Testing Bell’s inequality using Aharonov-Casher effect

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Abstract

We propose the Aharonov-Casher (AC) effect for four entangled spin-half particles carrying magnetic moments in the presence of impenetrable line charge. The four particle state undergoes AC phase shift in two causally disconnected region which can show up in the correlations between different spin states of distant particles. This correlation can violate Bell’s inequality, thus displaying the non-locality for four particle entangled states in an objective way.

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Microphysical reality is certainly different from the reality that we are familiar with at a classical level. This is best illustrated with Bell’s inequality [1] which expresses the constraint on the correlation experiments when locality and reality criterion are taken for granted. Quantum mechanical entangled states violate Bell-like inequalities [2] showing the incompatibility of quantum theory with any classical type local, realistic hidden variable theory. For entangled states the principle of separability is not satisfied, where separability means that two systems that belong to space-like separated regions are independent systems. Recent experiments of Aspect et al [3] using photon states show that in coincidence experiments the Bell inequality is violated, thus, favouring the non-local aspect of the quantum theory. This non-locality, however, can not be used to send signals instantaneously because that would violate one of the basic principles of relativity.

There is another kind of non-locality in quantum theory, which comes from the observable consequences of the existence of gauge fields. The wavefunction of the quantum system picks up a non-local phase when the particle travels a force free region but containing a non-zero gauge field. The famous example is Aharonov-Bohm (AB) effect [4] where a charged particle encircling a long solenoid acquires a phase proportional to the flux enclosed by the solenoid. Similarly, when the role of charge and magnetic flux is interchanged we have the Aharonov-Casher (AC) effect [5]. This effect is sometimes referred to as Anadan-Aharonov-Casher (AAC) effect, because Anandan [6] had independently predicted the same prior to Aharonov and Casher. Further using a covariant treatment of electrodynamics Anandan [7] has obtained the same result and the topological nature of the effect was clarified. In AC effect, when a particle of non-zero magnetic moment goes around a line charge it acquires a phase shift proportional to the charge-density. The AB effect has been clearly verified experimentally [8] demonstrating the non-local and topological aspects of the phase shift. The AC effect in the same spirit can be regarded as a non-local and topological effect although it is somewhat different to that of AB effect [9]. Recently, there has been arguments suggesting not to call the AC effect as a non-local and topological one [10] because the effect can be understood using interaction of magnetic moment with the local Maxwell field.

In this paper, we would like to demonstrate the AC effect for four-particle entangled states and using the AC phase shift we will show that the probabilities of joint measurements for distant particles are precisely the ones that violate Bell’s inequality. A similar scheme based on phase transformation of spin states on distant pairs has been recently proposed by Hacyan [11]. The phase shifts are supposed to occur under the presence of local magnetic fields in distant regions exerting force on various particles. However, in the present scheme, we exploit the topological nature of the AC effect where distant particles experience no force even though the electric field is not absent. Thus the violation of the Bell inequality based on AC effect would strengthen the non-locality of quantum world in contrast to phase shift
base on local (in presence of force) interactions.

Following the work of Hacyan [11], consider two pair of spin-$\frac{1}{2}$ particles in their respective singlet states $|0, 0 >_{12}$ and $|0, 0 >_{23}$. Let the particles 1 and 2 carrying magnetic moments $\mu_1$ and $\mu_2$ are emitted from a source in which the total spin $S = 0$ at some point $C$. Similarly, let the particles 3 and 4 carrying magnetic moments $\mu_3$ and $\mu_4$ are emitted from another source with total spin $S = 0$ at a point $D$. These particles are arranged in such a way that 1 and 4 meet at some point $A$ and 2 and 3 meet at some point $B$, which are quite far from each other and in fact can be causally disconnected. Imagine now the situation where the spin-1/2 particles are moving in a plane (x-y) with their magnetic moments aligned perpendicular to the plane of the motion and there is an impenetrable line charge aligned perpendicular to the plane, also. The path of these particles enclose the line charge and each of them experiences the electric field due to the line charge. But the force on these particles vanishes identically with proper alignments.

