Small Instantons and Weak Scale String Theory

Karim Benakli and Yaron Oz

Theory Division, CERN
CH-1211, Geneva 23, Switzerland

Abstract

We consider heterotic string compactifications to four dimensions when instantons shrink to zero size. If the standard model gauge group originates from the new gauge symmetry associated with the small instantons singularity, then the weakly or strongly coupled heterotic string scales can be taken to be arbitrarily low. The $SO(32)$ and $E_8 \times E_8$ gauge groups can then be very weakly coupled even at the string scale and behave as non-abelian global symmetries. We comment on a possible role of small instantons in supersymmetry breaking.

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1 Introduction

The string scale, compactification scale, and Planck scale and the relations between them, are of great importance for understanding the dynamics of string theory and its phenomenological applications. These relations depend strongly on the choice of the string vacuum [1]. In this note we consider heterotic string compactifications to four dimensions when instantons shrink to zero size [2, 3, 4] in weakly and strongly coupled [5] limits of heterotic strings. We discuss some aspects of the resulting four-dimensional $\mathcal{N} = 1$ supersymmetric gauge theories.

One of the main aims of our study is to illustrate how the heterotic string scale can be made arbitrarily low. Lowering the compactification scale [6], the string scale [7] and quantum gravity scale [8] to TeV or to intermediate energies [9, 10] allows new perspectives on phenomenological applications of string theory. It was argued that this is possible for Type I string theory in [11], for the Hořava–Witten compactifications of M-theory in [9] and for Type II string theory in [12] (see also [13]).

The paper is organized as follows. In the next section we will briefly discuss heterotic string compactification. In section 3 we will consider the singularities when $SO(32)$ instantons shrink to zero size and discuss phenomenological aspects of the four-dimensional $\mathcal{N} = 1$ supersymmetric gauge theories. We show that if the standard model gauge group originates from the new gauge symmetry associated with the small instantons singularity, then both the weakly and strongly coupled heterotic string scales can be taken to be arbitrarily low. The $SO(32)$ gauge group can be very weakly coupled even at the string scale and play the role of a non-abelian global symmetry. In section 4 we will consider the singularities when $E_8 \times E_8$ instantons shrink to zero size. We comment on a possible role of the small instantons in supersymmetry breaking.

2 Heterotic string compactification

Consider $E_8 \times E_8$ and the $SO(32)$ heterotic string theory with a ten–dimensional string coupling $\lambda_H = \exp(\phi_H)$ and a string scale $l_H$. At low energies the effective ten–dimensional description is in terms of a super Yang-Mills theory coupled to supergravity. The ten dimensional gauge coupling $g_{10}$ and the gravitational coupling $\kappa_{10}$ are

$$g_{10}^2 = \frac{1}{4} \lambda_H^2 r_H^6, \quad \kappa_{10} = \frac{1}{8} \lambda_H^2 r_H^8.$$  \hspace{1cm} (2.1)

We will be interested in a compactification of weakly coupled heterotic string theory on a Calabi–Yau 3-fold. Concretely, we will consider Calabi–Yau 3-folds of the form of a $K3$
fibration over a $P^1$ base. The four-dimensional gravitational coupling $\kappa_4$ is related to the Newton constant $G_N$ \(^1\) by $\kappa_4^2 = 8\pi G_N$. The latter is given by

$$
G_N = \frac{\lambda_H^2 l_H^8}{64\pi \langle V_{K3} V_{P^1} \rangle},
$$

(2.2)

where $V_{K3}$ and $V_{P^1}$ are the volumes of the $K3$ and of the $P^1$ base spaces respectively. The brackets stand for the average over the compact space. The tree-level four-dimensional gauge coupling constant of an unbroken subgroup of $SO(32)$ and $E_8 \times E_8$ is given by:

$$
\alpha_H \equiv \frac{g_4^2}{4\pi} = \frac{\lambda_H^2 l_H^6}{16\pi f \langle V_{K3} V_{P^1} \rangle},
$$

(2.3)

with $f$ standing for the different normalization of the traces in the gauge kinetic term. Below we ignore the model dependence arising from the factor $f$ and take $f = 1$. These lead to:

$$
\alpha_H = 4 \frac{l_p^2}{l_H^2}.
$$

(2.4)

As is clear from (2.4), requiring a gauge coupling $\alpha_H$ of order 1 implies that $l_H$ is of order $l_p$. In [14] it was suggested to use one-loop–modified relations with the inclusion of threshold correction due to $d$ large dimensions with size $R$. In such a case, while $\lambda_H$ governs the strength of gravitational interactions, the gauge interaction are governed by:

$$
\alpha_{H\text{ one-loop}} = \frac{\alpha_H}{1 + c\alpha_H (R/l_H)^d} = 4 \frac{l_p^2}{l_H^2 (1 + 4c l_p^2 R^d / l_H^d)},
$$

(2.5)

with $c$ a constant containing the beta-function coefficient. We see that for large $l_H$ a coupling of order 1 is obtained if the denominator is correspondingly small. Here $R$ is bounded to be smaller than $\sim \text{TeV}^{-1}$ and $d = 1, 2$ for the supersymmetric case and up to 6 for the non-supersymmetric case. This leads roughly to bounds of a string scale higher than $\sim 10^{11}$ GeV in the first case and $\sim 10^7$ GeV in the second one.

