Abstract

We discuss local \textit{R}-symmetry as a potentially powerful new model building tool. We first review and clarify that a $U(1)$ \textit{R}-symmetry can only be gauged in local and not in global supersymmetry. We determine the anomaly-cancellation conditions for the gauged \textit{R}-symmetry. For the standard superpotential these equations have no solution, independently of how many Standard Model singlets are added to the model. There is also no solution when we increase the number of families and the number of pairs of Higgs doublets. When the Green-Schwarz mechanism is employed to cancel the anomalies, solutions only exist for a large number of singlets. We find many anomaly-free family-independent models with an extra $SU(3)_c$ octet chiral superfield. We consider in detail the conditions for an anomaly-free family dependent $U(1)_R$ and find solutions with one, two, three and four extra singlets. Only with three and four extra singlets do we naturally obtain sfermion masses of order the weak-scale. For these solutions we consider the spontaneous breaking of supersymmetry and the \textit{R}-symmetry in the context of local supersymmetry. In general the $U(1)_R$ gauge group is broken at or close to the Planck scale. We consider the effects of the \textit{R}-symmetry on baryon- and lepton-number violation in supersymmetry. There is no logical connection between a conserved \textit{R}-symmetry and conserved \textit{R}-parity. For conserved \textit{R}-symmetry we have models for all possibilities of conserved or broken \textit{R}-parity. Most models predict dominant effects which could be observed at HERA.

1 Introduction

Supersymmetry combines fields of different spin into supermultiplets. It includes the special possibility of a symmetry which distinguishes between the fermionic and the bosonic component of a $N = 1$ supersymmetric superfield. Such symmetries are called \textit{R}-symmetries.
and they are particular to supersymmetry. As such, they deserve special attention when considering the implications of supersymmetry. R-parity can be thought of as a discrete R-symmetry and has been widely discussed in the context of the minimal supersymmetric standard model and its extensions. There is also a considerable amount of literature on global R-symmetries and their phenomenological implications. It is the purpose of this paper to reconsider local R-symmetries in the context of local supersymmetry and to make first steps towards a realistic model. It is similar in spirit to [1] where we considered a non-R U(1)′.

R-symmetries were first introduced in global supersymmetry by Salam and Strathdee [2] and by Fayet [3] in order to enforce global lepton- or baryon-number. In the following years, the discrete symmetry R-parity [1] has been imposed to prohibit all dimension four lepton- and baryon-number violating interactions which arise in the supersymmetric extension of the Standard Model. Global R-invariance has been proposed as a solution to the strong CP-problem [4], the mu problem [5, 6, 7], and the problem of the neutron electric dipole moment [5, 9]. Global R-invariance prohibits tree-level gaugino masses. This leads to the interesting possibility that the gaugino masses are generated radiatively or through a dynamical mechanism and thus predicted [10, 8, 11]. If the global R-symmetry remains unbroken to low energies [8] then only the electroweak gaugino masses can be generated after SU(2)_L × U(1)_Y breaking. (A bi-scalar mu-term must be generated or inserted by hand in the soft-susy breaking sector.) The radiative gluino mass is very light [10, 11] and excluded [12]. A heavy gluino can be obtained by adding an SU(3)_c octet chiral superfield [8]. One then loses any prediction for the gluino mass. This is not very natural but it is consistent with experiment. However, the potential of the scalar component of the octet is necessarily unrestricted and typically breaks SU(3)_c. If the global R-invariance is spontaneously broken [11] one has an unwanted light pseudo Goldstone boson. The gaugino masses can still be generated radiatively and the gluino is light [11]. One can add explicit R-breaking terms which give mass to the axion. However, if these terms are large this renders the R-symmetry meaningless. Recently global R-symmetries have been seen to arise in so-called generic models of global supersymmetry breaking [13]. The problems of the axion from R-breaking are resolved when embedded into local supersymmetry through explicit breaking terms [14]. Thus models with global R-symmetry suffer from an axion or a light gluino problem.

Beyond the immediate phenomenological problem of constructing a model with global R-symmetry there is a more fundamental problem. Supersymmetry breaking is necessarily embedded in local supersymmetry. Local supersymmetry automatically includes gravity and global symmetries are most likely broken by quantum gravity effects [15]. Thus at low energies we do not expect global symmetries such as baryon- or lepton-number to be fundamental symmetries of nature but only symmetries of the low-energy effective Lagrangian. At high-energy we expect all relevant symmetries to be gauge symmetries. We shall thus
investigate whether an \( R \)-symmetry can be gauged. We provide a new local symmetry as a model-building tool.

Our paper is structured as follows. In Section 2 we show that an \( R \)-symmetry can only be gauged in local supersymmetry. In Section 3 we then consider the conditions for an anomaly-free gauged \( R \)-symmetry and find several solutions. In Section 4 we discuss the spontaneous breaking of the \( R \)-symmetry and of supersymmetry. We find the important result that the \( R \)-symmetry is always broken at or near the Planck scale. In Section 5 we consider the implications for \( R \)-parity violation. In Section 6 we offer our conclusions and an outlook.

2 \( R \)-invariant Supersymmetric Theories

Below we first discuss \( R \)-symmetries in global supersymmetry and in the following subsection extend the discussion to local supersymmetric theories, where \( R \)-symmetries have not been as widely discussed.

2.1 Global Supersymmetry

For globally supersymmetric theories the global \( R \)-transformations are defined as a transformation on the superfields \[ V_k(x, \theta, \bar{\theta}) \rightarrow V_k(x, \theta e^{-i\alpha}, \bar{\theta} e^{i\alpha}), \]
\[ S_i(x, \theta, \bar{\theta}) \rightarrow e^{i\alpha} S_i(x, \theta e^{-i\alpha}, \bar{\theta} e^{i\alpha}), \]
where \( V_k \) is a gauge vector multiplet with components \( V_\mu^k, \lambda_k \), and \( D_k \) and \( S_i \) are left-handed chiral superfields with components \( z_i, \chi_i \), and \( F_i \). Thus the Grassman coordinates \( \theta, \bar{\theta} \) have non-trivial \( R \)-charge
\[ \theta \rightarrow e^{-i\alpha} \theta, \quad \bar{\theta} \rightarrow e^{i\alpha} \bar{\theta}, \quad \int d\theta \rightarrow e^{i\alpha} \int d\theta, \quad \int d\bar{\theta} \rightarrow e^{-i\alpha} \int d\bar{\theta}. \]

The latter two transformations hold since for Grassman variables integration is like differentiation. The \( R \)-transformations act on the components of the superfields as
\[ V_\mu^k \rightarrow V_\mu^k, \]
\[ (\lambda_k)_L \rightarrow \exp(-i\alpha)(\lambda_k)_L, \]
\[ (\lambda_k)_R \rightarrow \exp(i\alpha)(\lambda_k)_R, \]
\[ D_k \rightarrow D_k, \quad z_i \rightarrow \exp(i n_i \alpha) z_i, \quad \chi_i \rightarrow \exp(i \gamma_5 (n_i - 1) \alpha) \chi_i, \]
\[ F_i \rightarrow \exp(i (n_i - 2) \alpha) F_i. \]
and $R$-transformations are more general than lepton- or baryon-number. The action for the superpotential

$$\int d^2\theta g(S_i), \quad (2.4)$$

is invariant provided that the superpotential transforms as

$$g(S_i) \rightarrow e^{-2i\alpha}g(S_i), \quad (2.5)$$

under the transformation (2.1). Here we have made use of (2.2). We see that the superpotential transforms non-trivially. This is one essential fact of $R$-symmetries.

The kinetic terms of the vector and scalar multiplets are of the form

$$\int d^2\theta d^2\bar{\theta} \left[ (Se^{2\tilde{V}})^i S_i \right] + \int d^2\theta W^\beta W_\beta + \int d^2\bar{\theta} \bar{W}_\bar{\beta} \bar{W}^\bar{\beta}, \quad (2.6)$$

where

$$W_\beta = D^2(e^{-V} D_\beta e^V), \quad \bar{W}_\bar{\beta} = D^2(e^{-V} D_\bar{\beta} e^V), \quad (2.7)$$

are automatically invariant under (2.1), i.e.

$$W_\beta \rightarrow e^{-i\alpha} W_\beta, \quad \bar{W}_\bar{\beta} \rightarrow e^{i\alpha} \bar{W}_\bar{\beta}. \quad (2.8)$$

In this paper, we focus on the possibility of locally $R$-symmetric theories. However, as we now discuss it is not possible in globally supersymmetric theories to promote the global $R$-invariance to a local one. An easy way to see this is to notice that when the $R$-parameter $\alpha$ becomes $x$-dependent then the transformations (2.2) change to

$$\theta \rightarrow \theta e^{-i\alpha(x)}, \quad \bar{\theta} \rightarrow \bar{\theta} e^{i\alpha(x)}, \quad (2.9)$$

which is a special form of a local superspace transformation.

In more detail, one can also see this from Eq.(2.3) which implies that all gauginos have $R$-charge, including the $R$-gauginos. If the $R$-symmetry is to become a local symmetry then the $R$ gauge vector boson $V_R^\mu$ will have to couple to the $R$-gauginos $\lambda_R^i$ in the form

$$\mathcal{L} \sim \bar{\lambda}_R^i (\partial_\mu - ig_R V_R^\mu) \gamma^\mu \lambda_R^i + \bar{\lambda}_R^i (\partial_\mu + ig_R V_R^\mu) \gamma^\mu \lambda_R^i, \quad (2.10)$$

since the opposite chirality gauginos have non-trivial and opposite $R$-charge (2.3). The above equation implies the coupling

$$\mathcal{L} \sim g_R \bar{\lambda}_R^i \gamma^\mu \gamma^5 \lambda_R^i V_R^\mu, \quad (2.11)$$

in the Lagrangian which is an axial interaction and is not present in the action (2.6). In order to construct a supersymmetric Lagrangian containing (2.11) we must consider its supersymmetric transformation. It contains the term

$$g_R \epsilon^{\mu\rho\sigma\nu} \epsilon_{\gamma\mu} \lambda_R F^R_{\nu\rho} V^R_\sigma = \epsilon^{\mu\rho\sigma\nu} \delta V^R_{\nu} V^R_\mu F^R_{\rho\sigma}, \quad (2.12)$$

1The lower index $(L, R)$ on the gaugino $\lambda$ is the chirality and the upper index $R$ indicates the gauge group.
2We make use of the identity $\gamma_\alpha \gamma_\beta \gamma_\lambda = g_{\alpha\beta} \gamma_\lambda + g_{\beta\lambda} \gamma_\alpha - g_{\alpha\lambda} \gamma_\beta + i \epsilon_{\mu\alpha\beta\lambda} \gamma^\mu \gamma^5$. 


since the supersymmetric variation of the gaugino term \( \delta \lambda^R \) contains \( \gamma^{\mu\nu} \epsilon F^R_{\mu\nu} \). \( \epsilon \) is the infinitesimal parameter of the supersymmetry transformation. The above term can not be cancelled without departing from the setting of global supersymmetry.

