The origin of the helicity hemispheric sign rule reversals in the mean-field solar-type dynamo

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ABSTRACT
Observations of proxies of the magnetic helicity in the Sun over the past two solar cycles revealed reversals of the helicity hemispheric sign rule (negative in the North and positive in the South hemispheres). We apply the mean-field solar dynamo model to study the reversals of the magnetic helicity sign for the dynamo operating in the bulk of the solar convection zone. The evolution of the magnetic helicity is governed by the conservation law. We found that the reversal of the sign of the small-scale magnetic helicity follows the dynamo wave propagating inside the convection zone. Therefore, the spatial patterns of the magnetic helicity reversals reflect the processes which contribute to generation and evolution of the large-scale magnetic fields. At the surface, the patterns of the helicity sign reversals are determined by the magnetic helicity boundary conditions at the top of the convection zone. We demonstrate the impact of fluctuations in the dynamo parameters and variability in dynamo cycle amplitude on the reversals of the magnetic helicity sign rule. The obtained results suggest that the magnetic helicity of the large-scale axisymmetric field can be treated as an additional observational tracer for the solar dynamo and it probably can be used for the solar activity forecast as well.

Key words: dynamo – MHD – turbulence – Sun: activity – Sun: interior – Sun: magnetic fields.

1 INTRODUCTION
Vector magnetographic observations of the solar active regions (ARs) show that the distribution of the electric current helicity has a pronounced antisymmetry with respect to the solar equator (Seehafer 1990; Pevtsov, Canfield & Metcalf 1994, 1995; Bao & Zhang 1998; Kuzanyan, Zhang & Bao 2000; Hagino & Sakurai 2005; Zhang et al. 2010). This phenomenon is called the hemispheric sign rule of current helicity. By analysis of the photospheric vector magnetograms of ARs, it has been shown that the current helicity in the Northern hemisphere is mainly negative while in the Southern hemisphere it is positive. The same hemispheric sign rule was obtained from the synoptic magnetic field maps by Pevtsov & Latushko (2000) (see, also, Pevtsov, Canfield & Latushko 2001). Both kinds of observations deal with the line-of-sight part of current helicity, which can be identified with the total current helicity density using the assumption of the spatial isotropy of the current helicity distribution.

It is possible to relate the current helicity density to the magnetic helicity density taking into account the theoretical assumption on turbulent nature and isotropy of the magnetic fields (see, e.g., Moffatt 1978; Kleeorin & Rogachevskii 1999). Magnetic helicity is an integral of motion in magnetohydrodynamics (MHD; Woltjer 1958; Moffat 1969). This impacts the saturation of the magnetic field generation in the large-scale helical dynamos (Frisch et al. 1975; Kleeorin & Ruzmaikin 1982; Vainshtein & Kitchatinov 1983; Kleeorin et al. 2000; Brandenburg & Subramanian 2005). Thus, the information about the surface distribution of the current helicity density and about its evolution with the solar cycle may be important for our understanding of the dynamo processes inside the solar convection zone (Kleeorin et al. 2003; Choudhuri, Chatterjee & Nandy 2004; Zhang et al. 2012). It is also important for understanding the processes of the magnetic helicity transport from the convection zone to the corona (Berger & Ruzmaikin 2000; Brandenburg et al. 2011; Warnecke, Brandenburg & Mitra 2011).

The observations indicate departure from the hemispheric sign rule (Bao, Ai & Zhang 2000; Hagino & Sakurai 2005; Tiwari, Venkatakrishnan & Sankarasubramanian 2009). It was found that at some periods of the solar cycle, the hemispheric sign rule reverses to the opposite, at least at some latitudes and times (Zhang et al. 2010). It was realized that the properties of these reversals may...
be related to the kind of the dynamo operating in the Sun with the distribution of the kind of the dynamo process inside the convection zone, and with the types of the magnetic helicity loss involved in the dynamo (see, e.g., Sokoloff et al. 2006; Guererro, Chatterjee & Brandenburg 2010; Mitra et al. 2011; Pipin & Kosovichev 2011c; Zhang et al. 2012). These mechanisms do not exclude the local processes which may take part in the formation of the twisted magnetic field at the subsurface layers. Some of them were brought to notice in the literature and could be considered as alternative points of view to the problem (see, e.g., Longcope, Fisher & Pevtsov 1998; Kuzanyan, Pipin & Seehafer 2006; Pevtsov & Longcope 2007).

