Current and magnetic field dependences of a superconducting coplanar waveguide resonator

H. Kurokawa*, F. Nabeshima, and A. Maeda

Department of Basic Science, University of Tokyo, 3-8-1, Komaba, Meguro-Ku, Tokyo 153-8902, Japan

*E-mail: kurokawa@maeda1.c.u-tokyo.ac.jp

Received November 1, 2018; accepted December 17, 2018; published online February 11, 2019

We fabricated a superconducting coplanar waveguide resonator with leads for dc bias, which enables an ac conductivity measurement under dc bias. The current and magnetic field dependences of resonance properties were measured, and hysteretic behavior was observed as a function of the dc driving current. The observed shifts in the inverse of the quality factor, \( Q^{-1} \), and the center frequency, \( f_0 \), were explained by considering both motion of vortices and suppression of the order parameter by the dc current. Our investigation revealed that the strongly pinned vortices have a small influence on \( f_0 \) while they still affect \( Q^{-1} \). Our results indicate that the accurate understanding of the dynamics of driven vortices is indispensable for a control of the resonance properties with a high precision.

1. Introduction

Superconducting coplanar waveguide resonators become essential components in advanced measurement devices. They are integrated into various circuits for the manipulation of quantum bits\(^1,2\) used for studies of circuit quantum electrodynamics\(^3,4\) and for detection of photons from space\(^5\). For further advanced applications and versatility improvement, various techniques for manipulation of the resonance characteristics have been developed\(^6-14\).

Introduction of a dc bias current, \( I_{dc} \), to the resonator or application of a magnetic field were suggested to tune the resonance properties; the underlying mechanisms have been extensively investigated. The \( I_{dc} \) decreases the center frequency, \( f_0 \), of the resonator owing to the suppression of the order parameter of the superconductor\(^15,16\). In order to control \( f_0 \) by the \( I_{dc} \), several architectures were proposed\(^12,13\). For example, Ref. 12 proposed a half wavelength filter, and investigated the effect of the \( I_{dc} \) on the resonance properties. Li and Kycia fabricated a more adaptable quarter-wavelength filter incorporated into a resonator\(^13\). Operation of resonators under magnetic fields is required in hybrid systems consisting of ultra-cold atoms, molecules, single electron, and superconducting resonator\(^17-19\). The application of magnetic fields causes an additional loss due to the vortices penetrating the resonator, and induces hysteresis in the magnetic field dependences of \( f_0 \) and the quality factor, \( Q \)\(^11,20,21\), leading to the performance degradation of the resonator. In order to minimize these undesirable effects, fabrication of a slot or antidots was proposed. It can reduce the loss by trapping the vortices inside\(^20\).

In addition, if we operate the resonator under both driving current and the magnetic field, the vortices are subject to the driving force by the current, so that the resonance properties would exhibit complex behaviors. The complex nonequilibrium motions of vortices emerge owing to the interactions between driven vortices and pinning sites\(^22,23\). Therefore, the understanding of the influence of the \( I_{dc} \) under a magnetic field is of interest\(^24,25\) in fundamental physics as well as in applied physics. However, to the best of our knowledge, there are no reports about the resonance characteristics in the presence of both \( I_{dc} \) and magnetic fields perpendicular to the resonator. Therefore, we investigated the current and magnetic field dependences of \( Q \) and \( f_0 \) of a superconducting resonator.

In this study, we fabricated a superconducting coplanar waveguide resonator with leads for the introduction of the dc bias current. The dc current dependences of \( f_0 \) and \( Q^{-1} \) were measured with and without a magnetic field perpendicular to the resonator. A hysteretic behavior was observed in the current cycle. In particular, the behavior of \( Q^{-1} \) was complex, in contrast to that of \( f_0 \). These behaviors were explained considering the motion of vortices under the pinning. Our results confirmed that it is essential to consider the dynamics of vortices for the precise control of the resonance characteristics of the superconducting resonator.

2. Experimental method

The \( \lambda/2 \) superconducting coplanar transmission-line resonator was fabricated using a Nb film. The film was deposited on a \( 5 \times 5 \times 0.5 \text{ mm}^3 \) MgO substrate by the radio-frequency sputtering. The deposition rate was monitored by a quartz etching technique. The patterned film was etched by reactive ion etching using CF\(_4\) gas.

