Research Article

DFT-Based Channel Estimation with Symmetric Extension for OFDMA Systems

Yi Wang,1,2 Lihua Li,1,2 Ping Zhang,1,2 and Zemin Liu1,2

1 Key Laboratory of Universal Wireless Communications, Beijing University of Posts and Telecommunication, Ministry of Education, Beijing 100876, China
2 Wireless Technology Innovation Institute, Beijing University of Posts and Telecommunication, Beijing 100876, China

Correspondence should be addressed to Yi Wang, wangyi81@gmail.com

Received 31 July 2008; Revised 10 November 2008; Accepted 18 January 2009

Recommended by Yan Zhang

A novel partial frequency response channel estimator is proposed for OFDMA systems. First, the partial frequency response is obtained by least square (LS) method. The conventional discrete Fourier transform (DFT) method will eliminate the noise in time domain. However, after inverse discrete Fourier transform (IDFT) of partial frequency response, the channel impulse response will leak to all taps. As the leakage power and noise are mixed up, the conventional method will not only eliminate the noise, but also lose the useful leaked channel impulse response and result in mean square error (MSE) floor. In order to reduce MSE of the conventional DFT estimator, we have proposed the novel symmetric extension method to reduce the leakage power. The estimates of partial frequency response are extended symmetrically. After IDFT of the symmetric extended signal, the leakage power of channel impulse response is self-cancelled efficiently. Then, the noise power can be eliminated with very small leakage power loss. The computational complexity is very small, and the simulation results show that the accuracy of our estimator has increased significantly compared with the conventional DFT-based channel estimator.

Copyright © 2009 Yi Wang et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

1. Introduction

The orthogonal frequency-division multiplexing (OFDM) is an effective technique for combating multipath fading and for high-bit-rate transmission over mobile wireless channels. In OFDM system, the entire channel is divided into many narrow subchannels, which are transmitted in parallel, thereby increasing the symbol duration and reducing the ISI.

Channel estimation has been successfully used to improve the performance of OFDM systems. It is crucial for diversity combination, coherent detection, and space-time coding. Various OFDM channel estimation schemes have been proposed in literature. The LS or the linear minimum mean square error (LMMSE) estimation was proposed in [1]. Reference [2] also proposed a low-complexity LMMSE estimation method by partitioning off channel covariance matrix into some small matrices on the basis of coherent bandwidth. However, these modified LMMSE methods still have quite high-computational complexity for practical implementation and require exact channel covariance matrices. Reference [3] introduced additional DFT processing to obtain the frequency response of LS-estimated channel. In contrast to the frequency-domain estimation, the transform-domain estimation method uses the time-domain properties of channels. Since a channel impulse response is not longer than the guard interval in OFDM system, the LS and the LMMSE were modified in [4, 5] by limiting the number of channel taps in time domain. References [6, 7] showed the performance of various channel estimation methods and yielded that the DFT-based estimation can achieve significant performance benefits if the maximum channel delay is known. References [8–11] improved upon this idea by considering only the most significant channel taps. Reference [12] further investigated how to eliminate the noise on the insignificant taps by optimal threshold.

However, in many applications such as OFDMA system, only the estimates of partial frequency response are available, and the estimate of channel impulse response in time domain
cannot be obtained from the conventional DFT method. After IDFT of partial frequency response, the channel impulse response will leak to all taps in time domain. As the noise and leakage power are mixed up, the conventional DFT method will not only eliminate the noise, but also lose the useful channel leakage power and result in MSE floor. We have proposed the novel symmetric extension method to reduce the leakage power. The mathematic expression of the MSE of the conventional DFT estimator and the upper bound of the MSE of our proposed estimator are derived in this paper.

The rest of the paper is organized as follows. Section 2 describes the system model and briefly introduces the statistics of mobile wireless channel. Section 3 proposes the novel channel-estimation approach for OFDMA systems. Section 4 presents computer simulation results to demonstrate the effectiveness of the proposed estimation approach. Finally, conclusion is given in Section 5.

