The top quark mass and flavor mixing
in a Seesaw model of Quark Masses

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Abstract

The top quark mass and the flavor mixing are studied in the context of a Seesaw model of Quark Masses based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$. Six isosinglet quarks are introduced to give rise to the mass hierarchy of ordinary quarks. In this scheme, we reexamine a mechanism for the generation of the top quark mass. It is shown that, in order to prevent the Seesaw mechanism to act for the top quark, the mass parameter of its isosinglet partner must be much smaller than the breaking scale of $SU(2)_R$. As a result the fourth lightest up quark must have a mass of the order of the breaking scale of $SU(2)_R$, and a large mixing between the right-handed top quark and its singlet partner occurs. We also show that this mechanism is compatible with the mass spectrum of light quarks and their flavor mixing.

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1 Introduction

The Seesaw mechanism \cite{1}, \cite{2} was initially invented to explain the smallness of neutrino masses. Also, in a different framework, the smallness of masses of quarks other than the top quark compared to the scale of the electroweak symmetry breaking can be explained by a Seesaw Model of Quark Masses \cite{3}, \cite{4}, \cite{5}, \cite{6}, \cite{7}. In the context of $SU(2)_L \times SU(2)_R \times U(1)$ models including isosinglet quarks, it has been shown that the light ordinary quark masses are proportional to the breaking scales of $SU(2)_L$, $(\eta_L)$ and $SU(2)_R$, $(\eta_R)$ and are inversely proportional to the isosinglet quark mass $M$, i.e., $O\left(\frac{\eta_L \eta_R}{M}\right)$. Conventionally, the isosinglet quark mass $M$ is assumed to be much larger than $\eta_R$. This assumption leads to an explanation for the smallness of quark masses compared to the scale of the electroweak symmetry breaking. Though the Seesaw mechanism explains the smallness of the mass of the five flavors from the up quark to the bottom quark, it has not been shown that the top quark with a mass of $O(\eta_L)$ can be incorporated into the same scheme. In this letter, we study the top quark mass as well as the mass hierarchy of the up and down quark sectors in the context of the Seesaw mechanism. We show that the mass formulae for the light quarks proposed before is not valid for the top quark and must be replaced by a new one. The mass hierarchy of the light quarks and flavor mixing are studied in the same context by ref. \cite{5}, \cite{6}, \cite{7} under the assumption that the isosinglet quark diagonal mass parameter is much larger than the breaking scale of $SU(2)_R$. However, under the same assumption the top quark mass would also be much smaller than the breaking scale of $SU(2)_L$ unless the corresponding Yukawa coupling between isosinglet and isodoublet quarks is chosen to be very large. As we show later, if the diagonal mass parameter for the isosinglet partner of the top quark is much smaller than $\eta_R$, the Seesaw mechanism does not act for the top quark and its mass can be kept at the scale of $\eta_L$ without introducing a large Yukawa coupling. In this case the heavier mass eigenstate is as light as $\eta_R$ rather than being given by the singlet mass parameter. Because one of the mass eigenstates is as light as $\eta_R$, the flavor mixing and the stability of the light quark masses against the inclusion of flavor off-diagonal Yukawa couplings is a non-trivial problem, we study the light quark mass spectrum and flavor mixing taking into consideration the special rôle played by the top quark. We obtain the approximate mass formulae for quark masses and show that the mass hierarchy is stable against flavor mixing.

This paper is organized as follows. In section 2, we present the results for the diagonalization of the mass matrix for the top quark and its isosinglet partner. The mixing between singlet and doublet quarks is discussed both for left- and right-handed chiralities. In section 3, the mass spectrum of both light and heavy quarks is obtained by introducing flavor off-diagonal couplings. In section 4, the mixing angles are obtained. In section 5, we summarize the results.
2 The top quark mass and singlet-doublet mixing in a Seesaw model

