Weak decays of the axial-vector tetraquark $T_{bb;\bar{u}\bar{d}}$

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The weak decays of the axial-vector tetraquark $T_{bb;\bar{u}\bar{d}}^-$ to the scalar state $Z_{bc;\bar{u}\bar{d}}^{0}$ are investigated using the QCD three-point sum rule approach. In order to explore the process $T_{bb;\bar{u}\bar{d}}^− \rightarrow Z_{bc;\bar{u}\bar{d}}^0 l\bar{\nu}_l$ we first recalculate the spectroscopic parameters of the tetraquark $T_{bb;\bar{u}\bar{d}}^-$ as well as find the mass and coupling of the scalar four-quark system $Z_{bc;\bar{u}\bar{d}}^0$, which are important ingredients of calculations. The spectroscopic parameters of these tetraquarks are computed in the framework of the QCD two-point sum rule method by taking into account various condensates up to dimension ten. The mass of the $T_{bb;\bar{u}\bar{d}}$ state is found to be $m = (10035 \pm 120)$ MeV, which demonstrates that it is stable against the strong and electromagnetic decays. The full width $\Gamma$ and mean lifetime $\tau$ of $T_{bb;\bar{u}\bar{d}}^-$ are evaluated using its semileptonic decay channels $T_{bb;\bar{u}\bar{d}}^- \rightarrow Z_{bc;\bar{u}\bar{d}}^0 l\bar{\nu}_l$, $l = e, \mu$ and $\tau$. The obtained results, $\Gamma = (7.17 \pm 1.17) \times 10^{-8}$ MeV and $\tau = 9.18_{-1.79}^{+1.81}$ fs, can be useful for experimental investigation of the doubly-heavy tetraquarks.

I. INTRODUCTION

Assumptions about the existence of four-quark bound states (tetraquarks) were made in an early stage of QCD and aimed to explain some of unusual features of the meson spectroscopy. Thus, the nonet of light scalar mesons was considered as bound states of four light quarks rather than being composed of a quark and an antiquark as in the standard models of the mesons. The problems of stability of heavy and heavy-light tetraquarks were also among questions addressed in these studies [1–4].

Due to impressive experimental discoveries and theoretical progress of past fifteen years the study of multi-quark hadrons became an integral part of high energy physics. During this developing and growing period various difficulties in experimental studies, in classification and theoretical interpretation of numerous tetraquarks were successfully overcome [5–8].

But there are still problems in the physics of the exotic hadrons that are not fully solved: the identification of the tetraquark resonances and their stability are among these questions. It is known that the first charmonium-like resonances observed experimentally were interpreted not only as the tetraquarks but also as excited states of the conventional charmonium. Fortunately, there are different classes of tetraquarks which cannot be identified as charmonia or bottomonia states. Indeed, charged resonances carrying one or two units of electric charge, states containing two and more open quark flavors can easily be distinguished from charmonium- or bottomonium-like structures. All of the resonances observed in various experiments and classified as tetraquarks are unstable with respect to strong interactions. They lie either above the open charm (bottom) thresholds or are very close to them. Strong decays of four-quark compounds run through their breaking up to two conventional mesons. Because quarks required for creation of these mesons exist already in the master particles width of such states is rather large. Namely dissociation into two mesons are main strong decay channels of the unstable tetraquarks.

It is natural that theoretical explorations of stable four-quark systems and their experimental discovery remain on the agenda of the particle physics. The tetraquarks built of heavy $c$ or $b$ diquarks and light antidiquarks are real candidates to such states. Their studies have a long history: In fact, the class of exotic mesons $QQQQ$ and $QQQQ$ were studied in Ref. [4, 9, 10], where a potential model with the additive pairwise interaction was used to search for stable tetraquarks. It was demonstrated that in the context of this approach the exotic mesons composed of only heavy quarks are unstable. But the tetraquarks $QQQQ$ may form the stable compounds provided the ratio $m_Q/m_a$ is large. The same conclusions were made in Ref. [11], in which the only constraint imposed on the confining potential was its finiteness when two particles come close together. In accordance with results of this article the isoscalar $J^P = 1^+$ tetraquark $T_{bb;\bar{u}\bar{d}}^-$ resides below the two B-meson threshold and hence, can decay only weakly. At the same time, the tetraquarks $T_{cc;\bar{q}\bar{q}}$ and $T_{bc;\bar{q}\bar{q}}$ may exist as unstable or stable bound states. The stability of the $QQQQ$ compounds in the limit $m_Q \rightarrow \infty$ was studied in Ref. [12], as well.

The various theoretical models, starting from the chiral and dynamical quark models and ending by the relativistic quark model, were used to study properties and compute masses of the $T_{QQ}$ states [13–17]. The masses of the axial-vector states $T_{QQ;\bar{u}\bar{d}}$ were extracted also from the two-point sum rules [18]. In accordance with results of Ref. [18] the mass of the tetraquark $T_{bb;\bar{u}\bar{d}}^-$ amounts to $10.2 \pm 0.3$ GeV, and is below the open bottom threshold. In the context of the same method parameters of the $QQQQ$ states with the spin-parity $0^−$, $0^+$, $1^−$ and $1^+$ were evaluated in Ref. [19]. The production mechanisms
of the $T_{cc}$ tetraquarks such as the heavy ion and proton-proton collisions or the electron-positron annihilations, their possible decay channels were also addressed in the literature [20–23].

