2D Metal-Insulator transition as a percolation transition

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By carefully analyzing the low temperature density dependence of 2D conductivity in undoped high mobility n-GaAs heterostructures, we conclude that the 2D metal-insulator transition in this system is a density inhomogeneity driven percolation transition due to the breakdown of screening in the random charged impurity disorder background. In particular, our measured conductivity exponent of \( \sim 1.4 \) approaches the 2D percolation exponent value of 4/3 at low temperatures and our experimental data are inconsistent with there being a zero-temperature quantum critical point in our system.

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Ever since the pioneering observation by Kravchenko et al. of an apparent two-dimensional (2D) metal-insulator transition (MIT) in Si-MOSFET inversion layers, the nature of the transition has remained a controversial enigma. The 2D MIT has been claimed by some, most notably by Kravchenko and collaborators, to be an interaction driven and carrier density tuned T=0 quantum phase transition whereas others have argued that it is a disorder-driven crossover phenomenon between weakly and strongly localized 2D electron states. Density dependent scanning studies of the 2D chemical potential, surface acoustic wave measurements on low density 2D electrons, and compressibility measurements, as analyzed by direct numerical simulations of the energetics of the 2D disordered system, indicate very strong density inhomogeneities in the 2D system around the critical density \( n_c \) for the putative 2D MIT. Also, the critical density \( n_c \) is highly system-dependent and varies very strongly with the impurity disorder in the 2D system. In relatively highly disordered Si MOSFETs the typical \( n_c \approx 10^{11}\text{cm}^{-2} \) and in very high quality 2D n-GaAs heterostructures (the subject matter of our work presented in this paper) the typical \( n_c \sim 10^9\text{cm}^{-2} \). The strong dependence of \( n_c \) on disorder, the long range nature of the charged impurity disorder potential in 2D semiconductor structures, the observed density inhomogeneities around \( n_c \), all taken together, suggest a percolation-type transition underlying the 2D MIT. Indeed such a percolation transition for 2D systems was discussed by Efros fifteen years ago, and has recently been suggested in the context of the 2D MIT phenomena by several authors. In this Letter we provide rather compelling experimental evidence in support of the 2D MIT being a percolation transition, at least for our experimental system.

The basic picture of the percolation transition for 2D MIT is simple and highly physically motivated. As the carrier density \( n \) is lowered in a 2D system, screening becomes progressively weaker and strongly nonlinear. A small decrease in \( n \) leads to a large decrease in screening, and eventually to a highly inhomogeneous 2D system as the electron gas at low enough carrier densities becomes unable to screen the disorder potential with the individual charged impurity scattering potentials getting unscreened and their random distribution in the 2D system. This gives rise to the well-known random “hill-and-valley” potential landscape with the 2D carriers repelled from the potential hills and accumulating at the potential valleys in contrast to the high carrier density homogeneous situation. Once these “depletion (or denuded) zones”, associated with the disorder potential hills are numerous enough to prevent percolating conducting paths to span the 2D system connecting the “valley” regions, an effective 2D MIT transition takes place with the system being an effective metal (insulator) for \( n > (\sim) n_c \) where \( n_c \) is the critical percolation density. This percolation picture is particularly germane to 2D semiconductor systems because the disorder potential here arises from the presence of random charged impurities in the system making electronic screening a key ingredient in the effective disorder seen by the carrier system. Such a density-driven percolation transition is the hallmark of the long-range Coulomb disorder potential in the system since the Coulomb disorder can be tuned by changing the carrier density through the nonlinear screening mechanism. The key qualitative features of the 2D MIT phenomenon, namely the correlation between \( n_c \) and the sample quality as well as the density inhomogeneity around \( n_c \), are entirely consistent with a percolation transition in the screened Coulomb disorder potential.

Motivated by these percolation considerations we have carefully analyzed the density dependent conductivity \( \sigma(n) \) in ultra-high mobility n-GaAs undoped heterostructures which have demonstrated the lowest value of \( n_c \) \( (\sim 2\times10^9\text{cm}^{-2}) \) for 2D MIT so far in the literature. Most of the 2D MIT literature has emphasized the strong temperature dependence in the effective metallic \( (n > n_c) \) phase, which is now well-understood as a manifestation of the strong temperature dependence of the effective...
Coulombic disorder seen by the low-density 2D carriers through the temperature dependence of screening. In this Letter, we analyze the power law behavior of \( \sigma(n) \) in the context of density dependent screening and a percolation transition. We have also carried out a detailed Boltzmann theory calculation [13] using realistic quasi-2D parameters appropriate for our system, assuming the carrier transport to be dominated by charged impurities distributed randomly at the interface – the main physics being the temperature and wave vector dependent linear screening of the long-range Coulomb scattering potential of the charged impurities. We have earlier shown [14] that such a screening description provides an excellent qualitative agreement with experiment for the temperature dependent transport properties of our system.

