ENERGETICS AND LUMINOSITY FUNCTION OF GAMMA-RAY BURSTS

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ABSTRACT

Gamma-ray bursts (GRBs) are believed to be some catastrophic event in which material is ejected at a relativistic velocity, and internal collisions within this ejecta produce the observed $\gamma$-ray flash. The angular size of a causally connected region within a relativistic flow is of the order the angular width of the relativistic beaming, $\gamma^{-1}$. Thus, different observers along different lines of sight could see drastically different fluxes from the same burst. Specifically, we propose that the most energetic bursts correspond to exceptionally bright spots along the line of sight on colliding shells and do not represent much larger energy release in the explosion. The energy budget for an average GRB in this model is, however, same as in the uniform shell model. We calculate the distribution function of the observed fluence for random angular-fluctuation of ejecta. We find that the width of the distribution function for the observed fluence is about 2 orders of magnitude if the number of shells ejected along different lines of sight is 10 or less. The distribution function becomes narrower if number of shells along typical lines of sight increases. The analysis of the $\gamma$-ray fluence and afterglow emissions for GRBs with known redshifts provides support for our model, i.e., the large width of GRB luminosity function is not due to a large spread in the energy release but instead is due to large angular fluctuations in ejected material. We outline several observational tests of this model. In particular, for $\delta$-function energy distribution in explosions we predict little correlation between the $\gamma$-ray fluence and the afterglow emission as in fact is observed. We predict that the early (minutes-to-hours) afterglow would depict large temporal fluctuations whose amplitude decreases with time. Finally, we predict that there should be many weak bursts with about average afterglow luminosity in this scenario.

Subject headings: gamma rays: bursts — gamma rays: theory

1. INTRODUCTION

The improvement in the determination of angular position of gamma-ray bursts (GRBs) by the Dutch-Italian satellite, BeppoSAX, has led to measurement of distances to eight GRBs and has lent support for the relativistic shock model (cf. Costa et al. 1997; van Paradijs et al. 1997; Bond 1997; Frail et al. 1997). Unlike previous expectations these observations revealed that the GRB luminosity function is very broad; the width of the fluence distribution is about 2 orders of magnitude in energy. Assuming isotropic emission, one finds that the energy of the most energetic bursts is larger than $10^{54}$ ergs. The recently observed evidence for beaming with an opening angle of a few degrees reduces the energy estimate by a factor of 100 (Kulkarni et al. 1999; Sari, Piran, & Halpren 1999; Harrison et al. 1999). However, when taking into consideration the relatively low efficiency of conversion of kinetic energy to $\gamma$-rays, one finds that the total kinetic energy required is, after all, of order $10^{54}$ ergs even after the beaming correction (Kumar 1999). This energy is too large to be released by a compact solar mass object. According to the internal-external shock model comparable amount of energy should be released during the GRB phase and the afterglow. However, quite generally, only a fraction of the energy emitted as $\gamma$-rays is seen in the afterglow (mostly as X-rays). Moreover, there appears to be weak or little correlation between the gamma-ray fluence and the afterglow flux.

All the above mentioned phenomena could be unrelated. However, within the framework of relativistic internal shocks model (Narayan, Paczyński, & Piran 1992; Paczyński & Xu 1994; Rees & Mészáros 1994; Sari & Piran 1997a, 1997b) we suggest a possible connection: these properties could all be manifestations of large angular inhomogeneity of the relativistic ejecta. The size of a causally connected region of a shell of radius $R$ moving with Lorentz factor $\gamma$ is $\sim R/\gamma$. Because of relativistic beaming this is also the size of the region visible to a distant observer. During the GRB the angular size of these regions (<0.01 rad) is significantly smaller than the inferred angular width of the ejecta, $\delta \theta$ ~ a few degrees. There are therefore $(\gamma \delta \theta)^2$ causally disconnected regions within this cone. Thus the observed gamma-ray luminosity seen by different observers from the same burst could fluctuate strongly due to small-scale inhomogeneities in the emitting regions. Unless one is careful, one would overestimate the energy release in gamma-rays in cases in which a hot spot has been observed.\(^1\) At a later time when the Lorentz factor of the ejecta has become smaller and the size of causally connected regions is larger the dispersion of the afterglow luminosity seen along different lines of sight should be smaller as well. The emission at this stage yields a better estimate of the overall total energy involved.

