Studying Semi-leptonic $b \to (s, d)\nu\bar{\nu}$ Decays in the MSSM without R-parity

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Abstract

We present a complete study of R-parity violating supersymmetric effects in thirteen exclusive and inclusive semi-leptonic $b \to (s, d)\nu\bar{\nu}$ decays, including $B_u^+ \to K^{(*)+}\nu\bar{\nu}$, $B_d^0 \to K^{(*)0}\nu\bar{\nu}$, $B_s^0 \to \phi\nu\bar{\nu}$, $B_d^0 \to \pi^0(\rho^0)\nu\bar{\nu}$, $B_u^+ \to \pi^+(\rho^+)\nu\bar{\nu}$, $B_s^0 \to K^{(*)0}\nu\bar{\nu}$ and $B \to X_{s,d}\nu\bar{\nu}$ decay modes, and we find those thirteen modes are very sensitive to the constrained R-parity violating couplings. We derive stringent bounds on relevant R-parity violating couplings, which are based on all existent experimental upper limits of involved semi-leptonic decays. In addition, we also investigate the sensitivities of the branching ratios and di-neutrino invariant mass spectra to the survived R-parity violating coupling spaces. Since the experimental bounds would become much better soon through Super-B, we expect that future experiments will greatly strengthen our bounds.

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1 Introduction

The flavor changing neutral current (FCNC) processes are forbidden at tree level and occur at the lowest order only through one-loop diagrams in the standard model (SM). On the other hand, FCNC processes are very sensitive to possible new physics (NP) scenarios beyond the SM, and provide a unique source of constraints on some NP scenarios which predict a large change of these processes. And thus, the measurement of these processes has a very good chance to reveal NP beyond the SM. Therefore, they are widely recognized as a powerful tool to make stringent test of the SM.

Rare $B$ decays with a $\nu\bar{\nu}$ pair in the final state, as such FCNC examples, can be investigated through the large missing energy associated with the two neutrinos. On the other hand, experimental search of semi-leptonic $b \rightarrow (s,d)\nu\bar{\nu}$ decays is a hard task. At present, only the upper bounds have been set by the BABAR, Belle, DELPHI and ALEPH collaborations. We summarize here experimental upper limits for semi-leptonic $b \rightarrow s\nu\bar{\nu}$ and $b \rightarrow d\nu\bar{\nu}$ decays at the 90% C.L. in Eq. (1) and Eq. (2), respectively.

\begin{align}
B(B^0_d \rightarrow K^0\nu\bar{\nu}) &< 160 \times 10^{-6} \quad [1], \\
B(B^0_d \rightarrow K^{*0}\nu\bar{\nu}) &< 120 \times 10^{-6} \quad [2], \\
B(B^0_s \rightarrow \phi\nu\bar{\nu}) &< 5400 \times 10^{-6} \quad [3], \\
B(B_s \rightarrow X_s\nu\bar{\nu}) &< 640 \times 10^{-6} \quad [4], \\
B(B^0_d \rightarrow \pi^0\nu\bar{\nu}) &< 220 \times 10^{-6} \quad [1], \\
B(B^+_u \rightarrow \pi^+\nu\bar{\nu}) &< 100 \times 10^{-6} \quad [5], \\
B(B^+_d \rightarrow \rho^0\nu\bar{\nu}) &< 440 \times 10^{-6} \quad [1], \\
B(B^+_u \rightarrow \rho^+\nu\bar{\nu}) &< 150 \times 10^{-6} \quad [1].
\end{align}

Theoretically, $b \rightarrow (s,d)\nu\bar{\nu}$ decays are very clean processes, which are sensitive to several possible sources of NP [11]. The NP effects in $b \rightarrow (s,d)\nu\bar{\nu}$ decays have been investigated by many authors (see e.g., Refs. [6, 7, 8, 9, 10, 11, 12, 13, 14]). Supersymmetry is one of the most widely discussed options of NP, in both its R-parity conserving and R-parity violating (RPV) incarnations [15, 16]. In recent papers we have presented detailed study of charged Higgs effects and RPV effects in rare exclusive $b \rightarrow u\ell\nu\bar{\ell}$ and $b \rightarrow c\bar{c}s(d)$ decays [17, 18]. In the minimal supersymmetric standard model (MSSM) [19, 20], with R-parity conservation, the new contributions to the $b \rightarrow s\nu\bar{\nu}$ transition have been discussed (for instance, see Refs. [21, 22, 23]). In this work, we will concentrate on RPV effects in the exclusive and inclusive semi-leptonic $b \rightarrow (s,d)\nu\bar{\nu}$ decays. From the latest experimental data given in Eqs. (1-2) and the theoretical parameters with uncertainties, we will derive the new conservative upper limits.
on the relevant RPV coupling products. Moreover, we will also investigate how survived RPV coupling spaces can affect on the branching ratios and di-neutrino invariant mass (i.e. missing mass) spectra in these semi-leptonic $b \rightarrow (s, d)\nu \bar{\nu}$ decays. We find these observables are still very sensitive to survived RPV coupling spaces.

Our letter is organized as follows: In Sec. 2, we review the effective Hamiltonian for $b \rightarrow (s, d)\nu \bar{\nu}$ transitions and define the observables that can in principle be measured in these decays. In Sec. 3, we deal with the numerical results. We display the constrained parameter spaces which satisfy all the available experimental upper limits of the $b \rightarrow (s, d)\nu \bar{\nu}$, and then, we investigate the sensitivities of the branching ratios and di-neutrino invariant mass spectra to the survived RPV coupling spaces in those decays. We conclude in Sec. 4.

