Determining the Effect of Anthropogenic Loading on the Environmental State of a Surface Source of Water Supply

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1. Introduction

At present, even for a fairly large surface water object, the effectiveness of the forecasting models of water qualitative composition is largely determined by the completeness and adequacy of the source information. A review of approaches to using such models is given in scientific papers [1, 2]. Traditional forecasting methods, developed for using mod-
ern information and computing tools, are typically not effective in solving complex practical tasks on forecasting the state of a specific water object, taking into consideration all its features. For each surface water object, its hydrodynamic models should be constructed, fully adapted to its peculiarities. In this case, a model should meet the following requirements:

1. Represent the basic regularities in the distribution of pollutants ($Ps$) in water objects.
2. The calculation of $Ps$ transformation should be conducted in a relatively short time and at the economical use of the necessary resources.
3. The model parameter identification methods must be specified.
4. The accuracy of calculation results should be compared with the accuracy of the source data, as well as the possible accuracy of control over the process of $Ps$ scattering under natural conditions.

The construction of diagrams for the process of adequate-diffusion impurities transfer must take into consideration the main defining factors of the $Ps$ transfer process in the examined surface water object. These include the character of a water regime, its hydraulic, morphological, and hydrochemical characteristics, conditions of wastewater discharge, the factors and conditions of self-purification of the surface water object.

Therefore, it is a relevant research task to build and implement a mathematical model based on the modern system of collecting and analyzing source information according to data on the hydrochemical analysis of a surface source water, which should make it possible to perform calculations using computer equipment. It should be interpreted both in the current water use and in the process of implementing integrated water resources management.

2. Literature review and problem statement

The issue related to water quality in natural surface water objects through the development of appropriate norms of water management has been the subject of multilateral scientific studies over the past decade. A typical example of using such standards is the indicators of maximum allowable concentrations (MAC) of pollutants and the maximum allowable discharges to surface water objects, based on them, as well as the index of water pollution, all of which have a series of significant shortcomings.

When applying a MAC normative index, one can know only the quality composition of water at a given site of the surface water object. It can also be used to ration water use for drinking, household, or recreational purposes (for example, to feed water supply systems, bathing) or fish farming (at a given site). Yet, while using the normative indicator of MAC, nothing can be said about the reasons that led to its formation. Therefore, it is quite problematic to rely on a given indicator in assessing the ecological state of a water object and its catchment area, and when planning measures to improve this condition, without the knowledge of the natural component of pollution. In addition, a given indicator does not make it possible to trace the transformation of a polluting substance or compound along a watercourse length.

As one knows, the transformation of any $Ps$ or compound depends on many factors, which include:

- chemical (decay, binding to other substances, deposition);
- physical (transfer to another aggregate state, adsorption, coagulation);
- hydrodynamic (transfer by currents and scattering in the process of turbulent diffusion);
- biological (accumulation and transfer by organisms), that is, due to “pure breeding”;
- processes inside a water reservoir.

The prediction of $Ps$ transformation in surface water objects employs a large number of different models considered in international scientific works. Thus, paper [3] addresses the issue of modeling the impact of hazardous $Ps$ under the conditions of a surface water object, but it does not account for the influence of watercourses on a change in their composition. Study [4] reports a model to develop a comprehensive procedure for analyzing the overall nitrogen accumulation in a tank with drinking water based on the SWAT model. The proposed approach is aimed at determining the content of a single pollutant and ignores the possible transformational influence of other $Ps$ available in the water tank. The comparative analysis of SWAT and InVEST models for determining the spatial models of the hydrology of ecosystems in a surface water basin was carried out in [5]. However, the cited work did not consider the issue of the possible influence of a water object on the examined processes.

