Application of Least-Squares Method to Time Series Analysis for 4DPM Matrix

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Abstract. In the past 10 years, considerable attention has been paid to PMBOKM, and it has combined with specific examples. However, quantitative methods are rarely used for analysis. We select Han River Estuary Waterway Engineering Project as a study case. Based on the relationship between the stages of the project management system, it tries to analyse the first five stages of matrix time series of 4DPM in the three major forms by the least squares method, and the method is divided into four steps to solve the matrix. In addition, the matrix is subject to function changes and transpose processing, and then solved by the least squares method too. According to the set solution method, the matrix can be solved, and eight cases can be obtained. By comparing the errors, under the condition of transposed matrix, the function of the most suitable for matrix transmission can be found. So, it is known that the method is feasible and effective in 4DPM model matrix analysis.

1. Introduction

Project management body of knowledge (PMBOK) has been continuously improved since it was first published in 1986. On the basis of this system theory, Chinese professionals have developed a set of four-dimensional project management system, namely 4DPM. The system covers four dimensions, it includes process management as longitudinal axis A, content management of horizontal axis B, height management of vertical axis C, and form management of micro axis D. It is to add a time dimension on the basis of three-dimensional space [1].

What is the research situation of PMBOK? Many scholars have conducted research on various aspects of it. For example, Wang Liguo and Di yachan analyzed the differences of PMBOK, ICB and PRINCE2 knowledge systems, and compared them from the aspects of production background, guiding ideology, content arrangement, and so on [2]. Wang Mengying and Ning Yan put forward suggestions, it was about the limitations of PMBOK in the early management, scenario and other aspects of the project [3]. Zhou Jimeng built a four-dimensional construction project management system [4]. According to the actual situation of T company, Cong ping built an enterprise-level project management system [5]. Ma Fangmei optimized the customer service knowledge base of Qinghai mobile and applied PMBOK [6]. For the scientific and technological projects of petroleum enterprises, Wang Xuesong applied PMBOK in it and pointed out the aspects that should be strengthened in the
future [7]. Jantanee Dumrak investigated a linkage between project management knowledge areas and sustainable outcomes from four RHD projects in Thailand [8]. Shahron Williams van Rooij scaffold the project-based learning with PMBOK [9]. Benjamin Kokea and Robert C Moehlerb considered the green value to project management [10]. From existing literatures, it can be seen that PMBOK is widely used, but it is mostly combined with specific cases and only stays at the macro theoretical level.

Considering the actual needs of project management, it is necessary to apply quantitative analysis method, so it is important to use appropriate quantitative analysis method to improve and enrich the theoretical system of 4DPM.

2. The Overview of 4DPM Knowledge System

2.1. The General Model of 4DPM

4DPM is at the outline level, and mainly used in macro system construction and micro-system application. It is rich in content and characteristics, and its benchmark is the outline and framework. The object of the system is to compatible with various specific management technologies better and promote the development of various application-specific projects. Its main areas of knowledge include project overall management, project process management, project content management, project height management, project form management, project key management and project management capabilities, and the first six knowledge areas are the main six systems of it, too [11]. The three-dimensional coordinate map of 4DPM is shown in figure 1.

![Figure 1. 3D map of 4DPM.](image-url)

2.2. The Element Construction of 4DPM

In order to gain a deeper understanding of 4DPM, and then introduce its main system. The key contents are as follows.

The overall management of the project is the coordinate structure management based on the four-dimensional space (axis A, B, C and D). The process can be divided into the following seven steps. It includes establish a spatial coordinate system, determine the environment space of the project, draw the horizontal coordinate of the process, draw the vertical coordinate of the elements, draw the high-level coordinate, establish the total space of the project and form the spatial graph.
There are many small projects in the big project and it reflects in the different period. The process management can be divided into nine stages. It includes initiation phase, preparation stage, initial stage, implementation stage, end stage, monitoring stage, sleep stage, prefect stage and derivative stage.

Project content management can be divided into nine categories. They are product management, equipment management, technical management, development management, economic management, safety management, safeguard management, organization management and information management.

The project height of 4DPM can be divided into nine levels. It consists of overall decision making, system decision making, overall command, system command, integrated execution, professional execution, professional operation, compound operation and independent operation.

The project is divided into nine periods according to time. It consists of gestation period, formation period, growth period, vigorous period, mature period, stable period, atrophy period, termination period and curing period, and differ in 3D model with different colours.