The gauge group of the model is $SU(2)_L \times SU(2)_R \times U(1)$ and the ordinary quarks, isosinglet quarks and the relevant Higgs scalars are assigned to the following gauge group representations:

$$\psi^i_L = \begin{pmatrix} u^i \\ d^i \end{pmatrix}_L : (2, 1, 1/3),$$
$$\psi^i_R = \begin{pmatrix} u^i \\ d^i \end{pmatrix}_R : (1, 2, 1/3),$$
$$U^i_{L,R} : (1, 1, 4/3),$$
$$D^i_{L,R} : (1, 1, -2/3),$$
$$\phi_L : (2, 1, 1),$$
$$\phi_R : (1, 2, 1),$$

where $i = 1, 2, 3$. By introducing an isodoublet Higgs $\phi_L$ ($\phi_R$) for $SU(2)_L$ ($SU(2)_R$), the Yukawa interaction between doublet and singlet quarks and the mass term for the singlet quarks of the model are given by

$$\mathcal{L}_y = -y^i_{LD} \bar{\psi}^i_L \phi_L D^i_R - y^i_{LU} \bar{\psi}^i_L \phi_L U^i_R - y^i_{RD} \bar{\psi}^i_R \phi_R D^i_L - y^i_{RU} \bar{\psi}^i_R \phi_R U^i_L + (h.c.) - \bar{U}^i M^i_U U^i - \bar{D}^i M^i_D D^i,$$

(2)

where $i$ and $j$ are summed over from 1 to 3; $y_{L(R)}D(U)$ is the strength of the Yukawa coupling between the down(up) type left-handed {right-handed} isodoublet quark and isosinglet quark; $y_{L(R)}$ are $3 \times 3$ matrices; $M^i_U (M^i_D)$ are given by $M^1_U = M_U$, $M^2_U = M_C$, $M^3_U = M_T$, $M^1_D = M_D$, $M^2_D = M_S$ and $M^3_D = M_B$. Without loss of generality, the singlet quark mass matrix can be transformed into a real diagonal matrix through a bi-unitary transformation. The scale of the singlet mass parameter is going to be set later.

We first focus on the case in which the flavor mixing is absent and study the top quark sector. The mass eigenstates and eigenvalues for the top quark and its isosinglet partner are obtained by diagonalizing the two by two matrix:

$$\begin{pmatrix} \bar{t} \\ \bar{m} \end{pmatrix}_L \begin{pmatrix} 0 & y_{L} \eta_L \\ y_{R} \eta_R & M_T \end{pmatrix} \begin{pmatrix} t \\ T \end{pmatrix}_R = \begin{pmatrix} \bar{t}^m \\ \bar{m}^m \end{pmatrix}_L \begin{pmatrix} m_t & 0 \\ 0 & m_T \end{pmatrix} \begin{pmatrix} t^m \\ T^m \end{pmatrix}_R.$$

(3)
The mass eigenvalues and the mixing angles for the left-handed chiralities are obtained by diagonalizing $MM^\dagger$:

$$MM^\dagger = \begin{pmatrix} |y_L|^2 \eta_L^2 & y_L \eta_L M_T \\ y_L^* \eta_L^* M_T & M_T^2 + |y_R|^2 \eta_R^2 \end{pmatrix},$$

where $\eta_{L(R)}$ are the vacuum expectation values of the neutral Higgs particles,

$$\langle \phi_L \rangle = \begin{pmatrix} 0 \\ \eta_L \end{pmatrix}, \quad \langle \phi_R \rangle = \begin{pmatrix} 0 \\ \eta_R \end{pmatrix}. \tag{5}$$

The vacuum expectation values are related to the masses of the charged $SU(2)_{L(R)}$ gauge bosons by

$$M_L^2 = \frac{1}{2} g^2 \eta_L^2, \quad M_R^2 = \frac{1}{2} g^2 \eta_R^2, \tag{6}$$

where $g$ is the $SU(2)_{L(R)}$ gauge coupling constant. Therefore the mass eigenvalues are written in terms of the gauge boson masses as:

$$m_t \approx \sqrt{2} \left( \frac{|y_L y_R|}{g^2} \right) M_L \frac{1}{\sqrt{\left( \frac{|y_R|}{g} \right)^2 + \left( \frac{M_T}{\sqrt{2} M_R} \right)^2}}, \tag{7}$$

$$m_T \approx \sqrt{2} M_R \sqrt{\left( \frac{|y_R|}{g} \right)^2 + \left( \frac{M_T}{\sqrt{2} M_R} \right)^2}. \tag{8}$$

