Inter-Plane Inter-Satellite Connectivity in Dense LEO Constellations

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Abstract

With numerous ongoing deployments owned by private companies and startups, dense satellite constellations, with hundreds or even thousands of small satellites deployed in low Earth orbit (LEO), will play a major role in the near future of wireless communications. In addition, the 3rd Generation Partnership Project (3GPP) has ongoing efforts to integrate satellites into 5G and beyond-5G networks. Nevertheless, there are numerous challenges that must be overcome to fully exploit the connectivity capabilities of satellite constellations. These challenges are mainly a consequence of the low capabilities of individual small satellites, along with their high orbital speeds and small coverage due to the low altitude of deployment. In particular, inter-plane inter-satellite links (ISLs), which connect satellites from different orbital planes, are greatly dynamic and may be considerably affected by the Doppler shift. In this paper, we present a framework and the corresponding algorithms for the dynamic establishment of the inter-plane ISLs in LEO constellations. Our results show that the proposed algorithms increase 1) the sum of rates in the constellation by up to 81% with respect to the state-of-the-art benchmark schemes and 2) the resource efficiency by up to 700% when compared to random resource allocation.

I. INTRODUCTION

There is an unprecedented interest from the industry and international agencies on dense satellite constellations deployed in low Earth orbit (LEO). Due to their relatively low altitude of deployment, between 500 and 2000 km over the Earth’s surface, LEO constellations are able to provide global coverage and design flexibility, with reduced propagation delays in the ground-to-satellite links (GSLs) when compared to higher orbits. This combination of characteristics makes them an appealing option to support two of the three main use cases for 5G: massive
machine-type communications (mMTC) and enhanced mobile broadband (eMBB). In addition, LEO constellations can be used for ultra-reliable communications (URC) with high availability (up to 99.99% for LEO constellations [1]) and reliability in combination with slightly relaxed latency requirements, in the order of a few tens of milliseconds [1]–[3], in contrast to the commonly used 1 ms requirement of terrestrial 5G. Hence, LEO constellations are envisioned to be integrated into 5G and beyond-5G wireless networks with the purpose to dramatically extend cellular coverage, serve as a global backbone, and offload the cellular base stations in problematic hot spots [1], [2], [4], [5].

On the downside, when compared to satellites in higher orbits, the orbital velocities of LEO satellites are much greater (i.e., orbiting the Earth around 16 times a day) and the ground coverage of an individual LEO satellite is greatly reduced due to the low altitude of deployment. As a result, dense deployments are needed to provide global and continuous coverage. This later aspect, in combination with the necessity of reducing the overall cost of deployment of the constellation, fosters the use of physically small satellites with low individual computing and connectivity capabilities. Consequently, the antenna design (e.g., antenna/beam steering capabilities) and the communication protocols must be kept simple.

On the other hand, the high orbital velocities, of up to 7.6 km/s, lead to short orbital periods (i.e., the time it takes for the satellites to complete one rotation around the Earth) of around 90 minutes. This creates frequent changes in the satellite network topology and complicates the communication between satellites moving at different velocities and/or directions. LEO constellations are typically organized in groups of satellites that follow the same trajectory, one after the other, called orbital planes. Typically, small satellites in a constellation possess four antennas for inter-satellite communication and, depending on the budget, size, and weight constraints of the mission, an equal or lower number of transceivers. Communication between satellites of the same orbital plane occurs through the intra-plane inter-satellite links (ISLs), using the antennas located at both sides of the roll axis. Intra-plane ISLs are rather stable due to the almost constant distance between neighboring satellites from a same orbital plane, called intra-plane distance. On the other hand, communication between satellites in different orbital planes occurs through the inter-plane ISLs, using the antennas located at the positive + and negative − sides of the pitch axis. These ISLs are highly dynamic due to the different velocity vectors of the satellites. The characteristics described above, along with the satellite axes, are illustrated in Fig. 1 for a typical Walker star constellation [6]. Note that, in Fig. 1, the white
and black satellites are orbiting the Earth in opposite directions, which results in large relative velocities between them. The ISLs between these orbital planes are known as cross-seam ISLs, where the Doppler effect is considerably large. As a consequence, the cross-seam ISLs may not be implemented, as in the upcoming Kepler constellation [7]; hence are not illustrated in Fig. 1.

During the past decades, numerous of studies have investigated the connectivity aspects of satellite constellations [8]–[10]. However, most of them focus on the ground-to-satellite, satellite-to-ground, and intra-plane inter-satellite links, while only a few focus on inter-plane communication, even though it is essential to fully unleash the potential of LEO satellite constellations and facilitate their successful integration with 5G.

Furthermore, most of the theoretical research on inter-satellite communication and routing considers a perfectly symmetric constellation, where orbital planes are deployed at the same altitude and at evenly-spaced latitudes (see Fig. 1). Naturally, a perfect symmetry greatly simplifies the ISL communication. For example, [9] focuses on the optimal design of a completely symmetrical Walker star constellation. The objective is to maximize the coverage and throughput in the ISLs while minimizing the cost of deployment. Ekici et al. propose a routing algorithm for a perfectly symmetric Walker star constellation that exploits the horizontal alignment of the satellites to form rings [10]. These rings are paths, perpendicular to the orbital planes, that connect satellites at different orbital planes but at similar latitudes. Once these rings are formed, the algorithm uses decision maps to identify the path with the minimal propagation delay.
Nevertheless, slight asymmetries are needed in real LEO deployments to minimize the risk of collisions between the satellites at the intersections of the orbital planes (located in the poles in Fig. 1). These asymmetries are introduced in commercial dense LEO deployments such as OneWeb, SpaceX Starlink, and TeleSat in the form of slightly different altitudes of deployment, typically in the order of a few kilometers. These differences, commonly referred to as orbital separation, greatly reduce the probability of collision between satellites [11], [12] and eliminate the need for active station-keeping to maintain sufficient separation between satellites, which has a great cost in terms of propellant usage [13]. On the downside, orbital separations lead to slight differences in the orbital periods at the orbital planes, which greatly complicates inter-plane connectivity. For instance, these slight differences make it impossible to use fixed tables to establish the ISLs. Instead, the ISLs have to be established on-the-fly.

In this paper, we formulate the establishment of unicast inter-plane ISLs in dense LEO constellations as a dynamic matching problem and propose a framework to maximize the sum of the data rates selected for communication. Our approach is applicable to any satellite constellation regardless of its geometry and/or symmetry. Therefore, it can also be used in the initial deployment phases of a constellation, where only a few satellites and orbital planes have been deployed. Note that this paper and our previous work [14] are one of the few that focus on the establishment of inter-plane ISLs to maximize the achievable data rates. Instead, most of the approaches in the literature related to inter-plane ISLs focus on constellation design [9], on routing algorithms that are constellation geometry specific, assuming the ISLs are previously established in an optimal way [10], or on the achievable throughput in commercial constellation designs with fixed data rates [15].

Matching theory has been widely used to solve similar user association and resource allocation (RA) problems in wireless networks that involve a great number of agents and/or tasks [8], [16], [17]. Two of the most widely used matching algorithms are the Hungarian [18] and the deferred acceptance (DA) [19] algorithms. The Hungarian algorithm solves, exclusively, one-to-one matchings in bipartite graphs. That is, problems with a set of agents and a set of tasks, where each agent can be assigned to up to one task. On the other hand, the DA algorithm is more flexible and can solve one-to-one and many-to-one matches in bipartite graphs. That is, many-to-one matchings are a generalization of one-to-one matchings where each agent can be assigned to up to a given number of tasks, known as quota. Therefore, the quota is the maximum degree of an agent vertex after the matching.
Note that the potential inter-plane ISLs in the constellation can be modeled as a dynamic multi-partite graph \( G = (\mathcal{V}, \mathcal{E}) \), with \( P \in \mathbb{N}^+ \) orbital planes where \( \mathcal{V} = \{\mathcal{V}_1, \mathcal{V}_2, \ldots, \mathcal{V}_P\} \) is the set of vertices (i.e., satellites), \( \mathcal{V}_i \) is the set of satellites in orbital plane \( i \), and \( \mathcal{E} \) is the set of edges (i.e., inter-plane ISL links). Next, let \( K \) be the number of orthogonal wireless resources. The establishment of inter-plane ISLs encompasses the selection of a set of transceiver pairs and the allocation of wireless resources to the these pairs. To find the optimal solution, these tasks must be performed concurrently, selecting transceiver pairs and allocating resources simultaneously, by considering all possible transceiver pairs and resource allocations. However, finding the optimal solution is excessively complex.

