TOPOLOGIES AND APPROXIMATION OPERATORS INDUCED BY BINARY RELATIONS

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Abstract. Rough set theory is an important mathematical tool for dealing with uncertain or vague information. This paper studies some new topologies induced by a binary relation on universe with respect to neighborhood operators. Moreover, the relations among them are studied. In addition, lower and upper approximations of rough sets using the binary relation with respect to neighborhood operators are studied and examples are given.

1. Introduction

Rough set theory was introduced by Pawlak as a mathematical tool to process information with uncertainty and vagueness [6]. The rough set theory deals with the approximation of sets for classification of objects through equivalence relations. Important applications of the rough set theory have been applied in many fields, for example in medical science, data analysis, knowledge discovery in database [7, 8, 11].

The original rough set theory is based on equivalence relations, but for practical use, needs to some extensions on original rough set concept. This is to replace the equivalent relation by a general binary relation [13, 14, 11]. Topology is one of the most important subjects in mathematics. Many authors studied relationship between rough sets and topologies based on binary relations [9, 13, 14]. In this paper, we proposed and studied connections between topologies generated using successor, predecessor, successor-and-predecessor, and successor-or-predecessor neighborhood operators as a subbase by various binary relations on a universe, respectively. In addition to this, we investigate connection between lower and upper approximation operators using successor, predecessor, successor-and-predecessor, and successor-or-predecessor neighborhood operators by various binary relations on a universe, respectively. Moreover, we give several examples for a better understanding of the subject.

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2. Preliminaries

In this section, we shall briefly review basic concepts and relational propositions of the relation based rough sets and topology. For more details, we refer to \[13, 14, 9\].

2.1. Basic properties relation based rough approximations and neighborhood operators.

In this paper, we always assume that $U$ is a finite universe, i.e., a non-empty finite set of objects, $R$ is a binary relation on $U$, i.e., a subset of $U^2 = U \times U$. $R$ is serial if for each $x \in U$, there exists $y \in U$ such that $(x, y) \in R$; $R$ is inverse serial if for each $x \in U$, there exists $y \in U$ such that $(y, x) \in R$; $R$ is reflexive if for each $x \in U$, $(x, x) \in R$; $R$ is symmetric if for all $x, y \in U$, $(x, y) \in R$ implies $(y, x) \in R$; $R$ is transitive if for all $x, y, z \in U$, $(x, y) \in R$ and $(y, z) \in R$ imply $(x, z) \in R$ [9].

$R$ is called a pre-order (relation) if $R$ is both reflexive and transitive; $R$ is called a similarity (or, tolerance) relation if $R$ is both reflexive and symmetric; $R$ is called an equivalence relation if $R$ is reflexive, symmetric and transitive [9].

Given a universe $U$ and a binary relation $R$ on $U$, $x, y \in U$, the sets

$$R_s(x) = \{ y \in U | (x, y) \in R \},$$
$$R_p(x) = \{ y \in U | (y, x) \in R \},$$
$$R_{s \land p}(x) = \{ y \in U | (x, y) \in R \text{ and } (y, x) \in R \} = R_s(x) \cap R_p(x),$$
$$R_{s \lor p}(x) = \{ y \in U | (x, y) \in R \text{ or } (y, x) \in R \} = R_s(x) \cup R_p(x)$$

are called the successor, predecessor, successor-and-predecessor, and successor-or-predecessor neighborhood of $x$, respectively, and the following four set-valued operators from $U$ to the power set $P(U)$

$$R_s : x \mapsto R_s(x),$$
$$R_p : x \mapsto R_p(x),$$
$$R_{s \land p} : x \mapsto R_{s \land p}(x),$$
$$R_{s \lor p} : x \mapsto R_{s \lor p}(x)$$

are called the successor, predecessor, successor-and-predecessor, and successor-or-predecessor neighborhood operators, respectively. Relationships between these neighborhood systems can be expressed as:

$$R_{s \land p}(x) \subseteq R_s(x) \subseteq R_{s \lor p}(x),$$
$$R_{s \land p}(x) \subseteq R_p(x) \subseteq R_{s \lor p}(x)$$

[14, 9].

