Extremely Inclined Orbit of the S-type Planet $\gamma$ Cep Ab Induced by the Eccentric Kozai–Lidov Mechanism

Xiumin Huang$^{1,2}$ and Jianghui Ji$^{1,2,3}$

1 CAS Key Laboratory of Planetary Sciences, Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210023, People’s Republic of China
2 School of Astronomy and Space Science, University of Science and Technology of China, Hefei 230026, People’s Republic of China
3 CAS Center for Excellence in Comparative Planetology, Hefei 230026, People’s Republic of China

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Abstract

$\gamma$ Cep Ab is a typical S-type planet, which occupies a nearly perpendicular planetary orbit relative to the binary. Here, we use a Markov Chain Monte Carlo sampler to conduct a full N-body fitting and derive self-consistent orbital solutions for this hierarchical system. We then employ the eccentric Kozai–Lidov (EKL) mechanism to explain the extremely inclined orbit of the S-type planet $\gamma$ Cep Ab. The EKL mechanism plays an essential part in our exploration of the significant oscillations of the mutual inclination $i_{\text{mut}}$ between the planet and the secondary star. We perform a qualitative analysis and use extensive numerical integrations to investigate the flip conditions and timescales of $\gamma$ Cep Ab’s orbit. When the planetary mass is $15 \, M_{\text{Jup}}$, the planet can reach $i_{\text{mut}} \sim 113^\circ$ given the critical initial conditions of $i_{\text{mut}} < 60^\circ$ and $e_1 < 0.7$. The timescale for the first orbital flip decreases with the increase of the perturbation Hamiltonian. The flipping orbits of $\gamma$ Cep Ab are confirmed to have a large possibility of remaining stable, based on surfaces of section and the secular stability criterion. Furthermore, we extend the application of EKL to general S-type planetary systems with $a_1/a_2 < 0.1$, where the most intense excitation of $i_{\text{mut}}$ occurs when $a_1/a_2 = 0.1$ and $e_2 \sim 0.8$, and the variation in planetary mass mainly affects the flip possibility where $e_1 \leq 0.3$.

Unified Astronomy Thesaurus concepts: Exoplanet dynamics (490)

1. Introduction

As of today, more than 200 exoplanets have been discovered in binary systems, consisting of circumbinary (P-type) planets and satellite-like (S-type) orbiting bodies (Schwarz et al. 2016). The P-type planets detected by the Kepler Space Telescope are largely coplanar with the binary, where the mutual inclination is less than $2^\circ.5$ (Kostov et al. 2014). The catalog of exoplanets in binary systems$^4$ reports that only 26% of S-type planets are detected with an orbital inclination of $\sim 90^\circ$ through transit observations, while over 50% of this population are observed by radial velocity (RV) without determined inclinations. Therefore, the distribution of the orbital inclinations of planets in binaries plays a significant role in estimating their occurrence rate and understanding their evolution (Armstrong et al. 2014; Gong & Ji 2018).

Figure 1 shows the distribution of the semimajor axes (SMAs) of the planets and secondaries of S-type systems. Here, the blue dashed line denotes an upper limit on the SMA of the secondary, where the perturbation from the secondary is very significant. The red dotted–dashed line indicates the limit on the SMA ratio $a_1/a_2 = 0.1$, where $a_1$ and $a_2$ are the SMAs of the planet and secondary, respectively. In particular, three potentially inclined S-type planets, HD 199944 Ab, HD 196885 Ab, and $\gamma$ Cep Ab, as detected by RV, are labeled in Figure 1.

The planet $\gamma$ Cep Ab is one of the best-known S-type planets in close binary systems. The RV signal of the planet in $\gamma$ Cep was first measured in 1988 (Campbell et al. 1988), but observation errors and the influence of the secondary star made the planetary detection confusing. Hatzes et al. (2003) revealed the planetary companion to $\gamma$ Cep A with high-precision RV measurements from 1981 to 2002. The in situ formation of this planet does not seem to be likely, given the truncation model of the protoplanetary disk (Artemowicz & Lubow 1994) and the dynamical instability of the planetesimals in the presence of the star companion $\gamma$ Cep B (e.g., Jang-Condell et al. 2008; Giuppone et al. 2011). Martí & Beaugé (2012) have suggested that the binary orbital configuration of this system was established after the formation of the planet, as a result of a scattering scenario for the third fly-by star, with $\gamma$ Cep AB then forming the close binary configuration.

$\gamma$ Cep Ab occupies a nearly perpendicular orbit relative to the binary (Reffert & Quirrenbach 2011), which presents a tremendous challenge to the migration scenario of a protoplanetary disk with angular momentum exchange. However, close binaries may have a remarkable influence on the formation and evolution of S-type planets, through dynamical perturbations (Xie et al. 2010). Andrade-İnes et al. (2016) studied large parameter spaces of S-type planets to determine the applicability of the disturbing regime up to the second order, including the orbital stability, mean motion resonances, and short-period oscillations. With double stars having a relatively high occurrence rate of 50% (Tokovinin 1997), it would be natural and very likely for a hierarchical triple system hosting an S-type planet to occur, consisting of an inner close binary and a third object in a distant outer orbit.

In the pioneering work of Kozai (1962) and Lidov (1962), secular dynamics were applied to a hierarchical triple system, indicating that an inclined test particle with an inclination of $39^\circ \lesssim i \lesssim 141^\circ$ would lead to periodic oscillations of

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4 http://www.univie.ac.at/adg/schwarz/multiple.html
incline and eccentricity. This is now called the Kozai–Lidov (KL) mechanism. However, the fundamental theories and phenomena of the KL mechanism had been investigated and published by von Zeipel (1910) before the middle of the twentieth century. Afterward, the KL mechanism for hierarchical triple systems was extensively investigated (Harrington 1968; Lee & Peale 2003; Naoz et al. 2013; Teyssandier et al. 2013; Lei et al. 2018; Lei 2019; Tan et al. 2020; Lei 2021) in a wide range of circumstances. Ford et al. (2000) and Eggleton & Kiseleva-Eggleton (2001) showed that the KL mechanism plays an essential part in the secular evolution of hierarchical triple-star systems, and explored the interaction of tidal friction with Kozai cycles in a triple-star system. Such a scenario is then further employed to explain observed close binaries (Fabrycky & Tremaine 2007; Perets & Fabrycky 2009; Thompson 2011; Shappee & Thompson 2013; Naoz & Fabrycky 2014).

Li et al. (2014a) systematically explored the KL mechanism and characterized the parameter space that allows for large-amplitude oscillations in eccentricity and inclination under the test particle limit. As the outer perturber is eccentric and the SMA ratio of the inner orbit and outer orbit meets \( \alpha = a_1/a_2 \ll 1 \), the polynomial in the classical outer perturbation equation can be expanded to the third order. This allows the octupole-level term, which can give rise to the eccentric KL (EKL) mechanism (Lithwick & Naoz 2011; Naoz 2016). In addition, Li et al. (2014b) confirmed that sufficient initial mutual inclination could produce extremely large eccentricities and flips of the inner orbit, i.e., the orbital inclination transforms between \( I_1 < 90^\circ \) and \( I_1 > 90^\circ \) for originally prograde or retrograde orbits.

Additionally, for perturbed orbits with initial low eccentricity and high inclination, the orbital flips induced by the EKL mechanism have been demonstrated to be a kind of resonance, with the libration of a critical angle (Sidorenko 2018). Recently, Lei (2022) systematically studied the EKL mechanism, by analyzing the orbital flipping and its parameterization. The analytical averaging theory interprets the orbit flips as being solutions around polar periodic orbits or kinds of resonant trajectories.

This work aims to explore the inclination evolution and secular stability of potentially inclined S-type planetary systems under the EKL mechanism, particularly for the orbital flip in the \( \gamma \) Cep Ab B system. We conduct a Markov Chain Monte Carlo (MCMC) search using the RV data to derive the best-fitting orbital solution of \( \gamma \) Cep Ab, suggesting that its orbital plane is nearly perpendicular to that of the secondary star. We conclude that such high mutual inclination can provide substantial evidence of orbital flips with different timescales under specific conditions of planetary mass, eccentricity, and mutual inclination, implying that \( \gamma \) Cep Ab may have flipped due to EKL. When the planetary mass is \( 15 \, M_{\text{Jup}} \), \( \gamma \) Cep Ab can easily reach the targeted mutual inclination above \( 120^\circ \) with critical initial conditions of \( i_{\text{init}} < 60^\circ \) and \( e_1 < 0.7 \). Moreover, the flipping cases of \( \gamma \) Cep Ab are proved to be stable from the stability index and Poincaré surfaces of section. Finally, we explore the parameter spaces of planetary mass, SMA, and eccentricities to provide a theoretical prediction when searching for potentially inclined S-type planets in close binary systems.

