Dynamical Evolution of Galaxy Groups.
A comparison of two approaches.

García-Gómez, C.1, Athanassoula, E.2 and Garijo, A.1

Abstract. In this paper we test the performance of explicit simulations of groups of galaxies, (i.e simulations in which each galaxy is treated as a mass point and the physics of the interactions is modelled by specific analytical prescriptions for merging conditions), by comparing them with fully self-consistent simulations starting from identical initial conditions. The quality of the explicit simulations is very unequal. For some prescriptions the results are in complete disagreement with the self-consistent simulations. The inclusion of other dynamical effects like dynamical friction gives, in some cases, better agreement. We also propose a new merging criterion, which, combined with dynamical friction, gives much better agreement with self-consistent simulations in a variety of initial conditions, but even this criterion has a limited range of applicability.

1 Introduction.

Fully self-consistent N-body simulations, where each galaxy is represented by a large number of particles, are a useful, albeit expensive, tool for studying the evolution of galaxy groups and clusters. However, for simulations of large clusters of galaxies, like the Coma cluster, the necessary computing time is prohibitive. As a substitute people have considered explicit simulations, in which each galaxy is represented by a single point and the physics of the interactions is modelled by explicit prescriptions for merging conditions. In particular, a variety of recipes are explored for the conditions the two galaxies must fulfill in order to merge. In general, these merging conditions are based on self-consistent simulations of two-galaxy collisions, and do not include the tidal forces between the galaxies or collisions involving more than two galaxies. It is thus not a priori certain that they will perform well in simulations of group or cluster evolution. In some cases (Merritt, 1983; Richstone and Malumuth, 1983; Mamon 1987), the authors also introduce other effects like dynamical friction and tidal forces from the background. The main advantage of this type of approach is that it is inexpensive in computing time and therefore allows one to explore a wide parameter space. In any case, a considerable fraction of the results on the dynamics of galaxy groups are based on the explicit approach. We may cite works by Jones and Efstathiou (1979), Roos and Norman (1979), Aarseth and Fall (1980), Cooper and Miller (1981), Roos (1981), Roos and Aarseth (1982), Merritt (1983), Richstone and Malumuth

1Universitat Rovira i Virgili. E.T.S.E. Carretera Salou s/n. 43006 Tarragona Spain
2Observatoire de Marseille, 13248 Marseille Cedex 4, France
(1983), Malumuth and Richstone (1984), Saarinen and Valtonen (1985), Mamon (1987), 
Navarro et al. (1987) and Schindler and Bühringer (1993).

Not many self-consistent simulations of groups with more than 10 galaxies can be found 
in the literature. We can cite the articles by Carnevalli et al. (1981), Ishizawa et al. (1983), 
Ishizawa (1986), Rhee and Roos (1990), Barnes (1992), Funato et al. (1993) and Bode et 
al. (1994). The first works of this kind used Aarseth’s (1971) N-body code and a limited 
number of points, typically 10 − 20, to represent each galaxy, and only recently it has 
become possible to use the order of 1000 particles per galaxy.

Our aim is to compare the two approaches to see whether, and under what conditions, 
one can use explicit simulations and have confidence in the results. For this purpose, we 
have evolved a set of initial conditions in two different ways. One way is to use an N-body 
code where physics is included explicitly, the other, to use self-consistent simulations and 
a treecode (Barnes and Hut 1986, Hernquist 1987 for a vectorised version), representing 
each galaxy either by 100 or by 900 points. In section 2 we describe our initial conditions 
and the different merging criteria used so far in the literature. In section 3 we compare 
the results of fully self-consistent numerical simulations to those of explicit simulations 
made with the various merging criteria, both without (section 3.1) and with dynamical 
friction (section 3.2). This comparison led us to propose a new merging criterion (section 
3.3), whose performance we also compare with the fully self-consistent simulations. In 
this section we consider only groups with no common all-encompassing dark matter halo. 
Simulations including such a halo are presented in section 4, where again we compare the 
results of self-consistent and explicit simulations. We summarise and discuss our results in 
section 5.

