Spontaneous baryogenesis in flat directions

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We discuss a spontaneous baryogenesis mechanism in flat directions. After identifying the Nambu-Goldstone mode which derivatively couples to the associated U(1) current and rotates due to the A-term, we show that spontaneous baryogenesis can be naturally realized in the context of the flat direction. As applications, we discuss two scenarios of baryogenesis in detail. One is baryogenesis in a flat direction with a vanishing $B-L$ charge, especially, with neither baryon nor lepton charge, which was recently proposed by Chiba and the present authors. The other is a baryogenesis scenario compatible with a large lepton asymmetry.

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I. INTRODUCTION

Baryon asymmetry is a great mystery in cosmology and particle physics. The recent observations of the cosmic microwave background anisotropy by the Wilkinson Microwave Anisotropy Probe (WMAP) show that the baryon to entropy ratio is $n_B/s \sim 9 \times 10^{-11}$, which roughly coincides with the value inferred from big bang nucleosynthesis (BBN). Although many scenarios have been proposed so far to explain such an asymmetry, a spontaneous baryogenesis mechanism proposed by Cohen and Kaplan is special in that it works even in thermal equilibrium. The other scenarios require a deviation from thermal equilibrium, which often imposes severe constraints on the scenario.

The supersymmetric theory is one of the most attractive extensions of standard model particle physics because it stabilizes the electroweak scale against radiative corrections and realizes the unification of standard gauge couplings. In the supersymmetric theory, flat directions are ubiquitous and their existence distinguishes supersymmetric theories from ordinary ones. In particular, during inflation, a flat direction receives a mass squared proportional to the Hubble parameter squared, so that the flat direction acquires a large expectation value for the negative mass squared. As inflation ends and the Hubble parameter becomes small enough, the flat direction starts to oscillate and rotate due to the so-called A-term.

Associated with such dynamics of the flat direction, there are two different sources of baryon and/or lepton asymmetries. First, in the case that the flat direction carries the baryon and/or lepton numbers, the rotation due to the A-term implies that baryon and/or lepton asymmetries are generated as a condensate of the flat direction. After the decay of the flat direction, such asymmetries are released to the ordinary quarks and leptons. This is the so-called Affleck-Dine (AD) mechanism. Another source is the coupling between the phase of the flat direction and the baryon and/or lepton current. In fact, as shown later, the phase of the flat direction couples to such a current derivatively. Then, the rotation of the flat direction due to the A-term leads to CPT violation, so that baryon and/or lepton asymmetries are generated for light particles if the current violating interactions are in good thermal equilibrium. This is actually the realization of the spontaneous baryogenesis proposed by Cohen and Kaplan.

Until now, in almost all research, only the first source has been considered. However, the AD mechanism applies only to flat directions with nonzero $B-L$ charge because otherwise sphaleron effects wash out the produced baryon asymmetry. Recently, it was shown that, if Q-balls are formed, the AD mechanism can be applied to flat directions with vanishing $B-L$ charge. This is because Q-balls can protect the $B+L$ asymmetry from the sphaleron effects. However, the AD mechanism does not work at all for flat directions with neither baryon nor lepton charge, that is, $B = L = 0$.

On the other hand, very recently, Chiba and the present authors showed that baryogenesis is possible even for such a flat direction in the context of a second source if the flat direction has another charge. In this paper, we investigate the second source in detail, that is, a spontaneous baryogenesis mechanism in flat directions. Spontaneous baryogenesis in another context is considered in [11, 12]. In Sec. II, we identify the Nambu-Goldstone (NG) mode which derivatively couples to the associated U(1) current and rotates due to the A-term, and we give a general discussion on spontaneous baryogenesis in flat directions. In Sec III, as applications, we concentrate on two scenarios of baryogenesis in particu-
lar. One is baryogenesis in a flat direction with vanishing $B - L$ charge, especially, with neither baryon nor lepton charge. As listed in Ref. [13], in fact, there are many flat directions in which the $B - L$ charge vanishes. The other is a baryogenesis scenario compatible with a large lepton asymmetry. The baryon-to-entropy ratio inferred from recent results of WMAP roughly coincides with that inferred from BBN. However, according to a detailed analysis [14], the best fit value of the effective number of neutrino species $N_{\nu}$ is significantly smaller than 3.0. Of course, $N_{\nu} = 3.0$ is consistent at $\sim 2\sigma$, and such discordance may be completely removed as observations are further improved and the errors are reduced. However it is probable that such small discrepancies are genuine and suggest additional physics in BBN and the CMB. One interesting possibility eliminating such discrepancies is the presence of a large and positive lepton asymmetry of electron type [15], so we discuss such a possibility in this context. Finally, we devote Sec. IV to discussion and conclusions.

II. SPONTANEOUS BARYOGENESIS IN FLAT DIRECTION

A. Review of spontaneous baryogenesis

First, we shall explain how spontaneous baryogenesis works. For simplicity, we assume that the system has only baryon symmetry, $U(1)_B$. The extension to the case with several global $U(1)$ symmetries will be discussed later. Let us consider a scalar field $a$, which is derivatively coupled to the baryon current:

$$\mathcal{L}_{\text{eff}} = -\frac{\partial_{\mu} a}{M} J_{\mu}^B,$$

where $M$ is a cutoff scale. The baryonic current is given by

$$J_{\mu}^B = \sum_i B_i j_{\mu}^i,$$

$$j_{\mu}^i = \begin{cases} \psi_i\gamma^\mu\psi_i, & \text{for fermions,} \\ i(\bar{\varphi}_i\partial_{\mu}\varphi_i^* - \varphi_i^*\partial_{\mu}\varphi_i), & \text{for bosons}, \end{cases}$$

where $B_i$ is the baryon number of the $i$th field. If $\partial_{\mu}a$ takes a nonvanishing classical value, the above interaction induces spontaneous CPT violation since $\partial_{\mu}a$ is odd under CPT transformations. Assuming that the scalar field $a$ is homogeneous, we have

$$\mathcal{L}_{\text{eff}} = -\frac{\dot{a}}{M} n_B$$

$$\equiv \sum_i \mu_i n_i,$$

where the overdot denotes differentiation with respect to time, and $n_B$ is the baryon number density. Also we define the chemical potential $\mu_i \equiv -\dot{a}B_i/M$, and the number density of the $i$th field, $n_i \equiv j_{\mu}^i$. Thus, if $U(1)_B$ violating operators are in thermal equilibrium, as the thermal and chemical equilibrium states change, the number density $n_i$ for the $i$th field follows the chemical potential $\mu_i$.

