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To cite this article: G Lo Bianco et al 2011 J. Phys.: Conf. Ser. 267 012054

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E0 decay of the $0^+_2$ levels in $^{156}$Dy and $^{160}$Er

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Abstract.

The branching between the E0 $0^+_2 \rightarrow 0^+_1$ and the E2 $0^+_2 \rightarrow 2^+_1$ transitions in $^{156}$Dy and $^{160}$Er were measured following the $\varepsilon$ decay of $^{156}$Ho and $^{160}$Tm. A potential of the 4th order of the deformation parameter $\beta$, the “Lo Bianco potential”, was chosen to describe the $U(5) - SU(3)$ first order shape phase transition, covering the whole transitional path. The comparison of the excitation spectra and the measured ratios of reduced transition probabilities, $X(E0/E2)$, with the calculations, indicates that $^{156}$Dy is in the spherical region, while $^{160}$Er is located in the deformed region, but quite close to the critical point.

1. Introduction

In the geometrical description of collective nuclear motion there are three limiting cases, corresponding to the harmonic vibrator, the symmetrically deformed rotor and the triaxial rotor. Each of them is associated with a particular nuclear shape, spherical, axial-ellipsoidal, and triaxial. The transition from a spherical harmonic vibrator to an axially deformed rotor was described analytically by Iachello, introducing a dynamic symmetry, denoted $X(5)$ [1]. It is interesting in this context to investigate the E0 transition strength, in the decay of the first excited $0^+$ state, because this quantity is known to have particularly large values in the transitional region between spherical and deformed nuclei [2, 3]. This fact can be explained by strong mixing of states with different deformation [4], or within the framework of the Interacting Boson Model (IBM) [5], by mixing of states with different number of bosons [3].

Electric monopole, E0, transitions are forbidden by the $\gamma$-decay selection rules and can occur only via the emission of atomic electrons. Electron emission is the only process by which a $0^+$ state can decay to another $0^+$ state. If the nuclear spin is non-zero electric quadrupole E2, and E0 transitions compete with each other. The E0 transition probability is factorized into electron and nuclear terms [6],

$$W = \Omega \rho^2(E0),$$

where $\Omega$ represents all the “non-nuclear” contributions and can be calculated from several models [7, 8, 9]. The nuclear structure information is contained in $\rho^2(E0)$ and can be directly related to different models.

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Considering a simple collective geometrical model, Rasmussen estimated the $E^0$ transition probability and the dimensionless ratio of the reduced $E^0$ and $E^2$ transition probabilities [10]

$$X(E^0/E^2; 0^+_2 \rightarrow 0^+_1) = \rho^2(E^0; 0^+_2 \rightarrow 0^+_1) \cdot \beta^2 R_0^4 = 4\beta^2,$$

(2)

and demonstrated that it is proportional to the deformation, $\beta$. This ratio can be compared with the experiment as [11]:

$$X(E^0/E^2; 0^+_2 \rightarrow 0^+_1) = 2.54 \cdot 10^9 A^{4/3} E^5 \alpha_K(E2)/\Omega,$$

(3)

where $E_\gamma$ is the energy of the $E^2$ transition in MeV and $\alpha_K(E2)$ is the $K$-conversion coefficient for $E^2$ transition. The quantity $q^2$ is measured in the experiment as

$$q^2 = \frac{A_{e,K}(E0)}{\epsilon_{e,K}(E0)} \cdot \frac{\epsilon_{e,K}(E2)}{A_{e,K}(E2)},$$

(4)

where $A_{e,K}(E\lambda)$ is the intensity of the $K$ line for the corresponding transition and $\epsilon_{e,K}(E\lambda)$ is the efficiency of the $\beta$ spectrometer.

We report a study of the $E^0$ strength in two nuclei, $^{156}$Dy and $^{160}$Er. The nucleus $^{156}$Dy is considered as a good candidate for $X(5)$ symmetry [12], while $^{160}$Er, which has a structural parameter $R_{4/2} = E(4^+_1)/E(2^+_1) = 3.1$ lies, on the transition path between the critical point of the vibrator-rotor shape phase transition and the rigid rotor limit.

