We study the interaction of an equal mass binary with an isothermal circumbinary disk motivated by the evidence of the formation of massive black hole binaries surrounded by gas, after a major merger of gas-rich galaxies. We focus on the torques that the binary produces on the disk and how the exchange of angular momentum can drive the gap formation on it. We propose that the angular momentum exchange between the binary and the disk is through the gravitational interaction of the binary and a (tidally formed) global non-axisymmetric perturbation in the disk. Using this interaction, we derive an analytic criterion for the gap formation in the disk that can be expressed either via the characteristic velocities of the binary–disk system or in terms of the structural parameters \( h/a \) and \( M(<r)/M_{\text{bin}} \). Using numerical simulations we show that the simulations where the binary opens a gap in the disk and the simulations where the disk does not have a gap are distributed into two well separated regions. Our analytic criterion predicts a shape of the threshold between these two regions that is consistent with our simulations and the other ones in the literature. We propose an analogy between the regime without (with) a gap in the disk and the Type I (Type II) migration that is observed in simulations of planet–disk interaction (extreme mass ratio binary), emphasizing that the interaction that drives the formation of a gap on the disk is different in the regime that we analyze (comparable mass binary).

Key words: binaries: general – black hole physics – galaxies: nuclei – hydrodynamics – methods: numerical

Online-only material: color figures

1. INTRODUCTION

It is well known that galaxies are systems that can strongly interact gravitationally with each other and in some cases even merge. Also, it is widely accepted that galaxies with a significant bulge host a massive black hole (MBH) at their centers (Richstone et al. 1998). Therefore, it is natural to conclude that these MBHs will interact after a major merger (Milosavljevic & Merritt 2001, 2003) and possibly collide. The possible coalescence of MBHs was first considered by Begelman et al. (1980) in a study of the long-term evolution of a black hole binary at the center of a dense stellar system. Initially, dynamical friction brings the two black holes toward the center of the system and the resulting MBH binary continues to shrink via three-body interactions with the surrounding stars. This three-body interaction tends to eject stars from the central region, causing the merger to eventually stall (the “last parsec” problem) unless some additional mechanism is able to either extract angular momentum from the MBH binary or refill the stellar “loss-cone” (Khan et al. 2011).

There is theoretical (Barnes & Hernquist 1992, 1996; Mihos & Hernquist 1996; Barnes 2002; Mayer et al. 2007, 2010) and observational (Sanders & Mirabel 1996; Downes & Solomon 1998) evidence that in the mergers of gas-rich galaxies a large amount of gas can reach the central regions of the newly formed system. This gas induces the formation of an MBH binary (Kazantzidis et al. 2005; Mayr et al. 2007; Chapon et al. 2011) and can drive the final coalescence of the binary (Escala et al. 2005; Dotti et al. 2006; Cuadra et al. 2009; Roedig et al. 2011; Escala & del Valle 2011).

In all these studies that follow the evolution of a binary embedded in a gas environment, it is assumed that the gas lies in a disk (circumbinary disk) and the binary has a non-extreme mass ratio \( (q \sim 1) \). However, they find very different timescales of coalescence for the binary because they explore very different disk-to-binary mass ratios \( (M_{\text{disk}}/M_{\text{bin}}) \). When the mass of the disk is much greater than the binary mass it is found that the coalescence time of the binary is on the order of a few initial orbital times (Escala et al. 2005; Dotti et al. 2006). On the other hand, for disks with masses that are negligible compared to the binary mass, the coalescence time is on the order of several thousands of local orbital times (Artymowicz & Lubow 1994; Ivanov et al. 1999; Armitage & Natarajan 2002; Milosavljevic & Phinney 2005), which for \( M_{\text{BH}} \gtrsim 10^7 M_\odot \) is even longer than the Hubble time (Cuadra et al. 2009).

These two regimes of fast (few orbital times) and slow (several thousands of orbital times) migrations are also observed in simulations of planets embedded in circumstellar disks (e.g., extreme mass ratios; Ward & Hourigan 1989; Ward 1997; Bate et al. 2003; Armitage & Rice 2005; Baruteau & Masset 2012; Kocsis et al. 2012a, 2012b). In simulations of planet migration in protoplanetary disks \( (q \ll 1) \), these two regimes are defined as Type I and Type II migration. In the Type I regime the perturbation induced by the planet in the circumstellar disk remains small and permits a fast migration of the planet (of the order of a few orbital times) with a characteristic timescale that is inversely proportional to the planet’s mass \( (t_{\text{migration}} \propto M_p^{-1}) \). When the Hill radius of the planet is greater than the local pressure scale height \( (R_{\text{Hill}} \gg h) \) the perturbation induced by the planet in the disk becomes important and a gap begins to form, leading to coupled evolution of the planet and the disk on a viscous timescale and therefore, a much longer migration time (Type II migration). In this paper we will extend the same terminology to the case of non-extreme mass ratio binaries \( (q \sim 1) \) interacting with a disk by referring to the fast (slow) coalescence of the binary as Type I (II) migration.

