Fatigue strength assessment of track rollers

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Abstract. The paper presents a general concept for fatigue strength assessment in rolling contact between track roller and rail. The multi-axial stress state is analysed using the critical plane method. Depending on the orientation of the section plane, different load cycle combinations result with different stress-amplitude and mean-stress combinations. The Haigh diagram outlines the fatigue strength of the material. The load cycles in all section planes are plotted there and the safety factor can be determined directly. In comparison to other empirical concepts, similar values are obtained for the allowable maximum contact pressure. The advantage of this method is its general applicability, without any restrictions regarding the geometry or material.

1. Introduction

The bearing and movement of many mobile steel structures is realised with track rollers and rails. Important areas of application are in the field of conveyor technology (cranes, conveyor systems and large open cast mining equipment) and in hydraulic steel construction. In the case of sufficiently often repeated overruns (fatigue loading), in addition to the static strength assessment, a fatigue strength assessment for so-called rolling contact fatigue must also be carried out by the design engineer.

The work presented is limited to slow-running track rollers, which are not driven, thus without torque transmission. In this case, mainly pure rolling without slip between the two parts in contact occurs and the contact theory according to Hertz is applicable in good approximation.

The design of the roller-rail pairing according to standards is carried out in various industries with empirically derived formulas, which depend on the field of application. However, these cannot easily be transferred to other fields of application. For example, the standards for cylindrical gears (DIN ISO 281) or spur gears (DIN 3990) allow design against cyclic flank pressure, but assuming good lubrication in the contact area. There is so far no general approach for the analysis of rolling contact fatigue problems.

The stress state around the contact area of two spherical elastic bodies is multi-axial with rotating directions of principal stresses, when the bodies are moving. The present paper outlines a procedure for the strength analysis of rolling contact fatigue problems using of the critical plane method, which is a general and thus component-independent approach. The results are compared with those from different design rules and norms as a reference.
2. Stress state in roller and rail

2.1 Hertzian contact stress

2.1.1 Assumptions

The calculation of the stresses at contact of elastic, solids bodies dates back to H. Hertz (1881) [1]. Due to elastic deformation, an initial point or linear contact becomes a contact surface when normal forces are transmitted between two bodies. Both, the size of the contact surface and the pressure between the bodies increases nonlinearly as the normal force increases. The analytical solutions from Hertz are based on assumptions which are discussed below:

- Homogeneous, isotropic and linear elastic material behaviour: is fulfilled, as no plastic deformations are permissible when designing against fatigue in the roller-rail contact
- Body at rest: is approximately fulfilled with slow-running wheels (quasi-static stress-states)
- Only normal forces/no friction: is the case for non-driven wheels, since no torque is transmitted
- Contact surfaces flat and small compared to other body dimensions: fulfilled for typical roller-radii.

The following considerations refer to the case of contact between a spherical roller and a half-plane, as shown in Fig. 1. R is the radius of the roller and \( R_k \) the head radius.

2.1.2 Analytical solution

In the contact area is elliptical with the half-axes a and b. Its size depends on the ratio of the radii, the acting normal force F and the elastic material parameters (E, v).

\[
a = \xi \cdot \sqrt{\frac{3(1-v^2)F}{E(1/R+1/R_k)}},
\]

\[
b = \eta \cdot \sqrt{\frac{3(1-v^2)F}{E(1/R+1/R_k)}}.
\]

The transcendent functions \( \xi \) and \( \eta \) depend on the ratio of the radii and are provided in many table books, of for instance in [2]. The maximum Hertzian pressure in the middle of the contact surface is:

\[
p_{\text{max}} = \frac{3F}{2\pi ab}.
\]
Multi-axial stress states occur around the contact area. However, all stress-tensor components are proportional to $p_{\text{max}}$, which is why the maximum contact pressure is usually used to characterise the stresses. Loading and coordinates used in the following are shown in Fig. 2.

![Figure 2. Loading and coordinates](image)

### 2.1.3 Calculation example using FEM

The stress state is demonstrated with an example track-roller geometry using FEM calculations. The parameters used here are as follows:

- Material 42CrMo4 (ultimate tensile strength: $R_m = 900$ MPa)
- Load $F = 416$ kN
- Radius of roller $R = 250$ mm
- Head radius of roller $R_h/R = 15$

The FE model utilises the double symmetry and this consist of $\frac{1}{4}$ of the geometry. The mesh is shown in Fig. 3. Due to the high stress gradients at the contact, a very fine discretization with small elements is necessary.

The simulation was carried out with the software ANSYS (version 18.0) using elements with quadratic shape functions. The contact was modelled as free of friction and the contact algorithm Augmented-Lagrange was applied.

