Predicting Feasible Modifications of Ce$_2$ON$_2$ Using a Combination of Global Optimization and Data Mining

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Abstract Using a combination of global optimization and data mining, we identify feasible modifications of an ionic Ce-O-N ceramic compound, with composition Ce$_2$ON$_2$, that should at least be metastable at $T = 0$ K. The energy landscape of Ce$_2$ON$_2$ has been explored for various pressures using empirical potentials followed by ab initio level optimizations, and a multitude of structure candidates has been analyzed. The structure of the energetically lowest modification among these candidates at standard pressure, $\alpha$-Ce$_2$ON$_2$, is predicted to be similar to the AlCo$_2$Pr$_2$ structure type.

Keywords Ce-O-N compounds · computational studies · data mining · global optimization · structure prediction

1 Introduction

An important method for controlling the properties of oxide photocatalysts and tuning their band gaps is nitrogen doping.$^{[1]}$ Similarly, stoichiometric oxynitrides can have useful optical properties such as photocatalytic TaON$^{[2]}$ and yellow–red pigments in the solid solution series CaTaO$_2$N-LaTaON$_2$.$^{[2]}$ Furthermore, incorporation of nitrogen into ZrO$_2$ has been reported as an alternative method for cation doping that increases the anion vacancy concentration and stabilizes the cubic or tetragonal form at room temperature.$^{[3]}$ The nitrogen doped oxides show good mechanical, catalytical, and optical properties and have been reported as superionic conductors.$^{[3]}$

Another important ceramic material is pure ceria (CeO$_2$), which has important technological applications, e.g., as catalysts, in electrolyte solid oxide fuel cells, oxygen storage components, mechanical polishing for

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To predict possible (meta)stable structure candidates in the ceramic compound, which should be accessible to different oxidation states (ward due to the fact that cerium forms compounds with two accessible to an experimental synthesis is not straightforward. Clearly, finding a stable ionic Ce-O-N ceramic compound would be very interesting from a scientific and technological point of view, especially since this kind of material might possess many of the advanced properties mentioned above.

However, identifying (meta)stable Ce-O-N compounds accessible to an experimental synthesis is not straightforward due to the fact that cerium forms compounds with two different oxidation states (+III) and (+IV), as well as with mixed valence. In the literature there exists only one known Ce-O-N compound, Ce(NO)3 containing the complex anion (NO)1−, which crystallizes in the monoclinic C2/c space group (no. 15). However, cerium is in the oxidation state (+III) in this compound. One of the most common compounds with cerium in the oxidation state (+IV) is pure ceria, CeO2, exhibiting cubic symmetry (Fm-3m, no. 225) and fluorite type of structure. Another common pure oxide of cerium is Ce2O3, containing Ce in oxidation state (+III); here, we also find Ce4+/Ce3+ mixed valence compounds, depending on physical parameters such as temperature, the presence of other ions and the partial pressure of oxygen. Similarly, CeN is a common nitride of cerium having Ce in oxidation state (+III), as well as Ce4+/Ce3+ mixed valence symmetry (Fm-3m, no. 225) and a rock salt type of structure.

In this study, we focus on one specific composition in the Ce-O-N system, the not-yet-synthesized ionic cerium oxynitride Ce2ON2 compound, predicting its structure using a two-pronged approach of global exploration of its energy landscape and data mining. Both methods find the same lowest energy structure, suggesting that we have been able to identify a feasible at least metastable Ce2ON2 ceramic compound, which should be accessible to synthesis.

