Towards measuring the Archimedes force of vacuum

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Abstract

We discuss the force exerted by the gravitational field on a Casimir cavity in terms of Archimedes’ force of vacuum, we identify the force that can be tested against observation and we show that the present technology makes it possible to perform the first experimental tests. We motivate the use of suitable high-Tc superconductors as modulators of Archimedes’ force. We analyze the possibility of using gravitational wave interferometers as detectors of the force, transported through an optical spring from the Archimedes vacuum force apparatus to the gravitational interferometers test masses to maintain the two systems well separated. We also analyze the use of balances to actuate and detect the force, we compare different solutions and discuss the most important experimental issues.

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I. INTRODUCTION

One of the striking and longstanding problems of fundamental physics is the irreconcilability among the two main theories of last century, General Relativity and Quantum Theory. A manifestation of this tension is the value that quantum field theory attributes to the vacuum energy density, enormously larger than the value constrained from General Relativity by considering the radius of our universe. This problem, known as the cosmological constant problem [1], has been faced over the last decades with deep theoretical investigations, following also the evolution of the most important quantum gravity theories, like string theories, loop quantum gravity and many others [2–4]. None of the theoretical efforts has so far succeeded in finding a consensual solution, so that it is still questionable whether vacuum energy does interact with gravity, and what is its contribution to the cosmological constant [5, 6]. In spite of the common belief by the scientific community in the existence of an interaction between vacuum energy and gravity, not a single experimental test of this interaction exists.

About a decade ago, it was pointed out that a possible way to verify the interaction of vacuum fluctuations with gravity was to weight a (suitably realized, layered) rigid Casimir cavity [7]. At that time it was yet unclear whether Casimir energy could be modulated in a rigid cavity. Furthermore, the most important macroscopic detectors of exceedingly small forces, the gravitational wave detectors with which we compared our force, were still under construction. Nowadays, thanks to many activities in the various fields mentioned, the situation has been remarkably improved so that it is possible to step from the initial idealistic experiment to a road map towards the measurement of the effect.

The paper is organized as follows. In Sec. II the theory of the experiment is recalled and discussed to clearly identify the measured quantity. In Sec. III, the need for superconductors as actuators of the modulation of Casimir stress-energy tensor is proposed. In particular, the use of High-Tc superconductors is pointed out, in the light of what is presently possible and what can be expected for the near future. In particular, the force exerted by gravity on a multi-layer superconductor Casimir-cavity system will be considered. In Sec. IV the expected force is compared with the sensitivity of advanced gravitational wave detectors: an optical technique to link the force on the Casimir-test mass with the test mass of gravitational wave detectors is presented and the noises discussed in details. In Sec. V the possibility to
perform the measurement in the superconductors’ transition-favored low-frequency regime is discussed by analyzing the use of a suitable seismic isolated balance. The comparison of the two experimental ways is discussed in light of the most critical experimental issues.

II. THEORETICAL ASPECTS

Let us consider a rigid Casimir cavity in a weak gravitational field, like the one, for instance, of a laboratory at rest on the surface of the earth. To first order, the reference system is the Fermi system for which, neglecting rotations, one can write the line element as:

\[ ds^2 = -(1 + 2A_j x^j)(dx^0)^2 + \delta_{jk} dx^j dx^k + O_{\alpha\beta}(|x|^2) dx^\alpha dx^\beta. \]  

(2.1)

The term \( c^2 \vec{A} \) is the observer’s acceleration with respect to the local freely falling frame. It has components \((0, 0, |\vec{g}|)\), where \( g \) is the gravitational acceleration. \( c \) is the speed of light. The term \(-2A_j x^j\) is proportional to distance along the acceleration direction; \( x^3 \), which we also denote by \( z \), is positive in the upwards direction.

The force exerted by the gravitational field on a rigid Casimir cavity, with plates of proper area \( \mathcal{A} \), separated by the proper distance \( a \) and placed orthogonal to the gravitational acceleration \( \vec{g} \), has been calculated in different ways \[7–13\]. In particular, Ref. \[9\] and successively Ref. \[13\] clarified that, if the total force acting on an extended body is defined as the sum of red-shifted force densities, then the expected relation among weight of a body at rest in a weak gravitational field and its mass is recovered, and it is independent of the spatial stress-energy tensor components. (This definition is not the only possible one: for example, in Ref. \[14\], a definition of total force as the sum of all force densities leads to a re-definition of mass, defined as the total force over the acceleration given to a body, that contains explicitly the spatial stress-energy components.)

