EFFECTIVE INNER RADIUS OF TILTED BLACK HOLE ACCRETION DISKS

P. CHRIS FRAGILE
Department of Physics and Astronomy, College of Charleston, Charleston, SC 29424, USA; fragilep@cofc.edu
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ABSTRACT

One of the primary means of determining the spin $a$ of an astrophysical black hole is by actually measuring the inner radius $r_{in}$ of a surrounding accretion disk and using that to infer $a$. By comparing a number of different estimates of $r_{in}$ from simulations of tilted accretion disks with differing black hole spins, we show that such a procedure can give quite wrong answers. Over the range $0 \leq a/M \leq 0.9$, we find that, for moderately thick disks ($H/r \sim 0.2$) with modest tilt ($15^\circ$), $r_{in}$ is nearly independent of spin. This result is likely dependent on tilt, such that for larger tilts, it may even be that $r_{in}$ would increase with increasing spin. In the opposite limit, we confirm through numerical simulations of untitled disks that, in the limit of zero tilt, $r_{in}$ recovers approximately the expected dependence on $a$.

Key words: accretion, accretion disks – black hole physics – galaxies: active – X-rays: stars

1. INTRODUCTION

Classical, astrophysical black holes have only two defining properties: mass and angular momentum (or spin). As in other astrophysical contexts, the mass of a black hole can straightforwardly be determined by the application of Kepler’s Third Law, provided the black hole is orbited by another observable object and the distance to the system is reasonably well known (at least to within a factor of sin $i$, where $i$ is the inclination of the binary orbit relative to the observer’s line of sight).

The spin $a = Jc/GM$, however, is a very different matter. So far, no direct measure of spin has been proposed, leaving only indirect means of inferring it. A commonly used method is based on making some measure of the “inner radius” of the accretion disk. In general relativity, stable circular orbits are not permitted all the way down to the “surface” of the black hole, but are instead restricted to lie outside the marginally stable limit $r_{ms}$. Beginning with Novikov & Thorne (1973), it has commonly been assumed that the accretion disk will observe a similar restriction and truncate at $r_{ms}$. Because $r_{ms}$ has a well known monotonic dependence on the spin of the black hole (Bardeen et al. 1972), its association with an observed inner radius of the accretion disk is then used to infer the spin of the black hole. The observation of the inner radius can come either from modeling of the continuum emission in the thermally dominant state (e.g., Shafee et al. 2006; Davis et al. 2006) or from measuring relativistically broadened reflection features (e.g., Wilms et al. 2001).

However, it has been shown that there are many difficulties with such procedures. First, one has many choices for how to define the effective inner radius $r_{in}$ of the accretion disk, and some of these can vary considerably (factors of 2–3) from $r_{ms}$ (Krolik & Hawley 2002). Even if one restricts oneself to such observationally relevant measures as the “radiation edge” (the innermost radius from which significant luminosity emerges) or “reflection edge” (the smallest radius capable of producing significant X-ray reflection features), significant deviations may be observed (Krolik & Hawley 2002; Beckwith et al. 2008). For instance, Agol & Krolik (2000) showed that significant emission can emerge from inside $r_{ms}$, thus precluding a direct correlation with $r_{in}$. Similarly, Reynolds & Begelman (1997) showed that the reflection edge is not necessarily tied to $r_{ms}$.

Nevertheless, a strong result for untitled disks (disks that lie approximately in the symmetry plane of their black hole spacetimes and have their angular momenta aligned with the spins of their black holes) is that $r_{in}$ should gradually decrease with increasing black hole spin, i.e., as $a/M \rightarrow 1$. Therefore, it might at least be hoped that a study of $r_{in}$ for a large ensemble of accreting black holes might yield some information about the range and distribution of black hole spins represented in the ensemble.

However, we show in this Letter that there is a further complicating factor. Based on numerical simulations, we show that the effective inner radius of tilted disks (disks that do not have their angular momenta aligned with the spins of their black holes) yield very different results for $r_{in}$ than their untitled counterparts for all dynamical measures we have considered. A disk could be tilted for many reasons. In stellar mass binaries, the orientation of the outer disk is fixed by the binary orbit, whereas the orientation of the black hole is determined by how it became part of the system. If the black hole formed from a member of a preexisting binary through a supernova, then the black hole could be tilted if the explosion were asymmetric. If the black hole joined the binary through multi-body interactions, such as binary capture or replacement, then there would have been no preexisting symmetry, so the resulting system would nearly always harbor a tilted black hole. This same argument can be extended to active galactic nucleus in which merger events reorient the central black hole or its fuel supply and result in repeated tilted configurations. Tilted disks are subject to additional torques due to differential Lense–Thirring precession (Bardeen & Petterson 1975).