Definition 1. [14] Let $R$ be a binary relation on $U$. The ordered pair $(U, R)$ is called a (generalized) approximation space based on the relation $R$. For $X \subseteq U$, the lower and upper approximations of $X$ with respect to $R_s(x), R_p(x), R_{s \land p}(x)$,
The concepts of topologies and approximation operators induced by binary relations are given.

\[ R_s(x) \text{ and } R_p(x) \] are respectively defined as follows:

\[
\text{apr}_{R_s}(X) = \{ x \in U | R_s(x) \subseteq X \},
\]

\[
\text{apr}_{R_p}(X) = \{ x \in U | R_p(x) \subseteq X \},
\]

\[
\text{apr}_{R_{s\lor p}}(X) = \{ x \in U | R_{s\lor p}(x) \subseteq X \},
\]

\[
\text{apr}_{R_{s\land p}}(X) = \{ x \in U | R_{s\land p}(x) \subseteq X \},
\]

\[
\text{apr}_{R_{s\lor p}}(X) = \{ x \in U | R_{s\lor p}(x) \subseteq X \},
\]

\[
\text{apr}_{R_{s\land p}}(X) = \{ x \in U | R_{s\land p}(x) \subseteq X \},
\]

\[
\text{apr}_{R_{s\lor p}}(X) = \{ x \in U | R_{s\lor p}(x) \subseteq X \},
\]

\[
\text{apr}_{R_{s\land p}}(X) = \{ x \in U | R_{s\land p}(x) \subseteq X \}.
\]

In Pawlak’s classical rough set theory for lower and upper approximations operators, an equivalence relation \( R \) is used. In this case, four neighborhood operators become the same, i.e.,

\[ R_s(x) = R_p(x) = R_{s\land p}(x) = R_{s\lor p}(x) = [x]_R, \]

where \([x]_R \) is the equivalence class containing \( x \).

**Proposition 2.** [13] For an arbitrary neighborhood operator in an approximation space \((U, R)\), the pair of approximation operators satisfy the following properties:

\[
(L0) \quad \text{apr}(X) = (\text{apr}(X^c))^c,
\]

\[
(U0) \quad \text{appr}(X) = (\text{apr}(X^c))^c,
\]

\[
(L1) \quad \text{apr}(U) = U,
\]

\[
(U1) \quad \text{apr}(\emptyset) = \emptyset,
\]

\[
(L2) \quad \text{apr}(X \cap Y) = \text{apr}(X) \cap \text{apr}(Y),
\]

\[
(U2) \quad \text{appr}(X \cup Y) = \text{appr}(X) \cup \text{appr}(Y).
\]

where \( X^c \) is the complement of \( X \) with respect to \( U \).

Moreover, if \( R \) is reflexive, then

\[
(L3) \quad \text{apr}(X) \subseteq X,
\]

\[
(U3) \quad X \subseteq \text{appr}(X).
\]

If \( R \) is symmetric, then

\[
(L4) \quad X \subseteq \text{apr}(\text{appr}(X)),
\]

\[
(U4) \quad \text{appr}(\text{appr}(X)) \subseteq X.
\]

If \( R \) is transitive, then

\[
(L5) \quad \text{apr}(X) \subseteq \text{apr}(\text{appr}(X)),
\]

\[
(U5) \quad \text{appr}(\text{appr}(X)) \subseteq \text{appr}(X).
\]

**2.2. The concept of a topological space.**

In this section, we give some basic information about the topology [2, 9].

**Definition 3.** [2] A topological space is a pair \((U, T)\) consisting of a set \( U \) and a set \( T \) of subsets of \( U \) (called "open sets"), such that the following axioms hold:
(A1) Any union of open sets is open.
(A2) The intersection of any two open sets is open.
(A3) \emptyset and \( U \) are open.

The pair \((U, T)\) speaks simply of a topological space \( U \).

**Definition 4.** [9] Let \( U \) be a topological space.

1. \( X \subseteq U \) is called closed when \( X^c \) is open.
2. \( X \subseteq U \) is called a neighborhood of \( x \in X \) if there is an open set \( V \) with \( x \in V \subseteq X \).
3. A point \( x \) of a set \( X \) is an interior point of \( X \) if \( X \) is a neighborhood of \( x \), and the set of all interior points of \( X \) is called the interior of \( X \). The interior of \( X \) is denoted by \( \overset{o}{X} \).
4. The closure of a subset \( X \) of a topological space \( U \) is the intersection of the family of all closed sets containing \( X \). The closure of \( X \) is denoted by \( \overset{\bar{}}{X} \).

