Radial oscillations in neutron stars from unified equation of states

Souhardya Sen,1,* Athul Kunjipurayil,1,† and Bharat Kumar1,‡

1Department of Physics & Astronomy, National Institute of Technology, Rourkela 769008, India

(Dated: May 5, 2022)

We study radial oscillations in non-rotating neutron stars by considering the unified equation of states (EoSs), which is consistent with finite nuclei and nuclear matter properties and supports the $\geq 2M_\odot$ star criterion. We solve the Sturm-Liouville problem to compute oscillation of $f$-mode frequency and their eigenfunction for neutron star modelled with six selected unified EoSs from two distinct Skyrme-Hartree Fock and Relativistic Mean-Field models. The calculated eigenfunction reveals the damped harmonic motion varying with the frequencies corresponding to EoS and with different central densities. We check the variation of $f$-mode frequency of oscillation with different quantities like central density, mass, average density and compactness of the neutron star. In particular, we derived an empirical relation for the frequency of $f$-mode scaled with central density as a function of the square of the average density and discovered a linear trend.

I. INTRODUCTION

Neutron stars (NS) are the collapsing cores of previously massive stars that form after supernova explosions [1]. A NS’s central density is predicted to be around ten times that of the nuclear saturation density ($\rho$). A NS’s central density is predicted to be around ten times that of the nuclear saturation density ($\rho_0 \approx 0.16$ fm$^{-3}$). Some unusual phases, such as meson condensation [2–4] or quark deconfinement [5–7], can be achieved at such high densities. It is impossible to achieve these circumstances on Earth. As a result, studying NS gives us unique insights into the physics of strongly interacting nuclear matter and phase transitions at ultra-high densities. Understanding NS also necessitates knowledge of several scientific fields like nuclear physics, particle physics, astrophysics as well as gravitational physics. Even while we know a lot about how an NS forms, we know relatively little about its internal composition. The one known fact in NS is that characteristics like its mass and radius are strongly influenced by the equation of state (EoS) of dense matter. Our overall objective in this field is to identify such properties that may be observed so that we can utilise the generated observational data to construct an accurate EoS, thereby bringing us one step closer to completely comprehending the interior of a NS.

NSs are shown to pulse with different quasi-normal modes (QNMs) in which infinitesimal perturbations induce oscillations whose amplitude decays exponentially with time. Dynamical instabilities like mass accretion and tidal forces from a nearby binary companion [8, 9], star-quakes generated by fissures in the crust [10], and supernova explosions are a few sources that might induce oscillations inside an NS [11]. QNMs are categorised into two main groups based on the motion of the pulsations: radial and non-radial oscillation, and each of them is further classified based on the restoring force that acts on the displaced mass element to bring the system back to equilibrium. These restoring forces maybe gravity ($g$-mode), pressure gradient ($p$-mode), rotation ($\gamma$-mode), magnetic fields, and centrifugal and Coriolis forces in rotating neutron stars, distinguished by their frequency range. Here, we focus our work on the fundamental $f$-mode for the radial oscillation.

Because QNMs are sensitive to the internal composition and EoS of dense matter, we analyse the internal structure of the star and identify the thermodynamic parameters of the NS's interior using asteroseismology for the study of stellar pulsations in general relativistic frame. The study of frequencies can thus assist us in indirectly probing within an NS and discovering how strongly interacting nuclear matter behaves at such high densities, thereby further constraining the choice of EoS [12–21].

Although radial oscillations cannot directly emit gravitational waves (GW), they can couple with non-radial oscillations, amplifying them and creating a stronger GW that might be detected [17, 22]. During the formation of hyper-massive NS through binary NS merger, a short gamma-ray burst is emitted, which can be modulated by the radial oscillations [23]. As a result, it is not only valuable in understanding the physics of dense nuclear matter inside an NS, but it also has some application in GW physics.

