INVESTIGATION OF SIGNALS DISTORTION DURING INTERPOLATION IN SDR TRANSMITTERS WITH QPSK MODULATION

Abstract: Software-defined radio (SDR) allows using software to set or modify working radio frequency parameters: communication standard, frequency range, modulation type, output power, etc. An interpolator is an integral part of the transmitter, which increases the sampling frequency of the modulated signal. Interpolators are built on the basis of integrated cascaded integral-comb (CIC) non-multiplier filters. CIC interpolation filters have uneven amplitude-frequency characteristic (AFC), so the CIC compensator filter is turned on for its alignment. The eye-diagrams of signals with different interpolation coefficients are investigated in the paper. The quality of the signal in the transmitter can be evaluated by the eye-diagram. Interpolation is accompanied by the effect of amplifying the signal, reducing the time of increase in pulse, reducing the signal quality factor. The jitter in the transmitter decreases when the interpolation factor is a multiple of the sampling rate of the information sequence of symbols. If the signal quality factor is \( Q > 7 \), the bit error rate has a \( BER < 10^{-9} \) value for practice.

Keywords: interpolation, modulation, filter, signal processing, jitter.

Formulation of the problem

SDR is the best technology for modern telecommunication applications. The SDR is suitable for replacement the typical analog equipment such as digital generators, mixers, modulators and demodulators, which allow to be configured and managed with software. This allows communication systems to become more flexible and dynamic in response to changes in the communication channel. In the modern digital communication system the sampling rate plays an important role. High-speed signal processing can be implemented by usage of digital converters with increasing or decreasing sampling rate [1-3].

SDR is a device where signal modulation parameters are specified in software using programmable logic or digital signal processors of general purpose. The signals are digitally generated, then digital signals transform to analog using a bandwidth DAC, and then increasing the frequency from the intermediate to the radio frequency is held. The receiver, similarly, uses a bandwidth analog-to-digital converter (ADC) which captures all the channels of the software radiocommunication node. The receiver selects, converts with lowering the frequency and demodulates the channel signal using the software on the general purpose processor.

The article is devoted to the study of filters used for increasing the sampling rate in the transmitter. The distortion of signals occurring during interpolation using of eye-diagrams is analyzed. The estimate of the jitter and the standard signals deviation is given.

Methods of mathematical modeling, statistical analysis, eye-diagrams are used in this work.
Analysis of recent research and publications

The structural scheme of the transmitter of the digital communication system is presented in Fig.1.

![Fig. 1. The transmitter’s block diagram of the digital communication system: DIS – is a digital information source; CCD – channel coding device; M – modulator; I – interpolator; PS – pulse shaper; FC – frequency converter; PA – power amplifier; A – antenna](image)

The channel coder performs interrupt-encoding. The modulator performs the low frequency modulation consisting of the common-mode and quadrature signal components formation. Then an interpolation operation is held so, the sampling rate increases. The pulse generator limits the spectrum of the signal and minimizes inter-symbol interference [4-6].

Quadrature phase manipulation or QPSK is a common method of modulation in satellite communications systems [4]. QPSK has a doubling rate compared to BPSK. In comparison with quadrature amplitude modulation of higher order (QAM), QPSK has a lower data rate, less error rate and a non-complex design of the receiver.

The bit stream of data is multiplexed into $I$ and $Q$ channels, bits are transmitted to the non-return code to zero. Then the signal is interpolated at $N$ times. To increase the sampling frequency, $N-1$ zeros are inserted between each count. The reason for the increase in the sampling rate is about to avoid the problems of overlaying the spectra. The signal then is filtered by any filter that satisfies due to the Nyquist theorem to eliminate intersymbol interference. A filter of square-root-raised-cosine filter (SRRC) is often used.

The SRRC filter is half of the shape of the elevated cosine filter. At the receiver there is a corresponding SRRC filter which together with SRRC transmitter filter generates the same pulse shape as the elevated cosine filter [5-7].

Block diagram of a transmitter that generates in-phase and quadrature components of the modulated signal is shown in Fig. 2 [1, 8].