The discovery of the doubly charmed baryon $\Xi_{cc}^{++} = ccu$ by the LHCb Collaboration [24] induced new investigations of double-charm and -bottom tetraquarks [25–29]. Thus, the masses of the states $T_{bb;cc}^+$ and $T_{cc;bb}^+$ were estimated once more in the framework of a phenomenological model in Ref. [27]. In accordance with results of this work the mass of the isoscalar axial-vector state $T_{bb;cc}^-$ is equal to $m = 10389 \pm 12$ MeV which is 215 MeV below the $B^-\overline{B}^0$ threshold and 170 MeV below the threshold for decay $B^-\overline{B}^0\gamma$. This means that the tetraquark $T_{bb;cc}^-$ is stable against the strong and electromagnetic decays and breaks up only weakly. At the same time, the mass of the double-charm $T_{cc;bb}^+$ state equals to $3882 \pm 12$ MeV, and is above the thresholds for both to $D^0D^+$ and $D^0D^+\gamma$ decays. The double-charm states $T_{cc;bc}^{++}$ and $T_{cc;bb}^{+++}$ that belong to the class of doubly charged tetraquarks were investigated recently in our work [30]. Because these particles carry two units of the electric charge they may exist only as diquark-antidiquark bound states, which is a source of an additional interest to them. They are above the $D_s^+D_s^{++}(2317)$ and $D^+D_s^{***}(2317)$ thresholds, and width of the strong decays $T_{cc;bc}^{++} \rightarrow D_s^+D_s^{***}(2317)$ and $T_{cc;bc}^{+++} \rightarrow D^+D_s^{***}(2317)$ allowed us to clarify them as relatively broad resonances.

In the light of recent progress made in the physics of double-heavy tetraquarks and expected stability of the $T_{bb;cc}^0$ state its weak decays appear to be very interesting for detailed analysis. The semileptonic decays of four-quark systems, when an initial tetraquark transforms to a final tetraquark and $\ell\nu$ or $b\nu$ leptons are relatively new topic in the physics of the exotic mesons. Thus, the decay of the axial-vector tetraquark $Z_\ell = [c\bar{s}][b\bar{c}]$ to a final state $X(4274)\ell\nu_l$ was studied in Ref. [31]. The widths of these decays, where $l = e, \mu$ and $\tau$, are very small, therefore transitions $Z_\ell \rightarrow X(4274)\ell\nu_l$ were classified in Ref. [31] as rare processes.

In the present work we are going to explore the semileptonic decays of the tetraquark $T_{bb;cc}^-\gamma$ and evaluate its full width and mean lifetime. The tetraquark $T_{bb;cc}^-\gamma$ undergoes the weak decay through the transition $b \rightarrow W^−c$. At the final state its decay products consist of $\ell\nu_l$ and diquark-antidiquark $Z_{bb;cc}^0 = [bc][\overline{d}\overline{u}]$ state (for simplicity, hereafter $Z_{bb;cc}^0$). The tetraquark $Z_{bb;cc}^0$ may decay to $B$ and $D$ mesons with appropriate masses and spin-parities provided its mass is larger than corresponding thresholds. In this scenario $Z_{bb;cc}^0$ decays strongly to the final conventional mesons. Otherwise, at the next stage $Z_{bb;cc}^0$ should decay due to weak or electromagnetic interactions. In the present work we restrict ourselves by considering the semileptonic decay of $T_{bb;cc}^-\gamma$ only to the scalar state $Z_{bb;cc}^0$.

It is remarkable that $Z_{bb;cc}^0$ is the open charm-bottom tetraquark and, at the same time, contains four quarks of different flavors. These two classes of tetraquarks were subjects of rather intensive studies: the masses and strong decays of the scalar and axial-vector tetraquarks $Z_\ell = [c\bar{q}][b\bar{q}]$ and $Z_\ell = [c\bar{s}][b\bar{c}]$ were calculated in Refs. [32] and [33], respectively (see, also references therein). Masses of the open charm-bottom tetraquarks $Z_{bc}$ with different quark contents and spin-parities were computed in Ref. [34]. Two years ago an information of the D0 Collaboration on evidence for the state known as $X(5568)$ [35] directed interest of physicists to compound systems of four distinct quarks. But later both the experimental and theoretical studies of the state $X(5568)$ led to controversial conclusions leaving the status of this tetraquark unclear. Therefore investigation of the process $T_{bb;cc}^-\gamma \rightarrow Z_{bc}^0\ell\nu_l$ can help one to answer questions not only on the features of the tetraquark $T_{bb;cc}^-$ but also to clarify a structure and properties of its decay’s products.

The spectroscopic parameters of $T_{bb;cc}^-\gamma$ and $Z_{bc}^0$ are an important input information to study the semileptonic decay under consideration. In the present work we calculate the masses and couplings of these tetraquarks by employing QCD sum rules obtained from analysis of the relevant two-point correlation functions. In computation of the correlation functions we take into account vacuum expectation values of quark, gluon and mixed local operators up to dimension ten. We are going to evaluate the width of the semileptonic decay $T_{bb;cc}^-\gamma \rightarrow Z_{bc}^0\ell\nu_l$ by applying the standard prescriptions of the QCD three-point sum rule method. Our aim here is to extract the sum rules for the weak form factors $G_i(q^2)$, $i = 0, 1, 2, 3$ and to compute their numerical values. This allow us to determine so-called fit functions $F_i(q^2)$, which coincide with $G_i(q^2)$, but can be extended to a whole region of momentum transfers, which is not accessible to the QCD sum rules. The functions $F_i(q^2)$ are applied to integrate the differential decay rate $d\Gamma/dq^2$ and find the partial width of the decay processes $\Gamma\left(T_{bb;cc}^-\gamma \rightarrow Z_{bc}^0\ell\nu_l\right)$, $l = e, \mu$ and $\tau$.

