Liénard–Wiechert potential of a heavy quark moving in QGP medium

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We investigate the nature of the complex retarded potential of a heavy quark moving in a hot and dense static deconfined nuclear medium. The well-known concept of the retarded potential in electrodynamics is extended to the context of the heavy-quark by modifying the static vacuum Cornell potential through Lorentz transformation to the static frame of the medium. The resulting potential in the vacuum is further corrected to incorporate the screening effect offered by the thermal medium. To do so, the retarded Cornell potential is modified by the dielectric function of the static QGP medium. We present the numerical results for the real and imaginary parts of the potential along with the analytical expression of potential approximated by a small velocity limit. Finally, we present the thermal width of quarkonia in the QGP medium derived using the imaginary part of the potential and study its dependence on velocity and temperature.

Keywords: Debye mass, heavy quark, retarded potential, QGP, heavy-ion collision, dielectric permittivity.

I. INTRODUCTION

The quark gluon-plasma (QGP) is the primordial matter at high temperature and density composed of elementary particles known as quarks, antiquarks, and gluons, free to move beyond the nucleon volume. This phase is believed to exist after the Big Bang when the universe was of the age of a few microseconds. During the evolution of the universe, i.e., on further cooling and spatial expansion, the QGP underwent a phase transition to the hadronic matter, i.e., confined state of quarks and gluons known as hadrons. Their dynamics could be governed by the laws of strong interactions where the corresponding theory is quantum chromodynamics (QCD). The advanced experimental facilities for heavy-ion collisions (HIC), such as the Relativistic Heavy Ion Collider at the Brookhaven National Laboratory and the Large Hadron Collider at CERN, provide us a unique opportunity to study the strongly interacting deconfined quark and gluon matter but within some limitations, viz., the QGP medium formed in these experiments has a very small size (~10 fm) and very short-lived (~10^{-23} sec) [1–4]. Therefore, the very short persistence of the medium created in HIC restricts the possibility of exploring it through external probes to quantitatively characterize its properties. Therefore, we mostly rely on internal probes to look through the created matter. In this context, the heavy quarks (charm and bottom) and their bound states, i.e., heavy quarkonia, have a huge significance [5–8]. They are mostly created, due to their higher masses, at the very early stages after the collisions and behave almost as an independent degree of freedom while passing through the several phases of the created matter. Though they get merely affected by the QGP medium while passing through it, resulting in distinctive signatures in their final yields observed at the detectors. In Ref. [9], Matsui and Satz suggested that the heavy quarkonium production would be suppressed in high-energy heavy-ion collisions due to the Debye screening offered by the plasma that reduces the effective interaction between constituent particles.

In a similar scenario, we want to invite some attention towards the velocity dependence on the potential of a heavy quark (HQ) moving in the QGP medium. To study the Cornell potential [10, 11] which is a linear combination of the Coulomb and linear potentials, plays an important role as it successfully takes into account the two crucial features of QCD, namely the asymptotic freedom (at a small distance or high energy) and the quark confinement (at a large distance or low energy) [12]. The Cornell potential also played an important role in the study of various aspects of the heavy quarkonia. That includes the transition between the confined and deconfined phases of matter [13] and the calculation of the masses of various heavy quarkonium states. The authors in Ref. [14] first applied the potential models to study various quarkonia states at finite temperatures. Later, the quarkonium spectral functions and the meson current correlators have also been obtained from potential models [15–22] and are compared to the first-principle lattice QCD calculations [23–25]. Additionally, it has also been studied that the imaginary part of the potential due to the interaction with the medium leads to the thermal dissociation width of quarkonia states [26, 27].

