Spin dynamics and magneto-optical response in charge-neutral tunnel-coupled quantum dots

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Abstract
We model the electron and hole spin dynamics in an undoped double quantum dot structure, considering the carrier tunneling between quantum dots. Taking the presence of an additional in-plane or tilted magnetic field into account, we enable the simulation of magneto-optical experiments, like the time-resolved Kerr rotation measurement, which are currently performed on such structures to probe the temporal spin dynamics. With our model, we reproduce the experimentally observed effect of the extension of the spin polarization lifetime caused by spatial charge separation, which may occur in structures of this type. Moreover, we provide a number of qualitative predictions concerning the necessary conditions for observation of this effect as well as about possible channels of its suppression, including the spin–orbit coupling, which leads to tunneling of carriers accompanied by a spin flip. We also consider the impact of the magnetic field tilting, which results in an interesting spin polarization dynamics.

Keywords: spin dynamics, magneto-optical response, double quantum dots

(Some figures may appear in colour only in the online journal)

1. Introduction

Due to the rapid development of spintronics [1, 2] there is a need for new types of spin manipulation systems of high quality. The exploration of spin dynamics and spin coherence in semiconductors is promising, thanks to the high level of solid-state-based experimental techniques. The area of possible applications of such research is wide, including the development of new devices, like logic circuits [3, 4], magnetic random access memories [5–7], semiconductor spin-based quantum computing [8, 9], and spin-transfer nano-oscillators [10].

The search for good structures, which could be the candidates for spin-based devices, started mostly with the investigation of the dynamics of conduction-band electrons in compound semiconductors without inversion center, for example, GaAs [11]. At that time, the hole spin states were not the focus of so much attention, mostly due to their sub-picosecond dephasing time in bulk GaAs [12, 13]. However, the picture changes if one explores the spin dynamics in nanostructures, where the reduction of the system dimensionality yields extension of the hole spin coherence time, up to a few picoseconds in p-doped quantum wells [14–16]. This can be further enhanced by an additional carrier localization occurring at low temperatures, e.g. on local interface potential fluctuations or some kind of trapping centres [17–21] as well as by formation of quantum dots (QDs). Additionally, hole spins are not affected by the contact-hyperfine-interaction-driven decoherence due to their p-like wave functions [22]. Finally, the high anisotropy of the hole g-factor in low-dimensional GaAs creates new possibilities for spin manipulation [23–25].

The possibility of the experimental investigation of spin dynamics in neutral nanostructures is limited by exciton recombination—which, in most cases, is much faster than the actual spin dynamics to be considered [26]. For that reason,
doped structures with persistently present resident electron/hole spins are commonly used and methods of resident spin polarization have been proposed [17–21, 27]. However, the usage of doped structures comes with a number of drawbacks, first of which is the lowering of the material quality (especially optical) caused by the doping as well as problems with proper doping (particularly p-type) of some materials, such as InGaAs [18]. Moreover, it has been shown that there is an intrinsic spin dephasing present in the initialization of resident spins caused both by the temporal excitation to charged excitonic complex states with subsequent recombination itself and by the phonon reservoir reaction to this carrier system evolution, which takes place during the laser pulse [28]. This may be important in future applications but affects also the results of current experiments [29], in particular those based on the resonant spin amplification effect [20, 30, 31], where the spin coherence is crucial for the formation of the observed signal. For that reason, the search for an undoped system with long-living spins was crucial. It brought a proposal and first realizations of double quantum well/dot structures, in which, after the optical excitation, the electron–hole pair is spatially separated due to the carrier tunneling between the nanostructures enhanced by the electric field [32, 33]. This leads to the extension of carrier lifetime by orders of magnitude (tens of microseconds instead of less than 5ns for direct exciton in QWs [34]), which removes the artificial upper limit on the spin lifetime.

In this work we present a theoretical modeling of the spin dynamics taking place in such a system at finite temperatures. We consider an undoped vertically stacked double quantum dot (DQD) system. The theory is also applicable to double quantum well systems, where an additional weak lateral confinement of carriers is present, as indicated by recent experimental observations [17–21]. The system is excited by a pulse of circularly polarized laser light, which generates electron–hole pairs in one of the QDs. For the simulation of magneto-optical experiments we also take into account the presence of a magnetic field. This allows us to simulate the time-resolved Kerr rotation (TRKR) signal, which is experimentally measured to probe the spin dynamics. We reproduce the effect of the extension of the spin polarization lifetime, which was observed experimentally [32, 33]. Then, we study the destructive impact of temperature and sample inhomogeneity on this effect. Moreover, we include the spin–orbit coupling effects, which give rise to the mixing of states with different angular momenta and, in consequence, to the probability of a spin-flip-accompanied tunneling of carriers. Finally, we also investigate the impact of the magnetic field tilting on the spin dynamics.

The paper is organized as follows. First, in section 2, we provide the theoretical framework of the investigated system and derive the spin dynamics model. The results, as well as their discussion, are presented in section 3. Finally, we conclude the paper in section 4. In the appendix, we present a derivation of the relative rate of tunneling with a spin flip caused by the spin–orbit interaction, which we use in section 2.

2. The model

We investigate spin dynamics in an undoped DQD system by the extension and modification of the model developed previously for trapped carriers in a single QW [20, 23]. The fundamental optical transition in the system consists in a generation of an electron–hole pair (exciton). We simulate the optical excitation by a pulse of circularly polarized laser light resonant to the transition in one of the QDs (assumed further to be the first one). In this manner, due to the selection rules governing such pumping in this type of structure, a selected bright exciton state is pumped.

In figure 1 we present a schematic energy diagram of the system and indicate the essential dynamical processes simulated in this work. The band structure is tilted due to a homogeneous electric field applied along the growth axis, which promotes the tunneling of the electron and the hole between the QDs. For simplicity the diagram corresponds to $T = 0 \text{ K}$, at which only one-way (energy preferable) tunneling for each of the carriers is possible. There are four possible radiative recombination processes in the system, each of them corresponding to the decay of one of the bright exciton states. The simulated system is placed in a homogeneous magnetic field, which causes the Larmor precession of spins. The diagram corresponds to the case of an in-plane magnetic field.

