Probing compact dark matter with gravitational wave fringes detected by the Einstein Telescope

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Unlike the electromagnetic radiation from astrophysical objects, gravitational waves (GWs) from binary star mergers have much longer wavelengths and are inherently consistent. For ground-based GW detectors, when the lens object between the source and the earth has mass \( \sim 1 \times 10^5 M_\odot \), the diffraction effect should be considered since the chirping wavelengths are comparable to the scale of the barrier (its Schwarzschild radius). The waveform will thus be distorted as the fringes. In this work, we show that signals from the third-generation GW detectors like the Einstein Telescope (ET) would be a smoking gun for probing the nature of compact dark matter (CDM) or primordial black holes. The constraint of the mass can be up to the level of \( M_{\text{CDM}} \sim 5 M_\odot \). For a null search of the fringes, one-year observation of ET can constrain the CDM density fraction to \( \sim 10^{-2} - 10^{-5} \) in the mass range \( M_{\text{CDM}} = 10 M_\odot - 100 M_\odot \).

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INTRODUCTION

Cosmological observations have indicated that a considerable part of the energy density in the current Universe consists of dark matter. However, we still know little about its nature and composition. One may conjecture that dark matter (or part of it) exists in the form of compact objects. Theoretical models of particle physics and cosmology include the massive compact halo objects (MACHOs) [1-4], primordial black holes (PBHs) [5, 6], axion mini-clusters [7], compact mini halos [8] and so on. Hereafter we take all of them as the compact dark matter (CDM). The mass of CDM could be as light as \( 10^{-7} M_\odot \) and as heavy as the first stars \( \sim 10^5 M_\odot \) [9].

Many efforts have been devoted to probing CDM with various approaches. While the wide stellar binaries could be perturbed by large-mass CDM (\( \gtrsim 100 M_\odot \)) [10], the microlensing observations of the stars can constrain the CDM in our galaxy with low-mass (\( \lesssim 10 M_\odot \)) [1, 11-14]. The cosmic microwave background (CMB) can also give constraints by lack of radiation as a result of accretion [15]. Other methods include supernova lensing [16], caustic crossing [17], ultra-faint dwarf galaxies [18] and microlensing of quasars [19]. The observational constraints specifically focus on two parameters of CDM: the fraction \( f_{\text{CDM}} \) of dark matter and the mass \( M_{\text{CDM}} \). Current constraints are generally quite weak for \( f_{\text{CDM}} \lesssim 0.1 \) and in some mass windows.

It is worth mentioning that the mass range \( 10 M_\odot \leq M_{\text{CDM}} \leq 100 M_\odot \) has been receiving lots of attention especially after LIGO/VIRGO published their first successful detection [25]. It was pointed out that GW150914 might be a signature of PBH dark matter [20, 21]. However, the existing astronomical constraints about the abundance of PBHs in this mass range are too weak [17, 22] and thus not sufficient to test this conjecture. Independent and more powerful methods are quite needed. New methods based on lensing of transient sources like the Gamma Ray Bursts (GRBs) [23], the fast radio bursts (FRBs) [24] were proposed to probe this mass window since CDM as the lenses would alter the observed signals appearing as echoes. Remarkably, lensing of FRBs is expected be very promising since the intrinsic duration of FRB is \( \sim \) msec, comparable to the time delay caused by the lens mass in this window. However, it will not be easy to apply this method in real data due to the unknown properties of FRBs. For example, the redshifts are quite uncertain for most of signals and it is degenerate between a split signal and the intrinsic structure.

Direct detection of GW by LIGO/VIRGO opened a new window for astronomy and cosmology [25]. The leasing of GW has been attracting the eyes of the community, especially in recent years [26-37]. GW is completely distinct from and complementary to the electromagnetic (EM) wave. The emitters of EM radiation are charged particles, so EM radiation is emitted within small regions due to the overall charge neutrality. EM wave has short wavelengths and emits independently from each part of the source. By contrast, GWs are emitted by the cumulative mass and the momentum of entire systems, so they have long wavelengths and are consistent inherently. Therefore, while lensing of light is described by geometric optics without considering the interference, lensing of GW should be described by wave optics in some cases [38-41]. The lens is like the diffraction barrier which distorts the GW waveform as the fringes. Lensing of GWs observed by advanced LIGO (aLIGO) was proposed to probe the CDM as the lenses [42]. While this idea is novel, the constraint power is quite weak for aLIGO, especially in the mass range...
10M⊙ ≤ M_{CDM} ≤ 100M⊙ [42]. Motivated by this, in this work, we consider the fringes observed by the third-generation ground-based detectors, for example, the Einstein Telescope (ET). We prove that ET could be the key to pin down some nature of dark matter.