The purpose of this paper is to analyse the origin of the current helicity sign rule reversals within the framework of solar mean-field dynamo models. In our study, we examine the dynamo distributed over the convection zone. In this model, the global dynamo wave is shaped by the subsurface shear layer (Pipin & Kosovichev 2011a). Our approach is a development of the results of the dynamo model of Pipin et al. (2013) which alleviates catastrophic quenching by consideration of total magnetic helicity conservation. We compare our results with the ones for the solar dynamo operating in an overshoot layer at the bottom of the solar convective zone (see Zhang et al. 2012). Our study confronts the results of theoretical modelling with available observational data from the Huairou Solar Observing Station of Chinese Academy of Sciences.

2 BASIC EQUATIONS

The details of the model can be found in our previous papers (see, e.g., Pipin, Sokoloff & Ussoskin 2012; Pipin, 2013). Here, we briefly outline the basic framework. We study the mean-field induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{E} + \mathbf{U} \times \mathbf{B}),$$

(1)

where $\mathbf{U}$ is the mean velocity (differential rotation); $\mathbf{B}$ is the axisymmetric magnetic field:

$$\mathbf{B} = e_\theta \mathbf{b} + \nabla \times \frac{Ae_\theta}{r \sin \theta} \mathbf{e}_\theta,$$

(2)

where $\theta$ is a polar angle and $r$ is a radial distance; $\mathbf{E} = \mathbf{u} \times \mathbf{b}$ is the mean electromotive force, with $\mathbf{u}$ and $\mathbf{b}$ being the fluctuating velocity and magnetic field, respectively. Using the mean-field MHD framework (Krause & Rüdiger 1980), we write the $\mathbf{E}$ as follows:

$$\mathbf{E}_i = \left( \alpha_{ij} + \gamma_{ij}^{(A)} \right) \mathbf{B}_j - \left( \eta_{ijk} + \eta_{ijk}^{(A)} \right) \nabla_i \mathbf{B}_k,$$

(3)

where the turbulent kinetic coefficients are the $\alpha$ effect, $\alpha_{ij}$, the turbulent pumping $\gamma_{ij}^{(A)}$, the anisotropic diffusivity $\eta_{ijk}$ and the $\delta$ dynamo effect (Rüdiger 1969), $\eta_{ijk}^{(A)}$. They depend on the parameters of the turbulent convection, such as the mean density and turbulent diffusivity stratification, and the Coriolis number $\Omega^2 = 2\tau \Omega_0$, where $\tau_c$ is the typical convective turnover time and $\Omega_0$ is the global angular velocity.

The $\alpha$ effect includes the hydrodynamic and magnetic helicity contributions,

$$\alpha_{ij} = \kappa \sin^2 \theta \alpha_{ij}^{(H)} + \alpha_{ij}^{(M)},$$

(4)

where the hydrodynamic part of the $\alpha$ effect is defined by $\alpha_{ij}^{(H)}$. Note that we introduced the latitudinal factor $\sin^2 \theta$ to bring the model to better agreement with observations. This is supported by theoretical calculations of the $\alpha$ effect for convective turbulent flows (Kleeorin & Rogachevskii 2003) and it is often employed by others in their dynamo models (see, e.g., Moss & Brooke 2000). The expressions for the turbulent kinetic coefficients $\alpha_{ij}^{(H)}$, $\gamma_{ij}^{(A)}$, $\eta_{ijk}$ and $\eta_{ijk}^{(A)}$ are given in Appendix A. The contribution of small-scale magnetic helicity $\mathcal{F} = \mathbf{a} \cdot \mathbf{b}$ (a is the fluctuating magnetic vector potential) to the $\alpha$ effect is defined as follows (see P08):

$$\alpha_{ij}^{(M)} = 2\gamma_{ij}^{(A)} \epsilon_{ijk} \mathcal{F}_k - 2\gamma_{ij}^{(A)} \epsilon_{ijk} \mathcal{F}_k - 2J_j^{(a)} \epsilon_{ijk} \mathcal{F}_k.$$

(5)

