Study of the possibility of sound stimulation of the instability of the flow of chemically reacting gas in the boundary layer

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Abstract. Stability of plane-parallel and weak curved chemically reacting gas flow is studied. It is shown that for endothermic processes instability is possible for high-frequency disturbances. Instability of flow is not associated with viscous properties of matter. Comparison with previous similar theoretical investigations is made.

1. Introduction
It is well known that turbulent flow in many ways much more preferable than laminar. So, for example, it is improving the profile shape due to later detachment of the turbulent flow wrapping around the bodies. Flows inside the channels have more filled profile, and heat transfer on a wall is much higher.
This is connected with the fact that in turbulent flow transport coefficients (viscosity, thermal conductivity) is higher than in laminar [1]. However it is important that turbulent flow would be developed, i.e. a characteristic scale, wavelength of perturbation would be low and the perturbation frequency would be high.
Study of influence of chemical processes in gas on flow stability and especially endothermic processes such as dissociation and ionization emerged from experimental and theoretical study of instability of flow behind bow shock wave in some polyatomic gases [2,3].
In the process of studying this effect it was understood, that in addition to influence of chemical processes on the effect, influence of several other factors should be taken into account. The first of these is the curvature of the current, because for fairly large curvature flow instability will happen even in the ideal gas.
The second factor is reduction of the ratio of specific heats of gas, as for specific heats equal to 1 the flow and shock wave become unstable [4]. Of course, perturbations wavelength should be taken into account also because for some wavelengths instability can manifest itself, while for others – can’t.
Boundary conditions on the surface of solids, such as surface roughness certainly affect the stability of gas movement, as in other tasks of mathematical physics. However, in this problem the internal cause of flow instability is investigated, for example, the state of matter, chemical reactions in it. In the investigated problem it is important to find out exactly how the internal processes may affect the appearance of instability. Setting recalls Orr–Summerfield task of flow instability owing to viscous properties of gas [5]. However accounting boundary conditions only will complicate the understanding of the causes of instability.
2. Setting of the problem

The proposed formulation of the problem considers plane layer of gas flow. The transverse distribution of gas dynamic parameters and their transverse derivative in it, as well as the power of release or absorb energy through chemical-physical reactions are characteristic parameters, changing from task to task. Boundary conditions are lacking. Viscosity of the gas is ignored so, because its accounting could complicate understanding, as accounting boundary conditions. Thus, the internal causes of instability are explored, which are not related to the influence of boundary conditions and viscosity. In this formulation, it is naturally to explore such equations as dispersion one. This problem, of course, is easier than the general setting, with accounting the boundary conditions \([6,7,8]\).

Incorporation of boundary conditions corresponds to the incorporation of the transverse velocity gradient in the layer, which occurs due to the zero velocity of the flow at the border. Without taking into account the transverse gradient, thus, emergence of instability cannot be realized. Curvature flow in persistent Cartesian coordinates also leads to a deviation from the vertical velocity profile. Therefore parameter of crookedness should be proportional to the deviation of transverse derivative speed profile from zero.

3. Previous results

In previous works, a method was proposed to study the stability of the flow of chemically reacting gas and plasma in the boundary layer by means of a transition from the spatial problem in the study of solutions of the equation of second order to the study of time instability. The two-dimensional problem of propagation of perturbations in the boundary layer is considered in contrast to the study of plasma instability in [9], where dispersion equations written in general form were studied for the case of transverse waves. In the proposed formulation, both transverse and longitudinal waves and waves at an angle to the direction of flow are considered.

In the method proposed in the present work, the problem was reduced to the study of the roots of the polynomial of degree 4 with respect to the dimensionless parameter \(\tilde{\varepsilon} = i \omega \rho (U-c)/(K_s \gamma)\), that depends on the 4 parameters: \(\tilde{\varepsilon} = 2abU'p(K^3 \gamma)^{-1}, \gamma = c_p/c_v, \kappa = \frac{d}{d_T}Q(K_s)^{-1} d = (\rho d_T Q(Td_T)^{-1}(K^2 = a^2+b^2, s^2 = \rho M^2 = \gamma P^2 U^2).

This approach makes it possible to observe all kinds of perturbation modes simultaneously, since all the roots of the polynomial can be calculated in a single computational process. This avoids the possibility of missing a particular fashion that is responsible for instability. Thus, the physics of instability is better understood.

Derivatives denoted with strokes are taken on the variable \(y\); \(p\) is the density of the gas, \(U\) is the velocity of the gas, \(p\) - pressure, \(T\) is temperature, \(c\) is the speed of sound, \(M\) is the Mach number of the flow, \(\gamma\) is the ratio of specific heats. Pressure perturbations \(\delta p = \pi(y)\exp[i(a(x - c)\gamma)],\) and \(\rho, U, P\) - distributions of temperature, velocity, density, and pressure across the layer; \(c\) is the speed of sound in the gas, \(a\) and \(b\) - wave numbers of disturbances along the flow (\(x\)-coordinate) and transverse the flow (\(y\) coordinate).

\(M\) - Mach number, calculated through characteristic values of \(U\) and \(T\), \(d = (\rho d_T Q(Td_T)^{-1}; i - \) the imaginary unit. One takes into account derivatives of the volumetric capacity of heat source (or heat absorption) as on the temperature and gas density - \(d_T Q, d_P Q\).

