The characteristics of junior high school students in pattern generalization

A M Firdaus1*, D Juniati2, and P Pradnyo2
1Universitas Muhammadiyah Makassar, Jalan Sultan Alauddin 259, Makassar 90222, Indonesia
2Doctoral Program Student of Mathematics Education, Universitas Negeri Surabaya, Indonesia
*andi.mulawakkan@unismuh.ac.id

Abstract. Much research on generalization of Algebra, but related to the generalization of the pattern is still lacking. In this study we characterizing middle school students generalization of pattern. The participants were 40 students grade 8 took the test with instruments that have been developed and analyzed students working. The findings indicate that students showed the two characterizing in generalization of patterns that: (1) Factual, (2) Symbolic. Possible reason are discussed and suggestions for teaching with generalization of patterns are presented.

1. Introduction

Generalizing patterns is often considered as a difficult topic for students to learn. The generalization of patterns is an important aspect of all levels in mathematics and it is included in every highlighted material in teaching mathematics with Dindyal [1]. Exploring patterns especially in middle school level is a developing pattern. The developing pattern is closely related to the sequence of mathematical objects such as numbers and images. Mulligan & Mitchelmore Patterns in mathematics are represented as a regularity that can be number, spatial, or logical relationships [2]. Tikekar research shows that when students are given a number pattern, each student has a different way of generalizing the pattern [3].

In terms of generalizing patterns, students can make a conclusion which depends on their understanding or concepts that students have as well as what ways they take to generalize patterns. This generalization process can be used to enrich the cognitive structure in the students’ mind since there is an activity to link all the concepts. Barbosa & Vale Generalization takes an important role in the mathematical activity, it is considered as an ability of mathematical thought in general [4]. When students are able to generalize a pattern, then the student is in a structural understanding and in accordance with the purpose of learning mathematics.

Study Barbosa, students also still have difficulty to determine patterns on a larger tribe [5]. For students, the formation of patterns in larger tribes appears more complicated than finding the results of the nearest tribe of the existing pattern. This means that students are able to find the closest tribe of the pattern by using a counting strategy (constructing/ making model to describe the situation and calculate the objects) or recursive, (describe the sequence using the same difference from the previous tribes), rather than search for general rules or generalizations a pattern for a larger tribe. Lannin for example, Students seems to have barriers to determining the number of stickers to be attached from a series of cubes stamped on each surface [6]. They just see the geometry object in front of their sights and does
not consider the context of the problem. So when doing the calculations in determining the next pattern, it seems that students do not pay attention to the context of the proposed problem.

Radford associated with the generalizing patterns, there are three levels of generalization, they are factual, contextual and symbolic [7]. Factual generalization is the basic level of generalization based on numerical understanding but allows students to overcome certain cases, for example, the increasing sequence, $1 + 2, 2 + 3$, the typical students can use this pattern on determining the next pattern. Factual generalization is often accompanied by the use of adverbs such as "next" or "always", including the rhythmic effects of speech and movement (e.g., pointing). Rivera on the other hand, symbolic generalization is standard algebraic generalization symbols [8]. Symbolic generalization is a type of generalization associated with algebraic objects or symbols which are not limited to a particular object. Radford in generalizing pattern, it not enough just to declare the general rules and order patterns [7]. The students are required to justify the general rule of the pattern with the symbol.

This makes a reference for researchers to conduct research at 8th grade of middle school in Makassar. To find out about how student awareness develops, how students use their innate ability to look at the patterns and then make generalizations, and what skills students should have to generalize properly. The purpose of this research is to characterize students’ generalization of the patterns in 8th grader of Middle school in Makassar.

2. Experimental Method
This research uses an explorative research with qualitative approach since the main data of this research is the characteristics of generalization of pattern conducted by middle school student and also researcher analyzes deeply in determining process generalization pattern [9]. The study was conducted by involving 40 students of 8th grade who are given the instruments that have been developed. After that, the researcher analyzes students’ work. Selection of the subject begins by setting the class of research, provide a generalization test pattern. Problems used in the generalization test of this pattern are selected from the problems that have been developed [8].

