Parameter Estimation of Switched Hammerstein Systems

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Abstract This paper deals with the parameter estimation problem of the Single-Input-Single-Output (SISO) switched Hammerstein system. Suppose that the switching law is arbitrary but can be observed online. All subsystems are parameterized and the Recursive Least Squares (RLS) algorithm is applied to estimate their parameters. To overcome the difficulty caused by coupling of data from different subsystems, the concept intrinsic switch is introduced. Two cases are considered: i) The input is taken to be a sequence of independent identically distributed (i.i.d.) random variables when identification is the only purpose; ii) A diminishingly excited signal is superimposed on the control when the adaptive control law is given. The strong consistency of the estimates in both cases is established and a simulation example is given to verify the theoretical analysis.

Key words SISO switched Hammerstein system, RLS algorithm, intrinsic switch, diminishing excitation, strong consistency.

1 Introduction

Because of importance in engineering applications, the identification and control of switched systems have been active research areas for years [1]. Concerning parameter identification of switched systems, a survey is given in [7].

The switched systems can roughly be divided into two classes: systems with an arbitrary switching mechanism and systems governed by a constrained switching law, such as the Markovian switching rule. In the existing literature there are many papers on Markov Jump Systems, see, e.g., [8] and the references therein. The Markov models are also considered in [2,3] for purposes of anomaly detection.

By using the algebraic geometry as the key tool and under the assumption that the number of subsystems, the subsystem orders, and the switching sequence are unknown, the author of [9] provides an algorithm to recursively estimate the unknown parameters of the discrete-time Switched

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Auto-Regressive eXogenous (SARX) model, and gives the algorithm a convergence analysis. However, in the convergence analysis given in [9] no unpredictable disturbance is taken into account, despite the examples given there are with noises. While the authors of [10] tackle the SARX model with noises; they suggest an algorithm that alternates between data designation to submodels and parameter update, but do not prove its convergence. Recently, in transportation community, Zhang et. al [4, 5, 6] leverage Least Squares (LS) methods to ensure flow conservation and estimate Origin-Destination (OD) flow demand matrices, which have been demonstrated pretty effective and efficient, thus motivating our current work to consider a recursive version of LS.

In this work, we consider parameter estimation of the Single-Input-Single-Output (SISO) switched Hammerstein system and assume that the switching law is arbitrary but can be observed online. We will handle two cases:

i) In the case where identifying the system is the only concern, we take the system input as a sequence of i.i.d. random variables. It is assumed that the nonlinear function of each subsystem can be expanded to a linear combination of continuous base functions.

ii) In the case where the adaptive control has been designed for the system, we apply the diminishing excitation technique [11] to recursively estimate the unknown parameters. In this case, we assume that the continuous base functions, a linear combination of which the nonlinear part of each subsystem can be expanded to, are monomials.

The rest of the paper is organized as follows. The problem is formulated in Section 2, and the parameter estimation algorithm is constructed in Section 3. In Section 4 we prove that the estimates given by the proposed algorithm are strongly consistent, and then we provide a simulation example in Section 5. Some concluding remarks are given in Section 6. Appendix at the end is used to load proof details.

2 Problem Formulation

The SISO switched Hammerstein system considered in the paper is presented in Fig. 1. It contains a finite number of Hammerstein subsystems, each of which consists of a static nonlinear $G(\cdot)$ followed by an ARX subsystem in cascade.

![Fig. 1. SISO Switched Hammerstein System](image)

We assume that there are $J$ subsystems, and consider the case where the switch mechanism is available. To be precise, the mapping $\lambda(\cdot)$

\[
\mathbb{N} \xrightarrow{\lambda} \{1, 2, \ldots, J\} \quad k \mapsto \lambda_k
\]

2
can be observed online, where $N$ represents the set of all nonnegative integers, and $\lambda_k$ denotes the serial number of the Hammerstein subsystem that operates at time $k$. Besides, the orders $p, q$ of all ARX subsystems are supposed to be the same and known. Moreover, $G_j (\cdot), \forall j \in \{1, \ldots, J\}$, can be expressed as a linear combination of $r$ basis functions: $g_1 (\cdot), \ldots, g_r (\cdot)$.

By setting

$$
A_{\lambda_k} (z) \triangleq 1 + a_1^{(\lambda_k)} z + \cdots + a_p^{(\lambda_k)} z^p,
$$

$$
B_{\lambda_k} (z) \triangleq b_1^{(\lambda_k)} + b_2^{(\lambda_k)} z + \cdots + b_q^{(\lambda_k)} z^{q-1},
$$

$$
G_{\lambda_k} (\cdot) = \sum_{l=1}^{r} c_l^{(\lambda_k)} g_l (\cdot),
$$

the system can be described as

$$
\begin{align*}
\begin{cases}
u_k = G_{\lambda_k} (u_k), \\
A_{\lambda_k} (z) y_{k+1} = B_{\lambda_k} (z) v_k + \xi_{k+1}, & k \geq 0; \\
u_k \not\equiv 0, v_k \not\equiv 0, \xi_{k+1} \not\equiv 0, y_{k+1} \not\equiv 0, & k < 0,
\end{cases}
\end{align*}
$$

(1)

where $u_k$ is the input, $v_k$ is the unmeasurable internal signal generated by $G_{\lambda_k} (\cdot)$, $y_k$ is the output, $\xi_k$ is the driven noise, and $z$ denotes the backward shift operator, $z y_k = y_{k-1}$.

