On fatigue life scatter under variable-amplitude load history

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Abstract. Fatigue life scatter under a variable amplitude load history is determined by many factors such as the amplitudes of the individual load cycles, the number of the load cycles with different load levels, and the differences between the load levels as well. Besides an brief overview on the studies and experiments on the scatter of fatigue life under variable amplitude load histories, the present paper conducted a series of fatigue tests, demonstrates the life scatters under different load histories, and discusses the dominant factors of fatigue life scatter. It shows that all the load cycles play their roles for the life scatter.

1. Introduction

Fatigue life is a random variable even under a deterministic load history \cite{1}. For probabilistic fatigue life predictions, P-S-N (stress - probabilistic life) curve based methods are usually applied \cite{2-5}. Such methods implicitly assume that the scatter of fatigue life associated with any of the stress levels in a variable amplitude load history exclusively depends on the individual stress level itself.

Fatigue life distribution of a structure under a variable amplitude load history (VAL) is considerably different from the fatigue life distribution under a constant amplitude load history (CAL) \cite{6-7}. A series of tests about fatigue life scatter under CAL and VAL of riveted lap joints illustrate that the standard deviation of the logarithmic life under CAL, \( \sigma_{\log N, CAL} \) is a function of the fatigue life \( N \) \cite{6}. The value of \( \sigma_{\log N, CAL} \) under CAL obviously increases for an increasing fatigue life. However, the results for the VAL tests illustrate that all values of \( \sigma_{\log N, VAL} \) for the VAL tests is of the same order of magnitude as \( \sigma_{\log N, CAL} \) in the CAL tests for the largest amplitude of the VAL loading. It suggests that the highest amplitude of the load spectrum is responsible for the amount of scatter \cite{6}. Similar observations on fatigue life scatter in tests with random or programmed sequences were reported \cite{8-9}.

2. Experiment results of fatigue life

To study fatigue life scatter under variable amplitude stress histories, fatigue life tests are conducted with specimens of T6-aluminum alloy. Tested fatigue life data are listed in Table 1 and Table 2. Where, \( s \) stands for the amplitude of the fully reversed cyclic stress applied to the specimens in the tests, \( \bar{N} \) stands for the mean of the sample lives, \( \sigma \) stands for the standard deviation, \( \bar{N}_{\log} \) and \( \sigma_{\log} \) stand for the mean and standard deviation of the logarithmic lives, respectively.
Table 1. Fatigue life (number of stress cycles) test results under constant amplitude cyclic stress.

| $s_1$=110MPa | $s_1$=120MPa | $s_1$=130MPa |
|--------------|--------------|--------------|
| 769910       | 557459       | 308178       |
| 624380       | 522350       | 431678       |
| 569366       | 439991       | 363086       |
| 913500       | 464849       | 279024       |
| 979900       | 443776       | 390145       |
| 2129232      | 720648       | 267924       |
| 764900       | 676961       | 405001       |
| 1003100      | 705894       | 294558       |
| 747900       | 701234       | 291682       |
| 1209610      | 706700       | 276299       |
| 1571600      | 741700       | 453721       |
| 1175200      | 971300       | 299652       |
| 636294       | 569300       | 981000       |
| 981000       | 351700       | 338300       |
| 925674       | 465200       | 461289       |

In Table 2, the stress history composed by 100 cycles of $s_3$ and subsequent cycles of $s_1$ (Figure 1a) is denoted by $s_3$-$s_1$-$100$, that composed by 1000 cycles of $s_3$ and subsequent cycles of $s_1$ (Figure 1b) is denoted by $s_3$-$s_1$-$1000$, and that composed by the blocks of “1000 cycles of $s_3$ - 1000 cycles of $s_2$ - 1000 cycles of $s_1$” (Figure 1c) is denoted by $s_3$-$s_2$-$s_1$-$1000$-....

