**$B_s$ mixing phase and lepton flavor violation in supersymmetric SU(5)**

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Abstract. The connection between $B_s$ mixing phase and lepton flavor violation is studied in a supersymmetric SU(5) theory, in the light of the latest measurements thereof. The $O(1)$ phase, preferring a non-vanishing squark mixing, generically implies $\tau \rightarrow (e + \mu) \gamma$ and $\mu \rightarrow e\gamma$. In addition to the facts already well-known, stresses are put on the role of gaugino to scalar mass ratio at the GUT scale and the possible modifications due to Planck-suppressed non-renormalizable operators.

The $B_s$ mixing phase, denoted by $\phi_s$, is a theoretically clean observable, and one can make a close connection between its data and a theory possibly involving new physics. In the SM, one has $\phi_s \simeq -2\eta\lambda^2 \simeq -0.04$. On the experimental side, one could observe an interesting tendency in both data from DØ [1] and CDF [2], that each result appeared to favor a negative $O(1)$ value of $\phi_s$. This tendency came to stand out after the UTfit collaboration, based on the two experiments, reported that their global fit showed a $3.7 \sigma$ discrepancy of $\phi_s$ from its SM value [3]. This deviation, however, has decreased to $2.5 \sigma$ after they updated their analysis including newly available experimental information from DØ [4]. The latest constrained fit reported by HFAG [5], is away from the SM prediction by $2.4 \sigma$. Still, it is too early to draw a definite conclusion. If the difference solidifies, it should be a clean indication of a new source of CP violation.

A supersymmetric extension of the SM has potential new sources of flavor and/or CP violation in its soft supersymmetry breaking terms. It might be conceivable that one of them is revealing its existence through the above anomaly. We employ the notation of mass insertion parameters, written in the form of $(\delta^d_{ij})_{AB}$ with the generation indices $i, j = 1, 2, 3$ and the chiralities $A, B = L, R$. We do not only use their usual definition at the weak scale, but also borrow the same notation to specify an off-diagonal element of the soft scalar mass matrix at $M_{\text{GUT}}$, the unification scale [6]. Being a transition between the second and the third families, $B_s$-$\bar{B}_s$ mixing is naturally associated with $(\delta^d_{23})_{AB}$. Among the four possibilities, the $LR$ and the $RL$ mass insertions tend to cause an unacceptable change in $B \rightarrow X_s\gamma$ before they can give an appreciable modification to $B_s$-$\bar{B}_s$ mixing [7]. Therefore, we focus on $LL$ and $RR$ mixings in what follows.

In this work, we work with a grand unified theory (GUT). Since a single GUT multiplet contains both (s)quarks and (s)leptons, flavor transitions in the two sectors are related. Then, one immediately arrives at the conclusion that the new source of $b \leftrightarrow s$ transition, needed to account for $\phi_s$, generically implies lepton flavor violation (LFV). We wish to consider this scenario in a model independent fashion taking SU(5) as the unified gauge group.
Figure 1. LFV and $B_s$ mixing as functions of $x \equiv M_{1/2}/m_0^2$, for fixed $M_{1/2} = 180$ GeV.

If one has a perfect alignment between the mass eigenstates of quarks and leptons, $(\delta^d_{ij})_{RR}$ implies the transition of $l_i \rightarrow l_j$. However, this straightforward correspondence may be broken by the inclusion of non-renormalizable terms into the superpotential as a solution to the wrong quark–lepton mass relations of the lighter two families. With the assumption that the cutoff scale of the GUT is two orders of magnitude higher than $M_{\text{GUT}}$, one can nevertheless constrain $(\delta^d_{23})_{RR}$ using the combined mode, $\tau \rightarrow (e + \mu) \gamma$, exploiting the fact that the breakdown of $b-\tau$ alignment is suppressed by $\cos \beta$ [6].

Another point to note is that $(\delta^d_{23})_{RR}$ leads not only to $\tau \rightarrow \mu \gamma$ but also to $\mu \rightarrow e \gamma$ thanks to the radiative correction from the top Yukawa coupling to the scalar mass terms of $10$ [8]. The $\mu \rightarrow e \gamma$ constraint is particularly important for low $m_0$ [6].