This article is organized in the following manner: In Sec. 11 we derive the QCD two-point sum rules for the masses and couplings of the tetraquarks $T_{bb;cc}^-\gamma$ and $Z_{bc}^0$, and carry out numerical computations to find their numerical values. In the next section QCD three-point correlation function is used to derive sum rules for the weak form factors $G_i(q^2)$. In this section we perform also numerical analysis of obtained sum rules and determine the fit functions, which allow us to evaluate the width of the semileptonic decay $T_{bb;cc}^-\gamma \rightarrow Z_{bc}^0\ell\nu_l$ and mean lifetime of the state $T_{bb;cc}^-\gamma$. The last section contains discussion of the obtained results and our short conclusions. The explicit expression of the decay rate $d\Gamma/dq^2$ is removed to the Appendix.
II. SPECTROSCOPIC PARAMETERS OF THE TETRAQUARKS $T_{bc,\overline{ss}}^-$ AND $Z_{bc}^0$

In this section we calculate the spectroscopic parameters of the tetraquarks $T_{bc,\overline{ss}}^-$ and $Z_{bc}^0$ by employing the QCD two-point sum rules extracted from analysis of the relevant correlation functions $\Pi_{\mu\nu}(p)$ and $\Pi(p)$.

The function $\Pi_{\mu\nu}(p)$ is defined by the following expression

$$\Pi_{\mu\nu}(p) = i \int d^4xe^{ip\cdot x}\langle 0|T\{J_\mu(x)J_\nu^\dagger(0)\}|0\rangle,$$  \hspace{1cm} (1)

where $J_\mu(x)$ is the interpolating current to the axial-vector tetraquark $T_{bc,\overline{ss}}^-$ composed of an axial-vector diquark and a scalar antidiquark. This current is given by the formula \[18\]

$$J_\mu(x) = b_a^T(x)C\gamma_\mu b_b(x)\pi_a(x)\gamma_5 C\bar{d}^T_b(x).$$ \hspace{1cm} (2)

Here $a$ and $b$ are the color indices and $C$ is the charge conjugation operator.

The correlation function $\Pi(p)$ for the scalar tetraquark $Z_{bc}^0$ has the form

$$\Pi(p) = i \int d^4xe^{ip\cdot x}\langle 0|T\{J^Z(x)J^{Z\dagger}(0)\}|0\rangle,$$ \hspace{1cm} (3)

where the current $J^Z(x)$ is defined as \[34\]

$$J^Z(x) = b_a^T(x)C\gamma_\mu c_a(x)\left[\pi_a(x)\gamma_5 C\bar{d}^T_b(x) - \bar{\pi}_b(x)\gamma_5 C\bar{d}^T_a(x)\right].$$ \hspace{1cm} (4)

From Eq. (3) it is clear that $Z_{bc}^0$ is the binding state of a scalar diquark and antidiquark, and belongs to the triplet representation of the color group.

We concentrate here on calculation of the parameters of the tetraquark $T_{bc,\overline{ss}}^-$ and provide only necessary expressions and final results for $Z_{bc}^0$. In accordance with QCD sum rule method one first has to express the correlation function $\Pi_{\mu\nu}(p)$ in terms of the tetraquarks' mass $m$ and coupling $f$, which form the phenomenological or physical side of the sum rules. We treat the tetraquark $T_{bc,\overline{ss}}^-$ as a ground-state particle in its class, therefore isolate only first term in $\Pi_{\mu\nu}^{\text{phys}}(p)$ that is given by the formula

$$\Pi_{\mu\nu}^{\text{phys}}(p) = \frac{\langle 0|J_\mu T(p)|T(p)J_\nu^\dagger(0)\rangle}{m^2 - p^2} + \ldots$$ \hspace{1cm} (5)

This expression is derived by saturating the correlation function Eq. (1) with a complete set of states with $J^P = 1^+$ and performing the integration over $x$. The dots here indicate contribution to $\Pi_{\mu\nu}^{\text{phys}}(p)$ of higher resonances and continuum states.

The function $\Pi_{\mu\nu}^{\text{phys}}(p)$ can be further simplified by introducing the matrix element

$$\langle 0|J_\mu T(p, \epsilon) = f\epsilon_{\mu},$$ \hspace{1cm} (6)

where $\epsilon_{\mu}$ is the polarization vector of the $T_{bc,\overline{ss}}^-$ state. It is not difficult to demonstrate that in terms of $m$ and $f$ the function takes the following form

$$\Pi_{\mu\nu}^{\text{phys}}(p) = \frac{m^2f^2}{m^2 - p^2} \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}\right) + \ldots$$ \hspace{1cm} (7)

To suppress the contribution arising from the higher resonances and continuum, we carry out the Borel transformation of the correlation function, which reads

$$\mathcal{B}\Pi_{\mu\nu}^{\text{phys}}(p) = m^2f^2e^{-m^2/M^2}\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{M^2}\right) + \ldots,$$ \hspace{1cm} (8)

where $M^2$ is the Borel parameter.