Our results indicate that for moderately thick ($H/r \sim 0.2$), tilted ($15^\circ$) disks, $r_{in}$ is independent of spin, or, in some measures, actually increases with increasing black hole spin. These results could have important consequences for observational efforts to constrain black hole spins.

2. NUMERICAL SIMULATIONS

The numerical data in this Letter were taken from simulations presented in Fragile et al. (2007) and Fragile et al. (2009), as well as two previously unpublished simulations. All of the simulations used the Cosmos++ GRMHD code (Anninos et al. 2005). The simulations presented in Fragile et al. (2007) and the two previously unpublished simulations used a spherical-
polar grid in the Kerr–Schild coordinates, with an effective grid resolution of 128³, except near the poles which were purposefully underresolved. The simulations from Fragile et al. (2009) used either this same spherical-polar grid or a “cubed-sphere” grid also in the Kerr–Schild coordinates, with a resolution of 128 × 64 × 64 on each of six logically connected blocks that are morphed into segments of a sphere. As such, all of these simulations used what was referred to as “high” spatial resolution in Fragile et al. (2007) and Fragile et al. (2009). In all cases, the simulations were initialized with an analytically solvable, time-steady, axisymmetric gas torus (De Villiers & Hawley 2003), threaded with a weak poloidal magnetic field with minimum $P_{\text{gas}}/P_{\text{mag}} = 10$ initially. The magnetorotational instability (MRI) arose naturally from the initial conditions. The simulations were all evolved for $\sim 7900M$, or $\sim 40$ orbits at $r = 10M$ in units with $G = c = 1$. Only data from the final $\sim 790M$ of the simulation are used in this Letter, ensuring that the disk is fully turbulent and transient effects from the initial conditions have died down. The principle variables that are studied in these simulations are the spin of the black hole, varying over the range $0 \leq a/M \leq 0.9$, and the initial tilt between the disk and the black hole, being either 0° or 15°. The different simulations are summarized in Table 1.

### 3. EFFECTIVE INNER RADIUS

As pointed out in Krolik & Hawley (2002), there are many possible definitions of $r_{\text{in}}$. In this Letter, we will ignore any that rely on knowledge of the radiation field, instead sticking with definitions based on dynamical fluid properties that are easily obtained directly from the simulations. We explore four such definitions.

To best represent the quasi-steady states achieved in the simulations, all data are time-averaged over the interval $t_{\text{min}} \approx 7100M$ to $t_{\text{max}} \approx 7900M$. To ensure all derived quantities are representative of the disk and not the low-density background gas, we apply density-weighted shell averaging, i.e., we present quantities of the form

$$
\langle Q \rangle_i = \frac{1}{\langle \rho \rangle_i} \int_{t_{\text{min}}}^{t_{\text{max}}} \int_{0}^{2\pi} \int_{0}^{\pi} \rho(r_i, \theta, \phi, t)Q(r_i, \theta, \phi, t) \times \sqrt{-g} \, d\theta d\phi dt,
$$

where

$$
\langle \rho \rangle_i = \int_{t_{\text{min}}}^{t_{\text{max}}} \left[ \frac{2\pi}{r_i^2} \int_{0}^{\pi} \rho(r_i, \theta, \phi, t) \sqrt{-g} \, d\theta d\phi \right] dt
$$

is the average density of a given radial shell $i$.