In order to clarify the measured quantity we consider the forces on each plate, expanded to first order in \( \epsilon \equiv \frac{ga}{c^2} \), derived in \[13\]. The quote \( Q_2 \) refers to the upper plate, the quote \( Q_1 \) refers to the lower:

\[ \vec{f}_{Q_2} \approx -\frac{\pi^2}{240} \frac{Ahc}{a^4} \left[ 1 - \frac{g}{c^2} \left( \frac{2}{3} a \right) \right] \hat{z}, \]

(2.2) while for the lower plate we get

\[ \vec{f}_{Q_1} \approx \frac{\pi^2}{240} \frac{Ahc}{a^4} \left[ 1 + \frac{g}{c^2} \left( \frac{2}{3} a \right) \right] \hat{z}. \]

(2.3)
If the system is rigid and kept at rest by the application of the force at a point, the total force, obtained by red-shifting the force up to the common point $Q_2$, is given by

$$\vec{f}_{Q_2} + r_{Q_2}(Q_1)\vec{f}_{Q_1}^{(C)} \approx F^{(C)} \left\{ -\left[ 1 - \frac{g}{c^2} \left( \frac{2}{3} a \right) \right] + \left[ 1 - \frac{g}{c^2} \left( 2 a \right) \right] \left[ 1 + \frac{g}{c^2} \left( \frac{2}{3} a \right) \right] \right\} \hat{z}$$

$$\approx \frac{1}{3} \frac{g}{c^2} F^{(C)} \hat{z} = \frac{E^{(C)}}{c^2} \hat{g}, \quad (2.4)$$

where $F^{(C)} = \frac{\pi^2 \frac{A}{40}}{\alpha^2}$ is the Casimir force and $E^{(C)} = -\frac{\pi^2 \frac{A}{720}}{\alpha^2}$ is the Casimir energy. This condition is the case of the experiment here proposed, where a rigid (multi)cavity system is suspended in the gravitational field of the earth. This is the force that must be tested against observation. It is directed upwards and it is equal to the weight of the modes of the vacuum that are removed from the cavity. Therefore it can be interpreted as an Archimedes buoyancy force in vacuum.

If the sum of the forces on the two plates is taken by simple addition (as it could be obtained by independently measuring the forces acting on the two plates of a nonrigid system) the result can be written as

$$\vec{F} \approx \left( \frac{|E^{(C)}|}{c^2} (g) + F^{(C)} \delta \phi \right) \hat{z}, \quad (2.5)$$

where $\frac{\delta g}{c^2} = \delta \phi$ has been explicitly written as the variation of the gravitational potential when passing from lower to upper plate. As expected, the force is the sum of two contributions: the vacuum weight part $\frac{E^{(C)}}{c^2} \hat{g}$ and the Casimir pressure difference, multiplied by the surface, $\mathcal{A}P^{(C)} \delta \phi$ when passing from one plate to the other. This difference in pressure is physical, and implies the red-shifting of vacuum density in the gravitational field. It is similar to the Tolman-Ehrenfest effect [15, 16] where the same dependence is found in the temperature of a gas at equilibrium in a gravitational field. As already said, this contribution cannot be tested in the present experiment, the cavity being rigid and the force being read off from the weight of the whole system.

III. SUPERCONDUCTORS

The measurement of the effect cannot be performed statically. This would make it necessary to compare the weight of the assembled cavity with the sum of the weights of its individual parts, a measure impossible to perform. Thus, it becomes necessary to modulate
the Casimir energy contained in the cavity to be weighted, so as to perform the measurement in a region of frequency where the macroscopic detectors of small forces have good sensitivity. Furthermore, to actually perform the measurement, the cavity should be a rigid body, so as to be weighted as a whole, and composed by a multilayer of many cavities to enhance the effect. A key point in modulation is that the energy supplied to the system should be at most of the same order of magnitude of the Casimir energy modulation, otherwise it will be extremely difficult to recover the Casimir contribution to the weight. Some recent techniques, as an example, even if very interesting for studying the Casimir force, cannot be applied in our case because the efficiency is very low: only a few parts on a billion of the energy supplied to the system are converted in Casimir energy variation.

