Brane Constant-roll Inflation

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Abstract

The scenario of constant-roll inflation in the frame of RSII brane gravity model is considered. Based on the scenario, the smallness of the second slow-roll parameter is released and it is assumed as a constant which could be of order of unity. Applying the Hamilton-Jacobi formalism, the constancy of the parameter gives a differential equation for the Hubble parameter which leads to an exact solution for the model. Reconsidering the perturbation equations clarify that there are some modified terms appearing in the amplitude of the scalar perturbations and in turn in the scalar spectral index and tensor-to-scalar ratio. Comparing the theoretical results of the model with observational data, the free parameters of the model are determined. Then, the consistency of the model with the swampland criteria is investigated for the obtained values of the free parameters. As the final step, the attractor behavior of the model is considered.
I. INTRODUCTION

Inflation is known as a phase of accelerated expansion in very early times where the universe undergoes an extreme expansion in short period of time. The scenario has received a observational support\cite{1-5}, which makes it as the cornerstone of the cosmology so that any cosmological model is incomplete without implying to the inflation. The first inflationary model was proposed in \cite{6} aiming to solve the problem of initial singularity. Then, in 1999, a new idea of inflationary universe was proposed to solve the horizon and flatness problems, known as old inflation. The idea was improved and its problems were resolved in \cite{7,8}, and then it was finalized in \cite{9} (the reader could refer to \cite{10-13} for more information about the historical detail).

So far, many inflationary models have been introduced based on the slow-roll assumptions where the scalar field slowly rolls down from the top of the potential toward the minimum. Non-canonical inflation \cite{14-22}, tachyon inflation \cite{23-26}, DBI inflation \cite{27-31}, G-inflation \cite{32-35}, and warm inflation \cite{36-43} are some of these inflationary models which are formulated based on the assumptions. The slow-roll inflation usually described by the slow-roll parameters which their smallness during inflation is guaranteed by a potential with almost a flat part \cite{44-47}. However, recently a different inflationary scenario has been proposed which goes beyond one of the slow-roll approximations. The start point was the article \cite{48}, where the author study the inflationary scenario in the case the potential is exactly flat. The non-Gaussianity of the model was studies in \cite{49}, and it was found that the non-Gaussianity could be of order of one, in contrast to the standard inflation where the non-Gaussianity is small and of order of the slow-roll parameters. In \cite{50}, the idea was improved in which the second slow-roll parameter was taken as a constant. The author could find an approximate solution for the model. Using Hamilton-Jacobi formalism \cite{51-57}, an exact solution for the model was obtained by solving the differential equation of the Hubble parameter, resulted from the constancy of the second slow-roll parameter \cite{58}. The scenario of the constant-roll inflation has received huge interest and has been considered for many inflationary models \cite{59-75}.

One of the alternative theory of the general theory is the higher dimensional theories
where the brane gravity is known as one of these theories inspired from string theory. Brane gravity provides an interesting picture of the universe. the universe with all standard particles are confined on a four-dimensional hyper space-time (brane) embedded in five dimensional space-time (bulk), and only gravity could propagate along the fifth dimension. The theory was put forth by Randall and Sundrum in 1999 [76]. Their first model of brane gravity was aimed to solve the problem of Hierarchy. A negative cosmological constant fills the bulk and the brane includes a negative tension. It was shown that a brane with negative tension results in a repulsive gravitational force which is not physical [77]. In their second model of gravity, the extra dimension could be large and the universe is described by a brane with positive brane tension [78]. Enormous number of researches have been devoted to the topic an many different aspect of this modified gravity have been considered [79–86]. Also, many inflationary models have been studied in the frame of brane gravity [87, 88]. However, so far and up to our knowledge, no model of constant-roll inflation has been performed in the frame of brane gravity. The scenario of constant-roll inflation in the four-dimensional models of gravity has led to interesting results, and it is expected the same result be acquired in the brane gravity model.