When one uses a low energy hadronic process to constrain $\delta$ parameters at $M_{\text{GUT}}$, the running effect of soft mass terms results in an interesting behavior, illustrated in Fig. 1. Suppose that the $\delta$’s and the gaugino mass $M_{1/2}$ are fixed at $M_{\text{GUT}}$. Imagine that one increases $m_0$, the common diagonal entries of soft squark mass matrix at $M_{\text{GUT}}$. As $m_0$ increases, $(\Delta^d_{ij})_{AB} \equiv m_0^2 \times (\delta^d_{ij})_{AB}$ grows as well, thereby exerting more and more influence on low energy flavor violation such as $B_s$ mixing. At some point, however, squark loop effects begin to decouple as the squarks become too heavy. For $B_s$–$\bar{B}_s$ mixing, this is around $x \equiv M_{1/2}^2/m_0^2 = 1/12$. This gaugino to scalar mass ratio could be regarded as a condition for optimizing the sensitivity of a hadronic process to flavor non-universality at $M_{\text{GUT}}$ [6]. The importance of this observation is more pronounced when one tries to compare hadronic and leptonic constraints since the latter is monotonically weakened as $m_0$ is being raised.

We restrict ourselves to $LL$ and $RR$ mixings of down-type squarks. We consider three scenarios: the $LL$ scenario, the $RR$ scenario, and the $LL = RR$ scenario. It should be noted that we set an $LL$ insertion, unless it is a scanning variable, to a value generated by RG running from the supersymmetry breaking mediation scale $M_s$ down to $M_{\text{GUT}}$, where $M_s$ is taken to be the reduced Planck scale. These boundary conditions are given at $M_{\text{GUT}}$ with which we solve one-loop RG equations down to the weak scale. The constraint from each observable is depicted on the complex plane of a GUT scale mass insertion. As for $\phi_s$, we use the 90% confidence level (CL) region from HFAG [5],

$$\phi_s \in [-1.26, -0.13] \cup [-3.00, -1.88].$$

For concreteness, we assume that there is an exact quark–lepton flavor alignment. Regarding
Figure 2. Constraints on \((\delta_{23}^{d})_{LL}\). For \(\tau \rightarrow \mu \gamma\), the thin circle is an upper bound from the prospective branching ratio limit, \(10^{-8}\). For \(\mu \rightarrow e \gamma\), the thin circle shows the projected bound on the branching ratio, \(10^{-13}\). A yellow region is allowed by \(\Delta M_s\), given 30% uncertainty in the \(\Delta B = 2\) matrix element, and a cyan region is further consistent with \(\phi_s\). Of the two sides of the \(S^R_{CP}\) curve, the excluded one is indicated by thin short lines.

\(\tau \rightarrow \mu \gamma\), it is straightforward to translate their bounds presented below to a case with quark–lepton misalignment discussed above—interpret \(B(\tau \rightarrow \mu \gamma)\) as \(B(\tau \rightarrow (e + \mu) \gamma)\). This prescription is applicable to all the three scenarios considered here. As for \(\mu \rightarrow e \gamma\), barring accidental cancellations, a contour does not need a modification in the \(RR\) and \(LL = RR\) scenarios, while we do not have a systematic way to account for a misalignment in the \(LL\) scenario. We fix \(M_{1/2} = 180\) GeV, which makes the gluino mass be 500 GeV at the weak scale, and then try two different values of \(m_0 = 220\) GeV and 600 GeV, corresponding to the right-handed down-type squark masses of 500 GeV and 750 GeV at the weak scale, respectively. The former \(m_0\) results in a benchmark case often considered in the literature, and the latter \(m_0\) optimizes the sensitivity of neutral meson mixing to \(\delta_s\)’s at the GUT scale. We use \(\tan \beta = 5\). Other choices of parameters are considered in Refs. [6], which also explain other details.

First, the \(LL\) mixing scenario is shown in Figs. 2. Of the two figures, Fig. 2 (a) is for lower \(m_0\). For this \(m_0\), one recognizes that the supersymmetric contribution to \(B_s - \bar{B}_s\) mixing is not enough to fit the \(\phi_s\) data even if one allows for an \(O(1)\) mass insertion. The dotted contour lines tell us that a maximal alteration in \(\phi_s\) that can be expected is about 0.1. They reveal that the other experimental constraints are not the primary reason why the \(LL\) mixing scenario with lower \(m_0\) is inadequate for making an \(O(1)\) change in \(\phi_s\). The mixing is simply unable to make an enough difference, due to the dilution of squark mixing by gluino mass contribution in the course of RG running down to the weak scale. In Fig. 2 (b), one can find cyan regions that lead to \(\phi_s\) within its 90% CL intervals. They involve an \(O(1)\) size of \((\delta_{23}^{d})_{LL}\). However, the supersymmetric disturbance is great also in \(B \rightarrow X_s \gamma\), which excludes the bulk of a cyan zone. The disturbance in this decay mode grows with \(\tan \beta\) [9], as does that in \(S^R_{CP}\). In this scenario, discovery of LFV seems to be difficult at a super \(B\) factory or MEG.