One possible way is to use superconductors. As we have shown both theoretically and experimentally, for suitable geometrical configuration and material choices, in a cavity having plates made of superconducting material, at the transition from normal to superconducting state the variation of Casimir energy can be comparable with the condensation energy. The superconductors that we used to investigate the possibility of Casimir energy modulation were of the first type, and in particular aluminum was used in the experiment: this choice is good for demonstrating the effect, and in particular to investigate how vacuum fluctuations can affect phase transitions. Nevertheless, the metals being good conductors also in normal state, the Casimir energy variation is too small to be detected in a weighting measurement: in this case the use of high-Tc superconductors is mandatory. Some properties are of particular interest: generally high-Tc superconductors, particularly cuprates, are by construction multilayered cavities, being composed by Cu-O planes, that perform the superconducting transition, separated by poorly conducting planes. Furthermore, in normal state, also the Cu-O planes are poor conductors, so that the variation of Casimir energy is high at the transition.

In these systems the contribution of Casimir energy is not yet deeply investigated. In the rest of this paper we will assume valid, in order to have a numerical basis of the discussion, the Kempf hypothesis that the whole condensation energy in cuprates results from Casimir energy. Notice that, as will be shown in Secs. III and IV, even if the contribution of Casimir energy were of order of just a few over a thousand of the total energy at the transition, we might ascertain whether it gravitates.

Under that hypothesis the Casimir energy in the superconducting state is estimated as a
fraction $\eta = 4.3 \times 10^{-4}$ of the perfectly-reflecting plates Casimir energy.

Notice that, as stated in [21], the separation among the plates being of order of 1 nm, the 
“Casimir” energy is dominated by plasmons (i.e. by the Van der Walls) energy with respect 
to vacuum energy. Thus, our assumption of Kempf’s hypothesis should be regarded also as 
a starting point for further investigations on high-Tc superconductors, to be performed in 
the near future, directed in two ways. First, consider superconductors with higher spacing 
among conducting planes; second, consider the possibility of building ad hoc separation 
among plates until the recovering of conditions already studied in previous measurements 
with metallic plates [18–20].

Under these conditions the modulation of the effect can be performed by applying a 
time dependent magnetic field that partially spoils the superconducting state. The actual 
measurement should be performed for vanishing applied field, otherwise the sample will not 
be isolated when measuring its weight. Thus, the sample should be in two different conditions 
of superconductivity, (i.e. with more/less regions where the sample is superconducting) both 
at vanishing applied field: this can be obtained by using hysteretic superconductors. Among 
them, as an example, the cuprates. The modulation can be performed also by temperature 
variation, in the low frequency regime defined in Sec. IV.

In the following sections the detection of small forces using the best of current optical 
techniques will be considered. The use of high-Tc superconductors in high sensitivity optical 
devices is a field yet to be investigated, in particular in macroscopic devices. Nevertheless, 
present superconductors can be deposited on quite large surface optical elements: YBCO is 
well deposited on aluminum (Al$_2$O$_3$) substrates, which are the best substrates also for optics 
at low temperature. Indeed, a 300 nm thick YBCO layer deposited on a 3-inches diameter, 5 
mm thick Al$_2$O$_3$ substrate produced by CERACO is presently under test in our laboratory. 
Furthermore, much larger thicknesses can be reached by using superconducting crystals.

IV. USE OF GRAVITATIONAL WAVE DETECTORS

The force exerted by the gravitational field when the Casimir energy contained in the 
superconductor system is modulated should be compared with the up-to-date technology in 
the detection of small forces in macroscopic systems. Two main ways might be followed. 
The first way is to make use of the present most sensitive apparatuses in the detection of
small forces, the gravitational wave detectors; the second is to go towards lower frequencies and use torsion pendulums. In the following we will consider first the use of gravitational wave detectors. The main reason to explore this way is the possibility of making use of a very well developed technology in force detection and seismic attenuation. Another not negligible reason is that money can be saved if a replica of many instruments and methods already available is avoided. In this case, it is necessary to recover an experimental method, discussed later, to apply a force on such detectors (only at a given frequency) without perturbing the gravitational wave measurement in the other frequencies of the spectrum. Our comparison can start with the present state of the art of gravitational wave detectors. Over the last decades, this field has known many impressive technical improvements and developments. The two most sensitive detectors of gravitational waves, LIGO and Virgo, have demonstrated the feasibility of all foreseen techniques, by reaching, and in some frequency regions superseding, the sensitivities expected for the first generation detectors [22, 23]. Moreover, many important techniques already compliant or extremely useful in the next generation detectors have been demonstrated worldwide, i.e. in LIGO [24], Virgo [26] or in the smaller-scale detectors like GEO [27] or still in development like Kagra [29]. In light of all this progress it is very reasonable to expect, for the second generation of such detectors, the so-called Advanced Detectors, presently under construction, to reach the design sensitivities in the next few years [30, 31].

