Quark and Gluon Angular Momentum in the Nucleon

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Abstract. Parton distributions in impact parameter space, which are obtained by Fourier transforming GPDs, exhibit a significant deviation from axial symmetry when the target and/or quark is transversely polarized. From this deformation, we present an intuitive derivation of the Ji relation. In a scalar diquark model and in QED, we compare the Ji and Jaffe-Manohar decompositions of the nucleon spin. Using the MIT bag model, we estimate spectator effects through the presence of the gluon vector potential in the definitions of the quark orbital angular momentum.

DISTRIBUTION OF QUARKS IN THE TRANSVERSE PLANE

In the case of transversely polarized quarks and/or nucleons, parton distributions in impact parameter space [1] show a significant transverse deformation. In the case of unpolarized quarks in a nucleon polarized in the $+\hat{x}$ direction, this deformation is described by the $\perp$ gradient of the Fourier transform of the GPD $E^q$ [2]

$$q_{q/p}(x, b_\perp) = \int \frac{d^2x_\perp}{(2\pi)^2} e^{-ib_\perp \cdot \Delta_\perp} H^q(x, 0, -\Delta_\perp^2) - \frac{1}{2M} \partial_y \int \frac{d^2x_\perp}{(2\pi)^2} e^{-ib_\perp \cdot \Delta_\perp} E^q(x, 0, -\Delta_\perp^2)$$

for quarks of flavor $q$. Since $E^q(x, 0, t)$ also arises in the decomposition of the Pauli form factor $F_2^q = \int_1^1 dx E^q(x, 0, t)$ for quarks with flavor $q$ (here it is always understood that charge factors have been taken out) w.r.t. $x$, this allows to relate the $\perp$ flavor dipole moment to the contribution from quarks with flavor $q$ to the nucleon anomalous magnetic moment (here it is always understood that charge factors have been taken out)

$$d^q \equiv \int d^2b_\perp q(x, b_\perp) b_y = \frac{1}{2M} F_2^q(0) = \frac{1}{2M} \kappa_{q/p}.$$  

Here $e_q \kappa_{q/p}$ is the contribution from flavor $q$ to the anomalous magnetic moment of the proton. Neglecting the contribution from heavier quarks to the nucleon anomalous magnetic moment, one can use the proton and neutron anomalous magnetic moment to solve for the contributions from $q = u, d$, yielding $\kappa_{u/p} \approx 1.67$ and $\kappa_{d/p} \approx -2.03$. The resulting deformation ($|d_q| \sim 0.1$fm) of impact parameter dependent PDFs in the transverse direction (fig. 1) is rather significant and it is in opposite directions for $u$ and $d$ quarks. The sideways displacement of the center of momentum for each quark flavor from the origin provides a very intuitive derivation of the Ji-relation [3] for the contribution from quarks with flavor $q$ to the nucleon angular momentum: Consider nucleon state that is an eigenstate under rotation about the $\hat{x}$-axis (e.g. a wave packet...
FIGURE 1. Distribution of the \( j^+ \) density for \( u \) and \( d \) quarks in the \( \perp \) plane (\( x = 0.3 \) is fixed) for a proton that is polarized in the \( x \) direction in the model from Ref. [2]. For other values of \( x \) the distortion looks similar.

Describing a nucleon polarized in the \( \hat{x} \) direction with \( \vec{p} = 0 \). For such a state, \( \langle T_0^{yy} \rangle = 0 = \langle T_{0z}^{yy} \rangle \) and \( \langle T_{q}^{yy} \rangle = -\langle T_{q}^{0z} \rangle \), and therefore \( \langle T_{q}^{++} \rangle = \langle T_{q}^{+0} \rangle = \langle J_{q}^{y} \rangle \). This result allows to relate the \( \perp \) shift of the center of momentum for quark flavor \( q \) to the angular momentum \( J_{q}^{y} \) carried by that quark flavor. The displacement of the \( \perp \) center of momentum is a sum of two effects:

- as discussed above, \( \langle T_{q}^{++} \rangle \) for a quark relative to the center of momentum of a transversely polarized nucleon (Fig. 1)
- however, already for a point-like transversely polarized spin \( \frac{1}{2} \) particle, the \( \perp \) center of momentum \( \langle T^{++} \rangle \) is shifted by \( \frac{1}{2} \) a Compton wavelength away from the origin (the center of the wave packet in the rest frame)

