ALGORITHMS FOR CONTROL OF LONGITUDINAL MOTION OF A TWO-WHEEL EXPERIMENTAL SAMPLE

The subject of study is the process of forming algorithms for controlling the angular and translational movements of a two-wheeled experimental sample (TWES). The aim is to develop approaches to the formation of control algorithms for the translational and angular movements of a non-stationary automatic control object. Tasks: to concretize the process of synthesis of a control algorithm by state according to the criterion of the minimum integral of the weighted error modulus for a linear mathematical description of an automatic control object in the state space. Form a block diagram of an automatic control system by the state. Improve the approach to the synthesis of output control algorithms for mathematical description in the frequency domain of short-period and long-period motions of TWES. Illustrate the peculiarity of the approach using a specific example of a TWES under control and disturbing influences. Develop a simulation scheme in the Simulink environment and investigate responses to external step influences. Develop an approach to the formation of control algorithms by the diagnosis of TWES as an object of automatic control. Describe the procedure and means of deep diagnostics of emergencies of TWES. Develop algorithms for restoring the operability of the automatic rational control system. Used methods are a method of state space, the method of relative functions, the method of transfer functions, the method of optimization by integral criterion, the method of synthesis by logarithmic asymptotic frequency characteristics, methods of diagnosing and restoring operability. The following Results: three approaches were formed to the formation of control algorithms of the angular and translational movements of the TWES using linear mathematical descriptions in the time and frequency domains. Conclusions. The scientific novelty lies in the formation of approaches to the combined control of angular and translational movements, considering the structural and parametric features of the mathematical descriptions of TWES.

Keywords: two-wheeled experimental sample; automatic control object; state control; exit control; control by diagnosis.

Introduction

Motivation. Unstable movements is typical for various classes of aircraft, both in the longitudinal and lateral planes. The study of such movements on the ordinary experimental samples and the search for ways to stabilize them is a productive way, as in theory and in practice of automation. In theory regarding design features of experimental samples allow to form effective algorithms providing a constructive compromise between stability and motion quality indicators. In theory considering of experimental samples design features allow to form effective algorithms providing a constructive compromise between stability and motion quality indicators. In practice implementation of digital control algorithms for such movements allow to achieve a rational distribution of control functions between an automatic control system hardware and software [1–4].

One of the ordinary unstable movements is movement in the longitudinal plane. There have been developed and manufactured a two-wheeled experimental sample for scientific and practical researches on the stabilization of unstable movements (fig. 1).

Models of motion of a two-wheeled experimental sample (TWES) as an object of automatic control were obtained as a result of analytical studies and are described in [5].

Research status. There are methods and tools for synthesizing control algorithms in the theory of auto-

Fig. 1. Two-wheeled experimental sample
matic control for various generalized mathematical models of unstable control objects, both in time and frequency domains [6].

The transition from general approaches to specific practical problems solution rises a number of issues that require additional research, considering the specifics of a control object and conditions for its functioning. Thus, in [1, 4], traditional PID controllers are used to control a two-wheeled robot, which do not consider the structural features of the mathematical description. In [7, 8], the PID controller on the microcontroller board is used only for a self-balancing two-wheeled robot. Adaptive control in work [9] is formed under conditions of indefinite dynamics of an inverted pendulum only to ensure stable motion. A nonlinear method for controlling an inverted wheeled pendulum is proposed in [10, 11], which leads to a large expenditure of resources for motion control.

As follows from the reviewed publications, when using mathematical models of physical objects, it becomes possible, considering both specific structural and parametric features, to form productive approaches to the development of control algorithms.

**Purpose and structure.** The article proposes constructive approaches to the formation of control algorithms for a two-wheeled experimental sample by using linear mathematical models of an automatic control object in the time and frequency domains. The approaches are focused on the combined control of angular and translational movements, considering the structural and parametric features of the TWES mathematical descriptions.

The first section of the article describes an approach to state control with an incompletely measurable state vector. An algorithm for recovering three components of the state vector using a third-order filter is described. According to the criterion of the minimum of the integral of the weighted modulus of the error, the optimal characteristic polynomial of the fifth order is formed, from which the values of the coefficients of the control algorithm are calculated. The block diagram of the TWES control system is presented.

In the second section, an approach based on the use of logarithmic asymptotic frequency characteristics is presented. On a specific example, the possibilities of synthesizing sequential correcting devices that provide the required quality of control of the TWES movements are illustrated. The results of modeling in the Simulink environment are presented.

The third section presents a new approach to the control of TWES under destabilizing influences. The approach is based on the control by diagnosis principle using. The algorithms of deep diagnostics for specific destabilizing influences and algorithms for the operability restoring, which ensure the rational control of the TWES, are described.

### 1. State control algorithms

The algebraic methods wide using in various studies has led to the appearance of the analysis and synthesis section in the state space in the modern control theory [6].