In this case the frequency region of highest sensitivity $S_F$ to the force lies in the range from 20 to 40 Hz; if a gravitational wave test mass of 42 Kg is considered, the value, in this region, is of order of $S_F \approx 10^{-13} N/\sqrt{Hz}$.

Glancing at future detectors, the so-called third-generation detectors, like ET, we see that they will benefit of low seismic sites, low temperature and suitably injected power for low-frequency detection. The expected sensitivity in the amplitude of the force will gain about two orders of magnitude, showing the region of best force sensitivity at frequencies slightly smaller than 10 Hz [32].

Two main conditions, in our opinion, constrain and define the use of gravitational wave detectors also for a measurement of the weight of vacuum. The first is that no modifications are allowed to the gravitational wave detector that in any case risk to reduce the gravitational wave sensitivity. In particular, no changes of the suspensions chain, of the payloads, of the actuators will be allowed: the system providing the force should be “external” and
FIG. 1: Sketch of the optical link. GW-EM: Gravitational wave detector’s test masses; AF-IM(EM): Archimedes Force apparatus input(end) mirror; GW-beam: laser beam of the gravitational wave detector. The AF cavity length is 12 meters, compatible with the present Virgo halls.

sufficiently far from the gravitational wave test masses so as to avoid introducing spurious signals. The second is that the vacuum weight force is vertical (i.e. orthogonal to the earth’s surface) while the gravitational detectors are designed to detect horizontal forces (i.e. almost parallel to the earth surface, with a small coupling factor with the vertical due to earth’s curvature and mechanical imperfections).

A possible way to face both points is to build an ad-hoc apparatus for exerting the Archimedes force of vacuum in a separate system, lying several meters from the gravitational wave detectors test masses, and then transport the force, from vertical to horizontal, via an optical spring, as sketched in FIG. 1 the gravitational wave test mass is part of an external detuned cavity, formed by a mirror vacuum-weight driven, a 45° mirror (suspended to the immediately upper stage) to bring the light from vertical to horizontal, the gravitational wave test mass, and a symmetric return system that closes the cavity.

The Vacuum driven mirror is optically contacted with a Al₂O₃ substrate disk covered by
250 µm of YBCO, for a total surface $S = 2 \times 0.23 \ m^2$, equal to the present ADV-Virgo beam splitter but quite lighter, about 5 kg, maintained at the working temperature of 4 K. The amplitude of force modulation $F_m$ can be evaluated as

$$F_m \approx \eta \frac{E_{(cp)}}{c^2} g \approx \eta N^2 \frac{Shc}{240} a^4 g \approx 10^{-15} N,$$

where $\eta$ is the reduction factor with respect to the perfectly reflecting plates Casimir $E_{(cp)}$ \cite{21}, $N = 1.6 \times 10^5$ is the total number of layers and $a = 1.17 \ nm$ is the conducting layers separation in YBCO. The use of an optical spring is based on the fact that the optical spring stiffness is proportional to the laser input power and to the square of the finesse, while the noise reintroduced in the system from power fluctuations is proportional to the input power and the finesse (i.e. to the light circulating inside the cavity). By using a sufficiently high finesse it is thus possible to maintain a high spring stiffness while keeping low the power circulating in the cavity, making sure that a negligible level of noise is introduced. Note that, if the gravitational wave is linked by an optical spring to other free masses, under the condition presently assumed of small distances, with respect to armlength, among the single gravitational wave test mass and the Archimedes force masses, and not considering the region of frequency around the optical spring resonance frequency, the displacement of the gravitational wave test mass will not change because all masses will accelerate at once: this is the reason why we are considering the effect of the Vacuum force as comparing the displacement induced in the gravitational wave test mass instead of comparing directly the force. Note that this statement also assumes that the masses of Archimedes’ force are small with respect to the gravitational wave test mass. Indeed, the latter is not free: it is linked by the arm-cavity optical spring to the rest of masses of the gravitational wave interferometer: if the Archimedes forces are small the system is not perturbed, otherwise a complete simulation should be performed, which is outside the aim of the present paper.