The electron subsystem is described in the spin-up, spin-down basis of states (in terms of spin projection onto the axis normal to the sample plane) $|\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle$, where arrows denote the spin orientation and the number in the subscript indicates one of the two QDs. Analogously, the basis states for the holes are $|\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle$. The whole system is described in the product basis which contains the additional ground state $|0\rangle$, to allow for a description of the electron–hole pair recombination process. For convenience, we distinguish the four bright exciton states and denote them by $|X_i^\alpha\rangle$, where $i = 1, 2$ corresponds to direct excitons $(|X_i^\alpha\rangle = |\uparrow\downarrow\rangle_1, |X_i^\alpha\rangle = |\downarrow\uparrow\rangle_2)$ and $i = 3, 4$ to indirect ones $(|X_i^\alpha\rangle = |\downarrow\uparrow\rangle_1, |X_i^\alpha\rangle = |\uparrow\downarrow\rangle_2)$ with the spin-flipped counterpart for each of them labeled by $|X_i^\beta\rangle$. The corresponding direct and indirect dark exciton states are labeled by $|X_i^\gamma\rangle$ and $|X_i^\delta\rangle$, i.e. $|X_i^\gamma\rangle = |\uparrow\downarrow\rangle, |X_i^\delta\rangle = |\downarrow\uparrow\rangle$, etc.

The evolution of the system, which is assumed to be in contact with a bosonic reservoir, is described in the density matrix formalism in terms of the reduced density matrix, $\rho = Tr_\mathcal{R}\rho$, where the trace is done over the degrees of freedom of the reservoir, $\rho = |\Psi\rangle\langle\Psi|$ is the density matrix of the full system, i.e. the considered subsystem and the reservoir, and $|\Psi\rangle$ is the wave function of the full system.

We model the optical excitation by a laser Hamiltonian written in the rotating frame picture with respect to zero-field exciton energies and in the rotating wave approximation. The pump pulse is circularly polarized ($\sigma_\pm$) and energetically coupled to the first QD. The pumping laser Hamiltonian is

$$H_p(t) = \frac{1}{2} \int e^{i\Delta t} |0\rangle\langle X_i^\alpha| + \text{H.c.},$$

where $\Delta$ is the pulse detuning from the direct exciton line and...
$f(t)$ is the pulse envelope. One finds the density matrix of the system after the pulse in the second order with respect to the pulse amplitude

$$
\rho(t) = \rho_0 - \frac{i}{\hbar} \int_{-\infty}^{\infty} dt \left[ H_p(t), \rho_0 \right] - \frac{1}{\hbar^2} \int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} dt'' \left[ H_p(t), [H_p(t'), \rho_0] \right],
$$

where $\rho_0$ is the initial density matrix, assumed here to be $\rho_0 = |0\rangle\langle 0|$. The subsequent spin dynamics is modeled using a Markovian master equation in the rotating frame picture with respect to zero-field energy gaps of QDs, but with the spin dynamics kept in the Schrödinger picture

$$
\dot{\rho}(t) = -\frac{i}{\hbar} [H_0, \rho(t)] + \sum_{i=1}^{2} (L^i_{\phi}(\rho(t)) + L^i_{h}(\rho(t)))
+ \sum_{j=1}^{2} \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \left[ L^j_{\phi}(\rho(t)) + \sum_{\zeta} L^j_{\phi,flip}(\rho(t)) \right]
+ \sum_{j=1}^{2} \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \left[ L^j_{h}(\rho(t)) + \sum_{\zeta} L^j_{h,flip}(\rho(t)) \right]
+ \sum_{i=1}^{4} L^i_{X}(\rho(t)),
$$

where $H_0 = H_X + H_B$ is the Hamiltonian describing exciton energies and electron and hole spins in the magnetic field and $L$s are Lindblad generators defined further. The exciton energy part is

$$
H_X = \sum_{i} E_X \sum_{\zeta} \langle X^i_{\zeta} | \chi_{\zeta} | X^i_{\zeta} \rangle,
$$

where $\zeta = b, b', d, d'$ and the energies, $E_X = E_{e_1} + E_{e_2} + E_D$, $E_X = E_{e_1} + E_{e_2} + E_D$, $E_{e_1} = E_{e_1} + E_{h_1} + E_{ID} + V$, $E_{e_2} = E_{e_2} + E_{h_2} + E_{ID} - V$, are composed of the single-particle electron and hole energies in QDs, $E_{e_1,2}$ and $E_{h_1,2}$, measured with respect to corresponding band-edges, direct and indirect exciton binding energies, $E_D$ and $E_{ID}$, and, in the case of indirect excitons, of the dipole energy $V$ arising from the external electric field (see figure 1). The Coulomb interactions between carriers as well as the presence of external electric field are effectively included in respective energy shifts. Both Coulomb interaction and electric field may also lead to the charge redistribution and consequently modify exciton radiative lifetime. This may be assumed to be implicitly present in the chosen radiative recombination rate, which will be introduced further in this section as a parameter of the model.

The Zeeman Hamiltonian reads

$$
H_B = -\frac{1}{2} \mu_B B \sum_{i=1}^{3} (\tilde{g}_h \sigma_i^h + \tilde{g}_e \sigma_i^e),
$$

where $\mu_B$ is the Bohr magneton, $\tilde{g}_h$ is the hole Landé tensor with neglected in-plane anisotropy, $\tilde{g}_h = \text{diag}(\tilde{g}_{h_1}, \tilde{g}_{h_2}, \tilde{g}_{h_3})$, where $\tilde{g}_{h_1}$ and $\tilde{g}_{h_2}$ are respectively the in-plane and axial

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**Figure 1.** A schematic diagram of the system at $T = 0$ K; essential dynamical processes depicted. Dashed arrows show the tunneling process directions, including spin-preserving (blue) and spin-flip-accompanied (green) tunneling with rates equal $T_{21}^{e,flip}$ and $T_{21}^{h,flip}$, accordingly. Red wavy arrows depict possible radiative recombination processes with rates $\gamma_{1,2}$. In the magnified part, an exemplary Zeeman splitting in the in-plane magnetic field is shown; dotted closed green loop depicts the Larmor spin precession between Zeeman eigenstates with the frequency $\omega_{e}$. Light blue ovals represent the direct and indirect exciton binding energies for two exemplary states, $V$ is the interdot dipole energy arising from external electric field.
components of \( \hat{b}_0 \), \( g_e \) is the electron Landé factor, which is assumed to be isotropic, and \( \sigma^e_0, \sigma^e_1 \) are the Pauli matrices for the hole and electron spins, respectively, in the 4th QD. We assume that the Landé factors for the hole and the electron are identical in both QDs and treat the hole as a pseudo-spin-1/2 system. For the magnetic field oriented in the XZ plane, \( B = B(\sin \theta, 0, \cos \theta) \), the electron spin is simply quantized along the magnetic field direction. One also easily finds the spin quantization axis for the hole, \( \hat{e}_h = (\sin \phi, \cos \phi, \hat{z}) \), where \( \tan \phi = (g_e / g_h) \tan \theta \), and the effective Landé factor for the hole is \( g_h = \sqrt{(g_h \sin \theta)^2 + (g_h \cos \theta)^2} \).