In order to understand the 2\textsuperscript{nd} effect, i.e. the \( \perp \) shift of the center of momentum for a \( \perp \) polarized spin \( \frac{1}{2} \) particle, let us consider ‘bag model’ [5] type wave functions\(^1\) for the wave packet of the target, but with a ‘bag radius’ \( R \) that will be sent to \( \infty \) at the end

\[
\psi = \left( \frac{f(r)}{\hat{\sigma} \cdot \hat{p}} \right) \chi \quad \text{with} \quad \chi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}.
\]

Since \( \psi^\dagger \partial_2 \psi \) is even under \( y \rightarrow -y \), \( i\bar{q} \gamma^0 \partial_2 q \) does not contribute to \( \langle T^{0z} \rangle = \langle i\bar{q} \gamma^0 \partial_2 + \gamma^2 \partial_0 q \rangle \). Using \( i\partial_0 \psi = E \psi \), one thus finds

\[
\langle T^{0z} b_y \rangle = E \int d^3 r \psi^\dagger \gamma^0 \gamma^z \psi y = E \int d^3 r \psi^\dagger \begin{pmatrix} 0 & \sigma^z \\ \sigma^z & 0 \end{pmatrix} \psi y = \frac{2E}{E + M} \int d^3 r \chi^\dagger \sigma^z \sigma^y \chi f(r)(-i)\partial^y f(r)y = \frac{E}{E + M} \int d^3 r f^2(r).
\]

\(^1\) The precise shape of the wave packet does not matter as long as its size is sent to \( \infty \).
In the limit when the bag radius goes to infinity \( E = M \) and \( \int d^3 r f^2(r) = 1 \) and therefore the 2\(^{nd}\) moment of \( \perp \) flavor dipole moment for this wave packet reads

\[
\langle T^{++}_q \rangle = \langle T^{0z}_q b_\perp \rangle = \frac{1}{2},
\]

(5)
i.e. for a a spherically symmetric, delocalized (Dirac-)wave packet with \( J_x = \frac{1}{2} \) centered around the origin the \( \perp \) center of momentum \( \frac{1}{M} \langle T^{++}_q b_\perp \rangle \) is 
not at origin, but at \( \frac{1}{2M} \)!

This ‘overall shift’ of the nucleon \( \perp \) center of momentum implies a contribution \( \frac{1}{2} \langle x_q \rangle \) to \( \langle T^{++}_q \rangle \) from quarks carrying momentum fraction \( \frac{1}{2} \langle x_q \rangle \), which gives rise to the first term in the Ji relation. In addition, Eq. (1) implies a shift of the center of momentum for quark flavor \( q \) relative to that of the nucleon, by \( \frac{1}{2M} \int dx x E(x, 0, 0) \). Combining these two effects yields the Ji relation [3]

\[
J_q = \langle T^{++}_q b_\perp \rangle = \frac{1}{2} \int dx x [q(x) + E_q(x, 0, 0)]
\]

(6)

Since the famous EMC experiments revealed that only a small fraction of the nucleon spin is due to quark spins [4], there has been a great interest in ‘solving the spin puzzle’, i.e. in decomposing the nucleon spin into contributions from quark/gluon spin and orbital degrees of freedom. In this effort, the Ji decomposition [3]

\[
\frac{1}{2} = \frac{1}{2} \sum_q \Delta q + \sum_q L^z_q + J^z_g
\]

(7)

appears to be very useful, as not only the quark spin contributions \( \Delta q \) but also the quark total angular momenta \( J_q \equiv \frac{1}{2} \Delta q + L^z_q \) (and by subtracting the spin piece also the quark orbital angular momenta \( L^z_q \)) entering this decomposition can be accessed experimentally through GPDs. The terms in (7) are defined as expectation values of the corresponding terms in the angular momentum tensor

\[
M^{0xy} = \sum_q \frac{1}{2} q^+ \Sigma^0 q + \sum_q q^+ \left( \vec{r} \times \vec{iD} \right)^z q + \left[ \vec{r} \times \left( \vec{E} \times \vec{B} \right) \right]^z
\]

(8)
in an appropriate nucleon wave packet. Here \( i\vec{D} = i\vec{\partial} - g\vec{A} \) is the gauge-covariant derivative. The main advantages of this decomposition are that each term can be expressed as the expectation value of a manifestly gauge invariant local operator and that \( J^z_g = \frac{1}{2} \Delta q + L^z_g \) can be related to GPDs (6) and is thus accessible in deeply virtual Compton scattering and meson production and can also be calculated in lattice QCD. However, due to the presence of interactions through the vector potential in the gauge covariant derivative \( L^z_g \) does not have a parton interpretation.

