A Physical Model for a Radiative, Convective Dusty Disk in AGN

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Abstract

An accretion disk in an Active Galactic Nucleus harbors and shields dust from external illumination. Our model shows that, at the midplane of the disk around an $M_{BH} = 10^7 M_\odot$ black hole, dust can exist at 0.1 pc from the black hole, compared to 0.5 pc outside of the disk where such self-shielding is constrained. We construct a physical model of a disk region approximately located between the radius of dust sublimation at the disk midplane and the radius at which dust sublimes at the disk surface. Our main conclusion is that, for a wide range of model parameters such as local accretion rate and/or opacity, the accretion disk’s own radiation pressure on dust significantly influences its vertical structure. In this region, convection plays an important role in the vertical transport of energy. When the local accretion rate exceeds $2.5 M_\odot/yr$, the 10$^{-2}$ pc scale disk is supercritical with respect to dust opacity. Such a disk puffs up and transforms from geometrically thin to slim. Our model fits into the narrative of a “failed wind” scenario of Czerny & Hryniewicz and the “compact torus” model of Baskin & Laor, incorporating them as variations of the radiative dusty disk model.

Unified Astronomy Thesaurus concepts: Active galactic nuclei (16); Galaxy accretion disks (562); Astrophysical dust processes (99)

1. Introduction

The radiative output of Active Galactic Nuclei (AGN) is powered by accretion of gas, and most of the gas potential energy is released in the inner part of an accretion disk, i.e., within a few hundred gravitational radii, $R_g \approx \frac{2.95 \times 10^2}{M_\odot} M_7$ cm, where $M_7$ is the black hole (BH) mass in units of $10^7 M_\odot$. However, several crucial observational characteristics of AGN, such as the broad-line region (BLR) and the obscuring torus, are shaped considerably further away, at few $\times 0.01$–0.5 pc from the black hole. It is also at approximately this distance from the BH the radiation flux is sufficiently diluted that dust grains can survive the illumination from the nucleus. The presence of dust results in a 10–100 fold increase of opacity compared with only gas, which leads to a dramatic increase of coupling between the radiation from the nucleus and gas.

The radius outside of which dust can survive forms a dust sublimation surface, a boundary between the inner, mostly dust-free region and the outer part often associated with the dusty torus. The latter is invoked to explain the dichotomy between two types of AGN, in which optically thick equatorial material blocks the direct view onto the broad-line region and the accretion disk in type 2 galaxies (Rowan-Robinson 1977; Antonucci 1984; Antonucci & Miller 1985; Urry & Padovani 1995).

Mid-infrared (MIR) interferometric observations of nearby AGN clearly point to the presence of dust (Jaffe et al. 2004; Raban et al. 2009; Tristram & Schartmann 2011; Tristram et al. 2014) at distances $\geq 0.1$ pc from the center. The relative numbers of type 1 and type 2 objects suggest that the obscuring material is geometrically thick.

The virial theorem predicts that, in order to be geometrically thick at a distance, $r \approx 1$ pc, the temperature of the obscuring gas should be $\sim 10^6 K$ for a $10^7 M_\odot$ BH. This is not compatible with survival of dust in the obscurer (Krolik & Begelman 1988), and hence is in conflict with the presence of dust inferred from IR observations. On the other hand, the temperature expected at the surface of a thin accretion disk is $\leq 1000$ K at $\sim 1$ pc from the center.

Near-infrared reverberation mapping (RM) and interferometry is generally consistent with putting the inner boundary of the torus to within the dust sublimation surface at 0.4–0.5 pc (i.e., Kaspi et al. 2000; Koshida et al. 2014). The location of the BLR relative to the center has been measured by RM to be $\sim$ few $\times 0.01$ pc (e.g., Peterson et al. 2004; Suga et al. 2006), with a relatively established dependence on the luminosity: $R_{BLR} \approx 0.1 L_{12}^{1/2}$ (Kaspi et al. 2005, 2007) in the luminosity range $10^{40} - 10^{48}$ erg s$^{-1}$.

Magnetic or/and radiation driving have been proposed as a mechanisms behind the formation of the BLR and the torus. A line-driven wind, i.e., a wind driven by the radiation pressure in UV lines (Proga & Kallman 2004; Murray et al. 2005), has a launching radius that is $\sim 10\times$ smaller—and correspondingly, characteristic line widths that are a factor of $\sim 10\times$ greater—than indicated by the maximum BLR line widths. A line-driven wind from an accretion disk is not massive enough to be the torus.

Magnetic fields, specifically large-scale magnetic fields, are an alternative or augmenting mechanism that can be the driving engine of the BLR and the torus. Semi-analytical or numerical models show that large-scale B-field can potentially support an AGN torus whether in a form a wind (e.g., Emmering et al. 1992; Konigl & Kartje 1994; Everett 2005; Keating et al. 2012) or a quasi-static torus with a nested magnetic structure (Lovelace et al. 1998; Dorodnitsyn et al. 2016; Chan & Krolik 2017).

With better understanding of how large-scale magnetic fields are transported by accretion, the models for tori driven by magnetohydrodynamics (MHD) are challenged, at least with regard to the ubiquity of the torus phenomenon across the AGN population. Self-consistent numerical simulations of a thin disk threaded by a net vertical magnetic flux (Zhu & Stone 2018) show that thin disks cannot both transport large-scale magnetic
fields (Lubow et al. 1994; Bisnovatyi-Kogan & Lovelace 2000, 2007) and simultaneously have a massive, polar, MHD-driven outflow. In addition, typical MHD flows have an approximate equipartition between magnetic energy density and gas energy. Thus, the characteristic temperature of the gas near the launching radius of a magnetically driven, gravitationally unbound outflow is also expected to be on the order of the virial temperature.

Czerny & Hryniewicz (2011) (hereafter CH11) suggested that the disk’s own radiation may produce sufficient radiation pressure on dust grains that would expel a failed wind from the outer accretion disk atmosphere. Such a failed wind then would be responsible for the formation of the broad-line region seen in Seyfert I galaxies. Developing this idea further, Baskin & Laor (2018) (BL18) concluded that the contribution from large graphite grains near $T \approx 2000$ K increases dust opacity, which results in an inflated compact, torus-like structure near the observed BLR radius.

In a simple case, when dust is arranged in a spherically symmetric shell, it cannot survive closer than the dust sublimation radius, $R_{\text{sub}}$. However, $R_{\text{sub}}$ depends on the AGN luminosity and dust composition and there are several characteristic values adopted in the literature. In approximate terms, $R_{\text{sub}} \approx 1.3 \sqrt{L/4\pi}$ pc, where $L_{46} = L(\text{erg s}^{-1})/10^{46}$, $T_{\text{sub}} = 1500$ K, and the grain size $a = 0.05 \mu\text{m}$ (e.g., Barvainis 1987). For larger graphite grains, $R_{\text{sub}} \approx 0.2 \sqrt{L/4\pi}$ pc (e.g., Baskin & Laor 2018). For an AGN with $10^{43}$ erg s$^{-1}$, $R_{\text{sub}}$ is correspondingly smaller: $R_{\text{sub}} \approx 10^{-2}$ pc (e.g., Netzer & Trakhtenbrot 2014).

If dust is contained in a cold and dense accretion disk it can survive much closer to the BH, down to $\sim 10^{-3} - 10^{-2}$ pc (BL18, CH11). The radial scaling of the effective surface temperature of the disk: $T_{\text{eff}} \propto R^{-3/2}$ guarantees that, beyond a certain radius, $R_{\text{sub}}$, $T$ drops below the dust sublimation temperature allowing survival of dust. This corresponds to a dramatic increase of the opacity. Radiation flux within the disk, i.e., excluding external illumination, $F_{\text{esc}}$, may be strong enough to produce a non-negligible radiation pressure $\propto (F_{\text{esc}}/\epsilon)\kappa_d$, where $\kappa_d$ is dust opacity. As suggested by CH11 and BL18, this can lead to the formation of failed winds or produce a “compact torus” at few $\times 10^{-2}$ pc.

Analytic models for a 2D radiation-supported pc-scale torus require significant, often critical simplifications. Krolik (2007) neglected gas pressure, and after making a number of additional parameterizations, obtained a model of a geometrically thick, radiation-supported torus. Dorodnitsyn et al. (2011) realized that a torus supported by rotation and the pressure of the reprocessed radiation cannot sustain a static equilibrium. Dorodnitsyn & Kallman (2012) and Namekata & Umemura (2016) addressed the response of a preexisting disk-like density distribution to the external illumination using 2D radiation hydrodynamic torus models. This demonstrated the existence of outflowing solutions, and a class of “dynamic torus” models was proposed.