The system has been simulated by using the Optickle code \cite{28}. The masses considered are equal to $m = 5 \ Kg$, the cavity Finesse $F = 6.0 \times 10^5$, the input power $I_0 = 0.16 \ mW$. The detuning is $\delta_\gamma = 0.3$ where, denoting as usual the laser frequency by $\omega_0$, the nearest cavity resonant frequency by $\omega_{res}$, the linewidth by $\gamma = \frac{c\pi}{2LF}$, the detuning is defined as $\delta_\gamma \equiv \frac{\omega_{res} - \omega_0}{\gamma}$. Under these assumptions the expected signal for an integration time of 6 months, a typical time-scale of a run, is given in Fig. 2.

The signal is above the sensitivity by two orders of magnitude at low frequency, while it
falls under the ADV sensitivity around 100 Hz. As expected, the power fluctuations of the Archimedes force cavity laser inject negligible noise. Indeed, the power inside the cavity is about 60 Watt, to be compared with the 0.6 MW of light circulating in the gravitational wave arm cavity. In conclusion, the use of the optical spring to transport the force from the actuator system to the gravitational wave mass makes it possible to displace the actuator by several meters and to use as detectors the ultimate-sensitivity gravitational wave detectors.

The suspension system of the Archimedes force system can be a replica of the gravitational wave masses and the cryogenic part can make use of the several experimental studies and realizations now making progress in the world [29]. In this way, the optical system reduces to the optical spring actuator, which is relatively simple, being just a laser suitably locked on the cavity.

Note that, if the same system were applied to the next generation of gravitational wave detectors, in particular ET low-frequency [32, 33], whose sensitivity around 8 Hz is about \( \tilde{S}_F \approx 10^{-15} N/\sqrt{Hz} \), a remarkable improvement is expected. This is shown in Fig. 3. The cavity considered to perform the optical spring is similar to the previous one, with masses of 10 kg and a larger finesse = 1.5 \( \times 10^6 \), that is not far from the present best values.

FIG. 2: Expected signal for the YBCO actuator described in the previous Section.
The input power is $P_{\text{in}} = 1.6 \times 10^{-4}$ (not critical). The input power noise has been taken as shot-noise limit of the input power, equivalent to the noise-power ratio (RIN) of about $5 \times 10^{-8} 1/\sqrt{\text{Hz}}$: the power noise is more critical in this case but is negligible, remaining an order of magnitude lower than the sensitivity. The Figure shows that with an integration time of 6 months, the signal-to-noise ratio of about $S/R = 10^4$ is reached. This means that a signal-to-noise ratio of 1000 might be reached in a couple of days.

With such high signal-to-noise ratios, measurements with different materials, different layers separations up to tens of nanometers would then be possible, allowing a complete campaign of studies. This possibility clarifies also our working case on the Kempf hypothesis. According to that recipe, all the condensation energy, both at small and larger layer separation, results from Casimir energy. With this sensitivity, considering the accuracy of the gravitational wave detectors, even if the contribution of Casimir energy were only a few parts over a thousand, we might test whether it gravitates.
V. USE OF TORSION PENDULUMS

The use of torsion pendulums could be favored by the possibility to go towards low frequencies. Indeed, the modulation of superconducting phase transitions in macroscopic bodies, even if we are not requiring to spoil completely the superconductivity, is expected to be easier at lower frequencies. Furthermore, we will consider here also the possibility of performing force modulation also by temperature modulation. We evaluate the thermal noise at the temperature working point of 100 K, near the YBCO transition temperature. The main experimental point that we face in going towards low frequencies is that a proper seismic attenuation system for balances, i.e. for vertical torsion pendulums, does not yet exist.