2. Experiment

The experiments were performed at the INFN Laboratori Nazionali del Sud (LNS) in Catania. Partial level schemes of $^{156}$Dy and $^{160}$Er are presented in Fig. 1 and Fig. 2, respectively.

![Figure 1. Partial level scheme of $^{156}$Dy.](image)

![Figure 2. Partial level scheme of $^{160}$Er.](image)

Levels in $^{156}$Dy were populated by the $^{156}$Er $\rightarrow$ $^{156}$Ho $\rightarrow$ $^{156}$Dy $\epsilon$ decay chain. $^{156}$Er was produced by the $^{148}$Sm($^{12}$C,4n) reaction at 73 MeV. The $^{12}$C beam was provided by the LNS tandem accelerator and the target was a self-supporting isotope-enriched foil having a thickness of $\sim 0.8$ mg/cm$^2$. Since the $^{156}$Er $\rightarrow$ $^{156}$Ho and the $^{156}$Ho $\rightarrow$ $^{156}$Dy decays have half lives of 19.5 min and 56 min, respectively, the experiment was performed by repeating cycles in which
the $^{148}$Sm target was irradiated for one hour and, after a 5 min delay, the decay of $^{156}$Ho was measured for one hour by detecting off-beam $\gamma$ rays and conversion electrons. In-beam spectra, produced by the prompt de-excitation of $^{156,157}$Er levels, were also collected during the irradiation periods for calibration purposes.

The $0^+_2$ state in $^{160}$Er was populated through the $^{160}$Yb $\rightarrow$ $^{160}$Tm $\rightarrow$ $^{160}$Er $\epsilon$ decay chain. $^{160}$Yb was produced by the $^{150}$Sm($^{14}$N,4n) reaction at 72 MeV. The target was a self-supporting isotope-enriched foil with a thickness of 0.612 mg/cm$^2$. The half lives of $^{160}$Yb and $^{160}$Tm are 4.8 min and 9.4 min, respectively. Therefore, a ten-minute on-off beam cycle was used. The data was collected in the beam-off intervals.

This technique has the advantage that the decay chain starts with an even-even nucleus and the $\beta$ decay passes through low spin states, thus avoiding possible branches through higher spin states in the parent doubly-odd nucleus. As a result, the population of non-yrast states in the daughter nucleus is enhanced and relatively clean spectra are obtained. Another advantage of the technique is that the measurement is done off-beam, which reduces the background of the electron spectra. The $\gamma$ rays were measured with a coaxial HPGe detector which was positioned at 90$^\circ$ degrees with respect to the beam. Conversion electrons were measured with a mini-orange spectrometer, consisting of a magnetic filter made by permanent Sm-Co magnets, shaped like orange slices, and a 3 mm-thick Si(Li) detector cooled at liquid-nitrogen temperature. The spectrometer at Catania is a replica of the Groningen device [13]. Such spectrometers have a magnetic field which is transverse to the flat trajectories of the electrons. The mini-orange spectrometer was positioned at 135$^\circ$ with respect to the beam, in backward direction. Its efficiency is strongly energy dependent. The transmission window can be varied by changing the number and type of the permanent magnets and by adjusting the distances between the magnets and the target and between the target and the Si(Li) detector.

The efficiency calibration of the electron spectrometer is crucial for the correct treatment of the data. For example, in the $^{160}$Er experiment, the $^{124}$Sn($^{12}$C,4n)$^{132}$Ba reaction at 60 MeV was used for simultaneous calibration of the HPGe detector and the mini-orange spectrometer, because in this case stretched $E2$ transitions are populated in the desired energy region of 650 – 850 keV [14]. In the case of $^{156}$Dy the calibration was done in two different ways: (i) the lines of the yrast band in $^{156}$Er (4n reaction channel) were used, and (ii) the relative efficiency of the mini-orange spectrometer for the 667- and 691-keV lines was derived from the measured electron lines. The efficiency for the $E0$ and the 684-keV transitions was obtained assuming a linear dependence (see Fig. 3 lower panel). The efficiency of the mini-orange spectrometer, $\epsilon_e$, as a function of the electron energy is obtained from the measured areas of the corresponding peaks in the electron ($A_e$) and the $\gamma$-ray spectra ($I_\gamma$), e.g. in the case of two transitions with energies $E_1$ and $E_2$

$$ \frac{\epsilon_e(E_1 - B)}{\epsilon_e(E_2 - B)} = \frac{A_e(E_1 - B)}{A_e(E_2 - B)} \times \frac{\alpha(E_2)I_\gamma(E_2)}{\alpha(E_1)I_\gamma(E_1)}, \quad (5) $$

where $B$ is the binding energy of the electron. The theoretical conversion coefficients for the stretched $E2$ are used [15].