We explore here the implications of this model. The model is presented in the next section, followed by calculation of the distribution of fluence. The observational

\(^1\) Note that as BeppoSAX can detect only rather strong bursts, the BeppoSAX sample might be biased toward cases in which such a hot spot has been seen.
aspects are discussed in § 3, and the main results and some predictions of the model are summarized in § 4.

2. THE PHYSICAL MODEL AND NUMERICAL SIMULATION

The calculations described in this section are numerical, but in § 2.2 we analyze a simple model analytically to gain some insight into the numerical results.

We consider successive, random, ejection of blobs of angular size $\gamma^{-1}$ into a cone with an opening angle $\delta \vartheta = 10^\circ$. To estimate the expected fluence distribution we have carried out Monte Carlo simulations of the observed emission from randomly ejected shells—corresponding to the random conditions expected in different causally disconnected regions. In each case we keep the overall energy ejected into a given cone of opening angle $10^\circ$ approximately the same—$10^{52} \text{ ergs}$. Changing the total energy in bursts causes a linear translation of the fluence distribution function and increase the afterglow luminosity. The Lorentz factor of each shell is assumed to be a random number uniformly distributed between and we take $\gamma_{\min} = 5$ and $\gamma_{\max} = 400$. The energy distribution of blobs is taken to be either log-normal with mean of $10^{52}/N_b \text{ ergs}$ and width (FWHM) in $\log_{10}(\text{energy})$ of 1, or a $\delta$-function distribution; $N_b$ is the number of blobs ejected in the explosion. The number of blobs ejected along a line of sight is also taken to be a random number uniformly distributed between $N_{\min}$ and $N_{\max}$. We have considered two different values for the average number of shells ejected: $(N_{\min} + N_{\max})/2$ equal to 5 and 40. The shells are ejected randomly over some time interval corresponding to the total duration of $30 \text{ s}$.

The calculation and the results described here apply to the case where shells have little or no angular fluctuation, and the radiation received by different observers located within $10^\circ$ cone is a smooth function. The distribution function for fluence in this case is significantly narrower compared to when shells have large angular fluctuations (see § 2.1).

We consider all possible shell collisions along a line of sight, and for each collision we follow the forward and reverse shocks propagating within the colliding shells. The thermodynamic properties of the shocked gas are calculated by solving the continuity of energy, momentum, and baryon number flux (see, e.g., Piran 1999). Shell collisions continue until mergers arrange the shell velocities as a monotonically increasing function of their distance from the center of explosion. We assume equipartition of energy between electrons, protons, and magnetic field (the energy fraction in magnetic field $\epsilon_B = 0.3$, and in the electrons $\epsilon_e = 0.3$). The distribution of electron number density is taken to be a power-law function of energy, $n_e(E) \propto E^{-p}$, with $p = 2.5$.

We calculate synchrotron emission from relativistic electrons and include the effect of electron cooling and inverse-synchrotron absorption on the power spectrum (e.g., Sari, Piran, & Narayan 1998). The synchrotron photons undergo inverse Compton scattering to produce the emergent spectrum (we calculate multiple Compton scatterings when the Compton $\gamma$-parameter is greater than 1). As photons emitted by one shell might undergo elastic collisions in other shells, we follow the trajectory of photons produced in shell collisions, along with the trajectory of different shells, to determine the energy and momentum deposited by photons in different shells and the resulting change to the bulk kinetic energy of shells (Kumar 1999).

The computed emergent power spectrum is integrated between 10 and $10^3 \text{ keV}$, in the observer frame, to determine fluence along different lines of sight. From the "observed" fluence we calculate the isotropic radiative energy in gamma-rays (it should be emphasized, however, that the radiation is highly anisotropic when shells are not uniform). The total energy in explosion is obtained by adding the energy of all the blobs in a cone of $10^\circ$ opening angle, which as stated previously is taken to be $10^{52} \text{ ergs}$.