The translational motion of a two-wheeled experimental sample as an object of automatic control in the state space [5] is represented using vector-matrix equations reflecting the structural features:

\[
\begin{bmatrix}
    x_1(t) \\
    x_2(t) \\
    x_3(t) \\
    x_4(t) \\
    x_5(t)
\end{bmatrix}
= \begin{bmatrix}
    a_{21} & 0 & 0 & 0 & 0 \\
    0 & a_{25} & 0 & 0 & 0 \\
    0 & 0 & 1 & 0 & 0 \\
    0 & 0 & 0 & 1 & 0 \\
    0 & 0 & a_{54} & a_{55} & 0
\end{bmatrix}
\begin{bmatrix}
    x_1(t) \\
    x_2(t) \\
    x_3(t) \\
    x_4(t) \\
    x_5(t)
\end{bmatrix}
+ \begin{bmatrix}
    0 \\
    0 \\
    0 \\
    0 \\
    b_{f2}
\end{bmatrix}
\begin{bmatrix}
    f(t) \\
    x_3(t) \\
    x_4(t) \\
    x_5(t)
\end{bmatrix}
+ \begin{bmatrix}
    0 \\
    0 \\
    0 \\
    0 \\
    0
\end{bmatrix}.
\]

In compact form, these equations looks like:

\[
\begin{bmatrix}
    x_1(t) \\
    x_2(t) \\
    x_3(t) \\
    x_4(t) \\
    x_5(t)
\end{bmatrix}
= \begin{bmatrix}
    a_{11} & 0 & 0 & 0 & 0 \\
    0 & a_{23} & 0 & 0 & 0 \\
    0 & 0 & c_{23} & 0 & 0 \\
    0 & 0 & 0 & c_{23} & 0 \\
    0 & 0 & 0 & 0 & c_{23}
\end{bmatrix}
\begin{bmatrix}
    x_1(t) \\
    x_2(t) \\
    x_3(t) \\
    x_4(t) \\
    x_5(t)
\end{bmatrix}
+ \begin{bmatrix}
    0 \\
    0 \\
    0 \\
    0 \\
    0
\end{bmatrix}.
\]

To form control algorithms by state, the automatic control object must satisfy the following conditions:

1. be completely manageable;
2. be fully observable;
3. the state vector \( x(t) \) measurable.

The analysis of the equations of motion of the TWES as an object of automatic control according to the criterion of R. Kalman, presented in [5], testifies to its complete controllability and observability. The third condition on the availability of measurement of all components of the state vector \( x(t) \) is not satisfied, which follows from the structure of Eq. (2), namely, only two components of the state vector \( x_1(t) \) and \( x_3(t) \) available to measurements \( y_1(t) \) and \( y_2(t) \).
Missing components of the state vector $x_2(t)$, $x_4(t)$ and $x_5(t)$ can be identified using various filters [6]. As applied to the structure of equation (1), the ordinary filter can be an incomplete order filter, since two components of the state vector are measurable. It is obvious from equation (1) that the initial conditions for the components $x_2(t)$, $x_4(t)$ and $x_5(t)$ is equal to zero. This circumstance allow to form the ordinary third order filter described by the following vector-matrix equation:

$$
\begin{bmatrix}
\dot{x}_2(t) \\
\dot{x}_4(t) \\
\dot{x}_5(t)
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & a_{25} \\
0 & 0 & 1 \\
0 & a_{54} & a_{55}
\end{bmatrix}
\begin{bmatrix}
\dot{x}_2(t) \\
\dot{x}_4(t) \\
\dot{x}_5(t)
\end{bmatrix} +
\begin{bmatrix}
a_{21} \\
0 \\
0
\end{bmatrix}
y_1(t) +
\begin{bmatrix}
b_{12} \\
b_5 \\
0
\end{bmatrix} u(t) +
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} f(t);
$$

where $\dot{x}_2(t)$, $\dot{x}_4(t)$ and $\dot{x}_5(t)$ – estimates of the corresponding components of the state vector.

Presence of disturbance inaccessible to measurement $f(t)$ in component assessment $\dot{x}_2(t)$ leads to its displacement by the amount $b_{12}f(t)$. In a two-wheeled experimental sample, the perturbing effect is constant and is due to the friction force [5]. With a certain quality of the contacting surfaces in some cases, the friction force can be neglected in comparison with the torque of the wheels. Considering Eq. (4), without the disturbing effect, the dynamics equations (1) with measurable state variables can be represented in the following form:

$$
\begin{bmatrix}
y_1(t) \\
\dot{x}_2(t) \\
y_3(t) \\
\dot{x}_4(t) \\
\dot{x}_5(t)
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a_{25} & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a_{54} & a_{55}
\end{bmatrix}
\begin{bmatrix}
y_1(t) \\
\dot{x}_2(t) \\
y_3(t) \\
\dot{x}_4(t) \\
\dot{x}_5(t)
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
b_5
\end{bmatrix} u(t); 
\begin{bmatrix}
y_1(t) \\
\dot{x}_2(t) \\
y_3(t) \\
\dot{x}_4(t) \\
\dot{x}_5(t)
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}.
$$

The equation representation in a compact form is:

$$
\dot{x}(t) = A^* x(t) + b u(t); \dot{x}(t_0) = 0, \quad (6)
$$

where $\dot{x}(t)$ – vector of the identified state of the object;

$A^*$ – a new matrix of coefficients.

