Topological phase transition in the quasiperiodic disordered Su-Schrieffer-Heeger chain

Tong Liu and Hao Guo

1Department of Physics, Southeast University, Nanjing 211189, China

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We study the stability of the topological phase in one-dimensional Su-Schrieffer-Heeger chain subject to the quasiperiodic hopping disorder. We investigate two different hopping disorder configurations, one is the Aubry-André quasiperiodic disorder without mobility edges and the other is the slowly varying quasiperiodic disorder with mobility edges. With the increment of the quasiperiodic disorder strength, the topological phase of the system transitions to a topologically trivial phase. Interestingly, we find the occurrence of the topological phase transition at the critical disorder strength which has an exact linear relation with the dimerization strength for both disorder configurations. We further investigate the localized property of the Su-Schrieffer-Heeger chain with the slowly varying quasiperiodic disorder, and identify that there exist mobility edges in the spectrum when the dimerization strength is unequal to 1. These interesting features of models will shed light on the study of interplay between topological and disordered systems.

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I. INTRODUCTION

Topological insulators (TIs), a unique class of electronic materials, have not only a bulk gap but also symmetry-protected gapless states localized at the sample boundaries. Topological features of TIs are expected to be immune to perturbations of the fluctuation and the environmental noise, and thereby have great potential applications in quantum information processing. Since the effect of disorder is inevitable in real materials, physical properties of topological systems in the presence of disorder have drawn considerable attention in the past decades. The combination of topology and disorder can induce rich novel quantum phenomena. For weak disorder, the robustness of topological states has been demonstrated for various topological systems, especially the quantum spin Hall states. With the increase of the disorder strength, the transition from topological phase to topologically trivial phase can occur. Besides destroying topological states, the medium strength disorder can induce the so called topological Anderson insulators (TAI), i.e., a topologically trivial state is driven into a topological state by disorder.

However, the direct observation of the influence of disorder on TIs is difficult due to the difficulty to precisely control the disorder in solid-state experiments. Benefited from the development in the manipulation of ultracold atoms, exploring both disorder and topology via the quantum simulation in artificial systems has become exercisable. In one-dimensional (1D) system the topological/trivial feature of an insulator is completely determined by the presence/absence of the chiral symmetry. The Su-Schrieffer-Heeger (SSH) model is one of the most simple and widely studied models belonging to the BDI symmetry class, which hosts two topologically distinguishable phases characterized by the winding number. Using a combination of Bloch oscillations and Ramsey interferometry, Ref. measures the Zak phase to study the topological feature of Bloch bands in a dimerized optical lattice described by 1D SSH model. A very recent work, in which a 1D chiral symmetric SSH chain with controllable off-diagonal (hopping) disorder is synthesized based on the laser-driven ultracold atoms, systematically explores the disorder effect on topology. They observe the robustness of topological state immune to weak random hopping disorder, and also the topologically non-trivial to trivial transition at very strong hopping disorder. More interestingly, they find the TAI phase in which the band structure of a topologically trivial chain is driven to a non-trivial one by adding the random hopping disorder.

On the other hand, the bichromatic optical lattice with incommensurate wavelengths have also attracted enormous attention in the study of Anderson localization. A notable work experimentally investigates the localization property of 1D Aubry-André (AA) model by use of ultracold atoms in the incommensurate/quasiperiodic optical lattice. The AA model has a self-dual symmetry and can undergo a sharp localization transition at the self-dual point without mobility edges. There also exists a class of systems with slowly varying quasiperiodic disorder which can host mobility edges. Motivated by the highly precise control in these ultracold atomic experiments, an interesting question can be raised: what is the fate of the topological non-trivial state when the quasiperiodic hopping disorder is introduced in 1D SSH chain, besides the random hopping disorder? To examine this question, we propose two quasiperiodic disordered SSH models with two different disorder configurations and investigate how the topological phase transition occurs.