2.1. Fluence Distribution and Burst Energetics

Figure 1 shows the isotropic $\gamma$-ray fluence distribution resulting from the calculation described above where the total energy in the explosion is $10^{52} \text{ ergs}$. The FWHM of $\log_{10}(\text{fluence})$ distribution function is about 2.0, i.e., 2 orders of magnitude in the linear energy scale, when the mean number of shells along the line of sight is 5, and the FWHM of the energy distribution of individual blobs is 1.0 (on $\log_{10}$ scale). Part of the width of the fluence distribution results from the energy distribution of blobs, and rest from the distribution of their Lorentz factor. The FWHM of the fluence distribution function is 1.2, i.e., a factor of 15 on linear scale, when the energy of blobs follows a $\delta$-function distribution. Clearly in this case fluence distribution is a result of the fluctuation in the radiative efficiency of internal shocks.
The width of the fluence distribution function increases with increasing number of blobs along the line of sight; the FWHM is only half an order of magnitude when the mean number of shells along a direction is 40 (Fig. 1). The width of the distribution function is not sensitive to burst duration, and moreover it is almost independent of the energy fraction in magnetic field as long as \( e_B \) is not so small that the shocks become almost adiabatic. It is also evident from Figure 1 that there are a number of bright spots in the cone containing the ejecta for which the observed isotropic gamma-ray fluence is \( 5 \times 10^{53} \) ergs.

The fluence distribution function for the model consisting of collisions of uniform shells is similar to the graph in Figure 1. There are two differences, however, between the uniform and the patchy shell models. One, the width of the fluence distribution function is smaller by a factor of \( \sim 10 \) for the uniform shell model with the same fixed total energy in explosion. The other difference is that the afterglow flux falls off smoothly in this model, whereas in the case of inhomogeneous shells the afterglow flux at early time, within the first hour of the explosion, has large-amplitude fluctuation. As discussed in § 4, this can be used to distinguish between the two models observationally.

We have assumed that consecutive ejection of shells is completely random. It is possible that the energies of blobs ejected along a given line sight might have a nonzero correlation. It is straightforward to modify our calculation to include this correlation. We find that even for a perfect correlation of blob energy and random distribution of Lorentz factor, the width of the distribution function for fluence is only slightly larger than the case where the correlation is zero (see § 2.2 for discussion).

The integral probability distribution function is also shown in Figure 1. Note that the probability of seeing a burst with isotropic fluence of \( 7 \times 10^{53} \) ergs is about 0.1\% (the total energy in explosion is fixed at \( 10^{52} \) ergs). Thus, a particularly bright burst such as GRB 990123, which was in the top 0.1\% of all BATSE bursts, could well have had total energy in explosion similar to an average BATSE burst. In the case of GRB 990123 we were perhaps looking at a very bright spot on the colliding shell surface.

This suggests that gamma-ray fluence is not a reliable measure of the total energy in explosion. A more promising way to estimate the energetics of explosion is to consider emission at a later time when a number of causally disconnected regions have merged—but the electrons are radiative—thereby reducing the dispersion seen along different lines of sight. The ratio of the synchrotron cooling and the dynamical time is \( t_{\text{cool}}/t_{\text{dyn}} \approx 6 \pi m_e c/(\sigma_T \gamma \zeta t_{\text{obs}} B^2 \gamma^2) \approx m_e/(\sigma_T \gamma \zeta m_\text{in} t_{\text{obs}} \gamma^2) \approx t_{\text{obs}}^{1/2} \), which becomes greater than 1 at \( t_{\text{obs}} \approx 1 \) hr. At this time \( \gamma \approx 30 \), and we expect about 20 disconnected regions to have merged and the dispersion of the radiative flux along different lines of sights to have decreased by a factor of about 4. Note that the energy of the ejecta has also dropped by a factor of about 4 at this time.