A possible way to reduce seismic noise at frequencies lower than 0.1 Hz is to hang the balance to a cascade formed by an inverted pendulum followed by a blades' isolation stage. The inverted pendulum is efficient in the two horizontal translational degrees of freedom and the rotation around the vertical axis, while the blades' stage is efficient in the vertical degree of freedom and in the rotations \[25, 26\]. The Virgo inverted pendulum has already demonstrated to have a resonant frequency of 0.03 mHz and work is ongoing to further reduce it to the value of 0.01 Hz. Also the blades' stage resonance can be tuned, by careful tuning of magnetic antispring stiffness, to similar values. The control of this top stage can be done either at very low frequency, with unity gain of the feedback lower than the resonance or in high bandwidth, with unity gain of about 1 Hz. Here we assume to close the loop in high gain and reach, at the suspension point of the balance, the electronic noise floor of the accelerometers \(a_s \approx 4 \times 10^{-10} m^2/s\sqrt{Hz}\), corresponding to the displacement noise of \(1nm/\sqrt{Hz}\) at 0.1 Hz, and flat for frequency less than 0.1 Hz \[25\]. To calculate the expected signal and noises at the balance, we have considered a balance having arms of length \(L = 0.1 \text{ m}\), a plate at each arm’s end of mass \(M = 0.4 \text{ kg}\), total mass \(M_b = 1.25 \text{ kg}\), moment of inertia \(I = 0.01 kg \text{ m}^2\), resonance frequency \(F_{res} = 5 \text{ mHz}\), with mechanical internal loss angle \(\phi = 10^{-6}\). The resonance value is higher than typical torsion pendulum (horizontal) ones already existing \[34\] and takes into account the feasibility of a real vertical balance: in particular, the resonance of 5 mHz corresponds to careful setting of the bending point distance from the balance center of mass of about \(hb \approx 1\mu m\).

The material to be used for the suspension fiber (and for the balance itself) cannot
be fused silica, which is the material of choice for the test masses of all first-generation gravitational wave detectors, because it has a high dissipation at low temperatures [35, 36]. Sapphire has already been proposed as alternative material also for the suspension fiber, and here we assume it is the final material [37]. The wire length considered in our simulation is of 1 m and the diameter $d = 50 \, \mu m$. The end plates have radius $R=0.15 \, m$, made by a sapphire substrate and one is coated with 250 $\mu m$ of YBCO on both faces: the force modulation on the plate is $F_a = 4 \times 10^{-16} \, N$. As expected, simulations show that the most critical noise is the seismic noise injected through the coupling transversal to tilt. The simulated transfer function is shown in Fig. 4 for the case of 5 mHz and for a very optimistic case, similar to torsion pendulum value, of 1 mHz to show that this parameter is critical for reaching a significant attenuation.

The tilt signal can in principle be read off in various ways. A high-sensitivity possibility is to use a second balance and read the ends’ differential displacements with a Michelson interferometer having a Fabry-Perot cavity at the ends of the balances’ arms. For an interferometer having arm cavity finesse $F_b = 100$, input power $P_b = 0.01 \, W$, the sensitivity.
FIG. 5: Expected signal and noises for the balance. In the region of frequencies $5 < f < 100$ mHz the signal is about two orders of magnitude above the noise.

is reported in Fig. 5 where the radiation pressure noise and shot noise are plotted. The signal (blue curve) is obtained by integrating for 6 months and is approximately two orders of magnitude larger than the total noise (black curve).

Under the assumption on seismic noise reduction the sensitivity is limited at low frequency by suspension thermal noise and by seismic noise for frequencies larger than 30 mHz. The radiation pressure noise and shot noise curves make sure that fundamental noises will not make it impossible to perform the measurement of the vacuum-gravity force. Nevertheless, the noise is so lower with respect to other noises that other tilt detection methods, even if more noisy, can be exploited if simpler. As an example, optical lever systems or capacitors used in torsion pendulums have already shown remarkable sensitivities; they are not yet fully compatible with our needs, but surely deserve careful study and attention [34, 38].

The corresponding signal and noises in $N/\sqrt{Hz}$ are plotted in Fig. 6. The coupling of suspension point acceleration $a_s$ can be interpreted as producing a moment of inertia
FIG. 6: Force signal and noises. The dashed line describes the noise of an optical lever detection system.

\[ M_s = M_b \cdot a_s \cdot h, \] equivalent to the noise force \( M_b \cdot a_s \cdot h/b/L, \) that again shows how the setting of the bending point is critical. The plot reports (dashed line) also the read-out noise of an optical lever demonstrated in [38]: such a system makes it mandatory to perform the measurement in the neighborhood of the resonance (at the price of slightly reducing the sensitivity), but leads to a remarkable simplification of the detection method.