For a comparable mass binary in a disk, as in the star–planet–disk system, the threshold between Type I and Type II migrations is determined by the formation of a gap in the disk. Our aim in this paper is to establish a gap-opening criterion
that allows us to differentiate these two migration regimes and determine in which binary–disk system is an efficient coalescence of the MBH binary expected. For that purpose, we will use a simple model for the binary–disk interaction to derive an analytic expression for the gap-opening criterion. We will test this criterion against a variety of simulations with different model parameters for the binary to disk mass ratio and thickness of the disk.

This paper is organized as follows. After the derivation of an analytic gap-opening criterion in Section 2, we present in Section 3 the numerical method and the set of numerical simulations that we use to test our analytic criterion. In Section 4 we describe how we identified the formation of a gap in our simulations and in Section 4.1 we test our analytic gap-opening criteria against the numerical simulations. All the results that we find are discussed in Section 5 and we also compare our results with the other results of in the literature. Also, in Section 5 we discuss the implications of our results in real astrophysical systems. Finally, our conclusions are presented in Section 6.

2. ANALYTIC GAP-OPENING CRITERIA

The interaction of a binary with a circumbinary disk in cases where the total gas mass is typically much smaller than the mass of the primary has been widely studied in the context of star/planet formation (Lin & Papaloizou 1979; Goldreich & Tremaine 1982; Takeuchi et al. 1996; Armitage & Rice 2005; Baruteau & Masset 2012). In these studies a linear approximation for the equations of motion of the system is used and it is found that the interaction between the secondary and the disk is controlled by the sum of the torques arising from the inner and outer Lindblad and corotation resonances. This approach applies to binaries where the primary is much more massive than the secondary (q = M2/M1 ≪ 1) and leads to predictions of the gap structure that are consistent with simulations within the same regime (Ivanov et al. 1999; Armitage & Natarajan 2002; Nelson & Papaloizou 2003; Haiman et al. 2009; Baruteau & Masset 2012). Because of the success of this analysis in the planetary regime (q ≪ 1), it has been extrapolated to other cases where q ∼ 1 (Artymowicz & Lubow 1994, 1996; Gunther & Kley 2002; MacFadyen & Milosavljevic 2008) without considering the global nonlinear perturbation that is produced by the binary gravitational field in the regime q ≈ 1 (Shi et al. 2012). This nonlinear perturbation breaks the validity of the linearization of the equation of motion.

In this paper we study the binary–disk interaction in the regime q ≈ 1 without any assumption of linearity. For this purpose we explore if it is possible, due to the tidal nature of the binary–disk interaction, that the gap-opening process can be described by the interaction of the binary with a strong non-axisymmetric perturbation on the disk instead of a resonant process that appears in the linear approximation. This type of interaction between a strong non-axisymmetric perturbation and a binary was investigated by Escala et al. (2004, 2005). They show that, when the spheres of gravitational influence (i.e., Hill spheres) of the two components of the binary overlap, a strong tidal non-axisymmetric perturbation is produced in the disk with an ellipsoidal geometry for an equal mass binary (q = 1). They also found that the symmetry axis of this strong non-axisymmetric perturbation is not coincident with the binary axis but lags behind it, producing a gravitational torque on the binary that is responsible for angular momentum transport from the binary to the disk.

In this paper we will restrict our analysis to the case of an equal mass binary (q = 1) interacting with a disk, leaving the case q ∼ 1 but q ≠ 1 to a companion paper. In the case q = 1, it is justified to assume that the strong non-axisymmetric perturbation produced in the disk has an ellipsoidal geometry. Therefore, we study the gravitational interaction between the binary and an ellipsoid to derive analytically the exchange of angular momentum between the disk and the binary. This binary–ellipsoid system can be treated as an equivalent one-body problem subject to an external gravitational potential caused by the ellipsoidal perturbation. The gravitational potential of a uniform ellipsoid is given by

$$\Phi(x, y, z) = \pi G \rho \left( a_0 x^2 + \beta_0 y^2 + y_0 z^2 + \chi_0 \right),$$