### 3.1. Surface Density

Perhaps the conceptually simplest definition of $r_{\text{in}}$ is where the surface density of the disk drops significantly from some relevant peak value, especially if this occurs over some relatively narrow radial range. Indeed, we generally find that the surface density drops rapidly from a local peak at $r \sim 10M$ (see, e.g., Figure 13 of Fragile et al. 2007). For concreteness we take $\Sigma(r_{\text{in}}) = \Sigma_{\text{max}}/3e$, where $\Sigma(r_i) \equiv \langle \rho \rangle_i/(\Omega_1H)$, and $\Sigma_{\text{max}} = \max[\Sigma(r)]$ within the interval $r_{\text{BH}} \leq r_i \leq 20M$. The local scale height of the disk is estimated from

$$
H = \frac{\rho}{|\nabla \rho|}.
$$

Note that this definition differs from what we used in Fragile et al. (2007) and gives slightly larger values for $H$, though the resulting profile is quite similar.

In Figure 1, we plot the resulting estimates of $r_{\text{in}}$ from each simulation. For illustration purposes, we also plot a line representing the value of $r_{\text{max}}$ as a function of spin. The most obvious result of this figure is that, although the untilted simulations (circles) follow the trend indicated by $r_{\text{in}}$ (though with systematically higher numerical values), the tilted simulations (diamonds) do not. In fact, instead of $r_{\text{in}}$ decreasing with increasing black hole spin, it increases slightly. We further emphasize this difference by fitting simple trend lines to the untilted and tilted data separately. Clearly, the data are following two very different trends. This is crucial since it means that any estimates of $a/M$ based on the tilted data could be very wrong. For instance, the 915H simulation (with a spin of $a/M = 0.9$ and a tilt of 15°) produces a value for $r_{\text{in}}$ that is almost identical to the 0H ($a/M = 0$) simulation.

### 3.2. Inflow Time

Another possible definition for $r_{\text{in}}$ is where the radial infall time $t_{\text{in}}(r_i) = r_i/ \langle V \rangle_i$ of material moving in through the disk becomes shorter than some factor of the local orbital time $t_{\text{orb}}(r) = 2\pi/\Omega$, where $\Omega = (M/r^3)^{1/2}/[1 + a(M/r)^{1/2}] \approx \Omega_{\text{ECP}}$ is the local orbital angular frequency. Specifically, if we take $t_{\text{in}}(r_{\text{in}}) = 3t_{\text{orb}}(r_{\text{in}})$, then Figure 2 shows the resulting estimates of $r_{\text{in}}$. The factor of 3 in our definition of $t_{\text{in}}(r_{\text{in}})$

### Table 1: Simulation Parameters

| Simulation | $a/M$ | Tilt Angle | Grid                     |
|------------|-------|------------|--------------------------|
| 0H         | 0     |            | Spherical-polar          |
| 315H       | 0.3   | 15°        | Spherical-polar          |
| 50H        | 0.5   | 0°         | Cubed-sphere             |
| 515H-S     | 0.5   | 15°        | Spherical-polar          |
| 515H-C     | 0.5   | 15°        | Cubed-sphere             |
| 715H       | 0.7   | 15°        | Spherical-polar          |
| 90H        | 0.9   | 0°         | Spherical-polar          |
| 915H       | 0.9   | 15°        | Spherical-polar          |

Notes.
- a Fragile et al. (2009).
- b Fragile et al. (2007).
is rather arbitrary (Krolik & Hawley 2002 used a value of 10), and is simply chosen for convenience. Clearly, if a smaller coefficient is used, all of the data points in Figure 2 would shift to smaller radii, closer to the event horizon; whereas, if a larger coefficient is used, all of the data points would move to larger radii. A similar caveat applies to all of the normalizations used in this Letter. The key point is that the trend lines would remain largely unchanged, and it is the trend lines we are most interested in, not specific numerical values of $r_{in}$.

Looking at the trend lines in Figure 2, we again find with this measure of $r_{in}$ that the untilted simulations closely follow the prediction of $r_{rms}$. The tilted simulations again deviate from this trend; instead they now show an almost flat trend line. By this definition of $r_{in}$, we take $V_{rms}(r_{in}) \equiv \langle V_r \rangle_r$, where $\langle V_r \rangle_r$ is defined over the interval $r_{in} \leq r \leq 15M$. For our purposes we take $V_{rms}(r_{in}) \equiv \langle V'_r \rangle_{r_{in}}$, where

$$V'_r(r_{in}) = \sqrt{\langle (V'_r)^2 \rangle_{r_{in}}}.$$  \hfill (4)

We see in Figure 3 that this definition of $r_{in}$ gives similar results to the previous two. Again, $r_{in}$ follows the expected trend for the untilted simulations, but appears to increase with spin for the tilted ones.