The mathematical models, constructed up to now, make it possible, depending on the type of equation, the study area, and the conditions for calculating the field of impurities concentration, to obtain one of the following solution methods:

- an exact analytical solution, which is very convenient for practice. However, it is quite difficult to solve in an explicit form even relatively simple problems on the stationary advection-diffusion. Difficulties pile up in the transition to problems related to the non-stationary convective diffusion; therefore, most practical tasks cannot be solved by a precise analytical method;
- approximate analytical methods, which make it possible to obtain a solution to a series of simple tasks on water quality prediction. However, when studying the processes of advection-diffusion transport in natural surface water objects with complex morphometry under variable boundary conditions, these methods require bulky computations;
- a method of hydraulic modeling is widely used in the study of processes involving the dilution of wastewater in water bodies. A significant drawback of a given method is the complexity and high cost of hydraulic models, as well as the experiment itself, the impossibility of accounting for all the factors acting in nature, and, in some cases, the inability to achieve dynamic similarity;
- numerical methods, implemented by using modern computers, make it possible to solve the most complex problems that cannot be solved either analytically or by methods of hydraulic modeling. These methods include the single- and multidimensional models of river currents.

When using one-dimensional models [4], their application to river streams leads to difficulties in finding the roughness coefficient, whose numerical value is set by a “sorting method” yielding a large error. To study the spread of a conservative additive along a river current, the transfer equations are additionally solved [6, 7]. It should be noted that when solving a mass transfer equation, in addition to taking into consideration the temperature and dynamic characteristics of receiving watercourses, one must take
into consideration the interaction of components with each other [8]. The most common models include GLEN (USA), DRM (USA), QUAL (USA), FINNECOC, ERAUSO (Sweden, Finland), FINEST (Estonia).

Paper [9] considered the two-dimensional (in the horizontal plane) models of the transfer of impurities but without taking into consideration the turbulent movement in a watercourse. Work [10] applied a two-dimensional (in the vertical plane) model of a pollutant’s propagation; study [11] provides examples of its improvement and the interpretation for actual water object conditions taking into consideration the turbulence of the current. Yet, the cited papers consider only the conditions of the local space. Such models make it possible to describe complex processes and to consider a wider class of applied tasks; they are implemented using the system of equations of “shallow water”.

The three-dimensional models that are examined in [12] are applied to conditions when the flow of water in a surface water object cannot be considered constant and smoothly volatile. To describe the three-dimensional pattern of currents, the authors use different modifications of the theory of “shallow water”. Such theories make it possible to calculate the hydrochemical regime of a water course depending on changes in the velocity of the flow current, temperature, the concentration of impurities, and other characteristics, both in its waters and in depth.

The use of numerical methods is currently limited for two reasons. First, the application of these methods requires a significant amount of source data and large computing resources. Second, the accuracy of the source data does not make it possible to use very powerful computational means; and when using characteristics, averaged only for the cross-section of the flow, the model’s advantages are compromised.

Even more complex is the issue of assessing the initial parameters of models. Thus, work [13] reported a comparative analysis that revealed that complex multicompont model parameters are more sensitive to the accuracy of the source parameters than simpler ones. Papers [14, 15] tested such models for the actual conditions of a water object by assigning the parameters for such models in a general case, based on retrospective, hydrochemical, hydrological, and hydrobiological observations.

Therefore, when building a system of operative forecasting and evaluation of anthropogenic stresses, at a sufficiently limited volume of the original information, first of all, hydrochemical, hydrobiological, it is advisable to use simpler models.

The main regulatory document governing water legal relations is the Water Code of Ukraine, which came into force on June 6, 1995. The Water Code of Ukraine, in particular, defines the legal basis for norms in the field of use and protection of water objects, which imply setting water consumption limits (water supply and drainage), the development and adoption of the related standards, norms, and rules. The types of standards are formulated in article 35, which defined the following:

- the environmental safety regulations of water use;
- the ecological water quality standard for surface and groundwater water arrays;
- the norms of maximum allowable discharge of pollutants;
- the industry-related technological standards for the formation of substances that are discharged into water objects;
- the technological norms of water use.

The above regulations make it possible to control the quality of water in natural watercourses and reservoirs, to maintain them at a level that makes it possible to ensure the safe use of water and the ecological state of territories.

The basic principles for establishing the standards are the regionality; temporality; the development of a program of water-protecting measures; regular monitoring and revision of standards, as well as their adjustment with the availability of financial resources.