Such models did not include the pressure from the radiation produced locally in the disk due to “viscous” dissipation. Although Dorodnitsyn et al. (2016) included effective viscosity for more realistic angular momentum transport, the pc scales they considered implied that the locally generated radiation flux is very small. Similarly, Chan & Krolik (2016, 2017) included magnetic fields and thus took into account more realistic angular momentum transport. The regions considered were roughly the same, implying the relative unimportance of locally generated radiation.

Previous work has demonstrated the likely importance of dust in the dynamics of the gas in the torus and BLR. However, there has not been an examination of the effects of dust on the internal structure of the accretion disk in the 0.1–0.01 pc scale region of AGN. In the following analysis, we ignore self-gravitating instabilities inside the disk. For the analysis of self-gravitating disk, but without the effects considered in the current work, see, e.g., Goodman (2003). The goal of this paper is to explore the region of the disk where its temperature is comparable to the dust sublimation temperature, $T_{\text{sub}}$, and how radiation pressure on dust grains defines the vertical structure of such a disk.

The structure of this paper is as follows. In Section 2, we begin to examine the effects of the disk’s own radiation pressure on dust on the disk vertical structure. In Section 3, we further derive properties of the Active Disk Region. In Section 4, we calculate a detailed analytical and numerical solution of the disk equations. In Section 5, we show that, for a wide range of parameters, the radiative dusty disk is convectively unstable. In Section 6 we discuss the effects of external irradiation. The results are summarized in Section 7, along with the limitations of our approach, and the ideas developed in this paper are put into the context of the broader AGN accretion disk physics. We conclude in Section 8.

2. Properties of AGN from pc to $10^{-2}$ pc Scales

In this section, we examine the effect of dust formation or survival and its dependence on the global parameters of the AGN. In doing so, we adopt standard assumptions about accretion in a thin disk around a BH (Shakura & Sunyaev 1973; Lynden-Bell & Pringle 1974).

2.1. Global Parameters

It is customary to express the total luminosity of the AGN in terms of the accretion rate,

$$L = \epsilon \dot{M} c^2,$$

which is equivalent to the definition of an accretion efficiency, $\epsilon = \dot{\alpha}$, a parameter that is approximately bounded between 0.057 for a nonrotating BH and 0.42 for a BH rotating at maximum efficiency. The mass-accretion rate, $\dot{M}$, in (1) corresponds to the innermost part of the disk where the most of the radiative output is produced. After assuming $\epsilon$, it is standard to equate (1) to the Eddington luminosity:

$$L_E = \frac{4\pi G M}{\kappa_e} = 1.25 \times 10^{37} M_7 \text{erg s}^{-1}.$$

Here, $M$ is the mass of the BH, and to scale accretion rate in terms of “Eddington” accretion rate,

$$\dot{M}_E = 0.22 \epsilon^{-1} \dot{M}_T M_7 \text{yr}^{-1},$$

where in (2) and (3), the following parameters are adopted: $\kappa_e = 0.4 \text{cm}^2 \text{g}^{-1}$ is the Thomson opacity; we scale the BH mass in units of $M_7 = M/10^7 M_\odot$; and we fix the efficiency of accretion $\epsilon = 0.1$.

The above picture is augmented with the assumption that enough medium is supplied to the AGN from galactic scales near the AGN outer radius, $R_{\text{AGN}}$. Notice that there are no
obvious physical reasons why accretion cannot proceed beyond $R_{\text{AGN}}$. It is customarily defined as a “sphere of influence” of the BH, i.e., a radius where the gravity from the BH dominates the gravitational field of the host galaxy:

$$R_{\text{AGN}} = \frac{GM}{\sigma_{\text{Big}}^2} \simeq 4.30 M_7 \left(\frac{\sigma_{\text{Big}} \text{ (km s}^{-1}\text{)}}{100}\right)^{-2} \text{ pc}$$

$$= 4.5 \times 10^8 \left(\frac{\sigma_{\text{Big}} \text{ (km s}^{-1}\text{)}}{100}\right)^{-2} R_g. \quad (4)$$

Here, $\sigma_{\text{Big}}$ is the stellar velocity dispersion in the bulge, and the last equality is given in terms of the Schwarzschild gravitational radius of the BH:

$$R_g = \frac{2GM}{c^2} = 2.95 \times 10^{12} M_7 \text{ cm.} \quad (5)$$

Hereafter, we reserve $R$ for the radius in physical units, and $r$ for the scaled radius: $r = R/y$.

A crude estimate for the vertical scale height of the cold disk at $R_{\text{AGN}}$ is done assuming the scaling for the disk height: $H \sim v_T/\Omega$, where $H$ is the half-thickness of the disk, $\Omega = (GM_{\text{BH}})^{-3/2}$ is the orbital velocity, and $v_T$ is the isothermal sound speed, $v_T = (R_{\text{gas}} T/\mu m)_{\text{iso}}^{1/2}$, where $\mu m$ is the mean molecular weight and $R_{\text{gas}}$ is the gas constant. Hereafter, while calculating disk properties, we neglect the disk self-gravity. In the following sections, we present a more precise calculation of the disk thickness, $H$. Here, a simple estimate of $H(R_{\text{AGN}})$ is obtained while neglecting external illumination and assuming the disk is not warped and that in the vertical direction it is supported by gas pressure with constant temperature $T$:

$$H/R_{\text{AGN}} \simeq 6.45 \times 10^{-3} \frac{\sigma_{\text{Big}}^4}{\text{100}} T_{\text{iso}}^{1/2}. \quad (6)$$

The disk is very thin, and consequently it intercepts only a small fraction of the radiation flux from the nucleus. Thus, the scale height of the disk atmosphere can be affected by heating by the radiation from the nucleus, $F_{\text{ext}}$, which depends on the attenuation history along the way from the inner parts of the disk to a pc-scale disk.

The radiative energy density generated locally in the disk can be compared with that from external illumination. The radiation flux at pc scales is dominated by the flux from the nucleus, $F_{\text{ext}}$, which is produced in the inner parts of the disk. However, the flux incident on the disk surface depends on its angular dependence, the manifestation of the limb darkening effect in the disk, and has a simple dependence on $\mu = \cos \theta$, where $\theta$ is the inclination angle from the normal to the disk (i.e., Sobolev 1975; Sunyaev & Titarchuk 1985):

$$F_{\text{ext}} \simeq 6 \times 10^9 f(\theta) M_{0.1,0.1} r_{0.1}^{-1} \text{ erg cm}^{-2} \text{ s}^{-1} \gg F_{\text{loc}}, \quad (7)$$

where $f(\theta) = \mu(2\mu + 1)$ is the angular dependence of the radiation flux. The local radiation flux generated in the disk, i.e., the normal flux at the disk’s photosphere reads

$$F_{\text{loc}} = \frac{3}{8\pi} \frac{GM}{r^3} \simeq 3.4 \times 10^4 r_{0.1}^{-3} M_{\text{loc},0.1} \times M_7 \text{ erg cm}^{-2} \text{ s}^{-1}, \quad (8)$$

and it follows that, until matter can spiral down to a fraction of a parsec, the release of the gravitational potential energy produces local radiative output that is negligible for the gas dynamics. Radiation flux, $F_{\text{ext}}$, depends on the mass-accretion rate in the inner disk, while $F_{\text{loc}}$ depends on local accretion rate, and in the following, we reserve $M$ for global and $M_{\text{loc}}$ for local accretion rates. In this paper, we are interested in how the vertical structure of the $r \sim 10^{-2}$ pc disk is changed in response to the pressure on dust of the locally generated radiation flux. We will see that this can affect the disk scale height at $r \sim 10^{-2}$ pc. Notice that the methods adopted in this paper do not allow us to reliably calculate the effects of the external irradiation. However, if the local irradiation flux is known, some of the approximate conclusions can be derived from the theory in Section 6.

