AN X-RAY SIZE-TEMPERATURE RELATION FOR GALAXY CLUSTERS: OBSERVATION AND SIMULATION

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ABSTRACT

We show that galaxy clusters conform to a tight relation between X-ray isophotal size $R_i$ and emission-weighted intracluster medium (ICM) temperature $\langle T_X \rangle$. The best-fit relation for 41 members of an X-ray flux-limited cluster sample is $\log R_i = (0.93 \pm 0.11) \log (\langle T_X \rangle/6 \mathrm{keV}) - (0.08 \pm 0.01)$; intrinsic scatter in size about the relation is 15%, and for 30 clusters with $\langle T_X \rangle > 4 \mathrm{keV}$, the scatter is reduced to 10%. The existence of the size-temperature (ST) relation indicates that the ICM structure is a well-behaved function of $\langle T_X \rangle$. We use an ensemble of gasdynamic simulations to demonstrate that a cluster population experiencing present-epoch growth nevertheless conforms to an ST relation with scatter similar to that observed; the simulations also exhibit a tight relation between $M_{\text{vir}}$ and $\langle T_X \rangle$, providing the suggestion that a similar relation holds for observed clusters. We use the scatter in $R_i$ to estimate limits on the rms variation in ICM mass fraction $\delta_{\text{ICM}}$ at constant $\langle T_X \rangle$: $\delta_{\text{ICM}} f_{\text{ICM}} \leq 22\%$ ($\leq 14\%$ for clusters with $\langle T_X \rangle > 4 \mathrm{keV}$). It appears that a mechanism like feedback from galactic winds, which introduces systematic structural changes in the ICM, is required to reproduce the observed slope of the ST relation.

Subject headings: galaxies; clusters: general — intergalactic medium — X-rays: galaxies

1. INTRODUCTION

Ongoing cluster growth at the present epoch is an observational fact (see, e.g., Fabricant et al. 1986; Fabricant, Kent, & Kurtz 1989; Zabludoff & Zaritsky 1995; Henry & Briel 1995; Mohr, Geller, & Wegner 1996); large studies employing varied techniques indicate that 30%–70% of clusters exhibit evidence for recent mergers (Geller & Beers 1982; Dressler & Shectman 1988; Jones & Forman 1992; Mohr et al. 1995). A theoretical framework for understanding this growth has emerged, and structure formation simulations within a range of favored cosmological models produce clusters that exhibit merger signatures similar in detail and frequency to those observed (Mohr et al. 1995; Buote & Xu 1997). Yet recent numerical work indicates that even though clusters are growing at the present epoch, they are, on average, regular objects with temperatures and masses tightly correlated through the virial theorem (Evrard, Metzler, & Navarro 1996; Schindler 1996; Roettiger, Burns, & Loken 1996). Direct observational tests of these results are difficult, but analyzing relations between intracluster medium (ICM) structural parameters and temperature provides an alternative means of testing cluster regularity. The well known X-ray luminosity-temperature ($L_X-\langle T_X \rangle$) relation (see, e.g., Smith, Mushotzky, & Serlemitsos 1979; Mitchell et al. 1979; David et al. 1993; Mushotzky & Scharf 1997) is one observational indicator of cluster regularity, but the scatter around the relation is quite large. The scatter is largely caused by varying degrees of excess core emission associated with cooling flows (Fabian et al. 1994).

Here we report a tight relation between cluster isophotal size $R_i$ and mean emission-weighted temperature $\langle T_X \rangle$ that we uncovered in an ongoing morphological analysis of a large, X-ray flux-limited cluster sample. This size-temperature (ST) relation for observed clusters indicates a high degree of regularity, suggests the existence of a mass-temperature correlation, and provides new constraints on theories of the interactions between galaxies and the ICM. This newly discovered ST relation may ultimately lead to more accurate estimates of cluster baryon fractions and tighter constraints on the cosmic density parameter $\Omega_0$ (see, e.g., White et al. 1993; Evrard 1997).

Section 2 contains a description of the data reduction and analysis and the observed ST relation. We then use an ensemble of numerical simulations to examine the ST relation in a cluster population experiencing present-epoch growth (§ 3). In § 4, we discuss sources of scatter about the relation, estimate upper limits on the variation in ICM mass fraction $f_{\text{ICM}}$, and discuss the slope of the ST relation. Section 5 contains a discussion of our conclusions.