From Eq. (2.3) it is clear that the \( R \)-symmetry generator \( R \) does not commute with the supersymmetry generator \( Q \). In the literature this is quoted as an argument that an \( R \)-symmetry can not be gauged. Explicitly we have \[ [Q, R] = i(\gamma_5)_{\alpha}^{\beta} Q_\beta. \] (2.13)

Thus \( R \)-symmetry is an extension of supersymmetry with the chiral generator and the extension is a graded Lie Algebra. If the \( R \) generator of a globally \( R \)-supersymmetric theory is promoted to a local symmetry then the above equation can only hold if the transformation parameters of the supersymmetry algebra are \( x \)-dependent, \( i.e. \) \( Q_\alpha \) is the generator of a local transformation.

Thus \( R \)-symmetries are intimately connected with supersymmetry: a \textit{locally} \( R \)-invariant theory can only be constructed in a \textit{locally} supersymmetric framework; in \textit{global} supersymmetry only \textit{global} \( R \)-symmetries can be constructed.

### 2.2 Local Supersymmetry

In local supersymmetry the field content is extended to include a spin 2 graviton and a spin \( \frac{3}{2} \) gravitino. From Eq. (2.3) we generalize the \( R \)-symmetry to the graviton multiplet as

\[
\begin{align*}
\epsilon^m_\mu & \rightarrow \epsilon^m_\mu, \\
\psi_\mu & \rightarrow \exp(-i\alpha \gamma_5) \psi_\mu.
\end{align*}
\] (2.14)

From the above and Eqs. (2.3, 2.10) we see that a possible \( R \)-gauge boson would couple axially to the gravitino, the gauginos, and the chiral fermions. It was first noticed by Freedman [17] that the axial gauge vector \textit{can} couple to the gauginos and the gravitinos in an invariant way in \textit{local} supersymmetry. The variation of (2.11) is then cancelled by the variation of the term

\[
e^{-1} \mathcal{L} = \frac{i}{\sqrt{2}} \psi_\rho \gamma^{\mu\nu} F^R_{\mu\nu} \gamma_5 \lambda^R,
\] (2.15)

in the action since \( \delta \psi_\mu \) contains \( g_R V^R_{\mu} \gamma_5 \epsilon \). Later, Das \textit{et al.} [18] extended the Fayet-Illiopoulos model [19] of global supersymmetry to local supersymmetry. They found that the abelian gauge theory was chiral and just that of Freedman [17]. These results [17, 18] were reproduced in [20] including the gravitational auxiliary fields. Stelle and West [21] then derived the action for the Fayet-Illiopoulos term in local supersymmetry in the superconformal framework

\[
\int d^4xd^4\theta E e^{-\xi g_R V^R_{\mu}},
\] (2.16)
where \( E \) is the superspace determinant and \( \xi \) is the constant of the FI-term. The expression (2.16) is invariant under the \( U(1)_R \) gauge transformations

\[
V^R \rightarrow V^R + \frac{i}{g_R} (\Lambda - \overline{\Lambda}), \quad \bar{D}_\alpha \Lambda = 0, \quad (2.17)
\]
\[
E \rightarrow E e^{i\xi(\Lambda - \overline{\Lambda})}. \quad (2.18)
\]

The superspace determinant transforms non-trivially. In Ref.[17] it was shown that this implies a \( U(1)_R \) charge \( \frac{3}{2}\xi \) for the gauginos and the gravitino. In the previous section we had a global \( R \)-charge +1 for the gauginos which corresponds to the choice \( \xi = \frac{2}{3} \).

Barbieri et al. [23] extended this analysis to include matter fields in a general superpotential. In the superconformal framework an invariant superpotential is constructed by introducing a compensating superconformal chiral multiplet \( S_0 \) which transforms under the \( U(1)_R \) gauge group as

\[
S_0 \rightarrow e^{+i\xi \Lambda} S_0, \quad \overline{S}_0 \rightarrow e^{-i\xi \overline{\Lambda}} \overline{S}_0, \quad (2.19)
\]
and such that the matter multiplets transform as

\[
S_i \rightarrow e^{-in_i \xi \Lambda} S_i, \quad \overline{S}_i \rightarrow e^{in_i \xi \overline{\Lambda}} \overline{S}_i, \quad (2.20)
\]
with \( U(1)_R \) charge \( n_i \). Then the action

\[
\left[ S_0^3 g(S_i) \right]_F , \quad (2.21)
\]
is invariant under the gauge transformations (2.7)-(2.9) provided

\[
g(S_i) \rightarrow e^{-3i\xi \Lambda} g(S_i). \quad (2.22)
\]
The superpotential has a net \( U(1)_R \) charge just as in Eq.(2.3). We obtain the same charge +2 again for the choice \( \xi = \frac{2}{3} \). Thus it is found that the generalization of the Fayet-Iliopoulos term to local supersymmetry leads to a gauged \( R \)-symmetry!

In Ref.[24] Ferrara et al. showed that any \( R \)-invariant gauged action can be put into the canonical form of local supersymmetry [25]. The most general Lagrangian with local \( R \)-symmetry (with not more than 2 derivatives for the component fields) and local supersymmetry is given by [24]

\[
\mathcal{L} = -\frac{1}{2} \left[ \overline{S}_0 e^{-\xi g_R V^R} S_0 \phi(S_i, (\overline{S}_i e^{n_i \xi g_R V^R} e^{2\phi V}) \right]_D + \left[ g(S_i) S_0^3 \right]_F - \left[ g(\alpha) W^\alpha W^\beta \right]_F - \left[ f_R(S_i) W^2_R \right]_F + h.c. , \quad (2.23)
\]
where \( W_R \) is the field strength of the vector multiplet \( V^R \), a propagating gauge field, and where the function \( \phi \) is invariant under (2.21). \( W^\alpha \) is the field strength of other (non \( R \)-)

\footnote{Note that the chiral fields \( S_i \) transform with \( \Lambda \) and not with \( \overline{\Lambda} \). Thus \( g(S_i) g^*(S_i) \) is not \( R \)-invariant.}
gauge groups, \textit{e.g.} of $SU(2)_L$. To convert (2.23) into the familiar supergravity form, we first rescale the compensating multiplet $S_0$

$$S_0 \rightarrow S_0 g^{-\frac{1}{3}}, \quad (2.24)$$

which reduces the first two terms in (2.22) to

$$-\frac{1}{2} \left[ \bar{S}_0 S_0 \frac{\phi(S_i, \bar{S}^i e^{n_i \xi g R e^{2\bar{g}V}})}{(g^*(S^i) e^{3\xi g R e^{2\bar{g}V} g(S_i)})^\frac{1}{4}} \right] + [S_0^3]_F. \quad (2.25)$$

Using the invariance of the denominator of the first term in (2.24) under gauge transformations (including the $R$-ones), the denominator can be rewritten in the form

$$\left( g^*(S^i) e^{3\xi g R e^{2\bar{g}V} g(S_i)} \right)^\frac{1}{4}, \quad (2.26)$$

provided that $g$ satisfies the property (2.22). Here $\bar{g}$ is the gauge coupling of the non-$R$ gauge groups. In the minimal formulation of supergravity the terms in (2.25) take the form

$$-\frac{3}{2} \left[ \bar{S}_0 S_0 e^{\frac{1}{4} \bar{g}^i} \right]_D + [S_0^3]_F, \quad (2.27)$$

which implies that

$$e^{\frac{1}{4} \bar{g}^i} = \frac{1}{3} \frac{\phi(S_i, \bar{S}^i e^{n_i \xi g R e^{2\bar{g}V}})}{(g(S_i) g^*(S^i) e^{3\xi g R e^{2\bar{g}V} g(S_i)})^\frac{1}{4}}. \quad (2.28)$$

Then all the results of [25] hold except covariant derivatives include $V_R^i$. The function $G(z_i, \bar{z}^i)$ can be expressed in the form

$$G(z_i, \bar{z}^i) = 3 \ln \left( \frac{1}{3} \phi(z_i, \bar{z}^i) \right) - \ln |g(z_i) g^*(z^i)|. \quad (2.29)$$

The non-invariance of the term $\ln |g(z_i) g^*(z^i)|$ under $R$-transformations implies the appearance of the Fayet-Illiopoulos term in the potential. This follows because the D-term of the $R$-multiplet

$$g_R G^i n_i z_i = g_R (3 \frac{\phi^i}{\phi} - \frac{g^i}{g}) n_i z_i. \quad (2.30)$$

has a constant piece as a consequence of the homogeneity of the superpotential $g$

$$n_i z_i g^i = 3 \xi g. \quad (2.31)$$

We will see in Section 4 that this term is very important when considering the scalar potential. It leads to a cosmological constant of order $\kappa^4$ which must be cancelled by an appropriate term. As we will see this fixes the scale of $U(1)_R$ -breaking.