2 Computational Details

To predict possible (meta)stable structure candidates in the not-yet-explored Ce2ON2 system, a two-pronged approach has been used, combining global optimization and data mining; details about previous successful combinations of these methods can be found elsewhere. In the first stage of the study, we have performed a global search of the energy landscape—actually: the (potential) enthalpy landscape (\( H_{pot} = E_{pot} + pV \))—of the Ce2ON2 compound for several pressures using simulated annealing as a global optimization algorithm, followed by local optimization of the obtained candidates. Specifically, the standard stochastic simulated annealing based on random Monte Carlo walks on the energy landscape (in total 144 runs, with 4 x 10^6 steps each), was supplemented by periodic stochastic quenches (i.e. random walks with temperature \( T = 0 \) K) of length 3000 steps each, from 100 stopping points along every simulated annealing trajectory, as implemented in the G42+ code. These global searches were performed for one (2 Ce, 1 O, 2 N), two (4 Ce, 2 O, 4 N), and three (6 Ce, 3 O, 6 N) formula units per variable periodically repeated simulation cell, for six different pressures (0, 0.016, 0.16, 1.6, 16, and 160 GPa). The random walk consisted of the following moves (% of Monte Carlo steps): random shifts or exchanges of atoms while keeping the cell fixed (75%), and random changes of cell parameters with (10%) or without (15%) co-moving atoms. Here, co-moving indicates that the fractional coordinates of the atoms are kept fixed while changing the cell parameters (and thus the Cartesian coordinates of the atoms change within the simulation cell), while during cell changes without co-moving atoms the Cartesian coordinates of the atoms are kept fixed within the cell and only empty slices are removed from the cell. In order to be able to perform the global searches with a reasonable computational effort, we have employed a fast computable robust empirical two-body potential consisting of Lennard-Jones \( (\varepsilon = 0.3 \text{ eV/atom}; \quad \sigma = \frac{r_i + r_j}{2}) \), with effective radii \( r(\text{Ce(IV)}) = 1.28 \text{ Å} \), \( r(\text{O(II)}) = 1.45 \text{ Å} \), and \( r(\text{N(III)}) = 1.54 \text{ Å} \) and exponentially damped (factor: \( \exp(-\mu r) \); \( \mu = 0.1/\text{Å} \)) Coulomb terms. The epsilon parameter in the LJ-potential and the effective radii and the damping factor were taken from preliminary test runs for the binary compounds CeO2 and CeN.

In the next step, we have performed data mining based explorations of the ICSD database in order to find possible structure candidates in the unknown Ce2ON2 system via analogy to known crystallographic structures. The reason for supplementing the global landscape exploration with a data-mining search for structure candidates is the fact that the global optimization is not assured of identifying all feasible candidates. Besides computation time limitations of the search algorithm, this is mainly due to the use of an empirical potential as energy function: Experience has shown, that on empirical energy landscapes important candidate structures may not be represented as large low-energy minima (of the kind a typical global optimization method would be designed to identify), even though they would be prominently present on an ab initio
energy landscape. On the other hand, the past thirty years have shown that many of the most relevant structure candidates (from our experience more than 50%) in a given chemical system exhibit a structure that is very similar to the crystal structure of some other compound observed in another chemical system (even if there is no obvious chemical relationship between these two chemical systems). Thus, data mining can serve as an important supplementary tool when trying to identify promising compounds in not-yet-synthetically explored chemical systems.

We have used the well-known KDD (knowledge discovery in databases) process, which involves selection, preprocessing, transformation, and interpretation/evaluation (or post-processing). In order to identify a sufficiently large number of structure candidates for the Ce2ON2 system, we have extracted all candidates with general formula A2BC2 that occur in the ICSD database,[34,35] followed by a prototype[36] analysis to reduce the plethora of candidates to a manageable number of distinct structure types. More details about the KDD process, and the combination of data mining with ab initio methods can be found elsewhere,[28,37,38] and in section 3.2.

Finally, density functional theory (DFT) ab initio calculations of the total energy, and local optimizations (including the cell parameters) of the structure candidates that had been identified during the global search with empirical potential or via data mining, were performed using the CRYSTAL17 code,[39] which is based on linear combinations of atomic orbitals. For the local optimization runs, we have employed analytical gradients.[40,41] Local optimizations are performed on the density functional theory level using the local density approximation (LDA). In the case of Ce3+ a pseudopotential[42] was used, together with a [4s4p2d3f] basis set, for O2− a [4s3p] basis set was used,[43,44] and for N3− [3s2p] an all-electron basis set based on Gaussian-type orbitals was employed.[45,46] The symmetries of the analyzed structures were determined using the SFND[47] and RGS[48] algorithms implemented in the program KPLOT.[49] Structures were visualized using the Vesta software.[50]

3 Results and Discussion

3.1 Global Searches and Energy Landscapes

Global searches were performed on the enthalpy landscapes for three numbers of formula units per simulation cell (Z = 1, 2, 3), and for six different pressures (0, 0.016, 0.16, 1.6, 16 and 160 GPa), yielding a total of 14,400 structure candidates. All of these structures were subjected to detailed statistical, structural, and crystallographic analysis, before submitting the most promising ones for local optimization on ab initio level.