2. OBSERVATIONS

The cluster sample contains members of the X-ray flux-limited group of 55 clusters defined by Edge et al. (1990). Archival ROSAT Position Sensitive Proportional Counter (PSPC) images of 47 of these clusters are available on-line through the High Energy Astrophysics Science Archive Research Center (HEASARC). We reduce these images using PROS and Snowden analysis software (Snowden et al. 1994). The final image for each observation is the sum of individually flat-fielded subimages from the Snowden bands $R4$ through $R7$, corresponding to photon energies of 0.44–2.04 keV. We exclude time intervals with master veto rates higher than 220 counts s$^{-1}$ and exclude other high-background time intervals (typically, high-background intervals total <5 minutes of exposure) whose inclusion would degrade the detection significance of a source 10% as bright as the background (Pildis, Bregman, & Evrard 1995). We combine multiple images of a cluster using the positions of bright X-ray point sources where possible, or alternatively, the image-header pointing positions. Finally, we remove obvious point sources from the unsmoothed images (we do...
not remove possible point-source components coincident with the extended emission typical of cluster cooling flows, as in NGC 1275, for example) and then apply Gaussian smoothing ($\sigma = 2$ pixels). The pixel scale is 14.947, and the effective angular resolution is FWHM $\approx 1.5$. We correct the cluster images for $(1 + z)^4$ cosmological dimming but do not make Galactic absorption or K-corrections (discussed in more detail below).

We define the cluster size $R_f$ using the area $A_f$ of the largest region enclosed by the isophote $I$.

$$R_f = \sqrt{A_f / \pi}. \quad (1)$$

This approach deals consistently with clusters that are not azimuthally symmetric because of recent mergers and produces a tighter relation than would azimuthal averaging; in addition, $I$ can be chosen so that $R_f$ is unaffected by the central surface-brightness excesses typical of cooling flows. The isophote $I$ is the cluster surface brightness, and we determine the X-ray background using annuli well outside the X-ray–bright cluster region (typically 38′ from the cluster center with $\sim 8′$ extent). We estimate $I_b$ as the (sigma clipped) mean value of the pixels within the annulus, and we use the width of the distribution around $I_b$ to estimate the uncertainty. For the isophotes $I$ we use, the uncertainty in $R_f$ is primarily caused by the background uncertainty. We conservatively estimate the uncertainties in $R_f$ to be $\sigma_R = (R_f - R_f)/2$, where $I_+ - I = I + I_\beta/10$. The internal estimate of the uncertainty in $I_b$ is significantly smaller than 10% in all cases.

We use published emission-weighted mean temperatures $\langle T_X \rangle$ for each cluster. There are 42 clusters in the Edge sample with PSPC images and published $\langle T_X \rangle$ with uncertainties. Roughly half of these temperatures are from Einstein MPC observations (David et al. 1993), but where possible we substitute more accurate Ginga, ASCA, or ROSAT PSPC temperatures (Ginga: Day et al. 1991; Allen et al. 1992; Johnstone et al. 1992; David et al. 1993; Hughes et al. 1993; Arnaud & Evrard 1997; ASCA: Henriksen & Markevitch 1996; Markevitch 1996; Matsuzawa et al. 1996; Tamura et al. 1996; Markevitch & Vikhlinin 1997; PSPC: David, Jones, & Forman 1996). We exclude the Virgo Cluster from our analysis because bright cluster emission extends beyond the 2° PSPC field of view. Thus, our final sample contains 41 clusters.

The observational data for this sample are displayed in Table 1; the cluster name, effective PSPC exposure time $t_{\text{exp}}$, background brightness $I_b$, emission-weighted mean temperature $\langle T_X \rangle$, and designation as cooling flow cluster

