Quadcopter Path Following Control Design Using Output Feedback with Command Generator Tracker LOS Based At Square Path

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Abstract. Quadcopter an unstable system, underactuated and nonlinear in quadcopter control research developments become an important focus of attention. In this study, following the path control method for position on the X and Y axis, used structure-Generator Tracker Command (CGT) is tested. Attitude control and position feedback quadcopter is compared using the optimal output. The addition of the $H\infty$ performance optimal output feedback control is used to maintain the stability and robustness of quadcopter. Iterative numerical techniques Linear Matrix Inequality (LMI) is used to find the gain controller. The following path control problems is solved using the method of LQ regulators with output feedback. Simulations show that the control system can follow the paths that have been defined in the form of a reference signal square shape. The result of the simulation suggest that the method which used can bring the yaw angle at the expected value algorithm. Quadcopter can do automatically following path with cross track error mean $X=0.5$ m and $Y=0.2$ m.

1. Introduction
As the development of methods used to deal with control problems in quadcopter [1], nonlinear control techniques is used for controlling hover. More back stepping nonlinear method can approach the characteristics of nonlinear systems. Thus, the calculation steps used more complicated iteration. Even with such complicated calculations, quadcopter capable of following the path corresponding references from the condition values of x, y and z. But when quadcopter by disruption of the constant wind then the response generated ripple and robust experience.

In addition, the use of a linear approach as one of the other control strategies in solving quadcopter control described in [2] - [5]. One way to obtain the optimal conditions quadcopter tracking controller is using Linear Quadratic Tracking (LQT). Furthermore, adding the models that quadcopter is able to follow the reference signal and minimize overshoot is applied in which a simple linear model approach is able to keep representing the nonlinear models of quadcopter. At the time quadcopter tracking, the controller is also able to reduce their interference. When tracking the z axis by interference, then the system can not handle distractions well when tracking [2].

Other conditions which are also described in tracking problems using step signal treated with the linear approach is the method of Linear Quadratic Regulator (LQR) by adding the integral effect / compensator [3]. Systems that are linearized first and added concept integrator/compensator. This system capable of making quadcopter tracking by using a reference signal form step without any overshoot although the time needed to reach steady state for a long time.
In [4], approached through linearization of nonlinear systems with state feedback gain is obtained by using the LMI pole placement. Furthermore, the use Takagi-Sugeno fuzzy approach was taken into account to implement a linear control algorithm on each linear subsystem. The simulation results of fuzzy control system Takagi-Sugeno show that the response system can perform exact trajectory tracking system. However, the response on the condition of roll and pitch angle cannot follow the reference signal and the oscillation occurs.

An advance improvement to be made to improve the system response to the quadcopter in tracking issue is using the optimal method that is optimal output feedback by adding the structure of the Command-Generator Tracker (CGT) [5], this improvement is capable of tracking in the form of a circle, square or helix/spiral with. The method which is used to obtain the reference signal tracking conformity will result the tracking error as small as possible. To maintain a stable attitude, use of output feedback controller that performs more reliable $\mathcal{H}_\infty$ so quadcopter in dealing with interference from outside.

In addition, the use of Static Output Feedback (SOF) [6] is used to overcome the limitations of a measurable state where SOF design has the advantage of requiring only measurable signals from the plant to be controlled. SOF controller used on the helicopter are $\mathcal{H}_\infty$ performance as controls for tracking the position and attitude. Loop shaping techniques used in tracking control design position with the Gain controller Ricatti searched using the solution equation. The control algorithm is presented produces a robust control against disturbance.

Representation of reference input in Command Generator Tracker (CGT) structure is discuss in [5], [7], and [8]. Reference signal that used in trajectory or following path can be from all kind of signal. this structure can guarantee zero steady-state tracking error. The arrangement of the heading of a plant to a path or reference signal is performed with Line of Sight (LOS) algorithm [9]-[10].

The aim of this paper is to design a control system using output feedback with a Command-Generator Tracker (CGT) based on Line of Sight (LOS) to do following path by arranged the heading. The path in this systems are circle and square. The quadcopter moves in following given path in hover condition.

2. Quadcopter Model

Physical modeling will be complex if there are no assumptions used to simplify the equation on a quadcopter. Quadcopter has 6 degrees of freedom (DOF) with 12 outputs, which is 6 out of 12 outputs determine attitude of quadcopter. The kinematic and dynamic quadcopter models are delivered base on Newton-Euler with some simplification assumptions. quadcopter structure which is assumed rigid and symmetry. Propeller structure which is assumed rigid also thrust and drag force is proportional to the squared of propeller velocity.