In topological space \( U \), the operator

\[
o : P(U) \to P(U), \quad X \mapsto \overset{o}{X}
\]

is an interior operator on \( U \) and for all \( X, Y \subseteq U \) the following properties hold:

I1) \( \overset{o}{U} = U \),
I2) \( \overset{o}{X} \subseteq X \),
I3) \( \left( \overset{o}{X} \right)^c = X \),
I4) \( \left( X \cap Y \right)^o = \left( \overset{o}{X} \right) \cap \left( \overset{o}{Y} \right) \).

In topological space \( U \), the operator

\[
\overline{\cdot} : P(U) \to P(U), \quad X \mapsto \overset{\bar{}}{X}
\]

is a closure operator on \( U \) and for all \( X, Y \subseteq U \) the following properties hold:

C1) \( \overset{\bar{}}{\emptyset} = \emptyset \),
C2) \( X \subseteq \overset{\bar{}}{X} \),
C3) \( \overset{\bar{}}{X^c} = X \),
C4) \( \overset{\bar{}}{X} \cup \overset{\bar{}}{Y} = \overset{\bar{}}{X \cup Y} \).

In a topological space \((U, T)\) a family \( \mathcal{B} \subseteq T \) of sets is called a base for the topology \( T \) if for each point \( x \) of the space, and each neighborhood \( X \) of \( x \), there is a member \( V \) of \( \mathcal{B} \) such that \( x \in V \subseteq X \). We know that a subfamily \( \mathcal{B} \) of a topology \( T \) is a base for \( T \) if and only if each member of \( T \) is the union of members of \( \mathcal{B} \). Moreover, \( \mathcal{B} \subseteq P(U) \) forms a base for some topologies on \( U \) if and only if \( \mathcal{B} \) satisfies the following conditions:

B1) \( U = \bigcup \{ B | B \in \mathcal{B} \} \),
B2) For every two members \( X \) and \( Y \) of \( \mathcal{B} \) and each point \( x \in X \cap Y \), there is \( Z \in \mathcal{B} \) such that \( x \in Z \subseteq X \cap Y \).

Also, a family \( \mathcal{S} \subseteq T \) of sets is a subbase for the topology \( T \) if the family of all finite intersections of members of \( \mathcal{S} \) is a base for \( T \). Moreover, \( \mathcal{S} \subseteq P(U) \) is a subbase for some topology on \( U \) if and only if \( \mathcal{S} \) satisfies the following condition:
3. Correspondence between generating topologies by relations

In this section, we investigate connections between topologies generated using successor, predecessor, successor-and-predecessor, and successor-or-predecessor neighborhood operators as a subbase by various binary relations on a universe, respectively.

Let $R$ be a binary relation on a given universe $U$. Sets

\[
S_i = \bigcup \{ R_i(x) | x \in U \}, \text{ where } i : s, p, s \wedge p \text{ and } s \vee p
\]

defining by successor, predecessor, successor-and-predecessor, and successor-or-predecessor neighborhood operators, respectively. If $S_i$, where $i : s, p, s \wedge p$ and $s \vee p$, forms a subbase for some topology on $U$, then the topology generated $S_i$, where $i : s, p, s \wedge p$ and $s \vee p$, denoted $T_i$, where $i : s, p, s \wedge p$ and $s \vee p$, respectively.

A basic problem is: when does $S_i$, where $i : s, p, s \wedge p$ and $s \vee p$, form a subbase for some topologies on $U$?

Our aim is to solve this problem completely. This problem was solved by the authors in [9] using the family $S_s$ forms a subbase for some topology on $U$ by the following theorem.

**Theorem 5.** [9] If $R$ is a binary relation on $U$, then $S_s$ forms a subbase for some topologies on $U$ if and only if $R$ is inverse serial.

**Remark 6.** It is clear that if $R$ is inverse serial, then

\[
U = \bigcup_{x \in U} R_s(x).
\]

This is the condition (S0). Moreover, the family $S_s$ is covering of $U$.

