A Rapid Power-iterative Root-MUSIC Estimator for Massive/Ultra-massive MIMO Receiver

Qijuan Jie, Xuehui Wang, Peng Chen, Yiwen Chen, Feng Shu, and Peng Zhang

Abstract—For a passive direction of arrival (DOA) measurement system using massive multiple input multiple output (MIMO), the complexity of the covariance matrix decomposition-based DOA measurement method is extremely high. To significantly reduce the computational complexity, two strategies are proposed. Firstly, a rapid power-iterative estimation of signal parameters via rotational invariance technique (RPI-ESPRIT) method is proposed, which not only reduces the complexity but also achieves good directional measurement results. However, the general complexity is still high. In order to further the complexity, a rapid power-iterative root Multiple Signal Classification (RPI-Root-MUSIC) method is proposed. Simulation results show that the two proposed methods outperform the classical DOA estimation method in terms of computational complexity. In particular, the lowest complexity achieved by the RPI-Root-MUSIC method is about two-order-magnitude lower than that of Root-MUSIC in terms of FLOP. In addition, it is verified that the initial vector and relative error have a substantial effect on the performance of computational complexity.

Index Terms—DOA measurement, power-iterative, massive MIMO, covariance matrix decomposition, computational complexity

I. INTRODUCTION

Direction of arrival (DOA) estimation is widely used in many engineering applications, including wireless communications, radar, and rescue and other emergency assistance devices [1]-[2]. DOA is a key technology, which plays an important role in many emerging applications, such as the Internet of Things (IoT), unmanned aerial vehicle (UAV) communications [3], millimeter-wave communication [4], and massive multiple-input multiple-output (MIMO) systems.

It is well-known that DOA estimation problems can be divided into two main categories: subspace-based methods and parametric methods. The beamforming method makes the output signal coherent, so that an estimated angle can be obtained by maximizing the average output power. The Multiple Signal Classification (MUSIC) was proposed in [5], which obtained the estimated angle with high accuracy by maximizing the spatial spectral function. However, the above two methods perform DOA estimation by linear exhaustive search, which have high computational complexity. To avoid spectral search in MUSIC, the authors in [6] proposed a MUSIC-based search-free method, which was a “super-resolution” estimation method called root-MUSIC [7]-[8]. In this method, the spectrum search is replaced by the root solution. Another well-known search-free method is estimation of signal parameters via rotational invariance technique (ESPRIT), which not only reduces the complexity but also achieves good directional measurement results [9]-[10].

However, due to the fact that the number of antennas tends to be massive in MIMO system, the computational complexity and circuit cost are too high for commercial applications. Therefore, hybrid analog and digital (HAD) beamforming structures using parametric method to estimate DOA have emerged, which can achieve a good balance between beamforming computation, circuit cost, and complexity, using parametric method to estimate DOA. Authors in [11] investigated the DOA estimation using HAD structure in the receiver part and proposed two phase alignment (PA) methods: HAD-PA and hybrid digital and analog PA (HDA-PA). Meanwhile, for this hybrid structure, a fast root-MUSIC-HDAPA method was proposed to achieve an approximate analytical solution and reduce the computational complexity. For the DF ambiguity problem caused by HAD MIMO, a fast ambiguity phase elimination method was proposed in [12], which uses only two data blocks to achieve DOA estimation. In [13], the DOA estimation problem in the case of 1-bit ADC was considered. It demonstrated that the MUSIC method could be directly applied in the case without additional preprocessing, while the system performance degradation was analyzed. Then, the DOA estimation performance of the low-resolution ADC structure was investigated in [14]. In recent years, deep learning network (DNN) has been applied to DOA estimation, authors proposed a DNN-based DOA and channel estimation schemes in [15]-[17], which achieved better performance.

Aiming at rapidly estimating and tracking the main subspaces and major components of a vector sequence in [18], a projection approximation and subspace tracking (PAST) method was proposed. Furthermore, the proposed PAST method in [18] was improved in [19]. It proved that the improved PAST method was better in both subspace estimation and computational complexity. In [20], an improved power iteration (PI) method for modal analysis was proposed. The simulation results showed that the method significantly reduced the number of unnecessary iterations with a faster computational speed.