2. System and Channel Model

Consider an OFDM system that has \( N \) subcarriers. The data stream is modulated by inverse fast Fourier transform (IFFT), and a guard interval is added for every OFDM symbol to eliminate ISI caused by multipath fading channel. At the receiver, with the \( k \)th OFDM symbol, the \( k \)th subcarrier of the received signal is denoted as

\[
Y_{k,i} = H_{k,i} \cdot X_{k,i} + N_{k,i},
\]

where \( X_{k,i} \) are the pilot subcarriers, for simplicity, it is assumed that \( |X_{k,i}| = 1 \), \( H_{k,i} \) represents the channel frequency response on the \( k \)th subcarrier. \( N_{k,i} \) is the AWGN with zero mean and variance of \( \sigma^2 \).

The complex baseband representation of the mobile wireless channel impulse response can be described by [13]

\[
h(t, \tau) = \sum_k y_k(t) c(\tau - \tau_k),
\]

where \( \tau_k \) is the delay of the \( k \)th path, \( y_k(t) \) is the corresponding complex amplitude, and \( c(t) \) is the shaping pulse. For OFDM systems with proper cyclic extension and timing, it has been shown in [14] that the channel frequency response can be expressed as

\[
H_k \triangleq \sum_{l=0}^{L-1} h_{ij} e^{-j(2\pi kl/N)},
\]

where \( h_{ij} \) for \( l = 0, 1, \ldots, L - 1 \) are WSS narrowband complex Gaussian processes. \( L \) is the number of multipath taps. The average power of \( h_{ij} \) and \( L \) depends on the delay profile and dispersion of the wireless channels.

3. Channel Estimation Based on Symmetric Extension

3.1. Conventional DFT Method. For simplicity, the index \( i \) is omitted in the following formulation. The LS channel estimator is denoted as

\[
\hat{H}_{LS}^k = \frac{Y_k}{X_k} = H_k + \frac{N_k}{X_k}, \quad 0 \leq k \leq N - 1.
\]

After IFFT, the time-domain expression of \( \hat{H}_{LS}^k \) is denoted as

\[
\hat{h}_{LS}^k = \frac{1}{N} \sum_{k=0}^{N-1} H_k^L e^{j(2\pi N/k)n} = \frac{1}{N} \sum_{k=0}^{N-1} \left( H_k + \frac{N_k}{X_k} \right) e^{j(2\pi N/k)n} = h_n + z_n,
\]

where \( h_n \) is the channel impulse response on the \( n \)th path \( z_n = (1/N) \sum_{k=0}^{N-1} (N_k/X_k) e^{j(2\pi N/k)n} \). Most mobile wireless channels are characterized by discrete multipath arrivals, that is, the magnitude of \( h_n \) for most \( n \) is zeros or very small; hence, these channel taps can be ignored. Assume \( L_{CP} \) denote the length of guard interval, then the maximum length of nonzero \( h_n \) is \( L_{GP} \), and \( h_n = 0 \) for \( L_{CP} < n \leq N - 1 \). In the conventional DFT method, in order to eliminate the noise,

\[
\hat{H}_{DFT}^k = \begin{cases} \hat{h}_{LS}^k, & n = 0, \ldots, L_{GP} - 1, \\ 0, & n = L_{GP}, \ldots, N - 1. \end{cases}
\]

The estimate of frequency response is denoted as

\[
H_k^{DFT} \triangleq \sum_{l=0}^{L-1} \hat{h}_{n}^{DFT} e^{-j(2\pi N/k)l}.
\]

The basic block diagram of DFT-based estimation is shown in Figure 1.

