Use of parabolic mirrors as guide splitters

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Abstract. The use of non-linear tapered neutron guides has been extended over the last years and there are many examples of their performance and potential in several neutron scattering facilities around the world. However, the potential of these geometries is not fully explored. On the other hand, the increasing demand of neutron scattering instruments creates the need of more space for more instruments. One way to obtain it is by guide splitting. In this report we propose a guide splitter using a parabolic mirror. We make an analytical study and Monte Carlo simulations in order to observe its properties, performance and potential. The advantages of this configuration are clear: the filtering of unwanted neutrons in a shorter space than the traditional curved guide sections, the low divergence of the obtained beam and the lack of making larger moderators. However, the main disadvantage is the non-uniform divergence distribution of the beam.

1. Introduction

The use of non-linear tapered neutron guides has been extended over the last years and it has been used in neutron sources all over the world [1-4]. Elliptical guides have shown outstanding neutron transport properties [5] and they also have been used as focusing devices. On the other hand, Parabolic guides have been also used as focusing devices in order to use them in small samples [6].

However, the potential of non-linear geometries for neutron transport has not been completely explored. One of the main issues when designing the neutron beam transport system is the space needed for instruments. The demand for using neutron scattering instruments has increased with time, and new powerful instruments are needed to meet this demand. One possible solution is splitting the guides. However, if straight guides are used, the splitting demands larger moderators in order to have decent neutron transport or smaller guides to fit the size of the moderator. The first solution would increase the maintenance costs of the moderator and the second one would increase the reflection losses.
In this report, we propose a novel design to split guides without using larger moderators. For this purpose we want to develop the potentials of parabolic geometry for neutron transportation. We will study the suitability of this design analytically and by means of Monte Carlo (MC) simulations.

2. Description of the beam splitter and coupling of a point source

Figure 1 shows the proposed beam splitter for a point source. The source will be at the focus of the parabolic mirror and will convert the most divergent neutrons coming from the source into a new neutron beam (called secondary guide thereafter), which, in the case of a point source, has zero divergence. The length of the parabolic mirror ($L$) will depend on its shape and the dimensions of the secondary guide, as it will be discussed later. On the other hand, in order to filter the unwanted neutrons, it is necessary to extend the lower part of the new guide ($L_1$ is the length of that extension) avoiding the direct view of the source in the guide. The rest of the neutrons will go to the main guide (it is not sketched in the figure).

The focus of the parabolic mirror will be at the origin, and therefore, the equation of the mirror will be defined with the following equation

$$y = \sqrt{4p(x + p)} \quad (1)$$

where $p$ is the distance between the focus and the edge of the parabola and it is a parameter related with the shape of the parabola. The different divergences of the neutrons before and after the reflection as a function of the point they reflect ($x$) will be defined as follows.

$$\theta_i(x) = \frac{\sqrt{4p(x + p)}}{x} \quad (2)$$
\[ \theta'(x) = \sqrt{\frac{p}{x + p}} \quad (3) \]
\[ \theta_r(x) = 0 \quad (4) \]

Where \( \theta \) is the angle of incidence of the neutron against the main guide direction at the source, \( \theta' \) is the angle of incidence of the neutron at the surface of the parabolic mirror and \( \theta_r \), which does not appear in fig. 1, is the angle of the reflected neutron against the direction of the secondary guide.

The limit in the reflection will be determined by the coating following the condition \( \theta'(x) \leq \theta_c(\lambda) \). As \( \theta' \) decreases with increasing \( x \), that also means that the coating in the mirror has to change accordingly in order to fix a maximum wavelength of the filtered neutrons. The \( m \) value of the coating as a function of the position will be defined by the following equation.

\[ m(x) = \frac{\theta'(x)}{\theta_{c,Ni}(\lambda)} = \frac{1}{\theta_{c,Ni}(\lambda)} \sqrt{\frac{p}{x + p}} \quad (5) \]

where \( \theta_{c,Ni}(\lambda) \) is the critical angle for the Ni/vacuum interface. Taking this into account, it is important to note that \( p \), which is related to the shape of the parabolic mirror, will be set according to the shortest desired wavelength (\( \lambda_{\text{min}} \)) and the maximum \( m \) value of the coating that can be used. In this case, the maximum \( m \) value will be at the entrance.

\[ p = \frac{\lambda_{\text{min}}^2 m^2 (x_{\text{ent}}) x_{\text{ent}}^2 \theta_{c,Ni}(1A)}{1 - \lambda_{\text{min}}^2 m^2 (x_{\text{ent}}) \theta_{c,Ni}(1A)} \quad (6) \]

\( \theta_{c,Ni}(1A) \) is the critical angle for the Ni/vacuum interface and for 1Å neutrons. Taking into account the usual values for all the magnitudes, \( p \) will be in the range of \( 10^{-4} \text{-} 10^{-5} \) m.