2 Theoretical Framework

The $b \rightarrow d_j \nu_{i'} \bar{\nu}_i$ ($j = 1, 2$ and $i, i' = e, \mu, \tau$) transitions can be described by the effective Hamiltonian,

$$
H_{\text{eff}}(b \rightarrow d_j \nu_{i'} \bar{\nu}_i) = C_L^{\nu'} \overline{b} \gamma_\mu (1 - \gamma_5) d_j \bar{\nu}_i \gamma^\mu (1 - \gamma_5) \nu_{i'} + C_R^{\nu'} \overline{b} \gamma_\mu (1 + \gamma_5) d_j \bar{\nu}_i \gamma^\mu (1 - \gamma_5) \nu_{i'} .
$$

(3)

In the SM, $b \rightarrow d_j \nu_{i'} \bar{\nu}_i$ proceeds via $W$ box and $Z$ penguin diagrams, therefore only purely left-handed currents $\overline{b} \gamma_\mu (1 - \gamma_5) d_j \bar{\nu}_i \gamma^\mu (1 - \gamma_5) \nu_{i'}$ are present. The corresponding left-handed coefficient reads

$$
C_{L, \text{SM}}^{\nu'} = \frac{G_F}{2\pi \sqrt{2}} V_{td} V_{tb}^* X(x_t) / \sin^2 \theta_W .
$$

where $G_F$ is the Fermi constant, $\alpha_e$ is the fine structure constant, $\theta_W$ is the Weinberg angle, and $V_{ij}$ are the CKM matrix elements. Function $X(x_t)$ is dominated by the short-distance dynamics associated with top quark exchange $\Box$, and has the theoretical uncertainty due to the error of top quark mass, whose explicit form can be found in Refs. [25, 26].

In supersymmetric models without R-parity [15, 16], extra trilinear RPV terms $\lambda'_{ijk} \hat{L}_i \hat{Q}_j \hat{D}_k$ are allowed in the superpotential [27]. Both left-handed and right-handed currents are present in $b \rightarrow d_j \nu_{i'} \bar{\nu}_i$ transition at the tree level in these models. Then the corresponding coefficients in Eq. (3) are written as

$$
C_L^{\nu'} = C_{L, \text{SM}}^{\nu'} - \sum_k \frac{\lambda'_{ijk} \lambda'_{i'jk}}{8 m_{d_k}^2} , \quad C_R^{\nu'} = \sum_k \frac{\lambda'_{ikj} \lambda'_{i'k3}}{8 m_{d_k}^2} .
$$

(4)

$\hat{L}$ and $\hat{Q}$ are the SU(2) doublet lepton and quark superfields, respectively, $\hat{D}^c$ are the singlet superfields, while $i, j$ and $k$ are generation indices and the superscript $c$ denotes a charge conjugate field.
RPV couplings $\lambda^\nu_{ijk}\lambda^\nu_{ljk}$ arise from right-handed squark exchanges, and $\lambda^\nu_{ijk}\lambda^\nu_{ij3}$ come from left-handed squark exchanges. Note that the RPV coupling coefficient $\lambda^\nu_{ijk}$ can be a complex in our convention, which is different from Ref. [6].

From the theoretical point of view, the inclusive semi-leptonic $b \rightarrow q \nu \bar{\nu}$ ($q = s, d$) decays are very clean processes, since both the perturbative $\alpha_s$ and the non-perturbative $1/m_b^2$ corrections are known to be small. Their dineutrino invariant mass distributions are given as following

$$
\frac{dB(B \rightarrow X_q \nu \nu \bar{\nu})}{ds_b} = \frac{\pi \kappa(0)}{16\pi^3 m_b^2} \left( |C^\nu_L|^2 + |C^\nu_R|^2 \right) \sqrt{\lambda(m_b^2, m_q^2, s_b)} \times \left[ 3s_b \left( m_b^2 + m_q^2 - s_b - 4m_b m_q \frac{\text{Re}(C^\nu_L C^\nu_R)}{|C^\nu_L|^2 + |C^\nu_R|^2} \right) + \lambda(m_b^2, m_q^2, s_b) \right],
$$

(5)

where $s_b = (p_b - p_q)^2$, $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$, and $\kappa(0) = 0.83$ represents the QCD correction to the $b \rightarrow q \nu \bar{\nu}$ matrix element [6, 28, 29]. We have summed over the neutrino flavors in Eq. (5).

In order to compute branching ratios of the exclusive semi-leptonic $b \rightarrow (s, d)\nu \bar{\nu}$ decays, we need the matrix elements of the effective hamiltonian between the states of the initial $B$ particle and the final particles $M, \nu, \bar{\nu}$. The hadronic matrix elements for $B \rightarrow P$ transition ($P$ is a pseudoscalar meson, $\pi$ or $K$) can be parameterized in terms of the form factors $f_+^P(s_B)$ and $f_0^P(s_B)$ as

$$
c_P\langle P(p)|\bar{u}\gamma_\mu b|B(p_B)\rangle = f_+^P(s_B)(p + p_B)_\mu + \left[ f_0^P(s_B) - f_+^P(s_B) \right] \frac{m_B^2 - m_P^2}{s_B} q_\mu,
$$