Table 1 presents the results of the global searches performed at various pressures for 1 formula unit (Z = 1) of Ce2ON2. After detailed analysis we have observed that most of the local minima correspond to structure candidates that crystallize in structures containing defects or strongly distorted coordination polyhedra, exhibiting space group Pm ($Pm$ (no. 1)). However, the empirical potential enthalpy landscape of Ce2ON2 with Z = 1 also contains a large variety of structure candidates with higher symmetry, at various pressures, ranging structurally from triclinic and monoclinic, to orthorhombic and tetragonal structures. The structure candidates with the lowest total energy (calculated using empirical potentials) show space group C2/m (no. 12). After further structure optimization on DFT level, these structures converted to a modification we denote as $z$-Ce2ON2 with space group $P-3m1$ (no.164). More details will be presented in section 3.3. However, here we note that since the $z$-modification exhibits the lowest total energy at both empirical and ab initio levels, the empirical potential appears to be quite suitable for describing the ionic compound Ce2ON2, and it is quite likely that we have identified the global energy minimum in the Ce2ON2 system.

It is interesting to note that with the increase of pressure, the number of distorted/defect structures decreases; at extremely high pressures of 160 GPa, the preferred modification is a tetragonal structure with space group $P4m2$ (no. 115). This structure is energetically favorable at empirical potential level, suggesting a phase transition might occur at high pressures. In this context, one should note that all our calculations refer to T = 0 K, and thus only the enthalpy and not the free enthalpy is used to judge the quality and relative stability of the candidates. After local ab initio optimization, the candidate structure remains in the original space group and is still energetically quite favorable; this structure has been denoted the $\delta$-Ce2ON2 modification. The next interesting candidate at empirical potential level exhibits space group $Immm$ (no. 71). After DFT structure optimization this structure remains in the original space group; however, energetically it is higher on ab initio level compared to other structure candidates. Still, most of the structure candidates with space groups C2 (no.5), $Pm$ (no.6), and $Immm$ (no. 71). This suggests that this candidate represents a large local miniminima basin—at least on empirical potential level—which we have denoted as the Ce2ON2-$n$-1 modification. This structure is visualized in Fig. 1(a) with unit cell parameters of $a = 3.45$ Å; $b = 3.19$ Å; $c = 12.76$ Å. Cerium is in 7-fold coordination (with atom–atom distances of $2 \times 2.39$ Å—O, $4 \times 2.41$ Å—N, $1 \times 2.26$ Å—N), where
the mean distance is 2.38 Å, which together with the sevenfold coordination of the Ce-atoms is expected and suggests a good structure candidate, in principle—at least on the level of the crystallographic analysis.

Another very interesting structure candidate among those with \( Z = 1 \) (Table 1) appears in the space group \( \text{Amm2} \) (no. 38); this structure is shown in Fig. 1(b). Again, after ab initio refinement, this structure becomes energetically unfavorable, and thus it is denoted as \( \text{Ce}_2\text{ON}_2\text{-nf-2} \) modification. Here, we note that structure candidates with space group \( \text{Cm} \) (no. 8) converted to this \( \text{Ce}_2\text{ON}_2\text{-nf-2} \) modification, confirming it as a large local minima basin on the energy landscape. In this structure, we find two different Ce coordination polyhedra: a sevenfold coordinated Ce-atom (with distances \( 2 \times 2.28 \) Å—O, \( 4 \times 2.44 \) Å—N, \( 1 \times 2.36 \) Å—N) and a six-fold coordinated Ce-atom (with distances \( 1 \times 2.17 \) Å—O, \( 4 \times 2.41 \) Å—N, \( 1 \times 2.22 \) Å—N), respectively.