If $M_T \gg M_R$, the lighter mass eigenvalue $m_t$ in Eq.(7) is reduced to the well known Seesaw formulae for light quarks, i.e., $O \left( \frac{M_T M_R |y_L y_R|}{g^2} \right)$. However, for the top quark, we should take the limit $M_T \ll M_R$. Then the suppression factor $\frac{M_R}{M_T}$ is now absent and its mass is given by

$$m_t \approx \sqrt{2} \frac{|y_L|}{g} M_L, \tag{9}$$

which is at the scale of $M_L$. On the other hand, the heaviest eigenvalue $m_T$ in Eq.(8) is

$$m_T \approx \sqrt{2} \frac{|y_R|}{g} M_R, \tag{10}$$

which is at the symmetry breaking scale of $SU(2)_R$ instead of being given by the singlet mass parameter $M_T$. To reproduce the value of the top quark mass in Eq.(9), the Yukawa coupling $y_L$ must be a few times larger than the gauge coupling $g$. We
assume that it is still within the perturbative region. There is a simple reason why
the top quark mass is determined by $M_L$ and its partner’s mass is proportional to $M_R$. To show this, let us ignore the mass term of the singlet quark and keep only
singlet-doublet mixing terms. Then the Yukawa term of the top quark and its singlet
partner is

$$\mathcal{L} = -y_L \eta_L \bar{t}_L T_R - y_R^* \eta_R \bar{T}_L t_R + (h.c.).$$

From Eq.(11) the mass term can be diagonalized by the following rotations,

$$\begin{pmatrix} t_m \\ T_m \end{pmatrix}_L = \begin{pmatrix} t \\ T \end{pmatrix}_L,$$

$$\begin{pmatrix} t_m \\ T_m \end{pmatrix}_R = \begin{pmatrix} T \\ t \end{pmatrix}_R.$$  

Then the mass for $t^m$ is $|y_L|\eta_L$ and the mass for $T^m$ is $|y_R|\eta_R$. We note that the
isosinglet and doublet mixing is maximal for the right-handed sector. The large
mixing between the top quark and its singlet partner predicted in this model should
be possible to observe in an experiment of $t\bar{t}$ production.

In the limit where flavor mixing is absent we obtain, from the Lagrangean of
Eq.(2), mass matrices for the pairs $(u,U)$, $(c,C)$, $(d,D)$, $(s,S)$ and $(b,B)$, generically
denoted by $(q,Q)$, similar to the one for the pairs $(t,T)$ in Eq.(3). The eigenvalues
are also given by expressions of the form Eq.(7) and Eq.(8) with $M_T$ replaced by the
corresponding mass parameter $M_Q$. In order for the Seesaw mechanism to act, so
that the masses we obtain for the light quarks are much smaller than $M_L$, we need
$M_Q \gg M_R$. Then Eq.(9) and Eq.(10) are replaced by

$$m_q \approx 2 \frac{|y_L y_R|}{g^2} M_L \left( \frac{M_R}{M_Q} \right),$$

$$m_Q \approx M_Q.$$  

If $M_R$ is fixed, we can estimate the order of magnitude of $M_Q$ for each case. In these
cases the mixing is suppressed by a factor of $(\frac{m}{M_Q})$ for the left-handed components
and $(\frac{M_R}{M_Q})$ for the right-handed ones.

In the next section we extend our analysis to the case where flavor mixing mass
terms are present.

