Ultrafast all-optical femtosecond soliton steering in \(\mathcal{PT}\)-symmetric fiber couplers

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We report a detailed study for the first time, to the best of our knowledge, on soliton steering dynamics in a \(\mathcal{PT}\)-symmetric directional coupler in the femtosecond domain. In this regime, the practical application of any real fiber coupler requires incorporation of higher-order perturbative effects such as higher-order dispersions including third-order dispersion and fourth-order dispersion, self-steepening, and intrapulse Raman scattering. With a high gain/loss, the combined effect of perturbations is found to stabilize the soliton pulse evolution in the coupler from the chaotic behavior of unperturbed evolution. This work demonstrates that it is possible to achieve efficient soliton steering, even in the femtosecond regime, at very low critical power at a relatively higher gain/loss coefficient in the \(\mathcal{PT}\)-symmetric directional coupler.

I. INTRODUCTION

In the domains of high-speed signal processing and ultrafast communication, all-optical switching devices are considered to be key elements that have drawn significant attention over the past few decades [1]. In this context, a dual-core nonlinear directional coupler featuring power-dependent switching has been studied extensively [2-4]. In recent times, the first experimental observation of \(\mathcal{PT}\)-symmetric effect [5] in linear waveguide directional coupler with balanced gain/loss has provided an opportunity for further research [6-8]. In this connection, several studies have shown that such couplers are beneficial for all-optical switching when operating in the nonlinear regime [9,11]. Furthermore, the stability of optical solitons in \(\mathcal{PT}\)-symmetric fiber couplers has been investigated semi-analytically and numerically, wherein exact stable states have been found [12,13].

Recently, it has been reported that the requirement of high critical power for optical solitons can be reduced sharply by introducing \(\mathcal{PT}\)-symmetry in a nonlinear directional coupler [14]. It is worthwhile to note that in Ref. [14], the \(\mathcal{PT}\)-symmetric system has been described by the well-known coupled nonlinear Schrodinger equation (NLSE). However, in the domain of ultra-short bright solitons ranging from few femtoseconds to 1 picosecond (ps), the practical application of any real fiber coupler requires incorporation of higher-order perturbative effects such as higher-order dispersions (HODs) including third-order dispersion (TOD) and fourth-order dispersion (FOD), self-steepening (SS), and intrapulse Raman scattering (IRS) [15,16] in the coupled NLSE. Also, it is of particular interest to explore the idea of \(\mathcal{PT}\)-symmetric fiber coupler for the femtosecond (fs) pulse switching and observe the switching dynamics under the effect of balanced gain and loss.

In this paper, we investigate the soliton steering dynamics in a \(\mathcal{PT}\)-symmetric directional coupler in the fs domain in great detail. We first explore the effects of individual higher-order perturbations (IRS, SS, HODs) on switching dynamics. Following that, we investigate the cumulative combined effects of all perturbations, which result in a significant improvement in the stability and switching of fs soliton against individual perturbations as well as unperturbed cases.

In an earlier work [17], it has been reported that in the context of conventional couplers IRS restabilizes the symmetric solitons at sufficiently large energies. However, in the case of a \(\mathcal{PT}\)-symmetric coupler, we show that a partly radiating solitons caused by high gain/loss values (although the pulse is low energy fundamental soliton) are stabilised in the presence of higher-order perturbations. This finding is new in the context of ultrafast soliton steering in a \(\mathcal{PT}\)-symmetric coupler, paving the way for stable and efficient fs all-optical switching at low energy than the conventional one.

