FORMATION OF THE WIDE ASYNCHRONOUS BINARY ASTEROID POPULATION

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ABSTRACT

We propose and analyze a new mechanism for the formation of the wide asynchronous binary population. These binary asteroids have wide semimajor axes relative to most near-Earth and main belt asteroid systems. Confirmed members have rapidly rotating primaries and satellites that are not tidally locked. Previously suggested formation mechanisms from impact ejecta, from planetary flybys, and directly from rotational fission events cannot satisfy all of the observations. The newly hypothesized mechanism works as follows: (1) these systems are formed from rotational fission, (2) their satellites are tidally locked, (3) their orbits are expanded by the binary Yarkovsky–O’Keefe–Radzievskii–Paddack (BYORP) effect, (4) their satellites desynchronize as a result of the adiabatic invariance between the libration of the secondary and the mutual orbit, and (5) the secondary avoids resynchronization because of the YORP effect. This seemingly complex chain of events is a natural pathway for binaries with satellites that have particular shapes, which define the BYORP effect torque that acts on the system. After detailing the theory, we analyze each of the wide asynchronous binary members and candidates to assess their most likely formation mechanism. Finally, we suggest possible future observations to check and constrain our hypothesis.

Key words: celestial mechanics – minor planets, asteroids: general – minor planets, asteroids: individual (317 Roxane, 1509 Esclangona, 1717 Arlon, 4674 Pauling, 17246, 22899, 32039, 51356, 1998 ST27)

1. INTRODUCTION

The observation that most small near-Earth and main belt asteroid binary systems have a rapidly rotating primary is one of the key pieces of evidence that led astronomers to more closely investigate rotational fission as the dominant binary formation mechanism (Margot et al. 2002). Scheeres (2007a) and Walsh et al. (2008) showed that the creation of binary asteroid systems is possible by means of rotational fission induced by the Yarkovsky–O’Keefe–Radzievskii–Paddack (YORP) effect. Jacobson & Scheeres (2011a) modeled the dynamics of rotationally fissioned asteroids and determined the properties of these newly created binaries, which include the primary always rotating rapidly and the secondary rotating at a different rate from the mutual orbit (typically faster). As observed in numerical experiments, the rotational-fission formation mechanism creates tight binary systems with a median mutual semimajor axis of \( a = 3.3R_p \) and a maximum of \( 17R_p \) (Jacobson & Scheeres 2011a).

After creation, tidal synchronization of the secondary is the fastest tidal process within the binary system (Goldreich & Sari 2009). For low mass ratio systems, \( q < 0.2 \), the primary can take more than an order of magnitude longer to synchronize than the secondary, and for given estimates of the relevant tidal parameters, tidal synchronization of the secondary is also faster than the dynamical or collisional lifetime of kilometer and subkilometer binary systems for the near-Earth or main belt asteroid populations, respectively. The outcome of this tidal process is a tidally locked secondary orbiting a still rapidly rotating primary. These singly synchronous binary systems are the most prevalent small \( (R_p < 10 \text{ km}) \) binary asteroid systems in either the near-Earth or main belt asteroid populations (Margot et al. 2002; Pravec et al. 2006).

Two effects, mutual body tides and the binary YORP (BYORP) effect, continue to evolve singly synchronous binaries. Each is described in detail in Section 2.1. According to the tidal–BYORP equilibrium hypothesis proposed in Jacobson & Scheeres (2011c), singly synchronous binary systems can evolve to a long-term, stable semimajor axis equilibrium if the torque from the BYORP effect on the mutual orbit is contractive (i.e., acting opposite to the always expansive tidal torque). However, if the BYORP torque is expansive, then the two torques grow the mutual orbit to the Hill radius, where the orbit is disrupted by distant third-body encounters (Čuk 2007; McMahon & Scheeres 2010a; Jacobson & Scheeres 2011c).

Here we propose that these expanding singly synchronous binary systems are also the source of the wide asynchronous binary asteroid population, which is described in detail in Section 1.1. There exists an adiabatic invariance between the size of the mutual orbit and the libration amplitude of the tidally locked secondary, and so as the mutual orbits of these systems expand, the libration amplitude increases. At some large semimajor axis, the libration amplitude reaches 90° and the secondary begins to circulate. This circulation turns off the BYORP effect, which at large binary separations is the only significant torque acting on the orbit, since tidal evolution is very weak at these distances. Once the secondary has begun rotating...
circulating, it is rotationally accelerated by the YORP effect. This process is similar to YORP acceleration of a single body (Bottke et al. 2006), since tides are so weak.

Since the strength of mutual body tides is inversely proportional to the semimajor axis to the sixth power (Goldreich & Sari 2009), tides are much less effective on wider binaries than they are on the more frequently observed tight binary population. For instance, consider two systems that are identical except for their semimajor axes. The first has the median semimajor axis \( a = 3.3R_p \) found for post-fission binary systems (Jacobson & Scheeres 2011a), and the second has a semimajor axis of \( a = 12R_p \). The second then takes \( \sim 10^3 \) times longer to tidally damp than the first. This damping timescale ratio is the same for the synchronization of each body and the circularization of the orbit. Considering that the eccentricity damping timescales are thought to be between \( \sim 10^4 \) and \( \sim 10^7 \) yr for tight binary systems (\( 2R_p < a < 8R_p \), Fang & Margot 2012b), the tightest wide asynchronous candidates (\( a = 12R_p \)) should have damping timescales between \( 10^5 \) and \( 10^{10} \) yr.

Figure 1 further motivates the identification of this particular class of small binary, the wide asynchronous binary, and the expanding singly synchronous formation mechanism described above. It shows that the radiative BYORP and YORP effects are much more important than the mutual body tidal torques for the wide asynchronous binaries. A handful of wide asynchronous candidates may have secondaries with tidal torques stronger than the YORP effect. These systems are discussed in Section 4.2 in the context of alternative formation mechanisms. The clear trend of increasing ratio with semimajor axis affirms that the tidal dependence on separation distance is the most important factor.

The wide asynchronous binary populations are very different from the singly synchronous asteroids, which, as Jacobson & Scheeres (2011c) proposed, are a result of tides and the BYORP effect having equivalent strengths but opposite directions. However, by considering that the BYORP effect and tides can act in unison rather than in opposition, it is possible to connect these two distinct populations. The key to the proposed hypothesis is the adiabatic invariance, which transforms outward orbit expansion into excited libration of the secondary. Since the BYORP effect drives the orbit expansion and the BYORP effect requires a synchronized (even if librating) secondary, the widening of these systems becomes the very event that strands them at wide separations. This mechanism for the creation of wide asynchronous binaries requires specific initial conditions, namely, a wider orbit than is typical after rotational fission (as can be seen in Figure 7 below). Since not all expanding singly synchronous binaries will start at these initial separations, some systems will expand to their Hill radius and become asteroid pairs. Future observational determination of the ratio of tight to wide binaries would constrain just how sensitive these initial conditions are.

It is important to note that not all asynchronous binary systems are wide. Tight asynchronous binary systems have semimajor axes \( a \lesssim 8R_p \), which is also the semimajor axis cutoff observed in the singly synchronous binary population. This tight population is consistent with either recent formation from rotational fission (i.e., synchronization of the secondary is currently underway) or strong planetary perturbations, which essentially reset the system and require the secondary to be synchronized again to the new, typically slower orbital period (Jacobson & Scheeres 2011a, 2012; Fang & Margot 2012a).

1.1. The Wide Asynchronous Binary Asteroid Population

The population of wide asynchronous binary systems includes four known members and five suspects. All candidates have semimajor axes \( a > 8R_p \),\(^9\) larger than any member of the singly synchronous population (e.g., 66391 (1999 KW4)), which have semimajor axes \( a < 8R_p \) and are distributed about a median of \( a = 4R_p \). Wide asynchronous binary systems are distinct from other wide binary systems that have doubly synchronous members and high mass ratios (e.g., 4951 Iwamoto (1990 BM)), since all wide asynchronous candidates have low mass ratios and at least one asynchronous member. Possessing a second asynchronous member is the distinction between being a suspect and full membership in the wide asynchronous class.

\(^9\) For convenience, when we wish to refer to all of them, we will call them all candidates, since we are considering them for this possible creation mechanism.

\(^{10}\) When comparing binary systems with one another, it is important to use properly scaled measurements. Here we are comparing the semimajor axes of the systems measured in radii of the primary.

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\( \text{Figure 1. Top: ratio of the orbital torque from the BYORP effect to the tidal torque on the orbit from the tidal bulge raised on the primary by the secondary as a function of the semimajor axis of each wide asynchronous candidate system and wide asynchronous pair in each triple system. Bottom: ratio of the YORP effect rotation torque and the tidal torque on the rotation of the secondary from the primary as a function of the semimajor axis of the same systems as the top figure. The horizontal line is at 1, where the two torques are equal. The equations for these torques are in Appendix A and the data for each system are listed in Tables 1 and 2. The uncertainty is dominated by the poorly known BYORP, YORP, and tidal parameters. From left to right, the systems are 51356 (2000 RY76), 155391 (2001 SN393), 1717 Arlon, 1998 ST7, 136617 (1999 CC), 317 Roxane, 32039 (2000 IO25), 1509 Esclangona, 22899 (1999 TO14), 3749 Balam, 17246 (2000 GL24), and 4674 Pauling. The lines are styled such that the three tightest wide binary systems (51356, Arlon, and 1998 ST7) are short-dashed, the two triple systems (Balam and 136617) are long-dashed while 155391, a tight triple system, is short-dash–long-dashed, and the rest are solid.} \)
These wide asynchronous binaries are also distinct from the tight asynchronous binaries (e.g., 35107 (1991 VH)), which have a similar distribution of semimajor axes to the singly synchronous population, with the same observed upper limit of 8 Rp. All known wide asynchronous members have Rp ≲ 10 km. The YORP effect is not efficient at driving asteroids to rotational fission above this size, because of competition with collisional evolution and the dynamical lifetimes of the systems. We limit the suspected members to this size range and note that the other binary asteroid classes hypothesized to form from YORP-induced rotational fission obey this size cutoff. To summarize, wide asynchronous binary candidates have low mass ratios, small sizes, and rapidly rotating primaries with the exception of 317 Roxane and possibly 1717 Arlon. These characteristics are identical to the singly synchronous binary population (Jacobson & Scheeres 2011c), but all of these systems have larger semimajor axes, and those with measured secondary rotation periods are asynchronous.

The names and properties of each wide asynchronous candidate are listed in Table 1. The next two columns are the heliocentric semimajor axis a⊙ and eccentricity e⊙, which are relevant for radiative torques. The orbits of these systems are very diverse and include near-Earth, Mars-crossing, and main belt asteroids. The mass ratio q is in Column 4. With the possible exception of 1717 Arlon, which could be significantly larger, all systems have mass ratios below or near the low mass ratio limit of q ∼ 0.2 as defined in Jacobson & Scheeres (2011a), where it was determined that with all other things being equal, low mass ratio binaries tidally synchronize their secondary members much more quickly than their primary members. The primary radius Rp is in Column 5. For many calculations, we use a spherical approximation for both bodies, so the secondary radius is Rs = q 1/3 Rp. Then the mutual semimajor axis a is given in both Rp and kilometers. For all of the equations in the text, a has been normalized to the primary radius. This normalization allows us to close the table.