3 Mass formulae including flavor mixing in the
Seesaw model

In this section we derive the mass formulae in the case with mixings among different
quark flavors. We start with the six by six mass matrix

$$M = \begin{pmatrix} 0 & \eta_L y_L \\ \eta_R y_R^* & M_{Diag} \end{pmatrix},$$

(16)
where $M_{\text{Diag}}$ is a three by three diagonal mass matrix,

$$
M_{\text{Diag}} = \begin{bmatrix} M_U & & \\ & M_C & \\ & & M_T \end{bmatrix},
$$

and $y_L$ and $y_R$ are rank 3 matrices. The eigenvalue equation for the quark masses is given by

$$
\det (MM^\dagger - \Lambda) = \det \left[ \eta_L^2 y_L y_L^\dagger - \Lambda \eta_L M_{\text{Diag}} y_L^\dagger \eta_L M_{\text{Diag}} y_L + M_{\text{Diag}}^2 - \Lambda \right] = 0.
$$

The equation which determines the eigenvalues of order of $\eta_L^2$ (or smaller than $\eta_L^2$) is reduced to a cubic equation:

$$
\det \left[ y_L \left\{ 1 - M_{\text{Diag}} (\eta_R^2 y_R y_R^\dagger + M_{\text{Diag}}^2)^{-1} M_{\text{Diag}} \right\} y_L^\dagger - \lambda \right] = 0,
$$

where we use the normalized eigenvalue $\lambda = \frac{\Lambda}{\eta_L^2}$. This equation determines the ordinary quark masses. We further use the expansion as follows,

$$
(\eta_R^2 y_R y_R^\dagger + M_{\text{Diag}}^2)^{-1} = \frac{1}{M_0} \left[ 1 - \left( \frac{1}{M_0} \Delta M^2 \frac{1}{M_0} \right) + \left( \frac{1}{M_0} \Delta M^2 \frac{1}{M_0} \right)^2 + \cdots \right] \frac{1}{M_0},
$$

where we assume $M_U, M_C \gg \eta_R \gg M_T$. As a result the unperturbed part of $(\eta_R^2 y_R y_R^\dagger + M_{\text{Diag}}^2)$ is given by $M_0^2$ rather than by $M_{\text{Diag}}^2$. Eq.(19) is expressed in the simple form,

$$
\det \left[ y_L Y y_L^\dagger - \lambda \right] = 0,
$$

where the leading terms of $Y$, obtained by using the expansion of Eq.(20), are as follows:

$$
Y_{11} = X_U^2 [(y_R^\dagger y_R)_{11} - \frac{(y_R^\dagger y_R)_{13}(y_R^\dagger y_R)_{31}}{(y_R^\dagger y_R)_{33}}],
$$

$$
Y_{12} = X_U X_C [(y_R^\dagger y_R)_{12} - \frac{(y_R^\dagger y_R)_{13}(y_R^\dagger y_R)_{32}}{(y_R^\dagger y_R)_{33}}],
$$

$$
Y_{13} = X_U X_T (y_R^\dagger y_R)_{13} \frac{1}{(y_R^\dagger y_R)_{33}},
$$

with $X_U$, $X_C$, and $X_T$ being the components of $X$. The terms are normalized with respect to the mass matrix $M_{\text{Diag}}$.
\[
Y_{22} = X_C^2 [(y_R^* y_R)_{22} - \frac{(y_R^* y_R)_{23} (y_R^* y_R)_{32}}{(y_R^* y_R)_{33}}],
\]
\[
Y_{23} = X_C X_T (y_R^* y_R)_{23},
\]
\[
Y_{33} = 1 - \frac{X_T^2}{(y_R^* y_R)_{33}} + \frac{X_T^4}{(y_R^* y_R)_{33}}.
\]

(22)

where \(Y^* = Y\) and \(X_U, X_C\) and \(X_T\) are given by
\[
X_U = \frac{\eta_R}{M_U},
\]
\[
X_C = \frac{\eta_R}{M_C},
\]
\[
X_T = \frac{M_T}{\eta_R \sqrt{(y_R^* y_R)_{33}}}.\]

(23)

The eigenvalue equation now becomes
\[
F(\lambda) = -\lambda^3 + \lambda^2 \text{Tr}[y_L^* y_L Y] - \gamma \lambda + |\det y_L|^2 \det Y = 0.
\]

(24)