We aim to study the HQ potential in a context where the QGP medium is static and uniform, and the HQs are moving with respect to the rest frame of the medium. It is similar to the situation of the retarded potential of a moving charged particle in electromagnetic plasma (EMP) or the general Liénard–Wiechert potential in the context of the QCD. The results obtained here for a static and uniform medium do not automatically refer to a clean suppression signal in a rapidly expanding QGP. But it serves the purpose of seeing the relative motion between an HQ and the QGP medium that breaks the
spherical symmetry of the potential. It further helps to understand the modification of the binding of quark and antiquark pairs that, in turn, modify the survival probabilities of quarkonia states observed in an asymmetric emission pattern called “anisotropic flow”. The motivation behind the current analysis is to study the effects of temperature, screening, and velocity on the retarded potential of the moving HQ in the static QGP medium and its angular dependence while in motion. In this article, we provide a framework to study the Liénard–Wiechert/retarded potential of an open heavy flavor that will lead to quarkonium bound states potential inside the QGP. To do so, we write the Cornell potential in a covariant form and then perform a Lorentz transformation to go to the static QGP frame where the HQ is moving. Later, we modify this potential using the dielectric permittivity of the QGP medium. There we observed both the real and imaginary parts of the retarded potential. In our framework, we studied the full angular dependence of the retarded potential and showed the corresponding plots in the results section. Along with the derivation of the analytical expression within the small velocity limit, we also present the full numerical results to compare and check for the validity of the assumption. Further, we use the imaginary part of the potential to calculate the thermal width of the quarkonia and study its dependence on velocity and temperature.

The paper is organized as follows. In section II, we present the derivation of the retarded potential of a HQ moving in the vacuum. In section III, we obtain the complex form of in-medium HQ retarded potential using the static dielectric permittivity of the QGP medium. Section IV is dedicated to the derivation of the potential at small velocities. In section V, we obtain the thermal width of both charmonium and bottomonium states. Section VI is the results section where we discuss the velocity, temperature, and angular dependence of the medium-modified retarded potential and thermal width. Section VII is devoted to a summary and conclusion of the present work. Natural units are used throughout the text with c = ℏ = 1. We use a bold typeface to indicate three-vectors and a regular font to indicate four-vectors. The centre dot depicts the four-vector scalar product with the formula $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$.

II. FORMALISM

Correspondence from the QED with theoretical consistency makes it easier to understand the hot QCD medium. Specifically, the QED plasma resembles QCD plasma (QGP) in some special cases, such as at the soft scale where the field fluctuation is of the order of $\sqrt{g}$ and small coupling [32]. Here, we are employing the analogy of Liénard–Wiechert potential from the electrodynamics and proceeding to obtain the retarded potential for the HQ inside the QGP medium. For that, we first consider the (static) Cornell potential for the HQ in the vacuum and write it in a covariant form. Next, we know that the four-potential in a particular frame can be transformed into any other frame using Lorentz transformations. The four-potential of a HQ (HQ) in its rest frame with the Cornell potential as the scalar part is given by

$$A^\mu_0 = \left( -\frac{\alpha}{r} + \sigma r, 0 \right),$$  \hspace{1cm} (1)

where $r = |r|$ is the the distance from HQ to the field point; $\alpha = C_F C_A$ with $C_F = (N_c^2 - 1)/2N_c$ and $C_A$ is the strong coupling constant; $\sigma$ is the string tension; and $N_c$ is the number of color degrees of freedom. The four-potential $A^\mu_0$ in Eq. (1) can be written in the covariant form by introducing the four-velocity $u^\mu_0 = (1, 0)$ in the rest frame of the HQ. So, in the rest of the HQ the four-potential is

$$A^\mu_0 = \left( -\frac{\alpha}{r_\nu u^\mu_0} + \sigma (r_\nu u^\mu_0), 0 \right),$$  \hspace{1cm} (2)

Note that $r_\nu u^\mu_0 = r$. Now, the Lorentz transformations of Eq. (2) to a frame where the HQ is moving with a velocity, $v$, is given as

$$A^\mu = \left( -\frac{\gamma_\nu \alpha_\nu}{r_\nu u^\mu} + \gamma_\sigma r_\nu u^\mu, \gamma_\nu v_\nu + \gamma_\sigma v \cdot r_\nu u^\mu \right),$$  \hspace{1cm} (3)

where $\gamma = 1/\sqrt{1 - v^2}$ is the Lorentz factor. Rewriting Eq. (3) in a more compact form, one gets

$$A^\mu = \left( -\frac{\alpha}{(r_\nu u^\mu)} + \sigma (r_\nu u^\mu) \right) u^\mu.$$  \hspace{1cm} (4)