\footnote{See the addendum in [25] for the special case where the superpotential vanishes.}
So far we have started with a superpotential $g$, holomorphic in $z_i$ and a Kähler potential $\phi$. When constructing an $R$-invariant Lagrangian we explicitly included terms coupling $V^R_\mu$ to all gauginos (including the $R$-gaugino) and the gravitino. We then showed how $g$ and $\phi$ can be combined to $G$. For illustration, we reverse this procedure and start with a $N=1$ locally supersymmetric action characterised by a function $G$ of the form (2.29) and where the superpotential $g(z_i)$ is homogeneous of degree 3 $\xi$. To obtain the physical couplings we perform the reverse chiral rotations

$$\psi_{\mu L} \rightarrow \left(\frac{g^*}{g}\right)^{1/4} \psi_{\mu L}, \quad \lambda^\alpha_L \rightarrow \left(\frac{g^*}{g}\right)^{1/4} \lambda^\alpha_L, \quad \chi_{Li} \rightarrow \left(\frac{g}{g^*}\right)^{1/4} \chi_{Li}.$$  

Then the coupling of the vector $V^R_\mu$ to the scalars is of the form

$$D_\mu z_i = \partial_\mu z_i - g_R n_i V^R_\mu z_i.$$  

(2.33)

The coupling to the spinors $\chi_i$ of the chiral superfields is given in terms of $G$ by

$$- \bar{\chi}_{Li} \slashed{D} z_i \chi_R(G^i_j + \frac{1}{2} G^i_k G^k_j) + G^i_j \bar{\chi}_{Li} \slashed{D} \chi_{Rj}.$$  

(2.34)

For normalized kinetic energies $G^i_j = - \frac{1}{2} \delta^i_j$ this gives an effective $V^R_\mu$ coupling to the spinors of the chiral superfields of

$$D_\mu \chi_{iL} = \partial_\mu \chi_{iL} - ig_R (n_i - \frac{3}{2} \xi) V^R_\mu \chi_{iL}.$$  

(2.35)

The gauginos and gravitinos have “naïve” weight zero in the $G$ formulation. This can be seen in particular from the rotations (2.32) which apparently change the weights of the fermionic fields. Thus the covariant derivative contains no gauge field $V^R_\mu$. However, the full coupling is given by

$$- \frac{1}{2} \bar{\lambda} \slashed{D} \lambda - \frac{1}{2} \bar{\lambda}_L \gamma_\mu \lambda_R G^i_j D^\mu z_i = - \frac{1}{2} \bar{\lambda} \gamma^\mu \left(D_\mu \lambda_L - ig_R \xi \frac{3}{2} V^R_\mu \lambda_L\right),$$  

(2.36)

$$e^{-1} e^{\mu\rho\sigma} \left(- \frac{1}{4} \bar{\psi}_\mu \gamma_5 \gamma_\nu D_\rho \psi_\sigma + \frac{1}{8} \bar{\psi}_\mu \gamma_\nu \gamma_\rho \psi_\sigma G^i_j D_\sigma z_i\right) =$$

$$- \frac{1}{4} e^{-1} e^{\mu\rho\sigma} \bar{\psi}_\mu \gamma_5 \gamma_\nu \left(D_\rho \psi_\sigma - ig_R \xi \frac{3}{2} V^R_\mu \psi_{\sigma L}\right).$$  

(2.37)

where we have made use of (2.31). Therefore, as before, the charge of the gauginos and gravitinos is $\frac{3}{2} \xi$. Note that this applies to all gauginos, including the $R$-gaugino itself. Thus there is a $\lambda^R \lambda^R V^R_\mu$ coupling even though $U(1)_R$ is an abelian gauge group. The spinors $\chi_{iL}$ have $R$-charge $(n_i - \frac{3}{2} \xi)$ and their scalar superpartners $z_i$ have charge $n_i$. These numbers coincide with the ones used in the global case for $\xi = \frac{2}{3}$ so that any term in the superpotential $g$ must satisfy $\sum n_i = 2$ and the superpotential can not contain a constant term because of (2.31). Throughout the rest of the paper we fix the convention to

$$\xi = \frac{2}{3}.$$  

(2.38)

\footnote{We neglect here couplings to other gauge fields.}
2.3 Superconformal Approach

To understand what made gauging the $R$-symmetry possible we consider the embedding in the superconformal approach. The superconformal group has the generators

\[(P_m, M_{mn}, K_m, D), \ (Q_\alpha, S_\alpha), \ A.\] \hfill (2.39)

The first set of four generators form the conformal group of translations, rotations, conformal boosts and dilatations. The second set of two are the fermionic generators of supersymmetry and the “superpartner” of $K_m$. The last (bosonic) generator $A$ is a continuous chiral $U(1)$ symmetry. However, there is no corresponding kinetic term and thus no propagating gauge boson.

Superconformal gravity is based on gauging the superconformal group and then adding constraints on the field strengths corresponding to $P_m$, $M_{mn}$, and $S_\alpha$. The constraints are solved for the $M_{mn}$, $K_m$, and $S_\alpha$ gauge fields and the transformations of the gauge fields are modified so that the constraints are preserved. In Eq.(2.23) we presented the superconformal action with an additional $U(1)_R$ gauge group denoted by $W_R$. This action thus contains two extra $U(1)$’s beyond those contained in the $W^\alpha$, namely $U(1)_R$ and the $U(1)_A$ of the superconformal group.

Multiplets transform under the full superconformal group. The compensating multiplet $S_0$ in (2.19) transforms under both the $U(1)_R$ and superconformal transformations. Under the chiral $A$ transformations $S_i$ transforms as

\[
\delta_A z_i = \frac{i}{2} n_i z_i \Lambda_A \tag{2.40}
\]

\[
\delta_A \chi_{iL} = \frac{i}{2} (n_i - \frac{3}{2} \xi) \chi_{iL} \Lambda_A \tag{2.41}
\]

where $z_i$ and $\chi_i$ are the scalar and spinor components of the chiral multiplets $S_i$. Reducing the superconformal to superpoincaré invariance is done by fixing the real and imaginary part of $z_0$, the spinor $\chi_0$ (the components of $S_0$) and $b_\mu$ (the gauge field of dilatation). This, however, will also break the $U(1)_R$ invariance. But a linear combination of $U(1)_R$ and the chiral generator $A$ will survive; the resulting group we again call $U(1)_R$. The transformed $S_0$ in (2.24) is neutral under the new $U(1)_R$ gauge group as can be seen from Eqs.(2.19) and (2.22). Fixing the superconformal gauge on the transformed $S_0$ breaks the superconformal group to the superpoincaré but leaves the $U(1)_R$ invariant.

3 Conditions for the Cancellation of Anomalies

3.1 Family Independent Gauged R-symmetry

We have seen in the last section that it is only possible to construct a gauged $R$-invariant theory in the framework of locally supersymmetric theories. To build a realistic model the
new $U(1)_R$ gauge symmetry should be anomaly-free. To be specific we shall take the $N = 1$ locally supersymmetric theory to have the gauge group

$$G_{SM} \times U(1)_R \equiv SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_R,$$

which is that of the Standard Model extended by $U(1)_R$. The matter chiral multiplets are taken to be the quarks, leptons and a pair of Higgs doublets with the addition of $G_{SM}$ singlets, $N$, and $z_m$. These multiplets are denoted by

$$L : (1, 2, -\frac{1}{2}, l), \quad \bar{E} : (1, 1, 1, e), \quad Q : (3, 2, \frac{1}{6}, q),$$

$$U : (3, 1, -\frac{2}{3}, u), \quad \bar{D} : (3, 1, \frac{1}{3}, d), \quad H : (1, 2, -\frac{1}{2}, h),$$

$$\bar{H} : (1, 2, \frac{1}{2}, \bar{h}), \quad N : (1, 1, 0, n), \quad z_m : (1, 1, 0, z_m),$$

where we have indicated in parentheses the $G_{SM}$, and $U(1)_R$ quantum numbers, respectively.

The $U(1)_R$ quantum numbers are for the chiral fermions. The bosons will have numbers shifted by one unit, e.g. for the slepton doublet it is $l + 1$ (cf. Eq.(2.3)).

We shall assume that the superpotential in the observable sector has the form

$$g^{(O)} = h_E^{ij} L_i \bar{E}_j H + h_D^{ij} Q_i \bar{D}_j H + h_U^{ij} Q_i \bar{U}_j H + h_N N H \bar{H},$$

where $h_E$, $h_D$, $h_U$ are the generation mixing Yukawa couplings and $h_N$ is an additional Yukawa coupling. So at this stage we assume the theory conserves $R$-parity. We have added the term $N H \bar{H}$ instead of $\mu H \bar{H}$ as in the MSSM, in order to incorporate a possible solution to the mu-problem via a vacuum expectation value $<N>$. The singlets $z_m$ only couple in the hidden sector. The only requirement that comes from $R$-invariance on the form of $g^{(O)}$ is that it should transform with a global phase under the $R$-transformations as in Eq.(2.22). This implies that

$$l + e + h = -1,$$

$$q + d + h = -1,$$

$$q + u + \bar{h} = -1,$$

$$n + h + \bar{h} = -1.$$ 

We have employed our convention (2.38). The $-1$ corresponds to $\sum n_i = 2$ since we are now considering the fermionic charges, which are shifted by 1, i.e. $\sum (n^f_i + 1) = 2$ as seen from Eq.(2.3). At this stage we have also assumed that the $R$-charges are family independent, e.g. $l_1 = l_2 = l_3 = l$.