The coefficients for each term of Eq.(24) are
\[
\text{Tr}[y_L^* y_L Y] \approx (y_L^* y_L)_{33},
\]
\[
\gamma \approx X_C^2 |\alpha_{R2}|^2 |\alpha_{L2}|^2 (y_L^* y_L)_{33},
\]
\[
\det Y \approx X_C^2 X_T^2 \left(|\alpha_{R1}|^2 |\alpha_{R2}|^2 - |\alpha_{R1}^* \cdot \alpha_{R2}|^2\right),
\]

(25)

where we are keeping the leading term for each coefficient. Here \(\alpha\) and \(\beta\) are complex vectors which are related to the three complex vectors \((y_1, y_2, y_3)\) of the matrix \(y_L\) and \(y_R\). To show the relation, let us first write \(y_L\) and \(y_R\) as follows,
\[
y_L = (y_{L1}, y_{L2}, y_{L3}),
\]
\[
y_R = (y_{R1}, y_{R2}, y_{R3}).
\]

(26)

Then \(\alpha\) and \(\beta\) are defined by
\[
\alpha_{L1} = y_{L1} - \frac{y_{L1}^* \cdot y_{L1}}{|y_{L1}|^2} y_{L3},
\]
\[
\alpha_{R1} = y_{R1} - \frac{y_{R1}^* \cdot y_{R1}}{|y_{R1}|^2} y_{R3},
\]
\[
\beta_{L1} = \alpha_{L1} - \frac{\alpha_{L2}^* \cdot \alpha_{L1}}{|\alpha_{L2}|^2} \alpha_{L2},
\]
\[
\beta_{R1} = \alpha_{R1} - \frac{\alpha_{R2}^* \cdot \alpha_{R1}}{|\alpha_{R2}|^2} \alpha_{R2}.
\]

(27)
where $I = 1, 2$. Geometrically $\alpha_I (I = 1, 2)$ are the projections of $y_I$ onto the plane perpendicular to the vector $y_3$, and $\beta_I$ is the projection of $\alpha_1$ onto the line perpendicular to $\alpha_2$ and $y_3$. To find the solutions of Eq. (24), we first set the order of magnitude of $X_U$ and $X_C$ as follows. If we neglect flavor mixing, in the Seesaw model, the up quark mass is given by $\frac{M_U}{M_L}$ with $y_{L(R)} \simeq g$. Therefore the Seesaw suppression factor $X_U = \frac{M_U}{M_L}$ is of the order of $\frac{m_u}{M_L} \simeq 10^{-5}$. With a similar argument, we have $X_C = \frac{M_C}{M_L} \simeq 10^{-2}$ by assuming that the strength of the Yukawa coupling does not depend significantly on the flavor. By setting the magnitude of $X_U$ and $X_C$ in this way, we obtain an approximate mass formula for the case of flavor mixing. The point is that in the approximation that the strengths of all Yukawa couplings do not depend significantly on the flavor, the eigenvalue equation Eq. (24) has the following form,

$$F(\lambda) = -\lambda^3 + O(1)\lambda^2 - O(X_U^2)\lambda + O(X_U^2 X_C^2) = 0.$$  

The solutions of Eq. (28) are hierarchical, i.e., $\lambda = O(X_U^2), O(X_C^2), O(1)$ with $X_U \ll X_C \ll 1$. Taking into account the structure of the Yukawa coupling strengths the solutions are of the form:

$$m_i^2 \simeq \left| y_{LU3} \right|^2 \eta_L^2,$$

$$m_C^2 \simeq X_C^2 \left| \alpha_{RU2} \right|^2 \left| \alpha_{LU2} \right|^2 \eta_L^2,$$

$$m_T^2 \simeq X_T^2 \left| \beta_{RU1} \right|^2 \left| \beta_{LU1} \right|^2 \eta_L^2.$$  

The heavier eigenvalues are also obtained as follows,

$$m_U^2 \simeq M_U^2,$$

$$m_C^2 \simeq M_C^2,$$

$$m_T^2 \simeq |y_{RU3}|^2 \eta_R^2.$$  

The mass formulae for the five light quarks are of the Seesaw type. They are proportional to $\eta_L$, $\eta_R$ and inversely proportional to the singlet quark mass. They are also proportional to the appropriate projection of the Yukawa couplings $y_{LU1}, y_{RU1} (I = 1, 2)$ for the up and the charm quarks and $y_{LD1}, y_{RD1} (I = 1, 2, 3)$ for the down type quarks.