### A. Conclusion

We have shown that it is by now possible to begin the experimental path to check against observation whether virtual photons do gravitate and to verify the Archimedes force of vacuum. Experiments must be carried out in various fields, like in deposition of thick layers of high-Tc superconductors in optical substrates, application of optical springs to connect different apparatuses, improvements in low-frequency seismic isolation: if these
improvements, not far from the present technological achievements, will be successful, a first answer will be given to one of the deepest and longest lasting problems of fundamental physics.

[1] S. Weinberg: The cosmological constant problem. Rev. Mod. Phys. 61, 1 (1989).
[2] C. Rovelli, Quantum Gravity (Cambridge University Press, Cambridge, 2004).
[3] B. S. DeWitt and G. Esposito: An introduction to quantum gravity, Int. J. Geom Methods Mod. Phys. 5 101, (2008); G. Esposito, An introduction to quantum gravity, arXiv:1108.3269 [hep-th].
[4] C. Kiefer, Quantum Gravity, International Series of Monographs on Physics, 155 (Clarendon Press, Oxford, 2012).
[5] E Bianchi, C Rovelli: “Is dark energy really a mystery?”, Nature, 466 (2010) 321.
[6] T. Padmanabhan: why does gravity ignore the vacuum energy? Int. J. Mod. Phys. D, 15, 2029 (2006).
[7] E. Calloni, L. Di Fiore, G. Esposito, L. Milano, and L. Rosa: Vacuum fluctuation force on a rigid Casimir cavity in a gravitational field. Phys. Lett. A 297, 328 (2002).
[8] G. Bimonte, E. Calloni, G. Esposito, and L. Rosa: Energy-momentum tensor for a Casimir apparatus in a weak gravitational field. Phys. Rev. D 74, 085011 (2006); erratum Phys. Rev. D 75, 049904 (2007); erratum Phys. Rev. D 75, 089901 (2007), erratum Phys. Rev. D 77, 109903 (2008).
[9] S. A. Fulling, K. A. Milton, P. Parashar, A. Romeo, K. V. Shajesh, and J. Wagner: How does Casimir energy fall?, Phys. Rev. D 76, 025004 (2007).
[10] K. A. Milton, S. A. Fulling, P. Parashar, A. Romeo, K. V. Shajesh, and J. A. Wagner: Gravitational and inertial mass of Casimir energy. J. Phys. A 41, 164052 (2008).
[11] K. V. Shajesh, K. A. Milton, P. Parashar, and J. A. Wagner: How does Casimir energy fall? III. Inertial forces on vacuum energy. J. Phys. A 41, 164058 (2008).
[12] K. A. Milton, K. V. Shajesh, S. A. Fulling, and P. Parashar: How does Casimir energy fall? IV. Gravitational interaction of regularized quantum vacuum energy. arXiv:1401.0784 [hep-th].
[13] G. Bimonte, E. Calloni, G. Esposito, and L. Rosa: Relativistic mechanics of Casimir apparatuses in a weak gravitational field. Phys. Rev. D 76, 025008 (2007).
[14] C.W. Misner, K.S. Thorne, J.A. Wheeler, *Gravitation* (Freeman, New York, 1973).

[15] R. C. Tolman and P. Ehrenfest: Temperature Equilibrium in a Static Gravitational Field. Phys. Rev. 36 (1930) 12, 1791.

[16] H. M. Haggard, C. Rovelli: Death and resurrection of the zeroth principle of thermodynamics Phys. Rev. D 87, 8, 084001 (2013).

[17] F. Chen, G. L. Klimchitskaya, V. M. Mostepanenko, and U. Mohideen: Demonstration of optically modulated dispersion forces, Opt. Express 15, 4823 (2007); Control of the Casimir force by the modification of dielectric properties with light. Phys. Rev. B76, 035338 (2007).

[18] A. A. Banishev, C.-C. Chang, R. Castillo-Garza, G. L. Klimchitskaya, V. M. Mostepanenko, and U. Mohideen: Modifying the Casimir force between indium thin oxide film and Au sphere, Phys. Rev. B 85, 045436 (2012)

[19] G. Bimonte, E. Calloni, G. Esposito, L. Milano, and L. Rosa: Towards Measuring Variations of Casimir Energy by a Superconducting Cavity. Phys. Rev. Lett. 94, 180402 (2005).