### 2.2. Analysis of a Simple Model to Understand the Numerical Results

We analyze a simple model that captures some of the features of the numerical result presented above. Let us consider collision of two shells. The energy and the Lorentz factor of the outer (denoted by subscript 1) and the inner shell (subscript 2) are given by a distribution function \( P(E, \gamma) \). The energy radiated when shells collide is some fraction of the total thermal energy produced in shell collision which is given by (cf. Kobayashi et al. 1998)

\[
E_{\text{col}} = (E_1 + E_2)
\times \left[ 1 - \left( E_1 \gamma_1 + E_2 \gamma_2 \right) \left( \frac{E_1^2}{\gamma_1^2} + \frac{E_2^2}{\gamma_2^2} + \frac{2E_1 E_2 \gamma_r}{\gamma_1 \gamma_2} \right)^{-1/2} \right],
\]

where \( \gamma_r = \gamma_1 \gamma_2 (1 - v_1 v_2) \approx \gamma_1 \gamma_2 (1 - \gamma_1^{-2} + \gamma_2^{-2})/2 \) is the relative Lorentz factor of collision of the shells. The thermal energy release is a function of \( \gamma_2/\gamma_1 \equiv \eta \), and the energy radiated in some frequency band is a fraction of \( E_{\text{col}} \). For instance, if electrons, magnetic field and protons are in equipartition, the energy radiated in 10–10^3 keV is roughly \( E_{\text{col}}/10 \) (Kumar 1999; Panaitescu, Spada, & Mészáros 1999). To simplify the analysis we calculate the bolometric fluence distribution function. The distribution in some other band is not at all different.

The bolometric distribution function is given by

\[
P_{\text{rad}}(E) = \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} d\gamma_1 \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} d\gamma_2 \int_{E_{\text{min}}}^{E_{\text{max}}} dE_1 \int_{E_{\text{min}}}^{E_{\text{max}}} dE_2 \times P_{1}(E_1) P_{2}(E_2) ,
\]

where \( E_{\text{min}} \approx F/[(\eta + 1)(\eta^{-1/2} - 1)^2] \) is the minimum energy of the outer shell, for a given \( \eta = \gamma_2/\gamma_1 \), so that the radiated energy is \( F \), and \( E_{\text{max}} \approx 2F\eta^{-1}(1 - \eta^{-1}) \) the minimum energy of the inner shell to yield fluence \( F \) in shell collision.

Let us assume that the distribution functions are separable and write them as \( P(E, \gamma) = P_{1}(E) P_{2}(\gamma) \). Substituting this into equation (2) we obtain

\[
P_{\text{rad}}(E) = \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} d\gamma_1 \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} d\gamma_2 \times P_{\text{rad}}(E_{\text{min}}) P_{\text{rad}}(E_{\text{max}}) P_{1}(\gamma_1) P_{2}(\gamma_2) ,
\]

where \( P_{\text{rad}}(E) \equiv \int dE P_{\text{rad}}(E) \).

Let us consider a particularly simple case where the distribution function for the inner and the outer shells are both constant in the intervals \( (0, E_{\text{max}}) \) and \( (\gamma_{\text{min}}, \gamma_{\text{max}}) \) and zero outside. The above equation can be easily integrated in this case, and we find that the differential distribution function,

\[
P_{\text{rad}}(E) \equiv dP_{\text{rad}}/dF, \text{ is approximately proportional to } F^{-1/2} \text{ for } F \leq E_{\text{max}}/2 \approx F_{\text{max}} \text{ becomes steeper for } F \approx F_{\text{max}} \text{ and is zero for } F > F_{\text{max}} \text{. It can also be shown that for } P_{\text{rad}}(E) \propto E^{3}, \text{ } P_{\text{rad}}(F) \propto F^{-1/2} \text{ for } \alpha \geq 0, \text{ and } P_{\text{rad}} \text{ falls off more steeply with } F \text{ for } \alpha < 0 \text{. For } P_{\text{rad}}(E) \propto \delta(E - E_0), \text{ } P_{\text{rad}}(F) \propto F^{-1/2} \text{ as well, i.e., the fluence distribution has a finite width even when the energies of all the blobs are identical.}