Instead, we propose the use of greedy algorithms, which find the exact same solution regardless on whether these tasks are performed: 1) *concurrently*, selecting one transceiver pair and, then, assigning one wireless resource at each iteration; or 2) *sequentially*, selecting the whole set of transceiver pairs and, then, assigning the resources to each of them. Hence, for the sake of clarity and simplicity, throughout the rest of the paper we divide the problem of inter-plane ISL establishment into two phases. The first phase is the transceiver matching, where satellite transceiver pairs are selected with the aim of maximizing the rates in an interference-free environment, that is, as a function of the signal-to-noise-ratio (SNR). Initial results in this line can be found in our previous work \[14\]. The second phase is RA, where orthogonal wireless resources are allocated to each of the selected transceiver pairs with the aim of maximizing the sum of rates as a function of the signal-to-interference-noise ratio (SINR). As part of our framework, we propose two greedy transceiver matching algorithms, derived from our previous work \[14\], and one algorithm for RA.

To capture the dynamics of the constellation, the inter-plane ISLs are established (maintained) periodically, where a centralized entity with relatively large computational capabilities creates new transceiver pairs and allocates resources once every \( T \) seconds. This entity can be, for example, a dedicated ground station, a cloud server, a big GEO satellite, or even a mobile edge computing (MEC) platform deployed on the ground or space. This entity uses the geometry of the constellation to calculate the positions of the satellites and solve the matching problem in advance, such that the solution is available in all the satellites before initiating the procedures to establish the ISLs. In the following, we describe some distinctive characteristics small satellites that determine the ISL connectivity and provide an illustrative example on how the inter-plane ISLs are established within a region of the constellation.
Recall from Fig. 1 that small satellites usually have two antennas for inter-plane communication, located at both sides of the pitch axis. As shown in Fig. 1, we denote the direction of an inter-plane antenna w.r.t. the pitch axis as $d \in \{-, +\}$. These can be controlled by one or two transceivers, denoted as $Q \in \{1, 2\}$, where the case with $Q = 1$ represents greatly restricted small or nanosatellites, for example, cubesats. Therefore, a maximum of $Q$ ISLs can be established per satellite. However, due to numerous factors such as the beamwidth and beam steering capabilities of the antennas, a transceiver can only be matched with another in the corresponding relative direction of its antenna $d \in \{-, +\}$. Next, let $d(u, v) \in \{-, 0, +\}$ be the relative direction of given satellite $v \in \mathcal{V}$ from $u$ w.r.t. the pitch axis, where $d(u, v) = 0$ means that satellite $v$ is located exactly in the orbital plane of $u$ (i.e., in front or behind) regardless of the altitude.

Therefore, the process for transceiver matching begins with the set of feasible inter-satellite transceiver pairs in the constellation. This is illustrated in Fig. 2a for $1, 2 \in \mathcal{V}_1$, $3, 4 \in \mathcal{V}_2$, $5, 6 \in \mathcal{V}_3$ and $N \in \mathcal{V}_P$, where $Q = 2$ and $K = 3$ wireless resources are available. Note that we assume throughout the paper that the cross-seam ISLs (i.e., between orbital planes 1 and $P$) cannot be established due to the large Doppler shift, hence, these have been removed, as in Fig. 1. The first task of the algorithm is to find the set of feasible transceiver pairs that maximize the sum of rates in an interference-free environment. For example, the set of feasible pairs for satellite 1 is $\{3, 4, 5, 6\}$ but not 4 since $d(3, 4) = 0$ nor $N$ due to the Doppler shift. Then, our example shows that, after the matching, satellites 3 and 4 have two pairs each; one at each side. Conversely, the satellites in orbital plane 1 only have one pair. The reason for this is two-fold: no transceiver pairs can be made with satellites in orbital plane $P$ and the quota (of 1) for the transceivers in orbital plane 2 and direction $d = -$ has been reached.

Next, orthogonal wireless resources are assigned to each selected transceiver pair as illustrated in Fig. 2b. Note that in our example we have four transceiver pairs and $K = 3$ wireless resources. Hence, at least one resource has to be shared between two transceiver pairs. In this case, resource 1 is assigned to $\{1, 3\}$ and $\{4, 6\}$ since the centralized entity (hypothetically) calculated that this allocation maximizes the rates when compared to other combinations, for example, getting $\{1, 3\}$ and $\{3, 5\}$ to share resource 3. Note that achievable rates change as the matching progresses, since interference levels at each wireless resource vary as these are assigned. Therefore, the RA problem is analogous to a matching with externalities. Borst et al. proposed a centralized dynamic path selection algorithm to solve a similar RA problem in small and non-dynamic terrestrial networks. However, the latter and other traditional matching algorithms cannot be
directly applied to our ISL establishment problem. We build on the previous example to describe some of the distinctive characteristics of our problem.

1) **The number of transceivers is not the quota of the satellites:** This is different from a simple many-to-one matching, where the quota defines the maximum possible degree of a vertex after the matching and meeting the quota is sufficient. Instead, the number of matches per satellite is subject to the number of transceivers and to their quota of 1. Nevertheless, for the case $Q = 1$, any antenna can be controlled by the transceiver.

2) **The transceivers in the same satellite form a couple that wants to be separated:** Traditional matching problems with couples focus on those that want to be matched to a same vertex. Instead, the transceivers in the same satellite may want to be assigned to different wireless resources, due to their proximity, so the electromagnetic radiation from one antenna does not cause interference to the other. Note that the level of interference between the inter-plane antennas depends on the self-interference cancellation capabilities of the satellite.

3) **Wireless resources are completely exchangeable and have infinite quota:** Once the transceiver pairs have been made, the $K$ orthogonal wireless resources are allocated. However, these are completely exchangeable at the beginning of the RA. Then, a distinction between these only occurs after the first assignment is made, as interference is now possible. This is observed in Fig 2b, where $K = 3$ and the first three transceiver pairs can be allocated to any three different resources. This exchangeability creates problems for the DA algorithm, which makes decisions based on the preference of the agents. That is, having the exact same characteristics, the first iteration must be made completely at random. Furthermore, the wireless resources have an infinite quota, so these can be assigned to any number of transceiver pairs. This latter feature makes it different to the traditional college admission problem, which define a quota [17], [19]. Instead, the number of transceiver pairs that share a resource is only limited by the increasing interference.

Therefore, due to the distinctive characteristics of the ISL establishment, the transceiver matching and RA algorithms represent the major contribution of this paper. Besides, these contributions, we analyze the tradeoffs between density of deployment, inter-plane ISL connectivity, transmission power, and interference.

Our results show that: 1) the proposed framework for ISL establishment increases the sum of rates in the constellation by 81% when compared to our benchmark, adapted from the work by
Fig. 2: Exemplary matching diagrams for the (a) inter-plane transceiver matching, and (c) resource allocation (RA) given $\mathcal{V} = \{\mathcal{V}_1, \mathcal{V}_2, \ldots, \mathcal{V}_P\}$, where satellites $1, 2 \in \mathcal{V}_1$, $3, 4 \in \mathcal{V}_2$, $5, 6 \in \mathcal{V}_3$, and $K = 3$ wireless resources are available.

Ekici et al. [10]; 2) the proposed RA algorithm increases the resource efficiency by up to 700% when compared to random allocation; and 3) given that our matching algorithms are combined with an appropriate constellation design, the propagation delay at 80% of the inter-plane ISLs is less than 10 ms. This goes in line with the technical recommendations from the 3GPP [1], which considers one-hop propagation delays of 10 ms as typical in LEO constellations and helps meet the requirements for a one-way link, defined to be 30 ms [4].

The rest of the paper is organized as follows. Section II presents the system model for the considered LEO constellation. Then, Section III provides a detailed description of the transceiver matching and RA problems. Next, Section IV describes the proposed matching algorithms and Section V presents an analysis on ISL connectivity that serves as a base to select the simulation parameters. Section VI presents the results on the achievable performance with the transceiver matching and RA algorithms. Section VII concludes the paper.

