spin wave based quantum storage of electronic spin with a ring array of nuclear spins

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We propose a solid state based protocol to implement the universal quantum storage for electronic spin qubit. The quantum memory in this scheme is the spin wave excitation in the ring array of nuclei in a quantum dot. We show that the quantum information carried by an arbitrary state of the electronic spin can be coherently mapped onto the spin wave excitations or the magnon states. With an appropriate external control, the stored quantum state in quantum memory can be read out reversibly. We also explore in detail the quantum decoherence mechanism due to the inhomogeneous couplings between the electronic spin and the nuclear spins.

INTRODUCTION

A practical protocol of information processing has to involve the storage and retrieval of information. In the fast-developing field of quantum information and quantum computation, a coherent storage process is particularly indispensable because the quantum information carried by some qubits manipulable is extremely fragile in usual and thus needs to be stored before decoherence happens \cite{1, 2}. Most existing schemes, e.g., \cite{3, 4}, concern about the quantum storage of photon state instead of a qubit (a two-level system) state while the latter is more necessary in the universal quantum computation. Recently, some investigations \cite{5, 6, 7} are devoted to explore universal quantum storage based on the solid state system since it is widely believed that the practical quantum computation should be based on the scalable solid state system.

Among them, a novel protocol \cite{5} was presented to store the quantum information of electronic spin in the ensemble \(N\) nuclei confined in semiconductor quantum dot. Physically this storage process can be described as a reversible map from the electronic spin state onto the collective spin state of the surrounding nuclear ensemble. Because of the long decoherence time of the nuclear spins – up to seconds \cite{8}, it can be regarded as a long-lived quantum memory for electronic spin qubit, in which the information stored in them can be robustly preserved.

To analyze the universal applicability of this protocol in practice, we found that \cite{8}, only in the low excitation with large \(N\) limit and the relatively homogeneous couplings of electron to the nuclei, can the many-nuclei system behave as single mode boson to serve as an efficient quantum memory. The large number of nuclei in such quantum memory implies that many nuclei crowd in a single quantum dot and the inter-nucleon interaction may not be neglected in this case. This is just the first motivation of the present paper to consider the influence of the inter-nucleon interaction on the quantum storage process.

On the other hand, the low excitation condition requires a ground state with all spins oriented. Usually it can only be prepared by applying a magnetic field polarizing all spins along a single direction. However, for a free nucleon ensemble without the external magnetic field, there is not such a preferred ground state with spins pointing to the same direction and thus the low excitation condition can not be satisfied automatically. But one can recognize that a ferromagnetic spin chain – the Heisenberg chain – usually has a spontaneous magnetization in association with the mechanism of spontaneous symmetry breaking \cite{9}, which naturally offers a polarization ground state. The intrinsic interaction between spins correlates the nuclei to form magnon, the collective excitation mode of spin wave, even without the external magnetic field. This observation also motivates us to explore the possibility of using a ferromagnetic quantum spin system, instead of the non-interacting nuclei ensemble, to serve as a robust quantum memory. With these considerations, in this paper, we present and study a protocol to implement quantum storage of the electronic spin state to a ring-shape array of interacting nuclei(Fig.1a). Under appropriate control of exchange interaction between the electron spin and nuclear spins by adjusting the overlap of their wave functions (Fig.1b), an arbitrary
quantum state of the electronic spin qubit, either pure or mixed state, can be coherently stored in the collective mode of the nuclear spin chain, the magnon, and read out in reverse. This paper is organized as follows: In Section II, we present the model of quantum memory element. The quantum storage process based on this model is illustrated in Section III and the inhomogeneity induced decoherence effect is analyzed in detail in Section IV. In Section V, conclusions and remarks are provided.

**QUANTUM MEMORY UNIT BASED ON THE NUCLEAR FERROMAGNETIC SPIN CHAIN SYSTEM**

To start with, we illustrate the configuration of our proposal for the solid-state based quantum memory in Fig.1a. $N$ nuclei are arranged in a circle within a quantum dot to form a spin ring array. A single electron is just localized in the center of the ring array surrounded by the nuclei. Because of the long decoherence time of the nuclei system, which is just the motivation of this protocol, we neglect the decoherence effect due to the damping of nuclei in the following discussions. The interaction of the nuclear spins is assumed to exist only between the nearest neighbors while the external magnetic field $B_0$ threads through the spin array.