The rest of the paper is organized as follows. In Sec. I, we investigate the topological phase transition in 1D SSH chain by adding the AA quasiperiodic disorder. In Sec. III, we investigate the topological phase transition in 1D SSH chain by adding the slowly varying quasiperiodic disorder. We also investigate the localized property (mobility edges) of this system besides the topological phase transition.
The SSH model describes a 1D chain of spinless fermions with alternating hopping strengths between the neighboring tight-binding lattices. For a chain with $L$ lattices and open boundary conditions, the Hamiltonian of the model is expressed as

$$
\hat{H} = -\sum_{i=1}^{L} \left( t_{i,i+1} \hat{c}_{i}^\dagger \hat{c}_{i+1} + H.c. \right),
$$

where $\hat{c}_{i}^\dagger$ ($\hat{c}_{i}$) is the fermion creation (annihilation) operator of the $i$-th lattice, $t_{i,i+1} = t + a(u_{i+1} - u_{i})$ is the nearest-neighbor hopping amplitude, $a$ is the displacement coupling constant and $u_{i}$ is the configuration coordinate for the displacement of the $i$-th lattice. In the ideal limit, the displacement is of the perfectly periodic configuration co-ordinate for the $i$-th lattice. In the ideal limit, the displacement is of the perfectly periodic situation with respect to a topologically trivial phase when the strength of the dimerization $\lambda$ exceeds a certain level. This is the ordinary SSH model. When $\lambda > 0$ the system is in the topological phase with the presence of twofold-degenerate zero-energy edge states at two ends, whereas when $\lambda < 0$ the system is in the topologically trivial phase without the presence of zero-energy edge states.

When the displacement becomes disordered, i.e., $u_{i+1} - u_{i} = (-1)^{i+1} \frac{\delta}{a}$, Eq. (1) can be written as

$$
\hat{H} = -\sum_{i=1}^{L} \left( t_{1} \hat{c}_{2i}^\dagger \hat{c}_{2i+1} + H.c. \right) - \sum_{i=1}^{\frac{L}{2}} \left( t_{2} \hat{c}_{2i+1}^\dagger \hat{c}_{2i+2} + H.c. \right),
$$

where the intracell hopping $t_{1} = t - \lambda - \delta_{i}$ and the intercell hopping $t_{2} = t + \lambda + \delta_{i}$ with $\delta_{i} = \delta \cos(2\pi \beta i + \phi)$ represent the AA quasiperiodic disorder, see Fig. 1 for illustration. The total Hamiltonian still maintains the chiral symmetry. In this paper we concentrate on the stability of the topological phase, so we only focus on the situation with $\lambda > 0$ hereafter. A typical choice of the parameters is $\delta > 0$, $\beta = (\sqrt{5} - 1)/2$ and $\phi = 0$. For convenience, $t = 1$ is set as the energy unit.

By numerically diagonalizing the Hamiltonian (2), we can get the eigenvalues and the wave-functions (denoted by $\psi$) of the system. We show the spectrum when $\lambda = 0.5$ under the open boundary condition in Fig. 1 where there is a regime with nonzero energy gaps in the range $\delta < 1.5$ and there are zero-energy modes in the midgap. As long as the chiral symmetry is preserved, zero-energy modes cannot be removed by the weak disorder. To show the difference between wave-functions with the zero-energy and nonzero-energy modes clearly, we plot the spatial distributions of wave-functions for the midmost excitations (the 5000th and 5001th eigenvalues) with different $\delta$'s. When $\delta = 1.4$, the energy gap is still finite, and the wave-functions with zero-energy modes are located at the left (right) end of the chain and decay very quickly away from the left (right) edge with no overlap, as shown in Fig. 1. However, when $\delta = 1.6$, the energy gap is closed, the zero-energy modes disappear and the amplitudes of the wave-functions of the midmost excitations overlap together and are located within a finite range of the whole chain. Therefore, these results demonstrate that the system can undergo a transition from a topological phase to a topologically trivial phase when the strength of the quasiperiodic disorder $\delta$ exceeds a certain level.

We now wonder if there exists a fixed value of $\delta$ which denotes the gap-closing point $\delta_{c}$. Due to the chiral symmetry, the eigenvalues appear in pairs. We arrange the eigenvalues in the ascending order and use $E^{n}$ to denote...
the smallest eigenvalue which is larger than the zero energy. Thus, $\Delta_g = 2E^*$ can explicitly determine the gap-closing point and denote the topological phase boundary between different topological phases due to the bulk-boundary correspondence. In Fig. 3 we plot the variation of energy gap $\Delta_g$ versus $\delta$ for different $\lambda$’s. It is clearly shown that there are finite gaps as $\delta$ is smaller than a critical value, i.e. $\delta < 1 + \lambda$, whereas the energy gaps vanish when $\delta > 1 + \lambda$. To visualize the result better we make a finite size analysis of the scaling behavior of $\Delta_g$ for $\lambda = 1$ as a function of the inverse of system size $1/L$ (Fig. 3). It exhibits an oscillating behavior at $\delta = 1.8$, 1.9, which indicates that the energy gap is finite in the regime with $\delta < 1 + \lambda$. Whereas the energy gap approaches zero at $\delta = 2.0$, 2.1, which indicates that $\delta = 1 + \lambda$ indeed denotes the gap-closing point. To strengthen the validity of our conclusion, we also systematically calculate the energy gap with other sets of $\delta$ and $\lambda$, and find the similar behaviors.