The measured $\gamma$-ray and conversion electron spectra, revealing the decay of the $0^+_2$ states in $^{156}$Dy and $^{160}$Er are presented in Fig. 3 and Fig. 4, respectively. Transitions belonging to $^{156}$Dy and $^{160}$Er are indicated with their energies in keV. $K$- and $L$-electron lines are indicated by $K$ or $L$ in the conversion electron spectra. Contaminant lines are indicated by $\star$. The arrows in the lower panels indicate the peak corresponding to the $E0$ transition. In both cases, no peaks at these positions were observed in the $\gamma$ spectra (indicated by arrows in the upper panels), which sets limits on the degree of contamination.
3. Results and discussion

The 676-keV E0 transition in $^{156}\text{Dy}$ is rather weak (see Fig. 3 lower panel). Therefore, possible sources of contamination were analysed carefully to avoid systematic errors. Four peaks that correspond to the K-electron lines of 667-, 676-, 684- and 691-keV were used in the analysis of the conversion electron spectra. Using the $\gamma$-ray intensities from the experiment and the conversion coefficients of Ref. [15], $q^2 = 1.97(70)$ was obtained. Estimating the electronic factor with the method of Kantele [9] as $\Omega_K = 4.05 \cdot 10^{10}$ s$^{-1}$, the ratio of the reduced transition probabilities is $X(E0/E2; 0_2^+ \rightarrow 0_1^+) = 0.045(17)$. The $\gamma$-ray intensities, which were measured by Caprio et al. [12], can also be used in the analysis, since excited states in $^{156}\text{Dy}$ were populated in exactly the same way in both experiments. They yield $q^2 = 2.17(74)$, in perfect agreement with the value above.

These values can be compared with results from a recent compilation of the E0 strength [11], where the authors re-evaluated the existing data. For $^{156}\text{Dy}$ the results are: $q^2 = 3(2)$ and $X(E0/E2; 0_2^+ \rightarrow 0_1^+) = 0.08(5)$. The results are in agreement with each other, but with the present experiment, the uncertainty was reduced considerably.

In the case of $^{160}\text{Er}$ more intense $0_2^+ \rightarrow 0_1^+$ transition was observed (see Fig. 4b). With the mini-orange efficiency calibration and the $\gamma$-ray intensities of the experiment and the conversion coefficients of Ref. [15], $q^2 = 4.3(7)$ was obtained. This number should be considered as a preliminary result, since the analysis continues. Estimating the electronic factor with the method of Kantele [9] as $\Omega_K = 7.04 \cdot 10^{10}$ s$^{-1}$, the ratio of the reduced transition probabilities is $X(E0/E2; 0_2^+ \rightarrow 0_1^+) = 0.18(4)$.

The $X(5)$ critical point symmetry [1] is a solution of the Bohr Hamiltonian with a special choice of the potential: $v(\beta, \gamma) = u(\beta) + v(\gamma)$. This potential allows an approximate separation of variables and then the potential in $\beta$ is chosen as an infinite square well. This choice comes from the fact that, using the coherent state formalisms in the IBM, one can obtain the potential that (only) at the critical point goes as $\sim \beta^4$ and therefore it can be approximated with an infinite square well [16]. In a similar way, for the $U(5) - SU(3)$ first order shape phase transition, a
more general potential in $\beta$ can be introduced, the “Lo Bianco potential”, that is parametrized as follows:

$$u(\beta) = V_0(\zeta \beta^4 - 2\zeta \beta_0^3 + (1 - \zeta)\beta_0^2\beta^2),$$

