Erratum: Galactic potential constraints from clustering in action space of combined stellar stream data

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This is an erratum to the paper ‘Galactic potential constraints from clustering in action space of combined stellar stream data’ (Reino et al. 2021). In the original paper, there were errors in our action-space probability density estimates when considering multiple streams in combination. The errors do not stem from our overall procedure but from a technical limitation of our density estimator, that was not known to us, or the author of the density estimator, a priori.

When estimating the probability densities to calculate the weighted Kullback–Leibler Divergence we artificially separated the streams from each other by shifting them in $L_z$ to heighten the sensitivity of our method. We deemed this to be equivalent to estimating the probability density of each stream separately after accounting for normalization. We have since reviewed this assumption and found that in the case of the EnLink algorithm which we used to calculate our densities, this assumption is not correct. In particular, EnLink induces a shear-like effect on to the densities of clusters with large separation. This effect is stronger the fewer points there are in a particular cluster. The shift that we applied to $L_z$ therefore caused the necessary condition for the shear-effect to distort the estimated densities. As a consequence all our weighted combined-stream results were incorrect. All individual stream results, and all unweighted combined-stream results, on the other hand, were correct and remain unchanged.

As an example of the density estimation artefact we discovered in EnLink, consider two populations of actions $J$ drawn from two-dimensional Gaussian distributions

$$N(J | \mu_1, \Sigma_1) = \frac{1}{2\pi |\Sigma_1|^{1/2}} \exp \left( -\frac{1}{2} (J - \mu_1)^T \Sigma_1^{-1} (J - \mu_1) \right)$$

with centroids $\mu_1 = (0, 0)$ and $\mu_2 = (15, 0)$, and covariance matrices $\Sigma_1 = \Sigma_2 = 5I$. Cluster 1 is sampled with $N_1 = 100$ points and cluster 2 is sampled with $N_2 = 500$ points.

Using EnLink to estimate the densities $p_j(J)$ of each cluster separately, and renormalizing the individual cluster density estimates so that the total probability in both clusters together integrates to 1, i.e.

$$p(J) = \sum_j \frac{N_j}{N} p_j(J)$$

with $N = N_1 + N_2$ and $N_j = 2$, gives the result shown in the upper left-hand panel of Fig. 1. EnLink was run with default settings, density method being ‘shr’ and number of nearest neighbors being 10. Although the recommended default is 32 or even 64 particles, we typically use 10 nearest neighbors, motivated by tests performed by Sanderson et al. 2020 where this smaller number of particles still recovered densities effectively.

By construction the clusters are separated by a few times $\Sigma_2$ (the size of the larger cluster). Applying the density estimator to the entire distribution as constructed, including both clusters, produces estimates that differ only fractionally from those estimated for each cluster individually (Fig. 1, upper right-hand panel). However, if we artificially shift cluster 2 by $40\sigma_2$ in one component of $J$, then the clusters are now separated by many times their size. This shift is similar in magnitude, relative to the size of the clusters, to the shift applied in our paper to guarantee that all clusters were distinct in action space. Using EnLink on this density distribution produces density estimates that are severely sheared for the cluster with fewer points, as shown in the lower left-hand panel of Fig. 1. The shear is aligned with the direction toward the location to which the cluster with more points was shifted for the density estimation: densities in the cluster with fewer points are over-estimated along this line and under-estimated away from it.

Finally, we can verify that the effect is due mainly to the number of points in the cluster, and not to its spread or compactness, by setting $N_1 = 500$ and $N_2 = 100$ for fixed $\mu_j$ and $\Sigma_j$, shifting cluster 2, and recalculating the densities with EnLink. In this case the shear is induced in cluster 2, which is now the one with fewer points (Fig. 1, lower right-hand panel).

