Can we learn anything new from polarization observables of the deuteron disintegration near threshold?

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We discuss polarization characteristics of the deuteron disintegration near the threshold energy. Due to the small relative energy of the outgoing np pair, the dominant amplitude for this process is a $1^+ → 0^+$ transition. Relativistic covariance requires only one electromagnetic transition form factor for this amplitude. Subsequently this leads to substantial simplifications in the formulas of the polarization observables and allows to draw conclusions independent of the details of the interaction.

\[
\begin{align*}
M_{f_1} &= i e^2 \bar{u}(k_e', s_e') \gamma^\mu u(k_e, s_e) \frac{1}{q^2} \langle np|j_\mu|DM\rangle, \\
\end{align*}
\]

where $u(k_e, s_e)$ denotes the free electron spinor with 4-momentum $k_e$ and spin $s_e$, and $q = k_e - k_e'$ is the 4-momentum transfer. The hadronic transition matrix element $\langle np|j_\mu|DM\rangle$ is from the deuteron state $|DM\rangle$ with 4-momentum $K$ and total angular momentum projection $M$ to the final np state with 4-momentum $P = K + q$, where $j_\mu$ is the electromagnetic current operator.

In the $^1S_0$ approximation, the covariant form of the matrix hadronic matrix element is due to a $1^+ → 0^+$ transition. It depends on one scalar function only (because of parity conservation), which is the electromagnetic transition form factor $V(s, q^2)$. This form factor contains all the structure information, viz. the deuteron and np-pair wave functions. It is defined via

\[
\langle np(^1S_0)|j_\mu|DM\rangle = i \epsilon_{\mu\nu\alpha\beta} \xi_M^\nu \xi_M^\beta K^\gamma V(s, q^2) \\
= \xi_M^\nu G_{\mu\nu} V(s, q^2),
\]

where $s = P^2$, and the deuteron polarization 4-vector $\xi_M$ has been introduced.

Note, that this particular simple form of the amplitude is only valid if the $^1S_0$-channel dominates the final state. In other words, the reaction is dominated by the $M1$ transition. The inclusion of other $NN$ final channels (e.g. $^3S_1 → ^3D_1, ^1P_1, \ldots$) would lead to more complicated expressions and to additional form factors in eq. (2).

The purpose of the present letter is to give unique observables that enable us to check experimentally the assumptions that lead to eq. (2). Some polarization observables turn out to be independent of the form factor $V(s, q^2)$. This will be shown below.

Using eq. (1) the differential cross section may be obtained in the standard way, see ref. [12]. Here it is useful to introduce leptonic $l_{\mu\nu}$ and hadronic $W_{\mu\nu}$ tensors. The differential cross section then reads:

\[
\frac{d^2\sigma}{dE_{e'}d\Omega_e} = \alpha^2 \frac{|k_{e'}|^2}{q^2} l_{\mu\nu} W_{\mu\nu},
\]

with $\alpha = e^2/4\pi$. The leptonic tensor is given by

\[
l_{\mu\nu} = 2(k_{e'\mu}k_{e\nu} + k_{e'\nu}k_{e\mu}) + q^2 g_{\mu\nu} + 2im_\epsilon \epsilon_{\mu\nu\alpha\beta} q^\alpha s_\epsilon^\beta.
\]
The hadronic tensor $W_{\mu\nu}$ has the following form:

$$W_{\mu\nu} = \left( np(1S_0) | j^{\mu} | DM \right) \langle DM | j^{\nu} | np(1S_0) \rangle$$

$$\times \left( \frac{2M}{2\pi} \right)^3 \int \delta(K + q - k_p - k_n) \frac{dk_p}{2Ep(2\pi)^3} \frac{dk_n}{2En(2\pi)^3}.$$  \hspace{1cm} (5)

Using the general form of the hadronic transition current eq. (3), the hadronic tensor may be written as

$$W_{\mu\nu} = R G^{\mu\alpha} \rho_{\alpha\beta} G^{\nu\beta} V^2(s, q^2),$$  \hspace{1cm} (6)

where $R$ is a purely kinematical factor. It reads

$$R = \frac{1}{8\pi^2} \left| \frac{p^*}{s} \right| \left| \frac{p^*}{s^*} \right| = \sqrt{s - m^2}. \hspace{1cm} (7)$$

In eq. (6) the density matrix $\rho_{\alpha\beta}$ of the deuteron is given by

$$\rho_{\alpha\beta} = \frac{1}{3} \left( -g_{\alpha\beta} + \frac{K_{\alpha} K_{\beta}}{M^2} \right) + \frac{1}{2M} \epsilon_{\alpha\beta\gamma\delta} K^\gamma \delta_{IJ}$$

$$- \frac{1}{2} \left( (W_{l_1})_{\alpha\rho} (W_{l_2})_{\beta}\rho + (W_{l_2})_{\alpha\rho} (W_{l_1})_{\beta}\rho \right)$$

$$- \frac{2}{3} \left( g_{l_1l_2} + \frac{K_{l_1} K_{l_2}}{M^2} \right) \left( -g_{\alpha\beta} + \frac{K_{\alpha} K_{\beta}}{M^2} \right) p_{D\lambda l_1} p_{D\lambda l_2},$$

(8)

where $(W_{l_1})_{\alpha\rho} = \epsilon_{\alpha\beta\gamma\delta} K^\gamma \delta_{IJ}$ is the differential cross section, $A_{\mu\nu}$ denotes unpolarized, vector polarized and tensor polarized cases, respectively.

