THEORY OF QUARK-GLUON PLASMA AND PHASE TRANSITION.

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Abstract

Nonperturbative picture of strong interacting quark-gluon plasma is given based on the systematic Field Correlator Method. Equation of state, phase transition in density-temperature plane is derived and compared to lattice data as well as subsequent thermodynamical quantities of QGP.

1 Introduction

The perturbative exploring of quark-gluon plasma (QGP) has some difficulties in describing the physics of QGP and phase transitions. However, it was realized 30 years ago that nonperturbative (np) vacuum fields are strong ([1]) and later it was predicted ([2]) and confirmed on the lattice ([3]) that the magnetic part of gluon condensate does not decrease at \( T > T_c \) and even grows as \( T^4 \) at large \( T \) ([4]).

Therefore it is natural to apply the np approach, the Field Correlator Method (FCM) ([5]) to the problem of QGP and phase transitions, which was done in a series of papers ([6]-[10]). As a result one obtains np equation of state (EoS) of QGP and the full picture of phase transition, including an unbiased prediction for the critical temperature \( T_c(\mu) \) for different number of flavors \( n_f \).

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2 Nonperturbative EoS of QGP

We split the gluonic field $A_\mu$ into the background field $B_\mu$ and the (valence gluon) quantum field $a_\mu$: $A_\mu = B_\mu + a_\mu$ both satisfying the periodic boundary conditions.

The partition function averaged both in perturbative and np fields is

$$Z(V,T) = \langle Z(B + a) \rangle_{B,a}$$

(1)

Exploring free energy $F(T,\mu) = -T \ln \langle Z(B) \rangle_B$ that contains perturbative and np interactions of quarks and gluons (which also includes creation and dissociation of bound states) we follow so-called Single Line Approximation (SLA). Namely, we assume that quark-gluon system for $T > T_c$ stays gauge invariant, as it was for $T < T_c$, and neglect all perturbative interactions in the first approximation. Nevertheless in SLA already exist a strong interaction of gluons (and quarks) with np vacuum fields. This interaction consists of colorelectric (CE) and colormagnetic (CM) parts. The CE part in deconfinement phase creates np self-energy contribution for every quark and gluon embedded in corresponding Polyakov line. An important point is that Polyakov line is computed from the gauge invariant $q\bar{q}$ (gg) Wilson loop, which for np $D_1^E$ interaction splits into individual quark (gluon) contributions. As for CM part - its consideration is beyond the SLA, because as has been recently shown in paper ([11]) strong CM fields are responsible for creation of bound states of white combinations of quarks and gluons.

To proceed with FCM we apply the nonabelian Stokes theorem and the Gaussian approximation to compute the Polyakov line in terms of np field correlators

$$L_{fund} = \frac{1}{N_c} tr \ P \exp \left( ig \int_0^\beta B_4(z) dz^4 \right) =$$

$$\frac{1}{N_c} tr \ \exp \left( -\frac{g^2}{2} \int_{S_u} \int_{S_u} d\sigma_{\mu\nu}(u) d\sigma_{\lambda\sigma}(v) D_{\mu\nu,\lambda\sigma} \right)$$

with

$$D_{\mu\nu,\lambda\sigma} \equiv g^2 \langle F_{\mu\nu}(u) \Phi(u,v) F_{\lambda\sigma}(v) \Phi(v,u) \rangle$$

(2)

$D_1^E$ and $D^E$ arise from CE field strengths:

$$\frac{1}{N_c} D_{0i,0k} = \delta_{ik} \left[ D^E + D_1^E + u_i^2 \frac{\partial D_1^E}{\partial \bar{u}^2} \right] + u_i u_k \frac{\partial D_1^E}{\partial \bar{u}^2}$$

(3)
As a result the Polyakov loop can be expressed in terms of "potentials" $V_1$ and $V_D$

$$L_{fund} = \exp \left( -\frac{V_1(T) + 2V_D}{2T} \right), \quad L_{adj} = (L_{fund})^{9/4}, \quad (4)$$

with $V_1(T) \equiv V_1(\infty, T), \quad V_D \equiv V_D(r^*, T) \quad (5)$

$$V_1(r, T) = \int_0^\infty d\nu (1 - \nu T) \int_0^r d\xi \xi D_1^E(\sqrt{\xi^2 + \nu^2}) \quad (5)$$

$$V_D(r, T) = 2 \int_0^\infty d\nu (1 - \nu T) \int_0^r d\xi (r - \xi) D_1^E(\sqrt{\xi^2 + \nu^2}) \quad (6)$$

In what follows we use the Polyakov line fit $\quad (9, 10)$

$$L_{fund} \left( x = \frac{T}{T_c}, T \right) = \exp \left( -\frac{.175\text{Gev}}{(1.35x - 1)T} \right) \quad (7)$$

The free energy $F(T)$ of quarks and gluons in SLA can be expressed as a sum over all Matsubara winding numbers $n$ with coefficients $L^n_{fund}$ and $L^n_{adj}$ for quarks and gluons respectively. For nonzero chemical potential $\mu$ one can keep $L_{fund, adj}$ independent of $\mu$, treating np fields as strong and unchanged by $\mu$ in the first approximation.