Here, $r_\mu$ is the position four-vector from the HQ $(t_r, \mathbf{x})$ at retarded time to some field point $(t, \mathbf{r})$, and $u^\mu = \gamma(1, v)$ with $v = |v|$. It is important to note that the two events which define $r_\nu$ are connected by a signal propagating at the velocity of the light. Therefore, the events have null separation and $r_\mu$ is a light-like vector. The modified form of the Cornell potential shown in Eq. (4) is similar to the form of the Liénard–Wiechert potential in the electrodynamics but the string part is missing there. Now we have,

$$r_\nu u^\nu = r_\gamma - \gamma r \cdot v = \gamma r (1 - \hat{r} \cdot v),$$  \hspace{1cm} (5)

where $\hat{r}$ is the unit vector along $r$. Then the scalar potential, i.e., the zeroth component of the four-potential can be written as

$$V_{\text{vac}}(r, v) = -\frac{\alpha}{r (1 - \hat{r} \cdot v)} + \gamma^2 \sigma r (1 - \hat{r} \cdot v).$$  \hspace{1cm} (6)

This formalism is valid even when the HQ velocity is nonuniform. The calculation in this section uses a sequence of independent Lorentz transformations, each performed at a different point along the trajectory of the particle. To fulfill our purpose of HQ traveling in QGP medium, we can take the velocity to be constant, and further, if we orient our z-axis along the direction of velocity, then at $t = 0$ [28],

$$r (1 - \hat{r} \cdot v) = \sqrt{z^2 + (1 - v^2)(x^2 + y^2)}.$$  \hspace{1cm} (7)
Here, for convenience, the HQ is set to pass through the origin at \( t = 0 \) and used the fact that \( \mathbf{r}' \) is a light-like vector. Now using Eq. (7) in Eq. (6) the retarded potential in Cartesian coordinates becomes,

\[
V_{\text{vac}}(x, y, z, v) = -\frac{\alpha}{\sqrt{z^2 + (1 - v^2)(x^2 + y^2)}} + \gamma^2 \sigma \frac{\alpha}{\sqrt{z^2 + (1 - v^2)(x^2 + y^2)}}.
\] (8)

The medium modification of the retarded potential can be done in the Fourier space by dividing the potential with the dielectric permittivity of the medium. Thus, the vacuum potential in Fourier space is obtained as

\[
V_{\text{vac}}(p_x, p_y, p_z, v) = -\frac{\alpha}{\pi p^2 + p_z^2 + (1 - v^2) p_z^2} - 2\sqrt{\frac{2}{\pi}} \frac{\sigma}{p^4 (1 - v^2 \cos^2 \theta)}.
\] (9)

In spherical polar coordinates, the above equation becomes

\[
V_{\text{vac}}(p, v) = -\frac{\alpha}{\pi p^2 (1 - v^2 \cos^2 \theta)} - 2\sqrt{\frac{2}{\pi}} \frac{\sigma}{p^4 (1 - v^2 \cos^2 \theta)}.
\] (10)

where \( \theta \) is the polar angle in momentum space, i.e., the angle between the \( p_z \) and \( \mathbf{p} \). This expression gives the retarded scalar potential of a moving quark in the vacuum. As discussed earlier, when a charged particle passes through a thermal medium, its properties are affected by the response of that medium. Therefore, when a HQ passes through the QGP medium (which is at rest in this scenario), the retarded potential associated with it will be affected by the response of the QGP medium. Therefore, next, we shall discuss the modification of the HQ potential given in Eq. (10) through the dielectric permittivity of the QGP medium in the Fourier space.

