A Model for the $^3$He($\vec{d}$, p)$^4$He Reaction at Intermediate Energies$^4$)

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Polarization correlation coefficients have been measured at RIKEN for the $^3$He($\vec{d}$, p)$^4$He reaction at intermediate energies. We propose a model for the ($\vec{d}$, p) reaction mechanism using the $pd$ elastic scattering amplitude which is rigorously determined by a Faddeev calculation and using modern NN forces. Our theoretical predictions for deuteron polarization observables $A_y$, $A_{yy}$, $A_{xx}$ and $A_{xz}$ at $E_d=140$, 200 and 270 MeV agree qualitatively in shape with the experimental data for the reaction $^3$He($\vec{d}$, p)$^4$He.

§1. Introduction

The measurement$^1$) of the $^3$He($\vec{d}$, p)$^4$He reaction at RIKEN aimed at an investigation of the high momentum components of the deuteron wave function and the d-state admixture linked to them. High precision data resulted for the polarization observables $A_y$, $A_{yy}$, $A_{xx}$ and $A_{xz}$. Out of them the linear combination $C_{\parallel} = 1 + \frac{1}{4}(A_{yy} + A_{xx}) + \frac{3}{4}(C_{y,y} + C_{x,x})$ has been formed$^1)$. The Dubna and Saturne groups also obtained the polarization correlation coefficient $C_{\parallel}$ built in this case from the measurements of $T_{20}$ and $\kappa_0$ in d + p backward scattering$^2)$ and from the inclusive deuteron breakup process$^3)$. The polarization correlation coefficient $C_{\parallel}$ at forward angles of the outgoing proton is directly related to the ratio of deuteron wavefunction components if one uses the plane wave impulse approximation (PWIA):

$$
C_{\parallel}(PWIA) \equiv \frac{9}{4} \frac{w^2(k_{pn})}{u^2(k_{pn}) + w^2(k_{pn})} \quad \text{(1.1)}
$$

Here $u$ and $w$ are the S-, D-wave components of the deuteron wavefunction, and $k_{pn}$ a kinematically fixed relative momentum of the $pn$ pair. These PWIA calculations are very poor in relation to the data$^1)$. This is shown in table I for $C_{\parallel}$. There we also exhibit the different D-state probabilities for the modern realistic NN potentials, CD-Bonn$^4)$, AV18$^5)$ and Nijmegen I,II and 93$^6)$. Clearly one needs a better calculation for the analysis of the $^3$He($\vec{d}$, p)$^4$He reaction.

$^1)$ Dedicated to Prof. Shinsho Oryu on the occasion of his 60th birthday

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A theoretical analysis has been reported by the SUT group\(^7\) based on a \(^3\)He-n-p and d-d-p three-cluster model. However, the evaluations performed up to now in this model lead only to a tiny deviation from the PWIA calculations just mentioned. Recently, the Hosei group\(^8\) analyzed \(T_{20}\) and \(\kappa_0\) with the \(^3\)He-n-p cluster model by an analogy between \(^3\)He and the proton \((T=1/2, S=1/2)\). They conclude that PWIA describes the global features of the experimental data.

In this letter we would like to introduce again a 3N model, which when evaluated correctly leads to a great similarity of various polarization observables to the ones found in the reaction \(^3\)He(\(\vec{d},p\))\(^4\)He.

Table I. The polarization correlation coefficient \(C_{\parallel}\) according to Eq. (1.1) in a simple PWIA model for different NN potentials and from an experimental value for the reaction \(^3\)He(\(\vec{d},p\))\(^4\)He. We also show the corresponding deuteron D-state probabilities.

| Potential     | \(C_{\parallel}(PWIA)\) | D-state Probability (%) |
|---------------|-------------------------|-------------------------|
| CD-Bonn\(^4\) | 0.645                   | 4.86                    |
| AV18\(^5\)    | 0.722                   | 5.78                    |
| Nijmegen 93\(^6\) | 0.710                   | 5.76                    |
| Nijmegen I\(^6\) | 0.712                   | 5.68                    |
| Nijmegen II\(^6\) | 0.726                   | 5.65                    |
| exp.\(^1\)    | 0.223 ± 0.044(statistical) ± 0.037(systematic) | - |

