Stimulation of Beta Decay due to a Bose-Einstein Condensate

J.J. Hope and C.M. Savage

Department of Physics and Theoretical Physics,
The Australian National University, Australian Capital Territory 0200, Australia.
Joseph.Hope@anu.edu.au

Abstract

Nuclear processes can be stimulated by the presence of a macroscopic number of bosons in one of the final states. We describe the conditions necessary to observe the atom-stimulation of a beta decay process. The stimulation may be observable if it becomes possible to produce a Bose-Einstein condensate with the order of $10^{14}$ atoms in a trap.

It has recently been noted that the rate of gamma ray emission from a nuclear process can be altered by the presence of a Bose-Einstein condensate (BEC) [1]. This process of atom-stimulated photon emission is similar to other proposals involving electronic transitions [2–5], except that it emphasises the fact that any process, including nuclear transitions, may be enhanced. We calculate the stimulation of the emission of a massive particle, in particular a beta particle, and discover that the necessary experimental conditions are less restrictive. The origin of this improvement is that the total momentum kick due to the neutrino and the beta particle may be zero.

A large number of atoms in a single quantum mechanical (internal and external) state has been produced in atomic traps in the recent experiments which have produced a Bose-Einstein condensate [6–8]. The high degree of quantum degeneracy in these systems may be used to stimulate any process which has a final product which is condensed. Any transition rate between two states is enhanced by a factor of $(N + 1)$ where $N$ is the number of bosons occupying the final condensed state. One of the interesting features of condensed matter physics is that under certain conditions, large collections of an even number of bound fermions (i.e. atoms) can act as bosons. This approximation is true when the Hamiltonian describing the fields may be written only in terms of field operators for the entire atoms, which will hold for nuclear reactions when there is negligible probability of atomic disruption. It would
be extremely interesting to test this fundamental assumption of condensed matter physics with such an exotic example. The stimulation of any kind of transition by a BEC has not been observed, except indirectly, in the formation of the condensate itself.

Recently, there have been several similar proposals to measure the effect of the Bose enhancement of photon emission in atomic systems using a BEC. These include the work on light scattering from a BEC [2–4], the stimulated enhancement of cross section (SECS) for excited state atoms moving through a BEC[5], and the emission of a gamma ray photon from a nuclear transition [1]. It is possible that the light scattering proposal and the SECS proposal may be realised with current condensates. The gamma ray proposal was found to require the construction of a condensate with $10^{14}$ atoms and have a wavefunction which was a metre long in one dimension. It also involved two of these condensates fired at each other with a very well defined relative momentum which was equal to the recoil kick of a gamma ray photon. This appears to be a difficult experimental task, and there is a strong likelihood that at such energies there would be sufficient coupling between the two condensates to destroy them. Atom-stimulation of a photon emission is also used to produce a build up of atoms in the ground state of a trap in some of the recent models for an atom laser [9–11]. The atom laser proposals have yet to be realised.

We analyse a different system in which the stimulation of a massive particle can be detected. We consider a radioactively unstable ion, $A$, that is held in a trap where it will decay into the stable atom, $B$, as well as a beta particle and an (anti)neutrino.

$$A \rightarrow B + \beta + \bar{\nu}$$  

(1)

The unstable particle $A$ must be an ion so that after the beta decay it will be a stable atom of the new species without having to capture or emit an electron. If there is a second trap which contains a condensate of $N$ atoms of $B$, then there will be an enhancement of the fraction of the nuclear decay which goes into the ground state of the trap, and therefore an enhancement of the overall decay rate. It is important to note that only a very small number of unstable ions of species $A$ are needed in the first trap. We let $\gamma$ be the spontaneous nuclear decay rate and $f$ be the fraction of the atoms which will be in the ground state of the trap after a spontaneous decay. The total decay rate $\gamma_{tot}$ is the sum of the spontaneous decay rate into the non-ground states and the stimulated decay into the ground state

$$\gamma_{tot} = \gamma(1 - f) + (N + 1)\gamma f = \gamma(1 + Nf).$$  

(2)

It is clear from this equation that for a BEC with a sufficiently large number
of atoms, the decay rate can be detectably increased, and that the fraction \( f \) will determine the critical size of such a BEC. We will now calculate this fraction for reasonable parameters.

