POSSIBILITY OF DETECTING MOONS OF PULSAR PLANETS THROUGH TIME-OF-ARRIVAL ANALYSIS

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ABSTRACT

The perturbation caused by planet-moon binarity on the time-of-arrival signal of a pulsar with an orbiting planet is derived for the case in which the orbits of the moon and the planet-moon barycenter are both circular and coplanar. The signal consists of two sinusoids with frequency \((2n_p - 3n_b)\) and \((2n_p - n_b)\), where \(n_p\) and \(n_b\) are the mean motions of the planet and moon around their barycenter, and the planet-moon system around the host, respectively. The amplitude of the signal is the fraction \(\sin I [9(M_p M_m)/16(M_p + M_m)]^2 [r/R]^6\) of the system crossing time \(R/c\), where \(M_p\) and \(M_m\) are the masses of the planet and moon, \(r\) is their orbital separation, \(R\) is the distance between the host pulsar and planet-moon barycenter, \(I\) is the inclination of the orbital plane of the planet, and \(c\) is the speed of light. The analysis is applied to the case of PSR B1620–26b, a pulsar planet, to constrain the orbital separation and mass of any possible moons. We find that a stable moon orbiting this pulsar planet could be detected, if its mass were \(\approx 5\%\) of its planet's mass, and if the planet-moon distance were \(\approx 2\%\) of the planet-pulsar separation.

Subject headings: planetary systems — pulsars: general — pulsars: individual (PSR B1620–26) — stars: oscillations

1. INTRODUCTION TO EXTRASOLAR MOONS

In the past decade and a half, over 300 extrasolar planets have been discovered.1 With the data expected to be produced by satellites such as COROT (Auvergne et al. 2003) and Kepler (Basri et al. 2005), it will be possible to find not only smaller planets, but moons of those planets as well (Szabó et al. 2006). As a result, the detectability of extrasolar moons is starting to be explored in terms of their effect on planetary microlensing (Han & Han 2002) and transit light curves (Sartoretti & Schneider 1999; Szabó et al. 2006; Simon et al. 2007). Upper limits have already been placed on the mass and radius of putative moons of the planets HD 209458b (Brown et al. 2001), OGLE-TR-113b (Gillon et al. 2006), and HD 189733b (Pont et al. 2007).

While the limitations of microlensing and the transit technique for detecting moons have been discussed and used in the literature, the limitations of other techniques such as the time-of-arrival (TOA) technique have not. This technique involves determining the variations in line-of-sight position to the host star, usually a pulsar, using the observed time of periodic events associated with that host. The aim of this analysis is to explore what the TOA signal of a planet-moon pair is, and relate it to the planetary systems that can give the most precise timing information, those around millisecond pulsars.

2. REVIEW OF PLANETARY DETECTION AROUND MILLISECOND PULSARS

The first planetary system outside the solar system was detected around the millisecond pulsar PSR 1257+12 (Wolszczan & Frail 1992). This detection was made by investigating periodic variations in the time of arrival of its radio pulses using a timing model. An example timing model for the case in which the planet’s orbit around the pulsar is circular is

\[
(t_p - t_0) = (T_p - T_0) + \Delta T_{\text{corr}} + \text{TOA}_{\text{pert},p}(M_p, M_m, R, I, \phi_0(0)),
\]

where \(t_0\) and \(t_p\) are the times at which the initial and \(N\)th pulses are emitted in the pulsar’s frame. \(T_0\) and \(T_p\) are the times of the initial and \(N\)th pulses received in the observatory’s frame, and the term \(\Delta T_{\text{corr}}\) acts to change the frame of reference from the observatory on Earth to the barycenter of the pulsar system (see Backer 1993). The final term represents the effect of a planet on the motion of the pulsar, where \(R\) is the planet-pulsar distance, \(I\) is the angle between the normal of the planet-pulsar orbit and the line-of-sight, \(M_p\) and \(M_m\) are the mass of the pulsar and the planet respectively, and \(\phi_0(0)\) is the initial angular position of the planet measured from the \(x\)-axis, about the system barycenter.

Currently, four planets around two millisecond pulsars have been discovered, three around PSR 1257+12 (Wolszczan & Frail 1992; Wolszczan 1994) and one around PSR B1620–26 (Backer et al. 1993). These four planets include one with mass 0.02 Earth masses, the lowest mass extrasolar planet known. This high timing precision of millisecond pulsars indicates that they are optimal targets for planet, and consequent moon searches.