| Cluster | $t_{\text{exp}}$ (ks) | $I_b$ ($10^{-4}$ counts s$^{-1}$ arcmin$^{-2}$) | $R_f$ ($h_{50}^{-1}$ Mpc) | $\langle T_X \rangle$ (keV) | Flow |
|---------|---------------------|--------------------------------|-----------------|-----------------|-----|
| A262    | 8.1                 | 3.31                           | 0.289           | 1.36            | X   |
| MKW 3s  | 9.2                 | 5.80                           | 0.485           | 3.00            | X   |
| A1060   | 14.6                | 5.34                           | 0.279           | 3.10            | X   |
| A2052   | 5.9                 | 6.06                           | 0.467           | 3.10            | X   |
| A1367   | 17.7                | 3.07                           | 0.512           | 3.50            |     |
| A4059   | 5.2                 | 2.90                           | 0.552           | 3.50            |     |
| A3526   | 3.0                 | 7.09                           | 0.317           | 3.54            | X   |
| A3562   | 18.5                | 4.19                           | 0.643           | 3.80            |     |
| A780    | 17.3                | 2.34                           | 0.649           | 3.80            | X   |
| A1999   | 12.6                | 3.34                           | 0.452           | 3.90            | X   |
| A496    | 8.1                 | 3.82                           | 0.586           | 3.91            | X   |
| A2063   | 9.6                 | 6.90                           | 0.502           | 4.10            | X   |
| A3112   | 7.2                 | 2.64                           | 0.700           | 4.10            | X   |
| Cyg A   | 8.7                 | 7.07                           | 0.835           | 4.10            | X   |
| A2147   | 0.9                 | 6.40                           | 0.743           | 4.40            |     |
| A2199   | 9.7                 | 2.79                           | 0.570           | 4.50            | X   |
| A3391   | 5.5                 | 3.24                           | 0.644           | 5.20            |     |
| A1795   | 24.8                | 3.09                           | 0.878           | 5.34            | X   |
| A3158   | 2.9                 | 2.60                           | 0.850           | 5.50            |     |
| A3558   | 27.8                | 4.75                           | 0.920           | 5.70            |     |
| A119    | 14.3                | 2.82                           | 0.761           | 5.90            |     |
| A3266   | 7.0                 | 3.35                           | 1.061           | 6.20            |     |
| A85     | 5.3                 | 3.18                           | 0.915           | 6.20            | X   |
| A426    | 4.4                 | 7.40                           | 0.836           | 6.33            | X   |
| A3667   | 11.3                | 4.39                           | 1.156           | 6.50            |     |
| A644    | 9.5                 | 2.18                           | 0.849           | 6.59            |     |
| A478    | 21.4                | 1.48                           | 0.973           | 6.84            | X   |
| A2244   | 2.9                 | 2.31                           | 0.978           | 7.10            |     |
| A2255   | 12.5                | 2.11                           | 1.030           | 7.30            |     |
| A399    | 6.4                 | 2.29                           | 1.096           | 7.40            |     |
| A2256   | 17.1                | 2.61                           | 1.030           | 7.51            |     |
| A3571   | 5.5                 | 4.98                           | 0.862           | 7.60            |     |
| A2029   | 11.9                | 5.94                           | 1.098           | 7.80            | X   |
| A1656   | 19.4                | 5.21                           | 0.951           | 8.21            |     |
| A745    | 9.1                 | 1.97                           | 0.917           | 8.50            | X   |
| A754    | 6.1                 | 2.41                           | 1.062           | 8.50            |     |
| A2142   | 4.8                 | 2.65                           | 1.337           | 8.68            | X   |
| A2319   | 2.9                 | 4.92                           | 1.303           | 9.12            |     |
| Ophi    | 3.7                 | 6.38                           | 0.839           | 9.80            |     |
| A1689   | 13.3                | 2.53                           | 1.422           | 10.10           |     |
| Tria     | 6.6                 | 4.20                           | 1.086           | 10.30           |     |
are listed. We designate those clusters with central cooling times significantly below 10 Gyr (Edge, Stewart, & Fabian 1992) as cooling flow clusters.

Figure 1 contains a plot of the 41 clusters in log $R_i$ and log $\langle T_X \rangle$ for $I = 1.93 \times 10^{-3}$ counts s$^{-1}$ arcmin$^{-2}$. A linear fit to the sample (minimizing the sum of the orthogonal residuals from the relation) yields

$$ \log R_i = (0.93 \pm 0.11) \log \frac{\langle T_X \rangle}{6 \text{ keV}} - (0.08 \pm 0.01). \quad (2) $$

The rms scatter in log $R_i$ about this relation is $\delta \log R_{i,\text{raw}} = 0.084$. The coefficient uncertainties reflect the half-width of the 68% confidence region determined by bootstrap resampling.

We analyze the ST relation in the PSPC sample over a factor of $8$ in surface brightness (see Table 2 and § 4.3); varying $I$ primarily changes the zero point $b$ of the ST relation. The integrity of the relation is preserved until $I$ reaches values comparable to the peak surface brightnesses of some clusters.