When the baryon number violating interactions decouple at last, the baryon number density is frozen at the value at that time [3]:

$$n_B(t_D) = \sum_i B_i \frac{g_i\kappa_i T_D^3}{6} \left\{ \mu_i + O\left(\frac{\mu_i}{T_D}\right)^3 \right\},$$

where $T_D$ is the decoupling temperature and $g_i$ represents the degrees of freedom of the corresponding field. Also, $\kappa_i$ is defined as

$$\kappa_i = \begin{cases} 1 & \text{for fermions,} \\ 2 & \text{for bosons}. \end{cases}$$

It should be emphasized that such asymmetries are realized only in the light fields that contribute to the energy of the universe as radiation.

B. Nambu-Goldstone bosons associated with flat directions

Here we show that derivative interactions like Eq. (1) are naturally present in the minimal supersymmetric standard model, which contains many flat directions. Since flat directions are composed of the standard model fields charged under several (global or local) $U(1)$ symmetries, the NG bosons associated with these symmetries are induced if a flat direction develops a nonzero vacuum expectation value (VEV). It is one of these NG bosons that realizes spontaneous baryogenesis through derivative interactions. The purpose of this section is to identify this NG boson, and derive the derivative interaction relevant for spontaneous baryogenesis.

A flat direction $X$ is specified by a holomorphic gauge-invariant polynomial:

$$X \equiv \prod_i^{N} \chi_i,$$

where $N$ superfields $\{\chi_i\}$ constitute the flat direction $X$, and we have suppressed the gauge and family indices with the understanding that the latin letter $i$ contains all the information to label those constituents. When $X$ has a nonzero expectation value, each constituent field also takes a nonzero expectation value

$$\langle \chi_i \rangle = \frac{f_i}{\sqrt{2}} e^{i\theta_i} = \frac{f_i}{\sqrt{2}} e^{iP_i/f_i}.$$

Here each $f_i/\sqrt{2}$ is the absolute value of the expectation value $\langle \chi_i \rangle$, and is related to every other due to the D-(F-)flat conditions. $P_i$ is a canonically normalized field corresponding to the phase component $\theta_i$. For the moment,
we will pay attention to the baryon ($B$) and lepton ($L$) symmetries, and consider flat directions whose expectation values break both symmetries.\footnote{Strictly speaking, we must take into account the weak hypercharge $Y$. However, it does not change our result, so we neglect its effect for simplicity.} Later we consider the case that the standard model particles are charged under extra global $U(1)$ symmetry, such as the $PQ$ symmetry. We define $U(1)_{A\pm}$, which are the two independent linear combinations of $U(1)_B$ and $U(1)_L$ symmetries:

\[
U(1)_{A+} : Q^+ = \cos \xi \ B + \sin \xi \ L, \\
U(1)_{A-} : Q^- = -\sin \xi \ B + \cos \xi \ L, \\
\tan \xi \equiv \frac{\sum_i L_i}{\sum_i B_i}.
\] (8)

Here $B$, $L$, and $Q^\pm$ are the charges of $U(1)_B$, $U(1)_L$, and $U(1)_{A\pm}$, respectively. We define the generators of $U(1)_B$, $U(1)_L$, and $U(1)_{A\pm}$ as $\Upsilon_B$, $\Upsilon_L$, and $\Upsilon_{A\pm}$. Following the argument in Refs. [16, 17], we express $\theta_i$ as

\[
\theta_i \equiv B_i \alpha_B + L_i \alpha_L \\
\equiv Q^+_i \alpha_+ + Q^-_i \alpha_-,
\] (9)

where $B_i$, $L_i$, and $Q^\pm_i$ are the corresponding charges of $\chi_i$, and $\alpha_B$, $\alpha_L$, and $\alpha_\pm$ are the angles conjugate to the generators $\Upsilon_B$, $\Upsilon_L$, and $\Upsilon_{A\pm}$, respectively. They are related as follows:

\[
\begin{pmatrix} \alpha_B \\ \alpha_L \end{pmatrix} = \begin{pmatrix} \cos \xi & -\sin \xi \\ \sin \xi & \cos \xi \end{pmatrix} \begin{pmatrix} \alpha_+ \\ \alpha_- \end{pmatrix}.
\] (10)

The NG bosons associated with the spontaneous breaking of $U(1)_B$, $U(1)_L$, and $U(1)_{A\pm}$ are denoted as $a_B$, $a_L$, and $a_\pm$. Finally, we define the $2 \times 2$ real matrix of decay constants $F$ as

\[
\begin{pmatrix} a_+ \\ a_- \end{pmatrix} = F \begin{pmatrix} \alpha_+ \\ \alpha_- \end{pmatrix}.
\] (11)

Since the kinetic terms of $a_\pm$ come from those of $\chi_i$ with $f_i$ fixed, $F$ is related to the amplitudes $f_i$:

\[
F^T F = \sum_i f_i^2 \begin{pmatrix} Q^+_i Q^+_i & Q^+_i Q^-_i \\ Q^-_i Q^+_i & Q^-_i Q^-_i \end{pmatrix}.
\] (12)

As shown below, $a_+$ becomes massive due to the existence of the $A$-term, while $a_-$ remains massless. In order to solve for $F$, it is necessary to know the effective potential for flat directions.