When \( a_1/a_2 \) is fixed, the flip occurs where \( e_1 \) and \( e_2 \) are both larger than 0.2, or \( e_1 < 0.2 \) and \( e_2 > 0.3 \). The flip likelihood of typical S-type planets is further addressed in order to search for inclined S-type planets induced by EKL (see Figure 1).

This work is structured as follows. In Section 2, the published high-precision RV data are employed to derive the orbital solution of \( \gamma \) Cep Ab, through a full N-body fitting with the MCMC sampler. Section 3 describes the KL and EKL mechanisms. Section 4 provides a qualitative analysis of the numerical results of the maximum inclination and orbital flip timescales under various initial conditions for \( \gamma \) Cep Ab. The stability of the flipping cases is mapped in the planes of \( (e_1, e_2) \) and \( (i_1, g_1) \), as well as the Poincaré surfaces of section. Section 5 extends EKL to more general S-type planets in order to investigate the flip conditions. In Section 6, we summarize the major outcomes.

2. Orbital Solutions of \( \gamma \) Cep Ab

2.1. N-body Fitting of the RV Data

The \( \gamma \) Cep system is known as a close binary at a distance of 13.79 pc (Hatzes et al. 2003), and it is a candidate target of the CHES mission (Ji et al. 2022). The primary star \( \gamma \) Cep A is a planet-hosting bright star of spectral type K1III–IV, with a stellar mass of \( m_0 = 1.40 \pm 0.12 \, M_\odot \) (Neuhäuser et al. 2007). Neuhäuser et al. (2007) presented a direct detection of the companion \( \gamma \) Cep B, where the parameters of the secondary star are \( m_2 = 0.409 \pm 0.018 \, M_\odot \), \( a_2 = 20.18 \pm 0.66 \, \text{au} \), \( I_2 = 119.3^\circ \), \( \Omega_2 = 18.04 \pm 0.2^\circ \), and the orbital period is \( P_2 = 67.5 \pm 1.4 \, \text{yr} \). Reffert & Quirrenbach (2011) conducted a fitting only for the inclination \( I_1 \) and the ascending node \( \Omega_1 \), adopting \( P, a, e, \) and \( K \) from the best-fitting solution from Butler et al. (2006). They thereby obtained the best-fitting values of \( I_1 = 57.7^\circ, \Omega_1 = 37.5^\circ \) or \( I_1 = 173.7^\circ, \Omega_1 = 356.7^\circ \) (see Table 1), where \( I_2 \) and \( \Omega_2 \) are constrained by Neuhäuser et al. (2007).

By using the most recent spectroscopic observations from the Canada–France–Hawaii Telescope (CFHT; Walker et al. 1992) and the McDonald Observatory Planetary Search (MOPS) program (Hatzes et al. 2003), two-Keplerian solutions of the \( \gamma \) Cep system have previously been given (Hatzes et al. 2003; Neuhäuser et al. 2007; Torres 2007). In these studies, an
uncoupled two-Keplerian orbital fitting approach is utilized with the standard nonlinear least-squares technique. However, for a hierarchical system with a massive secondary companion, the significant mutual interaction between the planet and the star in the pair is supposed to be considered under the Jacobi frame, in order for the real dynamics to be represented (Lee & Peale 2003).

Here, we solve the orbits of the planet and the star companion with the MCMC ensemble sampler *emcee* (Foreman-Mackey et al. 2013) in the full N-body model (Ford 2006; Nelson et al. 2016). 14 parameters of \( m_1 \), \( P_1 \), \( \sqrt{e_1} \sin \omega_1 \), \( \sqrt{e_1} \cos \omega_1 \), \( M_1 \), \( m_2 \), \( P_2 \), \( \sqrt{e_2} \sin \omega_2 \), \( \sqrt{e_2} \cos \omega_2 \), \( M_2 \), plus the RV offsets \( [RV_{0.1}, RV_{0.2}, RV_{0.3}, RV_{0.4}] \) of four time series (CFHT, MOPS I, MOPS II, and MOPS III), are adopted for fitting at the first observation epoch (HJD-2444754.129) in the Jacobi reference frame (Lee & Peale 2003). \( \sqrt{e} \sin \omega \) and \( \sqrt{e} \cos \omega \) are more efficient than \( e \) and \( \omega \) for low-eccentricity planets, and can avoid a situation of multiple solutions (Ford 2006).

We then utilize the N-body integrator *IAS15* (Rein & Spiegel 2015), which is an integrator with an adaptive step-size control, to compute the perturbed orbits of the planet and the secondary star in each step of the *emcee* sampler. The initial orbital inclinations and ascending nodes are assumed to be known (Reffert & Quirrenbach 2011): \( I_1 = 5^\circ.7 \), \( \Omega_1 = 37^\circ.5 \) or \( I_1 = 173^\circ.1 \), \( \Omega_1 = 356^\circ.7 \). Other initials include the planetary mass \( m_1 \) and the secondary mass \( m_2 \), the SMA \( a_1 \), the eccentricity \( e_1 \), and the argument of periastron \( \omega_1 \), which can be derived from the resultant fitting parameters. The integration precision is given as \( 10^{-15} \) and the minimum time step is set to 0.001 days. The theoretical RV signals of the observation epochs are calculated from the Jacobi orbital elements, with the RV semi-amplitude \( K_{1,2} \) being defined in

\[
K = \left( \frac{2\pi G}{P} \right)^{1/3} \left( \frac{m_1 \sin I}{m_0 + m_1} \right)^{2/3} \frac{1}{\sqrt{1 - e^2}},
\]

where \( P_{1,2}, I_{1,2}, \) and \( e_{1,2} \) are time-variable orbital elements at each observation epoch.

To derive the N-body best-fitting solutions for this system, we run 100,000 steps each for 28 walkers in the 14-dimensional parameter space. The mean acceptance fraction of the sampler walkers is 0.36, which is in the range from 0.2 to 0.5 that is suggested to produce representative samples (Foreman-Mackey et al. 2013), and the autocorrelation time for each fitting parameter ranges from 200 to 500 steps, which is smaller than our number of sampling steps to ensure the convergence of the MCMC chains. Here, the autocorrelation time represents the number of steps required for the chain to produce an independent sample (Foreman-Mackey et al. 2013).

Finally, we report two set of solutions of the minimum mass \( m \), the RV semi-amplitude \( K \), the SMA \( a \), the eccentricity \( e \), the argument of periastron \( \omega \), and the epoch of periastron passage \( T_0 \) in Table 1, with \( \chi^2 = 1.48 \) for \( I_1 = 5^\circ.7 \) and \( \chi^2 = 1.44 \) for \( I_1 = 173^\circ.1 \), respectively. In Figures 2 and 3, we plot the best-fitting RV signals from the N-body solutions (the black solid lines), using the measurements versus the epoch with \( \chi^2 = 1.44 \), with respect to \( RV_{0.1} = 1290.0 \pm 17.6 \) ms\(^{-1} \), \( RV_{0.2} = 2035.2 \pm 18.0 \) ms\(^{-1} \), \( RV_{0.3} = 2231.1 \pm 19.2 \) ms\(^{-1} \), and \( RV_{0.4} = 865.0 \pm 19.4 \) ms\(^{-1} \), respectively.

By subtracting the theoretical RV induced by the secondary, Figure 4 presents the Lomb–Scargle periodogram analysis of the planet b, where the strongest signal reveals an orbital period
2.2. Estimation of the Planetary Mass

To derive more reliable orbital solutions, it is necessary to utilize high-precision RV and astrometric measurements in the fitting procedure at the same time. As is well known, Gaia Data Release 2 (Gaia Collaboration et al. 2018) was first accessible in 2018, and the science team recently announced Gaia Early Data Release 3 (Gaia Collaboration et al. 2021). For most of the sources in the Gaia catalogs, sequential astrometric data are not yet available, and will be released in Gaia Data Release 4. Thus, improvements of the planetary masses from Gaia will be expected in the future.