Now we turn to collision of more than two shells. Let us assume that collision of any two shells gives rise to a distribution of fluence that is same as calculated above. This is a drastic simplification; nevertheless, it provides useful insight into the qualitative behavior of the fluence-distribution function calculated in § 2.2. The distribution function resulting from \( N \) collisions can be easily calculated and is given by

\[
P_{\text{rad}}(F) = \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} dk \int_{-\infty}^{\infty} dk' \exp (-ikF) \times \left[ \int_{0}^{\pi} dF P_{\text{rad}}(F') \exp (ikF') \right]^{N-1} ,
\]

where \( H(\tau) \) is the Heaviside step function.
where \( P'_{\text{rad}}(k) \) is the Fourier transform of \( P'_{\text{rad}}(F) \). For \( P'_{\text{rad}} \propto F^{-z} \), we find from the above equation that \( P'_{\text{rad}}(F) \propto F^{(1+|z|)N-1} \) for \( F \ll F_{\text{max}} \) and it is zero for \( F > NF_{\text{max}} \). The width of the distribution function on \( \log_{10} \) scale is \( 3/((1-|z|)N+2) \). The effective value of \( z \) is about 0.7 for the case where the energy spectrum of ejected shells is flat. The width calculated for the toy problem is somewhat smaller (for fixed \( N \)) than for the more realistic problem considered above because the energy produced in shell mergers decreases in successive mergers; hence, the effective \( N \) for the mergers is smaller than the number of shells expelled in the explosion. We show the distribution function for the model problem in Figure 2 for \( z = 0.5 \) and 0.7.

To determine the effect of correlated ejection of shells we set \( P_2(E_2, \gamma_2) = \delta(E_1 - E_2)P_2(\gamma_2) \) in equation (2). We find that for \( F \ll F_{\text{max}} \), \( P_2(F) \) is not very different from the case where shells were randomly ejected with no correlation, whereas for \( F \sim F_{\text{max}} \), the distribution function \( P_2(F) \) falls off somewhat more rapidly, which causes \( P_{\text{rad}}(N) \) to become a bit broader.

3. COMPARISON WITH OBSERVATIONS

Table 1 depicts a compilation of the observations of GRBs and their afterglow with known redshifts. Included in the table is the observed isotropic gamma-ray fluence (from Piran, Band, & Jimenez 1999), the X-ray luminosity after 5 hr (estimated from the published X-ray flux and the slope of the light curve), and the \( R \)-band magnitude 24 hr after the burst. To obtain a uniform sample we consider only bursts that have been observed by BATSE, for which there is a well-determined fit for the spectrum using the Band function (Band et al. 1993). It can be seen from the table that while the spread in the isotropic gamma-ray fluence is about two and a half orders of magnitude, the spread in the isotropic X-ray luminosity 5 hr after the burst, and the \( R \)-band optical luminosity at 1 day, is only about one and a half orders of magnitude.

These dispersions can be quantified. Consider first the six bursts with known redshifts and a well-determined BATSE fluences.\(^2\) Assuming a normal distribution of \( \log(E_\gamma) \), we find that the standard deviation of the logarithm of the (isotropic) energy emitted in gamma-rays, \( \sigma_{\gamma} \), is 0.87. The corresponding FWHM is 2. The average isotropic gamma-ray fluence is \( 1.4 \times 10^{43} \) ergs. The likelihood is larger than 0.05 of the maximal value within the range \( 0.45 < \sigma_{\gamma} < 2.4 \). Similar analysis for the isotropic X-ray luminosity 5 hr after the burst for six bursts yields \( \sigma_{\chi} = 0.58 \), and FWHM of 1.4. The average (isotropic) X-ray luminosity is \( 1.3 \times 10^{46} \) ergs s\(^{-1} \). A variance range for likelihood larger than 0.05 of the maximal likelihood is 0.35 < \( \sigma_{\chi} < 1.45 \). Piran et al. (1999) have analyzed a larger sample of data and confirm these results, i.e., \( \sigma_{\chi} \approx 0.5 \), which is significantly less than \( \sigma_{\gamma} \). The standard deviation of the logarithm of the \( R \)-band luminosity 24 hr after the burst is \( \sigma_{\chi} = 0.53 \). We note that we have not corrected the observed flux for extinction and

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2 Note that eight bursts appear in Table 1. However, the calculation of \( \sigma_{\chi} \) is based on a subset of six bursts for which there is gamma-ray data. Other subsets of six bursts are used to calculate \( \sigma_{\chi} \) and \( \sigma_{\chi} \).