II. SYSTEM MODEL

We consider a general case of a Walker star constellation, such as the one illustrated in Fig. 1, where $N$ satellites are evenly distributed in $P$ circular and evenly-spaced orbital planes. This constellation is classified as a $N/P/0$ Walker star and was selected due to its relatively constant inter-plane velocities, which simplifies design aspects in the physical layer to achieve stable inter-plane ISLs [21]. The satellites communicate through unicast inter-plane ISLs according to an arbitrary predefined multiple access method, such as TDMA, FDMA, CDMA, or OFDMA.
Each orbital plane $p \in \{1, 2, \ldots, P\}$ is deployed at a given altitude above the Earth’s surface $h_p$ km, at a given longitude $\epsilon_p$ radians, and consists of $N_p$ evenly-spaced satellites. The polar angle of a satellite $u$ (i.e., its angle w.r.t. the Earth’s north pole, the $z$-axis) is denoted as $\theta_u$. Besides, we define the function $p(u)$ to be the orbital plane in which satellite $u$ is deployed. Therefore, the position of satellite $u$ in spherical coordinates is denoted as $(h_{p(u)} + R_E, \epsilon_{p(u)}, \theta_u)$, where $R_E$ is the Earth’s radius.

Each satellite is equipped with a total of four antennas for unicast communication; two for inter-plane and two for intra-plane ISLs. The inter-plane antennas are positioned at each side of the pitch axis, hence, normal to the orbital plane as shown in Fig. [I]. We denote the direction of the inter-plane antennas as $d \in \{-, +\}$, where $d = -$ and $d = +$ correspond to the antennas placed at the negative (left) and positive (right) sides of the pitch axes. We consider two cases, where either one or two transceivers, respectively, are available for the inter-plane communication for all the satellites, namely $Q \in \{1, 2\}$. If $Q = 2$, every satellite in the constellation has two inter-plane transceivers, one for each antenna, and, if $Q = 1$, every satellite has only one transceiver, so communication is only possible with one inter-plane antenna at the time.

We model the constellation at any given time instant $t$ as a weighted undirected graph $G = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V}$ is the set of vertices (satellites), $\mathcal{E}$ is the set of undirected edges (inter-plane ISLs), and $w(e) \in \mathbb{R}$ is the weight of an edge $e \in \mathcal{E}$. Note that, even though $G$ is a dynamic graph due to the movement of the satellites, we observe the system at specific time instants, with period $T$, and omit the time index $t$ throughout the paper for notation simplicity. Graph $G$ is multi-partite with $P$ vertex classes $\mathcal{V}_1, \mathcal{V}_2, \ldots, \mathcal{V}_P$.

Inter-satellite communication occurs in a free-space environment. Therefore, it is mainly affected by the free-space path loss (FSPL) and the (thermal) noise power, which is assumed to be additive white Gaussian (AWGN) [22]. To characterize the inter-plane ISLs, we define the function $d(u, v) \in \{-, 0, +\}$ as the relative direction of satellite $v$ – the destination – w.r.t. satellite $u$ – the source –. For the particular case of the Walker star constellation, the latter can be obtained by rotating the axes by $-\epsilon_{p(u)}$ along the Earth’s rotation axis (i.e., $z$), so that the orbital plane $p(u)$ is positioned along in the $xz$-plane. By doing so, the relative direction can be calculated as the function

$$f_d(v, \epsilon_{p(u)}) = \sin \theta_v \sin(\epsilon_p(v) - \epsilon_{p(u)}),$$ (1)
and we denote the relative direction as

\[ d(u, v) = \begin{cases} 
- , & \text{if } f_d(v, \epsilon_{p(u)}) > 0 \\
+ , & \text{if } f_d(v, \epsilon_{p(u)}) < 0 \\
0 , & \text{otherwise} 
\end{cases} \quad (2) \]

Next, we denote \( l(u, v) \) as the slant range (i.e. line-of-sight distance) between two satellites \( u \) and \( v \) and is given as

\[ l(u, v) = \left( (h_p + R_E)^2 + (h_q + R_E)^2 - 2(h_p + R_E)(h_q + R_E) \cos \theta_u \cos \theta_v + \cos(\epsilon_p - \epsilon_q) \sin \theta_u \sin \theta_v \right)^{1/2}, \quad (3) \]

where \( R_E \) is the radius of the Earth. Building on this, the FSPL between \( u \) and \( v \) is given as

\[ L(u, v) = \left( \frac{4\pi l(u, v)f}{c} \right)^2, \quad (4) \]

where \( f \) is the carrier frequency and \( c = 2.998 \cdot 10^8 \) m/s is the speed of light.

In the following, we focus on defining the set of feasible edges \( \mathcal{E} \) in the graph. That is, the set of inter-plane transceiver pairs where communication is feasible. Naturally, \( d(u, v) = 0 \) if \( p(u) = p(v) \), hence, the intra-plane ISLs are excluded from \( \mathcal{E} \), that is \( \{u, v\} \notin \mathcal{E} : p(u) = p(v) \). The rest of the feasible pairs are determined by the Doppler shift and the existence of LoS.

To calculate the Doppler shift in the inter-plane ISL, we first calculate the orbital speed of the satellites in an orbital plane \( p \) as the function of \( h_p \)

\[ v(p) = \sqrt{\frac{G M_E}{(h_p + R_E)}} \text{ [m/s]}, \quad (5) \]

where \( M_E \) kilograms and \( R_E \) meters are the mass and radius of the Earth, respectively, and \( G \) is the gravitational constant. From there, the orbital period of plane \( p \) is calculated as

\[ T_p = 2\pi(h_p + R_E)v(p)^{-1} \text{ [s]}. \quad (6) \]

Next, we calculate the relative instantaneous velocity between \( u \) and \( v \) at time \( t \) as \( \Delta v(u, v, t) = \partial l(u, v, t)/\partial t \), where \( l(u, v) \) is given as in \( (3) \) for the polar angles \( \theta_u = 2\pi t/T_{p(u)} \) and \( \theta_v = 2\pi t/T_{p(v)} \). From there, the Doppler shift is

\[ \Delta f(u, v, t) = \frac{\Delta v(u, v, t)f}{c}. \quad (7) \]

Throughout this paper we assume the inter-plane transceivers are designed to compensate for the Doppler shift if both satellites are orbiting the Earth in the same direction, where the relative
orbital velocities are not excessive. Conversely, as done in the upcoming Kepler constellation [7], which has a Walker star geometry, we assume that the transceivers cannot compensate for the Doppler shift if the satellites are orbiting the Earth in opposite directions (i.e., cross-seam ISLs), where \( \max_t \Delta v(u,v,t) \approx v(p(u)) + v(p(v)) \). This occurs between the first \( p(u) = 1 \) (black) and last \( p(v) = P \) (white) orbital planes of our Walker star constellation illustrated in Fig. 1. Hence, the set of non-feasible edges due to a high Doppler shift is \( \{\{u,v\} : |p(u) - p(v)| = P - 1\} \).

As a reference, the maximum Doppler shift in cross-seam ISLs with \( f = 2.4 \text{ GHz} \) and \( P = 5 \) is \( \max_t (\Delta f(1,P)) = \max (\Delta v) f/c = 114.32 \text{ kHz} \), whereas for ISLs between \( p(u) = 1 \) and \( p(v) = 2 \) is \( \max (\Delta f(1,2)) = 36.99 \text{ kHz} \).

In addition, there exists a maximum slant range between two satellites \( u \) and \( v \), in orbital planes \( p = p(u) \) and \( q = p(v) \), due to the presence of the Earth, denoted as \( l_{\text{LoS}}^*(p,q) \). That is, the Earth blocks the LoS between \( u \) and \( v \) if \( l(u,v) > l_{\text{LoS}}^*(p,q) \). Assuming the Earth is perfectly spherical, the latter is given as

\[
l_{\text{LoS}}^*(p,q) = \sqrt{h_p (h_p + 2R_E)} + \sqrt{h_q (h_q + 2R_E)}.
\]

Hence, the set of edges with no line of sight (NLoS) is \( \{\{u,v\} : l(u,v) > l_{\text{LoS}}^*(p,q)\} \). Building on this, we set \( L(u,v) = \infty \) for all \( u,v \in \mathcal{V} \) s.t. \( l(u,v) < l_{\text{LoS}}^*(p(u),p(v)) \) and define the set of feasible edges is

\[
\mathcal{E} = \{\{u,v\} : |p(u) - p(v)| \notin \{0,P-1\}, l(u,v) > l_{\text{LoS}}^*(p(v),p(v))\}.
\]

Having defined the set of feasible edges, we move on to characterize the ISLs. For this, let \( G_{(u,v)}^d \) denote the normalized gain of the antenna in direction \( d \) of satellite \( u \) towards \( v \). The latter gain is a function of the beamwidth and the relative direction of the beam of antenna \( d \) in \( u \) w.r.t. the position of \( v \). Throughout this paper, we assume that the transmission power \( P_t \) is fixed for all satellites and that the (directional) antennas have perfect beam steering capabilities. Furthermore, a transceiver pair \( \{u,v\} \in \mathcal{E} \) can only be formed with antennas in the corresponding directions. Therefore, the antennas of a transmitter \( u \) and the intended receiver \( v \) are always aligned in the direction of maximum radiation, where the effective isotropic radiated power is

\[
\text{EIRP} = P_t \max G_{(u,v)}^d G_{(v,u)}^d.
\]