Recent lattice calculations of GPDs surprised in several ways [6]. First, the light quark orbital angular momentum (OAM) is consistent with \( L_u \approx -L_d \), i.e. \( L_u + L_d \approx 0 \), which would imply that \( J_g \approx \frac{1}{2} \cdot 0.7 \) represents the largest piece in the nucleon spin decomposition. Secondly, \( L_{u} \approx -0.15 \) and \( L_{d} \approx +0.15 \) in these calculations, i.e. the opposite signs from what one would expect from many quark models with relativistic
effects, as we will also illustrate in the following section. While the inclusion of still-omitted disconnected diagrams may change the sum \( L_u + L_d \), it does not affect the difference \( L_u - L_d \). In Ref. [7], it was pointed out that evolution from a quark model scale of few hundred MeV to the lattice scale of few GeV might explain the difference.

Jaffe and Manohar have proposed an alternative decomposition of the nucleon spin, which does have a partonic interpretation [8]

\[
\frac{1}{2} = \frac{1}{2} \sum_q \Delta q + \sum_q L^z_q + \frac{1}{2} \Delta G + L^z_g,
\]

and whose terms are defined as matrix elements of the corresponding terms in the +12 component of the angular momentum tensor

\[
M^{+12} = \frac{1}{2} \sum_q q^+_+ \gamma^5 q^+_+ + \sum_q q^+_+ \left( \bar{q} \times i \partial \right) q^+_+ + \epsilon^{+--ij} \epsilon^{ij} A^+ + 2 \epsilon^{+--ij} \left( \bar{q} \times i \partial \right) A^+.
\]

The first and third term in (9),(10) are the ‘intrinsic’ contributions (no factor of \( \vec{r} \times \)) to the nucleon’s angular momentum \( J^z = +\frac{1}{2} \) and have a physical interpretation as quark and gluon spin respectively, while the second and fourth term can be identified with the quark/gluon OAM. Here \( q^+_+ \equiv \frac{1}{2} \gamma^- \gamma^+ q \) is the dynamical component of the quark field operators, and light-cone gauge \( A^+ \equiv A^0 + A^z = 0 \) is implied. The residual gauge invariance can be fixed by imposing anti-periodic boundary conditions \( A_\perp (x_\perp, -\infty^-) = -A_\perp (x_\perp, -\infty^-) \) on the transverse components of the vector potential.

Only the \( \Delta q \) are common to both decompositions. While for a nucleon at rest the difference in the Dirac structure between \( L^z_q \) and \( L^z_{\bar{q}} \) plays no role [9], the appearance of the gluon vector potential in the operator defining \( L^z_{\bar{q}} \) implies that in general \( L^z_q \neq L^z_{\bar{q}} \). Other nucleon spin decompositions have been proposed in Refs. [10, 11].

### QUARK OAM IN THE SCALAR DIQUARK MODEL

In the scalar diquark model [12], the light-cone wave function for a ‘nucleon’ with spin \( \uparrow \) and the quark spin aligned and anti-aligned respectively reads [13]

\[
\psi_{\uparrow \frac{1}{2}} (x, k_\perp) = \left( M + \frac{m}{x} \right) \phi(x, k_\perp) \quad \psi_{\uparrow \frac{1}{2}} (x, k_\perp) = -\frac{k_1 + ik_2^2}{x} \phi(x, k_\perp)
\]

with \( \phi = \frac{g/\sqrt{1-x}}{M^2 - k_\perp^2 + m^2 - k_\perp^2 + \lambda x} \). Here \( g \) is the Yukawa coupling and \( M/m/\lambda \) are the masses of the ‘nucleon’/quark/diquark respectively. Furthermore \( x \) is the momentum fraction carried by the quark and \( k_\perp \equiv k_\perp x - k_\perp y \) represents the relative \( \perp \) momentum. Using these light-cone wave functions it is straightforward to calculate a variety of observables that appear in the context of nucleon spin physics. For example, since only \( \psi_{\uparrow \frac{1}{2}} \) carries (one positive) unit of OAM, the quark OAM according to JM is obtained as

