Abstract

It is shown that hyperon beta decay data can be well accommodated within the framework of Cabbibo’s SU(3) symmetric description if one allows for a small SU(3) symmetry breaking proportional to the mass difference between strange and nonstrange quarks. The $F/D$ ratio does not depend sensitively on the exact form of the symmetry-breaking, and the best fits are close to the value previously used in the analysis of deep inelastic scattering of electrons or muons on polarized nucleons. The total quark helicity and strange quark polarization in the nucleon are discussed.
The spin-dependent (Gamow-Teller) matrix-elements, for transitions between members of the baryon octet [1], acquired renewed interest after measurements were made of the deep inelastic scattering (DIS) of polarized leptons by polarized protons and neutrons [2–7], which provided valuable information about the spin structure of the nucleon. One of the most important quantities measured in polarized DIS is the longitudinal spin structure function $g_1$. In the quark parton model, the spin structure function $g_1$ is directly related to the quark spin densities: $\Delta u(x)$, $\Delta d(x)$, $\Delta s(x)$ etc. where $\Delta q(x) \equiv q_\uparrow(x) - q_\downarrow(x) + \bar{q}_\uparrow(x) - \bar{q}_\downarrow(x)$.

To deduce the various quark spin densities from the $g_1$ data, one usually assumes that baryons may be assigned to a SU(3) flavor octet and uses the relation between the quark spin densities and weak matrix elements $F$ and $D$ from hyperon semileptonic decays. By using the earlier $F/D$ value, the EMC data led to the unexpected conclusion [2] that the quarks carry at most a small part of the spin of either nucleon and furthermore, that there is a significant contribution from “strange” quarks, which necessarily come from the “sea” of quark-antiquark pairs. This has led to many different suggestions for resolution of what has come to be called the “spin crisis” [8–10]. Among these is a suggestion [11,12] that the conclusions may be distorted because the $F/D$ value obtained from the hyperon semileptonic decays are based on exact SU(3) flavor symmetry. SU(3) symmetry breaking effects may significantly change the value.

There are many works attempting to evaluate the SU(3)-breaking effects in the bag model or in the quark model, by applying center-of-mass corrections [13,14], or by including one-gluon exchange interactions [16,17] or both [18]. The size of the corrections depends on the model and assumptions used to describe the symmetry breaking effects and on the ‘existing’ data to be fitted. Some authors used their own data and concluded [19] that there is no signal for the breakdown of Cabbibo’s SU(3) symmetric description. According to Ref. [15], however, an overall fit to the existing data using broken SU(3) scheme is better than that from the assumption of perfect SU(3) symmetry. Another approach, using the chiral ef-
effective lagrangian for baryons, \[20\] calculated SU(3) symmetry breaking corrections to axial currents of the baryon octet arising from meson loops. The size of corrections was found to be surprisingly large (the loop correction is almost as large as the lowest order result) which should already have raised suspicion. In a subsequent paper \[20\], including the spin-3/2 baryon decuplet in the intermediate state, the meson loop correction to the axial currents is significantly reduced but still substantial ($\simeq 30 - 50\%$). However, corrections due to higher baryon resonances, which in principle should be included in the intermediate states, have been ignored in the calculation and may change the result still further. Thus it appears that the validity of Cabbibo’s SU(3) symmetric description is far from settled. Most recently, instead of model-dependent calculations, an approach \[21\] based on phenomenological analysis of hyperon beta decay data has been suggested to estimate the SU(3) symmetry breaking effects. The authors present evidence for a strong variation of the $F/D$ parameter between various transitions.

In section II, we consider another approach based on a general discussion \[24\] of SU(3) flavor symmetry and its possible breaking and show that the hyperon beta decay data are adequately represented by at most a small deviation from of Cabibbo’s SU(3) symmetric description, which can be well accommodated within the framework of the usual assumption of a small SU(3) breaking proportional to the mass-difference between strange and nonstrange quarks. In section III, the consequences for the quark spin distribution in the nucleon are discussed. A brief summary is given in section IV.