3.2. Partial Frequency Response by Conventional DFT. In OFDMA system, as the pilot only occupies part of total subcarriers, we can only get the estimates of partial frequency response, which is denoted as

\[
\hat{H}_k^{\text{partial}} = \hat{H}_{LS}^k, \quad k = 0, \ldots, M - 1,
\]
where $M$ is the length of partial frequency response. For simplicity, we consider $M_1 = 0$ in this paper. However, with only minor modification, the result discussed here is applicable to any $M_1$. The $M$ point IFFT result of $H_k^{\text{partial}}$ is denoted as

$$h_n^{\text{partial}} = \frac{1}{M} \sum_{k=0}^{M-1} H_k^{\text{partial}} e^{j(2\pi kn/M)}$$

$$= \frac{1}{M} \sum_{k=0}^{M-1} H_k e^{j(2\pi kn/M)} + \frac{1}{M} \sum_{k=0}^{M-1} N_k e^{j(2\pi kn/M)}$$

$$= h_n^{\text{partial}} + z_n^{\text{partial}},$$

where $z_n^{\text{partial}} = (1/M) \sum_{k=0}^{M-1} (N_k/X_k) e^{j(2\pi kn/M)}$, and $h_n^{\text{partial}}$ is denoted as

$$h_n^{\text{partial}} = \frac{1}{M} \sum_{k=0}^{M-1} H_k e^{j(2\pi kn/M)}$$

$$= \frac{1}{M} \sum_{k=0}^{M-1} \sum_{l=0}^{L-1} h_l e^{-j(2\pi kl/M)} e^{j(2\pi kn/M)}$$

$$= \frac{1}{M} \sum_{l=0}^{L-1} h_l C_{\text{partial}}(n, l, M, N),$$

where $C_{\text{partial}}(n, l, M, N) = \sum_{k=0}^{M-1} e^{j(2\pi kn(M-2\pi l/M)k)}$. From (10), it can be seen that the channel impulse response $h_n$ will leak to all taps of $h_n^{\text{partial}}$. The conventional DFT method is no longer applicable as $h_n^{\text{partial}}$ will be nonzero due to the power leakage; the noise and leakage power are mixed up. The elimination of noise will also cause the loss of useful channel impulse response leakage.

It is assumed that each path is an independent zero-mean complex Gaussian random process. The leakage power-to-noise power ratio (LNR) on the $n$th tap in the conventional DFT method can be denoted as

$$\text{LNR}_n^{\text{partial}} = \frac{E[|h_n^{\text{partial}}|^2]}{E[|z_n^{\text{partial}}|^2]} = \frac{\sum_{l=0}^{L-1} |C_{\text{partial}}(n, l, M, N)|^2}{M\sigma^2},$$

where $\sigma_l^2$ is the average power of the $l$th path. As the channel power mainly focuses on the low-frequency band, in order to eliminate the noise in high-frequency band, let $L_{\text{partial}}$ denote the threshold, and the noise is eliminated by the conventional DFT method,

$$h_n^{\text{partial}} = \begin{cases} h_n^{\text{partial}}, & 0 \leq n \leq L_{\text{partial}} - 1 \text{ or } M - L_{\text{partial}} \leq n \leq M - 1, \\ 0, & L_{\text{partial}} \leq n \leq M - 1 - L_{\text{partial}}. \end{cases}$$

The corresponding estimate of partial frequency response is denoted as

$$U_k^{\text{partial}} = \sum_{n=0}^{M-1} h_n^{\text{partial}} e^{-j(2\pi kn/M)}, \quad k = 0, \ldots, M - 1. \quad (13)$$

3.3. Partial Frequency Response Estimation by Symmetric Extension Method. As $H_k, 0 \leq k \leq N - 1$ are the samples of the continuous and periodic channel frequency response, in time domain, the IFFT result of $H_k, 0 \leq k \leq N - 1$ will only concentrate on a few taps. However, the IFFT result of the partial frequency response samples $H_k, 0 \leq k \leq M - 1$ will leak to all taps. This is because $H_k, 0 \leq k \leq M - 1$ are the samples of partial-frequency response, and after periodic expansion, the continuity of the signal is severely destroyed. If the leakage power is reduced significantly compared with the noise power, the noise still can be eliminated efficiently with very small loss of leakage power. Inspired by this, in order to reduce the leakage power, we have proposed the novel symmetric extension method to construct a new sequence with better continuity. $H_k^{\text{partial}}$ is extended with symmetric signal of its own, and the symmetrically extended signal is denoted as

$$H_k^{\text{symmetric}} = \begin{cases} H_k^{\text{partial}}, & 0 \leq k \leq M - 1, \\ H_{2M-1-k}, & M \leq k \leq 2M - 1. \end{cases} \quad (14)$$