![Figure 3. Finite element model using double symmetry](image)
Fig. 4 shows the stress distribution (which is normalised to the maximum contact pressure $p_{\text{max}} = 1216$ MPa) directly below the contact point in vertical direction.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{stress_distribution}
\caption{Stress distribution below contact point}
\end{figure}

Fig. 5 shows the distribution of all stress components in the longitudinal direction of the rail. The path along which the evaluation was performed is shown in Fig. 6. The curves in Fig. 5 are symmetrical except for the one of the shear stress. The normal stresses are all negative (compressive stresses) throughout the entire rollover process. The stress ratio is therefore $R=0$ (pulsating stresses). It is somewhat different for the shear stress component. It changes the sign during the rollover process and is alternating ($R=-1$). This is considered as key influence for fatigue damage.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{stress_distribution_longitudinal}
\caption{Stress distribution along path shown in Fig. 5}
\end{figure}

The analytical solution for this example results in $a = 30.5$ mm and $b = 5.2$ mm for the half-axes of the contact ellipse ($\xi = 2.82$ and $\eta = 0.485$) and thus to the maximum contact pressure of $p_{\text{max}} = 1240$ MPa. There is a very good agreement with the numerical finite element solution.
3. Evaluation with the critical plane approach

3.1 Stresses in different section planes

The stresses are evaluated in section planes with different angular orientation. Different normal and shear stresses result, depending on the angle of the plane. Fig. 7 shows the stress vector \( \vec{s} \) in a section plane with the angle orientations \( \alpha, \beta \) and \( \gamma \) in respect to the Cartesian coordinate system \( x, y, z \).

\[
\sigma = \vec{s} \cdot \vec{n} = \left( \sigma_x \cdot \cos\alpha + \tau_{xy} \cdot \cos\beta + \tau_{xz} \cdot \cos\gamma \right) \cdot \frac{1}{|\vec{n}|} \left( \begin{array}{c} \cos\alpha \\ \cos\beta \\ \cos\gamma \end{array} \right) \quad (4)
\]

The normal stress \( \sigma \) in the section plane corresponds to the projection of the stress vector onto the normal unit vector of the section plane.

The orthogonal part of the stress vector corresponds to the shear stress \( \tau \) in the section plane.
3.2 Equivalent stress

There are different approaches for the definition of an equivalent stress in the section plane. For brittle materials, only the normal stress is used. A general approach for brittle and ductile materials is the hypothesis of scaled normal stress [4], which is also being used in the fatigue postprocessing software FEMFAT. The stress tensor is scaled with the factor \( f \) prior to the evaluation of stresses in different section planes:

\[
 f = 1 + (1 - k) \cdot V.
\]  

(5)

The parameter \( V \) is the ratio of the smallest principal stress to the largest principal stress as absolute value and is therefore defined in the range \(-1 \leq V \leq +1\). The factor \( k \) corresponds to the ratio of the alternating fatigue strength for normal stress and shear stress (\( k = 1.0 \) for brittle metal materials and \( k = 1.73 \) for ductile steels).

3.3 Haigh diagram

In each section plane a closed load cycle with different values of the stress amplitude and mean stress occurs for one complete rollover process. These combinations are being evaluated with regard to the material’s fatigue strength.

The Haigh diagram shows the fatigue strength for different combinations of stress amplitude and mean stress. The determining factor is the mean stress sensitivity \( M \) [6]. It is estimated for steels according to [5] from the tensile strength:

\[
 M = 0.00035 \cdot \left( \frac{R_m}{MPa} \right) - 0.1
\]  

(6)

Figure 8. Haigh-Diagramm, as defined in [5]

The stresses of each section plane are plotted in the Haigh-diagram and the section plane critical for failure is the one with the least safety.

3.4 Exemplary calculation and comparison with other concepts

The evaluation is demonstrated on the example of a spherical wheel on a straight plane (maximum contact pressure \( p_{max} = 1216 \) MPa). The geometry and load parameters are stated in section 2.1.3.

The course of the principal stresses required for the scaling factor \( f \) is shown in Fig. 9. It is evaluated in the depth of the max. equivalent stress and \( x \) is the horizontal direction along the track with \( x = 0 \) in the centre of the contact area. The evaluation in the Haigh diagram is shown in Fig. 10. The safety factor against the limit line for fatigue strength corresponds to \( S = 1.25 \). Thus, the maximum endurable force that could be applied is \( F_{allow} = 520 \) kN or the maximum contact pressure \( p_{allow} = 1520 \) MPa (without safety: \( SF = 1 \)).
Figure 9. Course of principal stresses for one load cycle

Figure 10. Evaluation in Haigh diagram

Tab. 1 shows the calculated maximum endurable contact pressure in comparison to those from various other concepts in standards or literature on the subject.

Table 1. Comparison with different design rules and norms.

| Reference                                      | $p_{allowable}$ |
|-----------------------------------------------|-----------------|
| DIN 15070 (German standard)                   | 1531 MPa        |
| AS 1418-1 (Australian standard)               | 1369 MPa        |
| Warkenthin [7]                                | 1350 MPa        |
| TGL (former east German standard)             | 1683 MPa        |
| Nölke [8]                                     | 1028 MPa        |
| Current evaluation (critical plane approach)  | 1520 MPa        |
4. Conclusion

With the approach presented, the fatigue strength can be evaluated for rolling contact between roller and wheel (without torque transmission). A comparison with other concepts shows that the calculated value lies within the scatter band of the other concepts. The advantage of this approach is that it is not limited to certain machine elements and can therefore be used universally. The fatigue strength of the material as well as the mean stress sensitivity are needed as material strength parameters.

The concept is easily applicable for line contact as well. Possible areas of application are not only materials handling technology but also steel construction and hydraulic engineering.

References

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