Precoding Design for Multi-user MIMO Systems with Delay-Constrained and -Tolerant Users
Minsu Kim, Jeonghun Park, and Jemin Lee

Abstract

In both academia and industry, multi-user multiple-input multiple-output (MU-MIMO) techniques have shown enormous gains in spectral efficiency by exploiting spatial degrees of freedom. So far, an underlying assumption in most of the existing MU-MIMO design has been that all the users use infinite blocklength, so that they can achieve the Shannon capacity. This setup, however, is not suitable considering delay-constrained users whose blocklength tends to be finite. In this paper, we consider a heterogeneous setting in MU-MIMO systems where delay-constrained users and delay-tolerant users coexist, called a DCTU-MIMO network. To maximize the sum spectral efficiency in this system, we present the spectral efficiency for delay-tolerant users and provide a lower bound of the spectral efficiency for delay-constrained users. We consider an optimization problem that maximizes the sum spectral efficiency of delay-tolerant users while satisfying the latency constraint of delay-constrained users, and propose a generalized power iteration (GPI) precoding algorithm that finds a principal precoding vector. Furthermore, we extend a DCTU-MIMO network to the multiple time slots scenario and propose a recursive generalized power iteration precoding algorithm. In simulation results, we prove proposed methods outperform baseline schemes and present the effect of network parameters on the ergodic sum spectral efficiency.

Index Terms

Multi-user multiple-input multiple-output, ultra-reliable low-latency communication, finite blocklength, spectral efficiency, incremental redundancy hybrid automatic repeat request

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I. INTRODUCTION

In fifth generation (5G) communications, as the demand of real-time applications (e.g., virtual reality, smart healthcare, and autonomous driving) increases, the ultra-reliable low-latency communication (URLLC) has been considered as the emerging application scenario [2]–[6]. Especially, the URLLC is required to have high reliability (e.g., more than 99.999%), low end-to-end (E2E) latency (e.g., less than 1ms), and small packet size (e.g., 32 bytes) [7]. To meet those extreme requirements, several wireless communication techniques are being actively studied; a multiple-input multiple-output (MIMO) technique is one of them. As the MIMO techniques have shown enormous performance gains in the previous generations, it is also being expected that the MIMO has a crucial role to support the 5G URLLC. One key requirement to reap the high spectral efficiency of MIMO systems is a delicate design of linear precoding [8]–[10]. In general, it is known that the sum spectral efficiency optimization with respect to linear precoding is non-convex; thereby finding a global optimal solution is infeasible. Further, incorporating user scheduling, a problem becomes NP-hard.

For resolving this difficulty, in the past few decades, many researchers have focused on the design of the user scheduling and the precoding in a multi-user multiple-input multiple-output (MU-MIMO) network. The works in [11]–[14] maximized the sum spectral efficiency subject to the total power constraint using the zero-forcing (ZF) and maximum ratio transmission (MRT) precoding, which are classic methods. However, they only optimized the user scheduling for the two precoding methods, which are not optimal in general power regime. In [15], the beamforming, the power control, and the user scheduling were optimized to maximize the minimum weighted rate among all users. In [16], the authors maximized the sum rate by adopting a decoupled approach for designing the quantized codebook, the precoder, and the scheduler. Extending to the multi-cell MU-MIMO network, the works in [17], [18] presented the optimal user scheduling and precoding vector to maximize the weighted sum spectral efficiency for the multi-cell network. However, the aforementioned works in [15]–[18] fail to optimize the user scheduling and the precoding jointly.

To overcome these limitations, some works in [19]–[21] proposed the joint user scheduling and precoding design algorithm. In [19], the optimal user scheduling and precoding were presented for three objectives: the weighted sum spectral efficiency maximization, the minimum signal-to-interference-plus-noise ratio (SINR) maximization, and the power consumption minimization.
In [20], the user selection, the power allocation, and the precoding were optimized to maximize the weighted sum spectral efficiency with the imperfect channel state information at transmitter (CSIT) for the multi-cell network. In [21], the authors maximized the sum spectral efficiency of a cell-free massive MIMO network by jointly identifying a set of cooperative access points (APs), the precoding for beamforming and compression, and the power control. However, the works in [19]–[21] only considered delay-tolerant users whose blocklength was implicitly assumed to be infinite. Hence, the solutions, provided in those works, are not applicable to the MIMO network with a delay-constrained user, which is generally operating in the finite blocklength regime. The delay-constrained users are the ones required to receive the information within a target latency. It is essential to consider those delay-constrained users in the network design for the increasing real-time applications.

Recently, some works considering delay-constrained users have been actively studied, especially for the URLLC scenarios that require the extremely low E2E latency (e.g., less than 1ms). Many researchers tried to reduce the communication duration by decreasing the packet size, and consequently, the approximated maximal achievable rate in the finite blocklength regime, different from conventional Shannon capacity, was presented [22], [23]. In the view of the finite blocklength regime, the optimal resource allocation is studied according to network parameters in [24], [25]. In [24], the global optimal resource allocation ensuring the decoding error probability and the transmission delay was investigated. The quality of service (QoS)-constrained throughput with incremental redundancy hybrid automatic repeat request (IR-HARQ) was analyzed to control the transmit power allocation in [25]. Furthermore, a few works in [26]–[28] analyzed the sum rate in the finite blocklength regime when the MU-MIMO network is considered. In [26], the optimal pilot and payload transmission power was presented for both maximum ratio combining (MRC) and ZF methods to maximize the weighted uplink data rate with the imperfect channel state information (CSI). In [27], the optimal beamforming vector was presented to maximize the weighted sum rate subject to the QoS requirement of each user. In [28], the authors maximized the minimum rate among users for two cases: optimizing the beamforming vector and optimizing the power allocation with the regularized zero-forcing (RZF) beamforming. However, most of those works did not consider the MU-MIMO network (such as [22]–[25]), and some of those works considered delay-constrained users only in the MU-MIMO network (such as [26]–[28]). Since the base station (BS) supports various services (e.g., autonomous driving and ultra-high dimension (UHD) video) in current and future networks, we need to carefully design the MU-MIMO
network where delay-constrained users and delay-tolerant users coexist.

Therefore, in this work, we consider a general setup of the MU-MIMO network with delay-constrained and delay-tolerant users (DCTUs), which we denote it as the DCTU-MIMO network. The BS, equipped with multiple transmit antennas, simultaneously serves delay-tolerant users and delay-constrained users. We also consider the scenario that the IR-HARQ is adopted to enhance the communication reliability of users. To the best of our knowledge, this is the first work that considers not only the delay-tolerant users but also the delay-constrained users in the MU-MIMO network. In the DCTU-MIMO network, we analyze the spectral efficiencies for delay-constrained users and delay-tolerant users, respectively. We then formulate an optimization problem that maximizes the sum spectral efficiency of delay-tolerant users while satisfying the latency requirement of delay-constrained users. We finally propose a generalized power iteration (GPI) precoding algorithm, defined as the Delay-GPI, that provides a joint solution for the user scheduling and the precoding at the BS of the optimization problem. The main contributions of this paper can be summarized as follows.

- We develop the novel optimization framework of the DCTU-MIMO network without and with the IR-HARQ scheme in the presence of delay-constrained users as well as delay-tolerant users. Especially, we consider the finite blocklength coding for the delay-constrained users while the infinite blocklength coding is used for the delay-tolerant users.
- We analyze the spectral efficiency for two types of users without and with the IR-HARQ scheme. After deriving an upper bound on the channel dispersion for the interference channel without the IR-HARQ scheme, we also provide the lower bound on the spectral efficiency of the delay-constrained user as a Rayleigh quotient form, which is more tractable in the optimization.
- We consider the optimization problem that maximizes the sum spectral efficiency of delay-tolerant users while satisfying the latency constraint of delay-constrained users. We provide the first-order optimality condition of this problem and present the generalized power iteration precoding algorithm to find a feasible principal precoding vector that also satisfies the first-order optimality condition.
- We finally show that the proposed algorithm outperforms baseline methods in terms of the ergodic sum spectral efficiency. Especially, the proposed algorithm improves the ergodic sum spectral efficiency by allocating minimal transmission power that meets the communication latency requirement to delay-constrained users.
The remainder of this paper is organized as follows. We introduce a DCTU-MIMO network in Section II and analyze the spectral efficiency of delay-tolerant users and delay-constrained users in Section III. From the spectral efficiency analysis, for the DCTU-MIMO network without the IR-HARQ scheme, we formulate a sum spectral efficiency maximization problem in Section IV. Furthermore, we derive a first-order optimality condition and propose an algorithm to find a solution of this problem. In Section V, we formulate an optimization problem of the DCTU-MIMO network with the IR-HARQ scheme and provide a computationally efficient algorithm. In Section VI, we evaluate the performance of the DCTU-MIMO network according to network parameters and compare the ergodic sum spectral efficiency of the proposed algorithm with that of baseline methods. Finally, the conclusion is presented in Section VII.

**Notation:** The conjugate transpose of \( x \) is denoted by \( x^H \) and the inverse matrix of \( x \) is denoted by \( x^{-1} \). In addition, \( I_N \) is the identity matrix with size \( N \times N \) and \( \text{diag} (A_1, \cdots, A_N) \in \mathbb{C}^{N_K \times N_K} \) is a block diagonal matrix where \( A_k \in \mathbb{C}^{K \times K} \) is a square matrix.

### II. System Model

In this section, we first introduce a DCTU-MIMO network. We then explain the IR-HARQ scheme and describe the DCTU-MIMO network with IR-HARQ scheme.

#### A. Network Model without IR-HARQ scheme

As shown in Fig. 1, we consider a DCTU-MIMO network, where a BS equipped with \( N \) antennas transmits the downlink signal \( x \) to two types of users equipped with a single antenna. Here, the first type of user is the delay-tolerant user and the second type of user is the delay-constrained user. The delay-tolerant user means a user who wants to achieve high spectral efficiency without the target latency. On the contrary, the delay-constrained user means a user who needs to receive the downlink signal within the target latency. The numbers of delay-tolerant users and delay-constrained users are denoted as \( K_t \) and \( K_s \), respectively. For convenience, we then define the total user set as \( \mathcal{K} = \{1, 2, \cdots, K_t + K_s\} \), which is divided into the delay-tolerant user set \( \mathcal{K}_t = \{1, 2, \cdots, K_t\} \) and the delay-constrained user set \( \mathcal{K}_s = \{K_t + 1, K_t + 2, \cdots, K_t + K_s\} \) (i.e., \( \mathcal{K} = \mathcal{K}_t \cup \mathcal{K}_s \)). In this network, we express the downlink signal \( x \) as

\[
x = \sum_{k=1}^{K_t+K_s} u_k s_k
\]
where $\mathbf{u}_k$ is the precoding vector for the user $k$ and $s_k$ is the transmit symbol for the user $k$ with the average power $P = \mathbb{E}[|s_k|^2]$. We assume that the BS determines the precoding vector for one resource frame and knows the perfect CSI of all the users. From (1), the received signal at the user $k$ is given by

$$y_k = \mathbf{h}_k^H \mathbf{u}_k s_k + \sum_{i \neq k} \mathbf{h}_k^H \mathbf{u}_i s_i + z_k, \quad \forall k$$

where $z_k$ is the noise signal at the user $k$, distributed as $\mathcal{CN}(0, \sigma_k^2)$. In (2), $\mathbf{h}_k = [h_{k1}, h_{k2}, \ldots, h_{kN}] \in \mathbb{C}^{N \times 1}$ is the channel vector from the BS to the user $k$. The distribution of $\mathbf{h}_k$ is considered as the complex Gaussian, i.e., $\mathbf{h}_k \sim \mathcal{CN}(0, \mathbf{C}_k)$ where $\mathbf{C}_k = \mathbb{E}[\mathbf{h}_k \mathbf{h}_k^H] \in \mathbb{C}^{N \times N}$ is the spatial covariance matrix of the channel.