3. WHAT IS THE TOA PERTURBATION CAUSED BY A MOON?

In order to investigate the perturbation caused by planet-moon binarity, the timing model presented in equation (1) must be updated to include effects due to the presence of the moon. For simplicity, we consider here only systems in which both the orbit of the planet and moon around their common barycenter, and the orbit of the planet-moon barycenter around the
pulsar, are circular and lie in the same plane. The resulting updated model is
\[
(t_N - t_0) = (T_N - T_0) + \Delta T_{\text{corr}} + \text{TOA}_{\text{pert}, p}(M_p, M_m, R, I, \phi_p(0)) + \text{TOA}_{\text{pert}, pm}(M_p, M_m, r, R, I, \phi_p(0), \phi_m(0)).
\]  
(2)

We have explicitly modified TOA_{\text{pert}, pm} to indicate that it depends on the combined planet-moon mass, and included another term, TOA_{\text{pert}, pm} to account for planet-moon binarity. Here \(M_m\) is the mass of the moon, \(r\) is the distance between the planet and the moon, and \(\phi_p(0)\) is the initial angular position of the planet measured from the \(x\)-axis, about the planet-moon barycenter (see Fig. 1). TOA_{\text{pert}, pm} can be derived from \(\mathbf{R}\), the vector between the system barycenter and the pulsar, using
\[
\frac{1}{c} \int_0^t \int_0^{t'} \mathbf{R} \cdot n \, dt \, dt' = \text{TOA}_{\text{pert}, p} + \text{TOA}_{\text{pert}, pm},
\]  
(3)
where \(c\) is the speed of light and \(n\) is a unit vector pointing along the line of sight.

The governing equation for \(\mathbf{R}\), can be written as the sum of the zeroth-order term, which describes TOA_{\text{pert}, p}, and the tidal terms, which describe TOA_{\text{pert}, pm}:
\[
\frac{d^2 \mathbf{R}}{dt^2} = \frac{G(M_p + M_m)}{R^3} \mathbf{R} + \left[ -\frac{G(M_p + M_m)}{R^3} \mathbf{R} + \frac{GM_p}{R^2} (R + r_p) + \frac{GM_m}{R^2} (R + r_m) \right],
\]  
(4)

where the tidal terms have been collected into square brackets and noting that \(\mathbf{R} = -(M_p + M_m)/(M_p + M_m)R\), \(r_p = -M_p/(M_p + M_m)r\), and \(r_m = M_m/(M_p + M_m)r\), and \(\mathbf{R}, \mathbf{r}_p, \mathbf{r}_m\), and \(r\) are also shown in Figure 1. It can be seen after some algebra that
\[
\mathbf{R} + r_m = \left( R \cos \phi_p - \frac{M_p}{M_p + M_m} r \cos \phi_p \right) \mathbf{i} + \left( R \sin \phi_p - \frac{M_p}{M_p + M_m} r \sin \phi_p \right) \mathbf{j},
\]  
(5)
\[
\mathbf{R} + r_p = \left( R \cos \phi_p + \frac{M_m}{M_p + M_m} r \cos \phi_p \right) \mathbf{i} + \left( R \sin \phi_p + \frac{M_m}{M_p + M_m} r \sin \phi_p \right) \mathbf{j},
\]  
(6)

where \(\mathbf{i}\) and \(\mathbf{j}\) are defined in Figure 1, and \(\mathbf{i}\) is the direction to the line-of-sight, projected onto the plane of the orbit.

As the orbits are both circular and coplanar, we have that \(\phi_p(t) = n_p t + \phi_p(0)\) and \(\phi_m(t) = n_p t + \phi_m(0)\), where \(n_p\) and \(n_m\) are the constant mean motions of the two respective orbits. Substituting equations (5) and (6) into the last two terms of equation (4), assuming \(r \ll R\), and using the binomial expansion to order \(r^2/R^2\) gives
\[
\frac{GM_p}{|\mathbf{R} + r_m|^3} = \frac{GM_p}{R^3} \left[ 1 - \frac{3}{2} \frac{M_p}{(M_p + M_m)^2} \frac{r}{R} \cos \phi_p + \frac{15}{2} \frac{\cos^2 \phi_p}{(M_p + M_m)^2} \right],
\]  
(7)
\[
\frac{GM_m}{|\mathbf{R} + r_p|^3} = \frac{GM_m}{R^3} \left[ 1 - \frac{3}{2} \frac{M_m}{(M_p + M_m)^2} \frac{r}{R} \cos \phi_p + \frac{15}{2} \frac{\cos^2 \phi_p}{(M_p + M_m)^2} \right],
\]  
(8)

Substituting equations (5), (6), (7), and (8) into equation (4) gives, after simplification,
\[
\frac{d^2 \mathbf{R}}{dt^2} = \frac{G(M_p + M_m)}{R^3} \mathbf{R} + \left[ \frac{GM_p}{(M_p + M_m)^2} \frac{r}{R} \cos \phi_p \right] \mathbf{i} + \left[ \frac{GM_m}{(M_p + M_m)^2} \frac{r}{R} \cos \phi_p \right] \mathbf{j}.
\]  
(9)