The second part of the sum rule is given by the same correlation function $\Pi_{\mu\nu}(p)$, but expressed in terms of the quark propagators

$$\Pi_{\mu\nu}^{\text{OPE}}(p) = i \int d^4xe^{ip\cdot x}\left\{\text{Tr}\left[\gamma_5\tilde{S}^a_{\mu\nu}(x)\gamma_5 S_a^{b\dagger}(x)\right] - \text{Tr}\left[\gamma_5\tilde{S}^a_{b\mu}(x)\gamma_5 S_a^{b\dagger}(x)\right]\right\}.$$ \hspace{1cm} (9)

In Eq. (9) $S_a^{b\dagger}(x)$ and $S_a^b(x)$ are the $b$ and $q(u, d)$-quark propagators, explicit expression of which can be found, for example, in Ref. [31]. Here we introduce also the notation

$$\tilde{S}_{b(q)}(x) = CS_{b(q)}^T(x).$$ \hspace{1cm} (10)

The QCD sum rule can be extracted by using the same Lorentz structures in both of $\Pi_{\mu\nu}^{\text{phys}}(p)$ and $\Pi_{\mu\nu}^{\text{OPE}}(p)$. The structures $\sim g_{\mu\nu}$ are appropriate for our purposes, because they receive contributions only from spin-1 particles. The invariant amplitude $\Pi_{\mu\nu}^{\text{OPE}}(p^2)$ corresponding to this structure can be represented as the dispersion integral

$$\Pi_{\mu\nu}^{\text{OPE}}(p^2) = \int_{4m_q^2}^{\infty} \frac{\rho_{\text{OPE}}(s)}{s - p^2} ds + \ldots,$$ \hspace{1cm} (11)

where $\rho_{\text{OPE}}(s)$ is the two-point spectral density. It is proportional to an imaginary part of the structure $\sim g_{\mu\nu}$ in the function $\Pi_{\mu\nu}^{\text{phys}}(p)$. In the present work $\rho_{\text{OPE}}(s)$ is calculated by taking into account the quark, gluon and mixed vacuum condensates up to dimension ten.

By applying the Borel transformation to $\Pi_{\mu\nu}^{\text{OPE}}(p^2)$, equating the obtained expression with the relevant part of the function $\mathcal{B}\Pi_{\mu\nu}^{\text{phys}}(p)$, and performing the continuum subtraction we find the final sum rules. Then the mass of the $T_{bc,\overline{ss}}^-$ state can be evaluated from the sum rule

$$m^2 = \frac{\int_{4m_q^2}^{\infty} ds \rho_{\text{OPE}}(s)e^{-s/M^2}}{\int_{4m_q^2}^{\infty} ds \rho_{\text{OPE}}(s)e^{-s/M^2}} = \frac{f^4\pi}{4m_q^2} \int_{4m_q^2}^{\infty} ds \rho_{\text{OPE}}(s)e^{-s/(M^2)}.$$

whereas to find the coupling $f$ we employ the expression

$$f^2 = \frac{1}{4m_q^2} \int_{4m_q^2}^{\infty} ds \rho_{\text{OPE}}(s)\epsilon^2/(m^2 - s/M^2)^2.$$
Here $s_0$ is the continuum threshold parameter that separates the ground-state and continuum contributions from each other.

In the case of the scalar tetraquark $Z^0_{bc}$, there are some differences stemming from its spin-parity and from structure of the interpolating current. Thus, the matrix element $(0\langle J^Z|z(p))$ has the form

$$
(0\langle J^Z|z(p)) = f_Z m_Z,
$$

(14)

which is analogous to the matrix element of a conventional scalar meson. The correlation function $\Pi^{\text{OPE}}(p)$ is given by the expression

$$
\Pi^{\text{OPE}}(p) = i \int d^4xe^{ip \cdot x} \text{Tr} \left[ S^{ab'}_c(\gamma_5 S_{a'}^{ab})(\gamma_5 S_{b'}^{bc})(\gamma_5 S_{c'}^{cd})(\gamma_5 S_{d'}^{dc})(\gamma_5 S_{0}) \right]
$$

(15)

The remaining manipulations and final sum rules for $m_Z$ and $f_Z$ are similar to ones for the tetraquark $T^{bc}_{bb, uu}$. The obtained sum rules depend on the quark, gluon and mixed condensates numerical values of which are collected in Table I. This table contains also the masses of the $b$ and $c$-quarks, that appear in the sum rules as input parameters.

Besides, Eqs. (12) and (13) depend on auxiliary parameters $M^2$ and $s_0$ which should satisfy standard constraints of the sum rules computations. Our analysis proves that the working windows

$$
M^2 \in [9, 13] \text{ GeV}^2, \ s_0 \in [115, 120] \text{ GeV}^2,
$$

(16)

meet all restrictions imposed on $M^2$ and $s_0$. Thus, the maximum of the Borel parameter is determined from the minimal allowed value of the pole contribution (PC), that at $M^2 = 13 \text{ GeV}^2$ equals to 16% of the full correlation function. Within the region $M^2 \in [9, 13] \text{ GeV}^2$ the pole contribution varies from 59% till 16%. The lower limit of the Borel parameter is fixed from convergence of the operator product expansion (OPE) for the correlation function. In the present work we use the criterium

$$
R(M^2) = \frac{\Pi^{\text{Dim}(8+9+10)}(M^2, s_0)}{\Pi(M^2, s_0)} < 0.05,
$$

(17)

where $\Pi(M^2, s_0)$ is the Borel transformed and subtracted function $\Pi^{\text{OPE}}(p^2)$, and $\Pi^{\text{Dim}(8+9+10)}(M^2, s_0)$ is contribution of the last three terms in its expansion. At $M^2 = 9 \text{ GeV}^2$ the ratio $R$ is equal to $R(9 \text{ GeV}^2) = 0.01$ which ensures the excellent convergence of the sum rules. Moreover, at $M^2 = 9 \text{ GeV}^2$ the perturbative contribution amounts to 74% of the full result exceeding considerably the nonperturbative terms.

