Extracting Flavor from Quiver Gauge Theories

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Abstract. We consider a large class of models where an SU(5) gauge symmetry and a Froggatt-Nielsen (FN) Abelian flavor symmetry arise from a quiver gauge theory. Such quiver models are very restrictive and therefore have strong predictive power. In particular, under mild assumptions neutrino mass anarchy is predicted.

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INTRODUCTION - THE NEUTRINO FLAVOR PUZZLE

The flavor puzzle in the SM, namely the smallness and hierarchical structure of the charged fermions, hints towards new physics beyond the SM. In particular, these features are very suggestive that an approximate horizontal symmetry is at work. A solution that employs such a symmetry is the Froggatt-Nielsen (FN) mechanism [1]. The various generations carry different charges under a spontaneously broken Abelian symmetry. The breaking is communicated to the SM fermions via heavy fermions in vector-like representations and the ratio between the scale of spontaneous symmetry breaking and the mass scale of the vector-like fermions provides a small symmetry-breaking parameter. Yukawa couplings that break the FN symmetry are suppressed by powers of the breaking parameter, depending on their FN charge.

In recent years, the discovery of neutrino masses and the measurements of several neutrino flavor parameters have introduced a new twist to the flavor puzzle. Indeed, the measured dimensionless parameters (encoding the information on flavor physics) are all of order one, implying a possible anarchy in the neutrino sector [2]. The flavor puzzle must then also address the question: Why are the neutrino flavor parameters different from those of the quarks and charged leptons?

It is simple to write down a FN model which generates hierarchy in the charged fermions and anarchy in the neutrino sector. Consider for example, an SU(5) model with a $U(1)_{\text{FN}}$ horizontal symmetry under which the matter content is charged as $\bar{5}_i (0;0;0)$, $10_i (2;1;0)$, $H_u$, $H_d$ and the additional scalar, $S$, responsible for spontaneously breaking the $U(1)_{\text{FN}}$ has charge 1. It is straightforward to check that for $\lambda S i = M = \epsilon'$ 0 $\Delta_{5}$ the resulting mass matrices (leaving out order one coefficients):

$$
M_u \begin{pmatrix}
0 & \epsilon^4 & \epsilon^3 & \epsilon^2 & 1 \\
\epsilon^2 & \epsilon & 1 & 1 & 1 \\
\end{pmatrix}
$$

$$
M_d \begin{pmatrix}
0 & \epsilon^2 & \epsilon^2 & 1 \\
\epsilon & 1 & 1 & 1 \\
\end{pmatrix}
$$

$$
M_\nu \begin{pmatrix}
\lambda H_u i \epsilon^2 \\
\lambda H_d i \\
\frac{\lambda H_u^2}{M} \\
\end{pmatrix}
$$

agree with the measured values. Here and below, $M$ is the scale at which the breaking is communicated to the SM. On the other hand, it is just as simple to generate a different structure in the neutrino sector. Taking a different set of charges, say $10_i (4;2;1;0)$ and
\[ \delta \left( l \; 1 \; 0 \right), \] one finds that a different mass structure arises but which still agrees with the data for \( \epsilon' \approx 0.23 \) and different order one coefficients.

The reason for this freedom stems from the nature of the FN mechanism. Indeed, model building within the FN framework usually requires a value for the small symmetry-breaking parameter(s), and a set of FN charges for the fermion and Higgs fields. These choices determine the parametric suppression of masses and mixing angles. One then checks that the experimental data can be fitted with a reasonable choice of order-one coefficients for the various Yukawa couplings. Thus all FN predictions are subject to inherent limitations: 1) The FN charges are not dictated by the theory. 2) The value of the small parameter is not predicted. 3) There is no information on the \( O \left( l \right) \) coefficients. The predictive power of the FN framework is thus limited.

To make further progress, one would like to embed the FN mechanism in a framework where some or all of the inherent limitations described above are lifted. This may happen in string theory. Below we consider string-inspired models in which the FN mechanism is embedded in quiver gauge theories \[ \text{(3).} \] These theories arise at low energy as the effective theories on D-branes placed at singular geometries \[ \text{(4, 5).} \] The structure of these theories tightly constrains model building and hence the realization of the FN mechanism.

**EMBEDDING FN IN QUIVER GAUGE THEORIES**

A quiver diagram is an efficient way for describing the gauge theory obtained from the open string sector. For oriented strings, nodes in the quiver denote \( U \left( N \right) \) gauge factors and the fields are represented by directed lines connecting two such nodes. A line coming out of a node stands for a field in the fundamental representation, while a line going into a node represents a field in the antifundamental. A line starting and ending on the same node, describes a field in the adjoint representation of the corresponding \( U \left( N \right) \) factor.

The quiver diagram can be generalized to accommodate unoriented strings. Nodes represent \( U \left( N \right) \), \( SO \left( N \right) \) or \( Sp \left( N \right) \) gauge groups and the lines are no longer directed but instead, each line must be drawn with an arrow at each of the two ends indicating what is the representation of the corresponding string under each of the two gauge group factors. Unoriented strings with both ends coming out of the same set of branes may reside in either the symmetric or the antisymmetric combination of \( N \; N \).