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**Table 1**

| GRB      | Z    | \( F_{\gamma} \) \( \times 10^{-5} \) ergs cm\(^{-2} \) | \( F_{\chi} \) \( \times 10^{-5} \) ergs cm\(^{-2} \) | \( F_{\text{iso}} \) \( \times 10^{33} \) ergs | \( F_{\chi} \) (5 hr) \( \times 10^{-12} \) ergs cm\(^{-2} \) s\(^{-1} \) | \( L_{\chi} \) (5 hr) \( \times 10^{46} \) ergs s\(^{-1} \) | \( R \) (24 hr) | \( L_{\text{opt}} \) (24 hr) \( \times 10^{42} \) ergs s\(^{-1} \) |
|----------|------|-------------------------------------------------|---------------------------------|-----------------|---------------------------------|-----------------|-----------------|-----------------|
| 990510...| 1.619| 2.26                                           | 2.94                           | 1.5             | ...                             | ...             | 19.5            | 2.8             |
| 990123...| 1.6  | 26.8                                           | 34.9                           | 18              | 13.5                            | 8.9             | 20.5            | 1.1             |
| 980703...| 967  | 2.26                                           | 3.0                            | 0.54            | 5.9                             | 1.6             | 20.7            | 0.37            |
| 980613...| 1.096| ...                                             | ...                            | ...             | 0.82                            | 2.7             | 22.9            | 0.06            |
| 971214...| 3.412| 0.944                                          | 1.11                           | 3.2             | 1.97                            | 4.2             | 22.5            | 0.54            |
| 970828...| 0.958| 9.60                                           | 13.9                           | 3.0             | 5.57                            | 1.5             | ...             | ...             |
| 970508...| 0.853| 0.317                                          | 0.55                           | 0.054           | 1.10                            | 0.22            | 21.2            | 0.18            |
| 970228...| 0.695| ...                                             | ...                            | ...             | ...                             | ...             | 21.2            | 0.13            |

\(^a\) The total isotropic energy in gamma-rays, the X-ray luminosity after 5 hr in 2–10 keV, and the optical luminosity in the \( R \)-band after 1 day are calculated assuming a flat universe with \( \Omega_{\Lambda} = 0.7 \) and \( h = 60 \).

\(^b\) Observed fluence in 10 keV–2 MeV range (from Band et al. 1999).

\(^c\) Gamma-ray fluence after K-correction based on a detailed fit of the spectrum (from Band et al. 1999).

\(^d\) X-ray flux, 5 hr after the burst, in 2–10 keV energy band.

\(^e\) X-ray luminosity in 2–10 keV using a K-correction corresponding to a spectrum \( \propto \nu^{-0.75} \).

\(^f\) Using a K-correction corresponding to a spectrum \( \propto \nu^{-0.75} \). Note that there is no extinction correction.
thereby have overestimated $\sigma_R$. A prediction of the patchy shell model is that $\sigma_R$ should be smaller than $\sigma_X$. Current observations are consistent with this expectation; however, more accurate determination of $\sigma_R$ and larger number of afterglows are needed to improve the statistical significance of this result.

