2D buoyant convective flow in a square box containing an adiabatic solid block: Effect of aspect ratio of solid block and various thermal boundaries

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Abstract. A numerical analysis is performed to study the natural convective flow in a square enclosed space filled with air containing an adiabatic square solid block at the center. The convective flow in a square cavity has examined for three different behaviours of heating. The left wall is heated at 1, y and sin(πy) and the temperature of the right wall is frozen to 0. The adiabatic square block that located in the center of the cavity has various aspect ratios associated to the main cavity. The two-dimensional flow and thermal pattern in the square cavity are investigated for various thermal boundary conditions and aspect ratio of the solid block. The averaged energy transport diminishes on raising the aspect ratio of the solid body for all kind of thermal boundary conditions.

1. Introduction
Over the past decades, researchers have been interesting on the essential problem of convective heat transfer in a finite enclosed space because its applications are extensively used, such as in electronic system cooling, solar power technology and buildings insulation. Additionally, the manner of developing the device hardware technology rises the investigation of thermal convective flow problems. Many investigations have attempted to study heat transfer in two- and three-dimensional enclosures such as Ostrach [1], Khalifa [2], and Polezhayev et al. [3]. Moreover, the effects of geometrical objects inside the enclosures on the flow field and thermal properties have been studied by many researchers. The performance of flow and thermal features with adiabatic single and dual block squares in a two-dimensional cavity are studied numerically by Mahapatra et al. [4].

The thermal conditions enforced on the system is very important everywhere. Several thermal conditions on convective stream and energy transfer have been analysed in several studies, that is, non-uniform heating [5],[6],[7],[8], discrete heating [9],[10],[11], partial heating [12], sinusoidal heating [13],[14],[15],[16],[17],[18], linearly heating [19]. In the current study, we explore the effect of three different thermal boundary conditions on convective stream and energy transport in an enclosed space containing an adiabatic square solid block at the centre.

2. Mathematical Model
Fig.1 presents a graphic of the square enclosed box with an adiabatic square solid body at its centre. The length of the cavity is denoted by L while the length of an adiabatic square block
donated by $l$. The aspect ratio which is the dimensionless size of the solid square block is defined as $AR_{SB} = l/L$. The left wall of the cavity is assumed to have a higher temperature $T_h$, while, the right wall is presented as cold wall $T_c$. The other walls of the box are kept insulated. The square enclosed box is filled with air. In the cavity, the fluid flow is Newtonian, steady, laminar, uniform, and incompressible. Additionally, the thermo-physical features of the fluid considered to be constant.

\[ \text{Figure 1: A graphic for the examined cavity in the present study.} \]

The governing equations are represented by continuity, momentum which are obtained from Navier-Stokes equations, and energy equations. The momentum equations and energy equation are linked with the volume force term related to the Boussinesq approximation. The viscous dissipation is neglected. The dimensional form of the mathematical model is:

\[
\begin{align*}
    u_x + v_y &= 0, \\
    uu_x + vu_y &= -\frac{1}{\rho}p_x + \nu \nabla^2 u, \\
    uv_x + vv_y &= -\frac{1}{\rho}p_y + \nu \nabla^2 v + g\beta (T - T_c), \\
    uT_x + vT_y &= \frac{k}{\rho C_p} \nabla^2 u.
\end{align*}
\]

The conditions at walls of the enclosed box are given as follows:
on the left-side wall $u = v = 0$, $T = T_h$
on the right-side wall $u = v = 0$, $T = T_c$
on the adiabatic solid body $u = v = 0$, $\frac{\partial T}{\partial n} = 0$
on the top and bottom walls $u = v = 0$, $\frac{\partial T}{\partial y} = 0$

The governing models are converted into non-dimensional system according to the dimensionless variables given below,

\[
\begin{align*}
    X &= \frac{x}{L}, & Y &= \frac{y}{L}, & U &= \frac{uL}{\alpha}, & V &= \frac{vL}{\alpha} \\
    P &= \frac{pL^2}{\mu \alpha}, & \Theta &= \frac{T - T_c}{T_h - T_c}, & Ra &= \frac{g\beta (T - T_c) L^3}{\nu \alpha}, & Pr &= \frac{v}{\alpha}
\end{align*}
\]
Hence, the following is the non-dimensional equations,

\[ U_X + V_Y = 0, \]  
\[ UU_X + VU_Y = -P_X + \nu \nabla^2 U, \]  
\[ UV_X + VV_Y = -P_Y + \nu \nabla^2 V + \frac{Ra}{Pr} \Theta, \]  
\[ U\Theta_X + V\Theta_Y = \frac{1}{Pr} \nabla^2 \Theta. \]  