\section{Model}

For the \(^3\)He(\(\vec{d},p\))\(^4\)He reaction we assume a model which is based on a three-body reaction process. This is shown in Fig. 1. The wavefunctions for \(^3\)He and \(^4\)He take on maximal values if the momenta of the subclusters are zero in their respective rest systems. These are for \(^3\)He the momenta of \(p\) and \(d\) and for \(^4\)He the momenta of the two deuterons. This means that for the moving nuclei the subcluster momenta should be equal. Therefore to form the \(\alpha\)-particle with highest probability in the picture of Fig. 1 one has to assume that the two deuterons, \(d'\) and \(\vec{d}\), have equal momenta. Likewise for \(^3\)He one has to assume that the proton and deuteron, \(\vec{p}\) and \(\vec{d}\), have equal momenta. This turns out to be kinematically inconsistent. Therefore we make a choice and assume that only the two deuterons forming the \(\alpha\) particle have equal momenta. We justify this choice by the larger binding energy of the \(\alpha\) particle.

![Fig. 1. Diagram of the reaction mechanism](image)

It is easy to see that our basic assumption

\[
\vec{k}_d = \vec{k}_{d'} \tag{2.1}
\]
fixes the kinematics uniquely. It follows by simple kinematical arguments that
\[ \vec{k}_p^{lab} = \frac{1}{2} \vec{k}_p^{cm} - \frac{2}{5} \vec{k}_d^{lab} = -\vec{k}_d^{lab} \]  
(2.2)
Here the superscripts \( lab \) and \( cm \) denote the laboratory and 5-body cm systems, respectively. Further the total momentum of the picked up proton and the incoming deuteron in the lab system is
\[ \vec{K} = \frac{1}{2} \vec{k}_p^{cm} + \frac{3}{5} \vec{k}_d^{lab} \]  
(2.3)
Also we get the momentum of the picked up proton in the 3-body center of mass system (3CM) as
\[ \vec{k}_p^{3CM} = \frac{1}{3} \vec{k}_p^{cm} - \frac{3}{5} \vec{k}_d^{lab} \]  
(2.4)
and the 3CM energy as
\[ E_{3CM} = \frac{3}{4m} (\vec{k}_p^{3CM})^2 \]  
(2.5)
We show in Fig. 2 the relevant kinematics for the cm and the 3CM systems. From the relation
\[ \vec{k}_p^{3CM} = \frac{2}{5} \vec{k}_p^{3CM} - \frac{3}{5} \vec{k}_d^{lab} \]  
(2.6)
it follows under our condition, that the angles shown in Fig. 2 are related as
\[ \theta_{3CM} = \theta_p^{3CM} - \theta_d^{3CM} \]  
(2.7)
(note that \( \theta_p \equiv \theta_{3CM}^p = \theta_p^{cm} \)). The dependence of \( E_{3CM} \) on \( \theta_p^{cm} \) is illustrated in Fig. 3 for 3 deuteron energies. The scattering angle \( \theta_{3CM} \) is shown against \( \theta_p^{cm} \) in Fig. 4 again for the same 3 deuteron energies.

![Fig. 2. Scattering angles for the 5-body (cm) and 3-body (3CM) center of mass systems.](image)

Our claim is now that \( \mathcal{O}(E_d, \theta_p^{cm}) \approx \mathcal{O}_{pd}(E_{3CM}, \theta_{3CM}) \) where \( \mathcal{O}_{pd} \) are the elastic pd deuteron polarization observables and \( \mathcal{O} \) the ones for the reaction \( ^3\text{He}(d, p)^4\text{He} \).