One of the solutions to the dispersion equation of disturbances in flat-parallel weak curved reacting gas flow shows the instability for endothermic reactions, as for shortwave disturbances and for longwave disturbances, which disappear with the disappearance of crookedness. The detected effect depends on the ratio of specific heats of gas and on the curvature of the streamlined surfaces, as well as on the speed of release or absorption of energy due to physical and chemical processes. Instability has baroclinic nature, and for example, flows of air plasma unstable at high-frequency of disturbances, order 8 kHz or more, regardless of whether there is a bend of speed profile or not \((U'' = 0)\), as it is required for the barotropic instability in the theory of Orr-Summerfield.
Because the attenuation of ultrasonic frequency perturbations in plasma is great, stimulation of plasma flow instability needs to be realized through the outside influence of ultrasonic waves.

4. New results

The previous results were obtained after calculating the roots of the polynomial for the real components of the wave vector $a$ and $b$ and plotting the dependence of the roots on the dimensionless wavelength of perturbations. However, the components of the wave vector $K$: $a$ and $b$ cannot always be considered real, especially if the wave packets are considered, which is more natural. In addition, the dimensionless parameters depend on the dimensionless wavelength of perturbations. Changing the coordinates in which the graph is built did not allow to solve this problem, so the calculations of the roots of the modified equation were carried out:

$$\lambda^2 Z^4 + \lambda^2 Z^3 + Z^2 + [q + g] Z + q g = 0$$

(1)

Where three first parameters are changed:

$$\lambda = \frac{1}{\lambda} d, Q \left( \gamma M^2 \right)^{-1}, Z = i \alpha \rho (U - c) \gamma M^2 (T_d) Q^{-1}, g = \left[ 2ab (a^2 + b^2) \right] U' \rho \gamma M^2 (T_d) Q^{-1}, q = (1 - d) / \gamma$$

Figure 1 shows the results of calculations of the mode that corresponds to the instability ($Im Z > 0$) at $d = 0.1$, $\gamma = 1.2$, the parameter $g$ is real and equals $0.1$. The equation was solved in Maple using their own algorithms, descriptions of which they do not give. The accuracy is of 16 significant numbers. In the other three modes imaginary parts do not exceed zero, so they correspond to the stability of the flow. It is seen that the unstable mode exists for all values of the dimension wavelength $\lambda$.

Figure 1. The imaginary part of complex roots of equation (2) as function of dimensionless perturbations wavelength $\lambda$ for $d = 0.1$, $g = 0.1$, $\gamma = 1.2$. Results of calculation for $g = 0.1 + 0.1i$ (i — imaginary unit) almost do not differ from the results for $g = 0.1$. 
This result differs from the results of previous studies, where it was noted the existence of a stable region at mean values. At the same time, the increment of perturbation growth $|\text{Im} Z|$ at $\lambda > 2$ becomes so small that the instability does not manifest itself, and is possible only at large distances along the flow, at times much longer than the time of chemical reactions. The increment increases sharply as the dimensionless wavelength $\lambda$ decreases to zero. The graph of the increment from $\lambda$ in Figure 1 resembles the curve of increase of the increment at resonance.

It should be noted that at $\lambda \to 0$ (the perturbation frequency tends to infinity) there are two real roots: $Z = -g$ and $Z = -q$. Accordingly, the imaginary parts are zero and there is a stability of the flow. This contradiction is resolved as for resonance by taking into account the viscous properties of the medium, which increase with increasing frequency of disturbances.

The second problem to be studied is that, in reality, wave packets should be considered when studying the instability of flows. This leads to the fact that the components of the wave number $a$ and $b$ cannot be considered only as real quantities (see, for example, [10]). This is due to the fact that the waves of different amplitudes that make up the packets lead to spatial attenuation or increase of the wave packet. Therefore, the only parameter that should be considered and which depends on $a$ and $b$ is the coefficient of parameter $g$: 

$$f = \left[2ab(a^2 + b^2)\right]^{-1}.$$ 

It can be expressed in the complex variable $w = ab^{-1}$. If the complex $a$ and $b$ are equal, the variable $w$ as $f$ are equal to 1. Therefore, in an isotropic medium for wave packets propagating at an angle of 45° to the direction of the flow velocity, studies for real $g$ are also applicable.

It is easy to show that $w$ is real if $\text{Im} a(\text{Re} a)^{-1} = \text{Im} b(\text{Re} b)^{-1}$. Thus, studies with real $g$ are possible if the attenuation of the wave packet both vertically and horizontally are equal fractions of the wavelength, which is real for an isotropic medium. The effect of the complexity of the coefficient on the solution of the problem will be small if the module of the complex coefficient $f \ll 1$. This is the case if $|w| \gg 2$, that is, if $|ab^{-1}| \gg 2$, or $|ba^{-1}| \gg 2$, since $a$ and $b$ are equal in rights. This condition is fulfilled if the angle between the components of the wave number $K$ is noticeably less than 30°.

To assess the effect of the complexity of the coefficient in the general case, we can assume that the complex component $g$ is comparable with its real component. Calculations have been made of the roots of the polynomial for $g=0.1 + 0.1i$ (i is the imaginary unit).

In figure 1, the results of the calculations are not presented, since they do not differ from the results at $g=0.1$ in the scale selected in the figure, although there is some deviation.

5. Conclusions

The presented results corrected the earlier studies taking into account the dependence of the parameters on the dimensionless wavelength of the flow perturbations and taking into account the complexity of the wave number for wave packets. The results confirmed the main conclusion of previous studies that instability occurs for short-wave. Also, the conclusion about instability for long-wave disturbances is confirmed, and the increment of the increase in perturbations is very small. The results are different for wavelengths of the average value. They are also unstable and also with small increment.

In addition, it is concluded that it is necessary to study the problem taking into account the viscous properties of the medium, since otherwise it is impossible to explain the sharp increase to infinity when the perturbation length tends to zero.

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