Another motivation for picking the brane gravity comes to the swampland criteria [89–91]. The recently proposed swampland criteria are two conjectures which imposes a higher bound on the scalar field range and a lower bound on the gradient of the potential. These two conjectures are actually a measure to divide the consistent from the inconsistent effective field theory (EFT). The consistent EFT are a class of EFT that could formulate a quantum gravity in which the string theory is known as the best candidate. On the other hand we have inconsistent EFT which are in contradiction with string theory. The second swampland criterion states that $M_p|V/V| > c'$ (where $c'$ is of order of unity) is in direct tension with the inflation. Based the inflationary scenario, the first slow-roll parameter $\epsilon = M_p^2 V'^2 / 2V^2$ is smaller than one. The desire for building a model based on a consistent EFT implies that the model should satisfy the swampland criteria. Then, some of the inflationary model are ruled out by the criterion, but there are still some other inflationary models which could properly satisfy them. The scenario of inflation in the frame of the brane gravity is one of these inflationary models which is assumed that have the chance to survive the swampland criteria [92–99].
The model has bee organized as follow: The main evolution equations of the model are briefly expressed in Sec.II. The scenario of the constant-roll inflation is discussed in the frame of brane gravity in Sec.IV. The differential equation of the model is obtained and the exact solution of the model is presented and the main background parameters are obtained in terms of the scalar field. The perturbation equations of the model are considered in Sec.IV, where the perturbation parameters are derived for the model. The consistency with the swampland criteria and the attractive behavior are respectively discussed in Secs. V and VI. Finally the results are summarized in Sec.VII.

II. THE MODEL

The action for the brane world is given by

\[ S = \int d^5x \sqrt{-g} \left( \frac{M_5^3}{2} R + \Lambda_5 \right) + \int d^4x \sqrt{-h} (\mathcal{L} + \lambda) \]  

where the first integral represents the action of the bulk and the second one corresponds to the brane, \( R \) is the Ricci scalar related to the five-dimensional metric \( g_{AB} \), \( g \) and \( h \) denote the determinants of the metric on the five dimensional space and the brane, respectively, \( \Lambda_5 \) the five-dimensional cosmological constant, \( \mathcal{L} \) the lagrangian of the matter fields, and \( \lambda \) the brane tension.

Taking variation of the action with respect to the metric yields the field equation

\[ G_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + \left( \frac{8\pi}{M_4^2} \right) T_{\mu\nu} + \left( \frac{8\pi}{M_5^2} \right)^2 \Pi_{\mu\nu} - E_{\mu\nu} , \]  

Here \( T_{\mu\nu} \) is the energy-momentum tensor of the matter on the brane, \( \Pi_{\mu\nu} \) a tensor that includes the terms quadratic in \( T_{\mu\nu} \), and \( E_{\mu\nu} \) represents the projection of Weyl tensor on the brane which portray the effects of the bulk graviton on the dynamical evolution of the brane. Assuming the geometry of the universe to be described by a five-dimensional FriedmannLemaitreRobertsonWalker (FLRW) metric

\[ ds_5^2 = -dt^2 + a^2 \delta_{ij}dx^i dx^j + dy^2, \]  

the Friedmann equation reads

\[ H^2 = \frac{\Lambda_4}{3} + \left( \frac{8\pi}{3M_4^2} \right) \rho + \left( \frac{4\pi}{3M_5^2} \right)^2 \rho^2 + \frac{C}{a^4}. \]
with $\Lambda_4$ is the cosmological constant of the brane, and $C/a^4$ is known as the dark radiation. The five and four dimensional Planck masses in the above equation are related as $M_4 = \sqrt{\frac{3}{4\pi \lambda}} M_5^3$.

During inflation, the dark radiation terms gets diluted, and hence can be neglected. Also, here the RS fine-tuning is being used to set the four-dimensional cosmological constant to zero. Thus, the Friedmann equation gets reduced to

$$H^2 = \frac{8\pi}{3M_4^2} \rho \left(1 + \frac{\rho}{2\lambda}\right), \tag{5}$$

Since all the matter fields are confined on the brane, the conservation of energy in this expanding universe is the same as in standard cosmology, i.e.