Accretion in a thin disk far from a BH is slow. The freefall timescale is the shortest in the hierarchy of timescales: $t_{\text{dyn}} = 1.49 \times 10^2 r_{0.1}^{3/2} \text{ yr}$. However, the accretion timescale corresponds to the viscous timescale $t_{\text{visc}}$ in the disk, $t_{\text{visc}} \propto \tau_{\text{visc}} = \frac{R}{v_r} = 5.15 \times 10^7 r_{0.1}^{-1/2} T_{1500}^{-1} \alpha_{0.1}^{-1} \text{ yr}, \quad (9)$

where $\alpha < 1$ is the effective viscosity parameter, introduced by Shakura (1972), and $\alpha_{0.1} = \alpha/0.1$. A geometrically thin disk cools efficiently through radiative losses, and as the disk cooling time is much smaller than $t_{\text{visc}}$, i.e., $t_{\text{visc}} = 1/\alpha < t_{\text{visc}} = 1.492 \times 10^2 r_{0.1}^{-1} \alpha_{0.1}^{3/2} \text{ yr}$, it is approximately $t_{\text{dyn}} \lesssim t_{\text{visc}}$. From (9), it is clear that a buildup of matter (in a thin disk) at large radii cannot quickly propagate through the disk toward smaller radii. Hence, we allow the situation in which the local accretion rate exceeds the rate at the center, $M_{\text{loc}} \simeq M$. The role of the local accretion rate $M_{\text{loc}}$ is to define the local rate of energy production in the disk—and correspondingly, the local vertical radiation flux, $F_{\text{loc}}$, in (8).

### 3. Solution for the Disk Vertical Structure

In this section, we build a simple model of an accretion disk that takes into account the disk’s own radiation pressure associated with the formation of dust in the disk, and the corresponding changes to the physical properties of such a disk.

#### 3.1. Basic Equations

In this section, we describe the details of our model used to derive estimates in the previous sections. We adopt the $\alpha$–disk theory of Shakura & Sunyaev (1973) (henceforth SS73) and include the pressure of infrared (IR) radiation on dust in the disk interior. The radiation is produced by viscous dissipation and assumed to be in thermodynamic equilibrium with a gas–dust mixture. Dust particles are assumed to be fully coupled to the gas. In realistic disk atmospheres, dust temperature, $T_{\text{dust}}$, can significantly deviate from the gas temperature, $T_{\text{gas}}$. In the bulk of the disk, the dust is well-coupled to the gas, and we expect that the crude assumption:

$$T_{\text{gas}} = T_{\text{dust}} = T \quad (10)$$

leads to qualitatively correct results. The equation of state is that of a mixture of ideal gas and radiation:

$$P = P_g + P_r \quad (11)$$

where $P$ is the total pressure, $P_r$ is the radiation pressure, $P_r = aT^4/3, \quad (12)$
and $P_g$ is the gas pressure,
\[ P_g = \rho RT, \]
where $\alpha$ is the radiation constant, and to simplify notation such as in (13), the mean molecular weight, $\mu_m$, is absorbed in the definition of the gas constant, $R = R_{\text{gas}}/\mu_m$, and in the rest of the paper we adopt $\mu_m = 1$ (see also Glossary). It is important to note that, hereafter, except for Section 6, we neglect external irradiation and radiation pressure is calculated assuming thermodynamic equilibrium between gas and radiation. Analytic solutions for the limiting cases of $P_g \ll P_r$ and $P_g \gg P_r$ are derived in Shakura & Sunyaev (1973). Here, we will solve the disk equations for arbitrary $P_g$ and $P_r$, assuming constant opacity.

We assume that, in the ADR region, accretion proceeds near the equatorial plane,
\[ \dot{M} = 2\pi rv \Sigma. \]
To simplify notation in this section, $\dot{M} = \dot{M}_{\text{loc}}$ is the time averaged accretion rate, $v$ is the radial gas velocity, and $\Sigma$ is the surface density,
\[ \Sigma = \int_H^H \rho \, dz \approx 2H \rho, \]
where $H$ is the vertical scale height of the disk and $\rho = \rho (z = 0)$. That is, in a single-layer disk model, all height-dependent quantities are taken at the disk midplane,
\[ \nu \Sigma = \frac{1}{3\pi} \dot{M}, \]
where we neglected a factor related to the inner boundary condition, (i.e., $J(R)$ in (28)). The viscosity $\nu$ in (47) is proportional to the total pressure, $P$:
\[ \nu = \alpha \frac{P}{\Omega \rho}. \]

The vertical gravitational acceleration, $g_z$, is found from
\[ g_z = \Omega^2 \zeta. \]
The disk, throughout this paper, is assumed to be Keplerian, $\Omega = \Omega_k$, where
\[ \Omega_k = \sqrt{(GM \, R^{-3})^{1/2}}. \]
The only relevant component of the radiation flux in a pc-scale thin disk is the vertical flux $F$, which satisfies
\[ \frac{dF}{dz} = \frac{9}{4} \rho \, \nu \, q_v \equiv \frac{9}{4} \Omega^2 \rho \, \nu, \]
where $q_v (\text{erg s}^{-1} \, \text{g}^{-1})$ is the specific rate of viscous energy dissipation,
\[ q_v = \Omega^2 \nu. \]
Integrating Equation (20) between $-H$ and $+H$, one gets the vertical radiation flux from the disk surface:
\[ F^+ = \frac{3}{8\pi} \dot{M} \Omega^2. \]

An implicit assumption was made when integrating (20) to obtain (22). That is, after Equation (20) was rewritten as
\[ \frac{df}{dz} = \frac{9}{4} q_v, \]
where $f$ is the mass coordinate, and then integrated over height, it was assumed that the rate of specific viscous dissipation, $q_v$, is constant.

The radiation moment equation in a plane-parallel case valid for the geometrically thin disk is
\[ \frac{\partial E}{\partial \tau} = \frac{3}{c} F, \]
where $E$ and $F$ are radiation energy density and radiation flux and $\tau$ is the vertical optical depth.

The boundary condition for (24) is $E(\tau = 0) = 2F^+ / c$. Integrating (24) results in $E = (3F^+ / c)(\tau + 2/3)$ and
\[ \sigma_r T^4 = \frac{27}{64} \kappa \nu / \Omega^2 \Sigma^2, \]
where $\kappa$ is the opacity of the gas-dust mixture. Introducing the subscript “c” for midplane quantities. From (25), it follows that, if $\tau_c \gg 1$, then
\[ T_c \propto \tau_c^{1/4} T_s, \]

### 4. Dust Sublimation Region in a Disk

#### 4.1. The Inner and Outer Dust Sublimation Scales

In order to calculate the structure of a thin disk, the assumption was made in SS73 that viscous dissipation is proportional to the gas density $\rho$. Here, we assume that all the dissipated energy is transported vertically by radiation. This gives the following estimate for the temperature at the disk surface:
\[ T_s = 879 \left( \frac{M_s \dot{M}_{\text{loc},0.1}}{n_{\text{H},0.1}^3} \right)^{1/4} \text{K}, \]
where it was assumed that $T_s = T(\tau_{\text{phcoh}} = 2/3)$, in which $\tau_{\text{phcoh}}$ is the optical depth at the disk photosphere. When $R = 10^{-3}$ pc, the surface temperature reaches 1479 K and little dust can survive in the disk.

If external illumination is neglected, the vertical decrease of temperature within a disk ensures that the midplane temperature, $T_c$, is always greater than the surface temperature $T_s$. We approximate that $T_c$ is a factor of $\tau_c^{1/4}$ greater than $T_s$, where $T_c$ is the vertical optical depth of the disk (see Section 3). In general, $\tau_c$ should be calculated from the solution for the vertical structure of the disk. From the standard gas-pressuredust only $\alpha$-disk solution, we get (see SS73, Equation (2.19))
\[ T_c = \frac{1}{2} \left( \frac{3}{2} \right)^{1/4} \kappa^{1/5} J(R)^{2/5} \mu_{\text{m}}^{1/5} M_{\text{loc}}^{2/5} \]
\[ \times \pi^{-6} \Omega^{5/2} R^{-1/5} \alpha - \sigma_\text{b}^{-1/4} \]
\[ \approx 2 \times 10^{3} \kappa^{1/5} M_{\text{f}}^{1/10} M_{\text{loc},0.1}^{2/5} \nu_{0.1}^{-9/10} \text{K}. \]
Here, the factor $J(R)$ is related to the inner boundary condition near the BH, and for subparsec distances, it is $J \approx 1$ to a good
Increasing the mass of the BH, we obtain

\[ R_{\text{in}} \approx 5 \times 10^{-3} M_{\text{BH},0.1}^{1/3} T_{27}^{-4/3} \text{ pc} \]

\[ \approx 5 \times 10^{-3} M_{7}^{2/3} M_{\text{loc},0.1}^{1/3} T_{1500}^{-4/3} R_{\text{g}}, \] (29)

where \( T_{\text{sub},1500} \) is the dust sublimation temperature in units of 1500 K. Similarly, from (28), we define an outer sublimation radius, \( R_{\text{out}} \), as such a radius that, at \( R \approx R_{\text{out}} \), dust sublimes at the midplane:

\[ R_{\text{out}} \approx 0.14 M_{7}^{1/3} M_{\text{loc},0.1}^{1/3} T_{1500}^{-4/3} \text{ pc} \]

\[ \approx 1.5 \times 10^{5} M_{7}^{4/9} M_{\text{loc},0.1}^{1/9} T_{1500}^{-2/3} \frac{2}{3} R_{\text{g}}. \] (30)

Since \( R_{\text{out}} \) is found from \( T(\Sigma_{c}) = T_{\text{sub}} \), the disk’s column density, \( \Sigma_{c} = 2\rho_{0} x \), can be found from the solution for the disk.