Then the electron-nuclei system can be modelled by a Hamiltonian $H = H_e + H_n + H_{en}$. Here

$$H_e = -g_e \mu_B B_0 \sigma_z,$$

(1)
corresponds to the Zeeman energy of electron with the Lande $g$ factor $g_e$ and Bohr magneton $\mu_B$ in a magnetic field $B_0$ along the $z$-direction.

$$H_n = -g_n \mu_B B_0 \sum_{l=1}^{N} S_z^{(l)} - J \sum_{l=1}^{N} S^{(l)} \cdot S^{(l+1)}.$$  \hspace{1cm} (2)

represents the Hamiltonian for an ensemble of nuclear spins with the Lande $g$ factor $g_n$, the nuclear magneton $\mu_N$. $S^{(l)}$ is the nuclear spin of the $l$-th site. the first term of $H_n$ is the Zeeman energy of the nuclear spins and the second one is the exchange energy including dipole-dipole interaction between the nearest neighbor nuclear spins. The coupling strength $J > 0$ and this spin ensemble acts as a ferromagnetic spin chain.

$$H_{en} = \frac{\lambda}{2N} \left( \sum_{l=1}^{N} S_z^{(l)} + h.c. \right).$$ \hspace{1cm} (3)

is the hyperfine contact interaction between the $s$-state conduction electron and the nuclei in the quantum dot. The Pauli matrices $\sigma$ is the electronic spin. The magnitude of each coupling term $\propto \sigma_+ S_z^{(l)} + h.c.$ depends on the overlap of the "large" wave function and the "localized" wave function of each nucleon schematically illustrated in Fig.1b and is estimated to be in the order of $10^{-1}$GHz with the same parameter in ref (3). The electronic wave function is supposed to be cylindrical symmetric, e.g., the $s$-wave component. Thus the coupling coefficient $\lambda \propto |\psi(r)|^2$ defined by the electron wave function $\psi(r)$ is almost homogenous for all the $N$ nuclei in the ring array.

The denominator $N$ in Eq. (3) originates from the envelope normalization of the localized electron wave-function $\underline{E}^{\underline{R}}$.

After the Holstein-Primakoff transformation

$$S_z^{(l)} = a_l^+ a_l - s,$$

$$S_x^{(l)} = \left( \sqrt{2s - a_l^+ a_l} \right) a_l$$

(4)
in terms of bosonic annihilation operator $a_l$, the Hamiltonian $H_n$ can be expressed in the low excitation limit

$$\langle a_l^+ a_l \rangle \ll 2s,$$

(5)

with the bosonic excitations as

$$H_n = (2js - g_n \mu_n B_0) \sum_{l=1}^{N} a_l^+ a_l - Js \sum_{l=1}^{N} (a_l^+ a_{l+1} + h.c.)$$

(6)

while the Hamiltonian of hyperfine contact interaction can be re-written as

$$H_{en} = \sqrt{s \lambda / 2N} \left( \sum_{l=1}^{N} a_l^+ + h.c. \right).$$

(7)

which is modelled as a system with one spin and $N$ interacting bosons.

The discrete Fourier transformation

$$a_l = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} e^{i \frac{2 \pi k l}{N}} b_k,$$

(8)
is used to diagonalize the total Hamiltonian as

$$H = H_N + \sum_{k=1}^{N} \omega_k b_k^\dagger b_k$$

(9)

where

$$H_N = \omega_N b_N^\dagger b_N + \frac{\Omega}{2} \sigma_z + \lambda \sqrt{N s / 2} \left( \sigma_+ b_N + \sigma_- b_N^\dagger \right).$$

(10)
quantum computation as well \cite{10}. With various physical spin-boson interaction forms the basis for ion-trap based Bg the total symmetric sector of the $S_n$ (electronic spin can be implemented in the invariant subspace, there also exist other similar to the spin chain surface. Under certain approximations the $N$ magnons orient in the same direction perpendicular to the spin chain surface. Under certain approximation, there also exist other $H_{en}$-invariant subspaces $V_{kn}$ ($k = 1, 2, \cdots, N - 1$) in the $k$-th magnon mode similarly. Since the symmetric term $\sum_i a_i^\dagger (\sum_i a_i)$ only belongs to the total symmetric sector of the $S_n$ group, it can not change the states out of the same sector of $S_n$. Therefore, the $N$-th mode is singled out as the only coupling term.