Chandrasekhar investigated radial oscillations in NS, being the simplest mode of oscillation, for the first time in 1964 [24]. Following that, it was researched by other authors such as Harrison et al. [25] and Chanmugam [26], employing zero temperature EoS and finite temperature EoS for Proto-NS by Gondek et al. [27], and strange stars by Benvenuto & Horvath [28], Gondek & Zdunik [29]. Glass and Lindblom performed the first major investigation of radial oscillations in 1983 [30]. Their numerical results were later rectified by Väth & Chanmugam in 1992 [21] and re-examined by Kokkotas & Ruoff [13] using two alternative numerical approaches that included six more zero temperature EoS. Their results suggested that oscillations become unstable after the central density at which NS reaches its maximum mass. This was
due to the fact that they used the equilibrium adiabatic index in all of their equations. However, if different adiabatic indices connected to the physical circumstances inside NS are employed [27] and the slowness of weakly interacting processes is taken into account, stability can be extended beyond that central density [26, 31].

The \( f \)-mode frequency of oscillation is calculated in this work using six unified EoSs based on the Relativistic Mean Field (RMF) and Skyrme-Hartree-Fock (SHF) models [32, 33]. We solve the Sturm-Liouville eigenvalue problem [34–36] with the assumption that our NS is non-rotating, has a finite temperature and zero magnetic field. We base our theory on the fact that the oscillations are small enough to use linear theory. Thus, they are adiabatic and do not affect the chemical composition of the star [37, 38].

This work is structured as follows: in the upcoming Sec. II, we go over the theoretical formalism, starting with hydrostatic equilibrium and stellar structure equations in general relativity in Sub-sec. II A, and radial oscillation equations in Sub-sec. II B. In Sec. III, we will offer a quick summary of our chosen six EoSs and the reasons for selecting them. In Sec. IV, we provide our numerical results. Sub-sec. IV A corresponds to the computation of M-R, whereas Sub-sec. IV B shows the results of the boundary conditions. Then, in Sub-sec. IV C, we connect our calculated \( f \)-mode frequency to other global properties such as mass, central and average density, and compactness. In addition, we provide a fitting curve with the computed \( f \)-mode frequency. Finally, we summarise and conclude, as well as suggest opportunities for further improving our current work, in Sec. V.

II. THEORETICAL FORMALISM

The massive gravitational forces of a non-rotating NS’s interior allow only slight deviations from spherical symmetry, resulting in a nearly perfect formed sphere in its equilibrium condition. As a result, our assumption that the star is spherically symmetric is a reasonable approximation. The gravitational field of such a body is itself spherically symmetric and is given by the Schwarzschild metric\(^1\) in the form of [39]:

\[
ds^2 = -e^\nu c^2 dt^2 + e^\lambda dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2),
\]

where \( \lambda \equiv \lambda(r) \) and \( \nu \equiv \nu(r) \) following their own set of equations. Here, the energy-momentum tensor \( T_{\mu \nu} \) has the form of a perfect fluid:

\[
T_{\mu \nu} = (P + \epsilon) u_{\mu} u_{\nu} + P g_{\mu \nu},
\]

where \( P \) is the pressure, \( \epsilon \) represents the energy density and \( u_{\mu} \) is the covariant velocity. Since we have spherical symmetry and are only going to consider motion along radial direction, only the components \( u_0 \) and \( u_1 \) are non-zero.

A. Hydrostatic equilibrium equations

In a state of hydrostatic equilibrium, all quantities are time-independent. Therefore, even \( u_1 \) is zero. From Einstein’s field equations, using the Schwarzschild metric in eq. (1) in equilibrium and applying the boundary condition \( \lambda(r = 0) = 0 \), we get:

\[
e^\lambda(r) = \left( 1 - \frac{2Gm}{c^2 r} \right)^{-1},
\]

where the mass \( m \) can be integrated using:

\[
\frac{dm}{dr} = \frac{4\pi \epsilon r^2}{c^2}.
\]

Similarly, using the law of conservation of momentum, we get [40]:

\[
\frac{dv}{dr} = \frac{-2}{P + \epsilon} \frac{dP}{dr}.
\]

Finally, using eq. (5) and the Einstein’s field equations, we get:

\[
\frac{dP}{dr} = -\frac{P + \epsilon}{c^2} \left( \frac{G}{r^2} + \frac{4\pi G \rho P}{c^2} \right) \left( 1 - \frac{2Gm}{c^2 r} \right)^{-1}.
\]

Eqs. (3) and (5) define the behaviour of the metric functions inside the NS where \( r < R \). At the surface, i.e. at \( r = R \), they satisfy the boundary condition,

\[
e^{\nu(R)} = e^{-\lambda(R)} = \left( 1 - \frac{2Gm}{c^2 R} \right).
\]

Eq. (7) stays true even outside the star, where \( \dot{R} \) should be replaced by \( r \) for \( r > R \) as it attains the familiar form of the Schwarzschild solution.