The principal non-linear feedback of the large-scale magnetic field on the $\alpha$ effect is due to a dynamical quenching because of the generation of the magnetic helicity by the dynamo (Frisch et al. 1975; Kleeorin & Ruzmaikin 1982; Brandenburg & Subramanian 2005). The relation of magnetic helicity on the large and small scales (Blackman & Field 2002; Hubbard & Brandenburg 2012; Pipin 2013) is governed by the equation

$$\frac{\partial \mathcal{F}}{\partial t} = - \frac{\partial (\bar{\mathbf{A}} \cdot \mathbf{B})}{\partial t} - \eta \mathbf{B} \cdot \nabla \mathcal{F} - \nabla \times \mathcal{F},$$

(6)

where $\mathcal{F} = -\eta_\alpha \nabla \mathcal{F} = \mathcal{F}$ is the diffusive flux of the total magnetic helicity (Mitra et al. 2010) and $\eta_\alpha$ is the turbulent diffusion coefficient for the magnetic helicity. In this paper, we use $R_\alpha = 10^6$ and $\eta_\alpha = 0.1 \eta_T$, where $\eta_T$ is the turbulent diffusivity profile (see Appendix A). For the axisymmetric magnetic fields, the large-scale magnetic vector potential is

$$\bar{\mathbf{A}} = e_\theta T + r P = \frac{e_\theta}{r \sin \theta} A + r e_\theta P.$$

(7)

The toroidal part of the vector potential is governed by the dynamo equations. The poloidal part of the vector potential can be restored from the equation $\nabla \times (r P) = e_\theta B$. The scalars $A$ and $P$ are uniquely determined with proper initial conditions and normalization $\oint \vec{A} d\omega = \oint P d\omega = 0$, where integration is done over the solid angle (see, e.g., Krause & Rüdiger 1980). We start our simulations from a weak initial large-scale field of mixed parity with zero helicity density. We matched the potential field outside and the perfect conductivity at the bottom boundary with the standard boundary conditions. For the magnetic helicity, we employ $\mathcal{F}_T = 0$ at the bottom of the convection zone. This paper elaborates two kinds of the surface boundary conditions for the magnetic helicity:

$$\eta_\alpha \nabla_i (\mathcal{F} + \bar{\mathbf{A}} \cdot \mathbf{B}) = 0, \quad \text{at surface},$$

(8)

$$\eta_\alpha \nabla_i (\mathcal{F} + \bar{\mathbf{A}} \cdot \mathbf{B}) = 0, \quad \text{at surface},$$

(9)

We call the model that satisfies the boundary condition (8) as model B1, and similarly, model B2 is referred to equation (9).

The construction of the radial profiles for the turbulent coefficients, which are involved in the mean electromotive force, remains rather arbitrary for various kinds of the dynamo models. In our models, we use the solar convection zone model developed by Stix (2002). In this paper, we use the same profiles for the turbulent coefficients as in our previous papers (see Pipin 2013, and appendix therein).

3 RESULTS

Fig. 1 shows the snapshots of the magnetic field and magnetic helicity evolution in the North hemisphere for model B1 which uses equation (8). The qualitatively similar results can be obtained for model B2. Here we see that the spatial patterns of the small-scale magnetic field follow the evolution of the large-scale magnetic...
helicity and the latter propagates with the toroidal part of the dynamo wave from the bottom of the convection zone to the surface. The dynamo wave has the equatorial and the polar branches. Near the surface the equatorial branch dominates. The hemispheric helicity rule suggests that the small-scale helicity is negative at the North and positive at the South hemisphere. Fig. 1 shows that in the upper part of the convection zone, the helicity rule is valid in the most phases of the cycle. In the upper part of the convection zone, the reversal sign of the small-scale magnetic helicity regions appears at the high latitudes when the dynamo wave of the toroidal magnetic field comes to the subsurface shear layer. At the equatorial latitudes, the reversal sign of $\chi$ occurs at the decaying phases of the dynamo wave cycle. One can see that the signs of the large- and small-scale helicities are spatially related. Fig. 2 illustrates this for the time-latitude and the time-radius variations of the magnetic field and magnetic helicity. The figure also demonstrates the effect of the boundary condition change for the magnetic helicity. For the boundary condition (8), the regions with reversed sign of the small-scale magnetic helicity penetrate into the surface while the condition (9) quenches this penetration. We find that the patterns of the reversed sign of the magnetic helicity are located at the edges of the butterfly wings of the time–latitude diagrams for the large-scale toroidal magnetic field. The novel feature which is demonstrated by Fig. 2 is the time–latitude diagram for the large-scale magnetic helicity which is attributed to the axisymmetric magnetic field. It is seen that within the current model, its distribution is closely connected with the distribution of the small-scale helicity. It is believed that the current helicity of the surface magnetic field is the observational proxy for the magnetic helicity $\chi$. We note that our model uses the full information about the large-scale magnetic helicity.