with $a_0$, $\beta_0$, $y_0$, and $\chi_0$ constants given in Lamb (1879), which depends on the ratios between the principal axes of the ellipsoid. Because of the symmetry of the ellipsoidal perturbation observed in Escala et al. (2004, 2005), $y_0 = \beta_0$ and $\chi_0 = 0$. Therefore, the Lagrangian of the binary–ellipsoid system in cylindrical coordinates has the form

$$L = \frac{1}{2} \mu \left[ \left( \frac{dr}{dt} \right)^2 + r^2 \left( \frac{d\phi}{dt} \right)^2 + \left( \frac{dz}{dt} \right)^2 \right] + G \frac{\mu^2}{r} - \frac{\pi}{2} G \mu \rho (a_0 r^2 \cos^2(\Delta \phi) + \beta_0 r^2 \sin^2(\Delta \phi) + \beta_0),$$

with $\mu = M_1 M_2 / M_{\text{bin}}$ being the reduced mass of the binary and $\Delta \phi$ the angle between the major axis of the ellipsoid and the binary axis. This angle is approximately constant over the evolution of the system (Escala et al. 2004, 2005). From this Lagrangian we derive the Euler–Lagrange equation for the coordinate $\phi$,

$$\frac{dL_{\text{bin}}}{dt} = \mu \frac{d}{dt} \left( r^2 \frac{d\phi}{dt} \right) = -\pi r^2 \left( \frac{M_{\text{bin}}}{4} \right) \rho G (\beta_0 - a_0) \cos(\Delta \phi) \sin(\Delta \phi),$$

where we use $\mu = M_{\text{bin}}/4$. This equation expresses the decrease in the angular momentum of the binary $L_{\text{bin}}$ and therefore the injection of angular momentum (torque) from the binary to the disk.

To derive a criterion for the opening of a gap in the disk we compare the gap-opening timescale (determined by the torque that the binary exchanges over the disk) with the gap-closing timescales (Goldreich & Tremaine 1980). The angular momentum $\Delta L$ that must be added to the gas to open a gap of radius $\Delta r$ in a disk with thickness $h$ is of the order of $\Delta L \approx \rho (\Delta r^2) h v$. The torque that the binary exchanges over the disk is $\tau = -dL_{\text{bin}}/dt = r^2 \rho (M_{\text{bin}}/4) G \pi (\beta_0 - a_0) \cos(\Delta \phi) \sin(\Delta \phi)$. This torque injects an angular momentum $\Delta L$ on a timescale $\Delta t_{\text{open}} = \Delta L / \tau$ and therefore this is the characteristic timescale to open a gap. This tendency is opposed by viscous diffusion, which fills up a gap of order $\Delta r$ on a timescale $\Delta t_{\text{close}} = (\Delta r)^2 / \nu$ (Goldreich & Tremaine 1980) where $\nu$ is the turbulent viscosity of the gas that can be parameterized assuming the standard $\alpha$-prescription $\nu = \alpha_{\text{ss}} c_s h$ of Shakura & Sunyaev (1973) where $\alpha_{\text{ss}}$ is the dimensionless viscosity parameter and $c_s$ is the sound speed of the gas. Gap formation occurs when $\Delta t_{\text{open}} \lesssim \Delta t_{\text{close}}$. We also assume that $c_s / v \approx h / r$ with $v$ the circular velocity of the binary–ellipsoid system. Here, based on previous numerical results (Escala et al., 2004, 2005).
parameters of the ellipsoid (strongly related to the spin) are approximately constant due to the self-similar behavior (Escala et al. 2004, 2005), we assume that the binary and the ellipsoid (strongly related to the spin) are approximately constant due to the self-similar behavior (Escala et al. 2004, 2005). This formulation of the gap-opening criterion contains information on the relative weight of the characteristic rotational speed of the binary–disk system (v), with the characteristic rotational speed of the isolated binary (v_{bin}) and the thermal state of the disk (v_{th}).

Regardless of the exact geometry of the strong non-axisymmetric density perturbation, the gravitational torque from the perturbed background medium onto the binary will have the form \( \tau = \frac{r^2}{2} G \mu K \), where \( K \) is a parameter that depends on the geometry of the density perturbation. Therefore, the dimensionless function \( f \) has in general the form \( f = 2/K \alpha_{ss} \) that represents the relative strength between the gravitational and viscous torques.

If we assume that the binary–disk system is rotationally supported against the overall gravitational potential on a circular orbit, we can also express this same criterion in terms of the disk structural parameters and the mass of the binary–disk system:

\[
\frac{\Delta \theta_{open}}{\Delta \theta_{close}} = \frac{1}{f} \left( \frac{h}{r} \right)^3 \left[ 1 + 8 \frac{M(<r)}{M_{bin}} \right] \leq 1 ,
\]

where \( M(<r) \) is the total mass enclosed by the orbit of the binary.