II. MODEL: \(\mathcal{PT}\)-SYMMETRIC FIBER COUPLER

To discuss the fs pulse switching in a realistic \(\mathcal{PT}\)-symmetric fiber coupler, it is important to consider the generalized NLSE with an integral form of the nonlinearities [16]. The coupled-mode equations of the slowly-varying envelopes \(A_1, A_2(z,t)\) in the two channels of the fiber couplers can be written in normalized units of the form

\[\begin{align*}
\frac{dA_1}{dz} &= \mathcal{L} A_1 + \mathcal{M} A_1 A_2 + \mathcal{N} A_2^2 + \mathcal{O} A_1^3 + \mathcal{P} A_2^3, \\
\frac{dA_2}{dz} &= \mathcal{L} A_2 + \mathcal{M} A_2 A_1 + \mathcal{N} A_1^2 + \mathcal{O} A_2^3 + \mathcal{P} A_1^3,
\end{align*}\]

where \(\mathcal{L}, \mathcal{M}, \mathcal{N}, \mathcal{O}, \mathcal{P}\) are the linear and nonlinear coefficients of the NLSE. The coupling terms \(\mathcal{M}\) are included by writing the NLSE in the form of a coupled NLSE.

\[\begin{align*}
\frac{dA_1}{dz} &= \mathcal{L} A_1 + \mathcal{M} A_1 A_2 + \mathcal{N} A_2^2 + \mathcal{O} A_1^3 + \mathcal{P} A_2^3, \\
\frac{dA_2}{dz} &= \mathcal{L} A_2 + \mathcal{M} A_2 A_1 + \mathcal{N} A_1^2 + \mathcal{O} A_2^3 + \mathcal{P} A_1^3,
\end{align*}\]

FIG. 1. (Color online) Schematic diagrams of fiber couplers with (a) type 1 and (b) type 2 \(\mathcal{PT}\)-symmetric configurations.
where $u_{1,2}(z,\tau) = A_{1,2}/\sqrt{P_0}$, with $P_0$ being the peak power of the input pulse. The propagation distance ($z$) and time ($\tau$) variables are, respectively, normalized with new parameters as $\xi = z/L_D$ and $\tau = (t - zv_g^{-1})/t_0$, with $L_D = L_2/|\beta_2(\omega_0)|$, $t_0$ is the input pulse width, $v_g$ is the group-velocity of the pulse, and $\beta_2(\omega_0)$ being the group-velocity dispersion (GVD) parameter at the carrier frequency $\omega_0$. The inter-core linear coupling ($K$) and balanced gain/loss ($G$) parameters are scaled as $\kappa = K L_D$ and $\Gamma = G L_D$. Also, the terms $\delta_n = [\beta_n/(n!|\beta_2|t_0^{n-2})]$, $s = 1/(\omega t_0)$, and $R(\tau) = (1 - f_R)\delta(\tau) + f_R h_R(\tau)$ respectively denote the normalized parameters of HOD, SS or shock, and the nonlinear response function in the case of IRS, and $f_R h_R(\tau)$ being the fractional contribution of the delayed Raman response to nonlinear polarization, $\tau_1 = 12.2 \text{ fs}/t_0$, $\tau_2 = 32 \text{ fs}/t_0$, $\tau_b \approx 96 \text{ fs}/t_0$, and the relative contribution of the boson peak is included through $f_b = 0.21$.