We would like to stress the following points.

- The mass formulae for the five light quarks are of the Seesaw type. They are proportional to $\eta_L$, $\eta_R$ and inversely proportional to the singlet quark mass. They are also proportional to the appropriate projection of the Yukawa couplings $y_{LU1}, y_{RU1} (I = 1, 2)$ for the up and the charm quarks and $y_{LD1}, y_{RD1} (I = 1, 2, 3)$ for the down type quarks.
• The top quark mass \( m_t \) is proportional to \( \eta_L \) and the length of the vector \( y_{LU3} \). The other mass eigenvalue \( m_T \) is proportional to \( \eta_R \) and the length of \( y_{RU3} \).

• The masses of the remaining five heavier quarks are given to a good approximation by the mass parameters \( M_{U_i} (i = 1, 2) \) and \( M_{D_i} (i = 1, 2, 3) \).

4 Flavor mixing

In order to find the mixing angles (CKM matrix) among left (right) chiralities of quarks, it is convenient to perform a unitary transformation among ordinary quarks such that the singlet-doublet Yukawa couplings, i.e., \( y_L \) and \( y_R \) become triangular matrices,

\[
y_{L(R)U(D)} = U_{U(D)L(R)} \begin{bmatrix} \times & 0 & 0 \\ \times & \times & 0 \\ \times & \times & \times \end{bmatrix},
\]

where the \( U \)'s are unitary matrices. This decomposition can always be done as proved in the Appendix. Further we introduce a new basis denoted by \( u', d' \) which is related to the original basis by the unitary matrices,

\[
\begin{align*}
u'_L &= U_{UL}^\dagger u_L, \\
u'_R &= U_{UR}^\dagger u_R, \\
d'_L &= U_{DL}^\dagger d_L, \\
u'_R &= U_{DR}^\dagger d_R.
\end{align*}
\]

In this new basis, the \( 6 \times 6 \) matrix \( M \) is transformed into

\[
M' = \begin{bmatrix} U_L^\dagger & 0 \\ 0 & 1 \end{bmatrix} M \begin{bmatrix} U_R & 0 \\ 0 & 1 \end{bmatrix}.
\]

The explicit form of \( M' \) for the up type quarks is given by

\[
M' = \begin{bmatrix}
0 & 0 & 0 & \beta_{LU11}\eta_L & 0 & 0 \\
0 & 0 & 0 & \beta_{LU21}\eta_L & |\alpha_{LU2}|\eta_L & 0 \\
0 & 0 & 0 & \alpha_{LU31}\eta_L & \alpha_{LU32}\eta_L & |y_{LU3}|\eta_L \\
\beta_{RU11}\eta_R & \beta_{RU21}\eta_R & \alpha_{RU31}\eta_R & 0 & M_U & 0 \\
0 & |\alpha_{RU2}|\eta_R & \alpha_{RU32}\eta_R & 0 & M_C & 0 \\
0 & 0 & |y_{RU3}|\eta_R & 0 & 0 & M_T
\end{bmatrix},
\]

where \( |\beta_{LU11}| = |\beta_{LU1}| \) and \( |\beta_{RU11}| = |\beta_{RU1}| \); their phases can be obtained from the Appendix; \( \alpha_{LD2} \), \( y_{LD3} \), \( \alpha_{LU2} \) and \( y_{LU3} \) are defined by Eq.(26) and Eq.(27). In this form, \( u' \) only couples to \( U \). Therefore the up quark mass is determined by the mass of the heaviest singlet quark \( U \) and is not affected by the presence of the other
Table 1: Singlet and Doublet mixing angles