3. INITIAL CONDITIONS AND NUMERICAL METHOD

In this section we present simulations that follow the evolution of a binary embedded in a gaseous disk. We use a natural units system in our simulations: [mass] = 1, [distance] = 1, and we set the gravitational constant \( G = 1 \). In our internal units, the initial radius of the disk is \( R_{disk} = 30 \), and the total gas mass is \( M_{disk} = 33 \). The density of the disk is given by

\[
\rho(R, z) = \frac{\Sigma_0}{2 h_c} \frac{R_0}{R_c} \cosh^{-2} \left( \frac{z}{h_c} \right) \quad R \leq R_c
\]

and

\[
\rho(R, z) = \frac{\Sigma_0}{2 h_c} \frac{R_0}{R_c^2} \cosh^{-2} \left( \frac{z}{h_c} \frac{R_c}{R} \right) \quad R_c \leq R \leq R_{disk}
\]

where \( R_c = 3 \) and \( h_c \) are the radius and thickness of the central zone of the disk where the surface density is constant, respectively. With this density we obtain a surface density \( \Sigma(r) = \text{constant} \) for \( R < R_c \) and \( \Sigma(r) \propto r^{-1} \) if \( R > R_c \). The vertical distribution of the disk changes over the evolution of the system, but the initial vertical distribution chosen at least prevents an initial vertical collapse of the disk.

The binary system has an initial circular orbit of radius \( r_0 \), coplanar and corotating with the disk, a total mass \( M_{bin} \), and a mass ratio between the two components of the binary \( q = M_2/M_1 = 1 \). We explore the parameter space \( r_0 \in [0.5, 4] \), \( M_{bin} \in [1, 33] \), and \( h_c \in [0.8, 3] \) with 25 simulations (see Table 1).

In addition to the disk and the binary we include a fixed Plummer potential (Plummer 1911) that helps to stabilize the disk and also, when we apply our result to the study of SMBH binaries in Section 5, will mimic the existence of an external stellar component. The total mass of the Plummer potential is \( M_p = 6.6(\approx 12\%) \) of the total mass of the disk) and its scale radius is \( R_p = 19.13 \). The applicability of the gap-opening criterion derived in Section 2 remains valid even in the presence of this Plummer potential because its mass can be included in the total mass enclosed by the orbit of the binary \( M(<r) \).

We model the gaseous disk with a collection of \( 2 \times 10^5 \) SPH particles of gravitational softening 0.1. We use a stable (\( Q \geq 1 \)) isothermal disk to avoid fragmentation to simplify the testing of our analytic criterion derived in Section 2. In Table 1 we tabulate the minimum value of the Toomre parameter \( Q \) of each simulation. Since the total potential of the system (binary–disk Plummer) is non-axisymmetric and therefore lacks a well-defined circular velocity, we initially assume a symmetric representation of the binary potential to compute the circular velocity of the disk (see Appendix A for details). For the binary we use two collisionless particles with gravitational softening of 0.1. We run each simulation for 10–15 binary orbital times using the SPH code called GADGET2 (Springel 2005; Springel et al. 2001).

In addition, to test the numerical convergence of the results, we run simulations with one million SPH particles and find that the conclusions found in Section 4 for our analytic gap-opening criterion do not change (Appendix B).

4. RESULTS

In order to test our analytic gap-opening criterion against SPH simulations, we first need a criterion that determines numerically...
which of our simulations opens a gap in the disk and which ones do not. To determine whether a gap is opened or not, we explore the radial density profile of each simulation and the evolution of the binary separation.

If the binary opens a gap in the disk, it must have a flow of gas from the central to the outer regions of the disk. This flow produces a “pile up” of gas on the perimeter of the gap. This type of pile up is also expected to form during the evolution of a circumbinary disk that interacts with an unequal mass binary but which in this regime ($q \ll 1$) is associated with a drop in the density of the disk and not necessarily with the formation of a gap (Kocsis et al. 2012). The pile up formed over the evolution of our simulations ($q = 1$) is represented by a peak in the density profile and its maximum is correlated to the existence of a gap in the disk. In Figure 1 we show that the density plot has a peak or pile up if the disk has a gap. For the cases where the disk does not have a gap, this pile up is less prominent or does not exist.