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{6}$$

Using the above equation and taking the time derivative of Eq. (5), we obtain the second Friedmann equation

$$\dot{H} = -\frac{4\pi}{M_4^2} \left(1 + \frac{\rho}{\lambda}\right) (\rho + p). \tag{7}$$

Inflation is driven the inflaton, a scalar field $\phi$, that is confined on the brane and with energy density and pressure

$$\rho = \frac{\dot{\phi}^2}{2} + V(\phi), \quad p = \frac{\dot{\phi}^2}{2} - V(\phi) \tag{8}$$

and obeys the equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \tag{9}$$

It is widely common to consider the inflation at the energy scale where the energy density is larger than the tension of the brane, i.e. $\rho \gg \lambda$. Therefore, the above Friedmann equations are reduced to

$$H^2 = \left(\frac{4\pi}{3M_5^2}\right)^2 \rho^2, \quad \dot{H} = -3 \left(\frac{4\pi}{3M_5^2}\right) H\dot{\phi}^2 \tag{10}$$

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1 This is because its dependence on the scale factor is the same as the energy density of radiation.
III. CONSTANT-ROLL INFLATION

In slow-roll inflationary models, the inflaton rolls down its potential very slow which can be described in terms of dimensionless parameters
\[ \epsilon = -\frac{\dot{H}}{H^2} \quad \text{and} \quad \eta = -\frac{\ddot{\phi}}{H\dot{\phi}} \] (11)
which satisfy the conditions \( \epsilon < 1 \) and \( \eta < 1 \), known as slow-roll parameters (SRP)\(^{46}\). Another scenario is the constant-roll inflation where the second slow-roll parameter is assumed to be constant and can be of order of unity:
\[ \eta = -\frac{\ddot{\phi}}{H\dot{\phi}} = \beta = \text{constant} \] (12)
The fact that \( \eta \) is a constant results in a differential equation for the Hubble parameter that admits exact solution for the model. For that, we first obtain the time derivative of the scalar field from the second Friedmann equation by taking the Hubble parameter as a function of the scalar field, i.e. \( H := H(\phi) \), and write
\[ \dot{H} = \dot{\phi} H' \quad \Rightarrow \quad \dot{\phi} = -\frac{1}{3} \left( \frac{3M_5^3}{4\pi} \right) \frac{H'}{H} \] (13)
Then, it follows the following differential equation for the Hubble parameter
\[ HH'' - H'^2 - \tilde{\beta}H^3 = 0, \quad \tilde{\beta} = \frac{4\pi}{3M_5^3} \beta. \] (14)
and has a solution given by
\[ H(\phi) = \frac{-\alpha}{2\beta} \left[ 1 - \tanh \left( \frac{\sqrt{\alpha}}{2} (\phi + \phi_0) \right) \right] \] (15)
where \( \alpha \) and \( \phi_0 \) are constants of integration. Note that since the Hubble parameter is positive and the term \( \tanh \) is smaller than one, the constant \( \alpha \) must be negative.

Now that we have the expression of \( H(\phi) \), we can derive \( \dot{\phi}(\phi) \) and \( V(\phi) \), and we get
\[ \dot{\phi} = \frac{M_5^3\sqrt{\alpha}}{4\pi} \tanh \left( \frac{\sqrt{\alpha}}{2} (\phi + \phi_0) \right) \] (16)
\[ V(\phi) = \left( \frac{M_5^3}{4\pi} \right)^2 \frac{\alpha}{2} \left[ \frac{-3}{\beta} + \left( \frac{3}{\beta} - 1 \right) \tanh^2 \left( \frac{\sqrt{\alpha}}{2} (\phi + \phi_0) \right) \right]. \] (17)
By integrating the equation of \( \dot{\phi} \) above, gives the time evolution of scalar field as
\[ \phi(t) + \phi_0 = \frac{2}{\sqrt{\alpha}} \sinh \left[ \exp \left( \frac{M_5^3\alpha}{8\pi} (t + t_0) \right) \right] \] (18)
A. scalar field at horizon crossing time

The inflationary phase will come to an end when the SRP $\epsilon(\phi)$ becomes equal to unity, i.e.