The paper is organized as follows: In Section 2, we introduce the lensing of GW with wave optics description; In Section 3, we introduce the distributions of binary star mergers and the ET; The methodology and results are presented in Section 4; Finally, we summarize and make discussions in Section 5. Throughout this paper, we use the natural units of c = G = 1 in all equations.

**WAVE OPTICS DESCRIPTION**

For GW lensing, if the wavelength is much shorter than the lens mass scale, the GW travels along geodesic in the geometric optics limit as the light [43]. In this case, the interference of two signals would occur [44]. On the contrary, if the wavelength is much longer than the lens, it belongs to the wave optics limit. In the intermediate regime, diffraction effect should be considered, which is described by wave optics [35][39]. We consider the geometry of the compact DM is weak, the metric is given by

$$g_{\mu \nu} = g^{(L)}_{\mu \nu} + h_{\mu \nu}, \quad (1)$$

where $g^{(L)}_{\mu \nu}$ is determined by the Newtonian potential $(U)$ of the lens. $h_{\mu \nu}$ is the GW perturbation which can be separated with a scalar field and its polarization:

$$h_{\mu \nu} = \phi e_{\mu \nu}. \quad (2)$$

Since the gravity field is weak, the polarization is taken as constant. Thus we take GW as a scalar wave whose evolution is determined by

$$\partial_\mu (\sqrt{-g^{(L)}} g^{(L)}_{\mu \nu} \partial_\nu \phi) = 0. \quad (3)$$

Using Newtonian potential, we rewrite it for given frequency $f$ as

$$(\Delta + 4\pi^2 f^2) \tilde{\phi} = 16\pi^2 f^2 U \tilde{\phi}. \quad (4)$$

To quantify the impacts of the lens on the waveform, we follow [39] to define the dimensionless amplification factor as

$$F(f) = \tilde{\phi}^L(f)/\tilde{\phi}(f), \quad (5)$$

where $\tilde{\phi}^L$ and $\tilde{\phi}$ are the lensed and unlensed $(U = 0)$ amplitudes in frequency domain, respectively. As diffraction, the observed signal is the superposition of all possible waves on the lens plane that have different time delays corresponding to different phases. Therefore $F(f)$ is given by [39]

$$F(f) = \frac{D_s R_E^2(1 + z_l) f}{D_l D_t s} \int d^2x \ exp \left[ 2\pi i t_d(x, y) \right], \quad (6)$$

where $R_E$ is the Einstein radius, $x, y$ are the impact position in the lens plane and source position in units of $R_E$. $t_d$ is the travelling time for any path on the lens plane:

$$t_d(x, y) = \frac{D_s R_E^2(1 + z_l)}{D_l D_t s} \left[ \frac{1}{2} |x - y|^2 - \psi(x) + \phi_m(y) \right], \quad (7)$$

where $\psi(x)$ is dimensionless deflection potential and $\phi_m(y)$ is chosen such that the minimum arrival time is zero. The CDM can be regarded as a point mass, whose potential $\psi(x) = \ln x$ and $\phi_m(y) = 0.5 \ (x_m - y)^2 - \ln x_m$ with $x_m = 0.5 \ (y + \sqrt{y^2 + 4})$. In such case, Eq. 6 can be written as

$$F(f) = \exp \left\{ \frac{\pi w}{4} + \frac{i w}{2} \left[ \ln \left( \frac{w}{2} \right) - 2\phi_m(y) \right] \right\} \times \Gamma \left( 1 - \frac{i w}{2} \right) F_1 \left( \frac{i w}{2}, 1, \frac{i w}{2} y^2 \right), \quad (8)$$

where the dimensionless parameter $w = 8\pi M_{CDM}(1 + z_l) f$, which serves as a comparison between the barrier scale (Schwarzschild radius $R_s = 2M_{CDM}$) and the wavelength $[\lambda \simeq 2 \times 10^3(100Hz/f)M⊙]$.  