The difference in penetration of the magnetic helicity to the surface results in difference in the distribution of the effective $\alpha$ effect near the surface. This issue is recently discussed by Käpylä, Mantere & Brandenburg (2012), Pipin et al. (2013) and Pipin (2013). Fig. 3 shows the snapshots of the $\alpha_{\phi\phi}$ and the small-scale magnetic helicity profiles for the different phases of the cycle at the latitude 45°. We find that for model B1, the $\alpha$ effect can be negative at the certain phases of the cycle and it has the sharp positive profile near the surface. Model B2 has the negative $\alpha$ effect for $r > 0.92R$ with the sharp negative profile near the surface. The abrupt growth of the $\alpha$ effect amplitude near the surface is because of the factor $(\pi \ell^2)^{-1}$

Figure 1. Snapshots of the magnetic field and magnetic helicity evolution inside the convection zone. The top panel shows the field lines of the poloidal component of the mean magnetic field, and the toroidal magnetic field (varies ±1 kG) is shown by colour. The bottom panel shows the small-scale magnetic helicity density (contours) and the large-scale magnetic helicity density (colour). Both quantities vary with the same magnitude.
Figure 2. The left column shows the results for model B1. Panel (a) shows the time–latitude diagram for the current helicity (background image). Panel (b) shows the toroidal magnetic field variations at $r = 0.95R$. Panel (c) shows variations of the small-scale magnetic helicity and the toroidal magnetic field inside the convection zone at the latitude 30°. Panels (d)–(f) show the same results for model B2.

Figure 3. Panels (a) and (b) show variations of the $\alpha$ effect and the small-scale magnetic helicity at the latitude 45° for model B1 (equation 8), and panels (c) and (d) show the same for model B2 (equation 9).

in the definition, see equation (5). It remains the matter of the direct numerical simulations to justify the correct choice of the boundary condition for the magnetic helicity. Model B2 has the zero boundary condition for the derivative of the small-scale current helicity at the top, see equation (9). It is found that for the condition $\nabla \chi_{\text{r,max}} = 0$, the negative part of the alpha effect near the surface is stronger than the one in model B2.

3.1 Impact of dynamo fluctuations on the helicity patterns

The sign reversals of the helicity rule can be due to random fluctuations in the dynamo parameters and due to some random processes which generate the magnetic helicity independent of the large-scale dynamo. In this subsection, we examine the effect of fluctuations in the dynamo parameters on the magnetic helicity distribution variations. We exploit here a scenario (Moss et al. 2008; Usoskin, Sokoloff & Moss 2009; Pipin et al. 2012) with fluctuations of the $\alpha$ effect as a possible source of the solar activity cycle parameters from one cycle to another. We introduce a random non-symmetric about equator variations of the $\alpha$ effect, $C_{\alpha} = C_{\alpha}(1 + 0.2(\xi_N/\Theta_1(\mu) + \xi_S/\Theta_1(-\mu)))$, where $\mu = \cos \theta$, $\Theta_1$ is the Heaviside function and $|\xi_{S,N}| < 2\sigma(\xi_{S,N})$ is the random Gaussian noise with the randomly floating phase and with the mean memory time equals to the dynamo cycle length. In this subsection, model B1 is discussed as it shows the stronger reversals of the helicity rule than model B2.