In the following we use the fact that (2.3) does not apply to gauge sectors of weakly coupled heterotic string that do not descend from the ten-dimensional $SO(32)$ or $E_8 \times E_8$ gauge groups. Such gauge sectors arise from singularities in the moduli space of the compactification where extra massless particles are present and there is an extra gauge symmetry. We will consider the enhanced gauge symmetries associated with the singularities when $SO(32)$ \(^2\) or $E_8 \times E_8$ instantons shrink to zero size. This phenomenon cannot be seen in the conformal field theory description, but can be seen at a weakly coupled heterotic string description.

\(^1\) $G_N \equiv 1/M_p^2 \equiv l_p^2$ where $M_p \sim 10^{19}$ GeV is the four-dimensional Planck scale.

\(^2\) In six dimensions this has been considered in [7].
3 \textbf{SO(32) Small Instantons}

Consider first the case of the weakly coupled \textit{SO}(32) heterotic string theory compactified on a \textit{K}3 fibration over a \textit{P}1 base. We will work in the adiabatic limit of a large \textit{P}1 base, which simplifies the discussion but is not essential. In this limit we can use a local description as a compactification to six dimensions on the \textit{K}3 fibre. In order to specify a compactification of the heterotic string on a manifold \textit{M}, we have to choose a gauge bundle \textit{V}. One requirement from the compactification is that

\[ \frac{1}{2}p_1(V) = \frac{1}{2}p_1(M), \] (3.1)

where \( p_1 \) is the first Pontryagin number. For \textit{K}3 it means that we have to choose the gauge bundle to have instanton number 24. Witten argued [2] that at the singularity, associated with a collapse of \( k \) instantons at the same point in \textit{K}3, a new \textit{Sp}(\( k \)) gauge symmetry appears. In addition massless hypermultiplets appear. They consist of \((32, 2k)\) of the \textit{SO}(32) \times \textit{Sp}(k) gauge group and a massless hypermultiplet in the antisymmetric representation of \textit{Sp}(k), which is a singlet of \textit{SO}(32).

The gauge coupling of the new gauge group is different from the \textit{SO}(32) gauge coupling. In order to determine the new gauge coupling it is useful to consider the dual picture of the instantons collapse in Type I string theory at strong coupling. The analogue in the Type I picture of the collapse of \( k \) instantons at the same point is \( k \) coinciding five-branes. The enhanced gauge group is the five-branes world-volume gauge group and the massless hypermultiplets are the matter content of the world-volume gauge theory. They are obtained by analysing the zero modes of the open strings attached to the D5-branes [2]. The six-dimensional \textit{Sp}(\( k \)) gauge coupling is \( g_{D5}^2 = (2\pi)^3 \lambda_I l_I^2 \), where \( \lambda_I = \exp(\phi_I) \) is the ten-dimensional Type I string coupling and \( l_I \) is the Type I string scale. The duality map between Type I string theory and the heterotic string theory is \( \lambda_I = 1/\lambda_H, \quad l_I^2 = \lambda_H l_H^2 \). Thus, the six-dimensional gauge coupling of the small instanton sector at the heterotic side is given by \( g_{SI}^2 = (2\pi)^3 l_H^2 \).

We would now like to reduce the six dimensional \( \mathcal{N} = 1 \) supersymmetric gauge theory on the \textit{P}1 and obtain a four-dimensional \( \mathcal{N} = 1 \) gauge theory. On the Type I side this means wrapping the D5 branes on the base \textit{P}1. The spectrum of the four-dimensional theory can be obtained as in [15]. Here we have to to take into account the non-trivial fibration of the fibre \textit{K}3 over the base \textit{P}1, which leads to a twist of the Dirac operator on \textit{P}1. The reduction preserves one covariantly constant spinor, yielding \( \mathcal{N} = 1 \) supersymmetry in four dimensions. The \textit{Sp}(\( k \)) vector fields survive the reduction as well as some of the six-dimensional matter hypermultiplets. For instance when \( k = 1 \), \textit{Sp}(1) \sim \textit{SU}(2)
the 32 hypermultiplets in the fundamental representation of the SU(2) gauge group give rise to four doublets in four dimensions [15].