To control the object relative to the zero state, control actions are formed in accordance with the equation

$$
u(t) = k^T \dot{x}(t), \quad (7)
$$

where $k^T$ is a row vector of feedback coefficients. Substituting the expressions for control actions into equation (6) following equation is got:

$$
\dot{x}(t) = A^* \dot{x}(t) + b k^T \dot{x}(t) = D \dot{x}(t). \quad (8)
$$

A necessary condition for the operability of a closed control loop is to find the roots of the characteristic equation

$$
\det[I - D] = s^5 + d_4 s^4 + d_3 s^3 + d_2 s^2 + d_1 s + d_0 = 0 \quad (9)
$$

in the left half-plane of the complex automatic control system.

A sufficient condition for the operability of the automatic control system consists in meeting the requirements for the quality of the transient process: the time of the transient process, overshoot, accuracy and other indicators of the response to step input actions. A sufficient condition for operability imposes restrictions on the area of placement of roots in the left half plane of the plane $s$. The necessary condition for operability conflicts with the sufficient condition. This contradiction can be resolved through a compromise. The process of finding a compromise is usually iterative. The number of iterative steps can be significantly reduced by using the optimal integral quality estimates for the error and normalized transient characteristics per step input signal [6]. So, for the integral of the weighted modulus of error

$$
I_{\varepsilon} = \int_0^t |e(t)| dt, \quad (10)
$$

where $t$ – time;

$e(t)$ – error signal, determined by the difference between the input signal and the feedback signal.

The characteristic polynomial with optimal values of the coefficients is:

$$s^5 + 2.8a_h s^4 + 5.0a_h^2 s^3 + 5.5a_h^3 s^2 + 3.4a_h^4 s + a_h^5, \quad (11)
$$

where $a_h$ – natural frequency of closed loop oscillations.

Setting the natural vibration frequency $a_h$ the numerical values of the optimal coefficients are determined. The coefficients of the characteristic equation (9) depend on the unknown coefficients of the row vector $k^T$. Based on the following relationships:
the numerical values of the unknown coefficients are calculated. This is the first step. The second iterative step is modeling an automatic control system in the Simulink environment and obtaining responses to step input control and disturbance actions. The obtained transient characteristics provide information about both the nature of the reaction and the quality indicators. Varying the value $\omega_n$, the numerical values of the row vector $\kappa^i$ coefficients could be obtained providing an acceptable compromise and an optimal value of the integral criterion (10).

The automatic control system based on the TWES state can be represented graphically using a compact structural diagram (fig. 2).

The automatic control system consists of two sub-systems: an automatic control object (ACO) and an automatic control device (ACD).

ACO includes mathematical models of electric drives that process the control signal $u(t)$, mathematical model of the body on wheels, mathematical models of the angular position sensors of the body and the displacement sensor – vector $y(t)$. The signals from the sensors are sent to the ACD, where they are compared with the vector of the reference influences $y_s(t)$. Difference signal vector $\Delta y(t)$ goes to the module that implements the transformation in accordance with the vector $\kappa^i$. The same module receives signals from a filter that restores components inaccessible to measurement $\hat{x}_2(t)$, $\hat{x}_4(t)$ and $\hat{x}_5(t)$ state vectors $x(t)$. The resulting transformation in the module $\kappa^i$ signal $u(t)$ enters the input of the OAU, providing stable and angular and longitudinal movement of the TWES in accordance with the setting action $y_s(t)$ in forward motion with a balancing position of the body.

2. Output control algorithms

The classical theory of automatic control is based on a number of provisions. One of them is to use for solving the problem of synthesis of signals from sensors that measure physical variables of control objects. There are many methods for synthesizing output control algorithms based on the use of transfer functions that reflect the transformation properties of automatic control objects in the frequency domain.

Let us represent in terms of physical quantities the transfer functions of the TWES as an object of automatic control, provided that the mass of the entire sample is greater than the body mass. Transfer function, reflecting the relation of the body inclination angle image $\Theta(s)$ on the electric drives control action image, is described by the following expression:

$$W_i(s) = \frac{\Theta(s)}{U(s)} = \frac{1}{M/L} s^2 + \frac{1}{\kappa_M} \frac{L}{s^2 + \frac{RI}{L} \frac{k_e}{s + \frac{M}{L} \Theta(s)}}. \quad (13)$$

where $M$ – the mass of the entire TWES; $g$ – acceleration of gravity; the rest of the designations correspond to the designations adopted in the previous article [5].

