Implicit Generation and Generalization with Energy Based Models

Yilun Du and Igor Mordatch
Energy-Based Model

- Distribution defined by energy function

\[ p_\theta(x) = \frac{\exp(-E_\theta(x))}{Z(\theta)} \quad Z(\theta) = \int \exp(-E_\theta(x))dx \]

see [LeCun et al, 2006] for review
Energy-Based Model

- Distribution defined by energy function

\[ p_\theta(x) = \frac{\exp(-E_\theta(x))}{Z(\theta)} \]

- Train to maximize data likelihood

\[ \mathcal{L}_{\text{ML}}(\theta) = \mathbb{E}_{x \sim p_D}[-\log p_\theta(x)] \]
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\[ \mathcal{L}_{\text{ML}}(\theta) = \mathbb{E}_{x \sim p_D} [-\log p_\theta(x)] \]

• Gradient:

\[ \mathbb{E}_{x^+ \sim p_D} [\nabla_\theta E_\theta(x^+)] - \mathbb{E}_{x^- \sim p_\theta} [\nabla_\theta E_\theta(x^-)] \]

See [Turner, 2006] for derivation
Energy-Based Model

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- gradient:

\[ \mathbb{E}_{x^+ \sim p_D} [\nabla_\theta E_\theta(x^+)] - \mathbb{E}_{x^- \sim p_\theta} [\nabla_\theta E_\theta(x^-)] \]

- Generate model samples implicitly via stochastic optimization

\[ \tilde{x}^k = \tilde{x}^{k-1} - \frac{\lambda}{2} \nabla_x E_\theta(\tilde{x}^{k-1}) + \omega^k, \ \omega^k \sim \mathcal{N}(0, \lambda) \]

Langevin Dynamics [Welling and Teh, 2011]
Why Energy-Based Generative Models?

1 Implicit Generation
   - Flexibility
   - One Object to Learn
   - Compositional
   - Generic Initialization and Computation Time

2 Intriguing Properties
   - Robustness
   - Online Learning
Why Do EBMs Work Now?

More compute and modern deep learning practices

Faster Sampling

- Continuous gradient based sampling using Langevin Dynamics
- Replay buffer of past samples (similar to persistent CD)

Stability improvements

- Constrain Lipschitz constant of energy function (spectral norm)
- Smoother activations (swish)
- And others ...
Comparison to Other Generative Models

- Training Cost
- Sampling Speed

- SNGAN
- Glow
- PixelCNN++
- EBM
| Model                                         | Inception | FID   |
|-----------------------------------------------|-----------|-------|
| **CIFAR-10 Unconditional**                    |           |       |
| PixelCNN (Van Oord et al., 2016)              | 4.60      | 65.93 |
| PixelIQN (Ostrovski et al., 2018)             | 5.29      | 49.46 |
| EBM (single)                                  | 6.02      | 40.58 |
| DCGAN (Radford et al., 2016)                  | 6.40      | 37.11 |
| WGAN + GP (Gulrajani et al., 2017)            | 6.50      | 36.4  |
| EBM (10 historical ensemble)                  | 6.78      | 38.2  |
| SNGAN (Miyato et al., 2018)                   | **8.22**  | 21.7  |
| **CIFAR-10 Conditional**                      |           |       |
| Improved GAN                                  | 8.09      | -     |
| EBM (single)                                  | 8.30      | 37.9  |
| Spectral Normalization GAN                    | **8.59**  | 25.5  |
| **ImageNet 32x32 Conditional**                |           |       |
| PixelCNN                                      | 8.33      | 33.27 |
| PixelIQN                                      | 10.18     | 22.99 |
| EBM (single)                                  | **18.22** | 14.31 |
| **ImageNet 128x128 Conditional**              |           |       |
| ACGAN (Odena et al., 2017)                    | 28.5      | -     |
| EBM* (single)                                 | 28.6      | 43.7  |
| SNGAN                                         | **36.8**  | **27.62** |
Cross Class Mapping

[Images of various objects and their corresponding classes: Deer, Bird, Frog, Ship, Car, Airplane, Automobile, Truck, Ship, Deer]
Cross Class Mapping
Surprising Benefits of Energy-Based Models

• Robustness
• Continual Learning
• Compositionality
• Trajectory Modeling
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Out-of-Distribution Relative Likelihoods

Also observed by [Hendrycks et al 2018] and [Nalisnick et al 2019]
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Out-of-Distribution Generalization

- Following [Hendrycks and Gimpel, 2016]

| Model       | SVHN | Textures | Monochrome Uniform | Uniform | CIFAR10 Interpolation | Average |
|-------------|------|----------|--------------------|---------|-----------------------|---------|
| PixelCNN++  | 0.32 | 0.33     | 0.0                | 1.0     | 0.71                  | 0.47    |
| Glow        | 0.24 | 0.27     | 0.0                | 1.0     | 0.59                  | 0.42    |
| EBM (ours)  | 0.63 | 0.48     | 0.30               | 1.0     | 0.70                  | 0.62    |
Robust Classification