### 3.3. Turbulence Edge

Krolik & Hawley (2002) suggested another possible definition of $r_{in}$ as the radius where the local turbulent velocity $V_{turb}$ becomes short compared to the local infall velocity $\langle V' \rangle$. Normally in the bulk of the disk, MRI-driven turbulence produces velocity fluctuations that are substantially larger on average than the mean infall velocity. However, as disk material approaches the black hole, its radial infall velocity increases more rapidly than its turbulent velocity. For this measure, we define the inner radius as $(V_{turb}/\langle V' \rangle)(r_{in}) = (V_{turb}/\langle V' \rangle)_{max}/2e$, where $V_{turb}/\langle V' \rangle_{max}$ is defined over the interval $r_{BH} \leq r \leq 15M$. For our purposes we take $V_{turb}(r_{in}) \equiv V'_{rms}(r_{in})$, where

$$V'_{rms}(r_{in}) = \sqrt{\langle (V'_r)^2 \rangle_{r_{in}}}.$$  \hfill (5)

We see in Figure 3 that this definition of $r_{in}$ gives similar results to the previous two. Again, $r_{in}$ follows the expected trend for the untilted simulations, but appears to increase with spin for the tilted ones.

### 3.4. Magnetic Field Structure

As a final possible definition of $r_{in}$, we consider the radius where the local magnetic field structure measure from MRI-driven turbulence toward an evolution better described as flux-freezing. Although a purely turbulent process would not lead to any correlation, the linear properties of the MRI and the consistent orbital shear of the disk impose some modest correlations between $B_r$ and $B^\phi$, such that $a_{mag} \sim 0.001$ (Krolik & Hawley 2002), where

$$a_{mag} = \frac{|u'u^\phi|B_r^2 - B^\phi B^\phi|}{4\pi P}$$  \hfill (5)

is the dimensionless magnetic stress parameter. As the flux-freezing limit is approached, correlations between the magnetic field components become stronger, and $a_{mag} \rightarrow 1$. In this region, the transfer of energy to smaller scales becomes slower than the infall time. For our purposes, we take $a_{mag}(r_{in}) = 2e(a_{mag})_{min}$, where $(a_{mag})_{min}$ is defined over the interval $r_{BH} \leq r \leq 20M$. We see in Figure 4 that this definition of $r_{in}$ gives similar results to the previous ones. Again, $r_{in}$ follows the expected trend for the untilted simulations, but appears to be nearly independent of spin for the tilted ones.

Note in Figure 4 no datum represents the cubed-sphere simulation 515H-C. This is because it is very difficult on the cubed-sphere grid to properly recover the $B_r$ and $B^\phi$ components, especially close to the black hole for tilted disks.
Therefore, we were not able to recover a meaningful value for \(a_{\text{mag}}\) much inside \(\sim 8M\) for that simulation.

4. DISCUSSION

We have explored four possible measures of the effective inner radius of simulated black hole accretion disks, based on surface density, inflow and turbulent velocities, and magnetic field structure. With all four measures, we found two consistent trends: (1) for untilted simulations \(r_{\text{in}}\) closely follows the analytic form of \(r_{\text{rms}}\); whereas (2) for 15° tilted simulations \(r_{\text{in}}\) shows a flat or even increasing trend as \(a/M \rightarrow 1\).

The fact that Figures 1–4 all show remarkable similarities suggests that our results are not due to a poor choice of definition for \(r_{\text{in}}\). Also, although we have ignored the dependence of \(r_{\text{rms}}\) on inclination in making our plots, Figure 5 of Fragile et al. (2007) shows that the deviation would be quite small for \(i = 15°\), as would be appropriate for this work.

We can get some estimate of the uncertainties associated with our results by comparing data obtained from the two 515H simulations, one done on the spherical-polar grid and one on the cubed-sphere. For the first three measures of \(r_{\text{in}}\), where we were able to get estimates using both simulations, they agree remarkably well. Furthermore, for most of the measures the trend lines fit the data well, suggesting small uncertainties.