![Fig. 2. Block diagram of the automatic control system for the movement of TWES](image)
The transfer function of nonzero initial conditions of the body angular position is described by the following transfer function:

$$W_2(s) = \frac{\Theta(s)}{\Theta_0(s)} = \frac{s}{s^2 - \frac{g}{\ell}}.$$  \hspace{1cm} (14)

The nonzero initial conditions of the angular position of the TWES body represent a disturbing effect for translational motion, moreover, available for measurement using an angle sensor. This circumstance allow to use the perturbation control principle to parry nonzero angular positions of the body. This fact is described using the block diagram of the compensation circuit for non-zero angular positions of the body.

On the block diagram $W_c(s)$ - transfer function of the correcting element; $W_{s1}(s)$ – transfer function of the angle sensor.

For the practical implementation of the disturbance control principle, it is necessary that functional elements with transfer functions containing minimum phase links are used in the disturbance compensation circuit. Minimum-phase links are links whose poles and zeros are located in the left half-plane of the root plane. The minimum phase links have an unambiguous relationship between the frequency characteristics. Transfer functions $W_1(s)$ and $W_2(s)$ contain poles $s_1 = +\sqrt{\frac{g}{\ell}}$ located in the right half-plane of the root plane. Links with positive poles belong to the class of non-minimum phase links. Non-minimal-phase links are unstable links with a diverging transient characteristic and creating a large negative phase shift of the output signal relative to the input one. Thus, it is fundamentally impossible to stabilize the angular position of the body using the perturbation control principle.

To stabilize the angular position of the TWES body and its translational movement, the principle of deflection control has been used. For this purpose, it is necessary to form a structural diagram of the TWES as an object of automatic control. Let's describe functional relations using transfer functions.

Transfer function of electric drives, reflecting the relation of the wheels rotation angle image $\varphi(s)$ on the control action image $U(s)$, looks like this:

$$W_3(s) = \frac{\varphi(s)}{U(s)} = \frac{1}{L} \left( \frac{1}{s} + \frac{RI}{k_M} \frac{s}{s^2 + \frac{k_s}{k_M} \frac{k_2}{L}} \right).$$ \hspace{1cm} (15)

The transfer function of electric drives for the disturbing effect is described by the following expression:

$$W_4(s) = \frac{\varphi(s)}{F(s)} = \frac{1}{s} \left( \frac{I_{rs}}{1} - \frac{Rr}{s^2 + \frac{k_s}{k_M} \frac{k_2}{L}} \right).$$ \hspace{1cm} (16)

The transfer function reflecting the influence of electric drives on the angular position of the TWES body is:

$$W_5(s) = \frac{\varphi(s)}{\varphi(s)} = \frac{M \tau s^2}{I_k s^2 - P \ell}.$$ \hspace{1cm} (17)

Transfer function connecting the image of the traveled path $X(s)$ with the image of the angular change of the wheels of electric drives $\varphi(s)$, is described by an expression of the following form:

$$W_6(s) = \frac{X(s)}{\varphi(s)} = \frac{1}{M r}.$$ \hspace{1cm} (18)

The influence of the angular position of the body on the translational motion of the TWES can be represented using the transfer function:

$$W_7(s) = \frac{X(s)}{\Theta(s)} = \frac{mg}{M S \ell}.$$ \hspace{1cm} (19)

The disturbance has a negative effect on the distance traveled in accordance with the transfer function

$$W_8(s) = \frac{X(s)}{F(s)} = \frac{-1}{M S^2}.$$ \hspace{1cm} (20)

Fig. 3. Block diagram of the compensation circuit for non-zero angular positions of the TWES body.
Having received the transfer functions that describe all the relations between the variables of the TWES, the transfer functions have been developed for the body position angle sensor:

\[ W_8(s) = \frac{U_{s1}(s)}{\Theta(s)} = \kappa_{s1} \]  

and the distance traveled sensor

\[ W_9(s) = \frac{U_{s2}(s)}{X(s)} = \kappa_{s2} . \]  

The above transfer functions (14)-(21) allow to form a structural diagram of the TWES as an object of automatic control. The diagram is shown in Fig. 4.

The presence of disturbing influences \( F(s) \) and \( \Theta(s) \) to the object of automatic control necessitates the use of the deviation control principle for the control of the TWES. For the synthesis of control algorithms the classical method of logarithmic amplitude-frequency characteristics has been used.

In the automatic control object, as follows from the structural diagram, the angular deviation of the body is measured using appropriate sensors – image \( U_{s1}(s) \) and translational movement – image \( U_{s2}(s) \). This circumstance allow to produce separately the synthesis of algorithms for stabilizing the angular position of the body and the synthesis of algorithms for the translational movement of the TWES. The location of the graphs of changes in the amplitude and phase in the logarithmic coordinate system depends on the of the transfer functions parameters specific values. Therefore, the results of the synthesis of algorithms are presented for specific values of the parameters of the TWES given in Table 1.

