Dark Energy and Dark Matter in Stars Physic

Plamen Fiziev*
Jouynt Institute of Nuclear Research, Dubna, Russian Federation
E-mail: fiziev@theor.jinr.ru

We present the basic equations and relations for the relativistic static spherically symmetric stars (SSSS) in the model of minimal dilatonic gravity (MDG) which is locally equivalent to the f(R) theories of gravity and gives an alternative description of the effects of dark matter and dark energy. The results for the simplest form of the relativistic equation of state (EOS) of neutron matter are represented. Our approach overcomes the well-known difficulties of the physics of SSSS in the f(R) theories of gravity introducing two novel EOS for cosmological energy-pressure densities and dilaton energy-pressure densities, as well as proper boundary conditions.

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*Speaker.
1. Introduction

One of the most important lessons from the spectacular development of cosmology in the last fifteen years is the clear understanding that the Einstein general relativity (GR) and standard particle model (SPM) are insufficient to explain all observed phenomena in the Nature. There exist three possible ways for further development:

1) To add some new content of the Universe beyond the SPM, like dark matter and dark energy;
2) To change the theory of gravity. The simplest models are $f(R)$ [1] and MDG [2, 3];
3) Some mixture of these two possibilities is not excluded by the current observational data.

Dozens of models with different functions $f(R)$ exist, some of them dubbed valuable [1].

The situation in star physics is similar. More than sixty-year development so far has not solved the problem with realistic EOS of compact star matter. At present one can find several dozens of EOS in the literature.

A series of attempts to use $f(R)$ models of gravity adopted to star physics also exist [4]. No fully convincing final result was reached.

The main goal of the present paper is to create a clear physical basis for application of MDG in star physics and thus to facilitate construction of models, which permit unified treatment of the physical problems at very different scales: from laboratory scales and compact star scales to the scale of the visible Universe. Such a unified approach may give much more definite justification of our models using all available information for the physical phenomena at all reachable scales.

The MDG model was proposed and studied in [2, 3, 5, 6]. It describes a simple generalization of the Einstein general relativity (GR), based on the following action of the gravi-dilaton sector

$$\mathcal{A}_{g,\Phi} = \frac{c^3}{16\pi G_N} \int d^4x \sqrt{|g|} \left( \Phi R - 2\Lambda U(\Phi) \right).$$  \hspace{1cm} (1.1)

Without any relation with astrophysics and cosmology, it was studied by O’Hanlon, as early as in [2]. There the term "dilaton" for the field $\Phi$ was introduced. Formally, MDG resembles the Brans-Dicke theory with $\omega \equiv 0$, if the most important MDG-cosmological term in Eq. (1.1) is ignored. To some extent MDG is related with the $f(R)$ theories of gravity [1, 4]. In general, the $f(R)$ theories are also physically different, being only locally equivalent to MDG [5].

The values $\Phi \in (0, \infty)$ must be positive to avoid the physically unacceptable antigravity. The value $\Phi = 0$ yields an infinite gravitational factor and makes the Cauchy problem in MDG not well posed [7]. The value $\Phi = \infty$ turns off the gravity and must also be excluded, as well as $\Phi = 0$.

The scalar field $\Phi$ introduces a variable gravitational factor $G(\Phi) = G_N / \Phi = G_N g(\Phi)$ instead of the Newton constant $G_N$. The cosmological potential $U(\Phi)$ introduces a variable factor $\Lambda U(\Phi)$ instead of the constant $\Lambda$. In GR with cosmological constant $\Lambda$ we have $\Phi \equiv 1$, $g(\Phi) \equiv 1$, and $U(1) \equiv 1$. Due to its specific physical meaning, the field $\Phi$ has quite unusual properties.

The function $U(\Phi)$ defines the cosmological potential which must be a positive single valued function of the dilaton field $\Phi$ by astrophysical reasons. See [8] for all physical requirements on the cosmological potential $U(\Phi)$, necessary for a sound MDG model. There the class of withholding potentials was introduced. These confine dynamically the values of the dilaton $\Phi$ in the physical domain. It is hard to formulate such a property for the function $f(R)$ in a simple way.
Some more physical and astrophysical consequences of MDG are described in [3, 4, 5, 6, 7]. This model provides an alternative explanation of the observed astrophysical phenomena without introduction of dark energy and dark matter.

In the present article we give a correct formulation of a star problem in MDG and consider the simplest example of a physically consistent family of SSSS for an admissible cosmological potential \( U(\Phi) \). We explicitly show how the dilatonic field \( \Phi \) changes the structure of the compact stars and creates a specific dilatonic sphere around them, analogous to dark matter halo.