Figure 1(a) shows our superconducting resonator. The total length of the resonator was 8 mm, which was designed so that \( f_0 \) is 8 GHz. The width of the center conductor was 10 \( \mu \text{m} \), while the gap between the center conductor and ground was 5 \( \mu \text{m} \) [Fig. 1(b)], so that the characteristic impedance of the resonator was 50 \( \Omega \). The resonator was capacitively coupled to the input and output ac feeds through a gap of 4 \( \mu \text{m} \), with additional dc leads at the ends. The resonator couples to external circuits through the dc leads; these additional couplings limit \( Q \). The length of the dc lead was equal to \( \lambda/4 \); open stubs were attached at the root of the lead. They serve as a \( \lambda/2 \) bandstop filter, reducing the couplings to the external circuits, maintaining a relatively high \( Q \) despite the leads\(^13\). At 3 K, we realized a \( Q \) of 0 mA; 0 G of 9000, and \( f_0 \) (0 mA; 0 G) was approximately 7.9 GHz.

The fabricated resonator was connected to a printed circuit board by aluminum bonding wires; the circuit board was attached to the semi-rigid coaxial cables with subminiature A (SMA) connectors and subminiature P (SMP) connectors. The device was inserted into a PPMS (Quantum Design), and
was cooled down to 3 K. The magnetic field was applied perpendicular to the film. All the measurements were performed under the field-cooled conditions.

We fabricated three resonators with the same shape; they exhibited the qualitatively similar current dependences. Therefore, we present the results of only one resonator in the next section.

3. Results and discussions

We measured the dc $I$–$V$ characteristics of the resonator with the standard dc four-probe method; the critical current, $I_{dc}$, was 70 mA at 3 K, and 30 G, as shown in Fig. 2(a).

Figure 2(b) shows the resonance spectra, $S_2$, measured with the increase of $I_{dc}$ at 3 K, and 30 G. The current dependence of the spectra was clearly observed. The asymmetry in the spectra may originate from the nonlinearity due to the coupling to the dc leads.\(^\text{26}\) We fitted these spectra by Lorentzian functions and obtained the changes in $Q^{-1}$ and $f_0$ as a function of $I_{dc}$, $Q^{-1}$ and $f_0$ also exhibited hysteretic behaviors, depending on the current history.

Figures 3(a) and 3(b) show the current and magnetic field dependences of $Q^{-1}$ and $f_0$ with the increase of $I_{dc}$. As shown in Fig. 3(c), in the 1st current sweep, the $I_{dc}$ was swept from 2.4 mA to slightly below the depairing current, which was 78 mA. In the 2nd current sweep, the $I_{dc}$ was increased from 2.4 mA in the same manner. We performed measurements at 0, 20, 30, and 40 G. The resonance peak almost disappeared at higher magnetic fields.

With the increase of the magnetic field, more vortices penetrated the resonator, leading to the increase in $Q^{-1}$. Simultaneously, $f_0$ decreased with the increase in the magnetic field, consistent with the results by Ref. 21. In addition, the hysteretic shifts in $Q^{-1}$ and $f_0$ became larger with more vortices. Qualitatively similar hysteretic behaviors were observed at these magnetic fields; therefore, we focus on the current dependence at 30 G. In the 1st sweep, $Q^{-1}$ gradually increased with the $I_{dc}$ until 35 mA and then decreased, while $f_0$ monotonously decreased. In the 2nd sweep, $Q^{-1}$ slightly decreased in the lower-current regime and increased for $I_{dc}$ over 60 mA, while $f_0$ continued to decrease. In addition, by comparing the data of the 1st and 2nd sweeps, $Q^{-1}$ was smaller by $2.8 \times 10^{-4}$, while $f_0$ was larger by 0.26 MHz at 0 mA in the 2nd sweep.