On the other hand, we set

$$
\tilde{A}^{(k)} \triangleq \begin{bmatrix}
-a_1^{(\lambda_k)} & 1 & \cdots & 0 \\
\vdots & 0 & \ddots & \vdots \\
\vdots & \vdots & \ddots & 1 \\
-a_{l}^{(\lambda_{k+l-1})} & 0 & \cdots & 0
\end{bmatrix}_{h \times h},
$$

$$
\tilde{B}^{(k)} \triangleq \begin{bmatrix}
b_1^{(\lambda_{k})} & c_1^{(\lambda_{k})} & \cdots & b_r^{(\lambda_{k})} & c_r^{(\lambda_{k})} \\
\vdots & \ddots & \ddots & \vdots & \vdots \\
\vdots & \ddots & \ddots & \vdots & \vdots \\
b_h^{(\lambda_{k+h-1})} & c_1^{(\lambda_{k+h-1})} & \cdots & b_r^{(\lambda_{k+h-1})} & c_r^{(\lambda_{k+h-1})}
\end{bmatrix}_{h \times r},
$$

$$
C^\tau \triangleq [1 \ 0 \ \cdots \ 0]_{1 \times h}, \text{ and } \tilde{u}_k^{(k)} \triangleq [g_1 (u_k) \ \ldots \ g_r (u_k)]_{1 \times r}, \text{ where } h \triangleq \max \{p, q\}, a_l^{(\cdot)} \overset{\triangle}{=} 0, b_l^{(\cdot)} \overset{\triangle}{=} 0, \text{ for } l > p, m > q, j \in \{1, \ldots, J\}.$$

Then System \cite{16} can be expressed in the state space form as follows:

$$
\begin{align*}
x_{k+1} &= \tilde{A}^{(k)} x_k + \tilde{B}^{(k)} \tilde{u}_k + C \xi_{k+1}, \\
y_k &= C^\tau x_k, \\
x_0^\tau &= [y_0 \ 0 \ \cdots \ 0]_{1 \times h} = [0 \ 0 \ \cdots \ 0]_{1 \times h}.
\end{align*}
$$

Remark 1 It is seen that $\tilde{A}^{(k)}$ and $\tilde{B}^{(k)}$ take values in the finite sets, which will be denoted by $\{A^{(1)}, \ldots, A^{(s_1)}\}$ and $\{B^{(1)}, \ldots, B^{(s_2)}\}$, respectively.

We make the following assumption on the system.

(H0) For each $j \in \{1, 2, \ldots, J\}$, $\lambda^{-1} (\{j\})$ is an infinite subsequence of $N$, and

$$
\lambda^{-1} (\{j_1\}) \cap \lambda^{-1} (\{j_2\}) = \emptyset, \ \forall \ 1 \leq j_1 \neq j_2 \leq J,
$$

$$
\bigcup_{j=1}^{J} \lambda^{-1} (\{j\}) = N.
$$
Remark 2 By (H0) we preclude those subsystems that only operate for a finite number of times; this is reasonable when processing parameter identification task.

For System (1), the parameter estimation problem is to recursively estimate the unknown parameters \( a_1^{(j)}, \ldots, a_p^{(j)}, b_1^{(j)}, \ldots, b_q^{(j)}, c_1^{(j)}, \ldots, c_r^{(j)} \), \( \forall j \in \{1, \ldots, J\} \), based on the designed input \( \{u_k\}_{k=0}^\infty \) and the measured output \( \{y_k\}_{k=1}^\infty \).

3 Estimation Algorithm

Let \( j \in \{1, 2, \ldots, J\} \) be arbitrarily fixed. By (H0) we are able to write \( \lambda^{-1}(\{j\}) = \{k_l^{(j)}\}_{l=0}^\infty \) with \( k_l^{(j)} < k_s^{(j)} \) whenever \( 0 \leq l < s \). Clearly \( \{k_l^{(j)}\}_{l=0}^\infty \) denotes all the times at which the \( j \)-th Hammerstein subsystem operates; we have \( k_l^{(j)} \rightarrow \infty \) as \( t \rightarrow \infty \). It is worth noting that \( y_{k_l^{(j)}+1} \) is generated by the \( j \)-th subsystem, while \( y_{k_l^{(j)}-d} \), \( \forall d \in \{0, \ldots, p-1\} \), is not necessarily the output of the \( j \)-th subsystem.

Let us introduce a concept named intrinsic switch. Corresponding to \( [y_{k_l^{(j)}} \cdots y_{k_l^{(j)}+1-p}] \), we set \( n_l^{(j)} \triangleq [n_0^{(j)} \cdots n_{d_l^{(j)}-1}] \), where \( n_d^{(j)} \), \( d \in \{0, \ldots, p-1\} \) denotes the serial number of the Hammerstein subsystem that generates \( y_{k_l^{(j)}-d} \). It is seen that \( n_l^{(j)} \) is among \( J_p \triangleq K \) different combinations. From now on, we say an intrinsic switch occurs whenever \( n_l^{(j)} \) changes. Evidently, we may partition \( \{t\}_{t=0}^\infty \) into \( K \) subsequence \( \{t^{m}\}_{m=0}^\infty \), \( K = 1, \ldots, K \), such that for each \( \kappa \in \{1, \ldots, K\} \), \( n_l^{(j)}(m) \) is independent of \( m \). It is noticed that there exists at least one \( \kappa \in \{1, \ldots, K\} \) such that \( \{t^{m}\}_{m=0}^\infty \) is an infinite subsequence of \( \{t\}_{t=0}^\infty \).

Remark 3 The term “intrinsic switch” should be distinguished from “switch”; “switch” indicates the behavior that System (1) jumps from one Hammerstein subsystem to another.

By the notation introduced above, we know the \( j \)-th Hammerstein subsystem works by the following equation:

\[
(1 + a_1^{(j)} z + \cdots + a_p^{(j)} z^p) y_{k_l^{(j)}+1} = (b_1^{(j)} z + \cdots + b_q^{(j)} z^{q-1}) \sum_{t=1}^r c_{t}^{(j)} g_r(u_{k_l^{(j)}}) + \xi_{k_l^{(j)}+1}, \quad t = 0, 1, \ldots
\]

(3)

Denoting by

\[
\theta^{(j)} \triangleq [-a_1^{(j)} \cdots - a_p^{(j)} b_1^{(j)} c_1^{(j)} \cdots b_q^{(j)} c_r^{(j)} \cdots b_q^{(j)} c_r^{(j)}]^{\top}
\]

and

\[
\varphi_{t}^{(j)} \triangleq \left[ y_{k_l^{(j)}} \cdots y_{k_l^{(j)}+1-p} \begin{array}{c} g_1(u_{k_l^{(j)}}) \cdots g_r(u_{k_l^{(j)}+1-p}) \end{array} \right]^{\top}
\]

the parameters in the \( j \)-th regression subsystem and the regressor, respectively, we rewrite (3) as

\[
y_{k_l^{(j)}+1} = \theta^{(j)} \varphi_t^{(j)} + \xi_{k_l^{(j)}+1}, \quad t \geq 0.
\]

(4)
Let \( \{ \theta_t^{(i)} \}_{t \geq 1} \) be the estimates of \( \theta^{(i)} \). Set \( \theta_0^{(i)} \) arbitrarily and \( P_0^{(i)} = \alpha_0^{(i)} I \) with some \( \alpha_0^{(i)} \in (0, \frac{1}{c}) \).