Since the $U(1)_R$ gauge boson is a propagating gauge boson, we must consider the relevant anomaly conditions. These severely constrain the $R$-numbers appearing in Eq.(3.2). We shall require the $U(1)^3_R$ anomaly, and the mixed $U(1)_R - U(1)_Y$, $U(1)_R - SU(2)_L$, and $U(1)_R - SU(3)_C$ anomalies to vanish. The hypercharge anomalies are satisfied by our
choice of $U(1)_Y$ charges. The equations for the absence of the $U(1)_Y - U(1)_R$ anomalies give

$$C_1 \equiv Tr Y^2 R \equiv 0,$$
$$Tr Y R^2 = 0,$$
$$Tr R^3 = 0.$$  \tag{3.8, 3.9, 3.10}

These can be rewritten in terms of the $R$-quantum numbers as

$$3\left[ \frac{1}{2} l + e + \frac{1}{6} q + \frac{4}{3} u + \frac{1}{3} d \right] + \frac{1}{2} (h + \bar{h}) = 0,$$  \tag{3.11}
$$3\left[ -l^2 + e^2 + q^2 - 2u^2 + d^2 \right] - h^2 + \bar{h}^2 = 0,$$  \tag{3.12}
$$3\left[ 2l^3 + e^3 + 6q^3 + 3u^3 + 3d^3 \right] + 2h^3 + 2\bar{h}^3 + 16 + n^3 + \sum z_m^3 = 0.$$  \tag{3.13}

In the last equation the term $16 = 13 + 3$ is due to the 13 gauginos present in our model ($SU(3)_C : 8, SU(2)_L : 3, U(1)_Y : 1$, and $U(1)_R : 1$) as well as the gravitino. The gravitino contribution is three times that of a gaugino \cite{27}. As seen in Eqs. (2.36, 2.37), they all have $R$-charge 1. The absence of the mixed $U(1)_R - SU(2)_L$ anomalies implies the condition

$$C_2 \equiv Tr_{\{SU(2)\}} R = 0,$$  \tag{3.14}

where the trace is limited to the non-trivial $SU(2)$ multiplets. This is evaluated as

$$3\left[ \frac{1}{2} l + \frac{3}{2} q \right] + \frac{1}{2} (h + \bar{h}) + 2 = 0.$$  \tag{3.15}

The constant 2 is due to the $SU(2)$ gauginos. For an arbitrary group $SU(N)$ the trace over the product of the adjoint representation generators is just $N$. Similarly the absence of the mixed $U(1)_R - SU(3)_C$ anomalies implies

$$C_3 \equiv Tr_{\{SU(3)\}} R = 0,$$  \tag{3.16}

where now the trace is limited to non-trivial $SU(3)_C$ multiplets,

$$3\left[ q + \frac{1}{2} u + \frac{1}{2} d \right] + 3 = 0.$$  \tag{3.17}

The cancellation of the mixed gravitational anomaly \cite{26} requires

$$Tr R = 0,$$  \tag{3.18}

where the trace is taken over all states because of the universality of the gravitational coupling. This implies

$$3\left[ 2l + e + 6q + 3u + 3d \right] + 2(h + \bar{h}) - 8 + n + \sum z_m = 0.$$  \tag{3.19}
The term $-8 = 13 - 21$ is due to the 13 gauginos as well as the gravitino. In the gravitational anomaly the gravitino contribution is $-21$ times the gaugino contribution [27]. To solve the set of ten equations (3.4-3.7), (3.11-3.13), (3.15-3.17), and (3.19) we note that the seven Eqs. (3.4)-(3.6), (3.11), (3.12), (3.15), and (3.17) form a decoupled system with the seven unknowns $l, e, q, u, d, h$, and $\bar{h}$. It is straightforward to show that these equations are incompatible and do not have a solution. This is independent of whether we replace the $NH\bar{H}$ term by $\mu H\bar{H}$ in the superpotential. Therefore we conclude that when the $R$-numbers of the fields are family independent the $U(1)_R$ extension of the supersymmetric standard model is anomalous.

There are several ways around this problem of which we shall in turn discuss three. First, we shall consider whether the anomaly can be cancelled by the Green Schwarz mechanism [28]. Second, we shall consider adding additional fields which transform non-trivially under $G_{SM}$, and third we shall consider a family-dependent $U(1)_R$.

### 3.2 Green-Schwarz Anomaly Cancellation

The Green-Schwarz mechanism of anomaly cancellation relies on coupling the system to a linear multiplet $(B_{\mu\nu}, \phi, \chi)$ where $B_{\mu\nu}$ is an antisymmetric tensor. The field strength of $B_{\mu\nu}$ is given by

$$H = dB,$$

and

$$B = B_{\mu\nu} dx^\mu \wedge dx^\nu,$$

is a two-form. In order to cancel the mixed gauge anomalies the action for $B_{\mu\nu}$ is not given by $H^2$, which is gauge and Lorentz invariant, but instead by $\hat{H}^2$. $\hat{H}$ is classically gauge and Lorentz non-invariant and is given by

$$\hat{H} = H - \omega^{YM} - \omega^L.$$  

Here

$$\omega^{YM} = Tr(AdA + \frac{2}{3} A^3),$$

$$\omega^L = tr(\omega d\omega + \frac{2}{3} \omega^3),$$

$$A = g_a A_\mu^a dx^\mu.$$  

$Tr$ is a trace over the gauge group and $tr$ is a trace over the Lorentz Clifford algebra. $A$ is a one-form and $T^a$ are the generators of $SU(3)_c$, $SU(2)_L$, $U(1)_Y$, and $U(1)_R$, while

$$\omega = \frac{1}{2} \omega^{ab}_\mu \sigma_{ab} dx^\mu,$$

---

\[6\] We thank D. Castano, D. Freedman, and C. Manuel for pointing out to us the difference in the anomaly contribution of a gaugino and the gravitino in Eqs. (3.13,3.19). This was treated incorrectly in an earlier version of this paper.
is the spin connection one-form.

The non-invariant part of the gauge transformations of $\hat{H}^2$ are of exactly the same form as the mixed gauge anomalies $C_1, C_2,$ and $C_3$ of the previous subsection. The combined action is gauge invariant, i.e. the transformation of $\hat{H}^2$ cancels the mixed gauge anomalies, provided

$$\frac{C_1}{k_1} = \frac{C_2}{k_2} = \frac{C_3}{k_3}. \quad (3.27)$$

Here the $k_i$ are real constants which take into account the different normalization of the gauge group generators. In string theories the $k_i$ are the Kać-Moody levels of the gauge algebra. In almost all string models we have $k_2 = k_3$. Most string models have been constructed at level $k = 1$ for non-abelian groups. $k_1$ is not necessarily integer.

For $k_2 = k_3$ (not necessarily $= 1$) the anomaly cancellation conditions are compatible if and only if

$$C_2 = C_1 + 6. \quad (3.28)$$

As in reference [29], we can simplify the equations by assuming that

$$\frac{C_2}{C_1} = \frac{3}{5}, \quad (3.29)$$

which corresponds to the choice $\sin^2 \theta_w = \frac{3}{8}$ at the unification scale. In this case

$$C_1 = -15, \quad (3.30)$$
$$C_2 = C_3 = -9. \quad (3.31)$$

Then the anomaly cancellation equations can all be expressed in terms of one variable $l' = \frac{30}{7} \cdot l$ beyond the quantum numbers of the singlet fields $z_m$. The remaining equations are

$$-80 + \frac{3}{2} l' + \sum z_m = 0, \quad (3.32)$$
$$-\frac{8004}{9} - 24l' + \frac{19}{5} l'^2 + \frac{3}{8} l'^3 + \sum z_m^3 = 0, \quad (3.33)$$

where we have added the contribution of the linear multiplet ($-1$) to both equations.

These equations have no rational solution for zero or one singlet. We have performed a numerical scan for three singlets with charges $m/6$, and $m$ an integer between $-200$ and $200$ and found no solution. It is not clear whether the situation would improve if we allow for different but realistic values for $C_2/C_1$ as the equations become very complicated. String models have also been constructed for level-two Kać-Moody algebras. We have considered the cases $k_2 = 2k_3$ and $k_3 = 2k_2$. The equations are of similar form to those above. There are no rational solutions for one or two singlets.

We conclude that it is not possible to cancel the anomaly via the Green-Schwarz mechanism with a small number of singlets.
3.3 Non-Singlet Field Extensions

We want to briefly investigate what possible extensions of the field content could lead to an anomaly-free family independent $R$-symmetry. First, we consider fields which transform under the electroweak gauge group. In order to maintain the anomaly cancellation in the Standard Model we allow for extra generations ($N_g$ is the number of generations) and pairs of Higgs doublets $N_h$. The seven decoupled anomaly equations lead to the equation

$$N_g = \frac{3N_h}{N_h + 3}. \quad (3.34)$$

This has no positive integer solutions. We thus consider the case of an extra $SU(3)$-octet chiral superfield $O_c$ with $G_{SM} \times U(1)_R$ quantum numbers $(8, 1, 0, o_c)$. Octet extensions have also been considered, for example in [30, 3]. The anomaly equations for (3.17) and (3.19) change, they are now

$$3[q + \frac{1}{2}u + \frac{1}{2}d] + 3 + 3o_c = 0, \quad (3.35)$$
$$3[2l^3 + c^3 + 6q^3 + 3u^3 + 3d^3] + 2h^3 + 2\bar{h}^3 + 16 + n^3 + \sum z_m^3 + 8o_c^3 = 0, \quad (3.36)$$
$$3[2l + c + 6q + 3u + 3d] + 2(h + \bar{h}) - 8 + n + \sum z_m + 8o_c = 0. \quad (3.37)$$

The seven independent equations are now in eight variables and have a solution in terms of two variables which we choose to be $l$ and $e$,

$$h = -(l + e + 1), \quad \bar{h} = l + e - 1, \quad q = -\frac{2}{9} - \frac{1}{3}l, \quad (3.38)$$
$$d = \frac{2}{9} + \frac{4}{3}l + e, \quad u = \frac{2}{3} - \frac{2}{3}l - e, \quad n = 1, \quad o_c = -1.$$

The remaining equations involving the singlets are then given by

$$3(2l + e) - 19 + \sum z_i = 0, \quad (3.39)$$
$$3(2l + e)^3 + 13 + \sum z_i^3 = 0. \quad (3.40)$$

For zero or one singlet this has no rational solution. We performed a numerical scan for singlet charges between -20 and 20 in steps of 1/6. We found no solution for two or three singlets. We found many (sixty-six) solutions with four singlets. Here we present three solutions written in terms of the quantum numbers $(2l + e, z_1, z_2, z_3, z_4)$:

$$(1, -\frac{47}{3}, \frac{25}{3}, 3, 13), \quad (3.41)$$
$$\left(\frac{11}{3}, -11, \frac{7}{3}, \frac{25}{3}, \frac{25}{3}\right), \quad (3.42)$$
$$\left(\frac{1}{3}, -20, \frac{20}{3}, \frac{47}{3}, \frac{47}{3}\right). \quad (3.43)$$

Note in Eq.(3.38) that the fermionic component of the octet chiral superfield has $R$-charge $-1$ and thus the spin zero component has $R$-charge 0. Therefore when supersymmetry is
broken the soft-supersymmetry breaking terms in the scalar potential involving only the spin zero octet are unconstrained. Such a potential typically breaks $SU(3)_c$. We do not consider these solutions any further.\footnote{After we submitted this paper \cite{10} also considered coloured triplets $D = (3, 1, -\frac{1}{3})$, $\bar{D} = (\bar{3}, 1, \frac{1}{3})$ and obtain anomaly-free solutions.}