(a) $L_\infty$ robustness

(b) $L_2$ Robustness
Robust Classification

(recent follow-up submission at ICLR 2020 improves baseline EBM performance)
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Continual Learning: Split MNIST

| Method          | Memory | Incremental task learning | Incremental domain learning | Incremental class learning |
|-----------------|--------|----------------------------|----------------------------|---------------------------|
| Baselines       |        |                            |                            |                           |
| Adam            |        | 93.46 ± 2.01               | 55.16 ± 1.38               | 19.71 ± 0.08              |
| SGD             |        | 97.98 ± 0.09               | 63.20 ± 0.35               | 19.46 ± 0.04              |
| Adagrad         |        | 98.06 ± 0.53               | 58.08 ± 1.06               | 19.82 ± 0.09              |
| L2              |        | 98.18 ± 0.96               | 66.00 ± 3.73               | 22.52 ± 1.08              |
| Naive rehearsal | ✓      | 99.40 ± 0.08               | 95.16 ± 0.49               | 90.78 ± 0.85              |
| Naive rehearsal-C | ✓    | 99.57 ± 0.07               | 97.11 ± 0.34               | 95.59 ± 0.49              |
| Continual learning methods |        |                            |                            |                           |
| EWC             |        | 97.70 ± 0.81               | 58.85 ± 2.59               | 19.80 ± 0.05              |
| Online EWC      |        | 98.04 ± 1.10               | 57.33 ± 1.44               | 19.77 ± 0.04              |
| SI              |        | 98.56 ± 0.49               | 64.76 ± 3.09               | 19.67 ± 0.09              |
| MAS             |        | 99.22 ± 0.21               | 68.57 ± 6.85               | 19.52 ± 0.29              |
| LwF             |        | 99.60 ± 0.03               | 71.02 ± 1.26               | 24.17 ± 0.33              |
| GEM             | ✓      | 98.42 ± 0.10               | 96.16 ± 0.35               | 92.20 ± 0.12              |
| DGR             | ✓      | 99.47 ± 0.03               | 95.74 ± 0.23               | 91.24 ± 0.33              |
| RlF             | ✓      | 99.66 ± 0.03               | 97.31 ± 0.11               | 92.56 ± 0.21              |
| Offline (upper bound) |        | 99.52 ± 0.16               | 98.59 ± 0.15               | 97.53 ± 0.30              |

Evaluation by [Hsu et al, 2019]
## Continual Learning: Split MNIST

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EBM: 64.99 ± 4.27 (10 seeds)

Evaluation by [Hsu at al, 2019]
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Evaluation by [Hsu at al, 2019]

**EBM: 64.99 ± 4.27**

Would any generative model work instead? Doesn’t look like it:
**VAE: 40.04 ± 1.31**
Surprising Benefits of Energy-Based Models

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• Trajectory Modeling
Compositionality via Sum of EBMs

[Hinton, 1999]

Specify a concept by successively adding constraints
Compositionality via Sum of Energies

Specify a concept by successively adding constraints

Compositional Visual Generation with EBM$s$  [Du, Li, Mordatch, 2019]
Compositionality via Sum of Energies

Specify a concept by successively adding constraints

Compositional Visual Generation with EBM (Du, Li, Mordatch, 2019)
Compositionality via Sum of Energies

Specify a concept by successively adding constraints

Compositional Visual Generation with EBMs  [Du, Li, Mordatch, 2019]
Compositionality via Sum of Energies

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EBMs for Trajectory Modeling and Control

[Du, Lin, Mordatch, 2019]

• Train energy to model pairwise state transitions $s_t, s_{t+1}$
• Trajectory probability:

$$p_\theta(\tau) = p_\theta(s_1, s_2, \ldots, s_T) = \prod_{t=1}^{T-1} p_\theta(s_t, s_{t+1})$$

$$\propto \exp(- \sum_{t=1}^{T} E(s_t, s_{t+1}))$$
EBMs for Trajectory Modeling and Control

[Du, Lin, Mordatch, 2019]

- Train energy to model pairwise state transitions $s_t, s_{t+1}$
- Generate trajectories that achieve specific tasks:

$$p_\theta(s_2, \ldots, s_T|s_1, R) \propto \exp(-\sum_{t=1}^{T-1} E(s_t, s_{t+1}) - \sum_{t=1}^{T} R(s_t))$$

(similar to direct trajectory optimization)
EBMs for Control

| Data    | Model   | Particle | Maze     | Reacher   |
|---------|---------|----------|----------|-----------|
| Pretrained | EBM     | -5.14    | -72.07   | -19.38    |
|         | Action FF | -6.11    | -65.06   | -25.54    |
| Online  | EBM     | -20.38   | -162.97  | -29.87    |
|         | Action FF | -850.67  | -949.99  | -42.37    |
Source Code

• Images
  • https://github.com/openai/ebm_code_release

• Trajectories
  • https://github.com/yilundu/model_based_planning_ebm

• Compositionality
  • https://drive.google.com/file/d/138w7Oj8rQl_e40_RfZJq2WKWb41NgKn3

• Interactive Notebook
  • https://drive.google.com/file/d/1fCFRW_YtqQPSNoqznlh2b1L2baFgLz4W/view