Naturally, for isotropic antennas we have \( G_{(u,v)}^d = 1 \) for all \( d \), and \( u,v \in \mathcal{V} \). Recall that, in our setting, a wireless resource \( k \) is selected from a pool of \( K \) orthogonal wireless resources \( \{1,2,\ldots,K\} \) to be used for communication at each inter-plane ISL (i.e., for each transceiver
pair). These resources are orthogonal to those used for the intra-plane ISLs, so no interference between these can occur. Thus, we define the set of indicator variables \{a(u,v,k)\} s.t. \(a(u,v,k) = 1\) if satellite \(u \in V\) has an ongoing inter-plane transmission towards \(v\) with resource \(k\) and \(a(u,v,k) = 0\) otherwise. For notation simplicity, we assume that the \(K\) resources are sufficiently close in the frequency domain and we can remove the frequency dependency in the path loss. Hence, we denote the path loss between \(u\) and \(v\) with any \(k \in \{1, 2, \ldots, K\}\) simply as \(L(u, v)\). Building on this, we define the received signal strength (RSS) at \(v\), with the antenna in direction \(d\), from \(u\) with wireless resource \(k\) as

\[
\text{RSS}(u, v, d, k) = \sum_{i \in V} a(u,i,k)P_t G_{(u,i)}^d G_{(v,u)}^d L(u,v) \quad (11)
\]

That is, \(v\) may receive a signal from \(u\), even though the latter is transmitting to a different satellite \(i\) if \(l(u,v) < l_{\text{LoS}}(p(u), p(v))\). From there, we define the signal-to-noise ratio (SNR) for an ongoing transmission from \(u\) to \(v\) (i.e., given \(a(u,v,k) = 1\)) as

\[
\text{SNR}(u,v) = \text{SNR}(u,v,k \mid a(u,v,k) = 1) = \frac{EIRP}{k_B \tau B L(u,v)}, \quad (12)
\]

where \(k_B\) is the Boltzmann constant, \(\tau\) is the thermal noise in Kelvin, and \(B\) is the channel bandwidth in Hertz.

A physical interference model with constant noise power, based on the power capture model, is considered [23]. Let \(I(u,v,k) = \{a(i,j,k) : \{i, j\} \in E \setminus \{u, v\}\}\) be a permissible interference pattern for a transmission from \(u\) to \(v\) with resource \(k\). The latter serves to define the interference at \(v\) for an ongoing transmission from \(u\) with resource \(k\), that is, given \(a(u,v,k) = 1\), as

\[
I(u,v,k) = \sum_{i=1}^{N} \text{RSS}(i,v,d(v,u),k) - \text{SNR}(u,v) = \sum_{a(i,j,k) \in I(u,v,k)} \frac{a(i,j,k)P_t G_{(i,v)}^{d(i,j)} G_{(v,i)}^{d(v,u)}}{L(i,v)}. \quad (13)
\]

Then, the signal-to-interference-plus-noise ratio (SINR) for an ongoing transmission from \(u\) to \(v\) with resource \(k\) is

\[
\text{SINR}(u,v,k) = \frac{\text{RSS}(u,v,d(v,u),k)}{k_B \tau B + I(u,v,k)} = \frac{EIRP}{L(u,v)(k_B \tau B + I(u,v,k))}. \quad (14)
\]

Next, let \(R(u,v,k)\) be the data rate used for communication from \(u\) to \(v\) with resource \(k\). The latter is selected from an infinite set of possible rates to have zero outage probability \(P_{\text{out}} = 0\). Note that, ensuring \(P_{\text{out}} = 0\) is of utmost importance in satellite communications to avoid the use of feedback with high round-trip-times (RTTs) due to the long propagation delays. Note that, interference can be mitigated if the inter-plane ISL antennas in all the satellites combine
sufficiently narrow beams with precise beam steering or antenna pointing capabilities. In the latter case, all communications in the constellation occur in an interference-free environment, where \( I(u,v,k) = 0 \) for all \( u, v, \) and \( k \). Therefore, the maximum data rate that \( u \) can use to communicate with \( v \) with \( P_{out} = 0 \) is

\[
R_{\text{SNR}}^*(u,v) = \max R(u,v,k \mid I(u,v,k) = 0, P_{out} = 0) = B \log_2 (1 + \text{SNR}(u,v)).
\]  

(15)

Conversely, if \( \exists I(u,v,k) > 0 \) RA must take place to maximize the rates. Throughout this paper, we assume that the interference follows a Gaussian distribution and can be treated as AWGN [23]. Naturally, if the instantaneous values of all the elements in \( I(u,v,k) \) are known, the maximum data rate at which \( u \) can transmit to \( v \) can be selected to ensure \( P_{out} = 0 \) as

\[
\max R(u,v,k \mid I(u,v,k), P_{out} = 0) = B \log_2 \left( 1 + \text{SINR} \left( u,v,k \mid I(u,v,k) \right) \right).
\]  

(16)

However, selecting and achieving the data rate described by (16) in practice is infeasible as it requires 1) instantaneous and perfect knowledge of the interference, determined by the activity of all the interferers \( I(u,v,k) \), and 2) real-time and perfect adaptation of the rate. Instead, we consider a realistic scenario in which the rates are selected at the time the ISL is established to achieve \( P_{out} = 0 \) for any interference pattern that can be created by a permissible combination of \( I(u,v,k) \). A permissible combination is one that can be obtained by following the matching rules, explained in the following section. Formally, the rates are selected as

\[
R_{\text{SNR}}^*(u,v,k) = \max R \left( u,v,k \mid \max I(u,v,k), P_{out} = 0 \right)
\]

\[
= B \log_2 \left( 1 + \min I(u,v,k) \text{SINR} \left( u,v,k \mid I(u,v,k) \right) \right).
\]  

(17)

At this point, it is convenient to introduce \( R_{\text{min}} \), defined as the minimum acceptable rate to establish an ISL prior to RA. That is, an ISL between \( u \) and \( v \) can only be established if \( R_{\text{SNR}}^*(u,v) > R_{\text{min}} \). The latter represents, for example, the minimum rate required to complete the necessary handshakes between transceivers. Building on this, we calculate the minimum SNR to establish an ISL for all satellites as \( \gamma = 2^{R_{\text{min}}/B} - 1 \). Hence, an ISL between \( u \) and \( v \) can only be established if

\[
\text{EIRP} \geq L(u,v)k_B\tau B \left( 2^{R_{\text{min}}/B} - 1 \right)
\]  

(18)

Hereafter, we treat the maximum path loss (MPL) to achieve the desired \( R_{\text{SNR}}^*(u,v) = R_{\text{min}} \) as a design parameter. The latter is given as

\[
\text{MPL} = \frac{\text{EIRP}}{k_B\tau B \left( 2^{R_{\text{min}}/B} - 1 \right)}.
\]  

(19)
Table I enlists relevant notation introduced in this section and used throughout the paper.

### III. Problem Formulation

We consider the two-fold problem of 1) inter-plane transceiver matching and 2) orthogonal RA to the transceiver pairs. As described in the Introduction, the optimal solution can only be achieved by solving these problems jointly. However, separating them makes the problem tractable and allows us to define the following three different sets of indicator variables, whose value of 1 represents ‘True’ and 0 represents ‘False.’

- \( \{ x^d_v \} \): Indicates whether satellite \( v \in V \) has established an inter-plane ISL in direction \( d \).
- \( \{ x^d_v(k) \} \): Indicates whether satellite \( v \in V \) has established an inter-plane ISL in direction \( d \) with resource \( k \in \{1, 2, \ldots, K \} \). Hence, \( \sum_{k \in K} x^d_v(k) = x^d_v \in \{0, 1\} \).
- \( \{ x^k_{uv} \} \): Indicates whether an inter-plane ISL has been established between \( u \) and \( v \) with resource \( k \). Hence, \( x^k_{uv} = 1 \iff \left( x^{d(u,v)}_u(k) = 1 \text{ AND } x^{d(v,u)}_v(k) = 1 \right) \).