Table 1 summarizes our derived orbital parameters of the \( \gamma \) Cep system. The minimum planetary mass is fitted as 1.7420\(^{+0.0743}_{-0.0733} \) \( M_{\text{Jup}} \) or 1.6306\(^{+0.0698}_{-0.0699} \) \( M_{\text{Jup}} \). Note that the real mass of planet b is significantly dependent upon the accuracy of the RV observations and the dynamical integrations; here, we simply estimate the mass range of \( \gamma \) Cep Ab. When the observed inclination of the planet is \( i_1 \in [3^\circ, 8^\circ] \) (Reffert & Quirrenbach 2011), we obtain an estimated planetary mass \( m_1 \in [5, 26.6] M_{\text{Jup}} \), while for \( i_1 \in [166^\circ, 174^\circ] \) (Reffert & Quirrenbach 2011), \( m_1 \in [7.1, 26.2] M_{\text{Jup}} \) can be derived. In addition, based on the planetary mass limit of 16.9 \( M_{\text{Jup}} \) (Torres 2007), we here assume \( m_1 \in [5, 16.9] M_{\text{Jup}} \).

To determine the current observed mutual inclination \( i_{\text{mut}} \) in the \( \gamma \) Cep system, we use the law of cosines for angles of a spherical triangle (Gellert et al. 1977):

\[
\cos i_{\text{mut}} = \cos i_1 \cos i_2 + \sin i_1 \sin i_2 \cos(\Omega_1 - \Omega_2),
\]

deriving \( i_{\text{mut}} = 113.9^\circ \) for \( i_1 = 5^\circ \). Thus, a question naturally arises—what kind of dynamical process could trigger such high mutual inclinations of the two bodies, and could the EKL mechanism play a role in the secular evolution of \( \gamma \) Cep Ab? In the following section, we describe the EKL mechanism and explore the secular evolution of \( \gamma \) Cep Ab under EKL.

3. Dynamical Model

3.1. KL Mechanism

In the secular evolution of triple-body systems, the perturbation arising from the third body acts on a timescale much longer than its orbital period. One typical origin of secular resonance is from the perturbation potential from adjacent orbits. When the mass of the perturbed body is negligibly small in comparison to those bodies in a hierarchical system, the test particle approximation comes into play. At this time, if the perturbation of a circular outer orbit works, the Hamiltonian of this system can be expanded to the quadrupole level, known as the KL mechanism, as aforementioned.

The KL mechanism addresses the inclination and eccentricity of the inner test particle oscillating over secular evolution in a hierarchical system, where the test body and
the primary star are surrounded by a distant companion (von Zeipel 1910; Kozai 1962; Lidov 1962). With the secular approximation, the inner and outer orbits only exchange angular momentum, so the SMAs of the orbits do not change. When the SMA ratio $\alpha = a_1/a_2$ is a small parameter, the perturbation term of the complete Hamiltonian can be expanded as a power series in $\alpha$ (Naoz 2016):

$$H = \frac{k^2 m_0 m_1}{2a_1} + \frac{k^2 m_2 (m_0 + m_1)}{2a_2} + \frac{k^2}{a_2^2} \sum_{j=2}^{\infty} \alpha_j^2 M_j \left( \frac{a_1}{a_2} \right)^{j+1} P_j(\cos \Phi),$$

$$M_j = m_0 m_1 m_2 \frac{m_j^{j-1}-(-m_j)^{j-1}}{(m_0 + m_1)^j},$$

where $m_0$ is the mass of the primary, $m_1$ and $m_2$ are the masses of the inner and the outer body, $k^2$ is the gravitational constant (with the mass unit of $M_\odot$ and the length unit of au), $r_1$ is the distance between $m_0$ and $m_1$, $r_2$ is the distance between the center of mass of the inner binary and $m_2$, $P_j$ is the Legendre polynomial, and $\Phi$ is the angle between the vectors $r_1$ and $r_2$ (the subscripts $j = 1, 2$ represent the inner and outer orbits, respectively).

The invariable plane reference frame is imported here to define three important Delaunay elements (Valtonen & Karttunen 2006): $l, g, h$, and their conjugate momenta $L, G, H$, where $l, g,$ and $h$ are the notations of the mean anomaly $M$, the argument of periastron $\omega$, and the longitude of ascending node $\Omega$, respectively. As shown in Figure 6, $G_{tot}$ is the total angular momentum vector of the system, which is conserved over the secular evolution.

**Figure 5.** Corner diagram of our $N$-body best-fitting solutions, with $\chi^2 = 1.44$. Here, $m \sin I$, $P$, $\sqrt{e} \sin \omega$, $\sqrt{e} \cos \omega$, and $M$ with subscripts 1 and 2, denote the minimum mass, orbital period, eccentricity vector, and mean anomaly of $\gamma$ Cep Ab and $\gamma$ Cep AB, respectively. The figure illustrates the one- and two-dimensional projections of the posterior probability distributions of the parameters, with the structure directly showing the covariances between the two parameters. The distribution histograms of each parameter are listed on the diagonal.
The three conjugate momenta $L$, $G$, $H$ are expressed as (Naoz et al. 2013):

\[
L_1 = \frac{m_0 m_1}{m_0 + m_1} \sqrt{k^2(m_0 + m_1)} a_1,
\]

\[
L_2 = \frac{m_2(m_0 + m_1)}{m_0 + m_1 + m_2} \sqrt{k^2(m_0 + m_1 + m_2)} a_2,
\]

\[
G_1 = G_1 \sqrt{1 - e_1^2}, \quad G_2 = G_2 \sqrt{1 - e_2^2},
\]

\[
H_1 = G_1 \cos i_1, \quad H_2 = G_2 \cos i_2,
\]

\[
G_{\text{tot}} = H_1 + H_2,
\]

where $L$ is only determined by constant parameters, including the masses $m_0$, $m_1$, and $m_2$, the SMAs $a_1$, $a_2$, and the gravitational constant $k$. Thus, $L$ is a constant for a specific system in the evolution, while $G$ and $H$ are time-varying. $G$ represents the magnitude of each orbit’s angular momentum and $H$ is the component of $G$ along the z-axis.

According to the geometric relations and the assumption of $h_1 - h_2 = \pi$, the mutual inclination between the inner and outer orbit $i_{\text{mut}}$ could be derived as (Naoz et al. 2013):

\[
\cos i_{\text{mut}} = \cos(i_1 + i_2) = \frac{G_{\text{tot}}^2 - G_1^2 - G_2^2}{2G_1 G_2}.
\]

Generally, the equations of motion can be expressed by canonical relations of the three conjugate momenta, $L$, $G$, $H$, and the three Delaunay elements, $l$, $g$, $h$, as the mean anomaly can be eliminated under the double-averaged secular approximation, and $h_1$ and $h_2$ in equations of motion have been removed by the relation $h_1 - h_2 = \pi$. The time evolution for $\omega$, $e$, and $i$ can easily be derived from the reduced canonical relations (Naoz 2016):

\[
\frac{dG_j}{dt} = \frac{\partial H}{\partial g_j}, \quad \frac{dg_j}{dt} = -\frac{\partial H}{\partial G_j},
\]

where $j = 1, 2$. The original nonplanar three-body system can thus be reduced to a dynamical system of 2 degrees of freedom (dof).

In Naoz et al. (2013), $i_2$ is set to be 0, thus $i_{\text{mut}}$ is equal to $i_1$. In this work, we treat the orbital plane of the secondary as the invariable plane, where $i_{\text{mut}} = i_1$. In the following sections, we redefine the orbital flip of the planet $b$ as the variation of $i_{\text{mut}}$ around 90°, instead of the real observed orbital inclination.