These tests point to an issue related to the way EnLink groups nearby points to use in density estimation. Generally, EnLink can obtain reliable density estimates and effectively group particles together in regions where the distribution of points is significantly elongated in one direction, even if that direction is not aligned with...
a particular coordinate, by estimating the local Mahalanobis metric
\[ s^2(x_i, x_j) = (x_i - x_j)^T \Sigma^{-1}(x_i - x_j) \] (3)
at each point, smoothing over this metric, and using it to compute the
distance between particles in the distribution. This distance is then
used to determine which particles are the ‘nearest neighbors’ of a
given particle. Importantly, there is no maximum set on the shear
of the metric tensor, which is estimated iteratively using a binary space
partitioning scheme with a minimum of 10 particles in the leaf node
that is often comparable to the number in some of the smaller clusters.
With relatively sparsely sampled clusters separated by many times
their own sizes, the iterative solution for the Mahalanobis metric
sometimes appears to include particles from the edge of the other
cluster, driving the shear of the metric to unreasonably large values
and resulting in the biases shown in the bottom panels of Fig. 1.
The bias shows up preferentially in the cluster with fewer particles
because the particles are organized by partitioning the volume into
a K-D tree, constructed by subdividing along the different local
principal axes of the density distribution determined by finding the
eigensystem of the smoothed Mahalanobis metric. The top level
of partitioning starts by dividing the entire data set in half, so the
first division is most likely to be located in the cluster with the
most points. Thus the shearing of the metric will preferentially drag
estimates from the cluster with fewer points toward the most massive
one. This explanation suggests that placing an adjustable maximum
on the degree of shear allowed in the local Mahalanobis metric could
rectify this issue in EnLink.

To correct the results we construct a probability density function
\[ p_j(J_j | \xi, \omega_j) \] for each stream \( j \) individually from points \( J_j \).
We then normalize each \( p_j \) with \( \frac{N_j}{N} \) to keep the each \( p_j \) at the correct
relative size between the different streams. The weighted KLD1
[substituting equation (20) from the original paper] then becomes
\[ \text{wKLD1}(\xi) = \sum_{j=1}^{N} \sum_{i=1}^{N_j} w_j \log \frac{N_j}{N} \frac{p_j(J_j | \xi, \omega_j)}{p(J_j)} \bigg|_{\omega_i} \]. (4)

Similarly, the weighted KLD2 equation [substituting equation (26)
from the original paper] becomes
\[ \text{wKLD2}(\xi) = \sum_{j=1}^{N} \sum_{i=1}^{N_j} w_j \log \frac{N_j}{N} \frac{p_j(J_j | \xi_0, \omega_j)}{p(J_j | \xi_0, \omega_j)} \bigg|_{\omega_i} \]. (5)

This correction has a minor effect on our results. Here, we present
the rectified versions of the affected figures and tables. Figs 2, 3, 4,
5, 6, 7, 8, 9, 10, 11, 12 and Tables 1, 2, 3 are the correct versions
of figs 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, B1 and tables 1, 2, 3, respectively,
in the original paper. The new enclosed mass constraints we find are:
\( M(< 20 \text{ kpc}) = 2.22^{+0.13}_{-0.08} \) from the GD-1, Pal 5 and Orphan streams
and \( 4.10^{+2.15}_{-1.25} \times 10^{11} \text{ M}_\odot \) when also including the Helmi stream.
These values replace those originally reported in Reino et al. (2021).
As Figs 10 and 11 show, our corrected three-stream results are
in much better agreement with other recent measurements of
the Galactic profile. All conclusions made in the original paper still
apply.
Figure 2. Weighted combined data results for the single-component potential. The best-fitting point is marked with a pink cross, the grey points represent potentials that resulted in unbound stars and were therefore discarded, and other points are colour-coded according to their KLD2 value. The orange region shows $1\sigma$ contours. Top: results when combining all four streams. Bottom: results for the combination of GD-1, Orphan, and Pal 5.