**B. Network Model with IR-HARQ scheme**

In the DCTU-MIMO network, to increase the reliability of the delay-constrained users, the hybrid automatic repeat request (HARQ) is used, which is a combination of forward error correction (FEC) and automatic repeat request (ARQ) schemes, among which we adopt the IR-HARQ scheme [29]. The IR-HARQ scheme is one of the HARQ schemes that the erroneous decoded packets are combined with incremental redundancy after each retransmission. Specifically, when a BS transmits the downlink signal to a user $k$, the user $k$ sends an acknowledge (ACK) or a negative-acknowledge (NACK) message to the BS. If the BS receives a ACK message, the BS
transmits new downlink signal to the user \( k \). On the other hand, if the BS receives a NACK message, the BS retransmits the same downlink signal with additional redundancy bits and the user \( k \) combines the signal, received during retransmissions for the same downlink signal, and uses it for decoding. In this consideration, we assume a resource frame is divided into \( L \) time slots. The user scheduling for delay-tolerant users is done for \( L \) time slots, while that for delay-constrained users is done for \( T \) time slots (\( T < L \)) to meet the URLLC latency requirement.

Under the same circumstances as the DCTU-MIMO network considered in Section II-A, given the IR-HARQ scheme, the downlink signal at the \( t \)-th transmission round is expressed as

\[
x(t) = \sum_{k=1}^{K_t+K_s} u_k(t)s_k, \quad \forall k, t = 1, \cdots, T
\]

where \( u_k(t) \in \mathbb{C}^{N \times 1} \) is the precoding vector of the user \( k \) at the \( t \)-th transmission round. From (3), the received signal of the user \( k \) at the \( t \)-th transmission round is given by

\[
y_k(t) = h_k^H(t)u_k(t)s_k + \sum_{i \neq k}^{K_t+K_s} h_i^H(t)u_i(t)s_i + z_k, \quad \forall k, t = 1, \cdots, T
\]

where \( h_k(t) \in \mathbb{C}^{N \times 1} \) is the channel vector from the BS to the user \( k \) at the \( t \)-th transmission round. The distribution of \( h_k(t) \) is also considered as the complex Gaussian, i.e., \( h_k(t) \sim \mathcal{CN}(0, C_k(t)) \) where \( C_k(t) = \mathbb{E}[h_k(t)h_k^H(t)] \in \mathbb{C}^{N \times N} \) is the spatial covariance matrix of the channel at the \( t \)-th transmission round.

III. SPECTRAL EFFICIENCY ANALYSIS

In this section, we analyze the spectral efficiency without and with the IR-HARQ scheme. Firstly, the delay-tolerant users can be allocated to long enough blocklength because the BS allocates more time resources to the delay-tolerant users. Therefore, for the delay-tolerant users, we consider the spectral efficiency using the infinite blocklength coding, which follows the Shannon capacity. For delay-constrained users, unlike delay-tolerant users, we consider the spectral efficiency in the finite blocklength regime. This is because, to satisfy the URLLC latency requirement, the delay-constrained users are allocated small time resources.

A. Spectral Efficiency without IR-HARQ scheme

From (2), the spectral efficiency of the delay-tolerant user, which follows the Shannon capacity, can be expressed as

\[
R_k(\gamma_k) = \log_2(1 + \gamma_k), \quad k \in \mathcal{K}_t
\]
where $\gamma_k$ is the SINR of the user $k$, given by

$$\gamma_k = \frac{|h_k^H u_k|^2}{\sum_{i \neq k} |h_k^H u_i|^2 + \frac{\sigma_k^2}{P}}$$

(6)

for the noise signal power $\sigma_k^2$. To make the spectral efficiency as the Rayleigh quotient that is a suitable form for applying the GPI method, we define the network-wide precoding vector $u$ as

$$u = \begin{bmatrix} u_{1}, \ldots, u_{K_t}, \ldots, u_{K_t+1}, \ldots, u_{K_t+K_s} \end{bmatrix}^H \in \mathbb{C}^{N(K_t+K_s) \times 1}. \quad (7)$$

Since the spectral efficiency is an increasing function with the transmit power consumption $\|u\|$, we assume that the maximum transmit power is used, i.e., $\|u\| = 1$. Using (7), we rewrite the spectral efficiency of the delay-tolerant user as

$$R_k(u) = \log_2 \left( \frac{\sum_{i=1}^{K_t+K_s} u_i^H (h_k h_k^H) u_i + \frac{\sigma_k^2}{P}}{\sum_{i \neq k} |h_k^H u_i|^2 + \frac{\sigma_k^2}{P}} \right)$$

$$= \log_2 \left( \frac{u_i^H A_k u_i}{u_i^H B_k u} \right), \quad k \in K_t \quad (8)$$

where $A_k = \text{diag} (h_k h_k^H, \ldots, h_k h_k^H) + \frac{\sigma_k^2}{P} I_{N(K_t+K_s)}$ and $B_k = A_k - \text{diag} (0, \ldots, h_k h_k^H, \ldots, 0)$ is constructed by subtracting the $k$-th sub-block matrix from $A_k$.

For delay-constrained users, for given blocklength $m$ and target decoding error probability $\varepsilon_k$, the spectral efficiency is expressed as [23], [30]

$$R_k(\gamma_k) = \log_2 (1 + \gamma_k) - \sqrt{\frac{V(\gamma_k)}{m}} Q^{-1}(\varepsilon_k), \quad k \in K_s \quad (9)$$

where $V(\gamma_k)$ is the channel dispersion of the user $k$, which are given by

$$V(\gamma_k) = \frac{2\gamma_k}{1 + \gamma_k} \left( \log_2 e \right)^2, \quad k \in K_s. \quad (10)$$

Note that as the inter-user interference is considered, the channel dispersion $V(\gamma_k)$ cannot be defined in additive white Gaussian noise (AWGN) channel. Hence, we use the channel dispersion of Gaussian codebooks under the non-Gaussian noise and the nearest-neighbor decoding [31]. However, since the spectral efficiency of delay-constrained users consists of the mixture of the log function and the square root function, it is difficult to deal with an optimization problem that maximizes the sum spectral efficiency of delay-tolerant users under latency constraints. Hence, to make the spectral efficiency of delay-constrained users a tractable form, we present the upper bound of the channel dispersion using the similar approach in [26] in the following Lemma.
Lemma 1: For any given \( \bar{x} > 0 \), the upper bound of \( \sqrt{\frac{2x}{1+x}} \) can be given as
\[
\sqrt{\frac{2x}{1+x}} \leq \rho(\bar{x}) \ln(1 + x) + \eta(\bar{x}), \quad \forall x > 0
\]  
where \( \rho(\bar{x}) = \frac{1}{\sqrt{2x(1+\bar{x})}} \) and \( \eta(\bar{x}) = \sqrt{\frac{2x}{1+x}} - \rho(\bar{x}) \ln(1+\bar{x}) \). When \( x = \bar{x} \), the equality in (11) holds.

Proof: Define \( H(x) = \sqrt{\frac{2x}{1+x}} \) and \( G(x, \bar{x}) = \rho(\bar{x}) \ln(1 + x) + \eta(\bar{x}) \). Firstly, we can readily know that \( H(x) \) and \( G(x, \bar{x}) \) are the same at \( x = \bar{x} \) by substituting given \( \rho(\bar{x}) \) and \( \eta(\bar{x}) \) into \( G(x, \bar{x}) \), which proves the equality of (11). We then prove the inequality \( H(x) < G(x, \bar{x}) \) by defining a function \( F(x, \bar{x}) \triangleq G(x, \bar{x}) - H(x) \) and obtaining the first derivative of \( F(x, \bar{x}) \) with respect to \( x \). The first derivative of \( F(x, \bar{x}) \) with respect to \( x \) can be given by
\[
\frac{\partial F(x, \bar{x})}{\partial x} = \frac{(1 + \bar{x})\sqrt{2x\sqrt{(1+x)^3} - (1 + x)\sqrt{2\bar{x}\sqrt{(1+\bar{x})^3}}}}{(1 + x)\sqrt{2\bar{x}\sqrt{(1+x)^3}\sqrt{2\bar{x}\sqrt{(1+\bar{x})^3}}}}.
\]  
(12)
Since \( x \) and \( \bar{x} \) are positive values, the sign of \( \frac{\partial F(x, \bar{x})}{\partial x} \) is affected by the numerator in (12). The numerator can be represented by
\[
J(x, \bar{x}) = (1 + \bar{x})(1 + x) \left( \sqrt{2x\sqrt{(1+x)^3} - \sqrt{2\bar{x}\sqrt{(1+\bar{x})^3}}} \right).
\]  
(13)
From new, we will show that \( J(x, \bar{x}) < 0 \) when \( 0 < x < \bar{x} \) and \( J(x, \bar{x}) > 0 \) when \( x > \bar{x} \). First, let us define \( U(x) \triangleq \sqrt{2x\sqrt{1+x}} \) and obtain the first derivative of \( U(x) \) with respect to \( x \) as
\[
\frac{\partial U(x)}{\partial x} = \frac{1 + 2x}{\sqrt{2x(1+x)}}.
\]  
(14)
From (14), we can know that \( \frac{\partial U(x)}{\partial x} > 0 \) when \( x > 0 \). Hence, when \( 0 < x < \bar{x} \), we have \( J(x, \bar{x}) < 0 \) because \( U(x) < U(\bar{x}) \). Consequently, we have \( \frac{\partial F(x, \bar{x})}{\partial x} < 0 \), which means \( F(x, \bar{x}) \) is a monotonically decreasing function of \( x \). Hence, the inequality \( F(x, \bar{x}) < F(0, \bar{x}) = \eta(\bar{x}) \) can be satisfied. Here, since \( \eta(\bar{x}) > 0 \) for \( \bar{x} > 0 \) and \( F(\bar{x}, \bar{x}) = 0 \), we obtain \( F(x, \bar{x}) > 0 \) holds, which results in \( G(x, \bar{x}) > H(x) \) when \( 0 < x < \bar{x} \).