2 Initial conditions and merging criteria

We have considered five different initial conditions, labeled A, B, C, D and H, each for 
systems consisting of 50 galaxies. In simulations A, B, D and H the radial distances 
from the galaxy centers to the center of the group were picked at random between 0 and 
$R_{out}$. For simulation C the central part of the sphere contained no galaxy, i.e. the radial 
distances were picked between $0.5R_{out}$ and $R_{out}$. For simulations A to D all the mass is in 
the individual galaxies, while in simulation H we included a common live halo, centered on 
the center of the group, and containing half of the total mass. The halo density distribution 
is a Plummer one with a core radius equal to half $R_{out}$. Run A starts in free-fall, and we 
will often refer to it as the collapsing group. The velocity dispersions in the remaining 
three runs were chosen to be independent of radius, gaussian, isotropic, and such that the 
system of galaxies starts off in virial equilibrium. Simulation D is similar to B but more 
compact, as the radius of the sphere containing all the galaxies is half that of run B. The
Table 1: Initial conditions of the simulations

| Run | \( R_{\text{out}} \) | \(<\text{sep}>\) | \( \sigma_{cl}/\sigma_{\text{gal}} \) | \( t_{cr} \) | \( t_{\text{tot}}/t_{cr} \) |
|-----|----------------|--------------|----------------|----------|----------------|
| A   | 30            | 8.0          | 0.0            | 15.3     | 2.0            |
| B   | 20            | 6.8          | 1.4            | 4.5      | 6.7            |
| C   | 20            | 10.3         | 1.0            | 11.6     | 2.6            |
| D   | 10            | 3.4          | 1.9            | 1.6      | 18.7           |
| H   | 20            | 7.4          | 2.7            | 8.9      | 3.4            |

Particles in a given galaxy were initially taken to follow a Plummer distribution of core radius equal to 0.2 and of unit mass. When evolved in isolation, an individual galaxy first shows a low amplitude relaxation in the very first few time steps due to the fact that the simulations have a softening, while the analytical Plummer sphere does not. After that, and for a time equal to that during which the group simulations were run, the galaxies do not evolve any further. Thus, during that time, for the representations with 900 particles per galaxy, the radii containing 25%, 50% and 75% of the mass of the galaxy vary only by a couple of percent. For the representation with 100 particles the radii containing 25% and 50% of the mass vary by 4-5%, and only the radius containing 75% of the mass varies significantly, particularly in the later phases of the evolution.

More information on the initial conditions for the simulations is summarised in Table 1. Column 1 contains the name of the simulation, Column 2 gives \( R_{\text{out}} \), the radius of the sphere containing the group at the start of the simulation, Column 3 shows the initial mean separation between the galaxies and Column 4 gives the ratio between the initial velocity dispersion of the galaxies considered as point masses, \( \sigma_{cl} \), divided by the velocity dispersion of the particles within a single galaxy, \( \sigma_{\text{gal}} \). Column 5 contains the crossing time defined as

\[
t_{cr} = \left( \frac{2R_h^3}{GM} \right)^{1/2},
\]

where \( R_h \) represents the half mass radius. Finally, Column 6 contains the ratio between \( t_{\text{tot}} \), the total duration of the simulation and \( t_{cr} \). All through this paper our units are such that the gravitational constant \( G = 1 \).

The self-consistent simulations were run using the vectorised version (Hernquist 1988) of the Barnes-Hut tree algorithm (Barnes and Hut 1986), with a softening of 0.05 and an opening angle \( \theta = 0.7 \). In explicit simulations each galaxy is represented by a single point to which is associated a mass, an internal energy and a core radius. These parameters may change during the evolution of the system due to the different interactions suffered by the point-galaxies and we used the recipes of Aarseth and Fall (1980) to follow their time
evolution. The explicit simulations are of course much faster than the self-consistent ones. A complete self-consistent simulation with 900 points per galaxy took 521663 seconds in a Cray YMP 2L computer. The self-consistent simulation with 100 particles per galaxy lasted only the 5% of this time and the explicit simulation only 0.2%.

In order to compare the results of the different kind of simulations we consider the time evolution of the following global parameters of the groups:

1. Number of galaxies: \(N_{\text{gal}}\)

2. Half mass radius: \(R_h\), where \(M(R_h) = 1/2 \, M_{\text{tot}}\)

3. Three dimensional velocity dispersion:

\[
\sigma_v^2 = \frac{\sum_{i=1}^{N_{\text{gal}}} m_i \left| \mathbf{v}_i - <\mathbf{v}> \right|^2}{M_{\text{tot}} - m_i(t = 0)}, \quad \text{where} \quad <\mathbf{v}> = \sum_{i=1}^{N_{\text{gal}}} \frac{m_i \mathbf{v}_i}{M_{\text{tot}}},
\]

where all quantities are evaluated at each timestep, except for \(m_i(t = 0) = 1\) which is the mass of all individual galaxies at the start of the simulations and is taken to be \(m_i(t = 0) = 1\).