The uncertainties in $\langle T_X \rangle$ are large enough to contribute to the observed scatter. We estimate the intrinsic scatter in $\delta \log R_i$ by subtracting the temperature contribution in quadrature. We find $\Delta \delta \log R_i = 0.93$ rms ($\langle \sigma_i / \langle T_X \rangle \ln 10 \rangle = 0.054$. Thus, the intrinsic scatter is approximately $\delta \log R_{i,\text{int}} = 0.064$. Figure 1 gives the impression that the scatter increases at lower temperature; fitting only those 30 clusters with $\langle T_X \rangle > 4$ keV leads to an ST relation $R_i = (0.82 \pm 0.14) \log (\langle T_X \rangle / 6 \text{ keV}) - (0.07 \pm 0.02)$, with raw/intrinsic scatter of $0.067/0.041$. Thus, the slope of the ST relation for hot clusters is slightly shallower than that for the whole population, and the scatter is smaller by 50%.

3. Simulations

We further examine the ST relation using a set of 48 high-resolution $N$-body plus gasdynamics simulations, to be described in detail elsewhere (Mohr & Evrard 1997). In summary, the cluster simulations are carried out within four different cold dark matter (CDM) dominated cosmologies (1) standard CDM (SCDM; $\Omega = 1$, $\sigma_8 = 0.6$, $h = 0.5$), (2) open CDM (OCDM; $\Omega_0 = 0.3$, $\sigma_8 = 1.0$, $h = 0.8$), (3) low-density CDM (LCDM; $\Omega_0 = 0.3$, $\lambda_0 = 0.7$, $\sigma_8 = 1.0$, $h = 0.8$), and (4) an alternative CDM (ZCDM; $\Omega_0 = 1$, $\sigma_8 = 1.0$, $h = 0.5$) with power spectra consistent with a CDM $\Gamma = 0.24$ transfer function (Davis et al. 1985). Here $H_0 = 100 h$ km s$^{-1}$ Mpc and $\sigma_8$ is the power spectrum normalization in $8 h^{-1}$ Mpc spheres. The baryon density is a fixed fraction of the total $\Omega_b = 0.2 \Omega_m$. Within each of these models, we use two $128^3$ $N$-body–only simulations of cubic regions with scale ~400 Mpc to determine sites of cluster formation. Within these initial runs, the virial regions of clusters with Coma-like masses of $10^{15} M_\odot$ contain ~$10^3$ particles.

Using the $N$-body results for each cosmological model, we choose clusters for additional study. We zoom in on these clusters, resimulating them at higher resolution with gasdynamics and gravity on a $64^3$ $N$-body grid. The simulation scheme is P3MSPH (Evrard 1988), and radiative cooling is ignored. These clusters have masses that vary by an order of magnitude. The scale of the simulated region surrounding each cluster is in the range 50–100 Mpc, and varies as $M_\text{halo}^{1/3}$, where $M_\text{halo}$ is approximately the mass enclosed within the present-epoch turn around radius. Thus, the 48 simulated clusters in our final sample have similar fractional mass resolutions; spatial resolutions vary from 125 to 250 kpc. We create X-ray images and temperature maps for further analysis following procedures described in Evrard (1990).

### Table 2

| Source | $I$ (10⁻⁴ counts s⁻¹ arcmin⁻²) | $m$ | $a$ | $\delta \log R^*$ |
|--------|-------------------------------|-----|-----|------------------|
| PSPC... | 9.65                         | 0.886 (0.124) | -0.030 (0.013) | 0.074 |
|        | 19.3                         | 0.934 (0.109) | -0.080 (0.013) | 0.064 |
|        | 38.6                         | 1.047 (0.130) | -0.204 (0.014) | 0.069 |
|        | 77.2                         | 1.514 (0.382) | -0.356 (0.024) | 0.151 |
| SCDM... | 16.2                         | 0.699 (0.035) | 0.119 (0.013) | 0.042 |
| OCDM... | 66.4                         | 0.612 (0.025) | 0.111 (0.010) | 0.054 |
| LCDM... | 66.4                         | 0.721 (0.024) | 0.156 (0.007) | 0.035 |
| ZCDM... | 16.2                         | 0.808 (0.067) | 0.161 (0.022) | 0.069 |

* Intrinsic scatter (corrected for errors in $\langle T_X \rangle$)
reached half of their present epoch mass at look-back
different formation histories. On average, these clusters
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1997).
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scatter is typically somewhat smaller than the observed
value. Only ZCDM clusters have a higher scatter than our
estimate of the intrinsic scatter in the observed sample; the
higher scatter is caused by a few ongoing major mergers at high
. The scatter around the ST relation in the simulated
populations is caused by projections of clusters along the
line of sight, small equilibrium departures, and small (5%) intrinsic fractional \( f_{ICM} \) variations that exist because of dif-
ferent formation histories. On average, these clusters reached half of their present epoch mass at look-back
epochs 0.33, 0.35, 0.42, and 0.44\( t_0 \) for Z-, S-, O-, and LCMD models, respectively, where \( t_0 \) is the age of the universe in each model.
The S-, O-, and LCDM clusters are morphologically similar to observed clusters, and the ZCDM clusters exhibit somewhat more evidence for recent mergers than the other three models and the observed clusters (Mohr & Evrard 1997).