Flat directions are lifted by both supersymmetry-breaking effects and nonrenormalizable operators, although there are no classical potentials along flat directions in the supersymmetric limit. Since a flat direction is well described by a single complex scalar field $\Phi \equiv \phi/\sqrt{2} e^{i\theta}$, these effects induce a potential for $\Phi$. Here $\phi$ is the amplitude, whose expectation value is given by a typical value of $f_i$, and the phase $\theta$ is defined as

\[
\theta \equiv \frac{1}{N} \sum_i \theta_i = \frac{1}{N} \sum_i Q^+_i \alpha_+.
\] (13)

So the scalar field $\Phi$ is often defined as $\Phi^N \equiv X$ (as shorthand). Assuming gravity-mediated supersymmetry breaking, the flat direction is lifted by supersymmetry-breaking effects \cite{18},

\[
V_{\text{grav}} \simeq m_{\phi}^2 \left[ 1 + K \log \left( \frac{|\Phi|^2}{M_G^2} \right) \right] |\Phi|^2,
\] (14)

where $m_{\phi} \sim 1$ TeV is a soft mass, $K$ is a numerical coefficient of one-loop corrections, and $M_G$ is the reduced Planck mass. Moreover, assuming a nonrenormalizable operator in the superpotential of the form

\[
W_{\text{NR}} = \frac{X^n}{N_k M^{nk-3}} \equiv \frac{\Phi^n}{n M^{n-3}},
\] (15)

the flat direction is further lifted by the potential

\[
V_{\text{NR}} = |\Phi|^{2n-2} \frac{M^{2n-6}}{M_{\text{cutoff}}^2},
\] (16)

where $n \equiv N_k$ and $M$ is a cutoff scale. In fact, the nonrenormalizable superpotential not only lifts the potential but also gives the baryon and/or lepton number violating $A$-terms of the form

\[
V_A = a_m \frac{m_{3/2}}{N_k M^{nk-3}} X^k + \text{h.c.} \\
= a_m \frac{m_{3/2}}{n M^{n-3}} \Phi^n + \text{h.c.},
\] (17)

where $m_{3/2}$ is the gravitino mass, $a_m$ is a complex constant of order unity, and we assume a vanishing cosmological constant. With the redefinition of the phase of $\Phi$, $a_m$ can be real. Hereafter we adopt this convention.

On the other hand, it is also possible that nonrenormalizable operators in Eq. \cite{15} are forbidden by some R-symmetries. Then the effective potential is parabolic like Eq. \cite{14} up to some cutoff scale of the Kähler potential, $M_{\ast}$. The $A$-terms are also supplied by the Kähler potential as

\[
V_{A_k} = a_\ast \frac{m_{3/2}^2}{N_k M_{\ast}^{nk-3}} X^k + \text{h.c.} \\
= a_\ast \frac{m_{3/2}^2}{n M_{\ast}^{n-3}} \Phi^n + \text{h.c.},
\] (18)

where $a_\ast$ is taken to be a real constant of order unity. Note that the dependence of the $A$-term on the gravitino mass is different from that of the nonrenormalizable term in the superpotential.

During the inflationary epoch, a flat direction can easily acquire a large expectation value. Strictly speaking, if a flat direction has a four-point coupling to the inflaton...
in the Kähler potential with appropriate sign and magnitude, it has a negative mass squared proportional to the Hubble parameter squared [19],

\[ V_H = -c_H H^2 |\Phi|^2, \]  

(19)

where \( c_H \) is a positive constant of order unity. This negative mass term destabilizes the flat direction at the origin, and the flat direction rolls down toward the minimum of the potential. The position of the minimum depends on whether the nonrenormalizable superpotential exists or not. If it exists, the minimum is determined by the balance between the negative mass term \( V_H \) and the nonrenormalizable potential \( V_{\text{NR}} \). If not, it is fixed around the cutoff scale \( \sim M_* \). Thus, the minimum of the potential during inflation is given by

\[ \phi_{\text{min}} \sim \left\{ \frac{HM^{n-3}}{M_*} \right\}^{\frac{1}{n-2}} \text{with } W_{\text{NR}}, \]

(20)

Notice that the flat direction takes the nonzero expectation value given by Eq. \( (20) \) even after inflation ends, once the initial position is set during inflation. We would like to comment on thermal effects on flat directions. Since those fields that couple to the flat direction obtain a large mass of \( O(f\phi_{\text{min}}) \) where \( f \) represents the yukawa or gauge coupling constant, they must be out-of-equilibrium. Therefore we do not take account of thermal effects in the following.

Now we consider the implication of the A-term. As one can see, the phase \( \theta \) appears only in the A-term. In either case, it can be expressed as

\[ V_A = M_A^4 \cos \left[ k \alpha_a \right], \]  

(21)

where

\[ Q = \sqrt{\left( \sum_i B_i \right)^2 + \left( \sum_i L_i \right)^2} \]

(22)

and \( M_A \) denotes the energy scale of the A-term. This means that \( U(1)_{A+} \) is explicitly violated by the A-term, while \( U(1)_{A-} \) remains intact. Since \( a_+ \) becomes massive due to the interaction in Eq. \( (21) \), \( F \) takes the following form:

\[ F = \begin{pmatrix} v_a & 0 \\ f_{01} & f_{11} \end{pmatrix}. \]

(23)

Substituting this equation into Eq. \( (12) \), we have

\[ v_a^2 = \frac{\sum_i f_{i1}^2 Q_i^2 + \left( \sum_i f_{i1}^2 Q_i^+ Q_i^- \right)^2}{\sum_i f_{i1}^2 Q_i^{-2}}, \]

\[ f_{01} = \frac{\sum_i f_{i1}^2 Q_i^+ Q_i^-}{\sqrt{\sum_i f_{i1}^2 Q_i^{-2}}}, \]

\[ f_{11} = \frac{\sum_i f_{i1}^2 Q_i^2}{\sqrt{\sum_i f_{i1}^2 Q_i^{-2}}}, \]

