High-Accuracy Radar Processing Algorithms for OCDM RadCom System

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Abstract. In this paper, an orthogonal chirp-division multiplexing (OCDM) symbol-based radar signal processing approach used to estimate the distance and velocity of the target echo in the Radar-Communication (RadCom) system is introduced. And to improve the accuracy of radar resolution, two radar signal processing algorithms, respectively based on periodogram and multiple signal classification (MUSIC) are presented. Simulation results show that these algorithms can significantly improve the scatterer of the radar range profile.

1. Introduction

The OCDM radar and communication system proposed in [1] has been proved to be promising for distance and velocity estimation, while its verification can be found in [2]. The radar signal processing of the OCDM is the key to the system design. In practical applications, there are high requirements for the radar resolution index. It is essential to improve the radar resolution through algorithms and obtain better target estimation performance.

In this paper, we discuss the radar estimation algorithm and accuracy of the OCDM signal with only one target in the detection range. This simplification reduces applicability in real-world scenarios but provides a good starting point for more general research. We will introduce the basics of symbol-based OCDM radar signal processing in the following Section. Section 3 will explain two ways of the OCDM radar algorithms. Next, simulation parameters of the OCDM RadCom system will be given in Section 4, and results are then shown in Section 4. Section 5 concludes.

2. Symbol-based OCDM radar processing

The OCDM signal is designed for multiplexing a bank of chirps in the same period and bandwidth. In the OCDM RadCom system, a transmitting signal frame is composed of $M$ temporal symbols and $N$ sub-chirps. And the amplitude and phase of each chirp can be used for modulating communication information. The $n$th chirp is modulated with data $x(n) \in \mathcal{X}$, and $\mathcal{X}$ is a set of modulation mapping table (e.g.16QAM). Based on such a chirp basis, the waveform can be expressed as

$$s(t) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} e^{j \frac{\pi}{2} \mathcal{X}(mN + n)} \exp \left[ -j \frac{N}{T} \left( t - n \frac{T}{N} \right)^2 \right] \text{rect} \left( t - mT_{\text{OCDM}} \right)$$

(1)

where $\text{rect}(\cdot)$ is a rectangular window function, and $T$ is the symbol duration time. There is a guard interval $T_G$, then $T_{\text{OCDM}} = T + T_G$ is the duration of an OCDM symbol. For the radar processing in an
OCDM-based system, an advanced joint distance and Doppler estimation algorithm for OCDM signal is considered, which directly operates on the transmitted and received modulation symbols, instead of the baseband signals. As shown in figure 1, the Fourier transform results of the transmitted and received modulation symbols are $T_x/F_x$ and $R_x/F_x$. The matrix $T_x/F_x$ contains the communication information modulated by the transmitter that is seen as a random interference term for radar estimation. By dividing two matrices, the interference is eliminated, and the matrix $F_x$, which contains the radar distance and Doppler information, is obtained.

$$F_{n,m} = \left(\frac{R_{n,m}}{T_{n,m}}\right) = A_{n,m} \exp(j2\pi f_m T) \exp\left(-j2\pi n \frac{2R}{c_0 T}\right) + Z_0_{n,m}$$  \hspace{1cm} (2)

where $( )_{n,m}$ represents the $n$th element on the $m$th symbol in the matrix, and the matrix $A$ is the amplitude attenuation coefficient matrix introduced by channel propagation, and $Z_0$ is the noise matrix. Furthermore, $F$ can be express as vector multiplication form,

$$(F)_{n,m} = (A)_{n,m} \cdot (k_x \otimes k_v)_{n,m} + (W)_{n,m}$$  \hspace{1cm} (3)

with $\otimes$ referring to a dyadic product, and the corresponding radar distance and Doppler shift can be obtained by

$$k_x = \exp\left(-j2\pi n \frac{2R}{c_0 T}\right), n = 0,1,\ldots,N - 1$$

$$k_v = \exp\left(j2\pi \frac{2V_{rel} f_m}{c_0 m}\right), m = 0,1,\ldots,M - 1$$  \hspace{1cm} (4)

The estimation of $R$ and $v$ is thus equivalent to detect the fundamental frequency of discretely sampled sinusoids. To improve the accuracy of estimation, two methods are introduced in Section 3.