In our explicit simulations we consider, in a first stage, only merging between galaxies. In a second set of simulations we include also the effect of dynamical friction. In this way we can check the importance of both effects. Merging between galaxies is usually described in the literature using an explicit condition involving the separation and relative velocities of the pair of galaxies. If this condition is fulfilled, the two galaxies are merged in a single one in this timestep, taking into account the conservation of mass, energy and momentum (Aarseth & Fall 1980). If this condition is not fulfilled both galaxies survive and continue their motion.

We found in the literature various criteria which have been used to decide whether two galaxies are going to merge and we used all of them in turn in our explicit simulations. The condition of Roos and Norman (1979, hereafter condition RN) is:

\[
v(r_p) \leq 3.1 \sigma (1 - 0.3 \frac{r_p}{R_g}) \left( \frac{1 + m_2/m_1}{2} \right)^{1/4}
\]

where \(m_2 \leq m_1\) and \(r_p/R_g < 1\). \(r_p\) is the minimum separation between the galaxies, \(v(r_p)\) is their relative velocity at \(r_p\), and \(R_g\) is the larger of their radii. This criterion was obtained empirically from collisions between galaxies described by fewer than 100 particles.
Aarseth and Fall (1980, hereafter condition AF) used the criterion:

\[
\left[ \frac{r_p}{2.6(\epsilon_1 + \epsilon_2)} \right]^2 + \left[ \frac{v(r_p)}{1.16v_e(r_p)} \right]^2 \leq 1, \tag{3}
\]

which is a simple fit to the results of the simulations of van Albada and van Gorkom (1977), White (1978) and Roos and Norman (1979). The core radius of galaxy \( i \) is \( \epsilon_i \), while \( v_e(r_p) \) is the escape velocity of the system composed of the two galaxies before merging at pericenter:

\[
v_e^2(r_p) = 2G(m_1 + m_2)(r_p^2 + \epsilon_1^2 + \epsilon_2^2)^{-1/2}. \tag{4}
\]

Farouki and Shapiro (1982, hereafter condition FS) obtained a similar condition for the merging of two rotating galaxies with massive halos and spins aligned with the orbital angular momentum:

\[
\left[ \frac{r_p}{5.5(\epsilon_1 + \epsilon_2)} \right]^2 + \left[ \frac{v(r_p)}{1.1v_e(r_p)} \right]^2 \leq 1. \tag{5}
\]

This condition predicts more mergings than the criterion from Aarseth and Fall (1980) for two reasons. It favours collisions between galaxies further apart and it forces the spins to be aligned. This criterion is not directly applicable to our case, where we use initially nonrotating Plummer spheres, but we include it for the sake of completeness.

Finally Richstone and Malumuth (1983, hereafter condition RM) use the different criterion:

\[
r_p v(r_p) \leq \left[ \frac{8}{3} G^2 (m_1 < r_2^2 > + m_2 < r_1^2 >)(m_1 + m_2) \right]^{1/4}, \tag{6}
\]

which is a generalisation of a criterion proposed by Tremaine (1980) for the case of different masses. The value \( < r_i^2 > \) is the mean quadratic radius of a galaxy. For the case of a Plummer sphere \( < r^2 > = \epsilon^2/2 \), and this is the value we have used in our simulations.

To save computer time we do not need to apply the adopted merging criterion to all galaxy pairs at all times. Following Navarro et al. (1987), we check whether the condition is fulfilled only if the separation between two galaxies is smaller than \( 3(r_{h_1} + r_{h_2}) \), where \( r_{h_i} \) is the half mass radius of the galaxy \( i \). This separation is sufficiently large so that merging events are not missed, while speeding up considerably the computations.

As the simulations evolve a central giant "galaxy" is formed as a result of the mergings and/or tidal stripping of the galaxies in the group. Dynamical friction between this and the remaining individual galaxies influences the evolution and we have therefore included this effect in the explicit simulations, using the well known Chandrashekar (1943) formula.
for the deceleration:

\[ a_v = -\frac{4\pi G^2 m_{gal} \ln \Lambda \rho(r)}{v^3} F(v) v \]  

(7)

where

\[ F(v) = \text{erf}(X) - \frac{2X}{\sqrt{\pi}} e^{-X^2} \]  

(8)

and \( \text{erf}(X) \) is the error function, \( X = v/\sqrt{2}\sigma \), \( \sigma \) is the velocity dispersion of the objects in the background, and \( m_{gal} \) is the mass of the galaxy travelling at speed \( v \); \( \rho(r) \) is the density of the central galaxy, considered as a Plummer sphere, at the position of the secondary galaxy, \( r \) being the relative separation of their centers, and \( \Lambda = b_{max}/b_{min} \), where \( b_{max} \) and \( b_{min} \) are the maximum and minimum impact parameters of encounters contributing to the drag. When we include a common halo we apply Eq. (6) twice, once for the central giant “galaxy” and the other for the halo, adding these two accelerations.

