Correlations between Triaxial Shapes and Formation History of Dark Matter Halos

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ABSTRACT
The shape of dark matter haloes plays a critical role in constraining cosmology with upcoming large-scale structure surveys. In this paper, we study the correlations between the triaxial shapes and formation histories in dark matter haloes in the MultiDark Planck 2 N-body cosmological simulation. We find that halo ellipticity is strongly correlated with halo properties that serve as proxies of halo formation history, such as halo concentration and the peak-centroid offset. In particular, the correlation between halo ellipticity and halo concentration is nearly independent of the halo density peak height. We present a simple model for the correlation between halo ellipticity and concentration using conditional abundance matching, and provide fitting formulae for the multi-dimensional distributions of triaxial halo shape as a function of halo peak height. We apply our halo shape model to gauge the effects of halo ellipticity and orientation bias on the excess surface mass density profiles in cluster-size halos. Our model should be useful for exploring the impact of triaxial halo shape on cosmological constraints in upcoming weak lensing surveys of galaxy clusters.

1 INTRODUCTION

In the ΛCDM cosmological model, the fundamental building block of large-scale structure is the dark matter (DM) halo. The centers of halos are the deepest points in the gravitational potential of the Universe, and so DM halos of sufficient mass are natural sites of galaxy formation (White & Rees 1978; Blumenthal et al. 1984). Observations of cluster-mass halos contain rich information about cosmology (e.g., Allen et al. 2011), and so a long-standing goal of large-scale structure cosmology is to accurately model and characterize the abundance, spatial distribution, and internal structure of DM haloes.

The shape of DM halos is generically expected to be non-spherical due to the non-spherical shape of the Gaussian density peak of the primordial density field from which the halo forms (Doroshkevich 1970) and the directional nature of the merger and accretion of halos along filamentary structures in the cosmic web (Zel’Dovich 1970). Since DM halo shape depends upon both the initial density field and on halo assembly, the observed distribution of halo shapes can be used to test and validate the ΛCDM structure formation scenario (Kawahara 2010; Sereno et al. 2018). The shape of galaxy clusters can also serve as a probe of the fundamental particle nature of DM: models of self-interacting DM generically predict more spherical distributions of cluster mass relative to models in which DM is collisionless, an effect that becomes more pronounced in the inner regions (Yoshida et al. 2000; Spergel & Steinhardt 2000; Davé et al. 2001; Peter et al. 2013).

The non-spherical shape of DM haloes has important implications for the study of cosmology and astrophysics with observations of galaxy clusters. Specifically, the shapes of DM haloes affect the measurements of the mass and gas content in galaxy clusters, which are commonly assumed to be spherically symmetric. In particular, the non-spherical shape of haloes can lead to biases gravitational lensing estimates of cluster mass due to the elongation of the mass distribution along the line-of-sight (e.g. Meneghetti et al. 2010; Lee et al. 2018). This leads to what is known as orientation bias (Hamana et al. 2012; Dietrich et al. 2014). With orientation bias, elliptical clusters elongated along the line-of-sight are preferentially detected, with their masses over-estimated. Hydrostatic estimates of cluster masses can also be under- or over-estimated as a result of the failing of the spherical symmetry assumption (e.g. Buote & Humphrey 2012). The non-spherical shape of haloes is therefore a source of systematic uncertainty in cosmological constraints derived from a wide variety of measurements of galaxy clusters (e.g., Smith & Watts 2005; Battaglia et al. 2012).

Previous theoretical studies have already established the triaxial nature of DM haloes (e.g., Jing & Suto 2002), where DM halo shape can be well-approximated by a triaxial ellipsoid specified by two parameters: the ratios of minor- to major- and the intermediate-to-major axes. The distribution of halo shapes exhibits a clear dependence on halo mass, with higher mass haloes being more elliptical relative to haloes of lower mass; at fixed mass, DM haloes at higher redshift present more elliptical shapes relative to present-day halos (Allgood et al. 2006). The mass-dependence of the average halo shape is approximately universal (Bonamigo et al. 2015; Vega-Ferrero et al. 2017), but there is significant scatter in halo shape at fixed halo mass. The scatter of halo shape at fixed mass can largely be attributed to differences in the halo formation histo-
2 Definitions of Halo Shape