(6)

where the factor $c_P$ accounts for the flavor content of particles ($c_P = \sqrt{2}$ for $\pi^0$, and $c_P = 1$ for $\pi^-, K^-$) and $s_B = q^2$ ($q = p_B - p = p_\nu + p_{\bar{\nu}}$). For $B \rightarrow V$ transition ($V$ is a vector $K^*, \rho$ or $\phi$ meson) can be written in terms of five form factors

$$
c_V\langle V(p, \varepsilon^*)|\bar{u}\gamma_\mu (1 - \gamma_5)b|B(p_B)\rangle = \frac{2V(s_B)}{m_B + m_V} \varepsilon_{\mu\alpha\beta\varepsilon^*\nu} p_\beta \varepsilon^* p_\alpha
d - \frac{i}{2m_V} \left[ \varepsilon^*_\mu (m_B + m_V) A_1(s_B) - (p_B + p)_\mu (\varepsilon^* \cdot p_B) \frac{A_2(s_B)}{m_B + m_V} \right]
+ \frac{i q_\mu (\varepsilon^* \cdot p_B)}{s_B} \frac{2m_V}{A_3(s_B) - A_0(s_B)},
$$

(7)

where $c_V = \sqrt{2}$ for $\rho^0$, $c_V = 1$ for $\rho^-, K^{*-}, \phi$, and with the relation $A_3(s_B) = \frac{m_B + m_V}{2m_V} A_1(s_B) - \frac{m_B - m_V}{2m_V} A_2(s_B)$.

In terms of the effective Hamiltonian shown in Eq. (3) and the relevant form factors given in Eqs. (6, 7), the di-neutrino invariant mass distributions for $B \rightarrow P\nu\bar{\nu}$ and $B \rightarrow V\nu\bar{\nu}$ decays
can be written as \([7, 30]\)

\[
\frac{d B(B \to P \nu \bar{\nu})}{d s_B} = |C_L^\nu + C_R^\nu|^2 \frac{\tau_B m_B^2}{2\pi^3 G_F} \Lambda^{3/2}_P(s_B) \left[ f^P_+(s_B) \right]^2,
\]

\[
\frac{d B(B \to V \nu \bar{\nu})}{d s_B} = |C_L^\nu + C_R^\nu|^2 \frac{\tau_B m_B^2}{2\pi^3 G_F} \Lambda^{1/2}_V(s_B) \frac{8 s_B \lambda_V(s_B) V^2(s_B)}{(1 + \sqrt{r_V})^2}
\]

\[
+ |C_L^\nu - C_R^\nu|^2 \frac{\tau_B m_B^2}{2\pi^3 G_F} \Lambda^{1/2}_V(s_B) \frac{1}{r_V} \left( (1 + \sqrt{r_V})^2 (\lambda_V(s_B) + 12 r_V s_B) A_1^2(s_B) \right)
\]

\[
+ \frac{\lambda_M^2(s_B) A_2^2(s_B)}{(1 + \sqrt{r_V})^2} - 2 \lambda_V(s_B)(1 - r_V - s_B) A_1(s_B) A_2(s_B)
\]

where \(\lambda_M(s_B) = \lambda(1, r_M, s_B/m_B^2)\) with \(r_M = m_M^2/m_B^2\), and we have summed over the neutrino flavors.

For our numerical results, we use the relevant \(B \to P(V)\) form factors given in \([31]\). However, \(B_\to K\) form factors are not given in LCSR results \([31]\). After discussions with authors of Ref. \([31]\), we obtain them as \(F^{B_\to K}(s_B) = F^{B_\to K}(s_B) \left( \frac{F^{B_\to K}}{F^{B_\to K}(s_B)} \right)\). The uncertainties of form factors at \(s_B = 0\) induced by \(F(0)\) are considered, to be conservative, we adopt these uncertainties for full \(s_B\) range. The CKM matrix elements are taken from \([32]\), and masses and lifetimes are from Ref. \([33]\).

## 3 Numerical Results and Discussions

In this section, we summarize our numerical results and analysis in the semi-leptonic \(b \to (s, d) \nu \bar{\nu}\) decays. To be conservative, we use all input parameters which are varied randomly within \(1\sigma\) ranges in our numerical results. We use the average \(\tau_B = (\tau_B^+ + \tau_B^-)/2\) for the inclusive decays. When we study the RPV effects, we consider only one RPV coupling product contributions at one time, neglecting the interferences between different RPV coupling products, but keeping their interferences with the SM amplitude. We assume the masses of sfermions are 500 GeV. For other values of the sfermion masses, the bounds on the couplings in this paper can be easily obtained by scaling them by factor \(\bar{f}^2 \equiv (\frac{m_{\tilde{f}}}{500\ \text{GeV}})^2\).