(24)

where we have assumed that the \( Q_i^+ \)'s are not all zero. Thus we can identify the NG boson that becomes massive due to the existence of the A-term:

\[ a_+ = v_a \alpha_a + \frac{\theta}{N} \sum_i Q_i^+, \]

(25)

where we used Eq. \( (13) \) in the last equality. This equation illuminates the reason why a flat direction can be well described by a single complex scalar field \( \Phi \). Its amplitude represents the magnitude of the expectation value of the flat direction, and the phase corresponds to the dynamical NG mode whose motion is affected by the A-term. The equation of motion of \( a_+ \) is given by

\[ \ddot{a}_+ + 3H \dot{a}_+ - M_A^2 k \bar{Q} \sin \left( k \bar{Q} \frac{a_+}{v_a} \right) = 0, \]

(26)

where we assumed that the \( \{f_i\} \) are constant.\(^2\) Its velocity can be estimated as

\[ |\dot{a}_+| \sim \frac{k \bar{Q}}{H v_a} M_A^4, \]

(27)

where we used the slow-roll approximation because the inverse curvature scale of the potential is roughly \( \sqrt{m_3/2H} \ll H. \)\(^3\) We have also assumed that \( a_+ \) remains far from the extremum of the potential by \( O(v_a) \). The velocity plays an essential role for both the AD mechanism and spontaneous baryogenesis in our scenario. In fact, it generates the asymmetry of \( U(1)_{A+} \) as a condensate of the flat direction. The \( Q^+ \) number density \( n_+ \) can be calculated as

\[ n_+ = -\sum_i Q_i^+ f_{i1}^2 \dot{\theta}_i = -v_a \dot{a}_+, \]

(28)

while the \( Q^- \) number density \( n_- \) remains zero. In the AD mechanism, the baryon (or lepton) asymmetry comes from this \( U(1)_{A+} \) asymmetry.

With this background we turn now to an account of the derivative interactions. The NG mode \( a_+ \) transforms as \( a_+ \rightarrow a_+ + v_a \epsilon \) under the \( U(1)_{A+} \) transformation \( \alpha_a \rightarrow \alpha_a + \epsilon \). Since the \( \{\chi_i\} \) are the particles of the standard model, they participate in the following Yukawa superpotential of the standard model:

\[ W_{\text{SM}} = y_u Q H_u + y_d Q H_d + y_e e L H_d, \]

(29)

where \( Q \) and \( L \) are \( SU(2)_L \) doublet quarks and leptons, \( u, d, \) and \( e \) are \( SU(2)_L \) singlet quarks and leptons, and

\(^2\) As long as \( \dot{f}_i \lesssim H f_i \), the equation of motion is still valid up to a numerical factor in the viscosity term.

\(^3\) During inflation, the effective mass of \( a_+ \) is comparable to the Hubble scale due to the Hubble induced A-term so that quantum fluctuations of \( a_+ \) are negligible.
$H_u(H_d)$ is the up-(down-)type Higgs superfield. The Yukawa interactions above therefore depend on the NG mode $a_+$ when the flat direction has an expectation value. In order to obtain the interaction between the NG mode $a_+$ and the other charged fields, we define the $U(1)_{A^+}$ current as

$$J_{A^+}^\mu \equiv -\sum_{m'} \frac{\partial L}{\partial (\partial_\mu \chi_{m'})}\chi_{m'}, \quad (30)$$

where $m'$ denotes all fields with nonzero $U(1)_{A^+}$ charges, that is, $\chi_{m'}$ transforms under $U(1)_{A^+}$ symmetry as $\chi_{m'} \to \chi_{m'} + \epsilon \delta \chi_{m'}$ with $\delta \chi_{m'} = iQ_{m'}^+\chi_{m'}$. Then, the $U(1)_{A^+}$ current is given by

$$J_{A^+}^\mu = v_0 \partial^\mu a_+ + \sum_m Q^+_m j^\mu_m, \quad (31)$$

where $m$ denotes all fields with nonzero $U(1)_{A^+}$ charges except the NG mode $a_+$. Current conservation yields the equation of motion for $a_+$

$$\partial_\mu J_{A^+}^\mu = v_0 \partial^2 a_+ + \sum_m Q^+_m \partial j^\mu_m = 0. \quad (32)$$

Here the first term in the middle equation is the kinetic term for the NG mode $a_+$ and the second term can be derived from the following effective Lagrangian,

$$L_{\text{eff}} = -\sum_m \frac{Q^+_m}{v_0} (\partial_\mu a_+) j^\mu_m, \quad (33)$$

which yields the derivative interactions between the NG mode $a_+$ and the other charged fields. Although the index $m$ should be taken over all the charged fields except the NG mode $a_+$, we will concentrate on a derivative coupling of all light fields because the asymmetries are induced only in the light degrees of freedom. Hereafter the subscript $m$ denote the species of light fields unless otherwise stated. Note that the above discussion applies for the $U(1)_{A^-}$ symmetry in the same way, and that there also exists the derivative coupling of $a_-$ with the $U(1)_{A^-}$ current. However, it does not have any significant meaning, since $\dot{a}_-$ cannot have a nonzero classical value.

Up to this point we have considered the case that the system has only baryon and lepton symmetries. However, the essential points of our argument expressed so far are still valid in the general case with extra $U(1)$ symmetries. There exists only one NG boson which becomes massive due to the A-term. In order to extract it, we may need to take a superposition of the symmetries as shown in Eq. (8). It is always possible to rotate out the NG boson from the Yukawa interactions, leading to a derivative interaction with a current. In particular, Eqs. (25) and (32) are valid with the understanding that $U(1)_{A^+}$ is the symmetry violated by the A-term, and that $Q^+_m$ and $a_+$ represent the corresponding charge and NG boson, respectively.