The \(RR\) scenario is shown in Figs. 3. Comparing these figures with Figs. 2, one notices that an \(RR\) insertion gives more effect on \(B_s\) mixing than an \(LL\) insertion. This is because an \(LL\) insertion is induced by RG running from \(M_*\) down to the weak scale even in the \(RR\) scenario.
The presence of $(\delta_{d}^{d})_{LL}$ enhances the effect of $(\delta_{d}^{d})_{RR}$ on $B_{s} \rightarrow \overline{B_{s}}$ mixing, and $\phi_{d}$ can be easily pushed to its 90% probability region. For lower $m_{0}$, however, those regions favored by $\phi_{d}$ are excluded by the current bounds from $\tau \rightarrow \mu \gamma$ and $\mu \rightarrow e \gamma$ even for $\tan \beta$ as low as 5, as is shown in Fig. 3 (a). It seems to be hard to satisfy both $\phi_{d}$ and LFV with an RR insertion with low $m_{0}$. This should be contrasted with the LL scenario where LFV was not a major problem. Note that $\mu \rightarrow e \gamma$ occurs as well. This stems from the nonzero $(\delta_{d}^{d})_{RR}$ set as a boundary condition at $M_{GUT}$. This value is expected from the radiative correction from top Yukawa coupling and CKM mixing. Next, we switch to a higher value of $m_{0}$. Compared to Fig. 3 (a) with lower $m_{0}$, the case in Fig. 3 (b) needs a smaller size of mass insertion to give an enough contribution to $B_{s} \rightarrow \overline{B_{s}}$ mixing, to its phase in particular. In contrast, LFV is suppressed because of heavier sleptons. These two changes make it easier to fit $\phi_{d}$ with smaller LFV rates. However, an RR insertion is limited by $d_{n}$, the neutron EDM. A region allowed by $d_{n}$ and $\Delta M_{s}$ around the origin, is separated from the $\phi_{d}$ region. The band obaying $d_{n}$ can be rotated to overlap the cyan region by altering $(\delta_{2}^{d})_{LL}$ at $M_{GUT}$, since $d_{n}$ is influenced through the combination of $\text{Im}(\delta_{2}^{d})_{LL} \phi_{d}$. The presented plots are valid for the phase of $(\delta_{2}^{d})_{LL}$ equal to arg$(-V_{ts}^{*}V_{tb})$. Note that $B \rightarrow X_{s}\gamma$ is not very tight. This is because the supersymmetric amplitude does not interfere with the SM one. LFV and $d_{n}$ are enhanced for high tan$\beta$. Therefore, lowering tan$\beta$ helps satisfy LFV and $d_{n}$ as well as $\phi_{d}$. One can find that the region preferred by $\phi_{d}$ involves the $\tau \rightarrow \mu \gamma$ rate in the vicinity of the current upper limit. For example, fitting the central value of $\phi_{d}$ causes $B(\tau \rightarrow \mu \gamma)$ to be around $10^{-7}$ which is already ruled out by the Belle data. The area still surviving could be explored by current and future experiments. The magnitude of mass insertion accessible with the sensitivity of $10^{-8}$, attainable at a super $B$ factory, is depicted by a thin circle inside the current upper bound. The cyan region is also expected to bring about $\mu \rightarrow e \gamma$ at a rate that can be probed by MEG.