From Figure 1 it can be seen that
\[
n = \sin l \mathbf{i} + \cos l \mathbf{k}.
\]  
(10)
Writing $n_p$ in terms of $r$, using Kepler’s law, gives

$$\text{TOA}_{\text{pert,pm}} = - \sin I \frac{GM_p M_m}{c(M_m + M_p)} \frac{r^2}{R^4} \left[ \frac{3}{32n_p^2} \cos (\phi_p - 2\phi_p) + \frac{15}{32n_p^2} \cos (3\phi_p - 2\phi_p) \right],$$

(12)

where we are making no assumptions about the size of $M_m/M_p$.

A similar study was conducted by Schneider & Cabrera (2006) investigating the radial velocity perturbation due to an equal-mass pair of binary stars on a distant companion. Converting their radial velocity perturbation to a timing perturbation, setting $M_p = M_m$, and noting that their $a_i$ is equivalent to $r/2$, our results agree.

4. IS IT POSSIBLE TO DETECT MOONS OF PLANETS ORBITING MILLISECOND PULSARS?

To investigate whether or not it is possible to detect moons of pulsar planets, we simplify equation (13) by summing the amplitudes of the sinusoids, giving the maximum possible amplitude:

$$\max (\text{TOA}_{\text{pert,pm}}) = \frac{9}{16} \sin I \frac{M_p M_m}{c(M_m + M_p) R^4} \left( \frac{r}{R} \right)^5,$$

(14)

Thus, the size of the perturbation varies as $[M_p M_m/(M_m + M_p)]^2[1/r(R)]^5$ times the system crossing time, $R/c$. So, the best hope of a detectable signal occurs when the planet-moon pair widely are separated from each other, both quite massive, and very accurate timing data is available. For example, a stable system such as a 0.1 AU Jupiter-Jupiter binary located 5.2 AU from a host pulsar would produce a TOA$_{\text{pert,pm}}$ of amplitude 960 ns, which compares well with the 130 ns residuals obtained from one of the most stable millisecond pulsars, PSR J0437–4715 (van Straten et al. 2001).

To demonstrate this method, the expected maximum signals from a moon orbiting each of the four known pulsar planets was explored. It was found that in the case of PSR B1620–26b, signals that are in principle detectable could confirm or rule out certain configurations of moon mass and orbital parameters (see Fig. 2).

In the particular case of PSR B1620–26b, the perturbation signal will not match the signal shown in equation (13) due to the effect of its white dwarf companion. As a side project, this companion’s effect was investigated and found to be the introduction of additional perturbations on top of the TOA$_{\text{pert,pm}}$ and TOA$_{\text{pert,pm}}$ calculated. Consequently, the detection threshold represents an upper limit to the minimum detectable signal and the analysis is still valid.

Unfortunately, there are practical limits to the applicability of this method. They include discounting other systems that could produce similar signals, sensitivity limits due to intrinsic pulsar timing noise, and limits imposed by moon formation and stability.

First, other systems that could produce similar signals need to be investigated. Possible processes include pulsar precession (e.g., Akgün et al. 2006), periodic variation in the ISM (Scherer et al. 1997), gravitational waves (Dettweiler 1979), unmodeled interactions between planets (Laughlin & Chambers 2001), and other small planets. To help investigate the last two options, we plan on completing a more in-depth analysis of the per-
turbation signal of an extrasolar moon, including the effects of inclination and eccentricity.

Second, the noise floor of the system needs to be examined. The suitability of pulsars for signal detection is limited by two main noise sources, phase jitter and red timing noise (e.g., Cordes 1993). Phase jitter is error due to pulse-to-pulse variations and leads to statistically independent errors for each TOA measurement. Phase jitter decreases with increasing rotation rate (decreasing \( \dot{P} \)) due to the increase in the number of pulses sampled each integration. Red timing noise refers to noise for which neighboring TOA residuals are correlated. Red timing noise has been historically modeled as a random walk in phase, frequency, or frequency derivative (e.g., Boynton et al. 1972; Cordes 1980; Kopeikin 1997). Red noise is strongly dependent on \( P \). It has been proposed that red noise is due to non-homogeneous angular momentum transport either between components within the pulsar (e.g., Jones 1990) or between it and its environment (e.g., Cheng 1987). To illustrate the effect of these two noise sources, an estimate of the resulting TOA residuals as a function of \( P \) and \( P \) is shown in Figure 3. For comparison, the values of \( P \) and \( P \) of every pulsar listed in the ATNF Pulsar Catalogue\(^4\) (Manchester et al. 2005) are also included.