We also explore if the gap can be identified as a drop in the azimuthal mean density. Although in some cases the existence of a gap is correlated with a drop in the mean azimuthal density, in many other cases there are streams of gas or inner disks around individual components of the binary that hide the existence of a gap in an azimuthally averaged density plot. Therefore, we decided to use the peak in the density profile as our first indicator for the existence of a gap. In addition to this, we follow the evolution of the binary separation. This separation tells us whether or not the binary is migrating to the inner regions of the disk on a timescale comparable to its orbital time $t_{orb}$. In order for the binary to migrate it is necessary that there is efficient angular momentum transfer, which cannot be reached if the material that participates in this angular momentum exchange is diffused to the outer regions as in the open-gap case. Therefore, a binary that is migrating to the center of the disk on a timescale $\sim t_{orb}$ is only consistent with a disk that does not have a gap (Type I migration).

Considering the density profile and the possible migration of the binary to the inner region of the disk, we called a simulation opened at a given time $t$ if it has a peak or pile up in density with $\rho_{\text{peak}} \geq 0.015$ (or $\rho_{\text{peak}} < 0.015$) and if the binary separation does not decrease by more than 10% in an orbit. If a simulation at time $t$ has a maximum density of the pile up $\rho_{\text{peak}} \leq 0.015$ and the binary separation decreases by more than 10% in an orbit, we call it closed. Otherwise we call it indefinite.

We analyze our simulations at the times $t$ in which the binary completes 2, 3, 5, 7, 10, and 15 orbits. For all the simulations in each of these time intervals, we define it as opened, closed, or indefinite if the gap in the disk is open, closed, or indefinite, respectively.

### 4.1. Testing the Gap-opening Criterion

In order to test our analytic gap-opening criterion, we plot the velocity parameters of the system $(v_{bin}/v)^2$, $(c_s/v)^2$. This parameters are present in one of the two equivalent gap-opening
An important consequence of Figure 2 is that the extension of the standard gap-opening criterion derived for the planetary regime, \( q \ll 1 \) to the regime \( q \sim 1 \) does not predict the right shape of the interface line between the opened and closed simulations. For example, the standard gap opening criterion in Lin & Papaloizou (1986) Equation (4) for the case \( q = 1 \), the criterion for gap opening in Lin & Papaloizou (1986) can be expressed as \( (h/r)^3 \leq (1/40\alpha_\infty)^{3/5} \) which corresponds to a horizontal line in Figure 2, which clearly cannot explain the distribution of opened and closed simulations in Figure 2.

4.2. Determining an Average Value for \( f(\Delta\phi, \alpha_0, \beta_0, \alpha_\infty) \)

The interface line that was shown in the previous section separates the parameter space between the opened and closed simulations and can be interpreted as the critical case in which a simulation has equal opening and closing times (i.e., \( \Delta t_{\text{open}} = \Delta t_{\text{close}} \)). The slope of the interface line that separates the set of opened simulations from the set of closed simulations in the velocity variables graph (Figure 2) is

\[
m = \left( \frac{v}{v_{\text{bin}}} \right)^2 \left( \frac{c_s}{v} \right)^3 = \frac{h/r}{r} \left[ 1 + 8 \frac{M(<r)}{M_{\text{bin}}} \right]^{3/2}.
\]

We find a value for the slope of the interface line of approximately \( m = 0.095 \).

From Equation (4) for the case \( \Delta t_{\text{open}} = \Delta t_{\text{close}} \) we can estimate that the dimensionless function \( f(\Delta\phi, \alpha_0, \beta_0, \alpha_\infty) \) is.

Replacing this numerical value by \( f \) in Equations (4) and (5) we can express the gap-opening criterion for an equal mass binary as

\[
\left( \frac{v}{v_{\text{bin}}} \right)^2 \left( \frac{c_s}{v} \right)^3 \leq 0.095
\]

or

\[
\left( \frac{h}{r} \right)^3 \left[ 1 + 8 \frac{M(<r)}{M_{\text{bin}}} \right]^{3/2} \leq 0.095.
\]

As \( f \propto \alpha_\infty^{-1} \) any changes in the viscosity parameter \( \alpha_\infty \) will change the slope of the interface line of Figure 2. If we increase the value of \( \alpha_\infty \) the slope will be less steep and the number of closed simulations will increase. This is consistent with the fact that with an increase in viscosity, it will be harder for the binary to open a gap on the disk. In our simulations we can estimate \( \alpha_\infty \) from the value of the SPH parameter of artificial viscosity \( \alpha_{\text{ph}} \) (Artymowicz & Lubow 1994; Murray 1996; Lodato & Price 2010; Taylor & Miller 2012). The value that we estimate ranges between \( \alpha_{\text{ph}} \approx 0.008–0.016 \). However, independent of its exact value, the functional dependence of our gap-opening criterion remains unchanged.