### C. Symmetry breaking operators in thermal equilibrium

We would next like to focus on the subtleties associated with the fact that there are several $U(1)$ symmetries. In the simple case illustrated in the first part of this section, thermal and chemical equilibrium are attained for each field, with the chemical potential given by the coefficient of the number density in the derivative interaction. In general, this is not the case. The asymmetries generated through spontaneous baryogenesis crucially depend on the symmetry-breaking operators in thermal equilibrium. For simplicity, we consider the case with baryon and lepton symmetries. Assuming that sphaleron processes are effective at a later epoch, we concentrate on $B - L$ violating operators in thermal equilibrium. The final $B - L$ asymmetry is determined by the one with the lowest decoupling temperature, as long as $\mu T^2$ decreases more slowly than $a^{-3}$. Such an interaction can be characterized by the amount of $B$ and $L$ violation, $\Delta_B$ and $\Delta_L$, respectively. Since the baryon and lepton asymmetries are generated through this interaction, they are related as $n_B\Delta_L = n_L\Delta_B$, that is,

$$\sum_m \Xi_m n_m = 0, \quad (34)$$

where we defined $\Xi_m = B_m \Delta_L - L_m \Delta_B$. We would like to know the resultant $n_m$ induced by the derivative interaction Eq. (33) under this constraint. If it were not for the constraint, the following asymmetry would be generated:

$$\Pi_m(t_D) = \frac{\kappa_m g_m}{6} \bar{\mu}_m T^2_D, \quad (35)$$

where $\bar{\mu}_m \equiv -Q^+_m \dot{a}_+ / v_0$. It is worth noting that $\bar{\mu}_m$ cannot be interpreted to be the chemical potential of the $m$th field, if we take the constraint into account. The reason for this is not difficult to see. Due to the constraint, $n_m$ is not able to vary freely. In fact, only the projection of $\{\bar{\mu}_m\}$ onto the parameter plane perpendicular to $\{\Xi_m\}$ has physical meaning. That is to say, the resultant asymmetry should depend on $\bar{\mu}_m$ defined as

$$\bar{\mu}_m \equiv \bar{\mu}_m - \frac{(\bar{\mu} \cdot \Xi)}{\Xi^2} \Xi_m, \quad (36)$$

where we adopt the following shorthand.

$$Y^2 \equiv \sum_m \kappa_m g_m Y^2_m, \quad Y \cdot Z \equiv \sum_m \kappa_m g_m Y_m Z_m. \quad (37)$$

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4 Here we assume that the decoupling temperatures are not degenerate.
5 Hereafter we assume that the interactions which transmit the generated baryon and lepton numbers to other particles are in thermal equilibrium.
Then it is easy to show that \( \mu_m \) are invariant under the transformation \( \pi_m \rightarrow \pi_m + \alpha \Xi_m \) for an arbitrary constant \( \alpha \). If we require that \( \{ \pi_m \} \) take the form of thermal and chemical equilibrium, the \( \{ n_m \} \) are then uniquely determined by using \( \mu_m \):

\[
n_m(t_D) = \frac{k_m g_m}{6} \tilde{\mu}_m T_D^2.
\]

(38)

The resultant \( B - L \) number density is then given by

\[
n_{B-L}(t_D) = \sum_m (B_m - L_m)n_m = \left( \Delta_B - \Delta_L \right) \left( \mu_B \Delta_B + \mu_L \Delta_L \right)
\]

\[
\times \frac{B^2 L^2}{B^2 \Delta_B^2 + L^2 \Delta_L^2} T_D^2,
\]

(39)

where \( \mu_B \equiv -\cos \xi \dot{\alpha}_+ / v_a \) and \( \mu_L \equiv -\sin \xi \dot{\alpha}_+ / v_a \), so that \( \pi_m = \mu_B B_m + \mu_L L_m \). Thus, the following two conditions must be met for \( B - L \) asymmetry to be generated:

\[
\Delta_B - \Delta_L \neq 0, \quad \mu_B \Delta_B + \mu_L \Delta_L \neq 0.
\]

(40)

(41)

The meaning of the first condition is clear. The interaction in thermal equilibrium should, of course, violate \( B - L \) symmetry. In order to understand the second condition, we rewrite it as follows:

\[
\mu_B \Delta_B + \mu_L \Delta_L = -\left( \Delta_B \cos \xi + \Delta_L \sin \xi \right) \frac{\dot{\alpha}_+}{v_a}
\]

\[
= -\Delta_Q \frac{\dot{\alpha}_+}{v_a}.
\]

(42)

Hence, the second one means that interaction in thermal equilibrium must also violate \( U(1)_1 \) symmetry, so that the derivative interaction does induce some asymmetries. If \( \Delta_Q + 0 \), no asymmetries result since the broken symmetry in thermal equilibrium is then orthogonal to the \( U(1)_1 \) symmetry, for which nonzero chemical potential is induced by the derivative interaction. What needs to be emphasized at this juncture is that the asymmetries are induced even for fields that do not participate in derivative interactions. The reason for this is that the constraint Eq. (41) relates the fields with \( \pi_m \neq 0 \) to those with \( \pi_m = 0 \), so that the latter can feel the effective chemical potential. For example, it is even possible to set all the derivative interactions in the hidden sector, as long as there exists an interaction in thermal equilibrium that violates both the hidden symmetry and \( B - L \) symmetry. Then the asymmetry generated in the hidden sector also induces \( B - L \) asymmetry through such an interaction. We shall detail an application of this striking feature in the next section.

### III. APPLICATIONS

#### A. Baryogenesis in flat directions with \( B - L = 0 \)

As an application of the spontaneous baryogenesis discussed in the previous section, here we consider baryogenesis in those flat directions with \( B - L = 0 \) and \( B + L \neq 0 \). In the next subsection we consider baryogenesis in flat directions with \( B = L = 0 \). The AD mechanism generates the asymmetry of the \( U(1) \) charge, which is explicitly violated by the \( A \)-term. Hence it does not work for flat directions with \( B - L = 0 \), unless \( Q \)-balls are formed. Even if \( Q \)-balls are formed, they might decay and/or evaporate away before the electroweak phase transition. Then no baryon asymmetry results due to the sphaleron processes. As listed in Ref. [13], in fact, there are many flat directions that have vanishing \( B - L \) (e.g., \( Q Q Q L \), \( u u d e \)), and some of them have neither baryon nor lepton charges (for example, \( Q u Q d \), \( Q u L e \)). If such flat directions were selected during inflation, it is difficult to explain the baryon asymmetry in the present universe. Thus, it is intriguing to examine whether it is possible to realize baryogenesis even when a flat direction with \( B - L = 0 \) is selected.