After $2M$ point IFFT, the time-domain expression of $H_k^{\text{symmetric}}$ is denoted as $h_n^{\text{symmetric}}$:

$$h_n^{\text{symmetric}} = \frac{1}{2M} \sum_{k=0}^{2M-1} H_k^{\text{symmetric}} e^{j(2\pi kn/2M)}$$

$$= \frac{1}{2M} \sum_{k=0}^{M-1} H_k \{e^{j(2\pi kn/2M)} + e^{j(2\pi kn/2M)(2M-1-k)} \}
+ \frac{1}{2M} \sum_{k=0}^{M-1} N_k \{e^{j(2\pi kn/2M)} + e^{j(2\pi kn/2M)(2M-1-k)} \}
= h_n^{\text{symmetric}} + z_n^{\text{symmetric}},$$

(15)
Figure 3: Block diagram of our proposed symmetric extension DFT-based channel estimation.

where \( z_n^{\text{symmetric}} \) = \((1/2M) \sum_{k=0}^{M-1} (N_k/X_k)(e^{j(2\pi n/2M)k} + e^{j(2\pi n/2M)(2M–1–k)})\), and \( h_n^{\text{symmetric}} \) is denoted as

\[
h_n^{\text{symmetric}} = \frac{1}{2M} \sum_{k=0}^{2M-1} H_k^{\text{symmetric}} e^{j(2\pi n k / 2M)}
\]

\[
= \frac{1}{2M} \sum_{k=0}^{M-1} H_k (e^{j(2\pi n/2M)k} + e^{j(2\pi n/2M)(2M–1–k)})
\]

\[
= \frac{1}{2M} \sum_{k=0}^{M-1} \sum_{l=0}^{L-1} h_le^{-j(2\pi kl/N)} \times (e^{j(2\pi n/2M)k} + e^{j(2\pi n/2M)(2M–1–k)})
\]

\[
= \frac{1}{2M} \sum_{l=0}^{L-1} h_l C_{\text{symmetric}}(n, l, M, N),
\]

where \( C_{\text{symmetric}}(n, l, M, N) = \sum_{k=0}^{M-1} e^{-j(2\pi l N/N)} (e^{j(2\pi n/2M)k} + e^{j(2\pi n/2M)(2M–1–k)})\).

The leakage power-to-noise power ratio (LNR) on the \( n \)th tap can be denoted as

\[
\text{LNR}_n^{\text{symmetric}} = \frac{E[|h_n^{\text{symmetric}}|^2]}{E[|z_n^{\text{symmetric}}|^2]} = \frac{\sum_{k=0}^{L-1} \sigma_k^2 |C_{\text{symmetric}}(n, l, M, N)|^2}{2M \sigma^2}.
\]

Let \( L_{\text{symmetric}} \) denote the threshold. Using the conventional DFT method, the noise and leakage power is eliminated by

\[
g_n^{\text{symmetric}} = \begin{cases} h_n^{\text{symmetric}}, & 0 \leq n \leq L_{\text{symmetric}} - 1 \\ 0, & \text{or } 2M - L_{\text{symmetric}} \leq n \leq 2M - 1, \\ 0, & L_{\text{symmetric}} \leq n \leq 2M - 1 - L_{\text{symmetric}}. \end{cases}
\]

After 2M point FFT,

\[
G_k^{\text{symmetric}} = \sum_{n=0}^{2M-1} g_n^{\text{symmetric}} e^{-j(2\pi nk/2M)}, \quad k = 0, \ldots, 2M - 1.
\]

The corresponding estimate of partial frequency response is denoted as

\[
U_k^{\text{symmetric}} = G_k^{\text{symmetric}} + G_{2M-1-k}^{\text{symmetric}} / 2, \quad k = 0, \ldots, M - 1.
\]

The basic block diagram of our proposed symmetric extension DFT-based estimation is shown in Figure 3.