4.3. Transition from Closed Regime to Opened Regime

Only 1 of our 25 simulations (simulation C5 in Table 1) evolves from a state where the disk has no gap (closed) to a state where the disk has a gap (opened). The binary in this simulation initially migrates to the center of the disk driven by the action of the tidal torque. As the binary migrates to the center, it eventually reaches a separation distance where the torque exerted on the disk is enough to open a gap. It is important...
In our set of simulations we identify two regimes of binary–disk interactions: those with a gap (opened) and those without a gap (closed). We found that in general less massive and thinner disks are strongly perturbed by the binary and a gap is formed in them. On the other hand, an increase in the mass or thickness of the disk tends to close this gap and in some cases even precludes its formation. We also found that the possibility of formation of such a gap is more sensitive to the thickness than to the mass of the disk. This is in agreement with our analytic criterion, which has a linear dependence on the mass ratio $M_{\text{bin}}/M(<r)$ and a cubic dependence on the ratio between the pressure scale height of the disk and the binary separation ($h/a$).

We find good agreement between our test simulations and our analytic criterion; this supports the role of the interaction between the binary and a strong non-axisymmetric perturbation as the one responsible for the formation of the gap in the disk, instead of a resonant process. However, the resonances that operate in regions far from the binary where the linearization of its gravitational potential is a good approximation ($r \gg a$) play an important role in the later evolution of the gap, “clearing” its edges and driving the angular momentum transport from the binary to the disk when the gap is already formed (Artymowicz & Lubow 1994; MacFadyen & Milosavljevic 2008; Cuadra et al. 2009). In this figure we also plot the interface curve which for this parameter space has an inverse-cubic form. For the cases of simulations with non-self-gravitating disks (Artymowicz & Lubow 1994, 1996; MacFadyen & Milosavljevic 2008), we assume $M(<r)/M_{\text{bin}} = 0$. All the opened simulations that we find in the literature begin either with a gap in the disk ($M(<r)/M_{\text{bin}} = 0$) or the disk has mass that is low compared to the binary mass, but we do not find any simulations where the formation of a gap is studied for massive disks ($M(<r)/M_{\text{bin}} > 1$). In Figure 4 we can see that the simulations in which the gap begins with a gap, or it is opened over the evolution of the simulation (red open points I, II, and III), are below the interface curve, consistent with the gap-opening criterion. The closed simulations (squares...
in Figure 4) represent the final evolution of the simulation that begins with the same initial condition as that used by Escala et al. (2005) and Dotti et al. (2006). These simulations are above the interface line and are also consistent with the gap-opening criterion. Therefore, all the simulations from papers that we refer from the literature are consistent with the gap-opening criterion independently of the different equations of state that are used for the disks.

In the final evolution of the simulations from Escala et al. (2005), squares in Figure 4, it is found that although the binary shrinks it separation down to distances where its gravitational potential begins to dominate locally and the enclosed mass is comparable to or smaller than the binary mass ($M(< r)/M_{\text{bin}} < 1$), the binary fails to form a gap in the disk. This is because the scale height of the disk remains roughly constant as their separation $a$ (equals to $2r$) decreases, resulting in an increase in $h/a$ that compensates for the decrease in $M(< r)/M_{\text{bin}}$ in such a way that the system always lies above the threshold line $f = 0.095$.

We expect that a situation like the one found in Escala et al. (2005) should happen in galaxies with a gas-rich nucleus, such as nuclear disks in ULIRGs (Downes & Solomon 1998), Submillimeter Galaxies (Chapman et al. 2003, 2005; Takami et al. 2006; Swinbank et al. 2010), and in general, on protogalaxies at the early universe. This is because both observations (Downes & Solomon 1998; Genzel et al. 1998) and simulations (Mayer et al. 2010; Bournaud et al. 2011) found a typical disk scale height larger than several tenths of parsecs and gaseous masses larger than $10^9 M_\odot$ (more than an increase of most massive MBHs binaries). Moreover, we determine $f$ in our gap-opening criterion (Equation (4)) with simulations using $a_{ss} \approx 0.01$ ($f \propto a_{ss}^{-1}$), and gas in massive nuclear disks is expected to be globally unstable. In such a case, the torques are significantly larger, with $a_{ss}$ of order unity (Krumholz et al. 2007; Escala 2007). Therefore, in this situation we expect that to open a gap will be even harder than what we found in our simulations.