Table 2. Fatigue life (number of stress cycles) test results under variable amplitude cyclic stress.

| $s_3$-$s_1$-$100$ | $s_3$-$s_1$-$1000$ | $s_3$-$s_2$-$s_1$-$1000$-.... |
|-------------------|-------------------|-----------------------------|
| 1265100           | 2823283           | 434842                      |
| 1572400           | 952459            | 663826                      |
| 1078400           | 935230            | 516081                      |
| 1262600           | 833013            | 577579                      |
| 1350900           | 1139520           | 683739                      |
| 720372            | 1055669           | 522651                      |
| 1084101           | 816340            | 417702                      |
| 1263499           | 953100            | 417591                      |
| 876939            | 869500            | 396786                      |
| 1825076           | 1124800           | 564243                      |
| 600168            | 883100            | 862723                      |
| 851955            | 1178400           | 615395                      |
| 1246100           | 1615900           | 324360                      |
| 2406100           | 1480700           | 414791                      |

$N_{log} = 6.068, \sigma_{log} = 0.158$  $N_{log} = 6.048, \sigma_{log} = 0.146$  $N_{log} = 5.711, \sigma_{log} = 0.111$
The relationships between fatigue life/standard deviation and stress level, fitted with the test results, are shown in Table 3.

**Table 3. Life-stress relations fitted from test data.**

| Relationship                  | Regression equation          | Correlation coefficient |
|-------------------------------|------------------------------|-------------------------|
| cyclic stress - mean life     | $s = 1332.6 \times (2N)^{0.173}$ | 0.9999                  |
| cyclic stress - mean log-life | $s = -47.6 N_{log} + 394.4$     | 0.9999                  |
| cyclic stress - std of log-life| $\sigma_{log} = 0.491-0.003s$ | 0.9978                  |

3. Fatigue life scatter analysis
The test results show that the fatigue life scatter, characterized by the standard deviation of the logarithmic life, under variable amplitude history, is different from that under any of the constant amplitude load histories. Shown in Table 4 are the standard deviations of the logarithmic lives under different load histories, and the Std ratios as well.

**Table 4. Standard deviation of logarithmic life under different load history.**

| Load history | Std of log-life | Std rate          |
|--------------|-----------------|-------------------|
| $s_1=110$MPa | Std=0.154       | Std/Std=0.72      |
| $s_2=120$MPa | Std=0.126       | Std/Std=0.88      |
| $s_3=130$MPa | Std=0.093       | Std/Std=1.19      |
| $s_3(100C)-s_1$| Std=0.158 | Std/Std=1.03      |
| $s_3(1000C)-s_1$| Std=0.146 | Std/Std=0.95      |
| $s_3-s_2-s_1$,...| Std=0.111 | Std/Std=1.19      |

For the load history $s_3-1000>s_1$, the log-life scatter (standard deviation of logarithmic life) 0.111 is between that associated with load history $s_2$ (0.126) and that associated with $s_3$ (0.093). It illustrates that the higher level stress cycles dominate the life scatter, noticing that the cycle numbers
associated with the three cyclic stress levels are the same (1/3 of the total cycles each) in the load history. This experiment result is different from that described in the literature [6], as the standard deviation associated with the highest cyclic stress $s_3$ is 0.093, while the standard deviation of logarithmic life under the variable amplitude load history is 0.111. Especially, the effect of the limited number of high stress cycles $s_3$ on fatigue life scatter is slight in the load history $s_3(100)s_1$ and $s_3(1000)s_1$. For both of these two load histories, the log-life scatters are more or less the same as that under the dominant low level stress cycles of the cyclic stress $s_1$.

![Figure 2. Standard deviation of fatigue life - cyclic stress level.](image)

![Figure 3. Standard deviation of fatigue life – mean life.](image)

Figure 2 shows the relationship (the dotted line) between the standard deviation of fatigue life and the cyclic stress level under constant amplitude load history, as well as the standard deviations of the fatigue lives under the variable amplitude load histories with different high stress cycle numbers in term of the highest cyclic stress level, it demonstrates that the fatigue life scatter is not exclusively determined by the highest stress level in the load history.

Shown in Figure 3 are the relationship (the dotted line) between the standard deviation of fatigue life and the fatigue life associated with the constant amplitude load history, as well as the standard deviations
of the fatigue lives under the variable amplitude load histories with different high stress cycle numbers in term of the life cycles, it demonstrates that the fatigue life scatter is largely determined by the fatigue life, no matter of the highest stress level in the load history.

4. Conclusions
High cycle fatigue life experiment results of the 6T-aluminum alloy illustrate that, under a variable amplitude load history, fatigue life scatter is jointly determined by all the stress cycles. The situation is that all the cyclic stress levels are far below the material elasticity limit, and the fatigue lives are in the range of $2\times10^5$–$3\times10^6$ cycles. The fitted relations show that the fatigue life scatter is dominated by the life cycles, instead of the highest cyclic stress level.

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