For an ellipsoidal halo with major, intermediate, and minor axes of length $a$, $b$, and $c$, the shape of the halo can be described in terms of two parameters, ellipticity and prolaticity, defined as follows:

$$
e = \left(1 - \frac{(c/a)^2}{2L}\right)$$

$$p = \left(1 - \frac{2(b/a)^2 + (c/a)^2}{2L}\right)$$

where $L \equiv 1 + (b/a)^2 + (c/a)^2$. We furthermore define halo triaxiality, $T$, as:

$$T = \frac{1 - (b/a)^2}{1 - (c/a)^2} = \frac{1}{2} \left(1 + \frac{p}{e}\right).$$

The condition $a \geq b \geq c \geq 0$ implies that the domain of $e$ is the range $[0, 1/2]$, and $T$ is the range $[0, 1]$, while the domain of $p$ is $[-e, e]$. Note that there is a physical lower limit of $T \geq 2 - 1/(2e)$, otherwise $c/a$ will not be a real number.

Halos with $T \geq 2/3$ are called prolate and present an elongated, cigar-like shape, while oblate halos with $T \leq 1/3$ exhibit a flattened shape like a lentil or a disk. In the present work, we will build our model for halo shape using $e$ and $T$ as independent variables, but we note that the equations above make it straightforward to transform these quantities to other alternative shape variables that are in common usage.

3 Simulation

In this paper, we aim to study the effects of halo formation histories on halo shapes for group- and cluster-size halos. For this purpose, we analyze the MultiDark Planck 2 (MDPL2) cosmological simulation (Klypin et al. 2016) that contains a large number of high-resolution group- and cluster-size halos with well measured formation histories. The MDPL2 is a gravity-only $N$-body simulation of 3840$^3$ particles in a periodic box with $L_{\text{box}} = 1\,\text{Gpc}/h$, giving a particle mass resolution of $m_\text{p} \approx 1.51 \times 10^9\,h^{-1}\,M_\odot$. The MDPL2 was run with a flat cosmology similar to Planck Collaboration et al. (2014), with $h = 0.6777$, $\Omega_m = 0.307115$, $\Omega_\Lambda = 0.692885$, $\sigma_8 = 0.829$ and $n_s = 0.96$. We refer the reader to Klypin et al. (2016) for more details about the simulation. Throughout this work, we use the axis ratio measurements and proxies for halo assembly histories from the publicly available ROCKSTAR (Behroozi et al. 2013a,b; Rodríguez-Puebla et al. 2016) halo catalogs in the MDPL2 simulation; data products for MDPL2 are publicly available through the MultiDark Database (Riebe et al. 2013), and can downloaded from the CosmoSim website.\footnote{https://www.cosmosim.org}

We select distinct host halos\footnote{That is, ROCKSTAR subhalos for which upid=1.} with virial mass $M_{\text{vir}} \geq 10^{13}\,h^{-1}\,M_\odot$ at $z = 0.0, 0.5, 1.0$. Halos in MDPL2 in this mass range are resolved by at least 2000 particles within the virial radius, $R_{\text{vir}}$, such that their shape measurements are robust.

We use the axis-ratio measurements and halo formation parameters provided in the ROCKSTAR halo catalog. The axis ratios are computed from the substructure-excluded inertia tensor for DM particles within the virial radius

$$I_{ij} = \frac{1}{N} \sum x_i x_j,$$

where $x_i$ is the position in the direction $i = 1, 2, 3$ of the DM particle relative to the halo centre. The major, intermediate, and minor axes $(a, b, c)$ of the ellipsoid are then the square roots of the sorted eigenvalues of the inertia tensor.

The halo formation parameters we used in this paper are summarized in Table 1. In particular we examine the correlation of halo shape with the halo concentration, the virial ratio, the offset between the density peak and center of mass (scaled by the virial radius), the half-mass scale (scale factor at which the halo attains half of its present mass), the time since the halo attains half of its mass, as well as their mass accretion rates. We refer the reader to Behroozi et al. (2013a); Rodríguez-Puebla et al. (2016) for further details on how these halo properties are measured.