II. SU(3) SYMMETRY-BREAKING EFFECTS

In the quark model, which provides an explicit realization of Cabibbo’s theory connecting strangeness-conserving and strangeness-changing weak interactions, the primary weak current responsible for transitions between hadrons is:

$$j_\mu^W = \bar{q} \gamma^\mu (1 + \gamma_5) q \frac{\lambda_W}{2}$$

(1)
with

$$\lambda_W = [\lambda_1 + i\lambda_2]\cos \theta_c + [\lambda_4 + i\lambda_5]\sin \theta_c$$

where $\lambda_i$ (i=1,2,...8) denote the Gell-Mann matrices and $q$ represents the triplet (u, d, s) of basic quark fields. Eq.(1) requires that weak transition-elements necessarily transform as a component of an SU(3) octet. If baryons are assigned to a SU(3) octet, represented in matrix form by:

$$B^b_a = \begin{pmatrix} \frac{1}{\sqrt{2}} \Sigma^o + \frac{1}{\sqrt{6}} \Lambda^o & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}} \Sigma^o + \frac{1}{\sqrt{6}} \Lambda^o & n \\ \Xi^- & \Xi^o & -\frac{2}{\sqrt{6}} \Lambda^o \end{pmatrix}$$

the SU(3)-octet matrix elements between baryons can be written, in the symmetric limit (we are concerned only with the values for $q^2 \rightarrow 0$, i.e. zero four-momentum transfer ), as

$$DTr(\bar{B}\{\lambda_W, B\}_+) + FTr(\bar{B}[\lambda_W, B]_-)$$

which can also be written as $a_0 Tr(\bar{B}B\lambda_W) + b_0 Tr(\bar{B}\lambda_W B)$, with $a_0 = D - F$ and $b_0 = D + F$. However, the SU(3) flavor symmetry is only approximate for strangeness-changing processes. If SU(3) symmetry-breaking effects cannot be ignored, the expressions for the matrix-elements must be generalized.

We assume that the breaking of SU(3)-flavor symmetry is due to a term which transforms like the eighth generator of SU(3). This would be the case, for example, if SU(3) breaking arose entirely from a mass-difference between strange and (degenerate) non-strange quarks. To first order in the symmetry-breaking interaction, transforming like $\lambda_8$, the most general SU(3) structure of the weak matrix-elements between baryons can be written as

$$a_0 Tr(\bar{B}B\lambda_W) + b_0 Tr(\bar{B}\lambda_W B) + a Tr(\bar{B}B\{\lambda_W, \lambda_8\}_+) + b Tr(\bar{B}\{\lambda_W, \lambda_8\}_+ B)$$

$$+ d Tr(\bar{B}\lambda_8\lambda_W B) + k [Tr(\bar{B}\lambda_W)Tr(B\lambda_8) + Tr(\bar{B}\lambda_8)Tr(B\lambda_W)]/2$$

where the first two terms are the ones given in eq.(3) and the others are SU(3) symmetry-breaking corrections. The corresponding symmetry-breaking parameters $a$, $b$, $d$, and $k$ should be small relative to $a_0$ and $b_0$ for such a perturbative expansion to be valid. Vector
coupling constants are not affected to first order \[23,24\]. For the ratio of axial-vector to vector amplitudes, eq.(4) yields \[24\]

\[
\frac{G_A}{G_V} \rightarrow_n \rightarrow_p = F + D + 2b
\]

\[
\frac{G_A}{G_V} \rightarrow_\Lambda \rightarrow_p = F + D/3 + a/3 - 2b/3 - d/3 - k
\]

\[
\frac{G_A}{G_V} \rightarrow_{\Sigma^-} \rightarrow_n = F - D + a - d
\]

\[
\frac{G_A}{G_V} \rightarrow_{\Xi^-} \rightarrow_\Lambda = F - D/3 + 2a/3 - b/3 + 4d/3 + k
\]

where we have listed only those transitions for which these ratios are relatively well measured \[25\]:

\[
\frac{G_A}{G_V} \rightarrow_n \rightarrow_p = 1.2573 \pm 0.0028 \tag{9}
\]

\[
\frac{G_A}{G_V} \rightarrow_\Lambda \rightarrow_p = 0.718 \pm 0.015 \tag{10}
\]

\[
\frac{G_A}{G_V} \rightarrow_{\Sigma^-} \rightarrow_n = -0.340 \pm 0.017 \tag{11}
\]

\[
\frac{G_A}{G_V} \rightarrow_{\Xi^-} \rightarrow_\Lambda = 0.25 \pm 0.05 \tag{12}
\]