The quantities evaluated by means of the sum rules, in general, should not depend on the auxiliary parameters $M^2$ and $s_0$. But in calculations of the mass $m$ and coupling $f$ we observe a residual dependence on $M^2$ and $s_0$. Therefore, stability of extracted parameters, i.e., of $m$ and $f$ is necessary condition to fix the working windows for $M^2$ and $s_0$. In Fig. 1 as an example, we plot dependence of the mass of the tetraquark $T^{bb, uu}$ on the parameters $M^2$ and $s_0$ which is mild. The coupling $f$ is more sensitive to the choice of $M^2$ and $s_0$, nevertheless corresponding ambiguities do not exceed 30% of the results.

Our analysis for the mass and coupling of the tetraquark $T^{bb, uu}$ predicts:

$$
m = (10035 \pm 120) \text{ MeV}, \quad f = (1.38 \pm 0.25) \cdot 10^{-2} \text{ GeV}^4.
$$

(18)

The similar studies of the $Z^0_{bc}$ lead to the following results:

$$
m_Z = (6660 \pm 115) \text{ MeV}, \quad f_Z = (0.51 \pm 0.19) \cdot 10^{-2} \text{ GeV}^4,
$$

(19)

which have been obtained using the working regions

$$
M^2 \in [5.5, 6.5] \text{ GeV}^2, \ s_0 \in [53, 55] \text{ GeV}^2.
$$

(20)

It is worth noting that in calculations of $m_Z$ and $f_Z$ the pole contribution PC changes within limits 55% - 21%. Contribution of the last three terms to the corresponding correlation function at the point $M^2 = 5.5 \text{ GeV}^2$ amounts to 1.9% of the total result that guarantees the convergence of the sum rules. In Fig. 2 we depict the mass of the tetraquark $Z^0_{bc}$ as a function of $M^2$ and $s_0$.
As it was noted above, the mass of the state $T_{bb\bar{u}\bar{d}}^{-}$ was evaluated using different approaches in Refs. [18] and [25]. Our result for the mass $m$ is smaller than prediction obtained in Ref. [18] using the QCD sum rule method with dimension-eight accuracy: There is an overlapping region between these two predictions, but the central values differ from each other. This discrepancy is presumably connected with the accuracy of performed analysis, and with the choice of the working intervals for the parameters $M^2$ and $s_0$.

The recent model analysis (see Ref. [25]) leads to $m$ that is larger than the present result. Nevertheless, all calculations confirm that the tetraquark $T_{bb\bar{u}\bar{d}}^{-}$ is stable against the strong and electromagnetic decays and can dissociate only weakly.

### III. SEMILEPTONIC DECAY $T_{bb\bar{u}\bar{d}}^{-} \rightarrow Z_{bc}^{0} \ell \nu_{\ell}$

The semileptonic decay of the tetraquark $T_{bb\bar{u}\bar{d}}^{-}$ to the final state $Z_{bc}^{0} \ell \nu_{\ell}$ runs through the chain of transitions $b \rightarrow W^{-}c$ and $W^{-} \rightarrow \ell \nu_{\ell}$. As is seen from the previous section, the difference between the initial and final tetraquarks’ masses is large enough to make all decays $l = e, \mu$ and $\tau$ kinematically allowed processes.
At the tree-level the transition $b \rightarrow c$ can be described using the effective Hamiltonian

$$\mathcal{H}^{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{bc} \Gamma \mu \nu (1 - \gamma_5) b \gamma^\mu (1 - \gamma_5) v_l,$$  \hspace{1cm} (21)

where $G_F$ is the Fermi coupling constant and $V_{bc}$ is the corresponding element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. After sandwiching the $\mathcal{H}^{\text{eff}}$ between the initial and final tetraquarks and factoring out the lepton fields we get the matrix element of the current

$$J^{\mu^p}_{\mu^s} = \mathcal{G}_\mu \mu (1 - \gamma_5) b,$$  \hspace{1cm} (22)

in terms of the form factors $G_i(q^2)$ that parameterize the long-distance dynamics of the weak transition. \[36\]

$$\langle Z(p') | J^{\mu^p}_{\mu^s} | T(p, e) \rangle = \tilde{G}_0(q^2) \epsilon_+ + \tilde{G}_1(q^2) (e^p) P_\mu + \tilde{G}_2(q^2) (e^p) q_\mu + i \tilde{G}_3(q^2) \epsilon_{\mu \alpha \beta} e^p \rho^\alpha \rho^\beta.$$  \hspace{1cm} (23)

The scaled functions $\tilde{G}_i(q^2)$ above are connected with the dimensionless form factors $G_i(q^2)$ by the following equalities

$$\tilde{G}_0(q^2) = \tilde{m} G_0(q^2), \quad \tilde{G}_j(q^2) = \frac{G_j(q^2)}{\tilde{m}}, \quad j = 1, 2, 3.$$  \hspace{1cm} (24)

In Eqs. (23) and (24) \[37\] $\tilde{m} = m + m_Z$, $p$ and $e$ are the momentum and polarization vector of the tetraquark $T^{\pm, \mp, \pm}$, $p'$ is the momentum of the state $Z^{0, 0}_b$, $P_\mu = p'_\mu + p_\mu$, and $q_\mu = p_\mu - p'_\mu$ is the momentum transferred to the leptons. It is clear that $q^2$ changes within the limits $m_l^2 < q^2 < (m - m_Z)^2$, where $m_l$ is the mass of the lepton $l$. The form factors $G_i(q^2)$ are quantities which should be extracted from the sum rules which, in turn, are obtainable from analysis of the three-point correlation function

$$\Pi_{\mu \nu}(p, p') = i^2 \int d^4xd^4ye^{i(p'y - px)}$$

$$\times \{0 | T \{J^{\mu^p}_{\mu^s} (y), J_t^{\mu^p}_{\mu^s} (0), J^Z (x) \} | 0 \},$$  \hspace{1cm} (25)

where $J_t (x)$ and $J^Z (y)$ are the interpolating currents to the $T^{\pm, \mp, \pm}$ and $Z^{0, 0}_b$ states, respectively.