Table 1

| Asteroid System | a⊙ (AU) | e⊙ | q | Rp (km) | a (Rp) | Pp (Pd) | Pp (Pd) | Pp (Ps) | Pp (Ps) |
|-----------------|--------|----|---|---------|--------|---------|---------|---------|---------|
| 317 Roxanea     | 2.29   | 0.09| 0.023b | 9.4b | 27 | 257b | 3.5 | 8.2b | 19b |
| 1509 Esclangona (1938 YG)d | 1.87 | 0.03 | 0.036d | 4.3a | 49 | 210d | 1.4 | 3.3a | 2.8 | 6.6a |
| 1717 Arlon (1954 AC)g | 2.20 | 0.13 | >0.216 | 3.9b | 15 | 59b | 2.2 | 5.16∗ | 7.8 | 18.26∗ |
| 4674 Pauling (1989 JC)h | 1.86 | 0.07 | 0.033 | 2.2j | 116 | 250j | 1.1 | 2.5j | 4.8 | 11.1j |
| 17246 (2000 GL04)i | 2.84 | 0.02 | 0.064 | 2.3h | 99 | 228h | 1.7 | 4.0h | 2.6 | 11h |
| 22899 (1999 TO14)m | 2.84 | 0.08 | 0.033m | 2.7n | 67 | 182n | 1.4 | 3.3n | 4.8 | 11.1n |
| 32039 (2000 JO23)p | 2.22 | 0.28 | 0.275p | 1.3h | 32 | 42h | 1.4 | 3.3p | 4.8 | 11.1p |
| 51536 (2000 RY70)q | 1.81 | 0.11 | 0.009q | 1.2b | 9 | 11b | 1.1 | 2.6b | 2.6 | 6.0b |
| 1998 ST3r | 0.82 | 0.53 | 0.0034r | 0.28r | 16 | 4.5r | 1.3 | 3.1r | 2.6 | 6.0r |

Notes. Columns are the heliocentric semimajor axis a⊙, and eccentricity e⊙, mass ratio q, radius of the primary Rp, mutual semimajor axis a measured in both primary radii Rp, and kilometers, and rotation periods of the primary and the secondary measured in hours and the surface disruption period limit Pd = √π/ρG, where ρ is the density and G is the gravitational constant. Osculating orbital elements are from the JPL Horizons system. The published 1σ uncertainties are often below the precision reported in the table, but separation distances are sometimes projected distances from adaptive optics direct imaging and not true semimajor axes.

a Merline et al. (2009).
b Durda et al. (2010).
c Harris et al. (1992).
d Merline et al. (2003a).
e Marchis et al. (2012).
f Warner et al. (2010).
g Cooney et al. (2011).
h Warner et al. (2010).
i Merline et al. (2004).
j Pravec et al. (2012a).
k Warner et al. (2006).
l Tamblyn et al. (2004).
m Merline et al. (2003b).
n Masiero et al. (2011).
o Polishook et al. (2011).
p Pray et al. (2007).
q Warner & Pray (2013).
r Benner et al. (2003).
s Brozović et al. (2011).

In the original report by Cooney et al. (2006), it is not clear which period belongs to which body of 1717 Arlon. We have assigned the periods based on the common pattern of a more rapidly rotating primary on the basis of likely formation by rotational fission; however, P. Pravec and colleagues (private communication) are preparing to report that the periods may more likely belong to the other body as shown above. We also call attention to the large mass ratio of 1717 Arlon, which is only a lower limit. This binary may be very strange indeed.

<ref>http://www.asu.cas.cz/~asteroid/binastdata.htm</ref> . It is compiled according to methods described by Pravec & Harris (2007).

11 Statistics regarding binaries in this paragraph are from the 2011 July 1 binary asteroid parameter release at <ref>http://www.asu.cas.cz/~asteroid/binastdata.htm</ref> . It is compiled according to methods described by Pravec & Harris (2007) and is maintained by P. Pravec and collaborators.
to consider all of the systems simultaneously. The next two sets of columns contain the primary period $P_p$ and the secondary period $P_s$ in two different units: $P_d = 2\pi/\omega_d = \sqrt{3\pi/\rho G}$, which is the period surface disruption limit, and hours. If a period is near $1P_d$, then the body is spinning near its surface disruption spin limit.

### 1.2. Outer Members of the Triple Systems

There is another class of wide asynchronous satellites, and these are tertiary (outermost) members of triple asteroid systems. They are listed in Table 2. With the exception of an interior third body, they resemble the wide asynchronous binaries in every way: mass ratio, absolute size, semimajor axis, and rotation periods. The mass of the interior satellite can be larger or smaller than the exterior satellite whose properties are listed in the table. The interaction with that third member can substantially change the dynamics of the system, but in this work, we do not consider the third member.

Forming a triple asteroid system is obviously more complex than forming a binary system, but all three of these systems have rapidly rotating primaries, and that is strongly suggestive of rotational fission (Fang et al. 2011). Both satellites could form at the same time from a secondary fission event (Jacobson & Scheeres 2011a), or one satellite could form and, if it ends up in a large orbit, the primary could go through another YORP-induced rotational fission event, forming a stable binary interior to the original binary. Regardless of formation mechanism, the outer binary pair appears similar to the wide asynchronous population, and we will consider these systems as if they evolved similarly to the wide asynchronous population by the expanding singly synchronous hypothesis and just ignore the role of the interior tertiary member and, therefore, all possible perturbations such as resonances.

### 2. FORMING THE WIDE ASYNCHRONOUS POPULATION

Singly synchronous binary asteroid systems are the most numerous observed binary systems (Pravec & Harris 2007). If they occupy the hypothesized tidal--BYORP equilibrium (Jacobson & Scheeres 2011c), then they represent only half of all binary systems that have undergone tidal synchronization, since the direction of the BYORP torque on the secondary is nominally independent of tidal synchronization. We ask what happens to those systems for which the BYORP effect is expansive. From previous work, we know that systems with expanding BYORP and tidal torques quickly evolve outward (Goldreich & Sari 2009; Čuk & Nesvorný 2010; McMahon & Scheeres 2010a), but until now there has not been a comprehensive model of this evolution that considers both mechanisms and the role of the adiabatic invariance between the size of the mutual orbit and the libration of the secondary.

#### 2.1. Tidal and BYORP Evolution

In this section, we describe the outward mutual orbit evolution due to both tides and the BYORP effect. It is important to note that throughout, we assume that the orbit and spin poles are parallel. This assumption is in good agreement with observation (Pravec et al. 2012b) and is consistent with formation from a process, such as rotational fission, that requires a large amount of angular momentum. For singly synchronous systems, mutual body tides expand the semimajor axis. As the primary rotates beneath the secondary at a rate different from the orbital rate, it deforms, attempting to obtain a figure in equilibrium with the ever-changing gravitational potentials of both bodies. Assuming the primary is not made of perfectly elastic material, this deformation dissipates the spin energy of the primary, slowing it down, and the disturbed primary figure transfers angular momentum to the orbit. This transferred angular momentum expands the orbit. If the bodies are in an eccentric orbit, there are similar deformations of each body due to both the radial oscillations of the orbit and the difference between the eccentric and mean anomalies. Mutual body tides are parameterized by the tidal Love number $k$ and the tidal quality number $Q$, which respectively quantify the deformation and the dissipation of the body undergoing the tidal stresses (Murray & Dermott 2000). The tidal Love number of a perfectly rigid, nondeformable body is $k = 1$, and a body with no dissipation has an infinite tidal quality number. Both parameters are poorly known, but it has been shown that for binary systems with rubble-pile components, the combined effects of these tides are to damp the mutual orbit’s eccentricity and grow the semimajor axis of the system (Goldreich & Sari 2009; Jacobson & Scheeres 2011c).

The BYORP effect is an averaged radiative torque on the mutual orbit that acts on any system with a body in a spin–orbit resonance (Čuk & Burns 2005). Photons exchange momentum with the body as they are absorbed or emitted. When this change in momentum acts as a torque with a lever arm from the surface point of interaction to the barycenter of the binary asteroid

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### Table 2

| Asteroid System                  | $a_0$ (AU) | $e_0$ | $q$  | $R_p$ (km) | $a$ (km) | $P_p$ (hr) | $P_t$ (hr) |
|----------------------------------|------------|-------|------|------------|----------|------------|------------|
| 3749 Balam (1982 BG1)$^a$        | 2.24       | 0.11  | 0.009$^b$ | 3.3$^b$  | 88       | 289$^b$   | 1.2        |
| 136617 (1994 CC)$^b$             | 1.64       | 0.42  | 0.0035$^c$ | 0.52$^c$ | 19       | 6.1$^c$   | 1.0        |
| 153591 (2001 SN341)$^d$          | 1.99       | 0.48  | 0.026$^e$  | 1.3$^e$  | 13       | 16.6$^e$  | 1.5        |

**Notes.** Columns are as in Table 1. The mass ratio $q$ is the mass of the tertiary (outermost member) divided by the mass of the primary.

$^a$ Merline et al. (2002).
$^b$ Marchis et al. (2008a).
$^c$ Marchis et al. (2008b).
$^d$ Brozovic et al. (2009).
$^e$ Fang et al. (2011).
$^f$ Brozovic et al. (2011).
$^g$ Nolan et al. (2008).
$^h$ Oey (2009).
system, it can perturb the mutual orbit. In order for the sum of every small momentum change from each photon to be nonzero, the surface of the body must be in a repeating relationship with the binary barycenter (i.e., spin–orbit resonance), or else they will sum to zero over large times. The BYORP effect is the averaged outcome of all of these instantaneous torques. It can expand or contract the mutual orbit, depending on the shape and orientation of the body. This is quantified to first order in eccentricity by the BYORP coefficient $B$ (McMahon & Scheeres 2010b). A BYORP coefficient $B = 0$ represents a completely symmetric body. We define the coefficient so a positive BYORP coefficient corresponds to an expanding mutual orbit. McMahon & Scheeres (2010a) determined that the BYORP effect can double or halve the semimajor axis of a singly synchronous binary system such as 1999 KW$_4$ in $\sim(3-6) \times 10^4$ yr. The BYORP effect is less effective with larger libration angles, but it still operates in the same direction.

Jacobson & Scheeres (2011c) hypothesized that the prevalence of singly synchronous systems is due to an equilibrium in semimajor axis between the BYORP effect and mutual body tides. In this case, tides expand the system and the BYORP effect contracts it (i.e., negative BYORP coefficient, $B < 0$), driving the mutual orbit to an equilibrium between the two torques. Without exogenous interference, these systems are stable because tides damp eccentricity more strongly than the BYORP effect can excite it. The BYORP coefficient can also be positive, with presumably the same range of absolute values as the equilibrium systems in Jacobson & Scheeres (2011c) and estimated from shape models of McMahon & Scheeres (2012). For example, if the tidally locked secondary in any of the singly synchronous binary systems is flipped so that its near and far faces are reversed, then the BYORP coefficient would be of the same strength but opposite sign and so the system would expand.

When the BYORP torque is expansive, it and mutual body tides damp the eccentricity $e$. The time evolution of the eccentricity is thus the linear combination of the tidal evolution (Goldreich & Sari 2009) and the BYORP effect evolution (McMahon & Scheeres 2010a):

$$\dot{e} = -\frac{k_p}{Q} (A_T L + a^2 A_B) e a^{-13/2},$$

where $k_p$ is the tidal Love number of the primary; $Q$ is the tidal quality number; $a$ is the semimajor axis measured in primary radii $R_p$; and $A_T$, $L$, and $A_B$ are positive coefficients defined in Appendix B that depend on various properties of the binary system. Since both torques damp eccentricity, we expect systems evolving outward to have smaller eccentricities. This is in stark contrast to the very high eccentricities expected when a binary forms directly from rotational fission, the expected prevalence of high-eccentricity escaping eject binaries (EEBs), or excitation from planetary flybys. However, this does not mean that we expect expanding singly synchronous systems and the wide asynchronous binaries they produce to have zero eccentricity; sometimes these systems can expand outward and desynchronize faster than tides can completely damp the eccentricity. It is a challenge to evaluate this exactly, because of the uncertainty regarding the initial conditions of the system and the tidal and BYORP parameters, but we explore it in Section 3.2.

As mentioned above, a positive BYORP coefficient leads to growth of the mutual orbit. The rate of the growth of the mutual semimajor axis is again the linear addition of the tidal evolution (Goldreich & Sari 2009) and the BYORP effect evolution (McMahon & Scheeres 2010a):

$$\dot{a} = \frac{k_p}{Q} (A_T + a^7 A_B) a^{-11/2},$$

where the ratio of tidal parameters $k_p/Q$ has been extracted, so that the coefficient $A_T$ depends on the BYORP coefficient $B$, and the tidal parameters in the exact relationship found in the tidal–BYORP equilibrium, $B/Q/k_p = 2557 (R_p/1 \text{ km})$, and $A_T$ depends only on better measured system properties (Jacobson & Scheeres 2011c). The ratio $k_p/Q$ is the denominator of the unknown tidal ratio $X$ described below and is the primary source of uncertainty for both semimajor axis and eccentricity evolution equations. Since both time derivatives depend linearly on $k_p/Q$, it is conceptually easy to understand how changes in this ratio affect the evolution rates and their associated timescales.