Inspired by the idea of radar target detection, we considered a new SVD-based passive target detection model in [21], which achieved better detection performance. While the complexity of massive MIMO based on covariance matrix decom-
position method was extremely high. For example, when the number of antennas are closed to 10000, and the computational complexity was tera(T) FLOPs. Therefore, how to significantly reduce the computational complexity of direction finding for a fast and high-performance DOA estimation is an extremely challenging problem, which is also the key to the future application of massive MIMO DOA estimation. Therefore, in this paper, we have proposed a fast convergent RPI method to achieve high performance with low complexity. The main contributions in this paper are summarized as follows:

1) In order to significantly reduce the computational complexity of DOA estimation for a fast and high-performance direction finding, two power iteration-based DOA estimation methods are respectively proposed, which are called RPI-ESPRIT and RPI-Root-MUSIC. Here, the sampling covariance of the received signal vector is first computed. An initial vector subjected to power iteration is determined, which replaces the traditional EVD. The simulation results show that PI-ESPRIT can obtain better DOA accuracy and lower computational complexity than ESPRIT, while its overall computational complexity is still high. The overall effect of RPI-Root-MUSIC is better than that of Root-MUSIC.

2) To reduce the number of unnecessary iterations and obtain a faster computation speed, the optional initial vectors are selected which can converge to the desired results. In each iteration, it must be ensured that the initial vector is not orthogonal to the incident wave, and it is better to keep them away from orthogonality. Moreover, the computation result often has a great relationship with the relative error. When a good initial vector and relative error are determined, the results with fast convergence and less iterations can be achieved.

The remainder of this paper is organized as follows. Section II describes the system model of the rapid power-iterative Root-MUSIC estimator for massive/ultra-massive MIMO receiver. In Section III, two methods are proposed, and their performance and computational complexities are also analyzed. We present our simulation results in Section IV. Finally, we draw conclusions in Section V.

Notations: Throughout the paper, x and X in bold typeface are used to represent vectors and matrices, respectively, while scalars are presented in normal typeface, such as x. Signs (·)T and | · | represent conjugate transpose and modulus, respectively. I_N denotes the N × N identity matrix. Furthermore, E[·] represents the expectation operator, and x ∼ CN(m, Φ) denotes a circularly symmetric complex Gaussian stochastic vector with mean vector m and covariance matrix Φ. ẑ represents the estimated value of x.

II. SYSTEM MODEL

Fig. 1 sketches a system of the rapid power-iterative Root-MUSIC estimator for massive/ultra-massive MIMO receiver. A uniformly-spaced linear array (ULA) with N antennas is considered, and the narrow band signals x(t)ej2πfct from a far-field emitter will arrive at the array, where x(t) is the baseband signal. Then the antennas will capture the signal with different time delays depended on the DOAs. It is assumed that all signals are narrowband with the same carrier frequency f_c. Therefore, the propagation delay of the mth antenna element is expressed as

\[ \tau_m = \tau_0 - \left( d_m / c \right) \sin \theta_0, \quad m = 1, 2, \ldots, N, \]

where \( \tau_0 \) is the propagation delay from the emitter to a reference point on the array, \( \theta_0 \) is the direction of the emitter relative to the line perpendicular to the array. \( d_m \) is the distance from the mth array element to the reference point, and c is the speed of light. Without loss of generality, we can assume that \( \tau_0 = 0 \). Thus, \( \tau_m = -\left( d_m / c \right) \sin \theta_0, m = 1, 2, \ldots, N. \)

A single emitter from the DOA \( \theta_0 \) can be modeled by

\[ y(t) = a(\theta_0) s(t) + v(t), \]

where \( v(t) \sim \mathcal{CN}(0, \sigma_v^2 I_N) \) is the additive white Gaussian noise (AWGN) vector, and \( a(\theta_0) \) only composed by the phase difference of all antennas is called array manifold, defined by

\[ a(\theta_0) = [e^{j2\pi d_1 \sin \theta_0 / \lambda}, e^{j2\pi d_2 \sin \theta_0 / \lambda}, \ldots, e^{j2\pi d_N \sin \theta_0 / \lambda}]^T. \]