3. OCDM Radar Algorithms

3.1. Periodogram spectral estimation$^{[3-4]}$

The expression of the periodogram method for a discrete-time signal $s(k)$ with a length of $N$ is

$$\text{Per}_{x(k)}(f) = \frac{1}{N} \left| \sum_{k=0}^{N-1} s(k) \exp(-j2\pi f k) \right|^2 = \frac{1}{N} \left| \text{FFT}_N \{ s(k) \} \right|^2$$  \hspace{1cm} (5)

where FFT {} represents the Fourier transform of the signal. The column vector and row vector of the matrix $F$ can be estimated by the periodogram estimator to obtain the distance and speed respectively.

![Figure 1. OCDM RadCom system setup.](http://example.com/figure1.png)
Extend the above method to two dimensions, and the estimation algorithm for relative distance and velocity of the targets is as follows:

- Perform FFT of length $M_{Per}$ for each column of $F$.
- On the result matrix, perform an IFFT of length $N_{Per}$ for each row.
- Modulus-square and obtain a two-dimensional periodogram result

$$
\text{Per}_f (n, m) = \frac{1}{NM} \left| \sum_{q=0}^{N_{Per}-1} \left( \sum_{p=0}^{M_{Per}-1} (F)_{n,m} \exp \left( -j2\pi \frac{m}{M_{Per}} p \right) \right) \exp \left( j2\pi \frac{n}{N_{Per}} q \right) \right|^2
$$

- The peak value of the detection matrix $\hat{(n_m)}$ corresponding to the estimated distance and velocity of the target $F$ is

$$
\hat{R} = \frac{\hat{nc}_0 f}{2N_{per}} \text{ and } \hat{v} = \frac{\hat{nc}_0}{2fTM_{per}}
$$

The distance $R$ is always greater than or equal to zero. The radial velocity $v \geq 0$ indicates that the target moves towards the receiver, and $v < 0$ indicates that the target moves away from the receiver. Then the points $n$ and $m$ are

$$
n = 0, \ldots, N_{per} - 1 \text{ and } m = -\frac{M_{per}}{2}, \ldots, \frac{M_{per}}{2} - 1,
$$

In this way, it is ensured that the estimated result has physical meaning.

3.2. MUSIC-based estimation

The multiple signal classification (MUSIC) algorithm proposed by R. O. Schmidt is a super-resolution spatial spectrum estimation algorithm based on eigenvalue decomposition $[5]$. The important steps of the algorithm are briefly introduced here. The model of the received signal is seen as

$$
x(t) = As(t) + n(t)
$$

And the autocorrelation matrix of $x(t)$ is

$$
\tilde{R}_{xx} = \frac{1}{N} \sum_{i=0}^{N-1} x_i^H x_i
$$

Since the received signal and noise are independent of each other, the eigenvalue decomposition of the autocorrelation matrix $R_{xx} \in \mathbb{C}^{M \times M}$ can be converted to

$$
R_{xx} = E \Lambda E^T = E_S \Lambda_S E_S^T + E_N \Lambda_N E_N^T
$$

where $\Lambda$ is the diagonal matrix of eigenvalues. Arbitrary sine vector $s \left( e^{j\Omega} \right) = \left( 1, e^{j\Omega}, e^{j2\Omega}, \ldots, e^{j(M-1)\Omega} \right)^T$, of which the frequency is $\Omega = \Omega_1$, then it can be proved

$$
s^T E_N E_N s = 0
$$

Therefore, the peak search result of $P = \frac{1}{s^T E_N E_N s}$ is the estimated value of the spectrum.