### 2.2. Adiabatic Invariance

Using a model of a larger sphere for the primary and a smaller triaxial ellipsoid for the secondary, which is consistent with observations (Pravec & Harris 2007), we derive in Appendix C an adiabatic invariant relationship from the Hamiltonian dynamics between the libration amplitude and the semimajor axis (or mean motion) of the system. When a Hamiltonian dynamical system has been canonically transformed to action-angle coordinates, an adiabatic invariance is the conservation of the action (or phase-space volume) of the system. The invariance is conserved even when the parameters describing the system undergo changes, as long as those changes are slow compared with the periodic motion of the angle. The outward expansion of the system and the damping of the libration angle are both slow compared with the libration frequency (i.e., the angle).

The adiabatic invariance for this system is

$$J_\phi = \frac{4G I_\phi \omega_0 G}{a^{3/2}} \sqrt{\frac{3S}{1 + s}},$$

where $G = G(\sin^2 \Phi)$ is a function of complete elliptic integrals as given in Appendix C with $\sin^2 \Phi$, always as the argument, $\Phi$ is the libration amplitude, $G I_\phi$ is the maximum moment of inertia of the secondary, $S$ is a shape factor of the secondary with $S = 0$ an oblate body and $S$ increasing with prolateness toward 1, and $s = G I/\nu q a^2$ is the secondary perturbation term with $I = I_s/I_p$ the ratio of the secondary $I_s = M_s R_p^2$ and primary $I_p = M_p R_p^2$ moments of inertia. The secondary perturbation term grows with increasing mass ratio $q$ or when the moment of inertia of the secondary increases relative to the primary, and it also increases with decreasing semimajor axis; however, $s$ is always nearly zero (see Appendix C for further elaboration regarding all of these terms and the derivation itself).

An intuitive understanding of the libration amplitude growth can be developed by studying the adiabatic invariant of the system at two different times:

$$\frac{G(\sin^2 \Phi_1)}{G(\sin^2 \Phi_2)} = \frac{a_1^{1/2} (1 + s_1)^{1/2}}{a_2^{1/2} (1 + s_2)^{1/2}},$$

where the subscripts 1 and 2 indicate the two different times. This relationship can be simplified further by recognizing that $G(\sin^2 \Phi)$ is well approximated by $\sin^2 \Phi$ and that $s \sim 0$ regardless of semimajor axis. Then the relationship is simply

$$\frac{\sin^2 \Phi_1}{\sin^2 \Phi_2} \propto \frac{a_1^{3/2}}{a_2^{3/2}} = \frac{n_2}{n_1},$$

where

$$\frac{\sin^2 \Phi_1}{\sin^2 \Phi_2} \propto \frac{a_1^{3/2}}{a_2^{3/2}} = \frac{n_2}{n_1}.$$
From the relationships above, it is now clear that as the system expands, the libration amplitude increases as well.

If the libration amplitude of the secondary exceeds 90°, then the secondary will no longer be librating and will begin to circulate. This will turn off the BYORP effect, since the secondary will no longer be in a synchronous orbit. Since the tides on the primary are very weak at large semimajor axes, this effectively ends outward expansion (details are shown in Section 3.1 and, particularly, Figure 4). The mutual orbit is then stranded at this semimajor axis, since tides are so weak that the secondary cannot be expected to be relocked, and without the spin–orbit resonance the BYORP effect averages to zero. The YORP effect controls the rotational evolution of the asteroid, and this is discussed in Section 3.2, but the mutual orbit evolution is complete. Therefore, the semimajor axis at which the secondary begins circulating, \( a_{\text{circ}} \), is the same semimajor axis that observers measure, \( a_{\text{obs}} \), which are the semimajor axes in Tables 1 and 2.

To understand this evolution, we assume a given libration amplitude \( \Phi \) at some semimajor axis \( a \); then the semimajor axis \( a_{\text{circ}} \) at which the secondary begins to circulate (i.e., \( \Phi_{\text{circ}} = \pi/2 \)) is

\[
a_{\text{circ}} = a \sin^{-1} \frac{\Phi}{\Phi_{\text{circ}}}.
\]

In Figure 2, we show contours of \( a_{\text{circ}} \) at semimajor axes consistent with the range of observed wide asynchronous binaries. This analysis indicates that this is a plausible mechanism for creating the wide asynchronous population, since all of the observed singly synchronous binaries correspond to systems with small libration angles at semimajor axes consistent with formation from rotational fission.

We have found an intuitive understanding of the adiabatic invariant relationship, and the values reported above for \( a_{\text{init}} \) and \( \Phi_{\text{init}} \) are informative, but we are left wondering, “What are the real values for the initial semimajor axis and libration amplitude for each system? Do they match these predictions?”

The semimajor axis of the system at synchronization seems a natural choice for the initial semimajor axis. As discussed above, Jacobson & Scheeres (2011a) found that rotational fission forms stable binaries from \( 2R_p \) to \( 17R_p \), and possibly up to \( 34R_p \). Tidal synchronization of the secondary is much faster than any tidal evolution of the orbit (Goldreich & Sari 2009; Fang & Margot 2012b), so this range of semimajor axes seems appropriate as possible initial values. We note that numerical simulations only produced binaries in the subset between \( 2R_p \) and \( 17R_p \).

The initial libration amplitude is more difficult to ascertain. In fact, the model so far incorrectly asserts that the libration angle should start small. As the secondary is tidally locked, it does not immediately have a small free libration; instead, it has a libration amplitude near 90°. Tides from the primary acting on the free libration of the secondary dissipate energy and drive the free libration toward zero. In the model presented so far, the free libration grows as a result of the adiabatic invariance through tidally driven and BYORP effect-driven orbit expansion, and it grows without any interference. The model must be expanded to include the libration dissipation due to these tides on the secondary.

2.3. Tidal Libration Damping

We directly determine the evolution of the libration amplitude from the energy dissipation due to the primary’s raising tides on the free librations of the secondary by utilizing a formulation developed by Wisdom (2004, 2008). The derivation, with some explanation, is included in Appendix E. It uses a two-sphere model assuming a circular orbit and homogenous interiors. It is important to note that tides on the free libration do not secularly evolve the orbit. This is because the tidal bulge on the secondary oscillates from the leading to trailing hemisphere and back, so the angular momentum transfer to the orbit averages to zero.

From Appendix E, the tidal energy dissipation rate is

\[
\dot{E} = -\frac{2\pi k_s\rho_s^2\omega_l\Phi_s^2R_p^5q^{5/3}}{Q_la^6},
\]

where \( \Phi_s \) is the libration amplitude of the secondary and \( \omega_l \) is the libration frequency. The libration frequency is derived in Appendix F and is

\[
\omega_l = \frac{\pi q^2}{2k_s^2} \sqrt{3} (1 + s).
\]

For low mass ratio systems \( q < 0.2 \) and small libration amplitudes \( \Phi_s \ll 1 \), so the libration frequency is just proportional to the mean motion: \( \omega_l \approx \pi q^2 \sqrt{3} (1 + s) \).

In Appendix G, we use this rate of energy dissipation to calculate the time derivative of the libration amplitude due to tides:

\[
\dot{\Phi}_s = -\frac{A_L \Phi_s^2}{a^{5/2} K \sin 2\Phi_s} \left( \frac{k_s}{Q_t} \right),
\]

where \( A_L \) is a useful organizational coefficient reported in Appendix G. Note that the tidal parameters \( k_s \) and \( Q_t \) are the primary sources of uncertainty, and we have left them outside of the coefficient \( A_L \) on purpose. They are the numerator of the soon-to-be-introduced tidal ratio \( X \).

2.4. Libration Amplitude Evolution

By taking the time derivative of the adiabatic invariant, we determine the libration growth due to orbit expansion:

\[
\dot{\Phi}_s = \frac{a G A_L}{K \sin 2\Phi_s},
\]

where we have defined \( A_L \) as a convenient coefficient always nearly equal to 3 and \( K = K(\sin^2 \Phi_s) \) is the complete elliptic function of the first kind. A derivation can be found in Appendix D.
The libration amplitude evolution has two components, the tidal damping derived above and the growth term from the adiabatic invariance. We add the two components to obtain the combined effect:

\[ \Phi_s = \frac{\dot{a}a\Phi_s^2}{K\sin 2\Phi_s} \left[ \frac{A_g G}{a^2\Phi_s^2} - \frac{k_s A_L}{Q_k a^{11/2}} \right], \tag{11} \]

where \( \dot{a} \) is the rate of orbit expansion due to tides and the BYORP effect.

We change variables from the formulation above, switching from the time derivative of the libration amplitude to the derivative of the libration amplitude as a function of semimajor axis:

\[ \frac{d\Phi_s}{da} = \frac{a\Phi_s^2}{K\sin 2\Phi_s} \left[ \frac{A_g G}{a^2\Phi_s^2} - \frac{X A_I}{A_T + a^7 A_B} \right], \tag{12} \]

where each \( A \) is a coefficient,13 \( G \) and \( K \) are functions of complete elliptic functions with \( \sin^2 \Phi_s \) as the argument, and

\[ X = \left( \frac{k_s}{Q_k} \right) \left( \frac{Q_k}{k_p} \right). \tag{13} \]

which is a ratio of the tidal coefficients describing the mutual-orbit evolution due to tides raised by the secondary on the primary and the tidal coefficients describing the libration evolution due to tides raised by the primary on the secondary. \( X \) is the ratio of the normalized strengths of each tide normalized such that absolute scales such as size and separation found in the tidal equations are not considered. Note that the tidal Love and quality numbers are not necessarily size and frequency independent. There is much ongoing research into the determination of these values (e.g., Greenberg 2009; Goldreich & Sari 2009; Jacobson & Scheeres 2011c; Efroimsky 2012; Ferraz-Mello 2013).

Not only are the tidal parameters \( k \) and \( Q \) likely dependent on the physical properties of each body, but the tides on each body are fundamentally different. The tidal bulge raised by the secondary on the rapidly rotating primary moves through the body at a constant rate and direction characterized by the difference between the rotation frequency of the primary and the mean motion, if we assume a circular orbit and relaxed spin state. On a spherical and homogenous primary, the shape and amplitude of the bulge remain fixed while each element of the body is distorted as it rotates through the different parts of the tidal potential. However, the tidal bulge raised by the primary on the librating secondary changes shape and amplitude as the secondary’s rotation rate relative to its mean motion slows down and speeds up according to its librating motion. While we have characterized these tides using similarly named parameters, the tides themselves are fundamentally different, and it will require significant future geophysical and tidal modeling to assess estimates of these parameters. We can use observations to make a prediction regarding the value of \( X \) assuming that most of the wide asynchronous binaries experienced this expanding singly synchronous mechanism.

13 These coefficients are independent of the semimajor axis \( a \) if we assume that the secondary perturbation term \( s \sim 0 \). We do not make this assumption for the numerical calculations displayed in the figures, but throughout we notify the reader when it is useful to do so to better conceptually understand the math.

Figure 3. Term \( A_T + a^7 A_B \) plotted as a function of semimajor axis for all of the wide asynchronous systems. From top to bottom along the right side, the lines show 1998 ST27, 136617 (1994 CC), 51356 (2000 RY9), 153591 (2001 SN303), 4674 Pauling, 3749 Balam, 32039 (2000 JO23), 1509 Esclangona, 22899 (1999 TO13), 17246 (2000 GL74), 1717 Arlon, and 317 Roxane. The line styles are as in Figure 1.

3. COMPARISON WITH OBSERVATIONS

We now have a model that includes the complete libration amplitude evolution, and incredibly, it has only a single unknown parameter, \( X \). Observations provide each of the unknown coefficients necessary to establish \( A_T, A_B, A_s, \) and \( A_L \) with the exception of the shape factor of the secondary, \( S \). Since none of the secondaries of the wide asynchronous systems have published shape models, \( S \) cannot be calculated for any of these systems. Instead, we use the shape information corresponding to the secondary of 1999 KW\(_4\), one of the best-studied singly synchronous binary systems. It has a shape factor \( S = 0.207 \) (Ostro et al. 2006). For reference, an oblate asteroid has \( S = 0 \), and 1620 Geographos, the most elongated asteroid yet observed, has \( S = 0.768 \) (Ostro et al. 1996). Since 1999 KW\(_4\) is a singly synchronous binary, its shape factor is the most appropriate as an initial guess. The final results are insensitive to a factor of 2 or 3 uncertainty in the value of \( S \), since it appears only as a square root in \( A_L \).