4 Results

4.1 Distributions of Ellipticity and Triaxiality

To characterize the joint mass- and redshift-dependence of halo ellipticity, we use the halo density peak height, $\nu \equiv 1.686/(\sigma(M_{\text{vir}}, z))$, to represent the mass of the halo at a given redshift, where $\sigma(M_{\text{vir}}, z)$ is the mass density fluctuation of a halo of mass $M_{\text{vir}}$ at redshift $z$. Figure 1 shows the distributions of ellipticity $e$ at different peak height values for $M_{\text{vir}} \geq 10^{13}\,h^{-1}\,M_\odot$ at $z = 0.0, 0.5, 1.0$. Halos with larger peak height (i.e., more massive and lower redshift halos) tend to have slightly higher mean ellipticities, however there is quite significant overlap in the full distribution of shapes of halos in different mass bins. At any particular value of $\nu$, the distribution of halo ellipticity is well-described by the generalized gamma
distribution

\[
P(e|a, c) = \frac{|e| e^{ae-1} \exp(-e^c)}{\Gamma(a)},
\]

where \( \Gamma(z) = \int_0^\infty x^{z-1} e^{-x} dx \) is the gamma function, and \( a \) and \( c \) are two free parameters that govern the shape of the distribution. We thereby capture the mass- and redshift-dependence of the ellipticity distribution via the following calibration:

\[
a = 1.0 + 1.7(\log \nu / 0.25), \tag{5}
\]

\[
c = 3.1. \tag{6}
\]

In addition to ellipticity, we also need to specify triaxiality to completely describe the shape of an ellipsoidal halo. Figure 2 shows the distributions of triaxiality at fixed ellipticity, \( P(T|e) \), for all halos at \( z = 0, 0.5, 1.0 \). Halos with larger values of \( e \) also tend to have larger values of \( T \), meaning that highly elliptical halos tend to be preferentially prolate. The spread of the triaxiality distribution also depends on halo ellipticity, such that less elliptical halos tend to have a broader distribution of triaxiality.

For a population of halos with ellipticity \( e \), we model \( P(T|e) \), the conditional distribution of \( T \), with the beta distribution:

\[
P(T|e; \alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} T^{\alpha-1} (1-T)^{\beta-1}, \tag{7}
\]

where \( \Gamma(z) \) is the gamma function, and \( \alpha, \beta > 0 \) are two free parameters that govern the shape of the distribution. We find that the beta distribution parameters \( (\alpha, \beta) \) depend on halo ellipticity \( e \) in a manner that is well-described by the following calibration:

\[
\alpha = \beta \left( 1 - \frac{3.5e + 25e^2}{1 - 2e} \right), \tag{8}
\]

\[
\beta = 2.5. \tag{9}
\]
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The correlation coefficients dependent on halo mass differently for different halo formation proxies. While the magnitude of the correlation between $e$ and $c_{\text{vir}}$ decreases slightly with mass, the correlation between $e$ and the half-mass scale increases with mass, from having nearly no correlation at $M_{\text{vir}} \approx 10^{13} h^{-1} M_\odot$ with $\rho = -0.03$, to being mildly positively correlated for massive halos $M_{\text{vir}} \approx 10^{15} h^{-1} M_\odot$ with $\rho = 0.29$. Note that all of the above correlations are statistically significant, as they all have $p$-values much less than 0.05. Figure 3 shows the mass dependence of $\rho$.

Next we examine dependence of the correlation between halo ellipticity and the halo formation proxies on halo peak height $v$. Figure 4 shows the dependence of ellipticities on the halo formation proxies for different halo peak heights at redshifts $z = 0, 0.5, 1$. In general, higher peak height haloes (i.e., more massive haloes) have higher ellipticities, which is expected since they are still actively accreting mass from the surrounding and thus most ‘unrelaxed’ and elliptical. Different halo formation proxies show variations in their $\nu$ and redshift dependence. For “time”-based proxies: $a_{t/2}$, $\Delta t_{1/2}$, and $\Delta \omega_{1/2}$, their relations with ellipticity varies the most between halo masses and redshifts. On the other hand, “space”-based proxies: $c_{\text{vir}}$, $T/|U|$, $X_{\text{off}}$ show more “universal” behaviour in their mass and redshift dependence.