On the other hand, in mergers of gas-poor galaxies, we expect MBH binaries with less massive and thinner circumbinary disks in which a gap will be easily opened. In such cases, the MBH–disk interaction will be better described by Type II simulations (Artymowicz & Lubow 1994, 1996; MacFadyen & Milosavljevic 2008; Cuadra et al. 2009). However, in order to make concrete predictions, high-resolution galaxy merger simulations that include feedback processes from star formation and BH accretion are needed to study under which conditions the criteria given by Equation (4) are fulfilled or not and whether the binary–disk interaction will be Type I or II.

Our study also has implications for the formation of gaps around binary protostars, in which both Type I and II interactions can be present at different epochs of their formation. In simulations of early epochs in the formation of binary stars, for example, Boss (1982) and Bate & Bonnell (1997), who studied the evolution of binary protostellar seeds within a collapsing cloud, have found that the interaction of the binary with the ambient gas is through a non-axisymmetric overdensity like the one studied in this paper as Type I. On the other hand, in studies of the final stages of the formation of binary stars, simulations of protoplanetary disks around binary stars recurrently found gaps that produce disk truncation at $r_i \sim 3/2a$ (Artymowicz & Lubow 1994, 1996; Gunther & Kley 2002). Between both limiting epochs in the formation of binary stars, there must be a transition epoch that is given by the threshold condition studied in this paper (Equation (4)). When this transition occurs during binary formation, there must be implications for the final separations and masses of the binaries, since it corresponds to a dramatic transition in both the migration and accretion timescales.

6. CONCLUSIONS

We study the interaction of equal mass binaries with an isothermal gas disk. We focus on the torques that the binary produces on the disk and how this exchange of angular momentum can drive the formation of a gap on it.

We propose that the angular momentum exchange between the binary and the disk is through the gravitational interaction of the binary and a tidally formed strong non-axisymmetric perturbation in the disk. From this gravitational torque we derive an analytic criterion to determine whether or not the binary will open a gap in the disk.

The analytic gap-opening criterion that we derive depends on two dimensionless parameters ($(v_{\text{bin}}/v)^2$ and $(c_s/v)^3$), where $v$ is the rotational velocity of the binary–disk system, $v_{\text{bin}}$ is the rotational velocity of the isolated binary, and $c_s$ is the sound speed of the disk. We also show that, through a transformation of variables, the criterion can be rewritten into other two parameters: the first is a function of the enclosed to binary mass ratio $(1 + M(< r)/M_{\text{bin}})$ and the second is the cubic ratio between the disk thickness and the binary separation $(h/r)^3$.

Using SPH simulations we show that the simulation where the binary opened a gap in the disk (opened simulations) and the simulations where the disk does not have a gap (closed simulations) are distributed into two well separated regions in parameter space ($(v_{\text{bin}}/v)^2$, $(c_s/v)^3$). Our analytic gap-opening criterion predicts a linear shape of the threshold between these two regions and our SPH simulations allow us to determine the value of its slope. The value that we find for the slope is roughly 0.095.

To increase the confidence in our results even more, we test our analytic gap-opening criterion against simulations in the literature and we find a good agreement between our gap-opening criterion and these simulations.

Our simulations assume only one type of initial condition for the binary–disk system and we leave for another work the study of the possible effects of different initial conditions on the results. However, we emphasize that for an isolated binary–disk system the gap-opening/closing criterion depends on the local parameters at a given time (e.g., the velocity of the binary–ellipsoid system) but how this local parameter evolves through time depends on the initial conditions; for that reason, our plan in the future is to vary them.

Finally, we discuss how our results can be applied to the study of the formation of gaps on circumbinary gas disks around binary MBHs and protostars. We discuss the implications of the formation of a gap on the migration and accretion timescales of these systems. Despite the generality of the process that drives the formation of a gap, we remark that it is mandatory to explore how more realistic simulations could affect the gap-opening criterion.

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and its core radius is roughly at 0.6 because in this region the real enclosed mass is less than the mass that is assumed by the point mass distribution. This overestimation opens an artificial gap in the center of the disk. The sphere of constant density (3) also generates an overestimation of the velocity in the region r < r0 but this overestimation is less intense than the overestimation due to the point mass distribution, because, for the homogeneous sphere, the enclosed mass decreases with radius.

We select the sphere of constant density as the spherical mass distribution for computing the initial velocity, because it approximates the potential of the binary better and the system starts closer to a rotationally supported equilibrium. The system is of course not in perfect equilibrium and our approximation of the initial circular velocity leads to an initial readjustment of the density profile in about an orbital time. For that reason, we only analyze the simulations after two initial orbits of the binary, that is, after the system has relaxed into a quasi-equilibrium configuration.

