Renormalization Group Flow in BRST Invariant Open String $\sigma$-Model

Rui Neves

Área Departamental de Física, UCE, Universidade do Algarve

Campus de Gambelas, 8000 Faro, Portugal

rneves@mozart.ualg.pt

December 24, 2018

Abstract

The renormalization group flow in the theory space of a BRST invariant string $\sigma$-model is investigated. For the open bosonic string the non-perturbative off-shell effective action and its gauge symmetry properties are determined from $\beta$-functions defined by the local Weyl anomaly. The interactions are shown to explicitly break the free theory BRST invariance generating new non-linear gauge symmetries of the type present in Witten’s string field theory. In the Feynman-Siegel gauge the $\sigma$-model is shown to generate Witten’s structure of vertex couplings.

1 Introduction

Conformal invariance on the world-sheet is at the heart of string theory. One of its fundamental consequences is that the dynamics of strings is constrained to curved spacetime backgrounds which are solutions of the string equations of motion. These conformal symmetry conditions are equivalent to the vanishing of the non-linear $\sigma$-model $\beta$-functions and so classical string vacua as well as on-shell string scattering amplitudes are characteristics of the renormalization group (RG) fixed points. This was first discovered when

*Research supported by F.C.T.’s PRAXIS XXI Post-Doctoral Fellowship BPD/14137/97.
studying string dynamics on massless background fields \([1]\) and rapidly seen to hold when including condensates of other string modes \([2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]\).

The conformal fixed points belong to a RG flow in an infinite dimensional field theory space spanned by all the string \(\sigma\)-model couplings. As indicated by the investigations in the vicinity of the on-shell gaussian fixed point, the RG flow looks like a gradient flow generated by a background field effective action \(I\) and a theory space metric \(G^{ij}\) (see for example \([4, 5, 8, 10]\)). The \(\beta\)-function \(\beta^j\) corresponding to the coupling \(g^j\) is then given by

\[
\beta^j = G^{ij} \frac{\delta I}{\delta g^i}.
\]

Since the effective action \(I\) is to be considered an off-shell functional of all string couplings it may naturally be interpreted as the tree level action for a string field theory.

The precise non-perturbative definition of the possible spacetime effective actions \(I\) associated with the RG flow is still an open problem. At the free field theory level the early uncovering of the \(L_{-1}\) string gauge invariances has not yet been followed by an equally clear successful generation of the \(L_{-n}\), \(n \geq 2\) gauge symmetries \([6, 9, 10, 12, 13, 14, 15]\). At the level of the interactions a near mass-shell field redefinition ambiguity was found unavoidable when deducing the covariant form of gauge fixed actions \([16]\). Without the field redefinitions the non-perturbative interactions were shown to depend on the chosen regulator \([14, 18]\) and a scheme \([18]\) was found for which the structure of Witten’s string field theory (WSFT) and the associated non-linear gauge symmetry \([19, 20]\) are obtained from the \(\sigma\)-model RG flow.

In this letter we consider the introduction of ghost field couplings in the \(\sigma\)-model based on the approach of Jain and Jevicki \([13]\). To study we select the open bosonic string and the region of theory space corresponding to the tachyon \(T\), the photon \(A^\mu\) and the ghost field \(\alpha\). We construct a BRST invariant string \(\sigma\)-model and determine the non-perturbative off-shell effective action \(I\) and its gauge symmetry properties. To be able to do so we define the \(\beta\)-functions by the local Weyl anomaly calculated via the covariant heat kernel regulator. We also require a unique but non-local theory space metric \(G^{ij}\). We show that the interactions explicitly break the free field theory BRST invariance generating new non-linear gauge symmetries of the type present in WSFT. We also show
that in the Feynman-Siegel gauge $\alpha = 0$ the effective action $I$ has Witten’s structure of vertex couplings. Decoupling the ghost sector the theory space metric becomes trivial and the curved world-sheet $\sigma$-model for $T$ and $A^\mu$ still generates the structure of Witten’s theory without the need for the unsatisfactory regularization scheme recently used in the literature [18].