| Doublet-Singlet | $\tan \theta_q L \exp(i\phi_{qL})$ | $\tan \theta_q R \exp(i\phi_{qR})$ |
|-----------------|---------------------------------|---------------------------------|
| $u' - U$        | $\beta_{LU11}\eta_L/M_U$        | $\beta_{RU11}\eta_R/M_U$        |
| $c' - C$        | $|\alpha_{LU2}|\eta_L/M_C$      | $|\alpha_{RU2}|\eta_R/M_C$      |
| $t' - T$        | $y_{LU3}\eta_L M_T/|y_{RU3}|^2\eta_R$ | $y_{RU3}\eta_R/M_T$            |
| $d' - D$        | $\beta_{LD11}\eta_L/M_D$        | $\beta_{RD11}\eta_R/M_D$        |
| $s' - S$        | $|\alpha_{LD2}|\eta_L/M_S$      | $|\alpha_{RD2}|\eta_R/M_S$      |
| $b' - B$        | $y_{LD3}\eta_L/M_B$             | $y_{RD3}\eta_R/M_B$             |

The off-diagonal elements of $M'$ which do not appear in $M'^0$ can be treated as a perturbation. In this approximation, each doublet quark only couples to one of the singlet quarks. So the eigenvalues and eigenstates are obtained by diagonalizing two by two matrices. It is interesting to see that the eigenvalues of $M'^0$ agree with those obtained in Eq.(29) and Eq.(30) with the same assumption for the singlet quark mass parameters. In addition the mass eigenstates are defined by the following rotation between the doublet quark $q'$ and the singlet quark $Q$, 

$$
\left( \begin{array}{c}
q^0 \\
Q^m
\end{array} \right) = \left( \begin{array}{cc}
\cos \theta_q & -\sin \theta_q e^{-i\phi_q} \\
\sin \theta_q e^{i\phi_q} & \cos \theta_q
\end{array} \right) \left( \begin{array}{c}
q' \\
Q
\end{array} \right). 
$$

In Table 1, we show the mixing angles between singlet and doublet quarks for both chiralities. It can be seen that the singlet to doublet mixing for the five light quarks is strongly suppressed being at least of the order $\frac{\eta_R}{M}$ for right (left) -handed quarks.

For the top quark, there is a suppression of the left-handed mixing angles of the order $\frac{\eta_L M_T}{\eta_R}$ while the right-handed mixing angle is not suppressed. In the following analysis, we only keep the right-handed mixing angle for the top quark and its singlet partner and set the other singlet to doublet mixing angles to zero. In this approximation, the left-handed charged currents and the CKM matrix are given by

$$
\bar{j}_{\mu L} = \overline{u^m L_i} \gamma_{\mu} V_{Lij} d^m L_j, 
$$

(38)
where the CKM matrix is written, in terms of the unitary matrices defined before, as:

\[ V_L = U_{UL}^\dagger U_{DL}. \]  

(39)

Within this approximation, we do not have FCNC for the left-handed neutral isospin current. On the other hand, the right-handed charged currents are given by

\[ j_{\mu R}^- = u^m_R \gamma_\mu V_{R1j} d^m_{Rj} + c^m_R \gamma_\mu V_{R2j} d^m_{Rj} + \cos \theta_{tR} t^m_R \gamma_\mu V_{R3j} d^m_{Rj} + \sin \theta_{tR} T^m_R \gamma_\mu V_{R3j} d^m_{Rj}, \]

(40)

where \( V_R = U_{UR}^\dagger U_{DR} \) and \( \theta_{tR} = \tan^{-1}(|y_{RU3}|/M_T) \) as shown in Table 1. The right-handed neutral isospin currents are

\[ j^3_{\mu R} = \frac{1}{2} \left\{ u^m_R \gamma_\mu u^m_{\bar{R}} + c^m_R \gamma_\mu c^m_{\bar{R}} + (\cos \theta_{tR})^2 T^m_R \gamma_\mu t^m_{R} + (\sin \theta_{tR})^2 T^m_R \gamma_\mu T^m_{R} + \sin \theta_{tR} \cos \theta_{tR} t^m_R \gamma_\mu t^m_{R} + (h.c.) \right\}, \]

(41)

in both currents a sum over \( j \) running from 1 to 3 is implied. Because \( \theta_{tR} \rightarrow \frac{\pi}{2} \) as \( M_T \rightarrow 0 \), there is a large mixing for \( T^m_R \) in the right-handed charged current \( j^-_{\mu R} \) and the right-handed neutral current \( j^3_{\mu R} \).

