FRW Bulk Viscous Cosmology with Modified Chaplygin Gas in Flat Space

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Abstract In this paper we study FRW bulk viscous cosmology in presence of modified Chaplygin gas. We write modified Friedmann equations due to bulk viscosity and Chaplygin gas and obtain time-dependent energy density for the special case of flat space.

Keywords: FRW Cosmology; Bulk Viscosity; Modified Chaplygin Gas.

1 Introduction

It is found that our universe expands with acceleration (Riess et al. 1998; Perlmutter et al. 1999; Knop et al. 2003; Riess et al. 2004; Bennet et al. 2003). The accelerating expansion of the universe may be explained in context of the dark energy (Bamba et al. 2012). Due to negative pressure, the simplest way for modeling the dark energy is the Einstein’s cosmological constant. On the other hand the study of the cosmological constant is one of the important subject in the theoretical and experimental physics (Weinberg 1989; Padmanabhan 2003; Peebles and Ratra 2003; Nobbenhuis 2006). Another candidate for the dark energy is scalar-field dark energy model (Peebles and Ratra 1988; Ratra and Peebles 1988; Turner and White 1997; Caldwell 2002; Sen 2002; Feng et al. 2005; Guo et al. 2005; Wei 2005; Wei et al. 2007). However presence of a scalar field is not only requirement of the transition from a universe filled with matter to an exponentially expanding universe. Therefore, Chaplygin gas (Setare 2007) used as an exotic type of fluid, which is based on the recent observational fact that the equation of state parameter for dark energy can be less than $-1$. Important picture of Chaplygin gas may seen in context of holography (Setare 2007; Setare 2009) where a correspondence between the holographic dark energy and Chaplygin gas energy density proposed.

On the other hand we know that the viscosity plays an important role in the cosmology (Singh and Devi 2011; Singh and Kale 2011; Setare and Sheykhi 2010; Misner 1969). In another word, the presence of viscosity in the fluid introduces many interesting pictures in the dynamics of homogeneous cosmological models, which is used to study the evolution of universe. Already (Chatterjee and Bhui 1990) the exact solutions of the field equations for a five-dimensional space-time with viscous fluid obtained. In another work (Chatterjee and Bhui 1990) a cosmological model with viscous fluid in higher-dimensional space-time constructed. In the interesting work (Singh et al. 2004) the exact solutions of the field equations for a five-dimensional cosmological model with variable bulk viscosity obtained.

The isotropic homogeneous spatially flat cosmological model with bulk viscous fluid is also constructed (Murphy 1973). Then the bulk viscous cosmological models with constant bulk viscosity coefficient constructed (Bali and Dave 2002). In the recent work (Katore et al. 2011) the FRW bulk viscous cosmology considered and bulk viscous coefficient obtained in the flat space, and then extended to non-flat space (Saadat 2012). In this work we consider both bulk viscous effect and Chaplygin gas in FRW cosmology in flat space. Indeed we modify Friedmann equation due to Chaplygin gas which has bulk viscosity. We should here note that, this modified theory may be stable and energy-momentum conserved. Stability of this theory is discussed and appropriate condition to have stable theory is obtained.
2 Friedmann equations

The Friedmann-Robertson-Walker (FRW) universe in four-dimensional space-time is described by the following metric (Saha et al. 2012; Jamil et al. 2012),

\[ ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \]

(1)

where \( d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2 \), and \( a(t) \) represents the scale factor. The \( \theta \) and \( \phi \) parameters are the usual azimuthal and polar angles of spherical coordinates, with \( 0 \leq \theta \leq \pi \) and \( 0 \leq \phi < 2\pi \). The coordinates \((t, r, \theta, \phi)\) are called co-moving coordinates. Also, constant \( k \) denotes the curvature of the space. In this paper we consider the case of \( k = 0 \) only, which is corresponding to flat space. In that case the Einstein equation is given by,

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu} + g_{\mu\nu} \Lambda, \]