Eqs. (4) and (6) are collectively known as the Tolman–Oppenheimer–Volkoff (TOV) equations [41, 42]. These equations express the equilibrium at each step of the radius \( r \), between the internal pressure of the overlying material against the gravitational force of attraction. These equations can be interpreted if we consider a shell of radius \( r \) and thickness \( dr \), with a pressure difference of \( dp \) in the exterior with respect to its interior and evaluate the net force on each side. The only thing needed to solve the structure equations of neutron stars is the Equation of State (EoS) of dense matter, i.e., the relation between the pressure and the energy density, which enters the TOV equations. For a given EoS, the TOV equations can be integrated from the origin with the boundary conditions \( m(r = 0) = 0 \) and \( P(0) = P_c \), where \( P_c \) is the central pressure, until the pressure becomes zero. The point \( R \), where the pressure vanishes, provides the circumferential radius of the star. The integration of eq. (4) from zero to \( R \) gives its total mass \( m(R) = M \).

---

\(^1\) Here, we use the mostly positive signature \((-+++\)).
B. Radial oscillations equations

Chandrasekhar, in 1964, derived the equations for dynamical stability of radial oscillations in stellar models [24]. Here, we use a similar approach to derive the radial perturbation equations for NS oscillations. We assume that the oscillations are harmonic, \( X(r,t) = X(r)e^{i\omega t} \) [26, 27]. Next, we define two eigenfunctions as \( \xi \equiv \Delta r/r \) and \( \eta \equiv \Delta P/P, \Delta r \) and \( \Delta P \) being the radial displacement and the corresponding Lagrangian perturbation of pressure [21, 26, 43]. Then, for infinitesimal adiabatic perturbation in the form of radial oscillations, we have two coupled first order differential equations,

\[
\frac{d\xi}{dr} = -\left( \frac{3}{r} + \frac{dP}{dr} \frac{1}{P + \xi} \right) \xi - \left( \frac{1}{r\gamma} \right) \eta, \tag{8}
\]

\[
\frac{d\eta}{dr} = \left[ \frac{\omega^2 e^{\lambda - \omega}}{c^2} \left( \frac{P + \xi}{P} \right) r - \frac{4}{P} \frac{dP}{dr} \right] \xi
- \left[ \frac{8\pi G}{c^4} e^{\lambda} (P + \xi) r - \left( \frac{dP}{dr} \right)^2 \frac{r}{P(P + \xi)} \right] \xi \tag{9}
\]

\[ + \left[ \frac{dP}{dr} \frac{\xi}{P(P + \xi)} - \frac{4\pi G}{c^4} (P + \xi) e^{\lambda} \right] \eta, \]

where \( \gamma \) is the relativistic adiabatic index given by

\[ \gamma = \frac{dP}{d\xi} \left( 1 + \frac{\xi}{P} \right), \tag{10} \]

and \( \omega \) is the angular frequency of the oscillation or the eigenvalue. We can notice that eq. (8) blows up at \( r = 0 \). To keep it finite everywhere we have to make sure that the coefficient of \( 1/r \) vanishes as \( r \to 0 \). Hence, at \( r = 0 \),

\[ 3\xi \gamma + \eta = 0, \tag{11} \]

and \( \xi(r = 0) = 1 \), since we assume normalised eigenfunction. Also, in eq. (9) as \( r \to R \Rightarrow P \to 0 \Rightarrow P/\xi \to 0 \Rightarrow \xi/P \to \infty \). So similarly the coefficient of \( \xi/P \) should also vanish as \( r \to R \). Hence, at \( r = R \),

\[ \eta = -\left[ \left( 1 - \frac{2GM}{c^2r} \right)^{-1} \left( \frac{\omega^2 R^3}{GM} + \frac{GM}{c^2r} \right) + 4 \right] \xi. \tag{12} \]

Eqs. (11) and (12) together constitute the boundary conditions for the set of differential equations (8-9) at centre and at the surface of NS.

