Radiative Left-Right Dirac Neutrino Mass

Ernest Ma\textsuperscript{1} and Utpal Sarkar\textsuperscript{2}

\textsuperscript{1} Physics & Astronomy Department and Graduate Division, University of California, Riverside, California 92521, USA

\textsuperscript{2} Physics Department, Indian Institute of Technology, Kharagpur 721302, India

Abstract

We consider the conventional left-right gauge extension of the standard model of quarks and leptons without a scalar bidoublet. We study systematically how one-loop radiative Dirac neutrino masses may be obtained. In addition to two well-known cases from almost 30 years ago, we find two new scenarios with verifiable predictions.
Introduction:

To explain why neutrino masses are so small, one approach is to consider the case where they are forbidden at tree level and only arise radiatively. In the standard model (SM) without a singlet right-handed neutrino, the left-handed neutrino may only acquire a mass through a dimension-five operator \( \mathcal{L}_M \), i.e.

\[
\mathcal{L}_M = -\frac{\kappa_{ij}}{\Lambda} (\nu_i \phi^0 - l_i \phi^+) (\nu_j \phi^0 - l_j \phi^+) + H.c.
\] (1)

This means that neutrino masses are Majorana and suppressed by the large scale \( \Lambda \). In 1998, it was shown [2] how this operator may be realized in three and only three ways at tree level (establishing thus the nomenclature of Types I, II, and III seesaw), as well as in one loop, assuming only fermions and scalars in the loop. Whereas Majorana neutrino masses are theoretically desirable in this context, there is at present still no supporting experimental evidence from neutrinoless double beta decay. Perhaps neutrinos are Dirac particles after all, and lepton number is actually conserved, in which case the question remains as to why they are so small. A general study in the context of the SM has recently appeared [3].

In this paper we focus on the conventional left-right gauge model, which contains the right-handed neutrino in an \( SU(2)_R \) doublet and is thus a natural framework for considering Dirac neutrinos. To connect the \( SU(2)_L \) fermion doublet with the \( SU(2)_R \) fermion doublet, a scalar bidoublet is required. Suppose a bidoublet is absent [4], then there are no fermion masses in the minimal model. However, they may be generated by dimension-five operators, in analogy with Eq. (1). Specifically, Dirac neutrino masses come from the operator

\[
\mathcal{L}_D = -\frac{\kappa_{ij}}{\Lambda} (\bar{\nu}_i L \phi^0_L - \bar{l}_i L \phi^0_L) (\nu_j R \phi^0_R - l_j R \phi^0_R) + H.c.
\] (2)

These operators may be realized at tree level using heavy singlet quarks and leptons, i.e. the mechanism of Dirac seesaw. Suppose only quark and charged-lepton masses are obtained in this fashion. Neutrinos would then appear to be massless. However Eq. (2) may still be realized in one loop and neutrinos acquire small Dirac masses, as detailed below.
Four Generic Structures:

There are four and only four generic structures which realize Eq. (2), in exact analogy to how Eq. (1) is realized in Ref. [2]. The idea is very simple. To connect $\nu_L$ with $\nu_R$ in one loop, we need an internal fermion line and an internal scalar line. There are thus only four ways to do this.

- (A) Attach both $\phi_L$ and $\phi_R$ to the fermion line.
- (B) Attach both $\phi_L$ and $\phi_R$ to the scalar line.
- (C) Attach $\phi_L$ to the fermion line and $\phi_R$ to the scalar line.
- (D) Attach $\phi_L$ to the scalar line and $\phi_R$ to the fermion line.

Model (A):

A possible implementation of this idea is to add a charged scalar singlet $\chi^-$, as shown in Fig. 1. This was done many years ago [5] and it implies that charged leptons have seesaw masses from three heavy singlet leptons $E$. It is the left-right analog of the Zee model [6], but without the need for a second scalar doublet. Note that the $d$ quark may be used instead of $e$, then $\chi^-$ should be replaced with a colored scalar singlet with charge $-1/3$.

![Figure 1: Dirac neutrino mass in Model (A).](image-url)
Model (B):

To make the connection in this case, the heavy singlet lepton is again used, as shown in Fig. 2. This was also done many years ago [7]. A second scalar $SU(2)_L$ doublet $(\eta^+_L, \eta^0_L)$ as well as a second scalar $SU(2)_R$ doublet $(\eta^+_R, \eta^0_R)$ are needed, because the invariant quartic scalar coupling is required to be of the form

$$\left(\phi^+_L \eta^0_L - \phi^0_L \eta^+_L\right)^* \left(\phi^+_R \eta^0_R - \phi^0_R \eta^+_R\right).$$

Figure 2: Dirac neutrino mass in Model (B).