\[
L_q = \int_0^1 dx \int \frac{d^2 k_\perp}{16\pi^2} (1-x) \left| \psi_{\uparrow \frac{1}{2}} \right|^2,
\]
where the factor $(1-x)$ is the fraction of OAM carried by the quark with momentum fraction $x$ in a two-body system with one unit of OAM [9]. One can also use the above light-cone wave functions to calculate the GPDs that enter the Ji relation, to calculate

$$L_q \equiv \frac{1}{2} \int_0^1 dx \left[ xq(x) + xE(x,0,0) - \Delta q(x) \right]. \quad (13)$$

Using a Lorentz invariant regulator, such as a Pauli-Villars regulator, one finds that the two definitions for quark OAM agree, i.e. $L_q = L^q_2$ in the scalar diquark model, as was expected since $L^q_2$ in the scalar diquark model does not contain a gauge field term.

However, despite $L_q = L^q_2$, no such equality holds for the corresponding unintegrated quantities. If one defines $L_q(x)$ by Eq. (13) without the $x$ integral [14], one does not obtain the OAM distribution that would be obtained from (12) without $x$ integral (Fig. 2), which renders the interpretation of $L_q(x)$ as the quark OAM distribution questionable.

**ELECTRON OAM IN QED**

In QED, there are four polarization states in the $e\gamma$ Fock component. To lowest order, the respective Fock space amplitudes for a dressed electron with $J^z = +\frac{1}{2}$ read

$$\Psi_{+\frac{1}{2}+1}^+ (x, k_{\perp}) = \frac{k^1 - ik^2}{x(1-x)} \phi(x, k_{\perp}^2) \quad \Psi_{+\frac{1}{2}-1}^+ (x, k_{\perp}) = -\frac{k^1 + ik^2}{1-x} \phi(x, k_{\perp}^2)$$

$$\Psi_{-\frac{1}{2}+1}^- (x, k_{\perp}) = \left( \frac{m}{x} - m \right) \phi(x, k_{\perp}^2) \quad \Psi_{-\frac{1}{2}-1}^- (x, k_{\perp}) = 0 \quad (14)$$

with $\phi(x, k_{\perp}^2) = \frac{\sqrt{2}}{\sqrt{1-x}} \frac{e}{M^2 - k_{\perp}^2 + m^2 \frac{x}{1-x}}$. The label $\pm \frac{1}{2}$ represents the spin of the electron in the $e\gamma$ Fock component and $\pm 1$ that of the $\gamma$.

Using these light-cone wave functions, it is again straightforward to calculate the OAM of the electron in the JM [8] decomposition. Including a Pauli-Villars subtraction...
with a heavy ‘photon’ with mass $\Lambda$ one thus finds

$$\mathcal{L}_e^z = \int_0^1 dx \int \frac{d^2 k_+}{16\pi^2} (1 - x) \left[ |\Psi_{+\frac{1}{2}+1}^\uparrow|^2 - |\Psi_{+\frac{1}{2}-1}^\uparrow|^2 \right] \xrightarrow{\Lambda \rightarrow 0} - \frac{\alpha}{4\pi} \left[ \frac{4}{3} \log \frac{\Lambda^2}{m^2} - \frac{2}{9} \right]$$

(15)

Likewise, inserting the corresponding PDFs/GPDs from Ref. [13] into (13) yields

$$L_z^z = \frac{1}{2} \int_0^1 dx \left[ xq_e(x) + xE_e(x,0,0) - \Delta q_e(x) \right] \xrightarrow{\Lambda \rightarrow 0} - \frac{\alpha}{4\pi} \left[ \frac{4}{3} \log \frac{\Lambda^2}{m^2} + \frac{7}{9} \right].$$

(16)