Therefore, we define that the transceiver on satellite \( u \) can only transmit to \( v \) with a given resource \( k \) if the centralized entity has explicitly allocated \( k \) to it and if it uses the antenna in
the direction \( d(u, v) \). That is, \( \Pr[a_{(u,v,k)} = 1] > 0 \iff x^k_{uv} = 1 \). Furthermore, we have that, at any given time \( t \), up to one transceiver at each transceiver pair can be transmitting. That is, there may be the case that an ISL is established but the involved satellites are not communicating, but they cannot both transmit through the same ISL at the same time. Formally, we have that \( a_{(u,v,k)} + a_{(v,u,k)} \in \{0, 1\} \) for all \( x^k_{uv} = 1 \). Building on these restrictions, we formulate the following definition.

**Definition 1.** Permissible interference pattern: We define \( I_{(u,v,k)} \) to be a permissible interference pattern for a transmission from \( u \) to \( v \) with resource \( k \) and at any given time \( t \), given \( x^k_{uv} = 1 \) and \( a_{(u,v,k)} = 1 \), if up to one transceiver per established ISL is transmitting at that time. Hence,

\[
I_{(u,v,k)} = \{ a_{(i,j,k)} : \{i,j\} \in \mathcal{E} \setminus \{u,v\}, x^k_{ij} = 1 \text{ AND } a_{(i,j,k)} + a_{(j,i,k)} \in \{0, 1\} \forall x^k_{ij} = 1 \}.
\]

The objective of the two phases of our framework, namely transceiver matching and RA, is to maximize the rates for the inter-plane ISLs in the constellation. As defined by (15) and (17), we consider a realistic worst case scenario where neither the centralized entity nor the satellites have instantaneous knowledge of \( I_{(u,v,k)} \). Therefore, the rates at each ISL are selected to ensure \( P_{out} = 0 \) for any permissible interference pattern and at all times. We describe these two phases in the following.

### A. Transceiver matching

Finding antenna pairs for the satellites to communicate in the inter-plane ISL is a one-to-one canonical dynamic matching problem. The task is to populate the set \( \mathcal{M} \subseteq \mathcal{E} \) with the antenna pairs (i.e., edges) that maximize the sum of rates in the constellation prior to RA. Therefore, an interference-free environment is considered at this point and the matching problem is defined as

\[
\text{maximize } \sum_{(u,v) \in \mathcal{M}} R_{\text{SNR}}^*(u,v)
\]

subject to \( x^d_v \in \{0, 1\} \quad \forall v \in \mathcal{V}, d \in \{-, +\}\)

\[
x^d_v + x^d_w \leq Q \quad \forall v \in \mathcal{V}. \tag{20}
\]

As described in Section II, if sufficiently narrow beams are used in combination with pointing or beam steering, it is possible to ensure \( I(u,v,k) = 0 \) for all \( \{u,v\} \in \mathcal{M} \). In this latter case, solving (20) solves the ISL establishment problem, as any number of resources \( K \in \mathbb{N}^+ \) allows to directly use the rates \( R_{\text{SNR}}^*(u,v) \) for communication with \( P_{out} = 0 \). Conversely, if
∃\{u, v\} ∈ \mathcal{M} : I(u, v, k) > 0, RA must take place as described in the following to maximize the sum of rates in the constellation.

B. Resource allocation

Allocating resources for the communication through the inter-plane ISLs is a many-to-one dynamic matching problem where \( K \) resources must be assigned to the transceiver pairs (i.e., ISLs) in \( \mathcal{M} \) to maximize the sum of rates. In particular, we define a RA as \( A = \{(\{u, v\}, k) : \{u, v\} ∈ \mathcal{M}\} \), where each element \( (\{u, v\}, k) \) indicates that resource \( k \) was allocated to the ISL \( \{u, v\} \in \mathcal{M} \). Note that, every time a resource \( k \) is assigned to an ISL \( e \in \mathcal{M} \) (i.e., one by one), the interference to all \( \{u, v\} \) that had previously selected \( k \) may increase. Therefore, RA is a matching with externalities and the rates \( R^*_\text{SINR}(u, v, k) \) can only be selected at the end of the matching. Building on this, we formally define the RA problem as

\[
\begin{align*}
\text{maximize} & \quad \sum_{u=1}^{N} \sum_{v=1}^{N} \sum_{k=1}^{K} R^*_\text{SINR}(u, v, k)x_{uv}^k \\
\text{subject to} & \quad \sum_{k=1}^{K} x_{uv}^d(k) \in \{0, 1\} \quad \forall v \in \mathcal{V}, d \in \{-, +\} \\
& \quad \sum_{k=1}^{K} x^-_v(k) + x^+_v(k) \leq Q \quad \forall v \in \mathcal{V}(\mathcal{M}) \\
& \quad x^d_v - \sum_{k=1}^{K} x^d_v(k) = 0 \quad \forall v \in \mathcal{V}(\mathcal{M})
\end{align*}
\]

where \( \mathcal{V}(\mathcal{M}) \) is the set of vertices in the ISL matching \( \mathcal{M} \). Next, let \( M = |\mathcal{M}| \) and \( A = |A| \) be the number of elements in the transceiver matching and RA, respectively. Observe that the last constraint in (21) ensures that \( A = M \) at the end of the RA.

IV. Algorithms

Having a central entity with high processing power allows for the execution of complex algorithms without increasing the computational load in the satellites. On the downside, for the centralized matching to be feasible, the matching must be solved ahead of time and communicated to the whole constellation. Hence, the processing time and the communication overhead must be taken into account, so the satellites can complete the process in a timely manner. Because of this, the status of the buffer and, hence, the activity pattern of the satellites cannot be taken into
account in real time. A solution to this problem is having the satellites themselves – or even
the centralized entity – to make projections and infer the activity of the satellites ahead of time.
However, these approaches are out of the scope of this paper. Instead, we assume a general case
in which the centralized entity has no prior knowledge on the traffic pattern of the satellites.

In the following, we describe the centralized algorithms used to solve the transceiver matching
and RA for any value of $P \in \mathbb{N}^+$ and $Q \in \{1, 2\}$ and derive their complexity. For the latter,
recall that $M = |M|$ and $A = |A|$.

A. Transceiver matching

**Greedy Independent Experiments transceiver Matching (GIEM):** This is a greedy cen-
tralized matching algorithm, where the matching $M$ is solved every time from $M = \emptyset$ and
$\{x^d_v = 0\}$ by a centralized entity that knows the positions of the satellites at all times. At each
iteration, this entity adds the inter-plane ISL with the minimum weight to the matching and
removes the adjacent edges.

A possible implementation is to create an ordered queue $L = (\ell_1, \ell_2, \ldots)$ with elements
$\ell_i \in \mathcal{E}$ s.t. $L(\ell_i) \leq$ MPL to satisfy $L(\ell_i) \leq L(\ell_{i+1})$. Then, at each iteration, the first element $\ell_1$ in the
queue $L$ – the edge $\{u, v\}$ – is selected. Then, if $x^d_{u,v} = x^d_{v,u} = 0$, the edge is added to $M$,
the first element in $L$ is removed, and $x^d_{u,v} \leftarrow 1$ and $x^d_{v,u} \leftarrow 1$. This process is repeated until
the queue $L$ is empty. Algorithm 1 summarizes the GIEM algorithm.

To calculate the complexity of the GIEM algorithm, we first define $\mathcal{E}' = \{u, v \in \mathcal{V} : p(u) \neq p(v)\}$, where $\mathcal{E}' \supset \mathcal{E}$. Next, we calculate an upper bound for the number of feasible edges in $L$.

$$|L| \ll |\mathcal{E}'| = \frac{1}{2} \left[ N^2 - \sum_{p=1}^{P} N_p^2 \right].$$

Note that, for the case with $N_p = N/P$, we have $|\mathcal{E}'| = P N_P^2 (P - 1)$. The insertion of the
elements in $L$ has a maximum cost $O(|\mathcal{E}'|)$. Then, $L$ is sorted, which has a maximum cost
$O(\sqrt{|\mathcal{E}'|})$ with Quick Sort.

Then, at each iteration, two comparisons, one deletion (the first element in $L$), up to one
insertion in $M$, and up to two assignments are performed. All of these operations have a cost
$O(1)$ and $|\mathcal{E}'|$ iterations are performed. Therefore, the cost of the operations performed after the
list $L$ has been sorted is $O(|L|) \ll O(|\mathcal{E}'|) = O \left( P N_P^2 (P - 1) \right)$, which for $P = 2$ is one order
of magnitude lower than $O \left( N_P^3 \right)$, the complexity of the well-known Hungarian algorithm for
Algorithm 1: Algorithm for greedy independent experiments transceiver matching (GIEM).

