Optimal Bounds for the Probability of Failure of Sheet Metal Forming Processes of DP Steel

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In this contribution, the Optimal Uncertainty Quantification framework is extended to account for aleatory uncertainties, such that both epistemic and aleatory uncertainties can be quantified. The extended framework is demonstrated for an example of a forming process of a Dual-Phase steel sheet, for which optimal bounds on the probability of failure are computed under polymorphic uncertainties.

1 Introduction

One crucial step during uncertainty quantification (UQ) is the choice of the appropriate tools, as a frequent obstacle in UQ are uncerfified assumptions on the available data or the disregard of important information. A method to quantify the sharpest bounds on the probability of failure (PoF), is the Optimal Uncertainty Quantification (OUQ) proposed by Owhadi et al. [1], which has been applied to a bioinformatics problem in [2]. This framework is designed to treat each uncertain parameter as epistemic in the sense that full probability distributions are not known for the uncertain input parameters. We propose to extend this framework to incorporate aleatory uncertainties and demonstrate it with an example of a sheet metal forming process, which is subjected to polymorphic uncertainties.

2 Optimal Uncertainty Quantification for Polymorphic Uncertainties

The computations of the optimal, i.e. sharpest, bounds \( \mathcal{L} \) and \( \mathcal{U} \) are defined as

\[
\mathcal{L} \leq \mathbb{P}[g > g_{\text{max}}] \leq \mathcal{U} \quad \text{with} \quad \mathcal{L} := \inf_{(g, \mu) \in \mathcal{A}} \mu[g > g_{\text{max}}] \quad \text{and} \quad \mathcal{U} := \sup_{(g, \mu) \in \mathcal{A}} \mu[g > g_{\text{max}}],
\]

where \( \mu[g > g_{\text{max}}] \) denotes the probability of failure, here characterized as a single scalar (mechanical) quantity \( g \) exceeding an admissible value \( g_{\text{max}} \). The tuple \((g, \mu)\) denotes a realization of the set of admissible scenarios \( \mathcal{A} \), which is defined by the uncertain problem itself and the level of available information regarding the \( q \) uncertain parameters \( Y = [Y^1, Y^2, \ldots, Y^q]^T \) and \( g_{\text{max}} \). The level of knowledge of a single parameter is reflected by the number of known statistical moments of the distribution of this parameter. The optimization problems in (1) are infinite dimensional and thus difficult to solve.

However, the reduction theorem in the OUQ framework proves, that a suitable solution can be found if the uncertain parameters of the problem are discretized in terms of finite mixture distributions of Dirac measures. Then, the objective function can be written as

\[
\mu[g > g_{\text{max}}] = \sum_{i=0}^{n^Y} \ldots \sum_{j=0}^{n^q} \sum_{m=0}^{n^\mu_{\text{max}}} p_i^Y p_j^Y \ldots p_m^\mu \chi (g(Y), g_{\text{max}}) \quad \text{with} \quad \sum_{i=0}^{n^Y} p_i^Y = 1 \quad \text{and} \quad \sum_{m=0}^{n^\mu_{\text{max}}} p_m^\mu = 1,
\]

where \( p_i^Y \) denotes the \( k \)-th weight of the corresponding support point \( Y_m^k \) of the \( m \)-th uncertain parameter. The function \( \chi \) in (2) denotes the probability of failure under the given parameter combinations of \( Y \) and \( g_{\text{max}} \). In the original proposal of the OUQ framework this is a deterministic evaluation, such that \( \chi \) takes the form of the indicator function and returns 1 in case of failure and 0 otherwise. The mentioned levels of knowledge determine the number of terms \( n^Y \) of each parameter and are enforced as constraints during the optimization, as the (non-central) moment of order \( k \) of parameter \( Y^m \) is computed by \( E_k[Y^m] = \sum_{i=0}^{n^Y} p_i^Y (Y^m)^k \).

So far, the OUQ framework allows the incorporation of uncertain quantities of epistemic nature. We propose two different approaches to account for aleatory uncertainties as well: (i) a smooth transition from the analysis of epistemic to aleatory uncertainties with limited data can be performed by incorporating more and more moment constraints until the resulting bounds on the PoF converge. Thereby, accurate bounds on the PoF are identified for the case where aleatory uncertainties are included in terms of a finite but large number of moment constraints. (ii) Alternatively, if full distribution functions \( f_m \) of a number \( r \) of uncertain variables \( Y_m, m = 1 \ldots r \), are known, a numerical integration in the failure region can be performed.
Fig. 1: a) Schematic illustration of the considered sheet metal forming process and the corresponding construction of the surrogate model based on a neural network. b) Optimal bounds on the PoF for approach (i) in the first $8 \times 2$ color bars, and approach (ii) in the last $1 \times 2$ color bars. In addition to that, two different levels of information are considered for the friction coefficient, which is firstly constrained to be within bounds (left bars) and secondly additionally the mean is constrained within bounds (right bars). Within the color bars, the required computing is given.

e.g. in terms of Monte-Carlo integration schemes. For this purpose, the associated PoF resulting only from the aleatory uncertainties in (2) is given by

$$\chi = \int_{g > g_{\text{max}}} \prod_{m=1}^{r} f_m(\hat{Y}_m) d\hat{Y}. \tag{3}$$

Herein, $\hat{Y}$ consists of all aleatory uncertain quantities $\hat{Y}_m$. Note that here uncorrelated uncertain variables are considered and thus, (3) needs to be modified in case correlated variables are to be analyzed. The two approaches differ mainly in terms of the numerical treatment of the aleatory uncertainties, as approach (i) avoids the numerical evaluation of the integral in Eq. (3), whereas the OUQ optimization becomes more expensive due to additional degrees of freedom. On the other hand, the inverse arguments hold for approach (ii), such that evaluation of the best approach has to be done for each individual problem.

### 3 Exemplary Evaluation of a DP-steel forming simulation

The problem of interest is a forming process of an S-Rail, as depicted in Fig. 1a, which is simulated in terms of Finite Elements. Therein, two material parameters describing the hardening behavior are assumed to be known with a beta-distribution (c.f. [3]) and are thus treated as aleatory uncertainties. Furthermore, these two material parameters are strongly correlated and therefore only one parameter is considered as uncertainty, whilst the other parameter is computed based on the first one. Considered epistemic uncertainties are the failure criterion, here the Cockroft-Latham-criterion, and the friction coefficient between the sheet metal and the tools. The failure criterion is assumed to be known within bounds only, whereas the friction coefficient is investigated for two different levels of information: first, only bounds are considered and secondly, also the mean is assumed to be known within bounds. Since a direct optimization on the Finite Element computations is too expensive, a neural network as depicted in Fig. 1a was constructed and used as surrogate model within the optimization. The resulting bounds on the PoF for approach (i) are depicted in Fig. 1b for an increasing number of moment constraints on the material parameter by green and gray bars, in which the individual computing time is plotted. The results of approach (ii) are shown in the blue bars on the right side of the figure. It can be seen, that approach (i) seems to converge slowly, although the considered number of moment constraints is still not sufficient, as approach (ii) yields upper bounds of approximately half the magnitude. At the same time, the computing time is significantly smaller, such that approach (ii) is the favorable approach for this particular problem.

In conclusion, the extended OUQ offers a versatile framework for UQ of the PoF under polymorphic uncertainties.

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