As a result of calculations, the following transfer functions were obtained. For electric drives, due to the electromagnetic time constant and electromechanical time constant product littleness, the transfer function gets the following form:

\[ W_3(s) = \frac{\varphi(s)}{U(s)} = \frac{4.54}{s(0.0013 + 1)} . \]

For angular movement of the TWES body:

\[ W_4(s) = \frac{\Theta(s)}{\varphi(s)} = \frac{0.005s^2}{0.004s^2 - 1} . \]

---

**Table 1**

| Parameter                  | The quantity | Dimension   |
|----------------------------|--------------|-------------|
| M bogie weight             | 0.19         | kg          |
| m body weight              | 0.05         | kg          |
| \( I_k \) case moment of inertia | 2.82 \( 10^{-4} \) | kg·m² |
| I moment of inertia of wheels | 12.5 \( 10^{-4} \) | kg·m² |
| r wheel radius             | 0.03         | m           |
| \( \ell \) center of mass shoulder | 0.04 | m          |
| R winding resistance       | 2            | Ohm         |
| \( \kappa_e \) transmission ratio | 0.22 | V·sec   |
| \( \kappa_a \) electric motor transmission ratio | 5 \( 10^{-2} \) | kg·m²·sec² |
| L winding inductance       | 3 \( 10^{-3} \) | H           |
| \( \kappa_{s1} \) transmission ratio | one | V/deg |
| \( \kappa_{s2} \) transmission ratio | one | V/cm |

---

**Fig. 4. Structural diagram of TWES**
For the TWES trolley:
\[ W_5(s) = \frac{X(s)}{\varphi(s)} = 0.22. \]

To link the angular movement of the body with the translational movement
\[ W_6(s) = \frac{X(s)}{\Theta(s)} = \frac{-2.58}{s^2}. \]

Fig. 5 shows the logarithmic frequency characteristics for the three transfer functions \( W_3(s), W_4(s) \) and \( W_8(s) \) series connection describing the short-period motion of the TWES body.

Comparison of the available asymptotic logarithmic characteristic \( L_p \) with available phase characteristic \( \varphi_a \) indicates the unstable nature of the angular motion.

From the conditions for ensuring stable angular motion and the transient process quality, the desired asymptotic amplitude and phase characteristics \( L_d \) and \( \varphi_d \) are formed.

Graphical difference \( L_a - L_d \) allowed to form the asymptotic characteristic of the correcting device \( L_c \), on the basis of which the transfer function is formed in the following form:

\[ W_{10}(s) = \frac{U(s)}{U_{z1}(s)} = \frac{10(56s+1)(0.064s+1)(0.001s+1)}{s^2}. \]

This transfer function reflects the structure and parameters of the formation of the algorithm for controlling electric drives according to the signal from the angle sensor.

The long-period movement of the trolley, considering the stabilized short-period movement of the body, is represented by the positioned asymptotic \( L_d \) and phase \( \varphi_d \) characteristics in fig. 6.

The figure shows desired asymptotic \( L_a \) and phase \( \varphi_a \) characteristics formed according to the ensuring stability margins and quality indicators conditions. Difference characteristic \( L_c \) reflects the nature and parameters of the control algorithm based on signals from the displacement sensor. Transfer function describing \( L_c \) looks like this:

\[ W_{11}(s) = \frac{U(s)}{U_{z2}(s)} = \frac{2(3.6s+1)}{s(0.064s+1)(0.023s+1)}. \]

Fig. 7 shows the simulation scheme in the Simulink environment.

![Logarithmic characteristics of the open-loop SAS: a – LAFC; b – LPFC](image-url)
When modeling the disturbing influences, the ordinary transfer functions of proportional links were used in order to study the response of the TWES to more stringent operating conditions.

Fig. 8 shows the diagrams of signals of the control system when setting the control action on the translational movement of the trolley.

When specifying the translational movement of the trolley \( v(t) = 0.05 \cdot 1(t) \), the trolley (fig. 8, b) with overshoot after 8 sec moves by a given value. In this case, the initial deviation of the body \( \varphi \) after two oscillations it stabilizes by 8 sec.

Fig. 9 shows the response to the impulse disturbance of the body angular position.

It is obvious that the control system counteracts this disturbing effect in 8 s in the oscillatory process.

Fig. 10 shows the diagrams of signals in response to disturbance step function \( f(t) = 0.03 \cdot 1(t) \).
Fig. 8. Response to the setting action: a – $u_3(t)$; b – $x(t)$; c – $\Theta(t)$

Fig. 9. Reaction to disturbance $\Theta_0$: a – $\Theta_0(t)$; b – $x(t)$; c – $\Theta(t)$
The reactions obtained indicate the ability of the control system to cope with such a disturbing effect. So, the use of the synthesis method with the help of logarithmic asymptotic characteristics allow to form algorithms for controlling the oscillatory and translational movements of the TWES according to signals from the angle and displacement sensors.

3. Control algorithms by diagnosis

The use of the deviation control principle does not allow to fend off all the destabilizing effects on the TWES, in particular, the effects from the set D [5], this effect:

- $d_1$ – change in the inertial properties of electric drives;
- $d_2$ – reducing the torque of electric drives;
- $d_3$ – a decrease in the transmission coefficient of the first sensor;
- $d_4$ – decrease in the coefficient of the second sensor.