As the gas spirals to \( R \approx R_{\text{out}} \), the disk becomes hotter than the dust sublimation temperature \( T_{\text{sub}} \). Dust is destroyed first near the midplane, and then as \( r \) gets smaller, progressively above. Naturally, \( R_{\text{out}} > R_{\text{in}} \), defining the region within the disk where hot dust exists at some height in the disk. Notice that, at \( R = R_{\text{out}} \), the disk surface is quite cool: \( T_{c} \approx 156(M_{7} M_{\text{loc},0.1})^{-1/3} R_{\text{g}}^{-2/3} \text{ K} \). Such a difference between \( T_{c} \) and \( T_{1500} \) is essentially the result of the blanketing effect from the disk. Dependence of \( R_{\text{in}} \) and \( R_{\text{out}} \) on \( M_{7} \) follows if we assume the scaling \( M_{\text{loc}} \propto M \propto L_{\text{edd}} \propto M^{2/3} \):

\[ R_{\text{in}} \propto M^{2/3}, \] (31)

\[ R_{\text{out}} \propto M^{7/9}. \] (32)

Increasing the mass of the BH, we obtain \( R_{\text{in}} \approx 0.1 \text{ pc} \) and \( R_{\text{out}} \approx 5 \text{ pc} \) for \( M = 10^{9} M_{\odot} \). Thus, local radiation pressure on dust is important in an “Active Dusty Region” (ADR), which is approximately bounded at the outside by the dust sublimation radius on the disk midplane and on the inside by the dust sublimation radius at the disk surface.

The accretion rate near the BH, \( M_{7} \), determines the central luminosity \( L \) through (1), and thus the global dust sublimation radius \( R_{\text{sub}} \):

\[ R_{\text{sub}} = 0.13 \left( \frac{f(\theta) c_{0.1} M_{0.1}}{T_{1500}^{2/3}} \right)^{1/2} \text{ pc}, \] (33)

where \( f(\theta) = \mu(2\mu + 1) \) is the angular dependence of the radiation flux from the nucleus, i.e., (67). The result is the dust sublimation surface—which, generally speaking, is different from a simple spherically symmetric case such as (33) if calculated for \( f = 1 \). Notice that there is a difference between our definition of \( R_{\text{out}} \) in (30) and the definition of BL18. The latter define \( R_{\text{out}} \) as the dust sublimation radius for AGN as in (33) with \( f = 1 \) because their work studies the size of the BLR rather than the structure and properties of the disk itself. As long as energy is transported vertically via radiation, it follows from (30) and (29) that \( R_{\text{out}} \propto M_{7}^{1/9} R_{\text{in}} \). Another interesting scaling, \( R_{\text{sub}} / R_{\text{in}} \propto (M_{7} M_{\text{loc},0.1})^{1/2} \), follows from (30) and (33).

Without the shielding protection of the accretion disk, the fate of dust above such disk depends on whether it is inside or outside the AGN dust sublimation surface \( R_{\text{sub}}(\theta) \):

1. If \( R_{\text{sub}} \gtrsim R_{\text{in}} \), the dust above the disk does not survive.
2. If \( R_{\text{in}} \lesssim R_{\text{sub}} \lesssim R_{\text{out}} \), depending on \( M_{\text{loc}} \), the disk can be (1) thin or (2) thick/outflowing (this situation is illustrated in Figure 2).

When \( M_{\text{loc}} \) exceeds

\[ M_{\text{loc}}(R_{\text{sub}} = R_{\text{out}}) = 0.08 \left( \frac{M_{7}^{1/3} T_{1500}^{8/9} R_{\text{out}}^{2/9}}{c_{0.1}^{2/9} M_{0.1}^{1/2}} \right)^{-9/4} \times M_{0.1}^{-9/7} \text{ yr}^{-1}, \] (34)
4.2. Scale Height of the Radiation-pressure-supported Disk

Assuming a thin disk approximation, the equation for vertical balance reads:

\[
\frac{dP_z}{dz} = -\rho g_z + \rho \frac{F_{\text{loc}} \kappa}{c},
\]

where \( \kappa \) is the opacity of the accreting material, which is assumed to be comparable to that of dust, \( \kappa_d \).

In the case when radiation pressure dominates, the characteristic scale height of the disk follows from (35) after neglecting the contribution from the gas pressure:

\[
H/R \simeq \frac{3}{2} \frac{M_{\text{loc}}}{M_{\text{loc,cr}}}. \tag{36}
\]

Here, \( M_{\text{loc,cr}} \) is the familiar Eddington accretion rate, but calculated using dust opacity \( \kappa_d \):

\[
M_{\text{loc,cr}} = \frac{4\pi c R}{\kappa_d}. \tag{37}
\]

The importance of \( M_{\text{loc,cr}} \) is that, when \( M_{\text{loc}} \simeq M_{\text{loc,cr}} \), the disk is locally geometrically thick. Characteristic values of \( M_{\text{loc,cr}} \) depend on \( r \) and on \( \kappa_d \). As the local accretion rate directly influences the disk height, it is instructive to estimate \( M_{\text{loc,cr}} \) at the inner and outer scales of the ADR. In the region of the disk where dust still exists, the smallest possible value of \( M_{\text{loc,cr}} \) is possible at the smallest possible \( r \approx R_{\text{in}} \approx 0.01 \). The disk at \( R_{\text{in}} \) is geometrically thick when \( M_{\text{loc}} \) is comparable to

\[
M_{\text{loc,cr}} \simeq \left\{ \begin{array}{ll}
12.3 \kappa_1^{-1} r_{0.01} M_{\odot} \text{ yr}^{-1}, \\
2.5 \kappa_{50}^{-1} r_{0.01} M_{\odot} \text{ yr}^{-1},
\end{array} \right.
\]

where \( \kappa_s = \kappa / x \) is the opacity scaled in \( \text{cm}^2 \text{ g}^{-1} \) units of opacity. In the second case in (38), the dust opacity reflects the contribution from large graphite grains (BL18). If the disk is such that \( H/R \sim 1 \) at \( R_{\text{out}} \) and \( M_{\text{loc}} \) is correspondingly larger: \( 25 \kappa_{50}^{-1} M_{\odot} \text{ yr}^{-1} \).

For the same set of parameters, the critical value of \( \dot{M} \) calculated assuming only electron scattering opacity is \( M_{\text{crit}} \approx 0.2 M_{\odot} \text{ yr}^{-1} \). For the disk not to be super-Eddington globally, the excess mass of the gas \( \sim \text{few} M_{\odot} \text{ yr}^{-1} \) should be expelled via winds along the way toward the BH.

If the thick disk is supported by radiation pressure, the condition \( H/R \propto 1 \) can be recast as a consequence of the virial theorem for the disk temperature, namely \( T \simeq T_{\text{vir,r}} \), where \( T_{\text{vir,r}} \) is the “virial” temperature for the radiation dominated medium (Dorodnitsyn et al. 2011):

\[
T_{\text{vir,r}} = \left( \frac{GMp}{aR} \right)^{1/4} \simeq 1755.93 \left( \frac{M_{\odot}(n/10^3)}{r_{0.1}} \right)^{1/4} \text{K}, \tag{39}
\]

derived for a spherically symmetric shell.

4.3. Solution with Radiation Pressure

The solution for \( T \) for a gas-dominated disk was given in (28). As \( T \) approaches \( T_{\text{sub}} \), radiation pressure becomes important. When \( T_r > T_{\text{sub}} \), a gas-only layer at the midplane is enveloped from above by a layer of the dusty gas. From above, its temperature is bound by \( T^+ \leq T_{\text{sub}} \) (see also the Discussion), and the transition from gas-dominated layer to dust-dominated envelope is happening at the height \( H_g \). The estimate of scale height \( H_g \) follows from (35):

\[
H_g/R \simeq \left( \frac{P_r - P + \rho^\gamma \Omega^2}{\rho \gamma \Omega^2} \right)^{1/2} \simeq 5.4 \times 10^{-3}(T_{1500} r_{0.1}/M_{\odot})^{1/2}, \tag{40}
\]

i.e., the gas layer is very geometrically thin.