We found that the reversals of the helicity rule are stronger during the periods of the grand minimum which are also related to the periods of the strong hemispheric asymmetry in the magnetic activity. Fig. 4 shows variations of the integral parameters of the model for the near-surface magnetic field. In our results we show the parity index, which determines the symmetry of the toroidal magnetic field about equator, with the value $-1$ corresponding to the dipolar symmetry of the near-surface toroidal magnetic fields and the value 1 corresponding to the quadrupolar symmetry. The sunspot number was simulated following Pipin et al. (2012). We also show the integral magnetic helicity for each hemisphere. The magnitude of the helicity variations is in agreement with the observational constraints obtained by Berger & Ruzmaikin (2000). Variations of the magnetic helicity go in antiphase at the large and small scales, because it is prescribed by equation (6). Nevertheless, for each hemisphere, there is a difference between the evolution of $A \cdot B$ and $\chi = a \cdot b$. The small-scale helicity, $\chi$, does change the sign in a course of the solar cycle and the large-scale helicity $A \cdot B$ almost does not. This is similar to results shown in Fig. 2, where we see that reversals of the helicity rule are much stronger for the small-scale helicity than for the large-scale one. Another interesting result is that the maxima of the integral large-scale magnetic helicity approximately correspond to the maxima of the decay rate in the simulated sunspot activity (cf. Figs 4c–e). This is due to the oscillatory character of the dynamo and delay between the activity of the major components of the large-scale magnetic fields which are related to the toroidal magnetic field and the large-scale toroidal vector potential determining the poloidal magnetic field.

Finally, Fig. 5 shows a comparison of the results for the simulated time–latitude diagrams for the toroidal magnetic field and the magnetic helicity with results of the current helicity observations reported by Zhang et al. (2010). We used a systematic series of vector magnetographic observations of solar ARs by the 35 cm filter-type Solar Multi-Channel Telescope (SMCT) telescope at the Huairou
The integral characteristics of the magnetic activity near the surface, $r = 0.95R$. (a) The parity index determining the symmetry of the toroidal magnetic field about the equator, $−1$ corresponds to the dipolar symmetry and 1 to the quadrupolar; (b) the simulated sunspot number (SN); (c) the latitudinal integral of the large-scale magnetic helicity, the dashed line denotes the North hemisphere, the dash–dotted line denotes the South one; (d) the same as (c) for the small-scale magnetic helicity; (e) the same as (b) (the SN), but the solid line denotes the total SN, the dashed line is shows the SN for the North hemisphere and the dash–dotted line shows the SN for the South hemisphere.

The results of the dynamo model are shown for the period of the grand minimum. It is the same period as discussed for Figs 4(c)–(e) above. The simulated butterfly diagrams are in visible qualitative agreement with the observations. One can see that the model keeps the basic antisymmetry of helicity (negative in the North and positive in the South, i.e. the so-called hemispheric sign rule); however, with evolution it shows various deviations from perfect periodicity (e.g. longer cycles, suppression of activity, asymmetry in the phases of growth and decay.

Solar Observing Station of Chinese Academy of Sciences. The data set comprises 6205 individual magnetograms of ARs more or less homogeneously covering the 18 yr period of 1988–2005, which is almost two sunspot cycles. The data have been grouped and averaged into statistically significant subsamples in time–latitude bins (2 yr in time and 7° in heliolatitude); see Zhang et al. (2010) for details. We have subsequently smoothed the data by using standard IDL linear interpolation for retaining only global features of the time–latitude distribution of helicity.
4 DISCUSSION AND CONCLUSIONS

The available bulk of the current helicity data covers two activity cycles and the transition to the following activity cycle which has been quite unusual. The observed helicity butterfly diagrams demonstrate that the size of the areas with the opposite helicity signs in the later cycle differs substantially from the first one. We studied the origins of the reversals of the magnetic helicity sign in the mean-field solar dynamo. The evolution of the magnetic helicity in the model is subjected to the global constraint of the magnetic helicity conservation law. The non-linear feedback of the large-scale magnetic field to the $\alpha$ effect is described by dynamical quenching due to the constraint of magnetic helicity conservation. The magnetic helicity, $\chi$, is subjected to the conservation law.

In the model, the sign reversals of the small-scale magnetic helicity are always related to the sign reversals of the large-scale magnetic helicity. This is due to the magnetic helicity conservation constraint. The result develops the simple model by Xu et al. (2009). The idea was recently elaborated by Zhang et al. (2012) for the toroidal part of the current helicity. Our model employs the total large-scale magnetic helicity and not only its toroidal part. Taking into account equations (2) and (7), we get the large-scale magnetic helicity formula for the spherical coordinates:

$$\mathbf{A} \cdot \mathbf{B} = \frac{r A B}{r \sin \theta} + \frac{P \partial A}{r \sin \theta \partial \theta},$$