On the other hand, for \( x > \bar{x} \), we have \( U(x) > U(\bar{x}) \) and \( J(x, \bar{x}) > 0 \). Therefore, \( \frac{\partial F(x, \bar{x})}{\partial x} > 0 \), which signifies \( F(x, \bar{x}) \) is a monotonically increasing function of \( x \). Hence, we have \( F(x, \bar{x}) > \eta(\bar{x}) \) and \( G(x, \bar{x}) > H(x) \) when \( x > \bar{x} \). Therefore, when \( x > 0 \), \( G(x, \bar{x}) \) is always greater than \( H(x) \).

From Lemma 1 by using (11) into (9), for given \( \gamma_k \), the lower bound of the spectral efficiency for the delay-constrained user can be represented as
\[
R_k(\gamma_k) \geq \tilde{R}_k(\gamma_k, \bar{\gamma}_k) = \log_2(1 + \gamma_k) - \frac{Q^{-1}(\varepsilon_k)}{\sqrt{M}} \left\{ \rho(\bar{\gamma}_k) \log_2(1 + \gamma_k) + \eta(\bar{\gamma}_k) \log_2 e \right\}
\]  
\[
= \log_2 \left\{ \frac{\left( \sum_{i=1}^{K+K_i} \| \mathbf{h}^i_k \mathbf{u} \|^2 + \frac{\sigma^2}{P} \right)^{1-f_k(\bar{\gamma}_k)}}{\left( \sum_{i \neq k}^{K+K_i} \| \mathbf{h}^i_k \mathbf{u} \|^2 + \frac{\sigma^2}{P} \right)^{1-f_k(\bar{\gamma}_k)}} \right\} - g_k(\bar{\gamma}_k), \quad k \in K_s \]  
(15)
where \( \rho_k(\tilde{\gamma}_k) = \frac{1}{\sqrt{2\tilde{\gamma}_k(1+\tilde{\gamma}_k)}} \), \( \eta_k(\tilde{\gamma}_k) = \sqrt{\frac{2\tilde{\gamma}_k}{1+\tilde{\gamma}_k}} - \rho_k(\tilde{\gamma}_k) \ln(1+\tilde{\gamma}_k) \), \( f_k(\tilde{\gamma}_k) = \frac{Q^{-1}(\varepsilon_k)\rho_k(\tilde{\gamma}_k)}{\sqrt{m}} \), and \\
g_k(\tilde{\gamma}_k) = \frac{Q^{-1}(\varepsilon_k)\rho_k(\tilde{\gamma}_k)}{\sqrt{m}} \log_2 e. Further, using (7), we rewrite the lower bound of the spectral efficiency for the delay-constrained user as a function of \( u \) and \( \tilde{\gamma}_k \) as

\[
\hat{R}_k(u, \tilde{\gamma}_k) = \log_2 \left\{ \frac{u^H A_k u}{u^H B_k u} \right\}^{1-f_k(\tilde{\gamma}_k)} - g_k(\tilde{\gamma}_k), \quad k \in K_s.
\]

Using (16), we can also present the upper bound of the communication latency, given by

\[
\frac{D_s}{\hat{R}_k(u, \tilde{\gamma}_k)} = \frac{\log_2 \left\{ \frac{u^H A_k u}{u^H B_k u} \right\}^{1-f_k(\tilde{\gamma}_k)} - g_k(\tilde{\gamma}_k)}{k \in K_s}
\]

where \( D_s \) is the data size of the downlink signal.

**B. Spectral Efficiency with IR-HARQ scheme**

Let us define \( u_{IR} \) as the network-wide precoding vector of the IR-HARQ scheme, which can be expressed as \\

\[
u_{IR} = \begin{bmatrix} u^H_{IR}(1) & \cdots & u^H_{IR} \end{bmatrix} \in \mathbb{C}^{N(K_t+K_s)T \times 1},
\]

where \( u_{IR}(t) = [u^H(t) \cdots u^H_{K_t+K_s}(t)]^H \in \mathbb{C}^{N(K_t+K_s) \times 1} \) is the network-wide precoding vector of the IR-HARQ scheme at the \( t \)-th transmission round. From (4), for given \( T \) transmission rounds, since we use the maximum transmit power, i.e., \( \|u_{IR}\| = 1 \), spectral efficiencies of the delay-tolerant user and the delay-constrained user are expressed as \([22], [29]\)

\[
R^\text{IR}_k(u_{IR}) = \sum_{t=1}^{T} R_{k,t}(u_{IR}) = \sum_{t=1}^{T} \log_2 (1 + \gamma_k(t)) = \sum_{t=1}^{T} \log_2 \left( \frac{u_{IR}^H D_k(t) u_{IR}}{u_{IR}^H E_k(t) u_{IR}} \right), \quad k \in K_t,
\]

\[
R^\text{IR}_k(u_{IR}) = \sum_{t=1}^{T} \log_2 (1 + \gamma_k(t)) - \sqrt{\frac{\sum_{t=1}^{T} V(\gamma_k(t))}{m}} Q^{-1}(\varepsilon_k)
\]

\[
= \sum_{t=1}^{T} \log_2 \left( \frac{u_{IR}^H D_k(t) u_{IR}}{u_{IR}^H E_k(t) u_{IR}} \right) - \sqrt{\frac{\sum_{t=1}^{T} V(\gamma_k(t))}{m}} Q^{-1}(\varepsilon_k), \quad k \in K_s
\]

where \( R_{k,t}(u_{IR}) \) is the spectral efficiency of user \( k \) at the \( t \)-th transmission round and \( D_k(t) \) is given by

\[
D_k(t) = \begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
0 & \cdots & D_k(t) & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & \cdots & 0 \end{bmatrix} + \frac{\sigma^2}{P} I_{N(K_t+K_s)T} \in \mathbb{C}^{N(K_t+K_s)T \times N(K_t+K_s)T}
\]

(19)
where $D_k(t) = \text{diag}(h_k(t)h_k^H(t), \ldots, h_k(t)h_k^H(t)) \in \mathbb{C}^{N(K_t+K_s) \times N(K_t+K_s)}$ is $t$-th sub-block matrix of $D_k(t)$. In (18), $E_k(t) = D_k(t) - \text{diag}(0, \ldots, h_k(t)h_k^H(t), \ldots, 0) \in \mathbb{C}^{N(K_t+K_s)T \times N(K_t+K_s)T}$ is constructed by subtracting the $k$-th sub-block matrix from $D_k(t)$ and $\gamma_k(t)$ is the SINR of user $k$ at the $t$-th transmission round, given by

$$\gamma_k(t) = \frac{|h_k^H(t)u_k(t)|^2}{\sum_{i \neq k}^{K_t+K_s} |h_k^H(t)u_i(t)|^2 + \frac{\sigma^2}{P}}, \ \forall k, \ \forall t. \quad (20)$$

As mentioned in Section III-A, since the spectral efficiency of delay-constrained users consists of the mixture of the log function and the square root function, it is difficult to deal with the optimization problem that maximizes the sum spectral efficiency of delay-tolerant users under the latency constraint. Hence, to make the spectral efficiency of delay-constrained users as a tractable form, for any given $\tilde{\gamma}_k(t) > 0$, we provide the upper bound of $\sqrt{\sum_{t=1}^{T} \frac{2\gamma_k(t)}{1 + \gamma_k(t)}}$ as

$$\sqrt{\sum_{t=1}^{T} \frac{2\gamma_k(t)}{1 + \gamma_k(t)}} \leq \sum_{t=1}^{T} \sqrt{\frac{2\gamma_k(t)}{1 + \gamma_k(t)}} \leq \sum_{t=1}^{T} \{\rho(\gamma_k(t)) \ln(1 + \gamma_k(t)) + \eta(\gamma_k(t))\} \quad (21)$$

where (a) is due to the triangle inequality, (b) is obtained using (11), $\rho(\tilde{\gamma}_k(t)) = \frac{1}{\sqrt{2\gamma_k(t)(1 + \gamma_k(t))}}$, and $\eta(\tilde{\gamma}_k(t)) = \sqrt{\frac{2\gamma_k(t)}{1 + \gamma_k(t)}} - \rho(\tilde{\gamma}_k(t)) \ln(1 + \gamma_k(t))$. By using (21) into (18), we obtain the lower bound of the spectral efficiency for delay-constrained users as

$$R_k^{IR}(u_{IR}) \geq \tilde{R}_k^{IR}(u_{IR}, \tilde{\gamma}_k(1), \ldots, \tilde{\gamma}_k(T)) = \sum_{t=1}^{T} \left[ \log_2 \left( \frac{\left(\frac{u_{IR}^H D_k(t) u_{IR}}{u_{IR}^H E_k(t) u_{IR}}\right)^{1-f_{k,t}(\tilde{\gamma}_k(t))}}{u_{IR}^H E_k(t) u_{IR}} \right) - g_{k,t}(\tilde{\gamma}_k(t)) \right]$$

$$= \sum_{t=1}^{T} \tilde{R}_{k,t}(u_{IR}, \tilde{\gamma}_k(t)), \ k \in K_s \quad (22)$$

where $\tilde{R}_{k,t}(u_{IR}, \tilde{\gamma}_k(t))$ is the lower bound of spectral efficiency of user $k$ at the $t$-th transmission round, $f_{k,t}(\tilde{\gamma}_k(t)) = \frac{Q^{-1}(\epsilon_k) P_k(\tilde{\gamma}_k(t))}{\sqrt{m}}$, and $g_{k,t}(\tilde{\gamma}_k(t)) = \frac{Q^{-1}(\epsilon_k) P_k(\tilde{\gamma}_k(t))}{\sqrt{m}} \ln 2 e$.

However, since the BS does not know the future CSI of all the users in practice, we cannot know the exact spectral efficiency of future transmission rounds. Therefore, future transmission rounds, by estimating the channel covariance matrix from geometrical locations or the angle-of-arrival (AoA) of users, we obtain the approximation of the ergodic spectral efficiencies as

$$\mathbb{E}_{h_k(t)} [R_{k,t}(u_{IR})] = \mathbb{E}_{h_k(t)} \left[ \log_2 \left( 1 + \frac{|h_k^H(t)u_k(t)|^2}{\sum_{i \neq k}^{K_t+K_s} |h_k^H(t)u_i(t)|^2 + \frac{\sigma^2}{P}} \right) \right]$$

$$\approx \log_2 \left( 1 + \frac{\mathbb{E}_{h_k(t)} \left[ |h_k^H(t)u_k(t)|^2 \right]}{\mathbb{E}_{h_k(t)} \left[ \sum_{i \neq k}^{K_t+K_s} |h_k^H(t)u_i(t)|^2 \right] + \frac{\sigma^2}{P}} \right) = \log_2 \left( \frac{u_{IR}^H \tilde{D}_k(t) u_{IR}}{u_{IR}^H \tilde{E}_k(t) u_{IR}} \right) = \tilde{R}_{k,t}(u_{IR}), \ k \in K_t,
\[ \mathbb{E}_{h_k(t)} \left[ \hat{R}_{k,t}(u_{IR}, \tilde{\gamma}_k(t)) \right] \approx \log_2 \left\{ \left( \frac{u_{IR}^H \hat{D}_k(t) u_{IR}}{u_{IR}^H \hat{E}_k(t) u_{IR}} \right)^{1-f_{k,t}(\tilde{\gamma}_k(t))} \right\} - g_{k,t}(\tilde{\gamma}_k(t)) = \hat{R}_{k,t}(u_{IR}, \tilde{\gamma}_k(t)), \; k \in \mathcal{K}_s \] (23)

where (a) is from the Lemma 1 of [32]. In (23), \( \hat{D}_k(t) \) and \( \hat{E}_k(t) \) are obtained by replacing \( h_k(t)h_k^H(t) \) of \( D_k(t) \) and \( E_k(t) \) with \( C_k(t) \). Here, \( C_k(t) = \mathbb{E} [h_k(t)h_k^H(t)] \in \mathbb{C}^{N \times N} \) is the spatial covariance matrix of the channel of user \( k \) at the \( t \)-th transmission round.