To counter such destabilizing influences, the principle of control by diagnosis is applicable [10]. In the first article [5], the structural and signal properties of TWES were analyzed according to the corresponding diagnostic criteria, which allows us to proceed to the formation of algorithms for diagnosing TWES based on available measurement signals $u_{s1}(t)$ and $u_{s2}(t)$.

Control by diagnosis is associated with the need for digital implementation of diagnosis and recovery processes. The mathematical description (1) and (2) of the automatic control object in the nominal mode of operation using the Euler formula in discrete form are:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \\ x_5(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T_0 & 0 & 0 & 0 \\ a_{21}T_0 & 1 & 0 & 0 & a_{25}T_0 \\ 0 & 0 & 1 & T_0 & 0 \\ 0 & 0 & 0 & 1 & T_0 \\ 0 & 0 & 0 & a_{54}T_0 & (1+a_{55}T_0) \end{bmatrix} \times \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \\ x_5(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ u(k) \\ 0 \\ 0 \end{bmatrix} f(k),$$

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} c_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{23} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \\ x_5(k) \end{bmatrix},$$

where $x_i(k)$ – discrete values of state variables $x_i(t)$, $i=1,5$, in the arguments of which the quantization period is omitted to simplify the description $T_0$;

$u(k)$ and $y_i(k)$, $i=1,2$ – discrete values of the corresponding description variables in the state space.
The control system for the angular and translation movements of the TWES formed using the principle of control by deviation has the ability to fend off destabilizing influences in the "small", i.e. small deviations of the parameters from the nominal values, disturbing the effects of a certain limited value. Large destabilizing influences, such as a closed-loop control system in the set D, cannot be countered. Therefore, a two-level organization of the TWES control system is possible. At the first – lower level, a closed control system functions, and at the second – higher level, the automatic control object is diagnosed and, according to the received diagnosis, its operability is restored by countering the destabilizing effects of the set D.

The first diagnostic task is to detect destabilization. Destabilization can be detected by the deviations of the sensor signals from the set values in the steady-state operating mode of the closed-loop system using the predicate equation:

\[
z_0 = S_2 \left[ \Delta y_1(k) - \delta_1 \right] \vee S_2 \left[ \Delta y_2(k) - \delta_2 \right];
\]

\[
k = k_1, k_2; \quad p = 0.9,
\]

where \( S_2 \{ \} \) – a symbol of a two-digit predicate;
\( \delta_1 \) and \( \delta_2 \) – permissible threshold values;
\( \vee \) – disjunction symbol;
\( p \) – the coefficient of confidence.

Difference signals \( \Delta y_1(k), \Delta y_2(k) \) can also be obtained using the reference model, both of the entire closed-loop control system, and the reference model of the automatic control object. The reference model is identical by structure and parameters to the nominal model (23), (24). In a compact form it could be represented as:

\[
\dot{x}(k+1) = Ax(k) + Bu(k); \quad x(k_0) = x_0;
\]

\[
\dot{y}(k) = C\hat{x}(k),
\]

where \( \hat{x}(k) \) – 5-dimensional vector of the reference state of the object;
\( \hat{y}(k) \) – 2-dimensional vector of the reference output of the object.

After detecting destabilization, it is required to find its place, i.e. the constructive part of the TWES, in which a violation appeared in the functioning, i.e. there was a destabilization. Using a predicate of the form:

\[
z_1 = S_2 \left[ \Delta y_1(k) - \delta_1 \right]; \quad k = k_2, k_3; \quad p = 0.8
\]

the malfunction of the angle sensor is determined – the first sensor. Failure of the second sensor – the distance sensor is detected using a similar predicate

\[
z_2 = S_2 \left[ \Delta y_2(k) - \delta_2 \right]; \quad k = k_3, k_4; \quad p = 0.8.
\]

The place of destabilization in the electric drive is set according to these values of the predicates \( z_1 = 1 \) and \( z_2 = 1 \).

Distinguish destabilization in an electric drive associated with a change in inertial properties – \( d_1 \) and from a decrease in torque – \( d_2 \), can be done using a predicate equation of the following form:

\[
z_3 = S_2 \left[ \delta_3 - \varphi(k+3) \right], \quad k = k_5, k_6; \quad p = 0.7,
\]

where the argument function \( \varphi(k+3) \) formed from the corresponding diagnostic model linking the deviations of the moment of inertia of the drives \( \Delta I \) with state variables

\[
\Delta x_2(k+1) = \Delta x_2(k) + \frac{T_0}{M\rho} \hat{\xi}_3(k)\Delta I.
\]

Variables \( \Delta x_2(k) \) and \( \hat{\xi}_3(k) \) are inaccessible to direct measurement, therefore they must be expressed through sensor measurements. Then a discrete function characterizing the deviation \( \Delta I \) constancy on the diagnostic interval is:

\[
\varphi(k+3) = \left[ y_1(k+2) - 2y_1(k+1) + y_1(k) \right] \times
\]

\[
\times \left[ y_3(k+2) - 2y_3(k+1) + y_3(k) \right] -
\]

\[
- \left[ y_1(k+1) - 2y_1(k) + y_1(k-1) \right] \times
\]

\[
\times \left[ y_3(k+3) - 2y_3(k+2) + y_3(k+1) \right]
\]

Function \( \varphi(k+3) \) is developed for discrete values of sensor signals.