3.4. Performance Analysis. From (13), the MSE of the conventional DFT method without symmetric extension is written as

\[
\text{MSE}_{\text{partial}} = \frac{1}{M} E \left\{ \sum_{k=0}^{M-1} \left| U_k^{\text{partial}} - H_k \right|^2 \right\}. \tag{21}
\]

From (20), the MSE of our proposed estimator is

\[
\text{MSE}_{\text{symmetric}} = \frac{1}{M} E \left\{ \sum_{k=0}^{M-1} \left| U_k^{\text{symmetric}} - H_k \right|^2 \right\}. \tag{22}
\]

The estimation error of the conventional method is divided into two parts. The first part is that when \( L_{\text{partial}} \leq n \leq M - 1 - L_{\text{partial}} \), the leakage power \( h_{\text{partial}} \) is lost as it is forced to be zero. The second part is that when \( n < L_{\text{partial}} \) or \( n > M - 1 - L_{\text{partial}} \), the error is caused by AWGN. The estimation error can be written as

\[
\text{ERROR}_{\text{partial}} = h_{\text{partial}}^{\text{symmetric}} - g_{\text{partial}}^{\text{symmetric}}
\]

\[
= \begin{cases} h_{\text{partial}}^{\text{symmetric}}, & L_{\text{partial}} \leq n \leq M - 1 - L_{\text{partial}}, \\ 0, & \text{others}. \end{cases}
\]

\[
= \begin{cases} h_{\text{partial}}^{\text{symmetric}}, & L_{\text{partial}} \leq n \leq M - 1 - L_{\text{partial}}, \\ 0, & \text{others}. \end{cases}
\]
Similarly, the estimation error of our proposed method is also divided into two parts. It can be written as

\[
\text{ERROR}_{\text{symmetric}} = h_n^{\text{symmetric}} - g_n^{\text{symmetric}}
\]

\[
= \begin{cases} h_n^{\text{symmetric}}, & L_{\text{leakage}} \leq n \leq 2M - 1 - L_{\text{leakage}}, \\ z_n^{\text{symmetric}}, & \text{others}. \end{cases}
\]  

(24)

According to the Parseval theorem, (21) can be written as

\[
\text{MSE}_{\text{partial}} = \frac{1}{M} E \left \{ \sum_{k=0}^{M-1} \left\| c_{k}^{\text{partial}} - H_k \right\|^2 \right\}
\]

\[
= E \left \{ \sum_{n=0}^{M-1} \left\| h_n^{\text{partial}} - g_n^{\text{partial}} \right\|^2 \right\}
\]

\[
= E \left \{ \sum_{n=0}^{M-L_{\text{leakage}}} \left\| h_n^{\text{partial}} \right\|^2 \right\} + E \left \{ \sum_{n=M-L_{\text{leakage}}}^{M-1} \left\| g_n^{\text{partial}} \right\|^2 \right\}
\]

\[
+ E \left \{ \sum_{n=M-L_{\text{leakage}}}^{M-1} \left\| h_n^{\text{partial}} - g_n^{\text{partial}} \right\|^2 \right\}
\]

\[
= \frac{1}{M^2} \sum_{n=L_{\text{leakage}}}^{M-L_{\text{leakage}}} \sum_{l=0}^{L_{\text{leakage}}} \sigma_l^2 \left| C_{\text{partial}}(n,l,M,N) \right|^2
\]

\[
+ \frac{L_{\text{leakage}}}{M} \sigma^2
\]

\[
= \frac{1}{M^2} \sum_{n=L_{\text{leakage}}}^{M-L_{\text{leakage}}} \sum_{l=0}^{L_{\text{leakage}}} \sigma_l^2 \left| C_{\text{partial}}(n,l,M,N) \right|^2
\]

\[
+ 2 \frac{L_{\text{leakage}}}{M} \sigma^2.
\]