2 The BRST Invariant String $\sigma$-Model

As was proposed by Jain and Jevicki [13] the spacetime ghost background fields may be introduced in the string $\sigma$-model if the reparametrization ghosts are bosonized and then coupled to the matter sector. Considering the open bosonic string in $d$ dimensions and the theory space associated with $T$, $A^\mu$ and $\alpha$, the 2D field theory action $S$ is defined as $S = S_0 + S_B$ where

$$S_0 = \frac{1}{4\pi\alpha'} \int d^2 \tilde{s} \left( X^\mu \Delta X_\mu + Y \Delta Y \right) + \frac{Q}{4\pi \sqrt{\alpha'}} \left( \int d^2 \tilde{s} \tilde{R}Y + 2 \int d\tilde{s} \tilde{g} \tilde{k}_\parallel Y \right), \quad (2)$$

and

$$S_B = \frac{g}{2\sqrt{2\alpha'}} \int d\tilde{s} \left\{ \sqrt{2} T(X) + i \partial_{\tilde{t}} X^\mu A_\mu(X) + \frac{i}{Q} \partial_{\tilde{t}} Y \left[ \sqrt{\alpha'} \partial \cdot A(X) - \alpha(X) \right] \right\}. \quad (3)$$

As is clear this action has been constructed to be invariant under the following BRST transformation of the bare field couplings ($\Box = \partial^\mu \partial_\mu$)

$$\delta T(X) = 0,$$

$$\delta A^\mu(X) = \sqrt{\alpha'} \partial^\mu \Lambda(X),$$

$$\delta \alpha(X) = \alpha' \Box \Lambda(X). \quad (4)$$

This classical property is clearly a consequence of the $U(1)$ breaking background field interaction which couples the matter and ghost sectors. It is independent of the normalization constants chosen in the boundary action $S_B$ which are justified by the quantum theory.

In formulae (2), (3) and (4) $\alpha'$ is the standard Regge slope and $g$ the string coupling. On the world-sheet $X^\mu$ and $Y$ are taken to satisfy Neumann boundary conditions, $\partial_{\tilde{r}} X^\mu = 0 = \partial_{\tilde{r}} Y$. The world-sheet is a 2D surface with $\tilde{R}$ and $k_\parallel$ as its scalar and geodesic curvatures. Its metric is in the conformal gauge, $\tilde{g}_{ab}(\xi) = \exp \left[ 2\varphi(\xi) \right] \delta_{ab}$, where the coordinates
\[ \xi = (\tau, \zeta), \quad -\infty < \tau < +\infty, \quad 0 \leq \zeta < +\infty \] span the upper-half plane and \( \varphi \) is the Liouville field also satisfying Neumann boundary conditions \( \partial_\hat{n} \varphi = \exp[-\varphi(\tau)] \partial_\xi \varphi(\xi) |_{\xi = 0} = 0 \).

The integration elements are \( d^2 \tilde{s} = d^2 \xi \sqrt{g} = d^2 \xi \exp[2\varphi(\xi)] \) and \( d\tilde{s} = d\tau \exp[\varphi(\tau)] \).

The kinetic operator is the covariant laplacian \( \tilde{\Delta} = -(1/\sqrt{g}) \partial_a \sqrt{g} g^{ab} \partial_b = -\exp[-2\varphi(\xi)] \partial^2 \) and \( \tilde{\partial}_t = \exp[-\varphi(\tau)] \partial_\tau \) is the derivative tangent to the boundary. The 2D surface curvatures are linked by the Gauss-Bonnet theorem \( \int d^2 \tilde{s} \tilde{R} + 2 \int d\tilde{s}k_{\tilde{g}} = 4\pi \chi \), where for the disc topology \( \chi = 1 \).

The curvature coupling \( Q \) is determined by the local Weyl anomaly associated with the reparametrization ghosts of Polyakov’s open bosonic string [21, 22]. Integrating \( Y \) and using the covariant heat kernel regulator we obtain

\[
\delta_\rho \ln \left[ \left( \frac{\text{Det}' \tilde{\Delta}}{\int d^2 \tilde{s}} \right)^{-1/2} e^{\tilde{F}_Q} \right] = \frac{1 + 3Q^2}{24\pi} \left( \int d^2 \tilde{s} \tilde{R}_\rho + 2 \int d\tilde{s} k_{\tilde{g}} \rho \right) - Q^2 \chi \int d^2 \tilde{s} \rho \int d^2 \tilde{s} \tilde{J}_Q(\tilde{\xi}),
\]