Under a given approach, the temporal norm of the permissible influences on a water object in terms of Ps arrival should be understood as a comprehensive indicator, which must include the following:

- an indicator of the existing state of a water object in terms of the arrival of chemicals and their compounds – IES;
- an approximate necessary level of reduction of the harmful effect on a surface water object in terms of Ps inflow – ANLR;
- a programs of water-protecting measures – PWM;
- the period of implementing a PWM program and the achievement of the approximate level of harmful effects on a water object – PIP.

In turn, the methodology for restoring the water quality of surface water objects must include the assessment of the existing state; determining the approximate level of improvement of water quality; the development of water-protecting measures; the adoption of a possible term of program implementation taking into consideration the economic opportunities of a region and the results reached in the preliminary stage.

Thus, in the first stage of establishing the permissible impacts on a water object in terms of Ps inflow, it is necessary to construct a mathematical model that could be tested and applied under the conditions of an actual water object. At the same time, it should be easy to adapt for calculations using computer equipment.

### 3. The aim and objectives of the study

The aim of this study is to build a mathematical model to determine the approximate necessary level of reduction of the harmful effects on a surface water object (site) in terms of Ps inflow, based on the material balance equation.

To achieve the set aim, the following tasks have been solved:

- to perform the mathematical modeling of the dependence of an increase in the concentration of the j-th pollutant on an increase in the mass of the same substance within the watercourse limits;
- to suggest, based on the mathematical model, an optimum system of operative prediction and evaluation of anthropogenic loading.

### 4. Materials and methods for determining the optimum system of operational prediction and evaluation of anthropogenic loading

The object under consideration is the section of a river, bounded by the lower (i+1)-th and the upper (i)-th river stations (Fig. 1).

Thus, the mass of a pollutant, which enters a water object with concentrated discharges from water intake, is
The inflow of $Q_{i-1}$ and $Q_{i}$ through a similar one in the upper river $Q_{i}$, the mass flow rate of $P_{s}$ in the lower and upper river stations is $m_{i}$ and $m_{i}^{'}$, accordingly, the flow rate of a watercourse in the upper river station – via $Q_{j}$, in the bottom river station – via $Q_{j}$. The concentration of $P_{s}$ in the bottom river station is denoted via $C_{i}$, in the upper one – $C_{i}^{'}$.

The water flow rate in the lower river station consists of the transit flow rates through the upper river station ($Q_{i}$) and the flow rates formed within the selected area both through the concentrated and diffuse discharges ($\Delta Q_{m^{op}}$). Thus, the flow rate of water in the lower river station can be calculated as $Q_{i} = \Delta Q_{w} + \Delta Q_{m^{op}}$, in this case, $\Delta Q_{m^{op}} = \Delta Q_{m^{op}_i} = Q_{i} - Q_{i}$. Within the selected section of the river, the transformation of the content of the $j$-th pollutant in water occurs due to a series of processes.

First, due to the arrival of an additional amount of $P_{s}$ with the concentrated ($m_{i}^{'}; m_{i}^{''}; m_{i}^{'''}$) and diffusion ($m_{i}^{'''}; m_{i}^{''''}; m_{i}^{'''}$) discharges from the catchment area and its dilution due to an increase in the flow rate from the concentrated ($Q_{i}$; $Q_{i}^{'}$; $Q_{i}^{''}$) and diffusion ($q$) discharges, including underground.

Second, in natural surface water bodies, the processes of changing the content of $P_{s}$ are not limited to the “mechanical” arrival and dilution. The mainstream of the river between the upper $i$-th and the bottom $(i+1)$-th river stations (Fig. 1) continuously undergoes complex natural processes. These include the processes of the chemical, biological, physical-chemical nature, and others (chemical reactions, absorption, and release of ingredients by biotas, soils, adsorption, absorption, desorption, and many others). They all lead to the transformation of $P_{s}$.

Thus, in order to determine the optimal system of operational prediction and evaluation of anthropogenic loads on a surface water object, it is necessary to perform the mathematical modeling of the possible influence exerted by an increase in the mass of a pollutant on an increase in its concentrations within the watercourse areas that are considered.