3.2. EKL Mechanism in Nonrestricted Triple Systems

In the $\gamma$ Cep Ab B system, $e_2$ is close to 0.36 and the estimated maximum mass of the planet is near the deuterium-burning limit. Thus, $\gamma$ Cep Ab B is a nonrestricted triple system with an eccentric outer orbit, where the octupole-level terms in the Hamiltonian can become important, and the eccentricities of the two orbits are coupled and oscillate simultaneously over secular evolution. Naoz et al. (2013) called it the EKL mechanism in nonrestricted triple systems and derived the complete Hamiltonian of the system, including the octupole term in addition to the quadrupole term:

\[
H_3(\Delta h \rightarrow \pi) = H_{\text{quad}} + H_{\text{oct}}
\]

\[
= C_2 (2 + 3 \epsilon_1^2) (3 \cos^2 i_{\text{mut}} - 1) + 15 \epsilon_2^2 \sin^2 i_{\text{mut}} \cos(2g_2)
\]

\[
+ G_3 \epsilon_1 e_2 [A \cos \phi + 10 \epsilon_2 \sin^2 i_{\text{mut}} \sin g_1 \sin g_2],
\]

where

\[
C_2 = \frac{k^4}{16 (m_0 + m_1 + m_2)^5 (m_0 m_1)^3 L_2^4 G_1^4},
\]

\[
\epsilon_M = \left( \frac{m_0 - m_1}{m_0 + m_1} \right)^{2/3} \frac{e_2}{a_1} \frac{e_2}{a_2} (1 - e_2^2),
\]

\[
C_3 = \frac{15}{4} \epsilon_M C_2,
\]

\[
A = 4 + 3 \epsilon_1^2 - \frac{5}{2} B \sin i_{\text{mut}},
\]

\[
B = 2 + 5 \epsilon_1^2 - 7 \epsilon_2^2 \cos(2g_1),
\]

\[
\cos \phi = -\cos g_1 \cos g_2 - \cos i_{\text{mut}} \sin g_1 \sin g_2.
\]

The secular perturbation theory in this specific case is called the EKL mechanism, as previously mentioned. The time evolution for $\omega$, $e$, and $i_{\text{mut}}$ can be derived through Equation (8) as well. Here, $\epsilon_M$ can be used as an indicator to characterize the strength of the octupole-level effect on the system’s perturbation potential.

4. Secular Evolution of $\gamma$ Cep Ab

As we described in Section 1, the EKL mechanism plays a crucial role in the secular evolution of celestial bodies. Here, in the $\gamma$ Cep Ab B system, the inner planet $b$ is assumed to be a substellar object with a remarkable mass (Reffert & Quirrenbach 2011); thus, the test particle approximation is not applicable. With $a_1 = 2.1376$ au, $a_2 = 18.6217$ au, and the SMA ratio of planet $b$ and the secondary $\alpha_b \sim 0.1$, the perturbation term of the complete Hamiltonian can thus be expanded as a power series in $\alpha_b$. As the eccentricity of the outer orbit $e_2 \sim 0.4$, the EKL mechanism can play a significant part when exploring the secular evolution of the $\gamma$ Cep Ab B system. Here, we will investigate the mutual inclination
The contour maps of the total angular momentum $G_{\text{tot}}$ and the perturbation Hamiltonian $\mathcal{H}$ for $m_1 = 5$ and $G_{\text{tot}}$ in the initial conditions plane of $(e_2, i_{\text{mut}})$. The white dashed lines represent the constant values of $G_{\text{mut}}$ for $e_2 \in [0.35, 0.45]$, while the color bars on the right indicate the different values of the perturbation Hamiltonian. In panel (a), when $e_2$ selects five different values, there will be five level curves of $G_{\text{mut}}$. We also mark out the initial conditions for $G_{\text{tot}} = 1.75318$ (the black dashed lines) and $\mathcal{H} = -8.06 \times 10^{-6}$ (the solid line) with black dots, corresponding to data points with red error bars in Figure 8(a).

Figure 7. The contour maps of the total angular momentum $G_{\text{tot}}$ and the perturbation Hamiltonian $\mathcal{H}$ for $m_1 = 5$ and $15 G_{\text{tot}}$ in the initial conditions plane of $(e_1, i_{\text{mut}})$. The white dashed lines represent the constant values of $G_{\text{mut}}$ for $e_2 \in [0.35, 0.45]$, while the color bars on the right indicate the different values of the perturbation Hamiltonian. In panel (a), when $e_2$ selects five different values, there will be five level curves of $G_{\text{mut}}$. We also mark out the initial conditions for $G_{\text{tot}} = 1.75318$ (the black dashed lines) and $\mathcal{H} = -8.06 \times 10^{-6}$ (the solid line) with black dots, corresponding to data points with red error bars in Figure 8(a).

Figure 7. The contour maps of the total angular momentum $G_{\text{tot}}$ and the perturbation Hamiltonian $\mathcal{H}$ for $m_1 = 5$ and $15 G_{\text{tot}}$ in the initial conditions plane of $(e_1, i_{\text{mut}})$. The white dashed lines represent the constant values of $G_{\text{mut}}$ for $e_2 \in [0.35, 0.45]$, while the color bars on the right indicate the different values of the perturbation Hamiltonian. In panel (a), when $e_2$ selects five different values, there will be five level curves of $G_{\text{mut}}$. We also mark out the initial conditions for $G_{\text{tot}} = 1.75318$ (the black dashed lines) and $\mathcal{H} = -8.06 \times 10^{-6}$ (the solid line) with black dots, corresponding to data points with red error bars in Figure 8(a).

Oscillations of the $\gamma$ Cep Ab B system to examine whether planet b can achieve an extremely inclined orbit through EKL.

In this section, we are mainly concerned with the flip conditions and timescale for the first flip of $i_1$, which is equal to $i_{\text{mut}}$. The stability of the rolling-over orbits is further explored. To investigate the relative global dynamics of the system, we show several kinds of representative planes of initial conditions. In Section 4.1, the $(e_1, i_{\text{mut}})$ plane is first used to search the flip conditions. In Section 4.3, in order to obtain the global qualitative structure of the averaged 2 dof Hamiltonian system, we import the representative plane of $(e_1, e_2)$ (Michtchenko & Malhotra 2004) and the phase space of $(e_1, g_1)$ (Tan et al. 2020). We then apply the $(\sqrt{1 - e_1^2}, \sqrt{1 - e_2^2})$ plane as the representative plane for studying the relative global dynamical features of the $\gamma$ Cep system, and employ the $(e_1, g_1)$ plane to theoretically define quasiperiodic and circulating orbits.

4.1. Orbital Flip Conditions

The investigation of the amplitude of the inclination oscillation reveals that $\gamma$ Cep Ab could evolve into an extremely inclined orbit. However, one may encounter difficulties in accurately predicting the theoretical correlation between the extent of the orbital inclination excitation and the initial conditions. Katz et al. (2011) defined a complex function of the analytical critical flip condition by averaging over the quadrupole-level effect.

We provide the planetary mass $m_1 \in [5, 16.9] G_{\text{Jup}}$ in Section 2.2. Here, we assume the mass of $\gamma$ Cep Ab to be $m_1 \in [5, 9, 11, 15] G_{\text{Jup}}$, then calculate the ranges of the perturbation Hamiltonian $\mathcal{H}$ and the total angular momentum $G_{\text{tot}}$ for each value of $m_1$ with known parameters $\{m_0, m_2, a_1, a_2\}$ and $e_1 \in [0, 1], e_2 \in [0.35, 0.45]$, and $i_{\text{mut}} \in [0^\circ, 180^\circ]$. Furthermore, we set $g_1 = 0^\circ$ and $g_2 = 0^\circ$ in the initial conditions, since $g_1$ and $g_2$ can always go through either $0^\circ$ or $180^\circ$ over secular evolution.

The contour maps of the total angular momentum $G_{\text{tot}}$ and the perturbation Hamiltonian $\mathcal{H}$ are simultaneously plotted in the $(e_1, i_{\text{mut}})$ plane. In Figure 7, we show two examples of $m_1 \in [5, 15] G_{\text{Jup}}$ with $e_2 \in [0.35, 0.45]$, which is around the current observation and will not vary significantly over the evolution. For other planetary masses, the structures of the $\mathcal{H}$–$G_{\text{tot}}$ contour maps have similar features with different ranges of $G_{\text{tot}}$ and $\mathcal{H}$. When $m_1 = 5 G_{\text{Jup}}, G_{\text{tot}} \in [1.7050, 1.8025], \mathcal{H} \in [-1.05 \times 10^{-6}, 3.0 \times 10^{-7}], \text{and when } m_1 = 9 G_{\text{Jup}}, G_{\text{tot}} \in [1.6994, 1.8137], \mathcal{H} \in [-1.75 \times 10^{-6}, 5.0 \times 10^{-7}].$ When $m_1 = 11 G_{\text{Jup}}, G_{\text{tot}} \in [1.6975, 1.8187], \mathcal{H} \in [-2.10 \times 10^{-6}, 6.0 \times 10^{-7}], \text{and when } m_1 = 15 G_{\text{Jup}}, G_{\text{tot}} \in [1.696, 1.824] \text{ and } \mathcal{H} \in [-2.8 \times 10^{-6}, 8.0 \times 10^{-7}].$ The cross points of the $G_{\text{tot}}$ and $\mathcal{H}$ contours indicate all the possible initial conditions of $i_{\text{mut}}$ and $e_1$ over the secular evolution. The range of values of $G_{\text{tot}}$ and $\mathcal{H}$ in Figure 7 will change with variational planetary masses, while the structure of the contours will remain similar.

