Light Stop and the $b \to s\gamma$ Process

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Abstract

A constraint from $b \to s\gamma$ process to the minimal supersymmetric standard model is derived in the light stop region where the coupling between the lighter stop and the $Z^0$ boson vanishes due to the left-right mixing of the stop states. It is pointed out that although some region in the parameter space is excluded from this process there remains a large parameter space where the amplitude of the $b \to s\gamma$ is suppressed due to partial cancellation between different diagrams. Stop as light as 20 GeV is still viable from the $b \to s\gamma$ constraint.
Although supersymmetry (SUSY) has attracted much attention as a promising candidate of physics beyond the standard model of the elementary particle physics we do not have any direct evidence of superpartners which are predicted to exist in the supersymmetric theory. It is therefore very important to search for any direct and indirect information on superpartners in order to test the idea of supersymmetry.

In this respect the flavor changing neutral current (FCNC) process deserves a special attention. In the supersymmetric standard model the FCNC is induced by loop effects just as in the case of the standard model. The origin of the flavor mixing is present not only in the quark mass matrices but also in those of its superpartner, i.e. squark. Therefore, FCNC process is sensitive to the masses and mixings of the squark sector. In fact it is well known that the squark masses are severely constrained from the $K^0 - \bar{K}^0$ mixing for general class of supersymmetric standard models at least for the first two generations\cite{1}. This is one of motivations to consider so called minimal supergravity model where the constraint from the $K^0 - \bar{K}^0$ mixing can be avoided because all the scalars have a common SUSY soft breaking mass at the GUT scale\cite{2, 3}.

Another important FCNC process in the supersymmetric model is the $b \to s\gamma$ process. It is well known that in the standard model the inclusive branching ratio of this process is about $3 \times 10^{-4}$ after taking account of the QCD correction\cite{4, 5}. This is compared with the recent improved upper bound $5.4 \times 10^{-4}$ from the CLEO collaboration\cite{6}. In the supersymmetric theory we have four new contributions due to loops of (i) charged Higgs and up-type quark, (ii) chargino and up-type squark, (iii) gluino and down-type squark, (iv) neutralino and down-type squark. In the context of the minimal supergravity model it is known that the charged Higgs contribution as well as the chargino and gluino contribution is sizable and, for some choice of
parameters, can be comparable to the standard model contribution although the neutralino loop contribution (iv) is very small [7]-[12].

In this paper we study contribution to the $b \to s\gamma$ process from the light stop loop effect in the context of the minimal supersymmetric standard model. Since the stop sector can have a sizable left-right mixing it is possible that one of the two stop states becomes much lighter than other squarks. In fact it was pointed out in Ref. [13] that a stop lighter than 45 GeV is still allowed since we can arrange the squark mixing so that the lighter stop does not couple to the $Z^0$ boson. In such case the experimental lower bound of the stop mass is given from the TRISTAN experiments. The bound is about 27 GeV provided that the photino mass is not close to the stop mass [15]. It is also possible that such a light stop can escape detection at the Fermilab Tevatron as discussed in Ref. [16]. If we assume that the stop exists in such a low mass region we can expect that the $b \to s\gamma$ amplitude becomes in principle very large because the stop and chargino loop contribution can dominate this process, therefore the improved measurement can put a useful constraint to the relevant parameter space. This is especially important if we assume the GUT relations among the gaugino masses, since not only one of the stops but also at least one of the charginos is necessarily light. We investigate the contribution from (ii) in the light stop region taking account of the experimental constraints in the gaugino and higgsino parameter space. We will see that although some parameter space is excluded from the $b \to s\gamma$ measurement there still remain an region where a stop as light as 20 GeV is not constrained. This is due to a partial cancellation between two graphs involving the top Yukawa and the SU(2) gauge coupling constants.

We consider here the usual minimal supersymmetric standard model with three families of quarks and leptons and their supersymmetric partners. For the parameters of the gaugino and higgsino sector we assume the GUT con-
dition for the Majorana mass terms of the gauginos, then three gaugino mass parameters are related to each other in terms of gauge coupling constants. For FCNC processes the squark mass matrices are especially important. We take here that the following simplified form for the up-type squark mass matrix \[8\]:

\[
L_{u-squark} = (\tilde{u}_L \tilde{u}_R)^* \begin{pmatrix} V_L(u) & 0 \\ 0 & V_R(u) \end{pmatrix} \begin{pmatrix} m_{u_L}^2 \\ m_{c_L}^2 \\ m_{t_L}^2 \\ m_{u_R}^2 \\ m_{c_R}^2 \\ m_{t_R}^2 \end{pmatrix} \begin{pmatrix} V_L^\dagger(u) & 0 \\ 0 & V_R^\dagger(u) \end{pmatrix} \begin{pmatrix} \tilde{u}_L \\ \tilde{u}_R \end{pmatrix},
\]