On the energy landscape of \( \text{Ce}_2\text{ON}_2 \) with \( Z = 1 \), two additional structure candidates appear to be of interest, although they are ranked as energetically unfavorable after the local optimization with DFT. The first is an orthorhombic candidate showing space group \( \text{Pmm2} \) (no. 25), which is observed as a high-pressure modification on empirical potential level. It has been denoted the \( \text{Ce}_2\text{ON}_2\text{-nf-3} \) structure, with a 7-fold coordination of cerium as shown in Fig. 2(a). Finally, there is a structure candidate with a tetragonal symmetry, which is not often found on the energy landscape but possesses the highest symmetry of all candidates. It has been named the \( \text{Ce}_2\text{ON}_2\text{-nf-4} \) modification, appearing in space group \( \text{I4/mmm} \) (no. 139) with 6-fold coordination of Ce by O- and N-atoms (Fig. 2b).
Analogous statistical and crystallographic analysis of the enthalpy landscapes with 2 and 3 formula units at pressures up to 160 GPa are presented in the Supporting Information (Tables S1 and S2). We note that with the increase of the number of formula units we observe more local minima associated with defect structures or several different coordination polyhedra of Ce-atoms by O- and N-atoms. This is not unexpected since once the composition of the chemical system exceeds two types of atoms, the number of local minima that correspond to structures containing “defects” increases, and thus most of the structure candidates show $P1_1 \text{ (no. 1)}$ symmetry. However, we would like to highlight the energetically most favorable structure candidates from the $Z = 2, 3$ energy landscapes. For example, the global minimum observed for $Z = 1$ is again found as the lowest energy structure for $Z = 2$, showing space group $C2/m \text{ (no. 12)}$ (Table S1). As for $Z = 1$, after further structure optimization on ab initio level, these candidates converted to the $\alpha$-Ce$_2$ON$_2$ modification with space group $P-3m1 \text{ (no. 164)}$. In addition, two candidates denoted the $\beta$-Ce$_2$ON$_2$ modification with space group $Cmc2_1 \text{ (no. 36)}$ and the $\epsilon$-Ce$_2$ON$_2$ modification, showing space group $P4_22_2 \text{ (no. 94)}$, respectively, have been found on these energy landscapes as energetically favorable structures (see Supporting Information). More details regarding these structure candidates will be presented in section 3.3.

### 3.2 Data Mining Based Searches Using ICSD

The data-mining search employed the ICSD database, which contains 216,302 inorganic structures, out of which more than 80% have already been assigned to specific structure types (up to now, 9397 structure types are listed in the ICSD). The first step of selection and pre-processing has been performed in order to reduce the number of structure candidates to a more manageable size. The first goal was to find the minimum number of unique structure candidates (prototypes) in the ternary A$_2$BC$_2$ system. In our first run through the database, we have selected from the complete ICSD database (> 200,000 structures), the 75,056 structures which belong to the ternary systems. In the next step, we have further reduced the number of structure candidates to 4754 using chemical elements search and by keeping only structures having the A$_2$BC$_2$ formula. Finally, we have further filtered the results by introducing the prototype criterion, where we assign a unique prototype to all structures with the same symmetry group, which can be mapped into each other by an overall rescaling where the different atom types in the A$_2$BC$_2$ formulae are matched appropriately.

Using the prototype criterion to eliminate quasi-duplicate structures reduced the number of structures by about two orders of magnitude, yielding 29 unique structure candidates: AlCo$_2$Pr$_2$, AlFe$_2$B$_2$, BaCu$_2$S$_2$, BaZn$_2$P$_2$, Be$_2$CaGe$_2$, CaRh$_2$B$_2$, Ce$_2$BiO$_2$, Ce$_2$Ni$_2$Sn, CeAl$_2$Ga$_2$, CoCl$_2$(H$_2$O)$_2$, CoW$_2$B$_2$, CoZr$_2$Si$_2$, Cu$_3$BaO$_2$, Dy$_2$OS$_2$, Ge$_2$LaPt$_2$, K$_2$GaP$_2$, K$_2$MnS$_2$, K$_2$PdP$_2$, K$_2$PtS$_2$, LaB$_2$C$_2$, Li$_2$PdO$_2$, Na$_2$HgO$_2$, Na$_2$PtS$_2$, Sc$_2$O$_2$S, Si$_2$N$_2$O, Th$_2$TeN$_2$, Yb$_2$S$_2$O, Zn(CN)$_2$, Zr$_2$NiAs$_2$. Note that these prototypical structures can be used in any future study involving A$_2$BC$_2$ systems (for more details and references, see the Supporting Information).
After local optimization on ab initio level using DFT-LDA, most of the structure candidates obtained from data mining have become energetically unfavorable compared to those candidates obtained via the global optimization. Space group, unit cell parameters (Å) and atomic positions for the unfavorable Ce₂ON₂ modifications after LDA optimization are given in the Table S3.

However, there were two notable exceptions: when we start the ab initio optimization from the AlCo₂Pr₂[51] structure type, we obtain the lowest energy minimum among the data-mined candidates, which also corresponds to the α-Ce₂ON₂ modification determined via global optimization. A second energetically favorable structure, denoted the γ-Ce₂ON₂ modification, was obtained from minimizing the Dy₂Os₂[52] prototype. In both instances, the generic structure of the starting prototype candidate was only slightly changed during the local ab initio optimization.