(2)

where we assumed \( c = 1 \) and \( 8\pi G = 1 \). Also the energy-momentum tensor corresponding to the bulk viscous fluid and modified Chaplygin gas (Benamou 2002; Deb-nath et al. 2004; Xu et al. 2012; Bandyopadhyay 2012; Rudra et al. 2012; Rudra 2012) is given by the following relation,

\[ T_{\mu\nu} = (\rho + \bar{p})u_\mu u_\nu - \bar{p} g_{\mu\nu}, \]

(3)

where \( \rho \) is the energy density and \( u^\mu \) is the velocity vector with normalization condition \( u^\mu u_\mu = -1 \). Also, the total pressure and the proper pressure involve bulk viscosity coefficient \( \zeta \) and Hubble expansion parameter \( H = \dot{a}/a \) are given by the following equations,

\[ \dot{\rho} = p - 3 \zeta H, \]

(4)

and

\[ p = \gamma \rho - \frac{B}{\rho^\alpha}, \]

(5)

with \( B > 0 \) and \( 0 < \alpha \leq 1 \). The equation of state \( \gamma \) is one of the most important quantity to describe the features of dark energy models. It is clear that the parameter \( \zeta \) shows bulk viscosity and \( B \) shows effect of Chaplygin gas. Already (Mazumder et al. 2012) the dynamics of FRW cosmology with modified Chaplygin gas as the matter formulated. Then the nature of the critical points are studied by evaluating the eigenvalues of the linearized Jacobi matrix for the special case of \( \alpha = 0.6 \). In this paper we consider special case with \( \alpha = 0.5 \) and extend the previous work (Mazumder et al. 2012) to including bulk viscous coefficient.

In that case the Friedmann equations are given by,

\[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{\rho}{3}, \]

(6)

and

\[ 2\frac{\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 = -\bar{p}, \]

(7)

where dot denotes derivative with respect to cosmic time \( t \). The energy-momentum conservation law obtained as the following,

\[ \dot{\rho} + 3H(\rho + \bar{p}) = 0. \]

(8)

In the next section we try to obtain time-dependent density by using above equations.

3 Time-dependent density

Using the equations (4), (5) and (6) in the conservation relation (8) we have,

\[ \dot{\rho} + \sqrt{3}(\gamma + 1)\rho^{\frac{2}{\gamma}} - 3\zeta\rho - \sqrt{3}B = 0. \]

(9)

If we set \( \zeta = 0 \), then one can extract energy density depend on scale factor (Mazumder et al. 2012),

\[ \rho(a) = \left[ \frac{1}{\gamma + 1} \left( B + \frac{c}{\sqrt{a^{3(\gamma + 1)}}} \right) \right]^\frac{2}{\gamma}, \]

(10)

where \( c \) is an integration constant. Here we also consider bulk viscous coefficient and would like to obtain energy density depend on time. In order to solve equation (9) we use the following ansatz,

\[ \rho = \frac{A}{t^2} + \frac{E}{t} + ht + Ce^{bt}, \]

(11)

where constants \( A, E, h, C \) and \( b \) should be determined. Substituting relation (11) in the equation (9) gives us the following coefficients,

\[ h = \sqrt{3}B, \]

(12)

\[ A = \frac{4}{3(\gamma + 1)^2}, \]

(13)

\[ E = \frac{2\zeta}{(\gamma + 1)^2}, \]

(14)

\[ C = \frac{(\gamma + 1)^2}{4} \left[ \frac{8\sqrt{3}\zeta^2}{(\gamma + 1)^3} - \frac{3(\gamma + 1)^4}{16\zeta^2} \right], \]