The RLS algorithm estimating \( \theta^{(i)} \) is defined as follows:

\[
\theta_{t+1}^{(i)} = \theta_t^{(i)} + \tilde{a}_t^{(i)} P_t^{(i)} \varphi_t^{(i)} (y_{k_t^{(i)+1}} - \theta_t^{(i)} \varphi_t^{(i)}),
\]

\[
\tilde{a}_t^{(i)} = \frac{1}{1 + \varphi_t^{(i)\top} P_t^{(i)} \varphi_t^{(i)}},
\]

\[
P_{t+1}^{(i)} = P_t^{(i)} - \tilde{a}_t^{(i)} P_t^{(i)} \varphi_t^{(i)} \varphi_t^{(i)\top} P_t^{(i)},
\]

\[
\varphi_t^{(i)} = \begin{bmatrix} y_{k_t^{(i)}} & \cdots & y_{k_t^{(i)+1-p}} & g_1(u_{k_t^{(i)}}) & \cdots & g_r(u_{k_t^{(i)+1-q}}) \end{bmatrix}^\top.
\]

By (6) and (7) it follows that \( (P_t^{(i)})^{-1} = \sum_{i=0}^t \varphi_i^{(i)\top} \varphi_i^{(i)} + \frac{1}{\alpha_0^{(i)}} I \).

Let \( \{ \xi_k, \mathcal{F}_k \} \) be an adapted sequence of random matrices with \( \{ \xi_k, \mathcal{F}_k \} \) a martingale difference sequence with

\[
\sup_k \mathbb{E} \left[ |\xi_{k+1}|^\beta \right] \mathcal{F}_k < \infty \text{ a.s.}, \quad \beta \geq 2,
\]

where \( \{ \mathcal{F}_k \} \) is a sequence of nondecreasing sub \( \sigma \)-algebras of \( \mathcal{F} \).

(H1') \( \{ \xi_k, \mathcal{F}_k \} \) is a martingale difference sequence with

\[
\sup_k |\xi_k| \leq W < \infty \text{ a.s.},
\]

where \( W \) is a positive constant, and \( \{ \mathcal{F}_k \} \) is a sequence of nondecreasing sub \( \sigma \)-algebras of \( \mathcal{F} \).

(H2) \( \lim_{k \to \infty} \frac{1}{k} \sum_{i=1}^k \xi_i^2 = R_\xi > 0 \text{ a.s.}, \) where \( R_\xi \) is a constant.

(H3) \( \{ 1, g_1(\cdot), \ldots, g_r(\cdot) \} \) is linearly independent over some interval \([a, b]\), and \( g_l(\cdot), \forall l \in \{1, \ldots, r\} \), is continuous on \([a, b]\).

(H5) There exists a \( \gamma > 0 \) such that as \( t \to \infty, \sum_{i=0}^t y_i^{(i)\top} y_i^{(i)} = O(t^\gamma) \text{ a.s.}, \forall d \in \{0, \ldots, p - 1\} \).

(H5') There exists a finite positive integer \( \tilde{n} \) such that \( \|A^{(i_1)} A^{(i_2)} \cdots A^{(i_{\tilde{n}})}\| < 1, \forall A^{(i_m)} \in \{ A^{(i)}, \ldots, A^{(s_1)} \} \), for \( m = 1, \ldots, \tilde{n} \), where \( \|\cdot\| \) is the induced 1-norm:

\[
\|A\| \triangleq \max_{1 \leq d_2 \leq \tilde{t}_2} \sum_{d_1=1}^{\ell_1} |a_{d_1d_2}|, \forall A = (a_{d_1d_2})_{\ell_1 \times \ell_2} \in \mathbb{R}^{\ell_1 \times \ell_2}.
\]

**Remark 4** Note that (H5'), as well as (H5), is a condition concerning stability of System [1]. Stability of time-varying systems is discussed in [12] by introducing an assumption similar to (H5').

For convenience of citation, we list a lemma here:

**Lemma 1** (Theorem 2.8 of [11]) Let \( \{ X_k, \mathcal{G}_k \} \) be a matrix martingale difference sequence and let \( \{ M_k, \mathcal{G}_k \} \) be an adapted sequence of random matrices with \( \| M_k \| < \infty \text{ a.s.}, \forall k \geq 0 \). If

\[
\sup_k \mathbb{E} \left[ \|X_{k+1}\|^\alpha \right] \mathcal{G}_k < \infty \text{ a.s.}
\]
for some $\alpha \in (0, 2]$, then as $k \to \infty$

$$\sum_{i=0}^{k} M_{i} X_{i+1} = O \left( s_{k}(\alpha) \log^{\frac{1}{\alpha}} (s_{k}^{\alpha}(\alpha) + \epsilon) \right) \text{ a.s., } \forall \eta > 0, \tag{9}$$

where $s_{k}(\alpha) = \left( \sum_{i=0}^{k} \| M_{i} \|_{\alpha} \right)^{\frac{1}{\alpha}}$.

We give the convergence analysis of Algorithm (5)-(8) for two cases as follows.

### 4.1 Case I—Using the i.i.d.-Type Input

The i.i.d.-type input is taken satisfying:

(H4) $\{u_{k}\}$ is a sequence of i.i.d. random variables with density $p(\cdot)$, which is positive and continuous over $[a, b]$, and vanishes outside $[a, b]$. Besides, $\{u_{k}\}$ is independent of $\{\xi_{k}\}$.

Before proving our first result (Theorem 1), we need lemmas 2-5.

**Lemma 2** (Lemma 1 of [12]) If (H3) and (H4) hold, then

$$R \triangleq E[g_{1}(u_{k}) - \mu_{1} \cdots g_{r}(u_{k}) - \mu_{r}] > 0,$$

where $\mu_{l} \triangleq E g_{l}(u_{k}), \forall l \in \{1, \ldots, r\}$.