III. DIELECTRIC PERMITTIVITY AND MEDIUM MODIFICATION OF POTENTIAL

The medium modified potential in the coordinate space \( (V(r, v)) \) can be obtained [33, 34] by correcting the vacuum potential with dielectric permittivity encoding the medium screening effect in Fourier space followed by inverse Fourier transformation, i.e.,

\[
\int \frac{d^3 \mathbf{p}}{(2\pi)^{3/2}} (e^{i \mathbf{p} \cdot \mathbf{r}} - 1) V_{\text{vac}}(p, v) \epsilon(p),
\] (11)

where \( V_{\text{vac}}(p, v) \) is the Fourier transform of the potential in coordinate space and \( \epsilon(p) \) is the dielectric permittivity of the medium. Here, we subtract the \( \mathbf{r} \)-independent terms in order to renormalize the HQ free energy [35]. The inverse of the dielectric permittivity of the static QGP medium is given as \([27, 36]\),

\[
\epsilon^{-1}(p) = \frac{p^2}{p^2 + m_D^2} - i\pi T \frac{m_D^2 p}{(p^2 + m_D^2)^2}. \] (12)

where \( p = |\mathbf{p}| \) and \( m_D \) is the Debye mass of QGP medium obtained from the static limit of longitudinal polarisation tensor in the high-temperature limit \([37]\),

\[
m_D = T \sqrt{\frac{4\pi\alpha_s}{N_f/6 + N_c/3}}.
\] (13)

where \( N_f \) is the number of flavor degrees of freedom, \( N_c \) is the number of color degrees of freedom and \( T \) is the temperature of the medium. We use one-loop strong coupling \( \alpha_s \) as \([38–40]\),

\[
\alpha_s = \frac{12\pi}{11N_c - 2N_f} \log \left( \frac{\Lambda_{\overline{MS}}^2}{\Lambda^2} \right)
\] (14)

where \( \Lambda_{\overline{MS}} = 176 \text{MeV} \) is the QCD scale fixing factor and \( \Lambda = 2\pi T \).

In order to calculate the exact real part of the potential we decompose the potential into Coulombic part and string part then perform integration separately,

\[
\Re V(r, v) = \Re V_{\alpha}(r, v) + \Re V_{\sigma}(r, v).
\] (15)

The Coulombic part is written as

\[
\Re V_{\alpha}(r, v) = -\frac{\sqrt{2}}{\pi} \int \frac{d^3 \mathbf{p}}{(2\pi)^{3/2}} \left( \frac{\epsilon^{i \mathbf{p} \cdot \mathbf{r}} - 1}{p^2 + m_D^2} \frac{\alpha}{1 - v^2 \cos^2 \theta} ight)
\] (16)

\[
\Re V_{\sigma}(r, v) = -\frac{1}{(2\pi)^{3/2}} \frac{2\sigma}{p^2 (1 - v^2 \cos^2 \theta)^2},
\] (17)

where

\[
\mathbf{p} \cdot \mathbf{r} = rp [\sin \Theta \sin \theta \cos(\Phi - \phi) + \cos \Theta \cos \theta],
\]

where the angles \( \Theta \) and \( \phi \) are polar and azimuthal angles in Fourier space (momentum space), respectively; whereas the angles \( \theta \) and \( \Phi \) are polar and azimuthal angles in coordinate space. Since the velocity of the HQ is considered along the \( z \)-axis, \( \Theta \) represents the angle between the velocity \( \mathbf{v} \) and the position of the field point \( \mathbf{r} \). The integration over the azimuthal angle, \( \phi \) can be done analytically and one obtains,
\[\Re V(r,v) = -\frac{1}{\pi} \int \frac{\sin \theta d\theta dp}{p^2 + m_D^2} \left[ \frac{\alpha \left( m_D^2 + p^2 e^{ipr \cos \theta \cos \Theta} J_0(pr \sin \theta \sin \Theta) \right)}{1 - v^2 \cos^2 \theta} - \frac{2\sigma \left( 1 - e^{ipr \cos \theta \cos \Theta} J_0(pr \sin \theta \sin \Theta) \right)}{(1 - v^2 \cos^2 \theta)^2} \right],\]

where \( J_0 \) represents the Bessel’s function of the first kind. The integration over \( p \) and \( \theta \) in Eq. (18) can be computed numerically, and the real part of the potential is plotted in the figure. Similarly, the exact imaginary part of the potential is calculated by substituting the imaginary part of the dielectric function in Eq. (11),

\[\Im V(r,v) = \int \frac{d^3 p}{(2\pi)^{3/2}} (e^{ipr} - 1) \frac{\pi T m_D^2 p}{(p^2 + m_D^2)^{3/2}} \sqrt{\frac{2}{\pi}} \left[ \frac{\alpha}{p^2 (1 - v^2 \cos^2 \theta)} + \frac{2\sigma}{p^4 (1 - v^2 \cos^2 \theta)^2} \right],\]