Let us first discuss the SU(3) symmetry scheme. Fig.1 exhibits the results reported in eqs.(9)–(12) under the assumption that SU(3) symmetry-breaking effects are negligible, viz. all breaking parameters are zero: \(a = b = d = k = 0\) in eqs. (5)–(8). We see that the \((G_A/G_V)\) ratios for the best-measured transitions (9)-(11) yield, within the errors, a unique solution for \(F\) and \(D\). While the line corresponding to the central value of \((G_A/G_V)\) for the less accurately measured \(\Xi^- \rightarrow \Lambda\) transition does not pass exactly through the same \((F, D)\) point, a downward shift of \((G_A/G_V)\) \(\Xi^- \rightarrow \Lambda\) by an amount equal to the quoted error, is sufficient to bring it into agreement with the others. Hence it seems that no significant SU(3) symmetry-breaking effect is needed to describe the existing \((G_A/G_V)\) data. It is also interesting to note that the favored solution for \(F\) and \(D\) obtained from data (9)–(11) is not too different from that predicted by the static SU(6) symmetric model with suitable relativistic recoil corrections (\(\approx 25%\) reduction \[14\]).
While there does not seem to be any compelling evidence demanding the inclusion of SU(3) breaking effects, it may be worthwhile to see what is obtained if one takes the data, eqs.(9)-(12), at face value and seeks a solution allowing any one of the symmetry breaking parameters in eq.(4) to be non-zero. We search in the three-dimensional space $F$, $D$, $\epsilon$ (where $\epsilon$ denotes one of four possible small symmetry breaking parameters: $a$, $b$, $d$ or $k$) to find the minimum of the quantity $\chi^2$. The results are listed in Table I.

Table I: One-parameter fit

|                | b,d,k=0 | a,d,k=0 | a,b,d=0 | a,b,k=0 | exp. |
|----------------|---------|---------|---------|---------|------|
| $a$            | $-0.0024$ | $0.0027$ | $0.0123$ | $0.0297$ |      |
| $b$            | $0.4581$ | $0.4576$ | $0.4610$ | $0.4721$ |      |
| $d$            | $0.7992$ | $0.7943$ | $0.7963$ | $0.7852$ |      |
| $F/D$          | $0.573$  | $0.576$  | $0.579$  | $0.601$  |      |
| $(G_A/G_V)_{n\rightarrow p}$ | $1.2573$ | $1.2573$ | $1.2573$ | $1.2573$ | $1.2573 \pm 0.0028$ |
| $(G_A/G_V)_{\Lambda\rightarrow p}$ | $0.723$  | $0.721$  | $0.714$  | $0.724$  | $0.718 \pm 0.015$  |
| $(G_A/G_V)_{\Sigma^-\rightarrow n}$ | $-0.343$ | $-0.337$ | $-0.335$ | $-0.343$ | $-0.340 \pm 0.017$ |
| $(G_A/G_V)_{\Xi^-\rightarrow \Lambda}$ | $0.190$  | $0.192$  | $0.208$  | $0.250$  | $0.25 \pm 0.05$  |
| $\chi^2$       | $1.61$   | $1.42$   | $0.86$   | $0.20$   |      |