The basic assumption of ESPRIT algorithm is that there exist two identical subarrays and the spacing \( \Delta \) of the two subarrays is known [10]. Since the structures of the two subarrays are identical and the number of subarray elements is \( m \), for the same signal, the outputs of the two subarrays have only one phase difference \( \phi_i, i = 1, 2, \ldots, M. \)

The following assumes that the received data of the first subarray is \( X_1(t) \), and the received data of the second subarray is \( X_2(t) \). Combine the two subarrays

\[ \mathbf{X}(t) = \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} = \begin{bmatrix} A & A\Phi \end{bmatrix} \mathbf{S} + \mathbf{V} = \bar{\mathbf{A}} \mathbf{S} + \mathbf{V}, \]

Fig. 1. System model of the rapid power-iterative Root-MUSIC estimator for massive/ultra-massive MIMO receiver.
where $\Phi = diag(e^{j\phi_1}, e^{j\phi_2}, \ldots, e^{j\phi_M})$ is a rotation invariant matrix, which contains the direction of arrival information of the incoming wave signal, and $\phi_i = 2\pi d \sin \theta_i / \lambda$.

In practice, although the ideal covariance matrix $R$ cannot be obtained, its estimated value is given by

$$R = \frac{1}{K} \sum_{n=1}^{K} y(n)y^H(n),$$  

(5)

where $K$ is the number of sampling points.

Make the eigenvalue decomposition of $R$

$$R = \sum_{i=1}^{2m} \lambda_i e_i e_i^H = U_S \Sigma_S U_S^H + U_V \Sigma_V U_V^H.$$  

(6)

The CRLB is the minimum variance of DOA estimation errors. It can provide a useful characterization of the achievable accuracy of the systems. According to [1], for an $N$-element linear array we have

$$CRB = \frac{\lambda^2}{8\pi^2 KSNR \cos^2 \theta_d d^2}$$  

(7)

where

$$d^2 = \sum_{m=1}^{N} d_m^2$$  

(8)

and SNR is the signal-to-noise ratio of the signal received at each antenna.

III. PROPOSED A RAPID POWER-ITERATIVE ESTIMATOR FOR MASSIVE/ULTRA-MASSIVE MIMO RECEIVER

The Root-MUSIC proposed in [1] is a well-known subspace-based method for DOA estimation. By replacing the spectral search with finding roots of polynomials, it has a lower complexity without loss of performance. As a result, it is widely used in many scenarios and has attracted much attention over the years. Therefore, in this section, Root-MUSIC method is extended to a rapid power-iterative Root-MUSIC (RPI-Root-MUSIC) method, and we demonstrate that the DOA estimation complexity can be effectively reduced without eigenvalue-decomposition (EVD). Similarly, a rapid power-iterative ESPRIT (RPI-ESPRIT) method is compared [10]. Standard ESPRIT is also a high-resolution subspace-based method, but in contrast to MUSIC, it applies to the signal subspace rather than the noise subspace.

A. Proposed RPI-ESPRIT method

It assumes that the $N \times N$ covariance matrix $R_y$ has $N$ eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_N$ with an associated collection of linearly independent eigenvectors $\{u_1, u_2, \ldots, u_N\}$. Moreover, we assume that $R$ has precisely one eigenvalue $\lambda_1$, which is the largest in magnitude, i.e.,

$$|\lambda_1| > |\lambda_2| > |\lambda_3| > \cdots > |\lambda_N| \geq 0.$$  

(9)

There exists a random vector $v^0 \in \mathbb{R}^n$ that satisfies

$$v^0 = \sum_{k=1}^{N} \alpha_k u_k,$$  

(10)

where $\alpha_1, \alpha_2, \ldots, \alpha_N$ ($\alpha_1 \neq 0$) are scalars.