After integrating over \( \phi \) we obtain,

\[\Im V(r,v) = -m_D^2 T \int \frac{\sin \theta d\theta dp}{(p^2 + m_D^2)^{3/2}} \left[ \frac{\alpha p}{1 - v^2 \cos^2 \theta} + \frac{2\sigma}{p (1 - v^2 \cos^2 \theta)^2} \right] \times \left\{ 1 - e^{ipr \cos \theta \cos \Theta} J_0(pr \sin \theta \sin \Theta) \right\}.\]

Both the real and imaginary parts of the potentials are independent of \( \Phi \) after \( \phi \) integration, i.e., potential has axial symmetry about the z-axis. The rest of the integration is done numerically, and the results are plotted as shown in the figure for both imaginary and real parts of the potential.

**IV. POTENTIAL AT SMALL VELOCITIES**

The real and imaginary part of the potential as obtained in Eqs. (18) and (20) can be simplified at small velocities. Considering small velocity, one can expand
FIG. 3. Numerical results for real (left) and imaginary (right) parts of the potential, a comparison of Cornell and Coulomb potential.

FIG. 4. Numerical results for the angular variation of real (left) and imaginary (right) parts of the potential at different velocities. Here, $\Theta$ is in radian.

the $V_p(p,v)$ in Eq. (10) and keep terms up to $O(v^2)$ as,

$$ \frac{1}{1 - v^2 \cos^2 \theta} \approx 1 + v^2 \cos^2 \theta + O(v^4). \quad (21) $$

This approximation is valid for the case of a HQ or quarkonia moving in the QGP medium with relatively small velocity. Next, we can analytically perform the integration in Eq. (11) using the approximation given in Eq. (21) to obtain the real part and imaginary part of the potential.

The modified form of the potential Eq. (10) in small velocity limit is

$$ V_p(p,\theta,v) = - \sqrt{\frac{2}{\pi}} \frac{1}{p^2} \left( \alpha + \alpha v^2 \cos^2 \theta + \frac{2\sigma}{p^2} + \frac{4\sigma v^2 \cos^2 \theta}{p^2} \right). \quad (22) $$

Therefore the real part of the potential at small velocity is obtained as

$$ \Re V(p,v) = \int \frac{d^3p}{(2\pi)^{3/2}} \left( e^{i p r} - 1 \right) V_p(p,\theta,v) \Re [e^{-1}(p)]. \quad (23) $$

The inverse Fourier transform of Eq.(23) is easy to calculate in spherical polar coordinates with $\cos \theta = p_z/p$. Doing so, the real part of the potential is computed as,
FIG. 5. Numerical results for real potential at different temperatures and velocities, a comparison.

FIG. 6. Numerical results for imaginary potential at different temperatures and velocities, a comparison.

\[
\Re V(r, \theta, v) \approx -\frac{\alpha e^{-m_D r}}{r} - \alpha m_D - \frac{2\sigma}{m_D r} (1 - e^{-m_D r}) + 2\sigma m_D - \frac{\alpha v^2 m_D}{3} - \frac{\alpha v^2}{m_D^2 r^3} (1 - 3 \cos^2 \Theta) \\
+ \frac{\alpha v^2 e^{-m_D r}}{r} \left[ \frac{1}{m_D r} + \frac{1}{m_D^2 r^2} - \left( 1 + \frac{3}{m_D r} + \frac{3}{m_D^2 r^2} \right) \cos^2 \Theta \right] + 4\sigma v^2 \left( 1 - \cos^2 \Theta \right) \\
+ \frac{4\sigma v^2}{m_D^3} \left( 1 - 3 \cos^2 \Theta \right) + 4\sigma v^2 e^{-m_D r} \left[ \frac{\cos^2 \Theta}{m_D^2 r} + \frac{3 \cos^2 \Theta}{m_D^3 r^2} + \frac{3 \cos^2 \Theta}{m_D^4 r^3} - \frac{1}{m_D^3 r^3} \right]. \tag{24}
\]