The calculation of Zeeman eigenstates is straightforward and yields

\[
\begin{align*}
|+\psi\rangle &= \cos \frac{\theta}{2} |1\rangle + \sin \frac{\theta}{2} |\downarrow\rangle, \\
|-\psi\rangle &= -\sin \frac{\theta}{2} |1\rangle + \cos \frac{\theta}{2} |\downarrow\rangle,
\end{align*}
\]

for the electron subspace and analogously for holes

\[
\begin{align*}
|+\rho\rangle &= \cos \frac{\phi}{2} |\uparrow\rangle + \sin \frac{\phi}{2} |\downarrow\rangle, \\
|-\rho\rangle &= -\sin \frac{\phi}{2} |\uparrow\rangle + \cos \frac{\phi}{2} |\downarrow\rangle,
\end{align*}
\]

where the subspace is indicated in the superscript \( e/h \), \( i \) labels the QD, and \( + \) and \( - \) denote the upper and lower energy states respectively.

The Zeeman Hamiltonian may be rewritten using its eigenstates

\[
H_B = -\frac{\hbar}{2} \sum_{i=1}^{2} \omega_e \{(-\tilde{\psi})(-\tilde{\psi}) - (\tilde{+\psi})(\tilde{+\psi})\} \otimes 1 \]

\[
+ \omega_h \{(-\tilde{\rho})(-\tilde{\rho}) - (\tilde{-\rho})(\tilde{-\rho})\},
\]

where \( \omega_e = \mu_B g_e B / \hbar \), \( \omega_h = \mu_B g_h B / \hbar \) are the Larmor precession frequencies for the electron and the hole, respectively. The exciton energy Hamiltonian maintains its original diagonal form in this basis due to the spin degeneracy of excitonic energies.

We neglect here the electron–hole exchange terms that would define the excitonic spectrum (fine structure) in the absence of magnetic field [35]. In a relatively strong magnetic field considered here these terms would present only a small correction to the exciton Zeeman levels.

The dissipative dynamics of the system, including spin relaxation, decoherence, electron and hole tunneling between the QDs, and the exciton recombination, is described in the Markov limit (justified by relatively long timescales involved here) by the superoperators \( L^\dagger \) in the universal Lindblad form [23]. The first term, \( \sum_{i=1}^{2} L^\dagger_1 \rho(t) + L^\dagger_2 \rho(t) \) consists of the electron and hole dissipators for each QD and describes the spin relaxation and pure dephasing processes caused by the interaction between the investigated open system and its environment (e.g. phonon reservoir). The dissipators are of the form

\[
L^\dagger_1^e/\rho = \kappa_1^e \left( \sigma_1^e(\rho) \sigma_1^e - \frac{1}{2} \sigma_1^e \sigma_1^e, \rho \right),
\]

\[
+ \kappa_1^h \left( \sigma_1^h(\rho) \sigma_1^h - \frac{1}{2} \sigma_1^h \sigma_1^h, \rho \right),
\]

\[
+ \kappa_0^e \left( \sigma_0^e(\rho) \sigma_0^e - \frac{1}{2} \sigma_0^e \sigma_0^e, \rho \right),
\]

where \( \eta = e, h \) denotes the carrier type,

\[
\sigma_1^e = \frac{1}{\sqrt{2}} (\sigma_1 \uparrow \downarrow - \sigma_1 \downarrow \uparrow),
\]

\[
\sigma_1^h = \frac{1}{\sqrt{2}} (\sigma_1 \uparrow \downarrow - \sigma_1 \downarrow \uparrow),
\]

\[
\sigma_0 = \frac{1}{\sqrt{2}} (\sigma_0 \uparrow \downarrow - \sigma_0 \downarrow \uparrow),
\]

are the spin-flip and pure dephasing transition operators for the electron and the hole in the ith QD, \( \kappa_0^e \) the pure dephasing rates, and \( \kappa_1^e, \kappa_1^h \) are the corresponding spin-flip rates. At zero temperature, the transitions from lower to higher states are forbidden so that \( \kappa_0^e = 0 \). At non-zero temperatures they are connected by the relation

\[
\kappa_0^e = \exp \left( -\Delta E_0 / k_B T \right) \kappa_1^e,
\]

which guarantees the detailed balance condition at equilibrium at temperature \( T \), with \( \Delta E_0 \) denoting the Zeeman energy splittings, and \( k_B \) the Boltzmann constant. The spin flip and dephasing rates account for any spin decoherence mechanisms present in a given system, including spin–orbit effects. Note that the spin-flip dissipators would lead to bright-dark exciton transitions in the absence of magnetic fields, while for the spin dynamics in a magnetic field, as considered here, they will induce damping of the spin precession (see section 3).

The carrier tunneling is accounted for in the Markov limit by the following superoperators

\[
L^\dagger_{1/2} \rho = \frac{1}{2} \left( t_{1/2}^1(\rho) t_{2/1}^1(\rho) - t_{2/1}^1(\rho) t_{1/2}^1(\rho) \right),
\]

\[
+ \frac{1}{2} \left( t_{1/2}^1(\rho) t_{2/1}^1(\rho) - t_{2/1}^1(\rho) t_{1/2}^1(\rho) \right),
\]

where \( j = 1, 2 \) denotes the position of the other carrier which is not affected by the transition modeled here but affects the transition energy, due to the difference in the direct and indirect exciton binding energies,
operator acting in the subspace of the QD number of which is given in the subscript.