We denote the center of mass wavefunction of the unstable ion \( A \) by \( \Psi(x) \). We also denote the mean field of \( B \) when it is in the condensate by \( \Phi(x) \), and the net momentum kick produced by the beta particle and neutrino emission by \( \hbar k \). The fraction \( f \) of atoms which would decay spontaneously into the ground state of the trap for \( B \) is given by the overlap integral

\[
  f = \frac{1}{L} \int d^3k \ g(k) \left\| \int d^3x \ \Phi^*(x) \Psi(x) e^{i k x} \right\|^2,
\]

where \( g(k) d^3k \) is the probability of the momentum kick being between \( k \) and \( k + d^3k \), and \( L \) is the number of magnetic sublevels in the nucleus. The fraction \( f \) is reduced by this factor because the beta decay will randomise the nuclear magnetic moment.

The beta particle will be emitted at relativistic speeds, and will therefore have virtually no interaction with the orbital electrons. This is due to an extremely poor spatial overlap between the wavefunction of the outgoing beta particle and the bound electrons.

The momentum kick distribution \( g(k) \) can be found from the dynamics of the reaction and the momentum distribution of the emitted particles. Conservation of energy and momentum allow us to determine the magnitude of the kick \( k(p, \theta) = |k| \) given to the atom:

\[
  k(p, \theta) = \frac{1}{\hbar} \left[ \frac{\Delta E^2}{c^2} + 2 p^2 + m^2 c^2 + 2 p \cos(\theta) \frac{\Delta E}{c} \right. \\
  \left. - 2(\Delta E/c + p \cos(\theta)) \sqrt{p^2 + m^2 c^2} \right]^{1/2}
\]

where \( m \) is the mass of the beta particle, \( p \) is the momentum of the beta particle, \( \Delta E = Q + mc^2 \) is energy difference between the nuclear states of \( A \) and \( B \), and \( \theta \) is the angle between the beta particle and the neutrino.

The distribution of the momentum kick is isotropic, and it can be calculated from the momentum distribution of the beta particles:

\[
  g(k) = \int_0^{p_{max}} dp \int_0^{\pi} d\theta \ \delta(k(p, \theta) - k) \ G(p)
\]

where \( p_{max} = \sqrt{\Delta E^2/c^2 - m^2 c^2} \), and the function \( G(p) \) is the momentum
The distribution of the beta particles. This distribution is found from the density of states [12], and is given by \( G(p) = p^2(\Delta E - \sqrt{p^2 + m^2c^4})^2/N \), where \( N \) is simply a normalization factor. We can calculate this factor to be

\[
N = \frac{4}{5} c^2 \left( \frac{\Delta E^2}{c^2} - m^2 c^2 \right)^{3/2} + \frac{\Delta E^2}{c^2} \sqrt{\frac{\Delta E^2}{c^2} - m^2 c^2} \left( m^2 c^4 - 2\Delta E^2 \right) + \frac{4}{3} \left( \frac{\Delta E^2}{c^2} - m^2 c^2 \right)^{3/2} \left( \Delta E^2 + m^2 c^4 \right) + c^5 m^4 \Delta E \ln \frac{\Delta E + \sqrt{\Delta E^2 - m^2 c^4}}{mc^2}.
\]

We now consider a specific case to calculate the overlap integral, Eq. (3). We consider the case where both wavefunctions have a Gaussian form, which corresponds to the ground state of an atom trap containing non-interacting atoms. For simplicity, we assume that both wavefunctions are isotropic and identical.

\[
\Psi(x) = \Phi(x) = \left( \frac{1}{2\pi l^2} \right)^{4} \exp \left[ -\frac{x^2}{4l^2} \right], \quad (6)
\]

where \( l \) is related the size of ground state of the trap. We have assumed here that the traps containing the two species have been perfectly aligned.

The traps which have been used in BEC experiments so far have been magnetic, which could trap the two prepared species of atoms, although in general the size of the ground state of the trap would be different for each species. In an experiment, the traps will have to be chosen so that the shape of the wavefunctions of the two states are as similar as possible. This could be possible with dipole force traps [13], which could also be used to prepare the atoms \( A \) and \( B \) separately and then move them together by slowly moving the traps. The disadvantage with optical traps is that they may excite the atoms, causing losses through spontaneous emission. This may be avoided through using extremely large detunings or Raman transitions [14].