It is thus also a left-right analog of the Zee model, but without the charged scalar singlet. Without loss of generality, we have chosen in Fig. 2 $\langle \eta^0_{L,R} \rangle = 0$, so that $\eta^\pm_{L,R}$ are the physical charged scalars, whereas $\phi^\pm_{L,R}$ have become the longitudinal components of $W^\pm_{L,R}$.

If the heavy $E$ lepton is replaced by the heavy $D$ quark, then $\eta_L$ and $\eta_R$ are replaced by the corresponding scalar leptoquark doublets. In this case, charged leptons also obtain radiative masses from these same leptoquark doublets through the heavy $U$ quarks.

If we impose a discrete $Z_2$ symmetry such that $\eta_{L,R}$ and $E$ are odd, then this is a left-right analog of the well-known scotogenic model [8] of radiative seesaw neutrino mass through dark matter, as discussed recently [9].

Model (C):

This is a new proposal and requires the existence of an exotic $SU(2)_R$ scalar doublet
Again, if $d$ and $D$ are used instead of $e$ and $E$, $\chi$ and $\zeta$ are replaced with $\nu$ and $\nu_R$. Figure 3: Dirac neutrino mass in Model (C).

the corresponding singlet and doublet scalar leptoquarks respectively.

Model (D):
This is the companion to (C) and requires the existence of an exotic $SU(2)_L$ scalar doublet $(\zeta_L^+, \zeta_L^+)$. Note that if we have both $\zeta_L$ and $\zeta_R$, then Model (B) may be realized in Fig. 2.

by reversing the arrows of the internal lines and replacing $\eta_{L,R}$ with $\zeta_{L,R}$. Figure 4: Dirac neutrino mass in Model (D).
Discussion:

To discover which of the above mechanisms is truly responsible for the radiative generation of Dirac neutrino masses, the corresponding new particles in the loop would have to be observed experimentally. Of particular interest are the doubly charged scalars $\zeta_{L,R}^{++}$. It is of course well-known that a scalar triplet $(\xi^{++}, \xi^+, \xi^0)$ may couple to the doublet neutrinos directly and provide them with Majorana masses with a small $\langle \xi^0 \rangle$. In that case \cite{10}, the decays of $\xi^{++}$ would map out \cite{11} the elements of the $3 \times 3$ neutrino mass matrix. These decays have been searched for at the Large Hadron Collider (LHC). Assuming 100% (50%) branching fraction to $e^-_Le^-_L$, the ATLAS Collaboration has the preliminary \cite{12} lower bound of 570 (530) GeV on its mass, based on an integrated luminosity of 13.9 fb$^{-1}$ at 13 TeV. However, if $\xi$ is indeed responsible for the neutrino mass matrix, its branching fraction to $e^-_Le^-_L$ is proportional to the $ee$ entry of the $3 \times 3$ Majorana neutrino mass matrix and if that is zero, there will be no bound from the LHC in this mode. Note also that if the decay is to $e^-_Re^-_R$ instead, the ATLAS bound becomes 420 (380) GeV. Recently, it was shown \cite{13} that the pair production of doubly charged scalars may be enhanced by photon-photon fusion, resulting in an improvement of the above limits.

In the models we are concerned with, the doubly charged scalar $\zeta^{++}_L$ is part of an $SU(2)_L$ doublet, whereas $\zeta^{++}_R$ is part of an $SU(2)_R$ doublet, so it is an $SU(2)_L$ singlet. This is important because any $SU(2)_L$ doublet or triplet will contribute to the $S, T, U$ oblique parameters in precision electroweak measurements, but not a singlet. In the triplet Higgs model, these are important constraints \cite{14}. In this context, we note that the new particles of Model (C) are all SM singlets, i.e. $\chi^-$, $(\zeta^{++}_R, \zeta^+_R)$, $E_{L,R}$, and $(\phi^+_R, \phi^0_R)$, so they will not affect the oblique parameters.

**Details of Model (C):**

The charged leptons $e, \mu, \tau$ obtain masses through the heavy singlets $E_{1,2,3}$ in the $6 \times 6$ Dirac
mass matrix linking \((\bar{e}_L, \mu_L, \tau_L; \bar{E}_1 L, \bar{E}_2 L, \bar{E}_3 L)\) to \((e_R, \mu_R, \tau_R; E_1 R, E_2 R, E_3 R)\):

\[
\mathcal{M}_{eE} = \begin{pmatrix} 0 & \mathcal{M}_L \\ \mathcal{M}_R & \mathcal{M}_E \end{pmatrix},
\]

(4)

where \(\mathcal{M}_{L,R}\) are \(3 \times 3\) mass matrices proportional to \(\langle \phi^0_{L,R} \rangle\) respectively. Hence

\[
\mathcal{M}_e = \mathcal{M}_L \mathcal{M}_E^{-1} \mathcal{M}_R.
\]

(5)