In this research there are two characteristics of generalization the pattern, they are factual and symbolic. To facilitate the process of data transcription, factual subjects are given AF code and symbolic subjects are coded AS. The method of collecting data in this study is task method and interview method. While the data analysis techniques include analysis of generalization pattern assignment results and interviews. Results of student work and interviews are presented, validated, and analyzed and summarized.

3. Result and Discussion
The task of generalizing the pattern is presented in the form of drawings which include: 1) the task of generalizing the pattern on the pattern of the image, 2), and the generalization task of the pattern in the form of a contextual problem. The task of generalizing patterns designed such that students are asked to determine the formula for the number of circles in a rectangle to-n, and count the number of existing circle in the fifth rectangle and 100th. Here is the generalization task of the pattern on the number pattern [8].

![Figure 1. Embedded rectangular pattern generalization task](image-url)
1) How many circles are there in the fifth rectangle? How do you know for sure?
2) How many circles are there in the 100th rectangle? How do you know for sure?
3) Find a direct formula for the number of circles in the nth rectangle. Explain how you obtained.

The subject of AF observes the problem by observing and calculating the inner circle of the box. Based on the number of circles in the first, second, third, and forth boxes, the subject of this category organizes the case by writing the number of circles in the form of a sequence or listing to count the many circles in the box, making it easier for the subject to work on the case. The most common way of organizing a particular case is by; existing data or tables [10]. Another way, AF organizes the case by giving a circle sign to signify the similarity of the pattern to make it easier for the subject to solve a given problem.

AF observes and calculates the circles separately, he also organizes cases to facilitate himself to finish this case. The way to solve the problem is based on the influence of a given problem. Transformation of objects that the individual does as an external need either explicitly or from memory, there are some steps as a guide in performing the operation [11].

AF try hard to find and predict the pattern by calculating the difference from each pattern and thinking about the number of circles in the next box. AF notices the inner circle of the box, the addition of the first box to the second box is 2, the addition of the second box to the third box is 2, and the third box to the fourth box is 2. Based on that addition, AF states that the pattern is increased by 2, and predicts the circle on the 5th box is 10.

AF calculated the difference between the second box and the first box is 4-2 = 2, the third box and the second box are 6-4 = 2, the fourth box and the third box also the difference is 2. Then AF validates by considering the difference between each circle. For circle 2, 4, 6, 8 the difference is 2, where the second box is 4 circle with the first box is 2 circles. Based on these findings, AF found a new formula that is 4n-2n. Then AF looks for the number of circles in the 5th and 100th boxes by using the formula that has been found, for example, 5th box = 4(5)-2(5) = 10 and the 100th box = 4(100)-2(100) = 200.

AF is paying attention the formula to determine the number of circles is 4n-2n. Next, AF validates with some particular examples and state that the obtained formula is equal to the number of known circles. The generalization by AF is called a factual generalization. The factual t generalization is a basic level of generalization based on numerical perform, but allows students to overcome certain cases, for example increasing order is 1 plus 2, 2 plus 3 and it can be used in this pattern to determine the next pattern [7].

AS analyzes the process by looking at the difference or addition of the first box with the second box. The addition of the first box to the second box is 2, the second box to the third box is 2, and the third box to the fourth box is also 2. By looking at the addition, the US subject formulates the pattern in general to determine the number of circles in – nth box = 2n. Then AS looks for the number of circles in the 5th and 100th boxes by using the formula that has been found, for example, 5th box = 2 (5) = 10 and the 100th box = 2 (100) = 200.