The transitions \(b \to (s\ or\ d) \nu \bar{\nu}\) involve the same set of the RPV coupling products for every generation of neutrinos: For six semi-leptonic \(b \to s \nu \bar{\nu}\) decays, \(B^+_u \to K^{(*)+} \nu \bar{\nu}\), \(B^0_d \to K^{(*)0} \nu \bar{\nu}\), \(B^0_s \to \phi \nu \bar{\nu}\) and \(B \to X_s \nu \bar{\nu}\), there are two kinds of RPV coupling products, \(\lambda_{3k}^\nu \lambda_{ik}^\nu\) and \(\lambda_{3k}^\nu \lambda_{ik}^\nu\), which come from left-handed and right-handed squark exchanges, respectively. For seven semi-leptonic \(b \to d \nu \bar{\nu}\) decay modes, \(B^0_d \to \pi^0(\rho^0) \nu \bar{\nu}\), \(B^+_u \to \pi^+(\rho^+) \nu \bar{\nu}\), \(B^0_s \to K^{(*)+} \nu \bar{\nu}\)
Figure 1: Survived parameter spaces shown for the relevant RPV coupling products with 500 GeV sfermion masses constrained by semi-leptonic (a-b) $b \rightarrow s \nu \bar{\nu}$ and (c-d) $b \rightarrow d \nu \bar{\nu}$ decays, respectively, where $\phi_{RPV}$ denotes the RPV weak phase.

Table 1: Bounds on the relevant RPV coupling products constrained by semi-leptonic $b \rightarrow (s, d) \nu \bar{\nu}$ decays for 500 GeV sfermions.

| Couplings                  | Our bounds  | [Processes]                                      | Previous bounds [Processes] |
|----------------------------|-------------|--------------------------------------------------|-----------------------------|
| $|\lambda^e_{i3k} \lambda_{i'2k}|$ | $\leq 1.5 \times 10^{-2}$ | $B^+_s \rightarrow K^{(*)+} \nu \bar{\nu}, B^0_d \rightarrow K^{(*)0} \nu \bar{\nu}$ | $\leq 3.5 \times 10^{-2}$ $[B \rightarrow X_d \nu \bar{\nu}]$ [4, 6] |
| $|\lambda^e_{i'k2} \lambda_{ik3}|$ | $\leq 1.3 \times 10^{-2}$ | $B^+_s \rightarrow K^{(*)+} \nu \bar{\nu}, B^0_d \rightarrow K^{(*)0} \nu \bar{\nu}$ | $\leq 3.5 \times 10^{-2}$ $[B \rightarrow X_d \nu \bar{\nu}]$ [4, 6] |
| $|\lambda^e_{i'k1} \lambda_{ik1}|$ | $\leq 2.5 \times 10^{-2}$ | $B^+_s \rightarrow \pi^+(\rho^+) \nu \bar{\nu}, B^0_d \rightarrow \pi^0(\rho^0) \nu \bar{\nu}$ | \cdots \cdots |
| $|\lambda^e_{i'k1} \lambda_{ik3}|$ | $\leq 2.5 \times 10^{-2}$ | $B^+_s \rightarrow \pi^+(\rho^+) \nu \bar{\nu}, B^0_d \rightarrow \pi^0(\rho^0) \nu \bar{\nu}$ | \cdots \cdots |

and $B \rightarrow X_d \nu \bar{\nu}$, RPV coupling products $\lambda^e_{i3k} \lambda_{i'1k}$ and $\lambda^e_{i'k1} \lambda_{ik3}$ arise from left-handed and right-handed squark exchanges, respectively. We use the latest experimental upper limits from Refs. [1, 2, 3, 4, 5], which are listed in Eqs. (1-2), to constrain the relevant RPV coupling products. Our bounds on the four RPV coupling products are demonstrated in Fig. 1. In Fig. 1(a-b), we find that the $b \rightarrow s \nu \bar{\nu}$ decays give quite strong correlation between the moduli and the
RPV weak phases of $\lambda_{i3k}^{*} \lambda'_{i'2k}$ and $\lambda_{i'2k}^{*} \lambda_{i'3k}$ coupling products. Fig. (c-d) show that RPV weak phases of $\lambda_{i3k}^{*} \lambda'_{i'1k}$ and $\lambda_{i'1k}^{*} \lambda_{i'3k}$ are not restricted by current experimental upper limits of $B_d^0 \rightarrow \pi^0(\rho^0)\bar{\nu}\bar{\nu}$ and $B_u^+ \rightarrow \pi^+(\rho^+)\bar{\nu}\bar{\nu}$ decays, however, corresponding moduli are upper limited. The upper limits of the moduli for the relevant RPV coupling products are summarized in Table I. Our bounds on $\lambda_{i3k}^{*} \lambda'_{i'1k}$ and $\lambda_{i'1k}^{*} \lambda_{i'3k}$, which are mainly from the semi-leptonic $b \rightarrow s\nu\bar{\nu}$ experimental data, are stronger than ones obtained from the inclusive semi-leptonic $b \rightarrow s\nu\bar{\nu}$ decay [4, 6]. We obtain for the first time the bounds on $\lambda_{i3k}^{*} \lambda'_{i'1k}$ and $\lambda_{i'2k}^{*} \lambda_{i'3k}$ couplings from the $b \rightarrow d\nu\bar{\nu}$ transitions.