It is possible to combine eqs. (8) and (9) into one second order differential equation in \( \xi \) as was done by Chandrasekhar [24]. This equation is of the Sturm-Liouville type with \( \omega_0^2 < \omega_1^2 < \ldots < \omega_n^2 < \ldots \), where all the eigenvalues are real so that the solutions of the eigenfunctions are purely oscillatory and \( n \) denotes the number of nodes. In the case of imaginary \( \omega \), the solution would grow exponentially and hence lead to instability. In this paper, we focus our work on finding only the fundamental \( f \)-mode (\( n = 0 \) node) frequency and analysing its nature with respect to various NS properties.

III. EQUATION OF STATE

To solve the radial oscillation of the NS, we are selecting six unified EoSs based on RMF and SHF models with different parametrization [32, 33], viz:

1. NL3: The famous NL3 is based on non-linear interaction, where only the \( \sigma \)-meson self-coupling term is included while the cross-coupling terms are not taken into account [44].

2. IOPB: Interaction with higher-order couplings including self-coupling of \( \rho \)-mesons and \( \omega \)-\( \rho \) cross-coupling terms [45].

3. G3: Improved effective RMF interaction, including coupling from \( \delta \) and \( \rho \) mesons and several cross-coupling terms of \( \rho \)-meson. G3 model is very well suited for low density pure nuclear matter [46].

4. DD2: A density-dependent interaction with experimental values of proton and neutron mass \( m_p, m_n \). This model can provide an accurate description of the composition, and thermodynamic quantities over a large range of densities [47].

5. DDME2: Also, a effective mean-field interaction with density-dependent meson-nucleon couplings [48].

6. SLy4: Based on the Skyrme-Lyon model, this interaction is suitable for calculating the properties of neutron-rich matter. This model can describe both the NS crust as well as the liquid core [49].

![Fig. 1. Pressure in NS matter versus baryon density calculated using six unified EoSs [32, 33]](image-url)

Fig. 1 shows the pressure calculated with six unified EoSs NL3 [44], IOPB [45], G3 [46], DD2 [47], DDME2 [48], and SLy4 [49].
| EoS  | $M$ (M$_\odot$) | $R$ (km) | $f$ (kHz) | $R_{1.4}$ (km) | $C_{1.4}$ (km) | $f_{1.4}$ (kHz) |
|------|----------------|----------|-----------|---------------|----------------|---------------|
| DD2  | 2.418          | 11.824   | 2.325     | 13.123        | 0.157          | 3.568         |
| DDME2 | 2.482          | 12.028   | 2.303     | 13.164        | 0.157          | 3.713         |
| G3   | 1.997          | 10.912   | 2.331     | 12.564        | 0.164          | 3.266         |
| IOPB | 2.149          | 11.912   | 2.041     | 13.270        | 0.156          | 3.509         |
| NL3  | 2.774          | 13.244   | 2.115     | 14.579        | 0.142          | 2.941         |
| SLy4 | 2.048          | 10.068   | 2.973     | 11.673        | 0.177          | 3.617         |

TABLE I. For each of the six EoSs, the maximum mass $M$ is listed together with its corresponding radius $R$ and fundamental mode frequency $f$. For a canonical NS, the radius $R_{1.4}$, compactness $C_{1.4}$, and frequency $f_{1.4}$ are also provided.

Among these chosen EoSs, NL3 yields the stiffer EoS, followed by DDME2 and DD2. SLy4 is initially the softest at lower densities but becomes stiffer than the remaining two EoSs as density rises. On the contrary, IOPB starts off stiffer than G3 and SLy4 but becomes softer with increasing density. We are going to use these six EoSs for the calculation of radial oscillations and NS properties.