Spin dephasing effects have been widely studied in higher-dimensional systems, like quantum wells [36–40], where spin–orbit interactions lead to polarization decay via spatial motion and momentum scattering of the carriers. For carriers fully confined in QDs, the same spin–orbit couplings induce spin relaxation by a weak admixture of states with opposite spin to the energy eigenstates [41]. In our model, these processes are accounted for by the spin dephasing rates $\kappa_{\text{spin}}^{(0,+/−)}$. Additionally, the mixing of states with different angular momenta resulting from the spin–orbit coupling leads to a non-zero probability of tunneling with a spin flip, which we take into account by addition of appropriate Lindblad generators, $L_{\text{spin}}^{(0,+/−)}$, where $\zeta = \pm$, and the tunneling transitions are given by

$$t_{e(+/−)}^{(12/21)\zeta} = \left(i\frac{e}{\hbar}\right) \left(\begin{array}{c} - \frac{1}{2} \zeta \mathbb{1} \\mathbb{1} \end{array}\right) \otimes I_{\text{sub}},$$

$$t_{h(+/−)}^{(12/21)\zeta} = \left(i\frac{e}{\hbar}\right) \left(\begin{array}{c} \frac{1}{2} \zeta \mathbb{1} \\mathbb{1} \end{array}\right) \otimes I_{\text{sub}},$$

$$t_{v(+/−)}^{(12/21)\zeta} = \left(i\frac{1}{\hbar}\right) \left(\begin{array}{c} \frac{1}{2} \zeta \mathbb{1} \\mathbb{1} \end{array}\right) \otimes I_{\text{sub}},$$

$$t_{b(+/−)}^{(12/21)\zeta} = \left(i\frac{1}{\hbar}\right) \left(\begin{array}{c} \frac{1}{2} \zeta \mathbb{1} \\mathbb{1} \end{array}\right) \otimes I_{\text{sub}},$$

$$t_{v(+/−)}^{(12/21)\zeta} = \left(i\frac{1}{\hbar}\right) \left(\begin{array}{c} \frac{1}{2} \zeta \mathbb{1} \\mathbb{1} \end{array}\right) \otimes I_{\text{sub}},$$

and their rates labeled by $T_{\text{spin}}^{(12/21)\zeta}$. Note that tunneling processes in a given direction with opposite spin flips differ in transition energy here (by twice the Zeeman splitting) and need to be described by separate Lindblad terms. In the appendix we derive the relative rate of the tunneling with a spin flip as compared to the spin-preserving tunneling in a typical heterostructure for the case of electron tunneling, which is given in equation (A.4). For small self-assembled QDs this rate is negligibly small, however, in the case of weak in-plane confinement of carriers (natural QDs) at $B = 5$ T we estimate the ratio to be $T_{\text{spin}}/T_e \approx 1/20$. We use this value in our further investigation.

The last process in the modeled dissipative dynamics is the exciton radiative recombination described by the super-operators

$$\mathcal{L}_\text{X}[\rho] = \gamma_i\left(\sigma_{X^+}\rho\sigma_{X^+}^{\dagger} - \frac{1}{2}\{\sigma_{X^+}\rho\sigma_{X^+}^{\dagger}, \rho\} + \frac{1}{2}\gamma_i^{(0)}\left(\sigma_{X^+}\rho\sigma_{X^+}^{\dagger} - \frac{1}{2}\{\sigma_{X^+}\rho\sigma_{X^+}^{\dagger}, \rho\}\right) + \frac{1}{2}\gamma_i^{(0)}\left(\sigma_{X^+}\rho\sigma_{X^+}^{\dagger} - \frac{1}{2}\{\sigma_{X^+}\rho\sigma_{X^+}^{\dagger}, \rho\}\right)\right),$$

where $\gamma_i$ is the radiative decay rate of the $i$th bright excitonic state, $\gamma_i^{(0)}$ are the additional pure dephasing rates for each exciton, and the transition operators are

$$\sigma_{X^+} = |0\rangle \langle X^b|, \quad \sigma_{X^+} = |0\rangle \langle X^b|,$$

$$\sigma_{X^+}^{(0)} = |X^b\rangle \langle X^b| + |X^b\rangle \langle X^b| - 2 |0\rangle \langle 0|.$$

To obtain the direct correspondence with experimentally measured quantities we use the density matrix in the spin-up, spin-down product basis and construct dynamical variables, each being an average value of an appropriate operator

$$A(t) = \operatorname{Tr}(\hat{A}(\rho(t))) = \sum_{i\in B_0} \langle i| \hat{A}\rho(t)|i\rangle,$$

where the sum runs over a complete set of states. The dynamical variables of interest include the spin polarization for electrons and holes in each QD

$$S^{(i)}_e = \frac{1}{\sqrt{2}} \left(\sigma_i^{(0)} \otimes 1 + \frac{1}{\sqrt{2}} (|\uparrow\rangle \langle \uparrow| - |\downarrow\rangle \langle \downarrow|) \otimes 1\right),$$

$$S^{(i)}_h = \frac{1}{\sqrt{2}} \left(\sigma_i^{(0)} \otimes 1 + \frac{1}{\sqrt{2}} (|\uparrow\rangle \langle \uparrow| - |\downarrow\rangle \langle \downarrow|) \otimes 1\right),$$

and the electron and hole spin coherences

$$\hat{X}_e^{(i)} = \frac{1}{\sqrt{2}} (|\uparrow\rangle \langle \uparrow| + |\downarrow\rangle \langle \downarrow|) \otimes 1,$$

$$\hat{Y}_e^{(i)} = \frac{1}{\sqrt{2}} (|\uparrow\rangle \langle \uparrow| - |\downarrow\rangle \langle \downarrow|) \otimes 1,$$

$$\hat{X}_h^{(i)} = \frac{1}{\sqrt{2}} (|\uparrow\rangle \langle \uparrow| + |\downarrow\rangle \langle \downarrow|) \otimes 1,$$

$$\hat{Y}_h^{(i)} = \frac{1}{\sqrt{2}} (|\uparrow\rangle \langle \uparrow| - |\downarrow\rangle \langle \downarrow|) \otimes 1.$$

Finally the system is probed with a linearly polarized laser pulse, which is assumed to be resonant with the fundamental transition in the probed QD. We label the time instant just before the arrival of the probe pulse by $\tau$ and the subsequent instant of measurement by $\tau$.

It is possible to collect the signal only from one selected QD, which is achieved by the tuning of the probe pulse wavelength to the transition in the selected dot, so the optical response (TRKR signal), that is, the rotation of the polarization plane of the reflected or transmitted probe pulse, is proportional to the difference between the electron and hole spin polarization [23, 42] in the selected QD just before the probe pulse arrival,

$$\text{TRKR}(\tau) \propto \Delta \Sigma_i^{(i)}(\tau) = \Delta \Sigma_e^{(i)}(\tau) - \Delta \Sigma_h^{(i)}(\tau).$$

3. Results and discussion

The numerical solution of the equation of motion (3) allows us to calculate the time dependence of spin polarization, equation (7), in each of the QDs, which is proportional to the experimentally measured TRKR signal. In this section, we utilize this to investigate the essential features of spin dynamics in the discussed DQD system.