**Theorem 7.** If $R$ is a binary relation on $U$, then $S_p$ forms a subbase for some topologies on $U$ if and only if $R$ is serial.

**Proof.** If $R$ is serial, then

\[
U = \bigcup_{x \in U} R_p(x)
\]

and the family $S_p$ provides the condition (S0).

**Theorem 8.** If $R$ is a binary relation on $U$, then $S_{s \wedge p}$ forms a subbase for some topologies on $U$ if and only if $R$ is symmetric and serial or inverse serial.

**Proof.** Similar to Theorem 7.

**Theorem 9.** If $R$ is a binary relation on $U$, then $S_{s \vee p}$ forms a subbase for some topologies on $U$ if and only if $R$ is serial or inverse serial.

**Proof.** If $R$ is serial or inverse serial, then

\[
U = \bigcup_{x \in U} R_{s \vee p}(x)
\]

and the family $S_{s \vee p}$ provides the condition (S0).

Let $S_1$ and $S_2$ be covering of $U$. A partition $S_1$ is a finer than $S_2$, or is coarser than $S_1$, for each neighborhood operator in $S_1$ produced by $x$, is subset the neighborhood operator in $S_2$ by $x$. This relation is denoted as $S_1 \leq S_2$.

$S_1 \leq S_2 \iff$ if every set of $S_1$ is contained in some sets of $S_2$, for all $x \in U$. 

Proposition 10. Let $U$ be the universe and $R$ is general binary relation. Then follows as equivalent:
1) $R_i(x) \subseteq R_j(x)$,
2) $S_i \leq S_j$, for all $x \in U$, where $i, j : s, p, s \land p$ and $s \lor p$.

Proposition 11. Let $U$ be the universe and $R$ is a serial relation. Then the following conditions are provided:
1) $S_p \leq S_{s \lor p}$,
2) $T_p \leq T_{s \lor p}$.

Proof. (1) is clear from Proposition 10. 
(2) From Theorem 7 and Theorem 9, $S_p (S_{s \lor p})$ forms a subbase for $T_p (T_{s \lor p})$ topology on $U$, respectively. □

Proposition 12. Let $U$ be the universe and $R$ is a inverse serial relation. Then the following conditions are provided:
1) $S_s \leq S_{s \lor p}$,
2) $T_s \leq T_{s \lor p}$.

Proposition 13. Let $U$ be the universe and $R$ is a symmetric relation. Then $R$ is a serial relation $\iff$ $R$ is a inverse serial relation.

Proof. Suppose that $R$ is a symmetric relation.

$R$ is a serial relation $\iff \forall x \exists y [(x, y) \in R]$ 
$\iff \forall x \exists y [(y, x) \in R]$ 
$\iff R$ is a inverse serial relation. □

Proposition 14. Let $U$ be the universe and $R$ is a symmetric and a serial (or inverse serial) relation. Then the following conditions are provided:
1) $S_{s \land p} = S_s = S_p = S_{s \lor p}$,
2) $T_{s \land p} = T_s = T_p = T_{s \lor p}$.

Corollary 15. Let $U$ be the universe and $R$ is a tolerance (symmetric and reflexive) relation. Then, the following conditions are provided:
1) $S_{s \land p} = S_s = S_p = S_{s \lor p}$,
2) $T_{s \land p} = T_s = T_p = T_{s \lor p}$.

Proposition 16. Let $U$ be the universe and $R$ is a reflexive relation. Then, the following conditions are provided:
1) $S_{s \land p} \leq S_s, S_p \leq S_{s \lor p}$,
2) $T_{s \land p} \leq T_s, T_p \leq T_{s \lor p}$.

Corollary 17. Let $U$ be the universe and $R$ is a preorder (reflexive and transitive) relation. Then the following conditions are provided:
1) $S_{s \land p} \leq S_s, S_p \leq S_{s \lor p}$,
2) $T_{s \land p} \leq T_s, T_p \leq T_{s \lor p}$.

Remark 18. If $R$ is a preorder relation on $U$, $S_{s \land p} (S_s, S_p, S_{s \lor p})$ form a base for $T_{s \land p} (T_s, T_p, T_{s \lor p})$ topology on $U$, respectively.