(1)

where \(V_L(u)\) and \(V_R(u)\) are up-type quark mixing matrices, i.e.,

\[
V_R^\dagger(u) \cdot m_u \cdot V_L(u) = \text{diagonal.}
\]

In this form we have assumed that, apart from the left-right mixing term for the third generation, the up-squark mass matrix is diagonal in the basis where the corresponding quark is diagonal. If we solve the renormalization group equations of the squark mass matrices with a universal scalar mass at the GUT scale the above form is obtained as an approximate solution when one neglects the small off-diagonal components for the first and the second generations \[3\]. In the super GIM basis where \(\tilde{u}_L' = V_L^\dagger(u)\tilde{u}_L, \tilde{u}_R' = V_R^\dagger(u)\tilde{u}_R\)
only the stop sector has a mixing,
\[ L_{\text{stop}} = (\tilde{t}_L \quad \tilde{t}_R) \left( \begin{array}{cc} m_{t_L}^2 & a m_t \\ a^* m_t & m_{t_R}^2 \end{array} \right) \left( \begin{array}{c} \tilde{t}_L' \\ \tilde{t}_R' \end{array} \right), \]  
then the light and heavy stop eigenstates are given by
\[ \tilde{t}_1 = \cos \theta_t \tilde{t}_L' - \sin \theta_t \tilde{t}_R', \]
\[ \tilde{t}_2 = \sin \theta_t \tilde{t}_L' + \cos \theta_t \tilde{t}_R'. \]

Here we have assumed that the coefficient \( a \) is real for simplicity. Since the coupling of the light stop \( \tilde{t}_1 \) to the \( Z^0 \) boson is proportional to
\[ \frac{1}{2} \cos^2 \theta_t - \frac{2}{3} \sin^2 \theta_W, \]
this coupling vanishes when \( | \cos \theta_t | \sim 0.55 \). In this region even the stop lighter than 45 GeV is still allowed. Hereafter we will concentrate our consideration to this case. Notice that there are two distinct possibilities in which the above coupling vanishes, i.e. \( \tan \theta_t \) is positive or negative. This ambiguity comes from the two choices of the sign in the off-diagonal term in Eq.(3).

The calculation of \( b \to s \gamma \) branching ratio in the standard model is given in Ref. [4, 5] and the SUSY contributions to this process are described in Ref. [7]. In the standard model we first determine the weak effective Hamiltonian at the weak scale.
\[ H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) O_i, \]  
where the operators \( O_i \)'s are shown in Ref. [4] and \( V_{ij} \) is the \( i,j \) component of the Kobayashi-Maskawa matrix. Especially, the \( O_7 \) is a magnetic moment operator given by
\[ O_7 = \frac{e}{16\pi^2} m_b s_L \sigma^{\mu\nu} b_R F_{\mu\nu}, \]
and $O_8$ is a gluonic operator,

$$O_8 = \frac{g}{16\pi^2} m_b \bar{s}L\sigma^{\mu\nu} b_R G_{\mu\nu},$$

where $g$ is a strong coupling constant. The coefficients $C_7(m_W)$ and $C_8(m_W)$ are given from one loop calculation of $b \to s\gamma$ diagrams. The $b \to s\gamma$ amplitude is determined from the effective Hamiltonian at $\mu = m_b$ scale, and the QCD correction is taken into account by solving the renormalization group equations for the coefficient functions $C_i(\mu)$ from the $m_W$ scale to the $m_b$ scale. The inclusive branching ratio of $b \to s\gamma$ is then given by

$$Br(b \to s\gamma) = \frac{\Gamma(b \to s\gamma)}{\Gamma(b \to ce\bar{\nu})} Br(b \to ce\bar{\nu}),$$

where $\Gamma(b \to s\gamma)$ is determined from the effective Hamiltonian:

$$\Gamma(b \to s\gamma) = \frac{m_b^5}{64\pi^2} \frac{\alpha_W^2 \alpha}{m_W^2} |V_{ts}^* V_{tb}|^2 |C_{7\text{eff}}(m_b)|^2,$$