![Fig. 1. Schematic of change in water flow rate in a river, the mass flow rate, and the concentration of the $j$-th pollutant or its compound, at the river section from river station $i$ to river station $(i+1)$](image)

The dependence of an increase in the concentration of the $j$-th pollutant on a decrease in its mass

5. Results of studying the establishment of the dependence of an increase in the concentration of the $j$-th pollutant on a decrease in its mass

Consider the pollutant mass flow rate in the bottom river station ($m_{i}^{''}i$) through a similar one in the upper river station ($m_{i}^{'''}i$). The self-cleaning capacity of a water object is ($m_{i}^{'''}i$). The inflow of $P_{s}$ within the selected area (diffusion, with point discharge) is

$$m_{i+1, (i+1) (i+1)} = \sum_{j} Q_{j} \cdot C_{j}^{'}$$

where $N$ is the number of concentrated tributaries, that is, $m_{i+1, (i+1) (i+1)} = m_{i+1, (i+1) (i+1)} + m_{i+1, (i+1) (i+1)}^{''}$. Consider the $\Delta m_{i+1, (i+1) (i+1)} = m_{i+1, (i+1) (i+1)}^{''} - m_{i+1, (i+1) (i+1)}^{'''}$. equation (1) can then be re-recorded:

$$\Delta m_{i+1, (i+1) (i+1)}^{''} = m_{i+1, (i+1) (i+1)}^{''} - m_{i+1, (i+1) (i+1)}^{'''}.$$ (2)

It is quite difficult to solve equation (2) since it is not possible to separate the inflow of a pollutant within the selected area, as well as its self-purification. At the same time, as noted above, the formation of a quality water composition directly in a surface water object occurs under the influence of two interrelated processes. The first of which is the process of “pure” mechanical “dilution” of $P_{s}$. The second is the transformation of the hydrochemical regime under the influence of natural processes (chemical, biological, physical-chemical, and others).

Consider a special case of a change in the qualitative composition of watercourses, based on their assimilation ability, and which is most often used in the practice of hydrochemical calculations, – this is the case of the so-called “pure dilution”. It is characterized by the following conditions:

1. The change in concentrations of pollutants occurs only at the expense of their arrival and subsequent dilution within the selected area, that is,

$$\Delta C_{i+1}^{'} = \Delta C_{i+1, (i+1) (i+1)}^{'} = C_{i+1}^{'} - C_{i+1} = C_{i+1, (i+1) (i+1)}^{'} - C_{i+1}.$$ 2. The changes in the mass of pollutants and, consequently, their concentrations due to processes inside the water reservoir (self-purification, arrival from bottom deposits, hydrobiological processes, etc.). Since a change in their masses in the mainstream of the river does not occur within the selected area, then

$$\Delta m_{i+1, (i+1) (i+1)}^{'} = m_{i+1, (i+1) (i+1)}^{'} + \Delta m_{i+1, (i+1) (i+1)}^{''}.$$ 3. The flow rate in the lower river station consists of the transit flow rate included in the upper river station ($Q_{i}$) and the flow rate of those arriving within the examined area ($\Delta Q_{i+1, (i+1)}$), that is,

$$\Delta Q_{i+1}^{'} = \Delta Q_{i+1} + Q_{i}.$$ Thus, the equation for calculating the concentration of the $j$-th pollutant in the bottom $(i+1)$-th river station for the conditions of “pure dilution” can be written as follows:

$$C_{i+1} = \frac{m_{i+1}^{''} + \Delta m_{i+1, (i+1) (i+1)}^{''}}{Q_{i+1}}.$$ (3) and the gain of the concentrations of the same substance between the considered river stations, which is the difference in the concentrations in the bottom $(i+1)$-th ($C_{i+1}^{'}$) and
in the upper (i-th) river stations \( (C_i') \), or \( \Delta C_{i+1}^{i} = C_{i+1}' - C_i' \), would equal:
\[
\Delta C_{i+1}^{i} = C_{i+1}' - C_i' = \frac{m_i^j + \Delta m_{i+1}^{i,j} \cdot Q_{i+1}}{Q_{i+1}} - C_i'.
\] (4)

After some transformations of equation (4), we obtain the following equation:
\[
\Delta C_{i+1}^{i} = \frac{1}{Q_{i+1}} \cdot \Delta m_{i+1}^{i,j} \cdot (C_i') \left( 1 - \frac{Q_i}{Q_{i+1}} \right).
\] (5)

which describes the transformation of the considered pollutant for the case of “pure dilution”.