IV. RESULTS AND DISCUSSION

In the relativistic frame, the oscillation $f$-mode frequencies and global properties for six separate sets, each with its own EoS, are computed concurrently. Numerical integration is used to solve the set of coupled first order differential equations (4–6 and 8–9). For this, the Runge–Kutta method of fourth order is employed, and the solution is iterated until the boundary conditions (in eqs. 7,11 and 12) are fulfilled. The numerical results from our computation are presented in Table I for the maximum and canonical mass of the NS. The detailed computational calculations will be described in the upcoming subsections. In this study, we have considered linear frequency $\nu$ instead of angular frequency $\omega$. It is worth noting that the eigenfrequencies in Sec. II B were angular frequencies ($\omega$), but we consider linear frequencies ($\nu$) where $\omega = 2\pi \nu$ in our results since it is more prevalent in asteroseismology.

A. M-R relation

Computations begin at the star’s centre with the initial parameter central density $\rho_c$ as input and continue solving the TOV equations (4) and (6) until $P < 0$, to obtain the radius $R$ and mass $M$ of a NS. We repeat the procedure by changing $\rho_c$ in increments of almost $1.8 \times 10^{13}$ g cm$^{-3}$ and performing the same calculations to obtain the required Mass-Radius ($M - R$) plot for a single EoS. Here, in Fig. 2, we have the $M - R$ plot for our six different EoSs. In this plot, we have also included observational maximum mass constraints from the pulsars PSR J0740+6620 ($M = 2.14^{+0.09}_{-0.08}$ M$_\odot$) by Cromartie et al. [50] in the red band; and PSR J0348+0432 ($M = 2.01 \pm 0.04$ M$_\odot$) by Antoniadis et al. [51] in the blue band. We have also considered the old Neutron Star Interior Composition Explorer (NICER) data represented in the green boxes for the canonical neutron star. From the analysis of PSR J0030+0451 by Miller et al. [53] and Riley et al. [54], we get the measurements of mass $M$ and radius $R$ as $M = 1.44^{+0.15}_{-0.14}$ M$_\odot$; $R = 13.02^{+1.24}_{-1.06}$ km and $M = 1.34^{+0.15}_{-0.16}$ M$_\odot$; $R = 12.71^{+1.14}_{-1.19}$ km respectively. The new NICER data and X-ray Multi-Mirror (XMM) Newton data by Miller et al. [55] also corresponds to the measurements of radius of a canonical neutron star $R = 12.45 \pm 0.65$ km and maximum mass ($M = 2.08$ M$_\odot$) neutron star $R = 12.35 \pm 0.75$ km. These are represented by black horizontal lines. Lastly, we have included the data from the gravitational wave detection of a black hole and a compact object merger GW190814 by Abbott et al. [52], which tells us the secondary mass of the compact object to be $M = 2.59^{+0.08}_{-0.09}$ M$_\odot$.

Computational data with mass-radius data of our EoSs, we see that the stiffer NL3 EoS does not satisfy the maximum mass criterion of the NICER and pulsar data. However, it is marginally comparable with the GW190814 data. Most of our EoSs are compatible with the old NICER data. Only G3 is the one that satisfies both the NS maximum mass and canonical radius criteria of the pulsars.
B. Boundary condition of $\eta$

To calculate the boundary condition of $\eta$, we implement the fourth order Runge-Kutta method with step length $h = 4m$. Thermodynamic factors like density and pressure change extremely quickly towards the surface. This variation has an impact on the g-modes \([56]\). However, because we are only concerned with the $f$-mode, this step length gives adequate precision for all of the properties we intend to compute throughout the star. We begin our calculations by setting the value of $\omega$ to zero and then perform the necessary computations to obtain the final value of the eigenfunction $\eta$. Then, we compare how far it is from the boundary condition (BC) in eq. \((12)\). We denote the difference as BC and investigate how it varies as $\omega$ is increased by 0.1 rad sec$^{-1}$.