The system described by Eq. (1) is $\mathcal{PT}$-symmetric due to the presence of equal gain and loss ($\Gamma$) in the two waveguides. According to the non-Hermitian photonics, there exist three possible states for a $\mathcal{PT}$-symmetric system: unbroken ($\kappa > \Gamma$), broken ($\kappa < \Gamma$), and an exceptional point ($\kappa = \Gamma$) that indicates the phase-transition regime of $\mathcal{PT}$-symmetry [6]. In the case of fiber couplers (shown in Fig. 1), two types of configurations are specified based on the sign of $\Gamma$, namely type-1 (for $\Gamma > 0$) and type-2 (for $\Gamma < 0$) $\mathcal{PT}$-symmetric couplers as done in [14]. Earlier study in the context of $\mathcal{PT}$-symmetric soliton switching has confirmed that a $2\pi$ coupler with type-1 $\mathcal{PT}$ configuration exhibits richer steering dynamics, achieving low critical steering power ($P_{cr}$) while maintaining excellent transmission efficiency [14]. In view of this, we confine our analysis to the ‘type-1’ $2\pi$ $\mathcal{PT}$-coupler, for which we refer to as the ‘$\mathcal{PT}$-symmetric coupler’ throughout the text. Similarly, we restrict our system to work in the unbroken regime alone by fixing a condition of $\kappa > \Gamma$ as $\kappa = 1$ and $\Gamma = 0.5$. This is due to the fact that power controlled steering is limited within the unbroken $\mathcal{PT}$-symmetric regime as the soliton pulse exhibits severe instability in the broke $\mathcal{PT}$-symmetric regime [14]. Note that Eq. (1) contains all the higher-order terms that act as perturbations in the context of ultrashort (fs) pulse dynamics [16]. When the pulse duration is large enough (ps or larger), IRS and SS can be ignored. Also, if the input pulse is launched far away from the zero-dispersion frequency, HODs can be neglected. In such a case, Eq. (1) resembles the unperturbed coupled-mode equation for $\mathcal{PT}$-symmetric systems [13]. To investigate the soliton switching dynamics, the unperturbed equation is first numerically solved by launching $u_1(0,\tau) = \sqrt{P_0} \text{sech}(\tau)$ and $u_2(0,\tau) = 0$, where $P_0$ is the input peak power. The numerical simulations are carried out using the split-step Fourier method for coupled-mode equations [14, 16] incorporating with fourth-order Runge-Kutta algorithm. Next, we numerically solve the Eq. (1) taking into account all of the perturbations present in the system. The numerical investigations show that HODs and higher-order nonlinear phenomena have a significant impact on $\mathcal{PT}$-symmetric couplers. The study further reveals that for an ultrashort pulse propagation, the critical switching power $P_{cr}$ depends on $\delta_n > 2$ (HOD parameters), IRS, and $s$ (SS or shock effect), i.e., $P_{cr} \sim f(\delta_n, IRS, s)$. The strength of these perturbations can be tuned by varying the input pulse duration $t_0$. We extensively study the pulse dynamics for a wide range of $t_0$ (from 5 fs to 1 ps). In order to get into the detail switching dynamics, we first systematically analyze the individual perturbation effects. Then, the combined effects of all the higher-order perturbations are discussed in some details.

### III. IMPACT OF HIGHER-ORDER NONLINEAR EFFECTS

Higher-order perturbations on ultrashort pulses have been studied in various optical waveguides, and the effects on pulse dynamics are significant. Ultrashort optical soliton in fibers with intense peak power can excite higher-order nonlinear effects like SS and IRS. Under the influence of IRS, the central frequency of the soliton experiences a redshift [20]. IRS also influences the temporal dynamics of the soliton by imposing a temporal deceleration. In a single soliton case, the effect of IRS leads to Raman amplification and lasing. SS, on the other hand, creates an optical shock on the leading edge of the pulse, resulting in asymmetric spectral broadening [16]. In order to investigate the effect of IRS and SS on the soliton steering dynamics in a $2\pi$ $\mathcal{PT}$-symmetric fiber coupler, we first numerically solve Eq. (1) for a 10 fs input \text{sech} pulse in the presence of IRS only. The temporal evolutions are plotted in Figs. 2(a) and 2(b) for the two channels. Here, the power of a fs soliton launched into the first channel steers back and forth between the two channels and eventually exit from the second. The IRS-induced characteristic temporal decelerations are also evident, which differ from the unperturbed case represented by vertical dashed lines at the middle. From our study on the spatial evolution of power ($P_1$ and $P_2$) and transmission ($T_1$ and $T_2$) in the coupler (not shown here), we find that the IRS induces the soliton to acquire an additional relative phase difference. Unlike the conventional case where $P_1$ and $P_2$ are out of phase by $\pi$ [14], with the introduction of $\mathcal{PT}$-symmetry, they are $\pi/2$ phase apart.
However, $T_1$ and $T_2$ are completely out of phase by $\pi$. For a short optical pulse, in the absence of any perturbation, inclusion of $\mathcal{PT}$-symmetry revealed a critical switching power to be $P_{cr} = 1.37$ with a sharp transmission efficiency [dotted curve in Fig. 2(c)]. However, for a pulse with $t_0 = 10$ fs, the IRS perturbation present in the system increases the critical switching power to $P_{cr} = 1.75$ while maintaining the transmission efficiency at nearly 99%. As a result, above the critical switching power, almost all of the output energy will be shared by the first channel, with a negligible amount by the second channel [see Fig. 2(d)]. The transmission characteristics for the 2π conventional case with IRS perturbation are also depicted in Fig. 2(c) by light-solid lines for comparison, which shows significantly less efficient multiple steering switching compared to the $\mathcal{PT}$ counterpart. Additionally, we explore the role of the input pulse width in order to demonstrate the relationship between critical switching power and the gain/loss parameter in a 2π coupler. With the increase in the pulse width, the value of the critical switching power decreases significantly [shown in Fig. 2(e)]. For a lower value of $\Gamma$ ($\Gamma < 0.45$), two threshold power indicates dual steering inside the $\mathcal{PT}$-symmetric coupler. For further verification of how accurate this integral form of the IRS model of Eq. 1 with respect to the derivative form of the IRS [15], we plot the critical switching power as a function of gain/loss parameter in Fig. 2(f). For $\Gamma < 0.45$, the integral model works better for an ultrashort pulse as compared to the derivative model. However, both of these models appear to be equivalent once $\Gamma > 0.45$. The soliton switching dynamics is further modified by the influence of SS. For a 10fs pulse, we find that the critical switching power increases at lower values of $\Gamma$ as illustrated by the gray dot-dashed curve in Fig. 2(f). However, at higher values of $\Gamma$ the values of $P_{cr}$ are same for both IRS and SS.