We discuss the current dependences of $Q^{-1}$, and then discuss $f_0$ in terms of vortices. The change in $Q^{-1}$ is directly related to the change in the resistance of the resonator, $\Delta R = \Delta(Q^{-1})\omega_0 L$, where $\omega_0$ is the resonant angular frequency, and $L$ is the inductance of the resonator. Several factors contribute to $Q^{-1}$, dielectric loss, $Q_{\text{die}}$, conductor loss, $Q_{\text{con}}$, radiation loss, $Q_{\text{rad}}$, loss in the external circuits, $Q_{\text{ex}}$, and loss due to the vortices, $Q_v$. Assuming the $I_{dc}$ affects only the vortices, the changes in $Q^{-1}$ correspond to the changes in the resistivity of the vortices, and the increase in $Q^{-1}$ in the lower-current regime implies that the ac response of vortices became more resistive. This indicates that the vortices experienced a weaker pinning force owing to the $I_{dc}$. When we continued to increase the $I_{dc}$, the vortices could gradually move to more stable, strongly pinned sites. Therefore, the strongly pinned vortices responded to the ac current less resistively, leading to the decrease in $Q^{-1}$ above 40 mA.

![Fig. 1.](image) (Color online) Fabricated coplanar waveguide resonator with dc leads. (a) Optical image. The bright area is a Nb film. (b) Cross section. (c) Schematic representation of the resonator pattern.

![Fig. 2.](image) (Color online) (a) $I$–$V$ characteristics of the Nb film measured at 3 K, and 30 G. The blue dashed line is a guide for the eye. (b) Resonance spectra of the 7.9 GHz resonator measured under various dc currents at 3 K, and 30 G.
According to Fig. 2(a), macroscopic motion of vortices occurs over 70 mA owing to a dc electric field induced by the translational motion of vortices. Therefore, the increase in $Q^{-1}$ over 70 mA in the 2nd sweep may be relevant to the translational motion of vortices. Indeed, such motions of vortices can reduce the pinning force, leading to the more resistive response to the ac current. Figure 3(d) summarizes the proposed motion for a vortex based on the above experiments. It is worth noting that simple models of the ac response cannot explain this gradual increase in $Q^{-1}$. Gittleman and Rosenblum formulated the ac response of a vortex based on the phenomenological equation of motion of a vortex: $\eta v + kv = f_{ac} \phi_0$, where $\eta$ is the viscous drag coefficient, and $\phi_0$ is the flux quantum. The pinning constant, $k$, is the curvature of the pinning potential, $U_p$. If we apply $I_{ac}^0$, this model predicts that $U_p$ and $k = \frac{d^2U_p}{dx^2}$ becomes zero, as $U_p \to 0$ at $I_{ac}^0$. The vanishment of the pinning potential should lead to a purely resistive response, i.e., flux flow. Reference 27 also showed a sharp increase in the resistivity at $I^0_{dc}$ by a numerical calculation of the equation of motion. Instead, we observed a gradual increase rather than such a significant sudden increase around $I_{ac}^0$. A more elaborate theory is indispensable for a further understanding of the pinning and ac response of the dc driven vortices.

In addition to the changes mentioned above, we briefly discuss the subtle decrease in $Q^{-1}$ in the 2nd sweep. We considered that the anharmonicity in $U_p$ can explain this decrease. If $U_p$ was harmonic everywhere, $k$ would be unchanged irrespective of the position of the vortex within a pinning site, and $Q^{-1}$ would be independent of the $I_{ac}$. On the other hand, if the potential is anharmonic, $k$ depends on the displacement of the vortex. In such cases, the $k$ and pinning force can become larger as the vortex moves from its initial position. The larger pinning force reduces the ac resistance of the vortex, leading to the decrease in $Q^{-1}$ in the 2nd sweep.

Next, we consider the current dependence of $f_0$. With the increase of the dc current, $f_0$ continued to decrease in a parabolic manner. The observed parabolic current dependence of $f_0$ could be attributed to the current dependence of the order parameter of the superconductor. From the Ginzburg–Landau equation, in thin films, current suppresses the order parameter as $|\psi|^2 \propto 1 - C \xi^2$, where $C$ is a constant dependent only on temperature. The imaginary part of the complex conductivity, $\sigma_2$, is proportional to $|\psi|^2$. The change in the kinetic inductance, $\Delta L_k$, can be expressed as $\Delta L_k \propto \sigma_2(f) - \sigma_2(0) \propto \xi^2$; $\Delta L_k$ is equal to $-2(\Delta f/f_0)L$. Therefore, the shift in the center frequency, $\Delta f$, is proportional to the square of the current density, yielding the parabolic shift in $f_0$.