$$\epsilon(\phi_e) := 2\beta \frac{\tanh^2 \left( \frac{\sqrt{\alpha}}{2} (\phi_e + \phi_0) \right)}{1 - \tanh^2 \left( \frac{\sqrt{\alpha}}{2} (\phi_e + \phi_0) \right)} = 1$$  \hspace{1cm} (19)

where $\phi_e$ is the value of the field at the exit of inflation, which can be determined by solving the above algebraic equation. With this we can quantify the amount of inflation the universe underwent, corresponding to the number of e-fold from the beginning of inflation an the instant $t_i$ to the exit time $t_e$, and is given by

$$N = \int_{t_i}^{t_e} H dt = \int_{\phi(t_e) = \phi_i}^{\phi(t_e) = \phi_e} \frac{H}{\dot{\phi}} d\phi = -\frac{4\pi}{M_5^3} \int_{\phi_i}^{\phi_e} \frac{H^2}{H'} d\phi$$  \hspace{1cm} (20)

Substituting the solution we have obtained for the Hubble parameter, and after some manipulation we obtain

$$N = -\frac{4\pi}{M_5^3 \beta} \ln \left( \tanh \left[ \frac{\sqrt{\alpha}}{2} (\phi + \phi_0) \right] \right) \bigg|_{\phi_i}^{\phi_e} = \frac{2\pi}{M_5^3 \beta} \ln \left( \frac{\tanh^2 \left( \frac{\sqrt{\alpha}}{2} (\phi_i + \phi_0) \right)}{\tanh^2 \left( \frac{\sqrt{\alpha}}{2} (\phi_e + \phi_0) \right)} \right)$$  \hspace{1cm} (21)

or, equivalently

$$\tanh^2 \left[ \frac{\sqrt{\alpha}}{2} (\phi_i + \phi_0) \right] = \frac{e^{2\beta N}}{1 - 2\beta}$$  \hspace{1cm} (22)

IV. COSMOLOGICAL PERTURBATIONS

Now, we come to the question of the quantum perturbations, one of the most important predictions of inflation and represent the main test that we have for verifying any inflationary model. The perturbations are usually divided to three types: scalar, vector, and tensor. Vector perturbation are usually ignored as they depend on the inverse of the scale factor and get diluted rapidly during inflation. Scalar perturbations are the seed for large scale structure formation in the universe. The tensor perturbations describes the primordial gravitational waves which have not been detected yet and at present we have only an upper bound on the tensor-to-scalar ratio.

The study of the cosmological perturbation in the constant-roll inflation is a little different than in the slow-roll scenario. Since the second SRP, $\eta$, might be of order unity,
in calculating the scalar and tensor perturbations the terms $\eta^2$, $\epsilon \eta$, and $\epsilon \eta^2$ can not be ignored. In this regard, the whole perturbation equations involving the second SRP should be reconsidered. In the following subsections, we are going to reconsider both scalar and tensor perturbations for any possible modification.

A. Scalar perturbations

To derive the perturbation parameters we usually need to obtain the Mukhanov-Sasaki equation [45–47, 100–104]. For this matter, the action is computed up to the second order of the perturbation parameter. Following [45, 105], the spatially flat gauge is used in which, up to the leading order of $\epsilon$, the fluctuations in the geometry of the action could be ignored. Since the scalar field lives on the brane, we have the same perturbation equation as we have in the standard four-dimensional cosmology, that is

$$v''_k(\tau) + \left(k^2 - \frac{z''}{z}\right) v_k(\tau) = 0 \quad (23)$$

where again $z$ has the same definition as $z^2 = a^2 \dot{\phi}^2/H^2$. Therefore, after some algebraic manipulations, the term $z''/z$ in the above equation can be expressed as

$$\frac{z''}{z} = \frac{1}{\tau^2} \left(2 + 6 \epsilon - 3 \beta - 9 \epsilon \beta + \beta^2 + 2 \epsilon \beta^2\right). \quad (24)$$

Making the change of variables $x = -k \tau$ and $f_k = v_k/\sqrt{-\tau}$, Eq. (23) becomes a Bessel differential equation as

$$\frac{d^2 f_k}{dx^2} + \frac{1}{x} \frac{df_k}{dx} + \left(1 - \frac{\nu^2}{x^2}\right) f_k = 0, \quad (25)$$

where we have used

$$\frac{z''}{z} = \frac{\nu^2 - \frac{1}{4}}{\tau^2} \quad \Rightarrow \quad \nu^2 = \frac{9}{4} + 6 \epsilon - 3 \beta - 9 \epsilon \beta + \beta^2 + 2 \epsilon \beta^2 \quad (26)$$