Using three features \( z_1, z_2 \) and \( z_3 \) allows to recognize each of the five possible states of the automatic control object. This follows from the analysis of tab. 2.

| \( d_i \) | Signs \( z_j \) |
|---|---|---|
| | \( z_1 \) | \( z_2 \) | \( z_3 \) |
| 1 | one | one | one |
| 2 | one | one | 0 |
| 3 | one | 0 | 0 |
| 4 | 0 | one | 0 |
As follows from the analysis, the number of features is minimal for finding any place of destabilization from the set D. Using the tab. 2, a binary tree of destabilization search is formed.

To obtain a complete diagnosis, numerical values of the destabilization values are required, i.e. it is necessary to identify the types of destabilizing influences.

The magnitude of the deviation of the electric drives inertia moment \( \Delta I \) is calculated from the corresponding diagnostic model using the following equation:

\[
\Delta I = \frac{\Delta x_2(k+1) - \Delta x_2(k)}{M/r}.
\]

(33)

On the right side of the equation, state variables are used that are not measurable. Through the quantities available for measurement, the equation is transformed to the following form:

\[
\Delta I(k+1) = T_0 M/r \left[ y_1(k+1) - 2y_2(k) + y_1(k-1) \right] - \left[ y_2(k+1) - 2y_2(k) + y_2(k-1) \right].
\]

(34)

The ordinary algorithm for obtaining the arithmetic mean \( \Delta \hat{I} \) is:

\[
\Delta \hat{I} = \frac{1}{m} \sum_{k=1}^{m} \Delta I(k+1).
\]

(35)

Torque Reduction Amount \( \Delta \kappa_m \) corresponds to an equation of the following form:

\[
\Delta \kappa_m = \frac{\Delta x_3(k+1) - \Delta x_3(k)}{M/r}.
\]

(36)

Through the measurement results of the second sensor

\[
\Delta \kappa_m(k+3) = \frac{IL}{T_0} \left[ k_0 y_3(k+1) - u(k) \right] - 4y_3(k+2) - 2y_3(k+1) - 2y_3(k) + y_3(k-1).
\]

(37)

then the estimated value algorithm is:

\[
\Delta \hat{\kappa}_m = \frac{1}{m} \sum_{k=1}^{m} \Delta \kappa_m(k+3).
\]

(38)

The estimated value of the angle sensor gain reduction is calculated as follows:

\[
\Delta \hat{\kappa}_1 = \frac{1}{m} \sum_{k=1}^{m} \Delta y_1(k),
\]

(39)

where \( \hat{y}_1(k) \) – discrete values of the reference model.

Similarly for the second sensor

\[
\Delta \hat{\kappa}_2 = \frac{1}{m} \sum_{k=1}^{m} \Delta y_3(k),
\]

(40)

where \( \hat{y}_3(k) \) – discrete values of the third component of the state vector of the automatic control object reference model.

So, the algorithms allow to organize the process of operational diagnostics of ACO and to obtain a complete diagnosis: the moment of the destabilization appearance, the destabilized unit (place of destabilization) and the specific numerical value of the unit functioning efficiency decrease.

Fig. 11 shows the structure of the binary tree, which allow to diagnose ACO.

![Fig. 11. Binary tree diagnosing ACO](image)

The above diagnostic algorithms, in fact, represent the production knowledge base of emergency situations of the ACO. This knowledge base could be extended by appropriate new emerging cases handling.

After receiving a complete diagnosis, it is required to restore the ACO performance [10]. So, when electric drives inertia increase, which will affect a control system transient process time increase, it is possible to restore operability using following signal adjustment:

\[
u_a(k+1) = \frac{k_a}{T_0} \left[ y_1(k+1) - y(k) \right],
\]

(41)

where \( k_a \) – adjustment factor depending on \( \Delta \hat{I} \).

The destabilization caused by an electric drives torque decrease can also be corrected by means of a signal adjustment:

\[
u_a(k) = k_a y_1(k),
\]

(42)

where \( k_a \) – coefficient equal to \( \Delta \hat{\kappa}_m \).