### 3.4 Family Dependent Gauged $U(1)_R$ Symmetry

The Standard Model has three generations which are only distinguished by their mass. Clearly this structure requires an explanation. One possibility is that the difference in the families are explained by a horizontal symmetry at very high energies. Thus in general we expect at high energies the electron to have different gauge quantum numbers from the muon or the tau and similarly for the quarks. Only at low energies are the gauge quantum numbers in the effective theory family independent. We shall see in Section 4 that $R$-symmetries are broken close to the Planck scale. In accordance with this philosophy we thus expect the $R$-symmetry to be family-dependent as well. In this section we investigate the conditions for an anomaly-free family-dependent $R$-symmetry. We shall denote the $R$-quantum number of the matter fields by $e_i, l_i, q_i, u_i, d_i$, $i = 1, 2, 3$. Motivated by the successes of the work on symmetric mass matrices \cite{31} we shall assume a left-right symmetry for the matter fields

$$Q_R(\chi_{iR}^a) = Q_R(\chi_{iL}^a). \quad (3.44)$$

Here $a$ is a flavour index. In particular we have

$$e_i = l_i, \quad u_i = d_i = q_i, \quad i = 1, 2, 3. \quad (3.45)$$

Also motivated by the structure of the quark and lepton masses we shall assume that only the fields of the third generation enter the superpotential. The superpotential for the observable sector is then given by

$$g^{(O)} = h_{3E}^3 L_3 E_3 H + h_{3D}^3 Q_3 D_3 H + h_{3U}^3 Q_3 U_3 H + h_N N H H. \quad (3.46)$$

The masses for the first and second generation will be generated after the breaking of some symmetry, possibly the $R$-symmetry. We shall here not further consider the problem of fermion mass.

The anomaly cancellation equations will keep the same form as in Eqs. (3.4)-(3.19) but with the factor of 3 outside the quark’s and lepton’s contributions replaced by $\sum_{i=1}^{3}$ and Eqs. (3.4)-(3.6) hold only for the third generation. Making use of our assumption (3.44, 3.45) these equations reduce respectively to

$$\frac{3}{2}(l_1 + l_2 + l_3) + \frac{11}{6}(q_1 + q_2 + q_3) + \frac{1}{2}(h + \bar{h}) = 0, \quad (3.47)$$
\[ h^2 - h^2 = 0, \quad (3.48) \]
\[ 3(l_1^3 + l_2^3 + l_3^3) + 12(q_1^3 + q_2^3 + q_3^3) + 2h^3 + 2\bar{h}^3 - 8 + n^3 + \sum z_m^3 = 0, \quad (3.49) \]
\[ \frac{1}{2}(l_1 + l_2 + l_3) + \frac{3}{2}(q_1 + q_2 + q_3) + \frac{1}{2}(h + \bar{h}) + 2 = 0, \quad (3.50) \]
\[ 2(q_1 + q_2 + q_3) + 3 = 0, \quad (3.51) \]
\[ 3(l_1 + l_2 + l_3) + 12(q_1 + q_2 + q_3) + 2(h + \bar{h}) + 16 + n + \sum z_m = 0. \quad (3.52) \]

The requirement that the superpotential has homogeneous weight two gives

\[ 2l_3 + h = -1, \quad (3.53) \]
\[ n + h + \bar{h} = -1, \quad (3.54) \]
\[ 2q_3 + h = -1, \quad (3.55) \]
\[ 2q_3 + \bar{h} = -1. \quad (3.56) \]

Combining all these equations we get

\[ h = \bar{h} = -1, \quad q_3 = l_3 = 0, \quad (3.57) \]

The only remaining equations to solve are (3.52) and (3.49) which simplify to

\[ \frac{45}{2}l_1(l_1 - \frac{5}{2}) - 54q_1(q_1 + \frac{3}{2}) + \frac{155}{8} + \sum z_m^3 = 0, \quad (3.58) \]

\[ \sum z_m = \frac{43}{2}. \quad (3.59) \]

We see that at least one singlet must be added. For one extra singlet we find two independent solutions:

\[ (q_1, q_2, l_1, l_2) = \left(-\frac{76}{3}, \frac{143}{6}, -\frac{61}{2}, 33\right), \quad (3.60) \]
\[ (q_1, q_2, l_1, l_2) = \left(-\frac{46}{3}, \frac{83}{6}, -7, \frac{19}{2}\right). \quad (3.61) \]

Both solutions have \( z = \frac{43}{2} \). In the next section we shall discuss the breaking of supersymmetry and of the gauged R-symmetry. We will see that this solution is unsatisfactory in many respects. The charge of the singlet \( z \) is positive which leads to an unacceptable cosmological constant. We also see that some of the fermionic charges of the observable fields \( (q_i, l_i, h, \bar{h}) \) are less than \(-1\); the bosonic charges are then negative. The potential then requires fine-tuning in order to guarantee weak-scale sfermion masses.

For two additional singlets we find many solutions. The two solutions with the smallest \(|q_1|\) values are

\[ (q_1, q_2, l_1, l_2, z_1, z_2) = \left(-\frac{61}{3}, \frac{113}{6}, -1, \frac{7}{2}, -6, -\frac{55}{2}\right), \quad (3.62) \]
\[ (q_1, q_2, l_1, l_2, z_1, z_2) = \left(\frac{61}{3}, -\frac{131}{6}, -6, \frac{17}{2}, -7, \frac{57}{2}\right). \quad (3.63) \]
These solutions have negative singlet charges which makes it possible to cancel the cosmological constant. However, \( q_1 \) or \( q_2 < -1 \). We scanned the three singlet case for appropriate solutions and found one. The fermionic charges are given by

\[
\{(q_1, q_2, q_3); (l_1, l_2, l_3); (z_1, z_2, z_3)\} = \left\{\left(-1, \frac{-1}{2}, 0\right); \left(\frac{1}{2}, 2, 0\right); \left(-\frac{115}{3}, \frac{26}{3}, \frac{203}{6}\right)\right\}. \tag{3.64}
\]

There are three further physically distinct solutions obtained by the interchanges \( q_1 \leftrightarrow q_2 \) and \( l_1 \leftrightarrow l_2 \). We study this solution in more detail in the next two sections. However, this solution has very large singlet charges and has a gauge invariant hidden-sector superpotential with very large powers of the singlet fields. We thus also studied the four-singlet solutions.

For four singlets we find very many solutions. The solutions with observable field fermionic charges greater than \(-1\) can be classified in two sets of twelve and ten classes

\[
q_1 = -1, \quad l_1 = \frac{n}{6}, \quad n = -6, ..., 6, \; n \neq 0 \tag{3.65}
\]

\[
q_1 = -\frac{5}{6}, \quad l_1 = \frac{n}{6}, \quad n = -6, ..., 6, \; n \neq -4, 0, 4 \tag{3.66}
\]

Physically distinct solutions are again obtained by the interchange \( q_1 \leftrightarrow q_2 \) and \( l_1 \leftrightarrow l_2 \). In (3.65,3.66) we have disregarded the solutions where \( l_1 = 0 \). These lead to a further term \( L_1 \bar{H} E_1 \) in the superpotential in contradiction to our assumption of dominant third generation Higgs Yukawa couplings. As we will see in Section 5 the solutions with \( q_1 = -1 \) lead to the simultaneous presence of \( L_i Q_j \bar{D}_k \) and \( \bar{U}_l \bar{D}_m \bar{D}_n \) in the superpotential. In most cases this leads to an unacceptable level of proton decay. The exceptions are discussed in [32]. We thus focus on the solutions (3.66). In Table 1 we present the complete charges of one representative of each of the ten classes.

In the next section we shall discuss the breaking of supersymmetry and R-symmetry for the three singlet solution of Eq.(3.64) and the four-singlet solutions of Table 1.

## 4 Supersymmetry and R-symmetry Breaking

To have a realistic model both supersymmetry and \( R \)-symmetry must be broken at low energies. Since we have a locally supersymmetric theory, it is possible to break supersymmetry spontaneously. The easiest way is to utilize a hidden sector whose fields are singlets with respect to the Standard Model gauge group. Depending on whether the \( R \)-symmetry and supersymmetry are to be broken simultaneously or not, (the bosonic component of) these singlets would have or not have non-trivial \( R \)-numbers. In the case of a gauged \( R \)-symmetry we have shown that anomaly free models are not possible for leptons and quarks with family independent \( R \)-numbers. When we allow for family dependent \( R \)-numbers for the leptons and quarks while maintaining a left-right symmetry, we obtain many solutions, including (3.64,3.66).
The $R$-number of the superpotential is 2, and a Fayet-Iliopoulos term is necessarily present in the D-term of the scalar potential. The $g_R$ part of this is

$$g_R^2 \left( \frac{1}{3} \right)^2 \left( n_i z^i z_i + \frac{4}{\kappa^2} \right)^2,$$

and we have a cosmological constant of the order of the Planck scale. In a realistic model we must avoid giving the squarks and sleptons superheavy masses, otherwise supersymmetry would be irrelevant at low-energies. Thus to lowest order the condition

$$\langle n_i z^i z_i \rangle + \frac{4}{\kappa^2} = 0,$$

must be satisfied. As shown in [40], the first order corrections will of order $(\kappa m_s^2)^2$, where $m_s$ is the supersymmetry scale of order $10^2 - 10^3 GeV$. From Eq. (4.1) it should be clear that at least one chiral superfield must have negative (bosonic) $R$-charge. In a realistic model only the singlets should get a vev at the Planck scale. We thus have the necessary requirement of a negatively charged singlet (fermionic charge $< -1$). This lead us in the previous section to reject the one-singlet solution. In the general minimization of the potential we also expect negatively charged observable chiral superfields to get a vev. In the previous section we thus imposed the additional constraint that the observable fields have semi-positive bosonic charges. This lead us to the three- and four-singlet solutions.