The self-consistent simulations where analyzed as follows. First, we need to define the central giant “galaxy”, which we will refer to in this paragraph simply as the central object. In order to do so, we analyze at each timestep separately each subsystem composed of the particles that were bound at \( t = 0 \) in a single galaxy. Using the positions and velocities of these particles we discard from the subsystem all particles with positive energy relative to it and consider them as part of the central object. The particles that still form a bound subsystem will define the state of the galaxy at this timestep. If after this process a galaxy contains less than 10% of the particles it had at \( t = 0 \), we discard this subsystem as a galaxy and we add all its particles to the central object, thus considering that the initial galaxy has been definitely disrupted. For each of the remaining galaxies we use the 35% of its most bound particles to define its position and velocity. Finally, we also consider possible mergings between the remaining galaxies, as well as between these galaxies and the central object. Two galaxies were merged in a single one if the following conditions are satisfied:

\[ \Delta r < a(r_{c1} + r_{c2}) \]
\[ \Delta v < b(\sigma_1 + \sigma_2) \]

where \( r_{ci} \) is the radius of the sphere containing the 35% most bound particles and \( \sigma_i \) its velocity dispersion. The constants \( a = 1.4 \) and \( b = 0.6 \) were selected in order to have smooth central objects. The parameters of this object were calculated with just the 10% most bound particles and not with the 35% as with the rest of the galaxies. This ensured that we do not consider a merger between the central galaxy and another galaxy while they still form two separate objects. We finally used the positions and velocities of the remaining galaxies to define the global parameters of the system.

3 Simulations without common dark matter halo
3.1 Evolution without dynamical friction.

In Fig. 1 we show the evolution of the number of galaxies $N_{\text{gal}}$ as a function of time for all the simulations without distributed dark matter. In the first column we compare the self-consistent simulations with 900 and 100 particles per galaxy with the explicit simulations obtained with the AF and RM conditions. In the second column, the evolution of the number of galaxies in the self-consistent simulations is compared with the explicit simulations using the FS and RN conditions.

We note that the explicit simulations perform rather unequally. The results depend on the type of initial conditions and on the merging condition used to describe the interactions. Globally we can say that the AF and RM conditions seem to follow the time evolution of $N_{\text{gal}}$ much better than the FS and RN conditions. In the first stages of the evolution of the collapsing group (Run A), the less tightly bound and virialised group (Run B) and for Run C, which is a virialised group with no central mass concentration, both AF and RM conditions describe the time evolution of the number of galaxies rather well. This is not true, however, for Run D (tightly bound and virialised group), for which the AF condition overestimates the number of mergings from the start, while the RM condition does the opposite. As the evolution proceeds the discrepancies between the self-consistent simulations and the explicit simulations become more evident. For all initial conditions the FS and RN conditions overestimate the number of mergings from the start. The sole exception is the explicit simulation with the FS condition in the case of Run C, where the agreement with the self-consistent simulation is quite good.

For the time evolution of the half-mass radius, $R_h$, we find similar results. This can be seen in Fig. 2, where the panels refer to the same initial conditions as in Fig. 1. In general, the explicit simulations controlled by the AF and RM conditions show a better global behaviour than the simulations governed by the FS and RN conditions. This is due to the high number of mergings predicted by the latter conditions. In the case of Run A, all explicit simulations follow well the collapse phase. When most of the mass is accumulated in the central area, the number of encounters is relatively large and there are strong interactions with the giant central galaxy. At this moment, self-consistent and explicit simulations separate. The AF and RM conditions allow some galaxies to avoid merging with the giant galaxy in the first passage and the system experiences an expansion which is not shown in self-consistent simulations. On the other hand, the FS and RN conditions predict a much higher rate of mergers than the self-consistent simulations and we are left too early with only a single giant galaxy. In the case of Run B, the AF and RM conditions describe very well the state of the system during the first part of the simulations. As the simulation evolves, however, some galaxies reach the central parts where they suffer an hyperbolic encounter with the central mass concentration of the giant galaxy instead of merging with it, as is the case in the self-consistent simulations, because the merging criteria strongly disfavour merging in high speed collisions. This makes the
system expand, an effect which is not seen in the self-consistent simulations. This does not happen for Run C where there is no such central mass concentration and explicit and self-consistent simulations follow the same evolution, except for minor differences and a strong deviation for the case of condition RN. Run D is the most difficult case for the explicit simulations. In this situation galaxies move at higher speeds than in Run B or Run C. Surprisingly, in the case of self-consistent simulations, this does not make merging with the central object more difficult, as one might expect naively in the first instance. However, the RM conditions predict more hyperbolic encounters than the self-consistent simulations, giving strong oscillations of the half mass radius. On the other hand, the AF condition seem to describe the situation quite well. The number of galaxies predicted by the RN and FS conditions are well below the numbers predicted by the self-consistent simulations, again due to the high number of mergers predicted by these conditions.