\section*{Quantum Storage Process}

The process of quantum information storage of the electronic spin can be implemented in the invariant subspace with $N$-th magnon. Here, the frequency $\omega_N = g_n \mu_B B_0$ of boson mode and the level spacing $\Omega$ can be modulated by the external field $B_0$ simultaneously. Such spin-boson interaction forms the basis for ion-trap based quantum computation as well \cite{10}. With various physical systems, people have proposed several spin-boson models based protocols for the universal quantum information storage. For example, the nano-mechanical resonator interacting with Josephson junction (JJ) phase qubits \cite{11} can be considered for universal quantum memory. We have analyzed the progressive decoherence process of such spin-boson model with nano-mechanical resonator \cite{12}. It is just this idealized scheme that motivates us to seek another more practical protocol based on the collective bosonic excitation in various physical systems.

The quantum storage process is described as follows. Suppose the initial state of the total system is prepared so that there is no excitation in the $N$ nuclei at all while the electron is in an arbitrary state

$$\rho_e (0) = \sum_{n,m = \pm} \rho_{nm} |n\rangle \langle m|$$

where $|+\rangle$ ($|-\rangle$) still denotes the electronic spin up (down) state. The quantum information carried by the electron is represented by the coefficients $\rho_{nm}$. The initial state of the total system can then be written as

$$\rho (0) = \rho_e (0) \otimes |0_N\rangle \langle 0_N| \otimes \rho_e (0)$$

in terms of $\rho_e (0) = \{|0\rangle\}_N \lbrack \{|0\rangle\}_N \otimes \rho_e (0)\rbrack$.

(14)

The above results show that $H$ only contains the interaction of the $N$-th magnon mode with the electronic spin while the other $N - 1$ magnons are decoupled with it. Actually this is the consequence of the cylindrical symmetry induced by the homogeneity of couplings of the electronic spin to the nuclei. As noted in Eq. (2) and \cite{7}, the Hamiltonian is invariant under any permutation of the nuclear spins. In the terminology of group theory, the permutation symmetry of the nuclear spins. In the terminology of group theory, the permutation symmetrical group $S_n$ is the symmetry transformation group of the nuclear system and thus there exist $2N s$ two-dimensional $H_{en}$-invariant subspaces $V_{N,n}$ spanned by

$$\begin{align*}
|n, +\rangle &= |n\rangle_N \otimes |+\rangle, \\
|n + 1, -\rangle &= |n + 1\rangle_N \otimes |-\rangle
\end{align*}$$

where

$$|n\rangle_N = \frac{1}{\sqrt{n!}} \left( b_N^{\dagger}\right)^n |G\rangle$$

$$= \frac{1}{\sqrt{n! N^n}} \sum_{i=1}^{N} a_i \right) \left| G \right|$$

is a symmetric state in the spatial configuration of the nuclear ensemble, which is an symmetrical combination of all the bosonic excitation defined on the $N$-th magnon mode : $|+\rangle$ ($|-\rangle$) denotes the electronic spin up (down) state. $n = 0, 1, \cdots, 2s - 1$ is the total spin of the state $|n\rangle_N$; $|G\rangle$ denotes the ground state with all the nuclear spins oriented in the same direction perpendicular to the spin chain surface. Under certain approximation, there also exist other $H_{en}$-invariant subspaces $V_{kn}$ ($k = 1, 2, \cdots, N - 1$) in the $k$-th magnon mode similarly. Since the symmetric term $\sum_i a_i^\dagger (\sum_i a_i)$ only belongs to the total symmetric sector of the $S_n$ group, it can not change the states out of the same sector of $S_n$. Therefore, the $N$-th mode is singled out as the only coupling term.

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The difference between \( w_F \) and \( \rho_e (0) \) is only a diagonal unitary transformation

\[
U = \begin{pmatrix} 1 & 0 \\ 0 & e^{i \lambda t} \end{pmatrix}
\]  

is independent of the stored initial state \( \rho_e (0) \) and can be cancelled by redefining the computational basis. This means the quantum information carried by the electron has been transferred to the nuclei.