In the echo signal model of RadCom, the matrix $F$ that needs to be estimated is obtained based on the digital symbol domain.
The autoregressive model of distance $R$ and relative speed $v$ can be respectively obtained by matrix $F$

$$F = AS + N(t)$$

(12)

The estimation algorithm for distance and relative velocity of the targets is as follows:

- Firstly, calculate the covariance matrix $\hat{R}_r$ and $\hat{R}_v$.
- Eigenvectors form noise space and calculate the frequency $\hat{\Omega}_r$ and $\hat{\Omega}_v$.
- The corresponding estimated distance and velocity are

$$\hat{R} = \frac{\hat{\Omega}_r c_0}{4\pi} \quad \text{and} \quad \hat{v} = \frac{\hat{\Omega}_v c_0}{4\pi f_c T_{OCDM}}$$

(14)

4. Simulation

The estimation quality is mainly determined by the signal-to-noise ratio (SNR). In the case of one target,

$$SNR_{db} = -10\log_{10} \left( \frac{S}{N} \right)$$

(15)

Its value is influenced by a large number of physical parameters shown in Table 1.

| Variables $P_r$ | Transmit power |
|----------------|----------------|
| $G$            | Total transmit and receive antenna gain |
| $f_c$          | Radio frequency |
| $B$            | Signal bandwidth |
| $N_0$          | Total noise power density |
| $r$            | Relative distance between target and receiver |
| $\sigma$       | Target radar cross section (RCS) |

While the radio system does not change during operation, $r$ and $\sigma$ depend on the target. The received power is determined by

$$P_r = \frac{P G^2 c^2 \sigma}{(4\pi)^3 f_c^2 R^4}$$

(16)

The receiver induces noise with a total noise power density of $N_0$. Thus, total SNR is

$$SNR = \frac{P_r}{N_0 B} = \frac{P G^2 c_0^2 \sigma}{(4\pi)^3 f_c^2 N_0 B R^4}$$

(17)

It is pointed out that the signal configuration is part of the SNR equation and the dependency on distance and bandwidth of SNR is highlighted.
For the verification of the distance and Doppler processing and the quality of the estimators in Section 3, a simulation of the OCDM RadCom system has been implemented in MATLAB. The parameters of the OCDM signal are shown in Table 2.

| $T$  | $N$  | $M$  | $f_c$ | $T_g$ |
|------|------|------|-------|-------|
| 10.24μs | 1024 | 256  | 24GHz | $1/8T$ |

Table 2. OCDM signal parameters.

To be able to compare the results between the waveforms, we fixed the parameters in the simulation. The total noise figure was set to 20dB, and the RCS to 10m². Point scatterer target is set at $R = 15$m to get high receiver SNR. At the receiver, the received baseband signal is processed with the algorithm as described in equation (6) and (13). Figure 2 shows the cut of the radar image, in which normalized reflected power is used.

![Radar image calculated for object at $R = 15$m.](image1)

From figure 2, we can see that the MUSIC algorithm can perform super-resolution estimation on target information. In contrast, the resolution of the FFT algorithm and the periodogram estimation algorithm is not so much high. The difference between the two is that the periodogram estimation algorithm has a lower estimated spectrum sidelobes than the FFT method. The reason is that the square term of the period estimation algorithm suppresses the sidelobes.

For further comparison of the results between the waveforms, we use different footstep of distances. At every step, 100 simulations were run at random velocities, uniformly distributed within ±150 m/s.

![The general performance of the periodogram and MUSIC estimators in comparing to using FFT.](image2)
In figure 3, the estimated variance of the estimators is shown. As can be seen, the estimation accuracy of the periodogram estimator is similar to that of using FFT method. And the MUSIC algorithm is more suitable for high SNR situations. In comparison, FFT and periodogram methods have little effect on SNR. In low SNR situations, these two methods are preferred.

5. Conclusion
In this paper, the resolution accuracy of the radar processing method of the OCDM RadCom system is studied. The simulation results show that both periodogram spectral estimation and MUSIC-based estimation are feasible to improve the resolution and reduce the sidelobe of the radar estimators, which can better distinguish the target. And it is clearly shown that the super-resolution characteristic of the MUSIC algorithm has its limits, thus the method’s application scope is strictly restrained. While further research is certainly required to extend these results to the case of multiple targets, simulations suggest that the OCDM radar approach is a very promising one.

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