![Figure 1. All Figure for State of Quadcopter (e for Earth Frame and b for Body Frame Quadcopter)](image)

The modeling of translational axis lies on the earth coordinate. The transformation from body frame coordinate to earth frame coordinate is using rotation matrix. This matrix is obtained from multiplying rotational matrix times x, y, z axis, expressed by:

$$
\begin{bmatrix}
  c\theta s\psi & s\theta s\psi + c\theta c\phi d\psi & s\theta c\psi - s\phi d\psi \\
  c\phi s\theta & c\phi c\theta s\psi + s\phi d\psi & s\phi c\psi - c\phi d\psi \\
  s\theta & -s\phi d\psi & c\phi d\psi
\end{bmatrix}
$$

With $s$ is the notation of sin, $c$ is the notation of cos.
2.1. LQR Output Feedback with Command-Generator Tracker (CGT)

Command-Generator Tracker (CGT) is a control design structure that provided compensator according to system requirements to get small tracking errors for the desired reference input \( r(t) \). Linear time invariant is written in differential equation which The plant equation and output performance \( z(t) \) are:

\[
X(t) = Ax(t) + Bu(t) + Dw(t)
\]

\[
y(t) = Cx(t)
\]

\[
z(t) = Hx(t)
\]

The initial conditions of the reference signal \( r(t) \) is satisfied with the following differential equation:

\[
r^{(d)} + a_1r^{(d-1)} + \ldots + a_dr = 0
\]

with \( d \) as the order and \( a_i \) as coefficient.

In [7], CGT has made based on dynamic signal reference. However, to fulfill the desired design specification, designing CGT is not using all derivatives from reference signal.

In this control technique, the path following problem will be converted into regulator problem in which error is regulated to 0. To create the whole result of path following system, error tracking modified is necessary:

\[
\Delta(s)e = \Delta(s)r - \Delta(s)Hx = -H\xi
\]

\( \xi \) is modified state vector.

To express dynamic from \( \xi(t) \), (5) is operated in \( \Delta(p) \) to get (6), which \( p = d/dt \) in time domain. Define the dynamics by multiplying augmented that operated in \( \Delta(p) \) so that the modified system is as follows:

\[
\begin{align*}
\dot{\xi} &= \tilde{A}\xi + \tilde{B}u \\
\frac{d}{dt}[\xi] &= \begin{bmatrix} G & : & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & A \end{bmatrix}\begin{bmatrix} \xi \\ \mu \end{bmatrix} + \begin{bmatrix} 0 \\ \cdots \\ -H \end{bmatrix}\begin{bmatrix} \xi \end{bmatrix}
\end{align*}
\]

(7)

LQ regulator design implemented on this system (7). If the state goes to zero, then the tracking error \( e(t) \) will be vanished. So, state feedback output of the system is [7]- [8]:

\[
\tilde{u} = [K_\xi \ K_p] \begin{bmatrix} \xi \\ \mu \end{bmatrix}
\]

(8)

The control law [9] has solved using LQ regulator with output feedback problem. The solution of this problem adopted the concept as described in [6]-[7]. Therefore, the necessary conditions of the augmented system (7) for LQ regulator with output feedback are as follows:

\[
\frac{\partial H}{\partial S} = \tilde{A}_c^T P + P\tilde{A}_c + \tilde{C}_c^T \tilde{K} \tilde{R}\tilde{K} + Q = 0
\]

(9)

\[
\frac{\partial H}{\partial P} = \tilde{A}_c S + S\tilde{A}_c^T + X = 0
\]

(10)

\[
\frac{\partial H}{2 \partial K} = \tilde{R}\tilde{K}\tilde{S}\tilde{C}^T - \tilde{B}^T P \tilde{S}\tilde{C}^T = 0
\]

(11)

Which \( \tilde{A}_c = \tilde{A} - \tilde{B}\tilde{R}\tilde{C}^T \) and \( X = E[\{x(0)x(0)^T\}] \). If the weighting matrix \( R > 0 \) and \( \tilde{C}\tilde{S}\tilde{C}^T \) is a nonsingular, the output feedback gain \( K \) has the following form:

\[
K = R^{-1}\tilde{B}^T P \tilde{S}\tilde{C}^T (\tilde{C}\tilde{S}\tilde{C}^T)^{-1}
\]

(12)

By using LQ regulator method with output feedback on X and Y, the gain control value is obtained.  