We introduce to equation (5) the designation
\[
d_{i+1}^{i,j} = \frac{1}{Q_{i+1}},
\]
which makes it possible to conclude that it does not depend on the type of pollutant and is inversely proportional to the magnitude of watercourse flow rate in the finishing (i+1)-th river station. In addition,
\[
f_{i+1, i}^{i,j} = \frac{m_i^j - m_{i+1}^j \cdot Q_i}{Q_i Q_{i+1}} = (C_i') \left( 1 - \frac{Q_i}{Q_{i+1}} \right),
\]
equation (5) can then be transformed into a linear form:
\[
\Delta C_{i+1}^{i} = d_{i+1, i}^{i,j} \cdot \Delta m_{i+1}^{i,j} - f_{i+1, i}^{i,j}.\] (6)

Thus, by analyzing equation (6), it can be concluded that the numerical value of the angular coefficient \( d_{i+1, i}^{i,j} \) does not depend on the type of pollutant because it represents the inverse magnitude of the watercourse flow rate in the finishing (i+1)-th river station, that is,
\[
d_{i+1, i}^{i,j} = \frac{1}{Q_{i+1}}.
\]

It should also be noted that while, for the conditions of “pure dilution” within the selected area, the incoming flow rate is absent, that is, \( \Delta Q_{i+1, i}^{i, app} = 0 \), the water loss for evaporation and absorption is not great. Given this statement, one can make the assumption about the steady watercourse flow rate, that is, \( Q_i = Q_{i+1} + Q_{cont} \), and, as a consequence, the increase in flow rate, in this case, would equal zero. This allows us to conclude that the growth in the mass flow rate and concentration within the area analyzed is zero. That is, \( \Delta m_{i+1}^{i,j} = 0 \) and \( \Delta C_{i+1}^{i} = 0 \), and the linear dependence \( \Delta C_{i+1}^{i} = f (\Delta m_{i+1}^{i,j}) \), which describes “pure dilution” would pass through the coordinate origin.

**6. Discussion of results of studying the approximate necessary level of reduction in a pollutant adverse effect**

Consider an actual water object where the processes of self-purification and the inflow are inseparable and are almost always present. The actual balance of all processes for the real conditions is described by the following equation:
\[
\Delta m_{i+1}^{i,j} = \left( \Delta m_{i+1}^{i,j} \right)_{app} = \left( Q_i \cdot \Delta C_{i+1}^{i} \right)_{app}.
\] (7)

By using data on a retrospective analysis of the Dnieper river’s ecological status [16] as an actual surface water object, we treated them mathematically. The simulation of the processes that change the Ps concentration allows us to assert that under real conditions the dependence of the increase in the apparent concentration of the j-th pollutant on an increase in its mass \( (\Delta m_{i+1}^{i,j})_{app} \) within the watercourse sections, limited by the existing stationary river stations, is described by the linear equation of the following form:
\[
\Delta C_{i+1, i}^{i,j} = \Delta m_{i+1, i}^{i,j} \cdot \alpha_{i+1, i}^{i,j} - \beta_{i+1, i}^{i,j},
\] (8)

where \( \alpha_{i+1, i}^{i,j} \) and \( \beta_{i+1, i}^{i,j} \) are, accordingly, the actual masses and concentrations of the j-th pollutant in the upper and bottom watercourse river stations. Accordingly: \( \alpha_{i+1, i}^{i,j} \) and \( \beta_{i+1, i}^{i,j} \) are the values of the angular coefficient and a free term for this same substance. Numerical values of the indicators for the examined sections, the watercourse in general, and for different watercourses are unequal. They take into consideration both the historically formed conditions for the formation of quantitative and qualitative indicators and the uniqueness of the water basin within which their formation occurs. Fig. 2 shows, as an example, the dependence of \( \Delta C \) on the content of manganese along the length of the Dnieper river.

\[
\Delta C = f (\Delta m)\] (on the content of manganese along the length of the Dnieper river.