**IV. Precoding Design for DCTU-MIMO Network without IR-HARQ Scheme**

In this section, we first formulate an optimization problem that maximizes the spectral efficiency of delay-tolerant users while satisfying the communication latency constraint of delay-constrained users for a DCTU-MIMO network. We then derive the first-order optimality condition of the optimization problem and propose the computationally efficient algorithm to find a sub-optimal solution that satisfies the first-order optimality condition.

**A. Problem Formulation**

Under the communication latency constraint, the sum spectral efficiency maximization problem can be formulated as

\[
\text{maximize} \quad \sum_{k=1}^{K_s} R_k(u) \\
\text{subject to} \quad \tilde{R}_k(u, \tilde{\gamma}_k) \geq \frac{D_s}{\delta_k}, \quad k \in \mathcal{K}_s. \tag{24}
\]

where \( \delta_k \) is the communication latency requirement of user \( k \). Note that for tractability, the upper bound of the communication latency is used in the first constraint. This is reasonable since satisfying the latency requirement with the upper bound latency always guarantees that with the actual latency. We assume that the BS knows the predefined latency requirement of delay-constrained users. However, since the optimization problem is not convex, it is difficult to obtain the optimal solution for (24). Instead of that, we can obtain the sub-optimal solution by checking the first-order optimality conditions.

**B. Local Optimal Condition**

In the following Theorem we present the first-order optimality condition of the optimization problem in (24) for the precoding vector and the Lagrangian multiplier.
Theorem 1: Let $\phi(u, \lambda) = \frac{\prod_{k=1}^{K_1} u^H A_k u \prod_{k=K_1+1}^{K_i} u^H A_k u}{\prod_{k=1}^{K_1} u^H B_k u \prod_{k=K_1+1}^{K_i} u^H B_k u} \lambda_k^{(1-f_k(\tilde{\gamma}_k))}$ for the Lagrangian multiplier of user $k$, $\lambda_k$. The first-order optimality condition satisfies when

$$A(u, \lambda) u = \phi(u, \lambda) B(u, \lambda) u$$

(25)

where $A(u, \lambda)$ and $B(u, \lambda)$ are given by

$$A(u, \lambda) = \prod_{k=1}^{K_1} u^H A_k u \prod_{k=K_1+1}^{K_i} u^H A_k u \lambda_k^{(1-f_k(\tilde{\gamma}_k))} \left\{ \sum_{k=1}^{K_1} 2A_k u^H A_k u + \sum_{k=K_1+1}^{K_i} 2\lambda_k (1-f_k(\tilde{\gamma}_k)) A_k u^H A_k u \right\}$$

(26)

$$B(u, \lambda) = \prod_{k=1}^{K_1} u^H B_k u \prod_{k=K_1+1}^{K_i} u^H B_k u \lambda_k^{(1-f_k(\tilde{\gamma}_k))} \left\{ \sum_{k=1}^{K_1} 2B_k u^H B_k u + \sum_{k=K_1+1}^{K_i} 2\lambda_k (1-f_k(\tilde{\gamma}_k)) B_k u^H B_k u \right\}$$

(27)

The Lagrangian multipliers $\lambda_k$ are chosen so that $u$ satisfies

$$\log_2 \left\{ \left( \frac{u^H A_k u}{\bar{u}^H B_k u} \right)^{1-f_k(\tilde{\gamma}_k)} \right\} - g_k (\tilde{\gamma}_k) = \frac{D_s}{\delta_k}, \quad k \in \mathcal{K}_s. \quad (28)$$

If the equation (28) cannot be satisfied, $\lambda$ are chosen so that $u$ satisfies $\tilde{R}_k(u, \tilde{\gamma}_k) > \frac{D_s}{\delta_k}, \quad k \in \mathcal{K}_s$.

Proof: From (8), (17), and (24), we can define the Lagrangian function as

$$L(u, \lambda) = -\sum_{k=1}^{K_1} \log \left( \frac{u^H A_k u}{\bar{u}^H B_k u} \right) - \sum_{k=K_1+1}^{K_i} \lambda_k \left[ \log \left( \frac{u^H A_k u}{\bar{u}^H B_k u} \right)^{1-f_k(\tilde{\gamma}_k)} \right] - g_k (\tilde{\gamma}_k) - \frac{D_s}{\delta_k}$$

(29)

From (29), we first obtain $\frac{\partial L(u, \lambda)}{\partial u^H}$ as

$$\frac{\partial L(u, \lambda)}{\partial u^H} = -\frac{1}{\phi(u, \lambda) \ln 2} \frac{\partial \phi(u, \lambda)}{\partial u^H}.$$ 

(30)

In (30), since $\phi(u, \lambda) > 0$, $\frac{\partial L(u, \lambda)}{\partial u^H} = 0$ is equivalent to $\frac{\partial \phi(u, \lambda)}{\partial u^H} = 0$, which can be presented as

$$\frac{\partial \phi(u, \lambda)}{\partial u^H} = 0 \iff \phi(u, \lambda) \left\{ \nabla_{u^H} \phi_1(u) + \nabla_{u^H} \phi_2(u, \lambda) - \nabla_{u^H} \phi_3(u) - \nabla_{u^H} \phi_4(u, \lambda) \right\} = 0$$

$$\iff \phi(u, \lambda) \left[ \sum_{k=1}^{K_1} 2A_k u^H A_k u + \sum_{k=K_1+1}^{K_i} 2\lambda_k (1-f_k(\tilde{\gamma}_k)) A_k u^H A_k u - \lambda_k^{(1-f_k(\tilde{\gamma}_k))} B_k u^H B_k u \right] = 0$$

(31)
where \( \phi_1(u), \phi_2(u, \lambda), \phi_3(u), \) and \( \phi_4(u, \lambda) \) are given by
\[
\phi_1(u) = \prod_{k=1}^{K_t} u^HA_ku, \quad \phi_2(u, \lambda) = \prod_{k=K_t+1}^{K_t+K_s} \left(u^HA_ku\right)_{\lambda_k(1-f_k(\tilde{\gamma}_k))}, \\
\phi_3(u) = \prod_{k=1}^{K_t} u^HB_ku, \quad \phi_4(u, \lambda) = \prod_{k=K_t+1}^{K_t+K_s} \left(u^HB_ku\right)_{\lambda_k(1-f_k(\tilde{\gamma}_k))}.
\]

From (31), the first-order optimality condition with respect to \( u \), i.e., \( \frac{\partial \mathcal{L}(u, \lambda)}{\partial u} = 0 \), can be presented as (25).

In addition, by setting the first derivative of \( \mathcal{L}(u, \lambda) \) with respect to \( \lambda_k \) to zero, i.e., \( \frac{\partial \mathcal{L}(u, \lambda)}{\partial \lambda_k} = 0 \), we can readily obtain the first-order optimality condition with respect to \( \lambda_k \) as (28).

From Theorem 1, we can obtain the first-order optimality condition for the precoding vector, expressed as the generalized eigenvalue problem in (25). It means that any stationary point of the problem in (24) is the eigenvector of the matrix \( \bar{B}(u, \lambda)^{-1} \bar{A}(u, \lambda) \). Therefore, when we interpret \( \phi(u, \lambda) \) as the eigenvalue of the matrix \( \bar{B}(u, \lambda)^{-1} \bar{A}(u, \lambda) \) and treat \( u \) as the eigenvector corresponding to the eigenvalue, we can obtain an optimal solution of the problem in (24) by finding a principal eigenvector of the matrix \( \bar{B}(u, \lambda)^{-1} \bar{A}(u, \lambda) \). Although the conventional generalized eigenvalue problem can be solved, we cannot solve a generalized eigenvalue problem in (25) as the matrix \( \bar{B}(u, \lambda)^{-1} \bar{A}(u, \lambda) \) is also affected by \( u \). In spite of this difficulty, using the power iteration algorithm [33], we provide a computationally efficient algorithm that finds a converged feasible precoding vector that satisfies the first-order optimality condition in (25).

C. Precoding Algorithm

In this subsection, we develop an algorithm that finds the principal eigenvector of a generalized eigenvalue problem in (25) as presented in Algorithm 1. We denote a proposed algorithm as the generalized power iteration with the latency constraint (Delay-GPI). In the proposed algorithm, we iteratively obtain an optimal precoding vector \( u \). In the \( j \)-th iteration, for given \( u^{(j-1)} \) and \( \lambda^{(n)} \), we construct the matrices \( \bar{A} \left(u^{(j-1)}, \lambda^{(n)}\right) \) and \( \bar{B} \left(u^{(j-1)}, \lambda^{(n)}\right) \) using Theorem 1. We then update the precoding vector \( u^{(j)} = \left[ B \left(u^{(j-1)}, \lambda^{(n)}\right) \right]^{-1} \bar{A} \left(u^{(j-1)}, \lambda^{(n)}\right) u^{(j-1)} \) by using the GPI method [20] and normalize it as \( u^{(j)} = u^{(j)}/\|u^{(j)}\| \). Until the precoding vector converges to the principal eigenvector, i.e., \( \|u^{(j)} - u^{(j-1)}\| < \xi \) with a predetermined tolerance parameter \( \xi \), we repeat this process. From the convergent precoding vector \( u^{(j)} \), we check the latency requirements of delay-constrained users. If latency requirements are satisfied, i.e., \( \tilde{R}_{k}(u^{(j)}, \tilde{\gamma}_k) \geq \frac{D}{\delta_k}, \forall k \in K_s \), then the algorithm ends. Otherwise, we update Lagrangian multipliers to satisfy latency requirements.
**Algorithm 1 Delay-GPI**

**initialize:** $u^{(0)} = \text{RZF}, u^{(-1)} = 0, \lambda_k^{(0)}$, and $\xi$.

Set the iteration count $n = 0$ and $j = 0$.

**while** $\tilde{R}_k(u^{(j)}, \tilde{\gamma}_k) < \frac{D_s}{\delta_k}$, $\forall k \in K_s$ **do**

1. $n \leftarrow n + 1$.
2. $\lambda_k^{(n)} \leftarrow \left[\lambda_k^{(n-1)} + \Delta \lambda_k^{(n)}\right]^+$, $\forall k \in K_s$.
3. **while** $\|u^{(j)} - u^{(j-1)}\| > \xi$ **do**

   1. $j \leftarrow j + 1$.
   2. Create the matrices $\bar{A}\left(u^{(j-1)}, \lambda^{(n)}\right)$ and $\bar{B}\left(u^{(j-1)}, \lambda^{(n)}\right)$ by using (26) and (27).
   3. Update $u^{(j)} = \left[\bar{B}\left(u^{(j-1)}, \lambda^{(n)}\right)\right]^{-1} \bar{A}\left(u^{(j-1)}, \lambda^{(n)}\right) u^{(j-1)}$.
   4. Normalize $u^{(j)} = u^{(j)}/\|u^{(j)}\|$.