In the first part of our simulations we set an in-plane magnetic field, $B = 5$ T, and effective $g$-factor values taken from experimental data [43, 44] and equalling $g_e = 0.18$ and $g_h = 0.07$ for electrons and holes respectively. The electron and hole spin relaxation times for both QDs are set to $\tau_e = 1\text{s}$ and $\tau_h = 3\text{s}$, which are typical values for such
The recombination times for direct and indirect excitons are set to 100 ps and 10 ms respectively. We set the direct and indirect exciton binding energies to $E_\text{D} = -5 \text{ meV}$ and $E_\text{D} = -1 \text{ meV}$, the dipole energy $V = 15 \text{ meV}$, and the single-particle energy differences $\Delta_e = 10 \text{ meV}$ and $\Delta_h = 1 \text{ meV}$.

3.1. Tunneling time dependence

First, we investigate the extension of the spin polarization lifetime due to the carrier tunneling and spatial separation of the exciton. For this purpose, we simulate the TRKR signals obtained by probing both QDs under the optical pumping of the first QD at the low temperature of 1 K. In figure 2, we show the result of such simulation for various tunneling times, $t = h/\tau$, for two cases, electron or hole tunneling to the second QD, which correspond to the opposite directions of the electric field. In both cases, one can observe the effects of an interplay between the fast direct exciton recombination in the first QD and the tunneling of one of the carriers to the other QD. For slow tunneling there is no spin polarization created in the second QD, while the signal from the first QD is composed of both electron and hole spin oscillations with different periods (due to the difference in $g$ values) and it undergoes a fast exponential decay governed by the direct exciton recombination. Through intermediate situations, one arrives at the regime of dynamics dominated by the tunneling, which is dealt with when the tunneling time is comparable to or shorter than the direct exciton lifetime. One then observes the separation of carriers visible in the plot as single-frequency oscillations of the signals from the both QDs. The lifetime of the spin polarization in both QDs is extended, and after some initial time (close to the tunneling time) becomes governed by the slow indirect exciton recombination.

3.2. Temperature effects

While in the low temperature limit discussed above one dealt effectively only with one-way carrier tunneling, from the first to the second QD, at higher temperatures the probability of the tunneling in the opposite direction becomes non-zero, which should block the desired effect of exciton separation and shorten the spin polarization lifetime. A simulation of this effect is shown in figure 3, where at low temperature one can see the fast electron tunneling to the second QD (a condition similar to the green dashed line plotted for $\tau_e = 100 \text{ ps}$ in figure 2). When the temperature is risen, one can observe the damping of spin polarization oscillations in the second QD. Confirmation of the fact that one really deals here with back-tunneling can be found in the signal from the first QD, where for the time of the whole simulation a composition of two (electron and hole spin) oscillations is present. According to equation (4), this effect becomes relevant when the thermal energy, $kT$, is comparable with the difference between the carrier energies in the two QDs. The latter, set in this simulation to $\Delta E_\text{D} = 1 \text{ meV}$, is proportional to the applied electric field (its magnitude is of the order of $10^6 \text{ V m}^{-1}$ here), which to some extent can be used to escape from this unfavorable regime.

![Figure 2](image-url)

**Figure 2.** Simulated TRKR signal from the first QD (left panels) and second QD (right panels) at 1 K under pulsed excitation of QD1 in a 5 T in-plane magnetic field. The Landé factors are $g_e = 0.18$ and $\tilde{g}_h = 0.07$ for electrons and holes respectively. The recombination times of the direct and indirect excitons are 100 ps and 10 $\mu$s respectively. In the upper row of panels, the case of the electron tunneling with various tunneling times is shown. The bottom panels show analogous plots for hole tunneling.
3.3. Spin–orbit interaction effects

To estimate the impact of the spin–orbit interaction on the system dynamics we include the additional channel of electron tunneling accompanied by a spin flip with the rate estimated in the appendix, i.e., $T_{e,\text{flip}}/T_e = 1/20$. In figure 4 we present the comparison of the simulated signals at 1 K with and without the spin-flip-accompanied tunneling (and other parameters unchanged with respect to figure 3). Since there is nearly no back-tunneling at such a low temperature, we observe here a signal loss reaching a constant value in the second QD, due to a partial loss of spin coherence at the tunneling. The amount of this loss may be found to be proportional to the ratio of tunneling rates. At higher temperatures, when a two-way tunneling occurs, the process accompanied by a spin flip leads to an accumulative signal loss, which is, however, too small to be visible as a correction to figure 3. For that reason, in figure 5, we present the results for a few values of temperature with the dominant effect of radiative recombination disabled. We estimate the lifetime of the signal at 20 K to be about 830 ps, which is a small correction to the 1 ns electron spin lifetime put intrinsically in the model, and we find it negligible in the presence of the unavoidable recombination (compare with figure 3).

3.4. Sample inhomogeneity

Another disadvantageous issue, which cannot be avoided in the experiment, is the inhomogeneity of the QDs. Geometrical parameters of DQDs as well as their composition may change slightly from place to place in the ensemble on a sample, which causes, among others, the spatial fluctuations of the Landé factor values. To take this fact into account, we averaged the simulations of the TRKR signal over a normal distribution of in-plane g-factor component values for both holes and electrons, with standard deviations $\Delta g_h$ and $\Delta g_e$ respectively. The result is presented in figure 6, where one can observe the impact of this inhomogeneity separately for each
type of carrier. The spin polarization oscillations are increasingly dumped with rising standard deviation of $g$. The lifetime of the spin polarization for both types of carrier is shortened to approximately 0.5 ns for the standard deviation of the Landé factor equal $D = g_{15\%}$.

3.5. Tilted magnetic field

Another interesting problem is the impact of magnetic field tilting on the spin dynamics. To examine this effect, we set a constant magnetic field magnitude, $B = 5$ T, and vary the angle of its tilt from the Voigt geometry, $\alpha = \pi/2 - \theta$. The results of this simulation are presented in figure 7. Recent experimental reports [27, 47] show that the hole g-tensor anisotropy may be very high in In(Al)$_x$Ga$_{1-x}$As/GaAs material system nanostructures, with the ratio $g_{b}/g_{h}$ exceeding a factor of 10. According to these results, in our simulation the out-of-plane and in-plane components of the hole g-tensor are $g_{b} = 0.93$ and $g_{h} = 0.066$, respectively.