Since we are considering the flat direction with \( B = L \neq 0 \), \( Q^\pm \) and \( v_a \) defined in Eqs. (8) and (20) are given by

\[
Q^+ = \frac{B + L}{\sqrt{2}},
\]

\[
Q^- = -\frac{B + L}{\sqrt{2}},
\]

\[
v_a^2 = 2 \sum_i \frac{f_i^2 f_j^2 B_i^2 L_j \cos \phi_{\min i}}{f_i^2 (B_i^2 + L_j^2)} \sim \phi_{\min i}^2,
\]

(43)

where we have used \( f_i \sim \phi_{\min i} \) and \( B_i \sim L_i \sim 1 \) in the last line. The derivative interaction leads to the following:

\[
\mathcal{L}_{\text{eff}} = \sum_i \pi_i \eta_i,
\]

(44)

with

\[
\bar{\eta}_i = -\frac{B_i + L_i}{\sqrt{2}v_a} \dot{\alpha}_+ \sim -\frac{k Q}{\sqrt{2} m_{3/2}} (B_i + L_i),
\]

(45)

where we have used Eqs. (21) and (27) and assumed the existence of a nonrenormalizable term and that \( \dot{\alpha}_+ > 0 \).

We need to incorporate a \( B - L \) breaking operator in thermal equilibrium that also violates \( B + L \) symmetry, in order to realize spontaneous baryogenesis. The resultant \( B - L \) asymmetry can be estimated as Eq. (39) with

\[
\mu_B = \mu_L = -\frac{k Q}{\sqrt{2} m_{3/2}}.
\]

(46)

As an illustration, we consider the following \( B - L \) breaking operator:

\[
\mathcal{L}_B = \frac{g}{v} \bar{l} l H_u H_u + \text{h.c.},
\]

(47)

where \( v \) is the scale characterizing the interaction and may be identified with the heavy Majorana mass for the right-handed neutrino in the context of the seesaw mechanism. This interaction violates the lepton number by two, while the baryon number is intact: \( \Delta_L = 2 \) and
$\Delta_B = 0$, so we have $\Xi_m = 2B_m$. Then the effective chemical potential $\bar{\mu}_m$ can be calculated as

$$\bar{\mu}_m \simeq -\frac{kQ}{\sqrt{2}} m_{3/2} L_m.$$ (48)

Naturally, this result means that only lepton asymmetry is generated through spontaneous baryogenesis. Thus the $B - L$ asymmetry at decoupling is

$$\frac{n_{B-L}}{s} = \frac{15kQ}{4\sqrt{2}\pi^2} \frac{L^2 m_{3/2}}{g_* T_D},$$ (49)

where $g_*$ counts the effective degrees of freedom for relativistic particles. The baryon asymmetry is obtained through the sphaleron effects$^6$ as$^{20}$

$$\frac{n_B}{s} = \frac{30kQ}{23\sqrt{2}\pi^2} \frac{L^2 m_{3/2}}{g_* T_D}.$$ (50)

In order to estimate the baryon asymmetry, we need to know the typical value of $T_D$.

The lepton number violating rate of the interaction given by Eq. 17 is given by $\Gamma \sim 0.04 T^5/v^2$ $^{21}$. Then, the decoupling temperature is calculated as

$$T_D \sim 5 \times 10^{11} \text{GeV} \left(\frac{g_*}{200}\right)^{\frac{1}{2}} \left(\frac{v}{10^{14} \text{GeV}}\right)^{2},$$

$$\sim 7 \times 10^{11} \text{GeV} \left(\frac{g_*}{200}\right)^{\frac{1}{2}} \left(\frac{m_\nu}{1 \text{ eV}}\right)^{-2} \sin^4 \beta,$$ (51)

where $\tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle$, and $m_\nu$ is the neutrino mass related to $v$ by $m_\nu = 4 \langle H_u \rangle^2 / v$. Also we have assumed that the reheating process ends before the decoupling. Such a high reheating temperature might lead to the gravitino problem$^{22}$. For $m_{3/2} = 3 - 10$ TeV, the reheating temperature is constrained as $T_{RH} \lesssim 10^{12}$ GeV, assuming that the mass of the lightest supersymmetric particle (LSP) is $O(100 \text{ GeV})$ $^{22}$. If we take this bound seriously, the decoupling temperature $T_D$ must be less than $10^{12}$ GeV so that the lepton number violating interaction is in thermal equilibrium. For $m_{3/2} \sim 100$ GeV, the reheating temperature must be smaller than $T_{RH} \lesssim 10^8$ GeV. Alternatively, these constraints on the reheating temperature can be evaded by the introduction of a supersymmetric partner with a mass much lighter than 100 GeV. One such particle is the axino. In fact, it was shown that the reheating temperature is constrained rather loosely as $T_{RH} < 10^{15}$ GeV for $m_{3/2} \approx 100$ GeV, if the axino is the LSP and the gravitino is the next LSP$^{23}$.