(10)

where $B = \mathbf{B}_\theta = -\frac{\partial P}{\partial \theta}$ and $\mathbf{A}_r = \frac{1}{r^2 \sin \theta} \frac{\partial A}{\partial \theta}$. We have to notice that the magnetic helicity of the large-scale axisymmetric field can be restored from observational tracers of $\mathbf{B}_\theta$ and $\mathbf{B}_r$, either from the vector magnetograms (Seehafer 1990) or from the line-of-sight magnetic observations, e.g. using the method by Pevtsov et al. (2001). The toroidal part of the potential can be restored from the surface distribution of the $B_r$ as $A(\theta) = \int_0^\theta r^2 \sin \theta \mathbf{B}_r \, d\theta$ which is equivalent to the flux going outside of the Sun, and, similarly, we can restore the poloidal part of vector potential using $P = -\int_0^\theta \mathbf{B}_\theta \, d\theta$. Note that the total helicity remains zero because of the equatorial symmetry of the axisymmetric magnetic field. Following the comment after equation (7) we note that in the axisymmetric case the scalars $A$ and $P$ can be uniquely determined if we assume that the observed helicity density is produced by the dynamo from the initial state which has zero magnetic helicity density. This assumption may be too restrictive. In the case of the oscillating dynamo, we can define helicity relative to some initial state in the past. Note that we suggest to apply equation (10) to the surface magnetic field. In this case, we have an alternative option to use the Coulomb gauge for the observational data. Furthermore, there are restrictions on the behaviour of the toroidal field at infinity. The magnetic fields of the Sun are decaying far away from the domain where they are generated. The estimation of the real rate of decay would require an account of additional factors such as cancellation of magnetic energy in the chromosphere and corona, the impact of the solar wind etc., so we leave this option beyond the framework of our consideration. This issue requires further study.

Therefore, the observations can give information about the magnetic helicity of the large-scale magnetic fields of the Sun. Our results indicate (see Figs 4c–e) that the reversal of magnetic helicity and lower values of integral helicity may precede the lower amplitude of cyclic dynamo activity. Our results show (see Figs 4d and e) that the magnetic helicity can be used as a precursor for the sunspot cycle forecast because the low values of the large-scale asymmetry in the shape of wings on butterfly diagrams, etc.). It looks plausible that long and weak cycles are associated with larger areas of helicity of the sign opposite to the hemispheric sign rule.
magnetic helicity are preceded by the low maxima of the simulated sunspot activity.

We found that in the models the wave of the reversed magnetic helicity sign propagates from the bottom of the convection zone. A similar property was recently found by Warnecke et al. (2011) in direct numerical simulations. Models B1 and B2 show the possibility for the strongly positive as well as for the negative dynamical $\alpha$ effect near the top of the convection zone. The model employs the subsurface rotational shear having the negative radial gradient of the angular velocity. Therefore, in the case of B1, the dynamo wave penetrates closer to the equator than in model B2 because of the Parker–Yoshimura rule (Parker 1955; Yoshimura 1975). The numerical simulations demonstrate a similar effect (Käpylä et al. 2012).

Generally, the sign reversals of the magnetic helicity are stronger in model B1 than in model B2. These two models have different boundary conditions for the magnetic helicity at the top. In model B1, the diffusive flux of the large-scale helicity from the surface is balanced by a counterpart from the small-scale helicity. Therefore, integrating equation (8) from some level $r = r_0$ to the top $r = r_e$, we have $\nabla \cdot \mathbf{J} = \nabla \cdot \mathbf{J}_0 + \nabla \cdot \mathbf{J}_\alpha$, where we use the magnetic boundary conditions as well. Thus, in model B1, the boundary conditions support the penetration of the local helicity $\nabla \cdot \mathbf{J}_\alpha$ (governed by equation 6) from the depth to the surface. For the boundary condition (9), we have $\nabla \cdot \mathbf{J}_\alpha = 0$ at the top. This is the same as $\nabla \cdot \mathbf{J} = \nabla \cdot \mathbf{J}_\alpha = \nabla \cdot \mathbf{J}_\alpha = \mathbf{J}_\alpha$, where $\mathbf{J}_\alpha = \nabla \cdot \mathbf{B}$. Thus, at the near-surface level, in model B2 the small-scale magnetic helicity is determined by the profile of $\mathbf{J}_\alpha$ and not by equation (6). The further study requires clarification of the issue if the boundary conditions impact the sign reversals of the magnetic helicity. We can make conjecture that the change in the boundary conditions results in the larger or smaller sign reversals of the magnetic helicity at the surface.