In addition, $f_0$ slightly increased after the 1st sweep. In order to explain the hysteretic behavior of $f_0$, we calculated the change in the magnetic penetration depth, $\Delta \lambda = \lambda(I_{dc}^0 (mA); B) - \lambda(0 \text{mA}; 0 B)$. According to Ref. 29, $L$ is a sum of the magnetic inductance, $L_{m}$, and kinetic inductance of a superconductor, $L_k$, which can be represented as $L_k = \mu_0(\lambda_t^2/\omega d)g$, where $d$ and $w$ are the thickness and width of the center conductor, respectively, and $g$ is the geometric factor. We obtained: $\Delta \lambda = (\omega d/2\lambda \mu_0)\Delta L_k$ using the relation $\lambda = 1.05\sqrt{(\rho(T_c)/T_c) \times 10^{-3} \text{ m}}$ and $\lambda(t) = \lambda_0/\sqrt{1 - t^2}$, where $t$ is the reduced temperature.

Figure 4(a) shows the current dependence of $\Delta \lambda$ at each magnetic field. We also plotted $\Delta \lambda$ of the 2nd sweep as a function of $I_{dc}^0$ in Fig. 4(b); their slopes are shown in Fig. 4(c). $\Delta \lambda$ ($I_{dc}^0$) of the 2nd sweep increased linearly; the slopes were almost unchanged at these magnetic fields. The independence on the magnetic inductance suggests that only the suppression of $\psi$ by the dc current contributed to $\Delta \lambda(I_{dc}^0)$ in the 2nd sweep. On the other hand, the hysteresis between the 1st and 2nd sweeps should originate from the motion of vortices. A weakly pinned vortex is easily movable in a pinning potential. Therefore, the ac field easily penetrates the superconductor, leading to a longer $\lambda$. In contrast, strongly pinned vortices prevent the penetration of an ac field, which leads to a shorter $\lambda$. As discussed above, vortices are considered to be strongly pinned after the 1st sweep, which is consistent with the shorter $\lambda$ after the 1st sweep at each magnetic field. This model cannot explain the result of 40 G in which the 1st sweep almost coincided with the 2nd sweep. It may be due to the asymmetry of the spectra or complexities.

Fig. 3. (Color online) Current and magnetic fields dependences of (a) $Q^{-1}$ and (b) $f_0$ at 3 K. The squares represent the data in the 1st current sweep, while the circles represent those in the 2nd current sweep. (c) Schematic diagram for the current sweep. (d) Schematic diagram for the motion of the vortex under the bias current. A blue circle represents a vortex, while a gray line represents a pinning potential.
of pinning potential, which need further experiments in other samples to conclude.

4. Conclusions

We fabricated the superconducting coplanar waveguide resonator with dc leads, and measured the current and magnetic field dependences of the resonance characteristics, $Q^{-1}$ and $f_0$. We observed the current dependences of $Q^{-1}$ and $f_0$ together with hysteresis behaviors, considered to originate from both changes in the vortex configuration and suppression of the order parameter. The application of the $I_{dc}$ caused local motion of the vortices, which experienced a weaker pinning force, and then were gradually trapped to the more stable pinning sites. Therefore, $Q^{-1}$ increased in the lower-current regime, while in the higher-current regime, it decreased in the 1st sweep. The current dependence of $f_0$ was mainly attributed to the change of $\psi$ in the superconductor, and was insensitive to the magnetic field in the 2nd sweep. The device is sensitive to local environment around vortices. Our experiment can be a new method to investigate the interaction between pinning sites and vortices through microwave responses under $I_{dc}$. In addition, these results are valuable for designs of future devices and quantum measurements, which incorporate both current bias and magnetic field perpendicular to superconducting resonators.

Acknowledgments

This study was supported by the JSPS KAKENHI Grant Number JP16H00795.