We note that some quadratic RPV coupling combinations, which contribute to $b \rightarrow (s, d)\nu\bar{\nu}$ transitions, may also give contributions to $b \rightarrow (s, d)\gamma$ and $b \rightarrow (s, d)\ell^+\ell^-$ ($\ell = e, \mu, \tau$) processes. The decay $b \rightarrow s\gamma$ in the MSSM without R-parity has been shown in [34] to give weak constraints on relevant RPV coupling combinations, $|\lambda_{i3k}^{*} \lambda'_{i'2k}| \leq 2.25$ and $|\lambda_{i'2k}^{*} \lambda'_{i'3k}| \leq 0.87$ with 500 GeV sfermion masses. The RPV effects in $b \rightarrow (s, d)\ell^+\ell^-$ processes have been studies in Refs. [35, 36, 37, 38, 39, 40, 41, 42], and some upper limits of their RPV coupling combinations are about one order of magnitude stronger than ours from $b \rightarrow (s, d)\nu\bar{\nu}$. Here, we list the stronger upper limits from $b \rightarrow (s, d)\ell^+\ell^-$ processes with 500 GeV sfermions: $|\lambda_{i3k}^{*} \lambda'_{i'2k}| \leq 1.2 \times 10^{-3}$ ($i = 1, 2$) [37], $|\lambda_{i'2k}^{*} \lambda'_{i'3k}| \leq 6.7 \times 10^{-3}$ ($i \neq i'$) [38], $|\lambda_{i'1k}^{*} \lambda_{i'3k}| \leq 2.8 \times 10^{-3}$ [39], and $|\lambda_{i'2k}^{*} \lambda_{i'3k}| \leq 3.3 \times 10^{-3}$ [39]. In addition, single bounds of $\lambda'_{ijk}$ are obtained by many authors (for instance, see Refs. [40, 41, 42, 43, 44, 45, 46, 47]). We also note that some of the single $\lambda'$ couplings can generate sizable neutrino masses [41, 43]. Allanach et al. have obtained quite strong upper bound $|\lambda'_{ijj}| \leq 10^{-2}$ with 500 GeV sfermions in the RPV mSUGRA model, and Barbier et al. have gotten $|\lambda'_{i33}| \leq 4.4 \times 10^{-3}$. Furthermore, the $\lambda'_{111}$ coupling has been constrained as low as $|\lambda'_{111}| \leq 1.8 \times 10^{-2}$ by neutrino-less double beta decay [47]. If we now compare our combined bounds with the products of the single bounds, we find that our combined bounds are weaker one or two order(s) of magnitude than the products of the single bounds. However, it also should be noted that the parameter spaces of $\lambda'$ from neutrino masses can be evaded since several other parameters are usually involved in the extraction of the constraints [48]. Furthermore, the constraints on $\lambda'$ from neutrino masses would depend on the explicit neutrino masses models with trilinear couplings only, bilinear couplings only, or both [41].

Next, we will first explore the RPV MSSM effects by using our constrained RPV parameter
spaces, and then discuss the RPV effects after also considering previous stronger bounds in the semi-leptonic $b \rightarrow (s,d)\nu \bar{\nu}$ decays. Now using the survived RPV parameter spaces shown in Fig. 1, we explore the RPV MSSM effects in the semi-leptonic $b \rightarrow (s,d)\nu \bar{\nu}$ decays, which satisfy all experimental upper limits given in Eqs. (1-2). Our RPV MSSM predictions within the theoretical uncertainties of input parameters are given in Table 2 together with experimental upper limits and the SM predictions for a convenient comparison. In Table 2, the second and third columns give the experimental upper limits and the SM predictions, respectively, the forth column lists the effects of left-handed squark exchange coupling $\lambda^+_{i3k}\lambda^j_{i'jk}$, and the last column summaries the effects of coupling $\lambda^+_{i'jk}\lambda^j_{ik3}$ due to right-handed squark exchange. Main theoretical uncertainties of the SM predictions arise from the CKM matrix elements, Wilson coefficient and hadronic transition form factors (only for the exclusive decays). Comparing with experimental upper limits and the SM predictions, we find some salient features of numerical results of the RPV effects listed in Table 2

Table 2: The branching ratios of the semi-leptonic $b \rightarrow (s,d)\nu \bar{\nu}$ decays (in units of $10^{-6}$), and $j = 2(1)$ for the $b \rightarrow s(d)$ transition.

| Observable | Exp. Data | SM Predictions | MSSM w/ $\lambda^+_{i3k}\lambda^j_{i'jk}$ | MSSM w/ $\lambda^+_{i'jk}\lambda^j_{ik3}$ |
|------------|-----------|----------------|------------------------------------------|------------------------------------------|
| $B(B^0 \rightarrow K^0\nu \bar{\nu})$ | $< 160$ | [3.48, 6.55] | [0.14, 13.14] | [0.14, 13.07] |
| $B(B^+ \rightarrow K^+\nu \bar{\nu})$ | $< 14$ | [3.75, 7.04] | [0.15, 14.00] | [0.15, 14.00] |
| $B(B^0 \rightarrow K^{*0}\nu \bar{\nu})$ | $< 120$ | [6.98, 15.19] | [0.21, 46.14] | [5.16, 74.66] |
| $B(B^+ \rightarrow K^{*+}\nu \bar{\nu})$ | $< 80$ | [7.55, 16.35] | [0.22, 49.33] | [5.55, 80.00] |
| $B(B^0_s \rightarrow \phi\nu \bar{\nu})$ | $< 5400$ | [8.89, 18.85] | [0.36, 56.48] | [5.56, 161.17] |
| $B(B \rightarrow X_d\nu \bar{\nu})$ | $< 640$ | [31.15, 48.94] | [2.09, 142.30] | [31.65, 282.06] |
| $B(B^0 \rightarrow \pi^0\nu \bar{\nu})$ | $< 220$ | [0.05, 0.12] | [0.07, 47.00] | [0.01, 46.73] |
| $B(B^+ \rightarrow \pi^+\nu \bar{\nu})$ | $< 100$ | [0.11, 0.25] | [0.14, 100.00] | [0.02, 100.00] |
| $B(B^0_s \rightarrow K^0\nu \bar{\nu})$ | $\ldots$ | [0.11, 0.43] | [0.10, 165.05] | [0.04, 166.20] |
| $B(B^0 \rightarrow \rho^0\nu \bar{\nu})$ | $< 440$ | [0.10, 0.29] | [0.11, 70.48] | [0.12, 70.48] |
| $B(B^+ \rightarrow \rho^+\nu \bar{\nu})$ | $< 150$ | [0.22, 0.62] | [0.24, 150.00] | [0.26, 150.00] |
| $B(B^0_s \rightarrow K^{*0}\nu \bar{\nu})$ | $\ldots$ | [0.24, 0.62] | [0.30, 245.25] | [0.19, 238.58] |
| $B(B \rightarrow X_d\nu \bar{\nu})$ | $\ldots$ | [1.17, 2.23] | [1.62, 907.06] | [1.63, 932.43] |