From Fig. 3, we see that BC shows a damped harmonic motion with respect to frequency $\nu$. In the first scenario, we used a SLy4 EoS \([49]\), and varied the central density $\rho_c$ in increments of $1.8 \times 10^{14}$ g cm$^{-3}$ to obtain the plot (Fig. 3(a)). The first solution of BC gives us the $f$-mode frequency, and subsequent solutions give us the $p_1$, $p_2$ mode, and so on till it dies down where it becomes difficult to identify the higher modes of oscillation. With increasing $\rho_c$, we can see that the initial amplitude of BC decreases. Zooming in on the section where the curve initially intersects the $x$-axis to examine the behaviour of the $f$-mode frequency, we observe that as $\rho_c$ grows, the value of frequency drops. This inverse relation is valid for the density range in question and does not represent a general trend (as shown in Fig. 4 - sky blue curve). In Fig. 3(b), we also demonstrate how BC behaves when six different EoSs are used while keeping the central density constant. Here, we see that the stiffest NL3 EoS has the lowest $f$-mode frequency, whereas the softest EoS (at this density) - IOPB, has the next lowest one. There is no direct correlation, and hence, we can infer that an EoS’s stiffness is not a parameter on which the $f$-mode frequency depends.

C. $f$-mode frequency

In Fig. 4, $f$-mode frequency is plotted as a function of central density $\rho_c$. We can see that the frequency dies down after reaching its maximum value. This decline is a general trend that occurs regardless of the choice of EoS. As the particular stellar model approaches its dy- namic stability limit, indicated by the presence of a zero eigenfrequency, the value of $f$-mode frequency approaches zero \([57]\). Because of their small mass and non-relativistic approach, NS at high densities become nearly homogeneous \([21, 58, 59]\) and $\omega^2 = G\bar{\rho}(4 - 3\gamma)$, where $\bar{\rho}$ is the average density squared \([60]\). The figure also shows that softer EoS, such as SLy4 and G3, decay more slowly than stiffer ones. Furthermore, NS with softer EoS has a larger frequency value at a given $\rho_c$. Our chosen six EoSs attain their maximum frequency values within the central density range of 0.59 - 0.98 $\times 10^{15}$ g cm$^{-3}$, as represented by the grey band. Another intriguing fact is that three EoSs (DD2, IOPB and NL3), despite having varying stiffness, achieve their maximum frequency value (represented by solid circles) at the same central density, $\rho_c = 0.624 \times 10^{15}$ g cm$^{-3}$ (represented by a black dashed line).

When, we plot $f$-mode frequency vs average density, given by $\sqrt{M/R^3}$ in Fig. 5, we notice a similar trend where the frequency initially rises, reaches its maximum.
value and then decays due to the reasons mentioned above. Similar to Fig. 4, for NS with softer EoS like SLy4 and G3, $f$ falls slowly compared to those with stiffer ones. In this case, Our chosen six EoSs attain their maximum frequency values within the average density range of $0.0238 - 0.0296 \, M_{\odot}/c^{2}$, as shown by the grey band.

The pattern continues, when we plot $f$-mode frequency vs mass $M$ in Fig. 6 and $f$-mode frequency vs compactness $C$, which is defined by $GM/c^2R$, in Fig. 7. However, in these instances, the nature changes since NS with stiffer EoS, such as NL3, is pushed to the right, achieving its maximum frequency value later than those with softer EoS. This can be explained by the fact that stiffer EoS can sustain greater mass at the same central density and radius, as demonstrated by the $M - R$ plot in Fig. 2. The grey band in Fig. 6 is between the mass range $1.252 - 2.001 \, M_{\odot}$, and for Fig. 7, it is between $0.2159 - 0.2985$ compactness.

It was found that $f$-mode frequency scaled by central density $f\rho_c$, as a function of average density squared $M/R^3$, shows a linear trend. This is independent of the choice of EoS. When we use linear regression to fit the data, in the form of a scatter-plot, as shown in Fig. 8, with a solid grey thick line, the fitting formula comes up
V. CONCLUSION

In this study, we investigated radial oscillations of NS while considering six realistic EoSs based on the RMF and SHF models [32, 33]. We computed the mass and radius from the TOV equations for each EoS and discovered that they are compatible with astrophysical observational evidence from pulsars and GWs [50–55]. The radial oscillation equations were then solved considering infinitesimal adiabatic perturbation to calculate the $f$-mode frequency.