## 2. Basic equations and boundary conditions for SSSS in MDG

In units \( G_N = c = 1 \) the field equations of MDG can be written in the form:

\[
\begin{align*}
\Phi \dddot{R}_\alpha^\beta + \nabla_\alpha \ddot{\nabla}^\beta \Phi + 8\pi \dot{T}_\alpha^\beta &= 0, \quad (2.1a) \\
\Box \Phi + \Lambda V'(\Phi) &= \frac{8\pi}{3} T. \quad (2.1b)
\end{align*}
\]

Here \( T_\alpha^\beta \) is the energy-momentum tensor of the matter, \( \dot{X}_\alpha^\beta = X_\alpha^\beta - \frac{1}{4} X \delta_\alpha^\beta \) is the traceless part of the 4D-tensor \( X_\alpha^\beta \), \( X = X_\alpha^\alpha \) is its trace, the relation \( V'(\Phi) = \frac{2}{3} \left( \Phi U'(\Phi) - 2U(\Phi) \right) \) introduces the dilatonic potential \( V(\Phi) \), and the prime denotes differentiation with respect to the variable \( \Phi \).

In the problems under consideration, the space-time-interval is \( ds^2 = e^{\nu(r)} dt^2 - e^{\lambda(r)} dr^2 - r^2 d\Omega^2 \) [8], where \( r \) is the luminosity distance to the center of symmetry, and \( d\Omega^2 \) describes the space-interval on the unite sphere. Then, after some algebra one obtains the following basic results for a SSSS of the luminosity radius \( r^* \).

In the inner domain \( r \in [0, r^*] \) the SSSS structure is determined by the system:

\[
\begin{align*}
\frac{dm}{dr} &= 4\pi r^2 \epsilon_{eff}/\Phi, \quad (2.2a) \\
\frac{d\Phi}{dr} &= -4\pi r^3 p_\phi/\Delta, \quad (2.2b) \\
\frac{dp_\phi}{dr} &= -P_\phi r/(r^3 - 2\pi^3 p_\phi/\Phi), \quad (2.2c) \\
\frac{dp}{dr} &= -P + \epsilon m + 4\pi r^3 p_{eff}/\Phi, \quad (2.2d)
\end{align*}
\]

The four unknown functions are \( m(r), \Phi(r), p_\phi(r), \) and \( p(r) \). In Eqs. (2.2) \( \Delta = r - 2m - \frac{1}{4}\Lambda r^3 \), \( \epsilon_{eff} = \epsilon + \epsilon_\Lambda + \epsilon_\phi, \) \( p_{eff} = p + p_\Lambda + p_\phi \). We obtain also two additional EOS specific for MDG:

\[
\begin{align*}
\epsilon_\Lambda &= -p_\Lambda - \frac{\Lambda}{12\pi} \Phi : \quad \text{CEOS; (2.3a)} \\
\epsilon_\phi &= p - \frac{1}{3} \epsilon + \frac{\Lambda}{8\pi} \nu'(\Phi) + \frac{p_\phi m}{2} \frac{4\pi r^3 p_{eff}/\Phi}{\Delta - 2\pi^3 p_\phi/\Phi} : \quad \text{DEOS; (2.3b)} \\
\epsilon &= \epsilon(p) : \quad \text{MEOS. (2.3c)}
\end{align*}
\]

Equation (2.3a) is the EOS for the cosmological energy density \( \epsilon_\Lambda = \frac{\Lambda}{8\pi} \left( U(\Phi) - \Phi \right) \) and the cosmological pressure \( p_\Lambda = -\frac{\Delta}{6\pi} \left( U(\Phi) - \frac{\Delta}{4} \Phi \right) \), i.e. the cosmological EOS (CEOS).
3. Results for the simplest MEOS of neutron matter

The most idealized MEOS of neutron matter is the one from the GR-TOV model [8]: \( \varepsilon = \frac{1}{4\pi} (\sinh t - t) \), \( p = \frac{1}{12\pi} (\sinh t - 8 \sinh(t/2) + 3t) \). It describes the ideal Fermi neutron gas at zero temperature and facilitates our study of the pure-MDG-effects in SSSS, shown in Figs. 1–11.
As seen in Fig. 7, MDG-SSSS are lighter and more compact than GR-SSSS. As in GR, MDG-SSSS may be stable only until maximal mass is reached, see Figs. 6 and 7. The mass of the disphere $m_{\text{disp}}(r)$ outside MDG-SSSS exponentially goes to a constant: $m_{\text{disp}} \approx 0.1638 M_{\odot}$. The total mass of the object $m_{\text{total}} \approx 0.671 M_{\odot}$ is quite close to the mass of GR-SSSS $m_{\text{GR,max}} \approx 0.705 M_{\odot}$. 
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The MDG-dependences $2\varepsilon_\Phi^*(r^*)$, $p_\Phi^*(r^*)$. Maximal values are reached for smaller $r^*$ than in Fig. 7.

The specific MDG-dependences $\varepsilon_\Lambda^*(r^*)$ and $p_\Lambda^*(r^*)$ in accord with CEOS.

Unexpectedly large variations of the gravitational factor $g(r) = 1/\Phi(r)$ in MDG-SSSS.

An interesting problem is a model of a moving and rotating star in MDG. In it one can expect an asymmetric configuration with appearance of different centers of the star and its disphere, or even detachment of the parts of disphere. For similar effects at cosmological scales see [8].

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