Application of fiber model considering the shear effect in the analysis of nonlinear RC shear wall

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Abstract. Because of the good performance of the fiber model in the simulation of axial and bending coupling, the new fiber model considering the axial, bending and shear effects is established to analysis the nonlinear properties of RC shear wall. The way of the shear fiber distribution in the cross section consistent with the axial fiber and each shear fiber current state of stiffness get from the independent shear hysteretic rules, then establish tangent element stiffness matrix containing the axial, bending and shear deformation under complex loading paths. Two different cross-section fiber models is compared. Compiling corresponding calculation program to simulate a shear wall’s nonlinear performance under monotonic load and compare with the laboratory RC shear wall test results to verify the validity of the model.

1. Introduction
Presently, RC shear wall is the component primarily resists lateral loads acting on the high-rise buildings and super high-rise buildings. It was applied much in urban construction and the research of its seismic performance is paid highly attention by experts at home and abroad.

For reinforced concrete component like shear wall, researchers try to establish mathematic models to reflect the relative material mechanical properties in microscopic view. At the meantime, it can reflect the earthquake resistant behavior of component or structure. Fiber model is the analytical model that is the closest to real mechanical behavior of structures. The unit of fiber model is mainly applied to simulate beam and column, which ignores the effect of shear and simulates the bending of component better. In recent years, many researchers are trying to consider shear effect in fiber model to better simulate component like shear wall which cannot ignore shear effect. Although some results were achieved, there are not models that being widely accepted. Future research is required to get a relative accurate and easy model to analyze nonlinear shear wall.

2. Fiber Wall Model
Before analysis, some assumptions should be made: 1. Axle bending should satisfy the plane section assumption; 2. The effect of bond and slip between concrete and rebar is ignored; 3. Small transmogrification assumption; 4. The influences of geometric nonlinearity are not considered; 5. Shear, axial load and bending moment are not coupling.

2.1 Derivation of Stiffness Matrix
The mechanical behavior of shear wall units, cross section of shear wall and fiber need to be considered to derive the stiffness matrix of shear wall. The mechanical behavior of shear wall unit can be described by load vector, stiffness matrix and nodal displacement vector, namely $Q = K_{st} \cdot q$. 
When the rigid body displacement is not considered, the unit load vector \( \mathbf{q} = (\theta, \theta, u)^T \), whose element of the vector is the angle of rotation and the displacement in the axial direction of each end of beam and column. The displacement vector of unit is \( \mathbf{u} = (\mathbf{u}_x, \mathbf{u}_y, \mathbf{u}_z)^T \), where the element of the vector is the angular rotation and translation displacement in each axis. When the rigid body displacement is considered, the mechanical properties under rigid body displacement can be derived by geometric transformation matrix \( \mathbf{L}_{\text{ele}} \). The mechanical behavior of shear wall which is described by section force vector, section displacement vector and section stiffness matrix, namely \( \mathbf{D}(x) = \mathbf{k}_{\text{sec}} \cdot \mathbf{d}(x) \). Section force matrix is \( \mathbf{D}(x) = (\mathbf{M}(x), \mathbf{N}(x), \mathbf{Q}(x))^T \), where the element is bending moment, axial force and shear force. Section displacement matrix is \( \mathbf{d}(x) = (\mathbf{\phi}(x), \mathbf{\epsilon}(x), \mathbf{\gamma}(x))^T \), where the element is the curvature, axial strain and shear strain.

Considering axial force, bending moment and shear force separately when the force is acting on the fiber. When calculating axial force and bending moment, the geometry meshing and formula derivation is the same with unit fiber model of beam and column \(^1\). When considering shear force, the section shear strain is assumed as \( \mathbf{\gamma}(x) = \frac{3}{2} \left(1 - \frac{4y^2}{4h^2}\right) \mathbf{\gamma}(x) \). The reason of the assumption is that the shear stress on the shear wall is parabolic distribution in the traverse section. Where \( \mathbf{\gamma}(x) \) is the shear strain of ith fiber strip of the x section; \( y_i \) is the horizontal ordinate of ith center point of fiber strip; h is the transverse length of the section; \( \mathbf{\gamma}(x) \) is the shear strain of the section.