5 MODELING HALO ELLIPTICITY USING CONDITIONAL ABUNDANCE MATCHING

In this section we present a probabilistic model for the dependence of halo ellipticity upon halo assembly. The basis of our model is the Conditional Abundance Matching technique (CAM, Masaki et al. 2013; Hearin & Watson 2013), coupled together with the results in Section 4. While CAM has capability to additionally capture higher dimensional correlations (Hearin et al. 2019), here we focus on capturing the dependence of ellipticity upon halo concentration at fixed halo mass, as we found $c_{\text{vir}}$ be the secondary halo property with the strongest correlation with halo ellipticity. We note that the CAM formalism could similarly be applied to any other proxy for halo formation, provided that the $M_{\text{halo}}$-conditioned correlation between ellipticity and the halo formation is monotonic (see Hearin et al. 2014, section 4.3 for technical details).³

We implement the CAM ansatz to capture the correlation between halo ellipticity and concentration by computing the cumulative distribution function (CDF) of each quantity at fixed halo peak height. The correspondence between ellipticity and concentration is then based on matching the conditional CDF of each variable. We then introduce stochasticity in the relation using the correlation coefficient computed from the simulation, resulting in a relation between ellipticity and halo formation proxy that closely follows from what we measured in the simulation, as shown in Figure 5. The left panel compares the ellipticity-concentration relation between our model and the simulation, while the right panel compares the distribution of ellipticity at a given peak height. We compare the two distributions by performing a Kolmogorov-Smirnov two-sample test, finding $p$-values greater than 0.05 at all values of peak height, so that the simulated and modeled distributions are statistically indistinguishable at the $2\sigma$ level.

³ See the conditional_abunmatch function in Halotools (Hearin et al. 2017) for a publicly available implementation of CAM.

Figure 3. Dependence of the magnitude of the Spearman rank correlation coefficient $|\rho|$ between halo ellipticity $e$ and different halo mass accretion proxies (shown by different coloured lines), plotted as a function of halo mass for all halos at $z = 0, 0.5, 1.0$. Halo concentration $c_{\text{vir}}$ correlates most strongly with halo ellipticity, followed closely by $T/|U|$, $\Delta t_{1/2}$, and $X_{\text{off}}$ (see Table 1 for details); the correlation coefficient for $c_{\text{vir}}$ also shows the weakest mass dependence.

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Figure 4. Dependence of ellipticity on $c_{\text{vir}}$, $T/|U|$, $X_{\text{off}}$ (top row, left to right), $a_{1/2}$, $t_{1/2}$, and $\Delta \omega_{1/2}$ (bottom row, left to right), for different mass bins at $z = 0, 0.5, 1.0$ (shown as solid, dashed, and dot-dashed lines respectively). Error bars indicate 1σ error on the mean. Correlations with $c_{\text{vir}}$ and $X_{\text{off}}$ show only very weak dependence upon either peak height or redshift.

Figure 5. Left panel: Dependence of ellipticity upon $c_{\text{vir}}$, with error bars show 1σ variance amongst all halos at $z = 0, 0.5, 1.0$ Dashed lines show the prediction of the probabilistic model computed using Conditional Abundance Matching (CAM) with shaded regions indicating 1σ variance. Different colors indicate different halo peak height bins, $\log \nu$, as indicated in the legend. Right panel: Distribution of ellipticity in the same bins of peak height, $P(e; c_{\text{vir}}| \log \nu)$, directly measured from simulation (solid lines), and those in our CAM model (shaded regions).
6 SIMPLE MODEL OF HALO ELLIPTICITY AND ORIENTATION BIAS IN WEAK LENSING MEASUREMENTS

The distribution of the shapes of dark matter halos, and the alignment of these shapes with large-scale structure, has a significant impact on measurements of gravitational lensing (Schneider et al. 2012). In the context of estimating cluster masses, the relevant quantity is the excess surface mass density for the 1-halo term, defined as

\[ \Delta \Sigma(R_p) = \bar{\Sigma}(< R_p) - \Sigma(R_p), \] (10)

where

\[ \Sigma(R_p) = \frac{2}{\pi} \int_{R_p}^{\infty} \frac{\rho dz}{\sqrt{R_p^2 + z^2}}, \] (11)

\[ \bar{\Sigma}(< R_p) = \frac{1}{\pi R_p^2} \int_0^{R_p} 2\pi r dr \Sigma(r), \] (12)

where \( R_p \) is the projected radius perpendicular to the line-of-sight, \( z \) is the coordinate along the line-of-sight, and \( \rho \) is the 3D density profile.