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RPV coupling $\lambda_{\nu i'2k}^s$ is only constrained by the experimental upper limit of $\mathcal{B}(B_u^+ \rightarrow K^+\nu\bar{\nu})$, and bounds on this coupling constant obtained from other exclusive $b \rightarrow s\nu\bar{\nu}$ decays and inclusive $B \rightarrow X_s\nu\bar{\nu}$ are weaker than one obtained from $B_u^+ \rightarrow K^+\nu\bar{\nu}$ decay. Comparing with the SM predictions, we find contributions of $\lambda_{\nu i'2k}^s$ coupling could enlarge the allowed ranges of all relevant branching ratios, their upper limits are increased two or three times, and their lower limits are reduced more than one order.

2. The restrictions of $\lambda_{\nu'k2}^s\lambda_{ik3}^s$ come from the experimental upper limits of $\mathcal{B}(B_u^+ \rightarrow K^+\nu\bar{\nu})$ and $\mathcal{B}(B_u^+ \rightarrow K^*\nu\bar{\nu})$. The $\lambda_{\nu'k2}^s\lambda_{ik3}^s$ coupling effects are same as the effects of $\lambda_{\nu'k3}^s\lambda_{i'2k}^s$ coupling in $B^0_d \rightarrow K^0\nu\bar{\nu}$ and $B_u^+ \rightarrow K^+\nu\bar{\nu}$ decays. For $B^0_d \rightarrow K^*0\nu\bar{\nu}$, $B_u^+ \rightarrow K^*\nu\bar{\nu}$, $B_s^0 \rightarrow \phi\nu\bar{\nu}$ and $B \rightarrow X_s\nu\bar{\nu}$ decays, $\lambda_{\nu'k2}^s\lambda_{i'k3}^s$ coupling could obviously increase the allowed upper limits of these branching ratios.

3. RPV couplings $\lambda_{\nu i'1k}^s\lambda_{\nu'j1k}^s$ and $\lambda_{\nu'k1}^s\lambda_{i'k3}^s$ are constrained by the experimental upper limits of $\mathcal{B}(B_u^+ \rightarrow \pi^+\nu\bar{\nu})$ and $\mathcal{B}(B_u^+ \rightarrow \rho^+\nu\bar{\nu})$. All allowed upper limits of the relevant branching ratios could be significantly increased by both $\lambda_{\nu i'2k}^s\lambda_{\nu'1k}^s$ and $\lambda_{\nu'k1}^s\lambda_{i'k3}^s$ couplings. The upper bounds of RPV predictions for $\mathcal{B}(B^0_d \rightarrow \pi^0(\rho^0)\nu\bar{\nu})$ are about 6 times stronger than existing experimental limits. The allowed lower limits of $\mathcal{B}(B^0_d \rightarrow \pi^0\nu\bar{\nu})$, $\mathcal{B}(B_u^+ \rightarrow \pi^+\nu\bar{\nu})$ and $\mathcal{B}(B_s^0 \rightarrow K^0\nu\bar{\nu})$ could be evidently decreased by $\lambda_{\nu'k1}^s\lambda_{i'k3}^s$ coupling.

Next we want to illustrate briefly the sensitivities of relevant observables to RPV couplings. To this end, for each RPV coupling product, we can present the correlations of di-neutrino quantities to RPV couplings.

The effects of the RPV couplings $\lambda_{\nu i'2k}^s$ and $\lambda_{\nu'k2}^s\lambda_{i'k3}^s$ on $B \rightarrow X_s\nu\bar{\nu}$, $K^+(K^{*+})\nu\bar{\nu}$ decays are shown in Fig. 2 and Fig. 3 respectively. Now we turn to discuss plots of Fig. 2 in detail. Fig. 2 displays the effects of RPV couplings $\lambda_{\nu i'2k}^s$ from left-handed squark exchanges in $B \rightarrow X_s\nu\bar{\nu}$, $K^+(K^{*+})\nu\bar{\nu}$ decays. As shown in Fig. 2(a-c), $d\mathcal{B}(B \rightarrow X_s\nu\bar{\nu})/ds_b$ and $d\mathcal{B}(B_u^+ \rightarrow K^+(K^{*+})\nu\bar{\nu})/ds_B$ are obviously affected by $\lambda_{\nu i'2k}^s$ coupling, but the $\lambda_{\nu i'2k}^s$ contributions to them cannot be distinguished from the SM expectations. The scatter plots Fig. 2(d-f) and
Figure 2: The effects of RPV couplings $\lambda'_{i3k}^l \lambda'_{i'2k}$ from left-handed squark exchanges in $B \rightarrow X_s \nu \bar{\nu}, K^+ (K^{*+}) \nu \bar{\nu}$ decays. $B$ and $|\lambda'_{i3k}^l \lambda'_{i'2k}|$ are in units of $10^{-6}$ and $10^{-2}$, respectively.