Third, whether or not moons will be discovered depends on whether or not they exist in certain configurations, which depends on their formation history and orbital stability. Recent research suggests that there are physical mass limits for satellites of both gas giants (Canup & Ward 2006) and terrestrial planets (Wada & Kokubo 2006). Also, tidal and three-body effects can strongly affect the longevity of moons (Barnes & O’Brien 2002; Domingos et al. 2006; Atobe & Ida 2007).

Finally, while this method was investigated for the specific case of a pulsar host, this technique could also be applied to planets orbiting other clocklike hosts such as pulsating giant stars (Silvotti et al. 2007) and white dwarfs (Mullally et al. 2006).

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REFERENCES

Akgün T., Link, B., & Wasserman, I. 2006, MNRAS, 365, 653
Arzoumanian, Z., Nice, D. J., Taylor, J. H., & Thorsett, S. E. 1994, ApJ, 422, 671
Atobe, K., & Ida, S. 2007, Icarus, 188, 1
Auvergne, M., et al. 2003, Proc, SPIE, 4854, 170
Backer, D. C. 1993, in ASP Conf. Ser. 36, Planets around Pulsars, ed. J. A. Phillips & S. E. Thorsett (San Francisco: ASP), 11
Backer, D. C., Foster, R. S., & Sallmen, S. 1993, Nature, 365, 817
Barnes, J. W., & O’Brien, D. P. 2002, ApJ, 575, 1087
Basri, G., Borucki, W. J., & Kock, D. 2005, NewA Rev., 49, 478
Boynton, P. E., Groth, E. J., Hutchinson, D. P., Nanos, G. P. Jr., Partridge, R. B., & Wilkinson, D. T. 1972, ApJ, 175, 217
Brown, T. M., Charbonneau, D., Gilliland, R. L., Robert, L., Noyes, R. W., & Burrows, A. 2001, ApJ, 552, 699
Canup, R. M., & Ward, W. R. 2006, Nature, 441, 834
Cheng, K. S. 1987, ApJ, 321, 805
Cordes, J. M. 1980, ApJ, 237, 216
———. 1993, in ASP Conf. Ser. 36, Planets around Pulsars, ed. J. A. Phillips & S. E. Thorsett (San Francisco: ASP), 43
Detweiler, S. 1979, ApJ, 234, 1100
Domingos, R. G., Winter, O. C., & Yokohama, T. 2006, MNRAS, 373, 1227
Gillon, M., Pont, F., Moutou, C., Bouchy, F., Courbin, F., Sohy, S., & Magain, P. 2006, A&A, 459, 249
Han, C., & Han, W. 2002, ApJ, 580, 490
Holman, M. J., & Wiepert, P. A. 1999, AJ, 117, 621
Jones, P. B. 1990, MNRAS, 246, 364
Kopeikin, S. M. 1997, MNRAS, 288, 129
Laughlin, G., & Chambers, J. E. 2001, ApJ, 551, L109
Manchester, R. N., Hobbs, G. B., Teoh, A., & Hobbs, M. 2005, AJ, 129, 1993
Mullally, F., Winge, D. E., & Kepler, S. O. 2006, in ASP Conf. Ser. 352, New Horizons in Astronomy, ed. S. J. Kannappan (San Francisco: ASP), 265
Pont, F., et al. 2007, A&A, 476, 1347
Sartoretti, P., & Schneider, J. 1999, A&AS, 134, 553
Scherer, K., Fichtner, H., Anderson, J. D., & Lauer, E. L. 1997, Science, 278, 1919
Schneider, J., & Cabrera, J. 2006, A&A, 445, 1159
Sigurdsson, S., Richer, H., Hansen, B., Stairs, I., & Thorsett, S. 2003, Science, 301, 193
Silvotti, R., et al. 2007, Nature, 449, 189
Simon, A., Szatmáry, K., & Szabó, Gy. M. 2007, A&A, 470, 727
Szabó, Gy. M., Szatmary, K., Divéki, Zs., & Simon, A. 2006, A&A, 450, 395
Thorsett, S. E., Arzoumanian, Z., Camilo, F., & Lyne, A. G. 1999, ApJ, 523, 763
van Straten, W., Bailes, M., Kulkarni, S. R., Anderson, S. B., Manchester, R. N., & Sarkissian, J. 2001, Nature, 412, 158
Wada, K., & Kokubo, E. 2006, ApJ, 638, 1180
Wolszczan, A. 1994, Science, 264, 538
Wolszczan, A., & Frail, D. A. 1992, Nature, 355, 145

\(^4\) See http://www.atnf.csiro.au/research/pulsar/psrcat/.