The non-dimensional boundary conditions are given as follows:

on the left-side wall:  
\[ U = V = 0, \Theta = 1, y, \sin(\pi y) \]
on the right-side wall:  
\[ U = V = 0, \Theta = 0 \]
on the adiabatic body, top and bottom walls:  
\[ U = V = 0, \frac{\partial \Theta}{\partial n} = 0 \]

The volume of heat transfer rate inside the cavity is a significant factor in thermal system investigation. Thus, it can be evaluated by the Nusselt number along the hot wall. The local Nusselt number \( (Nu) \) is defined by the formula:

\[ Nu = \left. \frac{\partial \Theta}{\partial y} \right|_{x=0} \]

The average Nusselt number \( (\overline{Nu}) \) is described by the formula:

\[ \overline{Nu} = \int_0^1 Nu dy \]

The governing system with initial and edge conditions are solved by Finite Element Method (FEM). In this work, different grid sizes are examined such as 40 × 40, 60 × 60, 80 × 80, 100 × 100, and 120 × 120. The grid size 100 × 100, and 120 × 120 give less difference, so we chose this (100 × 100) grid distribution to study our problem. The convergence condition is taken as 10^6 in this study. The Nusselt number is calculated by the Trapezoidal rule.

3. Results and Discussions

A numerical simulation is performed to explore the effect of the aspect ratio of an adiabatic square block and several thermal edge conditions on the buoyant-convection in a square enclosed box. The range of aspect ratio of the solid body is taken from 0.0 to 0.8. Three different thermal boundary conditions are used here. Figure 2 shows the stream pattern for various values of aspect ratio of the adiabatic solid body and different thermal boundary conditions. The flow pattern apparently depicts that the thermal boundary affects the stream. In the case of \( \sin(\pi y) \) boundary condition, we observed a recirculating eddy near the left-top corner of the box. When increasing the aspect ratio of the solid body, the shear increases near the thermal wall.

Figure 3 depicts the thermal distributions inside the cavity for various \( AR_{SB} \) and thermal boundary conditions. The isothermal clearly shows that the convection mode is dominated for all values of \( AR_{SB} \) and all thermal boundary conditions. the strong boundary layers forms near the thermal walls for the isothermal heating case (\( \Theta = 1 \)). When linearly heating case (\( \Theta = y \)), the weak thermal stratum is shaped along the hot wall while a strong thermal stratum appears along the upper portion of the cold wall. We can see the reverse heat transfer near the bottom-left and top-left corners of the box for the sinusoidal heating case (\( \Theta = \sin(\pi y) \)), which results the lower averaged heat transfer rate for weak buoyancy cases.

The local Nusselt number for three thermal boundary conditions with \( AR_{SB} = 0.4, Ra = 10^3 \) and \( Ra = 10^6 \) is depicted in the figure 4. The profiles of local Nusselt number clearly demonstrate
that the local heat transfer is directly affected according to the thermal boundary imposed on the wall. The highest local heat transfer is observed near the top of the wall for linearly heating case with $Ra = 10^3$, whereas the highest local heat transfer attains near the bottom wall for constant heating case. In order to demonstrate the effect of averaged energy transport across the enclosed space, the averaged Nusselt number is plotted against the aspect ratio of solid body for various values of Rayleigh number and different thermal boundary conditions in figure 5. It is obvious
that the thermal conditions imposed at the boundary is directly affect the local heat transfer rate near the wall and it results finally the averaged heat transfer rate inside the enclosed space. The constant heating ($\Theta = 1$) provides the higher energy transport imposed on the wall. The linear and sinusoidal heating provide different phenomena on averaged heat transfer according to the size (aspect ratio) of the solid body. It is also observed that the averaged heat transfer rate diminishes on raising the aspect ratio of the adiabatic solid body for all kind of thermal
boundary conditions.

![Graphs showing local Nusselt number for different Ra values](image)

Figure 4: local Nusselt number for $AR_{SB} = 0.4$ and different $Ra$

![Graphs showing averaged Nusselt number for various AR_{SB} and Ra values](image)

Figure 5: (a-d) Averaged Nusselt number for various $AR_{SB}$ and $Ra$

4. Conclusions
The effect of the aspect ratio of an adiabatic square block and three kinds of thermal conditions at wall on the buoyant convection in a enclosed box is explored numerically. The following results are observed here.

(i) The constant heating ($\Theta = 1$) provides the higher energy transport across the enclosed space than other heating cases (linear and sinusoidal).
(ii) The averaged energy transport diminishes on raising the aspect ratio of the solid body for all kind of thermal boundary conditions.

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