Taking the vector $v^0$ as the initial vector of the RPI-ESPRIT method, and multiplying both sides of this equation by $R$, $R^2$, $\ldots$, $R^n$, $\ldots$ gives

$$v^1 = Rv^0 = R \sum_{k=1}^{N} \alpha_k u_k = \sum_{k=1}^{N} \alpha_k R_k u_k = \sum_{k=1}^{N} \alpha_k \lambda_k u_k,$$

$$v^2 = Rv^1 = R^2 v^0 = R^2 \sum_{k=1}^{N} \alpha_k u_k = \sum_{k=1}^{N} \alpha_k \lambda_k^2 u_k,$$

$$\ldots$$

$$v^n = R^n v^0 = R^n \sum_{k=1}^{N} \alpha_k u_k = \sum_{k=1}^{N} \alpha_k \lambda_k^n u_k.$$  

(11)

The $v^n$ can be rewritten as

$$v^n = \lambda_1^n (\alpha_1 u_1 + \sum_{k=2}^{N} \alpha_k (\lambda_1^{-n}) \lambda_k) u_k = \lambda_1^n (\alpha_1 u_1 + \varepsilon_n),$$  

(12)

where

$$\varepsilon_n = \sum_{k=2}^{N} \alpha_k (\lambda_1^{-n}) \lambda_k u_k.$$  

(13)

Since $|\lambda_1| > |\lambda_k|$ for all $k = 2, 3, \ldots, N$, we have

$$\lim_{n \to \infty} \varepsilon_n = \lim_{n \to \infty} \sum_{k=2}^{N} \alpha_k (\lambda_1^{-n}) \lambda_k u_k = 0,$$  

(14)

therefore,

$$\lim_{n \to \infty} v^n = \lim_{n \to \infty} \lambda_1^n \alpha_1 u_1.$$  

(15)

The eigenvector corresponding to the main eigenvalue $\lambda_1$ is

$$u_{\lambda_1} = \lim_{n \to \infty} \varepsilon_n = \lim_{n \to \infty} \lambda_1^n (\alpha_1 u_1 + \varepsilon_n) = \alpha_1 u_1,$$  

(16)

$\lambda_1$ is expressed by the limit of the ratio of the $i$th component of vector $v^{n+1}$ to the $i$th component of vector $v^n$, we have

$$\lim_{n \to \infty} (\varepsilon_{n+1})_i = \lim_{n \to \infty} \frac{(\lambda_1^{n+1} (\alpha_1 u_1 + \varepsilon_{n+1})_i)}{(\lambda_1^n (\alpha_1 u_1 + \varepsilon_n)_i)} = \lim_{n \to \infty} \frac{\lambda_1 [\alpha_1 (u_1)_i + (\varepsilon_{n+1})_i]}{\alpha_1 (u_1)_i + (\varepsilon_n)_i} = \lambda_1,$$  

(17)

That is, the estimator $\hat{\lambda}_1$ of $\lambda_1$ is

$$\hat{\lambda}_1 = \frac{(\varepsilon_{n+1})_i}{(\varepsilon_n)_i}.$$  

(18)

The key problem solved by the ESPRIT algorithm is the proper use of the translation-invariant property of the linear array, so that the eigenvalues of the rotation-invariant matrix can be found to estimate the signal incidence angle.

Based on (11) and (16), we perform PI method on the covariance matrix $R$, and use the calculated $u_{\lambda_1}$ and $u_{\lambda_2}$ to estimate the real signal subspace $U_{S1}$ and $U_{S2}$, we have

$$\Psi = (u_{\lambda_1})^H u_{\lambda_2} = (u_{\lambda_1})^H u_{\lambda_1}^{-1} u_{\lambda_1}^H u_{\lambda_2},$$  

(19)

From (16), we regard the obtained $u_{\lambda_1}$ as the signal subspace, So

$$I = u_{\lambda_1} u_{\lambda_1}^H + U_V U_V^H,$$  

(20)
We have
\[ U_V = I - u_{\lambda_i}(u_{\lambda_i}^H u_{\lambda_i})^{-1} u_{\lambda_i}^H, \] (21)
the spatial spectral function is defined as
\[ S(\theta) = \frac{1}{|a^H(\theta)U_V|^2}. \] (22)

The spectral peak of \( S(\theta) \) is the root of the function \( f(z) = 1 + \sum_{i=1}^{M} u_{\lambda_i}^H R u_{\lambda_i} \approx 0 \) closest to the unit circle. When the \( M \) roots closest to the unit circle are obtained as \( z_i, i = 1, 2, \cdots, M \), the estimated angle can be obtained as
\[ \hat{\theta}_i = \arcsin\left(\frac{\text{arg} z_i}{2\pi M N d}\right). \] (23)

**B. Selecting initial vector and relative error**

The initial vector \( v_0 \) has a direct effect on the speed of convergence, thereby the iterations are affected. Generally, the faster the convergence, the fewer iterations. Usually a randomly generated vector is selected as the initial iteration vector, but not all the optional initial vectors can converge to obtain the desired results. In each iteration, it must be ensured that the initial vector is not orthogonal to the incident wave, and it is better to keep them away from orthogonality.