Both $\mathcal{L}_e^z$ and $L_z^z$ are negative, regardless of the value of $\Lambda^2$ (as long as $\Lambda^2 > \lambda^2$). In the case of $\mathcal{L}_e^z$ the physical reason is a preference for the emission of photons/gluons with the spin parallel (as compared to anti-parallel) to the original quark spin [15] — resulting more likely in a state with negative OAM. In fact, when $x \to 0$, this preference reflects the more general principle of helicity retention [16], which favors the lead parton (i.e. the parton carrying most of the momentum) to carry a spin as close as possible to that of the parent. It is also encoded in the evolution equations derived in Ref. [14]. The divergent parts of $\mathcal{L}_e^z$ and $L_z^z$ are the same so that their difference is UV finite

$$\mathcal{L}_e^z - L_z^z \xrightarrow{\Lambda \rightarrow 0} \frac{\alpha}{4\pi}.$$  

(17)

Applying these results to a (massive) quark with $J^z = +\frac{1}{2}$ yields to $O(\alpha_s)$

$$\mathcal{L}_q^z - L_q^z = \frac{\alpha_s}{3\pi}.$$  

(18)

This result may have important phenomenological consequences. Recent lattice QCD calculations for GPDs yielded $L_q^z < 0$ and $L_q^z > 0$, which is opposite to the sign obtained in typical relativistic quark models — such as the MIT bag model.

In QCD, the gluon spin is experimentally accessible, but the gluon OAM $\mathcal{L}_g^z$ is not. On the other hand, the gluon (total) angular momentum $J_g^z$ appearing in the Ji decomposition is accessible, either indirectly (by subtraction, using quark GPDs), or directly, using gluon GPDs from lattice and/or deeply virtual $J/\psi$ production. Even though $\frac{1}{2}\Delta G$ and $J_g^z$ belong to two incommensurable decompositions of the nucleon spin, it is thus tempting to consider the difference between these two quantities, hoping to learn something about gluon OAM. Subtracting (9) from (7), it is straightforward to convince oneself that $J_g^z - \frac{1}{2}\Delta G = \mathcal{L}_g^z + \sum_q (\mathcal{L}_q^z - L_q^z)$, i.e. numerically $J_g^z - \frac{1}{2}\Delta G$ differs from $\mathcal{L}_g^z$ by the same amount that $\sum_q \mathcal{L}_q^z$ differs from $\sum_q L_q^z$. In our QED example, with

$$\Delta G = \int_0^1 dx \int \frac{d^2 k_+}{16\pi^2} \left[ |\Psi_{+\frac{1}{2}+1}^\uparrow|^2 - |\Psi_{+\frac{1}{2}-1}^\uparrow|^2 + |\Psi_{-\frac{1}{2}+1}^\uparrow|^2 - |\Psi_{-\frac{1}{2}-1}^\uparrow|^2 \right]$$

(19)

being the photon spin contribution, one thus finds (for $\Lambda \to 0, \Lambda \to \infty$)

$$J_g^z - \frac{1}{2}\Delta G = \mathcal{L}_g^z + \frac{\alpha}{4\pi}. $$

(20)
As was the case in (17), \( \frac{e}{4\pi} \) appears to be a small correction, but one needs to keep in mind that for an electron \( J^z_r, \Delta y, \) and \( \mathcal{L}^z_r \) are also only of order \( \alpha \).

## QUARK OAM IN THE BAG MODEL

Since quark models have, to lowest order in \( \alpha_s \), no vector potential, it makes perhaps more sense to identify the quark OAM from these models with \( \mathcal{L}^z_q \) rather than with the GPD-based \( L^z_q \). In Ref. [9] the difference \( \Delta^z_q \equiv \mathcal{L}^z_q - L^z_q \) was calculated to order \( \alpha \) for a single electron in QED and the result then also applied to a single quark in QCD. However, in QCD quarks are never alone and the question arises regarding the effects from ‘spectator currents’ on the orbital angular momentum of each quark.

In order to address this issue, we will in the following use the MIT bag model [5] to estimate \( \mathcal{O}(\alpha_s) \) corrections to the difference

\[
\Delta^z_q = L^z_q - \mathcal{L}^z_q = \langle q^+ (\vec{r} \times g\vec{A})^z_q \rangle.
\]  

(21)

The vector potential in (21) is calculated from the spectator currents, which are obtained by taking matrix elements in the corresponding ground state bag model wave functions. The vector potential resulting from these static currents is obtained by solving

\[
\vec{V}^z A^a(\vec{r}) = -\vec{J}^a(\vec{r}) = -\sum_q g \Psi^+_q(\vec{r}) \vec{A}^z_q \Psi_q(\vec{r})
\]