Another observation-provided parameter is the tidal–BYORP effect equilibrium parameter relationship \( B_t Q/k_p = 2557 \) (\( R_p/1 \) km) (Jacobson & Scheeres 2011c). This is the greatest source of uncertainty in the model outside of the parameter \( X \), since \( B_t Q/k_p \) has an uncertainty of more than an order of magnitude. We can study the influence of this uncertainty by noting that \( B_t Q/k_p \) appears as a linear factor in \( A_B \) and examining the role of \( A_B \) in Equation (12) in detail. In Figure 3, we show the denominator of the second term in the brackets in Equation (12), \( A_T + a^7 A_B \), as a function of semimajor axis. It is clear that for semimajor axes \( a \gtrsim 8R_p \), the term \( a^7 A_B \) dominates and the \( A_T \) term can be neglected. This is exactly what has been argued all along: tides are extraordinarily weak compared with the BYORP effect at large semimajor axes. Naturally, the turning point in Figure 3 for each of these systems is the semimajor axis equilibrium that the tidal–BYORP equilibrium theory predicts would exist if these systems were BYORP effect contractive rather than expansive. Encouragingly, all of the turning points are located interior to the observed orbits of the wide asynchronous candidates. Also, these points are in the same region of semimajor axis space that the observed singly synchronous binaries occupy, with the exception of 1998 ST27, whose predicted equilibrium is that of a very tight binary system, \( a \lesssim 2R_p \). Adjusting \( B_t Q/k_p \) moves the turning points. Decreasing or...
increasing $B_s Q / k_p$ moves the turning points outward or inward, respectively, although even a change by an order of magnitude does not move them very far, since the equilibrium varies as $(A_T / A_B)^{1/7}$.

When the binary system is tight (i.e., below the turning point), the evolution of the libration amplitude is insensitive to the choice of $B_s Q / k_p$. This can be understood as a regime dominated by the two sets of tides, the libration tides on the secondary damping the libration amplitude and the tidal expansion due to tides on the primary growing the amplitude through the adiabatic invariance.

When the wide asynchronous binaries are evolving exterior to the turning points, $A_T$ can be ignored. Then Equation (12) becomes simply $d\Phi_t / da \propto X / (B_s Q / k_p)$ when all other parameters are held constant. As we explore how the libration amplitude, the initial semimajor axis, and the circulation semimajor axis vary as a function of $X$, we can keep this relationship in mind to better understand how the uncertainty in $B_s Q / k_p$ affects the results.

### 3.1. Individual System Evolution

As explained in detail above, when the libration of the secondary reaches $90^\circ$, it begins to circulate. This circulation turns off the BYORP effect. Mutual body tides continue to act on the system, but these tides are very weak. Figure 4 shows the time it takes for each wide asynchronous candidate system to expand to its current observed semimajor axis from some distance interior to that orbit (indicated along the abscissa). The timescale is calculated by numerically integrating $\tau_{\text{exp}} = \int_{a_{\text{eq}}}^{a_{\text{obs}}}(Q/k_p)(a^{1/2}/A_T) da$, where the role of the tidal coefficients $Q$ and $k_p$ is highlighted because they are the major source of uncertainty. $Q/k_p$ is estimated from the tidal–BYORP effect equilibrium, assuming a BYORP coefficient of $B_s = 2 \times 10^{-2}$. From top to bottom, the lines show 3749 Balam, 4674 Pauling, 17246 (2000 GL74), 22899 (1999 TO14), 1509 Esclangona, 317 Roxane, 136617 (1994 CC), 32309 (2000 JO23), 1998 ST27, 153591 (2001 SN263), 1717 Arlon, and 51556 (2000 RY6). The line styles are as Figure 1.

At this semimajor axis, each of the curves in Figure 3 turns around, approaches an asymptote, and then follows that asymptote to the observed semimajor axis of the system. This asymptote is Equation (10) integrated. Using the same approximation that the secondary perturbation term $s \sim 0$, the asymptote is defined by

$$a = a_{\text{c}} \exp \left[ -\int_\phi^{\pi/2} \frac{K \sin 2\Phi_t}{3G} d\Phi_t \right].$$

If it is discovered from future observations that a wide binary is librating, then this asymptote may be used to assess the future evolution of the system. The previous equilibrium equation determines whether it is approaching the asymptote or the equilibrium. This could provide a significantly better understanding of the tidal ratio $X$ than we now possess. Studying such systems would be very useful. The 185851 (2000 DP107) system is the widest tight synchronous binary and an outlier in the tidal–BYORP equilibrium data set and, therefore, a candidate singly synchronous expanding system with a librating secondary (Jacobson & Scheeres 2011c).

Repeating this analysis with the outer pairs of the triple systems, we find similar results. These are shown in Figure 6. Each pair could have formed in the $2R_p$–$34R_p$ region and then expanded to its current orbit.

In each of Figures 5 and 6, we have only shown evolutionary paths corresponding to synchronization semimajor axes between $2R_p$ and $34R_p$. For systems with observed semimajor axes are shown for all nine candidate wide asynchronous binaries in Figure 5. We have chosen to show a selection of five possible evolutionary tracks for each system. Each track has a different tidal ratio $X$, and each is listed above the plot under the name of the system. These correspond to the lines from left to right. Under different assumptions for $X$, each system can form as a stable binary and tidally synchronize its secondary within the range of semimajor axes $(2R_p < a < 34R_p)$ consistent with the observed singly synchronous and tight asynchronous populations and the simulated stable binaries from Jacobson & Scheeres (2011a). Then the binary expands from those separations to their observed semimajor axes. During that expansion, the secondary remains tidally locked but freely librating.

A libration amplitude evolution pattern emerges across each of these systems. At the tidal synchronization semimajor axis, the libration amplitude is $90^\circ$. The libration amplitude declines because of tides as the system expands only a small amount. The slope of the decline is controlled by the second term in Equation (12): $-X A_T / (A_T + a^2 A_B)$. The shallower slopes at small semimajor axes are due to the relative similarity of the $A_T$ and $a^2 A_B$ terms at those small semimajor axes. For systems with larger tidal synchronization semimajor axes, the slope is much steeper because of the dominance of the $a^2 A_B$ term. Shallow or steep, the libration amplitude damps until the first and second terms of Equation (12), adiabatic growth and tidal damping, balance.

Equation (12) is highly nonlinear, so it is only possible to exactly solve for this equilibrium point numerically. However, we can approximately solve for this equilibrium semimajor axis, $a_{\text{eq}}$, by making the following assumptions: (1) the secondary perturbation term $s \sim 0$ and (2) $a$ is sufficiently large so that $A_T \ll a^2 A_B$. Then

$$a_{\text{eq}} = \left( \frac{X A_T \Phi_t^2}{A_A A_B G} \right)^{1/5}.$$  (14)
Figure 5. Possible evolutionary tracks for each of the nine candidate wide asynchronous binary systems. Each plot shows the libration amplitude as a function of semimajor axis for a specific system. The different lines represent a range of tidal ratios $X$, listed above each panel in the same left-to-right order as the lines. The tidal ratios were chosen for each system to demonstrate the range of initial semimajor axes that can lead to the observed semimajor axis of the system. The initial semimajor axes are constrained to be between $2R_p$ and $30R_p$, which are the limits from Jacobson & Scheeres (2011a). The dashed lines are an asymptote explained in the text.

Figure 6. Same as Figure 5, but for the candidate wide asynchronous triple systems.
and slows down relative to the mean motion, and so waxes and wanes as the rotation rate of the secondary speeds up.

If the circulation tide is weaker because it is not a constant tide moving through a body, then we might expect the libration tide to be.

The ratio of the normalized strength of the libration tide to the circulation tide, 

\[ X \]

is the ratio of the normalized strength of the libration tide to the circulation tide.

\[ X \]

depends on the tidal ratio.

\[ a \]

depends on the tidal ratio.

\[ \omega_{l} \]

and the secondary libration frequency.

\[ \omega_{p} \]

and the primary spin frequency. Since these frequencies change, assuming a constant tidal ratio across the entire evolutionary history is a zeroth-order assumption likely to be violated; however, we defend it as an approximation by noting that the equilibrium semimajor axis defined in Equation (14) is only weakly dependent on the tidal ratio.

\[ a_{eq} \propto X^{1/2} \]

and the postequilibrium asymptote is not dependent on the tidal ratio.

Figure 7 shows how the initial semimajor axis for each system depends on the tidal ratio. There is a one-to-one relationship between the tidal ratio and the synchronization (initial) semimajor axis for each system; as the tidal ratio increases, the initial semimajor axis increases.

From Figure 7, some intuition about the tidal ratio can be gleaned if we assume that the wide asynchronous binaries formed from this expanding singly synchronous mechanism. Before we begin to speculate, it is important to remember that it is very unlikely that the tidal ratio would have the same value across all systems. Likely it will depend on the absolute sizes of the binary members, their compositions, and the relevant frequencies: mean motion, primary spin rate, and secondary libration frequency. Since these frequencies change, assuming a constant tidal ratio across the entire evolutionary history is a zeroth-order assumption likely to be violated; however, we defend it as an approximation by noting that the equilibrium semimajor axis defined in Equation (14) is only weakly dependent on the tidal ratio.

Figure 7 shows that if \( X \sim 0.1 \), then 10 out of 12 systems start within the 2\( R_p \) to 17\( R_p \) range and expand to their observed semimajor axes. The two that do not are both tight systems (51356 (2000 RY76) and 1717 Arlon). And in general, if 0.01 < \( X < 1 \), then most systems start within the appropriate range and expand to their circulation semimajor axes. Since \( X \) is the ratio of the normalized strength of the libration tide to the circulation tide, then we might expect the libration tide to be weaker because it is not a constant tide moving through a body like the circulation tide. As described before, the libration tide waxes and wanes as the rotation rate of the secondary speeds up and slows down relative to the mean motion, and so \( X < 1 \) may be a naturally expected outcome.

It is unclear what the fraction of expanding singly synchronous systems that desynchronize is relative to the total number that expand out toward the Hill radius. Those asteroids that do not desynchronize before the Hill radius would be considered asteroid pairs after solar perturbations disrupt the mutual orbit. These are not the asteroid pairs commonly found (Pravec et al. 2010). Those pairs are created directly from the chaotic and inefficient process of forming a stable binary after a rotational fission event. Jacobson & Scheeres (2011a) estimates that only \( \sim 8\% \) of systems undergoing rotational fission result in a stable binary; the rest form asteroid pairs, some with more than one member. Of that small minority that form stable binaries, nominally half will have expansive BYORP torques and expand toward the Hill radius and disruption. This would create a population of asteroid pairs with one rapidly and one slowly rotating member, but only one such system for every 23 asteroid pairs that obey the relationship in Pravec et al. (2010). This ratio will be a difficult observation to make.

We have shown that it is possible to get from a stable binary formed from a rotational fission event to a wide asynchronous binary. The expanding singly synchronous mechanism works, but is it plausible? Part of its plausibility rests with determining the proper value of the tidal ratio, which will take significant future work. Let us assume that the tidal ratio takes a value that is consistent with the expanding singly synchronous mechanism; then the binary evolves to a wide asynchronous state, but does it do so on a reasonable timescale?

3.2. Evolutionary Phases and Timescales

The expanding singly synchronous mechanism can be broken into three evolutionary phases: first, the tidal synchronization of the secondary; second, the BYORP effect-driven expansion of the orbit; third, the YORP effect rotational acceleration of the secondary. For this mechanism to be plausible, wide asynchronous binaries must evolve through all three
The synchronization timescale is calculated by linearly adding the tidal and YORP effect torques (Goldreich & Sari 2009; Scheeres 2007b):

\[
\tau_{\text{synch}} = \left[ \frac{15k_p\omega_d^2}{4Qd_{\text{init}}^6} + \frac{Y_rH_2}{2\pi\rho R_p^2q^{2/3}} \right]^{-1} \left( \frac{\omega_d}{1.2} - \frac{\omega_d}{v^{1/2}d_{\text{init}}^{3/2}} \right).
\] (16)

where each secondary is assumed to be initially rotating near its surface disruption limit (1.2\(P_D\)), consistent with Jacobson & Scheeres (2011a), and the YORP coefficient \(Y_r\) is a dimensionless number that quantifies the asymmetry of the body. The YORP effect is a sum of radiative torques similar to the BYORP effect, but the lever arm of each torque is from the radiating or irradiated surface element to the body’s center of mass rather than the barycenter of the mutual orbit (Rubincam 2000). The YORP effect can torque the secondary in the same direction (assistively) or opposite direction (resistively) as the tidal torque. These two scenarios, along with the no-YORP effect scenario, are shown in Figure 8. The tidal parameters \(Q/k_p\) are estimated from the tidal–BYORP effect equilibrium by assuming a BYORP coefficient of \(B_r = 2 \times 10^{-7}\) (McMahon & Scheeres 2010a; Jacobson & Scheeres 2011c).