Equation (8) makes it possible to calculate the mass flow rate of the j-th pollutant or its concentration in any river station. In addition, it can additionally be used to simulate changes in the increase of both the mass flow rate of a pollutant and its concentration within the selected site, as well as in lower areas, in the development of water protection programs. These water protection programs allow the distribution of the planned standards of permissible influences (SPI) among water users located within this area.

It is known that Ps are divided into conservative and non-conservative. There is no comprehensive assessment of the degree of pollution of surface waters based on the hydrochemical indicators for the explicit categorization of substances into conservative and non-conservative ones. Therefore, such a division should, in many respects, be considered conditional because, depending on the wateriness of the year, the hydraulic, thermal, and hydrochemical regimes,
the “conservatism” or “non-conservatism” of $P$s are manifested in varying degrees. At the same time, the development methodology of SPIs implies that calculating them should include a non-conservative coefficient. Its numerical value must be determined on the basis of data from field observations or reference data and recalculated depending on water temperature and the current velocity. Given the meager amount or almost complete lack of data, determining the value of the non-conservative factor for a particular pollutant, or refining its reference value, is often impossible.

The following methodological approach to determining the $P$s conservatism is proposed. Fig. 3, 4 show the linear dependences that describe the transformation of $P$s in a river mainstream within the selected area of the watercourse, both for the conditions of “pure dilution” (line 1) and in natural water objects (line 2).

![Fig. 3. Transformation of non-conservative $P$s (biogens)](image)

Analysis of Fig. 3, 4 makes it possible to distinguish the following characteristic points:

- point II shows that when meeting these conditions in a real water object, the increase in the mass flow rate of the $j$-th pollutant occurs only due to a change in the flow rate within the selected area, while the concentration of this substance in this area remains constant;
- point III shows that when meeting these conditions in a real water object, the increase in the mass flow rate of the $j$-th pollutant occurs only due to changes in the concentration within the selected area, while the flow rate remains constant;
- point IV shows the ratio between the “pure dilution” and natural processes in water bodies, which depends on the type of pollutant.

At the equal values of an increase in the concentration of conservative substance, the increase in the mass flow rate for a real water object ($\Delta m_{i,app}$) would be less than that for the conditions of “pure dilution” ($\Delta m_{i,pd}$). That is, in a real water object, the increase in a concentration gain amplifies natural processes ($\Delta m_{i,pd}$).

Thus, the current idea about a self-purification capacity of a water object being considered as a “reserve” is not always justified. In particular, for non-conservative $P$s, we should expect an increase in the arrival of these substances due to secondary pollution. For conservative $P$s (Fig. 3), the correlation between the above processes turn out to be directly opposite.

Thus, one can state that a real surface water object, depending on the wateriness of the year, may demonstrate the following characteristic cases:

1) With the increase in flow rate, a change in the content of the $j$-th pollutant occurs only due to the predominance of the process of its arrival with the concentrated and diffusion discharges from the catchment area and its dilution due to the increase in the incoming flow rate.
2) With a reduction in the flow rate, the process of “mechanical” arrival and dilution decreases; the natural processes in water objects tend to dominate.

This, in turn, makes it possible to prove that the linear dependences, one of which describes the process of “pure” “mechanical” dilution, and the other – those processes that occur in a real water object, will be mutually intersecting lines (Fig. 3, 4).

Consider point IV – the intersection of two linear dependences at which “pure dilution” corresponds to the processes in a real water object. If one compares the overall form of equations (6) and (8), they practically coincide. Then the results of calculations based on these equations for the considered case, for the mean annual data, should also be equal to:

$$\begin{align*}
\Delta C_p &= \Delta C_{pd}, \\
\Delta m_p &= \Delta m_{pd}, \\
\Delta m_{i,app} &= 0.
\end{align*}$$

Based on this, we make an assumption. If the condition

$$a_{i,app}^{j} - a_{i,pd}^{j} = \pm \delta_Q,$$

is met, where $\delta_Q$ is the error of water flow rate measurement in a watercourse. The second condition:

$$b_{i,app}^{j} - b_{i,pd}^{j} = \pm \sqrt{\delta_C^2 + \delta_Q^2},$$

where $\delta_C$ is the error in determining the concentration of the $j$-th pollutant. Then a pollutant, which endures the meeting of these conditions is conservative; otherwise – non-conservative, for the specific conditions of a watercourse site.