2.2. LOS (Line of Sight) Law on Steering Equations
The enclosure based strategy is used to drive $e(t)$ toward zero, then direct the velocity vector to the point of intersection $P_{los}^n = [X_{los}, Y_{los}]^T$ to match the heading of the path, this path is implicitly defined by the order in which the waypoints has been defined. The path directly involved the determination, direct assignment $\chi(t)$.

$$\tan(\chi(t)) = \frac{\Delta y(t)}{\Delta x(t)} = \frac{y_{los} - y(t)}{x_{los} - x(t)}$$

(13)

The centre coordinate of a vehicle $\{b\}$ is defined $P_n = [X, Y]^T$ and used a circle with a radius $R > 0$, then drawn from the centre point of the vehicle $\{b\}$. That circle will cut a path into two points, which one of the dots is $(X_{los}, Y_{los})$. The calculation of two unknown LOS points $P_{los}^n = [X_{los}, Y_{los}]^T$ can be found by solving two equations below:

$$[x_{los} - x(t)]^2 + [y_{los} - y(t)]^2 = R^2$$

$$\tan(\alpha_k) = \frac{x_{k+1} - x_k}{y_{k+1} - y_k} = \frac{y_{los} - y_k}{x_{los} - x_k} = \text{constant}$$

(14)

Where the equation is the Pythagoras equation, whereas the equation shows the angle of the path to the axis of the earth $x$ is $\alpha$. The slope between the two waypoints is constant and also applies for the magnitude of the slope to the points between them. The process of calculating the enclosure analytically based on [9]-[10] can be explained by:

Argument 1: for $|\Delta x| > 0$, So the equation

$$y_{los} = \frac{\Delta y}{\Delta x}(x_{los} - x_k) + y_k$$

With $\Delta x := x_{k+1} - x_k$ and $\Delta y := y_{k+1} - y_k$ are difference position $x$ and $y$ between two waypoints.

3. Design Control System

Inner loops are used to stabilize roll, pitch, and yaw angles that guarantee performance. The controller is designed based on the Static Output Feedback (SOF) problem, in which the feedback output gain $K$ is obtained by completing the Linear Matrix Inequality (LMI) iterative algorithm. The LMI form that uses Schur Complement is

$$\begin{bmatrix}
    p_{A} + A^T P_{e} + Q + L_{e}^T R^{-1} L_{a} & P_{e} B & P_{e} D \\
    B^T P_{e} & R & 0 \\
    D^T P_{e} & 0 & -\gamma^T I
\end{bmatrix} \leq 0$$

(16)

The result of state system linearization is for roll model, pitch, and yaw, then it becomes

$$x_{inner} = [\phi \ p \ \nu \ \theta \ q \ \psi \ \gamma]^T$$

(17)

In the design of the rotation controller, the main variables that controlled are the angle of roll and pitch, also three speeds i.e. angular speed of roll, pitch, and yaw. Therefore, the output vector of this controller is

$$x_{inner} = [\phi \ \theta \ \psi \ p \ q]^T$$

$$\bar{u} = [K_{\phi} \ K_{p} \ C_{\phi \ p \ \psi \ q}]$$

(18)

Inertia moment for roll and pitch have the same value $(I_{xx} = I_{yy} = 0.03 \text{ kg m}^2)$, so the roll dynamic equals with the pitch dynamic. These are the models of state space linear roll/pitch.

The best result is obtained by the parameter value as follows:

$\gamma = 0.7$, $R = 10$ and $Q = \text{diag \{755, 3, 0.1\}}$.

with the iterative LMI feasibility method, the result are $P$, $K$, and $L$ matrix after the 12th iteration.

$$\begin{bmatrix}
    \phi \\
    \nu
\end{bmatrix} = \begin{bmatrix}
    0 & 1 & 0 \\
    0 & 0 & 800 \\
    0 & 0 & -15
\end{bmatrix} \Delta \nu; \ y_{p} = C_{\nu} \begin{bmatrix}
    0 & 1 \\
    0 & 0.1 \\
    15
\end{bmatrix} \nu = H_{\phi} = \begin{bmatrix}
    0.3 & 0 \\
    0 & 0
\end{bmatrix} \phi$$

(19)
The objective control position of path following can make the quadcopter follow the desired path when it is in hover condition. Therefore, the reference signal that given is for performing path following $x$ position, which is a sinusoidal signal with a frequency of 0.1047 rad / s or equal to 0.0167 Hz with a phase equal to -0.26 rad. Here is the reference of signal equation:

$$x_d = -\sin(0.05\pi)$$

If it modified using Laplace transform, then the obtained is:

$$\Delta(p)x_d = p^2 + 0.02 = 0$$

By substituting the reference model and the system model, the result of quadcopter dynamics for the $x$ axis is as follows:

$$\begin{bmatrix}
\dot{\varepsilon} \\
\ddot{\varepsilon} \\
\dot{x} \\
\ddot{x}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
-0.02 & 0 & -1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\varepsilon \\
\dot{\varepsilon} \\
x \\
\dot{x}
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
1 \\
0
\end{bmatrix}u_i$$