Furthermore, among the modifications with high energy, two interesting hexagonal structure candidates, which we call Ce₂ON₂-nf-5 and Ce₂ON₂-nf-7, have been identified from among the data-mined candidates, and subsequently optimized. Although both structures appear in the same space group (P6₃/mmc, no. 194), the Ce₂ON₂-nf-5 modification shows the Sc₂O₂S structure type,[53] while the Ce₂ON₂-nf-7 modification shows the Zr₂NiAs₂ structure type.[54] In addition, Ce₂ON₂-nf-7 is higher in energy than Ce₂ON₂-nf-5, or other feasible candidates obtained from data mining (Table 2). Finally, a tetragonal modification has been found using data mining and denoted as Ce₂ON₂-nf-6. This structure appears in the P4/nmm space group (no. 129) with Be₂CaGe₂ structure type;[55] however it is unlikely to be synthesized because of its very high energy. The five most relevant structure candidates from data mining based searches, with their energies after local ab initio relaxations, are presented in Table 2.

### Table 2: The most relevant structure candidates from data mining based searches, after local ab initio relaxations

| Modification  | Structure type     | Space group (no.) | Total energy, eV/atom | Total energy, kJ/mol-atoms |
|---------------|--------------------|-------------------|-----------------------|---------------------------|
| α-Ce₂ON₂     | AlCo₂Pr₂-like      | 164               | -6164.1571            | -594,748.70               |
| γ-Ce₂ON₂     | Dy₂Os₂             | 62                | -6164.0520            | -594,738.56               |
| Ce₂ON₂-nf-5  | Sc₂O₂S             | 194               | -6164.0031            | -594,733.84               |
| Ce₂ON₂-nf-6  | Be₂CaGe₂           | 129               | -6164.6085            | -594,695.77               |
| Ce₂ON₂-nf-7  | Zr₂NiAs₂           | 194               | -6163.3723            | -594,672.98               |

3.3 E(V)-Curves on LDA Level

Figure 3 presents the energy versus volume curves on ab initio level using the LDA functional of the five most relevant structure candidates in the Ce₂ON₂ system, while their structural informations are listed in Table 3. The α-modification of the Ce₂ON₂ compound exhibits trigonal space group P-3m1 (no.164) with an AlCo₂Pr₂-like structure after the local optimization. Cerium is 7-fold coordinated (atom–atom distances: 3 × 2.25 Å—N; 1 × 2.27 Å—N; and 3 × 2.57 Å—O), with a mean distance of about 2.39 Å, as expected from the typical ionic radii of the Ce-, O- and N-atoms (see Fig. 4a and Table 3).[56] Here, we note several points: (a) the predicted α-modification has been found as a distorted version with lower symmetry on the empirical potential energy landscape using global optimization; (b) using data mining based searches, the α-modification has been found starting from the AlCo₂Pr₂[51] prototype, and the local optimization resulted in a closely related structure type (AlCo₂Pr₂-like); (c) according to the E(V) curves, the α-Ce₂ON₂ modification is the unique equilibrium structure type and global energy minimum among the single phase compounds with composition Ce₂ON₂—at least up to moderately elevated pressures as shown in Fig. 3; (d) the α-modification was found via both global optimization and data mining as the lowest energy structure in both sets of structure candidates, and exhibited the lowest energy for both empirical potential and ab initio calculations.

The β-Ce₂ON₂ structure candidate exhibits the space group Cmc₂₁ (no. 36) and has also been found after the global search. Here, the Ce-atoms are coordinated by 7
anions, similar as in the $\alpha$-modification; however, cerium is surrounded with 5 nitrogen and 2 oxygen atoms (instead of 4 in the $\alpha$-modification). Full structural data is presented in Table 3 and the structure is depicted in Fig. 4(b). The $\beta$-modification is the energetically favorable stable structure in the effective negative pressure region where low-density structures are preferred, in principle (c.f. the region with large volumes, $V > 80 \text{ Å}^3$ in Fig. 3). This could indicate a possible synthesis route via, e.g., crystallization from an amorphous phase deposited from the gas phase, such as was recently used in the synthesis of a new gallium modification ($\beta'$-gallium).\cite{57}

The $\gamma$-Ce$_2$ON$_2$ modification possesses the $Pnma$ space group (no. 62), following the Dy$_2$OS$_2$\cite{52} prototype. This structure candidate has been found only after data mining based searches, and according to the total energy ranking it might be capable of existence perhaps as a metastable modification (Fig. 3 and Table 3). Structurally, the $\gamma$-modification is particularly interesting since cerium is found with two different coordination polyhedra. In the first, the Ce-atoms are seven-fold coordinated, with 1 oxygen and 6 nitrogen atoms, while in the second, the Ce-atoms are eight-fold coordinated, with 3 oxygen and 5 nitrogen atoms (see Fig. 4c).