The bosonic components of the only three-singlet solution are given by

$$(z_1, z_2, z_3) = \left( -\frac{112}{3}, 27, \frac{209}{6} \right), \quad q_1 = 0, \quad t_1 = \frac{3}{2}.$$  

(4.3)

We have chosen $z_1, z_2, z_3$ such that $z_1 < z_2 < z_3$. For a realistic model we must have $\langle z_1 \rangle \approx O(\frac{1}{\kappa})$. The most general polynomial with $R$-charge 2 for such three singlets is given by

$$g'(z_1, z_2, z_3) = \frac{1}{\kappa^3} \left( a_1 (\kappa z_1)^{10} (\kappa z_2)^{10} + a_2 (\kappa z_1)^{14} (\kappa z_2)^{16} + a_3 (\kappa z_1)^{33} (\kappa z_2)^{7} (\kappa z_3)^{30} + a_4 (\kappa z_1)^{41} (\kappa z_3)^{41} + ... \right).$$  

(4.4)

(4.5)

We have only introduced the Planck scale. We take the arbitrary parameters $a_k = O(1)$.

We can not break supersymmetry via the Polonyi mechanism since a constant is not $R$-invariant. Instead we find the above superpotential sufficient. When we take at least three non-zero parameters $a_k$ in $g'$ then it is possible to find solutions for which the total potential $V$ is positive semi-definite with the value zero at the minimum, and where the kinetic energy is minimal and of the form

$$y = \frac{\kappa^2}{2} z_i z^i + ...$$

(4.6)
D-term is also zero at the minimum. For this we must of course fine-tune the parameters $a_k$.

In this case the $R$-gauge vector boson mass is of the order of the Planck mass. The total superpotential is then taken to be of the form

$$G = g'(z_1, z_2, z_3) + g^{(O)}(S_i), \quad \text{(4.6)}$$

where $g^{(O)}$ is the observable sector superpotential which only depends on the Standard Model superfields $S_i$ and is given by (3.3).

The most general potential in a locally supersymmetric theory with chiral multiplets $S_a$ is

$$V = \frac{1}{\kappa^3} e^G \left( G^{-1}_{a} b G_{a} b - 3 \right) + \frac{1}{2} g^2 R e f^{-1}_{\alpha\beta} \left( G_{i}^a (T^\alpha z)_a \right) \left( G_{i}^b (T^\beta z)_b \right). \quad \text{(4.7)}$$

For the three-singlet model we thus obtain the dependence of the $R$-symmetry D-term on $z_1, z_2, z_3$ as

$$g_R^2 \frac{1}{8} \left( \frac{2}{3} \right)^2 \left( -\frac{112}{3} |z_1|^2 + 27 |z_2|^2 + \frac{209}{6} |z_3|^2 + \frac{4}{\kappa^2} \right)^2 \quad \text{(4.8)}$$

From the form of $g'$ it is clear that there is no symmetry in $z_1, z_2, z_3$ and their vevs will be unequal. For the D-term to vanish at the minimum we must have $|z_2| < \frac{112}{81}$, and $|z_3| < \frac{224}{209} |z_1|$. By fine-tuning the parameters $a_k$ it might be possible to arrange for $|z_2| \approx z_3 \approx \frac{1}{2} |z_1|$ so that $|z_1| \approx \frac{1}{\sqrt{3} \kappa}$. Then if we start with the natural Planck scale $\frac{1}{\kappa}$, the effective value of $g'$ will be $\frac{m_s}{\kappa^2}$, where $m_s = \frac{1}{\kappa} \left( \frac{1}{2} \right)^{21} \left( \frac{1}{2} \right)^{11}$ is of order $O(10^2 GeV)$.

To be honest we must stress that studying such potentials is a very difficult task and needs a careful analysis. We shall assume that $z_1, z_2, z_3 \approx O(\frac{1}{\kappa})$ with coefficients less than one, so that when these fields are integrated out one gets $\langle \kappa^2 g' \rangle = m_s$.

By integrating the hidden sector fields $z_1, z_2, z_3$ one obtains the effective potential as a function of the light fields $z_i$. The general form of the effective potential is $[35]$.

$$V_{\text{eff}} = |\hat{g}_{ji}|^2 + m_s^2 |z_i|^2 + m_s (z_i \hat{g}_{ji} + (A - 3) \hat{g} + h.c.) + \frac{1}{8} g^2 (\bar{z}^i (T^\alpha z)_i)^2, \quad \text{(4.9)}$$

where $\hat{g}$ is related to $g^{(0)}$ through a multiplicative factor depending on the details of the hidden sector. Similarly for $A$ and $m_s$ which is given by $m_s = \langle \kappa^2 g' \rangle$. In the observable sector the singlet $N$ has $R$-number 2, one can show that $N$ can acquire a $< N > = O(m_s)$ because it also contributes to the D-term. This can be seen explicitly by minimizing the effective potential

$$V = |\hat{g}_{ji}|^2 + m_s^2 |z_i|^2 + m_s (z_i \hat{g}_{ji} + (A - 3) \hat{g} + h.c.)$$

$$+ \frac{1}{8} g^2 (H^* \sigma^a H + \bar{H}^* \sigma^a \bar{H})^2 + \frac{1}{8} g^2 \left( H^* H - \bar{H}^* \bar{H} \right)^2$$

$$+ \frac{1}{18} g_R^2 \left| 2 |N|^2 + \sum_{j=1}^{3} q_j (|Q_j|^2 + |U_j|^2 + |\bar{D}_j|^2) + l_j (|L_j|^2 + |\bar{E}_j|^2) \right|^2. \quad \text{(4.10)}$$
The above potential has $R$-breaking terms present in $\hat{g}$ and $z^i \hat{g}_i$. Together with the terms in $m_s^2 |z_i|^2$ they break supersymmetry softly. We can use the tree-level effective action plus the renormalization group equations to find the radiative corrections and the $R$-breaking effects present.

The three singlet solution is problematic with the $\bar{U} \bar{D} \bar{D}$ couplings as will be clear in the next section. Therefore we must consider the four singlet solutions which we required to avoid such a problem. The superpotentials for the ten different classes are given in Table 2.

As before we have to tune the parameters $a_k$ so that the potential is positive definite and so that $|z_1|, ..., |z_4| \approx \mathcal{O}(\frac{1}{\kappa})$ with coefficients less than one so as to induce a scale such that $<\kappa^2 g'> = m_s = \mathcal{O}(10^2 \text{GeV})$. The effective potential takes the same form as in the three singlet case, but with different $R$-numbers for the squarks and sleptons.

It is possible to add direct gaugino masses because the action contains the term

$$e^G \bar{G}_i \bar{G}_i^{-1} k f_{\alpha \beta, k} \bar{\lambda}^\alpha_R \lambda^\beta_R$$

which for the canonical choice of the kinetic energy becomes

$$\frac{1}{4} e^{\frac{\kappa^2}{2} z^i z^i} (g^i + \frac{\kappa^2}{2} z^i \hat{g}) f_{\alpha \beta, i} \bar{\lambda}^\alpha_R \lambda^\beta_R$$

Thus if we choose

$$f_{\alpha \beta} = \delta_{\alpha \beta} f(z_i)$$

where, e.g. we can take

$$f(z_i) = g'(z_i)$$

which will induce direct gaugino masses of order $<\kappa^2 g'> = \mathcal{O}(m_s)$ at the tree level. It is clear that gaugino masses will also be induced by radiative corrections [10, 11].

5 Applications to $R$-parity Violation

When extending the Standard Model to supersymmetry new dimension four Yukawa couplings are allowed which violate baryon- and lepton-number. When determining our solutions to the anomaly equations we have explicitly assumed that the superpotential conserves $R$-parity and that all these terms were forbidden. However, this was merely a working assumption, since we are mainly interested in an anomaly-free supersymmetric model with a gauged $R$-symmetry and the superpotentials (3.3) (family-independent) or (3.46) (family-dependent) posed the minimal number of constraints. Whether $R_p$ is conserved or not should only depend on gauge symmetries at the high energy scale. Therefore, we now investigate which $R_p$ violating terms are allowed in the anomaly-free models (3.64,3.66).

In order to determine the allowed superpotential terms we must consider the charge combinations of the leptons and the quarks. We shall denote by $\vec{l} = (l_1, l_2, l_3)$, and $\vec{q} =$
the set of family dependent fermionic lepton and quark charges. For the three singlet model they are given in Eq.\[3.64\]. For the four singlet case we had twenty models with $\vec{q} = (-1, -\frac{1}{2}, 0)$ and ten models with $\vec{q} = (-\frac{5}{6}, -\frac{2}{3}, 0)$. The corresponding leptonic charges are given in Table 3. In all three- and four-singlet models $h = \bar{h} = -1$.

The possible dimension four terms are

$$L_i L_j \bar{E}_k, \quad L_i Q_j \bar{D}_k, \quad U_i \bar{D}_j \bar{D}_k, \quad \bar{\mu} L_i \bar{H},$$

where $\bar{\mu}$ is a dimensionful parameter. The indices $i, j, k$ are generation indices and we have suppressed the gauge group indices. In the first term we must have $i \neq j$ due to an anti-symmetry in the $SU(2)_L$ indices. Similarly, in the third term we must have $j \neq k$ due to the $SU(3)_c$ structure. We have included the last term because the symmetry $U(1)_R$ distinguishes between the leptonic superfields $L_i$ and the Higgs $H$ and thus cannot be rotated away. In our notation and with the left-right symmetry the $U(1)_R$ charges of the above terms are given by

$$Q_R(L_i L_j \bar{E}_k) = l_i + l_j + l_k \equiv -1,$$

$$Q_R(L_i Q_j \bar{D}_k) = l_i + q_j + q_k \equiv -1,$$

$$Q_R(U_i \bar{D}_j \bar{D}_k) = q_i + q_j + q_k \equiv -1,$$

$$Q_R(L_i \bar{H}) = l_i + \bar{h} \equiv 0.$$

The last equality in each line is the requirement on the fermionic charges for $U(1)_R$ gauge invariance. The $L_i \bar{H}$ term is different just because we are considering the fermionic charges. The superfield charges must add to +2 for all terms.