From (10) and (12), we make the \( v_0 \) as
\[ v_0 = \sum_{j=1}^{N} \frac{R_{ij}}{S(R)} \] (24)
where \( S(R) \) is the sum of all elements in matrix \( R \), \( S(R) = \sum_{i=1}^{N} \sum_{j=1}^{N} R_{ij} \).

The element distribution of initial vector \( v_0 \) in (24) is consistent with the distribution of matrix \( R \), while the distribution is uniform, which can keep them far away from orthogonality and speed up the convergence of iteration.

And the relative error \( \varepsilon \) as
\[ \varepsilon = \frac{|\varepsilon_n|}{|\alpha_1 u_1|} = \frac{\left| \sum_{k=2}^{N} \alpha_k \left(\frac{\lambda_k}{\lambda_1}\right)^n u_k \right|}{|\alpha_1 u_1|} \] (25)
and
\[ \left| \sum_{k=2}^{N} \alpha_k \left(\frac{\lambda_k}{\lambda_1}\right)^n \right| \leq \left| \sum_{k=2}^{N} \alpha_2 \left(\frac{\lambda_2}{\lambda_1}\right)^n \right| \] (26)

Therefore, we have
\[ \varepsilon \leq (N - 1) \left(\frac{\lambda_2}{\lambda_1}\right)^n \frac{\alpha_2}{\alpha_1} \] (27)
\[ \log_2 \varepsilon \leq \log_2(N - 1) + n \log_2 \left(\frac{\lambda_2}{\lambda_1}\right) + \log_2 \frac{\alpha_1}{\alpha_2} \]

Available from \( \alpha_1 \geq \alpha_2 \),
\[ \log_2 \varepsilon \leq \log_2(N - 1) + n \log_2 \left(\frac{\lambda_2}{\lambda_1}\right) \] (28)
Since \( |\lambda_1| > |\lambda_2| \),
\[ n \leq \frac{\log_2 \varepsilon - \log_2(N - 1)}{\log_2 \left(\frac{\lambda_2}{\lambda_1}\right)} \] (29)

When the iteration approaches the final convergence, the convergence speed will be relatively slow and stable.

The whole procedure is summarized in Algorithm 1.

**Algorithm 1  PI method on subspace**

*Input:* \( R \) and \( \varepsilon \)

*Output:* \( \lambda_1, v^n \) and \( n \)

*Initialization:* choose a initial vector \( v^0 \), and \( n=1 \).

**repeat**
1. \( n = n+1 \).
2. Update \( v^0 \), \( v^0 = v^1 \);
3. Update \( v^1 \);
**until** \( \|\text{max}(v^0) - \text{max}(v^1)\| \leq \varepsilon \).

*Return* \( \lambda_1, v^n \) and \( n \)

where \( n \) is the number of iterations, \( \lambda_1 \) is dominant eigenvalue, and \( v^n \) is the dominant eigenvector corresponding to \( \lambda_1 \).

Notice that in each iteration we compute a single matrix-vector multiplication \( (O(N^2)) \). We never perform matrix-matrix multiplication, which requires greater number of operations \( (O(N^3)) \). If the matrix \( R \) is sparse (only a small portion of the entries of \( A \) are non-zero), matrix-vector multiplication can be performed very efficiently. Therefore, the power method is practical even if \( N \) is very large, such as in Google’s Page Rank algorithm.

**C. Complexity analysis**

Now, analyze the computational complexity of the proposed RPI-Root-MUSIC method with complexity order \( NK \) floating point operations (FLOPs) as a complexity benchmark. Compared with traditional Root-MUSIC methods, the proposed method achieves a lower computational complexity, only need to with \( O(N^2K) \) FLOPs, as shown in the next section. Moreover, the traditional DOA estimation methods require an incremental computation about \( O(N^3 + N^2K) \) FLOPs, which includes computing covariance matrix and its EVD. In particular, as \( N \) tends to large-scale, its complexity becomes huge.