Figure 3: The effects of RPV couplings $\lambda'_{i'k2}^l \lambda_{i3k}^l$ due to right-handed squark exchanges in $B \rightarrow X_s \nu \bar{\nu}, K^+ (K^{*+}) \nu \bar{\nu}$ decays. $B$ and $|\lambda'_{i'k2}^l \lambda_{i3k}^l|$ are in units of $10^{-6}$ and $10^{-2}$, respectively.
Figure 4: The effects of RPV couplings $\lambda_{i3k}^{s} \lambda_{i1k}^{y}$ from left-handed squark exchanges in $B \rightarrow X_d \nu \bar{\nu}, \pi^+(\rho^+)\nu \bar{\nu}$ decays. $B$ and $|\lambda_{i3k}^{s} \lambda_{i1k}^{y}|$ are in units of $10^{-6}$ and $10^{-2}$, respectively.

Fig. 2(g-i) show $B(B \rightarrow X_s \nu \bar{\nu}, K^+(K^{*+})\nu \bar{\nu})$ correlated with $|\lambda_{i3k}^{s} \lambda_{i2k}^{y}|$ and its phase $\phi_{RPV}$, respectively. From Fig. 2(d-f), we see that $|\lambda_{i3k}^{s} \lambda_{i2k}^{y}|$, and they may have minima at $|\lambda_{i3k}^{s} \lambda_{i2k}^{y}| \approx 5 \times 10^{-3}$. Fig. 2(g-i) show that $B(B \rightarrow X_s \nu \bar{\nu}, K^+(K^{*+})\nu \bar{\nu})$ have some sensitivity to $|\lambda_{i3k}^{s} \lambda_{i2k}^{y}|$, and they may have minima at $|\lambda_{i3k}^{s} \lambda_{i2k}^{y}| \approx 5 \times 10^{-3}$. Fig. 2(g-i) show that $B(B \rightarrow X_s \nu \bar{\nu}, K^+(K^{*+})\nu \bar{\nu}) < 40 \times 10^{-6}$ and $B(B_u^+ \rightarrow K^+(K^{*+})\nu \bar{\nu}) < 5(10) \times 10^{-6}$. Fig. 3 shows RPV coupling $\lambda_{i2k}^{s} \lambda_{ik3}^{y}$ effects due to right-handed squark exchanges in $B \rightarrow X_s \nu \bar{\nu}, K^+(K^{*+})\nu \bar{\nu}$ decays. From Fig. 3(a-c), we can see the effects of $\lambda_{i2k}^{s} \lambda_{ik3}^{y}$ on $dB(B \rightarrow X_s \nu \bar{\nu}, K^+(K^{*+})\nu \bar{\nu})/ds_B$ are very similar to those of $\lambda_{i3k}^{s} \lambda_{i2k}^{y}$ shown in Fig. 2(a-c). As shown in Fig. 3(d,f,g,i), $B(B \rightarrow X_s \nu \bar{\nu}, K^+(K^{*+})\nu \bar{\nu})$ are also very sensitive to $\lambda_{i2k}^{s} \lambda_{ik3}^{y}$ coupling, and $B(B \rightarrow X_s \nu \bar{\nu}, K^+(K^{*+})\nu \bar{\nu})$ are obviously increasing with $|\lambda_{i2k}^{s} \lambda_{ik3}^{y}|$ but decreasing with $|\phi_{RPV}|$. Fig. 3(e,h) show that $B(B_u^+ \rightarrow K^+\nu \bar{\nu})$ is sensitive to $\lambda_{i3k}^{s} \lambda_{i2k}^{y}$ coupling, and it has minimum at $|\lambda_{i3k}^{s} \lambda_{i2k}^{y}| \approx 5 \times 10^{-3}$ and $\phi_{RPV} \approx 0^\circ$.

Since the branching ratios of the semi-leptonic $b \rightarrow d \nu \bar{\nu}$ decays are not sensitive to the relevant RPV weak phases, we will only show the correlations between the branching ratios and the moduli. Fig. 4 illustrates the contributions of RPV coupling $\lambda_{i3k}^{s} \lambda_{i1k}^{y}$ to $B \rightarrow X_d \nu \bar{\nu}, \pi^+(\rho^+)\nu \bar{\nu}$ decays. As shown in the two-dimensional scatter plots Fig. 4(a-c), $\lambda_{i3k}^{s} \lambda_{i1k}^{y}$ coupling may change the order of $dB(B \rightarrow X_d \nu \bar{\nu})/ds_b$ and $dB(B_u^+ \rightarrow \pi^+(\rho^+)\nu \bar{\nu})/ds_B$, and the $\lambda_{i3k}^{s} \lambda_{i1k}^{y}$ contributions to them are possibly distinguishable from the SM expectations at all $s_b(s_B)$ regions. Fig. 4(d-f) show $B(B \rightarrow X_d \nu \bar{\nu}, \pi^+(\rho^+)\nu \bar{\nu})$ correlated with $|\lambda_{i3k}^{s} \lambda_{i1k}^{y}|$, and we see that $B(B \rightarrow X_d \nu \bar{\nu}, \pi^+(\rho^+)\nu \bar{\nu})$ are greatly increasing with $|\lambda_{i3k}^{s} \lambda_{i1k}^{y}|$. The effects of RPV couplings $\lambda_{i2k}^{s} \lambda_{ik3}^{y}$ due to right-handed squark exchanges in $B \rightarrow X_d \nu \bar{\nu}, \pi^+(\rho^+)\nu \bar{\nu}$ decays are very similar...
to those of $\lambda^*_{i3k}\lambda'_{i1k}$ in these decays shown in Fig. 4 and we will not show the correlations between observables and RPV coupling $\lambda^*_{i1k}\lambda'_{i3k}$ again.