where the contributions associated with local renormalization counterterms have been omitted as they are ultimately tuned to zero. In the Weyl anomaly [2] we wrote

\[
\tilde{F}_Q = \frac{1}{4\pi \alpha'} \int d^2 \tilde{s}(\xi)d^2 \tilde{s}(\xi') \tilde{J}_Q(\xi, \xi') \tilde{G}_N(\xi, \xi'),
\]

where the functional current \( \tilde{J}_Q \) verifies

\[
\tilde{J}_Q = \frac{Q\sqrt{\alpha'}}{2} (\tilde{R} + 2\delta_B k_{\tilde{g}}), \quad \int d^2 \tilde{s} \tilde{J}_Q = 2\pi Q \sqrt{\alpha'} \chi
\]

and \( \tilde{G}_N \) is the Neumann laplacian Green’s function defined to satisfy

\[
\tilde{\Delta} \tilde{G}_N(\xi, \xi') = \frac{\delta^2(\xi - \xi')}{\sqrt{g}} - \frac{1}{\int d^2 \tilde{s}}, \quad \partial_\hat{n} \tilde{G}_N(\xi, \xi') = 0
\]

as well as to be symmetric in its arguments and orthogonal to the laplacian’s constant zero mode

\[
\int d^2 \tilde{s}(\xi) \tilde{G}_N(\xi, \xi') = 0.
\]

The non-local piece in anomaly [3] is generated by \( \tilde{G}_N \) due to the presence of the laplacian’s zero mode [22]. This term is zero for a fluctuating world-sheet with infinite area.
Only then the anomaly (5) may be equal to the Weyl anomaly associated with the integration of the reparametrization ghosts in Polyakov’s open bosonic string. This is the condition for bosonization in this theory and it further selects \( Q = \pm 3i \). Taking the infinite target space to have the critical 26 dimensions the local Weyl anomaly is also cancelled.

3 The Renormalization Group Flow \( \beta \)-Functions

The quantum features of the BRST invariant string \( \sigma \)-model we are going to analyze in this work are defined by the RG flow \( \beta \)-functions. Let us introduce them following the approach of Klebanov and Susskind [8]. A first point to be noted is that the Liouville mode \( \varphi \) takes the role played by the scale parameter \( t \). This is because the global scale transformations are now extended to the the Weyl transformations. Then the general Wilson RG equations for the set of renormalized fields \( g^j \) are the following variational equations in \( \varphi \)

\[
\beta^j = \frac{\delta g^j}{\delta \varphi} = \lambda_j g^j + \alpha^j_{kl} g^k g^l + \cdots.
\] (10)

In Eqs. (10) \( \lambda_j \) are anomalous dimensions and \( \alpha^j_{kl} \) vertex coefficients. The ellipsis represent higher order vertex terms in the weak field expansion (WFE) of the \( \beta \)-functions. Since the anomalous dimension matrix is supposed to be diagonalized there is no summation in \( j \). The solutions of Eqs. (10) in terms of the bare fields \( g^j(0) \) are

\[
g^j(\varphi) = \exp (\lambda_j \varphi) g^j(0) + \left\{ \exp \left[ (\lambda_k + \lambda_l)\varphi \right] - \exp (\lambda_j \varphi) \right\} \frac{\alpha^j_{kl}}{\lambda_k + \lambda_l - \lambda_j} g^k(0)g^l(0) + \cdots.
\] (11)

So, to find the \( \beta \)-functions we have to determine the renormalized fields \( \hat{T}, \hat{A}^\mu, \hat{\alpha} \) as functions of the bare fields \( T, A^\mu, \alpha \) and then compare with Eqs. (10) and (11). We start by separating \( X^\mu \) into a classical background \( X^\mu_0 \) and a quantum fluctuation \( \tilde{X}^\mu \); \( X^\mu = X^\mu_0 + \tilde{X}^\mu \), where naturally both fields obey Neumann boundary conditions. Taking the ghost coordinate \( Y \) as a pure quantum field we then consider in the WFE the \( \sigma \)-model partition function

\[
Z = \langle 1 - S_B + \frac{1}{2} S_B^2 + \cdots \rangle = \int \mathcal{D}\tilde{g}(\tilde{X}, Y) \exp \left( - S_0 \right) \left( 1 - S_B + \frac{1}{2} S_B^2 + \cdots \right)
\] (12)
and the 2D effective action \( S_{\text{eff}} = -\ln Z \).