Finally, in Fig. 3 we show similar comparisons, now for the three dimensional velocity dispersion. The larger number of mergers predicted by the FS and RN conditions nearly always gives lower velocity dispersions than the self-consistent simulations as well as strong oscillations due to small number statistics. On the other hand, the AF and RM conditions give a better general description of the evolution of the three dimensional velocity dispersion. This is specially true for Run A, where all the motion is nearly radial and only small discrepancies appear at the end of the simulations. For the case of Run B and the RM condition, the hyperbolic encounters which lead to a higher half mass radius of the system, give also higher velocity dispersions, because some galaxies which merge in the self-consistent simulations can escape in the explicit ones. The AF condition describes this time evolution much better. The velocity dispersion of Run C is well described for both conditions until shortly before the end of the simulation, when both conditions predict higher velocity dispersions than the self-consistent simulations. In the case of Run D the RM condition has again some difficulty in describing the behaviour of the self-consistent simulations. This is also due to the high number of large deflections of the secondary galaxies. The AF condition follows well the evolution of the three dimensional velocity dispersion in this situation.

We would like to note at this point that the self-consistent simulations with 100 particles per galaxy and with 900 particles per galaxy do not show major differences. The number of galaxies as a function of time does not change appreciably between these two simulations and this for all the initial conditions, i.e. both for virialised and collapsing groups. In this sense our results differ from those of van Kampen (1995), who found that the small virialised clumps formed during the simulations associated with the galaxies do not resist the passage through the central part of the cluster. This could be due to the somewhat lower number of particles per galaxy, since the typical galaxies in van Kampen’s simulations are composed of 10-50 points (van Kampen 1995).

Similarly good agreement between the 900 and 100 points per galaxy simulations is found for the velocity dispersion. Somewhat bigger differences, in particular for run B, can
be seen for the half-mass radius, but even these are not excessive.

### 3.2 Simulations with dynamical friction.

Figure 4 compares the time evolution of the number of galaxies in the self-consistent simulations and in the explicit simulations when the effect of dynamical friction is included. Since this slows down the galaxies and thus favours merging, the number of galaxies, $N_g$, will diminish faster. This is clearly seen in all the panels of Fig. 4. As can be seen from the left hand panels, this worsens the predictions of the RN and FS conditions. The right hand panels show that the agreement is now better for the RM condition, and worst for the AF one. For the case of Run A there is a systematic deviation between the AF condition and the self-consistent simulations. On the other hand, the RM condition which had, in the absence of dynamical friction, predicted a low number of mergings is, in this case, in much better agreement with the self-consistent case. The same can be said about Run B, while in Run C the effect of dynamical frictions is not noticeable. For the most difficult case, Run D, the AF condition falls below the results of the self-consistent simulations while the RM condition gives good agreement.

The evolution of the half mass radius is also affected by the inclusion of dynamical friction, as is shown in Fig. 5, where we plot the evolution of $R_h$ as a function of time. For the explicit simulations with the RN and FS conditions dynamical friction does not alter the strong disagreement with the self-consistent simulations. This happens because the explicit simulations with these conditions allow too many mergings and we are left with a single supergiant galaxy at the center of the system which contains a large fraction of the mass and some small satellites. On the other hand, there is now a much better agreement between the explicit simulations made with the AF and RM conditions and the self-consistent cases. For Run A neither condition shows a secondary bouncing of the system. The dynamical friction acts as a braking mechanism that favours merging between the secondary galaxies and the central one and a lower number of satellites survive in this situation. In Run B, the hyperbolic encounters of the satellite galaxies with the central giant are not present and there is no later expansion of the system as in the explicit simulations without dynamical friction. The explicit simulations with both the AF and RM conditions predict too small a half mass radius. For Run C, as there is no giant galaxy, dynamical friction is unimportant and all the simulations again show the same general behaviour. In Run D the galaxies move faster because the system is more tightly bound. The explicit simulations with the RM condition and no dynamical friction were not capable of describing the evolution of the self-consistent simulations. The inclusion of dynamical friction gives a much better agreement between these two simulations. On the other hand, the explicit simulations with the AF conditions seem to be systematically
below the predictions of the self-consistent simulations.