It is beyond of the scope of this paper to calculate the vertical structure of the disk with the gas-to-dust transition. Instead, we adopt a single-layer approximation with the temperature-dependent opacity (see further (52)). Thus, in the following, our goal is to calculate the average properties of a disk that is supported by an arbitrary combination of gas and radiation pressure: \( P = P_{\text{gas}} + aT^4/3 \).

Assuming the relation:

\[
\rho = \frac{\Sigma}{2H},
\]

then the total pressure, \( P \) (remember that the molecular weight is absorbed in the gas constant \( R \)) is

\[
P = \rho \left( RT + \frac{2aT^4H}{3\Sigma} \right). \tag{42}
\]

The approximate relation for the scale height follows from the equation for the vertical balance:

\[
\frac{dP}{dz} = -\rho g_z,
\]

which is equivalent to (35) after noticing that, in the diffusion approximation,

\[
F = -c/(\kappa \rho) dP_r/dz,
\]

and thus recovering:

\[
H^2 = \frac{P}{\Omega^2 \rho}. \tag{45}
\]

Equation (45) is the equivalent of the vertical balance equation. Substituting (42) and (41)–(45), there follows a useful relation:

\[
\Sigma = \frac{2aHT^4}{3(H^2\Omega^2 - RT)}. \tag{46}
\]

After substituting (42) and (41)–(17), there follows another relation:

\[
\nu = \frac{\alpha(2aHT^4 + 3R\Sigma T)}{3\Sigma \Omega}. \tag{47}
\]

The third useful relation, which we can get from a system of two equations for \( \Sigma \) and \( H \) (and \( T \)), is obtained by first inserting \( \nu \) from (25) into (16), repeating by inserting \( \nu \) from (47) to (16), and then solving these two equations for \( H \):

\[
H = \frac{4F^+ - cRT}{3a\Omega T^4} - \frac{F^+}{F^+}. \tag{48}
\]

We now can use (42) and (41) with (45)–(48), and after much algebra to obtain an equation for \( T \):

\[
4(F^+)^2\kappa(6a\alpha c^2\Omega^2 T^3 + (F^+)^2\kappa(3a\alpha T^4 - 4c\Omega)) - 9a^2\alpha^2 c^3 R^2 T^{10} \Omega^3 = 0. \tag{49}
\]
Equation (49) can be further simplified down to an equivalently rather compact form:

\[
1 - \frac{3a\alpha c R\Omega}{4(F^+)^2 \kappa} T^5 - \frac{3a\alpha \kappa}{4c\Omega} T^4 = 0. \tag{50}
\]

Thus, the solution for the disk temperature should be calculated from the nonlinear algebraic Equation (50). It can be shown that, if radiation pressure is neglected, the term in the brackets in (50) is left and the corresponding expression for the gas pressure alpha disk is recovered:

\[
T_c(P_g) = \frac{(3\kappa)^{1/5}(M)^{2/5}\Omega^{3/5}}{2^{4/5}\pi^{2/5}(a\alpha c)^{1/5}}, \tag{51}
\]

from which, for example, the numerical scaling (28) can be calculated.

### 4.4. Numerical Solution

At a given \( r \), with \( F^+ \) calculated from (22), \( \kappa \) from (52), and \( \Omega \) from (19), Equation (50) is solved numerically. However, some tricks are required to ensure numerical stability. First, we assume that, above \( T_{\text{sub}} \), the opacity switches from \( \kappa_d + \kappa_e \) to \( \kappa_e \), and we approximate \( \kappa(T) \) using a bridging formula:

\[
\kappa(T) = \frac{\kappa_d}{\exp \left( \frac{T - T_{\text{sub}}}{\Delta T} \right) + 1} + \kappa_e, \tag{52}
\]

where the bridging parameter \( \Delta T \) is fixed at \( \Delta T = 0.1 \) as well as \( \kappa_d = 10 \text{ cm}^2 \text{ g}^{-1} \). Then, to find the roots of Equation (50), the following procedure is adopted: an initial approximation for \( T_0 \) is found after \( \kappa(T_0) \) is initially estimated from (52), where \( T_0 \) is obtained for the gas-pressure-only solution (28). Then, \( T_0 \) is used as an initial guess to numerically solve (50) with (52) for the true value of \( T \). Finally, multiple roots of Equation (50) should be weeded out via checking that they produce a positive right-hand side in the Equation (45).

Once \( T \) is known, the surface density \( \Sigma \) is found from Equations (16), (42) and (11), and (41):

\[
\Sigma = \frac{64\pi\sigma}{9\kappa} \frac{T^4}{M\Omega^2}. \tag{53}
\]

The result of a numerical solution of the Equation (50) with respect to \( T \) is shown in Figure 3, where the disk model for \( P = P_g \) is compared with the model for \( P = P_g + P_r \).

While this figure more accurately illustrates the relation between \( R_{\text{in}} \) and \( R_{\text{out}} \), we still probably underestimate \( R_{\text{out}} \) because \( \kappa_d \) becomes important before \( T \) is approaching \( T_{\text{sub}} \). Thus, the region where the dust opacity changes the vertical structure of the disk is extended further away. From (29) and (30), one can see that the size of this region depends relatively weakly on \( M \). Solving Equation (50) for different values of parameters, we calculate the size of the active region, i.e., \( R_{\text{in}} \) and \( R_{\text{out}} \). Table 1 summarizes the results for the dependence of \( R_{\text{in}} \) and \( R_{\text{out}} \) on \( M_{\text{loc}} \) and column for \( M_{\text{BH}} \). Each pair is shown at the intersection of the corresponding row for \( M_{\text{loc}} \) and column for \( M_{\text{BH}} \).

### 5. Convection

If a gas element that is rising over a small distance, adiabatically and in pressure equilibrium with the environment, is found to be lighter when contrasted to its surroundings, then buoyancy force will keep propelling it further (Kippenhahn & Weigert 1994). The opposite situation corresponds to the Schwarzschild criterion for convective stability,

\[
\frac{dT}{dz}_{\text{ad}} > \frac{dT}{dz}_{\text{rad}}, \tag{54}
\]

where \( \frac{dT}{dz}_{\text{ad}} \) and \( \frac{dT}{dz}_{\text{rad}} \) are the adiabatic and radiative temperature gradients (both negative), respectively. In this section, we revisit assumptions about vertical transport of energy in the disk and show that, as soon as the midplane temperature exceeds the sublimation temperature, \( T \sim T_{\text{sub}} \), there is a certain range of radii between \( R_{\text{in}} \) and \( R_{\text{out}} \) where the
disk is convectively unstable. There are two reasons for convection: (1) As we will show, the regular Schwarzschild criterion for the radiative disk indicates that, in a wide range of parameters, transferring energy vertically by convection is preferred over radiation diffusion. (2) Strong temperature dependence of the opacity of dust leads to a sudden increase of the radiation pressure at some height above the midplane.

The second point follows from the fact that, in a disk in radiative equilibrium, $T(z)$ decreases from the midplane (cleared from dust at $T > T_{\text{sub}}$) toward higher $z$. A dramatic increase of the radiation pressure associated with an opacity jump follows. As long as $T_c > T_{\text{sub}}$, there exists $z_c(r)$ within a vertical column of the disk where there is a transition from dust-free opacity, $\kappa_{\text{dir}}$, to dust opacity, $\kappa_d$. Correspondingly, $|dT/dz|$ becomes very large at $z_c$, triggering convection, which works toward smoothing the vertical distribution of the entropy.

The convection establishes a new distribution of $\rho$ and $T$, such as to decrease $|dT/dz|$. Our simplified model discussed so far adopts vertically averaged quantities, and a more elaborate treatment of convection calls for more sophisticated methods. The latter is beyond the scope of this paper, and in our far adopts vertically averaged quantities, and a more elaborate criterion for the radiative disk indicates that, in a wide range of parameters, transferring energy vertically by convection is preferred over radiation diffusion.