At \( v = 0 \), the approximate real part of the potential in Eq. (24) becomes the more familiar screened Cornell potential where the angular dependence is also disappeared,

\[
\Re V(r, \theta, 0) = -\frac{\alpha r}{r} - \alpha m_D - \frac{2\sigma}{m_D r} (1 - e^{-m_D r}) + 2\sigma m_D - \frac{\alpha v^2 m_D}{3} - \frac{\alpha v^2}{m_D^2 r^3} (1 - 3 \cos^2 \Theta) \\
+ \frac{\alpha v^2}{m_D^3} \left( 1 - 3 \cos^2 \Theta \right). \tag{25}
\]

The screened Cornell potential at \( v = 0 \) further converges to the vacuum Cornell potential as \( m_D \to 0 \) with \( T \to 0 \), we have

\[
\Re V(r)_{v=0, T=0} = -\frac{\alpha}{r} + \sigma r. \tag{26}
\]

Similarly, the imaginary part of the retarded potential in the small velocity approximation can be obtained as,

\[
\Im V(p, \theta, v) = \int \frac{d^3 p}{(2\pi)^{3/2}} (e^{ip \cdot r} - 1)V(p, \theta, v) \text{Im}[\epsilon^{-1}(p)]. \tag{27}
\]

After performing the \( \phi \) and the \( \theta \) integration, we obtain the following results for the imaginary part of the poten-
tial. For the static case, the potential is isotropic, and it is obtained as

\[ \mathcal{W}_{\text{iso}} = -2\alpha T \int_0^\infty \frac{z}{(z^2 + 1)^2} \left[ 1 - \frac{\sin(m_D r z)}{m_D r z} \right] dz \]

\[ - \frac{4\alpha T}{m_D^2} \int_0^\infty \frac{dz}{z(z^2 + 1)^2} \left[ 1 - \frac{\sin(m_D r z)}{m_D r z} \right], \tag{28} \]

\[ \mathcal{W}^\parallel (v, r) = \mathcal{W}_{\text{iso}} + \frac{2}{3} v^2 T \left( \frac{\alpha}{3} \int_0^\infty \frac{zd\theta}{(z^2 + 1)^2} \left[ 1 - \frac{3\sin(m_D r z)}{m_D r z} - \frac{6\cos(m_D r z)}{m_D^2 r^2 z^2} + \frac{6\sin(m_D r z)}{m_D^2 r^3 z^3} \right] + \frac{4\alpha}{m_D^2} \int_0^\infty \frac{dz}{z(z^2 + 1)^2} \left[ 1 - \frac{3\sin(m_D r z)}{m_D r z} - \frac{6\cos(m_D r z)}{m_D^2 r^2 z^2} + \frac{6\sin(m_D r z)}{m_D^2 r^3 z^3} \right] \right), \tag{29} \]

\[ \mathcal{W}^\perp (v, r) = \mathcal{W}_{\text{iso}} + \frac{2}{3} v^2 T \left( \frac{\alpha}{3} \int_0^\infty \frac{zd\theta}{(z^2 + 1)^2} \left[ 1 + \frac{3\cos(m_D r z)}{m_D^2 r^2 z^2} - \frac{3\sin(m_D r z)}{m_D^2 r^3 z^3} \right] + \frac{4\alpha}{m_D^2} \int_0^\infty \frac{dz}{z(z^2 + 1)^2} \left[ 1 + \frac{3\cos(m_D r z)}{m_D^2 r^2 z^2} - \frac{3\sin(m_D r z)}{m_D^2 r^3 z^3} \right] \right). \tag{30} \]

Therefore we can write \( A(r, T, v) = \left[ \mathcal{W}^\parallel (v, r) + \mathcal{W}^\perp (v, r) \right] /2 \) and \( B(r, T, v) = \left[ \mathcal{W}^\parallel (v, r) - \mathcal{W}^\perp (v, r) \right] /2 \). It is evident from Eq. (29) and Eq. (30) that at \( v = 0 \), the approximate imaginary part of the potential will contain only the isotropic part given in Eq. (28), which also vanishes in the vacuum as \( T \to 0 \). That means only the Cornell potential given in Eq.(1) remains after taking the limit \( v \to 0 \) and \( T \to 0 \), the original potential we started with.