**BINARY STAR MERGERS AND THE EINSTEIN TELESCOPE**

Following our previous works [27][30], we consider three classes of the dual compact objects including neutron star binaries (NS-NSs), neutron star-black holes (NS-BHs) and black hole binaries (BH-BHs). We adopt the inspiral standard scenarios with two metallicity evolution conditions including “low-end” and “high-end” cases. Figure 1 illustrates the distributions of the yearly merger events of the three classes as a function of redshift. For the BH-BH systems, the masses for each BH are considered in the range $[80M⊙−H⊙]$, which contain the detailed population synthesis calculations predicted by the StarTrack evolutionary code[54]. We consider the standard scenarios with two metallicity evolution conditions including “low-end” and “high-end” cases. Figure 1 illustrates the distributions of the yearly merger events of the three classes as a function of redshift. For the BH-BH systems, the masses for each BH are considered in the range $[80M⊙−H⊙]$, which contain the detailed population synthesis calculations predicted by the StarTrack evolutionary code[54]. We consider the standard scenarios with two metallicity evolution conditions including “low-end” and “high-end” cases. Figure 1 illustrates the distributions of the yearly merger events of the three classes as a function of redshift. For the BH-BH systems, the masses for each BH are considered in the range $[80M⊙−H⊙]$, which contain the detailed population synthesis calculations predicted by the StarTrack evolutionary code[54]. We consider the standard scenarios with two metallicity evolution conditions including “low-end” and “high-end” cases. Figure 1 illustrates the distributions of the yearly merger events of the three classes as a function of redshift. For the BH-BH systems, the masses for each BH are considered in the range $[80M⊙−H⊙]$, which contain the detailed population synthesis calculations predicted by the StarTrack evolutionary code[54]. We consider the standard scenarios with two metallicity evolution conditions including “low-end” and “high-end” cases. Figure 1 illustrates the distributions of the yearly merger events of the three classes as a function of redshift. For the BH-BH systems, the masses for each BH are considered in the range $[80M⊙−H⊙]$, which contain the detailed population synthesis calculations predicted by the StarTrack evolutionary code[54]. We consider the standard scenarios with two metallicity evolution conditions including “low-end” and “high-end” cases. Figure 1 illustrates the distributions of the yearly merger events of the three classes as a function of redshift. For the BH-BH systems, the masses for each BH are considered in the range $[80M⊙−H⊙]$, which contain the detailed population synthesis calculations predicted by the StarTrack evolutionary code[54]. We consider the standard scenarios with two metallicity evolution conditions including “low-end” and “high-end” cases. Figure 1 illustrates the distributions of the yearly merger events of the three classes as a function of redshift. For the BH-BH systems, the masses for each BH are considered in the range $[80M⊙−H⊙]$, which contain the detailed population synthesis calculations predicted by the StarTrack evolutionary code[54]. We consider the standard scenarios with two metallicity evolution conditions including “low-end” and “high-end” cases. Figure 1 illustrates the distributions of the yearly merger events of the three classes as a function of redshift. For the BH-BH systems, the masses for each BH are considered in the range $[80M⊙−H⊙]$, which contain the detailed population synthesis calculations predicted by the StarTrack evolutionary code[54]. We consider the standard scenarios with two metallicity evolution conditions including “low-end” and “high-end” cases. Figure 1 illustrates the distributions of the yearly merger events of the three classes as a function of redshift. For the BH-BH systems, the masses for each BH are considered in the range $[80M⊙−H⊙]$, which contain the detailed population synthesis calculations predicted by the StarTrack evolutionary code[54]. We consider the standard scenarios with two metallicity evolution conditions including “low-end” and “high-end” cases. Figure 1 illustrates the distributions of the yearly merger events of the three classes as a function of redshift. For the BH-BH systems, the masses for each BH are considered in the range $[80M⊙−H⊙]$, which contain the detailed population synthesis calculations predicted by the StarTrack evolutionary code[54]. We consider the standard scenarios with two metallicity evolution conditions including “low-end” and “high-end” cases.
by three orders of magnitude promising tens to hundreds thousands of detections per year. ET consists of three Michelson interferometers with 10 km long arms, arranged to form an equilateral triangles. Two sensitivity estimates have been put forward for ET. One is the xylophone design (ET-D) in which one detector is arranged to form an equilateral triangles. Two sensitivity curves for aLIGO and ET. The lensing optical depth for a given GW event at \( z_s \) is the probability that the event falls into the perceptible region of any CDM along the line of sight:

\[
\tau(M_{CDM}, z_s, S) = \frac{1}{2} \int_0^{z_S} d\chi(z_L)(1+z_L)^2 n_L \sigma(M_{CDM}, z_L, S),
\]

and

\[
\sigma(M_{CDM}, z_L, S) = 4\pi M_{CDM} \frac{D_L D_{LS}}{D_S} y^{2 \max}(M_{CDM}, z_L, S),
\]

where \( S = \{M_z, d_L, \phi_0, F, S_n(f), f_0, f_1\} \) is determined by the GW source itself and the detector such that the lensed signal is detectable. Specifically, for detecting a GW signal, the signal-to-noise ratio (SNR) must be large enough:

\[
\sqrt{S_n(f)} = S_0 \left( 2.39 \times 10^{-27} x^{-15.64} + 0.349 x^{-2.145} + 1.76 x^{-0.12} + 0.409 x^{1.1} \right),
\]
To summarize our pipeline for determining the constraints on $f_{DM} - M_{CDM}$ from one-year observation by ET:

1. Set $f_{CDM} = 1$ as the benchmark at first;
2. Given $M_{CDM}$, for each event in the catalog, we calculate $y_{\text{max}}$ at different lens redshift $z_L$ that satisfy our criteria;
3. Get $\tau$ of the event according to Eq. (21);
4. Sum up the $\tau s$ of all events as the expected number of detectable lensed signals;
5. Scale $f_{CDM}$ such that the expected number of lensed signal is unit;
6. Repeat from Step 2 for different $M_{CDM}$.

The constraint results are shown in Fig. 3. The curves stand for the case where one lensed signal is expected to be detected. Therefore, if no lensed signal is found by ET, the parameter space above the curves will be excluded. As one can see, ET is able to constrain the mass down to $\sim 5M_\odot$, consistent with [52]. For mass range $M_{CDM} = 10M_\odot - 100M_\odot$, $f_{CDM}$ will be $10^{-2} - 10^{-5}$, and for mass $> 100M_\odot$, it gradually gets close to $\sim 10^{-5}$, which is the best constraint among the current methods. For comparison, we also consider aLIGO. The constraints are much weaker, and it is unavailable for mass $< 100M_\odot$. Our results are consistent with [42], though they claimed $M_{CDM} > 20M_\odot$ is detectable at the level of $f_{CDM} \sim 0.1$ for $SNR_{\text{test}} > 3$ and for the optimistic black hole model (see Fig. 5 therein). It would be interesting to compare our results with theirs since we use different binary star models. Despite the small differences, all the results should manifest that aLIGO is quite weak for this mass window. Therefore, for the purpose of detecting CDM in this range, ET will be more promising. We also compare our results with current constraints.

**CONCLUSION AND DISCUSSIONS**

Whether dark matter exists in the form of compact objects is one of the most interesting problems for physicists and cosmologists. Compact dark matter could play the role of the lens, leaving the fringes in the detected gravitational waves. We have proved that its mass and fraction can be well understood especially in the mass range $M_{CDM} = 10M_\odot - 100M_\odot$ by the Einstein Telescope. The density fraction can be constrained at the level of $\sim 10^{-2} - 10^{-5}$ in such range for a null search of lensed signals. It would be visible to add the lensing effects to the standard pipeline of searching GWs. Constraints in this mass window would supplement the...
knowledge of the nature and origin of dark matter since no evidence of CDM has been found in other windows currently.

In this work, note that a constant mass function was assumed as all the analysis did in the literature. In fact, CDM may have its own mass function, for example, a power-law form. Further works combining all the methods need to be done to distinguish the mass functions.

Our analysis is from a statistical point of view. We only take the information that a GW signal is lensed or not. Actually, once a lensed signal is confirmed, further information of the lens mass can be extracted based on the parameter estimation. The redshifted lens mass can be well constrained [50], thus the mass of CDM can be pinned down in a certain order of magnitude with an uncertainty coming from the lens redshift (0 < zL < zS), further reducing the parameter space of CDM.

In addition to ET, another proposed US-based third-generation GW detector is Cosmic Explorer (CE) which will keep L-shaped configuration with 40 km arm lengths. A network of two or more third-generation detectors would further enhance the constraints.

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