From Eqs. (3, 4, 5, 6) we calculate the fraction \( f \) of atoms spontaneously decaying into the ground state under our assumptions,

\[
f = \frac{\pi^2 \hbar^2}{8c} \frac{(\Delta E^2 - m^2 c^4)^3 (\Delta E^2 + m^2 c^4)}{L \Delta E^5 l^2 N}.
\]

This decreases with increasing \( \Delta E \), which means that the best reaction will have a low maximum kinetic energy \( Q \) for the emitted beta particle. Examination of the dependence of \( f \) on the mass \( m \) of the emitted particle shows
Table 1
Comparison of the stimulation of beta decay from different nuclei

| Initial nucleus | Final nucleus | Half life (min) | Q (MeV) | N required to double decay rate | Ions needed to produce 1 decay/min ($N = 10^{14}$) |
|-----------------|---------------|----------------|--------|-------------------------------|---------------------------------|
| $^{42}$K        | $^{42}$Ca (I=0) | 744            | 3.5211 | $1.7 \times 10^{14}$          | 670                             |
| $^{11}$C        | $^{11}$B (I=3/2) | 20.4           | 1.9821 | $2.4 \times 10^{14}$          | 21                              |
| $^{31}$Si       | $^{31}$P (I=1/2) | 157            | 1.4908 | $7.4 \times 10^{13}$          | 96                              |

that beta decay will be stimulated more than a different two body decay involving a heavier particle. From the inverse square dependence on $l$, we see that a small ground state wavefunction will also increase $f$. The size of the ground state wavefunction is determined by the strength of the trap. The experiments which currently have produced a BEC have had traps corresponding to $l \approx 2 - 6 \mu m$.

To obtain a reasonable signal from this experiment, it is necessary to have enough ions trapped so that a useful number of decays happen over the lifetime of the experiment. Since they must be trapped within such a small region, it will be difficult to get more than a thousand ions into the trap. The stimulation will also require a large number of atoms in the condensate. The density of atoms in the condensate cannot be increased indefinitely, however, as collisions and interatomic attraction will eventually destroy the condensate. The mean interatomic spacing should be larger than the scattering length of the atoms, which imposes a condition on $l$, the size of the ground state wavefunction. The required number of atoms varies as $l^2$, so the density will go down if larger wavefunctions are used. There is a possibility that the scattering length can be tuned with an applied magnetic field, which is true for cesium [15]. If this technique can be applied more generally, it would allow quite high atomic densities in the trap.

In the presence of a strong bias magnetic field, the electronic and nuclear spins are not coupled. The magnetic moment of the nucleus will be randomised, so the number $L$ of internal states available to the product atom $B$ will be $2I + 1$, where $I$ is the spin of the final nuclear state. We use this assumption when calculating the results in Table 1, in which we show the number of atoms required to double the expected rate of beta decay for several different nuclei. This table uses the parameter $l = 2 \mu m$ for the size of the trap. The reactions on this table were chosen so that less than a thousand ions were required in the trap to produce a sufficiently large signal. This restriction excluded several reactions which used atomic species which have already been trapped, and shows that the best results are found using species which have not yet been cooled below recoil limited temperatures.
The first reaction on this table was the $\beta^-$ decay of $^{42}$K to $^{42}$Ca, for which the decay rate will be doubled if there are $1.7 \times 10^{14}$ atoms in the BEC. If there are $10^{14}$ atoms in the BEC, then there will be a 60% increase in the decay rate, and this new rate will be one decay per minute when there are 670 ions in the trap. These results require extremely large condensates of exotic species, so it is unlikely that the stimulation of a nuclear process will be observed in the near future. If it becomes possible to produce a BEC with $10^{14}$ atoms, this feasibility study shows that it may indeed be possible to measure the stimulation of a beta decay process. The largest BEC produced so far has contained $5 \times 10^6$ atoms [16], which is more than two orders of magnitude larger than the first BEC produced less than a year earlier [6].

The analytic form, Eq. (7), of the result was found by choosing an explicit and simple form for the shape of the wavefunctions. This means that the quantitative results for particular experiments may vary, but this is not important for a proof of principle calculation. In particular, the ground state of a harmonic trap will be altered by the presence of interatomic interactions, which are modelled by the non-linear Schrödinger equation[17]. The maximum overlap will be obtained when the wavefunction of the unstable atoms is matched to the ground state wavefunction of the BEC. For repulsive interactions, this wavefunction will be spread slightly (for an example, see the calculation in reference [4]).

This letter has examined the feasibility of measuring a stimulation of beta decay by a large number of bosonic atoms in a single quantum mechanical state. It shows that while there is no fundamental obstacle to such an experiment, the required condensate population is eight orders of magnitude beyond that achieved so far.

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