We conclude that the discrepancy of \(r_{\text{in}}\) between tilted and untilted simulations is a robust result, and must, therefore, be rooted in some physical process. The most likely agent of such a process would seem to be the standing shocks aligned along the line of nodes between the black hole symmetry plane and disk midplane (Fragile & Blaes 2008). These features are not present in simulations of untitled disks. Such non-axisymmetric shocks can be quite efficient at extracting angular momentum from the gas flow, which would reduce the effect of the black hole spin on \(r_{\text{in}}\).

In this work, we have only considered two values of initial tilt: \(\beta_i = 0°\) and 15°. It stands to reason, however, that as \(\beta_i \rightarrow 0\) the inner radius would approach the untitled values obtained in this work. On the other hand, for \(\beta_i > 15°\), it may well be that the discrepancies in \(r_{\text{in}}\) would be even larger. In such a case, \(r_{\text{in}}\) might truly increase with increasing spin.

The results in this Letter then imply that, at least for moderately thick accretion disks (\(H/r \gtrsim 0.1\)), measurements of \(r_{\text{in}}\) are not reliable predictors of \(a\), unless one can independently confirm that the disk is not tilted. This could pose a problem for recent attempts to determine spin using so-called “Hard” state observations (Miller et al. 2006; Reis et al. 2008), where the flow is generally taken to be thick.

For very thin disks (\(H/r \lesssim 0.01\)), the effect of tilt is expected to be much different. Such disks are expected to be subject to the Bardeen–Petterson effect (Bardeen & Petterson 1975), where the midplane of the disk at small radii would be aligned with the symmetry plane of the black hole through the competing action of Lense–Thirring precession and viscous diffusion, while possibly leaving an outer disk that is tilted at large radii. Since the inner disk would be aligned with the symmetry plane of the black hole, the effective inner radius should be the same as for an untitled disk. This would apply for disks in the “Soft” or “Thermally Dominant” state, with luminosities well below the Eddington limit, which are likely to be thin. Attempts to estimate the black hole spin by modeling the disk in this state may be unaffected by the cautions raised in this Letter. However, for sources with disk luminosities near or above the Eddington limit (e.g., Middleton et al. 2006), the disk should again be thick, and the effect discussed here will apply. Even if the disk is thin, we reiterate that it is still important to independently fit for the inclination of the inner disk, since this may not be the same as the binary inclination.

Other methods of determining black hole spin, such as using quasi-periodic oscillation (QPO) frequencies, may not be affected by our results. For instance, QPO models based on resonant frequencies attributable to the black hole spacetime itself (e.g., Abramowicz & Kluzniak 2001) would likely not be strongly affected. In fact, the weak dependence of \(r_{\text{in}}\) on \(a\) detailed in this Letter actually helps one model of low-frequency QPOs based on the precession of a radially extended thick disk, by ensuring that the maximum frequency would not exceed \(\sim 10\) Hz regardless of black hole spin, as observed (Ingram et al. 2009). However, to date, no consensus model for QPOs has emerged, leaving us with few options for unambiguously determining black hole spin.

To end, we emphasize that it would be wrong to conclude from this Letter that all tilted black hole accretion disks should appear to have \(r_{\text{in}} \approx 6M\). Remember, the normalizations used in each of our plots were chosen simply to give values for the untilted disks that were reasonably similar to the values for \(r_{\text{rms}}\). There is no other physical motivation for choosing those normalizations, and a different choice would have led to different numerical values for \(r_{\text{in}}\). Further, we cannot really say how the dynamically determined \(r_{\text{in}}\) in this work would compare to values of \(r_{\text{in}}\) measured from continuum modeling or iron-line fitting without doing more work. Specifically, we would need to generate synthetic disk images from the simulation data, and measure a radiation or reflection edge. Such work is already underway using the relativistic ray tracing code of Dexter & Agol (2009). Even then, there is the problem that the original simulations did not properly account for all of the important physical processes occurring in the disk, notably heating due to dissipation of turbulent energy and radiative cooling. That will have to await future simulations.

What can be concluded from this Letter is that all tilted disks (with a tilt around 15°) should appear to have the same \(r_{\text{in}}\), independent of spin. Our results may further imply that measurement of a small value for \(r_{\text{in}}\) (without defining small) would require a rapidly rotating black hole and an untitled disk, whereas a large value of \(r_{\text{in}}\) would not necessarily mean that the black hole is spinning slowly; it could simply be tilted.

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