(15)
\[ b = \frac{\zeta \left[ \sqrt{3}(\gamma + 1)(B(\gamma + 1) - \frac{2}{3}\zeta^3) + \frac{27}{2}(\gamma + \frac{1}{3}) + \frac{9}{28}\zeta^4 + O(\zeta^n) \right]}{8(\gamma + 1)(\sqrt{3}\zeta^4 - \frac{27}{128}(\gamma + 1)^7)} \]  

where,

\[ O(\zeta^n) = \frac{189}{64}\gamma^2 + \frac{189}{32}\gamma^3 + \frac{945}{128}\gamma^4 + \frac{189}{32}\gamma^5 + \frac{189}{64}\gamma^6 + \frac{27}{32}\gamma^7 + \frac{27}{256}\gamma^8 \]  

If we neglect both bulk viscosity and presence of Chaplygin gas then,

\[ \rho = \frac{4}{3(\gamma + 1)^2t^2}, \]  

which is agree with results of previous works [Saadat 2012; Mazumder et al. 2012] where \( \rho \propto t^{-2} \) established. On the other hand for the large bulk viscosity coefficient one can find that \( b < 0 \) and hence \( \rho \propto \zeta/t \) obtained. Also for the case of infinitesimal \( \zeta \) one can obtain constant negative energy density. It is interesting to check late time behavior of density. In that case the last term of the equation (11) is dominant so one can say \( \rho \sim C \exp(bt) \). In the general case, equation (11) with coefficients (12)-(16) tells us that the energy density is decreasing function of time. Such behavior happen for the Hubble expansion parameter which is discussed in the next section.

### 4 Hubble and deceleration parameters

By using time-dependent density in the relation (6) one can obtain Hubble expansion parameter. In that case we draw plot of Hubble expansion parameter in the Fig. 1 for \( \gamma \approx 1/3 \). In that case the modified Chaplygin gas model describes the evolution of the universe from the radiation regime to the \( \Lambda \)-cold dark matter scenario, where the fluid behaves as a cosmological constant, so there is an accelerated expansion of the universe.

It is possible to study deceleration parameter of this theory which obtained by the following relation,

\[ q = -\left(1 + \frac{\dot{H}}{H^2}\right). \]  

Numerically, we draw deceleration parameter in terms of time in the Fig. 2. It shows that the deceleration parameter yields to \(-1\) at the late time. In the case of \( \zeta = 0.2 \) there is a maximum value of the deceleration parameter at \( t \sim 0.35 \). In other cases time-dependent of the deceleration parameter is completely decreasing.

### 5 Stability

It is important to investigate stability of this theory. There are several ways to do this. We use speed of sound in viscous fluid to study stability of our system (Setare 2007; Sadeghi et al. 2010). In that case there is the following condition to have stable theory,

\[ C_s^2 = \frac{d\rho}{d\rho} \geq 0. \]  

We should use equations (4), (5) and (11) to satisfy the relation (20). In the Fig. 3 we draw plot of \( C_s^2 \) for selected value of \( \zeta \). It shows that stability of theory is depend on viscosity. We find that for \( \zeta < 0.65 \) theory is completely stable. On the other hand, in the case of \( \zeta > 0.65 \) theory has unstable region. However at the late time, theory is completely stable and sound speed yields to constant value.
Fig. 3  Square of sound speed in terms of time for $B = 3.4$ and $\gamma = 0.3$. Solid, dotted, and dashed lines represent $\zeta = 0.65, 0.6, 0.1$ respectively.

6 Conclusion

In this work we studied the FRW bulk viscous cosmology with modified Chaplygin gas as the matter contained. We obtained the modified Friedmann equations due to bulk viscous and Chaplygin gas coefficients. Then, we tried to solve equations and found time-dependent energy density. Therefore, we could extract Hubble expansion and deceleration parameters. We studied stability of theory and found that stability of system strongly is depend on viscosity coefficient. However at the late time theory is stable and speed of sound has constant real value. For the future work it is possible to repeat calculation of this paper for the case of arbitrary $\alpha$ or non-flat universe where $k \neq 0$. It is also interesting to study thermodynamics of Chaplygin gas with bulk viscosity similar to the recent work (Setare and Sheykhi 2010).
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