As expected, it takes only a small non-zero value of any of these, to obtain a statistically satisfactory solution. The fifth column, with a $d$-type correction, shows the best agreement between the calculated and the measured $(G_A/G_V)$ ratios, and provides the only indication that inclusion of SU(3) breaking effects may be required. The best fits under the assumption that SU(3) symmetry breaking arises from terms of the type $a$ or $b$, yield values which, in view of the quoted errors, are indistinguishable from zero, i.e. do not call for any correction at all. Similarly, the evidence for non-zero $k$ is marginal.
From the results listed in Tables I, taking average of all the results, we obtain

$$< F > = 0.462, \quad < D > = 0.794, \quad < F/D > = 0.582$$ \hspace{1cm} (13)

These values are consistent with those previously used in the analysis of deep inelastic scattering on polarized nucleons \[26\]:

$$F = 0.459 \pm 0.008, \quad D = 0.798 \pm 0.008 \quad F/D = 0.575 \pm 0.016$$ \hspace{1cm} (14)

For illustration, Fig. 2 shows the best fit for $k$-type solution. Comparing Fig. 2 and Fig. 1, one sees that after inclusion of SU(3) breaking in Cabbibo’s scheme, the lines corresponding to $\Lambda \rightarrow p$ and $\Sigma^{-} \rightarrow n$ are both slightly shifted up and the only significant change is for the line corresponding to $\Xi^{-} \rightarrow \Lambda$. All lines now intersect at one point which gives a unique solution of $F$ and $D$ for a given parameter set. Similar discussion can be carried out for $a$-, $b$- and $d$-type solutions.

It may be noted that all SU(3) symmetry-breaking parameters considered in this paper are significantly smaller than the SU(3) symmetric parameters $F$ and $D$. Compared to the result given in \[21\], our $F/D$ value for a given symmetry breaking parameter set is unique for the known baryon decay modes. It suggests that the entire pattern of existing hyperon semileptonic decay data can be very well described in a framework which is basically SU(3) flavor symmetry with small SU(3) symmetry-breaking effects. Therefore no evidence of strong violation for SU(3) symmetry in hyperon beta decay data can be found.

III. QUARK SPIN DISTRIBUTIONS IN THE NUCLEON

As we mentioned in the introduction, the quark spin distributions deduced from the $g_1$ data depend on the $F/D$ ratio. In the QCD corrected quark parton model, we have

$$\Gamma_1^p \equiv \int_0^1 g_1^p(x) dx = \frac{C_{NS}}{18} \left[ 2\Delta u - \Delta d - \Delta s \right] + \frac{C_S}{9} \Delta \Sigma$$ \hspace{1cm} (15)

where $\Delta u = \int_0^1 \Delta u(x) dx$ and $\Delta \Sigma = \Delta u + \Delta d + \Delta s$ represents the fraction of the proton spin carried by all the quarks and antiquarks, i.e. the net total quark helicity. where
\[ C_{NS} = 1 - y - 3.5833y^2 - 20.2133y^3 - O(130)y^4 \] and \[ C_S = 1 - y/3 - 0.5495y^2 - O(2)y^3, \]
with \( y \equiv \alpha_s/\pi \), are QCD correction coefficients for nonsinglet and singlet terms \[27\]. To simplify the notation, we have omitted the variable \( Q^2 \) in the quantities listed above. It should be noted that the anomalous gluon contributions \[28\] and higher twist effects \[29\] are not included in (15). The former is still a subject of debate and the latter is expected to be only a small correction at the low \( Q^2 \) value (for example, the E142 \( \Gamma_1^n \) data). Combining (15) and the following two relations

\[
\left( \frac{G_A}{G_V} \right)_{n\rightarrow p} = F + D = \Delta u - \Delta d
\]

\[
\left( \frac{G_A}{G_V} \right)_{\Sigma\rightarrow n} = F - D = \Delta d - \Delta s
\]

one obtains

\[
\Gamma_1^{p(n)} = C_{NS}^{12} \left( \frac{G_A}{G_V} \right)_{n\rightarrow p} \left[ +(-)1 + \frac{R - 1/3}{R + 1} \right] + C_s^{\Delta} \Delta \Sigma \]

hence the data \( \Delta \Sigma \) and \( \Delta s \) deduced from \( \Gamma_1^{p} \) depend on \( F/D \) value used as input in (18).