**IV. Simulation results and discussion**

\[ RMSE = \sqrt{\frac{1}{L} \sum_{i=1}^{L} (\hat{\theta}_i - \theta_0)^2} \] (30)

In this section, we provide the theoretical performance simulation to analyze the performance of the massive MIMO DOA estimation systems, which are based on the power iteration method. Furthermore, the simulation of root mean square error (RMSE) using Root-MUSIC is conducted.

Moreover, we consider the effect of SNR, the number of antennas and the number of snapshots on the Root-MUSIC, ESPRIT in the digital ADC architecture. In each simulation figure, the number of estimation value \( L \) is 2000. Without loss of generality, we assume that a emitter is located in \( \theta_0 = \).
50°, the number of snapshots $K$ is 1000, antennas distance $d = \lambda/2$, and the number of antenna elements $N$ is 64 in the massive MIMO system.

Fig. 2 plots the RMSE curves using four different methods versus SNR with CRLB as the performance benchmark. Observing Fig. 2 it is clear that Root-MUSIC can achieve the associated CRLB, and the RPI-Root-MUSIC method yields the same results, but with much lower computational complexity. Meanwhile, the simulation results are the same when ESPRIT is compared to Root-MUSIC, and the complexity of the RPI-ESPRIT is much lower than that of the traditional ESPRIT.

Fig. 3 and Fig. 4 present the RMSE of four different methods versus the number of antennas $N$ and snapshots $K$. Without loss of generality, we assume that $N \in [16, 272]$ and $K \in [100, 3600]$. From the Fig. 3 and Fig. 4 it can be seen that the improved DOA estimation method based on the power iteration method not only achieves slightly better performance than the traditional method, but also effectively reduce the complexity.

To explore the influence of the selection of the initial vector on the number of iterations and the convergence speed, Fig. 5 shows the relationship between the optimal eigenvector value $\nu^n$ and the number of iterations $n$, given three different initial vectors $\nu^0$. When the initial vector $\nu^0$ is infinitely close to the signal subspace, only two iterations are needed to complete the convergence, and the result is similar to that of the initial vector selected in (24). In addition, when the $\nu^0$ obeys a random vector distribution, the required number of iterations $n$ is more, which requires an average of eight iterations to reach the convergence of the $\max(\nu^n)$.

Fig. 6 plots curves of complexity analysis versus the number of antennas $N$ with $K = 50$, SNR=0dB. From this figure, it can be seen that the complexity of all methods gradually increases as the total number of antennas increases. However, compared to the Root-MUSIC method, the computational complexity of our proposed RPI-Root-MUSIC method is about two-order-magnitude lower when $N=1024$. 

In this paper, in order to reduce the computational complexity of DOA estimation using massive MIMO and to obtain better estimated performance, two high-performance schemes, namely RPI-Root-MUSIC and RPI-ESPRIT were proposed. Simulation results show that the proposed two methods can harvest lower complexity than the classical algorithm. Because of excellent low complexity, the proposed methods is very attractive. Additionally, by determining good initial vectors and relative errors, results with fast convergence can be achieved.

**REFERENCES**

[1] T. E. Tuncer and B. Friedlander, *Classical and Modern Direction-of-Arrival Estimation*. Academic Press, 2009.

[2] F. Shi, S. Yang, J. Lu, and J. Li, “On impact of earth constraint on TDOA-based localization performance in passive multisatellite localization systems,” *IEEE Systems Journal*, vol. 12, no. 4, pp. 3861–3864, 2018.

[3] Y. Li, F. Shu, B. Shi, X. Cheng, Y. Song, and J. Wang, “Enhanced rss-based UAV localization via trajectory and multi-base stations,” *IEEE Commun. Lett.*, vol. 25, no. 6, pp. 1881–1885, 2021.

[4] J. Zhao, S. Ni, L. Yang, Z. Zhang, Y. Gong, and X. You, “Multiband cooperation for 5g hetnets: A promising network paradigm,” *IEEE Veh. Technol. Mag.*, vol. 14, no. 4, pp. 85–93, 2019.