From this analysis, it becomes clear that there are three important regimes. First, there are the asynchronous binaries with semimajor axes \(a \lesssim 8R_p\). In this regime, tides generally dominate the evolution of the secondary. Not only do tides synchronize each system, but they do so quickly, in typically less than a million years. In the second regime, the semimajor axes range between \(8R_p\) and \(17R_p\). The YORP effect and tidal torques are nearly the same. Resistive YORP torques can prevent synchronization within this range. Jacobson & Scheeres (2012) concluded that this resistive scenario is responsible for the observed tight asynchronous binary population. Typical timescales from synchronization are between a million and a hundred million years. Last, for the widest semimajor axes, \(a \gtrsim 17R_p\), the YORP effect torques are typically much stronger than the tidal torques, with the caveat that very symmetric secondaries could have nearly nonexistent YORP coefficients.

Wide asynchronous binaries that form directly from rotational fission exist in this range. These last two regimes can also be seen in Figure 1, where the torques are directly compared for each system’s observed semimajor axis.

Given the observed wide asynchronous candidates, the analysis above, and the analysis of the tidal ratio \(X\) in Section 3.1, it seems likely that the wide asynchronous progenitors synchronize in the first and second regimes, \(a \lesssim 17R_p\), and then desynchronize in the second and third regimes, \(a \gtrsim 8R_p\).

If this is so, then all three near-Earth asteroid candidates synchronized in \(\lesssim 10^6\) yr, and the other main belt candidates likely synchronized in \(\lesssim 10^8\) yr. The only exception is if the YORP effect is resistive, in which case for each YORP coefficient, there exists a semimajor axis above which the system will never synchronize. All observed tight asynchronous binary systems exist at semimajor axes large enough to be consistent with being greater than this special semimajor axis (Jacobson & Scheeres 2012). It is possible that systems such as 1717 Arlon and 51356 (2000 RY\(_7\)) have more in common with the tight asynchronous population than the other wide asynchronous candidates. They may have formed directly from rotational fission. Future work will focus specifically on the role of the YORP effect in binary asteroid systems, bringing all of these ideas together.
The second evolutionary phase is the orbit expansion. This is simply the numerical integration of Equation (2):

\[ \tau = \int_{a_{\text{init}}}^{a_{\text{obs}}} \left( \frac{Q}{k_p} \right) a^{11/2} \frac{d a}{A_T + a^2 A_B} \tag{17} \]

where the linear roles of the tidal coefficients \( Q \) and \( k_p \) are highlighted because they are the major source of uncertainty. The expanding singly synchronous timescales are shown in Figure 9. All three near-Earth asteroid candidates expand in \( \lesssim 10^6 \) yr, and the other main belt candidates expand in \( \lesssim 10^8 \) yr.

The third evolutionary phase is the YORP effect rotational acceleration of the secondary. In some sense, this is the complement of the first phase. However, instead of driving toward or away from synchronization, the YORP effect always drives the secondary away from synchronization. In fact, during the expansion phase the YORP effect is always active on the secondary and imparts an angular offset on the libration state of the secondary; however, this offset is very small with the exception of a nearly oblate secondary (Jacobson & Scheeres 2011b). This offset does determine from which side of the libration potential the secondary exits. If the YORP effect is “assistive” prior to synchronization, then the YORP torque will rotationally decelerate the secondary after desynchronization. After the angular momentum of the secondary has been sufficiently drained by the YORP effect, it would be possible for small impacts to knock the body into a non-principal-axis rotation state (Henych & Pravec 2013).

None of the observed wide asynchronous binaries have secondary periods consistent with a sub-Keplerian rotation rate or non-principal-axis rotation; however, observing such a slow (hundreds of hours) period or a period with multiple components is a considerable challenge. But because of the tangential YORP effect (Golubov & Krugly 2012), we expect most secondaries to have a “resistive” YORP effect torque prior to synchronization. Thus, they will rotationally accelerate after desynchronization. This sequence of events is consistent with all of the confirmed wide asynchronous binaries, assuming that the spin and orbit poles are all aligned. Regardless of the direction of the YORP effect, mutual body tides resist the YORP effect torque after desynchronization, and the significant difference between the first and third phases is the orbit expansion in the second phase. At a much wider orbit, the tides are significantly weaker.

Figure 10 shows the time necessary for the YORP effect to rotationally accelerate the secondary to its observed rotation rate, \( \omega_{\text{obs}} \), as a function of the YORP coefficient of the secondary, \( Y_s \). Typical values are \( \sim 10^{-2} \) (Scheeres 2007b). For systems without an observed rotation rate, we use the average rate of 5.5 hr. The timescale is

\[ \tau_{\text{YORP}} = \left[ \frac{Y_s H_{\odot}}{2 \pi \rho R_p^2 q^{2/3} \left[ 15 k_p \omega_0^2 \right]^{-1}} - \frac{\omega_{\text{obs}} - \omega_d}{\nu^{1/2} a_{\text{obs}}^{3/2}} \right] \tag{18} \]

where the tidal parameters \( Q/k_p \) are estimated from the tidal–BYORP effect equilibrium by assuming a BYORP coefficient of \( B_t = 2 \times 10^{-2} \) (McMahon & Scheeres 2010a; Jacobson & Scheeres 2011c).

Looking at Figure 10, the tight systems (51356 (2000 RY76), 153591 (2001 SN263), and 1717 Arlon) require large YORP coefficients in order to rotationally accelerate their secondaries to their observed (or, in the case of 51356 (2000 RY76), assumed) rotation rates. These systems also stood out in Figure 1. This suggests that these are systems are being tidally locked rather than accelerating away from synchronicity. Since all have semimajor axes \( a < 17 R_p \), they are candidates for direct formation by rotational fission or, in the case of triple system 153591 (2001 SN263), something more exotic. The 317 Roxane system needs to have a nonnegligible YORP coefficient in order to rotationally accelerate away from synchronicity. The other systems do not require particularly large YORP coefficients to be driven by the YORP effect away from synchronicity to their observed (or assumed) rotation rates in appropriate amounts of time.

Finally, we consider the combined timescales of all three phases. This is difficult to do without having measurements of the secondary YORP coefficients for each system, so we explore two scenarios: resistive and assistive, assuming a YORP coefficient for the secondary of \( Y_s = 10^{-2} \) in both cases. Then it is a simple sum of the timescales associated with each phase. The results are shown in Figure 11. We do not include 51356 (2000 RY76), 153591 (2001 SN263), or 1717 Arlon; each of these systems is likely synchronizing and not a candidate for the expanding singly synchronous mechanism.
BYORP effect. In their model, the secondary has not evolved significantly outward when it loses synchronous rotation (all examples are within $a < 8R_p$ and only evolve outward for a fraction of an $R_p$), a significantly different prediction from that of the currently discussed hypothesis.

It is crucial to note, however, that their model has significant differences from our comprehensive model. First, there is a missing term in the second-order gravity expansion, which will affect the fundamental resonances present in the system. Furthermore, the model used for the BYORP effect may be too simple. The BYORP effect is a challenge to model numerically because it is very slow compared with a single orbit. McMahon & Scheeres (2010b) used averaging theory to derive the secular evolution of the mutual orbit (we use this model above), while Čuk & Nesvorný (2010) made a direct, nonaveraged application of Gauss’s planetary equations but, to observe changes in a reasonable number of orbits, were forced to model the BYORP effect as a torque with a magnitude many times stronger than those predicted from the theory, which speeds up the evolution of the system. Last, the model presented above includes tides, which have been shown to be significant for the evolution of binary systems (Jacobson & Scheeres 2011c) and the YORP effect, neither of which were considered by Čuk & Nesvorný (2010). Because of these differences, the evolutionary mechanisms in that paper seem to be fundamentally different from our current model, which considers all currently identified evolutionary effects.

4.1. Alternative Formation Mechanisms

Right after the discovery of the first wide asynchronous binary system (1998 ST27; Benner et al. 2003), Durda et al. (2004) proposed a binary formation mechanism that would create wide asynchronous binaries from the debris of large impacts. EEBs form when two blocks of ejecta from a large impact event are on escape trajectories from the target asteroid but are moving slowly relative to each other. If they are moving slower than their relative escape velocities and have the right angular momentum relative to their mutual center of mass, then they enter a bound orbit. According to simulations, the EEB formation mechanism can make hundreds of widely separated binaries from a single impact. These simulated binaries can match the observed separation distances of the wide asynchronous population; however, EEBs are predicted to be low and high mass ratio binary systems. No high mass ratio wide asynchronous binaries have been confirmed (Durda et al. 2010); 1717 Arlon may be the first (P. Pravec, 2013, private communication).

Most strikingly, the EEBs are not expected to have particular spin states, yet six of the nine primaries of the wide asynchronous candidate systems have spin periods between $P_d$ and $2P_d$ (shown in Table 1), indicating near-critical rotation rates. They are piled up at rapid rotation rates similar to the primaries of the singly synchronous binary systems (Pravec et al. 2008). Of the other three candidates, only the primary of 317 Roxane is clearly not a rapid rotator; the primary of 1717 Arlon is near rapid rotation, and the primary rotation period of 17246 (2000 GL74) has not been measured.

\[ \frac{3}{2\pi^2} \left( \frac{A}{2} + \frac{B}{2} - C \right) \] (19)

in their notation.
These rapid primary rotation rates are consistent with formation from YORP-induced rotational fission (Margot et al. 2002; Scheeres 2007a). Polishook et al. (2011, p. 167) came to a similar conclusion when they studied a subsample of the wide asynchronous candidates included in this study. They argue,

The rotation periods of four out of the six objects measured by our group and others and presented here show that these suspected EEBs have fast rotation rates of 2.5−4 hr. Because of the small size of the components of these binary asteroids, linked with this fast spinning, we conclude that the rotational-fission mechanism, which is a result of the thermal YORP effect, is the most likely formation scenario. Moreover, scaling the YORP effect for these objects shows that its timescale is shorter than the estimated ages of the three relevant Hirayama families hosting these binary asteroids. Therefore, only the largest (D ∼ 19 km) suspected asteroid, 317 Roxane, could be, in fact, the only known EEB.

We note only that 317 Roxane is also consistent with the expanding singly synchronous hypothesis put forward in this paper.

There are two other formation mechanisms besides the EEB hypothesis that need to be addressed. First, simulations show that stable binaries can be formed at semimajor axes larger than 5\(R_p\), Jacobson & Scheeres (2011a) found that stable binaries with semimajor axes up to 17\(R_p\), as shown in Figure 12, could form after a rotational fission event. Binary stabilization occurs after a short (<100 yr) period of significant orbit transformation as a result of spin–orbit coupling. Jacobson & Scheeres (2011a) found a relationship, when binary systems do stabilize, between the semimajor axis \(a\) and the eccentricity: \(e = 0.284 \ln(a - 0.701)\). The \(a\)–\(e\) relationship is due to the near-conservation of both energy and angular momentum across all three reservoirs: the orbit and the two spin states. This conservation is approximate because solar tidal perturbations on the orbit can act as a small source or sink. Extrapolating this relationship out to an eccentricity of nearly 1, the widest binaries created directly from rotational fission are limited to obtaining a semimajor axis of \(\lesssim 34R_p\).

Creation of wide binaries directly from rotational fission is rare because of this spin–orbit coupling, since increased eccentricity is often associated with further increased spin–orbit coupling, leading to positive feedback. Most systems (∼92%) do not stabilize and eventually disrupt or collide (see Jacobson & Scheeres 2011a for more details). From these simulations, we can divide the wide asynchronous binaries into three groups: the widest, \(a > 34R_p\) (1509 Esclangona, 4674 Pauling, 17246 (2000 GL74), and 22899 (1999 TO14)), which could not be formed directly from a rotational fission event; the tightest (1717 Arlon, 51356 (2000 RY76), and 1998 ST 27), which are within the formation range of simulated stable binaries, \(a \lesssim 17R_p\); and the intermediate, \(17R_p \lesssim a \lesssim 34R_p\) (317 Roxane and 32039 (2000 JO23)), which according to a simple fit of the simulated data could be a direct outcome of rotational fission. This intermediate range of semimajor axes corresponds to very high initial eccentricities if these binaries formed directly from rotational fission.