**Input:** Set of feasible weighted edges $\mathcal{E}$

**Input:** Number of transceivers $Q$

1. $\mathcal{M} = \emptyset, \{x_u^d = 0\}$
2. Create $\mathcal{L} = (\ell_1, \ell_2, \ldots)$ with all $\{\ell_i\} \in \mathcal{E}$ s.t. $L(\ell_i) \leq MPL$ and $L(\ell_i) \leq L(\ell_{i+1})$ for all $i$.
3. while $\mathcal{L} \neq \emptyset$ do
4. $\{u,v\} \leftarrow \ell_1$
5. if $x_u^{d(u,v)} + x_v^{d(v,u)} = 0$ and $x_u^- + x_u^+ < Q$ and $x_v^- + x_v^+ < Q$ then
6. $\mathcal{M} \leftarrow \mathcal{M} \cup \{u,v\}$
7. $x_u^{d(u,v)} \leftarrow 1, x_v^{d(v,u)} \leftarrow 1$
8. end if
9. Delete $\ell_1$
10. end while

$P = 2$. However, the overall cost of the ISL matching algorithm is determined by the sorting of the list $O(|\mathcal{L}|^2) \ll O(|\mathcal{E}'|^2)$.

**Greedy Markovian transceiver matching (GMM):** This is a modified version of the GIEM matching where the inter-plane ISLs are maintained as long as possible. Let $\mathcal{M}(n)$ denote the $n$th realization of the transceiver matching. The starting point for the $n$th realization of the GMM is $\mathcal{M}(n) = \emptyset$ and its previous realization $\mathcal{M}(n-1)$. Then, the algorithm searches within $\mathcal{M}(n-1)$ to identify the transceiver pairs that are still feasible. These are the edges in the set $\{e \in \mathcal{M}(n-1) : L(e) \leq MPL\}$ and includes them, one by one, in $\mathcal{M}(n)$ if the quota of the antennas has not been reached. This can happen if there are changes in the relative direction of the satellites. After these pairs are added to $\mathcal{M}(n)$, the matching continues as described above for the GIEM. Algorithm 2 summarizes the GMM algorithm.

Note that, for the GMM we have an even smaller $|\mathcal{L}|$ than for the GIEM because the former initiates the search within the previous matching $\mathcal{M}(n-1)$. Therefore, the majority of the reduction in the execution time of the GMM algorithm when compared to the GIEM algorithm, which will be observed in Section VI, is due to sorting a smaller list.

**Geographical matching (GEO, benchmark):** This is an adaptation of the routing algorithm provided by Ekici et al. [10]. As illustrated in Fig. 3, the latitude is divided into $N_p$ regions called
Algorithm 2: Algorithm for greedy Markovian transceiver matching (GMM).

**Input:** Set of weighted edges $E$

**Input:** Number of transceivers $Q$

**Input:** Previous matching $M(n-1)$

1. $M(n) = \emptyset, \{x^d_v = 0\}$
2. for all $\{\{u, v\} \in M(n-1) : L(u, v) \leq MPL\}$ do
3. if $x^{d(u,v)}_u + x^{d(v,u)}_v = 0$ and $x^-_u + x^+_u < Q$ and $x^-_v + x^+_v < Q$ then
4. $M(n) \leftarrow M(n) \cup \{u, v\}$
5. $x^{d(u,v)}_u \leftarrow 1, x^{d(v,u)}_v \leftarrow 1$
6. end if
7. end for
8. Create $L = (\ell_1, \ell_2, \ldots)$ with all $\ell_i \in E \setminus M(n)$ s.t. $L(\ell_i) \leq MPL$ and $L(\ell_i) \leq L(\ell_{i+1})$ for all $i$.
9. while $L \neq \emptyset$ do
10. $\{u, v\} \leftarrow \ell_1$
11. if $x^{d(u,v)}_u + x^{d(v,u)}_v = 0$ and $x^-_u + x^+_u < Q$ and $x^-_v + x^+_v < Q$ then
12. $M(n) \leftarrow M(n) \cup \{u, v\}$
13. $x^{d(u,v)}_u \leftarrow 1, x^{d(v,u)}_v \leftarrow 1$
14. end if
15. Delete $\ell_1$
16. end while

**logical locations** of width $2\pi/N_p$. Then, satellites in neighbouring orbital planes in the same logical location are matched. Note that this algorithm is simple, yet it has numerous drawbacks, for example, is directly applicable for any $P \in \mathbb{N}$ and $Q = 2$ only if $N_p = N/P$ for all $p \in \{1, 2, \ldots, P\}$. Otherwise, some satellites would not be matched. Furthermore, an additional mechanism is needed to avoid fully disconnected orbital planes for $Q = 1$ (e.g., having no inter-plane ISLs between two orbital planes). That is, the algorithm does not adapt to not having inter-plane neighbors.

To derive the complexity of the GEO algorithm, we have that computing the logical location for each satellite requires $PN_p$ divisions and comparisons. Then, matching the satellites in
neighboring orbital planes and in the same logical location, given $Q$ has not been reached, has a cost of $N_p(P - 1)$. Therefore, the complexity of the GEO algorithm is $O(PN_p)$.

**B. Resource allocation (RA)**

Once the transceiver pairs have been formed, orthogonal wireless resources are assigned to maximize the sum of rates as a function of the SINR (i.e., considering the interference). Let $\mathcal{A} = \emptyset$ be the RA at the beginning of a realization, that is, immediately after the transceiver matching, and recall that $x_{uv}^k \in \{0, 1\}$ is the indicator variable of the RA for the ISL $\{u, v\} \in \mathcal{M}$ so that $x_{uv}^k = x_u^{d(u,v)}(k)x_v^{d(v,u)}(k)$. Hence, $x_{uv}^k = 1 \iff \{u, v\}, k) \in \mathcal{A}$, so we initialize $\{x_{uv}^k = 0\}$. Next, from (17), the centralized entity calculates the maximum accumulated rate in the RA (matching) $\mathcal{A}$ to ensure $P_{\text{out}} = 0$ for all $e \in \mathcal{E}(\mathcal{A})$ as

$$R^*_{\text{SINR}}(\mathcal{A}) = \sum_{(\{u, v\}, k) \in \mathcal{A}} R^*_{\text{SINR}}(u, v, k) + R^*_{\text{SINR}}(v, u, k).$$

(23)

Let $\mathcal{E}(\mathcal{A})$ be the set of edges in $\mathcal{A}$. At each iteration, an orthogonal resource $k^\star$ will be allocated to an ISL $\{u, v\} \in \mathcal{M} \setminus \mathcal{E}(\mathcal{A})$. For this, the centralized entity calculates $R^*_{\text{SINR}}(\mathcal{A} \cup \{u, v\}, k)$

for all $k$ and assigns

$$k^\star = \arg \max_k R^*_{\text{SINR}}(\mathcal{A} \cup \{u, v\}, k)$$

(24)

to the edge $\{u, v\}$. Recall that $R^*_{\text{SINR}}(u, v, k)$ is the maximum rate that can be selected for the ISL $\{u, v\}, k)$ to ensure zero outage probability. The latter is calculated by evaluating the SINR with all possible combinations of $\mathcal{I}(u,v,k)$ via exhaustive search. Hence, $x_u^{d(u,v)}(k^\star) \leftarrow 1$ and $x_v^{d(v,u)}(k^\star) \leftarrow 1$. This process is summarized in Algorithm [5].
Algorithm 3: Centralized algorithm for RA.

**Input:** Transceiver matching $\mathcal{M}$

**Input:** Set of resources $\{1, 2, \ldots, K\}$

1. $\mathcal{A} = \emptyset$
2. **while** $A < M$ **do**
3. Select an edge $\{u, v\} \in \mathcal{M} \setminus \mathcal{E}(A)$
4. Allocate $k^*$ to $\{u, v\}$, according to (24)
5. $\mathcal{A} \leftarrow \mathcal{A} \cup (\{u, v\}, k^*)$
6. $x_{uv}^{k^*} \leftarrow 1$, $x_{u}^{d(u,v)}(k^*) \leftarrow 1$, $x_{v}^{d(v,u)}(k^*) \leftarrow 1$
7. **end while**

To obtain the complexity of the RA algorithm, we have that, at each iteration, $O(KM^2)$ operations are performed to calculate $R_{\text{SINR}}^*(A \cup (\{u, v\}, k))$ for all $k$, followed by $K$ additions and comparisons to identify $k^*$. As one resource $k$ is assigned to one transceiver pair $\{u, v\} \in \mathcal{M}$ per iteration, the complexity of our RA algorithm is $O(KM^2)$.