**end while**

**end while**

**output:** $u^* = u^{(j)}$

**Remark 1:** (Precoding vector feasibility) For a given latency requirement $\delta_k$, according to network parameters (e.g., decoding error probability $\varepsilon_k$ and data size of transmit symbol $D_s$), the Delay-GPI cannot converge to a principal eigenvector. This is because when the channel gain of a delay-constrained user is small, the BS should transmit the downlink signal to the user with larger power. However, due to the transmit power constraint, the latency requirement may not be satisfied. Hence, depending on the channel gain and the latency requirement, we should use enough average transmit power $P$ to make the precoding vector design feasible.

**Remark 2:** (Algorithm complexity) As we use the GPI-based algorithm, the complexity of the Delay-GPI algorithm is determined by the computing for $\left[\bar{B}\left(u^{(j-1)}, \lambda^{(n)}\right)\right]^{-1}$ [20]. In our problem, the matrix $\left[\bar{B}\left(u^{(j-1)}, \lambda^{(n)}\right)\right]$ is a linear combination of $B_k\left(u^{(j-1)}\right)$, which is a block diagonal and symmetric matrix. Therefore, the algorithm requires the computational complexity order of $O\left(\frac{1}{3} (K_t + K_s) N^3\right)$ to obtain $\left[\bar{B}\left(u^{(j-1)}, \lambda^{(n)}\right)\right]^{-1}$. Furthermore, to satisfy the latency requirement and converge the precoding vector, we need a total computational complexity order of $O\left(\frac{1}{3} N_c N_p (K_t + K_s) N^3\right)$ where $N_c$ is the number of iterations for the latency constraint and $N_p$ is the number of iterations for the precoding vector convergence.

**Remark 3:** ( Imperfect CSIT) In this paper, we assume that the BS has the perfect knowledge
of CSI. However, due to the imperfect channel estimation and the pilot contamination, the BS can have limited knowledge of CSI. Even under such circumstances, by using the generalized mutual information, we can obtain a lower bound of the ergodic spectral efficiency as

$$R_{\text{imp}}^k(u) = \log_2 \left( \frac{\sum_{i=1}^{K_t + K_s} u_i^H (\hat{h}_k \hat{h}_k^H) u_i + \sum_{i=1}^{K_t + K_s} u_i^H \Phi_k u_i + \frac{\sigma^2_k}{P}}{\sum_{i \neq k}^{K_t + K_s} u_i^H (\hat{h}_k \hat{h}_k^H) u_i + \sum_{i=1}^{K_t + K_s} u_i^H \Phi_k u_i + \frac{\sigma^2_k}{P}} \right) = \log_2 \left( \frac{u_i^H \hat{A}_k \hat{h}_k^H u_i}{u_i^H \hat{B}_k \hat{h}_k^H u_i} \right), \quad k \in \mathcal{K}_t,$$

where $\hat{h}_k = h_k - e_k$ and $e_k = [e_{k1}^1, e_{k2}^2, \ldots, e_{kN}^N] \in \mathbb{C}^{N \times 1}$ is the channel estimation error vector, randomly distributed as the complex Gaussian, i.e., $e_k \sim \mathcal{CN}(0, \Phi_k)$ where $\Phi_k = \mathbb{E}[e_k e_k^H] \in \mathbb{C}^{N \times N}$ is the covariance matrix of the estimation error. In (33), $A_k^{\text{imp}} = \text{diag}\left( \hat{h}_k \hat{h}_k^H + \Phi_k, \ldots, \hat{h}_k \hat{h}_k^H + \Phi_k \right) + \frac{\sigma^2_k}{P} I_{N(K_t + K_s)}$ and $B_k^{\text{imp}} = A_k^{\text{imp}} - \text{diag}\left( 0, \ldots, \hat{h}_k \hat{h}_k^H, \ldots, 0 \right)$. By using (33) in (24), we can readily obtain the first-order optimality condition and present a sub-optimal solution of the imperfect CSIT case using the Delay-GPI.

V. PRECODING DESIGN FOR DCTU-MIMO NETWORK WITH IR-HARQ SCHEME

In this section, we formulate optimization problems that maximize the spectral efficiency of delay-tolerant users while satisfying the communication latency constraint of delay-constrained users for a DCTU-MIMO network with the IR-HARQ scheme. We then derive the first-order optimality condition of optimization problems and extend the proposed Delay-GPI for the design of precoding vectors in a DCTU-MIMO network with the IR-HARQ scheme.

A. Problem Formulation

Using (18), (22), and (23), we formulate the optimization problem for a DCTU-MIMO network with the IR-HARQ scheme at the first transmission round as

$$\begin{aligned}
\max_{u_{\text{IR}}} & \quad \sum_{k=1}^{K_t} R_{k,1}(u_{\text{IR}}) \\
\text{subject to} & \quad \tilde{R}_{k,1}(u_{\text{IR}}, \tilde{\gamma}_k(1)) \geq \frac{D_k}{\beta_k} - \sum_{t=2}^{T} \tilde{R}_{k,t}(u_{\text{IR}}, \tilde{\gamma}_k(t)), \quad k \in \mathcal{K}_s.
\end{aligned}$$

(34)

Furthermore, the optimization problem for a DCTU-MIMO network with the IR-HARQ at the $t_o$-th transmission round ($2 \leq t_o < T$) can be formulated as
maximize \( \sum_{k=1}^{K_t} \tilde{R}_{k,t_o} (\mathbf{u}_{IR,t_o}) \)

subject to \( \tilde{R}_{k,t_o} (\mathbf{u}_{IR,t_o}, \tilde{\gamma}_k(t_o)) \geq \frac{D_s}{\delta_k} - \sum_{t=1}^{T-1} \tilde{R}_{k,t} (\mathbf{u}_{IR,t_o}, \tilde{\gamma}_k(t)) - \sum_{t=1}^{t_o-1} \tilde{R}_k(t), \quad k \in \mathcal{K}_s \) \( \text{(35)} \)

where \( \mathbf{u}_{IR,t_o} = [\mathbf{u}_{IR}^H(t_o), \ldots, \mathbf{u}_{IR}^H(T)]^H, \mathbf{u}_{IR,1} = \mathbf{u}_{IR} \). \( \tilde{R}_k(t) = \tilde{R}_{k,t} (\mathbf{u}_{IR,t_o}, \tilde{\gamma}_k(t)) \) is the spectral efficiency of the delay-constrained user obtained at the \( t \)-th transmission round, and \( \tilde{R}_{k,t_o} (\mathbf{u}_{IR,t_o}, \tilde{\gamma}_k(t_o)) \) and \( \tilde{R}_{k,t} (\mathbf{u}_{IR,t_o}, \tilde{\gamma}_k(t)) \) are the spectral efficiency of the delay-constrained user at the \( t_o \)-th transmission round and the approximation of the ergodic spectral efficiency of the delay-constrained user at the \( t \)-th transmission round \((t_o + 1 \leq t \leq T)\), respectively, given by

\[
\tilde{R}_{k,t_o} (\mathbf{u}_{IR,t_o}, \tilde{\gamma}_k(t_o)) = \log_2 \left\{ \frac{\mathbf{u}_{IR,t_o}^H \mathbf{D}_{k,t_o} (t_o) \mathbf{u}_{IR,t_o}^H}{\mathbf{u}_{IR,t_o}^H \mathbf{E}_{k,t_o} (t_o) \mathbf{u}_{IR,t_o}^H} \right\}^{1-f_k(t_o)} - \gamma_k(t_o), \quad k \in \mathcal{K}_s, \quad \text{(36)}
\]

\[
\tilde{R}_{k,t} (\mathbf{u}_{IR,t_o}, \tilde{\gamma}_k(t)) = \log_2 \left\{ \frac{\mathbf{u}_{IR,t_o}^H \tilde{\mathbf{D}}_{k,t} (t) \mathbf{u}_{IR,t_o}^H}{\mathbf{u}_{IR,t_o}^H \tilde{\mathbf{E}}_{k,t} (t) \mathbf{u}_{IR,t_o}^H} \right\}^{1-f_k(t)} - \gamma_k(t), \quad k \in \mathcal{K}_s. \quad \text{(36)}
\]

In (36), \( \mathbf{D}_{k,t_o} (t_o) \) and \( \tilde{\mathbf{D}}_{k,t} (t) \) are given by

\[
\mathbf{D}_{k,t_o} (t_o) = \begin{bmatrix} D_{k,t_o} (t_o) & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} + \frac{\sigma_k^2}{P} \mathbf{I}_{N(K_t+K_s)(T-t_o+1)} \in \mathbb{C}^{N(K_t+K_s)(T-t_o+1) \times N(K_t+K_s)(T-t_o+1)},
\]

\[
\tilde{\mathbf{D}}_{k,t} (t) = \begin{bmatrix} \hat{D}_{k,t} (t) & 0 & \cdots & 0 \\ 0 & \hat{D}_{k,t} (t) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} + \frac{\sigma_k^2}{P} \mathbf{I}_{N(K_t+K_s)(T-t_o+1)} \in \mathbb{C}^{N(K_t+K_s)(T-t_o+1) \times N(K_t+K_s)(T-t_o+1)},
\]

where \( \mathbf{D}_{k,t_o}^0 (t_o) = \text{diag} (\mathbf{h}_k(t_o) \mathbf{h}_k^H(t_o), \ldots, \mathbf{h}_k(t_o) \mathbf{h}_k^H(t_o)) \in \mathbb{C}^{N(K_t+K_s) \times N(K_t+K_s)} \) and \( \tilde{\mathbf{D}}_{k,t}^t (t) = \text{diag} (\mathbf{C}_k(t), \ldots, \mathbf{C}_k(t)) \in \mathbb{C}^{N(K_t+K_s) \times N(K_t+K_s)} \) is \( t \)-th sub-block matrix of \( \tilde{\mathbf{D}}_{k,t} (t) \). In addition, \( \mathbf{E}_{k,t_o} (t_o) = \mathbf{D}_{k,t_o} (t_o) - \text{diag} (0, \ldots, \mathbf{h}_k(t_o) \mathbf{h}_k^H(t_o), \ldots, 0) \) is constructed by subtracting the \( k \)-th sub-block matrix from \( \mathbf{D}_{k,t_o}^0 (t_o) \) and \( \tilde{\mathbf{E}}_{k,t} (t) = \tilde{\mathbf{D}}_{k,t} (t) - \text{diag} (0, \ldots, \mathbf{C}_k(t), \ldots, 0) \) is constructed by subtracting the \( k \)-th sub-block matrix from \( \tilde{\mathbf{D}}_{k,t}^t (t) \). After \( T-1 \) transmission rounds, the optimization problem for a DCTU-MIMO network with the IR-HARQ at the last transmission round is expressed as
where \( u_{IR,T} = [u^H(T), \ldots, u_{K_t+K_s}^H(T)]^H \). However, the optimization problems in (34), (35), and (38) are not convex, so it is difficult to obtain the optimal solutions. Hence, we obtain the sub-optimal solution by checking the first-order optimality conditions.