For the three singlet solution we obtain the following $G_{SM} \times U(1)_R$ additional dimension-four terms

$$LL\bar{E} : \text{ none} \quad (5.6)$$

$$LQ\bar{D} : \quad L_1 Q_1 \bar{D}_2, \ L_1 Q_2 \bar{D}_1; \ L_3 Q_1 \bar{D}_3, \ L_3 Q_2 \bar{D}_2, \quad (5.7)$$

$$U \bar{D} \bar{D} : \quad \bar{U}_3 \bar{D}_1 \bar{D}_3, \ \bar{U}_2 \bar{D}_2 \bar{D}_3, \quad (5.8)$$

$LQ\bar{D}$ and $U \bar{D} \bar{D}$ terms together can lead to a dangerous level of proton decay. Recently Carlson, Roy and Sher [32] studied the proton decay rates from all possible combinations. They found that some of the above combinations are more weakly bound than expected. But for example the product of Yukawa couplings for the operators $\bar{U}_2 \bar{D}_2 \bar{D}_3$ and $LQ\bar{D}$ is restricted to be smaller than $10^{-9}$. We thus exclude the three singlet solution. Similarly we also exclude the four singlet solutions with $q_1 = -1$. This is the reason why in the previous section we restricted ourselves to the case $q_1 = -\frac{5}{6}$.

For the ten models of Table 1 we find the following sets of gauge invariant $R$-parity violating dimension-four terms

$$I : \quad L_1 L_3 \bar{E}_3, \ L_1 Q_3 \bar{D}_3$$

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When determining solutions to the anomaly equations we had an additional set of solutions under the interchange $l_1 \leftrightarrow l_2$ and $q_1 \leftrightarrow q_2$. We can thus obtain a further set of allowed $R$-parity violating models

$$I': L_2 L_3 \bar{E}_3, L_2 Q_3 \bar{D}_3$$
$$III': L_2 L_3 \bar{E}_2$$
$$IV': L_2 Q_2 \bar{D}_3, L_2 Q_3 \bar{D}_2$$
$$IV'': L_2 Q_1 \bar{D}_3, L_2 Q_3 \bar{D}_1$$
$$IV'''': L_1 Q_1 \bar{D}_3, L_1 Q_3 \bar{D}_1$$
$$V': L_1 Q_2 \bar{D}_3, L_1 Q_3 \bar{D}_2$$
$$V''': L_2 Q_1 \bar{D}_3, L_2 Q_3 \bar{D}_1$$
$$V'''': L_2 Q_2 \bar{D}_3, L_2 Q_3 \bar{D}_2$$
$$V': L_1 Q_1 \bar{D}_1$$
$$V'': L_2 Q_1 \bar{D}_1$$
$$V'''': L_2 Q_2 \bar{D}_2$$
$$V'''': L_2 Q_1 \bar{D}_2, L_2 Q_2 \bar{D}_1$$
$$X': L_2 \bar{H}$$

We find the interesting point that we have models with only $LL\bar{E}$ type couplings, others with only $L_i \bar{H}$ or $LQ\bar{D}$ couplings. We also have three sets $II, VI, IX$ where $R$-parity is conserved. Thus there is no logical connection between a conserved $R$-symmetry and the status of $R$-parity. They are independent concepts.

The $L_{1,2}\bar{H}$ term has a dimensionful coupling $\tilde{\mu}$ similar to the $\mu$ term of the MSSM. Its natural value in our local supersymmetric models is $\kappa^{-1}$. At low energies, we can rotate away this term and thus generate $LL\bar{E}, LQ\bar{D}$ interactions which are strongly constrained experimentally. These bounds translate into $\tilde{\mu} < \mathcal{O}(m_s)$. In order to avoid a further hierarchy problem we require the absence of $L_i \bar{H}$ terms and therefore exclude the models $X, X'$.

Interestingly enough, most of the models ($I, I', IV, ..., IV'''', V, ..., V'''', VII, ..., VII'''', VIII, VIII'$) predict sizeable $L_{1,2}Q_i \bar{D}_j$ interactions. The first set leads to resonant squark production at HERA which has been investigated in detail in [38]. This should be observable.
with an integrated luminosity of about 100 $pb^{-1}$ for squark masses below 275 $GeV$. The second set also lead to observable signals at HERA even for very small couplings as discussed in [39].

We point out that only in model I we have additional terms $L_1HN$. These conserve $R$-parity provided $N$ is interpreted as a right-handed neutrino. $L_1HN$ is a Dirac neutrino mass and requires a very small Yukawa coupling. We thus exclude model I.

It is interesting to note that even though for the Higgs Yukawa couplings the third generation is dominant this is not necessarily the case for the $R_p$ violating interactions.

6 Conclusion

The purpose of this paper has been to take a first step towards model building with a gauged $R$-symmetry. We have discussed in detail that an $R$-symmetry can only be gauged in local supersymmetry since it does not commute with the supersymmetry generator. We showed that electroweak extensions of the minimal superpotential do not lead to an anomaly-free theory, independently of the number of standard model singlets added. We found anomaly-free family-independent $R$-symmetric models by adding an $SU(3)_c$ octet chiral superfield. This however typically breaks $SU(3)_c$. We then discussed in detail the family-dependent anomaly-free $R$-symmetry. Making assumptions based on mass matrix considerations we found solutions with one, two, three and four additional singlets. We discarded the one- and two-singlet solutions based on the symmetry breaking pattern. For the three- and four-singlet solutions we analysed the gauge- and supersymmetry breaking. The $U(1)_R$ symmetry is necessarily broken near the Planck scale because of the Fayet-Illiopoulos term. This could naturally lead after symmetry breaking to an expansion parameter of order the Wolfenstein parameter which is required for a dynamical generation of the correct mass matrix structure. We generated the supersymmetry scale of order the weak-scale because of the large powers in the superpotential. The large powers were determined by the $R$-symmetry.

We have allowed for the possibility of a solution to the mu problem via an additional singlet. But there is no potential for this singlet. We shall show in [41] how a proper solution to the mu-problem can be obtained.

In the last section we discuss in detail the $R_p$ violating structure of our models. We find that $R$-symmetry and $R$-parity are disconnected concepts. We exclude a large class of our models because they lead to an unacceptable level of proton decay. The remaining solutions typically predict $LQ\bar{D}$ $R$-parity violation which could be observed at HERA.

We expect gauged $R$-symmetries to be a useful model-building tool in the future.

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**Note Added:** After submitting our paper we have received the related work of Castano, freedman and Manuel [10]. We have included a few comments concerning the connection to their work in text. In particular we have modified the D-term of the low-energy effective potential.

**References**

[1] A.H. Chamseddine and H. Dreiner, Nucl. Phys. B 447 (1995) 195; [hep-ph/9503454].

[2] A. Salam and J. Strathdee, Nucl. Phys. B 87 (1975) 85.

[3] P. Fayet, Nucl. Phys. B 90 (1975) 104.

[4] S. Dimopoulos and H. Georgi, Nucl. Phys. B 193 (1981) 150;  
   N. Sakai, Z. Phys. C 11 (1981) 153;  
   H. P. Nilles and S. Raby, Nucl. Phys. B 198 (1982) 102;  
   N. Sakai and T. Yanagida, Nucl. Phys. B 197 (1982) 533;  
   S. Dimopoulos, S. Raby and F. Wilczek, Phys. Lett. B 112 (1982) 133.

[5] W. Buchmüller, D. Wyler, Phys. Lett. B 121 (1983) 321.

[6] E. J. Chun, J.E. Kim and H.P. Nilles, Nucl. Phys. B 370 (1992) 105;  
   J. E. Kim and H. P. Nilles, Mod. Phys. Lett. A 9 (1994) 3575.

[7] M. Dine and D. A. MacIntire, Phys. Rev. D 46 (1992) 2594.

[8] L. Hall and L. Randall, Nucl. Phys. B 352 (1991) 289.

[9] I. Affleck and M. Dine, Phys. Lett. B 154 (1985) 368.

[10] R. Barbieri, L. Girardello and A. Masiero, Phys. Lett. B 127 (1983) 429;  R. Barbieri and L. Maiani, Nucl. Phys. B 243 (1984) 429.

[11] G. Farrar and A. Masiero, Rutgers Preprint RU-94-38, [hep-ph/9410401].

[12] A. M. Cooper-Sarkar et al, Phys Lett B 160 (1985) 212; Howard E. Haber in _Supersymmetry and Unification of Fundamental Interactions_, World Scientific, Singapore, 1993 and [hep-ph/9308235]; M. Barnett, talk given at SUSY95, Paris, May 1995.

[13] A. E. Nelson and N. Seiberg, Nucl. Phys. B 416 (1994) 46.
[14] J. Bagger, E. Poppitz and L. Randall, Nucl. Phys. B 426 (1994) 3.

[15] For a review see A. Strominger in the Proceedings of the Theoretical Advanced Study Institute Summer School, 1988.

[16] P. West, *Introduction to Supersymmetry and Supergravity*, World Scientific, 1986.

[17] D.Z. Freedman, Phys. Rev. D 15 (1977) 1173.

[18] A. Das, M. Fischler and M. Roček, Phys. Rev. D 16 (1977) 3427.

[19] P. Fayet, and J. Illiopoulos Phys. Lett. B 51 (1974) 461.

[20] B. De Wit and P. van Nieuwenhuizen, Nucl. Phys. B 139 (1978) 216.

[21] K.S. Stelle and P.C. West, Nucl. Phys. B 145 (1978) 175.

[22] For an overview of the superconformal tensor calculus see T. Kugo and S. Uehara, Nucl. Phys. B 226 (1983) 49.

[23] R. Barbieri, S. Ferrara, D.V. Nanopoulos, and K.S. Stelle, Phys. Lett. B 113 (1982) 219.

[24] S. Ferrara, L. Girardello, T. Kugo and A. van Proeyen, Nucl. Phys. B (1983) 191.

[25] E. Cremmer, S. Ferrara, L. Girardello, and A. van Proeyen, Nucl. Phys. B 212 (1983) 413.

[26] R. Delbourgo and A. Salam, Phys. lett. B 40 (1972) 381; T. Eguchi and P. Freund, Phys. Rev. Lett 37 (1976) 1251; L. Alvarez-Gaume and E. Witten, Nucl. Phys. B 234 (1983) 269.