To generate our model’s predictions for \( \Delta \Sigma \) profiles, we adapt the Monte Carlo method used in Sgrö et al. (2013) to calculate the integrals in Eqs. 11 and 12. First, under the assumption that each halo follows the spherical NFW density profile, we use \( N = 10^5 \) particles to sample the spherical density profile for given halo mass \( M_{\rm vir} \) and concentration \( c_{\rm vir} \). We then draw ellipticity \( e \) and triaxiality \( T \) values from the distributions from Equations (4) and (7) and transform \( e \) and \( T \) to the axis ratios \( c/a \) and \( b/a \). We then transform the coordinates of the particles with \( c/a \) and \( b/a \) under the condition that the volume of the halo is conserved under the transformation, i.e., \( R_{\rm vir} = (abc)^{1/3} \). Assuming that \( z \) is direction of the elongated axis, and the \( x \) direction points to the shortest axis, we have transformation \( z \rightarrow za^{2/3}/(bc)^{1/3}, y \rightarrow ya^{2/3}/(bc)^{1/3}(b/a), \) and \( x \rightarrow xa^{2/3}/(bc)^{1/3}(c/a) \).

To model halo orientation, we assume the cosine of the angle between the elongated axis and the line-of-sight \( \cos(\theta) \), follows the distribution

\[ P(\cos(\theta)) = \frac{\Gamma(1+\alpha)}{\Gamma(\alpha)} |\cos(\theta)|^\alpha, \] (13)

where \( \alpha > 1 \) is a parameter that controls the shape of the distribution, and thus controls the degree of orientation bias. When \( \alpha = 1 \), \( P(\cos(\theta)) \) is uniform, thus the elongated axes of the halos are randomly oriented. Increasing \( \alpha > 1 \) will lead to more halos having their elongated axes aligning with the line-of-sight. For \( \alpha \rightarrow \infty \), all halos are aligned with the line-of-sight. For any given value of \( \alpha \), we draw a random \( \cos(\theta) \) value from the distribution for each halo, and rotate the halo such that its elongated axis makes an angle \( \theta \) with the line-of-sight. Finally, we compute \( \Delta \Sigma \) by integrating over the particles along the line-of-sight cylinder with projected radius \( R_p \). To show how ellipticity and halo orientation can impact lensing measurements, we use the above technique to compute \( \Delta \Sigma \) for 8190 halos with mass \( M_{\rm vir} \in [0.9, 1.1] \times 10^{14} M_\odot \), taken from the MDPL2 simulation at redshift \( z = 0 \). Figure 6 shows how orientation bias impacts the lensing profiles of a stack of cluster-mass halos. In each panel, curves of the same color correspond to a different levels of orientation bias, with the middle panel visually demonstrating the distribution of orientation angles for different values of \( \alpha \). The left panel illustrates lensing profiles of the halo samples of different orientation bias; the right panel demonstrates the diversity of lensing signals amongst haloes in the sample by showing the distribution of \( \Delta \Sigma(R_{\rm vir}) \) amongst members of the stack.

The dashed black curve in each panel shows results for the case of spherical halos with the same halo mass and concentration distributions as the triaxial haloes. The dashed black and solid blue curves exactly overlap in the middle panel (i.e. they are not subject to any orientation bias), while their mean lensing profiles are discernible in the left panel, reflecting the nontrivial change to the lensing profile induced by halo ellipticity.

Unfortunately, there is a degeneracy between the strength of orientation bias, \( \alpha \), and the true underlying distribution of ellipticity. We demonstrate this degeneracy with the orange curve corresponding to \( \alpha = 2 \), which has a modest orientation bias of \( \langle \cos(\theta) \rangle = 2/3 \); from the left-hand panel we see that this sample of haloes presents a nearly indistinguishable mean lensing signal relative to a comparable stack of spherical halos (see also Meneхhetti et al. 2007; Dietrich et al. 2014, for closely related discussion on cluster lensing degeneracies). However, the model presented in this work is a global model for the distribution of halo shapes across mass and redshift, and by comparing the black and orange curves in the right-hand panel of Figure 6, we can see that the scatter in the lensing profiles is tighter for a stack of spherical halos relative to an elliptical stack with orientation bias. As discussed in §7, this suggests it may be possible to extract additional information about triaxial halo profiles by incorporating our model for halo shape distributions into prediction pipelines for multi-wavelength synthetic lightcones.