The main feature present in the simulation for non-zero tilt angles is the deviation from the symmetric oscillations around zero polarization, which is visible in the signal from the QD1 in the case of the electron tunneling and in the signal from the QD2 in the case of the tunneling of the hole, that is
in the regions dominated by the hole spin polarization. Such behavior is not observed in the signal composed mostly of the electron spin polarization. This asymmetric shape of the signal is caused by the presence of a non-precessing exponentially decaying component, which was theoretically predicted \[23\] and observed experimentally \[227\] in the spin dynamics in a single doped QW. In the tilted magnetic field, the spin precession takes place around an axis, which is also tilted from the QD plane. Unlike the case of exact Voigt geometry, where only the dynamics of the spin component perpendicular to the precession axis has its reflection in the simulated TRKR signal (precession damped by the spin dephasing), in a tilted field the spin component parallel to the quantization axis also has a non-zero projection on the growth axis and contributes to the signal. This makes the spin relaxation leading to thermalization between the Zeeman eigenstates (longitudinal decoherence) also visible in the simulated signal as the non-oscillating component, which contribution increases with $\alpha$.

While the electron spin is quantized along the magnetic field direction, the axis of the hole spin quantization is tilted by the angle, $\beta = \pi/2 - \phi$, which increases rapidly with $\alpha$ due to the strong hole $g$-tensor anisotropy (see the inset in figure 7). This explains why the non-precessing component of the spin dynamics is noticeable only in the signal composed mostly of the hole spin contribution. For $\alpha = 8^\circ$ the tilt of the hole quantization axis reaches already $\beta = 63^\circ$ and the signal is dominated by the non-precessing component. In the right panels of figure 7, one can observe the dynamics of the carrier that tunnels. For the hole, it is composed of the initial increase of the signal connected with the appearance of the carrier in the QD, the non-precessing exponentially decaying component and oscillations.

In a tilted field, the spin thermalization process for both types of carrier ends in the equilibrium state, which has a non-zero projection on the $z$ axis. This remaining spin polarization increases with the tilt angle of the quantization axis and is not visible for the electron in the scale of the figures. As a result one obtains a long-living (limited only by the slow indirect exciton recombination) non-oscillating hole spin polarization in one of the QDs.

While the electron oscillation frequency remains unchanged upon the tilt of the magnetic field due to its isotropic $g$-factor, this is not the case for holes, the effective Landé factor of which changes with the tilt angle. This results in the increased oscillation frequency.

In figure 8 we present an equivalent simulation, which differs only in the fact that a fast spin dephasing process is enabled for both types of carriers with the dephasing times equal $\tau_e^{(0)} = 1/\nu_e^{(0)} = 0.3$ ns and $\tau_h^{(0)} = 1$ ns for electrons and holes respectively. Notice that only the oscillating component of the signal is damped here, due to the coherence decay, while the non-precessing component behavior remains unaffected and follows the spin relaxation process, as expected.

4. Conclusions

We simulated the spin dynamics of optically generated carriers in the DQD system in the presence of magnetic field using Lindblad equations. The theoretical model developed...
here allows one to simulate the results of magneto-optical experiments performed on such systems, especially the signal obtained in the TRKR measurements. Our model properly reproduces the effect of the extension of the lifetime of the spin polarization caused by the charge separation due to the possibility of tunneling of a carrier to the other QD.

We have shown that the observed spin polarization depends essentially on the ratio of the carrier tunneling time to the direct exciton recombination time, which is a fact to be taken into account during the sample design process. Moreover, we predict that this advantageous effect can be suppressed by various factors. We show that the possibility of back-tunneling, which increases with temperature and becomes relevant when thermal energy is comparable with the difference between the energies of the direct and indirect exciton states, may totally block the considered effect. The impact of the ensemble inhomogeneity (QD geometry or composition) was also investigated, and was shown to cause damping of the spin polarization and therefore loss of magneto-optical signal to be observed in the experiment. In the presence of spin–orbit coupling the possibility of tunneling accompanied by a spin–flip process leads to damping of the signal from the target QD. However, we show that its impact is much weaker than the losses caused by the unavoidable recombination process, which makes the correction arising from the spin–orbit coupling negligible.

The consideration of the magnetic field tilting showed the presence of the non-oscillating component in the TRKR signal from one of the QDs and the resulting asymmetry of spin polarization oscillations, which is considerable when the signal is composed mostly of the hole spin contribution. The hole spin relaxation in such conditions leads to a long-living non-zero signal, with lifetime limited only by the slow indirect exciton recombination.

The proposed model provides a complete description of the TRKR response from a DQD sample as a function of the essential parameters of the system. Hence, it can be used to extract dynamical parameters, such as carrier g-factors, spin lifetimes and coherence times for investigated systems, from experimental data.

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Appendix

In this appendix we derive the rate of tunneling of the electron with a spin flip, which is used in section 2. Here, we perturbatively take into account the spin–orbit coupling, leading to the admixture of p-shell states with inverted spin to the ground state, and derive the effective phonon-mediated tunneling Hamiltonian for the case of electron
tunneling. In the absence of phonons, we deal with two uncoupled subspaces, \{ | Ds \rangle, | IDs \rangle, | Dp \rangle, | IDp \rangle \} and \{ | Ds \rangle, | IDs \rangle, | Dp \rangle, | IDp \rangle \}, where D and ID stand for direct and indirect exciton respectively, s and p denote the orbital state of the electron with projection of angular momentum, \( m = 0, \pm 1 \), respectively, and arrows symbolize the electron spin. We use the harmonic oscillator model for the electron states and obtain the Fock–Darwin Hamiltonian \( E_{D} = E_{D0} - \Delta \), \( E_{ID} = E_{D0} + \omega_{p} \), \( E_{Dp} = E_{D0} + \omega_{sp}^{D} \), \( \omega_{sp}^{D} = \Delta \), for the first subspace. In the second subspace, \( E_{ID} = E_{D0} + \mu_{B}B \) and the other relations are analogous. Here, \( \omega_{sp}^{D} = \hbar \omega_{B} \pm \hbar \omega_{B} \mu_{B} \). \( \omega_{B} = \left( \omega_{s}^{2} + \frac{eB}{m^{*}} \right)^{1/2} \), \( \omega_{sp}^{D} = cB/m^{*} \), and \( \omega_{p} \) is the zero-field oscillator frequency.