In our interpretation of the current helicity observations, we assume that the sunspots and solar ARs are produced from the large-scale toroidal magnetic field which resides somewhere deep in the solar convection zone. Our model, as well as some of others’ models (Zhang et al. 2012), shows that the helicity of the large-scale field is positive in the Northern hemisphere most of the time during the solar cycle. Observations show that current helicity of the AR has a negative sign during the most time of the solar cycle. Following the model given in this paper, this helicity can be interpreted as a result of magnetic helicity conservation which says that there should be an approximate balance between the amount of helicity that is generated on the large and small scales. Thus, the observations of the current helicity contain the proxy of the large-scale dynamo processes inside the solar convection zone.

The definition of large and small scales and practical application of the idea of scale separation and mean-field dynamo to available observational data are a complex problem. The helicity is however measurable for mainly strong fields of scales somehow larger than the basic granulation scale. Computation of helicity for quiet Sun regions requires further investigation.

The main results of this paper can be summarized as follows. The current model suggests that the reversal of the sign of the small-scale magnetic helicity follows the dynamo wave propagating inside the convection zone. This was also suggested by the numerical simulations (Warnecke et al. 2011). Therefore, the spatial patterns of the magnetic helicity reversals reflect the processes which contribute to generation and evolution of the large-scale magnetic fields. At the surface, the patterns of the helicity rule reversals are determined by the magnetic helicity boundary conditions at the top of the convection zone. The model suggests that the magnetic helicity of the large-scale axisymmetric field can be used as an additional observational tracer for the solar dynamo.

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APPENDIX A

Here we provide some details of the dynamo model that can also be found in Pipin et al. (2012), Pipin (2013) and P08. The hydrodynamic part of the tensor \( \alpha_{ij} \) is represented by \( \alpha_{ij}^{\text{MLT}} \) (P08):

\[
\alpha_{ij}^{\text{MLT}} = \delta_{ij} \left\{ 3 \eta_T \left( f_{i0}^{(\alpha)} (\mathbf{e} \cdot \mathbf{A}^{(\alpha)}) + f_{i1}^{(\alpha)} (\mathbf{e} \cdot \mathbf{A}^{(\alpha)}) \right) \right\} + e e_j \left\{ 3 \eta_T \left( f_5^{(\alpha)} (\mathbf{e} \cdot \mathbf{A}^{(\alpha)}) + f_4^{(\alpha)} (\mathbf{e} \cdot \mathbf{A}^{(\alpha)}) \right) \right\} + 3 \eta_T \left\{ (e A_{ij}^{(\alpha)} + e A_{ij}^{(\alpha)}) e^\alpha \right\} + (e A_{ij}^{(\alpha)} + e A_{ij}^{(\alpha)}) e^\alpha, \tag{A1}\]

where \( \mathbf{A}^{(\alpha)} = \nabla \log \mathbf{P} \) is the inverse density stratification height, \( \mathbf{A}^{(\alpha)} = \frac{1}{2} \nabla \log (\rho^{(\alpha)}) \) is the same for the turbulent diffusivity, \( \mathbf{e} = \Omega / |\Omega| \) is a unit vector along the axis of rotation. The turbulent pumping, \( \gamma_{ij}^{\text{MLT}} \), depends on mean density and turbulent diffusivity stratification, and on the Coriolis number \( \Omega^* = 2 \tau \Omega_0 \), where \( \tau \) is the typical convective turnover time and \( \Omega_0 \) is the global angular velocity. Following the results of P08, \( \gamma_{ij}^{\text{MLT}} \) is expressed as follows:

\[
\gamma_{ij}^{\text{MLT}} = 3 \eta_T \left\{ f_3^{(\alpha)} \Lambda_{ij}^{(\alpha)} + f_4^{(\alpha)} (\mathbf{e} \cdot \mathbf{A}^{(\alpha)}) e^\alpha \right\} e_{ij} + 3 \eta_T \left\{ f_3^{(\alpha)} e f_{10} e_{ij} + f_4^{(\alpha)} (\mathbf{e} \cdot \mathbf{A}^{(\alpha)}) e^\alpha \right\} e_{ij}. \tag{A2}\]

The effect of turbulent diffusivity, which is anisotropic due to the Coriolis force, is given by

\[
\eta_{ijk} = 3 \eta_T \left\{ 2 f_1^{(\alpha)} \Lambda_{ij}^{(\alpha)} f_2^{(\alpha)} e_{ij} \right\} e_{ij} - 2 f_1^{(\alpha)} e_{ijk} e_{ij} e_{ij}. \tag{A3}\]