An important part of the digital interface used for radio frequency communication systems is the frequency converter (FC). The FC main function is to convert one or more data channels from the main band format into a radio signal consisting of modulated carriers belonging to a set of one or more mentioned radio frequencies. This is achieved in two stages: increasing the sampling rate by interpolation, providing spectrum generation and suppressing interpolation images by filtration and shifting the spectrum of the signal to the desired carrier frequencies using a multiplier and a heterodyne [9, 10,11].
Fig. 2. The block diagram of the transmitter in a quadrature circuit:
M – modulator; CSDP – converter of serial data in parallel; I – interpolator;
PS – pulse shaper; DAC – digital-to-analog converter; MP – multiplier;
H – heterodyne; SD – summing device; PA – power amplifier; A – antenna

**CIC interpolation filters**

The interpolation unit CIC performs an increase in the sampling rate (interpolation) for the input signal using an integer coefficient. Cascaded filters of (CIC) integrator-comb are a class of linear FIR-filters that consist of a comb part and an integrator [6].

The digital filter with an ultimate impulse response is described by the expression:

$$y(n) = \sum_{i=0}^{N} b_i x(n - i),$$

where \(y(n)\) - is the signal at the input of the filter; \(x(n)\) - is the signal at the output of the filter; \(b_i\) - are the coefficients of the filter; \(N\) - is the order of the filter.

To synthesize the FIR-filter it is necessary to determine the coefficients \(b_i\). The block diagram of the third-order interpolator CIC filter is shown in Fig. 3.

Fig. 3. Structural diagram of the CIC interpolator: CF – is a comb filter; \(\uparrow\) – is the interpolator with interpolation coefficient \(R\); I - integrator

Comb filter has a transmission coefficient:

$$H_C(z) = 1 - z^{-M}.$$  \hspace{1cm} (2)

The integrator has a transfer function:

$$H_I(z) = \frac{1}{1 - z^{-1}}.$$ \hspace{1cm} (3)

The transmitter function of the CIC interpolator has the form:

$$H(z) = H_I^N(z)H_C^N(z) = \left(1 - z^{-RM}\right)^N \left(1 - z^{-1}\right)^N = \sum_{k=0}^{RM-1} \left[\sum_{k=0}^{RM-1} z^{-k}\right]^N,$$ \hspace{1cm} (4)

where \(-H_I(z)\) is a transfer function of the CIC filter integrator;
$H_C(z)$ - is a transfer function of the comb filter; $N$ - is the number of sections in the comb or in CIC filter integrator; $R$ - is a coefficient of interpolation; $M$ - is a differential delay.

Simulink-model of interpolation of harmonic signal with frequency of 1 Hz and sampling rate of 6 Hz is shown in Fig. 4. The CIC interpolation filter has an interpolation factor $R_1=2$. The CIC compensation interpolator filter has an interpolation coefficient $R_2=2$.

![Simulink-model of interpolation of harmonic signal](image)

**Fig. 4. Simulink-model of interpolation of harmonic signal**

The input signal has an oscillogram shown in Fig. 5.

![Oscillogram of input signal readings](image)

**Fig. 5. Oscillogram of input signal readings**

During the simulation 10 sec. have 61 countdown. Signal in output of the interpolators filters is shown in Fig. 6.

![Oscillogram of output signal readings](image)

**Fig. 6. Oscillogram of output signal readings**

During the simulation 10 sec. have 244 counts and the signal amplitude is doubled. Gain factor is $K = (R \cdot M)^N = 2$. 
AFC of CIC compensation interpolator filter is shown in fig. 7 [10].

The CIC compensation interpolator block uses the multiphase FIR interpolator as a compensator filter. CIC compensatory interpolators are multi-core FIR filters that can be enabled sequentially with CIC interpolator filters to reduce the disadvantages of CIC filters.