**Lemma 3** If (H1’s), (H3), (H4), and (H5’s) hold, then $y_{k} = O(1)$ a.s., as $k \to \infty$.

**Proof** The proof is straightforward since System (2), and thereby System (1), is a contraction mapping.

By $\lambda_{\max}^{(j)}(t)$ and $\lambda_{\min}^{(j)}(t)$ we denote the largest and smallest eigenvalue of $\left( P_{t+1}^{(j)} \right)^{-1}$, respectively. The following two lemmas are motivated by Theorems 4.1 and 6.2 in [11], respectively.

**Lemma 4** Assume that (H0) and (H1) hold, and that $u_{n}$ is $\mathcal{F}_{n}$-measurable for all $n \geq 0$. Then as $t \to \infty$ the convergence (or divergence) rate of the estimate given by Algorithm (5)–(8) is expressed by

$$\| \theta_{t+1}^{(j)} - \theta^{(j)} \|^{2} = O \left( \frac{\log \lambda_{\max}^{(j)}(t) (\log \log \lambda_{\max}^{(j)}(t))^{\delta(\beta-2)}}{\lambda_{\min}^{(j)}(t)} \right) \text{ a.s.}, \tag{10}$$

where $\delta(x) \triangleq \begin{cases} 0, & x \neq 0; \\ c, & x = 0, \end{cases}$ with arbitrary constant $c > 1$.

**Proof** Applying the same method as that used in the proof of Theorem 4.1 in [11], we arrive at the desired result.

**Lemma 5** If (H0)–(H4) hold, then the following assertions are true.

1) It holds that

$$\liminf_{t \to \infty} \frac{\lambda_{\min}^{(j)}(t)}{t} > 0 \text{ a.s.} \tag{11}$$

2) If, in addition, (H5) holds, then the RLS estimate given by Algorithm (5)–(8) is strongly consistent and has the following convergence rate:

$$\| \theta_{t+1}^{(j)} - \theta^{(j)} \|^{2} = O \left( \frac{\log t (\log \log t)^{\delta(\beta-2)}}{t} \right) \text{ a.s.}. \tag{12}$$
Proof Analogously to the proof of Theorem 3 in [12], which is motivated by the proof of Theorem 6.2 in [11], we give the detailed proof of the lemma in Appendix.

We are now in a position to give and prove our first theorem.

Theorem 1 If \((H0), (H1'), (H2)-(H4),\) and \((H5')\) hold, then the RLS estimate given by Algorithm 5 is strongly consistent and has the following convergence rate:

\[
\|\theta_{t+1}^{(i)} - \theta^{(i)}\|^2 = O((\log t)/t) \ a.s.
\]  

(13)

Proof Combining Lemmas 3 and 5 yields the theorem.

4.2 Case II—Integrating the Given Adaptive Control with a Diminishingly Excited Signal

Assume the following assumption holds:

\((H3')\) \(g_l(x) \triangleq x^l, \forall x \in \mathbb{R}, \forall l \in \{1, \ldots, r\}.\)

Let \(\{\xi_k\}\) be a sequence of i.i.d. random variables with continuous distribution, and let \(\{\xi_k\}\) be independent of \(\{\xi_k\}\) with \(E\xi_k = 0, E\xi_k^2 = 1,\) and \(|\xi_k| \leq \delta_0,\) where \(\delta_0 > 0\) is a constant. Define

\[
v_k^{(d)} \triangleq \frac{\xi_k}{k^{r/2}}
\]

(14)

with \(\epsilon > 0\) sufficiently small such that the interval \((\frac{1}{n}, 1 - (M + 1)r\epsilon]\) is nonempty, where \(M = Jp + q - 1.\)

Without loss of generality, we assume \(\{\mathcal{F}_k\}\) is rich enough such that \(\xi_k, v_k^{(d)} \in \mathcal{F}_k.\) Set \(\mathcal{F}'_{k-1} \triangleq \sigma\{\xi_i, 0 \leq i_1 \leq k, \xi_i, 0 \leq i_2 \leq k - 1\}.\)

Motivated by Theorem 6.2 in [11], we introduce the following hypothesis.

\((H4')\) The given adaptive control \(u_k^{(c)}\) is \(\mathcal{F}'_{k-1}\)-measurable, i.e., \(u_k^{(c)} \in \mathcal{F}'_{k-1},\) \(\forall k,\) and \(u_k^{(c)} = O(1)\) a.s., as \(k \rightarrow \infty.\)

The diminishing excitation technique suggests to take

\[u_k \triangleq u_k^{(c)} + v_k^{(d)}\]

(15)

as the actual input, where \(v_k^{(d)}\) is given by (14).

Define

\[
U(k) \triangleq \begin{bmatrix}
1 & 1 \\
C_2^{1}v_k^{(c)} & C_2^{1}v_k^{(c)} \\
\vdots & \vdots \\
C^{r-1}_r(u_k^{(c)})^{r-1} & C^{r-1}_r(u_k^{(c)})^{r-1} \\
\end{bmatrix}_{r \times r} \quad \text{and} \quad \tau_k \triangleq \begin{bmatrix}
v_k^{(d)} \\
\vdots \\
(v_k^{(d)})^r
\end{bmatrix}_{r \times 1}
\]

The following lemma is a corollary of Lemma 4 in [17].

Lemma 6 Let \(\{k_s\}_{s=0}^{\infty}\) be an infinite subsequence of \(\{k\}_{k=0}^{\infty}\) and let \(\{t_n\}_{n=0}^{\infty}\) be an infinite subsequence of \(\{t\}_{t=0}^{\infty}\). If \((H4')\) holds, then we have

\[
\frac{1}{t_n} \sum_{s=0}^{t_n} U(k_s) (\tau_{k_s} - E\tau_{k_s}) (\tau_{k_s} - E\tau_{k_s})^\top U^\top (k_s) \geq \tilde{c}_0 I \ \ a.s.
\]

(16)
for all large enough \( t_n \), where \( \delta_0 > 0 \) may depend on sample paths.

Proof Noticing (H4'), we obtain (16) by investigating its counterpart in [17] with \( s \) replaced by \( r \) and \( \delta \) set as 0.

Modified from Theorem 2 in [17], we have the following theorem in parallel to Theorem 1.