1) M. A. Sillanpaa, J. I. Park, and R. W. Simmonds, Nature 449, 438 (2007).
2) J. Clarke and F. K. Wilhelm, Nature 453, 1031 (2008).
3) A. Blais, R.-S. Huang, A. Walraff, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. A 69, 1 (2004).
4) A. Walraff, D. J. Schuster, A. Blais, L. Frunzio, R. S. Huang, J. Majer, S. Kumar, S. M. Girvin, and R. J. Schoelkopf, Nature 431, 162 (2004).
5) J. Q. You and F. Nori, Nature 474, 589 (2012).
6) B. A. Mazin, P. K. Day, H. G. LeDuc, A. Vayonakis, and J. Zmuidzinas, Proc. SPIE 4849, 283 (2002).
7) P. K. Day, H. G. LeDuc, B. A. Mazin, A. Vayonakis, and J. Zmuidzinas, Nature 425, 817 (2003).
8) J. Zmuidzinas and P. L. Richards, Proc. IEEE 92, 1597 (2004).
9) K. O. Osborn, J. A. Strong, A. J. Sirois, and R. W. Simmonds, IEEE Trans. Appl. Supercond. 17, 166 (2007).
10) M. Sandberg, C. M. Wilson, F. Persson, G. Johansson, V. Shumeiko, T. Duty, and P. Delsing, Appl. Phys. Lett. 92, 203501 (2008).
11) J. E. Healey, T. Lindstrom, M. S. Colclough, C. M. Muirhead, and A. Y. Tzalenchuk, Appl. Phys. Lett. 93, 043513 (2008).
12) F. Chen, A. J. Sirois, R. W. Simmonds, and A. J. Rimberg, Appl. Phys. Lett. 98, 132509 (2011).
13) S. X. Li and J. B. Kycia, Appl. Phys. Lett. 102, 242601 (2013).
14) Y. Hao, F. Rouzinol, and M. D. LaHaye, Appl. Phys. Lett. 105, 222603 (2014).
15) J. Gittelman, B. Rosenblum, T. E. Seidei, and A. W. Wicklund, Phys. Rev. 137, A527 (1965).
16) M. Lüftenegger, J. Appl. Phys. 81, 301 (1997).
17) J. Verdu, H. Zoubi, C. Koller, J. Majer, H. Ritsch, and J. Schmiedmayer, Phys. Rev. Lett. 103, 043603 (2008).
18) P. Rabl, D. DeMille, J. M. Doyle, M. D. Lukin, R. J. Schoelkopf, and P. Zoller, Phys. Rev. Lett. 97, 030503 (2006).
19) P. Bushev et al., Eur. Phys. J. D 63, 9 (2010).
20) C. Song, M. P. DeFeo, K. Yu, and B. L. T. Plourde, Appl. Phys. Lett. 95, 232501 (2009).
21) D. Bohner, T. Gaber, M. Kemmler, D. Koelle, R. Kleiner, S. Wünsch, and M. Siegel, Phys. Rev. B 86, 014517 (2012).
22) J. I. Gittelman and B. Rosenblum, J. Appl. Phys. 39, 2617 (1968).
23) A. M. Campbell and J. E. Evets, Adv. Phys. 50, 1249 (1972).
24) P. L. Doussal and T. Giamarchi, Phys. Rev. B 57, 11356 (1998).
25) E. Silva, N. Pompeo, and O. V. Dobrovolsky, Phys. Sci. Rev. 2, 1 (2017).
26) H. Zhang, H. Chang, and W. Yuan, Microsys. Nanosyst. 3, 17023 (2017).
27) V. Shklovskii and O. Dobrovolsky, Phys. Rev. B 78, 164526 (2008).
28) R. Prozorov, R. W. Giannetta, J. A. Schlueter, and P. Zoller, Phys. Rev. B 67, 184501 (2002).
29) K. Watanabe, K. Yoshida, T. Aoki, and S. Kohjuro, Jpn. J. Appl. Phys. 33, 5708 (1994).
30) T. F. Orlando, E. J. McNiff, S. Foner, and M. R. Beasley, Phys. Rev. B 19, 4545 (1979).
31) R. Willa, V. B. Geshkenbein, and G. Blatter, Phys. Rev. B 93, 064515 (2016).