The two samples used in this study are undoped GaAs/AlGaAs heterojunctions with field-induced carriers. From the surface, the epilayers consist of 60 nm n+ GaAs followed by a 600 nm Al\(_{0.3}\)Ga\(_{0.7}\)As barrier, and then 1000 nm of GaAs. The 2D electrons are induced from the ohmic contacts with a density that is proportional to the voltage on the n+ GaAs layer that serves as a top gate. Standard four terminal lock-in measurements are made to measure the resistance over a wide range of density. Conductivity of these square samples is determined using van der Pauw techniques. In Sample A the density spans a range of 0.15 to 7.5 \( \times \) \( 10^{10} \) cm\(^{-2} \) with a very high maximum mobility of 8.5 \( \times \) \( 10^{6} \) cm\(^2\)/Vs at high density and sample B spans a density range of 0.5 to 13 \( \times \) \( 10^{10} \) cm\(^{-2} \) with a lower mobility of 3.3 \( \times \) \( 10^{6} \) cm\(^2\)/Vs at high density.

The experimentally measured conductivity \( \sigma(n) \) is shown in Fig. 1 along with the calculated [13] \( \sigma(n) \) curves. We first discuss a few salient qualitative features of the density dependent transport properties of our results. At high densities (\( n \gtrsim 10^{10} \) cm\(^{-2} \)), the conductivity depends on the density approximately as \( \sigma \sim n^\alpha \) with \( \alpha \approx 1.6 \). We emphasize that this is not a strict power law since \( \sigma \equiv \alpha(n) \) depends weakly on the density. This high density behavior is completely consistent with our Boltzmann theory based calculations assuming the conductivity being limited by linearly screened charged impurity scattering. As \( n \) decreases, \( \sigma \) starts decreasing faster with decreasing density and the experimental conductivity exponent \( \alpha(n) \) becomes strongly density dependent with its value increasing substantially above the high-density value of \( \alpha \approx 1.6 \). At the lowest density for sample A, \( \alpha \approx 5 \). Although a part of this strong density dependence of \( \alpha(n) \) at low density can be understood as arising from the strong suppression in screening at low carrier densities, we find that screening in a *homogeneous* electron gas fails qualitatively in explaining the \( \sigma(n) \) behavior at low densities whereas it gives quantitatively accurate results at high densities. As has been found from direct numerical simulations [8, 9], homogeneous screening of charged impurity disorder breaks down at low carrier densities with the 2D system developing strong inhomogeneities leading to a percolation transition at \( n = n_c \).

For \( n < n_c \), the system is an insulator containing isolated puddles of electrons with no “metallic” conducting path spanning through the whole system.

The percolation scenario also naturally explains the qualitative aspects of the observed strong temperature dependence of the effective metallic phase in low-disorder high-quality samples for \( n \gtrsim n_c \) in the 2D MIT phenomenon. In particular, low disorder ensures a low value of \( n_c \), which then automatically leads to a strongly temperature dependent screening correction producing a temperature dependent conductivity as both \( T/T_F \) and \( \rho_{TF}/2k_F \) are effectively large for low densities [13]. In samples with large values of \( n_c \) (low-quality samples) the temperature dependent screening effects are weak in the effective metallic phase (\( n > n_c \)) since the carrier densities are high with the associated \( \rho_{TF}/2k_F \) and \( T/T_F \) being “small” [13].