**B. Local Optimal Condition**

In the following Theorem 2, we present the first-order optimality conditions of the optimization problem at the \( t_o \)-th transmission round (\( 2 \leq t_o < T \)) for the precoding vector and the Lagrangian multiplier. Let us define \( \phi^{IR}(u_{IR,t_o}, \lambda) \) as

\[
\phi^{IR}(u_{IR,t_o}, \lambda) = \frac{\phi_1^{IR}(u_{IR,t_o}) \phi_2^{IR}(u_{IR,t_o}, \lambda) \phi_3^{IR}(u_{IR,t_o}, \lambda) \phi_5^{IR}(u_{IR,t_o}, \lambda) \phi_6^{IR}(u_{IR,t_o}, \lambda)}{\phi_4^{IR}(u_{IR,t_o}, \lambda)}
\]

where

\[
\phi_1^{IR}(u_{IR,t_o}) = \prod_{k=1}^{K_t} u_{IR,t_o}^H D_{k,t_o}(t_o) u_{IR,t_o}, \quad \phi_2^{IR}(u_{IR,t_o}, \lambda) = \prod_{k=K_t+1}^{K_t+K_s} (u_{IR,t_o}^H D_{k,t_o}(t_o) u_{IR,t_o})^{\lambda_k(1-f_k,t_o(\gamma_k(t_o)))},
\]

\[
\phi_3^{IR}(u_{IR,t_o}, \lambda) = \prod_{k=K_t+1}^{K_t+K_s} \left\{ \prod_{t=t_o+1}^{T} (u_{IR,t_o}^H \tilde{D}_{k,t}(t) u_{IR,t_o})^{1-f_k,t(\gamma_k(t))} \right\}^{\lambda_k},
\]

\[
\phi_4^{IR}(u_{IR,t_o}, \lambda) = \prod_{k=1}^{K_t} u_{IR,t_o}^H E_{k,t_o}(t_o) u_{IR,t_o}, \quad \phi_5^{IR}(u_{IR,t_o}, \lambda) = \prod_{k=K_t+1}^{K_t+K_s} (u_{IR,t_o}^H E_{k,t_o}(t_o) u_{IR,t_o})^{\lambda_k(1-f_k,t_o(\gamma_k(t_o)))},
\]

\[
\phi_6^{IR}(u_{IR,t_o}, \lambda) = \prod_{k=K_t+1}^{K_t+K_s} \left\{ \prod_{t=t_o+1}^{T} (u_{IR,t_o}^H \tilde{E}_{k,t}(t) u_{IR,t_o})^{1-f_k,t(\gamma_k(t))} \right\}^{\lambda_k}.
\]

**Theorem 2:** The first-order optimality condition of the optimization problem in (35) at the \( t_o \)-th transmission round (\( 2 \leq t_o < T \)) satisfies when

\[
\tilde{A}^{IR}(u_{IR,t_o}, \lambda) u_{IR,t_o} = \phi^{IR}(u_{IR,t_o}, \lambda) B^{IR}(u_{IR,t_o}, \lambda) u_{IR,t_o}
\]

where \( \tilde{A}^{IR}(u_{IR,t_o}, \lambda) \) and \( B^{IR}(u_{IR,t_o}, \lambda) \) are given by
where optimality condition for the precoding vector by using the precoding vector is given by
\[ \sum_{t=1}^{T} \bar{D}_{k,t} \bar{u}_{IR,t} (t_o) \lambda_k (1 - f_{k,t} (\gamma_k(t))) \}

\[ \lambda_k \left\{ \sum_{t=1}^{K_t} \bar{u}_{IR,t} (t_o) \bar{D}_{k,t} (t_o) \right\} \}

\[ \bar{u}_{IR,t} (t_o) \bar{D}_{k,t} (t_o) \]

\[ \lambda_k \left\{ \sum_{t=1}^{K_t} \bar{u}_{IR,t} (t_o) \bar{D}_{k,t} (t_o) \right\} \}

\[ \lambda_k \left\{ \sum_{t=1}^{K_t} \bar{u}_{IR,t} (t_o) \bar{D}_{k,t} (t_o) \right\} \]

In addition, the Lagrangian multipliers \( \lambda \) are chosen so that \( u_{IR,t} \) satisfies
\[ \tilde{R}_{k,t} (u_{IR,t}, \gamma_k(t)) + \sum_{t=t_o+1}^{T} \tilde{R}_{k,t} (u_{IR,t}, \gamma_k(t)) + \sum_{t=1}^{t_o-1} \tilde{R}_{k,t} (u_{IR,t}, \gamma_k(t)) > \frac{D_s}{\delta_k}, \quad k \in K_s. \] (44)

If the equation (44) cannot be satisfied, \( \lambda \) are chosen so that \( u_{IR,t} \) satisfies \( \tilde{R}_{k,t} (u_{IR,t}, \gamma_k(t)) + \sum_{t=t_o+1}^{T} \tilde{R}_{k,t} (u_{IR,t}, \gamma_k(t)) + \sum_{t=1}^{t_o-1} \tilde{R}_{k,t} (u_{IR,t}, \gamma_k(t)) > \frac{D_s}{\delta_k}, \quad k \in K_s. \)

**Proof:** This can be readily proven using the approach in the proof of Theorem [1].

From Theorem [2] for the first transmission round \( t_o = 1 \), we can readily obtain the first-order optimality condition for the precoding vector by using \( t_o = 1 \) in (44). In addition, Lagrangian multipliers \( \lambda \) are chosen so that \( u_{IR,t} \) satisfies an equation in (44) with \( t_o = 1 \) and \( \sum_{t=1}^{t_o-1} \tilde{R}_{k,t} (u_{IR,t}, \gamma_k(t)) > \frac{D_s}{\delta_k}, \quad k \in K_s. \)

If the equation (44) with \( t_o = 1 \) and \( \sum_{t=1}^{t_o-1} \tilde{R}_{k,t} (u_{IR,t}, \gamma_k(t)) = 0 \) cannot be satisfied, \( \lambda \) are chosen so that \( u_{IR,t} \) satisfies \( \tilde{R}_{k,1} (u_{IR}, \gamma_k(1)) + \sum_{t=2}^{T} \tilde{R}_{k,t} (u_{IR}, \gamma_k(t)) > \frac{D_s}{\delta_k}, \quad k \in K_s. \)

In addition, at the last transmission round, since there is no future transmission round, the functions related to \( \tilde{R}_{k,t} (u_{IR,t}, \gamma_k(t)) \) (e.g., \( \phi_{IR} (u_{IR,t}, \lambda) \) and \( \phi_{IR} (u_{IR,t}, \lambda) \)) need to be excluded in (44). Therefore, the first-order optimality condition of the optimization problem in (38) for the precoding vector is given by

\[ \bar{A}^{IR} (u_{IR,T}, \lambda) \] (45)

where \( \phi^{IR} (u_{IR,T}, \lambda) = \frac{\phi^{IR} (u_{IR,T})}{\phi^{IR} (u_{IR,T}) / \phi^{IR} (u_{IR,T})} \). In (45), \( \bar{A}^{IR} (u_{IR,T}, \lambda) \) and \( \bar{A}^{IR} (u_{IR,T}, \lambda) \) are given by
\[
\begin{align*}
\bar{A}^{\text{IR}}(u_{\text{IR},T}, \lambda) &= \prod_{k=1}^{K_1} u_{\text{IR},T}^H D_{k,T}(T) u_{\text{IR},T} \prod_{k=K_1+1}^{K_1+K_s} (u_{\text{IR},T}^H D_{k,T}(T) u_{\text{IR},T})^\lambda_k (1 - f_{k,T}(\tilde{\gamma}_k(T))) \\
&\times \left\{ \sum_{k=1}^{K_1} \frac{2 D_{k,T}(T)}{u_{\text{IR},T}^H D_{k,T}(T) u_{\text{IR},T}} + \sum_{k=K_1+1}^{K_1+K_s} 2 \lambda_k \left(1 - f_{k,T}(\tilde{\gamma}_k(T))\right) D_{k,T}(T) \right\},
\end{align*}
\]

\[
\bar{B}^{\text{IR}}(u_{\text{IR},T}, \lambda) = \prod_{k=1}^{K_1} u_{\text{IR},T}^H E_{k,T}(T) u_{\text{IR},T} \prod_{k=K_1+1}^{K_1+K_s} (u_{\text{IR},T}^H E_{k,T}(T) u_{\text{IR},T})^\lambda_k (1 - f_{k,T}(\tilde{\gamma}_k(T))) \\
&\times \left\{ \sum_{k=1}^{K_1} \frac{2 E_{k,T}(T)}{u_{\text{IR},T}^H E_{k,T}(T) u_{\text{IR},T}} + \sum_{k=K_1+1}^{K_1+K_s} 2 \lambda_k \left(1 - f_{k,T}(\tilde{\gamma}_k(T))\right) E_{k,T}(T) \right\}.
\]

The Lagrangian multipliers \(\lambda\) are chosen so that \(u_{\text{IR}}\) satisfies

\[
\tilde{R}_{k,T}(u_{\text{IR},T}, \tilde{\gamma}_k(T)) + \sum_{t=1}^{T-1} \tilde{R}_k(t) = \frac{D_s}{\delta_k}, \quad k \in K_s.
\]  

If the equation (48) cannot be satisfied, \(\lambda\) are chosen so that \(u_{\text{IR},T}\) satisfies \(\tilde{R}_{k,T}(u_{\text{IR},T}, \tilde{\gamma}_k(T)) + \sum_{t=1}^{T-1} \tilde{R}_k(t) > \frac{D_s}{\delta_k}, \quad k \in K_s\).

C. Precoding Algorithm

In this subsection, by modifying the Algorithm 1, we develop an algorithm that finds the principal eigenvector of generalized eigenvalue problems in (44) and (45) as presented in Algorithm 2. We denote a proposed algorithm as the HARQ-GPI. Compared to the Delay-GPI, we recursively optimize the precoding vector of the IR-HARQ scheme. Specifically, in the \(t\)-th transmission round, when the precoding vector \(u_{\text{IR},t} = [u_{\text{IR}}^H(t), \cdots, u_{\text{IR}}^H(T)]^H\) converges to a principal eigenvector, we only stores the precoding vector of the corresponding transmission round \(u_{\text{IR}}(t) = [u_{\text{IR}}^H(t), \cdots, u_{\text{IR}}^H(K_t+K_s)]^H\) and the power ratio of \(\|u_{\text{IR}}(t)\|\) to \(\|u_{\text{IR},t}\|\). In the next transmission round, by eliminating \(u_{\text{IR}}(t)\), we optimize the new precoding vector \(u_{\text{IR},t+1} = [u_{\text{IR}}^H(t+1), \cdots, u_{\text{IR}}^H(T)]^H\). Finally, after optimizing a predecoding vector of the last transmission round, we multiply the power ratio occupied in each transmission round by the optimal predecoding vector of each transmission round and combine them into \(u_{\text{IR}}^*\), and normalize it.