From (24), (22) can be rewritten as

\[
\text{MSE}_{\text{symmetric}} = \frac{1}{M} E \left \{ \sum_{k=0}^{M-1} \left\| c_{k}^{\text{symmetric}} - H_k \right\|^2 \right\}
\]

\[
= \frac{1}{M} E \left \{ \sum_{k=0}^{M-1} \left\| g_k^{\text{symmetric}} - c_{2M-1-k}^{\text{symmetric}} - H_k \right\|^2 \right\}
\]

\[
= \frac{1}{4M^2} E \left \{ \sum_{k=0}^{M-1} \left\| g_{2M-1-k}^{\text{symmetric}} - H_k \right\|^2 \right\}
\]

\[
\leq \frac{1}{4M^2} E \left \{ \sum_{k=0}^{M-1} \left\| c_{k}^{\text{symmetric}} - H_k \right\|^2 + \left\| g_{2M-1-k}^{\text{symmetric}} - H_k \right\|^2 \right\}.
\]

(26)

According to the Parseval theorem,

\[
\frac{1}{2M} E \left \{ \sum_{k=0}^{M-1} \left\| g_k^{\text{symmetric}} - H_k \right\|^2 + \left\| G_{2M-1-k}^{\text{symmetric}} - H_k \right\|^2 \right\}
\]

\[
= E \left \{ \sum_{n=0}^{2M-1} \left\| h_n^{\text{symmetric}} - g_n^{\text{symmetric}} \right\|^2 \right\}
\]

\[
= E \left \{ \sum_{n=L_{\text{leakage}}}^{2M-1-L_{\text{leakage}}} \left\| h_n^{\text{symmetric}} \right\|^2 \right\}
\]

\[
+ E \left \{ \sum_{n=L_{\text{leakage}}}^{2M-1-L_{\text{leakage}}} \left\| g_n^{\text{symmetric}} \right\|^2 \right\}
\]

\[
+ E \left \{ \sum_{n=L_{\text{leakage}}}^{2M-1-L_{\text{leakage}}} \left\| h_n^{\text{symmetric}} - g_n^{\text{symmetric}} \right\|^2 \right\}
\]

\[
= \frac{1}{4M^2} \sum_{n=L_{\text{leakage}}}^{2M-1-L_{\text{leakage}}} \sum_{l=0}^{L_{\text{leakage}}} \sigma_l^2 \left| C_{\text{symmetric}}(n,l,M,N) \right|^2
\]

\[
+ \frac{L_{\text{leakage}}}{M} \sigma^2.
\]

From (26), (27), the upper bound of the MSE of our proposed estimator is

\[
\text{MSE}_{\text{upper symmetric}} = \frac{1}{4M^2} \sum_{n=L_{\text{leakage}}}^{2M-1-L_{\text{leakage}}} \sum_{l=0}^{L_{\text{leakage}}} \sigma_l^2 \left| C_{\text{symmetric}}(n,l,M,N) \right|^2
\]

\[
+ \frac{L_{\text{leakage}}}{M} \sigma^2.
\]

3.5. Estimator Complexity. The conventional DFT-based channel estimator is very attractive for its good performance and low complexity. Its main computation complexity is 2M point IFFT and FFT. Our proposed symmetric extension method also inherits the low complexity of the DFT estimator, and its main computation complexity is 2M point IFFT and FFT. As the complexity of FFT and IFFT is significantly reduced nowadays, our proposed method can provide a good tradeoff between performance and complexity.

4. Performance Results

We investigate the performance of our proposed estimator through computer simulation. An OFDMA system with \(N = 512\) subcarriers is considered the guard interval \(L_{\text{GP}} = 64\). The sampling rate is 7.68 MHz, and subcarrier frequency space is 15 kHz. A six-path channel model is used. The power profile is given by \(P = [-3.0, -2, -6, -8, -10] \text{ dB}\), and the delay profile after sampling is \(\tau = [0, 2, 4, 12, 18, 38]\). Each path is an independent zero-mean complex Gaussian random process.