To derive sum rules for the weak form factors we express the correlation function $\Pi_{\mu \nu}(p, p')$ in terms of the masses and couplings of the involved particles, and, by this way, determine the physical or phenomenological side of the sum rule $\Pi^{\text{phys}}_{\mu \nu}(p, p')$. We also calculate $\Pi_{\mu \nu}(p, p')$ using the interpolating currents and quark propagators, which lead to its expression in terms of quark, gluon and mixed vacuum condensates. By matching the obtained results and employing the assumption on the quark-hadron duality it is possible to extract sum rules and evaluate the physical parameters of interest.

The function $\Pi^{\text{phys}}_{\mu \nu}(p, p')$ can be easily written down in the form

$$\Pi^{\text{phys}}_{\mu \nu}(p, p') = \frac{\langle 0 | J^{\mu^p}_{\mu^s} (Z(p')) \langle Z(p') | J^Z (T(p, e)) \rangle}{(p^2 - m^2)(p'^2 - m_Z^2)}$$

$$\times \langle T(p, e), J_t^{\mu^p}_{\mu^s} | 0 \rangle + \ldots,$$  \hspace{1cm} (26)

where we take into account contribution arising only from the ground-state particles, and denote by the dots effects of the excited and continuum states.

The phenomenological side of the sum rules can be further simplified by rewriting the relevant matrix elements in terms of the tetraquarks’ parameters, and employing for $\langle Z(p') | J^Z (T(p, e)) \rangle$ its expression through the weak transition form factors $G_i(q^2)$. The matrix element of the tetraquarks $T^{\pm, \mp, \pm}$ and $Z^{0, 0}_b$ are known and given by Eqs. (6) and (14), respectively. The matrix element $\langle Z(p') | J^Z (T(p, e)) \rangle$ is modeled by means of the four transition form factors $G_i(q^2)$ which can be used for calculation all of three semileptonic decays.

Substituting the relevant matrix elements into Eq. (20), for $\Pi^{\text{phys}}_{\mu \nu}(p, p', q^2)$ we finally get

$$\Pi^{\text{phys}}_{\mu \nu}(p, p', q^2) = \frac{f m_f Z m}{(p^2 - m^2)(p'^2 - m_Z^2)}$$

$$\times \left\{ \tilde{G}_0 (q^2) \left( -g_{\mu \nu} + \frac{\rho_{\mu \rho} \rho_{\nu \sigma}}{m^2} \right) + \tilde{G}_1 (q^2) P_\mu + \tilde{G}_2 (q^2) q_\mu \right\}$$

$$-i \tilde{G}_3 (q^2) \epsilon_{\mu \alpha \beta} e^p \rho^\alpha \rho^\beta \} \ldots.$$  \hspace{1cm} (27)

The function $\Pi^{\text{OPE}}_{\mu \nu}(p, p')$ constitutes the second side of the sum rules and has the following form

$$\Pi^{\text{OPE}}_{\mu \nu}(p, p') = \int d^4xd^4ye^{i(p'y - px)} \left\{ \text{Tr} \left[ \tilde{G}_3 (x - y) \gamma_5 \tilde{S}_d^{b'} (x - y) \right] \right.$$

$$\times \gamma_5 \gamma_5 \gamma_5 \gamma_5 (x - y) \left[ \text{Tr} \left[ \gamma_\mu \tilde{S}_b^{a'} (y - x) \gamma_5 \tilde{S}_d^{b}(y) \gamma_5 (1 - \gamma_5) \right] \right.$$

$$\times \tilde{S}_b^{b'} (y - x) \right\} + \text{Tr} \left[ \gamma_\mu \tilde{S}_d^{a'} (x - y) \right.$$

$$\gamma_5 \tilde{S}_d^{b} (y - x) \gamma_5 \tilde{S}_b^{b} (x - y) \left\} \right.$$
It is convenient to present obtained sum rules in a single formula, through the functions $\tilde{G}_i(q^2)$,

$$\tilde{G}_i(M^2, s_0, q^2) = \frac{1}{16 \pi f_M f_Z m_Z} \int_{s_0}^{\infty} ds \int_{m_b + m_c}^{m_b + m_c} ds' \rho_i(s, s', q^2) q^2 / M_i^2 M_i^2 / (s^2 - s') / M_i^2,$$  

(29)

bearing in mind that they are connected to the dimensionless form factors $G_i(q^2)$ by Eq. (24). Here $M^2 = (M_1^2, M_2^2)$ are the Borel parameters, and $s_0 = (s_0, s'_0)$ are the continuum threshold parameters that separate the main contribution to the sum rules from the continuum effects. The sum rules (29) are written down using the spectral densities $\rho_i(s, s', q^2)$ which are proportional to the imaginary part of the corresponding invariant amplitudes in $\Pi_{\text{fit}}^{\text{OPE}}(p, p')$. They contain the perturbative and nonperturbative contributions, and are calculated with dimension-6 accuracy.