Before calculating these 3N observables we introduce one more approximation. Looking at Fig. 3 we see that \( E_{3CM} \) varies with \( \theta_p^{cm} \) and consequently for each \( \theta_p^{cm} \) one would have to solve the 3N Faddeev equation. We avoided that for that qualitative investigation and have chosen available Faddeev results at three energies which lie in the three energy bands for \( 0 < \theta_p^{cm} < 40^\circ \). They are \( E_{3CM} = 66.7, 100, 133 \) MeV corresponding to \( E_d = 140, 200, 270 \) MeV, respectively.
§3. Results

As NN potential we used AV18 in the Faddeev calculations. The operator $U$ for elastic scattering has the form (see, for instance, \textsuperscript{9})

$$U = PG_0^{-1} + PT \quad (3.1)$$

where $G_0$, $P$ and $T$ are the free 3N propagator, permutation operators and a partial 3N break-up operator, which is determined by a Faddeev equation. The first term, the famous nucleon exchange term, is essentially related to the PWIA mentioned in introduction. In order to see the importance of solving the Faddeev equation correctly and not just replacing $U$ by $PG_0^{-1}$ we compare the corresponding predictions for $A_{yy}$, $A_{xx}$ and $A_{xz}$ in Figs. 5-7. We see large differences especially above about 15 degrees. Trivially $A_y$ is identically zero using only the real term $PG_0^{-1}$.

The predictions of the full Faddeev solution are shown in Figs. 8-11 at $E_{3CM}=66.7$, 100 and 133 MeV, respectively. This should be compared to recent data\textsuperscript{10}. We see a behavior qualitatively similar to those data, especially for $A_y$. For the $A_y$ data
Fig. 7. The same as Fig. 5 for $A_{xz}$. There is no data point.

Fig. 8. The deuteron vector analyzing power $A_y$ for the pd elastic scattering for $E_{CM}=66.7$ (solid), 100 (long dashed) and 133 (short dashed) MeV as a function of $\theta^\text{cm}_p$, corresponding to the $^3\text{He}(\vec{d},p)^4\text{He}$ reaction for $E_d=140, 200, 270$ MeV, respectively.

Fig. 9. The same as in Fig. 8 for $A_{yy}$. The data point for $^3\text{He}(\vec{d},p)^4\text{He}$ reaction is from $^1$.

Fig. 10. The same as in Fig. 8 for $A_{xx}$. The data point for $^3\text{He}(\vec{d},p)^4\text{He}$ reaction is from $^1$.

Fig. 11. The same as in Fig. 8 for $A_{xz}$.

the minima shift to smaller $\theta^\text{cm}_p$ value with increasing energy like in Fig. 8. Also for $A_{yy}$ the qualitative behavior is similar in our model and the data, especially at the highest energy. For $A_{xx}$ the shapes are again very similar. In Fig. 9 and 10 we include one data point from $^1$. This shows that our absolute values are too high. For $A_{xz}$ shown in Fig. 11 there are not yet data.
§4. Summary and Outlook

We assumed that the reaction $^3\bar{H}e(d,p)^4He$ at forward angles is mainly driven by elastic pd scattering. In this model the deuteron picks up a proton from $^3He$, scatters elastically and combines then again with the spectator nucleons to an $\alpha$ particle. Our main assumption is that the momentum of the scattered deuteron equals the spectator momentum of the deuteron in $^3He$. This leads to a high probability to form the final $\alpha$-particle. The resulting spin-observables are in astonishingly good qualitative agreement with the data. Important thereby is, that the elastic pd amplitude is a full solution of the 3N Faddeev equation and not only a simple PWIA expression. This model should be generalized by the mechanism that also a neutron from $^3He$ can be picked up. In this case one has to use the nd break-up amplitude. Since the polarization of $^3He$ is carried by more than 90% by the neutron this second mechanism is of course mandatory for a description of $C_{x,x}$ and $C_{y,y}$. The proton pick-up alone is too poor for those spin correlation observables. Also we neglected the momentum distributions of the proton in $^3He$ and of the deuteron in the $\alpha$ particle. As an additional improvement the spin of the deuteron should be properly rotated for the deuteron polarization observables.

Based on the promising qualitative results achieved it appears worthwhile to improve and enrich the model along the lines mentioned.

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