Figures 4 and 5 show the comparison of LNR between the conventional DFT method and our proposed method. \(\sigma^2\) is normalized to 1, and \(M\) is set to 16 and 64. It should be
noted that the FFT length of the conventional DFT method is \( M \), while the FFT length of our proposed method is \( 2M \) due to the symmetric extension. That is why the two curves have different lengths. It is shown that \( \text{LNR}_{\text{partial}} \) is much larger than \( \text{LNR}_{\text{symmetric}} \). Compared with the conventional method, the leakage power is significantly self-cancelled by symmetric extension method.

Figure 6 shows the theoretical MSE of the conventional DFT method when \( M = 16 \). The MSE is calculated under \( \text{SNR} = 5 \text{dB}, 10 \text{dB}, \text{and} 20 \text{dB} \), respectively. The MSE is large when \( L_{\text{partial}} \) is small, this is because although most noise can be eliminated, the channel power \( h_{n \text{partial}} \) is also lost, and the MSE is mainly caused by the loss of \( h_{n \text{partial}} \). When \( L_{\text{partial}} \) is large, although the loss of \( h_{n \text{partial}} \) is small, the noise cannot be eliminated efficiently, and the MSE is mainly caused by the noise.

Figure 7 shows the upper bound of the MSE of our proposed method. Compared with Figure 6, the upper bound of the MSE of our proposed method is smaller than the MSE of the conventional DFT method. This is because in our proposed method the channel leakage is significantly reduced, and the elimination of noise will cause less channel leakage power loss.

Figure 8 shows the MSE performance comparison of different methods. \( M \) is set to 16. In the conventional DFT
Figure 8: Comparing MSE performance with proposed estimator, conventional DFT estimator, and LS estimator, when $M = 16$, $L_{\text{partial}} = 4.6$, and $L_{\text{symmetric}} = 8.12$.

Figure 9: Comparing MSE performance with proposed estimator, conventional DFT estimator, and LS estimator, when $M = 64$, $L_{\text{partial}} = 16.24$, and $L_{\text{symmetric}} = 32.48$.

method, $L_{\text{partial}}$ is set to 4 and 6 as the FFT length of our proposed method is doubled, and the corresponding threshold $L_{\text{symmetric}}$ is set to 8 and 12. When SNR is low, both the conventional DFT method and our proposed method can reduce the MSE. However, when SNR is higher than 15 dB, there is an evident MSE floor larger than $10^{-2}$ in the conventional DFT method. While in our proposed method, the MSE floor is eliminated efficiently. This is because when SNR is low, the MSE is mainly caused by the noise, not the loss of channel leakage power. When SNR is high, the MSE is mainly caused by the leakage power loss instead. As the leakage power is significantly reduced in our proposed symmetric extension method, even when SNR is high, the noise still can be eliminated at very small expense of channel leakage power loss. Figure 8 also shows the effect of threshold. It can be seen that when SNR is low, smaller threshold has better MSE performance than larger threshold, and when SNR is high, it has worse MSE performance. This is because with the decrease of threshold, more noise can be eliminated, but more channel leakage power will be lost, and with the increase of threshold, less channel leakage power will be lost, but less noise is eliminated.

Figure 9 shows the MSE performance when $M$ is set to 64, $L_{\text{partial}}$ is set to 16 and 24, and $L_{\text{symmetric}}$ is 32 and 48. The simulation result is similar to Figure 8. It proves that our method is effective for different values of $M$.

Figure 10 shows the raw BER performance with different channel estimation methods. Each subcarrier is modulated by 16 QAM. $M$ is set to 16, $L_{\text{partial}} = 4.6$, and $L_{\text{symmetric}} = 8.12$. The channel is equalized by zero-forcing algorithm. It can be seen that the BER with the conventional DFT channel estimator still encounters BER floor because of the channel estimation errors. While in our proposed symmetric extension method, as the accuracy of channel estimator is significantly increased, the BER performance is also improved.
5. Conclusion

A simple DFT-based channel estimation method with symmetric extension is proposed in this paper. In order to increase the estimation accuracy, the noise is eliminated in time domain. As both the noise and the channel impulse leakage power will be eliminated, we have proposed the novel symmetric extension method to reduce the channel leakage power. The noise can be efficiently eliminated with very small loss of channel leakage power. The simulation results show that, compared with the conventional DFT method, the MSE of our proposed method is significantly reduced.