These results show that the FWHM of gamma-ray energy distribution is wider by a factor of 5 than the X-ray afterglow luminosity distribution and is roughly consistent with the expected decrease in fluctuation amplitude by a factor of 7 based on the merger of causally disconnected regions ($\gamma$ decreases by a factor of about 7 at 5 hr). If the gamma-ray-emitting surface were uniform (not highly patchy as considered here) and the large width of the isotropic gamma-ray energy distribution were due to a wide distribution of the explosion energy (or the opening angle of the jet) then the distribution of afterglow luminosity in the X-ray and other wavelengths should have been wider than the gamma-ray energy distribution since the afterglow flux above the cooling frequency is $\log f_{\nu} = \frac{1}{2}(p + 2) \log E + (p - 1) \log \epsilon_{\nu} + \text{constant}$; where $p \approx 2.5$, $E$ is the energy in the explosion per unit solid angle so long as the opening angle of the ejecta is larger than $\gamma^{-1}(\theta_{\text{obs}})$, and the constant term includes the dependence on $\theta_{\text{obs}}$ and $\epsilon_{\nu}$. Assuming that $E$ and $\epsilon_{\nu}$ are uncorrelated, we expect the width of the afterglow luminosity, $\log f_{\nu}$, to be larger than the width of $E$ distribution by at least a factor of 1.1. Since the observed X-ray afterglow luminosity distribution is narrower by a factor of $\sim 1.5$ compared to gamma-ray flux (on log scale) this suggests that the distribution of $E$ is very narrow and the large width of the gamma-ray fluence distribution arises as a result of angular fluctuation in the gamma-ray emitting surface.

We have considered the total energy in different explosions to be the same. It is straightforward to generalize and apply our results to a realistic situation where the total energy release in an explosion is drawn from a distribution with nonzero width—the gamma-ray fluence distribution function in this case is the convolution of the distribution function for energy in the explosion and the function shown in Figure 1. Note that bursts with larger energy release will have brighter X-ray afterglow, and the peak of the gamma-ray fluence distribution will be shifted to larger energy, thereby giving a correlation between the X-ray luminosity and gamma-ray fluence.

To check whether the calculated fluence distribution function agrees with the log $N$–log $S$ (or peak flux) distribution observed by BATSE we converted the fluence to peak luminosity by dividing it by $T = 30$ s ($T$ is the mean "effective" duration of bursts). Using a flat cosmology with $H = 60$ km s$^{-1}$ Mpc$^{-1}$, a GRB rate that follows the star formation rate, and a spectral index of $\alpha = -1.5$ we calculated the log $N$–log $S$ distribution from our theoretically calculated fluence distribution function and found it to be in good agreement with the observed flux distribution of BATSE 4B catalog. The ratio of the total energy in the burst and its duration is the only free parameter used in this fit. Our results are in rough agreement with calculations that use power-law luminosity function (e.g., Schmidt 1999, Che 1999; and others). It turns out that just about any luminosity function which has a width of about 2 orders of magnitude or larger will fit the observed data.

4. CONCLUSIONS AND PREDICTIONS

A relativistic shell ejected in an explosion could have large angular fluctuation because regions on the shell separated by an angle greater than $\gamma^{-1}$ are causally disconnected. We have explored some consequences of the angular fluctuations in GRB explosions. Most of the results described here also apply to the model consisting of internal collisions of uniform shells.

We have modeled shells as consisting of independent blobs of angular size $\gamma^{-1}$. Because of relativistic beaming only a small patch of a shell, of angular size $\gamma^{-1} \sim$ the size of a blob, is visible to a distant observer.

The blobs are ejected in the explosion with some distribution of Lorentz factor and energy. We have calculated the spectrum of emergent photons which are produced by synchrotron plus inverse Compton processes in internal shocks when blobs undergo collisions, and find that the distribution of the observed gamma-ray fluence along different lines of sight is very broad (the total energy in the explosion is fixed at $10^{52}$ ergs). The width of the fluence distribution function depends on the width of the distribution of energy and the Lorentz factor of blobs; the FWHM of the $\log f_{\nu}$ (fluence) distribution function is 1.2 when the distribution of blob energy is a $\delta$-function, where as the width is 2 when the energy distribution of blobs, lognormal, has a FWHM of 1.

An intrinsic spread in the energy release in explosions, not considered here, will broaden the width of the observed gamma-ray fluence distribution. The variation of the total explosion energy from one GRB to another will also give rise to a nonzero correlation between the gamma-ray fluence and the X-ray afterglow luminosity, which appears to be at odds with observations.