(6)

with $0 \leq \zeta \leq 1$. When $\zeta = 0$ there is a spherical minimum, at the critical point with $\zeta = 1/2$ there are two coexisting minima (with a very small bump in between), one in $\beta = 0$ and the other at $\beta = \beta_0$, and at $\zeta = 1$ there is only a unique deformed minimum in $3/2\beta_0$. This potential has the virtue of covering the whole transitional path of the shape phase transition at the price of having three parameters instead of the parameter-free predictions of the $X(5)$ model. The parameters, especially $V_0$, can be adjusted according to phenomenology to reproduce a subset of low-lying energy levels. An optimization procedure, based on a random walk in the parameter space (similar to Ref. [17]), to find optimal values for the parameters in order to fit the experimental energy levels has been set up. Calculations for several Dy and Er nuclei have been performed. Results for the excitation spectra of $^{160}$Er are displayed in Fig. 5.

For the Dy isotopes, in the case of $^{154}$Dy a value of $\zeta \approx 0.4$ is found, together with a value of $\beta_0 \approx 0.3$; the interplay of these two parameters makes up a potential with a flat bottom part, but still on the spherical side of the phase transition. From the fits one can see that $^{156}$Dy is in the spherical region ($\zeta \approx 0.1$) and $^{158}$Dy is just after the critical point ($\zeta = 0.54$), but the deformed minimum is already winning. In fact, although $\zeta$ is just after 1/2, the combination of all three parameters gives a deformed potential and the wave functions are more pronounced at the deformations around the minimum. In the Erbium series of isotopes, $^{158}$Er is close to the spherical limit ($\zeta = 0.06$), $^{162}$Er is clearly deformed ($\zeta = 0.79$) and $^{160}$Er has a value of $\zeta = 0.53$ that locates it quite close to the critical point.

In the framework of the Rasmussen model [10], for $^{156}$Dy the experimental ratio of the reduced transition probabilities, $X(E0/E2)$, was found to correspond to a modest equilibrium deformation, $\beta = 0.11(6)$, while for $^{160}$Er a value of $\beta = 0.21(10)$ is obtained.
The matrix elements of the $E_0$ operator, $\langle f | T(E_0) | i \rangle$, between $0^+$ states in $^{156}$Dy and $^{160}$Er, which are calculated with the present model, are listed in table 1.

Table 1. Matrix elements of the $T(E_0)$ operator between $0^+$ states with the “Lo Bianco potential”.

| $A$ | $Z$ | $\langle 0^+_1 | T(E_0) | 0^+_2 \rangle$ | $\langle 0^+_1 | T(E_0) | 0^+_3 \rangle$ | $\langle 0^+_1 | T(E_0) | 0^+_3 \rangle$ |
|-----|-----|--------------------------------|--------------------------------|--------------------------------|
| $^{156}$Dy | 0.04279 | -0.00213 | -0.06419 |
| $^{160}$Er | 0.03311 | -0.00467 | -0.04464 |

The matrix elements allow a straightforward calculation of $\rho_2^2(E_0)$:

$$\rho_2^2(E_0) = \left( \frac{3}{4\pi} \right)^2 Z^2 |\langle f | T(E_0) | i \rangle |^2,$$

which can be compared with the experiment. It turns out that, in the cases where $\rho_2^2(E_0)$ is measured [11], the comparison is not very good. The value calculated in the collective model is about four times bigger than the measured quantity and this is true not only with our potential, but also for the confined beta-soft model [18]. One could wonder whether this is a shortcoming of the collective model, rather than attributing it to details of the potential.

In conclusion, we have measured the ratios of the reduced transition probabilities $X(E_0/E_2)$ in $^{156}$Dy and $^{160}$Er and compared them with calculations using a potential, which is a combination of quadratic, cubic and quartic powers of $\beta$. The results indicate that $^{156}$Dy is in the spherical region, while $^{160}$Er is located in the deformed region, but quite close to the critical point for the $U(5) - SU(3)$ first order shape phase transition.

Acknowledgments

We like to give a last farewell to Nanni Lo Bianco, who left us unexpectedly in September 2009. He has been an excellent teacher and collaborator for all of us. This work has been supported by the Italian INFN GAMMA collaboration and by the Bulgarian National Science Fund, contract DID-02/16.

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