The above calculation demonstrates that, by placing the electron near the nuclei and switching off the magnetic field, the quantum information carried by the electron spin can be mapped onto the magnetic field, the quantum information carried by the electron near the nuclei and switching off the magnetic field does not play a role in this storage process. After this preparation, the magnetic field can be turned off all through. The ferromagnetic interaction among spins spontaneously breaks the isotropically degenerate ground state into this special direction and prevents the total effective Hamiltonian \( H \) is decoupled from the interactions between the other \( N-1 \) magnons and the electronic spin, the above protocol is still valid for any other initial state of \( N-1 \) magnons such as the thermal state so long as the \( N \)-th mode is prepared in the vacuum state\(^8\).

For the experimental implementation of the above protocol, the manipulation induced perturbations need to be discussed. As it seems from the above illustrations, this proposal requires an instantaneous control of the interaction between the electronic spin and the nuclear spins. However it is known that the interaction can never be switched on and off instantaneously. Therefore the coupling coefficient \( \lambda \) should be an analytical function of time \( t \), that is, \( \lambda = \lambda (t) \). Actually the finite rise-and-fall time of the control pulse causes deviation from the ideal calculation in many cases. Fortunately, in our protocol, \textit{this instantaneous time control is not really an indispensable part as it seems.} From Eq. (10), we can see that the free Hamiltonians for the electronic spin and the \( N \)-th magnon are both zero. The total effective Hamiltonian

\[
H_N = \lambda(t) \hat{X}.
\]  

at different instant are still commutative and then the evolution operator can be exactly

\[
U(t) = \exp(-i \hat{X} \int_0^t \lambda(t') dt').
\]  

unlike the usual time order integral. Therefore, the time-dependence of the coupling term only leads to a modification of the operation time \( t_0 \). So long as the new operation time \( t_0' \) satisfying

\[
\int_0^{t_0'} \lambda(t) \, dt = \frac{\pi}{\lambda} \sqrt{\frac{N}{2s}}.
\]

\( \rho (t_0') \) is still factorized as the Eq. (15) to record the quantum information of the initial state exactly.

![FIG. 2: (color online) Turn on and off of the nuclei-electron interaction can be implemented by using an external electrical field \( E \) to drive the electron an external electrical field, which drive the electron moving upward and downward along the axis that perpendicular to the plane.](image)

Another difficulty in the implementation is due to the perturbation of magnetic field caused by the electron. To store (retrieve) the information of the electronic spin to the nuclear spin chain one needs to monitor the coupling strength. For example we can remove the electron along the axis perpendicular to the plane where the ring spins in the array locates (see Fig.2). To maintain the symmetry of interaction between the electron and nuclear ensemble, the axis threads through the center of the ring of spins. Generally, the motion of electron is not at a constant velocity. The accelerated motion generates a magnetic field according to Maxwell equation. For example, if we use an external electrical field \( E \) to drive the electron (see Fig. 2), the acceleration of the electron is proportional to the magnitude of \( E \). The additional magnetic force will change the states of the nuclear spin systems since both the ground state and energy spectrum of magnons depend on the external magnetic field. This perturbation might be fatal for the free spin chains but less serious for our correlated spin chain since the interaction obviates the random flips of the nuclear spins. Thus the effect of this manipulation induced perturbation is greatly suppressed and this kind of spin chain acts as a more robust quantum memory.

**DECOHERENCE INDUCED BY THE INHOMOGENEITY OF COUPLINGS**

So far we have discussed the ideal case with homogeneous coupling between the electron and the nuclei, that
is, the coupling coefficients are the same constant $\lambda$ for all the nuclear spins. However, the inhomogeneous effect of coupling coefficients has to be taken into account if we concern some cases beyond the strictly cylindrical symmetric effective coupling. In this case, the quantum decoherence induced by the so-called quantum leakage has been extensively investigated for the atomic ensemble based quantum memory [14]. We now discuss the similar problems for the magnon based quantum memory.

For the general case, $\lambda_k \propto |\psi(r_i)|^2$ vary with the positions of the nuclear spins where $\psi(r_i)$ is the wave function of the electron at site $r_i$. In this case, the Hamiltonian contains other terms besides the interaction between the spin and $N$-th mode boson, that is, the inhomogeneity induced interaction

$$V = \lambda \sqrt{\frac{s}{2N}} (\sigma + \sum_{k=1}^{N-1} \chi_k b_k + h.c.)$$

should be added in our model Hamiltonian $H$ with

$$\chi_k = \frac{\lambda}{\lambda N} \sum_{l=1}^{N} \exp[i 2 \pi k l / N]$$

are dimensionless coefficients representing the relative magnitude of coupling.