Now, Fourier transforming the bare background fields to momentum space generates a sum of standard gaussian integrals. When integrated these will produce extra dependence on the conformal mode \( \varphi \) due to the quantum Weyl anomalies associated with the functional contractions. Let us first consider the linear order of the WFE,

\[
<S_B> = \frac{g^2}{2\sqrt{2\alpha'}}\int_{-\infty}^{+\infty} d\tau \int d^{26}k \int \mathcal{D}_\delta(\bar{X}, Y) \exp \left\{ -S_0 + ik \cdot \left[ X_0(\tau) + \bar{X}(\tau) \right] \right\} L_B(k, \tau),
\]

(13)

where

\[
L_B(k, \tau) = \sqrt{2} \exp \left[ \varphi(\tau) \right] T(k) + i \left[ \dot{X}_0^\mu(\tau) + \dot{\bar{X}}^\mu(\tau) \right] A^\mu(k) + \frac{i}{Q} \dot{Y}(\tau) \left[ i\sqrt{\alpha'} \cdot A(k) - \alpha(k) \right]
\]

(14)

and the dot over the fields is the notation for \( \partial_\tau \). Here the extra dependence on \( \varphi \) comes from the heat kernel regulated Neumann Green’s function \( \tilde{G}_N^R \) at coincident points \(^{22}\),

\[
\delta_\rho \tilde{G}_N^R(\gamma, \tau) \bigg|_{\gamma = \tau} = \frac{1}{\pi} \rho(\tau),
\]

(15)

and also from its first derivative

\[
\delta_\rho \partial_\gamma \tilde{G}_N^R(\gamma, \tau) \bigg|_{\gamma = \tau} = \frac{1}{2\pi} \dot{\rho}(\tau).
\]

(16)

In the next order we need to evaluate the contractions,

\[
<\frac{1}{2}S_B^2> = \frac{g^2}{16\alpha'}\int_{-\infty}^{+\infty} d\tau_1 \int_{-\infty}^{+\infty} d\tau_2 \int d^{26}k_1 \int d^{26}k_2 \int \mathcal{D}_\delta(\bar{X}, Y) \exp(-S_0) \times \exp \left\{ ik_1 \cdot \left[ X_0(\tau_1) + \bar{X}(\tau_1) \right] + ik_2 \cdot \left[ X_0(\tau_2) + \bar{X}(\tau_2) \right] \right\} L_B(k_1, \tau_1)L_B(k_2, \tau_2),
\]

(17)

which are associated with the first two terms in the expansions of \( [\varphi(\tau_2), X_0^\mu(\tau_2)] \) around \( \tau_1 \),

\[
[\varphi(\tau_2), X_0^\mu(\tau_2)] = [\varphi(\tau_1), X_0^\mu(\tau_1)] + \left[ \dot{\varphi}(\tau_1), X_0^\mu(\tau_1) \right] (\tau_2 - \tau_1) + \cdots.
\]

(18)

All the terms with higher order derivatives of \( \varphi \) or \( X_0^\mu \) will be ignored as they only contribute to the renormalizations of higher level massive fields.

In all the relevant contractions there are Weyl anomalous divergences which introduce extra dependence on \( \varphi \). Besides those like \((15)\) and \((16)\) we also find divergent integrals of the form

\[
\int_{-\infty}^{+\infty} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \exp \left[ -\pi P(k_1, k_2)\tilde{G}_N(\tau_1, \tau_2) \right],
\]

(19)
where \( \tilde{G}_N(\tau_1, \tau_2) = G_N(\tau_1, \tau_2) = -(1/\pi) \ln(|\tau_1 - \tau_2|/\sqrt{\alpha'}) \). Noting that for non-coincident points the Neumann Green’s function does not have a local Weyl anomaly \([22]\) and using the covariant heat kernel result \([13]\) we obtain the following extra dependence on the Liouville mode \( \varphi \)

\[
- \sqrt{\alpha'} \int_{-\infty}^{+\infty} d\tau [P(k_1, k_2) + 1]^{-1} \exp \{- [P(k_1, k_2) + 1] \varphi(\tau) \}. \tag{20}
\]

Note as well that the integrals \((19)\) have a potential infrared divergence appearing as \(\tau_2 \rightarrow -\infty\). To remove it we impose the convergence condition \(P(k_1, k_2) + 1 < 0\) \([8]\).