As long as the gravitino problem is absent, the typical value of the decoupling temperature is $T_D \sim 10^{12}$ GeV. Hence we obtain the right amount of baryon asymmetry,

$$\frac{n_B}{s} \sim 0.1 \left(\frac{m_{3/2}}{3 \text{ TeV}}\right) \left(\frac{T_D}{10^{12} \text{ GeV}}\right)^{-1}.$$ (52)

**B. Baryogenesis in flat directions with $B = L = 0$**

Next we show that baryogenesis is possible even for flat directions with $B = L = 0$. Our strategy is as follows. Since $\sum_i B_i = \sum_i L_i = 0$, the baryon and lepton symmetries are not violated by the A-term. Therefore the corresponding NG bosons $a_B$ and $a_L$ remain massless. Note that there is a degree of freedom $a$ which becomes massive due to the A-term and develops a classical value $\dot{a} \neq 0$. However, it is not a NG boson since the system does not possess the corresponding symmetry, so it cannot have any derivative interactions with the currents. Now it is clear what must be added to the system. We need to incorporate an additional $U(1)$ symmetry which is explicitly violated by the A-term. Then the corresponding NG boson obtains a nonzero velocity, leading to the derivative interactions relevant for spontaneous baryogenesis.

In order to realize our scenario, the standard particles should be charged under another global symmetry in addition to the baryon and lepton symmetries. One of the famous examples is $PQ$ symmetry, which was introduced to solve the strong $CP$ problem of quantum chromodynamics$^{24}$. For definiteness, we adopt the supersymmetric Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) axion model$^7$ $^{25}^{26}$, but it is trivial to extend it to the case with general global $U(1)$ symmetries. Then, a flat direction with neither baryon nor lepton charge can have nonzero $PQ$ charges if we assign them properly to standard particles. In the same way as before, we express $\theta_i$ as

$$\theta_i = R_i \alpha_R + B_i \alpha_B + L_i \alpha_L,$$ (53)

where $R_i$ is the $PQ$ charge of $\chi_i$, and $\alpha_R$ is the angle conjugate to the generator $T_R$. Then the $A$-term can be written as

$$V_A = M_A^4 \cos [k R \alpha_R],$$ (54)

$^6$ If the weak gauge bosons become massive due to the large VEV of the flat direction, the sphaleron configuration is not excited before the decay of the flat direction. Even if the flat direction is composed of $SU(2)$ singlet fields, the sphaleron process might be out of equilibrium at decoupling, since the decoupling temperature is rather high ($\sim 10^{12}$ GeV as shown below). Hereafter we assume that this is the case.

$^7$ Here we assume that the $PQ$ scalar fields, which are responsible for the spontaneous $PQ$ symmetry breaking in the present universe, have negative mass squared of order $H$ during inflation. Then we can avoid the problem of axion domain walls. In addition, cold dark matter can also be explained by the axion in our scenario as a by-product of adopting the $PQ$ symmetry.
where $\mathcal{R} \equiv \sum_i R_i$. We define the NG boson corresponding to $PQ$ symmetry as

$$a_R = v_a \alpha_R,$$

where $v_a$ now takes the complicated form \[10\]

$$v_a^2 = \sum_i R_i^2 f_i^2 - \frac{(\sum_i R_i^2 f_i^2)^2}{\sum_i B_i^2 f_i^2} - \frac{(\sum_i R_i^2 f_i^2)^2}{\sum_i L_i^2 f_i^2},$$

$$\sim \phi_{\text{min}}^2.$$

(56)

We have assumed that $\phi_{\text{min}}$ is much larger than the breaking scale of the $PQ$ symmetry, $F_a$, in the present universe. In fact, the NG boson $a_R$ continuously transforms into the axion that is the phase component of the $PQ$ scalar field, after the flat direction starts to oscillate. Differentiation of $a_R$ with respect to time is likewise estimated as

$$|\dot{a}_R| \sim \frac{k R}{H v_a} M_A^4.$$

(57)

The time component of the derivative interaction then reads

$$L_{\text{eff}} = \sum_m \tilde{\mu}_m n_m$$

(58)

with

$$\tilde{\mu}_m = -\frac{R_m}{v_a} \dot{a}_R \sim -k R R_m m_{3/2},$$

(59)

where we have used Eqs. (56) and (57), and assumed the existence of a nonrenormalizable superpotential and $\dot{a}_R > 0$.

In order to estimate the resultant number density, we must take into account two constraints like Eq. (54), because the system now possesses three $U(1)$ symmetries. We assume that the $B - L$ violating interaction$^8$ in thermal equilibrium breaks the $PQ$, $B$, and $L$ symmetries by $\Delta R$, $\Delta B$, and $\Delta L$, respectively. The constraints are written as

$$\sum_m \Xi_m^{(i)} n_m = 0 \quad \text{for} \quad i = 1, 2, 3,$$

(60)

and two of them are independent. The resultant number density is given by

$$\tilde{n}_m(t_D) = \frac{\kappa v m_{3/2}}{6} \tilde{\mu}_m T_D^2,$$

(62)

where

$$\tilde{\mu}_m \equiv \frac{\Xi_m^{(1)}}{\beta^{(1)}} - \frac{\Xi_m^{(2)}}{\beta^{(2)}},$$

$$\beta^{(1)} = \frac{\Xi_m^{(1)} (\bar{P} \cdot \Xi_m^{(1)}) - \Xi_m^{(1)} (\bar{\Xi}_m^{(1)})}{\Xi_m^{(1)} - \Xi_m^{(1)}},$$

$$\beta^{(2)} = \frac{\Xi_m^{(2)} (\bar{P} \cdot \Xi_m^{(1)}) - \Xi_m^{(1)} (\bar{\Xi}_m^{(1)})}{\Xi_m^{(2)} - \Xi_m^{(2)}},$$

(63)