Once the shape of the mirror is determined, it is necessary to determine the length of the mirror (\( L \)) which will depend on the shape of the mirror and by the height of the secondary guide (\( H \)) using the following equation:

\[ H = 2\left( \sqrt{p(x_{\text{ent}} + L + p)} - \sqrt{p(x_{\text{ent}} + p)} \right) \quad (7) \]

From this equation we can obtain \( L \) as a function of the shape of the mirror and the height of the guide:

\[ L = \frac{H(4 \sqrt{p(x_{\text{ent}} + p)})}{4p} \quad (8) \]

Figure 2 shows an example of eq. (8) for \( x_{\text{ent}}=0.3 \) m. and two different values of the height of the guide. As shown in the figure, the length of the parabolic mirror has a strong dependence on \( p \) and on the height of the guide (\( H \)). Taking into account that usually the curved guide sections are around 20-40 m. long, there is a practical limit of \( p \), in which the length of the mirror is larger than the usual curved guide sections, and therefore, considering mirrors with a lower \( p \) has no sense.
Fig. 2: Relationship between the length of the parabolic mirror (L) and the shape of the parabola (see eq. (1)) using eq. (8) for \( x_{\text{ent}} = 0.3 \) m. and two different values of the height of the secondary guide (H).

The existence of a practical minimum \( p \) imposes a limit in the transport, although the size of the secondary guide is also important to determine this practical limit. The smaller the H, the smaller the minimum \( p \) of this practical limit. Taking into account that in the ideal case of a point source the divergence is zero, there is no problem on making H smaller as there will be no reflection loses in the guide.

Finally, as explained in fig. 1, the lower part of the guide has to be extended in order to avoid the direct view. The following equation will determine the length needed to avoid that:

\[
L_1 = \frac{(x_{\text{ent}} + L) \sqrt{p(x_{\text{ent}} + p)}}{\sqrt{p(x_{\text{ent}} + L + p)}}
\]  

Taking into account all the analytical study of the ideal case, the advantages of the proposed parabolic mirror guide splitter are obvious. First of all, it is not necessary to build a larger moderator to obtain another neutron beam. On the other hand, the divergence of the beam is zero in the ideal case, which means it can be ideally transported to long distances with no reflection losses.

3. Coupling of an extended source with the guide splitter

As shown in the former section, the ideal case shows the advantages of the use of a parabolic mirror as a guide splitter. However, this is not realistic as the sources are extended. On the other hand, it is convenient to have an extended source if the main guide is a straight guide. For this reason, we must study analytically the effect of extended sources in the proposed guide splitter.
Figure 3 shows the sketch of the guide splitter for an extended source. In this case, the reflected neutron beam is not going to have zero divergence as most of the neutrons will not come from the focus of the parabolic mirror. The equations for the incident and reflected angles will be as follows:

\[ \theta_i = \frac{\sqrt{4p(x + p) - y}}{x} \]  
\[ \theta_r'(x, y) = \frac{p}{\sqrt{x + p}} - \frac{y}{x} \]  
\[ \theta_r(x, y) = \frac{y}{x} \]

Where \( x \) will be the reflection point and \( y \) the origin of the neutrons at the source. As the origin of the system is in the focus of the parabola, there will be different situations depending on where from the source the neutrons are coming from.

In the case of the neutrons coming from the positive side of the source, we will find that \( \theta_r'(x, y) < \theta_r(\lambda) \) and \( \theta_r > 0 \). For these neutrons multiple reflections in the parabolic mirror are likely to happen. When that happens, the divergence of the neutron will become smaller with every reflection in the mirror. Therefore, this imposes an upper limit in the divergence of the neutron beam, which is also the limit where the multiple reflections start.

\[ \theta_{r,\text{max}} = \frac{2\left(\sqrt{p(x_{\text{ent}} + L + p)} - \sqrt{p(x_{\text{ent}} + L)}\right)}{L} \]  

Equalising eqs. (12) and (13) for \( x=x_{\text{ent}} \) we will obtain the upper limit on \( y \) for multiple reflections.
\[
 y_{\text{mult}}^+ = \frac{2x_{\text{ent}} \left( \sqrt{p(x_{\text{ent}} + L + p)} - \sqrt{p(x_{\text{ent}} + p)} \right)}{L} \tag{14}
\]