In general, the topological phase transition is characterized by the change of the topological invariant. Beyond the translational invariant system, it is more suitable to work with the transfer matrix formulation to identify the topological phase of a finite disordered chain. The transfer matrix formulation is designed to study the propagation of waves via a chain with arbitrary configurations. It thus offers a natural means of probing the localization nature of zero-energy modes and determining the topological characteristics of the SSH chain in the presence of the disorder. From the tight-binding Hamiltonian we can directly express the zero-energy transfer matrix $M$ in the site basis,

$$
\begin{pmatrix}
\hat{t}_{i+1,i} \hat{\psi}_i \\
\hat{\psi}_{i+1}
\end{pmatrix} = M_i
\begin{pmatrix}
\hat{t}_{i,i-1} \hat{\psi}_{i-1} \\
\hat{\psi}_i
\end{pmatrix},
$$

(3)

$$
M_i = \begin{pmatrix}
0 & \hat{t}_{i+1,i} \\
-1/\hat{t}_{i+1,i} & 0
\end{pmatrix}
$$

(4)

Waves at the two ends of the chain ($L$ is even) are related by the total transfer matrix

$$
M = M_L M_{L-1} \cdots M_2 M_1 = \begin{pmatrix}
Z & 0 \\
0 & 1/Z
\end{pmatrix}.
$$

(5)

Then we transform to a new basis with right-moving and left-moving waves separated in the upper and lower two components by means of the unitary transformation $\Omega^T \sigma_y \Omega^* = \sigma_z$ with $\sigma_y$ and $\sigma_z$ being the second and third Pauli matrices, and we get

$$
\mathcal{M} = \Omega^T M \Omega^* = \frac{1}{2Z} \begin{pmatrix}
Z^2 + 1 & Z^2 - 1 \\
Z^2 - 1 & Z^2 + 1
\end{pmatrix},
$$

(6)

where

$$
Z = (-1)^{L/2} \prod_{i=1}^{L/2} \frac{\hat{t}_{2i+1,2i} - 1}{\hat{t}_{2i,2i-1}}.
$$

(7)

We obtain the reflection amplitude $r$ and transmission amplitude $t$ from Eq (6)

$$
\begin{align*}
\hat{r} &= \frac{1 - Z^2}{1 + Z^2}, \\
\hat{t} &= \frac{2Z}{1 + Z^2}.
\end{align*}
$$

(8)

Recollecting Fig. 1 the spatial distributions of the wavefunctions of the midmost excitations for topologically distinct phases are completely different. The amplitudes associated with zero-energy modes in topological phase are

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**FIG. 3.** (Color online) $\Delta_g$ as a function of $\delta$ with various $\lambda$’s. The inset shows the finite size analysis of $\Delta_g$ near the critical point $\delta = 1 + \lambda$ ($\lambda = 1$ as an example).

**FIG. 4.** (Color online) The reflection probability $R$ and the transmission probability $T$ as a function of $\delta$ for systems with various $\lambda$’s.
exponentially localized at two ends of the chain with no overlap, whereas the amplitudes associated with topologically trivial phase overlap together and disperse within the whole chain. At the Fermi level (the zero energy), the transmission probability \( T = \text{t}t^* \) via the chain must be zero because there are no probability distributions of particles from one end to the other in the topological phase. Hence the reflection probability \( R = rr^* \) must be 1 due to the conservation of the probability current, i.e., a complete reflection process occurs. Whereas in topologically trivial phase both \( T \) and \( R \) of the Fermi level are finite and less than 1 due to finite probability distributions of particles within the whole chain.

Therefore, we can employ \( T \) and \( R \) as probes of the topological phase transition. In Fig. 4 we plot \( T \) and \( R \) of the midmost excitations for the various \( T \) illustrated, to the conservation of the probability current, i.e., a complete reflection process occurs. Whereas in topologically trivial phase both \( T \) and \( R \) of the Fermi level are finite and less than 1 due to finite probability distributions of particles within the whole chain.