Using \( \left( \frac{G_A}{G_V} \right)_{n\rightarrow p} = 1.254 \pm 0.006, F/D = 0.632 \pm 0.062, \) and \( \alpha_s = 0.27 \) the EMC data \[2\] \( (\Gamma_1^{p})_{exp} = 0.126 \pm 0.018 \) led to

\[
\Delta \Sigma = 0.12 \pm 0.17, \quad \Delta s = -0.19 \pm 0.06
\]

However, if instead, using \( < F/D > = 0.582 \pm 0.008 \) and the same \( C_{NS} = 1 - \alpha_s/\pi \) and \( C_S = 1 - \alpha_s/3\pi \) as used in the EMC analysis \[2\], one obtains

\[
\Delta \Sigma = 0.14 \pm 0.17, \quad \Delta s = -0.15 \pm 0.06
\]

One can see that by using a smaller \( < F/D > \) value, \( \Delta \Sigma \) increases and the magnitude of \( \Delta s \) decreases. However, in contrast to the change of \( \Delta s \), the total quark helicity \( \Delta \Sigma \) is not sensitive to \( < F/D > \). This is consistent with the result given by Lipkin \[30\]. On the other hand, if we use \( C_{NS} \) up to \( (\alpha_s/\pi)^4 \) and \( C_S \) up to \( (\alpha_s/\pi)^3 \) as given in \[27\], then (20) becomes

\[
\Delta \Sigma = 0.19 \pm 0.17, \quad \Delta s = -0.13 \pm 0.06
\]
Comparing (21) with (20), one sees that $\Delta \Sigma$ significantly increases after inclusion of higher order QCD radiative corrections, which are very important in spin analysis, especially at moderate $Q^2$ range where the experiments performed.

Most recently, E143 group obtained more accurate data of $g_1^p$ which gives $\Gamma_1^p = 0.125 \pm 0.003$ [31] with $\alpha_s = 0.35$. From this, one obtains

$$ \Delta \Sigma = 0.27 \pm 0.04, \quad \Delta s = -0.10 \pm 0.02. $$

(22)

The difference between the central values of $\Delta \Sigma$ (and $\Delta s$) in (22) and in (21) is due to that the data are taken at different $Q^2$ and they have different QCD correction coefficients $C_{NS}(Q^2)$ and $C_S(Q^2)$. In obtaining (22), $\alpha_s = 0.35$ has been used, but for (21) $\alpha_s = 0.27$ was used. However, considering that the errors in (21) are quite large, the results given in (22) and (21) are consistent within the errors.

To avoid possible ambiguity caused by $SU(3)$ symmetry breaking effects, we may choose to only use the $SU(2)$ symmetry result (16) and do not use (17). From (15) and (16), one can obtain a relation between $\Delta \Sigma$ and $\Delta s$

$$ c_1 \Delta \Sigma - c_2 \Delta s = \Gamma_1^p - c_3 $$

for the proton and similarly

$$ c_1 \Delta \Sigma - c_2 \Delta s = \Gamma_1^n + c_3 $$

for the neutron, where

$$ c_1 = \frac{C_{NS} + 4C_S}{36}, \quad c_2 = \frac{C_S}{12}, \quad c_3 = c_2 \left( \frac{G_A}{G_V} \right)_{n \rightarrow p} $$

(25)

Actually, (23) and (24) are not independent, because the Bjorken sum rule

$$ \Gamma_1^p - \Gamma_1^n = 2c_3 = \frac{C_S}{6} \left( \frac{G_A}{G_V} \right)_{n \rightarrow p} $$

(26)

Therefore one can not deduce the $\Delta \Sigma$ and $\Delta s$ separately, even we have both $g_1^p$ and $g_1^n$ data.