[20] G. Bimonte, D. Born, E. Calloni, G. Esposito, U. Huebner, E. Il’ichev, L. Rosa, F. Tafuri, and R. Vaglio: Low noise cryogenic system for the measurement of the Casimir energy in rigid cavities. J. Phys. A 41, 164023 (2008). A. Allocca, G. Bimonte, D. Born, E. Calloni, G. Esposito, U. Huebner, E. Il’ichev, L. Rosa, F. Tafuri: Results of Measuring the Influence of Casimir Energy on Superconducting Phase Transitions. Jour. Super and Novel Mag. 25, 8, 2557 (2012).

[21] A. Kempf: On the Casimir effect in the high-Tc cuprates. Journal of Physics A: Mathematical and Theoretical 41, 16, 164038 (2008)

[22] T. Accadia, F. Acernese, M. Alshourbagy, et al. Virgo: a laser interferometer to detect gravitational waves. Jour. of Instrumentation 7, P03012 (2012)

[23] J. Abadie, B. P Abbott, R. Abbott, et al. - Group Author(s): LIGO Sci Collaboration; Virgo Collaboration: All-sky search for gravitational-wave bursts in the second joint LIGO-Virgo run. Phys. Rev. D 85, 12, 122007 (2012)

[24] Aasi, J.; Abadie, J.; Abbott, B. P.; et al.: Enhanced sensitivity of the LIGO gravitational wave detector by using squeezed states of light. Nature Photonics 7, 8, 613-619 (2013)

[25] J. Harms, B. J. J. Slagmolen, R. X. Adhikari, M. Coleman Miller, M. Evans, et al: Low-
Frequency Terrestrial Gravitational-Wave Detectors Phys. Rev. D 88, 122003 (2013)

[26] S. Braccini, L. Barsotti, C. Bradaschia et al.: Measurement of the seismic attenuation performance of the VIRGO Superattenuator ASTROPARTICLE PHYSICS 23, 6, 557-565 (2005)

[27] Grote, H.; Danzmann, K.; Dooley, K. L.; et al.: First Long-Term Application of Squeezed States of Light in a Gravitational-Wave Observatory Source: Phys. Rev. Lett. 10, 18, 181101 (2013)

[28] M. Evans, Optickle LIGO Document T070260 (2007)

[29] Y. Aso, Y. Michimura, K. Somyra et al - Group Author(s): KAGRA Collaboration: Interferometer design of the KAGRA gravitational wave detector. Phys. Rev. D 88, 4, 043007 (2013)

[30] T Accadia et al - Virgo Collaboration: Status of the Virgo project. Class. Quantum Grav. 28 114002 (2011)

[31] H. Harry - for the LIGO collaboration: Advanced LIGO: the next generation of gravitational wave detectors. Class. Quantum Grav. 27 084006 (2010)

[32] Sathyaprakash, B.; Abernathy, M.; Acernese, F.; et al: Scientific objectives of Einstein Telescope. Class. Quantum Grav. 29 124013 (2012)

[33] S.Hild, S.Chelkowski, A.Freise, J.Franc, N.Morgado, R.Flaminio and R.DeSalvo: ”A Xylophone Configuration for a third Generation Gravitational Wave Detector” Class. Quantum Grav. 27 015003 (2010).

[34] A. Cavalleri, G. Ciani, R. Dolesi et al: Direct force measurements for testing the LISA Pathfinder gravitational reference sensor. Class. Quantum Grav. 26 094012 (2009)

[35] J. Wiedersich, S. V. Adichtchev, and E. Rssler: Spectral Shape of Relaxations in Silica Glass. Phys. Rev. Lett. 84, 2718 (2000)

[36] S. Rowan et al: Test mass materials for a new generation of gravitational wave detectors. Proceedings of SPIE Vol. 4856 (2003)

[37] K. Kuroda and the LCGT Collaboration: The status of LCGT. Class. Quantum Grav. 23 S215 (2006)

[38] R. De Rosa, L. Di Fiore, F. Garufi, et al: An optical readout system for the drag free control of the LISA spacecraft. Astroparticle Phys. 34, 6, 394 (2011)

[39] T. Accadia, F. Acernese, F. Antonucci et al: The seismic Superattenuators of the Virgo gravitational waves interferometer. Journ. of Low Frequency Noise, Vibration and active control