In general, the solutions to (25) are

$$f_k = c_1(k) H^{(1)}_{\nu}(-k \tau) + c_2(k) H^{(2)}_{\nu}(-k \tau) \quad (27)$$

Here $H^{(1)}_{\nu}$ and $H^{(2)}_{\nu}$ are the Hankel’s functions of the first and second kind, respectively, and $c_1(k)$ and $c_2(k)$ are arbitrary constants. Comparing the asymptotic behavior of the general
solution, with the solution of the equation in sub-horizon limit \((k\tau \ll 1)\), the constants are determined, and finally one could obtain the amplitude of the scalar perturbations as

\[
\mathcal{P}_s = A_s^2 \left( \frac{k}{aH} \right)^{3-2\nu}, \quad A_s^2 = \frac{1}{25\pi^2} \left( \frac{2^{\nu-3/2}\Gamma(\nu)}{\Gamma(3/2)} \right)^2 \left( \frac{H^2}{\phi} \right)^2.
\]  

(28)

from which we deduce the scalar spectral index \(n_s\) as

\[
n_s - 1 = 3 - 2\nu
\]

(29)

B. Tensor perturbations

The second SRP plays no role in the tensor perturbation equations, and hence the evolution equation for the tensor perturbation will have the same form as the scalar case. The amplitude of such perturbations have been calculated in the framework of the brane-world gravity, and is given by \([106, 107]\)

\[
A_T^2 = \frac{16\pi}{25M_p^2} \left( \frac{H}{2\pi} \right)^2 F^2(x),
\]

(30)

where

\[
F^2 = \left[ \sqrt{1 + x^2} - x^2 \sinh^{-1} \left( \frac{1}{x} \right) \right]^{-1}, \quad x \equiv H M_p \sqrt{\frac{3}{4\pi\lambda}}.
\]

(31)

In high energy regime, where \(x \gg 1\), one arrives at \(F(x) = 3x/2\) \([107, 108]\). The tensor perturbations are measured indirectly through the parameter \(r\), defined as the ratio of tensor perturbations to scalar perturbations, which can be determined using Eqs.\((28)\) and \((30)\) as

\[
r = \frac{3}{2} \left( \frac{\Gamma(3/2)}{2^{\nu-3/2}\Gamma(\nu)} \right)^2 \epsilon.
\]

(32)

Currently, the value of this parameter is not determined by the data, and only an upper bound \(r < 0.064\) \([3, 5]\).

V. OBSERVATIONAL CONSTRAINTS ON THE MODEL

To determine the free parameters of the model, we compute the amplitude of the scalar perturbations, scalar spectral index and tensor-to-scalar ratio at the time of horizon crossing and compare with the available observational data. First, by substituting the expression in
Eq. (22) into Eq. (19), the slow-roll parameter $\epsilon$ can be written in terms of the number of e-folds as

$$\epsilon = \frac{-2\beta e^{2\beta N}}{1 - 2\beta - e^{2\beta N}}.$$ \hfill (33)

Note that (from Eqs. (26), (29), and (32)) the scalar spectral index and tensor-to-scalar ratio depend only on $\beta$ and $N$ at the time of horizon crossing. Comparing the theoretical results for $n_s$ and $r$ with allowed values of the spectral index and tensor-to-scalar ratio given by Planck collaboration in the form of $r - n_s$ diagram, we extract the values of $(N, \beta)$ that are in agreement with the observational data. This is parameter space is shown in Fig. (1). Using the amplitude of the scalar perturbations, the other constant of the model, i.e. $\alpha$, is determined as

$$\alpha^3 = \left( \frac{\Gamma(3/2)}{2^{3/2} \Gamma(\nu)} \right)^2 \left( \frac{4\pi}{M_5^3} \right)^2 \left( 4\pi (2\beta)^3 A_s \epsilon \right).$$ \hfill (34)

To have numerical insight about the result of the model, Table II represents the values of $\alpha$, scalar spectral index, tensor-to-scalar ratio, and the energy scale for inflation for different values of $\beta$ and number of e-fold, taken from Fig. II.