Then we find

\[
S_{\text{eff}} = \frac{1}{4\pi\alpha'} \int d^2\xi \partial_{\alpha} X_0^{\mu} \partial_{\mu} X_0 + \frac{g}{2\sqrt{2\alpha'}} \int_{-\infty}^{+\infty} d\tau \int d^2k \exp \left[ ik \cdot X_0(\tau) \right] \hat{L}_B(k, \tau) + \cdots, \tag{21}
\]

with

\[
\hat{L}_B(k, \tau) = \sqrt{2} \hat{T}[k, \varphi(\tau)] + i \hat{X}\, \hat{A}_\mu[k, \varphi(\tau)], \tag{22}
\]

where the renormalized fields are

\[
\hat{T}[k, \varphi(\tau)] = \exp \left[ (1 - \alpha'k^2) \varphi(\tau) \right] \left\{ T(k) - \int_k \left[ \frac{T(k_1)T(k_2)}{2\alpha'k_1 \cdot k_2 + 1} + 2i A_\mu(k_1) \times \frac{k_\mu}{\sqrt{\alpha'k_2^2}} \alpha(k_2) + \frac{A_\mu(k_1) A_\nu(k_2)}{2\alpha'k_1 \cdot k_2 - 1} \left( \eta^{\mu\nu} - 2\alpha'k^\mu k^\nu_2 - Q^{-2} \alpha'k_1^\mu k_2^\nu \right) + \frac{k_1 \cdot k_2}{\alpha'k_1^2 k_2^2} \alpha(k_1) \alpha(k_2) \right] \right\},
\]

\[
\hat{A}_\mu[k, \varphi(\tau)] = \exp \left[ -\alpha'k^2 \varphi(\tau) \right] \left\{ A_\mu(k) - 2 \int_k \left[ \frac{T(k_1)A_\nu(k_2)}{2\alpha'k_1 \cdot k_2 + 1} \left( \eta^{\mu\nu} + 2\alpha'k^\mu k_1^\nu \right) - \frac{i T(k_1) k_\mu}{\sqrt{\alpha'k_1^2}} \alpha(k_2) \right] \right\},
\]

\[
\hat{\alpha}[k, \varphi(\tau)] = O(g), \tag{23}
\]

where we have written \( \int_k = -(g/2) \int d^2k_1 \int d^2k_2 \delta^{26}(k_1 + k_2 - k) \) and also \( A_\mu(k) = A_\mu(k) + ik^\mu \alpha(k)/(\sqrt{\alpha'k^2}) \). Note that \( \hat{\alpha} \) is a completely arbitrary \( O(g) \) function because we are free to add a total boundary derivative to the 2D effective action.

At this stage it becomes clear that \( T, A_\mu \) and \( \alpha \) are not the correct fields to define the \( \beta \)-functions because of the mixing of \( A_\mu \) and \( \alpha \). Instead we must consider \( T, A_\mu \) and \( \alpha \). The correspondent anomalous dimensions are \( \lambda_T(k) = 1 - \alpha'k^2 \), \( \lambda_{A_\mu}(k) = -\alpha'k^2 \) and \( \lambda_\alpha(k) = 0 \). Comparing with the solutions \((11)\) it is also clear that we are missing the contribution associated with \( \exp[(\lambda_k + \lambda_l) \varphi] \). This term is negligible when compared with
exp(λ_j φ) if λ_k + λ_l - λ_j << 0. In the high energy region of phase space \(-α' k_1^2 >> 0, -α' k_2^2 >> 0, -2α' k_1 \cdot k_2 >> 0\) this is always true and since \(P(k_1, k_2) + 1 \sim 2α' k_1 \cdot k_2\) the infrared divergence is removed. Thus we define the \(β\)-functions by analytic continuation from this region of momentum space. Our results are

\[
\beta_{\hat{T}}(k) = (1 - α' k^2) \hat{T}(k) + \int_k \left[ \hat{T}(k_1) \hat{T}(k_2) + \hat{A}_\mu(k_1) \hat{A}_\nu(k_2) (η^{\mu\nu} - 2α' k_2^\mu k_1^\nu) + \right.
\]