To further derive the evolution results of these general initial conditions, we uniformly choose specific values of $\mathcal{H}$ and $G_{\text{tot}}$ between the upper and lower limits in Figure 7. For a given $\mathcal{H}$, based on the conserved Hamiltonian and the total angular momentum shown in Equations (5) and (8) of the octupole perturbation theory, we choose at least 40 cases of initial $e_1$ and $i_{\text{mut}}$. Considering the conservation of energy and the total angular momentum, we further explore the extreme value of the orbital inclination and orbital stability under these selected initial conditions in detail.

In the non–test particle approximation under the classical KL mechanism, the eccentricity and inclination oscillate regularly over a well-defined timescale $t_{\text{quad}}$ (Antognini 2015):

$$t_{\text{quad}} \sim \frac{16}{15} \frac{a_2^2 (1 - e_2^2)^{1/2}}{a_1^{1/2} m_2 k} \sqrt{m_0 + m_1}.$$

This relationship was derived under consideration of the equation of motion of $\omega$, by integrating between the maximum and minimum eccentricities. Here, $t_{\text{quad}}$ can be applied to estimate the timescale in the EKL scenario.

According to the values of $m_0, m_1, m_2, e_2, a_1,$ and $a_2$, the quadrupole period $t_{\text{quad}}$ of the $\gamma$ Cep system is estimated to be $\sim 1000$ yr, which is consistent with our numerical simulation results. Here, we investigate the secular evolution of $\gamma$ Cep Ab.
by considering a diverse planetary mass and performing the simulation for 100 Myr (∼10^5 t_quad) using the RKF7(8) integrator. The observed orbital inclinations I_1 and I_2 are required to calculate the constant total angular momentum and Hamiltonian. Hereafter, i_1 and i_2 denote the angles of the orbital plane relative to the invariable plane. However, the true orbital inclinations of the inner and outer orbits over the evolution are not well known.

Panels (a)–(d) of Figure 8 each plots three sets of typical outcomes, with H_1, H_2, and H_3. If m_1 = 5 M_Jup, both the prograde and retrograde orbits will flip when e_1 < 0.5 for H_1 = -8.06 × 10^{-8}, e_1 > 0.65 for H_2 = -4.95 × 10^{-7}, and e_1 > 0.8 for H_3 = -6.08 × 10^{-7}. We note that most intense orbital flips take place when the initial i_mut is close to 90°.

Figure 8. Ranges of i_mut for m_1 = 5, 9, 11, and 15 M_Jup over a timescale of 100 Myr. The black dots represent the positions of the initial parameters {e_1,0, i_mut,0}. The upper and lower limits of the error bars represent the maximum and minimum values of the mutual orbital inclination i_mut, respectively. The different colors correspond to specific values of the perturbation Hamiltonian.

From the estimated m_1 sin l_1 in Table 1, and the law of cosines for the angles of a spherical triangle in Equation (6), if m_1 = 5 M_Jup and the observed inclination of planet b is roughly 20°, the mutual inclination i_mut ∼ 100° is l_2 = 119.7°. As a consequence, it is clear that under the selected values of H_1, H_2, and H_3, such a high mutual inclination could always be achieved for original prograde orbits. In fact, as shown in Figure 8, the evolution of the mutual inclination of prograde and retrograde orbits is approximately symmetric about 90°, which can also be obtained from Equation (8).

Similarly, if m_1 = 9 M_Jup, the observed inclination of planet b is nearly 11°, thus the mutual inclination i_mut is estimated to be 108°. When H_1 = -1.45 × 10^{-7}, e_1 ∼ 0.5, H_2 = -6.35 × 10^{-7}, e_1 ∼ 0.65, and H_3 = -1.22 × 10^{-6}, e_1 ∼ 0.8, the amplitude of inclination for the orbital flip could be raised up to about 180°.

When m_1 is equal to 11 M_Jup, the calculated inclination of γ Cep Ab is about 9° and the targeted mutual inclination i_mut ∼ 110° can be achieved when H_1 = -1.98 × 10^{-7}, with e_1 ∼ 0.5, H_2 = -8.56 × 10^{-7}, with e_1 ∼ 0.7, and H_3 =
Thus, the equilibrium value of the initial eccentricity for the orbital flips of $H_2$ in Figure 8(c) is larger than the values for Figures 8(a), (b), and (d).

When $m_1 = 15\, M_{\text{Jup}}$, the observed inclination of $\gamma$ Cep Ab is about $6^\circ$ and the critical conditions for the targeted mutual inclination $i_{\text{mut}} \sim 113^\circ$ are $H_2 = -2.4 \times 10^{-7}$, with $e_1 \sim 0.5$, $H_2 = -1.44 \times 10^{-6}$, with $e_1 \sim 0.65$, and $H_2 = -2.03 \times 10^{-6}$, with $e_1 \sim 0.8$. In Figure 8(d), for $H_2 = -1.44 \times 10^{-6}$, the orbital flip occurs when the initial $i_{\text{mut}} < 60^\circ$. Thus, it is easier for $\gamma$ Cep Ab to reach the targeted mutual inclination with $m_1 = 15\, M_{\text{Jup}}$, $e_1 < 0.7$, and the critical initial $i_{\text{mut}}$ being lower than $60^\circ$.

Above all, we conclude that the initial conditions for the orbital flips under investigation for the $\gamma$ Cep system are $e_1 > 0.5$ and $i_{\text{mut}} \in [60^\circ, 120^\circ]$, and that various planetary masses simply affect the critical eccentricity when $e_1 > 0.6$, with little influence on the maximum mutual inclination. The distribution tendency of the flipping conditions with various initial inclinations and eccentricities is similar to that in Lei (2022), while we perform the variation of the flipping conditions under different $m_1$, which affects the octupole-level factor $e_M$.

### 4.2. Orbital Flip Timescale

To obtain an in-depth understanding of the rolling-over orbits reported in Section 4.1, we will further explore the orbital flip timescale over secular evolution, which may rely on the initial conditions, e.g., $m_1$, $e_1$, and $i_{\text{mut}}$. The duration of the flip is critical for the observational possibility of potentially extremely inclined S-type planets that are transforming between prograde and retrograde orbits. An approximate analytical timescale for the first flip for a nonchaotic orbit when $m_1 \rightarrow 0$ has been given by Katz et al. (2011) and Antognini (2015):

$$t \sim \frac{128}{15\pi} \frac{a_2^3}{a_1^{3/2}} \frac{m_0}{k m_2} \sqrt{\frac{10}{\epsilon}} \frac{(1 - e_2)^{3/2}}{1 - e_2^2},$$

where

$$\epsilon \equiv \frac{e_2}{a_1} \frac{a_1}{1 - e_2}. \quad (16)$$

This theoretical flip timescale is effective when the initial conditions meet $e_1 \rightarrow 0$, $\omega_1 \rightarrow 0$, and $i_{\text{mut}} \rightarrow 90^\circ$. Note that the flip timescale for the circular test particle approximation is mainly determined by $e_2$ when the object masses and the orbital SMAs are fixed. Nevertheless, in the case of high-inclination oscillation, the timescale for the first flip is difficult to quantitatively assess, because this evolution is likely to be chaotic. In order to investigate the dependence of the orbital flip possibility on the integration timescale, we adopt the definition of the flip ratio $f = f_{\text{flip}}/f_{\text{total}}$ (Teyssandier et al. 2013) in order to describe the timescale for the first flip and the observational possibility of the orbital flip process over secular evolution, where $f_{\text{flip}}$ is the duration of the orbital flip process and $f_{\text{total}}$ represents the total evolution timescale. We extract the cases from Section 4.1 in which orbital flips occur in Figure 9. For these rolling-over orbits, the integrations stop when $e_1$ becomes larger than 0.9999 and the planet falls into the Roche limit of the primary star.

In Figure 9(a), we observe that when $H_2 = -8.06 \times 10^{-8}$, the orbits turn over after 30 Myr and take more than 80 Myr to arrive at the equilibrium $f$. However, the flips under $H_2$ and $H_3$ occur from the very beginning of the evolution, and reach the equilibrium value before 1 Myr. This phenomenon also shows up in Figures 9(b) and (c), indicating that the flip timescale decreases with the increase of $H$, under most circumstances. The oscillation timescale of $f$ over the flip procedure decreases with the rise of the perturbation Hamiltonian, as can be seen from the blue and green curves in the four subfigures.