**Theorem 2** If (H0), (H1'), (H2), and (H3')–(H5') hold, then the RLS estimate given by Algorithm (5)–(8) is strongly consistent and has the following convergence rate:

\[
\|\theta_{t+1}^{(j)} - \theta^{(j)}\|^2 = O \left( \frac{\log t}{t^\alpha} \right) \quad \text{a.s., \ \forall \alpha \in \left( \frac{1}{2}, 1 - (M + 1) r \right]}.
\] (17)

Proof (outline) Bearing a resemblance to the proof of Lemma 5 (see Appendix), for simplicity of notation, we omit the superscript \( (j) \). Reviewing the proofs of Lemma 5 and Theorem 1, we see that to prove the present theorem, it suffices to show

\[
\liminf_{t \to \infty} \frac{1}{t^\alpha} \lambda_{\min} \left( \sum_{i=0}^{t} f_i f_i^T \right) > 0 \quad \text{a.s., \ \forall \alpha \in \left( \frac{1}{2}, 1 - (M + 1) r \right]}
\] (18)

where \( f_i \triangleq \prod_{s=1}^{J} A_s (z) \varphi_i \). Applying once again the method of reduction to absurdity and the procedure of subsequence partitioning and seeking (see Remark 6 at the end of Appendix) as that used in the proof of Lemma 5, and reasoning similarly to the proof of Theorem 2 in [17] with Lemmas 1 and 6 used repeatedly, we obtain the expected result.

### 5 Simulation Example

Consider the following system:

\[
\begin{align*}
y_{2t} &= 1.1 y_{2t-1} - 0.28 y_{2t-2} + 0.5 u_{2t-1} - 1.5 u_{2t-1}^2 + 2 u_{2t-1}^3 \\
&\quad - 2(0.5 u_{2t-2} - 1.5 u_{2t-2}^2 + 2 u_{2t-2}^3) + \xi_{2t}, \\
y_{2t-1} &= 0.8 y_{2t-2} - 0.15 y_{2t-3} + 0.4 u_{2t-2} + 1.6 u_{2t-2}^2 - 0.8 u_{2t-2}^3 \\
&\quad - 3(0.4 u_{2t-3} + 1.6 u_{2t-3}^2 - 0.8 u_{2t-3}^3) + \xi_{2t-1}, \\
t &\geq 2.
\end{align*}
\] (19)

Let us verify (H5') for System (19) first. It is seen that

\[
A^{(1)} = \begin{pmatrix} 1.1 & 1 \\ -0.28 & 0 \end{pmatrix}, \quad A^{(2)} = \begin{pmatrix} 1.1 & 1 \\ -0.15 & 0 \end{pmatrix},
\]

\[
A^{(3)} = \begin{pmatrix} 0.8 & 1 \\ -0.15 & 0 \end{pmatrix}, \quad A^{(4)} = \begin{pmatrix} 0.8 & 1 \\ -0.28 & 0 \end{pmatrix}.
\]

Using MATLAB to calculate, we find that (H5') holds with \( \bar{n} = 9 \).

We now assign the noise \( \{\xi_k\} \) and the excitation source \( \{\varepsilon_k\} \), and set the initial values for Algorithm (5)–(8). Let \( \{\xi_k\}_{k \geq 3} \) be i.i.d. and uniformly distributed on \([-3, 3]\). Take \( \{\varepsilon_k\}_{k \geq 1} \) to be i.i.d. and uniformly distributed on \([-2, 2]\) and independent of \( \{\xi_k\} \). Set \( \theta_0^{(1)} = \theta_0^{(2)} = 0 \) and \( P_0^{(1)} = P_0^{(2)} = 0.2 I_8 \), where \( I_8 \) denotes the 8 x 8 identity matrix.

Two types of input are taken separately to serve the parameter estimation task:
Case I Set \( u_k \triangleq \varepsilon_k \). It is noticed that all the conditions \((H0), (H1'), and (H2)--(H4)\) are fulfilled. Thus, by Theorem 1, the estimate given by Algorithm \([5]--[8]\) is strongly consistent. On the other hand, using the designed input and the collected output to execute Algorithm \([5]--[8]\) twice, each running 2000 steps, we obtain the recursive estimation for the parameters of System \((19)\) as shown by Fig. 2.

![Recursive estimation of the parameters of the regression subsystem (1)](image1)

![Recursive estimation of the parameters of the regression subsystem (2)](image2)

**Fig. 2. Simulation results (I)**

Case II Disregarding the specific control cost, we suppose that \( u^{(c)}_k \triangleq \frac{1}{y_{k-1} + |y_{k-1}| + 1} \) is the given adaptive control at time \( k \). Set \( v^{(c)}_k \triangleq \frac{1}{y_{k-1} + |y_{k-1}| + 1} \) and \( v_k \triangleq u^{(c)}_k + v^{(c)}_k = \frac{1}{y_{k-1} + |y_{k-1}| + 1} + \frac{\varepsilon_k}{|y_{k-1}| + 1} \). Clearly, all the assumptions needed by Theorem 2 hold; hence, by Theorem 2, the estimate given by Algorithm \([5]--[8]\) is strongly consistent.

In this case, the corresponding simulation results are presented in Fig. 3.

![Recursive estimation of the parameters of the regression subsystem (1)](image3)

![Recursive estimation of the parameters of the regression subsystem (2)](image4)

**Fig. 3. Simulation results (II)**

It is seen that in either case the simulation outcome convincingly validates the theoretical
analysis.

**Remark 5** To derive the estimates of $b_{1}^{(1)}, b_{2}^{(1)}, c_{1}^{(1)}, c_{2}^{(1)}, c_{3}^{(1)}, b_{1}^{(2)}, b_{2}^{(2)}, c_{1}^{(2)}, c_{2}^{(2)}, c_{3}^{(2)}$ from the simulation results, we need to introduce appropriate identifiable conditions, see, e.g., [12] or [13] or [14] for details.

6 Concluding Remarks

In this study, we apply the RLS algorithm to estimate the parameters of each parameterized subsystem of the SISO switched Hammerstein system, and under reasonable conditions we establish the strong consistency of the estimates. Especially, in the second case, by using the diminishing excitation technique, we also cater to adaptive control demands. For further work, it is of interest to consider the closed-loop identification problems with control costs associated.