References

[1] O. Edfors, M. Sandell, J.-J. van de Beek, S. K. Wilson, and P. O. Borjesson, “OFDM channel estimation by singular value decomposition,” *IEEE Transactions on Communications*, vol. 46, no. 7, pp. 931–939, 1998.
[2] M. Noh, Y. Lee, and H. Park, "Low complexity LMMSE channel estimation for OFDM," *IEE Proceedings: Communications*, vol. 153, no. 5, pp. 645–650, 2006.
[3] Y. Zhao and A. Huang, “A novel channel estimation method for OFDM mobile communication systems based on pilot signals and transform-domain processing,” in *Proceedings of the 47th IEEE Vehicular Technology Conference (VTC '97)*, vol. 3, pp. 2089–2093, Phoenix, Ariz, USA, May 1997.
[4] J.-J. van de Beek, O. Edfors, M. Sandell, S. K. Wilson, and P. O. Borjesson, “On channel estimation in OFDM systems,” in *Proceedings of the 45th IEEE Vehicular Technology Conference (VTC ’95)*, vol. 2, pp. 815–819, Chicago, Ill, USA, July 1995.
[5] O. Edfors, M. Sandell, J.-J. van de Beek, S. K. Wilson, and P. O. Borjesson, “Analysis of DFT-based channel estimators for OFDM,” *Wireless Personal Communications*, vol. 12, no. 1, pp. 55–70, 2000.
[6] A. Dowler, A. Doufexi, and A. Nix, “Performance evaluation of channel estimation techniques for a mobile fourth generation wide area OFDM system,” in *Proceedings of the 56th IEEE Vehicular Technology Conference (VTC ‘02)*, vol. 4, pp. 2036–2040, Vancouver, Canada, September 2002.
[7] B. Yang, K. B. Letaief, R. S. Cheng, and Z. Cao, “Channel estimation for OFDM transmission in multipath fading channels based on parametric channel modeling,” *IEEE Transactions on Communications*, vol. 49, no. 3, pp. 467–479, 2001.
[8] H. Minn and V. K. Bhargava, “An investigation into time-domain approach for OFDM channel estimation,” *IEEE Transactions on Broadcasting*, vol. 46, no. 4, pp. 240–248, 2000.
[9] M. Raghavendra and K. Giridhar, “Improving channel estimation in OFDM systems for sparse multipath channels,” *IEEE Signal Processing Letters*, vol. 12, no. 1, pp. 52–55, 2005.
[10] O. Simeone, Y. Bar-Ness, and U. Spagnolini, “Pilot-based channel estimation for OFDM systems by tracking the delay-subspace,” *IEEE Transactions on Wireless Communications*, vol. 3, no. 1, pp. 315–325, 2004.
[11] J. Oliver, R. Aravind, and K. M. M. Prabhu, “Sparse channel estimation in OFDM systems by threshold-based pruning,” *Electronics Letters*, vol. 44, no. 13, pp. 830–832, 2008.
[12] W. Yi, L. Lihua, Z. Ping, and L. Zemin, “Optimal threshold for channel estimation in MIMO-OFDM system,” in *Proceedings of the IEEE International Conference on Communications (ICC ’08)*, pp. 4376–4380, Beijing, China, May 2008.
[13] R. Steele, *Mobile Radio Communications*, IEEE Press, New York, NY, USA, 1992.
[14] Y. Li, L. J. Cimini Jr., and N. R. Sollenberger, “Robust channel estimation for OFDM systems with rapid dispersive fading channels,” *IEEE Transactions on Communications*, vol. 46, no. 7, pp. 902–915, 1998.