\[ \Psi = \frac{1}{\sqrt{\pi a_0}} e^{-r/a_0}, \tag{33} \]

where \( a_0 = 2/(C_F m_Q a_s) \) is the Bohr radius corresponds to the quarkonia and \( m_Q \) is the quark mass. Note that one can get the exact wave function solving Schrödinger equation with the real part of the potential (18) and we intended to do that in near future. Substituting Eq. (33) in Eq. (32) gives

\[ \Gamma_{Q\bar{Q}}(v) = \frac{1}{\pi a_0} \int d^3r e^{-2r/a_0} \mathcal{W}_{\text{Mod}} (v, r, \Theta). \tag{34} \]

Here we obtain the exact results using full imaginary potential given in Eq. (20) as

\[ \Gamma_{Q\bar{Q}}(v) = \frac{2m_D^2 T}{a_0^2} \int dr \sin \theta d\theta \sin \theta e^{-2r/a_0} \left[ \frac{\alpha p}{(p^2 + m_D^2)^2} \left( \frac{1 - v^2 \cos^2 \theta}{1 - v^2} \right) + \frac{2\sigma}{p (1 - v^2 \cos^2 \theta)^2} \right] \]

\[ \times \left\{ 1 - e^{i p r \cos \theta \cos \Theta} \right\} J_0 (p r \sin \theta \sin \Theta), \]

\[ = \frac{a_0^2 m_D^2 T}{(a_0^2 m_D^2 - 4)^3} \left[ \frac{\alpha}{2v} \log \frac{1 + v}{1 - v} \left( 48 - 16a_0^2 m_D^2 + a_0^4 m_D^4 + 64 \log \frac{a_0 m_D}{2} \right) \right] + \frac{2\sigma}{m_D^2} \left( \frac{1}{1 - v^2} + \frac{1 + v}{2v} \log \frac{1 + v}{1 - v} \right) \left[ 16 - 16a_0^2 m_D^2 + 12a_0^4 m_D^2 \log \frac{a_0 m_D}{2} - a_0^4 m_D^2 \log \frac{a_0 m_D}{2} \right]. \tag{35} \]
We plot the thermal width of the charmonium and the bottomonium ground states as a function of temperature and velocity using the expression derived above, which are further discussed in the result sections.

VI. RESULTS

The HQ potential in the QGP medium is studied analytically and numerically with respect to various parameters, primarily the HQ velocity and angular dependence. The thermal width of the quarkonium ground state is derived, and its dependence on velocity and temperature is studied. The illustration of results in various plots used different temperature $T = 1.5T_c$, $2T_c$, and $2.5T_c$ where the crossover temperature, $T_c = 0.155\text{ GeV}$. The number of quark flavors $N_f = 3$ and $\sigma = 0.18\text{ GeV}^2$. The temperature dependence in the potential arises through the strong coupling ($T$-dependence in the potential) and dielectric function ($\epsilon$). Both parts are symmetric about the plane containing the particle and perpendicular to the direction of motion. We can observe that the potential and its variation are different in all three directions. Initially, the potential increases sharply and then saturate as the distance increases. The potential decreases with an increase in velocity at a very short distance, whereas at a large distance, the potential increases as velocity increases; this switching is more noticeable in the $\Theta = \pi/2$ case. In Fig. 1 we have made a comparison between Cornell potential and Coulomb potential ($\sigma = 0$ case) along the direction of motion of the HQ. Both the real and imaginary parts of the Coulomb potential have negligible variation with change in velocity, whereas Cornell potential drastically changes with velocity from static to relativistic case. This implies the velocity dependence of the Cornell potential is almost solely due to the string part of the potential.