VI. Numerical Results

In this section, we evaluate the sum spectral efficiency according to the network parameters (e.g., user number, blocklength, and transmission round). We use the one-ring model for the
Algorithm 2 HARQ-GPI

initialize: $u_{IR}^{(0)} = \text{RZF}$, $u_{IR}^{(-1)} = 0$, $\lambda_k^{(0)}$, and $\xi$.

Set the iteration count $n = 0$ and $j = 0$.

for $t = 1$ to $T$ do

while $\hat{R}_{k,t} \left( u_{IR,t}^{(j)} \right) + \sum_{i=t+1}^{T} \hat{R}_{k,i} \left( u_{IR,i}^{(j)} \right) + \sum_{i=1}^{t-1} R_k(i) < \frac{D}{\delta_k}$, $\forall k \in K_s$ do

$n \leftarrow n + 1$.

$\lambda_k^{(n)} \leftarrow \left[ \lambda_k^{(n-1)} + \Delta \lambda_k \right]$, $\forall k \in K_s$.

while $\|u_{IR,t}^{(j)} - u_{IR,t}^{(j-1)}\| > \xi$ do

$j \leftarrow j + 1$.

Create matrices $\bar{A}^{IR} \left( u_{IR,t}^{(j-1)} \right)$ and $\bar{B}^{IR} \left( u_{IR,t}^{(j-1)} \right)$ by using (42) and (43).

Update $u_{IR,t}^{(j)} = \left[ \bar{B}^{IR} \left( u_{IR,t}^{(j-1)} \right) \right]^{-1} \bar{A}^{IR} \left( u_{IR,t}^{(j-1)} \right) u_{IR,t}^{(j-1)}$.

Normalize $u_{IR,t}^{(j)} = u_{IR,t}^{(j)}/\|u_{IR,t}^{(j)}\|$.

end while

end while

Update $u_{IR}^*(t) = u_{IR}^{(j)}(t)$.

Store the power ratio of $\|u_{IR}^{(j)}(t)\|$ to $\|u_{IR,t}^{(j)}\|$ as $q(t)$.

Define $u_{IR}^{(j)}(t+1)$ by subtracting $u_{IR}^{(j)}(t)$ from $u_{IR}^{(j)}(t)$.

Normalize $u_{IR,t+1}^{(j)} = u_{IR,t+1}^{(j)}/\|u_{IR,t+1}^{(j)}\|$.

end for

Combine $u_{IR}^* = [u_{IR}^*(1)q(1), u_{IR}^*(2)(1-q(1))q(2), u_{IR}^*(3)(1-q(1))(1-q(2))q(3), \ldots]^H$.

Normalize $u_{IR}^* = u_{IR}^*/\|u_{IR}^*\|$.

output: $u_{IR}^*$

Spatial covariance matrix of the channel [34]. Specifically, we consider that the BS is equipped with uniform circular array with a circle of radius $\nu D$ where $\nu$ is the wavelength and $D = \sqrt{1-(\cos(2\pi/N))^2+\sin(2\pi/N)^2}$. From this, the channel correlation coefficient between the $n$-th antenna and the $m$-th antenna of the user $k$ is given by

\[
[C_{k,n,m}] = \frac{1}{2\Delta} \int_{\theta_k-\Delta}^{\theta_k+\Delta} \exp \left\{ -j \frac{2\pi}{\nu} \Psi(x)(r_n - r_m) \right\} dx
\]

(49)

where $\theta_k$ is the AoA of the user $k$, $\Delta$ is the angular spread, $\Psi(x) = [\cos(x), \sin(x)]$ is the wave vector for a planer wave impinging with the angle of $x$, and $r_n$ is the position vector for the $n$-th antenna of the BS. We then assume that the AoA of the user $k$ is determined by its spatial
TABLE I
PARAMETER VALUES IF NOT OTHERWISE SPECIFIED

| Parameters | Values | Parameters | Values |
|------------|--------|------------|--------|
| $\sigma^2$ [dB] | $-113$ | $\Delta$ | $\frac{\pi}{6}$ |
| $m$ | $100$ | $\varepsilon$ | $10^{-5}$ |
| $\xi$ | $0.05$ | $N$ | $8$ |
| $K_t$ | $3$ | $D_s$ [byte] | $32$ |

location, which is uniformly distributed in $(0, 2\pi]$. In addition, for a given channel realization, since the algorithm can fail, we define the sum spectral efficiency as

$$R_s = \begin{cases} \sum_{k=1}^{K_t} w_k R_k(u) + \sum_{k=K_t+1}^{K_t+K_s} w_k D_s \frac{\delta_k}{\delta_k}, & \text{if } u \text{ is feasible} \\ 0, & \text{otherwise} \end{cases}$$

where $w_k$ is the weight allocated to a user $k$. If the algorithm fails or the precoding vector is infeasible, we set the sum spectral efficiency for a given channel realization to zero. Furthermore, even though the algorithm performs well, since baseline schemes (e.g., MRT and RZF methods) may not satisfy the latency requirement, we modify the sum spectral efficiency as

$$R_s = \sum_{k=1}^{K_t} w_k R_k(u) + \sum_{k=K_t+1}^{K_t+K_s} p_{c,k} w_k D_s \frac{\delta_k}{\delta_k}$$

(50)

where $p_{c,k}$ is the binary variable, which indicates that the latency requirement of user $k$ is satisfied if $p_{c,k} = 1$; otherwise, $p_{c,k} = 0$. We denote the ergodic sum spectral efficiency as $\bar{R}_s = \mathbb{E} [R_s]$.

Unless otherwise specified, values of simulation parameters presented in Table I are used.

In this simulation, we compare the proposed algorithm with following methods:

- **Infinite-GPI**: when we solve the optimization problem in (24), we obtain the optimal precoding vector by using the first-order optimality conditions in (25) and (28) with $m = \infty$.
  However, the spectral efficiency of delay-constrained users with finite blocklength $m$ is still used when checking latency requirements of delay-constrained users.

- **MRT**: this method is a MRT precoding [35]. The precoding vector of user $k$ is the same as $u_k = h_k$. In this method, we assume $\tilde{\gamma}_k = \gamma_k$. 
RZF: this method is a RZF precoding [36]. The precoding vector of user \( k \) is given by

\[
\mathbf{u}_k = \left( \mathbf{H}_c \mathbf{H}_c^H + \frac{\sigma^2}{P} \mathbf{I}_N \right)^{-1} \mathbf{H}_c^H \tag{51}
\]

where \( \mathbf{H}_c = [\mathbf{h}_1, \cdots, \mathbf{h}_{K_t}, \mathbf{h}_{K_t+1}, \cdots, \mathbf{h}_{K_t+K_s}] \in \mathbb{C}^{N(K_t+K_s) \times 1} \). Suppose that \( \hat{\gamma}_k = \gamma_k \).

A. Performance analysis of DCTU-MIMO networks without IR-HARQ

In this subsection, we analyze the ergodic sum spectral efficiency depending on the network parameters. Note that the latency requirement \( \delta_k \) is the requirement for the spectral efficiency, not the actual data rate, obtained from the spectral efficiency by multiplying with the bandwidth. Hence, the actual latency requirement is \( \delta_k \) divided by the bandwidth. For example, \( \delta_k = 500 \) means the actual latency requirement is 0.1ms when the bandwidth is 5MHz.

Figure 2 presents the ergodic sum spectral efficiency \( \bar{R}_s \) as a function of \( P/\sigma^2 \) with \( K_t = 3, K_s = 2, \delta_4 = 250, \) and \( \delta_5 = 450 \) for different values of \( N \) and \( m \). Here, \( \bar{\gamma}_4 = 2.38, \bar{\gamma}_5 = 1.35, w_1 = w_2 = w_3 = 1, \) and \( w_4 = w_5 = 3 \). From this figure, we can know that the Delay-GPI outperforms MRT and RZF methods. This is because, since two baseline schemes cannot satisfy the latency requirement of delay-constrained users, the spectral efficiencies of delay-constrained users can be excluded. In addition, the Delay-GPI is much better than the Infinite-GPI at low \( P/\sigma^2 \). This is because, as the principal eigenvector of the Infinite-GPI is obtained by the first-order optimality condition with \( m = \infty \), the obtained...
principal eigenvector is not optimal and can be infeasible. However, at high $P/\sigma^2$, the Delay-GPI is slightly better than the Infinite-GPI because the probability of success of both algorithms is similar. We can also see that as the number of the BS antennas $N$ increases, the ergodic sum spectral efficiency increases due to larger diversity gain. Furthermore, we can know that as the blocklength $m$ increases, the ergodic sum spectral efficiency increases. This is because as $m$ increases, the effect of the channel dispersion decreases and the spectral efficiency of delay-constrained users increases. Hence, since the spectral efficiency of delay-constrained users increases with larger blocklength, the ergodic sum spectral efficiency increases by allocating more transmission power to the user that has well-conditioned channel gain.

Figure 3 presents the ergodic sum spectral efficiency $\bar{R}_s$ as a function of $P/\sigma^2$ with $N = 8$ and $K_t = 3$ for different values of $K_s$. In the case of $K_s = 1$, $\delta_4 = 250$. When $K_s = 2$, $\delta_4 = 250$ and $\delta_5 = 450$.

Figure 4 presents the ergodic sum spectral efficiency $\bar{R}_s$ as a function of $P/\sigma^2$ with $N = 8$, and $K_t = 3$ for different values of the number of delay-constrained users $K_s$. Here, when $K_s = 1$, $\delta_4 = 250$, $\tilde{\gamma}_4 = 2.38$, $w_1 = w_2 = w_3 = 1$, and $w_4 = 3$. On one hand, when $K_s = 2$, $\delta_4 = 250$, $\delta_5 = 450$, $\tilde{\gamma}_4 = 2.38$, $\tilde{\gamma}_5 = 1.35$, $w_1 = w_2 = w_3 = 1$, and $w_4 = w_5 = 3$. From this figure, we can see that as the number of delay-constrained users $K_s$ increases, the ergodic sum spectral efficiency decreases. This is because as $K_s$ increases, within limited total transmission power, more transmission power is allocated to delay-constrained users that may have ill-conditioned channel gain to satisfy the latency requirement of delay-constrained users. Therefore, more transmission power cannot be allocated to the user that has well-conditioned channel gain.

Figure 5 presents the ergodic sum spectral efficiency $\bar{R}_s$ as a function of $P/\sigma^2$ with $N = 8$,
Fig. 4. Ergodic sum spectral efficiency $\bar{R}_s$ as a function of $P/\sigma^2$ with $N = 8$, $K_t = 3$, $K_s = 2$, $\delta_4 = 250$, $\delta_5 = 450$, $\tilde{\gamma}_4 = 2.38$, and $\tilde{\gamma}_5 = 1.35$ for different values of $w_k$.