As we mentioned before, some of our combined bounds are weaker one or two order(s) of magnitude than the existing bounds. Now we are ready to discuss the RPV coupling effects after also considering relevant previous stronger bounds. From above analysis, we know that the left-handed squark exchange RPV couplings have not evident effects on the branching ratios of $B \to X_s\nu\bar{\nu}, K^+\nu\bar{\nu}, K^{*+}\nu\bar{\nu}$ decays if $|\lambda^*_{i3k}\lambda'_{i2k}| < 2.7 \times 10^{-3}$. $B(B \to X_s\nu\bar{\nu}, K^{*+}\nu\bar{\nu})$ will not be obviously affected by RPV couplings due to right-handed squark exchanges if $|\lambda^*_{i1k}\lambda'_{i3k}| < 1.7 \times 10^{-3}$, and $B(B \to K^+\nu\bar{\nu})$ will not be obviously affected if $|\lambda^*_{i1k}\lambda'_{i3k}| < 1.1 \times 10^{-3}$. As for $B \to X_d\nu\bar{\nu}, \pi^+(\rho^+)\nu\bar{\nu}$ decays, the RPV coupling contributions still can distinguish from the SM ones if $|\lambda^*_{i3k}\lambda'_{i1k}|$ and $|\lambda^*_{i1k}\lambda'_{i3k}|$ are larger than $6.1 \times 10^{-4}$. Then we can give a conclusion safely, the RPV couplings $\lambda^*_{i1k}\lambda'_{i2k}$ ($i = 1, 2$), which moduli are less than $1.2 \times 10^{-3}$ from $b \to s\ell^+\ell^-$ [37], give small contributions to $B \to X_s\nu\bar{\nu}, K^+\nu\bar{\nu}, K^{*+}\nu\bar{\nu}$ decays, and all other relevant couplings still can give remarkable contributions to the semi-leptonic $b \to (s, d)\nu\bar{\nu}$ decays after considering the existing bounds.

4 Summary

In this letter we have performed a brief study of the RPV coupling effects in supersymmetry from the exclusive and inclusive semi-leptonic $B$ decays with a $\nu\bar{\nu}$ pair, which include $B^+_u \to K^{(*)+}\nu\bar{\nu}$, $B^0_d \to K^{(*)0}\nu\bar{\nu}$, $B^0_s \to \phi\nu\bar{\nu}$, $B \to X_s\nu\bar{\nu}$, $B^0_d \to \pi^0(\rho^0)\nu\bar{\nu}$, $B^+_u \to \pi^+(\rho^+)\nu\bar{\nu}$, $B^0_s \to K^{(*)0}\nu\bar{\nu}$ and $B \to X_d\nu\bar{\nu}$ thirteen decay modes. Considering the theoretical uncertainties, we have obtained conservatively constrained parameter spaces of RPV coupling constants from the latest experimental upper limits. We found, at present, the strongest bounds on the relevant RPV couplings come from the exclusive decays. Furthermore, we also investigated the sensitivities of the di-neutrino invariant mass spectra and branching ratios to the survived R-parity violating coupling spaces.

We have found that, after satisfying all the current experimental upper limits, both left-handed and right-handed squark exchange RPV couplings still have significant effects on these di-neutrino invariant mass spectra and branching ratios. The RPV contributions are not easily distinguishable from the SM predictions in the di-neutrino invariant mass spectra of the semi-
leptonic $b \rightarrow s\nu\bar{\nu}$ decays, nevertheless, the di-neutrino invariant mass spectra of the semi-leptonic $b \rightarrow d\nu\bar{\nu}$ decays are very useful to distinguish the RPV coupling effects at all kinematic regions. The branching ratios of the semi-leptonic $b \rightarrow s\nu\bar{\nu}$ decays are sensitive to both moduli and phases of relevant RPV coupling products, and the branching ratios of the semi-leptonic $b \rightarrow d\nu\bar{\nu}$ decays are only very sensitive to the moduli of relevant RPV coupling products.

However, observing rare $B$ decays with a $\nu\bar{\nu}$ pair is experimentally very challenging because of the two missing neutrinos and (many) hadrons, and these decays can be searched for through the large missing energy events in $B$ decays. With an advent of Super-$B$ facilities [49], the prospects of measuring the branching ratios of the semi-leptonic $b \rightarrow s\nu\bar{\nu}$ decays in next decade could be highly realistic, and it’s also possible to observe $B^+ \rightarrow \pi^+\nu\bar{\nu}$ decay. We expect that future experiments will significantly strengthen the allowed parameter spaces for RPV couplings. Our predictions of RPV effects on related observables could be very useful for probing RPV supersymmetric effects in future experiments.

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