At the validation the – nth formula = 2n. AS uses a particular example of what is known in the problem that is the 3rd box, n = 2 (3) = 6, the base of validation related to particular example (for instance, validation based on second, third, and forth) in the problem, namely internal validation, whereas validation is based on particular example which is unknown in the problem (e.g. validation with the 5th, 6th, ..., images to -n) is called external validation.

The process by AS to produce the correct formula based on a particular example is a perfect result analysis. The subject then validates with a particular instance and then says true. Validation aims are to know the truth of the formula generated based on a particular example, that validates the formula is the determination of the truth of a particular case but not in general. The process of formulating and validating the formula occurs repeatedly until it produces the correct formula [12].

AS believes that the general formula for determining the number of the rectangle in the – nth box is 2n. The subject believes the generated general formula is correct after validation. By believing in the general formula, the subject has generalized the resulting formula. The generalization by AS is a
symbolic generalization. A symbolic generalization is a type of generalization associated with an algebraic object or symbol that is not limited to a particular object [13].

4. Conclusion
From these results, it can be seen that there are two characteristics of middle school students in generalizing patterns found by researchers that are factual and symbolic. The factual subject counts the difference between the second box and the first box is 4-2 = 2, the third box and the second box are 6-4 = 2, and for the fourth box and the third box, absolutely the difference is also 2. Then the factual subject validates by considering the difference of each circle. For circle 2, 4, 6, 8 the difference is 2, where the second box is 4 circles with the first box is 2 circles. Based on these findings the factual subject finds a new formula that is 4n-2n. Symbolic subject performs a process analysis to see the difference or addition of the first box with the second box. The addition of the first box to the second box is 2, the addition of the second box to the third box is 2, and the third box to the fourth box is also 2. By looking at the addition, the symbolic subject formulates the pattern in general to determine the number of circles in—nth box = 2n.

Acknowledgments
This work was supported by Lembaga Pengelola Dana Pendidikan (LPDP) Indonesian under grant no. 20161141080709.

References
[1] Dindyal J 2007 High school students’ use of patterns and generalisations Proc. 30th Annu. Conf. Math. Educ. Res. Gr. Australas. 1 236–45
[2] Mulligan J, Mulligan J and Mitchelmore M 2014 Awareness of pattern and structure in early mathematical development. Mathematics Education Research ... Awareness of Pattern and Structure in Early Mathematical Development Math. Educ. Res. J. 21 33–49
[3] Tikekar V G 2009 Deceptive patterns in mathematics [2 Int. J. Math. Sci. Educ. 2 13–21
[4] Barbosa A and Vale I 2015 Visualization in pattern generalization: Potential and Challenges J. Eur. Teach. Educ. Netw. 10 57–70
[5] Barbosa A N A, Vale I and Palhares P 2012 Pattern Tasks: Thinking Processes USED Vie rs ió Cl a Rev. Latinoam. Investig. en matemática Educ. 15 273–93
[6] Lannin J K, Barker D D and Townsend B E 2006 Recursive and explicit rules: How can we build student algebraic understanding? J. Math. Behav. 25 299–317
[7] Radford L, Socas M, Zazkis R and Liljedahl P 2011 Generalization of Patterns: The tension between algebraic thinking and algebraic notation Proc. 35th Conf. Int. Gr. Psychol. Math. Educ. 1 379–402
[8] Rivera F 2013 Teaching and Learning Patterns in School Mathematics
[9] Moleong L J 2012 Metode Penelitian Kualitatif (Edisi Revisi) (PT. Remaja Rosdakarya)
[10] Allen L G 2014 Mathematical Teaching An Alternative Approach Induction: Math. Teach. 94 500–4
[11] Dubinsky E 2001 Using a Theory of Learning in College TalUM 12 10–5
[12] Canadas M C and Castro E 2007 A proposal of categorisation for analysing inductive reasonong Pna 1 67–78
[13] Radford L 2003 Gestures, Speech, and the Sprouting of Signs: A Semiotic-Cultural Approach to Students’ Types of Generalization Math. Think. Learn. 5 1–36