(22)

for each color component \( a \) and where the summation is over the spectators (here we pick the gauge \( \vec{V} \cdot \vec{A} = 0 \), but to \( \mathcal{O}(\alpha_s) \) the result is actually gauge invariant — at least in the subclass of all gauges where all color components are treated (globally) \( \text{SU}(3) \)-symmetrically. I such gauges, matrix elements of operators of the type \( q^+ \Gamma q A^a \), where \( \Gamma \) is some Dirac matrix, and \( A^a \) is calculated to \( \mathcal{O}(\alpha_s) \), are proportional to the matrix elements of the corresponding abelian operators. It is thus sufficient to establish gauge invariance of \( q^+ \vec{r} \times \vec{A} q \) for abelian fields. The key observation is that the bag model wave functions contain no correlations between the positions of the quarks. Therefore, after eliminating the color in this calculation and introducing abelian currents, \( \Delta^z_q \) factorizes into the density of the active quark \( \psi^+_q(\vec{r}) \psi_q(\vec{r}) \) times \( (\vec{r} \times \vec{A} z = r A_\phi \). Writing the volume integral \( \int d^3 r \) in cylindrical coordinates, one can isolate the only \( \phi \)-dependent term \( r \int_0^{2\pi} d\phi A_\phi = \oint r \vec{A}(\vec{r}) \) as a closed loop integral with fixed \( r \) and \( z \). The closed loop integral is gauge invariant (its numerical value represents the color-magnetic flux through a circle with radius \( r \)) and so is the volume integral in which it enters.

The contribution from a spectator with \( j^z = s' \) to \( \langle q(\vec{r} \times \vec{A})^z q \rangle \) thus reads

\[
\Delta^z_{s'} = -\frac{2}{3} \frac{g^2}{4\pi} \int d^3 r d^3 r' \psi^+_s(\vec{r}) \psi_s(\vec{r}) \psi^+_s(\vec{r'}) \psi_s(\vec{r'}) \frac{(\vec{r} \times \vec{A})^z}{|\vec{r} - \vec{r'}|} \psi_{s'}(\vec{r'}).
\]

(23)

Note that \( \Delta_{s'} \) is independent of the angular momentum \( j^z = s \) of the ‘active quark’, since \( \psi^+_s(\vec{r}) \psi_s(\vec{r}) = \psi^+_{s+}(\vec{r}) \psi_{s-}(\vec{r}) \). However, it depends on the spin of the spectator since
the orientation of the vector potential entering (21) depends on the latter. For example, for 
\( s' = +\frac{1}{2} \), one finds

\[
\psi_s^\dagger(\vec{r}')(\vec{r} \times \vec{\alpha})^z \psi_s(\vec{r}') = |\mathcal{N}|^2 j_0(kr') j_1(kr') 2 \frac{xx' + yy'}{r'}
\]

and hence independent of the bag radius \([17]\)

\[
\Delta_{\pm \frac{1}{2}}^z = \pm \frac{2}{3} \alpha_s |\mathcal{N}|^4 \int_{r < R} d^3 r \int_{r' < R} d^3 r' \left[ j_0^2(kr) + j_1^2(kr) \right] 2 \frac{xx' + yy'}{|\vec{r} - \vec{r}'|} j_0(kr') j_1(kr') = \pm 0.78 \alpha_s.
\]

When the active quark has \( s \) aligned with that of the proton, the two spectators must 
have opposite \( s' \) and their contribution to \( \Delta \) cancels, i.e. \( \Delta \) is nonzero only in those wave 
function components where the active quark has \( s = -\frac{1}{2} \), in which case both spectators 
have \( s' = +\frac{1}{2} \). As a result, \( \Delta_{q/p}^z \) is equal to twice \( \Delta_{\mp \frac{1}{2}}^z \) times the probability to find that 
quark flavor with \( s = -\frac{1}{2} \) (which is \( \frac{1}{3} \) for \( q = u \) and \( \frac{2}{3} \) for \( q = d \), and hence

\[
\Delta_{u/p}^z = \frac{2}{3} \Delta_{\mp \frac{1}{2}}^z = -0.052 \alpha_s \quad \Delta_{d/p}^z = \frac{4}{3} \Delta_{\mp \frac{1}{2}}^z = -0.104 \alpha_s.
\]

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