In the scalar sector, in addition to the \(\Phi_{L,R}\) doublets, there are the charged singlet \(\chi^-\) and the exotic doublet \(\zeta = (\zeta^{++}_R, \zeta^+_R)\). From the structure of Fig. 3, it is clear that \(\chi^-\) carries lepton number \(L = +2\) and \(\zeta\) carries \(L = -2\). The most general scalar potential is then given by

\[
V = -\mu^2_L \Phi^\dagger_L \Phi_L - \mu^2_R \Phi^\dagger_R \Phi_R + \mu_1^2 \chi^+ \chi^- + \mu_2^2 \zeta^+ \zeta^- + [\mu_3 \chi^- (\Phi^\dagger_R \zeta) + H.c.]
\]

\[
+ \frac{1}{2} \lambda_L (\Phi^\dagger_L \Phi_L)^2 + \frac{1}{2} \lambda_R (\Phi^\dagger_R \Phi_R)^2 + \frac{1}{2} \lambda_L (\chi^+ \chi^-)^2 + \frac{1}{2} \lambda_L (\zeta^+ \zeta^-)^2
\]

\[
+ \lambda_{LR} (\Phi^\dagger_L \Phi_L) (\Phi^\dagger_R \Phi_R) + \lambda_{L\chi} (\Phi^\dagger_L \Phi_L) (\chi^+ \chi^-) + \lambda_{L\zeta} (\Phi^\dagger_L \Phi_L) (\zeta^+ \zeta^-)
\]

\[
+ \lambda_{R\chi} (\Phi^\dagger_R \Phi_R) (\chi^+ \chi^-) + \lambda_{R\zeta} (\Phi^\dagger_R \Phi_R) (\zeta^+ \zeta^-) + \lambda_{\chi\zeta} (\chi^+ \chi^-) (\zeta^+ \zeta^-).
\]

(6)

As the gauge symmetries \(SU(2)_{L,R}\) get broken by \(\langle \phi^0_{L,R} \rangle = v_{L,R}\), where

\[
\mu^2_L = \lambda_L v^2_L + \lambda_{LR} v^2_R,
\]

(7)

\[
\mu^2_R = \lambda_R v^2_R + \lambda_{LR} v^2_L,
\]

(8)

only \(\sqrt{2} Re(\phi^0_{L,R}) = h_{L,R}\) become physical fields, with

\[
m^2_{h_L} = 2\lambda_L v^2_L, \quad m^2_{h_R} = 2\lambda_R v^2_R.
\]

(9)

The other physical scalars are \(\chi^-, \zeta^{++}_R\), and \(\zeta^+_R\). The \(2 \times 2\) mass-squared matrix spanning \((\chi^\pm, \zeta^\pm_R)\) is of the form

\[
\mathcal{M}^2_{\chi, \zeta} = \begin{pmatrix} m^2_{\chi} & \mu_3 v_R \\ \mu_3 v_R & m^2_{\zeta^{++}} \end{pmatrix},
\]

(10)
where

\[ m_\chi^2 = \mu_1^2 + \lambda_L v_L^2 + \lambda_R v_R^2, \]
\[ m_{\zeta^{++}}^2 = \mu_2^2 + \lambda_L \zeta v_L^2 + \lambda_R \zeta v_R^2. \] (11)

(12)

Let the two mass eigenstates be

\[ h_1^\pm = \chi^\pm \cos \theta - \zeta^\pm R \sin \theta, \quad h_2^\pm = \chi^\pm \sin \theta + \zeta^\pm R \cos \theta, \] (13)

with \( m_1 < m_2 \), then Eq. (10) implies that \( m_1 < m_{\zeta^{++}} < m_2 \).

The decay of \( \zeta^{++}_R \) is possible through the allowed coupling \( \zeta^{++}_R l_i R E_{jR} \) and the conversion of \( E_R \) to \( l_R \) through the \( 6 \times 6 \) mass matrix of Eq. (4). This is thus a two-body decay with final state \( l_i R l_{jR} \), analogous to the Higgs triplet case for neutrino mass but with the opposite chirality. Another possible decay is to \( h_1^+ \) and a (virtual) \( W_R^+ \) which becomes \( l_i^+ \nu_i \) or \( \bar{d}_i u_i \), with \( h_1^+ \) decaying subsequently to \( l_j^+ \bar{\nu}_j \), as shown in Fig. 5. Whereas the two-body decay is suppressed by \( l - E \) mixing, the three-body decay is suppressed by \( \sin \theta \) of Eq. (13). If the latter is not small, then the amplitude of Fig. 5 may be significant or even dominant. Its signature is then \( \zeta^{++}_R \rightarrow l_i^+ l_j^+ \) + missing energy, or \( \zeta^{++}_R \rightarrow l^+ + 2 \) jets + missing energy. For \( \zeta^{\pm\pm}_R \) pair production at the LHC, a distinctive final state of 4 charged leptons + missing energy may be observed. Future dedicated analyses based on the decay chain shown in Fig. 5 would be necessary to extract information on the existence of \( \zeta^{++}_R \).
The Scalar Leptoquark Option:

The particles in the loop of Fig. 3, i.e. $e_L, E_R, \chi^-, \zeta_R^+$, may be replaced by their color counterparts, i.e. $d_L, D_R, \chi^{-1/3}, \zeta_R^{1/3}$. In that case, the distinctive feature of the model is the exotic scalar leptoquark doublet ($\zeta_R^{1/3}, \zeta_R^{1/3}$). The singlet-doublet mixing of $\chi^\pm 1/3$ with $\zeta_R^\pm 1/3$ is of the same form as Eq. (10). Hence $\zeta_R^{-4/3}$ decays to $l_i d_j$ through $d - D$ mixing, as well as $W_R^- (\nu_i d_j - l_i u_j)$ in analogy to Fig. 5. If the latter dominates, the signature is again not that of the conventional scalar leptoquark, and requires more detailed analysis of future LHC data to search for its existence.

Exchange of $W_L$ and $W_R$:

A radiative Dirac neutrino mass is generated through $W_L - W_R$ mixing, so that $m_\nu/m_l$ has a natural lower bound [15]. In the absence of a scalar bidoublet, $W_L$ and $W_R$ do not mix. However, once the $u$ and $d$ quarks obtain masses, mixing does occur [16, 17] in one loop. Together with the appearance of charged-lepton masses, Dirac neutrino masses are induced in two loops. These are presumably subdominant effects to our one-loop Dirac neutrino masses. In addition, the analog of the $2W$ exchange diagram [18] for Majorana neutrino mass in the SM also exists, as shown in Fig. 6. This shows that even if only one neutrino, say $\nu_1$, picks up a Dirac mass from Eq. (2), the other neutrinos $\nu_{2,3}$ will get nonzero radiative

Figure 6: Dirac neutrino mass from $W_{L,R}$ exchange.
masses as well, although in practice they are numerically negligible.

**Concluding Remarks**

In the context of the well-known left-right gauge model of quarks and leptons, we study the case where the scalar bidoublet is absent. Assuming that quark and charged leptons obtain their masses through their corresponding heavy singlets using the Dirac seesaw mechanism, we consider how one-loop Dirac neutrino masses may be obtained. We identify four generic scenarios, two of which were proposed almost 30 years ago. We focus on one of the two new scenarios, with a new charged singlet scalar $\chi^-$ and an exotic $SU(2)_R$ scalar doublet $(\zeta_{++}^R, \zeta_+^R)$. We show how $\zeta_{++}^R$ differs from the well-known doubly charged scalar in the Higgs triplet model of neutrino mass, and discuss its distinctive decay signature at the LHC. We also point out that there could be a scalar leptoquark variant of this radiative mechanism using the exotic doublet $(\zeta_{4/3}^R, \zeta_{1/3}^R)$.

**Acknowledgement**

The work of E.M. was supported in part by the U. S. Department of Energy under Grant No. [de-sc0008541] and the work of U.S. was supported by a research grant associated with the J. C. Bose Fellowship, DST, India.

**References**

[1] S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979).

[2] E. Ma, Phys. Rev. Lett. **81**, 1171 (1998).

[3] E. Ma and O. Popov, Phys. Lett. **B764**, 142 (2017).

[4] B. Brahmachari, E. Ma, and U. Sarkar, Phys. Rev. Lett. **91**, 011801 (2003).

[5] R. N. Mohapatra, Phys. Lett. **B201**, 517 (1988).
[6] A. Zee, Phys. Lett. B93, 389 (1980).

[7] E. Ma, Phys. Rev. Lett. 63, 1042 (1989).

[8] E. Ma, Phys. Rev. D73, 077301 (2006).

[9] D. Borah and A. Dasgupta, JCAP 1706, 003 (2017).

[10] E. Ma and U. Sarkar, Phys. Rev. Lett. 80, 5716 (1998).

[11] E. Ma, M. Raidal, and U. Sarkar, Phys. Rev. Lett. 85, 3769 (2000).

[12] ATLAS Collaboration, Report No. ATLAS-CONF-2016-051.

[13] K. S. Babu and S. Jana, Phys. Rev. D95, 055020 (2017).

[14] See for example D. Das and A. Santamaria, Phys. Rev. D94, 015015 (2016).

[15] E. Ma, Mod. Phys. Lett. A2, 63 (1987).

[16] K. S. Babu, D. Eichler, and R. N. Mohapatra, Phys. Lett. B226, 347 (1989).

[17] K. S. Babu and X.-G. He, Mod. Phys. Lett. A4, 61 (1989).

[18] K. S. Babu and E. Ma, Phys. Rev. Lett. 61, 674 (1988).