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### Table 1

| $M_{\text{in}}$ | $10^5$ | $10^6$ | $10^7$ | $10^8$ | $10^9$ |
|-----------------|--------|--------|--------|--------|--------|
| $M_{\text{BH}}$ | 0.01   | 10.53  | 0.79   | 0.86   | 0.96   |
| $M_{\text{in}}$ | 0.01   | 1.09   | 1.81   | 7.17   | 1.54   |
| $M_{\text{BH}}$ | 0.1    | 7.29   | 8.61   | 9.57   | 1.02   |
| $M_{\text{in}}$ | 0.1    | 4.15   | 8.94   | 1.93   | 4.15   |
| $M_{\text{BH}}$ | 1      | 8.61   | 9.57   | 1.02   | 2.25   |
| $M_{\text{in}}$ | 1      | 1.06   | 2.28   | 4.90   | 1.06   |
| $M_{\text{BH}}$ | 2      | 1.72   | 9.79   | 1.04   | 2.84   |
| $M_{\text{in}}$ | 2      | 1.37   | 2.94   | 6.34   | 1.36   |
| $M_{\text{BH}}$ | 10     | 9.57   | 1.02   | 2.25   | 4.91   |
| $M_{\text{in}}$ | 10     | 2.21   | 4.76   | 1.03   | 1.86   |

Notes. Left column: accretion rate, $M_{\text{in}}$. Upper row: mass of the black hole, $M_{\text{BH}}$. Each cell not belonging to the first row or the first column contains two numbers: the upper number is $R_{\text{in}}$, the lower is $R_{\text{out}}$.

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5.1. Convective Region

Our adopted fiducial set of parameters include $R = 0.1$ pc, $T_r = T_{\text{sub}} = 1500$ K, i.e., corresponding to the situation of the dust at exactly the dust sublimation temperature at the disk midplane. Estimating $dT/dz$ for $n = 1.84 \times 10^{14} T^{3}_{1500}$ cm$^{-3}$ corresponding to $P_r = P_{g}$ case (61), we have

$$
\frac{dT}{dz} \propto \frac{T_c - T_s}{H} \approx -2.7 \times 10^{-12} \text{ K cm}^{-1},
$$

where $T_s$ and $T_c = T(z = 0)$ are estimated from the disk model. This should be compared with the adiabatic gradient $\left(\frac{dT}{dz}\right)_{\text{ad}}$ estimated from:

$$
\frac{dT}{dz}_{\text{ad}} = -g_c \rho \frac{\partial T}{\partial P}_{s} = -g_c \rho H
$$

$$
\approx -1.3 \times 10^{-12} \text{ K cm}^{-1},
$$

where the disk height $H$ is calculated with the equation of state (11) taken into account. The above estimations are too crude to adopt them in a judgement on convective instability; however, it follows from (56) and (57) that, in the region of $R_{\text{in}} \lesssim r \lesssim R_{\text{out}}$, adiabatic $\left(\frac{dT}{dz}\right)_{\text{ad}}$ can have a magnitude similar to that of $\frac{dT}{dz}$, warranting further investigation.

For the mixture of gas and radiation, the convective flux is (Kippenhahn & Weigert 1994; Bisnovatyi-Kogan 2001)

$$
F_{\text{conv}} = \frac{1}{4} l^2 \rho C_p \left(\frac{g_s}{T}\right)^{1/2} (\Delta \nabla T)^{1/2} \sqrt{\frac{4P}{P_g}} + 1,
$$

where $l$ is the mixing length, and $C_p$ is the heat capacity at constant pressure for the mixture of gas and radiation:

$$
C_p = \mathcal{R} \left(\frac{4P}{P_g}\right)^2 + \frac{20P}{P_g} + \frac{5}{2}.
$$

The temperature excess of the convective element over its surroundings is represented by $(\Delta \nabla T)$, which is found from

$$
\Delta \nabla T = \left(\frac{\partial T}{\partial P}ight)_s \frac{dP}{dz} - \frac{dT}{dz} = -g_c \rho \left(\frac{\partial T}{\partial P}ight)_s - \frac{dT}{dz}.
$$

We adopt $l \approx \epsilon_0 H$ for the mixing length, where $H$ is the half-thickness of the disk and $\epsilon_0 \leq 1$ is the mixing length parameter, for which we adopt the value $\epsilon_0 = 0.1$. Adopting $T$ and $\rho$ from
the disk model from Section 4.3, we numerically calculate $F_{\text{conv}}$ from (58) and (60). The result is shown in Figure 4, where nondimensional $\left(\frac{dT}{dz}\right)_{\text{ad}}$ and $\left(\frac{dT}{dz}\right)_{\text{eq}}$ are plotted. Consider the situation when $r$ is decreasing (i.e., tracing the diagram from right to left): the curves cross when $\left(\frac{dT}{dz}\right)_{\text{ad}} > \left(\frac{dT}{dz}\right)_{\text{eq}}$ and the medium becomes convectively unstable. In the left column, the effect of accretion rate is shown: for $\dot{M}_{\text{loc}} = 0.1 M_\odot$ yr$^{-1}$, the ADR is convective at $R_{\text{in}} < r < 0.13$ pc. Increasing the accretion rate pushes the convective region further out: for $\dot{M}_{\text{loc}} = 2 M_\odot$ yr$^{-1}$, the convective region is at $R_{\text{in}} < r < 0.3$ pc. In the right column, one can see a similar effect if the mass of the BH is increased to $M_{\text{BH}} = 10^9 M_\odot$. Convection tends to establish isentropic distribution: $S = $ const, correspondingly changing the disk vertical structure (see the Appendix).

### 6. Effects of External Irradiation

The source of the illumination is the same disk (or corona) but located very far away, in the inner parts of the disk. In general, the effect of such an illumination depends on the whole history of interaction of such radiation with the disk/winds at smaller radii and on the unknown curvature of the disk surface. Thus, the problem of the disk surface irradiation is intrinsically nonlocal.

If a thin, cold, dusty disk is illuminated by the UV flux $F_{\text{ext}}$, due to very high UV opacity, the radiation is stopped immediately near the surface of the disk, heating dust to approximately $T_{\text{sub}}$. Provided $r > R_{\text{in}}$, the temperature in such an idealized cold disk is decreasing toward the equatorial plane.

The vertical component of radiation pressure, $g_{\text{ext}} \propto (\nabla T)_r$, then points downward and is balanced by the vertical gradient of the gas pressure at the characteristic number density, $n$:

$$n_{\text{eq}} = n(P_{\text{g}} = P_f) \approx 6.12 \times 10^{10} T_{1500}^3 \text{ cm}^{-3}.$$  

Here, $T_{1500}$ is the dust temperature scaled in units of $T_{\text{sub}} = 1500$ K.

In the region where $T < T_{\text{sub}}$ in the bulk of the disk but $T > T_{\text{sub}}$ in the midplane, one can think of a disk as approximately consisting of a gas layer near the midplane and a dusty envelope above the midplane. Blanketing by the dusty envelope is approximately taken into account by Equation (25). To estimate $\rho_f$ in a gas layer, one can take a solution for a disk with $P = P_f$ with $\kappa = \kappa_f$:

$$n_{\text{eq}} = 5.6 \times 10^{13} M_{\odot,0.1}^{2/5} (M_f r_{-2}^{-3})^{1/20} \alpha_{0.1}^{-7/10} \kappa_{10}^{-3/10},$$

where $r_{-2} = r/(0.01 \text{ pc})$. In the case when radiation pressure on dust is important in the bulk of the disk, $n_c \gg n_{\text{eq}}$, and the midplane pressure is dominated by $P_f$.

Without external illumination, the gas-only layer in the ADR can grow in height only from the bottom up. Illumination can result in the surface layers of the disk becoming gas-dominated as well. Notice that the photosphere for the UV illumination, $\tau_{\text{UV,ext}} \sim 1$, is generally expected to be located at higher $z$ than the photosphere of the disk: $\tau_{\text{IR, disk}} \sim 2/3$.

In the presence of illumination by the radiation flux, the boundary condition for the surface temperature of the disk, $T_s$, should be modified to

$$\sigma_B T_s^4 = F^+ + f(1 - A)F_n,$$

where $F_n$ is the component of the external flux normal to the surface of the disk, $A$ is the disk albedo, $f$ the attenuation and angular-dependent factor, and $F^+$ is found from (22). Solving...
radiation transfer Equation (24) with (63), we have
\[ \sigma_B T^4 = \frac{3}{4} \left( \tau + \frac{2}{3} \right) F + F_{\text{ext}}, \quad (64) \]
where \( F_{\text{ext}} = f(1 - A)F_n \) and the photosphere is placed at \( \tau = 2/3 \). The condition for the irradiation significantly influencing \( T_s \) follows from (64):
\[ F_{\text{ext}} \gtrsim \tau_c^2 F^+. \quad (65) \]

The solution for the AD gives \( \tau_c \approx 4 \times 10^5 \). The solution for the AD at 0.01 pc gives \( \hbar/r \approx 0.1 \). One can further calculate that \( (F_{\text{ext}}/F^+) \) is a factor of \( 2 \times 10^2 \) smaller than required by condition (65).