Now it is possible to observe that our algorithms lead to the same result regardless if they are implemented concurrently or sequentially. To illustrate this, assume that, these are performed concurrently. Therefore, at each step, the first element in $L$, denoted $\ell_1$ is selected. If the quota of the transceivers involved in edge $\{u, v\} \leftarrow \ell_1$ has not been reached, $k^*$ is calculated as in (24) and the tuple $(\{u, v\}, k)$ is added to $A$. This results in the same transceiver pairs and RA as with the sequential implementation described throughout this section.

Note that our algorithms may not result in the maximum rates that can be obtained by any greedy algorithm. These can only be obtained by an algorithm that continuously creates an ordered list with all elements $(\{u, v\}, k) \notin A$, where the weight of each edge is $R_{\text{SINR}}^*(A \cup (\{u, v\}, k))$. However, the complexity of this algorithm would be enormous, as $K$ different allocations for all possible transceiver pairs $\{u, v\} \in \mathcal{E} \setminus \mathcal{E}(A)$ must be considered. In addition, this list must be updated and sorted at each iteration.

V. EXPERIMENT DESIGN AND PARAMETER SELECTION

In this section, we present a basic study on the connectivity of the inter-plane ISLs to select appropriate simulation parameters. In particular, we are set to select the minimum value for the
EIRP that must be supported by the ISL transceivers to ensure that all the $N$ satellites have at least one possible inter-plane neighbor with which they can communicate at a rate higher than $R_{\min}$, at all times. Hereafter, we refer to this characteristic as full inter-plane ISL connectivity.

As a starting point, we define $L^*(P, N_p, f)$ as the maximum FSPL to the nearest inter-plane neighbor in a Walker star constellation. Next, we set $\text{MPL} = L^*(P, N_p, f)$ and substitute $L(u, v)$ with the latter in \((18)\) calculate the minimum EIRP that ensures full inter-plane ISL connectivity.

Recall that the polar angle of a satellite $u \in \mathcal{V}$ (i.e., w.r.t. the $z$-axis) is denoted as $\theta_u$. From there, let $v^* \in \mathcal{V}_q$ be the closest satellite in $q = p(v^*)$ to $u \in \mathcal{V}_p$. We have that
\[
v^* = \arg \min_{v \in \mathcal{V}_q} l(u, v) \iff \theta_{v^*} = [\theta_u - \pi/N_p, \theta_u + \pi/N_p] \quad \forall p, q \in \{1, 2, \ldots, P\}
\]
Therefore, we define $\Delta \theta = |\theta_u - \theta_{v^*}| \in [0, \pi/N_p]$. Besides, we observe that the difference in longitude between adjacent orbital planes is $\pi/P$. Building on this, we find the maximum slant range between two satellites $u$ and $v^*$ in adjacent orbital planes, namely $p = p(u)$ and $q = p(v^*) = (p + 1 \mod P)$ in a general Walker star constellation from \((3)\) as
\[
L^*(P, N_p) = \max_{\theta_u, \Delta \theta, p} \left( (h_{p} + R_E)^2 + (h_{q} + R_E)^2 - 2(h_{p} + R_E)(h_{q} + R_E) \right.
\]
\[
\times \left. \cos \theta_u \cos (\theta_u + \Delta \theta) + \cos (\epsilon_{p} - \epsilon_{q}) \sin \theta_u \sin (\theta_u + \Delta \theta) \right)^{1/2}
\]
From there, we introduce specific characteristics of our constellation. In particular, we consider that the lowest orbital plane is deployed at a typical altitude of 600 km and an orbital separation of 10 km between orbital planes. Therefore, the altitude of the orbital planes is given as $h_p = 600 + 10(p - 1)$ km for all $p \in \{1, 2, \ldots, P\}$. Besides, as described in Section \[II\] we assume that the cross-seam inter-plane ISLs are not implemented. Building on this, by using simple optimization techniques and due to the symmetry of the slant range (metric), it is easy to obtain the closed-form expression of one of the maxima of \((26)\). For example, a maximum is achieved at $\theta_u^* = \pi/2$, $\Delta \theta^* = \pi/N_p$, and $p^* = P - 1$, where we have
\[
L^*(P, N_p) = \left( (h_{P-1} + R_E)^2 + (h_{P} + R_E)^2 \right.
\]
\[
- 2(h_{P-1} + R_E)(h_{P} + R_E) \left( \cos \left( \frac{\pi}{P} \right) \sin \left( \frac{\pi(2 + N_p)}{2N_p} \right) \right) \right)^{1/2}.
\]
Then, $L^*(P, N_p, f)$ is obtained by substituting $l(u, v)$ with $L^*(P, N_p)$ in \((18)\). Then, we set
\[
\text{EIRP} = L^*(P, N_p, f) k_B \tau B \left( 2^{R_{\min}/B} - 1 \right).
\]
Throughout the rest of the paper we investigate the performance of the transceiver matching algorithms in two operation regimes: limited and full inter-plane ISL connectivity. To do so, we fix $f = 2.4$ GHz, $R_{\text{min}} = 10$ kbps, and $\text{MPL} = L^*(7, 40, f)$ dB, so that full connectivity is only guaranteed for $P \geq 7$ and conduct our analyses for $P \in \{5, 6, 7, 8\}$; these and other relevant parameters are listed in Table II. Note that this combination of $f$ and MPL, along with the conservatively selected $R_{\text{min}} = 10$, results in an EIRP $= 12.81$ W and a communication range of $l^*(7, 40) = 3527$ km, which is much shorter than the maximum LoS slant range between satellites in orbital planes 1 and 2, defined in [8], which is $l_{\text{LoS}}^*(1, 2) = \min_{p,q} l_{\text{LoS}}^*(p, q) = 5686.7$ km.

In our analyses, we consider that the the inter-plane transceivers have no self-interference cancellation capabilities. Therefore we set $L(v, v) = 1$ for all $v$ to enable the calculation of the interference $I(v, v, k)$ as described in (13). Besides, we consider two scenarios for the impact of the antenna design on interference. The first is an optimistic scenario where the antennas have sufficiently narrow beams and perfect beam steering capabilities. Therefore, the power towards the intended receiver is always EIRP $= 12.81$ W and any interference is avoided (i.e., s.t. $I(u, v, k)$ for all $u, v$, and $k$). The second is a worst case scenario, where the interference caused by isotropic antennas is considered. These scenarios correspond to the tight upper and lower bounds on performance for the conservative $R_{\text{min}} = 10$ kbps and the calculated EIRP. Note that greater values of $R_{\text{min}}$ can be selected, which leads to a higher, yet achievable, EIRP and to a greater sum of rates.
To obtain the results presented in the next section, a simulator of the constellation geometry that implements the algorithms described in Section [IV] was developed in Python 3 specifically for this task. Monte Carlo simulations were run on a PC with Ubuntu 18.04.2 LTS (64 bit), an Intel Core i7-7820HQ CPU, 2.9 GHz, and 16 GB RAM, whose clock precision of this platform is $10^{-7}$ s. In each experiment, the constellation is first rotated according to the period between consecutive matching realizations (i.e., observations) $T = 30$ seconds and, then, the transceiver matching and RA algorithms are executed. At least $N_{\text{sim}} = 1000$ experiments are executed, which gives a total simulation period of 30000 seconds. As a reference, the orbital period of the lowest orbital plane, deployed at $h_1 = 600$ km, is 5801 seconds. Consequently, approximately 193 matchings are performed per orbital period and each orbital plane experiences around five complete rotations throughout each simulation. This means that, between two matchings, the latitude of a satellite changes by less than $0.01\pi$. The $n$th realization of the transceiver matching and RA are hereafter denoted as $\mathcal{M}(n)$ and $\mathcal{A}(n)$, respectively.