For numerical computations of the weak form factors $G_i(M^2, s_0, q^2)$ one needs to fix various parameters. Values of some of them are collected in Table I, the masses and coupling constants of the tetraquarks $T_{bb\pi\pi}$ and $Z_{bc}$ have been evaluated in the previous section. In the present computations we impose on the auxiliary parameters $M^2$ and $s_0$ the same constraints as in the mass calculations.

To obtain the width of the decay $T_{bb\pi\pi} \to Z_{bc}^0 \pi_l$, one has to integrate the differential decay rate $d\Gamma / dq^2$ (for details, see Appendix) within allowed kinematical limits $m_i^2 \leq q^2 \leq (m - m_Z)^2$. It is clear that for light leptons $l = e, \mu$ the lower limit of the integral is considerably smaller than 1 GeV$^2$, but the perturbative calculations lead to reliable predictions for momentum transfers $q^2 > 1$ GeV$^2$. Therefore, we use the usual prescription and replace the weak form factors in the whole integration region by fit functions $F_i(q^2)$ which for perturbatively allowed values of $q^2$ coincide with $G_i(q^2)$.

There are various analytical expressions for the fit functions. In the present work we utilize

$$F_i(q^2) = f_i^0 \exp \left[ c_{i1} \frac{q^2}{m_{\text{fit}}^2} + c_{i2} \left( \frac{q^2}{m_{\text{fit}}^2} \right)^2 \right],$$  

(30)

where $f_i^0, c_{i1}, c_{i2}$ and $m_{\text{fit}}^2$ are fitting parameters. Values of these parameters are presented in Table III. Besides that, for the numerical calculations we need the Fermi coupling constant $G_F$ and CKM matrix element $|V_{bc}|$ for which we use:

$$G_F = 1.16637 \cdot 10^{-5} \text{ GeV}^{-2},$$  

$$|V_{bc}| = (41.2 \pm 1.0) \cdot 10^{-3}. $$  

(31)

As a result, for the decay width of the processes $T_{bb\pi\pi} \to Z_{bc}^0 l\bar{l}$, $l = e, \mu$ and $\tau$ we find

$$\Gamma \left( T_{bb\pi\pi} \to Z_{bc}^0 l\bar{l} \right) = (2.65 \pm 0.74) \cdot 10^{-8} \text{ MeV},$$  

$$\Gamma \left( T_{bb\pi\pi} \to Z_{bc}^0 \mu\bar{\mu} \right) = (2.64 \pm 0.73) \cdot 10^{-8} \text{ MeV},$$  

$$\Gamma \left( T_{bb\pi\pi} \to Z_{bc}^0 \tau\bar{\tau} \right) = (1.88 \pm 0.53) \cdot 10^{-8} \text{ MeV},$$  

(32)

which are the main results of the present work.

The partial decay widths from Eq. (32) can be used to estimate the full width and mean lifetime of the tetraquark $T_{bb\pi\pi}$

$$\Gamma = (7.17 \pm 1.17) \cdot 10^{-8} \text{ MeV},$$  

$$\tau = 9.18^{+1.79}_{-1.29} \cdot 10^{-15} \text{ s}. $$  

(33)

These predictions can be employed to explore the double-heavy tetraquarks.

### IV. ANALYSIS AND CONCLUSIONS

The spectroscopic parameters of the tetraquarks $T_{bb\pi\pi}^-$ and $Z_{bc}^0$, as well as the width of the semileptonic decay $T_{bb\pi\pi}^- \to Z_{bc}^0 l\bar{l}$ provide very interesting information on properties of four-quark states. Thus, the mass of the tetraquark $T_{bb\pi\pi}^-$ obtained in the present work confirms once more that it is stable against strong and electromagnetic decays, and can transform only weakly to tetraquark $Z_{bc}$ and a pair of leptons $l\bar{l}$. This conclusion is valid even to take into account uncertainties inherent to the sum rule computations. Our result for $m$ is smaller than the predictions made in Refs. 13 and 24, using the QCD sum rule method and phenomenological model estimations, respectively. The semileptonic decays $T_{bb\pi\pi}^- \to Z_{bc}^0 l\bar{l}$, where $l = e, \mu$ and $\tau$ have allowed us to evaluate the width of $T_{bb\pi\pi}^-$ and its mean lifetime $\tau$. Our result for $\tau = 9.18^{+1.79}_{-1.29}$ fs is considerably smaller than prediction of Ref. 23.

Another interesting result of this work is connected with parameters of the scalar tetraquark $Z_{bc}^0$, composed of the heavy diquark $bc$ and light antidiquark $\bar{\pi}$. In fact, the mass of this state $m_Z = (6660 \pm 115)$ MeV is considerably below the threshold $\approx 7145$ MeV for strong $S^-$ wave decays to conventional heavy $B^- D^+$ and $B_s^0 D^0$. 

| $F_i(q^2)$ | $f_i^0$ | $c_{i1}$ | $c_{i2}$ | $m_{\text{fit}}^2$ (GeV$^2$) |
|------------|--------|--------|--------|-----------------|
| $F_0(q^2)$ | 2.34   | 19.53  | -36.87 | 100.70          |
| $F_1(q^2)$ | -1.75  | 18.45  | -14.29 | 100.70          |
| $F_2(q^2)$ | 8.80   | 20.21  | -32.09 | 100.70          |
| $F_3(q^2)$ | 17.13  | 20.60  | -32.49 | 100.70          |

### TABLE II: The parameters of the fit functions $F_i(q^2)$. 

$T_{bb\pi\pi}^- \to Z_{bc}^0 l\bar{l}$, $l = e, \mu$ and $\tau$ we find

$$\Gamma \left( T_{bb\pi\pi}^- \to Z_{bc}^0 e\bar{\nu}_e \right) = (2.65 \pm 0.74) \cdot 10^{-8} \text{ MeV},$$  

$$\Gamma \left( T_{bb\pi\pi}^- \to Z_{bc}^0 \mu\bar{\nu}_\mu \right) = (2.64 \pm 0.73) \cdot 10^{-8} \text{ MeV},$$  

$$\Gamma \left( T_{bb\pi\pi}^- \to Z_{bc}^0 \tau\bar{\nu}_\tau \right) = (1.88 \pm 0.53) \cdot 10^{-8} \text{ MeV},$$  

(32)

which are the main results of the present work.