As can be seen in Fig. 6, the three dimensional velocity dispersion shows marked differences between the explicit simulations and the self-consistent ones. As was the case in the absence of dynamical friction the explicit simulations with the RN and FS conditions do not track well the self-consistent results. The dynamical friction effect is barely noticeable in this case, except for some tendency towards lower velocity dispersions. As the RN and FS conditions predict many mergings, we are left with a giant galaxy in the center and a low number of satellites orbiting around it. The dispersions are then low but they are more subject to fluctuations and have stronger oscillations. Including dynamical friction in the explicit simulations with the AF and RM conditions does not substantially improve their results as can be seen if we compare Fig. 6 with Fig. 3. For runs A and C the situation is further improved and the explicit simulations follow the self-consistent ones very well. Bigger differences between the explicit simulations with and without dynamical friction are found for the virialised groups (Run B and D). The values predicted by the AF condition are now always near the values obtained with the self-consistent simulations. However, this is not the case for the RM parametrization. For Run B, there are marked differences between these explicit simulations and the last phase of the self-consistent simulations. For the case of Run D the RM condition gives a systematically higher velocity dispersion than the self-consistent simulations.

### 3.3 A new merging criterion

As we have seen, none of the merging criteria proposed so far in the literature is capable of describing the time evolution of the global properties of groups of galaxies in the variety of situations considered in this paper. We can say that, in general, the AF and RM conditions perform better that the FS and RN ones, but even they fail to describe the evolution of some of the groups. This has motivated our search for a more adequate merging criterion.

We searched for a formula of a form similar to the one proposed by Aarseth and Fall (1980), namely:

$$\left[\frac{(m_1 + m_2) r_p}{a (m_1 \epsilon_1 + m_2 \epsilon_2)}\right]^2 + \left[\frac{v(r_p)}{b v_c(r_p)}\right]^2 \leq 1.$$  (9)

For the part concerning the velocities, we keep the same expression as in the Aarseth and Fall formula, which performs quite well in the case of the time evolution of the three dimensional velocity dispersions. For the part concerning the cores of the galaxies and the separation at pericenter we use a mass weighted expression with the aim of taking into account possible differences in collisions between galaxies of different masses as in the expression due to Richstone and Malumuth (1983). The constants $a$ and $b$ are free parameters and will be determined using the self-consistent simulations as a reference.
This expression can be viewed as the equation of the points within an ellipse centered at the origin in the plane defined by \((m_1 + m_2)r_p/(m_1\epsilon_1 + m_2\epsilon_2)\) and \(v(r_p)/v_e(r_p)\). Then \(a\) and \(b\) are the semimajor axes of this ellipse. Increasing the value of \(a\) means increasing the axis of the ellipse corresponding to the relative separation at pericenter and thus allowing mergings in more distant collisions. On the other hand, if we increase the value of \(b\) we allow merging in faster collisions. With this in mind, we fitted the values of \(a\) and \(b\) to the self-consistent simulations using as the basis for our exploration the values used by Aarseth and Fall (1980). After some trials and comparisons with the self-consistent simulations we obtained the following merging criterion:

\[
\left[ \frac{(m_1 + m_2)r_p}{2.5(m_1\epsilon_1 + m_2\epsilon_2)} \right]^2 + \left[ \frac{v(r_p)}{1.18v_e(r_p)} \right]^2 \leq 1.
\]  

The effect of this new criterion is shown in Figs. 7, Fig. 8 and Fig. 9, where we compare the time evolution of the global parameters of the self-consistent simulations with that of the explicit simulations using the AF and RM criteria and our new one. The dynamical friction with the most massive galaxy is also included in these cases.