Now, during the propagation of particles 1 and 2 from the source situated at point $C$ the spin states will undergo the AC phase shift as given by

$$|\pm >_1 \rightarrow \exp(\pm i \int_C^A (\mathbf{E} \times \mu_1).d\mathbf{r})|\pm >_1,$$

$$|\pm >_2 \rightarrow \exp(\pm i \int_C^B (\mathbf{E} \times \mu_2).d\mathbf{r})|\pm >_2$$

(1)

and similarly particles 3 and 4 emitted from point $D$ undergo AC phase shift as is given by

$$|\pm >_3 \rightarrow \exp(\pm i \int_D^B (\mathbf{E} \times \mu_3).d\mathbf{r})|\pm >_3,$$

$$|\pm >_4 \rightarrow \exp(\pm i \int_D^A (\mathbf{E} \times \mu_4).d\mathbf{r})|\pm >_4$$

(2)

where $\pm$ sign in phase factors reflect the fact that whether the magnetic moment is parallel or antiparallel to the line charge. Under the AC phase shifts, the singlet state $|0, 0 >_{12}$ transforms to

$$|0, 0 >_{12} \rightarrow \cos(\int_C^A (\mathbf{E} \times \mu_1).d\mathbf{r} - \int_C^B (\mathbf{E} \times \mu_2).d\mathbf{r})|0, 0 >_{12}$$

$$+ i \sin(\int_C^A (\mathbf{E} \times \mu_1).d\mathbf{r} - \int_C^B (\mathbf{E} \times \mu_2).d\mathbf{r})|1, 0 >_{12},$$

(3)

and similarly for the state $|0, 0 >_{34}$ we have

$$|0, 0 >_{34} \rightarrow \cos(\int_C^A (\mathbf{E} \times \mu_3).d\mathbf{r} - \int_C^B (\mathbf{E} \times \mu_4).d\mathbf{r})|0, 0 >_{34}$$

$$+ i \sin(\int_C^A (\mathbf{E} \times \mu_3).d\mathbf{r} - \int_C^B (\mathbf{E} \times \mu_4).d\mathbf{r})|1, 0 >_{34},$$

(4)

where $|1, 0 >$ is the triplet state with total spin one magnetic moment zero.
Thus introducing a line charge into the configuration of two pairs of entangled states is equivalent to making each pair a superposition of singlet and triplet states with distinct probability amplitudes. This is a very notable feature of the phase transformation of the spin states \( |\psi\rangle \). The probability of finding the singlet state \(|0,0\rangle\), for example, has changed from unity to \(\cos^2(f_C^A(E \times \mu_1).dr - f_C^B(E \times \mu_2).dr)\), which depends on the AC phase shift (depending on the path of the trajectories \( C - A \) and \( C - B \)) that the distant particles undergo.

Before the AC phase shift the state of the combined system is \(|0,0\rangle \otimes |0,0\rangle\). The state of the combined system after the spin states have undergone AC phase shift is given by (see also eq.(3) in [11])

\[
|\Psi_{TOTAL}\rangle = \frac{1}{2} \left[ - e^{i(\Phi_1 - \Phi_2 - \Phi_3 + \Phi_4)} |1,1; 1,1\rangle + e^{-i(\Phi_1 - \Phi_2 - \Phi_3 + \Phi_4)} |1,1; 1,1\rangle + \right.
\]

\[
\left. \cos(\Phi_A - \Phi_B)(|1,0; 1,0\rangle - |0,0; 0,0\rangle) + i \sin(\Phi_A - \Phi_B)(|1,0; 1,0\rangle - |1,0; 1,0\rangle) \right]\]

where we have denoted \( \Phi_1 = \int_C^A (E \times \mu_1).dr, \Phi_2 = \int_C^B (E \times \mu_2).dr, \Phi_3 = \int_D^B (E \times \mu_3).dr, \Phi_4 = \int_D^A (E \times \mu_4).dr \) and the relative phases at locations \( A \) and \( B \) are \( \Phi_A = (\Phi_1 - \Phi_4) \) and \( \Phi_B = (\Phi_2 - \Phi_3) \), respectively.

