The mixture-of-Gaussians parameterization of the distributional critic studied in this report does introduce new hyperparameters of its own. While we do not need to specify the $v_{min}/v_{max}$ and $num_{atoms}$ hyperparameters as in the categorical parameterization, we do specify the number, initial scale, and initial location of the mixture components. In all experiments we set the initial locations of all components to the origin, however we did carry out a sensitivity analysis on the number and initial scale of the Gaussian components. While performance seemed to be quite robust to the number of components as seen in Figure 7, it did seem to be somewhat sensitive to the choice of initial scale on the manipulator tasks. Indeed, Figure 8 shows performance on manipulator favouring small values of initial scale over those that are closer to unity.

6.2. Generalization to other policy optimization methods

Although this work focuses on the MPO algorithm for policy optimization, our results also suggest that the benefits discussed in this report generalize to other approaches that make use of a critic (e.g. DDPG). Recall that DDPG with a C51 distributional critic is the very competitive agent known as D4PG [Barth-Maron et al., 2018].

7. Discussion and future work

We have shown that fitting more distributional moments has lead to better performance; and our analysis strongly suggests that this enhancement is due to better representation learning. The question remains as to what makes this representation better. Could it be that the losses due to the additional moment has a regularizing effect on feature learning, making it more robust to time-varying distributions of returns and/or state-action pairs?

Our cross-entropy loss has been shown not to be a norm under which our distributional Bellman operator is a contraction. While this was a simple first attempt with positive empirical results, we would like to investigate ways of deriving a similarly simple approach from a loss that would guarantee contraction. For example, in the single Gaussian case, we could try the 2-Wasserstein distance, or explore the Cramér distance [Bellemare et al., 2017b; Rowland et al., 2018] as a general solution.

Figure 7 | Sensitivity analysis of the number of Gaussian mixture components on the selected suite of challenging DeepMind control suite tasks. Solid curves are means over three seeds. Performance is quite robust to the number of components, though it seems more than one are needed to reach the state-of-the-art on the humanoid tasks.

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Figure 8 | Sensitivity analysis of the initial scale of Gaussian mixture components on the selected suite of challenging DeepMind control suite tasks. The number of components was set to the default of 5 for this comparison. Solid curves are means over three seeds. Performance is relatively robust to the initial scale of components, though on manipulator tasks it seems important to initialize the scales to a very small number.

Figure 9 | Benchmark runs on a selection of control suite tasks. Comparing DDPG policy improvement with various policy evaluation approaches; showing median and quartiles over 5 seeds. This figure shows that our findings generalize to other policy improvement algorithms. Note that the C51 learning curve corresponds exactly to the D4PG agent.
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