Figure 3. The weighted combined data results for a single-component potential: marginalized single parameter distributions. The top panel shows the results from the combination of all four streams, and the bottom panel shows the results of the combination of GD-1, Orphan, and Pal 5 streams. The green points show the parameter values against the wKLD2 of the potential they belong to. The values of the parameters in the best-fitting potentials are marked with a pink cross. The black lines are drawn at wKLD2 = 0.5 that signifies the $1\sigma$ confidence interval. The light grey bars show the range of values that are accepted with $1\sigma$ confidence.

Figure 4. Comparison of best-fitting parameter values for the single-component potential with their $1\sigma$ confidence intervals for enclosed mass (left), scale length (middle), and flattening (right). Results for individual streams are labelled with the stream name. Combined results are labelled with ‘GD-1/Orphan/Pal 5’ for the three-stream combination and ‘All’ for the four-stream combination. In addition, the combined stream labels end with an ‘s’ or a ‘w’ for the standard and weighted analyses, respectively. We remind the reader that $e > 1$ corresponds to an oblate potential, while $e < 1$ corresponds to a prolate potential.
Figure 5. As in Fig. 2, but showing the weighted combined data results for the two-component potential. Top: results for our four-stream data set. Bottom: results for the combination of GD-1, Orphan, and Pal 5.

Figure 6. Comparison of the best-fitting parameter values for the two-component potential with their 1σ confidence intervals for enclosed mass (left), scale length (middle), and flattening (right). Results for individual streams are labelled with the stream name. Combined results are labelled with ‘GD-1/Orphan/Pal 5’ for the three-stream combination and ‘All’ for the four-stream combination. We remind the reader that $e > 1$ corresponds to an oblate potential in our convention.
Figure 7. The orbits for the GD-1 (top), Pal 5 (middle), and Orphan (bottom) streams. These shaded regions correspond to the allowed orbits of the stars whose current position has been marked with a dot of the same colour. The edges of the shaded regions are defined by the potentials that produce the highest and the lowest enclosed mass within 20 kpc among those that are within 1σ of the best-fitting results of the combined analysis of GD-1, Orphan, and Pal 5 streams. Other stars in the stream are shown with purple dots. Axes are in the Galactocentric frame.
Figure 8. Comparison of actions of GD-1 (orange points), Pal 5 (teal points), and Orphan (yellow points) produced by two different two-component potentials: the best-fitting potential of the three-stream combination and another potential. Left: KLD1 values of the three-stream combination as a function of enclosed mass and scale length of the outer component. The higher the KLD1 value, the more clustered the action space. Centre: action distribution of the best-fitting potential (black cross in left-hand panel). Right: action distribution of a different potential with KLD1 0.35 lower than the corresponding best fit (purple cross in left-hand panel). All Orphan stars have been shifted up by 2500 kpc s\(^{-1}\) in \(J_z\) for clarity on both panels.

Figure 9. Histograms of the current \(\lambda\) for stars in GD-1 and Pal 5 in different two-component potentials, normalized by the pericentre and apocentre positions of each star’s orbit to serve as a proxy for the orbital phase, with pericentre at 0 and apocentre at 1. Top left-hand panel: the approximate orbital phase of GD-1 stars assuming the best-fitting GD-1 potential. Top right-hand panel: the position of GD-1 stars assuming the potential best fit to the combined GD-1, Orphan, and Pal 5 data. Bottom left-hand panel: the position of Pal 5 stars assuming the best-fitting Pal 5 potential. Bottom right-hand panel: the position of Pal 5 stars assuming the potential best fit to the combined GD-1, Orphan, and Pal 5 data.
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Figure 10. Comparison of our two-component potential results with previous measurements of the enclosed mass at different radii. The light blue shaded area represents the combined results for all four streams (left-hand panel) and for GD-1, Orphan, and Pal 5 (right-hand panel). The darker shaded regions show the subset of the potentials that are compatible with the current measurements of the local standard of rest velocity. The black markers signify individual stream results at their respective average radii. The coloured markers show measurements of the enclosed mass by other authors. The markers showing the results of Koposov, Rix & Hogg (2010) and Newberg et al. (2010) are slightly offset from 20 and 50 kpc, respectively, for clarity.