In Fig. 2 we fit our measured conductivity \( \sigma(n) \) to the expected [8, 10] percolation “critical” behavior

\[
\sigma(n) = A(n - n_c)^\delta, \tag{1}
\]

where \( \delta \) is the conductivity percolation exponent characterizing the vanishing of the conductivity. Nonlinear curve fitting to Eq. (1) with proper weighting of the conductivity is performed for a range of the low density data to determine values of \( \delta, n_c, \) and \( A \). For sample A the data used in the fit is \( 1 \times 10^9 < n < 4 \times 10^9 \) cm\(^{-2} \), although the results do not depend sensitively on the exact range of data used in the fit. Note that for \( T = 0.05 \) K...
and \( n < 2.3 \times 10^9 \text{cm}^{-2} \) a large lock-in quadrature signal prevents measurement of the conductivity. For Sample B, the 'goodness' of the fit is more strongly dependent on the range of data used for fitting. For the data shown, the lowest density point is excluded from the fit, and the fitting range is \( 5 \times 10^9 < n < 8 \times 10^9 \text{cm}^{-2} \). The values of the relevant parameters are \( \delta = 1.4 \pm 0.1 \) and \( n_c = 0.18 \pm 0.01 \times 10^{10} \text{cm}^{-2} \) at \( T = 50 \text{ mK} \) for sample A and \( 1.5 \pm 0.1 \) and \( n_c = 0.28 \pm 0.02 \times 10^{10} \text{cm}^{-2} \) for sample B. These values of \( \delta \) are close to the known 2D percolation exponent of 4/3 [16].

In Fig. 3 we show the dependence of the critical exponent \( \delta(T) \) and the critical density \( n_c(T) \) of sample A. At low temperature, the experimental conductivity exponent \( \delta(T) \) approaches a value \( \delta \approx 1.4 \) (1.5 for sample B), which is close to the expected 2D percolation exponent. The decrease of \( \delta \) with \( T \) for \( T > 0.2K \) is caused by the increasing importance of phonon scattering which should be disregarded for our discussion of the 2D MIT itself. The increase of \( \delta \) (for \( T < 0.2 \text{ K} \)) and decrease of \( n_c \) with increasing temperature can be well-understood by adding a non-critical temperature dependent contribution \( f(T) \), where \( f(T \to 0) = 0 \) and \( f(T) \neq 0 \) for \( T \neq 0 \), to the right-hand side of Eq. 1 and realizing that at any finite \( T \) the system, by definition, is an “effective” metal since the conductivity is non-zero at all finite temperatures. This automatically implies that the effective \( \delta(T) \) increases with \( T \) and the effective \( n_c(T) \) decreases with \( T \), since \( \delta \) and \( n_c \) are strictly defined only at \( T = 0 \) and at all finite \( T \) the system tends to behave as an effective metallic phase – in fact, for sufficiently high values of \( T \) (\( \geq 1K \)) one cannot distinguish at all between the metallic and the insulating phase by looking at the conductivity. The fact that critical behavior manifests over about a decade of density and two decades of conductivity as well as our finding of a conductivity exponent consistent with the 2D percolation transition suggests that the 2D MIT in these low disorder n-GaAs systems is a percolation transition.

It is important to point out that the critical density \( n_c \) we obtain by the optimal fitting of experimentally measured \( \sigma(n) \) to the conductivity scaling formula of Eq. 1 is consistently lower than the corresponding estimate \( (n'_c) \) based on the sign of \( d\sigma/dT \) as is often done in the literature to distinguish an effective metal from the insulator. This is perfectly understandable based on the “quantum-classical crossover” scenario discussed in ref. [15] – at low carrier densities the effective metallic phase exhibits an insulating \( d\sigma/dT (> 0) \) sign down to rather low temperature, and therefore any determination of the critical density \( n'_c \) based purely on the sign of \( d\sigma/dT \) will produce \( n'_c > n_c \). Thus, the 2D MIT transition density cannot be ascertained by looking for the point \( d\sigma/dT|_{n=n'_c} = 0 \) – the sign of \( d\sigma/dT \) is not a good indicator for the critical density. Our analyses as shown in Figs. 1–3 imply that the transition itself is better understood by concentrating on the density dependence of \( \sigma(n) \) at fixed (and very low) values of temperature whence the 2D MIT can be studied approaching entirely from the metallic regime with the sign of \( d\sigma/dT \) playing no role in the analysis. The effect of electron-electron interaction (beyond just the nonlinear screening mechanism leading to the electron ‘puddles’ or ‘droplets’) is mainly to decrease the effective value of the critical density at which the percolation transition occurs, since the main effect of interaction is to ‘homogenize’ the density of the electron liquid in the process reducing the effect of density inhomogeneities arising from the random charged impu-
rity centers. Without interaction effects, percolation will occur at higher carrier densities.