The final formulas for pressure of qgp are $\quad (8, 10)$

$$p_q \equiv \frac{P_{SLA}^{SLA}}{T^4} = \frac{n_f}{\pi^2} \left[ \Phi_\nu \left( \frac{\mu - \frac{V_1}{2}}{T} \right) + \Phi_\nu \left( \frac{-\mu + \frac{V_1}{2}}{T} \right) \right] \quad (8)$$

where $\nu = m_q/T$ and

$$\Phi_\nu (a) = \int_0^\infty \frac{z^4}{\sqrt{z^2 + \nu^2}} \frac{1}{(e^{\sqrt{z^2 + \nu^2}} + 1)} \quad (9)$$

$$P_{gl} \equiv \frac{P_{gl}^{SLA}}{T^4} = \frac{8}{3\pi^2} \int_0^\infty \frac{z^3 dz}{e^{z^2/9V_1} - 1} \quad (10)$$

The energy density is $\varepsilon = \frac{T^2}{T^4} \frac{\partial P}{\partial T}$ and the speed of sound in plasma is $c_s^2 = \frac{\partial P}{\partial \varepsilon}$. In Fig.5 $c_s^2$ is shown calculated with the use of $(8, 10)$ and compared to lattice data for $\mu = 0$ from $(12)$. No lattice calculations has yet been done for sound speed at nonzero baryon density, though our theory allows to do that and as is shown in $(13)$ the result for $\mu > 0$ does not differ much from the case $\mu = 0$. 

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Figure 1: Fit (7) of Polyakov line for $n_f = 0$ and $n_f = 2$ (black curves) to the lattice data [12].

Figure 2: Analytic (8), (10) and lattice [12] curves for pressure of QGP with $n_f = 0, 2 + 1, 3$ from [10].

3 Phase transition

To obtain the curve of phase transition one needs to define pressure $P_I$ in the confined phase and $P_{II}$ in the deconfined phase, taking into account that vacuum energy density contributes to the free energy, and hence to the pressure:

$$P_I = |\varepsilon_{vac}| + P_{hardon}, \quad P_{II} = |\varepsilon_{vac}^{dec}| + (p_q + p_{gl})T^4. \quad (11)$$

Having formulas for pressure (which contain parameter of $L_{fund}(x)$ (7)) we may write for the phase transition curve $T_c(\mu)$:

$$T_c(\mu) = \left(\frac{(11 - \frac{2}{3}n_f)\Delta G_2}{32(p_q + p_{gl})}\right)^{1/4} \quad (12)$$

here $\Delta \varepsilon_{vac} = |\varepsilon_{vac} - \varepsilon_{vac}^{dec}| = (11 - \frac{2}{3}n_f)/32\Delta G_2$. In particular, for the expected value of $\Delta G_2/G_2(\text{stand}) \approx 0.4$ one obtains $T_c = 0.27$ GeV ($n_f = 0$), 0.19 GeV ($n_f = 2$), 0.17 GeV ($n_f = 3$) in good agreement with lattice data.

4 Summary

The EoS of QGP is written, where the only np input is the Polyakov line. It should be stressed, that only the modulus of the Polyakov line enters in
Figure 3: Analytic and lattice ([12]) curves for energy density of QGP with $n_f = 2 + 1$ and $n_f = 3$ from ([10]).

Figure 4: The phase transition curve $T_c(\mu)$ (in GeV) from ([9]) as function of quark chemical potential $\mu$ (in GeV) for $n_f = 2$ (upper curve) and $n_f = 3$ (lower curve) and $\Delta G_2 = 0.0034$ GeV$^4$ from ([9]).

Figure 5: Sound speed for $\mu = 0$ and $n_f = 3$ (blue dashed curve) compared to lattice data from ([12]).

EoS due to gauge invariance. The phase transition curve $T_c(\mu)$ and speed of sound $c_s^2(T)$ are obtained and agree well with lattice data. An important point of the work is that the only parameter used to receive the final physical quantities from the initial QCD Lagrangian is the Polyakov line taken from
lattice data, and is in agreement with analytic estimate for $T = T_c$ ([6]).

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