As physics in both subspaces is similar, we only need to investigate one of them by writing the tunneling Hamiltonian

\[
\hat{H}_{T} = - \sum_{\sigma = \pm, \sigma} \sum_{i} t_{i} | D\sigma \rangle \langle D\sigma | + H.c.,
\]

where \( t_{i} > 0 \) are the tunneling couplings and there is no coupling between \( s \) and \( p \) states due to the difference in their symmetry. The spin–orbit–Hamiltonian leading to the \( s-p \) mixing, which is dominant among other spin–orbital spin-flip mechanisms [41], reads

\[
\hat{H}_{so} = \xi \{ | Ds \rangle \langle Dp | + | IDs \rangle \langle IDp | \} + H.c.,
\]

with the coupling \( \xi = -\hbar \beta \hbar \alpha_{so} \), where \( \beta = \gamma (k_{s}^{2})/\hbar \approx 10 \text{ nm ps}^{-1} \) for typical GaAs heterostructures [41]. \( \beta_{p} = \hbar /\beta_{0} \), \( \beta_{0} = \sqrt{\hbar /m^{*}\omega_{B}} \), and we assume the coupling to be equal in both QDs. As the states are strongly localized along the \( \z \) axis, we neglect the spin–orbit–couplings between direct and indirect states.

Finally, the Hamiltonian is \( H = \hat{H}_{T} + \hat{H}_{so} \) and we assume \( \Delta \sim t_{i} ; \omega_{sp} \sim \Delta, \xi \). Treating \( \hat{H}_{so} \) as a small correction in a quasi-degenerate perturbation theory [48], we find the perturbed Hamiltonian

\[
\hat{H}' = TH^{\dagger}T; \ T = e^{-\beta}s; \ S = \lambda S_{1} + \lambda^{2}S_{2} + ..., \quad (A.1)
\]

where \( \lambda \) is a formal expansion parameter. In the first order, the only non-zero matrix elements are

\[
\langle ls | i S_{1} | lp \rangle = \langle ls | i S_{0} \rangle \langle lp | = - \lambda \xi \omega_{sp}^{D},
\]

and their Hermitian conjugates, where \( l = D, ID \), and further we assume \( \omega_{sp}^{D} \approx \alpha_{so} \) to be equal. The analogical formula holds in the second subspace.

The phonon bath is taken into account by the free phonon Hamiltonian, \( \hat{H}_{ph} = \sum_{k} \hbar \omega_{k} b_{k}^{\dagger} b_{k} \), where \( s \) denotes a phonon branch, and by the exciton–phonon coupling

\[
\hat{H}_{X-\text{ph}} = \sum_{k,i} \sum_{l = D, ID, \beta = s,p} \alpha_{\beta} F_{\beta}^{\mu}(k, s) b_{k, \beta} + b_{k, \beta}^{\dagger},
\]

where \( \alpha_{\beta} = \alpha_{s, \beta} = | \alpha_{\beta} | \langle \beta | F_{\beta}^{\mu}(k, s) = F_{\beta}^{\mu}(k, s) \rangle \) is the coupling constant, and due to the carrier localization there are no D-ID transitions. This Hamiltonian, after transformation (A.1) keeps its original form with the substitution \( \alpha_{s, \beta} \rightarrow \frac{\alpha_{s, \beta}}{\alpha_{so} T^{\mu}} \), which in the first order is \( \alpha_{so} \rightarrow \alpha_{so} + [S_{1}, \sigma_{so}] \), and we find the spin–orbit–induced correction to be

\[
\hat{H}'_{X-\text{ph}} \approx \sum_{\mu} \sum_{k,i} \epsilon_{\mu} \left[ F_{\mu}^{\mu}(k, s) \langle \mu \rangle - F_{\mu}^{\mu}(k, s) \right] \langle \mu \rangle \langle \mu \rangle \times | \mu \rangle | \mu \rangle (b_{k, \mu} + b_{k, \mu}^{\dagger}) + H.c.
\]

We assume here \( F_{\mu}^{\mu} \approx F_{\mu} \) (which holds exactly for Fock–Darwin states) and use an approximation \( \omega_{sp}^{D} \approx \omega_{sp} \approx 2 \left( \hbar \omega_{B}^{2} \right)^{2} \left( \omega_{B}^{2} + \mu_{B}B \right) \), where \( \omega_{sp}^{D} \) each corresponding to an electron in one of the QDs, are further assumed to be equal to the similarity of QDs, and the exciton–phonon coupling constants to differ only by a factor arising from the translation, i.e. \( F_{\mu}(k, s) \equiv e^{i k_{z}z} F_{\mu}(k, s) \), where \( z_{D} = -d/2 \), \( z_{ID} = d/2 \), and \( d \) denotes the interdot distance. Next, we diagonalize the tunneling coupling by defining the \( s \)-shell basis, \( | + \sigma \rangle = \cos \theta | Ds \rangle + \sin \theta | IDs \rangle, | - \sigma \rangle = \cos \theta | Ds \rangle - \sin \theta | IDs \rangle \), where \( \tan \theta = -2t_{0}/\Delta \) and \( \Delta_{k} = \pm \sqrt{t_{0}^{2} + (\Delta/2)^{2} - \Delta^{2}} \). The tunneling transitions accompanied by a spin flip are generated by the off-diagonal part of the Hamiltonian, which now reads

\[
\hat{H}_{X-\text{ph}} \approx \sum_{\mu} \sum_{k,i} F_{\mu}^{\mu}(k, s) \langle l S_{1} \rangle \langle l S_{1} \rangle (b_{k, \mu} + b_{k, \mu}^{\dagger}) + H.c.
\]

Finally, assuming that sums in (A.2) and (A.3) are of the same order of magnitude, we arrive at the ratio of tunneling rates equal

\[
\frac{T_{e, \text{ph}}}{T_{e}} = \left[ \frac{2\beta_{0} \left( \hbar /m^{*} + \mu_{B}B \right) \left( \hbar /m^{*} + \mu_{B}B \right)}{\hbar /m^{*} + \mu_{B}B} \right]^{2}.
\]
At $B = 5$ T, for small QDs, for which $\hbar \omega_0 \sim 50$ meV, $\beta_0 = \hbar \beta / \hbar_0 \sim 2$ meV, $g_\mu_B \ll \hbar / m^*$, and $\hbar e / m^* \sim 5$ meV the ratio is $T_{\text{flip}} / T_e \sim 8 \times 10^{-5}$. In the case of weak localization (natural QDs), we estimate $\hbar \omega_0 \sim 5$ meV, and $\beta_0 = \hbar \beta / \hbar_0 \sim 0.5$ meV, which leads to a ratio of tunneling rates on the order of $T_{\text{flip}} / T_e \sim 1/20$.