[27] N. K. Nielsen, M.T. Grisaru, H. Römer, and P. van Nieuwenhuizen, Nucl. Phys. B 140 (1978) 477; N. K. Nielsen and H. Römer, Phys. Lett. B 154 (1985) 141; the third reference in [26]; J. P. Derendinger, Phys. Lett. B (1985) 203.

[28] M. B. Green and J. H. Schwarz, Phys. Lett. B 149 (1984) 117.

[29] L. Ibanez, Phys. Lett. B 303 (1993) 55.

[30] S. Weinberg, Phys. Rev D 26 (1982) 287.

[31] G. Giudice, Mod Phys. Lett. A 7 (1992) 2429, H. Dreiner, G.K. Leontaris, and N.D. Tracas, Mod. Phys. Lett. A8 (1993) 2099; G. Anderson, S. Dimopoulos, L. Hall and S. Raby, Phys. Rev. D 47 (1993) 3702; P. Ramond, R.G. Roberts and G.G. Ross, Nucl. Phys. B 406 (1993) 19, L. Ibanez and G.G. Ross, Phys. Lett. B 332 (1994) 100;
H. Dreiner, G.K. Leontaris, S. Lola and G.G. Ross, Nucl. Phys. B 436 (1995) 461;
V. Jain and R. Shrock, Stony Brook preprint ITP-SB-94-55 and [hep-ph/9412367].

[32] C. E. Carlson, P. Roy and M. Sher, Preprint William & Mary WM-95-104, Tata Institute TIFR/TH/95-20, [hep-ph/9506328].

[33] J. Polonyi, Budapest preprint KFKI-93 (1977).

[34] A. H. Chamseddine, R. Arnowitt, and P. Nath, Phys. Rev. Lett. 50 (1983) 232.

[35] A. H. Chamseddine, R. Arnowitt and P. Nath, Phys. Rev. Lett. 49 (1982) 970.

[36] E. Cremmer, P. Fayet and L. Girardello, Phys. Lett. B 122 (1983) 41.

[37] V. Barger, T. Han and G. Giudice, Phys. Rev. D 40 (1989) 2987.

[38] J. Butterworth and H. Dreiner, Nucl. Phys. B 397 (1993) 3; Proc. of the 2nd HERA Workshop on Physics, 1991.

[39] H. Dreiner and P. Morawitz, Nucl. Phys. B 428 (1994) 31.

[40] D. Castano, D. Freedman, and C. Manuel, [hep-ph/9507397].

[41] A. Chamseddine and H. Dreiner, work in progress.
| I  | $l_1$ | $z_1$ | $z_2$ | $z_3$ | $z_4$ |
|----|-------|-------|-------|-------|-------|
|    | -1    | $-\frac{80}{3}$ | 7     | $\frac{53}{3}$ | $\frac{47}{2}$ |
| II | $-\frac{5}{6}$ | $-\frac{203}{6}$ | 21    | $\frac{185}{6}$ | $\frac{7}{2}$ |
| III| $-\frac{1}{2}$ | $-\frac{131}{2}$ | $\frac{95}{6}$ | $\frac{391}{6}$ | 6 |
| IV | $-\frac{1}{3}$ | -24    | $\frac{25}{2}$ | $\frac{131}{6}$ | $\frac{67}{6}$ |
| V  | $-\frac{1}{6}$ | $-\frac{133}{6}$ | $\frac{27}{2}$ | $\frac{115}{6}$ | 11 |
| VI | $\frac{1}{6}$ | $-\frac{263}{6}$ | $-\frac{25}{6}$ | 36     | $\frac{67}{2}$ |
| VII| $\frac{1}{3}$ | $-\frac{74}{3}$ | $\frac{33}{2}$ | $\frac{43}{2}$ | $\frac{49}{6}$ |
| VIII| $\frac{1}{2}$ | $-\frac{83}{2}$ | $\frac{55}{2}$ | 37     | $-\frac{3}{2}$ |
| IX | $\frac{5}{6}$ | $-\frac{278}{3}$ | $-\frac{17}{2}$ | $\frac{187}{6}$ | $\frac{183}{2}$ |
| X  | 1     | $-\frac{167}{2}$ | 24    | $\frac{497}{6}$ | $-\frac{11}{6}$ |

Table 1: Fermionic charges of the ten four-singlet solutions with $q_1 = -\frac{5}{6}$, $q_2 = -\frac{2}{3}$, $l_2 = \frac{5}{2} - l_1$, $q_3 = l_3 = 0$. 
| Class | Superpotential $\kappa^3 g'$ |
|-------|-------------------------------|
| I     | $(a_1(\kappa z_1)^2(\kappa z_2)^2(\kappa z_3)^2 + a_2(\kappa z_1)^7(\kappa z_2)^2(\kappa z_3)(\kappa z_4)^6 + a_3(\kappa z_1)^5(\kappa z_2)^2(\kappa z_3)^5(\kappa z_4)^4 + a_4(\kappa z_1)^9(\kappa z_2)^2(\kappa z_3)^9(\kappa z_4)^2)$ |
| II    | $(a_1(\kappa z_1)^4(\kappa z_2)^4(\kappa z_3)(\kappa z_4)^3 + a_2(\kappa z_1)^7(\kappa z_3)^7(\kappa z_4)^2 + a_3(\kappa z_1)^5(\kappa z_2)^2(\kappa z_3)^2(\kappa z_4)^13 + a_4(\kappa z_1)^6(\kappa z_2)^7(\kappa z_4)^10)$ |
| III   | $(a_1(\kappa z_1)^5(\kappa z_2)^11(\kappa z_3)^2(\kappa z_4) + a_2(\kappa z_1)^6(\kappa z_2)^{14}(\kappa z_3)^2(\kappa z_4)^3 + a_3(\kappa z_1)^5(\kappa z_3)^3(\kappa z_4)^{18} + a_4(\kappa z_1)^6(\kappa z_2)^3(\kappa z_3)^3(\kappa z_4)^{20})$ |
| IV    | $(a_1(\kappa z_1)^5(\kappa z_2)(\kappa z_3)^4(\kappa z_4) + a_2(\kappa z_1)^6(\kappa z_2)^4(\kappa z_4)^4 + a_3(\kappa z_1)^7(\kappa z_2)^7(\kappa z_3)^3 + a_4(\kappa z_1)^8(\kappa z_2)^6(\kappa z_3)^3(\kappa z_4)^3)$ |
| V     | $(a_1(\kappa z_1)^5(\kappa z_2)^3(\kappa z_3)^2(\kappa z_4)^2 + a_2(\kappa z_1)^7(\kappa z_2)^4(\kappa z_3)(\kappa z_4)^6 + a_3(\kappa z_1)^{10}(\kappa z_3)^{10}(\kappa z_4) + a_4(\kappa z_1)^{10}(\kappa z_2)^5(\kappa z_3)^7)$ |
| VI    | $(a_1(\kappa z_1)^2(\kappa z_2)^5(\kappa z_4)^3 + a_2(\kappa z_1)^6(\kappa z_3)^7 + a_3(\kappa z_1)^5(\kappa z_2)^8(\kappa z_4)^7 + a_4(\kappa z_1)^4(\kappa z_2)^{13}(\kappa z_3)^3(\kappa z_4)^3)$ |
| VII   | $(a_1(\kappa z_1)^4(\kappa z_2)^5(\kappa z_4) + a_2(\kappa z_1)^9(\kappa z_2)^2(\kappa z_3)^8 + a_3(\kappa z_1)^9(\kappa z_2)^3(\kappa z_3)^6(\kappa z_4)^3 + a_4(\kappa z_1)^9(\kappa z_2)^4(\kappa z_3)^4(\kappa z_4)^6)$ |
| VIII  | $(a_1(\kappa z_1)^3(\kappa z_2)^3(\kappa z_3) + a_2(\kappa z_1)^2(\kappa z_2)^3(\kappa z_4)^5 + a_3(\kappa z_1)^9(\kappa z_2)^2(\kappa z_3)^5(\kappa z_4)^4 + a_4(\kappa z_1)^6(\kappa z_2)^6(\kappa z_3)^2(\kappa z_4)^4)$ |
| IX    | $(a_1(\kappa z_1)^4(\kappa z_2)^5(\kappa z_3)^4(\kappa z_4)^3 + a_2(\kappa z_1)^6(\kappa z_2)^2(\kappa z_3)^9(\kappa z_4)^3 + a_3(\kappa z_1)(\kappa z_2)^{17}(\kappa z_3)^4(\kappa z_4) + a_4(\kappa z_1)^3(\kappa z_2)^{14}(\kappa z_3)^3(\kappa z_4))$ |
| X     | $(a_1(\kappa z_1)^4(\kappa z_3)^4(\kappa z_4)^4 + a_2(\kappa z_1)^{10}(\kappa z_2)^3(\kappa z_3)^9(\kappa z_4)^3 + a_3(\kappa z_1)^7(\kappa z_2)^{10}(\kappa z_3)^4(\kappa z_4)^7 + a_4(\kappa z_1)^9(\kappa z_3)^9(\kappa z_4)^{12})$ |

Table 2: $R$-invariant superpotentials for the ten different classes of anomaly-free solutions with $q_1 = -\frac{5\lambda}{6}$. We have only kept the lowest four terms.
Table 3: Leptonic fermionic charges of the ten four-singlet solutions with $q_1 = -\frac{5}{6}$.

| Model | Lepton Charges |
|-------|----------------|
| I     | $\vec{l} = (-1, \frac{7}{3}, 0)$ |
| II    | $\vec{l} = (-\frac{5}{6}, \frac{10}{3}, 0)$ |
| III   | $\vec{l} = (-\frac{1}{3}, 3, 0)$ |
| IV    | $\vec{l} = (-\frac{1}{3}, \frac{17}{6}, 0)$ |
| V     | $\vec{l} = (-\frac{1}{6}, \frac{5}{3}, 0)$ |
| VI    | $\vec{l} = (\frac{1}{6}, \frac{7}{3}, 0)$ |
| VII   | $\vec{l} = (\frac{1}{3}, \frac{13}{6}, 0)$ |
| VIII  | $\vec{l} = (\frac{1}{2}, 2, 0)$ |
| IX    | $\vec{l} = (\frac{5}{6}, \frac{5}{3}, 0)$ |
| X     | $\vec{l} = (1, \frac{3}{2}, 0)$ |