For a Gaussian distribution of $\lambda$, e.g.,

$$\lambda_l = \frac{\lambda}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(l-1)^2}{2\sigma^2}\right]$$

with width $\sigma$ and $\lambda_1 = \lambda$, the corresponding inhomogeneous coupling is depicted by

$$\chi_k = \frac{1}{N} \sum_{l=1}^{N-1} \exp\left[-\frac{(l-1)^2}{2\sigma^2} + i 2 \pi k l / N\right]$$

Fig. 3 shows the magnitude of $\chi_k$ for the different Gaussian distributions of $\lambda$ with different widths $\sigma$. It can be seen that the coupling is stronger for the modes near mode 1 and $N-1$ than those far away from them. When the interaction gets more homogenous (with larger $\sigma$), the coupling coefficients $\chi_k$ for all the 1 to $N-1$ modes become smaller. It can be imagined that when the distribution is completely homogeneous (with $\sigma \to \infty$), all the couplings with the $N-1$ magnon modes vanish and the Hamiltonian $H$ in Eq. 24 is obtained.

For the above model for quantum dissipation of two-level system [12], various approaches have been presented to deal with the dynamic evolution of the spin-boson model for different types of bath, such as the Markovian method [15] and the exact solutions in the Ohmic regime [16]. Most recently, DiVincenzo and Loss explored the calculation of the next order Born approximation for the dissipation of such spin-boson model [17]. In the following we will adopt a rather direct method to analyze the decoherence problem for two limiting case.

If $N$ is so large that the spectrum of the quantum memory is quasi-continuous, this model is similar to the "standard model" of quantum dissipation for the vacuum induced spontaneous emission [17]. The $N-1$ magnons will induce the quantum dissipation of the electronic spin with decay rate

$$\gamma = 2\pi \sum_{k=1}^{N} \frac{\lambda^2 |\chi_k|^2}{2N} \delta \left(\omega_k - 2\lambda \sqrt{\frac{s}{2N}}\right).$$

Let $|\Psi(t)\rangle$ be the ideal evolution governed by the expected Hamiltonian without dissipation while the realistic evolution $|\Psi'(t)\rangle$ governed by the Hamiltonian with dissipation term Eq. 25. Suppose the initial state of the electron is

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}} (|+\rangle + |\rangle),$$

we can obtain the analytical result of the fidelity

$$F(t) = |\langle \Psi(t) |\Psi'(t) \rangle| = \frac{1}{2} + \frac{1}{2} e^{-\frac{\gamma}{2}t} \times \sec \varphi \cos gt \cos (\Delta'_1 t + \varphi) + \sin gt \sin \Delta'_1 t,$$

where

$$\varphi = \arcsin \sqrt{\frac{2N\gamma^2}{\lambda^2 s}},$$

$$g = \lambda \sqrt{\frac{s}{2N}}, \Delta'_1 = \sqrt{g^2 - \gamma^2}.$$

Fig. 4 shows the curve of the fidelity $F(t)$ varying with time $t$. We can see that the fidelity exhibits exponential decay behavior with sinusoidal oscillation. At the instance that the quantum storage process is just finished, the fidelity is about $1 - \pi \gamma/8g$. Therefore, the deviation from the ideal case with homogeneous couplings is very small for $\gamma/g << 1$. Since the ring-shape spin array with inhomogeneous coupling is just equivalent to an arbitrary
Heisenberg spin chain in the large \( N \) limit, the above arguments means that an arbitrary Heisenberg chain can be used for quantum storage following the same strategy above if \( \gamma / g \) is small, i.e., the inhomogeneous effect is not very strong.

\[
\lambda \sqrt{\frac{s}{2N}} \left\langle \chi_k \right\rangle << 1 \tag{32}
\]

for the \( N - 1 \) magnon modes. As a consequence of this adiabatic elimination, the effective Hamiltonian

\[
H_{\text{eff}} = \sum_{k=1}^{N-1} \omega_k b_k^\dagger b_k + H_N
\]

\[
+ \sum_{k,k'=1}^{N-1} \Omega_{kk'} (b_k b_k^\dagger \mid e \rangle \langle e \mid - b_k^\dagger b_{k'} \mid g \rangle \langle g \mid) \tag{33}
\]

can be obtained by ignoring the fast oscillating terms where

\[
\Omega_{kk'} = \frac{\lambda^2 s (\omega_k + \omega_{k'})}{2N \omega_k \omega_{k'}} \tag{34}
\]