We also include a static Plummer potential to increase the stability of the disk. This static potential decreases the overestimation in the inner region of the disk, reducing the initial gas losses in this region and the artificial formation of a gap. In the edges of the disk the cutoff of the disk produces a pressure gradient that drives the expansion of the disk edges. This flow of gas is also reduced due to the gravitational influence of this static potential. We use a static Plummer potential that has a total mass of 12% of the disk mass initially enclosed by the orbit of the binary.

APPENDIX A

INITIAL CONDITIONS

In our set of simulations, the binary produces a strong non-axisymmetric component on the total gravitational field. Therefore, for the SPH particles near the binary there is no well-defined circular velocity v(r). This is an inherent problem for the initialization of this type of simulation and we chose to solve this by approximating the binary potential by a spherical mass distribution with the same total mass to compute the initial circular velocity. These spherical representations allow us to compute an initial circular velocity for all the particles of the disk, but the system will not start in perfect equilibrium.

We probe different spherical mass distributions to study the effect of the initial velocity on the evolution of the system and then we select the mass distribution that produces the minor overestimation or underestimation of the equilibrium velocity. The spherical mass distributions we study are (1) a spherical shell, (2) a point mass distribution, and (3) a sphere of constant density. The spherical shell (1) produces a discontinuity on the velocity that generates a ring-shaped gap due to an overestimation of the velocity in the region where the gap forms. The point mass distribution (2) overestimates the velocity for all the particles in the region r < r0 because in this region the real enclosed mass is less than the mass that is assumed by the point mass distribution. This overestimation opens an artificial gap in the center of the disk. The sphere of constant density (3) also generates an overestimation of the velocity in the region r < r0 but this overestimation is less intense than the overestimation due to the point mass distribution, because, for the homogeneous sphere, the enclosed mass decreases with radius.

We select the sphere of constant density as the spherical mass distribution for computing the initial velocity, because it approximates the potential of the binary better and the system starts closer to a rotationally supported equilibrium. The system is of course not in perfect equilibrium and our approximation of the initial circular velocity leads to an initial readjustment of the density profile in about an orbital time. For that reason, we only analyze the simulations after two initial orbits of the binary, that is, after the system has relaxed into a quasi-equilibrium configuration.

We also include a static Plummer potential to increase the stability of the disk. This static potential decreases the overestimation of the rotational velocity in the inner region of the disk, reducing the initial gas losses in this region and the artificial formation of a gap. In the edges of the disk the cutoff of the disk produces a pressure gradient that drives the expansion of the disk edges. This flow of gas is also reduced due to the gravitational influence of this static potential. We use a static Plummer potential that has a total mass of 12% of the disk mass and its core radius is roughly at 0.6 Rdisk. With this selection of parameters, the Plummer potential’s mass initially enclosed by the orbit of the binary is 50% of the total enclosed mass.

APPENDIX B

RESOLUTION STUDY

Our results are derived from the comparison of an analytic criterion for gap opening and SPH numerical simulations. In this figure we plot four simulations with the same initial condition but a different number of SPH particles: 2 × 10^5, 5 × 10^4, 8 × 10^5, and 1 × 10^6. The green line is the interface between the simulations where a gap is opened in the disk and the simulations where the disk does not have a gap. Although the evolution of the orbital parameters of the binary is not exactly the same for high- and low-resolution simulations, the global structure of the disk remains almost unchanged and the region of parameters that the low- and high-resolution simulations cover is very similar. The classification of these simulation remains the same for high and low resolution.

(A color version of this figure is available in the online journal.)
the simulations presented in Section 3, the disk is represented by a collection of $2 \times 10^5$ SPH particles. In order to check that we have the resolution required and that our results do not depend on that, we run four additional simulations with different numbers of SPH particles: $5 \times 10^5$, $8 \times 10^5$, and two with $10^6$.

Although the evolution of the orbital parameters of the binary is not exactly the same for high- and low-resolution simulations, the global structure of the disk remains almost practically unchanged. The region of parameters that the low- and high-resolution simulations cover, in the velocity spaces of the parameters, is very similar (Figure 5) and the classification of these simulations (as closed or opened) remains the same. For the initial condition that we run with $2 \times 10^2$, $5 \times 10^5$, $8 \times 10^5$, and $10^6$ we find that the values of the velocity parameters converge with resolution (Figure 6).

For the high-resolution simulations we find a small shift to higher values of $(c_s/v)^1$. This shift can change the value of the slope of the threshold between the closed and the opened simulations ($f$) by 10% or less. Therefore, our overall conclusions for the testing of our analytic gap-opening criterion against SPH simulations remain unchanged.

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