\[
+ \ 2i \hat{A}_\mu(k_1) \frac{k_2^\mu}{\sqrt{α' k_2^2}} \hat{α}(k_2) \left( α' k^2 - 1 - α' k_1^2 \right) - \frac{α'}{Q^2} k_1 \cdot \hat{A}(k_1) k_2 \cdot \hat{A}(k_2) -
\]

\[
- \frac{k_1 \cdot k_2}{α' k_1^2 k_2^2} \hat{α}(k_1) \hat{α}(k_2) \left( -α' k^2 + 1 \right) \bigg], \nonumber
\]

\[
\beta_{\hat{A}_\mu}(k) = -α' k^2 \hat{A}_\mu(k) + 2 \int_k \left[ \hat{T}(k_1) \hat{A}_\nu(k_2) (η^{\mu\nu} + 2α' k_2^\mu k_1^\nu) - \right.
\]

\[
- \ i \hat{T}(k_1) \frac{k_2^\mu}{\sqrt{α' k_2^2}} \hat{α}(k_2) \bigg], \nonumber
\]

\[
\beta_\hat{α}(k) = O(g).
\]

(24)

Up to linear order the \(β\)-functions are invariant under the BRST gauge transformation of the renormalized fields. With the introduction of the quadratic \(O(g)\) interactions this symmetry is dynamically broken.

4 The Effective Action and its Gauge Symmetry

Properties

To order \(g\) the \(β\)-functions define a gradient flow in the theory space of couplings. The associated effective action is equal to

\[
I = \int d^6 x \left[ \frac{1}{2} \hat{T}(α' \Box + 1) \hat{T} + \frac{1}{2} \hat{A}_\mu α' \Box \hat{A}_\mu - \sqrt{α' \hat{A}_\mu \partial_\mu \hat{α} - \frac{1}{2} \hat{α}^2 - \frac{g}{3!} \hat{T}^3 - \right.
\]

\[
- \ g \hat{T} \hat{A}_\mu \hat{A}_\mu - g \sqrt{α' \hat{α}} \hat{A}_\mu \partial_\mu \hat{α} + \frac{g \sqrt{α'}}{Q^2} \hat{T} \partial_\mu \hat{A} \cdot \hat{A} - g \alpha' \hat{T} \partial_\nu \hat{A}^\mu \partial_\mu \hat{A}^\nu - \right.
\]

\[
- \ \frac{g \alpha'}{2 Q^2} \hat{T} \partial \cdot \hat{A} \partial \cdot \hat{A} - \frac{g}{2 Q^2} \hat{T} \hat{α}^2 \bigg].
\]

(25)

This is only possible if the total boundary derivative inducing an arbitrary \(β\)-function for the ghost field \(\hat{α}\) is fixed to have the necessary complementary terms to those shown in the \(β\)-functions of \(\hat{T}\) and \(\hat{A}\). Although at the linear order the RG flow is associated with a unit metric in the space of couplings, when the non-linear interaction terms are introduced the metric develops an off-diagonal element \(G^\mu_{\hat{A} \hat{T}}\).
\[
G^{jl} = \begin{bmatrix}
1 & 0 & 0 \\
G_{\dot{A}\dot{T}} & \eta^{\mu\nu} & 0 \\
0 & 0 & 1
\end{bmatrix},
\]
(26)

where
\[
G_{\dot{A}\dot{T}} = \frac{1}{Q_2} \partial \cdot \dot{A} \partial^{\mu} + \left( 2 + \frac{1}{Q_2} \right) \partial^{\mu} \partial \cdot \dot{A} \right) (\alpha' \Box + 1)^{-1}.
\]
(27)

This metric is invertible without the need to neglect higher order terms. The inverse metric has the same form but with \(G_{\dot{A}\dot{T}}^{-1} = -G_{\dot{A}\dot{T}}^{ij} \). With this condition on the invertibility of \(G^{jl} \) the RG gradient flow is unique.