In Fig 5 we have made a comparison between velocity dependence and temperature dependence of the real part of the potential. It is interesting to note that velocity dependence is most prominent along the direction of the velocity of HQ for both real and imaginary parts of the potential. At low velocities, there is little variation in potential, but as velocity increases, the spherical symmetry breaks down, leading to an increase in anisotropy. The real part is minimum, and the imaginary part is maximum at $\Theta = \pi/2$ (for $\Theta = 1.57\text{ rad}$) direction. Therefore, the quarkonia are most likely to be oriented in a plane perpendicular to its direction of motion.

In Fig 6 we compare velocity dependence and temperature dependence of the imaginary potential is more sensitive to the velocity along the direction of motion, i.e., at $\Theta = 0$. We can observe that the effect of velocity decreases with an increase in temperature in the case of the real part of the potential, especially at $\Theta = 0$, i.e., the variation of potential with change in velocity is more at $T = 1.5T_c$ than $T = 2.5T_c$. In comparison to the static case, $v = 0$ potential changes at finite/high velocity. Our results show that the velocity dependence of the real part is as important as temperature dependence. Similarly, in Fig 6 we compare velocity dependence and temperature dependence of the imaginary potential.
The variation of the potential at different angles and different velocities is more or less the same. The potential changes rapidly along the direction of motion $\Theta = 0$ of the HQ than the perpendicular direction $\Theta = \pi/2$. Our results show that the HQ velocity, as well as the medium temperature, highly influence the HQ potential.

Figure 7 shows the variation of the thermal width with velocity (left) and temperature (right) of charmonium ($J/\Psi$) and bottomonium ($\Upsilon$) ground states. As we have already discussed that the magnitude of the imaginary part of the potential increases with velocity and temperature; hence, the thermal width also increases with velocity and temperature for both the quarkonia states. Note that the thermal width obtained here qualitatively agrees with the thermal width calculated in Ref. [44] considering next-to-leading order. The mass of the charm quark (taken $M_c = 1.27$ GeV) is less as compared to the bottom (taken $M_b = 4.18$ GeV), one can notice that the thermal width of $J/\Psi$ is higher than $\Upsilon$ for the same parameters. This preserves the fact that the lighter bound state, i.e., $\bar{c}c$ dissociates faster than the comparatively heavier one.

VII. SUMMARY AND CONCLUSION

In the current analysis, we have studied the potential of a moving HQ in a static QGP medium. First, we derived the retarded potential of a uniformly moving HQ in the vacuum following the analogy of the Liénard–Wiechert potential in the electrodynamics, where we performed Lorentz transformation on the static potential to find its form in a boosted frame. The resulting velocity and angular-dependent potential are further modified for the inclusion of the QGP medium screening effect. This has been done through the medium dielectric permittivity, a complex quantity, which leads to a complex potential. We presented the exact numerical results and derived the analytical expression in the small velocity limit for both the real and imaginary parts of the potential. We have shown in the plots the variation of potential with respect to several parameters such as distance from HQ, temperature, velocity, and also angular dependence. We have also presented a comparison of Coulombic and Cornell potential, considering the presence and absence of string terms. As expected, the Coulombic contribution does not satisfy the basic requirement of the strong interaction, i.e., repulsion between $q$ and $\bar{q}$ at short and attraction at a large distance. Next, it is observed that the motion of HQ through the QGP breaks down the spherical symmetry of the potential, and the anisotropy of the potential increases with the increase in velocity. It has also been noted that the velocity dependence of the potential is as important as the temperature dependence. The maximum variation of both the real and imaginary part of the potential from the corresponding static case is found to be along the direction of motion of the HQ. Finally, we obtained thermal width, which is found to increase with velocity and temperature. In all the plots, it has been consistently observed that the quark-antiquark pair bonding becomes weaker as the velocity increases due to the thermal width hike. Also, the real part of the potential becomes positive quickly as distance increases with velocity, especially along the direction of HQ motion, i.e., Debye sphere shrinks and deformed.

As a continuation of the present work, we would like to use the potential derived in this article to study the dynamics of the heavy quarkonia propagating in the QGP medium. The binding energy can be calculated by solving the Schrödinger equation using the real part of the potential. The velocity and the angular dependence of the potential are expected to modify the survival probabilities of the quarkonia. This will be a matter of investigation in the near future.

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