From (64), one can see that, if \( F^+ \tau_c \gg F_n \), the midplane temperature, \( T_{\text{mp}} \), is practically independent from external sources of heating (Lyutyi & Sunyaev 1976). The total attenuation factor in (63) is approximately \( 0.5 e^{-2/3} \approx 1/4 \) for \( A \approx 0.5 \).

Vertical structure of the disk is calculated from (20) and (24), yielding the vertical distribution of temperature in the illuminated disk:
\[ T = T_s \left( 1 - \frac{3}{2} \frac{\tau_c^2}{T_s^4} \left( \frac{\sigma}{\sigma_c} \right)^2 \right), \quad (66) \]
where we assumed the specific rate of viscous energy dissipation to be constant, \( \tau_c \approx \sigma_c \kappa_a \). Surface temperature as found from (63) can, on the other hand, be significantly influenced by the external radiation flux:
\[ F_{\text{ext}} \approx \delta (1 - A) \frac{L}{4 \pi r^2} \mu (2\mu + 1), \quad (67) \]
where \( L \) is calculated from (1) and the factor \( \delta \approx h/r \) is related to the angle between the surface of the disk and the direction of radiation flux (Meyer & Meyer-Hofmeister 1982; Spruit 1996). Estimating \( \delta \) from (36):
\[ \delta \approx \frac{3}{8\pi c R} \kappa_a \dot{M}. \quad (68) \]
The effect from the external flux is demonstrated in Figure 5, which shows \( T_s \) and \( T_e \) for \( A = 0.5 \). Not surprisingly, \( T_e \) is noticeably influenced by irradiation.

In case of the central radiation flux, the dominating component of radiation force is directed very nearly along the disk surface. The vertical radiation pressure in a single-scattering approximation scales in the same way as the gravitational force. Thus, illumination of a thin disk by the UV flux is unlikely to be responsible for “puffing up” of the AGN disk at pc scales. X-rays penetrate much deeper and can potentially lead to a much stronger “puffing up” due to IR pressure (Chang et al. 2007; Dorodnitsyn et al. 2016).

In general, we expect that there will be a subregion within the ADR where temperature distribution of the surface layers may become significantly different than implied by our calculations. Self-shielding and self-shadowing could be very important as well, especially if the disk bulges up near \( R_{\text{in}} \). However, due to the abovementioned very strong increase of the optical depth with height in a disk, the qualitative properties of ADR and estimates for \( R_{\text{in}} \) and \( R_{\text{out}} \) will approximately hold. This problem is, however, beyond the scope of the current work.

The predictive power of our simple calculations of the effect of the irradiation is limited. The attenuation of the radiation flux from the nucleus depends on the obscuring properties of winds at smaller radii and can be addressed only via global numerical modeling.

### 6.1. Dust above the Disk

Dust above the disk at \( R < R_{\text{sh}} \) is exposed to UV and X-ray illumination from the nucleus. In general, the gas–dust medium is a very efficient UV absorber. The unattenuated UV radiation is promptly stopped in a very thin layer, where it is further converted into IR as well as heating the gas. The UV opacity of the 0.1 \( \mu \)m grain is about its geometrical cross section, \( \kappa_{\text{UV}} \approx 10^{20} f_{\text{d},0.01} \text{cm}^2 \text{g}^{-1} \), where \( f_{\text{d},0.01} \) is the gas-to-dust mass ratio in \( 10^{-2} \). The thickness of such a “photospheric” layer is
\[ \delta_{\text{UV}}/r_{\text{sh}} \approx 5 \times 10^{-2} f_{\text{d},0.01} n_5^{-1} L_{46}^{-1/2}, \quad (69) \]

while the penetration length of X-rays is significantly higher:
\[ \delta_{\text{XR}}/r_{\text{sh}} \approx \begin{cases} 0.023, & 0.5 < E < 7 \text{ keV} \\ 0.014, & E > 7 \text{ keV} \end{cases} \times n_5^{-1} L_{46}^{-1/2}, \quad (70) \]

where \( \kappa_{\text{XR}} \) is the X-ray opacity consisting of photoionization and Compton cross sections. In general, \( \delta_{\text{XR}} \approx \frac{\kappa_{\text{UV}}}{\kappa_{\text{XR}}} \delta_{\text{UV}} \). We
adopt the photoionization cross section from Maloney et al. (1996): for \( 0.5 < E < 7 \) keV, we have \( \sigma_{\text{XR}} \approx 2.6 \times 10^{-22} \) cm\(^2\), and for \( E > 7 \) keV, we have \( \sigma_{\text{XR}} \approx 4.4 \times 10^{-22} \) cm\(^2\).

Within the UV conversion layer, the UV radiation excerts pressure on the order of \( P_{\text{UV}} \approx 0.03 \) dyn cm\(^{-2}\) on the dusty surface of the disk/torus. If, from within the disk, such a layer is supported entirely by the gas pressure, then, recalling that the equilibrium density at \( P_{\text{gas}} = P_{\text{UV}} \) is \( n \approx 10^{11} \) cm\(^{-3}\) (61), the UV penetration length can be estimated to be just \( 10^{-8} - 10^{-7} \) cm.

Illumination by an X-ray flux has a devastating effect on the dust in an optically thin region. The sudden exposure of the gas–dust slab would create a receding evaporation layer where gas is transitioning from a cold to a \( 10^4 \) K hot component (i.e., Dorodnitsyn et al. 2008). It is not until such an X-ray flux is sufficiently attenuated (i.e., \( \tau_{\text{UV}} \approx 1 \)) in the X-ray evaporative gas/wind that enough dust can survive and the UV conversion layer has a chance to actually settle.

The opacity of the gas–dust mixture in the UV and IR is dominated by the opacity of dust, \( \kappa_d \), with two major contributors: silicon at lower temperatures and carbon at higher temperatures. Such temperature dependence can be approximately described as

\[
\kappa_d = \kappa_0 \left( \frac{T}{T_{\text{sub}}} \right)^6 \quad \text{for } T < T_{\text{sub}},
\]

where \( \kappa_0 = 10 - 50 \) cm\(^2\) g\(^{-1}\) (Semenov et al. 2003), and \( n \approx 1 - 2 \), when \( T_{\text{g}} \approx T_{\text{r}} \). In general, the dust grain sublimation timescale, \( t_{\text{sub}} \), is very short. The mass of the dust grain can increase or decrease depending on \( \Delta P = P_{\text{cap}} - P_{\text{th}} \), where \( P_{\text{cap}} \) is the saturation vapor pressure and \( P_{\text{th}} \) is the partial pressure of the species, \( i \) (Phinney 1980). The dust grain sublimation timescale can be estimated as

\[
t_{\text{sub}} = \frac{m_{\text{gr}}}{m_{\text{cap}}},
\]

where \( m_{\text{gr}} \) is the mass of a dust grain, \( m_{\text{cap}} = 4\pi \mu_i^2 P_{\text{cap}} \sqrt{\frac{\mu_i m_{\text{amu}}}{2\pi k}} \) is the dust grain mass-los rate density, \( \mu_i \) is the molecular weight, and \( m_{\text{amu}} \) is the atomic mass unit. Since \( P_{\text{cap}} \sim \exp(-\text{few} \cdot 10^4/T) \), it is a very sensitive function of the gas temperature. The corresponding timescale is very short compared to \( t_{\text{in}}, t_{\text{dyn}} \). For example, evaluating (72) for amorphous silicon dust, it is just

\[
t_{\text{sub}}(\text{MgFeSiO}_{\text{eq}}) \approx 0.22 \text{ days},
\]

and \( t_{\text{sub}} \ll t_{\text{dyn}}, t_{\text{th}} \) indeed follows.

7. Discussion

It has been suggested by Czerny & Hryniewicz (2011) that the pressure of the disk’s own, local radiation on dust can drive large-scale “failed” winds. Baskin & Laor (2018) recalculated the dust opacity based on the inclusion of the new data for graphite grains. The same authors predicted in that, as a result of such enhanced opacity, such a disk can “bulge up” and form a compact torus at approximately few \( 10^{-2} \) pc. In this work, we focus on a generally much broader region where radiation pressure can impact the vertical structure of the accretion disk in AGN. As the dusty gas spirals from galactic scales toward the nucleus, it is generally quite cold, so the disk is very thin. Closer to the BH, gas heats up due to internal viscous dissipation, until such internally generated radiation starts to influence the vertical structure of the disk via radiation pressure on dust grains.