Then it is easy to show that $\tilde{\mu}_m$ are invariant under the transformation $\tilde{\mu}_m \rightarrow \tilde{\mu}_m + \alpha^{(1)} \Xi_m^{(1)} + \alpha^{(2)} \Xi_m^{(2)}$ for arbitrary constants $\alpha^{(1)}$ and $\alpha^{(2)}$. One can also check that the number density is invariant under the permutation $\Xi_m^{(1)} \rightarrow \Xi_m^{(2)} \rightarrow \Xi_m^{(3)} \rightarrow \Xi_m^{(1)}$, as long as the two conditions used in the definition of $\tilde{\mu}_m$ are independent. The $B - L$ number density is given by

$$n_{B-L}(t_D) = \sum_m (B_m - L_m) n_m(t_D)$$

$$= - (\Delta_B - \Delta_L) \frac{D k R m_{3/2} T_D^2}{C}$$

(66)

where we have defined

$$C \equiv \Delta_L^2 \left[ B^2 R^2 - (B \cdot R)^2 \right] + \Delta_B^2 \left[ L^2 R^2 - (L \cdot R)^2 \right]$$

$$+ \Delta_{\Delta B} L^2 B^2 - 2 \Delta R \left[ \Delta_B L^2 (B \cdot R) + \Delta_L B^2 (L \cdot R) \right]$$

$$+ 2 \Delta_{\Delta B} (L \cdot R) (B \cdot R),$$

$$D \equiv L^2 B^2 R^2 - L^2 (B \cdot R)^2 - B^2 (L \cdot R)^2$$

(67)

Thus, $B - L$ asymmetry is generated if and only if the interaction in thermal equilibrium violates both $B - L$ and $PQ$ symmetries, i.e., $\Delta_B - \Delta_L \neq 0$ and $\Delta R \neq 0$, the meanings of which are the same as before.

As an illustration we take the operator in Eq. (17), leading to $\Delta R = 2(R_L + R_{H_u})$ and $\Delta_B - \Delta_L = -2$. Then the baryon-to-entropy ratio is given by

$$\frac{n_B}{s} = \frac{120 k R (R_L + R_{H_u}) D m_{3/2}}{23 \pi^2 g_* C T_D}$$

$$\sim 3 \times 10^{-10} \left( \frac{m_{3/2}}{3 \text{TeV}} \right) \left( \frac{T_D}{10^{12}\text{GeV}} \right)^{-1},$$

(68)

where we have assumed that the $PQ$ charges are of the order of unity.

It should be noted that the constraint on the reheating temperature due to the gravitino problem is avoided, since the axino, the superpartner of the axion, naturally exists in our scenario.

\[\text{The $PQ$ symmetry is also violated by strong sphaleron processes. However, the strong sphaleron configuration is not excited as long as the SU(3)$_C$ is spontaneously broken by the large VEV of the flat direction. Fortunately, since all the $B = L = 0$ directions contain squarks, this is always the case. Furthermore, the $\mu$-term, which breaks $PQ$ symmetry, is out of equilibrium at $T_D \sim 10^{12}$ GeV. Hereafter we assume that the other interactions violate neither $PQ$ nor $B - L$ symmetry at decoupling.}\]
C. Large lepton asymmetry

As a final application we consider a scenario in which a large lepton asymmetry of electron type and a small baryon asymmetry are generated simultaneously. We assume that the system has only baryon and lepton symmetries throughout this subsection for simplicity. With an additional $U(1)$ symmetry, the main points of the following discussion are unchanged. In order to generate a positive and large lepton asymmetry of electron type, we choose a leptonic flat direction such as “$c_L^2 L_3 L_1$"; in which the subscripts denote the generation. We also assume that nonrenormalizable operators in the superpotential are forbidden by some R-symmetry. Then the NG boson $a_L$ becomes massive due to the A-term, while $a_B$ remains massless. Our strategy is as follows. In the first place, large lepton asymmetry is generated through the AD mechanism. Since the large lepton asymmetry is stored in the AD condensate until it decays at the decay temperature $T_\phi \sim 10$ GeV, it is protected from the sphaleron effects. On the other hand, a small baryon asymmetry is generated through spontaneous baryogenesis in a flat direction with the gravitino problem, the entropy production due to the decay of the AD field weakens the constraint considerably, so we can safely adopt the assumption.

Recently, it was pointed out that complete or partial equilibrium between all active neutrinos may be accomplished before BBN through neutrino oscillations in the presence of neutrino chemical potentials \cite{27}. However, even if complete equilibrium is realized, positive lepton asymmetry of electron type still survives, contrary to the scenario proposed in \cite{28} because the total lepton asymmetry is positive in this case.

Finally we comment on $Q$-balls. For general flat directions that include squarks, the numerical coefficient of one-loop corrections, $K$, is considered to be negative. It is known that the scalar field, which oscillates in the potential as Eq. (14) with negative $K$, experiences spatial instabilities, and deforms into non-topological solitons, $Q$-balls. However, $K$ can be positive if the $c_L LL$ direction includes the third generation and $\tan \beta$ is large, so here we have assumed this is the case. Thus we do not have to take $Q$-balls into account.

IV. DISCUSSION AND CONCLUSIONS

In this paper, we have discussed a spontaneous baryogenesis mechanism in flat directions. First of all, we have identified the Nambu-Goldstone mode, which derivatively couples to the associated $U(1)$ current and rotates due to the A-term. Such a derivative coupling and a rotation of the NG mode naturally realize spontaneous baryogenesis in the context of the flat direction if a current violating interaction exists. We gave a generic formula for the baryon asymmetry produced.

As concrete examples, we have investigated two scenarios of baryogenesis in detail. First of all, we considered spontaneous baryogenesis in a flat direction with vanishing $B - L$ charge. For such a flat direction, the AD mechanism does not work without $Q$-balls. On the other hand, we have shown that baryogenesis in such a flat direction can be easily realized in the context of
spontaneous baryogenesis. All we need is an interaction violating the $B - L$ symmetry. Such an interaction is, for example, given by the dimension five operator, which gives the Majorana masses of neutrinos. In particular, we have discussed spontaneous baryogenesis in a flat direction with neither baryon nor lepton charge, which was recently proposed by Chiba and the present authors. It is shown that baryogenesis is possible if we introduce another global symmetry such as the PQ symmetry. Finally, we discussed a scenario in which a positive and large lepton asymmetry of electron type is compatible with a positive and small baryon asymmetry. It is shown that it is possible to realize such a scenario and thereby remove any discrepancy of baryon asymmetry between those derived from BBN and the CMB.

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