7 Appendix

**Proof of Lemma 5** For simplicity of notation, we omit the superscript $(j)$ wherever it is used to indicate the serial number of the chosen subsystem.

Define $f_{i} \triangleq \prod_{s=1}^{j} A_{s}(z) \varphi_{i}$. By expanding $\prod_{s=1}^{j} A_{s}(z)$ as $\prod_{s=0}^{J} A_{s}(z) = \sum_{s=0}^{J} \nu_{s} z^{s}$ with $\nu_{0} \triangleq 1$, we have $f_{i} = \sum_{s=0}^{J} \nu_{s} \varphi_{i-s}$ and

$$f_{i}^{T} = \prod_{s=1}^{j} A_{s}(z) \left[ y_{k_{i}} \cdots y_{k_{i}+1-p} g_{1}(u_{k_{i}}) \cdots ight.$$

$$\left. g_{r}(u_{k_{i}}) \cdots g_{1}(u_{k_{i}+1-q}) \cdots g_{r}(u_{k_{i}+1-q}) \right] = \left[ \prod_{s=1}^{j} A_{s}(z) / A_{n_{d}^{(i)}}(z) \right] \left[ \prod_{s=1}^{j} A_{s}(z) / A_{n_{d}^{(i)}}(z) \right] \left[ \prod_{s=1}^{j} A_{s}(z) / A_{n_{d}^{(i)}}(z) \right] \left[ \prod_{s=1}^{j} A_{s}(z) / A_{n_{d}^{(i)}}(z) \right]$$

$$\left[ \prod_{s=1}^{j} A_{s}(z) / A_{n_{d}^{(i)}}(z) \right] \left[ \prod_{s=1}^{j} A_{s}(z) / A_{n_{d}^{(i)}}(z) \right] \left[ \prod_{s=1}^{j} A_{s}(z) / A_{n_{d}^{(i)}}(z) \right]$$

where for each $d \in \{0, \ldots, p-1\}$, by $n_{d}^{(i)}$ we denote the serial number of the Hammerstein subsystem that generates $y_{k_{i}-d}$. Clearly, $n_{d}^{(i)} \in \{1, \ldots, J\}$. 

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Using the Cauchy-Schwarz inequality, we see that

$$\lambda_{\min}\left(\sum_{i=0}^{t} f_i f_i^T\right) \leq \inf_{\|x\|=1} \sum_{i=0}^{t} (1 + Jp) \sum_{s=0}^{Jp} \nu_s^2 (x^T \varphi_{i-s})^2$$

$$\leq (1 + Jp) \left(\sum_{s=0}^{Jp} \nu_s^2\right) \lambda_{\min} \left(t\right).$$ \hspace{1cm} (21)

Thus, in order to prove \[(11)\], we need only to show that

$$\lim \inf_{t \to \infty} \frac{1}{t} \lambda_{\min}\left(\sum_{i=0}^{t} f_i f_i^T\right) > 0 \text{ a.s.}$$ \hspace{1cm} (22)

We use the method of reduction to absurdity. If \[(22)\] were not true, then there would exist a measurable set \(D\) such that

$$\lim \inf_{t \to \infty} \frac{1}{t} \lambda_{\min}\left(\sum_{i=0}^{t} f_i f_i^T\right) = 0, \forall \omega \in D.$$ \hspace{1cm} (23)

We arbitrarily choose \(\omega_0 \in D\) and fix it. By \[(23)\] we know that there exist a subsequence \(\{t_n\}_{n \geq 0}\) of \(\{t\}_{t \geq 0}\) and a sequence of vectors \(\{\eta_{t_n}\}_{n \geq 0}\) with \(\|\eta_{t_n}\| = 1\) such that on the sample path \(\omega_0\) we have

$$\lim_{n \to \infty} \frac{1}{t_n} \sum_{i=0}^{t_n} (\eta_{t_n} f_i)^2 = 0.$$ \hspace{1cm} (24)

Write \(\eta_{t_n}\) as

$$\eta_{t_n} \triangleq \begin{bmatrix} \alpha^{(0)}_{t_n} & \ldots & \alpha^{(p-1)}_{t_n} & \beta^{(1,1)}_{t_n} & \ldots & \beta^{(1,r)}_{t_n} & \ldots & \beta^{(q,1)}_{t_n} & \ldots & \beta^{(q,r)}_{t_n} \end{bmatrix}^T.$$ \hspace{1cm} (25)

The boundedness of \(\{\eta_{t_n}\}\) implies the existence of its convergent subsequence. We arbitrarily choose such a subsequence and still use the same notation as \(\{\eta_{t_n}\}\) to denote it; accordingly, we are able to write

$$\eta_{t_n} \xrightarrow{\text{n-}\to\infty} \eta \triangleq \begin{bmatrix} \alpha^{(0)} & \ldots & \alpha^{(p-1)} & \beta^{(1,1)} & \ldots & \beta^{(1,r)} & \ldots & \beta^{(q,1)} & \ldots & \beta^{(q,r)} \end{bmatrix}^T,$$ \hspace{1cm} (26)

where \(\|\eta\| = 1\).

From \[(20)\] and \[(25)\] we obtain
\[\eta_{tn} f_1 = \left\{ \alpha_{t_n}^{(0)} \prod_{i=1}^{J} A_s(z) B_{n_0}^{(i)}(z) z_{c_1} \{^n_{i}^{(0)} \} \cdots \alpha_{t_n}^{(0)} \prod_{i=1}^{J} A_s(z) B_{n_0}^{(i)}(z) z_{c_r} \{^n_{i}^{(r)} \} \right\} + \cdots + \left\{ \alpha_{t_n}^{(p-1)} \prod_{s=1}^{J} A_s(z) B_{n_0}^{(i)}(z) z_{c_1} \{^n_{i}^{(p-1)} \} \cdots \alpha_{t_n}^{(p-1)} \prod_{s=1}^{J} A_s(z) B_{n_0}^{(i)}(z) z_{c_r} \{^n_{i}^{(p-1)} \} \right\}
+ \cdots + \left\{ \beta_{t_n}^{(1,1)} \prod_{s=1}^{J} A_s(z) \cdots \beta_{t_n}^{(1,r)} \prod_{s=1}^{J} A_s(z) \right\} \cdot [g_1(u_{k_1}) \cdots g_r(u_{k_r}) \xi_{k_i}]^\top, \tag{27} \]