Fig. III portrays the behavior of the obtained potential versus the scalar field for different values of $\beta$ and $\alpha$. As it is illustrated, the potential rolls down from the top of the potential.

The crucial point for any inflationary models is to have a graceful exit from the inflation stage. Considering the behavior of the first SRP presents the required information about the inflationary times and its end. The evolution of $\epsilon$ versus the number of e-fold in depicted in Fig. IV, where it is realized that by approaching to the end of inflation the parameter $\epsilon$ grows up and reaches one.
VI. CONSISTENCY WITH THE SWAMPLAND CRITERIA

The recently proposed swampland criteria is actually a measure for separating the consistent EFT from the inconsistent EFT. The consistent EFTs are able to successfully formulate string theory which is known as one of the best candidates of the quantum gravity. Inflation is believed to be occurred in the energy scale below the Planck energy and people believe that it could be described by a low-energy effective field theory. Therefore, it is the natural desire to build the model based on a consistent EFT. In this regard, the consistency of the obtained result for the presented inflationary model with the swampland criteria is investigated in the following lines.

The first criterion regards to the distance conjecture so that it imposes an upper bound on the scalar field excursion in the field space as $\Delta \phi/M_p < c$ where $c$ is of order of one. The evolution of the term $\Delta \phi/M_p$ for the model is presented in Fig. 2, where one could find that

| $\beta$ | $N$ | $\alpha$ | $n_s$ | $r$ | $V^*$ |
|--------|-----|---------|-------|----|------|
| −0.011 | 76  | 4.92 $\times$ $10^{-33}$ | 0.9580 | 0.0072 | 2.22 $\times$ $10^{53}$ |
| −0.014 | 80  | 4.95 $\times$ $10^{-33}$ | 0.9589 | 0.0047 | 1.92 $\times$ $10^{53}$ |
| −0.007 | 80  | 4.16 $\times$ $10^{-33}$ | 0.9594 | 0.0096 | 2.45 $\times$ $10^{53}$ |
| −0.014 | 84  | 4.64 $\times$ $10^{-33}$ | 0.9604 | 0.0041 | 1.85 $\times$ $10^{53}$ |
| −0.010 | 84  | 4.32 $\times$ $10^{-33}$ | 0.9620 | 0.0065 | 2.15 $\times$ $10^{53}$ |
| −0.004 | 84  | 3.51 $\times$ $10^{-33}$ | 0.9592 | 0.0119 | 2.62 $\times$ $10^{53}$ |
| −0.009 | 88  | 4.01 $\times$ $10^{-33}$ | 0.9637 | 0.0066 | 2.16 $\times$ $10^{53}$ |

TABLE I. numerical results of the model
FIG. 3. Behavior of the first slow-roll parameter $\epsilon$ versus the number of e-fold.

$\Delta \phi/M_p$ is smaller than unity for the whole time of the inflation.

The second criterion is a de Sitter conjecture which imposes a lower bound on the gradient of the potential. It states that $M_p|V'/V| > c'$ where $c'$ is of order of unity (further investigation determines that the constant could be of order of 0.1 [92]). The evolution of $M_p|V'/V|$ is plotted in Fig.5 stating that it remains bigger than one during the inflationary times.

VII. ATTRACTOR BEHAVIOR

The last feature we are going to consider is the attractive behavior of the model. The solution of the model has been obtained in Sec.III, where we have used the Hamilton-Jacobi formalism. To investigate the attractor behavior of the solution, we follow similar procedure as in [47, 52]. Assuming homogenous perturbation in the Hubble parameter, i.e.
$$H(\phi) = H_0 + \delta H(\phi),$$ and substituting it into the Hamilton-Jacobi equation,

$$V(\phi) = \left( \frac{3M_5^3}{4\pi} \right) H(\phi) - \frac{1}{9} \left( \frac{3M_5^3}{4\pi} \right)^2 \frac{H'^2(\phi)}{H^2(\phi)},$$