In fact, the topological phase transition can be determined by the following mathematical conjecture. The limit of \( Z \) in Eq. (7) is expressed as,

\[
\lim_{L \to \infty} Z = \lim_{L \to \infty} (-1)^{L/2} \prod_{i=1}^{L/2} \frac{1 + \lambda + \delta \cos(2\pi \beta i)}{1 - \lambda - \delta \cos(2\pi \beta i)} \quad (9)
\]

It can be conjectured that if \( \delta < 1 + \lambda \) Eq. (9) diverges and accordingly \( R \to 1 \) and \( T \to 0 \) in the large \( L \) limit, whereas if \( \delta \geq 1 + \lambda \) Eq. (9) converges and accordingly \( 0 < R, T < 1 \). Although we cannot prove this mathematical conjecture, we believe that this problem may bring some interests to both the physical and mathematical communities.

III. THE SLOWLY VARYING QUASIPERIODIC DISORDER

In this section we investigate the effect of the slowly varying quasiperiodic disorder on the topological phase of 1D SSH chain. The expression of Hamiltonian \( H \) remains unchanged except the disorder term is replaced by \( \delta_i = \delta \cos(2\pi \beta v) \) with \( 0 < v < 1 \). This type of disorder is revealed by Sarma et al. 27,28, and can induce mobility edges located at the spectrum, unlike the AA disorder. Without loss of generality, we choose the parameter \( v = 0.5 \).

Following the similar procedure in the last section about the AA quasiperiodic disorder, we first investigate the location of the gap-closing point with the increasing of \( \delta \) for different \( \lambda \)'s. In Fig. 5 we plot the variation of the energy gap \( \Delta_g \). It is shown that there are finite gaps as \( \delta \) reaches a critical value: \( \lambda \). However, unlike the AA quasiperiodic disorder, the energy gap \( \Delta_g \) at this critical point still seems to be finite. In fact this is due to the finite-size effect, and we perform the finite-size analysis in the inset of Fig. 5. As shown, the scaling behaviors of \( \Delta_g \) when \( \lambda = 1 \) and \( \delta = 0.8, 0.9 \) vary smoothly, and \( \Delta_g \) is obviously finite in the large \( L \) limit. Whereas when \( \delta = 1.1, \Delta_g \) vanishes quickly as \( L \) increases. The interest situation is associated with the critical point \( \delta = 1 \) where \( \Delta_g \) seems to be finite when \( L \) is small, whereas the scaling behavior of \( \Delta_g \) exhibits a power-law decreasing trend as \( L \) increases. We numerically fit the varying curve and obtain the expression \( \Delta_g = 1.514L^{-0.3356} \). Hence \( \Delta_g \) vanishes at the critical point \( \delta = 1 \) as \( L \to \infty \), which
FIG. 7. (Color online) Eigenvalues and IPR as a function of the disorder strength $\delta$, with $\lambda = (a) \ 0.5$, (b) $1$, (c) $1.5$, (d) $2.5$. The red dots in the midgap represent nontrivial zero-energy modes. The blue solid lines represent the boundary between spatially localized and extended states, i.e., mobility edges $E_{\pm c}$. In the inset we plot the blow up of eigenvalues near $E = -2$.

Indicates that $\delta = \lambda$ denotes the gap-closing point. We systematically calculate the energy gap $\Delta_g$ with some other sets of $\delta$ and $\lambda$ and find all gap-closing points indeed stay at $\delta = \lambda$.

We also wonder if there exists a topological invariant in this slowly varying quasiperiodic disordered SSH chain by calculating the total transfer matrix. By implementing the same procedure, we plot the numerical results of $T$ and $R$ of the midmost excitations for $\lambda = 0.5$, 1.5 in Fig. (c). Surprisingly, $T$ always approaches 0 and $R$ always approaches 1 with the growth of the disorder strength $\delta$ even when $\delta > \lambda$. This result obviously differs from that by studying the gap-closing point. According to the bulk-boundary correspondence, zero-energy modes are protected by the energy gap, hence they vanish at the gap-closing point which is accompanied with the topological phase transition. To clarify this situation we plot the midmost excitation (the 5001th eigenvalue) in the inset of Fig. (c). It can be found that eigenvalues have almost the same numerical accuracy $10^{-16}$ with the increase of $\delta$, i.e., the slow varying disorder induces trivial zero-energy modes at the chain even if the energy gap closes. The similar phenomenon has been discovered by studying the topological phase transition for a one-dimensional $p$-wave superconductor in slowly varying incommensurate potentials. This is why the transfer matrix breaks down in this model since the validity of it is based on the assumption that the topological phase transition is accompanied with vanishing of zero-energy modes. Thus we have not found a topological invariant to denote the topological phase transition in the presence of the slowly varying quasiperiodic disorder, instead we conjecture that it oc-
the thermodynamic limit, while it is finite even if