where $C_{7\text{eff}}(m_b) = C_7(m_b) - \frac{1}{3}(C_5(m_b) + 3C_6(m_b))$ and $\Gamma(b \to ce\bar{\nu})$ is the semi-leptonic decay width of the b quark. The SUSY and other short distance effects give extra contributions to the $C_7(m_W)$. The contribution from the chargino and stop loop depends on the masses and mixings of the up-type squark sector given by Eq. (1) and the parameters of the chargino mass matrix, i.e. the SU(2) gaugino mass parameter $M_2$, the higgsino mass parameter $\mu$, and the vacuum angle $\tan \beta$. In the actual calculation, we have taken into account the 8x8 anomalous dimension matrix of Ref. [5] and the QCD $\Lambda$ parameter $\Lambda_{LLA}^{f=5} = 100\text{MeV}$ and $m_b = 4.5\text{GeV}$. We calculated the amplitude in the parameter space of $M_2$ and $\mu$ for different values of the stop mass.

\footnote{Strictly speaking, the initial condition of the coefficients is given at the scale of the particle’s mass inside the loop, not at the $m_W$ scale. However, the correction is small since we are interested in the case in which these masses are not very different from $m_W$.}
In Fig. 1 we show the ratio of the $b \to s\gamma$ amplitude of the chargino contribution to that of the standard model contribution. For the standard model contribution we take the top quark mass to be 150 GeV. The light stop mass is taken to be 40 GeV and the stop mixing angle is fixed in such a way that the stop and $Z^0$ coupling vanishes and $\tan \beta = 2$. The Fig. 1 (a) and (b) correspond to two choices of $\tan \theta_t$. The heavy stop mass and other up-type squark masses are taken to be 200 GeV. Since the amplitude is dominated by the lightest stop loop the result does not depend strongly on the choice of the other squarks’ masses. In the figures we have shown the excluded region in the $\mu - M_2$ space from the chargino and neutralino search experiments at LEP\[14\]. The conditions which we take here to determine the excluded region (I) are that the chargino mass is less than 45 GeV, the invisible width of the $Z^0$ boson to the lightest neutralino ($\chi$) pair is more than 22MeV, and the branching ratio of $Z^0 \to \chi\chi'$ and $Z^0 \to \chi'\chi'$ is larger than $5 \times 10^{-5}$ where $\chi'$ represents the neutralino other than the lightest one. Also we have excluded the region (II) where the lightest stop is lighter than the lightest neutralino which is assumed to be the lightest supersymmetric particle (LSP).

We see that although the amplitude becomes large in some parameter region there remains a large parameter space where the ratio is within $\pm 25\%$ of the standard model amplitude especially for the choice of $\tan \theta_t > 0$. In fact, in order to just satisfy the present CLEO’s bound ($5.4 \times 10^{-4}$) the amplitude from the chargino graph can be $-2.4 \sim 0.4$ times the standard model amplitude, therefore most of the parameter space in the figures is allowed. The situation would change if the experiment gives a finite value to the branching ratio since we can now exclude a large parameter space where the standard model and chargino contribution cancel each other. Even in that case, however, no strong constraint is obtained if the $\mu$ and $M_2$ are in the region where the amplitude is the same as that of the standard model within $\pm 25\%$.
This is true even if we take the stop mass as small as 20 GeV as shown in Fig. 2. Here we take \( \tan \beta = 1.8 \) and \( \tan \theta_t > 0 \). In this case the phenomenological allowed region is confined to the negative \( \mu \) region where the chargino contribution to the \( b \to s\gamma \) amplitude is also suppressed and the ratio is about -0.25 in the most of the parameter space shown here. For \( \tan \theta_t < 0 \) this ratio is +0.25 \( \sim +0.5 \) in the same region which corresponds to a branching ratio 1.5 \( \sim 2.2 \) times as large as that of the standard model.\(^2\)

We can understand this property by looking at the light stop contribution to \( C_7(m_W) \),

\[
C_7 = \frac{m_W^2}{m_{\tilde{t}_1}^2} \sum_j \left\{ |V_{j1}\cos \theta_t + V_{j2}\tilde{z}_t\sin \theta_t| f^{(1)}(x_{\chi_j\tilde{t}_1}) - (V_{j1}\cos \theta_t + V_{j2}\tilde{z}_t\sin \theta_t) U_{j2}\cos \theta_t z_{\chi_j} f^{(2)}(x_{\chi_j\tilde{t}_1}) \right\}, \tag{11}
\]

where

\[
f^{(1)}(x_{\chi_j\tilde{t}_1}) = F_1(x_{\chi_j\tilde{t}_1}) + \frac{2}{3} F_2(x_{\chi_j\tilde{t}_1}), \tag{12}\]
\[
f^{(2)}(x_{\chi_j\tilde{t}_1}) = F_3(x_{\chi_j\tilde{t}_1}) + \frac{2}{3} F_4(x_{\chi_j\tilde{t}_1}), \tag{13}\]
\[
z_t = \frac{m_t}{\sqrt{2} m_W \sin \beta}, \quad z_{\chi_j} = \frac{m_{\chi_j}}{\sqrt{2} m_W \cos \beta}, \tag{14}\]