References

[1] Žutić I, Fabian J and Das Sarma S 2004 Rev. Mod. Phys. 76 323–410
[2] Fabian J, Matos-Abiague A, Ertler C, Stano P and Žutić I 2007 Acta Phys. Slovaca 57 565
[3] Datta S and Das B 1990 Appl. Phys. Lett. 56 665
[4] Chioleiro A, Allia P, Chiiodoni A, Pirri F, Celegato F and Coisson M 2007 J. Appl. Phys. 101 123915
[5] Gallagher W J and Parkin S S P 2005 IBM J. Res. & Dev. 50 5
[6] Kikkawa J M, Smorchkova I P, Samarth N and Awschalom D D 1998 Science 277 1284–7
[7] Koutrou M, Ducournau Y, Heiss D, Bichler M, Schuh D, Abstreiter G and Finley J J 2004 Nature 432 81–4
[8] Loss D and DiVincenzo D P 1998 Phys. Rev. A 57 120–6
[9] Hawrylak P and Korkusinski M 2005 Solid State Commun. 136 508–12
[10] Kiselev S I, Sankey J C, Krivtsovov I N, Emley N C, Schoelkopf R J, Buhrman R A and Ralph D C 2003 Nature 425 380
[11] Wu M, Jiang J and Weng M 2010 Phys. Rep. 493 61–236
[12] Hilton D J and Tang C L 2002 Phys. Rev. Lett. 89 146601
[13] Shen K and Wu M W 2010 Phys. Rev. B 82 115205
[14] Damen T C, Vila I, Cunningham J E, Shah J and Sham L J 1991 Phys. Rev. Lett. 67 3432–5
[15] Schneider P et al 2004 J. Appl. Phys. 96 420
[16] Lü C, Cheng J L and Wu M W 2006 Phys. Rev. B 73 125314
[17] Sypherd M, Yakovlev D R, Greilich A, Miesiewicz J, Bayer M, Reuter D and Wieck A D 2007 Phys. Rev. Lett. 99 187401
[18] Korn T, Kugler M, Griesbeck M, Schultz R, Wagner A, Hirmer M, Gerl C, Schuh D, Wegscheider W and Schüller C 2011 New J. Phys. 12 043003
[19] Korn T 2010 Phys. Rep. 494 415
[20] Kugler M et al 2011 Phys. Rev. B 84 085327
[21] Studer M, Hirmer M, Schuh D, Wegscheider W, Emslin K and Salis G 2011 Phys. Rev. B 84 085328
[22] Fischer J, Coish W A, Bulaev D V and Loss D 2008 Phys. Rev. B 78 155329
[23] Machnikowski P and Kuhn T 2010 Phys. Rev. B 81 115306
[24] Andlauer T and Vogl P 2009 Phys. Rev. B 79 045307
[25] Kugler M et al 2009 Phys. Rev. B 80 035325
[26] Dutt M V G et al 2005 Phys. Rev. Lett. 94 227403
[27] Korzekwa K et al 2013 Phys. Rev. B 88 155303
[28] Gawełczyk M and Machnikowski P 2013 Phys. Rev. B 87 195315
[29] Varwig S, René A, Greilich A, Yakovlev D R, Reuter D, Wieck A D and Bayer M 2013 Phys. Rev. B 87 115307
[30] Kikkawa J M and Awschalom D D 1998 Phys. Rev. Lett. 80 4313–6
[31] Varwig S, Schwan A, Baranscheid D, Müller C, Greilich A, Yuqofović D R, Reuter D, Wieck A D and Bayer M 2012 Phys. Rev. B 86 075321
[32] Dawson P and Godfrey M J 2003 Phys. Rev. B 68 115326
[33] Gradi C, Kempf M, Schuh D, Bougeard D, Winkler R, Schüller C and Korn T 2014 Phys. Rev. B 90 165439
[34] Colocci M, Guarini M, Vinattieri A, Ferrari F, Deparis C, Massies J and Neu G 1990 Europhys. Lett. 12 417
[35] Bayer M et al 2002 Phys. Rev. B 65 195315
[36] Nitta J, Akazaki T, Takayanagi H and Enoki T 1997 Phys. Rev. Lett. 78 1335–8
[37] Engels G, Lange J, Schäpers T and Lüth H 1997 Phys. Rev. B 55 R1958–61
[38] Ambrosetti A, Pederiva F, Lipparini E and Gandolfi S 2009 Phys. Rev. B 80 125306
[39] Ambrosetti A, Silvestrelli P L, Pederiva F, Mitas L and Toigo F 2015 Phys. Rev. A 91 053622
[40] Ambrosetti A, Salasnich L and Silvestrelli P L 2016 J. Low Temp. Phys. 185 3–25
[41] Khaetskii A and Nazarov Y 2001 Phys. Rev. B 64 125316
[42] Yuqofović I A, Glazov M M, Ivenchenko E L and Efros A L 2009 Phys. Rev. B 80 104436
[43] Hernandez F G G, Gusev G M and Bakarov A K 2013 J. Phys.: Conf. Ser. 456 012015
[44] Arora A, Mandal A, Chakraborti S and Ghosh S 2013 J. Appl. Phys. 113 213505
[45] Yokota N, Tsuemi Y, Ikeda K and Kawaguchi H 2012 Appl. Phys. Expr. 5 124041
[46] Roussignol P, Ferreira R, Delalande C, Bastard G, Vinattieri A, Martinez-Pastor J, Carreres L, Colocci M, Palmier J and Etienne B 1994 Surf. Sci. 305 263–6
[47] Soldatov I V, Germanenko A V and Kozlova N 2014 Phys. E 57 193
[48] Cohen-Tannoudji C, Dupont-Roc J and Grynberg G 2004 Atom-Photon Interactions: Basic Processes and Applications (Weinheim: Wiley-VCH) ISBN 0471293369