The transmission coefficient decrease for the first sensor, which is an angle sensor, leads to corresponding output signal decrease. To restore measurement results \( \hat{y}_1(k) \) it is necessary to increase them by the value \( \Delta \hat{\kappa}_1 \hat{y}_1(k) \). Similarly, to restore the functionality of the second sensor, which is an displacement sensor giving the output signal \( \hat{y}_3(k) \), an additional signal \( \Delta \hat{\kappa}_2 \hat{y}_3(k) \) should be added. The diagnostics and recovery algorithms complex functioning provide rational control of the TWES in conditions of destabilizing influences.
Conclusion

As a result of the research carried out for the models of angular and translational movements of the TWES described in [5], three approaches to the formation of control algorithms for its balanced movement were proposed. The first approach is to use estimates of the state vector components that are not measurable, and the integral criterion of the weighted modulus of the error signal. The second approach is to use a synthesis method based on asymptotic logarithmic characteristics. The results of such a synthesis using specific prototype of TWES are presented. The third approach is based on the new principle of control by diagnosis, which makes it possible to form a TWES rational control system. Algorithms for deep diagnostics and flexible restoration of operability of the motion control system are described. 

The proposed approaches to the control algorithms’ analysis for the translational and angular movements of the TWES can be used at the development of control systems for two-wheeled transport robots used in the modern assembly manufacturing technological processes. Using of the described approaches in automating of transient modes control of new classes of autonomous aircrafts, such as convertiplanes and transport quadcopters, will make it possible to find compromise solutions between stability and quality.

The results obtained could be used as in the educational process as course and diploma projects, as at the stages of draft design of the unstable objects automatic control systems.

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АЛГОРИТМИ УПРАВЛЕННЯ ПРОДОЛЬНЫМ ДВИЖЕНИЕМ ДВУХКОЛЕСНОГО ЭКСПЕРИМЕНТАЛЬНОГО ОБРАЗЦА

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Предметом изучения в статье является процесс формирования алгоритмов управления продольным движением двухколесного экспериментального образца (ДЭО). Целью является разработка подходов к формированию алгоритмов управления продольным движением двухколесного экспериментального образца. Задачи: конкретизировать процесс синтеза алгоритма управления по состоянию на основе минимума интеграла от взвешенного модуля ошибки для линейного математического описания объекта автоматического управления в частотном пространстве. Сформировать структурную схему системы автоматического управления на основе минимума интеграла от взвешенного модуля ошибки для линейного математического описания объекта автоматического управления в частотном пространстве. Сформировать алгоритм управления по выходу для математического описания объекта автоматического управления в частотном пространстве. Сформировать алгоритм управления по выходу для математического описания объекта автоматического управления в частотном пространстве.

Выводы. Научная новизна заключается в формировании подходов к совместному управлению продольным движением двухколесного экспериментального образца (ДЭО) с учетом структурных и параметрических особенностей математических описаний ДЭО.

Ключевые слова: двухколесный экспериментальный образец; объект автоматического управления; управление по состоянию; управление по выходу; управление по диагнозу.
АЛГОРИТИМИ КЕРУВАННЯ ПОЗДОВЖНЬОГО РУХУ ДВОКОЛІСНОГО ЕКСПЕРИМЕНТАЛЬНОГО ЗРАЗКА

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Предметом вивчення в статті є процес формувания алгоритмів керування кутовим і поступальним рухами двоколісного експериментального зразка (ДЕЗ). Метою є розробка підходів до формувания алгоритмів керування поступальним і кутовим рухами нестаціонарного об’єкта автоматичного керування. Завдання: конкретизувати процес синтезу алгоритму керування станом за критерієм мінімуму інтеграла від зваженого модуля помилки для лінійного математичного опису об’єкта автоматичного керування в просторі станів. Сформувати структурну схему системи автоматичного керування станом. Удосконалити підхід до синтезу алгоритмів керування по виходу для математичного опису в частотній області короткоперіодичних і длиноперіодичних рухів ДЕЗ. Проілюструвати особливість підходу на конкретному прикладі ДЕЗ для керування часом; керування по виходу; керування за діагнозом.

Наукова новизна полягає у формуванні підходів до спільного керування кутовим і поступальним рухами з урахуванням структурних і параметричних особливостей математичних описів ДЕЗ. Розробити алгоритми відновлення працездатності системи автоматичного керування ДЕЗ з використанням лінійних математичних описів в тимчасовій і частотній областях.

Розробити підхід до формувания алгоритмів керування за діагнозом ДЕЗ як об’єкта автоматичного керування. Описати процедуру і засоби глибокого діагностикування позаштучних ситуацій ДЕЗ. Розробити алгоритми відновлення працездатності системи автоматичного радіоелектронногокомп’ютерних систем.

Алгоритми керування кутовим і поступальним рухами нестаціонарного об’єкта автоматичного керування є розробка підходів до формувания алгоритмів керування за діагнозом ДЕЗ як об’єкта автоматичного керування. Описати процедуру і засоби глибокого діагностикування позаштучних ситуацій ДЕЗ. Розробити алгоритми відновлення працездатності системи автоматичного радіоелектронного комп’ютерного систем.

Ключові слова: двоколісний експериментальний зразок; об’єкт автоматичного керування; керування станом; керування по виходу; керування за діагнозом.

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