which can be rewritten as
\[\eta_{tn} f_1 \triangleq \left[ \sum_{m=0}^{M} \tilde{h}_{t_n}^{(1,m)}(z) z^m \cdots \sum_{m=0}^{M} \tilde{h}_{t_n}^{(r,m)}(z) z^m \sum_{m=0}^{M} \tilde{h}_{t_n}^{(0,m)}(z) z^m \right] \cdot [g_1(u_{k_1}) \cdots g_r(u_{k_r}) \xi_{k_i}]^\top, \tag{28} \]
where \(M = Jp + q - 1,\)
\[\sum_{m=0}^{M} \tilde{h}_{t_n}^{(1,m)}(z) z^m = \sum_{m=0}^{p-1} \alpha_{t_n}^{(m)} \prod_{s=1}^{J} A_s(z) B_{n_0}^{(i)}(z) z^m + \sum_{m=0}^{q-1} \beta_{t_n}^{(m+1,t)} \prod_{s=1}^{J} A_s(z) z^m, \quad l = 1, \ldots, r, \tag{29} \]
and
\[\sum_{m=0}^{M} \tilde{h}_{t_n}^{(0,m)}(z) z^m = \sum_{m=0}^{p-1} \alpha_{t_n}^{(m)} \prod_{s=1}^{J} A_s(z) z^m. \tag{30} \]

Recalling the concept intrinsic switch introduced in Section 3, we see that there exist \(K\) subse-
quences \(\{i_s^{(s)}, s \geq 0\}, \kappa = 1, \ldots, K,\) of \(\{i_s^{(s)}, s \geq 0\}\) such that \(\{i_s^{(s_1)}, s \geq 0\} \cap \{i_s^{(s_2)}, s \geq 0\} = \emptyset, \forall \kappa \leq \kappa_1 \neq \kappa_2 \leq K, \bigcup_{\kappa=1}^{K} \{i_s^{(s)}, s \geq 0\} = \{i_s^{(s)}, s \geq 0\}, \text{ and } \left[ n_0^{(i_0^{(s_0)})} \cdots n_{p-1}^{(i_0^{(s_0)})} \right], \forall \kappa \in \{1, \ldots, K\},\) is independent of
Since for each \( d \in \{0, \ldots, p - 1\} \), \( n_d^{(s)} \) depends only on \( \kappa \), let us rewrite it as \( n_d^{(s)} \) from now on. Obviously, there exists at least one \( \kappa \in \{1, \ldots, K\} \) such that \( \{i_s^{(\kappa)}, s \geq 0\} \) is an infinite subsequence of \( \{i_i\}_{i=0}^{\infty} \). Without loss of generality, we may assume that for each \( \kappa \in \{1, \ldots, K\} \), \( \{i_s^{(\kappa)}, s \geq 0\} \) is an infinite subsequence of \( \{i_i\}_{i=0}^{\infty} \).

Let us rewrite (28) as

\[
\eta_t^{\kappa, f_{i_s^{(\kappa)}}} \triangleq \left[ \sum_{m=0}^{M} \tilde{h}_{t_n}^{(l,m)(s)} z^m \right] \left[ \sum_{m=0}^{M} \tilde{h}_{t_n}^{(r,m)(s)} z^m \right]
\]

by noticing that \( \tilde{h}_{t_n}^{(l,m)(s)} \), \( \forall l \in \{0, 1, \ldots, r\} \), is independent of \( s \).

Corresponding to (29) and (30), we have

\[
\sum_{m=0}^{M} h_t^{(l,m)(s)} z^m = \sum_{m=0}^{p-1} \alpha_{t_n}^{(m)} \prod_{s=1}^{J} A_s(z) B_{n_m}^{(s)}(z) z^{m+1} c_l^{(n_s)}
\]

\[
+ \sum_{m=0}^{q-1} \beta_{t_n}^{(m+1,l)} \prod_{s=1}^{J} A_s(z) z^m, \quad l = 1, \ldots, r, \quad \kappa = 1, \ldots, K,
\]

and

\[
\sum_{m=0}^{M} h_t^{(l,m)(s)} z^m = \sum_{m=0}^{p-1} \alpha_{t_n}^{(m)} \prod_{s=1}^{J} A_s(z) z^m, \quad \kappa = 1, \ldots, K,
\]

where \( h_t^{(l,m)(s)} \in \mathbb{R}, \forall l \in \{0, 1, \ldots, r\}, m \in \{0, \ldots, M\}, \kappa \in \{1, \ldots, K\} \). It is seen that \( \{h_t^{(l,m)(s)} : l \in \{0, 1, \ldots, r\}, m \in \{0, \ldots, M\}, \kappa \in \{1, \ldots, K\}\} \) is bounded.

We now derive from (24) that there exist \( \kappa \in \{1, \ldots, K\} \), an infinite subsequence of \( \{t_i\}_{i=0}^{\infty} \), and an infinite subsequence of \( \{t_n\}_{n=0}^{\infty} \), where the latter two are denoted by \( \{\tilde{t}_n^{(\kappa)}\}_{n=0}^{\infty} \) and \( \{\tilde{t}_n\}_{n=0}^{\infty} \), respectively, such that

\[
\lim_{n \to \infty} \frac{1}{\tilde{t}_n^{(\kappa)}} \sum_{s=0}^{\tilde{t}_n^{(\kappa)}} \left( \eta_{\tilde{t}_n^{(\kappa)}} f_{i_s^{(\kappa)}} \right)^2 = 0.
\]

Actually, it is obvious that there exist \( K \) infinite subsequences of \( \{t_i\}_{i=0}^{\infty} \), denoted by \( \{t^{(1)}_{n}\}_{n=0}^{\infty}, \ldots \), and \( \{t^{(K)}_{n}\}_{n=0}^{\infty} \), respectively, such that for each \( n \in \mathbb{N} \), it holds that

\[
\{i_s^{(\kappa_1)}\}_{s=0}^{\tilde{t}^{(\kappa_1)}_n} \cap \{i_s^{(\kappa_2)}\}_{s=0}^{\tilde{t}^{(\kappa_2)}_n} = \emptyset, \quad \forall 1 \leq \kappa_1 \neq \kappa_2 \leq K,
\]