3. Application of Least-Squares Method

3.1. Time Series Analysis for 4DPM Matrix

With the 4DPM knowledge theory system, 3D model is established. It includes n kinds of forms. According to the content of the project, selecting phases of it and using a 9×9 two-dimensional matrix represent. The area, corresponding to each cuboid, represents the points in the general 3D model. The time of each point determines the thickness of the rectangle. After cutting each stage, draw a two-dimensional matrix based on the thickness of the cuboid. This is 4DPM matrix.

Because the matrix is determined by the number of days, and the first five stages reflect the changing process that gradually advances over time. In order to explore whether it is a correlation between the stages, we try to analyze it by quantitative analysis, and then enrich and improve the 4DPM theory system.

3.2. Analysis Process of Least-Squares Method

The least squares method finds the best function match of the data by minimizing the sum of the squares of the errors. It not only solves questions to obtain unknown data easily, but also studies the relationship between multiple variables. And its applicable fields are mainly algebra and mathematics. The formula is $X^T X \beta = X^T y$.

Because of the special relationship between matrices, we use the least squares method $Ax = b$ to solve the matrix. The solution process of the least squares method is as follows. The first step is to establish the equations.

$$Ax = b$$ (1)

$A$ is a matrix of $m \times s$, $x$ is the column vector of $s \times 1$, $b$ is the column vector of $m \times 1$.

$$A = (a_{ij})_{m \times s}$$ (2)

$$x = (x_1, x_2, \ldots, x_s)^T$$ (3)

If the equation has no solution, it is called incompatibility. And then trying to find a set of numbers $x_1^0, x_2^0, \ldots, x_s^0$ to calculate $\delta$.

$$\delta = \sum_{i=1}^{n} (a_{i1} x_1 + a_{i2} x_2 + \ldots + a_{is} x_s - b_i)^2$$ (4)

In this paper, the model is $AX = B$. $A$ is the initial matrix, $B$ is the target matrix, and $X$ is the transfer matrix. Their specific forms are as follows.
The calculation step is divided into four steps. Firstly, according to the least squares equation \( Ax = b \), the matrix \( X \) can be split by column into \( (x_1, x_2, \ldots, x_n) \), the matrix \( B \) splits into \( (b_1, b_2, \ldots, b_n) \). \( A \) and \( B \) are known to solve \( X \). Secondly, transform for each element of \( A \) and \( B \). There are four function transformation formulas, including \( x, x + 1, \log_2(x + 2), e^{x+1} \). Then, applying the least squares method to solve it. Thirdly, the matrix obtained by the above change is performed row-column transformation to obtain a transposed matrix of time series, it is solved by the least square method described above. Fourthly, Matlab solves the least squares method, and then obtains \( X \). According to \( A \) and \( X \), the target prediction matrix \( B^* \) can be obtained by calculating \( AX \). Prediction accuracy is determined by the squared sum of the residuals of the each matrix element of \( B \) and \( B^* \). Eight cases can be obtained, and comparing the prediction accuracy.

4. Case Study

4.1. 3D Model and Matrices for the Case

We select Han River Estuary Waterway Engineering Project as a study case. According to the implementation of the project, dividing the completed part of the project into three forms, and investigating the process, content and height of each form at the same time. The model becomes visible, when using Matlab to image the data obtained from the survey.

Using Matlab to draw the image of the gestation stage, formation period and growth period. They are shown in figure 2, 3 and 4. The thickness of the cuboid is determined by the method described above, and it is the length of each region in the x-axis direction. So, it can determine five regions based on days. And the corresponding thickness of each region is 0.2, 0.4, 0.6, 0.8, 1.0. For example, the day of 0-15 is 0.2, 16-30 is 0.4, 31-50 is 0.6.

The thickness in the y-axis and z-axis directions is one in this model.

**Figure 2.** 3D model of the incubation period. **Figure 3.** 3D model of the formation period.
According to 3.1 and 3.2, the matrix can be determined. The value of it takes an integer and then magnifies five times. Finally, it gets an integer time series. Each row represents a hierarchy, and each column represents the content. The 15 matrices are as shown in figure 5.