According to the patchy shell model the emission surface of the expanding shells consists of bright patches, of angular width of order $\gamma^{-1}$, and dark patches; a bright GRB results not because of larger energy release in the explosion but instead when our line of sight intersects a bright spot on the expanding colliding shell, and thus the gamma-ray fluence is not a good measure of the total energy release in the explosion.

The fluence distribution function resulting from collisions of uniform shells is similar in shape but smaller in width by factor of $\sim 10$, for fixed total energy in explosion, compared to when shells have large angular fluctuations. For the subclass of GRBs which show short timescale variability, the observed gamma-ray fluence for the uniform shell model is proportional to the energy in explosion, and the width of the distribution function in this case is equal to the width of the total energy release in explosions.

The $\sim 1\%$ radiative efficiency of internal shocks (Kumar 1999; Panaitescu et al. 1999), in $10^{40}$ keV energy band requires total energy in explosion to be larger than the observed energy in gamma-ray photons by a factor of about 100. The finite opening angle for burst ejecta reduces the energy requirement by a factor of 10–100. The angular inhomogeneity of shells ejected in the explosion could further reduce the energy budget of the brightest bursts, such as GRB 990123, by a factor of $\sim 10$, thereby bringing down the total energy involved in the brightest observed
bursts to a value of order the energy in weaker bursts, i.e., \( \lesssim 10^{53} \) ergs. It should be noted that the energy requirement for an average GRB in the patchy shell and the uniform shell models are very similar; the largest difference, a factor of \( \sim 10 \) in the energy budget, arises only for the very brightest bursts.

An interesting result of this model is that in spite of the very wide observed luminosity function of GRB, the total energy in GRB explosions could be roughly compared in all bursts. This could have interesting implications on the nature of the inner engine.

There are several predictions of our model. First, the width of the distribution function in the X-ray afterglow flux should be significantly smaller than the spread of fluence seen in gamma-rays. Moreover, the dispersion of optical luminosity should be smaller than the X-ray luminosity, and the late time radio afterglow should have the smallest dispersion which reflects the variation of energy in GRB explosions. The gamma-ray, X-ray, and the optical data for GRBs with known redshifts are consistent with these expectations.

It should be noted that the observed decrease in the width of the isotropic luminosity distribution for the afterglow emissions, compared to the width of the isotropic gamma-ray fluence distribution, is contrary to what is expected if the width of the fluence distribution were a consequence of a wide distribution of energy release in GRBs (see § 3).

A second prediction is that the afterglow flux should show small-amplitude fluctuation with time if the energy distribution of blobs is not a \( \delta \)-function or shells are not uniform. For instance, the fluctuation amplitude 1 day after the explosion, when the FWHM of blob energy distribution is 10, is \( \sim 0.02 \) mag in the optical band and the characteristic variability timescale is \( \sim 1 \) day. The early afterglow light curve should show, however, larger fluctuations whose amplitude decreases in time. This prediction could be directly tested with the forthcoming quick response GRB missions, HETE II, Swift, and BALERINA, and thereby distinguish between the uniform and fluctuating shell models.

A third prediction is the existence of numerous weak bursts, which will arise from the low energy tail of the GRB luminosity function. The afterglow flux from these weak bursts should be comparable to the afterglow from stronger bursts; this may have implications to the rate of “orphan” afterglows.

A fourth prediction is that the fluence distribution of multipeaked bursts (which arise due to numerous collisions) would be narrower than the fluence distribution of bursts with only a few peaks. Most of the bursts detected by BeppoSAX, for which afterglow emission and redshifts have been measured, show light curves consisting of a few peaks which could arise as a result of collision of just a few shells. For such bursts we expect very wide fluence distribution as observed.

Finally, as the prompt optical and prompt X-ray emissions arise in regions which are moving with very high Lorentz factor (Sari & Piran 1999) we expect these emissions to also have a very wide luminosity function, whose width should be comparable to the GRB luminosity function, i.e., the prompt emission could be dominated by small hot spots and produce unusually large fluences in some cases. As mentioned earlier we expect temporal fluctuations with a decreasing amplitude in time during this stage.

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