4. ANALYSIS

The existence of the ST relation in this large, approxi-
ately flux-limited cluster sample demonstrates that the
ICM structure is a well-behaved function of \( \langle T_X \rangle \). After
correcting for measurement uncertainties, the scatter of the
clusters about the ST relation is similar to the scatter of
bright ellipticals around the fundamental plane (see, e.g.,
Jørgensen, Franx, & Kjærgaard 1996; Mohr & Wegner 1997). To further understand this relation, we examine
sources of scatter about it, and then we use the scatter to
place upper limits on variations in the ICM mass fraction
\( f_{ICM} \). We also examine the ST-relation slope, which provides
information about the physical processes affecting the ICM
structure.

4.1. Scatter about the ST Relation

There are several sources of scatter in the ST relation besides the measurement uncertainties discussed in § 2. These include variations in Galactic \( N_H \) absorption, departures from equilibrium, projections of physically unassociated clusters along the line of sight, and dark matter and ICM structural variations at constant \( \langle T_X \rangle \).

The contribution from variations in Galactic \( N_H \) is insig-
nificant. We use PROS tasks to calculate surface-brightness correction factors for Galactic absorption at 1 keV for our
cluster sample. We find that these correction factors have a
mean of 1.17 and an rms variation of 6.8% about this mean
value for the 41 clusters. Typical surface-brightness profiles
are rather steep, \( I(R) \propto R^{-3} \) (see, e.g., Jones & Forman
1984; Mohr et al. 1995), so \( \delta \log R = \frac{1}{2} \delta \log I \), and the
variation in Galactic absorption along the lines of sight to
these clusters is expected to contribute scatter at the level of
\( \sim 2.3\% \). Galactic absorption is approximately 45% more
effective at 0.5 keV, the low-energy end of our bandwidth
(and, of course, less effective at 2.0 keV), so the maximal
contribution to the ST-relation scatter from variations in
Galactic \( N_H \) is \( \sim 3.3\% \).

Departures from equilibrium contribute significantly to
the scatter. Two clusters of similar mass about to merge in
the plane of the sky will be 40% too large in \( R_i \) for their
\( \langle T_X \rangle \). The scatter about the ST relations exhibited by the
simulations described in § 3 is an indicator of the expected

\( \log(\langle R_h \rangle) \) vs \( \log(\langle T_X \rangle) \) for the ST relation for Figure 2 (left-hand panel) is a plot of the ST relation for the simulated clusters (with \( I = 1.62 \times 10^{-3} h_{70}^2 \) counts s\(^{-1}\) arcmin\(^{-2}\)). Clusters from the four cosmological models are plotted with different point styles; each appears three times from orthogonal projections. The best-fit relation for the SCDM model is \( \log R = 0.70 \log (\langle T_X \rangle/6 \text{ keV}) + 0.119 \), and the scatter around this fit is \( \delta \log R = 0.042 \); Table 2 contains the relations and scatter for all models; the level of scatter is typically somewhat smaller than the observed value. Only ZCDM clusters have a higher scatter than our estimate of the intrinsic scatter in the observed sample; the higher scatter is caused by a few ongoing major mergers at high \( \langle T_X \rangle \).

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simulations described in § 3 is an indicator of the expected
contribution from equilibrium departures and projection effects in cluster populations still merging at the present epoch.

Equilibrium dark matter and ICM structural variations at a particular temperature will also contribute to the ST-relation scatter. N-body simulations indicate that the dark matter structure of clusters is a regular function of mass and that mild variations among clusters with similar mass are attributable to different formation times (Navarro, Frenk, & White 1996). Again, major mergers present exceptions but do not dominate the local population. The ICM distribution of clusters with similar masses has not been as extensively studied as the dark matter, but the experiments shown here and below suggest that the ICM structure is a well-behaved function of cluster mass or temperature, even in the presence of galactic winds (Metzler & Evrard 1997). Other sources of variation in the ICM distribution or gas mass–fraction are possible. One source is radiative cooling, which leads to strong emission excesses or to cooling flows in cluster centers (Jones & Forman 1984; Fabian 1994).