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$$\bigcup_{\kappa=1}^{K} \{t^{(\kappa)}_{n}\}_{s=0}^{t_n} = \{i\}_{i=0}^{t_n},$$

$$\sum_{i=0}^{t_n} (\eta_{t_n}^{*} f_i)^2 = \sum_{\kappa=1}^{K} \sum_{s=0}^{t_n^{(\kappa)}} \left(\eta_{t_n^{(\kappa)}}^{*} f_{t^{(\kappa)}_{n}}\right)^2,$$  \hspace{1cm} (35)

and

$$t_n + 1 = \sum_{\kappa=1}^{K} t_n^{(\kappa)} + K.$$  

From (35) and (24) we see that

$$\frac{1}{t_n + 1} \sum_{i=0}^{t_n} (\eta_{t_n}^{*} f_i)^2 = \frac{1}{\sum_{\kappa=1}^{K} t_n^{(\kappa)} + K} \sum_{\kappa=1}^{K} \sum_{s=0}^{t_n^{(\kappa)}} \left(\eta_{t_n^{(\kappa)}}^{*} f_{t^{(\kappa)}_{n}}\right)^2 \xrightarrow{n \to \infty} 0. \hspace{1cm} (36)$$

We now show \((34)\). Assume the converse: For every \(\kappa \in \{1, \ldots, K\}\),

$$\liminf_{n \to \infty} \frac{1}{t_n^{(\kappa)}} \sum_{s=0}^{t_n^{(\kappa)}} \left(\eta_{t_n^{(\kappa)}}^{*} f_{t^{(\kappa)}_{s}}\right)^2 > 0. \hspace{1cm} (37)$$

Then there exist a positive constant \(c_0\) and a sufficiently large positive integer \(N\) such that

$$\sum_{s=0}^{t_n^{(\kappa)}} \left(\eta_{t_n^{(\kappa)}}^{*} f_{t^{(\kappa)}_{s}}\right)^2 \geq c_0 t_n^{(\kappa)}, \hspace{1cm} \forall \kappa \in \{1, \ldots, K\}, \forall n \geq N, \hspace{1cm} (38)$$

which leads to

$$\frac{1}{\sum_{\kappa=1}^{K} t_n^{(\kappa)} + K} \sum_{\kappa=1}^{K} \sum_{s=0}^{t_n^{(\kappa)}} \left(\eta_{t_n^{(\kappa)}}^{*} f_{t^{(\kappa)}_{s}}\right)^2 \geq \frac{c_0 \sum_{\kappa=1}^{K} t_n^{(\kappa)}}{\sum_{\kappa=1}^{K} t_n^{(\kappa)} + K}, \forall n \geq N$$

or

$$\liminf_{n \to \infty} \frac{1}{\sum_{\kappa=1}^{K} t_n^{(\kappa)} + K} \sum_{\kappa=1}^{K} \sum_{s=0}^{t_n^{(\kappa)}} \left(\eta_{t_n^{(\kappa)}}^{*} f_{t^{(\kappa)}_{s}}\right)^2 \geq c_0 > 0,$$

contradicting (36). Thus, (37) is not true, and we have shown that there exists a \(\kappa \in \{1, \ldots, K\}\) such that

$$\liminf_{n \to \infty} \frac{1}{t_n^{(\kappa)}} \sum_{s=0}^{t_n^{(\kappa)}} \left(\eta_{t_n^{(\kappa)}}^{*} f_{t^{(\kappa)}_{s}}\right)^2 = 0,$$

which implies (34).

From now on, let the \(\kappa\) in (34) be fixed. For simplicity of notation, we omit the superscript "" in (34) and thereafter:

$$\lim_{n \to \infty} \frac{1}{t_n^{(\kappa)}} \sum_{s=0}^{t_n^{(\kappa)}} \left(\eta_{t_n^{(\kappa)}}^{*} f_{t^{(\kappa)}_{s}}\right)^2 = 0. \hspace{1cm} (39)$$

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Recall Lemmas 1 and 2. Arguing similarly to the proof of Theorem 3 in [12], which is motivated by the proof of Theorem 6.2 in [11], we derive from (26), (31), (32), (33), and (39) that
\[ \eta \triangleq \begin{bmatrix} \alpha(0) & \cdots & \alpha(p-1) \\ \beta(1,1) & \cdots & \beta(1,r) \\ \vdots & \ddots & \vdots \\ \beta(q,1) & \cdots & \beta(q,r) \end{bmatrix}^T \tau = 0, \] (40)
which contradicts \( \|\eta\| = 1 \). Thus \( I \) is established.

We now prove \( 2) \).

To this end, recalling (H4), without loss of generality, for each \( n \geq 0 \), we may assume \( u_n \) is \( \mathcal{F}_n \)-measurable, and therefore by Lemma 4 and \( 1) \) of Lemma 5, we need only to show there exists a \( \varrho > 0 \) such that
\[ \lambda_{\max}(t) = O(t^\varrho) \quad a.s. \] (41)
In fact, by (H5) it follows that
\[
\begin{align*}
\lambda_{\max}(t) \leq & \text{tr}\left( \sum_{i=0}^{t} \varphi_i \varphi_i^T + \frac{1}{\alpha_0} I \right) \\
= & O\left( \text{tr} \sum_{i=0}^{t} \varphi_i \varphi_i^T \right) = O\left( \sum_{i=0}^{t} \|\varphi_i\|^2 \right) \\
= & O\left( \sum_{i=0}^{t} \sum_{m=0}^{p-1} y_{k_i-m}^2 + \sum_{i=0}^{t} \sum_{m=0}^{q-1} \sum_{l=1}^{r} (g_l(u_{k_i-m}))^2 \right) \\
= & O\left( O(t^\gamma) + O(t) \right) = O(t^\gamma) \quad a.s.,
\end{align*}
\] (42)
where \( \varrho \triangleq \max(\gamma, 1) \); hence, \( 2) \) is true and the proof of Lemma 5 is completed.

\textbf{Remark 6} It is observed that throughout the proof of Lemma 5, the procedure of deriving (31)–(34), which can be characterized as “subsequence partitioning and seeking,” plays an important role; combining this procedure with the existing techniques applied in the proofs of Theorem 6.2 in [11] and Theorem 3 in [12] leads to the desired result.

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