The BRST invariant kinetic terms in the effective action (25) are exactly those of WSFT. At the first non-linear interaction level this symmetry is explicitly broken. To see if any other invariance emerges we consider the variation of the interactions in Eq. (25) under the BRST transformation and study how new quadratic gauge transformations might produce a compensating change of the kinetic terms. We find that the effective action (25) is exactly invariant under the following \(\gamma\)-parameter family of non-linear gauge transformations
\[
\delta \dot{T} = \frac{g}{2} \left[ \gamma \dot{\alpha} \Lambda + \sqrt{\alpha'} \left( 2 \dot{A}^\mu \partial_{\mu} \Lambda - \gamma \Lambda \partial \cdot \dot{A} \right) \right],
\]
\[
\delta \dot{A}^{\mu} = \sqrt{\alpha'} \partial^{\mu} \Lambda - \frac{g \sqrt{\alpha'}}{2} \left[ \gamma \Lambda \partial^{\mu} \dot{T} + (\gamma + 2) \dot{T} \partial^{\mu} \Lambda \right],
\]
\[
\delta \dot{\alpha} = \alpha' \Box \Lambda + \frac{g \gamma}{2} \left( \dot{T} \Lambda - 2 \alpha' \partial^{\mu} \dot{T} \partial^{\mu} \Lambda - \alpha' \dot{T} \Box \Lambda \right).
\]
(28)

Though not equal in its detail to WSFT tree level symmetry [19, 20], the \(\sigma\)-model invariance (28) is of a similar structural type. That is most notable in the transformation laws of \(T\) and \(A^{\mu}\) where the symmetry operator structure is the same if \(\gamma \neq 0\). Missing in the \(\sigma\)-model are the exact operator coefficients and smearing factors of Witten’s interactions. As is already clear in Eq. (25) it is when the ghost field \(\dot{\alpha}\) enters into the interactions that the theories most differ. This should be noted as evidence that when probing deeper and deeper into the off-shell structure of this string \(\sigma\)-model we will see it diverge more and more from WSFT.

Thus it should not be too surprising that in the Feynman-Siegel gauge \(\dot{\alpha} = 0\) we are able to obtain Witten’s structure of vertex couplings [19, 20] thought not its exact coefficients,
\[ I_{FS} = \int d^2 x \left[ \frac{1}{2} \hat{T}(\alpha'\Box + 1)\hat{T} + \frac{1}{2} \hat{A}^\mu\alpha'\Box\hat{A}_\mu - \frac{g}{3!}\hat{T}^3 - \frac{g}{2}\hat{T}\hat{A}^\mu\hat{A}_\mu - \frac{g\alpha'}{Q^2}\hat{A}_\nu\partial_\mu\hat{T}\partial^\nu\hat{A}^\mu - \frac{g\alpha'}{2Q^2}\hat{A}^\mu\hat{A}^\nu\partial_\mu\partial_\nu \hat{T} \right]. \]

If we set \( \partial \cdot \hat{A} = \alpha \) the matter and ghost sectors of the 2D bare action decouple. Then the \( \sigma \)-model is projected onto the theory space spanned by \( T \) and \( A^\mu \). The effective action is just Eq. (25) with \( \partial \cdot \hat{A} = \hat{\alpha} \). The corresponding non-linear invariance follows from Eq. (28) with \( \gamma = 0 \),

\[
\begin{align*}
\delta \hat{T} &= g\sqrt{\alpha'}\hat{A}^\mu\partial_\mu \Lambda, \\
\delta \hat{A}^\mu &= \sqrt{\alpha'}(1 - g\hat{T})\partial^\mu \Lambda. 
\end{align*}
\]

Here the agreement with WSFT is almost complete. Only missing in the interactions are its gaussian smearing factors.

## 5 Conclusions

In this letter we have only considered the slice of theory space corresponding to \( T, A^\mu \) and \( \alpha \). These are the relevant and marginal perturbations about the on-shell gaussian fixed point. In a level truncation type scheme [20] they are the first levels in an infinite tower of string states which in a fully off-shell description must be considered together. The introduction of the higher massive fields can naturally be done along the lines of this work but it is outside our present scope. We hope to report on progress in this direction in a future paper.

It is important to be aware of the limitations of the present approach. Thought non-perturbative in \( \alpha' \) the WFE does not allow us to probe very deeply off-shell in the RG flow and is permissively near mass-shell field redefinitions [16, 23] which are bound to hide important physical phenomena. In this work we have introduced a purely off-shell ghost coupling and avoided the field redefinitions but have not gone beyond the WFE. As a consequence our results are just an indication (thought a strong one, we believe) that non-linear gauge symmetries and non-trivial string vacua of the type found in WSFT [20] also exist in the RG flow. Perhaps we might reach farther away with the Exact Renormalization Group approach [3, 10, 11].