We examine the effects of cooling flows by dividing our cluster sample into those with cooling times short enough (tcool significantly less than 10 Gyr) and those with longer cooling times (Edge et al. 1992). We find that the best-fit ST relation for the 18 cooling flow clusters (see Table 1) has a slope m = 0.93 ± 0.16 and a zero point b = −0.07 ± 0.02; the relation for the 23 non–cooling flow clusters has m = 1.00 ± 0.21 and b = −0.09 ± 0.02. The raw/intrinsic scatter about these two relationships is δ log Rvir = 0.086/0.071 for the cooling flow clusters and δ log Rvir = 0.083/0.061 for the non–cooling flow clusters. These two relations are indistinguishable and the scatter about each is comparable; thus, for the present sample of 41 clusters, there is no indication that the ST relation is sensitive to the presence of cooling flows.

4.2. Upper Limit on ICM Mass Fraction Variations

By assuming the observed scatter in the ST relation is caused solely by the varying ICM mass fraction fICM at constant ⟨Tgas⟩, we estimate an upper limit on these variations. The surface brightness profiles of both real and simulated clusters are generally well fitted with the standard isothermal β model,

\[ I(R) = A \Lambda \rho_0^3 R_c \left[ 1 + \left( R / R_c \right)^2 \right]^{\beta + 1/2}, \]

where \( \rho_0 \) is the central ICM density, \( \Lambda \) is the specific emissivity, \( R_c \) is a core radius, \( \beta \) is the variable slope, and \( A \) is a constant with some \( \beta \) dependence (Cavaliere & Fusco-Femiano 1978). We use this model as a guide to understanding the ST relation (but the relationship below can be independently of the \( \beta \) model). Within this framework, the ICM mass fraction is

\[ f_{\text{ICM}} = \frac{4 \pi \rho_0 R_c^3 \int_{r_{\text{vir}}}^{R_c} d\lambda \lambda^2 \left( 1 + \lambda^2 \right)^{\beta/2} \rho \left( R \right) M_{\text{vir}}}{4 \pi R_c^3 \int_{r_{\text{vir}}}^{R_c} d\lambda \lambda^2 \left( 1 + \lambda^2 \right)^{1/2}}, \]

where \( R_{\text{vir}} \) is the virial radius encompassing a fixed density contrast.

Outside the core, the surface brightness scales as

\[ I(R) = C f_{\text{ICM}}^2 R^{-6\beta + 1}, \]

where the constant \( C \) incorporates the ICM structural information of equation (4). If clusters of a given temperature are structurally similar, then the scatter in the isophotal radii at constant \( \langle T_{\text{gas}} \rangle \) places an upper limit on variations in the gas fraction \( \delta \log f_{\text{ICM}} \leq [(6\beta - 1)/2] \delta \log R_c \). This inequality can be broken if structural variations (variations in \( C \)) are anticorrelated with variations in \( f_{\text{ICM}} \); we assume here that this is not the case. Qualitatively, such an anticorrelation would require gas-poor clusters to be more centrally concentrated (and vice versa), a situation at odds with traditional expectations.

For the typical value \( \beta = 4/3 \) (Jones & Forman 1984; Mohr et al. 1995), \( \delta \log f_{\text{ICM}} \leq 1.5 \delta \log R_c \) and the intrinsic scatter in \( \log R_c \) limit fractional gas variations to \( \delta f_{\text{ICM}} \leq 22\% \). The limits drop to \( \delta f_{\text{ICM}} = 14\% \) for the 30 clusters hotter than 4 keV.

The scatter about the ST relation in the simulated clusters includes a contribution from variations in \( f_{\text{ICM}} \) within the virial radius. This contribution is rather small, \( \delta f_{\text{ICM}} \approx 4\% \) within \( r_{200} \), the radius within which the mean cluster density is 500 times the critical density. Thus, other effects such as departures from equilibrium and projections dominate the scatter. Assuming these effects exist to a similar degree in the observations, then the similar scatter in the simulations and the observed clusters with \( \langle T_{\text{gas}} \rangle > 4 \) keV indicates that gas-fraction variations within hot clusters are very small, perhaps as small as 4\%. This argument does not apply for cooler clusters, which exhibit larger scatter about the ST relation.

4.3. Slope of the ST Relation

The steeper observed slope \( m = 0.93 \pm 0.11 \), compared to \( 0.61 \leq m \leq 0.81 \) in the simulations, suggests structural differences in real clusters compared to this set of models. Radiative cooling is not included in the simulations, but the similarity of the ST relation in clusters with and without cooling flows indicates this is not the cause of the slope differences. Possible explanations include systematic trends in the overall gas fraction \( f_{\text{ICM}} \) and/or systematic variations in the ICM distribution (as measured by \( \beta \)) with \( \langle T_{\text{gas}} \rangle \).