Corollary 19. Let $U$ be the universe and $R$ is an equivalent relation. Then, the following conditions are provided:
1) $S_{s \land p} = S_s = S_p = S_{s \lor p}$,
2) $T_{s \land p} = T_s = T_p = T_{s \lor p}$.
Remark 20. In the case when $R$ is an equivalent relation on $U$, i.e., $(U, R)$ is a Pawlak approximation space. Moreover, the set $\mathcal{S}_a \wedge_p (\mathcal{S}_a, \mathcal{S}_a, \mathcal{S}_a)$ is a base for $T_s \wedge_p (T_s, T_p, T_s)$ topology on $U$. In these topologies, each neighborhood operator is one equivalence class for all $x \in U$.

4. Rough approximation operators induced by relations

In this section, we investigate connection between lower and upper approximation operators using successor, predecessor, successor-and-predecessor, and successor-or-predecessor neighborhood operators by various binary relations on a universe, respectively.

Proposition 21. Let $U$ be the universe and $R$ is a binary relation. Then, for lower and upper approximation operators of $X \subseteq U$, the following conditions are provided:

1) $\text{apr}_{\wedge_p} (X) \subseteq \text{apr}_p (X), \text{apr}_{\wedge_p} (X) \subseteq \text{apr}_{\wedge_p} (X)$,

2) $\text{apr}_{\wedge_p} (X) \subseteq \text{apr}_p (X), \text{apr}_{\wedge_p} (X) \subseteq \text{apr}_{\wedge_p} (X)$.

Proof. (1) Let $x \in \text{apr}_{\wedge_p} (X)$ for any $x \in U$. Since $R_{\wedge_p} (x) \subseteq X$ and $R_s (x) \subseteq R_{\wedge_p} (x)$ then $R_s (x) \subseteq X$ and so $x \in \text{apr}_p (X)$. Now, since $x \in \text{apr}_{\wedge_p} (X)$ and $R_{\wedge_p} (x) \subseteq R_s (x) \subseteq X$ then $x \in \text{apr}_{\wedge_p} (X)$. Therefore $\text{apr}_{\wedge_p} (X) \subseteq \text{apr}_{\wedge_p} (X)$.

Similarly, $\text{apr}_{\wedge_p} (X) \subseteq \text{apr}_{\wedge_p} (X)$.

(2) Let $x \in \text{apr}_{\wedge_p} (X)$ for any $x \in U$. Since $R_{\wedge_p} (x) \cap X \neq \emptyset$ and $R_{\wedge_p} (x) \subseteq R_s (x), R_p (x) \subseteq R_{\wedge_p} (x)$ then $R_{\wedge_p} (x) \cap X \neq \emptyset$ an so $x \in \text{apr}_{\wedge_p} (X)$. Therefore, $\text{apr}_{\wedge_p} (X) \subseteq \text{apr}_{\wedge_p} (X), \text{apr}_{\wedge_p} (X) \subseteq \text{apr}_{\wedge_p} (X)$.

Example 22. Let $U = \{a, b, c, d\}$ and

$$R = \{(a, a), (a, c), (b, c), (c, a), (c, d)\}$$

be a binary relation on $U$. Then,

$$R_s (a) = \{a, c\}, \ R_s (b) = \{c\}, \ R_s (c) = \{a, d\}, \ R_s (d) = \emptyset,$$

$$R_p (a) = \{a, c\}, \ R_p (b) = \emptyset, \ R_p (c) = \{a, b\}, \ R_p (d) = \{c\},$$

$$R_{\wedge_p} (a) = \{a, c\}, \ R_{\wedge_p} (b) = \emptyset, \ R_{\wedge_p} (c) = \{a\}, \ R_{\wedge_p} (d) = \emptyset,$$

$$R_{\vee_p} (a) = \{a, c\}, \ R_{\vee_p} (b) = \{c\}, \ R_{\vee_p} (c) = \{a, b, d\}, \ R_{\vee_p} (d) = \{c\}.$$

Let $X = \{a, c, d\}$. Then,

$$\text{apr}_s (X) = \{a, b, c, d\},$$

$$\text{apr}_p (X) = \{a, b, d\},$$

$$\text{apr}_{\wedge_p} (X) = \{a, b, c, d\},$$

$$\text{apr}_{\wedge_p} (X) = \{a, b, d\},$$

$$\text{apr}_p (X) = \{a, c, d\},$$

$$\text{apr}_{\wedge_p} (X) = \{a, c\},$$

$$\text{apr}_{\wedge_p} (X) = \{a, b, c, d\}.$$ 

Hence, note that $\text{apr}_{\wedge_p} (X) \supset \text{apr}_s (X)$. 