\[
F_1(x) = \frac{1}{12(x-1)^3} (x^3 - 6x^2 + 3x + 2 + 6x \log x), \tag{15}\]
\[
F_2(x) = \frac{1}{12(x-1)^3} (2x^3 + 3x^2 - 6x + 1 - 6x^2 \log x), \tag{16}\]
\[
F_3(x) = \frac{1}{2(x-1)^3} (x^2 - 4x + 3 + 2 \log x), \tag{17}\]

\(^2\)If the stop is as light as 20 GeV, the TRISTAN experiment becomes important. However, the sensitivity of the stop search in the single photon annihilation process is lost if the mass difference between the stop and the LSP neutralino is within a few GeV.\[^{[15]}\] The author thanks R. Enomoto for explaining this point.
\[ F_4(x) = \frac{1}{2(x-1)^3}(x^2 - 1 - 2x \log x), \]  
(18) 

and \( U \) and \( V \) are matrices which diagonalize the chargino sector.

\[ U^\dagger M_C V = \begin{pmatrix} m_{\chi_1} & 0 \\ 0 & m_{\chi_2} \end{pmatrix}, \]  
(19) 

\[ M_C = \begin{pmatrix} M_2 & \sqrt{2}M_W \sin \beta \\ \sqrt{2}M_W \cos \beta & \mu \end{pmatrix}. \]  
(20) 

We have used an abbreviation \( x_{\chi_ji} = \frac{m_{\chi_j}^2}{m_{\tilde{t}_i}^2} \). Numerically, \( f^{(2)} \) is much larger than \( f^{(1)} \), therefore the second term in Eq. (11) is more important. The two terms in the second term, i.e. \( V_{j1} \cos \theta_t \) and \( V_{j2} \sin \theta_t \) correspond to two diagrams which depend on the SU(2) gauge coupling constant and the top Yukawa coupling constant respectively as shown in Fig. 3. When \( \tan \theta_t > 0 \) the two contributions tend to cancel each other, then the stop contribution is suppressed. In fact a complete cancellation occurs along a line which passes near the origin of the \( \mu - M_2 \) space. On the other hand when \( \tan \theta_t < 0 \) we don’t see such cancellation and the amplitude is large in most of the parameter space.

We have calculated the \( b \to s\gamma \) amplitude in the light stop region where the coupling between the \( Z^0 \) and the light stop is suppressed. We see that the contribution to the \( b \to s\gamma \) process is also suppressed in a large parameter region for one choice of \( \tan \theta_t \) and therefore the light stop as light as 20 GeV is still allowed from the \( b \to s\gamma \) constraint. It is known that in the SUSY model other contributions such as the charged Higgs or gluino loop gives sizable effects to the amplitude. Since the chargino and gluino contributions can have either sign for the amplitude relative to the standard model and the charged Higgs contribution, it is possible that different SUSY contributions cancel each other and in such cases relatively light masses for the
charged Higgs and/or SUSY particles are allowed. What is remarkable in the light stop case is, however, that the chargino contribution itself is suppressed without relying on any cancellation with other SUSY contributions.

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Figure Captions

Fig.1  The chargino loop contribution to the $b \to s\gamma$ amplitude normalized to the standard model amplitude in the $M_2 - \mu$ space. The numbers in x and y axes are in GeV. The light stop mass is taken to be 40 GeV and other squarks' masses are 200 GeV and $\tan \beta = 2$. (a) and (b) correspond to the stop mixing angle $\tan \theta_t = 1.51$ and $\tan \theta_t = -1.51$ respectively. I is the region excluded from the chargino and neutralino search at LEP and II is the region where the stop becomes lighter than the lightest neutralino.

Fig.2  The same contour plot as Fig. 1 in the case that the light stop is 20 GeV and $\tan \beta=1.8$ and $\tan \theta_t = 1.51$. Other parameters and the meanings of the excluded region I, II are the same as in Fig. 1.

Fig.3  Two diagrams which contribute to the $b \to s\gamma$ amplitude. $y_b$, $y_t$, and $g_2$ represent the bottom Yukawa, top Yukawa and SU(2) gauge coupling constants respectively. The photon ($\gamma$) line can be attached either to the stop ($\tilde{t}_1$) or the chargino ($\chi^-$) line.
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