In Figure 9(d), there is a special flipping case for $H_2$ when $m_1 = 15\, M_{\text{Jup}}$. This rolling-over orbit has initial conditions of $e_1 = 0.64$ and $i_{\text{mut}} = 58.79^\circ$ in Figure 8(d). In comparison to the other cases under $H_2$ for different masses of planet, it can be concluded that a relatively low initial $i_{\text{mut}}$ below $60^\circ$ leads to a relatively large timescale for the first orbital flip, as well as more time to reach the equilibrium value of the flip ratio.

Moreover, we investigate the equilibrium value $f$ for the flip cases for $H_2$ and $H_3$. We find that the final $f$ of the dotted–dashed lines are all located above 0.5, while those of the solid lines are all lower than 0.5. Thus, the time duration of the flip process for those original retrograde orbits is larger. The maximum $f$ occurs in Figure 9(b), with $f \sim 0.65$ when $m_1 = 9\, M_{\text{Jup}}$, $H_2 = -6.35 \times 10^{-7}$, whereas the minimum $f$ occurs in Figure 9(d), with $f \sim 0.4$ when $m_1 = 15\, M_{\text{Jup}}$, $H_2 = -2.03 \times 10^{-6}$. Thus, the equilibrium value of the flip ratio under the EKL mechanism is related to the planetary mass, initial eccentricities, and mutual inclinations. When the equilibrium value of $f$ is closer to 0.5, the observational possibility of the retrograde orbit transforming from the prograde is higher.

### 4.3. Stability of Flipping Orbits

For a planetary system with given orbital parameters, the periodic orbits and stability are identified by the representative plane of $(e_1, e_2)$, the level curves in the $(e_1, g_1)$ plane, the Poincaré surface of section, and the long-term stability criterion. These methods are applied to our investigation of the stability of specific S-type planets in binary systems, while the perturbative treatment and the invariant manifolds characterize the stability with a fixed Hamiltonian (Lei 2022).

To derive a global and comprehensive view of the system dynamics, we first attempt to construct a parametrical analysis of the secular model. We first show the representative plane of $(e_1, e_2)$ by Michtchenko & Malhotra (2004), which was then followed by studies in secular dynamics and resonances (Libert & Henrard 2006, 2007; Henrard & Libert 2008; Libert & Henrard 2008). This approach relies on the secular averaging of the full Hamiltonian and can be applied to construct global phase portraits of the systematic variables, thereby detecting resonances and discerning periodic orbits in the $N$-body dynamics.

According to Michtchenko & Malhotra (2004), the secular motion of planetary systems is mainly decided by the global quantities of total energy and the angular momentum deficit. The phase structures are determined by two constants: $a_1/a_2$ and $m_1/m_2$. The description of the secular behavior of this system is derived by the distribution of the initial values of $e_1$, $e_2$, $g_1$, $g_2$, and $i_{\text{mut}}$. To simplify the model, we fix $m_1 = 15\, M_{\text{Jup}}$ and $i_{\text{mut}} = 60^\circ$ in Figure 10, $\sqrt{1 - \epsilon_1^2} \cos g_1 > 0$ with $g_1 = 0^\circ$, while $\sqrt{1 - \epsilon_1^2} \cos g_1 < 0$ with $g_1 = 180^\circ$, and $g_1$ can always go through $0^\circ$ and $180^\circ$ over the evolution. As seen from Figure 10, we may come to the conclusion that periodic orbits occur when $G_{\text{tot}}$ and $H$ have two cross points.
For the secular evolution of the γ Cep Ab B system, one couple of free variables is $e_1$ and $g_1$ in the 2 dof averaged problem. Thus, we can first easily perform the qualitative analysis in the $(e_1, g_1)$ plane (Tan et al. 2020). Given the limit of $G_{\text{tot}}$ in Section 4.1, we select and fix the value of $G_{\text{tot}}$ to present level curves of the Hamiltonian in the parameter space of $(e_1, g_1)$, as shown in Figure 11. The green stream lines for circulating orbits and the blue circles for resonant orbits are easy to distinguish. With the changes of the perturbation Hamiltonian, the dynamical structure transitions from circulation to libration and the evolution interval of $e_1$ vary. The smallest circles have the largest magnitude of $\mathcal{H}$, which indicates that the secular resonance effect is the most intense, as shown in Figure 12, with further details.

Figure 9. The evolution of the flip ratio for $m_1 = 5, 9, 11, \text{and } 15 M_{\text{Jup}}$ over a timescale of 100 Myr. As in Figure 8, $\mathcal{H}_i$, $\mathcal{H}_c$, and $\mathcal{H}_p$ correspond to the selected perturbation Hamiltonian. In each panel, the evolution before 2 Myr is given as a subfigure. In these subfigures, dotted-dashed lines with an equilibrium value $f > 0.5$ represent prograde orbits that have been transformed from retrograde orbits, while solid lines with $f < 0.5$ have been transformed from prograde orbits to retrograde orbits.

In addition to the representative plane of $(\sqrt{1 - e_1^2}, \sqrt{1 - e_2^2})$ and the level curves in the $(e_1, g_1)$ plane, we further investigate the stability of the rolling-over orbits under the selected Hamiltonian with the Poincaré surface of section. For a two-dimensional Hamiltonian system, the Poincaré surface of section is a commonly used method: under a given energy integral, the phase flow of the system is three-dimensional. By selecting a suitable section, the intersection point of the systematic phase flow and the section is projected in two dimensions. In other words, the Poincaré surface of section reflects the geometry of the system dynamics. Given the integration of energy (i.e., the octopole perturbation Hamiltonian) and the selection of the section (i.e., the $g_1 = 0$ section),
The surface of section can be generated by plotting intersection points in the $g_2 - e_2$ frame for the plane of $g_1 = 0$.

The surface of section enables us to identify the order of orbital resonance and the dynamical stability of the system through a series of geometric structures. For a given fixed value of the Hamiltonian, the orbital mode can be derived from only two orbital parameters. Various orbital modes, including resonance, circulation, and chaos, are distinguished by the shape of the point distribution in the section.

We present typical structures in the surfaces of section of the $\gamma$ Cep Ab B system in Figure 12. The surfaces of section are plotted in the $g_1 = 0$ plane for $m_1 = 5, 9, 11, 15 M_{\text{Jup}}$, with a corresponding perturbation Hamiltonian. To initially set $e_2$ and $g_2$ in the uniform grid, we solve other initial parameters, based on the conservation of the system’s energy and the total angular momentum. There are two kinds of regions in these plots: the closed circles are resonant orbits (quasiperiodic orbits), where $e_2$ and $g_2$ oscillate in the bounded regions, while the stream lines denote circulating orbits, where at least one of the parameters circulates.

Figure 12 simply exhibits the dynamical structure of the surfaces of section for $m_1 = 11$ and $15 M_{\text{Jup}}$. With increasing $H$, the structures in the surfaces of section appear to be diverse. As shown in Figures 12(a), (c), and (e), when $m_1 = 11 M_{\text{Jup}}$, the orbits with respect to $g_2 = 180^\circ$ disappear, whereas both orbits with fixed points of $g_2 = 0^\circ$ and $180^\circ$ remain. These libration regions perfectly match the orbits that would turn over. In Figures 12(b), (d), and (f), when $m_1 = 15 M_{\text{Jup}}$, the positions of the fixed angles of $g_2$ are the same as for $m_1 = 11 M_{\text{Jup}}$. Figure 12(a) shows that the orbital flips of $\gamma$ Cep Ab are dominated by the octupole-level resonance, and the oscillation amplitude of $e_2$ in these flipping cases is close to 0.01. In Figures 12(e) and (f), when $m_1 = 11 M_{\text{Jup}}$, $H = -1.7 \times 10^{-6}$, and $m_1 = 15 M_{\text{Jup}}$, $H = -2.03 \times 10^{-6}$, respectively, most of the orbits around the initial values of $e_2$ could turn over in the secular evolution. Additional simulations for $m_1 = 5$ and $9 M_{\text{Jup}}$ reveal similar structures, but with various oscillation intervals of $e_2$, when compared to above results.