\[
\lambda \neq 0
\]

are exactly located at

\[
\lambda = 0
\]

represent nontrivial zero-energy modes, which are the topo-

\[
\delta
\]

functions as a function of the disorder strength

\[
\lambda
\]

values of the system and IPR’s of the corresponding wave-

\[
\text{IPR}
\]

very large for a localized state. Figure 7 plots eigenval-

\[
E
\]

an extended state scales like

\[
\delta \text{ and } 1 ± 2
\]

The blow up of the inset shows the scaling behavior of IPR

\[
L
\]

for extended states.

\[
\text{IPR}
\]

occurs at the gap-closing point \( \delta = \lambda \).

Due to the slowly varying quasiperiodic disorder ac-

\[
\delta
\]

companied with mobility edges\(^{34–38}\), we wonder the loca-

\[
\text{mobility edges}
\]

in the spectrum when this type of disorder is introduced in 1D SSH chain. To clarify the localized property of this model we present numerical calculations for various sets of \((\lambda, \delta)\) to ensure that mobility edges in the spectrum are indeed located at \(E_{\pm c}\) and \(±2\).

To consolidate the results of mobility edges, we plot IPR of the corresponding wave-functions as a function of eigenvalues for various \(L\)'s in Fig. 8. As the eigenvalue varies, we find that IPR changes dramatically from the order of magnitude \(10^{-2}\) (a typical value for the localized states) to \(10^{-4}\) (a typical value for the extended states), or inversely at certain energies which exactly correspond to \(E_{\pm c}\) and \(±2\) as indicated by previous analysis. With the increase of \(L\), the magnitude of IPR of extended states becomes lower as shown in the inset, hence the change of IPR at these turning points becomes more abrupt. Thus, we deduce that there will be a jump at these turning points in the thermodynamic limit \(L \to \infty\). This jumping phenomenon of IPR indeed indicates that there exist mobility edges at \(E_{\pm c}\) and \(±2\) in the energy spectrum.

\[
\text{IPR}
\]

IV. CONCLUSIONS

In this work we have studied the topological phase transition in 1D SSH chain subject to two types of quasiperiodic hopping disorder, one is the AA quasiperiodic disorder \(\delta_i = \delta \cos(2\pi/\beta i)\) and the other is the slowly varying quasiperiodic disorder \(\delta_i = \delta \cos(2\pi/\beta i')\) with \(0 < v < 1\), we find following interesting features of two models.

(1) When \(\delta_i = \delta \cos(2\pi/\beta i)\), the gap-closing point is located at the critical disorder strength \(\delta = 1 + \lambda\). We also calculate the transmission probability and the reflection probability by applying the transfer matrix method, which clearly demonstrates the topological phase transition indeed occurs at \(\delta = 1 + \lambda\), i.e. the gap-closing point.

(2) When \(\delta_i = \delta \cos(2\pi/\beta i')\), the gap-closing point is located at the other critical disorder strength \(\delta = \lambda\). However, the transfer matrix breaks down in this model, and we conjecture that the topological phase transition

\[
\text{IPR}
\]
occurs at the location of the gap-closing point. We further investigate the localized property of this model, and identify that there exist mobility edges in the spectrum when the dimerization strength is unequal to 1.

In general the random disorder will shift the separation point between the topologically trivial and non-trivial phases of the translation-invariant system into a finite region under the finite size, and we may get a smooth curve instead of a step function with respect to the topological phase transition in the parameter space. However, by varying the parameters of these models the sharp topological phase transitions emerge. We believe that these interesting features and open questions (such as the convergence of Eq. (9)) will bring new perspectives to a wide range of topological and disordered systems.

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* Corresponding author: guohao.ph@seu.edu.cn
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