A crucial property of quiver gauge theories is that the \( U \left( l \right) \) charge at each \( U \left( N \right) \) = \( SU \left( N \right) \) \( U \left( l \right) \) gauge factor depends on the representation under the non-Abelian part of the group. In particular a fundamental of the \( SU \left( N \right) \) is charged \( +1 \) under the corresponding \( U \left( l \right) \) while an anti-fundamental is charged \( -1 \). This property strongly restricts the possible Abelian charges in a particular theory and therefore plays a crucial part in the embedding of the FN mechanism. Without going to any details we note that many of these \( U \left( l \right) \) factors are in fact global where the corresponding gauge fields obtain a mass through couplings to axions. Such a higgsing process occurs both for anomalous \( U(1) \)s through the generalized Green-Schwartz mechanism \[ \text{(4, 6, 7),} \] and upon compactification also to many non-anomalous \( U(1) \) factors \[ \text{(8).} \]

With the aid of the global \( U \left( l \right) \)s, embedding the FN is an easy matter. Here we restrict ourselves to a single \( U \left( l \right)_{\text{FN}} \) and therefore by assumption there is only one
figure 1. A D-brane construction with an SU(5) gauge group and a distinct U(1)FN. The $\bar{5}$-plets are strings stretching between the two stacks and are thus charged under the FN group, while the 10-plets connect only to the U(5) stack and have no U(1)FN charge.

field in the quiver, $S$, which spontaneously breaks the symmetry and obtains a small VEV, $hS = M = \varepsilon^3$. As shown in [3], under these assumptions and due to the relation between the abelian and non-abelian charges, the largest possible suppression in such FN models is $\varepsilon^3$. This result shows the strong predictive power that is added to the FN mechanism when embedded in string theory.

In fact these constraints are so strong that they typically pose phenomenological problems, essentially since the global U(1) symmetries of the SM or its GUT extensions are not directly related to the local gauge groups. Consider as an example, an SU(5) theory with the following interactions:

$$W = Y^d_{ij} H_d \ 10 \bar{5}_j + Y^u_{ij} H_u \ 10 \ 10^c \ (y^\nu_{ij} = M) \ H_u \ H \ 10 \ 10$$

As shown in figure 1, the 10-plets are generated by strings having both ends on the U(5) stack of branes. Thus these 10-plets have charge +2 under the corresponding U(1). Similarly, $H_u$ is a fundamental of SU(5) and therefore has a +1 charge under the U(1). With these charges no up-type masses are allowed. These problems are very generic in D-brane constructions.

There are three possible solutions to the above problem:

1. The particle content is extended in such a way that the symmetry is realized. For example, an extended higgs sector [9] may be introduced.
2. The U(1) is broken spontaneously [3]. The only way to do this without breaking the associated SU(N) gauge group is by considering a composite singlet composed of $N$ fundamentals to obtain a VEV.
3. The U(1) is broken by non-perturbative effects. Depending on the matter content, gauge theory instantons may be generated or otherwise stringy effects such as D-instantons may break the U(1) [3, 8].

These non-perturbative effects relax some of the constraints on the possible suppressions stated above and in particular allow an $\varepsilon^4$-suppression of the Yukawa couplings in the case of SU(5) GUT theory.
**SU (5) AND ANARCHY**

The problem discussed above also demonstrates that for the simple SU (5) theory all 10-plets have equal charges under the FN symmetry. Thus, this scenario gives rise to up mass anarchy and is therefore phenomenologically excluded. One therefore must consider product groups whereby an SU (5) GUT model we mean that there is a range of energy scales where the gauge group is SU (5), with matter fields that transform as 5, 5, and 10. Product groups are a generic prediction of such FN models [3].

The simplest models, on which we focus, have the following pattern of gauge symmetry breaking: SU (5) → SU (5) \( \times \) SU (5) \text{diag}. The breaking is carried out by the FN field which is charged \((+1; 1)\) under the corresponding \( U (l)_L \times U (l)_R \). More complicated breaking patterns have a similar hierarchical form, but involve extended particle content. The \( 5 \)-plets then transform under the SU (5) \( \times \) SU (5) \( \times \) SU (5) \text{diag} as either \( (5; 5)_0; 0 \) or \( (5; 5)_0; 0 \). The 10-plets transform as either \( (10; 5)_0; 2 \) or \( (5; 10)_0; 2 \) or \( (10; 5)_0; 1 \). The \( H_u (5) \) field transforms as \( (5; 5)_0; 1 \). The strongest mass hierarchy in the various fermion mass matrices appears in the up sector. A viable model must produce this mass hierarchy and it is straightforward to check that such hierarchy requires the use of all three possible charges for the 10-plets. One is therefore left with no freedom in the choice of charges for these fields.

For the down sector, since the \( 5 \) fields carry charges of either \( (1; 0) \) or \( (0; 1) \), at least two of them have the same FN charge. Thus, there must be at least quasi-anarchy in the neutrino sector. Such a situation has implications for the down sector: either one or all three down mass ratios are of the same order as the corresponding mixing angles (e.g. \( m_s = m_b \) \( \neq m_b \)). If one further requires the correct hierarchy in the down sector, one finds that all \( 5 \)-plets must have the same FN charge. Thus anarchy is obtained in the neutrino sector. We note that this prediction is independent of the choice for strong dynamics which generate up-type masses. From this point of view there is only one viable quiver which is unique. The quiver is shown in [3].

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