**IV. IMPACT OF HIGHER-ORDER DISPERSIONS**

In soliton propagation, HODs are of particular importance, and they could be considered as perturbations. For a given waveguide geometry, dispersion profile may vary rapidly with frequency, making HOD more pronounced. While the interplay between Kerr-nonlinearity and GVD produces stable solitonic structure in time and frequency domains, HODs lead to significant temporal and spectral distortion. More specifically, the soliton sheds energy in the form of dispersive waves (DWs) that produce isolated spectral peaks [16, 21, 22]. In this process, the temporal distribution of the soliton is affected by the generation of asymmetric (for TOD) and symmetric (for FOD) side-lobes. Here, we consider a $2\pi$ $\mathcal{PT}$-symmetric single-mode fiber coupler whose cross-sectional geometry and the GVD profile are shown in Fig. 3(a). For this GVD profile, the values of the second-order GVD, TOD and FOD at the launching wavelength $\lambda_0 = 1.55 \mu m$ are calculated as, $\beta_2 \approx -23.391 ps^2/km$, $\beta_3 \approx 0.13472 ps^3/km$, and $\beta_4 \approx -4.1128 \times 10^4 ps^4/km$, respectively. The back and forth temporal evolution of a fundamental soliton between two channels are illustrated in Figs. 3(b) and 3(c) in the presence of TOD by solving Eq. 4 for a 10fs input sech pulse. In both the figures, the presence of side-wings indicates the presence of DWs. Here the HOD is dominated by the TOD term, which results in asymmetric one-sided lobes. Like IRS perturbation, the critical switching power is also modified in the presence of HOD terms. Figure 3(d) shows that the critical switching power ($P_{cr}$) increases from that of the unperturbed case. It is also observed that FOD degrades the overall switching efficiency of the $\mathcal{PT}$-symmetric coupler. Next, we plot the critical switching power as a function of gain/loss coefficient in Fig. 3(e), where the $\beta_3$ produces the larger switching power than that of the $\beta_4$. Also, one can observe from Figs. 3(e) and 3(f) that the critical switching power is significantly increased for lower gain/loss values in the presence of HOD as compared to the higher-order nonlinearity. For further verification of how the strength of $\delta_3$ (this can be obtained for...
different waveguide geometry with different GVD profile or for a fixed GVD structure with varying $t_0$ affects the $P_{cr}$, we plot $P_{cr}$ as a function of $\delta_3$ in Fig. 3(f) for $\Gamma = 0.5$ and $\kappa = 1$. These figures suggest that for a fixed $\Gamma$ and $\kappa$, the TOD and FOD parameters significantly affect the critical switching power and the transmission efficiency.