The four-dimensional gauge coupling reads

$$\alpha_{Sp} = \frac{2 \pi^2 l_p^2}{V_{P^1}},$$

where $V_{P^1}$ is the volume of the base. The configuration where one identifies the standard model gauge group with the small instantons gauge sector will allow us to consider arbitrary low heterotic SO(32) string scale.

There are three dimensionless expansion parameters in the system that we require to be small in order for the weakly coupled description to be valid. The first is the expansion parameter of the perturbative string description in ten dimensions $\lambda_H/(2\pi)^5$ [16]:

$$\lambda_H^2/(2\pi)^5 = \frac{2 l_p^2 \langle V_{K3} V_{P1} \rangle}{\pi^4 l_H^2 \langle V_{P1} \rangle^6},$$

which we require to be smaller than 1 in order for the heterotic string to be weakly coupled in space-time. The second parameter is $\langle V_{K3} V_{P1} \rangle$, which we require to be smaller than 1 in order for the heterotic string to be weakly coupled on the world-sheet. The third parameter is $\alpha_{Sp}$ in (3.2), which we require to be smaller than 1 in order for the new gauge symmetry to be weakly coupled.

Let us now analyse these conditions for the validity of the weakly coupled description. We choose $K3$ such that $l_H^4/(\langle V_{K3} \rangle) < 1$ so that the small instanton picture is valid. We assume that $\langle V_{K3} V_{P1} \rangle \sim \langle V_{K3} \rangle \langle V_{P1} \rangle$. Together with the requirement that $\alpha_{Sp} \sim l_H^2/(V_{P1}^4)$ (3.2) be small, it guarantees that $\langle V_{K3} V_{P1} \rangle$ is small too. Finally, in order for $\lambda_H^2/(2\pi)^5$ to be small, we require that $\langle V_{K3} V_{P1} \rangle$ be small, namely a weakness of gravitational interactions is consistent with the weakly coupled description.

We can view the weakness of gravitational interactions as arising either from a large $K3$ volume or from a very small string coupling constant. For instance, taking $\alpha_{Sp} \sim 1/10$ as a rough estimate, the first possibility arises, with a choice:

$$\lambda_H \sim 1 \text{ and } \langle V_{K3} \rangle^{1/4} \sim 10, \ 10^3, \ 10^6 l_H \text{ for } l_H^{-1} \sim 10^{16}, \ 10^{11}, \ 10^4 \text{ GeV respectively.}$$

The second possibility arises, with a choice:

$$\langle V_{K3} \rangle^{1/4} \sim \text{few } l_H \text{ and } \lambda_H \sim 10^{-1}, \ 10^{-6}, \ 10^{-13}, \text{ for } l_H^{-1} \sim 10^{16}, \ 10^{11}, \ 10^4 \text{ GeV respectively.}$$

The above discussion is valid till the gauge coupling (3.2) is large $\alpha_{Sp} \sim \frac{l_H^2}{\langle V_{P1} \rangle} \sim 1$ and we cannot trust a perturbative analysis. Passing to the dual Type I description is
not useful either, since at these energies the heterotic $SO(32)$ gauge coupling $\alpha_{SO} = 4l_H^2$ is still weak and therefore the $SO(32)$ gauge coupling on the Type I side is large. At this energy scale the system is no longer four-dimensional and we probe the physics of six dimensions.

If $\lambda_H$ is chosen to be very small, at energies below the string scale, the unbroken part of the $SO(32)$ symmetry is very weakly coupled and it is seen from the $Sp(k)$ side as a non-abelian “global” symmetry. Such kinds of symmetries can be useful for phenomenological issues such as forbidding operators that could lead to proton decay or other exotic processes. On the other hand the gravitational interactions are still weak at the string scale. The main experimental signature would be the observation of effects due to the Kaluza–Klein modes of $P^1$. If one instead explains the weakness of gravitational interactions by a large $K3$ volume (as in Type I scenarios) then at energies of order $l_H^{-1}$ the $SO(32)$ symmetry coupling is of the same order as the one of $Sp(k)$ and cannot be viewed as a global symmetry. This is due to the sum of the contributions from the Kaluza–Klein states propagating in the $K3$. Moreover at the string scale the gravitational interactions are now of the same strength as the gauge ones.

A large class of models with various gauge groups and matter content for which the above discussion continues to hold can be obtained by shrinking instantons at ADE singularities of $K3$ [17]. On the Type I side these models are obtained by placing the five-branes at these singular points [18, 19]. In these models the gauge groups are products of the classical gauge groups $\prod_{i,j,k} SO(n_i) \times Sp(m_j) \times U(l_k)$ arranged according to quiver (moose) diagrams related to the extended Dynkin diagrams of the ADE groups.

Finally we note that instead of a compactification on the base $P^1$, we can reduce to four dimensions on two circles with non-trivial boundary conditions on the circles. This leads to $\mathcal{N} = 0$ gauge theories in four dimensions. The above discussion continues to hold at energies $E \ll 1/R$, where $R$ is the radius of the circles. At higher energies we will probe these two compact coordinates, by producing the associated Kaluza–Klein states.