Are these initially high eccentricities subsequently damped by tides? No, and we know this from calculating the tidal evolution of each observed wide asynchronous candidate by simultaneously integrating the tidal damping equation and the tidal semimajor axis expansion equation, which are identical to Equations (1) and (2) but with no second, BYORP effect, term. We display and discuss these equations in Section 2.1 and also in Appendix B. Figure 13 shows the eccentricity damping due to mutual body tides for each of the wide asynchronous systems at their observed semimajor axes, but with eccentricities given by the fitted \(a\)–\(e\) relationship found for systems formed directly from rotational fission as shown in Figure 12 (i.e., each model system is identical to its observed parameters except for the eccentricity). Only wide asynchronous systems with semimajor axes \(a \lesssim 34R_p\) are shown, since according to the \(a\)–\(e\) relationship, if \(a \gg 34R_p\) then those systems cannot form directly from rotational fission—their stable orbits would have eccentricities greater than 1.

For each of the wide asynchronous candidates, no significant tidal circularization of their orbits occurs within the relevant timescales for near-Earth asteroids (∼10 Myr) or main belt asteroids (∼100 Myr). Many do not damp significantly within the age of the solar system. Since uncertainty in \(k_p/Q\) translates linearly into an uncertainty in the damping timescale, if these systems are expected to damp, then the assumed values from the tidal–BYORP equilibrium are incorrect by more than two orders of magnitude. Alternatively, if these systems are
observed to have more circular orbits, then they did not form directly from rotational fission. The proposed expanding singly synchronous mechanism predicts more circular orbits than the direct formation and EEB mechanisms. The EEB mechanism can create circular orbits, but it preferentially creates a higher number of eccentric orbits (Dürda et al. 2004). Examining the mutual orbits of wide binaries will distinguish these formation mechanisms.

The last alternative formation mechanism is not so much a formation mechanism as an alteration mechanism. Planetary flybys can strongly change the mutual orbits of binary asteroids (Fang & Margot 2012a). These flybys can expand the orbit and usually incline the orbit plane, excite the eccentricity, or both. Strong changes to the mutual orbit can desynchronize the secondary (Jacobson & Scheeres 2012). Within the wide asynchronous candidate population, the orbit of 1998 ST27 crosses the orbit of every terrestrial planet, while the orbits of 32039 (2000 JO23) and 51356 (2000 RY76) barely intersect the orbit of Mars. The other six wide asynchronous candidates are not planet crossing.

We summarize the three alternative formation mechanisms as follows. (1) The EEB hypothesis could produce each of these binary systems, but it would not explain why two-thirds have rapidly rotating primaries consistent with formation by rotational fission. Also, the mass ratio distribution does not match the predictions, and all EEBs should be associated with a large asteroid family or collision. (2) Direct formation from rotational fission could produce this two-thirds of the wide asynchronous candidates, but it makes strong predictions regarding the eccentricity of these systems: they should be high. (3) Planetary flybys could alter already existing binary systems formed by rotational fission into wide asynchronous binary systems. Three of the nine systems have planet-crossing orbits, including one that crosses all four terrestrial planets. These formation mechanisms may be able to explain the existence of some of the wide asynchronous candidates, but four candidates (1509 Esclangona, 4674 Pauling, 17246 (2000 GL74), and 22899 (1999 TO14)) are not planet crossers and have very wide orbits and low mass ratios, and three have rapidly rotating primaries. Only the proposed expanding singly synchronous mechanism can explain these four systems and, possibly, the other five candidates. Overall, these formation mechanisms are not mutually exclusive, and some systems could be created by means of one of these already proposed mechanisms.

4.2. Individual Systems

If we examine each system in light of the four mechanisms that could produce wide asynchronous binaries—the expanding singly synchronous mechanism, EEBs (Dürda et al. 2004), direct formation from rotational fission (Jacobson & Scheeres 2011a), and planetary flybys (Jacobson & Scheeres 2012)—then we may be able to assess the likelihood of each mechanism for each binary. We have done some of this work already throughout the paper.

We identified in Section 4.1 the four systems (1509 Esclangona, 4674 Pauling, 17246 (2000 GL74), and 22899 (1999 TO14)) that could not have formed through the other three mechanisms. All three can successfully form by mean of the expanding singly synchronous mechanism and do so on a timescale consistent with their lifetimes. The expanding singly synchronous mechanism predicts that the mutual eccentricities of the orbits should be low, although not necessarily zero. It predicts that the unmeasured primary period of 17246 (2000 GL74) is rapid, like the other three systems. If it is not rapid, there is the possibility that 17246 (2000 GL74) is a doubly synchronous binary, since both periods are unknown. However, it has a low mass ratio and would be the lowest mass ratio doubly synchronous system observed. Therefore, we believe it unlikely to be doubly synchronized, since tidal locking of the secondary is much faster than tidal locking of the primary in low mass ratio systems (Jacobson & Scheeres 2011a). Only 1509 Esclangona has a measured asynchronous secondary. The expanding singly synchronous mechanism predicts that the other three should be asynchronous as well.

There exists the possibility that the secondaries of these three systems are synchronous. In that case, it is likely that the orbit is still expanding because of the BYORP effect. The expanding singly synchronous mechanism requires that the initial synchronization semimajor axis be within a certain range, or else the system will expand past its Hill radius. This expansion process takes time, especially in the main belt (> millions of years, according to Figure 9), and so discovering an expanding singly synchronous binary before it desynchronizes or reaches the Hill radius is possible. In fact, among the wide asynchronous candidates, all of those without measured secondary periods could be in this state, including 317 Roxane, 4674 Pauling, 17246 (2000 GL74), 22899 (1999 TO14), and 51356 (2000 RY76). Alternatively, systems such as 185851 (2000 DP107), which is an outlier among synchronous systems when considering the tidal–BYORP effect equilibrium, may fall into this category of currently expanding singly synchronous systems (Jacobson & Scheeres 2011c). Considering that the librations expected for expanding singly synchronous systems can be quite large, this motivates future observations of the libration states of synchronous secondaries.

The 32039 (2000 JO23) system has a rapidly rotating primary, so it is unlikely to be an EEB, but it does barely cross the orbit of Mars, so a Martian flyby could have modified its orbit. A flyby consistent with expanding the orbit to its observed size, \( a \sim 32 R_p \), from the frequently observed tight \( a \sim 4 R_p \) would be a rare event (Fang & Margot 2012a). The system is a confirmed wide asynchronous binary and so most likely formed by the expanding singly synchronous mechanism; however, it could also have formed directly from rotational fission. The test will be the eccentricity of the system. Direct formation would leave a very high eccentricity, \( e \sim 0.98 \) (see Figure 12), that would not be damped within the age of the solar system (see Figure 13), while the expanding singly synchronous mechanism should produce a much lower eccentricity.

The 1998 ST27 system would have formed very quickly if by the expanding singly synchronous mechanism, but its formation from this mechanism is unlikely, since the YORP effect would continue to accelerate the secondary until it was rapidly rotating, because it has such a small heliocentric orbit and absolute size. Since the heliocentric orbit of this system crosses the orbits of every terrestrial planet, it is more likely that this system has been excited by a planetary flyby and is now undergoing tidal synchronization. The system has an observed eccentricity \( \geq 0.3 \), which is consistent with formation from a flyby (Benner et al. 2003; Fang & Margot 2012a). Alternatively, it could have formed directly from rotational fission, but for such a wide binary, theory would predict an eccentricity \( \sim 0.77 \) (see Figure 13). Its eccentricity should not have damped significantly (see Figure 13). More precise observations of the orbit of 1998 ST27 would determine which mechanism is more likely, since very rare and close flybys are required in order to create such large eccentricities (Fang & Margot 2012a).
We find that two of the tightest of the wide asynchronous candidates (1717 Arlon and 51356 (2000 RY76)) could only have formed by means of the expanding singly synchronous mechanism if the tidal ratio $X$ is within a narrow range that does not overlap much with all of the other systems (see Figure 7). Furthermore, in Section 3.2 we find that 317 Roxane, 1717 Arlon, and 51356 (2000 RY76) require large YORP coefficients in order to rotationally accelerate to their observed rotation periods. None of these three systems crosses any planetary orbits. Unlike the other two, 51356 (2000 RY76) has a rapidly rotating primary, and so if its secondary is asynchronous, it likely formed directly from rotational fission. If so, this system would have initially had an eccentricity $e \sim 0.60$ and evolved according to Figure 13. This system is also a good candidate for being in the expanding singly synchronous state, if the secondary is discovered to be synchronous.

YORP-induced rotational fission is less effective for bodies as large as 317 Roxane (Jacobson et al. 2013), and 317 Roxane and 1717 Arlon are not associated with rapid rotation. Thus, neither is likely to have formed by a rotational-fission mechanism. Polishook et al. (2011) already concluded that 317 Roxane may be an EEB (Durda et al. 2004). The 1717 Arlon system may also have a high mass ratio. High-mass-ratio systems that start tight should be doubly synchronized as a result of tides, which suggests that 1717 Arlon formed at its current wide separation. All of this evidence supports a formation by the EEB mechanism. Both are near the Flora family, although neither Nesvorný (2012) nor Masiero et al. (2013) recognize it as a family member. Despite this lack of a confirmed relationship with a large collision event, this is the only consistent mechanism.

We hesitate to comment too much about individual triple systems, since the dynamics are so much richer. We wanted to carry them along in the analysis to support possible future work, but there is much to consider outside of the dynamics of just the outer pair. It is notable that Vachier et al. (2012) have identified possibly significant eccentricity in the outer pair of 3749 Balam. If real, this may rule out a simple story of binary creation, expansion, and then a second binary formation event within the outer pair. This may be an argument for a scenario as proposed in Jacobson & Scheeres (2011a), which suggested that triple systems can be formed in a single rotational fission event, although this did not occur among the numerical simulations, perhaps because of inadequate statistics. Regardless, much more work needs to be done to understand small triple asteroid systems.

5. CONCLUSIONS

We have developed a new formation mechanism to create the wide asynchronous binary population. A stable binary forms directly from rotational fission. The secondary synchronizes as a result of tides and then begins to librate. That libration is damped by tides; if the BYORP effect is expansive, the system widens and an adiabatic invariance adds energy to the libration state. Later the libration ceases as the secondary begins to circulate. This turns off the BYORP effect, essentially stranding the system on a wide orbit. The YORP effect rotationally accelerates the secondary. These events naturally follow one another and lead to the observed properties of the wide asynchronous population.

We are grateful for the Asteroid Lightcurve Database (Warner et al. 2009), which is maintained by Brian Warner and colleagues, and the binary asteroid parameters data at http://www.asu.cas.cz/~asteroid/binastdata.htm, which are compiled according to methods described by Pravec & Harris (2007) and maintained by P. Pravec and colleagues. Both databases were incredibly useful for preparing this work. S.A.J. would also like to acknowledge the NASA Earth and Space Science Fellowship, which supported him throughout graduate school.

APPENDIX A

DERIVATION OF THE TORQUE RATIOS IN FIGURE 1

In Figure 1, we compared two sets of torques. In the top panel, we show the torque on the orbit from the BYORP effect divided by the torque on the orbit from the tides on the primary due to the secondary:

$$\frac{\Gamma_B}{\Gamma_T} = \frac{H_0 a^7}{2\pi \rho_0 a_0^2 R_p^2 q^{7/3}} \left( \frac{B_i Q}{k_p} \right),$$  \hspace{1cm} (A1)

where $H_0 = F_{\odot} (a_0^2 \sqrt{1 - e_\odot^2})$ is the heliocentric parameter, including the solar radiation constant $F_{\odot}$, the semimajor axis $a_0$, and the eccentricity $e_\odot$, $a_\odot = \sqrt{4\pi G M_\odot}$ is the surface disruption spin limit, $\rho$ is the density, $a$ is the semimajor axis measured in primary radii $R_p$, and $q$ is the mass ratio (Goldreich & Sari 2009; McMahon & Scheeres 2010a). The collection of tidal and BYORP effect parameters $B_i Q / k_p = 2557 (R_p / 1 km)$ is determined by assuming that the singly synchronous binary population occupies a tidal–BYORP effect equilibrium. The rest of the values are measured directly and reported in Tables 1 and 2. The errors are dominated by the uncertainty in the tidal and BYORP effect parameters. We show two orders of magnitude for the estimated uncertainty in Figure 1.