We are able to show the damped harmonic motion with respect to the boundary conditions of one of the eigenfunctions. The $f$-mode frequency was calculated as a function of many astrophysical quantities such as central density, average density, mass, and compactness. We discovered that at a certain point, the frequency drops when the aforesaid values are increased. At softer EoS, the decline in frequency with central and average density is slower; and for a fixed central density, NS with softer EoS has a larger $f$-mode frequency value. In comparison, stiffer EoS achieve their maximum $f$ value at higher mass and compactness, as demonstrated in Fig. 6 and 7 respectively. We found a linear correlation when we computed the $f$-mode frequency scaled by central density with the square of the average density.

The next generation of ground-based GW detectors like the Einstein Telescope and the Cosmic Explorer are expected to have higher precision than that at present at Advanced LIGO [8, 61]. Such detections could provide measurements of frequency, among other things, which will give an observational opportunity to test our theory.

ACKNOWLEDGMENTS

B. K. acknowledge partial support from the Department of Science and Technology, Government of India with grant no. CRG/2021/000101.

[1] W. Baade and F. Zwicky, Phys. Rev. 46, 76 (1934).
[2] A. Migdal, Sov. Phys. JETP 34, 1184 (1972).
[3] A. B. Migdal, Soviet Physics Uspekhi 14, 813 (1972).
[4] M. Mannarelli, Particles 2, 411 (2019).
[5] K. Rajagopal and F. Wilczek, in At The Frontier of Particle Physics: Handbook of QCD (In 3 Volumes) (World Scientific, 2001) pp. 2061–2151.
[6] M. G. Alford, A. Schmitt, K. Rajagopal, and T. Schäfer, Reviews of Modern Physics 80, 1455 (2008).
[7] R. Anglani, R. Casalbuoni, M. Ciminale, N. Ippolito, R. Gatto, M. Mannarelli, and M. Ruggieri, Reviews of Modern Physics 86, 509 (2014).
[8] C. Chirenti, R. Gold, and M. C. Miller, The Astrophysical Journal 837, 67 (2017).
[9] T. Hinderer, A. Taracchini, F. Foucart, A. Buonanno, J. Steinhoff, M. Duez, L. E. Kidder, H. P. Pfeiffer, M. A. Scheel, B. Szilagyi, K. Hotokezaka, K. Kyutoku, M. Shibata, and C. W. Carpenter, Phys. Rev. Lett. 116, 181101 (2016).
[10] L. M. Franco, B. Link, and R. I. Epstein, The Astrophysical Journal 543, 987 (2000).
[11] D. Tsang, J. S. Read, T. Hinderer, A. L. Piro, and R. Bondarescu, Phys. Rev. Lett. 108, 011102 (2012).
[12] A. Brillante and J. N. Mishustin, EPL (Europhysics Letters) 105, 39001 (2014).
[13] K. Kokkotas and J. Ruoff, Astronomy & Astrophysics 366, 565 (2001).
[14] G. Miniutti, J. Pons, E. Berti, L. Gualtieri, and V. Ferrari, Monthly Notices of the Royal Astronomical Society 388, 389 (2003).
[15] G. Panotopoulos and I. Lopes, Phys. Rev. D 96, 083013 (2017).
[16] A. Passamonti, M. Bruni, L. Gualtieri, and C. F. Sopuerta, Physical Review D 71, 024022 (2005).
[17] A. Passamonti, M. Bruni, L. Gualtieri, A. Nagar, and C. F. Sopuerta, Physical Review D 73, 084010 (2006).
[18] Savonije, G. J., A&A 469, 1057 (2007).
[19] C. V. Flores, Z. B. Hall, and P. Jaijum, Phys. Rev. C 96, 065803 (2017).
[20] C. V. Flores and G. Lugones, Physical Review D 82, 063006 (2010).
[21] H. M. Váth and G. Chamamgum, The Astrophysics and Astronomy Journal 260, 250 (1992).
[22] A. Passamonti, N. Stergioulas, and A. Nagar, Phys. Rev.