The orbital flip criterion of $e_2$ can be evaluated with the conserved total angular momentum in Equation (5), when $i_2 = 0^\circ$ and $i_{\text{mut}} = 90^\circ$:

$$e_{2,\text{flip}} = \sqrt{1 - \left( \frac{G_{\text{tot}}}{L_2} \right)^2}. \quad (17)$$

We mark this flip criterion in Figure 12 with the horizontal red lines. The circles and stream lines crossing the horizontal red line represent those orbits that invert regularly, while the ellipses above the horizontal red line in Figures 12(a)–(b) stand for the orbits that do not invert. Here, we find that the structures in the phase portrait become much clearer with the decrease in the perturbation Hamiltonian $H$, where the chaotic orbits disappear, similar to those in Lei (2022).

Additionally, we compare the boundaries of the regions of orbital flips in the $(e_1, 0, i_{\text{mut}}, 0)$ space for the nonrestricted model with those of Figure 3 (Lei 2022). By analyzing the results in the Poincaré surfaces of section, the inner orbit appears to be more stable in our hierarchical system when the planetary eccentricity $0.5 < e_1, 0 < 0.6$, which is simply occupied by circulating and librating orbits.

In order to further confirm whether the orbital flip in $\gamma$ Cep Ab is stable, we adopt the long-term stability criterion given by Mardling & Aarseth (2001):

$$\frac{a_2}{a_1} > 2.8 \left( \frac{1 - \frac{m_2}{m_0 + m_1}}{\left( 1 + e_2 \right)^{2/5}} \left( \frac{1 - e_2}{1 - e_2} \right)^{2/5} \left( 1 - \frac{0.3i_{\text{mut}}}{180^\circ} \right) \right). \quad (18)$$

where $e_2$ and $i_{\text{mut}}$ are time-varying, and the constants $a_1, a_2, m_0, m_1$, and $m_2$ can be moved to the left side of the expression. We then derive the new expression of this criterion by a new variable $S$:

$$S = \frac{(1 + e_2)^{2/5}}{(1 - e_2)^{2/5}} \left( 1 - \frac{0.3i_{\text{mut}}}{180^\circ} \right). \quad (19)$$

For $m_1 \in [5, 15] M_{\text{Jup}}$, $a_1 = 2.14 \text{ au}$, $a_2 = 18.62 \text{ au}$, thus $S$ should meet the requirement $S < 2.807$ to maintain the stability of the system. The calculated maximum $S$ for the rolling-over orbits in Figures 12(a), (c), and (f) is 2.13, which is definitely within the stability criterion. In Figures 12(b), (d), and (f), the calculated maximum $S$ is 2.64, providing stable cases using Equation (19). Hence, the orbital flips in Figure 12 are stable, without the chaotic excitation of the binary’s eccentricity and inclination.

Comparing with previous work on the stability of the $\gamma$ Cep system, Haghhighipour (2004) conducted an extensive numerical study of the orbital stability of the $\gamma$ Cep system and suggested that the system could remain steady for $i_{\text{mut}} \in [0^\circ, 60^\circ]$ and $e_2 < 0.5$. This condition is also supported in this work, since we constrain the eccentricity $e_2 \in [0.35, 0.45]$, which is well consistent with our simulation results. Taking Figure 12(b) as an example, the critical value of the initial $i_{\text{mut}}$ that is necessary for orbital flips to occur is lower than 60°, indicating a stable initial status, according to Haghhighipour (2004), with regular oscillations of $e_1$ and $i_{\text{mut}}$ causing the system always to return to a stable situation. Satyal et al. (2013) examined the stability and quasiperiodicity of $\gamma$ Cep, and explored the orbital stability for various inclinations and binary eccentricities $e_2$ through the reliability comparison of chaos indicator. They demonstrated that the planet $\gamma$ Cep Ab can remain stable for $e_2$ being as high as 0.6 or for $i_2 \leq 25^\circ$. In this work, for rolling-
over orbits, we demonstrate that γ Cep Ab can also remain stable when $e_2 \sim 0.4$ and $i_{mut} \sim 90^\circ$. We further confirm in this work that there is a great possibility of the flipping cases of γ Cep Ab being locked into a Kozai resonance, based on the surfaces of section in Figure 12 and the long-term stability criterion $S$.

From the simulation results, we also see that the planetary eccentricity could be stirred up to 0.9999, due to secular perturbation from the binary, as close approaches may eject the planet out of the system, so that it would not be observed or its orbit could be shrunk, owing to tides over the evolution. While this process takes too much time to be observed on the timescale of the EKL mechanisms, we employ $i_{mut}$ to explore the stability of the system, as the evolutions of $e_1$ and $i_{mut}$ are coupled under this scenario. The extreme oscillations of inclination and eccentricity would enhance the rate at which the system would be brought into an unstable situation. Li et al. (2014a) showed that there could be chaotic behavior when the mutual inclination between the inner and outer orbit remained high.

The instability of S-type planets may be induced by large eccentricity excitations. The planet may fall into the region of Roche lobe and merge into the primary, or move away from the inner binary system (Eggleton et al. 1998; Kiseleva et al. 1998; Ford et al. 2000).

5. Maximum Mutual Inclinations of General S-type Systems

After exploring crucial issues about the inclination excitation of γ Cep Ab, we attempt to reveal the dynamical features of general S-type planets under the octupole-level secular resonance. For the secular evolution of general S-type planets, dynamical evolution should be extensively studied in a wider parameter space, thus more parameters are accounted for in this section.

As the distribution of the SMAs of S-type planets has been introduced in Section 1, $m_1$ refers to the mass of the S-type planet, while $m_2$ denotes that of the stellar companion. For general S-type planets in close binary systems, the parameter space of $m_1$, $a_1/a_2$, $e_1$, and $e_2$ should be enlarged. In this section, the mass of the primary star is set to be 1 $M_\odot$ and the mass of the secondary companion is assumed to be 0.3 $M_\odot$, on the basis of the average stellar masses of detected star binary–hosting S-type planets (Raghavan et al. 2010; Moe & Di Stefano 2017).

In this section, we do not discuss the effect of the initial inclinations; rather, we mainly focus on whether the S-type planets in these systems may have experienced orbital flips under the relative conditions derived from the previous sections of this work, therefore we take the initial mutual inclination $i_{mut}$ as 50°, which is a bit larger than the critical inclination of 39° for orbital flips in the classical KL theory. Moreover, we let $e_1$ and $e_2$ uniformly distribute over 0.0–0.8, and yield a grid of 16 $\times$ 16. For each set of initial parameters, we integrate the system for 100 Myr. The argument of periastron is set as $g_1 = 5^\circ$ and $g_2 = 0^\circ$, with longitudes of node $h_1 = 180^\circ$ and $h_2 = 0^\circ$.

In order to show the evolution of γ Cep Ab from the current observed position, and to enlarge the parameter spaces of $e_1$, $e_2$ to present more general results, Figure 13 plots the distribution map of the maximum $i_{mut}$ for $m_1 = 11$ and 15 $M_{Jup}$ for $a_1 = 2$ au, $a_2 = 20$ au. Both panels show that the orbital flips occur when $e_1$ and $e_2$ are both larger than 0.2, or $e_1 < 0.2$ and $e_2 > 0.3$. We mark the orbital position of γ Cep Ab with the yellow pentagram in Figure 13(b). The flip constrains of $e_1$ and $e_2$ reveal that γ Cep Ab still has a great possibility of maintaining the flipping orbit. We use the new variable $p_{flip}$ to describe the flip possibility of a total of 256 runs in each panel: in Figure 13(a), $p_{flip} = 0.301$, while in Figure 13(b), $p_{flip}$ is calculated to be 0.305, and the difference is mainly induced by the tiny changes in the flip cases in the region of $e_1 < 0.3$.

As we mentioned in Section 3.2, the SMA ratio $a_1/a_2$ is the specific element for evaluating the strength of the octupole-level effect when $m_0$, $m_1$, and $e_2$ are fixed. We further set $a_1/a_2$ as a new independent variable, we choose $a_1 = \{1, 2, 5, 10\}$ au and $a_2 = \{10, 20, 50, 100\}$ au, which are selected from the “Target Area” in Figure 1, thus $a_1/a_2 = \{0.01, 0.02, 0.04, 0.05, 0.1\}$. Additionally, we assume the S-type planetary mass to be 1 $M_{Jup}$ according to the detected average of the minimum masses of S-type planets.