Figure 11. Rotation curves corresponding to the weighted combined-stream results of the two-component potential model. The lighter shaded region shows the rotation curves for potentials within $1\sigma$ of the best fit for each data set. The purple, orange, teal, and yellow data points correspond to the results of the Helmi, GD-1, Pal 5, and Orphan streams, respectively: they show the rotation velocity of the best-fitting potential at the mean Galactocentric distance of the stars in that stream. The darker shaded region shows the subset of the rotation curves that are compatible with the current measurements of the local standard of rest velocity. For comparison, the dashed black and cyan lines are the rotation curves from the GALPY MWPOTENTIAL2014 (Bovy 2015) and McMillan (2017), respectively, and the grey dots represent the data from Eilers et al. (2019). This is the corrected version of the top panel of fig. 13 in the original paper.
Figure 12. Comparison of action space of GD-1 (top panels), Pal 5 (middle panels), and Orphan (bottom panels) produced by two different two-component potentials. The action of stars are coloured based on their energies. Left: KLD1 values of corresponding individual stream analysis given as a function of enclosed mass and scale length of the outer component. The higher the KLD1 value, the more clustered the action space. Centre: action distribution of the best-fitting individual-stream potentials (black cross in left-hand panel). Right: action distribution of the same stream in the best-fitting three-stream potential (purple cross in left-hand panel). The KLD1 value of the purple cross potential in a single-stream analysis is reduced by 0.86 (for GD-1), 0.23 (for Pal 5), and 0.16 (for Orphan) compared to the KLD1 value of the black cross potential.

Table 1. The individual and combined stream results for a single-component potential. Best-fit parameters are given with their 1σ confidence intervals. We remind the reader that e > 1 corresponds to an oblate potential, while e < 1 corresponds to a prolate potential.

| Streams         | \(N_{\ast}\) | \(M(<20\text{kpc}) \times 10^{11}\) (\(M_\odot\)) | \(a\) (kpc) | \(e\) | \(M_{\ast\ast} \times 10^{12}\) (\(M_\odot\)) |
|-----------------|---------------|-----------------------------------------------|--------------|-----|-----------------------------------------------|
| GD-1            | 69            | 4.73^{+0.33}_{-1.05}                           | 15.12^{+2.10}_{-10.11} | 0.88^{+0.07}_{-0.38} | 2.65^{+0.51}_{-1.88} |
| Helmi           | 401           | 9.06^{+3.20}_{-6.36}                           | 10.24^{+10.68}_{-5.23} | 0.95^{+0.79}_{-0.31} | 2.81^{+3.35}_{-2.39} |
| Pal 5           | 136           | 2.73^{+0.60}_{-0.74}                           | 20.92^{+4.50}_{-5.80} | 1.40^{+0.60}_{-0.19} | 1.86^{+1.30}_{-1.14} |
| Orphan          | 117           | 1.89^{+1.05}_{-0.60}                           | 27.12^{+2.35}_{-12.00} | 1.26^{+0.74}_{-0.17} | 2.35^{+1.43}_{-1.32} |
| GD-1/Orphan/Pal 5 standard | 332   | 2.60^{+0.39}_{-0.28}                           | 18.37^{+5.65}_{-2.24} | 1.45^{+0.25}_{-0.24} | 1.38^{+0.83}_{-0.35} |
| GD-1/Orphan/Pal 5 weighted | 322  | 2.89^{+0.28}_{-0.27}                           | 19.60^{+5.81}_{-4.48} | 1.40^{+0.33}_{-0.23} | 1.75^{+1.41}_{-0.72} |
| GD-1/Helmi/Orphan/Pal 5 standard | 723 | 5.01^{+2.37}_{-1.18}                           | 11.66^{+5.56}_{-3.23} | 1.86^{+0.14}_{-0.60} | 1.30^{+1.86}_{-0.44} |
| GD-1/Helmi/Orphan/Pal 5 weighted | 723  | 4.26^{+1.22}_{-0.57}                           | 17.21^{+5.11}_{-3.05} | 1.93^{+0.07}_{-0.63} | 1.75^{+1.41}_{-0.59} |
Table 2. The individual and combined stream results for a two-component potential. Best-fitting parameters are given with their 1σ confidence intervals. Note that $e > 1$ corresponds to an oblate potential in our convention.