In the bottom panel, we show the torque on the rotation state of the secondary from the YORP effect divided by the torque on the spin of the secondary from the tides on the primary due to the primary:

$$\frac{\Gamma_Y}{\Gamma_T} = \frac{2H_0 a^6}{15 \pi \rho_0 a_0^2 R_p^2 q^{7/3}} \left( \frac{Y_i Q}{k_p} \right),$$  \hspace{1cm} (A2)

where the YORP coefficient $Y_i$ is now a significant unknown (Scheeres 2007b). It is the same order of magnitude as the BYORP coefficient, $\sim 10^{-2}$. We show two orders of magnitude in uncertainty for the estimated ratios in Figure 1.

APPENDIX B

PARAMETERIZATION OF THE SEMIMAJOR AXIS AND ECCENTRICITY EVOLUTION EQUATIONS

The time evolution of the eccentricity and semimajor axis due to tides and the BYORP effect are the linear additions of each effect, since they are independent of each other. The tidal equations are identical to those found in Goldreich & Sari (2009), and the BYORP effect equations are identical to those found in McMahon & Scheeres (2010a). We have rearranged them in terms of a limited number of parameters for convenience throughout the paper. The semimajor axis and eccentricity evolution equations are

$$\dot{a} = \frac{k_p}{Q} \left( A_T + a^7 A_B \right) a^{-11/2},$$

$$\dot{e} = -\frac{k_p}{Q} \left( A_T L + a^7 A_B \right) ea^{-13/2}. \hspace{1cm} (B1)$$
The newly introduced parameters are
\[ A_T = \frac{3\omega_d q}{\sqrt{1/2}}, \quad A_B = \frac{3H_0}{2\pi\omega_d R_p^2 q^{1/3} \sqrt{1/2}} \left( \frac{B_s Q}{k_p} \right), \]
\[ L = \frac{28k_s - 19k_p q^{1/3}}{8k_p q^{1/3}} = \frac{7k_s}{2k_p q^{1/3}} - \frac{19}{8}, \quad (B2) \]
where \( \omega_d = \sqrt{4\pi\rho G^3/\tau} \) is the surface spin disruption rate, \( \tau \) is the density, \( G \) is the gravitational constant, \( q \) is the mass ratio, \( \nu = (1 + q)^{-1} \) is the fraction of the total mass in the primary, and \( H_0 = F_0/(a_0^2 \sqrt{1 - e_0^2}) \) contains the heliocentric dependencies including the solar radiation constant \( F_0 \), the semimajor axis \( a_0 \), and the eccentricity \( e_0 \). Similarly to Jacobson & Scheeres (2011c), the moments, \( k_\nu \), of inertia are related:

\[ \nu = q^3/\nu^2 (1 - 2q)^{-3/2}, \quad A = \frac{\nu^2 - 1}{\nu - 1} = \frac{\nu}{(1 - 2q)^{3/2}}, \]

**APPENDIX C**
**DERIVATION OF THE ADIABATIC INVARIANCE BETWEEN THE SEMIMAJOR AXIS AND THE LIBRATION AMPLITUDE**

We here use a simple model of a sphere and a triaxial ellipsoid in a mutual planar orbit (i.e., all spin and orbit poles are aligned). The primary is a sphere with radius \( R_p \) and mass \( M_p \). The secondary is a triaxial ellipsoid with mass \( M_s = qM_p \). \( q \) is the mass ratio. The secondary is spinning in its relaxed state about the direction of the shortest body semiaxis, \( \hat{z}_s \), and \( \hat{y}_s \) are the longest and intermediate body semiaxis directions, respectively. Thus, the moments of inertia are related:

\[ I_s = C I_s \geq I_s' = B I_s = A I_s, \]

where the inertia factor of the secondary is \( I_s = M_s R_s^2 \). Therefore, the mean radius of the secondary is \( r_s = \sqrt{(A + B - C)(B + C - A)(C + A - B)}/q^{1/3} R_p \).

The coordinate tracking the instantaneous separation distance between the centers of mass of the two bodies is \( r \), where \( r \) and \( \dot{r} \) are measured in primary radii \( R_p \) and primary radii per unit time, respectively. The angle between the instantaneous line connecting the two mass centers and an arbitrary fixed line in inertial space is \( \theta \). The spin angle of the \( n \)th body relative to the line connecting the centers is \( \phi_n \). Since the potential of the sphere is independent of its orientation, the relative spin angle of the primary sphere, \( \phi_p \), does not need to be tracked.

Given these definitions, the free kinetic and potential energies of the system are

\[ T = \frac{1}{2} \frac{\nuq}{r} \left[ r^2 + r^2 \dot{\theta}^2 + \frac{CT}{vq} \left( \dot{\theta} + \dot{\phi}_n \right)^2 \right], \quad (C1) \]
\[ V = -q \frac{\nu}{r} \frac{\omega_d^2}{r} \left[ 1 - \frac{\nu}{4qr^2} (A + B + C - 3C(1 + S \cos 2\phi_n)) \right]. \quad (C2) \]

Since \( I_p = M_p R_p^2 \) is the inertia factor of the primary, \( \nu = I_p/I_p = q^{3/2} \) is the ratio of the inertia factors, \( \nu = M_p/(M_p + M_s) = (1 + q)^{-1} \) is the fraction of the total mass in the primary, \( \omega_d = \sqrt{4\pi\rho G^3/\tau} \) is the surface spin disruption limit for a sphere of density \( \rho \), \( G \) is the gravitational constant, and \( S = (B - A)/C \) is the shape factor of the secondary. From the definition of the moments, \( S = 0 \) for an oblate secondary, \( S > 0 \) for a prolate secondary, and \( S \) increases with increasing protleness of the secondary but is limited to be less than 1. Similar energy equations are derived in Scheeres (2009).

Using the Lagrangian \( L = T - V \) and the three generalized coordinates \( (r, \dot{r}, \dot{\theta}) \), we transform to the Hamiltonian formulation of the system. The generalized momenta for each coordinate are

\[ p_r = \nuq I_p \dot{r}, \quad p_\theta = \nuq I_p r^2 \dot{\theta} + CI_s (\dot{\theta} + \dot{\phi}_n), \quad p_\phi = CI_s (\dot{\theta} + \dot{\phi}_n). \quad (C3) \]

The Hamiltonian \( H \) is

\[ H = \frac{p_\theta^2}{2CI_s} + \frac{1}{2\nuq I_p} \left[ p_r^2 + \left( p_\theta - p_{\phi_n} \right)^2 \right] + V. \quad (C4) \]

The instantaneous equations of motion for the system can be determined from this Hamiltonian for the generalized coordinates given above, but we can reduce the number of canonical pairs by introducing an integral of the motion. From the equations above, it is clear that the coordinate \( \theta \) is ignorable: \( \partial L/\partial \theta = 0 \). This is the conservation of angular momentum, and the conserved quantity is

\[ K = \frac{\partial L}{\partial \dot{\theta}} = \nuq I_p r^2 \dot{\theta} + CI_s (\dot{\phi}_n + \dot{\theta}) = p_\theta. \quad (C5) \]

The generalized momentum for the relative spin angle of the secondary, \( p_{\phi_n} \), can then be expressed solely in terms of its coordinate:

\[ p_{\phi_n} = CI_s \dot{\phi}_n \left[ 1 + \frac{CT}{vq r^2} \right]^{-1}. \quad (C6) \]

Considering the system that we wish to study, the changes in the instantaneous separation distance \( \delta r \) are very small compared with the instantaneous separation distance \( r \), so \( \delta r \ll r \). We make the approximation that \( r \approx 0 \). This implies that the orbit is circular, so \( r \approx a \), where \( a \) is the semimajor axis measured in radii of the primary. The semimajor axis will change over time as a result of mutual body tides and the BYORP effect, but that change is very slow compared with the orbit or libration periods, so we assume \( r = 0 \). Therefore, in the Hamiltonian system the generalized momenta \( p_r \) is zero. Furthermore, when

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15 This value differs slightly from that reported in Jacobson & Scheeres (2011c), since it uses updated values from the binary parameter release provided by P. Pravec and collaborators at http://www.ias.ox.ac.uk/~asteroid/binndata.htm using methods described in Pravec & Harris (2007) and applies a Lambertian correction (McMahon & Scheeres 2010a).
δr ≪ r the instantaneous rotation of the line connecting the two mass centers relative to inertial space is then the mean motion \( \dot{\theta} = n \). This approximation means that Kepler’s third law is valid: \( \nu a n^2 = \omega_2^2 \).

Collecting constant terms, the single degree of freedom Hamiltonian is

\[
H = H_0 + H_1 p_{\phi_s}^2 - H_2 \cos 2\phi_s ,
\]

where

\[
H_0 = -\frac{\nu q I_2 \omega_2^2}{a^2} \left[ 1 - \frac{r}{4qa^2} (A + B - 2C) \right] - \frac{K^2}{2
nu q I_2 a^2} (1 + s)^{-1},
\]

\[
H_1 = \frac{1 + s}{2CI_s},
\]

\[
H_2 = \frac{3SCL_s \omega_2^2}{4a^3}
\]

with \( s = CI_s/vqa^2 \) as the secondary perturbation term, which appears throughout the rest of these derivations and in many of the final expressions. This quantity approaches zero as the semimajor axis increases, the mass ratio decreases, and the secondary maximum moment of inertia decreases relative to the primary moment of inertia factor.

The Hamiltonian equations of motion are

\[
\dot{p}_{\phi_s} = -2H_2 \sin 2\phi_s , \quad \dot{\phi}_s = 2H_1 p_{\phi_s} .
\]

Let us identify some features of this system. First, there are stable equilibria at \( \phi_s = (0, \pm \pi) \), and \( p_{\phi_s} = 0 \). This corresponds to the long-axis relative equilibrium described in Bellerose & Scheeres (2008). There are also unstable equilibria at \( \phi_s = \pm \pi/2 \) and \( p_{\phi_s} = 0 \), which correspond to the short-axis relative equilibrium in Bellerose & Scheeres (2008). Second, there is a separatrix that divides the motion of the secondary between libration and circulation. This separatrix goes through the unstable equilibria. The value of the Hamiltonian at the separatrix is

\[
H_s = H_0 + H_2 .
\]

The secondary is on the separatrix if \( H = H_s \), librating if \( H < H_s \), and circulating if \( H > H_s \). Third, when the system is librating, \( H < H_s \), the Hamiltonian can be expressed in terms of the libration amplitude of the secondary, \( \Phi_s \), as it librates about \( \phi_s = 0 \). When the secondary is at the maximum libration angle \( \phi_s = \Phi_s \), the relative spin velocity is \( \dot{\phi}_s = 0 \), so the system is on the separatrix. Therefore, the Hamiltonian can also be expressed as

\[
H = H_0 - H_2 \cos 2\Phi_s = H_0 - 2H_2 \cos^2 \Phi_s .
\]

The second expression for the Hamiltonian clearly shows that the system is librating as long as \( |\Phi_s| < \pi/2 \). When \( |\Phi_s| = \pi/2 \), the system is on the separatrix. This paper explores this dynamical system in this librating regime under the influence of orbit expansion and tides.

The action can be integrated directly using the expression for the conjugate momentum above, but the result is a function of an incomplete elliptical integral of the second kind. It is more useful to transform the action by substituting in the variable \( \chi \) using the trigonometric relation \( \sin^2 \chi = \sin^2 \phi_s / \sin^2 \Phi_s \):

\[
\begin{align*}
J_{\phi_s} &= \frac{32H_2}{H_1} \int_0^{\Phi_s} \sin^2 \Phi_s \left[ 1 - \frac{\sin^2 \phi_s}{\sin^2 \Phi_s} \right] \sin \Phi_s \cos \Phi_s \cos^2 \chi d\chi ,
\end{align*}
\]

\[
d\phi_s = \sqrt{\frac{32H_2}{H_1} \int_0^{\Phi_s} \sin^2 \Phi_s \cos^2 \chi \sin \Phi_s \cos \Phi_s \cos^2 \chi d\chi} .
\]

Now we integrate the action using complete elliptic integrals:

\[
J_{\phi_s} = \frac{4CL_s \omega_2}{a^{3/2}} \sqrt{\frac{S}{1 + s}} G(\sin^2 \Phi_s) \sqrt{\frac{S}{1 + s}} .
\]

The generalized solution \( G(k^2) \) to this integral can be expressed in terms of complete elliptic functions of the first, \( K(k^2) \), and second, \( E(k^2) \), kinds:

\[
G(k^2) = \int_0^{\pi/2} \frac{k^2 \cos^2 x}{\sqrt{1 - k^2 \sin^2 x}} dx = E(k^2) - (1 - k^2) K(k^2) .
\]

**APPENDIX D**

**DERIVATION OF THE LIBRATION GROWTH DUE TO ORBIT EXPANSION**

To obtain a relationship between the time derivative of the semimajor axis and the libration amplitude, we take the time derivative of the square of the adiabatic invariance (Equation (C16)):

\[
\frac{K(\sin^2 \Phi_s) \sin 2\Phi_s}{G(\sin^2 \Phi_s)} \dot{\Phi}_s - \frac{3 + s}{1 + s} \left( \frac{d}{a} \right) = 0 .
\]

The orbit expansion \( \dot{a} \) is due to both tides and the BYORP effect:

\[
\dot{a} = \frac{3k_p \omega_d q}{Q_p a^{13/2}\nu^{1/2}} + \frac{3H_2 B_s a^{1/2}}{2\pi \omega_d R_2^{1/2} q^{1/2} R^{3/2} v^{1/2}} .
\]

It is important to note that this tidal orbit expansion is due not to the libration tides on the secondary but to the circulation tides on the primary. We assume a primary rotating prograde and rapidly (i.e., rotation rate faster than the mean motion).