For neutrons coming from the positive side of the source there is also another limit which will impose the maximum extent of the source in the upper part. If the neutron were coming from an upper position, those would not reflect on the mirror as one of the conditions for being reflected by the mirror is that \( \theta' \geq 0 \). If \( \theta' < 0 \), then the neutron would not reflect and would contribute to the direct view, which is something that must be avoided. The maximum extent of the source will be:

\[
 y_{\text{max}}^+ = \sqrt{ \frac{p}{x_{\text{ent}} + p} (x_{\text{ent}} + 2p) } \tag{15}
\]

For neutrons coming from the negative part of the source (\( y < 0 \)) \( \theta_r < 0 \) and there will be no multiple reflections. While in the positive part, the divergence of the neutrons is limited by the shape (\( p \)) and the position (\( x_{\text{ent}} \)) where the parabolic mirror starts, in the negative part it will be limited not only by the shape, but also by the coating, and will be wavelength dependent, as shown in the following equation:

\[
 y_{\text{max}}^- (x) = x \left( \sqrt{ \frac{p}{x + p} - \theta_r (\lambda) } \right) \tag{16}
\]

This means that the upper and lower part of the divergence of the reflected neutron beam will be different and therefore, the divergence distribution of the reflected beam will be asymmetric.

In summary, the study of the more realistic case of an extended source shows the main disadvantage of this concept: an asymmetric divergence distribution for the outgoing neutron beam.

4. MC simulations on real systems

MC simulations will give us a way to study the applicability of this concept in real systems. For that purpose we used the software VITESS for neutron scattering instrument simulation [7] and the sketch of the simulated system can be found in fig. 4. An important fact to take into account is that the parabolic mirror in real systems has not a perfect parabolic shape, but it will have a polygonal shape with 50 cm. long linear sections as an approximation a parabolic surface.

![Fig. 4: Sketch of simulated system.](image)
The width of the mirror and the secondary guide is 10 cm., and H=10 cm.. The coating of the parabolic mirror has been calculated using eq. (5), while the coating of the secondary guide is m=1. As both, the parabolic mirror and the extension of the lower part of the secondary guide (L1) are built with 50 cm. long sections (and therefore L1 has to be a multiple of 50 cm), the avoidance of the direct view is not perfect and the secondary guide must be extended by Lg.

We use two values of \( p \) in order to study the performance of this configuration as a function of the shape. Table 1 will show the parameters used in the simulation:

| Name | \( p \) (m) | L (m) | L1 (m) | \( y^\text{max} \) (cm) | Lg (m) |
|------|-------------|------|-------|----------------|-----|
| P1   | 3.47 \( \times \) 10\( -4 \) | 10.0 | 9.0   | 1.0            | 11.5|
| P2   | 5.43 \( \times \) 10\( -4 \) | 7.0  | 6.5   | 1.3            | 4.0 |

These parabolic mirror configurations will be compared with a system composed by the source and a slit system which will give a similar angular extent for the divergence as the parabolic mirror configuration (1° x 1°). This is achieved with a two 10 x 10 cm. slits which are 5.75 m. from each other. In this way, we would like to compare the results in terms of source brightness losses.

![Fig. 5: Neutron spectra of the different parabolic mirror configurations compared with the reference](image)

Figure 5 shows the spectrum for the different parabolic mirrors and for the reference system. The capability of the parabolic mirror to filter unwanted neutrons depends on its shape, when \( p \) increases the parabolic mirror filters neutrons with longer wavelength.
Figure 6 shows the vertical divergence distribution of the different parabolic mirror configurations and the reference. The divergence distribution of the parabolic mirror configurations is non-uniform while the reference is completely uniform. It is also evident that the upper and lower limits in the divergence distribution are different, as it was predicted in the former section.

Simulations confirm the advantages and disadvantages deducted from analytical calculations. With the proposed configuration it is possible to filter unwanted neutrons in a shorter space, and it is possible to split a guide without using a larger moderator. However, the divergence distribution of the upcoming beam is non uniform, and therefore, unattractive for neutron scattering instrument designers.

One of the possible solutions is to place a collimator after the guide. This would uniformise the divergence distribution of the beam at the cost of losing flux. Another possible solution to explore is the use of hybrid geometries [8, 9] which could improve the quality of the beam, i. e., the uniformity of the divergence distribution. This last solution will be subject of further studies.

5. Conclusions

We present a guide splitter using a parabolic mirror. From analytical calculations and Monte Carlo simulations it is deduced that this guide splitter shows several advantages:

- It is not necessary to build a larger moderator
- It filters unwanted neutrons in a shorter space compared to curved guide sections
- The beam is collimated in the reflection plane
However, the main disadvantage is:

- Non-uniform divergence distribution

Solutions to avoid this disadvantage are under study.

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