With output performance as follows

$$\tilde{y} = C\tilde{x} =
\begin{bmatrix}
J & 0
\end{bmatrix}
\begin{bmatrix}
\varepsilon \\
\dot{\varepsilon} \\
x \\
\dot{x}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\varepsilon \\
\dot{\varepsilon} \\
x \\
\dot{x}
\end{bmatrix}$$

The path which want to be assigned as a reference signal to the quadcopter is in circle shape (in a 3-dimensional field). Therefore, the reference signal that given for $Y$ position is:

$$Y_d = -1 \cos(0.05\pi)$$

The polynomial characteristic equation is

$$\Delta(p)y_d = p^3 + 0.02p = 0$$

The polynomial characteristic equation of the reference signal with orde $d=3$ is written in a matrix state space with 3×3 dimension. Since the following model CGT structure is a representation of the reference signal, not all derivatives of the reference signal are needed to be used in the system structure. So that in this design a modification can be written as follows:

$$\Delta(p)y_d = 0.02p = 0 \Rightarrow \dot{y}_d =
\begin{bmatrix}
0 & 1 \\
0 & -0.02
\end{bmatrix}y_d = Gy_d$$

thus, the modification of multiple augmented system for the dynamic position on the $Y$ axis is

$$\begin{bmatrix}
\dot{\varepsilon} \\
\ddot{\varepsilon} \\
\dot{y} \\
\ddot{y}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & -0.02 & -1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & -136.2683
\end{bmatrix}
\begin{bmatrix}
\varepsilon \\
\dot{\varepsilon} \\
y \\
\dot{y}
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
1 \\
0
\end{bmatrix}u_i$$
The output feedback gains $\tilde{K}$ for the $X, Y$ position controllers are obtained as follows:

$$
\tilde{K}_x = \begin{bmatrix}
-4.418 & -9.6604 & 9.4440 & 5.3433 \\
\end{bmatrix}
$$

$$
\tilde{K}_y = \begin{bmatrix}
15.9326 & 23.4931 & -15.8108 & -6.2181 \\
\end{bmatrix}
$$

(30)

4. Simulation Results

Matlab is a software that used to know the effectiveness of quadcopter path following. Quadcopter is simulated by testing its movement, whether it is capable to doing the following path for square path as a tracking reference for its movement. The heading of quadcopter is considered to ensure the smoothness movement and maintain its heading as shown in Figure 2.

With adding Line of Sight (LOS) algorithm to calculate the heading of quadcopter, therefore yaw angle ($\psi$) can be calculated too. By observing the yaw angle, quadcopter can keep up the reference signal from the output controller of the outer loop with an average cross track error of 0.0245 rad. The comparison between reference and the actual yaw angle of quadcopter, it can be concluded that the heading is maintained well can be seen in figure 3.

**Figure 2.** Quadcopter Path Following on X and Y-axis

**Figure 3.** Quadcopter Path Following on Yaw Angle.

Quadcopter position on the X-axis which is able to follow the reference can be seen in figure 4. Path following is start at t=15s and the delay is about 2 seconds, that cause error of 0.3 m however the quadcopter still can follow the reference path.

**Figure 4.** Quadcopter Path Following on X-axis.

**Figure 5.** Quadcopter Path Following on Y-axis.

**Figure 6.** Comparison between Reference and Actual Tour on Square Path.
The movement on the Y-axis where the quadcopter has a delayed about 0.4 seconds at the first waypoint and it cause an error path about 0.3 m but the quadcopter can return to the reference path in the Figure 5.

Square shaped reference signal is also added in to the system, as shown in Figure 6. The result is, the quadcopter keeps up follow the path, but there is a deviation occurred at X=0.5 m and Y=0.2 m. LOS algorithm assists the quadcopter controller by giving heading to turn at an angle of 90° smoothly and keeping on track to the reference.

5. Conclusion
The output feedback control method with $H_{\infty}$ performance is used in the inner loop while the Command-Generator Tracker structure controls the model following by adding the LOS algorithm used on the outer loop. The control system which using output feedback with command generator tracker based on LOS on the Path Following Quadcopter problems, it can provide performance as desired.

The quadcopter is able to follow the reference signal. Furthermore, the heading of the quadcopter is also able to follow according to the reference signal obtained from Line of Sight algorithm (LOS). The value of Integral Square Error (ISE) path following position is 0.3 m. The use of line of sight algorithm (LOS) assist to overcome quadcopter movement as it passes through 900 angles to become smoother with the average of cross track error is X=0.5 m and Y=0.2 m. The proposed control system quadcopter also can follow according to the reference signal in the square path.

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