We also include the non-linear generation effects which are induced by the large-scale current and the global rotation that is usually called the \( \Omega \times \mathbf{J} \) effect or the \( \delta \) dynamo effect (Rädler 1969). It is supported by the numerical simulations (Käpylä, Korpi & Brandenburg 2008; Schrinner 2011). P08 suggested that

\[
\eta_{ijk} = 3 \eta_T c_3 f_{ijk} \left\{ \tilde{\psi}^{(\alpha)} w_{ij} + \tilde{\psi}^{(\alpha)} \frac{\mathbf{B}_i \mathbf{B}_j}{\mathbf{B}^2} \right\}. \tag{A4}\]

where \( c_3 \) measures the amplitude of the \( \Omega \times \mathbf{J} \) effect and \( \tilde{\psi}^{(\alpha)} (\beta) \) are normalized versions of the magnetic quenching functions \( \tilde{\psi}^{(\alpha)} \) given in P08. They are defined as follows: \( \tilde{\psi}^{(\alpha)} (\beta) = \frac{\epsilon^{(\alpha)}}{\epsilon^{(\alpha)}} (\beta) \).

The last term in equation (A4) is the non-linear contribution to the \( \Omega \times \mathbf{J} \) effect. Its structure is the same as for the \( \alpha \) effect because the associated electromotive force is proportional to \( \frac{3}{2} \eta_T c_3 f_{ijk} \tilde{\psi}^{(\alpha)} \mathbf{B}_i \mathbf{B}_j (\mathbf{e} \cdot \nabla) \log \mathbf{B} \) (see the details in P08). Thus, this effect works similar to the \( \alpha \) effect that is excited by the non-linear buoyant instability of the large-scale magnetic field. The functions \( f_{ijkl} \) in equations (A3) depend on the Coriolis number. They can be found in P08 (see also Pipin & Kosovichev 2011c; Pipin & Sokoloff 2011).

The mixing length is defined as \( \ell = \alpha_{\text{MLT}} |\Lambda^{(\alpha)}|^{-1} \), where \( \Lambda^{(\alpha)} = \nabla \log \mathbf{P} \) is the inverse pressure variation height and \( \alpha_{\text{MLT}} = 2 \). The turbulent diffusivity is parametrized in the form, \( \eta_T = C_n \eta_T^{(0)} \), where \( \eta_T^{(0)} = \frac{\epsilon^{(\alpha)}}{\epsilon^{(\alpha)}} \) is the characteristic mixing-length turbulent diffusivity, \( \ell \) is the typical correlation length of the turbulence and \( C_n \) is a constant to control the efficiency of the large-scale magnetic field dragging by the turbulent flow. Also, we modify the mixing-length turbulent diffusivity by factor \( f_m(\tau) = 1 + \exp (50(\tau - \tau_0)) \), \( \tau_0 = 0.725 R_\odot \) to get the saturation of the turbulent parameters to the bottom of the convection zone. The latter is suggested by the numerical simulations (see, e.g., Ossendrijver, Stix & Brandenburg 2001; Ossendrijver et al. 2002; Käpylä, Korpi & Brandenburg 2008). The results do not change very much if we apply \( \Lambda^{(\alpha)} = C_\alpha \nabla \log (\eta_T^{(0)}) \) with \( C_\alpha \leq 0.5 \). For the greater \( C_\alpha \), we get the steady non-oscillating dynamo which is concentrated to the bottom of the convection zone. The purpose of introducing the additional parameters like \( C_\alpha = 0.5 \) and \( f_m(\tau) \) is to get the distribution of the \( \alpha \) effect closer to the result obtained in the numerical simulations.

The bottom of the integration domain is \( r_b = 0.715 R_\odot \) and the top of the integration domain is \( r_s = 0.99 R_\odot \). The choice of parameters in the dynamo is justified by our previous studies (Pipin & Kosovichev 2011b), where it has been shown that solar-type dynamos can be obtained for \( C_\alpha / C_\delta \geq 2 \). In those papers, we find an approximate threshold \( C_\alpha \approx 0.03 \) for a given value of the diffusivity dilution factor \( C_\alpha = 0.05 \). The latter was chosen to tune the solar cycle period.

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