Figure 14 shows the distribution of the maximum orbital inclination in the parameter space for general S-type planets with masses of 1 $M_{Jup}$. We show the value of the flip possibility

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**Figure 11.** Two examples of Hamiltonian level curves with $m_1 = 11 M_{Jup}$ and 15 $M_{Jup}$ in the $(e_1, g_1)$ plane, where $e_2$ and $G_{mut}$ are fixed. The blue circles and green curves are resonant and oscillating orbits with equilibrium points of 90° and 180°, respectively.
Figure 12. Surfaces of sections in the $g_1 = 0$ plane, with different masses of $\gamma$ Cep Ab, by varying the perturbation Hamiltonian $H(\Delta h \rightarrow \pi)$. Panels (a), (c), and (e): dynamical maps for $m_1 = 11 M_{\text{Jup}}$. Panels (b), (d), and (f): dynamical maps for $m_1 = 15 M_{\text{Jup}}$. 
$\rho_{\text{flip}}$ in the lower left corner of each panel. For panels (d), (g), (i), and (j) in Figure 14, with $a_1/a_2 = 0.1$, the critical eccentricities for the orbital inverting are $e_2 \sim 0.3$ when $e_1 \sim 0$, and $e_2 \sim 0.2$ when $e_1 > 0.4$. For panels (c) and (h) in Figure 14, with $a_1/a_2 = 0.05$, the critical eccentricities for the orbital turning over are $e_2 \sim 0.5$ when $e_1 \sim 0$, and $e_2 \sim 0.4$ when $e_1 > 0.5$; the $\rho_{\text{flip}}$ in these two panels are both larger than 0.1. It should be noticed that in Figure 14(f), with $a_1/a_2 = 0.04$, the critical $e_1$ and $e_2$ for the flips are both 0.1 higher than those in Figures 14(c) and (h), with $\rho_{\text{flip}} < 0.1$. For the remaining plots with $a_1/a_2 = 0.01$ or 0.02, the flip regions in the parameter space gradually disappear.

We first conclude from Figure 14 that the orbital flips occur more easily with the increase of $a_1/a_2$, which scales the octupole strength, with the flip possibility rising simultaneously. The other major point to note in Figure 14 is that when $a_1/a_2$ is fixed, the regions of the orbital flips in the panels on the diagonal get larger with the decrease of $a_2$, since the gravitational perturbation from the secondary star is also getting stronger.

Comparing the distribution maps of the maximum $i_{\text{mut}}$ for $m_1 = 11$ and $15 M_{\text{Jup}}$ in Figure 13 and that for $m_1 = 1 M_{\text{Jup}}$ in Figure 14(i), we notice that the flip possibilities in these three plots are 0.301, 0.305, and 0.297, respectively, thus the number of flipping cases changes only a little with planetary mass. Anderson et al. (2016) also investigated the ratio of all likely consequences of the inward migration of giants in stellar binaries under the octupole perturbation and the tidal dissipation. They found that the fraction of systems giving rise to either hot Jupiter formation or tidal disruption is constantly 11%–14%, having little variation with planetary mass, stellar type, or tidal dissipation strength. Nevertheless, we still obtain some new findings relating to the variation of planetary masses. As the planetary mass increases, the original flips in the region of $e_1 \leq 0.1$ and $e_2 \sim 0.6$ disappear, while there new flips emerge in the region of $e_1 \in [0.2, 0.3]$ and $e_2 \sim 0.8$.

In Michtchenko & Malhotra (2004), the theoretical analysis of the secular dynamics for three-body systems was performed within the space $(e_1, e_2)$, and the phase space structure depended upon the ratios of the planetary masses and their SMAs. After presenting analysis for a wide range of planetary masses and SMA ratios, they showed that when both the mass and SMA ratios are far from unity, the domains of the oscillation orbits decrease. In comparison, our results in Figures 13 and 14 are consistent with those of Michtchenko & Malhotra (2004). In Figure 12, we find that the reverting orbits are almost identical to the oscillating cases, thus the magnitude of the flip possibility $\rho_{\text{flip}}$ (Figures 13 and 14) could be expressed as the region within the $(e_1, e_2)$ plane that is dominated by oscillations. Thus, we conclude that the flip possibility $\rho_{\text{flip}}$ goes down with decreasing $a_1/a_2$ and $m_1/m_2$.

We further calculate the orbital flip maps of some specific potentially inclined S-type planets, employing their real minimum planetary masses. Here, we mark out possible orbital positions of the S-type planets HD 19994 Ab (Mayor et al. 2004) and HD 196885 Ab (Chauvin et al. 2011) in Figure 14, since the emergence of the flipping cases changes little with planetary mass for $e_1 \geq 0.3$. The yellow diamond in panel (a) approximately gives the observed parameters for the S-type planet HD 19994 Ab. HD 19994 Ab was discovered in 2003 by RV measurements, with a minimum mass $m_1 \sin I_1 = 1.68 M_{\text{Jup}}, a_1 = 1.42 \pm 0.01$ au, and $e_1 = 0.3 \pm 0.04$. The binary HD 19994 AB consists of a primary of $m_0 = 1.34 M_\odot$ and a secondary of $m_2 = 0.35 M_\odot, a_2 = 100$ au, and $e_2 = 0.26$. Figure 14(a) indicates that HD 19994 Ab has a high probability of retaining mutual inclination below 60° for the minimum mass.

Aside from HD 19994 Ab, we also discuss the flip possibility of HD 196885 Ab, with $m_1 \sin I_1 = 2.98 M_{\text{Jup}}, a_1 = 2.0$ au, and $e_1 = 0.48$. HD 196885 AB binary consists of a primary star with $m_0 = 1.33 M_\odot$ and a secondary with $m_2 = 0.55 M_\odot$ and $a_2 = 23$ au. The problem is that we do not know the eccentricity of the secondary star. Here, we make some assumptions about $e_2$: if $e_2 \geq 0.65$, then the orbit of HD 196885 Ab will roll over for $m_1 = 1 M_{\text{Jup}}$, otherwise, it will not flip. Since we have found that the planetary mass does not change the orbital flip possibility for $e_1 \geq 0.3$, this assumption remains valid as $m_1$ increases.
6. Conclusions and Discussion

In this work, we employ the nonrestricted EKL mechanism to shed light on the secular evolution of the inclined S-type planet γ Cep Ab. Using a wide range of parameters of SMA, eccentricity, and planetary mass, we perform numerical simulations in relation to the octupole-level effects to extensively investigate the orbital flip possibility of potentially inclined S-type planets in general systems. Here, we summarize the major results as follows:

1. We first derive the posterior distributions of the orbital parameters of γ Cep Ab and the star companion in the γ Cep system with more accurate estimation uncertainties in the N-body model, using the MCMC ensemble sampler and the initial parameters independently. The minimum planetary mass is further estimated.

2. We then employ the EKL mechanism to explain the origin of the high inclination of γ Cep Ab. With initial conditions selected from the \((e_1, i_{\text{mut}})\) plane, we show that when \(m_1 = 15 \, M_{\odot}\), it is easier for γ Cep Ab to reach the target \(i_{\text{mut}}\) over 113° when the initial \(i_{\text{mut}} < 60°\) and \(e_1 < 0.7\). Our investigation further indicates that relatively small values of \(H\) and low initial \(i_{\text{mut}}\) may lead to a slightly longer timescale for the first orbital flip. The libration and circulation regions in the \((e_1, g_1)\) plane and Poincaré surfaces of section, as well as the secular stability criterion, confirm that there is a great possibility for the flipping orbits of γ Cep Ab to remain stable.

3. This work further extends the application of the EKL mechanism to general S-type planets. We take \(m_1, a_1,\) and \(e_2\) as independent variables, and the orbital flips can be observed in the selected space of \(a_1\) and \(a_2\). The most
intense orbital inclination excitation occurs when \( \alpha_1/\alpha_2 = 0.1 \) and \( e_2 \sim 0.8 \). As the planetary mass increases, the original flips at \( e_1 \leq 0.1 \) and \( e_2 \sim 0.6 \) are suppressed, whereas the new flips emerge for \( e_1 \in [0.2, 0.3] \) and \( e_2 \sim 0.8 \), with little effect on the total flip possibility.

In this study, we mainly focus on the extremely high mutual inclination or transformation between prograde and retrograde orbits in binary systems over the timescale of secular resonance. Note that the EKL mechanism can trigger high orbits in binary systems over the timescale of secular inclination or transformation between prograde and retrograde.

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