![3D model of the growth period](image)

**Figure 4.** 3D model of the growth period.

**Figure 5.** Matrix time series.

### 4.2. Predictive Analyses for Matrix

The results of the eight prediction schemes, are shown in table 1, and calculated by Matlab programming.
Table 1. Forecast Error Table.

|   | x   | x(T) | x+1 | x+1(T) | e^x+1 | e^x+1(T) | log(x+2) | log(x+2)(T) |
|---|-----|------|-----|--------|-------|----------|----------|-------------|
| T1| 0.00| 0.00 | 0.00| 0.00   | 0.00  | 0.00     | 0.00     | 0.00        |
| T2| 105.06| 36.01| 77.29| 34.40  | 124.70| 64.78    | 93.12    | 87.04       |
| T3| 32.40| 9.64 | 32.42| 13.03  | 98.42 | 56.78    | 87.33    | 84.04       |
| T4| 53.20| 29.23| 40.88| 25.54  | 55.64 | 54.34    | 87.90    | 85.33       |
| T5| 21.15| 7.28 | 20.49| 7.77   | 28.25 | 21.41    | 86.39    | 82.78       |
| T6| 122.22| 33.34| 95.97| 22.11  | 162.47| 37.60    | 96.11    | 84.51       |
| T7| 38.30| 20.30| 33.09| 19.63  | 55.60 | 28.01    | 86.12    | 85.79       |
| T8| 10.73| 35.19| 11.14| 25.31  | 20.13 | 34.30    | 83.89    | 86.08       |
| T9| 47.00| 119.70| 45.73| 89.11  | 121.55| 149.78   | 89.02    | 95.26       |
| T10| 6.24 | 7.81 | 6.24 | 7.25   | 11.38 | 8.50     | 82.50    | 82.68       |
| T11| 65.68| 29.46| 54.09| 27.70  | 92.65 | 44.09    | 89.66    | 87.58       |
| T12| 19.37| 85.93| 26.07| 68.25  | 108.39| 129.83   | 87.51    | 89.82       |
| T13| 94.00| 111.07| 57.52| 81.58  | 76.98 | 125.49   | 91.74    | 94.66       |
| T14| 8.63 | 12.95| 13.85| 14.80  | 116.62| 42.84    | 85.28    | 84.22       |
| T15| 7.80 | 6.84 | 10.21| 6.29   | 31.73 | 11.20    | 81.70    | 83.14       |
| Mean| 42.12| 36.32| 35.00| 29.52  | 73.63 | 53.93    | 81.88    | 80.86       |

From the above table, the conclusions are as follow.

1. No matter what kind of function, the least square prediction error of T5, T10, and T15 is the smallest.
2. The error of the transposed matrix is smaller than the non-transposed matrix.
3. When the function is \( F(x) = x + 1 \), the error mean is the smallest.
4. When the function is \( F(x) = \log(x+2) \), the errors are almost equal and obviously abnormal with other functions. So, it should be discarded.

According to the above conclusions, the two error sequences of the logarithm function can round off, and the remaining others are drawn by Matlab, the image is as shown in figure 6.

![Figure 6. The image of time series error.](image-url)
Figure 6 depicts the data of the first six columns in table 1, each column is arranged in the order of red, yellow, green, cyan, blue, and purple. It can be seen that the green polyline and the cyan polyline have small fluctuations, and the error mean is small, but the cyan error is the smallest. Therefore, using cyan as the benchmark, and then it can find the error values of T6 and T11 are not only similar but also the values are small, which are 22.11 and 27.70 respectively. From the transfer matrix table, it can find the commonality between them. It is that the value is focused on the first row, and the last three rows are all zero.

The transfer matrix of T6 and T11 are as follows.

Through the above analysis, the matrix of the strong form initiation phase can be predicted by the code of Matlab. The matrix is N16. It can be seen that the value of TRAN10-11 as the transfer matrix is more realistic than others.

5. Summary and Conclusion

We describe the theoretical framework of the 4DPM knowledge system. And then according to the case, using Matlab to draw the three-dimensional mode of the first three forms. The matrix time series of the first five stages in each form are extracted, and then are transformed by the function. The function are $x\cdot x+1, \log_2(x+2), e^{x+1}$. The least squares method is used to solve the matrix. We compare the error of the results, and it is solved after function transformation and transposition processing. It is found that under the condition of transposed matrix, the function $F(x) = x+1$ is the most suitable for matrix transmission. Therefore, the state matrix of the next stage can be calculated by the method of predictive transfer. It is helpful to analyze 4DPM from the perspective of quantitative analysis.

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