The degree of a substance conservatism matter can be estimated, in the first approximation, based on the correlation coefficient ($R^2$) of experimental dependence (8). For conservative substances, $R^2$ is much lower than that for non-conservative ones.

By expressing, from the equation of the transformation module of the $j$-th pollutant (6), a gain in the mass flow rate...
\( (\Delta m_{i,t}^1 = M_{i,t}^1 \cdot f_{i,t}) \), and substituting into equation (8), we obtain:

\[
\Delta C_{i,t}^{\text{app}} = a_{i,t}^1 \cdot M_{i,t}^1 \cdot f_{i,t} - b_{i,t}^1.
\]  

(9)

Equation (9) links two interconnected processes: the formation of a hydrochemical regime of a water object within the selected area and its transformation directly in the water object itself. This allows us to reach this conclusion – the increase in the concentration of the \( j \)-th pollutant will be negative (self-purification processes will prevail over the arrival) in the following cases: at \( M_{i,t}^1 > 0 \) only if

\[
a_{i,t}^1 \cdot M_{i,t}^1 \cdot f_{i,t} < b_{i,t}^1,
\]

always at \( M_{i,t}^1 < 0 \).

If equation (9) is transformed into

\[
M_{i,t}^1 = \frac{\Delta C_{i,t}^{\text{app}}}{a_{i,t}^1 \cdot f_{i,t}} + \frac{b_{i,t}^1}{a_{i,t}^1 \cdot f_{i,t}},
\]

then, based on the processing of the multi-number, long enough series of average annual observations, one can be sure that the second term is actually the existing (current) regional “background” of a given pollutant. As for the first term, it will reflect the process of transforming the considered ingredient, both in the watercourse and its catchment area. The value of the first term should underlie a program of water protection measures (the purpose of ANLR) based on the planned change in the transformation module of the \( j \)-th pollutant.

The purpose of ANLR for a water object is to stabilize its qualitative composition at the existing level and, in the future, to improve it and return it to its natural condition. Assume that due to conducting water protection measures, the arrival of \( Ps \) decreases and, consequently, the concentration of the \( j \)-th pollutant along the watercourse length decreases, that is, the condition \( C_j > C_i^{\text{app}} \) is met (Fig. 4). Then, in an arbitrary river station \( \{ L_i \leq L \leq L_{i+1}\} \), located between the existing stationary ones, the concentration of any pollutant can be calculated from formula

\[
C_j = \left( \frac{C_{i+1}^{\text{app}} - C_i^{\text{app}}}{C_f} \right) L_{i+1} - L_i.
\]

If an arbitrarily selected river station coincides with those located above, that is, \( L = L_i \), then the power index in it turns to zero and \( C_{i+1} = C_i \).

In addition, as noted above, for two randomly selected \( Ps \) at a given site of the watercourse the dependences of an increase in concentrations on mass flow rates are described by the linear equations, as shown in Fig. 5.

Choose an arbitrary segment along the abscissa axis

\[
\Delta m_{i,t}^1 = \Delta m_{i,t} = \Delta m_{i,t}^2 \quad (\text{Fig. 5})
\]

and draw the normal to a given axis. We obtain, for a first substance, the ordinate \( \Delta C_j^{\text{app}} = \Delta m_{i,t} \cdot a_{i,t}^1 \cdot f_{i,t} - b_{i,t}^1 \).

An example is the use of equation (10) to calculate the content of sulfates in a watercourse. Based on the processing of average multi-year data, the linear dependence for sulfates over an area between posts 3 and 4 \[16\] takes the following form:

\[
\Delta C_1 = 0.022 \cdot \Delta m_{\text{sulfate}} - 0.001,
\]

and for chlorides:

\[
\Delta C_{\text{chloride}} = 0.0143 \cdot \Delta m_{\text{chloride}} - 0.033.
\]

Taking into consideration the above considerations, the dependence of the content of sulfates on chloride content can be represented as:

\[
\Delta C_{\text{sulfate}} = 1.559 \cdot \Delta m_{\text{chloride}} + 2.286.
\]

Equation (10) makes it possible, in the first approximation, to calculate an increase in the concentration of one pollutant on condition that the gain of the concentration of another pollutant is known.