4 $E_8 \times E_8$ Small Instantons

Consider now the case of $E_8 \times E_8$ heterotic string compactified on a $K3$ fibration over a $P^1$ base, in the adiabatic limit. Denote by $n_1, n_2$ the instanton numbers of the two $E_8$ groups. We have to choose the gauge bundle with $n_1 + n_2 = 24$. When we shrink some of the instantons to zero size we do not get a new gauge symmetry in six dimensions. Instead, we get massless tensor multiplets and hypermultiplets in six dimensions [3, 4]. The six-
dimensional tensor multiplet contains a 2-form field $B_{\mu\nu}$ which is self-dual $dB = *dB$. In the dual picture of M-theory compactified on $S^1/Z_2$, this process is viewed as placing M5-branes near one of the $E_8$ walls. There are tensionless strings that arise from membranes stretched between the M5-branes and the $E_8$ wall and couple to $B$. When we reduce on $P^1$ the tensor multiplets do not give rise to gauge fields but rather to matter multiplets. This is due to the fact that there are no 1-forms $\omega$ on $P^1$, which otherwise would enable us to decompose $dB = F \wedge \omega$ and obtain the gauge field strength $F$.

We can however obtain vectors fields in six dimensions and a large class of gauge groups and matter content by shrinking $E_8$ instantons at ADE singularities [17]. For instance, if we shrink $k$ instantons at $A_{n-1}$ singularity we get a gauge group $\prod_{i=2}^{n-1} SU(i) \times SU(n)^{k-2n+1} \times \prod_{j=2}^{n-1} SU(j)$ with bi-fundamental matter. The six-dimensional gauge couplings of these gauge groups is determined by vacuum expectation values (vev’s) $\langle \phi \rangle$ of scalars in particular tensor multiplets [17]. These scalars in six dimensions have dimension two and we can choose vev’s $\langle \phi \rangle \sim 1/l_H^2$. Upon reduction on $P^1$ we can identically repeat the discussion in the previous section for the weakly coupled heterotic strings case. For the Hořava–Witten compactifications an arbitrarily low scale can be obtained by taking all or some of the five dimensions transverse to the M5-brane large.

5 Discussion

We have argued that the tree-level gauge and gravitational couplings dependence on the string and compactification scales allow the latter to be arbitrarily low. In addition to the necessity for building realistic models, many important questions related to the dynamics of string theory remain to be addressed. For instance: How is supersymmetry broken?, Does this allow the size of the couplings to be small or the volumes large, as required for lower values of the string scales? and How do loop corrections modify our analysis?

Here we would like to briefly comment on some issues of supersymmetry breaking. An interesting possibility is to use the small instantons to break supersymmetry. For this we choose a gauge bundle with $(n_1, -n_2)$ instanton number such that $n_1 - n_2 = 24$ and shrink the $n_2$ anti-instantons. This configuration breaks supersymmetry completely. On the strongly coupled heterotic side, described by M-theory compactified on $S^1/Z_2$, this would correspond to placing $n_2$ anti-M5-branes on one of the walls. Supersymmetry is then broken at the eleven-dimensional Planck scale. This would need to be at TeV if we live on the anti-M5-branes, at intermediate energies if we live on the opposite wall, and somewhere in between if we live on the same wall (gauge mediation). This is the heterotic
the theory scenario corresponding to the proposal of [20] for type I vacua. Obviously the issue of stability has to be addressed in these models.

Another possibility is to break supersymmetry spontaneously by wrapping the M5-brane on the boundary wall around a non-supersymmetric cycle [21] (the use of five-branes in the bulk as hidden sector to break supersymmetry was suggested in [22]). This is useful, for instance, in scenarios with a non-standard embedding [23] where the volume on the hidden wall of the Calabi–Yau space is large, making the gauge interaction on the wall weaker than needed to induce non-perturbative effects that are able to break supersymmetry at desired scales. Also when small instantons are localized at different points of $K3$, they may act as hidden sectors as in the F-theory scenario of [24]. The observable and “hidden” sectors communicate through both gravity and the $SO(32)$ or $E_8 \times E_8$ gauge symmetries. Finally, for very low values of the string scale, one may replace in all models discussed above the base $P^1$ of the Calabi–Yau 3-fold by two circles with boundary conditions that break all the supersymmetries.

To summarize, we considered generic features of the gauge theories arising from small instantons and pointed out to some aspects that are of phenomenological relevance, such as a possibility to lower the string scale, extremely weakly coupled gauge symmetries that act as global non-abelian symmetries, and finally a possible role in the dynamics of supersymmetry breaking. These, we believe, deserve further studies.

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