The selected performance indicators to assess the performance of the transceiver matching algorithms are the empirical mean number of established inter-plane ISLs per satellite

$$\hat{\mu}_M = \frac{1}{N_{\text{sim}}} \sum_{n=1}^{N_{\text{sim}}} |\mathcal{M}(n)|$$

and the sum of rates as a function of the SNR (i.e., neglecting interference)

$$\mu_{R^*_{\text{SNR}}} (\mathcal{M}) = \frac{1}{N_{\text{sim}}} \sum_{n=1}^{N_{\text{sim}}} \sum_{\{u,v\} \in \mathcal{M}(n)} R^*_{\text{SNR}}(u, v) + R^*_{\text{SNR}}(v, u).$$

Then, the selected performance indicator to assess the performance of the RA algorithm is the ratio of normalized mean sum of rates

$$\hat{\mu}_{R^*_{\text{SNR}}} (\mathcal{A}) = \frac{1}{\mu_{R^*_{\text{SNR}}} (\mathcal{M})} \sum_{n=1}^{N_{\text{sim}}} R^*_{\text{SNR}}(\mathcal{A}(n)).$$

**VI. Results**

This section presents the most relevant results on the performance of the inter-plane transceiver matching and RA algorithm described in Section [IV] with the parameters listed in Table [II].

As a starting point, we illustrate the characteristics of our problem and the impact of the selected parameters (listed in Table [II]) in Fig. [4] which shows the number of matches per satellite after a typical realization of the GiEM algorithm for $Q = 2$. Note that the number of matches in planes 1 and $P$ (located in the extremes of Fig. [4]) is lower since cross-seam ISLs
Fig. 4: Frontal view of the constellation showing the number of matches per satellite after one representative realization of the GIEM algorithm given $\text{MPL} = L^*(7,40,2.4)$ and $Q = 2$ with (a) $P = 5$, (b) $P = 6$, (c) $P = 7$, and (d) $P = 8$.

are not implemented. It is easy to see that a greedy algorithm will start by establishing the ISL around the crossing points of the orbital planes (i.e., near the poles), where the shortest slant ranges occur. On the other hand, matches are only observed throughout the whole constellation for $P \geq 7$ because the EIRP was selected as in (27) for $P = 7$. Nevertheless, full inter-plane ISL connectivity is not necessary to reap some of the benefits of the inter-plane ISLs. For example, it can be seen in Fig. 4b that any satellite is within a few intra- and inter-plane ISL hops from each other. That is, if a direct inter-plane ISL is not available, the packets can be first routed through intra-plane ISLs towards the poles until an inter-plane ISL is available.

Next, we illustrate the performance of the transceiver matching algorithms in terms of the (normalized) mean number of inter-plane ISLs per satellite $\hat{\mu}_M$ in Fig. 5 and the mean sum of rates as a function of the SNR $\mu R_{SNR}(M)$ in Fig. 6. From Fig. 5 we observe that, in all cases with $Q = 2$, a slightly higher $\hat{\mu}_M$ is achieved with the GIEM and GMM algorithms than with the GEO algorithm. However, mixed results were obtained with $Q = 1$, where the GEO algorithm leads to more established ISLs with $P \in \{6,8\}$. These results were also observed for higher values of $P$ and with different configuration parameters. Therefore, this phenomenon may be attributed to the sensitivity of the algorithms to the change of parity of $P$ when $Q = 1$ in combination with the fact that cross-seam ISLs are not implemented. Note that, for the GEO algorithm, the maximum number of established inter-plane ISLs $Q(P-1)N_p/2$ is achieved only for $P = 8$ even though full ISL connectivity is guaranteed for $P \geq 7$.

Next, Fig 6 showcases the massive gains in the sum of rates provided by the GIEM algorithm when compared to the GMM and GEO algorithms. Specifically, even though the three matching
algorithms establish a comparable number of inter-plane ISLs, the sum of rates with the GIEM algorithm are usually between 38 and 100% higher than with the GMM algorithm and between 38 and 81% higher than with the GEO algorithm. These gains are even higher with $P = 5$, where the ISL connectivity is greatly limited to the polar regions (see Fig. 4).

To finalize the performance evaluation of the transceiver matching algorithms, Fig. 7 shows the empirical CDF of (a) the rates at each satellite $R_{\text{SNR}}(u, v)$ and (b) the propagation delay for $P = 7$. As it can be seen, there is a great difference between the selected rates of the different satellites in the constellation. For example, with the GIEM algorithm, almost 50% of the rates are lower than 20 kbps, 20% are above 100 kbps, and only 4% are higher than 1 Mbps. Note that, due to its logarithmic scale, Fig. 7b underemphasizes the great differences between the matching algorithms, which were previously observed in Fig. 6. Besides, it is observed that the propagation
Fig. 7: Empirical CDF of the (a) rates $R_{\text{SNR}}^*(u, v)$ and of the (b) propagation delay per inter-plane ISL for $P = 7$. The maximum propagation delay at $l^*(7, 40) = 3527$ km is 11.77 ms.

delay is less than 10 ms in more than 80% of the established ISLs and only slight differences are observed between the matching algorithms.

Now we move on to assess the performance of the RA algorithm. As described above, the results presented in Fig. 6 for the sum rates $\mu R_{\text{SNR}}^*(\mathcal{M})$ and in Fig. 7a for the rate at each satellite $R_{\text{SNR}}^*(u, v)$ correspond to the upper bound where the interference is zero. Therefore, we are interested in the minimum value of $K$ needed to achieve $\hat{\mu}_{R_{\text{SNR}}^*(\mathcal{A})} \approx 1$.

For this, we show $\hat{\mu}_{R_{\text{SNR}}^*(\mathcal{A})}$ for the RA algorithm in Fig. 8 along with that for two benchmark approaches: round-robin and random allocation. These three algorithms were applied after transceiver matching with the GIEM algorithm. In the round-robin approach, the $K$ orthogonal wireless resources are allocated one by one from the first to the last element of the transceiver matching at each realization $\mathcal{M}(n)$.

As it can be seen, our RA algorithm effectively mitigates the interference and clearly outperforms the two benchmark approaches. Specifically, only with $K = 4$ orthogonal resources, our RA algorithm is able to achieve $\hat{\mu}_{R_{\text{SNR}}^*(\mathcal{A})} > 0.95$ of the upper bound of the sum of the rates in the worst case scenario. On the other hand, to achieve this value, $K = 28$ and $K = 11$ are needed with the random and round-robin approaches, respectively. This means that our RA algorithm leads to a seven-fold and nearly three-fold increase in resource efficiency w.r.t. random and round-robin allocation, respectively.

Finally, we compare the complexity of the transceiver matching algorithms, along with that of the GIEM algorithm with RA in Fig. 9 for $Q = 2$ and $K = P = 7$. Clearly, with $O(PN_p)$, as derived in Section IV, the GEO algorithm is the least complex, followed by the GMM and the
Fig. 8: Normalized mean sum of rates with our RA algorithm, along with round-robin and random allocation after transceiver matching with the GIEM algorithm for $Q = 2$ and $P = 7$.

Fig. 9: CDF of the execution time for the GIEM, GMM, and GEO, matching algorithms, along with the GIEM algorithm with RA with $Q = 2$ and $K = P = 7$.

GIEM algorithms with $O \left( PN_p^2(P - 1) \right)$. Recall, that the difference in execution times between these two algorithms is mainly a result of sorting a shorter list with the GMM algorithm. Finally, we observed Section IV that the complexity of our RA algorithm is $O \left( KM^2 \right)$. We have observed that $M(n) \approx \mu_M \approx N_p(P - 1)$ for all $n$, with $Q = 2$ and $P = 7$. Therefore, we have that the complexity of our RA algorithm is $O \left( KN_p^2(P - 1)^2 \right)$. This is much higher than the complexity of the transceiver matching algorithms, performing in the order of $P - 1$ times more operations than the GIEM algorithm. Hence, the complexity of establishing the ISLs is mainly determined by the RA, as observed in Fig. 9.

VII. CONCLUSIONS

In this paper, we a framework to maximize the rates in the inter-plane ISLs of dense LEO constellations. In our approach, the transceiver pairs are first selected and, then, orthogonal wireless resources are allocated to mitigate interference. Furthermore, we provided a simple
approach to constellation design, where we calculated the minimum EIRP that is needed to guarantee that all satellites have at least one potential inter-plane neighbor at all times (full inter-plane ISL connectivity).

Our results show that solving the transceiver matching problem from scratch at each realization with our GIEM algorithm leads to the maximum sum of rates, which are up to 81% higher than with the other considered algorithms with full inter-plane ISL connectivity. Conversely, maintaining the ISLs for as long as possible with our GMM algorithm reduces the execution time but also the sum of rates with respect to the GIEM algorithm to a value that is comparable to the GEO algorithm, used as a benchmark.

Regarding resource allocation, we observed that our algorithm provides massive gains when compared to random and round-robin resource allocation. Specifically, 700% and 275% more resources are needed to achieve 95% of the upper bound on the sum of rates with random and round-robin allocation than with our RA algorithm.

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