The observed cluster sample provides evidence that shallower surface-brightness profiles lead to steeper ST relations. Table 2 lists the best-fit slope \( m \) and zero point \( b \) for each of the observed clusters over a factor of 8 variation in \( I \). Along with the expected zero-point decrease with brightening \( I \), there is a tendency for a steeper slope. This steepening is related to the shallower surface-brightness profiles (smaller effective \( \beta \)) at brighter isophotes. Simulations from the four cosmological models provide additional evidence that shallower surface-brightness profiles generally lead to steeper ST relation slopes; the mean values of \( \beta \) of the Z-, S-, L-, and OCDM models are 0.67, 0.73, 0.77, and 0.81 and the values of \( m \) are 0.81, 0.70, 0.72, and 0.61, respectively. Thus, the steeper observed ST relation may simply be an indication of a more extended ICM distribution in the observations than in the simulations.

Galactic winds are one physical process expected to introduce systematic ICM structural variations. In such a model, star formation within galaxies results in supernova (SN)–driven winds that expel gas, and the expelled gas has the orbital kinetic energy of its parent galaxy plus the energy imparted by the SNs. This nongravitational source of energy is expected to be more important in low–\( \langle T_{\text{gas}} \rangle \) clusters where the orbital energies are smaller. We examine the effects of galactic winds on the ST relation using an ensemble of clusters from simulations that include galaxy
feedback (Metzler & Evrard 1994, 1997). The simulated cluster ensemble contains two populations, one population of 18 clusters evolved consistent with an SCDM model and another population with the same initial conditions and cosmological parameters, but with galaxy feedback modeled by discrete gas ejection totalling half the initial galaxy mass between \( z = 4.5 \) and the present. This extreme ejection model is intended to estimate the maximal effect of galaxy feedback.

Figure 2 (right-hand panel) is a plot of the effects of galaxy feedback on the ST relation. The slope of the best-fit ST relation for the clusters with no feedback (dotted line) is \( m = 0.69 \), consistent with the SCDM slope in our simulations (see Table 2). The slope for clusters with simulated ejection (solid line) is \( m = 0.99 \), somewhat steeper than the slope of our PSPC sample. This demonstrates that galaxy feedback can cause the structural changes required to match the slope of the observed ST relation. Structural changes include shallower surface-brightness profiles (smaller \( \beta \)) and lower overall gas fractions with decreasing \( \langle T_X \rangle \), as discussed in Metzler & Evrard (1997). The right-hand panel of Figure 2 makes clear that, as expected, ejection has the greatest effect on the clusters with lowest mass.

The tendency for low-\( \langle T_X \rangle \) clusters to have shallower surface-brightness profiles is well known (see, e.g., Mohr et al. 1995); our sample of 41 clusters exhibits this general trend. Figure 3 contains a plot of the slope of the surface-brightness profile \( \beta_{\text{eff}} \) measured over the region of the cluster used to calculate \( R_\text{c} \). The Spearman rank correlation coefficient (Press et al. 1992) for \( \langle T_X \rangle \) and \( \beta_{\text{eff}} \) is \( r_s = 0.54 \) with a correlation significance of 99.97%. The term \( \beta_{\text{eff}} \) is a two-dimensional generalization of \( \beta \) and, like \( R_\text{c} \), is measured without azimuthal averaging; moreover, \( \beta_{\text{eff}} \) reflects the local slope of the surface-brightness profile and so is independent of the cluster core radius. Briefly, to calculate the values in Figure 3, we determine the area \( A_i \) enclosed by the isophote \( I_i = 1.29 \times 10^{-3} \text{ counts s}^{-1} \text{ arcmin}^{-2} \) (chosen to be somewhat fainter than the \( I \) used to calculate \( R_\text{c} \)), and then calculate the isophote \( I_i \) that encloses the area \( A_i \) that is chosen to be \( A_i/2 \). This provides two isophotes and two areas, so we then calculate the effective slope of the surface-brightness profile between \( I_i \) and \( I_j \) as

\[
\beta_{\text{eff}} = \frac{1}{3} \left[ \log \left( \frac{I_j/I_i}{A_j/A_i} \right) + 1 \right].
\]

This form follows directly from the expression \( I(R) \propto R^{-6.6\Delta \beta_{\text{eff}} + 1} \). The trend of falling \( \beta_{\text{eff}} \) with \( \langle T_X \rangle \) is qualitatively consistent with the effects of galaxy feedback.