In the original rough set theory, lower approximation of $X$ is a subset its upper approximation. In order to provide this condition, we need some properties to add binary relations.

**Proposition 23.** Let $U$ be the universe and $R$ is a binary relation. Then, for all $x \in U$

$$R \text{ is serial } \Rightarrow apr_p(X) \subseteq apr_s(X)$$

**Corollary 24.** Let $U$ be the universe and $R$ is a binary relation. Then, for all $x \in U$

$$R \text{ is serial } \Rightarrow apr_{p\lor s}(X) \subseteq apr_p(X) \subseteq apr_s(X) \subseteq apr_{p\lor s}(X)$$

**Proof.** Proof is clear from Proposition 21 and Proposition 23. □

**Proposition 25.** Let $U$ be the universe and $R$ is a binary relation. Then, for all $x \in U$

$$R \text{ is invers serial } \Rightarrow apr_{p\land s}(X) \subseteq apr_p(X)$$

**Proof.** Let $x \in apr_p(X).$ Then, $R_p(x) \subseteq X,$ which gives $R_p(x) \cap X = R_p(x) \neq \emptyset,$ that is, $x \in apr_p(X).$ □

**Corollary 26.** Let $U$ be the universe and $R$ is a binary relation. Then, for all $x \in U$

$$R \text{ is invers serial } \Rightarrow apr_{p\lor s}(X) \subseteq apr_{p\land s}(X) \subseteq apr_p(X) \subseteq apr_{p\lor s}(X).$$

**Proof.** Proof is clear from proposition 21 and proposition 25. □

**Proposition 27.** Let $U$ be the universe and $R$ is a binary relation. Then, for all $x \in U$

$$R \text{ is symmetric and serial (or invers serial) } \Rightarrow apr_{p\land s}(X) \subseteq apr_{p\lor s}(X).$$

**Proof.** Let $x \in apr_{p\land s}(X).$ Then, from proposition 14 $R_{p\land s}(x) = R_p(x)$ which gives $apr_{p\land s}(X) = apr_{p\lor s}(X).$ So, from proposition 24 $x \in apr_{p\land s}(X).$ □

**Proposition 28.** Let $U$ be the universe and $R$ is a binary relation. Then, for all $x \in U$

$$R \text{ is reflexive } \Rightarrow apr_p(X) \subseteq X \subseteq apr_p(X), \text { where } i: \ s, \ p\land s \text{ and } p\lor s, \text{ respectively.}$$

**Proof.** Proof is clear from (L3) and (U3). □

**Corollary 29.** Let $U$ be the universe and $R$ is a reflexive or preorder binary relation. Then, for all $x \in U$

1. $apr_{p\lor s}(X) \subseteq apr_p(X) \subseteq apr_{p\land s}(X) \subseteq X \subseteq apr_{p\land s}(X) \subseteq apr_p(X) \subseteq apr_{p\lor s}(X)$
2. $apr_{p\lor s}(X) \subseteq apr_{p\land s}(X) \subseteq apr_p(X) \subseteq X \subseteq apr_{p\land s}(X) \subseteq apr_{p\lor s}(X) \subseteq apr_p(X).$

**Proof.** Proof is clear from Proposition 23 and Proposition 28. □

**Proposition 30.** Let $U$ be the universe and $R$ is a tolerance or equivalent binary relation. Then, for all $x \in U$

$$apr_s(X) = apr_{p\lor s}(X) = apr_{p\land s}(X) = apr_{p\lor s}(X) \subseteq X \subseteq apr_p(X) \subseteq apr_{p\lor s}(X) \subseteq apr_{p\land s}(X).$$

**Proof.** Proof is clear from corollary 15 and proposition 28 respectively. □
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