5 Summary

We study a mechanism for the generation of the top quark mass and flavor mixing in the context of a Seesaw model for quark masses. When the mass parameter \( M_T \) for the isosinglet quark is smaller than the symmetry breaking scale of \( SU(2)_R \), the lightest of the two eigenstates is at the breaking scale of \( SU(2)_L \) and can be identified with the top quark, the heaviest one will be found at the symmetry breaking scale of \( SU(2)_R \). We also study the mass hierarchy of ordinary quarks by including the flavor mixing. The stability of the light quark masses against flavor mixing is explicitly shown. The dependence on the strength of the Yukawa couplings is nontrivial and we have given a simple geometrical interpretation for it. The singlet-doublet mixing is suppressed except in the case of the mixing between the right-handed top quark and its singlet partner. This large mixing angle is a characteristic of our model and may be checked in the top quark production experiments.

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Note Added

After completing our work, we have received a paper by Y. Koide and H. Fusaoka [8] in which the enhancement of the top quark mass in a Seesaw model is studied. They found that the top quark mass is enhanced to the electroweak symmetry breaking scale at the singular point \( b_f = -1/3 \) of their singlet quark mass matrix (democratic matrix + \( b_f \times \) unit matrix). Their singular point would correspond to \( M_T = 0 \) in our case. Ignoring flavor mixing, the mass scale we obtained for the fourth lightest up type quark together with the mass formula for the top quark coincides with their result at the singular point. Still, our approach differs considerably from theirs and the mass formulae and the flavor mixing we obtain for the light quarks are quite different. We attribute the mass hierarchy of the five light quarks to the mass hierarchy of the five heavy singlet quark mass parameters and assume that the Yukawa couplings among the singlet and doublet quarks do not depend significantly on the flavour whilst they start from a specific type of the singlet quark mass matrix and introduce flavor dependent Yukawa couplings. As a result our formulae for the five light quarks cannot be directly translated into their framework. Furthermore their model gives constraints on the CKM matrix unlike ours where we cannot make definite predictions.

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Appendix

We prove the decomposition of $y_L(R)$ into a unitary matrix and a triangular matrix. Let us write a rank 3 matrix $y$ with three complex vectors in $C^3$,

$$y = (y_1, y_2, y_3),$$  \tag{42}

where $y$ is the singlet-doublet Yukawa coupling $y_L(R)$ in our case. Choose three orthonormal vectors $U = (u_1, u_2, u_3)$ with $u_3 = \frac{y_3}{|y_3|}$. Multiplying $U^\dagger$ on the left hand side of $y$, we obtain

$$U^\dagger y = \begin{bmatrix} \alpha_{11} & \alpha_{12} & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & |y_3| \end{bmatrix},$$  \tag{43}

where $\alpha_{ij} = u_i^\dagger \cdot y_j$. Then we define two vectors in $C^2$,

$$\mathbf{\alpha}_1 = \begin{bmatrix} \alpha_{11} \\ \alpha_{21} \end{bmatrix},$$

$$\mathbf{\alpha}_2 = \begin{bmatrix} \alpha_{12} \\ \alpha_{22} \end{bmatrix}.$$  \tag{44}

We also define two orthonormal vectors $v_1$ and $v_2$ with $v_2 = \frac{\mathbf{\alpha}_2}{|\mathbf{\alpha}_2|}$ in $C^2$. With these two vectors, we can form another unitary matrix $V$,

$$V = \begin{bmatrix} v_1 & v_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$  \tag{45}

By multiplying $V^\dagger$ to $U^\dagger y$, we finally obtain a triangular form,

$$V^\dagger U^\dagger y = \begin{bmatrix} \beta_{11} & 0 & 0 \\ \beta_{21} & |\mathbf{\alpha}_2| & 0 \\ \alpha_{31} & \alpha_{32} & |y_3| \end{bmatrix},$$  \tag{46}

where $\beta_{ij} = v_i^\dagger \cdot \mathbf{\alpha}_j$. Then $y$ is written as the product of a unitary matrix and a triangular matrix.