The preceding results are based on the supposition that the quark and the lepton mass eigenstates are aligned to each other. With following modifications, they can be applied to cases where this alignment is disturbed by the Planck-suppressed non-renormalizable operators.
incorporated to reproduce masses of the lighter quarks and leptons. Interpret the branching ratio of $\tau \rightarrow \mu \gamma$ as that of $\tau \rightarrow (e + \mu) \gamma$. Obtain a new thick $\tau \rightarrow (e + \mu) \gamma$ ring by expanding the old thick $\tau \rightarrow \mu \gamma$ ring, in order to encompass the events of $\tau \rightarrow e \gamma$. For this, multiply the old radius by 1.9. Leave the thin $\tau \rightarrow (e + \mu) \gamma$ (namely former $\tau \rightarrow \mu \gamma$) circles untouched. The $\mu \rightarrow e \gamma$ contours should be kept as they are in the $RR$ scenario, and discarded in the $LL$ scenario. The net consequence of these operations is that the current upper bound from $\tau \rightarrow \mu \gamma$ has been relaxed by the factor of 1.9 and $\mu \rightarrow e \gamma$ has been disconnected from the $LL$ mixing. Even after these modifications, the upper bound from a LFV process is relaxed at most by the factor of two only in the $RR$ case. In the $LL$ case, these changes do not make a big difference since LFV constraints were not very important from the beginning.

The above restrictions on the $RR$ insertion, with slight modifications, can be applied to a popular scenario where the soft terms are flavor-blind at $M_s$ and large neutrino Yukawa couplings are the only source of flavor violation apart from the CKM mixing. In this case, the slepton mass matrix receives additional contribution running below $M_{\text{GUT}}$. Because of this, given the same $\delta$ at $M_{\text{GUT}}$, LFV rates are higher than in Fig. 3, and therefore one obtains tighter LFV bounds.

The last scenario where $(\delta_{23}^{d})_{LL} = (\delta_{23}^{d})_{RR}$ at $M_{\text{GUT}}$, is shown in Figs. 4. Comparison of Figs. 4 and Figs. 3 shows that the conflict between LFV and $\phi_s$ has been reduced here. Simultaneous presence of $LL$ and $RR$ mixings reinforces contribution to the $B_s$ mixing even with a smaller size of each insertion about 0.1, while the LFV bounds remain almost the same. Nonetheless, the LFV data shows a disagreement with $\phi_s$ for lower $m_0$, which grows severer for higher tan$\beta$. This is illustrated in Fig. 4 (a). Again, raising $m_0$ to the optimal point, one can enhance supersymmetric effects on $B_s$-$\bar{B}_s$ mixing while suppressing LFV. Fig. 4 (b) shows regions well inside the LFV bounds which lead to $\phi_s$ in perfect agreement with the latest fit. Part of those regions can satisfy $S_{CP}^\phi$ and $d_n$ as well. An area preferred by $\phi_s$ gives rise to $B(\tau \rightarrow \mu \gamma)$ around $10^{-8}$ or larger. The rate of $\mu \rightarrow e \gamma$ expected from the same area is around the sensitivity of MEG. In this scenario, a higher tan$\beta = 10$ is viable as well [6].
We summarize. We have examined three patterns of \((\delta_{23}^d)^{LL}\) and \((\delta_{23}^d)^{RR}\): \(LL\), \(RR\), and \(LL = RR\). For reconciling \(\phi_s\) with LFV, it greatly helps to choose the optimal value of the GUT scale gaugino to scalar mass ratio, in all these three scenarios. It appears that the most adequate to fit the current value of \(\phi_s\) is \(LL = RR\) among the three scenarios. The main difficulties for this purpose are \(B \to X_s \gamma\) and \(S^{\phi K}_{CP}\) in the \(LL\) scenario, and LFV and the neutron EDM in the \(RR\) scenario. Inclusion of Planck-suppressed non-renormalizable terms for fixing the quark–lepton mass relations, in general, affects a LFV bound. This alteration can be estimated by weakening a \(\tau \to \mu \gamma\) bound to that from \(\tau \to (e + \mu) \gamma\). In the two scenarios involving an \(RR\) mixing, this reduces the tension between LFV and \(\phi_s\) to some extent but not completely. In all cases, low \(\tan \beta\) loosens \(B \to X_s \gamma\), \(S^{\phi K}_{CP}\), and \(d_n\) as well as LFV, providing for more room to accommodate \(\phi_s\).

Acknowledgments

The author thanks P. Ko and Masahiro Yamaguchi for the pleasant collaboration. He acknowledges Research Grants funded jointly by the Italian Ministero dell’Istruzione, dell’Università e della Ricerca (MIUR), by the University of Padova and by the Istituto Nazionale di Fisica Nucleare (INFN) within the Astroparticle Physics Project and the FA51 INFN Research Project. This work was supported in part by the European Community Research Training Network UniverseNet under contract MRTN–CT–2006–035863.

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