7 DISCUSSION

We have provided a calibrated approximation to the full probability distribution of halo shape as a function of halo mass, concentration and redshift, \( P(e, T | M_{\rm vir}, z, c) \). Our model for the distribution of triaxial halo shapes has potential to extend the predictive power of otherwise conventional implementations of the halo model. Standard formulations of the halo model include ingredients to predict the abundance and internal structure of DM halos as a function of their mass and redshift. For a halo of a given mass, concentration and redshift, our model creates the capability to make a significantly richer set of predictions for the distribution and evolution of large-scale structure. Along similar lines, simulated halo catalogs may not have halo shape information available, particularly when generated by approximate N-body methods (e.g., Monaco et al. 2013; Izard et al. 2016; Feng et al. 2016); our model offers a straightforward way to augment such catalogs with physically realistic distributions of triaxial shapes.

We note several limitations of our analysis, which assumes that (1) halo internal structure is perfectly described by the NFW profile; (2) ellipticity and triaxiality are constant functions of halo radius; and (3) substructure effects can be neglected. Realistically, halo ellipticity is known to vary with radius and can be significantly affected by the presence of substructure. Moreover, observational systematics such as mis-centering and projection effects contribute additional scatter and bias to the lensing signal \( \Delta \Sigma \) that we do not account for here. Additionally, the current work is based on gravity-only simulations of a single \( \text{Planck} \)-like cosmology (Planck Collaboration et al. 2018). However, halo shape is known to depend on both cosmology (Ho et al. 2006) and baryonic effects such as radiative cooling, star formation, and feedback from supernovae and supermassive blackholes (Kazantzidis et al. 2004; Lau et al. 2012; Bryan et al. 2013; Suto et al. 2017; Chua et al. 2019; Chen et al. 2020).
Baryonic physics also impacts the relationship between halo concentration and halo assembly (Duffy et al. 2010), a key ingredient of our approach.

On the one hand, these caveats imply that our current calibration will not provide a high-accuracy reproduction of the distribution of halo shapes seen in hydrodynamical simulations of different cosmology. However, simulation-based predictions have capacity to capture a range of physical effects that challenge traditional halo-model approaches, and our model of triaxial halo shape has been formulated with such purposes in mind. In ongoing follow-up work, we will characterize how baryonic physics modify the distribution of halo shapes, quantifying these changes in terms of modifications to the parameters of the model presented here. We will similarly use suites of cosmological simulations to study how changes in cosmology manifest in changes to the distribution of halo shapes. Multi-wavelength lightcone maps constructed via our forward model thereby create an opportunity to derive cluster-based constraints on cosmology in a manner that is robust to systematic uncertainty in baryonic physics by self-calibrating model parameters that capture these effects.

8 CONCLUSIONS

In this work, we have investigated the dependence of the halo shape and formation proxies using the gravity-only MDPL2 N-body simulation. Our main findings are summarized as follows:

- In keeping with earlier work, we find that halo ellipticity shows strong dependence on halo formation history, exhibiting the strongest correlations with halo formation proxies such as halo concentration, virial ratio, and peak-centroid offset. Halo ellipticity is additionally correlated with the lookback time to the redshift at which the halo attained half of its present-day mass.
- Halo concentration shows a nearly universal relation with halo ellipticity that is only weakly dependent on halo peak height.
- We have developed and calibrated a probabilistic model for the dependence of halo shape on mass, redshift, and formation history, using Conditional Abundance Matching to capture multi-dimensional correlations.
  - We presented a Monte Carlo integration technique for modeling halo surface mass density profiles, $\Delta \Sigma$, with capability to incorporate distributions of halo ellipticity and orientation in a straightforward manner. We find that the scatter in $\Delta \Sigma$ carries a signature that may be used to break the degeneracy between halo ellipticity and orientation bias using a forward modeling approach enabled by our model.

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