In Fig. 7 we show the time evolution of the number of galaxies \(N_g\). In the first column, we repeat the comparison between the self-consistent simulations and the explicit simulations with the AF and RM criteria and dynamical friction. In the second column, we have the comparison between the self-consistent simulations and the explicit simulations with dynamical friction and our new merging criterion. As can be seen, while the explicit simulations with the RM criterion mimic quite well the self-consistent simulations, this is not true for the AF condition. On the other hand, our new criteria follows quite well the evolution of the number of galaxies given by the self-consistent simulations for all initial conditions.

In Fig. 8 we show the time evolution of the half mass radius. For the case of Run A both AF and RM conditions follow quite well the self-consistent simulations until the point of maximum collapse. After this point, the half mass radius given by these explicit simulations falls below the self-consistent case. Our new condition, however, follows the self-consistent simulations with 900 particles very well. For the case of Run B, the AF and RM conditions end below the self-consistent case. Our new criterion performs better, following the self-consistent simulations, but with some oscillations. For runs C and D we can say that all three criteria give similar results.

Figure 9 which gives the time evolution of the three dimensional velocity dispersion, is the most interesting one. We have seen that the AF and RM conditions give good results for the case of the collapsing group (Run A) and this is true also for our new criterion. However, the AF and RM explicit simulations do not work well for the case of a virialised group (Run B). The AF condition ends with a higher velocity dispersion and the RM with a smaller velocity dispersion compared to the self-consistent case; on the
other hand, our new criterion performs much better than either. This is specially true for the most difficult case, Run D, the virialised and tightly bound group. In this case our new criterion performs much better than the AF and RM criteria.

4 Simulations with a dark matter halo encompassing the whole group

Several observations suggest that clusters and groups of galaxies may contain much matter not bound to the galaxies. This led us to run a self-consistent simulation (Run H), where part of the mass of the system is distributed in a background. In the corresponding explicit simulations the background is included as a rigid Plummer potential with the same parameters as the live background in the initial conditions of the self-consistent simulation. The explicit simulations include dynamical friction with the most massive galaxy and with the Plummer halo.

The evolution of the group leads to a system where the central part of the galaxy distribution has contracted, while the outer one has expanded. This results in an increase of the half-mass radius and a lowering of the velocity dispersion, as shown in Fig. 10. The upper panels give the time evolution of the number of galaxies in the system $N_g$, the middle ones that of the half mass radius $R_h$ and the lower ones that of the three dimensional velocity dispersion. In the left panels the self-consistent simulations are compared to the explicit simulations with the AF and RM conditions and in the right panels with simulations using our new criterion. As we can see, the number of galaxies diminishes slower in simulations including a common halo than in the case of virialised simulations with no distributed dark matter. The AF and RM conditions underestimate the real number of mergers, and so, though to a lesser extent, does our new criterion. For the time evolution of the half mass radius there are strong discrepancies between the self-consistent simulations and the explicit simulations using any of the merging criteria including the new criterion proposed in the previous section.

The three dimensional velocity dispersion of the galaxies is well described by the explicit simulations using any of the merging criteria. This global parameter systematically decreases during the simulation as the galaxies that move faster near the center disappear and form the giant central object. The slope of this evolution flattens off toward the end of the simulations. This behaviour is not well followed by the explicit simulations using the AF or RM criterion. On the other hand, Fig. 10 shows that our new merging criterion is able to reproduce these minor details better.
5 Summary.

In this paper we compared self-consistent simulations of galaxy groups with simulations where the physics of the interactions is modelled by merger rules. We used two sets of self-consistent simulations, one in which the galaxies were modelled with 900 points and the other with 100 points. Insofar as the global dynamical parameters are concerned, the evolution of galaxy groups is similar in those two cases. This shows that simulations with a relatively low number of particles can be used to follow the evolution of global dynamical properties of groups or clusters. However, from the work of van Kampen (1995) it can be inferred that using lower than 100 points per galaxy can be dangerous.

As far as the explicit simulations are concerned, we show that the conditions used in the literature to simulate the merging between galaxies are of unequal quality. Of these conditions, in the case where there is neither dynamical friction nor tidal forces, the best are those of Tremaine (1980), modified for the case of different masses by Richstone and Malumuth (1983), and the one by Aarseth and Fall (1980). When we include dynamical friction effects the AF condition predicts too many mergers but still maintains good predictions for the rest of the global parameters. The condition proposed by Richstone and Malumuth (1980) does better as far as the number of galaxies and \( R_h \) are concerned, but considerably worse for the velocity dispersion.