It can be seen from the above Hamiltonian that, though the electronic spin qubit and the memory mode \( b_N \) do not exchange energy with the other \( N - 1 \) modes, the last term in \( H_{\text{eff}} \) implies that they can record the “which-way” information carried by the electronic qubit. Then it causes dephasing, and also leads to mixing of different magnon modes. However, if the initial state can be prepared in vacuum state for all the magnon modes, only the diagonal terms concerning the magnon modes remain. The deviation from the ideal case can also be estimated by the fidelity \( F(t) \)

\[
F(t) = \frac{1}{2} \left| \cos \Delta_1 t \cos \Delta_1' t - i \sin \Delta_1 t \cos \xi \right|
\]

\[+ \sin \xi \sin \Delta_1 t \sin \Delta_1' t + e^{i \Delta_1' t} \right| \tag{35}
\]

where

\[
\Omega' = -\frac{\lambda^2 s}{N} \sum_{k=1}^{N-1} |\chi_k|^2
\]

\[
\Delta_1' = \sqrt{\left( \frac{\Omega'}{2} \right)^2 + \frac{\lambda^2 s}{2N}} \tag{36}
\]

and

\[
\cos \xi = \frac{\Omega'}{2\Delta_1}, \sin \xi = \frac{\lambda}{\Delta_1} \sqrt{s/2N} \tag{37}
\]

Fig. 5 illustrates the time evolution of this fidelity \( F(t) \).

\[
F \left( t = \frac{\pi}{\lambda} \sqrt{\frac{N}{2s}} \right) \approx \frac{1}{2}
\]

Thus we can conclude that in the presence of inhomogeneity the fidelity is still high in the large \( N \) limit. Therefore, to suppress the decoherence induced by other modes of magnon, \( N \) should be made large. However, as noted from Eq. (11), the energy spectrum is related to \( N \) and it is quasi-continuous in the large \( N \) limit. If the energy spacing is smaller than \( k_B T \) (\( k_B \) is the Boltzmann constant), the thermal fluctuation will wash out the quantum coherent effect. This should be avoided by using a spin chain with

\[
N \leq \pi \left( \arcsin \sqrt{\frac{k_B T}{4Js}} \right)^{-1} \tag{38}
\]

The value of \( N \) is restricted by the two requirement and the optimum number of \( N \) can be found within this range considering practical parameters in experiments.

**CONCLUSIONS AND REMARKS**

In conclusion, we have proposed a novel protocol of universal quantum storage for the electronic spin qubit
based on the ring array of nuclear spin. Here, the quantum memory unit is the spin wave excitation (magnon) of the spin chain. We illustrated how the quantum information of an arbitrary state of a two-level system can be coherently mapped from the electron to the spin wave quantum memory.

The mechanism for quantum decoherence induced by the inhomogeneity of couplings between the electronic spin and the nuclear spins is also investigated. In the presence of inhomogeneity, the interaction with other magnon modes is involved and the time evolution of the whole system inevitably deviates from the ideal case. This results in the less-than-unity fidelity. What’s more, some other problems arise. For example, if the initial state is a superposition of the states $|\{n_k\}\rangle$ with different $\sum_k n_k$, this added term leads to the entanglement of the electronic spin and the magnon states so that the two parts cannot be factorized. Thus the quantum information storage cannot be implemented effectively. Therefore this inhomogeneity imposes a strict restriction on the initial state of magnon modes. Another obstacle is the $z$-component coupling between the nuclear spins and the single electron spin. In more general cases this coupling cannot be neglected. For the homogeneous couplings, if the $N-1$ magnons are prepared in a specific subspace spanned by $|\{n_k\}\rangle$ that $\sum_k n_k$ is the same, our protocol still works efficiently even including such $z$-component coupling. For the case with inhomogeneous couplings, this term results in the additional interactions among the magnons. This greatly complicates the analysis since the decoherence source now is a set of interacting bosons instead of the conventional independent modes.

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