It should be noted that the data $\Gamma_1^p$ and $\Gamma_1^n$ from the experimental measurements may not satisfy (26). Hence the r.h.s. of (23) can be different from that of (24).
To obtain $\Delta \Sigma$ and $\Delta s$ separately, we need another relation between these two quantities. This can be obtained from (16) and (17)

$$\Delta \Sigma - 3\Delta s = (\frac{G_A}{G_V})_{n \rightarrow p} + 2(\frac{G_A}{G_V})_{\Sigma^+ \rightarrow n}$$

(27)

Using most recent E143 data $\Gamma^p_1 = 0.125 \pm 0.003$ and $\Gamma^n_1 = -0.033 \pm 0.008$ [31], one obtains from (23) and (24)

$$\Delta \Sigma - 0.518\Delta s = 0.325 \pm 0.023 \quad E143 \text{ proton data}$$

(28)

and

$$\Delta \Sigma - 0.518\Delta s = 0.394 \pm 0.063 \quad E143 \text{ neutron data}$$

(29)

They are shown in Fig.3 (line 1 for E143 proton data and line 3 for E143 neutron data, where $Y \equiv \Delta \Sigma$ and $X \equiv \Delta s$). If we assume that there is no strange quark polarization, $\Delta s=0$ as predicted by the naive quark model, then $\Delta \Sigma = 0.33 \pm 0.02$ from the proton data and $\Delta \Sigma = 0.39 \pm 0.06$ from the neutron data. They are consistent within the errors (see line 1 and line 2 in Fig. 3). However, using SU(3) symmetry result eq.(27) and combining data (9) and (11), one obtains

$$\Delta \Sigma - 3\Delta s = 0.577 \pm 0.034$$

(30)

which is also shown in Fig.3 (line 2). One can see that the strange quark polarization would be negative. From Fig. 3, one obtains that the range of $\Delta s$ would be

$$\Delta s = -0.12 \rightarrow -0.04$$

(31)

if the SU(3) symmetry is imposed.

It should be noted that if one can trust the earlier $\nu - p$ and $\bar{\nu} - p$ elastic scattering data, $\Delta s = -0.15 \pm 0.09$ [32] (which gives $\Delta \Sigma \simeq 0.19$ for E143 proton data and $\Delta \Sigma \simeq 0.32$ for E143 neutron data), then the SU(3) symmetry relation (30) is not necessary.
IV. SUMMARY

From a general discussion of SU(3) symmetry and its breaking, we show that the hyperon beta decay data can be well accommodated within the framework of the usual Cabbibo’s SU(3) symmetric description with a small SU(3) symmetry breaking proportional to the mass difference between strange and nonstrange quarks. The F/D ratio is not far from the value previously used in the deep inelastic scattering analysis. Hence the result given by using SU(3) symmetry on hyperon beta decays will not be significantly disturbed by SU(3) symmetry breaking effects. It implies that the total quark helicity is still far below naive quark model expectation and the strange quark polarization seems to be negative provided the anomalous gluon contributions and higher twist effects are neglected.

After completion of this work, we saw the paper by Ratcliffe [33] which reached similar conclusion about SU(3) breaking.

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FIG. 1. $F - D$ relations determined by experimental values for various baryonic transitions, assuming no SU(3) breaking. Line 1: $n \rightarrow p$, line 2: $\Lambda \rightarrow p$, line 3: $\Sigma^- \rightarrow n$, line 4: $\Xi^- \rightarrow \Lambda$. 
FIG. 2. $F - D$ relations, as in Fig.1, allowing for $k$-type SU(3) breaking with $k = 0.0123$. Line 1: $F + D = 1.2573 \pm 0.0028$, line 2: $F + D/3 + k = 0.714 \pm 0.015$, line 3: $F - D = -0.335 \pm 0.017$, line 4: $F - D/3 - k = 0.208 \pm 0.050$. 
FIG. 3. Plot of total quark helicity $Y \equiv \Delta \Sigma$ and strange quark polarization $X \equiv \Delta s$ constrained by the E143 proton data (line 1: eq.(29)) and neutron data (line 3: eq.(30)), and SU(3) symmetry relation (line 2: eq.(34)).
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