Then, the argument goes as in [11]. One can calculate the joint probabilities of measuring the pairs of particles at locations \( A \) and \( B \) in states of total spin \( S = 1 \) or \( 0 \) with \( m = 0 \). For example, the probability of measuring the spin at \( A \) as 1 and at \( B \) as 1 is given by

\[
P(1,1) = |<\Psi_{TOTAL}|1,0;1,0\rangle|^2 = \frac{1}{4} \cos^2(\Phi_A - \Phi_B). \quad (6)
\]

Similarly we have probabilities \( P(0,0) = \frac{1}{4} \cos^2(\Phi_A - \Phi_B), P(1,0) = P(0,1) = \frac{1}{4} \sin^2(\Phi_A - \Phi_B) \). The correlation function defined from the above joint probabilities is given by

\[
E(\Phi_A, \Phi_B) = \frac{P(1,1) + P(0,0) - P(1,0) - P(0,1)}{P(1,1) + P(0,0) + P(1,0) + P(0,1)}
\]

\[
= \cos 2(\Phi_A - \Phi_B). \quad (7)
\]

This can be shown to violate the inhomogenous Bell’s inequality

\[
|E(\Phi_A, \Phi_B) - E(\Phi_A, \Phi_B') + E(\Phi_A', \Phi_B') + E(\Phi_A', \Phi_B')| \leq 2. \quad (8)
\]

by arranging different points \( A, A' \) and \( B, B' \) to meet the particles 1, 4 and 2, 3, respectively. Thus, using AC phase shift one can check the violation of Bell’s inequality thereby ruling out local realism.

Like in any other scheme for testing Bell’s inequality, here, we have figured out two locally controlled parameters on which the correlations depend. For example, in the proposed
scheme of Hacyan [11] the locally controlable parameters are the interaction time of the particles with the magnetic fields present at events $A$ and $B$, where one measures the total spin of two spin-half particles. In the present scheme the correlation $E(\Phi_A, \Phi_B) = \cos 2(\Phi_A - \Phi_B)$ depends on two parameters, namely the location of the “meeting points” $A$ and $B$, where particle 1 and 4 and 2 and 3 meet, respectively. As the relative phase $\Phi_A$ is given by $\Phi_A = \int_C^A (E \times \mu_1).dr - \int_D^A (E \times \mu_4).dr$, one can see that the first term depends on the path length of the trajectory $C - A$ and the second term on the trajectory $D - A$, respectively. For a given experimental set up the location of the sources $C$ and $D$ are fixed. So one can only change the location of the “meeting point” $A$. Therefore, $\Phi_A$ can be locally controlled by changing the location $A$. Similarly, we can argue that the relative phase shift $\Phi_B$ depends on the path $C - B$ and $D - B$ and can be controlled by changing the location $B$. Therefore, in the proposed scheme the correlation $E(\Phi_A, \Phi_B)$ can be locally controlled by changing the locations $A$ and $B$. For obtaining different AC phase shifts the magnetic moments of the particles 1 and 2 have to be different from the particles 3 and 4.

If the magnetic moments of all these particles have same numerical value, then the quantum correlation reduces to $E(\Phi_A, \Phi_B) = \cos 2(\oint (E \times \mu).dr) = \cos(2\mu \lambda)$, where $\mu$ is the projection of the magnetic moment along the line charge and $\lambda$ is the charge density (charge per unit length) on the line. The correlation depends exactly on the AC phase as in the usual AC effect, where a single particle in an interference setup encircles the line charge and acquires a phase equal to $\mu \lambda$. Here, the difference between entangled state AC effect and single particle displaying AC effect is that in former case non of the particles (either from pair $|0, 0 >_{12}$ or $|0, 0 >_{34}$) encircle the line charge. Yet, the four particle state acquires a AC phase as if a single entity has undergone a motion arround the line charge, producing the same effect purely because of entanglement. Since, there is no objective way to associate the total AC phase shift to one particle or the other of two pairs, we believe that the AC effect for entangled state is a non-local effect.

To conclude this note, we have proposed a scheme where two pairs of entangled states undergo the AC phase shift in the presence of an impenetrable line charge which can be manifested in joint correlation measurements of spin states that violates standard Bell’s inequality. These phase shifts can be locally controlled by changing the location of the “meeting points” of the distant particles. In the special case, the AC effect for two pairs of entangled states can be regarded purely as a non-local effect. In same line of thought one can also explore the non-local geometric phase [12] concept for entangled states [13] to test Bell’s inequality using two photon interference experiments [14].
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