V. COMBINED EFFECT OF HIGHER-ORDER PERTURBATIONS

So far, we have discussed in detail the individual perturbative effects of both higher-order nonlinearities and HODs on the switching dynamics of a fs pulse. Here, we analyze the combined effects of the above-stated higher-order perturbations on the switching dynamics. For this purpose, we plot the variation of critical switching power as a function of gain/loss parameter as shown in Fig. 3(g). It exhibits a similar trend as observed in the cases of the individual perturbations [Figs. 2(e) and 3(c)]. However, the combined effects of all the perturbations (which is the practical case for fs soliton in fibers) enhances the critical switching power for lower gain/loss value more than that of the individual perturbation. However, for higher gain/loss values, $\Gamma > 0.7$, the difference between both critical switching powers is minimum. Next, we plot the transmission energy over a range of input pump power at $\Gamma = 0.71$ in Fig. 3(h). We observe that although the critical power remains the same ($P_{cr} \approx 0.38$), the transmission efficiency improves considerably ($\sim 99\%$) from that of the individual perturbations. So, in the context of fs soliton steering in a $PT$-symmetric fiber coupler, we can achieve very low critical power with almost full energy transfer at a relatively higher gain/loss coefficient.

To validate the fact, we plot the temporal evolution of a fundamental soliton between two channels, as shown

![FIG. 3. (Color online) (a) GVD profile of a single-mode fiber. Here, the vertical dotted line indicates the launching wavelength, and the red-circle represents the location of the zero-GVD wavelength. The cross-sectional geometry of the fiber is also shown in the inset [23]. (b) and (c) soliton evolution in the two channels of $2\pi$ type-1 $PT$-symmetric coupler under the effect of TOD for $t_0 = 5$ fs. (d) Switching dynamics and (e) critical switching power $P_{cr}$ as a function of gain/loss parameter $\Gamma$ in the presence of TOD only, FOD only, and combined effects for $t_0 = 5$ fs. Here the normalized gain/loss parameter is taken as $\Gamma = 0.5$. (f) $P_{cr}$ vs TOD parameter $\delta_3$ for $\Gamma = 0.5$. (g) $P_{cr}$ as a function of $\Gamma$ under the combined influence of higher-order nonlinearity and HOD effects. (h) Switching dynamics inside the two channels of a type-1 $2\pi$ $PT$-symmetric fiber coupler. Here, dashed lines represent unperturbed case, and solid lines represent the effect of all perturbations.

![FIG. 4. (Color online) Soliton evolution in the two channels of $2\pi$ type-1 $PT$-symmetric coupler without perturbations (a), (b), and with all perturbations (c). (d) for $\kappa = 1$, $\Gamma = 0.65$, and $t_0 = 10$ fs. (e), (f) Represent soliton evolution in a convensional coupler with all perturbations present for $\kappa = 1$ and $t_0 = 10$ fs.](image-url)
in Fig. 4(a)-(b) (without perturbations) and Fig. 4(c)-(d) (combination of all perturbations). Also, we plot the same evolution for a conventional coupler considering all higher-order perturbations in Fig. 4(e)-(f) for comparison. These plots demonstrate that, with a high gain/loss, the combined effects of perturbations stabilize the pulse evolution from the chaotic behavior of unperturbed evolutions. This could be attributed to the fact that IRS and HOD have opposite phenomena on spectral power (IRS tries to redshift the spectrum and HOD, in this case, tries to blueshift the spectrum in the form of DWs) that try to self-organize the power flow to have more stable pulse evolution.

VI. CONCLUSIONS

In summary, we have demonstrated theoretically that while in conventional fiber coupler fs soliton steering is hard to realize as it takes higher critical pump power to switch the pulse due to various perturbative effects, in a PT-symmetric fiber coupler, these perturbations, in particular, IRS rather assists in efficient soliton steering, in the presence of high gain/loss. This work may open up plethora of applications and studies related to soliton steering and switching using PT-symmetric fiber coupler in high loss/gain regime.

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