The shape function \( b(x) \) of internal force was established by the relation of section force and unit force\(^2\). The shape function is derived by energy method and the matrix is

\[
b(x) = \begin{bmatrix} \frac{l-x}{l} & 0 & 0 \\ 0 & \frac{x}{l} & 0 \\ 1 - \frac{x}{l} & 1 & 0 \end{bmatrix}
\]

where \( l \) is the length of the unit. As the description of unit, section and the mechanics condition, the tangent line stiffness matrix of the section is

\[
k_{\text{sec}} = k(x) = \begin{bmatrix}
s_{\text{x}}(x)E_i \cdot A_i \cdot y_i & -s_{\text{x}}(x)E_i \cdot A_i \cdot y_i & 0 \\
-s_{\text{x}}(x)E_i \cdot A_i \cdot y_i & s_{\text{x}}(x)E_i \cdot A_i \cdot y_i & 0 \\
0 & 0 & 3\sum_{i=1}^{m} \left(1 - \frac{4y^2}{4h^2}\right) \frac{dV(\gamma)}{d\gamma}|_{\gamma(x)} \end{bmatrix}
\]

where \( E_i \) is the ith fiber’s tangential modulus under the current strain; \( A_i \) is the ith fiber’s section area; \( y_i \) is the ith fiber’s horizontal ordinate; \( n(x) \) is the quantity of fiber which is considered the axle bending. From the relation between section force and fiber force, the resistance under the current situation is

\[
D_{k}(x) = \begin{bmatrix}
-s_{\text{x}}(x)E_i \cdot A_i \cdot y_i & s_{\text{x}}(x)E_i \cdot A_i & \sum_{i=1}^{m} V(\gamma_i) \end{bmatrix}^T
\]

where \( \sigma_i \) is the axial stress of the ith axle bending fiber.

### 2.2 Constitutive relation

The concrete constitutive model in this paper is first raised by Kent and Park in 1973 \(^3\) and modified by Scott et al \(^4\). The concrete unload and reload model is combined with linear model and curvilinear model. Zhu bolong unloading curve is applied when the concrete is unloading. Linear model that
points to unloading point is applied when the concrete is reloaded. Please see the bibliography for details [5]. The skeleton curve of rebar is two – line model [5] under repetitive loading. The equivalent elasticity modulus is $E' = 0.01E$ when the stress in the rebar is beyond the elastic limit.

Presently, the shear hysteretic model is not well developed and the shear hysteretic model suggested by other reference is limited. The modified model by Takeda is applied in this paper as the shear subunit. Shear hysteretic model and hysteretic curve are the same models mentioned in the reference [6].

3. Analysis of examples

The shear wall in the reference [8] is shown below. The effectiveness of model in this paper and the FORTRAN programming is proofread by the model in the reference. The figure below is the section view and cutaway view. The concrete grade is C30 and the compressive strength is 28.3 Mpa by the material text. The yield strength of longitudinal reinforcement is 400Mpa and the elasticity modulus is 200000Mpa. The yield strength of stirrup is 400 Mpa. The pressure on the shear wall is 737KN. $m$ is 10 in this paper.

In the model calculating, the order of load application is: applying stable pressure on the top of shear wall; applying horizontal load which is growing gradually by calculating till the shear wall fails. According to the relation between longitudinal displacement on the top of shear wall and the shear curve, the comparison diagram is as follows

![The section view and cutaway view of shear wall](image)

Load – displacement diagram from model 1 and model 2:

![Model 1 Load – displacement diagram](image) ![Model 2 Pushover load – displacement diagram](image)

The curves in the two diagrams are similar at the beginning of loading by analyzing the two diagrams. When they are near the yield point, the programming diagram has relatively obvious yield
point. However, the yield point of experimental curve is not obvious. The overall error is within the accepted range because the programming curve basically matches the experimental curve. Both two models can be used in research the nonlinear RC shear wall.

Compare the two programming curves and the simulation effect, a diagram shows below.

![Displacement curve with two different models](image)

The two different curves have little but well-distributed different by comparing the two different models. The displacement of model 1 is larger, which represents that the stiffness is a bit small at the beginning. The yield load and the yield displacement of beam model has little different. The model 2 has good yield plateau while the model 1 does not have obvious yield plateau.

4. Summary

Traditional fiber model is modified in this paper. Shear effect is added into the model to simulate the mechanical behavior of the shear wall. Comparing the simulation and experiment, the modified fiber model can better simulate the nonlinear properties of RC shear wall under bending moment, pressure and shear force. The model with multiple shear fiber section can better simulate the nonlinear properties of shear wall.

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