$K_t = 3$, $K_s = 2$, $w_1 = w_2 = w_3 = 1$, $\delta_4 = 250$, $\delta_5 = 450$, $\tilde{\gamma}_4 = 2.38$, and $\tilde{\gamma}_5 = 1.35$ for different values of the weight allocated to delay-constrained users, i.e., $w_4$ and $w_5$. From this figure, we can know that when $w_4 = w_5 = 3$, the Delay-GPI is better than other methods in general power regime. We can also see that as $w_4$ and $w_5$ decreases (e.g., from $w_4 = w_5 = 3$ to $w_4 = w_5 = 1$), even though the difference between the Delay-GPI and other methods decreases, the Delay-GPI is better than other methods in most power regime. However, when $P/\sigma^2$ is small (e.g., $P/\sigma^2 < 2.5$dB), the MRT and RZF methods are slightly better than the Delay-GPI. This is because the definition of the sum spectral efficiency is different. Specifically, if the Delay-GPI fails, the sum spectral efficiency will be zero. On the other hand, the MRT and RZF methods achieve the sum spectral efficiency in (50) regardless of the algorithm failure (i.e., even when the latency constraint is not satisfied). For example, when $P/\sigma^2 = 0$dB, the sum spectral efficiency of the Delay-GPI becomes zero with the algorithm failure probability of 0.3. On the other hand, for the MRT and RZF methods, the sum of spectral efficiencies of delay-tolerant users is always achieved even if the delay-constrained users do not satisfy the latency requirement with the probabilities $p_{c,4} = 0.1$ and $p_{c,5} = 0.26$ in the MRT case. From these result, we can know that when a low power regime, even though the ergodic sum spectral efficiency of the Delay-GPI can be lower than that of the MRT and RZF methods, the Delay-GPI has the advantage of serving all users because it satisfies the latency constraint of the delay-constrained users that MRT and RZF methods may not satisfy.
B. Performance analysis of DCTU-MIMO networks with IR-HARQ

In this subsection, we analyze the effect of transmission rounds and angular spread on the ergodic sum spectral efficiency. We first introduce methods to compare with the HARQ-GPI.

- Perfect-GPI: the Perfect-GPI is assumed that a BS knows the perfect CSI of total transmission rounds in advance. This algorithm is designed to solve the following optimization:

\[
\begin{align*}
\text{maximize} & \quad \sum_{t=1}^{T} \sum_{k=1}^{K_t} R_{k,t}(\mathbf{u}_{IR}) \\
\text{subject to} & \quad \sum_{t=1}^{T} R_{k,t}(\mathbf{u}_{IR}) \geq \frac{D_s}{\delta_k}, \quad k \in \mathcal{K}_s. \quad (52)
\end{align*}
\]

The first-order optimality condition of the problem in (52) satisfies when

\[
\mathbf{A}^{IR}(\mathbf{u}_{IR}, \lambda) \mathbf{u}_{IR} = \phi^{IR}(\mathbf{u}_{IR}, \lambda) \mathbf{B}^{IR}(\mathbf{u}_{IR}, \lambda) \mathbf{u}_{IR}. \quad (53)
\]

In (53), \(\mathbf{A}^{IR}(\mathbf{u}_{IR}, \lambda)\) and \(\mathbf{B}^{IR}(\mathbf{u}_{IR}, \lambda)\) are given by

\[
\begin{align*}
\mathbf{A}^{IR}(\mathbf{u}_{IR}, \lambda) &= \prod_{t=1}^{T} \prod_{k=1}^{K_t} \mathbf{u}_{IR}^H D_k(t) \mathbf{u}_{IR} \prod_{k=K_t+1}^{K_t+K_s} \left\{ \prod_{t=1}^{T} (\mathbf{u}_{IR}^H D_k(t) \mathbf{u}_{IR})^{1-f_{k,t}(\tilde{\gamma}_k(t))} \right\}^\lambda_k \\
&\times \left\{ \sum_{k=1}^{K_t} \sum_{t=1}^{T} 2D_k(t) \mathbf{u}_{IR}^H D_k(t) \mathbf{u}_{IR} + \sum_{k=K_t+1}^{K_t+K_s} \sum_{t=1}^{T} \frac{2\lambda_k (1-f_{k,t}(\tilde{\gamma}_k(t))) D_k(t)}{\mathbf{u}_{IR}^H D_k(t) \mathbf{u}_{IR}} \right\}, \quad (54)
\end{align*}
\]

\[
\begin{align*}
\mathbf{B}^{IR}(\mathbf{u}_{IR}, \lambda) &= \prod_{t=1}^{T} \prod_{k=1}^{K_t} \mathbf{u}_{IR}^H E_k(t) \mathbf{u}_{IR} \prod_{k=K_t+1}^{K_t+K_s} \left\{ \prod_{t=1}^{T} (\mathbf{u}_{IR}^H E_k(t) \mathbf{u}_{IR})^{1-f_{k,t}(\tilde{\gamma}_k(t))} \right\}^\lambda_k \\
&\times \left\{ \sum_{k=1}^{K_t} \sum_{t=1}^{T} 2E_k(t) \mathbf{u}_{IR}^H E_k(t) \mathbf{u}_{IR} + \sum_{k=K_t+1}^{K_t+K_s} \sum_{t=1}^{T} \frac{2\lambda_k (1-f_{k,t}(\tilde{\gamma}_k(t))) E_k(t)}{\mathbf{u}_{IR}^H E_k(t) \mathbf{u}_{IR}} \right\}. \quad (55)
\end{align*}
\]

In addition, Lagrangian multipliers \(\lambda\) are chosen so that \(\mathbf{u}_{IR}\) satisfies \(\sum_{t=1}^{T} \tilde{R}_{k,t}(\mathbf{u}_{IR}, \tilde{\gamma}_k(t)) \geq \frac{D_s}{\delta_k}, \quad k \in \mathcal{K}_s.\) The algorithm procedure is the same as Algorithm 1.

- MRT/RZF: we assume a BS only knows the perfect CSI of the current transmission round. Hence, we perform a MRT/RZF precoding separately for each transmission round and obtain spectral efficiencies of each transmission round. After the last transmission round, we sum spectral efficiencies of delay-tolerant users and delay-constrained users for total transmission rounds, respectively, and check the latency requirement for delay-constrained users.

Furthermore, to make a fairness comparison of MRT and RZF methods with Perfect-GPI and HARQ-GPI, the transmit power at each transmission round of MRT and RZF methods is \(P/T^2\), obtained by dividing the total transmit power \(P\) by the square of total transmission rounds \(T^2\).
Figure 5 presents the ergodic sum spectral efficiency $\bar{R}_s$ as a function of $P/\sigma^2$ with $K_t = 3$, $K_s = 2$, $\delta_4 = 210$, and $\delta_5 = 320$ for different values of $N$ and $T$. Here, we use $w_1 = w_2 = w_3 = 1$, and $w_4 = w_5 = 3$. In addition, in the case of $T = 2$, $\tilde{\gamma}_4(1) = \tilde{\gamma}_4(2) = 1.7$ and $\tilde{\gamma}_5(1) = \tilde{\gamma}_5(2) = 1.15$, while when $T = 3$, $\tilde{\gamma}_4(1) = \tilde{\gamma}_4(2) = \tilde{\gamma}_4(3) = 1.45$ and $\tilde{\gamma}_5(1) = \tilde{\gamma}_5(2) = \tilde{\gamma}_5(3) = 1.05$. From this figure, we can know that the Perfect-GPI outperforms the HARQ-GPI. This is because the Perfect-GPI knows the perfect CSI of total transmission rounds, while the HARQ-GPI knows the prefect CSI of the current transmission round and the channel covariance matrix of the future transmission round. However, the HARQ-GPI is greater than MRT and RZF methods because the two baseline methods do not have the channel knowledge of future transmission rounds. We can also see that as the total transmission rounds $T$ increase, the ergodic sum spectral efficiency of the HARQ-GPI increases because of the additional time diversity gain. However, the ergodic sum spectral efficiency of MRT and RZF methods decreases with $T$ at small $P/\sigma^2$. This is because, as $T$ increases, the transmit power at each transmission round decreases, therefore, at small $P/\sigma^2$, even though the opportunity to transmit increases, the latency requirements of delay-constrained users do not be satisfied. Furthermore, we can know that as the number of BS antennas increases, the ergodic sum spectral efficiency increases due to the additional antenna diversity gain.

Figure 6 presents the ergodic sum spectral efficiency $\bar{R}_s$ as a function of $P/\sigma^2$ with $N = 8$, $K_t = 3$, $K_s = 2$, $T = 3$, $\delta_4 = 210$, and $\delta_5 = 320$ for different values of the angular spread.
Fig. 6. Ergodic sum spectral efficiency $\bar{R}_s$ as a function of $P/\sigma^2$ with $N = 8$, $K_t = 3$, $K_s = 2$, $T = 3$, $\delta_4 = 210$, and $\delta_5 = 320$ for different values of $\Delta$.

Here, we use $\tilde{\gamma}_4(1) = \tilde{\gamma}_4(2) = \tilde{\gamma}_4(3) = 1.45$, $\tilde{\gamma}_5(1) = \tilde{\gamma}_5(2) = \tilde{\gamma}_5(3) = 1.05$, $w_1 = w_2 = w_3 = 1$, and $w_4 = w_5 = 3$. From this figure, we can see that as the angular spread $\Delta$ decreases, the difference between the ergodic sum spectral efficiency of the Perfect-GPI and that of the HARQ-GPI reduces. This is because as $\Delta$ decreases, the inter-beam interference between adjacent antennas decreases. Therefore, since the approximated spectral efficiencies at the future transmission rounds increase, more transmission power can be allocated to the current transmission round that has well-conditioned channel gain. However, since the channel correlation between adjacent antennas increases with decreasing $\Delta$, it can be difficult to obtain the time diversity enough (e.g., RZF and Perfect-GPI at high $P/\sigma^2$). From this result, when we do not know the future CSI of users, it is advantageous to have a smaller angular spread.

VII. CONCLUSION

In this paper, we consider a DCTU-MIMO network, where a BS equipped with multiple transmit antennas simultaneously serves delay-constrained users as well as delay-tolerant users. After analyzing the spectral efficiency of two types of users, we present the lower bound of the spectral efficiency of the delay-constrained user to make it tractable in the optimization. We then formulate the sum spectral efficiency maximization problem satisfying the latency constraint of the delay-constrained users. Using the GPI precoding algorithm, we propose a computationally efficient algorithm (Delay-GPI) that finds a principal precoding vector that satisfies the first-order
optimality condition of the optimization problem. Furthermore, by dividing a resource frame into multiple time slots, we consider a DCTU-MIMO network with the IR-HARQ scheme and propose the HARQ-GPI algorithm to obtain the principal precoding vector. Finally, we show that the proposed algorithms are better than the baseline schemes. We also see that the less transmission power is allocated to delay-constrained users, the greater the ergodic sum spectral efficiency is achieved.

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