From here, it is simple to state the time derivative of the libration amplitude due to BYORP-driven and tidally driven orbit expansion:

\[
\dot{\Phi}_s = \frac{3 + s}{1 + s} \left( \frac{3k_p \omega_d q}{Q_p a^{13/2}\nu^{1/2}} + \frac{3H_2 B_s a^{1/2}}{2\pi \omega_d R_2^{1/2} q^{1/2} R^{3/2} v^{1/2}} \right) \times \frac{G(\sin^2 \Phi_s)}{K(\sin^2 \Phi_s) \sin 2\Phi_s} .
\]
We are searching for the energy dissipated within the rotation state of a binary asteroid member due to mutual body tides. To simplify this problem, we will decouple the system and, for the determination of the tidal energy dissipation only, treat each body as a sphere. This derivation follows those given by Wisdom (2004, 2008), and we repeat material to inform the reader. Wisdom derives tidal dissipation induced by various forced librations and nonzero obliquities; we derive the tides for a free libration in a circular orbit below. Deriving these free libration tides for an eccentric orbit requires more than one set of tidal responses as a result of the multiple, nonharmonic forcing frequencies and is left for future work.

The energy dissipated within the interior of a homogenous (constant density) body moving through the gravitational field of a point mass is the work done on each individual element within the body. The work can be expressed as the inner product of the tidal force on that element of the body $F_T$ with the displacement of the element $\delta x$, which is also the instantaneous velocity $v$ of the element over an instant of time $\delta t$: $\delta W_T = F_T \cdot \delta x = F_T \cdot v \delta t$, which we rearrange to express as a work rate done per element.

The tidal force $F_T$ is a negative gradient of the perturbing potential energy $V_T$, which is perturbation tidal potential $U_T$ multiplied by the mass of the element $dm = \rho dV$. The expression for the rate of work done on each element can be integrated over all volume elements of the body to determine the total rate of change in energy:

$$\dot{E}_T = \int \int \int_{\text{Body}} \dot{W}_T \ dV = -\rho \int \int \int_{\text{Body}} v \cdot \nabla U_T \ dV. \quad (E1)$$

Assuming that the body is incompressible, $\nabla \cdot v = 0$, we make a simple substitution using the product rule, $\nabla(U_T v) = v \cdot \nabla U_T$, and use Gauss’s theorem to express this volume integral as a surface integral:

$$\dot{E} = -\rho \int \int \int_{\text{Body}} \nabla(U_T v) \ dV = -\rho \int \int_{\text{Body}} U_T v \cdot n \ dS, \quad (E2)$$

where $dS$ is the area of the particular surface element and $n$ is the unit normal direction to that surface.

Love (1948) determined that the radial displacement height of a surface element $\Delta r = -hU_T / g$ is linearly dependent on the displacement Love number $h = 5/3k$, which is related to the potential Love number $k$ and the delayed tidal potential $U_T$ (delayed because the dissipative response lags the forcing) and inversely dependent on the surface acceleration due to gravity, $g$. The time rate of change of this displacement is, conveniently, $v \cdot n$. The energy dissipation can now be expressed directly as the response of the surface of the body to the tidal potential and its time derivative:

$$\dot{E} = \frac{\rho h}{g} \int \int_{\text{Body}} U_T \frac{d}{dt} (U_T) \ dS. \quad (E3)$$

For the problem at hand, the dissipation occurs in the secondary, so the tidal-raising perturbing potential is that of the primary:

$$U_T = -\frac{\alpha^2 R_s^2}{a^3} P_2 (\cos \alpha), \quad (E4)$$

where $P_2 (\cos \alpha)$ is the second Legendre polynomial and $\alpha$ is the angle at the center of the second perturbed body between the vector from the first to the center of the secondary body to the first, $\mathbf{a}$, and the vector from the center of the second body to the surface element, $s$. The length of each vector is known: $|a| = R_s$ and $|s| = a$, and we determine $\cos \alpha = a \cdot s / (a R_s)$. The vector from the center of the secondary to the center of the primary $\mathbf{a}$ in Cartesian coordinates is, for a circular orbit, $\mathbf{a} = (a \cos nt, a \sin nt, 0)$ with mean motion $n$.

We can consider a number of surface motions that the second body could be making, but for now we only consider free libration of the secondary in a circular mutual orbit. The body is rotating at the rate of the mean anomaly $nt$ and librating with a frequency $\omega_l$ and an amplitude $\Phi_l$. The surface rotation matrix $\mathcal{R}$ is

$$\mathcal{R} = \begin{pmatrix}
\cos (nt - \Phi_l \sin \omega_l t) & -\sin (nt - \Phi_l \sin \omega_l t) & 0 \\
\sin (nt - \Phi_l \sin \omega_l t) & \cos (nt - \Phi_l \sin \omega_l t) & 0 \\
0 & 0 & 1
\end{pmatrix}, \quad (E5)$$

where $\omega_l$ is the libration frequency as before. We expand the rotation matrix to first order in the libration-angle amplitude $\Phi_l$, and from these equations we determine the tidal potential $U_T$:

$$U_T = -\frac{\omega_l^2 R_s^2 a^{2/3}}{16a^3} \left[ 2 - 3P_2^2 + 3 \left(2 + P_2^2\right) \cos 2\theta \\
+ 6 \sin^2 \theta \left( (P_2^2 - 2) \cos 2\lambda + 2P_2 \right) \right]. \quad (E6)$$

To determine the delayed potential, Wisdom (2004, p. 488) says, “The delayed potential $U_T$ is found by replacing $nt$ by $nt + \Delta$ in the expression for $U_T$. This is equivalent to making the substitution $t \rightarrow t + \Delta / n$. For the systems considered by Wisdom (2004, 2008), the tidal forcing frequency is always $n$ or a rational factor of $n$ (e.g., $1/3$). Since we are considering a freely librating body in a circular orbit, the tidal forcing frequency is the libration frequency $\omega_l$. Thus, an appropriate substitution rule is $t \rightarrow t + \Delta t / \omega_l$. Here we have distinguished the tidal phase lags $\Delta_t$ and $\Delta$, since these correspond to the libration frequency and mean motion forcing frequencies, respectively. Likewise, we will distinguish the tidal quality numbers $Q_t$ and $Q$, which correspond to each forcing frequency.

16 Arbitrary shapes could be treated for either the potential of the tide-raising body, if it were expanded in spherical harmonics, or the surface of the secondary, if it were expanded in surface spherical harmonics. This may be future work.

17 If we were considering an eccentric orbit, there would be two fundamental forcing frequencies, $\omega_l$ and $n$, which would necessitate a separate treatment for each, since each excitation would have its own tidal phase lag ($\Delta_t$ and $\Delta$) and tidal dissipation number ($Q_t$ and $Q$).
After determining \( U_T \), we return to Equation (E3) and perform the surface integral on the secondary:

\[
\dot{E} = -\frac{2\pi k_s \rho_a^3 \omega \Phi_s R_s^3 q^{5/3}}{3a^6} (3 \sin \Delta_l + \sin (2\Delta_l + 2\omega_l t)) \nonumber
- 3 \sin (\Delta_l + 2\omega_l t)). \tag{E7}
\]

This is the instantaneous energy dissipation rate of free libration within the secondary due to tides.

Similarly to Wisdom (2004, 2008), we can remove the time dependence of this equation by averaging over a forcing (libration) period:

\[
\dot{E} = -\frac{2\pi k_s \rho_a^3 \omega \Phi_s R_s^3 q^{5/3}}{Q_l a^6}, \tag{E8}
\]

where we removed the tidal lag angle dependence using the canonical relationship \( \sin \Phi = \sin \Phi_s \). The tidal quality number is defined as \( Q_l = \frac{2\pi E_0}{\int \dot{E} dt} \), where \( E_0 \) is the peak energy stored in the system during a forcing cycle and \( \int \dot{E} dt \) is the energy dissipated over the cycle.

This theory shares the same difficulty as the Mignard (1979, 1980) model, in which the tidal bulge could potentially wrap around the body. We can determine a condition for this to occur and make sure that we are safely outside those bounds. If the maximum tidal bulge angle is \( \Delta_l < \pi / 2 \) before wrapping occurs, then this constrains the tidal quality number to \( Q_l > 1 \), which implies that the system must be underdamped in order for the bulge not to wrap. Advantageously, this relationship between the tidal forcing and response avoids the awkward tidal switching that occurs in a model with a constant tidal lag angle (e.g., Goldreich 1963). This discontinuity, which occurs when the tidal bulge angle (i.e., the spin angle \( \phi_s \)) goes through zero, causes the tidal bulge to jump across the body.

### APPENDIX F

**DERIVATION OF THE LIBRATION FREQUENCY**

We derive the libration frequency \( \omega_l \) from the definition of the libration period, \( T = 2\pi / \omega_l = \int d\phi_l / d\phi_s \), where \( \phi_l \) is the spin angle of the secondary. When \( \phi_s = 0 \), the longest axis, \( \xi_s \), of the secondary is aligned with the line connecting the mass centers of the two bodies. Using the Hamiltonian equations of motion (Equation (C11)) and the Hamiltonian written as a function of the libration angle amplitude \( \Phi_l \) (Equation (C14)), the libration period is

\[
T = \sqrt{\frac{2}{H_1 H_2} \int_0^{\Phi_l} \frac{1}{\sin \Phi_l} \left[ 1 - \frac{\sin^2 \phi_s}{\sin^2 \Phi_l} \right]^{-1}}
\]

\[
d\phi_l = \sqrt{\frac{2}{H_1 H_2} K(\sin^2 \Phi_l)}, \tag{F1}
\]

where \( K(k^2) \) is the complete elliptic function of the first kind. The integration is done directly after substituting the variable \( \chi \) using the trigonometric relation \( \sin^2 \chi = \sin^2 \phi_s / \sin^2 \Phi_l \).

The libration frequency \( \omega_l \) is

\[
\omega_l = \frac{\pi \sqrt{2H_1 H_2}}{K(\sin^2 \Phi_l)} = \frac{\pi \omega_d \sqrt{35} (1 + s)}{2K(\sin^2 \Phi_l) a^{3/2}}. \tag{F2}
\]

This averaging is acceptable as long as the orbital period is much shorter than the energy dissipation timescale, which is always the case here.

### APPENDIX G

**DERIVATION OF THE LIBRATION DAMPING DUE TO TIDES**

Using the model described in Appendix C, the energy of the system is the Hamiltonian \( E = H_0 - H_2 \cos 2\Phi_s \), which is described solely as a function of the libration amplitude \( \Phi_l \). It is important to remember that because of their symmetry, the dissipation of libration tides does not secularly couple the two bodies. We take the time derivative of energy equal to the energy dissipation result found at the end of Appendix E in Equation (E8). Thus, the time derivative of the libration amplitude due to tidal damping is

\[
\dot{\Phi}_l = -\frac{k_s \omega_0 \Phi_l^2}{Q_l a^3 \sin 2\Phi_l}. \tag{G2}
\]

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For low mass ratio systems, \( q \ll 1 \), the libration frequency is proportional to the mean motion: \( \omega_l \approx \pi \sqrt{35} / 2K(\sin^2 \Phi_l) \). For small libration angle amplitudes, \( \Phi_l \ll 1 \), the relationship is merely a function of the shape of the secondary: \( \omega_l \approx \pi \sqrt{35}/4 \).

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