10
Acknowledgements

We would like to thank Yuri Kubyshin, Paul Mansfield and Tim Morris for their support and encouragement. We also thank the Department of Physics at Indiana University, U.S.A., for kind hospitality in the early stages of this work.

References

[1] C. Lovelace, Phys. Lett. B 135 (1984) 75; Nucl. Phys. B 273 (1986) 413;
    C. Callan, D. Friedan, E. Martinec and M. Perry, Nucl. Phys. B 262 (1985) 593;
    A. Sen, Phys. Rev. D 32 (1985) 2102; Phys. Lett. B 55 (1985) 1846;
    E.S. Fradkin and A.A. Tseytlin, Phys. Lett. B 158 (1985) 316; Nucl. Phys. B 261
    (1985) 1; Phys. Lett. B 163 (1985) 123;
    D. Fridling and A. Jevicki, Phys. Lett. B 174 (1986) 75;
    C. Callan, I. Klebanov and M. Perry, Nucl. Phys. B 278 (1986) 78;
    A. Abouelsaood, C.G. Callan, C.R. Nappi and S.A. Yost, Nucl. Phys. B 280 (1987)
    599.

[2] S.R. Das and B. Sathiapalan, Phys. Rev. Lett. 56 (1986) 2664; Phys. Rev. Lett. 57
    (1986) 1511.

[3] R. Akhoury and Y. Okada, Phys. Lett. B 183 (1987) 65;
    C. Itoi and Y. Watabiki, Phys. Lett. B 198 (1987) 486;
    Y. Watabiki, Z. Phys. C 38 (1988) 411.

[4] A.B. Zamolodchikov, JETP Lett. 43 (1986) 730;
    A.M. Polyakov, Phys. Scr. T 15 (1987) 191.

[5] T. Banks and E. Martinec, Nucl. Phys. B 294 (1987) 733.

[6] B. Sathiapalan, Nucl. Phys. B 294 (1987) 747; B 326 (1989) 376; Int. J. Mod. Phys.
    A 10 (1995) 4501.

[7] R. Brustein, D. Nemeschanski and S. Yankielowicz, Nucl. Phys. B 301 (1988) 224.

[8] I. Klebanov and L. Susskind, Phys. Lett. B 200 (1988) 446.
[9] A.N. Redlich, Phys. Lett. B 213 (1988) 285.

[10] J. Hughes, J. Liu and J. Polchinski, Nucl. Phys. B 316 (1989) 15.

[11] U. Ellwanger and J. Fuchs, Nucl. Phys. B 312 (1989) 95;
    U. Ellwanger, Phys. Lett. B 243 (1990) 93.

[12] J. Labastida and M. Vozmediano, Nucl. Phys. B 312 (1989) 308.

[13] S. Jain and A. Jevicki, Phys. Lett. B 220 (1989) 379.

[14] I.L. Buchbinder, E.S. Fradkin, S.L. Lyakhovich and V.D. Pershin, Phys. Lett. B 304 (1993) 239;
    I.L. Buchbinder, V.A. Krykhtin and V.D. Pershin, Phys. Lett. B 348 (1995) 63.

[15] K. Bardakci and L.M. Bernardo, Nucl. Phys. B 505 (1997) 463;
    K. Bardakci, Nucl. Phys. B 524 (1998) 545.

[16] A.A Tseytlin, Phys. Lett. B 264 (1991) 311.

[17] J. Schnittger and U. Ellwanger, Int. J. Mod. Phys. A 9 (1994) 1821.

[18] V.A. Kostelecký, M. Perry and R. Potting, Phys. Rev. Lett. 84 (2000) 4541.

[19] E. Witten, Nucl. Phys. B 268 (1986) 253.

[20] V.A. Kostelecký and S. Samuel, Nucl. Phys. B 336 (1990) 263; Phys. Rev. Lett. 64 (1990) 2238.

[21] P. Mansfield, Nucl. Phys. B 306 (1988) 630.

[22] P. Mansfield and R. Neves, Nucl. Phys. B 479 (1996) 82.

[23] T. Banks, Nucl. Phys. B 361 (1991) 166.