Before concluding we emphasize that one obvious and immediate consequence of the 2D MIT being a disorder driven percolation transition in our system is that it is manifestly not a quantum critical phenomenon as has sometimes been suggested in the literature. We have, in fact, attempted to fit our density and temperature dependent conductivity $\sigma(n, T)$ to the usual quantum critical scaling form \[ \sigma(n, T) \sim F(T/T_0), \] where $F$ is the universal scaling function with $T_0 \equiv T_0(n) \sim |n - n_c|^\gamma$ where the exponent $\gamma \equiv \nu z$ is a product of the correlation exponent $\nu$ and the dynamical exponent $z$. We have not been able to obtain any kind of scaling collapse of our $\sigma(n, T)$ data as a function of $T/T_0$. The inconsistency of our results with quantum criticality is obvious from the percolation scaling fits to Eq. (1) that our data exhibit so well. The fact that our fitted percolation exponent $\delta$ and critical density $n_c$ both depend on temperature already indicates an absence of quantum critical scaling in the data. A more subtle point is that the usual quantum critical ‘fan’ diagram\[17\] indicates that the scaling regime should expand with increasing temperature (at low enough temperatures) since the quantum critical regime spreads out in density (around $n_c$) at higher values of $T$. The expected quantum critical ‘fan’ behavior is in strong disagreement with our observed behavior of $\sigma(n, T)$ where we find that the best scaling of $\sigma(n) \sim (n - n_c)^\delta$ occurs at our lowest measurement temperatures, and the scaling regime (i.e. the regime in carrier density) definitively and systematically shrinks as temperature increases in sharp contrast to the quantum scaling fan behavior. It cannot, of course, be ruled out that the quantum critical regime ends at temperatures way below our lowest measurement temperature ($\sim 50$ mK). We believe that our data and the percolation analyses presented in this paper rules out a quantum critical phenomenon underlying 2D MIT.

We conclude by emphasizing that we provide in this Letter compelling evidence involving both experimental transport data and detailed theoretical analyses in support of the 2D MIT phenomenon being a 2D percolation transition in our system where the carrier system breaks up into an inhomogeneous collection of disconnected local ‘droplets’ of electrons localized in the potential valleys of the long-range Coulombic disorder potential. Our measured conductivity exponent $\delta \approx 1.4 - 1.5$, approaches the expected 2D percolation exponent of $4/3$, and we see impressive scaling behavior in our data consistent with a percolation transition. The percolation transition occurs at a critical density $n_c$ considerably lower than the corresponding ‘critical’ density $n_c’$, one would infer for 2D MIT based just on the temperature dependence of $\sigma(n, T)$. By comparing with our theoretical calculations, we establish that the effective disorder underlying 2D MIT is the scattering from screened charged impurity centers randomly distributed near the semiconductor-insulator interface. As screening weakens at low carrier densities, the system breaks up into inhomogeneous carrier droplets separated by denuded or depleted zones, leading eventually to the percolation transition as was discussed by Efros some years ago\[8\]. Our transport data validates the existence of a percolation transition in 2D MIT but cannot really provide a qualitative description of the underlying percolation process – for example, we cannot rule out percolation scenarios which are somewhat different from the physically appealing (but perhaps somewhat simplistic) picture of Efros where the percolation transition occurs primarily through the depleted zones becoming numerous enough at low enough carrier densities. More experimental work, involving simultaneous measurements of compressibility, capacitance, transport, local chemical potential, and perhaps even local conductivity, is clearly required to better understand the details of the percolation mechanism and the nature of the inhomogeneous 2D carrier system. Our work also brings up important questions regarding the universal nature of 2D MIT itself – for example, we urge experimental analyses of $\sigma(n)$ at fixed low temperatures in other 2D systems to check whether this percolation scenario is universal or just applies to our ultra-high mobility 2D n-GaAs system. In this context it is perhaps quite interesting to point out that even the extensively studied 3D MIT in doped Si systems has been speculated\[18\] to be a possible inhomogeneity driven percolation transition (similar to what we find for 2D MIT in our work), and such a percolation transition may very well be a generic mechanism for metal-insulator transitions in semiconductors where screened charged impurity potential is the dominant disorder mechanism.

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