The partial decay widths from Eq. (32) can be used to estimate the full width and mean lifetime of the tetraquark $T_{bb\pi\pi}^-$. 

$$\Gamma = (7.17 \pm 1.17) \cdot 10^{-8} \text{ MeV},$$  

$$\tau = 9.18^{+1.79}_{-1.29} \cdot 10^{-15} \text{ s}. $$  

(33)

These predictions can be employed to explore the double-heavy tetraquarks.
mesons. Because the quark content \( Z_{bc}^0 \) cannot decay to a pair of heavy and light mesons, as well. These features differ qualitatively from the open charm-bottom scalar tetraquarks \( Z_q = [cq][\bar{b}\bar{d}] \) and \( Z_s = [cs][\bar{b}\bar{d}] \), which decay strongly to \( B_s \pi \) and \( B_c q \) mesons [32], and, in turn, cannot fall apart to two heavy mesons. In other words, the four-quark systems built of a heavy-heavy diquark and a light-light antidiquark are more stable than ones made of a heavy-light diquark and a heavy-light antidiquark. This is seen from comparison of the masses of the tetraquark \( Z_{bc}^0 \) and the state \( Z_q \) for which \( m_{Z_q} = (6.97 \pm 0.19) \) GeV.

A theoretical information on the decay properties of the state \( T_{bc, \pi}^- \) can be further improved by including in analyses its other weak decay channels. Investigation of the stable open charm-bottom tetraquarks \( Z_{bc}^0 \) with different quantum numbers is also among interesting topics of the exotic hadrons’ physics: By clarifying these problems we can deepen our understanding of multiquark systems.

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**Appendix: The decay rate \( d\Gamma/dq^2 \)**

This Appendix contains explicit expression for the decay rate \( d\Gamma/dq^2 \) necessary to calculate the width of the semileptonic decay \( T_{bc, \pi}^- \rightarrow Z_{bc}^0 \pi \). Calculations led to the following result:

\[
\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{3 \cdot 2^8 \pi^3 m^3} \left( \frac{q^2 - m_1^2}{q^2}\right) \lambda (m^2, m_Z^2, q^2) \left[ \sum_{i=0}^{1=3} \bar{G}_i(q^2)A_i(q^2) + \bar{G}_0(q^2) \bar{G}_1(q^2) A_{01}(q^2) + \bar{G}_1(q^2) \bar{G}_2(q^2) A_{12}(q^2) \right],
\]

(A.1)

In Eq. (A1) the functions \( A_i(q^2) \) and \( A_{ij}(q^2) \) are given by the expressions:

\[
\begin{align*}
A_0(q^2) &= \frac{1}{2m^4 q^4} \left[ q^4 (m^2 - m_Z^2)^2 - 4q^2 m^2 m_Z^2 - m_i^4 (m^2 - m_Z^2 + q^2)^2 + 2q^2 (3m^2 - m_Z^2) + q^2 \right], \\
A_1(q^2) &= \frac{1}{2m^4 q^4} \left[ m^4 + (m_Z^2 - q^2)^2 - 2m^2(m_Z^2 + q^2) \right] \left[ m_i^4 (m^2 - m_Z^2)^2 + q^2 m_i^4 (q^2 - 2m^2 - 2m_Z^2) \\
&\quad - q^2 \left[ m^4 + (m_Z^2 - q^2)^2 - 2m^2(m_Z^2 + q^2) \right] \right], \\
A_2(q^2) &= \frac{m_i^2}{2m^2 q^2} \left[ q^2 (m^2 - m_Z^2) \right] \left[ m^4 + (m_Z^2 - q^2)^2 - 2m^2(m_Z^2 + q^2) \right], \\
A_3(q^2) &= \frac{1}{2q^2} \left[ m_i^4 - q^4 \right] \left[ m^4 + (m_Z^2 - q^2)^2 - 2m^2(m_Z^2 + q^2) \right], \\
A_{01}(q^2) &= \frac{1}{m^2 q^4} \left[ q^4 (m^2 + m_Z^2 - m^2 - q^2) + m_i^4 (m^2 - m_Z^2) \right] \left[ m^4 + (m_Z^2 - q^2)^2 - 2m^2(m_Z^2 + q^2) \right], \\
A_{02}(q^2) &= \frac{m_i^2 (m^2 - q^2)}{m^2 q^2} \left[ m^4 + (m_Z^2 - q^2)^2 - 2m^2(m_Z^2 + q^2) \right], \\
A_{12}(q^2) &= \frac{m_i^2 (q^2 - m_i^2)(m^2 - m_Z^2)}{m^2 q^2} \left[ m^4 + (m_Z^2 - q^2)^2 - 2m^2(m_Z^2 + q^2) \right],
\end{align*}
\]

(A.2)

and

\[
\lambda (m^2, m_Z^2, q^2) = \left[ m^4 + m_Z^4 + q^4 - 2 (m_Z^2 q^2 + m^2 q^2 + m_Z^2 q^2) \right]^{1/2}.
\]

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