The disk midplane temperature increases toward smaller \( r \) as \( T_{\text{c}} \propto r^{-(y-10)}/r^{-1} \). At the radius \( R_{\text{sub}} \), the midplane temperature, \( T_{\text{c}} \), equals the temperature of dust sublimation \( T_{\text{sub}} \approx 1500 \) K. When \( T_{\text{c}} \) reaches several hundred K, the contribution from radiation pressure on dust increases. When \( T_{\text{c}} > T_{\text{sub}} \), the midplane is cleared from dust and the total pressure at the midplane is dominated by that of the gas. However, just above the midplane, as the temperature drops to \( T_{\text{c}} < T_{\text{sub}} \), opacity increases by approximately two orders of magnitude and so does the coupling between vertical radiation flux and the dusty gas. AGN accretion disks thus have two regions where radiation pressure is important: one close to the BH as predicted by the standard SS73 theory, and the other considerably further away, approximately at \( R_{\text{sub}}(T_{\text{c}} = T_{\text{sub}}) < R < R_{\text{out}}(T_{\text{c}} = T_{\text{sub}}) \).

The mass accretion rate in the ADR, i.e., at \( R_{\text{in}} \ll R \ll R_{\text{out}} \), does not need to correspond to the mass accretion rate derived from the bolometric luminosity of the nucleus, \( L \propto M \), where \( M \) is the accretion rate within inner parts of the accretion disk. The latter is approximately limited by the Eddington accretion rate: \( \dot{M}_{\text{Edd}} \approx 0.2 M_L \text{ yr}^{-1} \). The local production rate of radiation in a disk depends on the local mass-accretion rate, \( \dot{M}_{\text{loc}} \). Correspondingly, for the envelope of a disk to "puff up" and become thin, such a disk should have a local mass-accretion rate, \( \dot{M}_{\text{loc}} > \dot{M}_{\text{loc,Edd}} \), where \( \dot{M}_{\text{loc,Edd}} \) is the Eddington mass-accretion rate as calculated with respect to the opacity of dust. Depending on assumptions about dust, \( \dot{M}_{\text{loc,Edd}} \approx 2.4 \cdot 10^3 M_L \text{ yr}^{-1} \). If \( \dot{M}_{\text{loc}} \) in ADR is larger than \( \dot{M}_E \), the excess gas should be removed by the winds or participate in large-scale flows.

In this paper, we necessarily made many simplifications. For example, when establishing that ADR disk is highly convectively unstable, we did not quantitatively address the problem that the vertical structure of such a disk should be considerably altered by convection. Instead, we assumed that, in the convective layer, the convection is so efficient that it drives the equation of state to an isentropic one. In reality, we expect the total vertical flux of energy to have contributions from both convection and from radiation.

A model of an AGN subpc accretion disk relevant to the formation of the BLR was developed by Czerny et al. (2016). This included a simplified numerical model of a self-gravitating \( \alpha \)-disk, as well as the effects associated with the radiation pressure and self-gravity. Those authors did not explicitly find the parameters of the ADR region or consider a locally super-critical slim disk, but they also found the relative importance of the convective transport. Though our methods are significantly different, our conclusions about the importance of convection seems to be in agreement, warranting further investigation.

When considering disk irradiation and its influence on the disk structure, we only considered how such illumination changes the surface boundary condition for the radiation transfer problem. The quasi-1D approach gives qualitatively correct results, but more detailed treatment should include angular dependent effects, which can be important when disk becomes geometrically thick.

If, in presence of irradiation, dust at the surface is destroyed, the disk will have a region of a sandwich-like vertical structure: a gas layer near the midplane enveloped by a dusty layer which is itself enveloped in a gas-only surface layer. Due to the large opacity jump, the intermediate dust layer can be very convective. In the absence of self-shadowing, the value for
$R_{\text{in}}$ would be smaller and the size of the ADR would decrease. The outer radius of the ADR will, however, be approximately unchanged, because of the condition (65). On the other hand, self-shadowing can effectively work toward protecting the dust near the inner edge of the ADR.

Analogies between accretion disk physics and outflowing stellar atmospheres can be helpful. The case of a luminous star atmosphere with high radiation pressure in a continuum, in a regime in which the convection solution competes with the outflowing one, was calculated in Bisnovatyi-Kogan (1973). It was found that the increase of the radiation flux (in our case, the equivalent to the increase of $M_{\text{loc}}$) leads there to the transition from convective solution to an outflowing one, with some overlap between the two. In this paper, we did not calculate such outflowing solutions, but the stellar analogy provides some evidence that such a transition may happen. Detailed calculations in disk geometry are significantly more difficult, at a very minimum requiring multidimensional numerical simulations.

After the dusty gas is expelled from the disk, it is exposed to radiation pressure forces from the nucleus, which can be significantly greater than the vertical radiation pressure from the disk itself. Depending on $M$ and $M_{\text{loc}}$, different types of dusty outflows can be envisaged: from thin layered flows along the disk surface, to the large-scale but gravitationally bound “failed winds,” to polar hollow cone dusty outflows of different curvature. An example of such a wind is shown in Figure 2.

To produce massive outflow most efficiently, two factors should align: the local accretion rate should be greater than the local critical rate, $M_{\text{loc}} > M_{\text{loc,cr}}$, and the location of ADR should be such as $R_{\text{out}} > R_{\text{sub}}$. Self-regulation of accretion due to the disk’s own radiation pressure on dust can be important for the regulation of the SMBH growth through accretion and deserves further investigation.

8. Conclusions

Our results can be summarized as follows:

1. We have shown that there is a region in an AGN accretion disk in which local radiation pressure on dust can have a major effect on the disk vertical structure and dynamics.

2. Such an ADR is approximately bounded at large radius by the dust sublimation radius on the disk midplane and at small radius by the dust sublimation radius at the disk surface.

3. The outer boundary of ADR in the disk is approximately identified as the radius, $R_{\text{out}}$, where the temperature at the disk midplane equals the dust sublimation temperature, $T_{\text{sub}}$. For $M_{\text{BH}} = 10^7M_\odot$, $R_{\text{out}} \approx 0.1$ pc. At $R < R_{\text{out}}$, dust is cleared near the midplane and there is a dramatic jump of opacity along the vertical through the disk.

4. The inner boundary of ADR is located at the radius where dust completely disappears inside the disk, i.e., at $T = T_{\text{sub}}$, where $T$ is the disk surface temperature.

5. We have shown that ADR is strongly convectively unstable with significant vertical energy transport via convection. Convection results in effective cooling of the disk interior. It is also possible that the convection from the ADR provides the turbulence driver for the BLR.
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Appendix

Convective Disk

When energy is transported toward the surface of a disk via convection, it is often the case that $F_{\text{conv}} \gg F_{\text{rad}}$ where $F_{\text{conv}}$ and $F_{\text{rad}}$ are convective and radiation fluxes, respectively. Convection tends to establish isentropic distribution: $S = \text{const}$, where $S$ is found from (55). Thus, to describe a fully convective disk, one adopts a polytropic equation of state for radiation (Bisnovatyi-Kogan & Blinnikov 1977):

$$P = K \rho^{4/3}, \quad (A1)$$

where

$$K = \left( \frac{3 \beta^4}{256a} \right)^{1/3} \approx \text{const.} \quad (A2)$$

Inserting polytropic equation of state (A1) into Equation (43) gives:

$$\frac{dP}{dz} = -\Omega^2 K^{-3/4} \rho^{3/4}. \quad (A3)$$

Further solving this equation, one can eventually obtain the following simple relations:

$$\rho \approx \rho_k \left( 1 - \frac{z^3}{z_b} \right)^3, \quad (A4)$$

$$P \approx P_k \left( 1 - \frac{z^3}{z_b} \right)^4. \quad (A5)$$

When simplifying (A4) and (A5), we took into account that $P_b \ll P_k$, $\rho_b \ll \rho_k$, and adopting these relations for simplicity, the assumptions $P_b \approx 0$ and $\rho_b \approx 0$ at $z = z_b$, where $\rho_b$ and $P_b$ are the corresponding values at the boundary of the disk. A surface boundary condition follows from integrating (35) between $z_b$ and infinity: $P_b = z_b^2 \Omega^2 \tau_b / \kappa_{ib}$, where $\tau_b \approx 2/3$, and all properties of a polytropic disk can then be derived as in Bisnovatyi-Kogan & Blinnikov (1977).

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