[5] R. O. Schmidt, “A signal subspace approach to multiple emitter location and spectral estimation,” Ph.D. dissertation, Stanford Univ., 1981.

[6] A. Barabell, “Improving the resolution performance of eigenstructure-based direction-finding algorithms,” in *ICASSP ’83. IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 8, 1983, pp. 336–339.

[7] F. Yan, M. Jin, S. Liu, and X. Qiao, “Real-valued MUSIC for efficient direction estimation with arbitrary array geometries,” *IEEE Trans. Signal Process.*, vol. 62, no. 6, pp. 1548–1560, 2014.

[8] M. Pesavento, A. Gershman, and M. Haardt, “Unitary root-MUSIC with a real-valued eigendecomposition: a theoretical and experimental performance study,” *IEEE Trans. Signal Process.*, vol. 48, no. 5, pp. 1306–1314, 2000.

[9] A. Paulraj, R. Roy, and T. Kailath, “A subspace rotation approach to signal parameter estimation,” *Proceedings of the IEEE*, vol. 74, no. 7, pp. 1044–1046, 1986.

[10] R. Roy, A. Paulraj, and T. Kailath, “ESPRIT—a subspace rotation approach to estimation of parameters of cissoids in noise,” *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 34, no. 5, pp. 1340–1342, 1986.

[11] F. Shu, Y. Qin, T. Liu, L. Gui, Y. Zhang, J. Li, and Z. Han, “Low-complexity and high-resolution DOA estimation for hybrid analog and digital massive MIMO receive array,” *IEEE Trans. Commun.*, vol. 58, no. 6, pp. 2487–2501, 2010.

[12] B. Shi, N. Chen, J. Teng, Ying. Lu, F. Shu, J. Zou, J. Li, and J. Wang, “Fast ambiguous DOA elimination method of DOA measurement for hybrid massive MIMO receiver,” *Science China. Information Sciences*, vol. 9, no. 1, pp. 83–86, 2020.

[13] X. Huang and B. Liao, “One-Bit MUSIC,” *IEEE Signal Process. Lett.*, vol. 26, no. 7, pp. 961–965, 2019.

[14] B. Shi, N. Chen, X. Zhu, Y. Qian, Y. Zhang, F. Shu, and J. Wang, “Impact of low-resolution ADC on DOA estimation performance for massive MIMO receive array,” *IEEE Systems Journal*, pp. 1–4, 2022.

[15] H. Huang, J. Yang, H. Huang, Y. Song, and G. Gui, “Deep Learning for Super-Resolution Channel Estimation and DOA Estimation Based Massive MIMO System,” *IEEE Trans. Veh. Technol.*, vol. 67, no. 9, pp. 8549–8560, 2018.

[16] D. Hu, Y. Zhang, L. He, and J. Wu, “Low-Complexity Deep-Learning-Based DOA Estimation for Hybrid Massive MIMO Systems With Uniform Circular Arrays,” *IEEE Wireless Commun. Lett.*, vol. 9, no. 1, pp. 83–86, 2020.

[17] Z. Zhuang, L. Xu, J. Li, J. Hu, and J. Wang, “Machine-learning-based high-resolution DOA measurement and robust directional modulation for hybrid analog-digital massive MIMO transceiver,” *Science China. Information Sciences*, vol. 63, no. 8, 2020.

[18] K. Abed-Meraim, A. Chkeif, and Y. Hua, “Fast orthonormal PAST algorithm,” *IEEE Signal Process. Lett.*, vol. 7, no. 3, pp. 60–62, 2000.

[19] R. Badeau, B. David, and G. Richard, “Fast approximated power iteration subspace tracking,” *IEEE Trans. Signal Process.*, vol. 53, no. 8, pp. 2931–2941, 2005.

[20] Z. Li, H. Hu, Y. Zhou, and Z. He, “A rapid modal analysis method for harmonic resonance using modified power iteration,” *IEEE Trans. Power Del.*, pp. 1–1, 2016.

[21] Q. Jie, X. Zhan, F. Shu, Y. Ding, B. Shi, Y. Li, and J. Wang, “High-performance passive eigen-model-based detectors of single emitter using massive MIMO receivers,” *IEEE Wireless Commun. Lett.*, pp. 1–1, 2022.