Next, the $\delta$-Ce$_2$ON$_2$ modification has been found as the preferred modification at high pressures during global searches with empirical potential, where it exhibits the space group $P4m2$ (no. 115) with cerium coordinated by

### Table 3

| Modification and space group | LDA | Total energy, eV/atom (kJ/mol-atoms) | Difference, eV/atom (kJ/mol-atoms, %) |
|-----------------------------|-----|------------------------------------|-------------------------------------|
| $\alpha$-Ce$_2$ON$_2$       |     |                                    |                                     |
| P-3m1 (164)                 |     |                                    |                                     |
| $\beta$-Ce$_2$ON$_2$        |     |                                    |                                     |
| Cmc$2_1$ (36)               |     |                                    |                                     |
| $\gamma$-Ce$_2$ON$_2$       |     |                                    |                                     |
| $Pnma$ (62)                 |     |                                    |                                     |
| $\delta$-Ce$_2$ON$_2$       |     |                                    |                                     |
| $P-4m2$ (115)               |     |                                    |                                     |
| $\epsilon$-Ce$_2$ON$_2$     |     |                                    |                                     |
| $P4_2$2/m (137)             |     |                                    |                                     |

Analogous data for the most relevant but energetically unfavorable modifications is given in table S3 of the Supporting Information.
five nitrogen and two oxygen atoms (Fig. 4d). After DFT optimization, the structure appears to be at best a metastable phase, which would probably need elevated pressures and temperatures during synthesis because of its high energy (Fig. 3, Tables 1 and 3). However, for pressures relevant to the E(V) curves in Fig. 3, the $\alpha$-Ce$_2$ON$_2$ modification should still have the highest relative thermodynamical stability among the various modifications at $T = 0$ K. In this context, we note that in Fig. 3, we have not included the E(V) curves of those other candidates that might perhaps be competitive at extremely high pressures but are clearly irrelevant at standard pressure, such as Ce$_2$ON$_2$-nf-6 and Ce$_2$ON$_2$-nf-7: they exhibit eight- and nine-fold coordinated cerium ions, respectively, as one would expect for high-pressure structures, but have very unfavorable total energy values (Ce$_2$ON$_2$-nf-6: $-1132.5417$ Hartrees and Ce$_2$ON$_2$-nf-7: $-1132.4982$ Hartrees).

Fig. 4 The best five Ce$_2$ON$_2$ candidates found during the global energy landscape search and via the data mining: (a) $\alpha$-Ce$_2$ON$_2$ (S.G. 164); (b) $\beta$-Ce$_2$ON$_2$ (S.G. 36); (c) $\gamma$-Ce$_2$ON$_2$ (S.G. 62); (d) $\delta$-Ce$_2$ON$_2$ (S. G. 115) and (e) $\epsilon$-Ce$_2$ON$_2$ (S. G. 137). Notation as in Fig. 1
Finally, the \( \varepsilon \)-Ce\(_3\)ON\(_2\) modification, showing space group \( P4_{2}1_{2} \) (no. 94), has been found on the enthalpy landscape as an energetically favorable candidate at extremely high pressures of 160 GPa. After \textit{ab initio} local optimization at standard pressure, the \( \varepsilon \)-modification has converted to a possibly metastable modification in the space group \( P4_{2}1_{2}nmc \) (no. 137), with cerium 7-fold coordinated (Fig. 4e), although its energy is quite high, of course, compared to the other structure candidates.

We note that the \( \delta \)- and \( \varepsilon \)-modifications are closely related: they are energetically close, both indicate the need for high-pressure and high-temperature synthesis conditions if they are accessible at all, and both appear in tetragonal symmetry (Fig. 3, Tables 1 and 3). Furthermore, they are structurally related having the same coordination polyhedra, where the \( \varepsilon \)-modification creates another layer by tilting the same basic polyhedra, thus becoming a polytype of the \( \delta \)-modification (Fig. 4d and e). Finally, it is interesting to note that in each of the five relevant structure candidates, the cerium cations are seven-fold coordinated by the anions, where the only difference is the shape of the coordination polyhedra and the way they are connected.