leads to the following differential equation

$$\frac{\delta H'(\phi)}{\delta H(\phi)} = \left( 1 + \frac{9}{2} \left( \frac{4\pi}{3M_5^3} \right) \frac{H'^2_0}{H^2_0} \right) \frac{H'_0}{H_0}$$

where the equation has been obtained up to the first order of the perturbation term. Integrating comes to

$$\delta H(\phi) = \delta H_i \exp \left[ \int_{\phi_i}^{\phi} \left( 1 + \frac{9}{2} \left( \frac{4\pi}{3M_5^3} \right) \frac{H'^3_0(\phi)}{H'^2_0(\phi)} \frac{H'_0(\phi)}{H_0(\phi)} \right) d\phi \right]$$

The integrand is illustrated in Fig. 6 versus the scalar field. The curves portray the behavior of the integrand versus the scalar field during the inflationary times. The area between the curve and the x-axis displays the actual value of the integral in the power of the exponential term in Eq. (37). Inflation begins for smaller field and it ends at bigger fields. Therefore, as the times passes and approaches the end of inflation, the area under the curve is getting larger and larger and the integral becomes more and more negative. Then, the exponential term approaches to zero implying that the homogenous perturbation $\delta H(\phi)$ dies away with time, and the model possesses attractor behavior.

VIII. CONCLUSION

The constant-roll inflation was investigated in the frame of RSII brane gravity model. Based on this scenario, the universe and all the matter fields, including the inflaton, are
FIG. 6. The curves display the behavior of the integrand versus the scalar field during the inflationary time for different values of $\beta$ and $\alpha$.

confined to a brane with positive tension, where the brane is embedded in five-dimensional space-time. The modified gravity model results in a modified Friedmann equation which contains both linear and quadratic terms of the energy density. In the high energy limit, the quadratic term dominates, and consequently, the Hubble parameter becomes proportional to the energy density $\rho$, instead of $\sqrt{\rho}$. In this scenario, the inflaton rolls down its potential at a constant rate where the second slow-roll inflation parameter is taken to be constant which, in general, can be of order unity. Using the Hamilton-Jacobi approach, we derive a differential equation for the Hubble parameter. For our model, there is a non-linear second-order differential equation which gives an exact solution for the model. Finding the Hubble parameter in terms of the scalar field, the other background parameters, such as the time derivative of the scalar field and the potential, were derived in terms of the scalar field. The slow-roll parameter $\epsilon$ was also obtained in terms of the scalar field, which is used to infer the scalar field at the end of inflation through the relation $\epsilon(\phi_e) = 1$. The scalar field at the beginning of inflation was acquired from the expression of the number of e-fold.

Another consequence of this scenario appears in the perturbation parameters where one could find the modified terms mainly in the amplitude of the scalar perturbations, scalar spectral index, and tensor-to-scalar ratio. Since the second slow-roll parameter might not be small, the scalar perturbation equations were reconsidered, and the modified scalar power spectrum was derived. The tensor power spectrum is actually the same as the slow-roll in brane inflation because the second slow-roll inflation parameters plays no role in tensor perturbation equations.

Computing the perturbation parameters at the time of horizon crossing, the scalar spectral
index and tensor-to-scalar ratio are obtained only in terms of the constant $\beta$ (i.e. the second slow-roll parameter) and the number of e-fold. Comparing the theoretical results of the model with the Planck data, a set of the $(\beta, N)$ is found that four any point in this set, the model perfectly agrees with observational data. The other constant of the model, i.e. $\alpha$, is determined from the amplitude of the scalar perturbation where there is an exact value for the parameter based on data. Using this result, a numerical result of the model about the main parameters including the energy scale of inflation are presented. In the next step, the consistency of the model with the recently proposed swampland criteria is considered. We tried to find whether the model with the obtained free parameter could satisfy the conjectures. The conclusions about the scalar field range and the gradient of the potential determine that the model appropriately satisfies both swampland criteria.

Finally, in the Hamilton-Jacobi formalism, we derived the differential equation (up to the first order) describing the behavior of a homogenous perturbation for the Hubble parameter as function of the inflator field. We showed that the perturbation parameter reduces as time approaches the end of inflation, which indicates that the solution of the model has the attractor behavior.

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