| Streams               | $N_e$ | $M(<20\,\text{kpc}) \times 10^{11} \,(M_\odot)$ | $a_{\text{outer}}$ (kpc) | $\epsilon_{\text{outer}}$ | $a_{\text{inner}}$ (kpc) | $M_{\text{tot}} \times 10^{12} \,(M_\odot)$ | $k$ |
|-----------------------|-------|-----------------------------------------------|---------------------------|---------------------------|---------------------------|-----------------------------------------------|-----|
| GD-1                  | 69    | $5.64^{+0.25}_{-0.26}$                       | $16.46^{+46.64}_{-11.44}$ | $1.00^{+0.94}_{-0.00}$    | $4.69^{+0.32}_{-0.39}$    | $3.16^{+0.00}_{-0.26}$                        | 0.01^{+0.29}_{-0.00} |
| Helmi                 | 401   | $7.93^{+0.51}_{-0.14}$                       | $12.07^{+61.03}_{-7.06}$  | $1.00^{+0.86}_{-0.00}$    | $3.49^{+1.52}_{-2.49}$    | $2.80^{+0.36}_{-0.49}$                        | 0.02^{+0.28}_{-0.01}  |
| Pal 5                 | 136   | $2.01^{+0.23}_{-0.03}$                       | $27.59^{+14.61}_{-11.97}$ | $1.01^{+0.03}_{-0.01}$    | $5.01^{+0.00}_{-0.03}$    | $2.80^{+0.36}_{-0.97}$                        | 0.01^{+0.09}_{-0.00}  |
| Orphan                | 117   | $1.91^{+1.26}_{-0.84}$                       | $39.62^{+23.47}_{-24.00}$ | $1.00^{+0.04}_{-0.00}$    | $1.53^{+3.48}_{-0.53}$    | $3.19^{+0.00}_{-5.17}$                        | 0.04^{+0.21}_{-0.03}  |
| GD-1/Orphan/Pal 5 weighted | 322   | $2.22^{+0.10}_{-0.08}$                       | $29.06^{+8.57}_{-12.60}$  | $1.01^{+0.03}_{-0.01}$    | $5.01^{+0.00}_{-0.03}$    | $2.48^{+0.18}_{-1.28}$                        | 0.05^{+0.07}_{-0.04}  |
| GD-1/Helmi/Orphan/Pal 5 weighted | 723   | $4.10^{+0.05}_{-1.25}$                       | $63.10^{+0.00}_{-57.24}$  | $1.00^{+0.17}_{-0.00}$    | $5.01^{+0.00}_{-4.01}$    | $3.16^{+0.00}_{-2.33}$                        | 0.21^{+0.09}_{-0.20}  |

Table 3. KLD1 values for the best-fitting results from individual and combined streams for single-component (1-comp) and two-component (2-comp) potentials.

| Stream                  | 1-comp | 2-comp |
|-------------------------|--------|--------|
| GD-1                    | 17.37  | 16.64  |
| Helmi                   | 13.43  | 13.43  |
| Pal 5                   | 14.81  | 13.71  |
| Orphan                  | 9.85   | 9.84   |
| GD-1/Orphan/Pal 5 standard | 11.83 | –       |
| GD-1/Orphan/Pal 5 weighted | 12.13 | 11.91  |
| GD-1/Helmi/Orphan/Pal 5 standard | 11.57 | –       |
| GD-1/Helmi/Orphan/Pal 5 weighted | 11.39 | 11.37  |

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