1) If there are average multi-year data regarding the changes in the concentration of multiple \( Ps \), the dependence

\[
\Delta m_{i,t} = \Delta m_{i,t}^1 = \Delta m_{i,t}^2 \quad (\text{Fig. 5})
\]
of the increasing concentration \( \Delta C_{n+1,1} \) of any of them can be expressed via similar dependences of others:

\[
\Delta C_{n+1,1} = \frac{a_{n+1,1}}{a_{n,1}} \Delta C_{n,1} + \frac{a_{n+1,1}}{(n-1) a_{n,1}} \Delta C_{n,2} + \ldots + \frac{a_{n+1,1}}{(n-1) a_{n,1}} \Delta C_{n,n} + b_{n+1,1} \frac{a_{n+1,1}}{a_{n,1}} \left( \frac{b_{n+1,1}}{a_{n+1,1}} + \frac{b_{n+1,1}}{a_{n+1,1}} + \ldots + \frac{b_{n+1,1}}{a_{n+1,1}} \right)
\]

or

\[
\Delta C_{n+1,1} = \frac{a_{n+1,1}}{(n-1) a_{n,1}} \times \\
\left[ \frac{1}{a_{n+1,1}} (\Delta C_{n,1} - b_{n+1,1}) + \ldots + \frac{1}{a_{n+1,1}} (\Delta C_{n,n} - b_{n+1,1}) \right] + b_{n+1,1} \text{ at } n \geq 2. \tag{11}
\]

For example, for the considered area, the linear dependence for phosphates takes the form:

\[
\Delta C_{\text{phosphate}} = 0.019 \cdot \Delta m_{\text{phosphate}} - 0.020;
\]

for sulfates:

\[
\Delta C_{\text{sulfate}} = 0.022 \cdot \Delta m_{\text{sulfate}} - 0.001;
\]

for chlorides:

\[
\Delta C_{\text{chloride}} = 0.0143 \cdot \Delta m_{\text{chloride}} - 0.033.
\]

Then the dependence of the content of phosphates on sulfate content takes the form:

\[
\Delta C_{\text{phosphate}} = 0.066 \cdot \Delta C_{\text{sulfate}} + 0.422 \Delta C_{\text{chloride}} - 0.017.
\]

This makes it possible, in any intermediate river station, to calculate the transformation of one pollutant based on data for other \( Ps \).

3) The dependence of the sums of increases in the concentrations of several \( Ps \) \( \left( \sum_{i=1}^{N} \Delta C_{n+1,1}^{i} \right) \) on their components \( \left( \Delta C_{n+1,1}^{1}, \Delta C_{n+1,1}^{2}, \ldots, \Delta C_{n+1,1}^{N} \right) \) can be calculated from formula (11) believing that \( \sum_{i=1}^{N} \Delta C_{n+1,1}^{i} = \Delta C_{n+1,1} \).

Thus, the above dependences (10) and (11) make it possible, in any arbitrary river station, to calculate in the first approximation an increase in the concentrations of \( Ps \). This significantly reduces the material and temporal costs for determining the \( Ps \) content compared to traditional determination procedures.

The study reported in this paper is a continuation of the research into the environmental status of the Dnipro river basin. It seems promising to use the proposed model for a water reservoir’s conditions, within the watercourse basin.

7. Conclusions

1. The mathematical treatment of a long series of observations on the hydrochemical characteristics of the Dnipro river has shown that under actual conditions the dependence of an increase in the concentration of the \( j \)-th pollutant \( \Delta m_{\text{conc}(i, j)} \) in the mass of the same substance \( \Delta m_{\text{conc}(i, j)} \) in the sections of a watercourse, bounded by existing stationary river stations, is described by a linear equation.

2. We have derived the dependences that make it possible to calculate an increase in the \( Ps \) concentrations in any arbitrary river station. The analysis of these dependences has made it possible to establish the characteristic points that \( Ps \) possess regardless of their type. These points allow the prompt prediction of an increase in the mass flow rate of the considered pollutant and a change in the concentration of the same substance.

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