### 4 Discussion

Of course, it would be of interest to know whether \( \alpha \)-Ce\(_3\)ON\(_2\) would be stable against decomposition into e.g. Ce\((+IV)\)\(_2\)O\(_2\) and Ce\((+IV)\)\(_3\)N\(_4\), via 2 Ce\(_2\)ON\(_2\) \(\Rightarrow\) Ce\(_2\)O\(_2\) + Ce\(_3\)N\(_4\). While Ce\((+IV)\)O\(_2\) is well-known, no compound Ce\((+IV)\)\(_3\)N\(_4\) is known to exist, and thus, for a rough estimate of the likelihood of a decomposition, we have computed the \textit{ab initio} energy of a hypothetical Ce\((+IV)\)\(_3\)N\(_4\) modification in a typical A\(_2\)N\(_4\) structure type, Si\(_3\)N\(_4\) (also known as the Nb\(_7\)Te\(_4\) structure type). We find that \( \alpha \)-Ce\(_3\)ON\(_2\) is relatively stable against such a hypothetical decomposition, since the energy of 2 formula units of \( \alpha \)-Ce\(_3\)ON\(_2\) is lower than the energy of the sum of one formula unit of Ce\(_2\)O\(_2\) and Ce\(_3\)N\(_4\), \(-61,641.571\) eV \(-\) 16,986,2775 eV \(+\) \((-\) 44,653,2461 eV), i.e. the energy difference per atom is \( (2Efu(Ce2ON2) - (Efu(Ce2O3) - Eu(solid))/10 = 0.0046\) eV/atom, which again demonstrates the hypothetical compound’s relative stability against the outlined decomposition. From this we would conclude that our estimate suggests that \( \alpha \)-Ce\(_3\)ON\(_2\) should be at least metastable thermodynamically at non-zero temperatures even though it might lose valence compounds, is beyond the purview of the current investigation.

Nevertheless, as a first estimate, we have computed the relative stability of \( \alpha \)-Ce\(_2\)ON\(_2\) against decomposition into the solid elements, 2 Ce\(_{\text{solid}}\) + O\(_{\text{solid}}\) + 2 N\(_{\text{solid}}\), and against decomposition of two \( \alpha \)-Ce\(_2\)ON\(_2\) into Ce\(_2\)O\(_2\) + 3CeN + N\(_{\text{solid}}\). Of course, our calculations do not account for temperature, and thus these decompositions only refer to \( T = 0 \) K. Table 4 shows the energies for the compounds and elements involved in these estimates, computed on DFT level with CRYSTAL17, using the same basis sets for consistency.

First, we note that Ce\(_2\)O\(_2\) is stable against decomposition into Ce\(_2\)O\(_3\) and O\(_{\text{solid}}\) by comparing the energies of 2 formula units of Ce\(_2\)O\(_2\) with the sum of one formula unit of Ce\(_2\)O\(_3\) and O\(_{\text{solid}}\): 2 \( Efu(Ce2O2) = -33,972.5550\) eV \(<\) \( Efu(Ce2O3) + Eu(solid)\) \(=\) \(-31,935.2407\) eV \(+\) \((-\) 2030.9471 eV) \(=\) \(-33,966.1878\) eV, i.e. the energy difference per atom is \( (2Efu(Ce2O2) - (Efu(Ce2O3) + Eu(O)))/6 = -0.0162\) eV/atom \((-\) 102.39 \(\text{kJ/mol-atoms})\), and thus at \( T = 0 \) K, Ce\(_2\)O\(_2\) is relatively stable against the above decomposition at \( T = 0 \) K.

Similarly, we consider the hypothetical Ce\(_3\)N\(_4\) compound and its decomposition into Ce\(_N\) and N\(_{\text{solid}}\): \( Efu(Ce3N4) = -44,653.2461\) eV \(<\) 3 \( Efu(CeN) + Eu(N_{\text{solid}})\) \(=\) \(-44,653.2139\) eV, i.e. the energy difference per atom is \( (Efu(Ce3N4) - (3Efu(CeN) + Eu(N_{\text{solid}})))/7 = -0.0046\) eV/atom \((-\) 0.44 \(\text{kJ/mol-atoms})\), and thus, we conclude that at \( T = 0 \) K, the hypothetical Ce\(_3\)N\(_4\) should be marginally relatively stable against decomposition into Ce\(_N\) and N\(_{\text{solid}}\). Thus, it appears that the oxidation state Ce\((+IV)\) is preferred to Ce\((+III)\), at least within the context of the LCAO-basis DFT \textit{ab initio} calculations we have employed, in agreement with the common occurrence of Ce\(_2\)O\(_2\) in nature.