5. DISCUSSION

We use observations of the 41 members of an X-ray flux-limited cluster sample with PSPC observations and published \( \langle T_X \rangle \) (excluding Virgo) to demonstrate that nearby clusters conform to a tight relation between X-ray isophotal size and cluster temperature. The intrinsic scatter in size at fixed temperature is only 15% (10% for clusters with \( \langle T_X \rangle > 4 \text{ keV} \)). The existence of the ST relation indicates that the ICM structure outside the core regions is a well-behaved function of \( \langle T_X \rangle \) and suggests a tight correlation between \( \langle T_X \rangle \) and \( M_{\text{vir}} \), as seen in our simulations and those of others (Evrard et al. 1996; Schindler 1996; Roettiger et al. 1996).

The ST relation is significantly tighter than the \( L_X \times \langle T_X \rangle \) relation for this cluster sample. The scatter (corrected for temperature uncertainties) in 2–10 keV X-ray luminosity (David et al. 1993) around the best-fit relation for these 41 clusters is \( \delta L_X/L_X = 52\% \). (We use the same set of temperature measurements in both cases.) The factor of 3.5 larger scatter in the \( L_X \times \langle T_X \rangle \) relation reflects the sensitivity of \( L_X \) to cluster core properties (Fabian et al. 1994). When restricted to weak cooling flow clusters—those with inferred cooling flow rates below \( 100 \ M_\odot \text{ yr}^{-1} \)—the \( L_X \times \langle T_X \rangle \) relation displays a much smaller dispersion, and inferred gas-fraction variations from a sample of 24 clusters are consistent with the limits set by this analysis (Arnaud & Evrard 1997). The narrow scatter in the ST relation indicates that structural regularity exists outside the core at observationally viable surface-brightness levels.

Our cluster sample provides no evidence that cooling flows significantly affect the ST relation. The 13 clusters with central cooling times significantly below 10 Gyr exhibit best-fit slopes, zero points, and scatter that are statistically indistinguishable from those for the 18 other clusters. The zero-point differences between these two ST relations indicate that \( \langle T_X \rangle \) in cooling flow clusters is biased low by \( \sim 5\% \pm 7\% \), consistent with previous estimates of \( \sim 10\% \text{–} 20\% \) (Fabian et al. 1994).

The scatter around the ST relation is comparable to that of the half-light radius of elliptical galaxies around the fundamental plane (see, e.g., Jörgensen et al. 1996; Mohr & Wegner 1997); an obvious use of the ST relation is as a distance indicator. Knowledge of \( \langle T_X \rangle \) and an apparent isophotal size predicts the cluster distance with an uncertainty limited by the scatter in \( R_\text{c} \) (\( \sim 10\% \) for hot clusters). We are currently evaluating the promise of this relation as a distance indicator at intermediate redshift.

The scatter about the best-fit ST relation for observed clusters provides an upper limit on the rms variation in cluster ICM mass fraction at a common temperature \( \langle T_X \rangle \).
Ignoring possible correlations between cluster structural changes and $f_{\text{ICM}}$ variations, we find upper limits $\Delta f_{\text{ICM}} \leq 22\%$ ($\leq 14\%$ for $\langle T_X \rangle > 4$ keV).

We use 48 cluster simulations within four cosmological models to demonstrate that cluster populations experiencing growth at the present epoch exhibit ST relations with scatter similar to that observed above $\langle T_X \rangle > 4$ keV: $\delta R_l / R_l \approx 10\%$. Only a small contribution to the scatter in the simulations is from $f_{\text{ICM}}$ variations, suggesting that the ICM mass fraction in hot clusters may be limited to $\lesssim 5\%$ variation within their virial regions. Implications for hierarchical models of galaxy formation remain to be explored.

The slope of the ST relation for the PSPC clusters is steeper than the slope in our simulations. We use numerical simulations (Metzler & Evrard 1994, 1997) to show that galaxy feedback introduces the kinds of ICM structural changes required to steepen the ST relation. The changes include shallower surface-brightness profiles and lower $f_{\text{ICM}}$ with decreasing $\langle T_X \rangle$. A decrease in $\beta$ with $\langle T_X \rangle$ (see Fig. 3) and a reduction of $f_{\text{ICM}}$ in low $\langle T_X \rangle$ systems (David, Jones, & Forman 1995; dell’Antonio, Geller, & Fabricant 1995) have been observed. Galactic feedback is a promising mechanism for creating the observed structural differences between low- and high-$\langle T_X \rangle$ clusters.

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