- CIC filters-interpolators are used in areas that require high interpolation factors. These filters are popular in ASIC and FPGA because they do not have multipliers. CIC filters have two disadvantages:

\[
H_{ci}(\omega) = \text{abs} \left( \frac{\sin \left( \frac{M \omega}{2} \right)^n}{\sin \left( \frac{\omega}{2} \right)} \right)
\]

where:  
- \( n \) - is the number of filter sections;  
- \( M \) - is a differential delay;  
- \( \omega \) - is the normalized angular frequency.

- CIC filters have a wide transition area.

Compensatory interpolation filters have a reverse sinusoid bandwidth for correction of the CIC fall and a narrow transition width.

CIC filters have an amplitude characteristic that causes a recession in the area of radio frequency.

![Magnitude Response (dB)](image)

Fig. 7. AFC of CIC compensation interpolator filter

Pulse formation in electronics and telecommunications is a process of changing the shape of transmitted pulses. Its purpose is to make the transmitted signal better to use in the communication channel usually by limiting the signal bandwidth. The intersymbol interferences created by the channel can be controlled with filtering of transmitted pulses.
When processing a signal, a riser cosine filter (RRC) is often used as a transmit and receive filter in a digital communication system to perform an agreed filtering. This helps to minimize intersymbol interference. The combined response of such two filters is a filter with a raised cosine. It is called so because its frequency response $H_{rrc}(f)$ is the square root of the frequency response of an exponential cosine filter $H_{rc}(f)$:

$$H_{rc}(f) = H_{rrc}(f) \cdot H_{rc}(f),$$

or:

$$|H_{rrc}(f)| = \sqrt{|H_{rc}(f)|}.$$

In order to have minimal ISI (inter-symbolic interference), the overall response of the transmission filter, the channel response and the acceptance filter must fit Nyquist's criterion. A high-rise cosine filter is the most popular filter response that satisfies this criterion. Half of this filtration is performed on the transfer side and other half is on the reception side. The channel response on the receiving side may also be taken into account if it can be accurately estimated so, the overall response is a riser cosine filter.

Nyquist showed that the pulse characteristic will have zeros with evenly distributed intervals if the frequency characteristic has odd symmetry at the cutoff frequency. It's much easier to achieve. The impact of jitter can be minimized.

The filter with the raised cosine characteristic has a frequency transfer coefficient:

$$X_{rc}(f) = \begin{cases} 
T \left\{ \frac{T}{2} \left[ 1 + \cos \left( \frac{\pi T}{\beta} \left( |f| - \frac{1 - \beta}{2T} \right) \right) \right] \right. & \text{for } 0 \leq |f| \leq \frac{1 - \beta}{2T} \\
0 & \text{for } \frac{1 - \beta}{2T} \leq |f| \leq \frac{1 + \beta}{2T} \\
\left. \left( |f| > \frac{1 + \beta}{2T} \right) \right. & \text{for } |f| > \frac{1 + \beta}{2T}. 
\end{cases}$$

(6)

Is the decreasing factor ($0 \leq \beta \leq 1$) of the filter frequency response, it is a measure of redundancy bandwidth filter that strips exceeding Nyquist bandwidth. The AFC of the Nyquist filter is shown in Fig. 8.
Results of Experimental Studies

In fig. 9 is shown the studying of Simulink-model transmitter:

Random Integer Generator using as an input signal source and a QPSK Modulator - is as a modulator. To increase the sampling rate with a large interpolation factor, the CIC Interpolation unit is connected. To align the frequency response of the CIC filter, a CIC compensator unit is connected. Filter and the transmit channel with additive white Gaussian noise are included to form the pulse of Raised Cosine Transmit Filter.

The Random Integer Generator output signal is shown in Figure 10. AFC of CIC Interpolation filter is shown in fig. 11.

Signal sampling frequency is 1 Hz. The QPSK signal diagram without interpolation filters is depicted in Fig. 12. Eye-diagram of transmitter signal for filter
interpolation coefficient is: CIC Interpolation is $R_1=5$; CIC Compensator is $R_2=2$ is shown in Fig. 13.