Now, for the decomposition of \( \alpha \)-Ce\(_3\)ON\(_2\) into the elements (all in the solid state at \( T = 0 \) K!), we find \( Efu(\alpha-Ce3ON2) = -30,820.7855\) eV \(<\) 2 \( Efu(Ce) + Eu(O_{solid})\) \(+\) 2 \( Eu(N_{solid}) = -7563.4721\) eV, i.e. the energy difference per atom is \( (Efu(\alpha-Ce3ON2) - (2Efu(Ce) + Eu(O_{solid}) + 2 Eu(N_{solid}))/5 = -4651.4627\) eV/atom \((-\) 448,796.38 \(\text{kJ/mol-atoms})\). Thus \( \alpha \)-Ce\(_3\)ON\(_2\) is relatively stable against decomposition into the elements, at least close to \( T = 0 \) K. Similarly, for a decomposition into Ce\(_N\), Ce\(_2\)O\(_2\) and solid nitrogen, we find 2 \( Efu(\alpha-Ce3ON2) = -61,641.5710\) eV \(<\) \( Efu(Ce2O2) + 3 Efu(CeN) + Eu(N_{solid})\) \(=\) \(-61,639.4914\) eV, i.e. the energy difference per atom is \( (2Efu(\alpha-Ce3ON2) - (Efu(Ce2O2) + 3 Efu(CeN) + Eu(N_{solid}))/10 = -0.0280\) eV/atom \((-\) 20.07 \(\text{kJ/mol-atoms})\), which again demonstrates the hypothetical compound’s relative stability against the outlined decomposition. From this we would conclude that our estimate suggests that \( \alpha \)-Ce\(_3\)ON\(_2\) should be at least metastable thermodynamically at non-zero temperatures even though it might lose.
nitrogen and/or oxygen at temperatures, where these elements are gaseous. Figure S1 in the supplementary material shows a rudimentary $T = 0$ K (triangular) composition phase diagram, with the computed data points (hypothetical $\alpha$-Ce$_2$ON$_2$, hypothetical Ce$_3$N$_4$, CeN, CeO$_2$, solid nitrogen, solid oxygen, and metallic Ce).

Clearly, at standard conditions, both oxygen and nitrogen are gaseous, and thus there will be a large contribution to the free energy from the entropy, similar to the one for an ideal gas. This will influence all the decompositions given above. However, the focus of this study has been the prediction of feasible structure candidates for the compound Ce$_2$ON$_2$, as a first step towards the exploration of the Ce-O-N system, and the effects of temperature and thus the presence of oxygen and nitrogen in the gaseous phase on the stability of this compound goes beyond the purview of this investigation. Thus, we also do not make assertions about the absolute thermodynamic stability of these hypothetical modifications of a Ce$_2$ON$_2$ compound, but only refer to their relative thermodynamic stability.

## 5 Conclusions

In this study, we have explored the energy landscape of the hypothetical ionic cerium oxynitride Ce$_2$ON$_2$ ceramic, and predicted several feasible modifications for this compound for the first time. The discovery of new theoretical modifications in the predicted system has been achieved using global optimization and data mining, where the energy landscape has been explored for various pressures and different numbers of formula units in the simulation cell. The global optimization has been performed with empirical potentials, with subsequent local minimization on the ab initio level. From more than 14,000 structure candidates obtained after global optimization with simulated annealing, plus the 29 potential structure candidates found via data mining and prototyping, we have obtained five highly plausible low-energy modifications that might be realized as (meta)stable modifications, plus seven crystallographically and chemically interesting but energetically unfavorable structures.

Among the hypothetical Ce$_2$ON$_2$ solid modifications, the $\alpha$-Ce$_2$ON$_2$ modification, with an AlCo$_2$Pr$_2$-like structure, is predicted to be the one with the highest relative thermodynamic stability, at least at low temperatures, while at effective negative pressures, the $\beta$-Ce$_2$ON$_2$ modification should exhibit the highest relative thermodynamic stability, and be synthetically accessible. Two predicted orthorhombic modifications might be metastable, the already mentioned $\beta$-modification and the $\gamma$-modification. Finally, high temperatures combined with high pressures might favor the existence of the $\delta$- or the $\epsilon$-modification as metastable structures. Clearly, this is only a first step towards the full phase diagram in the Ce-O-N system, where one would include temperature and also investigate many more ternary, and even binary compositions. Nevertheless, this work provides a first glimpse of the potential structural richness of the Ce-O-N system.

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