With the general form of hadronic tensor $W_{\mu\nu}$ eq. (5) it is straightforward to calculate asymmetries of the deuteron disintegration near threshold. First we consider the spin correlation of the incident particles, e.g.

$$A = \frac{d\sigma(\uparrow, D) - d\sigma(\downarrow, D)}{d\sigma(\uparrow, D) + d\sigma(\downarrow, D)},$$

(10)

where $d\sigma$ is the differential cross section, $\uparrow$ ($\downarrow$) denotes the helicity $\lambda_e = +1(-1)$ of the incoming electron and $D$ the polarization state of the deuteron, which might be vector or tensor type. We assume the initial electron moving along the $Z$ axis, and $\theta_e$ is the electron scattering angle. The scattering plane of the electron is in the $XZ$ plane (see Fig. 1). Then the vectors $k_e$ and $k'_e$ obtain the following form:

$$k_e = (E_e, 0, 0, E_e),$$

$$k'_e = (E'_e, -E'_e \sin \theta_e, 0, E'_e \cos \theta_e).$$

(11)

First we consider the case of vector polarized deuterons. If the direction of the deuteron polarization is parallel to the $Z$–axis, then the correlation is given by

$$A_{\parallel} = \frac{3}{2} \kappa \frac{(E + E')(E - E' \cos \theta_e)}{(E + E')^2 - 2EE' \cos^2 \theta_e/2}.$$  \hspace{1cm} (12)

where $\kappa$ is the degree of polarization of the deuterons. Note, that the dependence on the form factor $V(s, q^2)$ disappears. For the case of the backward scattering ($\theta_e = 180^\circ$) eq. (12) even simplifies to

$$A_{\parallel} = \frac{3}{2} \kappa.$$  \hspace{1cm} (13)

If the polarization of the deuteron is parallel to the $X$–axis, then

$$A_{\perp} = \frac{3}{2} \kappa \frac{(E + E')E \sin \theta_e}{(E + E')^2 - 2EE' \cos^2 \theta_e/2}.$$  \hspace{1cm} (14)

These formulae may be generalized to arbitrary polarization direction of the deuteron given by the angles $(\vartheta, \varphi)$, viz.

$$A(\vartheta, \varphi) = \frac{3}{2} \kappa (E + E') \frac{[(E' \sin \vartheta \sin \varphi + (E - E') \cos \vartheta \cos \varphi)]}{(E + E')^2 - 2EE' \cos^2 \theta_e/2}.$$  \hspace{1cm} (15)

Now, consider the case of tensor polarization of the initial target. If the initial deuteron is only aligned due to a $p_{Dzz}$ component, then the cross section reads

$$d\sigma(p_{zz}) = d\sigma(1 + A_{zz} p_{Dzz}),$$

(16)

$$A_{zz} = \frac{4E_e^2 + 4E'_e^2 - 4E_e E'_e \cos \theta_e + 3E_e^2 \cos 2\theta_e}{4(E_e + E'_e)^2 - 2E_e E'_e \cos^2 \theta_e/2},$$

where $A_{zz}$ is the tensor analyzing power. For the backward scattering the analyzing power is

$$A_{zz} = 1.$$  \hspace{1cm} (17)

As a last example consider the scattering of polarized initial electrons from unpolarized deuterons. Then the polarization transfer is maximal and the polarization of the final electrons coincides with the polarization of initial electron beam. Note, that the same holds for the scattering of electrons from a structureless target (Coloumb scattering).
We have shown that it is possible to obtain rather simple expressions for the polarization observables in the reactions $\vec{e}(\vec{d}, np)\vec{e}$ and $\vec{e}(\vec{d}, np)\vec{e}$. These simple relations given in eqs. (12), (14), and (16) fully demonstrate the advantages of a covariant formalism and gives a positive answer to our initial question. In a nonrelativistic treatment usually utilized for disintegration at the threshold energy these relations are not so obvious. This is due to different corrections that have to be taken into account in the transition matrix elements. In fact, it is possible to check the assumption of the $^1S_0$ dominance in the final channel experimentally. Any deviation of the structure independent values given above from the experimental data can only be due to other components in the final state. Besides the $^1S_0$ dominance the relations are based on the general form of the transition amplitude, which is constructed using invariance principles only. It is important to note that the form factor $V(s, q^2)$ is a very complex function containing all the information of the $NN$–interaction, electromagnetic properties of the nucleons, etc. Nonetheless, this function vanishes in the final relations.

On the other hand, if no accidental cancellations occur between higher partial components, eqs. (13) and (14) may be used to calibrate the deuteron target, i.e. to determine the polarization degree of the deuteron target.

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[1] S. Auffret et al., Phys. Rev. Lett. 55 (1985) 1362
[2] R.G. Arnold et al., Phys. Rev. C42 (1990) R1
M. Frodyma et al., Phys. Rev. C47 (1993) 1599
[3] see e.g. W. Leidemann and H. Arenhövel, Nucl. Phys. A393, 385 (1983)
[4] J.F. Mathiot, Nucl. Phys. A412 (1984) 201
[5] J.A. Lock and L.L. Foldy, Ann. of Phys. 93 (1975) 276
[6] L.Ya. Glozman et al., Phys. Lett. B200 (1986) 406
[7] V.V. Burov, et al., JINR Rapid Communications 6(57) (1992) 9
[8] K. Tamura et al., Nucl. Phys. A536 (1992) 597
[9] C.F. Williamson et al., Few body sys. supp. 6 (1991) 212.
[10] J. Carbonell, V.A. Karmanov, Nucl. Phys. A589 (1995) 713.
[11] S.G. Bondarenko, V.V. Burov, M. Beyer, S.M. Dorkin, MPG-VT-UR 87/96, to be published.
[12] we use the conventions of J.D. Bjorken and S.D. Drell, Relativistic Quantum Mechanics (Mc Graw-Hill, Inc, 1964).