The total interpolation coefficient is 10. According to the eye-diagram it can be concluded that the period of bits receipt is 100 msec or 0.1 sec, which corresponds to the interpolation coefficient 10. The signal amplitude is about 2.5 V. The signal rise time is 80 msec.

In case of interpolation coefficient of filters the eye diagram is: CIC Interpolation is $R_1=50$; CIC Compensator is $R_2=2$ is shown in Fig. 14. The total interpolation coefficient in the transmitter model is 100 and the gain is $K=18$. Time of signal rise is 8 msec.

Eye diagram in case of interpolation coefficient of filters is: CIC Interpolation is $R_1=60$; CIC Compensator is $R_2=2$ is shown in Fig. 15. That is, the total interpolation coefficient in the transmitter model is 120. A sufficiently strong jitter is observed with defined interpolation factor.

The eye diagram is an oscillogram and the digital signal is periodically discretized and fed to the vertical scan of the oscilloscope to create it a well as the data rate is used to trigger the horizontal scan of the oscilloscope. It is a tool for evaluating the combined effect of channel noise and intersymbol noise on the performance of the pulse transmission system in the main band of frequencies. This is a synchronized superposition of all possible realizations of the signal which is considered in a specific transmission interval. Several performance indicators of system can be got by analyzing the mapping. The open-eye pattern corresponds to the minimum distortion of the signal. Distortion of the signal shape through intersymbolic noise and noise is manifested as the closure of the eye structure.
An important characteristic of communication signals, especially in high-speed systems, is the trembling of synchronization (jitter). Trembling of synchronization is defined as the deviation of the clock signal from the ideal clock signal. Jitter in the time field is deterministic and random. Examples of deterministic jitter are periodic jitter and intersymbol interference. The periodic jitter can be modeled as the sinusoid sum and intersymbol interference can be modeled as a sequence of Dirac's functions. Random jitter is modeled as a Gaussian function. The jitter that occurs in the communication system may have any combination of these components.

Quality factor (Q-factor) is an indicator of high-quality signal in the communication system. The coefficient Q is defined as the difference between the mean values of the two levels of the signal (the level for the bit "1" and the level for the "0" bit) is divided by the sum of the mean square deviations of noise at two levels of the signal. A higher number in the result means that the pulse is relatively free of noise.

\[
Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0},
\]

where \(\sigma_1\) and \(\sigma_0\) - is the mean square deviation of the logical zero and the logical unit; \(I_1\) and \(I_0\) - the level of the logical zero and the logical unit.

For the interpolation coefficient 10 (Fig. 13), the signal-to-noise ratio is 30 dB. In the eye-diagram it can be determined: \(I_1=2.5\) and \(I_0=0\), \(\sigma_1=0.05\) and \(\sigma_0=0.1\), \(Q=16.7\).

If the interpolation factor is 100 (Fig. 14), the signal-to-noise ratio is 30 dB. In the eye-diagram it can be determined: \(I_1=36\) and \(I_0=0\), \(\sigma_1=3\) and \(\sigma_0=0.1\), \(Q=11.6\).

If the interpolation factor is 120 (Fig. 15), the signal-to-noise ratio is 30 dB. In the eye-diagram it can be determined \(Q=3\).
The intensity of bit errors in the eye-diagram can be determined from the expression:

\[ BER = \frac{1}{Q \sqrt{2\pi}} e^{-\frac{Q^2}{2}}. \]  

(8)

The intensity dependence of bit errors on the signal quality factor is shown in fig.16.

![The intensity of bit errors from the Q-factor](image)

Fig.16. Dependence of the intensity of bit errors on the signal quality factor

From the received schedule it is possible to draw a conclusion that the intensity of bit errors becomes unacceptably high at low signal quality.

**Conclusion**

Analyzing the obtained eye-diagrams it can be concluded that the interpolation is accompanied by the effect of signal amplifying, reducing of the pulse rise time, reducing of the signal quality factor. The jitter in the transmitter decreases if the interpolation factor is a multiple of the sampling rate of the information sequence of symbols. If the signal quality factor is \( Q > 7 \), then the bit error rate has a \( BER < 10^{-9} \) value for practice.

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