As none of these criteria seems to be a good guide for the time evolution of the groups as compared with the self-consistent simulations, we have fitted a new criterion to the results of self-consistent simulations. This new criterion is:

\[
\left[ \frac{(m_1 + m_2)r_p}{2.5(m_1\epsilon_1 + m_2\epsilon_2)} \right]^2 + \left[ \frac{v(r_p)}{1.18v_p(r_p)} \right]^2 \leq 1, \tag{11}
\]

and is inspired in the expressions given by Aarseth and Fall (1980) and Richstone and Malumuth (1980). This new criterion mimics relatively well the time evolution of the global parameters of the groups in as wide a variety of situations as those presented by our simulations A to D. However it performed not so well in case H which has a common halo, but this can be explained by the different nature of the simulations implying that even this new criterion has only a limited range of applicability.

Our comparisons show that some of the older results on the dynamics of groups and clusters of galaxies should be viewed with caution. For instance, Roos (1981) studied the evolution of expanding systems of galaxies to simulate the evolving universe. As he used the RN criterion in his simulation the predicted merger rate can be too high. In the same way, when Roos and Aarseth (1982) used this criterion to study the evolution of the luminosity function of a cluster of galaxies, their final luminosity functions can be artificially peaked towards high luminosities. Similarly, Valtonen et al. (1984), Saarinen and Valtonen (1985) and Perea et al. (1990) use explicit simulations to criticize the virial
mass obtained for galaxy clusters. We have, however, seen that this kind of simulation is biased toward higher velocity dispersions. Finally, the explicit simulations on compact groups by Mamon (1987) using a diffuse intergalactic background may also be biased.

Thus we can conclude that there is no ideal substitute for fully self-consistent N-body simulations. However, in cases when one needs to look only at global quantities describing the system and is not interested in fine structure and details, a first exploration of parameter space can be done using explicit simulations and the criterion proposed in this paper. This performs particularly well in cases where the group has no common halo.

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Figure Captions.

**Fig. 1** Comparison of the time evolution of the number of galaxies in the self-consistent simulations with 100 particles per galaxy (thin line) and 900 particles per galaxy (thick line) with the explicit simulations for the same initial conditions and without dynamical friction. In the left panels we use the AF and RM merging conditions and in the right panels we use the FS and RN ones. The initial conditions of each simulation are described in Table 1.

**Fig. 2** Comparison of the time evolution of the half mass radius of the system in self-consistent simulations and in explicit simulations without dynamical friction for the same initial conditions. The symbols are as in Fig. 1.

**Fig. 3** Time evolution of the three dimensional velocity dispersion of the galaxies considered as point masses for the self-consistent and the explicit simulations. The symbols are as in Fig. 1.

**Fig. 4** Time evolution of the number of galaxies \(N_g\) in the self-consistent simulations compared with the evolution of this number in the explicit simulations with dynamical friction included. The thick lines correspond to the self-consistent simulations with 900 particles per galaxy and the thin lines to the simulations with 100 points per galaxy. In the first column, we show the comparison with the explicit simulations using the AF criterion and using the RM criterion. In the second column, we show the same comparisons with the explicit simulations using the FS condition and using the RN condition.

**Fig. 5** Same as for Fig. 4 but for the time evolution of the half mass radius of the system.

**Fig. 6** Same as for Fig. 5 but for the time evolution of the three dimensional velocity dispersion.

**Fig. 7** Comparison of the explicit simulations using the AF and the RM criteria with the explicit simulations using the new criterion. The performance of each criterion is compared with the self-consistent simulations. In the first column, we show the time evolution of the number of galaxies in the self-consistent simulations with 900 particles per galaxy (thick lines) and with 100 particles per galaxy (thin lines) compared with the explicit simulations using the AF criterion and using the RM criterion. In the second column, we compare the time evolution of \(N_g\) for the self-consistent simulations with the results of the explicit simulations using the new criterion. In all cases we include dynamical friction.

**Fig. 8** Same as Fig. 7 but for the time evolution of the half mass radius of the system.

**Fig. 9** Same as Fig. 7 but for the time evolution of the three dimensional velocity dispersion.
**Fig. 10** Time evolution of the global parameters of the simulations with distributed background. In both columns we show the evolution of $N_g$, $R_h$ and $\sigma(3D)$ for the self-consistent simulations with 450 particles per galaxy (thick lines) and for the self-consistent simulations with 100 particles per galaxy (thin lines). In the left panel these are compared with the explicit simulations with the AF condition and with the RM condition. In the right panel the self-consistent simulations are compared with the explicit simulations with our new criterion.
